تمرینهای فصل ۵

www.ASEC.blogfa.com

ero Space Engineering Center

پایگاه مهندسی هوافضا

Chapter 5:

P5-2:

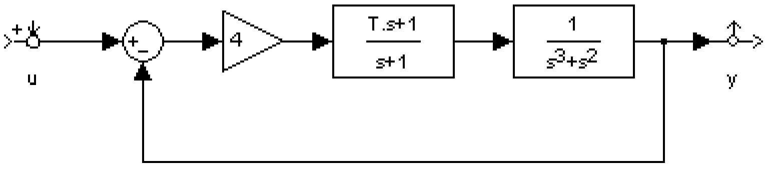

Figure 5-2

$$\frac{y}{u} = \frac{k(TD+1)}{D^4 + 2D^3 + D^2 + kTD + k}$$

By using the Routh method:

s^4	1	1	k
s^3	2	kT	0
s^2	$\frac{2-kT}{2}$	k	0
s^1	a	0	0

where

$$a = \frac{2kT - k^2T^2 - 4k}{2-kT}$$

so

$$kT \le 2 \Rightarrow k \le \frac{T}{2}$$

$$a \ge 0 \text{ or } a = 0$$

$$k = \frac{2T-4}{T^2} \Rightarrow \frac{dk}{dT} = 0 \rightarrow \frac{2T^2 - 2T(2T-4)}{T^4} = 0$$

$$T_{opt} = 4$$

P5-3:

$$s^3 + 7s^2 + 17s + k = 0$$

$$y = ae^{p_1t} + be^{p_2t} + ce^{p_3t}$$

$$T = \frac{1}{p_i}$$

Now, none of these $\frac{1}{p_i}$ must not be more than 0.5 sec

$$\begin{cases} p_1 + p_2 + p_3 = 7 \\ p_1 p_2 + p_1 p_3 + p_2 p_3 = 17 \quad \text{and } \forall p_i > 2 \\ p_1 p_2 p_3 = k \end{cases}$$

then the three poles can be found.

P5.4:

s^6	1	-7	8	16	0
s^5	-2	16	-32	0	0
s^4	1	-8	16	0	
s^3	$0 \text{N} 4$	$0 \text{N} -16$			
s^2	-8	16	0		
s	-8	0			
s^0	16				

$$\rightarrow \begin{cases} s^4 - 8s^2 + 16 = u \\ \rightarrow \dfrac{du}{ds} = 4s^3 - 16s \end{cases}$$

$$s^4 - 8s^2 + 16 = 0$$

$$(s^2 - 4)^2 = 0 \rightarrow s = \pm 2$$

$$(s^2 - 4)^2 (s - 1)^2 = 0$$

It has 4 positive poles.

P5.6

By calculate the closed loop transfer function and using the Routh method, it is clear that

$$\frac{T(-2k+1)+2}{T+2} \geq 0$$

$$k \leq \frac{1}{T} + 0.5$$

P5.7:

After calculating the c.l. transfer function and using Routh method, one of the array in the first column of Routh table will be

$$\frac{4p^2 + p - 32}{2p} \geq 0$$

so the numerator will be positive. It must be greater than -2.96.

P5.8:

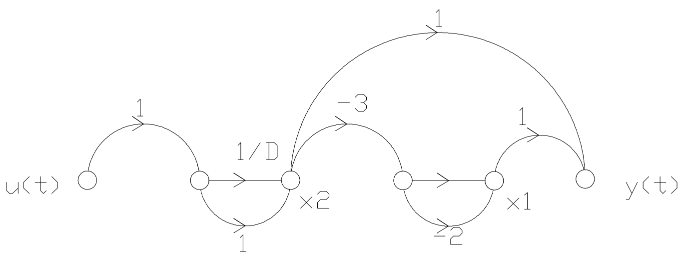

Figure 5.8

a)

$$\begin{cases} \dot{x}_1 = -2x_1 - 3x_2 \\ \dot{x}_2 = x_2 + u \\ y = x_1 + x_2 \end{cases} \Rightarrow \mathbf{A} = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

b)

$$G(s) = \frac{1}{s-1}\left(1 - \frac{3}{s+2}\right) = \frac{s-1}{(s-1)(s+2)} = \frac{1}{s+2}$$

c)

yes, system is stable.

d)

$$det(l\mathbf{I} - \mathbf{A}) = 0 \Rightarrow (l+2)(l-1) = 0 \Rightarrow \begin{cases} l_1 = 1 \\ l_2 = -2 \end{cases} \textit{not stable!!}$$

$$\begin{cases} l_1 = 1 \text{ and } v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ l_2 = -2 \text{ and } v_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{cases}$$

$$\mathbf{T} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \Rightarrow \mathbf{A}^* = \mathbf{T}^{-1}\mathbf{A}\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}, \mathbf{B}^* = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \mathbf{C}^* = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

system is not observable.

P5.12:

By using the Routh method it will clear that

$$\begin{cases} 5 = 2zw_n \\ k + 6 = w_n^2 \end{cases}$$

and then $w_n = 3.57, k = 6.7551$ the system is always stable.

P5.15:

$$G(s)=\frac{1}{(s+a)(s+b)}$$

in steady state con.

$$\text{as s} \to 0 \quad \frac{1}{ab}=1 \Rightarrow ab=1$$

then the characteristic equation is

$$s(s+a)(s+b)+k=0$$

By using the Routh method

$$\frac{(a+b)ab-2.5}{a+b}=0 \Rightarrow a+b=2.5\,.$$

P5.17:

By simplifying the block diagram, the characteristic equation will be

$$s+2k-1=0$$

so $k>0.5$.