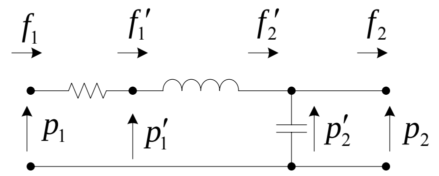
1



$p \equiv velocity$

$f \equiv force$

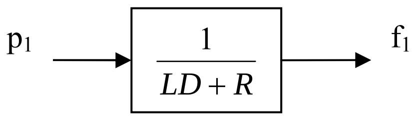
$R = 1/b, C = m, L = 1/K$

R L C

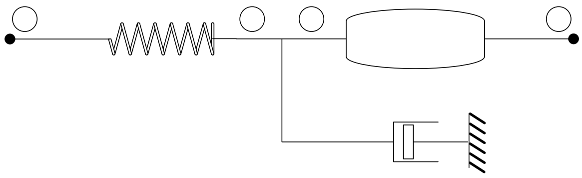
$$\left.\begin{aligned}\begin{bmatrix}p_1\\f_1\end{bmatrix} &= \begin{bmatrix}1 & R\\0 & 1\end{bmatrix}\begin{bmatrix}p_1'\\f_1'\end{bmatrix}\\ \begin{bmatrix}p_1'\\f_1'\end{bmatrix} &= \begin{bmatrix}1 & LD\\0 & 1\end{bmatrix}\begin{bmatrix}p_2'\\f_2'\end{bmatrix}\\ \begin{bmatrix}p_2'\\f_2'\end{bmatrix} &= \begin{bmatrix}1 & 0\\CD & 1\end{bmatrix}\begin{bmatrix}p_2\\f_2\end{bmatrix}\end{aligned}\right\} \Rightarrow \begin{bmatrix}p_1\\f_1\end{bmatrix} = \begin{bmatrix}1 & R\\0 & 1\end{bmatrix}\begin{bmatrix}1 & LD\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\CD & 1\end{bmatrix}\begin{bmatrix}p_2\\f_2\end{bmatrix}$$

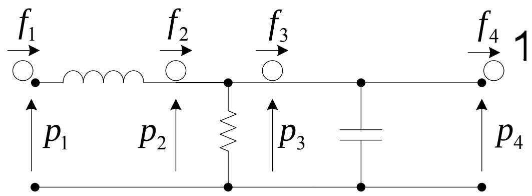
$$\Rightarrow \begin{bmatrix}p_1\\f_1\end{bmatrix} = \begin{bmatrix}1+(R+LD)CD & R+LD\\CD & 1\end{bmatrix}\begin{bmatrix}p_2\\f_2\end{bmatrix} \qquad (I)$$

$$\left.\begin{aligned}&\left.\begin{aligned}\xrightarrow{(I)} p_1 = (LCD^2 + RCD + 1)p_2 + (LD+R)f_2\\ p_2 = 0\end{aligned}\right\} \Rightarrow p_1 = (LD+R)f_2\\ &\left.\begin{aligned}\xrightarrow{(I)} f_1 = (CD)p_2 + f_2\\ p_2 = 0\end{aligned}\right\} \Rightarrow f_1 = f_2\end{aligned}\right\} \Rightarrow p_1 = (LD+R)f_1$$



2





1 2

K

$p \equiv velocity$

$f \equiv force$

$R = {}^{1}\!/_{b}, C = m, L = {}^{1}\!/_{K}$

$$\left.\begin{array}{l}\begin{bmatrix} p_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} 1 & LD \\ 0 & 1 \end{bmatrix}\begin{bmatrix} p_1' \\ f_1' \end{bmatrix} \\ \begin{bmatrix} p_1' \\ f_1' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ {}^{1}\!/_{R} & 1 \end{bmatrix}\begin{bmatrix} p_2' \\ f_2' \end{bmatrix} \\ \begin{bmatrix} p_2' \\ f_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ CD & 1 \end{bmatrix}\begin{bmatrix} p_2 \\ f_2 \end{bmatrix}\end{array}\right\} \Rightarrow \begin{bmatrix} p_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} 1 & LD \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ {}^{1}\!/_{R} & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ CD & 1 \end{bmatrix}\begin{bmatrix} p_4 \\ f_4 \end{bmatrix}$$

1 L 2

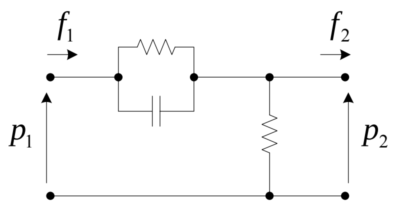
R

$$\Rightarrow \begin{bmatrix} p_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} 1 + \frac{LD}{R} + LCD^2 & LD \\ \frac{1}{R} + CD & 1 \end{bmatrix}\begin{bmatrix} p_4 \\ f_4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} p_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} 1 + \frac{b}{k}D + \frac{m}{k}D^2 & {}^{D}\!/_{k} \\ b + mD & 1 \end{bmatrix}\begin{bmatrix} p_4 \\ f_4 \end{bmatrix}$$

$$\Rightarrow \left| \begin{array}{l} f_1 = (b + mD)\,p_4 + f_4 \xrightarrow{p_4=0} f_1 = f_4 \\ p_1 = (1 + \frac{b}{k}D + \frac{m}{k}D^2)\,p_4 + ({}^{D}\!/_{k})\,f_4 \left\langle \begin{array}{l} \xrightarrow{p_4=0,\, f_1=f_4} p_1 = \frac{1}{k} f_1 \\ \xrightarrow{p_1=0} p_4 = \left(\frac{{}^{D}\!/_{k}}{1 + \frac{b}{k}D + \frac{m}{k}D^2}\right) f_4 \end{array}\right. \end{array}\right.$$

$$\boxed{\Rightarrow m\ddot{p}_4 + b\dot{p}_4 + kp_4 = \dot{f}_4}$$



3

R1

P1

C

$Z_1 \equiv \text{impedamce}(P_1P_2)$

$Y_1 \equiv \text{admittance}(P_3)$

$$\frac{1}{Z_1} = \frac{1}{R_1} + CD \Rightarrow Z_1 = \frac{R_1}{R_1CD+1}$$

$$Y_1 = {}^{1}\!/_{R_2}$$

$$\begin{bmatrix} p_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix}\begin{bmatrix} p_2 \\ f_2 \end{bmatrix} \Rightarrow \begin{bmatrix} p_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{R_1}{R_1CD+1} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \dfrac{1}{R_2} & 1 \end{bmatrix}\begin{bmatrix} p_2 \\ f_2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} p_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} 1 + \dfrac{R_1}{R_2} \times \dfrac{1}{R_1CD+1} & \dfrac{R_1}{R_1CD+1} \\ \dfrac{1}{R_2} & 1 \end{bmatrix}\begin{bmatrix} p_2 \\ f_2 \end{bmatrix} \quad \text{(الف)}$$

$$\left.\begin{aligned} p_1 &= \frac{R_2R_1CD + R_2 + R_1}{R_2(R_1CD+1)} p_2 + \frac{R_1}{R_1CD+1} f_2 \\ f_1 &= (\frac{1}{R_2})P_2 + f_2 \\ f_2 &= 0 \end{aligned}\right\} \Rightarrow$$

$$\frac{p_2}{p_1} = \frac{R_2(R_1CD+1)}{R_2R_1CD + R_2 + R_1} = \frac{R_2}{R_2+R_1} \times \frac{R_1CD+1}{\left(\dfrac{R_2R_1}{R_2+R_1}\right)CD+1} \Rightarrow$$

$$\begin{cases} a = \dfrac{R_2}{R_2+R_1} \\ T = R_1C \end{cases} \quad \text{(ب)}$$

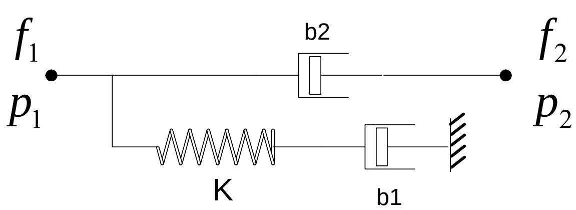
ج) سیستم مکانیکی معادل:

p = نیرو
f = سرعت

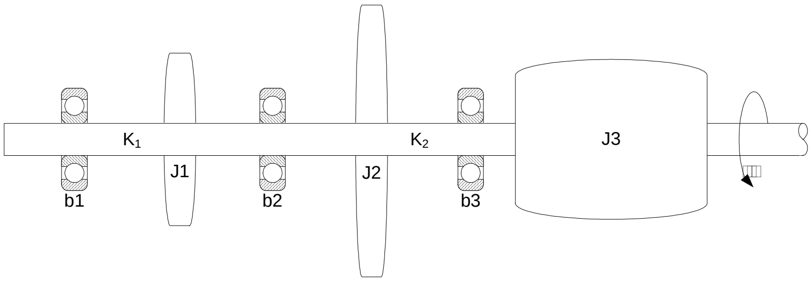
$R = {}^{1}\!/_{C}$

$b_1 = R_1$

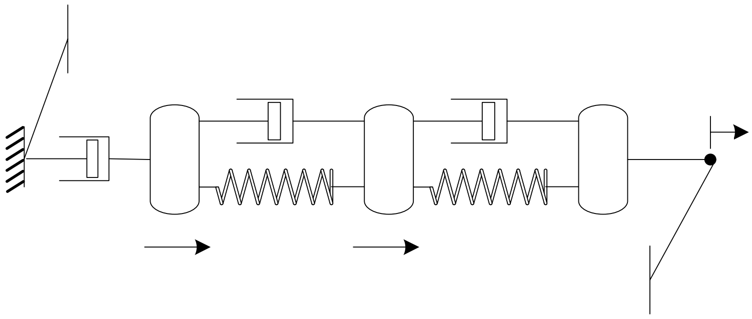
$b_2 = R_2$

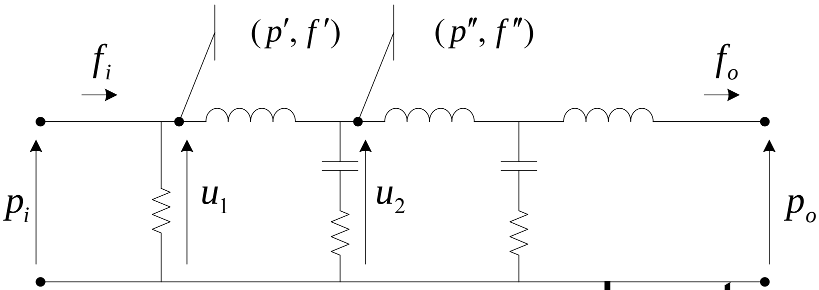


6



$$\begin{cases} J_1 = J_2 = J_3 = 1 \\ b_1 = b_2 = 1 \\ k_1 = k_2 = 1 \\ \omega = \Omega \\ \Omega = G_1(D)u_1 + G_2(D)u_2 \end{cases}$$





Input

b2

$$\begin{bmatrix} p_o \\ f_o \end{bmatrix} = \begin{bmatrix} 1 & L_3 D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{C_2 D}{1 + R_3 C_2 D} & 1 \end{bmatrix} \begin{bmatrix} 1 & L_2 D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p'' \\ f'' \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} p_o \\ f_o \end{bmatrix} = \begin{bmatrix} \dfrac{D^2 + D + 1}{D + 1} & D(\dfrac{D^2 + D + 1}{D + 1} + 1) \\ \dfrac{D}{D + 1} & \dfrac{D^2 + D + 1}{D + 1} \end{bmatrix} \begin{bmatrix} p'' \\ f'' \end{bmatrix}$$

$$\Rightarrow \begin{cases} p_o = (\dfrac{D^2 + D + 1}{D + 1})u_2 + D(\dfrac{D^2 + D + 1}{D + 1} + 1) f'' \\ f_o = (\dfrac{D}{D + 1})u_2 + (\dfrac{D^2 + D + 1}{D + 1}) f'' \qquad (I) \end{cases}$$

$$(III) \left| \begin{array}{l} G_3' = \dfrac{D}{D + 1} \\ G_4' = \dfrac{D^2 + D + 1}{D + 1} \end{array} \right.$$

m1

m

b1

K1

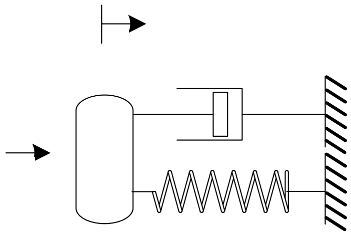
F1

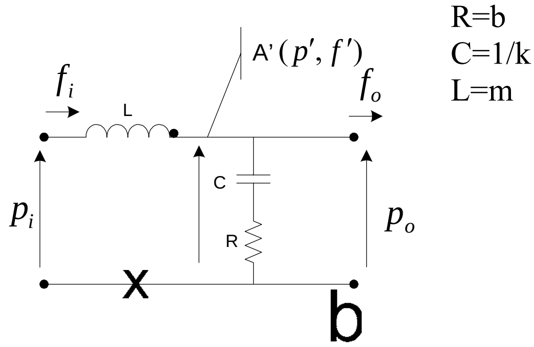
F

A'

L1

C1





$$p_i = F$$

$$x = f'/D$$

$$\tau = 1/Z ,\ Z = R + \frac{1}{CD} = b + \frac{k}{D}$$

F m1 d

k

$$\begin{bmatrix} p_i \\ \underbrace{f_i}_{f'} \end{bmatrix} = \begin{bmatrix} 1 & L_3 D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \tau & 1 \end{bmatrix} \begin{bmatrix} p_0 \\ \underbrace{f_0}_{0} \end{bmatrix}$$

$$\left.\begin{matrix} \begin{bmatrix} p_i \\ f_i \end{bmatrix} = \begin{bmatrix} 1 & LD \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p' \\ f' \end{bmatrix} \Rightarrow \begin{cases} p_i = p' + mDf' \\ f_i = f' \end{cases} \\ \begin{bmatrix} p' \\ f' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{D}{bD+k} & 1 \end{bmatrix} \begin{bmatrix} p_o \\ f_o \end{bmatrix} \Rightarrow \begin{cases} p' = p_o \\ f' = (\dfrac{D}{bD+k}) p_o + \underbrace{f_0}_{0} \end{cases} \end{matrix}\right\} \Rightarrow$$

$$p_i = (\frac{bD+k}{D} + mD) f' = (\frac{mD^2 + bD + k}{D}) f' \Rightarrow$$

$$\frac{x}{f} = \frac{\left(f'/D\right)}{p_i} = \frac{1}{mD^2 + bD + k}$$

$$\begin{bmatrix} p_o \\ f_o \end{bmatrix} = \begin{bmatrix} 1 & L_3 D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{\frac{1}{C_2 D}}{1 + R_3 C_2 D} & 1 \end{bmatrix} \begin{bmatrix} 1 & L_2 D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{\frac{1}{C_1 D}}{1 + R_2 C_1 D} & 1 \end{bmatrix} \begin{bmatrix} 1 & L_1 D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p' \\ f' \end{bmatrix}$$

$$\begin{bmatrix} p_o \\ f_o \end{bmatrix} = \begin{bmatrix} \dfrac{D^2+D+1}{D+1} + \dfrac{D^2}{D+1}(\dfrac{D^2+2D+2}{D+1}) & D(\dfrac{D^2+D+1}{D+1}) + D(\dfrac{D^2+2D+2}{D+1})(\dfrac{D^2+D+1}{D+1}) \\ \dfrac{D}{D+1} + \dfrac{D}{D+1}(\dfrac{D^2+D+1}{D+1}) & (\dfrac{D^2}{D+1})(\dfrac{D^2+D+1}{D+1})^2 \end{bmatrix} \begin{bmatrix} p' \\ f' \end{bmatrix}$$

$$\Rightarrow \begin{cases} p_o = (\dfrac{(D^2+D+1)(D+1) + D^2(D^2+2D+2)}{D^2+2D+1}) u_1 + D(\dfrac{(D^2+D+1)(D^2+3D+3)}{D^2+2D+1}) f' \\ f_o = D(\dfrac{D^2+2D+2}{D^2+2D+1}) u_1 + (\dfrac{D^2(D+1) + (D^2+D+1)^2}{D^2+2D+1}) f' \qquad (III) \end{cases}$$

$$(IV) \left| \begin{matrix} G_1' = D(\dfrac{D^2+2D+2}{D^2+2D+1}) \\ G_2' = (\dfrac{D^2(D+1) + (D^2+D+1)^2}{D^2+2D+1}) \end{matrix} \right.$$

اکنون باید f' و f'' را از روابط فوق حذف نماییم. برای این کار از اطلاعات نقطه ورودی استفاده می‌کنیم:

$$\left.\begin{array}{r}\begin{bmatrix}p_i\\ f_i\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1/R_1 & 1\end{bmatrix}\begin{bmatrix}p'\\ f'\end{bmatrix}\\ p_i=u_1\\ f_i=0\end{array}\right\}\Rightarrow f'=-u_1 \qquad (V)$$

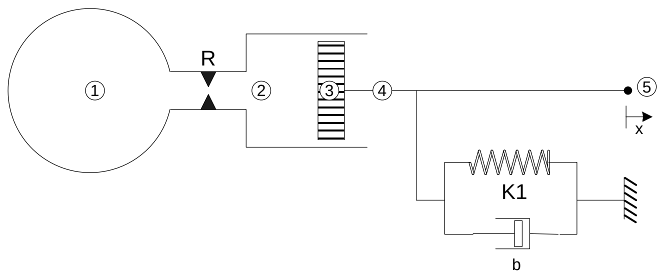
$$\begin{bmatrix}p_i\\ f_i\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1/R_1 & 1\end{bmatrix}\begin{bmatrix}1 & L_1D\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \dfrac{C_1D}{1+R_2C_1D} & 1\end{bmatrix}\begin{bmatrix}p''\\ f''\end{bmatrix}\Rightarrow$$

$$\left.\begin{array}{r}\begin{bmatrix}p_i\\ f_i\end{bmatrix}=\begin{bmatrix}\dfrac{D^2+D+1}{D+1} & D\\ D+1 & D+1\end{bmatrix}\begin{bmatrix}p''\\ f''\end{bmatrix}\\ p_i=u_1\\ f_i=0\end{array}\right\}\Rightarrow u_2=-f'' \qquad (VI)$$

$$\left.\begin{array}{r}I,II,V\Rightarrow f_o=G_1'u_1+G_2'u_1\\ III,IV,VI\Rightarrow f_o=G_1'u_1+G_2'u_1\\ f_o=\Omega\\ \Omega=G_1(D)u_1+G_2(D)u_2\end{array}\right\}\Rightarrow\left|\begin{array}{l}G_1=G_1'-G_2'\\ G_2=G_3'-G_4'\end{array}\right.$$

$$\boxed{\begin{array}{l}G_1=-\left(\dfrac{D^4+2D^3+2D^2+1}{D^2+2D+1}\right)\\ G_2=-\left(\dfrac{D^2+1}{D+1}\right)\end{array}}$$

7



$$\begin{cases} f_1 = f_2 \\ p_1 - p_2 = Rf_1 \end{cases}$$

$$\begin{cases} f_3 = f_4 \\ p_3 - p_4 = mDf_3 \end{cases}$$

$$\begin{cases} p_3 = p_2 A \\ f_3 = f_2 / A \end{cases}$$

$$\begin{cases} f_4 = f_5 \\ p_4 - p_5 = k\dfrac{f}{D} + bf_4 \end{cases}$$

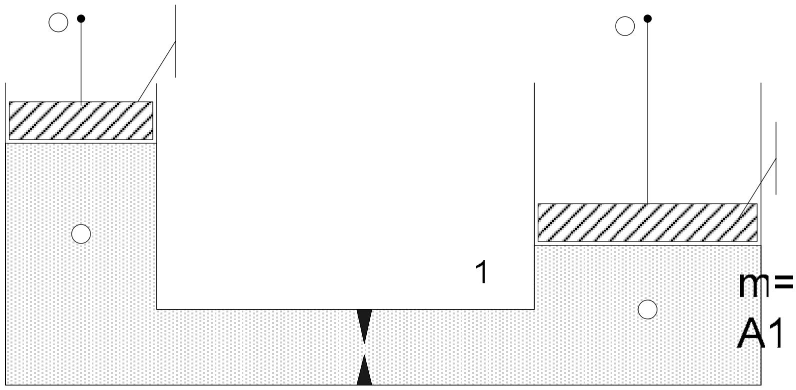
$$\left.\begin{aligned} \begin{bmatrix} p_1 \\ f_1 \end{bmatrix} &= \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_2 \\ f_2 \end{bmatrix} \\ \begin{bmatrix} p_2 \\ f_2 \end{bmatrix} &= \begin{bmatrix} 1/A & 0 \\ 0 & A \end{bmatrix} \begin{bmatrix} p_3 \\ f_3 \end{bmatrix} \\ \begin{bmatrix} p_3 \\ f_3 \end{bmatrix} &= \begin{bmatrix} 1 & mD \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_4 \\ f_4 \end{bmatrix} \\ \begin{bmatrix} p_4 \\ f_4 \end{bmatrix} &= \begin{bmatrix} 1 & \dfrac{k}{D} + b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_5 \\ f_5 \end{bmatrix} \end{aligned}\right\} \Rightarrow \begin{bmatrix} p_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/A & 0 \\ 0 & A \end{bmatrix} \begin{bmatrix} 1 & mD \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \dfrac{k}{D} + b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_5 \\ f_5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} p_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} 1/A & \dfrac{(mD + b + k/D)}{A} + RA \\ 0 & A \end{bmatrix} \begin{bmatrix} p_5 \\ f_5 \end{bmatrix} \xrightarrow{P_5 = 0} \left| \begin{aligned} & P_1 = \left[\frac{1}{A}(mD + b + k/D) + RA\right] f_5 \\ & f_1 = Af_5 \Rightarrow f_5 = f_1/A \end{aligned} \right\} \Rightarrow$$

$$P_1 = \frac{1}{A^2}\left[mD^2 + (b + RA^2)D + k\right] \underbrace{\left(f_1/D\right)}_{X(D)} \Rightarrow$$

$$\boxed{\frac{X(D)}{P(D)} = \frac{A^2}{mD^2 + (b + RA^2)D + k}}$$

11



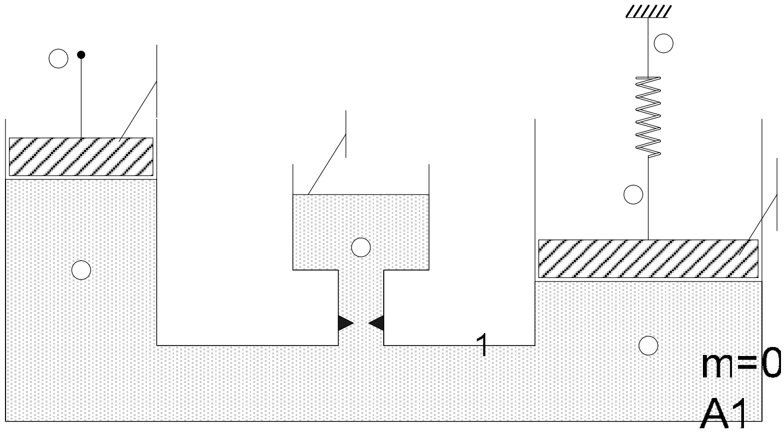
$$\left.\begin{aligned}
\begin{bmatrix} p_1 \\ f_1 \end{bmatrix} &= \begin{bmatrix} A_1 & 0 \\ 0 & 1/A_1 \end{bmatrix}\begin{bmatrix} p_3 \\ f_3 \end{bmatrix} \\
\begin{bmatrix} p_3 \\ f_3 \end{bmatrix} &= \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}\begin{bmatrix} p_4 \\ f_4 \end{bmatrix} \\
\begin{bmatrix} p_4 \\ f_4 \end{bmatrix} &= \begin{bmatrix} 1/A_2 & 0 \\ 0 & A_2 \end{bmatrix}\begin{bmatrix} p_2 \\ f_2 \end{bmatrix}
\end{aligned}\right\} \Rightarrow \begin{bmatrix} p_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & 1/A_1 \end{bmatrix}\begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1/A_2 & 0 \\ 0 & A_2 \end{bmatrix}\begin{bmatrix} p_2 \\ f_2 \end{bmatrix}$$

3

$$\boxed{\Rightarrow \begin{bmatrix} p_1 \\ f_1 \end{bmatrix} = \begin{bmatrix} A_1/A_2 & RA_1A_2 \\ 0 & A_2/A_1 \end{bmatrix}\begin{bmatrix} p_2 \\ f_2 \end{bmatrix}}$$

R

12



$$\begin{bmatrix}p_1\\f_1\end{bmatrix}=\begin{bmatrix}A_1 & 0\\0 & 1/A_1\end{bmatrix}\begin{bmatrix}p_3\\f_3\end{bmatrix}$$

$$\begin{bmatrix}p_3\\f_3\end{bmatrix}=\begin{bmatrix}1 & 0\\\underbrace{\dfrac{AD}{1+RAD}}_{ref:problem9} & 1\end{bmatrix}\begin{bmatrix}p_4\\f_4\end{bmatrix}$$

$$\begin{bmatrix}p_4\\f_4\end{bmatrix}=\begin{bmatrix}1/A_2 & 0\\0 & A_2\end{bmatrix}\begin{bmatrix}p_5\\f_5\end{bmatrix}$$

$$\begin{bmatrix}p_5\\f_5\end{bmatrix}=\begin{bmatrix}1 & mD\\0 & 1\end{bmatrix}\begin{bmatrix}p_2\\f_2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}p_1\\f_1\end{bmatrix}=\begin{bmatrix}A_1 & 0\\0 & 1/A_1\end{bmatrix}\begin{bmatrix}1 & 0\\\dfrac{AD}{1+RAD} & 1\end{bmatrix}\begin{bmatrix}1/A_2 & 0\\0 & A_2\end{bmatrix}\begin{bmatrix}1 & mD\\0 & 1\end{bmatrix}\begin{bmatrix}p_2\\f_2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}p_1\\f_1\end{bmatrix}=\begin{bmatrix}\dfrac{A_1}{A_2} & \dfrac{A_1}{A_2}mD\\\dfrac{A}{A_1A_2}\times\dfrac{D}{1+RAD} & \dfrac{A}{A_1A_2}\times\dfrac{mD^2}{1+RAD}+\dfrac{A_2}{A_1}\end{bmatrix}\begin{bmatrix}p_2\\f_2\end{bmatrix}\quad (I)$$

$$\begin{bmatrix}p_2\\f_2\end{bmatrix}=\begin{bmatrix}1 & 0\\D/k & 1\end{bmatrix}\begin{bmatrix}p_6\\\underbrace{f_6}_{0}\end{bmatrix}\Rightarrow\begin{cases}p_2=p_6\\f_2=\dfrac{D}{k}p_6=\dfrac{D}{k}p_2 \quad (II)\end{cases}$$

$$I,II\Rightarrow\begin{cases}P_1=\left(\dfrac{A_1}{A_2}\right)\left(1+\dfrac{m}{k}D^2\right)P_2\\f_1=\left(\dfrac{1}{A_1A_2(1+RAD)}\right)\left[AD+\dfrac{D}{k}\left(mAD^2+A_2{}^2(1+RAD)\right)\right]P_2\end{cases}$$