(1-3):

$$\begin{cases} A_1 \dot{x}_1 = U - \dfrac{x_1}{R_1} - \dfrac{x_1 - x_2}{R} \\ A_2 \dot{x}_2 = \dfrac{x_1 - x_2}{R} - \dfrac{x_2}{R_2} \end{cases}$$

$$\begin{cases} \dot{x}_1 = -\left(\dfrac{1}{A_1 R_1} + \dfrac{1}{A_1 R}\right) x_1 + \dfrac{1}{A_1 R} x_2 + \dfrac{1}{A_1} U \\ \dot{x}_2 = \dfrac{1}{A_2 R} x_1 - \left(\dfrac{1}{A_2 R_2} + \dfrac{1}{A_2 R}\right) x_2 \end{cases}$$

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\left(\dfrac{1}{A_1 R_1} + \dfrac{1}{A_1 R}\right) & \dfrac{1}{A_1 R} \\ \dfrac{1}{A_2 R} & -\left(\dfrac{1}{A_2 R_2} + \dfrac{1}{A_2 R}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{A_1} \\ 0 \end{bmatrix} U \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases}$$

So:

$$A = \begin{bmatrix} -\left(\dfrac{1}{A_1 R_1} + \dfrac{1}{A_1 R}\right) & \dfrac{1}{A_1 R} \\ \dfrac{1}{A_2 R} & -\left(\dfrac{1}{A_2 R_2} + \dfrac{1}{A_2 R}\right) \end{bmatrix} \qquad B = \begin{bmatrix} \dfrac{1}{A_1} \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D = 0$$

(2-3):

If: $A_1 = A_2 = 1$ $\quad R_1 = R_2 = R = 1$ $\quad G(s) = \dfrac{Y(s)}{U(s)} = ?$

if $y = x_1$:

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad c = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad d = 0$$

And : $G(s) = c(s)(sI - A)^{-1} b + d$

$$(sI - A) = \begin{bmatrix} s+2 & -1 \\ -1 & s+2 \end{bmatrix} \Rightarrow$$

$$(sI - A)^{-1} = \frac{adj(sI - A)}{\det(sI - A)}$$

$$(sI - A)^{-1} = \frac{1}{(s+2)^2 - 1} \begin{bmatrix} s+2 & 1 \\ 1 & s+2 \end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} \frac{s+2}{(s+3)(s+1)} & \frac{1}{(s+3)(s+1)} \\ \frac{1}{(s+3)(s+1)} & \frac{s+2}{(s+3)(s+1)} \end{bmatrix}$$

At last:

$$G(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{s+2}{(s+3)(s+1)} & \frac{1}{(s+3)(s+1)} \\ \frac{1}{(s+3)(s+1)} & \frac{s+2}{(s+3)(s+1)} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{s+2}{(s+3)(s+1)}$$

And if $y = x_1 - x_2$: $c = \begin{bmatrix} 1 & -1 \end{bmatrix}$

$$G(s) = \begin{bmatrix} 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{s+2}{(s+3)(s+1)} & \frac{1}{(s+3)(s+1)} \\ \frac{1}{(s+3)(s+1)} & \frac{s+2}{(s+3)(s+1)} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{(s+3)}$$

That :This System is not observable

(3-3):

$$\begin{cases} A_1 \dot{x}_1 = U_1 - \dfrac{x_1}{R_1} \\ A_2 \dot{x}_2 = U_2 + \dfrac{x_1}{R_1} - \dfrac{x_2}{R_2} \end{cases} \qquad \begin{cases} y_1 = x_2 \\ y_2 = \dfrac{d}{dt} x_2 \end{cases}$$

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{A_1R_1} & 0 \\ \dfrac{1}{A_2R_1} & -\dfrac{1}{A_2R_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{A_1} & 0 \\ 0 & \dfrac{1}{A_2} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \\ \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \dfrac{1}{R_1A_2} & -\dfrac{1}{R_2A_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{1}{A_2} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \end{cases}$$

So:

$$A = \begin{bmatrix} -\dfrac{1}{A_1R_1} & 0 \\ \dfrac{1}{A_2R_1} & -\dfrac{1}{A_2R_2} \end{bmatrix} \quad B = \begin{bmatrix} \dfrac{1}{A_1} & 0 \\ 0 & \dfrac{1}{A_2} \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ \dfrac{1}{R_1A_2} & -\dfrac{1}{R_2A_2} \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{1}{A_2} \end{bmatrix}$$

(4-3):

If: $A_1 = A_2 = 1 \qquad R_1 = 1 \qquad R_2 = \frac{1}{2} \qquad G(s) = \dfrac{Y(s)}{U(s)} = ?$

$$A = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

And : $G(s) = c(s)(sI - A)^{-1} b + d$

$$(sI - A)^{-1} = \begin{bmatrix} s+1 & 0 \\ -1 & s+2 \end{bmatrix}^{-1} = \frac{1}{(s+1)(s+2)} \begin{bmatrix} s+2 & 0 \\ 1 & s+1 \end{bmatrix}$$

$$G(s)=\frac{1}{(s+1)(s+2)}\begin{bmatrix}0 & 1\\ 1 & -2\end{bmatrix}\cdot\begin{bmatrix}s+2 & 0\\ 1 & s+1\end{bmatrix}\cdot\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$$

$$G(s)=\begin{bmatrix}\dfrac{1}{(s+1)(s+2)} & \dfrac{1}{(s+2)}\\ \dfrac{s}{(s+1)(s+2)} & \dfrac{s}{(s+2)}\end{bmatrix}$$

The second column is circled, with an arrow to: $\begin{cases}\textit{Unobservable}\\ \textit{or}\\ \textit{Uncontrolable}\end{cases}$

(5-3):

$$A=\begin{bmatrix}-2 & 1\\ 1 & -2\end{bmatrix}$$

$$\det(\lambda I-A)=0\Rightarrow$$

$$\begin{vmatrix}\lambda+2 & -1\\ -1 & \lambda+2\end{vmatrix}=0\Rightarrow$$

$$(\lambda+2)^2-1=0\Rightarrow\lambda^2+4\lambda+3=0\Rightarrow\begin{Bmatrix}\lambda_1=-1\\ \lambda_1=-3\end{Bmatrix}$$

$$(\lambda I-A).V=0$$

$$\begin{bmatrix}\lambda+2 & -1\\ -1 & \lambda+2\end{bmatrix}.\begin{bmatrix}V_1\\ V_2\end{bmatrix}=0\Rightarrow$$

$$\lambda=-1\Rightarrow\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}.\begin{bmatrix}V_1^1\\ V_2^1\end{bmatrix}=0\Rightarrow V^1=\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$\lambda=-3\Rightarrow\begin{bmatrix}-1 & -1\\ -1 & -1\end{bmatrix}.\begin{bmatrix}V_1^2\\ V_2^2\end{bmatrix}=0\Rightarrow V^2=\begin{bmatrix}1\\ -1\end{bmatrix}$$

(6-3):

$$A = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}$$

$$\det(\lambda I - A) = 0 \Rightarrow$$

$$\begin{vmatrix} \lambda+1 & 0 \\ -1 & \lambda+2 \end{vmatrix} = 0 \Rightarrow$$

$$(\lambda+1)(\lambda+2) = 0 \Rightarrow \begin{Bmatrix} \lambda_1 = -1 \\ \lambda_1 = -2 \end{Bmatrix}$$

$$(\lambda I - A).V = 0$$

$$\begin{bmatrix} \lambda+1 & 0 \\ -1 & \lambda+2 \end{bmatrix} . \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0 \Rightarrow$$

$$\lambda = -1 \Rightarrow \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix} . \begin{bmatrix} V_1^1 \\ V_2^1 \end{bmatrix} = 0 \Rightarrow V^1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda = -2 \Rightarrow \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix} . \begin{bmatrix} V_1^2 \\ V_2^2 \end{bmatrix} = 0 \Rightarrow V^2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

(7-3):

$$\begin{cases} m_1 c_1 \dot{x}_1 = u - \dfrac{x_1 - x_2}{R} - \dfrac{x_1 - x_3}{R} \\ m_2 c_2 \dot{x}_2 = \dfrac{x_1 - x_2}{R} - \dfrac{x_2 - x_3}{R} \\ m_3 c_3 \dot{x}_3 = \dfrac{x_1 - x_3}{R} + \dfrac{x_2 - x_3}{R} \end{cases}$$

$$A=\begin{bmatrix} \frac{-2}{m_1c_1R} & \frac{1}{m_1c_1R} & \frac{1}{m_1c_1R} \\ \frac{1}{m_2c_2R} & \frac{-2}{m_2c_2R} & \frac{1}{m_2c_2R} \\ \frac{1}{m_3c_3R} & \frac{1}{m_3c_3R} & \frac{-2}{m_3c_3R} \end{bmatrix}$$

If: $m_1c_1 = m_2c_2 = m_3c_3 = 1 \qquad R = 1$ **:** $A=\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

$$\det(\lambda I - A) = \begin{bmatrix} \lambda+2 & -1 & -1 \\ -1 & \lambda+2 & -1 \\ -1 & -1 & \lambda+2 \end{bmatrix} = \begin{bmatrix} \lambda+3 & -3-\lambda & 0 \\ -1 & \lambda+2 & -1 \\ -1 & -1 & \lambda+2 \end{bmatrix} = \begin{bmatrix} \lambda+3 & 0 & 0 \\ -1 & \lambda+1 & -1 \\ -1 & -2 & \lambda+2 \end{bmatrix}$$

$$\det(\lambda I - A) == (\lambda+3)\{(\lambda+1)(\lambda+2)-2\} = (\lambda+3)\lambda(\lambda+3)$$

$$if \ \det(\lambda I - A) = 0 \Rightarrow \begin{cases} \lambda_1 = 0 \\ \lambda_2 = -3 \\ \lambda_3 = -3 \end{cases}$$

$$G(s) = c(s)(sI - A)^{-1} b + d$$

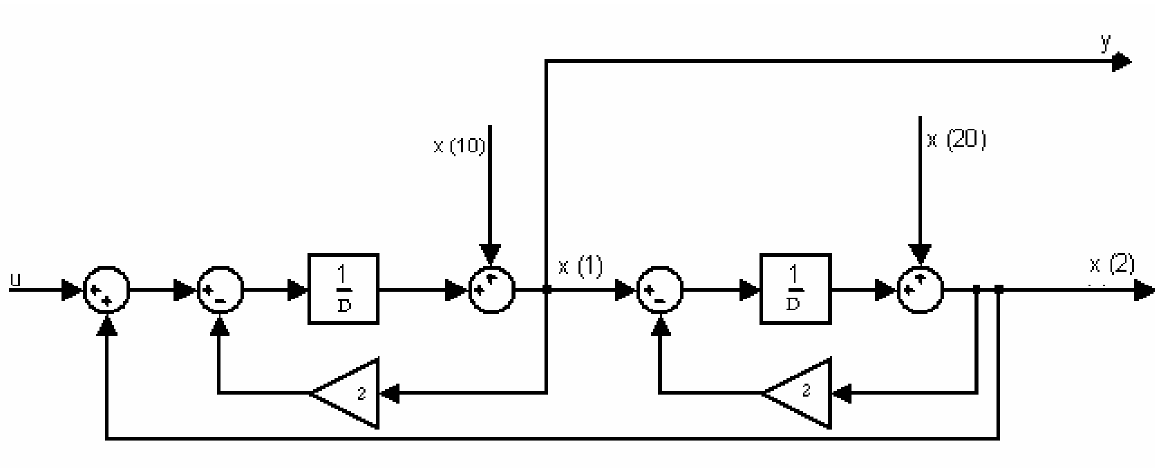
$$(sI - A)^{-1} = \begin{bmatrix} s+2 & -1 & -1 \\ -1 & s+2 & -1 \\ -1 & -1 & s+2 \end{bmatrix}^{-1} = \frac{1}{s^3 + 6s^2 + 9s} adj(sI - A)$$

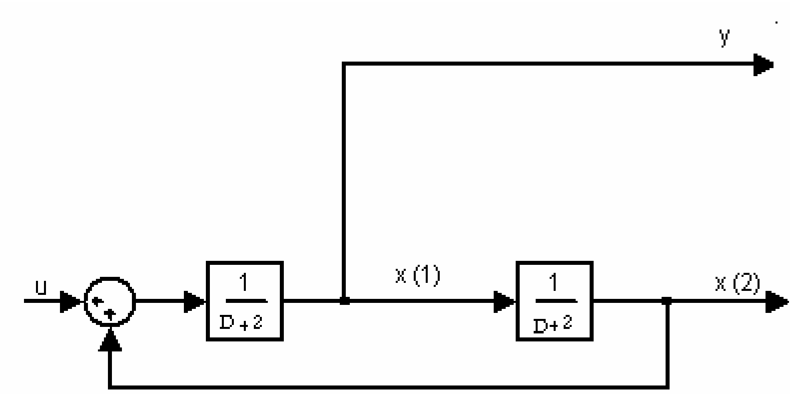
$$(sI - A)^{-1} = \frac{1}{s(s+3)^2} \begin{bmatrix} (s+1)(s+3) & (s+3) & (s+3) \\ (s+3) & (s+1)(s+3) & (s+3) \\ (s+3) & (s+3) & (s+1)(s+3) \end{bmatrix}$$

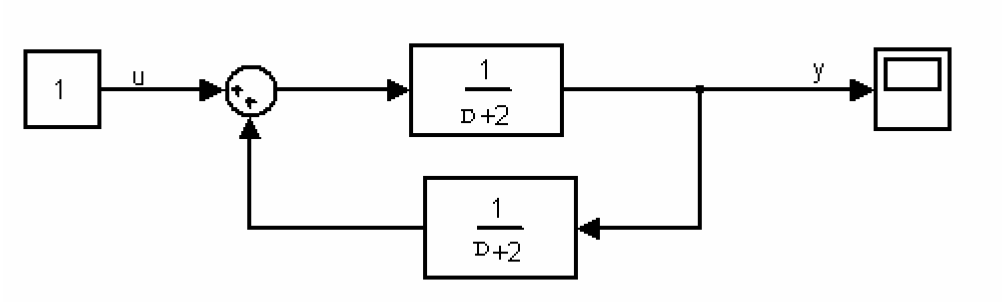
$$G(s) = \frac{1}{s(s+3)}$$

(8-3):

For f(1):







$$\frac{y}{u}=\frac{\frac{1}{D+2}}{1-\frac{1}{D+2}\cdot\frac{1}{D+2}}=\frac{D+2}{(D+2)^2-1}=\frac{D+2}{(D+1)(D+3)}$$

For f(2) and f(3):

$$\frac{y}{x_{10}} = \frac{D(D+2)}{(D+3)(D+1)} \qquad \frac{y}{x_{20}} = \frac{D}{(D+3)(D+1)}$$

And the other method:

$$y = x_1 = x_{10} + \frac{\dot{x}_1}{D} = x_{10} + \frac{1}{D}(u - 2x_1 + x_2)$$

$$x_2 = x_{20} + \frac{\dot{x}_2}{D} = x_{20} + \frac{1}{D}(x_1 - 2x_2)$$

$$x_2 = x_{20} + \frac{1}{D}(y - 2x_2) \rightarrow x_2 = \frac{x_{20} + y/D}{1 + 2/D}$$

$$y = x_{10} + \frac{1}{D}(u - 2y + \frac{x_{20} + y/D}{1 + 2/D})$$

$$y = x_{10} + \frac{1}{D}u - \frac{2}{D}y + \frac{1}{D}(\frac{Dx_{20} + y}{D+2})$$

$$y = x_{10} + \frac{1}{D}u - \frac{2}{D}y + \frac{Dx_{20}}{D(D+2)} + \frac{y}{D(D+2)}$$

$$\left[1 + \frac{2}{D} - \frac{1}{D(D+2)}\right]y = x_{10} + \frac{1}{D+2}x_{20} + \frac{1}{D}u$$

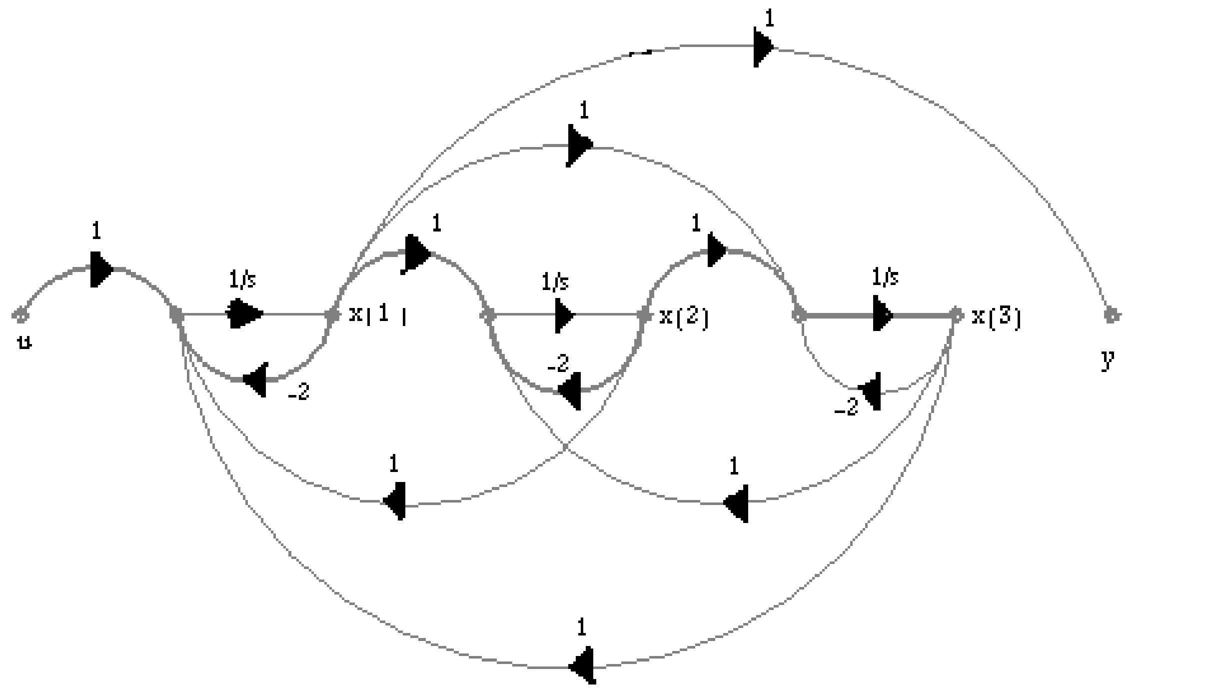
$$\left[\frac{D^2 + 4D + 3}{D(D+2)}\right]y = x_{10} + \frac{1}{D+2}x_{20} + \frac{1}{D}u$$

$$y = \frac{D(D+2)}{(D+1)(D+3)}x_{10} + \frac{D}{(D+1)(D+3)}x_{20} + \frac{(D+2)}{(D+1)(D+3)}u$$

....

(9-3):

$$\begin{cases} \dot{x}_1 = -2x_1 + x_2 + x_3 + u \\ \dot{x}_2 = x_1 - 2x_2 + x_3 \\ \dot{x}_3 = x_1 + x_2 - 2x_3 \\ y = x_3 \end{cases}$$



$$\begin{cases} (s+2)X_1 = X_2 + Y + U & (*) \\ (s+2)X_2 = X_1 + Y & (**) \\ (s+2)Y = X_1 + X_2 & (***) \end{cases}$$

$$** \Rightarrow \quad X_2 = \frac{1}{s+2}(X_1 + Y)$$

$$*** \Rightarrow \quad (s+2)Y = X_1 + \frac{1}{s+2}(X_1 + Y)$$

$$(s+2-\frac{1}{s+2})Y = (1+\frac{1}{s+2})X_1$$

$$\frac{(s+2)^2 - 1}{s+2}Y = \frac{s+2+1}{s+2}X_1$$

$$(s+3)(s+1)Y = (s+3)X_1$$

$$(s+1)Y = X_1$$

$$in\ **: X_2 = \frac{1}{s+2}((s+1)+1)Y = Y$$

$$in\ *: (s+2)(s+1)Y = Y + Y + U$$

$$s(s+3)Y = U$$

$$G(s) = \frac{1}{s(s+3)}$$

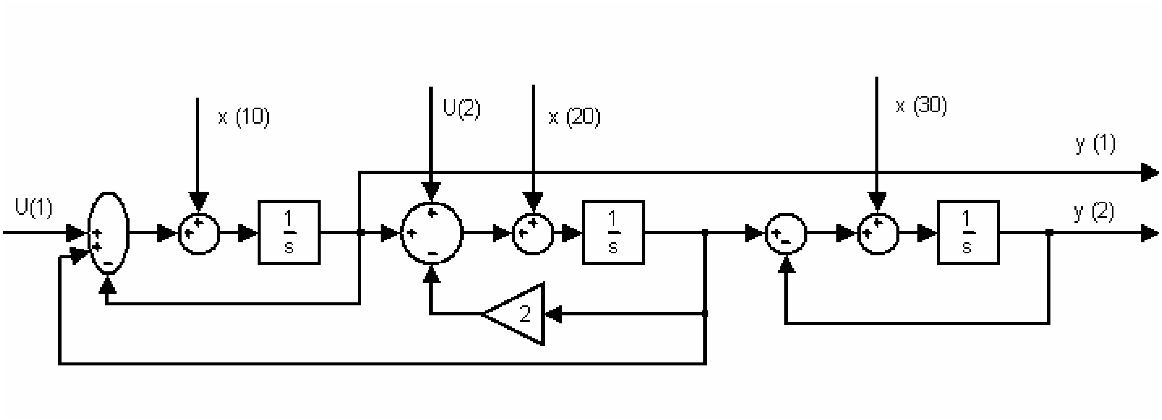
(10-3):

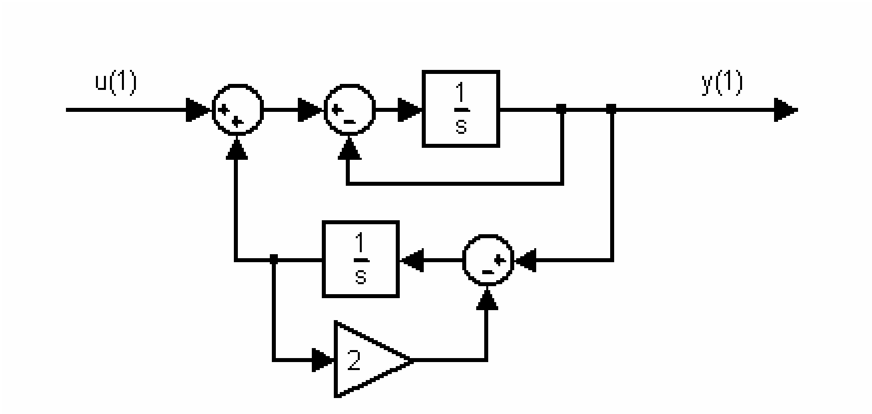
If: $A_1 = A_2 = A_3 = 1$ $\quad R_1 = R_2 = R_3 = 1$ $\quad \begin{cases} y_1 = x_1 \\ y_2 = x_3 \end{cases}$

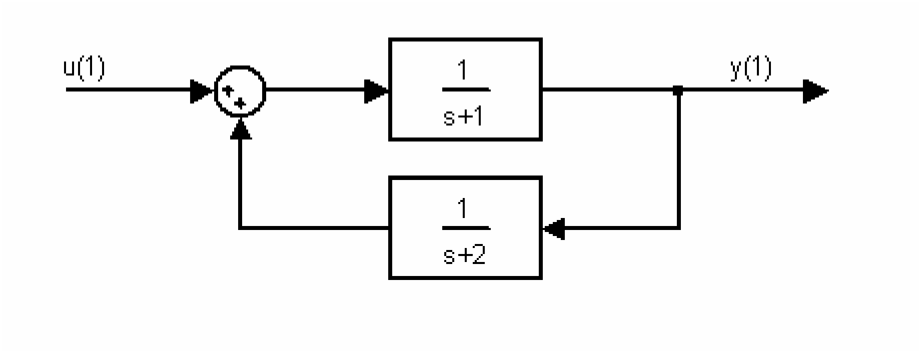
$$\begin{cases} A_1 \dot{x}_1 = U_1 - \dfrac{x_1 - x_2}{R_1} \\ A_2 \dot{x}_2 = U_2 + \dfrac{x_1 - x_2}{R} - \dfrac{x_2}{R_2} \\ A_3 \dot{x}_3 = \dfrac{x_2}{R_2} - \dfrac{x_3}{R_3} \end{cases}$$

$$\dot{x} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 1 & -1 \end{bmatrix} \cdot x + \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} .U \qquad \frac{Y(s)}{U(s)} = ? \qquad x_{10} = x_{20} = x_{30} = 0$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} .x$$







$$\frac{Y_1(s)}{U_1(s)} = \frac{\dfrac{1}{s+1}}{1-\dfrac{1}{(s+1)(s+2)}} = \frac{s+2}{(s+1)(s+2)-1} = \frac{s+2}{(s^2+3s+1}$$

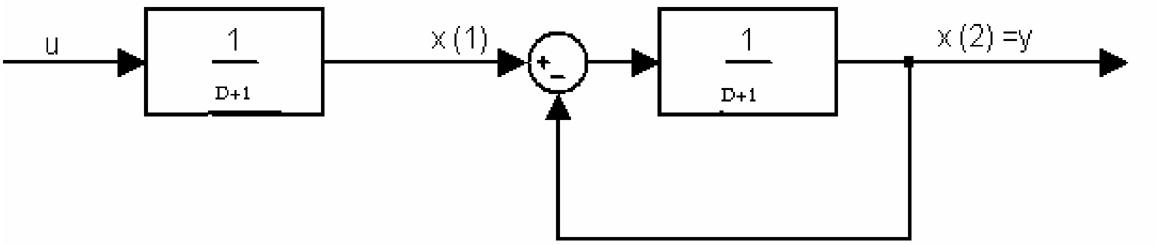
(11-3):

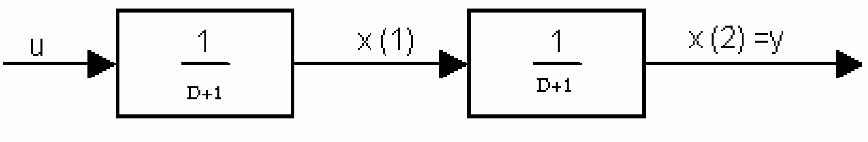
$u = (D+1)x_1 \rightarrow u = \dot{x}_1 + x_1 \rightarrow \dot{x}_1 = u - x_1$

$(x_1 - x_2).\dfrac{1}{(D+1)} = x_2 \rightarrow (x_1 - x_2) = (D+1).x_2 \rightarrow (x_1 - x_2) = \dot{x}_2 + x_2 \rightarrow \dot{x}_2 = x_1 - 2x_2$

So: $\begin{cases} \dot{x}_1 = u - x_1 \\ \dot{x}_2 = x_1 - 2x_2 \end{cases}$

$A = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}$ $\quad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad c = \begin{bmatrix} 0 & 1 \end{bmatrix}$ $\quad d = 0$





$\dfrac{y}{u} = \dfrac{1}{(D+1)(D+2)} \Rightarrow$

$G(s) = \dfrac{1}{(s+1)(s+2)}$

....

(12-3):

$$A = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}$$

$$\det(\lambda I - A) = 0 \Rightarrow$$

$$\begin{vmatrix} \lambda + 1 & 0 \\ -1 & \lambda + 2 \end{vmatrix} = 0 \Rightarrow$$

$$(\lambda + 1)(\lambda + 2) = 0 \Rightarrow \begin{Bmatrix} \lambda_1 = -1 \\ \lambda_1 = -2 \end{Bmatrix}$$

$$G(s) \rightarrow poles : (s+2)(s+1) = 0 \Rightarrow \begin{Bmatrix} s_1 = -1 \\ s_1 = -2 \end{Bmatrix}$$

(13-3):

$$\begin{cases} \dot{x}_1 = k_1 u + p_1 x_1 \\ \dot{x}_2 = k_2 u + p_2 x_1 \\ \dot{x}_3 = k_3 u + p_3 x_1 \\ y = x_1 + x_2 + x_3 \end{cases}$$

$$A = \begin{bmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{bmatrix} \qquad B = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \qquad D = 0$$

$$\text{Eigenvalue} : \mathrm{p}_1, \mathrm{p}_2, \mathrm{p}_3$$

Transfer function:

$$sX_1 = k_1U + p_1X_1 \Rightarrow \begin{cases} X_1 = \dfrac{k_1}{s-p_1}U \\ X_2 = \dfrac{k_2}{s-p_2}U \\ X_3 = \dfrac{k_3}{s-p_3}U \end{cases}$$

$$Y = \frac{k_1}{s-p_1}U + \frac{k_2}{s-p_2}U + \frac{k_3}{s-p_3}U$$

$$G(s) = \frac{Y(s)}{U(s)}$$

$$G(s) = \frac{k_1(s-p_2)(s-p_3) + k_2(s-p_1)(s-p_3) + k_3(s-p_1)(s-p_2)}{(s-p_1)(s-p_2)(s-p_3)}$$

(14-3):

$$\begin{Bmatrix} \dot{x}_1 = -2x_1 - 3x_2 \\ \dot{x}_2 = u + x_2 \\ y = x_1 + x_2 \end{Bmatrix}$$

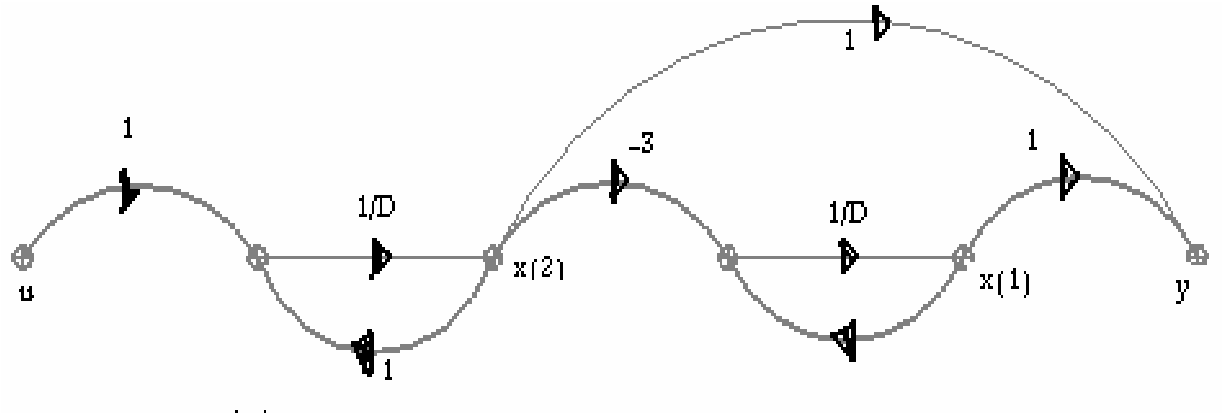
$$A = \begin{bmatrix} -2 & -3 \\ 0 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad c = \begin{bmatrix} 1 & 1 \end{bmatrix} \qquad d = 0$$

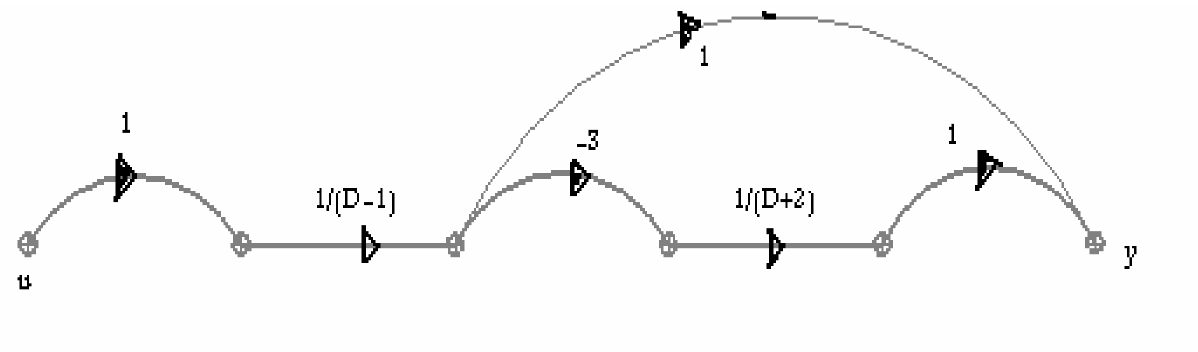
$$G(s) = c(sI - A)^{-1}b + d$$

$$(sI - A)^{-1} = \begin{bmatrix} s+2 & 3 \\ 0 & s-1 \end{bmatrix}^{-1} = \frac{1}{(s+2)(s-1)}\begin{bmatrix} s-1 & -3 \\ 0 & s+2 \end{bmatrix}$$

$$G(s) = \frac{1}{(s+2)(s-1)}\begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} s-1 & -3 \\ 0 & s+2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{(s+2)(s-1)}(s-1) = \frac{1}{s+2}$$

$$\det(\lambda I - A) = 0 \rightarrow \begin{vmatrix} \lambda + 2 & 3 \\ 0 & \lambda - 1 \end{vmatrix} = 0 \rightarrow \begin{cases} \lambda_1 = -2 \\ \lambda_2 = +1 \end{cases}$$





$$1 - \frac{3}{D+2} = \frac{D-1}{D+2}$$

$$\frac{y}{u} = \frac{1}{D-1} . \frac{D-1}{D+2} = \frac{1}{D+2}$$

Poles:-2
eigenvalues:1,-2

(15-3):

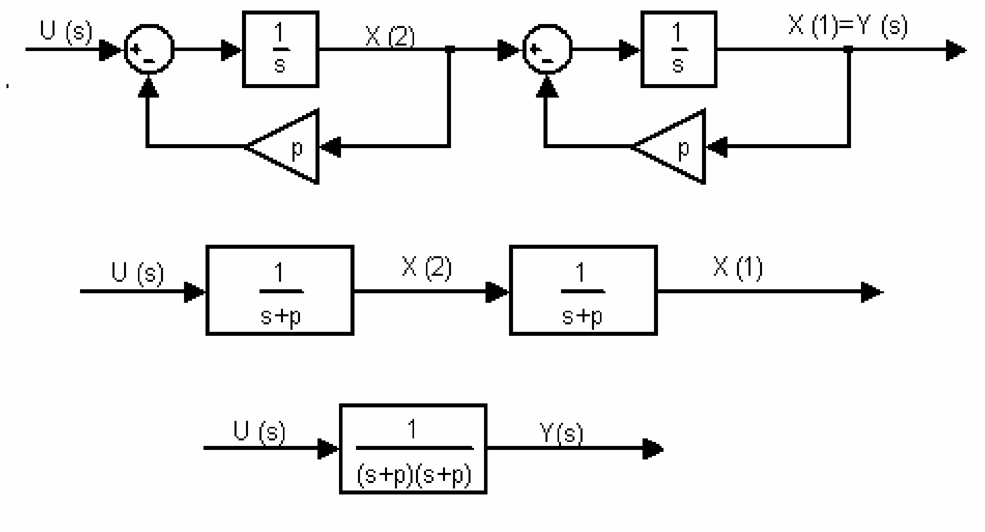
$$\begin{cases} \dot{x}_1 = -px_1 + x_2 \\ \dot{x}_2 = u - px_2 \\ y = x_1 \end{cases}$$

$$A = \begin{bmatrix} -p & +1 \\ 0 & -p \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad c = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad d = 0$$

$$G(s) = c(sI - A)^{-1}b + d$$

$$(sI - A)^{-1} = \begin{bmatrix} s+p & -1 \\ 0 & s+p \end{bmatrix}^{-1} = \frac{1}{(s+p)^2}\begin{bmatrix} s+p & +1 \\ 0 & s+p \end{bmatrix}$$

$$G(s) = \frac{1}{(s+p)^2}\begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} s+p & +1 \\ 0 & s+p \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{(s+p)^2}$$



$$G(s) = \frac{Y}{U} = \frac{1}{(s+p)^2}$$

(16-3):

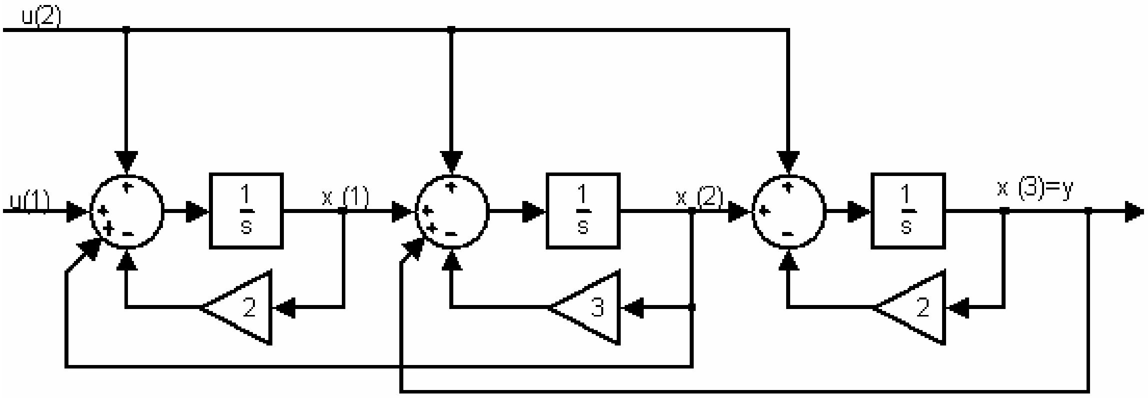
$$\begin{cases} c_1\dot{x}_1 = u_1 - \dfrac{x_1 - u_2}{R_1} - \dfrac{x_1 - x_2}{R'} \\ c_2\dot{x}_2 = \dfrac{x_1 - x_2}{R'} - \dfrac{x_2 - x_3}{R''} - \dfrac{x_2 - u_2}{R_2} \\ c_3\dot{x}_3 = \dfrac{x_2 - x_3}{R''} - \dfrac{x_3 - u_2}{R_3} \end{cases}$$

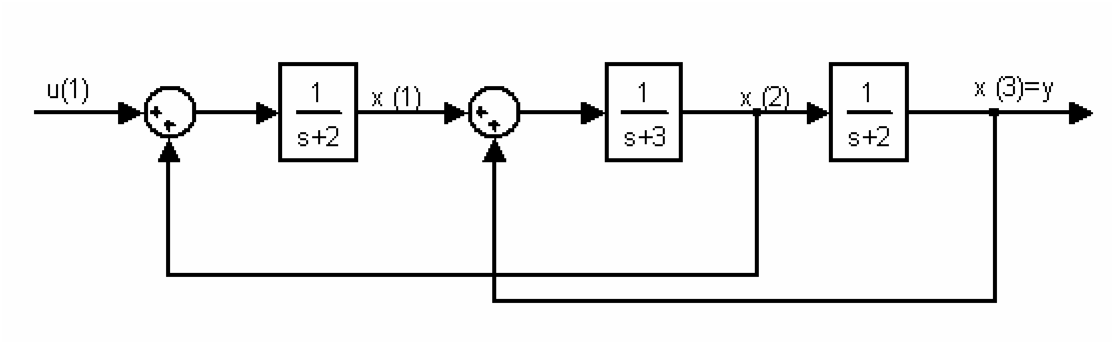
$$A = \begin{bmatrix} -(\dfrac{1}{c_1R} + \dfrac{1}{c_1R'}) & \dfrac{1}{c_1R'} & 0 \\ \dfrac{1}{c_2R'} & -(\dfrac{1}{c_2R'} + \dfrac{1}{c_2R_2} + \dfrac{1}{c_2R''}) & \dfrac{1}{c_2R''} \\ 0 & \dfrac{1}{c_3R''} & -(\dfrac{1}{c_3R''} + \dfrac{1}{c_3R_3}) \end{bmatrix}$$

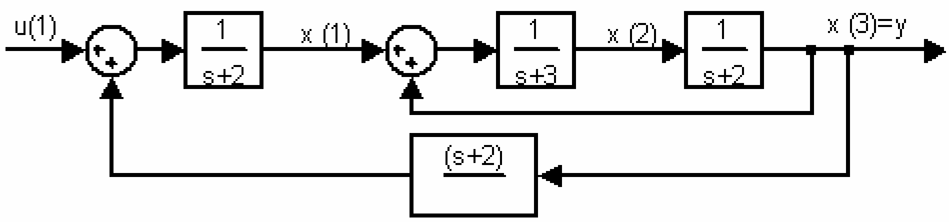
$$B = \begin{bmatrix} \dfrac{1}{c_1} & \dfrac{1}{c_1R_1} \\ 0 & \dfrac{1}{c_2R_2} \\ 0 & \dfrac{1}{c_3R_3} \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \qquad D = 0$$

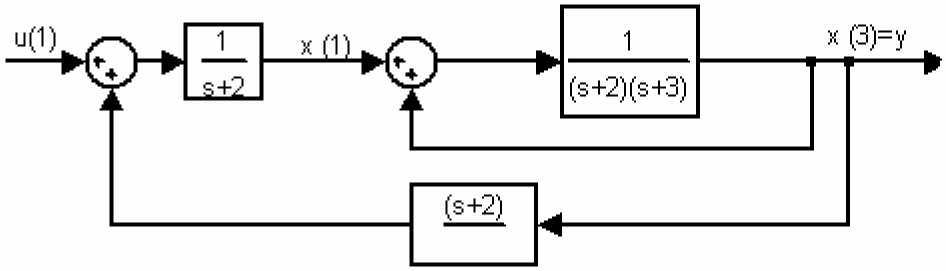
If all parameters=1:

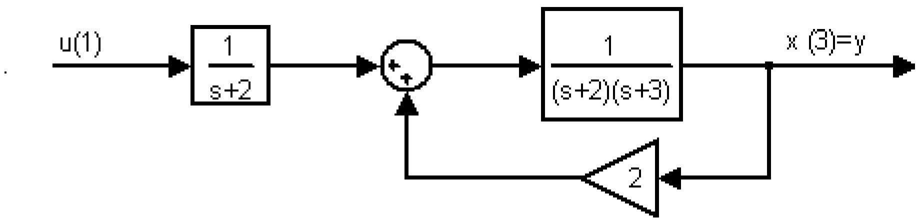
$$A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -3 & 1 \\ 0 & 1 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \qquad D = 0$$



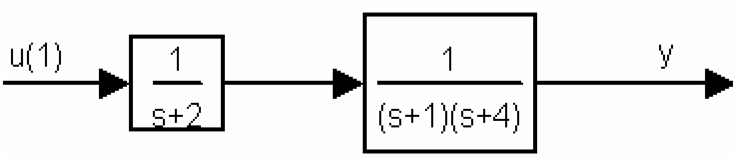








$$\frac{\dfrac{1}{(s+3)(s+2)}}{1-\dfrac{2}{(s+3)(s+2)}}=\frac{1}{s^2+5s+4}=\frac{1}{(s+1)(s+4)}$$



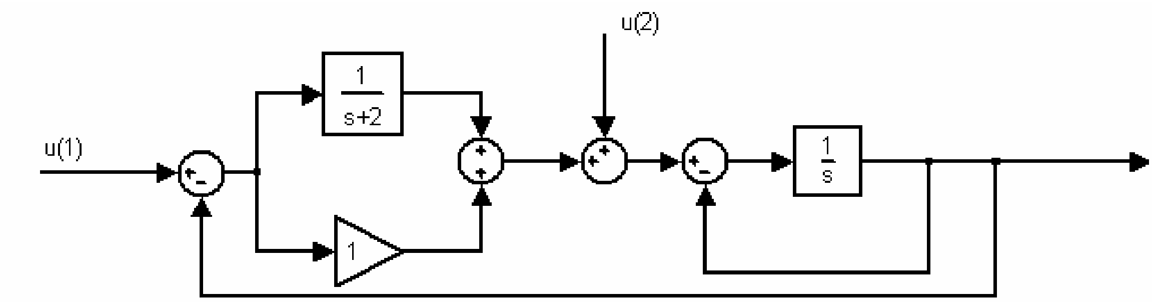
$$\frac{y}{u_1}=\frac{1}{(s+1)(s+2)(s+4)}$$

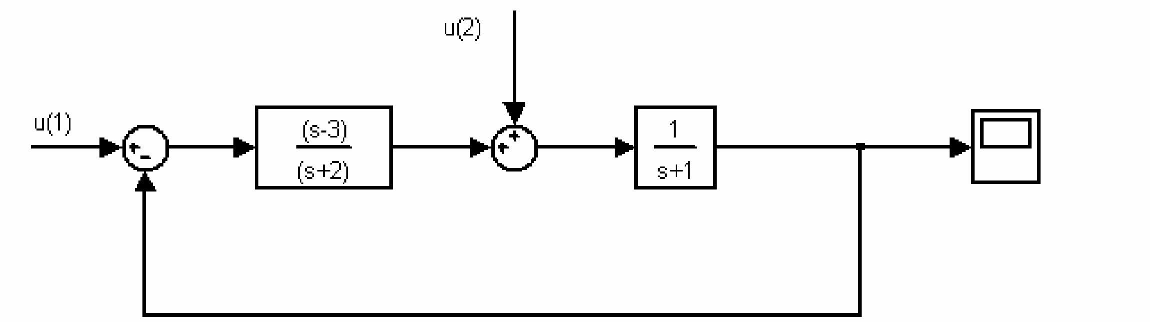
U2: and so on …

$$\frac{y}{u_2} = \frac{(s+2)(s+4)}{(s+1)(s+2)(s+4)}$$

....

(17-3):





$$G_1(s) = \frac{Y(s)}{U_1(s)} = \frac{\frac{s+3}{s+2}.\frac{1}{s+1}}{1+\frac{s+3}{s+2}.\frac{1}{s+1}} = \frac{s+3}{(s+2)(s+1)+(s+3)} = \frac{s+3}{s^2+4s+5}$$

$$G_2(s) = \frac{Y(s)}{U_2(s)} = \frac{\frac{1}{s+1}}{1+\frac{s+3}{s+2}.\frac{1}{s+1}} = \frac{s+2}{(s+2)(s+1)+(s+3)} = \frac{s+2}{s^2+4s+5}$$

$$Y(s) = G_1(s).U_1(s) + G_2(s).U_2(s)$$

(18-3):

$$\begin{cases} \dot{x}_1 = u_1 + x_1 \\ \dot{x}_2 = u_2 + x_1 + x_2 \\ \dot{x}_3 = x_2 + x_3 \\ y = x_3 \end{cases}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \end{bmatrix}$$