Problem 7.1

Given: The slope of the free surface of a steady wave in one-dimensional flow in a shallow liquid layer is described by the equation

$$\frac{\partial h}{\partial x} = -\frac{u}{g}\frac{\partial u}{\partial x}$$

Find: Nondimensionalize the equation (using length scale, L, and velocity scale, V_0)
Obtain the dimensionless groups that characterize this flow.

Solution:

To nondimensionalize the equation, all lengths are divided by the reference length, L, and all vecolities are divided by the reference velocity, V_0.

Denoting the nondimensional quantities by an asterisk,

$$h^* = \frac{h}{L} \quad , \; x^* = \frac{x}{L} \quad , \; u^* = \frac{u}{V_0}$$

Substituting into the governing equation

$$\frac{\partial (h^* L)}{\partial (x^* L)} = -\frac{V_0 u^*}{g}\frac{\partial (V_0 u^*)}{\partial (L x^*)}$$

$$\frac{\partial h^*}{\partial x^*} = -\frac{V_0^2}{gL}\frac{\partial u^*}{\partial x^*}$$

The dimensionless group is $\frac{V_0^2}{gL}$. This is the square of the Froude number.

Problem 7.2

Given: The propagation speed of small amplitude waves in a region of uniform depth is given by

$$c^2 = \left(\frac{\sigma}{\rho}\frac{2\pi}{\lambda} + \frac{g\lambda}{2\pi}\right)\tanh\frac{2\pi h}{\lambda}$$

where h is the depth of the undisturbed liquid
λ is the wavelength.

Find: Obtain the dimensionless groups that characterize the equation. (Use L as a characteristic length and V_0 as a characteristic velocity)

Solution:

$$c^2 = \left(\frac{\sigma}{\rho}\frac{2\pi}{\lambda} + \frac{g\lambda}{2\pi}\right)\tanh\frac{2\pi h}{\lambda}$$

To nondimensionalize the equation, all lengths are divided by L and all velocities are divided by V_0.

Denoting nondimensional quantities by an asterisk, then

$$\lambda^* = \frac{\lambda}{L} \qquad h^* = \frac{h}{L} \qquad c^* = \frac{c}{V_0}$$

Then

$$c^{*2}V_0^2 = \left(\frac{\sigma}{\rho}\frac{2\pi}{L\lambda^*} + \frac{g\lambda^* L}{2\pi}\right)\tanh\frac{2\pi h^* L}{\lambda^* L}$$

$$c^{*2} = \left(\frac{\sigma}{\rho L V_0^2}\frac{2\pi}{\lambda^*} + \frac{gL}{V_0^2}\frac{\lambda^*}{2\pi}\right)\tanh\frac{2\pi h^*}{\lambda^*}$$

$\therefore$ Dimensionless groups are $\dfrac{\sigma}{\rho L V_0^2}$, $\dfrac{gL}{V_0^2}$

Problem 7.3

7.3 The equation describing small amplitude vibration of a beam is

$$\rho A \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial x^4} = 0$$

where y is the beam deflection at location x and time t, ρ and E are the density and modulus of elasticity of the beam material, respectively, and A and I are the beam cross-section area and second moment of area, respectively. Use the beam length L, and frequency of vibration ω, to nondimensionalize this equation. Obtain the dimensionless groups that characterize the equation.

Given: Equation for beam

Find: Dimensionless groups

Solution:

Denoting nondimensional quantities by an asterisk

$$A^* = \frac{A}{L^2} \qquad y^* = \frac{y}{L} \qquad t^* = t\omega \qquad I^* = \frac{I}{L^4} \qquad x^* = \frac{x}{L}$$

Hence $$A = L^2 A^* \qquad y = L\, y^* \qquad t = \frac{t^*}{\omega} \qquad I = L^4 I^* \qquad x = L\, x^*$$

Substituting into the governing equation $$\rho L^2 L \omega^2 A^* \frac{\partial^2 y^*}{\partial t^{*2}} + EL^4 \frac{1}{L^4} LI^* \frac{\partial^4 y^*}{\partial x^{*4}} = 0$$

The final dimensionless equation is $$A^* \frac{\partial^2 y^*}{\partial t^{*2}} + \left(\frac{E}{\rho L^2 \omega^2}\right) I^* \frac{\partial^4 y^*}{\partial x^{*4}} = 0$$

The dimensionless group is $$\left(\frac{E}{\rho L^2 \omega^2}\right)$$

Problem 7.4

Given: One-dimensional, unsteady flow in a thin liquid layer is described by the equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial h}{\partial x}$$

Find: Nondimensionalize the equation (using length scale, L, and velocity scale, V_0)
Obtain the dimensionless groups that characterize this flow.

Solution:

To nondimensionalize the equation, all lengths are divided by the reference length, L, velocity is divided by the reference velocity, V_0, and time is divided by the ratio, L/V_0.

Denoting the nondimensional quantities by an asterisk,

$$x^* = \frac{x}{L}, \quad h^* = \frac{h}{L}, \quad u^* = \frac{u}{V_0}, \quad t^* = \frac{t}{L/V_0}$$

Substituting into the governing equation

$$\frac{\partial(V_0 u^*)}{\partial(Lt^*/V_0)} + u^* V_0 \frac{\partial(V_0 u^*)}{\partial(x^* L)} = -g\frac{\partial(h^* L)}{\partial(x^* L)}$$

$$\frac{V_0^2}{L}\frac{\partial u^*}{\partial t^*} + \frac{V_0^2}{L}u^*\frac{\partial u^*}{\partial x^*} = -g\frac{\partial h^*}{\partial x^*}$$

Multiplying through by L/V_0^2,

$$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} = -\frac{gL}{V_0^2}\frac{\partial h^*}{\partial x^*}$$

The dimensionless group is $\frac{gL}{V_0^2}$. This is one over the square of the Froude number.

Problem 7.5

Given: For steady, incompressible, two-dimensional flow, the Prandtl boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{-------- (1)}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \quad \text{--- (2)}$$

Find: Nondimensionalize these equations (using L and V_0 as characteristic length and velocity, respectively) and identify the resulting similarity parameters.

Solution:

Denoting nondimensional quantities by an asterisk.

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L} \quad u^* = \frac{u}{V_0} \quad v^* = \frac{v}{V_0}$$

Substituting into Eq. 1, we obtain

$$\frac{\partial(u^* V_0)}{\partial(x^* L)} + \frac{\partial(v^* V_0)}{\partial(y^* L)} = 0 = \frac{V_0}{L}\frac{\partial u^*}{\partial x^*} + \frac{V_0}{L}\frac{\partial v^*}{\partial y^*}$$

or

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

Consider each term in Eq. 2.

$$u\frac{\partial u}{\partial x} = u^* V_0 \frac{\partial(u^* V_0)}{\partial(x^* L)} = \frac{V_0^2}{L} u^* \frac{\partial u^*}{\partial x^*}$$

$$v\frac{\partial u}{\partial y} = v^* V_0 \frac{\partial(u^* V_0)}{\partial(y^* L)} = \frac{V_0^2}{L} v^* \frac{\partial u^*}{\partial x^*}$$

Leave 3rd term as is for the moment

$$\nu\frac{\partial^2 u}{\partial y^2} = \nu\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \nu\frac{\partial}{\partial y}\left(\frac{\partial u^* V_0}{\partial y^* L}\right) = \nu\frac{V_0}{L}\frac{\partial}{\partial y}\left(\frac{\partial u^*}{\partial y^*}\right) = \nu\frac{V_0}{L}\frac{\partial}{\partial(y^* L)}\frac{\partial u^*}{\partial y^*} = \nu\frac{V_0}{L^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

Substituting into Eq. 2

$$\frac{V_0^2}{L} u^* \frac{\partial u^*}{\partial x^*} + \frac{V_0^2}{L} v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{V_0}{L^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

Multiplying through by $\frac{L}{V_0^2}$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{L}{\rho V_0^2}\frac{\partial p}{\partial x} + \frac{\nu}{V_0 L}\frac{\partial^2 u^*}{\partial y^{*2}} = -\frac{L\,\partial\left(\frac{p}{\rho V_0^2}\right)}{\partial(x^* L)} + \frac{\nu}{V_0 L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

Define the non-dimensional pressure $p^* = \frac{p}{\rho V_0^2}$, then

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y} = -\frac{\partial p^*}{\partial x^*} + \frac{\nu}{V_0 L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

The similarity parameter is $\frac{\nu}{V_0 L} = \frac{1}{Re}$

Problem 7.6

7.6 In atmospheric studies the motion of the earth's atmosphere can sometimes be modeled with the equation

$$\frac{D\vec{V}}{Dt} + 2\vec{\Omega} \times \vec{V} = -\frac{1}{\rho}\nabla p$$

where $\vec{V}$ is the large-scale velocity of the atmosphere across the earth's surface, ∇p is the climatic pressure gradient, and $\vec{\Omega}$ is the earth's angular velocity. What is the meaning of the term $\vec{\Omega} \times \vec{V}$? Use the pressure difference, Δp, and typical length scale, L (which could, for example, be the magnitude of, and distance between, an atmospheric high and low, respectively), to nondimensionalize this equation. Obtain the dimensionless groups that characterize this flow.

Given: Equations for modeling atmospheric motion

Find: Non-dimensionalized equation; Dimensionless groups

Solution:

Recall that the total acceleration is

$$\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V}$$

Nondimensionalizing the velocity vector, pressure, angular velocity, spatial measure, and time, (using a typical velocity magnitude V and angular velocity magnitude Ω):

$$\vec{V}^* = \frac{\vec{V}}{V} \qquad p^* = \frac{p}{\Delta p} \qquad \vec{\Omega}^* = \frac{\vec{\Omega}}{\Omega} \qquad x^* = \frac{x}{L} \qquad t^* = t\frac{V}{L}$$

Hence

$$\vec{V} = V\vec{V}^* \qquad p = \Delta p\, p^* \qquad \vec{\Omega} = \Omega\vec{\Omega}^* \qquad x = L\,x^* \qquad t = \frac{L}{V}t^*$$

Substituting into the governing equation

$$V\frac{V}{L}\frac{\partial \vec{V}^*}{\partial t^*} + V\frac{V}{L}\vec{V}^* \cdot \nabla^* \vec{V}^* + 2\Omega V\,\vec{\Omega}^* \times \vec{V}^* = -\frac{1}{\rho}\frac{\Delta p}{L}\nabla p^*$$

The final dimensionless equation is

$$\frac{\partial \vec{V}^*}{\partial t^*} + \vec{V}^* \cdot \nabla^* \vec{V}^* + 2\left(\frac{\Omega L}{V}\right)\vec{\Omega}^* \times \vec{V} = -\frac{\Delta p}{\rho V^2}\nabla p^*$$

The dimensionless groups are

$$\frac{\Delta p}{\rho V^2} \qquad \frac{\Omega L}{V}$$

The second term on the left of the governing equation is the Coriolis force due to a rotating coordinate system. This is a very significant term in atmospheric studies, leading to such phenomena as geostrophic flow.

Problem 7.7

[2]

7.7 The equation describing motion of fluid in a pipe due to an applied pressure gradient, when the flow starts from rest, is

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)$$

Use the average velocity $\overline{V}$, pressure drop Δp, pipe length L, and diameter D to nondimensionalize this equation. Obtain the dimensionless groups that characterize this flow.

Given: Equations Describing pipe flow

Find: Non-dimensionalized equation; Dimensionless groups

Solution:

Nondimensionalizing the velocity, pressure, spatial measures, and time:

$$u^* = \frac{u}{\overline{V}} \qquad p^* = \frac{p}{\Delta p} \qquad x^* = \frac{x}{L} \qquad r^* = \frac{r}{L} \qquad t^* = t\frac{\overline{V}}{L}$$

Hence

$$u = \overline{V}\,u^* \qquad p = \Delta p\,p^* \qquad x = L\,x^* \qquad r = D\,r^* \qquad t = \frac{L}{\overline{V}}t^*$$

Substituting into the governing equation

$$\frac{\partial u}{\partial t} = \overline{V}\frac{\overline{V}}{L}\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\Delta p\frac{1}{L}\frac{\partial p^*}{\partial x^*} + \nu\overline{V}\frac{1}{D^2}\left(\frac{\partial^2 u^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right)$$

The final dimensionless equation is

$$\frac{\partial u^*}{\partial t^*} = -\frac{\Delta p}{\rho\overline{V}^2}\frac{\partial p^*}{\partial x^*} + \left(\frac{\nu}{D\overline{V}}\right)\left(\frac{L}{D}\right)\left(\frac{\partial^2 u^*}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial u^*}{\partial r^*}\right)$$

The dimensionless groups are

$$\frac{\Delta p}{\rho\overline{V}^2} \qquad \frac{\nu}{D\overline{V}} \qquad \frac{L}{D}$$

Problem 7.8

[2]

7.8 An unsteady, two dimensional, compressible, inviscid flow can be described by the equation

$$\frac{\partial^2\psi}{\partial t^2}+\frac{\partial}{\partial t}(u^2+v^2)+(u^2-c^2)\frac{\partial^2\psi}{\partial x^2}+(v^2-c^2)\frac{\partial^2\psi}{\partial y^2}+2uv\frac{\partial^2\psi}{\partial x\partial y}=0$$

where ψ is the stream function, u and v are the x and y components of velocity, respectively, c is the local speed of sound, and t is the time. Using L as a characteristic length and c_0 (the speed of sound at the stagnation point) to nondimensionalize this equation, obtain the dimensionless groups that characterize the equation.

Given: Equation for unsteady, 2D compressible, inviscid flow

Find: Dimensionless groups

Solution:

Denoting nondimensional quantities by an asterisk

$$x^*=\frac{x}{L}\qquad y^*=\frac{y}{L}\qquad u^*=\frac{u}{c_0}\qquad v^*=\frac{v}{c_0}\qquad c^*=\frac{c}{c_0}\qquad t^*=\frac{t\,c_0}{L}\qquad \psi^*=\frac{\psi}{L\,c_0}$$

Note that the stream function indicates volume flow rate/unit depth!

Hence

$$x=L\,x^*\qquad y=L\,y^*\qquad u=c_0\,u^*\qquad v=c_0\,v^*\qquad c=c_0\,c^*\qquad t=\frac{L\,t^*}{c_0}\qquad \psi=L\,c_0\,\psi^*$$

Substituting into the governing equation

$$\left(\frac{c_0^3}{L}\right)\frac{\partial^2\psi^*}{\partial t^{*2}}+\left(\frac{c_0^3}{L}\right)\frac{\partial\left(u^{*2}+v^{*2}\right)}{\partial t}+\left(\frac{c_0^3}{L}\right)\left(u^{*2}-c^{*2}\right)\frac{\partial^2\psi^*}{\partial x^{*2}}+\left(\frac{c_0^3}{L}\right)\left(v^{*2}-c^{*2}\right)\frac{\partial^2\psi^*}{\partial y^{*2}}+\left(\frac{c_0^3}{L}\right)2u^*v^*\frac{\partial^2\psi^*}{\partial x^*\partial y^*}=0$$

The final dimensionless equation is

$$\frac{\partial^2\psi^*}{\partial t^{*2}}+\frac{\partial\left(u^{*2}+v^{*2}\right)}{\partial t}+\left(u^{*2}-c^{*2}\right)\frac{\partial^2\psi^*}{\partial x^{*2}}+\left(v^{*2}-c^{*2}\right)\frac{\partial^2\psi^*}{\partial y^{*2}}+2u^*v^*\frac{\partial^2\psi^*}{\partial x^*\partial y^*}=0$$

No dimensionless group is needed for this equation!

Problem 7.9

Given: At low speeds, drag is independent of fluid density.

$$F = F(\mu, V, D)$$

Find: Appropriate dimensionless parameters.

Solution: Apply Buckingham Π procedure.

① $F \quad \mu \quad V \quad D$ — $n = 4$ parameters

② Select primary dimensions M, L, t.

③ $F \quad \mu \quad V \quad D$

$\frac{ML}{t^2} \quad \frac{M}{Lt} \quad \frac{L}{t} \quad L$ — $r = 3$ primary dimensions

④ μ, V, D — $m = r = 3$ repeating parameters

⑤ Then $n - m = 1$ dimensionless group will result. Setting up a dimensional equation,

$$\Pi_1 = \mu^a V^b D^c F$$

$$= \left(\frac{M}{Lt}\right)^a \left(\frac{L}{t}\right)^b (L)^c \frac{ML}{t^2} = M^0 L^0 t^0$$

Summing exponents,

$M: \quad a + 1 = 0$	$a = -1$
$L: \quad -a + b + c + 1 = 0$	$c = -1$
$t: \quad -a - b - 2 = 0$	$b = -1$

$$\therefore \Pi_1 = \frac{F}{\mu V D}$$ Π_1

⑥ Check, using F, L, t primary dimensions.

$$\Pi_1 = F \frac{L^2}{Ft} \frac{t}{L} \frac{1}{L} = [1] \quad \checkmark$$

Since the procedure produces only one dimensionless group, it must be a constant. Thus

$$\Pi_1 = \frac{F}{\mu V D} \quad \text{or} \quad F \propto \mu V D$$

Problem 7.10

7.10 At relatively high speeds the drag on an object is independent of fluid viscosity. Thus the aerodynamic drag force, F, on an automobile, is a function only of speed, V, air density ρ, and vehicle size, characterized by its frontal area A. Use dimensional analysis to determine how the drag force F depends on the speed V.

Given: That drag depends on speed, air density and frontal area

Find: How drag force depend on speed

Solution:

Apply the Buckingham Π procedure

① $F \quad V \quad \rho \quad A$ $\qquad$ $n = 4$ parameters

② Select primary dimensions M, L, t

③ $\begin{matrix} F & V & \rho & A \\ \frac{ML}{t^2} & \frac{L}{t} & \frac{M}{L^3} & L^2 \end{matrix}$ $\qquad$ $r = 3$ primary dimensions

④ $V \quad \rho \quad A$ $\qquad$ $m = r = 3$ repeat parameters

⑤ Then $n - m = 1$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = V^a \rho^b A^c F$$

$$= \left(\frac{L}{t}\right)^a \left(\frac{M}{L^3}\right)^b \left(L^2\right)^c \frac{ML}{t^2} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{rr|l} M: & b+1=0 & b=-1 \\ L: & a-3b+2c+1=0 & c=-1 \\ t: & -a-2=0 & a=-2 \end{array}$$

Hence

$$\Pi_1 = \frac{F}{\rho V^2 A}$$

⑥ Check using F, L, t as primary dimensions

$$\Pi_1 = \frac{F}{\frac{Ft^2}{L^4}\frac{L^2}{t^2}L^2} = [1]$$

The relation between drag force F and speed V must then be

$$F \propto \rho V^2 A \propto V^2$$

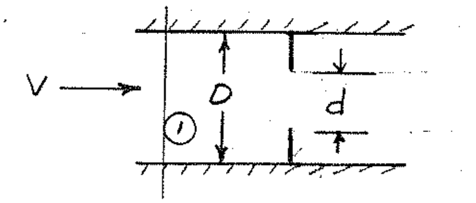
The drag is proportional to the *square* of the speed.

Problem 7.11

[2]

Given: Flow through an orifice plate

$$\Delta p = p_1 - p_2 = f(\rho, \mu, V, D, d)$$



Find: Dimensionless parameters.

Solution: Choose ρ, V, and D as repeating variables.

① Δp ρ μ V D d $n = 6$ parameters

② select primary dimensions M, L, t

③ Δp ρ μ V D d

$\frac{M}{Lt^2}$ $\frac{M}{L^3}$ $\frac{M}{Lt}$ $\frac{L}{t}$ L L $r = 3$ primary dimensions

④ ρ, V, D $m = r = 3$ repeating parameters

⑤ Then $n - m = 3$ dimensionless groups will result. Setting up dimensional equations,

$$\Pi_1 = \rho^a V^b D^c \Delta p = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \left(\frac{M}{Lt^2}\right) = M^0 L^0 t^0$$

Summing exponents,

M: $a + 1 = 0$ $a = -1$
L: $-3a + b + c - 1 = 0$
t: $-b - 2 = 0$ $b = -2$

$c = 1 - b + 3a = 0$

$$\therefore \Pi_1 = \frac{\Delta p}{\rho V^2}$$

$$\Pi_2 = \rho^a V^b D^c \mu = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \left(\frac{M}{Lt}\right) = M^0 L^0 t^0$$

Summing exponents,

M: $a + 1 = 0$ $a = -1$
L: $-3a + b + c - 1 = 0$
t: $-b - 1 = 0$ $b = -1$

$c = 1 - b + 3a = -1$

$$\therefore \Pi_2 = \frac{\mu}{\rho V D}$$

$$\Pi_3 = \rho^a V^b D^c d = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c L = M^0 L^0 t^0$$

M: $a + 0 = 0$ $a = 0$
L: $-3a + b + c + 1 = 0$
t: $-b + 0 = 0$ $b = 0$

$c = -1$; $\Pi_3 = \frac{d}{D}$

Thus $\Pi_1 = f(\Pi_2, \Pi_3)$ or $\frac{\Delta p}{\rho V^2} = f\left(\frac{\mu}{\rho V D}, \frac{d}{D}\right)$ ← $\frac{\Delta p}{\rho V^2}$

⑥ Check, using F, L, t $\Pi_1 = \frac{F}{L^2}\frac{L^4}{Ft^2}\frac{t^2}{L^2} = [1]$ ✓, $\Pi_2 = Re = [1]$ ✓, $\Pi_3 = \frac{L}{L} = [1]$ ✓

Problem 7.12

7.12 The speed, V, of a free-surface wave in shallow liquid is a function of depth, D, density, ρ, gravity, g, and surface tension, σ. Use dimensional analysis to find the functional dependence of V on the other variables. Express V in the simplest form possible.

Given: That speed of shallow waves depends on depth, density, gravity and surface tension

Find: Dimensionless groups; Simplest form of V

Solution:

Apply the Buckingham Π procedure

① $V \quad D \quad \rho \quad g \quad \sigma$ $\qquad n = 5$ parameters

② Select primary dimensions M, L, t

③ $$\begin{Bmatrix} V & D & \rho & g & \sigma \\ \frac{L}{t} & L & \frac{M}{L^3} & \frac{L}{t^2} & \frac{M}{t^2} \end{Bmatrix}$$ $r = 3$ primary dimensions

④ $g \quad \rho \quad D$ $\qquad m = r = 3$ repeat parameters

⑤ Then $n - m = 2$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = g^a \rho^b D^c V = \left(\frac{L}{t^2}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \frac{L}{t} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{ll|l} M: & b = 0 & b = 0 \\ L: & a - 3b + c + 1 = 0 & c = -\frac{1}{2} \\ t: & -2a - 1 = 0 & a = -\frac{1}{2} \end{array}$$
Hence $\Pi_1 = \dfrac{V}{\sqrt{gD}}$

$$\Pi_2 = g^a \rho^b D^c \sigma = \left(\frac{L}{t^2}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \frac{M}{t^2} = M^0 L^0 t^0$$

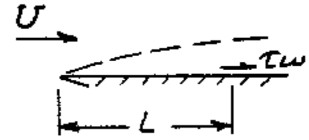
Summing exponents,
$$\begin{array}{ll|l} M: & b + 1 = 0 & b = -1 \\ L: & a - 3b + c = 0 & c = -2 \\ t: & -2a - 2 = 0 & a = -1 \end{array}$$
Hence $\Pi_2 = \dfrac{\sigma}{g\rho D^2}$

⑥ Check using F, L, t as primary dimensions $\qquad \Pi_1 = \dfrac{\frac{L}{t}}{\left(\frac{L}{t^2} L\right)^{\frac{1}{2}}} = [1] \qquad \Pi_2 = \dfrac{\frac{F}{L}}{\frac{L}{t^2}\frac{Ft^2}{L^4}L^2} = [1]$

The relation between drag force speed V is $\qquad \Pi_1 = f(\Pi_2) \qquad \dfrac{V}{\sqrt{gD}} = f\left(\dfrac{\sigma}{g\rho D^2}\right) \qquad V = \sqrt{gD}\, f\left(\dfrac{\sigma}{g\rho D^2}\right)$

Problem 7.13

Given: Wall shear stress, τ_w, in a boundary layer, depends on ρ, μ, L, and U.



Find: (a) Dimensionless groups.
(b) Express the functional relationship.

Solution: Step ① $\quad \tau_w \quad \rho \quad \mu \quad L \quad U \qquad n = 5$

Step ② Choose M, L, t. $\tau_w = \frac{F}{L^2} \times \frac{ML}{Ft^2} = \frac{M}{Lt^2}$

Step ③ $\quad \frac{M}{Lt^2} \quad \frac{M}{L^3} \quad \frac{M}{Lt} \quad L \quad \frac{L}{t} \qquad r = 3$

Step ④ Select ρ, L, U

Step ⑤ $\Pi_1 = \tau_w \rho^a L^b U^c = \frac{M}{Lt^2}\left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{L}{t}\right)^c = M^0 L^0 t^0$

$M: 0 = 1 + a \qquad a = -1$

$L: 0 = -1 - 3a + b + c \qquad b = 3a - c + 1 = 0$

$t: 0 = -2 - c \qquad c = -2$

$\Pi_1 = \frac{\tau_w}{\rho U^2}$ $\qquad \Pi_1$

$\Pi_2 = \mu \rho^a L^b U^c = \frac{M}{Lt}\left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{L}{t}\right)^c = M^0 L^0 t^0$

$M: 0 = 1 + a \qquad a = -1$

$L: 0 = -1 - 3a + b + c \qquad b = 3a - c + 1 = -1$

$t: 0 = -1 - c \qquad c = -1$

$\Pi_2 = \frac{\mu}{\rho U L}$ $\qquad \Pi_2$

Step ⑥: Check using F, L, t: $\rho = \frac{M}{L^3} \times \frac{Ft^2}{ML} = \frac{Ft^2}{L^4}$

$\Pi_1 = \frac{\tau_w}{\rho U^2} = \frac{F}{L^2} \frac{L^4}{Ft^2} \frac{t^2}{L^2} = \frac{FL^4t^2}{FL^4t^2} = 1$ ✓✓

$\Pi_2 = \frac{\mu}{\rho U L} = \frac{Ft}{L^2} \frac{L^4}{Ft^2} \frac{t}{L} \frac{1}{L} = \frac{FL^4t^2}{FL^4t^2} = 1$ ✓✓

The functional relationship is

$\Pi_1 = f(\Pi_2)$ $\qquad f$

Problem 7.14

Given: The boundary layer thickness, δ, on a smooth flat plate in incompressible flow without pressure gradient is a function of U (free stream velocity), ρ, μ, and x (distance)

Find: suitable dimensionless parameters

Solution: Apply Buckingham π-theorem

① $\delta \quad U \quad \rho \quad \mu \quad x \qquad n = 5$ parameters

② Select M, L, t as primary dimensions

③
δ	U	ρ	μ	x	
L	$\frac{L}{t}$	$\frac{M}{L^3}$	$\frac{M}{Lt}$	L	$r = 3$ primary dimensions

④ $\rho, U, x \qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 2$ dimensionless groups will result.

Setting up dimensional equations.

$\pi_1 = \rho^a U^b x^c \delta$

$M^0 L^0 t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c L$

Equating exponents,

M: $0 = a \qquad \therefore a = 0$

L: $0 = -3a + b + c + 1 \qquad c = -1$

t: $0 = -b \qquad \therefore b = 0$

$\therefore \pi_1 = \frac{\delta}{x}$

$\pi_2 = \rho^a U^b x^c \mu$

$M^0 L^0 t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{M}{Lt}$

Equating exponents,

M: $0 = a + 1 \qquad \therefore a = -1$

L: $0 = -3a + b + c - 1 \qquad c = -1$

t: $0 = -b - 1 \qquad \therefore b = -1$

$\pi_2 = \frac{\mu}{\rho U x}$

and $\frac{\delta}{x} = f\left(\frac{\rho U x}{\mu}\right)$ ← $\frac{\delta}{x}$

⑥ Check using F, L, t dimensions

$\pi_1 = \frac{L}{L} = [1]$ ✓

$\pi_2 = \frac{Ft}{L^2} \cdot \frac{L^4}{Ft^2} \frac{t}{L} \frac{1}{L} = [1]$ ✓

Problem 7.15

7.15 If an object is light enough it can be supported on the surface of a fluid by surface tension. Tests are to be done to investigate this phenomenon. The weight, W, supportable in this way depends on the object's perimeter, p, and the fluid's density, ρ, surface tension σ, and gravity, g. Determine the dimensionless parameters that characterize this problem.

Given: That light objects can be supported by surface tension

Find: Dimensionless groups

Solution:

Apply the Buckingham Π procedure

① $W \quad p \quad \rho \quad g \quad \sigma$ $\qquad n = 5$ parameters

② Select primary dimensions M, L, t

③ $$\left\{ \begin{matrix} W & p & \rho & g & \sigma \\ \dfrac{ML}{t^2} & L & \dfrac{M}{L^3} & \dfrac{L}{t^2} & \dfrac{M}{t^2} \end{matrix} \right\}$$ $r = 3$ primary dimensions

④ $g \quad \rho \quad p$ $\qquad m = r = 3$ repeat parameters

⑤ Then $n - m = 2$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = g^a \rho^b p^c W = \left(\frac{L}{t^2}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \frac{ML}{t^2} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{ll|l} M: & b + 1 = 0 & b = -1 \\ L: & a - 3b + c + 1 = 0 & c = -3 \\ t: & -2a - 2 = 0 & a = -1 \end{array}$$

Hence $$\Pi_1 = \frac{W}{g \rho p^3}$$

$$\Pi_2 = g^a \rho^b p^c \sigma = \left(\frac{L}{t^2}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \frac{M}{t^2} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{ll|l} M: & b + 1 = 0 & b = -1 \\ L: & a - 3b + c = 0 & c = -2 \\ t: & -2a - 2 = 0 & a = -1 \end{array}$$

Hence $$\Pi_2 = \frac{\sigma}{g \rho p^2}$$

⑥ Check using F, L, t as primary dimensions

$$\Pi_1 = \frac{F}{\dfrac{L}{t^2} \dfrac{Ft^2}{L^4} L^3} = [1] \qquad \Pi_2 = \frac{\dfrac{F}{L}}{\dfrac{L}{t^2} \dfrac{Ft^2}{L^4} L^2} = [1]$$

Note: Any combination of Π_1 and Π_2 is a Π group, e.g., $\dfrac{\Pi_1}{\Pi_2} = \dfrac{Wp}{\sigma}$, so Π_1 and Π_2 are not unique!

Given: The mean velocity, $\bar{u}$, for turbulent pipe or boundary layer flow, may be correlated in terms of the wall shear stress, τ_w, distance from the wall, y, and fluid properties, ρ and μ.

Find: (a) dimensionless parameter containing $\bar{u}$ and one containing y that are suitable for organizing experimental data.
(b) show that the result may be written as

$$\frac{\bar{u}}{u_*} = f\left(\frac{y u_*}{\nu}\right) \quad \text{where } u_* = (\tau_w/\rho)^{1/2}$$

Solution: Apply the Buckingham π-Theorem

① $\bar{u}$ $\quad \tau_w$ $\quad y$ $\quad \rho$ $\quad \mu$ $\qquad n = 5$ parameters

② Select M, L, t as primary dimensions

③ $\frac{L}{t}$ $\quad \frac{M}{Lt^2}$ $\quad L$ $\quad \frac{M}{L^3}$ $\quad \frac{M}{Lt}$

④ τ_w, y, ρ $\qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 2$ dimensionless groups will result

Setting up dimensional equations

$$\pi_1 = \tau_w^a y^b \rho^c \bar{u} \qquad\qquad \pi_2 = \tau_w^a y^b \rho^c \mu$$

$$M^0 L^0 t^0 = \left(\frac{M}{Lt^2}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{L}{t} \qquad\qquad M^0 L^0 t^0 = \left(\frac{M}{Lt^2}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{M}{Lt}$$

Summing exponents (for π_1):

M: $a + c = 0 \quad \therefore a = -c$

L: $-a + b - 3c + 1 = 0$

t: $-2a - 1 = 0 \quad \therefore a = -1/2$

$a = -1/2, \; c = 1/2, \; b = 0$

$$\pi_1 = \frac{\bar{u} \rho^{1/2}}{\tau_w^{1/2}} = \frac{\bar{u}}{\sqrt{\tau_w/\rho}}$$

Summing exponents (for π_2):

M: $a + c + 1 = 0 \quad \therefore c = -a - 1$

L: $-a + b - 3c - 1 = 0$

t: $-2a - 1 = 0 \quad \therefore a = -1/2$

$a = -1/2, \; c = -1/2, \; b = -1$

$$\pi_2 = \frac{\mu}{\tau_w^{1/2} \rho^{1/2} y} = \frac{\mu}{\rho y \sqrt{\frac{\tau_w}{\rho}}}$$

$$\pi_1 = f(\pi_2) \quad \text{or} \quad \frac{\bar{u}}{\sqrt{\tau_w/\rho}} = f\left(\frac{\mu}{\rho y \sqrt{\tau_w/\rho}}\right)$$

Since $\sqrt{\tau_w/\rho} = u_*$, then

$$\frac{\bar{u}}{u_*} = f\left(\frac{\mu}{\rho y u_*}\right) = f\left(\frac{\nu}{y u_*}\right) = g\left(\frac{y u_*}{\nu}\right) \qquad \frac{\bar{u}}{u_*}$$

Problem 7.17

Given: Velocity, V, of a free surface gravity wave in deep water is a function of λ (wavelength), D, ρ, and g

Find: Dependence of V on other variables.

Solution: Apply Buckingham Π-theorem

① $V \quad \lambda \quad D \quad \rho \quad g$ $\qquad n = 5$ parameters

② Select M, L, t as primary dimensions

③ $V \quad \lambda \quad D \quad \rho \quad g$

$\frac{L}{t} \quad L \quad L \quad \frac{M}{L^3} \quad \frac{L}{t^2}$ $\qquad r = 3$ primary dimensions

④ ρ, D, g $\qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 2$ dimensionless groups will result

Setting up dimensional equations

$\Pi_1 = \rho^a D^b g^c V$	$\Pi_2 = \rho^a D^b g^c \lambda$
$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t^2}\right)^c \frac{L}{t}$	$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t^2}\right)^c L$
Summing exponents,	Summing exponents,
$M: \quad a = 0$	$M: \quad a = 0$
$L: \quad -3a + b + c + 1 = 0$	$L: \quad -3a + b + c + 1 = 0$
$t: \quad -2c - 1 = 0$	$t: \quad -2c = 0$
i.e. $a = 0$	i.e. $a = 0$
$c = -\frac{1}{2}$	$c = 0$
$b = 3a - c - 1 = -\frac{1}{2}$	$b = 3a - c - 1 = -1$
$\therefore \Pi_1 = \frac{V}{\sqrt{gD}}$	$\therefore \Pi_2 = \frac{\lambda}{D}$

Thus $\frac{V}{\sqrt{gD}} = f\left(\frac{\lambda}{D}\right)$ or $V = \sqrt{gD}\, f\left(\frac{\lambda}{D}\right)$ ← V

⑥ Check using F, L, t

$\Pi_1 = \frac{L}{t} \cdot \frac{1}{\left(\frac{L}{t^2} L\right)^{1/2}} = [1]$ ✓ $\qquad \Pi_2 = \frac{L}{L} = [1]$

Problem 7.18

[2]

7.18 The torque, T, of a handheld automobile buffer is a function of rotational speed, ω, applied normal force, F, automobile surface roughness, e, buffing paste viscosity, μ, and surface tension, σ. Determine the dimensionless parameters that characterize this problem.

Given: That automobile buffer depends on several parameters

Find: Dimensionless groups

Solution:

Apply the Buckingham Π procedure

① $T \quad \omega \quad F \quad e \quad \mu \quad \sigma$ $\qquad n = 6$ parameters

② Select primary dimensions M, L, t

③ $$\left\{ \begin{matrix} T & \omega & F & e & \mu & \sigma \\ \frac{ML^2}{t^2} & \frac{1}{t} & \frac{ML}{t^2} & L & \frac{M}{Lt} & \frac{M}{t^2} \end{matrix} \right\}$$ $r = 3$ primary dimensions

④ $F \quad e \quad \omega$ $\qquad m = r = 3$ repeat parameters

⑤ Then $n - m = 3$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = F^a e^b \omega^c T = \left(\frac{ML}{t^2}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{ML^2}{t^2} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{ll|l} M: & a+1=0 & a=-1 \\ L: & a+b+2=0 & b=-1 \\ t: & -2a-c-2=0 & c=0 \end{array}$$

Hence $\Pi_1 = \dfrac{T}{Fe}$

$$\Pi_2 = F^a e^b \omega^c \mu = \left(\frac{ML}{t^2}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{Lt} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{ll|l} M: & a+1=0 & a=-1 \\ L: & a+b-1=0 & b=2 \\ t: & -2a-c-1=0 & c=1 \end{array}$$

Hence $\Pi_2 = \dfrac{\mu e^2 \omega}{F}$

$$\Pi_3 = F^a e^b \omega^c \sigma = \left(\frac{ML}{t^2}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{t^2} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{ll|l} M: & a+1=0 & a=-1 \\ L: & a+b=0 & b=1 \\ t: & -2a-c-2=0 & c=0 \end{array}$$

Hence $\Pi_3 = \dfrac{\sigma e}{F}$

⑥ Check using F, L, t as primary dimensions

$$\Pi_1 = \frac{FL}{FL} = [1] \qquad \Pi_2 = \frac{\frac{Ft}{L^2} L^2 \frac{1}{t}}{F} = [1] \qquad \Pi_3 = \frac{\frac{F}{L} L}{F} = [1]$$

Note: Any combination of Π_1, Π_2 and Π_3 is a Π group, e.g., $\frac{\Pi_1}{\Pi_2} = \frac{T}{\mu\omega e^3}$, so Π_1, Π_2 and Π_3 are not unique!

Problem 7.19

Given: Volume flow rate, Q, over a weir is a function of: upstream height, h, gravity, g, and channel width, b.

Find: Expression for Q (using dimensional analysis)

<u>Solution:</u> Apply Buckingham π-theorem

① List $\quad Q \quad h \quad g \quad b \qquad n = 4$ parameters

② Choose F, L, t as primary dimensions

③ Dimensions $\quad \frac{L^3}{t} \quad L \quad \frac{L}{t^2} \quad L$

④ Repeating variables $g, h \qquad m = r = 2$

⑤ Then $n - m = 2$ dimensionless groups will result

Setting up dimensional equations

$\pi_1 = g^a h^b Q$

$L^0 t^0 = \left(\frac{L}{t^2}\right)^a L^b \left(\frac{L^3}{t}\right)$

Equating exponents

$L: \quad 0 = a + b + 3$

$t \quad 0 = -2a - 1$

$\therefore a = -\frac{1}{2}$

$b = -2\frac{1}{2}$

$\therefore \pi_1 = \frac{Q}{g^{1/2} h^{2.5}}$

$\pi_1 = \frac{Q}{h^2 \sqrt{gh}}$

$\pi_2 = g^a h^c b$

$L^0 t^0 = \left(\frac{L}{t^2}\right)^a L^c L$

Equating exponents

$L: \quad 0 = a + c + 1$

$t: \quad 0 = -2a$

$\therefore a = 0$

$c = -1$

$\pi_2 = \frac{b}{h}$

(this is obvious by inspection)

Then

$$\frac{Q}{h^2 \sqrt{gh}} = f\left(\frac{b}{h}\right)$$

$$Q = h^2 \sqrt{gh}\; f\left(\frac{b}{h}\right)$$

Q

Problem 7.20

[2]

Given: Capillary waves form on a liquid free surface. The speed of the wave is a function of σ (surface tension), λ (the wave length) and ρ

Find: The wave speed as a function of the variables

Solution: Apply Buckingham Π - theorem

① $V \quad \sigma \quad \lambda \quad \rho$ $\qquad n = 4$ parameters

② Select M, L, t as primary dimensions

③ $V \quad \sigma \quad \lambda \quad \rho$

$\frac{L}{t} \quad \frac{M}{t^2} \quad L \quad \frac{M}{L^3}$ $\qquad r = 3$ primary dimensions

④ σ, λ, ρ $\qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 1$ dimensionless group will result

Setting up dimensional equation

$$\Pi_1 = \sigma^a \lambda^b \rho^c V$$

$$M^0 L^0 t^0 = \left(\frac{M}{t^2}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{L}{t}$$

Summing exponents

M: $a + c = 0$ $\qquad c = -a = \frac{1}{2}$

L: $b - 3c + 1 = 0$ $\qquad b = 3c - 1 = \frac{1}{2}$

t: $-2a - 1 = 0$ $\qquad \therefore a = -\frac{1}{2}$

$$\therefore \Pi_1 = \left(\frac{\rho\lambda}{\sigma}\right)^{\frac{1}{2}} V = \text{constant} \qquad \therefore V \propto \sqrt{\frac{\sigma}{\rho\lambda}}$$

V

⑥ Check using F, L, t

$$\Pi_1 = \left(\frac{Ft^2}{L^4} \cdot L \cdot \frac{L}{F}\right)^{1/2} \frac{L}{t} = [1] \checkmark$$

Problem 7.21

Given: Load-carrying capacity, W, (of a journal bearing) depends on: diameter, D; length, ℓ; clearance, c; angular speed, ω; lubricant viscosity, μ

Find: Dimensionless parameters that characterize the problem.

Solution: Apply Buckingham π - theorem

① List W D ℓ c ω μ n=6 parameters

② Choose F, L, t as primary dimensions

③ Dimensions F L L L $\frac{1}{t}$ $\frac{Ft}{L^2}$

④ Repeating variables D, ω, μ m = r = 3

⑤ Then n-m = 3 dimensionless groups will result

By inspection, $\pi_1 = \frac{\ell}{D}$ $\pi_2 = \frac{c}{D}$

Set up dimensional equation to determine π_3

$$\pi_3 = D^a \omega^b \mu^e W$$

$$F^0 L^0 t^0 = L^a \left(\frac{1}{t}\right)^b \left(\frac{Ft}{L^2}\right)^e F$$

Equating exponents: F $0 = e + 1$ $\therefore e = -1$

L $0 = a - 2e$ $\therefore a = -2$

t $0 = -b + e$ $\therefore b = -1$

and

$$\pi_3 = \frac{W}{D^2 \omega \mu}$$

⑥ Check using M, L, t dimensions

$$\pi_3 = \frac{ML}{t^2} \times \frac{1}{L^2} \times t \times \frac{Lt}{M} = [1] \checkmark$$

$$\therefore \frac{W}{D^2 \omega \mu} = f\left(\frac{\ell}{D}, \frac{c}{D}\right)$$ W

Problem 7.22 (In Excel)

7.22 The time, t, for oil to drain out of a viscosity calibration container depends on the fluid viscosity, μ, and density, ρ, the orifice diameter, d, and gravity, g. Use dimensional analysis to find the functional dependence of t on the other variables. Express t in the simplest possible form.

Given: That drain time depends on fluid viscosity and density, orifice diameter, and gravity

Find: Functional dependence of t on other variables

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is: $n = 5$
The number of primary dimensions is: $r = 3$
The number of repeat parameters is: $m = r = 3$
The number of Π groups is: $n - m = 2$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents a, b, and c for each.

REPEATING PARAMETERS: Choose ρ, g, d

	M	L	t
ρ	1	-3	
g		1	-2
d		1	

Π GROUPS:

	M	L	t
t	0	0	1

Π_1: $a = 0$, $b = 0.5$, $c = -0.5$

	M	L	t
μ	1	-1	-1

Π_2: $a = -1$, $b = -0.5$, $c = -1.5$

The following Π groups from Example 7.1 are not used:

M	L	t
0	0	0

Π_3: $a = 0$, $b = 0$, $c = 0$

M	L	t
0	0	0

Π_4: $a = 0$, $b = 0$, $c = 0$

Hence $\Pi_1 = t\sqrt{\frac{g}{d}}$ and $\Pi_2 = \frac{\mu}{\rho g^{\frac{1}{2}} d^{\frac{3}{2}}} \rightarrow \frac{\mu^2}{\rho^2 g d^3}$ with $\Pi_1 = f(\Pi_2)$

The final result is $t = \sqrt{\frac{d}{g}}\, f\left(\frac{\mu^2}{\rho^2 g d^3}\right)$

Problem 7.23

7.23 The power, $\mathscr{P}$, used by a vacuum cleaner is to be correlated with the amount of suction provided (indicated by the pressure drop, Δp, below the ambient room pressure). It also depends on impeller diameter, D, and width, d, motor speed, ω, air density, ρ, and cleaner inlet and exit widths, d_i and d_o, respectively. Determine the dimensionless parameters that characterize this problem.

Given: That the power of a vacuum depends on various parameters

Find: Dimensionless groups

Solution:

Apply the Buckingham Π procedure

① $\mathscr{P}$ Δp D d ω ρ d_i d_o $n = 8$ parameters

② Select primary dimensions M, L, t

③
$$\left\{\begin{array}{cccccccc} \mathscr{P} & \Delta p & D & d & \omega & \rho & d_i & d_o \\ \dfrac{ML^2}{t^3} & \dfrac{M}{Lt^2} & L & L & \dfrac{1}{t} & \dfrac{M}{L^3} & L & L \end{array}\right\}$$
$r = 3$ primary dimensions

④ ρ D ω $m = r = 3$ repeat parameters

⑤ Then $n - m = 5$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = \rho^a D^b \omega^c \mathscr{P} = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{ML^2}{t^3} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{lr|l} M: & a+1=0 & a=-1 \\ L: & -3a+b+2=0 & b=-5 \\ t: & -c-3=0 & c=-3 \end{array}$$

Hence $\Pi_1 = \dfrac{\mathscr{P}}{\rho D^5 \omega^3}$

$$\Pi_2 = \rho^a D^b \omega^c \Delta p = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{Lt^2} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{lr|l} M: & a+1=0 & a=-1 \\ L: & -3a+b-1=0 & b=-2 \\ t: & -c-2=0 & c=-2 \end{array}$$

Hence $\Pi_2 = \dfrac{\Delta p}{\rho D^2 \omega^2}$

The other Π groups can be found by inspection: $\Pi_3 = \dfrac{d}{D}$ $\Pi_4 = \dfrac{d_i}{D}$ $\Pi_5 = \dfrac{d_o}{D}$

⑥ Check using F, L, t as primary dimensions

$$\Pi_1 = \frac{\dfrac{FL}{t}}{\dfrac{Ft^2}{L^4} L^5 \dfrac{1}{t^3}} = [1] \qquad \Pi_2 = \frac{\dfrac{F}{L^2}}{\dfrac{Ft^2}{L^4} L^2 \dfrac{1}{t^2}} = [1] \qquad \Pi_3 = \Pi_4 = \Pi_5 = \frac{L}{L} = [1]$$

Note: Any combination of Π_1, Π_2 and Π_3 is a Π group, e.g., $\dfrac{\Pi_1}{\Pi_2} = \dfrac{\mathscr{P}}{\Delta p D^3 \omega}$, so the Π's are not unique!

Problem 7.24

Given: Power per unit cross-sectional area, E, transmitted by a sound wave, depends on wave speed, V, amplitude, r, frequency, n, and medium density, ρ.

Find: General form of dependence of E on the other variables.

Solution: Step ① $E \quad V \quad r \quad n \quad \rho \qquad n = 5$

Step ② Choose M, L, t. $E = \frac{P}{L^2} = \frac{FL}{t} \times \frac{1}{L^2} = \frac{F}{Lt} \times \frac{ML}{Ft^2} = \frac{M}{t^3}$

Step ③ $\frac{M}{t^3} \quad \frac{L}{t} \quad L \quad \frac{1}{t} \quad \frac{M}{L^3} \qquad r = 3$

Step ④ Choose ρ, V, r

Step ⑤ $\Pi_1 = \rho^a V^b r^c E = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c \frac{M}{t^3} = M^0 L^0 t^0$

$M: a + 1 = 0 \qquad a = -1$
$L: -3a + b + c = 0 \qquad c = 3a - b = 3(-1) - (-3) = 0$
$t: -b - 3 = 0 \qquad b = -3$

$\Pi_1 = \frac{E}{\rho V^3}$ ← Π_1

$\Pi_2 = \rho^a V^b r^c n = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c \frac{1}{t} = M^0 L^0 t^0$

$M: a + 0 = 0 \qquad a = 0$
$L: -3a + b + c = 0 \qquad c = 3a - b = 3(0) - (-1) = 1$
$t: -b - 1 = 0 \qquad b = -1$

$\Pi_2 = \frac{nr}{V}$ ← Π_2

Step ⑥ Check using FLt: $\rho = \frac{M}{L^3} \times \frac{Ft^2}{ML} = \frac{Ft^2}{L^4}$

$\Pi_1 = \frac{E}{\rho V^3} = \frac{FL}{tL^2} \frac{L^4}{Ft^2} \frac{t^3}{L^3} = \frac{FL^5 t^3}{FL^5 t^3} = 1$ ✓✓

$\Pi_2 = \frac{nr}{V} = \frac{1}{t} L \times \frac{t}{L} = \frac{Lt}{Lt} = 1$ ✓✓

Problem 7.25

Given: Draining of a tank from initial level, h_0.

Time, τ, depends on tank diameter, D, orifice diameter, d, acceleration of gravity, g, density, ρ, and viscosity, μ.

Find: (a) Number of dimensionless parameters
(b) Number of repeating variables.
(c) Π-parameter containing viscosity.

Solution: Step ① τ h_0 D d g ρ μ

Step ② Choose MLt system $(n = 7)$

Step ③ t L L L $\frac{L}{t^2}$ $\frac{M}{L^3}$ $\frac{M}{Lt}$

$(r=3)$

Then $n - r = 7 - 3 = 4$ parameters will result. ← Π's

Step ④ $r = 3$, so choose 3 variables: ρ, d, g ←

Step ⑤ $\Pi_1 = \rho^a d^b g^c \mu = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t^2}\right)^c \frac{M}{Lt} = M^0 L^0 t^0$

$M: a + 1 = 0 \qquad a = -1$

$L: -3a + b + c - 1 = 0 \qquad b = 3a - c + 1 = 3(-1) - (-\tfrac{1}{2}) + 1$

$t: -2c - 1 = 0 \qquad c = -\tfrac{1}{2} \qquad b = -\tfrac{3}{2}$

$$\Pi_1 = \frac{\mu}{\rho d^{3/2} g^{1/2}}$$ ← Π_1

Step ⑥ Check, using FLt system.

$\mu = \frac{Ft}{L^2}$; $\rho = \frac{M}{L^3} \times \frac{Ft^2}{ML} = \frac{Ft^2}{L^4}$

$$\Pi_1 = \frac{Ft}{L^2} \frac{L^4}{Ft^2} \frac{1}{L^{3/2}} \frac{t}{L^{1/2}} = \frac{FL^4 t^2}{FL^4 t^2} = 1 \quad ✓✓$$

Problem 7.26

[2]

Given: Power, $\mathcal{P}$, required to drive a fan depends on ρ, Q, D and ω.

Find: Dependence of $\mathcal{P}$ on other parameters.

Solution: Apply Buckingham Π procedure.

① $\mathcal{P} \quad \rho \quad Q \quad D \quad \omega \qquad n = 5$ parameters

② Choose primary dimensions M, L, t

③ $\mathcal{P} \quad \rho \quad Q \quad D \quad \omega$

$\frac{ML^2}{t^3} \quad \frac{M}{L^3} \quad \frac{L^3}{t} \quad L \quad \frac{1}{t} \qquad r = 3$ primary dimensions

④ $\rho, D, \omega \qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 2$ dimensionless groups will result. Setting up dimensional equations,

$$\Pi_1 = \rho^a D^b \omega^c \mathcal{P} = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \left(\frac{ML^2}{t^3}\right) = M^0 L^0 t^0$$

Summing exponents,

$M: \; a + 1 = 0 \qquad a = -1$
$L: \; -3a + b + 2 = 0 \qquad b = -5$
$t: \; -c - 3 = 0 \qquad c = -3$

$$\therefore \; \Pi_1 = \frac{\mathcal{P}}{\rho D^5 \omega^3}$$

$$\Pi_2 = \rho^d D^e \omega^f Q = \left(\frac{M}{L^3}\right)^d (L)^e \left(\frac{1}{t}\right)^f \left(\frac{L^3}{t}\right) = M^0 L^0 t^0$$

Summing exponents,

$M: \; d + 0 = 0 \qquad d = 0$
$L: \; -3d + e + 3 = 0 \qquad e = -3$
$t: \; -f - 1 = 0 \qquad f = -1$

$$\therefore \; \Pi_2 = \frac{Q}{D^3 \omega}$$

⑥ Check using primary dimensions F, L, t

$$\Pi_1 = \frac{FL}{t} \frac{L^4}{Ft^2} \frac{1}{L^5} t^3 = [1] \checkmark \qquad \Pi_2 = \frac{L^3}{t} \frac{1}{L^3} t = [1] \checkmark$$

Thus $\Pi_1 = f(\Pi_2)$, or $\dfrac{\mathcal{P}}{\rho D^5 \omega^3} = f\left(\dfrac{Q}{D^3 \omega}\right)$

$\mathcal{P}$

Problem 7.27

Given: Continuous belt moving vertically through a viscous liquid bath

The volume rate of liquid loss, Q, is a function of μ, ρ, g, h (thickness of liquid layer), and V

Find: form of dependence of Q on other variables.

Solution: Apply Buckingham π-theorem.

① $Q \quad \mu \quad \rho \quad g \quad h \quad V \qquad n = 6$ parameters

② Select M, L, t as primary dimensions

③
$$\begin{array}{cccccc} Q & \mu & \rho & g & h & V \\ \frac{L^3}{t} & \frac{M}{Lt} & \frac{M}{L^3} & \frac{L}{t^2} & L & \frac{L}{t} \end{array} \qquad r = 3 \text{ primary dimensions}$$

④ $\rho, V, h \qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 3$ dimensionless groups will result.

Setting up dimensional equations.

$\pi_1 = \rho^a V^b h^c Q$

$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{L^3}{t}$

Equating exponents,

M: $0 = a$

L: $0 = -3a + b + c + 3$

t: $0 = -b - 1$

i.e. $a = 0$, $b = -1$, $c = -2$

$\therefore \pi_1 = \frac{Q}{Vh^2}$

$\pi_2 = \rho^a V^b h^c \mu$

$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{M}{Lt}$

Equating exponents,

M: $0 = a + 1$

L: $0 = -3a + b + c - 1$

t: $0 = -b - 1$

i.e. $a = -1$, $b = -1$, $c = -1$

$\therefore \pi_2 = \frac{\mu}{\rho V h}$

$\pi_3 = \rho^a V^b h^c g$

$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{L}{t^2}$

Equating exponents,

M: $0 = a$

L: $0 = -3a + b + c + 1$

t: $0 = -b - 2$

i.e. $a = 0$, $b = -2$, $c = 1$

$\therefore \pi_3 = \frac{gh}{V^2}$

Then
$$\frac{Q}{Vh^2} = f\left(\frac{\rho V h}{\mu}, \frac{V^2}{gh}\right) \qquad Q$$

⑥ Check using F, L, t dimensions

$\pi_1 = \frac{L^3}{t} \cdot \frac{t}{L} \cdot \frac{1}{L^2} = [1]$ ✓ $\qquad \pi_2 = \frac{Ft}{L^2} \cdot \frac{L^4}{Ft^2} \cdot \frac{t}{L} \cdot \frac{1}{L} = [1]$ ✓ $\qquad \pi_3 = \frac{L}{t^2} \cdot L \cdot \frac{t^2}{L^2} = [1]$ ✓

Problem 7.28

Given: Water is drained from a tank of diameter D, through a smoothly rounded drain hole of diameter, d. The initial mass flow rate, $\dot{m}$, from the tank is written in functional form as

$$\dot{m} = \dot{m}(h_o, D, d, g, \rho, \mu)$$

where h_o is the initial water depth in the tank
g is the acceleration of gravity
ρ and μ are fluid properties.

Find: (a) the number of dimensionless groups required to correlate the data
(b) the number of repeating variables that must be selected to determine the dimensionless parameters.
(c) the π parameter that contains the fluid viscosity, μ.

Solution: Apply the Buckingham π-theorem

① List $\quad \dot{m} \quad h_o \quad D \quad d \quad g \quad \rho \quad \mu \qquad n = 7$ parameters

② Select M, L, t as primary dimensions

③ Dimensions $\quad \frac{M}{t} \quad L \quad L \quad L \quad \frac{L}{t^2} \quad \frac{M}{L^3} \quad \frac{M}{Lt} \qquad r = 3$ prim. dim

④ Choose repeating variables $\rho, d, g \qquad m = 3$ repeating parameters ← m

$\therefore$ expect $n - m = 7 - 3 = 4$ dimensionless parameters ← n

⑤ $\pi_1 = \rho^a d^b g^c \mu$

$$M^0 L^0 t^0 = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t^2}\right)^c \frac{M}{Lt}$$

t: $\quad 0 = -2c - 1 \qquad \therefore c = -\frac{1}{2}$
M: $\quad 0 = a + 1 \qquad \therefore a = -1$
L: $\quad 0 = -3a + b + c - 1 \qquad \therefore b = 3a - c + 1 = -\frac{3}{2}$

$$\pi_1 = \frac{\mu}{\rho d^{3/2} g^{1/2}} \quad \leftarrow \pi$$

⑥ Check $\pi_1 = \frac{Ft}{L^2} \times \frac{L^4}{Ft^2} \times \frac{1}{L^{3/2}} \times \frac{t}{L^{1/2}} = [1]$ ✓

Problem 7.29

Given: Diameter, d, of liquid droplets formed in fuel injection process is a function of ρ, μ, σ (surface tension), V, D.

Find: (a) number of dimensionless ratios required to characterize the process
(b) the dimensionless ratios.

Solution: Apply Buckingham π-theorem

① $d \quad \rho \quad \mu \quad \sigma \quad V \quad D$ $\qquad n = 6$ parameters

② Select M, L, t as primary dimensions

③ $d \quad \rho \quad \mu \quad \sigma \quad V \quad D$

$L \quad \frac{M}{L^3} \quad \frac{M}{Lt} \quad \frac{M}{t^2} \quad \frac{L}{t} \quad L$ $\qquad r = 3$ primary dimensions

④ ρ, D, V $\qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 3$ dimensionless groups will result.

Setting up dimensional equations

$\Pi_1 = \rho^a D^b V^c d$	$\Pi_2 = \rho^a D^b V^c \mu$	$\Pi_3 = \rho^a D^b V^c \sigma$
$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t}\right)^c L$	$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t}\right)^c \frac{M}{Lt}$	$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L}{t}\right)^c \frac{M}{t^2}$
Summing exponents,	Summing exponents	Summing exponents
$M: \; a = 0$	$M: \; a + 1 = 0$	$M: \; a + 1 = 0$
$L: \; -3a + b + c + 1 = 0$	$L: \; -3a + b + c - 1 = 0$	$L: \; -3a + b + c = 0$
$t: \; -c = 0$	$t: \; -c - 1 = 0$	$t: \; -c - 2 = 0$
i.e. $a = 0$, $c = 0$, $b = -1$	i.e. $a = -1$, $c = -1$, $b = 3a - c + 1 = -1$	i.e. $a = -1$, $c = -2$, $b = 3a - c = -1$
$\therefore \Pi_1 = \frac{d}{D}$	$\therefore \Pi_2 = \frac{\mu}{\rho V D}$	$\therefore \Pi_3 = \frac{\sigma}{\rho D V^2}$

⑥ Check using F, L, t dimensions

$\Pi_1 = \frac{L}{L} = [1]$ ✓ $\qquad \Pi_2 = \frac{Ft}{L^2} \cdot \frac{L^4}{Ft^2} \cdot \frac{t}{L} \cdot \frac{1}{L} = [1]$ ✓ $\qquad \Pi_3 = \frac{F}{L} \cdot \frac{L^4}{Ft^2} \cdot \frac{1}{L} \cdot \frac{t^2}{L^2} = [1]$ ✓

Problem 7.30 (In Excel)

7.30 The diameter, d, of the dots made by an ink jet printer depends on the ink viscosity, μ, density, ρ, and surface tension, σ, the nozzle diameter, D, the distance, L, of the nozzle from the paper surface, and the ink jet velocity, V. Use dimensional analysis to find the Π parameters that characterize the ink jet's behavior.

Given: That dot size depends on ink viscosity, density, and surface tension, and geometry

Find: Π groups

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is:	$n = 7$
The number of primary dimensions is:	$r = 3$
The number of repeat parameters is:	$m = r = 3$
The number of Π groups is:	$n - m = 4$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents a, b, and c for each.

REPEATING PARAMETERS: **Choose ρ, V, D**

	M	L	t
ρ	1	-3	
V		1	-1
D		1	

Π GROUPS:

	M	L	t
d	0	1	0

Π_1: $a = $ **0**, $b = $ **0**, $c = $ **-1**

	M	L	t
μ	1	-1	-1

Π_2: $a = $ **-1**, $b = $ **-1**, $c = $ **-1**

	M	L	t
σ	1	0	-2

Π_3: $a = $ **-1**, $b = $ **-2**, $c = $ **-1**

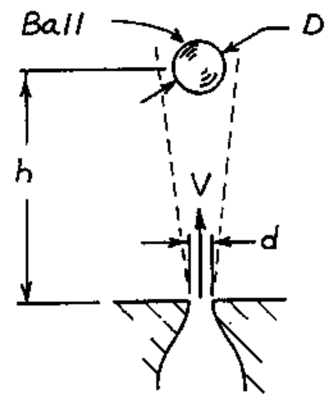
	M	L	t
L	0	1	0

Π_4: $a = $ **0**, $b = $ **0**, $c = $ **-1**

Hence $\Pi_1 = \dfrac{d}{D}$ $\quad \Pi_2 = \dfrac{\mu}{\rho V D} \rightarrow \dfrac{\rho V D}{\mu}$ $\quad \Pi_3 = \dfrac{\sigma}{\rho V^2 D}$ $\quad \Pi_4 = \dfrac{L}{D}$

Note that groups Π_1 and Π_4 can be obtained by inspection

Problem 7.31



Given: Ball in jet

$$h = h(d, D, \rho, V, \mu, W)$$

Find: Pi parameters

Solution: Apply Buckingham procedure

① $h \quad d \quad D \quad \rho \quad V \quad \mu \quad W \qquad n = 7$

② M, L, t

③ $L \quad L \quad L \quad \frac{M}{L^3} \quad \frac{L}{t} \quad \frac{M}{Lt} \quad \frac{ML}{t^2} \qquad m = 3 \qquad n - m = 7 - 3 = 4$ parameters

④ Choose ρ, V, d as repeating parameters.

⑤ $\rho^a V^b d^c W = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c \frac{ML}{t^2} = M^0 L^0 t^0$

$M: a + 1 = 0 \qquad a = -1$

$L: -3a + b + c + 1 = 0 \qquad c = -2$

$t: -b - 2 = 0 \qquad b = -2$

$$\Pi_1 = \frac{W}{\rho V^2 d^2} \qquad \Pi_1$$

⑥ Check: $F \times \frac{L^4}{Ft^2} \frac{t^2}{L^2} \frac{1}{L^2} = 1$ ✓✓

$$\Pi_2 = \rho^a V^b d^c \mu = \frac{\mu}{\rho V d}$$

$$\Pi_3 = \rho^a V^b d^c h = \frac{h}{d}$$

$$\Pi_4 = \rho^a V^b d^c D = \frac{D}{d}$$

Problem 7.32 (In Excel)

7.32 The terminal speed V of shipping boxes sliding down an incline on a layer of air (injected through numerous pinholes in the incline surface) depends on the box mass, m, and base area, A, gravity, g, the incline angle, θ, the air viscosity, μ, and the air layer thickness, δ. Use dimensional analysis to find the Π parameters that characterize this phenomenon.

Given: Speed depends on mass, area, gravity, slope, and air viscosity and thickness

Find: Π groups

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is:	$n = 7$
The number of primary dimensions is:	$r = 3$
The number of repeat parameters is:	$m = r = 3$
The number of Π groups is:	$n - m = 4$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents a, b, and c for each.

REPEATING PARAMETERS: Choose g, δ, m

	M	L	t
g		1	-2
δ		1	
m	1		

Π GROUPS:

	M	L	t
V	0	1	-1

Π_1: a = **-0.5**, b = **-0.5**, c = **0**

	M	L	t
μ	1	-1	-1

Π_2: a = **-0.5**, b = **1.5**, c = **-1**

	M	L	t
θ	0	0	0

Π_3: a = **0**, b = **0**, c = **0**

	M	L	t
A	0	2	0

Π_4: a = **0**, b = **-2**, c = **0**

Hence

$$\Pi_1 = \frac{V}{g^{\frac{1}{2}}\delta^{\frac{1}{2}}} \rightarrow \frac{V^2}{g\delta} \qquad \Pi_2 = \frac{\mu\delta^{\frac{3}{2}}}{g^{\frac{1}{2}}m} \rightarrow \frac{\mu^2\delta^3}{m^2 g} \qquad \Pi_3 = \theta \qquad \Pi_4 = \frac{A}{\delta^2}$$

Note that the Π_1, Π_3 and Π_4 groups can be obtained by inspection

Problem 7.33 (In Excel)

7.33 The diameter, d, of bubbles produced by a bubble-making toy depends on the soapy water viscosity, μ, density, ρ, and surface tension, σ, the ring diameter, D, and the pressure differential, Δp, generating the bubbles. Use dimensional analysis to find the Π parameters that characterize this phenomenon.

Given: Bubble size depends on viscosity, density, surface tension, geometry and pressure

Find: Π groups

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is:	$n = 6$
The number of primary dimensions is:	$r = 3$
The number of repeat parameters is:	$m = r = 3$
The number of Π groups is:	$n - m = 3$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents a, b, and c for each.

REPEATING PARAMETERS: **Choose ρ, Δp, D**

	M	L	t
ρ	1	-3	
Δp	1	-1	-2
D		1	

Π GROUPS:

	M	L	t
d	0	1	0

Π_1: a = **0**, b = **0**, c = **-1**

	M	L	t
μ	1	-1	-1

Π_2: a = **-0.5**, b = **-0.5**, c = **-1**

	M	L	t
σ	1	0	-2

Π_3: a = **0**, b = **-1**, c = **-1**

	M	L	t
	0	0	0

Π_4: a = **0**, b = **0**, c = **0**

Hence $$\Pi_1 = \frac{d}{D} \qquad \Pi_2 = \frac{\mu}{\rho^{\frac{1}{2}}\Delta p^{\frac{1}{2}} D} \rightarrow \frac{\mu^2}{\rho \Delta p D^2} \qquad \Pi_3 = \frac{\sigma}{D\Delta p}$$

Note that the Π_1 group can be obtained by inspection

Problem 7.34

7.34 A washing machine agitator is to be designed. The power, $\mathscr{P}$, required for the agitator is to be correlated with the amount of water used (indicated by the depth, H, of the water). It also depends on the agitator diameter, D, height, h, maximum angular velocity, ω_{max}, and frequency of oscillations, f, and water density, ρ, and viscosity, μ. Determine the dimensionless parameters that characterize this problem.

Given: That the power of a washing machine agitator depends on various parameters

Find: Dimensionless groups

Solution:

Apply the Buckingham Π procedure

① $\mathscr{P}$ H D h ω_{max} f ρ μ $n = 8$ parameters

② Select primary dimensions M, L, t

③
$$\left\{\begin{matrix} \mathscr{P} & H & D & h & \omega_{max} & f & \rho & \mu \\ \frac{ML^2}{t^3} & L & L & L & \frac{1}{t} & \frac{1}{t} & \frac{M}{L^3} & \frac{M}{Lt} \end{matrix}\right\}$$
$r = 3$ primary dimensions

④ ρ D ω_{max} $m = r = 3$ repeat parameters

⑤ Then $n - m = 5$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = \rho^a D^b \omega_{max}^c \mathscr{P} = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{ML^2}{t^3} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{aligned} M: &\quad a + 1 = 0 & a &= -1 \\ L: &\quad -3a + b + 2 = 0 & b &= -5 \\ t: &\quad -c - 3 = 0 & c &= -3 \end{aligned}$$

Hence $\Pi_1 = \dfrac{\mathscr{P}}{\rho D^5 \omega_{max}^3}$

$$\Pi_2 = \rho^a D^b \omega_{max}^c \mu = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{Lt} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{aligned} M: &\quad a + 1 = 0 & a &= -1 \\ L: &\quad -3a + b - 1 = 0 & b &= -2 \\ t: &\quad -c - 1 = 0 & c &= -1 \end{aligned}$$

Hence $\Pi_2 = \dfrac{\mu}{\rho D^2 \omega_{max}}$

The other Π groups can be found by inspection: $\Pi_3 = \dfrac{H}{D}$ $\Pi_4 = \dfrac{h}{D}$ $\Pi_5 = \dfrac{f}{\omega_{max}}$

⑥ Check using F, L, t as primary dimensions

$$\Pi_1 = \frac{\frac{FL}{t}}{\frac{Ft^2}{L^4} L^5 \frac{1}{t^3}} = [1] \qquad \Pi_2 = \frac{\frac{Ft}{L^2}}{\frac{Ft^2}{L^4} L^2 \frac{1}{t}} = [1] \qquad \Pi_3 = \Pi_4 = \Pi_5 = [1]$$

Note: Any combination of Π's is a Π group, e.g., $\dfrac{\Pi_1}{\Pi_2} = \dfrac{\mathscr{P}}{D^3 \omega_{max}^2 \mu}$, so the Π's are not unique!

Problem 7.35 (In Excel)

7.35 The time, t, for a flywheel, with moment of inertia, I, to reach angular velocity, ω, from rest, depends on the applied torque, T, and the following flywheel bearing properties: the oil viscosity, μ, gap, δ, diameter, D, and length, L. Use dimensional analysis to find the Π parameters that characterize this phenomenon.

Given: Time to speed up depends on inertia, speed, torque, oil viscosity and geometry

Find: Π groups

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is: $n = 8$

The number of primary dimensions is: $r = 3$

The number of repeat parameters is: $m = r = 3$

The number of Π groups is: $n - m = 5$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents *a* , *b* , and *c* for each.

REPEATING PARAMETERS: **Choose ω, D, T**

	M	L	t
ω			-1
D		1	
T	1	2	-2

Π GROUPS:

Two Π groups can be obtained by inspection: δ/D and L/D. The others are obtained below

	M	L	t
t	0	0	1

Π_1: a = **1**, b = **0**, c = **0**

	M	L	t
μ	1	-1	-1

Π_2: a = **1**, b = **3**, c = **-1**

	M	L	t
I	1	2	0

Π_3: a = **2**, b = **0**, c = **-1**

	M	L	t
	0	0	0

Π_4: a = **0**, b = **0**, c = **0**

Hence the Π groups are

$$t\omega \qquad \frac{\delta}{D} \qquad \frac{L}{D} \qquad \frac{\mu\omega D^3}{T} \qquad \frac{I\omega^2}{T}$$

Note that the Π_1 group can also be easily obtained by inspection

Problem 7.36

Given: Pressurized tank drained through a smooth nozzle, area A.

$$\dot{m} = \dot{m}(\Delta p, h, \rho, A, g)$$

Find: (a) Number of independent dimensionless parameters.
(b) Obtain the parameters.
(c) State the functional relationship for $\dot{m}$.

Solution: Apply the Buckingham Π-theorem.

① $\dot{m}$ Δp h ρ A g $n=6$ parameters

② Select M, L, t as primary dimensions

③ $\frac{M}{t}$ $\frac{M}{Lt^2}$ L $\frac{M}{L^3}$ L^2 $\frac{L}{t^2}$ $r=3$ primary dimensions

$m=r=3$

④ Choose ρ, A, g as repeating parameters.

⑤ Then $n-m = 6-3 = 3$ dimensionless parameters result. — $n-m$

Set up dimensional equations:

$\Pi_1 = \rho^a A^b g^c \dot{m}$	$\Pi_2 = \rho^a A^b g^c \Delta p$	$\Pi_3 = \rho^a A^b g^c h$
$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a (L^2)^b \left(\frac{L}{t^2}\right)^c \frac{M}{t}$	$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a (L^2)^b \left(\frac{L}{t^2}\right)^c \frac{M}{Lt^2}$	$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a (L^2)^b \left(\frac{L}{t^2}\right)^c L$
Equating exponents:	Equating exponents:	Equating exponents:
M: $a+1=0$ $\quad a=-1$	M: $a+1=0$ $\quad a=-1$	M: $a=0$ $\quad a=0$
L: $-3a+2b+c=0$	L: $-3a+2b+c-1=0$	L: $-3a+2b+c+1=0$
t: $-2c-1=0$ $\quad c=-\frac{1}{2}$	t: $-2c-2=0$ $\quad c=-1$	t: $-2c+0=0$ $\quad c=0$
$\therefore b = \frac{1}{2}(3a-c) = -\frac{5}{4}$	$\therefore b = \frac{1}{2}(1+3a-c) = -\frac{1}{2}$	$\therefore b = \frac{1}{2}(-1+3a-c) = -\frac{1}{2}$
$\Pi_1 = \frac{\dot{m}}{\rho A^{5/4} g^{1/2}}$	$\Pi_2 = \frac{\Delta p}{\rho A^{1/2} g}$	$\Pi_3 = \frac{h}{A^{1/2}}$

— Π's

⑥ Check using FLt dimensions: $\dot{m} = \frac{M}{t}\frac{Ft^2}{ML} = \frac{Ft}{L}$; $\rho = \frac{M}{L^3}\frac{Ft^2}{ML} = \frac{Ft^2}{L^4}$

$\Pi_1 = \frac{Ft}{L}\frac{L^4}{Ft^2}\frac{1}{L^{5/2}}\frac{t}{L^{1/2}} = [1]$ ✓✓	$\Pi_2 = \frac{F}{L^2}\frac{L^4}{Ft^2}\frac{1}{L}\frac{t^2}{L} = [1]$ ✓✓	$\Pi_3 = \frac{L}{L} = [1]$ ✓✓

Thus $\quad \Pi_1 = f(\Pi_2, \Pi_3) \qquad \frac{\dot{m}}{\rho A^{5/4} g^{1/2}} = f\left(\frac{\Delta p}{\rho A^{1/2} g}, \frac{h}{A^{1/2}}\right)$

or

$$\dot{m} = \rho A^{5/4} g^{1/2} f\left(\frac{\Delta p}{\rho A^{1/2} g}, \frac{h}{A^{1/2}}\right)$$

— $\dot{m}$

Problem 7.37

7.37 The ventilation in the clubhouse on a cruise ship is insufficient to clear cigarette smoke (the ship is not yet completely smoke-free). Tests are to be done to see if a larger extractor fan will work. The concentration of smoke, c (particles per cubic meter) depends on the number of smokers, N, the pressure drop produced by the fan, Δp, the fan diameter, D, motor speed, ω, the particle and air densities, ρ_p, and ρ, respectively, gravity, g, and air viscosity, μ. Determine the dimensionless parameters that characterize this problem.

Given: Ventilation system of cruise ship clubhouse

Find: Dimensionless groups

Solution:

Apply the Buckingham Π procedure

① $c \quad N \quad \Delta p \quad D \quad \omega \quad \rho_p \quad \rho \quad g \quad \mu$ $\qquad n = 9$ parameters

② Select primary dimensions M, L, t

③
$$\begin{Bmatrix} c & N & \Delta p & D & \omega & \rho_p & \rho & g & \mu \\ \dfrac{1}{L^3} & 1 & \dfrac{M}{Lt^2} & L & \dfrac{1}{t} & \dfrac{M}{L^3} & \dfrac{M}{L^3} & \dfrac{L}{t^2} & \dfrac{M}{Lt} \end{Bmatrix}$$
$r = 3$ primary dimensions

④ $\rho \quad D \quad \omega$ $\qquad m = r = 3$ repeat parameters

⑤ Then $n - m = 6$ dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = \rho^a D^b \omega^c \Delta p = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{Lt^2} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{rr|l} M: & a + 1 = 0 & a = -1 \\ L: & -3a + b - 1 = 0 & b = -2 \\ t: & -c - 2 = 0 & c = -2 \end{array}$$
Hence
$$\Pi_1 = \frac{\Delta p}{\rho D^2 \omega^2}$$

$$\Pi_2 = \rho^a D^b \omega^c \mu = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{Lt} = M^0 L^0 t^0$$

Summing exponents,
$$\begin{array}{rr|l} M: & a + 1 = 0 & a = -1 \\ L: & -3a + b - 1 = 0 & b = -2 \\ t: & -c - 1 = 0 & c = -1 \end{array}$$
Hence
$$\Pi_2 = \frac{\mu}{\rho D^2 \omega}$$

The other Π groups can be found by inspection: $\Pi_3 = cD^3 \qquad \Pi_4 = N \qquad \Pi_5 = \dfrac{\rho_p}{\rho} \qquad \Pi_6 = \dfrac{g}{D\omega^2}$

⑥ Check using F, L, t as primary dimensions

$$\Pi_1 = \frac{\dfrac{F}{L^2}}{\dfrac{Ft^2}{L^4} L^2 \dfrac{1}{t^2}} = [1] \qquad \Pi_2 = \frac{\dfrac{Ft}{L^2}}{\dfrac{Ft^2}{L^4} L^2 \dfrac{1}{t}} = [1] \qquad \Pi_3 = \Pi_4 = \Pi_5 = \Pi_6 = [1]$$

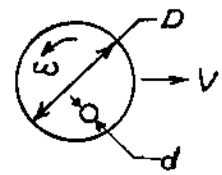
Note: Any combination of Π's is a Π group, e.g., $\dfrac{\Pi_1}{\Pi_2} = \dfrac{\Delta p}{\omega\mu}$, so the Π's are not unique!

Problem 7.38

[3]

Given: Aerodynamic torque on spinning ball,

$$T = f(V, \rho, \mu, D, \omega, d)$$



Find: Dimensionless parameters

Solution: Apply Buckingham procedure.

① List: $T \quad V \quad \rho \quad \mu \quad D \quad \omega \quad d \qquad n = 7$

② Choose M, L, t

③ $\frac{ML^2}{t^2} \quad \frac{L}{t} \quad \frac{M}{L^3} \quad \frac{M}{Lt} \quad L \quad \frac{1}{t} \quad L \qquad m = 3$

④ Choose ρ, V, D $\qquad n - m = 4$ parameters

⑤ $\Pi_1 = \rho^a V^b D^c T = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c \frac{ML^2}{t^2} = M^0 L^0 t^0$

$M: \; a + 1 = 0 \qquad a = -1$

$L: \; -3a + b + c + 2 = 0 \qquad c = -3 \qquad \Pi_1 = \frac{T}{\rho V^2 D^3}$ $\qquad \Pi_1$

$t: \; -b - 2 = 0 \qquad b = -2$

⑥ Check: $\Pi_1 = FL \times \frac{L^4}{Ft^2} \times \frac{t^2}{L^2} \times \frac{1}{L^3} = 1$ ✓✓

$$\Pi_2 = \frac{\mu}{\rho V D}$$

$$\Pi_3 = \frac{\omega D}{V}$$

$$\Pi_4 = \frac{d}{D}$$

$$\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4)$$

$$\frac{T}{\rho V^2 D^3} = f\left(\frac{\mu}{\rho V D}, \frac{\omega D}{V}, \frac{d}{D}\right)$$ $\qquad T$

Problem 7.39

Given: Power loss, $\mathcal{P}$, depends on: length, l, diameter, D; clearance, c; angular speed, ω; viscosity, μ; mean pressure, p.

Find: (a) Dimensionless parameters that characterize the problem
(b) Functional form of dependence of $\mathcal{P}$ on these parameters

Solution: Apply Buckingham π - theorem

① $\mathcal{P}$ l D c ω μ p $n=7$ parameters

② Select F, L, t as primary dimensions

③

$\mathcal{P}$	l	D	c	ω	μ	p
$\frac{FL}{t}$	L	L	L	$\frac{1}{t}$	$\frac{Ft}{L^2}$	$\frac{F}{L^2}$

④ D, ω, p $m=r=3$ repeating parameters

⑤ Then $n-m=4$ dimensionless groups will result.

Setting up dimensional equations

$\pi_1 = D^a \omega^b p^e \mathcal{P}$	$\pi_2 = D^a \omega^b p^e l$	$\pi_3 = D^a \omega^b p^e c$	$\pi_4 = D^a \omega^b p^e \mu$
$F^0L^0t^0 = L^a\left(\frac{1}{t}\right)^b\left(\frac{F}{L^2}\right)^e \frac{FL}{t}$	$F^0L^0t^0 = L^a\left(\frac{1}{t}\right)^b\left(\frac{F}{L^2}\right)^e L$	$F^0L^0t^0 = L^a\left(\frac{1}{t}\right)^b\left(\frac{F}{L^2}\right)^e L$	$F^0L^0t^0 = L^a\left(\frac{1}{t}\right)^b\left(\frac{F}{L^2}\right)^e \frac{Ft}{L^2}$
Equating exponents,	Equating exponents,	Equating exponents,	Equating exponents,
$F:\ 0 = e+1$	$F:\ 0 = e$	$F:\ 0 = e$	$F:\ 0 = e+1$
$L:\ 0 = a-2e+1$	$L:\ 0 = a-2e+1$	$L:\ 0 = a-2e+1$	$L:\ 0 = a-2e-2$
$t:\ 0 = -b-1$	$t:\ 0 = -b$	$t:\ 0 = -b$	$t:\ 0 = -b+1$
$\therefore\ e=-1$	$\therefore\ e=0$	$\therefore\ e=0$	$\therefore\ e=-1$
$a=-3$	$a=-1$	$a=-1$	$a=0$
$b=-1$	$b=0$	$b=0$	$b=1$
$\pi_1 = \frac{\mathcal{P}}{p\omega D^3}$	$\pi_2 = \frac{l}{D}$	$\pi_3 = \frac{c}{D}$	$\pi_4 = \frac{\mu\omega}{p}$

Then, $\frac{\mathcal{P}}{p\omega D^3} = f\left(\frac{\mu\omega}{p}, \frac{c}{D}, \frac{l}{D}\right)$ ← $\mathcal{P}$

⑥ Check using M, L, t dimensions

$\pi_1 = \frac{ML^2}{t^3} \times \frac{Lt^2}{M} \times t \times \frac{1}{L^3} = [1]$ ✓

$\pi_2 = \frac{L}{L} = [1]$ ✓ $\pi_3 = \frac{L}{L} = [1]$ ✓

$\pi_4 = \frac{M}{Lt} \times \frac{1}{t} \times \frac{Lt^2}{M} = [1]$ ✓

Problem 7.40

Given: Thrust, F_t, of a marine propeller is thought to depend on: ρ (water density), D (diameter), V (speed of advance), g (acceleration of gravity), ω (angular speed of propeller), p (pressure in the liquid), and μ (liquid viscosity)

Find: Dimensionless parameters that characterize propeller performance.

Solution: Apply Buckingham π - theorem

① List: $F_t \quad \rho \quad D \quad V \quad g \quad \omega \quad p \quad \mu \quad (n=8)$

② Choose M, L, t as primary dimensions

③ Dimensions: $\frac{ML}{t^2} \quad \frac{M}{L^3} \quad L \quad \frac{L}{t} \quad \frac{L}{t^2} \quad \frac{1}{t} \quad \frac{M}{Lt^2} \quad \frac{M}{Lt}$

④ Repeating variables $\rho, V, D \quad m = r = 3$

⑤ Then $n - m = 5$ dimensionless groups will result

Setting up dimensional equations

$$\pi_1 = \rho^a V^b D^c F_t$$

$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a\left(\frac{L}{t}\right)^b L^c \frac{ML}{t^2} \quad \begin{cases} M: 0 = a+1 \\ t: 0 = -b-2 \\ L: 0 = -3a+b+c+1 \end{cases} \quad \begin{matrix} a=-1 \\ b=-2 \\ c=-2 \end{matrix} \quad \therefore \pi_1 = \frac{F_t}{\rho V^2 D^2}$$

$$\pi_2 = \rho^a V^b D^c g$$

$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a\left(\frac{L}{t}\right)^b L^c \left(\frac{L}{t^2}\right) \quad \begin{cases} M: 0 = a \\ t: 0 = -b-2 \\ L: 0 = -3a+b+c+1 \end{cases} \quad \begin{matrix} a=0 \\ b=-2 \\ c=1 \end{matrix} \quad \therefore \pi_2 = \frac{gD}{V^2}$$

$$\pi_3 = \rho^a V^b D^c \omega$$

$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a\left(\frac{L}{t}\right)^b L^c \left(\frac{1}{t}\right) \quad \begin{cases} M: 0 = a \\ t: 0 = -b-1 \\ L: 0 = -3a+b+c \end{cases} \quad \begin{matrix} a=0 \\ b=-1 \\ c=1 \end{matrix} \quad \therefore \pi_3 = \frac{\omega D}{V}$$

$$\pi_4 = \rho^a V^b D^c p$$

$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a\left(\frac{L}{t}\right)^b L^c \frac{M}{Lt^2} \quad \begin{cases} M: 0 = a+1 \\ t: 0 = -b-2 \\ L: 0 = -3a+b+c-1 \end{cases} \quad \begin{matrix} a=-1 \\ b=-2 \\ c=0 \end{matrix} \quad \therefore \pi_4 = \frac{p}{\rho V^2}$$

$$\pi_5 = \rho^a V^b D^c \mu$$

$$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a\left(\frac{L}{t}\right)^b L^c \frac{M}{Lt} \quad \begin{cases} M: 0 = a+1 \\ t: 0 = -b-1 \\ L: 0 = -3a+b+c-1 \end{cases} \quad \begin{matrix} a=-1 \\ b=-1 \\ c=-1 \end{matrix} \quad \therefore \pi_5 = \frac{\mu}{\rho V D}$$

Dimensionless parameters are $\frac{F_t}{\rho V^2 D^2}, \frac{gD}{V^2}, \frac{\omega D}{V}, \frac{p}{\rho V^2}, \frac{\mu}{\rho V D}$ ←

⑥ Check using F, L, t

$\pi_1 = F \times \frac{L^4}{Ft^2} \times \frac{t^2}{L^2} \times \frac{1}{L^2} = [1]$ ✓ $\quad \pi_2 = \frac{L}{t^2} \times L \times \frac{t^2}{L^2} = [1]$ ✓

$\pi_3 = \frac{1}{t} \times L \times \frac{t}{L} = [1]$, $\quad \pi_4 = \frac{F}{L^2} \times \frac{L^4}{Ft^2} \times \frac{t^2}{L^2} = [1]$

$\pi_5 = \frac{1}{Re} = [1]$ ✓

Problem 7.41

Given: Fan-assisted convection oven; $\dot{Q}$ = heat transfer rate (energy/time).

$$\dot{Q} = f(c_p, \Theta, L, \rho, \mu, V)$$

Find: (a) Number of basic dimensions included in these variables.
(b) Number of Π-parameters.
(c) Obtain the parameters.

Solution: Apply the Buckingham Π-theorem.

① $\dot{Q}$ $\quad c_p$ $\quad \Theta$ $\quad L$ $\quad \rho$ $\quad \mu$ $\quad V$ $\quad n = 7$ parameters

② Select F, L, t, T (temperature) as primary dimensions.

③ $\frac{FL}{t}$ $\quad \frac{L^2}{t^2T}$ $\quad T$ $\quad L$ $\quad \frac{Ft^2}{L^4}$ $\quad \frac{Ft}{L^2}$ $\quad \frac{L}{t}$ $\quad r = 4$ primary dimensions

④ Choose ρ, V, L, Θ as repeating parameters. $\quad m = r = 4$

⑤ Then $n - m = 7 - 4 = 3$ dimensionless parameters result. $\quad n-m$

Set up dimensional equations:

$\Pi_1 = \rho^a V^b L^c \Theta^d \dot{Q}$

$F^0L^0t^0T^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{L}{t}\right)^b (L)^c (T)^d \frac{FL}{t}$

Equating exponents:

F: $a + 1 = 0$ $\quad a = -1$
L: $-4a + b + c + 1 = 0$
t: $2a - b - 1 = 0$ $\quad b = -3$
T: $d = 0$

$\therefore c = -1 + 4a - b = -2$

$$\Pi_1 = \frac{\dot{Q}}{\rho V^3 L^2}$$

$\Pi_2 = \rho^a V^b L^c \Theta^d c_p$

$F^0L^0t^0T^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{L}{t}\right)^b (L)^c (T)^d \frac{L^2}{t^2T}$

Equating exponents:

F: $a = 0$ $\quad a = 0$
L: $-4a + b + c + 2 = 0$
t: $2a - b - 2 = 0$ $\quad b = -2$
T: $d - 1 = 0$ $\quad d = 1$

$\therefore c = -2 + 4a - b = 0$

$$\Pi_2 = \frac{c_p \Theta}{V^2}$$

$\Pi_3 = \rho^a V^b L^c \Theta^d \mu$

$F^0L^0t^0T^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{L}{t}\right)^b (L)^c (T)^d \frac{Ft}{L^2}$

Equating exponents:

F: $a + 1 = 0$ $\quad a = -1$
L: $-4a + b + c - 2 = 0$
t: $2a - b + 1 = 0$ $\quad b = -1$
T: $d = 0$ $\quad d = 0$

$\therefore c = 2 + 4a - b = -1$

$$\Pi_3 = \frac{\mu}{\rho V L}$$

⑥ Check, using MLtT dimensions: $\dot{Q} = ML^2/t^3$; $\mu = M/Lt$

$\Pi_1 = \frac{ML^2}{t^3}\frac{L^3}{M}\frac{t^3}{L^3}\frac{1}{L^2} = [1]$ ✓✓ $\quad \Pi_2 = \frac{L^2}{t^2T}\,T\,\frac{t^2}{L^2} = [1]$ ✓✓ $\quad \Pi_3 = \frac{M}{Lt}\frac{L^3}{M}\frac{t}{L}\frac{1}{L} = [1]$ ✓✓

Thus

$$\Pi_1 = f(\Pi_2, \Pi_3) \qquad \frac{\dot{Q}}{\rho V^3 L^2} = f\left(\frac{c_p\Theta}{V^2}, \frac{\mu}{\rho V L}\right)$$

or

$$\dot{Q} = \rho V^3 L^2 f\left(\frac{c_p\Theta}{V^2}, \frac{\mu}{\rho V L}\right)$$

$\dot{Q}$

Problem 7.42

[3]

Given: Power, $\mathcal{P}$, required to drive a propeller is a function of V, D, ω (angular velocity), μ, ρ, and c (speed of sound)

Find: (a) number of dimensionless groups required to characterize situation
(b) the dimensionless groups

Solution: Apply Buckingham π-theorem

① $\mathcal{P}$ V D ω μ ρ c $\qquad n = 7$ parameters

② Select M, L, t as primary dimensions

③ $\mathcal{P}$ V D ω μ ρ c

$\frac{ML^2}{t^3}$ $\frac{L}{t}$ L $\frac{1}{t}$ $\frac{M}{Lt}$ $\frac{M}{L^3}$ $\frac{L}{t}$ $\qquad r = 3$ primary dimensions

④ V, D, ρ $\qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 4$ dimensionless groups will result.

Setting up dimensional equations:

$\pi_1 = V^a D^b \rho^c \mathcal{P}$

$M^0L^0t^0 = \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{ML^2}{t^3}$

Summing exponents,

M: $c + 1 = 0 \quad \therefore c = -1$

L: $a + b - 3c + 2 = 0$

t: $-a - 3 = 0 \quad \therefore a = -3$

$b = 3c - 2 - a = -2$

$\therefore \pi_1 = \frac{\mathcal{P}}{\rho D^2 V^3}$

$\pi_2 = V^a D^b \rho^c \omega$

$M^0L^0t^0 = \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{1}{t}$

Summing exponents,

M: $c = 0$

L: $a + b - 3c = 0$

t: $-a - 1 = 0 \quad \therefore a = -1$

$b = 3c - a = 1$

$\therefore \pi_2 = \frac{\omega D}{V}$

$\pi_3 = V^a D^b \rho^c \mu$

$M^0L^0t^0 = \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{M}{Lt}$

Summing exponents,

M: $c + 1 = 0 \quad \therefore c = -1$

L: $a + b - 3c - 1 = 0$

t: $-a - 1 = 0 \quad \therefore a = -1$

$b = 3c + 1 - a = -1$

$\therefore \pi_3 = \frac{\mu}{\rho D V}$

$\pi_4 = V^a D^b \rho^c c$

$M^0L^0t^0 = \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \frac{L}{t}$

Summing exponents,

M: $c = 0$

L: $a + b - 3c + 1 = 0$

t: $-a - 1 = 0 \quad \therefore a = -1$

$b = 3c - a - 1 = 0$

$\therefore \pi_4 = \frac{c}{V}$

Dimensionless groups are: $\frac{\mathcal{P}}{\rho D^2 V^3}, \frac{\omega D}{V}, \frac{\mu}{\rho D V}, \frac{c}{V}$

⑥ Check using F, L, t

$\pi_1 = \frac{FL}{t} \cdot \frac{L^4}{Ft^2} \frac{1}{L^2} \frac{t^3}{L^3} = [1]$ ✓ $\qquad \pi_2 = \frac{1}{t} L \cdot \frac{t}{L} = [1]$ ✓

$\pi_3 = \frac{1}{Re} = [1]$ ✓ $\qquad \pi_4 = \frac{L}{t} \cdot \frac{t}{L} = [1]$ ✓

Problem 7.43

7.43 The rate dT/dt at which the temperature T at the center of a rice kernel falls during a food technology process is critical—too high a value leads to cracking of the kernel, and too low a value makes the process slow and costly. The rate depends on the rice specific heat, c, thermal conductivity, k, and size, L, as well as the cooling air specific heat, c_p, density, ρ, viscosity, μ, and speed, V. How many basic dimensions are included in these variables? Determine the Π parameters for this problem.

Given: That the cooling rate depends on rice properties and air properties

Find: The Π groups

Solution:

Apply the Buckingham Π procedure

① $dT/dt \quad c \quad k \quad L \quad c_p \quad \rho \quad \mu \quad V$ $\qquad n = 8$ parameters

② Select primary dimensions M, L, t and T (temperature)

③ $$\begin{matrix} dT/dt & c & k & L & c_p & \rho & \mu & V \\ \dfrac{T}{t} & \dfrac{L^2}{t^2T} & \dfrac{ML}{t^2T} & L & \dfrac{L^2}{t^2T} & \dfrac{M}{L^3} & \dfrac{M}{Lt} & \dfrac{L}{t} \end{matrix}$$ $r = 4$ primary dimensions

④ $V \quad \rho \quad L \quad c_p$ $\qquad m = r = 4$ repeat parameters

Then $n - m = 4$ dimensionless groups will result. By inspection, one Π group is c/c_p. Setting up a dimensional equation,

$$\Pi_1 = V^a \rho^b L^c c_p^d \frac{dT}{dt} = \left(\frac{L}{t}\right)^a \left(\frac{M}{L^3}\right)^b (L)^c \left(\frac{L^2}{t^2T}\right)^d \frac{T}{t} = T^0 M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{lc|c} T: & -d+1=0 & d=1 \\ M: & b=0 & b=0 \\ L: & a-3b+c+2d=0 & a+c=-2 \rightarrow c=1 \\ t: & -a-2d-1=0 & a=-3 \end{array}$$

Hence $\quad \Pi_1 = \dfrac{dT}{dt}\dfrac{Lc_p}{V^3}$

By a similar process, we find $\qquad \Pi_2 = \dfrac{k}{\rho L^2 c_p} \qquad$ and $\qquad \Pi_3 = \dfrac{\mu}{\rho L V}$

Hence

$$\frac{dT}{dt}\frac{Lc_p}{V^3} = f\left(\frac{c}{c_p}, \frac{k}{\rho L^2 c_p}, \frac{\mu}{\rho L V}\right)$$

Problem 7.44

[4]

Given: Water hammer caused by sudden closure of valve in pipeline.

$$p_{max} = f(\rho, V_0, E_v)$$

Find: (a) How many dimensionless groups needed to characterize?
(b) Functional relationship in terms of Π groups.

Solution: Step ①: List p_{max} ρ V_0 E_v $n = 4$

Step ②: Choose M, L, t

Step ③: $\frac{M}{Lt^2}$ $\frac{M}{L^3}$ $\frac{L}{t}$ $\frac{M}{Lt^2}$

Check dimensional matrix:

	p_{max}	ρ	V_0	E_v
M	1	1	0	1
L	-1	-3	1	-1
t	-2	0	-1	-2

For this matrix, $r = 2$

Step ④: Choose ρ, V_0

Step ⑤: $\Pi_1 = \rho^a V_0^b p_{max} = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b \frac{M}{Lt^2} = M^0 L^0 t^0$

$M: a + 1 = 0$ $a = -1$
$L: -3a + b - 1 = 0$
$t: -b - 2 = 0$ $b = -2$ ✓

$$\Pi_1 = \frac{p_{max}}{\rho V_0^2}$$ Π_1

$\Pi_2 = \rho^a V_0^b E_v = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b \frac{M}{Lt^2} = M^0 L^0 t^0$

By inspection $$\Pi_2 = \frac{E_v}{\rho V_0^2}$$ Π_2

Step ⑥: Check using FLt: $\rho = \frac{M}{L^3} \times \frac{Ft^2}{ML} = \frac{Ft^2}{L^4}$

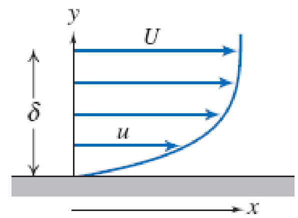
$$\Pi_1 = \frac{F}{L^2} \frac{L^4}{Ft^2} \frac{t^2}{L^2} = \frac{FL^4t^2}{FL^4t^2} = 1 \quad ✓✓$$

The functional relationship is $\Pi_1 = f(\Pi_2)$. Thus

$$\frac{p_{max}}{\rho V_0^2} = f\left(\frac{E_v}{\rho V_0^2}\right)$$ f

Problem 7.45

7.45 The fluid velocity u at any point in a boundary layer depends on the distance y of the point above the surface, the free-stream velocity U and free-stream velocity gradient dU/dx, the fluid kinematic viscosity ν, and the boundary layer thickness δ. How many dimensionless groups are required to describe this problem? Find: (a) two Π groups by inspection, (b) one Π that is a standard fluid mechanics group, and (c) any remaining Π groups using the Buckingham Pi theorem.



Given: Boundary layer profile

Find: Two Π groups by inspection; One Π that is a standard fluid mechanics group; Dimensionless groups

Solution:

Two obvious Π groups are u/U and y/δ. A dimensionless group common in fluid mechanics is $U\delta/\nu$ (Reynolds number)

Apply the Buckingham Π procedure

① u y U dU/dx ν δ $n = 6$ parameters

② Select primary dimensions M, L, t

③ $$\left\{ \begin{matrix} u & y & U & dU/dx & \nu & \delta \\ \frac{L}{t} & L & \frac{L}{t} & \frac{1}{t} & \frac{L^2}{t} & L \end{matrix} \right\}$$ $m = r = 3$ primary dimensions

④ U δ $m = r = 2$ repeat parameters

⑤ Then $n - m = 4$ dimensionless groups will result. We can easily do these by inspection

$$\Pi_1 = \frac{u}{U} \qquad \Pi_2 = \frac{y}{\delta} \qquad \Pi_3 = \frac{(dU/dy)\delta}{U} \qquad \Pi_4 = \frac{\nu}{\delta U}$$

⑥ Check using F, L, t as primary dimensions, is not really needed here

Note: Any combination of Π's can be used; they are not unique!

Problem 7.46

Given: Airship to operate at 20 m/sec in standard air.
Model built to 1/20 scale tested at same air temperature.
Model is tested at 75 m/sec

Find: (a) Criterion for dynamic similarity.
(b) Wind tunnel pressure.
(c) Prototype drag if drag force on model is 250 N.

Solution

Dimensional analysis predicts $\frac{F}{\rho V^2 L^2} = f\left(\frac{\rho V L}{\mu}\right)$

Consequently for similarity, $\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_p$

Since L is fixed, and $\mu_p = \mu_m$ (because T is the same)

$\rho_m = \rho_p \frac{V_p}{V_m} \frac{L_p}{L_m} \frac{\mu_m}{\mu_p} = \rho_p \frac{20}{75}(20)(1) = 5.33\,\rho_p$

From ideal gas law, $P = \rho R T$

$\therefore \frac{P_m}{P_p} = \frac{\rho_m}{\rho_p} = 5.33$ and $P_m = 5.33\,P_p = 5.33 \times 101\ \text{kPa} = 5.39 \times 10^5\ \text{Pa}$ P_m

From the force ratios,

$F_p = F_m \frac{\rho_p}{\rho_m} \frac{V_p^2}{V_m^2} \frac{L_p^2}{L_m^2} = F_m \frac{1}{5.33}\left(\frac{20}{75}\right)^2 (20)^2 = 5.34\,F_m$

Thus

$F_p = 5.34\,F_m = 5.34 \times 250\ \text{N} = 1.34\ \text{kN}$ F_p

Problem 7.47

7.47 The designers of a large tethered pollution-sampling balloon wish to know what the drag will be on the balloon for the maximum anticipated wind speed of 5 m/s (the air is assumed to be at 20°C). A $\frac{1}{20}$-scale model is built for testing in water at 20°C. What water speed is required to model the prototype? At this speed the model drag is measured to be 2 kN. What will be the corresponding drag on the prototype?

Given: Model scale for on balloon

Find: Required water model water speed; drag on protype based on model drag

Solution:

From Appendix A (inc. Fig. A.2) $\rho_{air} = 1.24 \cdot \frac{kg}{m^3}$ $\quad \mu_{air} = 1.8 \times 10^{-5} \cdot \frac{N \cdot s}{m^2}$ $\quad \rho_w = 999 \cdot \frac{kg}{m^3}$ $\quad \mu_w = 10^{-3} \cdot \frac{N \cdot s}{m^2}$

The given data is $V_{air} = 5 \cdot \frac{m}{s}$ $\quad L_{ratio} = 20$ $\quad F_w = 2 \cdot kN$

For dynamic similarity we assume $\frac{\rho_w \cdot V_w \cdot L_w}{\mu_w} = \frac{\rho_{air} \cdot V_{air} \cdot L_{air}}{\mu_{air}}$

Then $V_w = V_{air} \cdot \frac{\mu_w}{\mu_{air}} \cdot \frac{\rho_{air}}{\rho_w} \cdot \frac{L_{air}}{L_w} = V_{air} \cdot \frac{\mu_w}{\mu_{air}} \cdot \frac{\rho_{air}}{\rho_w} \cdot L_{ratio} = 5 \cdot \frac{m}{s} \times \left(\frac{10^{-3}}{1.8 \times 10^{-5}} \right) \times \left(\frac{1.24}{999} \right) \times 20$ $\quad V_w = 6.90 \frac{m}{s}$

For the same Reynolds numbers, the drag coefficients will be the same so we have $\frac{F_{air}}{\frac{1}{2} \cdot \rho_{air} \cdot A_{air} \cdot V_{air}^2} = \frac{F_w}{\frac{1}{2} \cdot \rho_w \cdot A_w \cdot V_w^2}$

where $\frac{A_{air}}{A_w} = \left(\frac{L_{air}}{L_w} \right)^2 = L_{ratio}^2$

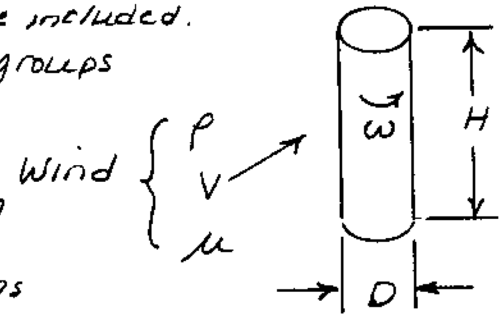
Hence the prototype drag is $F_{air} = F_w \cdot \frac{\rho_{air}}{\rho_w} \cdot L_{ratio}^2 \cdot \left(\frac{V_{air}}{V_w} \right)^2 = 2000 \cdot N \times \left(\frac{1.24}{999} \right) \times 20^2 \times \left(\frac{5}{6.9} \right)^2$ $\quad F_{air} = 522\,N$

Problem 7.48

[5]

Given: Vessel to be powered by rotating cylinder. Model to be tested to estimate power neededed to rotate cylinder.

Find: (a) Parameters that should be included.
(b) Important dimensionless groups



Solution: $\mathcal{P} = f(\rho, \omega, D, \mu, H, V)$

① $\rho \quad \omega \quad D \quad \mu \quad H \quad V \quad \mathcal{P} \qquad n = 7$

② Choose M, L, t as primary dimensions

③ $\frac{M}{L^3} \quad \frac{1}{t} \quad L \quad \frac{M}{Lt} \quad L \quad \frac{L}{t} \quad \frac{ML^2}{t^3} \qquad r = 3$ primary dimensions

④ $\rho, \omega, D \quad m = 3 \qquad m = r = 3$ repeating parameters

⑤ Then expect $n - m = 4$ dimensionless groups

$\Pi_1 = \rho^a \omega^b D^c \mathcal{P} = \left(\frac{M}{L^3}\right)^a \left(\frac{1}{t}\right)^b (L)^c \frac{ML^2}{t^3}$

$M: a + 1 = 0 \qquad a = -1$
$L: -3a + c + 2 = 0 \qquad c = -5$
$t: -b - 3 = 0 \qquad b = -3$

$\Pi_1 = \frac{\mathcal{P}}{\rho \omega^3 D^5}$ ← Π_1

$\Pi_2 = \rho^a \omega^b D^c V = \left(\frac{M}{L^3}\right)^a \left(\frac{1}{t}\right)^b (L)^c \frac{L}{t}$

$M: a + 0 = 0 \qquad a = 0$
$L: -3a + c + 1 = 0 \qquad c = -1$
$t: -b - 1 = 0 \qquad b = -1$

$\Pi_2 = \frac{V}{\omega D}$ ← Π_2

$\Pi_3 = \rho^a \omega^b D^c H$

By inspection $\Pi_3 = \frac{H}{D}$ ← Π_3

$\Pi_4 = \rho^a \omega^b D^c \mu = \left(\frac{M}{L^3}\right)^a \left(\frac{1}{t}\right)^b (L)^c \frac{M}{Lt}$

$M: a + 1 = 0 \qquad a = -1$
$L: -3a + c - 1 = 0 \qquad c = -2$
$t: -b - 1 = 0 \qquad b = -1$

$\Pi_4 = \frac{\mu}{\rho \omega D^2}$ ←

Thus $\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4)$ or $\frac{\mathcal{P}}{\rho \omega^3 D^5} = f\left(\frac{V}{\omega D}, \frac{H}{D}, \frac{\mu}{\rho \omega D^2}\right)$ ← $\frac{\mathcal{P}}{\rho \omega^3 D^5}$

⑥ Check, using F, L, t

$\Pi_1 = \frac{FL}{t} \frac{L^4}{Ft^2} \frac{t^3}{1} \frac{1}{L^5} = [1]$ ✓ $\qquad \Pi_2 = \frac{L}{t} \frac{t}{1} \frac{1}{L} = [1]$ ✓

$\Pi_3 = \frac{L}{L} = [1]$ ✓ $\qquad \Pi_4 = \frac{Ft}{L^2} \frac{L^4}{Ft^2} \frac{t}{1} \frac{1}{L^2} = [1]$ ✓

Problem 7.49

Given: Desire to match Reynolds number in two flows: one of air and one of water, using the same size model.

Find: Which flow must have the higher speed, and by how much.

Solution: Set $Re_w = \frac{\rho_w V_w L_w}{\mu_w} = Re_a = \frac{\rho_a V_a L_a}{\mu_a}$

Since $L_w = L_a$, then $\frac{V_a}{V_w} = \frac{\rho_w}{\rho_a} \frac{\mu_a}{\mu_w} = \frac{\nu_a}{\nu_w}$

From Tables A.8 and A.10, at 20°C, $\nu_w = 1.00 \times 10^{-6}\ m^2/s$ and $\nu_a = 1.51 \times 10^{-5}\ m^2/s$.

Thus $\frac{V_a}{V_w} = 1.51 \times 10^{-5} \frac{m^2}{s} \times \frac{s}{1.00 \times 10^{-6}\ m^2} = 15.1$

Therefore V_a must be larger than V_w. ← V_a

In fact, to match Re,

$$V_a = 15.1\ V_w$$ ← V_a

Given: Measurements of drag force are made on a model car in a towing tank filled with freshwater; $L_m/L_p = 1/5$. The dimensionless force ratio becomes constant at model test speeds above $V_m = 4$ m/s. At this speed the drag force on the model is $F_{D_m} = 182$ N.

Find: (a) State conditions required to assure dynamic similarity between model and prototype
(b) Determine required speed ratio V_m/V_p to assure dynamically similar conditions
(c) Calculate expected prototype drag when operating in air at speed, $V_p = 90$ km/hr

Solution:

(a) The flows must be geometrically and kinematically similar, and have equal Reynolds numbers to be dynamically similar.

- geometric similarity requires true model in all respects
- kinematic similarity requires same flow pattern, i.e. no free-surface effects or cavitation.
- the problem may be stated as $F_D = f(\rho, V, L, \mu)$. Dimensional analysis gives

$$\frac{F_D}{\rho V^2 L^2} = f\left(\frac{\mu}{\rho V L}\right) = g(Re).$$

(b) Matching Reynolds numbers between model & prototype flows gives

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p} \qquad \text{Assume } T = 20^\circ C$$

$$\frac{V_m}{V_p} = \frac{\nu_m}{\nu_p} \times \frac{L_p}{L_m} = 1\times10^{-6}\,\frac{m^2}{s} \times \frac{1}{1.51\times10^{-5}}\,\frac{s}{m^2} \times 5 = 0.331 \qquad \frac{V_m}{V_p}$$

(c) For dynamically similar conditions, $\left.\frac{F_D}{\rho V^2 L^2}\right)_m = \left.\frac{F_D}{\rho V^2 L^2}\right)_p$

$$\therefore F_{D_p} = F_{D_m}\,\frac{\rho_p}{\rho_m} \times \left(\frac{V_p}{V_m}\right)^2 \times \left(\frac{L_p}{L_m}\right)^2$$

$$= 182\,N \times \frac{1.20}{999} \times \left(\frac{90\,km}{hr} \times \frac{1000\,m}{km} \times \frac{hr}{3600\,s} \times \frac{s}{4\,m}\right)^2 (5)^2$$

$$F_{D_p} = 214\,N \qquad F_{D_p}$$

Problem 7.51

[2]

7.51 On a cruise ship, passengers complain about the noise emanating from the ship's propellers (probably due to turbulent flow effects between propeller and ship). You have been hired to find out the source of this noise. You will study the flow pattern around the propellers and have decided to use a 1:10 scale water tank. If the ship's propellers rotate at 125 rpm, estimate the model propeller rotation speed if a) the Froude number or b) the Reynolds number is the governing dimensionless group. Which is most likely to lead to the best modeling?

Given: Flow around ship's propeller

Find: Model propeller speed using Froude number and Reynolds number

Solution:

Basic equations $\quad Fr = \frac{V}{\sqrt{g \cdot L}} \qquad Re = \frac{V \cdot L}{\nu}$

Using the Froude number $\quad Fr_m = \frac{V_m}{\sqrt{g \cdot L_m}} = Fr_p = \frac{V_p}{\sqrt{g \cdot L_p}} \quad$ or $\quad \frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} \qquad (1)$

But the angular velocity is given by $\quad V = L \cdot \omega \quad$ so $\quad \frac{V_m}{V_p} = \frac{L_m}{L_p} \cdot \frac{\omega_m}{\omega_p} \qquad (2)$

Comparing Eqs. 1 and 2 $\quad \frac{L_m}{L_p} \cdot \frac{\omega_m}{\omega_p} = \sqrt{\frac{L_m}{L_p}} \qquad \frac{\omega_m}{\omega_p} = \sqrt{\frac{L_p}{L_m}}$

The model rotation speed is then $\quad \omega_m = \omega_p \cdot \sqrt{\frac{L_p}{L_m}} \qquad \omega_m = 125 \cdot rpm \times \sqrt{\frac{10}{1}} \qquad \omega_m = 395\, rpm$

Using the Reynolds number $\quad Re_m = \frac{V_m \cdot L_m}{\nu_m} = Re_p = \frac{V_p \cdot L_p}{\nu_p} \quad$ or $\quad \frac{V_m}{V_p} = \frac{L_p}{L_m} \cdot \frac{\nu_m}{\nu_p} = \frac{L_p}{L_m} \qquad (3)$

(We have assumed the viscosities of the sea water and model water are comparable)

Comparing Eqs. 2 and 3 $\quad \frac{L_m}{L_p} \cdot \frac{\omega_m}{\omega_p} = \frac{L_p}{L_m} \qquad \frac{\omega_m}{\omega_p} = \left(\frac{L_p}{L_m}\right)^2$

The model rotation speed is then $\quad \omega_m = \omega_p \cdot \left(\frac{L_p}{L_m}\right)^2 \qquad \omega_m = 125 \cdot rpm \times \left(\frac{10}{1}\right)^2 \qquad \omega_m = 12500\, rpm$

Of the two models, the Froude number appears most realistic; at 12,500 rpm serious cavitation will occur. Both flows will likely have high Reynolds numbers so that the flow becomes independent of Reynolds number; the Froude number is likely to be a good indicator of static pressure to dynamic pressure for this (although cavitation number would be better).

Problem 7.52

Given: Prototype torpedo, $D = 533$ mm, $l = 6.7$ m operates in water at a speed of 28 m/s. Model (1/5 scale) is to be tested in a wind tunnel. Maximum wind tunnel speed is 110 m/sec; $T = 20°C$; pressure is variable.
At dynamically similar test conditions, $F_{D\,model} = 618$ N

Find: (a) required wind tunnel pressure for dynamically similar test
(b) expected drag force on prototype

Solution:

Assume $F_D = F_D(V, D, \rho, \mu)$. From the Buckingham π-theorem, for $n = 5$, with $m = r = 3$, we would expect two dimensionless groups.

$$\frac{F}{\rho V^2 D^2} = f\left(\frac{\rho V D}{\mu}\right)$$

To attain dynamically similar model test, $\left.\frac{\rho V D}{\mu}\right)_m = \left.\frac{\rho V D}{\mu}\right)_p$

$$\therefore \quad \rho_m = \rho_p \frac{V_p}{V_m} \frac{D_p}{D_m} \frac{\mu_m}{\mu_p}$$

For air at 20°C $\mu_m = 1.81 \times 10^{-5}$ N·s/m²
water at 20°C $\mu_p = 1 \times 10^{-3}$ N·s/m²

$$\rho_m = 998 \frac{kg}{m^3} \times \frac{28}{110} \times 5 \times \frac{1.81 \times 10^{-5}}{1 \times 10^{-3}} = 23.0 \ kg/m^3$$

From the ideal gas equation of state,

$$p = \rho_m R T_m = 23.0 \frac{kg}{m^3} \times 287 \frac{N \cdot m}{kg \cdot K} \times 293 \ K = 1.93 \ MPa \ (abs)$$ ← p

For dynamically similar flows,

$$\left.\frac{F_D}{\rho V^2 D^2}\right)_m = \left.\frac{F_D}{\rho V^2 D^2}\right)_p$$

$$\therefore \quad F_{D_p} = F_{D_m} \frac{\rho_p}{\rho_m} \left(\frac{V_p}{V_m}\right)^2 \left(\frac{D_p}{D_m}\right)^2$$

$$= 618 \ N \times \frac{998}{23.0} \left(\frac{28}{110}\right)^2 (5)^2$$

$$F_{D_p} = 43.4 \ kN$$ ← F_{D_p}

Problem 7.53

Given: Drag force, F, of an airfoil at zero angle of attack is a function of ρ, μ, V, and L.

Model test conditions:

$\frac{L_m}{L_p} = \frac{1}{10}$ $\quad Re_m = 5.5 \times 10^6$ based on chord length

$T = 15°C$, $P = 10$ atmospheres

Prototype data: chord length, $L = 2m$

$T = 15°C \quad P = 101$ kPa

Find: (a) velocity, V_m, of model test

(b) corresponding prototype velocity.

Solution

Dimensional analysis predicts $\frac{F}{\rho V^2 L^2} = f\left(\frac{\rho V L}{\mu}\right)$

$Re_m = \left.\frac{\rho V L}{\mu}\right)_m$ and hence $V_m = \frac{Re_m \mu_m}{\rho_m L_m}$

To determine ρ_m assume air behaves as an ideal gas.

$\rho_m = \frac{P_m}{RT_m} = 10 \times 101 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287 \, N \cdot m} \times \frac{1}{288 K} = 12.2 \, kg/m^3$

From Table A.10, Appendix A, $\mu_m = 1.79 \times 10^{-5}$ N.s/m²

$V_m = \frac{Re_m \mu_m}{\rho_m L_m} = 5.5 \times 10^6 \times 1.79 \times 10^{-5} \frac{N \cdot sec}{m^2} \times \frac{m^3}{12.2 \, kg} \times \frac{1}{0.2m} \times \frac{kg \cdot m}{N \cdot sec^2}$

$V_m = 40.3$ m/s $\quad V_m$

For dynamic similarity $\left.\frac{\rho V L}{\mu}\right)_m = \left.\frac{\rho V L}{\mu}\right)_p$

$V_p = V_m \frac{\mu_p}{\mu_m} \frac{\rho_m}{\rho_p} \frac{L_m}{L_p} = V_m \frac{\mu_p}{\mu_m} \frac{P_m}{P_p} \frac{T_p}{T_m} \frac{L_m}{L_p}$

$V_p = 40.3 \frac{m}{s} \times (1) \times (10) \times (1) \times \left(\frac{1}{10}\right) = 40.3$ m/s $\quad V_p$

Problem 7.54

[2]

7.54 Consider a smooth sphere, of diameter D, immersed in a fluid moving with speed V. The drag force on a 10-ft diameter weather balloon in air moving at 5 ft/s is to be calculated from test data. The test is to be performed in water using a 2-in. diameter model. Under dynamically similar conditions, the model drag force is measured as 0.85 lbf. Evaluate the model test speed and the drag force expected on the full-scale balloon.

Given: Model of weather balloon

Find: Model test speed; drag force expected on full-scale balloon

Solution:

From Buckingham Π
$$\frac{F}{\rho \cdot V^2 \cdot D^2} = f\left(\frac{\nu}{V \cdot D}, \frac{V}{c}\right) = F(Re, M)$$

For similarity $\quad Re_p = Re_m \quad$ and $\quad M_p = M_m \quad$ (Mach number criterion satisified because M<<)

Hence
$$Re_p = \frac{V_p \cdot D_p}{\nu_p} = Re_m = \frac{V_m \cdot D_m}{\nu_m}$$

$$V_m = V_p \cdot \frac{\nu_m}{\nu_p} \cdot \frac{D_p}{D_m}$$

From Table A.7 at 68ºF $\quad \nu_m = 1.08 \times 10^{-5} \cdot \frac{ft^2}{s} \quad$ From Table A.9 at 68ºF $\quad \nu_p = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s}$

$$V_m = 5 \cdot \frac{ft}{s} \times \left(\frac{1.08 \times 10^{-5} \cdot \frac{ft^2}{s}}{1.62 \times 10^{-4} \cdot \frac{ft^2}{s}}\right) \times \left(\frac{10 \cdot ft}{\frac{1}{6} \cdot ft}\right) \qquad V_m = 20.0 \frac{ft}{s}$$

Then
$$\frac{F_m}{\rho_m \cdot V_m^2 \cdot D_m^2} = \frac{F_p}{\rho_p \cdot V_p^2 \cdot D_p^2} \qquad F_p = F_m \cdot \frac{\rho_p}{\rho_{m.}} \cdot \frac{V_p^2}{V_m^2} \cdot \frac{D_p^2}{D_m^2}$$

$$F_p = 0.85 \cdot lbf \times \left(\frac{0.00234 \cdot \frac{slug}{ft^3}}{1.94 \cdot \frac{slug}{ft^3}}\right) \times \left(\frac{5 \frac{ft}{s}}{20 \cdot \frac{ft}{s}}\right)^2 \times \left(\frac{10 \cdot ft}{\frac{1}{6} \cdot ft}\right)^2 \qquad F_p = 0.231\, lbf$$

Problem 7.55

7.55 An airplane wing, with chord length of 1.5 m and span of 9 m, is designed to move through standard air at a speed of 7.5 m/s. A $\frac{1}{10}$-scale model of this wing is to be tested in a water tunnel. What speed is necessary in the water tunnel to achieve dynamic similarity? What will be the ratio of forces measured in the model flow to those on the prototype wing?

Given: Model of wing

Find: Model test speed for dynamic similarity; ratio of model to prototype forces

Solution:

We would expect $F = F(l, s, V, \rho, \mu)$ where F is the force (lift or drag), l is the chord and s the span

From Buckingham Π
$$\frac{F}{\rho \cdot V^2 \cdot l \cdot s} = f\left(\frac{\rho \cdot V \cdot l}{\mu}, \frac{l}{s}\right)$$

For dynamic similarity
$$\frac{\rho_m \cdot V_m \cdot l_m}{\mu_m} = \frac{\rho_p \cdot V_p \cdot l_p}{\mu_p}$$

Hence
$$V_m = V_p \cdot \frac{\rho_p}{\rho_m} \cdot \frac{l_p}{l_m} \cdot \frac{\mu_m}{\mu_p}$$

From Table A.8 at 20°C $\mu_m = 1.01 \times 10^{-3} \cdot \frac{N \cdot s}{m^2}$ From Table A.10 at 20°C $\mu_p = 1.81 \times 10^{-5} \cdot \frac{N \cdot s}{m^2}$

$$V_m = 7.5 \cdot \frac{m}{s} \times \left(\frac{1.21 \cdot \frac{kg}{m^3}}{998 \cdot \frac{kg}{m^3}}\right) \times \left(\frac{10}{1}\right) \times \left(\frac{1.01 \times 10^{-3} \cdot \frac{N \cdot s}{m^2}}{1.81 \times 10^{-5} \cdot \frac{N \cdot s}{m^2}}\right) \qquad V_m = 5.07 \frac{m}{s}$$

Then
$$\frac{F_m}{\rho_m \cdot V_m^2 \cdot l_m \cdot s_m} = \frac{F_p}{\rho_p \cdot V_p^2 \cdot l_p \cdot s_p} \qquad \frac{F_m}{F_p} = \frac{\rho_m}{\rho_p} \cdot \frac{V_m^2}{V_p^2} \cdot \frac{l_m \cdot s_m}{l_p \cdot s_p} = \frac{998}{1.21} \times \left(\frac{5.07}{7.5}\right)^2 \times \frac{1}{10} \times \frac{1}{10} = 3.77$$

Problem 7.56

Given: Fluid dynamic characteristics of a golf ball are to be tested using a model in a wind tunnel.
dependent variables: F_D, F_L
independent variables should include ω • d (dimple depth)
Golf pro can hit prototype ($D = 1.68$ in) at $V = 240$ ft/s and $\omega = 9000$ rpm. Prototype is to be modeled in wind tunnel with $V = 80$ ft/s.

Find: (a) suitable dimensionless parameters
(b) required diameter of model
(c) required rotational speed of model

Solution: Assume the functional dependence to be given by

$$F_D = F_D(D, V, \omega, d, \rho, \mu) \quad \text{and} \quad F_L = F_L(D, V, \omega, d, \rho, \mu)$$

From the Buckingham π-theorem, for $n = 7$ and $m = r = 3$, we would expect four dimensionless groups

$$\frac{F_D}{\rho V^2 D^2} = f\left(\frac{\rho V D}{\mu}, \frac{\omega D}{V}, \frac{d}{D}\right) \quad \text{and} \quad \frac{F_L}{\rho V^2 D^2} = g\left(\frac{\rho V D}{\mu}, \frac{\omega D}{V}, \frac{d}{D}\right) \qquad \pi\text{'s}$$

To determine the required diameter of the model,

$$\left.\frac{\rho V D}{\mu}\right)_m = \left.\frac{\rho V D}{\mu}\right)_p \qquad \therefore D_m = \frac{\rho_p}{\rho_m}\frac{V_p}{V_m}\frac{\mu_m}{\mu_p}D_p = 1 \times \frac{240}{80} \times 1 \times D_p$$

$$D_m = 3 D_p = 3 \times 1.68 \text{ in} = 5.04 \text{ in.} \qquad D_m$$

To determine the required rotational speed of the model,

$$\left.\frac{\omega D}{V}\right)_m = \left.\frac{\omega D}{V}\right)_p \qquad \therefore \omega_m = \omega_p \frac{D_p}{D_m}\frac{V_m}{V_p} = \omega_p \frac{1}{3} \times \frac{80}{240} = \frac{1}{9}\omega_p$$

$$\omega_m = \frac{1}{9}\omega_p = \frac{1}{9} \times 9000 \text{ rpm} = 1000 \text{ rpm} \qquad \omega_m$$

Problem 7.57

[3]

7.57 A water pump with impeller diameter 60 cm is to be designed to move 0.4 m^3/s when running at 800 rpm. Testing is performed on a $\frac{1}{2}$ scale model running at 2000 rpm using air (20°C) as the fluid. For similar conditions (neglecting Reynolds number effects), what will be the model flow rate? If the model draws 75 W, what will be the power requirement of the prototype?

Given: Model of water pump

Find: Model flow rate for dynamic similarity (ignoring Re); Power of prototype

Solution:

From Buckingham Π $\quad \dfrac{Q}{\omega \cdot D^3} \quad$ and $\quad \dfrac{P}{\rho \cdot \omega^3 \cdot D^5}$

where Q is flow rate, ω is angular speed, d is diameter, and ρ is density (these Π groups will be discussed in Chapter 10

For dynamic similarity $\quad \dfrac{Q_m}{\omega_m \cdot D_m^{\,3}} = \dfrac{Q_p}{\omega_p \cdot D_p^{\,3}}$

Hence $\quad Q_m = Q_p \cdot \dfrac{\omega_m}{\omega_p} \cdot \left(\dfrac{D_m}{D_p}\right)^3$

$$Q_m = 0.4 \cdot \frac{m^3}{s} \times \left(\frac{2000}{800}\right) \times \left(\frac{1}{2}\right)^3 \qquad Q_m = 0.125 \frac{m^3}{s}$$

From Table A.8 at 20°C $\quad \rho_p = 998 \cdot \dfrac{kg}{m^3}$ $\quad$ From Table A.10 at 20°C $\quad \mu_m = 1.21 \cdot \dfrac{kg}{m^3}$

Then $\quad \dfrac{P_m}{\rho_m \cdot \omega_m^{\,3} \cdot D_m^{\,5}} = \dfrac{P_p}{\rho_p \cdot \omega_p^{\,3} \cdot D_p^{\,5}}$

$$P_p = P_m \cdot \frac{\rho_p}{\rho_m} \cdot \left(\frac{\omega_p}{\omega_m}\right)^3 \cdot \left(\frac{D_p}{D_m}\right)^5$$

$$P_p = 75 \cdot W \times \frac{998}{1.21} \times \left(\frac{800}{2000}\right)^3 \times \left(\frac{2}{1}\right)^5 \qquad P_p = 127\,kW$$

Problem 7.58

[2]

7.58 A model test is performed to determine the flight characteristics of a Frisbee. Dependent parameters are drag force, F_D, and lift force, F_L. The independent parameters should include angular speed, ω, and roughness height, h. Determine suitable dimensionless parameters and express the functional dependence among them. The test (using air) on a $\frac{1}{4}$-scale model Frisbee is to be geometrically, kinematically, and dynamically similar to the prototype. The prototype values are $V_p = 5$ m/s and $\omega_p = 100$ rpm. What values of V_m and ω_m should be used?

Given: Model of Frisbee

Find: Dimensionless parameters; Model speed and angular speed

Solution:

The functional dependence is $F = F(D, V, \omega, h, \rho, \mu)$ where F represents lift or drag

From Buckingham Π $\frac{F}{\rho \cdot V^2 \cdot D^2} = f\left(\frac{\rho \cdot V \cdot D}{\mu}, \frac{\omega \cdot D}{V}, \frac{h}{D}\right)$

For dynamic similarity $\frac{\rho_m \cdot V_m \cdot D_m}{\mu_m} = \frac{\rho_p \cdot V_p \cdot D_p}{\mu_p}$ $\quad V_m = V_p \cdot \frac{\rho_p}{\rho_m} \cdot \frac{D_p}{D_m} \cdot \frac{\mu_m}{\mu_p}$ $\quad V_m = 5 \cdot \frac{m}{s} \times (1) \times \left(\frac{4}{1}\right) \times (1)$ $\quad V_m = 20 \frac{m}{s}$

Also $\frac{\omega_m \cdot D_m}{V_m} = \frac{\omega_p \cdot D_p}{V_p}$ $\quad \omega_m = \omega_p \cdot \frac{D_p}{D_m} \cdot \frac{V_m}{V_p}$ $\quad \omega_m = 100 \cdot rpm \times \left(\frac{4}{1}\right) \times \left(\frac{20}{5}\right)$ $\quad \omega_m = 1600\,rpm$

Given: Model of hydrofoil boat (1:20 scale) is to be tested in water at 130°F. Prototype operates at speed of 60 knots in water at 45°F.
To model cavitation correctly, cavitation number must be duplicated.

Find: ambient pressure at which model test must be run.

Solution:

To duplicate the Froude number between model and prototype requires

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}} \quad \text{or} \quad \frac{V_m}{V_p} = \left(\frac{L_m}{L_p}\right)^{1/2} = \frac{1}{\sqrt{20}}$$

$$\text{and } V_m = \frac{1}{\sqrt{20}} V_p = \frac{1}{\sqrt{20}} 60 \text{ knot} = 13.4 \text{ knot}$$

For $Ca_m = Ca_p$, then

$$\left.\frac{p - p_v}{\frac{1}{2}\rho V^2}\right)_m = \left.\frac{p - p_v}{\frac{1}{2}\rho V^2}\right)_p$$

or

$$p_m = p_{v_m} + (p - p_v)_p \frac{V_m^2}{V_p^2} \qquad (\text{assuming } \rho_m \approx \rho_p)$$

and

$$p_m = p_{v_m} + (p - p_v)_p \cdot \frac{1}{20}$$

From the Table A.7, at $T = 130°F$ $p_{v_m} = 2.23$ psia

$T = 45 F$ $p_{v_p} = 0.15$ psia

$$\therefore p_m = 2.23 \text{ psia} + (14.7 - 0.15) \text{ psia} \cdot \frac{1}{20}$$

$$p_m = 2.96 \text{ psia}$$

p_m

Problem 7.60

[2]

7.60 SAE 10W oil at 25°C flowing in a 25-mm diameter horizontal pipe, at an average speed of 1 m/s, produces a pressure drop of 450 kPa (gage) over a 150-m length. Water at 15°C flows through the same pipe under dynamically similar conditions. Using the results of Example 7.2, calculate the average speed of the water flow and the corresponding pressure drop.

Given: Oil flow in pipe and dynamically similar water flow

Find: Average water speed and pressure drop

Solution:

From Example 7.2
$$\frac{\Delta p}{\rho \cdot V^2} = f\left(\frac{\mu}{\rho \cdot V \cdot D}, \frac{l}{D}, \frac{e}{D}\right)$$

For dynamic similarity
$$\frac{\mu_{H2O}}{\rho_{H2O} \cdot V_{H2O} \cdot D_{H2O}} = \frac{\mu_{Oil}}{\rho_{Oil} \cdot V_{Oil} \cdot D_{Oil}}$$
so
$$V_{H2O} = \frac{\mu_{H2O}}{\rho_{H2O}} \cdot \frac{\rho_{Oil}}{\mu_{Oil}} \cdot V_{oil} = \frac{\nu_{H2O}}{\nu_{Oil}} \cdot V_{Oil}$$

From Fig. A.3 at 25ºC $\nu_{Oil} = 8 \times 10^{-5} \cdot \frac{m^2}{s}$ From Table A.8 at 15ºC $\nu_{H2O} = 1.14 \times 10^{-6} \cdot \frac{m^2}{s}$

Hence
$$V_{H2O} = \frac{1.14 \times 10^{-6} \cdot \frac{m^2}{s}}{8 \times 10^{-5} \cdot \frac{m^2}{s}} \times 1 \cdot \frac{m}{s}$$
$$V_{H2O} = 0.0142 \frac{m}{s}$$

Then
$$\frac{\Delta p_{Oil}}{\rho_{Oil} \cdot V_{Oil}^2} = \frac{\Delta p_{H2O}}{\rho_{H2O} \cdot V_{H2O}^2}$$
$$\Delta p_{H2O} = \frac{\rho_{H2O} \cdot V_{H2O}^2}{\rho_{Oil} \cdot V_{Oil}^2} \cdot \Delta p_{Oil}$$

From Table A.2 $SG_{Oil} = 0.92$

$$\Delta p_{H2O} = \frac{1}{0.92} \times \left(\frac{0.0142}{1}\right)^2 \times 450 \cdot kPa$$
$$\Delta p_{H2O} = 98.6 \cdot Pa$$

Problem 7.61

[3]

Given: The frequency, f, of vortex shedding from the rear of a bluff cylinder is a function of ρ, V, d, μ

Two cylinders in standard air, $\frac{d_1}{d_2} = 2$

Find: (a) functional relationship for f, using dimensional analysis
(b) V_1/V_2 for dynamic similarity
(c) f_1/f_2

Solution: Apply Buckingham π theorem.

① $f \quad \rho \quad V \quad d \quad \mu$ $\qquad n = 5$ parameters

② Select M, L, t as primary dimensions

③ $f \quad \rho \quad V \quad d \quad \mu$

$\frac{1}{t} \quad \frac{M}{L^3} \quad \frac{L}{t} \quad L \quad \frac{M}{Lt}$ $\qquad r = 3$ primary dimensions

④ ρ, V, d $\qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 2$ dimensionless groups will result

Setting up dimensional equations

$\pi_1 = \rho^a V^b d^c f$

$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{1}{t}$

Equating exponents,

M: $0 = a$

L: $0 = -3a + b + c \qquad c = 1$

t: $0 = -b - 1 \qquad \therefore b = -1$

$\therefore \pi_1 = \frac{fd}{V}$

$\pi_2 = \rho^a V^b d^c \mu$

$M^0L^0t^0 = \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b L^c \frac{M}{Lt}$

Equating exponents,

M: $0 = a + 1 \qquad \therefore a = -1$

L: $0 = -3a + b + c - 1 \qquad c = -1$

t: $0 = -b - 1 \qquad \therefore b = -1$

$\therefore \pi_2 = \frac{\mu}{\rho V d}$

⑥ Check using F, L, t dimensions

$\pi_1 = \frac{1}{t} \cdot L \cdot \frac{t}{L} = [1]$ ✓ $\qquad \pi_2 = \frac{Ft}{L^2} \cdot \frac{L^4}{Ft^2} \cdot \frac{t}{L} \cdot \frac{1}{L} = [1]$ ✓

$$\therefore \quad \frac{fd}{V} = g\left(\frac{\rho V d}{\mu}\right)$$ $\qquad \frac{fd}{V}$

To achieve dynamic similarity between geometrically similar flows, we must duplicate all but one of the dimensionless groups

$\left.\frac{\rho V d}{\mu}\right)_1 = \left.\frac{\rho V d}{\mu}\right)_2 \Rightarrow \frac{V_1}{V_2} = \frac{\rho_2}{\rho_1} \frac{\mu_1}{\mu_2} \frac{d_2}{d_1} = 1 \times 1 \times \frac{1}{2} = \frac{1}{2}$ $\qquad V_1/V_2$

If $\left.\frac{\rho V d}{\mu}\right)_1 = \left.\frac{\rho V d}{\mu}\right)_2$, then $\left.\frac{fd}{V}\right)_1 = \left.\frac{fd}{V}\right)_2$

and $\frac{f_1}{f_2} = \frac{V_1}{V_2} \frac{d_2}{d_1} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\qquad f_1/f_2$

Problem 7.62

Given: $\frac{1}{8}$-scale model of tractor-trailer rig tested in pressurized wind tunnel.

$W = 0.305$ m $V = 75.0$ m/s
$H = 0.476$ m $F_D = 128$ N
$L = 2.48$ m $\rho = 3.23$ kg/m^3

Find: (a) Aerodynamic drag coefficient of model.
(b) Compare Reynolds number for model with prototype at $V = 55$ mph.
(c) Aerodynamic drag on prototype at $V = 55$ mph, with headwind, $V_w = 10$ mph.

Solution: Defining equations: $F_D = C_D A \frac{1}{2}\rho V^2$; $Re = \frac{\rho V L}{\mu}$

Then $C_{D_m} = \dfrac{F_{D_m}}{\frac{1}{2}\rho_m V_m^2 A_m}$

Assume $A_m = W_m H_m = 0.305\ m \times 0.476\ m = 0.145\ m^2$

$$C_{D_m} = 2 \times 128\ N \times \frac{m^3}{3.23\ kg} \times \frac{s^2}{(75)^2 m^2} \times \frac{1}{0.145\ m^2} \times \frac{kg \cdot m}{N \cdot s^2} = 0.0972$$

C_{Dm}

$$\frac{Re_m}{Re_p} = \frac{\rho_m V_m L_m}{\mu_m} \times \frac{\mu_p}{\rho_p V_p L_p} = \frac{\rho_m}{\rho_p}\frac{V_m}{V_p}\frac{L_m}{L_p} \quad (\text{assume air: } \mu_m = \mu_p)$$

For the prototype, $V_p = 55\frac{mi}{hr} \times 5280\frac{ft}{mi} \times \frac{hr}{3600\ s} \times 0.305\frac{m}{ft} = 24.6$ m/s

$$\frac{Re_m}{Re_p} = \left(\frac{3.23}{1.23}\right)\left(\frac{75.0}{24.6}\right)\left(\frac{1}{8}\right) = 1.00 \quad \therefore Re_m = Re_p$$

Re

Since $Re_m = Re_p$, then $C_{D_p} = C_{D_m}$, assuming geometric and kinematic similarity, so

$$F_{D_p} = C_{D_p} A_p \frac{1}{2}\rho_p (V_p + V_w)^2$$

With $V_w = 10$ mph, $V_p + V_w = \frac{65}{55} \times 24.6\ m/s = 29.1\ m/s$

Thus

$$F_{D_p} = 0.0972 \times (8)^2\, 0.145\ m^2 \times \frac{1}{2} \times 1.23\ \frac{kg}{m^3}\,(29.1)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m}$$

$F_{D_p} = 470$ N

F_{Dp}

Problem 7.63

7.63 On a cruise ship, passengers complain about the amount of smoke that becomes entrained behind the cylindrical smoke stack. You have been hired to study the flow pattern around the stack, and have decided to use a 1:12.5 scale model of the 4.75 m smoke stack. What range of wind tunnel speeds could you use if the ship speed for which the problem occurs is 15 knots to 25 knots?

Given: Flow around cruise ship smoke stack

Find: Range of wind tunnel speeds

Solution:

For dynamic similarity $\frac{V_m \cdot D_m}{\nu_m} = \frac{V_p \cdot D_p}{\nu_m}$ or $V_m = \frac{D_p}{D_m} \cdot V_p = \frac{1}{12.5} \cdot V_p = 0.08 \cdot V_p$

From Wikipedia $1 \cdot \text{knot} = 1.852 \frac{\text{km}}{\text{hr}} = 1.852 \cdot \frac{\text{km}}{\text{hr}} \times \frac{1000 \cdot \text{m}}{\text{km}} \times \frac{1 \cdot \text{hr}}{3600 \cdot \text{s}} = 0.514 \cdot \frac{\text{m}}{\text{s}}$

Hence for $V_p = 15 \cdot \text{knot} = 15 \cdot \text{knot} \times \frac{0.514 \cdot \frac{\text{m}}{\text{s}}}{1 \cdot \text{knot}}$ $\quad V_p = 7.72 \cdot \frac{\text{m}}{\text{s}}$ $\quad V_m = 0.08 \times 7.72 \cdot \frac{\text{m}}{\text{s}}$ $\quad V_m = 0.618 \frac{\text{m}}{\text{s}}$

$V_p = 25 \cdot \text{knot} = 25 \cdot \text{knot} \times \frac{0.514 \cdot \frac{\text{m}}{\text{s}}}{1 \cdot \text{knot}}$ $\quad V_p = 12.86 \cdot \frac{\text{m}}{\text{s}}$ $\quad V_m = 0.08 \times 12.86 \cdot \frac{\text{m}}{\text{s}}$ $\quad V_m = 1.03 \frac{\text{m}}{\text{s}}$

Problem 7.64

[2]

7.64 The aerodynamic behavior of a flying insect is to be investigated in a wind tunnel using a ten-times scale model. If the insect flaps its wings 50 times a second when flying at 4 ft/s, determine the wind tunnel air speed and wing oscillation frequency required for dynamic similarity. Do you expect that this would be a successful or practical model for generating an easily measurable wing lift? If not, can you suggest a different fluid (e.g., water, or air at a different pressure and/or temperature) that would produce a better modeling?

Given: Model of flying insect

Find: Wind tunnel speed and wing frequency; select a better model fluid

Solution:

For dynamic similarity the following dimensionless groups must be the same in the insect and model (these are Reynolds number and Strouhal number, and can be obtained from a Buckingham Π analysis)

$$\frac{V_{insect} \cdot L_{insect}}{\nu_{air}} = \frac{V_m \cdot L_m}{\nu_m} \qquad \frac{\omega_{insect} \cdot L_{insect}}{V_{insect}} = \frac{\omega_m \cdot L_m}{V_m}$$

From Table A.9 (68ºF) $\rho_{air} = 0.00234 \cdot \frac{slug}{ft^3}$ $\quad \nu_{air} = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s}$

The given data is $\omega_{insect} = 50 \cdot Hz$ $\quad V_{insect} = 4 \cdot \frac{ft}{s}$ $\quad \frac{L_{insect}}{L_m} = \frac{1}{10}$

Hence in the wind tunnel $V_m = V_{insect} \cdot \frac{L_{insect}}{L_m} \cdot \frac{\nu_m}{\nu_{air}} = V_{insect} \cdot \frac{L_{insect}}{L_m} = 4 \cdot \frac{ft}{s} \times \frac{1}{10}$ $\quad V_m = 0.4 \cdot \frac{ft}{s}$

Also $\omega_m = \omega_{insect} \cdot \frac{V_m}{V_{insect}} \cdot \frac{L_{insect}}{L_m} = 50 \cdot Hz \times \frac{0.4}{4} \times \frac{1}{10}$ $\quad \omega_m = 0.5 \cdot Hz$

It is unlikely measurable wing lift can be measured at such a low wing frequency (unless the measured lift was averaged, using an integrator circuit). Maybe try hot air (200ºF) for the model

For hot air try $\nu_{hot} = 2.4 \times 10^{-4} \cdot \frac{ft^2}{s}$ instead of $\nu_{air} = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s}$

Hence $\frac{V_{insect} \cdot L_{insect}}{\nu_{air}} = \frac{V_m \cdot L_m}{\nu_{hot}}$ $\quad V_m = V_{insect} \cdot \frac{L_{insect}}{L_m} \cdot \frac{\nu_{hot}}{\nu_{air}} = 4 \cdot \frac{ft}{s} \times \frac{1}{10} \times \frac{2.4 \times 10^{-4}}{1.62 \times 10^{-4}}$ $\quad V_m = 0.593 \cdot \frac{ft}{s}$

Also $\omega_m = \omega_{insect} \cdot \frac{V_m}{V_{insect}} \cdot \frac{L_{insect}}{L_m} = 50 \cdot Hz \times \frac{0.593}{4} \times \frac{1}{10}$ $\quad \omega_m = 0.741 \cdot Hz$

Hot air does not improve things much. Try modeling in wa $\nu_w = 1.08 \times 10^{-5} \cdot \frac{ft^2}{s}$

Hence $\frac{V_{insect} \cdot L_{insect}}{\nu_{air}} = \frac{V_m \cdot L_m}{\nu_w}$ $\quad V_m = V_{insect} \cdot \frac{L_{insect}}{L_m} \cdot \frac{\nu_w}{\nu_{air}} = 4 \cdot \frac{ft}{s} \times \frac{1}{10} \times \frac{1.08 \times 10^{-5}}{1.62 \times 10^{-4}}$ $\quad V_m = 0.0267 \cdot \frac{ft}{s}$

Also $\omega_m = \omega_{insect} \cdot \frac{V_m}{V_{insect}} \cdot \frac{L_{insect}}{L_m} = \omega_{insect} \cdot \frac{V_m}{V_{insect}} \cdot L_{ratio} = 50 \cdot Hz \times \frac{0.0267}{4} \times \frac{1}{10}$ $\quad \omega_m = 0.033 \cdot Hz$

This is even worse! It seems the best bet is hot (very hot) air for the wind tunnel. Alternatively, choose a much smaller wind tunnel model, e.g., a 2.5 X model would lead to V_m = 1.6 ft/s and ω_m = 8 Hz

Problem 7.65

Given: Model test of tractor-trailer rig in standard air.

$F_D = f(A, V, \rho, \mu)$; scale is 1:4 ; $A_m = 0.625\ m^2$

At $V_m = 89.6$ m/s, $F_D = 2.46$ kN

Find: (a) Dimensionless parameters.
(b) Conditions for dynamic similarity.
(c) Drag force on prototype at $V_p = 22.4$ m/s (no wind).
(d) Power to overcome aero drag.

Solution: ① $F_D \quad A \quad V \quad \rho \quad \mu$ | ② $M\,L\,t$

③ $\frac{ML}{t^2} \quad L^2 \quad \frac{L}{t} \quad \frac{M}{L^3} \quad \frac{M}{Lt}$ | ④ $\rho V A$

⑤ $\Pi_1 = \rho^a V^b A^c F_D = M^0 L^0 t^0$ $\qquad$ $\Pi_2 = \rho^a V^b A^c \mu$

M: $a + 1 = 0$	$a = -1$	M: $a + 1 = 0$	$a = -1$
L: $-3a + b + 2c + 1 = 0$	$c = -1$	L: $-3a + b + 2c - 1 = 0$	$c = -\frac{1}{2}$
t: $-b - 2 = 0$	$b = -2$	t: $-b - 1 = 0$	$b = -1$

$$\Pi_1 = \frac{F_D}{\rho V^2 A} \qquad \Pi_2 = \frac{\mu}{\rho V A^{1/2}}$$ Π_1, Π_2

⑥ $\Pi_1 = F \times \frac{L^4}{Ft^2} \times \frac{t^2}{L^2} \times \frac{1}{L^2} = 1$ ✓✓ $\qquad$ $\Pi_2 = \frac{Ft}{L^2} \times \frac{L^4}{Ft^2} \times \frac{t}{L} \times \frac{1}{L} = 1$ ✓✓

For dynamic similarity, must have geometric and kinematic similarity and $Re_m = Re_p$. Then $\left.\frac{F_D}{\rho V^2 A}\right)_m = \left.\frac{F_D}{\rho V^2 A}\right)_p$

For the prototype,

$$F_{D_p} = F_{D_m} \frac{\rho_p}{\rho_m} \left(\frac{V_p}{V_m}\right)^2 \frac{A_p}{A_m} = F_{D_m} \left(\frac{1.23}{1.23}\right)\left(\frac{22.4}{89.6}\right)^2 (4)^2 = F_{D_m} = 2.46\ \text{kN}$$ F_{D_p}

The power requirement is

$$P = F_{D_p} V_p = 2.46\ \text{kN} \times 22.4\ \frac{m}{s} \times \frac{W \cdot s}{N \cdot m} = 55.1\ \text{kW}\ (73.9\ \text{hp})$$ P

Problem 7.66

[2]

7.66 Tests are performed on a 1:5 scale boat model. What must be the kinematic viscosity of the model fluid if friction and wave drag phenomena are to be correctly modeled? The full size boat will be used in a freshwater lake where the average water temperature is 10°C.

Given: Model of boat

Find: Model kinematic viscosity for dynamic similarity

Solution:

For dynamic similarity $$\frac{V_m \cdot L_m}{\nu_m} = \frac{V_p \cdot L_p}{\nu_p} \quad (1) \qquad \frac{V_m}{\sqrt{g \cdot L_m}} = \frac{V_p}{\sqrt{g \cdot L_p}} \quad (2)$$

(from Buckingham Π; the first is the Reynolds number, the second the Froude number)

Hence from Eq 2 $$\frac{V_m}{V_p} = \sqrt{\frac{g \cdot L_m}{g \cdot L_p}} = \sqrt{\frac{L_m}{L_p}}$$

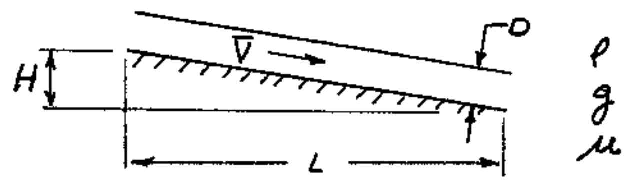
Using this in Eq 1 $$\nu_m = \nu_p \cdot \frac{V_m}{V_p} \cdot \frac{L_m}{L_p} = \nu_p \cdot \sqrt{\frac{L_m}{L_p}} \cdot \frac{L_m}{L_p} = \nu_p \cdot \left(\frac{L_m}{L_p}\right)^{\frac{3}{2}}$$

From Table A.8 at 10°C $$\nu_p = 1.3 \times 10^{-6} \cdot \frac{m^2}{s} \qquad \nu_m = 1.3 \times 10^{-6} \cdot \frac{m^2}{s} \times \left(\frac{1}{5}\right)^{\frac{3}{2}} \qquad \nu_m = 1.16 \times 10^{-7} \frac{m^2}{s}$$

Problem 7.67

Given: Model glacier using glycerine. Assume ice is Newtonian and 10^6 x as viscous.

$$\left.\begin{aligned} D &= 15\ m \\ H &= 1.5\ m \\ L &= 1850\ m \end{aligned}\right\} \text{model}$$



In lab test, model instructor reappears in $\tau = 9.6$ hr.

Find: (a) Develop suitable dimensionless parameters.
(b) Estimate time when instructor will reappear.

Solution: ① $\bar{V}$ $\quad \rho \quad g \quad \mu \quad D \quad H \quad L \qquad n = 7$

② MLt $\quad \frac{L}{t} \quad \frac{M}{L^3} \quad \frac{L}{t^2} \quad \frac{M}{Lt} \quad L \quad L \quad L \qquad m = r = 3$

④ Choose ρ, g, D as repeating variables: $n - m = 7 - 3 = 4$ parameters

⑤ $\Pi_1 = \rho^a g^b D^c \bar{V} = M^0 L^0 t^0$

M: $a + 0 = 0$ | $a = 0$
L: $-3a + b + c + 1 = 0$ | $c = -b - 1 = -\frac{1}{2}$
t: $-2b - 1 = 0$ | $b = -\frac{1}{2}$

$$\Pi_1 = \frac{\bar{V}}{\sqrt{gD}} \quad \text{(Froude no.)}$$

$\Pi_2 = \rho^a g^b D^c \mu = M^0 L^0 t^0$

M: $a + 1 = 0$ $\quad a = -1$
L: $-3a + b + c - 1 = 0$ $\quad c = 3a - b + 1 = -\frac{3}{2}$
t: $-2b - 1 = 0$ $\quad b = -\frac{1}{2}$

$$\Pi_2 = \frac{\mu}{\rho g^{1/2} D^{3/2}} \sim \frac{\mu}{\rho \sqrt{gD}\, D} \quad \text{(Reynolds no)}$$

$$\Pi_3 = \frac{H}{D}, \quad \Pi_4 = \frac{L}{D} \quad \text{(by inspection)}$$

⑥ Check: obvious from forms above. $\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4)$ ← geometric similarity

For dynamic similarity, $\Pi_{2m} = \Pi_{2p} = \dfrac{\mu_m}{\rho_m g_m^{1/2} D_m^{3/2}} = \dfrac{\mu_p}{\rho_p g_p^{1/2} D_p^{3/2}}$, so

$$\frac{D_m}{D_p} = \left(\frac{\mu_m}{\mu_p}\frac{\rho_p}{\rho_m}\right)^{2/3} = \left(\frac{\mu_m}{\mu_p}\frac{SG_p}{SG_m}\right)^{2/3} = \left(\frac{1}{10^6} \times \frac{0.92}{1.26}\right)^{2/3} = 8.11 \times 10^{-5}$$

{ $SG_{ice} = 0.92$ (Table A.1) ; $SG_{glycerin} = 1.26$ (A.2) }

So $\frac{L_m}{L_p} = 8.11 \times 10^{-5}$; $L_m = 8.11 \times 10^{-5} L_p = 8.11 \times 10^{-5} \times 1850\ m = 0.150\ m$

From Π_1, $\dfrac{\bar{V}_m}{\bar{V}_p} = \sqrt{\dfrac{D_m}{D_p}} = 9.00 \times 10^{-3}$

The time to reappear is $\tau = L/\bar{V}$, so $\tau_p = L_p/\bar{V}_p$, $\tau_m = L_m/\bar{V}_m$

$$\frac{\tau_p}{\tau_m} = \frac{L_p}{L_m}\frac{\bar{V}_m}{\bar{V}_p} = \frac{D_p}{D_m}\sqrt{\frac{D_m}{D_p}} = \sqrt{\frac{D_p}{D_m}} = \frac{1}{9.00 \times 10^{-3}} = 111$$

Thus $\tau_p = 111\, \tau_m = 111 \times 9.6\ hr = 1070\ hr$ (~45 days) $\qquad \tau_p$

{ The instructor will reappear before the semester ends! }

Problem 7.68

7.68 An automobile is to travel through standard air at 60 mph. To determine the pressure distribution, a $\frac{1}{5}$-scale model is to be tested in water. What factors must be considered to ensure kinematic similarity in the tests? Determine the water speed that should be used. What is the corresponding ratio of drag force between prototype and model flows? The lowest pressure coefficient is $C_p = -1.4$ at the location of the minimum static pressure on the surface. Estimate the minimum tunnel pressure required to avoid cavitation, if the onset of cavitation occurs at a cavitation number of 0.5.

Given: Model of automobile

Find: Factors for kinematic similarity; Model speed; ratio of protype and model drags; minimum pressure for no cavitation

Solution:

For dynamic similarity $$\frac{\rho_m \cdot V_m \cdot L_m}{\mu_m} = \frac{\rho_p \cdot V_p \cdot L_p}{\mu_p} \qquad V_m = V_p \cdot \frac{\rho_p}{\rho_m} \cdot \frac{L_p}{L_m} \cdot \frac{\mu_m}{\mu_p}$$

For air (Table A.9) and water (Table A.7) at 68ºF

$$\rho_p = 0.00234 \cdot \frac{\text{slug}}{\text{ft}^3} \qquad \mu_p = 3.79 \times 10^{-7} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$$

$$\rho_m = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3} \qquad \mu_m = 2.10 \times 10^{-5} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$$

$$V_m = 60 \cdot \text{mph} \times \frac{88 \cdot \frac{\text{ft}}{\text{s}}}{60 \cdot \text{mph}} \times \left(\frac{0.00234}{1.94}\right) \times \left(\frac{5}{1}\right) \times \left(\frac{2.10 \times 10^{-5}}{3.79 \times 10^{-7}}\right) \qquad V_m = 29.4 \cdot \frac{\text{ft}}{\text{s}}$$

Then $$\frac{F_m}{\rho_m \cdot V_m^2 \cdot L_m^2} = \frac{F_p}{\rho_p \cdot V_p^2 \cdot L_p^2}$$

Hence $$\frac{F_p}{F_m} = \frac{\rho_p \cdot V_p^2 \cdot L_p^2}{\rho_m \cdot V_m^2 \cdot L_m^2} = \left(\frac{0.00234}{1.94}\right) \times \left(\frac{88}{29.4}\right)^2 \times \left(\frac{5}{1}\right)^2 \qquad \frac{F_p}{F_m} = 0.270$$

For Ca = 0.5 $$\frac{p_{min} - p_v}{\frac{1}{2} \cdot \rho \cdot V^2} = 0.5 \qquad \text{so we get} \qquad p_{min} = p_v + \frac{1}{4} \cdot \rho \cdot V^2 \qquad \text{for the water tank}$$

From steam tables, for water at 68ºF $p_v = 0.339 \cdot \text{psi}$ so

$$p_{min} = 0.339 \cdot \text{psi} + \frac{1}{4} \times 1.94 \cdot \frac{\text{slug}}{\text{ft}^3} \times \left(29.4 \cdot \frac{\text{ft}}{\text{s}}\right)^2 \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \times \left(\frac{1 \cdot \text{ft}}{12 \cdot \text{in}}\right)^2 \qquad p_{min} = 3.25 \, \text{psi}$$

This is the minimum allowable pressure in the water tank; we can use it to find the required tank pressure

$$C_p = -1.4 = \frac{p_{min} - p_{tank}}{\frac{1}{2} \cdot \rho \cdot V^2} \qquad p_{tank} = p_{min} + \frac{1.4}{2} \cdot \rho \cdot V^2 = p_{min} + 0.7 \cdot \rho \cdot V^2$$

$$p_{tank} = 3.25 \cdot \text{psi} + 0.7 \times 1.94 \cdot \frac{\text{slug}}{\text{ft}^3} \times \left(29.4 \cdot \frac{\text{ft}}{\text{s}}\right)^2 \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \times \left(\frac{1 \cdot \text{ft}}{12 \cdot \text{in}}\right)^2 \qquad p_{tank} = 11.4 \, \text{psi}$$

Given: Submarine model (1:30 scale) to be tested in fresh water under two conditions:
(1) on the surface at 20 kt (prototype)
(2) far below the surface at 0.5 kt (prototype)

Find: (a) Speed for model test on surface
(b) Speed for model test submerged
(c) Ratio of full-scale to model drag force.

Solution: On the surface, match the Froude number, $Fr = \frac{V}{\sqrt{gL}}$

Thus $Fr_m = \frac{V_m}{\sqrt{gL_m}} = Fr_p = \frac{V_p}{\sqrt{gL_p}}$ or $V_m = V_p\sqrt{\frac{L_m}{L_p}}$

For 1:30 scale,

$$V_m = 20\,kt\sqrt{\frac{1}{30}} = 3.65\,kt$$

$$V_m = 3.65\,\frac{nm}{hr} \times 1852\,\frac{m}{nm} \times \frac{hr}{3600\,s} = 1.88\,m/s$$ V_m

Submerged, match the Reynolds number, $Re = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$

Thus $Re_m = \frac{V_m L_m}{\nu_m} = Re_p = \frac{V_p L_p}{\nu_p}$ or $V_m = V_p \frac{L_p}{L_m}\frac{\nu_m}{\nu_p}$

From Table A.2, for seawater, SG = 1.025 and $\mu = 1.08 \times 10^{-3}\,N\cdot s/m^2$ at 20°C, Thus

$$\nu_p = \frac{\mu_p}{\rho_p} = \frac{\mu_p}{SG\,\rho_{H_2O}} = 1.08\times10^{-3}\,\frac{N\cdot s}{m^2} \times \frac{m^3}{(1.025)\,1000\,kg} \times \frac{kg\cdot m}{N\cdot s^2} = 1.05\times10^{-6}\,\frac{m^2}{s}$$

From Table A.8, fresh water at 20°C has $\nu = 1.00\times10^{-6}\,m^2/s$.

For 1:30 scale

$$V_m = 0.5\,kt \times \frac{30}{1} \times 1.00\times10^{-6}\,\frac{m^2}{s} \times \frac{s}{1.05\times10^{-6}\,m^2} = 14.3\,kt$$

$$V_m = 14.3\,\frac{nm}{hr} \times 1852\,\frac{m}{nm} \times \frac{hr}{3600\,s} = 7.36\,m/s$$ V_m

Under dynamically similar conditions the drag coefficients, $C_D = \frac{F_D}{\rho V^2 A}$, will be identical. Thus

$$\frac{F_p}{\rho_p V_p^2 L_p^2} = \frac{F_m}{\rho_m V_m^2 L_m^2} \quad \text{or} \quad F_p = F_m \frac{\rho_p}{\rho_m}\frac{V_p^2}{V_m^2}\frac{L_p^2}{L_m^2}$$

$$F_p = F_m \frac{1.025}{0.999} \times \left(\frac{0.5}{14.3}\right)^2\left(\frac{30}{1}\right)^2 = 1.13 \text{ (submerged)},\ 2.77\times10^4 \text{ (surface)}$$ F_p/F_m

Problem 7.70

Given: The drag force on a circular cylinder immersed in a water flow can be expressed as

$$F_D = f(D, l, V, \rho, \mu)$$

The static pressure distribution on a circular cylinder can be expressed in terms of the dimensionless pressure coefficient

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho V^2}$$

At the location of minimum static pressure on the cylinder surface, $C_p = -2.4$. The onset of cavitation occurs at $Ca = 0.5$

Find: (a) expression for dimensionless drag force
(b) an estimate of maximum speed V at which cylinder could be towed in water (at p_{atm}) without causing cavitation

Solution:

$F_D = f(D, l, V, \rho, \mu)$. From the Buckingham π-theorem, for $n = 6$, with $m = r = 3$, we would expect three dimensionless groups.

$$\frac{F}{\rho V^2 D^2} = f_1\left(\frac{l}{D}, \frac{\rho V D}{\mu}\right)$$

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho V^2} \qquad Ca = \frac{p - p_v}{\frac{1}{2}\rho V^2}$$

For $C_{p_{min}} = -2.4$, $p_{min} - p_\infty = \frac{1}{2}\rho V_{max}^2 (C_{p_{min}})$ $\therefore p_{min} = p_\infty + \frac{1}{2}\rho V_{max}^2 C_{p_{min}}$

For $Ca = \frac{1}{2}$, $p_{min} - p_v = \frac{1}{2}\rho V_{max}^2\, Ca$ $\therefore p_{min} = p_v + \frac{1}{2}\rho V_{max}^2\, Ca$

Equating expressions for p_{min},

$$p_\infty + \frac{1}{2}\rho V_{max}^2 C_{p_{min}} = p_v + \frac{1}{2}\rho V_{max}^2\, Ca$$

$$\frac{1}{2}\rho V_{max}^2 \left[Ca - C_{p_{min}}\right] = p_\infty - p_v$$

$$V_{max} = \left\{\frac{2(p_\infty - p_v)}{\rho\left[Ca - C_{p_{min}}\right]}\right\}^{1/2}$$

For water at 68°F (Table A.7), $p_v = 0.339$ psia

$$\therefore V_{max} = \left\{2 \times (14.7 - 0.339)\frac{\text{lbf}}{\text{in}^2} \times \frac{\text{ft}^3}{1.94\,\text{slug}} \times \frac{1}{[0.5 - (-2.4)]} \times \frac{144\,\text{in}^2}{\text{ft}^2} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right\}^{1/2}$$

$V_{max} = 27.1$ ft/s (8.26 m/s) ←

Problem 7.71

[3]

7.71 A $\frac{1}{10}$-scale model of a tractor-trailer rig is tested in a wind tunnel. The model frontal area is $A_m = 0.1\ \text{m}^2$. When tested at $V_m = 75$ m/s in standard air, the measured drag force is $F_D = 350$ N. Evaluate the drag coefficient for the model conditions given. Assuming that the drag coefficient is the same for model and prototype, calculate the drag force on a prototype rig at a highway speed of 90 km/hr. Determine the air speed at which a model should be tested to ensure dynamically similar results if the prototype speed is 90 km/hr. Is this air speed practical? Why or why not?

Given:
Model of tractor-trailer truck

Find:
Drag coefficient; Drag on prototype; Model speed for dynamic similarity

Solution:

For kinematic similarity we need to ensure the geometries of model and prototype are similar, as is the incoming flow field

The drag coefficient is
$$C_D = \frac{F_m}{\frac{1}{2}\cdot\rho_m\cdot V_m^{\,2}\cdot A_m}$$

For air (Table A.10) at 20ºC
$$\rho_m = 1.21\cdot\frac{kg}{m^3} \qquad \mu_p = 1.81\times10^{-5}\cdot\frac{N\cdot s}{m^2}$$

$$C_D = 2\times350\cdot N\times\frac{m^3}{1.21\cdot kg}\times\left(\frac{s}{75\cdot m}\right)^2\times\frac{1}{0.1\cdot m^2}\times\frac{N\cdot s^2}{kg\cdot m} \qquad C_D = 1.028$$

This is the drag coefficient for model and prototype

For the rig
$$F_p = \frac{1}{2}\cdot\rho_p\cdot V_p^{\,2}\cdot A_p\cdot C_D \quad \text{with} \quad \frac{A_p}{A_m} = \left(\frac{L_p}{L_m}\right)^2 = 100 \qquad A_p = 10\cdot m^2$$

$$F_p = \frac{1}{2}\times1.21\cdot\frac{kg}{m^3}\times\left(90\cdot\frac{km}{hr}\times\frac{1000\cdot m}{1\cdot km}\times\frac{1\cdot hr}{3600\cdot s}\right)^2\times10\cdot m^2\times1.028\times\frac{N\cdot s^2}{kg\cdot m} \qquad F_p = 3.89\,kN$$

For dynamic similarity
$$\frac{\rho_m\cdot V_m\cdot L_m}{\mu_m} = \frac{\rho_p\cdot V_p\cdot L_p}{\mu_p} \qquad V_m = V_p\cdot\frac{\rho_p}{\rho_m}\cdot\frac{L_p}{L_m}\cdot\frac{\mu_m}{\mu_p} = V_p\cdot\frac{L_p}{L_m}$$

$$V_m = 90\cdot\frac{km}{hr}\times\frac{1000\cdot m}{1\cdot km}\times\frac{1\cdot hr}{3600\cdot s}\times\frac{10}{1} \qquad V_m = 250\,\frac{m}{s}$$

For air at standard conditions, the speed of sound is $c = \sqrt{k\cdot R\cdot T}$

$$c = \sqrt{1.40\times286.9\cdot\frac{N\cdot m}{kg\cdot K}\times(20+273)\cdot K\times\frac{kg\cdot m}{s^2\cdot N}} \qquad c = 343\,\frac{m}{s}$$

Hence we have
$$M = \frac{V_m}{c} = \frac{250}{343} = 0.729$$
which indicates compressibility is significant - this model speed is impractical (and unnecessary)

Problem 7.72

Given: Circular container partially filled with water is rotated about its axis at constant angular velocity, ω.

The velocity V_θ is a function of: location, r, time from start, τ, angular velocity, ω, density, ρ, and viscosity, μ.

Water is replaced with honey and cylinder is rotated at the same value of ω.

Find: (a) dimensionless parameters that characterize the problem.
(b) Determine whether honey will attain steady state motion as quickly as water.
(c) Explain why Re would not be an important parameter in scaling the steady state motion of the liquid.

Solution:

$$V_\theta = V_\theta(\omega, r, \tau, \rho, \mu)$$

From the Buckingham π-theorem, for $n=6$ and $m=r=3$, we would expect three dimensionless groups.

$$\frac{V_\theta}{\omega r} = f\left(\frac{\mu}{\rho \omega r^2}, \omega \tau\right)$$

$\frac{V_\theta}{\omega r}$

From the above results $\pi_2 = \frac{\mu}{\rho \omega r^2}$ contains the fluid properties ρ, μ.

$\pi_3 = \omega \tau$ contains the time τ

$$\pi_2 \pi_3 = \frac{\mu}{\rho \omega r^2}\omega \tau = \frac{\mu \tau}{\rho r^2} = \frac{\nu \tau}{r^2} \qquad \text{where } \nu = \mu/\rho$$

For steady flow at the same radius

$$\left.\frac{\nu \tau}{r^2}\right)_{Honey} = \left.\frac{\nu \tau}{r^2}\right)_{water}$$

$$\therefore \tau_H = \frac{\nu_{water}}{\nu_{Honey}} \tau_{water}$$

Since $\nu_{Honey} > \nu_{water}$ ($\mu_{Honey} > \mu_{water}$ and $\rho_H \approx \rho_w$)

$$\tau_H < \tau_{water}$$

τ_H

At steady state conditions, we have solid body rotation there are no viscous forces. Hence Re is not important.

Problem 7.73

Given: Recommended procedures for wind tunnel tests of trucks & buses suggest:

$A_{model} / A_{test\ section} < 0.05$

$h_{model} / h_{test\ section} < 0.30$ (h = height)

w_{model} at max yaw ($20°$) $/ w_{test\ sect} < 0.30$ (w = projected width)

$V_{max} < 300$ ft/s

Wind tunnel test section is $h = 1.5$ ft, $w = 2$ ft.

Prototype has: $h = 13.5$ ft, $w = 8$ ft, Length = 65 ft.

Find: (a) scale ratio of largest model that meets the recommended criteria.

(b) Use results of Ex. Prob 7.5 to assess whether an adequate value of Re can be achieved in the test facility.

Solution:

Let s = scale ratio. Then $h_m = s h_p$, $w_m = s w_p$, $l_m = s l_p$

(1) height criteria

$$h_m = 0.30\, h_{test\ section} = 0.3(1.5\text{ ft}) = 0.45\text{ ft}$$

$$s = \frac{h_m}{h_p} = \frac{0.45\text{ ft}}{13.5\text{ ft}} = 0.0333 \qquad \left\{ \frac{1}{s} = 30 \right\}$$

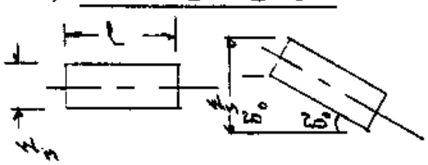
(2) frontal area criteria

$$A_{model} = 0.05\, A_{test\ sect} = 0.05 \times 1.5\text{ ft} \times 2\text{ ft} = 0.15\text{ ft}^2$$

$$A_{model} = s^2 A_p = s^2 [13.5\text{ ft} \times 8\text{ ft}] = s^2 (108)\text{ ft}^2 = 0.15$$

$$\therefore s = \left(\frac{0.15}{108}\right)^{1/2} = 0.0373 \qquad \left\{ \frac{1}{s} = 26.8 \right\}$$

(3) width criteria



$$w_{m_{20°}} = l_m \sin 20° + w_m \cos 20°$$

$$= s(l_p \sin 20° + w_p \cos 20°)$$

$$w_{m_{20°}} = s[65 \sin 20° + 8 \cos 20°]\text{ ft} = 29.7\, s\text{ ft}$$

From constraint, $w_{m_{20°}} = 0.30\, w_{test\ sect} = 0.30(2\text{ ft}) = 0.6\text{ ft}$

$\therefore 0.6\text{ ft} = 29.7 s\text{ ft}$ and $s = 0.0202$ $\left\{ \frac{1}{s} = 49.5 \right\}$

The width criteria is the most stringent $\therefore s = \frac{1}{50}$

Model $= \frac{1}{50}$ Prototype ←

From Ex. Prob 7.5, C_D = const for $Re > 4 \times 10^5$

with $Re = \frac{\rho V w}{\mu} = \frac{V w}{\nu}$ Standard air $\nu = 1.57 \times 10^{-4}$ ft²/s

For current model test, $Re = 300\frac{\text{ft}}{\text{sec}} \times \left(\frac{1}{50} \times 8\text{ ft}\right) \times \frac{\text{s}}{1.57 \times 10^{-4}\text{ ft}^2} = 3.06 \times 10^5$

$\therefore$ Adequate Re cannot be achieved

Problem 7.74

Given: Power, P, to drive a fan depends on ρ, Q, D, and ω.

Condition	D (mm)	Q (m^3/s)	ω (rpm)
1	200	0.4	2400
2	400	?	1850

Find: Volume flow rate at Condition 2, for dynamic similarity.

Solution: step ① $P \quad \rho \quad Q \quad D \quad \omega$

step ② MLt ③: $\frac{ML^2}{t^3} \quad \frac{M}{L^3} \quad \frac{L^3}{t} \quad L \quad \frac{1}{t}$ ④ ρ, ω, D

⑤ $\Pi_1 = \rho^a \omega^b D^c P = M^0 L^0 t^0$ | $\Pi_2 = \rho^a \omega^b D^c Q = M^0 L^0 t^0$

$M: a + 1 = 0 \quad | \quad a = -1$

$L: -3a + c + 2 = 0 \quad | \quad c = 3a - 2 = -5$

$t: -b - 3 = 0 \quad | \quad b = -3$

$$\Pi_1 = \frac{P}{\rho \omega^3 D^5}$$

$M: a + 0 = 0 \quad | \quad a = 0$

$L: -3a + c + 3 = 0 \quad | \quad c = -3$

$t: -b - 1 = 0 \quad | \quad b = -1$

$$\Pi_2 = \frac{Q}{\omega D^3}$$

⑥ $\Pi_1 = \frac{FL}{t} \times \frac{L^4}{Ft^2} \times t^3 \times \frac{1}{L^5} = \frac{FL^5t^3}{FL^5t^3} = 1$ ✓✓ $\quad \Pi_2 = \frac{L^3}{t} \times t \times \frac{1}{L^3} = \frac{L^3 t}{L^3 t} = 1$ ✓✓

Thus $\Pi_1 = f(\Pi_2)$ or $\frac{P}{\rho \omega^3 D^5} = f\left(\frac{Q}{\omega D^3}\right)$

For dynamic similarity, need geometric and kinematic similarity and

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}$$

Thus

$$Q_2 = Q_1 \frac{\omega_2}{\omega_1}\left(\frac{D_2}{D_1}\right)^3 = 0.4\ m^3/s\ \frac{1850\ rpm}{2400\ rpm}\left(\frac{200\ mm}{400\ mm}\right)^3 = 2.47\ m^3/s$$

Q_2

Problem 7.75 (In Excel)

[3]

7.75 Over a certain range of air speeds, V, the lift, F_L, produced by a model of a complete aircraft in a wind tunnel depends on the air speed, air density, ρ, and a characteristic length (the wing base chord length, $c = 150$ mm). The following experimental data is obtained for air at standard atmospheric conditions:

V (m/s)	10	15	20	25	30	35	40	45	50
F_L (N)	2.2	4.8	8.7	13.3	19.6	26.5	34.5	43.8	54

Plot the lift versus speed curve. By using *Excel* to perform a trend-line analysis on this curve, generate and plot data for the lift produced by the prototype, which has a wing base chord length of 5 m, over a speed range of 75 m/s to 250 m/s.

Given: Data on model of aircraft

Find: Plot of lift vs speed of model; also of prototype

Solution:

V_m (m/s)	10	15	20	25	30	35	40	45	50
F_m (N)	2.2	4.8	8.7	13.3	19.6	26.5	34.5	43.8	54.0

This data can be fit to

$$F_m = \frac{1}{2}\cdot\rho\cdot A_m\cdot C_D\cdot V_m^2 \qquad \text{or} \qquad F_m = k_m\cdot V_m^2$$

From the trendline, we see that

$k_m = 0.0219$ N/(m/s)^2

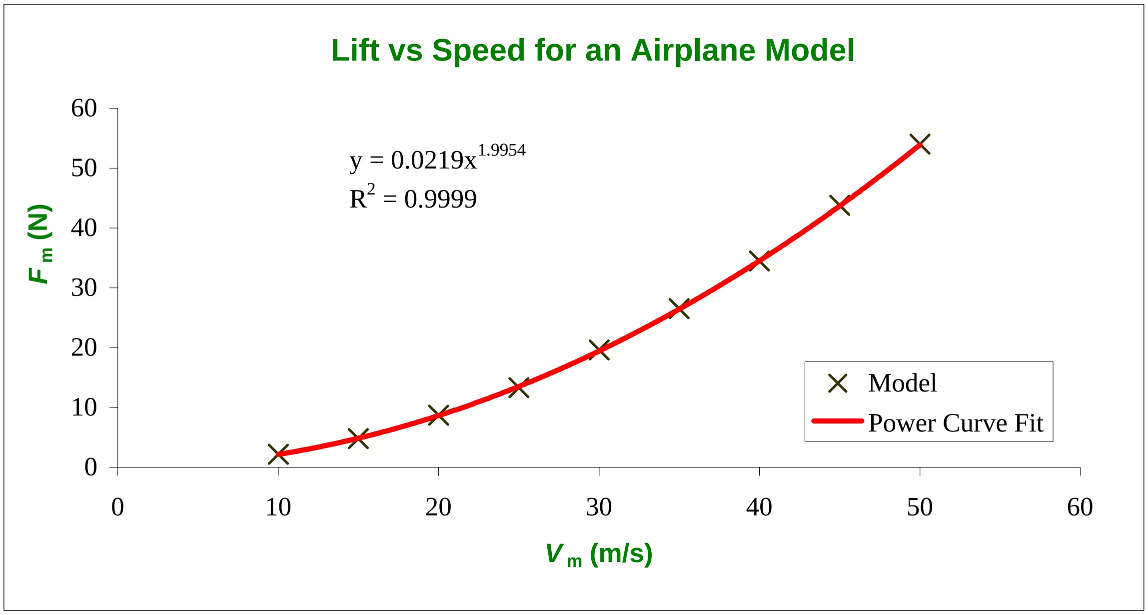
(And note that the power is 1.9954 or 2.00 to three signifcant figures, confirming the relation is quadratic)

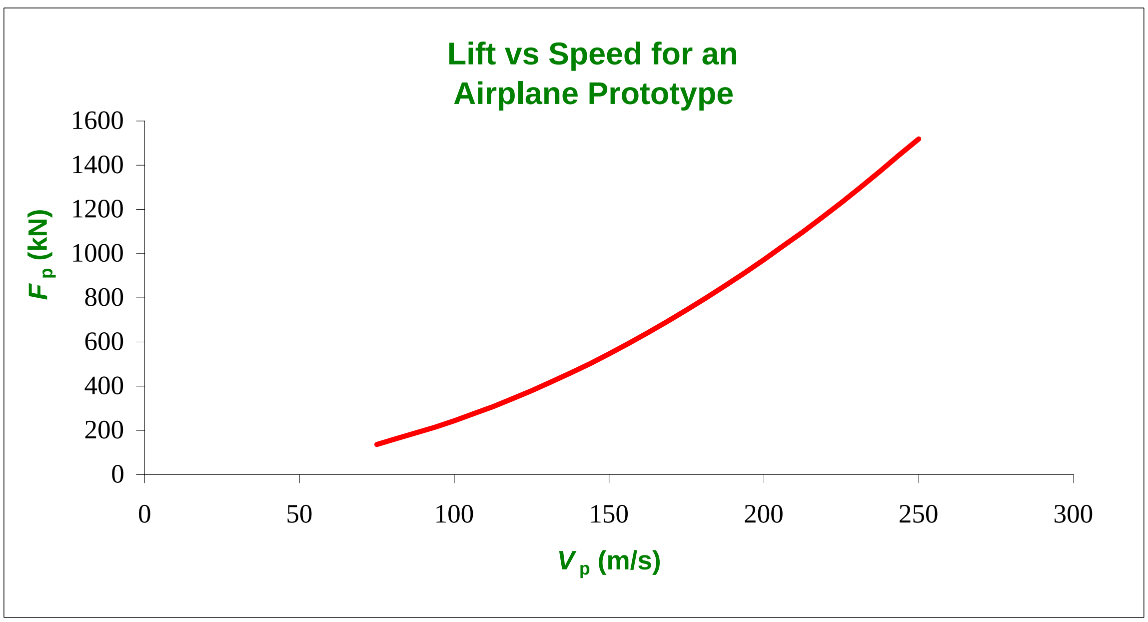
Also, $k_p = 1110\ k_m$

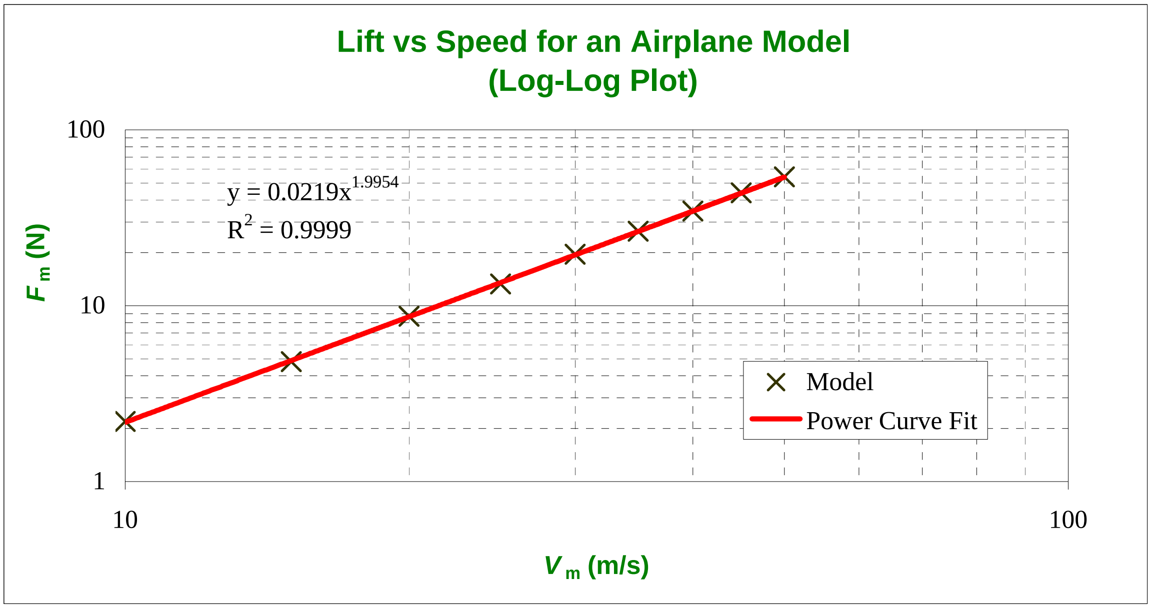
Hence,

$k_p = 24.3$ N/(m/s)^2 $\qquad F_p = k_p V_m^2$

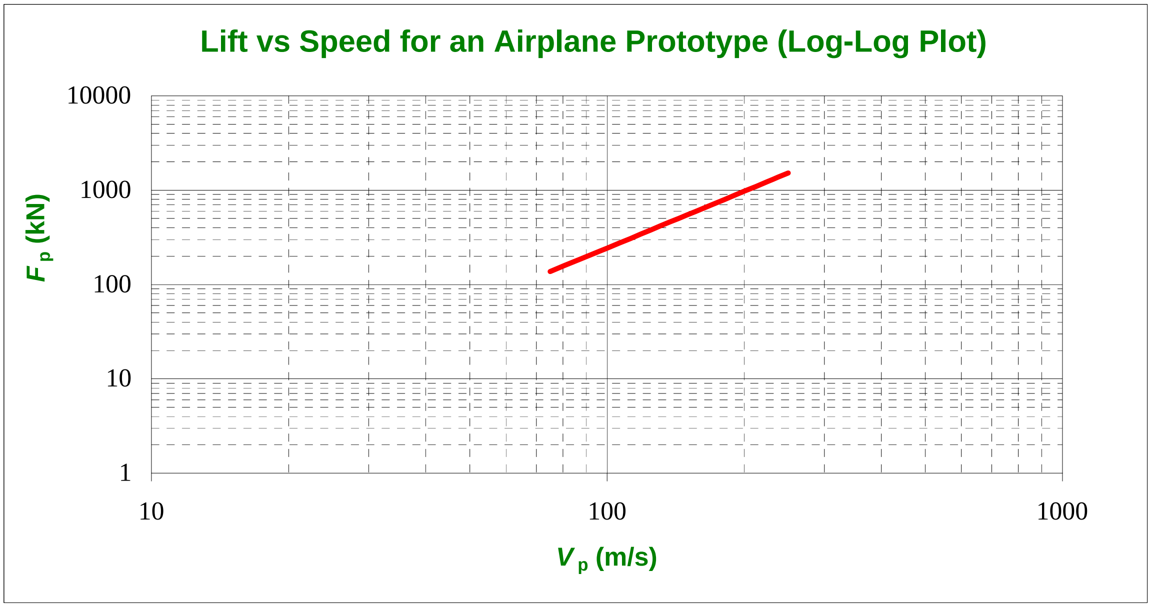
V_p (m/s)	75	100	125	150	175	200	225	250
F_p (kN) (Trendline)	137	243	380	547	744	972	1231	1519


Lift vs Speed for an Airplane Model
y = 0.0219x^1.9954
R^2 = 0.9999
F m (N)
V m (m/s)
0
10
20
30
40
50
60
Model
Power Curve Fit


Lift vs Speed for an
Airplane Prototype
F p (kN)
V p (m/s)
0
200
400
600
800
1000
1200
1400
1600
0
50
100
150
200
250
300



Lift vs Speed for an Airplane Model
(Log-Log Plot)
y = 0.0219x^1.9954
R^2 = 0.9999
Model
Power Curve Fit
F m (N)
V m (m/s)
100
10
1
10
100



Lift vs Speed for an Airplane Prototype (Log-Log Plot)
F p (kN)
V p (m/s)
10000
1000
100
10
1
10
100
1000

Problem 7.76

Given: Information relating to geometrically similar model test of centrifugal pump:

	Variable	Prototype	Model
Pressure Rise	Δp		29.3 kPa
Volume Flow Rate	Q	1.25 m³/min	
Density	ρ	800 kg/m³	999 kg/m³
Angular Speed	ω	183 rad/s	367 rad/s
Diameter	D	150 mm	50 mm

Find: Missing values for dynamically similar conditions.

Solution: Apply Buckingham Π-theorem. Assume $\Delta p = f(Q, \rho, \omega, D)$

① $\Delta p \quad Q \quad \rho \quad \omega \quad D \qquad n = 5$ parameters

② Choose M, L, t as fundamental dimensions.

③ $\frac{M}{Lt^2} \quad \frac{L^3}{t} \quad \frac{M}{L^3} \quad \frac{1}{t} \quad L \qquad r = 3$ primary dimensions

④ Let ρ, ω, and D be repeating variables. $\quad m = r = 3$

⑤ Then $n - m = 5 - 3 = 2$ dimensionless parameters result. ⑥ Check:

$$\Pi_1 = \rho^a \omega^b D^c \Delta p = \left(\frac{M}{L^3}\right)^a \left(\frac{1}{t}\right)^b (L)^c \frac{M}{Lt^2} = M^0 L^0 t^0$$

$$\left.\begin{matrix} M: a + 1 = 0 & a = -1 \\ L: -3a + c - 1 = 0 & c = -2 \\ t: -b - 2 = 0 & b = -2 \end{matrix}\right\} \Pi_1 = \frac{\Delta p}{\rho \omega^2 D^2} \qquad \Pi_1 = \frac{F}{L^2}\frac{L^4}{Ft^2}\frac{t^2}{1}\frac{1}{L^2} = [1] \checkmark\checkmark$$

$$\Pi_2 = \rho^a \omega^b D^c Q = \left(\frac{M}{L^3}\right)^a \left(\frac{1}{t}\right)^b (L)^c \frac{L^3}{t} = M^0 L^0 t^0$$

$$\left.\begin{matrix} M: a = 0 & a = 0 \\ L: -3a + c + 3 = 0 & c = -3 \\ t: -b - 1 = 0 & b = -1 \end{matrix}\right\} \Pi_2 = \frac{Q}{\omega D^3} \qquad \Pi_2 = \frac{L^3}{t}\frac{t}{1}\frac{1}{L^3} = [1] \checkmark\checkmark$$

Thus $\Pi_1 = f(\Pi_2)$ for this situation. Flows are geometrically similar. Assume kinematic similarity. Then for dynamic similarity, if $\Pi_{2m} = \Pi_{2p}$ then $\Pi_{1m} = \Pi_{1p}$.

$$\Pi_{2m} = \frac{Q_m}{\omega_m D_m^3} = \Pi_{2p} = \frac{Q_p}{\omega_p D_p^3}; \quad Q_m = Q_p \left(\frac{\omega_m}{\omega_p}\right)\left(\frac{D_m}{D_p}\right)^3 = Q_p \left(\frac{367}{183}\right)\left(\frac{50}{150}\right)^3 = 0.0743\, Q_p$$

$$Q_m = 0.0743 \times 1.25 \frac{m^3}{min} = 0.0928 \ m^3/min$$ Q_m

$$\Pi_{1m} = \frac{\Delta p_m}{\rho_m \omega_m^2 D_m^2} = \Pi_{1p} = \frac{\Delta p_p}{\rho_p \omega_p^2 D_p^2}; \quad \Delta p_p = \Delta p_m \frac{\rho_p}{\rho_m}\left(\frac{\omega_p}{\omega_m}\right)^2 \left(\frac{D_p}{D_m}\right)^2$$

$$\Delta p_p = \Delta p_m \left(\frac{800}{999}\right)\left(\frac{183}{367}\right)^2 \left(\frac{150}{50}\right)^2 = 1.79 \times 29.3 \text{ kPa} = 52.5 \text{ kPa}$$ Δp_p

{This result neglects any effect of viscosity.}

Problem 7.77

[3]

7.77 Tests are performed on a 1-m long ship model in a water tank. Results obtained (after doing some data analysis) are as follows:

V (m/s)	3	6	9	12	15	18	20
D_{Wave} (N)	0	0.125	0.5	1.5	3	4	5.5
$D_{Friction}$ (N)	0.1	0.35	0.75	1.25	2	2.75	3.25

The assumption is that wave drag modeling is done using the Froude number, and friction drag by the Reynolds number. The full size ship will be 50 m long when built. Estimate the total drag when it is cruising at 15 knots, and at 20 knots, in a freshwater lake.

For drag we can use $C_D = \dfrac{D}{\frac{1}{2}\rho V^2 A}$ As a suitable scaling area for A we use L^2 $C_D = \dfrac{D}{\frac{1}{2}\rho V^2 L^2}$

Model: $L =$ 1 m

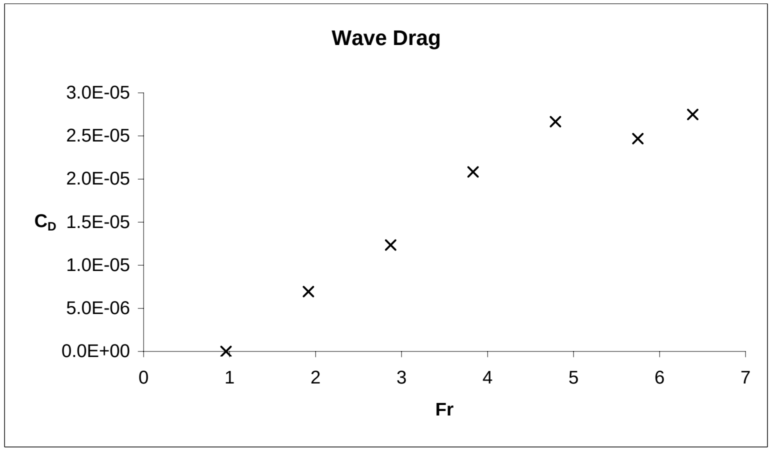
For water $\rho =$ 1000 kg/m^3

$\mu =$ 1.01E-03 N·s/m^2

The data is:

V (m/s)	3	6	9	12	15	18	20
D_{Wave} (N)	0	0.125	0.5	1.5	3	4	5.5
$D_{Friction}$ (N)	0.1	0.35	0.75	1.25	2	2.75	3.25

Fr	0.958	1.916	2.873	3.831	4.789	5.747	6.386
Re	2.97E+06	5.94E+06	8.91E+06	1.19E+07	1.49E+07	1.78E+07	1.98E+07
$C_{D(Wave)}$	0.00E+00	6.94E-06	1.23E-05	2.08E-05	2.67E-05	2.47E-05	2.75E-05
$C_{D(Friction)}$	2.22E-05	1.94E-05	1.85E-05	1.74E-05	1.78E-05	1.70E-05	1.63E-05

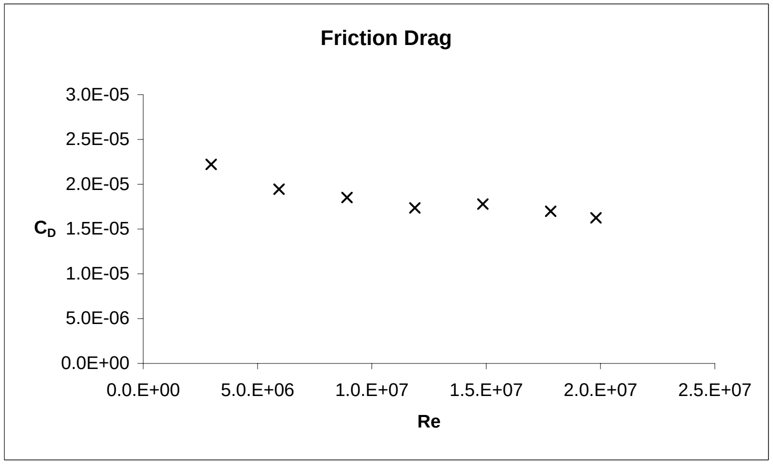


The friction drag coefficient becomes a constant, as expected, at high Re.
The wave drag coefficient appears to be linear with Fr, over most values

Ship: $L =$ 50 m

V (knot)	15	20
V (m/s)	7.72	10.29
Fr	0.348	0.465
Re	3.82E+08	5.09E+08

$$D = \frac{1}{2}\rho V^2 L^2 C_D$$



Hence for the ship we have very high Re, and low Fr.
From the graph we see the friction C_D levels out at about 1.75 x 10^{-5}
From the graph we see the wave C_D is negligibly small

$C_{D(Wave)}$	0	0
$C_{D(Friction)}$	1.75E-05	1.75E-05

D_{Wave} (N)	0	0
$D_{Friction}$ (N)	1303	2316

D_{Total} (N)	1303	2316

Problem 7.78 (In Excel)

7.78 A centrifugal water pump running at speed $\omega = 750$ rpm has the following data for flow rate Q and pressure head Δp:

Q (m³/hr)	0	100	150	200	250	300	325	350
Δp (kPa)	361	349	328	293	230	145	114	59

The pressure head Δp is a function of flow rate, Q, and speed, ω, and also impeller diameter, D, and water density, ρ. Plot the pressure head versus flow rate curve. Find the two Π parameters for this problem, and from the above data plot one against the other. By using *Excel* to perform a trendline analysis on this latter curve, generate and plot data for pressure head versus flow rate for impeller speeds of 500 rpm and 1000 rpm.

Given: Data on centrifugal water pump

Find: Π groups; plot pressure head vs flow rate for range of speeds

Solution:

We will use the workbook of Example 7.1, modified for the current problem

The number of parameters is:	$n = 5$
The number of primary dimensions is:	$r = 3$
The number of repeat parameters is:	$m = r = 3$
The number of Π groups is:	$n - m = 2$

Enter the dimensions (**M**, **L**, **t**) of
the repeating parameters, and of up to
four other parameters (for up to four Π groups).
The spreadsheet will compute the exponents *a*, *b*, and *c* for each.

REPEATING PARAMETERS: Choose ρ, *g*, *d*

	M	L	t
ρ	1	-3	
ω			-1
D		1	

Π GROUPS:

	M	L	t
Δp	1	-1	-2

Π_1: $a = -1$, $b = -2$, $c = -2$

	M	L	t
Q	0	3	-1

Π_2: $a = 0$, $b = -1$, $c = -3$

The following Π groups from Example 7.1 are not used:

	M	L	t
	0	0	0

Π_3: $a = 0$, $b = 0$, $c = 0$

	M	L	t
	0	0	0

Π_4: $a = 0$, $b = 0$, $c = 0$

Hence $\Pi_1 = \dfrac{\Delta p}{\rho\omega^2 D^2}$ and $\Pi_2 = \dfrac{Q}{\omega D^3}$ with $\Pi_1 = f(\Pi_2)$.

Based on the plotted data, it looks like the relation between Π_1 and Π_2 may be parabolic

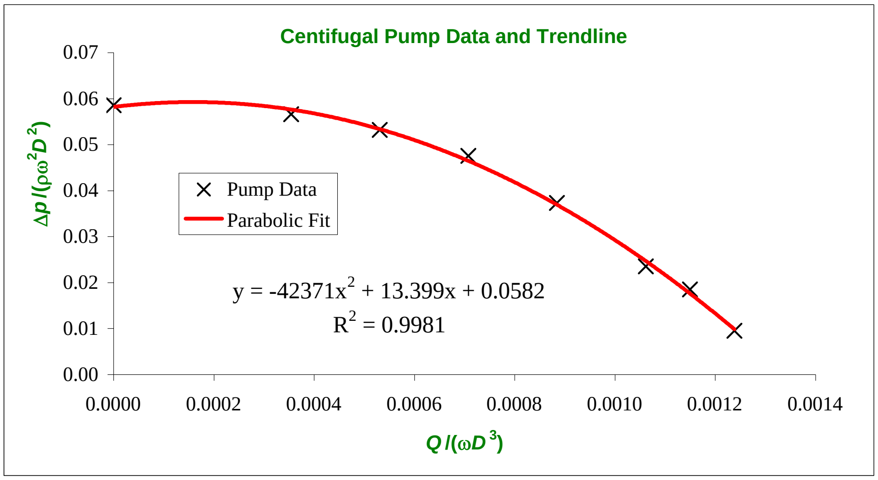
Hence $$\frac{\Delta p}{\rho\omega^2 D^2} = a + b\left(\frac{Q}{\omega D^3}\right) + c\left(\frac{Q}{\omega D^3}\right)^2$$

The data is

Q (m³/hr)	0	100	150	200	250	300	325	350
Δp (kPa)	361	349	328	293	230	145	114	59

ρ = 999 kg/m^3
ω = 750 rpm
D = 1 m (*D* is not given; use *D* = 1 m as a scale)

$Q/(\omega D^3)$	0.00000	0.000354	0.000531	0.000707	0.000884	0.00106	0.00115	0.00124
$\Delta p/(\rho\omega^2 D^2)$	0.0586	0.0566	0.0532	0.0475	0.0373	0.0235	0.0185	0.00957



From the *Trendline* analysis

a = 0.0582
b = 13.4
c = -42371

and $$\Delta p = \rho\omega^2 D^2\left[a + b\left(\frac{Q}{\omega D^3}\right) + c\left(\frac{Q}{\omega D^3}\right)^2\right]$$

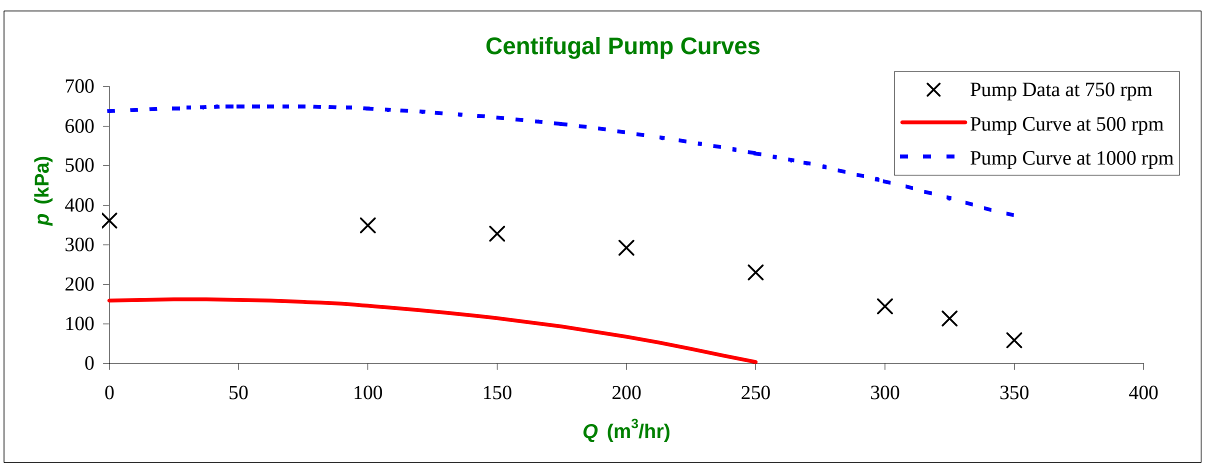
Finally, data at 500 and 1000 rpm can be calculated and plotted

ω = 500 rpm

Q (m^3/hr)	0	25	50	75	100	150	200	250
Δp (kPa)	159	162	161	156	146	115	68	4

ω = 1000 rpm

Q (m^3/hr)	0	25	50	100	175	250	300	350
Δp (kPa)	638	645	649	644	606	531	460	374



Problem 7.79

[3]

7.79 An axial-flow pump is required to deliver 0.75 m^3/s of water at a head of 15 J/kg. The diameter of the rotor is 0.25 m, and it is to be driven at 500 rpm. The prototype is to be modeled on a small test apparatus having a 2.25 kW, 1000 rpm power supply. For similar performance between the prototype and the model, calculate the head, volume flow rate, and diameter of the model.

Given: Model of water pump

Find: Model head, flow rate and diameter

Solution:

From Buckingham Π
$$\frac{h}{\omega^2 \cdot D^2} = f\left(\frac{Q}{\omega \cdot D^3}, \frac{\rho \cdot \omega \cdot D^2}{\mu}\right) \quad \text{and} \quad \frac{P}{\omega^3 \cdot D^5} = f\left(\frac{Q}{\omega \cdot D^3}, \frac{\rho \cdot \omega \cdot D^2}{\mu}\right)$$

Neglecting viscous effects
$$\frac{Q_m}{\omega_m \cdot D_m^3} = \frac{Q_p}{\omega_p \cdot D_p^3} \quad \text{then} \quad \frac{h_m}{\omega_m^2 \cdot D_m^2} = \frac{h_p}{\omega_p^2 \cdot D_p^2} \quad \text{and} \quad \frac{P_m}{\omega_m^3 \cdot D_m^5} = \frac{P_p}{\omega_p^3 \cdot D_p^5}$$

Hence if
$$\frac{Q_m}{Q_p} = \frac{\omega_m}{\omega_p} \cdot \left(\frac{D_m}{D_p}\right)^3 = \frac{1000}{500} \cdot \left(\frac{D_m}{D_p}\right)^3 = 2 \cdot \left(\frac{D_m}{D_p}\right)^3 \qquad (1)$$

then
$$\frac{h_m}{h_p} = \frac{\omega_m^2}{\omega_p^2} \cdot \frac{D_m^2}{D_p^2} = \left(\frac{1000}{500}\right)^2 \cdot \frac{D_m^2}{D_p^2} = 4 \cdot \frac{D_m^2}{D_p^2} \qquad (2)$$

and
$$\frac{P_m}{P_p} = \frac{\omega_m^3}{\omega_p^3} \cdot \frac{D_m^5}{D_p^5} = \left(\frac{1000}{500}\right)^3 \cdot \frac{D_m^5}{D_p^5} = 8 \cdot \frac{D_m^5}{D_p^5} \qquad (3)$$

We can find P_p from
$$P_p = \rho \cdot Q \cdot h = 1000 \cdot \frac{kg}{m^3} \times 0.75 \cdot \frac{m^3}{s} \times 15 \cdot \frac{J}{kg} = 11.25 \cdot kW$$

From Eq 3
$$\frac{P_m}{P_p} = 8 \cdot \frac{D_m^5}{D_p^5} \quad \text{so} \quad D_m = D_p \cdot \left(\frac{1}{8} \cdot \frac{P_m}{P_p}\right)^{\frac{1}{5}} \quad D_m = 0.25 \cdot m \times \left(\frac{1}{8} \times \frac{2.25}{11.25}\right)^{\frac{1}{5}} \quad D_m = 0.120\,m$$

From Eq 1
$$\frac{Q_m}{Q_p} = 2 \cdot \left(\frac{D_m}{D_p}\right)^3 \quad \text{so} \quad Q_m = Q_p \cdot 2 \cdot \left(\frac{D_m}{D_p}\right)^3 \quad Q_m = 0.75 \cdot \frac{m^3}{s} \times 2 \times \left(\frac{0.12}{0.25}\right)^3 \quad Q_m = 0.166\,\frac{m^3}{s}$$

From Eq 2
$$\frac{h_m}{h_p} = 4 \cdot \left(\frac{D_m}{D_p}\right)^2 \quad \text{so} \quad h_m = h_p \cdot 4 \cdot \left(\frac{D_m}{D_p}\right)^2 \quad h_m = 15 \cdot \frac{J}{kg} \times 4 \times \left(\frac{0.12}{0.25}\right)^2 \quad h_m = 13.8\,\frac{J}{kg}$$

Problem 7.80

7.80 A model propeller 2 ft in diameter is tested in a wind tunnel. Air approaches the propeller at 150 ft/s when it rotates at 2000 rpm. The thrust and torque measured under these conditions are 25 lbf and 7.5 lbf•ft, respectively. A prototype 10 times as large as the model is to be built. At a dynamically similar operating point, the approach air speed is to be 400 ft/s. Calculate the speed, thrust, and torque of the prototype propeller under these conditions, neglecting the effect of viscosity but including density.

Given: Data on model propeller

Find: Speed, thrust and torque on prototype

Solution:

There are two problems here: Determine $F_t = f_1(D, \omega, V, \mu, \rho)$ and also $T = f_2(D, \omega, V, \mu, \rho)$. Since μ is to be ignored, do not select it as a repeat parameter; instead select D, ω, ρ as repeats.

Apply the Buckingham Π procedure

① $F_t \quad D \quad \omega \quad V \quad \mu \quad \rho$ $\qquad$ n = 6 parameters

② Select primary dimensions M, L, t

③ $\left\{ \begin{array}{cccccc} F_t & D & \omega & V & \mu & \rho \\ \frac{ML}{t^2} & L & \frac{1}{t} & \frac{L}{t} & \frac{M}{Lt} & \frac{M}{L^3} \end{array} \right\}$ $\qquad$ r = 3 primary dimensions

④ $\rho \quad D \quad \omega$ $\qquad$ $m = r$ = 3 repeat parameters

⑤ Then $n - m$ = 5 dimensionless groups will result. Setting up a dimensional equation,

$$\Pi_1 = \rho^a D^b \omega^c F_t = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{ML}{t^2} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{ll|l} M: & a + 1 = 0 & a = -1 \\ L: & -3a + b + 1 = 0 & b = -4 \\ t: & -c - 2 = 0 & c = -2 \end{array}$$

Hence $\Pi_1 = \dfrac{F_t}{\rho D^4 \omega^2}$

$$\Pi_2 = \rho^a D^b \omega^c V = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{L}{t} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{ll|l} M: & a = 0 & a = 0 \\ L: & -3a + b + 1 = 0 & b = -1 \\ t: & -c - 1 = 0 & c = -1 \end{array}$$

Hence $\Pi_2 = \dfrac{V}{D\omega}$

$$\Pi_3 = \rho^a D^b \omega^c \mu = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{M}{Lt} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{rl|l} M: & a+1=0 & a=-1 \\ L: & -3a+b-1=0 & b=-2 \\ t: & -c-1=0 & c=-1 \end{array}$$

Hence $\Pi_3 = \dfrac{\mu}{\rho D^2 \omega}$

⑥ Check using *F*, *L*, *t* as primary dimensions

$$\Pi_1 = \frac{F}{\frac{Ft^2}{L^4} L^4 \frac{1}{t^2}} = [1] \qquad \Pi_2 = \frac{\frac{L}{t}}{L\frac{1}{t}} = [1] \qquad \Pi_3 = \frac{\frac{Ft}{L^2}}{\frac{Ft^2}{L^4} L^2 \frac{1}{t}} = [1]$$

Then $\Pi_1 = f_1(\Pi_2, \Pi_3)$ $\qquad \dfrac{F_t}{\rho D^4 \omega^2} = f_1\left(\dfrac{V}{D\omega}, \dfrac{\mu}{\rho D^2 \omega}\right)$

If viscous effects are neglected $\dfrac{F_t}{\rho D^4 \omega^2} = g_1\left(\dfrac{V}{D\omega}\right)$

For dynamic similarity $\dfrac{V_m}{D_m \omega_m} = \dfrac{V_p}{D_p \omega_p}$

so $\omega_p = \dfrac{D_m}{D_p}\dfrac{V_p}{V_m}\omega_m = \left(\dfrac{1}{10}\right) \times \left(\dfrac{400}{150}\right) \times 2000\,\text{rpm} = 533\,\text{rpm}$

Under these conditions $\dfrac{F_{t_m}}{\rho D_m^4 \omega_m^2} = \dfrac{F_{t_p}}{\rho D_p^4 \omega_p^2}$ (assuming $\rho_m = \rho_p$)

or $F_{t_p} = \dfrac{D_p^4}{D_m^4}\dfrac{\omega_p^2}{\omega_m^2} F_{t_m} = \left(\dfrac{10}{1}\right)^4 \times \left(\dfrac{533}{2000}\right)^2 \times 25\,\text{lbf} = 1.78 \times 10^4\ \text{lbf}$

For the torque we can avoid repeating a lot of the work

$$\Pi_4 = \rho^a D^b \omega^c T = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \frac{ML^2}{t^2} = M^0 L^0 t^0$$

Summing exponents,

$$\begin{array}{rl|l} M: & a+1=0 & a=-1 \\ L: & -3a+b+2=0 & b=-5 \\ t: & -c-2=0 & c=-2 \end{array}$$

Hence $\Pi_4 = \dfrac{T}{\rho D^5 \omega^2}$

Then $\Pi_4 = f_2(\Pi_2, \Pi_3)$ $\qquad \dfrac{T}{\rho D^5 \omega^2} = f_2\left(\dfrac{V}{D\omega}, \dfrac{\mu}{\rho D^2 \omega}\right)$

If viscous effects are neglected $\dfrac{T}{\rho D^5 \omega^2} = g_2\left(\dfrac{V}{D\omega}\right)$

For dynamic similarity $\dfrac{T_m}{\rho D_m^5 \omega_m^2} = \dfrac{T_p}{\rho D_p^5 \omega_p^2}$

or $T_p = \dfrac{D_p^5}{D_m^5}\dfrac{\omega_p^2}{\omega_m^2} T_m = \left(\dfrac{10}{1}\right)^5 \times \left(\dfrac{533}{2000}\right)^2 \times 7.5\,\text{lbf}\cdot\text{ft} = 5.33 \times 10^4\ \text{lbf}\cdot\text{ft}$

Problem 7.81

Given: For a marine propeller (see Problem 7.40) the thrust force, F_t, is

$$F_t = F_t(\rho, D, V, g, \omega, p, \mu)$$

Neglecting viscous effects, and pressure, then

$$F_t = F_t(\rho, D, V, g, \omega)$$

Assume that Torque, T, and power, $\mathcal{P}$, depend on same parameters

$$T = T(\rho, D, V, g, \omega)$$

$$\mathcal{P} = \mathcal{P}(\rho, D, V, g, \omega)$$

Find: Derive scaling "laws" for propellers that relate F_t, T, and $\mathcal{P}$ to other variables.

Solution: Apply Buckingham π-theorem

① $\rho \quad D \quad V \quad g \quad \omega \quad F_t \quad T \quad \mathcal{P}$

② Choose F, L, t as primary dimensions

③ $\frac{Ft^2}{L^4} \quad L \quad \frac{L}{t} \quad \frac{L}{t^2} \quad \frac{1}{t} \quad F \quad FL \quad \frac{FL}{t}$

④ Repeating variables ρ, ω, D

⑤ Then $n - m = 5$ dimensionless groups (2 independent, 3 dependent)

Setting up dimensional equations

$$\pi_1 = \rho^a \omega^b D^c V \qquad F^0L^0t^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{1}{t}\right)^b L^c \frac{L}{t} \qquad \begin{cases} F: & 0 = a \\ t: & 0 = 2a - b - 1 \\ L: & 0 = -4a + c + 1 \end{cases} \quad \begin{matrix} a = 0 \\ b = -1 \\ c = -1 \end{matrix} \qquad \therefore \pi_1 = \frac{V}{\omega D}$$

$$\pi_2 = \rho^a \omega^b D^c g \qquad F^0L^0t^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{1}{t}\right)^b L^c \frac{L}{t^2} \qquad \begin{cases} F: & 0 = a \\ t: & 0 = 2a - b - 2 \\ L: & 0 = -4a + c + 1 \end{cases} \quad \begin{matrix} a = 0 \\ b = -2 \\ c = -1 \end{matrix} \qquad \therefore \pi_2 = \frac{g}{\omega^2 D}$$

$$\pi_3 = \rho^a \omega^b D^c F_t \qquad F^0L^0t^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{1}{t}\right)^b L^c F \qquad \begin{cases} F: & 0 = a + 1 \\ t: & 0 = 2a - b \\ L: & 0 = -4a + c \end{cases} \quad \begin{matrix} a = -1 \\ b = -2 \\ c = -4 \end{matrix} \qquad \therefore \pi_3 = \frac{F_t}{\rho \omega^2 D^4}$$

$$\pi_4 = \rho^a \omega^b D^c T \qquad F^0L^0t^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{1}{t}\right)^b L^c FL \qquad \begin{cases} F: & 0 = a + 1 \\ t: & 0 = 2a - b \\ L: & 0 = -4a + c + 1 \end{cases} \quad \begin{matrix} a = -1 \\ b = -2 \\ c = -5 \end{matrix} \qquad \therefore \pi_4 = \frac{T}{\rho \omega^2 D^5}$$

$$\pi_5 = \rho^a \omega^b D^c \mathcal{P} \qquad F^0L^0t^0 = \left(\frac{Ft^2}{L^4}\right)^a \left(\frac{1}{t}\right)^b L^c \frac{FL}{t} \qquad \begin{cases} F: & 0 = a + 1 \\ t: & 0 = 2a - b - 1 \\ L: & 0 = -4a + c + 1 \end{cases} \quad \begin{matrix} a = -1 \\ b = -3 \\ c = -5 \end{matrix} \qquad \therefore \pi_5 = \frac{\mathcal{P}}{\rho \omega^3 D^5}$$

Then scaling "laws" are

$$\frac{F_t}{\rho \omega^2 D^4} = f_1\left(\frac{V}{\omega D}, \frac{g}{\omega^2 D}\right)$$

$$\frac{T}{\rho \omega^2 D^5} = f_2\left(\frac{V}{\omega D}, \frac{g}{\omega^2 D}\right)$$

$$\frac{\mathcal{P}}{\rho \omega^3 D^5} = f_3\left(\frac{V}{\omega D}, \frac{g}{\omega^2 D}\right)$$

Problem 7.82

[2]

7.82 Water drops are produced by a mechanism that it is believed follows the pattern $d_p = D(We)^{-3/5}$. In this formula, d_p is the drop size, D is proportional to a length scale, and We is the Weber number. In scaling up, if the large-scale characteristic length scale was increased by 10 and the large-scale velocity decreased by a factor of 4, how would the small- and large-scale drops differ from each other for the same material, for example, water?

Given: Water drop mechanism

Find: Difference between small and large scale drops

Solution:

Given relation

$$d = D\cdot(We)^{-\frac{3}{5}} = D\cdot\left(\frac{\rho\cdot V^2\cdot D}{\sigma}\right)^{-\frac{3}{5}}$$

For dynamic similarity

$$\frac{d_m}{d_p} = \frac{D_m\cdot\left(\frac{\rho\cdot V_m^{\ 2}\cdot D_m}{\sigma}\right)^{-\frac{3}{5}}}{D_p\cdot\left(\frac{\rho\cdot V_p^{\ 2}\cdot D_p}{\sigma}\right)^{-\frac{3}{5}}} = \left(\frac{D_m}{D_p}\right)^{\frac{2}{5}}\cdot\left(\frac{V_m}{V_p}\right)^{-\frac{6}{5}}$$

where d_p stands for $d_{prototype}$ not the original d_p!

Hence

$$\frac{d_m}{d_p} = \left(\frac{1}{10}\right)^{\frac{2}{5}}\times\left(\frac{4}{1}\right)^{-\frac{6}{5}}$$

$$\frac{d_m}{d_p} = 0.075$$

The small scale droplets are 7.5% of the size of the large scale

Given: The kinetic energy ratio is a figure of merit defined as the ratio of kinetic energy flux in a wind tunnel test section to the drive power.

Find: an estimate of the kinetic energy ratio for the 40×80 wind tunnel at NASA-Ames.

Solution:

From text (p.319), for NASA-Ames tunnel:

$$A = 40\,\text{ft} \times 80\,\text{ft} = 3200\,\text{ft}^2 \quad , \quad \mathcal{P} = 125{,}000\,\text{hp}$$

$$V_{max} = 300\,\frac{\text{kmi}}{\text{hr}} \times 6080\,\frac{\text{ft}}{\text{k-mi}} \times \frac{\text{hr}}{3600\,\text{s}} = 507\,\text{ft/s}$$

$$\text{K.E ratio} = \frac{\text{K.E flux}}{\text{Power in}} = \frac{\dot{m}\frac{V^2}{2}}{\mathcal{P}} = \frac{\rho V A V^2}{2\mathcal{P}} = \frac{\rho V^3 A}{2\mathcal{P}}$$

Assuming standard air,

$$\text{K.E. ratio} = \frac{1}{2} \times 0.00238\,\frac{\text{slug}}{\text{ft}^3} \times (507)^3\,\frac{\text{ft}^3}{\text{s}^3} \times 3200\,\text{ft}^2 \times \frac{1}{125{,}000\,\text{hp}} \times \frac{\text{hp}\cdot\text{s}}{550\,\text{ft}\cdot\text{lbf}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

$$\text{K.E ratio} = 7.22$$

Given: A 1:16 scale model of a 20m long truck is tested in a wind tunnel at speed $V_m = 80$ m/s. The axial pressure gradient at this speed is $dh/dx = -1.2$ mm H_2O/m. The frontal area of the prototype is $A_p = 10\,m^2$. $C_D = 0.85$

Find: (a) Estimate the horizontal buoyancy correction
(b) Express the correction as a fraction of the measured C_D.

Solution:

The horizontal buoyancy force, F_B, is the difference in the pressure force between the front and back of the model due to the pressure gradient in the tunnel

$$F_B = (P_f - P_b)A = \rho_w g \frac{dh}{dx} L_m A_m \qquad (\Delta P = \rho_w g \Delta h)$$

$$L_m = \frac{L_p}{16} \qquad A_m = \frac{A_p}{(16)^2}$$

$$\therefore F_B = 999\,\frac{kg}{m^3} \times 9.81\,\frac{m}{s^2} \times (-1.2)\times 10^{-3}\,\frac{m}{m} \times \frac{20m}{16} \times \frac{10m^2}{(16)^2} \times \frac{N.s^2}{kg.m}$$

$$F_B = -0.574\,N$$ F_B

The horizontal buoyancy correction should be added to the measured drag force on the model.

The measured drag force on the model is given by

$$F_{Dm} = \frac{1}{2}\rho V^2 A_m C_D = \frac{1}{2}\rho V^2 \frac{A_p}{(16)^2} C_D$$

Assume air at standard conditions, $\rho = 1.23$ kg/m^3

$$F_{Dm} = \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (80)^2\,\frac{m^2}{s^2} \times \frac{10m^2}{(16)^2} \times 0.85 \times \frac{N.s^2}{kg.m}$$

$$F_{Dm} = 131\,N$$

$$\frac{F_B}{F_{Dm}} = \frac{-0.574}{131} = -4.38\times 10^{-3} = -0.44\%$$ F_B/F_D

Problem 7.85

Given: Wind tunnel test of 1:16 model bus in standard air.

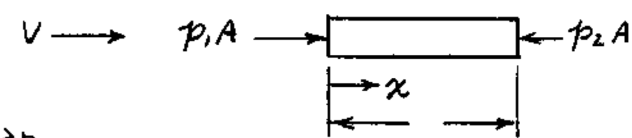
$W = 152$ mm $V = 26.5$ m/s Pressure gradient:
$H = 200$ mm $F_D = 6.09$ N $\frac{dp}{dx} = -11.8$ N/m²/m
$L = 762$ mm (measured)

Find: (a) Estimate the horizontal buoyancy correction.
(b) Calculate the corrected model drag coefficient.
(c) Evaluate the drag force on the prototype at 100 km/hr on a calm day.

Solution: Apply definitions

Computing equations: $C_D \equiv \frac{F_D}{\frac{1}{2}\rho V^2 A}$ Assume $A = WH$

The buoyancy force will be



$$F_B = p_1 A - p_2 A = (p_1 - p_2)A$$

But $p_2 = p_1 + \frac{\partial p}{\partial x}\Delta x + \cdots \approx p_1 + \frac{\partial p}{\partial x} L$

Therefore $p_1 - p_2 = -\frac{\partial p}{\partial x} L$, and $F_B \approx -\frac{\partial p}{\partial x} LA = -\frac{\partial p}{\partial x} LWH$

$$F_B \approx -(-11.8)\frac{N}{m^3} \times 0.762\,m \times 0.152\,m \times 0.200\,m = 0.273\ N \text{ (to right)}$$ F_B

The corrected drag force is

$$F_{D_c} = F_{D_m} - F_B = (6.09 - 0.273)\,N = 5.82\,N$$

The corrected model drag coefficient is

$$C_{D_m} = \frac{F_{D_c}}{\frac{1}{2}\rho V^2 A} = 2 \times 5.82\,N \times \frac{m^3}{1.23\,kg} \times \frac{s^2}{(26.5)^2 m^2} \times \frac{1}{(0.200)(0.152)m^2} \times \frac{kg \cdot m}{N \cdot s^2} = 0.443$$ C_D

Assume the test was conducted at high enough Reynolds number so $C_{D_p} = C_{D_m}$. Then

$$F_{D_p} = C_{D_p} A_p \frac{1}{2}\rho V_p^2$$

$$= \frac{1}{2} \times 0.443 \times 0.200(16)\,m \times 0.152(16)\,m \times 1.23\frac{kg}{m^3} \times \left[100\frac{km}{hr} \times 1000\frac{m}{km} \times \frac{hr}{3600\,s}\right]^2 \frac{N \cdot s^2}{kg \cdot m}$$

$F_{D_p} = 1.64$ kN (prototype at 100 km/hr) F_{D_p}

{Rolling resistance must be included to obtain the total tractive effort needed to propel the full-scale vehicle.}

Problem 7.86

7.86 Frequently one observes a flag on a pole flapping in the wind. Explain why this occurs.

Given: Flapping flag on a flagpole

Find: Explanation of the flappinh

Solution:

Open-Ended Problem Statement: Frequently one observes a flag on a pole "flapping" in the wind. Explain why this occurs. What dimensionless parameters might characterize the phenomenon? Why?

Discussion: The natural wind contains significant fluctuations in air speed and direction. These fluctuations tend to disturb the flag from an initially plane position.

When the flag is bent or curved from the plane position, the flow nearby must follow its contour. Flow over a convex surface tends to be faster, and have lower pressure, than flow over a concave curved surface. The resulting pressure forces tend to exaggerate the curvature of the flag. The result is a seemingly random "flapping" motion of the flag.

The rope or chain used to raise the flag may also flap in the wind. It is much more likely to exhibit a periodic motion than the flag itself. The rope is quite close to the flag pole, where it is influenced by any vortices shed from the pole. If the Reynolds number is such that periodic vortices are shed from the pole, they will tend to make the rope move with the same frequency. This accounts for the periodic thump of a rope or clank of a chain against the pole.

The vortex shedding phenomenon is characterized by the Strouhal number, $St = fD/V_\infty$, where f is the vortex shedding frequency, D is the pole diameter, and D is the wind speed. The Strouhal number is constant at approximately 0.2 over a broad range of Reynolds numbers.

Problem 7.87

Open-Ended Problem Statement: Explore the variation in wave propagation speed given by the equation of Problem 7.2 for a free-surface flow of water. Find the operating depth to minimize the speed of capillary waves (waves with small wavelength, also called *ripples*). First assume wavelength is much smaller than water depth. Then explore the effect of depth. What depth do you recommend for a water table used to visualize compressible-flow wave phenomena? What is the effect of reducing surface tension by adding a surfactant?

Discussion: The equation given in Problem 7.2 contains three terms. The first term contains surface tension and gives a speed inversely proportional to wavelength. This term will be important when small wavelengths are considered.

The second term contains gravity and gives a speed proportional to wavelength. This term will be important when long wavelengths are considered.

The argument of the hyperbolic tangent is proportional to water depth and inversely proportional to wavelength. For small wavelengths, this term should approach unity since the hyperbolic tangent of a large number approaches one.

See the spreadsheet for numerical values and a plot.

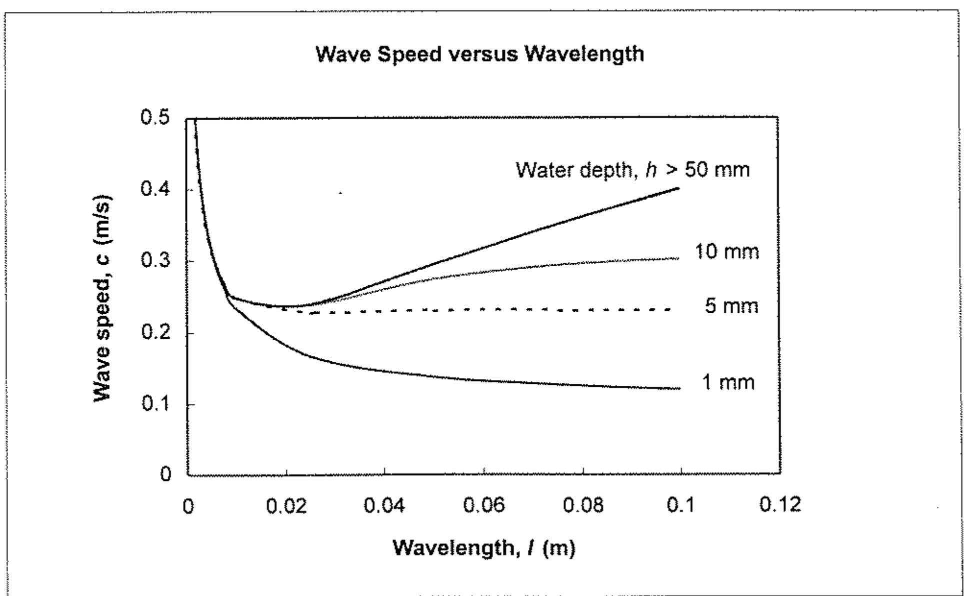
Problem 7.87

Input Parameters:

g =	9.81	m/s^2	Acceleration of gravity
h =	0.01	m	Liquid depth (for hyperbolic tangent calculation)
ρ =	999	kg/m^3	Liquid density
σ =	0.0728	N/m	Surface tension

Calculated Values:

Wavelength, λ (m)	tanh (---) (h = 10 mm)	h (m) = 0.001	0.005	0.01	0.05	0.1	0.5
		Wave Speed, c (m/s)					
0.00185	1.00	0.500	0.500	0.500	0.500	0.500	0.500
0.003	1.00	0.396	0.397	0.397	0.397	0.397	0.397
0.005	1.00	0.313	0.315	0.315	0.315	0.315	0.315
0.0075	1.00	0.263	0.270	0.270	0.270	0.270	0.270
0.01	1.00	0.233	0.248	0.248	0.248	0.248	0.248
0.025	0.987	0.167	0.227	0.238	0.239	0.239	0.239
0.05	0.850	0.138	0.229	0.275	0.295	0.295	0.295
0.075	0.685	0.126	0.229	0.294	0.351	0.351	0.351
0.1	0.557	0.120	0.228	0.303	0.400	0.401	0.401
0.2	0.304	0.110	0.226	0.312	0.537	0.560	0.561
0.5	0.125	0.104	0.223	0.314	0.660	0.815	0.884
0.75	0.0836	0.102	0.223	0.314	0.681	0.896	1.08
1	0.0627	0.101	0.222	0.314	0.690	0.933	1.25
2	0.0314	0.100	0.222	0.314	0.698	0.975	1.69
5	0.0126	0.100	0.222	0.313	0.700	0.988	2.09
7.5	0.00838	0.0994	0.222	0.313	0.700	0.989	2.15
10	0.00628	0.0993	0.222	0.313	0.700	0.990	2.18
Froude Speed, $(gh)^{1/2}$ (m/s)		0.0990	0.221	0.313	0.700	0.990	2.21



Problem 8.1

8.1 Standard air enters a 6-in. diameter duct. Find the volume flow rate at which the flow becomes turbulent. At this flow rate, estimate the entrance length required to establish fully developed flow.

Given: Air entering duct

Find: Flow rate for turbulence; Entrance length

Solution:

The governing equations are $Re = \frac{V \cdot D}{\nu}$ $\quad Re_{crit} = 2300$ $\quad Q = \frac{\pi}{4} \cdot D^2 \cdot V$

The given data is $D = 6 \cdot in$ From Table A.9 $\nu = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s}$

$L_{laminar} = 0.06 \cdot Re_{crit} \cdot D$ or, for turbulent, $L_{turb} = 25D$ to $40D$

Hence $Re_{crit} = \frac{\frac{Q}{\frac{\pi}{4} \cdot D^2} \cdot D}{\nu}$ or $Q = \frac{Re_{crit} \cdot \pi \cdot \nu \cdot D}{4}$

$$Q = 2300 \times \frac{\pi}{4} \times 1.62 \times 10^{-4} \cdot \frac{ft^2}{s} \times \frac{1}{2} \cdot ft \qquad Q = 0.146 \cdot \frac{ft^3}{s}$$

For laminar flow $L_{laminar} = 0.06 \cdot Re_{crit} \cdot D$ $\quad L_{laminar} = 0.06 \times 2300 \times 6 \cdot in$ $\quad L_{laminar} = 69.0 \cdot ft$

For turbulent flow $L_{min} = 25 \cdot D$ $\quad L_{min} = 12.5 \cdot ft$ $\quad L_{max} = 40 \cdot D$ $\quad L_{max} = 20 \cdot ft$

Given: Incompressible flow in a circular channel.
$Re = 1800$ in a section where the channel diameter is $D = 10$ mm.

Find: (i) general expression for Re in terms of
(a) volume flow rate, Q, and channel diameter, D
(b) mass flow rate, $\dot{m}$, and channel diameter, D.
(ii) Re for same flow rate and $D = 6$ mm.

Solution:

Assume steady, incompressible flow

Definitions: $Re = \frac{\rho D \bar{V}}{\mu}$, $Q = A\bar{V}$, $\dot{m} = \rho A \bar{V}$ and $A = \frac{\pi D^2}{4}$

Then,

$$Re = \frac{\rho D \bar{V}}{\mu} = \frac{\rho D}{\mu}\frac{Q}{A} = \frac{\rho D}{\mu}\frac{Q4}{\pi D^2} = \frac{4Q}{\pi D}\frac{\rho}{\mu} = \frac{4Q}{\pi D \nu} \quad \text{Re}$$

Also

$$Re = \frac{\rho D \bar{V}}{\mu} = \frac{D}{\mu}\frac{\rho \bar{V} A}{A} = \frac{D}{\mu}\frac{\dot{m}4}{\pi D^2} = \frac{4\dot{m}}{\pi D \mu} \quad \text{Re}$$

From Eq (i) a

$$Q = \frac{\pi D \nu Re}{4}$$

Then for same flow rate in sections with different channel diameter,

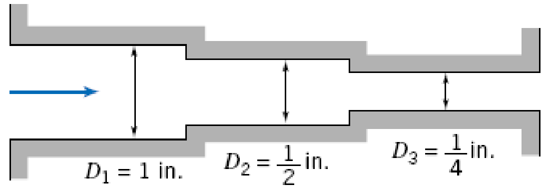
$$D_1 Re_1 = D_2 Re_2$$

$$Re_2 = \frac{D_1}{D_2} Re_1 = \frac{10\,\text{mm}}{6\,\text{mm}} \times 1800 = 3000 \quad Re_2$$

Problem 8.3

[3]

8.3 Standard air flows in a pipe system in which diameter is decreased in two stages from 1 in., to $\frac{1}{2}$ in., to $\frac{1}{4}$ in. Each section is 5 ft long. As the flow rate is increased, which section will become turbulent first? Determine the flow rates at which one, two, then all three sections first become turbulent. At each of these flow rates, determine which sections, if any, attain fully developed flow.



Given: Air entering pipe system

Find: Flow rate for turbulence in each section; Which become fully developed

Solution:

From Table A.9 $\quad \nu = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s}$

The given data is $\quad L = 5 \cdot ft \quad D_1 = 1 \cdot in \quad D_2 = \frac{1}{2} \cdot in \quad D_3 = \frac{1}{4} \cdot in$

The critical Reynolds number is $\quad Re_{crit} = 2300$

Writing the Reynolds number as a function of flow rate

$$Re = \frac{V \cdot D}{\nu} = \frac{Q}{\frac{\pi}{4} \cdot D^2} \cdot \frac{D}{\nu} \qquad \text{or} \qquad Q = \frac{Re \cdot \pi \cdot \nu \cdot D}{4}$$

Then the flow rates for turbulence to begin in each section of pipe are

$$Q_1 = \frac{Re_{crit} \cdot \pi \cdot \nu \cdot D_1}{4} \qquad Q_1 = 2300 \times \frac{\pi}{4} \times 1.62 \times 10^{-4} \cdot \frac{ft^2}{s} \times \frac{1}{12} \cdot ft \qquad Q_1 = 0.0244 \frac{ft^3}{s}$$

$$Q_2 = \frac{Re_{crit} \cdot \pi \cdot \nu \cdot D_2}{4} \qquad Q_2 = 0.0122 \frac{ft^3}{s} \qquad Q_3 = \frac{Re_{crit} \cdot \pi \cdot \nu \cdot D_3}{4} \qquad Q_3 = 0.00610 \frac{ft^3}{s}$$

Hence, **smallest** pipe becomes turbulent first, then second, then the largest.

For the smallest pipe transitioning to turbulence (Q_3)

For pipe 3 $\quad Re_3 = 2300 \quad L_{laminar} = 0.06 \cdot Re_3 \cdot D_3 \quad L_{laminar} = 2.87\,ft \quad L_{laminar} < L$: Not fully developed

or, for turbulent, $\quad L_{min} = 25 \cdot D_3 \quad L_{min} = 0.521\,ft \quad L_{max} = 40 \cdot D_3 \quad L_{max} = 0.833\,ft \quad L_{max/min} < L$: Not fully developed

For pipes 1 and 2 $\quad L_{laminar} = 0.06 \cdot \left(\frac{4 \cdot Q_3}{\pi \cdot \nu \cdot D_1}\right) \cdot D_1 \quad L_{laminar} = 2.87\,ft \quad L_{laminar} < L$: Not fully developed

$L_{laminar} = 0.06 \cdot \left(\frac{4 \cdot Q_3}{\pi \cdot \nu \cdot D_2}\right) \cdot D_2 \quad L_{laminar} = 2.87\,ft \quad L_{laminar} < L$: Not fully developed

For the middle pipe transitioning to turbulence (Q_2)

For pipe 2	$Re_2 = 2300$	$L_{laminar} = 0.06 \cdot Re_2 \cdot D_2$	$L_{laminar} = 5.75\,ft$	
				$L_{laminar}$ > L: Fully developed
or, for turbulent,	$L_{min} = 25 \cdot D_2$	$L_{min} = 1.04\,ft$	$L_{max} = 40 \cdot D_2$	$L_{max} = 1.67\,ft$
				$L_{max/min}$ < L: Not fully developed
For pipes 1 and 3	$L_1 = 0.06 \cdot \left(\frac{4 \cdot Q_2}{\pi \cdot \nu \cdot D_1}\right) \cdot D_1$		$L_1 = 5.75\,ft$	
	$L_{3min} = 25 \cdot D_3$	$L_{3min} = 0.521\,ft$	$L_{3max} = 40 \cdot D_3$	$L_{3max} = 0.833\,ft$
				$L_{max/min}$ < L: Not fully developed

For the large pipe transitioning to turbulence (Q_1)

For pipe 1	$Re_1 = 2300$	$L_{laminar} = 0.06 \cdot Re_1 \cdot D_1$	$L_{laminar} = 11.5\,ft$	
				$L_{laminar}$ > L: Fully developed
or, for turbulent,	$L_{min} = 25 \cdot D_1$	$L_{min} = 2.08\,ft$	$L_{max} = 40 \cdot D_1$	$L_{max} = 3.33\,ft$
				$L_{max/min}$ < L: Not fully developed
For pipes 2 and 3	$L_{2min} = 25 \cdot D_2$	$L_{2min} = 1.04\,ft$	$L_{2max} = 40 \cdot D_2$	$L_{2max} = 1.67\,ft$
				$L_{max/min}$ < L: Not fully developed
	$L_{3min} = 25 \cdot D_3$	$L_{3min} = 0.521\,ft$	$L_{3max} = 40 \cdot D_3$	$L_{3max} = 0.833\,ft$
				$L_{max/min}$ < L: Not fully developed

Problem 8.4

8.4 For flow in circular tubes, transition to turbulence usually occurs around $Re \approx 2300$. Investigate the circumstances under which the flows of (a) standard air and (b) water at 15°C become turbulent. On log-log graphs, plot: the average velocity, the volume flow rate, and the mass flow rate, at which turbulence first occurs, as functions of tube diameter.

Given: That transition to turbulence occurs at about *Re* = 2300

Find: Plots of average velocity and volume and mass flow rates for turbulence for air and water

Solution:

From Tables A.8 and A.10 $\quad \rho_{air} = 1.23 \cdot \frac{kg}{m^3} \quad \nu_{air} = 1.45 \times 10^{-5} \cdot \frac{m^2}{s} \quad \rho_w = 999 \cdot \frac{kg}{m^3} \quad \nu_w = 1.14 \times 10^{-6} \cdot \frac{m^2}{s}$

The governing equations are $\quad Re = \frac{V \cdot D}{\nu} \quad Re_{crit} = 2300$

For the average velocity $\quad V = \frac{Re_{crit} \cdot \nu}{D}$

Hence for air $\quad V_{air} = \frac{2300 \times 1.45 \times 10^{-5} \cdot \frac{m^2}{s}}{D} \quad V_{air} = \frac{0.0334 \cdot \frac{m^2}{s}}{D}$

For water $\quad V_w = \frac{2300 \times 1.14 \times 10^{-6} \cdot \frac{m^2}{s}}{D} \quad V_w = \frac{0.00262 \cdot \frac{m^2}{s}}{D}$

For the volume flow rates $\quad Q = A \cdot V = \frac{\pi}{4} \cdot D^2 \cdot V = \frac{\pi}{4} \cdot D^2 \cdot \frac{Re_{crit} \cdot \nu}{D} = \frac{\pi \cdot Re_{crit} \cdot \nu}{4} \cdot D$

Hence for air $\quad Q_{air} = \frac{\pi}{4} \times 2300 \times 1.45 \cdot 10^{-5} \cdot \frac{m^2}{s} \cdot D \quad Q_{air} = 0.0262 \cdot \frac{m^2}{s} \times D$

For water $\quad Q_w = \frac{\pi}{4} \times 2300 \times 1.14 \cdot 10^{-6} \cdot \frac{m^2}{s} \cdot D \quad Q_w = 0.00206 \cdot \frac{m^2}{s} \times D$

Finally, the mass flow rates are obtained from volume flow rates

$m_{air} = \rho_{air} \cdot Q_{air} \quad m_{air} = 0.0322 \cdot \frac{kg}{m \cdot s} \times D$

$m_w = \rho_w \cdot Q_w \quad m_w = 2.06 \cdot \frac{kg}{m \cdot s} \times D$

These results are plotted in the associated *Excel* workbook

Problem 8.4

8.4 For flow in circular tubes, transition to turbulence usually occurs around $Re \approx 2300$. Investigate the circumstances under which the flows of (a) standard air and (b) water at 15°C become turbulent. On log-log graphs, plot: the average velocity, the volume flow rate, and the mass flow rate, at which turbulence first occurs, as functions of tube diameter.

Given: That transition to turbulence occurs at about $Re = 2300$

Find: Plots of average velocity and volume and mass flow rates for turbulence for air and water

Solution:

The relations needed are

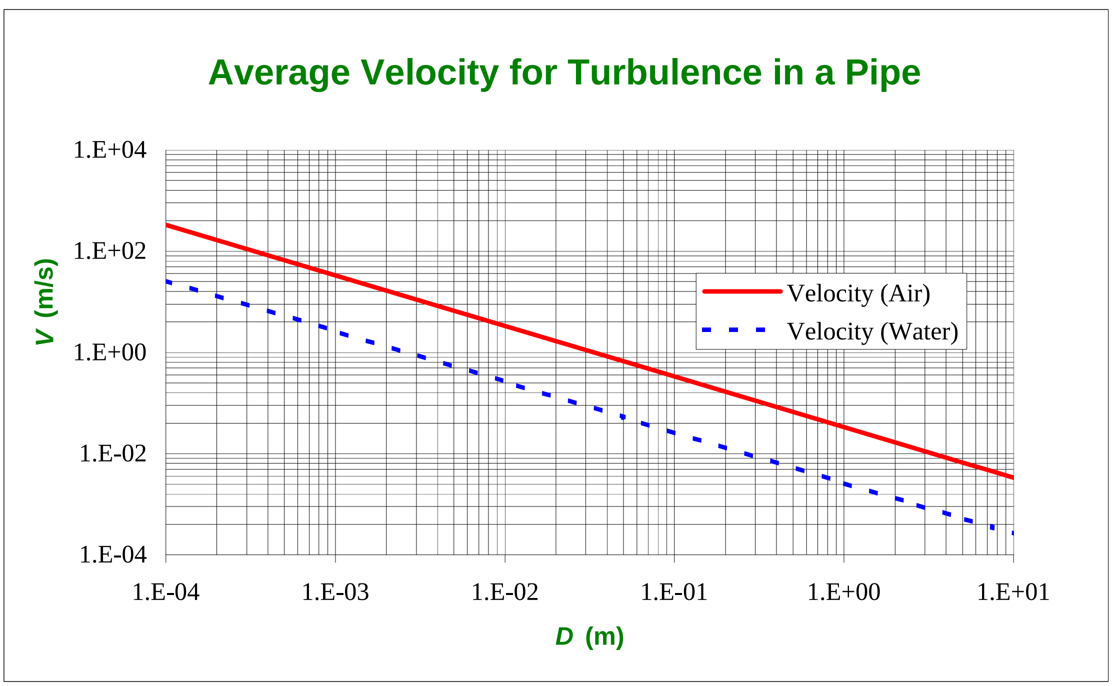
$$Re_{crit} = 2300 \qquad V = \frac{Re_{crit} \cdot \nu}{D} \qquad Q = \frac{\pi \cdot Re_{crit} \cdot \nu}{4} \cdot D \qquad m_{rate} = \rho \cdot Q$$

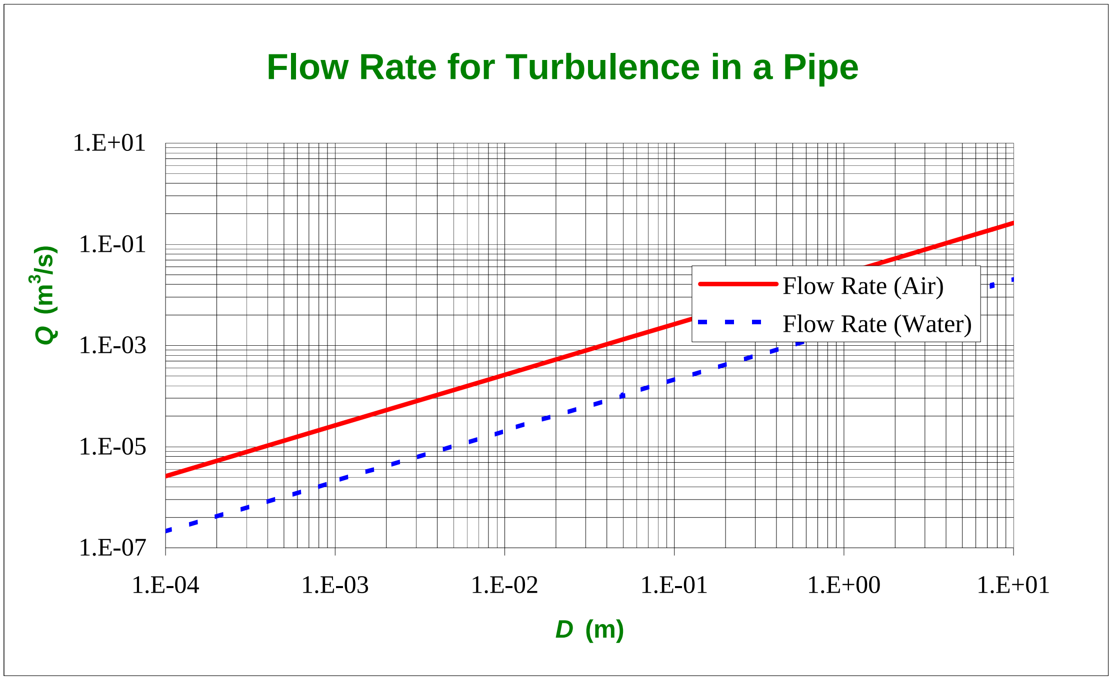
From Tables A.8 and A.10 the data required is

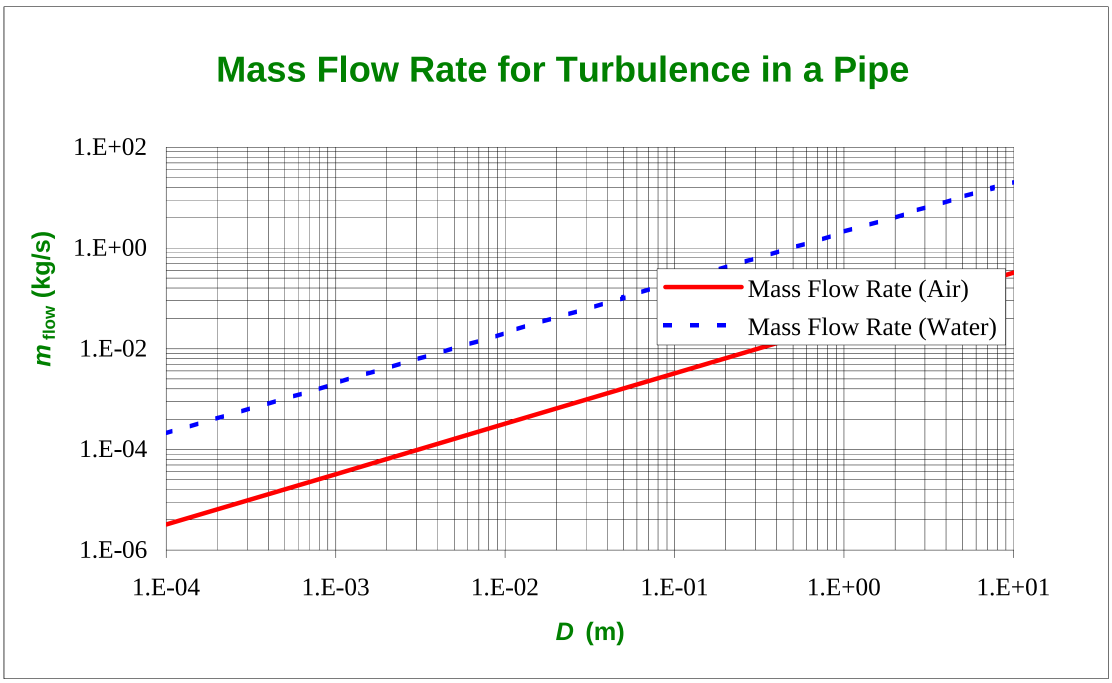
$\rho_{air} =$ 1.23 kg/m^3 $\qquad \nu_{air} =$ 1.45E-05 m^2/s

$\rho_w =$ 999 kg/m^3 $\qquad \nu_w =$ 1.14E-06 m^2/s

***D* (m)**	0.0001	0.001	0.01	0.05	1.0	2.5	5.0	7.5	10.0
V $_{air}$ **(m/s)**	333.500	33.350	3.335	0.667	3.34E-02	1.33E-02	6.67E-03	4.45E-03	3.34E-03
V $_w$ **(m/s)**	26.2	2.62	0.262	5.24E-02	2.62E-03	1.05E-03	5.24E-04	3.50E-04	2.62E-04
Q $_{air}$ **(m^3/s)**	2.62E-06	2.62E-05	2.62E-04	1.31E-03	2.62E-02	6.55E-02	1.31E-01	1.96E-01	2.62E-01
Q $_w$ **(m^3/s)**	2.06E-07	2.06E-06	2.06E-05	1.03E-04	2.06E-03	5.15E-03	1.03E-02	1.54E-02	2.06E-02
m $_{air}$ **(kg/s)**	3.22E-06	3.22E-05	3.22E-04	1.61E-03	3.22E-02	8.05E-02	1.61E-01	2.42E-01	3.22E-01
m $_w$ **(kg/s)**	2.06E-04	2.06E-03	2.06E-02	1.03E-01	2.06E+00	5.14E+00	1.03E+01	1.54E+01	2.06E+01




Flow Rate for Turbulence in a Pipe
Q (m^3/s)
1.E+01
1.E-01
1.E-03
1.E-05
1.E-07
1.E-04
1.E-03
1.E-02
1.E-01
1.E+00
1.E+01
D (m)
Flow Rate (Air)
Flow Rate (Water)


Mass Flow Rate for Turbulence in a Pipe
m flow (kg/s)
1.E+02
1.E+00
1.E-02
1.E-04
1.E-06
1.E-04
1.E-03
1.E-02
1.E-01
1.E+00
1.E+01
D (m)
Mass Flow Rate (Air)
Mass Flow Rate (Water)

Problem 8.5

Given: Laminar flow in the entrance section of a pipe shown schematically in Fig. 8.1.

Find: Sketch centerline velocity, static pressure, and wall shear stress as functions of distance along the pipe. Explain significant features of the plots, comparing them with fully developed flow. Can the Bernoulli equation be applied anywhere in the flow field? If so, where? Explain briefly.

Discussion: The centerline velocity, static pressure, and wall shear stress variations are sketched on the next page. Each variation sketch is aligned vertically with the corresponding sections of the developing pipe flow in Fig. 8.1.

Boundary layers grow on the tube wall, reducing the velocity near the wall. The velocity reduction becomes more pronounced farther downstream. Consequently the centerline velocity must increase in the streamwise direction to carry the same mass flow rate across each section of the tube. (When laminar flow becomes fully developed, the centerline velocity becomes twice the average velocity at any cross-section.)

Frictional effects are concentrated within the boundary layers. The boundary layers do not join at the tube centerline for some distance along the tube. Therefore in the center region outside the boundary layers flow may still be considered to behave as though it were inviscid.

Flow outside the boundary layers is steady, frictionless, incompressible, and along a streamline. These are the restrictions required to apply the Bernoulli equation. Therefore the Bernoulli equation may be applied as a reasonable model for the actual flow outside the boundary layers. The Bernoulli equation predicts that pressure decreases as flow speed increases.

After the boundary layers merge at the centerline of the channel the entire flow is affected by friction. Therefore it is no longer possible to apply the Bernoulli equation.

When flow becomes fully developed the rate of change of pressure with distance becomes constant. In the entrance region the pressure falls more rapidly; the increased pressure gradient is caused by increased shear stress at the wall (larger than for fully developed flow) and by the developing velocity profile, which causes momentum flux to increase.

In fully developed flow the pressure curve becomes linear; the pressure drops the same amount for each length along the tube. The pressure distribution curve at the end of the entrance length becomes asymptotic to the linear variation for fully developed flow.

The wall shear stress initially is large, because the boundary layers are thin. The shear stress decreases as the boundary layers become thicker. At the end of the entrance length the shear stress asymptotically approaches the constant value for fully developed flow.

Problem 8.5

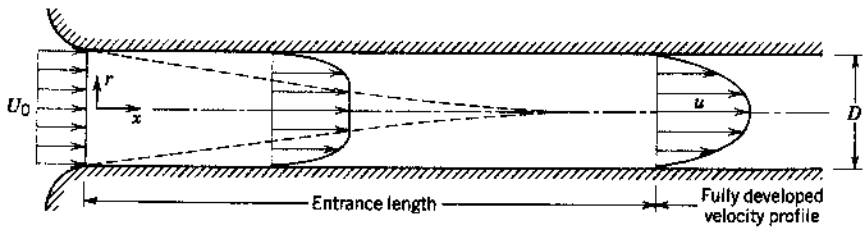
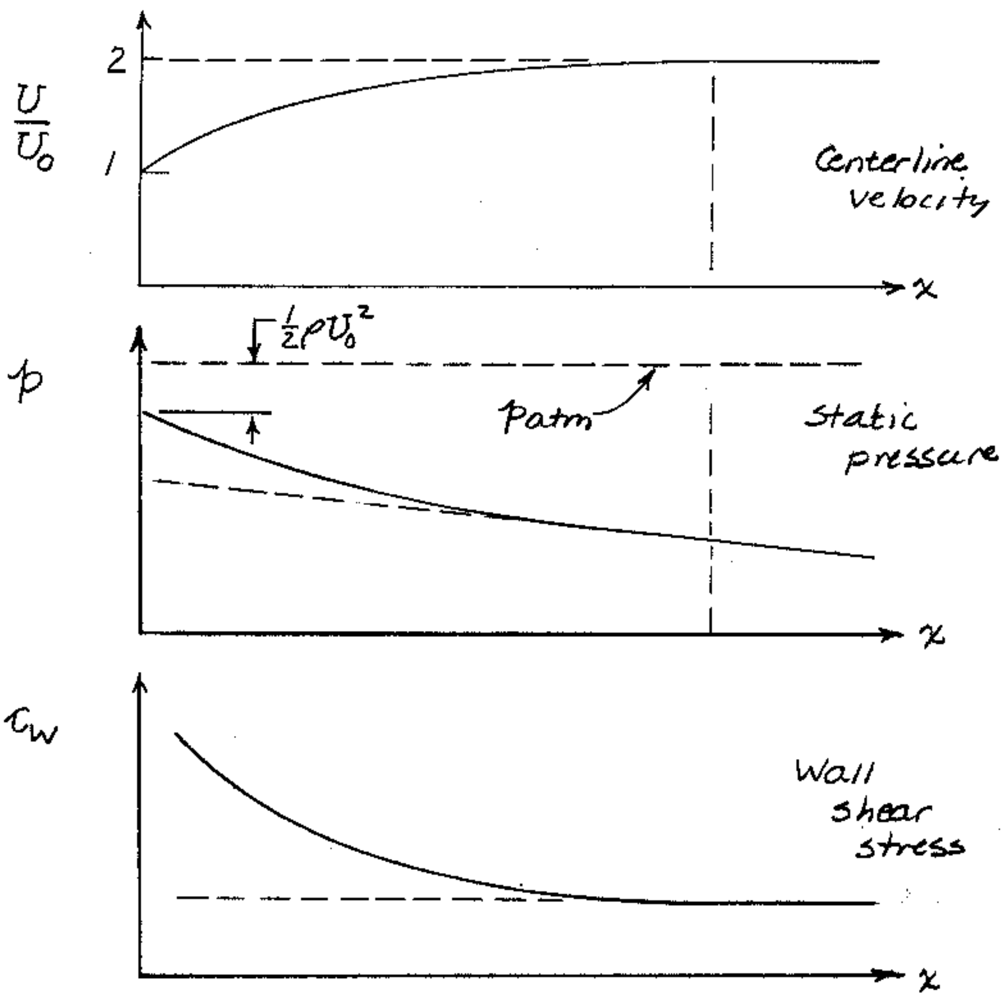


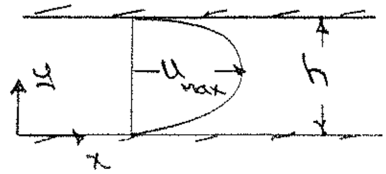
Fig. 8.1 Flow in the entrance region of a pipe.



Problem 8.6

Given: Incompressible flow between parallel plates with

$$u = U_{max}(Ay^2 + By + C)$$



Find: (a) constants A, B, C using appropriate boundary conditions
(b) Q per unit depth b.
(c) $\bar{V}/U_{max}$

Solution:

(a) Available boundary conditions: (1) $y=0$, $u=0$
(2) $y=h$, $u=0$
(3) $y=h/2$, $u=U_{max}$

From B.C (1) $u(0) = 0 = U_{max} C \quad \therefore \quad C = 0$ ← C

From B.C (2) $u(h) = 0 = U_{max}(Ah^2 + Bh)$ ....(i)

From B.C (3) $u(h/2) = U_{max} = U_{max}\left(A\frac{h^2}{4} + B\frac{h}{2}\right)$ ...(ii)

From Eq (i), $B = -Ah$. Substituting into Eq (ii) gives

$$U_{max} = U_{max}\left(A\frac{h^2}{4} - A\frac{h^2}{2}\right) \quad \therefore \quad A = -\frac{4}{h^2}$$ ← A

and $B = -Ah = \frac{4}{h}$ ← B

Then

$$u = U_{max}(Ay^2 + By + C) = U_{max}\left(-4\frac{y^2}{h^2} + 4\frac{y}{h}\right) = 4U_{max}\left[\frac{y}{h} - \left(\frac{y}{h}\right)^2\right]$$

(b) $$Q = \int_0^h u\,b\,dy = \int_0^h 4U_{max}\left[\frac{y}{h} - \frac{y^2}{h^2}\right]b\,dy = 4U_{max}b\left[\frac{y^2}{2h} - \frac{y^3}{3h^2}\right]_0^h$$

$$Q = 4bU_{max}\left[\frac{h}{2} - \frac{h}{3}\right] = \frac{2}{3}U_{max}bh$$

$$Q/b = \frac{2}{3}U_{max}h$$ ← Q/b

(c) Since $Q = \bar{V}A = \bar{V}bh$

$$\frac{Q}{b} = \bar{V}h = \frac{2}{3}U_{max}h$$

and

$$\frac{\bar{V}}{U_{max}} = \frac{2}{3}$$ ← $\frac{\bar{V}}{U_{max}}$

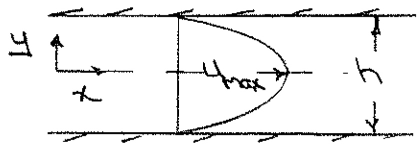
Problem 8.7

[2]

Given: Velocity profile for flow between stationary parallel plates.

$$u = a(h^2/4 - y^2)$$

where a = constant



Find: Ratio $\bar{V}/u_{max}$

Solution: First find u_{max}, by setting $\frac{du}{dy} = 0$

$$\frac{du}{dy} = -2ay \quad ; \quad \frac{du}{dy} = 0 \text{ at } y = 0$$

$$u_{max} = u(0) = a\frac{h^2}{4}$$

From the definition of $\bar{V}$,

$$\bar{V} = \frac{Q}{A} = \frac{1}{A}\int u\,dA = \frac{1}{h}\int_{-h/2}^{h/2} u\,dy$$

$$= \frac{1}{h}\int_{-h/2}^{h/2} a\left(\frac{h^2}{4} - y^2\right) = \frac{a}{h}\left[\frac{h^2 y}{4} - \frac{y^3}{3}\right]_{-h/2}^{h/2}$$

$$\bar{V} = \frac{a}{h}\left[\left(\frac{h^3}{8} - \frac{h^3}{24}\right) - \left(-\frac{h^3}{8} + \frac{h^3}{24}\right)\right] = \frac{a}{h}\left[\frac{h^3}{4} - \frac{h^3}{12}\right]$$

$$\bar{V} = \frac{1}{6}ah^2$$

and

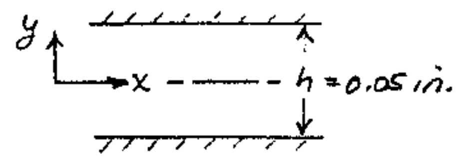
$$\frac{\bar{V}}{u_{max}} = \frac{ah^2}{6}\,\frac{4}{ah^2} = \frac{2}{3}$$

$\frac{\bar{V}}{u_{max}}$

Problem 8.8

Given: Fully developed laminar flow between parallel plates.

$\mu = 2.40 \times 10^{-5} \frac{lbf \cdot s}{ft^2}$; $\frac{\partial p}{\partial x} = -4 \frac{lbf}{ft^3}$



Find: (a) Derive and plot equation for shear stress versus y.
(b) Maximum shear stress.

Solution: From Eq. 8.7, with a = h, $u = -\frac{h^2}{8\mu}\frac{\partial p}{\partial x}\left[1 - \left(\frac{2y}{h}\right)^2\right]$

By symmetry, the origin for y must be located at the channel centerline. Apply Newton's law of viscosity.

$$\tau_{yx} = \mu \frac{du}{dy}$$

Assumption: Newtonian fluid

Then
$$\tau_{yx} = \mu \frac{d}{dy}\left\{-\frac{h^2}{8\mu}\frac{\partial p}{\partial x}\left[1 - \left(\frac{2y}{h}\right)^2\right]\right\} = y\frac{\partial p}{\partial x}$$ τ_{yx}

For $u > 0$, $\partial p/\partial x < 0$. Thus $\tau_{yx} < 0$ for $y > 0$ and $\tau_{yx} > 0$ for $y < 0$.

On the upper plate (a minus y surface), $\tau_{yx} < 0$, so shear stress acts to the right.

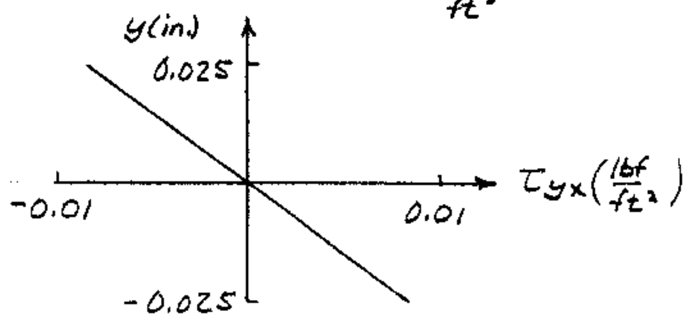
On the lower plate (a plus y surface), $\tau_{yx} > 0$, so shear stress acts to the right.

The maximum stress occurs when $y = \pm h/2$. Thus

$$\tau_{max} = \tau_{yx}\left(\frac{h}{2}\right) = \frac{h}{2}\frac{\partial p}{\partial x} = \frac{1}{2} \times 0.05\,in. \times \frac{ft}{12\,in.} \times \left(-4.0\frac{lbf}{ft^3}\right) = -0.00835\frac{lbf}{ft^2}$$ τ_{max}

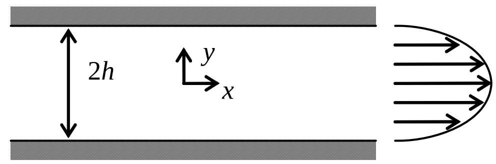
or $\tau_{max} = \tau_{yx}\left(-\frac{h}{2}\right) = 0.00835\frac{lbf}{ft^2}$

Plot:



Problem 8.9

8.9 Viscous oil flows steadily between parallel plates. The flow is fully developed and laminar. The pressure gradient is 1.25 kPa/m and the channel half-width is $h = 1.5$ mm. Calculate the magnitude and direction of the wall shear stress at the upper plate surface. Find the volume flow rate through the channel ($\mu = 0.50\ \mathrm{N \cdot s/m^2}$).



Given: Laminar flow between flat plates

Find: Shear stress on upper plate; Volume flow rate per width

Solution:

Basic equation $\tau_{yx} = \mu \cdot \frac{du}{dy}$ $\quad u(y) = -\frac{h^2}{2\cdot\mu}\cdot\frac{dp}{dx}\cdot\left[1-\left(\frac{y}{h}\right)^2\right]$ (from Eq. 8.7)

Then $\tau_{yx} = \frac{-h^2}{2}\cdot\frac{dp}{dx}\cdot\left(-\frac{2\cdot y}{h^2}\right) = -y\cdot\frac{dp}{dx}$

At the upper surface $y = h$ $\quad \tau_{yx} = -1.5\cdot\mathrm{mm}\times\frac{1\cdot\mathrm{m}}{1000\cdot\mathrm{mm}}\times 1.25\times10^3\cdot\frac{\mathrm{N}}{\mathrm{m^2\cdot m}}$ $\quad \tau_{yx} = -1.88\,\mathrm{Pa}$

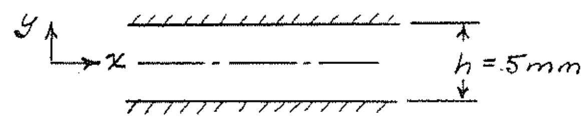
The volume flow rate is $Q = \int u\,dA = \int_{-h}^{h} u\cdot b\,dy = -\frac{h^2\cdot b}{2\cdot\mu}\cdot\frac{dp}{dx}\cdot\int_{-h}^{h}\left[1-\left(\frac{y}{h}\right)^2\right]dy$ $\quad Q = -\frac{2\cdot h^3\cdot b}{3\cdot\mu}\cdot\frac{dp}{dx}$

$$\frac{Q}{b} = -\frac{2}{3}\times\left(1.5\cdot\mathrm{mm}\times\frac{1\cdot\mathrm{m}}{1000\cdot\mathrm{mm}}\right)^3\times1.25\times10^3\cdot\frac{\mathrm{N}}{\mathrm{m^2\cdot m}}\times\frac{\mathrm{m^2}}{0.5\cdot\mathrm{N\cdot s}}$$ $\quad \frac{Q}{b} = -5.63\times10^{-6}\,\frac{\mathrm{m^2}}{\mathrm{s}}$

Problem 8.10

Given: Laminar, fully developed flow between parallel plates

$\mu = 0.5 \frac{N \cdot s}{m^2}$; $\frac{\partial p}{\partial x} = -1000 \frac{N}{m^3}$



Find: (a) Shear stress on upper plate.
(b) Volume flow rate per unit width.

Width = b

Solution: From Eq. 8.7 with $a = h$, $u = -\frac{h^2}{8\mu}\frac{\partial p}{\partial x}\left[1 - \left(\frac{2y}{h}\right)^2\right]$

Then

$$\tau_{yx} = \mu \frac{du}{dy} = -\frac{h^2}{8}\frac{\partial p}{\partial x}\left(-\frac{8y}{h^2}\right) = y\frac{\partial p}{\partial x}$$

At upper surface, $y = h/2$, and

$$\tau_{yx} = \frac{0.005\ m}{2} \times -1000 \frac{N}{m^3} = -2.5\ N/m^2$$

The upper plate is a negative y surface. Thus since $\tau_{yx} < 0$, stress acts to right, in +x direction. τ_{yx}

The volume flow rate is

$$Q = \int_A u\,dA = \int_{-h/2}^{h/2} u\,b\,dy = 2\int_0^{h/2} u\,b\,dy = 2\left(\frac{h}{2}\right)b\int_0^1 u\,d\left(\frac{2y}{h}\right)$$

or

$$\frac{Q}{b} = h\int_0^1 u\,d\eta \quad \text{where } \eta = \frac{2y}{h} \text{ and } u = -\frac{h^2}{8\mu}\frac{\partial p}{\partial x}(1-\eta^2)$$

Thus

$$\frac{Q}{b} = h\int_0^1 -\frac{h^2}{8\mu}\frac{\partial p}{\partial x}(1-\eta^2)\,d\eta = -\frac{h^3}{8\mu}\frac{\partial p}{\partial x}\left(\eta - \frac{1}{3}\eta^3\right)\Big|_0^1 = -\frac{h^3}{12\mu}\frac{\partial p}{\partial x}$$

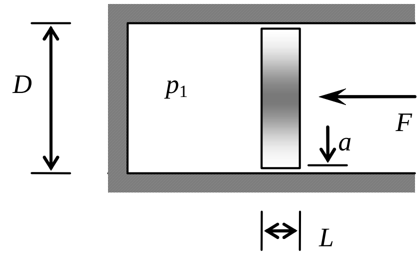
$$\frac{Q}{b} = -\frac{1}{12} \times (0.005)^3 m^3 \times \frac{m^2}{0.5\ N \cdot s} \times -1000 \frac{N}{m^3} = 20.8 \times 10^{-6}\ m^2/s$$

Q/b

Note $u > 0$, so flow is from left to right.

Problem 8.11

8.11 Oil is confined in a 4-in. diameter cylinder by a piston having a radial clearance of 0.001 in. and a length of 2 in. A steady force of 4500 lbf is applied to the piston. Assume the properties of SAE 30 oil at 120°F. Estimate the rate at which oil leaks past the piston.



Given: Piston cylinder assembly

Find: Rate of oil leak

Solution:

Basic equation $\frac{Q}{l} = \frac{a^3 \cdot \Delta p}{12 \cdot \mu \cdot L}$ $\quad Q = \frac{\pi \cdot D \cdot a^3 \cdot \Delta p}{12 \cdot \mu \cdot L}$ (from Eq. 8.6c; we assume laminar flow and verify this is correct after solving)

For the system $\Delta p = p_1 - p_{atm} = \frac{F}{A} = \frac{4 \cdot F}{\pi \cdot D^2}$

$$\Delta p = \frac{4}{\pi} \times 4500 \cdot \text{lbf} \times \left(\frac{1}{4 \cdot \text{in}} \times \frac{12 \cdot \text{in}}{1 \cdot \text{ft}}\right)^2 \qquad \Delta p = 358 \cdot \text{psi}$$

At 120ºF (about 50ºC), from Fig. A.2 $\quad \mu = 0.06 \times 0.0209 \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2} \qquad \mu = 1.25 \times 10^{-3} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$

$$Q = \frac{\pi}{12} \times 4 \cdot \text{in} \times \left(0.001 \cdot \text{in} \times \frac{1 \cdot \text{ft}}{12 \cdot \text{in}}\right)^3 \times 358 \cdot \frac{\text{lbf}}{\text{in}^2} \times \frac{144 \cdot \text{in}^2}{1 \cdot \text{ft}^2} \times \frac{\text{ft}^2}{1.25 \times 10^{-3} \text{lbf} \cdot \text{s}} \times \frac{1}{2 \cdot \text{in}} \qquad Q = 1.25 \times 10^{-5} \cdot \frac{\text{ft}^3}{\text{s}} \qquad Q = 0.0216 \cdot \frac{\text{in}^3}{\text{s}}$$

Check Re: $\quad V = \frac{Q}{A} = \frac{Q}{a \cdot \pi \cdot D} \qquad V = \frac{1}{\pi} \times 1.25 \times 10^{-5} \frac{\text{ft}^3}{\text{s}} \times \frac{1}{.001 \cdot \text{in}} \times \frac{1}{4 \cdot \text{in}} \times \left(\frac{12 \cdot \text{in}}{1 \cdot \text{ft}}\right)^2 \qquad V = 0.143 \cdot \frac{\text{ft}}{\text{s}}$

$$Re = \frac{V \cdot a}{\nu} \qquad \nu = 6 \times 10^{-5} \times 10.8 \frac{\text{ft}^2}{\text{s}} \qquad \nu = 6.48 \times 10^{-4} \cdot \frac{\text{ft}^2}{\text{s}}$$ (at 120ºF, from Fig. A.3)

$$Re = 0.143 \cdot \frac{\text{ft}}{\text{s}} \times 0.001 \cdot \text{in} \times \frac{1 \cdot \text{ft}}{12 \cdot \text{in}} \times \frac{\text{s}}{6.48 \times 10^{-4} \text{ft}^2} \qquad Re = 0.0184$$ so flow is very much laminar

The speed of the piston is approximately

$$V_p = \frac{Q}{\left(\frac{\pi \cdot D^2}{4}\right)} \qquad V_p = \frac{4}{\pi} \times 1.25 \times 10^{-5} \frac{\text{ft}^3}{\text{s}} \times \left(\frac{1}{4 \cdot \text{in}} \times \frac{12 \cdot \text{in}}{1 \cdot \text{ft}}\right)^2 \qquad V_p = 1.432 \times 10^{-4} \cdot \frac{\text{ft}}{\text{s}}$$

The piston motion is negligible so our assumption of flow between parallel plates is reasonable

Problem 8.12

Given: Hydraulic jack supports supports a load of 9000 kg

piston diameter $D = 100$ mm

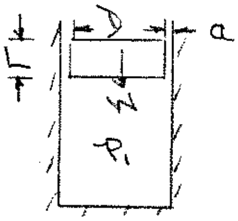
radial clearance $a = 0.05$ mm

piston length $L = 120$ mm

Fluid has viscosity of SAE 30 oil at 30°C

Find: Leakage rate of fluid past the piston

Solution:



Model the flow as steady, fully developed laminar flow between stationary parallel plates, i.e., neglect motion of the piston.

Then, the leakage flow rate can be evaluated from Eq. 8.6c (in the text)

$$\frac{Q}{l} = \frac{a^3 \Delta p}{12 \mu L} \quad \text{where } l = \pi D$$

From Fig. A.2 at $T = 30°C$, $\mu = 3.0 \times 10^{-1}$ N.s/m²

$$\Delta p = p_1 - p_{atm} \quad \text{and} \quad p_1 = \frac{W}{A} = \frac{Mg}{A} = \frac{4Mg}{\pi D^2}$$

$$p_1 = \frac{4}{\pi} \times 9000\,\text{kg} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times \frac{1}{(0.1\,\text{m})^2} \times \frac{\text{N.s}^2}{\text{kg.m}} = 11.2\,\text{MPa}$$

$$Q = \frac{\pi D a^3 \Delta p}{12 \mu L} = \frac{\pi}{12} \times (0.1\,\text{m}) \times (5 \times 10^{-5}\,\text{m})^3 \times 11.2 \times 10^6\,\frac{\text{N}}{\text{m}^2} \times \frac{1}{0.3}\,\frac{\text{m}^2}{\text{N.s}} \times \frac{1}{0.12\,\text{m}}$$

$$Q = 1.01 \times 10^{-6}\,\text{m}^3/\text{s} = 1.01 \times 10^{-3}\,\text{L/s} \qquad Q$$

Check $Re = \frac{\rho a \bar{V}}{\mu} = \frac{a \bar{V}}{\nu}$ where $\nu = 2.8 \times 10^{-4}$ m²/s (Fig. A.3)

$$\bar{V} = \frac{Q}{A} = \frac{Q}{al} = \frac{Q}{a \pi D} = \frac{1}{\pi} \times 1.01 \times 10^{-6}\,\frac{\text{m}^3}{\text{s}} \times \frac{1}{5 \times 10^{-5}\,\text{m}} \times \frac{1}{0.1\,\text{m}} = 0.0643\,\text{m/s}$$

$$Re = \frac{a \bar{V}}{\nu} = 5 \times 10^{-5}\,\text{m} \times 0.0643\,\frac{\text{m}}{\text{s}} \times \frac{1}{2.8 \times 10^{-4}}\,\frac{\text{s}}{\text{m}^2} = 0.011$$

$\therefore$ flow is definitely laminar

Piston moving down at speed v displaces liquid at rate Q where

$$Q = \frac{\pi D^2}{4} v$$

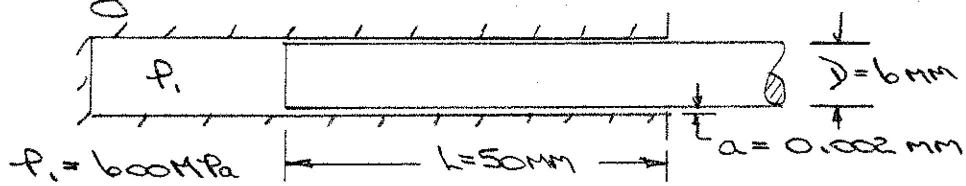
Then

$$v = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 1.01 \times 10^{-6}\,\frac{\text{m}^3}{\text{s}} \times \frac{1}{(0.1\,\text{m})^2} = 1.29 \times 10^{-4}\,\text{m/s}$$

Since $\frac{v}{\bar{V}} = \frac{1.29 \times 10^{-4}\,\text{m/s}}{0.0643\,\text{m/s}} = 2.0 \times 10^{-3}$, motion of piston can be neglected.

Problem 8.13

Given: Piston-cylinder device with SAE 10W oil at 35°



Find: Leakage flow rate

Solution: Computing equation: $\frac{Q}{l} = \frac{a^3 \Delta p}{12 \mu L}$ (8.6c)

Assumptions: (1) Laminar flow
(2) Fully developed flow ($L \gg a$)

For SAE 10W oil at 35°C, $\mu = 3.8 \times 10^{-2}$ N·s/m² (Fig. A.2)

For this configuration, $l = \pi D$, since $a \ll D$. Then

$$Q = \frac{a^3 \Delta p\, l}{12 \mu L} = \frac{\pi a^3 \Delta p\, D}{12 \mu L}$$

$$Q = \frac{\pi}{12} \times (2 \times 10^{-6}\, m)^3 \times 6 \times 10^8 \frac{N}{m^2} \times 0.006\, m \times \frac{1}{3.8 \times 10^{-2}} \frac{m^2}{N \cdot s} \times \frac{1}{0.05\, m}$$

$$Q = 3.97 \times 10^{-9}\, m^3/s = 3.97 \times 10^{-6}\, L/s$$ Q

Check Re to assure laminar flow

$$\bar{V} = \frac{Q}{A} = \frac{Q}{\pi D a} = \frac{1}{\pi} \times 3.97 \times 10^{-9} \frac{m^3}{s} \times \frac{1}{0.006\, m} \times \frac{1}{2 \times 10^{-6}\, m} = 0.105\ m/s$$

SG = 0.88 (Table A.2); $\rho = SG\, \rho_{H_2O}$

$$Re = \frac{\rho \bar{V} a}{\mu} = \frac{SG\, \rho_{H_2O} \bar{V} a}{\mu}$$

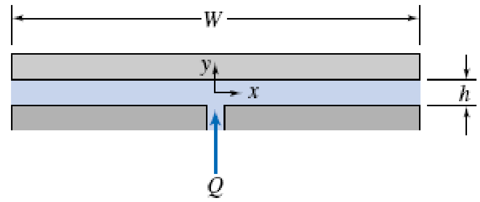
$$= 0.88 \times 999 \frac{kg}{m^3} \times 0.105 \frac{m}{s} \times 2 \times 10^{-6}\, m \times \frac{1}{3.8 \times 10^{-2}} \frac{m^2}{N \cdot s}$$

Re = 0.005 ≪ 2300 so flow is definitely laminar!

Problem 8.14

[3]

8.14 A hydrostatic bearing is to support a load of 50,000 N per meter of length perpendicular to the diagram. The bearing is supplied with SAE 30 oil at 35°C and 700 kPa (gage) through the central slit. Since the oil is viscous and the gap is small, the flow may be considered fully developed. Calculate (a) the required width of the bearing pad, (b) the resulting pressure gradient, *dp/dx*, and (c) the gap height, if $Q = 1$ mL/min per meter of length.



Given:
Hydrostatic bearing

Find:
Required pad width; Pressure gradient; Gap height

Solution:

For a laminar flow (we will verify this assumption later), the pressure gradient is consta $p(x) = p_i \cdot \left(1 - \frac{2 \cdot x}{W}\right)$

where p_i = 700 kPa is the inlet pressure (gage)

Hence the total force in the y direction due to pressure is $F = b \cdot \int p\,dx$ where b is the pad width into the paper

$$F = b \cdot \int_{-\frac{W}{2}}^{\frac{W}{2}} p_i \cdot \left(1 - \frac{2 \cdot x}{W}\right) dx \qquad F = p_i \cdot \frac{b \cdot W}{2}$$

This must be equal to the applied load F. Hence $W = \frac{2}{p_i} \cdot \frac{F}{b}$ $\qquad W = 2 \times \frac{m^2}{700 \times 10^3 \cdot N} \times \frac{50000 \cdot N}{m}$ $\qquad W = 0.143\,m$

The pressure gradient is then $\frac{dp}{dx} = -\frac{\Delta p}{\frac{W}{2}} = -\frac{2 \cdot \Delta p}{W} = -2 \times \frac{700 \times 10^3 \cdot N}{m^2} \times \frac{1}{0.143 \cdot m} = -9.79 \cdot \frac{MPa}{m}$

The flow rate is given $\frac{Q}{l} = -\frac{h^3}{12 \cdot \mu} \cdot \left(\frac{dp}{dx}\right)$ (Eq. 8.6c)

Hence, for h we have $h = \left(-\frac{12 \cdot \mu \cdot \frac{Q}{l}}{\frac{dp}{dx}}\right)^{\frac{1}{3}}$ At 35°C, from Fig. A.2 $\mu = 0.15 \cdot \frac{N \cdot s}{m^2}$

$$h = \left[-12 \times \left(-\frac{m^3}{9.79 \times 10^6 \cdot N}\right) \times 0.15 \cdot \frac{N \cdot s}{m^2} \times \frac{1 \cdot mL}{min \cdot m} \times \frac{10^{-6} \cdot m^3}{1 \cdot mL} \times \frac{1 \cdot min}{60 \cdot s}\right]^{\frac{1}{3}} \qquad h = 1.452 \times 10^{-5} m$$

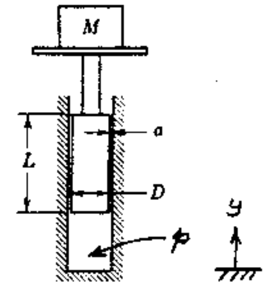
Check Re: $Re = \frac{V \cdot D}{\nu} = \frac{D}{\nu} \cdot \frac{Q}{A} = \frac{h}{\nu} \cdot \frac{Q}{b \cdot h} = \frac{1}{\nu} \cdot \frac{Q}{l}$ $\qquad \nu = 1.6 \times 10^{-4} \frac{m^2}{s}$ (at 35°C, from Fig. A.3)

$$Re = \frac{s}{1.6 \times 10^{-4} \cdot m^2} \times \frac{1 \cdot mL}{min \cdot m} \times \frac{10^{-6} \cdot m^3}{1 \cdot mL} \times \frac{1 \cdot min}{60 \cdot s} \qquad Re = 1.04 \times 10^{-4}$$

so flow is very much laminar

Problem 8.15

Given: Piston-cylinder device, as shown.



$D = 6\ mm \quad L = 25\ mm$

Liquid is SAE 30 oil at 20°C.

Find: (a) M to develop $p = 1.5$ MPa (gage)
(b) Leakage flow rate in terms of a
(c) Maximum a to provide < 1 mm/min movement.

Solution: The mass may be found from a force balance on the piston.

$$\Sigma F_y = \frac{\pi D^2}{4}(p - p_{atm}) - Mg = 0 \qquad \text{so} \quad M = \frac{\pi D^2}{4g} p_{gage}$$

$$M = \frac{\pi}{4} \times (0.006)^2 m^2 \times 1.5\times10^6 \frac{N}{m^2} \times \frac{s^2}{9.81\ m} \times \frac{kg \cdot m}{N \cdot s^2} = 4.32\ kg$$ M

The leakage flow rate may be evaluated for flow between flat plates. From Eq. 8.6c, neglecting motion of the piston,

$$\frac{Q}{l} = \frac{a^3 \Delta p}{12 \mu L} \qquad \text{or, since } l = \pi D, \quad Q = \frac{\pi}{12} \frac{a^3 \Delta p D}{\mu L} \sim a^3$$ Q

The piston, moving downward at speed, v, displaces liquid at rate

$$Q = \frac{\pi D^2}{4} v = \frac{\pi}{4}(0.006)^2 m^2 \times 0.001 \frac{m}{min} \times \frac{min}{60\ s} = 4.71\times10^{-10}\ m^3/s$$

Then, with $\mu = 0.42\ N\cdot sec/m^2$ (at 20°C, Fig. A.2),

$$a = \left[\frac{12 \mu Q L}{\pi D \Delta p}\right]^{1/3} = \left[\frac{12}{\pi} \times 0.42 \frac{N \cdot s}{m^2} \times 4.71\times10^{-10} \frac{m^3}{s} \times 0.025\ m \times \frac{1}{0.006\ m} \times \frac{m^2}{1.5\times10^6\ N}\right]^{1/3}$$

$$a = 1.28\times10^{-5}\ m \quad (12.8\ \mu m)$$ a

Check assumptions: $\bar{V} = \frac{Q}{A} = \frac{Q}{\pi D a} = \frac{1}{\pi} \times 4.71\times10^{-10} \frac{m^3}{s} \times \frac{1}{0.006\ m} \times \frac{1}{1.28\times10^{-5}\ m} = 1.95 \frac{mm}{s}$

Thus $\frac{v}{\bar{V}} = 1 \frac{mm}{min} \times \frac{sec}{1.95\ mm} \times \frac{min}{60\ s} = 0.00855 < 0.01$

Therefore piston motion is negligible.

Also $Re = \frac{\bar{V} a}{\nu}$; $\nu = \frac{\mu}{\rho} = \frac{\mu}{SG\, \rho_{H_2O}}$. From Table A.2 (Appendix A), SG = 0.92

$$\nu = 0.42 \frac{N \cdot s}{m^2} \times \frac{m^3}{(0.92)1000\ kg} \times \frac{kg \cdot m}{N \cdot s^2} = 4.57\times10^{-4}\ m^2/s$$

$$Re = 1.95\times10^{-3} \frac{m}{s} \times 1.28\times10^{-5} m \times \frac{s}{4.57\times10^{-4}\ m^2} = 5.46\times10^{-5} \lll 1$$

Therefore flow is surely laminar!

Problem 8.16

8.16 In Section 8-2 we derived the velocity profile between parallel plates (Eq. 8.5) by using a differential control volume. Instead, following the procedure we used in Example 5.9, derive Eq. 8.5 by starting with the Navier-Stokes equations (Eqs. 5.27). Be sure to state all assumptions.

Given: Navier-Stokes Equations

Find: Derivation of Eq. 8.5

Solution:

The Navier-Stokes equations are

$$\overset{4}{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y} + \overset{3}{\frac{\partial w}{\partial z}} = 0 \tag{5.1c}$$

$$\rho\left(\overset{1}{\frac{\partial u}{\partial t}} + u\overset{4}{\frac{\partial u}{\partial x}} + v\overset{5}{\frac{\partial u}{\partial y}} + w\overset{3}{\frac{\partial u}{\partial z}}\right) = \overset{6}{\rho g_x} - \frac{\partial p}{\partial x} + \mu\left(\overset{4}{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \overset{3}{\frac{\partial^2 u}{\partial z^2}}\right) \tag{5.27a}$$

$$\rho\left(\overset{1}{\frac{\partial v}{\partial t}} + u\overset{4}{\frac{\partial v}{\partial x}} + v\overset{5}{\frac{\partial v}{\partial y}} + w\overset{3}{\frac{\partial v}{\partial z}}\right) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\overset{4}{\frac{\partial^2 v}{\partial x^2}} + \overset{5}{\frac{\partial^2 v}{\partial y^2}} + \overset{3}{\frac{\partial^2 v}{\partial z^2}}\right) \tag{5.27b}$$

$$\rho\left(\overset{1}{\frac{\partial w}{\partial t}} + u\overset{3}{\frac{\partial w}{\partial x}} + v\overset{3}{\frac{\partial w}{\partial y}} + w\overset{3}{\frac{\partial w}{\partial z}}\right) = \overset{3}{\rho g_z} - \overset{3}{\frac{\partial p}{\partial z}} + \mu\left(\overset{3}{\frac{\partial^2 w}{\partial x^2}} + \overset{3}{\frac{\partial^2 w}{\partial y^2}} + \overset{3}{\frac{\partial^2 w}{\partial z^2}}\right) \tag{5.27c}$$

The following assumptions have been applied:

(1) Steady flow (given).
(2) Incompressible flow; ρ = constant.
(3) No flow or variation of properties in the z direction; w= 0 and $\partial/\partial z = 0$.
(4) Fully developed flow, so no properties except pressure p vary in the x direction; $\partial/\partial x = 0$.
(5) See analysis below.
(6) No body force in the x direction; $g_x = 0$

Assumption (1) eliminates time variations in any fluid property. Assumption (2) eliminates space variations in density. Assumption (3) states that there is no z component of velocity and no property variations in the z direction. All terms in the z component of the Navier–Stokes equation cancel. After assumption (4) is applied, the continuity equation reduces to $\partial v/\partial y = 0$. Assumptions (3) and (4) also indicate that $\partial v/\partial z = 0$ and $\partial v/\partial x = 0$. Therefore v must be constant. Since v is zero at the solid surface, then v must be zero everywhere. The fact that $v = 0$ reduces the Navier–Stokes equations further, as indicated by (5). Hence for the y direction

$$\frac{\partial p}{\partial y} = \rho g$$

which indicates a hydrostatic variation of pressure. In the x direction, after assumption (6) we obtain

$$\mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial p}{\partial x} = 0$$

Integrating twice

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}y^2 + \frac{c_1}{\mu}y + c_2$$

To evaluate the constants, c_1 and c_2, we must apply the boundary conditions. At $y = 0$, $u = 0$. Consequently, $c_2 = 0$. At $y = a$, $u = 0$. Hence

$$0 = \frac{1}{2\mu}\frac{\partial p}{\partial x}a^2 + \frac{c_1}{\mu}a$$

which gives

$$c_1 = -\frac{1}{2\mu}\frac{\partial p}{\partial x}a$$

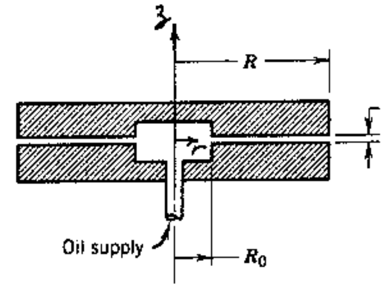
and finally

$$u = \frac{a^2}{2\mu}\frac{\partial p}{\partial x}\left[\left(\frac{y}{a}\right)^2 - \left(\frac{y}{a}\right)\right]$$

Problem 8.17

Given: Viscous flow in narrow gap between parallel disks, as shown.

Flow rate is Q, accelerations are small.
Velocity profile same as fully developed.



Find: (a) Expression for $\bar{V}(r)$, (b) dp/dr in gap
(c) Expression for $p(r)$.
(d) Show net force to hold upper plate is

$$F = \frac{3\mu Q R^2}{h^3}\left[1-\left(\frac{R_0}{R}\right)^2\right]$$

Solution: From the definition of mean velocity, $Q = \bar{V}2\pi r h$ so $\bar{V} = \frac{Q}{2\pi r h}$ $\quad \bar{V}(r)$

The pressure change with radius can be evaluated by analogy to Eq. 8.6b

$$\frac{Q}{l} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)h^3 \quad \text{with } l = 2\pi r \quad \text{so } \frac{Q}{2\pi r} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial r}\right)h^3$$

Thus

$$\frac{dp}{dr} = -\frac{6\mu Q}{\pi h^3 r}$$ $\quad \frac{dp}{dr}$

Integrating to find $p(r)$,

$$\int_p^{p_{atm}} dp = p_{atm} - p = \int_r^R -\frac{6\mu Q}{\pi h^3 r}dr = -\frac{6\mu Q}{\pi h^3}\ln r\Big]_r^R = \frac{6\mu Q}{\pi h^3}\ln(r/R)$$

Thus $p(r) = p_{atm} - \frac{6\mu Q}{\pi h^3}\ln(r/R) \quad (R_0 < r < R) \; ; \; p = p_0 \quad r < R_0$ $\quad p(r)$

The force on the upper plate is $dF_z = (p(r) - p_{atm})2\pi r dr$

Integrating and using gage pressures (note $p_{0g} = -\frac{6\mu Q}{\pi h^3}\ln\left(\frac{R_0}{R}\right)$)

$$F_z = p_0\pi R_0^2 + \int_{R_0}^R p(r)\,2\pi r\,dr = p_0\pi R_0^2 + 2\pi R^2\int_{R_0/R}^1 p(r)\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right)$$

$$= p_0\pi R_0^2 + 2\pi R^2\int_{R_0/R}^1 -\frac{6\mu Q}{\pi h^3}\ln\left(\frac{r}{R}\right)\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right) = p_0\pi R_0^2 - \frac{12\mu Q R^2}{h^3}\left(\frac{r}{R}\right)^2\left[\frac{1}{2}\ln\left(\frac{r}{R}\right) - \frac{1}{4}\right]_{R_0/R}^1$$

$$= p_0\pi R_0^2 - \frac{12\mu Q R^2}{h^3}\left\{(1)\left[\frac{1}{2}(0) - \frac{1}{4}\right] - \left(\frac{R_0}{R}\right)^2\left[\frac{1}{2}\ln\left(\frac{R_0}{R}\right) - \frac{1}{4}\right]\right\}$$

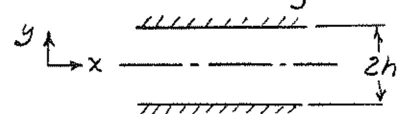
$$= -\frac{6\mu Q R^2}{h^3}\left(\frac{R_0}{R}\right)^2\ln\left(\frac{R_0}{R}\right) - \frac{6\mu Q R^2}{h^3}\left[-\frac{1}{2} - \left(\frac{R_0}{R}\right)^2\ln\left(\frac{R_0}{R}\right) + \frac{1}{2}\left(\frac{R_0}{R}\right)^2\right]$$

$$F_z = \frac{3\mu Q R^2}{h^3}\left[1 - \left(\frac{R_0}{R}\right)^2\right]$$ $\quad F_z$

Problem 8.18

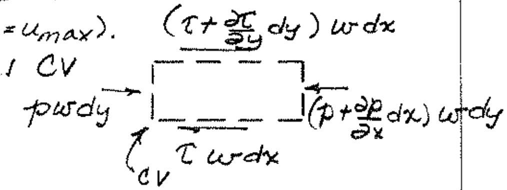
Given: Power-law model for non-Newtonian liquid, $\tau_{yx} = k\left(\frac{du}{dy}\right)^n$

Find: Show $u = \left(\frac{h}{k}\frac{\Delta p}{L}\right)^{1/n}\frac{nh}{n+1}\left[1-\left(\frac{y}{h}\right)^{\frac{n+1}{n}}\right]$ for fully developed laminar flow between plates.



Plot: Profiles u/U vs. y/h for $n = 0.7, 1.0,$ and 1.3 ($U = u_{max}$).

Solution: Apply momentum equation to differential CV



Basic equation:

$$F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$$

($F_{B_x} = 0(1)$, $\frac{\partial}{\partial t}\int_{CV} = 0(2)$, $\int_{CS} = 0(3)$)

Assumptions: (1) Horizontal flow
(2) Steady flow
(3) Fully developed flow

Then

$$p\,w\,dy + \left(\tau + \frac{\partial \tau}{\partial y}dy\right)w\,dx - \left(p + \frac{\partial p}{\partial x}dx\right)w\,dy - \tau\, w\,dx = 0 \quad \text{or} \quad \frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}$$

Since $\tau = \tau(y)$ and $p = p(x)$, then $\frac{d\tau}{dy} = \frac{\partial p}{\partial x} = \text{constant}$ and $\tau = y\frac{\partial p}{\partial x}$ or

$$\tau_{yx} = k\left(\frac{du}{dy}\right)^n = y\frac{\partial p}{\partial x} = -y\frac{\Delta p}{L}$$

Thus

$$\frac{du}{dy} = -\left(\frac{1}{k}\frac{\Delta p}{L}\right)^{1/n} y^{1/n}$$

Integrating

$$u = -\left(\frac{1}{k}\frac{\Delta p}{L}\right)^{1/n}\frac{1}{1/n+1}\, y^{1/n+1} + c = -\left(\frac{1}{k}\frac{\Delta p}{L}\right)^{\frac{1}{n}}\frac{n}{n+1}\, y^{\frac{n+1}{n}} + c$$

But $u = 0$ at $y = h$, so

$$c = \left(\frac{1}{k}\frac{\Delta p}{L}\right)^{\frac{1}{n}}\frac{n}{n+1}\, h^{\frac{n+1}{n}}$$

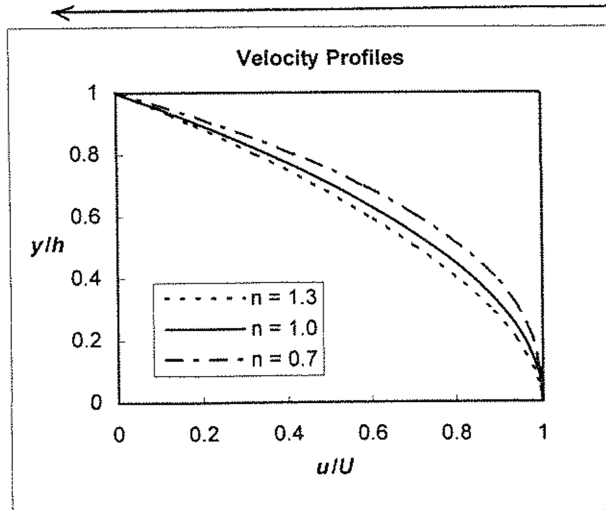
and

$$u = \left(\frac{1}{k}\frac{\Delta p}{L}\right)^{\frac{1}{n}}\frac{n}{n+1}\, h^{\frac{n+1}{n}}\left[1-\left(\frac{y}{h}\right)^{\frac{n+1}{n}}\right]$$

or

$$u = \left(\frac{h}{k}\frac{\Delta p}{L}\right)^{\frac{1}{n}}\frac{nh}{n+1}\left[1-\left(\frac{y}{h}\right)^{\frac{n+1}{n}}\right]$$

	$n = 0.7$	$n = 1.0$	$n = 1.3$
y/h	u/U	u/U	u/U
0	1	1	1
0.03	1.000	0.999	0.998
0.06	0.999	0.996	0.993
0.1	0.996	0.990	0.983
0.2	0.980	0.960	0.942
0.3	0.946	0.910	0.881
0.4	0.892	0.840	0.802
0.5	0.814	0.750	0.707
0.6	0.711	0.640	0.595
0.7	0.580	0.510	0.468
0.8	0.418	0.360	0.326
0.9	0.226	0.190	0.170
1	0	0	0



Problem 8.19

8.19 Using the profile of Problem 8.18, show that the flow rate for fully developed laminar flow of a power-law fluid between stationary parallel plates may be written as

$$Q = \left(\frac{h}{k}\frac{\Delta p}{L}\right)^{1/n} \frac{2nwh^2}{2n+1}$$

Here w is the plate width. In such an experimental setup the following data on applied pressure difference Δp and flow rate Q were obtained:

Δp (kPa)	10	20	30	40	50	60	70	80	90	100
Q (L/min)	0.451	0.759	1.01	1.15	1.41	1.57	1.66	1.85	2.05	2.25

Determine if the fluid is pseudoplastic or dilatant, and obtain an experimental value for n.

Given: Laminar velocity profile of power-law fluid flow between parallel plates

Find: Expression for flow rate; from data determine the type of fluid

Solution:

The velocity profile is
$$u = \left(\frac{h}{k}\cdot\frac{\Delta p}{L}\right)^{\frac{1}{n}}\cdot\frac{n\cdot h}{n+1}\cdot\left[1-\left(\frac{y}{h}\right)^{\frac{n+1}{n}}\right]$$

The flow rate is then
$$Q = w\cdot\int_{-h}^{h} u\,dy$$
or, because the flow is symmetric
$$Q = 2\cdot w\cdot\int_{0}^{h} u\,dy$$

The integral is computed as
$$\int 1-\left(\frac{y}{h}\right)^{\frac{n+1}{n}} dy = y\cdot\left[1-\frac{n}{2\cdot n+1}\cdot\left(\frac{y}{h}\right)^{\frac{2\cdot n+1}{n}}\right]$$

Using this with the limits
$$Q = 2\cdot w\cdot\left(\frac{h}{k}\cdot\frac{\Delta p}{L}\right)^{\frac{1}{n}}\cdot\frac{n\cdot h}{n+1}\cdot h\cdot\left[1-\frac{n}{2\cdot n+1}\cdot(1)^{\frac{2\cdot n+1}{n}}\right]$$
$$Q = \left(\frac{h}{k}\cdot\frac{\Delta p}{L}\right)^{\frac{1}{n}}\cdot\frac{2\cdot n\cdot w\cdot h^2}{2\cdot n+1}$$

The associated *Excel* spreadsheet shows computation of *n*.

Problem 8.19

8.19 Using the profile of Problem 8.18, show that the flow rate for fully developed laminar flow of a power-law fluid between stationary parallel plates may be written as

$$Q = \left(\frac{h}{k}\frac{\Delta p}{L}\right)^{1/n} \frac{2nwh^2}{2n+1}$$

Here w is the plate width. In such an experimental setup the following data on applied pressure difference Δp and flow rate Q were obtained:

Δp (kPa)	10	20	30	40	50	60	70	80	90	100
Q (L/min)	0.451	0.759	1.01	1.15	1.41	1.57	1.66	1.85	2.05	2.25

Determine if the fluid is pseudoplastic or dilatant, and obtain an experimental value for n.

Given: Laminar velocity profile of power-law fluid flow between parallel plates

Find: Expression for flow rate; from data determine the type of fluid

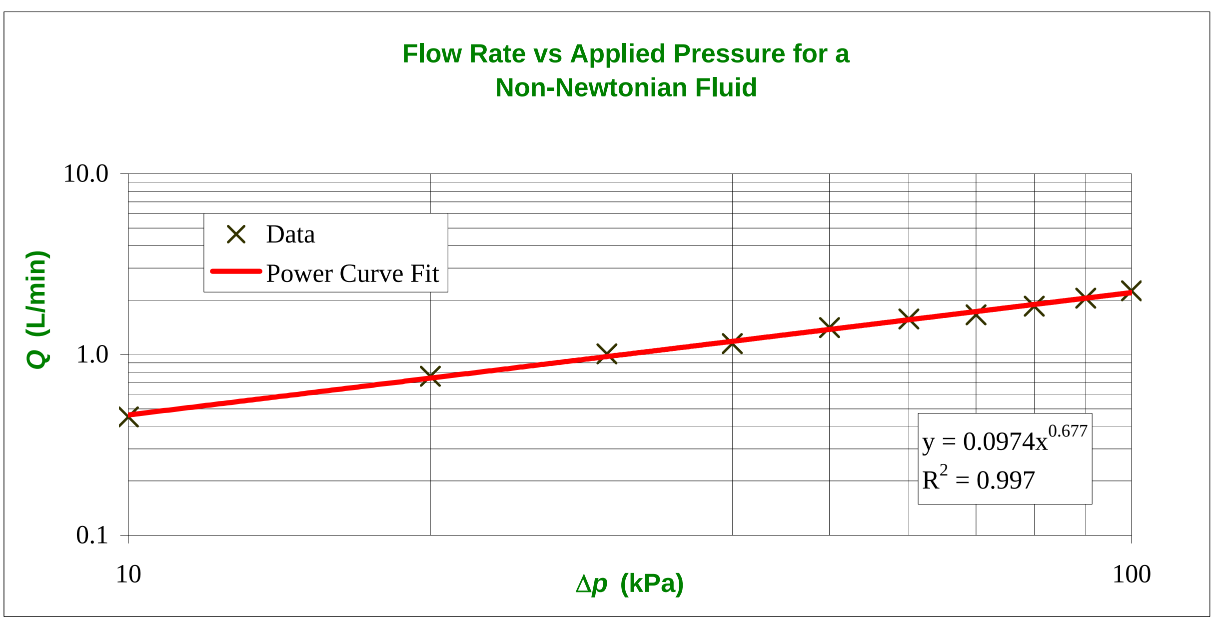
Solution:

The data is

Δp (kPa)	10	20	30	40	50	60	70	80	90	100
Q (L/min)	0.451	0.759	1.01	1.15	1.41	1.57	1.66	1.85	2.05	2.25

This must be fitted to $Q = \left(\frac{h}{k}\cdot\frac{\Delta p}{L}\right)^{\frac{1}{n}} \cdot \frac{2\cdot n\cdot w\cdot h^2}{2\cdot n+1}$ or $Q = k\cdot\Delta p^{\frac{1}{n}}$

We can fit a power curve to the data



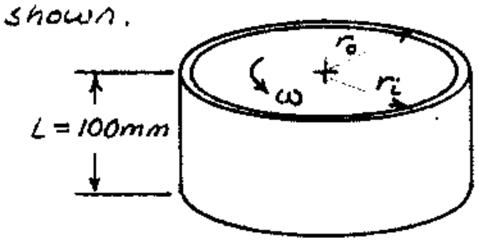
Hence $1/n =$ 0.677 $n =$ 1.48

Problem 8.20

[2]

Given: Sealed journal bearing rotating as shown.

$r_o = 26\ mm,\ r_i = 25\ mm$

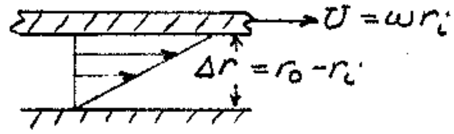


Gap contains oil in laminar motion with linear velocity profile.

$\omega = 2800\ rpm$ and Torque, $T = 0.2\ N\cdot m$

Find: (a) Viscosity of oil
(b) Will torque increase or decrease with time? Why?

Solution: "Unfold" bearing since gap is small, and consider as flow between parallel plates. Apply Newton's law of viscosity.



Basic equation: $\tau_{yx} = \mu \frac{du}{dy}$

Assumption: Linear velocity profile

Then $\tau_{yx} = \mu \frac{U}{\Delta r} = \frac{\mu \omega r_i}{\Delta r}$

and

$$T = r_i\ (2\pi r_i L \tau_{yx}) = 2\pi r_i^2 L \tau_{yx} = \frac{2\pi \mu \omega r_i^3 L}{\Delta r}$$

Solving, $\mu = \frac{\Delta r\, T}{2\pi \omega r_i^3 L}$

$$\mu = \frac{1}{2\pi} \times 0.001\, m \times 0.2\, N\cdot m \times \frac{min}{2800\, rev} \times \frac{1}{(0.025)^3 m^3} \times \frac{1}{0.1\, m} \times \frac{rev}{2\pi\, rad} \times \frac{60\, s}{min}$$

$$\mu = 0.0695\ N\cdot s / m^2$$ μ

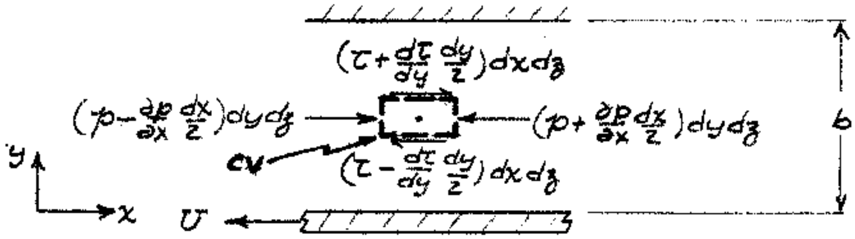
Bearing is sealed, so oil temperature will increase as energy is dissipated by friction. For liquids, μ decreases as T increases. Thus torque will decrease, since it is proportional to μ.

Problem 8.21

Given: Water at 60°C flows between large flat plates.

$U = 0.3\ m/s$

$b = 3\ mm$



Find: Pressure gradient required for zero net flow at a section.

Solution: Apply momentum equation using CV and coordinates shown.

Basic equations:

$$F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\rho\, dV + \int_{CS} u\rho \vec{V}\cdot d\vec{A}, \quad \tau = \tau_{yx} = \mu\frac{du}{dy}$$

($F_{B_x} = 0(1)$, $\frac{\partial}{\partial t}\int = 0(2)$, $\int_{CS} = 0(3)$)

Assumptions: (1) $F_{Bx} = 0$
(2) Steady flow
(3) Fully-developed flow
(4) Newtonian fluid

Then $F_{Sx} = 0$. Substituting the force terms (see page 315 for details) gives

$$\frac{\partial p}{\partial x} = \frac{d\tau_{yx}}{dy} = \frac{d}{dy}\left(\mu\frac{du}{dy}\right) = \mu\frac{d^2u}{dy^2} \quad \text{or} \quad \frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

Integrating twice,

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}y^2 + c_1 y + c_2$$

To evaluate the constants c_1 and c_2, we must use the boundary conditions. At $y=0$, $u=-U$, so $c_2 = -U$. At $y=b$, $u=0$, so

$$0 = \frac{1}{2\mu}\frac{\partial p}{\partial x}b^2 + c_1 b - U \quad \text{or} \quad c_1 = \frac{U}{b} - \frac{1}{2\mu}\frac{\partial p}{\partial x}b$$

Thus

$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - by) + U\left(\frac{y}{b} - 1\right)$$

To find the flowrate, we integrate

$$\frac{Q}{w} = \int_0^b u\,dy = \int_0^b\left[\frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - by) + U\left(\frac{y}{b} - 1\right)\right]dy = -\frac{1}{12\mu}\frac{\partial p}{\partial x}b^3 - \frac{Ub}{2}$$

For $Q = 0$, with $\mu = 4.63\times10^{-4}\ \frac{N\cdot s}{m^2}$ from Table A.8,

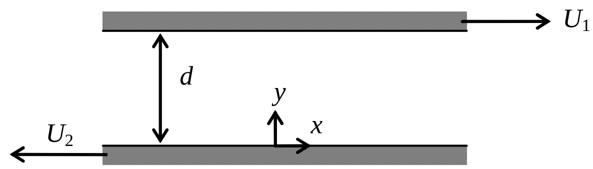
$$\frac{\partial p}{\partial x} = -\frac{6U\mu}{b^2} = -6\times 0.3\frac{m}{s}\times 4.63\times10^{-4}\frac{N\cdot s}{m^2}\times\frac{1}{(0.003)^2 m^2} = -92.6\ N/m^2\cdot m \qquad \frac{\partial p}{\partial x}$$

Thus pressure must **decrease** in x direction for zero net flowrate.

Problem 8.22

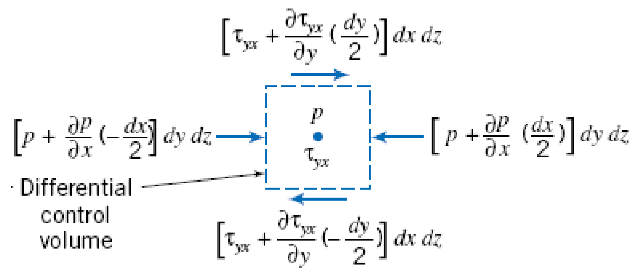
[2]

8.22 Consider fully developed laminar flow between infinite parallel plates separated by gap width $d = 10$ mm. The upper plate moves to the right with speed $U_2 = 0.5$ m/s; the lower plate moves to the left with speed $U_1 = 0.25$ m/s. The pressure gradient in the direction of flow is zero. Develop an expression for the velocity distribution in the gap. Find the volume flow rate per unit depth passing a given cross-section.



Given: Laminar flow between moving plates

Find: Expression for velocity; Volume flow rate per depth



Solution:

Using the analysis of Section 8-2, the sum of forces in the x direction is

$$\left[\tau + \frac{\partial}{\partial y}\tau\cdot\frac{dy}{2} - \left(\tau - \frac{\partial}{\partial y}\tau\cdot\frac{dy}{2}\right)\right]\cdot b\cdot dx + \left(p - \frac{\partial}{\partial x}p\cdot\frac{dx}{2} - p + \frac{\partial}{\partial x}p\cdot\frac{dx}{2}\right)\cdot b\cdot dy = 0$$

Simplifying $\quad \frac{d\tau}{dy} = \frac{dp}{dx} = 0 \quad$ or $\quad \mu\cdot\frac{d^2u}{dy^2} = 0$

Integrating twice $\quad u = c_1\cdot y + c_2$

Boundary conditions: $\quad u(0) = -U_1 \quad c_2 = -U_1 \quad u(y = d) = U_2 \quad c_1 = \frac{U_1 + U_2}{d}$

Hence $\quad u(y) = \left(U_1 + U_2\right)\cdot\frac{y}{d} - U_1 \quad u(y) = 75\cdot y - 0.25 \quad$ (u in m/s, y in m)

The volume flow rate is $\quad Q = \int u\, dA = b\cdot\int u\, dy \quad Q = b\cdot\int_0^d \left[\left(U_1 + U_2\right)\cdot\frac{y}{d} - U_1\right] dx$

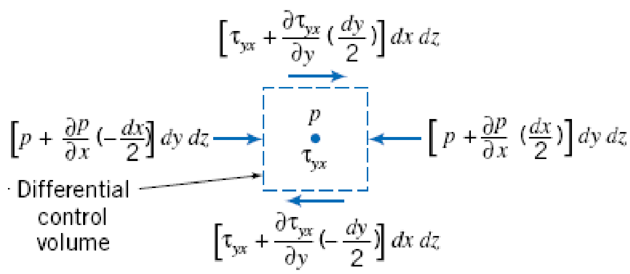
$$Q = b\cdot d\cdot\frac{\left(U_2 - U_1\right)}{2} \qquad \frac{Q}{b} = 10\cdot mm \times \frac{1\cdot m}{1000\cdot mm} \times \frac{1}{2} \times (0.5 - 0.25) \times \frac{m}{s} \qquad Q = 0.00125\,\frac{\frac{m^3}{s}}{m}$$

Problem 8.23

8.23 Two immiscible fluids are contained between infinite parallel plates. The plates are separated by distance $2h$, and the two fluid layers are of equal thickness h; the dynamic viscosity of the upper fluid is three times that of the lower fluid. If the lower plate is stationary and the upper plate moves at constant speed $U = 20$ ft/s, what is the velocity at the interface? Assume laminar flows, and that the pressure gradient in the direction of flow is zero.

Given: Laminar flow of two fluids between plates

Find: Velocity at the interface



Solution:

Using the analysis of Section 8-2, the sum of forces in the x direction is

$$\left[\tau + \frac{\partial}{\partial y}\tau\cdot\frac{dy}{2} - \left(\tau - \frac{\partial}{\partial y}\tau\cdot\frac{dy}{2}\right)\right]\cdot b\cdot dx + \left(p - \frac{\partial}{\partial x}p\cdot\frac{dx}{2} - p + \frac{\partial}{\partial x}p\cdot\frac{dx}{2}\right)\cdot b\cdot dy = 0$$

Simplifying $\qquad \frac{d\tau}{dy} = \frac{dp}{dx} = 0 \qquad$ or $\qquad \mu\cdot\frac{d^2u}{dy^2} = 0$

Applying this to fluid 1 (lower fluid) and fluid 2 (upper fluid), integrating twice yields $\qquad u_1 = c_1\cdot y + c_2 \qquad u_2 = c_3\cdot y + c_4$

We need four BCs. Three are obvious $\qquad y = 0 \quad u_1 = 0 \qquad y = h \quad u_1 = u_2 \qquad y = 2\cdot h \quad u_2 = U$

The fourth BC comes from the fact that the stress at the interface generated by each fluid is the same

$$y = h \qquad \mu_1\cdot\frac{du_1}{dy} = \mu_2\cdot\frac{du_2}{dy}$$

Using these four BCs $\qquad 0 = c_2 \qquad c_1\cdot h + c_2 = c_3\cdot h + c_4 \qquad U = c_3\cdot 2\cdot h + c_4 \qquad \mu_1\cdot c_1 = \mu_2\cdot c_3$

Hence $\qquad c_2 = 0$

From the 2nd and 3rd equations $\qquad c_1\cdot h - U = -c_3\cdot h \qquad$ and $\qquad \mu_1\cdot c_1 = \mu_2\cdot c_3$

Hence $\qquad c_1\cdot h - U = -c_3\cdot h = -\frac{\mu_1}{\mu_2}\cdot h\cdot c_1 \qquad c_1 = \frac{U}{h\cdot\left(1 + \frac{\mu_1}{\mu_2}\right)}$

Hence for fluid 1 (we do not need to complete the analysis for fluid 2) $\qquad u_1 = \frac{U}{h\cdot\left(1 + \frac{\mu_1}{\mu_2}\right)}\cdot y$

Evaluating this at $y = h$, where $u_1 = u_{interface}$ $\qquad u_{interface} = \frac{20\cdot\frac{ft}{s}}{\left(1 + \frac{1}{3}\right)} \qquad u_{interface} = 15\cdot\frac{ft}{s}$

Problem 8.24

8.24 Two immiscible fluids are contained between infinite parallel plates. The plates are separated by distance $2h$, and the two fluid layers are of equal thickness $h = 2.5$ mm. The dynamic viscosity of the upper fluid is twice that of the lower fluid, which is $\mu_{lower} = 0.5\ \mathrm{N \cdot s/m^2}$. If the plates are stationary and the applied pressure gradient is $-1000\ \mathrm{N/m^2/m}$, find the velocity at the interface. What is the maximum velocity of the flow? Plot the velocity distribution.

Given: Properties of two fluids flowing between parallel plates; applied pressure gradient

Find: Velocity at the interface; maximum velocity; plot velocity distribution

Solution:

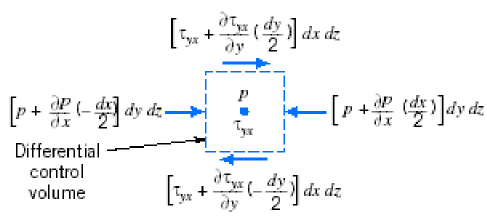
Given data $\quad k = \dfrac{dp}{dx} = -1000 \cdot \dfrac{\mathrm{Pa}}{\mathrm{m}} \qquad h = 2.5 \cdot \mathrm{mm}$

$$\mu_1 = 0.5 \cdot \frac{\mathrm{N \cdot s}}{\mathrm{m^2}} \qquad \mu_2 = 2 \cdot \mu_1 \qquad \mu_2 = 1 \cdot \frac{\mathrm{N \cdot s}}{\mathrm{m^2}}$$

(Lower fluid is fluid 1; upper is fluid 2)

Following the analysis of Section 8-2, analyse the forces on a differential CV of either fluid

The net force is zero for steady flow, so



$$\left[\tau + \frac{d\tau}{dy} \cdot \frac{dy}{2} - \left(\tau - \frac{d\tau}{dy} \cdot \frac{dy}{2}\right)\right] \cdot dx \cdot dz + \left[p - \frac{dp}{dx} \cdot \frac{dx}{2} - \left(p + \frac{dp}{dx} \cdot \frac{dx}{2}\right)\right] \cdot dy \cdot dz = 0$$

Simplifying $\quad \dfrac{d\tau}{dy} = \dfrac{dp}{dx} = k \qquad$ so for each fluid $\quad \mu \cdot \dfrac{d^2}{dy^2} u = k$

Applying this to fluid 1 (lower fluid) and fluid 2 (upper fluid), integrating twice yields

$$u_1 = \frac{k}{2 \cdot \mu_1} \cdot y^2 + c_1 \cdot y + c_2 \qquad u_2 = \frac{k}{2 \cdot \mu_2} \cdot y^2 + c_3 \cdot y + c_4$$

For convenience the origin of coordinates is placed at the centerline

We need four BCs. Three are obvious

$y = -h \qquad u_1 = 0 \qquad (1)$

$y = 0 \qquad u_1 = u_2 \qquad (2)$

$y = h \qquad u_2 = 0 \qquad (3)$

The fourth BC comes from the fact that the stress at the interface generated by each fluid is the same

$$y = 0 \qquad \mu_1 \cdot \frac{du_1}{dy} = \mu_2 \cdot \frac{du_2}{dy} \qquad (4)$$

Using these four BCs

$$0 = \frac{k}{2\cdot\mu_1}\cdot h^2 - c_1\cdot h + c_2$$

$$c_2 = c_4$$

$$0 = \frac{k}{2\cdot\mu_2}\cdot h^2 + c_3\cdot h + c_4$$

$$\mu_1\cdot c_1 = \mu_2\cdot c_3$$

Hence, after some algebra

$$c_1 = \frac{k\cdot h}{2\cdot\mu_1}\cdot\frac{(\mu_2 - \mu_1)}{(\mu_2 + \mu_1)} \qquad c_2 = c_4 = -\frac{k\cdot h^2}{\mu_2 + \mu_1} \qquad c_3 = \frac{k\cdot h}{2\cdot\mu_2}\cdot\frac{(\mu_2 - \mu_1)}{(\mu_2 + \mu_1)}$$

The velocity distributions are then

$$u_1 = \frac{k}{2\cdot\mu_1}\cdot\left[y^2 + y\cdot h\cdot\frac{(\mu_2 - \mu_1)}{(\mu_2 + \mu_1)}\right] - \frac{k\cdot h^2}{\mu_2 + \mu_1} \qquad u_2 = \frac{k}{2\cdot\mu_2}\cdot\left[y^2 + y\cdot h\cdot\frac{(\mu_2 - \mu_1)}{(\mu_2 + \mu_1)}\right] - \frac{k\cdot h^2}{\mu_2 + \mu_1}$$

Evaluating either velocity at *y* = 0, gives the velocity at the interface

$$u_{interface} = -\frac{k\cdot h^2}{\mu_2 + \mu_1} \qquad u_{interface} = 4.17 \times 10^{-3}\frac{m}{s}$$

The plots of these velocity distributions are shown in the associated *Excel* workbook, as is the determination of the maximum velocity.

From *Excel* $$u_{max} = 4.34 \times 10^{-3}\cdot\frac{m}{s}$$

Problem 8.24

8.24 Two immiscible fluids are contained between infinite parallel plates. The plates are separated by distance $2h$, and the two fluid layers are of equal thickness $h = 2.5$ mm. The dynamic viscosity of the upper fluid is twice that of the lower fluid, which is $\mu_{lower} = 0.5\ N \cdot s/m^2$. If the plates are stationary and the applied pressure gradient is $-1000\ N/m^2/m$, find the velocity at the interface. What is the maximum velocity of the flow? Plot the velocity distribution.

Given: Properties of two fluids flowing between parallel plates; applied pressure gradient

Find: Velocity at the interface; maximum velocity; plot velocity distribution

Solution:

The data is

k =	-1000	Pa/m
h =	2.5	mm
μ_1 =	0.5	$N.s/m^2$
μ_2 =	1.0	$N.s/m^2$

The velocity distribution is

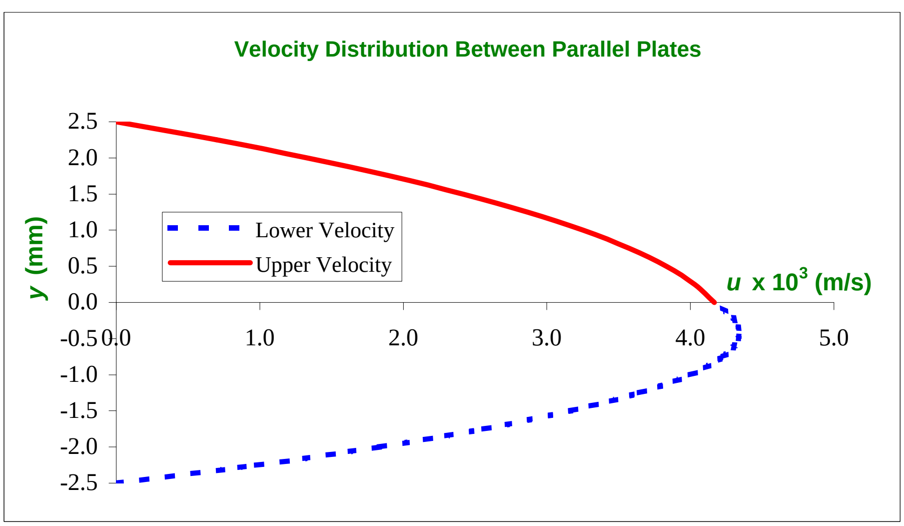
$$u_1 = \frac{k}{2 \cdot \mu_1} \cdot \left[y^2 + y \cdot h \cdot \frac{(\mu_2 - \mu_1)}{(\mu_2 + \mu_1)} \right] - \frac{k \cdot h^2}{\mu_2 + \mu_1}$$

$$u_2 = \frac{k}{2 \cdot \mu_2} \cdot \left[y^2 + y \cdot h \cdot \frac{(\mu_2 - \mu_1)}{(\mu_2 + \mu_1)} \right] - \frac{k \cdot h^2}{\mu_2 + \mu_1}$$

y (mm)	$u_1 \times 10^3$ (m/s)	$u_2 \times 10^3$ (m/s)
-2.50	0.000	NA
-2.25	0.979	NA
-2.00	1.83	NA
-1.75	2.56	NA
-1.50	3.17	NA
-1.25	3.65	NA
-1.00	4.00	NA
-0.75	4.23	NA
-0.50	4.33	NA
-0.25	4.31	NA
0.00	4.17	4.17
0.25	NA	4.03
0.50	NA	3.83
0.75	NA	3.57
1.00	NA	3.25
1.25	NA	2.86
1.50	NA	2.42
1.75	NA	1.91
2.00	NA	1.33
2.25	NA	0.698
2.50	NA	0.000

The lower fluid has the highest velocity
We can use *Solver* to find the maximum
(Or we could differentiate to find the maximum)

y (mm)	$u_{max} \times 10^3$ (m/s)
-0.417	4.34



Problem 8.25

[2]

8.25 The dimensionless velocity profile for fully developed laminar flow between infinite parallel plates with the upper plate moving at constant speed U is shown in Fig. 8.6. Find the pressure gradient $\partial p/\partial x$ at which (a) the upper plate and (b) the lower plate experience zero shear stress, in terms of U, a, and μ. Plot the dimensionless velocity profiles for these cases.

Given: Velocity profile between parallel plates

Find: Pressure gradients for zero stress at upper/lower plates; plot

Solution:

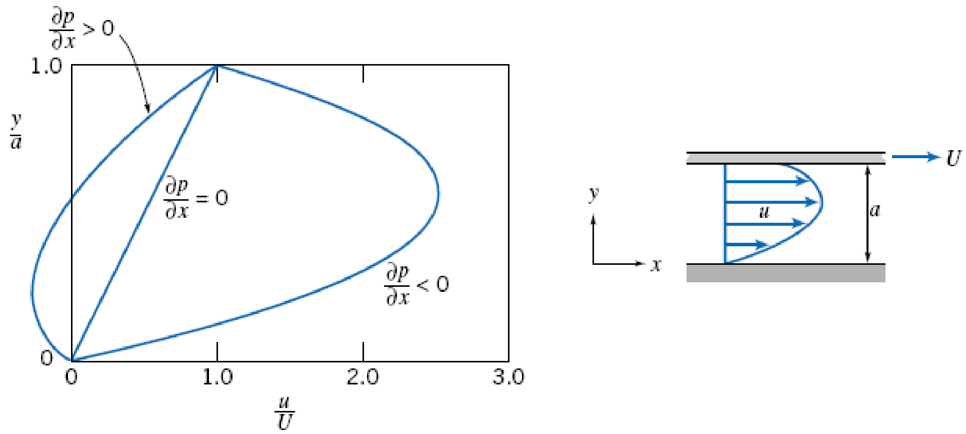


Fig. 8.6 Dimensionless velocity profile for fully developed laminar flow between infinite parallel plates: upper plate moving with constant speed, U.

From Eq. 8.8, the velocity distribution is
$$u = \frac{U \cdot y}{a} + \frac{a^2}{2 \cdot \mu} \cdot \left(\frac{\partial}{\partial x} p\right) \cdot \left[\left(\frac{y}{a}\right)^2 - \frac{y}{a}\right]$$

The shear stress is
$$\tau_{yx} = \mu \cdot \frac{du}{dy} = \mu \cdot \frac{U}{a} + \frac{a^2}{2} \cdot \left(\frac{\partial}{\partial x} p\right) \cdot \left(2 \cdot \frac{y}{a^2} - \frac{1}{a}\right)$$

(a) For $\tau_{yx} = 0$ at $y = a$
$$0 = \mu \cdot \frac{U}{a} + \frac{a}{2} \cdot \frac{\partial}{\partial x} p \qquad \frac{\partial}{\partial x} p = -\frac{2 \cdot U \cdot \mu}{a^2}$$

The velocity distribution is then
$$u = \frac{U \cdot y}{a} - \frac{a^2}{2 \cdot \mu} \cdot \frac{2 \cdot U \cdot \mu}{a^2} \cdot \left[\left(\frac{y}{a}\right)^2 - \frac{y}{a}\right] \qquad \frac{u}{U} = 2 \cdot \frac{y}{a} - \left(\frac{y}{a}\right)^2$$

(b) For $\tau_{yx} = 0$ at $y = 0$
$$0 = \mu \cdot \frac{U}{a} - \frac{a}{2} \cdot \frac{\partial}{\partial x} p \qquad \frac{\partial}{\partial x} p = \frac{2 \cdot U \cdot \mu}{a^2}$$

The velocity distribution is then
$$u = \frac{U \cdot y}{a} + \frac{a^2}{2 \cdot \mu} \cdot \frac{2 \cdot U \cdot \mu}{a^2} \cdot \left[\left(\frac{y}{a}\right)^2 - \frac{y}{a}\right] \qquad \frac{u}{U} = \left(\frac{y}{a}\right)^2$$

The velocity distributions are plotted in the associated *Excel* workbook

Problem 8.25

8.25 The dimensionless velocity profile for fully developed laminar flow between infinite parallel plates with the upper plate moving at constant speed U is shown in Fig. 8.6. Find the pressure gradient $\partial p/\partial x$ at which (a) the upper plate and (b) the lower plate experience zero shear stress, in terms of U, a, and μ. Plot the dimensionless velocity profiles for these cases.

Given: Velocity profile between parallel plates

Find: Pressure gradients for zero stress at upper/lower plates; plot

Solution:

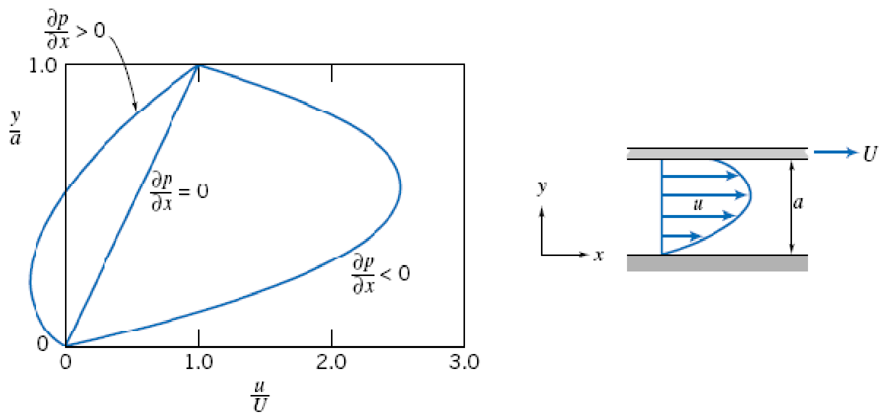
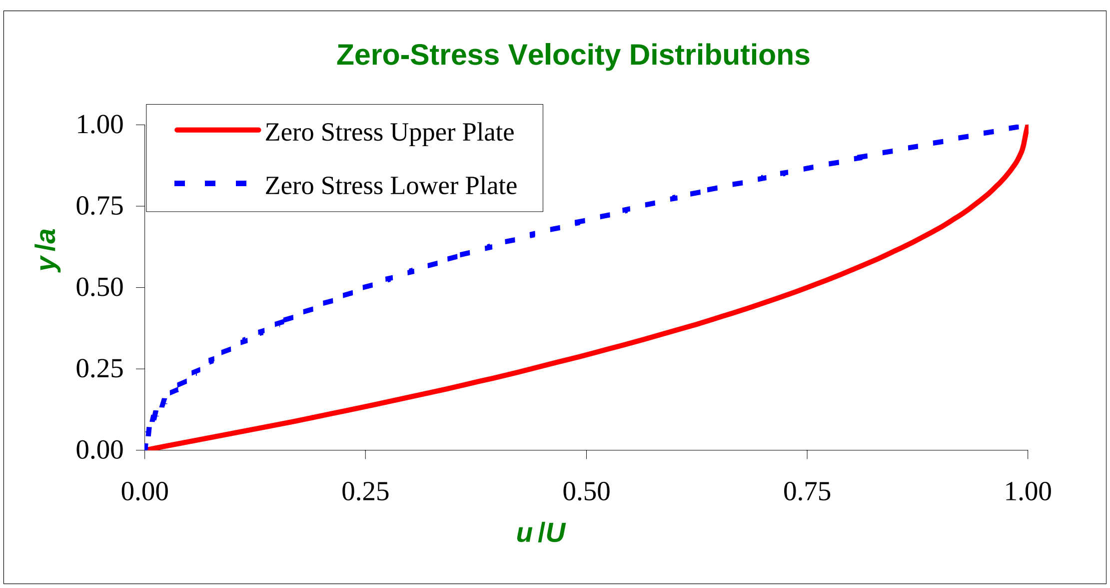


Fig. 8.6 Dimensionless velocity profile for fully developed laminar flow between infinite parallel plates: upper plate moving with constant speed, U.

(a) For zero shear stress at upper plate $\qquad \dfrac{u}{U} = 2\cdot\dfrac{y}{a} - \left(\dfrac{y}{a}\right)^2$

(b) For zero shear stress at lower plate $\qquad \dfrac{u}{U} = \left(\dfrac{y}{a}\right)^2$

y/a	(a) u/U	(b) u/U
0.0	0.000	0.000
0.1	0.190	0.010
0.2	0.360	0.040
0.3	0.510	0.090
0.4	0.640	0.160
0.5	0.750	0.250
0.6	0.840	0.360
0.7	0.910	0.490
0.8	0.960	0.640
0.9	0.990	0.810
1.0	1.00	1.000



Problem 8.26

[2]

8.26 The record-read head for a computer disk-drive memory storage system rides above the spinning disk on a very thin film of air (the film thickness is 0.25 μm). The head location is 25 mm from the disk centerline; the disk spins at 8500 rpm. The record-read head is 5 mm square. For standard air in the gap between the head and disk, determine (a) the Reynolds number of the flow, (b) the viscous shear stress, and (c) the power required to overcome viscous shear.

Given: Computer disk drive

Find: Flow Reynolds number; Shear stress; Power required

Solution:

For a distance R from the center of a disk spinning at speed ω

$$V = R\cdot\omega \qquad V = 25\cdot mm \times \frac{1\cdot m}{1000\cdot mm} \times 8500\cdot rpm \times \frac{2\cdot\pi\cdot rad}{rev} \times \frac{1\cdot min}{60\cdot s} \qquad V = 22.3\cdot\frac{m}{s}$$

The gap Reynolds number is $Re = \frac{\rho\cdot V\cdot a}{\mu} = \frac{V\cdot a}{\nu}$ $\qquad \nu = 1.45 \times 10^{-5}\cdot\frac{m^2}{s}$ from Table A.10 at 15ºC

$$Re = 22.3\cdot\frac{m}{s} \times 0.25 \times 10^{-6}\cdot m \times \frac{s}{1.45 \times 10^{-5}\cdot m^2} \qquad Re = 0.384$$

The flow is definitely laminar

The shear stress is then $\tau = \mu\cdot\frac{du}{dy} = \mu\cdot\frac{V}{a}$ $\qquad \mu = 1.79 \times 10^{-5}\cdot\frac{N\cdot s}{m^2}$ from Table A.10 at 15ºC

$$\tau = 1.79 \times 10^{-5}\cdot\frac{N\cdot s}{m^2} \times 22.3\cdot\frac{m}{s} \times \frac{1}{0.25 \times 10^{-6}\cdot m} \qquad \tau = 1.60\cdot kPa$$

The power required is $P = T\cdot\omega$ where torque T is given by $T = \tau\cdot A\cdot R$ with $A = (5\cdot mm)^2$ $\qquad A = 2.5 \times 10^{-5}\,m^2$

$$P = \tau\cdot A\cdot R\cdot\omega \qquad P = 1600\cdot\frac{N}{m^2} \times 2.5 \times 10^{-5}\cdot m^2 \times 25\cdot mm \times \frac{1\cdot m}{1000\cdot mm} \times 8500\cdot rpm \times \frac{2\cdot\pi\cdot rad}{rev} \times \frac{1\cdot min}{60\cdot s} \qquad P = 0.890\,W$$

Problem 8.27

Given: Steady, incompressible, fully developed laminar flow down an incline (of angle θ). Velocity profile (Example Problem 5.9) is

$$u = \frac{\rho g \sin\theta}{\mu}\left(hy - y^2/2\right)$$

Find: Kinematic viscosity ν of liquid for $h = 0.8$ mm, $\theta = 30°$ and $u_{max} = 15.7$ mm/s

Plot: the velocity profile

Solution:

$$u = \frac{\rho g \sin\theta}{\mu}\left(hy - y^2/2\right) = \frac{g\sin\theta}{\nu}\left(hy - y^2/2\right)$$

$u = u_{max}$ at $y = h$

$$\therefore\ u_{max} = \frac{g\sin\theta}{\nu}\left(h^2 - h^2/2\right) = \frac{g\sin\theta\, h^2}{2\nu}$$

and

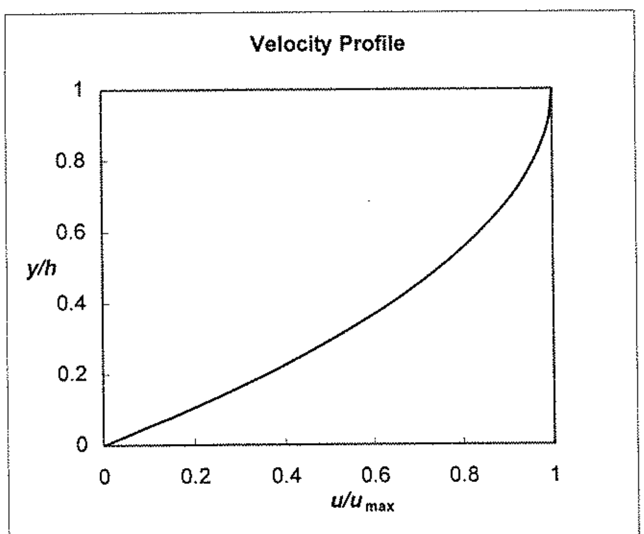
$$\nu = \frac{g\sin\theta\, h^2}{2u_{max}} = \frac{\sin 30°}{2} \times 9.81\,\frac{m}{s^2} \times \left(0.8\times10^{-3}\,m\right)^2 \times \frac{s}{15.7\times10^{-3}\,m}$$

$$\nu = 1.00\times10^{-4}\ m^2/s \qquad \nu$$

Plot

$$\frac{u}{u_{max}} = \frac{g\sin\theta}{\nu}\left(hy - y^2/2\right) \times \frac{2\nu}{g\sin\theta\, h^2} = 2\frac{y}{h} - \left(\frac{y}{h}\right)^2$$

u/u_{max}	y/h
0	0
0.0396	0.02
0.098	0.05
0.190	0.1
0.360	0.2
0.510	0.3
0.640	0.4
0.750	0.5
0.840	0.6
0.910	0.7
0.960	0.8
0.990	0.9
1.00	1.0



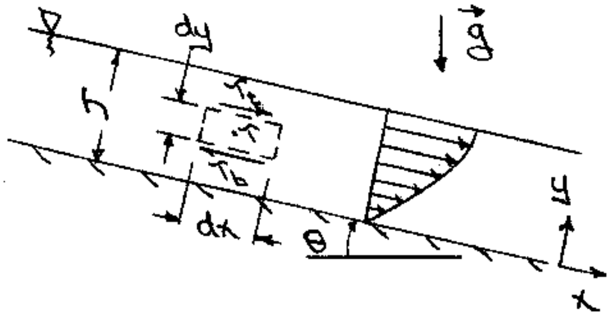
Problem 8.28

Given: Fully developed, laminar flow of an incompressible liquid down an inclined surface. The thickness, h, of the liquid layer is constant.

Find: (a) the velocity profile by use of a suitably chosen differential control volume. (b) volume flow rate, Q/w

Solution: Flow is fully developed, so $u = u(y)$ and $\tau = \tau(y)$.

Expand τ in a Taylor series about the center of the differential CV



$$\tau_t = \tau + \frac{d\tau}{dy}\frac{dy}{2}$$

$$\tau_b = \tau + \frac{d\tau}{dy}\left(-\frac{dy}{2}\right)$$

The boundary conditions on the velocity profile are:

@ $y = 0$, $u = 0$ (no slip).

@ $y = h$, $\frac{du}{dy} = 0$ (no shear stress).

Apply the x component of the momentum equation to the differential CV shown

$$F_{S_x} + F_{B_x} = \cancel{\frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall}^{\,0(1)} + \cancel{\int_{CS} u\,\rho\vec{V}\cdot d\vec{A}}^{\,0(2)}$$

Assumptions: (1) steady flow

(2) fully developed flow, so u and τ are functions of y only

(3) no variation of pressure in the x direction

Then

$$F_{S_x} + F_{B_x} = 0 = \left(\tau + \frac{d\tau}{dy}\frac{dy}{2}\right)dx\,dz - \left(\tau - \frac{d\tau}{dy}\frac{dy}{2}\right)dx\,dz + \rho g \sin\theta\, dx\,dy\,dz$$

or

$$\frac{d\tau}{dy} = -\rho g \sin\theta$$

Integrating, $\tau = -\rho g \sin\theta\, y + c_1$

But $\tau = 0$ @ $y = h$, $\therefore$ $c_1 = \rho g \sin\theta\, h$, and

$$\frac{du}{dy} = \frac{\rho g \sin\theta}{\mu}(h - y)$$

Integrating again,

$$u = \frac{\rho g \sin\theta}{\mu}\left(hy - \frac{y^2}{2}\right) + c_2$$

At $y = 0$, $u = 0$, so $c_2 = 0$ and hence

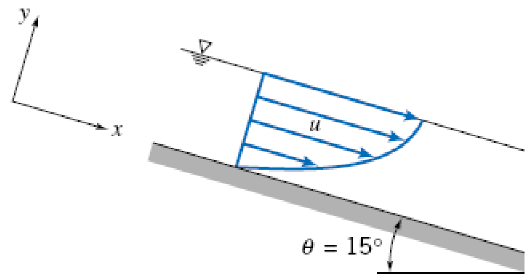
$$u = \frac{\rho g \sin\theta}{\mu}\left(hy - \frac{y^2}{2}\right) \longleftarrow u$$

$$Q/w = \int_0^h u\,dy = \frac{\rho g \sin\theta}{\mu}\int_0^h \left(hy - \frac{y^2}{2}\right)dy = \frac{\rho g \sin\theta}{\mu}\left[\frac{hy^2}{2} - \frac{y^3}{6}\right]_0^h$$

$$Q/w = \rho g \sin\theta\, h^3 / 3\mu \longleftarrow Q/w$$

Problem 8.29

8.29 The velocity distribution for flow of a thin viscous film down an inclined plane surface was developed in Example 5.9. Consider a film 7 mm thick, of liquid with SG = 1.2 and dynamic viscosity of 1.60 N·s/m^2. Derive an expression for the shear stress distribution within the film. Calculate the maximum shear stress within the film and indicate its direction. Evaluate the volume flow rate in the film, in mm^3/s per millimeter of surface width. Calculate the film Reynolds number based on average velocity.



Given: Velocity distribution on incline

Find: Expression for shear stress; Maximum shear; volume flow rate/mm width; Reynolds number

Solution:

From Example 5.9 $$u(y) = \frac{\rho\cdot g\cdot \sin(\theta)}{\mu}\cdot\left(h\cdot y - \frac{y^2}{2}\right)$$

For the shear stress $$\tau = \mu\cdot\frac{du}{dy} = \rho\cdot g\cdot\sin(\theta)\cdot(h - y)$$

τ is a maximum at y = 0 $$\tau_{max} = \rho\cdot g\cdot\sin(\theta)\cdot h = SG\cdot\rho_{H2O}\cdot g\cdot\sin(\theta)\cdot h$$

$$\tau_{max} = 1.2\times 1000\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times\sin(15\cdot deg)\times 0.007\cdot m\times\frac{N\cdot s^2}{kg\cdot m} \qquad \tau_{max} = 21.3\,Pa$$

This stress is in the x direction on the wall

The flow rate is $$Q = \int u\,dA = w\cdot\int_0^h u(y)\,dy = w\cdot\int_0^h \frac{\rho\cdot g\cdot\sin(\theta)}{\mu}\cdot\left(h\cdot y - \frac{y^2}{2}\right)dy \qquad Q = \frac{\rho\cdot g\cdot\sin(\theta)\cdot w\cdot h^3}{3\cdot\mu}$$

$$\frac{Q}{w} = \frac{1}{3}\times 1.2\times 1000\frac{kg}{m^3}\times 9.81\cdot\frac{m}{s^2}\times\sin(15\cdot deg)\times(0.007\cdot m)^3\times\frac{m^2}{1.60\cdot N\cdot s}\cdot\frac{N\cdot s^2}{kg\cdot m} = 2.18\times 10^{-4}\frac{\frac{m^3}{s}}{m} \qquad \frac{Q}{w} = 217\frac{\frac{mm^3}{s}}{mm}$$

The average velocity is $$V = \frac{Q}{A} = \frac{Q}{w\cdot h} \qquad V = 217\cdot\frac{\frac{mm^3}{s}}{mm}\times\frac{1}{7\cdot mm} \qquad V = 31.0\frac{mm}{s}$$

The gap Reynolds number is $$Re = \frac{\rho\cdot V\cdot h}{\mu}$$

$$Re = 1.2\times 1000\frac{kg}{m^3}\times 31\cdot\frac{mm}{s}\times 7\cdot mm\times\frac{m^2}{1.60\cdot N\cdot s}\times\left(\frac{1\cdot m}{1000\cdot mm}\right)^2 \qquad Re = 0.163$$

The flow is definitely laminar

Problem 8.30

8.30 Two immiscible fluids of equal density are flowing down a surface inclined at a 30° angle. The two fluid layers are of equal thickness $h = 2.5$ mm; the kinematic viscosity of the upper fluid is twice that of the lower fluid, which is $\nu_{lower} = 2 \times 10^{-4}$ m²/s. Find the velocity at the interface and the velocity at the free surface. Plot the velocity distribution.

Given: Data on flow of liquids down an incline

Find: Velocity at interface; velocity at free surface; plot

Solution:

Given data $h = 2.5 \cdot mm$ $\theta = 30 \cdot deg$ $\nu_1 = 2 \times 10^{-4} \cdot \frac{m^2}{s}$ $\nu_2 = 2 \cdot \nu_1$

(The lower fluid is designated fluid 1, the upper fluid 2)

From Example 5.9 (or Exanple 8.3 with *g* replaced with *g*sinθ), a free body analysis leads to (for either fluid)

$$\frac{d^2}{dy^2} u = -\frac{\rho \cdot g \cdot \sin(\theta)}{\mu}$$

Applying this to fluid 1 (lower fluid) and fluid 2 (upper fluid), integrating twice yields

$$u_1 = -\frac{\rho \cdot g \cdot \sin(\theta)}{2 \cdot \mu_1} \cdot y^2 + c_1 \cdot y + c_2 \qquad u_2 = -\frac{\rho \cdot g \cdot \sin(\theta)}{2 \cdot \mu_2} \cdot y^2 + c_3 \cdot y + c_4$$

We need four BCs. Two are obvious $y = 0$ $u_1 = 0$ (1)

$y = h$ $u_1 = u_2$ (2)

The third BC comes from the fact that there is no shear stress at the free surface

$y = 2 \cdot h$ $\mu_2 \cdot \frac{du_2}{dy} = 0$ (3)

The fourth BC comes from the fact that the stress at the interface generated by each fluid is the same

$y = h$ $\mu_1 \cdot \frac{du_1}{dy} = \mu_2 \cdot \frac{du_2}{dy}$ (4)

Using these four BCs $c_2 = 0$

$$-\frac{\rho \cdot g \cdot \sin(\theta)}{2 \cdot \mu_1} \cdot h^2 + c_1 \cdot h + c_2 = -\frac{\rho \cdot g \cdot \sin(\theta)}{2 \cdot \mu_2} \cdot h^2 + c_3 \cdot h + c_4$$

$$-\rho \cdot g \cdot \sin(\theta) \cdot 2 \cdot h + \mu_2 \cdot c_3 = 0$$

$$-\rho \cdot g \cdot \sin(\theta) \cdot h + \mu_1 \cdot c_1 = -\rho \cdot g \cdot \sin(\theta) \cdot h + \mu_2 \cdot c_3$$

Hence, after some algebra

$$c_1 = \frac{2\cdot\rho\cdot g\cdot\sin(\theta)\cdot h}{\mu_1} \qquad c_2 = 0$$

$$c_3 = \frac{2\cdot\rho\cdot g\cdot\sin(\theta)\cdot h}{\mu_2} \qquad c_4 = 3\cdot\rho\cdot g\cdot\sin(\theta)\cdot h^2\cdot\frac{(\mu_2 - \mu_1)}{2\cdot\mu_1\cdot\mu_2}$$

The velocity distributions are then

$$u_1 = \frac{\rho\cdot g\cdot\sin(\theta)}{2\cdot\mu_1}\cdot\left(4\cdot y\cdot h - y^2\right) \qquad u_2 = \frac{\rho\cdot g\cdot\sin(\theta)}{2\cdot\mu_2}\cdot\left[3\cdot h^2\cdot\frac{(\mu_2 - \mu_1)}{\mu_1} + 4\cdot y\cdot h - y^2\right]$$

Rewriting in terms of ν_1 and ν_2 (ρ is constant and equal for both fluids)

$$u_1 = \frac{g\cdot\sin(\theta)}{2\cdot\nu_1}\cdot\left(4\cdot y\cdot h - y^2\right) \qquad u_2 = \frac{g\cdot\sin(\theta)}{2\cdot\nu_2}\cdot\left[3\cdot h^2\cdot\frac{(\nu_2 - \nu_1)}{\nu_1} + 4\cdot y\cdot h - y^2\right]$$

(Note that these result in the same expression if $\nu_1 = \nu_2$, i.e., if we have one fluid)

Evaluating either velocity at $y = h$, gives the velocity at the interface

$$u_{interface} = \frac{3\cdot g\cdot h^2\cdot\sin(\theta)}{2\cdot\nu_1} \qquad u_{interface} = 0.23\,\frac{m}{s}$$

Evaluating u_2 at $y = 2h$ gives the velocity at the free surface

$$u_{freesurface} = g\cdot h^2\cdot\sin(\theta)\cdot\frac{(3\cdot\nu_2 + \nu_1)}{2\cdot\nu_1\cdot\nu_2} \qquad u_{freesurface} = 0.268\,\frac{m}{s}$$

The velocity distributions are plotted in the associated *Excel* workbook

Problem 8.30

8.30 Two immiscible fluids of equal density are flowing down a surface inclined at a 30° angle. The two fluid layers are of equal thickness $h = 2.5$ mm; the kinematic viscosity of the upper fluid is twice that of the lower fluid, which is $\nu_{lower} = 2 \times 10^{-4}$ m²/s. Find the velocity at the interface and the velocity at the free surface. Plot the velocity distribution.

Given: Data on flow of liquids down an incline

Find: Velocity at interface; velocity at free surface; plot

Solution:

h = 2.5 mm

θ = 30 deg

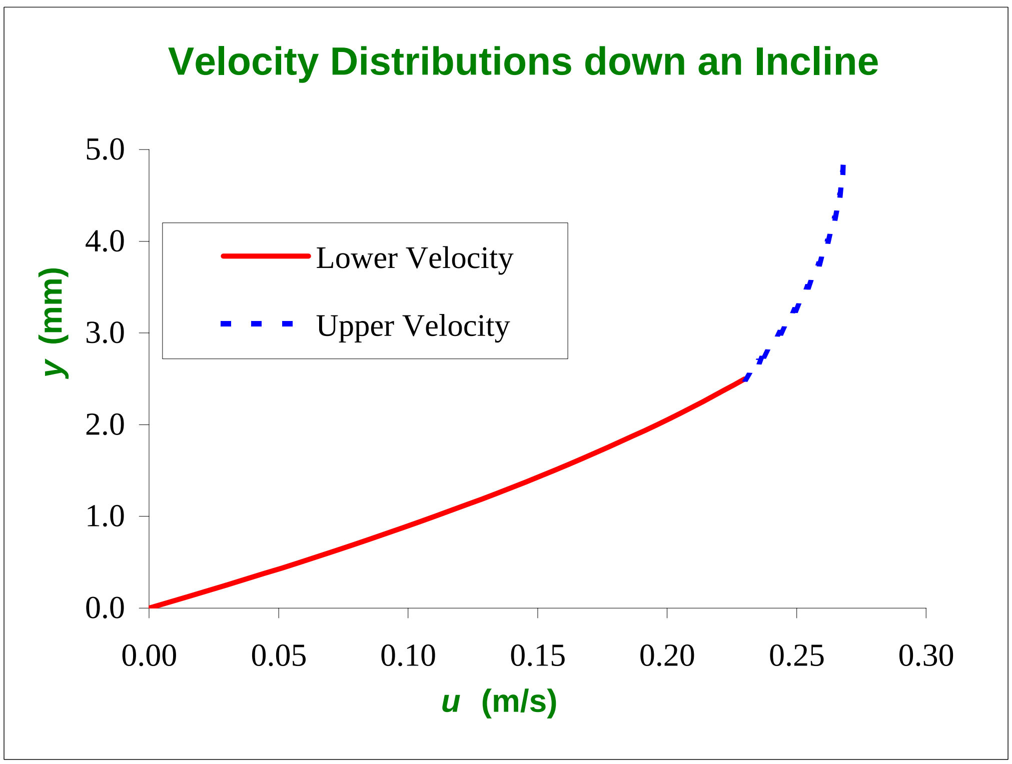
ν_1 = 2.00E-04 m^2/s

ν_2 = 4.00E-04 m^2/s

$$u_1 = \frac{g \cdot \sin(\theta)}{2 \cdot \nu_1} \cdot \left(4 \cdot y \cdot h - y^2\right)$$

$$u_2 = \frac{g \cdot \sin(\theta)}{2 \cdot \nu_2} \cdot \left[3 \cdot h^2 \cdot \frac{(\nu_2 - \nu_1)}{\nu_1} + 4 \cdot y \cdot h - y^2\right]$$

y (mm)	u_1 (m/s)	u_2 (m/s)
0.000	0.000	
0.250	0.0299	
0.500	0.0582	
0.750	0.0851	
1.000	0.110	
1.250	0.134	
1.500	0.156	
1.750	0.177	
2.000	0.196	
2.250	0.214	
2.500	0.230	0.230
2.750		0.237
3.000		0.244
3.250		0.249
3.500		0.254
3.750		0.259
4.000		0.262
4.250		0.265
4.500		0.267
4.750		0.268
5.000		0.268



Problem 8.31

8.31 Consider fully developed flow between parallel plates with the upper plate moving at $U = 5$ ft/s; the spacing between the plates is $a = 0.1$ in. Determine the flow rate per unit depth for the case of zero pressure gradient. If the fluid is air, evaluate the shear stress on the lower plate and plot the shear stress distribution across the channel for the zero pressure gradient case. Will the flow rate increase or decrease if the pressure gradient is adverse? Determine the pressure gradient that will give zero shear stress at $y = 0.25a$. Plot the shear stress distribution across the channel for the latter case.

Given: Flow between parallel plates

Find: Shear stress on lower plate; Plot shear stress; Flow rate for pressure gradient; Pressure gradient for zero shear; Plot

Solution:

From Section 8-2 $\quad u(y) = \frac{U\cdot y}{a} + \frac{a^2}{2\cdot\mu}\cdot\frac{dp}{dx}\cdot\left[\left(\frac{y}{a}\right)^2 - \frac{y}{a}\right]$

For dp/dx = 0 $\quad u = U\cdot\frac{y}{a} \qquad \frac{Q}{l} = \int_0^a u(y)\,dy = w\cdot\int_0^a U\cdot\frac{y}{a}\,dy = \frac{U\cdot a}{2} \qquad Q = \frac{1}{2}\times 5\cdot\frac{ft}{s}\times\frac{0.1}{12}\cdot ft \qquad Q = 0.0208\,\frac{\frac{ft^3}{s}}{ft}$

For the shear stress $\quad \tau = \mu\cdot\frac{du}{dy} = \frac{\mu\cdot U}{a}$ when dp/dx = 0 $\qquad \mu = 3.79\times 10^{-7}\cdot\frac{lbf\cdot s}{ft^2}$ (Table A.9)

The shear stress is constant - no need to plot!

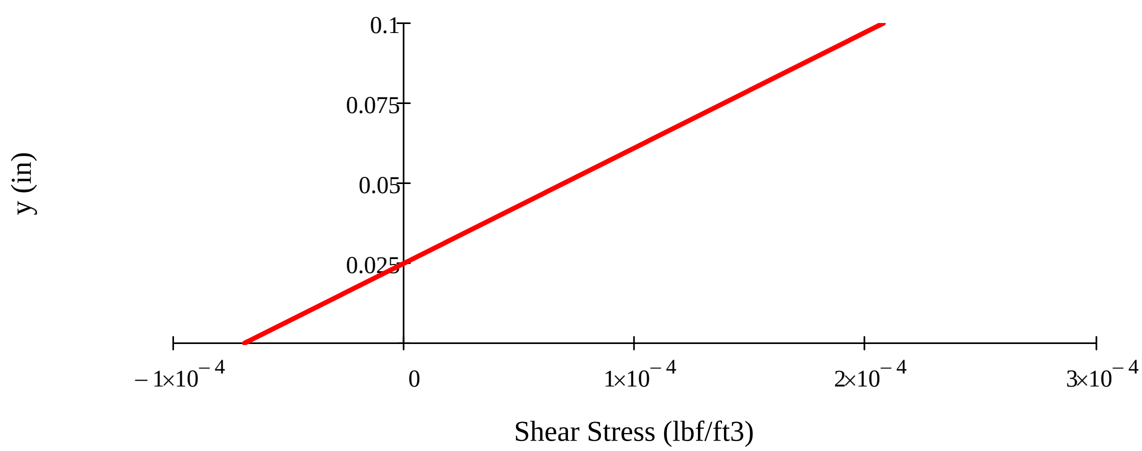
$$\tau = 3.79\times 10^{-7}\cdot\frac{lbf\cdot s}{ft^2}\times 5\cdot\frac{ft}{s}\times\frac{12}{0.1\cdot ft}\times\left(\frac{1\cdot ft}{12\cdot in}\right)^2 \qquad \tau = 1.58\times 10^{-6}\,psi$$

Q will decrease if dp/dx > 0; it will increase if dp/dx < 0.

For non- zero dp/dx: $\quad \tau = \mu\cdot\frac{du}{dy} = \frac{\mu\cdot U}{a} + a\cdot\frac{dp}{dx}\cdot\left(\frac{y}{a} - \frac{1}{2}\right)$

At y = 0.25a, we get $\quad \tau(y = 0.25\cdot a) = \mu\cdot\frac{U}{a} + a\cdot\frac{dp}{dx}\cdot\left(\frac{1}{4} - \frac{1}{2}\right) = \mu\cdot\frac{U}{a} - \frac{a}{4}\cdot\frac{dp}{dx}$

Hence this stress is zero when $\quad \frac{dp}{dx} = \frac{4\cdot\mu\cdot U}{a^2} = 4\times 3.79\times 10^{-7}\cdot\frac{lbf\cdot s}{ft^2}\times 5\cdot\frac{ft}{s}\times\left(\frac{12}{0.1\cdot ft}\right)^2 = 0.109\cdot\frac{\frac{lbf}{ft^2}}{ft} = 7.58\times 10^{-4}\,\frac{psi}{ft}$



Problem 8.32

8.32 Water at 15°C flows between parallel plates with gap width $b = 2.5$ mm. The upper plate moves with speed $U = 0.25$ m/s in the positive x direction. The pressure gradient is $\partial p/\partial x = -175$ Pa/m. Locate the point of maximum velocity and determine its magnitude (let $y = 0$ at the bottom plate). Determine the volume of flow that passes a given cross-section (x = constant) in 10 s. Plot the velocity and shear stress distributions.

Given: Flow between parallel plates

Find: Location and magnitude of maximum velocity; Volume flow in 10 s; Plot velocity and shear stress

Solution:

From Section 8-2
$$u(y) = \frac{U \cdot y}{b} + \frac{b^2}{2\cdot\mu}\cdot\frac{dp}{dx}\cdot\left[\left(\frac{y}{b}\right)^2 - \frac{y}{b}\right]$$

For u_{max} set du/dx = 0
$$\frac{du}{dy} = 0 = \frac{U}{b} + \frac{b^2}{2\cdot\mu}\cdot\frac{dp}{dx}\cdot\left(\frac{2\cdot y}{b^2} - \frac{1}{a}\right) = \frac{U}{b} + \frac{1}{2\cdot\mu}\cdot\frac{dp}{dx}\cdot(2\cdot y - b)$$

Hence $u = u_{max}$ at $y = \dfrac{b}{2} - \dfrac{\mu\cdot U}{b\cdot\frac{dp}{dx}}$

From Table A.8 at 15ºC $\mu = 1.14\times 10^{-3}\cdot\dfrac{N\cdot s}{m^2}$

$$y = \frac{0.0025\cdot m}{2} - 1.14\times 10^{-3}\cdot\frac{N\cdot s}{m^2}\times 0.25\cdot\frac{m}{s}\times\frac{1}{0.0025\cdot m}\times\left(-\frac{m^3}{175\cdot N}\right)$$

$y = 1.90\times 10^{-3}\cdot m$ $\qquad$ $y = 1.90\cdot mm$

Hence
$$u_{max} = \frac{U\cdot y}{b} + \frac{b^2}{2\cdot\mu}\cdot\frac{dp}{dx}\cdot\left[\left(\frac{y}{b}\right)^2 - \frac{y}{b}\right]$$
with y = 1.90 mm

$$u_{max} = 0.25\cdot\frac{m}{s}\times\left(\frac{1.90}{2.5}\right) + \frac{1}{2}\times(0.0025\cdot m)^2\times\frac{m^2}{1.14\times 10^{-3}\cdot N\cdot s}\times\left(-\frac{175\cdot N}{m^3}\right)\times\left[\left(\frac{1.90}{2.5}\right)^2 - \left(\frac{1.90}{2.5}\right)\right]$$

$u_{max} = 0.278\dfrac{m}{s}$

$$\frac{Q}{w} = \int_0^b u(y)\,dy = w\cdot\int_0^b\left[\frac{U\cdot y}{b} + \frac{b^2}{2\cdot\mu}\cdot\frac{dp}{dx}\cdot\left[\left(\frac{y}{b}\right)^2 - \frac{y}{b}\right]\right]dy = \frac{U\cdot b}{2} - \frac{b^3}{12\cdot\mu}\cdot\frac{dp}{dx}$$

$$\frac{Q}{w} = \frac{1}{2}\times 0.25\cdot\frac{m}{s}\times 0.0025\cdot m - \frac{1}{12}\times(0.0025\cdot m)^3\times\frac{m^2}{1.14\times 10^{-3}\cdot N\cdot s}\times\left(-\frac{175\cdot N}{m^3}\right)$$

$\dfrac{Q}{w} = 5.12\times 10^{-4}\,\dfrac{m^2}{s}$

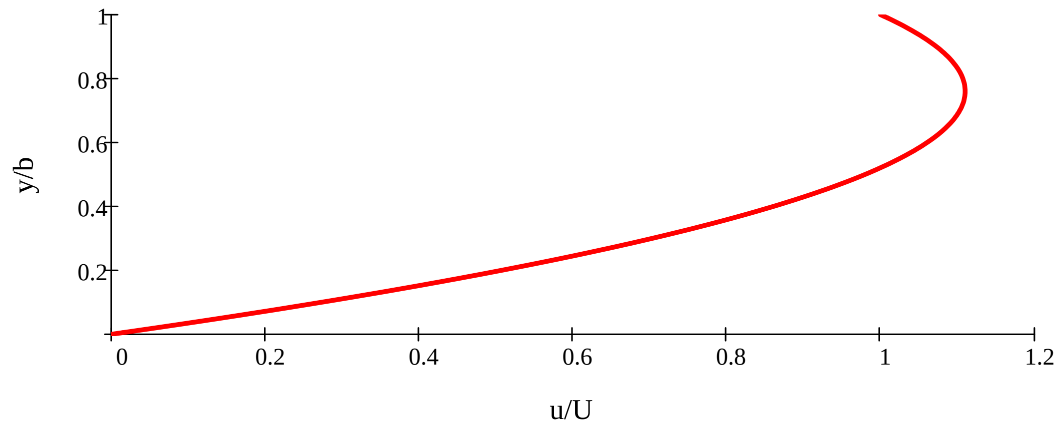
$$Flow = \frac{Q}{w}\cdot\Delta t = 5.12\times 10^{-4}\,\frac{m^2}{s}\times 10\cdot s$$

$$Flow = 5.12\times 10^{-3}\,m^2 = 5.12\times 10^{-3}\,\frac{m^3}{m}$$

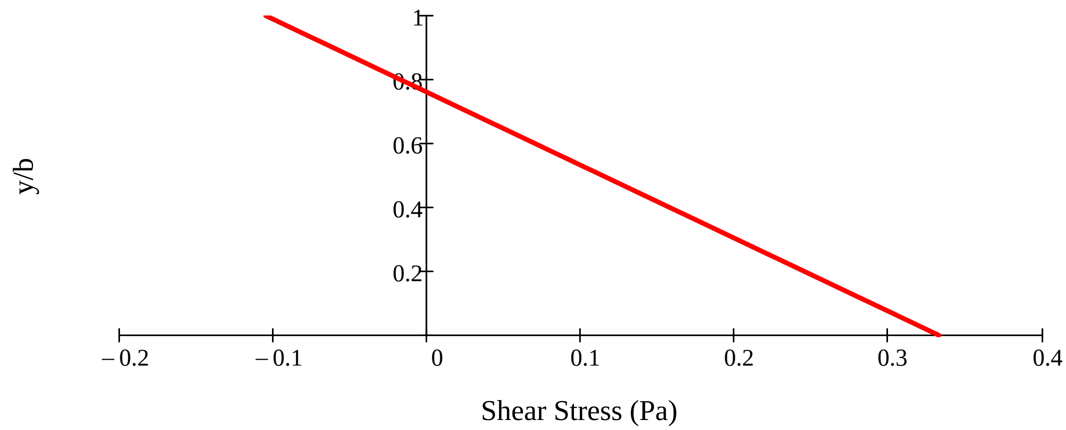
The velocity profile is
$$\frac{u}{U} = \frac{y}{b} + \frac{b^2}{2\cdot\mu\cdot U}\cdot\frac{dp}{dx}\cdot\left[\left(\frac{y}{b}\right)^2 - \frac{y}{b}\right]$$

For the shear stress
$$\tau = \mu\cdot\frac{du}{dy} = \mu\cdot\frac{U}{b} + \frac{b}{2}\cdot\frac{dp}{dx}\cdot\left[2\cdot\left(\frac{y}{b}\right) - 1\right]$$

The graphs on the next page can be plotted in *Excel*



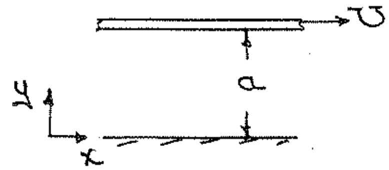
y/b
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
1.2
u/U



y/b
1
0.8
0.6
0.4
0.2
– 0.2
– 0.1
0
0.1
0.2
0.3
0.4
Shear Stress (Pa)

Given: Velocity profile for fully developed laminar flow of air between parallel plates

$$u = \frac{Uy}{a} + \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y}{a}\right)^2 - \left(\frac{y}{a}\right)\right]$$



$U = 2$ m/s $\quad a = 2.5$ mm

Find: (a) pressure gradient for which net flow is zero; plot expected $u(y)$ and $\tau_{yx}(y)$

(b) expected $u(y)$ and $\tau_{yx}(y)$ for case where $u = 2U$ at $y/a = 0.5$

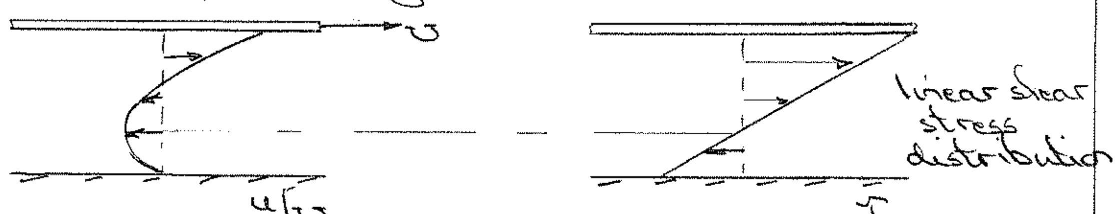
Solution:

Computing equations:
$$Q/l = \frac{Ua}{2} - \frac{a^3}{12\mu}\left(\frac{\partial p}{\partial x}\right) \quad (8.9b)$$
$$\tau_{yx} = \mu\frac{U}{a} + a\left(\frac{\partial p}{\partial x}\right)\left[\frac{y}{a} - \frac{1}{2}\right] \quad (8.9a)$$

For $Q=0$, from Eq. 8.9b (assuming $T = 15°C$)

$$\frac{\partial p}{\partial x} = \frac{6\mu U}{a^2} = 6 \times 1.79\times10^{-5}\,\frac{N\cdot s}{m^2} \times 2\,\frac{m}{s} \times \frac{1}{(2.5\times10^{-3}\,m)^2} = 34.4\ N/m^2/m \quad \frac{\partial p}{\partial x}$$

For this adverse pressure gradient



(b) For $u = 2U$ at $y/a = 0.5$

$$2U = 0.5U + \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\frac{1}{4} - \frac{1}{2}\right] \quad \text{and} \quad \frac{3}{2}U = -\frac{a^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)$$

$$\frac{\partial p}{\partial x} = -\frac{12U\mu}{a^2} = -12 \times 2\,\frac{m}{s} \times 1.79\times10^{-5}\,\frac{N\cdot s}{m^2} \times \frac{1}{(2.5\times10^{-3}\,m)^2} = -68.7\ N/m^2/m$$

$$\tau = \mu\frac{U}{a} + a\left(\frac{\partial p}{\partial x}\right)\left[\frac{y}{a} - \frac{1}{2}\right] \quad \{\text{shear stress is linear}\}$$

$y=0$ $\quad \tau = \mu\frac{U}{a} - \frac{a}{2}\left(\frac{\partial p}{\partial x}\right) = 1.79\times10^{-5}\,\frac{N\cdot s}{m^2} \times 2\,\frac{m}{s} \times \frac{1}{2.5\times10^{-3}\,m} - \frac{2.5\times10^{-3}\,m}{2}\left(-68.7\,\frac{N}{m^3}\right) = 0.10\,\frac{N}{m^2}$

$y=a$ $\quad \tau = \mu\frac{U}{a} + \frac{a}{2}\left(\frac{\partial p}{\partial x}\right) = -0.0716\ N/m^2$

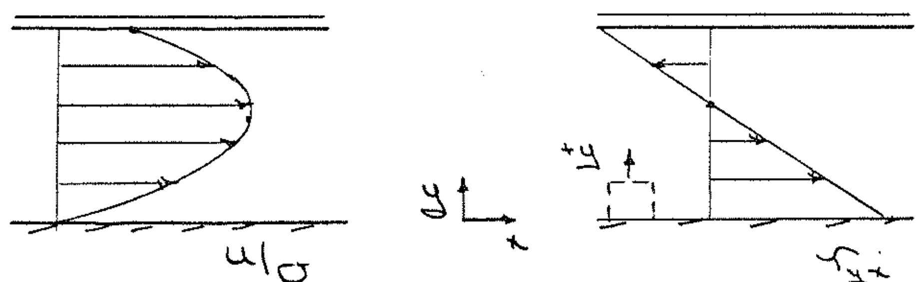
Note that the point of zero shear stress is not at $y/a = 0.5$ and hence $y/a = 0.5$ is not the location of maximum velocity. Maximum velocity occurs at $y/a > 0.5$.

To find the location of zero shear set $\tau_{yx} = 0$, then

$$0 = \frac{\mu U}{a} + a\left(\frac{\partial p}{\partial x}\right)\left(\frac{y}{a} - \frac{1}{2}\right) \quad \text{and} \quad \frac{y}{a} = \frac{1}{2} - \frac{\mu U}{a^2\left(\frac{\partial p}{\partial x}\right)}$$

$$\frac{y}{a} = 0.5 - 1.79 \times 10^{-5} \frac{N \cdot s}{m^2} \times \frac{2 \cdot m}{s} \times \frac{1}{(2.5 \times 10^{-3} m)^2} \times \frac{1}{(-68.7)} \frac{m^3}{N} = 0.583$$

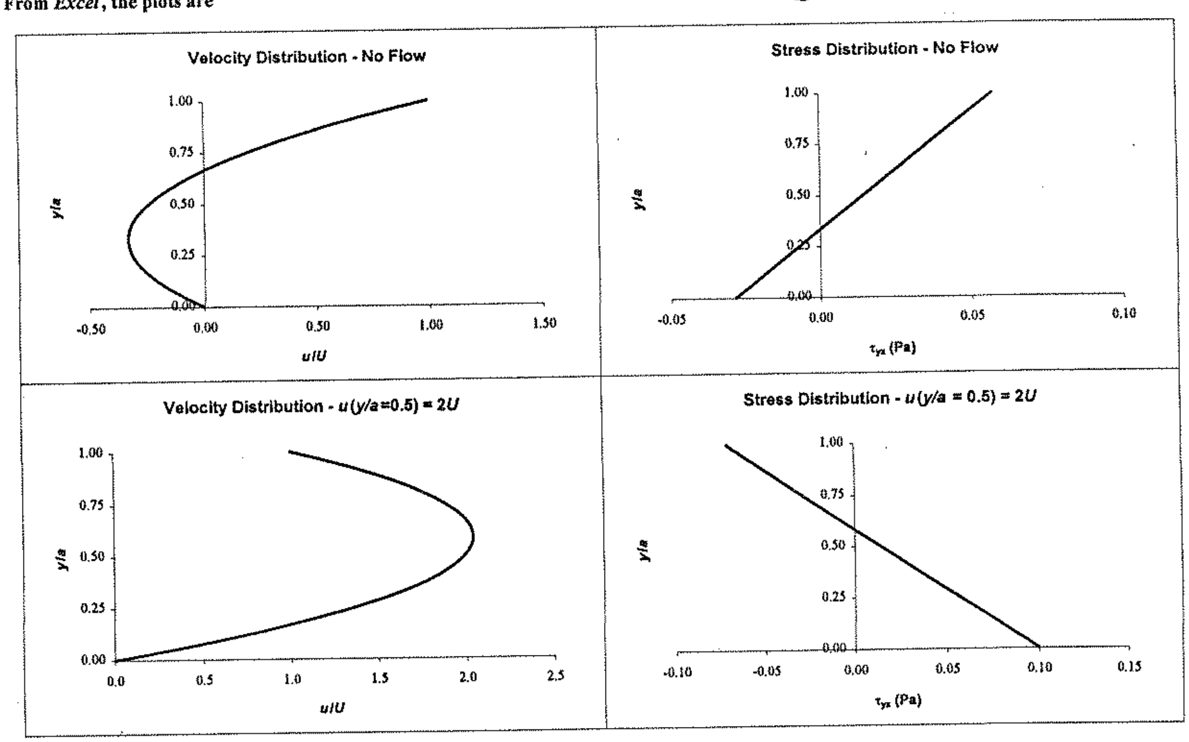
For this case ($u = 2U$ at $y/a = 0.5$) the velocity and shear stress distributions would be as follows



The shear stress is positive ($du/dy > 0$) below $y/a = 0.583$; positive stress acts in positive x direction on a positive y surface.

The shear stress is negative ($du/dy < 0$) above $y/a = 0.583$; negative stress acts in the negative x direction on a positive y surface.

From *Excel*, the plots are

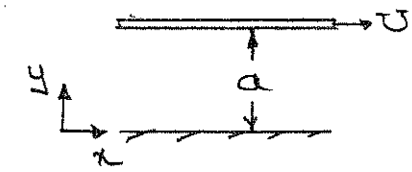


Problem 8.34

Given: Velocity profile for fully developed laminar flow of water between parallel plates

$$u = \frac{Uy}{a} + \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y}{a}\right)^2 - \frac{y}{a}\right]$$

$U = 2\ \text{m/s} \qquad a = 2.5\ \text{mm}$



Find: (a) Volume flow rate for zero pressure gradient.
(b) shear stress on lower plate; sketch $\tau(y)$
(c) effect of mild adverse pressure gradient on Q
(d) pressure gradient for zero shear at $y/a = 0.25$; sketch $\tau(y)$.

Solution:

Computing equations: $\tau_{yx} = \mu\frac{U}{a} + a\left(\frac{\partial p}{\partial x}\right)\left[\frac{y}{a} - \frac{1}{2}\right]$ (8.9a)

$\frac{Q}{l} = \frac{Ua}{2} - \frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^3$ (8.9b)

For $\partial p/\partial x = 0$, $\frac{Q}{l} = \frac{Ua}{2} = \frac{1}{2}\times 2\frac{\text{m}}{\text{s}}\times 2.5\times10^{-3}\,\text{m} = 2.50\times10^{-3}\ \text{m}^3/\text{s}/\text{m}$ — Q

The shear stress is $\tau_{yx} = \mu\frac{U}{a}$ {At 15°C, $\mu = 1.14\times10^{-3}\ \text{N}\cdot\text{s}/\text{m}^2$ (Table A.8)}

$\tau_{yx} = 1.14\times10^{-3}\frac{\text{N}\cdot\text{s}}{\text{m}^2}\times 2\frac{\text{m}}{\text{s}}\times\frac{1}{2.5\times10^{-3}\,\text{m}} = 0.912\ \text{N/m}^2$ ← τ_{yx}

The shear stress is constant across the channel (curve 1 below)

For $\partial p/\partial x > 0$, Eq. 8.9b indicates that Q will decrease.

For $\tau = 0$ at $y/a = 0.25$

$$\tau_{yx} = 0 = \mu\frac{U}{a} + a\left(\frac{\partial p}{\partial x}\right)\left[\frac{1}{4} - \frac{1}{2}\right] = \mu\frac{U}{a} - \frac{a}{4}\left(\frac{\partial p}{\partial x}\right)$$

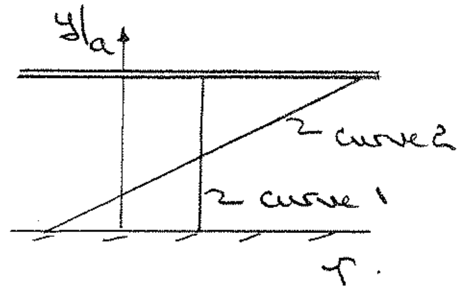
$$\frac{\partial p}{\partial x} = \frac{4\mu U}{a^2} = 4\times1.14\times10^{-3}\frac{\text{N}\cdot\text{s}}{\text{m}^2}\times 2\frac{\text{m}}{\text{s}}\times\frac{1}{(2.5\times10^{-3}\,\text{m})^2} = 1.46\ \text{kN/m}^2/\text{m}$$ ← $\frac{\partial p}{\partial x}$

For this pressure gradient

$$\tau_{yx} = 1.14\times10^{-3}\frac{\text{N}\cdot\text{s}}{\text{m}^2}\times 2\frac{\text{m}}{\text{s}}\times\frac{1}{2.5\times10^{-3}\,\text{m}} + 2.5\times10^{-3}\,\text{m}\times1.46\times10^{3}\frac{\text{N}}{\text{m}^3}\left[\frac{y}{a} - 0.5\right]$$

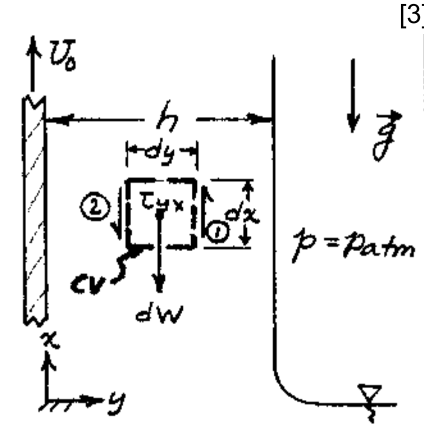
$$\tau_{yx} = 0.912\ \text{N/m}^2 + 3.65\ \frac{\text{N}}{\text{m}^2}\left(\frac{y}{a} - 0.5\right)$$

$\left.\tau_{yx}\right)_{y=0} = -0.913\ \text{N/m}^2$
$\left.\tau_{yx}\right)_{y=a} = 2.74\ \text{N/m}^2$ } curve 2



Problem 8.35

[3]



Bath: ρ, μ

Given: Belt moving steadily through bath as shown.

Assume zero shear at film/air surface, and no pressure forces.

Find: (a) Boundary conditions for velocity at $y=0$, $y=h$.
(b) Velocity profile.

Solution: Choose CV $dx\,dy\,dz$ as shown.
Apply x component of momentum equation.

Basic equations:

$$F_{S_x} + F_{B_x} = \overset{=0(2)}{\frac{\partial}{\partial t}\int_{CV} u\rho\, dV} + \overset{=0(3)}{\int_{CS} u\rho \vec{V}\cdot d\vec{A}} \quad ; \quad \tau_{yx} = \mu \frac{du}{dy} = \tau$$

Assumptions: (1) F_{S_x} due to shear forces only
(2) Steady flow
(3) Fully-developed flow

Then

$$F_{S_x} + F_{B_x} = F_{①} - F_{②} + F_{B_x} = \left(\tau + \frac{d\tau}{dy}\frac{dy}{2}\right)dx\,dz - \left(\tau - \frac{d\tau}{dy}\frac{dy}{2}\right)dx\,dz - \rho g\, dx\,dy\,dz = 0$$

or $\frac{d\tau}{dy} = \rho g$. Integrating

$$\tau = \rho g y + c_1 = \mu \frac{du}{dy} \quad \text{or} \quad \frac{du}{dy} = \frac{\rho g y}{\mu} + \frac{c_1}{\mu}. \text{ Integrating again,}$$

$$u = \frac{\rho g y^2}{2\mu} + \frac{c_1}{\mu} y + c_2$$

To evaluate the constants c_1 and c_2, apply the boundary conditions:

At $y=0$, $u = U_0$, so $c_2 = U_0$

At $y=h$, $\tau = 0$, so $\frac{du}{dy} = 0$, and $c_1 = -\rho g h$ ← BC

Substituting,

$$u = \frac{\rho g y^2}{2\mu} - \frac{\rho g h y}{\mu} + U_0 = \frac{\rho g}{\mu}\left(\frac{y^2}{2} - hy\right) + U_0$$ ← u

{Note that at $y = h$,

$$u = \frac{\rho g}{\mu}\left(-\frac{h^2}{2}\right) + U_0 \neq 0$$

Thus the solution is determined only when U_0 and h are known.}

Problem 8.36

[2]

8.36 In Example 8.3 we derived the velocity profile for laminar flow on a vertical wall by using a differential control volume. Instead, following the procedure we used in Example 5.9, derive the velocity profile by starting with the Navier-Stokes equations (Eqs. 5.27). Be sure to state all assumptions.

Given: Navier-Stokes Equations

Find: Derivation of Eq. 8.5

Solution:

The Navier-Stokes equations are (using the coordinates of Example 8.3, so that x is vertical, y is horizontal)

$$\cancelto{4}{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y} + \cancelto{3}{\frac{\partial w}{\partial z}} = 0 \qquad (5.1c)$$

$$\rho\left(\cancelto{1}{\frac{\partial u}{\partial t}} + u\cancelto{4}{\frac{\partial u}{\partial x}} + v\cancelto{5}{\frac{\partial u}{\partial y}} + w\cancelto{3}{\frac{\partial u}{\partial z}}\right) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\cancelto{4}{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancelto{3}{\frac{\partial^2 u}{\partial z^2}}\right) \qquad (5.27a)$$

$$\rho\left(\cancelto{1}{\frac{\partial v}{\partial t}} + u\cancelto{4}{\frac{\partial v}{\partial x}} + v\cancelto{5}{\frac{\partial v}{\partial y}} + w\cancelto{3}{\frac{\partial v}{\partial z}}\right) = \cancelto{6}{\rho g_y} - \frac{\partial p}{\partial y} + \mu\left(\cancelto{4}{\frac{\partial^2 v}{\partial x^2}} + \cancelto{5}{\frac{\partial^2 v}{\partial y^2}} + \cancelto{3}{\frac{\partial^2 v}{\partial z^2}}\right) \qquad (5.27b)$$

$$\rho\left(\cancelto{1}{\frac{\partial w}{\partial t}} + u\cancelto{3}{\frac{\partial w}{\partial x}} + v\cancelto{3}{\frac{\partial w}{\partial y}} + w\cancelto{3}{\frac{\partial w}{\partial z}}\right) = \cancelto{3}{\rho g_z} - \cancelto{3}{\frac{\partial p}{\partial z}} + \mu\left(\cancelto{3}{\frac{\partial^2 w}{\partial x^2}} + \cancelto{3}{\frac{\partial^2 w}{\partial y^2}} + \cancelto{3}{\frac{\partial^2 w}{\partial z^2}}\right) \qquad (5.27c)$$

The following assumptions have been applied:

(1) Steady flow (given).
(2) Incompressible flow; ρ = constant.
(3) No flow or variation of properties in the z direction; w= 0 and $\partial/\partial z = 0$.
(4) Fully developed flow, so no properties except possibly pressure p vary in the x direction; $\partial/\partial x = 0$.
(5) See analysis below.
(6) No body force in the y direction; $g_y = 0$

Assumption (1) eliminates time variations in any fluid property. Assumption (2) eliminates space variations in density. Assumption (3) states that there is no z component of velocity and no property variations in the z direction. All terms in the z component of the Navier–Stokes equation cancel. After assumption (4) is applied, the continuity equation reduces to $\partial v/\partial y = 0$. Assumptions (3) and (4) also indicate that $\partial v/\partial z = 0$ and $\partial v/\partial x = 0$. Therefore v must be constant. Since v is zero at the solid surface, then v must be zero everywhere. The fact that $v = 0$ reduces the Navier–Stokes equations further, as indicated by (5). Hence for the y direction

$$\frac{\partial p}{\partial y} = 0$$

which indicates the pressure is a constant across the layer. However, at the free surface $p = p_{atm}$ = constant. Hence we conclude that p = constant throughout the fluid, and so

$$\frac{\partial p}{\partial x} = 0$$

In the *x* direction, we obtain

$$\mu \frac{\partial^2 u}{\partial y^2} + \rho g = 0$$

Integrating twice

$$u = -\frac{1}{2\mu}\rho g y^2 + \frac{c_1}{\mu} y + c_2$$

To evaluate the constants, c_1 and c_2, we must apply the boundary conditions. At $y = 0$, $u = 0$. Consequently, $c_2 = 0$. At $y = a$, $du/dy = 0$ (we assume air friction is negligible). Hence

$$\tau(y = \delta) = \mu \frac{du}{dy}\bigg|_{y=\delta} = -\frac{1}{\mu}\rho g \delta + \frac{c_1}{\mu} = 0$$

which gives

$$c_1 = \rho g \delta$$

and finally

$$u = -\frac{1}{2\mu}\rho g y^2 + \frac{\rho g}{\mu} y = \frac{\rho g}{\mu}\delta^2 \left[\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^2 \right]$$

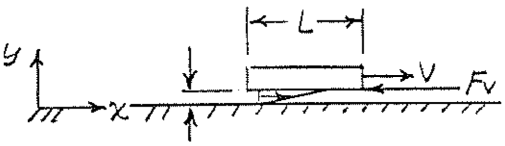
Problem 8.37

[4]

Given: Microchip supported on air film, on a horizontal surface. Chips are $L = 11.7$ mm long, $w = 9.35$ mm wide, and have mass $m = 0.325$ g. The air film is $h = 0.125$ mm thick. The initial speed of the chips is $V_0 = 1.75$ mm/s; they slow from viscous shear.

Find: (a) Differential equation for chip motion during deceleration.
(b) Time required for chip to lose 5 percent of V_0.
(c) Plot of chip speed vs. time, with labels and comments.

Solution: Apply Newton's law of viscosity



Basic equations: $\tau_{yx} = \mu \dfrac{du}{dy}$

$$F_V = \tau A \qquad \sum F = m a_x$$

Assume: (1) Newtonian fluid
(2) Linear velocity profile in narrow gap
(3) Air at STP

Then

$$\tau_{yx} = \mu \frac{du}{dy} = \mu \frac{V}{h}; \quad F_V = \tau A = \mu \frac{V}{h} wL = \frac{\mu V w L}{h}$$

The free-body diagram for the chip is

$$\sum F_x = -F_V = -\frac{\mu V w L}{h} = m\frac{dV}{dt}; \quad \frac{dV}{V} = -\frac{\mu w L}{m h}\, dt$$ D.E.

Integrating, $\displaystyle\int_{V_0}^{V} \frac{dV}{V} = \ln \frac{V}{V_0} = -\frac{\mu w L}{m h} t$

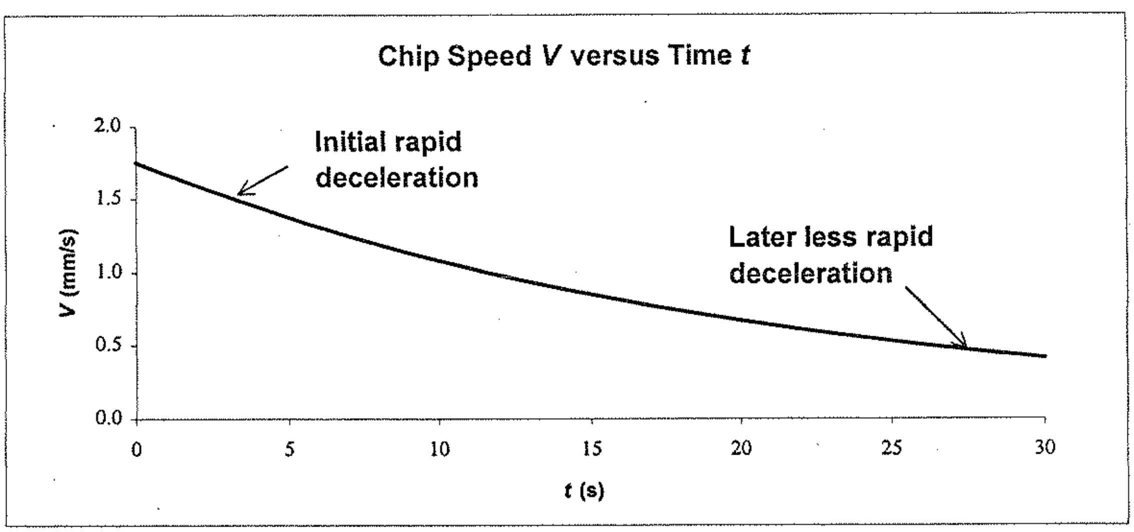
Thus

$$t = -\frac{m h}{\mu w L} \ln \frac{V}{V_0}$$

$$t = -0.325\,\text{g} \times 0.125\,\text{mm} \times \frac{\text{m}\cdot\text{s}}{1.79\times10^{-5}\,\text{kg}} \times \frac{1}{9.35\,\text{mm}} \times \frac{1}{11.7\,\text{mm}} \times \ln 0.95 \times \frac{\text{kg}}{1000\,\text{g}} \times \frac{1000\,\text{mm}}{\text{m}}$$

$t = 1.06$ s t

From *Excel*, the plot of speed vs. time is:



Problem 8.38

Given: Free-surface waves begin to form on a laminar liquid film flowing down an inclined surface whenever the Reynolds number, based on mass flow per unit width of film, is larger than about 33.

Find: Estimate of the maximum thickness of a laminar film of water that remains free from waves while flowing down a vertical surface.

Solution: The mass flow rate is $\dot{m} = \rho \bar{V} A = \rho \bar{V} w \delta$, so $\dot{m}/w = \rho \bar{V} \delta$.

Thus

$$Re = \frac{\rho \bar{V} \delta}{\mu} = \frac{\bar{V}\delta}{\nu} = 33 \text{ (maximum)}$$

Using the result for average velocity from Example 8.3

$$\bar{V} = \frac{\rho g \delta^2}{3\mu}$$

Thus

$$\frac{\rho \bar{V} \delta}{\mu} = \frac{\rho^2 g \delta^3}{3\mu^2} = 33$$

Solving for δ,

$$\delta = \left[\frac{99\mu^2}{\rho^2 g}\right]^{1/3}$$

At $T = 20°C$, $\mu = 1.00 \times 10^{-3}$ kg/m·s and $\rho = 998$ kg/m³ (Table A.8). Substituting,

$$\delta = \left[99 \times (1.00\times10^{-3})^2 \frac{kg^2}{m^2\cdot s^2} \times \frac{m^6}{(998)^2\, kg^2} \times \frac{s^2}{9.81\, m}\right]^{1/3}$$

$$\delta = 2.16 \times 10^{-4}\ m \quad \text{or} \quad 0.216\ mm$$

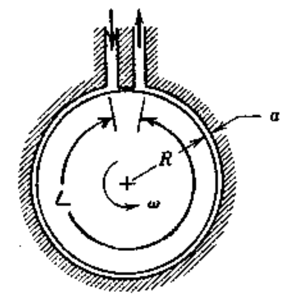
δ_{max}

Problem 8.39

[5]

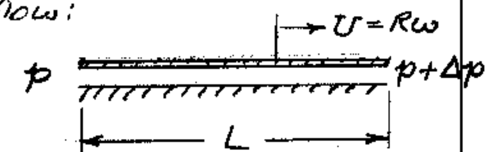
Given: Viscous-shear pump, as shown.

b = width normal to diagram; $a \ll R$



Find: Performance characteristics
(a) Pressure differential
(b) Input power
(c) Efficiency
as functions of volume flow rate.

Solution: Since $a \ll R$, unwrap to form flow between parallel plates. Apply Eqs. 8.9 to fully developed flow:



Volume flow rate is $\frac{Q}{b} = \frac{Ua}{2} - \frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^3$

Substituting $U = R\omega$ and $\frac{\partial p}{\partial x} = \frac{\Delta p}{L}$, then

$$\Delta p = \frac{12\mu L}{a^3}\left(\frac{\omega R a}{2} - \frac{Q}{b}\right) = \frac{6\mu L R\omega}{a^2}\left(1 - \frac{2Q}{abR\omega}\right)$$ Δp

Torque is $T = \tau R(bL) = RLb\tau$. Power is $P = T\omega$. From Eq. 8.9a, at $y = a$,

$$P = RLb\omega\left[\mu\frac{R\omega}{a} + \frac{\Delta p}{L}\frac{a}{2}\right] = RLb\omega\left[\mu\frac{R\omega}{a} + \frac{6\mu L R\omega}{a^2}\left(1 - \frac{2Q}{abR\omega}\right)\frac{a}{2L}\right]$$

$$P = RLb\omega\left[\frac{\mu R\omega}{a}\left(4 - \frac{6Q}{abR\omega}\right)\right] = \frac{\mu L b (R\omega)^2}{a}\left(4 - \frac{6Q}{abR\omega}\right)$$ P

Output power is $Q\Delta p$, so efficiency is

$$\eta = \frac{Q\Delta p}{P} = \frac{6\mu Q L R\omega}{a^2}\left(1 - \frac{2Q}{abR\omega}\right)\frac{a}{\mu L b (R\omega)^2}\frac{1}{\left(4 - \frac{6Q}{abR\omega}\right)}$$

$$\eta = \frac{6Q}{abR\omega}\frac{\left(1 - \frac{2Q}{abR\omega}\right)}{\left(4 - \frac{6Q}{abR\omega}\right)}$$ η

Problem 8.40 (In Excel)

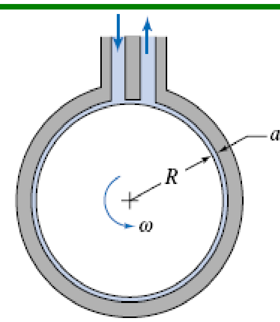
8.40 The efficiency of the viscous-shear pump of Fig. P8.39 is given by

$$\eta = 6q\frac{(1-2q)}{(4-6q)}$$

where $q = Q/abR\omega$ is a dimensionless flow rate (Q is the flow rate at pressure differential Δp, and b is the depth normal to the diagram). Plot the efficiency versus dimensionless flow rate, and find the flow rate for maximum efficiency. Explain why the efficiency peaks, and why it is zero at certain values of q.

Given: Expression for efficiency

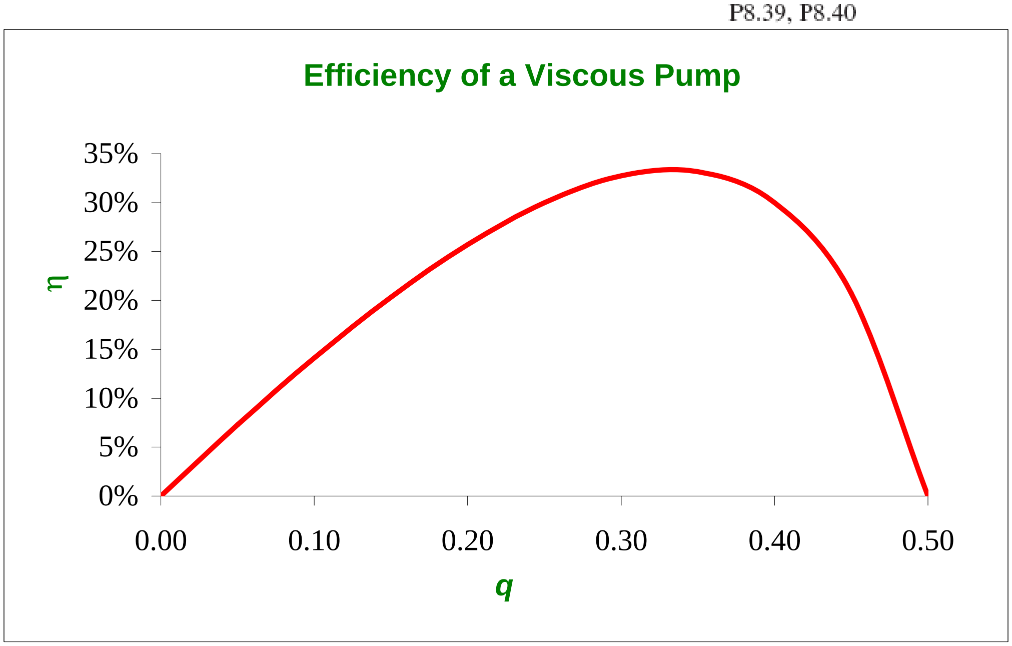
Find: Plot; find flow rate for maximum efficiency; explain curve



P8.39, P8.40

Solution:

q	η
0.00	0.0%
0.05	7.30%
0.10	14.1%
0.15	20.3%
0.20	25.7%
0.25	30.0%
0.30	32.7%
0.35	33.2%
0.40	30.0%
0.45	20.8%
0.50	0.0%

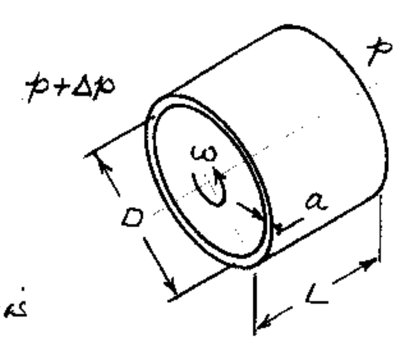


For the maximum efficiency point we can use *Solver* (or alternatively differentiate)

q	η
0.333	33.3%

The efficiency is zero at zero flow rate because there is no output at all
The efficiency is zero at maximum flow rate $\Delta p = 0$ so there is no output
The efficiency must therefore peak somewhere between these extremes

Problem 8.41



Given: Annular gap seal as shown.

Power required to pump oil, P_p.

Power to overcome viscous dissipation, P_v.

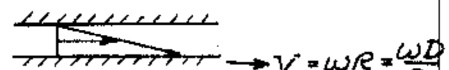
Find: (a) Expressions for P_p, P_v

(b) Show total power minimized when a is chosen so that $P_v = 3P_p$.

Solution: Apply Eqs. 8.6 and 8.9 for flow between parallel plates.

Assumptions: (1) $a \ll D$, so unfold to flat plates

(2) No pressure gradient circumferentially



The viscous power is the product of viscous torque times ω:

$$P_v = T\omega = \tau(2\pi R L)R\omega = \mu\frac{V}{a}\left(2\pi\frac{D}{2}L\right)\frac{D}{2}\omega = \mu\frac{\omega D}{2a}\pi D L\frac{D}{2}\omega = \frac{\pi\mu\omega^2 D^3 L}{4a}$$ P_v

The pump power is the product of flow rate times pressure drop.

$$P_p = Q\Delta p$$

From Eq. 8.6c, $Q = \dfrac{l a^3\Delta p}{12\mu L} = \dfrac{\pi D a^3 \Delta p}{12\mu L}$, so $P_p = \dfrac{\pi D a^3 \Delta p^2}{12\mu L}$ P_p

The total power required is $P_T = P_v + P_p = \dfrac{\pi\mu\omega^2 D^3 L}{4a} + \dfrac{\pi D a^3 \Delta p^2}{12\mu L}$

It may be minimized by setting $\dfrac{dP_T}{da} = 0$. Thus

$$\frac{dP_T}{da} = -\frac{\pi\mu\omega^2 D^3 L}{4a^2} + \frac{\pi D a^2 \Delta p^2}{4\mu L} = 0 \qquad (1)$$

This can be written

$$\frac{dP_T}{da} = -\frac{1}{a}P_v + \frac{3}{a}P_p = 0$$

which is satisfied when $3P_p - P_v = 0$ or $P_v = 3P_p$ Optimum

Equation 1 also can be solved for a at optimum conditions:

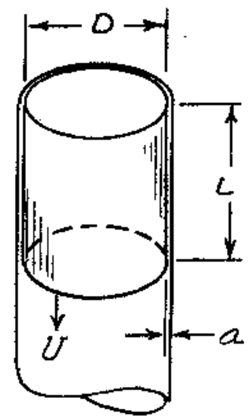
$$a^4 = \frac{\mu^2\omega^2 D^2 L^2}{\Delta p^2} \quad \text{or} \quad a^2 = \frac{\mu\omega D L}{\Delta p} \quad \text{or} \quad \frac{a}{D} = \sqrt{\frac{\mu\omega L}{D\Delta p}} \text{ (optimum)}$$

Problem 8.42

Given: "Viscous timer," consisting of a cylindrical mass inside a circular tube filled with viscous liquid, creating a narrow annular gap.

Find: (a) The flow field created when the mass falls under gravity.
(b) Whether this would make a satisfactory timer, and if so, for what range of time intervals.
(c) Effect of temperature change on measured time interval.

Solution: Apply conservation of mass to a CV enclosing the cylinder and the moving mass:



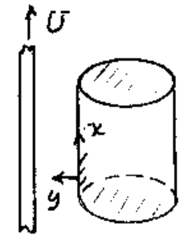
Then: $Q = U\frac{\pi D^2}{4} = \bar{V}\,\pi D a = \bar{V} l a$ (1)

Assume: (1) Gap is narrow, $a \ll D$
(2) Unroll gap so flat, $l = \pi D$
(3) Steady flow
(4) Fully developed laminar flow

Under these assumptions, the flow field in the gap is that for flow between parallel plates with one plate moving.

Flow Field

Place coordinates on the moving mass:

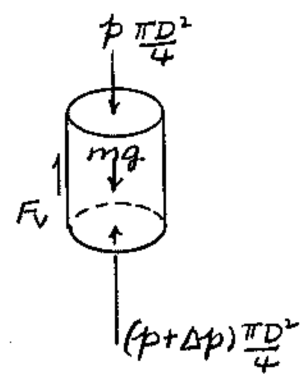


Then the volume flow rate (Eq. 8.9b) is

$$\frac{Q}{l} = \frac{Q}{\pi D} = \frac{Ua}{2} - \frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^3$$

But $\frac{\partial p}{\partial x} = -\frac{\Delta p_v}{L}$, where Δp_v is the pressure drop driving viscous flow, so

$$\frac{Q}{l} = \frac{Ua}{2} - \frac{1}{12\mu}\left(-\frac{\Delta p_v}{L}\right)a^3 = \frac{Ua}{2} + \frac{\Delta p_v a^3}{12\mu L} \qquad (2)$$



The pressure change across the moving mass is

$$\Delta p = \rho_l g L + \Delta p_v \qquad (3)$$

Summing forces on the moving mass gives

$$\Sigma F_x = \Delta p\,\frac{\pi D^2}{4} - mg + F_v = m\frac{dU}{dt} = 0\,(3)$$

But $mg = \rho_m \frac{\pi D^2}{4} L$ and $F_v = \tau_s \pi D L$

From Eq. 8.9a, $\tau_s = \mu\frac{U}{a} - \frac{a}{2}\left(\frac{\partial p}{\partial x}\right) = \mu\frac{U}{a} + \frac{a}{2}\frac{\Delta p_v}{L}$

Substituting, $\Delta p\,\frac{\pi D^2}{4} - \rho_m \frac{\pi D^2}{4} L g + \left[\mu\frac{U}{a} + \frac{a}{2}\frac{\Delta p_v}{L}\right]\pi D L = 0$

or $\Delta p = \rho_m g L - \left[\mu\frac{U}{a} + \frac{a}{2}\frac{\Delta p_v}{L}\right]\frac{4L}{D}$ (4)

Combining Eqs. 1 and 2 gives $\frac{UD}{4} = \frac{Ua}{2} + \frac{\Delta p_v a^3}{12\mu L}$

Thus

$$\Delta p_v = \frac{12\mu L}{a^3}\left[\frac{UD}{4} - \frac{Ua}{2}\right] = \frac{3\mu U L D}{a^3} \qquad (5)$$

($a \ll D$) Δp_v

Combining Eqs. 3 and 4 gives $\Delta p = \rho_\ell g L + \Delta p_v = \rho_m g L - \left[\mu \frac{U}{a} + \frac{a}{2}\frac{\Delta p_v}{L}\right]\frac{4L}{D}$

Using Eq. 5,

$$\rho_\ell g L + \frac{3\mu U L D}{a^3} = \rho_m g L - \mu \frac{U}{a}\frac{4L}{D} - \frac{a}{2}\frac{3\mu U L D}{L a^3}\frac{4L}{D}$$

Simplifying and re-arranging,

$$(\rho_m - \rho_\ell) g L = \frac{3\mu U L D}{a^3} + \frac{4\mu U L}{a D} + \frac{6\mu U L}{a^2} \approx \frac{3\mu U L D}{a^3}$$

Finally, using $\rho = SG\,\rho_{H_2O}$,

$$U = \frac{(SG_m - SG_\ell)\rho_{H_2O}\, g\, a^3}{3\mu D}$$

U

The time interval for the mass to move distance H is

$$\Delta t = \frac{H}{U} = \frac{3\mu D}{(SG_m - SG_\ell)\rho_{H_2O}\, g\, a^3} \qquad (6)$$

Δt

Equation 6 shows that the time interval for the mass to fall any distance *H* is proportional to liquid viscosity μ and inversely proportional to gap width *a* cubed. A temperature change would affect the diameter of the measuring tube and the diameter of the falling mass. A temperature change also would affect the viscosity of the liquid in the tube.

Speed of the falling mass is proportional to the cube of gap width. If the coefficient of thermal expansion of the falling mass were greater than that of the glass measuring tube (which seems likely), then the width of the annular gap would decrease with increasing temperature. This would tend to slow the falling mass. The total amount of thermal expansion would depend on the diameter of the mass and tube. The effect on gap width would be greater, the larger the tube diameter compared to the initial gap width.

It might be possible to "tailor" the thermal expansion coefficient of the cylinder, by using a suitable material, to closely match that of the falling mass. Then there would be no differential thermal expansion between the mass and tube, and changes in temperature would not affect the gap width.

Speed of the falling mass is inversely proportional to liquid viscosity. Liquid viscosity decreases sharply as temperature increases (the viscosity of SAE 30 oil drops more than 10 percent as its temperature increases from 20°C to 25°C, see Fig. A.2). This would tend to increase the speed of the falling mass.

The entire device could be maintained at constant temperature.

Problem 8.43

[5]

Open-Ended Design Problem: Automotive design is tending toward all-wheel drive to improve vehicle performance and safety when traction is poor. An all-wheel drive vehicle must have an interaxle differential to allow operation on dry roads. Numerous vehicles are being built using multiplate viscous drives for interaxle differentials. Perform the analysis and design needed to define the torque transmitted by the differential for a given speed difference, in terms of the design parameters. Identify suitable dimensions for a viscous differential to transmit a torque of 150 N · m at a speed loss of 125 rpm, using lubricant with the properties of SAE 30 oil. Discuss how to find the minimum material cost for the differential, if the plate cost per square meter is constant.

Solution: From Problem 2.59, $dT = r\,dF = r\tau\,dA$

But $\tau = \mu \frac{du}{dy} = \mu \frac{u}{h} = \mu \frac{r\Delta\omega}{h}$; $dA = 2\pi r\,dr$

Thus $dT = r\mu \frac{r\Delta\omega}{h} 2\pi r\,dr = \frac{2\pi\mu\Delta\omega}{h} r^3\,dr$; $T = \frac{\pi\mu\Delta\omega}{2h}\left[R_o^4 - R_i^4\right]$

or $T = \frac{\pi\mu\Delta\omega}{2h} R^4(1-\alpha^4)$ where $\alpha = R_i/R$ — T

This value is per gap. Each rotor has 2 gaps to a housing. For n gaps

$$T_n = \frac{n\pi\mu\Delta\omega}{2h} R^4(1-\alpha^4) \qquad (1)$$ — T_n

From Eq. 1, assuming $\mu = 0.18$ kg/m·s (Fig. A.2) and $\alpha = \frac{1}{2}$, so $1-\alpha^4 = 1 - \frac{1}{16} \approx 1$, then

$$\frac{nR^4}{h} = \frac{2T_n}{\pi\mu\Delta\omega} = \frac{2}{\pi} \times 150\ \text{N}\cdot\text{m} \times \frac{\text{m}^2}{0.18\ \text{N}\cdot\text{s}} \times \frac{\text{min}}{125\ \text{rev}} \times \frac{\text{rev}}{2\pi\ \text{rad}} \times 60\frac{\text{s}}{\text{min}} = 40.5\ \text{m}^3 = C$$

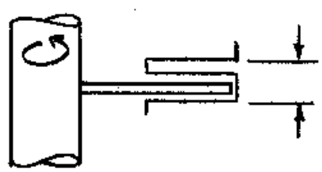
or

$$R^4 = C\frac{h}{n}$$

For $n = 100$ and $h = 0.2$ mm, $R^4 = 40.5\ \text{m}^3 \times 0.0002\ \text{m} \times \frac{1}{100} = 8.11 \times 10^{-5}\ \text{m}^4$

$$R = \left[8.11\times 10^{-5}\right]^{1/4} \text{m} = 0.0949\ \text{m} \quad (\text{or } D \approx 190\ \text{mm})$$ — R

The stack length might be



≈ 2.5 mm for $n = 2$, or 125 mm for $n = 100$

Problem 8.44

8.44 A journal bearing consists of a shaft of diameter $D = 50$ mm and length $L = 1$ m (moment of inertia $I = 0.055$ kg · m^2) installed symmetrically in a stationary housing such that the annular gap is $\delta = 1$ mm. The fluid in the gap has viscosity $\mu = 0.1$ N · s/m^2. If the shaft is given an initial angular velocity of $\omega = 60$ rpm, determine the time for the shaft to slow to 10 rpm.

Given: Data on a journal bearing

Find: Time for the bearing to slow to 10 rpm

Solution:

The given data is $D = 50 \cdot mm$ $\quad L = 1 \cdot m$ $\quad I = 0.055 \cdot kg \cdot m^2$ $\quad \delta = 1 \cdot mm$

$\mu = 0.1 \cdot \frac{N \cdot s}{m^2}$ $\quad \omega_i = 60 \cdot rpm$ $\quad \omega_f = 10 \cdot rpm$

The equation of motion for the slowing bearing is $I \cdot \alpha = Torque = -\tau \cdot A \cdot \frac{D}{2}$

where α is the angular acceleration and τ is the viscous stress, and $A = \pi \cdot D \cdot L$ is the surface area of the bearing

As in Example 8.2 the stress is given by $\tau = \mu \cdot \frac{U}{\delta} = \frac{\mu \cdot D \cdot \omega}{2 \cdot \delta}$

where *U* and ω are the instantaneous linear and angular velocities.

Hence $I \cdot \alpha = I \cdot \frac{d\omega}{dt} = -\frac{\mu \cdot D \cdot \omega}{2 \cdot \delta} \cdot \pi \cdot D \cdot L \cdot \frac{D}{2} = -\frac{\mu \cdot \pi \cdot D^3 \cdot L}{4 \cdot \delta} \cdot \omega$

Separating variables $\frac{d\omega}{\omega} = -\frac{\mu \cdot \pi \cdot D^3 \cdot L}{4 \cdot \delta \cdot I} \cdot dt$

Integrating and using IC $\omega = \omega_0$ $\quad \omega(t) = \omega_i \cdot e^{-\frac{\mu \cdot \pi \cdot D^3 \cdot L}{4 \cdot \delta \cdot I} \cdot t}$

The time to slow down to $\omega_f = 10$ rpm is obtained from solving $\quad \omega_f = \omega_i \cdot e^{-\frac{\mu \cdot \pi \cdot D^3 \cdot L}{4 \cdot \delta \cdot I} \cdot t}$

so $\quad t = -\frac{4 \cdot \delta \cdot I}{\mu \cdot \pi \cdot D^3 \cdot L} \cdot \ln\left(\frac{\omega_f}{\omega_i}\right)$ $\quad t = 10\,s$

Problem 8.45

[2]

Given: Fully developed laminar flow in a pipe, with

$$u = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{r}{R}\right)^2\right]$$

Find: Radius from pipe axis at which u equals the average velocity, $\bar{V}$.

Solution: First determine $\bar{V}$.

$$\bar{V} = \frac{Q}{A} = \frac{1}{\pi R^2}\int_A u\,dA = \frac{1}{\pi R^2}\int_0^R \left\{-\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{r}{R}\right)^2\right]\right\} 2\pi r\,dr$$

$$= -\frac{R^2}{2\mu}\frac{\partial p}{\partial x}\int_0^1\left[1-\left(\frac{r}{R}\right)^2\right]\left(\frac{r}{R}\right)d\left(\frac{r}{R}\right) = -\frac{R^2}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1}{2}\left(\frac{r}{R}\right)^2-\frac{1}{4}\left(\frac{r}{R}\right)^4\right]_0^1$$

$$\bar{V} = -\frac{R^2}{8\mu}\frac{\partial p}{\partial x}$$

Then $u = \bar{V}$ when

$$u = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{r}{R}\right)^2\right] = \bar{V} = -\frac{R^2}{8\mu}\frac{\partial p}{\partial x}$$

or

$$1-\left(\frac{r}{R}\right)^2 = \frac{1}{2}$$

or

$$\left(\frac{r}{R}\right)^2 = \frac{1}{2}$$

$$r = \frac{R}{\sqrt{2}} = 0.707\,R$$

r

Problem 8.46

Given: Water and SAE 10W oil flowing at 40°C through a 6 mm tube.

Find, for each fluid:

(a) The maximum flowrate for laminar flow.
(b) The corresponding pressure gradient.

Solution: Laminar flow is expected for $Re \le 2300$. Expressing this in terms of flowrate,

$$Re = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} = \frac{QD}{A\nu} = \frac{4}{\pi D^2}\frac{QD}{\nu} = \frac{4Q}{\pi \nu D} \quad \text{or} \quad Q = \frac{\pi \nu D Re}{4}$$

Thus

$$Q_{max} = \frac{\pi \nu D Re_{max}}{4} = \frac{\pi}{4} \times 2300 \times 0.006\,m \times \nu\,\frac{m^2}{s} = 10.8\,\nu\left(\frac{m^3}{s}\right)$$

Also, $Q = -\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial x}$ for laminar flow, according to Eq. 8.13b. Then

$$\frac{\partial p}{\partial x} = -\frac{8\mu Q}{\pi R^4} = -\frac{128\,\mu Q}{\pi D^4}$$

so

$$\frac{\partial p}{\partial x} = -\frac{128}{\pi} \times \mu\,\frac{N\cdot s}{m^2} \times Q\,\frac{m^3}{s} \times \frac{1}{(0.006)^4\,m^4} = -3.14 \times 10^{10}\,\mu Q\,\left(\frac{N}{m^3}\right)$$

Using data from Appendix A, at 40°C,

Fluid	$\nu\left(\frac{m^2}{s}\right)$	$Q\left(\frac{m^3}{s}\right)$	$\mu\left(\frac{N\cdot s}{m^2}\right)$	$\mu Q\,(N\cdot m)$	$\frac{\partial p}{\partial x}\left(\frac{N}{m^3}\right)$
Water	6.57×10^{-7}	7.10×10^{-6}	6.51×10^{-4}	4.62×10^{-9}	-145
SAE 10W oil	3.8×10^{-5}	4.10×10^{-4}	3.4×10^{-2}	1.39×10^{-5}	-4.36×10^{5}

$\frac{\partial p}{\partial x}$

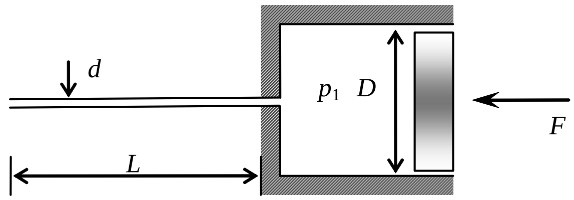
Q_{ma}

$\left\{ \text{Note } Q \sim \nu = \frac{\mu}{\rho} \text{ and } \frac{\partial p}{\partial x} \sim \mu Q \sim \frac{\mu^2}{\rho}. \right\}$

Problem 8.47

[2]

8.47 A hypodermic needle, with inside diameter $d = 0.005$ in. and length $L = 1$ in., is used to inject saline solution with viscosity five times that of water. The plunger diameter is $D = 0.375$ in.; the maximum force that can be exerted by a thumb on the plunger is $F = 7.5$ lbf. Estimate the volume flow rate of saline that can be produced.



Given: Hyperdermic needle

Find: Volume flow rate of saline

Solution:

Basic equation $Q = \frac{\pi \cdot \Delta p \cdot d^4}{128 \cdot \mu \cdot L}$ (Eq. 8.13c; we assume laminar flow and verify this is correct after solving)

For the system $\Delta p = p_1 - p_{atm} = \frac{F}{A} = \frac{4 \cdot F}{\pi \cdot D^2}$

$$\Delta p = \frac{4}{\pi} \times 7.5 \cdot lbf \times \left(\frac{1}{0.375 \cdot in} \times \frac{12 \cdot in}{1 \cdot ft}\right)^2 \qquad \Delta p = 67.9 \cdot psi$$

At 68°F, from Table A.7 $\mu_{H2O} = 2.1 \times 10^{-5} \cdot \frac{lbf \cdot s}{ft^2}$ $\mu = 5 \cdot \mu_{H2O}$ $\mu = 1.05 \times 10^{-4} \cdot \frac{lbf \cdot s}{ft^2}$

$$Q = \frac{\pi}{128} \times 67.9 \cdot \frac{lbf}{in^2} \times \frac{144 \cdot in^2}{1 \cdot ft^2} \times \left(0.005 \cdot in \times \frac{1 \cdot ft}{12 \cdot in}\right)^4 \times \frac{ft^2}{1.05 \times 10^{-4}\ lbf \cdot s} \times \frac{1}{1 \cdot in} \times \frac{12 \cdot in}{1 \cdot ft}$$

$$Q = 8.27 \times 10^{-7} \cdot \frac{ft^3}{s} \qquad Q = 1.43 \times 10^{-3} \cdot \frac{in^3}{s} \qquad Q = 0.0857 \cdot \frac{in^3}{min}$$

Check Re: $V = \frac{Q}{A} = \frac{Q}{\frac{\pi \cdot d^2}{4}}$ $V = \frac{4}{\pi} \times 8.27 \times 10^{-7} \frac{ft^3}{s} \times \left(\frac{1}{.005 \cdot in}\right)^2 \times \left(\frac{12 \cdot in}{1 \cdot ft}\right)^2$ $V = 6.07 \cdot \frac{ft}{s}$

$Re = \frac{\rho \cdot V \cdot d}{\mu}$ $\rho = 1.94 \cdot \frac{slug}{ft^3}$ (assuming saline is close to water)

$$Re = 1.94 \cdot \frac{slug}{ft^3} \times 6.07 \cdot \frac{ft}{s} \times 0.005 \cdot in \times \frac{1 \cdot ft}{12 \cdot in} \times \frac{ft^2}{1.05 \times 10^{-4} \cdot lbf \cdot s} \times \frac{slug \cdot ft}{s^2 \cdot lbf} \qquad Re = 46.7$$

Flow is laminar

Problem 8.48

8.48 In engineering science there are often analogies to be made between disparate phenomena. For example, the applied pressure difference Δp and corresponding volume flow rate Q in a tube can be compared to the applied DC voltage V across and current I through an electrical resistor, respectively. By analogy, find a formula for the "resistance" of laminar flow of fluid of viscosity μ in a tube length of L and diameter D, corresponding to electrical resistance R. For a tube 100 mm long with inside diameter 0.3 mm, find the maximum flow rate and pressure difference for which this analogy will hold for (a) kerosine and (b) castor oil (both at 40°C). When the flow exceeds this maximum, why does the analogy fail?

Given: Data on a tube

Find: "Resistance" of tube; maximum flow rate and pressure difference for which electrical analogy holds for (a) kerosine and (b) castor oil

Solution:

The given data is $L = 100\cdot mm$ $\quad D = 0.3\cdot mm$

From Fig. A.2 and Table A.2

Kerosene: $\mu = 1.1\times 10^{-3}\cdot\frac{N\cdot s}{m^2}$ $\quad \rho = 0.82\times 990\cdot\frac{kg}{m^3} = 812\cdot\frac{kg}{m^3}$

Castor oil: $\mu = 0.25\cdot\frac{N\cdot s}{m^2}$ $\quad \rho = 2.11\times 990\cdot\frac{kg}{m^3} = 2090\cdot\frac{kg}{m^3}$

For an electrical resistor $V = R\cdot I$ (1)

The governing equation for the flow rate for laminar flow in a tube is Eq. 8.13c

$$Q = \frac{\pi\cdot\Delta p\cdot D^4}{128\cdot\mu\cdot L}$$

or

$$\Delta p = \frac{128\cdot\mu\cdot L}{\pi\cdot D^4}\cdot Q \qquad (2)$$

By analogy, current *I* is represented by flow rate *Q*, and voltage *V* by pressure drop Δp.
Comparing Eqs. (1) and (2), the "resistance" of the tube is

$$R = \frac{128\cdot\mu\cdot L}{\pi\cdot D^4}$$

The "resistance" of a tube is directly proportional to fluid viscosity and pipe length, and strongly dependent on the inverse of diameter

The analogy is only valid for $Re < 2300$ or $\frac{\rho\cdot V\cdot D}{\mu} < 2300$

Writing this constraint in terms of flow rate $\frac{\rho\cdot\frac{Q}{\frac{\pi}{4}\cdot D^2}\cdot D}{\mu} < 2300$ or $Q_{max} = \frac{2300\cdot\mu\cdot\pi\cdot D}{4\cdot\rho}$

The corresponding maximum pressure gradient is then obtained from Eq. (2)

$$\Delta p_{max} = \frac{128 \cdot \mu \cdot L}{\pi \cdot D^4} \cdot Q_{max} = \frac{32 \cdot 2300 \cdot \mu^2 \cdot L}{\rho \cdot D^3}$$

(a) For kerosine $Q_{max} = 7.34 \times 10^{-7} \frac{m^3}{s}$ $\Delta p_{max} = 406\,kPa$

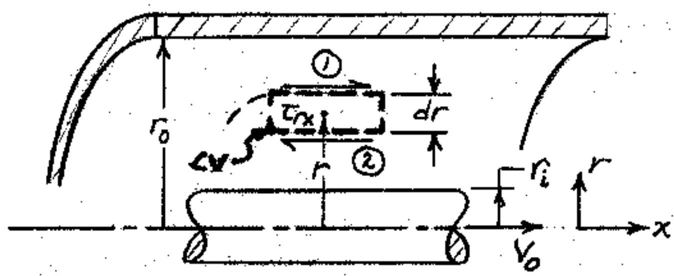
(b) For castor oil $Q_{max} = 6.49 \times 10^{-5} \frac{m^3}{s}$ $\Delta p_{max} = 8156\,MPa$

The analogy fails when Re > 2300 because the flow becomes turbulent, and "resistance" to flow is then no longer linear with flow rate

Problem 8.49

Given: Fully-developed laminar flow in an annulus as shown. The inner section is stationary; the outer moves at V_0.

Assume $\frac{\partial p}{\partial x} = 0$.



Find: (a) $\tau(r)$ in terms of c_1.
(b) $V(r)$ in terms of c_1, c_2.
(c) Evaluate c_1, c_2.

Solution: Apply x component of momentum equation, using annular CV shown.

Basic Equations: $F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$; $\tau_{rx} = \mu \frac{du}{dr} = \tau$ (with $F_{Bx} = 0(1)$, $\frac{\partial}{\partial t} = 0(2)$, $\int_{CS} = 0(3)$)

Assumptions: (1) $F_{Bx} = 0$
(2) steady flow
(3) Fully-developed flow

Then

$$F_{Sx} = F_{①} - F_{②} = \left(\tau + \frac{d\tau}{dr}\frac{dr}{2}\right)2\pi\left(r + \frac{dr}{2}\right)dx - \left(\tau - \frac{d\tau}{dr}\frac{dr}{2}\right)2\pi\left(r - \frac{dr}{2}\right)dx = 0$$

Neglecting products of differentials, this reduces to

$$\tau + r\frac{d\tau}{dr} = 0 \quad \text{or} \quad \frac{d}{dr}(r\tau) = 0$$

Thus $r\tau = c_1$ or $\tau = \frac{c_1}{r}$ $\qquad \tau(r)$

But $\tau = \mu\frac{du}{dr}$, so $\frac{du}{dr} = \frac{c_1}{\mu r}$

and $u = \frac{c_1}{\mu}\ln r + c_2$ $\qquad u(r)$

To evaluate constants c_1 and c_2, use boundary conditions.

At $r = r_i$, $u = V_0$, so $V_0 = \frac{c_1}{\mu}\ln r_i + c_2$

At $r = r_0$, $u = 0$, so $0 = \frac{c_1}{\mu}\ln r_0 + c_2$ and $c_2 = -\frac{c_1}{\mu}\ln r_0$

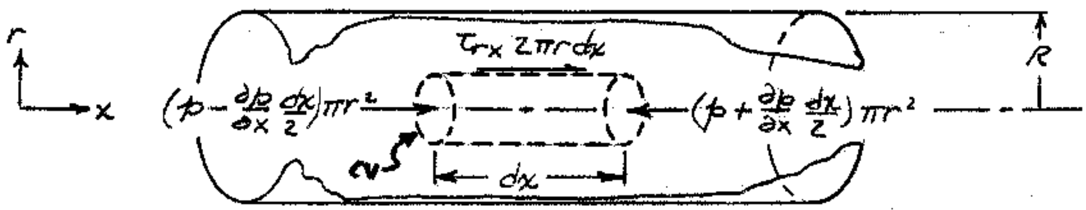
Thus, subtracting, $V_0 = \frac{c_1}{\mu}\ln\left(\frac{r_i}{r_0}\right)$ or $c_1 = \frac{\mu V_0}{\ln(r_i/r_0)}$ so $c_2 = -\frac{V_0 \ln r_0}{\ln(r_i/r_0)}$

Finally

$$u = \frac{V_0}{\ln(r_i/r_0)}(\ln r - \ln r_i) = V_0\frac{\ln(r/r_0)}{\ln(r_i/r_0)}$$ $\qquad u(r)$

Problem 8.50

Given: Fully-developed laminar flow in a circular pipe, with cylindrical control volume as shown.



Find: (a) Forces acting on CV.
(b) Expression for velocity distribution.

Solution: The forces on a CV of radius r are shown above.

Apply the x component of momentum to CV shown.

Basic equations:

$$F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho \vec{V}\cdot d\vec{A} \quad , \quad \tau_{rx} = \mu \frac{du}{dr}$$

($F_{B_x} = 0(1)$, $\frac{\partial}{\partial t} = 0(2)$, $\int_{CS} = 0(3)$)

Assumptions: (1) $F_{B_x} = 0$
(2) Steady flow
(3) Fully-developed flow

Then

$$F_{S_x} = (p - \frac{\partial p}{\partial x}\frac{dx}{2})\pi r^2 + \tau_{rx}\, 2\pi r dx - (p + \frac{\partial p}{\partial x}\frac{dx}{2})\pi r^2 = 0$$

Cancelling and combining terms,

$$-r\frac{\partial p}{\partial x} + 2\tau_{rx} = 0 \qquad \text{or} \qquad \tau_{rx} = \mu\frac{du}{dr} = \frac{r}{2}\frac{\partial p}{\partial x}$$

Thus $\frac{du}{dr} = \frac{r}{2\mu}\frac{\partial p}{\partial x}$

and

$$u = \frac{r^2}{4\mu}\frac{\partial p}{\partial x} + c_1$$

To evaluate c_1, apply the boundary condition $u = 0$ at $r = R$. Thus

$$c_1 = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x}$$

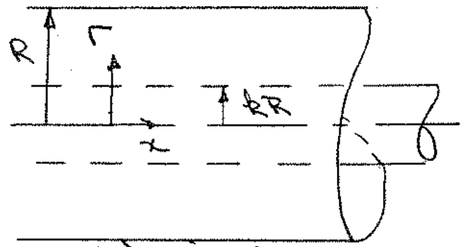
and

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x}(r^2 - R^2) = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

u

which is identical to Eq. 8.12.

Given: Fully developed laminar flow with pressure gradient, $\partial p/\partial x$, in the annulus shown



(a) Show that the velocity profile is given by $u = -\frac{R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left[1-\left(\frac{r}{R}\right)^2 + \frac{(1-k^2)}{\ln(1/k)}\ln\frac{r}{R}\right]$

(b) Obtain an expression for the location ($\alpha = r/R$) of maximum u as a function of k.

(c) Plot α vs k.

(d) Compare limiting case, $k \to 0$, with flow in circular pipe.

Solution: We may use the results of the differential control volume analysis of Section 8-3 to write

$$u = \frac{r^2}{4\mu}\frac{\partial p}{\partial x} + \frac{c_1}{\mu}\ln r + c_2 \qquad \text{----(1)}$$

The boundary conditions are $u=0$ at $r=R$
$u=0$ at $r=kR$.

Substituting the boundary conditions

$$0 = \frac{R^2}{4\mu}\frac{\partial p}{\partial x} + \frac{c_1}{\mu}\ln R + c_2 \qquad \text{----(2)}$$

$$0 = \frac{k^2R^2}{4\mu}\frac{\partial p}{\partial x} + \frac{c_1}{\mu}\ln kR + c_2 \qquad \text{----(3)}$$

Subtracting, $0 = \frac{R^2}{4\mu}\frac{\partial p}{\partial x}(1-k^2) + \frac{c_1}{\mu}(\ln R - \ln kR)$

$$\therefore\quad c_1 = -\frac{R^2}{4}\frac{\partial p}{\partial x}\frac{(1-k^2)}{\ln(1/k)}$$

From Eq. 2

$$c_2 = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x} + \frac{R^2}{4\mu}\frac{\partial p}{\partial x}\frac{(1-k^2)}{\ln(1/k)}\ln R$$

Substituting for c_1 and c_2 into Eq. 1 gives

$$u = \frac{r^2}{4\mu}\frac{\partial p}{\partial x} - \frac{R^2}{4\mu}\frac{\partial p}{\partial x}\frac{(1-k^2)}{\ln(1/k)}\ln r - \frac{R^2}{4\mu}\frac{\partial p}{\partial x} + \frac{R^2}{4\mu}\frac{\partial p}{\partial x}\frac{(1-k^2)}{\ln(1/k)}\ln R$$

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x}\left[r^2 - R^2 - \frac{R^2(1-k^2)}{\ln(1/k)}(\ln r - \ln R)\right]$$

$$u = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{r}{R}\right)^2 + \frac{(1-k^2)}{\ln(1/k)}\ln\frac{r}{R}\right] \longleftarrow u$$

To locate max u, set $\tau_{rx} = \mu\frac{du}{dr} = 0$

$$\tau_{rx} = \mu\frac{du}{dr} = -\frac{R^2}{4}\frac{\partial p}{\partial x}\left[-\frac{2r}{R^2} + \frac{(1-k^2)}{\ln(1/k)}\frac{1}{r}\right]$$

Problem 8.51

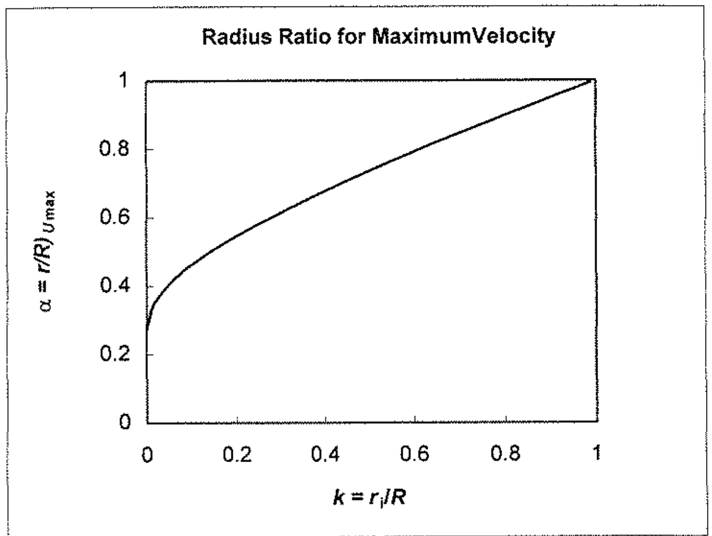
Set $\tau_{rx} = 0$ at $r = \alpha R$. Then

$$0 = -\frac{2\alpha R}{R^2} + \frac{(1-k^2)}{\ln(1/k)}\,\frac{1}{\alpha R}$$

and

$$\alpha = \left[\frac{1}{2}\,\frac{(1-k^2)}{\ln(1/k)}\right]^{1/2} \longleftarrow \alpha = \alpha(k)$$

$k = r_i/R$	$\alpha = r/R)_{u\,max}$
0.001	0.269
0.01	0.329
0.02	0.357
0.05	0.408
0.08	0.444
0.1	0.464
0.2	0.546
0.4	0.677
0.6	0.791
0.8	0.898
0.95	0.975
0.99	0.995



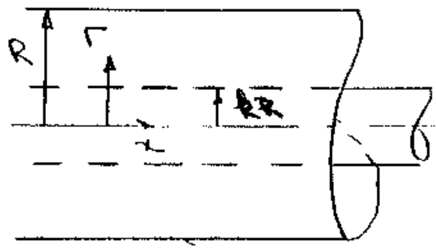
For $k \to 0$, $\alpha \to 0$ and $u = -\frac{R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left[1-\left(\frac{r}{R}\right)^2\right]$

This agrees with the results for flow in a circular pipe

As $k \to 1.0$, $\alpha \to 1.0$ and the flow behaves like flow between stationary infinite parallel plates.

Problem 8.52

Given: Fully developed laminar flow in the annulus shown, with pressure gradient $\partial p/\partial x$.
The velocity profile is given by



$$u = -\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\left[1-\left(\frac{r}{R}\right)^2 + \frac{(1-k^2)}{\ln(1/k)}\ln\frac{r}{R}\right]$$

(a) Show that the volume flow rate is given by

$$Q = -\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial x}\left[(1-k^4) - \frac{(1-k^2)^2}{\ln(1/k)}\right]$$

(b) Obtain an expression for the average velocity

(c) Compare limiting case, $k \to 0$, with flow in a circular pipe.

Solution: The volume flow rate is given by

$$Q = \int u\,dA = \int_{kR}^{R} u\,2\pi r\,dr = 2\pi\int_{kR}^{R} u\,r\,dr$$

$$= 2\pi\left(-\frac{R^2}{4\mu}\frac{\partial p}{\partial x}\right)\int_{kR}^{R}\left[r - \frac{r^3}{R^2} + \frac{(1-k^2)}{\ln(1/k)}\,r\ln\frac{r}{R}\right]dr$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\int_{k}^{1}\left[\frac{r}{R} - \left(\frac{r}{R}\right)^3 + \frac{(1-k^2)}{\ln(1/k)}\frac{r}{R}\ln\frac{r}{R}\right]d\left(\frac{r}{R}\right)$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4 + \frac{(1-k^2)}{\ln(1/k)}\left\{\left(\frac{r}{R}\right)^2\left[\frac{1}{2}\ln\left(\frac{r}{R}\right) - \frac{1}{4}\right]\right\}\right]_{k}^{1}$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1}{2} - \frac{k^2}{2} - \frac{1}{4} + \frac{k^4}{4} + \frac{(1-k^2)}{\ln(1/k)}\left\{-\frac{1}{4} - k^2\left[\frac{1}{2}\ln k - \frac{1}{4}\right]\right\}\right]$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1}{4} - \frac{k^2}{2} + \frac{k^4}{4} + \frac{(1-k^2)}{\ln(1/k)}\left\{-\frac{1}{4} + \frac{k^2}{4} - k^2\frac{1}{2}\ln k\right\}\right]$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1-2k^2+k^4}{4} + \frac{(1-k^2)}{\ln(1/k)}\frac{(k^2-1)}{4} - \frac{(1-k^2)}{\ln(1/k)}k^2\frac{1}{2}\ln k\right]$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1-2k^2+k^4}{4} - \frac{(1-k^2)^2}{4\ln(1/k)} + \frac{k^2-k^4}{2}\right]$$

$$= -\frac{\pi R^4}{2\mu}\frac{\partial p}{\partial x}\left[\frac{1-2k^2+k^4+2k^2-2k^4}{4} - \frac{(1-k^2)^2}{4\ln(1/k)}\right]$$

$$Q = -\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial x}\left[(1-k^4) - \frac{(1-k^2)^2}{\ln(1/k)}\right] \quad Q$$

The average velocity, $\bar{V} = \frac{Q}{A}$

The area is given by

$$A = \int dA = \int 2\pi r\,dr = 2\pi R^2 \int_k^1 \frac{r}{R}\, d\left(\frac{r}{R}\right)$$

$$A = 2\pi R^2 \left[\frac{1}{2}\left(\frac{r}{R}\right)^2 \right]_k^1 = 2\pi R^2 \times \frac{1}{2}(1-k^2) = \pi R^2(1-k^2)$$

Thus

$$\bar{V} = \frac{Q}{A} = -\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial x} \times \frac{1}{\pi R^2}\left[\frac{(1-k^4)}{(1-k^2)} - \frac{(1-k^2)}{\ln(1/k)} \right]$$

$$\bar{V} = -\frac{R^2}{8\mu}\frac{\partial p}{\partial x}\left[\frac{(1-k^4)}{(1-k^2)} - \frac{(1-k^2)}{\ln(1/k)} \right] \longleftarrow \bar{V}$$

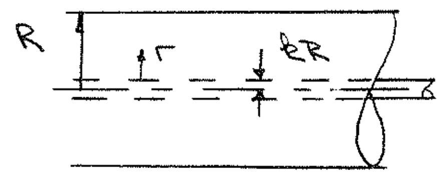
For $k \to 0$

$$Q = -\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial x} \quad \text{and} \quad \bar{V} = -\frac{R^2}{8\mu}\frac{\partial p}{\partial x}$$

These agree with the results for flow in a circular pipe.

Problem 8.53

Given: Fully developed laminar flow in a circular pipe is converted to flow in an annulus by insertion of a thin wire along the centerline



(a) Use results of Problem 8.52 to obtain an expression for the percent change in pressure drop as a function of radius ratio k.

(b) Plot percent change in Δp vs k for $0.001 \le k \le 0.10$

Solution: The results of problem 8.48 give

$$Q = -\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial x}\left[(1-k^4) - \frac{(1-k^2)^2}{\ln(1/k)}\right]$$

Thus

$$\frac{\Delta p}{L} = -\frac{\partial p}{\partial x} = \frac{8\mu Q}{\pi R^4} \times \frac{1}{\left[(1-k^4) - \frac{(1-k^2)^2}{\ln(1/k)}\right]}$$

For $k=0$, $\frac{\Delta p}{L} = \frac{8\mu Q}{\pi R^4}$

$$\text{Percent change} = \frac{\Delta p/L - \Delta p/L)_{k=0}}{\Delta p/L)_{k=0}} = \frac{1}{\left[(1-k^4) - \frac{(1-k^2)^2}{\ln(1/k)}\right]} - 1$$

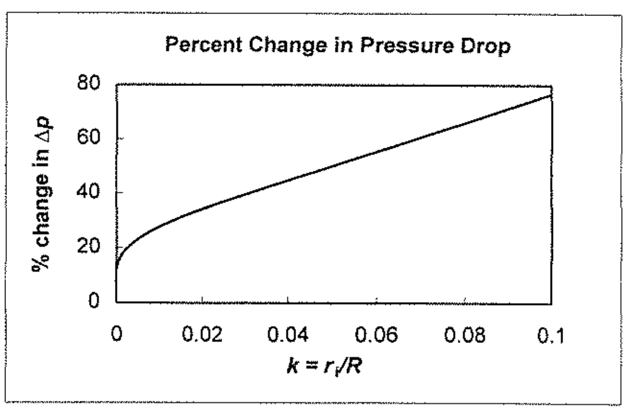
$$\%\text{ change} = \frac{1 - \left[(1-k^4) - \frac{(1-k^2)}{\ln(1/k)}\right]}{\left[(1-k^4) - \frac{(1-k^2)}{\ln(1/k)}\right]}$$

For small k,

$$\%\text{ change} = \frac{1-\left[1 - \frac{1}{\ln(1/k)}\right]}{\left[1 - \frac{1}{\ln(1/k)}\right]} = \frac{1-\left[1 + \frac{1}{\ln k}\right]}{\left[1 + \frac{1}{\ln k}\right]} = \frac{-\frac{1}{\ln k}}{\left[1 + \frac{1}{\ln k}\right]}$$

$$\%\text{ change} = -\frac{1}{\ln k\left(1 + \frac{1}{\ln k}\right)} \times 100 \qquad \leftarrow \%\text{ change}$$

$k = r_i/R$	% change in Δp
0.0001	12.2
0.0002	13.3
0.0005	15.1
0.001	16.9
0.002	19.2
0.005	23.3
0.01	27.7
0.02	34.3
0.05	50.1
0.1	76.8



The plot shows that even the smallest of wires causes a significant increase in pressure drop for a given flow rate.

Problem 8.54

[3]

8.54 In a food industry plant two immiscible fluids are pumped through a tube such that fluid 1 ($\mu_1 = 0.02$ lbf • s/ft^2) forms an inner core and fluid 2 ($\mu_2 = 0.03$ lbf • s/ft^2) forms an outer annulus. The tube has $D = 0.2$ in. diameter and length $L = 50$ ft. Derive and plot the velocity distribution if the applied pressure difference, Δp, is 1 psi.

Given: Two-fluid flow in tube

Find: Velocity distribution; Plot

Solution:

Given data $\quad D = 0.2 \cdot in \quad L = 50 \cdot ft \quad \Delta p = -1 \cdot psi \quad \mu_1 = 0.02 \cdot \frac{lbf \cdot s}{ft^2} \quad \mu_2 = 0.03 \cdot \frac{lbf \cdot s}{ft^2}$

From Section 8-3 for flow in a pipe, Eq. 8.11 can be applied to either fluid

$$u = \frac{r^2}{4 \cdot \mu} \cdot \left(\frac{\partial}{\partial x} p\right) + \frac{c_1}{\mu} \cdot \ln(r) + c_2$$

Applying this to fluid 1 (inner fluid) and fluid 2 (outer fluid)

$$u_1 = \frac{r^2}{4 \cdot \mu_1} \cdot \frac{\Delta p}{L} + \frac{c_1}{\mu_1} \cdot \ln(r) + c_2 \qquad u_2 = \frac{r^2}{4 \cdot \mu_2} \cdot \frac{\Delta p}{L} + \frac{c_3}{\mu_2} \cdot \ln(r) + c_4$$

We need four BCs. Two are obvious $\quad r = \frac{D}{2} \quad u_2 = 0 \quad (1) \qquad r = \frac{D}{4} \quad u_1 = u_2 \quad (2)$

The third BC comes from the fact that the axis is a line of symmetry

$$r = 0 \qquad \frac{du_1}{dr} = 0 \qquad (3)$$

The fourth BC comes from the fact that the stress at the interface generated by each fluid is the same

$$r = \frac{D}{4} \qquad \mu_1 \cdot \frac{du_1}{dr} = \mu_2 \cdot \frac{du_2}{dr} \qquad (4)$$

Using these four BCs $\quad \frac{\left(\frac{D}{2}\right)^2}{4 \cdot \mu_2} \cdot \frac{\Delta p}{L} + \frac{c_3}{\mu_2} \cdot \ln\left(\frac{D}{2}\right) + c_4 = 0 \qquad \frac{\left(\frac{D}{4}\right)^2}{4 \cdot \mu_1} \cdot \frac{\Delta p}{L} + \frac{c_1}{\mu_1} \cdot \ln\left(\frac{D}{4}\right) + c_2 = \frac{\left(\frac{D}{4}\right)^2}{4 \cdot \mu_2} \cdot \frac{\Delta p}{L} + \frac{c_3}{\mu_2} \cdot \ln\left(\frac{D}{4}\right) + c_4$

$$\lim_{r \to 0} \frac{c_1}{\mu_1 \cdot r} = 0 \qquad \frac{D}{8} \cdot \frac{\Delta p}{L} + \frac{4 \cdot c_1}{D} = \frac{D}{8} \cdot \frac{\Delta p}{L} + \frac{4 \cdot c_3}{D}$$

Hence, after some algebra

$c_1 = 0$ (To avoid singularity) $\qquad c_2 = -\frac{D^2 \cdot \Delta p}{64 \cdot L} \frac{(\mu_2 + 3 \cdot \mu_1)}{\mu_1 \cdot \mu_2} \qquad c_3 = 0 \qquad c_4 = -\frac{D^2 \cdot \Delta p}{16 \cdot L \cdot \mu_2}$

The velocity distributions are then

$$u_1(r) = \frac{\Delta p}{4 \cdot \mu_1 \cdot L} \cdot \left[r^2 - \left(\frac{D}{2}\right)^2 \cdot \frac{(\mu_2 + 3 \cdot \mu_1)}{4 \cdot \mu_2}\right] \qquad u_2(r) = \frac{\Delta p}{4 \cdot \mu_2 \cdot L} \cdot \left[r^2 - \left(\frac{D}{2}\right)^2\right]$$

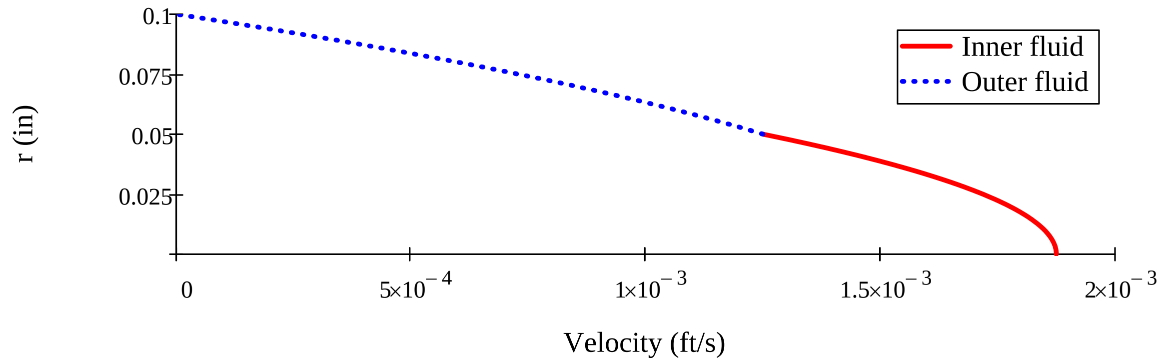
(Note that these result in the same expression if $\mu_1 = \mu_2$, i.e., if we have one fluid)

Evaluating either velocity at $r = D/4$ gives the velocity at the interface

$$u_{interface} = -\frac{3 \cdot D^2 \cdot \Delta p}{64 \cdot \mu_2 \cdot L} \qquad u_{interface} = -\frac{3}{64} \times \left(\frac{0.2}{12} \cdot ft\right)^2 \times \left(-1 \cdot \frac{lbf}{in^2}\right) \times \frac{144 \cdot in^2}{1 \cdot ft^2} \times \frac{ft^2}{0.03 \cdot lbf \cdot s} \times \frac{1}{50 \cdot ft} \qquad u_{interface} = 1.25 \times 10^{-3} \frac{ft}{s}$$

Evaluating u_1 at $r = 0$ gives the maximum velocity

$$u_{max} = -\frac{D^2 \cdot \Delta p \cdot (\mu_2 + 3 \cdot \mu_1)}{64 \cdot \mu_1 \cdot \mu_2 \cdot L} \qquad u_{max} = -\frac{1}{64} \times \left(\frac{0.2}{12} \cdot ft\right)^2 \times \left(-1 \cdot \frac{lbf}{in^2}\right) \times \frac{0.03 + 3 \times 0.02}{0.02 \times 0.03} \cdot \frac{ft^2}{lbf \cdot s} \times \frac{1}{50 \cdot ft} \qquad u_{max} = 1.88 \times 10^{-3} \frac{ft}{s}$$



The velocity distributions can be plotted in *Excel*

Problem 8.55

8.55 A horizontal pipe carries fluid in fully developed turbulent flow. The static pressure difference measured between two sections is 35 kPa. The distance between the sections is 10 m and the pipe diameter is 150 mm. Calculate the shear stress, τ_w, that acts on the walls.

Given: Turbulent pipe flow

Find: Wall shear stress

Solution:

Basic equation $$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$ (Eq. 4.18a)

Assumptions 1) Horizontal pipe 2) Steady flow 3) Fully developed flow

With these assumptions the x momentum equation becomes

$$p_1\cdot\frac{\pi\cdot D^2}{4} + \tau_w\cdot\pi\cdot D\cdot L - p_2\cdot\frac{\pi\cdot D^2}{4} = 0$$ or $$\tau_w = \frac{(p_2 - p_1)\cdot D}{4\cdot L} = -\frac{\Delta p\cdot D}{4\cdot L}$$

$$\tau_w = -\frac{1}{4}\times 35\times 10^3\cdot\frac{N}{m^2}\times 150\cdot mm\times\frac{1\cdot m}{1000\cdot mm}\times\frac{1}{10\cdot m}$$ $$\tau_w = -131\,Pa$$

Since τ_w is negative it acts to the left on the fluid, to the right on the pipe wall

Problem 8.56

8.56 One end of a horizontal pipe is attached using glue to a pressurized tank containing liquid, and the other has a cap attached. The inside diameter of the pipe is 2.5 cm, and the tank pressure is 250 kPa (gage). Find the force the glue must withstand, and the force it must withstand when the cap is off and the liquid is discharging to atmosphere.

Given: Pipe glued to tank

Find: Force glue must hold when cap is on and off

Solution:

Basic equation $$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, dV + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A} \qquad \text{(Eq. 4.18a)}$$

First solve when the cap is on. In this static case

$$F_{glue} = \frac{\pi \cdot D^2}{4} \cdot p_1$$ where p_1 is the tank pressure

Second, solve for when flow is occuring:

Assumptions 1) Horizontal pipe 2) Steady flow 3) Fully developed flow

With these assumptions the x momentum equation becomes

$$p_1 \cdot \frac{\pi \cdot D^2}{4} + \tau_w \cdot \pi \cdot D \cdot L - p_2 \cdot \frac{\pi \cdot D^2}{4} = 0$$

Here p_1 is again the tank pressure and p_2 is the pressure at the pipe exit; the pipe exit pressure is p_{atm} = 0 kPa gage. Hence

$$F_{pipe} = F_{glue} = -\tau_w \cdot \pi \cdot D \cdot L = \frac{\pi \cdot D^2}{4} \cdot p_1$$

We conclude that in each case the force on the glue is the same! When the cap is on the glue has to withstand the tank pressure; when the cap is off, the glue has to hold the pipe in place against the friction of the fluid on the pipe, which is equal in magnitude to the pressure drop.

$$F_{glue} = \frac{\pi}{4} \times \left(2.5 \cdot cm \times \frac{1 \cdot m}{100 \cdot cm}\right)^2 \times 250 \times 10^3 \cdot \frac{N}{m^2} \qquad F_{glue} = 123\,N$$

Problem 8.57

[2]

8.57 The pressure drop between two taps separated in the streamwise direction by 30 ft in a horizontal, fully developed channel flow of water is 1 psi. The cross-section of the channel is a 1 in. × $9\frac{1}{2}$ in. rectangle. Calculate the average wall shear stress.

Given: Flow through channel

Find: Average wall stress

Solution:

Basic equation $$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV\!\!\!/ + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$ (Eq. 4.18a)

Assumptions 1) Horizontal pipe 2) Steady flow 3) Fully developed flow

With these assumptions the x momentum equation becomes

$$p_1\cdot W\cdot H + \tau_w\cdot 2\cdot L\cdot(W+H) - p_2\cdot W\cdot H = 0$$ or $$\tau_w = (p_2 - p_1)\cdot\frac{W\cdot H}{2\cdot(W+H)\cdot L}$$ $$\tau_w = -\Delta p\cdot\frac{\frac{H}{L}}{2\cdot\left(1+\frac{H}{W}\right)}$$

$$\tau_w = -\frac{1}{2}\times 1\cdot\frac{lbf}{in^2}\times\frac{144\cdot in^2}{ft^2}\times\frac{1\cdot in\times\frac{1\cdot ft}{12\cdot in}}{30\cdot ft}\times\left(\frac{1}{1+\frac{9.5\cdot in\times\frac{1\cdot ft}{12\cdot in}}{30\cdot ft}}\right)$$ $$\tau_w = -0.195\frac{lbf}{ft^2}$$ $$\tau_w = -1.35\times 10^{-3}\,psi$$

Since $\tau_w < 0$, it acts to the left on the fluid, to the right on the channel wall

Problem 8.58

[2]

8.58 Kerosine is pumped through a smooth tube with inside diameter $D = 30$ mm at close to the critical Reynolds number. The flow is unstable and fluctuates between laminar and turbulent states, causing the pressure gradient to intermittently change from approximately -4.5 kPa/m to -11 kPa/m. Which pressure gradient corresponds to laminar, and which to turbulent, flow? For each flow, compute the shear stress at the tube wall, and sketch the shear stress distributions.

Given: Data on pressure drops in flow in a tube

Find: Which pressure drop is laminar flow, which turbulent

Solution:

Given data $\frac{\partial}{\partial x}p_1 = -4.5\cdot\frac{kPa}{m}$ $\qquad$ $\frac{\partial}{\partial x}p_2 = -11\cdot\frac{kPa}{m}$ $\qquad$ $D = 30\cdot mm$

From Section 8-4, a force balance on a section of fluid leads to

$$\tau_w = -\frac{R}{2}\cdot\frac{\partial}{\partial x}p = -\frac{D}{4}\cdot\frac{\partial}{\partial x}p$$

Hence for the two cases

$$\tau_{w1} = -\frac{D}{4}\cdot\frac{\partial}{\partial x}p_1 \qquad \tau_{w1} = 33.8\,Pa$$

$$\tau_{w2} = -\frac{D}{4}\cdot\frac{\partial}{\partial x}p_2 \qquad \tau_{w2} = 82.5\,Pa$$

Because both flows are at the same nominal flow rate, the higher pressure drop must correspond to the turbulent flow, because, as indicated in Section 8-4, turbulent flows experience additional stresses. Also indicated in Section 8-4 is that for both flows the shear stress varies from zero at the centerline to the maximums computed above at the walls.

The stress distributions are linear in both cases: Maximum at the walls and zero at the centerline.

Problem 8.59

[3]

Given: Liquid with viscosity and density of water in laminar flow in a smooth capillary tube. $D = 0.25\ mm$, $L = 50\ mm$.

Find: (a) Maximum volume flow rate.
(b) Pressure drop to produce this flow rate.
(c) Corresponding wall shear stress.

Solution: Flow will be laminar for $Re < 2300$.

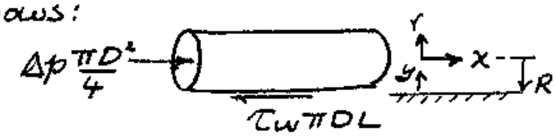
$$Re = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} = \frac{Q}{A}\frac{D}{\nu} = \frac{4Q}{\pi D^2}\frac{D}{\nu} = \frac{4Q}{\pi \nu D} < 2300$$

Thus (at $T = 20°C$)

$$Q < \frac{2300 \pi \nu D}{4} = \frac{2300\pi}{4} \times 1.0 \times 10^{-6}\frac{m^2}{s} \times 0.00025\ m = 4.52 \times 10^{-7}\ m^3/s$$ Q

(This flow rate corresponds to 27.1 mL/min.)

A force balance on a fluid element shows:



$$\sum F_x = \Delta p \frac{\pi D^2}{4} - \tau_w \pi D L = 0$$

or

$$\Delta p = \tau_w \frac{4L}{D}$$

For laminar pipe flow, $u = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$, from Eq. 8.14. Thus

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big)_{y=0} = -\mu \frac{\partial u}{\partial r}\Big)_{r=R} = -\mu u_{max}\left(-\frac{2r}{R^2}\right)_{r=R} = \frac{2\mu u_{max}}{R}$$

But $u_{max} = 2\bar{V}$, so $\tau_w = \frac{2\mu 2\bar{V}}{D/2} = \frac{8\mu\bar{V}}{D} = \frac{8\rho\nu\bar{V}}{D}$

Also

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 4.52 \times 10^{-7}\frac{m^3}{s} \times \frac{1}{(0.00025)^2 m^2} = 9.21\ m/s$$

Thus

$$\tau_w = 8 \times 999\frac{kg}{m^3} \times 1.0 \times 10^{-6}\frac{m^2}{s} \times 9.21\frac{m}{s} \times \frac{1}{0.00025\ m} \times \frac{N \cdot s^2}{kg \cdot m} = 294\ N/m^2\ (294\ Pa)$$ τ_w

and

$$\Delta p = 4 \times 0.05\ m \times \frac{1}{0.00025\ m} \times 294\frac{N}{m^2} = 235\ kPa$$ Δp

Problem 8.60

Given: Velocity profiles for pipe flow

$\frac{u}{U} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$ (turbulent); $\frac{u}{U} = 1 - \left(\frac{r}{R}\right)^2$ (laminar)

Find: (a) value of r/R at which $u = \bar{V}$ for each profile.

Plot: r/R vs n for $6 \le n \le 10$

Solution:

Definition: $\bar{V} = \frac{Q}{A} = \frac{1}{A}\int u\,dA$

For laminar flow, $\bar{V} = \frac{1}{\pi R^2}\int_0^R U\left[1 - \left(\frac{r}{R}\right)^2\right] 2\pi r\,dr = 2U\int_0^1 \left[1 - \left(\frac{r}{R}\right)^2\right]\frac{r}{R}\,d\left(\frac{r}{R}\right)$

$$\bar{V} = 2U\left[\frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4\right]_0^1 = \frac{U}{2}$$

Thus $u = \bar{V}$ when $1 - \left(\frac{r}{R}\right)^2 = \frac{\bar{V}}{U} = \frac{1}{2}$ or $\frac{r}{R} = 0.707$ ← laminar

For turbulent flow, $\bar{V} = \frac{1}{\pi R^2}\int_0^R U\left(1 - \frac{r}{R}\right)^{\frac{1}{n}} 2\pi r\,dr$

$$\bar{V} = 2U\int_0^1 \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} \frac{r}{R}\,d\left(\frac{r}{R}\right)$$

To integrate let $m = 1 - \frac{r}{R}$. Then $\frac{r}{R} = 1 - m$, $d\left(\frac{r}{R}\right) = -dm$

and

$$\bar{V} = 2U\int_1^0 m^{\frac{1}{n}}(1-m)(-dm) = 2U\int_0^1 \left(m^{\frac{1}{n}} - m^{1+\frac{1}{n}}\right)dm$$

$$= 2U\left[\frac{n}{(n+1)} m^{\frac{1}{n}+1} - \frac{n}{2n+1} m^{2+\frac{1}{n}}\right]_0^1 = 2U\left[\frac{n}{n+1} - \frac{n}{2n+1}\right]$$

$$\bar{V} = 2U\left[\frac{n(2n+1) - n(n+1)}{(n+1)(2n+1)}\right] = U\frac{2n^2}{(n+1)(2n+1)} \quad \text{.......... (8.24)}$$

For $n = 7$, $\bar{V} = U\frac{2(7)^2}{8 \times 15} = 0.817U$

Thus $u = \bar{V}$ when $\left(1 - \frac{r}{R}\right)^{1/7} = 0.817$ or $\frac{r}{R} = 1 - (0.817)^7 = 0.758$ ← turb

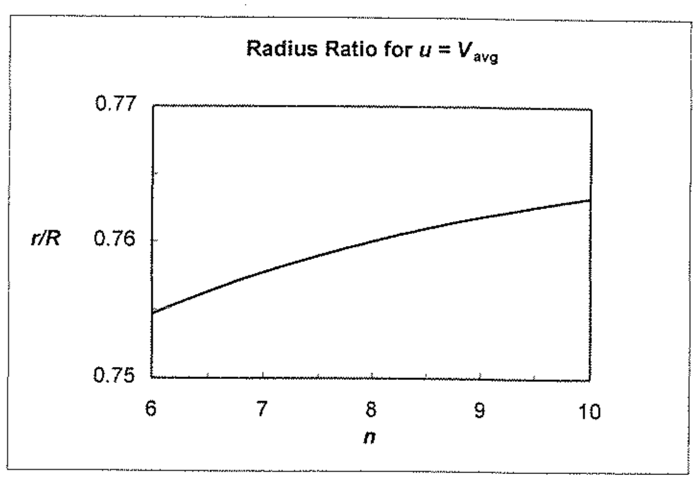
From Eq 8.24, $u = \bar{V}$ when

$$\left(1 - \frac{r}{R}\right)^{\frac{1}{n}} = \frac{2n^2}{(n+1)(2n+1)}$$

or

$$\frac{r}{R} = 1 - \left[\frac{2n^2}{(n+1)(2n+1)}\right]^n$$

r/R is plotted vs n.



Problem 8.61 [3]

8.61 Laufer [5] measured the following data for mean velocity in fully developed turbulent pipe flow at $Re_U = 50,000$:

$\bar{u}/U$	0.996	0.981	0.963	0.937	0.907	0.866	0.831
y/r	0.898	0.794	0.691	0.588	0.486	0.383	0.280
$\bar{u}/U$	0.792	0.742	0.700	0.650	0.619	0.551	
y/R	0.216	0.154	0.093	0.062	0.041	0.024	

In addition, Laufer measured the following data for mean velocity in fully developed turbulent pipe flow at $Re_U = 500{,}000$:

$\bar{u}/U$	0.997	0.988	0.975	0.959	0.934	0.908
y/R	0.898	0.794	0.691	0.588	0.486	0.383
$\bar{u}/U$	0.874	0.847	0.818	0.771	0.736	0.690
y/R	0.280	0.216	0.154	0.093	0.062	0.037

Using *Excel*'s trendline analysis, fit each set of data to the "power-law" profile for turbulent flow, Eq. 8.22, and obtain a value of n for each set. Do the data tend to confirm the validity of Eq. 8.22? Plot the data and their corresponding trendlines on the same graph.

Given: Data on mean velocity in fully developed turbulent flow

Find: Trendlines for each set; values of n for each set; plot

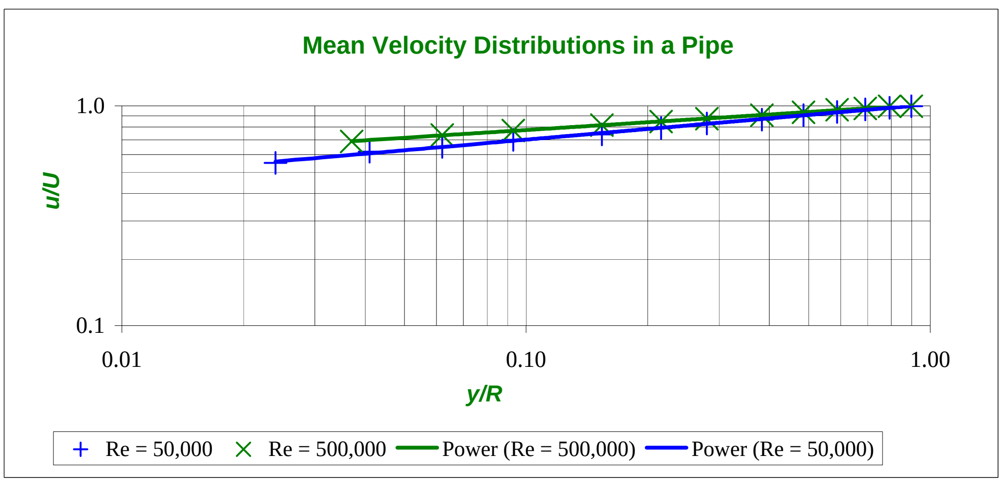
Solution:

y/R	*u/U*
0.898	0.996
0.794	0.981
0.691	0.963
0.588	0.937
0.486	0.907
0.383	0.866
0.280	0.831
0.216	0.792
0.154	0.742
0.093	0.700
0.062	0.650
0.041	0.619
0.024	0.551

y/R	*u/U*
0.898	0.997
0.794	0.998
0.691	0.975
0.588	0.959
0.486	0.934
0.383	0.908
0.280	0.874
0.216	0.847
0.154	0.818
0.093	0.771
0.062	0.736
0.037	0.690

Equation 8.22 is

$$\frac{\bar{u}}{U} = \left(\frac{y}{R}\right)^{1/n} = \left(1 - \frac{y}{R}\right)^{1/n}$$



Applying the *Trendline* analysis to each set of data:

At Re = 50,000

$u/U = 1.017(y/R)^{0.161}$
with R^2 = 0.998 (high confidence)

Hence $1/n$ = 0.161
n = 6.21

At Re = 500,000

$u/U = 1.017(y/R)^{0.117}$
with R^2 = 0.999 (high confidence)

Hence $1/n$ = 0.117
n = 8.55

Both sets of data tend to confirm the validity of Eq. 8.22

Problem 8.62

Given: Power-law exponent n as a function of Re_U and ratio $\bar{V}/U$ as a function of n.

$$n = -1.7 + 1.8 \log Re_U \qquad (8.23)$$

$$\bar{V}/U = \frac{2n^2}{(n+1)(2n+1)} \qquad (8.24)$$

Plot: $\bar{V}/U$ vs $Re_{\bar{V}}$

Solution:

Prepare a Table of values

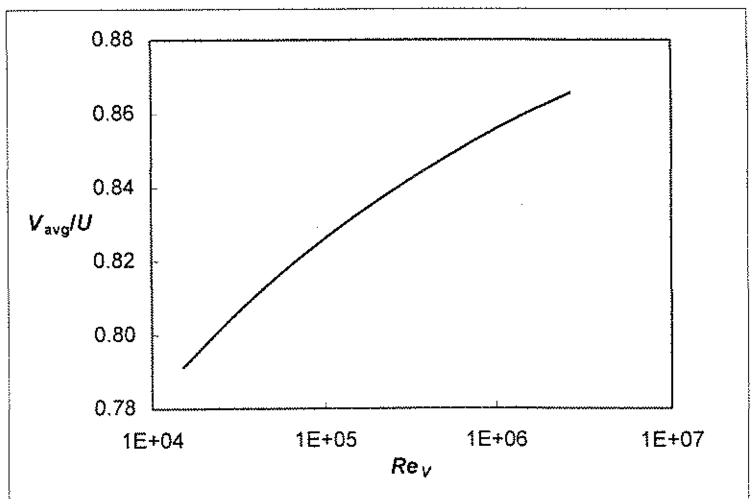
Re_U

n from Eq 8.23

$\bar{V}/U$ from Eq 8.24

$$Re_{\bar{V}} = \frac{\bar{V}}{U} \times Re_U$$

Re_U	n	Re_V	V_{avg}/U
1.90E+04	6.00	1.50E+04	0.791
3.60E+04	6.50	2.90E+04	0.805
6.85E+04	7.00	5.59E+04	0.817
1.29E+05	7.50	1.07E+05	0.827
2.45E+05	8.00	2.05E+05	0.837
4.65E+05	8.50	3.93E+05	0.845
8.80E+05	9.00	7.50E+05	0.853
1.67E+06	9.50	1.44E+06	0.860
3.16E+06	10.0	2.74E+06	0.866



Problem 8.63

Given: Velocity profiles for pipe flow:

$\frac{u}{U} = 1 - \left(\frac{r}{R}\right)^2$ (laminar); $\frac{u}{U} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$ (turbulent)

Momentum coefficient, β, where $\beta \dot{m} \bar{V} = \int_A u \rho u dA$

Find: (a) β for laminar profile.

(b) β for turbulent profile with $n = 7$

Plot: β vs n for turbulent profile over range $6 \le n \le 10$, and compare with laminar profile

Solution:

$$\beta = \frac{1}{\dot{m}\bar{V}} \int_A u \rho u dA = \frac{1}{\rho \bar{V} \pi R^2 \bar{V}} \int_0^R u \rho u 2\pi r dr$$

Noting that $\frac{u}{U} = f(r/R)$

$$\beta = \frac{1}{\rho \pi R^2} \left[\frac{U}{\bar{V}}\right]^2 \int_0^R \left(\frac{u}{U}\right)^2 2\rho\pi r dr = 2\left[\frac{U}{\bar{V}}\right]^2 \int_0^1 \left(\frac{u}{U}\right)^2 \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) \quad \text{--- } \beta \text{ ---}$$

For laminar flow, $\frac{u}{U} = 1 - \left(\frac{r}{R}\right)^2$, so $\left(\frac{u}{U}\right)^2 = 1 - 2\left(\frac{r}{R}\right)^2 + \left(\frac{r}{R}\right)^4$, and

$$\beta = 2\left[\frac{U}{\bar{V}}\right]^2 \int_0^1 \left[\left(\frac{r}{R}\right) - 2\left(\frac{r}{R}\right)^3 + \left(\frac{r}{R}\right)^5\right] d\left(\frac{r}{R}\right) = 2\left[\frac{U}{\bar{V}}\right]^2 \left[\frac{1}{2} - \frac{1}{2} + \frac{1}{6}\right]$$

$\beta = \frac{1}{3}\left[\frac{U}{\bar{V}}\right]^2$. For this case $U = 2\bar{V}$ so

$$\beta = \frac{1}{3}[2]^2 = \frac{4}{3} \longleftarrow \beta_{laminar}$$

For turbulent flow, $\frac{u}{U} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$, so $\left(\frac{u}{U}\right)^2 = \left(1 - \frac{r}{R}\right)^{\frac{2}{n}}$, and

$$\beta = 2\left[\frac{U}{\bar{V}}\right]^2 \int_0^1 \left(1 - \frac{r}{R}\right)^{\frac{2}{n}} \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

To integrate, let $m = 1 - \frac{r}{R}$. Then $\frac{r}{R} = 1 - m$, $d\left(\frac{r}{R}\right) = -dm$, so

$$\beta = 2\left[\frac{U}{\bar{V}}\right]^2 \int_1^0 m^{\frac{2}{n}} (1-m)(-dm) = 2\left[\frac{U}{\bar{V}}\right]^2 \int_0^1 \left(m^{\frac{2}{n}} - m^{1+\frac{2}{n}}\right) dm$$

$$\beta = 2\left[\frac{U}{\bar{V}}\right]^2 \left[\frac{n}{(n+2)} m^{\frac{2}{n}+1} - \frac{n}{(2n+2)} m^{2+\frac{2}{n}}\right]_0^1 = 2\left[\frac{U}{\bar{V}}\right]^2 \left[\frac{n}{(n+2)} - \frac{n}{(2n+2)}\right]$$

$$\beta = 2\left[\frac{U}{\bar{V}}\right]^2 \left[\frac{(2n+2)n - (n+2)n}{(n+2)(2n+2)}\right] = 2\left[\frac{U}{\bar{V}}\right]^2 \left[\frac{n^2}{(n+2)(2n+2)}\right] \quad \text{--- (1)}$$

From Eq. 8.24, $\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)}$

For $n = 7$, $\frac{\bar{V}}{U} = 0.817$, so

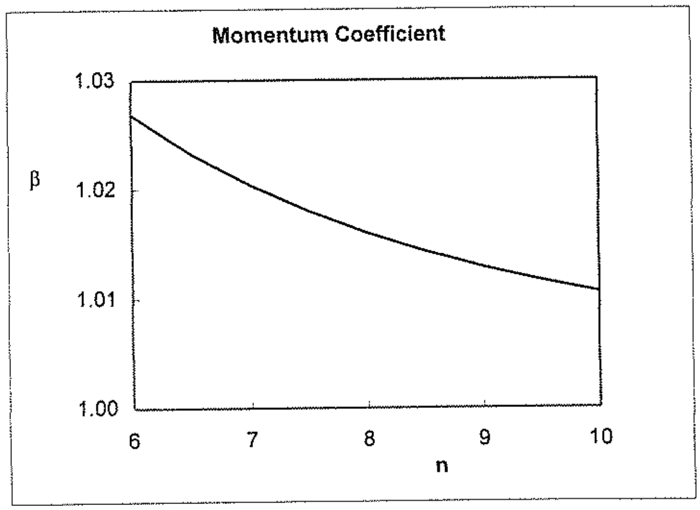
$$\beta = \left[\frac{1}{0.817}\right]^2 \frac{2(7)^2}{(9)(16)} = 1.02 \longleftarrow \beta_{turb}$$

Problem 8.63

To plot β vs n

- $$\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)} \quad \text{.......} (8.24)$$
- $$\beta = \left[\frac{U}{\bar{V}}\right]^2 \frac{n^2}{(n+2)(n+1)}$$
- $$\beta = \frac{(n+1)(2n+1)^2}{4n^2(n+2)}$$

n	β
6.0	1.027
6.5	1.023
7.0	1.020
7.5	1.018
8.0	1.016
8.5	1.014
9.0	1.013
9.5	1.012
10.0	1.011



Problem 8.64 [3]

8.64 Consider fully developed laminar flow of water between stationary parallel plates. The maximum flow speed, plate spacing, and width are 20 ft/s, 0.075 in. and 1.25 in. respectively. Find the kinetic energy coefficient, α.

Given: Laminar flow between parallel plates

Find: Kinetic energy coefficient, α

Solution:

Basic Equation: The kinetic energy coefficient, α is given by

$$\alpha = \frac{\int_A \rho V^3 dA}{\dot{m}\bar{V}^2} \qquad (8.26b)$$

From Section 8-2, for flow between parallel plates

$$u = u_{max}\left[1 - \left(\frac{y}{a/2}\right)^2\right] = \frac{3}{2}\bar{V}\left[1 - \left(\frac{y}{a/2}\right)^2\right]$$

since $u_{max} = \frac{3}{2}\bar{V}$.

Substituting

$$\alpha = \frac{\int_A \rho V^3 dA}{\dot{m}\bar{V}^2} = \frac{\int_A \rho u^3 dA}{\rho\bar{V}A\bar{V}^2} = \frac{1}{A}\int_A \left(\frac{u}{\bar{V}}\right)^3 dA = \frac{1}{wa}\int_{-\frac{a}{2}}^{\frac{a}{2}} \left(\frac{u}{\bar{V}}\right)^3 w\,dy = \frac{2}{a}\int_0^{\frac{a}{2}} \left(\frac{u}{\bar{V}}\right)^3 dy$$

Then

$$\alpha = \frac{2}{a}\frac{a}{2}\int_0^1 \left(\frac{u}{u_{max}}\right)^3 \left(\frac{u_{max}}{\bar{V}}\right)^3 d\left(\frac{y}{a/2}\right) = \left(\frac{3}{2}\right)^3 \int_0^1 \left(1-\eta^2\right)^3 d\eta$$

where $\eta = \frac{y}{a/2}$

Evaluating,

$$\left(1-\eta^2\right)^3 = 1 - 3\eta^2 + 3\eta^4 - \eta^6$$

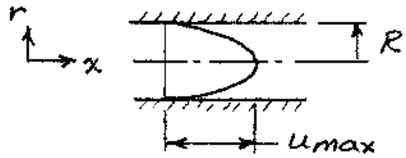
The integral is then

$$\alpha = \left(\frac{3}{2}\right)^3 \int_0^1 \left(1 - 3\eta^2 + 3\eta^4 - \eta^6\right) d\eta = \left(\frac{3}{2}\right)^3 \left[\eta - \eta^3 + \frac{3}{5}\eta^5 - \frac{1}{7}\eta^7\right]_0^1 = \frac{27}{8}\frac{16}{35} = 1.54$$

Problem 8.65

[3]

Given: Fully developed laminar flow in a circular tube.



Find: Kinetic energy coefficient, α.

Solution: Apply definition of kinetic energy coefficient,

$$\alpha = \frac{\int_A \rho V^3 dA}{\dot{m}\bar{V}^2}, \quad \dot{m} = \rho \bar{V} A \qquad (8.26b)$$

From the analysis of Section 8-3, for flow in a circular tube,

$$u = u_{max}\left[1-\left(\frac{r}{R}\right)^2\right] = 2\bar{V}\left[1-\left(\frac{r}{R}\right)^2\right] \quad \text{since } u_{max} = 2\bar{V}$$

Substituting into Eq. 8.25b,

$$\alpha = \frac{\int_A \rho V^3 dA}{\dot{m}\bar{V}^2} = \frac{\int_A \rho u^3 dA}{\rho \bar{V} A \bar{V}^2} = \frac{1}{A}\int_A \left(\frac{u}{\bar{V}}\right)^3 dA = \frac{1}{\pi R^2}\int_0^R \left(\frac{u}{\bar{V}}\right)^3 2\pi r\,dr = 2\int_0^1 \left(\frac{u}{\bar{V}}\right)^3 \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

Then

$$\alpha = 2\int_0^1 \left(\frac{u}{u_{max}}\right)^3 \left(\frac{u_{max}}{\bar{V}}\right)^3 \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = 2(2)^3 \int_0^1 (1-\eta^2)^3 \eta\, d\eta \quad \text{where } \eta = \frac{r}{R}$$

Evaluating,

$$(1-\eta^2)^3 \eta = \eta - 3\eta^3 + 3\eta^5 - \eta^7$$

The integral is

$$\int_0^1 (1-\eta^2)^3 \eta\, d\eta = \left[\frac{\eta^2}{2} - \frac{3}{4}\eta^4 + \frac{3}{6}\eta^6 - \frac{1}{8}\eta^8\right]_0^1 = \frac{1}{2} - \frac{3}{4} + \frac{1}{2} - \frac{1}{8} = \frac{1}{8}$$

Substituting,

$$\alpha = 16\int_0^1 (1-\eta^2)^3 \eta\, d\eta = 16 \times \frac{1}{8} = 2$$

α

Problem 8.66

8.66 Show that the kinetic energy coefficient, α, for the "power law" turbulent velocity profile of Eq. 8.22 is given by Eq. 8.27. Plot α as a function of $Re_{\bar{V}}$, for $Re_{\bar{V}} = 1 \times 10^4$ to 1×10^7. When analyzing pipe flow problems it is common practice to assume $\alpha \approx 1$. Plot the error associated with this assumption as a function of $Re_{\bar{V}}$, for $Re_{\bar{V}} = 1 \times 10^4$ to 1×10^7.

Given: Definition of kinetic energy correction coefficient α

Find: α for the power-law velocity profile; plot

Solution:

Equation 8.26b is

$$\alpha = \frac{\int \rho \cdot V^3\, dA}{m_{rate} \cdot V_{av}^2}$$

where V is the velocity, m_{rate} is the mass flow rate and V_{av} is the average velocity

For the power-law profile (Eq. 8.22)

$$V = U \cdot \left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$$

For the mass flow rate

$$m_{rate} = \rho \cdot \pi \cdot R^2 \cdot V_{av}$$

Hence the denominator of Eq. 8.26b is

$$m_{rate} \cdot V_{av}^2 = \rho \cdot \pi \cdot R^2 \cdot V_{av}^3$$

We next must evaluate the numerator of Eq. 8.26b

$$\int \rho \cdot V^3\, dA = \int \rho \cdot 2 \cdot \pi \cdot r \cdot U^3 \cdot \left(1 - \frac{r}{R}\right)^{\frac{3}{n}} dr$$

$$\int_0^R \rho \cdot 2 \cdot \pi \cdot r \cdot U^3 \cdot \left(1 - \frac{r}{R}\right)^{\frac{3}{n}} dr = \frac{2 \cdot \pi \cdot \rho \cdot R^2 \cdot n^2 \cdot U^3}{(3 + n) \cdot (3 + 2 \cdot n)}$$

To integrate substitute

$$m = 1 - \frac{r}{R} \qquad dm = -\frac{dr}{R}$$

Then

$$r = R \cdot (1 - m) \qquad dr = -R \cdot dm$$

$$\int_0^R \rho \cdot 2 \cdot \pi \cdot r \cdot U^3 \cdot \left(1 - \frac{r}{R}\right)^{\frac{3}{n}} dr = -\int_1^0 \rho \cdot 2 \cdot \pi \cdot R \cdot (1 - m) \cdot m^{\frac{3}{n}} \cdot R\, dm$$

Hence $$\int \rho \cdot V^3\, dA = \int_0^1 \rho \cdot 2 \cdot \pi \cdot R \cdot \left(m^{\frac{3}{n}} - m^{\frac{3}{n}+1} \right) \cdot R\, dm$$

$$\int \rho \cdot V^3\, dA = \frac{2 \cdot R^2 \cdot n^2 \cdot \rho \cdot \pi \cdot U^3}{(3+n) \cdot (3+2 \cdot n)}$$

Putting all these results together $$\alpha = \frac{\int \rho \cdot V^3\, dA}{m_{rate} \cdot V_{av}^{\;2}} = \frac{\frac{2 \cdot R^2 \cdot n^2 \cdot \rho \cdot \pi \cdot U^3}{(3+n) \cdot (3+2 \cdot n)}}{\rho \cdot \pi \cdot R^2 \cdot V_{av}^{\;3}}$$

$$\alpha = \left(\frac{U}{V_{av}} \right)^3 \cdot \frac{2 \cdot n^2}{(3+n) \cdot (3+2 \cdot n)}$$

To plot α versus Re_{Vav} we use the following parametric relations

$$n = -1.7 + 1.8 \cdot \log\left(Re_u\right) \qquad \text{(Eq. 8.23)}$$

$$\frac{V_{av}}{U} = \frac{2 \cdot n^2}{(n+1) \cdot (2 \cdot n+1)} \qquad \text{(Eq. 8.24)}$$

$$Re_{Vav} = \frac{V_{av}}{U} \cdot Re_U$$

$$\alpha = \left(\frac{U}{V_{av}} \right)^3 \cdot \frac{2 \cdot n^2}{(3+n) \cdot (3+2 \cdot n)} \qquad \text{(Eq. 8.27)}$$

A value of Re_U leads to a value for n; this leads to a value for V_{av}/U; these lead to a value for Re_{Vav} and α

The plots of α, and the error in assuming α = 1, versus Re_{Vav} are shown in the associated *Excel* workbook

Problem 8.66

8.66 Show that the kinetic energy coefficient, α, for the "power law" turbulent velocity profile of Eq. 8.22 is given by Eq. 8.27. Plot α as a function of $Re_{\bar{V}}$, for $Re_{\bar{V}} = 1 \times 10^4$ to 1×10^7. When analyzing pipe flow problems it is common practice to assume $\alpha \approx 1$. Plot the error associated with this assumption as a function of $Re_{\bar{V}}$, for $Re_{\bar{V}} = 1 \times 10^4$ to 1×10^7.

Given: Definition of kinetic energy correction coefficient α

Find: α for the power-law velocity profile; plot

Solution:

$$n = -1.7 + 1.8 \cdot \log(Re_u) \qquad \text{(Eq. 8.23)}$$

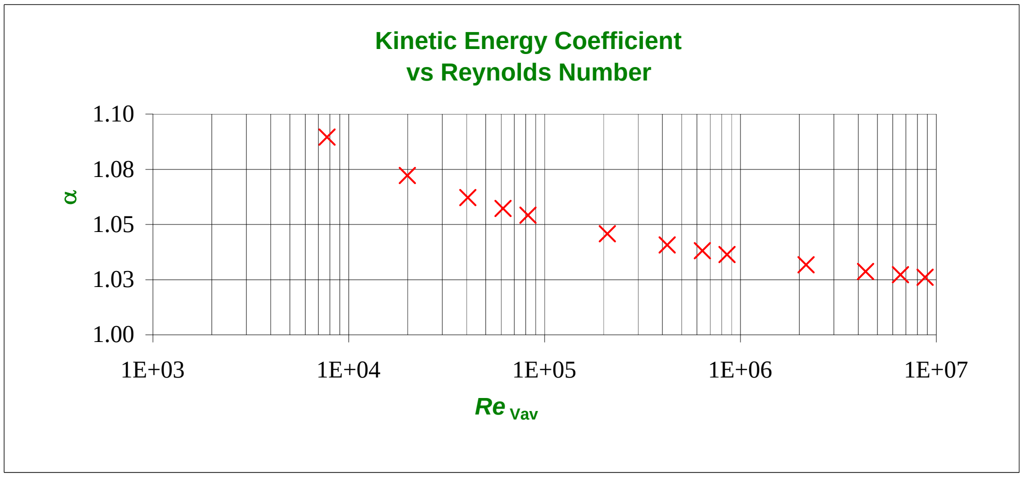
$$\frac{V_{av}}{U} = \frac{2 \cdot n^2}{(n+1) \cdot (2 \cdot n + 1)} \qquad \text{(Eq. 8.24)}$$

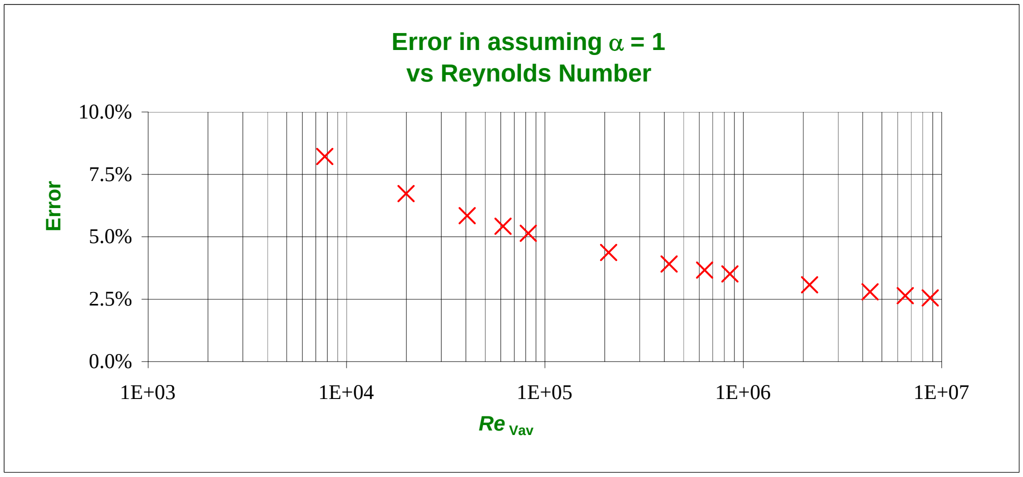
$$Re_{Vav} = \frac{V_{av}}{U} \cdot Re_U$$

$$\alpha = \left(\frac{U}{V_{av}}\right)^3 \cdot \frac{2 \cdot n^2}{(3+n) \cdot (3 + 2 \cdot n)} \qquad \text{(Eq. 8.27)}$$

this leads to a value for V_{av}/U;
these lead to a value for Re_{Vav} and α

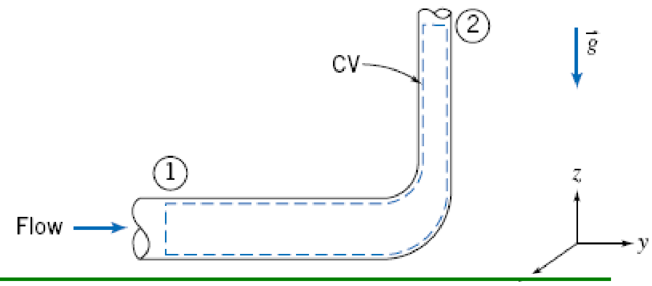
Re_U	n	V_{av}/U	Re_{Vav}	α	α Error
1.00E+04	5.50	0.776	7.76E+03	1.09	8.2%
2.50E+04	6.22	0.797	1.99E+04	1.07	6.7%
5.00E+04	6.76	0.811	4.06E+04	1.06	5.9%
7.50E+04	7.08	0.818	6.14E+04	1.06	5.4%
1.00E+05	7.30	0.823	8.23E+04	1.05	5.1%
2.50E+05	8.02	0.837	2.09E+05	1.05	4.4%
5.00E+05	8.56	0.846	4.23E+05	1.04	3.9%
7.50E+05	8.88	0.851	6.38E+05	1.04	3.7%
1.00E+06	9.10	0.854	8.54E+05	1.04	3.5%
2.50E+06	9.82	0.864	2.16E+06	1.03	3.1%
5.00E+06	10.4	0.870	4.35E+06	1.03	2.8%
7.50E+06	10.7	0.873	6.55E+06	1.03	2.6%
1.00E+07	10.9	0.876	8.76E+06	1.03	2.5%





Problem 8.67

8.67 Measurements are made for the flow configuration shown in Fig. 8.12. At the inlet, section (1), the pressure is 70 kPa (gage), the average velocity is 1.75 m/s, and the elevation is 2.25 m. At the outlet, section (2), the pressure, average velocity, and elevation are 45 kPa (gage), 3.5 m/s, and 3 m, respectively. Calculate the head loss in meters. Convert to units of energy per unit mass.



Given: Data on flow through elbow

Find: Head loss

Solution:

Basic equation
$$\left(\frac{p_1}{\rho\cdot g}+\alpha\cdot\frac{V_1^{\,2}}{2\cdot g}+z_1\right)-\left(\frac{p_2}{\rho\cdot g}+\alpha\cdot\frac{V_2^{\,2}}{2\cdot g}+z_2\right)=\frac{h_{lT}}{g}=H_{lT}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1

Then
$$H_{lT}=\frac{p_1-p_2}{\rho\cdot g}+\frac{V_1^{\,2}-V_2^{\,2}}{2\cdot g}+z_1-z_2$$

$$H_{lT} = (70-45)\times 10^3\cdot\frac{N}{m^2}\times\frac{m^3}{1000\cdot kg}\times\frac{kg\cdot m}{s^2\cdot N}\times\frac{s^2}{9.81\cdot m}+\frac{1}{2}\times\left(1.75^2-3.5^2\right)\cdot\left(\frac{m}{s}\right)^2\times\frac{s^2}{9.81\cdot m}+(2.25-3)\cdot m$$ $$H_{lT} = 1.33\,m$$

In terms of energy/mass $\quad h_{lT} = g\cdot H_{lT}$

$$h_{lT} = 9.81\cdot\frac{m}{s^2}\times 1.33\cdot m\times\frac{N\cdot s^2}{kg\cdot m}$$ $$h_{lT} = 13.0\cdot\frac{N\cdot m}{kg}$$

Problem 8.68

[2]

8.68 Water flows in a horizontal constant-area pipe; the pipe diameter is 50 mm and the average flow speed is 1.5 m/s. At the pipe inlet the gage pressure is 588 kPa, and the outlet is at atmospheric pressure. Determine the head loss in the pipe. If the pipe is now aligned so that the outlet is 25 m above the inlet, what will the inlet pressure need to be to maintain the same flow rate? If the pipe is now aligned so that the outlet is 25 m below the inlet, what will the inlet pressure need to be to maintain the same flow rate? Finally, how much lower than the inlet must the outlet be so that the same flow rate is maintained if both ends of the pipe are at atmospheric pressure (i.e., gravity feed)?

Given:
Data on flow in a pipe

Find:
Head loss for horizontal pipe; inlet pressure for different alignments; slope for gravity feed

Solution:

Given or available data $D = 50 \cdot mm$ $\qquad \rho = 1000 \cdot \frac{kg}{m^3}$

The governing equation between inlet (1) and exit (2) is

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_{lT} \qquad (8.29)$$

Horizontal pipe data $p_1 = 588 \cdot kPa$ $\qquad p_2 = 0 \cdot kPa$ $\qquad$ (Gage pressures)

$z_1 = z_2$ $\qquad V_1 = V_2$

Equation 8.29 becomes $h_{lT} = \frac{p_1 - p_2}{\rho}$ $\qquad h_{lT} = 588 \cdot \frac{J}{kg}$

For an inclined pipe with the same flow rate, the head loss will be the same as above; in addition we have the following new data

$z_1 = 0 \cdot m$ $\qquad z_2 = 25 \cdot m$

Equation 8.29 becomes $p_1 = p_2 + \rho \cdot g \cdot (z_2 - z_1) + \rho \cdot h_{lT}$ $\qquad p_1 = 833 \cdot kPa$

For a declining pipe with the same flow rate, the head loss will be the same as above; in addition we have the following new data

$z_1 = 0 \cdot m$ $\qquad z_2 = -25 \cdot m$

Equation 8.29 becomes $p_1 = p_2 + \rho \cdot g \cdot (z_2 - z_1) + \rho \cdot h_{lT}$ $\qquad p_1 = 343 \cdot kPa$

For a gravity feed with the same flow rate, the head loss will be the same as above; in addition we have the following new data

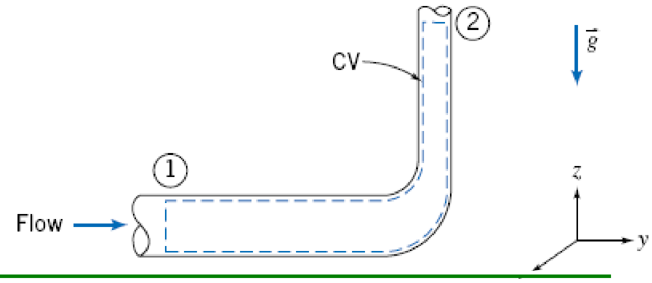
$p_1 = 0 \cdot kPa$ $\qquad$ (Gage)

Equation 8.29 becomes $z_2 = z_1 - \frac{h_{lT}}{g}$ $\qquad z_2 = -60\,m$

Problem 8.69

[2]

8.69 For the flow configuration of Fig. 8.12, it is known that the head loss is 1 m. The pressure drop from inlet to outlet is 50 kPa, the velocity doubles from inlet to outlet, and the elevation increase is 2 m. Compute the inlet water velocity.



Given: Data on flow through elbow

Find: Inlet velocity

Solution:

Basic equation
$$\left(\frac{p_1}{\rho\cdot g}+\alpha\cdot\frac{V_1^2}{2\cdot g}+z_1\right)-\left(\frac{p_2}{\rho\cdot g}+\alpha\cdot\frac{V_2^2}{2\cdot g}+z_2\right)=\frac{h_{lT}}{g}=H_{lT}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1

Then
$$V_2^2-V_1^2=(2\cdot V_1)^2-V_1^2=3\cdot V_1^2=\frac{2\cdot(p_1-p_2)}{\rho}+2\cdot g\cdot(z_1-z_2)-2\cdot g\cdot H_{lT}$$

$$V_1=\sqrt{\frac{2}{3}\cdot\left[\frac{(p_1-p_2)}{\rho}+g\cdot(z_1-z_2)-g\cdot H_{lT}\right]}$$

$$V_1=\sqrt{\frac{2}{3}\times\left[50\times10^3\cdot\frac{N}{m^2}\times\frac{m^3}{1000\cdot kg}\times\frac{kg\cdot m}{s^2\cdot N}+\frac{9.81\cdot m}{s^2}\times(-2)\cdot m-9.81\cdot\frac{m}{s^2}\times1\cdot m\right]}$$

$$V_1=3.70\,\frac{m}{s}$$

Problem 8.70

8.70 Consider the pipe flow from the water tower of Example 8.7. After another 10 years the pipe roughness has increased such that the flow is fully turbulent and $f = 0.04$. Find by how much the flow rate is decreased.

Given: Increased friction factor for water tower flow

Find: How much flow is decreased

Solution:

Basic equation from Example 8.7 $$V_2 = \sqrt{\frac{2 \cdot g \cdot (z_1 - z_2)}{f \cdot \left(\frac{L}{D} + 8\right) + 1}}$$

where $L = 680 \cdot ft$ $\quad D = 4 \cdot in$ $\quad z_1 - z_2 = 80 \cdot ft$

With f = 0.0308, we obtain $V_2 = 8.97 \cdot \frac{ft}{s}$ and Q = 351 gpm

We need to recompute with f = 0.04 $$V_2 = \sqrt{2 \times 32.2 \cdot \frac{ft}{s^2} \times 80 \cdot ft \times \frac{1}{0.04 \cdot \left(\frac{680}{\frac{4}{12}} + 8\right) + 1}}$$ $$V_2 = 7.88 \frac{ft}{s}$$

Hence $$Q = V_2 \cdot A = V_2 \cdot \frac{\pi \cdot D^2}{4}$$

$$Q = 7.88 \cdot \frac{ft}{s} \times \frac{\pi}{4} \times \left(\frac{4}{12} \cdot ft\right)^2 \times \frac{7.48 \cdot gal}{1 \cdot ft^3} \times \frac{60 \cdot s}{1 \cdot min}$$ $$Q = 309\,gpm$$

(From Table G.2 1 ft^3 = 7.48 gal)

Hence the flow is decreased by $(351 - 309) \cdot gpm = 42\,gpm$

Problem 8.71

8.71 Consider the pipe flow from the water tower of Problem 8.70. To increase delivery, the pipe length is reduced from 600 ft to 300 ft (the flow is still fully turbulent and $f \approx 0.04$). What is the flow rate?

Given: Increased friction factor for water tower flow, and reduced length

Find: How much flow is decreased

Solution:

Basic equation from Example 8.7

$$V_2 = \sqrt{\frac{2\cdot g\cdot\left(z_1 - z_2\right)}{f\cdot\left(\frac{L}{D} + 8\right) + 1}}$$

where now we have $L = 380\cdot ft$ $\quad D = 4\cdot in$ $\quad z_1 - z_2 = 80\cdot ft$

We need to recompute with f = 0.04

$$V_2 = \sqrt{2 \times 32.2\cdot\frac{ft}{s^2} \times 80\cdot ft \times \frac{1}{0.04\cdot\left(\frac{380}{\frac{4}{12}} + 8\right) + 1}} \qquad V_2 = 10.5\frac{ft}{s}$$

Hence

$$Q = V_2\cdot A = V_2\cdot\frac{\pi\cdot D^2}{4}$$

$$Q = 10.5\cdot\frac{ft}{s} \times \frac{\pi}{4} \times \left(\frac{4}{12}\cdot ft\right)^2 \times \frac{7.48\cdot gal}{1\cdot ft^3} \times \frac{60\cdot s}{1\cdot min} \qquad Q = 411\,gpm$$

(From Table G.2 1 ft^3 = 7.48 gal)

Problem 8.72

[2]

8.72 The average flow speed in a constant-diameter section of the Alaskan pipeline is 2.5 m/s. At the inlet, the pressure is 8.25 MPa (gage) and the elevation is 45 m; at the outlet, the pressure is 350 kPa (gage) and the elevation is 115 m. Calculate the head loss in this section of pipeline.

Given: Data on flow through Alaskan pipeline

Find: Head loss

Solution:

Basic equation

$$\left(\frac{p_1}{\rho_{oil}\cdot g} + \alpha\cdot\frac{V_1^2}{2\cdot g} + z_1\right) - \left(\frac{p_2}{\rho_{oil}\cdot g} + \alpha\cdot\frac{V_2^2}{2\cdot g} + z_2\right) = \frac{h_{lT}}{g} = H_{lT}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) SG = 0.9 (Table A.2)

Then

$$H_{lT} = \frac{p_1 - p_2}{SG_{oil}\cdot\rho_{H2O}\cdot g} + z_1 - z_2$$

$$H_{lT} = (8250 - 350)\times 10^3\cdot\frac{N}{m^2}\times\frac{1}{0.9}\times\frac{m^3}{1000\cdot kg}\times\frac{kg\cdot m}{s^2\cdot N}\times\frac{s^2}{9.81\cdot m} + (45 - 115)\cdot m \qquad H_{lT} = 825\,m$$

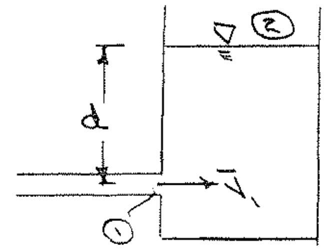
In terms of energy/mass

$$h_{lT} = g\cdot H_{lT} \qquad h_{lT} = 9.81\cdot\frac{m}{s^2}\times 825\cdot m\times\frac{N\cdot s^2}{kg\cdot m} \qquad h_{lT} = 8.09\cdot\frac{kN\cdot m}{kg}$$

Problem 8.73

Given: Water flows from a horizontal tube into a very large tank as shown.

$d = 2.5\,m$, $h_{\ell} = 2\,J/kg$



Find: Average flow speed in tube

Solution:

Apply definition of head loss, Eq 8.29,

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) = h_{\ell T}$$

At free surface, $V_2 = 0$, $p_2 = p_{atm}$

At tube discharge $p_1 = \rho g d$, $z_1 = 0$. Assume $\alpha_1 \approx 1$

Then

$$g d + \frac{\bar{V}_1^2}{2} - g d = h_{\ell T}$$

$$\bar{V}_1^2 = 2 h_{\ell T} = 2 \times 2 \frac{N \cdot m}{kg} \times \frac{kg \cdot m}{N \cdot s^2} = 4\,m^2/s^2$$

$$\bar{V}_1 = 2\,m/s$$

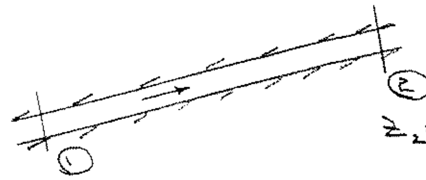
$\bar{V}_1$

Problem 8.74

[2]

Given: Section of Alaskan pipeline with conditions shown

$h_{l_{1\to 2}} = 6.9$ kJ/kg



Find: outlet pressure, p_2

$z_2 = 115$ m

$p_1 = 8.5$ MPa

$z_1 = 45$ m

Solution:

Computing equation: $\left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T}$ (8.29)

Assumptions: (1) incompressible flow, so $\bar{V}_1 = \bar{V}_2$

(2) fully developed so $\alpha_1 = \alpha_2$

(3) SG crude oil = 0.90 (Table A.2)

Then

$$p_2 = p_1 + \rho g(z_1 - z_2) - \rho h_{l_T}$$

$$= 8.5\times10^6 \, N/m^2 + 0.9\times999\,\frac{kg}{m^3}\times9.81\,\frac{m}{s^2}\times(-70\,m)\times\frac{N\cdot s^2}{kg\cdot m}$$

$$- 0.9\times999\,\frac{kg}{m^3}\times6.9\times10^3\,\frac{N\cdot m}{kg}$$

$p_2 = 1.68$ MPa ← p_2

Problem 8.75

Given: Water flow at $Q = 3\,gpm$ through a horizontal 5/8 in. diameter garden hose. Pressure drop in $L = 50\,ft$ is 12.3 psi.

Find: Head loss

Solution: Computing equation is

$$h_{lT} = \left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right)$$

(with the $\bar{V}^2/2$ terms marked (1) and the gz terms marked (3))

Assumptions: (1) Incompressible flow, so $\bar{V}_1 = \bar{V}_2$

(2) Fully developed so $\alpha_1 = \alpha_2$

(3) Horizontal, so $z_1 = z_2$

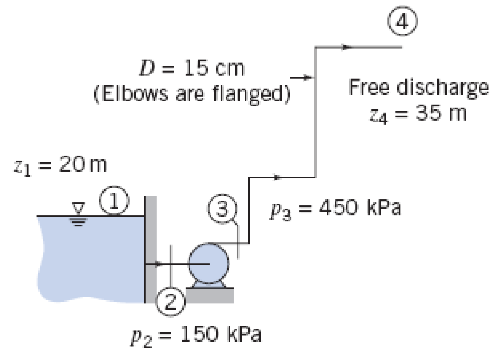
Then $h_{lT} = \frac{p_1 - p_2}{\rho} = 12.3 \frac{lbf}{in.^2} \times \frac{ft^3}{1.94\,slug} \times 144 \frac{in.^2}{ft^2} \times \frac{slug \cdot ft}{lbf \cdot s^2}$

$h_{lT} = 913\ ft^2/s^2$ ← h_{lT}

Also $H_{lT} = \frac{h_{lT}}{g} = 913 \frac{ft^2}{s^2} \times \frac{s^2}{32.2\,ft} = 28.4\ ft$ ← H_{lT}

Problem 8.76

8.76 Water is pumped at the rate of 0.075 m^3/s from a reservoir 20 m above a pump to a free discharge 35 m above the pump. The pressure on the intake side of the pump is 150 kPa and the pressure on the discharge side is 450 kPa. All pipes are commercial steel of 15 cm diameter. Determine (a) the head supplied by the pump and (b) the total head loss between the pump and point of free discharge.



Given: Data on flow from reservoir

Find: Head from pump; head loss

Solution:

Basic equations

$$\left(\frac{p_3}{\rho \cdot g} + \alpha \cdot \frac{V_3^2}{2 \cdot g} + z_3\right) - \left(\frac{p_4}{\rho \cdot g} + \alpha \cdot \frac{V_4^2}{2 \cdot g} + z_4\right) = \frac{h_{lT}}{g} = H_{lT}$$ for flow from 3 to 4

$$\left(\frac{p_3}{\rho \cdot g} + \alpha \cdot \frac{V_3^2}{2 \cdot g} + z_3\right) - \left(\frac{p_2}{\rho \cdot g} + \alpha \cdot \frac{V_2^2}{2 \cdot g} + z_2\right) = \frac{\Delta h_{pump}}{g} = H_{pump}$$ for flow from 2 to 3

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) $V_2 = V_3 = V_4$ (constant area pipe)

Then for the pump $$H_{pump} = \frac{p_3 - p_2}{\rho \cdot g}$$

$$H_{pump} = (450 - 150) \times 10^3 \cdot \frac{N}{m^2} \times \frac{m^3}{1000 \cdot kg} \times \frac{kg \cdot m}{s^2 \cdot N} \times \frac{s^2}{9.81 \cdot m}$$ $$H_{pump} = 30.6\,m$$

In terms of energy/mass $$h_{pump} = g \cdot H_{pump}$$ $$h_{pump} = 9.81 \cdot \frac{m}{s^2} \times 30.6 \cdot m \times \frac{N \cdot s^2}{kg \cdot m}$$ $$h_{pump} = 300 \cdot \frac{N \cdot m}{kg}$$

For the head loss from 3 to 4 $$H_{lT} = \frac{p_3 - p_4}{\rho \cdot g} + z_3 - z_4$$

$$H_{lT} = (450 - 0) \times 10^3 \cdot \frac{N}{m^2} \times \frac{m^3}{1000 \cdot kg} \times \frac{kg \cdot m}{s^2 \cdot N} \times \frac{s^2}{9.81 \cdot m} + (0 - 35) \cdot m$$ $$H_{lT} = 10.9\,m$$

In terms of energy/mass $$h_{lT} = g \cdot H_{lT}$$ $$h_{lT} = 9.81 \cdot \frac{m}{s^2} \times 10.9 \cdot m \times \frac{N \cdot s^2}{kg \cdot m}$$ $$h_{lT} = 107 \cdot \frac{N \cdot m}{kg}$$

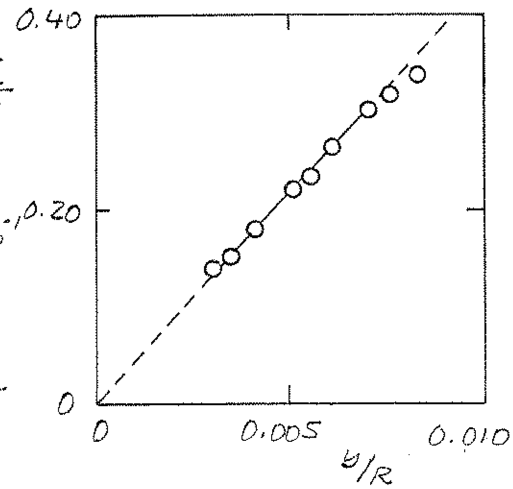
Problem 8.77

Given: Data measured in fully developed turbulent pipe flow at $Re_U = 50{,}000$ in air:

$\frac{\bar{u}}{U}$	0.343	0.318	0.300	0.264	0.228	0.221	0.179	0.152	0.140
$\frac{y}{R}$	0.0082	0.0075	0.0071	0.0061	0.0055	0.0051	0.0041	0.0034	0.0030

$U = 9.8$ ft/s and $R = 4.86$ in.

Find: (a) Evaluate best-fit value of $d\bar{u}/dy$ from plot.
(b) $\tau_w = \mu\, d\bar{u}/dy$
(c) τ_w calculated from friction factor.



Solution: "Best-fit" slope is $\left\{\begin{matrix}\text{from analysis}\\ \text{in Excel file}\end{matrix}\right\}$

$$\frac{d(\bar{u}/U)}{d(y/R)} \approx \frac{\Delta(\bar{u}/U)}{\Delta(y/R)} = 39.8$$

$$\frac{d\bar{u}}{dy} = \frac{U\, d(\bar{u}/U)}{R\, d(y/R)} = 39.8 \times 9.8\frac{\text{ft}}{\text{s}} \times \frac{1}{4.86\,\text{in.}} \times 12\frac{\text{in.}}{\text{ft}} = 963\ \text{s}^{-1}$$ $\frac{d\bar{u}}{dy}$

For standard air, $\mu = 3.72\times10^{-7}$ lbf·s / ft², so

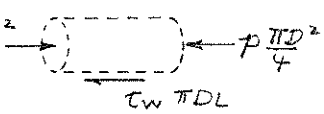
$$\tau_w = \mu\frac{d\bar{u}}{dy} = 3.72\times10^{-7}\,\frac{\text{lbf}\cdot\text{s}}{\text{ft}^2} \times \frac{963}{\text{s}} = 3.58\times10^{-4}\ \text{lbf/ft}^2$$ τ_w

Friction factor is $f = f(Re, e/D)$. For $Re_U = 50{,}000$, $n = 6.8$ from Eq. 8.23. Then from Eq. 8.24,

$$\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)} = 0.812 \quad \text{and} \quad Re_{\bar{V}} = 0.812\, Re_U = 0.812 \times 50{,}000 = 40{,}600$$

Assuming smooth pipe, $f = 0.0219$ from Eq. 8.37

Balancing forces on a fluid element: $(p+\Delta p)\frac{\pi D^2}{4}$



Then $(p+\Delta p)\frac{\pi D^2}{4} - \tau_w \pi D L - p\frac{\pi D^2}{4} = 0$

$$\tau_w = \frac{R}{2}\frac{\Delta p}{L} = \frac{D}{4L} f \frac{L}{D}\rho\frac{\bar{V}^2}{2} = \frac{f}{8}\rho\bar{V}^2\ ;\quad \bar{V} = 0.812U = 0.812 \times 9.8\frac{\text{ft}}{\text{sec}} = 7.96\ \text{ft/sec}$$

Substituting,

$$\tau_w = \frac{0.0219}{8} \times 0.00238\frac{\text{slug}}{\text{ft}^3} \times (7.96)^2\frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} = 4.13\times10^{-4}\ \text{lbf/ft}^2$$ τ_w

The result calculated from the friction factor is 15% higher than that evaluated graphically!

Problem 8.78

Given: Small-diameter (i.d. = 0.5 mm) capillary tube made from drawn aluminum is used in place of an expansion valve in a home refrigerator

Find: corresponding relative roughness; with regard to fluid flow, can tube be considered "smooth"?

Solution:

For drawn tubing, from Table 8.1, $e = 0.0015$ mm

Then with $D = 0.5$ mm, $\frac{e}{D} = \frac{0.0015}{0.5} = 0.003$

Looking at the Moody diagram (Fig. 8.13), it is clear that this tube cannot be considered smooth for turbulent flow through the tube.

For laminar flow ($Re < 2300$) the relative roughness has no effect on the flow.

Problem 8.79

[2]

8.79 A smooth, 75-mm diameter pipe carries water (65°C) horizontally. When the mass flow rate is 0.075 kg/s, the pressure drop is measured to be 7.5 Pa per 100 m of pipe. Based on these measurements, what is the friction factor? What is the Reynolds number? Does this Reynolds number generally indicate laminar or turbulent flow? Is the flow actually laminar or turbulent?

Given: Data on flow in a pipe

Find: Friction factor; Reynolds number; if flow is laminar or turbulent

Solution:

Given data $D = 75\cdot mm$ $\frac{\Delta p}{L} = 0.075\cdot\frac{Pa}{m}$ $m_{rate} = 0.075\cdot\frac{kg}{s}$

From Appendix A $\rho = 1000\cdot\frac{kg}{m^3}$ $\mu = 4\cdot 10^{-4}\cdot\frac{N\cdot s}{m^2}$

The governing equations between inlet (1) and exit (2) are

$$\left(\frac{p_1}{\rho} + \alpha_1\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_l \qquad (8.29)$$

$$h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \qquad (8.34)$$

For a constant area pipe $V_1 = V_2 = V$

Hence Eqs. 8.29 and 8.34 become $f = \frac{2\cdot D}{L\cdot V^2}\cdot\frac{(p_1 - p_2)}{\rho} = \frac{2\cdot D}{\rho\cdot V^2}\cdot\frac{\Delta p}{L}$

For the velocity $V = \frac{m_{rate}}{\rho\cdot\frac{\pi}{4}\cdot D^2}$ $V = 0.017\frac{m}{s}$

Hence $f = \frac{2\cdot D}{\rho\cdot V^2}\cdot\frac{\Delta p}{L}$ $f = 0.0390$

The Reynolds number is $Re = \frac{\rho\cdot V\cdot D}{\mu}$ $Re = 3183$

This Reynolds number indicates the flow is turbulent.

(From Eq. 8.37, at this Reynolds number the friction factor for a smooth pipe is *f* = 0.043; the friction factor computed above thus indicates that, within experimental error, the flow corresponds to turbulent flow in a smooth pipe)

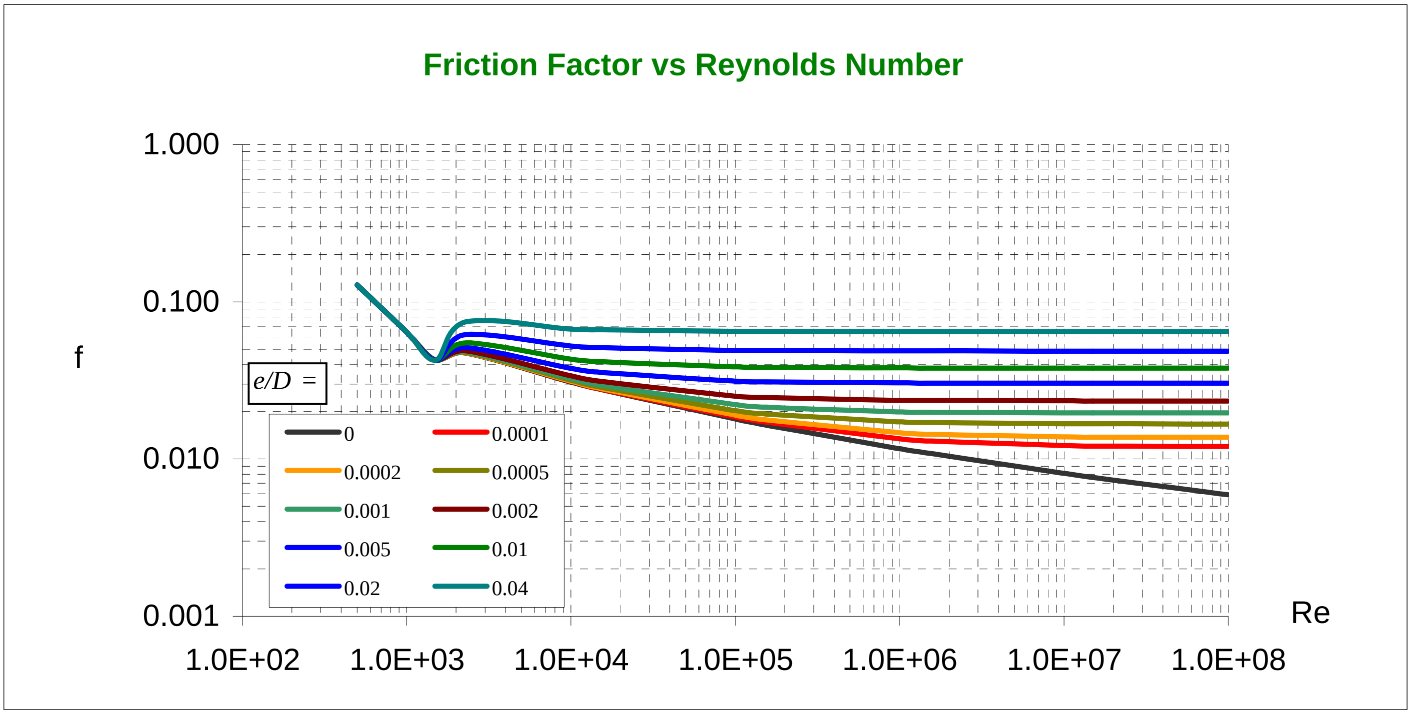
Problem 8.80

8.80 Using Eqs. 8.36 and 8.37, generate the Moody chart of Fig. 8.13.

Solution:

Using the add-in function *Friction factor* from the web site

e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.04
Re	*f*									
500	0.1280	0.1280	0.1280	0.1280	0.1280	0.1280	0.1280	0.1280	0.1280	0.1280
1.00E+03	0.0640	0.0640	0.0640	0.0640	0.0640	0.0640	0.0640	0.0640	0.0640	0.0640
1.50E+03	0.0427	0.0427	0.0427	0.0427	0.0427	0.0427	0.0427	0.0427	0.0427	0.0427
2.30E+03	0.0473	0.0474	0.0474	0.0477	0.0481	0.0489	0.0512	0.0549	0.0619	0.0747
1.00E+04	0.0309	0.0310	0.0312	0.0316	0.0324	0.0338	0.0376	0.0431	0.0523	0.0672
1.50E+04	0.0278	0.0280	0.0282	0.0287	0.0296	0.0313	0.0356	0.0415	0.0511	0.0664
1.00E+05	0.0180	0.0185	0.0190	0.0203	0.0222	0.0251	0.0313	0.0385	0.0490	0.0649
1.50E+05	0.0166	0.0172	0.0178	0.0194	0.0214	0.0246	0.0310	0.0383	0.0489	0.0648
1.00E+06	0.0116	0.0134	0.0147	0.0172	0.0199	0.0236	0.0305	0.0380	0.0487	0.0647
1.50E+06	0.0109	0.0130	0.0144	0.0170	0.0198	0.0235	0.0304	0.0379	0.0487	0.0647
1.00E+07	0.0081	0.0122	0.0138	0.0168	0.0197	0.0234	0.0304	0.0379	0.0486	0.0647
1.50E+07	0.0076	0.0121	0.0138	0.0167	0.0197	0.0234	0.0304	0.0379	0.0486	0.0647
1.00E+08	0.0059	0.0120	0.0137	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0647



Problem 8.81

8.81 The Colebrook equation (Eq. 8.37) for computing the turbulent friction factor is implicit in f. An explicit expression [30] that gives reasonable accuracy is

$$f_0 = 0.25\left[\log\left(\frac{e/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^{-2}$$

Compare the accuracy of this expression for f with Eq. 8.37 by computing the percentage discrepancy as a function of Re and e/D, for $Re = 10^4$ to 10^8, and $e/D = 0$, 0.0001, 0.001, 0.01, and 0.05. What is the maximum discrepancy for these Re and e/D values? Plot f against Re with e/D as a parameter.

Using the above formula for f_0, and Eq. 8.37 for f_1

e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.05
Re	f_0									
1.00E+04	0.0310	0.0311	0.0313	0.0318	0.0327	0.0342	0.0383	0.0440	0.0534	0.0750
2.50E+04	0.0244	0.0247	0.0250	0.0258	0.0270	0.0291	0.0342	0.0407	0.0508	0.0731
5.00E+04	0.0208	0.0212	0.0216	0.0226	0.0242	0.0268	0.0325	0.0395	0.0498	0.0724
7.50E+04	0.0190	0.0195	0.0200	0.0212	0.0230	0.0258	0.0319	0.0390	0.0494	0.0721
1.00E+05	0.0179	0.0185	0.0190	0.0204	0.0223	0.0253	0.0316	0.0388	0.0493	0.0720
2.50E+05	0.0149	0.0158	0.0167	0.0186	0.0209	0.0243	0.0309	0.0383	0.0489	0.0717
5.00E+05	0.0131	0.0145	0.0155	0.0178	0.0204	0.0239	0.0307	0.0381	0.0488	0.0717
7.50E+05	0.0122	0.0139	0.0150	0.0175	0.0201	0.0238	0.0306	0.0380	0.0487	0.0716
1.00E+06	0.0116	0.0135	0.0148	0.0173	0.0200	0.0237	0.0305	0.0380	0.0487	0.0716
5.00E+06	0.0090	0.0124	0.0140	0.0168	0.0197	0.0235	0.0304	0.0379	0.0487	0.0716
1.00E+07	0.0081	0.0122	0.0139	0.0168	0.0197	0.0235	0.0304	0.0379	0.0486	0.0716
5.00E+07	0.0066	0.0120	0.0138	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0716
1.00E+08	0.0060	0.0120	0.0137	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0716

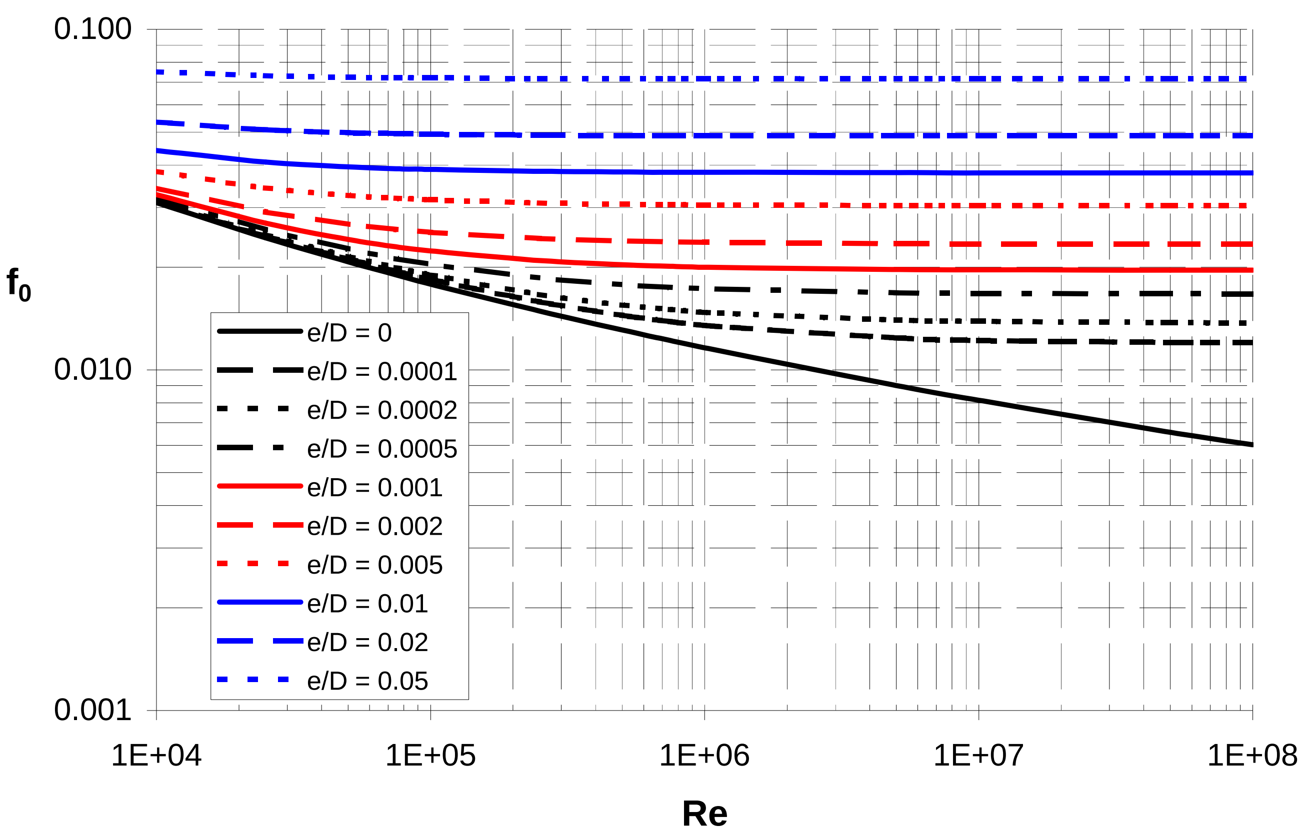
Using the add-in function *Friction factor* from the Web

e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.05
Re	f									
1.00E+04	0.0309	0.0310	0.0312	0.0316	0.0324	0.0338	0.0376	0.0431	0.0523	0.0738
2.50E+04	0.0245	0.0248	0.0250	0.0257	0.0268	0.0288	0.0337	0.0402	0.0502	0.0725
5.00E+04	0.0209	0.0212	0.0216	0.0226	0.0240	0.0265	0.0322	0.0391	0.0494	0.0720
7.50E+04	0.0191	0.0196	0.0200	0.0212	0.0228	0.0256	0.0316	0.0387	0.0492	0.0719
1.00E+05	0.0180	0.0185	0.0190	0.0203	0.0222	0.0251	0.0313	0.0385	0.0490	0.0718
2.50E+05	0.0150	0.0158	0.0166	0.0185	0.0208	0.0241	0.0308	0.0381	0.0488	0.0716
5.00E+05	0.0132	0.0144	0.0154	0.0177	0.0202	0.0238	0.0306	0.0380	0.0487	0.0716
7.50E+05	0.0122	0.0138	0.0150	0.0174	0.0200	0.0237	0.0305	0.0380	0.0487	0.0716
1.00E+06	0.0116	0.0134	0.0147	0.0172	0.0199	0.0236	0.0305	0.0380	0.0487	0.0716
5.00E+06	0.0090	0.0123	0.0139	0.0168	0.0197	0.0235	0.0304	0.0379	0.0486	0.0716
1.00E+07	0.0081	0.0122	0.0138	0.0168	0.0197	0.0234	0.0304	0.0379	0.0486	0.0716
5.00E+07	0.0065	0.0120	0.0138	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0716
1.00E+08	0.0059	0.0120	0.0137	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0716

The error can now be computed

e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.05
Re	**Error (%)**									
1.00E+04	0.29%	0.36%	0.43%	0.61%	0.88%	1.27%	1.86%	**2.12%**	2.08%	1.68%
2.50E+04	0.39%	0.24%	0.11%	0.21%	0.60%	1.04%	1.42%	1.41%	1.21%	0.87%
5.00E+04	0.63%	0.39%	0.19%	0.25%	0.67%	1.00%	1.11%	0.98%	0.77%	0.52%
7.50E+04	0.69%	0.38%	0.13%	0.35%	0.73%	0.95%	0.93%	0.77%	0.58%	0.38%
1.00E+05	0.71%	0.33%	0.06%	0.43%	0.76%	0.90%	0.81%	0.64%	0.47%	0.30%
2.50E+05	0.65%	0.04%	0.28%	0.64%	0.72%	0.66%	0.48%	0.35%	0.24%	0.14%
5.00E+05	0.52%	0.26%	0.51%	0.64%	0.59%	0.47%	0.31%	0.21%	0.14%	0.08%
7.50E+05	0.41%	0.41%	0.58%	0.59%	0.50%	0.37%	0.23%	0.15%	0.10%	0.06%
1.00E+06	0.33%	0.49%	0.60%	0.54%	0.43%	0.31%	0.19%	0.12%	0.08%	0.05%
5.00E+06	0.22%	0.51%	0.39%	0.24%	0.16%	0.10%	0.06%	0.03%	0.02%	0.01%
1.00E+07	0.49%	0.39%	0.27%	0.15%	0.10%	0.06%	0.03%	0.02%	0.01%	0.01%
5.00E+07	1.15%	0.15%	0.09%	0.05%	0.03%	0.02%	0.01%	0.01%	0.00%	0.00%
1.00E+08	1.44%	0.09%	0.06%	0.03%	0.02%	0.01%	0.00%	0.00%	0.00%	0.00%

The maximum discrepancy is 2.12% at *Re* = 10,000 and *e/D* = 0.01



Problem 8.82

8.82 We saw in Section 8-7 that instead of the implicit Colebrook equation (Eq. 8.37) for computing the turbulent friction factor f, an explicit expression that gives reasonable accuracy is

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{e/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$$

Compare the accuracy of this expression for f with Eq. 8.37 by computing the percentage discrepancy as a function of Re and e/D, for $Re = 10^4$ to 10^8, and $e/D = 0$, 0.0001, 0.001, 0.01, and 0.05. What is the maximum discrepancy for these Re and e/D values? Plot f against Re with e/D as a parameter.

Using the above formula for f_0, and Eq. 8.37 for f_1

e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.05

Re	f_0									
1.00E+04	0.0309	0.0310	0.0311	0.0315	0.0322	0.0335	0.0374	0.0430	0.0524	0.0741
2.50E+04	0.0244	0.0245	0.0248	0.0254	0.0265	0.0285	0.0336	0.0401	0.0502	0.0727
5.00E+04	0.0207	0.0210	0.0213	0.0223	0.0237	0.0263	0.0321	0.0391	0.0495	0.0722
7.50E+04	0.0189	0.0193	0.0197	0.0209	0.0226	0.0254	0.0316	0.0387	0.0492	0.0720
1.00E+05	0.0178	0.0183	0.0187	0.0201	0.0220	0.0250	0.0313	0.0385	0.0491	0.0719
2.50E+05	0.0148	0.0156	0.0164	0.0183	0.0207	0.0241	0.0308	0.0382	0.0489	0.0718
5.00E+05	0.0131	0.0143	0.0153	0.0176	0.0202	0.0238	0.0306	0.0381	0.0488	0.0717
7.50E+05	0.0122	0.0137	0.0148	0.0173	0.0200	0.0237	0.0305	0.0381	0.0488	0.0717
1.00E+06	0.0116	0.0133	0.0146	0.0172	0.0199	0.0236	0.0305	0.0380	0.0488	0.0717
5.00E+06	0.0090	0.0123	0.0139	0.0168	0.0197	0.0235	0.0304	0.0380	0.0487	0.0717
1.00E+07	0.0081	0.0122	0.0139	0.0168	0.0197	0.0235	0.0304	0.0380	0.0487	0.0717
5.00E+07	0.0066	0.0120	0.0138	0.0167	0.0197	0.0235	0.0304	0.0380	0.0487	0.0717
1.00E+08	0.0060	0.0120	0.0138	0.0167	0.0197	0.0235	0.0304	0.0380	0.0487	0.0717

Using the add-in function *Friction factor* from the Web

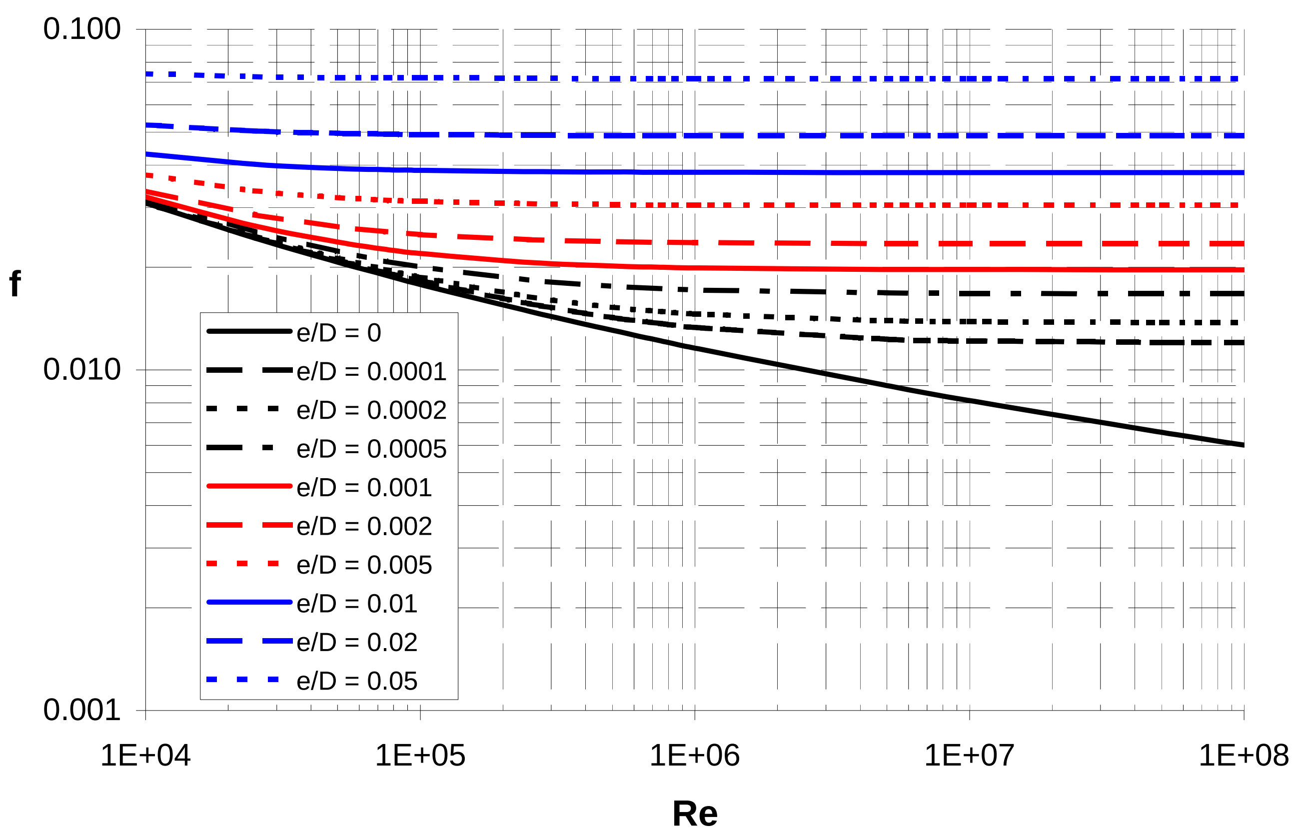
e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.05

Re	f									
1.00E+04	0.0309	0.0310	0.0312	0.0316	0.0324	0.0338	0.0376	0.0431	0.0523	0.0738
2.50E+04	0.0245	0.0248	0.0250	0.0257	0.0268	0.0288	0.0337	0.0402	0.0502	0.0725
5.00E+04	0.0209	0.0212	0.0216	0.0226	0.0240	0.0265	0.0322	0.0391	0.0494	0.0720
7.50E+04	0.0191	0.0196	0.0200	0.0212	0.0228	0.0256	0.0316	0.0387	0.0492	0.0719
1.00E+05	0.0180	0.0185	0.0190	0.0203	0.0222	0.0251	0.0313	0.0385	0.0490	0.0718
2.50E+05	0.0150	0.0158	0.0166	0.0185	0.0208	0.0241	0.0308	0.0381	0.0488	0.0716
5.00E+05	0.0132	0.0144	0.0154	0.0177	0.0202	0.0238	0.0306	0.0380	0.0487	0.0716
7.50E+05	0.0122	0.0138	0.0150	0.0174	0.0200	0.0237	0.0305	0.0380	0.0487	0.0716
1.00E+06	0.0116	0.0134	0.0147	0.0172	0.0199	0.0236	0.0305	0.0380	0.0487	0.0716
5.00E+06	0.0090	0.0123	0.0139	0.0168	0.0197	0.0235	0.0304	0.0379	0.0486	0.0716
1.00E+07	0.0081	0.0122	0.0138	0.0168	0.0197	0.0234	0.0304	0.0379	0.0486	0.0716
5.00E+07	0.0065	0.0120	0.0138	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0716
1.00E+08	0.0059	0.0120	0.0137	0.0167	0.0196	0.0234	0.0304	0.0379	0.0486	0.0716

The error can now be computed

e/D =	0	0.0001	0.0002	0.0005	0.001	0.002	0.005	0.01	0.02	0.05
Re	Error (%)									
1.00E+04	0.01%	0.15%	0.26%	0.46%	0.64%	0.73%	0.55%	0.19%	0.17%	0.43%
2.50E+04	0.63%	0.88%	1.02%	1.20%	1.22%	1.03%	0.51%	0.11%	0.14%	0.29%
5.00E+04	0.85%	1.19%	1.32%	1.38%	1.21%	0.84%	0.28%	0.00%	0.16%	0.24%
7.50E+04	0.90%	1.30%	1.40%	1.35%	1.07%	0.65%	0.16%	0.06%	0.17%	0.23%
1.00E+05	0.92%	1.34%	**1.42%**	1.28%	0.94%	0.52%	0.09%	0.09%	0.18%	0.22%
2.50E+05	0.84%	1.33%	1.25%	0.85%	0.47%	0.16%	0.07%	0.15%	0.19%	0.21%
5.00E+05	0.70%	1.16%	0.93%	0.48%	0.19%	0.00%	0.13%	0.18%	0.20%	0.20%
7.50E+05	0.59%	0.99%	0.72%	0.30%	0.07%	0.07%	0.16%	0.18%	0.20%	0.20%
1.00E+06	0.50%	0.86%	0.57%	0.20%	0.01%	0.10%	0.17%	0.19%	0.20%	0.20%
5.00E+06	0.07%	0.17%	0.01%	0.11%	0.15%	0.18%	0.19%	0.20%	0.20%	0.20%
1.00E+07	0.35%	0.00%	0.09%	0.15%	0.18%	0.19%	0.20%	0.20%	0.20%	0.20%
5.00E+07	1.02%	0.16%	0.18%	0.19%	0.20%	0.20%	0.20%	0.20%	0.20%	0.20%
1.00E+08	1.31%	0.18%	0.19%	0.20%	0.20%	0.20%	0.20%	0.20%	0.20%	0.20%

The maximum discrepancy is 1.42% at *Re* = 100,000 and *e/D* = 0.0002



Problem 8.83

[2]

Given: Moody diagram gives <u>Darcy</u> friction factor, f.

<u>Fanning</u> friction factor is $f_F \equiv \dfrac{\tau_w}{\frac{1}{2}\rho \bar{V}^2}$

Find: Relate Darcy and Fanning friction factors for fully developed pipe flow. Show $f = 4f_F$.

<u>Solution</u>: Consider cylindrical CV containing fluid in pipe; apply force balance, definition of f.

Basic equations: $\sum F_x = 0$

$$\Delta p = f \frac{L}{D} \rho \frac{\bar{V}^2}{2}$$

From the force balance,

$$(p+\Delta p)\frac{\pi D^2}{4} - \tau_w \pi D L - p\frac{\pi D^2}{4} = 0 \quad \text{or} \quad \tau_w = \frac{D}{4}\frac{\Delta p}{L}$$

Substituting,

$$\tau_w = \frac{D}{4L} f \frac{L}{D} \rho \frac{\bar{V}^2}{2} = f \frac{\rho \bar{V}^2}{8}$$

But

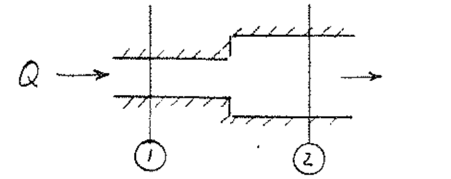
$$f_F \equiv \frac{\tau_w}{\frac{1}{2}\rho \bar{V}^2} = \frac{f \rho \bar{V}^2}{8} \frac{2}{\rho \bar{V}^2} = \frac{f}{4}$$

f_F

Problem 8.84

Given: Water flow through sudden enlargement from 25 mm to 50 mm diameter. $Q = 1.25$ liters per minute.

Find: Pressure rise across enlargement.
Comparison with value for frictionless flow.



Solution: Apply energy equation for pipe flow.

$D_1 = 25\text{ mm}$ $D_2 = 50\text{ mm}$

Computing equation: $$\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2 + h_{l_T}$$

$$h_{l_T} = K \frac{\bar{V}_1^2}{2}$$

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) Uniform flow at each section: $\alpha_1 = \alpha_2 = 1$
(4) Horizontal section

Then

$$p_2 - p_1 = \frac{\rho}{2}\left(\bar{V}_1^2 - \bar{V}_2^2\right) - \rho h_{l_{T_{12}}}$$

From continuity, $\bar{V}_1 A_1 = \bar{V}_2 A_2$, so $\bar{V}_2 = \bar{V}_1 \frac{A_1}{A_2} = \bar{V}_1\left(\frac{D_1}{D_2}\right)^2$; $\bar{V}_2^2 = \bar{V}_1^2\left(\frac{D_1}{D_2}\right)^4$

From Fig. 8.14, at $AR = \left(\frac{D_1}{D_2}\right)^2 = \frac{1}{4}$, $K = 0.56$.

$$\bar{V}_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4}{\pi} \times 1.25 \frac{\text{L}}{\text{s}} \times 10^{-3} \frac{\text{m}^3}{\text{L}} \times \frac{1}{(25 \times 10^{-3})^2 \text{m}^2} = 2.55 \text{ m/s}$$

Substituting,

$$p_2 - p_1 = \frac{\rho \bar{V}_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right] - K \frac{\rho \bar{V}_1^2}{2} = \frac{1}{2}\rho \bar{V}_1^2\left[1 - \left(\frac{D_1}{D_2}\right)^4 - K\right]$$

$$= \frac{1}{2} \times 999 \frac{\text{kg}}{\text{m}^3} \times (2.55)^2 \frac{\text{m}^2}{\text{s}^2}\left[1 - \left(\frac{1}{2}\right)^4 - 0.56\right] \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$$p_2 - p_1 = 1.22 \text{ kPa}$$ Δp_a

For frictionless flow, $K = 0$, and

$$p_2 - p_1 = \frac{1}{2}\rho \bar{V}_1^2\left[1 - \left(\frac{D_1}{D_2}\right)^4\right] = 3.04 \text{ kPa}$$ Δp_f

Thus $$\frac{\Delta p_{actual}}{\Delta p_{frictionless}} = \frac{1.22}{3.04} = 0.403 \text{ or } 40.3\%$$ Ratio

Problem 8.85

8.85 Water flows at 0.003 mm^3/s through a gradual contraction, in which the pipe diameter is reduced from 5 cm to 2.5 cm, with a 120° included angle. If the pressure before the contraction is 200 kPa, estimate the pressure after the contraction. Recompute the answer if the included angle is changed to 180° (a sudden contraction).

Given: Flow through gradual contraction

Find: Pressure after contraction; compare to sudden contraction

Solution:

Basic equations
$$\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_{lm} \qquad h_{lm} = K\cdot\frac{V_2^2}{2} \qquad Q = V\cdot A$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Horizontal

For an included angle of 120^o and an area ratio $\frac{A_2}{A_1} = \left(\frac{D_2}{D_1}\right)^2 = \left(\frac{2.5}{5}\right)^2 = 0.25$ we find from Table 8.. $K = 0.27$

Hence the energy equation becomes $\left(\frac{p_1}{\rho} + \frac{V_1^2}{2}\right) - \left(\frac{p_2}{\rho} + \frac{V_2^2}{2}\right) = K\cdot\frac{V_2^2}{2}$ with $V_1 = \frac{4\cdot Q}{\pi\cdot D_1^2}$ $\quad V_2 = \frac{4\cdot Q}{\pi\cdot D_2^2}$

$$p_2 = p_1 - \frac{\rho}{2}\cdot\left[(1+K)\cdot V_2^2 - V_1^2\right] = p_2 - \frac{8\cdot\rho\cdot Q^2}{\pi^2}\cdot\left[\frac{(1+K)}{D_2^4} - \frac{1}{D_1^4}\right]$$

$$p_2 = 200\times10^3\cdot\frac{N}{m^2} - \frac{8}{\pi^2}\times1000\cdot\frac{kg}{m^3}\times\left[\frac{0.003\cdot mm^3}{s}\cdot\left(\frac{1\cdot m}{1000\cdot mm}\right)^3\right]^2\times\left[(1+0.27)\times\frac{1}{(0.025\cdot m)^4} - \frac{1}{(0.05\cdot m)^4}\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$p_2 = 200\cdot kPa$ No change because the flow rate is miniscule!

Repeating the above analysis for an included angle of 180º (sudden contraction) $K = 0.41$

$$p_2 = 200\times10^3\cdot\frac{N}{m^2} - \frac{8}{\pi^2}\times1000\cdot\frac{kg}{m^3}\times\left[\frac{0.003\cdot mm^3}{s}\cdot\left(\frac{1\cdot m}{1000\cdot mm}\right)^3\right]^2\times\left[(1+0.41)\times\frac{1}{(0.025\cdot m)^4} - \frac{1}{(0.05\cdot m)^4}\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$p_2 = 200\cdot kPa$ No change because the flow rate is miniscule!

The flow rate has a typo: it is much too small, and should be $Q = 0.003\cdot\frac{m^3}{s}$ not $Q = 0.003\cdot\frac{mm^3}{s}$

$$p_2 = 200\times10^3\cdot\frac{N}{m^2} - \frac{8}{\pi^2}\times1000\cdot\frac{kg}{m^3}\times\left(\frac{0.003\cdot m^3}{s}\right)^2\times\left[(1+0.27)\times\frac{1}{(0.025\cdot m)^4} - \frac{1}{(0.05\cdot m)^4}\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$p_2 = 177\cdot kPa$

Repeating the above analysis for an included angle of 180º (sudden contraction) $K = 0.41$

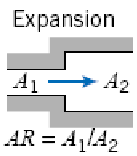
$$p_2 = 200\times10^3\cdot\frac{N}{m^2} - \frac{8}{\pi^2}\times1000\cdot\frac{kg}{m^3}\times\left(\frac{0.003\cdot m^3}{s}\right)^2\times\left[(1+0.41)\times\frac{1}{(0.025\cdot m)^4} - \frac{1}{(0.05\cdot m)^4}\right]\times\frac{N\cdot s^2}{kg\cdot m}$$

$p_2 = 175\cdot kPa$

There is slightly more loss in the sudden contraction

Problem 8.86

8.86 Air at standard conditions flows through a sudden expansion in a circular duct. The upstream and downstream duct diameters are 75 mm and 225 mm, respectively. The pressure downstream is 5 mm of water higher than that upstream. Determine the average speed of the air approaching the expansion and the volume flow rate.



Given: Flow through sudden expansion

Find: Inlet speed; Volume flow rate

Solution:

Basic equations $\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_{lm}$ $\quad h_{lm} = K\cdot\frac{V_1^2}{2}$ $\quad Q = V\cdot A$ $\quad \Delta p = \rho_{H2O}\cdot g\cdot\Delta h$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Horizontal

Hence the energy equation becomes

$$\left(\frac{p_1}{\rho} + \frac{V_1^2}{2}\right) - \left(\frac{p_2}{\rho} + \frac{V_2^2}{2}\right) = K\cdot\frac{V_1^2}{2}$$

From continuity $V_2 = V_1\cdot\frac{A_1}{A_2} = V_1\cdot AR$

Hence

$$\left(\frac{p_1}{\rho} + \frac{V_1^2}{2}\right) - \left(\frac{p_2}{\rho} + \frac{V_1^2\cdot AR^2}{2}\right) = K\cdot\frac{V_1^2}{2}$$

Solving for V_1 $\quad V_1 = \sqrt{\frac{2\cdot(p_2 - p_1)}{\rho\cdot(1 - AR^2 - K)}}$ $\quad AR = \left(\frac{D_1}{D_2}\right)^2 = \left(\frac{75}{225}\right)^2 = 0.111$ $\quad$ so from Fig. 8.14 $\quad K = 0.8$

Also

$$p_2 - p_1 = \rho_{H2O}\cdot g\cdot\Delta h = 1000\cdot\frac{kg}{m^3} \times 9.81\cdot\frac{m}{s^2} \times \frac{5}{1000}\cdot m \times \frac{N\cdot s^2}{kg\cdot m} = 49.1\cdot Pa$$

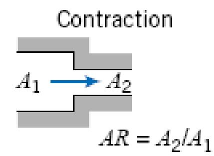
Hence

$$V_1 = \sqrt{2 \times 49.1\cdot\frac{N}{m^2} \times \frac{m^3}{1.23\cdot kg} \times \frac{1}{\left(1 - 0.111^2 - 0.8\right)} \times \frac{kg\cdot m}{N\cdot s^2}} \qquad V_1 = 20.6\frac{m}{s}$$

$$Q = V_1\cdot A_1 = \frac{\pi\cdot D_1^2}{4}\cdot V_1 \qquad Q = \frac{\pi}{4} \times \left(\frac{75}{1000}\cdot m\right)^2 \times 20.6\cdot\frac{m}{s} \qquad Q = 0.0910\cdot\frac{m^3}{s} \qquad Q = 5.46\cdot\frac{m^3}{min}$$

Problem 8.87

8.87 Water flows through a 2-in. diameter tube that suddenly contracts to 1 in. diameter. The pressure drop across the contraction is 0.5 psi. Determine the volume flow rate.



Given: Flow through sudden contraction

Find: Volume flow rate

Solution:

Basic equations $\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_{lm}$ $\quad h_{lm} = K\cdot\frac{V_2^2}{2}$ $\quad Q = V\cdot A$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Horizontal

Hence the energy equation becomes

$$\left(\frac{p_1}{\rho} + \frac{V_1^2}{2}\right) - \left(\frac{p_2}{\rho} + \frac{V_2^2}{2}\right) = K\cdot\frac{V_2^2}{2}$$

From continuity $V_1 = V_2\cdot\frac{A_2}{A_1} = V_2\cdot AR$

Hence $$\left(\frac{p_1}{\rho} + \frac{V_2^2\cdot AR^2}{2}\right) - \left(\frac{p_2}{\rho} + \frac{V_2^2}{2}\right) = K\cdot\frac{V_2^2}{2}$$

Solving for V_2 $\quad V_2 = \sqrt{\frac{2\cdot(p_1 - p_2)}{\rho\cdot(1 - AR^2 + K)}}$ $\quad AR = \left(\frac{D_2}{D_1}\right)^2 = \left(\frac{1}{2}\right)^2 = 0.25$ $\quad$ so from Fig. 8.14 $\quad K = 0.4$

Hence $$V_2 = \sqrt{2 \times 0.5\cdot\frac{lbf}{in^2} \times \left(\frac{12\cdot in}{1\cdot ft}\right)^2 \times \frac{ft^3}{1.94\cdot slug} \times \frac{1}{\left(1 - 0.25^2 + 0.4\right)} \times \frac{slug\cdot ft}{lbf\cdot s^2}} \qquad V_2 = 7.45\cdot\frac{ft}{s}$$

$$Q = V_2\cdot A_2 = \frac{\pi\cdot D_2^2}{4}\cdot V_2 \qquad Q = \frac{\pi}{4} \times \left(\frac{1}{12}\cdot ft\right)^2 \times 7.45\cdot\frac{ft}{s} \qquad Q = 0.0406\cdot\frac{ft^3}{s} \qquad Q = 2.44\cdot\frac{ft^3}{min} \qquad Q = 18.2\,gpm$$

Problem 8.88

8.88 In an undergraduate laboratory you have been assigned the task of developing a crude flow meter for measuring the flow in a 400-mm diameter water pipe system. You are to install a 200-mm diameter section of pipe, and a water manometer to measure the pressure drop at the sudden contraction. Derive an expression for the theoretical calibration constant k in $Q = k\sqrt{\Delta h}$, where Q is the volume flow rate in L/min, and Δh is the manometer deflection in mm. Plot the theoretical calibration curve for a flow rate range of 0 to 200 L/min. Would you expect this to be a practical device for measuring flow rate?

Given: Data on a pipe sudden contraction

Find: Theoretical calibration constant; plot

Solution:

Given data $D_1 = 400 \cdot mm$ $D_2 = 200 \cdot mm$

The governing equations between inlet (1) and exit (2) are

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_l \qquad (8.29)$$

where

$$h_l = K \cdot \frac{V_2^2}{2} \qquad (8.40a)$$

Hence the pressure drop is (assuming $\alpha = 1$)

$$\Delta p = p_1 - p_2 = \rho \cdot \left(\frac{V_2^2}{2} - \frac{V_1^2}{2} + K \cdot \frac{V_2^2}{2}\right)$$

For the sudden contraction

$$V_1 \cdot \frac{\pi}{4} \cdot D_1^2 = V_2 \cdot \frac{\pi}{4} \cdot D_2^2 = Q$$

or

$$V_2 = V_1 \cdot \left(\frac{D_1}{D_2}\right)^2$$

so

$$\Delta p = \frac{\rho \cdot V_1^2}{2} \cdot \left[\left(\frac{D_1}{D_2}\right)^4 (1 + K) - 1\right]$$

For the pressure drop we can use the manometer equation

$$\Delta p = \rho \cdot g \cdot \Delta h$$

Hence

$$\rho \cdot g \cdot \Delta h = \frac{\rho \cdot V_1^2}{2} \cdot \left[\left(\frac{D_1}{D_2}\right)^4 (1 + K) - 1\right]$$

In terms of flow rate *Q*

$$\rho \cdot g \cdot \Delta h = \frac{\rho}{2} \cdot \frac{Q^2}{\left(\frac{\pi}{4} \cdot D_1^2\right)^2} \cdot \left[\left(\frac{D_1}{D_2}\right)^4 (1 + K) - 1\right]$$

or

$$g \cdot \Delta h = \frac{8 \cdot Q^2}{\pi^2 \cdot D_1^4} \cdot \left[\left(\frac{D_1}{D_2}\right)^4 (1 + K) - 1\right]$$

Hence for flow rate *Q* we find

$$Q = k \cdot \sqrt{\Delta h}$$

where

$$k = \sqrt{\frac{g \cdot \pi^2 \cdot D_1^4}{8 \cdot \left[\left(\frac{D_1}{D_2}\right)^4 (1 + K) - 1\right]}}$$

For *K*, we need the aspect ratio *AR*

$$AR = \left(\frac{D_2}{D_1}\right)^2 \qquad AR = 0.25$$

From Fig. 8.15

$$K = 0.4$$

Using this in the expression for *k*, with the other given values

$$k = \sqrt{\frac{g \cdot \pi^2 \cdot D_1^4}{8 \cdot \left[\left(\frac{D_1}{D_2}\right)^4 (1 + K) - 1\right]}} = 0.12 \cdot \frac{m^{\frac{5}{2}}}{s}$$

For Δh in mm and *Q* in L/min

$$k = 228 \frac{\frac{L}{min}}{mm^{\frac{1}{2}}}$$

The plot of theoretical *Q* versus flow rate Δh is shown in the associated *Excel* workbook

8.88 In an undergraduate laboratory you have been assigned the task of developing a crude flow meter for measuring the flow in a 400-mm diameter water pipe system. You are to install a 200-mm diameter section of pipe, and a water manometer to measure the pressure drop at the sudden contraction. Derive an expression for the theoretical calibration constant k in $Q = k\sqrt{\Delta h}$, where Q is the volume flow rate in L/min, and Δh is the manometer deflection in mm. Plot the theoretical calibration curve for a flow rate range of 0 to 200 L/min. Would you expect this to be a practical device for measuring flow rate?

Given: Data on a pipe sudden contraction

Find: Theoretical calibration constant; plot

Solution:

$D_1 =$ 400 mm

$D_1 =$ 200 mm

$K =$ 0.4

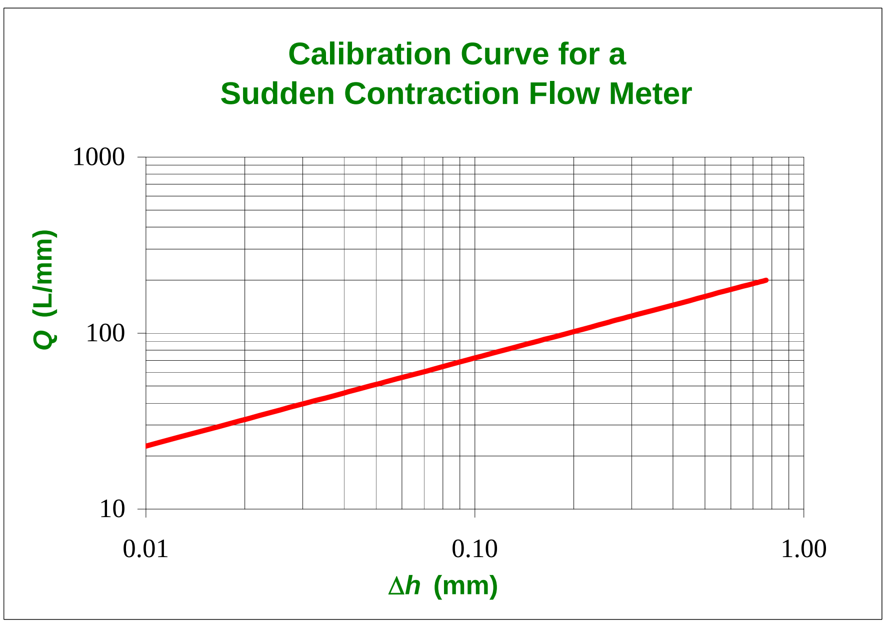
$k =$ 228 L/min/mm$^{1/2}$

$$Q = k\cdot\sqrt{\Delta h}$$

$$k = \sqrt{\frac{g\cdot\pi^2\cdot D_1^{\,4}}{8\cdot\left[\left(\frac{D_1}{D_2}\right)^4\cdot(1+K) - 1\right]}}$$

The values for Δh are quite low; this would not be a good meter - it is not sensitive enough. In addition, it is non-linear.

Δh (mm)	Q (L/min)
0.010	23
0.020	32
0.030	40
0.040	46
0.050	51
0.075	63
0.100	72
0.150	88
0.200	102
0.250	114
0.300	125
0.400	144
0.500	161
0.600	177
0.700	191
0.767	200

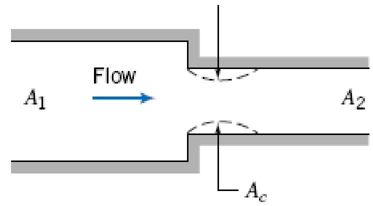


Problem 8.89

8.89 Flow through a sudden contraction is shown. The minimum flow area at the vena contracta is given in terms of the area ratio by the contraction coefficient [31],

$$C_c = \frac{A_c}{A_2} = 0.62 + 0.38\left(\frac{A_2}{A_1}\right)^3$$

The loss in a sudden contraction is mostly a result of the vena contracta: The fluid accelerates into the contraction, there is flow separation (as shown by the dashed lines), and the vena contracta acts as a miniature sudden expansion with significant secondary flow losses. Use these assumptions to obtain and plot estimates of the minor loss coefficient for a sudden contraction, and compare with the data presented in Fig. 8.15.



Given: Contraction coefficient for sudden contraction

Find: Expression for minor head loss; compare with Fig. 8.15; plot

Solution:

We analyse the loss at the "sudden expansion" at the vena contracta

The governing CV equations (mass, momentum, and energy) are

$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad (4.12)$$

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A} \qquad (4.18a)$$

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\,\rho\, d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A} \qquad (4.56)$$

Assume: 1) Steady flow 2) Incompressible flow 3) Uniform flow at each section 4) Horizontal: no body force 5) No shaft work 6) Neglect viscous friction 7) Neglect gravity

The mass equation becomes $\quad V_c\cdot A_c = V_2\cdot A_2 \qquad (1)$

The momentum equation becomes $\quad p_c\cdot A_2 - p_2\cdot A_2 = V_c\cdot\left(-\rho\cdot V_c\cdot A_c\right) + V_2\cdot\left(\rho\cdot V_2\cdot A_2\right)$

or (using Eq. 1) $\quad p_c - p_2 = \rho\cdot V_c\cdot\frac{A_c}{A_2}\cdot\left(V_2 - V_c\right) \qquad (2)$

The energy equation becomes $\quad Q_{rate} = \left(u_c + \frac{p_c}{\rho} + V_c^{\,2}\right)\cdot\left(-\rho\cdot V_c\cdot A_c\right) + \left(u_2 + \frac{p_2}{\rho} + V_2^{\,2}\right)\cdot\left(\rho\cdot V_2\cdot A_2\right)$

or (using Eq. 1) $\quad h_{lm} = u_2 - u_c - \frac{Q_{rate}}{m_{rate}} = \frac{V_c^{\,2} - V_2^{\,2}}{2} + \frac{p_c - p_2}{\rho} \qquad (3)$

Combining Eqs. 2 and 3

$$h_{lm} = \frac{V_c^2 - V_2^2}{2} + V_c \cdot \frac{A_c}{A_2} \cdot (V_2 - V_c)$$

$$h_{lm} = \frac{V_c^2}{2} \cdot \left[1 - \left(\frac{V_2}{V_c}\right)^2\right] + V_c^2 \cdot \frac{A_c}{A_2} \cdot \left[\left(\frac{V_2}{V_c}\right) - 1\right]$$

From Eq. 1

$$C_c = \frac{A_c}{A_2} = \frac{V_2}{V_c}$$

Hence

$$h_{lm} = \frac{V_c^2}{2} \cdot \left(1 - C_c^2\right) + V_c^2 \cdot C_c \cdot \left(C_c - 1\right)$$

$$h_{lm} = \frac{V_c^2}{2} \cdot \left(1 - C_c^2 + 2 \cdot C_c^2 - 2 \cdot C_c\right)$$

$$h_{lm} = \frac{V_c^2}{2} \cdot \left(1 - C_c\right)^2 \qquad (4)$$

But we have

$$h_{lm} = K \cdot \frac{V_2^2}{2} = K \cdot \frac{V_c^2}{2} \cdot \left(\frac{V_2}{V_c}\right)^2 = K \cdot \frac{V_c^2}{2} \cdot C_c^2 \qquad (5)$$

Hence, comparing Eqs. 4 and 5

$$K = \frac{\left(1 - C_c\right)^2}{C_c^2}$$

So, finally

$$K = \left(\frac{1}{C_c} - 1\right)^2$$

where

$$C_c = 0.62 + 0.38 \cdot \left(\frac{A_2}{A_1}\right)^3$$

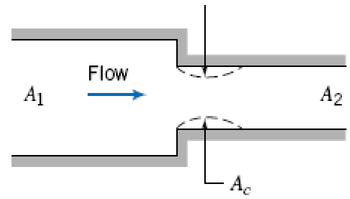
This result, and the curve of Fig. 8.15, are shown in the associated *Excel* workbook. The agreement is reasonable.

Problem 8.89

8.89 Flow through a sudden contraction is shown. The minimum flow area at the vena contracta is given in terms of the area ratio by the contraction coefficient [31],

$$C_c = \frac{A_c}{A_2} = 0.62 + 0.38\left(\frac{A_2}{A_1}\right)^3$$

The loss in a sudden contraction is mostly a result of the vena contracta: The fluid accelerates into the contraction, there is flow separation (as shown by the dashed lines), and the vena contracta acts as a miniature sudden expansion with significant secondary flow losses. Use these assumptions to obtain and plot estimates of the minor loss coefficient for a sudden contraction, and compare with the data presented in Fig. 8.15.



Given: Contraction coefficient for sudden contraction

Find: Expression for minor head loss; compare with Fig. 8.15; plot

Solution:

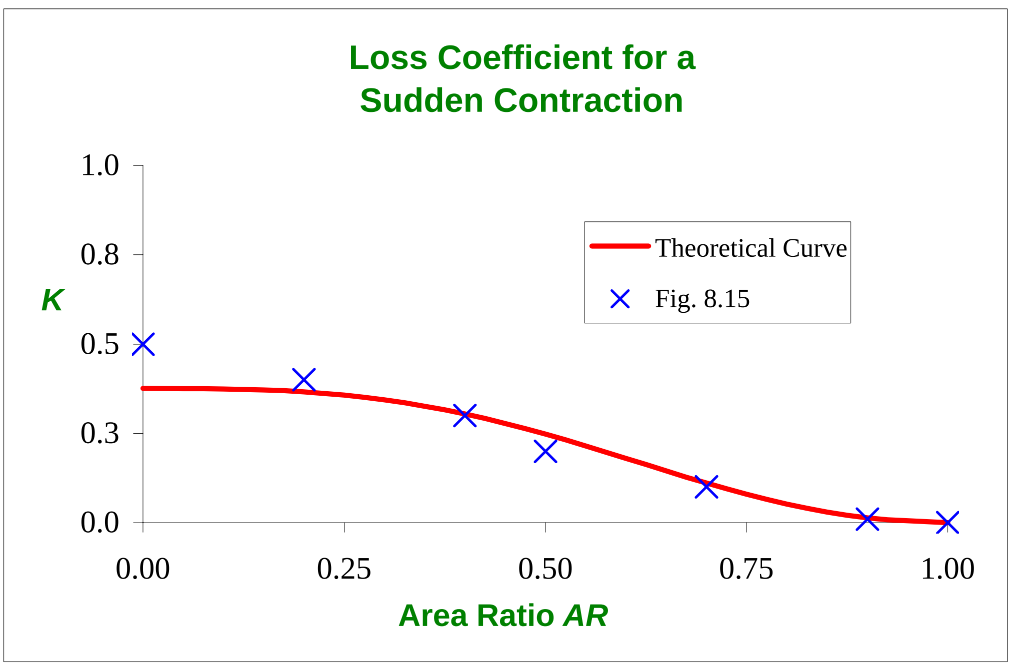
The CV analysis le

$$K = \left(\frac{1}{C_c} - 1\right)^2$$

$$C_c = 0.62 + 0.38 \cdot \left(\frac{A_2}{A_1}\right)^3$$

A_2/A_1	K_{CV}	$K_{\text{Fig. 8.15}}$
0.0	0.376	0.50
0.1	0.374	
0.2	0.366	0.40
0.3	0.344	
0.4	0.305	0.30
0.5	0.248	0.20
0.6	0.180	
0.7	0.111	0.10
0.8	0.052	
0.9	0.013	0.01
1.0	0.000	0.00

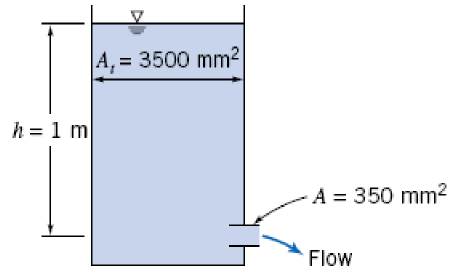
(Data from Fig. 8.15 is "eyeballed")
Agreement is reasonable



Problem 8.90

[2]

8.90 Water flows from the tank shown through a very short pipe. Assume the flow is quasi-steady. Estimate the flow rate at the instant shown. How could you improve the flow system if a larger flow rate were desired?



Given: Flow through short pipe

Find: Volume flow rate; How to improve flow rate

Solution:

Basic equations $\left(\frac{p_1}{\rho} + \alpha \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_{lT}$ $\quad h_{lT} = h_l + h_{lm} = f \cdot \frac{L}{D} \cdot \frac{V_2^2}{2} + K \cdot \frac{V_2^2}{2}$ $\quad Q = V \cdot A$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) L << so ignore h_l 5) Reentrant

Hence between the free surface (Point 1) and the exit (2) the energy equation becomes

$$\frac{V_1^2}{2} + g \cdot z_1 - \frac{V_2^2}{2} = K \cdot \frac{V_2^2}{2}$$

From continuity $V_1 = V_2 \cdot \frac{A_2}{A_1}$

Hence $\frac{V_2^2}{2} \cdot \left(\frac{A_2}{A_1}\right)^2 + g \cdot h - \frac{V_2^2}{2} = K \cdot \frac{V_2^2}{2}$

Solving for V_2 $\quad V_2 = \sqrt{\frac{2 \cdot g \cdot h}{\left[1 + K - \left(\frac{A_2}{A_1}\right)^2\right]}}$ $\quad$ and from Table 8.2 $\quad K = 0.78$

Hence $\quad V_2 = \sqrt{2 \times 9.81 \cdot \frac{m}{s^2} \times 1 \cdot m \times \frac{1}{\left[1 + 0.78 - \left(\frac{350}{3500}\right)^2\right]}}$ $\quad V_2 = 3.33 \frac{m}{s}$

$Q = V_2 \cdot A_2 \quad Q = 3.33 \cdot \frac{m}{s} \times 350 \cdot mm^2 \times \left(\frac{1 \cdot m}{1000 \cdot mm}\right)^2 \quad Q = 1.17 \times 10^{-3} \frac{m^3}{s} \quad Q = 0.070 \frac{m^3}{min}$

The flow rate could be increased by (1) rounding the entrance and/or (2) adding a diffuser (both somewhat expensive)

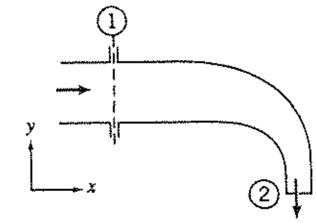
Problem 8.91

Given: Consider again flow through the elbow analyzed in Example Problem 4.6

$p_1 = 221\ kPa \qquad A_1 = 0.01\ m^2$

$V_2 = 16\ m/s \qquad A_2 = 0.0025\ m^2$

$p_2 = p_{atm}$



Find: Minor head loss coefficient for the elbow

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) = h_{l_T} = h_l + h_{l_m}$ (with z terms crossed out, $p_2 = 0$ (4), $h_l = 0$)

Assumptions: (1) $\alpha_1 = \alpha_2 = 1$

(2) neglect Δz

(3) uniform, incompressible flow so $\bar{V}_1 A_1 = \bar{V}_2 A_2$

(4) use gage pressures

From continuity $\bar{V}_1 = \bar{V}_2 \frac{A_2}{A_1} = 16\ \frac{m}{s} \times \frac{0.0025\ m^2}{0.01\ m^2} = 4\ m/s$

Then

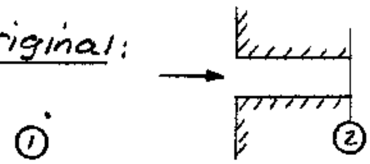
$$h_{l_m} = \frac{p_{1g}}{\rho} + \frac{\bar{V}_1^2}{2} - \frac{\bar{V}_2^2}{2} = (221-101)10^3\ \frac{N}{m^2} \times \frac{m^3}{999\ kg} \times \frac{kg \cdot m}{N \cdot s^2} + \frac{1}{2}\left[(4)^2 - (16)^2\right]\frac{m^2}{s^2}$$

$h_{l_m} = 0.120\ m^2/s^2$

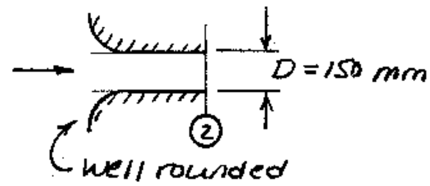
But $h_{l_m} = K \frac{\bar{V}_2^2}{2}$; $K = \frac{2 h_{l_m}}{\bar{V}_2^2} = 2 \times 0.120\ \frac{m^2}{s^2} \times \frac{s^2}{(16)^2\ m^2} = 9.38 \times 10^{-4}$ $\qquad K$

Given: Air flow from a clean room through a duct of 150 mm diameter.

Original:



Modified:



$h_1 - h_2 = 2.5 \text{ mm } H_2O$

Friction losses negligible, compared to inlet and exit losses.

Find: Increase in volume flow rate for modified duct.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equations:

$$\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2 + h_{l_T} \quad (\bar{V}_1 \approx 0\ (1);\ z \text{ terms cancel})$$

$$h_{l_T} = h_l + h_{l_m}\ (h_l \approx 0\ (4))\ ; \quad h_{l_m} = K_{ent} \frac{\bar{V}_2^2}{2}\ ; \quad \Delta p = \rho_{H_2O}\, g\, \Delta h$$

Assumptions: (1) $\bar{V}_1 \approx 0$ (3) Uniform flow at exit
(2) Neglect elevation changes (4) Neglect frictional losses

Then

$$\frac{\Delta p}{\rho} = \frac{p_1 - p_2}{\rho} = \frac{\bar{V}_2^2}{2} + K_{ent}\frac{\bar{V}_2^2}{2} = \frac{\bar{V}_2^2}{2}(1 + K_{ent}) = \frac{\rho_{H_2O}\, g\, \Delta h}{\rho}$$

or

$$\bar{V}_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho(1 + K_{ent})}} = \sqrt{\frac{2 \rho_{H_2O}\, g\, \Delta h}{\rho(1 + K_{ent})}}$$

From Table 8.2, $K_{ent} = 0.5$ for square-edged, $K_{ent} = 0.04$ for rounded entrance.

$$\bar{V}_2 = \sqrt{\frac{2}{1.50} \times 999 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{s}^2} \times 0.0025 \text{ m} \times \frac{\text{m}^3}{1.23 \text{ kg}}} = 5.15 \text{ m/s}$$

$$\bar{V}_2 (\text{modified}) = \sqrt{\frac{2}{1.04} \times 999 \frac{\text{kg}}{\text{m}^3} \times 9.81 \frac{\text{m}}{\text{s}^2} \times 0.0025 \text{ m} \times \frac{\text{m}^3}{1.23 \text{ kg}}} = 6.19 \text{ m/s}$$

Since $Q = \bar{V} A$, then

$$\Delta Q = (\bar{V}_{2m} - \bar{V}_2) A = (6.19 - 5.15) \frac{\text{m}}{\text{s}} \times \frac{\pi}{4} (0.150)^2 \text{m}^2 = 0.0184 \text{ m}^3/\text{s}$$ ΔQ

The percentage improvement is

$$\% = \frac{\Delta Q}{Q} \times 100 = \frac{\bar{V}_{2m} - \bar{V}_2}{\bar{V}_2} \times 100 = \frac{6.19 - 5.15}{5.15} \times 100 = 20.2 \text{ percent}$$

Problem 8.93

[3]

8.93 A water tank (open to the atmosphere) contains water to a depth of 10 ft. A $\frac{1}{2}$-in. diameter hole is punched in the bottom. Modeling the hole as square-edged, estimate the flow rate (gpm) exiting the tank. If you were to stick a short section of pipe into the hole, by how much would the flow rate change? If instead you were to machine the inside of the hole to give it a rounded edge ($r = 0.01$ in.), by how much would the flow rate change?

Given: Flow out of water tank

Find: Volume flow rate using hole; Using short pipe section; Using rounded edge

Solution:

Basic equations
$$\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^{\,2}}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^{\,2}}{2} + g\cdot z_2\right) = h_{lT} \qquad h_{lT} = h_l + h_{lm} = f\cdot\frac{L}{D}\cdot\frac{V_2^{\,2}}{2} + K\cdot\frac{V_2^{\,2}}{2} \qquad Q = V\cdot A$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) V_l << 5) L << so $h_l = 0$

Hence for all three cases, between the free surface (Point 1) and the exit (2) the energy equation becomes

$$g\cdot z_1 - \frac{V_2^{\,2}}{2} = K\cdot\frac{V_2^{\,2}}{2}$$

Solving for V_2
$$V_2 = \sqrt{\frac{2\cdot g\cdot h}{(1+K)}}$$

From Table 8.2 $K_{hole} = 0.5$ for a hole (assumed to be square-edged) $K_{pipe} = 0.78$ for a short pipe (rentrant)

Also, for a rounded edge $\frac{r}{D} = \frac{0.01\cdot in}{0.5\cdot in} = 0.02$ so from Table 8.2 $K_{round} = 0.28$

Hence for the hole
$$V_2 = \sqrt{2\times 32.2\cdot\frac{ft}{s^2}\times 10\cdot ft\times\frac{1}{(1+0.5)}} \qquad V_2 = 20.7\cdot\frac{ft}{s}$$

$$Q = V_2\cdot A_2 \qquad Q = 20.7\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{0.5}{12}\cdot ft\right)^2\times\frac{7.48\cdot gal}{1\cdot ft^3}\times\frac{60\cdot s}{1\cdot min} \qquad Q = 12.7\cdot gpm$$

Hence for the pipe
$$V_2 = \sqrt{2\times 32.2\cdot\frac{ft}{s^2}\times 10\cdot ft\times\frac{1}{(1+0.78)}} \qquad V_2 = 19.0\cdot\frac{ft}{s}$$

$$Q = V_2\cdot A_2 \qquad Q = 19.0\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{0.5}{12}\cdot ft\right)^2\times\frac{7.48\cdot gal}{1\cdot ft^3}\times\frac{60\cdot s}{1\cdot min} \qquad Q = 11.6\cdot gpm$$

Hence the change in flow rate is $11.6 - 12.7 = -1.1\cdot gpm$ The pipe leads to a LOWER flow rate

Hence for the rounded
$$V_2 = \sqrt{2\times 32.2\cdot\frac{ft}{s^2}\times 10\cdot ft\times\frac{1}{(1+0.28)}} \qquad V_2 = 22.4\cdot\frac{ft}{s}$$

$$Q = V_2\cdot A_2 \qquad Q = 22.4\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{0.5}{12}\cdot ft\right)^2\times\frac{7.48\cdot gal}{1\cdot ft^3}\times\frac{60\cdot s}{1\cdot min} \qquad Q = 13.7\cdot gpm$$

Hence the change in flow rate is $13.7 - 12.7 = 1.0\cdot gpm$ The rounded edge leads to a HIGHER flow rate

Problem 8.94

[2]

8.94 A conical diffuser is used to expand a pipe flow from a diameter of 100 mm to a diameter of 150 mm. Find the minimum length of the diffuser if we want a loss coefficient (a) $K_{\text{diffuser}} \leq 0.2$, (b) $K_{\text{diffuser}} \leq 0.35$.

Given: Data on inlet and exit diameters of diffuser

Find: Minimum lengths to satisfy requirements

Solution:

Given data $\quad D_1 = 100 \cdot mm \quad D_2 = 150 \cdot mm$

The governing equations for the diffuser are

$$h_{lm} = K \cdot \frac{V_1^2}{2} = \left(C_{pi} - C_p\right) \cdot \frac{V_1^2}{2} \qquad (8.44)$$

and

$$C_{pi} = 1 - \frac{1}{AR^2} \qquad (8.42)$$

Combining these we obtain an expression for the loss coefficient *K*

$$K = 1 - \frac{1}{AR^2} - C_p \qquad (1)$$

The area ratio *AR* is $\quad AR = \left(\frac{D_2}{D_1}\right)^2 \quad AR = 2.25$

The pressure recovery coefficient C_p is obtained from Eq. 1 above once we select *K*; then, with C_p and *AR* specified, the minimum value of N/R_1 (where *N* is the length and R_1 is the inlet radius) can be read from Fig. 8.15

(a) $\quad K = 0.2 \quad C_p = 1 - \frac{1}{AR^2} - K \quad C_p = 0.602$

From Fig. 8.15 $\quad \frac{N}{R_1} = 5.5 \quad R_1 = \frac{D_1}{2} \quad R_1 = 50 \cdot mm$

$N = 5.5 \cdot R_1 \quad N = 275 \cdot mm$

(b) $\quad K = 0.35 \quad C_p = 1 - \frac{1}{AR^2} - K \quad C_p = 0.452$

From Fig. 8.15 $\quad \frac{N}{R_1} = 3$

$N = 3 \cdot R_1 \quad N = 150 \cdot mm$

Problem 8.95

[3]

8.95 A conical diffuser of length 6 in. is used to expand a pipe flow from a diameter of 2 in. to a diameter of 3.5 in. For a water flow rate of 750 gal/min, estimate the static pressure rise. What is the approximate value of the loss coefficient?

Given: Data on geometry of conical diffuser; flow rate

Find: Static pressure rise; loss coefficient

Solution:

Basic equations $C_p = \dfrac{p_2 - p_1}{\frac{1}{2}\cdot\rho\cdot V_1^2}$ (8.41) $h_{lm} = K\cdot\dfrac{V_1^2}{2} = \left(C_{pi} - C_p\right)\cdot\dfrac{V_1^2}{2}$ (8.44) $C_{pi} = 1 - \dfrac{1}{AR^2}$ (8.42)

Given data $D_1 = 2\cdot in$ $D_2 = 3.5\cdot in$ $N = 6\cdot in$ (N = length) $Q = 750\cdot gpm$

From Eq. 8.41 $\Delta p = p_2 - p_1 = \dfrac{1}{2}\cdot\rho\cdot V_1^2\cdot C_p$ (1)

Combining Eqs. 8.44 and 8.42 we obtain an expression for the loss coefficient *K* $K = 1 - \dfrac{1}{AR^2} - C_p$ (2)

The pressure recovery coefficient C_p for use in Eqs. 1 and 2 above is obtained from Fig. 8.15 once compute *AR* and the dimensionless length N/R_1 (where R_1 is the inlet radius)

The aspect ratio *AR* is $AR = \left(\dfrac{D_2}{D_1}\right)^2$ $AR = \left(\dfrac{3.5}{2}\right)^2$ $AR = 3.06$

$R_1 = \dfrac{D_1}{2}$ $R_1 = 1\cdot in$ Hence $\dfrac{N}{R_1} = 6$

From Fig. 8.15, with *AR* = 3.06 and the dimensionless length N/R_1 = 6, we find $C_p = 0.6$

To complete the calculations we need V_1 $V_1 = \dfrac{Q}{\frac{\pi}{4}\cdot D_1^2}$ $V_1 = \dfrac{4}{\pi}\times 750\cdot\dfrac{gal}{min}\times\dfrac{1\cdot ft^3}{7.48\cdot gal}\times\dfrac{1\cdot min}{60\cdot s}\times\left(\dfrac{1}{\frac{2}{12}\cdot ft}\right)^2$ $V_1 = 76.6\cdot\dfrac{ft}{s}$

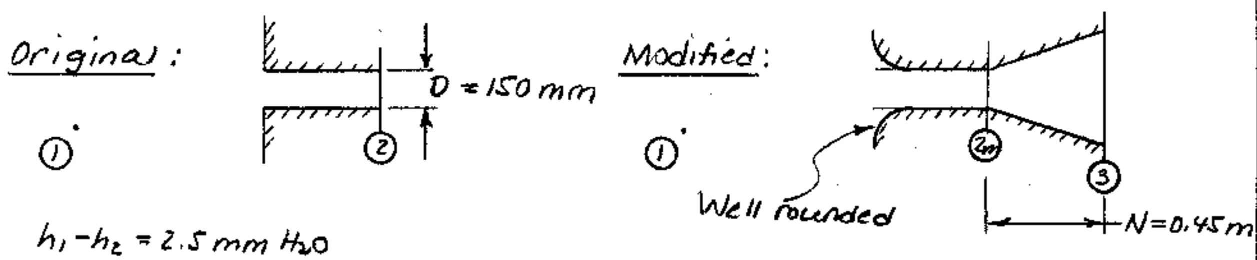
We can now compute the pressure rise and loss coefficient from Eqs. 1 and 2 $\Delta p = \dfrac{1}{2}\cdot\rho\cdot V_1^2\cdot C_p$

$$\Delta p = \frac{1}{2}\times 1.94\cdot\frac{slug}{ft^3}\times\left(76.6\cdot\frac{ft}{s}\right)^2\times 0.6\times\frac{lbf\cdot s^2}{slug\cdot ft}\times\left(\frac{1\cdot ft}{12\cdot in}\right)^2 \qquad \Delta p = 23.7\cdot psi$$

$$K = 1 - \frac{1}{AR^2} - C_p \qquad K = 1 - \frac{1}{3.06^2} - 0.6 \qquad K = 0.293$$

Problem 8.96

Given: Air flow from a clean room through a duct of 150 mm diameter.



$h_1 - h_2 = 2.5 \text{ mm } H_2O$

$h_1 - h_3 = 2.5 \text{ mm } H_2O$

Neglect friction losses compared to "minor" losses.

Find: (a) Area ratio and angle for optimum conical diffuser.
(b) Flow rate for modified system.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equations:

$$\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2 + h_{\ell T} \quad \text{(or to section 3)}$$

($\bar{V}_1 \approx 0$ (1); z_1, z_2 cancelled)

$$h_{\ell T} = h_\ell + h_{\ell m}; \quad h_{\ell m} = K_{ent} + h_{\ell\, diffuser}; \quad \Delta p = \rho_{H_2O} g \Delta h$$

($h_\ell \approx 0$ (4))

$$\text{From Eq. 8.42, } h_{\ell\, diffuser} = \frac{\bar{V}_2^2}{2}\left[1 - \frac{1}{AR^2} - C_p\right]$$

Assumptions: (1) $\bar{V}_1 \approx 0$
(2) Neglect Δz
(3) Uniform flow at each section
(4) Neglect frictional losses

For the original system, $\frac{p_1 - p_2}{\rho} = \frac{\bar{V}_2^2}{2} + K_{ent}\frac{\bar{V}_2^2}{2} = 1.5\frac{\bar{V}_2^2}{2} = \frac{\rho_{H_2O} g \Delta h}{\rho}$ $\quad (K_{ent} = 0.5)$

Thus

$$\bar{V}_2 = \sqrt{\frac{2}{1.5}\frac{\rho_{H_2O} g \Delta h}{\rho}} = \sqrt{\frac{2}{1.5} \times 999\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times 0.0025\, m \times \frac{m^3}{1.23\, kg}} = 5.15 \text{ m/s}$$

For the modified system, $\frac{p_1 - p_3}{\rho} = \frac{\bar{V}_3^2}{2} + K_{ent}\frac{\bar{V}_2^2}{2} + \frac{\bar{V}_2^2}{2}\left[1 - \frac{1}{AR^2} - C_p\right] = \frac{\bar{V}_2^2}{2}\left[1 + K_{ent} - C_p\right]$

since $\bar{V}_3^2 = \bar{V}_2^2 \frac{1}{AR^2}$. Thus the best diffuser has the highest C_p.

From Fig. 8.16, $C_p = f(N/R_1, AR)$. $N/R_1 = 2N/D_1 = 2 \times \frac{0.45\, m}{0.15\, m} = 6$. From the figure, the best diffuser is

$$C_p \approx 0.62 \text{ at } AR \approx 2.7 \text{ and } 2\phi \approx 12 \text{ deg}$$

AR, …

For the modified system,

$$\bar{V}_2 = \sqrt{\frac{2}{1 + K_{ent} - C_p}\frac{\rho_{H_2O} g \Delta h}{\rho}} = \sqrt{\frac{2}{1 + 0.04 - 0.62} \times 999\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times 0.0025\, m \times \frac{m^3}{1.23\, kg}} = 9.74 \text{ m/s}$$

and

$$Q = \bar{V}_2 A_2 = 9.74\frac{m}{s} \times \frac{\pi}{4}(0.15)^2 m^2 = 0.172 \text{ m}^3/\text{s}$$

Q

{The improvement is $\frac{Q_m - Q}{Q} \times 100 = \frac{\bar{V}_m - \bar{V}}{\bar{V}} \times 100 = \frac{9.74 - 5.15}{5.15} \times 100 = 89.1$ percent more}

Problem 8.97

8.97 By applying the basic equations to a control volume starting at the expansion and ending downstream, analyze flow through a sudden expansion (assume the inlet pressure p_1 acts on the area A_2 at the expansion). Develop an expression for and plot the minor head loss across the expansion as a function of area ratio, and compare with the data of Fig. 8.15.

Given: Sudden expansion

Find: Expression for minor head loss; compare with Fig. 8.15; plot

Solution:

The governing CV equations (mass, momentum, and energy) are

$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad (4.12)$$

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A} \qquad (4.18a)$$

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\,\rho\, d\forall + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A} \qquad (4.56)$$

Assume: 1) Steady flow 2) Incompressible flow 3) Uniform flow at each section 4) Horizontal: no body force 5) No shaft work 6) Neglect viscous friction 7) Neglect gravity

The mass equation becomes
$$V_1\cdot A_1 = V_2\cdot A_2 \qquad (1)$$

The momentum equation becomes
$$p_1\cdot A_2 - p_2\cdot A_2 = V_1\cdot(-\rho\cdot V_1\cdot A_1) + V_2\cdot(\rho\cdot V_2\cdot A_2)$$

or (using Eq. 1)
$$p_1 - p_2 = \rho\cdot V_1\cdot\frac{A_1}{A_2}\cdot(V_2 - V_1) \qquad (2)$$

The energy equation becomes
$$Q_{rate} = \left(u_1 + \frac{p_1}{\rho} + V_1^{\,2}\right)\cdot(-\rho\cdot V_1\cdot A_1) + \left(u_2 + \frac{p_2}{\rho} + V_2^{\,2}\right)\cdot(\rho\cdot V_2\cdot A_2)$$

or (using Eq. 1)
$$h_{lm} = u_2 - u_1 - \frac{Q_{rate}}{m_{rate}} = \frac{V_1^{\,2} - V_2^{\,2}}{2} + \frac{p_1 - p_2}{\rho} \qquad (3)$$

Combining Eqs. 2 and 3
$$h_{lm} = \frac{V_1^{\,2} - V_2^{\,2}}{2} + V_1\cdot\frac{A_1}{A_2}\cdot(V_2 - V_1)$$

$$h_{lm} = \frac{V_1^{\,2}}{2}\cdot\left[1 - \left(\frac{V_2}{V_1}\right)^2\right] + V_1^{\,2}\cdot\frac{A_1}{A_2}\cdot\left[\left(\frac{V_2}{V_1}\right) - 1\right]$$

From Eq. 1 $$AR = \frac{A_1}{A_2} = \frac{V_2}{V_1}$$

Hence $$h_{lm} = \frac{V_1^2}{2}\cdot\left(1 - AR^2\right) + V_1^2\cdot AR\cdot(AR - 1)$$

$$h_{lm} = \frac{V_1^2}{2}\cdot\left(1 - AR^2 + 2\cdot AR^2 - 2\cdot AR\right)$$

$$h_{lm} = K\cdot\frac{V_1^2}{2} = (1 - AR)^2\cdot\frac{V_1^2}{2}$$

Finally $$K = (1 - AR)^2$$

This result, and the curve of Fig. 8.15, are shown in the associated *Excel* workbook. The agreement is excellent

Problem 8.97

8.97 By applying the basic equations to a control volume starting at the expansion and ending downstream, analyze flow through a sudden expansion (assume the inlet pressure p_1 acts on the area A_2 at the expansion). Develop an expression for and plot the minor head loss across the expansion as a function of area ratio, and compare with the data of Fig. 8.15.

Given: Sudden expansion

Find: Expression for minor head loss; compare with Fig. 8.15; plot

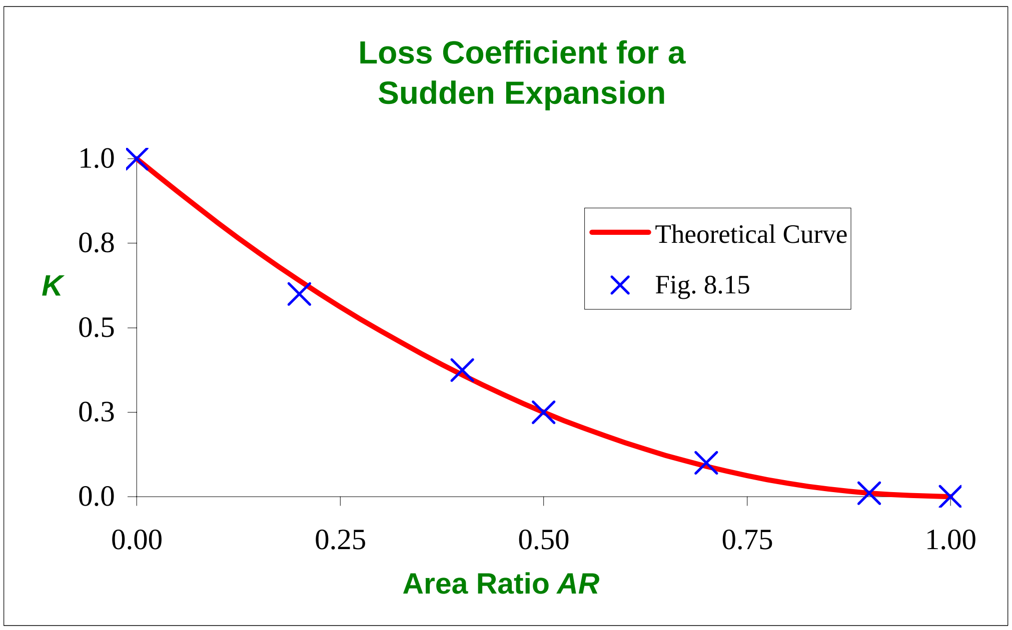
Solution:

From the CV analysis

$$K = (1 - AR)^2$$

AR	K_{CV}	$K_{Fig.\ 8.15}$
0.0	1.00	1.00
0.1	0.81	
0.2	0.64	0.60
0.3	0.49	
0.4	0.36	0.38
0.5	0.25	0.25
0.6	0.16	
0.7	0.09	0.10
0.8	0.04	
0.9	0.01	0.01
1.0	0.00	0.00

(Data from Fig. 8.15 is "eyeballed")
Agreement is excellent



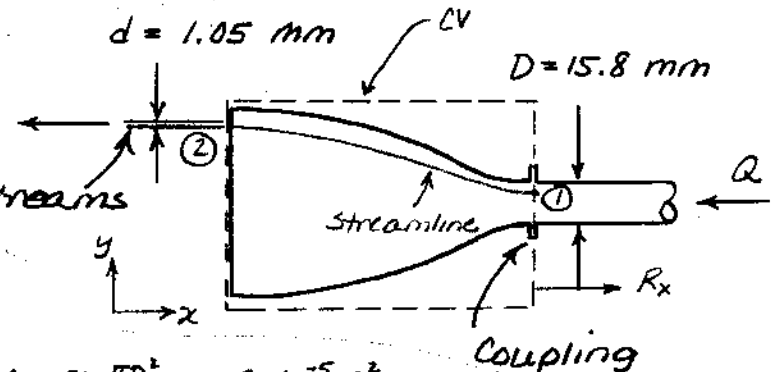
Problem 8.98

Given: Water at 45°C enters a shower head through a circular tube with 15.8 mm inside diameter. The water leaves in 24 streams, each of 1.05 mm diameter. The volume flow rate is 5.67 L/min.

Find:
(a) Estimate of the minimum water pressure needed at the inlet to the shower head.
(b) Force needed to hold the shower head onto the end of the circular tube, indicating clearly whether this is a compression or a tension force.

Solution: Apply the energy equation for steady, incompressible pipe flow, and the x component of momentum, using the CV shown.

Assume: (1) Steady flow
(2) Incompressible flow
(3) Neglect changes in z
(4) Uniform flow: $\alpha_1 = \alpha_2 \approx 1$
(5) Use gage pressures



Then

$$\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1 - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} = h_l + h_{l_m}$$

($p_2 = 0$ (gage), $h_l = 0$)

$$A_2 = 24\,\frac{\pi D_2^2}{4} = 2.08\times10^{-5}\,m^2 \qquad A_1 = \frac{\pi D_1^2}{4} = 1.96\times10^{-4}\,m^2$$

$$\bar{V}_1 = \frac{Q}{A_1} = 5.67\,\frac{L}{min}\times\frac{1}{1.96\times10^{-4}\,m^2}\times\frac{m^3}{1000\,L}\times\frac{min}{60\,s} = 0.487\,m/s$$

$$\bar{V}_2 = \bar{V}_1\frac{A_1}{A_2} = 0.487\,\frac{m}{s}\times\frac{1.96\times10^{-4}\,m^2}{2.08\times10^{-5}\,m^2} = 4.59\,m/s$$

Use $K = 0.5$, for a square-edged orifice, $\rho = 990\,kg/m^3$ (Table A.8). Then

$$p_1 = \frac{\rho}{2}\left(\bar{V}_2^2 + K\bar{V}_2^2 - \bar{V}_1^2\right) = \frac{\rho}{2}\left[(1+K)\bar{V}_2^2 - \bar{V}_1^2\right]$$

$$p_1 = \frac{1}{2}\times 990\,\frac{kg}{m^3}\left[(1+0.5)(4.59)^2 - (0.487)^2\right]\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} = 15.5\,kPa\ (gage)$$ p_1

Use momentum to find force:

Basic equation: $$F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

($F_{B_x} = 0$ (6), $\frac{\partial}{\partial t} = 0$ (1))

Assume: (6) $F_{B_x} = 0$

Step ①: Then $$R_x - p_{1g}A_1 = u_1\{-\rho Q\} + u_2\{+\rho Q\} = -V_1\{-\rho Q\} + (-V_2)\{+\rho Q\} = \rho Q(V_1 - V_2)$$

Step ②: $u_1 = -V_1 \qquad u_2 = -V_2$

$$R_x = p_{1g}A_1 + \rho Q(V_1 - V_2) = 15.5\times10^3\,\frac{N}{m^2}\times1.96\times10^{-4}\,m^2 + 990\,\frac{kg}{m^3}\times5.67\,\frac{L}{min}\times(0.487 - 4.59)\,\frac{m}{s}\times\frac{m^3}{1000\,L}\times\frac{min}{60\,s}$$

$R_x = 2.65\ N$ (in direction shown, i.e., <u>tension</u>) R_x

Problem 8.99

[2]

8.99 Analyze flow through a sudden expansion to obtain an expression for the upstream average velocity $\overline{V}_1$ in terms of the pressure change $\Delta p = p_2 - p_1$, area ratio AR, fluid density ρ, and loss coefficient K. If the flow were frictionless, would the flow rate indicated by a measured pressure change be higher or lower than a real flow, and why? Conversely, if the flow were frictionless, would a given flow generate a larger or smaller pressure change, and why?

Given: Sudden expansion

Find: Expression for upstream average velocity

Solution:

The governing equation is

$$\left(\frac{p_1}{\rho} + \alpha_1\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_{lT} \qquad (8.29)$$

$$h_{lT} = h_l + K\cdot\frac{V^2}{2}$$

Assume: 1) Steady flow 2) Incompressible flow 3) $h_l = 0$ 4) $\alpha_1 = \alpha_2 = 1$ 5) Neglect gravity

The mass equation is $V_1\cdot A_1 = V_2\cdot A_2$ so $V_2 = V_1\cdot\frac{A_1}{A_2}$

$$V_2 = AR\cdot V_1 \qquad (1)$$

Equation 8.29 becomes

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} = \frac{p_1}{\rho} + \frac{V_1^2}{2} + K\cdot\frac{V_1^2}{2}$$

or (using Eq. 1)

$$\frac{\Delta p}{\rho} = \frac{p_2 - p_1}{\rho} = \frac{V_1^2}{2}\cdot\left(1 - AR^2 - K\right)$$

Solving for V_1

$$V_1 = \sqrt{\frac{2\cdot\Delta p}{\rho\cdot\left(1 - AR^2 - K\right)}}$$

If the flow were frictionless, $K = 0$, so

$$V_{inviscid} = \sqrt{\frac{2\cdot\Delta p}{\rho\cdot\left(1 - AR^2\right)}} < V_1$$

Hence the flow rate indicated by a given Δp would be lower

If the flow were frictionless, $K = 0$, so

$$\Delta p_{invscid} = \frac{V_1^2}{2}\cdot\left(1 - AR^2\right)$$

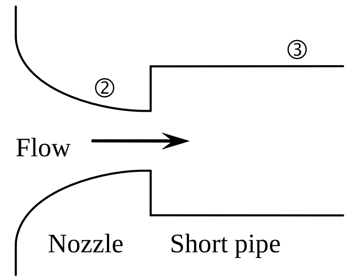
compared to

$$\Delta p = \frac{V_1^2}{2}\cdot\left(1 - AR^2 - K\right)$$

Hence a given flow rate would generate a larger Δp for inviscid flow

Problem 8.100

8.100 Water discharges to atmosphere from a large reservoir through a moderately rounded horizontal nozzle of 1 in. diameter. The free surface is 5 ft above the nozzle exit plane. Calculate the change in flow rate when a short section of 2-in. diameter pipe is attached to the end of the nozzle to form a sudden expansion. Determine the location and estimate the magnitude of the minimum pressure with the sudden expansion in place. If the flow were frictionless (with the sudden expansion in place), would the minimum pressure be higher, lower, or the same? Would the flow rate be higher, lower, or the same?



Given: Flow out of water tank through a nozzle

Find: Change in flow rate when short pipe section is added; Minimum pressure; Effect of frictionless flow

Solution:

Basic equations
$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT}\qquad h_{lT}=h_l+h_{lm}=f\cdot\frac{L}{D}\cdot\frac{V_2^2}{2}+K\cdot\frac{V_2^2}{2}\qquad Q=V\cdot A$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) V_l << 5) L << so $h_l = 0$

Hence for the nozzle case, between the free surface (Point 1) and the exit (2) the energy equation becomes

$$g\cdot z_1-\frac{V_2^2}{2}=K_{nozzle}\cdot\frac{V_2^2}{2}$$

Solving for V_2
$$V_2=\sqrt{\frac{2\cdot g\cdot z_1}{\left(1+K_{nozzle}\right)}}$$

For a rounded edge, we choose the first value from Table 8.2 $\qquad K_{nozzle} = 0.28$

Hence
$$V_2=\sqrt{2\times 32.2\cdot\frac{ft}{s^2}\times 5\cdot ft\times\frac{1}{(1+0.28)}}\qquad V_2=15.9\cdot\frac{ft}{s}$$

$$Q=V_2\cdot A_2\qquad Q=15.9\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{0.5}{12}\cdot ft\right)^2\times\frac{7.48\cdot gal}{1\cdot ft^3}\times\frac{60\cdot s}{1\cdot min}\qquad Q=9.73\cdot gpm\qquad Q=0.0217\frac{ft^3}{s}$$

When a small piece of pipe is added the energy equation between the free surface (Point 1) and the exit (3) becomes

$$g\cdot z_1-\frac{V_3^2}{2}=K_{nozzle}\cdot\frac{V_2^2}{2}+K_e\cdot\frac{V_2^2}{2}$$

From continuity
$$V_3=V_2\cdot\frac{A_2}{A_3}=V_2\cdot AR$$

Solving for V_2
$$V_2=\sqrt{\frac{2\cdot g\cdot z_1}{\left(AR^2+K_{nozzle}+K_e\right)}}$$

We need the AR for the sudden expansion
$$AR=\frac{A_2}{A_3}=\left(\frac{D_2}{D_3}\right)^2=\left(\frac{1}{2}\right)^2=0.25$$

From Fig. 8.14 for AR = 0.25 $\qquad K_e = 0.6$

Hence $$V_2 = \sqrt{2 \times 32.2 \cdot \frac{ft}{s^2} \times 5 \cdot ft \times \frac{1}{\left(0.25^2 + 0.28 + 0.6\right)}}$$ $$V_2 = 18.5 \cdot \frac{ft}{s}$$

$$Q = V_2 \cdot A_2 \qquad Q = 18.5 \cdot \frac{ft}{s} \times \frac{\pi}{4} \times \left(\frac{0.5}{12} \cdot ft\right)^2 \times \frac{7.48 \cdot gal}{1 \cdot ft^3} \times \frac{60 \cdot s}{1 \cdot min} \qquad Q = 11.32 \cdot gpm \qquad Q = 0.0252 \frac{ft^3}{s}$$

Comparing results we see the flow increases from 0.0217 ft^3/s to 0.0252 ft^3/s $$\frac{\Delta Q}{Q} = \frac{0.0252 - 0.0217}{0.0217} = 16.1 \cdot \%$$

The flow increases because the effect of the pipe is to allow an exit pressure at the nozzle LESS than atmospheric!

The minimum pressure point will now be at Point 2 (it was atmospheric before adding the small pipe). The energy equation between 1 and 2 is

$$g \cdot z_1 - \left(\frac{p_2}{\rho} + \frac{V_2^2}{2}\right) = K_{nozzle} \cdot \frac{V_2^2}{2}$$

Solving for p_2 $$p_2 = \rho \cdot \left[g \cdot z_1 - \frac{V_2^2}{2} \cdot \left(K_{nozzle} + 1\right)\right]$$

Hence $$p_2 = 1.94 \cdot \frac{slug}{ft^3} \times \left[32.2 \cdot \frac{ft}{s^2} \times 5 \cdot ft - \frac{1}{2} \times \left(18.5 \cdot \frac{ft}{s}\right)^2 \times (0.28 + 1)\right] \times \frac{lbf \cdot s^2}{slug \cdot ft} \qquad p_2 = -113 \frac{lbf}{ft^2} \qquad p_2 = -0.782\,psi$$

If the flow were frictionless the the two loss coeffcients would be zero. Instead of $$V_2 = \sqrt{\frac{2 \cdot g \cdot z_1}{\left(AR^2 + K_{nozzle} + K_e\right)}}$$

We'd have $$V_2 = \sqrt{\frac{2 \cdot g \cdot z_1}{AR^2}}$$ which is larger

If V_2 is larger, then p_2, through Bernoulli, would be lower (more negative)

Problem 8.101

Given: Steady flow of water from a large tank through a length of smooth plastic tubing, with $D = 3.18$ mm and $L = 15.3$ m.

Find: (a) Maximum volume flow rate for laminar flow.
(b) Estimate maximum water level in tank for laminar flow ($\alpha = 2$ and $K_{ent} = 1.4$)

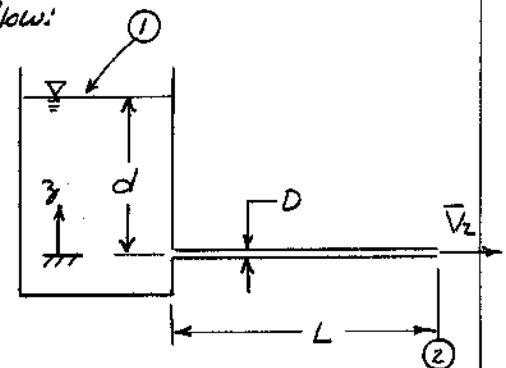
Solution: Assume water at 20°C. From Table A.8, $\rho = 998\ kg/m^3$, $\nu = 1.00 \times 10^{-6}\ m^2/s$.

$$Re = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} \le 2300\ ;\ \bar{V}_{max} = \frac{2300\nu}{D} = 2300 \times 1.00 \times 10^{-6} \frac{m^2}{s} \times \frac{1}{0.00318\ m} = 0.723\ m/s$$ $\bar{V}$

$$Q = \bar{V} A\ ;\ A = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.00318)^2 m^2 = 7.94 \times 10^{-6}\ m^2$$

$$Q = 0.723 \frac{m}{s} \times 7.94 \times 10^{-6}\ m^2 = 5.74 \times 10^{-6} \frac{m^3}{s} \times 10^3 \frac{L}{m^3} \times 60 \frac{s}{min} = 0.345\ L/min$$ Q

Apply energy equation for steady, ρ = constant pipe flow:



Computing Equation:
$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) = h_{\ell T}$$
($\bar{V}_1 \approx 0$, $z_2 = 0$)

$$h_{\ell T} = h_{\ell m} + h_\ell$$

Assumptions: (1) $p_1 = p_2 = p_{atm}$
(2) $\bar{V}_1 \approx 0$
(3) $K_{ent} = 1.4$ (given)

Then
$$g d = \alpha_2 \frac{\bar{V}_2^2}{2} + K_{ent} \frac{\bar{V}_2^2}{2} + f \frac{L}{D} \frac{\bar{V}_2^2}{2} \quad \text{or} \quad d = \frac{\bar{V}_2^2}{2g}\left(\alpha_2 + K_{ent} + f \frac{L}{D}\right)$$

For laminar flow, $f = \frac{64}{Re} = \frac{64}{2300} = 0.0278$. Substituting

$$d = \frac{1}{2} \times (0.723)^2 \frac{m^2}{s^2} \times \frac{s^2}{9.81\ m}\left(2.0 + 1.4 + 0.0278\ \frac{15.3\ m}{0.00318\ m}\right)$$

$$d = 3.65\ m$$ d

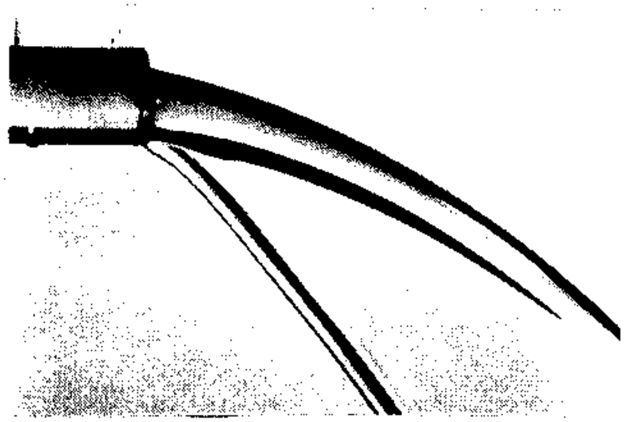
Problem 8.102

Open-Ended Problem Statement: You are asked to compare the behavior of fully developed laminar flow and fully developed turbulent flow in a horizontal pipe under different conditions. For the same flow rate, which will have the larger centerline velocity? Why? If the pipe discharges to atmosphere what would you expect the trajectory of the discharge stream to look like (for the same flow rate)? Sketch your expectations for each case. For the same flow rate, which flow would give the larger wall shear stress? Why? Sketch the shear stress distribution τ/τ_w as a function of radius for each flow. For the same Reynolds number, which flow would have the larger pressure drop per unit length? Why? For a given imposed pressure differential, which flow would have the larger flow rate? Why?

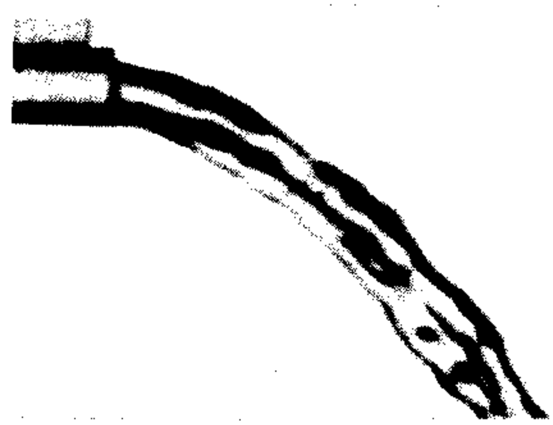
Discussion: In the following fully developed laminar flow and fully developed turbulent flow in a pipe are compared:

(a) For the same flow rate, laminar flow has the higher maximum velocity, because the turbulent velocity profile is more blunt.

(b) The trajectory of the discharge stream spreads out for laminar flow because of the large variation in velocity across the pipe exit. For turbulent flow the exit profile is more nearly uniform (except for the region adjacent to the wall) and hence the trajectory is more uniform. Since centerline velocity is larger for laminar flow, liquid travels the greatest horizontal distance. Trajectories for the two flow cases are shown below:



(i) Laminar flow



(ii) Turbulent flow

(c) For the same flow rate (same mean velocity), turbulent flow has larger wall shear stress because of the larger velocity gradient at the pipe wall. For fully developed flow the pressure force driving the flow is balanced by the shear force at the wall.

(d) Shear stress varies linearly with radius for both flow cases, from its maximum value at the wall to zero at the pipe centerline.

(e) For the same Reynolds number, turbulent flow has a larger pressure drop per unit length because the friction factor is larger.

(f) For a given pressure drop (per unit length), laminar flow has the larger flow rate (larger mean velocity), because it has the smaller friction factor.

The two flow cases are compared in the NCFMF video *Turbulence*, in which R. W. Stewart uses a clever experimental setup to contrast the two flow regimes at constant volume flow rate by varying the liquid viscosity. The trajectories of the liquid streams leaving the end of the pipe are particularly well shown.

Problem 8.103

[3]

8.103 Estimate the minimum level in the water tank of Problem 8.101 such that the flow will be turbulent.

Given: Data on water flow from a tank/tubing system

Find: Minimum tank level for turbulent flow

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_{lT} = \sum_{major} h_l + \sum_{minor} h_{lm} \qquad (8.29)$$

$$Re = \frac{\rho \cdot V \cdot D}{\mu} \qquad h_l = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \quad (8.34) \qquad h_{lm} = K \cdot \frac{V^2}{2} \quad (8.40a) \qquad h_{lm} = f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2} \quad (8.40b)$$

$$f = \frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes

$$g \cdot d - \alpha \cdot \frac{V^2}{2} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} + K \cdot \frac{V^2}{2}$$

This can be solved expicitly for height *d*, or solved using *Solver*

Given data:

L = 15.3 m
D = 3.18 mm
K_{ent} = 1.4
α = 2

Tabulated or graphical data:

ν = 1.00E-06 m^2/s
ρ = 998 kg/m^3
(Appendix A)

Computed results:

Re = 2300 (Transition Re)
V = 0.723 m/s
α = 1 (Turbulent)
f = 0.0473 (Turbulent)

d = 6.13 m (Vary d to minimize error in energy equation)

Energy equation:
(Using *Solver*)

Left (m^2/s)	Right (m^2/s)	Error
59.9	59.9	0.00%

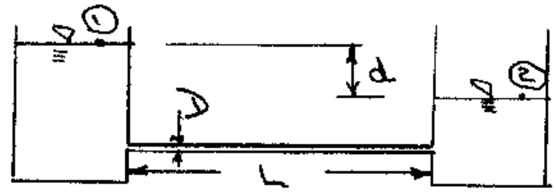
Note that we used α = 1 (turbulent); using α = 2 (laminar) gives d = 6.16 m

Problem 8.104

Given: System for measuring pressure drop for water flow in smooth tube as shown

$D = 15.9\,mm \quad L = 3.56\,m$

square-edged entrance to pipe



Find: (a) Volume flow rate needed for turbulent flow in pipe
(b) reservoir height differential needed for turbulent pipe flow

Solution:

Flow will be turbulent for $Re_D > 2300$

$$Re_D = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} = \frac{Q D}{A \nu} = \frac{Q D}{\nu}\frac{4}{\pi D^2} = \frac{4Q}{\pi \nu D} \quad \text{so } Q = \frac{\pi \nu D\, Re}{4}$$

Assume $T = 20°C$, $\nu = 1.00 \times 10^{-6}\, m^2/s$ (Table A.8)

For $Re = 2300$,

$$Q = \frac{\pi}{4} \times 1.0 \times 10^{-6} \frac{m^2}{s} \times 15.9 \times 10^{-3} m \times 2300 = 2.87 \times 10^{-5}\, m^3/s$$ — Q

Basic equations: $$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) = h_{l_T} \quad (8.29)$$

$$h_{l_T} = h_l + h_{l_m} \qquad h_l = f \frac{L}{D}\frac{\bar{V}^2}{2}, \quad h_{l_m} = K \frac{\bar{V}^2}{2}$$

Assumptions: (1) $p_1 = p_2 = p_{atm}$ (2) $\bar{V}_1 = \bar{V}_2 = 0$
(3) $K_{ent} = 0.5$ (Table 8.2), $K_{exit} = 1.0$

Then,
$$z_1 - z_2 = \frac{\bar{V}^2}{2g}\left[f \frac{L}{D} + K_{ent} + K_{exit}\right] \quad (1)$$

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 2.87 \times 10^{-5} \frac{m^3}{s} \times \frac{1}{(15.9 \times 10^{-3} m)^2} = 0.145\, m/s$$

For turbulent flow in a smooth pipe at $Re = 2300$,
$f = 0.05$ (Fig 8.13)

From Eq. 1
$$d = z_1 - z_2 = \frac{(0.145)^2}{2}\frac{m^2}{s^2} \times \frac{s^2}{9.81\, m}\left[0.05 \times \frac{3.56 \times 10^3}{15.9} + 0.5 + 1.0\right]$$

$d = 0.0136\, m$ or $13.6\, mm$ — d

Problem 8.105

8.105 As discussed in Problem 8.48, the applied pressure difference, Δp, and corresponding volume flow rate, Q, for laminar flow in a tube can be compared to the applied DC voltage V across, and current I through, an electrical resistor, respectively. Investigate whether or not this analogy is valid for turbulent flow by plotting the "resistance" $\Delta p/Q$ as a function of Q for turbulent flow of kerosine (at 40°C) in a tube 100 mm long with inside diameter 0.3 mm.

Given:

Data on a tube

Find:

"Resistance" of tube for flow of kerosine; plot

Solution:

The given data is $L = 100 \cdot mm$ $D = 0.3 \cdot mm$

From Fig. A.2 and Table A.2 $\mu = 1.1 \times 10^{-3} \cdot \frac{N \cdot s}{m^2}$ $\rho = 0.82 \times 990 \cdot \frac{kg}{m^3} = 812 \cdot \frac{kg}{m^3}$ (Kerosene)

For an electrical resistor $V = R \cdot I$ (1)

The governing equations for turbulent flow are

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_l \qquad (8.29)$$

$$h_l = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \quad (8.34) \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad (8.37)$$

Simplifying Eqs. 8.29 and 8.34 for a horizontal, constant-area pipe

$$\frac{p_1 - p_2}{\rho} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} = f \cdot \frac{L}{D} \cdot \frac{\left(\frac{Q}{\frac{\pi}{4} \cdot D^2}\right)^2}{2} \quad \text{or} \quad \Delta p = \frac{8 \cdot \rho \cdot f \cdot L}{\pi^2 \cdot D^5} \cdot Q^2 \qquad (2)$$

By analogy, current I is represented by flow rate Q, and voltage V by pressure drop Δp.
Comparing Eqs. (1) and (2), the "resistance" of the tube is

$$R = \frac{\Delta p}{Q} = \frac{8 \cdot \rho \cdot f \cdot L \cdot Q}{\pi^2 \cdot D^5}$$

The "resistance" of a tube is not constant, but is proportional to the "current" Q! Actually, the dependence is not quite linear, because f decreases slightly (and nonlinearly) with Q. The analogy fails!

The analogy is hence invalid for $Re > 2300$ or $\frac{\rho \cdot V \cdot D}{\mu} > 2300$

Writing this constraint in terms of flow rate $\frac{\rho \cdot \frac{Q}{\frac{\pi}{4} \cdot D^2} \cdot D}{\mu} > 2300$ or $Q > \frac{2300 \cdot \mu \cdot \pi \cdot D}{4 \cdot \rho}$

Flow rate above which analogy fails $Q = 7.34 \times 10^{-7} \frac{m^3}{s}$

The plot of "resistance" versus flow rate is shown in the associated *Excel* workbook

Problem 8.105

[3]

8.105 As discussed in Problem 8.48, the applied pressure difference, Δp, and corresponding volume flow rate, Q, for laminar flow in a tube can be compared to the applied DC voltage V across, and current I through, an electrical resistor, respectively. Investigate whether or not this analogy is valid for turbulent flow by plotting the "resistance" $\Delta p/Q$ as a function of Q for turbulent flow of kerosine (at 40°C) in a tube 100 mm long with inside diameter 0.3 mm.

Given: Data on a tube

Find: "Resistance" of tube for flow of kerosine; plot

Solution:

The "resistance" is $$R = \frac{\Delta p}{Q} = \frac{8 \cdot \rho \cdot f \cdot L \cdot Q}{\pi^2 \cdot D^5}$$

The "resistance" of a tube is not constant, but is proportional to the "current" Q! Actually, the dependence is not quite linear, because f decreases slightly (and nonlinearly) with Q. The analogy fails!

Given data:

L = 100 mm
D = 0.3 mm

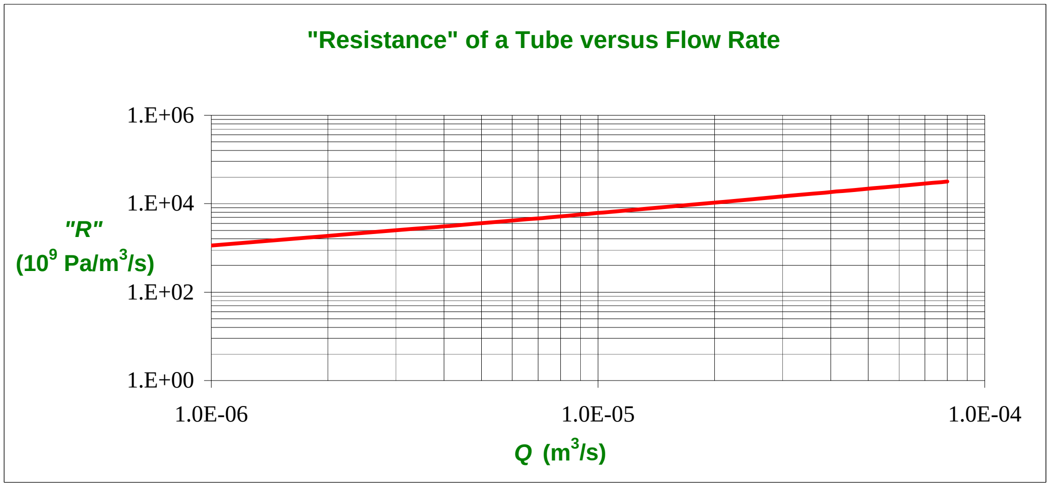
Tabulated or graphical data:

μ = 1.01E-03 $N.s/m^2$
SG_{ker} = 0.82
ρ_w = 990 kg/m^3
ρ = 812 kg/m^3
(Appendix A)

Computed results:

Q (m^3/s)	V (m/s)	Re	f	"R" (10^9 $Pa/m^3/s$)
1.0E-06	14.1	3.4E+03	0.0419	1133
2.0E-06	28.3	6.8E+03	0.0343	1855
4.0E-06	56.6	1.4E+04	0.0285	3085
6.0E-06	84.9	2.0E+04	0.0257	4182
8.0E-06	113.2	2.7E+04	0.0240	5202
1.0E-05	141.5	3.4E+04	0.0228	6171
2.0E-05	282.9	6.8E+04	0.0195	10568
4.0E-05	565.9	1.4E+05	0.0169	18279
6.0E-05	848.8	2.0E+05	0.0156	25292
8.0E-05	1131.8	2.7E+05	0.0147	31900

The "resistance" is not constant; the analogy is invalid for turbulent flow



Problem 8.106

8.106 Plot the required reservoir depth of water to create flow in a smooth tube of diameter 10 mm and length 100 m, for a flow rate range of 1 L/min through 10 L/min.

Given: Data on tube geometry

Find: Plot of reservoir depth as a function of flow rate

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_{lT} = \sum_{major} h_l + \sum_{minor} h_{lm} \qquad (8.29)$$

$$Re = \frac{\rho \cdot V \cdot D}{\mu} \qquad h_l = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \quad (8.34) \qquad h_{lm} = K \cdot \frac{V^2}{2} \quad (8.40a) \qquad h_{lm} = f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2} \quad (8.40b)$$

$$f = \frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes

$$g \cdot d - \alpha \cdot \frac{V^2}{2} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} + K \cdot \frac{V^2}{2}$$

This can be solved expicitly for height *d*, or solved using *Solver*

$$d = \frac{V^2}{2 \cdot g} \cdot \left(\alpha + f \cdot \frac{L}{D} + K\right)$$

Given data:

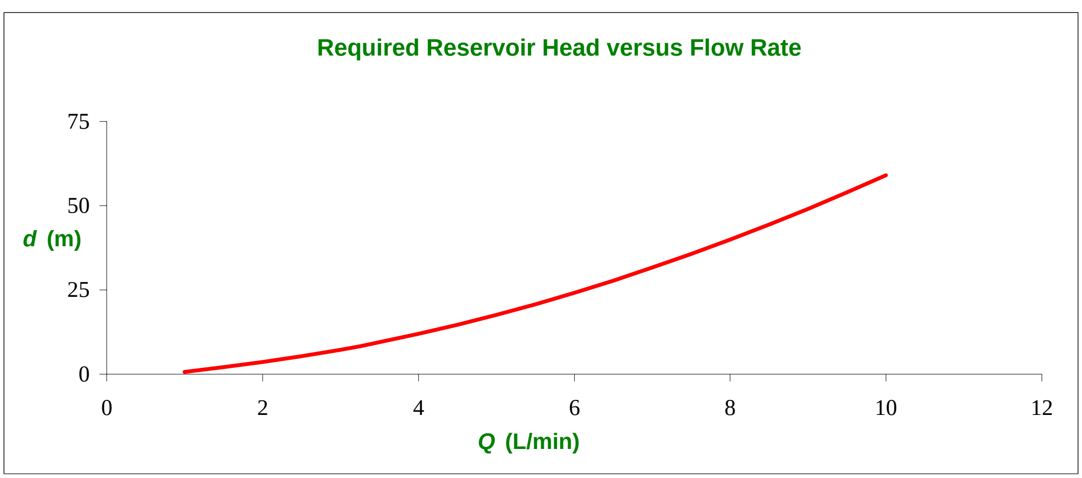
L = 100 m
D = 10 mm
α = 1 (All flows turbulent)

Tabulated or graphical data:

μ = 1.01E-03 N.s/m^2
ρ = 998 kg/m^3
(Table A.8)
K_{ent} = 0.5 (Square-edged)
(Table 8.2)

Computed results:

Q (L/min)	*V* (m/s)	*Re*	*f*	*d* (m)
1	0.2	2.1E+03	0.0305	0.704
2	0.4	4.2E+03	0.0394	3.63
3	0.6	6.3E+03	0.0350	7.27
4	0.8	8.4E+03	0.0324	11.9
5	1.1	1.0E+04	0.0305	17.6
6	1.3	1.3E+04	0.0291	24.2
7	1.5	1.5E+04	0.0280	31.6
8	1.7	1.7E+04	0.0270	39.9
9	1.9	1.9E+04	0.0263	49.1
10	2.1	2.1E+04	0.0256	59.1



Problem 8.107

8.107 Oil with kinematic viscosity $\nu = 0.00005\ \text{m}^2/\text{s}$ flows at $0.003\ \text{m}^3/\text{s}$ in a 25-m long horizontal steel pipe of 4 cm diameter. By what percentage ratio will the energy loss increase if the same flow rate is maintained while the pipe diameter is reduced to 1 cm?

Given: Flow of oil in a pipe

Find: Percentage change in loss if diameter is reduced

Solution:

Basic equations $\quad h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \qquad f = \frac{64}{Re}$ Laminar $\qquad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ Turbulent

Here $\quad V = \frac{Q}{A} = \frac{4\cdot Q}{\pi\cdot D^2} \qquad V = \frac{4}{\pi}\times 0.003\cdot\frac{m^3}{s}\times\left(\frac{1}{0.04\cdot m}\right)^2 \qquad V = 2.39\frac{m}{s}$

Then $\quad Re = \frac{V\cdot D}{\nu} \qquad Re = 2.39\cdot\frac{m}{s}\times 0.04\cdot m\times\frac{s}{0.00005\cdot m^2} \qquad Re = 1912$

The flow is LAMINAR $\quad h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \qquad h_l = \frac{64}{Re}\cdot\frac{L}{D}\cdot\frac{V^2}{2} \qquad h_l = \frac{64}{1912}\times\frac{25\cdot m}{0.04\cdot m}\times\frac{\left(2.39\cdot\frac{m}{s}\right)^2}{2} \qquad h_l = 643\cdot\frac{ft^2}{s^2}$

When the diameter is reduced

$$V = \frac{Q}{A} = \frac{4\cdot Q}{\pi\cdot D^2} \qquad V = \frac{4}{\pi}\times 0.003\cdot\frac{m^3}{s}\times\left(\frac{1}{0.01\cdot m}\right)^2 \qquad V = 38.2\frac{m}{s}$$

$$Re = \frac{V\cdot D}{\nu} \qquad Re = 38.2\cdot\frac{m}{s}\times 0.01\cdot m\times\frac{s}{0.00005\cdot m^2} \qquad Re = 7640$$

The flow is TURBULENT For a steel pipe, from table 8.1 $\qquad e = 0.046\cdot mm$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0389$

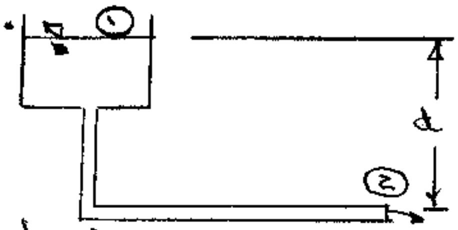
$$h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \qquad h_l = 0.0389\times\frac{25\cdot m}{0.01\cdot m}\times\frac{\left(38.2\cdot\frac{m}{s}\right)^2}{2} \qquad h_l = 7.64\times 10^5\cdot\frac{ft^2}{s^2}$$

The increase in loss is $\quad \dfrac{7.64\times 10^5\ \frac{ft^2}{s^2}}{643\ \frac{ft^2}{s^2}} = 1188$

This is a HUGH increase! As a percentage increase of 118800%. Hence choice of diameter is very important! The increase is because the diameter reduces by a factor of four and the velocity therefore increases by a factor of 16, and is squared!

Problem 8.108

Given: System for measuring pressure drop for water flow in smooth pipe supplies water from an overhead constant-head tank.



System includes:

- square-edged entrance
- two 45° standard elbows
- two 90° standard elbows
- fully open gate valve
- pipe length $L = 9.8$ m, diameter $D = 15.9$ mm

Find: elevation of water surface in supply tank above pipe discharge needed to achieve $Re_D = 10^5$.

Solution:

$Re = \frac{\rho D \bar{V}}{\mu} = \frac{D\bar{V}}{\nu}$ Assume $T = 20°C$, $\nu = 1.00 \times 10^{-6}\ m^2/s$ (Table A.8).

For $Re = 10^5$, $\bar{V} = \frac{Re\,\nu}{D} = 10^5 \times 1.0 \times 10^{-6} \frac{m^2}{s} \times \frac{1}{15.9 \times 10^{-3}\ m} = 6.29\ m/s$

Basic equations: $\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T}$ (8.29)

$h_{l_T} = h_l + h_{l_m}$, $h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$, $h_{l_m} = f\frac{\bar{V}^2}{2}\sum\frac{L_e}{D} + K_{ent}\frac{\bar{V}^2}{2}$

Assumptions: (1) $p_1 = p_2 = p_{atm}$ (2) $\bar{V}_1 = 0$ (3) $\alpha_2 = 1.0$

Then,

$$g(z_1 - z_2) = \frac{\bar{V}_2^2}{2} + f\frac{L}{D}\frac{\bar{V}^2}{2} + f\frac{\bar{V}^2}{2}\sum\left(\frac{L_e}{D}\right) + K_{ent}\frac{\bar{V}^2}{2}$$

$$d = (z_1 - z_2) = \frac{\bar{V}_2^2}{2g}\left[1 + f\frac{L}{D} + 2f\left(\frac{L_e}{D}\right)_{45°el} + 2f\left(\frac{L_e}{D}\right)_{90°el} + f\left(\frac{L_e}{D}\right)_{gv} + K_{ent}\right]$$

From Table 8.2 $K_{ent} = 0.5$

From Table 8.4 $(L_e/D)_{45°el} = 16$, $(L_e/D)_{90°el} = 30$, $(L_e/D)_{gv} = 8$

For $Re = 10^5$ in smooth pipe, $f = 0.018$ (Fig. 8.13)

Thus

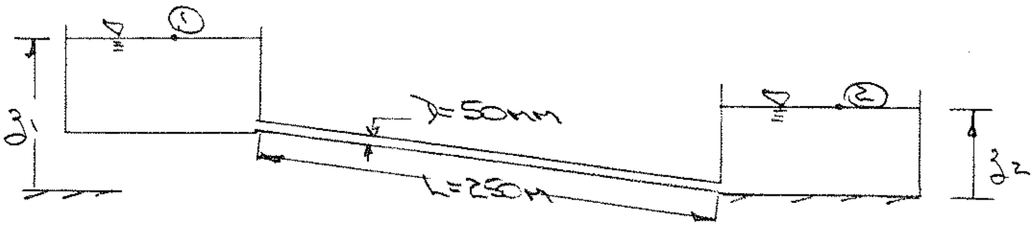
$$d = \frac{1}{2} \times (6.29)^2 \frac{m^2}{s^2} \times \frac{s^2}{9.81\,m}\left[1 + 0.018 \times \frac{9.8 \times 10^3}{15.9} + 2(0.018)16 + 2(0.018)30 + 0.018(8) + 0.5\right]$$

$d = 29.0\ m$ ← d

This value of d indicates that it will not be possible to obtain a value of $Re = 10^5$ in the flow system. The maximum value of Re will be considerably less than 10^5.

Problem 8.109

Given: Water flow by gravity between two reservoirs through straight galvanized iron pipe. Required flow rate is Q



Plot: required elevation difference Δz vs Q for $0 \le Q \le 0.01\ m^3/s$

Estimate: fraction of Δz due to minor losses

Plot: (a) Δz and (b) minor loss/total loss versus Q

Solution:

Apply the energy equation for steady incompressible flow between sections ① and ②.

Basic equations: $\left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T}$ (8.29)

$h_{l_T} = h_l + h_{lm}$; $h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$; $h_{lm} = K_{ent}\frac{\bar{V}^2}{2} + K_{exit}\frac{\bar{V}^2}{2}$

Assumptions: (1) $p_1 = p_2 = p_{atm}$ (given)

(2) $\bar{V}_1 = \bar{V}_2 \approx 0$

(3) square edged entrance

For square edged entrance (Table 8.2) $K_{ent} = 0.5$; also $K_{exit} = 1.0$

For water at 20°C, $\nu = 1.00 \times 10^{-6}\ m^2/s$ (Table A.8)

$$Re = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V}D}{\nu} = \frac{Q}{A}\frac{D}{\nu} = \frac{QD}{\nu}\cdot\frac{4}{\pi D^2} = \frac{4Q}{\pi \nu D} \quad \text{------} \quad (1)$$

To plot Δz vs Q

$$\bar{V} = \frac{4Q}{\pi D^2} = 509\, Q\ (m^3/s)$$

$$\Delta z = \frac{\bar{V}^2}{2g}\left[K_{ent} + K_{exit} + f\frac{L}{D}\right] = \frac{\bar{V}^2}{2g}\left[1.5 + 5000 f\right]$$

where $f = f(Re,\ e/D = 0.003)$

$$\frac{h_{lm}}{h_{l_T}} = \frac{K_{ent} + K_{exit}}{K_{ent} + K_{exit} + f\frac{L}{D}} = \frac{1.5}{1.5 + 5000f}$$

The ratio h_{lm}/h_{l_T} increases with increasing Re because f decreases with increasing Re.

Problem 8.109 [3]

8.109 Water is to flow by gravity from one reservoir to a lower one through a straight, inclined galvanized iron pipe. The pipe diameter is 50 mm, and the total length is 250 m. Each reservoir is open to the atmosphere. Plot the required elevation difference Δz as a function of flow rate Q, for Q ranging from 0 to 0.01 m^3/s. Estimate the fraction of Δz due to minor losses.

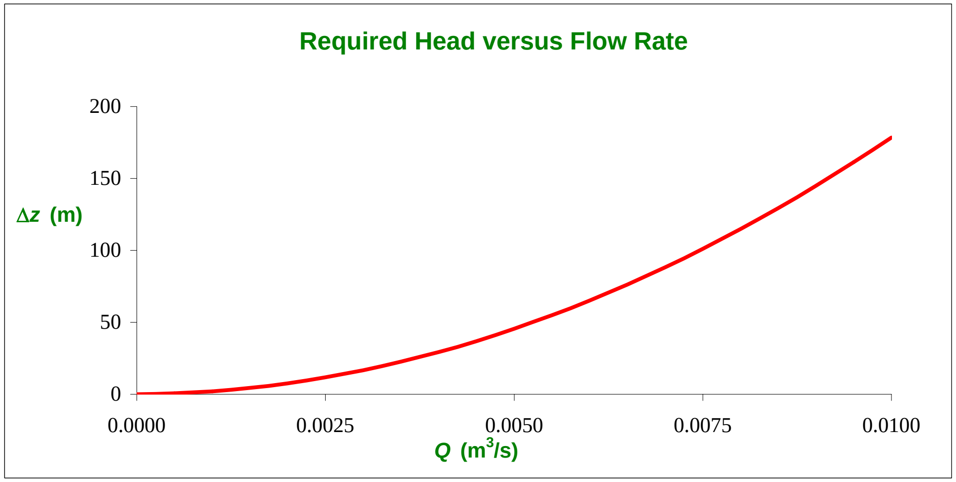
Given: Data on reservoir/pipe system

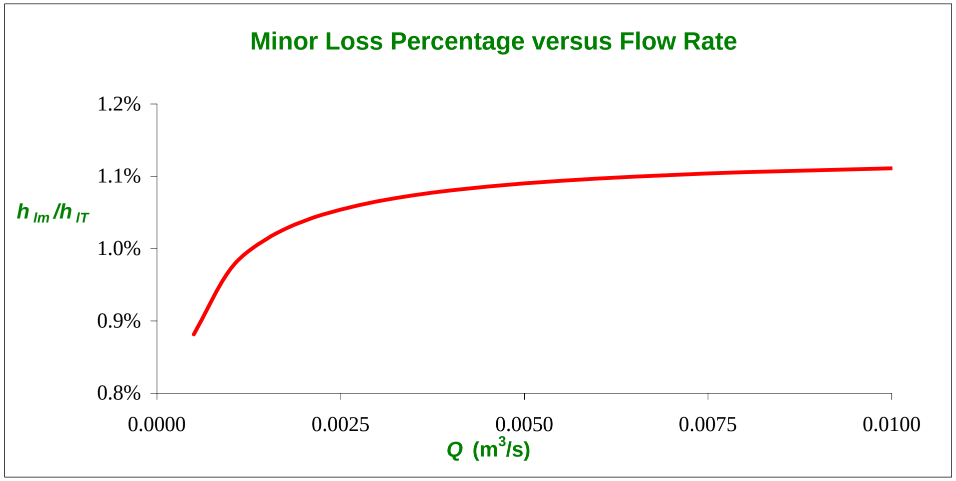
Find: Plot elevation as a function of flow rate; fraction due to minor losses

Solution:

L = 250 m
D = 50 mm
e/D = 0.003
K_{ent} = 0.5
K_{exit} = 1.0
ν = 1.01E-06 m^2/s

Q (m^3/s)	V (m/s)	Re	f	Δz (m)	h_{lm}/h_{lT}
0.0000	0.000	0.00E+00		0.000	
0.0005	0.255	1.26E+04	0.0337	0.562	0.882%
0.0010	0.509	2.52E+04	0.0306	2.04	0.972%
0.0015	0.764	3.78E+04	0.0293	4.40	1.01%
0.0020	1.02	5.04E+04	0.0286	7.64	1.04%
0.0025	1.27	6.30E+04	0.0282	11.8	1.05%
0.0030	1.53	7.56E+04	0.0279	16.7	1.07%
0.0035	1.78	8.82E+04	0.0276	22.6	1.07%
0.0040	2.04	1.01E+05	0.0275	29.4	1.08%
0.0045	2.29	1.13E+05	0.0273	37.0	1.09%
0.0050	2.55	1.26E+05	0.0272	45.5	1.09%
0.0055	2.80	1.39E+05	0.0271	54.8	1.09%
0.0060	3.06	1.51E+05	0.0270	65.1	1.10%
0.0065	3.31	1.64E+05	0.0270	76.2	1.10%
0.0070	3.57	1.76E+05	0.0269	88.2	1.10%
0.0075	3.82	1.89E+05	0.0269	101	1.10%
0.0080	4.07	2.02E+05	0.0268	115	1.11%
0.0085	4.33	2.14E+05	0.0268	129	1.11%
0.0090	4.58	2.27E+05	0.0268	145	1.11%
0.0095	4.84	2.40E+05	0.0267	161	1.11%
0.0100	5.09	2.52E+05	0.0267	179	1.11%

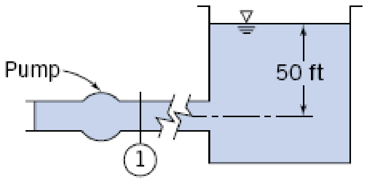




Problem 8.110

[2]

8.110 Water from a pump flows through a 9-in. diameter commercial steel pipe for a distance of 4 miles from the pump discharge to a reservoir open to the atmosphere. The level of the water in the reservoir is 50 ft above the pump discharge, and the average speed of the water in the pipe is 10 ft/s. Calculate the pressure at the pump discharge.



Given: Flow from pump to reservoir

Find: Pressure at pump discharge

Solution:

Basic equations

$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT} \qquad h_{lT}=h_l+h_{lm}=f\cdot\frac{L}{D}\cdot\frac{V_1^2}{2}+K_{exit}\cdot\frac{V_1^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) V_2 <<

Hence the energy equation between Point 1 and the free surface (Point 2) becomes

$$\left(\frac{p_1}{\rho}+\frac{V^2}{2}\right)-\left(g\cdot z_2\right)=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+K_{exit}\cdot\frac{V^2}{2}$$

Solving for p_1

$$p_1=\rho\cdot\left(g\cdot z_2-\frac{V^2}{2}+f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+K_{exit}\cdot\frac{V^2}{2}\right)$$

From Table A.7 (68ºF) $\quad \rho=1.94\cdot\frac{slug}{ft^3} \qquad \nu=1.08\times10^{-5}\cdot\frac{ft^2}{s}$

$$Re=\frac{V\cdot D}{\nu} \qquad Re=10\cdot\frac{ft}{s}\times\frac{9}{12}\cdot ft\times\frac{s}{1.08\times10^{-5}\cdot ft^2} \qquad Re=6.94\times10^5 \qquad \text{Turbulent}$$

For commercial steel pipe $e=0.00015\cdot ft$ (Table 8.1) so $\frac{e}{D}=0.0002$

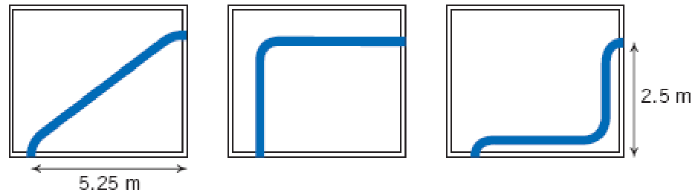
Flow is turbulent: Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f=0.0150$

For the exit $\quad K_{exit}=1.0 \quad$ so we find $\quad p_1=\rho\cdot\left(g\cdot z_2+f\cdot\frac{L}{D}\cdot\frac{V^2}{2}\right)$

$$p_1=1.94\cdot\frac{slug}{ft^3}\times\left[32.2\cdot\frac{ft}{s^2}\times50\cdot ft+.0150\times\frac{4\cdot mile}{0.75\cdot ft}\times\frac{5280\cdot ft}{1mile}\times\frac{1}{2}\times\left(10\cdot\frac{ft}{s}\right)^2\right]\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad p_1=4.41\times10^4\cdot\frac{lbf}{ft^2} \qquad p_1=306\cdot psi$$

Problem 8.111

8.111 A 5-cm diameter potable water line is to be run through a maintenance room in a commercial building. Three possible layouts for the water line are proposed, as shown. Which is the best option, based on minimizing losses? Assume galvanized iron, and a flow rate of 350 L/min.



(a) Two miter bends (b) A standard elbow (c) Three standard elbows

Given:

Flow through three different layouts

Find:

Which has minimum loss

Solution:

Basic equations

$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT} \qquad h_{lT}=h_l+h_{lm}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+\sum_{Minor}\left(f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}\right)$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) Ignore additional length of elbows

For a flow rate of $Q = 350\cdot\frac{L}{min}$ $\quad V=\frac{Q}{A}=\frac{4\cdot Q}{\pi\cdot D^2}$ $\quad V=\frac{4}{\pi}\times 350\cdot\frac{L}{min}\times\frac{0.001\cdot m^3}{1\cdot L}\times\frac{1\cdot min}{60\cdot s}\times\left(\frac{1}{0.05\cdot m}\right)^2$ $\quad V = 2.97\frac{m}{s}$

For water at 20ºC $\quad \nu = 1.01\times 10^{-6}\cdot\frac{m^2}{s}$ $\quad Re=\frac{V\cdot D}{\nu}$ $\quad Re = 2.97\cdot\frac{m}{s}\times 0.05\cdot m\times\frac{s}{1.01\times 10^{-6}\cdot m^2}$ $\quad Re = 1.47\times 10^5$

Flow is turbulent. From Table 8.1 $\quad e = 0.15\cdot mm$ $\quad \frac{e}{D}=6.56\times 10^{-4}$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f = 0.0201$

For Case (a) $\quad L=\sqrt{5.25^2+2.5^2}\cdot m$ $\quad L = 5.81\,m$ $\quad$ Two 45º miter bends (Fig. 8.16), for each $\frac{L_e}{D}=13$

Hence the energy equation is $\quad \frac{p_1}{\rho}-\frac{p_2}{\rho}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+2\cdot f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}$

Solving for Δp $\quad \Delta p=p_1-p_2=\rho\cdot f\cdot\frac{V^2}{2}\cdot\left(\frac{L}{D}+2\cdot\frac{L_e}{D}\right)$

$$\Delta p = 1000\cdot\frac{kg}{m^3}\times .0201\times\left(2.97\cdot\frac{m}{s}\right)^2\times\left(\frac{5.81}{0.05}+2\cdot 13\right)\times\frac{N\cdot s^2}{kg\cdot m} \qquad \Delta p = 25.2\,kPa$$

For Case (b) $\quad L=(5.25+2.5)\cdot m$ $\quad L = 7.75\,m$ $\quad$ One standard 90º elbow (Table 8.4) $\frac{L_e}{D}=30$

Hence the energy equation is $\quad \frac{p_1}{\rho}-\frac{p_2}{\rho}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}$

Solving for Δp $\quad \Delta p=p_1-p_2=\rho\cdot f\cdot\frac{V^2}{2}\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)$

$$\Delta p = 1000 \cdot \frac{kg}{m^3} \times .0201 \times \left(2.97 \cdot \frac{m}{s}\right)^2 \times \left(\frac{7.75}{0.05} + 30\right) \times \frac{N \cdot s^2}{kg \cdot m} \qquad \Delta p = 32.8\,kPa$$

For Case (c) $\quad L = (5.25 + 2.5) \cdot m \quad L = 7.75\,m \quad$ Three standard 90º elbows, for each $\frac{L_e}{D} = 30$

Hence the energy equation is $\quad \frac{p_1}{\rho} - \frac{p_2}{\rho} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} + 3 \cdot f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2}$

Solving for Δp $\quad \Delta p = p_1 - p_2 = \rho \cdot f \cdot \frac{V^2}{2} \cdot \left(\frac{L}{D} + 3 \cdot \frac{L_e}{D}\right)$

$$\Delta p = 1000 \cdot \frac{kg}{m^3} \times .0201 \times \left(2.97 \cdot \frac{m}{s}\right)^2 \times \left(\frac{7.75}{0.05} + 3 \times 30\right) \times \frac{N \cdot s^2}{kg \cdot m} \qquad \Delta p = 43.4\,kPa$$

Hence we conclude Case (a) is the best and Case (c) is the worst

Problem 8.112

Given: Air at a flowrate of 35 m^3/min at standard conditions in a smooth duct 0.3 m square.

Find: Pressure drop in mm H_2O per 30 m of horizontal duct.

Solution: Apply the energy equation for steady, incompressible flow that is uniform at each section. Use hydraulic diameter.

Basic equation: $$\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + g z_2 + f\frac{L}{D_h}\frac{\bar{V}^2}{2} + h_{\ell m}\,; \quad D_h = \frac{4A}{P_w}$$

(The $\bar{V}^2/2$ terms are struck out by (1), the gz terms by (2), and $h_{\ell m} = 0$ by (3).)

Assumptions: (1) $\bar{V}_1 = \bar{V}_2$

(2) Horizontal

(3) $h_{\ell m} = 0$

Then

$$\Delta p = p_1 - p_2 = f\frac{L}{D_h}\,\rho\frac{\bar{V}^2}{2}$$

From continuity, $$\bar{V} = \frac{Q}{A} = 35\frac{m^3}{min} \times \frac{1}{(0.3)^2 m^2} \times \frac{min}{60\,sec} = 6.48\ m/s$$

$$D_h = \frac{4A}{P_w} = 4 \times (0.3)^2 m^2 \times \frac{1}{4(0.3)m} = 0.3\,m\ ; \quad \nu = 1.45 \times 10^{-5}\ m^2/s \ \text{(Table A.10)}$$

$$Re = \frac{\bar{V} D_h}{\nu} = 6.48\frac{m}{s} \times 0.3\,m \times \frac{s}{1.46 \times 10^{-5} m^2} = 1.33 \times 10^5$$

$$f = 0.017 \quad \text{(Fig. 8.13)}$$

Then $$\Delta p = \frac{0.017}{2} \times \frac{30m}{0.3m} \times 1.23\frac{kg}{m^3} \times (6.48)^2\frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 43.9\ N/m^2$$ Δp

For a manometer, $\Delta p = \rho_{H_2O}\, g\, \Delta h$

$$\Delta h = \frac{\Delta p}{\rho_{H_2O}\, g} = 43.9\frac{N}{m^2} \times \frac{m^3}{999\,kg} \times \frac{s^2}{9.81\,m} \times \frac{kg \cdot m}{N \cdot s^2} = 0.00448\ m$$

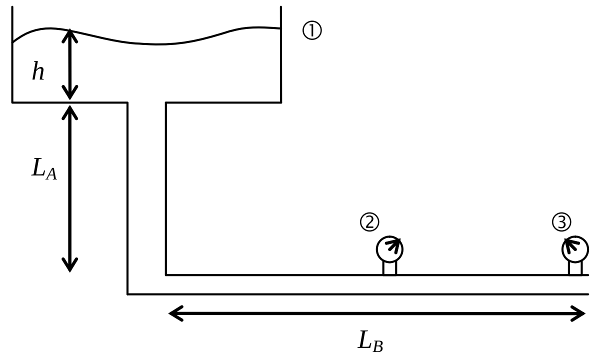
Thus

$\Delta h = 4.48$ mm H_2O (per 30 m of duct) Δh

(This is Δp expressed in mm of water.)

Problem 8.113

8.113 A pipe friction experiment is to be designed, using water, to reach a Reynolds number of 100,000. The system will use 5 cm smooth PVC pipe from a constant-head tank to the flow bench and 20 m of smooth 2.5 cm PVC line mounted horizontally for the test section. The water level in the constant-head tank is 0.5 m above the entrance to the 5 cm PVC line. Determine the required average speed of water in the 2.5 cm pipe. Estimate the feasibility of using a constant-head tank. Calculate the pressure difference expected between taps 5 m apart in the horizontal test section.



Given: Pipe friction experiment

Find: Required average speed; Estimate feasibility of constant head tank; Pressure drop over 5 m

Solution:

Basic equations $$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT} \qquad h_{lT}=h_A+h_B=f_A\cdot\frac{L_A}{D_A}\cdot\frac{V_A^2}{2}+f_B\cdot\frac{L_B}{D_B}\cdot\frac{V_B^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) Ignore minor losses

We wish to have $Re_B = 10^5$

Hence, from $Re_B=\frac{V_B\cdot D_B}{\nu}$ $\quad V_B=\frac{Re_B\cdot\nu}{D_B}$ $\quad$ and for water at 20ºC $\quad \nu = 1.01\times 10^{-6}\cdot\frac{m^2}{s}$

$$V_B = 10^5\times 1.01\times 10^{-6}\cdot\frac{m^2}{s}\times\frac{1}{0.025\cdot m} \qquad V_B = 4.04\frac{m}{s}$$

We will also need $V_A = V_B\cdot\left(\frac{D_B}{D_A}\right)^2$ $\quad V_A = 4.04\cdot\frac{m}{s}\times\left(\frac{2.5}{5}\right)^2$ $\quad V_A = 1.01\frac{m}{s}$

$$Re_A=\frac{V_A\cdot D_A}{\nu} \qquad Re_A = 1.01\cdot\frac{m}{s}\times 0.05\cdot m\times\frac{s}{1.01\times 10^{-6}\cdot m^2} \qquad Re_A = 5\times 10^4$$

Both tubes have turbulent flow

For PVC pipe (from Googling!) $e = 0.0015\cdot mm$

For tube A Given $$\frac{1}{\sqrt{f_A}} = -2.0\cdot\log\left(\frac{\frac{e}{D_A}}{3.7}+\frac{2.51}{Re_A\cdot\sqrt{f_A}}\right) \qquad f_A = 0.0210$$

For tube B Given $$\frac{1}{\sqrt{f_B}} = -2.0\cdot\log\left(\frac{\frac{e}{D_B}}{3.7}+\frac{2.51}{Re_B\cdot\sqrt{f_B}}\right) \qquad f_B = 0.0183$$

Applying the energy equation between Points 1 and 3

$$g\cdot\left(L_A+h\right)-\frac{V_B^2}{2} = f_A\cdot\frac{L_A}{D_A}\cdot\frac{V_A^2}{2}+f_B\cdot\frac{L_B}{D_B}\cdot\frac{V_B^2}{2}$$

Solving for L_A

$$L_A = \frac{\frac{V_B^2}{2}\cdot\left(1 + f_B\cdot\frac{L_B}{D_B}\right) - g\cdot h}{\left(g - \frac{f_A}{D_A}\cdot\frac{V_A^2}{2}\right)}$$

$$L_A = \frac{\frac{1}{2}\times\left(4.04\cdot\frac{m}{s}\right)^2\times\left(1 + 0.0183\times\frac{20}{0.025}\right) - 9.81\cdot\frac{m}{s^2}\times 0.5\cdot m}{9.81\cdot\frac{m}{s^2} - \frac{0.0210}{2}\times\frac{1}{0.05\cdot m}\times\left(1.01\cdot\frac{m}{s}\right)^2} \qquad L_A = 12.8\,m$$

Most ceilings are about 3.5 m or 4 m, so this height is IMPRACTICAL

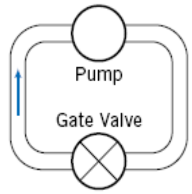
Applying the energy equation between Points 2 and 3

$$\left(\frac{p_2}{\rho} + \frac{V_B^2}{2}\right) - \left(\frac{p_3}{\rho} + \frac{V_B^2}{2}\right) = f_B\cdot\frac{L}{D_B}\cdot\frac{V_B^2}{2} \qquad \text{or} \qquad \Delta p = \rho\cdot f_B\cdot\frac{L}{D_B}\cdot\frac{V_B^2}{2}$$

$$\Delta p = 1000\cdot\frac{kg}{m^3}\times\frac{0.0183}{2}\times\frac{5\cdot m}{0.025\cdot m}\times\left(4.04\cdot\frac{m}{s}\right)^2\times\frac{N\cdot s^2}{kg\cdot m} \qquad \Delta p = 29.9\cdot kPa$$

Problem 8.114

8.114 A system for testing variable-output pumps consists of the pump, four standard elbows, and an open gate valve forming a closed circuit as shown. The circuit is to absorb the energy added by the pump. The tubing is 75-mm diameter cast iron, and the total length of the circuit is 20 m. Plot the pressure difference required from the pump for water flow rates Q ranging from 0.01 m^3/s to 0.06 m^3/s.



Given: Data on circuit

Find: Plot pressure difference for a range of flow rates

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT}=\sum_{major}h_l+\sum_{minor}h_{lm} \qquad (8.29)$$

$$Re=\frac{\rho\cdot V\cdot D}{\mu} \qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \quad (8.34) \qquad h_{lm}=K\cdot\frac{V^2}{2} \quad (8.40a) \qquad h_{lm}=f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} \quad (8.40b)$$

$$f=\frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes for the circuit (1 = pump inlet, 2 = pump outlet)

$$\frac{p_1-p_2}{\rho}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+4\cdot f\cdot L_{elbow}\cdot\frac{V^2}{2}+f\cdot L_{valve}\cdot\frac{V^2}{2} \qquad \text{or} \qquad \Delta p=\rho\cdot f\cdot\frac{V^2}{2}\cdot\left(\frac{L}{D}+4\cdot\frac{L_{elbow}}{D}+\frac{L_{valve}}{D}\right)$$

Given data:

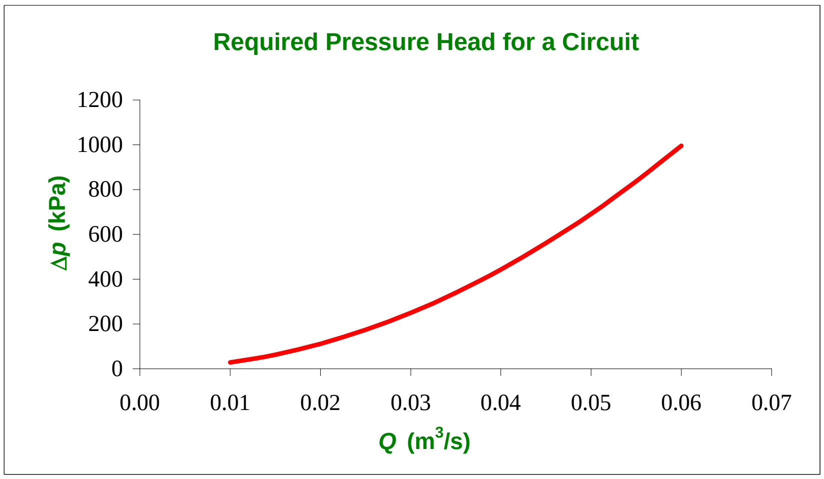
L = 20 m
D = 75 mm

Tabulated or graphical data:

e = 0.26 mm
(Table 8.1)
μ = 1.00E-03 $N.s/m^2$
ρ = 999 kg/m^3
(Appendix A)
Gate valve L_e/D = 8
Elbow L_e/D = 30
(Table 8.4)

Computed results:

Q (m^3/s)	V (m/s)	Re	f	Δp (kPa)
0.010	2.26	1.70E+05	0.0280	28.3
0.015	3.40	2.54E+05	0.0277	63.1
0.020	4.53	3.39E+05	0.0276	112
0.025	5.66	4.24E+05	0.0276	174
0.030	6.79	5.09E+05	0.0275	250
0.035	7.92	5.94E+05	0.0275	340
0.040	9.05	6.78E+05	0.0274	444
0.045	10.2	7.63E+05	0.0274	561
0.050	11.3	8.48E+05	0.0274	692
0.055	12.4	9.33E+05	0.0274	837
0.060	13.6	1.02E+06	0.0274	996



Problem 8.115

8.115 Consider flow of standard air at 1250 ft^3/min. Compare the pressure drop per unit length of a round duct with that for rectangular ducts of aspect ratio 1, 2, and 3. Assume that all ducts are smooth, with cross-sectional areas of 1 ft^2.

Given: Same flow rate in various ducts

Find: Pressure drops of each compared to round duct

Solution:

Basic equations $\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^{\ 2}}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^{\ 2}}{2} + g\cdot z_2\right) = h_l$ $\quad D_h = \frac{4\cdot A}{P_w}$ $\quad e = 0$ (Smooth)

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) Ignore minor losses

The energy equation simplifies to

$$\Delta p = p_1 - p_2 = \rho\cdot f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2} \qquad \text{or} \qquad \frac{\Delta p}{L} = \rho\cdot\frac{f}{D_h}\cdot\frac{V^2}{2}$$

But we have $V = \frac{Q}{A}$ $\quad V = 1250\cdot\frac{ft^3}{min}\times\frac{1\cdot min}{60\cdot s}\times\frac{1}{1\cdot ft^2}$ $\quad V = 20.8\frac{ft}{s}$

From Table A.9 $\nu = 1.62\times10^{-4}\cdot\frac{ft^2}{s}$ $\quad \rho = 0.00234\cdot\frac{slug}{ft^3}$ $\quad$ at 68ºF

Hence $Re = \frac{V\cdot D_h}{\nu}$ $\quad Re = 20.8\cdot\frac{ft}{s}\times\frac{s}{1.62\times10^{-4}\cdot ft^2}\times D_h = 1.284\times10^5\cdot D_h$ $\quad$ (D_h in ft)

For a round duct $D_h = D = \sqrt{\frac{4\cdot A}{\pi}}$ $\quad D_h = \sqrt{\frac{4}{\pi}\times 1\cdot ft^2}$ $\quad D_h = 1.13\,ft$

For a rectangular duct $D_h = \frac{4\cdot A}{P_w} = \frac{4\cdot b\cdot h}{2\cdot(b+h)} = \frac{2\cdot h\cdot ar}{1+ar}$ $\quad$ where $\quad ar = \frac{b}{h}$

But $h = \frac{b}{ar}$ $\quad$ so $\quad h^2 = \frac{b\cdot h}{ar} = \frac{A}{ar}$ $\quad$ or $\quad h = \sqrt{\frac{A}{ar}}$ $\quad$ and $\quad D_h = \frac{2\cdot\sqrt{ar}}{1+ar}\cdot\sqrt{A}$

The results are:

Round $D_h = 1.13\cdot ft$ $\quad Re = 1.284\times10^5\cdot\frac{1}{ft}\cdot D_h$ $\quad Re = 1.45\times10^5$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D_h}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f = 0.0167$ $\quad \frac{\Delta p}{L} = \rho\cdot\frac{f}{D_h}\cdot\frac{V^2}{2}$ $\quad \frac{\Delta p}{L} = 7.51\times10^{-3}\cdot\frac{lbf}{ft^3}$

$ar = 1$ $\quad D_h = \frac{2\cdot\sqrt{ar}}{1+ar}\cdot\sqrt{A}$ $\quad D_h = 1\,ft$ $\quad Re = 1.284\times10^5\cdot\frac{1}{ft}\cdot D_h$ $\quad Re = 1.28\times10^5$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D_h}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f = 0.0171$ $\quad \frac{\Delta p}{L} = \rho\cdot\frac{f}{D_h}\cdot\frac{V^2}{2}$ $\quad \frac{\Delta p}{L} = 8.68\times10^{-3}\cdot\frac{lbf}{ft^3}$

Hence the square duct experiences a percentage increase in pressure drop of $\dfrac{8.68\times10^{-3}-7.51\times10^{-3}}{7.51\times10^{-3}} = 15.6\%$

$ar = 2$ $\quad D_h = \dfrac{2\cdot\sqrt{ar}}{1+ar}\cdot\sqrt{A}$ $\quad D_h = 0.943\,\text{ft}$ $\quad Re = 1.284\times10^{5}\cdot\dfrac{1}{\text{ft}}\cdot D_h$ $\quad Re = 1.21\times10^{5}$

Given $\quad \dfrac{1}{\sqrt{f}} = -2.0\cdot\log\left(\dfrac{\frac{e}{D_h}}{3.7}+\dfrac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f = 0.0173$ $\quad \dfrac{\Delta p}{L} = \rho\cdot\dfrac{f}{D_h}\cdot\dfrac{V^2}{2}$ $\quad \dfrac{\Delta p}{L} = 9.32\times10^{-3}\cdot\dfrac{\text{lbf}}{\text{ft}^3}$

Hence the 2 x 1 duct experiences a percentage increase in pressure drop of $\dfrac{9.32\times10^{-3}-7.51\times10^{-3}}{7.51\times10^{-3}} = 24.1\%$

$ar = 3$ $\quad D_h = \dfrac{2\cdot\sqrt{ar}}{1+ar}\cdot\sqrt{A}$ $\quad D_h = 0.866\,\text{ft}$ $\quad Re = 1.284\times10^{5}\cdot\dfrac{1}{\text{ft}}\cdot D_h$ $\quad Re = 1.11\times10^{5}$

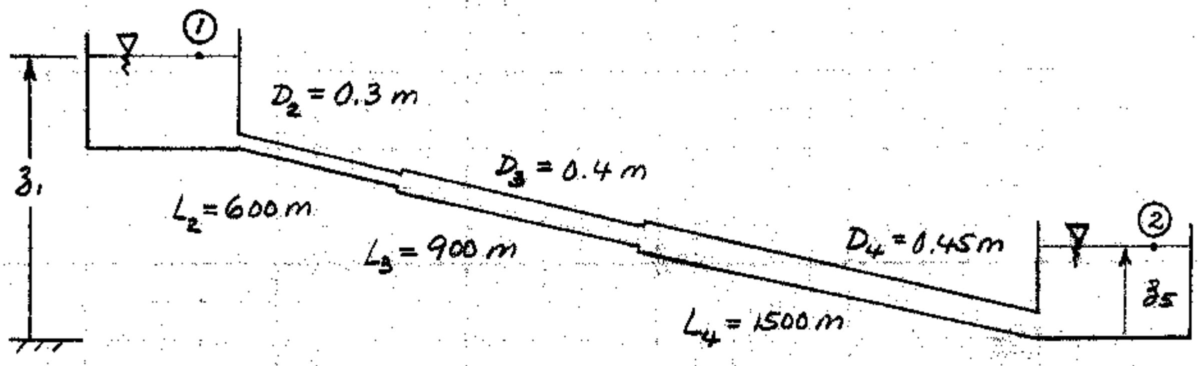
Given $\quad \dfrac{1}{\sqrt{f}} = -2.0\cdot\log\left(\dfrac{\frac{e}{D_h}}{3.7}+\dfrac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f = 0.0176$ $\quad \dfrac{\Delta p}{L} = \rho\cdot\dfrac{f}{D_h}\cdot\dfrac{V^2}{2}$ $\quad \dfrac{\Delta p}{L} = 0.01\cdot\dfrac{\text{lbf}}{\text{ft}^3}$

Hence the 3 x 1 duct experiences a percentage increase in pressure drop of $\dfrac{0.01-7.51\times10^{-3}}{7.51\times10^{-3}} = 33.2\%$

Note that f varies only about 7%; the large change in $\Delta p/L$ is primarily due to the $1/D_h$ factor

Problem 8.116

Given: Reservoirs connected by three clean, cast iron pipes in series. The flow is water at $0.11\ m^3/s$ and $15°C$.



Find: Elevation difference, $z_1 - z_5$

Solution: Apply the energy equation for steady, incompressible flow that is uniform at each section.

Basic equations: $\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + g z_1 = \frac{p_5}{\rho} + \frac{\bar{V}_5^2}{2} + g z_5 + h_{lT}$ (terms cancelled by (1), (2))

$$h_{lT} = \sum f \frac{L}{D} \frac{\bar{V}^2}{2} + h_{lm}\ ; \quad h_{lm} = K_{ent} \frac{\bar{V}_2^2}{2} + \sum h_{l_{exp}} + K_{exit} \frac{\bar{V}_4^2}{2}$$

($\sum h_{l_{exp}} \simeq 0$ (3))

Assumptions: (1) $p_1 = p_5 = p_{atm}$
(2) $\bar{V}_1 = \bar{V}_5 \simeq 0$
(3) Neglect $h_{l_{exp}}$ at pipe joints (note all minor losses are probably small due to long lengths of straight pipe sections, but we will check)

For non-smooth pipe, $f = f(Re, e/D)$. $\mu = 1.1 \times 10^{-3}\ N\cdot s/m^2$ from Table A.8.

Section ②: $e/D_2 = 0.26\ mm / 300\ mm = 0.00087$ (for cast iron, $e = 0.26\ mm$, Table 8.1)

$$\bar{V}_2 = \frac{Q}{A_2} = 0.11 \frac{m^3}{s} \times \frac{4}{\pi} \frac{1}{(0.3)^2 m^2} = 1.56\ m/s$$

$$Re_2 = \frac{\rho \bar{V}_2 D_2}{\mu} = 999 \frac{kg}{m^3} \times 1.56 \frac{m}{s} \times 0.3\ m \times \frac{m^2}{1.14 \times 10^{-3}\ N\cdot s} \times \frac{N\cdot s^2}{kg\cdot m} = 4.10 \times 10^5$$

From Fig 8.13, $f_2 = 0.020$

Section ③: $e/D_3 = 0.00065$

$$\bar{V}_3 = \frac{Q}{A_3} = 0.11 \frac{m^3}{s} \times \frac{4}{\pi} \frac{1}{(0.4)^2 m^2} = 0.875\ m/s$$

$$Re_3 = \frac{\rho \bar{V}_3 D_3}{\mu} = 999 \frac{kg}{m^3} \times 0.875 \frac{m}{s} \times 0.4\ m \times \frac{m^2}{1.14 \times 10^{-3}\ N\cdot s} \times \frac{N\cdot s^2}{kg\cdot m} = 3.07 \times 10^5$$

From Fig. 8.13, $f_3 = 0.019$

Section ④: $e/D_4 = 0.00058$

Problem 8.116

$$\bar{V}_4 = \frac{Q}{A_4} = 0.11\,\frac{m^3}{s} \times \frac{4}{\pi}\,\frac{1}{(0.45)^2 m^2} = 0.692\ m/s$$

$$Re_4 = \frac{\rho \bar{V}_4 D_4}{\mu} = 999\,\frac{kg}{m^3} \times 0.692\,\frac{m}{s} \times 0.45\,m \times \frac{m^2}{1.14\times10^{-3}\,N\cdot s} \times \frac{N\cdot s^2}{kg\cdot m} = 2.73\times10^5$$

From Fig. 8.13, $f_4 = 0.0185$

Then
$$\sum f\frac{L}{D}\frac{\bar{V}^2}{2} = 0.020 \times \frac{600\,m}{0.3\,m} \times \frac{1}{2}(1.56)^2\frac{m^2}{s^2} + 0.019 \times \frac{900\,m}{0.4\,m} \times \frac{1}{2}(0.875)^2\frac{m^2}{s^2}$$

$$+\,0.0185 \times \frac{1500\,m}{0.45\,m} \times \frac{1}{2}(0.692)^2\frac{m^2}{s^2} = 79.8\ m^2/s^2$$

The minor loss coefficients are $K_{ent} = 0.5$ (Table 8.2) and $K_{exit} = 1.0$. Thus,

$$h_{\ell m} = K_{ent}\frac{\bar{V}_2^2}{2} + K_{exit}\frac{\bar{V}_4^2}{2}$$

$$h_{\ell m} = 0.5 \times \frac{1}{2} \times (1.56)^2\frac{m^2}{s^2} + 1.0 \times \frac{1}{2} \times (0.692)^2\frac{m^2}{s^2} = 0.848\ m^2/s^2$$

Therefore minor losses are roughly 1 percent of the frictional losses, so they may be neglected. Thus from the energy equation

$$z_1 - z_5 = \sum f\frac{L}{D}\frac{\bar{V}^2}{2g} = 79.8\,\frac{m^2}{s^2} \times \frac{s^2}{9.81\,m} = 8.13\ m$$

$z_1 - z_5$

Problem 8.117

8.117 Water, at volume flow rate $Q = 0.75\ \text{ft}^3/\text{s}$, is delivered by a fire hose and nozzle assembly. The hose ($L = 250$ ft, $D = 3$ in and $e/D = 0.004$) is made up of four 60 ft sections joined by couplings. The entrance is square-edged; the minor loss coefficient for each coupling is $K_c = 0.5$, based on mean velocity through the hose. The nozzle loss coefficient is $K_n = 0.02$, based on velocity in the exit jet, of $D_2 = 1$ in. diameter. Estimate the supply pressure required at this flow rate.

Given: Flow through fire hose and nozzle

Find: Supply pressure

Solution:

Basic equations

$$\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_{lT} \qquad h_{lT} = h_l + h_{lm} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} + \sum_{\text{Minor}}\left(K\cdot\frac{V^2}{2}\right)$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) $p_2 = p_{atm}$ so $p_2 = 0$ gage

Hence the energy equation between Point 1 at the supply and the nozzle exit (Point n); let the velocity in the hose be V

$$\frac{p_1}{\rho} - \frac{V_n^2}{2} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} + \left(K_e + 4\cdot K_c\right)\cdot\frac{V^2}{2} + K_n\cdot\frac{V_n^2}{2}$$

From continuity

$$V_n = \left(\frac{D}{D_2}\right)^2\cdot V \qquad \text{and} \qquad V = \frac{Q}{A} = \frac{4\cdot Q}{\pi\cdot D^2} \qquad V = \frac{4}{\pi}\times 0.75\cdot\frac{\text{ft}^3}{\text{s}}\times\frac{1}{\left(\frac{1}{4}\cdot\text{ft}\right)^2} \qquad V = 15.3\frac{\text{ft}}{\text{s}}$$

Solving for p_1

$$p_1 = \frac{\rho\cdot V^2}{2}\cdot\left[f\cdot\frac{L}{D} + K_e + 4\cdot K_c + \left(\frac{D}{D_2}\right)^4\cdot\left(1 + K_n\right)\right]$$

From Table A.7 (68ºF)

$$\rho = 1.94\cdot\frac{\text{slug}}{\text{ft}^3} \qquad \nu = 1.08\times 10^{-5}\cdot\frac{\text{ft}^2}{\text{s}}$$

$$Re = \frac{V\cdot D}{\nu} \qquad Re = 15.3\cdot\frac{\text{ft}}{\text{s}}\times\frac{3}{12}\cdot\text{ft}\times\frac{\text{s}}{1.08\times 10^{-5}\cdot\text{ft}^2} \qquad Re = 3.54\times 10^5 \qquad \text{Turbulent}$$

For the hose

$$\frac{e}{D} = 0.004$$

Flow is turbulent: Given

$$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0287$$

$$p_1 = \frac{1}{2}\times 1.94\cdot\frac{\text{slug}}{\text{ft}^3}\times\left(15.3\cdot\frac{\text{ft}}{\text{s}}\right)^2\times\left[0.0287\times\frac{250}{\frac{1}{4}} + 0.5 + 4\times 0.5 + \left(\frac{3}{1}\right)^4\times(1 + 0.02)\right]\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

$$p_1 = 2.58\times 10^4\cdot\frac{\text{lbf}}{\text{ft}^2} \qquad p_1 = 179\cdot\text{psi}$$

Problem 8.118

8.118 Data were obtained from measurements on a vertical section of old, corroded, galvanized iron pipe of 25 mm inside diameter. At one section the pressure was $p_1 = 700$ kPa (gage); at a second section, 6 m lower, the pressure was $p_2 = 525$ kPa (gage). The volume flow rate of water was 0.2 m³/min. Estimate the relative roughness of the pipe. What percent savings in pumping power would result if the pipe were restored to its new, clean relative roughness?

Given: Flow down corroded iron pipe

Find: Pipe roughness; Power savings with new pipe

Solution:

Basic equations

$$\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_l \qquad h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) No minor losses

Hence the energy equation becomes

$$\left(\frac{p_1}{\rho} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + g\cdot z_2\right) = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

and $V = \frac{Q}{A} = \frac{4\cdot Q}{\pi\cdot D^2}$ $\qquad V = \frac{4}{\pi}\times 0.2\cdot\frac{m^3}{min}\times\frac{1\cdot min}{60\cdot s}\times\frac{1}{(0.025\cdot m)^2}$ $\qquad V = 6.79\frac{m}{s}$

In this problem we can compute directly f and Re, and hnece obtain e/D

Solving for f $\qquad f = \frac{2\cdot D}{L\cdot V^2}\cdot\left(\frac{p_1 - p_2}{\rho} + g(z_1 - z_2)\right)$

$$f = 2\times\frac{0.025}{6}\times\left(\frac{s}{6.79\cdot m}\right)^2\times\left[(700 - 525)\times 10^3\cdot\frac{N}{m^2}\times\frac{m^3}{1000\cdot kg}\times\frac{kg\cdot m}{s^2\cdot N} + 9.81\cdot\frac{m}{s^2}\times 6\cdot m\right] \qquad f = 0.0423$$

From Table A.8 (20ºF) $\quad \nu = 1.01\times 10^{-6}\cdot\frac{m^2}{s}$ $\qquad Re = \frac{V\cdot D}{\nu}$ $\qquad Re = 6.79\cdot\frac{m}{s}\times 0.025\cdot m\times\frac{s}{1.01\times 10^{-6}\cdot m^2}$ $\qquad Re = 1.68\times 10^5$

Flow is turbulent: Given $\qquad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\qquad \frac{e}{D} = 0.0134$

New pipe (Table 8.1) $\quad e = 0.15\cdot mm$ $\qquad \frac{e}{D} = 0.006$

Given $\qquad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\qquad f = 0.0326$

In this problem $\qquad \Delta p = p_1 - p_2 = \rho\cdot\left[g\cdot(z_2 - z_1) + f\cdot\frac{L}{D}\cdot\frac{V^2}{2}\right]$

Hence $\qquad \Delta p_{new} = 1000\cdot\frac{kg}{m^3}\times\left[9.81\cdot\frac{m}{s^2}\times(-6\cdot m) + \frac{0.0326}{2}\times\frac{6}{0.025}\times\left(6.79\cdot\frac{m}{s}\right)^2\right]\times\frac{N\cdot s^2}{kg\cdot m}$ $\qquad \Delta p_{new} = 121\cdot kPa$

Compared to $\Delta p_{old} = 175\cdot kPa$ we find $\qquad \frac{\Delta p_{old} - \Delta p_{new}}{\Delta p_{old}} = 30.6\cdot\%$

Problem 8.119

8.119 Flow in a tube may alternate between laminar and turbulent states for Reynolds numbers in the transition zone. Design a bench-top experiment consisting of a constant-head cylindrical transparent plastic tank with depth graduations, and a length of plastic tubing (assumed smooth) attached at the base of the tank through which the water flows to a measuring container. Select tank and tubing dimensions so that the system is compact, but will operate in the transition zone range. Design the experiment so that you can easily increase the tank head from a low range (laminar flow) through transition to turbulent flow, and vice versa. (Write instructions for students on recognizing when the flow is laminar or turbulent.) Generate plots (on the same graph) of tank depth against Reynolds number, assuming laminar or turbulent flow.

Given: Proposal for bench top experiment

Find: Design it; Plot tank depth versus Re

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1\cdot\frac{V_1^{\,2}}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\cdot\frac{V_2^{\,2}}{2} + g\cdot z_2\right) = h_{lT} = \sum_{major} h_l + \sum_{minor} h_{lm} \qquad (8.29)$$

$$Re = \frac{\rho\cdot V\cdot D}{\mu} \qquad h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \quad (8.34) \qquad h_{lm} = K\cdot\frac{V^2}{2} \quad (8.40a) \qquad h_{lm} = f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} \quad (8.40b)$$

$$f = \frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes

$$g\cdot H - \alpha\cdot\frac{V^2}{2} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} + K\cdot\frac{V^2}{2}$$

This can be solved explicity for reservoir height H

$$H = \frac{V^2}{2\cdot g}\cdot\left(\alpha + f\cdot\frac{L}{D} + K\right)$$

Choose data:

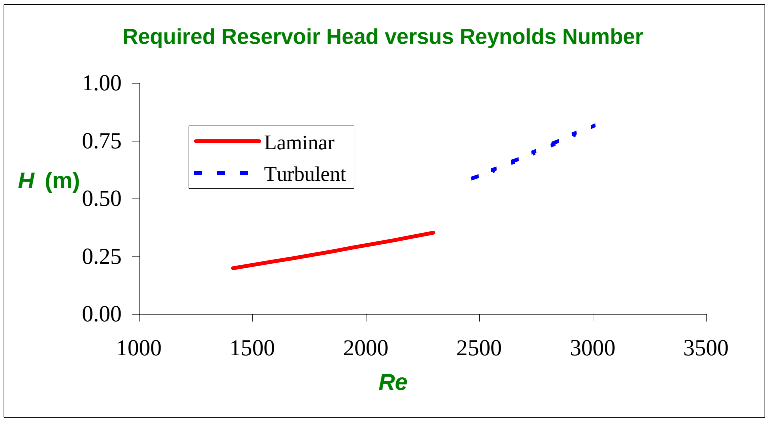
L = 1.0 m
D = 3.0 mm
e = 0.0 mm
α = 2 (Laminar)
= 1 (Turbulent)

Tabulated or graphical data:

μ = 1.00E-03 N.s/m^2
ρ = 999 kg/m^3
(Appendix A)
K_{ent} = 0.5 (Square-edged)
(Table 8.2)

Computed results:

Q (L/min)	*V* (m/s)	*Re*	Regime	*f*	*H* (m)
0.200	0.472	1413	Laminar	0.0453	0.199
0.225	0.531	1590	Laminar	0.0403	0.228
0.250	0.589	1767	Laminar	0.0362	0.258
0.275	0.648	1943	Laminar	0.0329	0.289
0.300	0.707	2120	Laminar	0.0302	0.320
0.325	0.766	2297	Laminar	0.0279	0.353
0.350	0.825	2473	Turbulent	0.0462	0.587
0.375	0.884	2650	Turbulent	0.0452	0.660
0.400	0.943	2827	Turbulent	0.0443	0.738
0.425	1.002	3003	Turbulent	0.0435	0.819
0.450	1.061	3180	Turbulent	0.0428	0.904



The flow rates are realistic, and could easily be measured using a tank/timer system
The head required is also realistic for a small-scale laboratory experiment
Around Re = 2300 the flow may oscillate between laminar and turbulent:
Once turbulence is triggered (when H > 0.353 m), the resistance to flow increases requiring H >0.587 m to maintain; hence the flow reverts to laminar, only to trip over again to turbulent! This behavior will be visible: the exit flow will switch back and forth between smooth (laminar) and chaotic (turbulent)

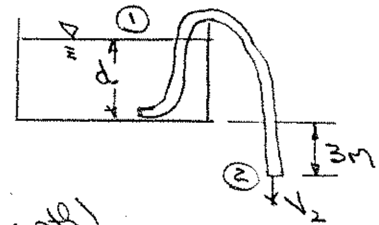
Problem 8.120

Given: Small swimming pool is drained using a garden hose.

Hose: $D = 20\,\text{mm}$, $L = 30\,\text{m}$

$e = 0.2\,\text{mm}$

$\bar{V}_2 = 1.2\,\text{m/s}$



Find: Water depth at instant shown.
If the flow were inviscid (at this depth) what would be the velocity

Solution:

Apply the energy equation for steady incompressible flow between sections ① and ②

Basic equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) = h_{l_T} \qquad (8.29)$$

$$h_{l_T} = h_l + h_{l_m}\ ; \quad h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}\ ; \quad h_{l_m} = K_{ent} \frac{\bar{V}^2}{2}$$

Assumptions: (1) $p_1 = p_2 = p_{atm}$

(2) $\bar{V}_1 = 0$, $\alpha_2 = 1.0$

(3) square edged entrance

Then

$$z_1 - z_2 = d + 3\,\text{m} = f \frac{L}{D} \frac{\bar{V}^2}{2g} + K_{ent} \frac{\bar{V}^2}{2g} + \frac{\bar{V}_2^2}{2g} = \frac{\bar{V}^2}{2g}\left[f \frac{L}{D} + K_{ent} + 1\right]$$

$$\therefore\ d = \frac{\bar{V}^2}{2g}\left[f \frac{L}{D} + K_{ent} + 1\right] - 3\,\text{m} \qquad (1)$$

For square edged entrance (Table 8.2) $K_{ent} = 0.5$

$$Re = \frac{\rho \bar{V} D}{\mu} = \frac{\bar{V} D}{\nu} = 0.020\,\text{m} \times 1.2\,\frac{\text{m}}{\text{s}} \times \frac{1}{1.00 \times 10^{-6}}\,\frac{\text{s}}{\text{m}^2} = 2.4 \times 10^4 \qquad \left\{\begin{array}{l}\text{assume } T = 20^\circ\text{C} \\ \text{Table A.8}\end{array}\right\}$$

$e/D = 0.2/20 = 0.01$. From Fig. 8.13, $f = 0.04$

Then from Eq. 1

$$d = \frac{(1.2)^2}{2} \frac{\text{m}^2}{\text{s}^2} \times \frac{1}{9.81}\frac{\text{s}^2}{\text{m}}\left[0.04 \times \frac{30}{0.02} + 0.5 + 1\right] - 3\,\text{m} = 1.51\,\text{m} \qquad d$$

For frictionless flow, $h_{l_T} = f \frac{L}{D} \frac{\bar{V}^2}{2} + K_{ent} \frac{\bar{V}^2}{2} = 0$ and Eq. 1 gives

$$d = \frac{\bar{V}^2}{2g} - 3\,\text{m}$$

and

$$\bar{V} = \left[2g\,(d + 3\,\text{m})\right]^{1/2} = \left[2 \times 9.81\,\frac{\text{m}}{\text{s}^2}\,(1.51 + 3)\,\text{m}\right]^{1/2}$$

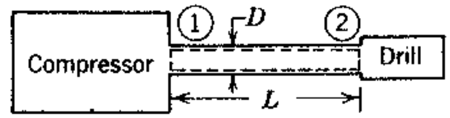
$$\bar{V} = 9.41\,\text{m/s} \qquad \bar{V}_{inviscid}$$

Given: Air flow through a line, of length L and diameter $D = 40$ mm.

$p_1 = 670$ kPa(g) $p_2 = 650$ kPa (g)

$T_1 = 40°C$ $\dot{m} = 0.25$ kg/s

$\rho \approx$ constant



Find: Allowable length of hose

Solution:

Computing equation: $\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) = h_{l_T} = h_l + h_{l_m}$ (8.29)

where $h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$ $\quad h_{l_m} = K \frac{\bar{V}^2}{2}$

For $\rho = c$, then $\bar{V}_1 = \bar{V}_2$, since $A_1 = A_2$. Since p_1 and p_2 are given, neglect minor losses. Assume $\alpha_1 = \alpha_2$ and neglect elevation changes. Then Eq. 8.29 can be written as

$$\frac{p_1 - p_2}{\rho} = f \frac{L}{D} \frac{\bar{V}^2}{2} \quad \text{or} \quad L = \frac{(p_1 - p_2)}{\rho} \frac{2D}{f \bar{V}^2}$$

The density is

$$\rho = \rho_1 = \frac{p_1}{RT_1} = 7.91 \times 10^5 \frac{N}{m^2} \times \frac{kg \cdot K}{287 N \cdot m} \times \frac{1}{313 K} = 8.81 \text{ kg/m}^3$$

From continuity

$$\bar{V} = \frac{\dot{m}}{\rho A} = \frac{4 \dot{m}}{\pi \rho D^2} = \frac{4}{\pi} \times 0.25 \frac{kg}{sec} \times \frac{m^3}{8.81 kg} \times \frac{1}{(0.04)^2 m^2} = 22.6 \text{ m/sec}$$

For air at 40°C, $\mu = 1.91 \times 10^{-5}$ kg/m·s (Table A.10), so

$$Re = \frac{\rho \bar{V} D}{\mu} = 8.81 \frac{kg}{m^3} \times 22.6 \frac{m}{sec} \times 0.04 m \times \frac{m \cdot sec}{1.91 \times 10^{-5} kg} = 4.17 \times 10^5$$

Assume smooth pipe; then from Fig. 8.13, $f = 0.0134$

Substituting gives

$$L = \frac{(p_1 - p_2)}{\rho} \frac{2D}{f \bar{V}^2}$$

$$= 20 \times 10^3 \frac{N}{m^2} \times 2 \times 0.04 m \times \frac{m^3}{8.81 kg} \times \frac{1}{0.0134} \times \frac{sec^2}{(22.6)^2 m^2} \times \frac{kg \cdot m}{N \cdot sec^2}$$

$L = 26.5$ m ← L

Problem 8.122

[3]

8.122 What flow rate (gpm) will be produced in a 4-in. diameter water pipe for which there is a pressure drop of 40 psi over a 300 ft length? The pipe roughness is 0.01 ft. The water is at 68°F.

Given: Flow in horizontal pipe

Find: Flow rate

Solution:

Basic equations $\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^{\ 2}}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^{\ 2}}{2} + g\cdot z_2\right) = h_l \qquad h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) No minor losses

Hence the energy equation becomes

$$\frac{p_1}{\rho} - \frac{p_2}{\rho} = \frac{\Delta p}{\rho} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Solving for V $\quad V = \sqrt{\frac{2\cdot D\cdot\Delta p}{L\cdot\rho\cdot f}} \qquad V = \frac{k}{\sqrt{f}} \qquad (1)$

$$k = \sqrt{\frac{2\cdot D\cdot\Delta p}{L\cdot\rho}} \qquad k = \sqrt{2\times\frac{\frac{1}{3}}{300}\times 40\cdot\frac{lbf}{in^2}\times\left(\frac{12\cdot in}{1\cdot ft}\right)^2\times\frac{ft^3}{1.94\cdot slug}\times\frac{slug\,ft}{s^2\cdot lbf}} \qquad k = 2.57\cdot\frac{ft}{s}$$

We also have $\quad Re = \frac{V\cdot D}{\nu} \quad$ or $\quad Re = c\cdot V \quad (2) \quad$ where $\quad c = \frac{D}{\nu}$

From Table A.7 (68ºF) $\quad \nu = 1.08\times 10^{-5}\cdot\frac{ft^2}{s} \qquad c = \frac{1}{3}\cdot ft\times\frac{s}{1.08\times 10^{-5}\cdot ft^2} \qquad c = 3.09\times 10^4\cdot\frac{s}{ft}$

In addition $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad (3)$

Equations 1, 2 and 3 form a set of simultaneous equations for V, Re and f

Make a guess for f $\quad f = 0.1 \quad$ then $\quad V = \frac{k}{\sqrt{f}} \qquad V = 8.12\cdot\frac{ft}{s} \qquad Re = c\cdot V \qquad Re = 2.51\times 10^5$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0573 \qquad V = \frac{k}{\sqrt{f}} \qquad V = 10.7\cdot\frac{ft}{s} \qquad Re = c\cdot V \qquad Re = 3.31\times 10^5$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0573 \qquad V = \frac{k}{\sqrt{f}} \qquad V = 10.7\cdot\frac{ft}{s} \qquad Re = c\cdot V \qquad Re = 3.31\times 10^5$

The flow rate is then $\quad Q = V\cdot\frac{\pi\cdot D^2}{4} \qquad Q = 10.7\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{1}{3}\cdot ft\right)^2\times\frac{7.48\cdot gal}{1\cdot ft^3}\times\frac{60\cdot s}{1\cdot min} \qquad Q = 419\cdot gpm$

Note that we could use *Excel*'s *Solver* for this problem

Problem 8.123

8.123 When you drink you beverage with a straw, you need to overcome both gravity and friction in the straw. Estimate the fraction of the total effort you put into quenching your thirst of each factor, making suitable assumptions about the liquid and straw properties, and your drinking rate (for example, how long it would take you to drink a 12 oz drink if you drank it all in one go (quite a feat with a straw). Is the flow laminar or turbulent? (Ignore minor losses.)

Given: Drinking of a beverage

Find: Fraction of effort of drinking of friction and gravity

Solution:

Basic equations $$\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_l \qquad h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) No minor losses

Hence the energy equation becomes, between the bottom of the straw (Point 1) and top (Point 2)

$$g\cdot z_1 - \left(\frac{p_2}{\rho} + g\cdot z_2\right) = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$ where p_2 is the gage pressure in the mouth

The negative gage pressure the mouth must create is therefore due to two parts

$$p_{grav} = -\rho\cdot g\cdot\left(z_2 - z_1\right) \qquad p_{fric} = -\rho\cdot f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Assuming a person can drink 12 fluid ounces in 5 s $$Q = \frac{\frac{12}{128}\cdot gal}{5\cdot s} \times \frac{1\cdot ft^3}{7.48\cdot gal} \qquad Q = 2.51\times 10^{-3}\frac{ft^3}{s}$$

Assuming a straw is 6 in long diameter 0.2 in, with roughness $e = 5\times 10^{-5}in$ (from Googling!)

$$V = \frac{4\cdot Q}{\pi\cdot D^2} \qquad V = \frac{4}{\pi}\times 2.51\times 10^{-3}\frac{ft^3}{s}\times\left(\frac{1}{0.2\cdot in}\times\frac{12\cdot in}{1\cdot ft}\right)^2 \qquad V = 11.5\frac{ft}{s}$$

From Table A.7 (68ºF) $\nu = 1.08\times 10^{-5}\cdot\frac{ft^2}{s}$ (for water, but close enough)

$$Re = \frac{V\cdot D}{\nu} \qquad Re = 11.5\cdot\frac{ft}{s}\times\frac{0.2}{12}\cdot ft\times\frac{s}{1.08\times 10^{-5}ft^2} \qquad Re = 1.775\times 10^4$$

Given $$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0272$$

Then $$p_{grav} = -1.94\cdot\frac{slug}{ft^3}\times 32.2\cdot\frac{ft}{s^2}\times\frac{1}{2}\cdot ft\times\frac{lbf\cdot s^2}{slug\,ft} \qquad p_{grav} = -31.2\frac{lbf}{ft^2} \qquad p_{grav} = -0.217\,psi$$

and $$p_{fric} = -1.94\cdot\frac{slug}{ft^3}\times 0.0272\times\frac{6}{0.2}\times\frac{1}{2}\times\left(11.5\cdot\frac{ft}{s}\right)^2\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad p_{fric} = -105\frac{lbf}{ft^2} \qquad p_{fric} = -0.727\,psi$$

Hence the fraction due to friction is $\frac{p_{fric}}{p_{fric} + p_{grav}} = 77\%$ and gravity is $\frac{p_{grav}}{p_{fric} + p_{grav}} = 23\%$

These results will vary depending on assumptions, but it seems friction is significant!

Problem 8.124

[2]

Given: Gasoline flow in a horizontal pipeline at 15°C. The distance and pressure drop between pumping stations are 13 km and 1.4 MPa, respectively. The pipe is 0.6 m in diameter. Its roughness corresponds to galvanized iron.

Find: Volume flow rate.

Solution: Apply the energy equation for steady, incompressible flow that is uniform at each section.

Basic equation: $\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2 + h_{l_T}$; $h_{l_T} = f\frac{L}{D}\frac{\bar{V}^2}{2} + h_{l_m}$

(crossed-out terms: $\alpha_1\frac{\bar{V}_1^2}{2}$ = 0(1), gz_1 = 0(2), $\alpha_2\frac{\bar{V}_2^2}{2}$ = 0(1), gz_2 = 0(2), h_{l_m} = 0(1))

Assumptions: (1) Constant area pipe, so $\bar{V}_1 = \bar{V}_2$, $h_{l_m} = 0$
(2) Level, so $z_1 = z_2$

Thus

$$\frac{p_1 - p_2}{\rho} = f\frac{L}{D}\frac{\bar{V}^2}{2} \quad \text{or} \quad \bar{V} = \left[\frac{2D(p_1 - p_2)}{\rho f L}\right]^{\frac{1}{2}}$$

But $f = f(Re, e/D)$, and the Reynolds number is not known. Therefore iteration is required. Choose f in the fully-rough zone. From Table 8.1, $e = 0.15$ mm; $e/D = 0.00025$. Then from Fig. 8.13, $f \simeq 0.014$. {From Eq. 8.37, using Excel's solver, $f = 0.014$} Then,

$$\bar{V} = \left[2 \times 0.6\,m \times 1.4 \times 10^6 \frac{N}{m^2} \times \frac{m^3}{(0.72)1000\,kg} \times \frac{1}{0.014} \times \frac{1}{13 \times 10^3\,m} \times \frac{kg \cdot m}{N \cdot s^2}\right]^{\frac{1}{2}}$$

{SG = 0.72, Table A.2}

$\bar{V} = 3.58$ m/s

Now compute Re and check on guess for f. Choose $\mu \simeq 5 \times 10^{-4}$ N·s / m² (Fig. A.2).*

$$Re = \frac{\rho \bar{V} D}{\mu} = (0.72)1000 \frac{kg}{m^3} \times 3.58 \frac{m}{s} \times 0.6\,m \times \frac{m^2}{5 \times 10^{-4}\,N \cdot s} \times \frac{N \cdot s^2}{kg \cdot m} = 3.09 \times 10^6$$

Checking on Fig. 8.13, flow is essentially in the fully-rough zone, and initial guess for f was okay. Thus

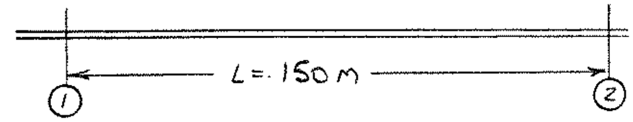
$$Q = \bar{V}A = 3.58 \frac{m}{s} \times \frac{\pi}{4}(0.6)^2 m^2 = 1.01 \; m^3/s$$

Q

* Note gasoline is between heptane and octane.

Problem 8.125

Given: Steady flow of water in 5 in. diameter, horizontal, cast-iron pipe.



$D = 125\text{ mm}$ $\quad L = 150\text{ m}$ $\quad \Delta p = p_1 - p_2 = 150\text{ kPa}$

Find: Volume flow rate.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation:
$$\left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) + h_{lT}$$
$$h_{lT} = h_l + h_{lm} = f\frac{L}{D}\frac{\bar{V}^2}{2} + K\frac{\bar{V}^2}{2}$$

Assumptions: (1) Fully developed flow: $\alpha_1\bar{V}_1^2 = \alpha_2\bar{V}_2^2$
(2) Horizontal: $z_1 = z_2$
(3) Constant area, so $K = 0$

Then
$$\frac{\Delta p}{\rho} = h_{lT} = f\frac{L}{D}\frac{\bar{V}^2}{2} \quad \text{so} \quad \bar{V} = \sqrt{\frac{2\Delta p D}{\rho f L}}$$

Since flow rate (hence Re and f) are unknown, must iterate. Guess a trial value of f in the fully rough zone. From Table 8.1, e = 0.26 mm

Then $e/D = \frac{0.26}{125} = 0.0021$. Then from Eq. 8.37*, $f = 0.0237$ for $Re \geq 6\times10^5$

$$\bar{V} = \left[2\times150\times10^3\frac{N}{m^2}\times0.125\,m\times\frac{m^3}{999\,kg}\times\frac{1}{0.0237}\times\frac{1}{150\,m}\times\frac{kg\cdot m}{N\cdot s^2}\right]^{1/2} = 3.25\,m/s$$

and, checking Re, with $\nu = 1.14\times10^{-6}\,m^2/s$ at $T = 15°C$ (Table A.8),

$$Re = \frac{\bar{V}D}{\nu} = 3.25\frac{m}{s}\times0.125\,m\times\frac{s}{1.14\times10^{-6}\,m^2} = 3.56\times10^5$$

The friction factor at this Re is still $f = 0.0242$ (2% error), so convergence is ok.

$$Q = \bar{V}A = 3.25\frac{m}{s}\times\frac{\pi}{4}\times(0.125\,m)^2 = 0.0399\,m^3/s$$ Q

Using $f = 0242$, $\bar{V} = 3.22\,m/s$ and $Q = 0.0395\,m^3/s$

* Value of f = 0.237 obtained using Excel's Solver (or Goal Seek)

Problem 8.126

Given: Steady flow of water through a cast iron pipe of diameter $D = 125$ mm. The pressure drop over a length of pipe, $L = 150$ m is $p_1 - p_2 = 150$ kPa. Section 2 is located 15 m above section 1.

Find: the volume flow rate, Q.

Solution: Apply the energy equation for steady, incompressible pipe flow

Computing equation:

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} \quad (1)$$

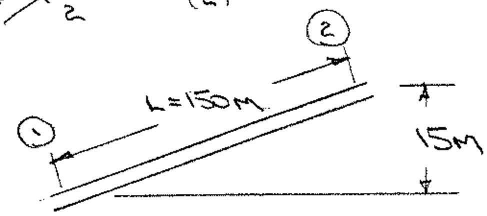
$$h_{l_T} = h_l + h_{l_m} = f\frac{L}{D}\frac{\bar{V}^2}{2} + K\frac{\bar{V}^2}{2} \approx 0 \,(4) \quad (2)$$

Assumptions: (1) $\bar{V}_1 = \bar{V}_2$ from continuity

(2) $\alpha_1 = \alpha_2$

(3) $z_2 - z_1 = 15$ m

(4) neglect minor losses



For cast iron pipe with $D = 125$ mm $\frac{\varepsilon}{D} = 0.0021$ ($\varepsilon = 0.26$ mm, Table 8.1)

Since $f = f(Re)$ and $\bar{V}$ is unknown, iteration will be required

From Eqs (1) and (2)

$$\left(\frac{p_1}{\rho} + gz_1\right) - \left(\frac{p_2}{\rho} + gz_2\right) = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

Then

$$f\bar{V}^2 = \frac{2D}{L}\left[\frac{(p_1 - p_2)}{\rho} + g(z_1 - z_2)\right]$$

$$f\bar{V}^2 = 2 \times \frac{0.125\text{m}}{150\text{m}}\left[150 \times 10^3 \frac{\text{N}}{\text{m}^2} \times \frac{\text{m}^3}{999\text{kg}} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right] + 9.81\frac{\text{m}}{\text{s}^2} \times (-15\text{m})$$

$$f\bar{V}^2 = 0.005 \text{ m}^2/\text{s}^2$$

Assume flow in fully rough region, $f = 0.0237$, then $\bar{V} = 0.46$ m/s

Check Re. Assume $T = 15°$C, $\nu = 1.14 \times 10^{-6}$ m²/s (Table A.8)

Then $Re = \frac{D\bar{V}}{\nu} = 0.125\text{m} \times 0.46\frac{\text{m}}{\text{s}} \times \frac{1}{1.14 \times 10^{-6}}\frac{\text{s}}{\text{m}^2} = 50{,}400$

From Eq. 8.37 with $Re = 50{,}400$, $e/D = 0.0021$, then using Excel's Solver (or Goal Seek)

$f = 0.0267$ and $\bar{V} = 0.433$ m/s

With this value of $\bar{V}$, $Re = 47{,}500$, $f = 0.0268$, $\bar{V} = 0.432$ m/s

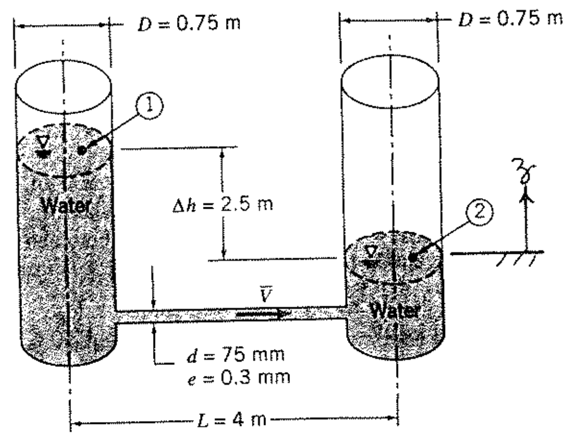
Then

$$Q = A\bar{V} = \frac{\pi D^2}{4}\bar{V} = \frac{\pi}{4}(0.125\text{m})^2 \times 0.432\frac{\text{m}}{\text{s}} = 0.0053 \text{ m}^3/\text{s} \qquad Q$$

Problem 8.127

Given: Two open standpipes shown. Water flows by gravity.

Find: Estimate of rate of change of water level in left standpipe.



Solution: Apply the energy equation for quasi-steady, incompressible pipe flow.

Computing equation:

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)=h_{lT};\quad h_{lT}=h_l+h_{lm}=\left[f\frac{(L-D)}{d}+K_{ent}+K_{exit}\right]\frac{\bar{V}^2}{2}$$

($z_2 = 0$)

From continuity, $A_1 V_1 = A_p \bar{V}$

Assumptions: (1) Neglect unsteady effects
(2) Incompressible flow
(3) $p_1 = p_2 = p_{atm}$
(4) $\bar{V}_1 = \bar{V}_2$ since diameters are equal

Then $g\Delta h = h_{lT} = \left[f\frac{(L-D)}{d}+K_{ent}+K_{exit}\right]\frac{\bar{V}^2}{2}$

Flow rate (hence speed) is unknown, so assume flow is in fully rough zone.

$\frac{e}{D} = \frac{0.3}{75} = 0.004$, so $f \approx 0.0285$ from Eq. 8.37 (using Excel's Solver or Goal Seek)

From Table 8.2, $K_{ent} = 0.5$; from Fig. 8.15, $K_{exit} = 1$. Then

$$\bar{V} = \left[\frac{2g\Delta h}{f\frac{(L-D)}{d}+K_{ent}+K_{exit}}\right]^{\frac{1}{2}} = \left[\frac{2\times 9.81\,\frac{m}{s^2}\times 2.5\,m}{0.028\left(\frac{4-0.75}{0.075}\right)+0.5+1.0}\right]^{\frac{1}{2}} = 4.23\ m/s$$

Check Re and f. For water at 20°C, $\nu = 1.00\times10^{-6}\ m^2/s$ (Table A.8)

$$Re = \frac{\bar{V}d}{\nu} = 4.23\,\frac{m}{s}\times 0.075\,m\times\frac{s}{1.00\times10^{-6}\,m^2} = 3.18\times10^5$$

From Equation 8.37, $f \approx 0.0288$, so this is satisfactory agreement. (~1%)

$$V_1 = \frac{A_p}{A_1}V_p = \left(\frac{d}{D}\right)^2 V_p = \left(\frac{0.075}{0.75}\right)^2\times 4.23\,\frac{m}{s} = 0.0423\ m/s\ \text{(down)}$$

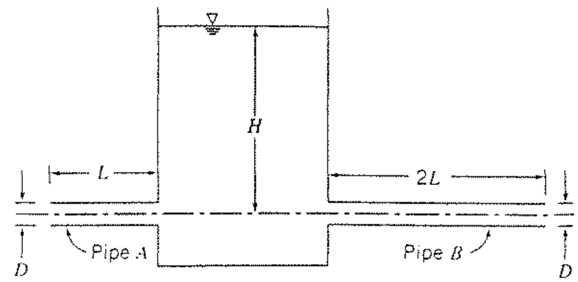
The water level in the left tank falls at about 42.3 mm/s

Problem 8.128

Given: Two galvanized iron pipes connected to large water reservoir as shown.

Determine: (a) which pipe will pass the larger flow rate (without calculations); justify

(b) the larger flow rate if $H = 10\,m$, $D = 50\,mm$, $L = 50\,m$



Solution:

Flow through each pipe is governed by the energy equation for steady incompressible flow

Basic equations: $$\left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} \qquad (8.29)$$

$$h_{l_T} = h_l + h_{l_m} = f\frac{L}{D}\frac{\bar{V}^2}{2} + K_{ent}\frac{\bar{V}^2}{2}$$

Assumptions: (1) $p_1 = p_2 = p_3 = p_{atm}$

(2) $\bar{V}_1 = 0$, $\alpha_2 = \alpha_3 = 1.0$

Then

$$g(z_1 - z_2) = h_{l_T} + \frac{\bar{V}_2^2}{2} = \frac{\bar{V}_2^2}{2}\left[f\frac{L}{D} + K_{ent} + 1\right] \qquad \text{(pipe A)}$$

$$g(z_1 - z_3) = h_{l_T} + \frac{\bar{V}_3^2}{2} = \frac{\bar{V}_3^2}{2}\left[f\frac{2L}{D} + K_{ent} + 1\right] \qquad \text{(pipe B)}$$

Since $z_1 - z_2 = z_1 - z_3$, then $\bar{V}_2 > \bar{V}_3$ and $Q_A > Q_B$

Through pipe A $\qquad gH = \frac{\bar{V}_2^2}{2}\left[f\frac{L}{D} + K_{ent} + 1\right]$

From Table 8.1 $e = 0.15\,m$ $\therefore e/D = 0.15/50 = 0.003$

Assume water at 20°C, $\nu = 1.00 \times 10^{-6}\,m^2/s$ (Table A.8)

Choose friction factor $f = 0.0263^*$ (in fully rough region)

Then $$\bar{V}_2 = \left\{\frac{2gH}{\left[f\frac{L}{D} + K_{ent} + 1\right]}\right\}^{1/2} = \left\{2 \times 9.81\frac{m}{s^2} \times 10\,m \times \frac{1}{\left[0.0263 \times \frac{50}{0.05} + 0.5 + 1.0\right]}\right\}$$

$$\bar{V}_2 = 2.66\,m/s$$

Check $Re = \frac{D\bar{V}}{\nu} = 0.05\,m \times 2.66\frac{m}{s} \times \frac{1}{1.00 \times 10^{-6}}\frac{s}{m^2} = 1.33 \times 10^5$

At this Re, $f = 0.0272^*$ and $\bar{V}_2 = 2.62\,m/s$

$$Q = A\bar{V} = \frac{\pi D^2}{4}\bar{V} = \frac{\pi}{4} \times (0.05\,m)^2 \times 2.62\frac{m}{s} = 5.14 \times 10^{-3}\,m^3/s \qquad Q_A$$

* Value obtained from Eq. 8.37, using Excel Solver (or Goal Seek)

Problem 8.129

8.129 Galvanized iron drainpipes of diameter 7.5 cm are located at the four corners of a building, but three of them become clogged with debris. Find the rate of downpour (cm/min) at which the single functioning drainpipe can no longer drain the roof. The building roof area is 500 m^2, and the height is 5 m. Assume the drainpipes are the same height as the building, and that both ends are open to atmosphere. Ignore minor losses.

Given: Galvanized drainpipe

Find: Maximum downpour it can handle

Solution:

Basic equations

$$\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_l \qquad h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) No minor losses

Hence the energy equation becomes

$$g\cdot z_1 - g\cdot z_2 = g\cdot(z_1 - z_2) = g\cdot h = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \qquad h = L$$

Solving for V

$$V = \sqrt{\frac{2\cdot D\cdot g\cdot h}{L\cdot f}} = \sqrt{\frac{2\cdot D\cdot g}{f}} \qquad V = \frac{k}{\sqrt{f}} \qquad (1)$$

$$k = \sqrt{2\cdot D\cdot g} \qquad k = \sqrt{2\times 0.075\cdot m\times 9.81\cdot\frac{m}{s^2}} \qquad k = 1.21\frac{m}{s}$$

We also have

$$Re = \frac{V\cdot D}{\nu} \quad \text{or} \quad Re = c\cdot V \qquad (2) \qquad \text{where} \quad c = \frac{D}{\nu}$$

From Table A.7 (20ºC)

$$\nu = 1.01\times 10^{-6}\cdot\frac{m^2}{s} \qquad c = 0.075\cdot m\times\frac{s}{1.01\times 10^{-6}\cdot m^2} \qquad c = 7.43\times 10^4\cdot\frac{s}{m}$$

In addition

$$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad (3) \qquad e = 0.15\,mm \qquad \text{(Table 8.1)}$$

Equations 1, 2 and 3 form a set of simultaneous equations for V, Re and f

Make a guess for f $\quad f = 0.01 \quad$ then $\quad V = \frac{k}{\sqrt{f}} \qquad V = 12.13\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 9.01\times 10^5$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0236 \qquad V = \frac{k}{\sqrt{f}} \qquad V = 7.90\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 5.86\times 10^5$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0237 \qquad V = \frac{k}{\sqrt{f}} \qquad V = 7.88\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 5.85\times 10^5$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0237 \qquad V = \frac{k}{\sqrt{f}} \qquad V = 7.88\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 5.85\times 10^5$

The flow rate is then $Q = V \cdot \frac{\pi \cdot D^2}{4}$ $Q = 7.88 \cdot \frac{m}{s} \times \frac{\pi}{4} \times (0.075 \cdot m)^2$ $Q = 0.0348 \cdot \frac{m^3}{s}$

The downpour rate is then $\frac{Q}{A_{roof}} = \frac{0.0348 \cdot \frac{m^3}{s}}{500 \cdot m^2} \times \frac{100 \cdot cm}{1 \cdot m} \times \frac{60 \cdot s}{1 \cdot min} = 0.418 \cdot \frac{cm}{min}$ The drain can handle 0.418 cm/min

Note that we could use *Excel*'s *Solver* for this problem

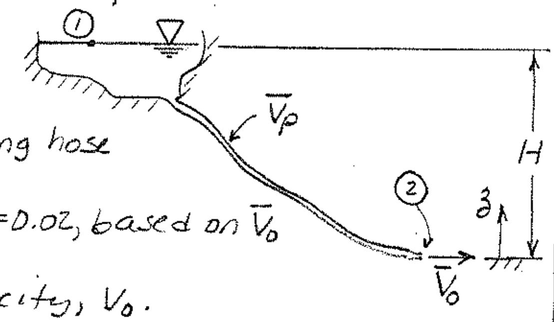
Problem 8.130

Given: Site for hydraulic mining, $H = 300\ m$, $L = 900\ m$.

Hose with $D = 75\ mm$, $e/D = 0.01$.

Couplings, $\frac{L_e}{D} = 20$, every 10 m along hose

Nozzle diameter, $d = 25\ mm$; $K = 0.02$, based on $\bar{V}_0$



Find: (a) Estimate maximum outlet velocity, V_0.
(b) Determine maximum force of jet on rock face.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) = h_{l_T}$$ (with $p_1 = 0(1)$, $\bar{V}_1 = 0(2)$, $p_2 = 0(3)$)

Assume: (1) $p_1 = 0$; (2) $\bar{V}_1 = 0$; (3) $p_2 = 0$; (4) $\alpha_2 = 1$; (5) $z_2 = 0$; (6) Fully-rough zone

Then $$gH = h_{l_T} + \frac{\bar{V}_2^2}{2} = f \frac{L}{D} \frac{\bar{V}_p^2}{2} + f \times 90 \frac{L_e}{D} \frac{\bar{V}_p^2}{2} + K \frac{\bar{V}_0^2}{2} + \frac{\bar{V}_2^2}{2}$$

From continuity $\bar{V}_p A_p = \bar{V}_0 A_0$; $\bar{V}_2 = \bar{V}_0 \frac{A_0}{A_2}$; $\bar{V}_2^2 = \bar{V}_0^2 \left(\frac{A_0}{A_2}\right)^2 = \bar{V}_0^2 \left(\frac{d}{D}\right)^4$

Substituting, $$gH = \left[f\left(\frac{L}{D} + 90 \frac{L_e}{D}\right)\left(\frac{d}{D}\right)^4 + 1 + K\right] \frac{\bar{V}_0^2}{2}$$

$$\bar{V}_0 = \left[\frac{2gH}{f\left(\frac{L}{D} + 90 \frac{L_e}{D}\right)\left(\frac{d}{D}\right)^4 + 1 + K}\right]^{1/2}$$; in fully-rough zone ($\frac{e}{D} = 0.01$), $f = 0.038^*$ (Eq. 8.37)

$$\bar{V}_0 = \left[2 \times 9.81 \frac{m}{s^2} \times 300\ m \times \frac{1}{0.038\left(\frac{900\ m}{0.075\ m} + 90\ (20)\right)\left(\frac{0.025}{0.075}\right)^4 + 1 + 0.02}\right]^{1/2} = 28.0\ m/s\ (\text{est.})$$

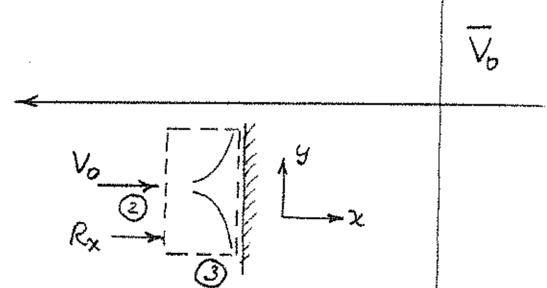
Check for fully-rough flow zone:

$Re = \frac{\bar{V}_p D}{\nu}$; $\bar{V}_p = \bar{V}_0 \left(\frac{d}{D}\right)^4 = 28.0 \frac{m}{s} \left(\frac{1}{3}\right)^4 = 0.346\ m/s$ {Assume $T = 20°C$}

$Re = 0.346 \frac{m}{sec} \times 0.075\ m \times \frac{1}{1 \times 10^{-6}\ m^2} = 2.60 \times 10^4$; at $\frac{e}{D} = 0.01$, $f = 0.040$ (Eq. 8.37)

The new estimate is

$$\bar{V}_0 = \sqrt{\frac{0.038}{0.040}}\ \bar{V}_0\ (\text{est}) = \sqrt{\frac{0.038}{0.040}}\ 28.0 \frac{m}{s} = 27.3\ m/s$$



Apply momentum to find force: CV is shown.

$$F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u \rho\, d\forall + \int_{CS} u \rho \vec{V} \cdot d\vec{A}$$

Assumptions: (1) No pressure forces
(2) $F_{B_x} = 0$
(3) Steady flow

Problem 8.130

(4) Uniform flow at each cross-section

Then

$$R_x = u_2\{-\rho \bar{V}_0 A_0\} + u_3\{+\rho \bar{V}_0 A_0\}$$

$$u_2 = \bar{V}_0 \qquad u_3 = 0$$

$$R_x = -\rho \bar{V}_0^2 A_0$$

The force <u>on</u> the rock face is

$$K_x = -R_x = \rho \bar{V}_0^2 A_0$$

$$= 999 \frac{kg}{m^3} \times (27.3)^2 \frac{m^2}{s^2} \times \frac{\pi}{4}(0.025)^2 m^2 \times \frac{N \cdot s^2}{kg \cdot m}$$

$$K_x = 365 \text{ N (to right)}$$

K_x

* Values of f obtained from Eq. 8.37 using Excel's Solver (or Goal Seek)

8.131 Investigate the effect of tube roughness on flow rate by computing the flow generated by a pressure difference $\Delta p = 100$ kPa applied to a length $L = 100$ m of tubing, with diameter $D = 25$ mm. Plot the flow rate against tube relative roughness e/D for e/D ranging from 0 to 0.05 (this could be replicated experimentally by progressively roughening the tube surface). Is it possible that this tubing could be roughened so much that the flow could be slowed to a laminar flow rate?

Given: Flow in a tube

Find: Effect of tube roughness on flow rate; Plot

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_{lT} = \sum_{major} h_l + \sum_{minor} h_{lm} \qquad (8.29)$$

$$Re = \frac{\rho\cdot V\cdot D}{\mu} \qquad h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \quad (8.34) \qquad h_{lm} = K\cdot\frac{V^2}{2} \quad (8.40a) \qquad h_{lm} = f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} \quad (8.40b)$$

$$f = \frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes for flow in a tube

$$p_1 - p_2 = \Delta p = \rho\cdot f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

This cannot be solved explicitly for velocity V, (and hence flow rate Q) because f depends on V; solution for a given relative roughness e/D requires iteration (or use of *Solver*)

Fluid is not specified: use water

Given data:

$\Delta p =$ 100 kPa

$D =$ 25 mm

$L =$ 100 m

Tabulated or graphical data:

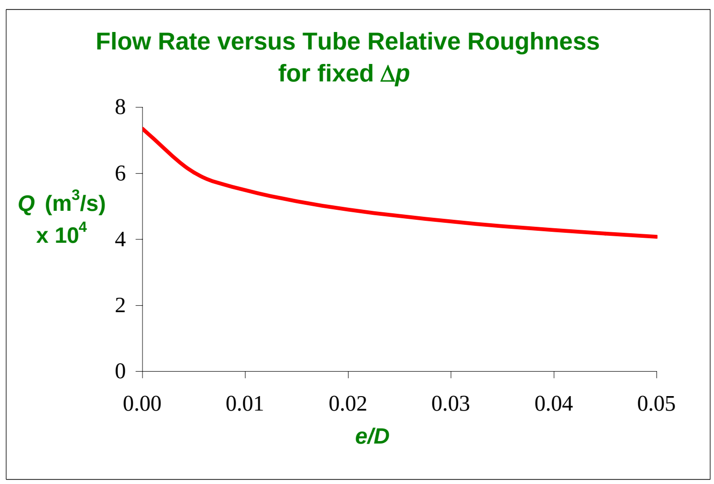
$\mu =$ 1.00E-03 N.s/m^2

$\rho =$ 999 kg/m^3

(Water - Appendix A)

Computed results:

e/D	V (m/s)	Q (m^3/s) x 10^4	Re	Regime	f	Δp (kPa)	Error
0.000	1.50	7.35	37408	Turbulent	0.0223	100	0.0%
0.005	1.23	6.03	30670	Turbulent	0.0332	100	0.0%
0.010	1.12	5.49	27953	Turbulent	0.0400	100	0.0%
0.015	1.05	5.15	26221	Turbulent	0.0454	100	0.0%
0.020	0.999	4.90	24947	Turbulent	0.0502	100	0.0%
0.025	0.959	4.71	23939	Turbulent	0.0545	100	0.0%
0.030	0.925	4.54	23105	Turbulent	0.0585	100	0.0%
0.035	0.897	4.40	22396	Turbulent	0.0623	100	0.0%
0.040	0.872	4.28	21774	Turbulent	0.0659	100	0.0%
0.045	0.850	4.17	21224	Turbulent	0.0693	100	0.0%
0.050	0.830	4.07	20730	Turbulent	0.0727	100	0.0%



It is not possible to roughen the tube sufficiently to slow the flow down to a laminar flow for this Δp. Even a relative roughness of 0.5 (a physical impossibility!) would not work.

Problem 8.132

8.132 Investigate the effect of tube length on water flow rate by computing the flow generated by a pressure difference $\Delta p = 100$ kPa applied to a length L of smooth tubing, of diameter $D = 25$ mm. Plot the flow rate against tube length for flow ranging from low speed laminar to fully turbulent.

Given: Flow in a tube

Find: Effect of tube length on flow rate; Plot

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_{lT} = \sum_{major} h_l + \sum_{minor} h_{lm} \qquad (8.29)$$

$$Re = \frac{\rho \cdot V \cdot D}{\mu} \qquad h_l = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \quad (8.34) \qquad h_{lm} = K \cdot \frac{V^2}{2} \quad (8.40a) \qquad h_{lm} = f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2} \quad (8.40b)$$

$$f = \frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes for flow in a tube

$$p_1 - p_2 = \Delta p = \rho \cdot f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$$

This cannot be solved explicitly for velocity V, (and hence flow rate Q) because f depends on V; solution for a given L requires iteration (or use of *Solver*)

Fluid is not specified: use water

Given data:

$\Delta p =$ 100 m

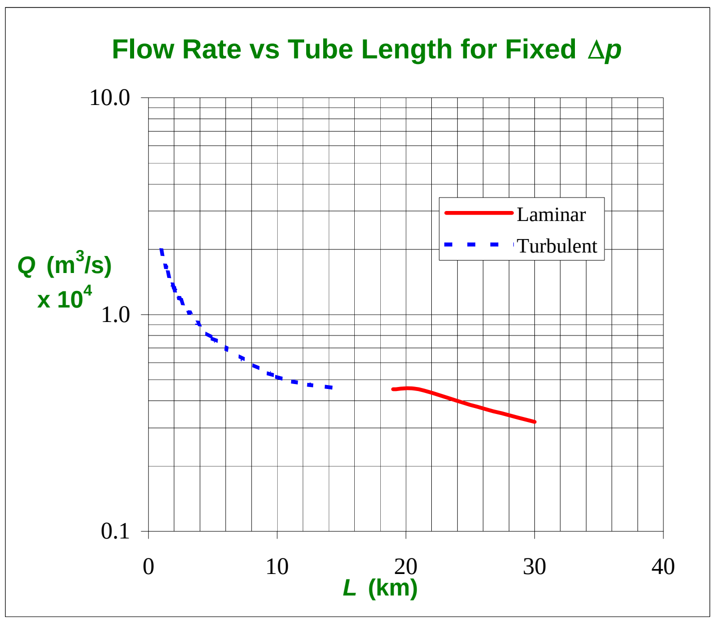
$D =$ 25 mm

Tabulated or graphical data:

$\mu =$ 1.00E-03 $N.s/m^2$

$\rho =$ 999 kg/m^3

(Water - Appendix A)



Computed results:

L (km)	V (m/s)	Q (m^3/s) x 10^4	Re	Regime	f	Δp (kPa)	Error
1.0	0.40	1.98	10063	Turbulent	0.0308	100	0.0%
1.5	0.319	1.56	7962	Turbulent	0.0328	100	0.0%
2.0	0.270	1.32	6739	Turbulent	0.0344	100	0.0%
2.5	0.237	1.16	5919	Turbulent	0.0356	100	0.0%
5.0	0.158	0.776	3948	Turbulent	0.0401	100	0.0%
10	0.105	0.516	2623	Turbulent	0.0454	100	0.0%
15	0.092	0.452	2300	Turbulent	0.0473	120	20.2%
19	0.092	0.452	2300	Laminar	0.0278	90	10.4%
21	0.092	0.452	2300	Laminar	0.0278	99	1.0%
25	0.078	0.383	1951	Laminar	0.0328	100	0.0%
30	0.065	0.320	1626	Laminar	0.0394	100	0.0%

The "critical" length of tube is between 15 and 20 km.
For this range, the fluid is making a transition between laminar and turbulent flow, and is quite unstable. In this range the flow oscillates between laminar and turbulent; no consistent solution is found (i.e., an Re corresponding to turbulent flow needs an f assuming laminar to produce the Δp required, and vice versa!)

More realistic numbers (e.g., tube length) are obtained for a fluid such as SAE 10W oil (The graph will remain the same except for scale)

Problem 8.133

8.133 For the pipe flow into a reservoir of Example 8.5 consider the effect of pipe roughness on flow rate, assuming the pressure of the pump is maintained at 153 kPa. Plot the flow rate against pipe roughness ranging from smooth ($e = 0$) to very rough ($e = 3.75$ mm). Also consider the effect of pipe length (again assuming the pump always produces 153 kPa) for smooth pipe. Plot the flow rate against pipe length for $L = 100$ m through $L = 1000$ m.

Given: Flow from a reservoir

Find: Effect of pipe roughness and pipe length on flow rate; Plot

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_{lT} = \sum_{major} h_l + \sum_{minor} h_{lm} \qquad (8.29)$$

$$Re = \frac{\rho \cdot V \cdot D}{\mu} \qquad h_l = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \quad (8.34) \qquad h_{lm} = K \cdot \frac{V^2}{2} \quad (8.40a) \qquad h_{lm} = f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2} \quad (8.40b)$$

$$f = \frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes for this flow (see Example 8.5)

$$p_{pump} = \Delta p = \rho \cdot \left(g \cdot d + f \cdot \frac{L}{D} \cdot \frac{V^2}{2}\right)$$

We need to solve this for velocity V, (and hence flow rate Q) as a function of roughness e, then length L. This cannot be solved explicitly for velocity V, (and hence flow rate Q) because f depends on V; solution for a given relative roughness e/D or length L requires iteration (or use of *Solver*)

Given data:

Δp = 153 kPa
D = 75 mm
L = 100 m

Tabulated or graphical data:

μ = 1.00E-03 N.s/m^2
ρ = 999 kg/m^3
(Water - Appendix A)

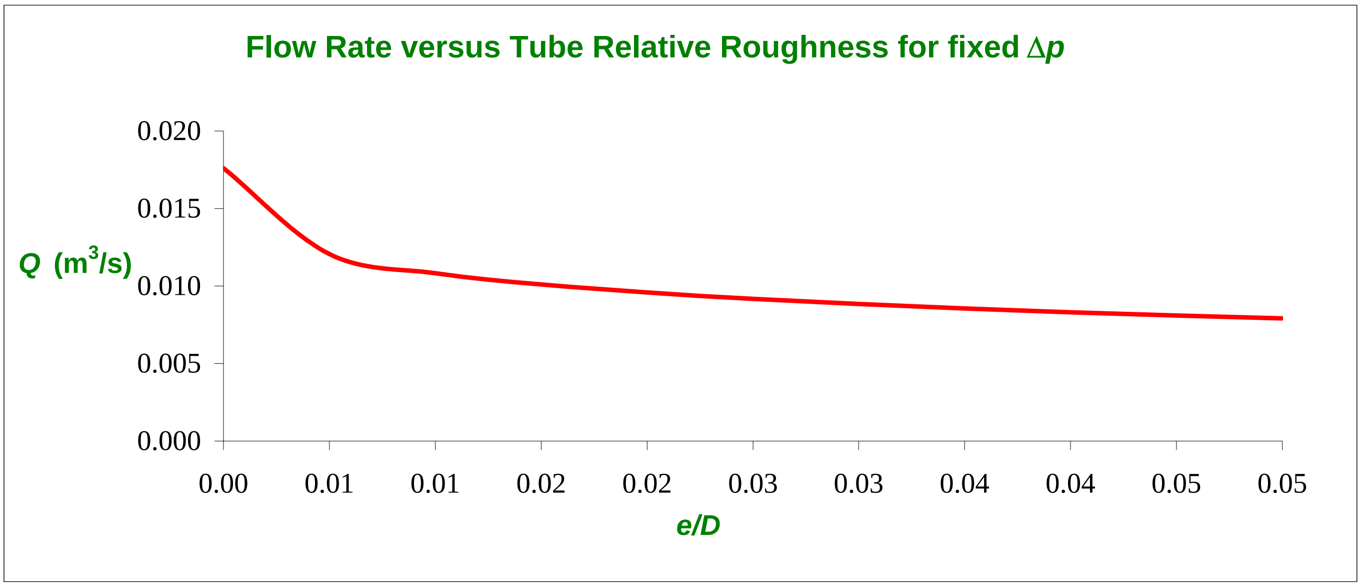
Computed results:

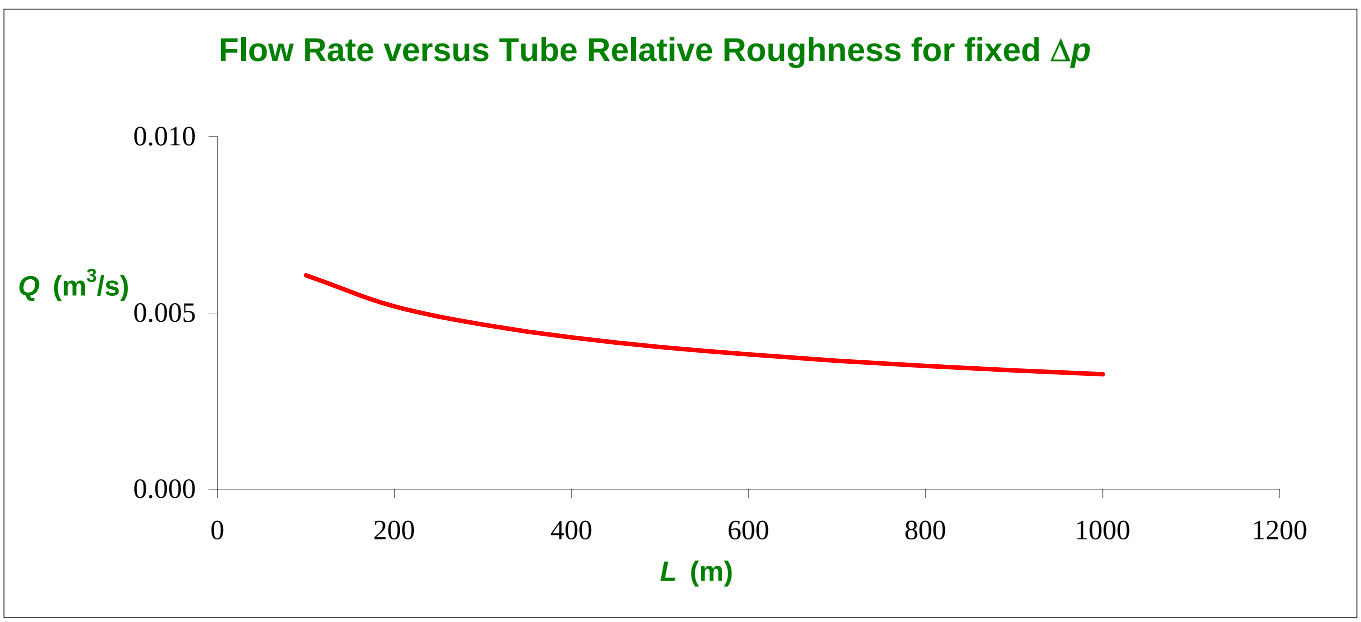
e/D	*V* (m/s)	*Q* (m^3/s)	*Re*	Regime	*f*	Δp (kPa)	Error
0.000	3.98	0.0176	2.98E+05	Turbulent	0.0145	153	0.0%
0.005	2.73	0.0121	2.05E+05	Turbulent	0.0308	153	0.0%
0.010	2.45	0.0108	1.84E+05	Turbulent	0.0382	153	0.0%
0.015	2.29	0.0101	1.71E+05	Turbulent	0.0440	153	0.0%
0.020	2.168	0.00958	1.62E+05	Turbulent	0.0489	153	0.0%
0.025	2.076	0.00917	1.56E+05	Turbulent	0.0533	153	0.0%
0.030	2.001	0.00884	1.50E+05	Turbulent	0.0574	153	0.0%
0.035	1.937	0.00856	1.45E+05	Turbulent	0.0612	153	0.0%
0.040	1.882	0.00832	1.41E+05	Turbulent	0.0649	153	0.0%
0.045	1.833	0.00810	1.37E+05	Turbulent	0.0683	153	0.0%
0.050	1.790	0.00791	1.34E+05	Turbulent	0.0717	153	0.0%

It is not possible to roughen the tube sufficiently to slow the flow down to a laminar flow for this Δp.

Computed results:

L (m)	V (m/s)	Q (m^3/s)	*Re*	Regime	f	Δp (kPa)	Error
100	1.37	0.00606	1.03E+05	Turbulent	0.1219	153	0.0%
200	1.175	0.00519	8.80E+04	Turbulent	0.0833	153	0.0%
300	1.056	0.00467	7.92E+04	Turbulent	0.0686	153	0.0%
400	0.975	0.00431	7.30E+04	Turbulent	0.0604	153	0.0%
500	0.913	0.004036	6.84E+04	Turbulent	0.0551	153	0.0%
600	0.865	0.003821	6.48E+04	Turbulent	0.0512	153	0.0%
700	0.825	0.003645	6.18E+04	Turbulent	0.0482	153	0.0%
800	0.791	0.003496	5.93E+04	Turbulent	0.0459	153	0.0%
900	0.762	0.003368	5.71E+04	Turbulent	0.0439	153	0.0%
1000	0.737	0.003257	5.52E+04	Turbulent	0.0423	153	0.0%





Problem 8.134

8.134 Water for a fire protection system is supplied from a water tower through a 150 mm cast-iron pipe. A pressure gage at a fire hydrant indicates 600 kPa when no water is flowing. The total pipe length between the elevated tank and the hydrant is 200 m. Determine the height of the water tower above the hydrant. Calculate the maximum volume flow rate that can be achieved when the system is flushed by opening the hydrant wide (assume minor losses are 10 percent of major losses at this condition). When a fire hose is attached to the hydrant, the volume flow rate is 0.75 m^3/min. Determine the reading of the pressure gage at this flow condition.

Given: System for fire protection

Find: Height of water tower; Maximum flow rate; Pressure gage reading

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT}=\sum_{major}h_l+\sum_{minor}h_{lm}\qquad(8.29)$$

$$Re=\frac{\rho\cdot V\cdot D}{\mu}\qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}\quad(8.34)\qquad h_{lm}=0.1\cdot h_l\qquad h_{lm}=f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}\quad(8.40b)$$

$$f=\frac{64}{Re}\quad(8.36)\quad\text{(Laminar)}\qquad\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right)\quad(8.37)\quad\text{(Turbulent)}$$

For no flow the energy equation (Eq. 8.29) applied between the water tower free surface (state 1; height *H*) and the pressure gage is

$$g\cdot H=\frac{p_2}{\rho}\qquad\text{or}\qquad H=\frac{p_2}{\rho\cdot g}\qquad(1)$$

The energy equation (Eq. 8.29) becomes, for maximum flow (and α = 1)

$$g\cdot H-\frac{V^2}{2}=h_{lT}=(1+0.1)\cdot h_l\qquad\text{or}\qquad g\cdot H=\frac{V^2}{2}\cdot\left(1+1.1\cdot f\cdot\frac{L}{D}\right)\qquad(2)$$

This can be solved for *V* (and hence *Q*) by iterating, or by using *Solver*

The energy equation (Eq. 8.29) becomes, for restricted flow

$$g\cdot H-\frac{p_2}{\rho}+\frac{V^2}{2}=h_{lT}=(1+0.1)\cdot h_l\qquad p_2=\rho\cdot g\cdot H-\rho\cdot\frac{V^2}{2}\cdot\left(1+1.1\cdot\rho\cdot f\cdot\frac{L}{D}\right)\qquad(3)$$

Given data:

p_2 = 600 kPa (Closed)

D = 150 mm

L = 200 m

Q = 0.75 m^3/min (Open)

Tabulated or graphical data:

e = 0.26 mm (Table 8.1)

μ = 1.00E-03 N.s/m^2

ρ = 999 kg/m^3 (Water - Appendix A)

Computed results:

Closed:

H = 61.2 m (Eq. 1)

Fully open:

V = 5.91 m/s

Re = 8.85E+05

f = 0.0228

Eq. 2, solved by varying *V* using *Solver* :

Left (m^2/s)	Right (m^2/s)	Error
601	601	0%

Q = 0.104 m^3/s

Partially open:

Q = 0.75 m^3/min

V = 0.71 m/s

Re = 1.06E+05

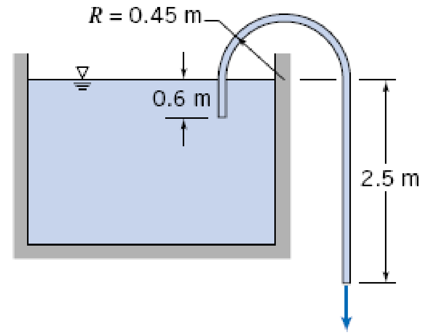
f = 0.0243

p_2 = 591 kPa (Eq. 3)

Problem 8.135

[3]

8.135 The siphon shown is fabricated from 50 mm i.d. drawn aluminum tubing. The liquid is water at 15°C. Compute the volume flow rate through the siphon. Estimate the minimum pressure inside the tube.



Given: Syphon system

Find: Flow rate; Minimum pressure

Solution:

Basic equations
$$\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_{lT} \qquad h_{lT} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} + h_{lm}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1

Hence the energy equation applied between the tank free surface (Point 1) and the tube exit (Point 2, z = 0) becomes

$$g\cdot z_1 - \frac{V_2^2}{2} = g\cdot z_1 - \frac{V^2}{2} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} + K_{ent}\cdot\frac{V^2}{2} + f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}$$

From Table 8.2 for reentrant entrance $K_{ent} = 0.78$

For the bend $\frac{R}{D} = 9$ so from Fig. 8.16 $\frac{L_e}{D} = 28$ for a 90º bend so for a 180º bend $\frac{L_e}{D} = 56$

Solving for V
$$V = \sqrt{\frac{2\cdot g\cdot h}{\left[1 + K_{ent} + f\cdot\left(\frac{L}{D} + \frac{L_e}{D}\right)\right]}} \qquad (1) \qquad \text{and} \qquad h = 2.5\cdot m$$

The two lengths are $L_e = 56\cdot D$ $\quad L_e = 2.8\,m$ $\quad L = (0.6 + \pi\cdot 0.45 + 2.5)\cdot m$ $\quad L = 4.51\,m$

We also have $Re = \frac{V\cdot D}{\nu}$ or $Re = c\cdot V$ (2) where $c = \frac{D}{\nu}$

From Table A.7 (15ºC) $\nu = 1.14\times 10^{-6}\cdot\frac{m^2}{s}$ $\quad c = 0.05\cdot m\times\frac{s}{1.14\times 10^{-6}\cdot m^2}$ $\quad c = 4.39\times 10^4\cdot\frac{s}{m}$

In addition
$$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad (3) \qquad e = 0.0015\,mm \quad \text{(Table 8.1)}$$

Equations 1, 2 and 3 form a set of simultaneous equations for V, Re and f

Make a guess for f $\quad f = 0.01$ then

$$V = \sqrt{\frac{2\cdot g\cdot h}{\left[1 + K_{ent} + f\cdot\left(\frac{L}{D} + \frac{L_e}{D}\right)\right]}} \qquad V = 3.89\,\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 1.71\times 10^5$$

Given $\frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right)$ $\quad f = 0.0164$

$V = \sqrt{\frac{2 \cdot g \cdot h}{\left[1 + K_{ent} + f \cdot \left(\frac{L}{D} + \frac{L_e}{D}\right)\right]}}$ $\quad V = 3.43 \frac{m}{s}$ $\quad Re = c \cdot V$ $\quad Re = 1.50 \times 10^5$

Given $\frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right)$ $\quad f = 0.0168$

$V = \sqrt{\frac{2 \cdot g \cdot h}{\left[1 + K_{ent} + f \cdot \left(\frac{L}{D} + \frac{L_e}{D}\right)\right]}}$ $\quad V = 3.40 \frac{m}{s}$ $\quad Re = c \cdot V$ $\quad Re = 1.49 \times 10^5$

Given $\frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right)$ $\quad f = 0.0168$

$V = \sqrt{\frac{2 \cdot g \cdot h}{\left[1 + K_{ent} + f \cdot \left(\frac{L}{D} + \frac{L_e}{D}\right)\right]}}$ $\quad V = 3.40 \frac{m}{s}$ $\quad Re = c \cdot V$ $\quad Re = 1.49 \times 10^5$

Note that we could use *Excel*'s *Solver* for this problem

The minimum pressure occurs at the top of the curve (Point 3). Applying the energy equation between Points 1 and 3

$$g \cdot z_1 - \left(\frac{p_3}{\rho} + \frac{V_3^{\,2}}{2} + g \cdot z_3\right) = g \cdot z_1 - \left(\frac{p_3}{\rho} + \frac{V^2}{2} + g \cdot z_3\right) = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} + K_{ent} \cdot \frac{V^2}{2} + f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2}$$

where we have $\frac{L_e}{D} = 28$ for the first 90º of the bend, and $L = \left(0.6 + \frac{\pi \times 0.45}{2}\right) \cdot m$ $\quad L = 1.31\,m$

$$p_3 = \rho \cdot \left[g \cdot \left(z_1 - z_3\right) - \frac{V^2}{2} \cdot \left[1 + K_{ent} + f \cdot \left(\frac{L}{D} + \frac{L_e}{D}\right)\right]\right]$$

$$p_3 = 1000 \cdot \frac{kg}{m^3} \times \left[9.81 \cdot \frac{m}{s^2} \times (-0.45 \cdot m) - \left(3.4 \cdot \frac{m}{s}\right)^2 \cdot \left[1 + 0.78 + 0.0168 \cdot \left(\frac{1.31}{0.05} + 28\right)\right]\right] \times \frac{N \cdot s^2}{kg \cdot m}$$ $\quad p_3 = -35.5\,kPa$

Problem 8.136

8.136 A large open water tank has a horizontal 2.5 cm diameter cast iron drainpipe of length 1.5 m attached at its base, If the depth of water is 3.5 m, find the flow rate (m^3/hr) if the pipe entrance is a) reentrant, b) square-edged and c) rounded (r = 3.75 mm).

Given: Tank with drainpipe

Find: Flow rate for rentrant, square-edged, and rounded entrances

Solution:

Basic equations
$$\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_{lT} \qquad h_{lT} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} + K_{ent}\cdot\frac{V^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1

Hence the energy equation applied between the tank free surface (Point 1) and the pipe exit (Point 2, z = 0) becomes

$$g\cdot z_1 - \frac{V_2^2}{2} = g\cdot z_1 - \frac{V^2}{2} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} + K_{ent}\cdot\frac{V^2}{2}$$

Solving for V
$$V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + K_{ent} + f\cdot\frac{L}{D}\right)}} \qquad (1) \qquad \text{and} \qquad h = (1.5 + 3.5)\cdot m \qquad h = 5\,m$$

We also have
$$Re = \frac{V\cdot D}{\nu} \quad \text{or} \quad Re = c\cdot V \quad (2) \qquad \text{where} \qquad c = \frac{D}{\nu}$$

From Table A.7 (20ºC)
$$\nu = 1.01\times 10^{-6}\cdot\frac{m^2}{s} \qquad c = 0.025\cdot m \times \frac{s}{1.01\times 10^{-6}\cdot m^2} \qquad c = 2.48\times 10^4\cdot\frac{s}{m}$$

In addition
$$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad (3) \qquad e = 0.26\cdot mm \qquad \text{(Table 8.1)}$$

Equations 1, 2 and 3 form a set of simultaneous equations for V, Re and f

For a reentrant entrance, from Table 8.2 $\quad K_{ent} = 0.78$

Make a guess for f $\quad f = 0.01 \quad$ then

$$V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + K_{ent} + f\cdot\frac{L}{D}\right)}} \qquad V = 6.42\,\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 1.59\times 10^5$$

Given
$$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0388$$

$$V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + K_{ent} + f\cdot\frac{L}{D}\right)}} \qquad V = 4.89\,\frac{m}{s} \qquad Re = c\cdot V \qquad Re = 1.21\times 10^5$$

Given
$$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0389$$

$$V = \sqrt{\frac{2 \cdot g \cdot h}{\left(1 + K_{ent} + f \cdot \frac{L}{D}\right)}} \qquad V = 4.88 \frac{m}{s} \qquad Re = c \cdot V \qquad Re = 1.21 \times 10^5$$

Given $\quad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \qquad f = 0.0389$

$$V = \sqrt{\frac{2 \cdot g \cdot h}{\left(1 + K_{ent} + f \cdot \frac{L}{D}\right)}} \qquad V = 4.88 \frac{m}{s} \qquad Re = c \cdot V \qquad Re = 1.21 \times 10^5$$

Note that we could use *Excel*'s *Solver* for this problem

The flow rate is then $\quad Q = V \cdot \frac{\pi \cdot D^2}{4} \qquad Q = 4.88 \cdot \frac{m}{s} \times \frac{\pi}{4} \times (0.025 \cdot m)^2 \qquad Q = 2.4 \times 10^{-3} \cdot \frac{m^3}{s} \qquad Q = 8.62 \cdot \frac{m^3}{hr}$

For a square-edged entrance, from Table 8.2 $\quad K_{ent} = 0.5$

Make a guess for f $\quad f = 0.01 \quad$ then

$$V = \sqrt{\frac{2 \cdot g \cdot h}{\left(1 + K_{ent} + f \cdot \frac{L}{D}\right)}} \qquad V = 6.83 \frac{m}{s} \qquad Re = c \cdot V \qquad Re = 1.69 \times 10^5$$

Given $\quad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \qquad f = 0.0388$

$$V = \sqrt{\frac{2 \cdot g \cdot h}{\left(1 + K_{ent} + f \cdot \frac{L}{D}\right)}} \qquad V = 5.06 \frac{m}{s} \qquad Re = c \cdot V \qquad Re = 1.25 \times 10^5$$

Given $\quad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \qquad f = 0.0389$

$$V = \sqrt{\frac{2 \cdot g \cdot h}{\left(1 + K_{ent} + f \cdot \frac{L}{D}\right)}} \qquad V = 5.06 \frac{m}{s} \qquad Re = c \cdot V \qquad Re = 1.25 \times 10^5$$

The flow rate is then $\quad Q = V \cdot \frac{\pi \cdot D^2}{4} \qquad Q = 5.06 \cdot \frac{m}{s} \times \frac{\pi}{4} \times (0.025 \cdot m)^2 \qquad Q = 2.48 \times 10^{-3} \cdot \frac{m^3}{s} \qquad Q = 8.94 \cdot \frac{m^3}{hr}$

For a rounded entrance, from Table 8.2 $\quad \frac{r}{D} = \frac{3.75}{25} = 0.15 \qquad K_{ent} = 0.04$

Make a guess for f $\quad f = 0.01 \quad$ then

$$V = \sqrt{\frac{2 \cdot g \cdot h}{\left(1 + K_{ent} + f \cdot \frac{L}{D}\right)}} \qquad V = 7.73 \frac{m}{s} \qquad Re = c \cdot V \qquad Re = 1.91 \times 10^5$$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0387$

$V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + K_{ent} + f\cdot\frac{L}{D}\right)}}$ $V = 5.40\frac{m}{s}$ $Re = c\cdot V$ $Re = 1.34\times 10^5$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0389$

$V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + K_{ent} + f\cdot\frac{L}{D}\right)}}$ $V = 5.39\frac{m}{s}$ $Re = c\cdot V$ $Re = 1.34\times 10^5$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0389$

$V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + K_{ent} + f\cdot\frac{L}{D}\right)}}$ $V = 5.39\frac{m}{s}$ $Re = c\cdot V$ $Re = 1.34\times 10^5$

Note that we could use *Excel*'s *Solver* for this problem

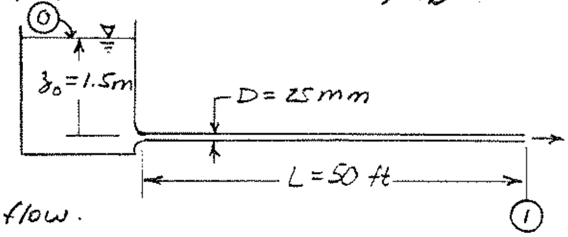
The flow rate is then $Q = V\cdot\frac{\pi\cdot D^2}{4}$ $Q = 5.39\cdot\frac{m}{s}\times\frac{\pi}{4}\times(0.025\cdot m)^2$ $Q = 2.65\times 10^{-3}\cdot\frac{m^3}{s}$ $Q = 9.52\cdot\frac{m^3}{hr}$

In summary: Renentrant: $Q = 8.62\cdot\frac{m^3}{hr}$ Square-edged: $Q = 8.94\cdot\frac{m^3}{hr}$ Rounded: $Q = 9.52\cdot\frac{m^3}{hr}$

Problem 8.137

Given: Roman water supply system from Example Problem 8.10, but with 50 foot length of straight pipe with $D = 25\ mm$, $e/D = 0.01$.

Find: (a) Flow rate delivered.
(b) Effect of adding a diffuser.



Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation:

$$\frac{p_0}{\rho} + \alpha_0\frac{\bar{V}_0^2}{2} + gz_0 = \frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1 + h_{\ell T}\ ; \quad h_{\ell T} = \left(f\frac{L}{D} + K_{ent}\right)\frac{\bar{V}_1^2}{2}$$

Assumptions: (1) $p_0 = p_1 = p_{atm}$ (3) $\alpha_1 \approx 1$
(2) $\bar{V}_1 \approx 0$ (4) $K_{ent} = 0.04$

Then

$$gz_0 = \frac{\bar{V}_1^2}{2} + \left(f\frac{L}{D} + K_{ent}\right)\frac{\bar{V}_1^2}{2} \quad \text{or} \quad \bar{V}_1 = \sqrt{\frac{2gz_0}{1 + f\frac{L}{D} + K}}$$

For $e/D = 0.01$, $f = 0.038$ from Eq. 8.37*, so

$$\bar{V}_1 = \left[2 \times 9.81\frac{m}{s^2} \times 1.5\,m \times \frac{1}{1 + 0.038 \times 50\,ft \times \frac{1}{25\,mm} \times 304.8\frac{mm}{ft} + 0.04}\right]^{1/2} = 1.10\ m/s$$

Checking, assuming $T = 20°C$,

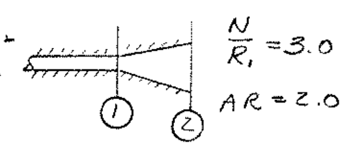
$$Re = \frac{\bar{V}D}{\nu} = 1.10\frac{m}{sec} \times 0.025\,m \times \frac{sec}{1.0\times10^{-6}\,m^2} = 2.75\times10^4\ ; \text{ from Eq. 8.37*}, f \approx 0.040, \text{ so}$$

$$\bar{V}_1 = \left[2 \times 9.81\frac{m}{s^2} \times 1.5\,m \times \frac{1}{1 + 0.040 \times 50\,ft \times \frac{1}{25\,mm} \times 304.8\frac{mm}{ft} + 0.04}\right]^{1/2} = 1.08\ m/s$$

$$Q = \bar{V}_1 A = 1.08\frac{m}{s} \times \frac{\pi}{4}(0.025)^2 m^2 = 5.30\times10^{-4}\ m^3/s \quad \text{(no diffuser)}$$ Q

The diffuser would increase head loss by $K_{diffuser} = 0.3$ (see Example 8.10), but would reduce $\bar{V}_2$ to $\frac{1}{2}\bar{V}_1$. The energy equation would be

$$gz_0 = \frac{\bar{V}_2^2}{2} + \left(f\frac{L}{D} + K_{ent} + K_{diff}\right)\frac{\bar{V}_1^2}{2} = \left(\frac{1}{4} + f\frac{L}{D} + K_{ent} + K_d\right)\frac{\bar{V}_1^2}{2}$$



Thus

$$\bar{V}_1 = \sqrt{\frac{2gz_0}{0.25 + f\frac{L}{D} + K_{ent} + K_{diff}}}$$

$$\bar{V}_1 = \left[2 \times 9.81\frac{m}{s^2} \times 1.5\,m \times \frac{1}{0.25 + 0.040 \times 50\,ft \times \frac{1}{25\,mm} \times 304.8\frac{mm}{ft} + 0.04 + 0.3}\right]^{1/2} = 1.09\ m/s$$

and

$$Q = \bar{V}_1 A = 1.09\frac{m}{s} \times \frac{\pi}{4}(0.025)^2 m^2 = 5.35\times10^{-4}\ m^3/s \quad \text{(with diffuser)}$$ Q

{The diffuser increases flow rate only slightly (~1 percent), because loss is dominated by fL/D.}

* Values of f obtained using Excel's Solver (or Goal Seek)

Problem 8.138

8.138 A 7500 gallon tank of kerosine is to be emptied by a gravity feed using a drain hose of diameter 1 in., roughness 0.01 in., and length 50 ft. The top of the tank is open to the atmosphere and the hose exits to an open chamber. If the kerosense level is initially 10 ft above the drain exit, estimate (by assuming steady flow) the initial drainage rate. Estimate the flow rate when the kerosene level is down to 5 ft, and down to 1 ft. Based on these three estimates, make a rough estimate of the time it took to drain to the 1 ft level.

Given: Tank with drain hose

Find: Flow rate at different instants; Estimate of drain time

Solution:

Basic equations $\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_l$ $\qquad h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α is approximately 1 4) Ignore minor loss at entrance (L >>; verify later)

Hence the energy equation applied between the tank free surface (Point 1) and the hose exit (Point 2, z = 0) becomes

$$g\cdot z_1 - \frac{V_2^2}{2} = g\cdot z_1 - \frac{V^2}{2} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Solving for V $\qquad V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + f\cdot\frac{L}{D}\right)}}$ $\qquad$ (1) $\qquad$ and $\qquad h = 10\cdot ft$ $\qquad$ initially

We also have $\qquad Re = \frac{V\cdot D}{\nu}$ $\qquad$ or $\qquad Re = c\cdot V$ $\quad$ (2) $\qquad$ where $\qquad c = \frac{D}{\nu}$

From Fig. A.2 (20ºC) $\qquad \nu = 1.8\times 10^{-6}\cdot\frac{m^2}{s}\times\frac{10.8\cdot\frac{ft^2}{s}}{1\cdot\frac{m^2}{s}}$ $\qquad \nu = 1.94\times 10^{-5}\frac{ft^2}{s}$

$$c = \frac{1}{12}\cdot ft\times\frac{s}{1.94\times 10^{-5}\cdot ft^2} \qquad c = 4.30\times 10^3\cdot\frac{s}{ft}$$

In addition $\qquad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\qquad$ (3) $\qquad$ with $\qquad e = 0.01\cdot in$ $\qquad D = 1\,in$

Equations 1, 2 and 3 form a set of simultaneous equations for V, Re and f

Make a guess for f $\qquad f = 0.01$ $\qquad$ then

$V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + f\cdot\frac{L}{D}\right)}}$ $\qquad V = 9.59\cdot\frac{ft}{s}$ $\qquad Re = c\cdot V$ $\qquad Re = 4.12\times 10^4$

Given $\qquad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\qquad f = 0.0393$ $\qquad V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + f\cdot\frac{L}{D}\right)}}$ $\qquad V = 5.12\cdot\frac{ft}{s}$ $\qquad Re = c\cdot V$ $\qquad Re = 2.20\times 10^4$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0405$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + f\cdot\frac{L}{D}\right)}}$ $V = 5.04\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 2.17\times 10^4$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0405$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + f\cdot\frac{L}{D}\right)}}$ $V = 5.04\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 2.17\times 10^4$

Note that we could use *Excel*'s *Solver* for this problem   Note: $f\cdot\frac{L}{D} = 24.3$ $K_e = 0.5$ $h_{lm} < h_l$

The flow rate is then $Q = V\cdot\frac{\pi\cdot D^2}{4}$ $Q = 5.04\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{1}{12}\cdot ft\right)^2$ $Q = 0.0275\cdot\frac{ft^3}{s}$ $Q = 12.3\cdot gpm$

Next we recompute everything for $h = 5\cdot ft$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0405$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + f\cdot\frac{L}{D}\right)}}$ $V = 3.57\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 1.53\times 10^4$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0415$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + f\cdot\frac{L}{D}\right)}}$ $V = 3.52\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 1.51\times 10^4$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0415$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + f\cdot\frac{L}{D}\right)}}$ $V = 3.52\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 1.51\times 10^4$

The flow rate is then $Q = V\cdot\frac{\pi\cdot D^2}{4}$ $Q = 3.52\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{1}{12}\cdot ft\right)^2$ $Q = 0.0192\cdot\frac{ft^3}{s}$ $Q = 8.62\cdot gpm$

Next we recompute everything for $h = 1\cdot ft$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0415$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + f\cdot\frac{L}{D}\right)}}$ $V = 1.58\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 6.77\times 10^3$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0452$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + f\cdot\frac{L}{D}\right)}}$ $V = 1.51\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 6.50\times 10^3$

Given $\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $f = 0.0454$ $V = \sqrt{\frac{2\cdot g\cdot h}{\left(1 + f\cdot\frac{L}{D}\right)}}$ $V = 1.51\cdot\frac{ft}{s}$ $Re = c\cdot V$ $Re = 6.48\times 10^3$

The flow rate is then $Q = V\cdot\frac{\pi\cdot D^2}{4}$ $Q = 1.51\cdot\frac{ft}{s}\times\frac{\pi}{4}\times\left(\frac{1}{12}\cdot ft\right)^2$ $Q = 0.00824\cdot\frac{ft^3}{s}$ $Q = 3.70\cdot gpm$

Initially we have dQ/dt = -12.3 gpm, then -8.62 gpm, then -3.70 gpm. These occur at h = 10 ft, 5 ft and 1 ft. The corresponding volumes in the tank are then Q = 7500 gal, 3750 gal, and 750 gal. Using *Excel* we can fit a power trendline to the dQ/dt versus Q data to find, approximately

$\frac{dQ}{dt} = -0.12\cdot Q^{\frac{1}{2}}$ where dQ/dt is in gpm and t is min. Solving this with initial condition Q = 7500 gpm when t = 0 gives

$t = \frac{1}{0.06}\cdot\left(\sqrt{7500} - \sqrt{Q}\right)$ Hence, when Q = 750 gal (h = 1 ft) $t = \frac{1}{0.06}\cdot\left(\sqrt{7500} - \sqrt{750}\right)\cdot min$ $t = 987\,min$ $t = 16.4\,hr$

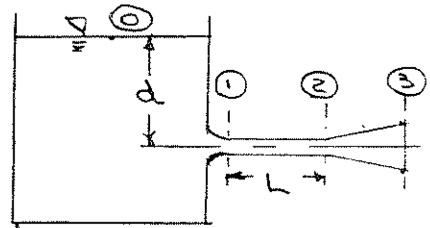
Problem 8.139

Given: Pipe of length L inserted between the nozzle (attached to the water main) and diffuser of Example Problem 8.10.

$D_1 = 25\text{ mm}$, $K_{ent} = 0.04$, $d = 1.5\text{ m}$

Diffuser: $N/R_1 = 3.0$, $AR = 2.0$

$K_{diff} = 0.3$



Flow with nozzle alone:

$Q_i = 2.61 \times 10^{-3}\ \text{m}^3/\text{s}$, $\bar{V}_1 = 5.32\ \text{m/s}$

Flow with nozzle and diffuser (L=0) $Q_d = 3.47\times 10^{-3}\ \text{m}^3/\text{s}$

Find: Length (L) of pipe with $e/D = 0.01$ required to give flow rate Q_i with diffuser in place; compare with commissioner's requirement of L = 50 ft (15.2 m)

Plot: Q/Q_i vs L/D

Solution:

Apply the energy equation for steady, incompressible flow between the water surface and the diffuser discharge.

Basic equations:
$$\left(\frac{p_0}{\rho} + \alpha_0\frac{\bar{V}_0^2}{2} + gz_0\right) - \left(\frac{p_3}{\rho} + \alpha_3\frac{\bar{V}_3^2}{2} + gz_3\right) = h_{l_T} \qquad (8.29)$$
$$h_{l_T} = h_l + h_{l_m} \ ; \ h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \ , \ h_{l_m} = (K_{ent} + K_{diff})\frac{\bar{V}_2^2}{2}$$

Assumptions: (1) $p_0 = p_3 = p_{atm}$

(2) $\bar{V}_0 = 0$, $\alpha_3 = 1.0$

(3) water @ 20°C, $\nu = 1.00\times 10^{-6}\ \text{m}^2/\text{s}$

Then,
$$g(z_0 - z_3) = gd = f\frac{L}{D}\frac{\bar{V}^2}{2} + (K_{ent} + K_{diff})\frac{\bar{V}_2^2}{2} + \frac{\bar{V}_3^2}{2}$$

From continuity $A_2\bar{V}_2 = A_3\bar{V}_3 \qquad \therefore \bar{V}_3 = \frac{\bar{V}_2}{AR}$
$$gd = f\frac{L}{D}\frac{\bar{V}_2^2}{2} + \left(K_{ent} + K_{diff} + \frac{1}{AR^2}\right)\frac{\bar{V}_2^2}{2} = f\frac{L}{D}\frac{\bar{V}_2^2}{2} + 0.59\frac{\bar{V}_2^2}{2} \quad \text{--- (1)}$$

and
$$L = \frac{D}{f}\left[\frac{2gd}{\bar{V}_2^2} - 0.590\right] \quad \text{--- (2)}$$

For $\bar{V}_1 = 5.32\ \text{m/s}$, $Re = \frac{D\bar{V}}{\nu} = 0.025\text{ m} \times 5.32\frac{\text{m}}{\text{s}} \times \frac{\text{s}}{1.00\times10^{-6}\ \text{m}^2} = 1.33\times 10^5$

with $e/D = 0.01$, $f = 0.038$ (Fig. 8.13) and
$$L = \frac{0.025\text{ m}}{0.038}\left[2\times 9.81\frac{\text{m}}{\text{s}^2}\times 1.5\text{ m} \times \frac{\text{s}^2}{(5.32)^2\ \text{m}^2} - 0.590\right] = 0.296\text{ m} \qquad L$$

$(0.971\text{ ft}, L/D = 11.8)$

This is significantly less than the 50 ft required by the water commissioner. He was extremely conservative.

Note that $Q/Q_i = \bar{V}/\bar{V}_i$ where $V_i = 5.32\ \text{m/s}$

Increasing L reduces $\bar{V}$

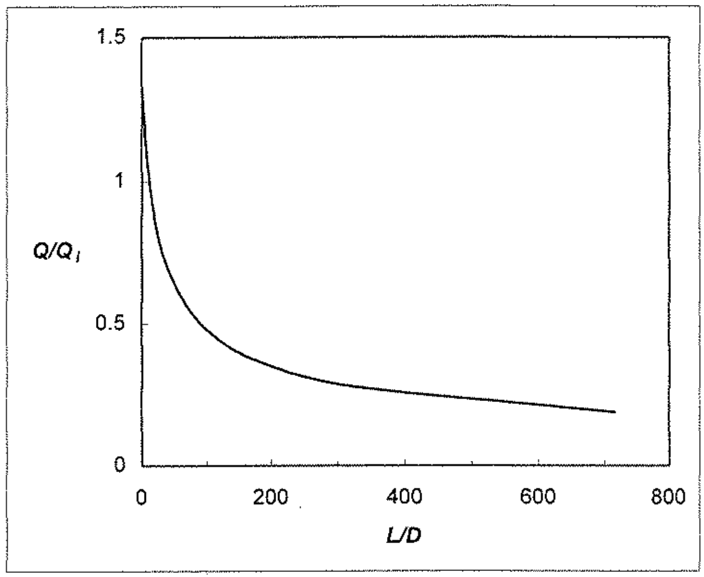
With $L = 0$ $\quad Q/Q_i = 1.33$

$L = 0.226\ m$ $(L/D = 11.8)$ $\quad Q/Q_i = 1.00$

As L is increased $\bar{V}_2$ (and hence Re) will decrease; the friction factor will increase slightly from 0.038.

The plot of Q/Q_i $(\bar{V}/\bar{V}_i)$ is best done by assuming values of $\bar{V}_1$ and solving Eq. 2 for L.

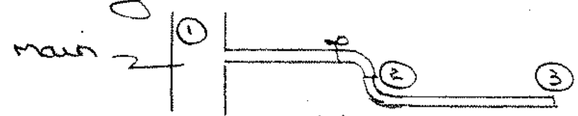
V_{avg} (m/s)	Re (---)	f (---)	L/D (---)	V/V_i (---)
7.06	1.77E+05	0.0382	0.0	1.33
5.32	1.33E+05	0.0384	11.7	1.00
5.0	1.25E+05	0.0384	15.3	0.940
4.5	1.13E+05	0.0384	22.5	0.846
4.0	1.00E+05	0.0385	32.4	0.752
3.5	8.75E+04	0.0386	47.0	0.658
3.0	7.50E+04	0.0387	69.3	0.564
2.5	6.25E+04	0.0388	106	0.470
2.0	5.00E+04	0.0391	173	0.376
1.5	3.75E+04	0.0394	317	0.282
1.0	2.50E+04	0.0402	718	0.188



Given: Water flow from spigot (at 60°F) through an old hose with $D = 0.75$ in and $e = 0.022$ in. Pressure at main remains constant at 50 psig; pressure at spigot varies with flow rate.
One 50 ft. length of hose delivers 15 gpm

Find: (a) pressure at spigot (psig) for this case.
(b) delivery with two 50-ft lengths of hose connected.

Solution:



Apply the energy equation for steady, incompressible flow between the spigot ② and the hose discharge ③

Basic equations: $\left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) - \left(\frac{p_3}{\rho} + \alpha_3\frac{\bar{V}_3^2}{2} + gz_3\right) = h_{l_T}$ (8.29)

$$h_{l_T} = h_l + h_{l_m}, \quad h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

Assumptions: (1) $p_3 = p_{atm}$
(2) $\bar{V}_2 = \bar{V}_3$, $\alpha_2 = \alpha_3 = 1.0$
(3) $z_2 = z_3$
(4) Turbulent flow so $\Delta p_{1\to2} \propto Q^2$

Then $p_2 = \rho f \frac{L}{D}\frac{\bar{V}^2}{2}$ — — — — — — — — — — — (1)

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 15\frac{gal}{min} \times \frac{min}{60\,s} \times \frac{ft^3}{7.48\,gal} \times \left(\frac{12}{0.75}\right)^2 \frac{1}{ft^2} = 10.9\ ft/s$$

$$Re = \frac{D\bar{V}}{\nu} = \frac{0.75}{12}ft \times 10.9\frac{ft}{s} \times \frac{s}{1.21\times10^{-5}\,ft^2} = 5.63\times10^4 \quad \{\nu \text{ from Table A.7}\}$$

$$e/D = 0.022/0.75 = 0.0293$$

From Eq. 8.37 $f = 0.056^*$. From Eq. 1,

$$p_2 = 1.94\frac{slug}{ft^3} \times 0.056 \times \frac{50\,ft}{0.75\,in} \times \frac{12\,in}{ft} \times \frac{1}{2} \times (10.9)^2\frac{ft^2}{s^2} \times \frac{lbf\cdot s^2}{slug\cdot ft} \times \frac{ft^2}{144\,in^2}$$

$p_2 = 35.9$ psi gage ← p_2

The pressure drop from the main ① to the spigot ② is proportional to the square of the flow rate. Obtain the loss coefficient using the energy equation between ① and ②.

$$\left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) = K\frac{\bar{V}_2^2}{2}$$

Assumptions: (4) $\bar{V}_1 = 0$ (5) $z_1 = z_2$

$$p_1 - p_2 = \rho\left[K\frac{\bar{V}_2^2}{2} + \frac{\bar{V}_2^2}{2}\right] = \rho\frac{\bar{V}_2^2}{2}[K+1]$$

$$K = \frac{\Delta p}{\frac{1}{2}\rho\bar{V}_2^2} - 1 = 2(50-35.9)144\frac{lbf}{ft^2} \times \frac{ft^3}{1.94\,slug \times (10.9)^2 ft^2} \frac{s^2}{} \times \frac{slug\cdot ft}{lbf\cdot s^2} - 1$$

$$K = 16.6$$

* Value of f obtained using Excel's Solver (or Goal Seek).

To find the delivery with two hoses, again apply the energy equation from the main ① to the end ④ of the second hose

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_4}{\rho} + \alpha_4 \frac{\bar{V}_4^2}{2} + g z_4\right) = f \frac{L_{24}}{D} \frac{\bar{V}_4^2}{2} + K \frac{\bar{V}_4^2}{2}$$

$p_4 = p_{atm}$, $z_1 = z_4$, $\bar{V}_1 = 0$, $\alpha_4 = 1$.

$$\frac{p_1 - p_{atm}}{\rho} = \frac{p_{1g}}{\rho} = \frac{\bar{V}_4^2}{2}\left(f \frac{L_{24}}{D} + K + 1\right) \quad \text{and}$$

$$\bar{V}_4 = \left[\frac{2 p_{1g}}{\rho\left(2f \frac{L}{D} + K + 1\right)}\right]^{1/2}$$

Delivery will be reduced somewhat with two lengths of hose, but f will not change much. Assume $f \approx 0.036$ and check

$$\bar{V}_4 = \left[2 \times 50 \frac{lbf}{in^2} \times 144 \frac{in^2}{ft^2} \times \frac{ft^3}{1.94\ slug} \times \frac{1}{\left(0.036 \times \frac{100\ ft}{0.75\ in} \times \frac{12\ in}{ft} + 16.6 + 1\right)} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{1/2}$$

$\bar{V}_4 = 8.32$ ft/s.

Checking,

$$Re = \frac{\bar{V}D}{\nu} = \frac{0.75}{12} ft \times 8.32 \frac{ft}{s} \times \frac{s}{1.21 \times 10^{-5}\ ft^2} = 4.30 \times 10^4 \text{ , so } f \approx 0.036$$

Thus with two hoses,

$$Q = \bar{V}A = 8.32 \frac{ft}{s} \times \frac{\pi}{4} \times \left(\frac{0.75}{12}\right)^2 ft^2 \times 7.48 \frac{gal}{ft^3} \times 60 \frac{s}{min} = 11.5 \text{ gpm} \qquad Q$$

{Similar calculations could be performed using any desired number of hose lengths.}

Problem 8.141

8.141 Your boss, from the "old school," claims that for pipe flow the flow rate, $Q \propto \sqrt{\Delta p}$, where Δp is the pressure difference driving the flow. You dispute this, so perform some calculations. You take a 1-in. diameter commercial steel pipe and assume an initial flow rate of 1.25 gal/min of water. You then increase the applied pressure in equal increments and compute the new flow rates so you can plot Q versus Δp, as computed by you and your boss. Plot the two curves on the same graph. Was your boss right?

Applying the energy equation between inlet and exit:

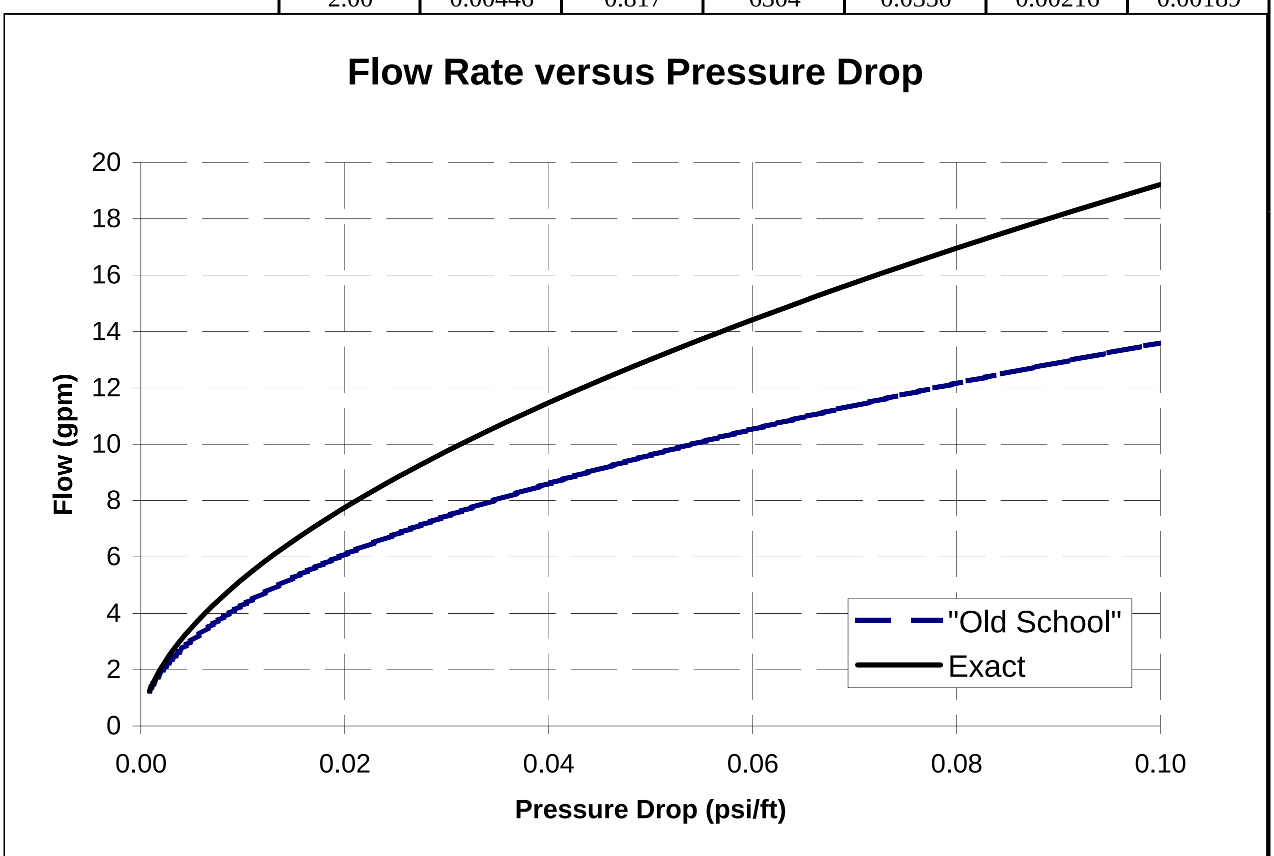
$$\frac{\Delta p}{\rho} = f\,\frac{L}{D}\frac{V^2}{2} \quad \text{or} \quad \frac{\Delta p}{L} = \frac{\rho f}{D}\frac{V^2}{2}$$

"Old school":

$$\frac{\Delta p}{L} = \left(\frac{\Delta p}{L}\right)_0 \left(\frac{Q_0}{Q}\right)^2$$

D = 1 in
e = 0.00015 ft
ν = 1.08E-05 ft^2/s
ρ = 1.94 $slug/ft^3$

Q (gpm)	Q (ft^3/s)	V (ft/s)	Re	f	Δp (old school) (psi)	Δp (psi/ft)
1.25	0.00279	0.511	3940	0.0401	0.00085	0.00085
1.50	0.00334	0.613	4728	0.0380	0.00122	0.00115
1.75	0.00390	0.715	5516	0.0364	0.00166	0.00150
2.00	0.00446	0.817	6304	0.0350	0.00216	0.00189
8.75	0.01950	3.575	27582	0.0240	0.04142	0.02477
9.00	0.02005	3.677	28370	0.0238	0.04382	0.02604



Your boss was wrong!

Problem 8.142

8.142 A hydraulic press is powered by a remote high-pressure pump. The gage pressure at the pump outlet is 3000 psi, whereas the pressure required for the press is 2750 psi (gage), at a flow rate of 0.02 ft^3/s. The press and pump are connected by 165 ft of smooth, drawn steel tubing. The fluid is SAE 10W oil at 100°F. Determine the minimum tubing diameter that may be used.

Given: Hydraulic press system

Find: Minimum required diameter of tubing

Solution:

Basic equations

$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_l \qquad h_l = f\cdot\frac{L}{D}\cdot\frac{V_2^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Ignore minor losses

The flow rate is low and it's oil, so try assuming laminar flow. Then, from Eq. 8.13c

$$\Delta p = \frac{128\cdot\mu\cdot Q\cdot L}{\pi\cdot D^4} \qquad \text{or} \qquad D = \left(\frac{128\cdot\mu\cdot Q\cdot L}{\pi\cdot\Delta p}\right)^{\frac{1}{4}}$$

For SAE 10W oil at 100°F (Fig. A.2, 38°C) $\mu = 3.5\times10^{-2}\cdot\frac{N\cdot s}{m^2}\times\frac{0.0209\cdot\frac{lbf\cdot s}{ft^2}}{1\cdot\frac{N\cdot s}{m^2}}$ $\qquad \mu = 7.32\times10^{-4}\frac{lbf\cdot s}{ft^2}$

Hence $D = \left[\frac{128}{\pi}\times7.32\times10^{-4}\frac{lbf\cdot s}{ft^2}\times0.02\cdot\frac{ft^3}{s}\times165\cdot ft\times\frac{in^2}{(3000-2750)\cdot lbf}\times\left(\frac{1\cdot ft}{12\cdot in}\right)^2\right]^{\frac{1}{4}}$ $\qquad D = 0.0407\,ft \qquad D = 0.488\,in$

Check Re to assure flow is laminar $V = \frac{Q}{A} = \frac{4\cdot Q}{\pi\cdot D^2}$ $\qquad V = \frac{4}{\pi}\times0.02\cdot\frac{ft^3}{s}\times\left(\frac{12}{0.488}\cdot\frac{1}{ft}\right)^2$ $\qquad V = 15.4\frac{ft}{s}$

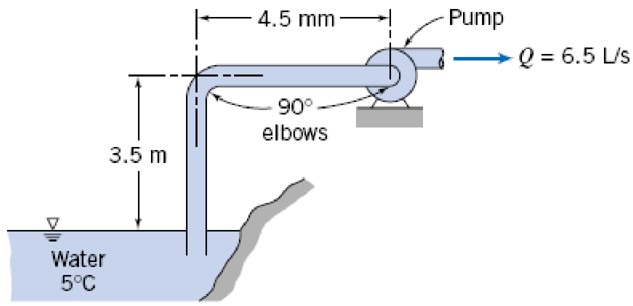
From Table A.2 $SG_{oil} = 0.92$ so $Re = \frac{SG_{oil}\cdot\rho_{H2O}\cdot V\cdot D}{\mu}$

$$Re = 0.92\times1.94\cdot\frac{slug}{ft^3}\times15.4\cdot\frac{ft}{s}\times\frac{0.488}{12}\cdot ft\times\frac{ft^2}{7.32\times10^{-4}\,lbf\cdot s}\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad Re = 1527$$

Hence the flow is laminar, Re < 2300. The minimum diameter is 0.488 in, so 0.5 in ID tube should be chosen

Problem 8.143

8.143 A pump is located 4.5 m to one side of, and 3.5 m above a reservoir. The pump is designed for a flow rate of 6 L/s. For satisfactory operation, the static pressure at the pump inlet must not be lower than −6 m of water gage. Determine the smallest standard commercial steel pipe that will give the required performance.



Given: Flow out of reservoir by pump

Find: Smallest pipe needed

Solution:

Basic equations $$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_{lT} \qquad h_{lT}=h_l+h_{lm}=f\cdot\frac{L}{D}\cdot\frac{V_2^2}{2}+K_{ent}\cdot\frac{V_2^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V_2^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) V_l <<

Hence for flow between the free surface (Point 1) and the pump inlet (2) the energy equation becomes

$$-\frac{p_2}{\rho}-g\cdot z_2-\frac{V_2^2}{2}=-\frac{p_2}{\rho}-g\cdot z_2-\frac{V^2}{2}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+K_{ent}\cdot\frac{V^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} \quad \text{and} \quad p=\rho\cdot g\cdot h$$

Solving for $h_2 = p_2/\rho g$ $$h_2=-z_2-\frac{V^2}{2\cdot g}\cdot\left[f\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)+K_{ent}\right] \qquad (1)$$

From Table 8.2 $K_{ent} = 0.78$ for rentrant, and from Table 8.4 two standard elbows lead to $\frac{L_e}{D}=2\times 30=60$

We also have $e = 0.046\cdot mm$ (Table 8.1) $\nu = 1.51\times 10^{-6}\cdot\frac{m^2}{s}$ (Table A.8)

and we are given $Q=6\cdot\frac{L}{s}$ $Q=6\times 10^{-3}\frac{m^3}{s}$ $z_2 = 3.5\cdot m$ $L = (3.5+4.5)\cdot m$ $L = 8\,m$ $h_2 = -6\cdot m$

Equation 1 is tricky because D is unknown, so V is unknown (even though Q is known), L/D and L_e/D are unknown, and Re and hence f are unknown! We COULD set up *Excel* to solve Eq 1, the Reynolds number, and f, simultaneously by varying D, but here we try guesses:

$$D=2.5\cdot cm \quad V=\frac{4\cdot Q}{\pi\cdot D^2} \qquad V=12.2\frac{m}{s} \qquad Re=\frac{V\cdot D}{\nu} \qquad Re=2.02\times 10^5$$

Given $$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f=0.0238$$

$$h_2=-z_2-\frac{V^2}{2\cdot g}\cdot\left[f\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)+K_{ent}\right] \qquad h_2=-78.45\,m \quad \text{but we need -6 m!}$$

$$D=5\cdot cm \quad V=\frac{4\cdot Q}{\pi\cdot D^2} \qquad V=3.06\frac{m}{s} \qquad Re=\frac{V\cdot D}{\nu} \qquad Re=1.01\times 10^5$$

Given $$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f=0.0219$$

$$h_2 = -z_2 - \frac{V^2}{2 \cdot g} \cdot \left[f \cdot \left(\frac{L}{D} + \frac{L_e}{D} \right) + K_{ent} \right] \qquad h_2 = -6.16\,m \qquad \text{but we need -6 m!}$$

$$D = 5.1 \cdot cm \qquad V = \frac{4 \cdot Q}{\pi \cdot D^2} \qquad V = 2.94 \frac{m}{s} \qquad Re = \frac{V \cdot D}{\nu} \qquad Re = 9.92 \times 10^4$$

Given

$$\frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}} \right) \qquad f = 0.0219$$

$$h_2 = -z_2 - \frac{V^2}{2 \cdot g} \cdot \left[f \cdot \left(\frac{L}{D} + \frac{L_e}{D} \right) + K_{ent} \right] \qquad h_2 = -5.93\,m$$

To within 1%, we can use 5-5.1 cm tubing; this corresponds to standard 2 in pipe.

Problem 8.144

8.144 Determine the minimum size smooth rectangular duct with an aspect ratio of 2 that will pass 2850 cfm of standard air with a head loss of 1.25 in. of water per 100 ft of duct.

Given: Flow of air in rectangular duct

Find: Minimum required size

Solution:

Basic equations
$$\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_l \qquad h_l=f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2} \qquad D_h=\frac{4\cdot A}{P_w}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Ignore minor losses

Hence for flow between the inlet (Point 1) and the exit (2) the energy equation becomes

$$\frac{p_1}{\rho}-\frac{p_2}{\rho}=\frac{\Delta p}{\rho}=f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2} \qquad \text{and} \qquad \Delta p=\rho_{H2O}\cdot g\cdot\Delta h$$

For a rectangular duct
$$D_h=\frac{4\cdot b\cdot h}{2\cdot(b+h)}=\frac{2\cdot h^2\cdot ar}{h\cdot(1+ar)}=\frac{2\cdot h\cdot ar}{1+ar} \qquad \text{and also} \qquad A=b\cdot h=h^2\cdot\frac{b}{h}=h^2\cdot ar$$

Hence
$$\Delta p=\rho\cdot f\cdot L\cdot\frac{V^2}{2}\cdot\frac{(1+ar)}{2\cdot h\cdot ar}=\rho\cdot f\cdot L\cdot\frac{Q^2}{2\cdot A^2}\cdot\frac{(1+ar)}{2\cdot h\cdot ar}=\frac{\rho\cdot f\cdot L\cdot Q^2}{4}\cdot\frac{(1+ar)}{ar^3}\cdot\frac{1}{h^5}$$

Solving for h
$$h=\left[\frac{\rho\cdot f\cdot L\cdot Q^2}{4\cdot\Delta p}\cdot\frac{(1+ar)}{ar^3}\right]^{\frac{1}{5}} \qquad (1)$$

We are given
$$Q=2850\cdot\frac{ft^3}{min} \qquad L=100\cdot ft \qquad e=0\cdot ft \qquad ar=2$$

and also
$$\Delta p=\rho_{H2O}\cdot g\cdot\Delta h \qquad \Delta p=1.94\cdot\frac{slug}{ft^3}\times 32.2\frac{ft}{s^2}\times\frac{1.25}{12}\cdot ft\times\frac{lbf\cdot s^2}{slug\cdot ft} \qquad \Delta p=6.51\cdot\frac{lbf}{ft^2}$$

$$\rho=0.00234\cdot\frac{slug}{ft^3} \qquad \nu=1.62\times 10^{-4}\cdot\frac{ft^2}{s} \qquad \text{(Table A.9)}$$

Equation 1 is tricky because h is unknown, so D_h is unknown, hence V is unknown (even though Q is known), and Re and hence f are unknown! We COULD set up *Excel* to solve Eq 1, the Reynolds number, and f, simmultaneously by varying h, but here we try guesses:

$$f=0.01 \qquad h=\left[\frac{\rho\cdot f\cdot L\cdot Q^2}{4\cdot\Delta p}\cdot\frac{(1+ar)}{ar^3}\right]^{\frac{1}{5}} \qquad h=0.597\cdot ft \qquad V=\frac{Q}{h^2\cdot ar} \qquad V=66.6\cdot\frac{ft}{s}$$

$$D_h=\frac{2\cdot h\cdot ar}{1+ar} \qquad D_h=0.796\cdot ft \qquad Re=\frac{V\cdot D_h}{\nu} \qquad Re=3.27\times 10^5$$

Given
$$\frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D_h}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f=0.0142$$

$$h = \left[\frac{\rho \cdot f \cdot L \cdot Q^2}{4 \cdot \Delta p} \cdot \frac{(1 + ar)}{ar^3}\right]^{\frac{1}{5}} \qquad h = 0.641 \cdot ft \qquad V = \frac{Q}{h^2 \cdot ar} \qquad V = 57.8 \cdot \frac{ft}{s}$$

$$D_h = \frac{2 \cdot h \cdot ar}{1 + ar} \qquad D_h = 0.855 \cdot ft \qquad Re = \frac{V \cdot D_h}{\nu} \qquad Re = 3.05 \times 10^5$$

Given $\qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D_h}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \qquad f = 0.0144$

$$h = \left[\frac{\rho \cdot f \cdot L \cdot Q^2}{4 \cdot \Delta p} \cdot \frac{(1 + ar)}{ar^3}\right]^{\frac{1}{5}} \qquad h = 0.643 \cdot ft \qquad V = \frac{Q}{h^2 \cdot ar} \qquad V = 57.5 \cdot \frac{ft}{s}$$

$$D_h = \frac{2 \cdot h \cdot ar}{1 + ar} \qquad D_h = 0.857 \cdot ft \qquad Re = \frac{V \cdot D_h}{\nu} \qquad Re = 3.04 \times 10^5$$

Given $\qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D_h}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \qquad f = 0.0144$

$$h = \left[\frac{\rho \cdot f \cdot L \cdot Q^2}{4 \cdot \Delta p} \cdot \frac{(1 + ar)}{ar^3}\right]^{\frac{1}{5}} \qquad h = 0.643 \cdot ft \qquad V = \frac{Q}{h^2 \cdot ar} \qquad V = 57.5 \cdot \frac{ft}{s}$$

$$D_h = \frac{2 \cdot h \cdot ar}{1 + ar} \qquad D_h = 0.857 \cdot ft \qquad Re = \frac{V \cdot D_h}{\nu} \qquad Re = 3.04 \times 10^5$$

In this process h and f have converged to a solution. The minimum dimensions are 0.642 ft by 1.28 ft, or 7.71 in by 15.4 in

Problem 8.145

Given: New industrial plant requires water supply of $5.7\ m^3/min$. The gage pressure at the main, 50 m from the plant, is 800 kPa. The supply line will have 4 elbows in a total length of 65 m. Pressure in the plant must be at least 500 kPa (gage).

Find: Minimum line size of galvanized iron to install.

Solution: Apply the energy equation for steady, incompressible flow that is uniform at each section ($\alpha \simeq 1$).

Basic equation: $$\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + g z_2 + f \frac{L}{D} \frac{\bar{V}^2}{2} + h_{\ell m}$$ (velocity terms cancel by (2), elevation terms by (3))

Assumptions: (1) $p_1 - p_2 \leq 300\ kPa = \Delta p$

(2) Fully developed flow in constant-area pipe, $\bar{V}_1 = \bar{V}_2 = \bar{V}$

(3) $z_1 = z_2$

(4) $h_{\ell m} = 4\left(\frac{L_e}{D}\right)_{elbow} \frac{\bar{V}^2}{2} = 120 \frac{\bar{V}^2}{2}$ ($\frac{L_e}{D} = 30$, from Table 8.5)

Then

$$\frac{\Delta p}{\rho} = f\left(\frac{L}{D} + 120\right)\frac{\bar{V}^2}{2} \quad \text{or} \quad \Delta p = \rho f \left(\frac{L}{D} + 120\right)\frac{\bar{V}^2}{2}$$

Since D is unknown, iteration is required. The calculating equations are:

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 5.7 \frac{m^3}{min} \times \frac{1}{D^2 m^2} \times \frac{min}{60\ s} = \frac{0.121}{D^2} (m/s)$$

$$Re = \frac{\bar{V} D}{\nu} = \frac{4Q}{\pi \nu D} = \frac{4}{\pi} \times 5.7 \frac{m^3}{min} \times \frac{s}{1.14 \times 10^{-6}\ m^2} \times \frac{1}{D\ m} \times \frac{min}{60\ s} = \frac{1.06 \times 10^5}{D} \quad (T = 15°C)$$

$e = 0.15\ mm$ (Table 8.1), f from Eq. 8.37*, L = 65 m. D from Table 8.5.

D (nom.)	D (m)	$\bar{V}$ (m/s)	Re (−)	e/D (−)	f (−)	L/D (−)	Δp (kPa)
3	0.0779	19.9	1.36×10^6	0.0019	0.024	834	4530
5	0.128	7.39	8.29×10^5	0.0012	0.021	508	360
6	0.154	5.10	6.89×10^5	0.001	0.020	422	141

Pipe friction calculations are accurate only within about ±10 percent. Line resistance (and consequently Δp) will increase with age.

Recommend installation of 6 in. (nominal) line. D

* Values of f obtained using Excel's Solver (or Goal Seek)

Problem 8.146

[4]

8.146 Air at 20°C flows in a horizontal square cross-section duct made from commercial steel. The duct is 25 m long. What size (length of a side) duct is required to convey 2 m^3/s of air with a pressure drop of 1.5 cm H_2O?

Given: Flow of air in square duct

Find: Minimum required size

Solution:

Basic equations
$$\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_l \qquad h_l = f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2} \qquad D_h = \frac{4\cdot A}{P_w}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Ignore minor losses

Hence for flow between the inlet (Point 1) and the exit (2) the energy equation becomes

$$\frac{p_1}{\rho} - \frac{p_2}{\rho} = \frac{\Delta p}{\rho} = f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2} \qquad \text{and} \qquad \Delta p = \rho_{H2O}\cdot g\cdot\Delta h$$

For a square duct
$$D_h = \frac{4\cdot h\cdot h}{2\cdot(h+h)} = h \qquad \text{and also} \qquad A = h\cdot h = h^2$$

Hence
$$\Delta p = \rho\cdot f\cdot L\cdot\frac{V^2}{2\cdot h} = \rho\cdot f\cdot L\cdot\frac{Q^2}{2\cdot h\cdot A^2} = \frac{\rho\cdot f\cdot L\cdot Q^2}{2\cdot h^5}$$

Solving for h
$$h = \left(\frac{\rho\cdot f\cdot L\cdot Q^2}{2\cdot\Delta p}\right)^{\frac{1}{5}} \qquad (1)$$

We are given
$$Q = 2\cdot\frac{m^3}{s} \qquad L = 25\cdot m \qquad e = 0.046\cdot mm \qquad \text{(Table 8.1)}$$

and also
$$\Delta p = \rho_{H2O}\cdot g\cdot\Delta h \qquad \Delta p = 1000\cdot\frac{kg}{m^3}\times 9.81\frac{m}{s^2}\times 0.015\cdot m\times\frac{N\cdot s^2}{kg\cdot m} \qquad \Delta p = 147\,Pa$$

$$\rho = 1.21\cdot\frac{kg}{m^3} \qquad \nu = 1.50\times 10^{-5}\cdot\frac{m^2}{s} \qquad \text{(Table A.10)}$$

Equation 1 is tricky because h is unknown, so D_h is unknown, hence V is unknown (even though Q is known), and Re and hence f are unknown! We COULD set up *Excel* to solve Eq 1, the Reynolds number, and f, simmultaneously by varying h, but here we try guesses:

$$f = 0.01 \qquad h = \left(\frac{\rho\cdot f\cdot L\cdot Q^2}{2\cdot\Delta p}\right)^{\frac{1}{5}} \qquad h = 0.333\,m \qquad V = \frac{Q}{h^2} \qquad V = 18.0\cdot\frac{m}{s}$$

$$D_h = h \qquad D_h = 0.333\,m \qquad Re = \frac{V\cdot D_h}{\nu} \qquad Re = 4.00\times 10^5$$

Given
$$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D_h}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0152$$

$$h = \left(\frac{\rho \cdot f \cdot L \cdot Q^2}{2 \cdot \Delta p}\right)^{\frac{1}{5}} \qquad h = 0.362\,m \qquad V = \frac{Q}{h^2} \qquad V = 15.2\,\frac{m}{s}$$

$$D_h = h \qquad D_h = 0.362 \cdot m \qquad Re = \frac{V \cdot D_h}{\nu} \qquad Re = 3.68 \times 10^5$$

Given $\quad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D_h}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \qquad f = 0.0153$

$$h = \left(\frac{\rho \cdot f \cdot L \cdot Q^2}{2 \cdot \Delta p}\right)^{\frac{1}{5}} \qquad h = 0.363\,m \qquad V = \frac{Q}{h^2} \qquad V = 15.2\,\frac{m}{s}$$

In this process h and f have converged to a solution. The minimum dimensions are 0.363 m by 0.363 m, or 36.3 cm by 36.3 cm

Problem 8.147

8.147 Investigate the effect of tube diameter on water flow rate by computing the flow generated by a pressure difference, $\Delta p = 100$ kPa, applied to a length $L = 100$ m of smooth tubing. Plot the flow rate against tube diameter for a range that includes laminar and turbulent flow.

Given: Flow in a tube

Find: Effect of diameter; Plot flow rate versus diameter

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_l \qquad (8.29)$$

$$Re = \frac{\rho \cdot V \cdot D}{\mu} \qquad h_l = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \qquad (8.34)$$

$$f = \frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes for flow in a tube

$$p_1 - p_2 = \Delta p = \rho \cdot f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$$

This cannot be solved explicitly for velocity V (and hence flow rate Q), because f depends on V; solution for a given diameter D requires iteration (or use of *Solver*)

Fluid is not specified: use water (basic trends in plot apply to any fluid)

Given data:

Δp = 100 kPa

L = 100 m

Tabulated or graphical data:

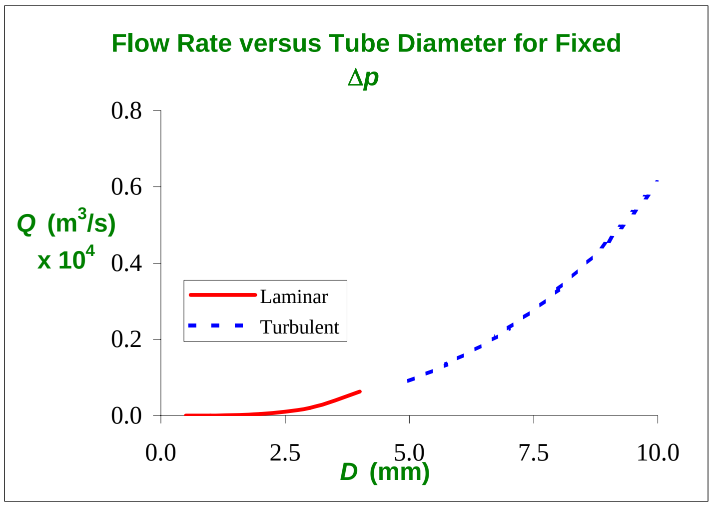
μ = 1.00E-03 N.s/m^2

ρ = 999 kg/m^3

(Water - Appendix A)

Computed results:

D (mm)	V (m/s)	Q (m^3/s) x 10^4	Re	Regime	f	Δp (kPa)	Error
0.5	0.00781	0.0000153	4	Laminar	16.4	100	0.0%
1.0	0.0312	0.000245	31	Laminar	2.05	100	0.0%
2.0	0.125	0.00393	250	Laminar	0.256	100	0.0%
3.0	0.281	0.0199	843	Laminar	0.0759	100	0.0%
4.0	0.500	0.0628	1998	Laminar	0.0320	100	0.0%
5.0	0.460	0.0904	2300	Turbulent	0.0473	100	0.2%
6.0	0.530	0.150	3177	Turbulent	0.0428	100	0.0%
7.0	0.596	0.229	4169	Turbulent	0.0394	100	0.0%
8.0	0.659	0.331	5270	Turbulent	0.0368	100	0.0%
9.0	0.720	0.458	6474	Turbulent	0.0348	100	0.0%
10.0	0.778	0.611	7776	Turbulent	0.0330	100	0.0%



Problem 8.148

8.148 What diameter water pipe is required to handle 1200 gpm and a 50 psi pressure drop? The pipe length is 500 ft, and roughness is 0.01 ft. The water is at 68°F.

Given:
Flow of water in circular pipe

Find:
Minimum required diameter

Solution:

Basic equations $$\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^{\,2}}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^{\,2}}{2} + g\cdot z_2\right) = h_l \qquad h_l = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} \qquad \text{and also} \qquad A = \frac{\pi\cdot D^2}{4}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) Ignore minor losses

Hence for flow between the inlet (Point 1) and the exit (2) the energy equation becomes

$$\frac{p_1}{\rho} - \frac{p_2}{\rho} = \frac{\Delta p}{\rho} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

Hence $$\Delta p = \rho\cdot f\cdot\frac{L}{D}\cdot\frac{V^2}{2} = \rho\cdot f\cdot\frac{L}{D}\cdot\frac{Q^2}{2\cdot A^2} = \frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot D^5}$$

Solving for D $$D = \left(\frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot\Delta p}\right)^{\frac{1}{5}} \qquad (1)$$

We are given $Q = 1200\cdot gpm$ $\quad L = 500\cdot ft$ $\quad e = 0.01\cdot ft$ $\quad \Delta p = 50\cdot psi$

and also $\rho = 1.94\cdot\frac{slug}{ft^3}$ $\quad \nu = 1.08\times 10^{-5}\cdot\frac{ft^2}{s}$ (Table A.7)

Equation 1 is tricky because D is unknown, hence V is unknown (even though Q is known), and Re and hence f are unknown!
We COULD set up *Excel* to solve Eq 1, the Reynolds number, and f, simultaneously by varying D, but here we try guesses:

$f = 0.01$ $\quad D = \left(\frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot\Delta p}\right)^{\frac{1}{5}}$ $\quad D = 0.379\cdot ft$ $\quad V = \frac{4\cdot Q}{\pi\cdot D^2}$ $\quad V = 23.7\cdot\frac{ft}{s}$ $\quad Re = \frac{V\cdot D}{\nu}$ $\quad Re = 8.32\times 10^5$

Given $$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0543$$

$D = \left(\frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot\Delta p}\right)^{\frac{1}{5}}$ $\quad D = 0.531\cdot ft$ $\quad V = \frac{4\cdot Q}{\pi\cdot D^2}$ $\quad V = 12.1\cdot\frac{ft}{s}$ $\quad Re = \frac{V\cdot D}{\nu}$ $\quad Re = 5.93\times 10^5$

Given $$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0476$$

$$D = \left(\frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot\Delta p}\right)^{\frac{1}{5}} \qquad D = 0.518\cdot ft \qquad V = \frac{4\cdot Q}{\pi\cdot D^2} \qquad V = 12.7\cdot\frac{ft}{s} \qquad Re = \frac{V\cdot D}{\nu} \qquad Re = 6.09\times 10^5$$

Given $$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0481$$

$$D = \left(\frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot\Delta p}\right)^{\frac{1}{5}} \qquad D = 0.519\cdot ft \qquad V = \frac{4\cdot Q}{\pi\cdot D^2} \qquad V = 12.7\cdot\frac{ft}{s} \qquad Re = \frac{V\cdot D}{\nu} \qquad Re = 6.08\times 10^5$$

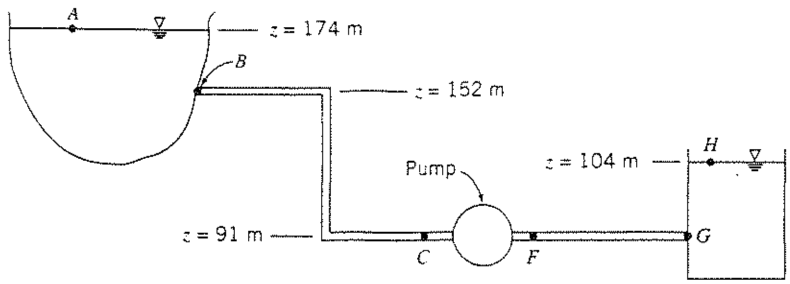
Given $$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0480$$

$$D = \left(\frac{8\cdot\rho\cdot f\cdot L\cdot Q^2}{\pi^2\cdot\Delta p}\right)^{\frac{1}{5}} \qquad D = 0.519\cdot ft \qquad V = \frac{4\cdot Q}{\pi\cdot D^2} \qquad V = 12.7\cdot\frac{ft}{s} \qquad Re = \frac{V\cdot D}{\nu} \qquad Re = 6.08\times 10^5$$

In this process D and f have converged to a solution. The minimum diameter is 0.519 ft or 6.22 in

Problem 8.149

Given: Portion of water supply system designed to provide $Q = 1310$ L/s at $T = 20°C$.



System B → C
- square edged entrance
- 3 gate valves
- 4 45° elbows
- 2 90° elbows
- 760 m pipe
- $p_C = 197$ kPa gage

System F → G
- 760 m pipe
- 2 gate valves
- 4 90° elbows

All pipe is cast iron, $D = 508$ mm

Find: (a) average velocity in pipeline
(b) gage pressure p_F
(c) shear stress on pipe centerline at C
(d) power input to pump if efficiency $\eta = 80\%$
(e) wall shear stress at G.

Solution:

Since $Q = A\bar{V}$, $\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 1310\frac{L}{s} \times \frac{1}{(0.508m)^2} \times 10^{-3}\frac{m^3}{L} = 6.46$ m/s $\quad \bar{V}$

To determine the pressure at point F, apply the energy equation for steady, incompressible flow between F and G.

Basic equation: $\left(\frac{p_F}{\rho} + \alpha\frac{\bar{V}_F^2}{2} + gz_F\right) - \left(\frac{p_H}{\rho} + \alpha\frac{\bar{V}_H^2}{2} + gz_H\right) = h_{l_T}$ (8.29)

$h_{l_T} = h_l + h_{l_m}$, $h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$, $h_{l_m} = \frac{\bar{V}^2}{2}\sum f\frac{L_e}{D} + \frac{\bar{V}^2}{2}K_{exit}$

Assume: (1) $\bar{V}_H = 0$ (large storage tank) (2) $p_H = p_{atm}$
(3) $\alpha_F \approx 1.0$

Then $\frac{p_F}{\rho} = h_{l_T} + g(z_H - z_F) - \frac{\bar{V}_F^2}{2}$

$$\frac{p_F}{\rho} = h_{l_{F-G}} + 2h_{l_{m\,gv}} + 4h_{l_{m\,90°el}} + h_{l_{m\,exit}} + g(z_H - z_F) - \frac{\bar{V}_F^2}{2}$$

$$\frac{p_F}{\rho} = f\frac{L}{D}\frac{\bar{V}^2}{2} + 2f\left(\frac{L_e}{D}\right)_{gv}\frac{\bar{V}^2}{2} + 4f\left(\frac{L_e}{D}\right)_{90°el}\frac{\bar{V}^2}{2} + K_{exit}\frac{\bar{V}^2}{2} + g(z_H - z_F) - \frac{\bar{V}_F^2}{2} \quad \text{---(1)}$$

From Table 8.4 $\left(\frac{L_e}{D}\right)_{gv} = 8$, $\left(\frac{L_e}{D}\right)_{90°el} = 30$; also $K_{exit} = 1$

$Re = \frac{\bar{V}D}{\nu} = 0.508\,m \times 6.46\frac{m}{s} \times \frac{s}{1.00\times10^{-6}\,m^2} = 3.28\times10^6$ (ν from Table A.8)

From Table 8.1, $e = 0.26$ mm $\therefore e/D = 0.00051$

From Eq. 8.37, $f = 0.017$ (using Excel's Solver [or Goal Seek])

From Eq (1)

$$\frac{p_F}{\rho} = f\frac{\bar{V}^2}{2}\left[\frac{L}{D} + 2\left(\frac{L_e}{D}\right)_{gv} + 4\left(\frac{L_e}{D}\right)_{90°el}\right] + g(z_H - z_F)$$

$$\frac{p_F}{\rho} = f\frac{\bar{V}^2}{2}\left[\frac{760}{0.508} + 2(8) + 4(30)\right] + g(z_H - z_F) = f\frac{\bar{V}^2}{2}(1630) + g(z_H - z_F)$$

$$p_F = \rho\left[1630\, f\frac{\bar{V}^2}{2} + g(z_H - z_F)\right]$$

$$= 999\frac{kg}{m^3}\left[\frac{1630}{2} \times 0.017 \times (6.46)^2\frac{m^2}{s^2} + 9.81\frac{m}{s^2}(104 - 91)m\right] \times \frac{N \cdot s^2}{kg \cdot m}$$

$p_F = 705$ kPa (gage) ← p_F

For fully developed flow in a pipe $\quad \tau = \frac{r}{2}\frac{\partial p}{\partial x} \qquad (8.15)$

At the pipe centerline, $\tau = 0$ ← $\tau_{\mathcal{C}}$

To determine the power input to the fluid apply the energy equation across the pump. Assuming 100% efficiency

$$\frac{\dot{W}_{pump}}{\dot{m}} = \left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_{discharge} - \left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_{suction} \qquad (8.47)$$

$$\dot{W}_{pump} = \left(\frac{p_F}{\rho} - \frac{p_C}{\rho}\right)\rho A\bar{V} = (p_F - p_C)Q$$

$$\dot{W}_{pump} = (705 - 197) \times 10^3\frac{N}{m^2} \times 1310\frac{L}{s} \times 10^{-3}\frac{m^3}{L} = 6.65 \times 10^5\frac{N \cdot m}{s}$$

The actual pump input, $\dot{W}_{pump})_{act} = \dot{W}_{pump})_{ideal}/\eta$

$\dot{W}_{pump})_{actual} = 8.32 \times 10^5$ N·m/s $= 832$ kW ← $\dot{W}_{actual}$

From Eq. 8.15 $\quad \tau_w = \frac{R}{2}\frac{\partial p}{\partial x}$

Along the pipe from F to G $\quad \frac{\Delta p}{\rho} = f\frac{L}{D}\frac{\bar{V}^2}{2}$

$$\therefore \frac{\partial p}{\partial x} = \frac{\Delta p}{L} = \rho\frac{f}{D}\frac{\bar{V}^2}{2} = 999\frac{kg}{m^3} \times \frac{0.017}{0.508\,m} \times \frac{1}{2} \times (6.46)^2\frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m}$$

$\frac{\partial p}{\partial x} = 698$ N/m²/m

$$\therefore \tau_w = \frac{R}{2}\frac{\partial p}{\partial x} = \frac{0.254\,m}{2} \times 698\frac{N}{m^3} = 88.6\ N/m^2$$ ← τ_w

Problem 8.150

Given: An air-pipe friction experiment utilizes smooth brass tube, $D = 63.5$ mm, $L = 1.52$ m.
At one flow condition $\Delta p = 12.3$ mm merian red oil, $U_{cl} = 23.1$ m/s.

Find: (a) $Re_{\bar{V}}$
(b) friction factor f; compare with value for Fig. 8.13.

Solution:

Apply the energy equation for steady, incompressible flow along the pipe

Basic equation: $\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) = h_{l_T}$ (8.29)

$$h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$$

Computing equation: $\frac{\bar{V}}{U} = \frac{2n^2}{(n+1)(2n+1)}$ (8.24)

Assumptions: (1) power law profile, $n = 7$
(2) $\alpha_1 = \alpha_2$, $z_1 = z_2$
(3) air at $T = 15^\circ C$ $\nu = 1.46 \times 10^{-5}$ m²/s (Table A.10)

From Eq. 8.24 with $n = 7$

$$\frac{\bar{V}}{U} = \frac{2(7)^2}{(8)(15)} = 0.817$$

$$Re_{\bar{V}} = \frac{D\bar{V}}{\nu} = 0.0635\,\text{m} \times 0.817 \times 23.1\,\frac{\text{m}}{\text{s}} \times \frac{\text{s}}{1.45 \times 10^{-5}\,\text{m}^2} = 8.26 \times 10^4 \quad Re_{\bar{V}}$$

From Eq. 8.29 $\Delta p / \rho = f \frac{L}{D} \frac{\bar{V}^2}{2}$

and

$$f = \frac{\Delta p}{\rho \frac{L}{D} \frac{\bar{V}^2}{2}} = \frac{\rho_{oil}\, g\, \Delta h}{\rho_{air} \frac{L}{D} \frac{\bar{V}^2}{2}} = \frac{2\,\rho_{H_2O}\, SG_{oil}\, g\, \Delta h}{\rho_{air} \frac{L}{D} \bar{V}^2} \quad \left\{\begin{matrix} SG = 0.827 \\ \text{Table A.1} \end{matrix}\right\}$$

$$f = 2 \times \frac{10^3}{1.23} \times 0.827 \times 9.81\,\frac{\text{m}}{\text{s}^2} \times 0.0123\,\text{m} \times \frac{0.0635\,\text{m}}{1.52\,\text{m}} \times \frac{\text{s}^2}{(0.817 \times 23.1)^2\,\text{m}^2}$$

$$f = 0.0190 \quad f$$

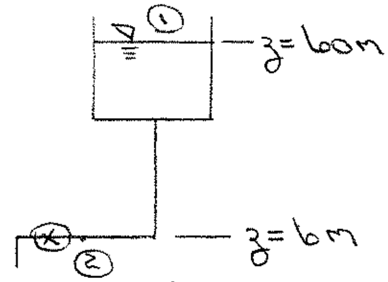
From Eq. 8.37 at $Re = 8.26 \times 10^4$ for smooth tube, $f = 0.0187$

The value of f is obtained using Excel's Solver (or Goal Seek)

Problem *8.151

Given: Oil flowing from a large tank on a hill to a tanker at the wharf. In stopping the flow, valve on wharf at such a rate that $p_2 = 1$ MPa is maintained in the line immediately upstream of the valve. Assume:

Length of line from tank to valve	3 km
Inside diameter of line	200 mm
Elevation of oil surface in tank	60 m
Elevation of valve on wharf	6 m
Instantaneous flow rate	2.5 m³/min
Head loss in line (exclusive of valve being closed) at this rate of flow	23 m of oil
Specific gravity of oil	0.88



Find: the initial instantaneous rate of change of volume flow rate.

Solution: For unsteady flow with friction, we modify the unsteady Bernoulli equation (Eq. 6.21) to include a head loss term.

Computing equation:
$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 + \int_1^2 \frac{\partial V_s}{\partial t}\,ds + h_\ell$$

Assume: (1) $V_1 \approx 0$ (2) $p_1 = p_{atm}$ (3) ρ = constant

Then
$$\int_1^2 \frac{\partial V_s}{\partial t}\,ds = \frac{p_1 - p_2}{\rho} + g(z_1 - z_2) - h_\ell - \frac{V_2^2}{2}$$

If we neglect velocity in the tank except for small region near the inlet to the pipe, then
$$\int_1^2 \frac{\partial V_s}{\partial t}\,ds \approx \int_0^L \frac{\partial V_s}{\partial t}\,ds$$
. Since $V_s = V_2$ everywhere, then
$$\int_0^L \frac{\partial V_s}{\partial t}\,ds = L\frac{dV_2}{dt}$$
and
$$\frac{dV_2}{dt} = \frac{1}{L}\left[\frac{p_1 - p_2}{\rho} + g(z_1 - z_2) - h_\ell - \frac{V_2^2}{2}\right], \quad V_2 = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$

Note $h_\ell = h_\ell(V)$ and hence this result can only be used to obtain the initial instantaneous rate of change of flow velocity

$$\therefore \left.\frac{dV_2}{dt}\right)_{initial} = \frac{1}{3\times10^3\,m}\left[10^6\frac{N}{m^2} \times \frac{m^3}{999\,kg} \times \frac{1}{0.88}\frac{kg\cdot m}{N\cdot s^2} + 9.81\frac{m}{s^2} \times 54\,m - 23\,m \times 9.81\frac{m}{s^2} - \frac{1}{2}\left\{\frac{4}{\pi} \times 2.5\frac{m^3}{min} \times \frac{1}{(0.2\,m)^2} \times \frac{1\,min}{60\,s}\right\}^2\right]$$

$$\left.\frac{dV_2}{dt}\right)_{initial} = -0.278\ m/s/s$$

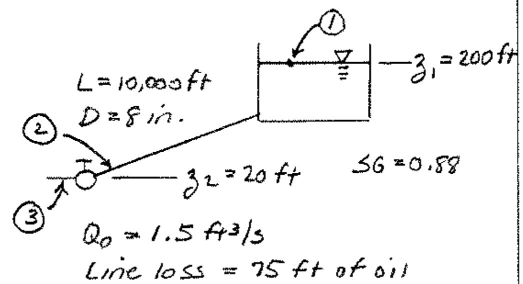
The instantaneous rate of change of volume flow rate is
$$dQ/dt = \frac{d}{dt}(AV) = A\frac{dV}{dt} = \frac{\pi D^2}{4}\frac{dV}{dt}$$
$$dQ/dt = \frac{\pi}{4}(0.2\,m)^2 \times \left(-0.278\frac{m/s}{s}\right) \times \frac{60\,s}{min} = -0.524\ m^3/s/min \leftarrow dQ/dt$$

Problem *8.152

Given: Problem 8.151 describes a situation in which flow in a long pipeline from a hilltop tank is slowed gradually to avoid a large pressure rise.

Find: Expansion of this analysis to predict and plot the closing schedule (valve loss coefficient versus time) needed to maintain the maximum pressure at the valve at or below a given value throughout the process of stopping the flow from the tank.

Solution: Apply the unsteady Bernoulli equation with a head loss term added.



Computing equation:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 + \int_1^2 \frac{\partial V}{\partial t}\,ds + h_{lT}$$

($p_1 \approx 0$, $V_1^2/2 \approx 0$ (1))

Assume: (1) $V_1 \approx 0$ (3) ρ = constant
(2) $p_1 = p_{atm}$

At the initial condition, $V = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times \left(\frac{12}{8}\right)^2 \frac{1}{ft^2} \times 1.5\,\frac{ft^3}{s} = 4.30\ ft/s$

$$H_{lT} = 75\,ft = \frac{h_{lT}}{g} = f\frac{L}{D}\frac{V^2}{2g}\,;\quad f\frac{L}{D} = H_{lT}\frac{2g}{V^2} = 2 \times 75\,ft \times 32.2\,\frac{ft}{s^2} \times \frac{s^2}{(4.30)^2\,ft^2} = 261$$

Neglecting velocity in tank, $\int_1^2 \frac{\partial V}{\partial t}\,ds \approx \int_0^L \frac{\partial V}{\partial t}\,ds = \frac{dV}{dt}L$

Thus $$\frac{dV}{dt} = \frac{1}{L}\left[-\frac{p_2}{\rho} + g(z_1 - z_2) - f\frac{L}{D}\frac{V^2}{2} - \frac{V^2}{2}\right]$$

Substituting values,

$$\frac{dV}{dt} = \frac{1}{10{,}000\,ft}\left[-150\,\frac{lbf}{in.^2} \times \frac{ft^3}{(0.88)\,1.94\,slug} \times 144\,\frac{in.^2}{ft^2} \times \frac{slug\cdot ft}{lbf\cdot s^2} + 32.2\,\frac{ft}{s^2}(200-20)\,ft - (261+1)\frac{V^2}{2}\right]$$

$$\frac{dV}{dt} = -0.686\,\frac{ft}{s^2} - 0.0131\,V^2 = -(a^2 + b^2V^2)\,;\quad a = \sqrt{0.686} = 0.828\,;\ V \text{ in ft/s}$$

$$b = \sqrt{0.0131} = 0.114$$

Separating variables and integrating

$$\int_{V_0}^{V} \frac{dV}{a^2 + b^2 t} = \frac{1}{ab}\tan^{-1}\frac{bV}{a}\Big]_{V_0}^{V} = \frac{1}{ab}\left[\tan^{-1}\frac{bV}{a} - \tan^{-1}\frac{bV_0}{a}\right] = -\int_0^t dt = -t$$

Thus

$$\tan^{-1}\frac{bV}{a} = -abt + \tan^{-1}\frac{bV_0}{a} \quad \text{or} \quad V = \frac{a}{b}\tan\left[\tan^{-1}\frac{bV_0}{a} - abt\right]$$ ← V(t)

The pressure must drop across the valve:

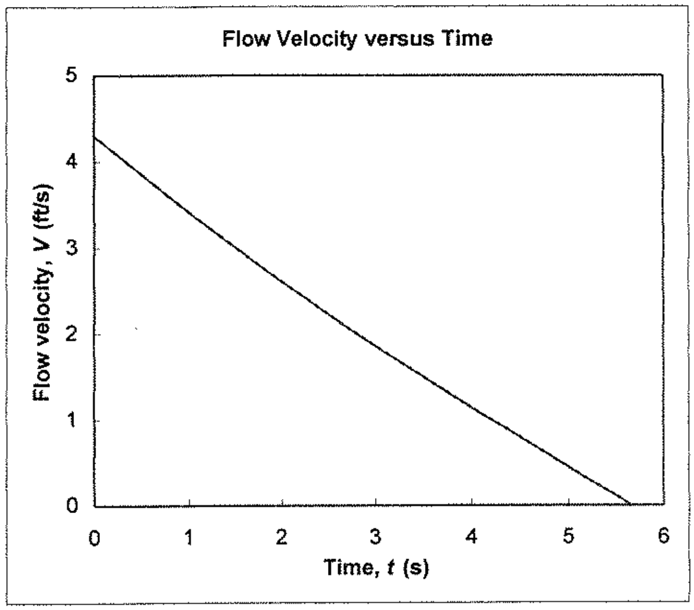
$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 - \left(\frac{p_3}{\rho} + \frac{V_3^2}{2} + gz_3\right) = h_{lT} = K_V\frac{V^2}{2} \quad \text{or} \quad K_V = \frac{2(p_2 - p_3)}{\rho V^2} \approx \frac{2p_2}{\rho V^2}$$

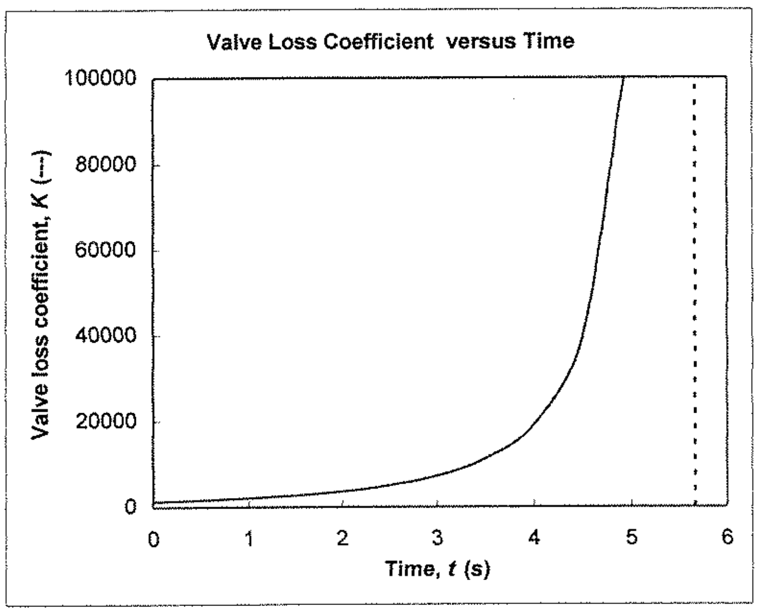
At $t = 0$, $$K_V = 2 \times 150\,\frac{lbf}{in.^2} \times \frac{ft^3}{(0.88)\,1.94\,slug} \times \frac{s^2}{(4.3)^2\,ft^2} \times 144\,\frac{in.^2}{ft^2} \times \frac{slug\cdot ft}{lbf\cdot s^2} = 1{,}370 \quad (t = 0)$$ ← $K_V(0)$

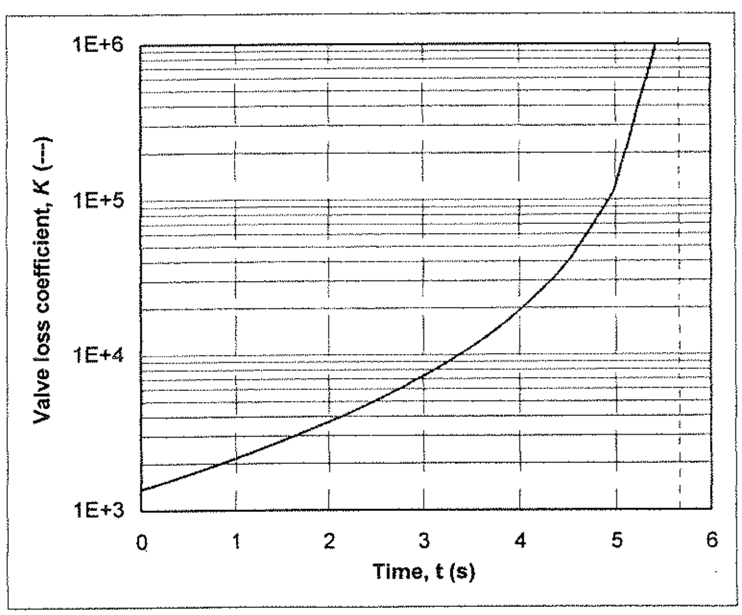
Calculations and plots are shown on the spreadsheet, next page.

Problem *8.152

	a =	0.828	(---)
	b =	0.114	(---)
	V_0 =	4.3	(ft/s)
t (s)	***V*** (ft/s)	***K*** (---)	
0.0	4.300	1.37E+3	
0.5	3.849	1.71E+3	
1.0	3.421	2.16E+3	
1.5	3.011	2.79E+3	
2.0	2.616	3.70E+3	
2.5	2.235	5.07E+3	
3.0	1.865	7.28E+3	
3.5	1.504	1.12E+4	
4.0	1.150	1.92E+4	
4.5	0.801	3.95E+4	
4.93	0.503	1.00E+5	
5.0	0.455	1.22E+5	
5.431	0.159	1.00E+6	
5.5	0.112	2.03E+6	
5.663	0.000	2.92E+13	







Problem 8.153

8.153 A pump draws water at a steady flow rate of 25 lbm/s through a piping system. The pressure on the suction side of the pump is −2.5 psig. The pump outlet pressure is 50 psig. The inlet pipe diameter is 3 in.; the outlet pipe diameter is 2 in. The pump efficiency is 70 percent. Calculate the power required to drive the pump.

Given: Flow through water pump

Find: Power required

Solution:

Basic equations
$$h_{pump} = \left(\frac{p_d}{\rho} + \frac{V_d^2}{2} + g\cdot z_d\right) - \left(\frac{p_s}{\rho} + \frac{V_s^2}{2} + g\cdot z_s\right) \qquad V = \frac{Q}{A} = \frac{4\cdot Q}{\pi\cdot D^2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Hence for the inlet
$$V_s = \frac{4}{\pi}\times 25\cdot\frac{lbm}{s}\times\frac{1\cdot slug}{32.2\cdot lbm}\times\frac{ft^3}{1.94\cdot slug}\times\left(\frac{12}{3}\cdot\frac{1}{ft}\right)^2 \qquad V_s = 8.15\frac{ft}{s} \qquad p_s = -2.5\cdot psi$$

For the outlet
$$V_d = \frac{4}{\pi}\times 25\cdot\frac{lbm}{s}\times\frac{1\cdot slug}{32.2\cdot lbm}\times\frac{ft^3}{1.94\cdot slug}\times\left(\frac{12}{2}\cdot\frac{1}{ft}\right)^2 \qquad V_d = 18.3\frac{ft}{s} \qquad p_d = 50\cdot psi$$

Then
$$h_{pump} = \frac{p_d - p_s}{\rho} + \frac{V_d^2 - V_s^2}{2} \qquad \text{and} \qquad W_{pump} = m_{pump}\cdot h_{pump}$$

$$W_{pump} = m_{pump}\cdot\left(\frac{p_d - p_s}{\rho} + \frac{V_d^2 - V_s^2}{2}\right)$$

Note that the software cannot render a dot, so the power is W_{pump} and mass flow rate is m_{pump}!

$$W_{pump} = 25\cdot\frac{lbm}{s}\times\frac{1\cdot slug}{32.2\cdot lbm}\times\left[(50 - -2.5)\cdot\frac{lbf}{in^2}\times\left(\frac{12\cdot in}{1\cdot ft}\right)^2\times\frac{ft^3}{1.94\cdot slug} + \frac{1}{2}\times\left(18.3^2 - 8.15^2\right)\cdot\left(\frac{ft}{s}\right)^2\times\frac{lbf\cdot s^2}{slug\cdot ft}\right]\times\frac{1\cdot hp}{550\cdot\frac{ft\cdot lbf}{s}}$$

$$W_{pump} = 5.69\,hp \qquad \text{For an efficiency of} \quad \eta = 70\cdot\% \qquad W_{required} = \frac{W_{pump}}{\eta} \qquad W_{required} = 8.13\,hp$$

Problem 8.154

[1]

8.154 The pressure rise across a water pump is 75 kPa when the volume flow rate is 25 L/s. If the pump efficiency is 80 percent, determine the power input to the pump.

Given: Flow through water pump

Find: Power required

Solution:

Basic equations $$h_{pump} = \left(\frac{p_d}{\rho} + \frac{V_d^2}{2} + g\cdot z_d\right) - \left(\frac{p_s}{\rho} + \frac{V_s^2}{2} + g\cdot z_s\right) \qquad V = \frac{Q}{A} = \frac{4\cdot Q}{\pi\cdot D^2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

In this case we assume $D_s = D_d$ so $V_s = V_d$

Then $$h_{pump} = \frac{p_d - p_s}{\rho} = \frac{\Delta p}{\rho} \qquad \text{and} \qquad W_{pump} = m_{pump}\cdot h_{pump}$$

$$W_{pump} = m_{pump}\cdot\frac{\Delta p}{\rho} = \rho\cdot Q\cdot\frac{\Delta p}{\rho} = Q\cdot\Delta p$$

Note that the software cannot render a dot, so the power is W_{pump} and mass flow rate is m_{pump}!

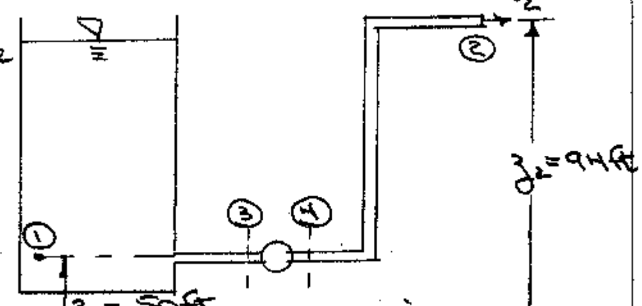
$$W_{pump} = 25\cdot\frac{L}{s}\times\frac{0.001\cdot m^3}{1\cdot L}\times 75\times 10^3\cdot\frac{N}{m^2} \qquad W_{pump} = 1.88\,kW$$

For an efficiency of $\eta = 80\cdot\%$ $$W_{required} = \frac{W_{pump}}{\eta} \qquad W_{required} = 2.34\cdot kW$$

Problem 8.155

Given: Pump in piping system shown moves $Q = 0.439\ \text{ft}^3/\text{s}$ of water.

System includes:
- $L = 290$ ft galvanized pipe, $D = 2.5$ in. (nominal)
- 2 gate valves (open)
- 1 angle valve (open)
- 7 standard 90° elbows
- 1 square edge entrance
- 1 free discharge



Find: pressure rise, $p_4 - p_3$, across pump.

Solution:

Computing equation: $\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) + \Delta h_{pump} = h_{l_T}$ (8.48) [with $\bar{V}_1$ term ≈ 0 (1), p_2 term ≈ 0 (2)]

$$h_{l_T} = h_l + h_{l_m}, \quad h_l = f \frac{L}{D}\frac{\bar{V}^2}{2}, \quad h_{l_m} = \frac{\bar{V}^2}{2}\left\{\sum f\left(\frac{L_e}{D}\right) + \sum K\right\}$$

Assumptions: (1) $\bar{V}_1 = 0$ (2) $p_2 = p_{atm}$ (3) $\alpha_2 = 1.0$ (4) $T = 60°\text{F}$

Then,

$$\Delta h_{pump} = h_{l_T} + g(z_2 - z_1) + \frac{\bar{V}_2^2}{2} - \frac{p_1}{\rho} \quad \text{......(1)}$$

$$h_{l_T} = \frac{\bar{V}_2^2}{2}\left[f\frac{L}{D} + 2f\left(\frac{L_e}{D}\right)_{gv} + f\left(\frac{L_e}{D}\right)_{av} + 7f\left(\frac{L_e}{D}\right)_{90°el} + K_{ent}\right] \quad \text{......(2)}$$

From Table 8.4 $L_e/D)_{gv} = 8$, $L_e/D)_{av} = 150$, $L_e/D)_{90°el} = 30$

From Table 8.2 $K_e = 0.5$. From Table 8.5 $D = 2.47$ in

From Table 8.1 $e = 0.0005$ ft $\therefore e/D = 0.0005 \times \frac{12}{2.47} = 0.0024$

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 0.439\,\frac{\text{ft}^3}{\text{s}} \times \left(\frac{12}{2.47\,\text{ft}}\right)^2 = 13.2\ \text{ft/s}$$

$$Re = \frac{D\bar{V}}{\nu} = \frac{2.47\,\text{ft}}{12} \times 13.2\,\frac{\text{ft}}{\text{s}} \times \frac{\text{s}}{1.21\times10^{-5}\,\text{ft}^2} = 2.25\times10^5 \quad \{\nu \text{ from Table A.7}\}$$

From Fig. 8.13, $f = 0.025$.

From Eq. 2

$$h_{l_T} = \frac{1}{2}\times(13.2)^2\frac{\text{ft}^2}{\text{s}^2}\left[0.025 \times \frac{290\times12}{2.47} + 2(0.025)(8) + (0.025)(150) + 7(0.025)(30) + 0.5\right]$$

$h_{l_T} = 3930\ \text{ft}^2/\text{s}^2$. Then from Eq. 1

$$\Delta h_{pump} = 3930\,\frac{\text{ft}^2}{\text{s}^2} + 32.2\,\frac{\text{ft}}{\text{s}^2}(44\,\text{ft}) + \frac{1}{2}(13.2)^2\frac{\text{ft}^2}{\text{s}^2} - 20\,\frac{\text{lbf}}{\text{in}^2}\times\frac{\text{ft}^3}{1.94\,\text{slug}}\times\frac{144\,\text{in}^2}{\text{ft}^2}\times\frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}$$

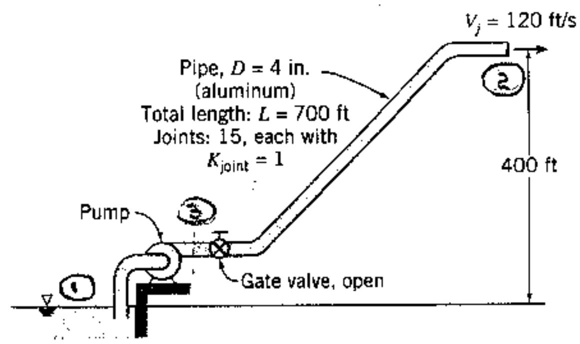
$$\Delta h_{pump} = 3950\ \text{ft}^2/\text{s}^2$$

Apply the energy equation across the pump

$$\Delta h_{pump} = \left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_{discharge} - \left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_{suction} \quad (8.47)$$

$$\Delta p = \rho\,\Delta h_{pump} = 1.94\,\frac{\text{slug}}{\text{ft}^3}\times 3950\,\frac{\text{ft}^2}{\text{s}^2}\times\frac{\text{lbf}\cdot\text{s}^2}{\text{ft}\cdot\text{slug}}\times\frac{\text{ft}^2}{144\,\text{in}^2} = 53.2\ \text{lbf/in}^2 \quad \underline{(p_4 - p_3)}$$

Problem 8.156



Given: Cooling water supply system
$Q = 600$ gpm
$\eta_{pump} = 0.70$

Find: (a) minimum pressure needed at pump outlet
(b) power requirement

Solution:

Computing equations: $\left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) + \Delta h_{pump} = h_{l_T}$ (8.48)

$h_{l_T} = h_l + h_{l_m}$, $h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$, $h_{l_m} = \frac{\bar{V}^2}{2}\left(\Sigma K + \Sigma f\left(\frac{L_e}{D}\right)\right)$

Assumptions: (1) $\bar{V}_1 = 0$ (2) $\alpha_2 = \alpha_3 = 1$ (3) $p_1 = p_2 = p_{atm}$

Then

$$\Delta h_{pump} = gz_2 + \frac{\bar{V}_2^2}{2} + f\frac{L}{D}\frac{\bar{V}^2}{2} + \frac{\bar{V}^2}{2}\left[K_{ent} + f\left(\frac{L_e}{D}\right)_{90°el} + 2f\left(\frac{L_e}{D}\right)_{45°el} + 15K_J\right] \quad (1)$$

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 600\frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,sec} \times \frac{1}{(0.333\,ft)^2} = 15.3\ ft/s$$

$$Re = \frac{D\bar{V}}{\nu} = \frac{1}{3}ft \times 15.3\frac{ft}{s} \times 1.24\times10^{5}\frac{s}{ft^2} = 4.11\times10^5 \qquad \{\nu \text{ at } T = 60°F, \text{ Table A.7}\}$$

Table 8.1, $e = 5\times10^{-6}$ ft (drawn tubing) $\therefore e/D = 1.5\times10^{-5}$

From Fig. 8.13, $f = 0.0135$

From Table 8.1, $K_{ent} = 0.78$

From Table 8.4, $L_e/D)_{g.v} = 8$, $L_e/D)_{90°el} = 30$, $L_e/D)_{45°el} = 16$

Then from Eq. (1)

$$\Delta h_{pump} = 32.2\frac{ft}{s^2} \times 400\,ft + \frac{1}{2}(120)^2\frac{ft^2}{s^2} + 0.0135 \times \frac{700}{0.333} \times \frac{1}{2} \times (15.3)^2\frac{ft^2}{s^2}$$
$$+ \frac{1}{2}(15.3)^2\frac{ft^2}{s^2}\left[0.78 + 0.0135(30) + 2\times0.0135(16) + 15(1)\right]$$

$\Delta h_{pump} = 2.53\times10^4\ ft^2/s^2$

The theoretical power input to the pump is given by $\dot{W}_{pump} = \dot{m}\,\Delta h_{pump}$

From the definition of efficiency, $\eta = \dot{W}_{theor}/\dot{W}_{act}$, then

$$\dot{W}_{act} = \frac{\dot{m}\,\Delta h_{pump}}{\eta} = \frac{\rho Q\,\Delta h_{pump}}{\eta}$$

$$\dot{W}_{act} = \frac{1.94}{0.7}\frac{slug}{ft^3} \times 600\frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} \times 2.53\times10^4\frac{ft^2}{s^2} \times \frac{lbf\cdot s^2}{ft\cdot slug} \times \frac{hp\cdot s}{550\,ft\cdot lbf} = \underline{170\,hp} \quad \dot{W}_{in}$$

The discharge pressure from the pump is obtained by applying Eq. 8.48 between sections ① and ③ neglecting losses in the inlet section, elevation change, and kinetic energy at ③

$$p_3 - p_A = \rho\,\Delta h_{pump} = 1.94\frac{slug}{ft^3} \times 2.53\times10^4\frac{ft^2}{s^2} \times \frac{lbf\cdot s^2}{ft\cdot s} \times \frac{ft^2}{144\,in^2} = \underline{341\,psi} \quad p_3$$

Problem 8.157

Given: Water supply system requires $Q = 100$ gpm pumped to reservoir at elevation of 340 m. Water pressure at pump inlet (street level) is 400 kPa (gage). Piping is to be commercial steel; $\bar{V}_{max} = 3.5$ m/s

Find: (a) Minimum pipe diameter
(b) pressure rise across the pump
(c) minimum power needed to drive the pump

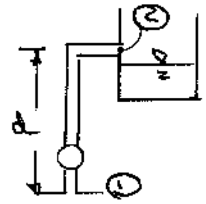
Solution:

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} \quad \therefore D = \left[\frac{4Q}{\pi \bar{V}}\right]^{1/2} = \left[\frac{4}{\pi} \times 100\frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} \times \left(\frac{0.305\,m}{ft}\right)^3 \times \frac{s}{3.5\,m}\right]^{1/2}$$

$$D = 0.048\,m = 48\,mm \qquad D$$

Computing equations: $\left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) + \Delta h_{pump} = h_{l_T}$ (8.48)

$$h_{l_T} = h_l + h_{l_m}, \quad h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$$



Assume: (1) $\alpha_1 = \alpha_2$ (2) $p_2 = p_{atm}$
(3) minor losses are negligible.
(4) water at 20°C ($\nu = 1\times10^{-6}$ m²/s, Table A.8).

Then

$$\Delta h_{pump} = h_l - \frac{p_1}{\rho} + g(z_2 - z_1) = f\frac{L}{D}\frac{\bar{V}^2}{2} - \frac{p_1}{\rho} + gd \quad \ldots\ldots(1)$$

From Table 8.1, $e = 0.046$ mm $\therefore e/D = 0.046/48 = 0.00096$

$$Re = \frac{D\bar{V}}{\nu} = 0.048\,m \times 3.5\frac{m}{s} \times \frac{1}{1\times10^{-6}}\frac{s}{m^2} = 1.68\times10^5$$

From Fig. 8.13, $f = 0.021$. Then from Eq. 1

$$\Delta h_{pump} = 0.021 \times \frac{340}{0.048} \times \frac{1}{2}(3.5)^2\frac{m^2}{s^2} - 400\times10^3\frac{N}{m^2} \times \frac{m^3}{998\,kg} \times \frac{kg\cdot m}{N\cdot s^2} + 9.81\frac{m}{s^2} \times 340\,m$$

$$\Delta h_{pump} = 3{,}850\ m^2/s^2 \quad \text{(This is head added to fluid).}$$

$$\Delta h_{pump} = \frac{\dot{W}_{pump}}{\dot{m}} = \left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_{discharge} - \left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_{suction} \quad (8.47)$$

Assume: (5) $\bar{V}_{disch} = \bar{V}_{suct}$; $z_{discharge} \approx z_{suction}$

$$\Delta p = \rho\,\Delta h = 998\frac{kg}{m^3} \times 3850\frac{m^2}{s^2} \times \frac{N\cdot s^2}{kg\cdot m} = 3840\ kPa \qquad \Delta p$$

Also from Eq. 8.47

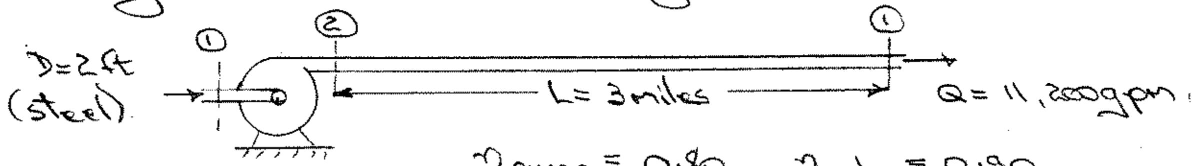
$$\dot{W}_{pump} = \dot{m}\,\Delta h_{pump} = \rho Q\,\Delta h_{pump}$$

$$\dot{W}_{pump} = 998\frac{kg}{m^3} \times 100\frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \left(\frac{0.305\,m}{ft}\right)^3 \times \frac{min}{60\,s} \times 3850\frac{m^2}{s^2} \times \frac{N\cdot s^2}{kg\cdot m}$$

$$\dot{W}_{pump} = 24.3\ kW\ (32.6\ hp) \qquad \dot{W}_{pump}$$

Problem 8.158

Given: Chilled-water pipe system for campus air conditioning makes a loop of length L = 3 miles.



$\eta_{pump} = 0.80$, $\eta_{motor} = 0.90$

$C = \$0.12 / (kw.hr)$

Find: (a) the pressure drop, $p_2 - p_1$
(b) rate of energy addition to the water
(c) daily cost of electrical energy for pumping

Solution:

Apply energy equation for steady, incompressible pipe flow from pump discharge around loop to pump inlet

Computing equations: $\left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) - \left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) = h_{\ell_T}$ (8.29)

$h_{\ell_T} = h_\ell + h_{\ell m}$ (3) $\qquad h_\ell = f \frac{L}{D} \frac{\bar{V}^2}{2}$

Assumptions: (1) $\alpha_1 = \alpha_2$, (2) $z_1 = z_2$ (3) neglect minor losses

Then $p_2 - p_1 = f \frac{L}{D} \rho \frac{\bar{V}^2}{2}$, $\bar{V} = \frac{Q}{A} = \frac{11{,}200\,gal}{min} \times \frac{4}{\pi (2)^2 ft^2} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} = 7.94\,ft/s$

Assume T = 50°F , so $\nu = 1.40 \times 10^{-5}\, ft^2/s$

$Re = \frac{D\bar{V}}{\nu} = 2\,ft \times 7.94 \frac{ft}{s} \times \frac{s}{1.40 \times 10^{-5}\, ft^2} = 1.13 \times 10^6$

From Table 8.1, e = 0.00015 ft ; ∴ e/D = 0.000075. Then, from Fig. 8.13, f = 0.013 , and

$\Delta p = p_2 - p_1 = 0.013 \times \frac{3\,mi}{2\,ft} \times \frac{5280\,ft}{mi} \times 1.94 \frac{slug}{ft^3} \times \frac{1}{2} \times (7.94)^2 \frac{ft^2}{s^2} \times \frac{lbf \cdot s^2}{ft \cdot slug} \times \frac{ft^2}{144\,in^2}$

$\Delta p = 43.7$ psi. ← Δp

To determine the energy per unit mass applied by the pump

$\frac{\dot{W}_{pump}}{\dot{m}} = \left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_{discharge} - \left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_{suction}$ (8.45)

$\dot{W}_{pump} = \dot{m} \frac{\Delta p}{\rho} = Q \Delta p$

$\dot{W}_{pump} = 11{,}200 \frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} \times 43.7 \frac{lbf}{in^2} \times \frac{144\,in^2}{ft^2} \times \frac{hp \cdot s}{550\,ft \cdot lbf} = 286\,hp$ ← $\dot{W}_{pump}$

The actual energy required to run the pump is

$P = \frac{\dot{W}_{pump}}{\eta_{pump}\,\eta_{motor}} = 286\,hp \times \frac{1}{0.80} \times \frac{1}{0.90} = 397\,hp.$

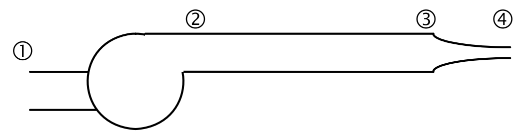
The daily cost is

$C = \frac{\$0.12}{kw.hr} \times 397\,hp \times 0.746 \frac{kw}{hp} \times \frac{24\,hr}{day} = \$853$

Problem 8.159

[4]

8.159 A fire nozzle is supplied through 100 m of 3.5-cm diameter, smooth, rubber-lined hose. Water from a hydrant is supplied to a booster pump on board the pumper truck at 350 kPa (gage). At design conditions, the pressure at the nozzle inlet is 700 kPa (gage), and the pressure drop along the hose is 750 kPa per 100 m of length. Determine (a) the design flow rate, (b) the nozzle exit velocity, assuming no losses in the nozzle, and (c) the power required to drive the booster pump, if its efficiency is 70 percent.



Given:
Fire nozzle/pump system

Find:
Design flow rate; nozzle exit velocity; pump power needed

Solution:

Basic equations
$$\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)-\left(\frac{p_3}{\rho}+\alpha\cdot\frac{V_3^2}{2}+g\cdot z_3\right)=h_l \qquad h_l=f\cdot\frac{L}{D}\cdot\frac{V_2^2}{2}\qquad \text{for the hose}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 2 and 3 is approximately 1 4) No minor loss

$$\frac{p_3}{\rho}+\frac{V_3^2}{2}+g\cdot z_3=\frac{p_4}{\rho}+\frac{V_4^2}{2}+g\cdot z_4$$
for the nozzle (assuming Bernoulli applies)

$$\left(\frac{p_2}{\rho}+\alpha\cdot\frac{V_2^2}{2}+g\cdot z_2\right)-\left(\frac{p_1}{\rho}+\alpha\cdot\frac{V_1^2}{2}+g\cdot z_1\right)=h_{pump}$$
for the pump

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) No minor loss

Hence for the hose $\frac{\Delta p}{\rho}=\frac{p_2-p_3}{\rho}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$ or $V=\sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$

We need to iterate to solve this for V because f is unknown until Re is known. This can be done using *Excel*'s Solver, but here:

$\Delta p = 750\cdot kPa$ $\quad L = 100\cdot m$ $\quad e = 0$ $\quad D = 3.5\cdot cm$ $\quad \rho = 1000\cdot\frac{kg}{m^3}$ $\quad \nu = 1.01\times10^{-6}\cdot\frac{m^2}{s}$

Make a guess for f $\quad f = 0.01 \quad V=\sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}} \quad V = 7.25\frac{m}{s} \quad Re=\frac{V\cdot D}{\nu} \quad Re = 2.51\times10^5$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \quad f = 0.0150$

$V=\sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}} \quad V = 5.92\frac{m}{s} \quad Re=\frac{V\cdot D}{\nu} \quad Re = 2.05\times10^5$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \quad f = 0.0156$

$V=\sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}} \quad V = 5.81\frac{m}{s} \quad Re=\frac{V\cdot D}{\nu} \quad Re = 2.01\times10^5$

Given $\quad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \quad f = 0.0156$

$V = \sqrt{\frac{2 \cdot \Delta p \cdot D}{\rho \cdot f \cdot L}}$ $\quad V = 5.80 \frac{m}{s}$ $\quad Re = \frac{V \cdot D}{\nu}$ $\quad Re = 2.01 \times 10^{5}$

$Q = V \cdot A = \frac{\pi \cdot D^{2}}{4} \cdot V$ $\quad Q = \frac{\pi}{4} \times (0.035 \cdot m)^{2} \times 5.80 \cdot \frac{m}{s}$ $\quad Q = 5.58 \times 10^{-3} \frac{m^{3}}{s}$ $\quad Q = 0.335 \frac{m^{3}}{min}$

For the nozzle $\quad \frac{p_3}{\rho} + \frac{V_3^{2}}{2} + g \cdot z_3 = \frac{p_4}{\rho} + \frac{V_4^{2}}{2} + g \cdot z_4$ $\quad$ so $\quad V_4 = \sqrt{\frac{2 \cdot (p_3 - p_4)}{\rho} + V_3^{2}}$

$V_4 = \sqrt{2 \times 700 \times 10^{3} \cdot \frac{N}{m^{2}} \times \frac{m^{3}}{1000 \cdot kg} \times \frac{kg \cdot m}{s^{2} \cdot N} + \left(5.80 \cdot \frac{m}{s}\right)^{2}}$ $\quad V_4 = 37.9 \frac{m}{s}$

For the pump $\quad \left(\frac{p_2}{\rho} + \alpha \cdot \frac{V_2^{2}}{2} + g \cdot z_2\right) - \left(\frac{p_1}{\rho} + \alpha \cdot \frac{V_1^{2}}{2} + g \cdot z_1\right) = h_{pump}$ $\quad$ so $\quad h_{pump} = \frac{p_2 - p_1}{\rho}$

$p_1 = 350 \cdot kPa$ $\quad p_2 = 700 \cdot kPa + 750 \cdot kPa$ $\quad p_2 = 1450\,kPa$

The pump power is $\quad P_{pump} = m_{pump} \cdot h_{pump}$ $\quad$ where P_{pump} and m_{pump} are the pump power and mass flow rate (software cannot render a dot!)

$P_{pump} = \rho \cdot Q \cdot \frac{(p_2 - p_1)}{\rho} = Q \cdot (p_2 - p_1)$ $\quad P_{pump} = 5.58 \times 10^{-3} \cdot \frac{m^{3}}{s} \times (1450 - 350) \times 10^{3} \cdot \frac{N}{m^{2}}$ $\quad P_{pump} = 6.14\,kW$

$P_{required} = \frac{P_{pump}}{\eta}$ $\quad P_{required} = \frac{6.14 \cdot kW}{70 \cdot \%}$ $\quad P_{required} = 8.77\,kW$

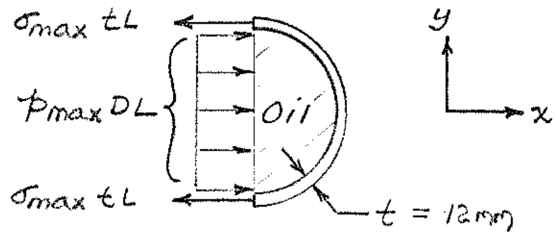
Problem 8.160

Given: Heavy crude oil (SG = 0.925) pumped through a level pipeline at a rate of 400,000 barrels per day (1 bbl = 42 gal). Pipe is 600 mm in diameter with 12 mm wall thickness. Maximum allowable stress in pipe wall is 275 MPa. Minimum pressure in oil is 500 kPa ($\nu = 1.0\times10^{-4}\ m^2/s$) Pipeline is steel.

Find: (a) Maximum allowable spacing between pumping stations.
(b) Power added to oil at each pumping station.

Solution: First find the maximum pressure allowable in pipe.
Consider a free body diagram of a segment of length, L:

Basic equation: $\sum F_x = 0$



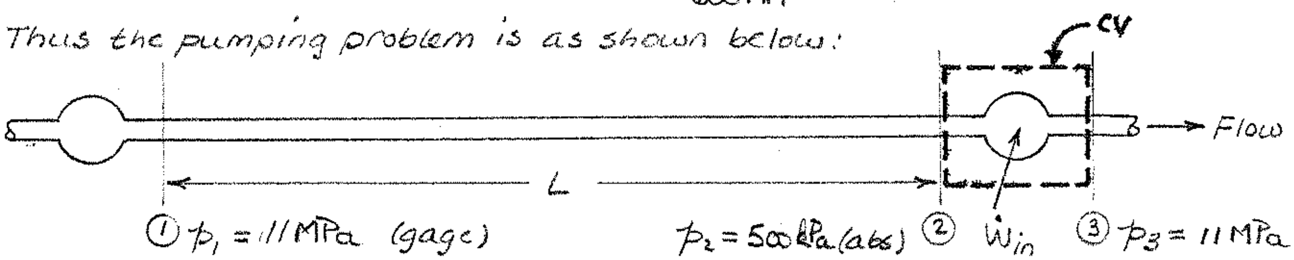
Assumption: Neglect hydrostatic pressure variation, and atmospheric pressure

Then

$$\sum F_x = p_{max} DL - 2\sigma_{max} tL = 0$$

$$p_{max} = 2\sigma_{max}\frac{t}{D} = 2 \times 275\ MPa \times \frac{12mm}{600mm} = 11\ MPa\ (gage)$$

Thus the pumping problem is as shown below:



To find L, apply the energy equation for steady, incompressible flow that is uniform at each section.

Basic equation: $$\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2 + h_{lT}\ ;\ h_{lT} = f\frac{L}{D}\frac{\bar{V}^2}{2} + h_{lm}$$
(cancellations marked (1), (2) on the kinetic energy and elevation terms; $h_{lm} = 0\ (3)$)

Assumptions: (1) $\bar{V}_1 = \bar{V}_2$
(2) $z_1 = z_2$ (level)
(3) $h_{lm} = 0$, since straight, constant area pipe

Then

$$f\frac{L}{D}\frac{\bar{V}^2}{2} = \frac{p_1 - p_2}{\rho} \quad or \quad L = \frac{D}{f}\left(\frac{p_1 - p_2}{\rho}\right)\frac{2}{\bar{V}^2} \quad \text{------------ (1)}$$

$$\bar{V} = \frac{Q}{A} = 4\times10^5\frac{bbl}{day}\times\frac{day}{24hr}\times\frac{hr}{3600s}\times 42\frac{gal}{bbl}\times 4\frac{qt}{gal}\times 9.46\times10^{-4}\frac{m^3}{qt}\times\frac{4}{\pi}\frac{1}{(0.6m)^2} = 2.60\ m/s$$

$f = f(Re, e/D)$. From Table 8.1, $e = 0.046mm$, so $e/D = 7.7\times10^{-5}$. Reynolds number is

$$Re = \frac{\rho\bar{V}D}{\mu} = \frac{\bar{V}D}{\nu} = 2.6\frac{m}{s}\times 0.6m\times\frac{s}{1.0\times10^{-4}\ m^2} = 1.56\times10^4$$

From Eq 8.37, $f = 0.0277$ (Using Excel's Solver or Goal Seek)

Thus, substituting into Eq. 1

$$L = \frac{0.6m}{0.0277}\left[11\times10^6 \frac{N}{m^2} - (500-101)\times10^3 \frac{N}{m^2}\right] \times \frac{m^3}{(0.925)999\,kg} \times 2 \times \frac{s^2}{(2.16)^2 m^2} \times \frac{kg\cdot m}{N\cdot s^2}$$

$$L = 72.8\ km$$

L

To find pump power delivered to the oil, apply the energy equation to the CV shown, between sections ② and ③

$$\left(\frac{p}{\rho} + \alpha\frac{\bar{V}^2}{2} + gz\right)_{discharge} - \left(\frac{p}{\rho} + \alpha\frac{\bar{V}^2}{2} + gz\right)_{suction} = \frac{\dot{W}_{pump}}{\dot{m}} = \Delta h_{pump} \qquad (8.45)$$

Since $\bar{V}$ = constant and elevation change is small, this reduces to

$$\Delta h_{pump} = \frac{p_3 - p_2}{\rho}$$

$$= [11\times10^6 - (500-101)\times10^3]\frac{N}{m^2} \times \frac{m^3}{(0.925)999\,kg} \times \frac{kg\cdot m}{N\cdot s^2}$$

$$\Delta h_{pump} = 1.15\times10^4\ m^2/s^2$$

The mass flow rate is

$$\dot{m} = \rho Q = (0.925)999\frac{kg}{m^3} \times 400{,}000\frac{bbl}{day} \times 42\frac{gal}{bbl} \times 9.46\times10^{-4}\frac{m^3}{qt} \times \frac{4\,qt}{gal} \times \frac{day}{24\,hr} \times \frac{hr}{3600\,s}$$

$$\dot{m} = 680\ kg/s$$

The power added to the oil is

$$\dot{W}_{pump} = \dot{m}\,\Delta h_{pump}$$

$$= 680\frac{kg}{s} \times 1.15\times10^4\frac{m^2}{s^2} \times \frac{N\cdot s^2}{kg\cdot m}$$

$$\dot{W}_{pump} = 7{,}730\ kW$$

$\dot{W}_{pum}$

Note pump efficiency does not affect the power that must be added to the oil.

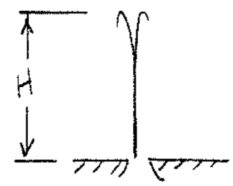
Problem 8.161

Given: Fountain on Purdue's Engineering Mall has

$Q = 550$ gpm and $H = 10$ m (32.8 ft)

Find: Estimate of annual cost to operate the fountain.

Solution: Model fountain as a vertical jet (this will give maximum cost).



Computing equations:

$$C(\$/yr) = C_e\left(\frac{\$}{kW\cdot hr}\right) \times \mathcal{P}_{motor}(kW) \times N(hr/yr)$$

Assume $C_e = \$0.12/kW\cdot hr$

$$\mathcal{P}_{motor} = \frac{\mathcal{P}_{hydraulic}}{\eta_{pump}\eta_{motor}} \; ; \; \eta_{motor} = 0.9, \eta_{pump} = 0.8$$

$$\mathcal{P}_{hydraulic} = Q\Delta p$$

$$N = 365 \frac{days}{yr} \times 24 \frac{hr}{day} = 8{,}760 \; hr/yr$$

The minimum required Δp is $\rho g H$, so

$$\Delta p = 1.94 \frac{slug}{ft^3} \times 32.2 \frac{ft}{s^2} \times 32.8 \; ft \times \frac{lbf \cdot s^2}{slug \cdot ft} = 2.05 \times 10^3 \; lbf/ft^2$$

Combining,

$$C = \frac{\$0.12}{kW\cdot hr} \times \frac{1}{0.8(0.9)} \times 550 \frac{gal}{min} \times 2.05\times10^3 \frac{lbf}{ft^2} \times 8{,}760 \frac{hr}{yr} \times \frac{ft^3}{7.48 \; gal} \times \frac{hp\cdot min}{33{,}000 \; ft\cdot lbf} \times 0.746 \frac{kW}{hp}$$

$$C = \$4980/yr$$

The fountain does not operate year-round. It might be more fair to say $C \approx \$13$ per day of operation.

Problem 8.162

Given: Petroleum products transported long distances by pipeline, e.g., the Alaskan pipeline (see Example Problem 8.6).

Find: (a) Estimate of energy needed to pump typical petroleum product, expressed as a fraction of throughput energy carried by pipeline.

(b) Statement and critical assessment of assumptions.

Solution: From Example Problem 8.6, for the Alaskan pipeline, $Q = 1.6 \times 10^6$ bpd.

Thus $$Q = 1.6\times10^6 \frac{bbl}{day} \times 42\frac{gal}{bbl} \times \frac{ft^3}{7.48\,gal} \times \frac{day}{24\,hr} \times \frac{hr}{3600\,s} = 104\,ft^3/s$$

and
$$\dot{m} = \rho Q = SG\,\rho_{H_2O} Q = 0.93 \times 1.94\frac{slug}{ft^3} \times 104\frac{ft^3}{s} = 188\ slug/s$$

The energy content of a typical petroleum product is about 18,000 Btu/lbm, so the throughput energy is

$$\dot{E} = e\dot{m} = 18{,}000\frac{Btu}{lbm} \times 188\frac{slug}{s} \times 32.2\frac{lbm}{slug} = 1.09\times10^8\ Btu/s$$ $\dot{E}$

From Example Problem 8.6, each pumping station requires 36,800 hp, and they are located $L = 120$ mi apart.

The entire pipeline is about 750 mi long. Thus there must be $N = 750/120$ or about $N = 7$ pumping stations. Thus the total energy required to pump must be

$$\mathcal{P} = N\dot{W} = 7\ stations \times 36{,}800\frac{hp}{station} = 258{,}000\ hp$$ $\mathcal{P}$

Expressed as a fraction of throughput energy

$$\frac{\mathcal{P}}{\dot{E}} = 258{,}000\,hp \times \frac{s}{1.09\times10^8\,Btu} \times 2545\frac{Btu}{hp\cdot hr} \times \frac{hr}{3600\,s} = 1.67\times10^{-3} \text{ or } 0.00167$$

Thus about 0.167% of energy is used for transporting petroleum.

The assumptions outlined above appear reasonable. The computed result is probably accurate within ±10%.

A more universal metric would be energy per unit mass and distance, e.g., energy per ton-mile of transport.

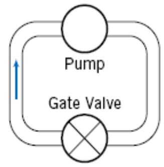
$$\frac{E}{M\cdot L} = \frac{E/t}{M/t\cdot L} = \frac{\mathcal{P}}{\dot{m}L} = 36{,}800\,hp \times \frac{s}{188\,slug} \times \frac{1}{120\,mi} \times 2545\frac{Btu}{hp\cdot hr} \times \frac{slug}{32.2\,lbm} \times 2000\frac{lbm}{ton} \times \frac{hr}{3600\,s}$$

Thus
$$e = \frac{\mathcal{P}}{\dot{m}L} = 71.6\ Btu/ton\cdot mi$$ e

This specific metric allows direct comparison with other modes of transport.

Problem 8.163

8.163 The pump testing system of Problem 8.114 is run with a pump that generates a pressure difference given by $\Delta p = 750 - 15 \times 10^4 Q^2$ where Δp is in kPa, and the generated flow rate is Q m³/s. Find the water flow rate, pressure difference, and power supplied to the pump if it is 70 percent efficient.



Given: Flow in a pump testing system

Find: Flow rate; Pressure difference; Power

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_{lT} = \sum_{major} h_l + \sum_{minor} h_{lm} \quad (8.29)$$

$$Re = \frac{\rho \cdot V \cdot D}{\mu} \qquad h_l = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \quad (8.34) \qquad h_{lm} = f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2} \quad (8.40b)$$

$$f = \frac{64}{Re} \quad (8.36) \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad (8.37) \quad \text{(Turbulent)}$$

The energy equation (Eq. 8.29) becomes for the circuit (1 = pump outlet, 2 = pump inlet)

$$\frac{p_1 - p_2}{\rho} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} + 4 \cdot f \cdot L_{elbow} \cdot \frac{V^2}{2} + f \cdot L_{valve} \cdot \frac{V^2}{2}$$

or

$$\Delta p = \rho \cdot f \cdot \frac{V^2}{2} \cdot \left(\frac{L}{D} + 4 \cdot \frac{L_{elbow}}{D} + \frac{L_{valve}}{D}\right) \quad (1)$$

This must be matched to the pump characteristic equation; at steady state, the pressure generated by the pump just equals that lost to friction in the circuit

$$\Delta p = 750 - 15 \times 10^4 \cdot Q^2 \quad (2)$$

Finally, the power supplied to the pump, efficiency η, is

$$Power = \frac{Q \cdot \Delta p}{\eta} \quad (3)$$

Given data:

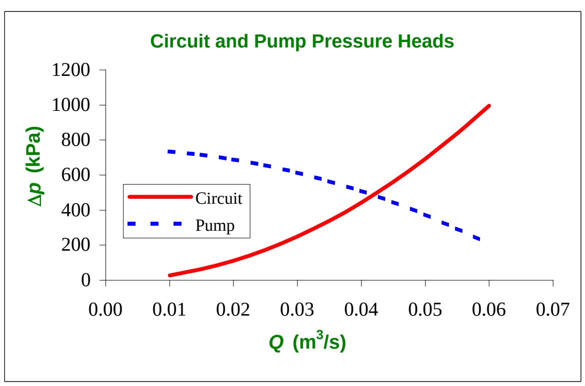
L = 20 m
D = 75 mm
η_{pump} = 70%

Tabulated or graphical data:

e = 0.26 mm
(Table 8.1)
μ = 1.00E-03 N.s/m²
ρ = 999 kg/m³
(Appendix A)
Gate valve L_e/D = 8
Elbow L_e/D = 30
(Table 8.4)

Computed results:

Q (m³/s)	V (m/s)	Re	f	Δp (kPa) (Eq 1)	Δp (kPa) (Eq 2)
0.010	2.26	1.70E+05	0.0280	28.3	735
0.015	3.40	2.54E+05	0.0277	63.1	716
0.020	4.53	3.39E+05	0.0276	112	690
0.025	5.66	4.24E+05	0.0276	174	656
0.030	6.79	5.09E+05	0.0275	250	615
0.035	7.92	5.94E+05	0.0275	340	566
0.040	9.05	6.78E+05	0.0274	444	510
0.045	10.2	7.63E+05	0.0274	561	446
0.050	11.3	8.48E+05	0.0274	692	375
0.055	12.4	9.33E+05	0.0274	837	296
0.060	13.6	1.02E+06	0.0274	996	210



Q (m³/s)	V (m/s)	Re	f	Δp (kPa) (Eq 1)	Δp (kPa) (Eq 2)	Error	
0.0419	9.48	7.11E+05	0.0274	487	487	0	Using *Solver*!

Power = 29.1 kW (Eq. 3)

Problem 8.164 Equations

8.164 A water pump can generate a pressure difference Δp (psi) given by $\Delta p = 145 - 0.1\,Q^2$, where the flow rate is Q ft^3/s. It supplies a pipe of diameter 20 in., roughness 0.5 in., and length 2500 ft. Find the flow rate, pressure difference, and the power supplied to the pump if it is 70 percent efficient. If the pipe were replaced with one of roughness 0.25 in., how much would the flow increase, and what would the required power be?

Given: Pump/pipe system

Find: Flow rate, pressure drop, and power supplied; Effect of roughness

Solution:

$$Re = \frac{\rho \cdot V \cdot D}{\mu} \qquad \left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_{lT} - \Delta h_{pump} \qquad h_{lT} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$$

$$f = \frac{64}{Re} \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad \text{(Turbulent)}$$

The energy equation becomes for the system (1 = pipe inlet, 2 = pipe outlet)

$$\Delta h_{pump} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \quad \text{or} \quad \Delta p_{pump} = \rho \cdot f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \qquad (1)$$

This must be matched to the pump characteristic equation; at steady state, the pressure generated by the pump just equals that lost to friction in the circuit

$$\Delta p_{pump} = 145 - 0.1 \cdot Q^2 \qquad (2)$$

Finally, the power supplied to the pump, efficiency η, is

$$Power = \frac{Q \cdot \Delta p}{\eta} \qquad (3)$$

Tabulated or graphical data:

μ = 2.10E-05 lbf·s/ft^2
ρ = 1.94 slug/ft^3
(Appendix A)

Given data:

L = 2500 ft
D = 20 in
η_{pump} = 70%

Computed results: e = 0.5 in

Q (ft^3/s)	V (ft/s)	Re	f	Δp (psi) (Eq 1)	Δp (psi) (Eq 2)	
10	4.58	7.06E+05	0.0531	11.3	135.0	
12	5.50	8.47E+05	0.0531	16.2	130.6	
14	6.42	9.88E+05	0.0531	22.1	125.4	
16	7.33	1.13E+06	0.0531	28.9	119.4	
18	8.25	1.27E+06	0.0531	36.5	112.6	
20	9.17	1.41E+06	0.0531	45.1	105.0	
22	10.08	1.55E+06	0.0531	54.6	96.6	
24	11.00	1.69E+06	0.0531	64.9	87.4	
26	11.92	1.83E+06	0.0531	76.2	77.4	
28	12.83	1.98E+06	0.0531	88.4	66.6	
30	13.75	2.12E+06	0.0531	101.4	55.0	
						Error
26.1	12.0	1.84E+06	0.0531	76.8	76.8	0.00

Using *Solver* !

Power = 750 hp (Eq. 3)

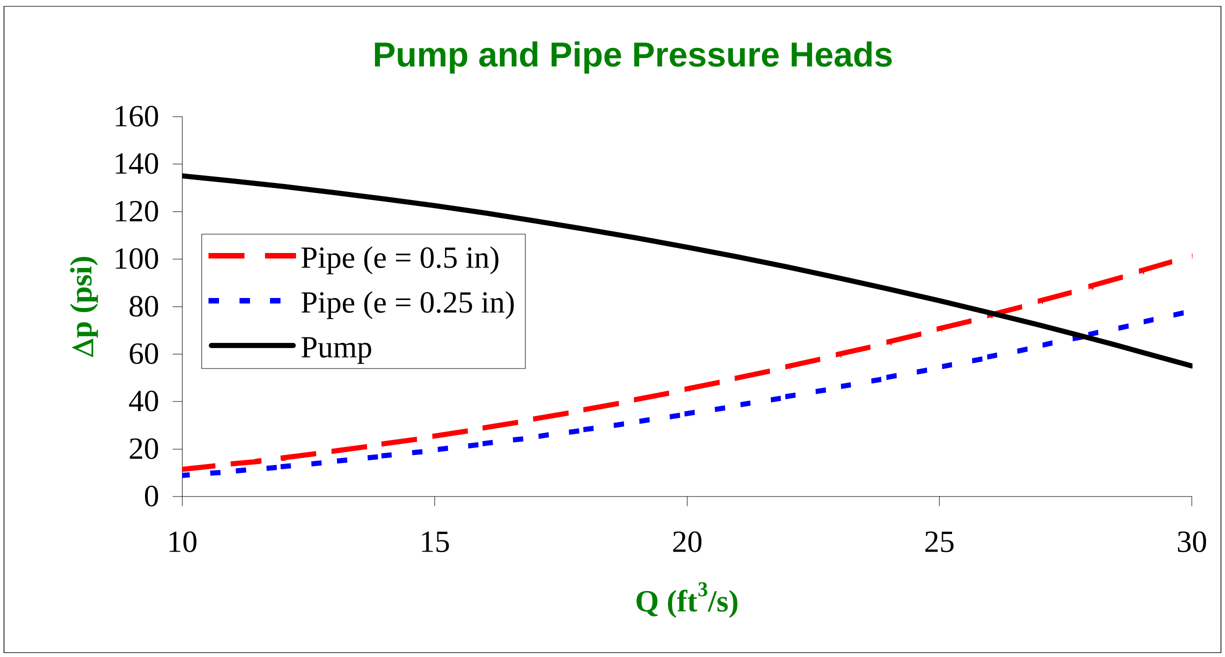
Repeating, with smoother pipe

Computed results: *e* = 0.25 in

Q (ft^3/s)	V (ft/s)	Re	f	Δp (psi) (Eq 1)	Δp (psi) (Eq 2)	
10	4.58	7.06E+05	0.0410	8.71	135.0	
12	5.50	8.47E+05	0.0410	12.5	130.6	
14	6.42	9.88E+05	0.0410	17.1	125.4	
16	7.33	1.13E+06	0.0410	22.3	119.4	
18	8.25	1.27E+06	0.0410	28.2	112.6	
20	9.17	1.41E+06	0.0410	34.8	105.0	
22	10.08	1.55E+06	0.0410	42.1	96.6	
24	11.00	1.69E+06	0.0410	50.1	87.4	
26	11.92	1.83E+06	0.0410	58.8	77.4	
28	12.83	1.98E+06	0.0410	68.2	66.6	
30	13.75	2.12E+06	0.0410	78.3	55.0	
						Error
27.8	12.8	1.97E+06	0.0410	67.4	67.4	0.00

Using *Solver* !

Power = 702 hp (Eq. 3)



Problem 8.165 Equations

[3]

8.165 A square cross-section duct (0.5 m × 0.5 m × 30 m) is used to convey air ($\rho = 1.1\ kg/m^3$) into a clean room in an electronics manufacturing facility. The air is supplied by a fan and passes through a filter installed in the duct. The duct friction factor is $f = 0.03$, the filter has a loss coefficient of $K = 12$, and the clean room is kept at a positive gage pressure of 50 Pa. The fan performance is given by $\Delta p = 1020 - 25Q - 30\,Q^2$, where Δp (Pa) is the pressure generated by the fan at flow rate Q (m^3/s). Determine the flow rate delivered to the room.

Given: Fan/duct system

Find: Flow rate

Solution:

$$\left(\frac{p_1}{\rho} + \alpha_1\cdot\frac{V_1^{\,2}}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\cdot\frac{V_2^{\,2}}{2} + g\cdot z_2\right) = h_{lT} - \Delta h_{fan} \qquad h_{lT} = f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2} + K\cdot\frac{V^2}{2} \qquad f = 0.03$$

The energy equation becomes for the system (1 = duct inlet, 2 = duct outlet)

$$\Delta h_{fan} = f\cdot\frac{L}{D_h}\cdot\frac{V^2}{2} + K\cdot\frac{V^2}{2} \quad \text{or} \quad \Delta p_{pump} = \frac{\rho\cdot V^2}{2}\cdot\left(f\cdot\frac{L}{D_h} + K\right) \quad (1) \qquad \text{where} \quad D_h = \frac{4\cdot A}{P_w} = \frac{4\cdot h^2}{4\cdot h} = h$$

This must be matched to the fan characteristic equation; at steady state, the pressure generated by the fan just equals that lost to friction in the circuit

$$\Delta p_{fan} = 1020 - 25\cdot Q - 30\cdot Q^2 \qquad (2)$$

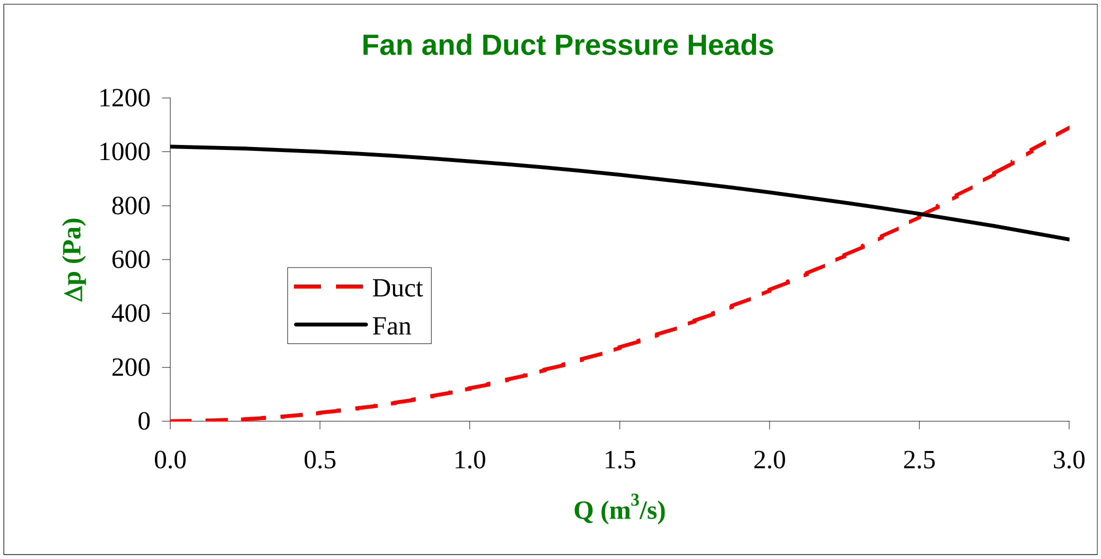
Given data:

L =	30	m
D_h =	0.5	m
K =	12	
f =	0.03	
ρ =	1.1	kg/m^3

Computed results:

Q (m^3/s)	V (m/s)	Δp (Pa) (Eq 1)	Δp (Pa) (Eq 2)
0.00	0.00	0	1020
0.25	1.00	8	1012
0.50	2.00	30	1000
0.75	3.00	68	984
1.00	4.00	121	965
1.25	5.00	190	942
1.50	6.00	273	915
1.75	7.00	372	884
2.00	8.00	486	850
2.25	9.00	615	812
2.50	10.00	759	770
2.75	11.00	918	724
3.00	12.00	1093	675

				Error	
2.51	10.06	768	768	0.00	Using *Solver* !



Problem 8.166

Given: Fan with outlet dimensions of 8 × 16 in. Head vs Capacity curve is approximately

$$H\,(\text{in. } H_2O) = 30 - 10^{-7}\,[Q\,(\text{ft}^3/\text{min})]^2$$

Find: Air flow rate delivered into a 200 ft. length of straight 8 × 16 in. duct.

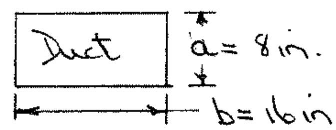
Solution:

Basic equation: $\left(\frac{p_1}{\rho g} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1\right) - \left(\frac{p_2}{\rho g} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2\right) = H_{l_T}$ (8.30)

(terms $\alpha_1 \frac{\bar{V}_1^2}{2g}$, z_1, $\alpha_2 \frac{\bar{V}_2^2}{2g}$, z_2 struck out, marked (1), (2))

$$H_{l_T} = f\frac{L}{D_h}\frac{\bar{V}^2}{2g} + h_{l_m} \quad ; \quad D_h = \frac{4A}{P_w}$$

Assumptions: (1) $\bar{V}_1 = \bar{V}_2$, $\alpha_1 = \alpha_2 \simeq 1$
(2) $z_1 = z_2$
(3) $h_{l_m} = 0$



$$A = ab = \frac{8}{12}\text{ft} \times \frac{16}{12}\text{ft} = 0.889\ \text{ft}^2$$

$$D_h = \frac{4A}{P_w} = \frac{4A}{2(a+b)} = \frac{2 \times 0.889\ \text{ft}^2}{(2/3 + 4/3)\ \text{ft}} = 0.889\ \text{ft}$$

From Eq. 8.30 $\quad \Delta p = f\frac{L}{D}\rho_{air}\frac{\bar{V}^2}{2} = f\frac{L}{D}\frac{\rho}{2}\frac{Q^2}{A^2} = \gamma_{H_2O} H_{duct}$

where H_{duct} is the pressure drop in head of water.

$$H_{duct} = \frac{f L \rho_{air} Q^2}{2\gamma_{H_2O} D_h A^2} = \frac{f}{2} \times \frac{200\ \text{ft}}{0.889\ \text{ft}} \times 0.00238\frac{\text{slug}}{\text{ft}^3} \times \frac{\text{ft}^3}{62.4\ \text{lbf}} \times \frac{1}{(0.889)^2\ \text{ft}^4} \times \left[Q\frac{\text{ft}^3}{\text{min}} \times \frac{\text{min}}{60\ \text{s}}\right]^2 \times \frac{12\ \text{in}}{\text{ft}}$$

$$H_{duct} = 1.81 \times 10^{-5} f Q^2 \quad (\text{where } H \text{ is in in. } H_2O) \quad \text{.........(1)}$$

For a smooth duct, $f = f(Re)$

$$Re = \frac{\bar{V} D_h}{\nu} = \frac{D_h Q}{\nu A} \qquad \text{For } T = 68°F, \text{ from Table A.9, } \nu = 1.62 \times 10^{-4}\ \text{ft}^2/\text{s}$$

$$Re = \frac{0.889\ \text{ft}}{0.889\ \text{ft}^2 \times 1.62 \times 10^{-4}} \frac{\text{s}}{\text{ft}^2} \times Q\frac{\text{ft}^3}{\text{min}} \times \frac{\text{min}}{60\ \text{s}} = 103\ Q$$

To determine the air flow rate delivered we need to determine the operating point of the fan.

The operating point is at the intersection of the

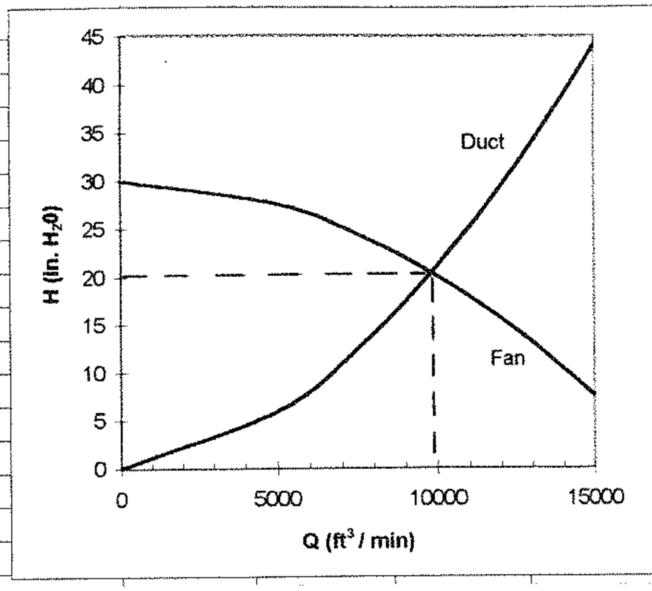
- fan head capacity curve, and the
- system curve (the head loss in the duct)

This is shown on the plot below.

Note that the friction factor f is determined from the Colebrook equation (8.37a) using Eq. 8.37b for the initial estimate of f.

Problem 8.166

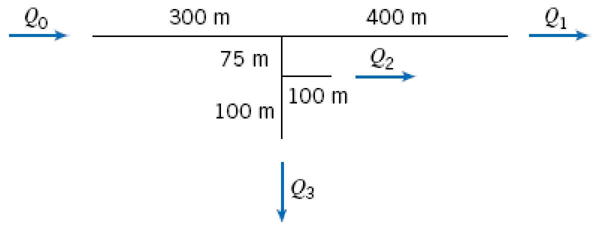
Q	Re	H (fan)	f_0	f	H (duct)
(ft^3/min)	(-)	(in.H_2O)	(-)	(-)	(in.H_2O)
0		30			0
5,000	5.15E+05	27.5	0.0130	0.0131	5.9
7,500	7.73E+05	24.4	0.0121	0.0122	12.4
10,000	1.03E+06	20.0	0.0115	0.0116	21.0
12,500	1.29E+06	14.4	0.0111	0.0112	31.6
15,000	1.55E+06	7.5	0.0108	0.0108	44.1



Operating point
$Q \approx 10{,}000\ ft^3/min$
$H = 20\ in.\ H_2O$

Problem *8.167 [5]

*8.167 The water pipe system shown is constructed from 75 mm galvanized iron pipe. Minor losses may be neglected. The inlet is at 250 kPa (gage), and all exits are at atmospheric pressure. Find the flow rates Q_0, Q_1, Q_2, and Q_3. If the flow in the 400 m branch is closed off ($Q_1 = 0$), find the increase in flows Q_2, and Q_3.



Given: Pipe system

Find: Flow in each branch; Effect of shutting 400 m branch

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^{\ 2}}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^{\ 2}}{2} + g \cdot z_2\right) = h_l \qquad (8.29) \qquad h_{lT} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \qquad (8.34)$$

$$f = \frac{64}{Re} \quad \text{(Laminar)} \quad (8.36) \qquad \frac{1}{f^{0.5}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot f^{0.5}}\right) \quad \text{(Turbulent)} \quad (8.37)$$

The energy equation (Eq. 8.29) can be simplified to $\Delta p = \rho \cdot f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$

This can be written for each pipe section

In addition we have the following contraints

$$Q_0 = Q_1 + Q_4 \qquad (1)$$

$$Q_4 = Q_2 + Q_3 \qquad (2)$$

$$\Delta p = \Delta p_0 + \Delta p_1 \qquad (3)$$

$$\Delta p = \Delta p_0 + \Delta p_4 + \Delta p_2 \qquad (4)$$

$$\Delta p_2 = \Delta p_3 \qquad (5)$$

(Pipe 4 is the 75 m unlabeled section)
We have 5 unknown flow rates (or, equivalently, velocities) and five equations

The workbook for Example 8.11 is modified for use in this problem

Pipe Data:

Pipe	*L* (m)	*D* (mm)	*e* (mm)
0	300	75	0.15
1	400	75	0.15
2	100	75	0.15
3	100	75	0.15
4	75	75	0.15

Fluid Properties:

ρ = 999 kg/m^3
μ = 0.001 $N.s/m^2$

Available Head:

Δp = 250 kPa

Flows:

Q_0 (m^3/s)	Q_1 (m^3/s)	Q_2 (m^3/s)	Q_3 (m^3/s)	Q_4 (m^3/s)
0.00928	0.00306	0.00311	0.00311	0.00623

V_0 (m/s)	V_1 (m/s)	V_2 (m/s)	V_3 (m/s)	V_4 (m/s)
2.10	0.692	0.705	0.705	1.41

Re_0	Re_1	Re_2	Re_3	Re_4
1.57E+05	5.18E+04	5.28E+04	5.28E+04	1.06E+05

f_0	f_1	f_2	f_3	f_4
0.0245	0.0264	0.0264	0.0264	0.0250

Heads:

Δp_0 (kPa)	Δp_1 (kPa)	Δp_2 (kPa)	Δp_3 (kPa)	Δp_4 (kPa)
216.4	33.7	8.7	8.7	24.8

Constraints:

(1) $Q_0 = Q_1 + Q_4$
0.00%

(2) $Q_4 = Q_2 + Q_3$
0.01%

(3) $\Delta p = \Delta p_0 + \Delta p_1$
0.03%

(4) $\Delta p = \Delta p_0 + \Delta p_4 + \Delta p_2$
0.01%

(5) $\Delta p_2 = \Delta p_3$
0.00%

Error: **0.05%** Vary Q_0, Q_1, Q_2, Q_3 and Q_4 using *Solver* to minimize total error

Problem 8.168

*8.168 A cast-iron pipe system consists of a 150 ft section of water pipe, after which the flow branches into two 150 ft sections, which then meet in a final 150 ft section. Minor losses may be neglected. All sections are 1.5 in. diameter, except one of the two branches, which is 1 in. diameter. If the applied pressure across the system is 50 psi, find the overall flow rate and the flow rates in each of the two branches.

Given: Water pipe system

Find: Flow rates

Solution:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^{\,2}}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^{\,2}}{2} + g \cdot z_2\right) = h_l \qquad h_{lT} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$$

$$f = \frac{64}{Re} \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \quad \text{(Turbulent)}$$

The energy equation can be simplified to $\Delta p = \rho \cdot f \cdot \frac{L}{D} \cdot \frac{V^2}{2}$

This can be written for each pipe section

Pipe A (first section) $$\Delta p_A = \rho \cdot f_A \cdot \frac{L_A}{D_A} \cdot \frac{V_A^{\,2}}{2} \qquad (1)$$

Pipe B (1.5 in branch) $$\Delta p_B = \rho \cdot f_B \cdot \frac{L_B}{D_B} \cdot \frac{V_B^{\,2}}{2} \qquad (2)$$

Pipe C (1 in branch) $$\Delta p_C = \rho \cdot f_C \cdot \frac{L_C}{D_C} \cdot \frac{V_C^{\,2}}{2} \qquad (3)$$

Pipe D (last section) $$\Delta p_D = \rho \cdot f_D \cdot \frac{L_D}{D_D} \cdot \frac{V_D^{\,2}}{2} \qquad (4)$$

In addition we have the following contraints

$$Q_A = Q_D \qquad (5)$$

$$Q_A = Q_B + Q_C \qquad (6)$$

$$\Delta p = \Delta p_A + \Delta p_B + \Delta p_D \qquad (7)$$

$$\Delta p_B = \Delta p_C \qquad (8)$$

We have 4 unknown flow rates (or, equivalently, velocities) and four equations (5 - 8); Eqs 1 - 4 relate pressure drops to flow rates (velocities)

The workbook for Example Problem 8.11 is modified for use in this problem

Pipe Data:

Pipe	L (ft)	D (in)	e (ft)
A	150	1.5	0.00085
B	150	1.5	0.00085
C	150	1	0.00085
D	150	1.5	0.00085

Fluid Properties:

ρ = 1.94 slug/ft^3

μ = 2.10E-05 lbf·s/ft^2

Available Head:

Δp = 50 psi

Flows:

Q_A (ft^3/s)	Q_B (ft^3/s)	Q_C (ft^3/s)	Q_D (ft^3/s)
0.103	0.077	0.026	0.103

V_A (ft/s)	V_B (ft/s)	V_C (ft/s)	V_D (ft/s)
8.41	6.28	4.78	8.41

Re_A	Re_B	Re_C	Re_D
9.71E+04	7.25E+04	3.68E+04	9.71E+04

f_A	f_B	f_C	f_D
0.0342	0.0345	0.0397	0.0342

Heads:

Δp_A (psi)	Δp_B (psi)	Δp_C (psi)	Δp_D (psi)
19.5	11.0	11.0	19.5

Constraints:

(5) $Q_A = Q_D$

0.00%

(6) $Q_A = Q_B + Q_C$

0.05%

(7) $\Delta p = \Delta p_A + \Delta p_B + \Delta p_D$

0.00%

(8) $\Delta p_B = \Delta p_C$

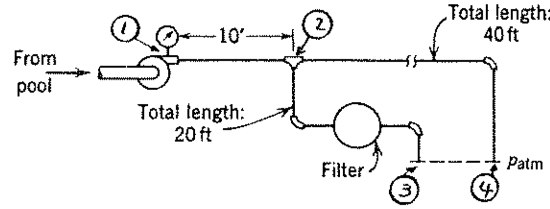
0.00%

Error: **0.05%**

Vary Q_A, Q_B, Q_C, and Q_D using *Solver* to minimize total error

Problem *8.169

Given: Partial-flow filtration system;



Pipes are 3/4 in. nominal PVC (smooth plastic) with $D = 0.824$ in.

Pump delivers 30 gpm at 75°F.

Filter pressure drop is $\Delta p(\text{psi}) = 0.6[Q(\text{gpm})]^2$.

Find: (a) Pressure at pump outlet.
(b) Flow rate through each branch of system.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2 + h_{\ell T}$; $h_{\ell T} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2}$

Assumptions: (1) $\alpha_1\bar{V}_1^2 = \alpha_2\bar{V}_2^2$; (2) $z_1 = z_2$, (3) $h_{\ell m} = 0$ for $1 \to 2$, (4) Ignore "tee" at ②

The flow rate is $Q_{12} = 30$ gpm ($0.0668\ \text{ft}^3/\text{sec}$), so $\bar{V} = \frac{Q}{A} = 18.0$ ft/sec. Then

$$Re = \frac{\bar{V}D}{\nu} = 18.0\frac{\text{ft}}{\text{sec}} \times \left(\frac{0.824}{12}\right)\text{ft} \times \frac{\text{sec}}{1.0\times10^{-5}\ \text{ft}^2} = 1.24\times10^5,\ \text{so } f = 0.017$$

$$\Delta p_{12} = f\frac{L}{D}\rho\frac{\bar{V}^2}{2} = 0.017 \times \frac{10\ \text{ft}}{0.824\ \text{in.}} \times \frac{1}{2} \times 1.94\frac{\text{slug}}{\text{ft}^3} \times (18.0)^2\frac{\text{ft}^2}{\text{sec}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \times \frac{\text{ft}}{12\ \text{in.}} = 5.40\ \text{psi}$$

Branch flow rates are unknown, but flow split must produce the same drop in each branch. Solve by iteration to obtain

$Q_{23} = 5.2$ gpm; $\bar{V}_{23} = 3.12$ ft/s; $Re = 2.15\times10^4$, and $f = 0.025$*

$$\Delta p_{23} = f\left(\frac{L}{D} + 2\frac{L_e}{D}\right)\rho\frac{\bar{V}^2}{2} + 0.6Q^2$$

$$\Delta p_{23} = 0.025\left[\frac{240}{0.824} + 2(30)\right]\frac{1}{2}\times1.94\frac{\text{slug}}{\text{ft}^3}\times(3.12)^2\frac{\text{ft}^2}{\text{s}^2}\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\times\frac{\text{ft}^2}{144\ \text{in.}^2} + 0.6(5.2)^2\frac{\text{lbf}}{\text{in.}^2} = 16.8\ \text{psi}$$

$Q_{24} = 24.8$ gpm; $\bar{V}_{24} = 14.9$ ft/s; $Re = 1.03\times10^5$, and $f = 0.018$

$$\Delta p_{24} = f\left(\frac{L}{D} + \frac{L_e}{D}\right)\rho\frac{\bar{V}^2}{2} = 0.018\left(\frac{480}{0.824} + 30\right)\frac{1}{2}\times1.94\frac{\text{slug}}{\text{ft}^3}\times(14.9)^2\frac{\text{ft}^2}{\text{s}^2}\times\frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\times\frac{\text{ft}^2}{144\ \text{in}^2} = 16.5\ \text{psi}$$

The pump outlet pressure is

$$\Delta p_{pump} = \Delta p_{12} + \Delta p_{23} = (5.4 + 16.8)\ \text{psi} = 22.2\ \text{psi}$$ Δp

The branch flow rates are

$Q_{23} \approx 5.2$ gpm Q_{23}

$Q_{24} \approx 24.8$ gpm Q_{24}

* Value of f obtained from Eq 8.37 using Excel's solver (or Goal Seek)

Problem 8.170

Open-Ended Problem Statement: Why does the shower temperature change when a toilet is flushed? Sketch pressure curves for the hot and cold water supply systems to explain what happens.

Discussion: Assume the pressure in the water main servicing the dwelling remains constant. The hot and cold water flow rates reaching the shower are controlled by valve(s) in the shower. Assuming a water heater temperature of 140°F, a cold water temperature of 60°F, and a shower water temperature of 100°F, the hot and cold flow rates must be equal. The two water streams mix before reaching the shower head, then spray out into the shower itself at 100°F.

Supply curves and system curves for the hot and cold water streams are shown below. Diagram *a* is the cold water system and diagram *b* is the hot water system. The numerical values are representative of an actual system.

In general the supply curves for the hot and cold streams are not the same. The difference is caused by the two systems having different pipe lengths and different fittings.

Each stream operates at the flow rate where the curves intersect. An equal flow split is accomplished by adjusting the shower valves to vary their resistances.

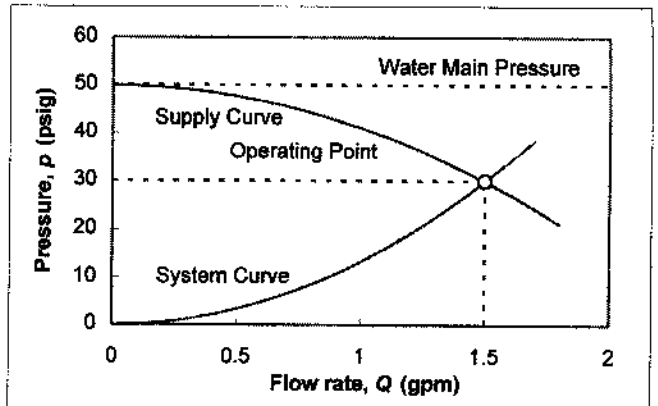
Flushing the toilet temporarily increases the flow rate of cold water to the bathroom. This reduces the cold water supply pressure reaching the shower. The system curves do not change because the valve settings stay the same. Therefore the flow rate of cold water must decrease to again match the supply and system curves (diagram *c*).

When the flow rate of cold water decreases the shower temperature increases, as experience testifies!

Problem 8.170

(a) Cold water system:

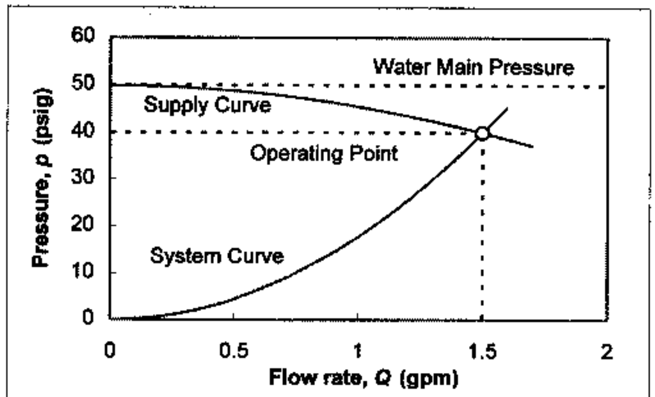
Q (gpm)	System Curve p (psig)	Supply Curve p (psig)
0	0.00	50.0
0.2	0.53	49.6
0.4	2.13	48.6
0.6	4.80	46.8
0.8	8.53	44.3
1.0	13.3	41.1
1.2	19.2	37.2
1.4	26.1	32.6
1.6	34.1	27.2
1.7	38.5	24.3
1.8		21.2



(a) Cold water system curves

(b) Hot water system:

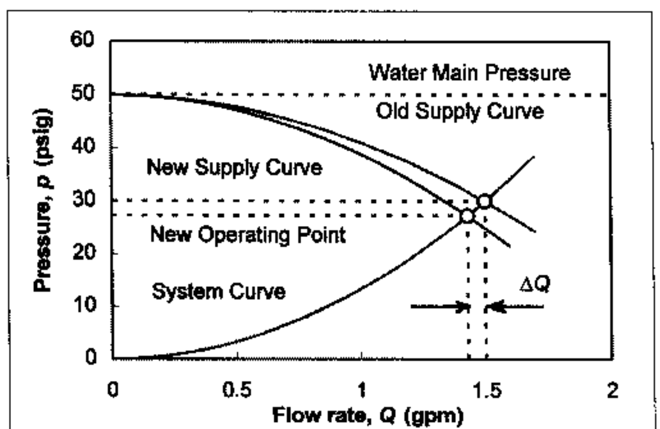
Q (gpm)	System Curve p (psig)	Supply Curve p (psig)
0	0.00	50.0
0.2	0.71	49.8
0.4	2.84	49.3
0.6	6.40	48.4
0.8	11.38	47.2
1.0	17.78	45.6
1.2	25.60	43.6
1.4	34.84	41.3
1.6	45.51	38.6
1.7		37.2
1.8		



(b) Hot water system curves

(c) Cold water system: toilet flush

Q (gpm)	System Curve p (psig)	Old Supply Curve p (psig)	New Supply Curve p (psig)
0	0.00	50.0	50.0
0.2	0.53	49.6	49.6
0.4	2.13	48.6	48.2
0.6	4.80	46.8	46.0
0.8	8.53	44.3	42.9
1.0	13.3	41.1	38.9
1.2	19.2	37.2	34.0
1.4	26.1	32.6	28.2
1.430	27.27	31.8	27.28
1.6	34.1	27.2	21.6
1.7	38.5	24.3	
1.8			



(c) Cold water system curves: toilet flush

Problem 8.171

8.171 Water at 65°C flows through a 75-mm diameter orifice installed in a 150-mm i.d. pipe. The flow rate is 20 L/s. Determine the pressure difference between the corner taps.

Given:
Flow through an orifice

Find:
Pressure drop

Solution:

Basic equation $$m_{actual} = K \cdot A_t \cdot \sqrt{2 \cdot \rho \cdot (p_1 - p_2)} = K \cdot A_t \cdot \sqrt{2 \cdot \rho \cdot \Delta p}$$

Note that m_{actual} is mass flow rate (the software cannot render a dot!)

For the flow coefficient $$K = K\left(Re_{D1}, \frac{D_t}{D_1}\right)$$

At 65°C,(Table A.8) $$\rho = 980 \cdot \frac{kg}{m^3} \qquad \nu = 4.40 \times 10^{-7} \cdot \frac{m^2}{s}$$

$$V = \frac{Q}{A} \qquad V = \frac{4}{\pi} \times \frac{1}{(0.15 \cdot m)^2} \times 20 \cdot \frac{L}{s} \times \frac{0.001 \cdot m^3}{1 \cdot L} \qquad V = 1.13 \frac{m}{s}$$

$$Re_{D1} = \frac{V \cdot D}{\nu} \qquad Re_{D1} = 1.13 \cdot \frac{m}{s} \times 0.15 \cdot m \times \frac{s}{4.40 \times 10^{-7} \cdot m^2} \qquad Re_{D1} = 3.85 \times 10^5$$

$$\beta = \frac{D_t}{D_1} \qquad \beta = \frac{75}{150} \qquad \beta = 0.5$$

From Fig. 8.20 $K = 0.624$

Then $$\Delta p = \left(\frac{m_{actual}}{K \cdot A_t}\right)^2 \cdot \frac{1}{2 \cdot \rho} = \left(\frac{\rho \cdot Q}{K \cdot A_t}\right)^2 \cdot \frac{1}{2 \cdot \rho} = \frac{\rho}{2} \cdot \left(\frac{Q}{K \cdot A_t}\right)^2$$

$$\Delta p = \frac{1}{2} \times 980 \cdot \frac{kg}{m^3} \times \left[20 \cdot \frac{L}{s} \times \frac{0.001 \cdot m^3}{1 \cdot L} \times \frac{1}{0.624} \times \frac{4}{\pi} \times \frac{1}{(0.075 \cdot m)^2}\right]^2 \qquad \Delta p = 25.8 \, kPa$$

Problem 8.172

[2]

Given: Square-edged orifice, $d_t = 100$ mm, used to meter air flow in a 150 mm i.d. line. The pressure upstream of the orifice is $p_1 = 600$ kPa. The pressure drop across the orifice is $\Delta p = 750$ mm H_2O. The air temperature is 25°C

Find: the volume flow rate of air in the line

Solution: Apply analysis of section 8-10; data from Fig. 8.23 apply

Computing equation: $$\dot{m}_{actual} = K A_t \sqrt{2\rho(p_1 - p_2)} \qquad (8.56)$$

Since $\dot{m} = \rho Q$, then

$$Q = \frac{\dot{m}}{\rho} = K A_t \sqrt{\frac{2(p_1 - p_2)}{\rho}}$$

$$p_1 - p_2 = 750 \text{ mm } H_2O = \rho g \Delta h_{H_2O} = 999 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 0.75 m \times \frac{N \cdot s^2}{kg \cdot m} = 7.35 \text{ kPa}$$

For this small Δp, the assumption of incompressible flow is certainly valid

$$\rho = \frac{p_1}{RT} = 701 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287 J} \times \frac{1}{298 K} \times \frac{J}{N \cdot m} = 8.20 \text{ kg/m}^3$$

The flow coefficient $K = K(Re_{D_1}, \frac{D_t}{D_1})$

Assume $Re > 2 \times 10^5$. For $\beta = \frac{D_t}{D_1} = \frac{2}{3}$, from Fig. 8.20, $K = 0.675$

$$Q = K A_t \sqrt{\frac{2(p_1 - p_2)}{\rho}} = 0.675 \frac{\pi}{4} (0.1 m)^2 \left[2 \times 7350 \frac{N}{m^2} \times \frac{m^3}{8.20 kg} \times \frac{kg \cdot m}{N \cdot s^2} \right]^{1/2}$$

$Q = 0.224 \text{ m}^3/s$ ← Q

Check Re. At $T = 25°C$ $\quad \mu = 1.84 \times 10^{-5}$ N·s/m² (Table A.10)

$$Re = \frac{\rho D V}{\mu} = \frac{\rho D Q}{\mu A} = \frac{\rho D Q}{\mu} \frac{4}{\pi D^2} = \frac{4 \rho Q}{\pi \mu D}$$

$$Re = \frac{4}{\pi} \times 8.2 \frac{kg}{m^3} \times 0.224 \frac{m^3}{s} \times \frac{1}{1.84 \times 10^{-5}} \frac{m^2}{N \cdot s} \times \frac{1}{0.15 m} \times \frac{N \cdot s^2}{kg \cdot m}$$

$Re = 8.47 \times 10^5$ ✓ assumption is valid

Problem 8.173

8.173 A venturi meter with a 30-in. diameter throat is placed in a 6-in.-diameter line carrying water at 75°F. The pressure drop between the upstream tap and the venturi throat is 12 in. of mercury. Compute the rate of flow.

Given: Flow through a venturi meter (NOTE: Throat is obviously 3 in not 30 in!)

Find: Flow rate

Solution:

Basic equation
$$m_{actual} = \frac{C \cdot A_t}{\sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot (p_1 - p_2)} = \frac{C \cdot A_t}{\sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot \Delta p}$$

Note that m_{actual} is mass flow rate (the software cannot render a dot!)

For $Re_{D1} > 2 \times 10^5$, $0.980 < C < 0.995$. Assume C = 0.99, then check Re

$$\beta = \frac{D_t}{D_1} \qquad \beta = \frac{3}{6} \qquad \beta = 0.5$$

Also
$$\Delta p = \rho_{Hg} \cdot g \cdot \Delta h = SG_{Hg} \cdot \rho \cdot g \cdot \Delta h$$

Then
$$Q = \frac{m_{actual}}{\rho} = \frac{C \cdot A_t}{\rho \cdot \sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot \Delta p} = \frac{\pi \cdot C \cdot D_t^{\,2}}{4 \cdot \rho \cdot \sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot SG_{Hg} \cdot \rho \cdot g \cdot \Delta h} = \frac{\pi \cdot C \cdot D_t^{\,2}}{4 \cdot \sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot SG_{Hg} \cdot g \cdot \Delta h}$$

$$Q = \frac{\pi}{4 \times \sqrt{1-0.5^4}} \times 0.99 \times \left(\frac{1}{4} \cdot ft\right)^2 \times \sqrt{2 \times 13.6 \times 32.2 \cdot \frac{ft}{s^2} \times 1 \cdot ft} \qquad Q = 1.49 \cdot \frac{ft^3}{s}$$

Hence
$$V = \frac{Q}{A} = \frac{4 \cdot Q}{\pi \cdot D_1^{\,2}} \qquad V = \frac{4}{\pi} \times \frac{1}{\left(\frac{1}{2} \cdot ft\right)^2} \times 1.49 \cdot \frac{ft^3}{s} \qquad V = 7.59 \cdot \frac{ft}{s}$$

At 75°F,(Table A.7)
$$\nu = 9.96 \times 10^{-6} \cdot \frac{ft^2}{s}$$

$$Re_{D1} = \frac{V \cdot D_1}{\nu} \qquad Re_{D1} = 7.59 \cdot \frac{ft}{s} \times \frac{1}{2} \cdot ft \times \frac{s}{9.96 \times 10^{-6} \cdot ft^2} \qquad Re_{D1} = 3.81 \times 10^5$$

Thus $Re_{D1} > 2 \times 10^5$. The volume flow rate is
$$Q = 1.49 \cdot \frac{ft^3}{s}$$

Problem 8.174

8.174 A smooth 200 m pipe, 100 mm diameter connects two reservoirs (the entrance and exit of the pipe are sharp-edged). At the midpoint of the pipe is an orifice plate with diameter 40 mm. If the water levels in the reservoirs differ by 30 m, estimate the pressure differential indicated by the orifice plate and the flow rate.

Given: Reservoir-pipe system

Find: Orifice plate pressure difference; Flow rate

Solution:

Governing equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_{lT} = h_l + \Sigma h_{lm} \qquad (8.29)$$

$$h_l = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} \quad (8.34) \qquad h_{lm} = K \cdot \frac{V^2}{2} \quad (8.40a)$$

$$f = \frac{64}{Re} \quad \text{(Laminar)} \quad (8.36) \qquad \frac{1}{f^{0.5}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot f^{0.5}}\right) \quad \text{(Turbulent)} \quad (8.37)$$

There are three minor losses: at the entrance; at the orifice plate; at the exit. For each $h_{lm} = K \cdot \frac{V^2}{2}$

The energy equation (Eq. 8.29) becomes ($\alpha = 1$)

$$g \cdot \Delta H = \frac{V^2}{2} \cdot \left(f \cdot \frac{L}{D} + K_{ent} + K_{orifice} + K_{exit}\right) \qquad (1)$$

(ΔH is the difference in reservoir heights)

This cannot be solved for V (and hence Q) because f depends on V; we can solve by manually iterating, or by using *Solver*

The tricky part to this problem is that the orifice loss coefficient $K_{orifice}$ is given in Fig. 8.23 as a percentage of pressure differential Δp across the orifice, which is unknown until V is known!

The mass flow rate is given by

$$m_{rate} = K \cdot A_t \cdot \sqrt{2 \cdot \rho \cdot \Delta p} \qquad (2)$$

where K is the orifice flow coefficient, A_t is the orifice area, and Δp is the pressure drop across the orifice

Equations 1 and 2 form a set for solving for TWO unknowns: the pressure drop Δp across the orifice (leading to a value for $K_{orifice}$) and the velocity V. The easiest way to do this is by using *Solver*

Given data:

ΔH =	30	m
L =	200	m
D =	100	mm
D_t =	40	mm
β =	0.40	

Tabulated or graphical data:

K_{ent} =	0.50	(Fig. 8.14)
K_{exit} =	1.00	(Fig. 8.14)
Loss at orifice =	80%	(Fig. 8.23)
μ =	0.001	$N.s/m^2$
ρ =	999	kg/m^3
	(Water - Appendix A)	

Computed results:

Orifice loss coefficient:

K = 0.61
(Fig. 8.20
Assuming high Re)

Flow system:

V = 2.25 m/s
Q = 0.0176 m^3/s
Re = 2.24E+05
f = 0.0153

Orifice pressure drop

Δp = 265 kPa

Eq. 1, solved by varying V AND Δp, using *Solver*:

Left (m^2/s)	Right (m^2/s)	Error
294	293	0.5%

Eq. 2 and $m_{rate} = \rho Q$ compared, varying V AND Δp

	(From Q)	(From Eq. 2)	Error
m_{rate} (kg/s) =	17.6	17.6	0.0%

Total Error	0.5%

Procedure using *Solver*:
a) Guess at V and Δp
b) Compute error in Eq. 1
c) Compute error in mass flow rate
d) Minimize total error
e) Minimize total error by varying V and Δp

Problem 8.175

Given: Flow of gasoline through a venturi meter.

$$SG = 0.73,\ D_1 = 2.0\ \text{in.},\ D_t = 1.0\ \text{in.},\ \Delta h = 380\ \text{mm Hg}$$

Find: Volume flow rate of gasoline.

Solution: Apply the analysis of Section 8-10.3.

Computing equations:

$$\dot{m}_{actual} = \frac{C A_t}{\sqrt{1-\beta^4}}\sqrt{2\rho(p_1 - p_2)} \qquad (8.52)$$

$$C = 0.99 \text{ for } Re_{D_1} > 2\times10^5$$

For the manometer, $\Delta p = \rho_{Hg}\, g\, \Delta h = SG_{Hg}\, \rho_{H_2O}\, g\, \Delta h$

Then

$$Q = \frac{\dot{m}}{\rho} = \frac{C A_t}{\sqrt{1-\beta^4}}\sqrt{\frac{2\Delta p}{\rho}} = \frac{C A_t}{\sqrt{1-\beta^4}}\sqrt{\frac{2 SG_{Hg}\, \rho_{H_2O}\, g\, \Delta h}{SG_{gas}\, \rho_{H_2O}}} = \frac{C A_t}{\sqrt{1-\beta^4}}\sqrt{\frac{2 SG_{Hg}\, g\, \Delta h}{SG_{gas}}}$$

$$Q = \frac{0.99}{\sqrt{1-(0.5)^4}}\frac{\pi}{4}(0.0254)^2\,m^2\sqrt{2\times\frac{13.6}{0.73}\times 9.81\frac{m}{s^2}\times 0.38\,m} = 0.00611\ m^3/s$$

Now check Reynolds number:

$$\bar{V}_1 = \frac{Q}{A_1} = 0.00611\frac{m^3}{s}\times\frac{4}{\pi(0.0508)^2 m^2} = 3.01\ m/s$$

Assume viscosity midway between octane and heptane at 20°C. From Fig. A.1,

$$\mu \approx 5.0\times10^{-4}\ N\cdot s/m^2$$

$$Re_{D_1} = \frac{\rho \bar{V}_1 D_1}{\mu} = (0.73)\,1000\frac{kg}{m^3}\times 3.01\frac{m}{s}\times 0.0508\,m\times\frac{m^2}{5.0\times10^{-4}\,N\cdot s}\times\frac{N\cdot s^2}{kg\cdot m} = 2.23\times10^5$$

Thus assumption that $C = 0.99$ is okay

$$Q = 0.00611\ m^3/s$$

Q

Problem 8.176

[2]

8.176 Consider a horizontal 50 × 25 mm venturi with water flow. For a differential pressure of 150 kPa, calculate the volume flow rate.

Given: Flow through an venturi meter

Find: Flow rate

Solution:

Basic equation
$$m_{actual} = \frac{C \cdot A_t}{\sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot (p_1 - p_2)} = \frac{C \cdot A_t}{\sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot \Delta p}$$

Note that m_{actual} is mass flow rate (the software cannot render a dot!)

For $Re_{D1} > 2 \times 10^5$, $0.980 < C < 0.995$. Assume C = 0.99, then check Re

$$\beta = \frac{D_t}{D_1} \qquad \beta = \frac{25}{50} \qquad \beta = 0.5$$

Then
$$Q = \frac{m_{actual}}{\rho} = \frac{C \cdot A_t}{\rho \cdot \sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot \Delta p} = \frac{\pi \cdot C \cdot D_t^{\,2}}{4 \cdot \sqrt{1-\beta^4}} \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho}}$$

$$Q = \frac{\pi}{4 \times \sqrt{1-0.5^4}} \times 0.99 \times (0.025 \cdot m)^2 \times \sqrt{2 \times 150 \times 10^3 \cdot \frac{N}{m^2} \times \frac{m^3}{1000 \cdot kg} \times \frac{kg \cdot m}{s^2 \cdot N}} \qquad Q = 8.69 \times 10^{-3} \frac{m^3}{s}$$

Hence
$$V = \frac{Q}{A} = \frac{4 \cdot Q}{\pi \cdot D_1^{\,2}} \qquad V = \frac{4}{\pi} \times \frac{1}{(0.05 \cdot m)^2} \times 8.69 \times 10^{-3} \frac{m^3}{s} \qquad V = 4.43 \frac{m}{s}$$

At 20ºC (Table A.8) $\nu = 1.01 \times 10^{-6} \cdot \frac{m^2}{s}$

$$Re_{D1} = \frac{V \cdot D}{\nu} \qquad Re_{D1} = 4.43 \cdot \frac{m}{s} \times 0.05 \cdot m \times \frac{s}{1.01 \times 10^{-6} \cdot m^2} \qquad Re_{D1} = 2.19 \times 10^5$$

Thus $Re_{D1} > 2 \times 10^5$. The volume flow rate is $Q = 8.69 \times 10^{-3} \frac{m^3}{s}$ $\qquad Q = 0.522 \cdot \frac{m^3}{min}$

Problem 8.177

Given: Test of 1.6 ℓ internal combustion engine at 6000 rpm.
Meter air with flow nozzle, $\Delta h \le 0.25$ m. Manometer reads to ± 0.5 mm of water.

Find: (a) Flow nozzle diameter required.
(b) Minimum rate of air flow that can be measured ±2 percent.

Solution: Apply computing equation for flow nozzle.

Computing equation: $\dot{m} = KA_t\sqrt{2\rho(p_1 - p_2)}$ (8.54)

Assumptions: (1) $K = 0.97$ (Section 8-10.2b.)
(2) $\beta = 0$ (nozzle inlet is from atmosphere)
(3) Four-stroke cycle engine with 100 percent volumetric efficiency (+/rev = displacement/2)
(4) Standard air

Then

$$\dot{m} = \rho Q = 1.23\,\frac{kg}{m^3}\times\frac{1.6\,\ell}{2\,rev}\times 6000\,\frac{rev}{min}\times\frac{m^3}{1000\,\ell}\times\frac{min}{60\,s} = 0.0984\ kg/s$$

Solving for A_t,

$$A_t = \frac{\dot{m}}{K\sqrt{2\rho\Delta p}} = \frac{\dot{m}}{K\sqrt{2\rho\rho_{H_2O}g\Delta h}}$$

$$A_t = 0.0984\,\frac{kg}{s}\times\frac{1}{0.97}\left[\frac{1}{2}\times\frac{m^3}{1.23\,kg}\times\frac{m^3}{999\,kg}\times\frac{s^2}{9.81\,m}\times\frac{1}{0.25\,m}\right]^{\frac{1}{2}} = 1.31\times10^{-3}\,m^2$$

$$A_t = \frac{\pi D_t^2}{4}\ ;\ D_t = \sqrt{\frac{4A_t}{\pi}} = 40.8\ mm$$ D_t

The allowable error is ±2 percent, or ±0.02. As discussed in Appendix E, the square-root relationship halves the experimental uncertainty. Thus

$$e = \pm 0.02 \text{ when } e_{\Delta h} = \pm 0.04\ ;\ \Delta h_{min} = \frac{\pm 0.5\,mm}{\pm 0.04} = 12.5\ mm$$

$$\dot{m}_{min} \simeq \dot{m}\sqrt{\frac{\Delta h_{min}}{\Delta h}} = 0.0984\,\frac{kg}{s}\sqrt{\frac{12.5\,mm}{250\,mm}} = 0.0220\ kg/s$$ $\dot{m}_{min}$

The air flow rate could be measured with ±2 percent accuracy down to about

$$\omega = 6000\,rpm\,\frac{0.0220}{0.0984} = 1340\ rpm$$

with this setup.

Problem 8.178

8.178 Air flows through the venturi meter described in Problem 8.173. Assume that the upstream pressure is 60 psi, and that the temperature is everywhere constant at 68°F. Determine the maximum possible mass flow rate of air for which the assumption of incompressible flow is a valid engineering approximation. Compute the corresponding differential pressure reading on a mercury manometer.

Given: Flow through a venturi meter (NOTE: Throat is obviously 3 in not 30 in!)

Find: Maximum flow rate for incompressible flow; Pressure reading

Solution:

Basic equation
$$m_{actual} = \frac{C \cdot A_t}{\sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot (p_1 - p_2)} = \frac{C \cdot A_t}{\sqrt{1-\beta^4}} \cdot \sqrt{2 \cdot \rho \cdot \Delta p}$$

Note that m_{actual} is mass flow rate (the software cannot render a dot!)

Assumptions: 1) Neglect density change 2) Use ideal gas equation for density

Then
$$\rho = \frac{p}{R_{air} \cdot T} \qquad \rho = 60 \cdot \frac{lbf}{in^2} \times \left(\frac{12 \cdot in}{1 \cdot ft}\right)^2 \times \frac{lbm \cdot R}{53.33 \cdot ft \cdot lbf} \times \frac{1 \cdot slug}{32.2 \cdot lbm} \cdot \frac{1}{(68+460) \cdot R} \qquad \rho = 9.53 \times 10^{-3} \cdot \frac{slug}{ft^3}$$

For incompressible flow V must be less than about 100 m/s or 330 ft/s at the throat. Hence

$$m_{actual} = \rho \cdot V_2 \cdot A_2 \qquad m_{actual} = 9.53 \times 10^{-3} \frac{slug}{ft^3} \times 330 \cdot \frac{ft}{s} \times \frac{\pi}{4} \times \left(\frac{1}{4} \cdot ft\right)^2 \qquad m_{actual} = 0.154 \cdot \frac{slug}{s}$$

$$\beta = \frac{D_t}{D_1} \qquad \beta = \frac{3}{6} \qquad \beta = 0.5$$

Also
$$\Delta p = \rho_{Hg} \cdot g \cdot \Delta h \qquad \Delta h = \frac{\Delta p}{\rho_{Hg} \cdot g}$$

and in addition
$$\Delta p = \frac{1}{2 \cdot \rho} \cdot \left(\frac{m_{actual}}{C \cdot A_t}\right)^2 \cdot \left(1-\beta^4\right) \qquad \text{so} \qquad \Delta h = \frac{\left(1-\beta^4\right)}{2 \cdot \rho \cdot \rho_{Hg} \cdot g} \cdot \left(\frac{m_{actual}}{C \cdot A_t}\right)^2$$

For $Re_{D1} > 2 \times 10^5$, $0.980 < C < 0.995$. Assume C = 0.99, then check Re

$$\Delta h = \frac{\left(1-0.5^4\right)}{2} \times \frac{ft^3}{9.53 \times 10^{-3}\ slug} \times \frac{ft^3}{13.6 \cdot 1.94 \cdot slug} \times \frac{s^2}{32.2 \cdot ft} \times \left[0.154 \frac{slug}{s} \times \frac{1}{0.99} \times \frac{4}{\pi} \times \left(\frac{4}{1 \cdot ft}\right)^2\right]^2 \qquad \Delta h = 0.581 \cdot ft \qquad \Delta h = 6.98 \cdot in$$

Hence
$$V = \frac{Q}{A} = \frac{4 \cdot m_{actual}}{\pi \cdot \rho \cdot D_1^2} \qquad V = \frac{4}{\pi} \times \frac{ft^3}{9.53 \times 10^{-3}\ slug} \times \frac{1}{\left(\frac{1}{2} \cdot ft\right)^2} \times 0.154 \frac{slug}{s} \qquad V = 82.3 \cdot \frac{ft}{s}$$

At 68°F,(Table A.7) $\nu = 1.08 \times 10^{-5} \cdot \frac{ft^2}{s}$

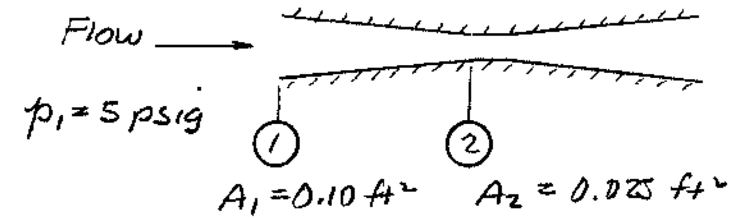
$$Re_{D1} = \frac{V \cdot D_1}{\nu} \qquad Re_{D1} = 82.3 \cdot \frac{ft}{s} \times \frac{1}{2} \cdot ft \times \frac{s}{1.08 \times 10^{-5} \cdot ft^2} \qquad Re_{D1} = 3.81 \times 10^6$$

Thus $Re_{D1} > 2 \times 10^5$. The mass flow rate is $m_{actual} = 0.154 \frac{slug}{s}$ and pressure $\Delta h = 6.98\, in$ Hg

Problem 8.179

[4]

Given: Water at 70°F flows through a Venturi.



Find: Estimate the maximum flow rate with no cavitation. (Express answer in cfs.)

Solution: Apply flowmeter equation.

Computing equation: $\dot{m} = \frac{C A_t}{\sqrt{1-\beta^4}}\sqrt{2\rho(p_1 - p_2)}$; $\beta^2 = A_t/A_1$

Assume $C = 0.99$ for $Re_{D_1} \geq 2\times10^5$.

Cavitation occurs when $p_2 \leq p_v$. From Steam table, $p_v = 0.363$ psia at 70 F. Thus

$$p_1 - p_2 = (14.7 + 5.0) - 0.363 = 19.3 \text{ psi}$$

and

$$\dot{m} = 0.99 \times 0.025\,\text{ft}^2 \times \frac{1}{\sqrt{1-(0.025/0.1)^2}}\left[2 \times 1.94\,\frac{\text{slug}}{\text{ft}^3} \times 19.3\,\frac{\text{lbf}}{\text{in.}^2} \times 144\,\frac{\text{in.}^2}{\text{ft}^2} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right]^{1/2}$$

$$\dot{m} = 2.65 \text{ slug/s}$$

But $\dot{m} = \rho \bar{V} A = \rho Q$, so

$$Q = \frac{\dot{m}}{\rho} = 2.65\,\frac{\text{slug}}{\text{sec}} \times \frac{\text{ft}^3}{1.94\,\text{slug}} = 1.37\,\text{ft}^3/\text{s}$$ Q

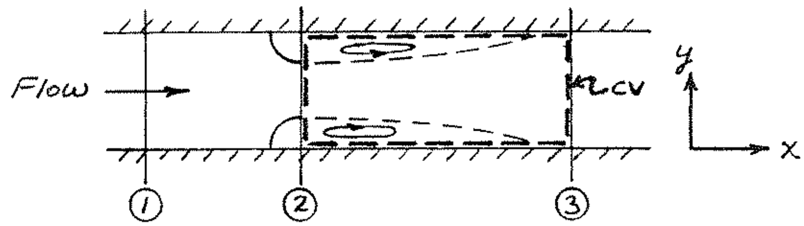
{Note $Q = 1.37\,\frac{\text{ft}^3}{\text{s}} \times 7.48\,\frac{\text{gal}}{\text{ft}^3} \times 60\,\frac{\text{s}}{\text{min}} = 613$ gpm.}

At 70 F, $\nu = 1.05\times10^{-5}$ ft²/s (Table A.7). $Re_{D_1} = \frac{\bar{V}_1 D_1}{\nu}$ $A_1 = \frac{\pi D_1^2}{4}$; so

$D_1 = \sqrt{\frac{4A_1}{\pi}} = \sqrt{\frac{4}{\pi}\times 0.1\,\text{ft}^2} = 0.357$ ft (4.28 in.) ; $\bar{V}_1 = \frac{Q}{A_1} = 1.37\,\frac{\text{ft}^3}{\text{s}} \times \frac{1}{0.1\,\text{ft}^2} = 13.7$ ft/s

Then $Re_{D_1} = 13.7\,\frac{\text{ft}}{\text{s}} \times 0.357\,\text{ft} \times \frac{\text{s}}{1.05\times10^{-5}\,\text{ft}^2} = 4.66\times10^5$, so $C = 0.99$ is okay. ✓✓

Given: Flow nozzle installation in pipe as shown.



Find: Head loss between sections ① and ③, expressed in coefficient form, $C_\ell = \frac{p_1 - p_3}{p_1 - p_2}$, Show $C_\ell = \frac{1 - A_2/A_1}{1 + A_2/A_1}$

Plot: C_ℓ vs. D_2/D_1

Solution: Apply the Bernoulli, continuity, momentum and energy equations, using the CV shown.

Basic equations:

$$\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2 \quad (4)$$

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A} \quad (=0(1))$$

$$F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u \rho \, d\forall + \int_{CS} u \rho \vec{V} \cdot d\vec{A} \quad (=0(5), =0(1))$$

$$\dot{Q} + \dot{W}_s = \frac{\partial}{\partial t}\int_{CV} e \rho \, d\forall + \int_{CS} \left(u + \frac{\bar{V}^2}{2} + gz + \frac{p}{\rho}\right) \rho \vec{V} \cdot d\vec{A} \quad (=0(6), =0(1), (4))$$

Assumptions:
(1) Steady flow
(2) Incompressible flow
(3) No friction between ① and ②
(4) Neglect elevation terms
(5) $F_{B_x} = 0$
(6) $\dot{W}_s = 0$
(7) Uniform flow at each section

From continuity,

$$Q = \bar{V}_1 A_1 = \bar{V}_2 A_2 = \bar{V}_3 A_3$$

Apply Bernoulli along a streamline from ① to ②, noting $A_1 = A_3$,

$$\frac{p_1 - p_2}{\rho} = \frac{\bar{V}_2^2 - \bar{V}_1^2}{2} = \frac{\bar{V}_2^2}{2}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] = \frac{\bar{V}_2^2}{2}\left[1 - \left(\frac{A_2}{A_3}\right)^2\right]$$

From momentum, and using continuity,

$$F_{S_x} = p_2 A_1 - p_3 A_3 = \bar{V}_2\{-|\rho \bar{V}_2 A_2|\} + \bar{V}_3\{+|\rho \bar{V}_3 A_3|\} = (\bar{V}_3 - \bar{V}_2)\rho \bar{V}_3 A_3$$

or

$$\frac{p_3 - p_2}{\rho} = \bar{V}_3(\bar{V}_2 - \bar{V}_3) = \bar{V}_2 \frac{A_2}{A_3}\left[\bar{V}_2 - \bar{V}_2 \frac{A_2}{A_3}\right] = \bar{V}_2^2 \frac{A_2}{A_3}\left(1 - \frac{A_2}{A_3}\right)$$

From energy,

$$\dot{Q} = \left(u_2 + \frac{\bar{V}_2^2}{2} + \frac{p_2}{\rho}\right)\{-|\rho \bar{V}_2 A_2|\} + \left(u_3 + \frac{\bar{V}_3^2}{2} + \frac{p_3}{\rho}\right)\{|\rho \bar{V}_3 A_3|\}$$

or $h_{l_{23}} = u_3 - u_2 - \frac{\dot{Q}}{\dot{m}} = \frac{\bar{V}_2^2 - \bar{V}_3^2}{2} - \frac{p_3 - p_2}{\rho} = \frac{\bar{V}_2^2}{2}\left[1 - \left(\frac{A_2}{A_3}\right)^2\right] - \frac{p_3 - p_2}{\rho}$

But $h_{l_{12}} \simeq 0$ by assumption (3), so $h_{l_{13}} \approx h_{l_{23}}$ and using momentum

$$h_{l_{13}} \simeq \frac{\bar{V}_2^2}{2}\left[1 - \left(\frac{A_2}{A_3}\right)^2\right] - \bar{V}_2^2 \frac{A_2}{A_3}\left(1 - \frac{A_2}{A_3}\right)$$

After a little algebra, this may be written

$$h_{l_{13}} \simeq \frac{\bar{V}_2^2}{2}\left(1 - \frac{A_2}{A_3}\right)^2$$

Dividing by $(p_1 - p_2)/\rho$, a loss coefficient is derived as

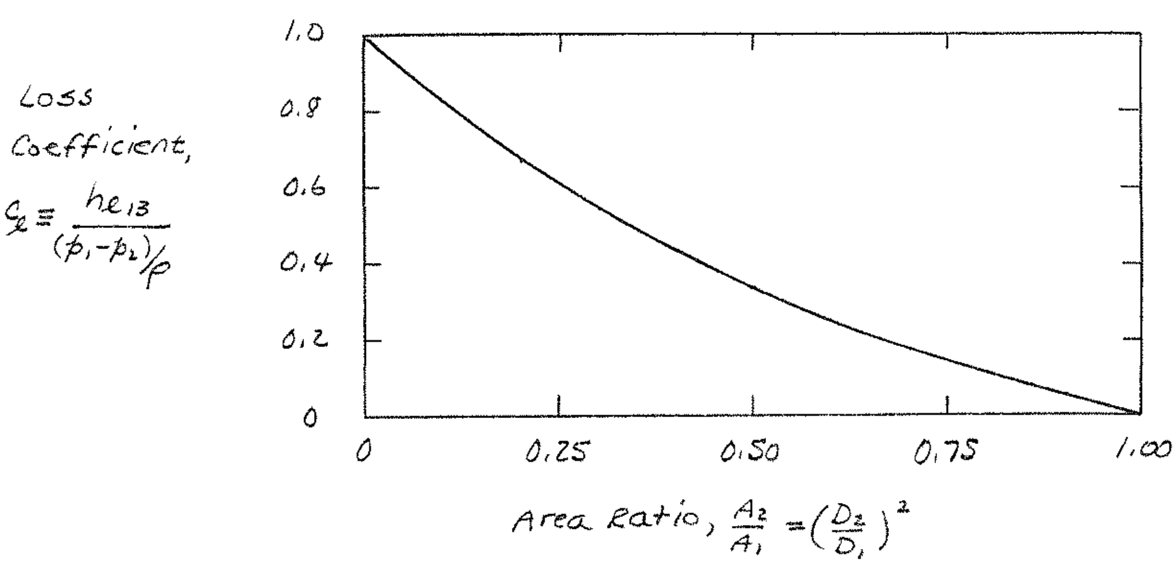
$$C_l = \frac{h_{l_{13}}}{(p_1 - p_2)/\rho} = \frac{\frac{\bar{V}_2^2}{2}\left(1 - \frac{A_2}{A_3}\right)^2}{\frac{\bar{V}_2^2}{2}\left[1 - \left(\frac{A_2}{A_3}\right)^2\right]} = \frac{\left(1 - A_2/A_3\right)^2}{\left[1 - \left(A_2/A_3\right)^2\right]}$$

But $1 - \left(\frac{A_2}{A_3}\right)^2 = \left(1 + \frac{A_2}{A_3}\right)\left(1 - \frac{A_2}{A_3}\right)$, so

$$C_l = \frac{h_{l_{13}}}{(p_1 - p_2)/\rho} = \frac{1 - A_2/A_3}{1 + A_2/A_3}$$

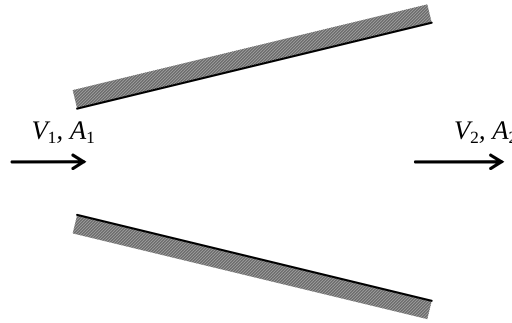
C_l

Plotting:



Problem 8.181

8.181 Derive Eq. 8.42, the pressure loss coefficient for a diffuser assuming ideal (frictionless) flow.



Given: Flow through a diffuser

Find: Derivation of Eq. 8.42

Solution:

Basic equations $\quad C_p = \dfrac{p_2 - p_1}{\frac{1}{2}\cdot\rho\cdot V_1^{\,2}} \qquad \dfrac{p_1}{\rho} + \dfrac{V_1^{\,2}}{2} + g\cdot z_1 = \dfrac{p_2}{\rho} + \dfrac{V_2^{\,2}}{2} + g\cdot z_2 \qquad Q = V\cdot A$

Assumptions: 1) All the assumptions of the Bernoulli equation 2) Horizontal flow 3) No flow separation

From Bernoulli $\quad \dfrac{p_2 - p_1}{\rho} = \dfrac{V_1^{\,2}}{2} - \dfrac{V_2^{\,2}}{2} = \dfrac{V_1^{\,2}}{2} - \dfrac{V_1^{\,2}}{2}\cdot\left(\dfrac{A_1}{A_2}\right)^2 \quad$ using continuity

Hence $\quad C_p = \dfrac{p_2 - p_1}{\frac{1}{2}\cdot\rho\cdot V_1^{\,2}} = \dfrac{1}{\frac{1}{2}\cdot V_1^{\,2}}\cdot\left[\dfrac{V_1^{\,2}}{2} - \dfrac{V_1^{\,2}}{2}\cdot\left(\dfrac{A_1}{A_2}\right)^2\right] = 1 - \left(\dfrac{A_1}{A_2}\right)^2$

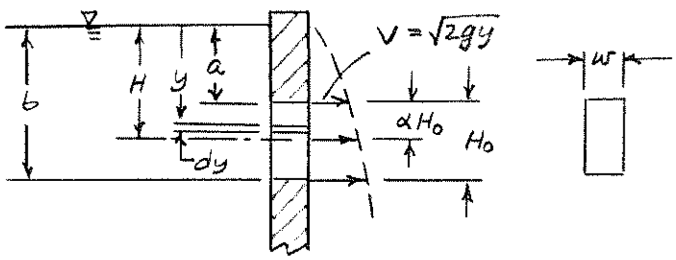
Finally $\quad C_p = 1 - \dfrac{1}{AR^2} \quad$ which is Eq. 8.42.

This result is not realistic as a real diffuser is very likely to have flow separation

Problem 8.182

Open-Ended Problem Statement: In some western states, water for mining and irrigation was sold by the "miner's inch," the rate at which water flows through an opening in a vertical plank of 1 in.^{2} area, up to 4 in. tall, under a head of 6 to 9 in. Develop an equation to predict the flow rate through such an orifice. Specify clearly the aspect ratio of the opening, thickness of the plank, and datum level for measurement of head (top, bottom, or middle of the opening). Show that the unit of measure varies from 38.4 (in Colorado) to 50 (in Arizona, Idaho, Nevada, and Utah) miner's inches equal to 1 ft^3/s.

Analysis: The geometry of the opening in a vertical plank is shown. The analysis includes the effect on flow speed of the variation in water depth vertically across the opening.



Aspect ratio, $ar \equiv \frac{H_0}{w}$; area, $A = w H_0 = 1 \text{ in.}^2$

$\alpha = 0, \frac{1}{2}, 1$

$$Q_{geom} = \int V\,dA = \int_a^b \sqrt{2gy}\, w\,dy = w\sqrt{2g}\left[\frac{2}{3} y^{3/2}\right]_a^b = \frac{2}{3} w a \sqrt{2ga}\left[\left(\frac{b}{a}\right)^{3/2} - 1\right]$$

For $ar = 1$, $\alpha = 0$, $a = H = 9$ in., $b = 10.0$ in., $w = 1.0$ in.

$$Q_{geom} = \frac{2}{3} \times 1.0 \text{ in.} \times 9 \text{ in.} \left[2 \times 32.2 \frac{\text{ft}}{\text{s}^2} \times 9 \text{ in.} \times \frac{\text{ft}}{12 \text{ in.}}\right]^{\frac{1}{2}} \left[\left(\frac{10}{9}\right)^{3/2} - 1\right] \frac{\text{ft}^2}{144 \text{ in.}^2} = 0.0496 \frac{\text{ft}^3}{\text{s}}$$

$Q_{actual} = 0.6\, Q_{geom} = 0.0297 \text{ ft}^3/\text{s}$; thus $1/0.0297 = 33.6$ MI $= 1$ cfs

MI

Numerical results are presented in the spread sheet on the next page.

Discussion: All results assume a *vena contracta* in the liquid jet leaving the opening, reducing the effective flow area to 60 percent of the geometric area of the opening.

The calculated unit of measure varies from 31.3 to 52.4 miner's inch per cubic foot of water flow per second. This range encompasses the 38.4 and 50 values given in the problem statement.

Trends may be summarized as follows. The largest flow rate occurs when datum H is measured to the top of the opening in the vertical plank. This gives the deepest submergence and thus the highest flow speeds through the opening.

When $ar = 1$, the opening is square; when $ar = 16$, the opening is 4 inches tall and 1/4 inch wide. Increasing ar from 1 to 16 increases the flow rate through the opening when H is measured to the top of the opening, because it increases the submergence of the lower portion of the opening, thus increasing the flow speeds. When H is measured to the center of the opening ar has almost no effect on flow rate. When H is measured to the bottom of the opening, increasing ar reduces the flow rate. For this case, the depth of the opening decreases as ar becomes larger.

Plank thickness does not affect calculated flow rates since a *vena contracta* is assumed. In this flow model, water separates from the interior edges of the opening in the vertical plank. Only if the plank were several inches thick might the stream reattach and affect the flow rate.

The actual relationship between Q_{flow} and Q_{geom} might be a weak function of aspect ratio. The flow separates from all four edges of the opening in the vertical plank. At large ar, contraction on the narrow ends of the stream has a relatively small effect on flow area. As ar approaches 1 the effect becomes more pronounced, but would need to be measured experimentally. Assuming a constant 60 percent area fraction certainly gives reasonable trends.

Problem 8.182

Computation of "Miner's Inch" in Engineering Units:

a =	depth to top of opening	(in.)
ar =	aspect ratio of opening	(---)
A =	area of opening	1 in.^{2}
b =	depth to bottom of opening	(in.)
H =	nominal head	(in.)
H_0 =	height of opening	(in.)
MI =	"miner's inch"	(mixed)
Q =	volume flow rate	(ft^3/s)
w =	width of opening	(in.)

Assume $Q_{flow} = 0.6 \times Q_{geometric}$ to account for contraction of the stream leaving the opening.

(a) Measure H to top of opening:

H	ar	H_0	a	b	w	Q_{geom}	Q_{flow}	MI/cfs
9	1	1.00	9.00	10.0	1.00	0.0496	0.0297	33.6
9	2	1.41	9.00	10.4	0.707	0.0501	0.0301	33.3
9	4	2.00	9.00	11.0	0.500	0.0509	0.0305	32.8
9	8	2.83	9.00	11.8	0.354	0.0519	0.0311	32.1
9	16	4.00	9.00	13.0	0.250	0.0533	0.0320	31.3
6	1	1.00	6.00	7.00	1.00	0.0410	0.0246	40.6
6	2	1.41	6.00	7.41	0.707	0.0416	0.0250	40.0
6	4	2.00	6.00	8.00	0.500	0.0425	0.0255	39.2
6	8	2.83	6.00	8.83	0.354	0.0437	0.0262	38.1
6	16	4.00	6.00	10.0	0.250	0.0454	0.0272	36.7

(b) Measure H to middle of opening:

H	ar	H_0	a	b	w	Q_{geom}	Q_{flow}	MI/cfs
9	1	1.00	8.50	9.50	1.00	0.0483	0.0290	34.5
9	2	1.41	8.29	9.71	0.707	0.0483	0.0290	34.5
9	4	2.00	8.00	10.0	0.500	0.0482	0.0289	34.6
9	8	2.83	7.59	10.4	0.354	0.0482	0.0289	34.6
9	16	4.00	7.00	11.0	0.250	0.0482	0.0289	34.6
6	1	1.00	5.50	6.50	1.00	0.0394	0.0236	42.3
6	2	1.41	5.29	6.71	0.707	0.0394	0.0236	42.3
6	4	2.00	5.00	7.00	0.500	0.0394	0.0236	42.3
6	8	2.83	4.59	7.41	0.354	0.0393	0.0236	42.4
6	16	4.00	4.00	8.00	0.250	0.0392	0.0235	42.5

(c) Measure H to bottom of opening:

H	ar	H_0	a	b	w	Q_{geom}	Q_{flow}	MI/cfs
9	1	1.00	8.00	9.00	1.00	0.0469	0.0281	35.5
9	2	1.41	7.59	9.00	0.707	0.0463	0.0278	36.0
9	4	2.00	7.00	9.00	0.500	0.0455	0.0273	36.7
9	8	2.83	6.17	9.00	0.354	0.0442	0.0265	37.7
9	16	4.00	5.00	9.00	0.250	0.0424	0.0254	39.3
6	1	1.00	5.00	6.00	1.00	0.0377	0.0226	44.2
6	2	1.41	4.59	6.00	0.707	0.0370	0.0222	45.1
6	4	2.00	4.00	6.00	0.500	0.0359	0.0215	46.4
6	8	2.83	3.17	6.00	0.354	0.0343	0.0206	48.6
6	16	4.00	2.00	6.00	0.250	0.0318	0.0191	52.4

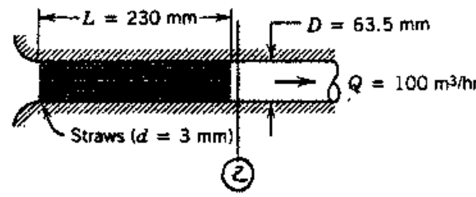
Problem 8.183

Given: Pipe-flow experiment with flow straightener made from straws.

Find: (a) Reynolds number for flow in each straw. $K_{ent} = 1.4$

(b) Friction factor for flow in each straw. $\alpha = 2.0$

(c) Gage pressure at exit from straws.



Solution: Apply energy equation for steady, incompressible pipe flow.

Computing equation:

$$\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2 + h_{lT}$$

$$h_{lT} = h_l + h_{lm} = f \frac{L}{D} \frac{\bar{V}^2}{2} + K_{ent} \frac{\bar{V}^2}{2} = \left(f \frac{L}{D} + K_{ent}\right) \frac{\bar{V}^2}{2}$$

Assumptions: (1) Flow from atmosphere; $p_1 = p_{atm}$, $\bar{V}_1 \approx 0$

(2) Horizontal

(3) Neglect thickness of straws

Then

$$\bar{V}_2 = \frac{Q}{A} = 100 \frac{m^3}{hr} \times \frac{4}{\pi (0.0635)^2 m^2} \times \frac{hr}{3600\ s} = 8.77\ m/s$$

$$Re_d = \frac{\bar{V}_2 d}{\nu} = 8.77 \frac{m}{sec} \times 0.003\ m \times \frac{sec}{1.46 \times 10^{-5} m^2} = 1800$$ Re_d

For laminar flow,

$$f = \frac{64}{Re} = \frac{64}{1800} = 0.0356$$ f

The gage pressure at ② is

$$p_{2g} = -\rho \frac{\bar{V}_2^2}{2}\left(\alpha_2 + K_{ent} + f \frac{L}{D}\right)$$

$$= -\frac{1}{2} \times 1.23 \frac{kg}{m^3} \times (8.77)^2 \frac{m^2}{s^2} \left(2.0 + 1.4 + 0.0356 \times \frac{230\ mm}{3\ mm}\right) \frac{N \cdot s^2}{kg \cdot m}$$

$$p_{2g} = -290\ N/m^2 \text{ (gage)}$$ p_{2g}

This pressure drop is equivalent to

$$\Delta h = \frac{\Delta p}{\rho_{H_2O}\, g} = 290 \frac{N}{m^2} \times \frac{m^3}{999\ kg} \times \frac{s^2}{9.81\ m} \times \frac{kg \cdot m}{N \cdot s^2} = 29.6\ mm\ H_2O$$

Comments: (1) This pressure drop is large enough to measure readily. The straws could be used as a flowmeter.

(2) Straws would eliminate any swirl from the flow.

Problem 8.184

Given: Volume flow rate in a circular duct is to be measured using a "Pitot traverse," by measuring the velocity in each of several area segments across the duct, then summing.

Find: Comment on the way the traverse should be set up. Quantify and plot the expected error in measurement of flow rate as a function of the number of radial locations used in the traverse.

Solution: First divide the duct cross section into segments of equal area. Then measure velocity at the mean area of each segment.

Assume flow is turbulent, and that the velocity profile is well represented by the 1/7-power profile. From Eq. 8.24 the ratio of average flow velocity to centerline velocity is 0.817.

Distinguish two cases, depending on whether velocity is measured at the centerline.

Case 1: Measure velocity at the duct centerline, plus at (k - 1) other locations.

For $k = 1$, the sole measurement is at the duct centerline. This measures the centerline velocity U, which is 1/0.817 = 1.22 times the average flow velocity $\bar{\ }$. Thus the volume flow rate estimated by this 1-point measurement is 22 percent larger than the true value.

For $k = 2$, the duct is divided into two segments of equal area. The centerline velocity is measured and assigned the half of the duct area surrounding the centerline. The second measurement point is located at the midpoint of the remaining half of the duct area. Thus this point is located at the radius that encloses 3/4 of the duct area, or $r_2/R = (3/4)^{1/2} = 0.866$, as shown on the attached spreadsheet. The velocity ratio at this point is $\bar{u}/U = 0.92$. Averaging the segmental flow rates gives (1.22 + 0.92)/2 = 1.07. Thus the volume flow rate estimated by this 2-point measurement is 7 percent high.

For $k = 3$, the duct is divided into three portions of equal area. The centerline velocity is measured and assigned the one-third of the duct area surrounding the centerline. The second measurement point is located at the midpoint of the second one-third of the duct area. This point is located at the radius that encloses half the duct area, or at $r_2/R = (1/2)^{1/2} = 0.707$. The third measurement point is located at the midpoint of the third one-third of the duct area. This point is located at the radius enclosing 5/6 of the duct area, or at $r_3/R = (5/6)^{1/2} = 0.913$.

Results of calculations for $k = 4$ and 5 are also given on the spreadsheet.

Case 2: Measure velocity at k locations, not including the centerline.

For $k = 1$, the radius is chosen at half the duct area. Thus $r_1/R = (1/2)^{1/2} = 0.707$, $\bar{u}/U = 0.839$, and $\bar{u}/\bar{\ } = 1.03$, or about 3 percent too high, as shown on the spreadsheet.

For $k = 2$, the duct is divided into two equal areas. The first measurement is made at the midpoint of the inner area, where the radius includes one fourth of the total area. The second is made at the midpoint of the outer area, where the radius includes three fourths of the total duct area. The results are shown; the flow rate estimate is high by about 1.4 percent.

For $k = 3$, the duct is divided into three equal areas. The first measurement is made at the midpoint of the inner 1/3 of the duct area, where the radius includes 1/6 of the total area. The second is made at the midpoint of the second 1/3 of the duct area, where the radius includes 1/2 of the total duct area. The third is made at the midpoint of the third 1/3 of the duct area, where the radius includes 5/6 of the total duct area. The results are shown; the flow rate estimate is high by about 0.9 percent.

Results of calculations for $k = 4$ and 5 also are given on the spreadsheet.

Remarkably, Case 2 gives less than 2 percent error for any number of locations.

Problem 8.184

$V_{bar}/U =$ 0.817 $n =$ 7 $k =$ Number of measurement points

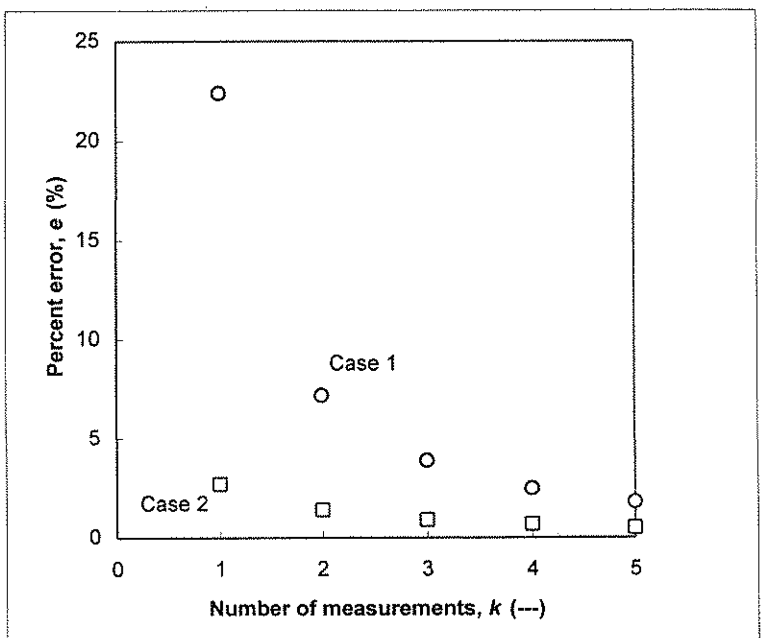
Case 1: Measure at centerline plus at (k - 1) other locations

k	i	r_i/R	u/U	u/V_{bar}	(%) Error
1	1	0.000	1.000	1.22	22.4
2	1	0.000	1.000	1.22	
	2	0.866	0.750	0.92	
				1.07	7.2
3	1	0.000	1.000	1.22	
	2	0.707	0.839	1.03	
	3	0.913	0.706	0.864	
				1.04	3.9
4	1	0.000	1.000	1.22	
	2	0.612	0.873	1.07	
	3	0.791	0.800	0.98	
	4	0.935	0.676	0.828	
				1.03	2.5
5	1	0.000	1.000	1.22	
	2	0.548	0.893	1.09	
	3	0.707	0.839	1.03	
	4	0.837	0.772	0.945	
	5	0.949	0.654	0.801	
				1.02	1.8

Case 2: Measure at k locations not including the centerline

k	i	r_i/R	u/U	u/V_{bar}	(%) Error
1	1	0.707	0.839	1.03	2.7
2	1	0.500	0.906	1.11	
	2	0.866	0.750	0.92	
				1.01	1.4
3	1	0.408	0.928	1.14	
	2	0.707	0.839	1.03	
	3	0.913	0.706	0.86	
				1.01	0.9
4	1	0.354	0.940	1.15	
	2	0.612	0.873	1.07	
	3	0.791	0.800	0.98	
	4	0.935	0.676	0.83	
				1.01	0.7
5	1	0.316	0.947	1.16	
	2	0.548	0.893	1.09	
	3	0.707	0.839	1.03	
	4	0.837	0.772	0.95	
	5	0.949	0.654	0.80	
				1.01	0.5

	Case 1	Case 2
k	e (%)	e (%)
1	22.4	2.7
2	7.2	1.4
3	3.9	0.9
4	2.5	0.7
5	1.8	0.5



Problem 8.185

Open-Ended Problem Statement: The chilled-water pipeline system that provides air conditioning for the Purdue University campus is described in Problem 8.158. The pipe diameter is selected to minimize total cost (capital cost plus operating cost). Annualized costs are compared, since capital cost occurs once and operating cost continues for the life of the system. The optimum diameter depends on both cost factors and operating conditions; the analysis must be repeated when these variables change. Perform a pipeline optimization analysis. Solve Problem 8.158 arranging your calculations to study the effect of pipe diameter on annual pumping cost. (Assume friction factor remains constant.) Obtain an expression for total annual cost per unit delivery (e.g., dollars per cubic meter), assuming construction cost varies as the square of pipe diameter. Obtain an analytic relation for the pipe diameter that yields minimum total cost per unit delivery. Assume the present chilled-water pipeline was optimized for a 20-year life with 5 percent annual interest. Repeat the optimization for a design to operate at 30 percent larger flow rate. Plot the annual cost for electrical energy for pumping and the capital cost, using the flow conditions of Problem 8.158, with pipe diameter varied from 300 to 900 mm. Show how the diameter may be chosen to minimize total cost. How sensitive are the results to interest rate?

(From Problem 8.158: The pipe makes a loop 3 miles in length. The pipe diameter is 2 ft and the material is steel. The maximum design volume flow rate is 11,200 gpm. The circulating pump is driven by an electric motor. The efficiencies of pump and motor are $\eta_p = 0.80$ and $\eta_m = 0.90$, respectively. Electricity cost is \$0.067/(kW·hr).)

Analysis: From Problem 8.158, the electrical energy for pumping costs \$174,000 per year for 11,200 gallons per minute circulation. The present line, with D = 24 in., is optimized for this flow rate. $\dot{W} = Q\Delta p$, so $\dot{W}/Q = \Delta p$.

The optimum pipe diameter minimizes total annualized cost, for construction and operation of the pipeline, $C_t = C_c + C_p$. Construction cost C_c is a one-time cost. Annualized pumping cost C_p is computed by summing the present worth of each annual pumping cost over the lifetime of the pipeline. For 20 years at 5 percent per year, spwf = 13.1 (see spreadsheet). Costs may be expressed in terms of diameter as

$$C_t = C_c + C_p = K_c D^2 + \frac{K_p}{D^5} \qquad (1)$$

For the optimum diameter, $dC_t/dD = 2K_c D - 5K_p D^{-6} = 0$, so

$$K_c = \frac{5K_p}{2D^7} = \frac{5C_p}{2D^2} = \frac{5}{2} \times (13.1)\$174{,}000 \times \frac{1}{(24)^2\,\text{in.}^2} = \$9890/\text{in.}^2$$ K_c

From Eq. 1,

$$K_p = C_p D^5 = (13.1)\$174{,}000 \times (24)^5\,\text{in.}^5 = 1.81 \times 10^{13}\ \$\cdot\text{in.}^5$$ K_p

Calculations with these values are shown on the spreadsheet.

To optimize at a new, larger flow rate, note $C_p \sim \Delta p \sim f\frac{L}{D}\frac{\rho\bar{V}^2}{2} = f\frac{L}{D}\frac{\rho}{2}\left(\frac{Q}{A}\right)^2 \sim f\frac{Q^2}{D^5}$

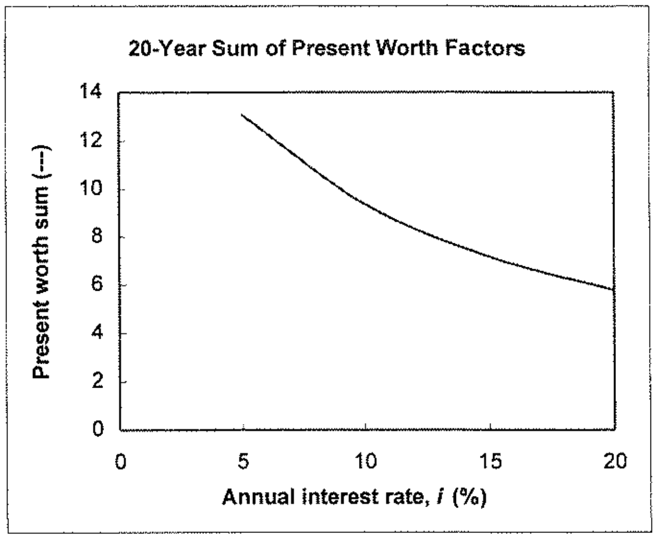
Thus

$K_p(\text{new}) = K_p(\text{old})\left(\frac{Q_{new}}{Q_{old}}\right)^2 = (1.3)^2 K_p(\text{old}) = 3.06 \times 10^{13}\ \$\cdot\text{in.}^5$ The new optimum is at $D \approx 25.9$ in., as shown on the second plot.

Results are not too sensitive to interest rate; only K_p varies. $D_{opt} \to 25$ in. for $i = 15\%$.

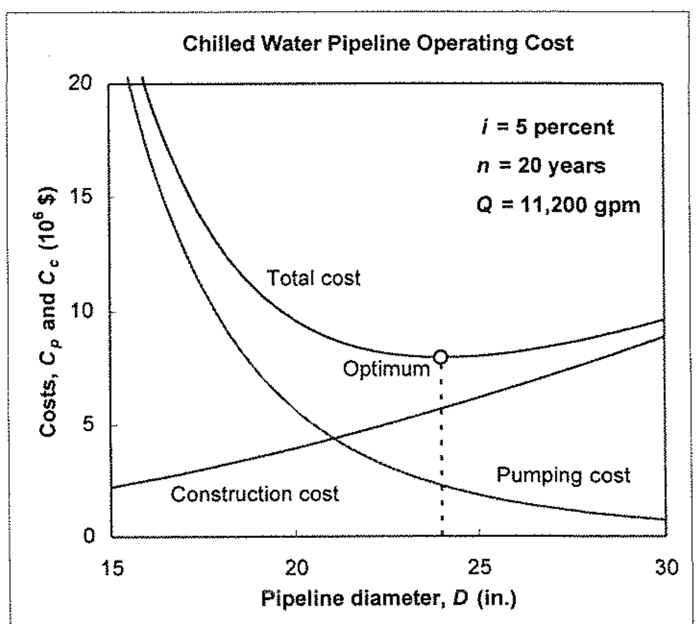
Problem 8.185

	Annual interest rate (%)			
i =	5	10	15	20
Year	*pwf*	*pwf*	*pwf*	*pwf*
1	1.00	1.00	1.00	1.00
2	0.952	0.909	0.870	0.833
3	0.907	0.826	0.756	0.694
4	0.864	0.751	0.658	0.579
5	0.823	0.683	0.572	0.482
6	0.784	0.621	0.497	0.402
7	0.746	0.564	0.432	0.335
8	0.711	0.513	0.376	0.279
9	0.677	0.467	0.327	0.233
10	0.645	0.424	0.284	0.194
11	0.614	0.386	0.247	0.162
12	0.585	0.350	0.215	0.135
13	0.557	0.319	0.187	0.112
14	0.530	0.290	0.163	0.0935
15	0.505	0.263	0.141	0.0779
16	0.481	0.239	0.123	0.0649
17	0.458	0.218	0.107	0.0541
18	0.436	0.198	0.0929	0.0451
19	0.416	0.180	0.0808	0.0376
20	0.396	0.164	0.0703	0.0313
Sum:	**13.1**	**9.4**	**7.2**	**5.8**



K_c =	9,890	\$/in.^{2}	Cost of construction per diameter squared
K_p =	1.81E+13	\$*in.^{5}	Present worth 20-yr cost of pumping 11,200 gpm

Pipe Diameter, D (in.)	Cost of Pumping, C_p (10^6 \$)	Cost to Construct, C_c (10^6 \$)	Total Cost, C_t (10^6 \$)
15	23.9	2.23	26.1
16	17.3	2.53	19.8
17	12.8	2.86	15.6
18	9.59	3.20	12.8
19	7.32	3.57	10.9
20	5.67	3.96	9.62
21	4.44	4.36	8.80
22	3.52	4.79	8.30
23	2.82	5.23	8.05
24	2.28	5.70	7.97
25	1.86	6.18	8.04
26	1.53	6.69	8.21
27	1.26	7.21	8.47
28	1.05	7.75	8.81
29	0.884	8.32	9.20
30	0.746	8.90	9.65

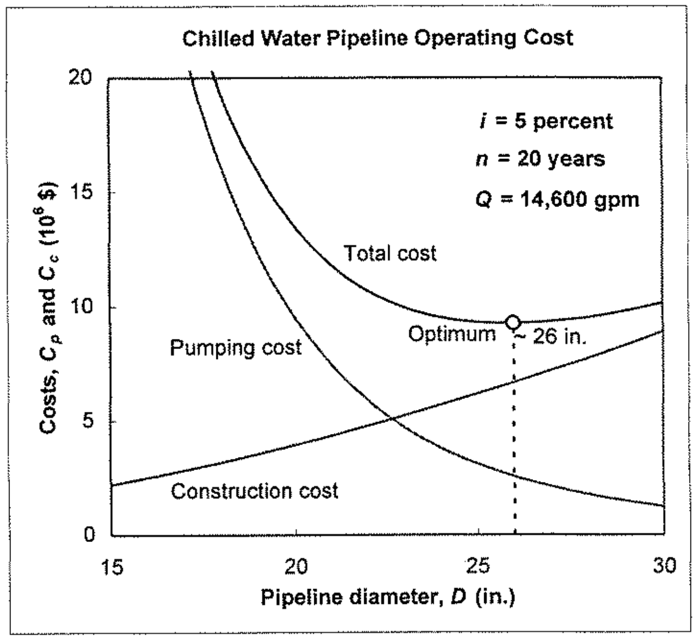


Problem 8.185

K_c = 9,890 $/in.^{2} Cost of construction per diameter squared

K_p = 3.06E+13 $*in.^{5} Present worth 20-yr cost of pumping 14,600 gpm

Pipe Diameter, D (in.)	Cost of Pumping, C_p (10^6 $)	Cost to Construct, C_c (10^6 $)	Total Cost, C_t (10^6 $)
15	40.3	2.23	42.6
16	29.2	2.53	31.8
17	21.6	2.86	24.4
18	16.2	3.20	19.4
19	12.4	3.57	15.9
20	9.57	3.96	13.53
21	7.50	4.36	11.86
22	5.95	4.79	10.73
23	4.76	5.23	9.99
24	3.85	5.70	9.54
25	3.14	6.18	9.32
26	2.58	6.69	9.26
27	2.14	7.21	9.35
28	1.78	7.75	9.53
29	1.49	8.32	9.81
30	1.26	8.90	10.2



Problem 9.1

[2]

9.1 A model of a river towboat is to be tested at 1:18 scale. The boat is designed to travel at 3.5 m/s in fresh water at 10°C. Estimate the distance from the bow where transition occurs. Where should transition be stimulated on the model towboat?

Given: Model of riverboat

Find: Distance at which transition occurs

Solution:

Basic equation $Re_x = \frac{\rho \cdot U \cdot x}{\mu} = \frac{U \cdot x}{\nu}$ and transition occurs at about $Re_x = 5 \times 10^5$

For water at 10ºC $\nu = 1.30 \times 10^{-6} \cdot \frac{m^2}{s}$ (Table A.8) and we are given $U = 3.5 \cdot \frac{m}{s}$

Hence $x_p = \frac{\nu \cdot Re_x}{U}$ $x_p = 0.186\,m$ $x_p = 18.6 \cdot cm$

For the model $x_m = \frac{x_p}{18}$ $x_m = 0.0103\,m$ $x_m = 10.3 \cdot mm$

Problem 9.2

9.2 The roof of a minivan is approximated as a horizontal flat plate. Plot the length of the laminar boundary layer as a function of minivan speed, *V*, as the minivan accelerates from 10 mph to 90 mph.

Given: Minivan traveling at various speeds

Find: Plot of boundary layer length as function of speed

Solution:

Governing equations:

The critical Reynolds number for transition to turbulence is

$$Re_{crit} = \rho V L_{crit}/\mu = 500000$$

The critical length is then

$$L_{crit} = 500000\mu/V\rho$$

Tabulated or graphical data:

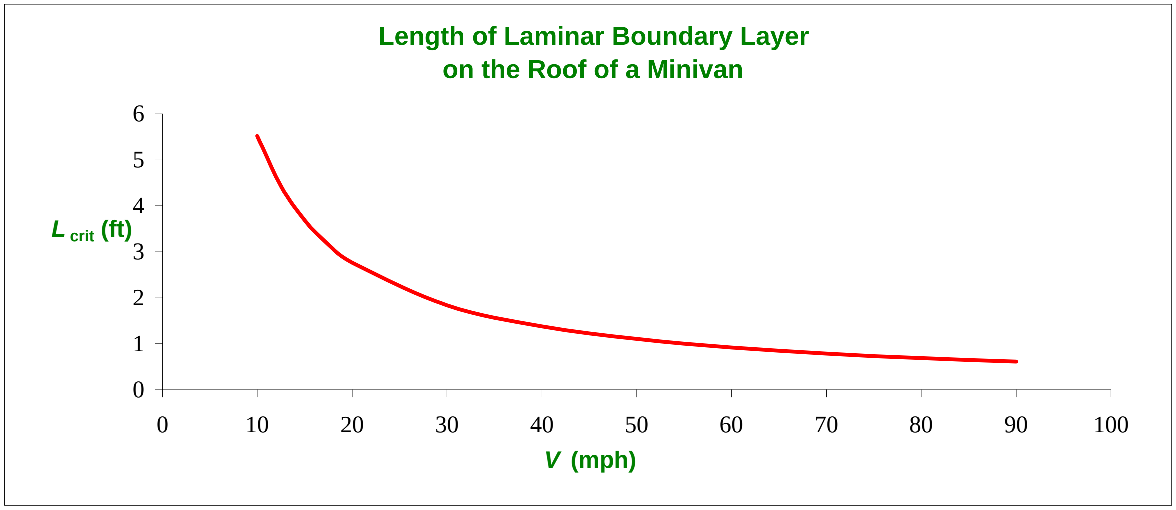
$\mu =$ 3.79E-07 lbf.s/ft^2

$\rho =$ 0.00234 slug/ft^3

(Table A.9, 68°F)

Computed results:

V (mph)	L_{crit} (ft)
10	5.52
13	4.42
15	3.68
18	3.16
20	2.76
30	1.84
40	1.38
50	1.10
60	0.920
70	0.789
80	0.690
90	0.614



Problem 9.3

9.3 The takeoff speed of a Boeing 757 is 260 km/hr. At approximately what distance will the boundary layer on the wings become turbulent? If it cruises at 850 km/hr at 10,000 m, at approximately what distance will the boundary layer on the wings now become turbulent?

Given: Boeing 757

Find: Point at which transition occurs; Same point at 10,000 m

Solution:

Basic equation $Re_x = \frac{\rho \cdot U \cdot x}{\mu} = \frac{U \cdot x}{\nu}$ and transition occurs at about $Re_x = 5 \times 10^5$

For air at 20ºC $\nu = 1.50 \times 10^{-5} \cdot \frac{m^2}{s}$ (Table A.10) and we are given $U = 260 \cdot \frac{km}{hr}$

Hence $x_p = \frac{\nu \cdot Re_x}{U}$ $x_p = 0.104\,m$ $x_p = 10.4\,cm$

At 10,000 m $T = 223.3 \cdot K$ (Table A.3) $T = -49.8\,°C$

We need to estimate ν or μ at this temperature. From Appendix A-3

$$\mu = \frac{b \cdot \sqrt{T}}{1 + \frac{S}{T}} \qquad b = 1.458 \times 10^{-6} \cdot \frac{kg}{m \cdot s \cdot K^{\frac{1}{2}}} \qquad S = 110.4 \cdot K$$

Hence $\mu = \frac{b \cdot \sqrt{T}}{1 + \frac{S}{T}}$ $\mu = 1.458 \times 10^{-5} \frac{N \cdot s}{m^2}$

For air at 10,000 m (Table A.3)

$$\frac{\rho}{\rho_{SL}} = 0.3376 \qquad \rho_{SL} = 1.225 \cdot \frac{kg}{m^3} \qquad \rho = 0.3376 \cdot \rho_{SL} \qquad \rho = 0.414 \frac{kg}{m^3}$$

$\nu = \frac{\mu}{\rho}$ $\nu = 3.53 \times 10^{-5} \frac{m^2}{s}$ and we are given $U = 850 \cdot \frac{km}{hr}$

Hence $x_p = \frac{\nu \cdot Re_x}{U}$ $x_p = 0.0747\,m$ $x_p = 7.47\,cm$

Problem 9.4

9.4 For flow around a sphere the boundary layer becomes turbulent around $Re_D \approx 2.5 \times 10^5$. Find the speeds at which (a) an American golf ball ($D = 1.68$ in.), (b) a British golf ball ($D = 41.1$ mm), and (c) a soccer ball ($D = 8.75$ in.) develop turbulent boundary layers. Assume standard atmospheric conditions.

Given: Flow around American and British golf balls, and soccer ball

Find: Speed at which boundary layer becomes turbulent

Solution:

Basic equation $Re_D = \frac{\rho \cdot U \cdot D}{\mu} = \frac{U \cdot D}{\nu}$ and transition occurs at about $Re_D = 2.5 \times 10^5$

For air $\nu = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s}$ (Table A.9)

For the American golf ball $D = 1.68 \cdot in$ Hence $U = \frac{\nu \cdot Re_D}{D}$ $U = 289 \cdot \frac{ft}{s}$ $U = 197\,mph$ $U = 88.2\,\frac{m}{s}$

For the British golf ball $D = 41.1 \cdot mm$ Hence $U = \frac{\nu \cdot Re_D}{D}$ $U = 300 \cdot \frac{ft}{s}$ $U = 205\,mph$ $U = 91.5\,\frac{m}{s}$

For soccer ball $D = 8.75 \cdot in$ Hence $U = \frac{\nu \cdot Re_D}{D}$ $U = 55.5 \cdot \frac{ft}{s}$ $U = 37.9\,mph$ $U = 16.9\,\frac{m}{s}$

Problem 9.5

9.5 A student is to design an experiment involving dragging a sphere through a tank of fluid to illustrate (a) "creeping flow" ($Re_D < 1$) and (b) flow for which the boundary layer becomes turbulent ($Re_D \approx 2.5 \times 10^5$). She proposes to use a smooth sphere of diameter 1 cm in SAE 10 oil at room temperature. Is this realistic for both cases? If either case is unrealistic, select an alternative reasonable sphere diameter and common fluid for that case.

Given: Experiment with 1 cm diameter sphere in SAE 10 oil

Find: Reasonableness of two flow extremes

Solution:

Basic equation $Re_D = \frac{\rho \cdot U \cdot D}{\mu} = \frac{U \cdot D}{\nu}$ and transition occurs at about

For SAE 10 $\nu = 1.1 \times 10^{-4} \cdot \frac{m^2}{s}$ (Fig. A.3 at 20ºC) and $D = 1 \cdot cm$

For $Re_D = 1$ we find $U = \frac{\nu \cdot Re_D}{D}$ $U = 0.011 \cdot \frac{m}{s}$ $U = 1.10 \cdot \frac{cm}{s}$ which is reasonable

For $Re_D = 2.5 \times 10^5$ $U = \frac{\nu \cdot Re_D}{D}$ $U = 2750 \frac{m}{s}$ which is much too high!

Note that for $Re_D = 2.5 \times 10^5$ we need to increase the sphere diameter D by a factor of about 1000, or reduce the viscosity ν b the same factor, or some combination of these. One possible solution is

For water $\nu = 1.01 \times 10^{-6} \cdot \frac{m^2}{s}$ (Table A.8 at 20ºC) and $D = 10 \cdot cm$

For $Re_D = 2.5 \times 10^5$ we find $U = \frac{\nu \cdot Re_D}{D}$ $U = 2.52 \cdot \frac{m}{s}$ which is reasonable

Hence one solution is to use a 10 cm diameter sphere in a water tank.

Problem 9.6

[2]

9.6 A 4 ft × 8 ft sheet of plywood is attached to the roof of your vehicle after being purchased at the hardware store. At what speed (mph) will the boundary layer first start becoming turbulent? At what speed is about 90% of the boundary layer turbulent?

Given: Sheet of plywood attached to the roof of a car

Find: Speed at which boundary layer becomes turbulent; Speed at which 90% is turbulent

Solution:

Basic equation $Re_x = \frac{\rho \cdot U \cdot x}{\mu} = \frac{U \cdot x}{\nu}$ and transition occurs at about $Re_x = 5 \times 10^5$

For air $\nu = 1.62 \times 10^{-4} \cdot \frac{ft^2}{s}$ (Table A.9)

For the plywood $x = 8 \cdot ft$ Hence $U = \frac{\nu \cdot Re_x}{x}$ $U = 10.1 \cdot \frac{ft}{s}$ $U = 6.90 \cdot mph$

When 90% of the boundary layer is turbulent $x = 0.1 \times 8 \cdot ft$ Hence $U = \frac{\nu \cdot Re_x}{x}$ $U = 101 \cdot \frac{ft}{s}$ $U = 69.0 \cdot mph$

Problem 9.7

[2]

9.7 The extent of the laminar boundary layer on the surface of an aircraft or missile varies with altitude. For a given speed, will the laminar boundary-layer length increase or decrease with altitude? Why? Plot the ratio of laminar boundary-layer length at altitude z, to boundary-layer length at sea level, as a function of z, up to altitude $z = 30$ km, for a standard atmosphere.

Given: Aircraft or missile at various altitudes

Find: Plot of boundary layer length as function of altitude

Solution:

Governing equations:

The critical Reynolds number for transition to turbulence is

$$Re_{crit} = \rho U L_{crit}/\mu = 500000$$

The critical length is then

$$L_{crit} = 500000\mu/U\rho$$

Let L_0 be the length at sea level (density ρ_0 and viscosity μ_0). Then

$$L_{crit}/L_0 = (\mu/\mu_0)/(\rho/\rho_0)$$

The viscosity of air increases with temperature so generally decreases with elevation;
the density also decreases with elevation, but much more rapidly.
Hence we expect that the length ratio increases with elevation

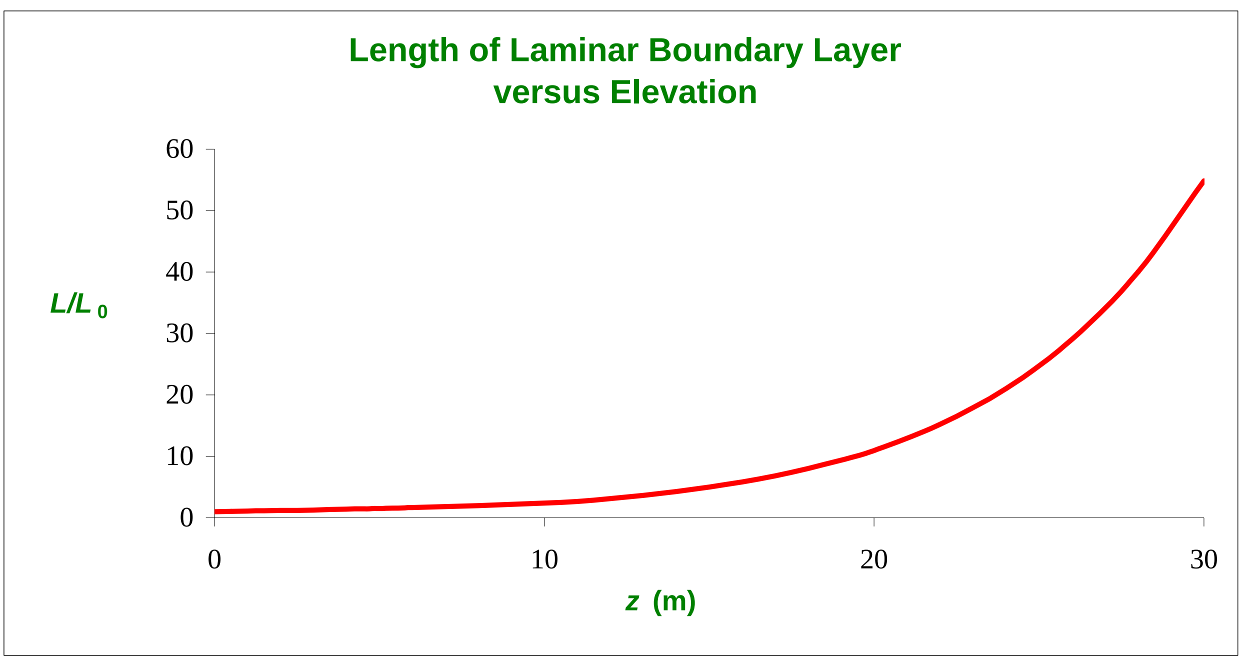
For the density ρ, we use data from Table A.3.
For the viscosity μ, we use the Sutherland correlation (Eq. A.1)

$$\mu = bT^{1/2}/(1+S/T)$$

b = 1.46E-06 kg/m.s.K$^{1/2}$
S = 110.4 K

Computed results:

z (km)	T (K)	ρ/ρ_0	μ/μ_0	L_{crit}/L_0
0.0	288.2	1.0000	1.000	1.000
0.5	284.9	0.9529	0.991	1.04
1.0	281.7	0.9075	0.982	1.08
1.5	278.4	0.8638	0.973	1.13
2.0	275.2	0.8217	0.965	1.17
2.5	271.9	0.7812	0.955	1.22
3.0	268.7	0.7423	0.947	1.28
3.5	265.4	0.7048	0.937	1.33
4.0	262.2	0.6689	0.928	1.39
4.5	258.9	0.6343	0.919	1.45
5.0	255.7	0.6012	0.910	1.51
6.0	249.2	0.5389	0.891	1.65
7.0	242.7	0.4817	0.872	1.81
8.0	236.2	0.4292	0.853	1.99
9.0	229.7	0.3813	0.834	2.19
10.0	223.3	0.3376	0.815	2.41
11.0	216.8	0.2978	0.795	2.67
12.0	216.7	0.2546	0.795	3.12
13.0	216.7	0.2176	0.795	3.65
14.0	216.7	0.1860	0.795	4.27
15.0	216.7	0.1590	0.795	5.00
16.0	216.7	0.1359	0.795	5.85
17.0	216.7	0.1162	0.795	6.84
18.0	216.7	0.0993	0.795	8.00
19.0	216.7	0.0849	0.795	9.36
20.0	216.7	0.0726	0.795	10.9
22.0	218.6	0.0527	0.800	15.2
24.0	220.6	0.0383	0.806	21.0
26.0	222.5	0.0280	0.812	29.0
28.0	224.5	0.0205	0.818	40.0
30.0	226.5	0.0150	0.824	54.8



Problem 9.8

[2]

9.8 Plot on one graph the length of the laminar boundary layer on a flat plate, as a function of freestream velocity, for (a) water and standard air at (b) sea level and (c) 10 km altitude. Use log-log axes, and compute data for the boundary-layer length ranging from 0.01 m to 10 m.

Given: Laminar boundary layer (air & water)

Find: Plot of boundary layer length as function of speed (at various altitudes for air)

Solution:

Governing equations:

The critical Reynolds number for transition to turbulence is

$$Re_{crit} = UL_{crit}/\mu = 500000$$

The critical length is then

$$L_{crit} = 500000\mu/U\rho$$

For air at sea level and 10 km, we can use tabulated data for density ρ from Table A.3.
For the viscosity μ, use the Sutherland correlation (Eq. A.1)

$$\mu = bT^{1/2}/(1+S/T)$$

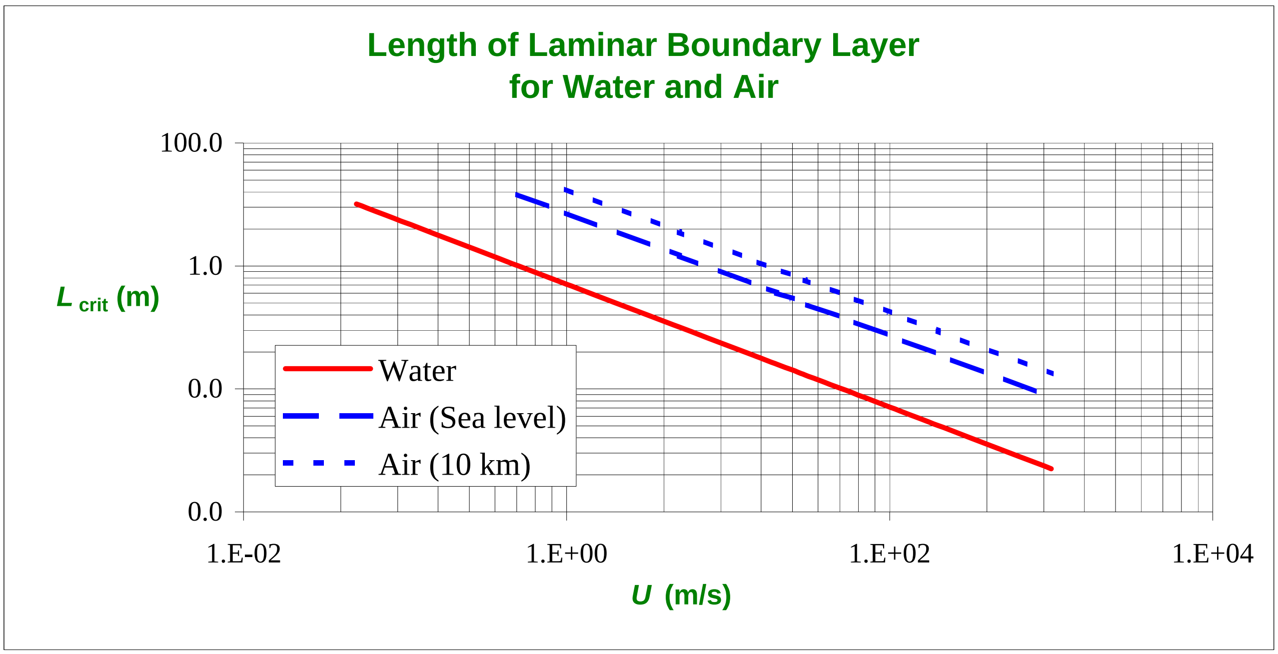
b = 1.46E-06 kg/m.s.K$^{1/2}$

S = 110.4 K

Air (sea level, T = 288.2 K):			Air (10 K, T = 223.3 K):			Water (20^{o}C):		
ρ =	1.225	kg/m^3	ρ =	0.414	kg/m^3	ρ =	998	slug/ft^3
	(Table A.3)			(Table A.3)		μ =	1.01E-03	N.s/m^2
μ =	1.79E-05	N.s/m^2	μ =	1.46E-05	N.s/m^2		(Table A.8)	
	(Sutherland)			(Sutherland)				

Computed results:

U (m/s)	Water L_{crit} (m)	Air (Sea level) L_{crit} (m)	Air (10 km) L_{crit} (m)
0.05	10.12	146.09	352.53
0.10	5.06	73.05	176.26
0.5	1.01	14.61	35.25
1.0	0.506	7.30	17.63
5.0	0.101	1.46	3.53
15	0.0337	0.487	1.18
20	0.0253	0.365	0.881
25	0.0202	0.292	0.705
30	0.0169	0.243	0.588
50	0.0101	0.146	0.353
100	0.00506	0.0730	0.176
200	0.00253	0.0365	0.0881
1000	0.00051	0.0073	0.0176



Problem 9.9

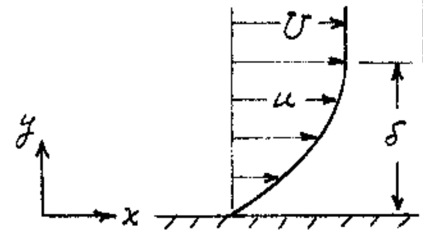
[2]

Given: Sinusoidal velocity profile for laminar boundary layer

$$u = A\sin(By) + C$$

Find: (a) State three applicable boundary conditions.
(b) Evaluate A, B and C.

Solution: For the boundary layer, at



(1) $y = 0, \quad u = 0$ (no slip)

(2) $y = \delta, \quad u = U$

(3) $\dfrac{\partial u}{\partial y} = 0$ (no shear stress)

B.C.

Applying these boundary conditions,

(1) $u(0) = A\sin(0) + C = 0 \quad \therefore C = 0$

(2) $u(\delta) = A\sin(B\delta) = U$

$$\frac{\partial u}{\partial y} = AB\cos(By)$$

(3) $\left.\dfrac{\partial u}{\partial y}\right)_{y=\delta} = AB\cos(B\delta) = 0 \quad \therefore B\delta = \dfrac{\pi}{2} \quad \text{or } B = \dfrac{\pi}{2\delta}$

Then from (2), $A\sin(B\delta) = A\sin(\frac{\pi}{2}) = A = U$, and then

$$u = U\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

$$A = U, \quad B = \frac{\pi}{2\delta}, \quad C = 0$$

A, B, C

Problem 9.10

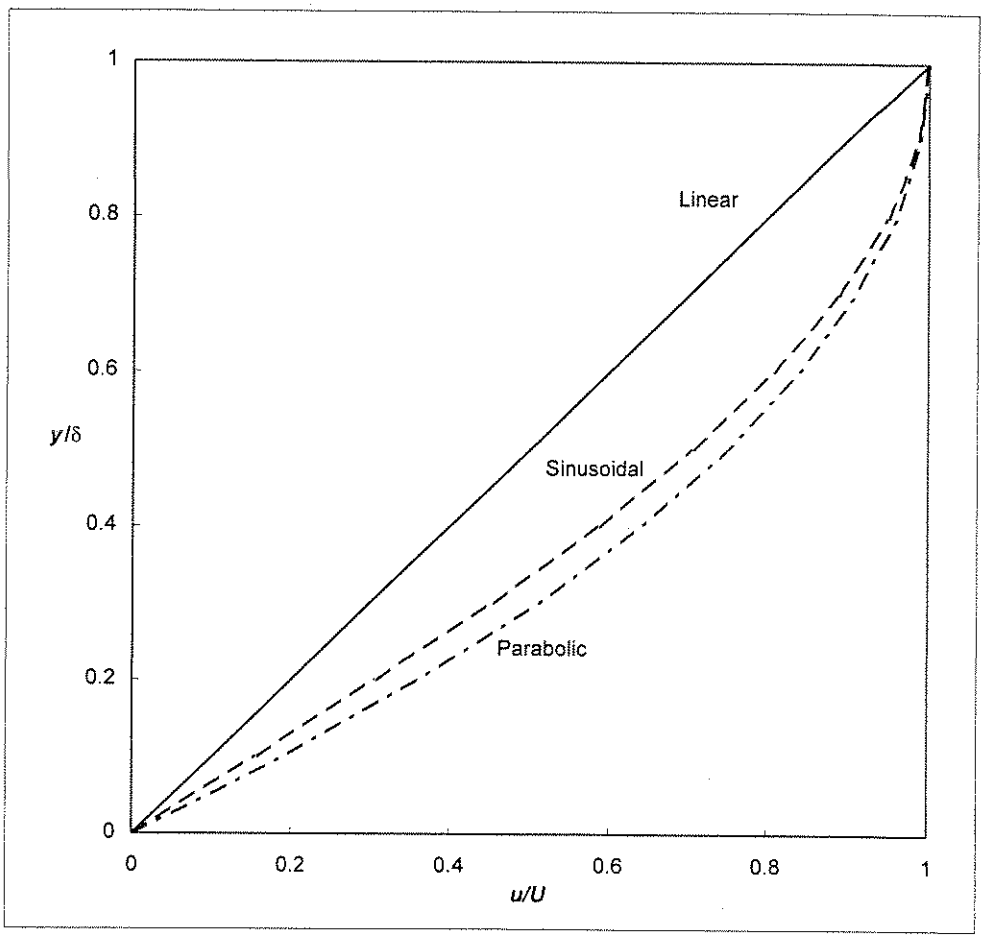
Given: Linear, parabolic, and sinusoidal velocity profiles for laminar boundary layer.

Linear $\frac{u}{U} = \frac{y}{\delta}$

Parabolic $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ Sinusoidal $\frac{u}{U} = \sin \frac{\pi}{2}\left(\frac{y}{\delta}\right)$

Find: Compare shapes by plotting $\frac{y}{\delta}$ vs. $\frac{u}{U}$.

Solution:



Problem 9.11

9.11 An approximation for the velocity profile in a laminar boundary layer is

$$\frac{u}{U}=\frac{3}{2}\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Does this expression satisfy boundary conditions applicable to the laminar boundary-layer velocity profile? Evaluate δ^*/δ and θ/δ.

Given: Laminar boundary layer profile

Find: If it satisfies BC's; Evaluate δ^*/δ and θ/δ

Solution:

The boundary layer equation is $\quad \dfrac{u}{U}=\dfrac{3}{2}\dfrac{y}{\delta}-\dfrac{1}{2}\left(\dfrac{y}{\delta}\right)^3$ for which $u = U$ at $y = \delta$

The BC's are $\quad u(0)=0 \qquad \left.\dfrac{du}{dy}\right|_{y=\delta}=0$

At $y = 0$ $\quad \dfrac{u}{U}=\dfrac{3}{2}(0)-\dfrac{1}{2}(0)^3=0$

At $y = \delta$ $\quad \dfrac{du}{dy}=U\left(\dfrac{3}{2}\dfrac{1}{\delta}-\dfrac{3}{2}\dfrac{y^2}{\delta^3}\right)\Bigg|_{y=\delta}=U\left(\dfrac{3}{2}\dfrac{1}{\delta}-\dfrac{3}{2}\dfrac{\delta^2}{\delta^3}\right)=0$

For δ^*: $\quad \delta^*=\displaystyle\int_0^\infty\left(1-\frac{u}{U}\right)dy=\int_0^\delta\left(1-\frac{u}{U}\right)dy$

Then $\quad \dfrac{\delta^*}{\delta}=\dfrac{1}{\delta}\displaystyle\int_0^\delta\left(1-\frac{u}{U}\right)dy=\int_0^1\left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right)=\int_0^1\left(1-\frac{u}{U}\right)d\eta$

with $\quad \dfrac{u}{U}=\dfrac{3}{2}\eta-\dfrac{1}{2}\eta^3$

Hence $\quad \dfrac{\delta^*}{\delta}=\displaystyle\int_0^1\left(1-\frac{u}{U}\right)d\eta=\int_0^1\left(1-\frac{3}{2}\eta+\frac{1}{2}\eta^3\right)d\eta=\left[\eta-\frac{3}{4}\eta^2+\frac{1}{8}\eta^4\right]_0^1=\frac{3}{8}=0.375$

For θ: $\quad \theta=\displaystyle\int_0^\infty\frac{u}{U}\left(1-\frac{u}{U}\right)dy=\int_0^\delta\frac{u}{U}\left(1-\frac{u}{U}\right)dy$

Then $\quad \dfrac{\theta}{\delta}=\dfrac{1}{\delta}\displaystyle\int_0^\delta\frac{u}{U}\left(1-\frac{u}{U}\right)dy=\int_0^1\frac{u}{U}\left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right)=\int_0^1\frac{u}{U}\left(1-\frac{u}{U}\right)d\eta$

Hence $\quad \dfrac{\theta}{\delta}=\displaystyle\int_0^1\frac{u}{U}\left(1-\frac{u}{U}\right)d\eta=\int_0^1\left(\frac{3}{2}\eta-\frac{1}{2}\eta^3\right)\left(1-\frac{3}{2}\eta+\frac{1}{2}\eta^3\right)d\eta=\int_0^1\left(\frac{3}{2}\eta-\frac{9}{4}\eta^2-\frac{1}{2}\eta^3+\frac{3}{2}\eta^4-\frac{1}{4}\eta^6\right)d\eta$

$$\frac{\theta}{\delta}=\left[\frac{3}{4}\eta^2-\frac{3}{4}\eta^3-\frac{1}{8}\eta^4+\frac{3}{10}\eta^5-\frac{1}{28}\eta^7\right]_0^1=\frac{39}{280}=0.139$$

Problem 9.12

9.12 An approximation for the velocity profile in a laminar boundary layer is

$$\frac{u}{U} = 2\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$$

Does this expression satisfy boundary conditions applicable to the laminar boundary-layer velocity profile? Evaluate δ^*/δ and θ/δ.

Given: Laminar boundary layer profile

Find: If it satisfies BC's; Evaluate δ^*/δ and θ/δ

Solution:

The boundary layer equation is $\frac{u}{U} = 2\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$ for which $u = U$ at $y = \delta$

The BC's are $u(0) = 0 \qquad \left.\frac{du}{dy}\right|_{y=\delta} = 0$

At $y = 0$ $\qquad \frac{u}{U} = 2(0) - 2(0)^3 + (0)^4 = 0$

At $y = \delta$ $\qquad \frac{du}{dy} = U\left(2\frac{1}{\delta} - 6\frac{y^2}{\delta^3} + 4\frac{y^3}{\delta^4}\right)\Bigg|_{y=\delta} = U\left(2\frac{1}{\delta} - 6\frac{\delta^2}{\delta^3} + 4\frac{\delta^3}{\delta^4}\right) = 0$

For δ^*: $\qquad \delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right)dy = \int_0^\delta \left(1 - \frac{u}{U}\right)dy$

Then $\qquad \frac{\delta^*}{\delta} = \frac{1}{\delta}\int_0^\delta \left(1 - \frac{u}{U}\right)dy = \int_0^1 \left(1 - \frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \int_0^1 \left(1 - \frac{u}{U}\right)d\eta$

with $\qquad \frac{u}{U} = 2\eta - 2\eta^3 + \eta^4$

Hence $\qquad \frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{u}{U}\right)d\eta = \int_0^1 \left(1 - 2\eta + 2\eta^3 - \eta^4\right)d\eta = \left[\eta - \eta^2 + \frac{1}{2}\eta^4 - \frac{1}{5}\eta^5\right]_0^1 = \frac{3}{10} = 0.3$

For θ: $\qquad \theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy$

Then $\qquad \frac{\theta}{\delta} = \frac{1}{\delta}\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right)d\eta$

Hence

$$\frac{\theta}{\delta} = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right)d\eta = \int_0^1 \left(2\eta - \eta^3 + \eta^4\right)\left(1 - 2\eta + \eta^3 - \eta^4\right)d\eta = \int_0^1 \left(2\eta - 4\eta^2 - 2\eta^3 + 9\eta^4 - 4\eta^5 - 4\eta^6 + 4\eta^7 - \eta^8\right)d\eta$$

$$\frac{\theta}{\delta} = \left[\eta^2 - \frac{4}{3}\eta^3 - \frac{1}{2}\eta^4 + \frac{9}{5}\eta^5 - \frac{4}{7}\eta^7 + \frac{1}{2}\eta^8 - \frac{1}{9}\eta^9 \right]_0^1 = \frac{37}{315} = 0.117$$

Problem 9.13

9.13 A simplistic laminar boundary-layer model is

$$\frac{u}{U}=\sqrt{2}\frac{y}{\delta} \qquad 0<y\le\frac{\delta}{2}$$

$$\frac{u}{U}=(2-\sqrt{2})\frac{y}{\delta}+(\sqrt{2}-1) \qquad \frac{\delta}{2}<y\le\delta$$

Does this expression satisfy boundary conditions applicable to the laminar boundary-layer velocity profile? Evaluate δ^*/δ and θ/δ.

Given: Laminar boundary layer profile

Find: If it satisfies BC's; Evaluate δ^*/δ and θ/δ

Solution:

The boundary layer equation is
$$\frac{u}{U}=\sqrt{2}\frac{y}{\delta} \qquad 0<y<\frac{\delta}{2}$$
$$\frac{u}{U}=\left(2-\sqrt{2}\right)\frac{y}{\delta}+\left(\sqrt{2}-1\right) \qquad \frac{\delta}{2}<y<\delta$$ for which $u = U$ at $y = \delta$

The BC's are
$$u(0)=0 \qquad \left.\frac{du}{dy}\right|_{y=\delta}=0$$

At $y = 0$
$$\frac{u}{U}=\sqrt{2}(0)=0$$

At $y = \delta$
$$\frac{du}{dy}=U\left[\left(2-\sqrt{2}\right)\frac{1}{\delta}\right]_{y=\delta}\neq 0$$ so it fails the outer BC.

This simplistic distribution is a piecewise linear profile: The first half of the layer has velocity gradient $\sqrt{2}\frac{U}{\delta}=1.414\frac{U}{\delta}$, and the second half has velocity gradient $\left(2-\sqrt{2}\right)\frac{U}{\delta}=0.586\frac{U}{\delta}$. At $y = \delta$, we make another transition to zero velocity gradient.

For δ^*:
$$\delta^*=\int_0^\infty\left(1-\frac{u}{U}\right)dy=\int_0^\delta\left(1-\frac{u}{U}\right)dy$$

Then
$$\frac{\delta^*}{\delta}=\frac{1}{\delta}\int_0^\delta\left(1-\frac{u}{U}\right)dy=\int_0^1\left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right)=\int_0^1\left(1-\frac{u}{U}\right)d\eta$$

with
$$\frac{u}{U}=\sqrt{2}\eta \qquad 0<\eta<\frac{1}{2}$$
$$\frac{u}{U}=\left(2-\sqrt{2}\right)\eta+\left(\sqrt{2}-1\right) \qquad \frac{1}{2}<\eta<1$$

Hence

$$\frac{\delta^*}{\delta}=\int_0^1\left(1-\frac{u}{U}\right)d\eta=\int_0^{1/2}\left(1-\sqrt{2}\eta\right)d\eta+\int_{1/2}^1\left[1-\left(2-\sqrt{2}\right)\eta-\left(\sqrt{2}-1\right)\right]d\eta=\left[\frac{1}{2\sqrt{2}}\left(\sqrt{2}\eta-1\right)^2\right]_0^{1/2}+\left[\frac{1}{2}(\eta-1)^2\left(\sqrt{2}-2\right)\right]_{1/2}^1$$

$$\frac{\delta *}{\delta} = \left[\frac{1}{2} - \frac{\sqrt{2}}{8}\right] + \left[\frac{1}{4} - \frac{\sqrt{2}}{8}\right] = \frac{3}{4} - \frac{\sqrt{2}}{4} = 0.396$$

For θ:

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy$$

Then

$$\frac{\theta}{\delta} = \frac{1}{\delta}\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right)d\eta$$

Hence, after a LOT of work

$$\frac{\theta}{\delta} = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right)d\eta = \int_0^{1/2} \sqrt{2}\eta\left(1 - \sqrt{2}\eta\right)d\eta + \int_{1/2}^1 \left[\left(\left(2 - \sqrt{2}\right)\eta + \left(\sqrt{2} - 1\right)\right)\left(1 - \left(2 - \sqrt{2}\right)\eta - \left(\sqrt{2} - 1\right)\right)\right]d\eta$$

$$\frac{\theta}{\delta} = \left[\sqrt{2}\eta^2\left(\frac{\sqrt{2}\eta}{3} - \frac{1}{2}\right)\right]_0^{1/2} + \left[\left(\frac{1}{3}\left(\sqrt{2} - 2\right)\left(\eta - 1\right) - \frac{1}{2}\right)\left(\sqrt{2} - 2\right)\left(\eta - 1\right)^2\right]_{1/2}^1 = \frac{\sqrt{2}}{8} - \frac{1}{12} + \frac{\sqrt{2}}{24} = \frac{\sqrt{2}}{6} - \frac{1}{12} = 0.152$$

Problem 9.14

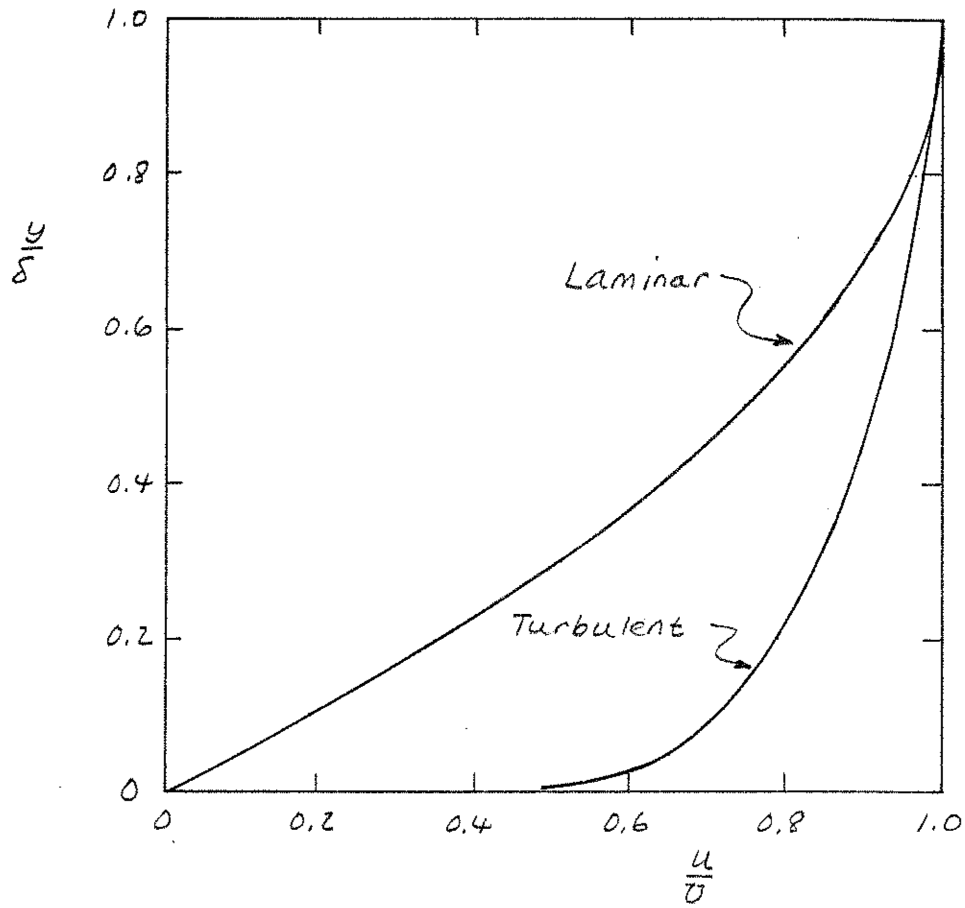
[2]

Given: "Power law" velocity profile for turbulent boundary layer and parabolic profile for laminar boundary layer,

"Power law" $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$ Parabolic $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

Find: Compare shapes by plotting $\frac{y}{\delta}$ vs. $\frac{u}{U}$.

Solution:



Note that the "power law" profile gives an infinite value of du/dy at y=0, since

$$\frac{du}{dy} = \frac{d\left(\frac{u}{U}\right)}{d\left(\frac{y}{\delta}\right)} = \frac{1}{7}\frac{1}{\left(\frac{y}{\delta}\right)^{6/7}} \rightarrow \infty \text{ as } \frac{y}{\delta} \rightarrow 0.$$

Problem 9.15

Given: Linear, parabolic, and sinusoidal profiles used to represent the laminar boundary layer velocity profile.

Evaluate: the ratio θ/δ for each profile.

Solution:

Definition: $\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy$ (9.2)

then, $\theta/\delta = \frac{1}{\delta}\int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\eta$

Linear profile $\frac{u}{U} = y/\delta = \eta$

$$\theta/\delta = \int_0^1 \eta(1-\eta)\,d\eta = \int_0^1 (\eta-\eta^2)\,d\eta = \left[\frac{1}{2}\eta^2 - \frac{1}{3}\eta^3\right]_0^1 = \frac{1}{6} = 0.167$$

Parabolic profile $\frac{u}{U} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 = 2\eta - \eta^2$

$$\theta/\delta = \int_0^1 (2\eta-\eta^2)(1-2\eta+\eta^2)\,d\eta = \int_0^1 (2\eta - 5\eta^2 + 4\eta^3 - \eta^4)\,d\eta$$

$$\theta/\delta = \left[\eta^2 - \frac{5}{3}\eta^3 + \eta^4 - \frac{1}{5}\eta^5\right]_0^1 = \left[1 - \frac{5}{3} + 1 - \frac{1}{5}\right] = \frac{2}{15} = 0.133$$

Sinusoidal profile $\frac{u}{U} = \sin\frac{\pi}{2}\frac{y}{\delta} = \sin\frac{\pi}{2}\eta$

$$\theta/\delta = \int_0^1 \sin\frac{\pi}{2}\eta\left(1-\sin\frac{\pi}{2}\eta\right)d\eta = \int_0^1 \left(\sin\frac{\pi}{2}\eta - \sin^2\frac{\pi}{2}\eta\right)d\eta$$

$$\theta/\delta = \left[-\frac{2}{\pi}\cos\frac{\pi}{2}\eta - \frac{2}{\pi}\left\{\frac{\pi\eta}{4} - \frac{1}{4}\sin\pi\eta\right\}\right]_0^1 = -0-\left(-\frac{2}{\pi}\right) - \frac{2}{\pi}\left(\frac{\pi}{4}\right) - 0$$

$$\theta/\delta = \frac{2}{\pi} - \frac{1}{2} = 0.137$$

Summarizing:

Profile	Expression	θ/δ
Linear	$u/U = \eta$	0.167
Parabolic	$u/U = 2\eta - \eta^2$	0.133
Sinusoidal	$u/U = \sin\frac{\pi}{2}\eta$	0.137

Problem 9.16

Given: Linear, parabolic, and sinusoidal velocity profiles for laminar boundary layer,

(a) linear $u/U = y/\delta$

(b) parabolic $u/U = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$

(c) sinusoidal $u/U = \sin\frac{\pi}{2}\frac{y}{\delta}$

Find: ratio δ^*/δ for each profile.

Solution:

Definition: $\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right)dy \approx \int_0^\delta \left(1 - \frac{u}{U}\right)dy$ (9.1)

Then, $\frac{\delta^*}{\delta} = \frac{1}{\delta}\int_0^\delta \left(1 - \frac{u}{U}\right)dy = \int_0^1 \left(1 - \frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \int_0^1 \left(1 - \frac{u}{U}\right)d\eta$

(a) Linear profile $u/U = y/\delta = \eta$

$\delta^*/\delta = \int_0^1 \left(1 - \frac{u}{U}\right)d\eta = \int_0^1 (1-\eta)\,d\eta = \left[\eta - \frac{1}{2}\eta^2\right]_0^1 = \frac{1}{2}$

(b) Parabolic profile $u/U = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 = 2\eta - \eta^2$

$\delta^*/\delta = \int_0^1 \left(1 - \frac{u}{U}\right)d\eta = \int_0^1 (1 - 2\eta + \eta^2)\,d\eta = \left[\eta - \eta^2 + \frac{1}{3}\eta^3\right]_0^1 = \frac{1}{3}$

(c) Sinusoidal profile $u/U = \sin\frac{\pi}{2}\frac{y}{\delta} = \sin\frac{\pi}{2}\eta$

$\delta^*/\delta = \int_0^1 \left(1 - \frac{u}{U}\right)d\eta = \int_0^1 \left(1 - \sin\frac{\pi}{2}\eta\right)d\eta = \left[\eta + \frac{2}{\pi}\cos\frac{\pi}{2}\eta\right]_0^1 = 1 - \frac{2}{\pi} = 0.363$

Problem 9.17

Given: "Power-law" velocity profile for turbulent boundary layer and parabolic profile for laminar boundary layer,

"Power-law" $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$ Parabolic $\frac{u}{U} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$

Evaluate: (and compare) ratios δ^*/δ and θ/δ for each profile

Solution:

Definitions: $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$ (9.1)

$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$ (9.2)

Then $\delta^*/\delta = \frac{1}{\delta}\int_0^\delta \left(1-\frac{u}{U}\right)dy = \int_0^1 \left(1-\frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \int_0^1 \left(1-\frac{u}{U}\right) d\eta$

For the "power-law" profile

$\delta^*/\delta = \int_0^1 \left(1-\eta^{1/7}\right) d\eta = \left[\eta - \frac{7}{8}\eta^{8/7}\right]_0^1 = \frac{1}{8}$

For the parabolic profile

$\delta^*/\delta = \int_0^1 \left(1-2\eta+\eta^2\right) d\eta = \left[\eta - \eta^2 + \frac{\eta^3}{3}\right]_0^1 = \frac{1}{3}$

thus $\delta^*/\delta \text{ (turbulent)} = \frac{3}{8}\, \delta^*/\delta \text{ (laminar)}$ ______ δ^*/δ

Also $\theta/\delta = \frac{1}{\delta}\int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\eta$.

For the "power-law" profile

$\theta/\delta = \int_0^1 \eta^{1/7}\left(1-\eta^{1/7}\right)d\eta = \int_0^1 \left(\eta^{1/7} - \eta^{2/7}\right)d\eta = \left[\frac{7}{8}\eta^{8/7} - \frac{7}{9}\eta^{9/7}\right]_0^1 = \frac{7}{72}$

For the parabolic profile

$\theta/\delta = \int_0^1 \left(2\eta-\eta^2\right)\left(1-2\eta+\eta^2\right)d\eta = \int_0^1 \left(2\eta - 5\eta^2 + 4\eta^3 - \eta^4\right)d\eta$

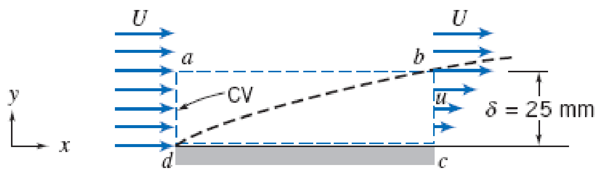
$\theta/\delta = \left[\eta^2 - \frac{5}{3}\eta^3 + \eta^4 - \frac{\eta^5}{5}\right]_0^1 = \left[1 - \frac{5}{3} + 1 - \frac{1}{5}\right] = \frac{2}{15}$

thus $\theta/\delta \text{ (turbulent)} = 0.729\, \theta/\delta \text{ (laminar)}$ ______ θ/δ

Profile	δ^*/δ	θ/δ
Power-law	0.125	0.0972
Parabolic	0.333	0.133

Problem 9.18

9.18 A fluid, with density $\rho = 800\ \text{kg/m}^3$, flows at $U = 3$ m/s over a flat plate 3 m long and 1 m wide. At the trailing edge the boundary-layer thickness is $\delta = 25$ mm. Assume the velocity profile is linear, as shown, and that the flow is two-dimensional (flow conditions are independent of z). Using control volume *abcd*, shown by dashed lines, compute the mass flow rate across surface *ab*. Determine the drag force on the upper surface of the plate. Explain how this (viscous) drag can be computed from the given data even though we do not know the fluid viscosity (see Problem 9.41).



Given: Data on fluid and boundary layer geometry

Find: Mass flow rate across *ab*; Drag

Solution:

The given data is $\rho = 800\cdot\frac{\text{kg}}{\text{m}^3}$ $\quad U = 3\cdot\frac{\text{m}}{\text{s}}$ $\quad L = 3\cdot\text{m}$ $\quad \delta = 25\cdot\text{mm}$ $\quad b = 1\cdot\text{m}$

Governing equations:

Mass $$\frac{\partial}{\partial t}\int_{CV}\rho\, dV + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad (4.12)$$

Momentum $$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV}\vec{V}\rho\, dV + \int_{CS}\vec{V}\rho\vec{V}\cdot d\vec{A} \qquad (4.17a)$$

Assumptions: (1) Steady flow (2) No pressure force (3) No body force in *x* direction (4) Uniform flow at *a*

Applying these to the CV *abcd*

Mass $$(-\rho\cdot U\cdot b\cdot\delta) + \int_0^\delta \rho\cdot u\cdot b\, dy + m_{ab} = 0$$

For the boundary layer $\frac{u}{U} = \frac{y}{\delta} = \eta$ $\qquad \frac{dy}{\delta} = d\eta$

Hence $$m_{ab} = \rho\cdot U\cdot b\cdot\delta - \int_0^1 \rho\cdot U\cdot\eta\cdot\delta\, dy = \rho\cdot U\cdot b\cdot\delta - \frac{1}{2}\cdot\rho\cdot U\cdot b\cdot\delta$$

$$m_{ab} = \frac{1}{2}\cdot\rho\cdot U\cdot b\cdot\delta \qquad m_{ab} = 30\frac{\text{kg}}{\text{s}}$$

Momentum $$R_x = U\cdot(-\rho\cdot U\cdot\delta) + m_{ab}\cdot u_{ab} + \int_0^\delta u\cdot\rho\cdot u\cdot b\, dy$$

Note that $u_{ab} = U$ and $\int_0^\delta u\cdot\rho\cdot u\cdot b\, dy = \int_0^1 \rho\cdot U^2\cdot b\cdot\delta\cdot\eta^2\, d\eta$

$$R_x = -\rho\cdot U^2\cdot b\cdot\delta + \frac{1}{2}\cdot\rho\cdot U\cdot b\cdot\delta\cdot U + \int_0^1 \rho\cdot U^2\cdot b\cdot\delta\cdot\eta^2\, dy$$

$$R_x = -\rho\cdot U^2\cdot b\cdot\delta + \frac{1}{2}\cdot\rho\cdot U^2\cdot\delta + \frac{1}{3}\cdot\rho\cdot U^2\cdot\delta \qquad R_x = -\frac{1}{6}\cdot\rho\cdot U^2\cdot b\cdot\delta \qquad R_x = -30\,\text{N}$$

We are able to compute the boundary layer drag even though we do not know the viscosity because it is the viscosity that creates the boundary layer in the first place

Problem 9.19

9.19 The flat plate of Problem 9.18 is turned so that the 1 m side is parallel to the flow (the width becomes 3 m). Should we expect that the drag increases or decreases? Why? The trailing edge boundary-layer thickness is now $\delta = 14$ mm. Assume again that the velocity profile is linear, and that the flow is two-dimensional (flow conditions are independent of z). Repeat the analysis of Problem 9.18.

Given: Data on fluid and boundary layer geometry

Find: Mass flow rate across *ab*; Drag; Compare to Problem 9.18

Solution:

The given data is $\rho = 800 \cdot \frac{kg}{m^3}$ $\quad U = 3 \cdot \frac{m}{s}$ $\quad L = 1 \cdot m$ $\quad \delta = 14 \cdot mm$ $\quad b = 3 \cdot m$

Governing equations:

Mass
$$\frac{\partial}{\partial t}\int_{CV} \rho \, dV + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0 \qquad (4.12)$$

Momentum
$$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V} \rho \, dV + \int_{CS} \vec{V} \rho \vec{V} \cdot d\vec{A} \qquad (4.17a)$$

Assumptions: (1) Steady flow (2) No pressure force (3) No body force in x direction (4) Uniform flow at *a*

Applying these to the CV *abcd*

Mass
$$(-\rho \cdot U \cdot b \cdot \delta) + \int_0^{\delta} \rho \cdot u \cdot b \, dy + m_{ab} = 0$$

For the boundary layer
$$\frac{u}{U} = \frac{y}{\delta} = \eta \qquad \frac{dy}{\delta} = d\eta$$

Hence
$$m_{ab} = \rho \cdot U \cdot b \cdot \delta - \int_0^1 \rho \cdot U \cdot \eta \cdot \delta \, dy = \rho \cdot U \cdot b \cdot \delta - \frac{1}{2} \cdot \rho \cdot U \cdot b \cdot \delta$$

$$m_{ab} = \frac{1}{2} \cdot \rho \cdot U \cdot b \cdot \delta \qquad m_{ab} = 50.4 \frac{kg}{s}$$

Momentum
$$R_x = U \cdot (-\rho \cdot U \cdot \delta) + m_{ab} \cdot u_{ab} + \int_0^{\delta} u \cdot \rho \cdot u \cdot b \, dy$$

Note that $u_{ab} = U$ and $\int_0^{\delta} u \cdot \rho \cdot u \cdot b \, dy = \int_0^1 \rho \cdot U^2 \cdot b \cdot \delta \cdot \eta^2 \, d\eta$

$$R_x = -\rho \cdot U^2 \cdot b \cdot \delta + \frac{1}{2} \cdot \rho \cdot U \cdot b \cdot \delta \cdot U + \int_0^1 \rho \cdot U^2 \cdot b \cdot \delta \cdot \eta^2 \, dy$$

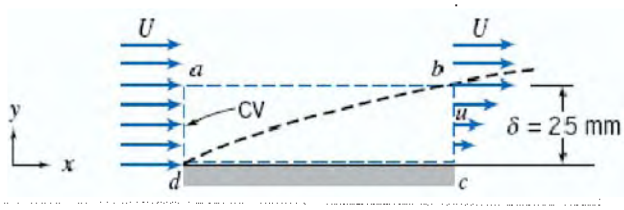
$$R_x = -\rho \cdot U^2 \cdot b \cdot \delta + \frac{1}{2} \cdot \rho \cdot U^2 \cdot \delta + \frac{1}{3} \cdot \rho \cdot U^2 \cdot \delta$$

$$R_x = -\frac{1}{6} \cdot \rho \cdot U^2 \cdot b \cdot \delta \qquad R_x = -50.4 \, N$$

We should expect the drag to be larger than for Problem 9.18 because the viscous friction is mostly concentrated near the leading edge (which is only 1 m wide in Problem 9.18 but 3 m here). The reason viscous stress is highest at the front region is that the boundary layer is very small (δ <<) so $\tau = \mu du/dy \sim \mu U/\delta$ >>

Problem 9.20

Given: Fluid flow over a thin flat plate of width, $b = 1.0$ m. Flow is two-dimensional. Assume that in the boundary layer the velocity profile is parabolic. (The plate is 3 m long.)



At bc, $\frac{u}{U} = 2\eta - \eta^2$; $\eta = \frac{y}{\delta}$

$U = 3.0$ m/s

$\rho = 800$ kg/m³

Find: (a) Mass flowrate across ab.
(b) x component (and direction) of force needed to hold plate.

Solution: Apply the continuity and x component momentum equations.

Basic equations: $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$ ($\frac{\partial}{\partial t}$ term $= 0(1)$)

$$F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho \vec{V} \cdot d\vec{A}$$

($F_{Bx} = 0(3)$, $\frac{\partial}{\partial t}$ term $= 0(1)$)

Assumptions: (1) Steady flow
(2) No pressure forces
(3) $F_{Bx} = 0$
(4) Uniform flow at da

Then
$$0 = \{-|\rho U b \delta|\} + \int_0^\delta \rho u b\, dy + \dot{m}_{ab}$$

But $\int_0^\delta \rho u b\, dy = \rho U b \delta \int_0^1 (2\eta - \eta^2)\, d\eta = \rho U b \delta \left[\eta^2 - \frac{1}{3}\eta^3\right]_0^1 = \frac{2}{3}\rho U b \delta$

Thus $\dot{m}_{ab} = \rho U b \delta - \frac{2}{3}\rho U b \delta = \frac{1}{3}\rho U b \delta$

$$\dot{m}_{ab} = \frac{1}{3} \times 800\,\frac{\text{kg}}{\text{m}^3} \times 3\,\frac{\text{m}}{\text{s}} \times 1\,\text{m} \times 0.025\,\text{m} = 20\ \text{kg/s}$$ $\dot{m}_{ab}$

From momentum,

$$R_x = u_{da}\{-|\rho U b \delta|\} + u_{ab}\,\dot{m}_{ab} + \int_0^\delta u \rho u b\, dy \; ; \; u_{da} = u_{ab} = U$$

But $\int_0^\delta u \rho u b\, dy = \rho U^2 b \delta \int_0^1 (2\eta - \eta^2)^2\, d\eta = \rho U^2 b \delta \left[\frac{4}{3}\eta^3 - \eta^4 + \frac{1}{5}\eta^5\right]_0^1 = \frac{8}{15}\rho U^2 b \delta$

Thus $R_x = -\rho U^2 b \delta + \frac{1}{3}\rho U^2 b \delta + \frac{8}{15}\rho U^2 b \delta = -\frac{2}{15}\rho U^2 b \delta$

$$R_x = -\frac{2}{15} \times 800\,\frac{\text{kg}}{\text{m}^3} \times (3)^2\,\frac{\text{m}^2}{\text{s}^2} \times 1\,\text{m} \times 0.025\,\text{m} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} = -24\ \text{N}$$

This force must be applied to the control volume by the plate. Thus to hold the plate,

$F_x = R_x = -24$ (to the left) F_x

Problem 9.21

9.21 The test section of a low speed wind tunnel is 1.5 meters long, preceded by a nozzle and with a diffuser at the outlet. The tunnel cross-section is 20 cm × 20 cm. The wind tunnel is to operate with 40°C air and have a design velocity of 50 m/s in the test section. A potential problem with such a wind tunnel is boundary-layer blockage. The boundary-layer displacement thickness reduces the effective cross-sectional area (the test area, in which we have uniform flow), and in addition the uniform flow will be accelerated. If these effects are pronounced, we end up with a smaller useful test cross section with a velocity somewhat higher than anticipated. If the boundary-layer thickness is 10 mm at the entrance and 25 mm at the exit, and the boundary-layer velocity profile is given by $u/U = (y/\delta)^{1/7}$, estimate the displacement thickness at the end of the test section and the percent change in the uniform velocity between the inlet and outlet.

Given:
Data on wind tunnel and boundary layers

Find:
Displacement thickness at exit; Percent change in uniform velocity through test section

Solution:

The solution involves using mass conservation in the inviscid core, allowing for the fact that as the boundary layer grows it reduces the size of the core. One approach would be to integrate the 1/7 law velocity profile to compute the mass flow in the boundary layer; an easier approach is to simply use the displacement thickness!

Basic equations $$\frac{\partial}{\partial t}\int_{CV}\rho\, d\mathcal{V} + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \quad (4.12) \qquad \delta_{disp} = \int_0^{\delta}\left(1-\frac{u}{U}\right)dy$$

Assumptions: 1) Steady flow 2) Incompressible 3) No friction outside boundary layer 4) Flow along streamline 5) Horizontal

For this flow $\rho\cdot U\cdot A = \text{const}$ and $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$

The design data is $U_{design} = 50\cdot\frac{m}{s}$ $\quad w = 20\cdot cm$ $\quad h = 20\cdot cm$ $\quad A_{design} = w\cdot h$ $\quad A_{design} = 0.04\,m^2$

The volume flow rate is $Q = U_{design}\cdot A_{design}$ $\quad Q = 2\,\frac{m^3}{s}$

We also have $\delta_{in} = 10\cdot mm$ $\quad \delta_{exit} = 25\cdot mm$

Hence $$\delta_{disp} = \int_0^{\delta}\left(1-\frac{u}{U}\right)dy = \int_0^{\delta}\left[1-\left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right]dy = \delta\cdot\int_0^1\left(1-\eta^{\frac{1}{7}}\right)d\eta \quad \text{where} \quad \eta = \frac{y}{\delta} \qquad \delta_{disp} = \frac{\delta}{8}$$

Hence at the inlet and exit

$\delta_{dispin} = \frac{\delta_{in}}{8}$ $\quad \delta_{dispin} = 1.25\cdot mm$ $\quad \delta_{dispexit} = \frac{\delta_{exit}}{8}$ $\quad \delta_{dispexit} = 3.125\cdot mm$

Hence the areas are $A_{in} = (w - 2\cdot\delta_{dispin})\cdot(h - 2\cdot\delta_{dispin})$ $\quad A_{in} = 0.0390\cdot m^2$

$A_{exit} = (w - 2\cdot\delta_{dispexit})\cdot(h - 2\cdot\delta_{dispexit})$ $\quad A_{exit} = 0.0375\cdot m^2$

Applying mass conservation between "design" conditions and the inlet

$$(-\rho\cdot U_{design}\cdot A_{design}) + (\rho\cdot U_{in}\cdot A_{in}) = 0$$

or $U_{in} = U_{design}\cdot\frac{A_{design}}{A_{in}}$ $\quad U_{in} = 51.3\frac{m}{s}$

Also $U_{exit} = U_{design}\cdot\frac{A_{design}}{A_{exit}}$ $\quad U_{exit} = 53.3\frac{m}{s}$

The percent change in uniform velocity is then $\frac{U_{exit} - U_{in}}{U_{in}} = 3.91\cdot\%$ The exit displacement thickness is $\delta_{dispexit} = 3.125\cdot mm$

Problem 9.22

[2]

9.22 Laboratory wind tunnels have test sections 25 cm square and 50 cm long. With nominal air speed $U_1 = 25$ m/s at the test section inlet, turbulent boundary layers form on the top, bottom, and side walls of the tunnel. The boundary-layer thickness is $\delta_1 = 20$ mm at the inlet and $\delta_2 = 30$ mm at the outlet from the test section. The boundary-layer velocity profiles are of power-law form, with $u/U = (y/\delta)^{1/7}$. Evaluate the freestream velocity, U_2, at the exit from the wind-tunnel test section. Determine the change in static pressure along the test section.

Given: Data on wind tunnel and boundary layers

Find: Uniform velocity at exit; Change in static pressure through the test section

Solution:

Basic equations $$\frac{\partial}{\partial t}\int_{CV}\rho\, dV + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \quad (4.12) \qquad \delta_{disp} = \int_0^{\delta}\left(1-\frac{u}{U}\right)dy \qquad \frac{p}{\rho}+\frac{V^2}{2}+g\cdot z = const$$

Assumptions: 1) Steady flow 2) Incompressible 3) No friction outside boundary layer 4) Flow along streamline 5) Horizontal

For this flow $\rho\cdot U\cdot A = const$ and $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$

The given data is $U_1 = 25\cdot\frac{m}{s}$ $h = 25\cdot cm$ $A = h^2$ $A = 625\cdot cm^2$

We also have $\delta_1 = 20\cdot mm$ $\delta_2 = 30\cdot mm$

Hence $$\delta_{disp} = \int_0^{\delta}\left(1-\frac{u}{U}\right)dy = \int_0^{\delta}\left[1-\left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right]dy = \delta\cdot\int_0^{1}\left(1-\eta^{\frac{1}{7}}\right)d\eta$$ where $\eta = \frac{y}{\delta}$ $\quad \delta_{disp} = \frac{\delta}{8}$

Hence at the inlet and exit

$\delta_{disp1} = \frac{\delta_1}{8}$ $\delta_{disp1} = 2.5\cdot mm$ $\delta_{disp2} = \frac{\delta_2}{8}$ $\delta_{disp2} = 3.75\cdot mm$

Hence the areas are $A_1 = \left(h - 2\cdot\delta_{disp1}\right)^2$ $A_1 = 600\cdot cm^2$

$A_2 = \left(h - 2\cdot\delta_{disp2}\right)^2$ $A_2 = 588\cdot cm^2$

Applying mass conservation between Points 1 and 2

$$\left(-\rho\cdot U_1\cdot A_1\right) + \left(\rho\cdot U_2\cdot A_2\right) = 0 \quad \text{or} \quad U_2 = U_1\cdot\frac{A_1}{A_2} \qquad U_2 = 25.52\frac{m}{s}$$

The pressure change is found from Bernoulli $\frac{p_1}{\rho}+\frac{U_1^2}{2} = \frac{p_2}{\rho}+\frac{U_2^2}{2}$ with $\rho = 1.21\cdot\frac{kg}{m^3}$

Hence $\Delta p = \frac{\rho}{2}\cdot\left(U_1^2 - U_2^2\right)$ $\Delta p = -15.8\,Pa$ The pressure drops slightly through the test section

Problem 9.23

[2]

9.23 Air flows in a horizontal cylindrical duct of diameter D = 100 mm. At a section a few meters from the entrance, the turbulent boundary layer is of thickness δ_1 = 5.25 mm, and the velocity in the inviscid central core is U_1 = 12.5 m/s. Farther downstream the boundary layer is of thickness δ_2 = 24 mm. The velocity profile in the boundary layer is approximated well by the $\frac{1}{7}$-power expression. Find the velocity, U_2, in the inviscid central core at the second section, and the pressure drop between the two sections.

Given: Data on boundary layer in a cylindrical duct

Find: Velocity U_2 in the inviscid core at location 2; Pressure drop

Solution:

The solution involves using mass conservation in the inviscid core, allowing for the fact that as the boundary layer grows it reduces the size the core. One approach would be to integrate the 1/7 law velocity profile to compute the mass flow in the boundary layer; an easier approa is to simply use the displacement thickness!

The given or available data (from Appendix A) is

$\rho = 1.23 \cdot \frac{kg}{m^3}$ $U_1 = 12.5 \cdot \frac{m}{s}$ $D = 100 \cdot mm$ $\delta_1 = 5.25 \cdot mm$ $\delta_2 = 24 \cdot mm$

Governing equations:

Mass $\frac{\partial}{\partial t}\int_{CV} \rho \, d\mathcal{V} + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$ (4.12)

Bernoulli $\frac{p}{\rho} + \frac{V^2}{2} + g \cdot z = constant$ (4.24)

The displacement thicknesses can be computed from boundary layer thicknesses using Eq. 9.1

$$\delta_{disp} = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy = \delta \cdot \int_0^1 \left(1 - \eta^{\frac{1}{7}}\right) d\eta = \frac{\delta}{8}$$

Hence at locations 1 and 2 $\delta_{disp1} = \frac{\delta_1}{8}$ $\delta_{disp1} = 0.656 \cdot mm$ $\delta_{disp2} = \frac{\delta_2}{8}$ $\delta_{disp2} = 3 \cdot mm$

Applying mass conservation at locations 1 and 2 $(-\rho \cdot U_1 \cdot A_1) + (\rho \cdot U_2 \cdot A_2) = 0$ or $U_2 = U_1 \cdot \frac{A_1}{A_2}$

The two areas are given by the duct cross section area minus the displacement boundary layer

$A_1 = \frac{\pi}{4} \cdot (D - 2 \cdot \delta_{disp1})^2$ $A_1 = 7.65 \times 10^{-3} m^2$ $A_2 = \frac{\pi}{4} \cdot (D - 2 \cdot \delta_{disp2})^2$ $A_2 = 6.94 \times 10^{-3} m^2$

Hence $U_2 = U_1 \cdot \frac{A_1}{A_2}$ $U_2 = 13.8 \frac{m}{s}$

For the pressure drop we can apply Bernoulli to locations 1 and 2 to find $p_1 - p_2 = \Delta p = \frac{\rho}{2} \cdot \left(U_2^2 - U_1^2\right)$ $\Delta p = 20.6 \, Pa$

Problem 9.24

[2]

9.24 The square test section of a small laboratory wind tunnel has sides of width $W = 12$ in. At one measurement location, the turbulent boundary layers on the tunnel walls are $\delta_1 = 0.4$ in. thick. The velocity profile is approximated well by the $\frac{1}{7}$-power expression. At this location the freestream air speed is $U_1 = 60$ ft/s, and the static pressure is $p_1 = -1$ in. H_2O (gage). At a second measurement location downstream, the boundary-layer thickness is $\delta_2 = 0.5$ in. Evaluate the air speed in the freestream at the second section. Calculate the difference in static pressure from section (1) to section (2).

Given: Data on wind tunnel and boundary layers

Find: Uniform velocity at Point 2; Change in static pressure through the test section

Solution:

Basic equations $\frac{\partial}{\partial t}\int_{CV}\rho\, dV + \int_{CS}\rho \vec{V}\cdot d\vec{A} = 0$ (4.12) $\delta_{disp} = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$ $\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{const}$

Assumptions: 1) Steady flow 2) Incompressible 3) No friction outside boundary layer 4) Flow along streamline 5) Horizontal

For this flow $\rho\cdot U\cdot A = \text{const}$ and $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$

The given data is $U_1 = 60\cdot\frac{ft}{s}$ $W = 12\cdot in$ $A = W^2$ $A = 144\cdot in^2$

We also have $\delta_1 = 0.4\cdot in$ $\delta_2 = 0.5\cdot in$

Hence $\delta_{disp} = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left[1 - \left(\frac{y}{\delta}\right)^{\frac{1}{7}}\right] dy = \delta\cdot\int_0^1 \left(1 - \eta^{\frac{1}{7}}\right) d\eta$ where $\eta = \frac{y}{\delta}$ $\delta_{disp} = \frac{\delta}{8}$

Hence at the inlet and exit

$\delta_{disp1} = \frac{\delta_1}{8}$ $\delta_{disp1} = 0.050\cdot in$ $\delta_{disp2} = \frac{\delta_2}{8}$ $\delta_{disp2} = 0.0625\cdot in$

Hence the areas are $A_1 = \left(W - 2\cdot\delta_{disp1}\right)^2$ $A_1 = 142\cdot in^2$

$A_2 = \left(W - 2\cdot\delta_{disp2}\right)^2$ $A_2 = 141\cdot in^2$

Applying mass conservation between Points 1 and 2

$\left(-\rho\cdot U_1\cdot A_1\right) + \left(\rho\cdot U_2\cdot A_2\right) = 0$ or $U_2 = U_1\cdot\frac{A_1}{A_2}$ $U_2 = 60.25\cdot\frac{ft}{s}$

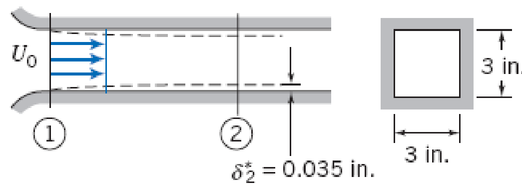
The pressure change is found from Bernoulli $\frac{p_1}{\rho} + \frac{U_1^2}{2} = \frac{p_2}{\rho} + \frac{U_2^2}{2}$ with $\rho = 0.00234\cdot\frac{slug}{ft^3}$

Hence $\Delta p = \frac{\rho}{2}\cdot\left(U_1^2 - U_2^2\right)$ $\Delta p = -2.47\times 10^{-4}\cdot psi$ $\Delta p = -0.0356\cdot\frac{lbf}{ft^2}$

In terms of inches of water $\rho_{H2O} = 1.94\cdot\frac{slug}{ft^3}$ $\Delta h = \frac{\Delta p}{\rho_{H2O}\cdot g}$ $\Delta h = -0.00684\cdot in$

Problem 9.25

9.25 Air flows in the entrance region of a square duct, as shown. The velocity is uniform, $U_0 = 100$ ft/s, and the duct is 3 in. square. At a section 1 ft downstream from the entrance, the displacement thickness, δ^*, on each wall measures 0.035 in. Determine the pressure change between sections ① and ②.



Given:
Data on wind tunnel and boundary layers

Find:
Pressure change between points 1 and 2

Solution:

Basic equations

$$\frac{\partial}{\partial t}\int_{CV}\rho\, dV + \int_{CS}\rho \vec{V}\cdot d\vec{A} = 0 \qquad (4.12) \qquad \frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{const}$$

Assumptions: 1) Steady flow 2) Incompressible 3) No friction outside boundary layer 4) Flow along streamline 5) Horizontal

For this flow $\rho\cdot U\cdot A = \text{const}$

The given data is $U_0 = 100\cdot\frac{ft}{s}$ $\quad U_1 = U_0 \quad h = 3\cdot in \quad A_1 = h^2 \quad A_1 = 9\cdot in^2$

We also have $\delta_{disp2} = 0.035\cdot in$

Hence at the Point 2 $A_2 = \left(h - 2\cdot\delta_{disp2}\right)^2 \qquad A_2 = 8.58\cdot in^2$

Applying mass conservation between Points 1 and 2

$$\left(-\rho\cdot U_1\cdot A_1\right) + \left(\rho\cdot U_2\cdot A_2\right) = 0 \quad \text{or} \quad U_2 = U_1\cdot\frac{A_1}{A_2} \qquad U_2 = 105\cdot\frac{ft}{s}$$

The pressure change is found from Bernoulli $\frac{p_1}{\rho} + \frac{U_1^2}{2} = \frac{p_2}{\rho} + \frac{U_2^2}{2}$ with $\rho = 0.00234\cdot\frac{slug}{ft^3}$

Hence $\Delta p = \frac{\rho}{2}\cdot\left(U_1^2 - U_2^2\right) \qquad \Delta p = -8.05\times 10^{-3}\cdot psi \qquad \Delta p = -1.16\cdot\frac{lbf}{ft^2}$

The pressure drops by a small amount as the air accelerates

Problem 9.26

Given: Developing flow of air in flat horizontal duct.

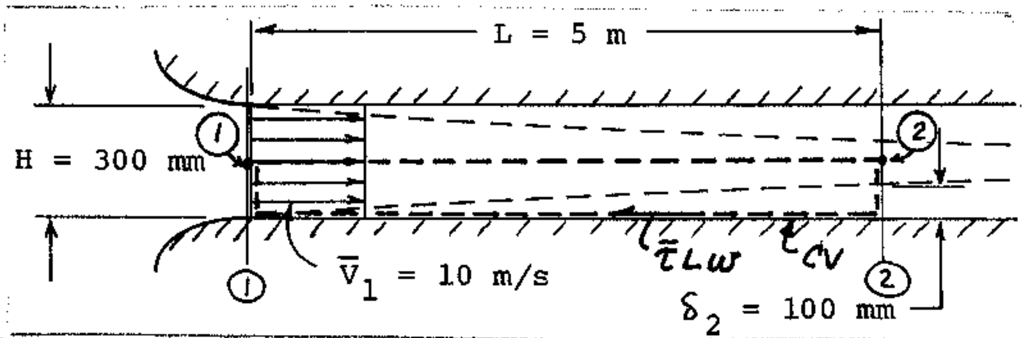
Assume $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$ in boundary layers.

Find: (a) Show $\delta^*/\delta = 1/8$

(b) Evaluate p_2

(c) Calculate average wall shear stress.

Solution:



Assume flow is steady and incompressible, and that frictional effects are negligible outside the boundary layers.

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy = \delta\int_0^1 \left(1-\frac{u}{U}\right)d\eta = \delta\int_0^1 \left(1-\eta^{1/7}\right)d\eta = \delta\left(\eta - \frac{7}{8}\eta^{8/7}\right)\Big|_0^1 = \delta/8$$ δ^*

From continuity $V_1 A_1 = V_1 w H = V_2 A_2 = V_2 w (H - 2\delta_2^*)$

$$V_2 = V_1 \frac{H}{H-2\delta_2^*} = 10\,\frac{m}{s}\,\frac{300\ mm}{(300-25)\ mm} = 10.9\ m/s$$

From Bernoulli, since z = constant, $\frac{p_0}{\rho} + \frac{V_0^2}{2} = \frac{p}{\rho} + \frac{V^2}{2}$ along a streamline. (with $\frac{V_0^2}{2}$ crossed out)

$$p_{1g} = p_1 - p_0 = -\frac{1}{2}\rho V_1^2 = -\frac{1}{2}\times 1.23\,\frac{kg}{m^3}\times(10)^2\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} = -61.5\ Pa$$

$$p_{2g} = p_2 - p_0 = -\frac{1}{2}\rho V_2^2 = -\frac{1}{2}\times 1.23\,\frac{kg}{m^3}\times(10.9)^2\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} = -73.1\ Pa$$ p_{2g}

Apply momentum using CV shown:

$$\text{BE}\quad F_{S_x} + \cancel{F_{B_x}}^{=0} = \cancel{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall}^{=0} + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$$

$$(p_1 - p_2)w\frac{H}{2} - \bar{\tau} w L = \bar{V}_1\left\{-\rho\bar{V}_1\frac{H}{2}w\right\} + \underbrace{\int_0^{\delta_2} u\rho u w\, dy}_{\rho V_2^2\delta_2 w\int_0^1 \eta^{2/7}d\eta = \rho V_2^2\left(\frac{7}{9}\delta_2\right)w} + V_2\left\{+\rho V_2\left(\frac{H}{2}-\delta_2\right)w\right\}$$

$$\bar{\tau} w L = (p_1 - p_2)w\frac{H}{2} + \rho V_1^2\frac{H}{2}w - \rho V_2^2\left(\frac{H}{2} - \frac{2}{9}\delta_2\right)w$$

$$\bar{\tau} = \frac{1}{L}\left[(p_1-p_2)\frac{H}{2} + \rho V_1^2\frac{H}{2} - \rho V_2^2\left(\frac{H}{2}-\frac{2}{9}\delta_2\right)\right]$$

$$= \frac{1}{5\ m}\left[11.6\,\frac{N}{m^2}\times 0.15\ m + 1.23\,\frac{kg}{m^3}\times(10)^2\frac{m^2}{s^2}\times 0.15\ m\times\frac{N\cdot s^2}{kg\cdot m} - 1.23\,\frac{kg}{m^3}\times(10.9)^2\frac{m^2}{s^2}(0.15-0.0222)\ m\times\frac{N\cdot s^2}{kg\cdot m}\right]$$

$$\bar{\tau} = 0.300\ N/m^2$$ $\bar{\tau}$

Problem 9.27

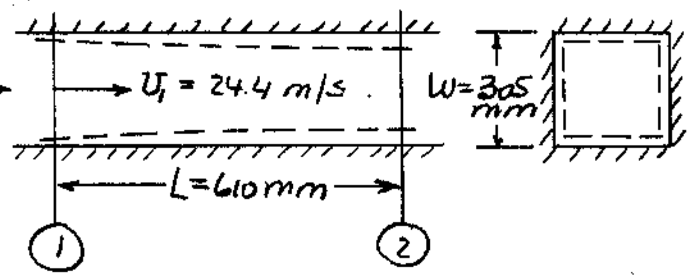
Given: Air flow in laboratory wind tunnel test section.

$h_{l_{0-1}} = -6.5$ mm H_2O

$\delta_1 = 20.3$ mm

$\delta_2 = 25.4$ mm

$\frac{\bar{u}}{U} = \left(\frac{y}{\delta}\right)^{1/7}$ in BLs



Find: (a) Freestream speed at exit, U_2.
(b) Pressure at exit, p_2.

Solution: Apply displacement thickness, continuity, Bernoulli eqs.

Computing equations: $\frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{u}{U}\right) dy$; $0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$ (first term $= 0(1)$)

$$\frac{p_1}{\rho} + \frac{U_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{U_2^2}{2} + g z_2 \quad (8)$$

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) No friction (outside BL)
(4) Along a streamline
(5) Uniform flow (outside BL)
(6) Same BL on four walls
(7) Neglect corner effects
(8) Neglect Δz

Then

$$\frac{\delta^*}{\delta} = \int_0^1 (1 - \lambda^{1/7}) d\lambda = \left(\lambda - \frac{7}{8}\lambda^{8/7}\right)\Big|_0^1 = 1 - \frac{7}{8} = \frac{1}{8} \quad (\lambda = y/\delta)$$

$$\delta_1^* = \frac{1}{8}\delta_1 = \frac{1}{8} \times 20.3 \text{ mm} = 2.54 \text{ mm} ; \delta_2^* = \frac{1}{8}\delta_2 = \frac{1}{8} \times 25.4 \text{ mm} = 3.18 \text{ mm}$$

From continuity, $U_1 A_1 = U_1 (W - 2\delta_1^*)^2 = U_2 A_2 = U_2 (W - 2\delta_2^*)^2$

$$U_2 = U_1 \left(\frac{W - 2\delta_1^*}{W - 2\delta_2^*}\right)^2 = 24.4 \frac{m}{s} \left[\frac{300 - 2(2.54)}{300 - 2(3.18)}\right]^2 = 24.6 \text{ m/s}$$

From Bernoulli, $\frac{p_1}{\rho} + \frac{U_1^2}{2} = \frac{p_2}{\rho} + \frac{U_2^2}{2}$

$$p_2 - p_1 = \frac{\rho}{2}(U_1^2 - U_2^2) = \frac{1}{2} \times 1.23 \frac{kg}{m^3} \left[(24.4)^2 - (24.6)^2\right] \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = -6.03 \text{ N/m}^2$$

Since $\Delta p = \rho g \Delta h$, then $\Delta h = \Delta p / \rho g$

$$p_2 - p_1 = -6.03 \frac{N}{m^2} \times \frac{m^3}{999 \text{ kg}} \times \frac{s^2}{9.81 \text{ m}} \times \frac{kg \cdot m}{N \cdot s^2} = -0.615 \text{ mm } H_2O$$

From ambient to ①: $\left(\frac{p_0}{\rho} + \frac{U_0^2}{2}\right) - \left(\frac{p_1}{\rho} + \frac{U_1^2}{2}\right) = h_{l_T}$ ($p_0 = p_{atm} \approx 0$, $U_0 \approx 0$)

$$p_1 = -\rho h_{l_T} - \frac{1}{2}\rho U_1^2$$

$$p_1 = -6.5 \text{ mm } H_2O - \frac{1}{2} \times 1.23 \frac{kg}{m^3} \times (24.4)^2 \frac{m^2}{s^2} \times \frac{m^3}{999 \text{ kg}} \times \frac{s^2}{9.81 \text{ m}} = -43.9 \text{ mm } H_2O$$

$$p_2 = p_1 + (p_2 - p_1) = -43.9 - 0.615 \text{ mm } H_2O = -44.5 \text{ mm } H_2O$$

p_2

Problem 9.28

9.28 Flow of air develops in a horizontal cylindrical duct, of diameter $D = 400$ mm, following a well-rounded entrance. A turbulent boundary grows on the duct wall, but the flow is not yet fully developed. Assume that the velocity profile in the boundary layer is $u/U = (y/\delta)^{1/7}$. The inlet flow is at $\overline{V} = 15$ m/s at section ①. At section ②, the boundary-layer thickness is $\delta_2 = 100$ mm. Evaluate the static gage pressure at section ②, located at $L = 6$ m. Find the average wall shear stress.

Given: Data on fluid and boundary layer geometry

Find: Gage pressure at location 2; average wall stress

Solution:

The solution involves using mass conservation in the inviscid core, allowing for the fact that as the boundary layer grows it reduces the size core. One approach would be to integrate the 1/7 law velocity profile to compute the mass flow in the boundary layer; an easier approach i simply use the displacement thickness!

The average wall stress can be estimated using the momentum equation for a CV

The given and available (from Appendix A) data is

$$\rho = 1.23\cdot\frac{kg}{m^3} \qquad U_1 = 15\cdot\frac{m}{s} \qquad L = 6\cdot m \qquad D = 400\cdot mm \qquad \delta_2 = 100\cdot mm$$

Governing equations:

Mass
$$\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0 \qquad (4.12)$$

Momentum
$$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV}\vec{V}\,\rho\, d\forall + \int_{CS}\vec{V}\,\rho\vec{V}\cdot d\vec{A} \qquad (4.17a)$$

Bernoulli
$$\frac{p}{\rho} + \frac{V^2}{2} + g\cdot z = \text{constant} \qquad (4.24)$$

Assumptions: (1) Steady flow (2) No pressure force (3) No body force in *x* direction

The displacement thickness at location 2 can be computed from boundary layer thickness using Eq. 9.1

$$\delta_{disp2} = \int_0^{\delta_2}\left(1 - \frac{u}{U}\right)dy = \delta_2\cdot\int_0^1\left(1 - \eta^{\frac{1}{7}}\right)d\eta = \frac{\delta_2}{8}$$

Hence
$$\delta_{disp2} = \frac{\delta_2}{8} \qquad \delta_{disp2} = 12.5\,mm$$

Applying mass conservation at locations 1 and 2
$$(-\rho\cdot U_1\cdot A_1) + (\rho\cdot U_2\cdot A_2) = 0 \qquad \text{or} \qquad U_2 = U_1\cdot\frac{A_1}{A_2}$$

$$A_1 = \frac{\pi}{4}\cdot D^2 \qquad A_1 = 0.126\,m^2$$

The area at location 2 is given by the duct cross section area minus the displacement boundary layer

$$A_2 = \frac{\pi}{4}\cdot(D - 2\cdot\delta_{disp2})^2 \qquad A_2 = 0.11\,m^2$$

Hence $U_2 = U_1 \cdot \frac{A_1}{A_2}$ $U_2 = 17.1 \frac{m}{s}$

For the pressure change we can apply Bernoulli to locations 1 and 2 to find

$$p_1 - p_2 = \Delta p = \frac{\rho}{2} \cdot \left(U_2^{\,2} - U_1^{\,2}\right) \qquad \Delta p = 40.8\,\text{Pa} \qquad p_2 = -\Delta p$$

Hence $p_2(\text{gage}) = p_1(\text{gage}) - \Delta p$ $p_2 = -40.8\,\text{Pa}$

For the average wall shear stress we use the momentum equation, simplified for this problem

$$\Delta p \cdot A_1 - \tau \cdot \pi \cdot D \cdot L = -\rho \cdot U_1^{\,2} \cdot A_1 + \rho \cdot U_2^{\,2} \cdot \frac{\pi}{4} \cdot \left(D - 2 \cdot \delta_2\right)^2 + \rho \cdot \int_{\frac{D}{2}-\delta_2}^{\frac{D}{2}} 2 \cdot \pi \cdot r \cdot u^2 \, dr$$

where $u(r) = U_2 \cdot \left(\frac{y}{\delta_2}\right)^{\frac{1}{7}}$ and $r = \frac{D}{2} - y$ $dr = -dy$

The integral is

$$\rho \cdot \int_{\frac{D}{2}-\delta_2}^{\frac{D}{2}} 2 \cdot \pi \cdot r \cdot u^2 \, dr = -2 \cdot \pi \cdot \rho \cdot U_2^{\,2} \cdot \int_{\delta_2}^{0} \left(\frac{D}{2} - y\right) \cdot \left(\frac{y}{\delta_2}\right)^{\frac{2}{7}} dy$$

$$\rho \cdot \int_{\frac{D}{2}-\delta_2}^{\frac{D}{2}} 2 \cdot \pi \cdot r \cdot u^2 \, dr = 7 \cdot \pi \cdot \rho \cdot U_2^{\,2} \cdot \delta_2 \cdot \left(\frac{D}{9} - \frac{\delta_2}{8}\right)$$

Hence

$$\tau = \frac{\Delta p \cdot A_1 + \rho \cdot U_1^{\,2} \cdot A_1 - \rho \cdot U_2^{\,2} \cdot \frac{\pi}{4} \cdot \left(D - 2 \cdot \delta_2\right)^2 - 7 \cdot \pi \cdot \rho \cdot U_2^{\,2} \cdot \delta_2 \cdot \left(\frac{D}{9} - \frac{\delta_2}{8}\right)}{\pi \cdot D \cdot L}$$

$$\tau = 0.461\,\text{Pa}$$

Problem 9.29

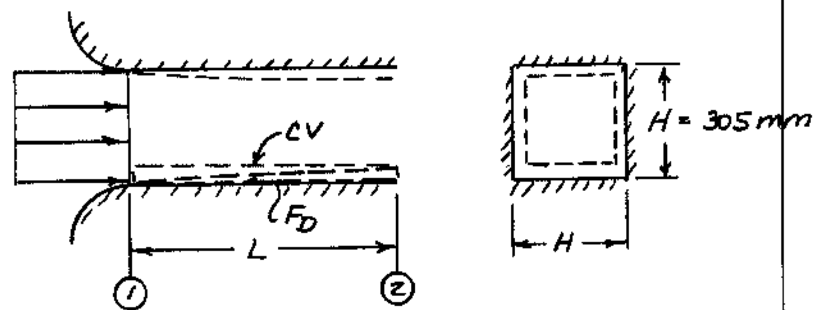
Given: Air flow into wind tunnel contraction and test section as shown.

$U_1 = 50.2\ m/s$

$\delta_2 = 20.3\ mm$

$H = 305\ mm$

$L = 609\ mm$



Find: (a) δ_2^*

(b) $p_2 - p_1$

(c) Estimate total drag force caused by friction on each wall.

Solution: Assume turbulent BL with 1/7-power profile, $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7} = \lambda^{1/7}$

By definition $\frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \int_0^1 (1 - \lambda^{1/7})\, d\lambda = \lambda - \frac{7}{8}\lambda^{8/7} = 1/8$

Thus $\delta_2^* = \frac{1}{8}\delta_2 = \frac{1}{8} \times 20.3\ mm = 2.54\ mm$ ← δ_2^*

From continuity, $U_1 A_1 = U_1 H^2 = U_2 A_2 = U_2 (H - 2\delta_2^*)^2$

$$U_2 = U_1 \frac{H^2}{(H - 2\delta_2^*)^2} = 50.2\ \frac{m}{s} \times \frac{(305)^2}{(305 - 2 \times 2.54)^2} = 51.9\ m/s$$

Apply the Bernoulli equation to the steady, incompressible, frictionless flow along a streamline outside the boundary layers:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 \;;\; z_2 = z_1, \text{ so } p_2 - p_1 = \frac{\rho}{2}(U_1^2 - U_2^2)$$

So

$$p_2 - p_1 = \frac{1}{2} \times 1.23\ \frac{kg}{m^3} \left[(50.2)^2 - (51.9)^2\right] \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = -107\ N/m^2$$ ← $p_2 - p_1$

Choose the CV shown to evaluate drag caused by friction

Basic equation: $F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u \rho\, d\forall + \int_{CS} u \rho \vec{V} \cdot d\vec{A}$ (F_{B_x} = 0(1), $\frac{\partial}{\partial t}$ = 0(2))

Assume: (1) Horizontal, so $F_{B_x} = 0$

(2) Steady flow

Then

$$p_1 H \delta_2 - F_D - p_2 H \delta_2 = U_1\{-\rho U_1 H \delta_2\} + \bar{U}\{+\dot{m}_{top}\} + \int_0^{\delta_2} u \rho u H\, dy$$

or $F_D = (p_1 - p_2) H \delta_2 + \rho U_1^2 H \delta_2 - \bar{U}\dot{m}_{top} - \int_0^{\delta_2} u \rho u H\, dy$ (1)

From conservation of mass, $\dot{m}_{top} = \dot{m}_1 - \dot{m}_2 = \rho U_1 H \delta_2 - \int_0^{\delta_2} \rho u H\, dy$ (2)

Evaluating the integrals, $\int_0^{\delta_2} \rho u H\, dy = \rho U_2 H \delta_2 \int_0^1 \lambda^{1/7} d\lambda = \frac{7}{8}\rho U_2 H \delta_2$

and $\int_0^{\delta_2} u \rho u H\, dy = \rho U_2^2 H \delta_2 \int_0^1 \lambda^{2/7} d\lambda = \frac{7}{9}\rho U_2^2 H \delta_2$

Thus $\int_0^{\delta_1} \rho u H dy = \frac{7}{8}\rho U_2 H \delta_2 = \frac{7}{8} \times 1.23 \frac{kg}{m^3} \times 51.9 \frac{m}{s} \times 0.305 m \times 0.0203 m = 0.346 \; kg/s$

$\rho U_1 H \delta_2 = 1.23 \frac{kg}{m^3} \times 50.2 \frac{m}{s} \times 0.305 m \times 0.0203 m = 0.382 \; kg/s$

From Eq. 2, $\dot{m}_{top} = (0.382 - 0.346) \frac{kg}{s} = 0.0364 \; kg/s$

Also $\int_0^{\delta_2} u \rho u H dy = \frac{7}{9}\rho U_2^2 H \delta_2 = \frac{7}{9} \times 1.23 \frac{kg}{m^3} \times (51.9)^2 \frac{m^2}{s^2} \times 0.305 m \times 0.0203 m \times \frac{N \cdot s^2}{kg \cdot m} = 16.0 \; N$

and $\bar{U} \approx \frac{1}{2}(U_1 + U_2) = \frac{1}{2}(50.2 + 51.9)\frac{m}{s} = 51.1 \; m/s$

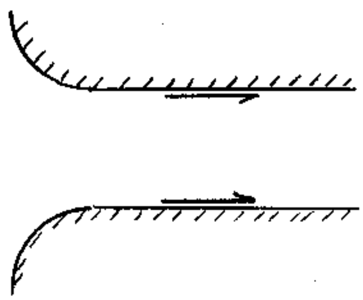
From Eq. 1, $(p_1 - p_2) H \delta_2 = 107 \frac{N}{m^2} \times 0.305 m \times 0.0203 m = 0.662 \; N$

$\rho U_1^2 H \delta_2 = 1.23 \frac{kg}{m^3} \times (50.2)^2 \frac{m^2}{s^2} \times 0.305 m \times 0.0203 m \times \frac{N \cdot s^2}{kg \cdot m} = 19.2 \; N$

Finally, substituting into Eq. 1

$F_D = 0.662 \; N + 19.2 \; N - 51.1 \frac{m}{s} \times 0.0364 \frac{kg}{s} \times \frac{N \cdot s^2}{kg \cdot m} - 16.0 \; N = 2.00 \; N$ ← F_D

The viscous drag force acts on the CV in the direction shown. The viscous drag force on the wall of the test section is equal and opposite:



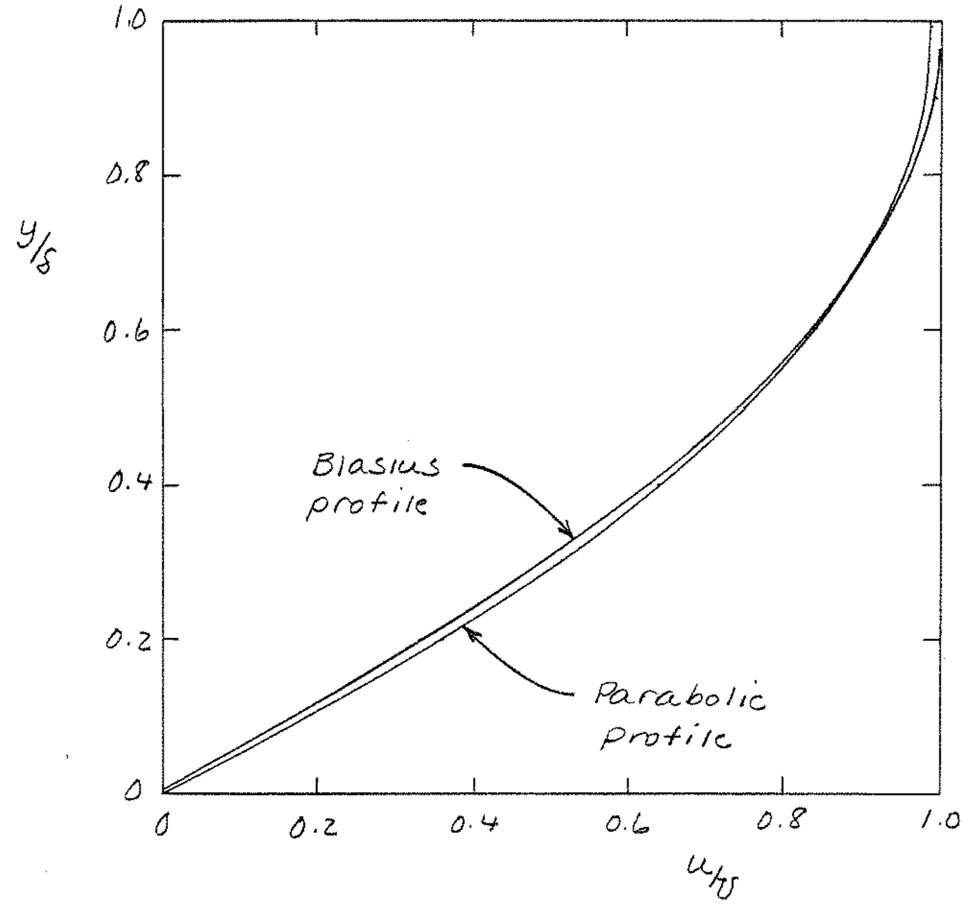
Viscous drag forces on walls of test section

Problem *9.30

Given: Blasius exact solution for laminar boundary-layer flow.

Find: Plot and compare to parabolic profile, $\frac{u}{U} = 2\eta - \eta^2$.

Solution: The Blasius solution is given in Table 9.1; it is plotted below.



Problem *9.31

Given: Numerical results of Blasius for laminar boundary-layer flow.

Find: (a) Evaluate τ distribution.
(b) Plot τ/τ_w versus y/δ.
(c) Compare results from the sinusoidal profile, $\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$

Solution: For the Blasius solution, $u = U\frac{df}{d\eta} = Uf'(\eta)$, and $\eta = y\sqrt{\frac{U}{\nu x}}$

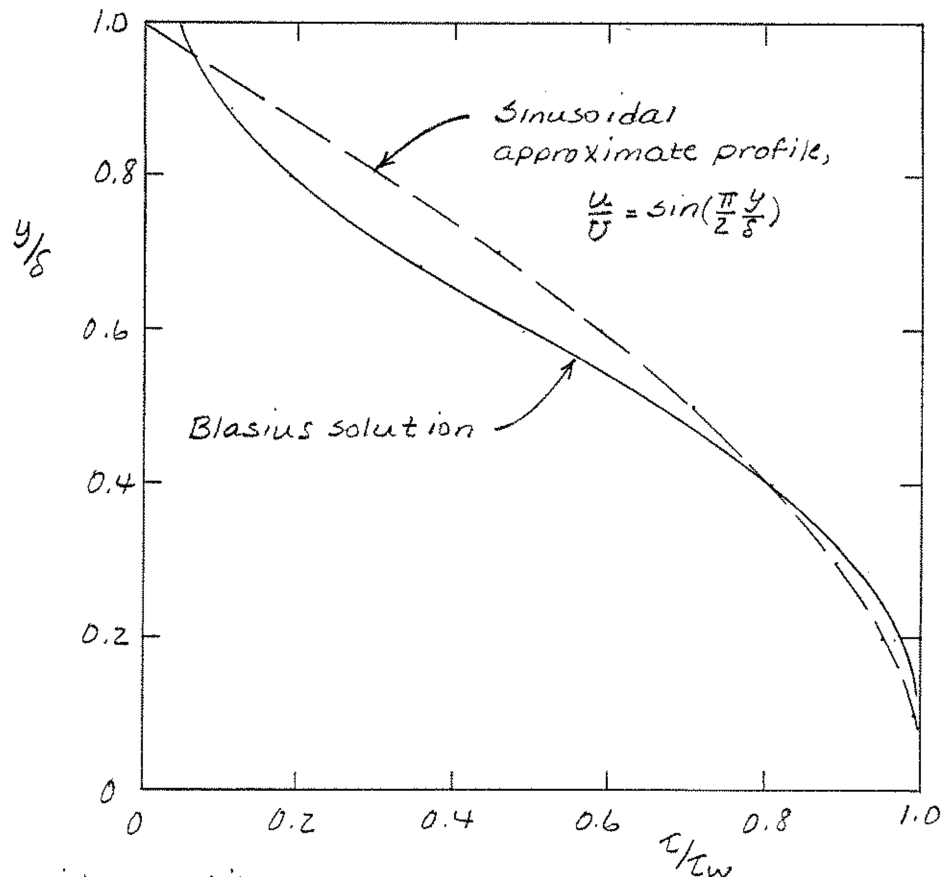
Computing equation: $\tau = \mu \frac{\partial u}{\partial y}$

For the Blasius profile, $\tau = \mu \frac{\partial}{\partial y}\left[U\frac{df}{d\eta}\right] = \mu U \frac{d^2 f}{d\eta^2}\frac{\partial \eta}{\partial y} = \mu U f''(\eta)\sqrt{\frac{U}{\nu x}}$

Thus $\frac{\tau}{\rho U^2} = \frac{\mu}{\rho U} f''(\eta)\sqrt{\frac{U}{\nu x}} = \frac{f''(\eta)}{\sqrt{Re_x}}$; $\tau \sim f''(\eta)$ τ

From the above equation, $\frac{\tau}{\tau_w} = \frac{f''(\eta)}{f''(0)} = \frac{f''(\eta)}{0.33206}$

Since $y = \delta$ at $\eta = 5$, then $y/\delta = \eta/5$. Plotting:



For the sinusoidal profile,

$$\tau = \mu\frac{\partial u}{\partial y} = \frac{\mu U}{\delta}\frac{d(u/U)}{d(y/\delta)} = \frac{\mu U}{\delta}\frac{\pi}{2}\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) \; ; \; \tau_w = \frac{\mu U}{\delta}\frac{\pi}{2}$$

Thus

$$\tau/\tau_w = \cos\left(\frac{\pi}{2}\frac{y}{\delta}\right) \quad \text{(sinusoidal profile)}$$ τ/τ_w

Problem *9.32

Given: Numerical results of Blasius for laminar boundary-layer flow.

Find: (a) Evaluate τ distribution.
(b) Plot τ/τ_w versus y/δ.
(c) Compare results from the parabolic profile, $\frac{u}{U} = 2\eta - \eta^2$

Solution: For the Blasius solution, $u = U\frac{df}{d\eta} = Uf'(\eta)$, and $\eta = y\sqrt{\frac{U}{\nu x}}$

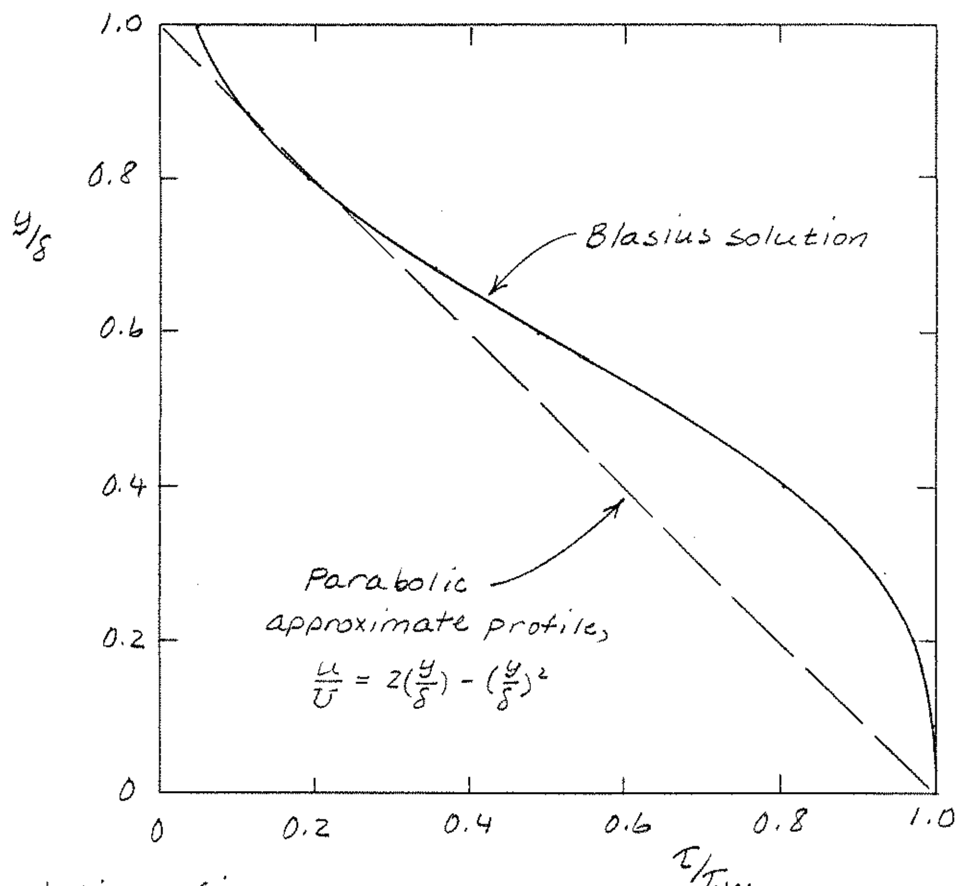
Computing equation: $\tau = \mu \frac{\partial u}{\partial y}$

For the Blasius profile, $\tau = \mu \frac{\partial}{\partial y}\left[U\frac{df}{d\eta}\right] = \mu U \frac{d^2 f}{d\eta^2}\frac{\partial \eta}{\partial y} = \mu U f''(\eta)\sqrt{\frac{U}{\nu x}}$

Thus $\frac{\tau}{\rho U^2} = \frac{\mu}{\rho U} f''(\eta)\sqrt{\frac{U}{\nu x}} = \frac{f''(\eta)}{\sqrt{Re_x}}$; $\tau \sim f''(\eta)$ ← τ

From above equation, $\frac{\tau}{\tau_w} = \frac{f''(\eta)}{f''(0)} = \frac{f''(\eta)}{0.33206}$

Since $y = \delta$ at $\eta = 5$, then $y/\delta = \eta/5$. Plotting:



For the parabolic profile,

$$\tau = \mu\frac{\partial u}{\partial y} = \frac{\mu U}{\delta}\frac{d(u/U)}{d(y/\delta)} = \frac{\mu U}{\delta}\left[2 - 2\left(\frac{y}{\delta}\right)\right] \; ; \; \tau_w = \frac{\mu U}{\delta}[2]$$

Thus

$$\tau/\tau_w = 1 - \frac{y}{\delta} \quad \text{(parabolic profile)}$$ ← τ/τ_w

Problem *9.33

Given: Numerical results of Blasius for laminar boundary-layer flow.

Find: Plot v/U versus y/δ for $Re_x = 10^5$.

Solution: For the Blasius solution, $\psi = \sqrt{U\nu x}\, f(\eta)$ and $\eta = y\sqrt{\frac{U}{\nu x}}$

From the stream function, $v = -\frac{\partial \psi}{\partial x} = -\left[\frac{1}{2}\sqrt{\frac{\nu U}{x}}\, f(\eta) + \sqrt{U\nu x}\,\frac{df}{d\eta}\frac{\partial \eta}{\partial x}\right]$

But $\frac{\partial \eta}{\partial x} = -\frac{1}{2}\frac{y}{x}\sqrt{\frac{U}{\nu x}} = -\frac{1}{2}\frac{\eta}{x}$

Thus $v = -\frac{1}{2}\sqrt{\frac{U\nu}{x}}\, f(\eta) - \sqrt{U\nu x}\,\frac{df}{d\eta}\left(-\frac{1}{2}\frac{\eta}{x}\right) = \frac{1}{2}\sqrt{\frac{U\nu}{x}}\left[\eta f'(\eta) - f(\eta)\right]$

and

$$\frac{v}{U} = \frac{1}{2}\sqrt{\frac{\nu}{Ux}}\left[\eta f'(\eta) - f(\eta)\right] = \frac{\eta f'(\eta) - f(\eta)}{2\sqrt{Re_x}}$$

$\frac{v}{U}$

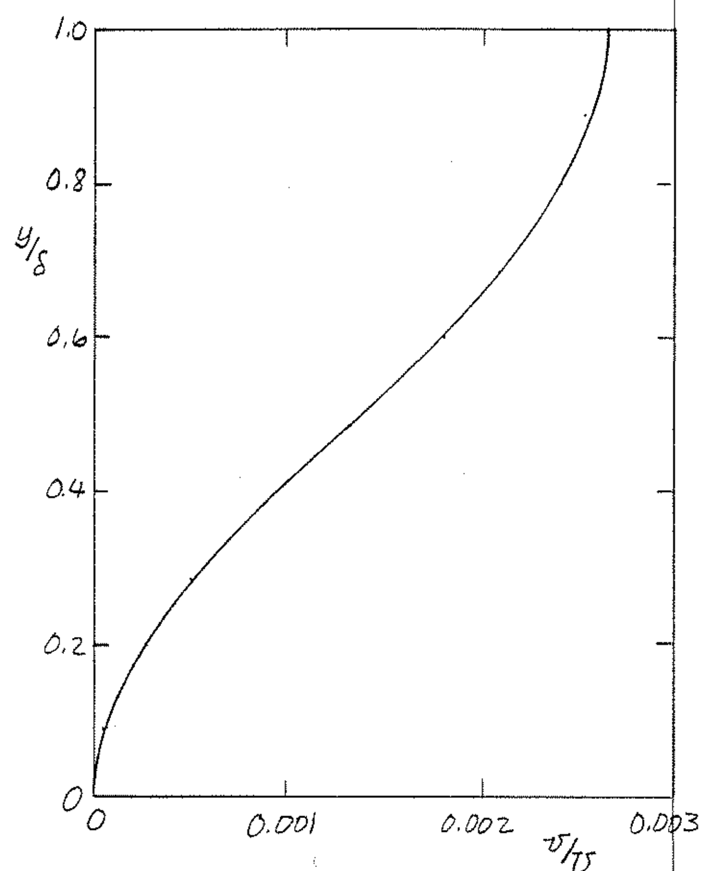
Also

$$\frac{y}{\delta} = \frac{y}{5\sqrt{\frac{\nu x}{U}}} = \frac{\eta}{5}$$

Tabulate from Table 9.1:

η	$\eta f'(\eta) - f(\eta)$	v/U
0	0	0
0.4	0.0265	4.20×10^{-5}
1.0	0.164	2.60×10^{-4}
1.4	0.316	4.99×10^{-4}
2.0	0.610	9.64×10^{-4}
2.4	0.827	1.31×10^{-3}
3.0	1.14	1.80×10^{-3}
3.4	1.32	2.09×10^{-3}
4.0	1.52	2.40×10^{-3}
4.4	1.60	2.53×10^{-3}
5.0	1.67	2.65×10^{-3}

Plot:



Given: Blasius solution to boundary layer equations gives

$$v = \frac{1}{2}\sqrt{\frac{\nu U}{x}}\left(\eta f' - f\right)$$

where $\psi = f(\eta)\sqrt{\nu x U}$ and $\eta = y\sqrt{\frac{U}{\nu x}}$

Find: (a) Verify expression for v

(b) Obtain an expression for a_x

Plot: a_x vs η to determine maximum a_x for given x.

Solution: From the definition of ψ, $v = -\partial\psi/\partial x$

$$v = -\frac{\partial \psi}{\partial x} = -\left[\sqrt{\nu x U}\,\frac{\partial f}{\partial x} + f\,\frac{1}{2}\sqrt{\frac{\nu U}{x}}\right] = -\left[\sqrt{\nu x U}\,\frac{df}{d\eta}\frac{\partial \eta}{\partial x} + f\,\frac{1}{2}\sqrt{\frac{\nu U}{x}}\right]$$

$$\frac{\partial \eta}{\partial x} = y\sqrt{\frac{U}{\nu}}\left(-\frac{1}{2x^{3/2}}\right) = -\frac{y}{2x}\sqrt{\frac{U}{\nu x}} = -\frac{\eta}{2x}$$

$$\therefore v = -\left[\sqrt{\nu x U}\, f'\left(-\frac{\eta}{2x}\right) + f\,\frac{1}{2}\sqrt{\frac{\nu U}{x}}\right] = \frac{1}{2}\sqrt{\frac{\nu U}{x}}\left(\eta f' - f\right) \longleftarrow v$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \qquad \text{where } u = U f'$$

$$\frac{\partial u}{\partial x} = U\frac{df'}{d\eta}\frac{\partial \eta}{\partial x} = U f''\left(-\frac{\eta}{2x}\right) = -\frac{1}{2}\frac{\eta U f''}{x}$$

$$\frac{\partial u}{\partial y} = U\frac{df'}{d\eta}\frac{\partial \eta}{\partial y} = U f''\sqrt{\frac{U}{\nu x}}$$

Then

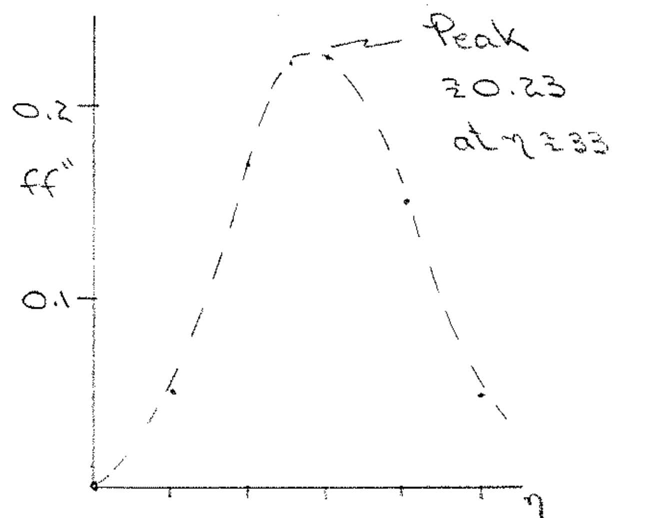
$$a_x = U f'\left(-\frac{1}{2}\frac{\eta U f''}{x}\right) + \frac{1}{2}\sqrt{\frac{\nu U}{x}}\left(\eta f' - f\right) U f''\sqrt{\frac{U}{\nu x}}$$

$$= -\frac{1}{2}\frac{U^2}{x}\eta f' f'' + \frac{1}{2}\frac{U^2}{x}\left(\eta f' f'' - f f''\right)$$

$$a_x = -\frac{U^2}{2x} f f'' \longleftarrow a_x$$

For given x, $a_{x\,max}$ for max ff''

η	f	f''	ff''
0	0	0.3321	0
1	0.1656	0.3230	0.053
2	0.6500	0.2668	0.173
3	1.3968	0.1614	0.225
4	2.3057	0.0642	0.148
5	3.2833	0.0159	0.052



From plot $ff''|_{max} \approx 0.23$

$$\therefore a_{x\,max} \approx -0.115\,\frac{U^2}{x} \quad (\text{at } \eta \approx 3) \longleftarrow a_{x\,max}$$

Problem 9.35

9.35 Numerical results of the Blasius solution to the Prandtl boundary-layer equations are presented in Table 9.1. Consider steady, incompressible flow of standard air over a flat plate at free-stream speed $U = 15$ ft/s. At $x = 7.5$ in., estimate the distance from the surface at which $u = 0.95\,U$. Evaluate the slope of the streamline through this point. Obtain an algebraic expression for the local skin friction, $\tau_w(x)$. Obtain an algebraic expression for the total skin friction drag force on the plate. Evaluate the momentum thickness at $L = 3$ ft.

Given: Blasius solution for laminar boundary layer

Find: Point at which u = 0.95U; Slope of streamline; expression for skin friction coefficient and total drag; Momentum thicknes

Solution:

Basic equation: Use results of Blasius solution (Table 9.1 on the web), and $\eta = y\cdot\sqrt{\frac{\nu\cdot x}{U}}$

$$f' = \frac{u}{U} = 0.9130 \qquad \text{at} \qquad \eta = 3.5$$

$$f' = \frac{u}{U} = 0.9555 \qquad \text{at} \qquad \eta = 4.0$$

Hence by linear interpolation, when $f' = 0.95$ $\qquad \eta = 3.5 + \frac{(4-3.5)}{(0.9555-0.9310)}\cdot(0.95-0.9310) \qquad \eta = 3.89$

From Table A.9 at 68°F $\qquad \nu = 1.62\times10^{-4}\cdot\frac{ft^2}{s}$ and $U = 15\cdot\frac{ft}{s} \qquad x = 7.5\cdot in$

Hence $\qquad y = \eta\cdot\sqrt{\frac{\nu\cdot x}{U}} \qquad y = 0.121\,in$

The streamline slope is given by $\qquad \frac{dy}{dx} = \frac{v}{u}$ where $u = U\cdot f'$ and $v = \frac{1}{2}\cdot\sqrt{\frac{\nu\cdot U}{x}}\cdot(\eta\cdot f' - f)$

$$\frac{dy}{dx} = \frac{1}{2}\cdot\sqrt{\frac{\nu\cdot U}{x}}\cdot(\eta\cdot f' - f)\cdot\frac{1}{U\cdot f'} = \frac{1}{2}\cdot\sqrt{\frac{\nu}{U\cdot x}}\cdot\frac{(\eta\cdot f' - f)}{f'} = \frac{1}{2\cdot\sqrt{Re_x}}\cdot\frac{(\eta\cdot f' - f)}{f'}$$

We have $\qquad Re_x = \frac{U\cdot x}{\nu} \qquad Re_x = 5.79\times10^4$

From the Blasius solution (Table 9.1 on the web)

$$f = 1.8377 \qquad \text{at} \qquad \eta = 3.5$$

$$f = 2.3057 \qquad \text{at} \qquad \eta = 4.0$$

Hence by linear interpolation $\qquad f = 1.8377 + \frac{(2.3057-1.8377)}{(4.0-3.5)}\cdot(3.89-3.5) \qquad f = 2.2027$

$$\frac{dy}{dx} = \frac{1}{2\cdot\sqrt{Re_x}}\cdot\frac{(\eta\cdot f' - f)}{f'} = 0.00326$$

The shear stress is $\qquad \tau_w = \mu\cdot\left(\frac{\partial}{\partial y}u + \frac{\partial}{\partial x}v\right) = \mu\cdot\frac{\partial}{\partial y}u$ at y = 0 (v = 0 at the wall for all x, so the derivative is zero there)

$$\tau_w = \mu\cdot U\cdot\sqrt{\frac{U}{\nu\cdot x}}\cdot\frac{d^2f}{d\eta^2}$$ and at η = 0 $\qquad \frac{d^2f}{d\eta^2} = 0.3321$ (from Table 9.1)

$$\tau_w = 0.3321 \cdot U \cdot \sqrt{\frac{\rho \cdot U \cdot \mu}{x}} \qquad \tau_w = 0.3321 \cdot \rho \cdot U^2 \cdot \sqrt{\frac{\mu}{\rho \cdot U \cdot x}} = 0.3321 \cdot \frac{\rho \cdot U^2}{\sqrt{Re_x}}$$

The friction drag is $F_D = \int \tau_w \, dA = \int_0^L \tau_w \cdot b \, dx$ where b is the plate width

$$F_D = \int_0^L 0.3321 \cdot \frac{\rho \cdot U^2}{\sqrt{Re_x}} \cdot b \, dx = 0.3321 \cdot \rho \cdot U^2 \cdot \sqrt{\frac{\nu}{U}} \cdot \int_0^L \frac{1}{x^{\frac{1}{2}}} \, dx$$

$$F_D = 0.3321 \cdot \rho \cdot U^2 \cdot \sqrt{\frac{\nu}{U}} \cdot b \cdot 2 \cdot L^{\frac{1}{2}} \qquad F_D = \rho \cdot U^2 \cdot b \cdot L \cdot \frac{0.6642}{\sqrt{Re_L}}$$

For the momentum integral $\frac{\tau_w}{\rho \cdot U^2} = \frac{d\theta}{dx}$ or $d\theta = \frac{\tau_w}{\rho \cdot U^2} \cdot dx$

$$\theta_L = \frac{1}{\rho \cdot U^2} \cdot \int_0^L \tau_w \, dx = \frac{1}{\rho \cdot U^2} \cdot \frac{F_D}{b} = \frac{0.6642 \cdot L}{\sqrt{Re_L}}$$

We have $L = 3 \cdot ft$ $\quad Re_L = \frac{U \cdot L}{\nu}$ $\quad Re_L = 2.78 \times 10^5$

$$\theta_L = \frac{0.6642 \cdot L}{\sqrt{Re_L}} \qquad \theta_L = 0.0454 \, in$$

Problem *9.36

[5]

*9.36 The Blasius exact solution involves solving a nonlinear equation, Eq. 9.11, with initial and boundary conditions given by Eq. 9.12. Set up an *Excel* workbook to obtain a numerical solution of this system. The workbook should consist of columns for η, f, f', and f''. The rows should consist of values of these, with a suitable step size for η (e.g., for 1000 rows the step size for η would be 0.01 to generate data through $\eta = 10$, to go a little beyond the data in Table 9.1). The values of f and f' for the first row are zero (from the initial conditions, Eq. 9.12); a guess value is needed for f'' (try 0.5). Subsequent row values for f, f', and f'' can be obtained from previous row values using Euler's finite difference method for approximating first derivatives (and Eq. 9.11). Finally, a solution can be found by using *Excel*'s *Goal Seek* or *Solver* functions to vary the initial value of f'' until $f' = 1$ for large η (e.g., $\eta = 10$, boundary condition of Eq. 9.12). Plot the results. Note: Because Euler's method is relatively crude, the results will agree with Blasius' only to within about 1%.

Given: Blasius nonlinear equation

Find: Blasius solution using Excel

Solution:

The equation to be solved is

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0 \qquad (9.11)$$

The boundary conditions are

$$f = 0 \quad \text{and} \quad \frac{df}{d\eta} = 0 \quad \text{at} \quad \eta = 0$$

$$f' = \frac{df}{d\eta} = 1 \quad \text{at} \quad \eta \to \infty \qquad (9.12)$$

Recall that these somewhat abstract variables are related to physically meaningful variables:

$$\frac{u}{U} = f'$$

and

$$\eta = y\sqrt{\frac{U}{\nu x}} \propto \frac{y}{\delta}$$

Using Euler's numerical method

$$f_{n+1} \approx f_n + \Delta\eta\, f'_n \qquad (1)$$

$$f'_{n+1} \approx f'_n + \Delta\eta\, f''_n \qquad (2)$$

$$f''_{n+1} \approx f''_n + \Delta\eta\, f'''_n$$

In these equations, the subscripts refer to the n^{th} discrete value of the variables, and $\Delta\eta = 10/N$ is the step size for η (N is the total number of steps).

But from Eq. 9.11

$$f''' = -\frac{1}{2} f\, f''$$

so the last of the three equations is

$$f''_{n+1} \approx f''_n + \Delta\eta\left(-\frac{1}{2} f_n\, f''_n\right) \qquad (3)$$

Equations 1 through 3 form a complete set for computing f, f', f''. All we need is the starting condition for each. From Eqs. 9.12

$$f_0 = 0 \quad \text{and} \quad f'_0 = 0$$

We do NOT have a starting condition for f''! Instead we must choose (using *Solver*) f''_0 so that the last condition of Eqs. 9.12 is met:

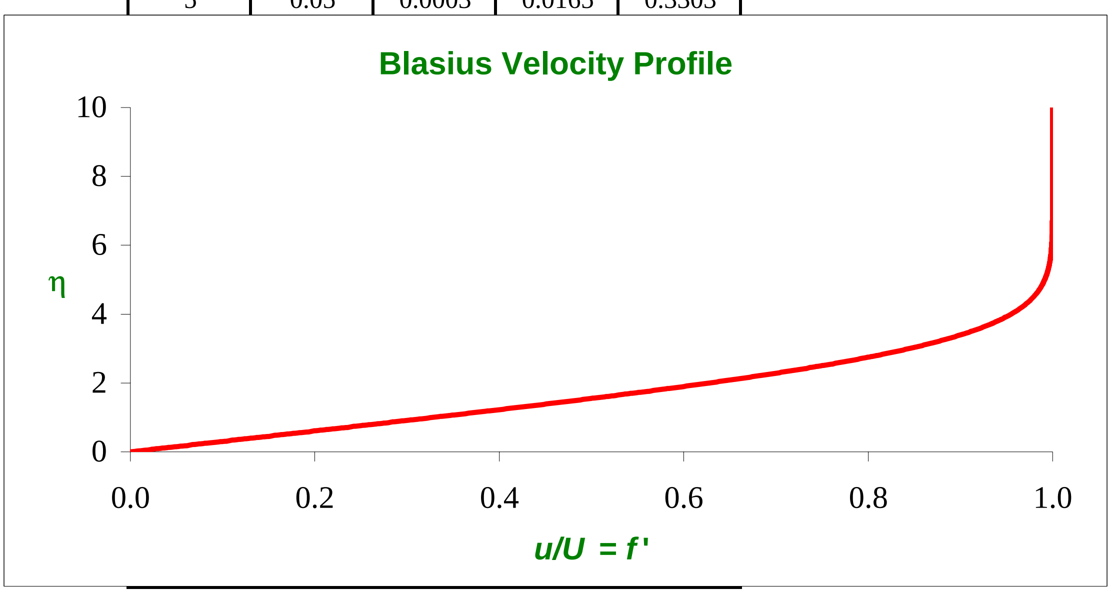
$$f'_N = 1$$

Computations (only the first few lines of 1000 are shown):

$\Delta\eta =$ 0.01

Make a guess for the first f''; use Solver to vary it until $f'N = 1$

Count	η	***f***	***f'***	***f''***
0	0.00	0.0000	0.0000	0.3303
1	0.01	0.0000	0.0033	0.3303
2	0.02	0.0000	0.0066	0.3303
3	0.03	0.0001	0.0099	0.3303
4	0.04	0.0002	0.0132	0.3303
5	0.05	0.0003	0.0165	0.3303



Problem 9.37

[2]

9.37 Consider flow of air over a flat plate. On one graph, plot the laminar boundary-layer thickness as a function of distance along the plate (up to transition) for freestream speeds $U = 1$ m/s, 2 m/s, 3 m/s, 4 m/s, 5 m/s, and 10 m/s.

Given: Data on flow over flat plate

Find: Plot of laminar thickness at various speeds

Solution:

Governing equations:

$$\frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}} \quad (9.21) \qquad \text{and} \qquad Re_x = \frac{U \cdot x}{\nu} \qquad \text{so} \qquad \delta = 5.48 \cdot \sqrt{\frac{\nu \cdot x}{U}}$$

The critical Reynolds number is $\qquad Re_{crit} = 500000$

Hence, for velocity U the critical length x_{crit} is $\qquad x_{crit} = 500000 \cdot \frac{\nu}{U}$

Tabulated or graphical data:

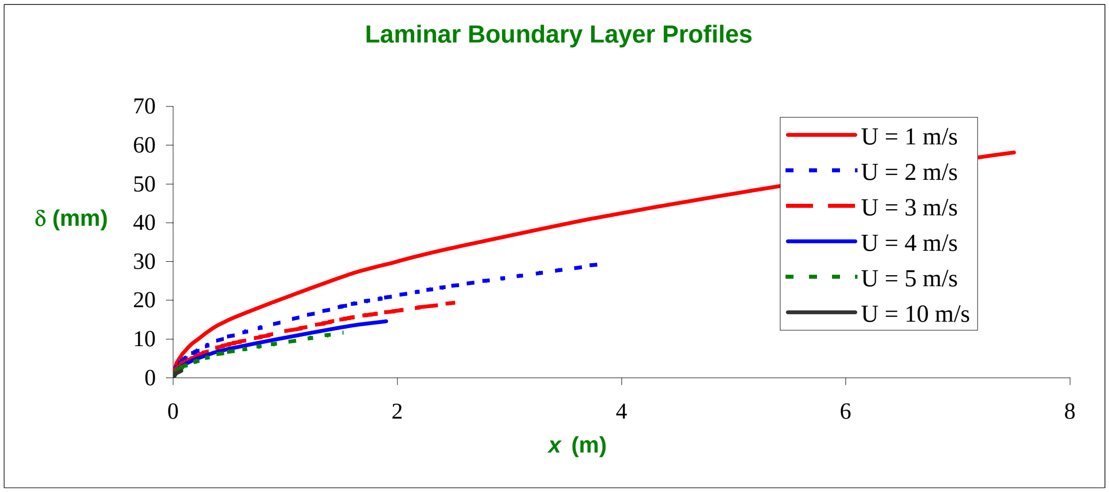
$\nu =$ 1.50E-05 m^2/s

(Table A.10, 20°C)

Computed results:

U (m/s)	1	2	3	4	5	10
x_{crit} (m)	7.5	3.8	2.5	1.9	1.5	0.75

x (m)	δ (mm)	δ (mm)	δ (mm)	δ (mm)	δ (mm)	δ (mm)
0.000	0.00	0.00	0.00	0.00	0.00	0.00
0.025	3.36	2.37	1.94	1.68	1.50	1.06
0.050	4.75	3.36	2.74	2.37	2.12	1.50
0.075	5.81	4.11	3.36	2.91	2.60	1.84
0.100	6.71	4.75	3.87	3.36	3.00	
0.2	9.49	6.71	5.48	4.75	4.24	
0.5	15.01	10.61	8.66	7.50	6.71	
1.5	25.99	18.38	15.01	13.00	11.62	
1.9	29.26	20.69	16.89	14.63		
2.5	33.56	23.73	19.37			
3.8	41.37	29.26				
5.0	47.46					
6.0	51.99					
7.5	58.12					



Problem 9.38

*9.38 A thin flat plate, $L = 0.25$ m long and $b = 1$ m wide, is installed in a water tunnel as a splitter. The freestream speed is $U = 1.75$ m/s and the velocity profile in the boundary layer is approximated as parabolic. Plot δ, δ^*, and τ_w versus x/L for the plate.

Given: Parabolic solution for laminar boundary layer

Find: Plot of δ, δ^*, and τ_w versus x/L

Solution:

Basic equations:

$$\frac{u}{U} = 2\cdot\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \qquad \frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}} \qquad c_f = \frac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2} = \frac{0.730}{\sqrt{Re_x}}$$

Hence

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right)dy = \delta\int_0^1 \left(1 - \frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \delta\int_0^1 \left(1 - 2 + \eta^2\right)d\eta = \delta\left[\eta - \eta^2 + \frac{1}{3}\eta^3\right]_0^1 = \frac{\delta}{3}$$

Tabulated or graphical data: Given data:

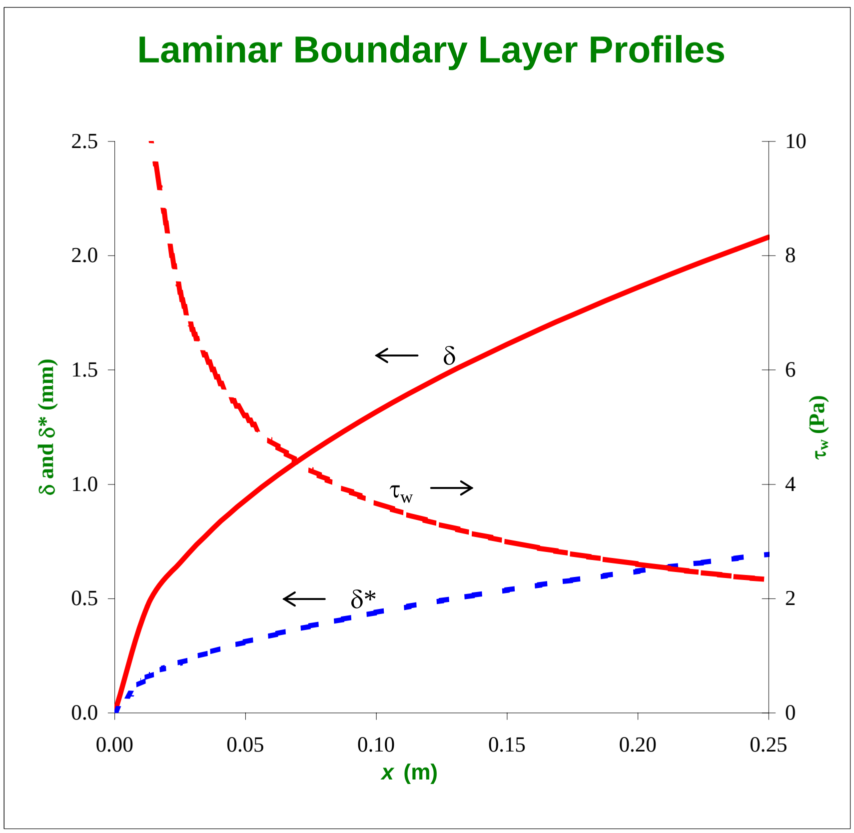
$\nu =$ 1.01E-06 m^2/s (Table A.8, 20°C)

$L =$ 0.25 m

$U =$ 1.75 m/s

Computed results:

x (m)	Re_x	δ (mm)	δ^* (mm)	τ_w (Pa)
0.0000	0.00.E+00	0.000	0.000	
0.0125	2.17.E+04	0.465	0.155	10.40
0.0250	4.33.E+04	0.658	0.219	7.36
0.0375	6.50.E+04	0.806	0.269	6.01
0.0500	8.66.E+04	0.931	0.310	5.20
0.0625	1.08.E+05	1.041	0.347	4.65
0.0750	1.30.E+05	1.140	0.380	4.25
0.0875	1.52.E+05	1.231	0.410	3.93
0.1000	1.73.E+05	1.317	0.439	3.68
0.1125	1.95.E+05	1.396	0.465	3.47
0.1250	2.17.E+05	1.472	0.491	3.29
0.1375	2.38.E+05	1.544	0.515	3.14
0.1500	2.60.E+05	1.612	0.537	3.00
0.1625	2.82.E+05	1.678	0.559	2.89
0.1750	3.03.E+05	1.742	0.581	2.78
0.1875	3.25.E+05	1.803	0.601	2.69
0.2000	3.47.E+05	1.862	0.621	2.60
0.2125	3.68.E+05	1.919	0.640	2.52
0.2250	3.90.E+05	1.975	0.658	2.45
0.2375	4.12.E+05	2.029	0.676	2.39
0.2500	4.33.E+05	**2.082**	**0.694**	**2.33**



Problem 9.39

[2]

9.39 Consider flow over the splitter plate of Problem 9.38. Show algebraically that the total drag force on one side of the splitter plate may be written $F_D = \rho U^2 \theta_L b$. Evaluate θ_L and the total drag for the given conditions.

Given: Parabolic solution for laminar boundary layer

Find: Derivation of F_D; Evaluate F_D and θ_L

Solution:

Basic equations: $\frac{u}{U} = 2\cdot\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ $\qquad \frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}}$ $\qquad \frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) + \delta^* U \frac{dU}{dx}$

$L = 0.25\cdot m \qquad b = 1\cdot m \qquad U = 1.75\cdot\frac{m}{s} \qquad \rho = 1000\cdot\frac{kg}{m^3}$

Assumptions: 1) Flat plate so $\frac{\partial}{\partial x}p = 0$, and U = const 2) δ is a function of x only 3) Incompressible

The momentum integral equation then simplifies to $\frac{\tau_w}{\rho} = \frac{d}{dx}\left(U^2\cdot\theta\right)$ where $\theta = \int_0^\delta \frac{u}{U}\cdot\left(1 - \frac{u}{U}\right) dy$

For U = const $\qquad \tau_w = \rho\cdot U^2\cdot\frac{d\theta}{dx}$

The drag force is then $\qquad F_D = \int \tau_w\, dA = \int_0^L \tau_w\cdot b\, dx = \int_0^L \rho\cdot U^2\cdot\frac{d\theta}{dx}\cdot b\, dx = \rho\cdot U^2\cdot b\cdot\int_0^{\theta_L} 1\, d\theta \qquad F_D = \rho\cdot U^2\cdot b\cdot\theta_L$

For the given profile $\qquad \frac{\theta}{\delta} = \int_0^1 \frac{u}{U}\cdot\left(1 - \frac{u}{U}\right) d\eta = \int_0^1 \left(2\cdot\eta - \eta^2\right)\cdot\left(1 - 2\cdot\eta + \eta^2\right) d\eta = \int_0^1 \left(2\cdot\eta - 5\cdot\eta^2 + 4\cdot\eta^3 - \eta^4\right) d\eta = \frac{2}{15}$

$\theta = \frac{2}{15}\cdot\delta$

From Table A.8 at 20ºC $\qquad \nu = 1.01\times10^{-6}\cdot\frac{m^2}{s} \qquad Re_L = \frac{U\cdot L}{\nu} \qquad Re_L = 4.332\times10^5$

$\delta_L = L\cdot\frac{5.48}{\sqrt{Re_L}} \qquad \delta_L = 2.08\,mm$

$\theta_L = \frac{2}{15}\cdot\delta_L \qquad \theta_L = 0.278\,mm$

$F_D = \rho\cdot U^2\cdot b\cdot\theta_L \qquad F_D = 0.850\,N$

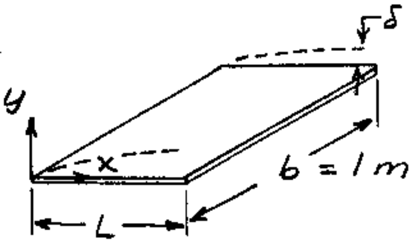
Problem 9.40

[2]

Given: Thin flat plate in water tunnel.

$U = 1.6\ m/s \longrightarrow$

$L = 0.3\ m$



Velocity profile is $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

Find: Total drag force on plate due to skin friction.

Solution: Check $Re_L = \frac{UL}{\nu} = 1.6\frac{m}{s} \times 0.3\ m \times \frac{s}{10^{-6}\ m^2} = 4.8 \times 10^5$, so laminar.

Viscous drag for two sides of plate is (Assume T = 20°C.)

$$Drag = 2\int_0^L \tau_w\, b\, dx \qquad (1)$$

From the definition, $\tau_w = \mu \frac{\partial u}{\partial y}\Big)_{y=0} = \frac{\mu U}{\delta}\frac{d(u/U)}{d(y/\delta)}\Big]_{\eta=0} = \frac{\mu U}{\delta}(2 - 2\eta)\Big]_{\eta=0} = \frac{2\mu U}{\delta}$

Since $\frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}}$, $\delta = \frac{5.48x}{\sqrt{Re_x}} = \frac{5.48}{\sqrt{U/\nu}}x^{1/2}$ $\qquad \mu = \rho\nu$

Substituting into Eq. 1,

$$F_D = 2\int_0^L \frac{2\mu U}{\delta} b\,dx = \frac{4}{5.48} b\mu U\sqrt{\frac{U}{\nu}}\int_0^L \frac{dx}{x^{1/2}} = \frac{4b\mu U}{5.48}\sqrt{\frac{U}{\nu}}\left[2x^{1/2}\right]_0^L = \frac{8b\mu U}{5.48}\sqrt{\frac{UL}{\nu}}$$

$$= \frac{8}{5.48} \times 1.0\ m \times 999\frac{kg}{m^3} \times 10^{-6}\frac{m^2}{s} \times 1.6\frac{m}{s} \times \left[1.6\frac{m}{s} \times 0.3\ m \times \frac{s}{10^{-6}\ m^2}\right]^{1/2} \frac{N \cdot s^2}{kg \cdot m}$$

$F_D = 1.62\ N$

F_D

Problem 9.41

9.41 In Problems 9.18 and 9.19 the drag on the upper surface of a flat plate with flow (fluid density $\rho = 800\ \text{kg/m}^3$) at freestream speed $U = 3$ m/s, was determined from momentum flux calculations. The drag was determined for the plate with its long edge (3 m) and its short edge (1 m) parallel to the flow. If the fluid viscosity $\mu = 0.02\ \text{N}\cdot\text{s/m}^2$, compute the drag using boundary-layer equations.

Given: Data on fluid and plate geometry

Find: Drag at both orientations using boundary layer equation

Solution:

The given data is $\rho = 800\cdot\frac{\text{kg}}{\text{m}^3}$ $\mu = 0.02\cdot\frac{\text{N}\cdot\text{s}}{\text{m}^2}$ $U = 3\cdot\frac{\text{m}}{\text{s}}$ $L = 3\cdot\text{m}$ $b = 1\cdot\text{m}$

First determine the nature of the boundary layer $Re_L = \frac{\rho\cdot U\cdot L}{\mu}$ $Re_L = 3.6\times 10^5$

The maximum Reynolds number is less than the critical value of 5×10^5

Hence:

Governing equations: $c_f = \frac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ (9.22) $c_f = \frac{0.730}{\sqrt{Re_x}}$ (9.23)

The drag (one side) is $F_D = \int_0^L \tau_w\cdot b\, dx$

Using Eqs. 9.22 and 9.23 $F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot b\cdot\int_0^L \frac{0.73}{\sqrt{\frac{\rho\cdot U\cdot x}{\mu}}}\, dx$

$F_D = 0.73\cdot b\cdot\sqrt{\mu\cdot L\cdot\rho\cdot U^3}$ $F_D = 26.3\,\text{N}$ (Compare to 30 N for Problem 9.18)

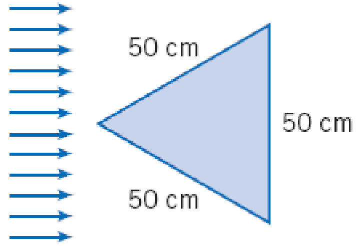
Repeating for $L = 1\cdot\text{m}$ $b = 3\cdot\text{m}$

$F_D = 0.73\cdot b\cdot\sqrt{\mu\cdot L\cdot\rho\cdot U^3}$ $F_D = 45.5\,\text{N}$ (Compare to 50.4 N for Problem 9.19)

Problem 9.42

[3]

9.42 Assume laminar boundary-layer flow to estimate the drag on the plate shown when it is placed parallel to a 5-m/s air flow. The air is at 20°C and 1 atm.



Given: Triangular plate

Find: Drag

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\quad c_f = \dfrac{0.730}{\sqrt{Re_x}}$

$L = 0.50\cdot cm\cdot\dfrac{\sqrt{3}}{2}$ $\quad L = 0.433\cdot cm$ $\quad W = 50\cdot cm$ $\quad U = 5\cdot\dfrac{m}{s}$

From Table A.10 at 20°C $\quad \nu = 1.50\times 10^{-5}\cdot\dfrac{m^2}{s}$ $\quad \rho = 1.21\cdot\dfrac{kg}{m^3}$

First determine the nature of the boundary layer $\quad Re_L = \dfrac{U\cdot L}{\nu}$ $\quad Re_L = 1443$ $\quad$ so definitely laminar

The drag (one side) is $\quad F_D = \displaystyle\int \tau_w\, dA$ $\quad F_D = \displaystyle\int_0^L \tau_w\cdot w(x)\, dx$ $\quad w(x) = W\cdot\dfrac{x}{L}$

We also have $\quad \tau_w = c_f\cdot\dfrac{1}{2}\cdot\rho\cdot U^2 = \dfrac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.730}{\sqrt{Re_x}}$

Hence

$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot\frac{W}{L}\cdot\int_0^L \frac{0.730\cdot x}{\sqrt{\frac{U\cdot x}{\nu}}}\, dx = \frac{0.730}{2}\cdot\rho\cdot U^{\frac{3}{2}}\cdot\frac{W}{L}\cdot\sqrt{\nu}\cdot\int_0^L x^{\frac{1}{2}}\, dx$$

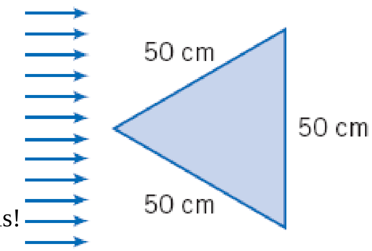
The integral is $\quad \displaystyle\int_0^L x^{\frac{1}{2}}\, dx = \frac{2}{3}\cdot L^{\frac{3}{2}}$ $\quad$ so $\quad F_D = 0.243\cdot\rho\cdot W\cdot\sqrt{\nu\cdot L\cdot U^3}$ $\quad F_D = 4.19\times 10^{-4} N$

Note: For two-sided solution $\quad 2\cdot F_D = 8.38\times 10^{-4} N$

Problem 9.43

[3]

9.43 Assume laminar boundary-layer flow to estimate the drag on the plate shown when it is placed parallel to a 5-m/s air flow, except that the base rather than the tip faces the flow. Would you expect this to be larger than, the same as, or lower than the drag for Problem 9.42?



Given: Triangular plate

Find: Drag

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\qquad c_f = \dfrac{0.730}{\sqrt{Re_x}}$

$L = 0.50\cdot cm\cdot\dfrac{\sqrt{3}}{2}$ $\qquad L = 0.433\cdot cm$ $\qquad W = 50\cdot cm$ $\qquad U = 5\cdot\dfrac{m}{s}$

From Table A.10 at 20ºC $\quad \nu = 1.50\times 10^{-5}\cdot\dfrac{m^2}{s}$ $\qquad \rho = 1.21\cdot\dfrac{kg}{m^3}$

First determine the nature of the boundary layer $\quad Re_L = \dfrac{U\cdot L}{\nu}$ $\qquad Re_L = 1443$ $\qquad$ so definitely laminar

The drag (one side) is $\quad F_D = \int \tau_w\, dA$ $\qquad F_D = \int_0^L \tau_w\cdot w(x)\, dx$ $\qquad w(x) = W\cdot\left(1 - \dfrac{x}{L}\right)$

We also have $\quad \tau_w = c_f\cdot\dfrac{1}{2}\cdot\rho\cdot U^2 = \dfrac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.730}{\sqrt{Re_x}}$

Hence

$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot W\cdot\int_0^L \frac{0.730\cdot\left(1-\frac{x}{L}\right)}{\sqrt{\frac{U\cdot x}{\nu}}}\, dx = \frac{0.730}{2}\cdot\rho\cdot U^{\frac{3}{2}}\cdot W\cdot\sqrt{\nu}\cdot\int_0^L \left(x^{-\frac{1}{2}} - \frac{x^{\frac{1}{2}}}{L}\right) dx$$

The integral is

$$\int_0^L \left(x^{-\frac{1}{2}} - \frac{x^{\frac{1}{2}}}{L}\right) dx = 2\cdot L^{\frac{1}{2}} - \frac{2}{3}\cdot\frac{L^{\frac{3}{2}}}{L} = \frac{4}{3}\cdot\sqrt{L}$$

$$F_D = 0.487\cdot\rho\cdot W\cdot\sqrt{\nu\cdot L\cdot U^3}$$

$F_D = 8.40\times 10^{-4}\, N$

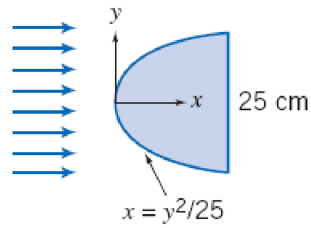
Note: For two-sided solution $\quad 2\cdot F_D = 1.68\times 10^{-3}\, N$

The drag is much higher (twice as much) compared to Problem 9.42. This is because τ_w is largest near the leading edge and falls off rapidly; in this problem the widest area is also at the front

Problem 9.44

[3]

9.44 Assume laminar boundary-layer flow to estimate the drag on the plate shown when it is placed parallel to a 7.5-m/s air flow. The air is at 20°C and 1 atm. (Note that the shape is given by $x = y^2/25$, where x and y are in cm.)



Given: Parabolic plate

Find: Drag

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\quad c_f = \dfrac{0.730}{\sqrt{Re_x}}$

$W = 25\cdot cm$ $\quad L = \dfrac{\left(\frac{W}{2}\right)^2}{25\cdot cm}$ $\quad L = 6.25\cdot cm$ $\quad U = 7.5\cdot\dfrac{m}{s}$

Note: "y" is the equation of the upper and lower surfaces, so y = W/2 at x = L

From Table A.10 at 20ºC $\quad \nu = 1.50\times 10^{-5}\cdot\dfrac{m^2}{s}$ $\quad \rho = 1.21\cdot\dfrac{kg}{m^3}$

First determine the nature of the boundary layer $\quad Re_L = \dfrac{U\cdot L}{\nu}$ $\quad Re_L = 3.12\times 10^4$ so just laminar

The drag (one side) is $\quad F_D = \int \tau_w\, dA$ $\quad F_D = \int_0^L \tau_w\cdot w(x)\, dx$ $\quad w(x) = W\cdot\sqrt{\dfrac{x}{L}}$

We also have $\quad \tau_w = c_f\cdot\frac{1}{2}\cdot\rho\cdot U^2 = \frac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.730}{\sqrt{Re_x}}$

Hence

$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot W\cdot\int_0^L \frac{0.730\cdot\sqrt{\frac{x}{L}}}{\sqrt{\frac{U\cdot x}{\nu}}}\, dx = \frac{0.730}{2}\cdot\rho\cdot U^{\frac{3}{2}}\cdot W\cdot\sqrt{\frac{\nu}{L}}\cdot\int_0^L 1\, dx$$

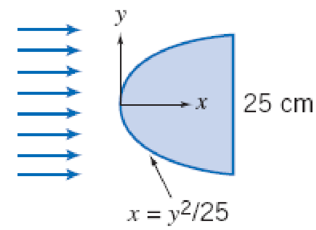
$$F_D = 0.365\cdot\rho\cdot W\cdot\sqrt{\nu\cdot L\cdot U^3}$$ $\quad F_D = 2.20\times 10^{-3}\,N$

Note: For two-sided solution $\quad 2\cdot F_D = 4.39\times 10^{-3}\,N$

Problem 9.45

9.45 Assume laminar boundary-layer flow to estimate the drag on the plate shown when it is placed parallel to a 7.5-m/s air flow, except that the base rather than the tip faces the flow. Would you expect this to be larger than, the same as, or lower than the drag for Problem 9.44?

Note: Plate is now reversed!



Given: Parabolic plate

Find: Drag

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\qquad c_f = \dfrac{0.730}{\sqrt{Re_x}}$

$W = 25\cdot cm \qquad L = \dfrac{\left(\frac{W}{2}\right)^2}{25\cdot cm} \qquad L = 6.25\cdot cm \qquad U = 7.5\cdot\dfrac{m}{s}$

Note: "y" is the equation of the upper and lower surfaces, so y = W/2 at x = 0

From Table A.10 at 20ºC $\quad \nu = 1.50\times 10^{-5}\cdot\dfrac{m^2}{s} \qquad \rho = 1.21\cdot\dfrac{kg}{m^3}$

First determine the nature of the boundary layer $\quad Re_L = \dfrac{U\cdot L}{\nu} \qquad Re_L = 3.12\times 10^4 \quad$ so just laminar

The drag (one side) is $\quad F_D = \int \tau_w\, dA \qquad F_D = \int_0^L \tau_w\cdot w(x)\, dx \qquad w(x) = W\cdot\sqrt{1-\dfrac{x}{L}}$

We also have $\quad \tau_w = c_f\cdot\dfrac{1}{2}\cdot\rho\cdot U^2 = \dfrac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.730}{\sqrt{Re_x}}$

Hence

$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot W\cdot\int_0^L \frac{0.730\cdot\sqrt{1-\frac{x}{L}}}{\sqrt{\frac{U\cdot x}{\nu}}}\,dx = \frac{0.730}{2}\cdot\rho\cdot U^{\frac{3}{2}}\cdot W\cdot\sqrt{\nu}\cdot\int_0^L \sqrt{\frac{1}{x}-\frac{1}{L}}\,dx$$

The tricky integral is (this might be easier to do numerically!)

$$\int \sqrt{\frac{1}{x}-\frac{1}{L}}\,dx = \sqrt{x-\frac{x^2}{L}} - \frac{i}{2}\cdot\sqrt{L}\cdot\ln\left(\frac{\sqrt{-L-x}-\sqrt{x}}{\sqrt{-L-x}+\sqrt{x}}\right)$$

so

$$\int_0^L \sqrt{\frac{1}{x}-\frac{1}{L}}\,dx = 0.393\cdot\sqrt{m}$$

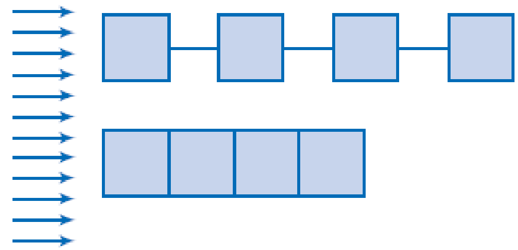
$$F_D = \frac{0.730}{2}\cdot\rho\cdot U^{\frac{3}{2}}\cdot W\cdot\sqrt{\nu}\cdot\int_0^L \sqrt{\frac{1}{x}-\frac{1}{L}}\,dx \qquad F_D = 3.45\times 10^{-3}\,N$$

Note: For two-sided solution $\qquad 2\cdot F_D = 6.9\times 10^{-3}\,N$

The drag is much higher compared to Problem 9.44. This is because τ_w is largest near the leading edge and falls off rapidly; in this problem the widest area is also at the front

Problem 9.46

9.46 Assume laminar boundary-layer flow to estimate the drag on four square plates (each 7.5 cm × 7.5 cm) placed parallel to a 1-m/s water flow, for the two configurations shown. Before calculating, which configuration do you expect to experience the lowest drag? Assume the plates attached with string are far enough apart for wake effects to be negligible, and that the water is at 20°C.



Given: Pattern of flat plates

Find: Drag on separate and composite plates

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\qquad c_f = \dfrac{0.730}{\sqrt{Re_x}}$

For separate plates $L = 7.5\cdot cm$ $\qquad W = 7.5\cdot cm$ $\qquad U = 1\cdot\dfrac{m}{s}$

From Table A.8 at 20ºC $\nu = 1.01\times 10^{-6}\cdot\dfrac{m^2}{s}$ $\qquad \rho = 998\cdot\dfrac{kg}{m^3}$

First determine the nature of the boundary layer $Re_L = \dfrac{U\cdot L}{\nu}$ $\qquad Re_L = 7.43\times 10^4$ so definitely laminar

The drag (one side) is $F_D = \displaystyle\int \tau_w\, dA$ $\qquad F_D = \displaystyle\int_0^L \tau_w\cdot W\, dx$

We also have $\tau_w = c_f\cdot\frac{1}{2}\cdot\rho\cdot U^2 = \frac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.730}{\sqrt{Re_x}}$

Hence

$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot W\cdot\int_0^L \frac{0.730}{\sqrt{\frac{U\cdot x}{\nu}}}\, dx = \frac{0.730}{2}\cdot\rho\cdot U^{\frac{3}{2}}\cdot W\cdot\sqrt{\nu}\cdot\int_0^L x^{-\frac{1}{2}}\, dx$$

The integral is $\displaystyle\int_0^L x^{-\frac{1}{2}}\, dx = 2\cdot L^{\frac{1}{2}}$ so $F_D = 0.730\cdot\rho\cdot W\cdot\sqrt{\nu\cdot L\cdot U^3}$ $\qquad F_D = 0.0150\,N$

This is the drag on one plate. The total drag is then $F_{Total} = 4\cdot F_D$ $\qquad F_{Total} = 0.0602\,N$

For both sides: $2\cdot F_{Total} = 0.120\,N$

For the composite plate $L = 4\times 7.5\cdot cm$ $\qquad L = 0.30\,m$

$F_{Composite} = 0.730\cdot\rho\cdot W\cdot\sqrt{\nu\cdot L\cdot U^3}$ $\qquad F_{Composite} = 0.0301\,N$

For both sides: $2\cdot F_{Composite} = 0.0602\,N$

The drag is much lower on the composite compared to the separate plates. This is because τ_w is largest near the leading edges and falls off rapidly; in this problem the separate plates experience leading edges four times!

Problem 9.47

Given: Laminar boundary layer in std. air, sinusoidal profile.
Conditions: $U = 3.2$ m/s, $L = 1.8$ m, $b = 0.9$ m.

Find: Plot δ, δ^*, and τ_w vs. x/L.

<u>Solution</u>: Apply momentum integral equation for steady, incompressible flow.

($U \longrightarrow$; $\longleftarrow L = 1.8$ m $\longrightarrow$)

Computing equations: $\tau_w = \rho U^2 \dfrac{d\theta}{dx}$, $\dfrac{\delta^*}{\delta} = \displaystyle\int_0^1 \left(1 - \frac{u}{U}\right) d\lambda$, $\dfrac{\theta}{\delta} = \displaystyle\int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\lambda$

For the sinusoidal profile,

$$\frac{\theta}{\delta} = \int_0^1 \sin\frac{\pi}{2}\lambda\left(1 - \sin\frac{\pi}{2}\lambda\right) d\lambda = \int_0^1 \left(\sin\frac{\pi}{2}\lambda - \sin^2\frac{\pi}{2}\lambda\right) d\lambda$$

$$= \int_0^1 \left(\sin\frac{\pi}{2}\lambda - \frac{1 - \cos\pi\lambda}{2}\right) d\lambda = \left[-\frac{2}{\pi}\cos\frac{\pi}{2}\lambda - \frac{\lambda}{2} + \frac{1}{2\pi}\sin\pi\lambda\right]_0^1$$

$$\frac{\theta}{\delta} = -\frac{2}{\pi}(0-1) - \frac{1}{2} + \frac{1}{2\pi}(0-0) = \frac{2}{\pi} - \frac{1}{2} = \frac{4-\pi}{2\pi} = 0.137$$

Also $\tau_w = \mu \left.\dfrac{du}{dy}\right)_{y=0} = \mu U \dfrac{d}{dy}\sin\left(\dfrac{\pi}{2}\dfrac{y}{\delta}\right)\Big]_{y=0} = \dfrac{\pi \mu U}{2\delta}\left(\cos\dfrac{\pi}{2}\dfrac{y}{\delta}\right)\Big]_{y=0} = \dfrac{\pi\mu U}{2\delta}$

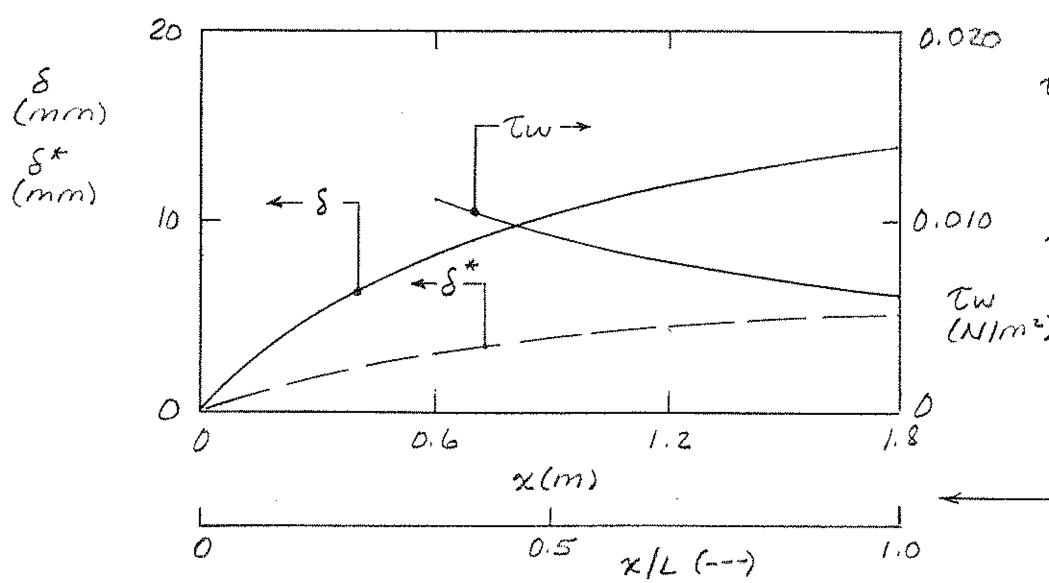
Thus $\tau_w = \rho U^2 \dfrac{d\theta}{dx} = \dfrac{\pi \mu U}{2\delta} = \rho U^2 \left(\dfrac{4-\pi}{2\pi}\right)\dfrac{d\delta}{dx}$ or $\delta\, d\delta = \dfrac{\pi^2}{4-\pi}\dfrac{\mu}{\rho U}dx$

Integrating, $\dfrac{\delta^2}{2} = \dfrac{\pi^2}{4-\pi}\dfrac{\mu}{\rho U}x$ so $\dfrac{\delta}{x} = \sqrt{\dfrac{2\pi^2}{4-\pi}}\sqrt{\dfrac{\mu}{\rho U x}} = \dfrac{4.80}{\sqrt{Re_x}}$ $\longleftarrow$ δ/x

Also $\dfrac{\delta^*}{\delta} = \displaystyle\int_0^1 \left(1 - \frac{u}{U}\right) d\lambda = \int_0^1 \left(1 - \sin\frac{\pi}{2}\lambda\right) d\lambda = \left(\lambda + \frac{2}{\pi}\cos\frac{\pi}{2}\lambda\right)\Big|_0^1 = 1 - \frac{2}{\pi} = 0.363$ $\longleftarrow$ δ^*/δ

Tabulate: $Re_x = \dfrac{Ux}{\nu} = 3.2\,\dfrac{m}{s} \cdot x\,m \times \dfrac{s}{1.46 \times 10^{-5}\,m^2} = 2.19 \times 10^5\,x$

x (m)	Re_x (---)	$1/\sqrt{Re_x}$ (---)	δ (mm)	δ^* (mm)	τ_w (N/m²)
0.6	1.31×10^5	0.00276	7.95	2.89	0.0114
1.2	2.63×10^5	0.00195	11.2	4.07	0.00804
1.8	3.94×10^5	0.00159	13.7	4.97	0.00656



$\tau_w = \dfrac{\pi}{2}\dfrac{\mu}{\rho U \delta}\rho U^2$

$= \dfrac{1}{2}\rho U^2 \dfrac{\mu}{\rho U x}\dfrac{x}{\delta}\pi$

$\tau_w = \dfrac{\pi}{4.80\sqrt{Re_x}}\dfrac{1}{2}\rho U^2$

$\longleftarrow$ Plot

Problem 9.48

Given: Laminar boundary layer flow with velocity profile, $\frac{u}{U} = \frac{y}{\delta} = \eta$.

Find: Expressions for δ/x, C_f, using the momentum integral equation.

Solution: The momentum integral equation is

Computing equation: $$-\delta \frac{\partial p}{\partial x} - \tau_w = \frac{\partial}{\partial x}\int_0^\delta u\rho u\, dy - U\frac{\partial}{\partial x}\int_0^\delta \rho u\, dy$$ (with $\frac{\partial p}{\partial x} = 0(1)$)

Assumptions: (1) Flat plate, so U = constant and $\frac{\partial p}{\partial x} = 0$
(2) δ is a function of x only, and $\delta = 0$ at $x = 0$
(3) Incompressible flow

Then

$$\tau_w = U\frac{\partial}{\partial x}\int_0^\delta \rho u\, dy - \frac{\partial}{\partial x}\int_0^\delta u\rho u\, dy = \frac{\partial}{\partial x}\int_0^\delta \rho u(U-u)\, dy$$

or $$\tau_w = \rho U^2 \frac{\partial \delta}{\partial x}\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \rho U^2 \frac{d\delta}{dx}\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right) d\eta = \rho U^2 \beta \frac{d\delta}{dx}$$

Now use the given velocity profile:

$$\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right) d\eta = \int_0^1 \eta(1-\eta)\, d\eta = \left[\frac{1}{2}\eta^2 - \frac{1}{3}\eta^3\right]_0^1 = \frac{1}{6} = \beta$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu U}{\delta}\left.\frac{\partial(u/U)}{\partial(y/\delta)}\right]_{y/\delta=0} = \frac{\mu U}{\delta}\left.\frac{d(u/U)}{d\eta}\right]_{\eta=0} = \frac{\mu U}{\delta}$$

Substituting for β and τ_w,

$$\frac{\mu U}{\delta} = \rho U^2 \frac{d\delta}{dx}\left(\frac{1}{6}\right) \quad \text{or} \quad \delta\, d\delta = \frac{6\mu}{\rho U}\, dx$$

Integrating, $\frac{\delta^2}{2} = \frac{6\mu}{\rho U}x + c$, but $c = 0$ since $\delta = 0$ at $x = 0$. Thus

$$\delta = \sqrt{\frac{12\mu}{\rho U}x}$$

or

$$\frac{\delta}{x} = \sqrt{\frac{12\mu}{\rho U x}} = \frac{3.46}{\sqrt{Re_x}}$$ $\frac{\delta}{x}$

Also

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{\mu\frac{U}{\delta}}{\frac{1}{2}\rho U^2} = \frac{2\mu}{\rho U\delta} = 2\times\frac{\mu}{\rho U x}\,\frac{x}{\delta} = \frac{2}{Re_x}\,\frac{\sqrt{Re_x}}{3.46}$$

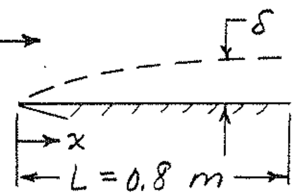
$$C_f = \frac{0.577}{\sqrt{Re_x}}$$ C_f

Problem 9.49

Given: Laminar boundary-layer flow of air, conditions as shown.

Assume $\frac{u}{U} = \frac{y}{\delta}$ $\quad U = 5.3$ m/s $\quad$ Assume STP.

Find: Plot δ, δ^*, and τ_w vs. x/L.



Solution: Apply the momentum integral equation, Eq. 9.19.

Computing equations: $\tau_w = \rho U^2 \frac{d\delta}{dx}\beta \qquad \tau_w = \mu \left.\frac{\partial u}{\partial y}\right)_{y=0}$

Assume: (1) steady flow, (2) Incompressible flow, (3) zero pressure gradient

Thus $\tau_w = \mu \left.\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu U}{\delta} \left.\frac{d(u/U)}{d(y/\delta)}\right]_{y=0} = \frac{\mu U}{\delta}(1) = \frac{\mu U}{\delta}$; $\frac{\delta^*}{\delta} = \int_0^1 (1-\lambda)\,d\lambda = \frac{1}{2}$

$$\beta = \frac{\theta}{\delta} = \int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \int_0^1 \lambda(1-\lambda)\,d\lambda = \left[\frac{\lambda^2}{2}-\frac{\lambda^3}{3}\right]_0^1 = \frac{1}{6}$$

Substituting into the MIE, $\frac{\mu U}{\delta} = \rho U^2 \frac{d\delta}{dx}\left(\frac{1}{6}\right)$ so $\delta\, d\delta = 6\frac{\mu}{\rho U}dx$

Integrating, $\delta^2 = 12\frac{\mu}{\rho U}x + c$ so $\frac{\delta}{x} = \frac{\sqrt{12}}{\sqrt{Re_x}} = \frac{3.46}{\sqrt{Re_x}}$; $\frac{\delta^*}{x} = \frac{1.73}{\sqrt{Re_x}}$

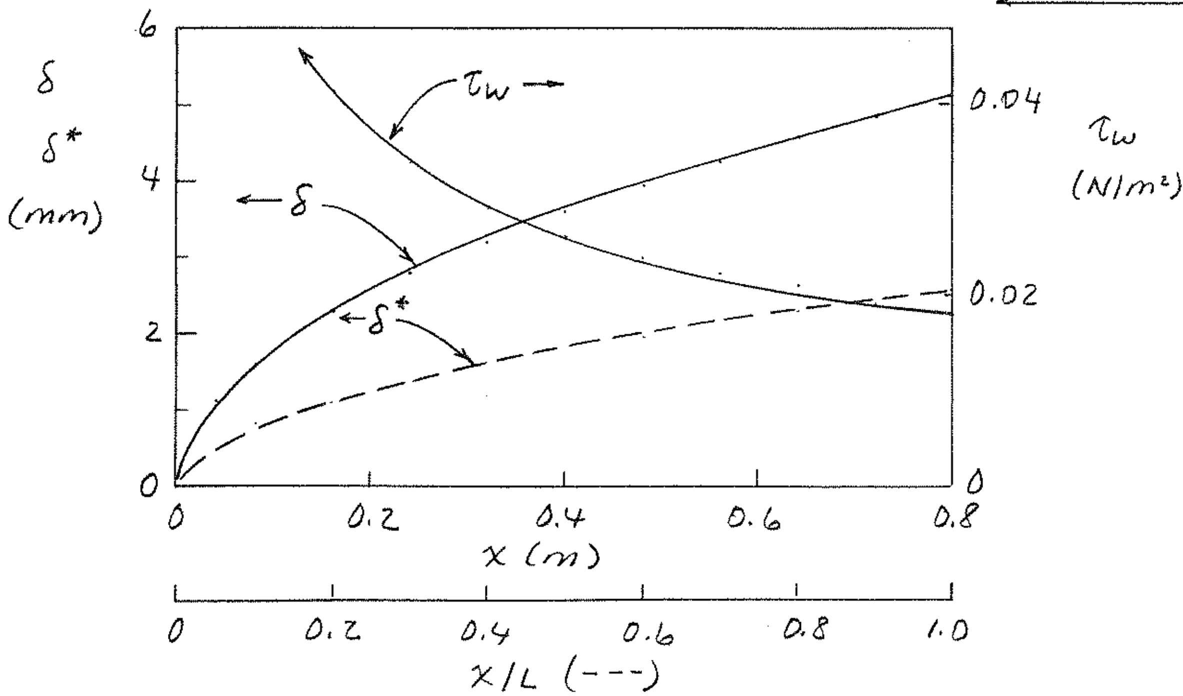
$$\tau_w = \frac{\mu U}{\delta} = \mu U \frac{\sqrt{Re_x}}{3.46\,x} = \frac{\rho \nu U}{x}\frac{\sqrt{Re_x}}{3.46} \; ; \quad C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.578}{\sqrt{Re_x}}$$

At $x = L$, $Re_L = \frac{UL}{\nu} = 5.3\,\frac{m}{s} \times 0.8\,m \times \frac{s}{1.46\times10^{-5}\,m^2} = 2.90\times10^5$, so laminar

$$\frac{\delta}{L} = \frac{3.46}{\sqrt{2.92\times10^5}} = 0.00643 \; ; \; \delta_L = 0.00643L = 0.00643 \times 800\text{ mm} = 5.14\text{ mm} \qquad \delta_L$$

$$\delta^* = \frac{1}{2}\delta, \text{ so } \delta_L^* = \frac{1}{2}\times 5.14\text{ mm} = 2.57\text{ mm} \qquad \delta_L^*$$

$$\tau_w(x=L) = \frac{0.578}{\sqrt{2.90\times10^5}} \times \frac{1}{2} \times 1.23\,\frac{kg}{m^3}\,(5.3)^2\frac{m^2}{sec^2} \times \frac{N\cdot sec^2}{kg\cdot m} = 0.0185\text{ N/m}^2 \qquad \tau_{wL}$$



Problem 9.50

Given: Laminar boundary layer forms on flat plate of length $L = 0.8\,m$ and width $b = 1.9\,m$. Free stream velocity, $U = 5.3\,m/s$. Velocity profile in the boundary layer is linear. Standard air

Find: (a) algebraic expression for $\tau_w(x)$
(b) algebraic expression for F_D
(c) magnitude of F_D.

Solution: Apply the momentum integral equation, Eq 9.19

Computing eqs. $\tau_w = \rho U^2 \frac{d\theta}{dx}$, $\theta = \delta \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right)$

For a linear profile $\theta = \delta \int_0^1 \eta(1-\eta)\,d\eta = \delta\left[\frac{\eta^2}{2} - \frac{\eta^3}{3}\right]_0^1 = \frac{1}{6}\delta$

$\therefore \tau_w = \mu \left.\frac{du}{dy}\right)_{y=0} = \mu \frac{U}{\delta} \left.\frac{d(u/U)}{d\eta}\right]_{\eta=0} = \frac{\mu U}{\delta} = \rho U^2 \frac{1}{6}\frac{d\delta}{dx}$

Separating variables and integrating gives

$$\delta\, d\delta = 6 \frac{\mu}{\rho U} dx$$

Then $\frac{\delta^2}{2} = \frac{6\mu}{\rho U} x + c$. Since $\delta = 0$ at $x = 0$ then

$$\delta = \left[\frac{12 \mu x}{\rho U}\right]^{1/2} = 3.46 \left[\frac{\mu x}{\rho U}\right]^{1/2} \quad \text{or} \quad \frac{\delta}{x} = \frac{3.46}{\sqrt{Re_x}}$$

$$\tau_w = \mu \frac{U}{\delta} = \frac{1}{3.46} \frac{\mu^{1/2} \rho^{1/2} U^{3/2}}{x^{1/2}} = 0.289 \frac{U\mu}{x} \sqrt{Re_x} \longleftarrow \tau_w(x)$$

The drag force is given by

$$F_D = \int \tau_w\, dA = \int_0^L \tau_w\, b\, dx = b \int_0^L \rho U^2 \frac{d\theta}{dx} dx = b \int_0^{\theta_L} \rho U^2 d\theta$$

$$F_D = \rho U^2 b\, \theta_L \longleftarrow F_D$$

where $\theta_L = \frac{1}{6}\delta_L$ and $\delta_L = \frac{3.46 L}{\sqrt{Re_L}}$

For given conditions

$$Re_L = \frac{UL}{\nu} = 5.3 \frac{m}{s} \times 0.8\,m \times \frac{s}{1.46 \times 10^{-5}\, m^2} = 2.90 \times 10^5$$

$$\delta_L = \frac{3.46 L}{\sqrt{Re_L}} = 3.46 \times 0.8\,m \times \frac{1}{(2.90 \times 10^5)^{1/2}} = 5.14\,mm$$

$$\theta_L = \frac{1}{6}\delta_L = 0.857\,mm$$

$$F_D = \rho U^2 b \theta_L = 1.23 \frac{kg}{m^3} \times (5.3)^2 \frac{m^2}{s^2} \times 1.9\,m \times 0.854 \times 10^{-3}\,m \times \frac{N \cdot s^2}{kg \cdot m}$$

$$F_D = 5.63 \times 10^{-2}\,N \longleftarrow F_D$$

Problem 9.51

[3]

9.51 Water at 10°C flows over a flat plate at a speed of 0.8 m/s. The plate is 0.35 m long and 1 m wide. The boundary layer on each surface of the plate is laminar. Assume that the velocity profile may be approximated as linear. Determine the drag force on the plate.

Given: Water flow over flat plate

Find: Drag on plate for linear boundary layer

Solution:

Basic equations: $F_D = 2\cdot\int \tau_w\, dA$ $\quad \tau_w = \mu\cdot\frac{du}{dy}$ at y = 0, and also $\tau_w = \rho\cdot U^2\cdot\frac{d\delta}{dx}\cdot\int_0^1 \frac{u}{U}\cdot\left(1-\frac{u}{U}\right)d\eta$

$L = 0.35\cdot m$ $\quad W = 1\cdot m$ $\quad U = 0.8\cdot\frac{m}{s}$

From Table A.8 at 10ºC $\nu = 1.30\times10^{-6}\cdot\frac{m^2}{s}$ $\quad \rho = 1000\cdot\frac{kg}{m^3}$

First determine the nature of the boundary layer $Re_L = \frac{U\cdot L}{\nu}$ $\quad Re_L = 2.15\times10^5$ so laminar

The velocity profile is $u = U\cdot\frac{y}{\delta} = U\cdot\eta$

Hence $\tau_w = \mu\cdot\frac{du}{dy} = \mu\cdot\frac{U}{\delta}$ (1) but we need δ(x)

We also have $\tau_w = \rho\cdot U^2\cdot\frac{d\delta}{dx}\cdot\int_0^1 \frac{u}{U}\cdot\left(1-\frac{u}{U}\right)d\eta = \rho\cdot U^2\cdot\frac{d\delta}{dx}\cdot\int_0^1 \eta\cdot(1-\eta)\,d\eta$

The integral is $\int_0^1\left(\eta-\eta^2\right)dx = \frac{1}{6}$ so $\tau_w = \rho\cdot U^2\cdot\frac{d\delta}{dx} = \frac{1}{6}\cdot\rho\cdot U^2\cdot\frac{d\delta}{dx}$ (2)

Comparing Eqs 1 and 2 $\tau_w = \mu\cdot\frac{U}{\delta} = \frac{1}{6}\cdot\rho\cdot U^2\cdot\frac{d\delta}{dx}$

Separating variables $\delta\cdot d\delta = \frac{6\cdot\mu}{\rho\cdot U}\cdot dx$ or $\frac{\delta^2}{2} = \frac{6\cdot\mu}{\rho\cdot U}\cdot x + c$ but δ(0) = 0 so c = 0

Hence $\delta = \sqrt{\frac{12\cdot\mu}{\rho\cdot U}\cdot x}$ or $\frac{\delta}{x} = \sqrt{\frac{12}{Re_x}} = \frac{3.46}{Re_x}$

Then $F_D = 2\cdot\int \tau_w\, dA = 2\cdot W\cdot\int_0^L \mu\cdot\frac{U}{\delta}\,dx = 2\cdot W\cdot\int_0^L \mu\cdot U\cdot\sqrt{\frac{\rho\cdot U}{12\cdot\mu}}\cdot x^{-\frac{1}{2}}\,dx = \frac{\mu\cdot W\cdot U}{\sqrt{3}}\cdot\sqrt{\frac{U}{\nu}}\cdot\int_0^L x^{-\frac{1}{2}}\,dx$

The integral is $\int_0^L x^{-\frac{1}{2}}\,dx = 2\cdot\sqrt{L}$ so $F_D = \frac{2\cdot\mu\cdot W\cdot U}{\sqrt{3}}\cdot\sqrt{\frac{U\cdot L}{\nu}}$

$F_D = \frac{2}{\sqrt{3}}\cdot\rho\cdot W\cdot\sqrt{\nu\cdot L\cdot U^3}$ $\quad F_D = 0.557\,N$

Problem 9.52

[3]

9.52 Standard air flows from the atmosphere into the wide, flat channel shown. Laminar boundary layers form on the top and bottom walls of the channel (ignore boundary-layer effects on the side walls). Assume the boundary layers behave as on a flat plate, with linear velocity profiles. At any axial distance from the inlet, the static pressure is uniform across the channel. Assume uniform flow at section (1). Indicate where the Bernoulli equation can be applied in this flow field. Find the static pressure (gage) and the displacement thickness at section (2). Plot the stagnation pressure (gage) across the channel at section (2), and explain the result. Find the static pressure (gage) at section (1) and compare to the static pressure (gage) at section (2).

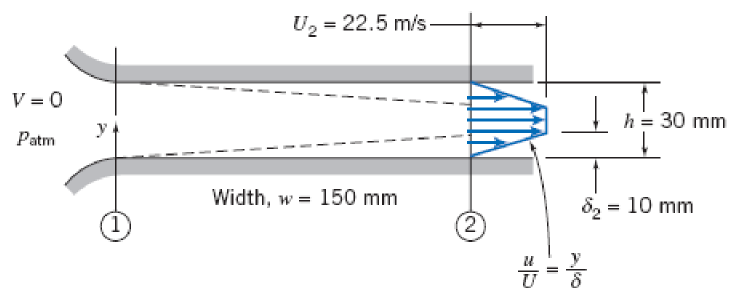
Given: Data on flow in a channel

Find: Static pressures; plot of stagnation pressure

Solution:

The given data is $h = 30 \cdot mm$ $\quad \delta_2 = 10 \cdot mm$ $\quad U_2 = 22.5 \cdot \frac{m}{s}$ $\quad w = 1 \cdot m$ (Arbitrary)

Appendix A $\rho = 1.23 \cdot \frac{kg}{m^3}$



Governing equations

Mass $\quad \frac{\partial}{\partial t}\int_{CV} \rho \, dV + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0 \quad$ (4.12)

Before entering the duct, and in the the inviscid core, the Bernoulli equation holds

$$\frac{p}{\rho} + \frac{V^2}{2} + g \cdot z = \text{constant} \qquad (4.24)$$

Assumptions: (1) Steady flow (2) No body force in *x* direction

For a linear velocity profile, from Table 9.2 the displacement thickness at location 2 is

$$\delta_{disp2} = \frac{\delta_2}{2} \qquad \delta_{disp2} = 5\,mm$$

From the definition of the displacement thickness, to compute the flow rate, the uniform flow at location 2 is assumed to take place in the entire duct, minus the displacement thicknesses at top and bottom

$$A_2 = w \cdot \left(h - 2 \cdot \delta_{disp2}\right) \qquad A_2 = 0.02\,m^2$$

Then $\quad Q = A_2 \cdot U_2 \qquad Q = 0.45 \frac{m^3}{s}$

Mass conservation (Eq. 4.12) leads to U_2

$$U_1 \cdot A_1 = U_2 \cdot A_2 \qquad \text{where} \qquad A_1 = w \cdot h \qquad A_1 = 0.03\,m^2$$

$$U_1 = \frac{A_2}{A_1} \cdot U_2 \qquad U_1 = 15\,\frac{m}{s}$$

The Bernoull equation applied between atmosphere and location 1 is

$$\frac{p_{atm}}{\rho} = \frac{p_1}{\rho} + \frac{U_1^{\,2}}{2}$$

or, working in gage pressures

$$p_1 = -\frac{1}{2} \cdot \rho \cdot U_1^{\,2} \qquad p_1 = -138\,Pa$$

(Static pressure)

Similarly, between atmosphere and location 2 (gage pressures)

$$p_2 = -\frac{1}{2} \cdot \rho \cdot U_2^{\,2} \qquad p_2 = -311\,Pa$$

(Static pressure)

The static pressure falls continuously in the entrance region as the fluid in the central core accelerates into a decreasing core

The stagnation pressure at location 2 (measured, e.g., with a Pitot tube as in Eq. 6.12), is indicated by an application of the Bernoulli equation at a point

$$\frac{p_t}{\rho} = \frac{p}{\rho} + \frac{u^2}{2}$$

where p_t is the total or stagnation pressure, $p = p_2$ is the static pressure, and u is the local velocity, given by

$$\frac{u}{U_2} = \frac{y}{\delta_2} \qquad y \le \delta_2$$

$$u = U_2 \qquad \delta_2 < y \le \frac{h}{2}$$

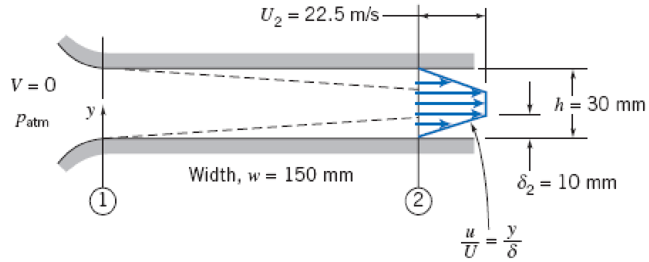
(Flow and pressure distibutions are symmetric about centerline)

Hence $$p_t = p_2 + \frac{1}{2} \cdot \rho \cdot u^2$$

The plot of stagnation pressure is shown in the associated *Excel* workbook

Problem 9.52 (In Excel)

9.52 Standard air flows from the atmosphere into the wide, flat channel shown. Laminar boundary layers form on the top and bottom walls of the channel (ignore boundary-layer effects on the side walls). Assume the boundary layers behave as on a flat plate, with linear velocity profiles. At any axial distance from the inlet, the static pressure is uniform across the channel. Assume uniform flow at section ①. Indicate where the Bernoulli equation can be applied in this flow field. Find the static pressure (gage) and the displacement thickness at section ②. Plot the stagnation pressure (gage) across the channel at section ②, and explain the result. Find the static pressure (gage) at section ① and compare to the static pressure (gage) at section ②.



Given: Data on flow in a channel

Find: Static pressures; plot of stagnation pressure

Solution:

Given data:

h = 30 mm

U_2 = 22.5 m/s

δ_2 = 10 mm

ρ = 1.23 kg/m^3

p_2 = -311 Pa

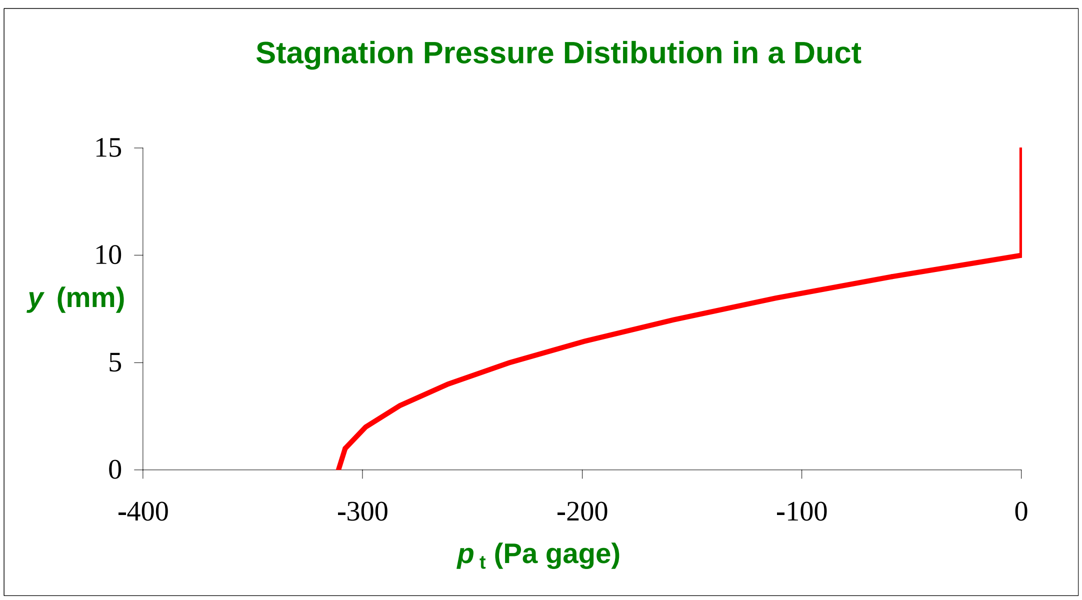
The relevant equations are:

$$\frac{u}{U_2} = \frac{y}{\delta_2} \qquad y \le \delta_2$$

$$u = U_2 \qquad \delta_2 < y \le \frac{h}{2}$$

$$p_t = p_2 + \frac{1}{2}\cdot\rho\cdot u^2$$

y (mm)	u (m/s)	p_t (Pa)
0.0	0.00	-311.00
1.0	2.25	-307.89
2.0	4.50	-298.55
3.0	6.75	-282.98
4.0	9.00	-261.19
5.0	11.25	-233.16
6.0	13.50	-198.92
7.0	15.75	-158.44
8.0	18.00	-111.74
9.0	20.25	-58.81
10.0	22.50	0.34
11.0	22.50	0.34
12.0	22.50	0.34
13.0	22.50	0.34
14.0	22.50	0.34
15.0	22.50	0.34



The stagnation pressure indicates total mechanical energy - the curve indicates significant loss close to the walls and no loss of energy in the central core.

Problem 9.53

[3]

9.53 Consider flow of air over a flat plate of length 5 m. On one graph, plot the boundary-layer thickness as a function of distance along the plate for free-stream speed $U = 10$ m/s assuming (a) a completely laminar boundary layer, (b) a completely turbulent boundary layer, and (c) a laminar boundary layer that becomes turbulent at $Re_x = 5 \times 10^5$. Use *Excel's Goal Seek* or *Solver* to find the speeds U for which transition occurs at the trailing edge, and at x = 4 m, 3 m, 2 m, and 1 m.

Given: Data on flow over a flat plate

Find: Plot of laminar and turbulent boundary layer; Speeds for transition at trailing edge

Solution:

For laminar flow

$$\frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}} \quad (9.21) \qquad \text{and} \qquad Re_x = \frac{U \cdot x}{\nu} \qquad \text{so} \qquad \delta = 5.48 \cdot \sqrt{\frac{\nu \cdot x}{U}} \qquad (1)$$

The critical Reynolds number is $Re_{crit} = 500000$

Hence, for velocity U the critical length x_{crit} is

$$x_{crit} = 500000 \cdot \frac{\nu}{U} \qquad (2)$$

For turbulent flow

$$\frac{\delta}{x} = \frac{0.382}{Re_x^{\frac{1}{5}}} \quad (9.26) \qquad \text{so} \qquad \delta = 0.382 \cdot \left(\frac{\nu}{U}\right)^{\frac{1}{5}} \cdot x^{\frac{4}{5}} \qquad (3)$$

For (a) completely laminar flow Eq. 1 holds; for (b) completely turbulent flow Eq. 3 holds; for (c) transitional flow Eq.1 or 3 holds depending on x_{crit} in Eq. 2

Given data:

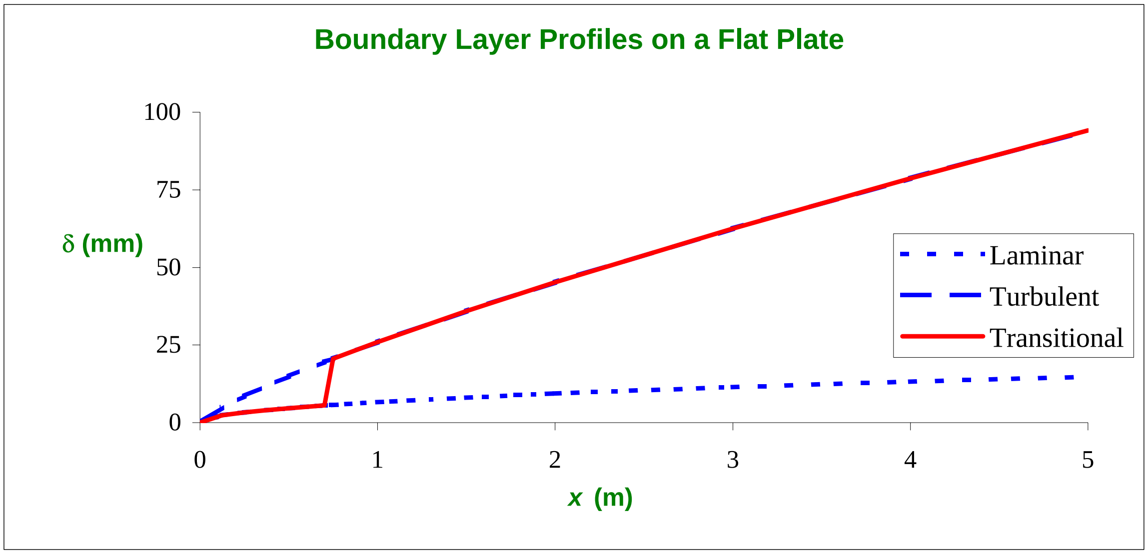
U = 10 m/s
L = 5 m

Tabulated data:

ν = 1.45E-05 m^2/s
(Table A.10)

Computed results:

x (m)	*Re* $_x$	(a) Laminar δ (mm)	(b) Turbulent δ (mm)	(c) Transition δ (mm)
0.00	0.00E+00	0.00	0.00	0.00
0.125	8.62E+04	2.33	4.92	2.33
0.250	1.72E+05	3.30	8.56	3.30
0.375	2.59E+05	4.04	11.8	4.04
0.500	3.45E+05	4.67	14.9	4.67
0.700	4.83E+05	5.52	19.5	5.5
0.75	5.17E+05	5.71	20.6	20.6
1.00	6.90E+05	6.60	26.0	26.0
1.50	1.03E+06	8.08	35.9	35.9
2.00	1.38E+06	9.3	45.2	45.2
3.00	2.07E+06	11.4	62.5	62.5
4.00	2.76E+06	13.2	78.7	78.7
5.00	3.45E+06	14.8	94.1	94.1



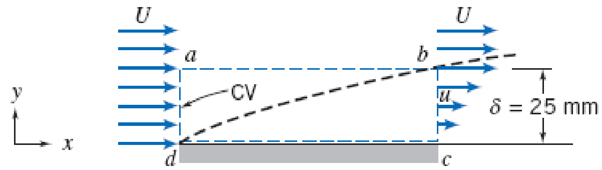
The speeds *U* at which transition occurs at specific points are shown below

x_{trans} (m)	*U* (m/s)
5	1.45
4	1.81
3	2.42
2	3.63
1	7.25

Problem 9.54

[3]

*9.54 A developing boundary layer of standard air on a flat plate is shown in Fig. P9.18. The free-stream flow outside the boundary layer is undisturbed with $U = 165$ ft/s. The plate is 10 ft wide perpendicular to the diagram. Assume flow in the boundary layer is turbulent, with a $\frac{1}{7}$-power velocity profile, and that $\delta = 0.75$ in. at surface bc. Calculate the mass flow rate across surface ad and the mass flux across surface ab. Evaluate the x momentum flux across surface bc. Determine the drag force exerted on the flat plate between d and c. Estimate the distance from the leading edge at which transition from laminar to turbulent flow may be expected.



Note: Figure data applies to problem 9.18 only

Given: Data on fluid and turbulent boundary layer

Find: Mass flow rate across *ab*; Momentum flux across *bc*; Distance at which turbulence occurs

Solution:

Basic equations: Mass

$$\frac{\partial}{\partial t}\int_{CV}\rho\, dV + \int_{CS}\rho\vec{V}\cdot d\vec{A} = 0$$

Momentum

$$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, dV + \int_{CS} u\,\rho\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Steady flow 2) No pressure force 3) No body force in *x* direction 4) Uniform flow at *ab*

The given or available data (Table A.9) is

$U = 165\cdot\frac{ft}{s}$ $\qquad \delta = 0.75\cdot in$ $\qquad b = 10\cdot ft$ $\qquad \rho = 0.00234\cdot\frac{slug}{ft^3}$ $\qquad \nu = 1.62\times 10^{-4}\cdot\frac{ft^2}{s}$

Consider CV *abcd* $\qquad m_{ad} = -\rho\cdot U\cdot b\cdot\delta \qquad m_{ad} = -0.241\,\frac{slug}{s}$ $\qquad$ (Note: Software cannot render a dot)

Mass $\qquad m_{ad} + \int_0^{\delta}\rho\cdot u\cdot b\, dy + m_{ab} = 0$ $\qquad$ and in the boundary layer $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}} = \eta^{\frac{1}{7}}$ $\qquad dy = d\eta\cdot\delta$

Hence $\qquad m_{.ab} = \rho\cdot U\cdot b\cdot\delta - \int_0^1 \rho\cdot U\cdot\eta^{\frac{1}{7}}\cdot\delta\, d\eta = \rho\cdot U\cdot b\cdot\delta - \frac{7}{8}\cdot\rho\cdot U\cdot b\cdot\delta$ $\qquad m_{ab} = \frac{1}{8}\cdot\rho\cdot U\cdot b\cdot\delta$ $\qquad m_{ab} = 0.0302\,\frac{slug}{s}$

The momentum flux across *bc* is

$$mf_{bc} = \int_0^{\delta} u\cdot\rho\cdot\vec{V}\, d\vec{A} = \int_0^{\delta} u\cdot\rho\cdot u\cdot b\, dy = \int_0^1 \rho\cdot U^2\cdot b\cdot\delta\cdot\eta^{\frac{2}{7}}\, d\eta = \rho\cdot U^2\cdot b\cdot\delta\cdot\frac{7}{9}$$

$mf_{bc} = \frac{7}{9}\cdot\rho\cdot U^2\cdot b\cdot\delta$ $\qquad mf_{bc} = 31\,\frac{slug\cdot ft}{s^2}$

From momentum $\qquad -R_x = U\cdot(-\rho\cdot U\cdot\delta) + m_{ab}\cdot u_{ab} + mf_{bc}$ $\qquad R_x = \rho\cdot U^2\cdot b\cdot\delta - m_{ab}\cdot U - mf_{bc}$ $\qquad R_x = 3.87\,lbf$

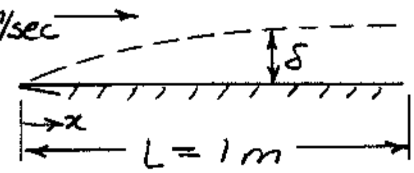
Transition occurs at $\qquad Re_x = 5\times 10^5$ $\qquad$ and $\qquad Re_x = \frac{U\cdot x}{\nu}$ $\qquad x_{trans} = \frac{Re_x\cdot\nu}{U}$ $\qquad x_{trans} = 0.491\,ft$

Problem 9.55

[3]

Given: Turbulent boundary-layer flow of water, $\frac{1}{7}$-power profile.

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$



{Conditions of Example Problem 9.4.}

Find: (a) Expression for wall shear stress, τ_w.
(b) Integrate to obtain expression for drag force, F_D.
(c) Evaluate for the conditions shown.

Solution: Apply results from the momentum integral equation.

Computing equation: $$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0594}{(Re_x)^{1/5}} \qquad (9.27)$$

Solving for τ_w,

$$\tau_w = \frac{1}{2}\rho U^2 \frac{0.0594}{(Re_x)^{1/5}} = 0.0594\left(\frac{1}{2}\rho U^2\right)\left(\frac{U}{\nu}\right)^{-1/5} x^{-1/5}$$ $\tau_w(x)$

Integrating to obtain F_D,

$$F_D = \int_0^L \tau_w \, b\,dx = \int_0^L \underbrace{0.0594\left(\frac{1}{2}\rho U^2\right)\left(\frac{U}{\nu}\right)^{-1/5}}_{C} x^{-1/5}\, b\,dx = Cb\int_0^L x^{-1/5}dx$$

$$= Cb\,\frac{5}{4}x^{4/5}\Big]_0^L = \frac{5}{4}CbL^{4/5} = \frac{5}{4}CbL\,L^{-1/5}$$

$$F_D = \frac{5}{4}(0.0594)\frac{1}{2}\rho U^2 bL\left(\frac{U}{\nu}\right)^{-1/5}L^{-1/5} = \frac{1}{2}\rho U^2 bL\,\frac{0.0721}{(Re_L)^{1/5}}$$ F_D

Evaluating, with $b = 1$ m,

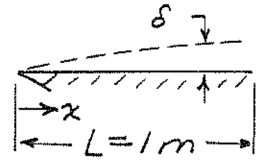
$$Re_L = \frac{UL}{\nu} = 1\,\frac{m}{s} \times 1\,m \times \frac{s}{1\times 10^{-6}\,m^2} = 1.00\times 10^6 \quad (T = 20°C,\ \text{Table A.8})$$

$$F_D = \frac{1}{2}\times 999\,\frac{kg}{m^3}\times (1)^2\frac{m^2}{s^2}\times 1\,m\times 1\,m\times \frac{0.0743}{(10^6)^{1/5}}\times\frac{N\cdot s^2}{kg\cdot m} = 2.34\,N$$ F_D

Problem 9.56

Given: Turbulent boundary-layer flow of water, conditions of Example Problem 9.4.

Assume $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$



Find: Plot δ, δ^*, and τ_w versus distance.

Solution: Apply the results of Example Problem 9.4.

Computing equations: $\frac{\delta}{x} = \frac{0.382}{(Re_x)^{1/5}}$; $\frac{\delta^*}{\delta} = \frac{1}{8}$; $C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0594}{(Re_x)^{1/5}}$

Assume: (1) BL turbulent from $x = 0$ (i.e. tripped)

For conditions given: $Re_L = \frac{UL}{\nu} = \frac{1\,m}{s} \times 1\,m \times \frac{s}{1\times10^{-6}\,m^2} = 10^6$

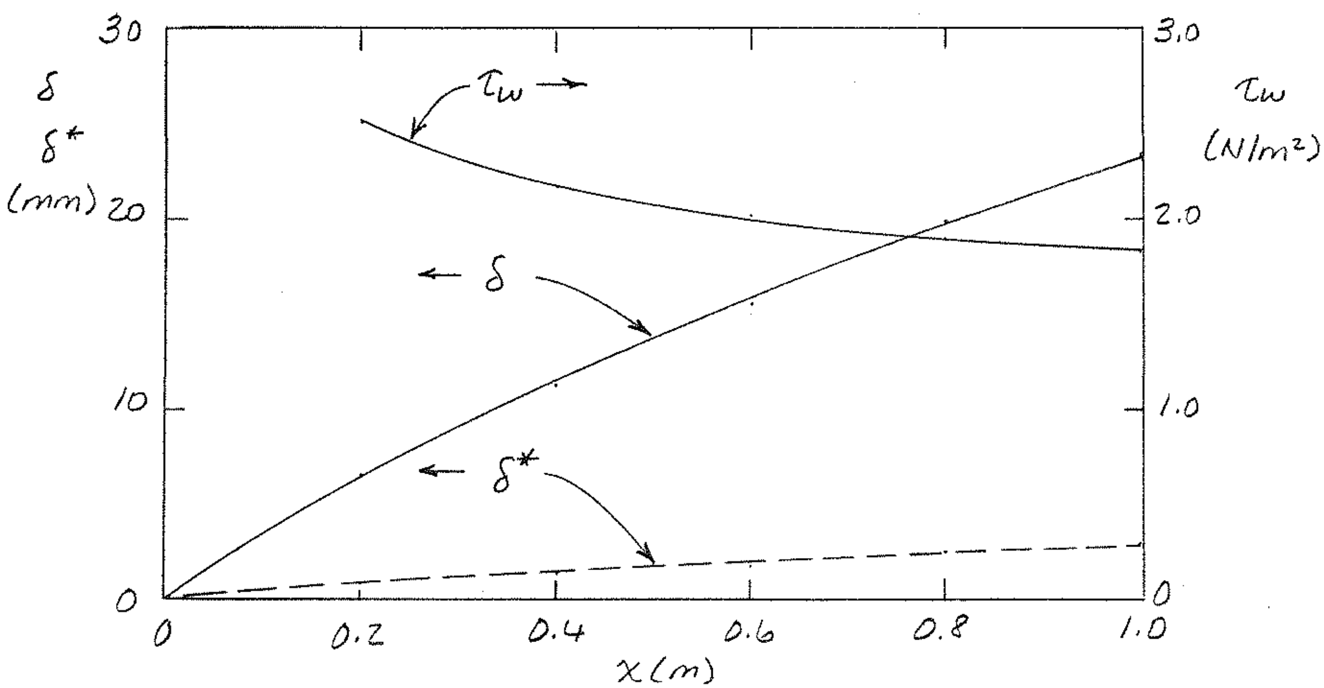
$$q = \frac{1}{2}\rho U^2 = \frac{1}{2} \times 999\,\frac{kg}{m^3} \times (1)^2\frac{m^2}{s^2} \times \frac{N\cdot s^2}{kg\cdot m} = 500\,N/m^2$$

$$\tau_w = 0.0594 \times 500\,\frac{N}{m^2} \times \frac{1}{(Re_x)^{1/5}} = \frac{29.7}{(Re_x)^{1/5}}$$

Tabulate:

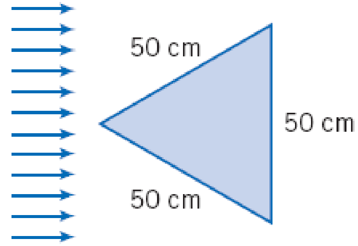
x (m)	Re_x (---)	$Re_x^{1/5}$ (---)	δ (mm)	δ^* (mm)	τ_w (N/m²)
0.2	2×10^5	11.5	6.64	0.830	2.58
0.4	4×10^5	13.2	11.6	1.45	2.25
0.6	6×10^5	14.3	16.0	2.00	2.08
0.8	8×10^5	15.2	20.1	2.51	1.95
1.0	10×10^5	15.8	24.2	3.03	1.88

Plot:



Problem 9.57

9.57 Repeat Problem 9.42, except that the air flow is now at 25 m/s (assume turbulent boundary-layer flow).



Given: Triangular plate

Find: Drag

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\quad c_f = \dfrac{0.0594}{Re_x^{\frac{1}{5}}}$

$L = 0.50\cdot cm\cdot\dfrac{\sqrt{3}}{2}$ $\quad L = 0.433\cdot cm$ $\quad W = 50\cdot cm$ $\quad U = 25\cdot\dfrac{m}{s}$

From Table A.10 at 20ºC $\quad \nu = 1.50\times 10^{-5}\cdot\dfrac{m^2}{s}$ $\quad \rho = 1.21\cdot\dfrac{kg}{m^3}$

First determine the nature of the boundary layer $\quad Re_L = \dfrac{U\cdot L}{\nu}$ $\quad Re_L = 7217$ $\quad$ so definitely still laminar, but we are told to assume turbulent!

The drag (one side) is $\quad F_D = \displaystyle\int \tau_w\,dA$ $\quad F_D = \displaystyle\int_0^L \tau_w\cdot w(x)\,dx$ $\quad w(x) = W\cdot\dfrac{x}{L}$

We also have $\quad \tau_w = c_f\cdot\dfrac{1}{2}\cdot\rho\cdot U^2 = \dfrac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.0594}{Re_x^{\frac{1}{5}}}$

Hence

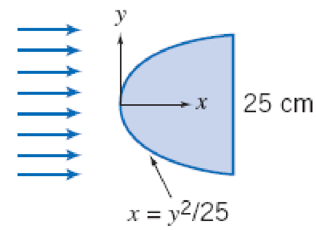
$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot\frac{W}{L}\cdot\int_0^L \frac{0.0594\cdot x}{\left(\frac{U\cdot x}{\nu}\right)^{\frac{1}{5}}}\,dx = \frac{0.0594}{2}\cdot\rho\cdot U^{\frac{9}{5}}\cdot\frac{W}{L}\cdot\nu^{\frac{1}{5}}\cdot\int_0^L x^{\frac{4}{5}}\,dx$$

The integral is $\quad \displaystyle\int_0^L x^{\frac{4}{5}}\,dx = \frac{5}{9}\cdot L^{\frac{9}{5}}$ $\quad$ so $\quad F_D = 0.0165\cdot\rho\cdot W\cdot\left(L^4\cdot\nu\cdot U^9\right)^{\frac{1}{5}}$ $\quad F_D = 4.57\times 10^{-3}\,N$

Note: For two-sided solution $\quad 2\cdot F_D = 9.14\times 10^{-3}\,N$

Problem 9.58

9.58 Repeat Problem 9.44, except that the air flow is now at 25 m/s (assume turbulent boundary-layer flow).



Given: Parabolic plate

Find: Drag

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\quad c_f = \dfrac{0.0594}{Re_x^{\frac{1}{5}}}$

$W = 25\cdot cm$ $\quad L = \dfrac{\left(\frac{W}{2}\right)^2}{25\cdot cm}$ $\quad L = 6.25\cdot cm$ $\quad U = 25\cdot\dfrac{m}{s}$

Note: "y" is the equation of the upper and lower surfaces, so y = W/2 at x = L

From Table A.10 at 20ºC $\quad \nu = 1.50\times 10^{-5}\cdot\dfrac{m^2}{s}$ $\quad \rho = 1.21\cdot\dfrac{kg}{m^3}$

First determine the nature of the boundary layer $\quad Re_L = \dfrac{U\cdot L}{\nu}$ $\quad Re_L = 1.04\times 10^5$ $\quad$ so still laminar, but we are told to assume turbulent!

The drag (one side) is $\quad F_D = \int \tau_w\, dA$ $\quad F_D = \int_0^L \tau_w\cdot w(x)\, dx$ $\quad w(x) = W\cdot\sqrt{\dfrac{x}{L}}$

We also have $\quad \tau_w = c_f\cdot\frac{1}{2}\cdot\rho\cdot U^2 = \frac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.0594}{Re_x^{\frac{1}{5}}}$

Hence

$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot W\cdot\int_0^L \frac{0.0594\cdot\sqrt{\frac{x}{L}}}{\left(\frac{U\cdot x}{\nu}\right)^{\frac{1}{5}}}\, dx = \frac{0.0594}{2}\cdot\rho\cdot U^{\frac{9}{5}}\cdot W\cdot L^{-\frac{1}{2}}\cdot\nu^{\frac{1}{5}}\cdot\int_0^L x^{\frac{3}{10}}\, dx$$

$$F_D = 0.0228\cdot\rho\cdot W\cdot\left(\nu\cdot L^4\cdot U^9\right)^{\frac{1}{5}}$$

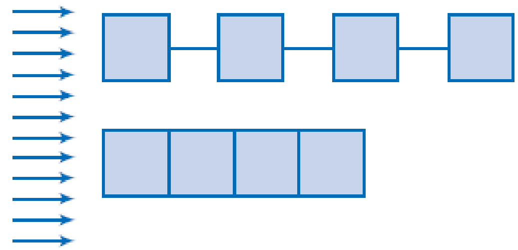
$F_D = 0.0267\, N$

Note: For two-sided solution $\quad 2\cdot F_D = 0.0534\, N$

Problem 9.59

[3]

9.59 Repeat Problem 9.46, except that the water flow is now at 10 m/s (assume turbulent boundary-layer flow).



Given: Pattern of flat plates

Find: Drag on separate and composite plates

Solution:

Basic equations: $c_f = \dfrac{\tau_w}{\frac{1}{2}\cdot\rho\cdot U^2}$ $\qquad c_f = \dfrac{0.0594}{Re_x^{\frac{1}{5}}}$

For separate plates $\quad L = 7.5\cdot cm \qquad W = 7.5\cdot cm \qquad U = 10\cdot\dfrac{m}{s}$

From Table A.8 at 20ºC $\quad \nu = 1.01\times 10^{-6}\cdot\dfrac{m^2}{s} \qquad \rho = 998\cdot\dfrac{kg}{m^3}$

First determine the nature of the boundary layer $\quad Re_L = \dfrac{U\cdot L}{\nu} \qquad Re_L = 7.43\times 10^5 \quad$ so turbulent

The drag (one side) is $\quad F_D = \displaystyle\int \tau_w\, dA \qquad F_D = \int_0^L \tau_w\cdot W\, dx$

We also have $\quad \tau_w = c_f\cdot\dfrac{1}{2}\cdot\rho\cdot U^2 = \dfrac{1}{2}\cdot\rho\cdot U^2\cdot\dfrac{0.0594}{Re_x^{\frac{1}{5}}}$

Hence
$$F_D = \frac{1}{2}\cdot\rho\cdot U^2\cdot W\cdot\int_0^L \frac{0.0594}{\left(\frac{U\cdot x}{\nu}\right)^{\frac{1}{5}}}\,dx = \frac{0.0594}{2}\cdot\rho\cdot U^{\frac{9}{5}}\cdot W\cdot\nu^{\frac{1}{5}}\cdot\int_0^L x^{-\frac{1}{5}}\,dx$$

The integral is $\quad \displaystyle\int_0^L x^{-\frac{1}{5}}\,dx = \frac{5}{4}\cdot L^{\frac{4}{5}} \quad$ so $\quad F_D = 0.371\cdot\rho\cdot W\cdot\left(\nu\cdot L^4\cdot U^9\right)^{\frac{1}{5}} \qquad F_D = 13.9\,N$

This is the drag on one plate. The total drag is then $\quad F_{Total} = 4\cdot F_D \qquad F_{Total} = 55.8\,N$

For both sides: $\quad 2\cdot F_{Total} = 112\,N$

For the composite plate $L = 4 \times 7.5 \cdot cm$ $L = 0.30\,m$

$$F_{Composite} = 0.371 \cdot \rho \cdot W \cdot \left(\nu \cdot L^4 \cdot U^9\right)^{\frac{1}{5}}$$ $$F_{Composite} = 42.3\,N$$

For both sides: $2 \cdot F_{Composite} = 84.6\,N$

The drag is much lower on the composite compared to the separate plates. This is because τ_w is largest near the leading edges and falls off rapidly; in this problem the separate plates experience leading edges four times!

Problem 9.60

[3]

Given: Turbulent boundary layer with velocity profile, $\frac{u}{U} = \eta^{1/6}$; $\eta = \frac{y}{\delta}$.

Find: Expressions for δ/x, C_f, using momentum integral equation. Compare with results of "1/7-power" profile, Section 9-5.2.

Solution: The momentum integral equation is

Computing equation: $$-\delta \frac{\partial p}{\partial x} - \tau_w = \frac{\partial}{\partial x}\int_0^\delta u\rho u\,dy - U\frac{\partial}{\partial x}\int_0^\delta \rho u\,dy \qquad \left(\frac{\partial p}{\partial x} = 0(1)\right)$$

Assumptions: (1) Flat plate, so U = constant and $\frac{\partial p}{\partial x} = 0$
(2) δ is a function of x only; $\delta = 0$ at $x = 0$
(3) Incompressible flow
(4) $\tau_w = 0.0233\,\rho U^2 (\nu/U\delta)^{1/4}$

Then

$$\tau_w = U\frac{\partial}{\partial x}\int_0^\delta \rho u\,dy - \int_0^\delta u\rho u\,dy = \frac{\partial}{\partial x}\int_0^\delta \rho u(U-u)\,dy$$

or

$$\tau_w = \rho U^2 \frac{d\delta}{dx}\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \rho U^2 \frac{d\delta}{dx}\beta$$

Evaluating β,

$$\beta = \int_0^1 \eta^{1/6}(1-\eta^{1/6})\,d\eta = \left[\frac{6}{7}\eta^{7/6} - \frac{6}{8}\eta^{8/6}\right]_0^1 = \frac{6}{56}$$

Substituting

$$0.0233\,\rho U^2\left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}} = \rho U^2\frac{d\delta}{dx}\beta \quad \text{or} \quad \delta^{1/4}d\delta = \frac{0.0233}{\beta}\left(\frac{\nu}{U}\right)^{\frac{1}{4}}dx$$

Integrating $$\frac{4}{5}\delta^{5/4} = \frac{0.0225}{\beta}\left(\frac{\nu}{U}\right)^{\frac{1}{4}}x + c, \text{ but } c=0, \text{ since } \delta = 0 \text{ at } x=0.$$

Thus $$\delta = \left[\frac{5}{4}\,\frac{0.0233}{\beta}\left(\frac{\nu}{U}\right)^{\frac{1}{4}}x\right]^{4/5} = 0.353\left(\frac{\nu}{U}\right)^{\frac{1}{5}}x^{\frac{4}{5}}$$

and

$$\frac{\delta}{x} = 0.353\left(\frac{\nu}{Ux}\right)^{\frac{1}{5}} = \frac{0.353}{(Re_x)^{1/5}}$$ $\frac{\delta}{x}$

Also

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0233\,\rho U^2\left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}}}{\frac{1}{2}\rho U^2} = 0.0466\left(\frac{\nu}{Ux}\right)^{\frac{1}{4}}\left(\frac{x}{\delta}\right)^{\frac{1}{4}}$$

$$= 0.0466\left(\frac{\nu}{Ux}\right)^{\frac{1}{4}}\left(\frac{1}{0.353}\right)^{\frac{1}{4}}\left[\left(\frac{Ux}{\nu}\right)^{\frac{1}{5}}\right]^{\frac{1}{4}}$$

$$C_f = \frac{0.0605}{(Re_x)^{1/5}}$$ C_f

Comparing:

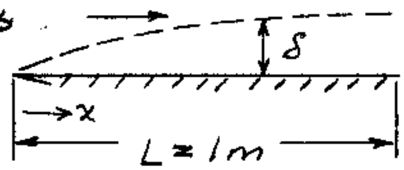
	$\delta/x\,(Re_x)^{1/5}$	$C_f\,(Re_x)^{1/5}$
1/6-power	0.353	0.0605
1/7-power	0.382	0.0594

Problem 9.61

[3]

Given: Turbulent boundary-layer flow of water, $\frac{1}{6}$-power profile.

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/6} \qquad U = 1 \text{ m/s}$$



{Flow conditions of Example Problem 9.4.}

Find: (a) Expression for the ratio, δ/x
(b) Expression for the skin friction coefficient, C_f.
(c) Evaluate F_D for the conditions shown.

Solution: Apply the momentum integral equation.

Computing equation: $$\tau_w = \rho U^2 \frac{d\theta}{dx} = \rho U^2 \frac{d\delta}{dx} \beta \qquad (9.19)$$

Assumptions: (1) Flat plate, so U = constant and $\frac{\partial p}{\partial x} = 0$
(2) $\delta = \delta(x)$ only; $\delta = 0$ at $x = 0$
(3) Incompressible flow
(4) Wall shear correlation, $\tau_w = 0.0233 \rho U^2 \left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}}$ (9.25)

Now $\beta = \frac{\theta}{\delta} = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \int_0^1 \lambda^{1/6}(1 - \lambda^{1/6})\,d\lambda = \int_0^1 (\lambda^{1/6} - \lambda^{2/6})\,d\lambda \; ; \; \lambda = \frac{y}{\delta}$

$$\beta = \frac{6}{7}\lambda^{7/6} - \frac{6}{8}\lambda^{8/6}\Big]_0^1 = \frac{6(8) - 6(7)}{7(8)} = \frac{6}{56} = 0.107$$

Substituting into the MIE,

$$0.0233 \rho U^2 \left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}} = \rho U^2 \frac{d\delta}{dx}(0.107) \quad \text{or} \quad \delta^{1/4} d\delta = 0.218 \left(\frac{\nu}{U}\right)^{\frac{1}{4}} dx$$

Integrating, $\frac{4}{5}\delta^{5/4} = 0.218\left(\frac{\nu}{U}\right)^{\frac{1}{4}} x + c$, but $c = 0$, since $\delta(0) = 0$.

Thus $$\delta = \left[0.218\left(\frac{5}{4}\right)\left(\frac{\nu}{U}\right)^{1/4} x\right]^{4/5} = 0.353\left(\frac{\nu}{U}\right)^{1/5} x^{4/5}$$

and $$\frac{\delta}{x} = 0.353\left(\frac{\nu}{U}\right)^{1/5} x^{-1/5} = \frac{0.353}{(Re_x)^{1/5}}$$ δ/x

From the wall shear correlation,

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0233 \rho U^2 \left(\frac{\nu}{U\delta}\right)^{1/4}}{\frac{1}{2}\rho U^2} = 0.0466\left(\frac{\nu}{U\delta}\right)^{1/4} = 0.0466\left(\frac{\nu}{Ux}\right)^{1/4}\left(\frac{x}{\delta}\right)^{1/4}$$

$$C_f = 0.0466\left(\frac{\nu}{Ux}\right)^{1/4}\left[\frac{(Ux/\nu)^{1/5}}{0.353}\right]^{1/4} = \frac{0.0612}{(Re_x)^{1/5}}$$ C_f

The drag force is $F_D = \int_0^L \tau_w \, b\,dx = \int_0^L 0.0612 \, \frac{1}{2}\rho U^2 \left(\frac{\nu}{U}\right)^{1/5} x^{-1/5} b\,dx$

$$F_D = 0.0612 \, \frac{1}{2}\rho U^2 \left(\frac{\nu}{U}\right)^{1/5} b \int_0^L x^{-1/5} dx = 0.0765 \, \frac{1}{2}\rho U^2 bL \left(\frac{\nu}{UL}\right)^{1/5}$$

$$F_D = \frac{0.0765}{2} \times 999 \frac{\text{kg}}{\text{m}^3} (1)^2 \frac{\text{m}^2}{\text{s}^2} \times 1\text{m} \times 1\text{m} \times \left(\frac{1}{10^6}\right)^{1/5} \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 2.41 \text{ N}$$ F_D

Problem 9.62

Given: Turbulent boundary layer with velocity profile, $\frac{u}{U} = \eta^{1/8}$; $\eta = \frac{y}{\delta}$.

Find: Expressions for δ/x, C_f, using momentum integral equation. Compare with results of "1/7-power" profile, section 9-5.2.

Solution: The momentum integral equation is

Computing equation: $$-\delta\frac{\partial p}{\partial x} - \tau_w = \frac{\partial}{\partial x}\int_0^\delta u\rho u\,dy - U\frac{\partial}{\partial x}\int_0^\delta \rho u\,dy$$

Assumptions: (1) Flat plate, so U = constant and $\frac{\partial p}{\partial x} = 0$
(2) δ is a function of x only; $\delta = 0$ at $x = 0$
(3) Incompressible flow
(4) $\tau_w = 0.0233\,\rho U^2(\nu/U\delta)^{1/4}$

Then
$$\tau_w = U\frac{\partial}{\partial x}\int_0^\delta \rho u\,dy - \int_0^\delta u\rho u\,dy = \rho U^2\frac{d\delta}{dx}\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\eta = \rho U^2\frac{d\delta}{dx}\beta$$

Evaluating β,
$$\beta = \int_0^1 \eta^{1/8}(1-\eta^{1/8})\,d\eta = \left[\frac{8}{9}\eta^{9/8} - \frac{8}{10}\eta^{10/8}\right]_0^1 = \frac{8}{90}$$

Substituting,
$$0.0233\,\rho U^2\left(\frac{\nu}{U\delta}\right)^{\frac{1}{4}} = \rho U^2\frac{d\delta}{dx}\beta \quad \text{or} \quad \delta^{1/4}d\delta = \frac{0.0233}{\beta}\left(\frac{\nu}{U}\right)^{\frac{1}{4}}dx$$

Integrating $$\frac{4}{5}\delta^{5/4} = \frac{0.0225}{\beta}\left(\frac{\nu}{U}\right)^{\frac{1}{4}}x + c, \text{ but } c = 0, \text{ since } \delta = 0 \text{ at } x = 0.$$

Thus $$\delta = \left[\frac{5}{4}\,\frac{0.0233}{\beta}\left(\frac{\nu}{U}\right)^{\frac{1}{4}}x\right]^{4/5} = 0.410\left(\frac{\nu}{U}\right)^{\frac{1}{5}}x^{4/5}$$

and $$\frac{\delta}{x} = 0.410\left(\frac{\nu}{Ux}\right)^{1/5} = \frac{0.410}{(Re_x)^{1/5}}$$ $\frac{\delta}{x}$

Also
$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0233\,\rho U^2\left(\frac{\nu}{U\delta}\right)^{1/4}}{\frac{1}{2}\rho U^2} = 0.0466\left(\frac{\nu}{Ux}\right)^{\frac{1}{4}}\left(\frac{x}{\delta}\right)^{\frac{1}{4}}$$

$$C_f = 0.0466\left(\frac{\nu}{Ux}\right)^{\frac{1}{4}}\left(\frac{1}{0.410}\right)^{\frac{1}{4}}\left[\left(\frac{Ux}{\nu}\right)^{\frac{1}{5}}\right]^{\frac{1}{4}} = \frac{0.0582}{(Re_x)^{1/5}}$$ C_f

Comparing:

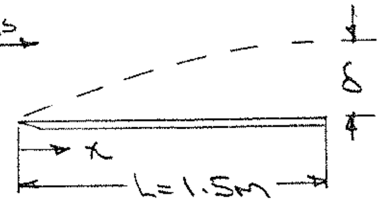
Profile	$\frac{\delta}{x}(Re_x)^{1/5}$	$C_f\,(Re_x)^{1/5}$
1/8-power	0.410	0.0582
1/7-power	0.382	0.0594

Problem 9.63

Given: Air flow over smooth flat plate as shown; width $b = 0.8$ m.

BL tripped, so turbulent $U = 20$ m/s

Velocity profile is 1/7-power.



Find: (a) δ at $x = L$, (b) τ_w at $x = L$

(c) Drag on portion $0.5\text{ m} < x < L$.

Solution:

Computing equations: $\dfrac{\delta}{x} = \dfrac{0.382}{Re_x^{1/5}}$ $\quad c_f = \dfrac{\tau_w}{\frac{1}{2}\rho U^2} = \dfrac{0.0594}{Re_x^{1/5}}$

Assumptions: (1) steady flow, (2) incompressible flow

(3) zero pressure gradient.

(4) standard air ($T = 15°C$)

$$Re_L = \frac{UL}{\nu} = 20\,\frac{m}{s} \times 1.5\,m \times \frac{s}{1.46\times10^{-5}\,m^2} = 2.06\times10^6$$

$$\delta_L = \frac{0.382\,L}{Re_L^{1/5}} = 0.382 \times 1.5\,m \times \frac{1}{(2.06\times10^6)^{0.2}} = 31.3\text{ mm} \quad \longleftarrow \delta_L$$

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0594}{Re_x^{1/5}} \qquad \therefore\ \tau_w = \frac{1}{2}\rho U^2\,\frac{0.0594}{Re_x^{1/5}}$$

$$\tau_{w_L} = \frac{1}{2}\times 1.23\,\frac{kg}{m^3}\times(20)^2\,\frac{m^2}{s^2}\times\frac{0.0594}{(2.06\times10^6)^{0.2}}\times\frac{N\cdot s^2}{kg\cdot m} = 0.798\ N/m^2 \quad \longleftarrow \tau_{w_L}$$

The drag force is given by

$$F_D = \int_{x_1}^{L} \tau_w b\,dx = \frac{1}{2}\rho U^2 b \int_{x_1}^{L} \frac{0.0594}{Re_x^{1/5}}\,dx = \frac{1}{2}\rho U^2 b\,\frac{0.0594}{(U/\nu)^{1/5}} \int_{x_1}^{L} \frac{dx}{x^{1/5}}$$

$$= \frac{1}{2}\rho U^2 b\,\frac{0.0594}{(U/\nu)^{1/5}}\left[\frac{5}{4}x^{4/5}\right]_{x_1}^{L} = \frac{1}{2}\rho U^2 b\,\frac{0.0594}{(U/\nu)^{1/5}}\,\frac{5}{4}\left[\frac{x}{x^{1/5}}\right]_{x_1}^{L}$$

$$F_D = \frac{1}{2}\rho U^2 b\,\frac{5}{4}\left[L\,c_{f_L} - x_1\,c_{f_{x_1}}\right]$$

For $x_1 = 0.5$ m, $Re_{x_1} = 6.85\times10^5$, and $c_{f_{x_1}} = 0.00404$

For $x = L = 1.5$ m, $Re_L = 2.06\times10^6$, and $c_{f_L} = 0.00324$

Substituting,

$$F_D = \frac{1}{2}\times 1.23\,\frac{kg}{m^3}\times(20)^2\,\frac{m^2}{s^2}\times 0.8\,m\times\frac{5}{4}\times\frac{N\cdot s^2}{kg\cdot m}\left[1.5\,m\times 0.00324 - 0.5\,m\times 0.00404\right]$$

$$F_D = 0.700\ N \quad \longleftarrow F_D$$

Alternate solution: $F_D = \rho U^2 b(\theta_L - \theta_{x_1})$

For 1/7-power, $\theta = \frac{7}{72}\delta$. $\theta_L = 3.04$ mm; $\delta_{x_1} = 13$ mm, $\theta_{x_1} = 1.26$ mm

$$F_D = 1.23\,\frac{kg}{m^3}\times(20)^2\,\frac{m^2}{s^2}\times 0.8\,m\times\frac{N\cdot s^2}{kg\cdot m}(0.00304 - 0.00126)\,m = 0.700\ N$$

Problem 9.64

Given: Air at standard conditions flows at 10 m/s over a flat plate.

Find: δ and τ_w at a point 1 m from leading edge for
(a) completely laminar flow (parabolic velocity profile)
(b) completely turbulent flow (1/7-power velocity profile)

Solution

Computing equations:

	Laminar Flow	Turbulent Flow
	$\frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}}$	$\frac{\delta}{x} = \frac{0.382}{Re_x^{1/5}}$
	$C_f = \frac{0.730}{\sqrt{Re_x}}$	$C_f = \frac{0.0594}{Re_x^{1/5}}$

For standard air, $\rho = 1.23\ kg/m^3$, $\nu = 1.46 \times 10^{-5}\ m^2/s$ (Table A.10)
the Reynolds number is

$$Re_x = \frac{Ux}{\nu} = 10\,\frac{m}{s} \times 1\,m \times \frac{s}{1.46\times10^{-5}\,m^2} = 6.85\times10^5$$

For laminar flow

$$\delta = \frac{5.48\,x}{\sqrt{Re_x}} = 5.48\times1\,m \times \frac{1}{(6.85\times10^5)^{1/2}} = 6.62\ mm \quad \longleftarrow \delta_{lam}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.730}{\sqrt{Re_x}} \quad \therefore \tau_w = \frac{1}{2}\rho U^2\,\frac{0.730}{\sqrt{Re_x}}$$

$$\tau_w = \frac{1}{2}\times1.23\,\frac{kg}{m^3}\times(10)^2\,\frac{m^2}{s^2}\times\frac{0.730}{(6.85\times10^5)^{1/2}}\times\frac{N\cdot s^2}{kg\cdot m} = 0.054\ N/m^2 \quad \tau_{lam}$$

For turbulent flow

$$\delta = \frac{0.382\,x}{Re_x^{1/5}} = 0.382\times1\,m\times\frac{1}{(6.85\times10^5)^{0.2}} = 26.0\ mm \quad \longleftarrow \delta_{turb}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.0594}{Re_x^{1/5}} \quad \therefore \tau_w = \frac{1}{2}\rho U^2\,\frac{0.0594}{Re_x^{1/5}}$$

$$\tau_w = \frac{1}{2}\times1.23\,\frac{kg}{m^3}\times(10)^2\,\frac{m^2}{s^2}\times\frac{0.0594}{(6.85\times10^5)^{0.2}}\times\frac{N\cdot s^2}{kg\cdot m} = 0.249\ N/m^2 \quad \longleftarrow \tau_{turb}$$

Comparing,

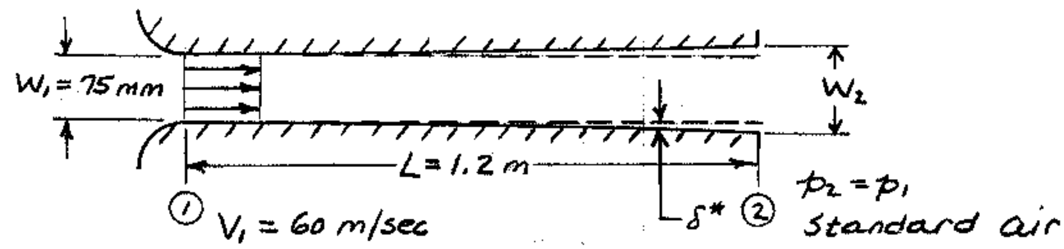
$$\frac{\delta_{turb}}{\delta_{lam}} = 3.93 \quad \text{and} \quad \frac{\tau_{w\,turb}}{\tau_{w,lam}} = 4.58$$

Thus the turbulent boundary layer has a much larger skin friction which causes it to grow more rapidly

Problem 9.65

[3]

Given: Incompressible flow of air through a plane-wall diffuser. Diffuser walls diverge slightly to accommodate the boundary layer development, so there is no pressure gradient. Assume flat plate boundary layer development.



Find: (a) Explain why Bernoulli equation is applicable to this flow.
(b) Exit width, W_2.

Solution: The Bernoulli equation may be applied along a streamline in any steady, incompressible flow in the absence of friction. The given flow is steady and incompressible. Frictional effects are confined to the thin wall boundary layers. Therefore the Bernoulli equation may be applied along any streamline in the core flow outside the boundary layers. (Since there is no streamline curvature, the pressure is uniform across sections ① and ②.)

Basic equations: $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$ (first term $= 0(1)$)

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2$$ (4), (3)

Assumptions: (1) Steady flow
(2) Turbulent, "1/7-power" boundary layer from entrance
(3) $z_1 = z_2$
(4) $p_1 = p_2$

Then from Bernoulli, $V_1 = V_2$, and from continuity,

$$0 = \{-|\rho V_1 A_1|\} + \{|\rho V_2 A_{2,eff}|\} \quad \text{or} \quad A_{2,eff} = (W_2 - 2\delta_2^*) b = W_1 b$$

Use the analysis of Section 9-5.2: $\frac{\delta}{x} = \frac{0.382}{Re_x^{1/5}}$ or $\frac{\delta}{L} = \frac{0.382}{Re_L^{1/5}}$

$$Re_L = \frac{\rho V L}{\mu} = \frac{VL}{\nu} = 60\,\frac{m}{s} \times 1.2\,m \times \frac{s}{1.46 \times 10^{-5}\,m^2} = 4.93 \times 10^6; \quad \frac{\delta_2}{L} = 0.0175$$

or $\delta_2 = 0.017\ L = (0.0171)\,1.2\,m = 0.0205\,m$, or $20.5\,mm$. $\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$, so

$$\delta_2^* = \delta_2 \int_0^1 \left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \delta \int_0^1 \left(1 - \eta^{1/7}\right) d\eta = \delta \left[\eta - \frac{7}{8}\eta^{8/7}\right]_0^1 = \frac{1}{8}\delta = 2.63\,mm$$

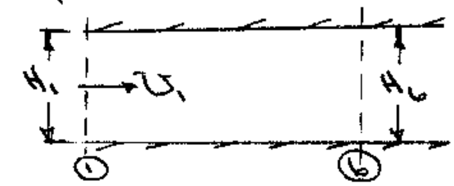
Then $W_2 = W_1 + 2\delta_2^* = 75\,mm + 2(2.63\,mm) = 80.3\,mm$ ← W_2

Problem 9.66

Given: Wind tunnel with flexible upper wall and constant width $W_1 = 305$ mm. Tunnel height adjusted to give zero pressure gradient. Wall boundary layers represented by 1/7 power profile. At two sections in the tunnel

① $H_1 = 305$ mm, $\delta_1 = 12.2$ mm
$U_1 = 26.5$ m/s

② $\delta_6 = 16.6$ mm



Find: (a) the height, H_6
(b) equivalent length of flat plate to give $\delta_1 = 12.2$ mm
(c) estimate the distance between sections ① and ⑥

Solution: To determine the height H_6 use the continuity equation and the concept of δ^*

From continuity, $A_1U_1 = A_6U_6$ where A is the effective flow area. Since $U_1 = U_6$ for zero pressure gradient, $A_1 = A_6$

$$\therefore (W - \delta_1^*)(H_1 - \delta_1^*) = (W - \delta_6^*)(H_6 - \delta_6^*)$$

and

$$H_6 = \frac{(W - \delta_1^*)(H_1 - \delta_1^*)}{(W - \delta_6^*)} + \delta_6^*$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \delta \int_0^1 \left(1 - \frac{u}{U}\right) d(y/\delta) = \delta \int_0^1 \left(1 - \frac{u}{U}\right) d\eta$$

$$\delta^* = \delta \int_0^1 (1 - \eta^{1/7})\, d\eta = \delta \left[\eta - \frac{7}{8}\eta^{8/7}\right]_0^1 = \frac{\delta}{8}$$

Substituting into the expression for H_6

$$H_6 = \frac{(305 - \frac{1}{8} \times 12.2)(305 - \frac{1}{8} \times 12.2)\,\text{mm}}{(305 - \frac{1}{8} \times 16.6)} + 16.6\,\text{mm} = 321\,\text{mm} \quad \longleftarrow H_6$$

For flat plate turbulent boundary with 1/7 power law profile

$$\frac{\delta}{x} = \frac{0.370}{Re_x^{1/5}} \quad (9.26) \qquad \therefore \delta = 0.370 \left(\frac{\nu}{U}\right)^{1/5} x^{4/5}$$

Then

$$x = \left[\frac{\delta}{0.370}\right]^{5/4} \left(\frac{U}{\nu}\right)^{1/4}$$

At section ①, $\delta = 12.2$ mm

$$x_1 = \left[\frac{0.0122\,\text{m}}{0.370}\right]^{1.25} \left(26.5\,\frac{\text{m}}{\text{s}} \times \frac{\text{s}}{1.45 \times 10^{-5}\,\text{m}^2}\right)^{0.25} = 0.517\,\text{m} \quad \longleftarrow L_{eq}$$

At section ⑥, $\delta = 16.6$ mm

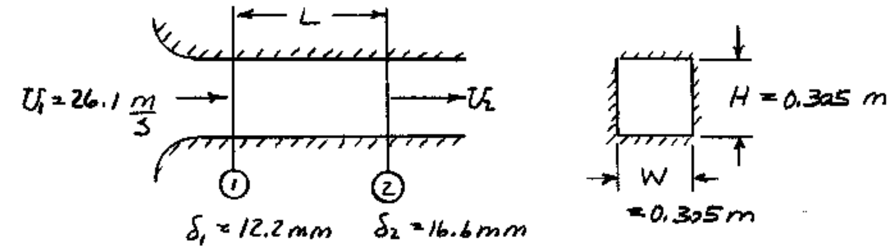
$$x_6 = \left[\frac{0.0166\,\text{m}}{0.370}\right]^{1.25} \left(26.5\,\frac{\text{m}}{\text{s}} \times \frac{\text{s}}{1.45 \times 10^{-5}\,\text{m}^2}\right)^{0.25} = 0.759\,\text{m}$$

Approximate distance $L_6 = x_6 - x_1$

$$= 0.759\,\text{m} - 0.517\,\text{m} = 0.242\,\text{m} \quad \longleftarrow L_6$$

Problem 9.67

Given: Small laboratory wind tunnels with square test sections.



Boundary layers are turbulent with 1/7-power velocity profiles.

Find: (a) Change in static pressure between ① and ②.
(b) Estimate of distance L.

Solution: For a 1/7-power profile in the turbulent boundary layer, $\delta^* = \delta/8$.
Therefore $\delta_1^* = 12.2/8 = 1.53$ mm and $\delta_2^* = 16.6/8 = 2.08$ mm.

Apply conservation of mass: $U_1 A_1 = U_2 A_2$; $A = W - 2\delta^*$

$$U_2 = U_1\left[\frac{305 - 2(1.53)}{305 - 2(2.08)}\right]^2 = 1.00733\, U_1$$

From Bernoulli (for steady, incompressible, inviscid flow along a streamline),

$$\frac{p_1}{\rho} + \frac{U_1^2}{2} = \frac{p_2}{\rho} + \frac{U_2^2}{2}\ ;\quad p_1 - p_2 = \frac{\rho}{2}(U_2^2 - U_1^2) = \frac{\rho U_1^2}{2}\left[(1.00733)^2 - 1\right]$$

$$p_1 - p_2 = \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (26.1)^2 \frac{m^2}{s^2}\left[(1.00733)^2 - 1\right]\frac{N \cdot s^2}{kg \cdot m} = 6.16\ N/m^2$$

$p_1 - p$

For a turbulent boundary layer, $\frac{\delta}{x} = \frac{0.382}{Re_x^{1/5}} = \frac{0.382}{\left(\frac{Ux}{\nu}\right)^{1/5}} = 0.382\left(\frac{\nu}{U}\right)^{1/5}\frac{1}{x^{1/5}}$

Thus $\delta = 0.382\left(\frac{\nu}{U}\right)^{1/5} x^{4/5}$ or $x = \left(\frac{\delta}{0.382}\right)^{5/4}\left(\frac{U}{\nu}\right)^{1/4}$

For standard air $\nu = 1.46 \times 10^{-5}\ m^2/s$ (Table A.10). Thus

$$x_1 = \left(\frac{0.0122\ m}{0.382}\right)^{5/4}\left(26.1\,\frac{m}{s} \times \frac{s}{1.46 \times 10^{-5}\ m^2}\right)^{1/4} = 0.494\ m$$

$$x_2 = \left(\frac{0.0166\ m}{0.382}\right)^{5/4}\left(26.3\,\frac{m}{s} \times \frac{s}{1.46 \times 10^{-5}\ m^2}\right)^{1/4} = 0.727\ m$$

Then

$$L_{12} = x_2 - x_1 = (0.727 - 0.494)\ m = 0.233\ m \text{ or } 233\ mm$$

L_{12}

{Note that 6 significant figures were carried in the factor 1.00733 so that 3 would remain in 1.00733 − 1 = 0.733.}

Problem 9.68

9.68 Air flows in a cylindrical duct of diameter $D = 150$ mm. At section (1), the turbulent boundary layer is of thickness $\delta_1 = 10$ mm, and the velocity in the inviscid central core is $U_1 = 25$ m/s. Further downstream, at section (2), the boundary layer is of thickness $\delta_2 = 30$ mm. The velocity profile in the boundary layer is approximated well by the $\frac{1}{7}$-power expression. Find the velocity, U_2, in the inviscid central core at the second section, and the pressure drop between the two sections. Does the magnitude of the pressure drop indicate that we are justified in approximating the flow between sections (1) and (2) as one with zero pressure gradient? Estimate the length of duct between sections (1) and (2). Estimate the distance downstream from section (1) at which the boundary-layer thickness is $\delta = 20$ mm.

Given: Data on flow in a duct

Find: Velocity at location 2; pressure drop; length of duct; position at which boundary layer is 20 mm

Solution:

The given data is $D = 150 \cdot \text{mm}$ $\quad \delta_1 = 10 \cdot \text{mm}$ $\quad \delta_2 = 30 \cdot \text{mm}$ $\quad U_1 = 25 \cdot \frac{\text{m}}{\text{s}}$

Table A.10 $\rho = 1.23 \cdot \frac{\text{kg}}{\text{m}^3}$ $\quad \nu = 1.45 \times 10^{-5} \cdot \frac{\text{m}^2}{\text{s}}$

Governing equations

Mass
$$\frac{\partial}{\partial t}\int_{CV} \rho \, dV + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0 \qquad (4.12)$$

In the boundary layer
$$\frac{\delta}{x} = \frac{0.382}{\text{Re}_x^{\frac{1}{5}}} \qquad (9.26)$$

In the the inviscid core, the Bernoulli equation holds

$$\frac{p}{\rho} + \frac{V^2}{2} + g \cdot z = \text{constant} \qquad (4.24)$$

Assumptions: (1) Steady flow (2) No body force (gravity) in *x* direction

For a 1/7-power law profile, from Example 9.4 the displacement thickness is $\delta_{disp} = \frac{\delta}{8}$

Hence $\delta_{disp1} = \frac{\delta_1}{8}$ $\qquad \delta_{disp1} = 1.25\,\text{mm}$

$\delta_{disp2} = \frac{\delta_2}{8}$ $\qquad \delta_{disp2} = 3.75\,\text{mm}$

From the definition of the displacement thickness, to compute the flow rate, the uniform flow at locations 1 and 2 is assumed to take place in the entire duct, minus the displacement thicknesses

$A_1 = \frac{\pi}{4} \cdot \left(D - 2 \cdot \delta_{disp1}\right)^2$ $\qquad A_1 = 0.0171\,\text{m}^2$

$$A_2 = \frac{\pi}{4}\cdot\left(D - 2\cdot\delta_{disp2}\right)^2 \qquad A_2 = 0.0159\,m^2$$

Mass conservation (Eq. 4.12) leads to U_2

$$\left(-\rho\cdot U_1\cdot A_1\right) + \left(\rho\cdot U_2\cdot A_2\right) = 0 \qquad \text{or} \qquad U_2 = U_1\cdot\frac{A_1}{A_2} \qquad U_2 = 26.8\,\frac{m}{s}$$

The Bernoulli equation applied between locations 1 and 2 is

$$\frac{p_1}{\rho} + \frac{U_1^{\,2}}{2} = \frac{p_2}{\rho} + \frac{U_2^{\,2}}{2}$$

or the pressure drop is $p_1 - p_2 = \Delta p = \frac{\rho}{2}\cdot\left(U_2^{\,2} - U_1^{\,2}\right)$ $\qquad \Delta p = 56.9\,Pa$ (Depends on ρ value selected)

The static pressure falls continuously in the entrance region as the fluid in the central core accelerates into a decreasing core.

If we assume the stagnation pressure is atmospheric, a change in pressure of about 60 Pa is not significant; in addition, the velocity changes about 5%, again not a large change to within engineering accuracy

To compute distances corresponding to boundary layer thicknesses, rearrange Eq.9.26

$$\frac{\delta}{x} = \frac{0.382}{Re_x^{\frac{1}{5}}} = 0.382\cdot\left(\frac{\nu}{U\cdot x}\right)^{\frac{1}{5}} \qquad \text{so} \qquad x = \left(\frac{\delta}{0.382}\right)^{\frac{5}{4}}\cdot\left(\frac{U}{\nu}\right)^{\frac{1}{4}}$$

Applying this equation to locations 1 and 2 (using $U = U_1$ or U_2 as approximations)

$$x_1 = \left(\frac{\delta_1}{0.382}\right)^{\frac{5}{4}}\cdot\left(\frac{U_1}{\nu}\right)^{\frac{1}{4}} \qquad x_1 = 0.382\,m$$

$$x_2 = \left(\frac{\delta_2}{0.382}\right)^{\frac{5}{4}}\cdot\left(\frac{U_2}{\nu}\right)^{\frac{1}{4}} \qquad x_2 = 1.533\,m$$

$x_2 - x_1 = 1.15\,m$ (Depends on ν value selected)

For location 3 $\qquad \delta_3 = 20\cdot mm \qquad \delta_{disp3} = \frac{\delta_3}{8} \qquad \delta_{disp3} = 2.5\,mm$

$$A_3 = \frac{\pi}{4}\cdot\left(D - 2\cdot\delta_{disp3}\right)^2 \qquad A_3 = 0.017\,m^2$$

$$U_3 = U_1\cdot\frac{A_1}{A_3} \qquad U_3 = 25.9\,\frac{m}{s}$$

$$x_3 = \left(\frac{\delta_3}{0.382}\right)^{\frac{5}{4}}\cdot\left(\frac{U_2}{\nu}\right)^{\frac{1}{4}} \qquad x_3 = 0.923\,m$$

$x_3 - x_1 = 0.542\,m$ (Depends on ν value selected)

Problem 9.69

9.69 Perform a cost-effectiveness analysis on a typical large tanker used for transporting petroleum. Determine, as a percentage of the petroleum cargo, the amount of petroleum that is consumed in traveling a distance of 2000 miles. Use data from Example 9.5, and the following: Assume the petroleum cargo constitutes 75% of the total weight, the propeller efficiency is 70%, the wave drag and power to run auxiliary equipment constitute losses equivalent to an additional 20%, the engines have a thermal efficiency of 40%, and the petroleum energy is 20,000 Btu/lbm. Also compare the performance of this tanker to that of the Alaskan Pipeline, which requires about 120 Btu of energy for each ton-mile of petroleum delivery.

Given: Data on a large tanker

Find: Cost effectiveness of tanker; compare to Alaska pipeline

Solution:

The given data is $L = 360 \cdot m$ $B = 70 \cdot m$ $D = 25 \cdot m$ $\rho = 1020 \cdot \frac{kg}{m^3}$ $U = 6.69 \cdot \frac{m}{s}$ $x = 2000 \cdot mi$

$P = 9.7 \cdot MW$ $P = 1.30 \times 10^4 hp$ (Power consumed by drag)

The power to the propeller is $P_{prop} = \frac{P}{70 \cdot \%}$ $P_{prop} = 1.86 \times 10^4 hp$

The shaft power is $P_s = 120\% \cdot P_{prop}$ $P_s = 2.23 \times 10^4 hp$

The efficiency of the engines is $\eta = 40 \cdot \%$

Hence the heat supplied to the engines is $Q = \frac{P_s}{\eta}$ $Q = 1.42 \times 10^8 \frac{BTU}{hr}$

The journey time is $t = \frac{x}{U}$ $t = 134 hr$

The total energy consumed is $Q_{total} = Q \cdot t$ $Q_{total} = 1.9 \times 10^{10} BTU$

From buoyancy the total ship weight equals the displaced seawater volume

$M_{ship} \cdot g = \rho \cdot g \cdot L \cdot B \cdot D$ $M_{ship} = \rho \cdot L \cdot B \cdot D$ $M_{ship} = 1.42 \times 10^9 lb$

Hence the mass of oil is $M_{oil} = 75\% \cdot M_{ship}$ $M_{oil} = 1.06 \times 10^9 lb$

The chemical energy stored in the petroleum is $q = 20000 \cdot \frac{BTU}{lb}$

The total chemical energy is $E = q \cdot M_{oil}$ $E = 2.13 \times 10^{13} BTU$

The equivalent percentage of petroleum cargo used is then $\frac{Q_{total}}{E} = 0.089\%$

The Alaska pipeline uses $e_{pipeline} = 120 \cdot \frac{BTU}{ton \cdot mi}$ but for the ship $e_{ship} = \frac{Q_{total}}{M_{oil} \cdot x}$ $e_{ship} = 17.8 \frac{BTU}{ton \cdot mi}$

The ship uses only about 15% of the energy of the pipeline!

Problem 9.70

[3]

9.70 Consider the linear, sinusoidal, and parabolic laminar boundary-layer approximations of Problem 9.10. Compare the momentum fluxes of these profiles. Which is most likely to separate first when encountering an adverse pressure gradient?

Given: Linear, sinusoidal and parabolic velocity profiles

Find: Momentum fluxes

Solution:

The momentum flux is given by $$mf = \int_0^\delta \rho \cdot u^2 \cdot w \, dy$$

where *w* is the width of the boundary layer

For a linear velocity profile $$\frac{u}{U} = \frac{y}{\delta} = \eta \qquad (1)$$

For a sinusoidal velocity profile $$\frac{u}{U} = \sin\left(\frac{\pi}{2} \cdot \frac{y}{\delta}\right) = \sin\left(\frac{\pi}{2} \cdot \eta\right) \qquad (2)$$

For a parabolic velocity profile $$\frac{u}{U} = 2 \cdot \left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 = 2 \cdot \eta - (\eta)^2 \qquad (3)$$

For each of these $u = U \cdot f(\eta)$ $\qquad y = \delta \cdot \eta$

Using these in the momentum flux equation $$mf = \rho \cdot U^2 \cdot \delta \cdot w \cdot \int_0^1 f(\eta)^2 \, d\eta \qquad (4)$$

For the linear profile Eqs. 1 and 4 give $$mf = \rho \cdot U^2 \cdot \delta \cdot w \cdot \int_0^1 \eta^2 \, d\eta \qquad mf = \frac{1}{3} \cdot \rho \cdot U^2 \cdot \delta \cdot w$$

For the sinusoidal profile Eqs. 2 and 4 give $$mf = \rho \cdot U^2 \cdot \delta \cdot w \cdot \int_0^1 \sin\left(\frac{\pi}{2} \cdot \eta\right)^2 d\eta \qquad mf = \frac{1}{2} \cdot \rho \cdot U^2 \cdot \delta \cdot w$$

For the parabolic profile Eqs. 3 and 4 give $$mf = \rho \cdot U^2 \cdot \delta \cdot w \cdot \int_0^1 \left[2 \cdot \eta - (\eta)^2\right]^2 d\eta \qquad mf = \frac{8}{15} \cdot \rho \cdot U^2 \cdot \delta \cdot w$$

The linear profile has the smallest momentum, so would be most likely to separate

Problem *9.71

*9.71 Table 9.1 shows the numerical results obtained from Blasius exact solution of the laminar boundary-layer equations. Plot the velocity distribution (note that from Eq. 9.13 we see that $\eta \approx 5.0\frac{y}{\delta}$). On the same graph, plot the turbulent velocity distribution given by the $\frac{1}{7}$-power expression of Eq. 9.24. Which is most likely to separate first when encountering an adverse pressure gradient? To justify your answer, compare the momentum fluxes of these profiles (the laminar data can be integrated using a numerical method such as Simpson's rule).

Given: Laminar (Blasius) and turbulent (1/7 - power) velocity distributions

Find: Plot of distributions; momentum fluxes

Solution:

The momentum flux is given by $mf = \int_0^{\delta} \rho \cdot u^2\, dy$ per unit width of the boundary layer

Using the substitutions $\frac{u}{U} = f(\eta)$ $\frac{y}{\delta} = \eta$

the momentum flux becomes $mf = \rho \cdot U^2 \cdot \delta \cdot \int_0^1 f(\eta)^2\, d\eta$

For the Blasius solution a numerical evaluation (a Simpson's rule) of the integral is needed

$$mf_{lam} = \rho \cdot U^2 \cdot \delta \cdot \frac{\Delta\eta}{3} \cdot \left(f\left(\eta_0\right)^2 + 4 \cdot f\left(\eta_1\right)^2 + 2 \cdot f\left(\eta_2\right)^2 + f\left(\eta_N\right)^2\right)$$

where $\Delta\eta$ is the step size and N the number of steps

The result for the Blasius profile is $mf_{lam} = 0.525 \cdot \rho \cdot U^2 \cdot \delta$

For a 1/7 power velocity profile $mf_{turb} = \rho \cdot U^2 \cdot \delta \cdot \int_0^1 \eta^{\frac{2}{7}}\, d\eta$ $mf_{turb} = \frac{7}{9} \cdot \rho \cdot U^2 \cdot \delta$

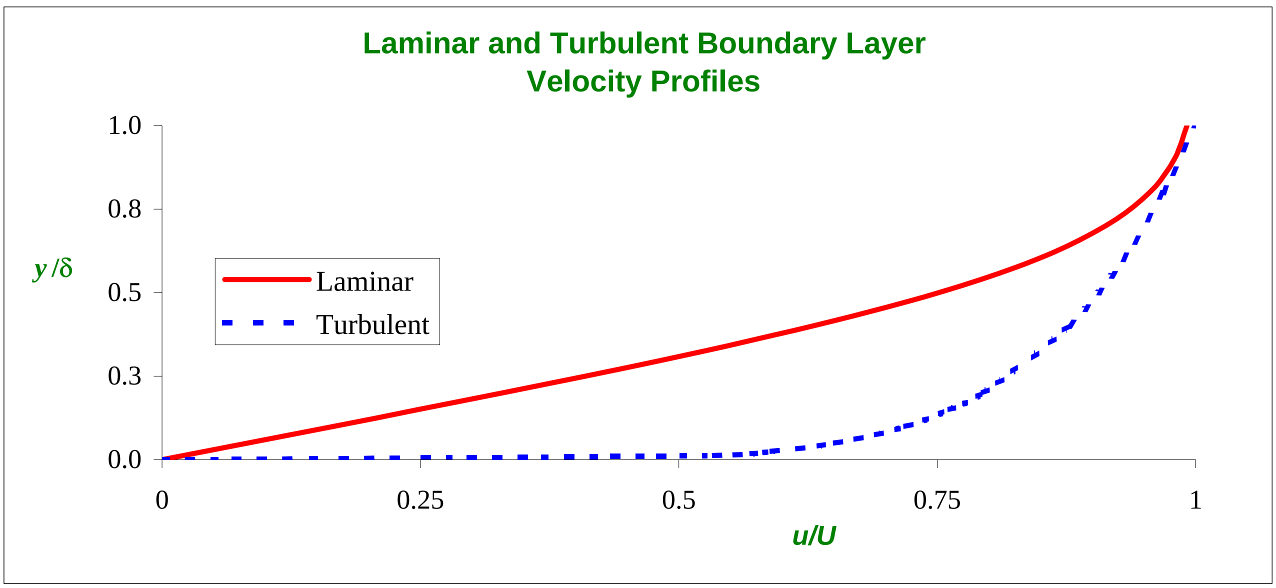
The laminar boundary has less momentum, so will separate first when encountering an adverse pressure gradient

Computed results:

η	Laminar u/U	Weight w	Weight x $(u/U)^2$
0.0	0.000	1	0.00
0.5	0.166	4	0.11
1.0	0.330	2	0.22
1.5	0.487	4	0.95
2.0	0.630	2	0.79
2.5	0.751	4	2.26
3.0	0.846	2	1.43
3.5	0.913	4	3.33
4.0	0.956	2	1.83
4.5	0.980	4	3.84
5.0	0.992	1	0.98
		Simpsons':	**0.525**

(Table 9.1) (Simpsons Rule)

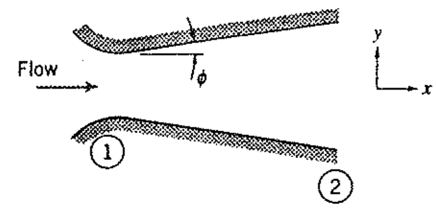
$y/\delta = \eta$	Turbulent u/U
0.0	0.00
0.0125	0.53
0.025	0.59
0.050	0.65
0.10	0.72
0.15	0.76
0.2	0.79
0.4	0.88
0.6	0.93
0.8	0.97
1.0	1.00



Problem 9.72

Given: Flow through plane-wall diffuser, as shown. We wish to compare the behavior of inviscid and viscous fluids

Find: (a) For an inviscid fluid, describe flow pattern and pressure distribution as ϕ is increased from $\phi = 0$
(b) Include viscous (boundary layer effects)
(c) Which fluid will have the highest exit pressure?



Solution:

For the inviscid fluid,

- with $\phi = 0$ (straight channel) there will be no change in velocity, and hence no pressure gradient.
- as ϕ is increased, the velocity decreases and hence the pressure increases (from the Bernoulli equation along the channel.

For the viscous fluid:

- with $\phi = 0$, boundary layers will form along the channel walls reducing the effective flow area. Thus to satisfy continuity for incompressible flow, the centerline velocity must increase and the pressure will drop along the channel.
- as ϕ is increased, the adverse pressure gradient increases. This causes an increased rate of boundary layer growth. If ϕ is too large, the flow will separate from one (or both) walls.

The inviscid fluid will have the highest exit pressure. (The pressure gradient with the real fluid is reduced by boundary layer development for all values of ϕ.)

Problem 9.73

[3] Part 1/2

Given: Laminar boundary layer with velocity profile

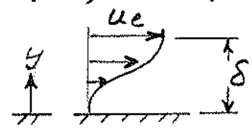
$$\frac{u}{u_e} = a + b\eta + c\eta^2 + d\eta^3 \quad ; \quad \eta = \frac{y}{\delta}$$

Separation occurs when τ_w becomes zero.

Find: (a) Four boundary conditions for this laminar velocity profile.
(b) Evaluate constants a, b, c, and d.
(c) Calculate shape parameter, H, at separation.

Plot: Profile and compare with parabolic.

Solution: The profile shape will be:



Boundary conditions are:

$$y = 0: \ u = 0 \ \text{and} \ \tau = \mu\frac{du}{dy} = 0 \ ; \ y = \delta: \ u = u_e \ \text{and} \ \tau = \mu\frac{du}{dy} = 0$$ BC

Applying boundary conditions:

$$y = 0: \eta = 0 \qquad \frac{u}{u_e} = 0 = (a + b\eta + c\eta^2 + d\eta^3)_{\eta=0} = a \qquad a = 0$$ a

$$\frac{du}{dy} = \frac{u_e}{\delta}\frac{d(u/u_e)}{d\eta} = 0 = (b + 2c\eta + 3c\eta^2)_{\eta=0} = b \qquad b = 0$$ b

$$y = \delta: \eta = 1 \qquad \frac{u}{u_e} = 1 = (c\eta^2 + d\eta^3)_{\eta=1} \ ; \quad c + d = 1$$

$$\frac{d(u/u_e)}{d\eta} = 0 = (2c\eta + 3d\eta^2)_{\eta=1} ; \quad 2c + 3d = 0$$

Solving

$$d = 1 - c \ ; \quad 2c + 3(1-c) = 2c + 3 - 3c = 3 - c = 0 \qquad c = 3$$ c

$$d = 1 - c = 1 - 3 = -2 \qquad d = -2$$ d

The velocity profile is $\frac{u}{u_e} = 3\eta^2 - 2\eta^3$. $H \equiv \delta^*/\theta = \frac{\delta^*}{\delta}\frac{\delta}{\theta}$, so

$$\frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{u}{u_e}\right)d\eta = \int_0^1 (1 - 3\eta^2 + 2\eta^3)\,d\eta = \left[\eta - \eta^3 + \frac{2}{4}\eta^4\right]_0^1 = \frac{1}{2}$$

$$\frac{\theta}{\delta} = \int_0^1 \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right)d\eta = \int_0^1 (3\eta^2 - 2\eta^3)(1 - 3\eta^2 + 2\eta^3)\,d\eta$$

$$= \int_0^1 (3\eta^2 - 2\eta^3 - 9\eta^4 + 12\eta^5 - 4\eta^6)\,d\eta$$

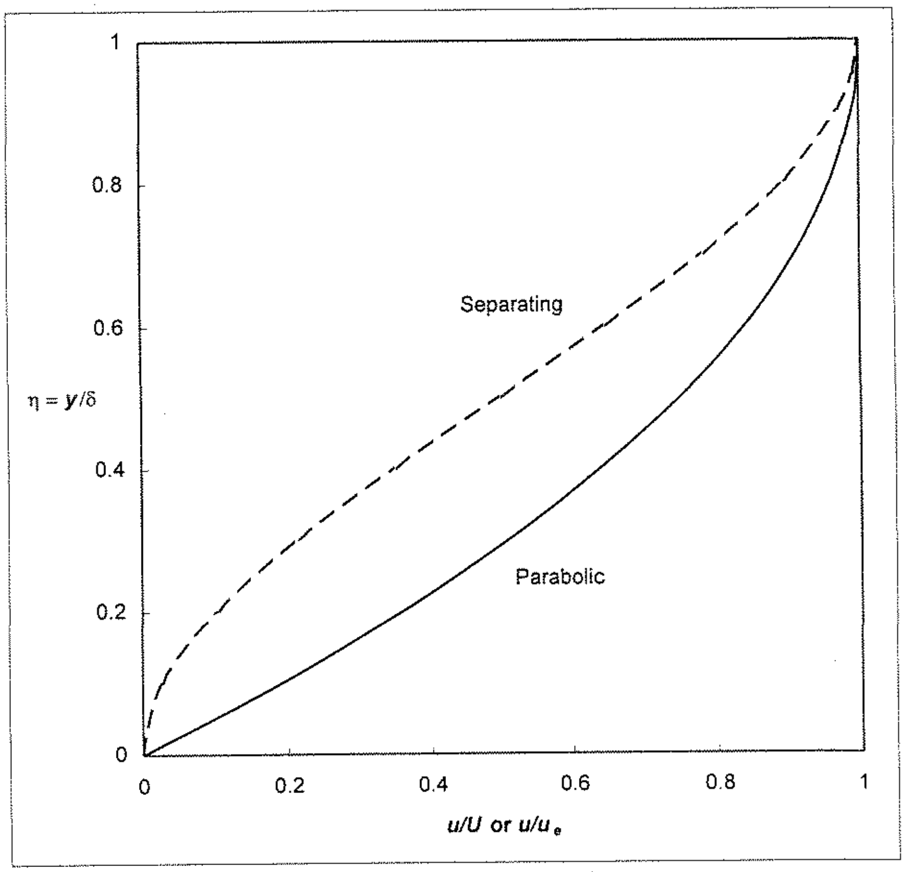
$$\frac{\theta}{\delta} = \left[\eta^3 - \frac{2}{4}\eta^4 - \frac{9}{5}\eta^5 + \frac{12}{6}\eta^6 - \frac{4}{7}\eta^7\right]_0^1 = \frac{9}{70}$$

Thus

$$H = \frac{1}{2} \times \frac{70}{9} = \frac{70}{18} = \frac{35}{9} = 3.89$$ H

Problem 9.73

Separating:	η (---)	$\frac{u}{u_e}$ (---)	Flat plate:	$\frac{u}{U}$ (---)
$\frac{u}{u_e} = 3\eta^2 - 2\eta^3$	1.0	1.00	$\frac{u}{U} = 2\eta - \eta^2$	1.00
	0.9	0.972		0.990
	0.8	0.896		0.960
	0.7	0.784		0.910
	0.6	0.648		0.840
	0.5	0.500		0.750
	0.4	0.352		0.640
	0.3	0.216		0.510
	0.2	0.104		0.360
	0.1	0.028		0.190



Problem 9.74

Open-Ended Problem Statement: For flow over a flat plate with zero pressure gradient, will the shear stress increase, decrease, or remain constant along the plate? Justify your answer. Does the momentum flux increase, decrease, or remain constant as the flow proceeds along the plate? Justify your answer. Compare the behavior of laminar flow and turbulent flow (both from the leading edge) over a flat plate. At a given distance from the leading edge, which flow will have the larger boundary-layer thickness? Does your answer depend on the distance along the plate? How would you justify your answer?

Discussion: Shear stress decreases along the plate because the freestream flow speed remains constant while the boundary-layer thickness increases.

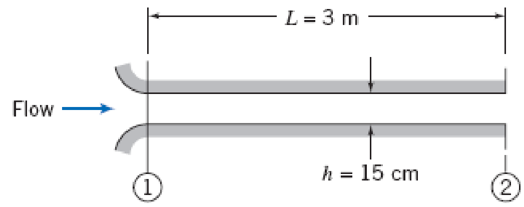
The momentum flux decreases as the flow proceeds along the plate. Momentum thickness θ (actually proportional to the defect in momentum within the boundary layer) increases, showing that momentum flux decreases. The force that must be applied to hold the plate stationary reduces the momentum flux of the stream and boundary layer.

The laminar boundary layer has less shear stress than the turbulent boundary layer. Therefore laminar boundary-layer flow from the leading edge produces a thinner boundary layer and less shear stress everywhere along the plate than a turbulent boundary layer from the leading edge.

Since both boundary layers continue to grow with increasing distance from the leading edge, and the turbulent boundary layer continues to grow more rapidly because of its higher shear stress, this comparison will be the same no matter the distance from the leading edge.

Problem 9.75

9.75 Cooling air is supplied through the wide, flat channel shown. For minimum noise and disturbance of the outlet flow, laminar boundary layers must be maintained on the channel walls. Estimate the maximum inlet flow speed at which the outlet flow will be laminar. Assuming parabolic velocity profiles in the laminar boundary layers, evaluate the pressure drop, $p_1 - p_2$. Express your answer in inches of water.



Given: Channel flow with laminar boundary layers

Find: Maximum inlet speed for laminar exit; Pressure drop for parabolic velocity in boundary layers

Solution:

Basic equations: $Re_{trans} = 5 \times 10^5$ $\quad \frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}}$ $\quad \frac{p}{\rho} + \frac{V^2}{2} + g \cdot z = const$

Assumptions: 1) Steady flow 2) Incompressible 3) z = constant

From Table A.10 at 20ºC $\quad \nu = 1.50 \times 10^{-5} \cdot \frac{m^2}{s}$ $\quad \rho = 1.21 \cdot \frac{kg}{m^3}$ $\quad L = 3 \cdot m$ $\quad h = 15 \cdot cm$

Then $\quad Re_{trans} = \frac{U_{max} \cdot L}{\nu}$ $\quad U_{max} = \frac{Re_{trans} \cdot \nu}{L}$ $\quad U_{max} = 2.50 \frac{m}{s}$ $\quad U_1 = U_{max}$ $\quad U_1 = 2.50 \frac{m}{s}$

For $\quad Re_{trans} = 5 \times 10^5$ $\quad \delta_2 = L \cdot \frac{5.48}{\sqrt{Re_{trans}}}$ $\quad \delta_2 = 0.0232\,m$

For a parabolic profile $\quad \frac{\delta_{disp}}{\delta} = \int_0^1 \left(1 - \frac{u}{U}\right) d\lambda = \int_0^1 \left(1 - 2 \cdot \lambda + \lambda^2\right) d\lambda = \frac{1}{3}$ $\quad$ where δ_{trans} is the displacement thickness

$\delta_{disp2} = \frac{1}{3} \cdot \delta_2$ $\quad \delta_{disp2} = 0.00775\,m$

From continuity $\quad U_1 \cdot w \cdot h = U_2 \cdot w \cdot \left(h - 2 \cdot \delta_{disp2}\right)$ $\quad U_2 = U_1 \cdot \frac{h}{h - 2 \cdot \delta_{disp2}}$ $\quad U_2 = 2.79 \frac{m}{s}$

Since the boundary layers do not meet Bernoulli applies in the core

$$\frac{p_1}{\rho} + \frac{U_1^2}{2} = \frac{p_2}{\rho} + \frac{U_2^2}{2} \qquad \Delta p = p_1 - p_2 = \frac{\rho}{2} \cdot \left(U_2^2 - U_1^2\right)$$

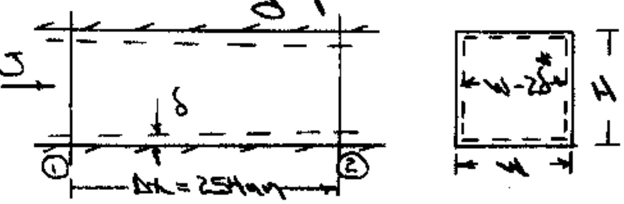
$$\Delta p = \frac{\rho}{2} \cdot \left(U_2^2 - U_1^2\right) \qquad \Delta p = 0.922\,Pa$$

From hydrostatics $\quad \Delta p = \rho_{H2O} \cdot g \cdot \Delta h$ $\quad$ with $\quad \rho_{H2O} = 1000 \cdot \frac{kg}{m^3}$

$$\Delta h = \frac{\Delta p}{\rho_{H2O} \cdot g} \qquad \Delta h = 0.0940\,mm \qquad \Delta h = 0.00370\,in$$

Given: Wind tunnel with square cross section, $W_1 = H_1 = 305$ mm. At inlet section ①, $U_1 = 24.5$ m/s, $\delta_1 = 9.75$ mm with a "1/7-power" turbulent velocity profile

$$\left(\frac{dp}{dx}\right)_1 = -0.035 \frac{\text{mm}\,H_2O}{\text{mm}}$$



Find: (a) reduction in flow area at ① caused by boundary layer
(b) $d\theta/dx$ at ①
(c) estimate of θ_2

Solution: Apply continuity and the momentum integral eqs.

Computing eqs.: $\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2} - (H+2)\frac{\theta}{U}\frac{dU}{dx}$; $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7} = \eta^{1/7}$

$$H = \frac{\delta^*}{\theta}, \quad \frac{\tau_w}{\rho U^2} = 0.0233\left(\frac{\nu}{U\delta}\right)^{1/4}$$

Assumptions: (1) steady flow
(2) uniform flow outside boundary layer.
(3) standard air

$$\%\text{ reduction in flow area} = \frac{A_{eff} - A}{A} = \frac{(W-2\delta^*)(H-2\delta^*) - WH}{WH}$$

$$\frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \int_0^1 \left(1 - \eta^{1/7}\right) d\eta = \left[\eta - \frac{7}{8}\eta^{8/7}\right]_0^1 = \frac{1}{8}$$

$$\therefore \delta_1^* = \frac{1}{8}\delta_1 = \frac{1}{8} \times 9.75\,\text{mm} = 1.22\,\text{mm}$$

$$\therefore \text{area reduction} = \left(1 - \frac{2\delta^*}{W}\right)\left(1 - \frac{2\delta^*}{H}\right) - 1 = \left[1 - \frac{2\times1.22}{305}\right]\left[1 - \frac{2\times1.22}{305}\right] - 1$$

area reduction $= -0.0159$ (-1.59%) ← %

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2} - (H+2)\frac{\theta}{U}\frac{dU}{dx}$$

$$\frac{\tau_w}{\rho U^2} = 0.0233\left(\frac{\nu}{U\delta}\right)^{1/4} = 0.0233\left[1.45\times10^{-5}\frac{m^2}{s} \times \frac{s}{24.5\,m} \times \frac{1}{9.75\times10^{-3}\,m}\right]^{0.25}$$

$\frac{\tau_w}{\rho U^2} = 0.00206$ ← – –

$$\frac{\theta}{\delta} = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right) d\left(\frac{y}{\delta}\right) = \int_0^1 \eta^{1/7}\left(1 - \eta^{1/7}\right) d\eta = \left[\frac{7}{8}\eta^{8/7} - \frac{7}{9}\eta^{9/7}\right]_0^1 = \frac{7}{72}$$

$\theta = \frac{7}{72}\delta = \frac{7}{72} \times 9.75\,\text{mm} = 0.948$ mm ← – –

$H = \frac{\delta^*}{\theta} = \frac{\frac{1}{8}\delta}{7/72\,\delta} = \frac{9}{7} = 1.29$ – – –

Outside the boundary layer, $p + \frac{1}{2}\rho U^2 = \text{constant}$

Then, $\frac{dp}{dx} = -\rho U \frac{dU}{dx}$ and $\frac{1}{U}\frac{dU}{dx} = -\frac{1}{\rho U^2}\frac{dp}{dx} = -\frac{1}{\rho U^2}\frac{d\,\rho_w g h}{dx}$

$$\frac{1}{U}\frac{dU}{dx} = -\frac{m^3}{1.23\,kg} \times \frac{1}{(24.5)^2}\frac{s^2}{m^2} \times 999\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times (-0.035)\frac{mm}{mm}$$

$$\frac{1}{U}\frac{dU}{dx} = 0.465\ m^{-1}$$

Substituting

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2} - (H+2)\frac{\theta}{U}\frac{dU}{dx}$$

$$= 0.00206 - (1.29+2)\ 0.948\,mm \times \frac{0.465}{m} \times \frac{m}{10^3\,mm}$$

$$\frac{d\theta}{dx} = 0.00206 - 0.00145 = 0.00061\ mm/mm$$

$$d\theta/dx = 0.61\ mm/m \qquad d\theta/dx$$

$$\theta_2 \approx \theta_1 + \left.\frac{d\theta}{dx}\right)_1 \Delta x$$

$$= 0.948 + 0.61\frac{mm}{m} \times 0.254\,m$$

$$\theta_2 \approx 1.10\ mm \qquad \theta_2$$

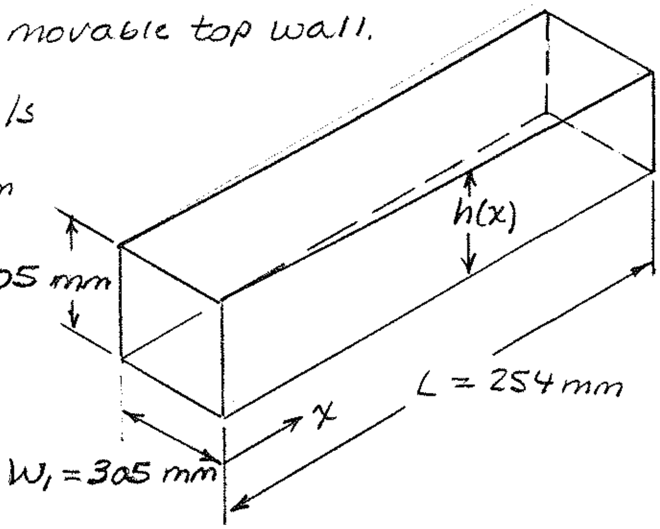
Given: Wind tunnel with movable top wall.

At inlet: $U_1 = 24.5$ m/s

$\delta_1 = 9.75$ mm

Assume 1/7-power turbulent BL development,

δ = constant

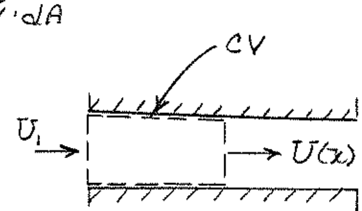


Find: (a) Velocity distribution to make δ = const, (b) $h(x)$ from 0 to L.

Solution: Calculate required pressure gradient ($dp/dx = -\rho U \, dU/dx$) from the momentum integral equation, then integrate to find $U(x)$, use continuity to solve for $h(x)$:

Computing equations:
$$\frac{\tau_w}{\rho U^2} = \frac{d\theta}{dx} + (H+2)\frac{\theta}{U}\frac{dU}{dx} \qquad (9.27)$$
(with $d\theta/dx = 0(4)$)

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$$
(with $\frac{\partial}{\partial t} = 0(1)$)



Assumptions: (1) Steady flow
(2) Incompressible flow
(3) Uniform flow outside BL
(4) Constant BL thickness:

$$\delta^* = \frac{1}{8}\delta \text{ and } \theta = \frac{7}{72}\delta; \; H = \frac{\delta^*}{\theta} = \frac{72}{(7)8} = 1.29 \text{ for 1/7-power profile}$$

Then from the momentum integral equation

$$\frac{\tau_w}{\rho U^2} = (H+2)\frac{\theta}{U}\frac{dU}{dx}$$

To integrate, we must make an assumption about τ_w.

Case 1: Assume τ_w = constant, and rearrange

Then $U\,dU = \dfrac{\tau_w}{\rho\theta(H+2)}\,dx$

Integrating, $\frac{1}{2}U^2\Big]_1^2 = \dfrac{\tau_w}{\rho\theta(H+2)}x$ or $\frac{1}{2}U_2^2 - \frac{1}{2}U_1^2 = \dfrac{\tau_w}{\rho\theta(H+2)}x$

so $$\frac{U}{U_1} = \left[1 + \frac{2\tau_w}{\rho U_1^2}\frac{x}{\theta(H+2)}\right]^{\frac{1}{2}} = \left[1 + \frac{C_f}{\theta(H+2)}x\right]^{\frac{1}{2}} \longleftarrow U(x)$$

Case 2: Assume $\tau_w \neq$ constant

Then $\tau_w = 0.0233 \rho U^2 \left(\frac{\nu}{U\delta}\right)^{1/4}$ (Eq. 9.25)

Substituting and rearranging

$$\frac{\tau_w}{\rho U^2} = 0.0233 \left(\frac{\nu}{\delta}\right)^{1/4} \frac{1}{U^{1/4}} = (H+2)\frac{\theta}{U}\frac{dU}{dx}$$

or $$\frac{dU}{U^{0.75}} = 0.0233 \left(\frac{\nu}{\delta}\right)^{1/4} \frac{dx}{(H+2)\theta}$$

Integrating, $$4U^{1/4}\Big]_1^2 = 0.0233 \left(\frac{\nu}{\delta}\right)^{1/4} \frac{x}{(H+2)\theta}$$

or $$\frac{U}{U_1} = \left[1 + 0.00583 \left(\frac{\nu}{U_1\delta}\right)^{\frac{1}{4}} \frac{x}{(H+2)\theta}\right]^4$$ $U(x)$

From continuity $U_1 A_1 = UA = U_1(W_1 - 2\delta_1^*)(H_1 - 2\delta_1^*) = U(W_1 - 2\delta_1^*)(h - 2\delta_1^*)$

Thus $A/A_1 = (U/U_1)^{-1}$ and $\frac{h - 2\delta_1^*}{H - 2\delta_1^*} = \frac{U_1}{U} = \frac{h/H - 2\delta_1^*/H}{1 - 2\delta_1^*/H}$

$$\frac{h}{W} = \left(1 - 2\frac{\delta_1^*}{h_1}\right)\frac{U_1}{U} + \frac{2\delta_1^*}{h_1}$$ $h(x)$

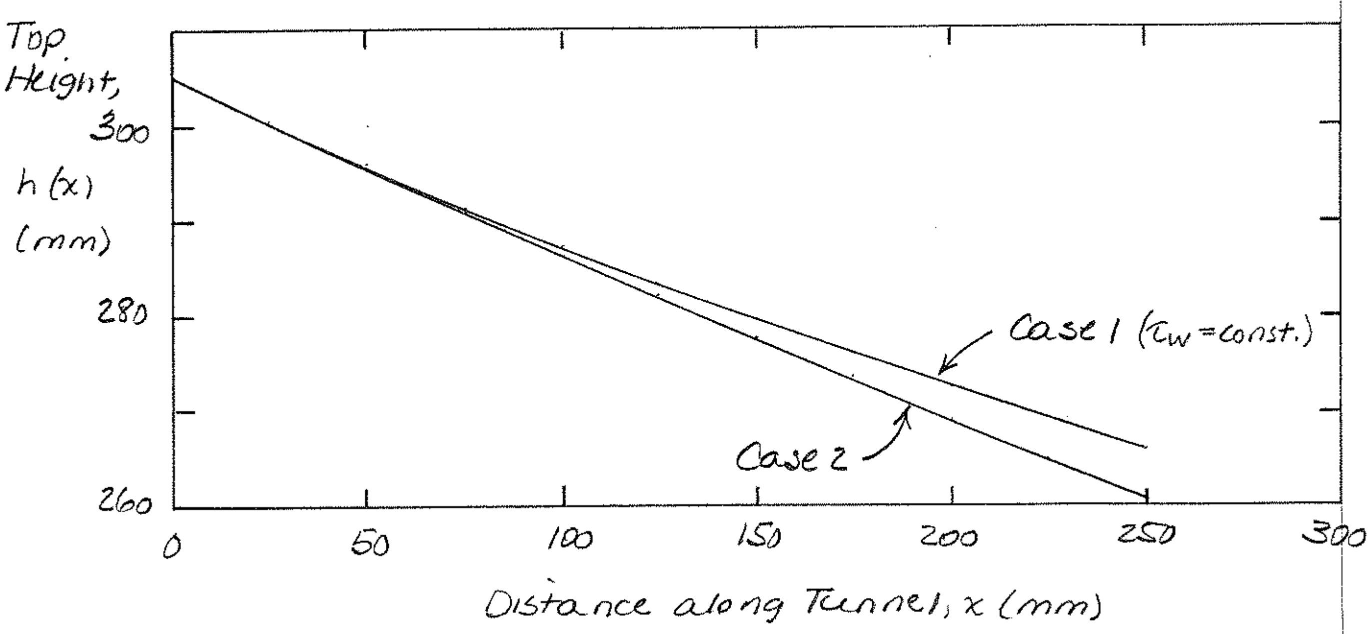
Evaluating: $\delta_1 = 9.75$ mm, $\delta_1^* = \frac{1}{8}\delta_1 = 1.22$ mm, $\theta_1 = \frac{7}{72}\delta_1 = 0.948$ mm

$$\frac{1}{2}\rho U_1^2 = \frac{1}{2} \times 1.23 \frac{kg}{m^3} \times (24.5)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 369\ N/m^2$$

$$Re_{\delta_1} = \frac{U_1 \delta_1}{\nu} = 24.5 \frac{m}{s} \times 0.00975\ m \times \frac{s}{1.46 \times 10^{-5} m^2} = 1.64 \times 10^4$$

$$C_f = 0.0466\ Re_{\delta_1}^{-1/4} = 0.0466(0.0883) = 0.00411$$

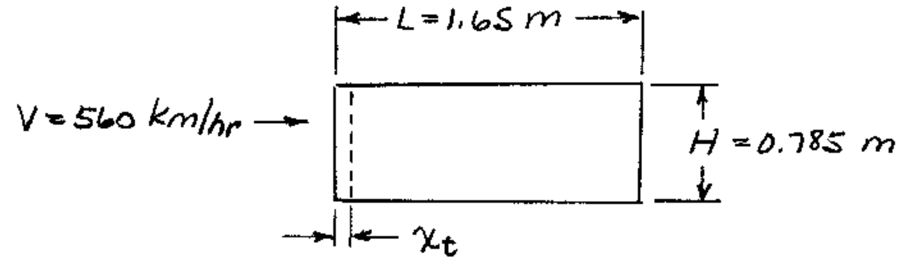
Plot results:



Problem 9.78

Given: Stabilizing fin on Bonneville land speed record auto.

$z = 1{,}340$ m



Find: (a) Evaluate Re_L

(b) Location of x_t

(c) Power to overcome skin friction drag on fin.

<u>Solution</u>: Assume standard atmosphere, so $T = 279$ K, $\rho/\rho_0 = 0.877$ (Table A.3); $\mu = 1.79 \times 10^{-5}$ kg/m·s (Table A.7). Then

$$Re_L = \frac{\rho V L}{\mu} = (0.877)\,1.23\,\frac{kg}{m^3} \times 560\times10^3\,\frac{m}{hr} \times 1.65\,m \times \frac{m\cdot s}{1.79\times10^{-5}\,kg} \times \frac{hr}{3600\,s}$$

$$Re_L = 1.55 \times 10^7$$ Re_L

Assume transition occurs at $Re_x = 500{,}000$. Then

$$\frac{x_t}{L} = \frac{Re_{x_t}}{Re_L} = \frac{500{,}000}{1.55\times10^7} = 0.0323$$

$$x_t = 0.0323\,L = 0.0323 \times 1.65\,m = 0.0532\,m = 53.2\,mm$$ x_t

Calculate drag force using C_D from Fig. 9.8: $F_D = C_D A \frac{1}{2}\rho V^2$

$$C_D = 0.0029 \text{ (Fig. 9.8)}; \quad A = 2LH = 2 \times 1.65\,m \times 0.785\,m = 2.59\,m^2 \text{ (2 sides)}$$

$$\frac{1}{2}\rho V^2 = \frac{1}{2} \times (0.877)\,1.23\,\frac{kg}{m^3}\left(560\times10^3\,\frac{m}{hr} \times \frac{hr}{3600\,s}\right)^2 \frac{N\cdot s^2}{kg\cdot m} = 1.31\times10^4\,N/m^2$$

$$F_D = 0.0029 \times 2.59\,m^2 \times 1.31\times10^4\,\frac{N}{m^2} = 98.4\,N \text{ (skin friction drag on fin)}$$

The power required is

$$P = F_D V = 98.4\,N \times 560\times10^3\,\frac{m}{hr} \times \frac{hr}{3600\,s} \times \frac{W\cdot s}{N\cdot m} = 15.3\,kW$$ P

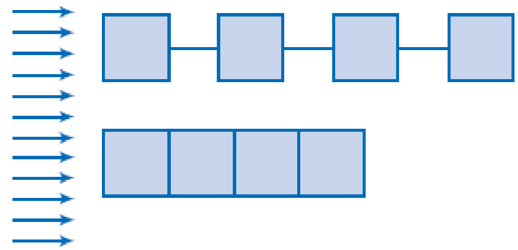
Check using Eq. 9.37b:

$$C_D = \frac{0.455}{(\log Re_L)^{2.58}} - \frac{1610}{Re_L} = 0.00270$$

This is slightly less than from the graph, but reasonable agreement.

Problem 9.79

9.79 Repeat Problem 9.46, except that the water flow is now at 10 m/s (use formulas for C_D from Section 9-7).



Given: Pattern of flat plates

Find: Drag on separate and composite plates

Solution:

Basic equations: $$C_D = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$$

For separate plates $L = 7.5\cdot cm$ $\quad W = 7.5\cdot cm$ $\quad A = W\cdot L$ $\quad A = 5.625\times 10^{-3}\,m^2$ $\quad V = 10\cdot\frac{m}{s}$

From Table A.8 at 20ºC $\nu = 1.01\times 10^{-6}\cdot\frac{m^2}{s}$ $\quad \rho = 998\cdot\frac{kg}{m^3}$

First determine the Reynolds number $Re_L = \frac{V\cdot L}{\nu}$ $\quad Re_L = 7.43\times 10^5$ so use Eq. 9.34

$$C_D = \frac{0.0742}{Re_L^{\frac{1}{5}}} \qquad C_D = 0.00497$$

The drag (one side) is then $F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot V^2\cdot A$ $\quad F_D = 1.39\,N$

This is the drag on one plate. The total drag is then $F_{Total} = 4\cdot F_D$ $\quad F_{Total} = 5.58\,N$

For both sides: $2\cdot F_{Total} = 11.2\,N$

For the composite plate $L = 4\times 7.5\cdot cm$ $\quad L = 0.300\,m$ $\quad A = W\cdot L$ $\quad A = 0.0225\,m^2$

First determine the Reylolds number $Re_L = \frac{V\cdot L}{\nu}$ $\quad Re_L = 2.97\times 10^6$ so use Eq. 9.34

$$C_D = \frac{0.0742}{Re_L^{\frac{1}{5}}} \qquad C_D = 0.00377$$

The drag (one side) is then $F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot V^2\cdot A$ $\quad F_D = 4.23\,N$ For both sides: $2\cdot F_D = 8.46\,N$

The drag is much lower on the composite compared to the separate plates. This is because τ_w is largest near the leading edges and falls off rapidly; in this problem the separate plates experience leading edges four times!

Problem 9.80

Given: Towboat model at 1:13.5 scale to be tested in towing tank.
Dimensions are:

Length, 11.1 ft; Beam, 3.11 ft; Draft, 0.62 ft

Model displacement in fresh water is 1200 lbf.

Find: (a) Estimate average length of wetted surface on hull.
(b) Skin friction drag force on prototype at V = 8 mph.

Solution: Represent the towboat as a rectangular solid of length, $\bar{L}$, with the displacement of the boat. From buoyancy,

$$W = \rho g \forall = \rho g \bar{L} B D \quad \text{or} \quad \bar{L} = \frac{W}{\rho g B D}$$

$$\bar{L} = 1200\,\text{lbf} \times \frac{\text{ft}^3}{1.94\,\text{slug}} \times \frac{\text{sec}^2}{32.2\,\text{ft}} \times \frac{1}{3.11\,\text{ft}} \times \frac{1}{0.62\,\text{ft}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{sec}^2} = 9.96\,\text{ft}$$ $\bar{L}$

For the prototype,

$$\bar{L}_p = 13.5\,\bar{L}_m = 13.5 \times 9.96\,\text{ft} = 134\,\text{ft}$$

The Reynolds number is (Table A.7)

$$Re_L = \frac{VL}{\nu} = 8\,\frac{\text{mi}}{\text{hr}} \times 134\,\text{ft} \times \frac{\text{s}}{1.08\times10^{-5}\,\text{ft}^2} \times 5280\,\frac{\text{ft}}{\text{mi}} \times \frac{\text{hr}}{3600\,\text{s}} = 1.46\times10^{8} \qquad (T = 68^\circ\text{F})$$

Thus flow is predominantly turbulent. Apply turbulent flow analysis.

Computing equations: $F_D = C_D A \frac{1}{2}\rho U^2 \qquad C_D = \dfrac{0.455}{(\log_{10} Re_L)^{2.58}} - \dfrac{1610}{Re_L} \qquad (9.37b)$

The wetted area is $A = \bar{L}(B + 2D)$.

$$C_D = \frac{0.455}{(\log_{10} 1.46\times10^{8})^{2.58}} - \frac{1610}{1.46\times10^{8}} = 0.00214$$

Substituting,

$$F_D = 0.00214 \times 134\,\text{ft} \times 13.5(3.11 + 2\times0.62)\,\text{ft} \times \frac{1}{2} \times 1.94\,\frac{\text{slug}}{\text{ft}^3}\left(8\,\frac{\text{mi}}{\text{hr}} \times 5280\,\frac{\text{ft}}{\text{mi}} \times \frac{\text{hr}}{3600\,\text{s}}\right)^2 \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$$

$F_D = 2250\,\text{lbf}$ (skin friction only) F_D

Problem 9.81

9.81 A jet transport aircraft cruises at 40,000 ft altitude in steady level flight at 500 mph. Model the aircraft fuselage as a circular cylinder with diameter $D = 12$ ft and length $L = 125$ ft. Neglecting compressibility effects, estimate the skin friction drag force on the fuselage. Evaluate the power needed to overcome this force.

Given: Aircraft cruising at 40,000 ft

Find: Skin friction drag force; Power required

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$

We "unwrap" the cylinder to obtain an equivalent flat plate

$L = 125\cdot ft$ $\quad D = 12\cdot ft$ $\quad A = L\cdot\pi\cdot D$ $\quad A = 4712\cdot ft^2$ $\quad V = 500\cdot mph$

From Table A.3, with $z = 40000\cdot ft$ $\quad z = 12192\,m$

For $z = 12000\cdot m$ $\quad \dfrac{\rho}{\rho_{SL}} = 0.2546$ $\quad$ with $\quad \rho_{SL} = 0.002377\cdot\dfrac{slug}{ft^3}$

$z = 13000\cdot m$ $\quad \dfrac{\rho}{\rho_{SL}} = 0.2176$

Hence at $z = 12192\,m$ $\quad \dfrac{\rho}{\rho_{SL}} = 0.2546 + \dfrac{(0.2176 - 0.2546)}{(1300 - 12000)}\cdot(12192 - 12000) = 0.255$

$\rho = 0.255\cdot\rho_{SL}$ $\quad \rho = 0.000606\cdot\dfrac{slug}{ft^3}$ $\quad$ and also $\quad T = 216.7\cdot K$

From Appendix A-3 $\quad \mu = \dfrac{b\cdot T^{\frac{1}{2}}}{1 + \frac{S}{T}}$ $\quad$ with $\quad b = 1.458\times10^{-6}\cdot\dfrac{kg}{m\cdot s\cdot K^{\frac{1}{2}}}$ $\quad S = 110.4\cdot K$

Hence $\quad \mu = \dfrac{b\cdot T^{\frac{1}{2}}}{1 + \frac{S}{T}}$ $\quad \mu = 1.42\times10^{-5}\cdot\dfrac{N\cdot s}{m^2}$ $\quad \mu = 2.97\times10^{-7}\cdot\dfrac{lbf\cdot s}{ft^2}$

Next we need the Reynolds number $\quad Re_L = \dfrac{\rho\cdot V\cdot L}{\mu}$ $\quad Re_L = 1.87\times10^{8}$ $\quad$ so use Eq. 9.35

$C_D = \dfrac{0.455}{\log(Re_L)^{2.58}}$ $\quad C_D = 0.00195$

The drag is then $\quad F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot V^2\cdot A$ $\quad F_D = 1500\cdot lbf$

The power consumed is $\quad P = F_D\cdot V$ $\quad P = 1.100\times10^{6}\cdot\dfrac{ft\cdot lbf}{s}$ $\quad P = 1999\cdot hp$

Problem 9.82

Given: Model barge tested at 1:13.5 scale; prototype speed, $V_p = 8\,mph$.

Dimensions are: Length, $L = 22\,ft$
Beam, $B = 4.0\,ft$
Draft, $D = 0.667\,ft$

Find: (a) Model test speed, V_m
(b) Boundary layers laminar or turbulent on prototype?
(c) Where position BL trips on model?
(d) Estimate skin friction drag for model and prototype.

Solution: Test should be run so $Fr_m = Fr_p = V_m/\sqrt{gL_m} = V_p/\sqrt{gL_p}$

Thus $V_m = V_p\sqrt{L_m/L_p} = 8\,mph\sqrt{1/13.5} = 2.18\,mph$ ← V_m

$$Re_p = \frac{V_p L_p}{\nu} = (8)(1.47)\frac{ft}{s} \times (22)(13.5)\,ft \times \frac{s}{1.08\times10^{-5}\,ft^2} = 3.24\times10^8 \quad (T = 68°F,\ \text{Table A.7})$$

Therefore boundary-layer flow is <u>turbulent</u>. Transition would occur at $Re_{x_t} \approx 5\times10^5$, so

$$x_t/L = \frac{5\times10^5}{3.24\times10^8} = 0.00154;\quad x_t = 0.00154\,L_m = 0.0339\,ft \text{ from front}$$ ←

The wetted area is $A = L(B+2D)$. Assume for turbulent BL flow

$$C_D = 1.25\,C_f = 1.25 \times \frac{0.0594}{(Re_L)^{1/5}} = \frac{0.0743}{(Re_L)^{1/5}}$$

For the model, $L = 22\,ft$, and

$$Re_m = \frac{V_m L_m}{\nu} = (2.18)(1.47)\frac{ft}{sec} \times 22\,ft \times \frac{s}{1.08\times10^{-5}\,ft^2} = 6.53\times10^6$$

$$C_{D_m} = \frac{0.0743}{(6.53\times10^6)^{1/5}} = 0.00322$$

For the model

$$F_{D_m} = C_{D_m}\tfrac{1}{2}\rho V_m^2 A_m = \frac{0.00322}{2} \times 1.94\frac{slug}{ft^3} \times [2.18(1.47)]^2\frac{ft^2}{s^2} \times [22(4+2(0.667))]\,ft^2 \times \frac{lbf\cdot s^2}{slug\cdot ft}$$

$F_{D_m} = 3.77\,lbf$ ← F_{D_m}

For the prototype,

$$Re_p = 3.24\times10^8,\quad C_{D_p} = 0.00148,\quad A_p = (13.5)^2 A_m = 21{,}400\,ft^2$$

$$F_{D_p} = C_{D_p}\tfrac{1}{2}\rho V_p^2 A_p = \frac{0.00148}{2} \times 1.94\frac{slug}{ft^3}\,[(8)(1.47)]^2\frac{ft^2}{s^2} \times 21{,}400\,ft^2 \times \frac{lbf\cdot s^2}{slug\cdot ft} = 4{,}250\,lbf$$ ← F_{D_p}

Problem 9.83

9.83 A flat-bottomed barge, 80 ft long and 35 ft wide, submerged to a depth of 5 ft, is to be pushed up a river (the river water is at 60°F). Estimate and plot the power required to overcome skin friction for speeds ranging up to 15 mph.

Given: Barge pushed upriver

Find: Power required to overcome friction; Plot power versus speed

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot U^2\cdot A}$ (9.32) $\quad C_D = \dfrac{0.455}{\left(\log\left(Re_L\right)\right)^{2.58}} - \dfrac{1610}{Re_L}$ (9.37b) $\quad Re_L = \dfrac{U\cdot L}{\nu}$

From Eq. 9.32 $\quad F_D = C_D\cdot A\cdot\frac{1}{2}\cdot\rho\cdot U^2$ and $\quad A = L\cdot(B + 2\cdot D)$

The power consumed is $\quad P = F_D\cdot U \quad P = C_D\cdot A\cdot\frac{1}{2}\cdot\rho\cdot U^3$

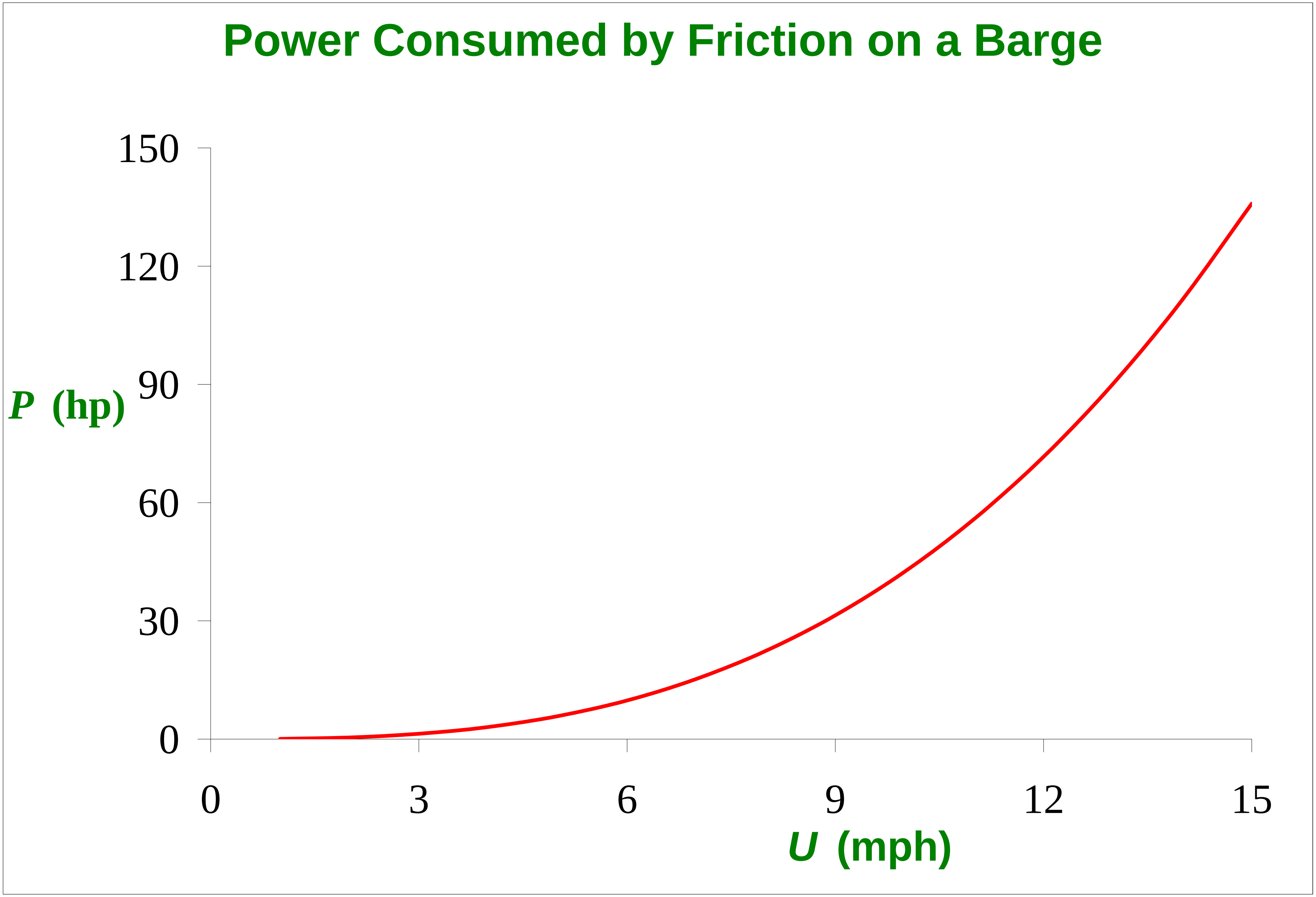
Given data:

L =	80	ft	
B =	35	ft	
D =	5	ft	
ν =	1.21E-05	ft^2/s	(Table A.7)
ρ =	1.94	$slug/ft^3$	(Table A.7)

Computed results:

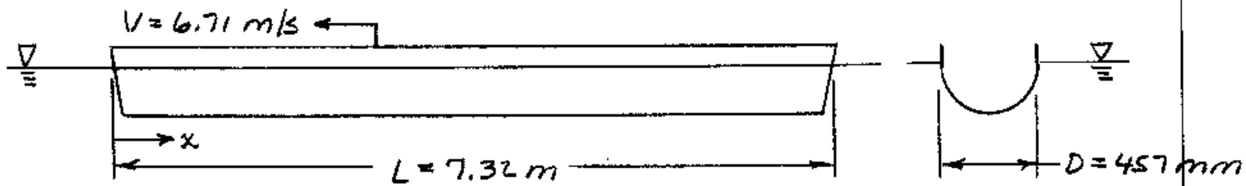
A = 3600 ft^2

U **(mph)**	***Re*** $_L$	***C*** $_D$	***P*** **(hp)**
1	9.70E+06	0.00285	0.0571
2	1.94E+07	0.00262	0.421
3	2.91E+07	0.00249	1.35
4	3.88E+07	0.00240	3.1
5	4.85E+07	0.00233	5.8
6	5.82E+07	0.00227	9.8
7	6.79E+07	0.00222	15
8	7.76E+07	0.00219	22
9	8.73E+07	0.00215	31
10	9.70E+07	0.00212	42
11	1.07E+08	0.00209	56
12	1.16E+08	0.00207	72
13	1.26E+08	0.00205	90
14	1.36E+08	0.00203	111
15	1.45E+08	0.00201	136


Power Consumed by Friction on a Barge
P (hp)
150
120
90
60
30
0
0
3
6
9
12
15
U (mph)

Problem 9.84

Given: Racing shell of Purdue crew, approximated as half a cylinder.



Find: (a) Location of transition in boundary layers on hull.
(b) Thickness of TBL at rear of hull.
(c) Total skin friction drag on hull.

Solution: Assume flow behaves as on a flat plate, with $Re_{x,t} = 500{,}000$.

$$Re_{x,t} = \frac{Vx_t}{\nu} = 500{,}000\ ;\ x_t = \frac{500{,}000\,\nu}{V} = 5\times10^5 \times 1\times10^{-6}\frac{m^2}{s}\times\frac{s}{6.71\ m} = 0.0745\ m \qquad x_t$$

($\nu = 1\times10^{-6}\ m^2/s$ for water at 20°C, Table A.8). Thus LBL is only 1% of L.

For the turbulent boundary layer $\frac{\delta}{x} = \frac{0.382}{Re_x^{1/5}}$, so $\delta = \frac{0.382}{Re_L^{1/5}}L$

$$Re_L = \frac{VL}{\nu} = 6.71\ \frac{m}{s}\times 7.32\ m\times\frac{s}{1.00\times10^{-6}\ m^2} = 4.91\times10^7$$

$$\delta = 0.382\times\frac{1}{(4.91\times10^7)^{1/5}}\times 7.32\ m = 0.0810\ m \qquad \delta$$

The drag force is $F_D = C_D A \frac{1}{2}\rho V^2$.

$$A \approx wL = \frac{\pi D}{2}L = \frac{\pi}{2}\times 0.457\ m\times 7.32\ m = 5.25\ m^2$$

Since $10^7 \le Re_L < 10^9$, then $C_D = \frac{0.455}{(\log Re_L)^{2.58}} = 0.00237$

Then

$$F_D = 0.00237\times 5.25\ m^2\times\frac{1}{2}\times 999\ \frac{kg}{m^3}\times(6.71)^2\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m} = 280\ N \qquad F_D$$

{ Note the rowers must produce an average power of

$$\mathcal{P} = F_D V = 280\ N\times 6.71\ \frac{m}{s}\times\frac{W\cdot s}{N\cdot m} = 1.88\ kW$$

to move the shell at this speed. }

Problem 9.85

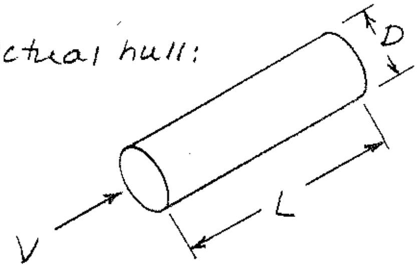
Given: Nuclear submarine, cruising submerged at $V = 27$ kt.

Assume hull is a circular cylinder, $D = 11.0$ m, and $L = 107$ m.

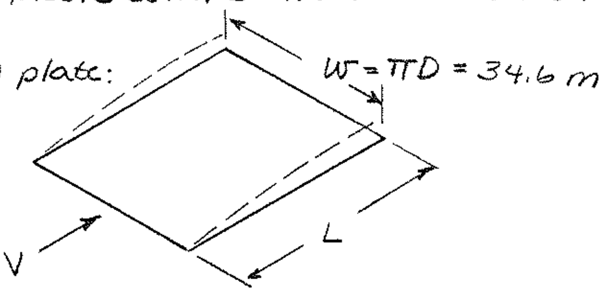
Find: (a) Estimate percentage of hull length with laminar BL.
(b) Calculate drag due to skin friction.
(c) Estimate power consumed

Solution: Treat hull as a flat plate with same wetted area.

Actual hull:



Model plate:



Computing equations: $Re_{x_t} = 500{,}000 \qquad C_D = \dfrac{0.455}{(\log_{10} Re_L)^{2.58}}$

For seawater, $\nu = 1.05 \times 10^{-6}$ m²/sec (Table A.2), so $(T = 20°C)$

$$Re_L = \frac{VL}{\nu} = 27\,\frac{nm}{hr} \times 6076\,\frac{ft}{nm} \times 0.305\,\frac{m}{ft} \times \frac{hr}{3600\,s} \times 107\,m \times \frac{s}{1.05 \times 10^{-6}\,m^2} = 1.42 \times 10^9$$

Thus $\dfrac{x_t}{L} = \dfrac{Re_{x_t}}{Re_L} = \dfrac{500{,}000}{1.42 \times 10^9} = 3.52 \times 10^{-4}$ or $x_t = 0.0352\%$ of L %L

Neglect laminar BL; assume flow is completely turbulent.

$$C_D = \frac{0.455}{(\log_{10} Re_L)^{2.58}} = \frac{0.455}{(9.15)^{2.58}} = 0.00150 \;;\; A = wL = 34.6\,m \times 107\,m = 3.70 \times 10^3\,m^2$$

$$q = \frac{1}{2}\rho V^2 = \frac{1}{2} \times 1025\,\frac{kg}{m^3}\left(\frac{27(6076)(0.305)}{3600}\right)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 99.0\,kPa$$

$$F_D = C_D\, q\, A = 0.00150 \times 99.0 \times 10^3\,\frac{N}{m^2} \times 3.70 \times 10^3\,m^2 = 5.49 \times 10^5\,N$$ F_D

$$\mathcal{P} = F_D V = 5.49 \times 10^5\,N \times 27\,\frac{nm}{hr} \times 6076\,\frac{ft}{nm} \times 0.305\,\frac{m}{ft} \times \frac{hr}{3600\,s} = 7.63\,MW$$

Problem 9.86

9.86 A sheet of plastic material 10 mm thick, with specific gravity SG = 1.5, is dropped into a large tank containing water. The sheet is 0.5 m by 1 m. Estimate the terminal speed of the sheet as it falls with (a) the short side vertical and (b) the long side vertical. Assume that the drag is due only to skin friction, and that the boundary layers are turbulent from the leading edge.

Given: Plastic sheet falling in water

Find: Terminal speed both ways

Solution:

Basic equations: $\Sigma F_y = 0$ for terminal speed $\quad C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A} \quad C_D = \dfrac{0.0742}{Re_L^{\frac{1}{5}}}$ (9.34) (assuming $5 \times 10^5 < Re_L < 10^7$)

$h = 10\cdot mm \quad W = 1\cdot m \quad L = 0.5\cdot m \quad A = W\cdot L \quad SG = 1.5$

From Table A.8 at 20°C $\quad \nu = 1.01 \times 10^{-6}\cdot\dfrac{m^2}{s} \quad \rho = 998\cdot\dfrac{kg}{m^3}$ for water

Hence $\quad F_D + F_{buoyancy} - W = 0 \quad F_D = W - F_{buoyancy} = \rho\cdot g\cdot h\cdot A\cdot(SG - 1)$

Also $$F_D = 2\cdot C_D\cdot A\cdot\frac{1}{2}\cdot\rho\cdot V^2 = 2\cdot\frac{0.0742}{Re_L^{\frac{1}{5}}}\cdot A\cdot\frac{1}{2}\cdot\rho\cdot V^2 = \frac{0.0742}{\left(\frac{V\cdot L}{\nu}\right)^{\frac{1}{5}}}\cdot W\cdot L\cdot\rho\cdot V^2 = 0.0742\cdot W\cdot L^{\frac{4}{5}}\cdot\nu^{\frac{1}{5}}\cdot\frac{1}{2}\cdot\rho\cdot V^{\frac{9}{5}}$$

Note that we double F_D because we have two sides!

Hence $$\rho_{H2O}\cdot g\cdot h\cdot W\cdot L\cdot(SG - 1) = 0.0742\cdot W\cdot L^{\frac{4}{5}}\cdot\nu^{\frac{1}{5}}\cdot\frac{1}{2}\cdot\rho\cdot V^{\frac{9}{5}}$$

Solving for V $$V = \left[\frac{g\cdot h\cdot(SG - 1)}{0.0742}\cdot\left(\frac{L}{\nu}\right)^{\frac{1}{5}}\right]^{\frac{5}{9}} \qquad V = 3.41\,\frac{m}{s}$$

Check the Reynolds number $\quad Re_L = \dfrac{V\cdot L}{\nu} \qquad Re_L = 1.69 \times 10^6 \quad$ Hence Eq. 9.34 is reasonable

Repeating for $\quad L = 1\cdot m \qquad V = \left[\dfrac{g\cdot h\cdot(SG - 1)}{0.0742}\cdot\left(\dfrac{L}{\nu}\right)^{\frac{1}{5}}\right]^{\frac{5}{9}} \qquad V = 3.68\,\dfrac{m}{s}$

Check the Reynolds number $\quad Re_L = \dfrac{V\cdot L}{\nu} \qquad Re_L = 3.65 \times 10^6 \quad$ Eq. 9.34 is still reasonable

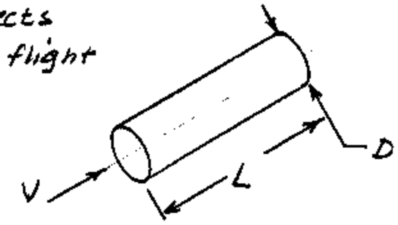
The short side vertical orientation falls more slowly because the largest friction is at the region of the leading edge (τ tails off as the boundary layer progresses); its leading edge area is larger. Note that neither orientation is likely - the plate will flip around in a chaotic manner

Problem 9.87

Given: 600-seat jet transport proposed by Airbus Industrie. Fuselage has length $L = 70$ m and diameter $D = 7.5$ m. Aircraft operates 14 hr per day, 6 days per week, cruising at $V = 257$ m/s ($M = 0.87$) at $z = 12$ km. The thrust specific fuel consumption (TSFC) is 0.06 kg/N·hr.

Find: (a) Estimate of skin friction drag on fuselage.
(b) Annual fuel saved by 1% reduction in drag by modifying surface.

Solution: Assume: (1) BL behaves as though on flat plate, $A = \pi DL = 1650\ m^2$
(2) Neglect compressibility effects
(3) All fuel consumed in cruise flight



Need Reynolds number

$$Re_L = \frac{\rho V L}{\mu}$$

From Table A.3, $T = 216.7$ K and $\rho/\rho_{SL} = 0.2546$; $\rho = 0.2546 \times 1.23 \frac{kg}{m^3} = 0.313\ kg/m^3$

From Eq. A.1, $\mu = 1.458 \times 10^{-6} \frac{kg}{m \cdot s \cdot K^{1/2}} \times (216.7)^{3/2} K^{3/2} \times \frac{1}{(110.4 + 216.7)K} = 1.42 \times 10^{-5} \frac{kg}{m \cdot s}$

Thus

$$Re_L = 0.313 \frac{kg}{m^3} \times 257 \frac{m}{s} \times 70\ m \times \frac{m \cdot s}{1.42 \times 10^{-5}\ kg} = 3.97 \times 10^8$$

From Eq. 9.35 ($Re_L < 10^9$),

$$C_D = \frac{0.455}{(\log Re_L)^{2.58}} = \frac{0.455}{(\log 3.97 \times 10^8)^{2.58}} = 0.0177$$

and

$$F_D = C_D A \tfrac{1}{2} \rho V^2 = \frac{1}{2} \times 0.00177 \times 1650\ m^2 \times 0.313 \frac{kg}{m^3} \times (257)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 3.02 \times 10^4\ N$$ F_D

Then $\Delta F_D = 0.01\ F_D = 3.02 \times 10^2\ N = 302\ N$

and $\Delta F_C = \Delta F_D \times TSFC \times t$

$$\Delta F_C = 302\ N \times 0.06 \frac{kg}{N \cdot hr} \times 365 \frac{day}{yr} \times 14 \frac{hr}{day} \times \frac{6}{7} = 7.94 \times 10^4\ kg/yr$$ ΔF_C

The specific gravity of jet fuel (kerosine) is about 0.82 (Table A.2). Thus

$$\Delta \forall = \frac{\Delta F_C}{\rho} = \frac{\Delta F_C}{SG\ \rho_{H_2O}}$$

$$\Delta \forall = 7.94 \times 10^4 \frac{kg}{yr} \times \frac{m^3}{(0.82)\ 1000\ kg} \times \frac{ft^3}{(0.305)^3\ m^3} \times 7.48 \frac{gal}{ft^3} = 2.55 \times 10^4\ gal/yr$$

$$\Delta C \approx 2.55 \times 10^4 \frac{gal}{yr} \times \frac{\$1}{gal} \approx \$25{,}000$$

This is a substantial saving per aircraft. The cost saving for a fleet would be impressive.

Problem 9.88

Given: Supertanker with 600,000 metric ton displacement.
Length, $L = 300$ m; Beam, $b = 80$ m; Draft, $D = 25$ m
Ship steams at 14 kt in seawater at 4°C

Estimate: (a) BL thickness at stern of ship.
(b) total skin-friction drag.
(c) power required to overcome skin-friction drag.

Solution:

Apply results of momentum integral analysis (Section 9-5.2) and correlations for drag coefficient (Section 9-7.1)

Computing equations: $\delta/x = \dfrac{0.382}{Re_x^{1/5}}$ (9.26)

$$C_D = \frac{0.455}{(\log Re_L)^{2.58}} - \frac{1610}{Re_L} \quad (9.37b)$$

Assumptions: (1) boundary layers behave as on a flat plate.
(2) 1/7-power turbulent velocity profiles

For seawater at 4°C, $SG = 1.025$, $\nu = 1.05\,\nu_{water}$ (Table A.2)
At 4°C, $\nu_{water} = 1.55\times10^{-6}$ m²/s (Table A.8).

$$V = 14\,\frac{nm}{hr}\times\frac{1852\,m}{nm}\times\frac{hr}{3600\,s} = 7.20\ m/s.$$

$$Re_L = \frac{VL}{\nu} = 7.20\,\frac{m}{s}\times 300\,m\times\frac{s}{1.05\times1.55\times10^{-6}\,m^2} = 1.33\times10^9$$

Thus

$$\delta_L = \frac{0.382\times300\,m}{(1.33\times10^9)^{0.20}} = 1.72\ m \quad \delta_L$$

Model drag area as in Example Problem 9.5

$$A = (b+2D)L = [80+2(25)]\,m\times300\,m = 3.90\times10^4\ m^2$$

$$C_D = \frac{0.455}{(\log 1.33\times10^9)^{2.58}} - \frac{1610}{Re_L} = 0.00151$$

$$F_D = C_D A \tfrac{1}{2}\rho V^2 = 0.00151\times3.90\times10^4\,m^2\times\frac{1}{2}\times(1.025)\times10^3\,\frac{kg}{m^3}\times(7.20)^2\,\frac{m^2}{s^2}\times\frac{N\cdot s^2}{kg\cdot m}$$

$$F_D = 1.56\ MN \quad F_D$$

(This is skin-friction drag; wave drag cannot be estimated.)

The power is

$$\mathcal{P} = F_D V = 1.56\times10^6\,N\times7.20\,\frac{m}{s}\times\frac{W\cdot s}{N\cdot m} = 11.2\ MW \quad \mathcal{P}$$

Problem 9.89

9.89 In Section 7-6 the wave resistance and viscous resistance on a model and prototype ship were discussed. For the prototype, L = 409 ft and A = 19,500 ft^2. From the data of Figs. 7.2 and 7.3, plot on one graph the wave, viscous, and total resistance (lbf) experienced by the prototype, as a function of speed. Plot a similar graph for the model. Discuss your results. Finally, plot the power (hp) required for the prototype ship to overcome the total resistance.

Given: "Resistance" data on a ship

Find: Plot of wave, viscous and total drag (protoype and model); Power required by prototype

Solution:

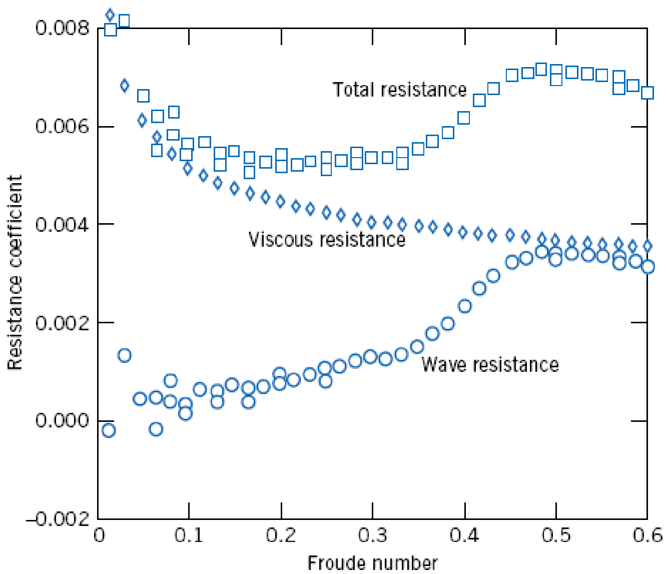


Fig. 7.2 Data from test of 1:80 scale model of U.S. Navy guided missile frigate *Oliver Hazard Perry* (FFG-7). (Data from U.S. Naval Academy Hydromechanics Laboratory, courtesy of Professor Bruce Johnson.)

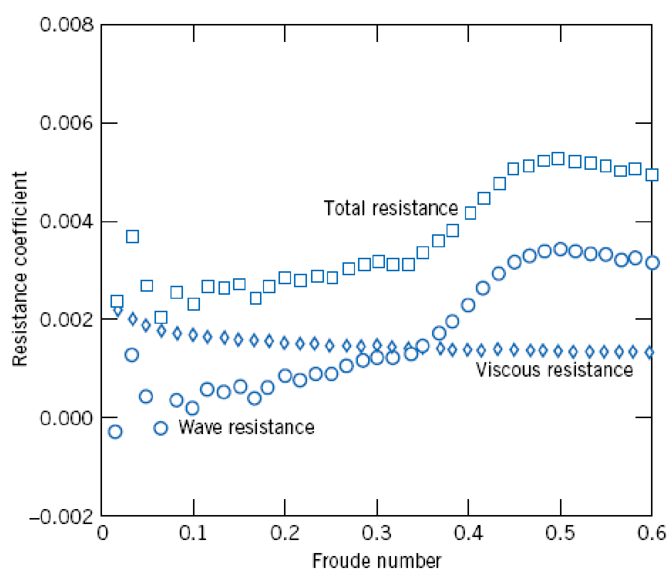


Fig. 7.3 Resistance of full-scale ship predicted from model test results. (Data from U.S. Naval Academy Hydromechanics Laboratory, courtesy of Professor Bruce Johnson.)

Governing equation: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot U^2\cdot A}$ (9.32)

$Fr = \dfrac{U}{\sqrt{gL}}$

From Eq. 9.32 $F_D = C_D\cdot A\cdot\frac{1}{2}\cdot\rho\cdot U^2$

This applies to each component of the drag (wave and viscous) as well as to the total

The power consumed is $P = F_D\cdot U$ $P = C_D\cdot A\cdot\frac{1}{2}\cdot\rho\cdot U^3$

From the Froude number $U = Fr\cdot\sqrt{gL}$

The solution technique is: For each speed Fr value from the graph, compute U; compute the drag from the corresponding "resistance" value from the graph

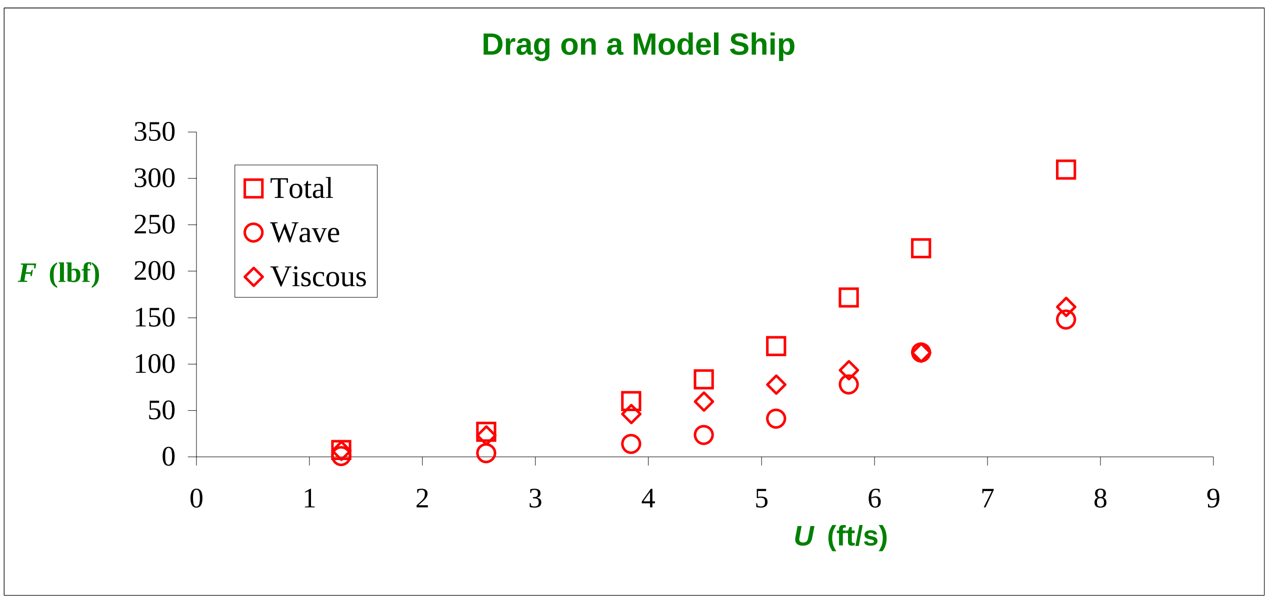
Given data:

L_p =	409	ft	
A_p =	19500	ft^2	
L_m =	5.11	ft (1/80 scale)	
A_m =	3.05	ft^2	
SG =	1.025		(Table A.2)
μ =	2.26E-05	$lbf.s/ft^2$	(Table A.2)
ρ =	1023	$slug/ft^3$	

Computed results:

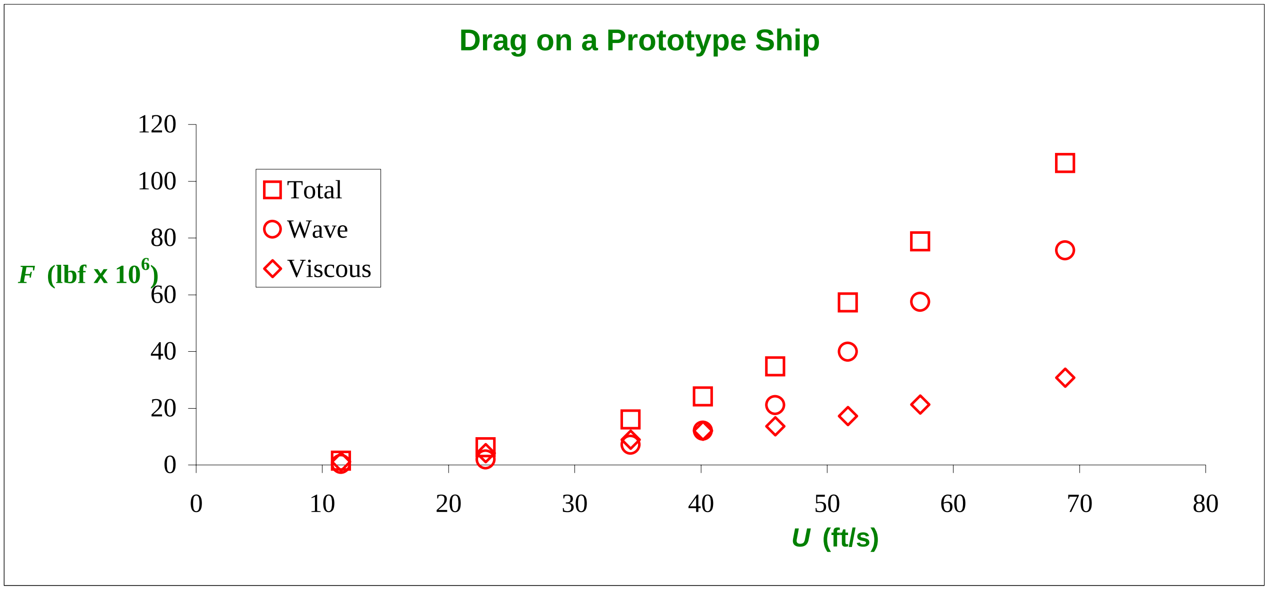
Model

Fr	Wave "Resistance"	Viscous "Resistance"	Total "Resistance"	*U* (ft/s)	Wave Drag (lbf)	Viscous Drag (lbf)	Total Drag (lbf)
0.10	0.00050	0.0052	0.0057	1.28	0.641	6.67	7.31
0.20	0.00075	0.0045	0.0053	2.57	3.85	23.1	26.9
0.30	0.00120	0.0040	0.0052	3.85	13.9	46.2	60.0
0.35	0.00150	0.0038	0.0053	4.49	23.6	59.7	83.3
0.40	0.00200	0.0038	0.0058	5.13	41.0	78.0	119
0.45	0.00300	0.0036	0.0066	5.77	77.9	93.5	171
0.50	0.00350	0.0035	0.0070	6.42	112	112	224
0.60	0.00320	0.0035	0.0067	7.70	148	162	309



Prototype

Fr	Wave "Resistance"	Viscous "Resistance"	Total "Resistance"	*U* (ft/s)	Wave Drag (lbf x 10^6)	Viscous Drag (lbf x 10^6)	Total Drag (lbf x 10^6)
0.10	0.00050	0.0017	0.0022	11.5	0.328	1.12	1.44
0.20	0.00075	0.0016	0.0024	23.0	1.97	4.20	6.17
0.30	0.00120	0.0015	0.0027	34.4	7.09	8.87	16.0
0.35	0.00150	0.0015	0.0030	40.2	12.1	12.1	24.1
0.40	0.00200	0.0013	0.0033	45.9	21.0	13.7	34.7
0.45	0.00300	0.0013	0.0043	51.6	39.9	17.3	57.2
0.50	0.00350	0.0013	0.0048	57.4	57.5	21.3	78.8
0.60	0.00320	0.0013	0.0045	68.9	75.7	30.7	106



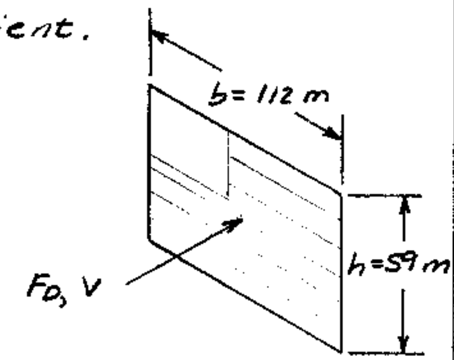
For the prototype wave resistance is a much more significant factor at high speeds!

Problem 9.90

Given: Flag, 59 m high and 112 m wide, mounted vertically.

Find: Force on flag in 16 km/hr wind. Was failure a surprise?

Solution: Apply definition of drag coefficient.



Computing equation: $F_D = C_D \frac{1}{2} \rho V^2 A$

Assumptions: (1) Flag acts as flat plate
(2) Standard air

The aspect ratio is $\frac{b}{h} = \frac{112\,m}{59\,m} = 1.9.$

From Fig. 9.10, $C_D \simeq 1.15$. Then

$$F_D = 1.15 \times \frac{1}{2} \times 1.23 \frac{kg}{m^3} \left(16 \times 10^3 \frac{m}{hr} \times \frac{hr}{3600\,s} \right)^2 112\,m \times 59\,m \times \frac{N \cdot s^2}{kg \cdot m}$$

$F_D = 92.3$ kN ← F_D

The flag failure should have been expected. This is a large force.

Problem 9.91

[3]

*9.91 Fishing net is made of 1/32-in. diameter nylon thread assembled in a rectangular pattern. The horizontal and vertical distances between adjacent thread centerlines are 3/8 in. Estimate the drag on a 5 ft × 40 ft section of this net when it is dragged (perpendicular to the flow) through 60°F water at 7 knots. What is the power required to maintain this motion?

Given: Fishing net

Find: Drag; Power to maintain motion

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$

We convert the net into an equivalent cylinder (we assume each segment does not interfere with its neighbors)

$L = 40\cdot ft$ $W = 5\cdot ft$ $d = \frac{1}{32}\cdot in$ Spacing: $D = \frac{3}{8}\cdot in$ $V = 7\cdot knot$ $V = 11.8\,\frac{ft}{s}$

Total number of threads of length L is $n_1 = \frac{W}{D}$ $n_1 = 160$ Total length $L_1 = n_1\cdot L$ $L_1 = 6400\,ft$

Total number of threads of length W is $n_2 = \frac{L}{D}$ $n_2 = 1280$ Total length $L_2 = n_2\cdot W$ $L_2 = 6400\,ft$

Total length of thread $L_T = L_1 + L_2$ $L_T = 12800\,ft$ $L_T = 2.42\,mile$ A lot!

The frontal area is then $A = L_T\cdot d$ $A = 33.3\,ft^2$ Note that $L\cdot W = 200\,ft^2$

From Table A.7 $\rho = 1.94\cdot\frac{slug}{ft^3}$ $\nu = 1.21\times 10^{-5}\cdot\frac{ft^2}{s}$

The Reynolds number is $Re_d = \frac{V\cdot d}{\nu}$ $Re_d = 2543$

For a cylinder in a crossflow at this Reynolds number, from Fig. 9.13, approximately $C_D = 0.8$

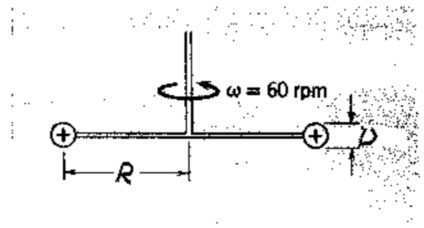
Hence $F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot V^2\cdot A$ $F_D = 3611\,lbf$

The power required is $P = F_D\cdot V$ $P = 42658\,\frac{ft\cdot lbf}{s}$ $P = 77.6\,hp$

Problem 9.92

[2]

Given: Rotary mixer, constructed as shown: The mixer is rotated in brine, SG = 1.1

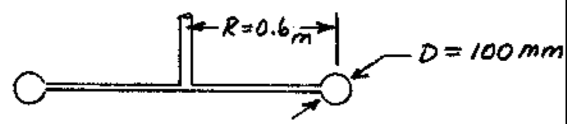


Neglect motion of liquid and drag on rods.

Find: (a) Torque
(b) Horsepower required to drive mixer.

Solution: Use drag coefficient data from Table 9.2, and

Basic equations: $T = 2RF_D$



$\mathbb{P} = T\omega$

$C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$

Thus $V = R\omega = 0.6\,m \times 60\,\frac{rev}{min} \times 2\pi\,\frac{rad}{rev} \times \frac{min}{60\,s} = 3.77\ m/s$

From Table 9.2, $C_D = 1.17$ for a disk, so neglecting drag of rods,

$$F_D = \frac{1}{2}\rho V^2 \frac{\pi D^2}{4} C_D = \frac{\pi}{8} \times (1.1)\,999\,\frac{kg}{m^3} \times (3.77)^2\,\frac{m^2}{s^2} \times (0.1)^2\,m^2 \times 1.17 \times \frac{N\cdot s^2}{kg\cdot m} = 71.8\ N$$

Then

$$T = 2RF_D = 2 \times 0.6\,m \times 71.8\,N = 86.2\ N\cdot m$$ T

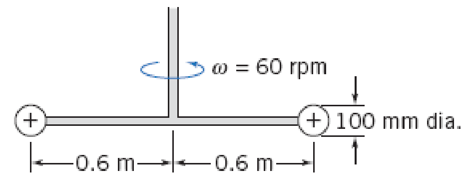
and

$$\mathbb{P} = T\omega = 86.2\,N\cdot m \times 60\,\frac{rev}{min} \times 2\pi\,\frac{rad}{rev} \times \frac{min}{60\,s} \times \frac{W\cdot s}{N\cdot m} = 542\ W$$ $\mathbb{P}$

Problem 9.93

[3]

9.93 As a young design engineer you decide to make the rotary mixer look more "cool" by replacing the disks with rings. The rings may have the added benefit of making the mixer mix more effectively. If the mixer absorbs 350 W at 60 rpm, redesign the device. There is a design constraint that the outer diameter of the rings not exceed 125 mm.



Given: Data on a rotary mixer

Find: New design dimensions

Solution:

The given data or available data is

$R = 0.6\cdot m$ $\quad P = 350\cdot W$ $\quad \omega = 60\cdot rpm$ $\quad \rho = 1099\cdot\frac{kg}{m^3}$

For a ring, from Table 9.3 $\quad C_D = 1.2$

The torque at the specified power and speed is

$$T = \frac{P}{\omega} \qquad T = 55.7\,N\cdot m$$

The drag on *each* ring is then $\quad F_D = \frac{1}{2}\cdot\frac{T}{R} \qquad F_D = 46.4\,N$

The linear velocity of each ring is $\quad V = R\cdot\omega \qquad V = 3.77\frac{m}{s}$

The drag and velocity of each ring are related using the definition of drag coefficient

$$C_D = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$$

Solving for the ring area $\quad A = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot C_D} \qquad A = 4.95\times 10^{-3}\,m^2$

But $\quad A = \frac{\pi}{4}\cdot\left(d_o^{\,2} - d_i^{\,2}\right)$

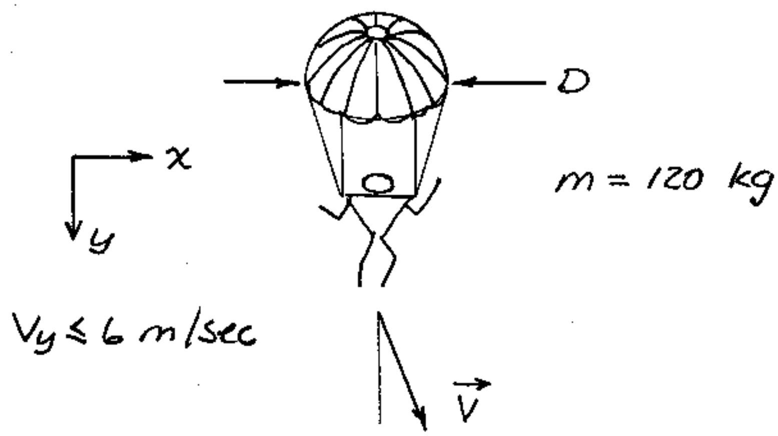
The outer diameter is $\quad d_o = 125\cdot mm$

Hence the inner diameter is $\quad d_i = \sqrt{d_o^{\,2} - \frac{4\cdot A}{\pi}} \qquad d_i = 96.5\,mm$

Problem 9.94

[2]

Given: Parachute and man with total mass, $m = 120$ kg.



Find: Minimum diameter, D.

Solution: Apply Newton's second law of motion, definition of drag coefficient.

Computing equations: $\sum F_y = mg - F_D = ma_y$; $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$

Assumptions: (1) Standard air
(2) Parachute behaves as open hemisphere
(3) V_y = constant; $a_y = 0$

Then $\sum F_y = mg - F_D = 0$ or $F_D = C_D A \frac{1}{2}\rho V_y^2 = mg$

$$A = \frac{\pi D^2}{4} \quad \text{so} \quad mg = C_D \frac{\pi D^2}{8} \rho V_y^2$$

$$D^2 = \frac{8mg}{\pi C_D \rho V_y^2} \quad \text{and} \quad D = \sqrt{\frac{8mg}{\pi C_D \rho V_y^2}}$$

From Table 9.3, $C_D = 1.42$ for an open hemisphere.

$$D = \left[\frac{8}{\pi} \times 120\ \text{kg} \times 9.81\ \frac{\text{m}}{\text{s}^2} \times \frac{1}{1.42} \times \frac{\text{m}^3}{1.23\ \text{kg}} \times \frac{\text{s}^2}{(6)^2\ \text{m}^2}\right]^{1/2}$$

$D = 6.90$ m D

Problem 9.95

9.95 An emergency braking parachute system on a military aircraft consists of a large parachute of diameter 6 m. If the airplane mass is 8500 kg, and it lands at 400 km/hr, find the time and distance at which the airplane is slowed to 100 km/hr by the parachute alone. Plot the aircraft speed versus distance and versus time. What is the maximum "g-force" experienced? An engineer proposes that less space would be taken up by replacing the large parachute with three non-interfering parachutes each of diameter 3.75 m. What effect would this have on the time and distance to slow to 100 km/hr?

Given: Data on airplane and parachute

Find: Time and distance to slow down; plot speed against distance and time; maximum "g"'s

Solution:

Newton's second law for the aircraft is

$$M \cdot \frac{dV}{dt} = -C_D \cdot \frac{1}{2} \cdot \rho \cdot A \cdot V^2$$

where A and C_D are the single parachute area and drag coefficient

Separating variables

$$\frac{dV}{V^2} = -\frac{C_D \cdot \rho \cdot A}{2 \cdot M} \cdot dt$$

Integrating, with IC $V = V_i$

$$V(t) = \frac{V_i}{1 + \frac{C_D \cdot \rho \cdot A}{2 \cdot M} \cdot V_i \cdot t} \qquad (1)$$

Integrating again with respect to t

$$x(t) = \frac{2 \cdot M}{C_D \cdot \rho \cdot A} \cdot \ln\left(1 + \frac{C_D \cdot \rho \cdot A}{2 \cdot M} \cdot V_i \cdot t\right) \qquad (2)$$

Eliminating t from Eqs. 1 and 2

$$x = \frac{2 \cdot M}{C_D \cdot \rho \cdot A} \cdot \ln\left(\frac{V_i}{V}\right) \qquad (3)$$

To find the time and distance to slow down to 100 km/hr, Eqs. 1 and 3 are solved with V = 100 km/hr (or use *Goal Seek*)

The "g"'s are given by

$$\frac{\frac{dV}{dt}}{g} = \frac{-C_D \cdot \rho \cdot A \cdot V^2}{2 \cdot M \cdot g}$$

which has a maximum at the initial instant ($V = V_i$)

For three parachutes, the analysis is the same except A is replaced with $3A$. leading to

$$V(t) = \frac{V_i}{1 + \frac{3 \cdot C_D \cdot \rho \cdot A}{2 \cdot M} \cdot V_i \cdot t}$$

$$x(t) = \frac{2 \cdot M}{3 \cdot C_D \cdot \rho \cdot A} \cdot \ln\left(1 + \frac{3 \cdot C_D \cdot \rho \cdot A}{2 \cdot M} \cdot V_i \cdot t\right)$$

Given data:

M = 8500 kg

V_i = 400 km/hr

V_f = 100 km/hr

C_D = 1.42 (Table 9.3)

ρ = 1.23 kg/m^3

Single: D = 6 m   Triple: D = 3.75 m

Computed results:

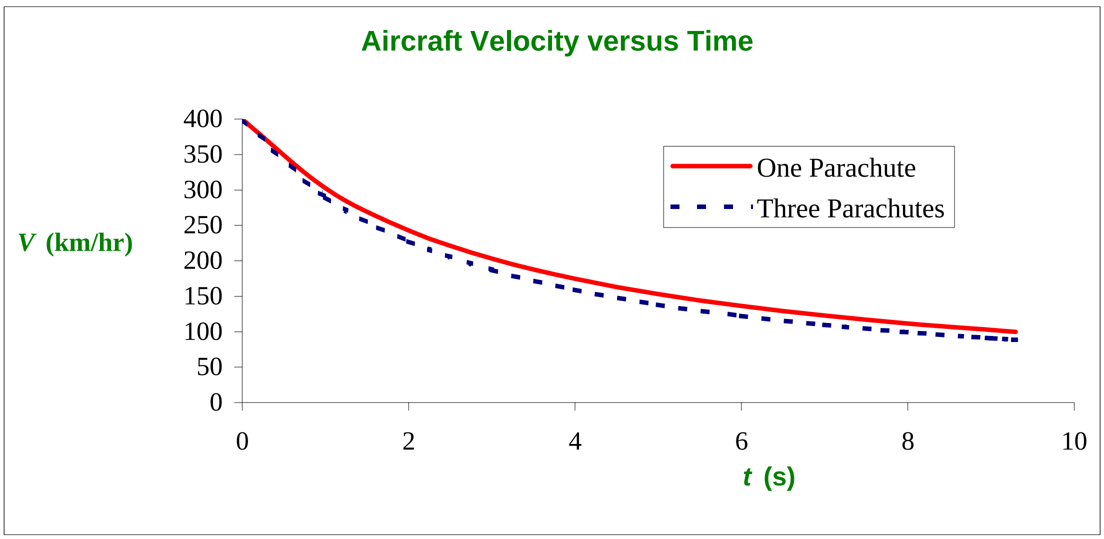
A = 28.3 m^2

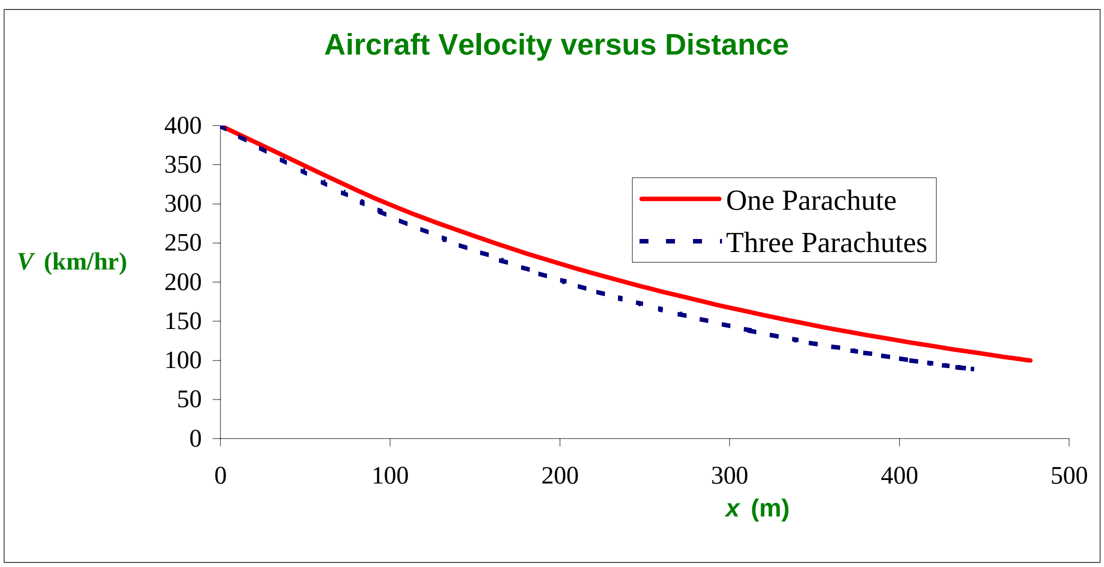
t (s)	x (m)	V (km/hr)
0.0	0.0	400
1.0	96.3	302
2.0	171	243
3.0	233	203
4.0	285	175
5.0	331	153
6.0	371	136
7.0	407	123
8.0	439	112
9.0	469	102
9.29	477	100

"g"'s = -3.66 Max

A = 11.0 m^2

t (s)	x (m)	V (km/hr)
0.0	0.0	400
1.0	94.2	290
2.0	165	228
3.0	223	187
4.0	271	159
5.0	312	138
6.0	348	122
7.0	380	110
7.93	407	100
9.0	436	91
9.3	443	89





Problem 9.96

9.96 As a young design engineer you are asked to design an emergency braking parachute system for use with a military aircraft of mass 9500 kg. The plane lands at 350 km/hr, and the parachute system alone must slow the airplane to 100 km/hr in less than 1200 m. Find the minimum diameter required for a single parachute, and for three non-interfering parachutes. Plot the airplane speed versus distance and versus time. What is the maximum "g-force" experienced?

Given: Data on airplane landing

Find: Single and three-parachute sizes; plot speed against distance and time; maximum "g"s

Solution:

Newton's second law for the aircraft is
$$M\cdot\frac{dV}{dt} = -C_D\cdot\frac{1}{2}\cdot\rho\cdot A\cdot V^2$$

where A and CD are the single parachute area and drag coefficient

Separating variables
$$\frac{dV}{V^2} = -\frac{C_D\cdot\rho\cdot A}{2\cdot M}\cdot dt$$

Integrating, with IC $V = V_i$
$$V(t) = \frac{V_i}{1 + \frac{C_D\cdot\rho\cdot A}{2\cdot M}\cdot V_i\cdot t} \qquad (1)$$

Integrating again with respect to t
$$x(t) = \frac{2\cdot M}{C_D\cdot\rho\cdot A}\cdot\ln\left(1 + \frac{C_D\cdot\rho\cdot A}{2\cdot M}\cdot V_i\cdot t\right) \qquad (2)$$

Eliminating t from Eqs. 1 and 2
$$x = \frac{2\cdot M}{C_D\cdot\rho\cdot A}\cdot\ln\left(\frac{V_i}{V}\right) \qquad (3)$$

To find the minimum parachute area we must solve Eq 3 for A with $x = x_f$ when $V = V_f$
$$A = \frac{2\cdot M}{C_D\cdot\rho\cdot x_f}\cdot\ln\left(\frac{V_i}{V_f}\right) \qquad (4)$$

For three parachutes, the analysis is the same except A is replaced with $3A$, leading to
$$A = \frac{2\cdot M}{3\cdot C_D\cdot\rho\cdot x_f}\cdot\ln\left(\frac{V_i}{V_f}\right) \qquad (5)$$

The "g"'s are given by
$$\frac{\frac{dV}{dt}}{g} = \frac{-C_D\cdot\rho\cdot A\cdot V^2}{2\cdot M\cdot g}$$

which has a maximum at the initial instant ($V = V_i$)

Given data:

M = 9500 kg
V_i = 350 km/hr
V_f = 100 km/hr
x_f = 1200 m
C_D = 1.42 (Table 9.3)
ρ = 1.23 kg/m^3

Computed results:

Single:

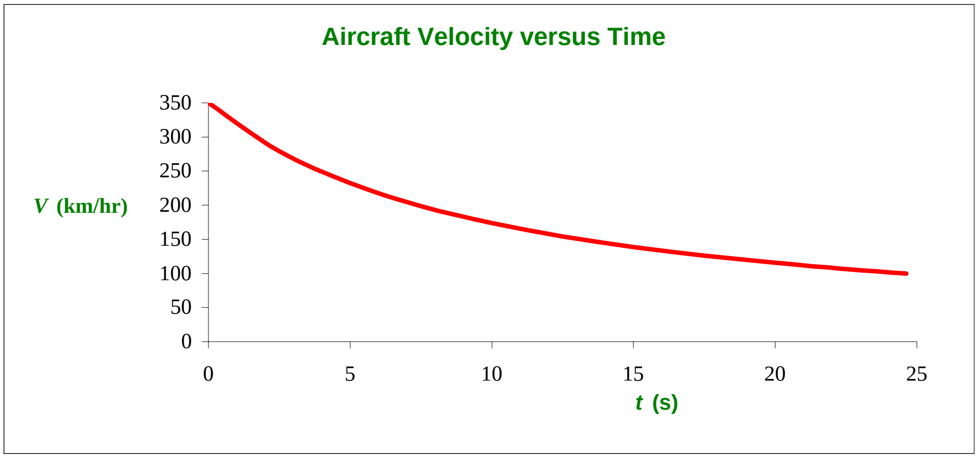
A = 11.4 m^2
D = 3.80 m

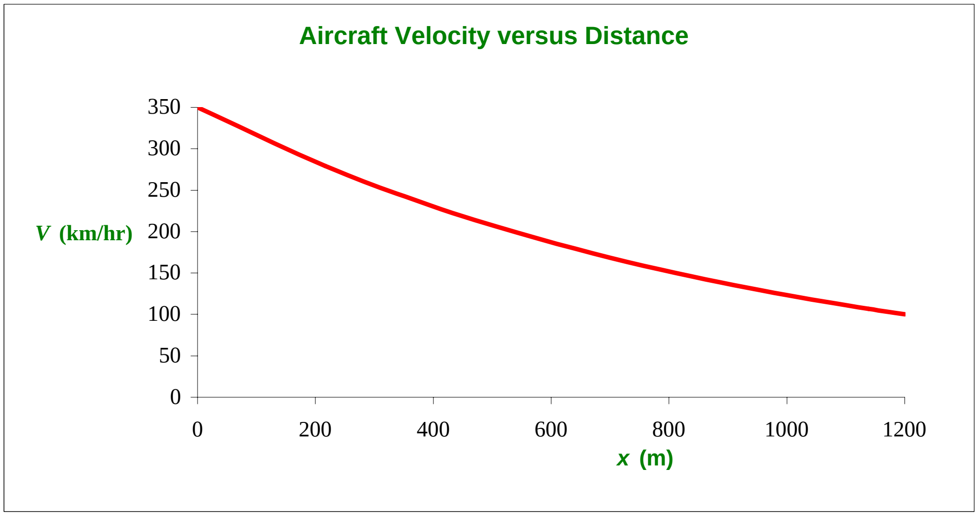
Triple:

A = 3.8 m^2
D = 2.20 m

"g"'s = -1.01 Max

t (s)	x (m)	V (km/hr)
0.00	0.0	350
2.50	216.6	279
5.00	393.2	232
7.50	542.2	199
10.0	671.1	174
12.5	784.7	154
15.0	886.3	139
17.5	978.1	126
20.0	1061.9	116
22.5	1138.9	107
24.6	1200.0	100





Problem 9.97

Given: Windmills to be made from surplus 55-gal oil drums.

For a drum, $D = 24$ in., $H = 29$ in.

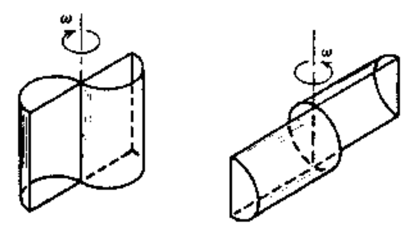
Find: Which configuration would be better, why, and by how much?

Solution: Sum moments about pivot, neglecting friction, interference.

Configuration A:

$$\Sigma M = \frac{D}{2} F_u - \frac{D}{2} F_d = \frac{D}{2}(F_u - F_d)$$

$$\Sigma M = \frac{D}{2}(C_{Du} - C_{Dd}) A \frac{1}{2} \rho V^2$$



Configuration (A) (B)

Configuration B:

$$\Sigma M = \frac{H}{2} F_u - \frac{H}{2} F_d = \frac{H}{2}(F_u - F_d)$$

$$\Sigma M = \frac{H}{2}(C_{Du} - C_{Dd}) A \frac{1}{2} \rho V^2$$

Performance of B will be better because $H > D$.

$$\frac{H - D}{D} = \frac{29 - 24}{24} = 0.208 \quad \text{or} \quad 20.8 \text{ percent improvement!}$$

Problem 9.98

[2]

Given: Bike and rider with $M = 100\ kg$, $A = 0.46\ m^2$, and negligible rolling resistance, has terminal speed, $V_t = 15\ m/s$, on a hill with 8 percent grade. Drag coefficient estimated as $C_D \approx 1.2$.

Find: (a) Verify this calculation of drag coefficient.
(b) Distance for bike and rider to slow from 15 to 10 m/sec after reaching level road.

<u>Solution</u>: Treat the bike and rider as a system. From a free-body diagram,

$$\Sigma F_x = F_G - F_D = mg\sin\theta - C_D A \tfrac{1}{2}\rho V^2 = m a_x \qquad \theta = \tan^{-1}(0.08) = 4.57°$$

At terminal speed, $a_x = 0$. Then $mg\sin\theta = C_D A \frac{1}{2}\rho V_t^2$, so

$$C_D = \frac{2mg\sin\theta}{A\rho V_t^2} = 2 \times 100\,kg \times 9.81\frac{m}{s^2} \times \sin 4.57° \times \frac{1}{0.46\,m^2} \times \frac{m^3}{1.23\,kg} \times \frac{s^2}{(15)^2 m^2} = 1.23$$ C_D

Thus $C_D \approx 1.2$ is correct.

On a flat surface, $\Sigma F_x = -F_D = -C_D A \frac{1}{2}\rho V^2 = m\frac{dV}{dt} = mV\frac{dV}{ds}$

Thus $mV\frac{dV}{ds} = -C_D A \frac{1}{2}\rho V^2$

or

$$ds = -\frac{2m}{C_D\rho A}\frac{dV}{V}$$

Integrating

$$s - s_0 = \int_{s_0}^{s} ds = -\frac{2m}{C_D\rho A}\int_{V_0}^{V}\frac{dV}{V} = -\frac{2m}{C_D\rho A}\ln V\Big]_{V_0}^{V} = -\frac{2m}{C_D\rho A}\ln\left(\frac{V}{V_0}\right)$$

(with $s_0 = 0$)

$$s = -2 \times 100\,kg \times \frac{1}{1.23} \times \frac{m^3}{1.23\,kg} \times \frac{1}{0.46\,m^2}\ln\frac{10}{15} = 117\,m$$ s

Problem 9.99

[2]

Given: Ballistic data for 44 magnum revolver bullet:

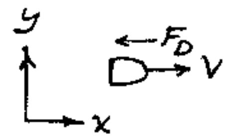
$V_i = 250$ m/s, $V_f = 210$ m/s over $\Delta x = 150$ m; $D = 11.2$ mm; $m = 15.6$ g

Find: Evaluate average drag coefficient.

Solution: Apply Newton's second law, definition of drag coefficient.

Computing equation: $F_D = \bar{C}_D A \frac{1}{2}\rho V^2$

Basic equation: $\sum F_x = m a_x = m\frac{dV}{dt} = mV\frac{dV}{dx}$



From the free-body diagram, $\sum F_x = -F_D$, so

$$mV\frac{dV}{dx} = -F_D = -\bar{C}_D A \frac{1}{2}\rho V^2$$

Thus

$$\frac{dV}{V} = -\frac{\bar{C}_D A \rho}{2m}\,dx$$

Integrating

$$\int_{V_i}^{V_f}\frac{dV}{V} = \ln V\Big]_{V_i}^{V_f} = \ln\frac{V_f}{V_i} = -\frac{\bar{C}_D A\rho}{2m}\Delta x$$

Solving, using density of standard air,

$$\bar{C}_D = -\frac{2m}{\rho A\Delta x}\ln\frac{V_f}{V_i}$$

$$= -2\times 0.0156\ \text{kg}\times\frac{\text{m}^3}{1.23\ \text{kg}}\times\frac{4}{\pi(0.0112)^2\,\text{m}^2}\times\frac{1}{150\ \text{m}}\ln\left(\frac{210}{250}\right)$$

$\bar{C}_D = 0.299$ ← $\bar{C}_D$

Problem 9.100

[2]

9.100 A cyclist is able to attain a maximum speed of 30 km/hr on a calm day. The total mass of rider and bike is 65 kg. The rolling resistance of the tires is $F_R = 7.5$ N, and the drag coefficient and frontal area are $C_D = 1.2$ and $A = 0.25$ m^2. The cyclist bets that today, even though there is a headwind of 10 km/hr, she can maintain a speed of 24 km/hr. She also bets that, cycling with wind support, she can attain a top speed of 40 km/hr. Which, if any, bets does she win?

Given: Data on cyclist performance on a calm day

Find: Performance hindered and aided by wind

Solution:

The given data or available data is

$F_R = 7.5 \cdot N$ $\qquad M = 65 \cdot kg$ $\qquad A = 0.25 \cdot m^2$

$C_D = 1.2$ $\qquad \rho = 1.23 \cdot \frac{kg}{m^3}$ $\qquad V = 30 \cdot \frac{km}{hr}$

The governing equation is $\qquad F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$ $\qquad F_D = 12.8\,N$

The power steady power generated by the cyclist is

$P = \left(F_D + F_R\right) \cdot V$ $\qquad P = 169\,W$ $\qquad P = 0.227\,hp$

Now, with a headwind we have $\qquad V_w = 10 \cdot \frac{km}{hr}$ $\qquad V = 24 \cdot \frac{km}{hr}$

The aerodynamic drag is greater because of the greater effective wind speed

$F_D = \frac{1}{2} \cdot \rho \cdot A \cdot \left(V + V_w\right)^2 \cdot C_D$ $\qquad F_D = 16.5\,N$

The power required is that needed to overcome the total force $F_D + F_R$, moving at the cyclist's speed

$P = V \cdot \left(F_D + F_R\right)$ $\qquad P = 160\,W$

This is less than the power she can generate $\qquad$ She wins the bet!

With the wind supporting her the effective wind speed is substantially lower

$V_W = 10 \cdot \frac{km}{hr}$ $\qquad V = 40 \cdot \frac{km}{hr}$

$F_D = \frac{1}{2} \cdot \rho \cdot A \cdot \left(V - V_W\right)^2 \cdot C_D$ $\qquad F_D = 12.8\,N$

The power required is that needed to overcome the total force $F_D + F_R$, moving at the cyclist's speed

$P = V \cdot \left(F_D + F_R\right)$ $\qquad P = 226\,W$

This is more than the power she can generate $\qquad$ She loses the bet

Problem 9.101

9.101 Consider the cyclist in Problem 9.100. Determine the maximum speeds she is actually able to attain today (with the 10 km/hr wind) cycling into the wind, and cycling with the wind. If she were to replace the tires with high-tech ones that had a rolling resistance of only 3.5 N, determine her maximum speed on a calm day, cycling into the wind, and cycling with the wind. If she in addition attaches an aerodynamic fairing that reduces the drag coefficient to $C_D = 0.9$, what will be her new maximum speeds?

Given: Data on cyclist performance on a calm day

Find: Performance hindered and aided by wind; repeat with high-tech tires; with fairing

Solution:

The given data or available data is

$F_R = 7.5 \cdot N$ $\quad M = 65 \cdot kg$ $\quad A = 0.25 \cdot m^2$

$C_D = 1.2$ $\quad \rho = 1.23 \cdot \frac{kg}{m^3}$ $\quad V = 30 \cdot \frac{km}{hr}$

The governing equation is $\quad F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D \quad F_D = 12.8 N$

Power steady power generated by the cyclist is $\quad P = (F_D + F_R) \cdot V \quad P = 169 W \quad P = 0.227 \cdot hp$

Now, with a headwind we have $\quad V_w = 10 \cdot \frac{km}{hr}$

The aerodynamic drag is greater because of the greater effective wind speed

$$F_D = \frac{1}{2} \cdot \rho \cdot A \cdot (V + V_w)^2 \cdot C_D \qquad (1)$$

The power required is that needed to overcome the total force $F_D + F_R$, moving at the cyclist's speed is

$$P = V \cdot (F_D + F_R) \qquad (2)$$

Combining Eqs 1 and 2 we obtain an expression for the cyclist's maximum speed V cycling into a headwind (where P = 169 W is the cyclist's power)

Cycling into the wind: $$P = \left[F_R + \frac{1}{2} \cdot \rho \cdot A \cdot (V + V_w)^2 \cdot C_D\right] \cdot V \qquad (3)$$

This is a cubic equation for V; it can be solved analytically, or by iterating. It is convenient to use *Excel*'s *Goal Seek* (or *Solver*). From the associated *Excel* workbook

From *Solver* $\quad V = 24.7 \cdot \frac{km}{hr}$

By a similar reasoning:

Cycling with the wind: $$P = \left[F_R + \frac{1}{2} \cdot \rho \cdot A \cdot (V - V_w)^2 \cdot C_D\right] \cdot V \qquad (4)$$

From *Solver* $V = 35.8 \cdot \frac{km}{hr}$

With improved tires $F_R = 3.5 \cdot N$

Maximum speed on a calm day is obtained from $P = \left(F_R + \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D\right) \cdot V$

This is a again a cubic equation for *V*; it can be solved analytically, or by iterating. It is convenient to use *Excel*'s *Goal Seek* (or *Solver*). From the associated *Excel* workbook

From *Solver* $V = 32.6 \cdot \frac{km}{hr}$

Equations 3 and 4 are repeated for the case of improved tires

From *Solver* Against the wind $V = 26.8 \cdot \frac{km}{hr}$ With the wind $V = 39.1 \cdot \frac{km}{hr}$

For improved tires and fairing, from *Solver*

$V = 35.7 \cdot \frac{km}{hr}$ Against the wind $V = 29.8 \cdot \frac{km}{hr}$ With the wind $V = 42.1 \cdot \frac{km}{hr}$

Problem 9.101 (In Excel)

9.101 Consider the cyclist in Problem 9.100. Determine the maximum speeds she is actually able to attain today (with the 10 km/hr wind) cycling into the wind, and cycling with the wind. If she were to replace the tires with high-tech ones that had a rolling resistance of only 3.5 N, determine her maximum speed on a calm day, cycling into the wind, and cycling with the wind. If she in addition attaches an aerodynamic fairing that reduces the drag coefficient to $C_D = 0.9$, what will be her new maximum speeds?

Given: Data on cyclist performance on a calm day

Find: Performance hindered and aided by wind; repeat with high-tech tires; with fairing

Solution:

Given data:

$F_R =$	7.5	N
$M =$	65	kg
$A =$	0.25	m^2
$C_D =$	1.2	
$\rho =$	1.23	kg/m^3
$V =$	30	km/hr
$V_w =$	10	km/hr

Computed results:

$$F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$$

$F_D =$ 12.8 N

$$P = (F_D + F_R) \cdot V$$

$P =$ 169 W

Cycling into the wind:

$$P = \left[F_R + \frac{1}{2} \cdot \rho \cdot A \cdot (V + V_w)^2 \cdot C_D\right] \cdot V$$

	Left (W)	Right (W)	Error	*V* (km/hr)
Using *Solver* :	169	169	0%	24.7

Cycling with the wind:

$$P = \left[F_R + \frac{1}{2} \cdot \rho \cdot A \cdot (V - V_w)^2 \cdot C_D\right] \cdot V$$

	Left (W)	Right (W)	Error	*V* (km/hr)
Using *Solver* :	169	169	0%	35.8

With improved tires:

$F_R =$ 3.5 N

$$P = \left(F_R + \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D\right) \cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	32.6

Cycling into the wind: $$P = \left[F_R + \frac{1}{2} \cdot \rho \cdot A \cdot \left(V + V_w\right)^2 \cdot C_D\right] \cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	26.8

Cycling with the wind: $$P = \left[F_R + \frac{1}{2} \cdot \rho \cdot A \cdot \left(V - V_w\right)^2 \cdot C_D\right] \cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	39.1

With improved tires and fairing:

$F_R =$ 3.5 N

$C_D =$ 0.9

$$P = \left(F_R + \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D\right) \cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	35.7

Cycling into the wind: $$P = \left[F_R + \frac{1}{2} \cdot \rho \cdot A \cdot \left(V + V_w\right)^2 \cdot C_D\right] \cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	29.8

Cycling with the wind: $$P = \left[F_R + \frac{1}{2} \cdot \rho \cdot A \cdot \left(V - V_w\right)^2 \cdot C_D\right] \cdot V$$

Using *Solver* :

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	42.1

Problem 9.102

9.102 Consider the cyclist in Problem 9.100. She is having a bad day, because she has to climb a hill with a 5° slope. What is the speed she is able to attain? What is the maximum speed if there is also a headwind of 10 km/hr? She reaches the top of the hill, and turns around and heads down the hill. If she still pedals as hard as possible, what will be her top speed (when it is calm, and when the wind is present)? What will be her maximum speed if she decides to coast down the hill (with and without the aid of the wind)?

Given: Data on cyclist performance on a calm day

Find: Performance on a hill with and without wind

Solution:

The given data or available data is

$F_R = 7.5 \cdot N$ $\quad M = 65 \cdot kg$ $\quad A = 0.25 \cdot m^2$

$C_D = 1.2$ $\quad \rho = 1.23 \cdot \frac{kg}{m^3}$ $\quad V = 30 \cdot \frac{km}{hr}$

The governing equation is $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$ $\quad F_D = 12.8\,N$

Power steady power generated by the cyclist is $P = \left(F_D + F_R\right) \cdot V$ $\quad P = 169\,W$ $\quad P = 0.227\,hp$

Riding up the hill (no wind) $\quad \theta = 5 \cdot deg$

For steady speed the cyclist's power is consumed by working against the net force (rolling resistance, darg, and gravity)

Cycling up the hill: $$P = \left(F_R + \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D + M \cdot g \cdot \sin(\theta)\right) \cdot V$$

This is a cubic equation for the speed which can be solved analytically, or by iteration, or using *Excel*'s *Goal Seek* or *Solver*. The solution is obtained from the associated *Excel* workbook

From *Solver* $\quad V = 9.47 \cdot \frac{km}{hr}$

Now, with a headwind we have $\quad V_w = 10 \cdot \frac{km}{hr}$

The aerodynamic drag is greater because of the greater effective wind speed

$$F_D = \frac{1}{2} \cdot \rho \cdot A \cdot \left(V + V_w\right)^2 \cdot C_D$$

The power required is that needed to overcome the total force (rolling resistance, drag, and gravity) moving at the cyclist's speed is

Uphill against the wind: $$P = \left[F_R + \frac{1}{2} \cdot \rho \cdot A \cdot \left(V + V_w\right)^2 \cdot C_D + M \cdot g \cdot \sin(\theta)\right] \cdot V$$

This is again a cubic equation for *V*

From *Solver* $\quad V = 8.94 \cdot \frac{km}{hr}$

Pedalling downhill (no wind) gravity helps increase the speed; the maximum speed is obtained from

Cycling down the hill: $$P = \left(F_R + \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D - M\cdot g\cdot \sin(\theta)\right)\cdot V$$

This cubic equation for *V* is solved in the associated *Excel* workbook

From *Solver* $$V = 63.6\cdot\frac{km}{hr}$$

Pedalling downhill (wind assisted) gravity helps increase the speed; the maximum speed is obtained from

Wind-assisted downhill: $$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V - V_w\right)^2\cdot C_D - M\cdot g\cdot \sin(\theta)\right]\cdot V$$

This cubic equation for *V* is solved in the associated *Excel* workbook

From *Solver* $$V = 73.0\cdot\frac{km}{hr}$$

Freewheeling downhill, the maximum speed is obtained from the fact that the net force is zero

Freewheeling downhill: $$F_R + \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D - M\cdot g\cdot \sin(\theta) = 0$$

$$V = \sqrt{\frac{M\cdot g\cdot \sin(\theta) - F_R}{\frac{1}{2}\cdot\rho\cdot A\cdot C_D}} \qquad V = 58.1\,\frac{km}{hr}$$

Wind assisted: $$F_R + \frac{1}{2}\cdot\rho\cdot A\cdot\left(V - V_w\right)^2\cdot C_D - M\cdot g\cdot \sin(\theta) = 0$$

$$V = V_w + \sqrt{\frac{M\cdot g\cdot \sin(\theta) - F_R}{\frac{1}{2}\cdot\rho\cdot A\cdot C_D}} \qquad V = 68.1\,\frac{km}{hr}$$

Problem 9.102 (In Excel)

9.102 Consider the cyclist in Problem 9.100. She is having a bad day, because she has to climb a hill with a 5° slope. What is the speed she is able to attain? What is the maximum speed if there is also a headwind of 10 km/hr? She reaches the top of the hill, and turns around and heads down the hill. If she still pedals as hard as possible, what will be her top speed (when it is calm, and when the wind is present)? What will be her maximum speed if she decides to coast down the hill (with and without the aid of the wind)?

Given: Data on cyclist performance on a calm day

Find: Performance on a hill with and without wind

Solution:

Given data:

F_R =	7.5	N
M =	65	kg
A =	0.25	m^2
C_D =	1.2	
ρ =	1.23	kg/m^3
V =	30	km/hr
V_w =	10	km/hr
θ =	5	deg

Computed results:

$$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$$ F_D = 12.8 N

$$P = (F_D + F_R)\cdot V$$ P = 169 W

Cycling up the hill: $$P = \left(F_R + \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D + M\cdot g\cdot\sin(\theta)\right)\cdot V$$

Using *Solver*:

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	9.47

Uphill against the wind: $$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot(V + V_w)^2\cdot C_D + M\cdot g\cdot\sin(\theta)\right]\cdot V$$

Using *Solver*:

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	8.94

Cycling down the hill: $$P = \left(F_R + \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D - M\cdot g\cdot\sin(\theta)\right)\cdot V$$

Using *Solver*:

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	63.6

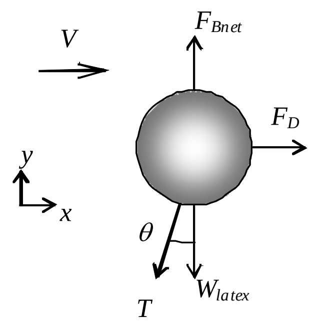
Wind-assisted downhill: $$P = \left[F_R + \frac{1}{2}\cdot\rho\cdot A\cdot(V - V_w)^2\cdot C_D - M\cdot g\cdot\sin(\theta)\right]\cdot V$$

Using *Solver*:

Left (W)	Right (W)	Error	*V* (km/hr)
169	169	0%	73.0

Problem *9.103

*9.103 At a surprise party for a friend you've tied a series of 9-in. diameter helium balloons to a flagpole, each tied with a short string. The first one is tied 3 ft above the ground, and the other eight are tied at 3 ft. spacings, so the last is tied at a height of 63 ft. Being quite a nerdy engineer, you notice that in the steady wind, each balloon is blown by the wind so it looks like the angles the strings make with the vertical are about 5°, 10°, 20°, 30°, 35°, 45°, 50°, 60° and 65°. Estimate and plot the wind velocity profile for the 63 ft. range. Assume the helium is at 70°F and 1.5 psig, and that each balloon is made of 1/10 oz. of latex.



Given: Series of party balloons

Find: Wind velocity profile; Plot

Note: Flagpole is actually 27 ft tall, not 63 ft!

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$ $\quad F_B = \rho_{air}\cdot g\cdot Vol$ $\quad \Sigma\vec{F} = 0$

The above figure applies to each balloon

For the horizontal forces $\quad F_D - T\cdot\sin(\theta) = 0 \quad$ (1)

For the vertical forces $\quad -T\cdot\cos(\theta) + F_{Bnet} - W_{latex} = 0 \quad$ (2)

Here $\quad F_{Bnet} = F_B - W = \left(\rho_{air} - \rho_{He}\right)\cdot g\cdot\dfrac{\pi\cdot D^3}{6}$

$D = 9\cdot in$ $\quad M_{latex} = \dfrac{1}{10}\cdot oz$ $\quad W_{latex} = M_{latex}\cdot g$ $\quad W_{latex} = 0.00625\,lbf$

We have (Table A.6) $\quad R_{He} = 386.1\cdot\dfrac{ft\cdot lbf}{lbm\cdot R}$ $\quad p_{He} = 16.2\cdot psi$ $\quad T_{He} = 530\cdot R$ $\quad \rho_{He} = \dfrac{p_{He}}{R_{He}\cdot T_{He}}$ $\quad \rho_{He} = 0.000354\,\dfrac{slug}{ft^3}$

$R_{air} = 53.33\cdot\dfrac{ft\cdot lbf}{lbm\cdot R}$ $\quad p_{air} = 14.7\cdot psi$ $\quad T_{air} = 530\cdot R$ $\quad \rho_{air} = \dfrac{p_{air}}{R_{air}\cdot T_{air}}$ $\quad \rho_{air} = 0.00233\,\dfrac{slug}{ft^3}$

$F_{Bnet} = \left(\rho_{air} - \rho_{He}\right)\cdot g\cdot\dfrac{\pi\cdot D^3}{6}$ $\quad F_{Bnet} = 0.0140\,lbf$

Applying Eqs 1 and 2 to the top balloon, for which $\quad \theta = 65\cdot deg$

$$F_D = T\cdot\sin(\theta) = \frac{F_{Bnet} - W_{latex}}{\cos(\theta)}\cdot\sin(\theta)$$

Hence $\quad F_D = \left(F_{Bnet} - W_{latex}\right)\cdot\tan(\theta)$ $\quad F_D = 0.0167\,lbf$

But we have $\quad F_D = C_D\cdot\dfrac{1}{2}\cdot\rho_{air}\cdot V^2\cdot A = C_D\cdot\dfrac{1}{2}\cdot\rho_{air}\cdot V^2\cdot\dfrac{\pi\cdot D^2}{4}$ with $C_D = 0.4$ from Fig. 9.11 (we will check Re later)

$V = \sqrt{\dfrac{8\cdot F_D}{C_D\cdot\rho_{air}\cdot\pi\cdot D^2}}$ $\quad V = 9.00\,\dfrac{ft}{s}$

From Table A.9 $\quad \nu = 1.63\times10^{-4}\cdot\dfrac{ft^2}{s}$ The Reynolds number is $Re_d = \dfrac{V\cdot D}{\nu}$ $\quad Re_d = 4.14\times10^4$ We are okay!

For the next balloon $\theta = 60 \cdot deg$ $F_D = (F_{Bnet} - W_{latex}) \cdot \tan(\theta)$ $F_D = 0.0135\,lbf$ with $C_D = 0.4$

$$V = \sqrt{\frac{8 \cdot F_D}{C_D \cdot \rho_{air} \cdot \pi \cdot D^2}} \qquad V = 8.09\,\frac{ft}{s}$$

The Reynolds number is $Re_d = \frac{V \cdot D}{\nu}$ $Re_d = 3.72 \times 10^4$ We are okay!

For the next balloon $\theta = 50 \cdot deg$ $F_D = (F_{Bnet} - W_{latex}) \cdot \tan(\theta)$ $F_D = 0.00927\,lbf$ with $C_D = 0.4$

$$V = \sqrt{\frac{8 \cdot F_D}{C_D \cdot \rho_{air} \cdot \pi \cdot D^2}} \qquad V = 6.71\,\frac{ft}{s}$$

The Reynolds number is $Re_d = \frac{V \cdot D}{\nu}$ $Re_d = 3.09 \times 10^4$ We are okay!

For the next balloon $\theta = 45 \cdot deg$ $F_D = (F_{Bnet} - W_{latex}) \cdot \tan(\theta)$ $F_D = 0.00777\,lbf$ with $C_D = 0.4$

$$V = \sqrt{\frac{8 \cdot F_D}{C_D \cdot \rho_{air} \cdot \pi \cdot D^2}} \qquad V = 6.15\,\frac{ft}{s}$$

The Reynolds number is $Re_d = \frac{V \cdot D}{\nu}$ $Re_d = 2.83 \times 10^4$ We are okay!

For the next balloon $\theta = 35 \cdot deg$ $F_D = (F_{Bnet} - W_{latex}) \cdot \tan(\theta)$ $F_D = 0.00544\,lbf$ with $C_D = 0.4$

$$V = \sqrt{\frac{8 \cdot F_D}{C_D \cdot \rho_{air} \cdot \pi \cdot D^2}} \qquad V = 5.14\,\frac{ft}{s}$$

The Reynolds number is $Re_d = \frac{V \cdot D}{\nu}$ $Re_d = 2.37 \times 10^4$ We are okay!

For the next balloon $\theta = 30 \cdot deg$ $F_D = (F_{Bnet} - W_{latex}) \cdot \tan(\theta)$ $F_D = 0.00449\,lbf$ with $C_D = 0.4$

$$V = \sqrt{\frac{8 \cdot F_D}{C_D \cdot \rho_{air} \cdot \pi \cdot D^2}} \qquad V = 4.67\,\frac{ft}{s}$$

The Reynolds number is $Re_d = \frac{V \cdot D}{\nu}$ $Re_d = 2.15 \times 10^4$ We are okay!

For the next balloon $\theta = 20 \cdot deg$ $F_D = (F_{Bnet} - W_{latex}) \cdot \tan(\theta)$ $F_D = 0.00283\,lbf$ with $C_D = 0.4$

$$V = \sqrt{\frac{8 \cdot F_D}{C_D \cdot \rho_{air} \cdot \pi \cdot D^2}} \qquad V = 3.71\,\frac{ft}{s}$$

The Reynolds number is $Re_d = \frac{V \cdot D}{\nu}$ $Re_d = 1.71 \times 10^4$ We are okay!

For the next balloon $\theta = 10 \cdot deg$ $F_D = (F_{Bnet} - W_{latex}) \cdot \tan(\theta)$ $F_D = 0.00137\,lbf$ with $C_D = 0.4$

$$V = \sqrt{\frac{8 \cdot F_D}{C_D \cdot \rho_{air} \cdot \pi \cdot D^2}} \qquad V = 2.58\,\frac{ft}{s}$$

The Reynolds number is $Re_d = \frac{V \cdot D}{\nu}$ $Re_d = 1.19 \times 10^4$ We are okay!

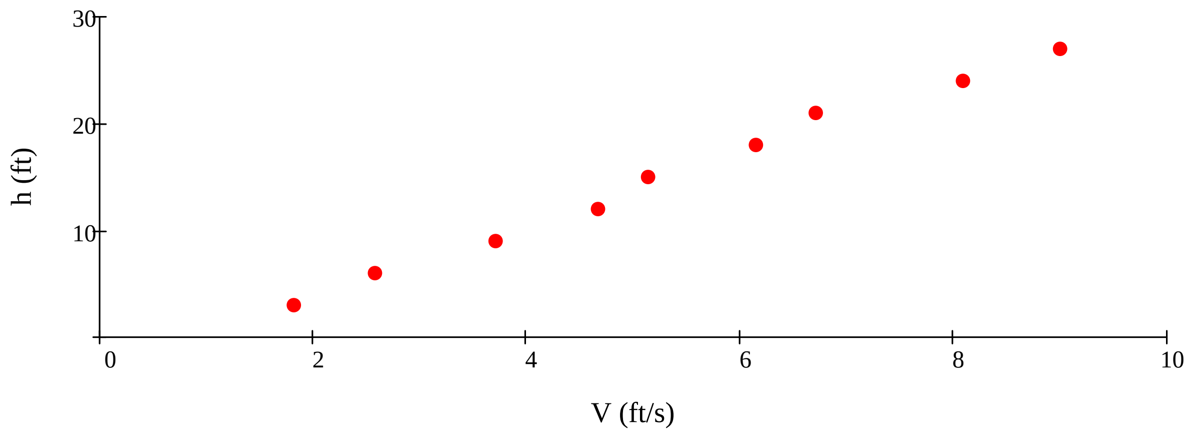
For the next balloon $\theta = 5 \cdot deg$ $F_D = (F_{Bnet} - W_{latex}) \cdot \tan(\theta)$ $F_D = 0.000680\,lbf$ with $C_D = 0.4$

$$V = \sqrt{\frac{8 \cdot F_D}{C_D \cdot \rho_{air} \cdot \pi \cdot D^2}} \qquad V = 1.82\,\frac{ft}{s}$$

The Reynolds number is $Re_d = \frac{V \cdot D}{\nu}$ $\qquad Re_d = 8367.80$ $\qquad$ We are okay!

In summary we have $V = (1.82 \;\; 2.58 \;\; 3.71 \;\; 4.67 \;\; 5.14 \;\; 6.15 \;\; 6.71 \;\; 8.09 \;\; 9.00) \cdot \frac{ft}{s}$

$h = (3 \;\; 6 \;\; 9 \;\; 12 \;\; 15 \;\; 18 \;\; 21 \;\; 24 \;\; 27) \cdot ft$

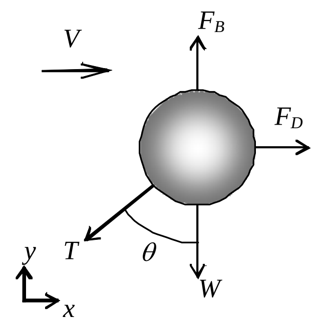


This problem is ideal for computing and plotting in *Excel*

Problem 9.104

[2]

9.104 A 1-ft diameter hollow plastic sphere containing pollution test equipment is being dragged through the Hudson River in New York by a diver riding an underwater jet device. The sphere (with an effective specific gravity of SG = 0.25) is fully submerged, and is tethered to the diver by a thin 4-ft long wire. What is the relative velocity of the diver and sphere if the angle the wire makes with the horizontal is 45°? The water is at 50°F.



Given: Sphere dragged through river

Find: Relative velocity of sphere

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$ $\quad F_B = \rho\cdot g\cdot Vol$ $\quad \Sigma\vec{F} = 0$

The above figure applies to the sphere

For the horizontal forces $\quad F_D - T\cdot\sin(\theta) = 0 \qquad (1)$

For the vertical forces $\quad -T\cdot\cos(\theta) + F_B - W = 0 \qquad (2)$

Here $\quad D = 1\cdot ft \quad SG = 0.25 \quad$ and from Table A.7 $\quad \nu = 1.41\times10^{-5}\cdot\dfrac{ft^2}{s} \quad \rho = 1.94\cdot\dfrac{slug}{ft^3}$

Applying Eqs 1 and 2 to the sphere, for which $\quad \theta = 45\cdot deg$

$$F_D = T\cdot\sin(\theta) = \frac{F_B - W}{\cos(\theta)}\cdot\sin(\theta) = \rho\cdot g\cdot Vol\cdot(1 - SG)\cdot\tan(\theta)$$

Hence $\quad F_D = \rho\cdot g\cdot\dfrac{\pi\cdot D^3}{6}\cdot(1 - SG)\cdot\tan(\theta) \qquad F_D = 24.5\cdot lbf$

But we have $\quad F_D = C_D\cdot\dfrac{1}{2}\cdot\rho\cdot V^2\cdot A = C_D\cdot\dfrac{1}{2}\cdot\rho\cdot V^2\cdot\dfrac{\pi\cdot D^2}{4} \quad$ with $\quad C_D = 0.4 \quad$ from Fig. 9.11 (we will check Re later)

$$V = \sqrt{\frac{8\cdot F_D}{C_D\cdot\rho\cdot\pi\cdot D^2}} \qquad V = 8.97\cdot\frac{ft}{s}$$

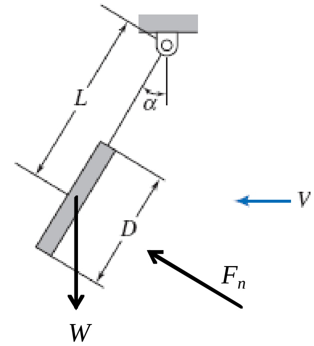
The Reynolds number is $\quad Re_d = \dfrac{V\cdot D}{\nu} \qquad Re_d = 6.36\times10^5 \quad$ A bit off from Fig 9.11

Try $\quad C_D = 0.15 \qquad V = \sqrt{\dfrac{8\cdot F_D}{C_D\cdot\rho\cdot\pi\cdot D^2}} \qquad V = 14.65\cdot\dfrac{ft}{s}$

The Reynolds number is $\quad Re_d = \dfrac{V\cdot D}{\nu} \qquad Re_d = 1.04\times10^6 \quad$ A good fit with Fig 9.11 (extreme right of graph)

Problem 9.105

*9.105 A circular disk is hung in an air stream from a pivoted strut as shown. In a wind-tunnel experiment, performed in air at 15 m/s with a 25-mm diameter disk, α was measured at 10°. For these conditions determine the mass of the disk. Assume the drag coefficient for the disk applies when the component of wind speed normal to the disk is used. Assume drag on the strut and friction in the pivot are negligible. Plot a theoretical curve of α as a function of air speed.



Given: Circular disk in wind

Find: Mass of disk; Plot α versus V

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$ $\qquad \Sigma\vec{M} = 0$

Summing moments at the pivot $W\cdot L\cdot\sin(\alpha) - F_n\cdot L = 0$ and $F_n = \frac{1}{2}\cdot\rho\cdot V_n^{\,2}\cdot A\cdot C_D$

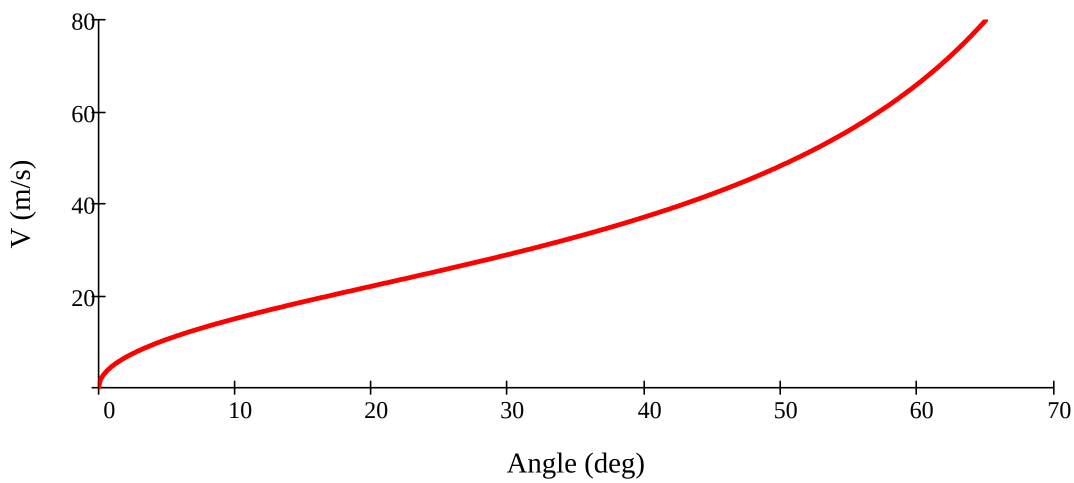
Hence $M\cdot g\cdot\sin(\alpha) = \frac{1}{2}\cdot\rho\cdot(V\cdot\cos(\alpha))^2\cdot\dfrac{\pi\cdot D^2}{4}\cdot C_D$

The data is $\rho = 1.225\cdot\dfrac{kg}{m^3}$ $\quad V = 15\cdot\dfrac{m}{s}$ $\quad D = 25\cdot mm$ $\quad \alpha = 10\cdot deg$ $\quad C_D = 1.17$ (Table 9.3)

$$M = \frac{\pi\cdot\rho\cdot V^2\cdot\cos(\alpha)^2\cdot D^2\cdot C_D}{8\cdot g\cdot\sin(\alpha)} \qquad M = 0.0451\,kg$$

Rearranging $V = \sqrt{\dfrac{8\cdot M\cdot g}{\pi\cdot\rho\cdot D^2\cdot C_D}}\cdot\sqrt{\dfrac{\tan(\alpha)}{\cos(\alpha)}}$ $\qquad V = 35.5\cdot\dfrac{m}{s}\cdot\sqrt{\dfrac{\tan(\alpha)}{\cos(\alpha)}}$

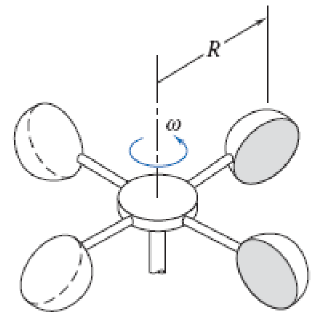
We can plot this by choosing α and computing V



This graph can be easily plotted in *Excel*

Problem 9.106

9.106 An anemometer to measure wind speed is made from four hemispherical cups of 50 mm diameter, as shown. The center of each cup is placed at $R = 80$ mm from the pivot. Find the theoretical calibration constant k in the calibration equation $V = k\omega$, where V (km/hr) is the wind speed and ω (rpm) is the rotation speed. In your analysis base the torque calculations on the drag generated at the instant when two of the cups are orthogonal, and the other two cups are parallel, and ignore friction in the bearings. Explain why, in the absence of friction, at any given wind speed, the anemometer runs at constant speed rather than accelerating without limit. If the actual anemometer bearing has (constant) friction such that the anemometer needs a minimum wind speed of 1 km/hr to begin rotating, compare the rotation speeds with and without friction for $V = 10$ km/hr.



Given: Data on dimensions of anemometer

Find: Calibration constant; compare to actual with friction

Solution:

The given data or available data is $D = 50 \cdot mm$ $\quad R = 80 \cdot mm$ $\quad \rho = 1.23 \cdot \frac{kg}{m^3}$

The drag coefficients for a cup with open end facing the airflow and a cup with open end facing downstream are, respectively, from Table 9

$$C_{Dopen} = 1.42 \qquad C_{Dnotopen} = 0.38$$

The equation for computing drag is $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$ (1)

where $A = \frac{\pi \cdot D^2}{4}$ $\qquad A = 1.96 \times 10^{-3} m^2$

Assuming steady speed ω at steady wind speed *V* the sum of moments will be zero. The two cups that are momentarily parallel to the flow will exert no moment; the two cups with open end facing and not facing the flow will exert a moment beacuse of their drag forces. For eac the drag is based on Eq. 1 (with the *relative* velocity used!). In addition, friction of the anemometer is neglected

$$\Sigma M = 0 = \left[\frac{1}{2} \cdot \rho \cdot A \cdot (V - R \cdot \omega)^2 \cdot C_{Dopen}\right] \cdot R - \left[\frac{1}{2} \cdot \rho \cdot A \cdot (V + R \cdot \omega)^2 \cdot C_{Dnotopen}\right] \cdot R$$

or $$(V - R \cdot \omega)^2 \cdot C_{Dopen} = (V + R \cdot \omega)^2 \cdot C_{Dnotopen}$$

This indicates that the anemometer reaches a steady speed even in the abscence of friction because it is the *relative* velocity on each cup that matters: the cup that has a higher drag coefficient has a lower relative velocity

Rearranging for $k = \frac{V}{\omega}$ $\qquad \left(\frac{V}{\omega} - R\right)^2 \cdot C_{Dopen} = \left(\frac{V}{\omega} + R\right)^2 \cdot C_{Dnotopen}$

Hence $k = \dfrac{\left(1 + \sqrt{\dfrac{C_{Dnotopen}}{C_{Dopen}}}\right)}{\left(1 - \sqrt{\dfrac{C_{Dnotopen}}{C_{Dopen}}}\right)} \cdot R$ $\quad k = 0.251\,m \quad k = 0.0948\,\dfrac{\frac{km}{hr}}{rpm}$

For the actual anemometer (with friction), we first need to determine the torque produced when the anemometer is stationary but about to rotate

Minimum wind for rotation is $V_{min} = 1 \cdot \dfrac{km}{hr}$

The torque produced at this wind speed is

$$T_f = \left(\frac{1}{2} \cdot \rho \cdot A \cdot V_{min}{}^2 \cdot C_{Dopen}\right) \cdot R - \left(\frac{1}{2} \cdot \rho \cdot A \cdot V_{min}{}^2 \cdot C_{Dnotopen}\right) \cdot R$$

$$T_f = 7.75 \times 10^{-6}\,N \cdot m$$

A moment balance at wind speed *V*, including this friction, is

$$\Sigma M = 0 = \left[\frac{1}{2} \cdot \rho \cdot A \cdot (V - R \cdot \omega)^2 \cdot C_{Dopen}\right] \cdot R - \left[\frac{1}{2} \cdot \rho \cdot A \cdot (V + R \cdot \omega)^2 \cdot C_{Dnotopen}\right] \cdot R - T_f$$

or $(V - R \cdot \omega)^2 \cdot C_{Dopen} - (V + R \cdot \omega)^2 \cdot C_{Dnotopen} = \dfrac{2 \cdot T_f}{R \cdot \rho \cdot A}$

This quadratic equation is to be solved for ω when $V = 10 \cdot \dfrac{km}{hr}$

After considerable calculations $\omega = 104\,rpm$

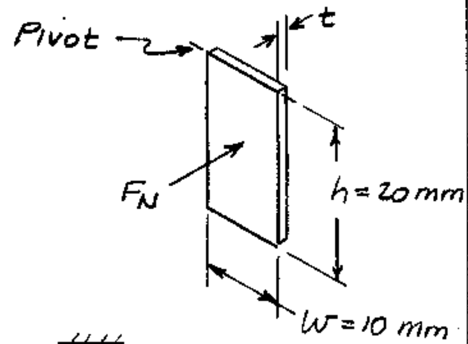
This must be compared to the rotation for a frictionless model, given by

$$\omega_{frictionless} = \frac{V}{k} \qquad \omega_{frictionless} = 105\,rpm$$

The error in neglecting friction is $\left|\dfrac{\omega - \omega_{frictionless}}{\omega}\right| = 1.12\,\%$

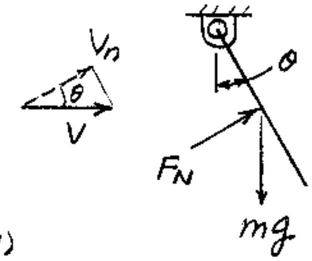
Problem 9.107

Given: Single-vane anemometer made from brass plate, t thick, with $h = 20$ mm and $w = 10$ mm.



Find: (a) Relationship for wind speed as a function of deflection angle, θ.
(b) Plate thickness to give $\theta = 30°$ at $V = 10$ m/s.

Solution: Sum moments about pivot.



$$\Sigma M = F_N \frac{h}{2} - mg\frac{h}{2}\sin\theta = 0$$

$$F_N = C_D A \frac{1}{2}\rho V_n^2 = mg\sin\theta$$

$$C_D A \frac{1}{2}\rho V^2 \cos^2\theta = mg\sin\theta \qquad (1)$$

$$V = \left[\frac{2mg\sin\theta}{C_D A \rho \cos^2\theta}\right]^{1/2}$$ V

From plate geometry, $m = \rho w h t = SG\rho_{H_2O} w h t$. From Eq. 1,

$$SG\rho_{H_2O} g w h t \sin\theta = C_D A \frac{1}{2}\rho V^2 \cos^2\theta \qquad \{\text{From Table A.1, } SG = 8.55 \text{ for brass.}\}$$

$$t = \frac{C_D A \rho V^2 \cos^2\theta}{2SG\rho_{H_2O} w h \sin\theta g} = \frac{C_D \rho V^2 \cos^2\theta}{2SG\rho_{H_2O}\sin\theta g} \qquad \text{since } A = wh$$

From Fig. 9.10, $C_D = 1.2$ at $b/h = 2.0$, so

$$t = \frac{1.2}{8.55} \times \frac{1.23}{2}\frac{\text{kg}}{\text{m}^3} \times (10)^2\frac{\text{m}^2}{\text{s}^2} \times \cos^2(30°) \times \frac{\text{m}^3}{999\text{ kg}} \times \frac{1}{\sin(30°)} \times \frac{\text{s}^2}{9.81\text{ m}} \times 1000\frac{\text{mm}}{\text{m}}$$

$$t = 1.30 \text{ mm}$$ t

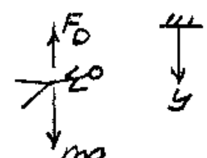
Problem 9.108

Given: Experimental data for a sky diver with $M = 75$ kg:

Prone, spread-eagled	$C_D A = 0.85\ m^2$
Vertical fall	$C_D A = 0.11\ m^2$

Find: Estimate time and distance needed to reach 95 percent of terminal speed at 3000 m altitude on a standard day.

Solution: From Table A.3, $\rho/\rho_{SL} = 0.7423$ at 3000 m altitude.
Consider free-body diagram of sky diver:



$$\Sigma F_y = mg - F_D = mg - C_D A \tfrac{1}{2}\rho V^2 = m a_y = m\frac{dV}{dt} = mV\frac{dV}{dy}$$

At terminal speed, $a_y = 0$. Then $mg = C_D A \frac{1}{2}\rho V_t^2$

so $V_t^2 = \dfrac{2mg}{C_D A \rho}$. From above, $\dfrac{1}{g}\dfrac{dV}{dt} = \dfrac{V_t}{g}\dfrac{d(V/V_t)}{dt} = 1 - \dfrac{C_D A \rho V^2}{2mg} = 1 - \left(\dfrac{V}{V_t}\right)^2$

Thus $$\frac{d(V/V_t)}{1-(V/V_t)^2} = \frac{g}{V_t}\,dt$$

Integrating $$\int_0^{0.95} \frac{d(V/V_t)}{1-(V/V_t)^2} = \tanh^{-1}\left(\frac{V}{V_t}\right)\Big]_0^{0.95} = 1.83 = \frac{gt}{V_t}\ ;\ t = \frac{1.83\,V_t}{g}$$ t

Also $$1 - \left(\frac{V}{V_t}\right)^2 = \frac{V}{g}\frac{dV}{dy} = \frac{V_t^2}{g}\left(\frac{V}{V_t}\right)\frac{d(V/V_t)}{dy} \quad \text{or} \quad \frac{(V/V_t)\,d(V/V_t)}{1-(V/V_t)^2} = \frac{g}{V_t^2}\,dy$$

Integrating, $$\int_0^{0.95} \frac{(V/V_t)\,d(V/V_t)}{1-(V/V_t)^2} = -\frac{1}{2}\ln\left[1-(V/V_t)^2\right]_0^{0.95} = 1.16 = \frac{g}{V_t^2}\,y\ ;\ y = \frac{1.16\,V_t^2}{g}$$ y

Calculating for $C_D A = 0.85\ m^2$:

$$V_t = \left[2 \times 75\,\text{kg} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times \frac{1}{0.85\,\text{m}^2} \times \frac{\text{m}^3}{(0.7423)\,1.23\,\text{kg}}\right]^{1/2} = 43.5\ \text{m/s}$$

$$t = \frac{1.83\,V_t}{g} = 1.83 \times 43.5\,\frac{\text{m}}{\text{s}} \times \frac{\text{s}^2}{9.81\,\text{m}} = 8.11\ \text{s}$$

$$y = \frac{1.16\,V_t^2}{g} = 1.16\,(43.5)^2\,\frac{\text{m}^2}{\text{s}^2} \times \frac{\text{s}}{9.81\,\text{m}} = 224\ \text{m}$$

Tabulating:

Position	V_t (m/s)	t (s)	y (m)
Prone	43.5	8.11	224
Vertical*	121	22.6	1,730

* These are estimates; density would vary significantly during this fall.

Problem 9.109

Given: F-4 aircraft slowed by dual parachutes, each 12 ft in diameter. Craft weighs 32,000 lbf, lands at 160 kt. Neglect drag of aircraft; brakes not applied.

Find: Time required to decelerate to 100 kt.

Solution: Apply Newton's second law of motion, definition of C_D.

Basic equations: $\Sigma F_x = ma_x$

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

Then

$$\Sigma F_x = -2F_D = -C_D \rho V^2 A = ma_x = \frac{W}{g}\frac{dV}{dt} \qquad (1)$$

or $\frac{dV}{V^2} = -\frac{C_D \rho g A}{W} dt$

Integrating,

$$\int_{V_i}^{V_f} \frac{dV}{V^2} = -\frac{1}{V}\Big]_{V_i}^{V_f} = \frac{1}{V_i} - \frac{1}{V_f} = \int_0^t -\frac{C_D \rho g A}{W} dt = -\frac{C_D \rho g A}{W} t$$

or

$$t = \frac{W}{C_D \rho g A}\left[\frac{1}{V_f} - \frac{1}{V_i}\right]$$

Since two chutes (assume hemispheres), $A = 2\left(\frac{\pi D^2}{4}\right) = \frac{\pi D^2}{2}$

From Table 9.3, $C_D = 1.42$ for hemisphere facing stream. For standard air, $\rho g = \gamma \simeq 0.075$ lbf/ft^3, and

$$t = 32{,}000\,\text{lbf} \times \frac{1}{1.42} \times \frac{\text{ft}^3}{0.075\,\text{lbf}} \times \frac{2}{\pi} \times \frac{1}{(12)^2\,\text{ft}^2}\left[\frac{1}{100} - \frac{1}{160}\right]\frac{\text{hr}}{\text{nm}} \times 3600\frac{\text{s}}{\text{hr}} \times \frac{\text{nm}}{6080\,\text{ft}}$$

or

$$t = 2.45\ \text{s}$$ t

To find distance, set $a_x = \frac{dV}{dt} = V\frac{dV}{dx}$. Then, from Eq. 1,

$$-2C_D \rho \frac{V^2}{2} A = \frac{W}{g} V \frac{dV}{dx}$$

and $\frac{dV}{V} = -\frac{C_D A \rho g}{W} dx$

Integrating,

$$\int_{V_i}^{V_f} \frac{dV}{V} = \ln V\Big]_{V_i}^{V_f} = \ln\frac{V_f}{V_i} = -\frac{C_D A \rho g}{W} x \quad \text{or} \quad x = -\frac{W}{C_D A \rho g}\ln\frac{V_f}{V_i}$$

Thus

$$x = -\frac{1}{1.42} \times 32{,}000\,\text{lbf} \times \frac{2}{\pi (12)^2\,\text{ft}^2} \times \frac{\text{ft}^3}{0.075\,\text{lbf}} \times \ln\left(\frac{100}{160}\right) = 624\ \text{ft}$$ x

Problem 9.110

Given: Land speed record vehicle at Bonneville Salt Flats, elevation 4400 ft. Engine power, $\mathbb{P} = 500$ hp, frontal area, $A = 15\ ft^2$, and $C_D = 0.15$.

Find: Theoretical maximum speed (a) in still air, (b) 20 mph head wind.

<u>Solution</u>: Apply definitions of power, drag coefficient.

Computing equations: $\mathbb{P} = F_D V$, $C_D = \dfrac{F_D}{\frac{1}{2}\rho (V+V_w)^2 A}$

Assumptions: (1) Neglect rolling drag
(2) $\rho \simeq 0.878\,\rho_0$ (Table A.3)

For no wind case, $V_w = 0$, and

$$\mathbb{P} = F_D V = C_D \tfrac{1}{2}\rho V^2 A V = C_D \tfrac{1}{2}\rho V^3 A$$

$$V = \left[\frac{2\mathbb{P}}{\rho C_D A}\right]^{1/3} = \left[2 \times 500\ hp \times \frac{ft^3}{(0.878)0.00238\ slug} \times \frac{1}{0.15} \times \frac{1}{15\ ft^2} \times 550\,\frac{ft \cdot lbf}{hp \cdot s} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{1/3}$$

$V = 489$ ft/s (333 mph) ← V

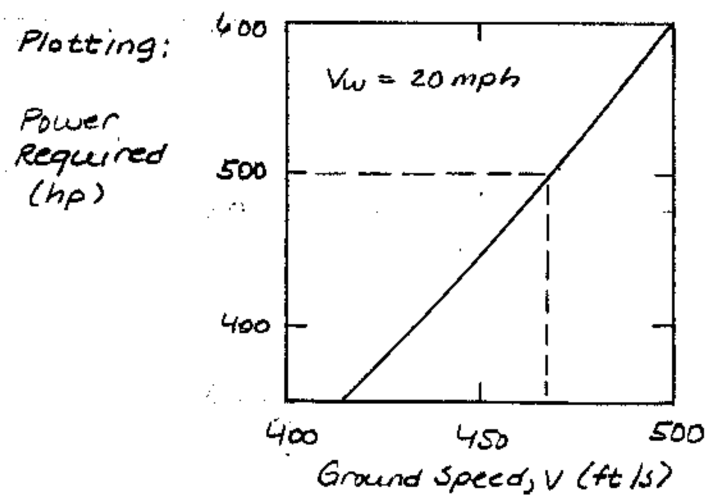
With a head wind,

$$\mathbb{P} = F_D V = C_D \tfrac{1}{2}\rho (V+V_w)^2 A V \quad or \quad \mathbb{P}(hp) = 4.27 \times 10^{-6} (V+V_w)^2 V \ (ft^3/s^3)$$

This can be solved by iteration. Using $V_w = 20$ mph or 29.3 ft/s,

V (ft/s)	$\mathbb{P}$ (hp)
400	315
470	501
500	599

Plotting:



From the plot, $V \simeq 468$ ft/s (319 mph) ← V

{ Note that the maximum speed is not reduced by 20 mph when wind is present, because drag is nonlinear. }

Problem 9.111

[2]

9.111 Compare and plot the power (hp) required by a typical large American sedan of the 1970s and a current midsize sedan to overcome aerodynamic drag versus road speed in standard air, for a speed range of 20 mph to 100 mph. Use the following as representative values:

	Weight (lbf)	Drag Coefficient	Frontal Area (ft^2)
1970's Sedan	4500	0.5	24
Current Sedan	3500	0.3	20

If rolling resistance is 1.5 percent of curb weight, determine for each vehicle the speed at which the aerodynamic force exceeds frictional resistance.

Given: Data on 1970's and current sedans

Find: Plot of power versus speed; Speeds at which aerodynamic drag exceeds rolling drag

Solution:

Basic equation: $$C_D = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A}$$

The aerodynamic drag is $F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot V^2\cdot A$ The rolling resistance is $F_R = 0.015\cdot W$

Total resistance $F_T = F_D + F_R$

	1970's Sedan		Current Sedan	
W =	4500	lbf	3500	lbf
C_D =	0.5		0.3	
A =	24	ft^2	20	ft^2
ρ =	0.00234	$slug/ft^3$	(Table A.9)	

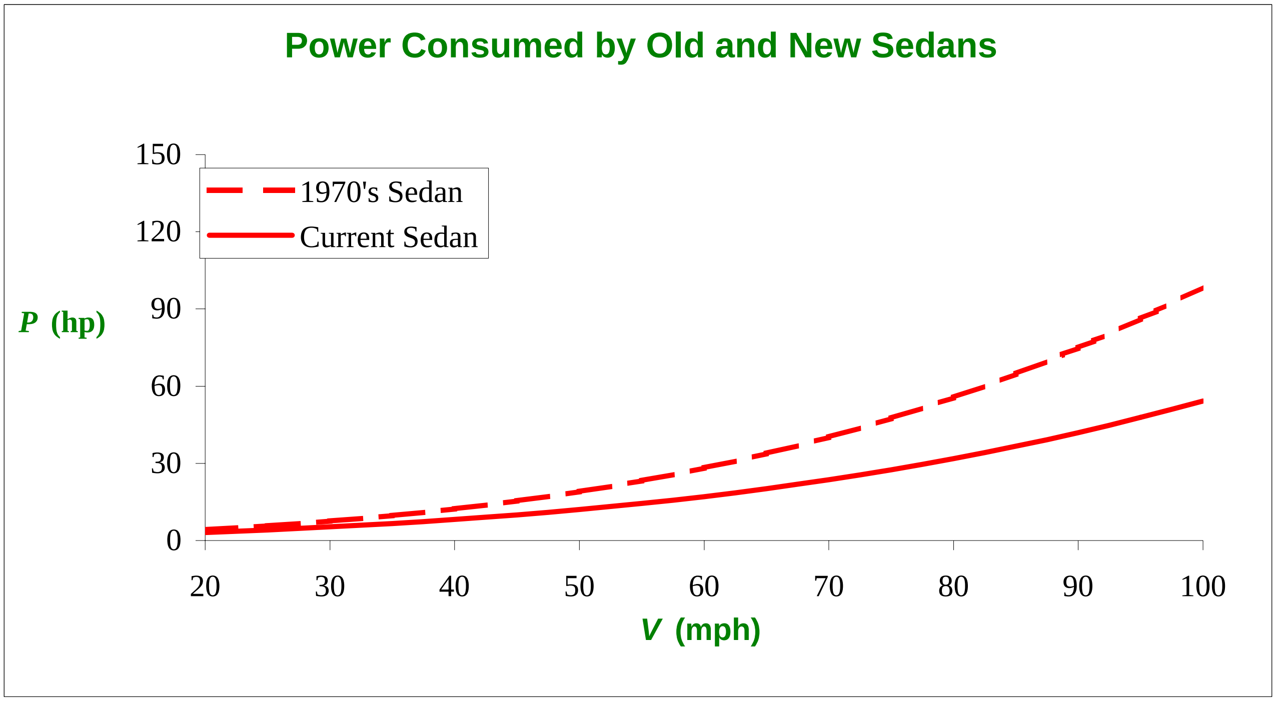
Computed results:

	1970's Sedan			Current Sedan		
V (mph)	F_D (lbf)	F_T (lbf)	P (hp)	F_D (lbf)	F_T (lbf)	P (hp)
20	12.1	79.6	4.24	6.04	58.5	3.12
25	18.9	86.4	5.76	9.44	61.9	4.13
30	27.2	94.7	7.57	13.6	66.1	5.29
35	37.0	104	9.75	18.5	71.0	6.63
40	48.3	116	12.4	24.2	76.7	8.18
45	61.2	129	15.4	30.6	83.1	10.0
50	75.5	143	19.1	37.8	90.3	12.0
55	91.4	159	23.3	45.7	98.2	14.4
60	109	176	28.2	54.4	107	17.1
65	128	195	33.8	63.8	116	20.2
70	148	215	40.2	74.0	126	23.6
75	170	237	47.5	84.9	137	27.5
80	193	261	55.6	96.6	149	31.8
85	218	286	64.8	109	162	36.6
90	245	312	74.9	122	175	42.0
95	273	340	86.2	136	189	47.8
100	302	370	98.5	151	204	54.3

V (mph)	F_D (lbf)	F_R (lbf)
47.3	67.5	67.5

V (mph)	F_D (lbf)	F_R (lbf)
59.0	52.5	52.5

The two speeds above were obtained using *Solver*



Problem 9.112

[3]

9.112 A bus travels at 50 mph in standard air. The frontal area of the vehicle is 80 ft^2, and the drag coefficient is 0.95. How much power is required to overcome aerodynamic drag? Estimate the maximum speed of the bus if the engine is rated at 450 hp. A young engineer proposes adding fairings on the front and rear of the bus to reduce the drag coefficient. Tests indicate that this would reduce the drag coefficient to 0.85 without changing the frontal area. What would be the required power at 50 mph, and the new top speed? If the fuel cost for the bus is currently $200/day, how long would the modification take to pay for itself if it costs $4,500 to install?

Given: Data on a bus

Find: Power to overcome drag; Maximum speed; Recompute with new fairing; Time for fairing to pay for itself

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad P = F_D\cdot V$

The given data or available data is $V = 50\cdot mph$ $\quad V = 73.3\frac{ft}{s}$ $\quad A = 80\cdot ft^2$ $\quad C_D = 0.95$ $\quad \rho = 0.00234\cdot\frac{slug}{ft^3}$

$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad F_D = 478\,lbf$ $\quad P = F_D\cdot V$ $\quad P = 3.51\times10^4\frac{ft\cdot lbf}{s}$ $\quad P = 63.8\,hp$

The power available is $P_{max} = 450\cdot hp$

The maximum speed corresponding to this maximum power is obtained from

$P_{max} = \left(\frac{1}{2}\cdot\rho\cdot A\cdot V_{max}{}^2\cdot C_D\right)\cdot V_{max}$ or $V_{max} = \left(\frac{P_{max}}{\frac{1}{2}\cdot\rho\cdot A\cdot C_D}\right)^{\frac{1}{3}}$ $\quad V_{max} = 141\frac{ft}{s}$ $\quad V_{max} = 95.9\,mph$

We repeat these calculations with the new fairing, for which $C_D = 0.85$

$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad F_D = 428\,lbf$ $\quad P_{new} = F_D\cdot V$ $\quad P_{new} = 3.14\times10^4\frac{ft\cdot lbf}{s}$ $\quad P_{new} = 57.0\,hp$

The maximum speed is now $V_{max} = \left(\frac{P_{max}}{\frac{1}{2}\cdot\rho\cdot A\cdot C_D}\right)^{\frac{1}{3}}$ $\quad V_{max} = 146\frac{ft}{s}$ $\quad V_{max} = 99.5\,mph$

The initial cost of the fairing is $Cost = 4500\cdot dollars$ $\quad$ The fuel cost is $Cost_{day} = 200\cdot\frac{dollars}{day}$

The cost per day is reduced by improvement in the bus performance at 50 mph $Gain = \frac{P_{new}}{P}$ $\quad Gain = 89.5\,\%$

The new cost per day is then $Cost_{daynew} = Gain\cdot Cost_{day}$ $\quad Cost_{daynew} = 179\frac{dollars}{day}$

Hence the savings per day is $Saving = Cost_{day} - Cost_{daynew}$ $\quad Saving = 21.1\frac{dollars}{day}$

The initial cost will be paid for in $\tau = \frac{Cost}{Saving}$ $\quad \tau = 7.02\,month$

Problem 9.113

Given: Tractor-trailer rig, with $A = 102\ ft^2$; $C_D = 0.9$. Rolling resistance is 6 lbf per 1000 lbf; $W = 72{,}000$ lbf. BSFC is 0.34 lbm/hp·hr, $\eta_d = 0.92$, and $\rho = 6.9$ lbm/gal. Truck travels 120,000 mi/yr.

Find: (a) Estimate fuel economy at 55 mph.
(b) Fuel saved by air deflector that reduces C_D by 15 percent.

Solution: Tractive force is $F_T = F_R + F_D$ ← aerodynamic force; F_R ← rolling resistance force

Engine power is $P_e = \dfrac{P_T}{\eta_d} = \dfrac{F_T V}{\eta_d}$

Thus

$$F_R = C_R W = 0.006 \times 72{,}000\ lbf = 432\ lbf$$

$$F_D = C_D A \tfrac{1}{2}\rho V^2 \qquad V = 55\ \frac{mi}{hr} \times 5280\ \frac{ft}{mi} \times \frac{hr}{3600\ s} = 80.7\ ft/s$$

$$F_D = 0.9 \times 102\ ft^2 \times \frac{1}{2} \times 0.00238\ \frac{slug}{ft^3} \times (80.7)^2 \frac{ft^2}{s^2} \times \frac{lbf \cdot s^2}{slug \cdot ft} = 711\ lbf$$

so

$$F_T = F_R + F_D = 432 + 711 = 1140\ lbf$$

$$P_e = 1140\ lbf \times 80.7\ \frac{ft}{s} \times \frac{1}{0.92} \times \frac{hp \cdot s}{550\ ft \cdot lbf} = 182\ hp$$

Finally

$$FE = \frac{\rho V}{P_e\, BSFC} = 6.9\ \frac{lbm}{gal} \times 55\ \frac{mi}{hr} \times \frac{1}{182\ hp} \times \frac{hp \cdot hr}{0.34\ lbm} = 6.13\ mi/gal$$ FE

With the air deflector,

$$F_D = (1 - 0.15)\, 711\ lbf = 604\ lbf$$

$$F_T = F_R + F_D = 432 + 604 = 1040\ lbf$$

$$P_e = 1040\ lbf \times 80.7\ \frac{ft}{s} \times \frac{1}{0.92} \times \frac{hp \cdot s}{550\ ft \cdot lbf} = 166\ hp$$

and $$FE = 6.9\ \frac{lbm}{gal} \times 55\ \frac{mi}{hr} \times \frac{1}{166\ hp} \times \frac{hp \cdot hr}{0.34\ lbm} = 6.72\ mi/gal$$

The fuel saving would be

$$\Delta Q = \left(\frac{1}{FE}\Big)_{without} - \frac{1}{FE_{with}}\right) \text{mileage} = \left(\frac{1}{6.13} - \frac{1}{6.72}\right) \frac{gal}{mi} \times 120{,}000\ \frac{mi}{yr} = 1720\ gal/yr$$ Q

The percentage saving would be

$$\frac{\Delta Q}{Q} = \frac{\left(\frac{1}{FE_{without}} - \frac{1}{FE_{with}}\right)}{\frac{1}{FE}\Big)_{without}} = 0.0878 \text{ or } 8.78 \text{ percent savings}$$

Problem 9.114

9.114 A 165 hp sports car of frontal area 18.5 ft^2, with a drag coefficient of 0.32, requires 12 hp to cruise at 55 mph. At what speed does aerodynamic drag first exceed rolling resistance? (The rolling resistance is 1% of the car weight, and the car mass is 2750 lb.) Find the drivetrain efficiency. What is the maximum acceleration at 55 mph? What is the maximum speed? Which redesign will lead to a higher maximum speed: improving the drive train efficiency by 5% from its current value, reducing the drag coefficient to 0.29, or reducing the rolling resistance to 0.93% of the car weight?

Given: Data on a sports car

Find: Speed for aerodynamic drag to exceed rolling resistance; maximum speed & acceleration at 55 mph; Redesign change that has greatest effect

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad P = F_D\cdot V$

The given data or available data is $M = 2750\cdot lbm$ $\quad A = 18.5\cdot ft^2$ $\quad C_D = 0.32$

$P_{engine} = 165\cdot hp$ $\quad F_R = 0.01\times M\cdot g$ $\quad \rho = 0.00234\cdot\frac{slug}{ft^3}$

The rolling resistance is then $F_R = 27.5\,lbf$

To find the speed at which aerodynamic drag first equals rolling resistance, set the two forces equal $\frac{1}{2}\cdot\rho\cdot V^2\cdot A\cdot C_D = F_R$

Hence $V = \sqrt{\frac{2\cdot F_R}{\rho\cdot A\cdot C_D}}$ $\quad V = 63.0\frac{ft}{s}$ $\quad V = 43.0\,mph$

To find the drive train efficiency we use the data at a speed of 55 mph $V = 55\cdot mph$ $\quad V = 80.7\frac{ft}{s}$ $\quad P_{engine} = 12\cdot hp$

The aerodynamic drag at this speed is $F_D = \frac{1}{2}\cdot\rho\cdot V^2\cdot A\cdot C_D$ $\quad F_D = 45.1\,lbf$

The power consumed by drag and rolling resistance at this speed is $P_{used} = \left(F_D + F_R\right)\cdot V$ $\quad P_{used} = 10.6\,hp$

Hence the drive train efficiency is $\eta = \frac{P_{used}}{P_{engine}}$ $\quad \eta = 88.7\%$

The acceleration is obtained from Newton's second lav $M\cdot a = \Sigma F = T - F_R - F_D$

where *T* is the thrust produced by the engine, given by $T = \frac{P}{V}$

The maximum acceleration at 55 mph is when we have maximum thrust, when full engine power is used. $P_{engine} = 165\cdot hp$

Because of drive train inefficiencies the maximum power at the wheels $P_{max} = \eta\cdot P_{engine}$ $\quad P_{max} = 146\,hp$

Hence the maximum thrust is $T_{max} = \frac{P_{max}}{V}$ $\quad T_{max} = 998\,lbf$

The maximum acceleration at 55 mph is then $a_{max} = \frac{T_{max} - F_D - F_R}{M}$ $\quad a_{max} = 10.8\frac{ft}{s^2}$

The maximum speed is obtained when the maximum engine power is just balanced by power consumed by drag and rolling resistance

For maximum speed: $$P_{max} = \left(\frac{1}{2}\cdot\rho\cdot V_{max}^{\;2}\cdot A\cdot C_D + F_R\right)\cdot V_{max}$$

This is a cubic equation that can be solved by iteration or by using *Excel*'s *Goal Seek* or *Solver* $V_{max} = 150\,mph$

We are to evaluate several possible improvements:

For improved drive train $\eta = \eta + 5\cdot\%$ $\eta = 93.7\%$ $P_{max} = \eta\cdot P_{engine}$ $P_{max} = 155\,hp$

$$P_{max} = \left(\frac{1}{2}\cdot\rho\cdot V_{max}^{\;2}\cdot A\cdot C_D + F_R\right)\cdot V_{max}$$

Solving the cubic (using *Solver*) $V_{max} = 153\,mph$

Improved drag coefficient: $C_{Dnew} = 0.29$

$$P_{max} = \left(\frac{1}{2}\cdot\rho\cdot V_{max}^{\;2}\cdot A\cdot C_{Dnew} + F_R\right)\cdot V_{max}$$

Solving the cubic (using *Solver*) $V_{max} = 158\,mph$ This is the best option!

Reduced rolling resistance: $F_{Rnew} = 0.93\cdot\%\cdot M\cdot g$ $F_{Rnew} = 25.6\,lbf$

$$P_{max} = \left(\frac{1}{2}\cdot\rho\cdot V_{max}^{\;2}\cdot A\cdot C_D + F_{Rnew}\right)\cdot V_{max}$$

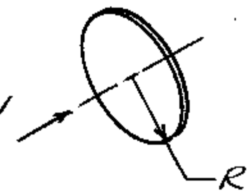
Solving the cubic (using *Solver*) $V_{max} = 154\,mph$

Problem 9.115

Given: Round, thin disk of radius, R, with pressure data:

$$C_p = 1-\left(\frac{r}{R}\right)^6 \text{ (front)}$$

$$C_p = -0.42 \text{ (rear)}$$



Find: Calculate the drag coefficient, C_D.

Solution: Computing equations are

$$C_p = \frac{p-p_\infty}{\frac{1}{2}\rho V^2} \qquad C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} \qquad A = \pi R^2$$

Assumptions: (1) Steady, incompressible flow
(2) Neglect skin friction drag (disk thin, edge area small)

Then $$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = \frac{\int_A (p_f - p_r)\,dA}{\frac{1}{2}\rho V^2 \pi R^2} = \frac{\int_0^R (p_f - p_r)\, 2\pi r\,dr}{\frac{1}{2}\rho V^2 \pi R^2}$$

From defn of C_p, $p_f = p_\infty + C_{p_f}\frac{1}{2}\rho V^2$, $p_r = p_\infty + C_{p_r}\frac{1}{2}\rho V^2$

and $$p_f - p_r = (C_{p_f} - C_{p_r})\frac{1}{2}\rho V^2$$

Substituting,

$$C_D = \frac{\frac{1}{2}\rho V^2 \int_0^R (C_{p_f} - C_{p_r})\, 2\pi r\,dr}{\frac{1}{2}\rho V^2 \pi R^2} = \frac{2}{R^2}\int_0^R \left[1-\left(\frac{r}{R}\right)^6 + 0.42\right] r\,dr$$

$$= 2\int_0^1 \left[1.42 - \left(\frac{r}{R}\right)^7\right]\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = 2\left[\frac{1.42}{2}\left(\frac{r}{R}\right)^2 - \frac{1}{8}\left(\frac{r}{R}\right)^8\right]_0^1$$

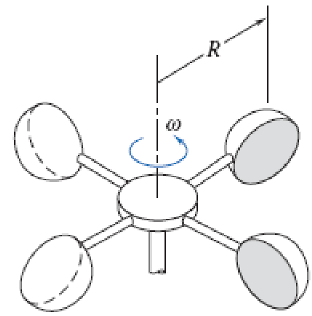
$$= 2\,(0.710 - 0.125)$$

$$C_D = 1.17$$ C_D

Problem 9.116

[5]

9.116 Repeat the analysis for the frictionless anemometer of Problem 9.106, except this time base the torque calculations on the more realistic model that the average torque is obtained by integrating, over one revolution, the instantaneous torque generated by each cup (i.e., as the cup's orientation to the wind varies).



Given: Data on dimensions of anemometer

Find: Calibration constant

Solution:

The given data or available data is $D = 50\cdot mm$ $R = 80\cdot mm$ $\rho = 1.23\cdot\frac{kg}{m^3}$

The drag coefficients for a cup with open end facing the airflow and a cup with open end facing downstream are, respectively, from Table 9

$C_{Dopen} = 1.42$ $C_{Dnotopen} = 0.38$

Assume the anemometer achieves steady speed ω due to steady wind speed *V*

The goal is to find the calibration constant *k*, defined by $k = \frac{V}{\omega}$

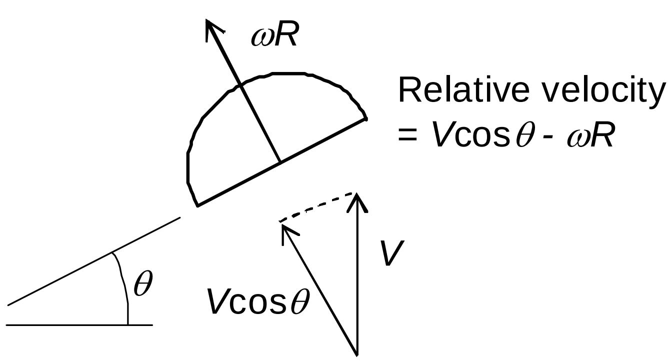
We will analyse each cup separately, with the following assumptions

1) Drag is based on the instantaneous normal component of velocity (we ignore possible effects on drag coefficient of velocity component parallel to the cup)

2) Each cup is assumed unaffected by the others - as if it were the only object present

3) Swirl is neglected

4) Effects of struts is neglected



In this more sophisticated analysis we need to compute the instantaneous normal relative velocity.
From the sketch, when a cup is at angle θ, the normal component of relative velocity is

$$V_n = V\cdot\cos(\theta) - \omega\cdot R \qquad (1)$$

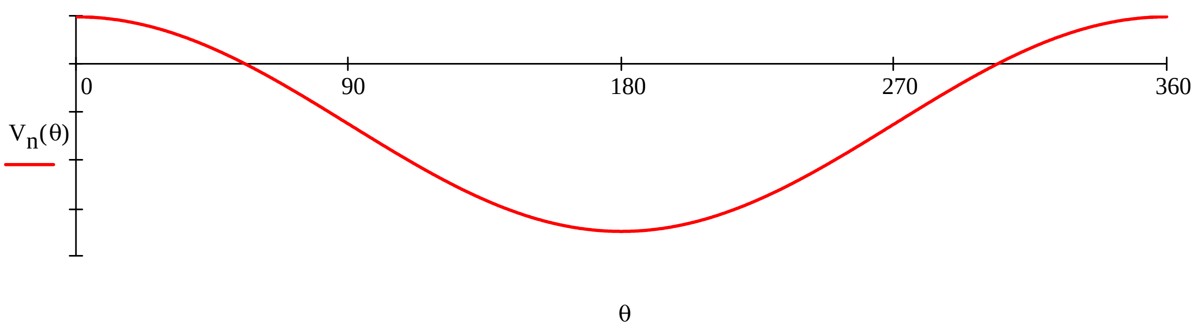
The relative velocity is sometimes positive sometimes negatiive. From Eq. 1, this is determined by

$$\theta_c = \mathrm{acos}\left(\frac{\omega\cdot R}{V}\right) \qquad (2)$$

For $0 < \theta < \theta_c$ $\quad V_n > 0$

$\theta_c < \theta < 2\cdot\pi - \theta_c$ $\quad V_n < 0$

$\theta_c < \theta < 2\cdot\pi$ $\quad V_n > 0$



The equation for computing drag is $$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V_n^{\;2}\cdot C_D \qquad (3)$$

where $A = \frac{\pi\cdot D^2}{4}$ $\qquad A = 1.96\times 10^{-3}\,m^2$

In Eq. 3, the drag coefficient, and whether the drag is postive or negative, depend on the sign of the relative velocity

For $0 < \theta < \theta_c$ $\quad C_D = C_{Dopen}$ $\quad F_D > 0$

$\theta_c < \theta < 2\cdot\pi - \theta_c$ $\quad C_D = C_{Dnotopen}$ $\quad F_D < 0$

$\theta_c < \theta < 2\cdot\pi$ $\quad C_D = C_{Dopen}$ $\quad F_D > 0$

The torque is $$T = F_D\cdot R = \frac{1}{2}\cdot\rho\cdot A\cdot V_n^{\;2}\cdot C_D\cdot R$$

The average torque is $$T_{av} = \frac{1}{2\cdot\pi}\cdot\int_{\theta}^{2\cdot\pi} T\,d\theta = \frac{1}{\pi}\cdot\int_{\theta}^{\pi} T\,d\theta$$

where we have taken advantage of symmetry

Evaluating this, allowing for changes when $\theta = \theta_c$ $$T_{av} = \frac{1}{\pi}\cdot\int_{\theta}^{\theta_c}\frac{1}{2}\cdot\rho\cdot A\cdot V_n^{\;2}\cdot C_{Dopen}\cdot R\,d\theta - \frac{1}{\pi}\cdot\int_{\theta_c}^{\pi}\frac{1}{2}\cdot\rho\cdot A\cdot V_n^{\;2}\cdot C_{Dnotopen}\cdot R\,d\theta$$

Using Eq. 1

$$T_{av} = \frac{\rho\cdot A\cdot R}{2\cdot\pi}\cdot\left[C_{Dopen}\cdot\int_{\theta}^{\theta_c}(V\cdot\cos(\theta)-\omega\cdot R)^2\,d\theta - C_{Dnotopen}\cdot\int_{\theta_c}^{\pi}(V\cdot\cos(\theta)-\omega\cdot R)^2\,d\theta\right]$$

$$T_{av} = \frac{\rho\cdot A\cdot R\cdot\omega^2}{2\cdot\pi}\cdot\left[C_{Dopen}\cdot\int_{\theta}^{\theta_c}\left(\frac{V}{\omega}\cdot\cos(\theta)-R\right)^2 d\theta - C_{Dnotopen}\cdot\int_{\theta_c}^{\pi}\left(\frac{V}{\omega}\cdot\cos(\theta)-R\right)^2 d\theta\right]$$

and note that $\frac{V}{\omega} = k$

The integral is $\int (k\cdot\cos(\theta)-R)^2\,d\theta = k^2\cdot\left(\frac{1}{2}\cdot\cos(\theta)\cdot\sin(\theta)+\frac{1}{2}\cdot\theta\right) - 2\cdot k\cdot R\cdot\sin(\theta) + R^2\cdot\theta$

For convenience define $f(\theta) = k^2\cdot\left(\frac{1}{2}\cdot\cos(\theta)\cdot\sin(\theta)+\frac{1}{2}\cdot\theta\right) - 2\cdot k\cdot R\cdot\sin(\theta) + R^2\cdot\theta$

Hence $T_{av} = \frac{\rho\cdot A\cdot R}{2\cdot\pi}\cdot\left[C_{Dopen}\cdot f(\theta_c) - C_{Dnotopen}\cdot\left(f(\pi)-f(\theta_c)\right)\right]$

For steady state conditions the torque (of each cup, and of all the cups) is zero. Hence

$$C_{Dopen}\cdot f(\theta_c) - C_{Dnotopen}\cdot\left(f(\pi)-f(\theta_c)\right) = 0$$

or $f(\theta_c) = \frac{C_{Dnotopen}}{C_{Dopen}+C_{Dnotopen}}\cdot f(\pi)$

Hence $k^2\cdot\left(\frac{1}{2}\cdot\cos(\theta_c)\cdot\sin(\theta_c)+\frac{1}{2}\cdot\theta_c\right) - 2\cdot k\cdot R\cdot\sin(\theta_c) + R^2\cdot\theta_c = \frac{C_{Dnotopen}}{C_{Dopen}+C_{Dnotopen}}\cdot\left(k^2\cdot\frac{\pi}{2}+R^2\cdot\pi\right)$

Recall from Eq 2 that $\theta_c = \operatorname{acos}\left(\frac{\omega\cdot R}{V}\right)$ or $\theta_c = \operatorname{acos}\left(\frac{R}{k}\right)$

Hence $k^2\cdot\left(\frac{1}{2}\cdot\frac{R}{k}\cdot\sin\left(\operatorname{acos}\left(\frac{R}{k}\right)\right)+\frac{1}{2}\cdot\operatorname{acos}\left(\frac{R}{k}\right)\right) - 2\cdot k\cdot R\cdot\sin\left(\operatorname{acos}\left(\frac{R}{k}\right)\right) + R^2\cdot\operatorname{acos}\left(\frac{R}{k}\right) = \frac{C_{Dnotopen}}{C_{Dopen}+C_{Dnotopen}}\cdot\left(k^2\cdot\frac{\pi}{2}+R^2\cdot\pi\right)$

This equation is to be solved for the coefficient *k*. The equation is highly nonlinear; it can be solved by iteration or using *Excel*'s *Goal Seek* or *Solver*

From the associated *Excel* workbook

$k = 0.316\cdot m$ $\qquad$ $k = 0.119\cdot\frac{\frac{km}{hr}}{rpm}$

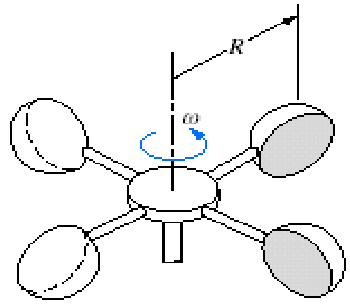
Problem 9.116 (In Excel)

[5]

9.116 Repeat the analysis for the frictionless anemometer of Problem 9.106, except this time base the torque calculations on the more realistic model that the average torque is obtained by integrating, over one revolution, the instantaneous torque generated by each cup (i.e., as the cup's orientation to the wind varies).

Given: Data on dimensions of anemometer

Find: Calibration constant



Solution:

Given data:

D =	50	mm
R =	80	mm
C_{Dopen} =	1.42	
$C_{Dnotopen}$ =	0.38	

$$k^2\cdot\left(\frac{1}{2}\cdot\frac{R}{k}\cdot\sin\left(\operatorname{acos}\left(\frac{R}{k}\right)\right)+\frac{1}{2}\cdot\operatorname{acos}\left(\frac{R}{k}\right)\right)-2\cdot k\cdot R\cdot\sin\left(\operatorname{acos}\left(\frac{R}{k}\right)\right)+R^2\cdot\operatorname{acos}\left(\frac{R}{k}\right)=\frac{C_{Dnotopen}}{C_{Dopen}+C_{Dnotopen}}\cdot\left(k^2\cdot\frac{\pi}{2}+R^2\cdot\pi\right)$$

Use *Solver* to find *k* to make the error zero!

k (mm)	Left	Right	Error
315.85	37325.8	37326	0%

k =	0.316	m
k =	0.119	km/hr/rpm

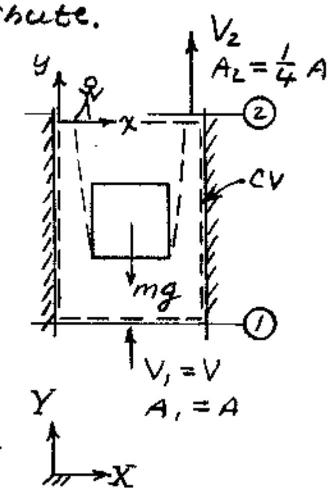
Problem 9.117

Given: Object of mass, m, falling in air down mail chute.

Motion is steady.

Wake area at ② is 3/4 of chute area.

Pressure is constant in wake.



Find: Expression for terminal speed of object.

Solution: Choose a CV moving with the object.
Apply continuity, Bernoulli, and y momentum.

Basic equations:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A} \quad (=0(1))$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2$$

$$F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v \rho\, d\forall + \int_{CS} v \rho \vec{V}\cdot d\vec{A} \quad (=0(1))$$

Assumptions: (1) Steady flow relative to CV
(2) Incompressible flow
(3) Neglect friction
(4) Flow along a streamline
(5) Uniform flow and pressures at ① and ②.
(6) Neglect Δz
(7) No net flow in wake

From continuity,

$$0 = \{-\rho V_1 A_1\} + \{\rho V_2 A_2\} \quad \text{so} \quad V_2 = V_1 \frac{A_1}{A_2} = V\frac{A}{A_2}$$

From Bernoulli

$$p_1 - p_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 = \frac{1}{2}\rho V^2\left[\left(\frac{V_2}{V}\right)^2 - 1\right] = \frac{1}{2}\rho V^2\left[\left(\frac{A}{A_2}\right)^2 - 1\right]$$

From momentum

$$p_1 A - p_2 A - mg = v_1\{-\rho V_1 A_1\} + v_2\{+\rho V_2 A_2\} = \rho V A (V_2 - V) = \rho V^2 A\left(\frac{V_2}{V} - 1\right)$$

$$v_1 = V \qquad v_2 = V_2$$

or

$$(p_1 - p_2)A - mg = \rho V^2 A\left(\frac{A}{A_2} - 1\right)$$

Substituting for $p_1 - p_2$

$$\frac{1}{2}\rho V^2 A\left[\left(\frac{A}{A_2}\right)^2 - 1\right] - mg = \rho V^2 A\left(\frac{A}{A_2} - 1\right) \quad \text{or} \quad mg = \frac{1}{2}\rho V^2 A\left[\left(\frac{A}{A_2}\right)^2 - 2\left(\frac{A}{A_2}\right) + 1\right]$$

Thus

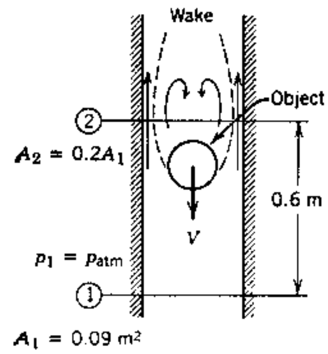
$$V = \left[\frac{2mg}{\rho A}\,\frac{1}{\left(\frac{A}{A_2}\right)^2 - 2\left(\frac{A}{A_2}\right) + 1}\right]^{1/2}$$

V

where A_2 is the net flow area at Section ②.

Problem 9.118

[4]



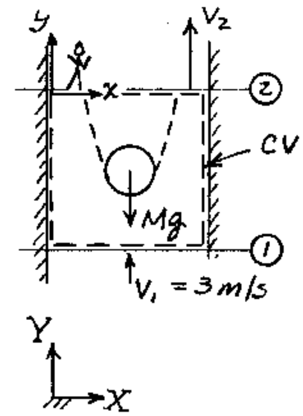
Given: Object falling in air down chute.

$V = 3\ m/s$

Frictional effects negligible.

Find: (a) Flow speed, V_2, relative to object.
(b) Static pressure, p_2.
(c) Mass of object.

Solution: Choose a CV moving with object. Apply continuity, Bernoulli, and y momentum.



Basic equations:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A} \qquad (=0(1))$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2$$

$$F_{S_y} + F_{B_y} = \frac{\partial}{\partial t}\int_{CV} v \rho\, d\forall + \int_{CS} v \rho \vec{V}\cdot d\vec{A} \qquad (=0(1))$$

Assumptions: (1) Steady flow relative to CV
(2) Incompressible flow
(3) Neglect friction
(4) Flow along a streamline
(5) Neglect Δz
(6) Uniform flow and pressures at ① and ②.
(7) No net flow in wake

Then from continuity,

$$0 = \{-\rho V_1 A_1\} + \{+\rho V_2 A_2\} \quad \text{so} \quad V_2 = V_1 \frac{A_1}{A_2} = 3\,\frac{m}{s} \times \frac{1}{0.2} = 15\ m/s$$ V_2

From Bernoulli,

$$p_2 = p_1 + \tfrac{1}{2}\rho V_1^2 - \tfrac{1}{2}\rho V_2^2 = p_1 + \tfrac{1}{2}\rho V_1^2\left[1 - \left(\frac{V_2}{V_1}\right)^2\right]$$

$$p_2(\text{gage}) = \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (3)^2\frac{m^2}{s^2}\left[1 - \left(\frac{1}{0.2}\right)^2\right]\frac{N\cdot s^2}{kg\cdot m} = -133\ N/m^2\ (\text{gage})$$ p_2

From momentum

$$p_1 A_1 - p_2 A_2 - Mg = v_1\{-\rho V_1 A_1\} + v_2\{+\rho V_2 A_2\} = \rho V_1 A_1 (V_2 - V_1) = \rho V_1^2 A_1 \left(\frac{V_2}{V_1} - 1\right)$$

$$v_1 = V_1 \qquad v_2 = V_2$$

Thus

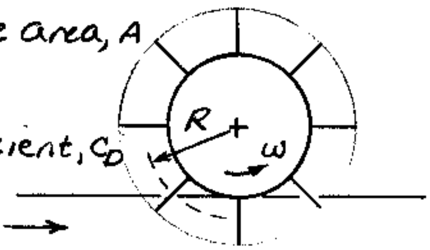
$$M = \frac{A_1}{g}\left[p_1 - p_2 - \rho V_1^2\left(\frac{V_2}{V_1} - 1\right)\right]$$

$$M = 0.09\ m^2 \times \frac{s^2}{9.81\ m}\left[133\,\frac{N}{m^2} \times \frac{kg\cdot m}{N\cdot s^2} - 1.23\,\frac{kg}{m^3} \times (3)^2\frac{m^2}{s^2}\left(\frac{1}{0.2} - 1\right)\right] = 0.814\ kg$$ M

Problem 9.119

Given: Paddle wheel immersed in river current as shown.

Assume only one paddle equivalent is immersed at a time.



Find: Expressions for (a) force, (b) torque, and (c) power produced by the wheel, and find optimum angular speed.

Solution: Computing equations $F_D = C_D A \frac{1}{2}\rho V_{rel}^2$

$$T = F_D R \, , \quad P = T\omega$$

Assumptions: (1) Neglect air resistance, since $\rho_{air} \ll \rho_{water}$
(2) Use velocity relative to the paddle

Thus $V_{rel} = V - U = V - R\omega$

$$F_D = C_D A \frac{1}{2}\rho V_{rel}^2 = C_D A \frac{1}{2}\rho (V-U)^2$$ F_D

The torque is

$$T = F_D R = C_D A \frac{1}{2}\rho (V-U)^2 R$$ T

The power is

$$P = T\omega = C_D A \frac{1}{2}\rho (V-U)^2 R\omega = C_D A \frac{1}{2}\rho (V-U)^2 U$$ P

To optimize power, set $\frac{dP}{dU} = 0$

$$\frac{dP}{dU} = C_D A \frac{1}{2}\rho \left[2(V-U)U(-1) + (V-U)^2\right] = 0$$

Cancelling a factor $(V-U)$ gives $-2U + V - U = 0$ or $V - 3U = 0$

Thus $U = R\omega = \frac{V}{3}$ so

$$\omega_{opt} = \frac{V}{3R}$$ ω_{opt}

Problem 9.120

[2]

9.120 A light plane tows an advertising banner over a football stadium on a Saturday afternoon. The banner is 4 ft tall and 45 ft long. According to Hoerner [16], the drag coefficient based on area (Lh) for such a banner is approximated by $C_D = 0.05\ L/h$, where L is the banner length and h is the banner height. Estimate the power required to tow the banner at $V = 55$ mph. Compare with the drag of a rigid flat plate. Why is the drag larger for the banner?

Given: Data on advertising banner

Find: Power to tow banner; Compare to flat plate; Explain discrepancy

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad P = F_D\cdot V$

The given data or available data is $V = 55\cdot mph$ $\quad V = 80.7\cdot\frac{ft}{s}$ $\quad L = 45\cdot ft$ $\quad h = 4\cdot ft$ $\quad \rho = 0.00234\cdot\frac{slug}{ft^3}$

$A = L\cdot h$ $\quad A = 180\cdot ft^2$ $\quad C_D = 0.05\cdot\frac{L}{h}$ $\quad C_D = 0.563$

$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad F_D = 771\cdot lbf$ $\quad P = F_D\cdot V$ $\quad P = 6.22\times 10^4\cdot\frac{ft\cdot lbf}{s}$ $\quad P = 113\cdot hp$

For a flate plate, check Re $\quad \nu = 1.62\times 10^{-4}\cdot\frac{ft^2}{s}$ $\quad$ (Table A.9, 69ºF)

$Re_L = \frac{V\cdot L}{\nu}$ $\quad Re_L = 2.241\times 10^7$ $\quad$ so flow is fully turbulent. Hence use Eq 9.37b

$$C_D = \frac{0.455}{\log\left(Re_L\right)^{2.58}} - \frac{1610}{Re_L} \quad C_D = 0.00258$$

$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad F_D = 3.53\cdot lbf$

This is the drag on one side. The total drag is then $2\cdot F_D = 7.06\cdot lbf$. This is VERY much less than the banner drag. The banner drag allows for banner flutter and other secondary motion which induces significant form drag.

Problem 9.121

[1]

9.121 The antenna on a car is 10 mm in diameter and 1.8 m long. Estimate the bending moment that tends to snap it off if the car is driven at 120 km/hr on a standard day.

Given:
Data on car antenna

Find:
Bending moment

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$

The given or available data is $V = 120\cdot\frac{km}{hr}$ $\quad V = 33.3\cdot\frac{m}{s}$ $\quad L = 1.8\cdot m$ $\quad D = 10\cdot mm$

$A = L\cdot D$ $\quad A = 0.018\,m^2$

$\rho = 1.225\cdot\frac{kg}{m^3}$ $\quad \nu = 1.50\times 10^{-5}\cdot\frac{m^2}{s}$ $\quad$ (Table A.10, 20ºC)

For a cylinder, check Re $\quad Re = \frac{V\cdot D}{\nu}$ $\quad Re = 2.22\times 10^4$

From Fig. 9.13 $\quad C_D = 1.0$ $\quad F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad F_D = 12.3\,N$

The bending moment is then $\quad M = F_D\cdot\frac{L}{2}$ $\quad M = 11.0\cdot N\cdot m$

Problem 9.122

9.122 A large three-blade horizontal axis wind turbine (HAWT) can be damaged if the wind speed is too high. To avoid this, the blades of the turbine can be oriented so that they are parallel to the flow. Find the bending moment at the base of each blade when the wind speed is 45 m/s. Model each blade as a flat plate 35 m wide and 0.45 m long.

Given: Data on wind turbine blade

Find: Bending moment

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$

The given or available data is $V = 45\cdot\frac{m}{s}$ $\quad L = 0.45\cdot m$ $\quad W = 35\cdot m$

$A = L\cdot W$ $\quad A = 15.75\,m^2$

$\rho = 1.225\cdot\frac{kg}{m^3}$ $\quad \nu = 1.50\times 10^{-5}\cdot\frac{m^2}{s}$ $\quad$ (Table A.10, 20ºC)

For a flat plate, check Re $\quad Re_L = \frac{V\cdot L}{\nu}$ $\quad Re_L = 1.35\times 10^6$ $\quad$ so use Eq. 9.37a

$C_D = \frac{0.0742}{Re_L^{\frac{1}{5}}} - \frac{1740}{Re_L}$ $\quad C_D = 0.00312$

$F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad F_D = 61.0\,N$

The bending moment is then $\quad M = F_D\cdot\frac{W}{2}$ $\quad M = 1067\cdot N\cdot m$

Problem 9.123

9.123 The HAWT of Problem 9.122 is not self-starting. The generator is used as an electric motor to get the turbine up to the operating speed of 20 rpm. To make this easier, the blades are aligned so they lie in the plane of rotation. Assuming an overall efficiency of motor and drive train of 65%, find the power required to maintain the turbine at the operating speed. As an approximation, model each blade as a series of flat plates (the outer region of each blade moves at a significantly higher speed than the inner region).

Given: Data on wind turbine blade

Find: Power required to maintain operating speed

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$

The given or available data is $\omega = 20\cdot rpm$ $\quad L = 0.45\cdot m$ $\quad w = 35\cdot m$

$\rho = 1.225\cdot\frac{kg}{m^3}$ $\quad \nu = 1.50\times 10^{-5}\cdot\frac{m^2}{s}$ (Table A.10, 20ºC)

The velocity is a function of radial position, $V(r) = r\cdot\omega$, so Re varies from 0 to $Re_{max} = \frac{V(w)\cdot L}{\nu}$ $\quad Re_{max} = 2.20\times 10^6$

The transition Reynolds number is 500,000 which therefore occurs at about 1/4 of the maximum radial distance; the boundary layer is laminar for the first quarter of the blade. We approximate the entire blade as turbulent - the first 1/4 of the blade will not exert much moment in any event

Hence $Re(r) = \frac{L}{\nu}\cdot V(r) = \frac{L\cdot\omega}{\nu}\cdot r$

Using Eq. 9.37a

$$C_D = \frac{0.0742}{Re_L^{\frac{1}{5}}} - \frac{1740}{Re_L} = \frac{0.0742}{\left(\frac{L\cdot\omega}{\nu}\cdot r\right)^{\frac{1}{5}}} - \frac{1740}{\frac{L\cdot\omega}{\nu}\cdot r} = 0.0742\cdot\left(\frac{\nu}{L\cdot\omega}\right)^{\frac{1}{5}}\cdot r^{-\frac{1}{5}} - 1740\cdot\left(\frac{\nu}{L\cdot\omega}\right)\cdot r^{-1}$$

The drag on a differential area is $dF_D = \frac{1}{2}\cdot\rho\cdot dA\cdot V^2\cdot C_D = \frac{1}{2}\cdot\rho\cdot L\cdot V^2\cdot C_D\cdot dr$ $\quad$ The bending moment is then $dM = dF_D\cdot r$

Hence $M = \int 1\,dM = \int_0^w \frac{1}{2}\cdot\rho\cdot L\cdot V^2\cdot C_D\cdot r\,dr$

$$M = \int_0^w \frac{1}{2}\cdot\rho\cdot L\cdot\omega^2\cdot r^3\cdot\left[0.0742\cdot\left(\frac{\nu}{L\cdot\omega}\right)^{\frac{1}{5}}\cdot r^{-\frac{1}{5}} - 1740\cdot\left(\frac{\nu}{L\cdot\omega}\right)\cdot r^{-1}\right]dr$$

$$M = \frac{1}{2}\cdot\rho\cdot L\cdot\omega^2\cdot\int_0^w \left[0.0742\cdot\left(\frac{\nu}{L\cdot\omega}\right)^{\frac{1}{5}}\cdot r^{\frac{14}{5}} - 1740\cdot\left(\frac{\nu}{L\cdot\omega}\right)\cdot r^2\right]dr$$

$$M = \frac{1}{2}\cdot\rho\cdot L\cdot\omega^2\cdot\left[\frac{5\cdot 0.0742}{19}\cdot\left(\frac{\nu}{L\cdot\omega}\right)^{\frac{1}{5}}\cdot w^{\frac{19}{5}} - \frac{1740}{3}\cdot\left(\frac{\nu}{L\cdot\omega}\right)\cdot w^3\right]$$

$M = 1.43\cdot kN\cdot m$ $\quad$ Hence the power is $\quad P = M\cdot\omega$ $\quad P = 3.00\,kW$

Problem 9.124

[2]

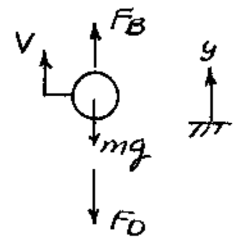
Given: Small droplets of oil (SG = 0.85) rising in water.

Find: (a) Relationship for terminal speed, V_t (m/s), as a function of droplet diameter, D (in mm).

(b) Range of D for which Stokes flow is a reasonable assumption.

Solution: Draw free-body diagram of droplet, apply Newton's second law.

Basic equation: $\Sigma F_y = -mg + F_B - F_D = ma_y$



Assume: Stokes' drag law, $F_D = 3\pi\mu V D$, for Re < 10

Then $-\rho \forall g + \rho_{H_2O} \forall g - 3\pi\mu V_t D = 0$ at terminal speed, V_t.

Solving, $V_t = \dfrac{(\rho_{H_2O} - \rho_o)\forall g}{3\pi\mu D} = \rho_{H_2O}(1 - SG_o)\dfrac{\pi D^3}{6}\dfrac{g}{3\pi\mu D} = \dfrac{(1-SG_o)D^2 g}{18\nu}$

Evaluating,

$$V_t (m/s) = \frac{(1-0.85)}{18} \times D^2\,mm^2 \times 9.81\,\frac{m}{s} \times \frac{s}{1.00\,10^{-6}\,m^2} \times \frac{m^2}{10^6\,mm^2} \qquad (T = 20°C)$$

$$V_t (m/s) = 0.0818\,[D(mm)]^2$$

V_t

For Stokes flow, Re < 1, so

$$Re = \frac{\rho V_t D}{\mu} = \frac{V_t D}{\nu} = \frac{(1-SG_o)D^3 g}{18\,\nu^2} \le 1$$

Thus

$$D^3 \le \frac{18\,\nu^2}{(1-SG_o)g} \quad \text{or} \quad D \le \left[\frac{18\,\nu^2}{(1-SG_o)g}\right]^{1/3}$$

Evaluating,

$$D \le \left[\frac{18}{(1-0.85)}(1.00\times10^{-6})^2\frac{m^4}{s^2} \times \frac{s^2}{9.81\,m}\right]^{1/3} = 2.31\times10^{-4}\,m \;(0.231\,mm)$$

D

Thus Stokes' flow will be a valid assumption for D < 0.231 mm.

Problem 9.125

Given: Wind tunnel with standard air drawn in.

Sphere with $D = 30$ mm on a force balance.

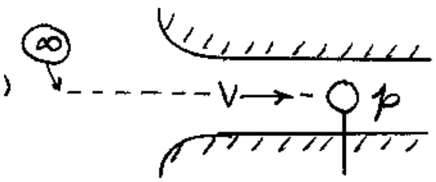
Static pressure in tunnel, $p = -40$ mm (oil, SG = 0.85)

Find: (a) Freestream air speed

(b) Reynolds number for flow over sphere

(c) Drag force on sphere.

Solution: Apply Bernoulli



$$p_\infty + \frac{1}{2}\rho V_\infty^2 + \rho g z_\infty = p + \frac{1}{2}\rho V^2 + \rho g z$$

($V_\infty^2 \approx 0$ (6); $\rho g z$ terms (5))

Assume: (1) Steady flow

(2) Incompressible flow

(3) Flow along a streamline

(4) No friction (neglect honeycomb and/or screens

(5) Neglect z

(6) $V_\infty \approx 0$

Then $p_\infty = p + \frac{1}{2}\rho V^2$ or $V = \sqrt{\frac{2(p_\infty - p)}{\rho}}$

But $p_\infty - p = -\rho_{oil} g \Delta h = -SG\,\rho_{H_2O}\, g \Delta h$

$$V = \left[\frac{-2\,SG\,\rho_{H_2O}\, g\,\Delta h}{\rho}\right]^{\frac{1}{2}}$$

$$V = \left[-2(0.85) \times 1000\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times (-0.04\,m) \times \frac{m^3}{1.23\,kg}\right]^{\frac{1}{2}} = 23.3\,m/s$$ V

and

$$Re = \frac{VD}{\nu} = 23.3\frac{m}{s} \times 0.03\,m \times \frac{s}{1.45 \times 10^{-5}\,m^2} = 48{,}200$$ Re

Re is subcritical; BLs are laminar, and $C_D = 0.47$

$$F_D = C_D A \frac{1}{2}\rho V^2 \qquad A = \frac{\pi D^2}{4} = 7.07 \times 10^{-4}\,m^2$$

$$= 0.47 \times 7.07 \times 10^{-4}\,m^2 \times \frac{1}{2} \times 1.23\frac{kg}{m^3} \times (23.3)^2\frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m}$$

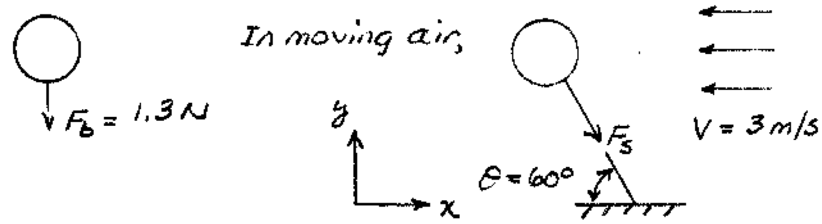
$$F_D = 0.111\,N$$ F_D

Problem 9.126

[2]

Given: Spherical hydrogen-filled balloon, $D = 0.6$ m, in standard air.

In stationary air, In moving air,



Find: Drag coefficient of balloon.

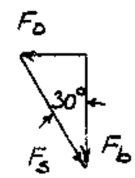
Solution: Apply Newton's second law of motion, definition of C_D.

Basic equations: $\sum F_x = m a_x^{\;=0(1)}$, $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$

Assumptions: (1) Balloon is stationary

(2) F_b (buoyancy of balloon) = 1.3 N

Then a free body diagram gives



From the diagram, $F_D = F_s \tan 30°$

$$F_D = 1.3\text{ N} \times 0.577 = 0.750\text{ N}$$

For standard air, $\rho = 1.23$ kg/m³. $A = \pi D^2/4$, so

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = \frac{8}{\pi}\frac{F_D}{\rho V^2 D^2}$$

$$C_D = \frac{8}{\pi} \times 0.750\,\text{N} \times \frac{\text{m}^3}{1.23\,\text{kg}} \times \frac{\text{s}^2}{(3)^2\,\text{m}^2} \times \frac{1}{(0.6)^2\,\text{m}^2} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2} = 0.479$$

C_D

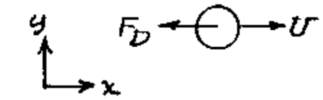
Problem 9.127

[2]

Given: Field hockey ball with $D = 73$ mm and $m = 160$ g, leaving stick at $U_0 = 50$ m/s. Ball is smooth sphere.

Find: Estimate distance traveled in horizontal flight to reduce speed of ball 10 percent.

<u>Solution</u>: Apply Newton's second law of motion:



Basic equation: $\Sigma F_x = ma_x = m\dfrac{dU}{dt} = mU\dfrac{dU}{dx}$

Thus $-F_D = -C_D A \frac{1}{2}\rho U^2 = mU\dfrac{dU}{dx}$ or $dx = -\dfrac{2m}{C_D A \rho}\dfrac{dU}{U}$

Check Re to find C_D (use ν at $T = 15°C$ from Table A.10):

$$Re \le \frac{U_0 D}{\nu} = 50\frac{m}{s} \times 0.075\,m \times \frac{s}{1.46\times10^{-5}\,m^2} = 2.57\times10^5 \qquad \text{(standard air)}$$

From Fig. 9.11, flow is <u>subcritical</u> and $C_D \simeq 0.47 =$ constant.

Thus

$$x = \int_0^x dx = \int_{U_0}^{U} -\frac{2m}{C_D A\rho}\frac{dU}{U} = -\frac{2m}{C_D A \rho}\ln U\Big]_{U_0}^{U} = -\frac{2m}{C_D A\rho}\ln\left(\frac{U}{U_0}\right)$$

or

$$x = -2\times 0.160\,kg \times \frac{1}{0.47}\times\frac{4}{\pi(0.073)^2\,m^2}\times\frac{m^3}{1.23\,kg}\ln(0.9) = 13.9\,m$$

x

Problem 9.128

9.128 Compute the terminal speed of a 3-mm diameter raindrop (assume spherical) in standard air.

Given: 3 mm raindrop

Find: Terminal speed

Solution:

Basic equation: $F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad\Sigma F = 0$

Given or available data is $D = 3\cdot mm$ $\quad\rho_{H2O} = 1000\cdot\frac{kg}{m^3}$ $\quad\rho_{air} = 1.225\cdot\frac{kg}{m^3}$ $\quad\nu = 1.50\times 10^{-5}\cdot\frac{m^2}{s}$ (Table A.10, 20ºC)

Summing vertical forces $M\cdot g - F_D = M\cdot g - \frac{1}{2}\cdot\rho_{air}\cdot A\cdot V^2\cdot C_D = 0$ Buoyancy is negligible

$M = \rho_{H2O}\cdot\frac{\pi\cdot D^3}{6}$ $\quad M = 1.41\times 10^{-5}kg$ $\quad A = \frac{\pi\cdot D^2}{4}$ $\quad A = 7.07\times 10^{-6}m^2$

Assume the drag coefficient is in the flat region of Fig. 9.11 and verify Re later $\quad C_D = 0.4$

$$V = \sqrt{\frac{2\cdot M\cdot g}{C_D\cdot\rho_{air}\cdot A}} \qquad V = 8.95\frac{m}{s}$$

Check Re $\quad Re = \frac{V\cdot D}{\nu}$ $\quad Re = 1.79\times 10^3$ which does place us in the flat region of the curve

Actual raindrops are not quite spherical, so their speed will only be approximated by this result

Problem 9.129

[3]

Given: Small sphere falling through castor oil at 20 C; $D = 6$ mm.

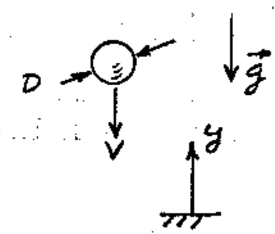
Terminal speed is 60 mm/s.

Find: (a) Compute C_D for given sphere.
(b) Density of sphere.
(c) Compare terminal speed in water.

Solution: Apply Newton's second law of motion, definition of C_D.

Basic equations: $\Sigma F_y = m a_y$

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$



Assume $Re < 1$, so Stokes flow, $F_D = 3\pi\mu DV$.

From the definition, noting A is the frontal area, $A = \frac{\pi D^2}{4}$,

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = \frac{3\pi\mu DV}{\frac{1}{2}\rho V^2 \frac{\pi D^2}{4}} = \frac{24\mu}{\rho VD} = \frac{24}{Re}$$

For castor oil at 20 C, $\mu = 0.9\ \text{N}\cdot\text{s}/\text{m}^2$ (Fig. A.2) and $SG \approx 0.97$ (Table A.2).

$$Re = \frac{SG\,\rho_{H_2O} VD}{\mu} = (0.97)\,999\,\frac{\text{kg}}{\text{m}^3} \times 0.06\,\frac{\text{m}}{\text{s}} \times 0.006\,\text{m} \times \frac{\text{m}^2}{0.9\,\text{N}\cdot\text{s}} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 0.388 < 1 \quad ✓✓$$

$$C_D = \frac{24}{Re} = \frac{24}{0.388} = 61.9$$ C_D

From Newton's second law, $F_D + F_{buoyancy} - mg = 0$ or $F_D = mg - F_b$. Thus

$$F_D = C_D \tfrac{1}{2}\rho V^2 A = mg - F_b = (SG_s - SG_o)\rho_{H_2O} g \forall \text{ or } SG_s = SG_o + \frac{C_D \frac{1}{2}\rho V^2 A}{\rho_{H_2O} g \forall} = SG_o\left(1 + \frac{C_D V^2 A}{2g\forall}\right)$$

But $\frac{A}{\forall} = \frac{\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{4r} = \frac{3}{2D}$, so $SG_s = SG_o\left(1 + \frac{3}{4}\frac{C_D V^2}{gD}\right)$

$$SG_s = 0.97\left(1 + \frac{3}{4} \times 61.9 \times (0.06)^2 \frac{\text{m}^2}{\text{s}^2} \cdot \frac{\text{s}^2}{9.81\,\text{m}} \times \frac{1}{0.006\,\text{m}}\right) = 3.72, \text{ so } \rho_s = 3720\ \frac{\text{kg}}{\text{m}^3}$$ ρ_s

In water, the net weight and drag must balance, or

$$F_D = \tfrac{1}{2}\rho V^2 C_D A = W - F_b = \gamma_{H_2O}(SG_s - 1)\forall \quad \text{or} \quad V = \left[\frac{4(SG-1)gD}{3C_D}\right]^{\frac{1}{2}}$$

However, C_D is a function of Re, so iteration is needed. From Fig. 9.11, $C_D \approx 0.4$ over a range of Re. Using $C_D = 0.4$,

$$V = \left[\frac{4}{3}\left(\frac{2.72}{0.4}\right) 9.81\,\frac{\text{m}}{\text{s}^2} \times 0.006\,\text{m}\right]^{\frac{1}{2}} = 0.731\ \text{m/s}$$ V

Check Re:

$$Re = \frac{VD}{\nu} = 0.731\,\frac{\text{m}}{\text{s}} \times 0.006\,\text{m} \times \frac{\text{s}}{1.1 \times 10^{-6}\,\text{m}^2} = 3990 \quad \text{ok!}$$

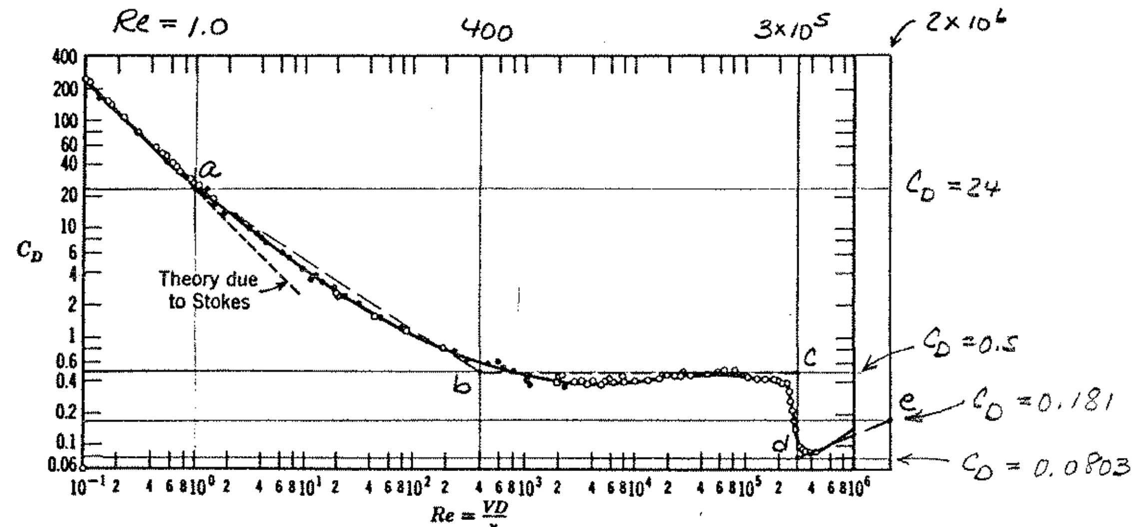
Problem 9.130

Given: Curve fit for drag coefficient of a sphere versus Re:

$C_D = 24/Re$	$Re \leq 1$
$C_D = 24/Re^{0.646}$	$1 < Re \leq 400$
$C_D = 0.5$	$400 < Re \leq 3\times10^5$
$C_D = 0.000366\,Re^{0.4275}$	$3\times10^5 < Re \leq 2\times10^6$
$C_D = 0.18$	$Re > 2\times10^6$

Find: Use data from Fig. 9.11 to evaluate the maximum error between the curve fit and experimental data.

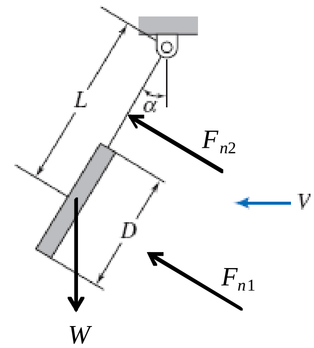
Solution: The curve-fit segments are plotted on Fig. 9.11 below:



The maximum significant error occurs in the region where C_D is modeled as equal to the constant value, $C_D = 0.5$. The curve fit appears to be about 10 percent high in the region from $Re \simeq 10^3$ to $Re \simeq 10^4$.

Problem 9.131

9.131 Problem 9.105 showed a circular disk hung in an air stream from a cylindrical strut. Assume the strut is L = 40 mm long and d = 3 mm in diameter. Solve Problem 9.105 including the effect of drag on the support.



Given: Circular disk in wind

Find: Mass of disk; Plot α versus V

Solution:

Basic equations: $$C_D = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot V^2\cdot A} \qquad \Sigma \vec{M} = 0$$

Summing moments at the pivot $$W\cdot L\cdot \sin(\alpha) - F_{n1}\cdot L - \frac{1}{2}\cdot\left(L - \frac{D}{2}\right)\cdot F_{n2} = 0 \quad (1)$$ and for each normal drag $$F_n = \frac{1}{2}\cdot\rho\cdot V_n^2\cdot A\cdot C_D$$

Assume 1) No pivot friction 2) C_D is valid for $V_n = V\cos(\alpha)$

The data is $\rho = 1.225\cdot\frac{kg}{m^3}$ $\quad \mu = 1.8\times 10^{-5}\cdot\frac{N\cdot s}{m^2}$ $\quad V = 15\cdot\frac{m}{s}$

$D = 25\cdot mm$ $\quad d = 3\cdot mm$ $\quad L = 40\cdot mm$ $\quad \alpha = 10\cdot deg$

$C_{D1} = 1.17$ (Table 9.3) $\quad Re_d = \frac{\rho\cdot V\cdot d}{\mu}$ $\quad Re_d = 3063$ so from Fig. 9.13 $\quad C_{D2} = 0.9$

Hence $$F_{n1} = \frac{1}{2}\cdot\rho\cdot(V\cdot\cos(\alpha))^2\cdot\frac{\pi\cdot D^2}{4}\cdot C_{D1} \qquad F_{n1} = 0.077\,N$$

$$F_{n2} = \frac{1}{2}\cdot\rho\cdot(V\cdot\cos(\alpha))^2\cdot\left(L - \frac{D}{2}\right)\cdot d\cdot C_{D2} \qquad F_{n2} = 0.00992\,N$$

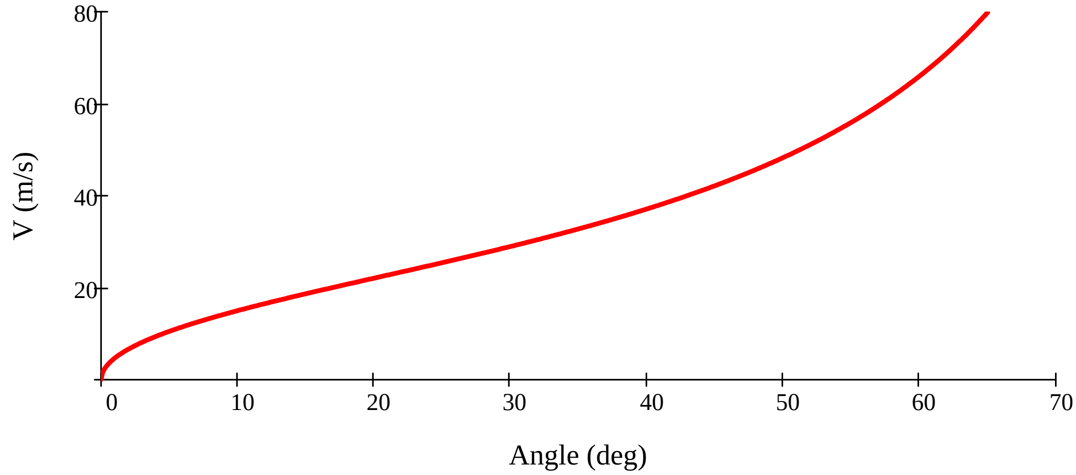
The drag on the support is much less than on the disk (and moment even less), so results will not be much different from those of Problem !

Hence Eq. 1 becomes $$M\cdot L\cdot g\cdot\sin(\alpha) = L\cdot\frac{1}{2}\cdot\rho\cdot(V\cdot\cos(\alpha))^2\cdot\frac{\pi\cdot D^2}{4}\cdot C_{D1} + \frac{1}{2}\cdot\left(L - \frac{D}{2}\right)\cdot\left[\frac{1}{2}\cdot\rho\cdot(V\cdot\cos(\alpha))^2\cdot\left(L - \frac{D}{2}\right)\cdot d\cdot C_{D2}\right]$$

$$M = \frac{\rho\cdot V^2\cdot\cos(\alpha)^2}{4\cdot g\cdot\sin(\alpha)}\cdot\left[\frac{1}{2}\cdot\pi\cdot D^2\cdot C_{D1} + \left(1 - \frac{D}{2\cdot L}\right)\cdot\left(L - \frac{D}{2}\right)\cdot d\cdot C_{D2}\right] \qquad M = 0.0471\,kg$$

Rearranging $$V = \sqrt{\frac{4\cdot M\cdot g}{\rho}}\cdot\sqrt{\frac{\tan(\alpha)}{\cos(\alpha)}}\cdot\frac{1}{\sqrt{\left[\frac{1}{2}\cdot\pi\cdot D^2\cdot C_{D1}+\left(1-\frac{D}{2\cdot L}\right)\cdot\left(L-\frac{D}{2}\right)\cdot d\cdot C_{D2}\right]}} \qquad V = 35.5\cdot\frac{m}{s}\cdot\sqrt{\frac{\tan(\alpha)}{\cos(\alpha)}}$$

We can plot this by choosing α and computing V



This graph can be easily plotted in *Excel*

Problem 9.132

9.132 A tennis ball with a mass of 57 g and diameter of 64 mm is dropped in standard sea level air. Calculate the terminal velocity of the ball. Assuming as an approximation that the drag coefficient remains constant at its terminal-velocity value, estimate the time and distance required for the ball to reach 95% of its terminal speed.

Given: Data on a tennis ball

Find: Terminal speed time and distance to reach 95% of terminal speed

Solution:

The given data or available data is $M = 57 \cdot gm$ $\quad D = 64 \cdot mm$ $\quad \nu = 1.45 \cdot 10^{-5} \cdot \frac{m^2}{s}$ $\quad \rho = 1.23 \cdot \frac{kg}{m^3}$

Then $A = \frac{\pi \cdot D^2}{4}$ $\quad A = 3.22 \times 10^{-3} m^2$

Assuming high Reynolds number $C_D = 0.5$ (from Fig. 9.11)

At terminal speed drag equals weight $F_D = M \cdot g$

The drag at speed *V* is given by $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$

Hence the terminal speed is $V_t = \sqrt{\frac{M \cdot g}{\frac{1}{2} \cdot \rho \cdot A \cdot C_D}}$ $\quad V_t = 23.8 \frac{m}{s}$

Check the Reynolds number $Re = \frac{V_t \cdot D}{\nu}$ $\quad Re = 1.05 \times 10^5$ Check!

For motion before terminal speed Newton's second law applies

$$M \cdot a = M \cdot \frac{dV}{dt} = M \cdot g - \frac{1}{2} \cdot \rho \cdot V^2 \cdot A \cdot C_D \quad \text{or} \quad \frac{d}{dt} V = g - k \cdot V^2 \quad \text{where} \quad k = \frac{\rho \cdot A \cdot C_D}{2 \cdot M} \quad k = 0.0174 \frac{1}{m}$$

Separating variables $\int_0^V \frac{1}{g - k \cdot V^2} dV = t$ $\quad \int \frac{1}{g - k \cdot V^2} dV = \frac{1}{\sqrt{g \cdot k}} \cdot \operatorname{atanh}\left(\sqrt{\frac{k}{g}} \cdot V\right)$

Hence $V(t) = \sqrt{\frac{g}{k}} \cdot \tanh\left(\sqrt{g \cdot k} \cdot t\right)$

Evaluating at $V = 0.95V_t$ $\quad 0.95 \cdot V_t = \sqrt{\frac{g}{k}} \cdot \tanh\left(\sqrt{g \cdot k} \cdot t\right)$ $\quad t = \frac{1}{\sqrt{g \cdot k}} \cdot \operatorname{atanh}\left(0.95 \cdot V_t \cdot \sqrt{\frac{k}{g}}\right)$ $\quad t = 4.44 s$

For distance *x* versus time, integrate $\frac{dx}{dt} = \sqrt{\frac{g}{k}} \cdot \tanh\left(\sqrt{g \cdot k} \cdot t\right)$ $\quad x = \int_0^t \sqrt{\frac{g}{k}} \cdot \tanh\left(\sqrt{g \cdot k} \cdot t\right) dt$

Note that $\displaystyle\int \tanh(a\cdot t)\,dt = \frac{1}{a}\cdot\ln(\cosh(a\cdot t))$

Hence $x(t) = \dfrac{1}{k}\cdot\ln\left(\cosh\left(\sqrt{g\cdot k}\cdot t\right)\right)$

Evaluating at $V = 0.95V_t$ $t = 4.44\,s$ so $x(t) = 67.1\,m$

Problem 9.133

9.133 A model airfoil of chord 15 cm and span 60 cm is placed in a wind tunnel with an air flow of 30 m/s (the air is at 20°C). It is mounted on a cylindrical support rod 2 cm in diameter and 25 cm tall. Instruments at the base of the rod indicate a vertical force of 50 N and a horizontal force of 6 N. Calculate the lift and drag coefficients of the airfoil.

Given: Data on model airfoil

Find: Lift and drag coefficients

Solution:

Basic equation: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad C_L = \dfrac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ where A is plan area for airfoil, frontal area for rod

Given or available data is $D = 2\cdot cm$ $\quad L = 25\cdot cm$ (Rod) $\quad b = 60\cdot cm$ $\quad c = 15\cdot cm$ (Airfoil)

$V = 30\cdot\frac{m}{s}$ $\quad F_L = 50\cdot N$ $\quad F_H = 6\cdot N$

Note that the horizontal force F_H is due to drag on the airfoil AND on the rod

$\rho = 1.225\cdot\frac{kg}{m^3}$ $\quad \nu = 1.50\times 10^{-5}\cdot\frac{m^2}{s}$ (Table A.10, 20ºC)

For the rod $\quad Re_{rod} = \frac{V\cdot D}{\nu}$ $\quad Re_{rod} = 4\times 10^4$ so from Fig. 9.13 $\quad C_{Drod} = 1.0$

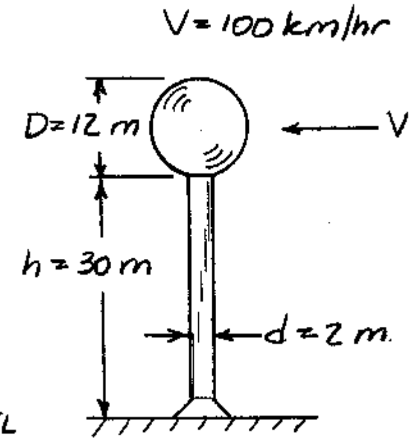
$A_{rod} = L\cdot D$ $\quad A_{rod} = 5\times 10^{-3}\,m^2$

$F_{Drod} = C_{Drod}\cdot\frac{1}{2}\cdot\rho\cdot A_{rod}\cdot V^2$ $\quad F_{Drod} = 2.76\,N$

Hence for the airfoil $\quad A = b\cdot c$ $\quad F_D = F_H - F_{Drod}$ $\quad F_D = 3.24\,N$

$C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad C_D = 0.0654$ $\quad C_L = \dfrac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad C_L = 1.01$ $\quad \dfrac{C_L}{C_D} = 15.4$

Problem 9.134



Given: Water tower as shown.

Standard day.

Find: Estimate bending moment at base of tower.

<u>Solution</u>: Apply definition of C_D, sum moments about base.

Computing equations: $C_D \equiv \frac{F_D}{\frac{1}{2}\rho V^2 A}$; $\Sigma \overset{\curvearrowleft +}{M} = \Sigma F L$

Assumptions: (1) F_{D_S} acts at center of sphere; F_{D_C} at center of cylinder
(2) Neglect interference between sphere and cylinder.

Then $M = F_{D_S}\left(h + \frac{D}{2}\right) + F_{D_C}\left(\frac{h}{2}\right)$ $\qquad A_S = \frac{\pi D^2}{4} = \frac{\pi}{4}(12)^2\, m^2 = 113\ m^2$

$A_C = hd = 30\ m \times 2\ m = 60\ m^2$

$$V = 100\ \frac{km}{hr} \times 1000\ \frac{m}{km} \times \frac{hr}{3600\ s} = 27.8\ m/s$$

$$q = \frac{1}{2}\rho V^2 = \frac{1}{2} \times 1.23\ \frac{kg}{m^3} \times (27.8)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 475\ N/m^2$$

$C_D = C_D(Re)$. For standard air (Table A.10), $\nu = 1.46 \times 10^{-5}\ m^2/s$, so

$$Re_S = \frac{VD}{\nu} = 27.8\ \frac{m}{s} \times 12\ m \times \frac{s}{1.46 \times 10^{-5}\ m^2} = 2.28 \times 10^7$$

This Re is too large for Fig. 9.11. Thus guess $C_{D_S} = 0.18$ (Problem 9.125).

$$F_{D_S} = C_{D_S}\, q\, A_S = 0.18 \times 475\ \frac{N}{m^2} \times 113\ m^2 = 9.66\ kN$$

$$Re_C = \frac{Vd}{\nu} = 27.8\ \frac{m}{s} \times 2.0\ m \times \frac{s}{1.46 \times 10^{-5}\ m^2} = 3.81 \times 10^6$$

This Re is too large for Fig. 9.13. Thus guess $C_{D_C} \simeq 0.4$.

$$F_{D_C} = C_{D_C}\, q\, A_C = 0.4 \times 475\ \frac{N}{m^2} \times 60\ m^2 = 11.4\ kN$$

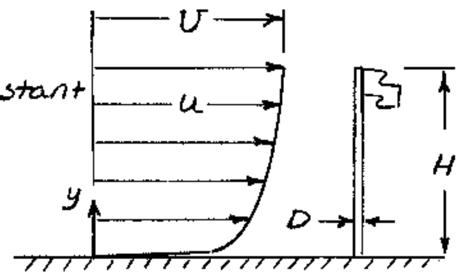
The moment is

$$M = 9.66\ kN\left(30\ m + \frac{12\ m}{2}\right) + 11.4\ kN\left(\frac{30\ m}{2}\right) = 519\ kN \cdot m$$ M

Problem 9.135

Given: Cylindrical flag pole of height, H.

Wind-speed profile, $\frac{u}{U} = \left(\frac{y}{H}\right)^{1/7}$; C_D constant



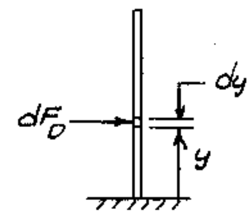
Find: (a) Drag force
(b) Bending moment
(c) Compare with values for uniform profile, U.

Solution: Apply definition of drag coefficient, $C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$

Assume: (1) C_D = constant
(2) C_D same as circular cylinder

On an element of the pole,

$$dF_D = C_D \tfrac{1}{2}\rho u^2 dA = C_D \tfrac{1}{2}\rho U^2 \left(\frac{y}{H}\right)^{2/7} D\,dy$$



Thus

$$F_D = \int_0^H dF_D = \int_0^H C_D \tfrac{1}{2}\rho U^2 \left(\frac{y}{H}\right)^{2/7} D\,dy$$

$$F_D = C_D \tfrac{1}{2}\rho U^2 DH \int_0^1 \left(\frac{y}{H}\right)^{2/7} d\left(\frac{y}{H}\right) = C_D \tfrac{1}{2}\rho U^2 DH \left[\frac{7}{9}\left(\frac{y}{H}\right)^{9/7}\right]_0^1 = \frac{7}{9} C_D \tfrac{1}{2}\rho U^2 DH$$ F_D

On an element of the pole,

$$dM = y\,dF_D = y\,C_D \tfrac{1}{2}\rho u^2 dA = y\,C_D \tfrac{1}{2}\rho u^2 D\,dy$$

Thus

$$M = \int_0^H dM = \int_0^H y\,C_D \tfrac{1}{2}\rho U^2 \left(\frac{y}{H}\right)^{2/7} D\,dy = C_D \tfrac{1}{2}\rho U^2 DH^2 \int_0^1 \left(\frac{y}{H}\right)\left(\frac{y}{H}\right)^{2/7} d\left(\frac{y}{H}\right)$$

$$M = C_D \tfrac{1}{2}\rho U^2 DH^2 \int_0^1 \left(\frac{y}{H}\right)^{9/7} d\left(\frac{y}{H}\right) = C_D \tfrac{1}{2}\rho U^2 DH^2 \left[\frac{7}{16}\left(\frac{y}{H}\right)^{16/7}\right]_0^1 = \frac{7}{16} C_D \tfrac{1}{2}\rho U^2 DH^2$$ M

Comparing,

$$\frac{F_D(\text{1/7-profile})}{F_D(\text{uniform})} = \frac{\frac{7}{9} C_D \frac{1}{2}\rho U^2 DH}{C_D \frac{1}{2}\rho U^2 DH} = \frac{7}{9}$$

$$\frac{M(\text{1/7-profile})}{M(\text{uniform})} = \frac{\frac{7}{16} C_D \frac{1}{2}\rho U^2 DH^2}{C_D \frac{1}{2}\rho U^2 DH \frac{H}{2}} = \frac{7/16}{1/2} = \frac{7}{8}$$ $\frac{M}{M_U}$

Problem 9.136

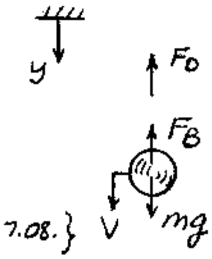
Given: Cast-iron "12-pounder" ($m = 12$ lbm) cannon ball rolls off ship and sinks in ocean where depth is $d = 1000$ m.

Find: Estimate time elapsed before cannon ball hits sea bottom.

Solution: Apply Newton's second law of motion, definition of C_D.

Computing equations:

$$\Sigma F_y = m a_y \qquad C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$



First find diameter of ball. In air,

$$W = mg = \rho \forall g = SG \rho_{H_2O} \frac{\pi D^3}{6} g = 12 \text{ lbf} \quad \{\text{From Table A.1, } SG = 7.08.\}$$

Thus

$$D = \left[\frac{6W}{\pi SG \rho_{H_2O} g}\right]^{1/3} = \left[\frac{6}{\pi} \times 12 \text{ lbf} \times \frac{1}{7.08} \times \frac{ft^3}{1.94 \text{ slug}} \times \frac{s^2}{32.2 \text{ ft}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot s^2}\right]^{1/3} = 0.373 \text{ ft}$$

or

$$D = 0.373 \text{ ft} \times 0.3048 \frac{m}{ft} = 0.114 \text{ m}$$ D

At terminal speed, $V = V_t$, and $a_y = 0$. Summing forces,

$$mg - F_B - F_D = SG \rho_{H_2O} \forall g - SG_{sw} \rho_{H_2O} \forall g - C_D A \frac{1}{2} \rho V_t^2 = 0$$

or

$$V_t = \left[\frac{2(SG_{ci} - SG_{sw}) \rho_{H_2O} \forall g}{C_D \, SG_{sw} \rho_{H_2O} A}\right]^{1/2}$$

Introducing $\forall = \frac{\pi D^3}{6}$ and $A = \frac{\pi D^2}{4}$, then

$$V_t = \left[\frac{4}{3} \frac{(SG_{ci}/SG_{sw} - 1) D g}{C_D}\right]^{1/2} = \left[\frac{4}{3} (7.08/1.025 - 1)\, 0.114 \text{ m} \times 9.81 \frac{m}{s^2} \times \frac{1}{C_D}\right]^{1/2} = \frac{2.97}{\sqrt{C_D}} \text{ m/s}$$

Choose $C_D = 0.47$ from flat range of curve:

$$V_t = \frac{2.97}{\sqrt{0.47}} \text{ m/sec} = 4.33 \text{ m/s} \qquad \{\text{At } T = 20°C, \nu_{sw} = 1.05 \nu_{H_2O} \text{ (Table A.2).}\}$$

Then $Re = \frac{\rho V_t D}{\mu} = \frac{V_t D}{\nu_{sw}} = 4.33 \frac{m}{s} \times 0.114 \text{ m} \times \frac{s}{1.05 \times 10^{-6} m^2} = 4.70 \times 10^5$

This is a supercritical Re, so choose $C_D \approx 0.09$ (Fig. 9.11). Then

$$V_t = \frac{2.97}{\sqrt{0.09}} \text{ m/s} = 9.90 \text{ m/s}$$ V_t

Then $Re = 9.90 \frac{m}{s} \times 0.114 \text{ m} \times \frac{s}{1.05 \times 10^{-6} m^2} = 1.07 \times 10^6$

From Fig. 9.11, $C_D \approx 0.14$. Therefore $V_t = \frac{2.97}{\sqrt{0.14}} = 7.94$ m/s

$$t = \frac{d}{V_t} = 1000 \text{ m} \times \frac{s}{7.94 \text{ m}} = 126 \text{ s}$$ t

Problem 9.137

[4]

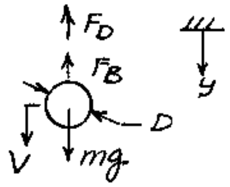
Given: Stokes drag law for smooth spheres, $F_D = 3\pi\mu VD$, to be verified experimentally by dropping steel balls in glycerin.

Find: (a) Largest steel ball for which $Re < 1$.
(b) Height of glycerine column needed to reach 95 percent of terminal speed.

<u>Solution</u>: Draw free-body diagram of ball, apply Newton's second law.

Basic equation: $\Sigma F_y = mg - F_B - F_D = m\frac{dV}{dt} = mV\frac{dV}{dy}$ (1)

$\forall = \frac{\pi D^3}{6}$ $\quad m = \rho_s \forall \quad F_B = \rho_g \forall g \quad F_D = 3\pi\mu VD$



At terminal speed, V_t, acceleration is zero. Thus

$$\rho_s \forall g - \rho_g \forall g - 3\pi\mu V_t D = 0 \quad \text{or} \quad V_t = (\rho_s - \rho_g)\frac{\pi D^3}{6} g \frac{1}{3\pi\mu D} = \frac{(\rho_s - \rho_g)D^2 g}{18\mu}$$

$$\text{or} \quad V_t = \frac{(\rho_s/\rho_g - 1)\rho_g D^2 g}{18\mu} = \frac{(SG_s/SG_g - 1)D^2 g}{18\nu} = \frac{(7.8/1.26 - 1)D^2 g}{18\nu} = 0.288\, D^2 g/\nu$$

(from Table A.2, $SG_g = 1.26$). Stokes' drag law holds for $Re < 1$. Thus

$$Re = \frac{\rho V_t D}{\mu} = \frac{V_t D}{\nu} = \frac{0.288 D^3 g}{\nu^2} \le 1 \quad \text{or} \quad D^3 \le \frac{1}{0.288}\frac{\nu^2}{g} \quad \text{or} \quad D \le \left[\frac{3.47\nu^2}{g}\right]^{1/3}$$

Assuming $T = 20°C$, then from Fig. A.3, $\nu = 0.0012\ m^2/s$, so

$$D \le \left[3.47 \times (0.0012)^2 \frac{m^4}{s^2} \times \frac{s^2}{9.81\ m}\right]^{1/3} = 0.00799\ m \quad (7.99\ mm)$$ D

From Eq. 1,

$$\rho_s \forall g - \rho_g \forall g - 3\pi\mu VD = \rho_s \forall V\frac{dV}{dy}$$

Dividing by $(\rho_s - \rho_g)\forall g$ gives

$$1 - \frac{3\pi\mu D}{(\rho_s - \rho_g)\frac{\pi D^3}{6}g} V = 1 - \frac{V}{V_t} = \frac{\rho_s \forall}{(\rho_s - \rho_g)\forall g} V\frac{dV}{dy} = \left(\frac{\rho_s}{\rho_s - \rho_g}\right)\frac{V}{g}\frac{dV}{dy} = \left(\frac{\rho_s}{\rho_s - \rho_g}\right)\frac{V_t^2}{g}\frac{V}{V_t}\frac{d(V/V_t)}{dy}$$

Separating variables, $dy = \left(\frac{\rho_s}{\rho_s - \rho_g}\right)\frac{V_t^2}{g}\frac{(V/V_t)\,d(V/V_t)}{1 - V/V_t} = \left(\frac{\rho_s}{\rho_s - \rho_g}\right)\frac{V_t^2}{g}\frac{r\,dr}{1 - r}$

Integrating, $$\int_0^{0.95}\frac{r\,dr}{1-r} = \int_1^{0.05}\frac{(1-x)(-dx)}{x} = \int_1^{0.05} dx - \int\frac{dx}{x} = x - \ln x\Big]_1^{0.05} = -0.95 - \ln(0.05)$$

Thus

$$y = 2.05\left(\frac{\rho_s}{\rho_s - \rho_g}\right)\frac{V_t^2}{g} = 2.05\left(\frac{SG_s}{SG_s - SG_g}\right)\frac{V_t^2}{g}$$

But $V_t = 0.288\frac{D^2 g}{\nu} = 0.288 \times (0.0172)^2 m^2 \times 9.81\frac{m}{s^2} \times \frac{s}{0.0012\ m^2} = 0.697\ m/s$

so

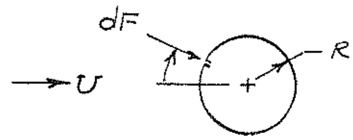
$$y = 2.05 \times \frac{7.8}{(7.8 - 1.26)} \times (0.697)^2 \frac{m^2}{s^2} \times \frac{s^2}{9.81\ m} = 0.121\ m \quad (121\ mm)$$ y

Problem 9.138

Given: Measured data for pressure difference versus angle for flow around a circular cylinder at $Re = 80{,}000$.

Find: (a) Estimate C_D for this flow.
(b) Compare with data from Fig. 9.13; explain any difference.

Solution: Consider the geometry sketched.
Apply the definition of drag coefficient.



Computing equation: $C_D = \dfrac{F_D}{\frac{1}{2}\rho U^2 A}$

Assumption: Neglect viscous force; $dF_D = dF\cos\theta = p\,dA\cos\theta = p\,wR\,d\theta\cos\theta$

Then $F_D = \int_A dF_D = \int_0^{2\pi} p\,wR\,d\theta\cos\theta = 2\int_0^{\pi} p\,wR\,d\theta\cos\theta = \int_0^{\pi} p\cos\theta\,(w2R)\,d\theta$

Since $\int_0^{\pi} p_\infty\cos\theta\,d\theta = 0$, then $F_D = \int_0^{\pi}(p-p_\infty)\cos\theta\,(w2R)\,d\theta$

The stagnation pressure is $p(0) - p_\infty = \frac{1}{2}\rho U^2$; so

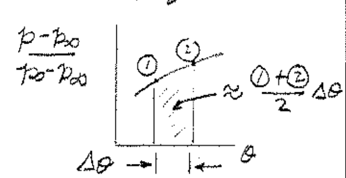
$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A} = \frac{\int_0^{\pi}(p-p_\infty)\cos\theta\,(w2R)\,d\theta}{(p_0-p_\infty)(w2R)} = \int_0^{\pi}\left(\frac{p-p_\infty}{p_0-p_\infty}\right)\cos\theta\,d\theta$$

Tabulating:

θ (deg)	$p-p_\infty$ (in. H_2O)	$\frac{p-p_\infty}{p_0-p_\infty}$	$\left(\frac{p-p_\infty}{p_0-p_\infty}\right)\cos\theta$
0	0.52	1.00	1.00
10	0.46	0.885	0.871
20	0.33	0.635	0.596
30	0.13	0.250	0.217
40	−0.13	−0.250	−0.192
50	−0.40	−0.769	−0.494
60	−0.57	−1.10	−0.548
70	−0.60	−1.15	−0.395
80	−0.55	−1.06	−0.184
90	−0.52	−1.00	0.00
100	−0.51	−0.981	0.170
110	−0.52	−1.00	0.342
120	−0.52	−1.00	0.500
130	−0.52	−1.00	0.643
140	−0.52	−1.00	0.766
150	−0.52	−1.00	0.866
160	−0.54	−1.04	0.976
170	−0.54	−1.04	1.02
180	−0.54	−1.04	1.04

$\Sigma = 7.194$

Trapezoidal rule:



Half of the end points are subtracted to avoid double counting.

$$C_D \approx \left\{\Sigma\left(\frac{p-p_\infty}{p_0-p_\infty}\right)\cos\theta - \tfrac{1}{2}(\,)_0 - \tfrac{1}{2}(\,)_{180}\right\}\Delta\theta = \left\{7.194 - \tfrac{1}{2}(1.00+1.04)\right\} 10\,\text{deg} \times \frac{\pi\,\text{rad}}{180\,\text{deg}} = 1.08 \qquad C_D$$

From Fig. 9.13, $C_D \approx 1.2$. The difference is due to skin friction effects.

Problem 9.139

[4]

9.139 Consider the tennis ball of Problem 9.132. Use the equations for drag coefficient given in Problem 9.130, and a numerical integration scheme (e.g., Simpson's rule) to compute the time and distance required for the ball to reach 95% of its terminal speed.

Given: Data on a tennis ball

Find: Terminal speed time and distance to reach 95% of terminal speed

Solution:

The given data or available data is $M = 57 \cdot gm$ $\quad D = 64 \cdot mm$ $\quad \nu = 1.45 \cdot 10^{-5} \cdot \frac{m^2}{s}$ $\quad \rho = 1.23 \cdot \frac{kg}{m^3}$

Then $A = \frac{\pi \cdot D^2}{4}$ $\quad A = 3.22 \times 10^{-3} m^2$

From Problem 9.130

$C_D = \frac{24}{Re}$ $\quad Re \le 1$

$C_D = \frac{24}{Re^{0.646}}$ $\quad 1 < Re \le 400$

$C_D = 0.5$ $\quad 400 < Re \le 3 \times 10^5$

$C_D = 0.000366 \cdot Re^{0.4275}$ $\quad 3 \times 10^5 < Re \le 2 \times 10^6$

$C_D = 0.18$ $\quad Re > 2 \times 10^6$

At terminal speed drag equals weight $F_D = M \cdot g$

The drag at speed *V* is given by $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$

Assume $C_D = 0.5$

Hence the terminal speed is $V_t = \sqrt{\frac{M \cdot g}{\frac{1}{2} \cdot \rho \cdot A \cdot C_D}}$ $\quad V_t = 23.8 \frac{m}{s}$

Check the Reynolds number $Re = \frac{V_t \cdot D}{\nu}$ $\quad Re = 1.05 \times 10^5$

This is consistent with the tabulated C_D values!

For motion before terminal speed, Newton's second law is $M \cdot a = M \cdot \frac{dV}{dt} = M \cdot g - \frac{1}{2} \cdot \rho \cdot V^2 \cdot A \cdot C_D$

Hence the time to reach 95% of terminal speed is obtained by separating variables and integrating

$$t = \int_0^{0.95 \cdot V_t} \frac{1}{g - \frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2} \, dV$$

For the distance to reach terminal speed Newton's second law is written in the form

$$M \cdot a = M \cdot V \cdot \frac{dV}{dx} = M \cdot g - \frac{1}{2} \cdot \rho \cdot V^2 \cdot A \cdot C_D$$

Hence the distance to reach 95% of terminal speed is obtained by separating variables and integrating

$$x = \int_0^{0.95 \cdot V_t} \frac{V}{g - \frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2} \, dV$$

These integrals are quite difficult because the drag coefficient varies with Reynolds number, which varies with speed. They are best evaluated numerically. A form of Simpson's Rule is

$$\int f(V)\, dV = \frac{\Delta V}{3} \cdot \left(f\left(V_0\right) + 4 \cdot f\left(V_1\right) + 2 \cdot f\left(V_2\right) + 4 \cdot f\left(V_3\right) + f\left(V_N\right)\right)$$

where ΔV is the step size, and V_0, V_1 etc., are the velocities at points 0, 1, ... N.

Here $V_0 = 0$ $V_N = 0.95 \cdot V_t$ $\Delta V = \frac{0.95 \cdot V_t}{N}$

From the associated *Excel* workbook $t = 4.69 \cdot s$ $x = 70.9 \cdot m$

These results compare to 4.44 s and 67.1 m from Problem 9.132, which assumed the drag coefficient was constant and analytically integrated. Note that the drag coefficient IS essentially constant, so numerical integration was not really necessary!

Problem 9.139 (In Excel)

9.139 Consider the tennis ball of Problem 9.132. Use the equations for drag coefficient given in Problem 9.130, and a numerical integration scheme (e.g., Simpson's rule) to compute the time and distance required for the ball to reach 95% of its terminal speed.

Given: Data on a tennis ball

Find: Terminal speed time and distance to reach 95% of terminal speed

Solution:

$$t = \int_0^{0.95 \cdot V_t} \frac{1}{g - \frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2} \, dV \qquad x = \int_0^{0.95 \cdot V_t} \frac{V}{g - \frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2} \, dV$$

Given data:

M = 57 gm

ρ = 1.23 kg/m^3

D = 64 mm

C_D = 0.5 (Fig. 9.11)

ν = 1.45E-05 m^2/s

$$C_D = \frac{24}{Re} \qquad Re \le 1$$

$$C_D = \frac{24}{Re^{0.646}} \qquad 1 < Re \le 400$$

$$C_D = 0.5 \qquad 400 < Re \le 3 \times 10^5$$

$$C_D = 0.000366 \cdot Re^{0.4275} \qquad 3 \times 10^5 < Re \le 2 \times 10^6$$

$$C_D = 0.18 \qquad Re > 2 \times 10^6$$

Computed results:

A = 0.00322 m^2

V_t = 23.8 m/s

N = 20

ΔV = 1.19 m/s

For the time:

V (m/s)	Re	C_D	W	$f(V)$	$W \times f(V)$
0	0	5438	1	0.102	0.102
1.13	4985	0.500	4	0.102	0.409
2.26	9969	0.500	2	0.103	0.206
3.39	14954	0.500	4	0.104	0.416
4.52	19938	0.500	2	0.106	0.212
5.65	24923	0.500	4	0.108	0.432
6.78	29908	0.500	2	0.111	0.222
7.91	34892	0.500	4	0.115	0.458
9.03	39877	0.500	2	0.119	0.238
10.2	44861	0.500	4	0.125	0.499
11.3	49846	0.500	2	0.132	0.263
12.4	54831	0.500	4	0.140	0.561
13.6	59815	0.500	2	0.151	0.302
14.7	64800	0.500	4	0.165	0.659
15.8	69784	0.500	2	0.183	0.366
16.9	74769	0.500	4	0.207	0.828
18.1	79754	0.500	2	0.241	0.483
19.2	84738	0.500	4	0.293	1.17
20.3	89723	0.500	2	0.379	0.758
21.5	94707	0.500	4	0.550	2.20
22.6	99692	0.500	1	1.05	1.05

For the distance:

$f(V)$	$W \times f(V)$
0.00	0.000
0.115	0.462
0.232	0.465
0.353	1.41
0.478	0.955
0.610	2.44
0.752	1.50
0.906	3.62
1.08	2.15
1.27	5.07
1.49	2.97
1.74	6.97
2.05	4.09
2.42	9.68
2.89	5.78
3.51	14.03
4.36	8.72
5.62	22.5
7.70	15.4
11.8	47.2
23.6	23.6

Total time: 4.69 s
(This compares to 4.44s for the exact result)

Total distance: 70.9 m
(This compares to 67.1 m for the exact result)

Note that C_D is basically constant, so analytical result of Problem 9.132 is accurate!

Problem 9.140

9.140 The air bubble of Problem 3.11 expands as it rises in water. Find the time it takes for the bubble to reach the surface. Repeat for bubbles of diameter 5 mm and 15 mm. Compute and plot the depth of the bubbles as a function of time.

Given: Data on an air bubble

Find: Time to reach surface; plot depth as function of time; repeat for different sizes

Solution:

The given data or available data is $d_0 = 0.3\cdot in$ $\quad h = 100\cdot ft$ $\quad \rho_w = 1000\cdot\frac{kg}{m^3}$ $\quad SG = 1.025$ (Table A.2)

$\rho = SG\cdot\rho_w$ $\quad \nu = 1.05 \times 8.03 \times 10^{-7}\cdot\frac{m^2}{s}$ (Tables A.2 & A.8) $\quad p_{atm} = 101\cdot kPa$

The density of air is negligible compared to that of water, so Newton's second law is applicable with negligible *MdV/dt*

$$M\cdot\frac{dV}{dt} = 0 = \Sigma F = F_B - F_D \quad \text{or} \quad F_B = F_D \qquad (1)$$

where F_B is the buoyancy force and F_D is the drag (upwards is positive x)

$$F_B = \rho\cdot Vol\cdot g \qquad F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D \qquad (2)$$

For a sphere, assuming high Reynolds number, from Fig. 9.11 $C_D = 0.5$

The volume of the sphere increases as the bubble rises and experiences decreased pressure. Assuming the air is an isothermal idea gas

$$p_0\cdot Vol_0 = p\cdot Vol$$

where p_0 and Vol_0 are the initial pressure and volume (at depth h), and p and Vol are the pressure and volume at any depth

$$p_0 = p_{atm} + \rho\cdot g\cdot h \qquad p = p_{atm} + \rho\cdot g\cdot(h - x)$$

Hence

$$\left(p_{atm} + \rho\cdot g\cdot h\right)\cdot\frac{\pi}{6}\cdot d_0^{\,3} = \left[p_{atm} + \rho\cdot g\cdot(h - x)\right]\cdot\frac{\pi}{6}\cdot d^3$$

$$d = d_0\cdot\sqrt[3]{\frac{\left(p_{atm} + \rho\cdot g\cdot h\right)}{\left[p_{atm} + \rho\cdot g\cdot(h - x)\right]}} \qquad (3)$$

For example, at the free surface ($x = h$) $\quad d = 12.1\,mm$

Combining Eqs. 1, 2 and 3 $\quad \rho\cdot\frac{\pi}{6}\cdot d^3 = \frac{1}{2}\cdot\rho\cdot\frac{\pi}{4}\cdot d^2\cdot V^2\cdot C_D$

$$V = \sqrt{\frac{4\cdot g\cdot d}{3\cdot C_D}} \qquad V = \sqrt{\frac{4\cdot g\cdot d_0}{3\cdot C_D}\cdot\left[\frac{\left(p_{atm} + \rho\cdot g\cdot h\right)}{\left[p_{atm} + \rho\cdot g\cdot(h - x)\right]}\right]^{\frac{1}{6}}}$$

Strictly speaking, to obtain x as a function of t we would have to integrate this expression ($V = dx/dt$).

However, evaluating V at depth h (x = 0) and at the free surface ($x = h$)

$x = 0 \qquad V_0 = 0.446 \frac{m}{s}$

$x = h \qquad V = 0.563 \frac{m}{s}$

we see that the velocity varies slightly. Hence, instead of integrating we use the approximation dx = Vdt where dx is an increment of displacement and dt is an increment of time. (This amounts to numerically integrating)

Note that the Reynolds number at the initial depth (the smallest Re) is $Re_0 = \frac{V_0 \cdot d_0}{\nu}$ $\qquad Re_0 = 4034$

so our use of C_D = 0.5 from Fig. 9.11 is reasonable

The plots of depth versus time are shown in the associated *Excel* workbook

The results are

$d_0 = 0.3 \cdot in \qquad t = 63.4 \cdot s$

$d_0 = 5 \cdot mm \qquad t = 77.8 \cdot s$

$d_0 = 15 \cdot mm \qquad t = 45.1 \cdot s$

Problem 9.140 (In Excel)

[4]

9.140 The air bubble of Problem 3.11 expands as it rises in water. Find the time it takes for the bubble to reach the surface. Repeat for bubbles of diameter 5 mm and 15 mm. Compute and plot the depth of the bubbles as a function of time.

Given: Data on an air bubble

Find: Time to reach surface; plot depth as function of time; repeat for different sizes

Solution:

The equation is $dx = V \cdot dt$ where $V = \sqrt{\frac{4 \cdot g \cdot d_0}{3 \cdot C_D} \cdot \left[\frac{(p_{atm} + \rho \cdot g \cdot h)}{[p_{atm} + \rho \cdot g \cdot (h - x)]}\right]^{\frac{1}{6}}}$

Given data:

h = 100 ft

h = 30.5 m

ρ_w = 1000 kg/m^3

SG = 1.025 Table A.2)

C_D = 0.5 Fig. 9.11)

ρ = 1025 kg/m^3

p_{atm} = 101 kPa

Computed results:

d_0 = 0.3 in

d_0 = 7.62 mm

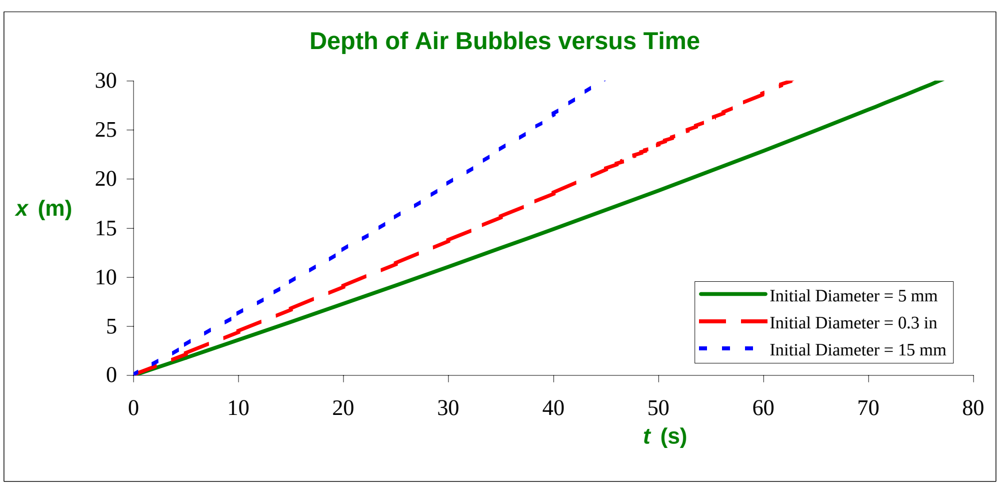
t (s)	*x* (m)	*V* (m/s)
0	0	0.446
5	2.23	0.451
10	4.49	0.455
15	6.76	0.460
20	9.1	0.466
25	11.4	0.472
30	13.8	0.478
35	16.1	0.486
40	18.6	0.494
45	21.0	0.504
50	23.6	0.516
63.4	30.5	0.563

d_0 = 5 mm

t (s)	*x* (m)	*V* (m/s)
0	0	0.362
5	1.81	0.364
10	3.63	0.367
15	5.47	0.371
20	7.32	0.374
25	9.19	0.377
30	11.1	0.381
35	13.0	0.386
40	14.9	0.390
45	16.9	0.396
50	18.8	0.401
55	20.8	0.408
60	22.9	0.415
65	25.0	0.424
70	27.1	0.435
75	29.3	0.448
77.8	30.5	0.456

d_0 = 15 mm

t (s)	*x* (m)	*V* (m/s)
0.0	0	0.626
5.0	3.13	0.635
10.0	6.31	0.644
15.0	9.53	0.655
20.0	12.8	0.667
25.0	16.1	0.682
30.0	19.5	0.699
35.0	23.0	0.721
40.0	26.6	0.749
45.1	30.5	0.790



Problem 9.141

9.141 Consider the tennis ball of Problem 9.132. Suppose it is hit so that it has an initial upward speed of 50 m/s. Estimate the maximum height of the ball, assuming (a) a constant drag coefficient and (b) using the equations for drag coefficient given in Problem 9.130, and a numerical integration scheme (e.g., a Simpson's rule).

Given: Data on a tennis ball

Find: Maximum height

Solution:

The given data or available data is $M = 57 \cdot gm$ $\quad D = 64 \cdot mm$ $\quad V_i = 50 \cdot \frac{m}{s}$ $\quad \nu = 1.45 \cdot 10^{-5} \cdot \frac{m^2}{s}$ $\quad \rho = 1.23 \cdot \frac{kg}{m^3}$

Then $A = \frac{\pi \cdot D^2}{4}$ $\quad A = 3.22 \times 10^{-3}\, m^2$

From Problem 9.130

$C_D = \frac{24}{Re}$ $\quad Re \le 1$

$C_D = \frac{24}{Re^{0.646}}$ $\quad 1 < Re \le 400$

$C_D = 0.5$ $\quad 400 < Re \le 3 \times 10^5$

$C_D = 0.000366 \cdot Re^{0.4275}$ $\quad 3 \times 10^5 < Re \le 2 \times 10^6$

$C_D = 0.18$ $\quad Re > 2 \times 10^6$

The drag at speed *V* is given by $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$

For motion before terminal speed, Newton's second law (*x* upwards) is $M \cdot a = M \cdot \frac{dV}{dt} = -\frac{1}{2} \cdot \rho \cdot V^2 \cdot A \cdot C_D - M \cdot g$

For the maximum height Newton's second law is written in the form $M \cdot a = M \cdot V \cdot \frac{dV}{dx} = -\frac{1}{2} \cdot \rho \cdot V^2 \cdot A \cdot C_D - M \cdot g$

Hence the maximum height is

$$x_{max} = \int_{V_i}^{0} \frac{V}{-\frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2 - g}\, dV = \int_{0}^{V_i} \frac{V}{\frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2 + g}\, dV$$

This integral is quite difficult because the drag coefficient varies with Reynolds number, which varies with speed. It is best evaluated numerically. A form of Simpson's Rule is

$$\int f(V)\, dV = \frac{\Delta V}{3} \cdot \left(f\left(V_0\right) + 4 \cdot f\left(V_1\right) + 2 \cdot f\left(V_2\right) + 4 \cdot f\left(V_3\right) + f\left(V_N\right)\right)$$

where ΔV is the step size, and V_0, V_1 etc., are the velocities at points 0, 1, ... *N*.

Here $V_0 = 0$ $V_N = V_i$ $\Delta V = -\frac{V_i}{N}$

From the associated *Excel* workbook $x_{max} = 48.7 \cdot m$

If we assume $C_D = 0.5$

the integral
$$x_{max} = \int_0^{V_i} \frac{V}{\frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2 + g}\, dV$$

becomes
$$x_{max} = \frac{M}{\rho \cdot A \cdot C_D} \cdot \ln\left(\frac{\rho \cdot A \cdot C_D}{2 \cdot M \cdot g} \cdot V_i^{\,2} + 1\right)$$
$x_{max} = 48.7\,m$

The two results agree very closely! This is because the integrand does not vary much after the first few steps so the numerical integral is accurate, and the analytic solution assumes $C_D = 0.5$, which it essentially does!

Problem 9.141 (In Excel)

9.141 Consider the tennis ball of Problem 9.132. Suppose it is hit so that it has an initial upward speed of 50 m/s. Estimate the maximum height of the ball, assuming (a) a constant drag coefficient and (b) using the equations for drag coefficient given in Problem 9.130, and a numerical integration scheme (e.g., a Simpson's rule).

Given: Data on a tennis ball

Find: Maximum height

Solution:

The equation is
$$x_{max} = \int_{V_i}^{0} \frac{V}{-\frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2 - g} dV = \int_{0}^{V_i} \frac{V}{\frac{\rho \cdot A \cdot C_D}{2 \cdot M} \cdot V^2 + g} dV$$

Given data:

M =	57	gm
V_0 =	50.0	m/s
ρ =	1.23	kg/m^3
D =	64	mm
C_D =	0.5	(Fig. 9.11)
ν =	1.45E-05	m^2/s

$C_D = \frac{24}{Re}$ $Re \le 1$

$C_D = \frac{24}{Re^{0.646}}$ $1 < Re \le 400$

$C_D = 0.5$ $400 < Re \le 3 \times 10^5$

$C_D = 0.000366 \cdot Re^{0.4275}$ $3 \times 10^5 < Re \le 2 \times 10^6$

$C_D = 0.18$ $Re > 2 \times 10^6$

Computed results:

A =	0.00322	m^2
N =	20	
ΔV =	2.50	m/s

V (m/s)	Re	C_D	W	$f(V)$	$W \times f(V)$
0.0	0	0.000	1	0.000	0.000
2.5	11034	0.500	4	0.252	1.01
5.0	22069	0.500	2	0.488	0.976
7.5	33103	0.500	4	0.695	2.78
10.0	44138	0.500	2	0.866	1.73
12.5	55172	0.500	4	1.00	3.99
15.0	66207	0.500	2	1.09	2.19
17.5	77241	0.500	4	1.16	4.63
20.0	88276	0.500	2	1.19	2.39
22.5	99310	0.500	4	1.21	4.84
25.0	110345	0.500	2	1.21	2.42
27.5	121379	0.500	4	1.20	4.80
30.0	132414	0.500	2	1.18	2.36
32.5	143448	0.500	4	1.15	4.62
35.0	154483	0.500	2	1.13	2.25
37.5	165517	0.500	4	1.10	4.38
40.0	176552	0.500	2	1.06	2.13
42.5	187586	0.500	4	1.03	4.13
45.0	198621	0.500	2	1.00	2.00
47.5	209655	0.500	4	0.970	3.88
50.0	220690	0.500	1	0.940	0.940

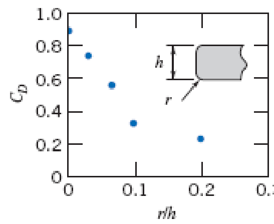
Maximum height: 48.7 m

(This is the same as the exact result)

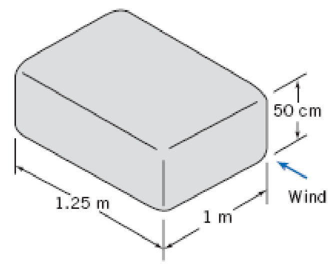
Note that C_D is basically constant, so analytical result of Problem 9.132 is accurate!

Problem 9.142

9.142 Approximate dimensions of a rented rooftop carrier are shown. Estimate the drag force on the carrier (r = 10 cm) at 100 km/hr. If the drivetrain efficiency of the vehicle is 0.85 and the brake specific fuel consumption of its engine is 0.3 kg/(kW • hr), estimate the additional rate of fuel consumption due to the carrier. Compute the effect on fuel economy if the auto achieves 12.75 km/L without the carrier. The rental company offers you a cheaper, square-edged carrier at a price \$5 less than the current carrier. Estimate the extra cost of using this carrier instead of the round-edged one for a 750 km trip, assuming fuel is \$3.50 per gallon. Is the cheaper carrier really cheaper?



Drag coefficient v. radius ratio [37]



Given:
Data on rooftop carrier

Find:
Drag on carrier; Additional fuel used; Effect on economy; Effect of "cheaper" carrier

Solution:

Basic equation: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$

Given or available data is $w = 1\cdot m$ $\quad h = 50\cdot cm$ $\quad r = 10\cdot cm$ $\quad \eta_d = 85\cdot\%$

$V = 100\cdot\dfrac{km}{hr}$ $\quad V = 27.8\dfrac{m}{s}$ $\quad FE = 12.75\cdot\dfrac{km}{L}$ $\quad FE = 30.0\dfrac{mi}{gal}$

$\rho_{H2O} = 1000\cdot\dfrac{kg}{m^3}$ $\quad A = w\cdot h$ $\quad A = 0.5\,m^2$ $\quad BSFC = 0.3\cdot\dfrac{kg}{kW\cdot hr}$

$\rho = 1.225\cdot\dfrac{kg}{m^3}$ $\quad \nu = 1.50\times10^{-5}\cdot\dfrac{m^2}{s}$ $\quad$ (Table A.10, 20ºF)

From the diagram $\dfrac{r}{h} = 0.2$ so $C_D = 0.25$ $\quad F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot A\cdot V^2$ $\quad F_D = 59.1\,N$

Additional power is $\Delta P = \dfrac{F_D\cdot V}{\eta_d}$ $\quad \Delta P = 1.93\,kW$

Additional fuel is $\Delta FC = BSFC\cdot\Delta P$ $\quad \Delta FC = 1.61\times10^{-4}\dfrac{kg}{s}$ $\quad \Delta FC = 0.00965\dfrac{kg}{min}$

Fuel consumption of the car only is (with $SG_{gas} = 0.72$ from Table A.2)

$FC = \dfrac{V}{FE}\cdot SG_{gas}\cdot\rho_{H2O}$ $\quad FC = 1.57\times10^{-3}\dfrac{kg}{s}$ $\quad FC = 0.0941\dfrac{kg}{min}$

The total fuel consumption is then $FC_T = FC + \Delta FC$ $\quad FC_T = 1.73\times10^{-3}\dfrac{kg}{s}$ $\quad FC_T = 0.104\dfrac{kg}{min}$

Fuel economy with the carrier is $FE = \dfrac{V}{FC_T}\cdot SG_{gas}\cdot\rho_{H2O}$ $\quad FE = 11.6\dfrac{km}{L}$ $\quad FE = 27.2\dfrac{mi}{gal}$

For the square-edged: $\dfrac{r}{h} = 0$ so $C_D = 0.9$ $\quad F_D = C_D\cdot\frac{1}{2}\cdot\rho\cdot A\cdot V^2$ $\quad F_D = 213\,N$

Additional power is $\Delta P = \dfrac{F_D\cdot V}{\eta_d}$ $\quad \Delta P = 6.95\,kW$

Additional fuel is $\Delta FC = BSFC \cdot \Delta P$ $\quad \Delta FC = 5.79 \times 10^{-4} \frac{kg}{s}$ $\quad \Delta FC = 0.0348 \frac{kg}{min}$

The total fuel consumption is then $FC_T = FC + \Delta FC$ $\quad FC_T = 2.148 \times 10^{-3} \frac{kg}{s}$ $\quad FC_T = 0.129 \frac{kg}{min}$

Fuel economy withy the carrier is now $FE = \frac{V}{FC_T} \cdot SG_{gas} \cdot \rho_{H2O}$ $\quad FE = 9.3 \frac{km}{L}$ $\quad FE = 21.9 \frac{mi}{gal}$

The cost of the trip of distance $d = 750 \cdot km$ for fuel costing $p = \frac{\$ \cdot 3.50}{gal}$ with a rental $discount = \$ \cdot 5$ less than the rounded carrier is then

$$Cost = \frac{d}{FE} \cdot p - discount \qquad Cost = 69.47\,\$$$ plus the rental fee

The cost of the trip of with the rounded carrier ($FE = 11.6 \cdot \frac{km}{L}$) is then

$$Cost = \frac{d}{FE} \cdot p \qquad Cost = 59.78\,\$$$ plus the rental fee

Hence the "cheaper" carrier is more expensive (AND the environment is significantly more damaged!)

Problem 9.143

Given: Coastdown test data from level road, calm day, measured for vehicle with $W = 25{,}000$ lbf and $A = 79\ \text{ft}^2$. $F_D(5\ \text{mph}) \ll F_D(55\ \text{mph})$.

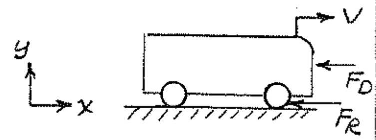
V (mph)	5	55
$\frac{dV}{dt}$ (mph/s)	−0.150	−0.475

Find: Aerodynamic drag coefficient for this vehicle.
Speed at which F_D first exceeds F_R

Solution: Apply Newton's second law of motion, definition of C_D.

Computing equations:

$$\sum F_x = m a_x \qquad C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$



Summing forces, $-F_R - F_D = m a_x$

At 55 mph $\quad -F_R - F_{D55} = m a_{x55}$

At 5 mph $\quad -F_R - F_{D5} = m a_{x5}$ ($F_{D5} \approx 0$)

Subtracting, obtain $\quad -F_{D55} = m(a_{x55} - a_{x5}) = m\left[\frac{dV}{dt}\Big)_{55} - \frac{dV}{dt}\Big)_5\right] = -C_D A \rho \frac{V^2}{2}$

Thus

$$C_D = \frac{m\left[\frac{dV}{dt}\Big)_5 - \frac{dV}{dt}\Big)_{55}\right]}{\frac{1}{2}\rho V^2 A} = \frac{2W\left[\frac{dV}{dt}\Big)_5 - \frac{dV}{dt}\Big)_{55}\right]}{\rho g V^2 A}$$

Evaluating, assuming standard air, with $\rho g = 0.0765\ \text{lbf/ft}^3$

$$C_D = 2 \times 25000\ \text{lbf} \times \frac{\text{ft}^3}{0.0765\ \text{lbf}} \times \frac{\text{hr}^2}{(55)^2\ \text{mi}^2} \times \frac{1}{79\ \text{ft}^2} \times [-0.150 - (-0.475)]\frac{\text{mi}}{\text{hr}\cdot\text{s}} \times \frac{\text{mi}}{5280\ \text{ft}} \times 3600\ \frac{\text{s}}{\text{hr}}$$

$$C_D = 0.606$$

C_D

At $V = 5$ mph, $F_D \approx 0 \quad \therefore \; -F_R = m a_{x5}$

$$F_R = -m a_{x5} = -\frac{25000\ \text{lbf}}{32.2\ \text{ft}}\ \text{s}^2 \times -0.150\ \frac{\text{mi}}{\text{hr}\cdot\text{s}} \times \frac{\text{hr}}{3600\ \text{s}} \times \frac{5280\ \text{ft}}{\text{mi}} = 171\ \text{lbf}$$

For $F_R = F_D = C_D \frac{1}{2}\rho V^2 A$, then $V = \left[\frac{2F_R}{\rho A C_D}\right]^{1/2}$

$$V = \left[2 \times 171\ \text{lbf} \times \frac{\text{ft}^3}{0.00238\ \text{slug}} \times \frac{1}{79\ \text{ft}^2} \times \frac{1}{0.606} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right]^{1/2}$$

$V = 54.8\ \frac{\text{ft}}{\text{s}} = 37.4$ mph.

$V_{F_R = F_D}$

Problem 9.144

[4]

Given: Spherical sonar transducer with $D = 0.375$ m, to be towed in seawater, fully submerged, at $V = 31$ kt.

To avoid cavitation, minimum pressure on transducer surface must be > 30 kPa (abs).

Find: (a) Hydrodynamic drag force on transducer.
(b) Minimum depth of submergence.

Solution: $V = 31.1 \frac{nm}{hr} \times 1852 \frac{m}{nm} \times \frac{hr}{3600\,s} = 16.0$ m/s

$$q = \frac{1}{2}\rho V^2 = \frac{1}{2} \times (1.025)\, 1000 \frac{kg}{m^3} \times (16.0)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} \times \frac{Pa \cdot m^2}{N} = 131 \text{ kPa}$$

$$Re = \frac{VD}{\nu} = 16.0 \frac{m}{s} \times 0.375\, m \times \frac{1}{(1.08)\, 1 \times 10^{-6}\, m^2} = 5.56 \times 10^6$$

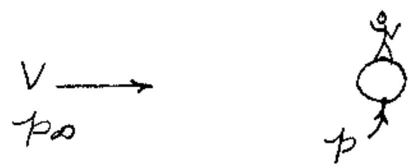
Therefore flow over sphere is supercritical; from Fig. 9.11, $C_D \approx 0.18$

$$F_D = C_D A \frac{1}{2}\rho V^2 \qquad A = \frac{\pi D^2}{4} = \frac{\pi}{4} \times (0.375)^2 m^2 = 0.110 \text{ m}^2$$

$$F_D = 0.18 \times 0.110\, m^2 \times 131 \times 10^3 \frac{N}{m^2} = 2.59 \text{ kN}$$ F_D

From Fig. 9.12, the minimum pressure on a sphere with supercritical flow is $C_p \approx -1.2$

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho V^2} = \frac{p - p_\infty}{q} = -1.2$$



or $p_\infty = p - C_p q$

$$= 30 \text{ kPa (abs)} - (-1.2)\, 131 \text{ kPa}$$

$$p_\infty \text{(abs)} = 187 \text{ kPa (abs)}$$

Thus

$$p_\infty \text{(gage)} = p_\infty \text{(abs)} - p_{atm} = (187 - 101) \text{ kPa} = 86.2 \text{ kPa}$$

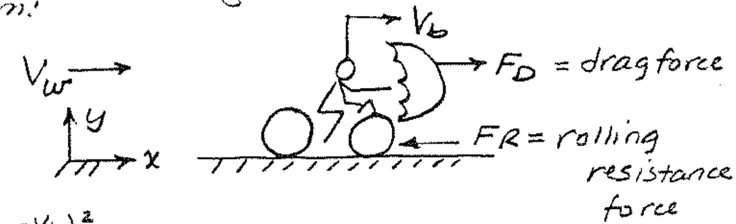
But $p_\infty \text{(gage)} = \rho g h$, so

$$h = \frac{p_\infty \text{(gage)}}{\rho g} = 86.2 \times 10^3 \frac{N}{m^2} \times \frac{m^3}{(1.025)\, 1000\, kg} \times \frac{s^2}{9.81\, m} \times \frac{kg \cdot m}{N \cdot s^2} = 8.57 \text{ m}$$ h

Problem 9.145

Open-Ended Problem Statement: While walking across campus one windy day, Floyd Fluids speculates about using an umbrella as a "sail" to propel a bicycle along the sidewalk. Develop an algebraic expression for the speed a bike could reach on level ground with the umbrella "propulsion system." The frontal area of the bike and rider is estimated as 0.3 m^2. $C_D = 1.2$ Evaluate the bike speed that could be achieved with an umbrella 1.22 m in diameter in a wind that blows at 24 km/hr. Discuss the practicality of this propulsion system.
Assume rolling resistance is 0.75% of weight ($m = 75$ kg)

Analysis: *Draw a free-body diagram:*



Sum forces in x direction:

$$\Sigma F_x = F_D - F_R = 0$$

But $F_D = (C_{Du} A_u + C_{Db} A_b)\frac{1}{2}\rho(V_w - V_b)^2$

$$A_u = \frac{\pi D_u^2}{4} = \frac{\pi}{4}(1.22)^2 m^2 = 1.17\ m^2$$

$$F_R = C_R\, mg$$

Choose $C_{Du} = 1.42$ (Table 9.3).

$$C_R = 0.75\%,\ m = 75\ kg, \text{ so } F_R = 0.0075 \times 75\ kg \times 9.81 \frac{m}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 5.52\ N$$

Then $$V_b = V_w - \left[\frac{2F_R}{\rho(C_{Du}A_u + C_{Db}A_b)}\right]^{\frac{1}{2}}$$ V_b

But

$$V_w = 24 \frac{km}{hr} \times 1000 \frac{m}{km} \times \frac{hr}{3600\, s} = 6.67\ m/s$$

$$V_b = 6.67 \frac{m}{s} - \left[2 \times 5.52\ N \times \frac{m^3}{1.23\ kg} \frac{1}{(1.42)1.17\ m^2 + (1.20)0.3\ m^2} \times \frac{kg \cdot m}{N \cdot s^2}\right]^{\frac{1}{2}}$$

$$V_b = 6.67 \frac{m}{s} - 2.11 \frac{m}{s} = 4.56 \frac{m}{s} \text{ or } 16.4 \frac{km}{hr}$$ V_b

Thus Floyd's bicycle (with the umbrella propelling it) travels at 68.3% wind speed.

{Without the umbrella, $V_b = 1.68 \frac{m}{s}$ or $6.04 \frac{km}{hr}$, by setting $C_{Du} = 0$ above.}

Discussion: Floyd is confused about his fluid mechanics principles if he thinks he can exceed the wind speed. It is impossible to obtain a propulsive force from aerodynamic drag unless the bicycle is moving more slowly than the wind. The drag force must be sufficient to overcome the rolling resistance of the bike and rider. At equilibrium speed the drag force and rolling resistance force must be equal and opposite.

The only benefit could be achieved by adding drag force more rapidly than rolling resistance. An umbrella, with its relatively high drag and low weight, is ideal for this purpose.

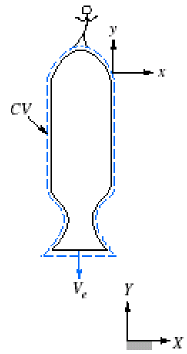
However, one would somehow have to hold the umbrella perpendicular to the wind while riding the bike. This would be dangerous at best, especially if the bike had hand-activated brakes.

Since the umbrella must be held perpendicular to the wind, it would be very effective at blocking the rider's view of the road ahead!

In summary, this "system" of propulsion appears quite impractical.

Problem 9.146

9.146 Motion of a small rocket was analyzed in Example 4.12 assuming negligible aerodynamic drag. This was not realistic at the final calculated speed of 369 m/s. Use Euler's finite difference method for approximating the first derivatives, in an *Excel* workbook, to solve the equation of motion for the rocket. Plot the rocket speed as a function of time, assuming $C_D = 0.3$ and a rocket diameter of 700 mm. Compare with the results for $C_D = 0$.



Given: Data on a rocket

Find: Plot of rocket speed with and without drag

Solution:

From Example 4.12, with the addition of drag the momentum equation becomes

$$F_{B_y} + F_{S_y} - \int_{\text{CV}} a_{rf_y} \rho d\Psi = \frac{\partial}{\partial t}\int_{\text{CV}} v_{xyz} \rho d\Psi + \int_{\text{CV}} v_{xyz} \rho \vec{V}_{xyz} \cdot d\vec{A}$$

where the surface force is

$$F_{S_y} = -\frac{1}{2}\rho A V^2 C_{\text{D}}$$

Following the analysis of the example problem, we end up with

$$\frac{dV_{\text{CV}}}{dt} = \frac{V_e \dot{m}_e - \frac{1}{2}\rho A V_{\text{CV}}{}^2 C_{\text{D}}}{M_0 - \dot{m}_e t} - g$$

This can be written (dropping the subscript for convenience)

$$\frac{dV}{dt} = f(V,t) \quad (1)$$

where

$$f(V,t) = \frac{V_e \dot{m}_e - \frac{1}{2}\rho A V^2 C_{\text{D}}}{M_0 - \dot{m}_e t} - g \quad (2)$$

Equation 1 is a differential equation for speed *V*.

It can be solved using Euler's numerical method

$$V_{\text{n+1}} \approx V_{\text{n}} + \Delta t\, f_{\text{n}}$$

where $V_{\text{n+1}}$ and V_{n} are the n + 1$^{\text{th}}$ and n$^{\text{th}}$ values of *V*, f_{n} is the function given by Eq. 2 evaluated at the n$^{\text{th}}$ step, and Δt is the time step.

The initial condition is $V_0 = 0 \;\text{ at }\; t = 0$

Given or available data:

$M_0 =$ 400 kg

$m_e =$ 5 kg/s

$V_e =$ 3500 m/s

$\rho =$ 1.23 kg/m^3

$D =$ 700 mm

$C_D =$ 0.3

Computed results:

$A =$ 0.385 m^2

$N =$ 20

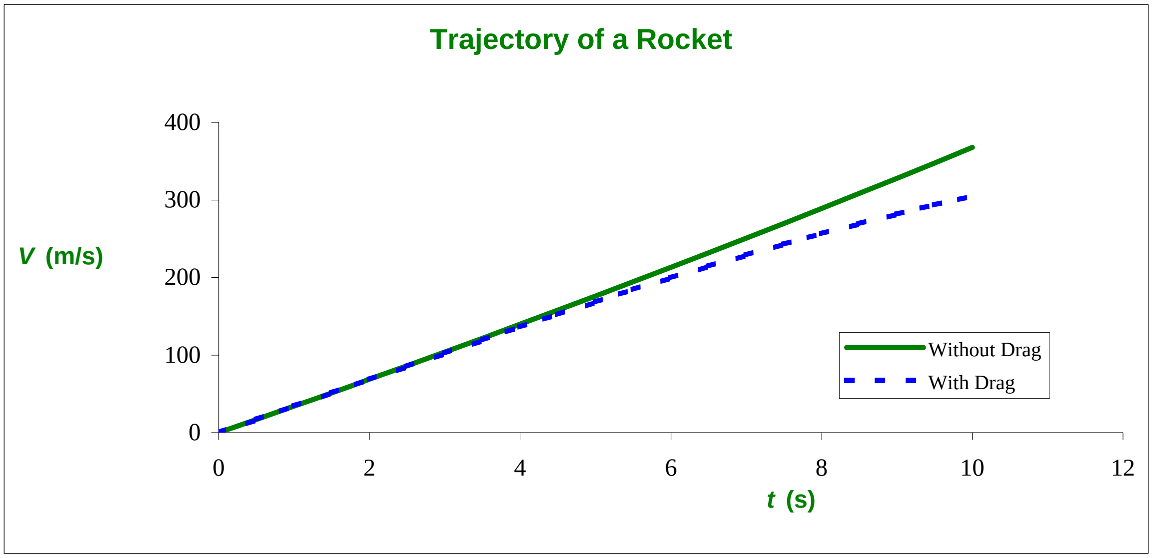
$\Delta t =$ 0.50 s

With drag:

n	t_n (s)	V_n (m/s)	f_n	V_{n+1} (m/s)
0	0.0	0.0	33.9	17.0
1	0.5	17.0	34.2	34.1
2	1.0	34.1	34.3	51.2
3	1.5	51.2	34.3	68.3
4	2.0	68.3	34.2	85.5
5	2.5	85.5	34.0	102
6	3.0	102	33.7	119
7	3.5	119	33.3	136
8	4.0	136	32.8	152
9	4.5	152	32.2	168
10	5.0	168	31.5	184
11	5.5	184	30.7	200
12	6.0	200	29.8	214
13	6.5	214	28.9	229
14	7.0	229	27.9	243
15	7.5	243	26.9	256
16	8.0	256	25.8	269
17	8.5	269	24.7	282
18	9.0	282	23.6	293
19	9.5	293	22.5	305
20	10.0	305	21.4	315

Without drag:

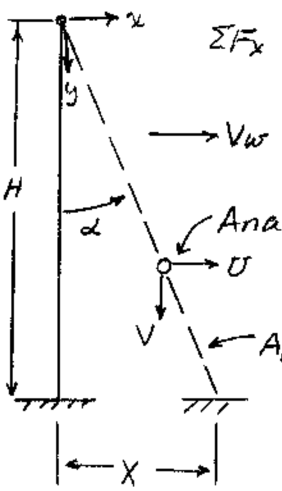
V_n (m/s)	f_n	V_{n+1} (m/s)
0.0	33.9	17.0
17.0	34.2	34.1
34.1	34.5	51.3
51.3	34.8	68.7
68.7	35.1	86.2
86.2	35.4	104
104	35.6	122
122	35.9	140
140	36.2	158
158	36.5	176
176	36.9	195
195	37.2	213
213	37.5	232
232	37.8	251
251	38.1	270
270	38.5	289
289	38.8	308
308	39.1	328
328	39.5	348
348	39.8	368
368	40.2	388



Problem 9.147

Open-Ended Problem Statement: Towers for television transmitters may be up to 500 m in height. In the winter, ice forms on structural members. When the ice thaws, chunks break off and fall to the ground. How far from the base of a tower would you recommend placing a fence to limit danger to pedestrians from falling ice chunks?

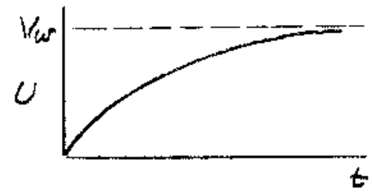
Analysis: An ice chunk detaching from a tower starts at rest, falls by gravity, and simultaneously is blown sideways by wind. Because drag is proportional to relative speed squared, it may be treated in separate x and y components.



$$\Sigma F_x = F_D = m\frac{dU}{dt} = C_D A \frac{1}{2}\rho (V_w - U)^2 = m\frac{dU}{dt} = -m\frac{d(V_w - U)}{dt} \qquad (1)$$

This equation is solved in Example Problem 4.11. The result is

$$\frac{U}{V_w} = \frac{bt}{1+bt}$$



$$b = \frac{\rho C_D A}{2m} V_w$$

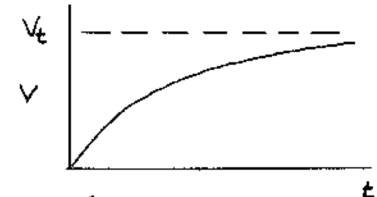
U quickly approaches V_w

$$\Sigma F_y = mg - F_D = m\frac{dV}{dt} = mV\frac{dV}{dy} = mg - C_D A \frac{1}{2}\rho V^2 = mg - kV^2 \qquad (2)$$

This equation is solved in Example Problem 1.2. The result is

$$V = \left\{\frac{mg}{k}\left(1 - e^{-\frac{2k}{m}y}\right)\right\}^{1/2} \quad \text{or} \quad \frac{V}{V_t} = 1 - e^{-2\frac{k}{m}y}$$

$$V_t = \left[\frac{mg}{k}\right]^{\frac{1}{2}}$$



Model the chunk as a geometric sphere, but with larger C_D because of jagged edges. Both b and V_t depend on diameter; $V_t = [8.11\,D(mm)]^{\frac{1}{2}}$; or 11.0 m/s for D = 15 mm.

Thus a reasonable approximation is falling at V_t and moving sideways at V_w. For V_w = 10 mph (4.47 m/s), and V_t = 11.0 m/s, then $\alpha = \tan^{-1}(4.47/11.0) = 22.1°$, and $X = H \tan 22.1° = 203$ m. X

To more precisely compute distance, obtain x(t) and y(t) by solving Eqs. 1 and 2 numerically, and plot the particle path.

Discussion: Because towers may be very tall, ice chunks can travel long distances from the base even in moderate winds. Considerable area around the base of a tower must be fenced to keep personnel on the ground safe from falling ice.

The analysis in this problem would be accurate if the drag-area product $C_D A$ for an ice chunk were known precisely. However, the size of the structural members and the thickness of the ice coating are both unknown. Therefore it is difficult to choose the most probable drag-area product. We recommend you bracket the sizes of known ice chunks, pick a reasonable range of drag coefficients, and then use the analysis to develop guidelines for the safety of personnel.

Problem 9.148

Open-Ended Problem Statement: Wiffle™ balls made from light plastic with numerous holes are used to practice baseball and golf. Explain the purpose of the holes and why they work. Explain how you could test your hypothesis experimentally.

Discussion: The basic concept of the Wiffle ball is a low-mass, high-drag configuration that can be hit or struck with full force, but will not fly fast or far. Thus the Wiffle ball can be used for practice in a limited space.

The low mass is achieved by making the ball of relatively thin plastic material. This gives it low mass for its size, and is a step toward making the drag force relatively high compared to the weight of the ball.

Even higher drag force is achieved by perforating the surface of the Wiffle ball with numerous large holes. These holes further reduce the mass of the Wiffle ball.

In the sub-critical flow regime (below $Re_D \approx 2 \times 10^5$) skin friction drag accounts for less than 5 percent of the total drag of a sphere. The holes increase the skin friction drag of the ball by allowing boundary-layer fluid to escape into the interior of the ball. Each new bit of surface then sees essentially a new boundary layer developing, with attendant high shear stress.

Pressure drag accounts for the majority of the drag of a sphere at any Reynolds number above about 1000. The holes disrupt the flow pattern around the ball and probably trigger early separation. This ensures that the ball remains in the high-drag sub-critical flow regime no matter what its actual Reynolds number.

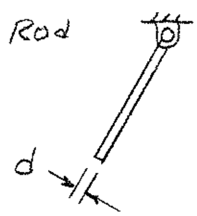
This hypothesis could be tested experimentally by comparing the performance of two balls, one with holes and one without. (The balls should have nearly the same mass and diameter.) With the help of an assistant, drop the balls from some height (for example, down a stairwell). After each ball has reached terminal speed, measure the time required for it to fall through a fixed distance. Then calculate and compare the drag coefficients for the two balls. If the drag coefficient for the ball with holes is significantly larger than for the ball without holes, the hypothesis is confirmed.

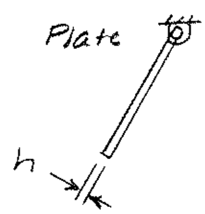
Several balls of each type might be evaluated experimentally to obtain an idea of the Reynolds number dependence of the results.

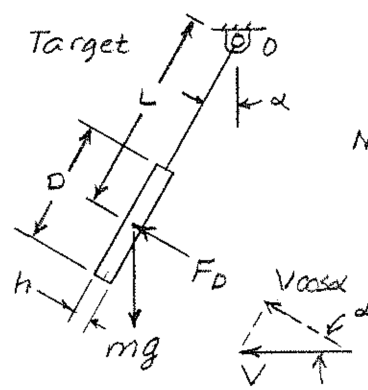
Problem 9.149

Open-Ended Problem Statement: Design a wind anemometer that uses aerodynamic drag to move or deflect a member or linkage, producing an output that can be related to wind speed, for the range from 1 to 10 m/s in standard air. Consider three alternative design concepts. Select the best concept and prepare a detailed design. Specify the shape, size, and material for each component. Quantify the relation between wind speed and anemometer output. Present results as a "calibration curve" of anemometer output versus wind speed. Discuss reasons why you rejected the alternative designs and chose your final design concept.

Analysis: The "target" concept was chosen for analysis. The drag force acting on the target is calculated, then moments are summed about the pivot (see Problem 9.105). The results are:







Neglect drag on strut.

$$\Sigma M_O = mg\,L\sin\alpha - F_D L = 0$$

$$F_D = C_D A \tfrac{1}{2}\rho (V\cos\alpha)^2 = C_D A \tfrac{1}{2}\rho V^2 \cos^2\alpha$$

So $$C_D A \tfrac{1}{2}\rho V^2 \cos^2\alpha = mg\sin\alpha = \rho_m \forall_m g \sin\alpha = SG\,\rho_{H_2O} A h g \sin\alpha \qquad (1)$$

Assume $\alpha = 60°$ at highest wind speed (when $V = 10$ m/s; $q = \tfrac{1}{2}\rho V^2 = 61.5$ N/m²).

Then $$\frac{\sin\alpha}{\cos^2\alpha} = \frac{C_D A q}{SG\,\rho_{H_2O} A g h} = \frac{C_D q}{SG\,\rho_{H_2O} g h} = f(\alpha) \qquad \text{At } \alpha = 60°,\ f(\alpha) = 3.46$$

$$h = \frac{C_D q}{SG\,\rho_{H_2O} g f(\alpha)} = 1.2 \times 61.5\,\frac{N}{m^2} \times \frac{m^3}{(SG)\,1000\,kg} \times \frac{s^2}{9.81\,m} \times \frac{1}{3.46} \times \frac{kg\cdot m}{N\cdot s^2} = \frac{0.00217\,m}{SG} = \frac{2.17\,mm}{SG}$$

Choose aluminum, with $SG = 2.64$ (Table A.1). Then

$$h = \frac{2.17\,mm}{2.64} = 0.822\,mm$$ h

Solve Eq. 1 for velocity

$$V = \left[\frac{2\,SG\,\rho_{H_2O} h g}{\rho C_D} \times f(\alpha)\right]^{\frac{1}{2}} \qquad \text{(see plot)}$$ V

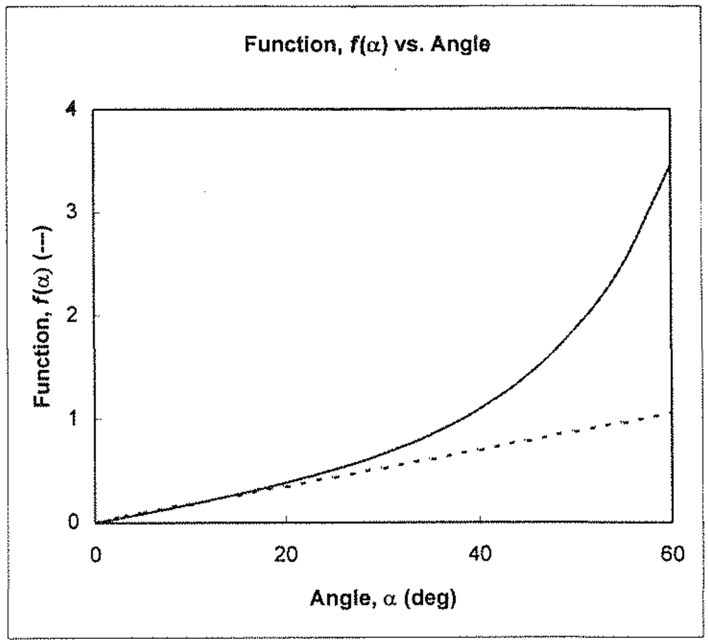
Discussion: Concepts considered included a manometer that sensed stagnation pressure, a parallelogram linkage supporting a vertical target, and bending of a thin member in the air stream. The three final concepts chosen were variations on the theme of a single hanging member supported from a single pivot, and were chosen for their simplicity.

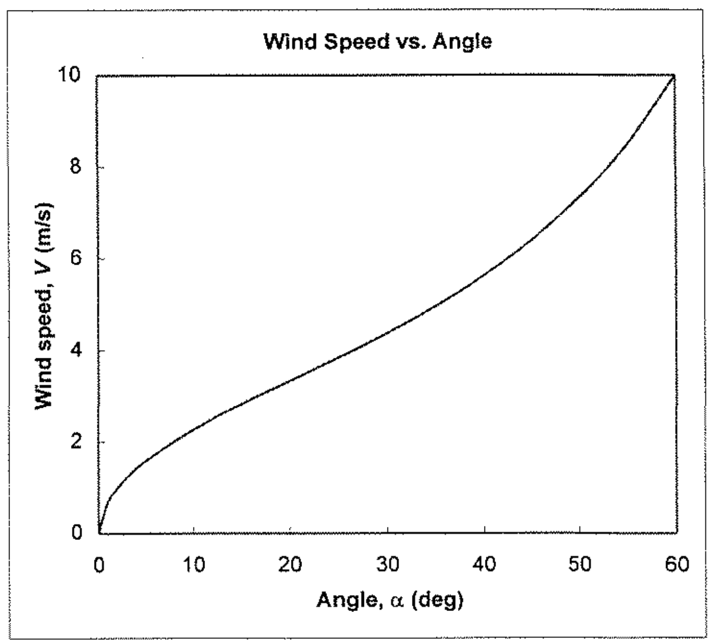
The major advantage of the target concept is that different materials can be used for the rod and target; this concept can be tailored to give the largest deflection angle for a given wind speed. Therefore this device should be capable of the most accurate indication at low wind speeds.

Drag force on the target is assumed to depend on the component of wind velocity acting normal to the target. This model could be improved by using actual experimental data for the drag coefficient of a disk at angle of attack.

Problem 9.149

α (deg)	α (rad)	*f*(α) (---)	*V* (m/s)
0	0	0	0
1	0.0175	0.0175	0.71
2	0.0349	0.0349	1.00
3	0.0524	0.0525	1.23
5	0.0873	0.0878	1.59
10	0.175	0.179	2.27
15	0.262	0.277	2.83
20	0.349	0.387	3.34
25	0.436	0.515	3.85
30	0.524	0.667	4.39
35	0.611	0.855	4.97
40	0.698	1.10	5.62
45	0.785	1.41	6.39
50	0.873	1.85	7.31
55	0.960	2.49	8.47
60	1.05	3.46	10.0





Open-Ended Problem Statement: The "shot tower," used to produce spherical lead shot, has been recognized as a mechanical engineering landmark. In a shot tower, molten lead is dropped from a high tower; as the lead solidifies, surface tension pulls each shot into a spherical shape. Discuss the possibility of increasing the "hang time," or of using a shorter tower, by dropping molten lead into an air stream that is moving upward. Support your discussion with appropriate calculations.

Analysis: This problem may be analyzed parametrically, in terms of shot diameter. Consider the range from "bird shot" of about 1 mm to musket balls of about 15 mm diameter to illustrate the results.

Analysis with still air: Terminal speed is reached when aerodynamic drag force exactly equals the weight of the shot. The first plot shows terminal speed versus diameter of lead shot.

The solution for shot speed versus distance traveled, with no upward air movement, parallels the solution of Example Problem 1.2, which gives the fraction of terminal speed reached in a tower of specified height. For any tower height (choose 50 m to illustrate the results) the fraction of terminal speed reached decreases with increasing shot diameter (see the second plot).

The solution for hang time versus diameter is shown in the third plot.

Analysis with upward flow of air: The solution for shot speed versus distance traveled is more complex when air in the shot tower flows upward. Introducing upward air flow in the tower increases drag force compared to shot weight. Therefore the shot accelerate more slowly in the upward flow. It is possible to obtain an analytical solution, but the result is so complex that it is difficult to interpret. Results can be obtained for specific cases by integrating the differential equations numerically.

The solution for shot speed versus time also is more complex when air flows upward. Again numerical integration can be used to obtain results for specific cases.

Outline of Procedure: Derive a differential equation for shot acceleration from a free-body diagram. Integrate once to obtain shot velocity as a function of time. Integrate again to obtain shot position as a function of time.

From the results of the second integration, identify the "hang time" when the shot reaches the bottom of the tower. Plot hang time versus diameter and compare with results for the case with no upward air flow.

Set the upward air flow velocity to zero and compare numerical results with the analytical results for the case without flow to validate your model.

Discussion: The terminal speed reached by small shot in still air is quite low. Therefore, the "hang time" of small shot can be increased significantly by providing upward flow of air at reasonable speed in the tower.

Larger shot have higher terminal speeds. However, the higher terminal speed does not reduce hang time much because the large shot reach only a smaller fraction of their terminal speed in the 50 m tower height.

Introducing upward flow of air in the shot tower increases the drag force and results in slower acceleration of the shot. Therefore the hang time is increased. The increase in hang time allows more time for cooling, and should result in the production of more nearly spherical shot.

Problem 9.150

Input data:

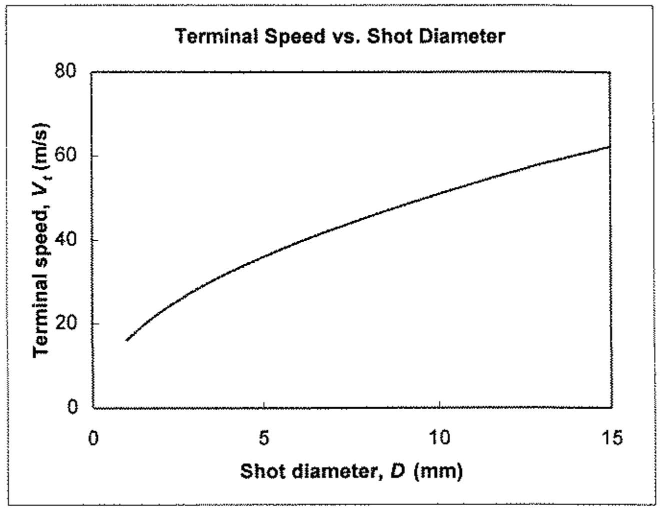
	C_D =	0.47	(---)	Drag coefficient of sphere
	SG_s =	11.4	(---)	Specific gravity of (lead) shot
	Δz =	50	m	Height of shot tower
	ρ_{air} =	1.23	kg/m^3	Density of air

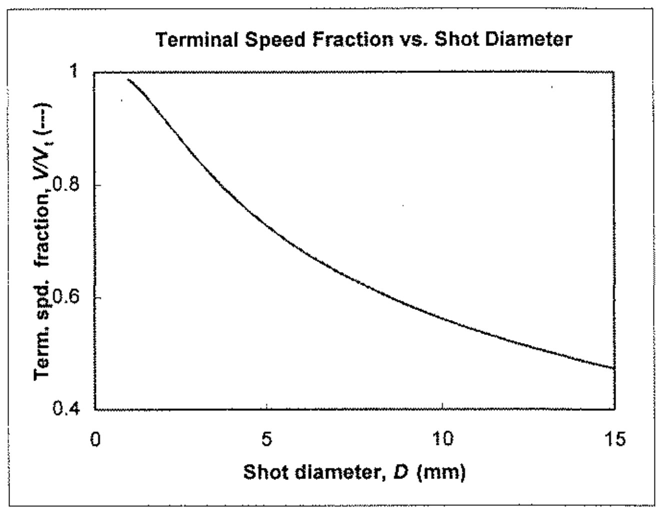
Calculated parameters:

k/D^2 =	2.27E-07	kg/m-mm^2	Drag factor, $F_D = kV^2$
m/D^3 =	5.969E-06	kg/mm^3	Mass of shot

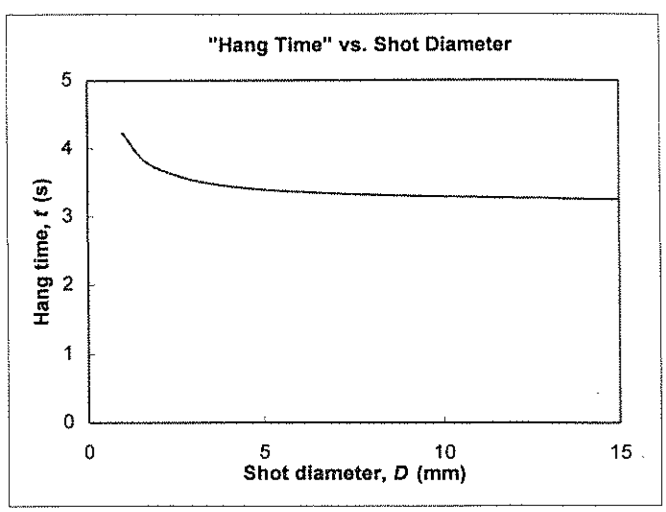
(1) Shot falling in still air:

D (mm)	V_t (m/s)	V/V_t (---)
1	16.1	0.989
1.5	19.7	0.960
2	22.7	0.922
3	27.8	0.848
4	32.1	0.783
5	35.9	0.730
6	39.3	0.685
7	42.5	0.647
8	45.4	0.615
9	48.2	0.587
10	50.8	0.562
11	53.3	0.541
12	55.6	0.521
13	57.9	0.504
14	60.1	0.488
15	62.2	0.473





D (mm)	V_t (m/s)	V/V_t (---)	t (s)
1	16.1	0.989	4.24
1.5	19.7	0.960	3.89
2	22.7	0.922	3.71
3	27.8	0.848	3.54
4	32.1	0.783	3.45
5	35.9	0.730	3.40
6	39.3	0.685	3.36
7	42.5	0.647	3.34
8	45.4	0.615	3.32
9	48.2	0.587	3.31
10	50.8	0.562	3.29
11	53.3	0.541	3.29
12	55.6	0.521	3.28
13	57.9	0.504	3.27
14	60.1	0.488	3.27
15	62.2	0.473	3.26



Problem 9.151

9.151 An antique airplane carries 50 m of external guy wires stretched normal to the direction of motion. The wire diameter is 5 mm. Estimate the maximum power saving that results from an optimum streamlining of the wires at a plane speed of 175 km/hr in standard air at sea level.

Given:
Antique airplane guy wires

Find:
Maximum power saving using optimum streamlining

Solution:

Basic equation: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad P = F_D\cdot V$

Given or available data is $L = 50\cdot m$ $\quad D = 5\cdot mm$ $\quad V = 175\cdot\dfrac{km}{hr}$ $\quad V = 48.6\dfrac{m}{s}$

$A = L\cdot D$ $\quad A = 0.25\,m^2$

$\rho = 1.21\cdot\dfrac{kg}{m^3}$ $\quad \nu = 1.50\times 10^{-5}\cdot\dfrac{m^2}{s}$ $\quad$ (Table A.10, 20ºC)

The Reynolds number is $Re = \dfrac{V\cdot D}{\nu}$ $\quad Re = 1.62\times 10^4$ $\quad$ so from Fig. 9.13 $\quad C_D = 1.0$

Hence $P = \left(C_D\cdot\frac{1}{2}\cdot\rho\cdot A\cdot V^2\right)\cdot V$ $\quad P = 17.4\cdot kW$ $\quad$ with standard wires

Figure 9.19 suggests we could reduce the drag coefficient to $C_D = 0.06$

Hence $P_{faired} = \left(C_D\cdot\frac{1}{2}\cdot\rho\cdot A\cdot V^2\right)\cdot V$ $\quad P_{faired} = 1.04\cdot kW$

The maximum power saving is then $\Delta P = P - P_{faired}$ $\quad \Delta P = 16.3\cdot kW$

Thus $\dfrac{\Delta P}{P} = 94\cdot\%$ $\quad$ which is a HUGE savings! It's amazing the antique planes flew!

Problem 9.152

Open-Ended Problem Statement: Why do modern guns have rifled barrels?

Discussion: Almost all projectiles fired by modern guns have smoothly rounded noses and abruptly tapered ("boat-tailed") or square rear ends. The minimum drag for these shapes is obtained when the projectile travels with its axis parallel to the direction of motion and its nose pointed forward.

Rifling in a gun barrel imparts spin about the longitudinal axis of the projectile. This rotation about the longitudinal axis causes the projectile to act as a gyroscope and stabilizes it during flight to keep its nose pointed in the direction of motion.

Early smoothbore guns primarily used ball projectiles. The balls were spherical and molded from lead. Since the ball shape was spherical and had no preferred orientation, no benefit would have been achieved from rifling that caused spin. Therefore the gun barrels were bored smooth, i.e., without rifling grooves, hence these guns were called "smoothbore" guns.

Problem 9.153

Open-Ended Problem Statement: Why is it possible to kick a football farther in a spiral motion than in an end-over-end tumbling motion?

Discussion: A football has a prolate spheroid shape. It is almost circular when viewed from the front (parallel to the major axis), and longer and more elliptical when viewed from the side (along a minor axis). The football has more frontal area when traveling with the major axis perpendicular to the motion that when it is "spiraling" with the major axis parallel to the direction of travel.

The drag coefficient of the ball when parallel to the flow in spiral motion undoubtedly is less than when perpendicular to the flow. As a rough approximation, the perpendicular drag coefficient might be similar to that of a cylinder ($C_D = 1.2$), whereas the spiral drag coefficient probably is less (perhaps $C_D \approx 0.2$ - 0.3) than that of a sphere ($C_D = 0.5$). Thus the drag coefficient when traveling with the long axis perpendicular to the flow may be 4 to 6 times as large as when traveling in spiral motion with the long axis parallel to the flow. The difference in the drag-area product C_DA will be even larger.

In tumbling motion the drag-area product varies cyclically between the two extremes we have discussed. On average the drag-area product for the tumbling ball is considerably larger, perhaps 2 to 3 times as large, as when the ball is in spiral motion. Therefore the maximum range (travel distance) that can be achieved with tumbling motion is much less than that for spiral motion.

Also, a well kicked or thrown spiral is a thing of beauty. Perhaps function follows form here!

Problem 9.154

Given: Aircraft with NACA 23012 section airfoils and effective lift area, $A = 25\ m^2$. Maximum flap setting corresponds to condition ② in Fig. 9.23. Takeoff speed is 150 kph. Neglect added lift due to ground effect.

Find: (a) Maximum gross mass at takeoff speed in Denver ($z = 1.61$ km).
(b) Minimum takeoff speed in Denver

Solution: Apply definition of lift coefficient.

Basic equation: $C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 A_p}$

Assumption: Lift force must equal gravity force at takeoff.

$$F_L = mg = C_D A_p \tfrac{1}{2}\rho V^2$$

For maximum mass, need maximum lift, so use $C_{L,max}$:

$$m_{max} = \frac{C_{L,max} A_p \rho V^2}{2g}$$

From Fig. 9.23, $C_{L,max} = 2.67$ for condition ②. Then for std. air,

$$m_{max} = \frac{2.67}{2} \times 25\ m^2 \times 1.23\frac{kg}{m^3}\left(150\times10^3 \frac{m}{hr}\times\frac{hr}{3600\ s}\right)^2 \frac{s^2}{9.81\ m}$$

$$m_{max} = 7260\ kg$$ m_{max}

{This represents the maximum mass theoretically possible when the aircraft is on the verge of stalling. To attempt takeoff at such a large mass would be ill-advised.}

(b) In Denver, $z = 1.61$ km. From Table A.3, at $z = 1.61$ km, $\rho/\rho_0 = 0.855$.

At the same gross mass, the lift force remains the same. Thus

$$F_{L_0} = C_D A \tfrac{1}{2}\rho_0 V_0^2 = F_{L_D} = C_D A \tfrac{1}{2}\rho_D V_D^2 \quad \text{or} \quad \rho_0 V_0^2 = \rho_D V_D^2$$

and

$$V_D = V_0\left(\frac{\rho_0}{\rho_D}\right)^{1/2} = 150\ kph\left(\frac{1}{0.855}\right)^{1/2} = 162\ kph$$ V_D

{The takeoff speed must increase about 8 percent.}

Problem 9.155

Open-Ended Problem Statement: How do cab-mounted wind deflectors for tractor-trailer trucks work? Explain using diagrams of the flow pattern around the truck and pressure distribution on the surface of the truck.

Discussion: Consider both the cab and the trailer flow patterns and pressure distributions in no-wind and crosswind situations.

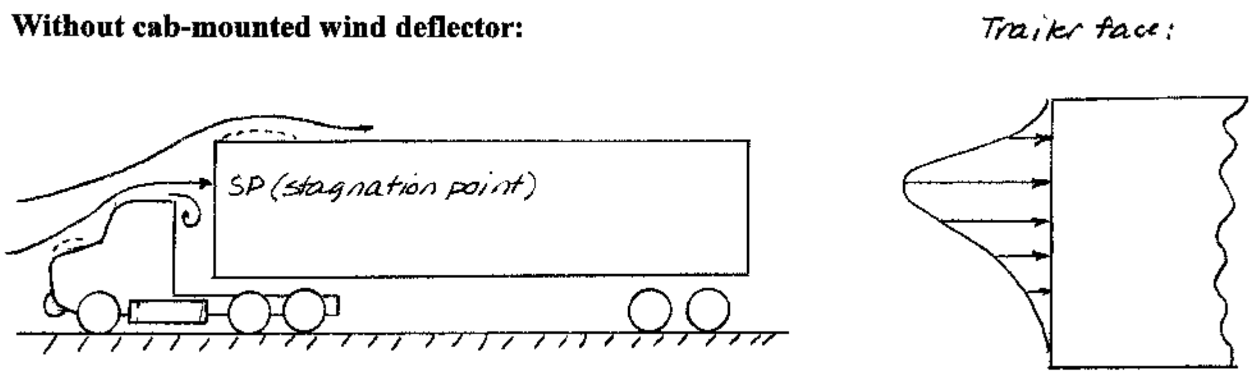
No-wind situation: Without the deflector, flow separation from the roof and sides of the tractor creates a low-pressure wake and high drag force on the tractor. (Flow patterns and pressure distributions on the tractor and the front of the trailer are sketched below.) The cab-mounted deflector reduces the pressures on the front of the tractor, thus reducing the aerodynamic drag force on the tractor.

Without the deflector, high-speed air separates from the roof of the tractor and impinges on the vertical front face of the trailer. The cab-mounted deflector reduces the amount of high-speed air hitting the front of the trailer, reducing the net aerodynamic drag force on the trailer. (Ideally air from the deflector flows smoothly along the top and sides of the trailer.)

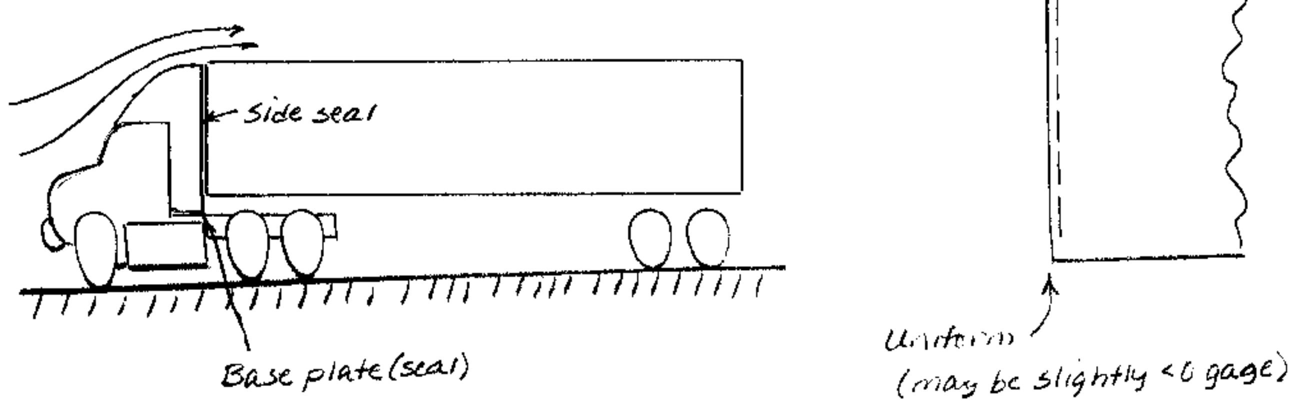
Crosswind situation: Without the deflector, flow separates from the lee side of the tractor, altering the pressure field and increasing the drag on the tractor. The cab-mounted deflector, especially in combination with side seals, minimizes the increase in drag by reducing the amount of separation around the tractor.

Without the deflector, the front face of the trailer is impacted by the high-speed air from the freestream flow. Massive separation occurs on the lee side of the trailer, thus altering the pressure field and increasing the drag on the trailer. With the cab-mounted deflector, the amount of high-speed air impacting the trailer is markedly reduced. This alters the flow pattern and minimizes the increase in drag caused by the crosswind.

Without cab-mounted wind deflector:



With cab-mounted wind deflector:



Problem 9.156

9.156 An aircraft is in level flight at 225 km/hr through air at standard conditions. The lift coefficient at this speed is 0.45 and the drag coefficient is 0.065. The mass of the aircraft is 900 kg. Calculate the effective lift area for the craft, and the required engine thrust and power.

Given: Aircraft in level flight

Find: Effective lift area; Engine thrust and power

Solution:

Basic equation: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad C_L = \dfrac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad P = T\cdot V$

For level, constant speed $\quad F_D = T \quad F_L = W$

Given or available data is $\quad V = 225\cdot\dfrac{km}{hr} \quad V = 62.5\dfrac{m}{s} \quad C_L = 0.45 \quad C_D = 0.065 \quad M = 900\cdot kg$

$\rho = 1.21\cdot\dfrac{kg}{m^3}$ (Table A.10, 20ºC)

Hence $\quad F_L = C_L\cdot\dfrac{1}{2}\cdot\rho\cdot A\cdot V^2 = M\cdot g \quad A = \dfrac{2\cdot M\cdot g}{C_L\cdot\rho\cdot V^2} \quad A = 8.30\,m^2$

Also $\quad \dfrac{F_L}{F_D} = \dfrac{C_L}{C_D} \quad F_L = M\cdot g \quad F_L = 8826\,N \quad F_D = F_L\cdot\dfrac{C_D}{C_L} \quad F_D = 1275\,N$

$T = F_D \quad T = 1275\,N$

The power required is then $\quad P = T\cdot V \quad P = 79.7\,kW$

Problem 9.157

Given: Hydrofoil craft with effective foil area, $A = 0.7\ m^2$ and mass, $m = 1800$ kg. Foils have $C_L = 1.6$ and $C_D = 0.5$. Neglect induced drag.

Find: (a) Minimum speed to support craft on foils.
(b) Power required at this speed.
(c) Maximum speed if 110 kW is available.

Solution: Apply definitions of lift, drag coefficients and power.

Computing equations: $C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 A}$; $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$; $\mathcal{P} = F_D V$

Assumptions: (1) Lift force equals gravity force.
(2) Neglect induced drag.

Then

$$F_L = mg = C_L A \tfrac{1}{2}\rho V^2 \quad \text{so} \quad V = \left[\frac{2mg}{C_L \rho A}\right]^{1/2}$$

Minimum speed is

$$V_{min} = \left[\frac{2}{1.6} \times 1800\ \text{kg} \times 9.81\ \frac{\text{m}}{\text{s}^2} \times \frac{\text{m}^3}{999\ \text{kg}} \times \frac{1}{0.7\ \text{m}^2}\right] = 5.62\ \text{m/s} \quad (10.9\ \text{kt})$$

V_{min}

The drag force at any speed is

$$F_D = C_D A \tfrac{1}{2}\rho V^2 \quad \text{so} \quad F_D = \frac{C_D}{C_L} F_L = \frac{C_D}{C_L} mg$$

and

$$\mathcal{P} = F_D V = \frac{C_D}{C_L} mg V$$

The minimum power is

$$\mathcal{P}_{min} = \frac{C_D}{C_L} mg V_{min} = \frac{0.5}{1.6} \times 1800\ \text{kg} \times 9.81\ \frac{\text{m}}{\text{s}^2} \times 5.62\ \frac{\text{m}}{\text{s}} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}$$

$$\mathcal{P}_{min} = 31.0 \times 10^3\ \frac{\text{N} \cdot \text{m}}{\text{s}} \times \frac{\text{W} \cdot \text{s}}{\text{N} \cdot \text{m}} = 31.0\ \text{kW}$$

$\mathcal{P}_{min}$

As speed increases, the craft will ride higher in the water, decreasing the lifting area such that $F_L = mg$. Thus

$$\mathcal{P}_{max} = \frac{C_D}{C_L} mg V_{max} \quad \text{or} \quad V_{max} = \frac{C_L}{C_D} \frac{\mathcal{P}_{max}}{mg}$$

Assuming C_D/C_L remains constant,

$$V_{max} = \frac{1.6}{0.5} \times 110 \times 10^3\ \text{W} \times \frac{1}{1800\ \text{kg}} \times \frac{\text{s}^2}{9.81\ \text{m}} \times \frac{\text{N} \cdot \text{m}}{\text{W} \cdot \text{s}} \times \frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2} = 19.9\ \text{m/s} \quad (38.7\ \text{kt})$$

V_{max}

Problem 9.158

[2]

9.158 A high school project involves building a model ultralight airplane. Some of the students propose making an airfoil from a sheet of plastic 1.5 m long by 2 m wide at an angle of attack of 12°. At this airfoil's aspect ratio and angle of attack the lift and drag coefficients are $C_L = 0.72$ and $C_D = 0.17$. If the airplane is designed to fly at 12 m/s, what is the maximum total payload? What will be the required power to maintain flight? Does this proposal seem feasible?

Given: Data on an airfoil

Find: Maximum payload; power required

Solution:

The given data or available data is $\rho = 1.23 \cdot \frac{kg}{m^3}$ $L = 1.5 \cdot m$ $w = 2 \cdot m$ $V = 12 \cdot \frac{m}{s}$ $C_L = 0.72$ $C_D = 0.17$

Then $A = w \cdot L$ $A = 3\,m^2$

The governing equations for steady flight are $W = F_L$ and $T = F_D$

where *W* is the model total weight and *T* is the thrust

The lift is given by $F_L = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_L$ $F_L = 191\,N$ $F_L = 43 \cdot lbf$

The payload is then given by $W = M \cdot g = F_L$

or $M = \frac{F_L}{g}$ $M = 19.5\,kg$ $M = 43 \cdot lb$

The drag is given by $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$ $F_D = 45.2\,N$ $F_D = 10.2 \cdot lbf$

Engine thrust required $T = F_D$ $T = 45.2\,N$

The power required is $P = T \cdot V$ $P = 542\,W$ $P = 0.727 \cdot hp$

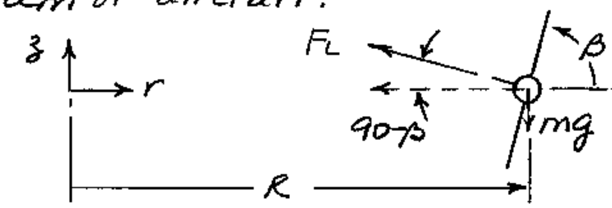
The model ultralight is just feasible: it is possible to find an engine that can produce about 1 hp that weighs less than about 45 lb

Problem 9.159

Given: USAF F-16 with $A_p = 27.9\ m^2$ and $C_{L,max} = 1.6$, at maximum gross mass of $M = 11{,}600$ kg. Turn flown level with aircraft banked.

Find: (a) Minimum speed in standard air for $a_t = 5g$.
(b) Corresponding radius.
(c) Discuss effect of altitude.

Solution: Draw free-body diagram of aircraft:



Computing equations:

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A_p} \qquad a_r = -\frac{V^2}{R}$$

Assume: (1) Standard air, $\rho = 1.23\ kg/m^3$
(2) Pilot feels a_t along F_L

Then $F_L = ma_t = C_L A \frac{1}{2}\rho V^2$ or $V = \sqrt{\frac{2ma_t}{C_L A \rho}}$ is minimum at $C_{L,max}$.

$$V_{min} = \left[2 \times 11{,}600\ kg \times (5)\, 9.81 \frac{m}{s^2} \times \frac{1}{1.6} \times \frac{1}{27.9\ m^2} \times \frac{m^3}{1.23\ kg}\right]^{1/2}$$

$V_{min} = 144$ m/s (minimum speed) ← V_{min}

Need a_r to find V. Sum forces vertically

$$\Sigma F_z = F_L \sin(90-\beta) - mg = ma_z = 0$$

$$\sin(90-\beta) = \cos\beta = \frac{mg}{F_L} = \frac{mg}{5mg} = \frac{1}{5}\ ;\ \beta = \cos^{-1}(\tfrac{1}{5}) = 78.5°$$

Sum forces radially:

$$\Sigma F_r = -F_L \cos(90-\beta) = ma_r = m\left(-\frac{V^2}{R}\right)$$

$$R = \frac{mV^2}{F_L \sin\beta} = \frac{mV^2}{5mg\sin\beta} = \frac{V^2}{5g\sin\beta}$$

$$= \frac{1}{5} \times \frac{(144)^2 m^2}{s^2} \times \frac{s^2}{9.81\ m} \times \frac{1}{\sin 78.5°}$$

$R = 431$ m ← R

As altitude increases, density decreases, and V is raised. This also increases R. At $z = 15$ km, $\rho/\rho_0 = 0.159$. Thus

$$\frac{V}{V_0} = \sqrt{\frac{\rho_0}{\rho}} = 2.51 \quad \text{and} \quad \frac{R}{R_0} = \frac{V^2}{V_0^2} = 6.29$$

Problem 9.160

9.160 A light airplane, with mass $M = 1000$ kg, has a conventional-section (NACA 23015) wing of planform area $A = 10\ m^2$. Find the angle of attack of the wing for a cruising speed of $V = 63$ m/s. What is the required power? Find the maximum instantaneous vertical "g force" experienced at cruising speed if the angle of attack is suddenly increased.

Given: Data on a light airplane

Find: Angle of attack of wing; power required; maximum "g" force

Solution:

The given data or available data is $\rho = 1.23\cdot\frac{kg}{m^3}$ $\quad M = 1000\cdot kg$ $\quad A = 10\cdot m^2$

$V = 63\cdot\frac{m}{s}$ $\quad C_L = 0.72$ $\quad C_D = 0.17$

The governing equations for steady flight are $\quad W = M\cdot g = F_L$ $\quad T = F_D$

where *W* is the weight *T* is the engine thrust

The lift coeffcient is given by $\quad F_L = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_d$

Hence the required lift coefficient is $\quad C_L = \frac{M\cdot g}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad C_L = 0.402$

From Fig 9.17, for at this lift coefficient $\quad \alpha = 3\cdot deg$

and the drag coefficient at this angle of attack is $\quad C_D = 0.0065$

(Note that this does NOT allow for aspect ratio effects on lift and drag!)

Hence the drag is $\quad F_D = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_D$ $\quad F_D = 159\,N$

and $\quad T = F_D$ $\quad T = 159\,N$

The power required is then $\quad P = T\cdot V$ $\quad P = 10\,kW$

The maximum "g"'s occur when the angle of attack is suddenly increased to produce the maximum lift

From Fig. 9.17 $\quad C_{L.max} = 1.72$

$F_{Lmax} = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_{L.max}$ $\quad F_{Lmax} = 42\,kN$

The maximum "g"s are given by application of Newton's second law

$$M\cdot a_{perp} = F_{Lmax}$$

where a_{perp} is the acceleration perpendicular to the flight direction

Hence $a_{perp} = \frac{F_{Lmax}}{M}$ $a_{perp} = 42\,\frac{m}{s^2}$

In terms of "g"s $\frac{a_{perp}}{g} = 4.28$

Note that this result occurs when the airplane is banking at 90^o, i.e, when the airplane is flying momentarily in a circular flight path in the horizontal plane. For a straight horizontal flight path Newton's second law is

$$M \cdot a_{perp} = F_{Lmax} - M \cdot g$$

Hence $a_{perp} = \frac{F_{Lmax}}{M} - g$ $a_{perp} = 32.2\,\frac{m}{s^2}$

In terms of "g"s $\frac{a_{perp}}{g} = 3.28$

Problem 9.161

[3]

9.161 The teacher of the students designing the airplane of Problem 9.158 is not happy with the idea of using a sheet of plastic for the airfoil. He asks the students to evaluate the expected maximum total payload, and required power to maintain flight, if the sheet of plastic is replaced with a conventional section (NACA 23015) airfoil with the same aspect ratio and angle of attack. What are the results of the analysis?

Given: Data on an airfoil

Find: Maximum payload; power required

Solution:

The given data or available data is $V = 12 \cdot \frac{m}{s}$ $\quad \rho = 1.23 \cdot \frac{kg}{m^3}$ $\quad c = 1.5 \cdot m$ $\quad b = 2 \cdot m$

Then the area is $A = b \cdot c$ $\quad A = 3\,m^2$

and the aspect ratio is $ar = \frac{b}{c}$ $\quad ar = 1.33$

The governing equations for steady flight are

$W = F_L$ and $T = F_D$

where W is the model total weight and T is the thrust

At a 12° angle of attack, from Fig. 9.17 $\quad C_L = 1.4$ $\quad C_{Di} = 0.012$

where C_{Di} is the section drag coefficient

The wing drag coefficient is given by Eq. 9.42 $\quad C_D = C_{Di} + \frac{C_L^2}{\pi \cdot ar}$ $\quad C_D = 0.48$

The lift is given by $F_L = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_L$ $\quad F_L = 372\,N$ $\quad F_L = 83.6\,lbf$

The payload is then given by $W = M \cdot g = F_L$

or $M = \frac{F_L}{g}$ $\quad M = 37.9\,kg$ $\quad M = 83.6\,lb$

The drag is given by $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$ $\quad F_D = 127.5\,N$ $\quad F_D = 28.7\,lbf$

Engine thrust required $T = F_D$ $\quad T = 127.5\,N$

The power required is $P = T \cdot V$ $\quad P = 1.53\,kW$ $\quad P = 2.05\,hp$

NOTE: Strictly speaking we have TWO extremely stubby wings, so a recalculation of drag effects (lift is unaffected) gives

$b = 1 \cdot m$ $\quad c = 1.5\,m$

and $A = b \cdot c$ $A = 1.5\,m^2$ $ar = \frac{b}{c}$ $ar = 0.667$

so the wing drag coefficient is $C_D = C_{Di} + \frac{{C_L}^2}{\pi \cdot ar}$ $C_D = 0.948$

The drag is $F_D = 2 \cdot \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$ $F_D = 252\,N$ $F_D = 56.6\,lbf$

Engine thrust is $T = F_D$ $T = 252\,N$

The power required is $P = T \cdot V$ $P = 3.02\,kW$ $P = 4.05\,hp$

Problem 9.162

Given: Light plane with NACA 23015 airfoil, $S = 10\ m$, $C = 1.8\ m$, cruises at $V = 225\ km/hr$ near sea level on a standard day.

Find: Determine cruise speed with NACA 66_2-215 section airfoil.

Solution: Apply definitions of coefficients, use data from Fig. 9.19.

Computing equations: $C_D = C_{D,\infty} + C_{D,i} = C_{D,\infty} + \frac{C_L^2}{\pi ar}$

From a free-body diagram $F_L = W$, $\mathcal{P} = F_D V/\eta_p$

From Fig. 9.19, recognize airfoils should operate near design lift coefficients. Thus assume:

Section	C_L	$C_{D,\infty}$
23015	0.3	0.0062
66_2-215	0.2	0.0031

$$ar = \frac{S}{C} = \frac{10\ m}{1.8\ m} = 5.56$$

Thus $C_{D_{old}} \approx 0.0062 + \frac{(0.3)^2}{\pi(5.56)} = 0.0062 + 0.00515 = 0.0114$

$$C_{D_{new}} \approx 0.0031 + \frac{(0.2)^2}{\pi(5.56)} = 0.0031 + 0.00229 = 0.00539$$

Since for level flight,

$$\mathcal{P} = F_D V/\eta_p = \frac{C_D A \frac{1}{2}\rho V^2 V}{\eta_p}, \text{ then } V = \left[\frac{2\eta_p \mathcal{P}}{C_D A}\right]^{\frac{1}{3}}$$

Assuming $\eta_p \mathcal{P}$ remains constant, then $V_{new} = V_{old}\left[\frac{C_{D,old}}{C_{D,new}}\right]^{\frac{1}{3}}$

$$V_{new} \approx 225\ \frac{km}{hr}\left[\frac{0.0114}{0.00539}\right]^{\frac{1}{3}} = 289\ km/hr$$ V_{new}

Check assumption on C_L: Since $F_L = W = C_L A \frac{1}{2}\rho V^2$, then

$$C_{L_{new}} \approx C_{L_{old}}\left[\frac{V_{old}}{V_{new}}\right]^2 = 0.3\left[\frac{225}{289}\right]^2 = 0.182$$

Therefore the above estimate for new cruise speed is probably conservative.

Problem 9.163

Given: Boeing 727 aircraft, with NACA 23012 section, $A_p = 1600\ ft^2$, and effective aspect ratio, $ar = 6.5$. Aircraft flies at $V = 150$ kt, with $W = 175{,}000$ lbf.

Find: Estimate thrust needed to maintain steady, level flight.

Solution: For steady, level flight, thrust equals drag and lift equals weight.

Computing equations: $F_L = W = C_L \frac{1}{2}\rho V^2 A$ (1)

$$F_D = T = C_D \frac{1}{2}\rho V^2 A \quad (2)$$

$$C_D = C_{D,\infty} + C_{D,i} = C_{D,0} + \frac{C_L^2}{\pi ar}$$

Assumptions: (1) Standard air
(2) Data from Fig. 9.23 apply

$$V = 150 \frac{nm}{hr} \times 6076 \frac{ft}{nm} \times \frac{hr}{3600\ sec} = 253\ ft/sec$$

$$q = \frac{1}{2}\rho V^2 = \frac{1}{2} \times 0.00238 \frac{slug}{ft^3} \times (253)^2 \frac{ft^2}{s^2} \times \frac{lbf \cdot s^2}{slug \cdot ft} = 76.2\ lbf/ft^2$$

From Eq. 1, $C_L = \frac{W}{qA} = 175000\ lbf \times \frac{ft^2}{76.2\ lbf} \times \frac{1}{1600\ ft^2} = 1.44$

From Fig. 9.23, this corresponds to operation with a single slot open, and $C_{D,0} \cong 0.04$. Thus

$$C_D = C_{D,0} + \frac{C_L^2}{\pi ar} = 0.04 + \frac{(1.44)^2}{\pi(6.5)} = 0.142$$

To find thrust, note

$$\frac{T}{F_L} = \frac{C_D}{C_L}\frac{qA}{qA} = \frac{C_D}{C_L} = \frac{0.142}{1.44} = 0.0986$$

Thus

$$T = F_L \frac{C_D}{C_L} = W \frac{C_D}{C_L} = 175{,}000\ lbf \times 0.0986 = 17{,}300\ lbf$$

T

Problem 9.164

9.164 Instead of a new laminar-flow airfoil, a redesign of the light airplane of Problem 9.162 is proposed in which the current conventional airfoil section is replaced with another conventional airfoil section of the same area, but with aspect ratio $AR = 8$. Determine the cruising speed that could be achieved with this new airfoil for the same power.

Given: Data on an airfoil

Find: Maximum payload; power required

Solution:

The given data or available data is $V_{old} = 225 \cdot \frac{m}{s}$ $\quad \rho = 1.23 \cdot \frac{kg}{m^3}$ $\quad A = 180 \cdot m^2$ $\quad ar_{old} = \frac{10}{1.8}$ $\quad ar_{old} = 5.56$

Assuming the old airfoil operates at close to design lift, from Fig. 9.19 $\quad C_L = 0.3 \quad C_{Di} = 0.0062$ (C_{Di} is the old airfoil's section drag coefficient)

Then $$C_{Dold} = C_{Di} + \frac{C_L^{\,2}}{\pi \cdot ar_{old}} \qquad C_{Dold} = 0.0114$$

The new wing aspect ratio is $ar_{new} = 8$

Hence $$C_{Dnew} = C_{Di} + \frac{C_L^{\,2}}{\pi \cdot ar_{new}} \qquad C_{Dnew} = 0.00978$$

The power required is $$P = T \cdot V = F_D \cdot V = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D \cdot V$$

If the old and new designs have the same available power, then

$$\frac{1}{2} \cdot \rho \cdot A \cdot V_{new}^{\,2} \cdot C_{Dnew} \cdot V_{new} = \frac{1}{2} \cdot \rho \cdot A \cdot V_{old}^{\,2} \cdot C_{Dold} \cdot V_{old}$$

or $$V_{new} = V_{old} \cdot \sqrt[3]{\frac{C_{Dold}}{C_{Dnew}}} \qquad V_{new} = 236 \frac{m}{s}$$

Problem 9.165

[3]

9.165 An airplane with mass of 10,000 lb is flown at constant elevation and speed on a circular path at 150 mph. The flight circle has a radius of 3,250 ft. The plane has lifting area of 225 ft^2 and is fitted with NACA 23015 section airfoils with effective aspect ratio of 7. Estimate the drag on the aircraft and the power required.

Given: Aircraft in circular flight

Find: Drag and power

Solution:

Basic equations: $C_D = \dfrac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad C_L = \dfrac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad P = F_D\cdot V$ $\quad \Sigma\vec{F} = M\cdot\vec{a}$

The given data or available data are

$\rho = 0.002377\cdot\dfrac{\text{slug}}{\text{ft}^3}$ $\quad R = 3250\cdot\text{ft}$ $\quad M = 10000\cdot\text{lbm}$ $\quad M = 311\cdot\text{slug}$

$V = 150\cdot\text{mph}$ $\quad V = 220\cdot\dfrac{\text{ft}}{\text{s}}$ $\quad A = 225\cdot\text{ft}^2$ $\quad ar = 7$

Assuming the aircraft is flying banked at angle β, the vertical force balance is

$$F_L\cdot\cos(\beta) - M\cdot g = 0 \quad \text{or} \quad \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L\cdot\cos(\beta) = M\cdot g \qquad (1)$$

The horizontal force balance is

$$-F_L\cdot\sin(\beta) = M\cdot a_r = -\frac{M\cdot V^2}{R} \quad \text{or} \quad \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L\cdot\sin(\beta) = \frac{M\cdot V^2}{R} \qquad (2)$$

Equations 1 and 2 enable the bank angle β to be found $\quad \tan(\beta) = \dfrac{V^2}{R\cdot g}$ $\quad \beta = \text{atan}\left(\dfrac{V^2}{R\cdot g}\right)$ $\quad \beta = 24.8\cdot\text{deg}$

Then from Eq 1 $\quad F_L = \dfrac{M\cdot g}{\cos(\beta)}$ $\quad F_L = 1.10\times 10^4\cdot\text{lbf}$

Hence $\quad C_L = \dfrac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\quad C_L = 0.851$

For the section, $C_{Dinf} = 0.0075$ at $C_L = 0.851$ (from Fig. 9.19), so $\quad C_D = C_{Dinf} + \dfrac{C_L^{\,2}}{\pi\cdot ar}$ $\quad C_D = 0.040$

Hence $\quad F_D = F_L\cdot\dfrac{C_D}{C_L}$ $\quad F_D = 524\cdot\text{lbf}$

The power is $\quad P = F_D\cdot V$ $\quad P = 1.15\times 10^5\cdot\dfrac{\text{ft}\cdot\text{lbf}}{\text{s}}$ $\quad P = 209\cdot\text{hp}$

Problem 9.166

9.166 Find the minimum and maximum speeds at which the airplane of Problem 9.165 can fly on a 3,250 ft radius circular flight path, and estimate the drag on the aircraft and power required at these extremes.

Given:
Aircraft in circular flight

Find:
Maximum and minimum speeds; Drag and power at these extremes

Solution:

Basic equations:

$$C_D = \frac{F_D}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2} \qquad C_L = \frac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2} \qquad P = F_D\cdot V \qquad \Sigma\cdot\vec{F} = M\cdot\vec{a}$$

The given data or available data are

$$\rho = 0.002377\cdot\frac{slug}{ft^3} \qquad R = 3250\cdot ft \qquad M = 10000\cdot lbm \qquad M = 311\cdot slug$$

$$A = 225\cdot ft^2 \qquad ar = 7$$

The minimum velocity will be when the wing is at its maximum lift condition. From Fig . 9. 17 or Fig. 9.19

$$C_L = 1.72 \qquad C_{Dinf} = 0.02$$

where C_{Dinf} is the section drag coefficient

The wing drag coefficient is then $\quad C_D = C_{Dinf} + \frac{C_L^2}{\pi\cdot ar} \qquad C_D = 0.155$

Assuming the aircraft is flying banked at angle β, the vertical force balance is

$$F_L\cdot\cos(\beta) - M\cdot g = 0 \qquad \text{or} \qquad \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L\cdot\cos(\beta) = M\cdot g \qquad (1)$$

The horizontal force balance is

$$-F_L\cdot\sin(\beta) = M\cdot a_r = -\frac{M\cdot V^2}{R} \qquad \text{or} \qquad \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L\cdot\sin(\beta) = \frac{M\cdot V^2}{R} \qquad (2)$$

Equations 1 and 2 enable the bank angle β and the velocity V to be determined

$$\sin(\beta)^2 + \cos(\beta)^2 = \left(\frac{\frac{M\cdot V^2}{R}}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L}\right)^2 + \left(\frac{M\cdot g}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L}\right)^2 = 1$$

or

$$\frac{M^2\cdot V^4}{R^2} + M^2\cdot g^2 = \frac{\rho^2\cdot A^2\cdot V^4\cdot C_L^2}{4}$$

$$V = \sqrt[4]{\frac{M^2\cdot g^2}{\frac{\rho^2\cdot A^2\cdot C_L^2}{4} - \frac{M^2}{R^2}}} \qquad V = 149\frac{ft}{s} \qquad V = 102\,mph$$

$$\tan(\beta) = \frac{V^2}{R\cdot g} \qquad \beta = atan\left(\frac{V^2}{R\cdot g}\right) \qquad \beta = 12.0\,deg$$

The drag is then $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$ $F_D = 918\,\text{lbf}$

The power required to overcome drag is $P = F_D \cdot V$ $P = 1.37 \times 10^5 \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $P = 249\,\text{hp}$

The analysis is repeated for the maximum speed case, when the lift/drag coefficient is at its minimum value. From Fig. 9.19, reasonable values are

$C_L = 0.3$ $C_{Dinf} = \frac{C_L}{47.6}$ corresponding to α = 2º (Fig. 9.17)

The wing drag coefficient is then $C_D = C_{Dinf} + \frac{{C_L}^2}{\pi \cdot ar}$ $C_D = 0.0104$

From Eqs. 1 and 2 $V = \sqrt[4]{\frac{M^2 \cdot g^2}{\frac{\rho^2 \cdot A^2 \cdot {C_L}^2}{4} - \frac{M^2}{R^2}}}$ $V = (309.9 + 309.9i) \frac{\text{ft}}{\text{s}}$ Obviously unrealistic (lift is just too low, and angle of attack is too low to generate sufficient lift)

We try instead a larger, more reasonable, angle of attack

$C_L = 0.55$ $C_{Dinf} = 0.0065$ corresponding to α = 4º (Fig. 9.17)

The wing drag coefficient is then $C_D = C_{Dinf} + \frac{{C_L}^2}{\pi \cdot ar}$ $C_D = 0.0203$

From Eqs. 1 and 2 $V = \sqrt[4]{\frac{M^2 \cdot g^2}{\frac{\rho^2 \cdot A^2 \cdot {C_L}^2}{4} - \frac{M^2}{R^2}}}$ $V = 91.2 \frac{\text{m}}{\text{s}}$ $V = 204\,\text{mph}$

$\tan(\beta) = \frac{V^2}{R \cdot g}$ $\beta = \text{atan}\left(\frac{V^2}{R \cdot g}\right)$ $\beta = 40.6\,\text{deg}$

The drag is then $F_D = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_D$ $F_D = 485\,\text{lbf}$

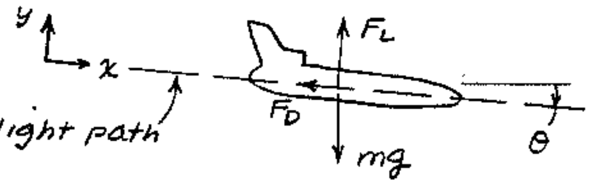
The power required to overcome drag is $P = F_D \cdot V$ $P = 1.45 \times 10^5 \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $P = 264\,\text{hp}$

Problem 9.167

Given: Unpowered flight with lift, drag, and weight in equilibrium.

Find: (a) Show glide slope angle is $\tan\theta = C_D/C_L$
(b) Evaluate minimum glide slope angle for Boeing 727-200 of Example Problem 9.8.
(c) Glide distance from altitude of 10 km on a standard day.

Solution: Consider free-body diagram:



Sum forces along (x) and normal to (y) flight path:

$$\Sigma F_x = -F_D + mg\sin\theta = 0 \qquad mg\sin\theta = F_D$$
$$\Sigma F_y = F_L - mg\cos\theta = 0 \qquad mg\cos\theta = F_L$$
$$\left.\right\} \tan\theta = \frac{F_D}{F_L} = \frac{C_D}{C_L}$$

$\tan\theta$

Use relationships from Section 9-8:

Computing equation: $C_D = C_{D,0} + \dfrac{C_L^2}{\pi ar} = C_{D,0} + C_{D,i}$

Thus $$\frac{C_D}{C_L} = \frac{C_{D,0}}{C_L} + \frac{C_L}{\pi ar} \qquad (1)$$

To minimize, set $d(C_D/C_L)/dC_L = 0$

$$\frac{d}{dC_L}\left(\frac{C_D}{C_L}\right) = (-1)\frac{C_{D,0}}{C_L^2} + \frac{1}{\pi ar} = 0 \quad \text{when} \quad C_{D,0} = \frac{C_L^2}{\pi ar} = C_{D,i}$$

From Example Problem 9.8, $C_{D,0} = 0.0182$ and $ar = 6.5$. Thus optimum is

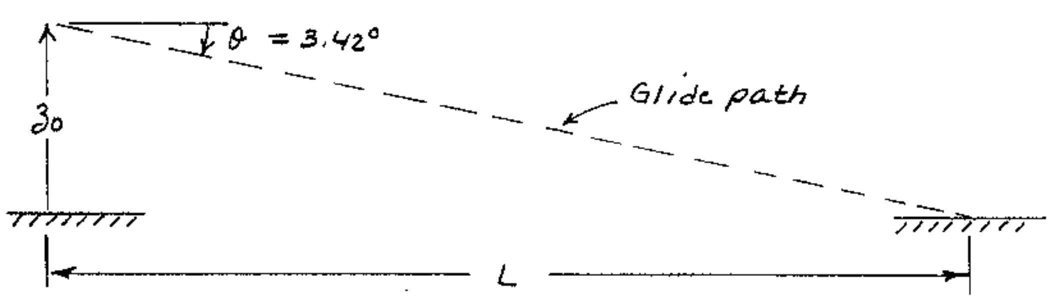
$$C_L = (\pi ar\, C_{D,0})^{1/2} = [\pi(6.5)\,0.0182]^{1/2} = 0.610$$

and from Eq. 1,

$$\frac{C_D}{C_L} = \frac{0.0182}{0.61} + \frac{0.61}{\pi(6.5)} = 0.0597 = \tan\theta\ ;\ \theta = \tan^{-1}(0.0597) = 3.42°$$

θ

Note θ is independent of atmospheric conditions. Thus θ = constant



$$\frac{z_0}{L} = \tan\theta\ ;\ L = \frac{z_0}{\tan\theta} = \frac{10\text{ km}}{0.0597} = 168\text{ km}$$

L

Problem 9.168

Given: Chaparral 2F with rear-mounted airfoil having span, s = 6 ft, and chord, c = 1 ft. Lift and drag coefficients same as conventional section in Fig. 9.17. Consider V = 120 mph (calm day), $\alpha = -12°$ (down).

Find: (a) Maximum downward force.
(b) Maximum increase in deceleration force.

Solution: Apply definitions of C_L and C_D.

Computing equations: $C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A}$ $\quad C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = C_{D,\infty} + \frac{C_L^2}{\pi ar}$ $\quad ar = \frac{s}{c}$

From Fig. 9.17, at $\alpha = 12°$, $C_L = 1.4$ and $C_{D,\infty} = 0.013$. Thus, since $\alpha = -12°$;

$$F_L = -C_L A \frac{1}{2}\rho V^2 \qquad A = sc = 6\ ft^2$$

$$= -1.4 \times 6\ ft^2 \times \frac{1}{2} \times 0.00238 \frac{slug}{ft^3} \times \left(120 \frac{mi}{hr} \times 5280 \frac{ft}{mi} \times \frac{hr}{3600\ s}\right)^2 \frac{lbf \cdot s^2}{slug \cdot ft}$$

$$F_L = -310\ lbf \text{ (downward force)}$$

F_L

Then $F_D = F_L \frac{C_D}{C_L} = \frac{0.013 + \frac{(1.4)^2}{\pi(6)}}{1.4} \times 310\ lbf = 25.9\ lbf$

Braking thrust increases as drag increases and as normal force increases tire adhesion (friction). Thus

$$\Delta F_B = \mu_k F_L + F_D$$

For $\mu_k = 1.0$ (probably conservative for racing tires),

$$\Delta F_B = 1.0 \times 310\ lbf + 25.9\ lbf = 336\ lbf$$

ΔF_B

Problem 9.169

9.169 Some cars come with a "spoiler," a wing section mounted on the rear of the vehicle that salespeople sometimes claim significantly increases traction of the tires at highway speeds. Investigate the validity of this claim. Are these devices really just cosmetic?

Given: Car spoiler

Find: Whether they are effective

Solution:

To perform the investigation, consider some typical data

For the spoiler, assume $b = 4 \cdot ft$ $\quad c = 6 \cdot in$ $\quad \rho = 1.23 \cdot \frac{kg}{m^3}$ $\quad A = b \cdot c$ $\quad A = 2\,ft^2$

From Fig. 9.17 a reasonable lift coefficient for a conventional airfoil section is $C_L = 1.4$

Assume the car speed is $V = 55 \cdot mph$

Hence the "negative lift" is $F_L = \frac{1}{2} \cdot \rho \cdot A \cdot V^2 \cdot C_L$ $\quad F_L = 21.7\,lbf$

This is a relatively minor negative lift force (about four bags of sugar); it is not likely to produce a noticeable difference in car traction

The picture gets worse at 30 mph: $F_L = 6.5\,lbf$

For a race car, such as that shown on the cover of the text, typical data might be

$b = 5 \cdot ft$ $\quad c = 18 \cdot in$ $\quad A = b \cdot c$ $\quad A = 7.5\,ft^2$ $\quad V = 200 \cdot mph$

In this case: $F_L = 1078\,lbf$

Hence, for a race car, a spoiler can generate very significant negative lift!

Given: Man-powered aircraft, the Gossamer Condor:

$W/A = 0.4\ lbf/ft^2 \quad W = 200\ lbf \quad ar = 17 \quad F_D = 6\ lbf$ at 12 mph

Pilot could sustain 0.39 hp for 2 hr.

Find: (a) Minimum power to fly aircraft.
(b) Compare to pilot output capability.

Solution: Apply relationships from Section 9-8:

Computing equations: $W = F_L = C_L A \frac{1}{2}\rho V^2 \qquad P = VF_D$

$$T = F_D = C_D A \tfrac{1}{2}\rho V^2 \;;\; C_D = C_{D,0} + \frac{C_L^2}{\pi ar}$$

The task is to find V to minimize P:

$$P = VF_D = V(C_D A \tfrac{1}{2}\rho V^2) = \left(C_{D,0} + \frac{C_L^2}{\pi ar}\right) A \tfrac{1}{2}\rho V^3 \qquad (1)$$

But C_L varies with aircraft speed:

$$F_L = W = C_L A \tfrac{1}{2}\rho V^2 \;;\quad C_L = \frac{2W}{\rho V^2 A} \;;\quad C_L^2 = \left(\frac{2W}{\rho A}\right)^2 \frac{1}{V^4}$$

Substituting into Eq. 1,

$$P = \left[C_{D,0} + \frac{1}{\pi ar}\left(\frac{2W}{\rho A}\right)^2 \frac{1}{V^4}\right] A \tfrac{1}{2}\rho V^3$$

To minimize power, set $dP/dV = 0$. Then

$$\frac{dP}{dV} = C_{D,0} A \tfrac{1}{2}\rho V^2 (3) + (-1)\frac{1}{\pi ar}\left(\frac{2W}{\rho A}\right)^2 \frac{1}{V^2} A \tfrac{1}{2}\rho = 0$$

Thus at minimum power,

$$3C_{D,0} = \frac{1}{\pi ar}\left(\frac{2W}{\rho A}\right)^2 \frac{1}{V^4} = \frac{C_L^2}{\pi ar} = C_{D,i} \quad \text{or} \quad C_{D,i} = 3C_{D,0} \quad \text{minimum power}$$

From given data, at 12 mph (17.6 ft/s):

$$\tfrac{1}{2}\rho V^2 = \tfrac{1}{2} \times 0.00238\frac{slug}{ft^3} \times (17.6)^2 \frac{ft^2}{s^2} \times \frac{lbf \cdot s^2}{slug \cdot ft} = 0.369\ lbf/ft^2$$

$$C_L = \frac{F_L}{\tfrac{1}{2}\rho V^2 A} = \frac{W/A}{\tfrac{1}{2}\rho V^2} = 0.4\frac{lbf}{ft^2} \times \frac{ft^2}{0.369\ lbf} = 1.08 \;;\; A = \frac{W}{W/A} = 200\ lbf \times \frac{ft^2}{0.4\ lbf} = 500\ ft^2$$

$$C_D = \frac{F_D}{\tfrac{1}{2}\rho V^2 A} = 6\ lbf \times \frac{ft^2}{0.369\ lbf} \times \frac{1}{500\ ft^2} = 0.0325$$

$$C_{D,0} = C_D - C_{D,i} = C_D - \frac{C_L^2}{\pi ar} = 0.0325 - \frac{(1.08)^2}{\pi(17)} = 0.0107 \quad (C_{D,0} = \text{constant})$$

At flight speed for minimum power, $C_{D,i} = 3C_{D,0}$

Thus at minimum power

$$C_{D,i} = \frac{C_L^2}{\pi ar} = 3C_{D,0} = 3(0.0107) = 0.0321$$

so

$$C_L = (0.0321\,\pi\,ar)^{1/2} = 1.31 \quad \text{(minimum power)}$$

since

$$F_L = W = C_L A \tfrac{1}{2}\rho V^2$$

then

$$V = \sqrt{\frac{2W}{C_L \rho A}} \quad \text{and} \quad \frac{V_{min}}{V} = \sqrt{\frac{C_L}{C_{L\,min}}}$$

Thus at minimum power

$$V_{min} = 12\text{ mph}\sqrt{\frac{1.08}{1.31}} = 10.9\text{ mph } (16.0\text{ ft/s}) \quad \text{(minimum power)}$$ V_{min}

The power requirement would be

$$P_{pilot} = \frac{P_{flight}}{\eta_{drive}\,\eta_{prop}}$$

$$P_{flight} = VF_D = V C_D A \tfrac{1}{2}\rho V^2 = (C_{D,0} + 3C_{D,0}) A \tfrac{1}{2}\rho V^3 = 2C_{D,0} A \rho V^3$$

$$= 2(0.0107)\,500\text{ ft}^2 \times 0.00238\frac{\text{slug}}{\text{ft}^3} \times (16.0)^3\frac{\text{ft}^3}{\text{s}^3} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \times \frac{\text{hp}\cdot\text{s}}{550\text{ ft}\cdot\text{lbf}}$$

$$P_{flight} = 0.190\text{ hp} \quad \text{(power for flight)}$$

If $\eta_{drive} = 0.9$ and $\eta_{prop} = 0.7$,

$$P_{pilot} \approx \frac{0.190}{(0.9)(0.7)} = 0.302\text{ hp} \quad \text{(minimum power)}$$ P_{pilot}

Thus $P_{pilot} < 0.39$ hp!

Problem 9.171

Open-Ended Problem Statement: How does a Frisbee™ fly? What causes it to curve left or right? What is the effect of spin on its flight?

Discussion: When viewed from the side, the Frisbee shape has a rounded upper surface and a flat bottom surface. Such a shape is capable of generating lift as it travels through air.

When a Frisbee is not spinning, the lift vector probably acts slightly forward of the maximum thickness on the profile. When spinning, the motion of the surface likely affects the development and separation of the boundary layers. This may displace the center of lift slightly to the right or left of center, depending on the direction of spin.

A Frisbee is not stable when thrown without spin: it will tend to tumble as it moves through the air. Spin is used to stabilize the motion (just as a spinning gyroscope tends to remain upright). The combination of spin and the off-center lift vector cause the Frisbee to precess as a gyroscope. Therefore its spin axis can change from vertical while in flight, causing the flight path to curve right or left.

The Frisbee also can be thrown intentionally to curve right or left. This is done by inclining the spin axis so that it is not vertical at launch. When the spin axis is inclined to the left (as seen by the thrower), the Frisbee drifts to the left along a more-or-less constant radius path. Inclining the spin axis to the right causes the opposite effect.

Problem 9.172

Open-Ended Problem Statement: Roadside signs tend to oscillate in a twisting motion when a strong wind blows. Discuss the phenomena that must occur to cause this behavior.

Discussion: Many roadside signs are mounted on a single post formed from stamped steel. The post has an open "C" cross-section, which provides little torsional rigidity. Wind gusts can excite oscillations in a sign, which acts as a flat plate at an angle of attack relative to the oncoming wind. When at an angle of attack, a plate develops both a lift force and a moment that tends to twist the sign farther from its equilibrium position. While the sign twists, the post provides a resisting torque as a result of being twisted from its equilibrium position.

As the sign twists, the angle of attack relative to the oncoming air increases. An overshoot phenomenon called dynamic stall allows the flow to remain attached and the angle of attack to grow larger before stall occurs than if the change in angle of attack had been slow and gradual. Once stall occurs, the lift force and moment decrease, and the motion is no longer forced. Then the sign tends to return to its undisturbed position.

The moment of inertia of the sign causes it to overshoot the equilibrium position. The sign continues beyond equilibrium and develops a lift force and a moment tending to move it farther past equilibrium. The process repeats, with growing amplitude, until a more-or-less steady-state oscillation is reached.

The sign and post form a spring-mass-damper mechanical system. The sign is the mass, the post is the spring, and hysteresis and aerodynamic resistance to oscillation provide the damping. The "steady" oscillation occurs near the natural frequency of the system.

At steady state, the rate at which energy is added to the sign by the gusting wind exactly balances the rate at which energy is dissipated by hysteresis in the sign motion and its supporting post. The oscillations can continue almost indefinitely, and with considerable amplitude, as can be observed on a windy day. In some cases the oscillations lead to fatigue failure of the sign post.

Problem 9.173

Open-Ended Problem Statement: An automobile travels down the road with a bicycle attached to a carrier across the rear of the trunk. The bicycle wheels rotate slowly. Explain why and in what direction the rotation occurs.

Discussion: All objects moving in ground effect generate lift (the air flows over the top faster than over the bottom because of the shape of the automobile). Any object that produces lift carries with it a bound vortex that creates circulation about the profile that accompanies lift.

The bound vortex creates two trailing vortices, one on each side of the car, which rotate in opposite directions as they follow in the wake of the automobile. When viewed from the rear of the auto, the left side trailing vortex rotates clockwise and the right side trailing vortex rotates counterclockwise.

The swirl in the trailing vortex motion is responsible for the motion of the bicycle wheels (check it out on your next auto trip during the summer months!). The swirl causes shear stresses that tend to rotate the bicycle wheels in the same senses as the trailing vortices. Again viewing from the rear of the auto, the left wheel rotates clockwise and the right wheel counterclockwise.

(Sometimes the rear wheel of the bicycle cannot freewheel. In this case only the front wheel turns slowly as the car drives down the road.)

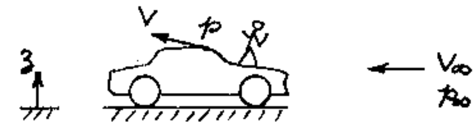
Problem 9.174

Given: Air moving over automobile, as shown in Fig. 9.25.

Find: (a) Estimate pressure reduction in car when a window is "cracked" while traveling at $V_0 = 100$ km/hr.
(b) Air speed in freestream near window opening.

Solution: Apply the Bernoulli equation and pressure coefficient definition.

Basic equations: $\frac{p_\infty}{\rho} + \frac{V_\infty^2}{2} + g z_\infty = \frac{p}{\rho} + \frac{V^2}{2} + g z$ $\qquad C_p = \frac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2}$



Assumptions: (1) Steady flow seen from auto
(2) Incompressible flow
(3) No friction
(4) Flow along a streamline
(5) Neglect changes in elevation

From Fig. 9.25, C_p near driver's window ranges between -1.23 and -0.40.

At $V_\infty = 100$ km/hr,

$$q = \frac{1}{2}\rho V_\infty^2 = \frac{1}{2} \times 1.23 \frac{\text{kg}}{\text{m}^3} \times \left[100 \frac{\text{km}}{\text{hr}} \times 1000 \frac{\text{m}}{\text{km}} \times \frac{\text{hr}}{3600\,\text{s}}\right]^2 \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} = 475\ \text{N/m}^2$$

Thus the pressures outside may be between

$$p - p_\infty = C_p \frac{1}{2}\rho V_\infty^2 = -1.23 \times 475 \frac{\text{N}}{\text{m}^2} = -584\ \text{N/m}^2\ (\text{gage})$$

and $\quad p - p_\infty = -0.40 \times 475 \frac{\text{N}}{\text{m}^2} = -190\ \text{N/m}^2\ (\text{gage})$ $\qquad$ p

From the Bernoulli equation,

$$\frac{V^2}{2} = \frac{V_\infty^2}{2} + \frac{p_\infty - p}{\rho} = \frac{V_\infty^2}{2}\left(1 - \frac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2}\right) = \frac{V_\infty^2}{2}(1 - C_p)$$

Thus

$$V = V_\infty \sqrt{1 - C_p}$$

The local flow speeds range from

$$V = V_\infty \sqrt{1 - (-1.23)} = 100 \frac{\text{km}}{\text{hr}} \sqrt{2.23} = 149\ \text{km/hr} \quad (41.5\ \text{m/s})$$

$$V = V_\infty \sqrt{1 - (-0.40)} = 100 \frac{\text{km}}{\text{hr}} \sqrt{1.40} = 118\ \text{km/hr} \quad (32.9\ \text{m/s})$$ $\qquad$ V

Thus local flow speeds are significantly higher than V_∞.

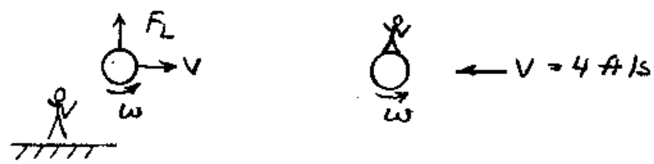
Problem 9.175

[2]

Given: Classroom demonstration of lift on spinning cylinder.

$L = 10$ in., $D = 2$ in.

$\omega = 300$ rpm



Find: Estimate lift force acting on cylinder.

Solution: Apply definition of lift coefficient, data from Fig. 9.29.

Computing equation: $F_L = C_D A \frac{1}{2} \rho V^2$

From Fig. 9.29, $C_L = C_L(\omega D/2V)$

$$\omega = 300 \frac{\text{rev}}{\text{min}} \times 2\pi \frac{\text{rad}}{\text{rev}} \times \frac{\text{min}}{60\text{ s}} = 31.4 \text{ rad/s}$$

$$\frac{\omega D}{2V} = \frac{1}{2} \times 31.4 \frac{\text{rad}}{\text{s}} \times 2 \text{ in.} \times \frac{\text{s}}{4 \text{ ft}} \times \frac{\text{ft}}{12 \text{ in.}} = 0.654$$

There is a data band in Fig. 9.29. The highest value is $C_L \approx 1.1$

$$A = DL = 2 \text{ in.} \times 10 \text{ in.} \times \frac{\text{ft}^2}{144 \text{ in.}^2} = 0.139 \text{ ft}^2$$

For standard atmosphere conditions

$$F_L = 1.1 \times 0.139 \text{ ft}^2 \times \frac{1}{2} \times 0.00238 \frac{\text{slug}}{\text{ft}^3} \times (4)^2 \frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} = 0.00291 \text{ lbf}$$ F_L

{This is quite a small force, but the speed is low.}

Problem 9.176

Given: Golf ball with mass, $m = 48\,g$, and diameter, $D = 43\,mm$, is hit from a sand trap with speed, $V = 20\,m/sec$, and backspin, $\omega = 2000\,rpm$.

Find: (a) Lift and drag forces acting on ball.
(b) Express as fractions of mg.

Solution: Use data from Fig. 9.28 for lift and drag coefficients.

Computing equations: $F_L = C_L A \frac{1}{2}\rho V^2 \qquad F_D = C_D A \frac{1}{2}\rho V^2$

At $V = 20\,m/sec$, then $q = \frac{1}{2}\rho V^2 = \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (20)^2 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} = 246\,N/m^2$

From Fig. 9.28, $C_L = C_L\left(Re, \frac{\omega D}{2V}\right)$

$$Re = \frac{VD}{\nu} = 20\,\frac{m}{s} \times 0.043\,m \times \frac{s}{1.45 \times 10^{-5}\,m^2} = 5.93 \times 10^4 \text{ (assume close to } 1.26 \times 10^5\text{)}$$

$$\frac{\omega D}{2V} = \frac{1}{2} \times 2000\,\frac{rev}{min} \times 0.043\,m \times \frac{s}{20\,m} \times 2\pi\,\frac{rad}{rev} \times \frac{min}{60\,s} = 0.225$$

Then $C_L \simeq 0.23$ and $F_L = C_L A \frac{1}{2}\rho V^2 = C_L A q$

$$F_L = 0.23 \times \frac{\pi (0.043)^2}{4} m^2 \times 246\,\frac{N}{m^2} = 0.0822\,N$$

F_L

From Fig. 9.28 $C_D = 0.31$, so

$$F_D = \frac{C_D}{C_L} F_L = \frac{0.31}{0.23} \times 0.0822\,N = 0.111\,N$$

F_D

For the ball, $mg = 0.048\,kg \times 9.81\,\frac{m}{sec^2} \times \frac{N \cdot sec^2}{kg \cdot m} = 0.471\,N$

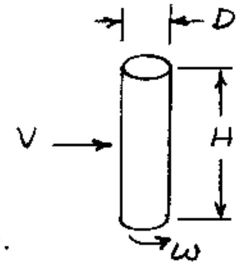
Thus
$$\frac{F_L}{mg} = \frac{0.0822\,N}{0.471\,N} = 0.175$$

$$\frac{F_D}{mg} = \frac{0.111\,N}{0.471\,N} = 0.236$$

Ratios

Problem 9.177

Given: Rotating cylinders for ship propulsion.



$D = 3\ m,\ H = 15\ m,\ N \leq 750\ rpm$

Find: (a) Calculate maximum lift and drag forces on each cylinder at $V = 50\ km/hr$.
(b) Compare to force at optimum L/D.
(c) Estimate power needed to spin rotor at $N = 750\ rpm$.

Solution: Apply definitions of coefficients, data from Fig. 9.29.

Computing equations: $C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 A} \qquad C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A} \qquad A = DH$

Coefficients are functions of spin ratio, $\omega D/2V$.

$$\omega = 750\,\frac{rev}{min} \times 2\pi\,\frac{rad}{rev} \times \frac{min}{60\ s} = 78.5\ rad/s\ ; \quad V = 50\,\frac{km}{hr} \times 1000\,\frac{m}{km} \times \frac{hr}{3600\ s} = 13.9\,\frac{m}{s}$$

$$\frac{\omega D}{2V} = \frac{1}{2} \times 78.5\,\frac{rad}{s} \times 3\ m \times \frac{s}{13.9\ m} = 8.47\ ; \quad q = \frac{1}{2}\rho V^2 = \frac{1}{2} \times 1.23\,\frac{kg}{m^3} \times (13.9)^2\,\frac{m^2}{s^2} = 119\ N/m^2$$

This is above the range of data shown in Fig. 9.29, so ω could be less.
Choose $C_L \approx 9.5$, $C_D \approx 3.5$, $A = DH = 45\ m^2$

$$F_L = 9.5 \times 45\ m^2 \times 119\,\frac{N}{m^2} = 50.9\ kN$$ F_L

$$F_D = 3.5 \times 45\ m^2 \times 119\,\frac{N}{m^2} = 18.7\ kN$$ F_D

From inspection of Fig. 9.29, this appears close to the optimum L/D.
Then

$$F = \left[F_L^2 + F_D^2\right]^{\frac{1}{2}} = \left[(50.9)^2 + (18.7)^2\right]^{\frac{1}{2}} kN = 54.2\ kN$$ F

Torque will be product of mean shear stress, $\bar{\tau}$, area, and radius.

$$T = \bar{\tau} A R = \bar{\tau}(\pi D H) R = \frac{\pi \bar{\tau} D^2 H}{2}$$

Estimate (very roughly) $\bar{\tau}$ from flat plate correlation. Assume

$$Re = \frac{(V+U)D}{\nu} = \frac{(V+\omega R)D}{\nu} = (13.9 + 118)\,\frac{m}{s} \times 3\ m \times \frac{s}{1.45 \times 10^{-5}\ m^2} = 2.73 \times 10^7$$

From Fig. 9.8, $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A} = 0.003\ ; \quad \bar{\tau} = \dfrac{F_D}{A} = C_D\, q = 0.003 \times 119\,\dfrac{N}{m^2} = 0.357\,\dfrac{N}{m^2}$

Then

$$T = \frac{\pi}{2} \times 0.357\,\frac{N}{m^2} \times (3)^2 m^2 \times 15\ m = 75.7\ N\cdot m$$

and

$$\mathcal{P} = \omega T = 78.5\,\frac{rad}{s} \times 75.7\ N\cdot m \times \frac{W\cdot s}{N\cdot m} = 5.94\ kW$$ $\mathcal{P}$

{The power estimate is gross only. More specific information is needed to design with confidence.}

Problem 9.178

[3]

Given: American and British golf balls (Problems 1.11 and 1.12), dimensions below. Hit from tee at $V = 85$ m/sec, with backspin, $N = 9000$ rpm.

Find: (a) Evaluate lift and drag forces on each ball (express as fractions of body force).
(b) Estimate radius of curvature of trajectory.
(c) Which ball would have the longer range?

Solution: Apply definitions of lift and drag coefficients, data from Fig. 9.28.

Computing equations: $C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 A} = \dfrac{F_L}{qA}$; $C_D = \dfrac{F_D}{qA}$; $q = \frac{1}{2}\rho V^2$; $A = \dfrac{\pi D^2}{4}$

The parameters are Re_D and $\omega D/2V$; tabulate results:

Ball	m (oz)	D (in.)	$\frac{\omega D}{2V}$ (---)	Re (---)	C_L (---)	C_D (---)	F_L (N)	F_D (N)
American	1.62	1.68	0.236	2.50×10^5	0.27	0.32	1.71	2.03
British	1.62	1.62	0.228	2.41×10^5	0.26	0.31	1.53	1.83

Taking ratios for the American ball: British ball:

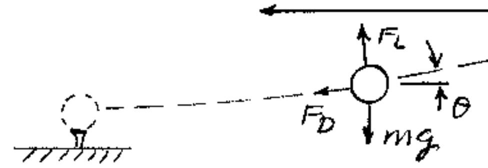
$$F_L/mg = 1.71\ \text{N} \times \frac{1}{1.62\ \text{oz}} \times \frac{16\ \text{oz}}{\text{lbf}} \times \frac{\text{lbf}}{4.448\ \text{N}} = 3.80 \qquad 3.40$$

$$F_D/mg \qquad = 4.51 \qquad 4.07$$

F_L/mg

F_D/mg

Draw FBD to compute trajectory:



$$\Sigma F_{\perp \text{ to path}} = F_L - mg\cos\theta = m\frac{V^2}{R}$$

Assume θ small, so $\cos\theta \approx 1$. Then

$$R \approx \frac{mV^2}{F_L - mg} = \frac{V^2/g}{F_L/mg - 1} = \frac{1}{3.80 - 1} \times (85)^2 \frac{m^2}{s^2} \times \frac{s^2}{9.81\ m} = 263\ m \quad (\text{American})$$

$$= \frac{1}{3.40 - 1} \times (85)^2 \frac{m^2}{s^2} \times \frac{s^2}{9.81\ m} = 307\ m \quad (\text{British})$$

R

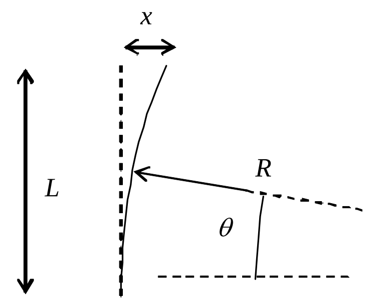
(Note because $F_L/mg > 1$, the balls actually rise!)

Drag probably is more important than lift in affecting range of a drive. Therefore one probably would expect the British ball to carry farther.

Range

Problem 9.179

9.179 A baseball pitcher throws a ball at 80 mph. Home plate is 60 ft away from the pitcher's mound. What spin should be placed on the ball for maximum horizontal deviation from a straight path? (A baseball has a mass of 5 oz and a circumference of 9 in.) How far will the ball deviate from a straight line?



Given: Baseball pitch

Find: Spin on the ball

Solution:

Basic equations: $C_L = \dfrac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\qquad \Sigma\cdot\vec{F} = M\cdot\vec{a}$

The given or available data is $\rho = 0.00234\cdot\dfrac{\text{slug}}{\text{ft}^3}$ $\qquad \nu = 1.62\times 10^{-4}\cdot\dfrac{\text{ft}^2}{\text{s}}$ $\qquad L = 60\cdot\text{ft}$

$M = 5\cdot\text{oz}$ $\qquad C = 9\cdot\text{in}$ $\qquad D = \dfrac{C}{\pi}$ $\qquad D = 2.86\,\text{in}$ $\qquad A = \dfrac{\pi\cdot D^2}{4}$ $\qquad A = 6.45\,\text{in}^2$ $\qquad V = 80\cdot\text{mph}$

Compute the Reynolds number $\qquad Re = \dfrac{V\cdot D}{\nu}$ $\qquad Re = 1.73\times 10^5$

This Reynolds number is slightly beyond the range of Fig. 9.27; we use Fig. 9.27 as a rough estimate

The ball follows a trajectory defined by Newton's second law. In the horizontal plane (*x* coordinate)

$$F_L = M\cdot a_R = M\cdot a_x = M\cdot\frac{V^2}{R} \qquad \text{and} \qquad F_L = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L$$

where R is the instantaneous radius of curvature of the trajectory

From Eq 1 we see the ball trajectory has the smallest radius (i.e. it curves the most) when CL is as large as possible.
From Fig. 9.27 we see this is when $C_L = 0.4$

Solving for R $\qquad R = \dfrac{2\cdot M}{C_L\cdot A\cdot\rho}$ $\qquad$ (1) $\qquad R = 463.6\,\text{ft}$

Also, from Fig. 9.27 $\qquad \dfrac{\omega\cdot D}{2\cdot V} = 1.5$ $\qquad$ to $\qquad \dfrac{\omega\cdot D}{2\cdot V} = 1.8$ $\qquad$ defines the best range

Hence $\qquad \omega = 1.5\cdot\dfrac{2\cdot V}{D}$ $\qquad \omega = 14080\,\text{rpm}$ $\qquad \omega = 1.8\cdot\dfrac{2\cdot V}{D}$ $\qquad \omega = 16896\,\text{rpm}$

From the trajectory geometry $\qquad x + R\cdot\cos(\theta) = R$ $\qquad$ where $\qquad \sin(\theta) = \dfrac{L}{R}$

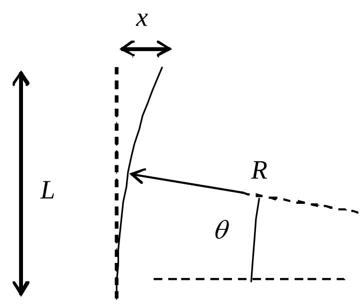
Hence $\qquad x + R\cdot\sqrt{1-\left(\dfrac{L}{R}\right)^2} = R$

Solving for x $\qquad x = R - R\cdot\sqrt{1-\left(\dfrac{L}{R}\right)^2}$ $\qquad x = 3.90\,\text{ft}$

Problem 9.180

[4]

9.180 A soccer player takes a free kick. Over a distance of 10 m, the ball veers to the right by about 1 m. Estimate the spin the player's kick put on the ball if its speed is 30 m/s. The ball has a mass of 420 gm and has a circumference of 70 cm.



Given:
Soccer free kick

Find:
Spin on the ball

Solution:

Basic equations: $C_L = \dfrac{F_L}{\frac{1}{2}\cdot\rho\cdot A\cdot V^2}$ $\qquad \Sigma\vec{F} = M\cdot\vec{a}$

The given or available data is $\rho = 1.21\cdot\dfrac{kg}{m^3}$ $\quad \nu = 1.50\cdot 10^{-5}\cdot\dfrac{m^2}{s}$ $\quad L = 10\cdot m$ $\quad x = 1\cdot m$

$M = 420\cdot gm$ $\quad C = 70\cdot cm$ $\quad D = \dfrac{C}{\pi}$ $\quad D = 22.3\,cm$ $\quad A = \dfrac{\pi\cdot D^2}{4}$ $\quad A = 0.0390\,m^2$ $\quad V = 30\cdot\dfrac{m}{s}$

Compute the Reynolds number $\quad Re = \dfrac{V\cdot D}{\nu}$ $\quad Re = 4.46\times 10^5$

This Reynolds number is beyond the range of Fig. 9.27; however, we use Fig. 9.27 as a rough estimate

The ball follows a trajectory defined by Newton's second law. In the horizontal plane (*x* coordinate)

$$F_L = M\cdot a_R = M\cdot a_x = M\cdot\frac{V^2}{R} \quad \text{and} \quad F_L = \frac{1}{2}\cdot\rho\cdot A\cdot V^2\cdot C_L$$

where R is the instantaneous radius of curvature of the trajectory

Hence, solving for R $\quad R = \dfrac{2\cdot M}{C_L\cdot A\cdot\rho}$ $\quad$ (1)

From the trajectory geometry $\quad x + R\cdot\cos(\theta) = R$ $\quad$ where $\quad \sin(\theta) = \dfrac{L}{R}$

Hence $\quad x + R\cdot\sqrt{1-\left(\dfrac{L}{R}\right)^2} = R$

Solving for *R* $\quad R = \dfrac{\left(L^2 + x^2\right)}{2\cdot x}$ $\quad R = 50.5\,m$

Hence, from Eq 1 $\quad C_L = \dfrac{2\cdot M}{R\cdot A\cdot\rho}$ $\quad C_L = 0.353$

For this lift coefficient, from Fig. 9.27 $\quad \dfrac{\omega\cdot D}{2\cdot V} = 1.2$

Hence $\quad \omega = 1.2\cdot\dfrac{2\cdot V}{D}$ $\quad \omega = 3086\,rpm$

(And of course, Beckham still kind of rules!)

Problem 10.1

Given: Impeller dimensions of Example Problem 10.1:

		Inlet ①	Outlet ②
$Q = 150$ gpm	Radius, r (in.)		2.0
$N = 3450$ rpm	Blade width, b (in.)		0.383
	Blade angle, β (deg)		60°, 70°, 80°, 85°

Construct: velocity diagram for exit flow leaving tangent to the blade.

Find: (a) ideal head rise, (b) mechanical power input

Solution:

Computing equations:

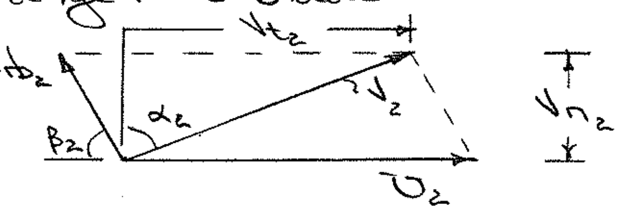
$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \quad (10.2b)$$

$$H = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \quad (10.2c)$$

(the $U_1 V_{t_1}$ terms crossed out: $0^{(1)}$)

Assumptions: (1) axial inlet flow (given) so $V_{t_1} = 0$
(2) at blade outlet, flow is uniform and leaves tangent to blade

Exit velocity diagram:



From continuity, $V_{n_2} = \dfrac{Q}{2\pi r_2 b_2} = V_{rb_2}\sin\beta_2 \quad \therefore V_{rb_2} = \dfrac{V_{n_2}}{\sin\beta_2}$

From geometry, $V_{t_2} = U_2 - V_{rb_2}\cos\beta_2 = U_2 - \dfrac{V_{n_2}}{\sin\beta_2}\cos\beta_2 = U_2 - V_{n_2}\cot\beta_2$

Substituting numerical values, for $\beta = 60°$

$$U_2 = \omega r_2 = 3450\,\frac{\text{rev}}{\text{min}} \times 2\pi\,\frac{\text{rad}}{\text{rev}} \times \frac{\text{min}}{60\,\text{s}} \times 2.0\,\text{in} \times \frac{\text{ft}}{12\,\text{in}} = 60.2\,\text{ft/s}$$

$$V_{n_2} = \frac{Q}{2\pi r_2 b_2} = \frac{1}{2\pi} \times 150\,\frac{\text{gal}}{\text{min}} \times \frac{\text{min}}{60\,\text{s}} \times \frac{\text{ft}^3}{7.48\,\text{gal}} \times \frac{1}{2.0\,\text{in}} \times \frac{1}{0.383\,\text{in}} \times \frac{144\,\text{in}^2}{\text{ft}^2} = 10.0\,\text{ft/s}$$

$$V_{t_2} = U_2 - V_{n_2}\cot\beta_2 = 60.2\,\frac{\text{ft}}{\text{s}} - 10.0\,\frac{\text{ft}}{\text{s}}\cot 60° = 54.4\,\text{ft/s}$$

$$\dot{m} = \rho Q = 1.94\,\frac{\text{slug}}{\text{ft}^3} \times 150\,\frac{\text{gal}}{\text{min}} \times \frac{\text{min}}{60\,\text{s}} \times \frac{\text{ft}^3}{7.48\,\text{gal}} = 0.648\,\text{slug/s}$$

$$H = \frac{1}{g}U_2 V_{t_2} = \frac{\text{s}^2}{32.2\,\text{ft}} \times 60.2\,\frac{\text{ft}}{\text{s}} \times 54.4\,\frac{\text{ft}}{\text{s}} = 102\,\text{ft}$$ ← H ($\beta = 60°$)

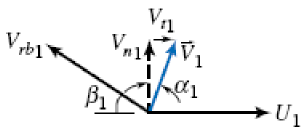
$$\dot{W}_m = \dot{m}U_2 V_{t_2} = \dot{m}gH = 0.648\,\frac{\text{slug}}{\text{s}} \times 32.2\,\frac{\text{ft}}{\text{s}^2} \times 102\,\text{ft} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} \times \frac{\text{hp}\cdot\text{s}}{550\,\text{ft}\cdot\text{lbf}}$$

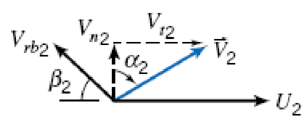
$$\dot{W}_m = 3.87\,\text{hp}$$ ← $\dot{W}_m$ ($\beta = 60°$)

For $\beta = 70°$, $V_{t_2} = 56.6$ ft/s, $H = 106$ ft, $\dot{W}_m = 4.02$ hp ←
$\beta = 80°$, $V_{t_2} = 58.4$ ft/s, $H = 109$ ft, $\dot{W}_m = 4.15$ hp ←
$\beta = 85°$, $V_{t_2} = 59.3$ ft/s, $H = 111$ ft, $\dot{W}_m = 4.21$ hp ←

Problem 10.2

10.2 The geometry of a centrifugal water pump is $r_1 = 4$ in., $r_2 = 7.5$ in., $b_1 = b_2 = 1.5$ in., $\beta_1 = 30°$, $\beta_2 = 20°$, and it runs at speed 1500 rpm. Estimate the discharge required for axial entry, the horsepower generated in the water, and the head produced.





Given: Geometry of centrifugal pump

Find: Estimate discharge for axial entry; Head

Solution:

Basic equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ (Eq. 10.2b)

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$$ (Eq. 10.2c)

The given or available data is

$\rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$ $\quad r_1 = 4 \cdot \text{in}$ $\quad r_2 = 7.5 \cdot \text{in}$ $\quad b_1 = 1.5 \cdot \text{in}$ $\quad b_2 = 1.5 \cdot \text{in}$

$\omega = 1500 \cdot \text{rpm}$ $\quad \beta_1 = 30 \cdot \text{deg}$ $\quad \beta_2 = 20 \cdot \text{deg}$

From continuity $\quad V_n = \frac{Q}{2 \cdot \pi \cdot r \cdot b} = V_{rb} \cdot \sin(\beta)$ $\quad V_{rb} = \frac{V_n}{\sin(\beta)}$

From geometry $\quad V_t = U - V_{rb} \cdot \cos(\beta) = U - \frac{V_n}{\sin(\beta)} \cdot \cos(\beta) = U - \frac{Q}{2 \cdot \pi \cdot r \cdot b} \cdot \cot(\beta)$

For an axial entry $\quad V_{t1} = 0$ $\quad$ so $\quad U_1 - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1} \cdot \cot(\beta_1) = 0$

Using given data $\quad U_1 = \omega \cdot r_1$ $\quad U_1 = 52.4 \cdot \frac{\text{ft}}{\text{s}}$

Hence $\quad Q = 2 \cdot \pi \cdot r_1 \cdot b_1 \cdot U_1 \cdot \tan(\beta_1)$ $\quad Q = 7.91 \cdot \frac{\text{ft}^3}{\text{s}}$ $\quad Q = 3552 \cdot \text{gpm}$

To find the power we need U_2, V_{t2}, and m_{rate}

The mass flow rate is $\quad m_{rate} = \rho \cdot Q$ $\quad m_{rate} = 15.4 \cdot \frac{\text{slug}}{\text{s}}$

$U_2 = \omega \cdot r_2$ $\quad U_2 = 98.2 \cdot \frac{\text{ft}}{\text{s}}$

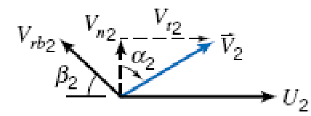
$V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2)$ $\quad V_{t2} = 53.9 \cdot \frac{\text{ft}}{\text{s}}$

Hence $\quad W_m = (U_2 \cdot V_{t2} - U_1 \cdot V_{t1}) \cdot m_{rate}$ $\quad W_m = 81212 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $\quad W_m = 148 \cdot \text{hp}$

The head is $\quad H = \frac{W_m}{m_{rate} \cdot g}$ $\quad H = 164 \cdot \text{ft}$

Problem 10.3

10.3 A centrifugal pump running at 3500 rpm pumps water at a rate of 150 gpm. The water enters axially, and leaves the impeller at 17.5 ft/s relative to the blades, which are radial at the exit. If the pump requires 6.75 hp, and is 67 percent efficient, estimate the basic dimensions (impeller exit diameter and width), using the Euler turbomachine equation.



Given:
Data on centrifugal pump

Find:
Estimate basic dimensions

Solution:

Basic equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ (Eq. 10.2b, directly derived from the Euler turbomachine equation)

The given or available data is

$\rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$ $\quad Q = 150 \cdot \text{gpm}$ $\quad Q = 0.334 \frac{\text{ft}^3}{\text{s}}$ $\quad W_{in} = 6.75 \cdot \text{hp}$ $\quad \eta = 67 \cdot \%$

$\omega = 3500 \cdot \text{rpm}$ $\quad V_{rb2} = 17.5 \cdot \frac{\text{ft}}{\text{s}}$ $\quad \beta_2 = 90 \cdot \text{deg}$

For an axial inlet $\quad V_{t1} = 0$

From the outlet geometry $\quad V_{t2} = U_2 - V_{rb2} \cdot \cos(\beta_2) = U_2$ $\quad$ and $\quad U_2 = r_2 \cdot \omega$

Hence, in Eq. 10.2b $\quad W_m = U_2^2 \cdot m_{rate} = r_2^2 \cdot \omega^2 \cdot m_{rate}$

with $\quad W_m = \eta \cdot W_{in}$ $\quad W_m = 4.52 \, \text{hp}$

and $\quad m_{rate} = \rho \cdot Q$ $\quad m_{rate} = 0.648 \frac{\text{slug}}{\text{s}}$

Hence $\quad r_2 = \sqrt{\frac{W_m}{m_{rate} \cdot \omega^2}}$ $\quad r_2 = 0.169 \, \text{ft}$ $\quad r_2 = 2.03 \, \text{in}$

Also $\quad V_{n2} = V_{rb2} \cdot \sin(\beta_2)$ $\quad V_{n2} = 17.5 \frac{\text{ft}}{\text{s}}$

From continuity $\quad V_{n2} = \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2}$

Hence $\quad b_2 = \frac{Q}{2 \cdot \pi \cdot r_2 \cdot V_{n2}}$ $\quad b_2 = 0.0180 \, \text{ft}$ $\quad b_2 = 0.216 \, \text{in}$

Problem 10.4

Given: Dimensions of a centrifugal pump impeller:
$N = 750$ rpm

	Inlet ①	Outlet ②
Radius, r (mm)	175	500
Blade width, b (mm)	50	30
Blade angle, β (deg)	65	70

Find: (a) theoretical head, (b) mechanical power input for water flow rate of $Q = 0.75\ m^3/s$.

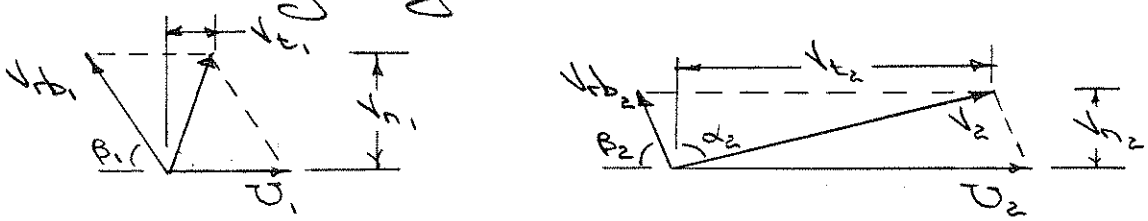
Solution:

Computing equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ (10.2b)

$H = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$ (10.2c)

Assume: (1) uniform flow at blade inlet and outlet
(2) flow enters and leaves tangent to the blade

Draw velocity diagrams:



From continuity, $V_n = \frac{Q}{2\pi r b} = V_{rb}\sin\beta \quad \therefore V_{rb} = \frac{V_n}{\sin\beta}$

From geometry, $V_t = U - V_{rb}\cos\beta = U - \frac{V_n}{\sin\beta}\cos\beta = U - \frac{Q}{2\pi r b}\cot\beta$

Substituting numerical values,

$\omega = 750 \frac{rev}{min} \times 2\pi \frac{rad}{rev} \times \frac{min}{60\,s} = 78.5\ rad/s$

$U_1 = \omega r_1 = 78.5 \frac{rad}{s} \times 0.175\,m = 13.7\ m/s \quad ; \quad U_2 = 39.3\ m/s$

$V_{t_1} = U_1 - \frac{Q}{2\pi r_1 b_1}\cot\beta_1 = 13.7 \frac{m}{s} - \frac{0.75}{2\pi}\frac{m^3}{s} \times \frac{1}{0.175\,m} \times \frac{\cot 65°}{0.05\,m} = 7.34\ m/s$

$V_{t_2} = 39.3 \frac{m}{s} - \frac{0.75}{2\pi}\frac{m^3}{s} \times \frac{1}{0.50\,m} \times \frac{\cot 70°}{0.03\,m} = 36.4\ m/s$

$H = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) = \frac{s^2}{9.81\,m}\left(39.3\frac{m}{s} \times 36.4\frac{m}{s} - 13.7\frac{m}{s} \times 7.34\frac{m}{s}\right) = 135\ m$ ← H

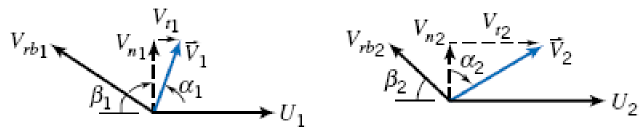
$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} = gH\rho Q = 9.81\frac{m}{s^2} \times 135\,m \times 999\frac{kg}{m^3} \times 0.75\frac{m^3}{s} \times \frac{N\,s^2}{kg\,m} \times \frac{W\,s}{N\,m}$

$\dot{W}_m = 994\ kW$ ← $\dot{W}_m$

Problem 10.5

10.5 Dimensions of a centrifugal pump impeller are

Parameter	Inlet, Section (1)	Outlet, Section (2)
Radius, r (in.)	3	9.75
Blade width, b (in.)	1.5	1.125
Blade angle, β (deg)	60	70



The pump is driven at 1250 rpm while pumping water. Calculate the theoretical head and mechanical power input if the flow rate is 1500 gpm.

Given: Geometry of centrifugal pump

Find: Theoretical head; Power input for given flow rate

Solution:

Basic equations:

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \qquad \text{(Eq. 10.2b)}$$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad \text{(Eq. 10.2c)}$$

The given or available data is

$\rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$ $\quad r_1 = 3 \cdot \text{in}$ $\quad r_2 = 9.75 \cdot \text{in}$ $\quad b_1 = 1.5 \cdot \text{in}$ $\quad b_2 = 1.125 \cdot \text{in}$

$\omega = 1250 \cdot \text{rpm}$ $\quad \beta_1 = 60 \cdot \text{deg}$ $\quad \beta_2 = 70 \cdot \text{deg}$ $\quad Q = 1500 \cdot \text{gpm}$ $\quad Q = 3.34 \frac{\text{ft}^3}{\text{s}}$

From continuity $\quad V_n = \frac{Q}{2 \cdot \pi \cdot r \cdot b} = V_{rb} \cdot \sin(\beta) \qquad V_{rb} = \frac{V_n}{\sin(\beta)}$

From geometry $\quad V_t = U - V_{rb} \cdot \cos(\beta) = U - \frac{V_n}{\sin(\beta)} \cdot \cos(\beta) = U - \frac{Q}{2 \cdot \pi \cdot r \cdot b} \cdot \cot(\beta)$

Using given data $\quad U_1 = \omega \cdot r_1 \qquad U_1 = 32.7 \frac{\text{ft}}{\text{s}} \qquad U_2 = \omega \cdot r_2 \qquad U_2 = 106.4 \frac{\text{ft}}{\text{s}}$

$V_{t1} = U_1 - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1} \cdot \cot(\beta_1) \qquad V_{t1} = 22.9 \frac{\text{ft}}{\text{s}}$

$V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2) \qquad V_{t2} = 104 \frac{\text{ft}}{\text{s}}$

The mass flow rate is $\quad m_{rate} = \rho \cdot Q \qquad m_{rate} = 6.48 \frac{\text{slug}}{\text{s}}$

Hence $\quad W_m = (U_2 \cdot V_{t2} - U_1 \cdot V_{t1}) \cdot m_{rate} \qquad W_m = 66728 \frac{\text{ft} \cdot \text{lbf}}{\text{s}} \qquad W_m = 121\,\text{hp}$

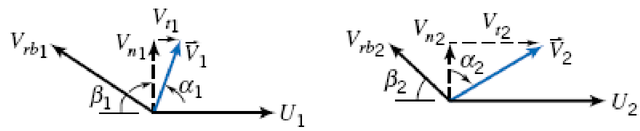
The head is $\quad H = \frac{W_m}{m_{rate} \cdot g} \qquad H = 320\,\text{ft}$

Problem 10.6

10.6 Dimensions of a centrifugal pump impeller are

Parameter	Inlet, Section (1)	Outlet, Section (2)
Radius, r (in.)	15	45
Blade width, b (in.)	4.75	3.25
Blade angle, β (deg)	40	60

The pump is driven at 575 rpm and the fluid is water. Calculate the theoretical head and mechanical power if the flow rate is 80,000 gpm.



Given: Geometry of centrifugal pump

Find: Theoretical head; Power input for given flow rate

Solution:

Basic equations:

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \qquad \text{(Eq. 10.2b)}$$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad \text{(Eq. 10.2c)}$$

The given or available data is

$\rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$ $\quad r_1 = 15 \cdot \text{in}$ $\quad r_2 = 45 \cdot \text{in}$ $\quad b_1 = 4.75 \cdot \text{in}$ $\quad b_2 = 3.25 \cdot \text{in}$

$\omega = 575 \cdot \text{rpm}$ $\quad \beta_1 = 40 \cdot \text{deg}$ $\quad \beta_2 = 60 \cdot \text{deg}$ $\quad Q = 80000 \cdot \text{gpm}$ $\quad Q = 178 \frac{\text{ft}^3}{\text{s}}$

From continuity $\quad V_n = \frac{Q}{2 \cdot \pi \cdot r \cdot b} = V_{rb} \cdot \sin(\beta)$ $\quad V_{rb} = \frac{V_n}{\sin(\beta)}$

From geometry $\quad V_t = U - V_{rb} \cdot \cos(\beta) = U - \frac{V_n}{\sin(\beta)} \cdot \cos(\beta) = U - \frac{Q}{2 \cdot \pi \cdot r \cdot b} \cdot \cot(\beta)$

Using given data $\quad U_1 = \omega \cdot r_1$ $\quad U_1 = 75.3 \frac{\text{ft}}{\text{s}}$ $\quad U_2 = \omega \cdot r_2$ $\quad U_2 = 226 \frac{\text{ft}}{\text{s}}$

$V_{t1} = U_1 - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1} \cdot \cot(\beta_1)$ $\quad V_{t1} = 6.94 \frac{\text{ft}}{\text{s}}$

$V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2)$ $\quad V_{t2} = 210 \frac{\text{ft}}{\text{s}}$

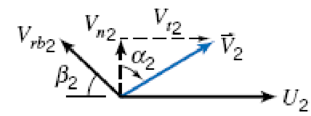
The mass flow rate is $\quad m_{rate} = \rho \cdot Q$ $\quad m_{rate} = 346 \frac{\text{slug}}{\text{s}}$

Hence $\quad W_m = (U_2 \cdot V_{t2} - U_1 \cdot V_{t1}) \cdot m_{rate}$ $\quad W_m = 1.62 \times 10^7 \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $\quad W_m = 2.94 \times 10^4 \text{hp}$

The head is $\quad H = \frac{W_m}{m_{rate} \cdot g}$ $\quad H = 1455 \text{ ft}$

Problem 10.7

10.7 A centrifugal water pump, with 15 cm diameter impeller and axial inlet flow, is driven at 1750 rpm. The impeller vanes are backward-curved ($\beta_2 = 65°$) and have axial width $b_2 = 2$ cm. For a volume flow rate of 225 m^3/hr determine the theoretical head rise and power input to the pump.



Given: Geometry of centrifugal pump

Find: Theoretical head; Power input for given flow rate

Solution:

Basic equations:

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \qquad \text{(Eq. 10.2b)}$$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad \text{(Eq. 10.2c)}$$

The given or available data is

$\rho = 1000 \cdot \frac{kg}{m^3}$ $\quad r_2 = 7.5 \cdot cm$ $\quad b_2 = 2 \cdot cm$ $\quad \beta_2 = 65 \cdot deg$

$\omega = 1750 \cdot rpm$ $\quad Q = 225 \cdot \frac{m^3}{hr}$ $\quad Q = 0.0625 \frac{m^3}{s}$

From continuity $\quad V_{n2} = \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2}$ $\quad V_{n2} = 6.63 \frac{m}{s}$

From geometry $\quad V_{t2} = U_2 - V_{rb2} \cdot \cos(\beta_2) = U_2 - \frac{V_{n2}}{\sin(\beta_2)} \cdot \cos(\beta_2)$

Using given data $\quad U_2 = \omega \cdot r_2$ $\quad U_2 = 13.7 \frac{m}{s}$

Hence $\quad V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2)$ $\quad V_{t2} = 10.7 \frac{m}{s}$ $\quad V_{t1} = 0$ (axial inlet)

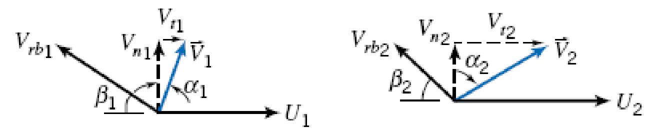
The mass flow rate is $\quad m_{rate} = \rho \cdot Q$ $\quad m_{rate} = 62.5 \frac{kg}{s}$

Hence $\quad W_m = U_2 \cdot V_{t2} \cdot m_{rate}$ $\quad W_m = 9.15 \, kW$

The head is $\quad H = \frac{W_m}{m_{rate} \cdot g}$ $\quad H = 14.9 \, m$

Problem 10.8

10.8 For the impeller of Problem 10.5, determine the rotational speed for which the tangential component of the inlet velocity is zero if the volume flow rate is 4000 gpm. Calculate the theoretical head and mechanical power input.



Given: Geometry of centrifugal pump

Find: Rotational speed for zero inlet velocity; Theoretical head; Power input

Solution:

Basic equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ (Eq. 10.2b)

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$$ (Eq. 10.2c)

The given or available data is

$\rho = 1.94 \cdot \frac{slug}{ft^3}$ $r_1 = 3 \cdot in$ $r_2 = 9.75 \cdot in$ $b_1 = 1.5 \cdot in$ $b_2 = 1.125 \cdot in$

$\beta_1 = 60 \cdot deg$ $\beta_2 = 70 \cdot deg$ $Q = 4000 \cdot gpm$ $Q = 8.91 \cdot \frac{ft^3}{s}$

From continuity $V_n = \frac{Q}{2 \cdot \pi \cdot r \cdot b} = V_{rb} \cdot \sin(\beta)$ $V_{rb} = \frac{V_n}{\sin(\beta)}$

From geometry $V_t = U - V_{rb} \cdot \cos(\beta) = U - \frac{V_n}{\sin(\beta)} \cdot \cos(\beta) = U - \frac{Q}{2 \cdot \pi \cdot r \cdot b} \cdot \cot(\beta)$

For $V_{t1} = 0$ we get $U_1 - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1} \cdot \cot(\beta_1) = 0$ or $\omega \cdot r_1 - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1} \cdot \cot(\beta_1) = 0$

Hence, solving for ω $\omega = \frac{Q}{2 \cdot \pi \cdot r_1^2 \cdot b_1} \cdot \cot(\beta_1)$ $\omega = 105 \frac{rad}{s}$ $\omega = 1001\, rpm$

We can now find U_2 $U_2 = \omega \cdot r_2$ $U_2 = 85.2 \cdot \frac{ft}{s}$

$V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2)$ $V_{t2} = 78.4 \cdot \frac{ft}{s}$

The mass flow rate is $m_{rate} = \rho \cdot Q$ $m_{rate} = 17.3 \cdot \frac{slug}{s}$

Hence Eq 10.2b becomes $W_m = U_2 \cdot V_{t2} \cdot m_{rate}$ $W_m = 1.15 \times 10^5 \cdot \frac{ft \cdot lbf}{s}$ $W_m = 210 \cdot hp$

The head is $H = \frac{W_m}{m_{rate} \cdot g}$ $H = 208 \cdot ft$

Problem 10.9

[2]

10.9 Consider the geometry of the idealized centrifugal pump described in Problem 10.11. Draw inlet and outlet velocity diagrams assuming b = constant. Calculate the inlet blade angles required for "shockless" entry flow at the design flow rate. Evaluate the theoretical power input to the pump at the design flow rate.

Given: Geometry of centrifugal pump

Find: Draw inlet and exit velocity diagrams; Inlet blade angle; Power

Solution:

Basic equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ $\qquad V_n = \dfrac{Q}{2\cdot\pi\cdot r\cdot b}$

The given or available data is

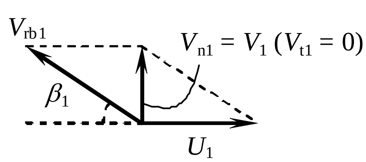
$R_1 = 1\cdot\text{in}$ $\qquad R_2 = 7.5\cdot\text{in}$ $\qquad b_2 = 0.375\cdot\text{in}$ $\qquad \omega = 2000\cdot\text{rpm}$

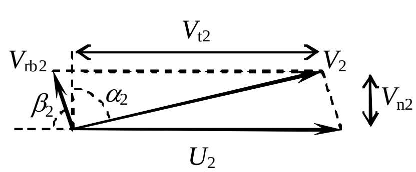
$\rho = 1.94\cdot\dfrac{\text{slug}}{\text{ft}^3}$ $\qquad Q = 800\cdot\text{gpm}$ $\qquad Q = 1.8\cdot\dfrac{\text{ft}^3}{\text{s}}$ $\qquad \beta_2 = 75\cdot\text{deg}$

$U_1 = \omega\cdot R_1$ $\qquad U_1 = 17.5\cdot\dfrac{\text{ft}}{\text{s}}$ $\qquad U_2 = \omega\cdot R_2$ $\qquad U_2 = 131\cdot\dfrac{\text{ft}}{\text{s}}$

$V_{n2} = \dfrac{Q}{2\cdot\pi\cdot R_2\cdot b_2}$ $\qquad V_{n2} = 14.5\cdot\dfrac{\text{ft}}{\text{s}}$ $\qquad V_{n1} = \dfrac{R_2}{R_1}\cdot V_{n2}$ $\qquad V_{n1} = 109\cdot\dfrac{\text{ft}}{\text{s}}$

Velocity diagrams:





Then $\qquad \beta_1 = \text{atan}\left(\dfrac{V_{n1}}{U_1}\right)$ $\qquad \beta_1 = 80.9\cdot\text{deg}$ $\qquad$ (Essentially radial entry)

From geometry $\qquad V_{t1} = U_1 - V_{n1}\cdot\cos(\beta_1)$ $\qquad V_{t1} = 0.2198\cdot\dfrac{\text{ft}}{\text{s}}$ $\qquad V_{t2} = U_2 - V_{n2}\cdot\cos(\beta_2)$ $\qquad V_{t2} = 127.1\cdot\dfrac{\text{ft}}{\text{s}}$

Then $\qquad W_m = (U_2\cdot V_{t2} - U_1\cdot V_{t1})\cdot\rho\cdot Q$ $\qquad W_m = 5.75\times 10^4\cdot\dfrac{\text{ft}\cdot\text{lbf}}{\text{s}}$ $\qquad W_m = 105\cdot\text{hp}$

Note: In earlier printings the flow rate was given as 8000 gpm not 800 gpm; water at 1089 ft/s would be quite dangerous!

Problem 10.10

Given: Dimensions of a centrifugal pump impeller:
$N = 750$ rpm

	Inlet ①	Outlet ②
Radius, r (mm)	175	500
Blade width, b (mm)	50	30
Blade angle, β (deg)	65	70

Find: (a) volume flow rate Q for which $V_{t_1} = 0$, (b) theoretical head, and (c) mechanical power input

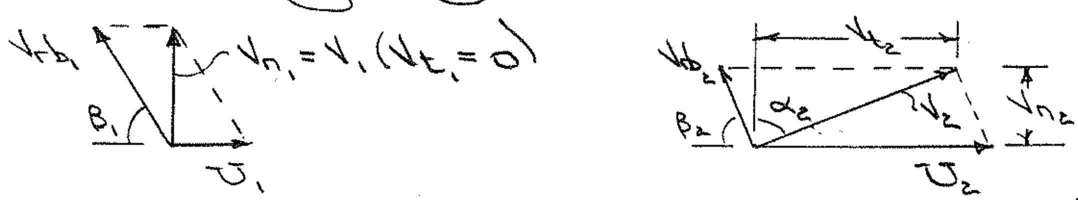
Solution:

Computing equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ (10.2b) [V_{t_1} term crossed out: 0(3)]

$H = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$ (10.2c) [V_{t_1} term crossed out: 0(3)]

Assume: (1) uniform flow at blade inlet and outlet
(2) flow enters and leaves tangent to the blade
(3) $V_{t_1} = 0$ (given)

Draw velocity diagrams:



From continuity, $V_n = \frac{Q}{2\pi r b} = V_{rb}\sin\beta \quad \therefore V_{rb} = \frac{V_n}{\sin\beta}$

From geometry, $V_t = U - V_{rb}\cos\beta = U - \frac{V_n}{\sin\beta}\cos\beta = U - \frac{Q}{2\pi r b}\cot\beta$

For $V_{t_1} = 0$, then $U_1 - \frac{Q}{2\pi r_1 b_1}\cot\beta_1 = 0$ and $Q = \frac{2\pi r_1 b_1 U_1}{\cot\beta_1}$

Substituting numerical values

$U_1 = \omega_1 r_1 = 750 \frac{\text{rev}}{\text{min}} \times 2\pi \frac{\text{rad}}{\text{rev}} \times \frac{\text{min}}{60\,\text{s}} \times 0.175\,\text{m} = 13.7\,\text{m/s}; \; U_2 = 39.3\,\text{m/s}$

$Q = \frac{2\pi}{\cot 65°} \times 0.175\,\text{m} \times 0.050\,\text{m} \times 13.7\,\frac{\text{m}}{\text{s}} = 1.62\,\text{m}^3/\text{s}$ ← Q

$V_{t_2} = U_2 - \frac{Q}{2\pi r_2 b_2}\cot\beta_2 = 39.3\,\frac{\text{m}}{\text{s}} - 1.62\,\frac{\text{m}^3}{\text{s}} \times \frac{1}{2\pi} \times \frac{1}{0.50\,\text{m}} \times \frac{\cot 70°}{0.03\,\text{m}} = 33.0\,\text{m/s}$

$H = \frac{1}{g} U_2 V_{t_2} = \frac{\text{s}^2}{9.81\,\text{m}} \times 39.3\,\frac{\text{m}}{\text{s}} \times 33.0\,\frac{\text{m}}{\text{s}} = 132\,\text{m}$ ← H

$\dot{W}_m = \dot{m} U_2 V_{t_2} = \rho Q g H = 999\,\frac{\text{kg}}{\text{m}^3} \times 1.62\,\frac{\text{m}^3}{\text{s}} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times 132\,\text{m} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} \times \frac{\text{W}\cdot\text{s}}{\text{N}\cdot\text{m}}$

$\dot{W}_m = 2{,}100\,\text{kW}$ ←

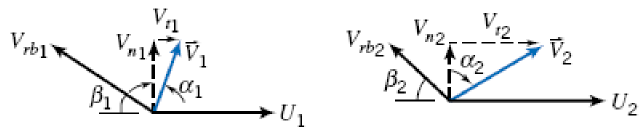
Problem 10.11

[3]

10.11 Consider a centrifugal pump whose geometry and flow conditions are

Impeller inlet radius, R_1	1 in.
Impeller outlet radius, R_2	7.5 in.
Impeller outlet width, b_2	0.375 in.
Design speed, N	2000 rpm
Design flow rate, Q	8000 gpm
Backward-curved vanes (outlet blade angle), β_2	75°
Required flow rate range	50–150% of design

Assume ideal pump behavior with 100 percent efficiency. Find the shutoff head. Calculate the absolute and relative discharge velocities, the total head, and the theoretical power required at the design flow rate.



Note: Earlier printings had 8000 gpm; it is actually 800 gpm!

Given: Geometry of centrifugal pump

Find: Shutoff head; Absolute and relative exit velocitiesTheoretical head; Power input

Solution:

Basic equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ (Eq. 10.2b)

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$$ (Eq. 10.2c)

The given or available data is

$\rho = 1.94\cdot\frac{slug}{ft^3}$ $R_1 = 1\cdot in$ $R_2 = 7.5\cdot in$ $b_2 = 0.375\cdot in$

$\omega = 2000\cdot rpm$ $\beta_2 = 75\cdot deg$ $Q = 800\cdot gpm$ $Q = 1.8\cdot\frac{ft^3}{s}$

At the exit $U_2 = \omega\cdot R_2$ $U_2 = 131\cdot\frac{ft}{s}$

At shutoff $V_{t2} = U_2$ $V_{t2} = 131\cdot\frac{ft}{s}$ $H_0 = \frac{1}{g}\cdot(U_2\cdot V_{t2})$ $H_0 = 533\cdot ft$

At design. from continuity $V_{n2} = \frac{Q}{2\cdot\pi\cdot R_2\cdot b_2}$ $V_{n2} = 15\cdot\frac{ft}{s}$

From the velocity diagram $V_{n2} = V_{rb2}\cdot\sin(\beta_2)$ $V_{rb2} = \frac{V_{n2}}{\sin(\beta_2)}$ $V_{rb2} = 15.0\cdot\frac{ft}{s}$

$V_{t2} = U_2 - V_{n2}\cdot\cot(\beta_2)$ $V_{t2} = 127.0\cdot\frac{ft}{s}$

Hence we obtain $V_2 = \sqrt{V_{n2}^2 + V_{t2}^2}$ $V_2 = 128\cdot\frac{ft}{s}$

with (see sketch above) $\alpha_2 = \operatorname{atan}\left(\frac{V_{t2}}{V_{n2}}\right)$ $\alpha_2 = 83.5\cdot deg$

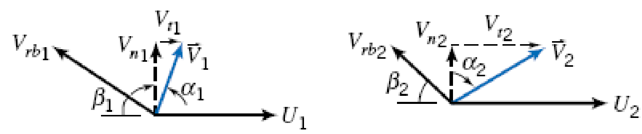
For $V_{t1} = 0$ we get $W_m = U_2\cdot V_{t2}\cdot\rho\cdot Q$ $W_m = 5.75\times 10^4\cdot\frac{ft\cdot lbf}{s}$ $W_m = 105\cdot hp$

$H = \frac{W_m}{\rho\cdot Q\cdot g}$ $H = 517\cdot ft$

Problem 10.12

[2]

10.12 For the impeller of Problem 10.6, determine the inlet blade angle for which the tangential component of the inlet velocity is zero if the volume flow rate is 125,000 gpm. Calculate the theoretical head and mechanical power input.



Given: Geometry of centrifugal pump

Find: Inlet blade angle for no tangential inlet velocity at 125,000 gpm; Head; Power

Solution:

Basic equations:

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \qquad \text{(Eq. 10.2b)}$$

$$H = \frac{\dot{W}_m}{\dot{m} g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad \text{(Eq. 10.2c)}$$

The given or available data is

$\rho = 1.94\cdot\frac{\text{slug}}{\text{ft}^3}$ $\quad r_1 = 15\cdot\text{in}$ $\quad r_2 = 45\cdot\text{in}$ $\quad b_1 = 4.75\cdot\text{in}$ $\quad b_2 = 3.25\cdot\text{in}$

$\omega = 575\cdot\text{rpm}$ $\quad \beta_2 = 60\cdot\text{deg}$ $\quad Q = 125000\cdot\text{gpm}$ $\quad Q = 279\frac{\text{ft}^3}{\text{s}}$

From continuity $\qquad V_n = \frac{Q}{2\cdot\pi\cdot r\cdot b} = V_{rb}\cdot\sin(\beta) \qquad V_{rb} = \frac{V_n}{\sin(\beta)}$

From geometry $\qquad V_t = U - V_{rb}\cdot\cos(\beta) = U - \frac{V_n}{\sin(\beta)}\cdot\cos(\beta) = U - \frac{Q}{2\cdot\pi\cdot r\cdot b}\cdot\cot(\beta)$

For $V_{t1} = 0$ we obtain $\qquad U_1 - \frac{Q}{2\cdot\pi\cdot r_1\cdot b_1}\cdot\cot(\beta_1) = 0 \qquad$ or $\qquad \cot(\beta_1) = \frac{2\cdot\pi\cdot r_1\cdot b_1\cdot U_1}{Q}$

Using given data $\qquad U_1 = \omega\cdot r_1 \qquad U_1 = 75.3\cdot\frac{\text{ft}}{\text{s}}$

Hence $\qquad \beta_1 = \text{acot}\left(\frac{2\cdot\pi\cdot r_1\cdot b_1\cdot U_1}{Q}\right) \qquad \beta_1 = 50\,\text{deg}$

Also $\qquad U_2 = \omega\cdot r_2 \qquad U_2 = 226\cdot\frac{\text{ft}}{\text{s}}$

$$V_{t2} = U_2 - \frac{Q}{2\cdot\pi\cdot r_2\cdot b_2}\cdot\cot(\beta_2) \qquad V_{t2} = 201\cdot\frac{\text{ft}}{\text{s}}$$

The mass flow rate is $\qquad m_{rate} = \rho\cdot Q \qquad m_{rate} = 540\cdot\frac{\text{slug}}{\text{s}}$

Hence $\qquad W_m = (U_2\cdot V_{t2} - U_1\cdot V_{t1})\cdot m_{rate} \qquad W_m = 2.45\times 10^7\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{s}} \qquad W_m = 44497\cdot\text{hp}$

The head is $\qquad H = \frac{W_m}{m_{rate}\cdot g} \qquad H = 1408\cdot\text{ft}$

Problem 10.13

Given: Impeller dimensions of Example Problem 10.1: $Q = 150$ gpm

$D_1 = 1.25$ in.

$b = 0.383$ in.

$N = 3450$ rpm

Find: Construct velocity diagram for shockless flow at the impeller inlet.

Investigate effects on inlet flow angle of:
(a) variations in impeller width
(b) variations in inlet swirl velocity

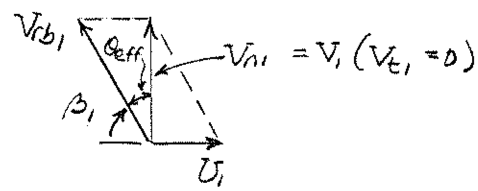
<u>Solution</u>: $Q = 150\,\frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} = 0.334\ ft^3/sec$; $r_1 = 0.0521$ ft

$b = 0.0319$ ft ; $\omega = 3450\,\frac{rev}{min} \times 2\pi\,\frac{rad}{rev} \times \frac{min}{60\,s} = 361$ rad/s

From continuity, $V_{n1} = \frac{Q}{2\pi r_1 b_1} = \frac{1}{2\pi} \times 0.334\,\frac{ft^3}{s} \times \frac{1}{0.0521\,ft} \times \frac{1}{0.0319\,ft} = 32.0$ ft/s

$U_1 = \omega r_1 = 361\,\frac{rad}{s} \times 0.0521$ ft $= 18.8$ ft/s

$\beta_1 = \tan^{-1}\frac{V_{n1}}{U_1} = \tan^{-1}\left(\frac{32.0}{18.8}\right) = 59.6°$



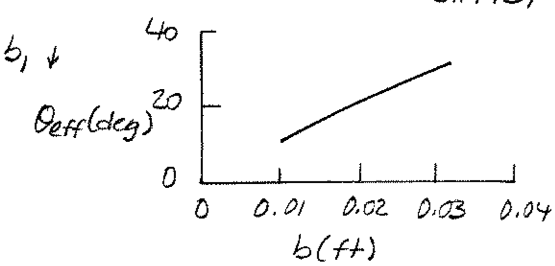
Thus for radial vanes,

$\theta_{eff} = \frac{\pi}{2} - \beta_1 = 90° - 59.6° = 30.4°$ ← θ_{eff}

To change θ_{eff}: (a) Vary b with no inlet swirl: $V_1 = V_{n1} = \frac{Q}{2\pi r_1 b_1}$

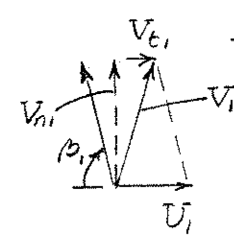
$\beta_1 = \tan^{-1}\frac{Q}{2\pi r_1 b_1 U_1}$ so $\beta_1 \uparrow$ as $b_1 \downarrow$

$\theta_{eff} = 90° - \beta_1$



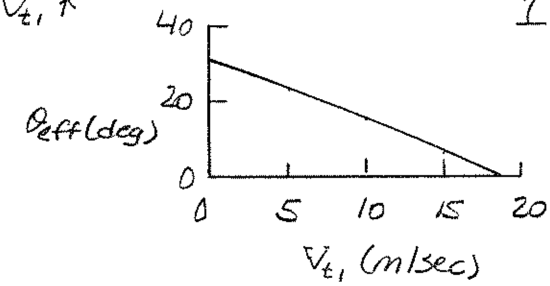
← b

(b) Vary inlet swirl (V_{t1}) with $b = 0.0319$ ft:



$\beta_1 = \tan^{-1}\frac{V_{n1}}{U_1 - V_{t1}}$ so $\beta_1 \uparrow$ as $V_{t1} \uparrow$

$\theta_{eff} = 90° - \beta_1$



← V_{t1}

Problem 10.14

10.14 A centrifugal pump runs at 1750 rpm while pumping water at a rate of 50 L/s. The water enters axially, and leaves tangential to the impeller blades. The impeller exit diameter and width are 300 mm and 10 mm, respectively. If the pump requires 45 kW, and is 75 percent efficient, estimate the exit angle of the impeller blades.

Given: Data on a centrifugal pump

Find: Estimate exit angle of impeller blades

Solution:

The given or available data is $\rho = 999 \cdot \frac{kg}{m^3}$ $\quad Q = 50 \cdot \frac{L}{s}$ $\quad W_{in} = 45 \cdot kW$ $\quad \eta = 75 \cdot \%$

$\omega = 1750 \cdot rpm$ $\quad b_2 = 10 \cdot mm$ $\quad D = 300 \cdot mm$

The governing equation (derived directly from the Euler turbomachine equation) is $W_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ (10.2b)

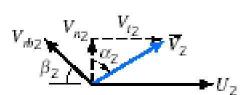
For an axial inlet $V_{t1} = 0$ hence $V_{t2} = \frac{W_m}{U_2 \cdot \rho \cdot Q}$

We have $U_2 = \frac{D}{2} \cdot \omega$ $\quad U_2 = 27.5 \frac{m}{s}$ and $W_m = \eta \cdot W_{in}$ $\quad W_m = 33.8\,kW$

Hence $V_{t2} = \frac{W_m}{U_2 \cdot \rho \cdot Q}$ $\quad V_{t2} = 24.6 \frac{m}{s}$

From continuity $V_{n2} = \frac{Q}{\pi \cdot D \cdot b_2}$ $\quad V_{n2} = 5.31 \frac{m}{s}$

With the exit velocities determined, β can be determined from exit geometry



$$\tan(\beta) = \frac{V_{n2}}{U_2 - V_{t2}} \quad \text{or} \quad \beta = \operatorname{atan}\left(\frac{V_{n2}}{U_2 - V_{t2}}\right) \qquad \beta = 61.3\,deg$$

Problem 10.15

Given: Centrifugal water-pump designed for $N = 1300$ rpm; $\eta = 0.75$, $Q = 35$ L/s.

$r_1 = 100$ mm $r_2 = 175$ mm

$b_1 = 10$ mm $b_2 = 7.5$ mm

$\beta_2 = 40°$

(a) Draw the inlet and outlet velocity diagrams
(b) Find inlet blade angle so $V_{t_1} = 0$
(c) Determine the outlet absolute flow angle (measured w.r.t. normal).
(d) Evaluate hydraulic power and head.

Solution: Apply continuity and the Euler turbomachine equation.

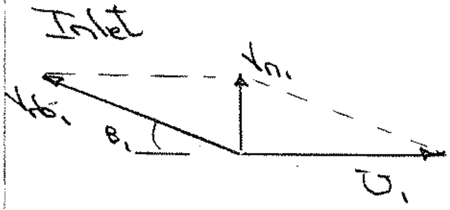
Computing equations: $V_n = \frac{Q}{2\pi r b}$ $\dot{W}_m = \rho Q (U_2 V_{t_2} - U_1 V_{t_1})$

$\omega = 1300 \frac{\text{rev}}{\text{min}} \times 2\pi \frac{\text{rad}}{\text{rev}} \times \frac{\text{min}}{60\,\text{s}} = 136$ rad/s ; $U_1 = 13.6$ m/s, $U_2 = 23.8$ m/s

$V_{n_1} = \frac{Q}{2\pi r_1 b_1} = \frac{1}{2\pi} \times 35 \frac{\text{L}}{\text{s}} \times \frac{\text{m}^3}{10^3\,\text{L}} \times \frac{1}{0.1\,\text{m}} \times \frac{1}{0.01\,\text{m}} = 5.57$ m/s.

$V_{n_2} = \frac{r_1 b_1}{r_2 b_2} V_{n_1} = \frac{100}{175} \times \frac{10}{7.5} \times 5.57$ m/s $= 4.24$ m/s.

Inlet

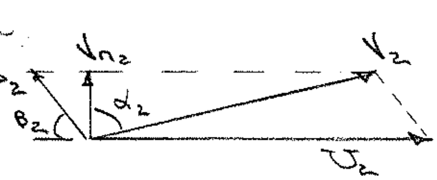


$\tan\beta_1 = \frac{V_{n_1}}{U_1}$

$\beta_1 = \tan^{-1}\left(\frac{5.57}{13.6}\right)$

$\beta_1 = 22.3°$ ——— β_1

Outlet



From the outlet diagram, $V_{t_2} = U_2 - V_{n_2}\cot\beta_2 = 23.8 \frac{\text{m}}{\text{s}} - 4.24 \frac{\text{m}}{\text{s}} \times \frac{1}{\tan 40°}$

$V_{t_2} = 18.8$ m/s

$\alpha_2 = \tan^{-1} \frac{V_{t_2}}{V_{n_2}} = \tan^{-1}\left(\frac{18.8}{4.24}\right) = 77.3°$ ——— α_2

$\dot{W}_m = \rho Q (U_2 V_{t_2} - U_1 V_{t_1}) = 999 \frac{\text{kg}}{\text{m}^3} \times 35 \frac{\text{L}}{\text{s}} \times \frac{\text{m}^3}{10^3\,\text{L}} \left[23.8 \frac{\text{m}}{\text{s}} \times 18.8 \frac{\text{m}}{\text{s}} - 0\right] \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} \times 0.75$

$\dot{W}_m = 11.7$ kW ——— $\dot{W}_m$

$H = \frac{\dot{W}_m}{\rho g Q} = \frac{U_2 V_{t_2} - U_1 V_{t_1}}{g} = 23.8 \frac{\text{m}}{\text{s}} \times 18.8 \frac{\text{m}}{\text{s}} \times \frac{\text{s}}{9.81\,\text{m}} \times 0.75 = 34.2$ m ——— H

Problem 10.16

10.16 Repeat the analysis for determining the optimum speed for an impulse turbine of Example 10.5, using the Euler turbomachine equation.

Given: Impulse turbibe

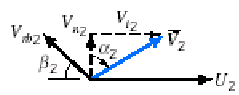
Find: Optimum speed using the Euler turbomachine equation

Solution:

The governing equation is the Euler turbomachine equation

$$T_{shaft} = (r_2 V_{t_2} - r_1 V_{t_1})\dot{m} \qquad (10.1c)$$

In terms of the notation of Example 10.5, for a stationary CV



$r_1 = r_2 = R$ $\quad U_1 = U_2 = U$ $\quad V_{t1} = V - U$ $\quad V_{t2} = (V - U)\cdot\cos(\theta)$ and $m_{flow} = \rho\cdot Q$

Hence $\quad T_{shaft} = [R\cdot(V - U)\cdot\cos(\theta) - R\cdot(V - U)]\cdot\rho\cdot Q$ $\quad T_{out} = T_{shaft} = \rho\cdot Q\cdot R\cdot(V - U)\cdot(1 - \cos(\theta))$

The power is $\quad W_{out} = \omega\cdot T_{out} = \rho\cdot Q\cdot R\cdot\omega\cdot(V - U)\cdot(1 - \cos(\theta))$ $\quad W_{out} = \rho\cdot Q\cdot U\cdot(V - U)\cdot(1 - \cos(\theta))$

These results are identical to those of Example 10.5. The proof that maximum power is when $U = V/2$ is hence also the same and will not be repeated here.

Problem 10.17

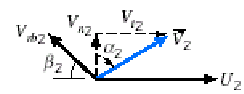
10.17 A centrifugal water pump designed to operate at 1200 rpm has dimensions

Parameter	Inlet	Outlet
Radius, *r* (mm)	90	150
Blade width, *b* (mm)	10	7.5
Blade angle, *β* (deg)	25	45

Determine the flow rate at which the entering velocity has no tangential component. Draw the outlet velocity diagram, and determine the outlet absolute flow angle (measured relative to the normal direction) at this flow rate. Evaluate the hydraulic power delivered by the pump if its efficiency is 70 percent. Determine the head developed by the pump.

Given: Data on a centrifugal pump

Find: Flow rate for zero inlet tangential velocity; outlet flow angle; power; head developed



Solution:

The given or available data is $\rho = 999\cdot\frac{kg}{m^3}$ $\omega = 1200\cdot rpm$ $\eta = 70\cdot\%$

$r_1 = 90\cdot mm$ $b_1 = 10\cdot mm$ $\beta_1 = 25\cdot deg$ $r_2 = 150\cdot mm$ $b_2 = 7.5\cdot mm$ $\beta_2 = 45\cdot deg$

The governing equations (derived directly from the Euler turbomachine equation) are

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \qquad (10.2b)$$

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad (10.2c)$$

We also have from geometry $\alpha_2 = \operatorname{atan}\left(\frac{V_{t2}}{V_{n2}}\right)$ (1)

From geometry $V_{t1} = 0 = U_1 - V_{rb1}\cdot\cos(\beta_1) = r_1\cdot\omega\cdot -\frac{V_{n1}}{\sin(\beta_1)}\cdot\cos(\beta_1)$

and from continuity $V_{n1} = \frac{Q}{2\cdot\pi\cdot r_1\cdot b_1}$

Hence $r_1\cdot\omega - \frac{Q}{2\cdot\pi\cdot r_1\cdot b_1\cdot\tan(\beta_1)} = 0$ $Q = 2\cdot\pi\cdot r_1^{\,2}\cdot b_1\cdot\omega\cdot\tan(\beta_1)$ $Q = 29.8\frac{L}{s}$ $Q = 0.0298\frac{m^3}{s}$

The power, head and absolute angle α at the exit are obtained from direct computation using Eqs. 10.2b, 10.2c, and 1 above

$U_1 = r_1\cdot\omega$ $U_1 = 11.3\frac{m}{s}$ $U_2 = r_2\cdot\omega$ $U_2 = 18.8\frac{m}{s}$ $V_{t1} = 0\cdot\frac{m}{s}$

From geometry $V_{t2} = U_2 - V_{rb2}\cdot\cos(\beta_2) = r_2\cdot\omega\cdot -\frac{V_{n2}}{\sin(\beta_2)}\cdot\cos(\beta_2)$

and from continuity $V_{n2} = \frac{Q}{2\cdot\pi\cdot r_2\cdot b_2}$ $V_{n2} = 4.22\frac{m}{s}$

Hence $V_{t2} = r_2 \cdot \omega - \frac{V_{n2}}{\tan(\beta_2)}$ $V_{t2} = 14.6 \frac{m}{s}$

Using these results in Eq. 1 $\alpha_2 = \text{atan}\left(\frac{V_{t2}}{V_{n2}}\right)$ $\alpha_2 = 73.9\,\text{deg}$

Using them in Eq. 10.2b $W_m = (U_2 \cdot V_{t2} - U_1 \cdot V_{t1}) \cdot \rho \cdot Q$ $W_m = 8.22\,\text{kW}$

Using them in Eq. 10.2c $H = \frac{1}{g} \cdot (U_2 \cdot V_{t2} - U_1 \cdot V_{t1})$ $H = 28.1\,\text{m}$

This is the power and head assuming no inefficiency; with $\eta = 70\%$, we have (from Eq. 10.8c)

$W_h = \eta \cdot W_m$ $W_h = 5.75\,\text{kW}$

$H_p = \eta \cdot H$ $H_p = 19.7\,\text{m}$

(This last result can also be obtained from Eq. 10.8a $W_h = \rho \cdot Q \cdot g \cdot H_p$)

Problem 10.18

[1]

10.18 Gasoline is pumped by a centrifugal pump. When the flow rate is 0.025 m^3/s, the pump requires 15 kW input, and its efficiency is 85 percent. Calculate the pressure rise produced by the pump. Express this result as (a) ft of water and (b) ft of gasoline.

Given: Data on centrifugal pump

Find: Pressure rise; Express as ft of water and gasoline

Solution:

Basic equations: $\eta = \frac{\rho \cdot Q \cdot g \cdot H}{W_m}$

The given or available data is $\rho_w = 1000 \cdot \frac{kg}{m^3}$ $Q = 0.025 \cdot \frac{m^3}{s}$ $W_m = 15 \cdot kW$ $\eta = 85 \cdot \%$

Solving for H $H = \frac{\eta \cdot W_m}{\rho_w \cdot Q \cdot g}$ $H = 52.0\,m$ $H = 171\,ft$

For gasoline, from Table A.2 $SG = 0.72$ $H_g = \frac{\eta \cdot W_m}{SG \cdot \rho_w \cdot Q \cdot g}$ $H_g = 72.2\,m$ $H_g = 237\,ft$

Problem 10.19

[3]

10.19 A centrifugal pump designed to deliver water at 30 L/s has dimensions

Parameter	Inlet	Outlet
Radius, r (mm)	75	150
Blade width, b (mm)	7.5	6.25
Blade angle, β (deg)	25	40

Draw the inlet velocity diagram. Determine the design speed if the entering velocity has no tangential component. Draw the outlet velocity diagram. Determine the outlet absolute flow angle (measured relative to the normal direction). Evaluate the theoretical head developed by the pump. Estimate the minimum mechanical power delivered to the pump.

Given:

Geometry of centrifugal pump

Find:

Draw inlet velocity diagram; Design speed for no inlet tangential velocity; Outlet angle; Head; Power

Solution:

Basic equations:

$$\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m} \qquad \text{(Eq. 10.2b)}$$

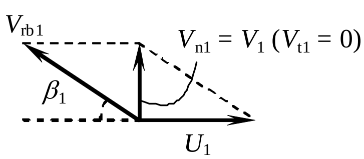
$$H = \frac{\dot{W}_m}{\dot{m} g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad \text{(Eq. 10.2c)}$$

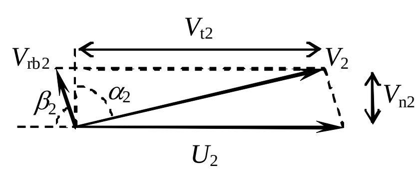
The given or available data is

$r_1 = 75 \cdot \text{mm}$ $\quad r_2 = 150 \cdot \text{mm}$ $\quad b_1 = 7.5 \cdot \text{mm}$ $\quad b_2 = 6.25 \cdot \text{mm}$ $\quad \beta_1 = 25 \cdot \text{deg}$ $\quad \beta_2 = 40 \cdot \text{deg}$

$\rho = 1000 \cdot \frac{\text{kg}}{\text{m}^3}$ $\quad Q = 30 \cdot \frac{\text{L}}{\text{s}}$ $\quad Q = 0.030 \cdot \frac{\text{m}^3}{\text{s}}$

Velocity diagrams:





From continuity $\quad V_n = \frac{Q}{2 \cdot \pi \cdot r \cdot b} = V_{rb} \cdot \sin(\beta)$ $\quad V_{rb} = \frac{V_n}{\sin(\beta)}$ $\quad \frac{V_{n1}}{V_{n2}} = \frac{A_1}{A_2} = \frac{r_1 \cdot b_1}{r_2 \cdot b_2}$

From geometry $\quad V_t = U - V_{rb} \cdot \cos(\beta) = U - \frac{V_n}{\sin(\beta)} \cdot \cos(\beta) = U - \frac{Q}{2 \cdot \pi \cdot r \cdot b} \cdot \cot(\beta)$

For $V_{t1} = 0$ we obtain $\quad U_1 - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1} \cdot \cot(\beta_1) = 0$ $\quad$ or $\quad \omega \cdot r_1 - \frac{Q}{2 \cdot \pi \cdot r_1 \cdot b_1} \cdot \cot(\beta_1) = 0$

Solving for ω $\quad \omega = \frac{Q}{2 \cdot \pi \cdot r_1^{\,2} \cdot b_1} \cdot \cot(\beta_1)$ $\quad \omega = 243 \frac{\text{rad}}{\text{s}}$ $\quad \omega = 2318\,\text{rpm}$

Hence $\quad U_1 = \omega \cdot r_1$ $\quad U_1 = 18.2 \frac{\text{m}}{\text{s}}$ $\quad U_2 = \omega \cdot r_2$ $\quad U_2 = 36.4 \frac{\text{m}}{\text{s}}$

$V_{n2} = \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2}$ $\quad V_{n2} = 5.09 \frac{\text{m}}{\text{s}}$ $\quad V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2)$ $\quad V_{t2} = 30.3 \frac{\text{m}}{\text{s}}$

From the sketch $\alpha_2 = \operatorname{atan}\left(\frac{V_{t2}}{V_{n2}}\right)$ $\alpha_2 = 80.5\,\text{deg}$

Hence $W_m = U_2 \cdot V_{t2} \cdot \rho \cdot Q$ $W_m = 33.1 \cdot \text{kW}$

The head is $H = \frac{W_m}{\rho \cdot Q \cdot g}$ $H = 113\,\text{m}$

Problem 10.20

Given: Centrifugal pump operating with water, at shutoff.

Actual head rise is 70 percent of theoretical.

Find: (a) Prepare log-log plot of impeller radius versus theoretical head rise, with standard motor speeds as parameters.
(b) Explain how this plot might be used for preliminary design.

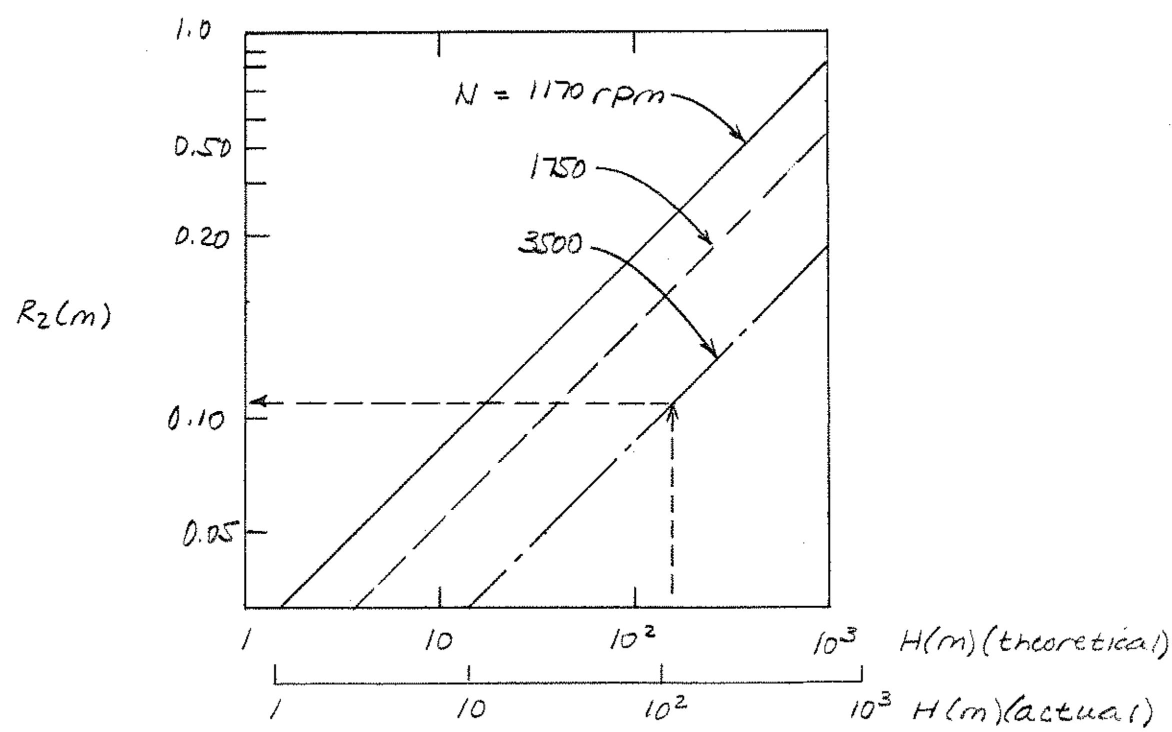
Solution: Apply the Euler turbomachine equation.

Computing equation: $H = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1})$ ($V_{t_1} \approx 0$ (2))

Assumptions: (1) No through flow, (2) Neglect V_{t_1}

Then $H = \frac{1}{g}(\omega R_2 \omega R_2) = \frac{\omega^2 R_2^2}{g}$ or $\log H = 2\log\omega + 2\log R_2 - \log g$

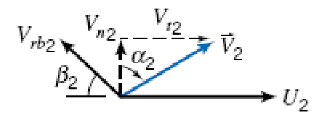
These will be straight lines on a plot of $\log R_2$ vs. $\log H$ (at constant ω):



For a given application, enter the abscissa with the desired head, move up to the desired driver speed, then move left to the ordinate and read the required impeller radius. The example (--- line) illustrates.

Problem 10.21

10.21 In the water pump of Problem 10.7, the pump casing acts as a diffuser, which converts 60 percent of the absolute velocity head at the impeller outlet to static pressure rise. The head loss through the pump suction and discharge channels is 0.75 times the radial component of velocity head leaving the impeller. Estimate the volume flow rate, head rise, power input, and pump efficiency at the maximum efficiency point. Assume the torque to overcome bearing, seal, and spin losses is 10 percent of the ideal torque at $Q = 0.065\ m^3/s$.



Given:

Geometry of centrifugal pump with diffuser casing

Find:

Flow rate; Theoretical head; Power; Pump efficiency at maximum efficiency point

Solution:

Basic equations: $\dot{W}_m = (U_2 V_{t_2} - U_1 V_{t_1})\dot{m}$ (Eq. 10.2b)

$$H = \frac{\dot{W}_m}{\dot{m} g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \quad \text{(Eq. 10.2c)}$$

The given or available data is

$\rho = 1000 \cdot \frac{kg}{m^3}$ $\quad r_2 = 7.5 \cdot cm$ $\quad b_2 = 2 \cdot cm$ $\quad \beta_2 = 65 \cdot deg$

$\omega = 1750 \cdot rpm$ $\quad \omega = 183 \cdot \frac{rad}{s}$

Using given data $\quad U_2 = \omega \cdot r_2 \quad U_2 = 13.7 \frac{m}{s}$

Illustrate the procedure with $\quad Q = 0.065 \cdot \frac{m^3}{s}$

From continuity $\quad V_{n2} = \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \quad V_{n2} = 6.9 \frac{m}{s}$

From geometry $\quad V_{t2} = U_2 - V_{rb2} \cdot \cos(\beta_2) = U_2 - \frac{V_{n2}}{\sin(\beta_2)} \cdot \cos(\beta_2)$

Hence $\quad V_{t2} = U_2 - \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2} \cdot \cot(\beta_2) \quad V_{t2} = 10.5 \frac{m}{s} \quad V_{t1} = 0$ (axial inlet)

$V_2 = \sqrt{V_{n2}^2 + V_{t2}^2} \quad V_2 = 12.6 \frac{m}{s}$

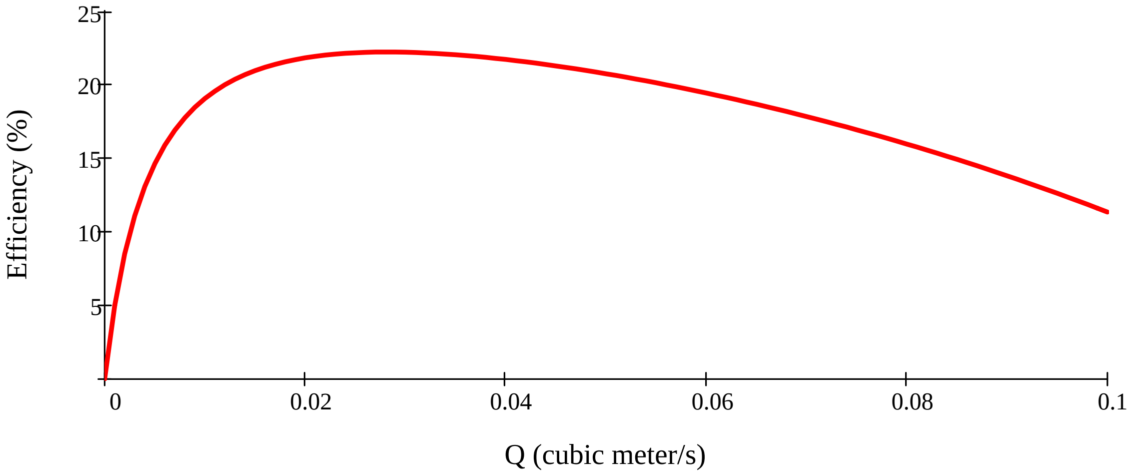
$H_{ideal} = \frac{U_2 \cdot V_{t2}}{g} \quad H_{ideal} = 14.8 \cdot m$

$$T_{friction} = 10 \cdot \% \cdot \frac{W_{mideal}}{\omega} = 10 \cdot \% \cdot \frac{\rho \cdot Q \cdot H_{ideal}}{\omega}$$

$T_{friction} = 10 \cdot \% \cdot \frac{Q \cdot \rho \cdot g \cdot H_{ideal}}{\omega} \quad T_{friction} = 5.13\,N \cdot m$

$$H_{actual} = 60\cdot\%\cdot\frac{V_2^{\ 2}}{2\cdot g} - 0.75\cdot\frac{V_{n2}^{\ 2}}{2\cdot g} \qquad H_{actual} = 3.03\,m$$

$$\eta = \frac{Q\cdot\rho\cdot g\cdot H_{actual}}{Q\cdot\rho\cdot g\cdot H_{ideal} + \omega\cdot T_{friction}} \qquad \eta = 18.7\,\%$$



The above graph can be plotted in Excel. In addition, Solver can be used to vary Q to maximize η. The results are

$Q = 0.0282\,\frac{m^3}{s}$ $\qquad \eta = 22.2\,\%$ $\qquad H_{ideal} = 17.3\,m$ $\qquad H_{actual} = 4.60\,m$

$$W_m = Q\cdot\rho\cdot g\cdot H_{ideal} + \omega\cdot T_{friction} \qquad W_m = 5.72\,kW$$

Problem 10.22

Given: Performance curves (Appendix D) for Peerless 4AE12 pump at 1750 and 3550 nominal rpm, with a 12.12 in. impeller.

Find: Obtain and plot curve-fits for total head vs. delivery at each speed for this pump.

Solution: Tabulate data from Figs. D.4 (1750 rpm) and D.5 (3550 rpm):

1750 rpm:

Q (gpm)	0	200	400	600
H (ft)	155	150	137	106

Curve-fit: $\hat{H}(ft) = 156 - 1.36\times 10^{-4}[Q(gpm)]^2$; $r^2 = 0.994$

$\hat{H}$ (ft)	156	151	134	107

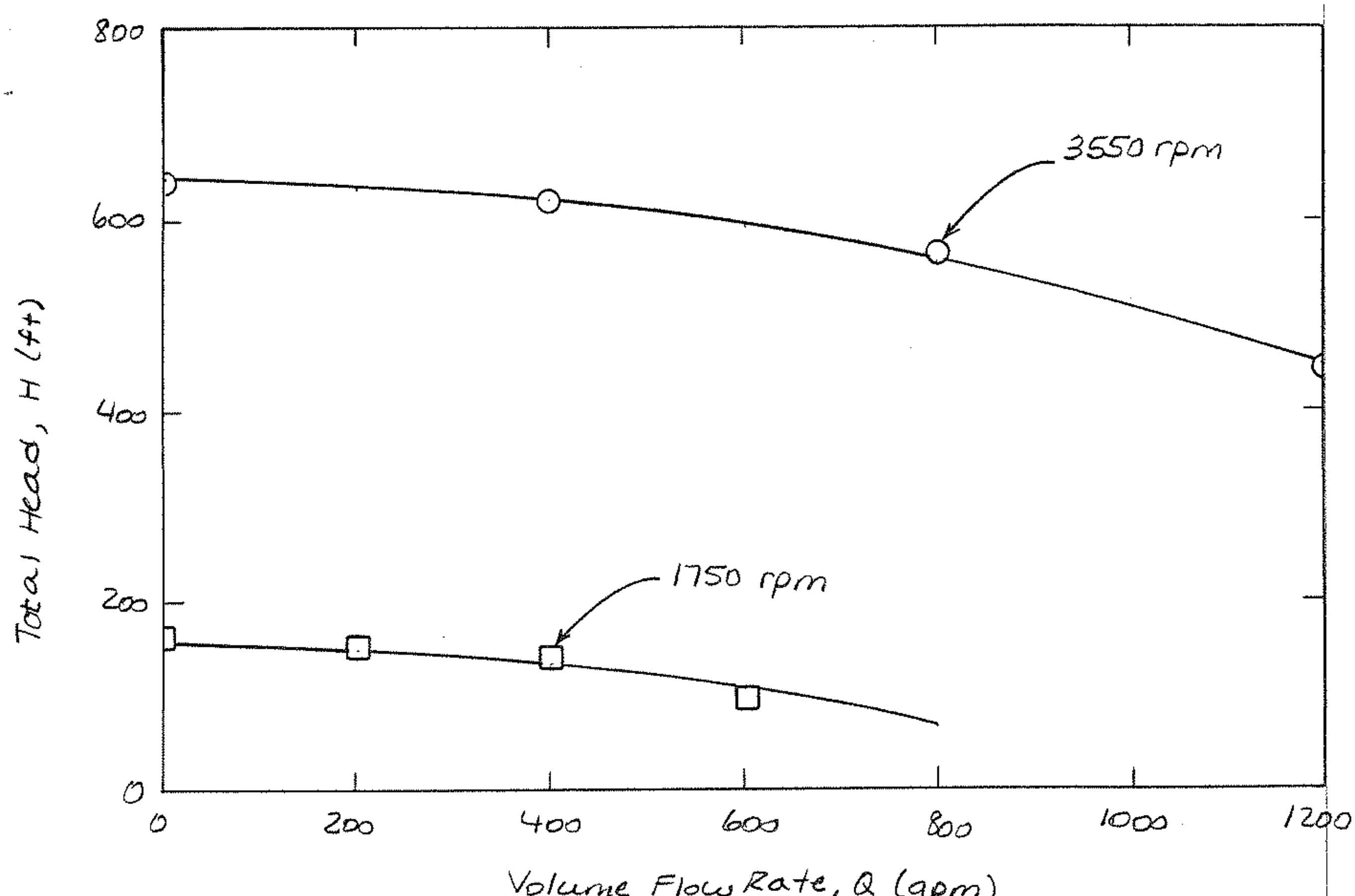
3550 rpm:

Q (gpm)	0	400	800	1200
H (ft)	635	620	565	445

Curve-fit: $\hat{H}(ft) = 641 - 1.33\times 10^{-4}[Q(gpm)]^2$; $r^2 = 0.994$

$\hat{H}$ (ft)	641	619	556	449

Plot:



Problem 10.23

Given: Performance curves (Appendix D) for Peerless 16A 18B pump at 705 and 880 nominal rpm, with an 18.0 in. diameter impeller.

Find: Obtain and plot curve-fits for total head versus delivery at each speed for this pump.

Solution: Tabulate data from Figs. D.9 (705 rpm) and D.10 (880 rpm):

705 rpm:

Q (gpm)	0	2000	4000	6000	8000
H (ft)	59	56	50	43	32

Curve-fit: $\hat{H}(ft) = 57.8 - 4.09 \times 10^{-7} [Q(gpm)]^2$; $r^2 = 0.994$

$\hat{H}$ (ft)	57.8	56.2	51.3	43.1	31.6

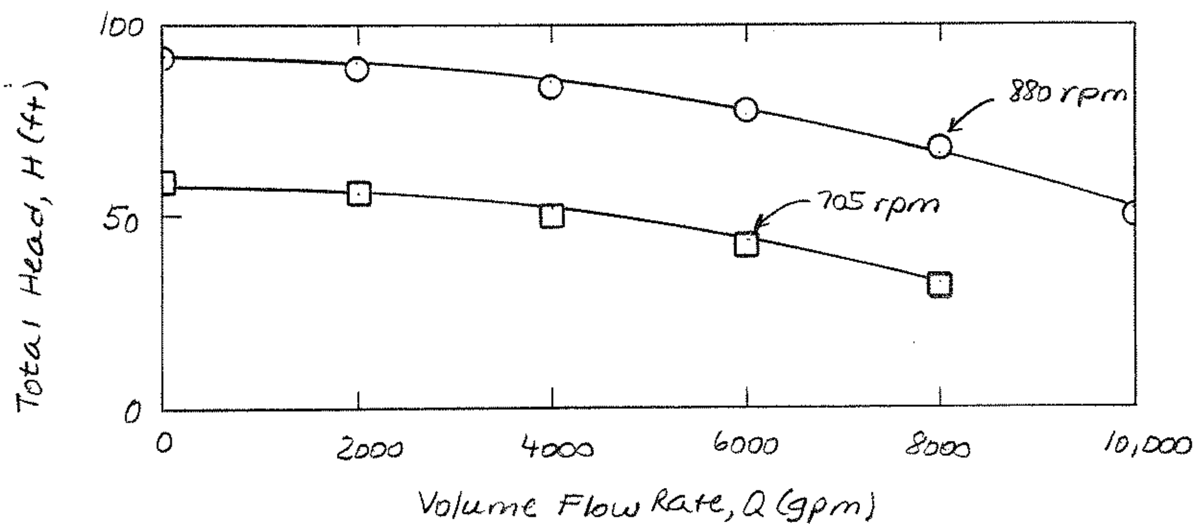
880 rpm:

Q (gpm)	0	2000	4000	6000	8000	10,000
H (ft)	92	89	84	78	68	50

Curve-fit: $\hat{H}(ft) = 91.5 - 4.01 \times 10^{-7} [Q(gpm)]^2$; $r^2 = 0.992$

$\hat{H}$ (ft)	91.5	89.9	85.1	77.1	65.9	51.5

Plot:



Problem 10.24

10.24 Data from tests of a suction pump operated at 1750 rpm with a 35-cm diameter impeller are

Flow rate, Q (m^3/s $\times$ 10^3)	17	26	38	45	63
Total head, H (m)	60	59	54	50	37
Power input, $\mathscr{P}$ (kW)	19	22	26	30	34

Plot the performance curves for this pump; include a curve of efficiency versus volume flow rate. Locate the best efficiency point and specify the pump rating at this point.

Given: Data on suction pump

Find: Plot of performance curves; Best efficiency point

Solution:

Basic equations: $\eta_p = \dfrac{P_h}{P_m}$ $\qquad P_h = \rho \cdot Q \cdot g \cdot H$ (Note: Software cannot render a dot!)

$\rho = 1000 \ \ \text{kg/m}^3$

Fitting a 2nd order polynomial to each set of data we find

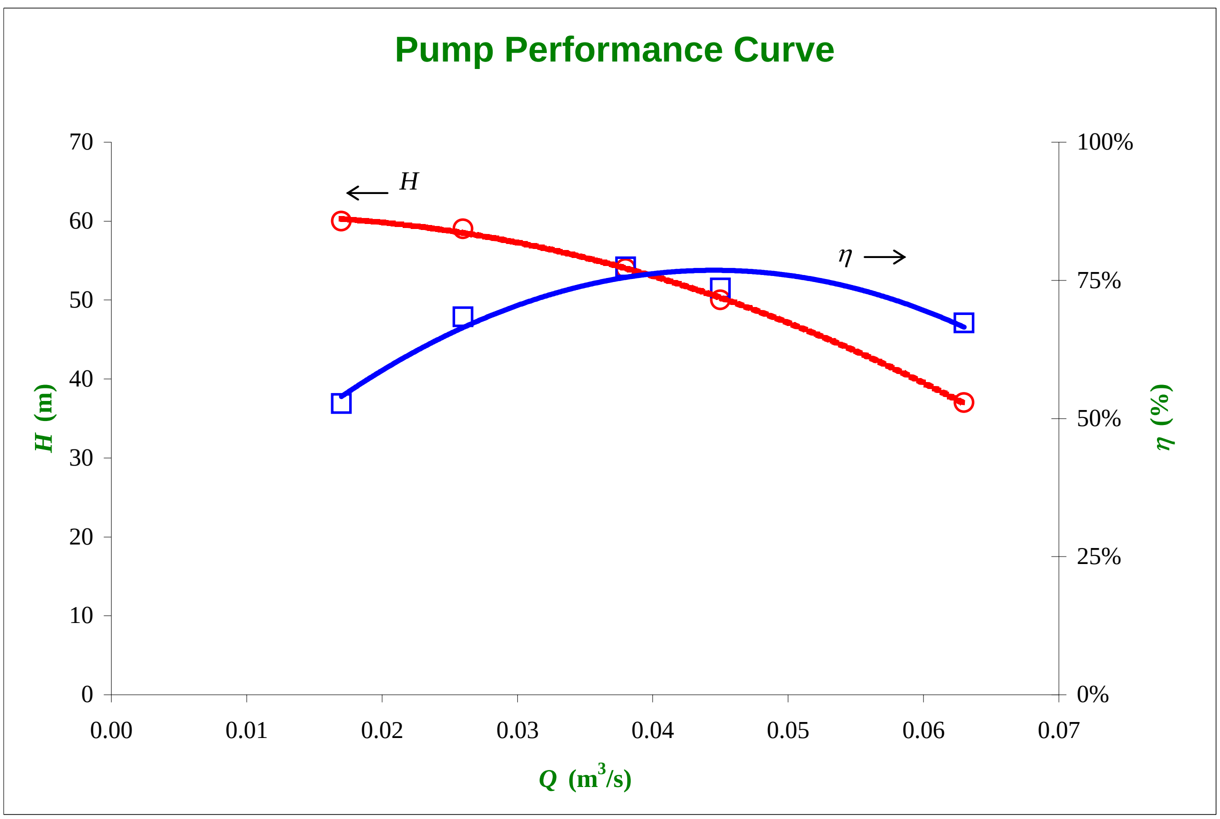
$H = -8440Q^2 + 167Q + 59.9$

$\eta = -302Q^2 + 26.9Q + 0.170$

Q (m^3/s)	H (m)	$\mathscr{P}_m$ (kW)	$\mathscr{P}_h$ (kW)	η (%)
0.017	60	19	10.0	52.7%
0.026	59	22	15.0	68.4%
0.038	54	26	20.1	77.4%
0.045	50	30	22.1	73.6%
0.063	37	34	22.9	67.3%

Finally, we use Solver to maximize η by varying Q:

Q (m^3/s)	H (m)	η (%)
0.045	50.6	76.9%



Problem 10.25

10.25 Data from tests of a suction pump operated at 1750 rpm with a 35-cm diameter impeller are

Flow rate, Q ($m^3/s \times 10^3$)	18	28	35	50	58	81
Total head, H (m)	62	62	61	57	53	41
Power input, $\mathscr{P}$ (kW)	22	26	30	34	37	45

Plot the performance curves for this pump; include a curve of efficiency versus volume flow rate. Locate the best efficiency point and specify the pump rating at this point.

Given: Data on suction pump

Find: Plot of performance curves; Best efficiency point

Solution:

Basic equations: $\eta_p = \frac{P_h}{P_m}$ $\qquad P_h = \rho \cdot Q \cdot g \cdot H$ (Note: Software cannot render a dot!)

$\rho = 1000 \quad kg/m^3$

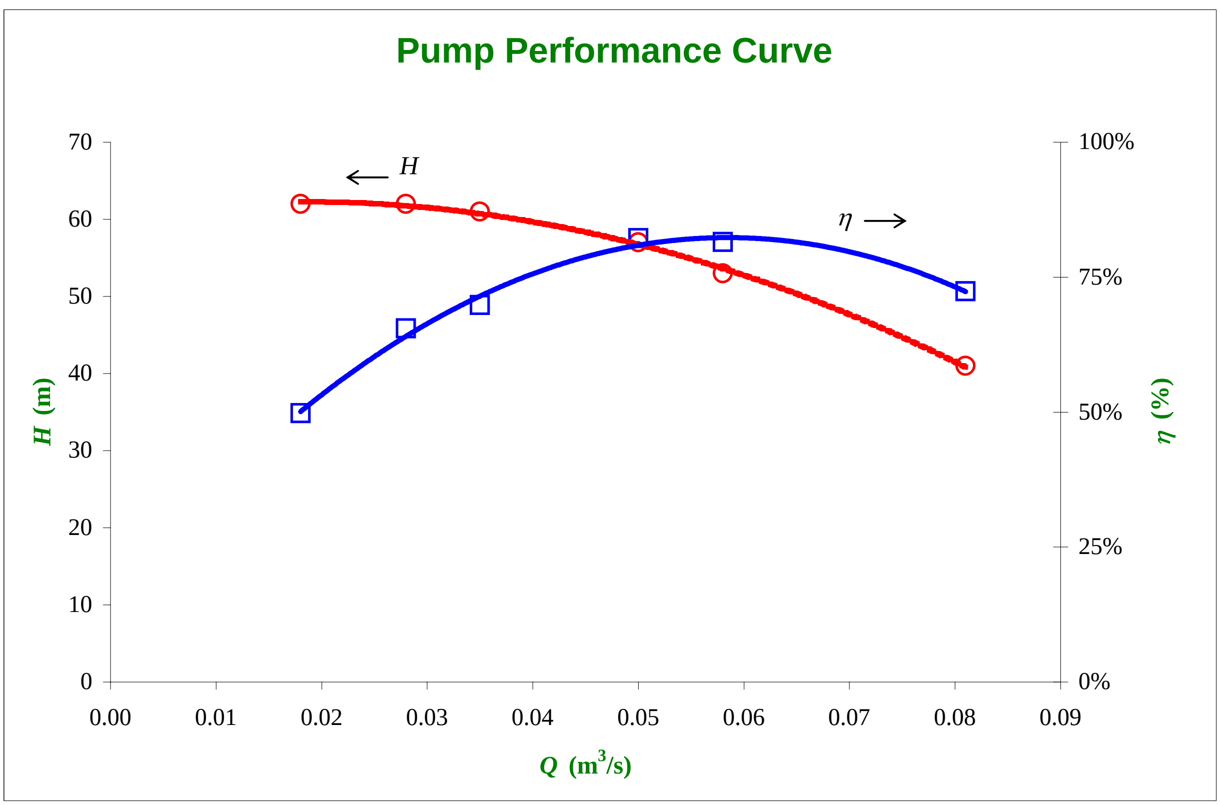
Q (m^3/s)	H (m)	$\mathscr{P}_m$ (kW)	$\mathscr{P}_h$ (kW)	η (%)
0.018	62	22	10.9	49.8%
0.028	62	26	17.0	65.5%
0.035	61	30	20.9	69.8%
0.050	57	34	28.0	82.2%
0.058	53	37	30.2	81.5%
0.081	41	45	32.6	72.4%

Fitting a 2nd order polynomial to each set of data we find

$$H = -5404Q^2 + 194Q + 60.5$$
$$\eta = -197Q^2 + 23.0Q + 0.150$$

Finally, we use Solver to maximize η by varying Q:

Q (m^3/s)	H (m)	η (%)
0.058	53.4	82.1%



Problem 10.26

10.26 Data measured during tests of a centrifugal pump at 2750 rpm are

Parameter	Inlet, Section (1)	Outlet, Section (2)
Gage pressure, p (psi)	17.5	75
Elevation above datum, z (ft)	8.25	30
Average speed of flow, $\bar{V}$ (ft/s)	9	12

The flow rate is 65 gpm and the torque applied to the pump shaft is 6.25 lbf • ft. Evaluate the total dynamic heads at the pump inlet and outlet, the hydraulic power input to the fluid, and the pump efficiency. Specify the electric motor size needed to drive the pump. If the electric motor efficiency is 85 percent, calculate the electric power requirement.

Given: Data on centrifugal pump

Find: Dynamic head at inlet and exit; Hydraulic power input; Pump efficiency; Motor size; Electric power required

Solution:

Basic equations: $\dot{W}_h = \rho Q g H_p$ (Eq. 10.8a)

$$H_p = \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g} + z\right)_{\text{discharge}} - \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g} + z\right)_{\text{suction}} \quad \text{(Eq. 10.8b)}$$

$$\eta_p = \frac{\dot{W}_h}{\dot{W}_m} = \frac{\rho Q g H_p}{\omega T} \quad \text{(Eq. 10.8c)}$$

The given or available data is

$\rho = 1.94\cdot\frac{\text{slug}}{\text{ft}^3}$ $\quad \omega = 2750\cdot\text{rpm}$ $\quad \eta_e = 85\cdot\%$ $\quad Q = 65\cdot\text{gpm}$ $\quad Q = 0.145\cdot\frac{\text{ft}^3}{\text{s}}$ $\quad T = 6.25\cdot\text{lbf}\cdot\text{ft}$

$p_1 = 17.5\cdot\text{psi}$ $\quad z_1 = 8.25\cdot\text{ft}$ $\quad V_1 = 9\cdot\frac{\text{ft}}{\text{s}}$ $\quad p_2 = 75\cdot\text{psi}$ $\quad z_2 = 30\cdot\text{ft}$ $\quad V_2 = 12\cdot\frac{\text{ft}}{\text{s}}$

Then $H_{p1} = \frac{p_1}{\rho\cdot g} + \frac{V_1^{\,2}}{2\cdot g} + z_1$ $\quad H_{p1} = 49.9\cdot\text{ft}$ $\quad H_{p2} = \frac{p_2}{\rho\cdot g} + \frac{V_2^{\,2}}{2\cdot g} + z_2$ $\quad H_{p2} = 205\cdot\text{ft}$

Also, from Eq. 10.8a $\quad W_h = \rho\cdot g\cdot Q\cdot\left(H_{p2} - H_{p1}\right)$ $\quad W_h = 1405\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{s}}$ $\quad W_h = 2.55\cdot\text{hp}$

The mechanical power in is $\quad W_m = \omega\cdot T$ $\quad W_m = 1800\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{s}}$ $\quad W_m = 3.27\cdot\text{hp}$

We need a 3.5 hp motor

From Eq. 10.8c $\quad \eta_p = \frac{W_h}{W_m}$ $\quad \eta_p = 78.0\cdot\%$

The input power is then $\quad W_e = \frac{W_m}{\eta_e}$ $\quad W_e = 2117\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{s}}$ $\quad W_e = 3.85\cdot\text{hp}$ $\quad W_e = 2.87\cdot\text{kW}$

Problem 10.27

10.27 Data measured during tests of a centrifugal pump driven at 3000 rpm are

Parameter	Inlet, Section (1)	Outlet, Section (2)
Gage pressure, p (psi)	12.5	
Elevation above datum, z (ft)	6.5	32.5
Average speed of flow, $\bar{V}$ (ft/s)	6.5	15

The flow rate is 65 gpm and the torque applied to the pump shaft is 4.75 lbf • ft. The pump efficiency is 75 percent, and the electric motor efficiency is 85 percent. Find the electric power required, and the gage pressure at section (2).

Given:
Data on centrifugal pump

Find:
Electric power required; gage pressure at exit

Solution:

Basic equations: $\dot{W}_h = \rho Q g H_p$ (Eq. 10.8a)

$$H_p = \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g} + z\right)_{\text{discharge}} - \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g} + z\right)_{\text{suction}} \quad \text{(Eq. 10.8b)}$$

$$\eta_p = \frac{\dot{W}_h}{\dot{W}_m} = \frac{\rho Q g H_p}{\omega T} \quad \text{(Eq. 10.8c)}$$

The given or available data is

$\rho = 1.94 \cdot \frac{\text{slug}}{\text{ft}^3}$ $\omega = 3000 \cdot \text{rpm}$ $\eta_p = 75 \cdot \%$ $\eta_e = 85 \cdot \%$ $Q = 65 \cdot \text{gpm}$ $Q = 0.145 \cdot \frac{\text{ft}^3}{\text{s}}$

$T = 4.75 \cdot \text{lbf} \cdot \text{ft}$ $p_1 = 12.5 \cdot \text{psi}$ $z_1 = 6.5 \cdot \text{ft}$ $V_1 = 6.5 \cdot \frac{\text{ft}}{\text{s}}$ $z_2 = 32.5 \cdot \text{ft}$ $V_2 = 15 \cdot \frac{\text{ft}}{\text{s}}$

From Eq. 10.8c $H_p = \frac{\omega \cdot T \cdot \eta_p}{\rho \cdot Q \cdot g}$ $H_p = 124 \cdot \text{ft}$

Hence, from Eq. 10.8b $p_2 = p_1 + \frac{\rho}{2} \cdot \left(V_1^{\,2} - V_2^{\,2}\right) + \rho \cdot g \cdot \left(z_1 - z_2\right) + \rho \cdot g \cdot H_p$ $p_2 = 53.7 \cdot \text{psi}$

Also $W_h = \rho \cdot g \cdot Q \cdot H_p$ $W_h = 1119 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $W_h = 2.03 \cdot \text{hp}$

The shaft work is then $W_m = \frac{W_h}{\eta_p}$ $W_m = 1492 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $W_m = 2.71 \cdot \text{hp}$

Hence, electrical input is $W_e = \frac{W_m}{\eta_e}$ $W_e = 1756 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{s}}$ $W_e = 2.38 \cdot \text{kW}$

Problem 10.28

[2]

Write the turbine specific speed in terms of the power coefficient and the head coefficient

Solution:

$N_s = \omega P^{1/2} / \rho^{1/2} h^{5/4}$ 10.18a

Power coefficient $\pi_3 = \dfrac{P}{\rho \omega^3 D^5}$

Head coefficient $\pi_2 = \dfrac{h}{\omega^2 D^2}$

Then

$$N_s = \left[\frac{P}{\rho \omega^3 D^5}\right]^{1/2} \left[\frac{\omega^2 D^2}{h}\right]^{5/4} = \frac{P^{1/2}}{\rho^{1/2} \omega^{3/2} D^{5/2}} \times \frac{\omega^{5/2} D^{5/2}}{h^{5/4}} = \frac{P^{1/2} \omega}{\rho^{1/2} h^{5/4}}$$

$N_s = \pi_3^{1/2} / \pi_2^{5/4}$ ← N_s

Problem 10.29

Write the pump specific speed in terms of the flow coefficient and the head coefficient

Solution:

$$N_s = \frac{\omega Q^{1/2}}{h^{3/4}} \quad \text{.......} \quad (7.6a)$$

Flow coefficient $\pi_1 = \frac{Q}{\omega D^3}$

Head coefficient $\pi_2 = \frac{h}{\omega^2 D^2}$

Then

$$N_s = \left[\frac{Q}{\omega D^3}\right]^{1/2} \cdot \left[\frac{\omega^2 D^2}{h}\right]^{3/4} = \frac{Q^{1/2}}{\omega^{1/2} D^{3/2}} \cdot \frac{\omega^{3/2} D^{3/2}}{h^{3/4}} = \frac{\omega Q^{1/2}}{h^{3/4}}$$

$$N_s = \pi_1^{1/2} / \pi_2^{3/4}$$

N_s

Problem 10.30

[2]

Given: Kilogram force ≡ force exerted on 1 kg in standard gravity.

Metric horsepower (hpm) ≡ 75 m·kgf/s.

Find: (a) Develop a conversion relating hpm to U.S. hp.
(b) Relate specific speed for a hydraulic turbine -- expressed in units of rpm, hpm, and m -- to the specific speed calculated in U.S. customary units.

Solution:

$$1\ hp(U.S.) = 550\frac{ft\cdot lbf}{s}\times 0.305\frac{m}{ft}\times 0.4536\frac{kgf}{lbf}\times\frac{hpm\cdot s}{75\ m\cdot kgf} = 1.01\ hpm$$ hpm

$$N_{s\,cu} = \frac{N(\mathcal{P})^{1/2}}{(h)^{5/4}} = \frac{N(rpm)[\mathcal{P}(hpUS)]^{1/2}}{[h(ft)]^{5/4}}$$

$$= N(rpm)\frac{[\mathcal{P}(hpUS)]^{1/2}}{[\mathcal{P}(hpm)]^{1/2}}\times[\mathcal{P}(hpm)]^{1/2}\times\frac{[h(m)]^{5/4}}{[h(ft)]^{5/4}}\times\frac{1}{[h(m)]^{5/4}}$$

$$= N(rpm)\frac{[\mathcal{P}(hpm)]^{1/2}}{[h(m)]^{5/4}}\times\left[\frac{\mathcal{P}(hpUS)}{\mathcal{P}(hpm)}\right]^{1/2}\times\left[\frac{h(m)}{h(ft)}\right]^{5/4}$$

$$= N_s(rpm, hpm, m)\times(1.01)^{1/2}\times(0.305)^{5/4}$$

$$N_{s\,cu} = 0.228\ N_s(rpm, hpm, m)$$ Ns

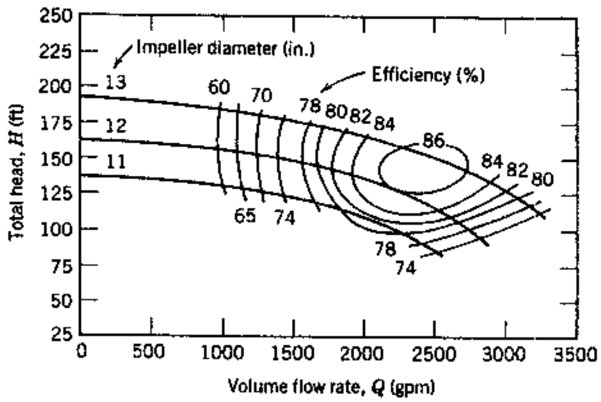
Check: $N = 1\ rpm,\ \mathcal{P} = 1\ hp,\ h = 1\ ft;\ N_s\ (USCS) = 1$

$$N = 1\ rpm,\ \mathcal{P} = 1\ hpm,\ h = 1\ m;\ N_s = \frac{(1)(\frac{1}{1.01})^{1/2}}{(0.305)^{5/4}} = 4.39$$

$$\frac{N_s(USCS)}{N_s(rpm, hpm, m)} = \frac{1}{4.39} = 0.228 \checkmark\checkmark$$

Problem 10.31

Given: Typical performance curves for a centrifugal pump, tested with three different impeller diameters:



Find: (a) Specify the flow rate and head at the best efficiency point (BEP) with a 12 in. diameter impeller.
(b) Scale these data to predict the BEP for 11 in. and 13 in. diameter impellers.
(c) Comment on the accuracy of the scaling procedure.

Solution: From the graph, BEP occurs for the 12 in. impeller at

$Q \simeq 2200\ gpm$ and $H \simeq 130\ ft$ — BEP_{12}

From Section 10-4.3, scaling rules are

$$Q_2 = Q_1\left(\frac{D_2}{D_1}\right)^3 ;\quad Q_{11} = 2200\,gpm\left(\frac{11}{12}\right)^3 = 1690\ gpm$$

$$Q_{13} = 2200\ gpm\left(\frac{13}{12}\right)^3 = 2800\ gpm$$

$$H_2 = H_1\left(\frac{D_2}{D_1}\right)^2 ;\quad H_{11} = 130\,ft\left(\frac{11}{12}\right)^2 = 109\ ft$$

$$H_{13} = 130\ ft\left(\frac{13}{12}\right)^2 = 153\ ft$$

Thus BEP_{11} is at $Q = 1690\ gpm,\ H = 109\ ft$ — BEP_{11}

BEP_{13} is at $Q = 2800\ gpm,\ H = 153\ ft$ — BEP_{13}

The complete scaling rules tend to move the volume flow rate too far. Accuracy would be improved using $Q_2 = Q_1(D_2/D_1)^2$, since the impeller width does not change, and $H_2 = H_1(D_2/D_1)^2$, since $H \simeq V^2$. With these modified rules

$(Q_{11}, H_{11}) = 1850\,gpm,\ 109\ ft$ and $(Q_{13}, H_{13}) = 2580\,gpm,\ 153\ ft$

These modified scaling points are closer to the measured BEPs.

Problem 10.32

[2]

10.32 A small centrifugal pump, when tested at $N = 2875$ rpm with water, delivered $Q = 0.016\ \text{m}^3/\text{s}$ and $H = 40$ m at its best efficiency point ($\eta = 0.70$). Determine the specific speed of the pump at this test condition. Sketch the impeller shape you expect. Compute the required power input to the pump.

Given: Data on small centrifugal pump

Find: Specific speed; Sketch impeller shape; Required power input

Solution:

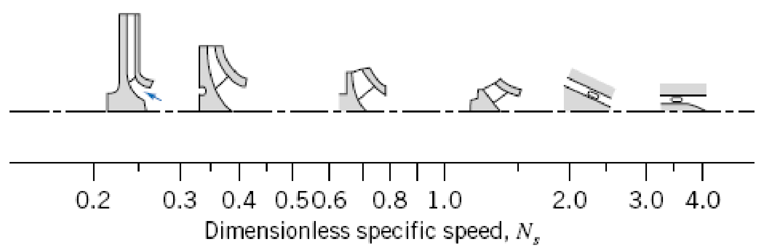
Basic equation: $N_S = \dfrac{\omega Q^{1/2}}{h^{3/4}}$ (Eq. 10.22b) $\qquad \eta_p = \dfrac{\dot{W}_h}{\dot{W}_m} = \dfrac{\rho Q g H_p}{\omega T}$ (Eq. 10.8c)

The given or available data is

$$\rho = 1000 \cdot \frac{\text{kg}}{\text{m}^3} \qquad \omega = 2875 \cdot \text{rpm} \qquad \eta_p = 70 \cdot \% \qquad Q = 0.016 \cdot \frac{\text{m}^3}{\text{s}} \qquad H = 40 \cdot \text{m}$$

Hence $\quad h = g \cdot H \qquad h = 392 \dfrac{\text{m}^2}{\text{s}^2} \qquad$ (H is energy/weight. h is energy/mass)

Then $\quad N_S = \dfrac{\omega \cdot Q^{\frac{1}{2}}}{h^{\frac{3}{4}}} \qquad N_S = 0.432$



From the figure we see the impeller will be centrifugal

The power input is (from Eq. 10.8c) $\quad W_m = \dfrac{W_h}{\eta_p} \qquad W_m = \dfrac{\rho \cdot Q \cdot g \cdot H}{\eta_p} \qquad W_m = 8.97\,\text{kW}$

Problem 10.33

10.33 A pump with $D = 500$ mm delivers $Q = 0.725\ m^3/s$ of water at $H = 10$ m at its best efficiency point. If the specific speed of the pump is 1.74, and the required input power is 90 kW, determine the shutoff head, H_0, and best efficiency, η. What type of pump is this? If the pump is now run at 900 rpm, by scaling the performance curve, estimate the new flow rate, head, shutoff head, and required power.

Given: Data on a pump

Find: Shutoff head; best efficiency; type of pump; flow rate, head, shutoff head and power at 900 rpm

Solution:

The given or available data is

$$\rho = 999 \cdot \frac{kg}{m^3} \qquad N_s = 1.74 \qquad D = 500 \cdot mm \qquad Q = 0.725 \cdot \frac{m^3}{s} \qquad H = 10 \cdot m \qquad W_m = 90 \cdot kW \qquad \omega' = 900 \cdot rpm$$

The governing equations are

$$W_h = \rho \cdot Q \cdot g \cdot H \qquad (10.8a)$$

$$N_s = \omega \cdot Q^{\frac{1}{2}} h^{\frac{3}{4}} \qquad (7.16a)$$

$$H_0 = C_1 = \frac{U_2^2}{g} \qquad \text{(From Eq. 10.7b)}$$

Similarity rules

$$\frac{Q_1}{\omega_1 \cdot D_1^3} = \frac{Q_2}{\omega_2 \cdot D_2^3} \qquad (10.19a) \qquad \frac{h_1}{\omega_1^2 \cdot D_1^2} = \frac{h2}{\omega_2^2 \cdot D_2^2} \qquad (10.19b) \qquad \frac{P_1}{\rho_1 \cdot \omega_1^3 \cdot D_1^5} = \frac{P_2}{\rho_2 \cdot \omega_2^3 \cdot D_2^5} \qquad (10.19a)$$

$$h = g \cdot H \qquad h = 98.1 \frac{J}{kg}$$

Hence from Eq. 7.16a

$$\omega = \frac{N_s \cdot h^{\frac{3}{4}}}{Q^{\frac{1}{2}}} \qquad \omega = 608\,rpm \qquad \omega = 63.7 \frac{rad}{s}$$

From Eq. 10.8a

$$W_h = \rho \cdot Q \cdot g \cdot H \qquad W_h = 71\,kW$$

$$\eta_p = \frac{W_h}{W_m} \qquad \eta_p = 78.9\,\%$$

The shutoff head is given by

$$H_0 = \frac{U_2^2}{g} \qquad \text{(From Eq. 10.7b)}$$

$$U_2 = \frac{D}{2}\cdot\omega \qquad U_2 = 15.9\,\frac{m}{s}$$

Hence $$H_0 = \frac{U_2^{\,2}}{g} \qquad H_0 = 25.8\,m$$

From Eq. 10.19a (with $D_1 = D_2$) $$\frac{Q_1}{\omega_1} = \frac{Q_2}{\omega_2} \quad \text{or} \quad \frac{Q}{\omega} = \frac{Q'}{\omega'} \qquad Q' = Q\cdot\frac{\omega'}{\omega} \qquad Q' = 1.07\,\frac{m^3}{s}$$

From Eq. 10.19b (with $D_1 = D_2$) $$\frac{h_1}{\omega_1^{\,2}} = \frac{h_2}{\omega_2^{\,2}} \quad \text{or} \quad \frac{H}{\omega^2} = \frac{H'}{\omega'^2} \qquad H' = H\cdot\left(\frac{\omega'}{\omega}\right)^2 \qquad H' = 21.9\,m$$

Also $$\frac{H_0}{\omega^2} = \frac{H'_0}{\omega'^2} \qquad H'_0 = H_0\cdot\left(\frac{\omega'}{\omega}\right)^2 \qquad H'_0 = 56.6\,m$$

(Alternatively, we could have used $H'_0 = \dfrac{U'_2{}^2}{g}$)

From Eq. 10.19c (with $D_1 = D_2$) $$\frac{P_1}{\rho\cdot\omega_1^{\,3}} = \frac{P_2}{\rho\cdot\omega_2^{\,3}} \quad \text{or} \quad \frac{W_m}{\omega^3} = \frac{W'_m}{\omega'^3} \qquad W'_m = W_m\cdot\left(\frac{\omega'}{\omega}\right)^3 \qquad W'_m = 292\,kW$$

Problem 10.34

Given: Mixed-flow pump at BEP ($\eta = 0.85$) has $D = 400$ mm, and delivers $Q = 1.20\ m^3/s$ at $H = 50$ m when operating at $N = 1500$ rpm.

(a) Calculate the specific speed of this pump
(b) Estimate the required power input
(c) Determine the curve-fit parameters using BEP and shutoff points.
(d) Scale the performance curve to estimate the flow, head, efficiency, and power required at 750 rpm.

Solution: In SI units, $N = 157$ rad/s, $Q = 1.20\ m^3/s$, and $h = gH = 490\ m^2/s^2$

$$N_s = \frac{\omega Q^{1/2}}{h^{3/4}} = 157 \frac{rad}{s} \times (1.20)^{1/2} \frac{m^{3/2}}{s^{1/2}} \times \frac{s^{3/2}}{(490)^{3/4}\ m^{3/2}} = 1.65 \quad N_s$$

$$\dot{W}_m = \frac{\dot{W}_h}{\eta} = \frac{\rho g Q H}{\eta} = \frac{\rho Q h}{\eta}$$

$$\dot{W}_m = \frac{999}{0.85} \frac{kg}{m^3} \times 1.20 \frac{m^3}{s} \times 490 \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m} \times \frac{kW \cdot s}{10^3\ N \cdot m} = 691\ kW \quad \dot{W}_m$$

At shutoff, $V_{t_2} = U_2$, so $H_0 = \frac{U_2^2}{g} = \frac{(\omega R_2)^2}{g}$

$$H_0 = \left(157 \frac{rad}{s} \times \frac{0.40}{2} m\right)^2 \times \frac{s^2}{9.81\ m} = 100\ m$$

Thus, $H = H_0 - AQ^2$ or $A = (H_0 - H)/Q^2$

$$A = (100 - 50)\ m \times \frac{1}{(1.20)^2} \frac{s^2}{m^6} = 34.7\ m^{-5} s^2$$

or

$$H_m = 100 - 34.7\left[Q(m^3/s)\right]^2 \quad (1500\ rpm) \quad H$$

At 750 rpm, $H_0' = \left(\frac{N'}{N}\right)^2 H_0 = \left(\frac{750}{1500}\right)^2 100 = 25\ m$, and $A' = A$

Thus

$$H'(m) = 25 - 34.7\left[Q'(m^3/s)\right]^2 \quad (750\ rpm) \quad H$$

At BEP, $Q' = Q\left(\frac{\omega'}{\omega}\right) = 1.20 \frac{m^3}{s}\left(\frac{750}{1500}\right) = 0.60\ m^3/s$

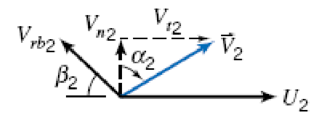
$$H' = H\left(\frac{\omega'}{\omega}\right)^2 = 50\ m \left(\frac{750}{1500}\right)^2 = 12.5\ m$$

$$\eta' = \eta = 0.85$$

$$P' = P\left(\frac{\omega'}{\omega}\right)^3 = 691\ kW \left(\frac{750}{1500}\right)^3 = 86.4\ kW \quad 750\ rpm$$

Problem 10.35

10.35 A centrifugal water pump operates at 1750 rpm; the impeller has backward-curved vanes with $\beta_2 = 60°$ and $b_2 = 1.25$ cm. At a flow rate of 0.025 m^3/s, the radial outlet velocity is $V_{n_2} = 3.5$ m/s. Estimate the head this pump could deliver at 1150 rpm.



Given: Data on centrifugal pump

Find: Head at 1150 rpm

Solution:

Basic equation:

$$H = \frac{\dot{W}_m}{\dot{m}g} = \frac{1}{g}(U_2 V_{t_2} - U_1 V_{t_1}) \qquad \text{(Eq. 10.2c)}$$

The given or available data is

$$\rho = 1000 \cdot \frac{kg}{m^3} \qquad Q = 0.025 \cdot \frac{m^3}{s} \qquad \beta_2 = 60 \cdot deg \qquad b_2 = 1.25 \cdot cm$$

$$\omega = 1750 \cdot rpm \qquad \omega' = 1150 \cdot rpm \qquad V_{n2} = 3.5 \cdot \frac{m}{s}$$

From continuity $$V_{n2} = \frac{Q}{2 \cdot \pi \cdot r_2 \cdot b_2}$$

Hence $$r_2 = \frac{Q}{2 \cdot \pi \cdot b_2 \cdot V_{n2}} \qquad r_2 = 0.0909\,m \qquad r_2 = 9.09\,cm$$

Then $$V'_{n2} = \frac{\omega'}{\omega} \cdot V_{n2} \qquad V'_{n2} = 2.30\,\frac{m}{s}$$

Also $$U'_2 = \omega' \cdot r_2 \qquad U'_2 = 11.0\,\frac{m}{s}$$

From the outlet geometry $$V'_{t2} = U'_2 - V'_{n2} \cdot \cos(\beta_2) \qquad V'_{t2} = 9.80\,\frac{m}{s}$$

Finally $$H' = \frac{U'_2 \cdot V'_{t2}}{g} \qquad H' = 10.9\,m$$

Problem 10.36

10.36 A pumping system must be specified for a lift station at a wastewater treatment facility. The average flow rate is 110 million liters per day and the required lift is 10 m. Non-clogging impellers must be used; about 65 percent efficiency is expected. For convenient installation, electric motors of 37.5 kW or less are desired. Determine the number of motor/pump units needed and recommend an appropriate operating speed.

Given: Data on pumping system

Find: Number of pumps needed; Operating speed

Solution:

Basic equations: $W_h = \rho \cdot Q \cdot g \cdot H \qquad \eta_p = \frac{W_h}{W_m}$

The given or available data is

$\rho = 1000 \cdot \frac{kg}{m^3} \qquad Q_{total} = 110 \times 10^6 \cdot \frac{L}{day} \qquad Q_{total} = 1.273 \frac{m^3}{s} \qquad H = 10 \cdot m \qquad \eta = 65 \cdot \%$

Then for the system $W_h = \rho \cdot Q_{total} \cdot g \cdot H \qquad W_h = 125 \cdot kW$

The required total power is $W_m = \frac{W_h}{\eta} \qquad W_m = 192 \cdot kW$

Hence the total number of pumps must be $\frac{192}{37.5} = 5.12$, or at least six pumps

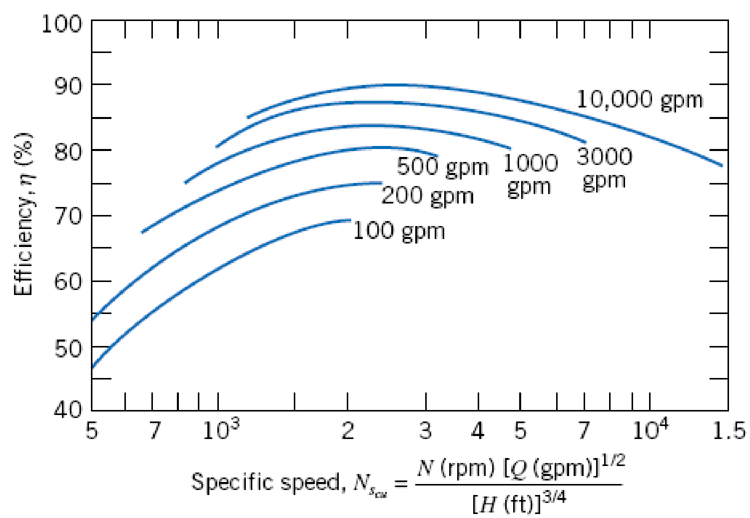
The flow rate per pump will then be $Q = \frac{Q_{total}}{6} \qquad Q = 0.212 \frac{m^3}{s} \qquad Q = 212 \cdot \frac{L}{s}$

From Fig. 10.15 the peak effiiciency is at a specific speed of about

$N_{Scu} = 2000$

We also need $H = 32.8 \cdot ft \qquad Q = 3363 \cdot gpm$

Hence $N = N_{Scu} \cdot \frac{H^{\frac{3}{4}}}{Q^{\frac{1}{2}}} \qquad N = 473$



The nearest standard speed to $N = 473$ rpm should be used

Problem 10.37

10.37 A set of seven 35 hp motor-pump units is used to deliver water through an elevation of 50 ft. The efficiency of the pumps is specified to be 60 percent. Estimate the delivery (gallons per day) and select an appropriate operating speed.

Given: Data on pumping system

Find: Total delivery; Operating speed

Solution:

Basic equations: $W_h = \rho \cdot Q \cdot g \cdot H$ $\quad \eta_p = \frac{W_h}{W_m}$

The given or available data is

$\rho = 1000 \cdot \frac{kg}{m^3}$ $\quad W_m = 35 \cdot hp$ $\quad H = 50 \cdot ft$ $\quad \eta = 60 \cdot \%$

Then for the system $\quad W_{mTotal} = 7 \cdot W_m$ $\quad W_{mTotal} = 245 \cdot hp$

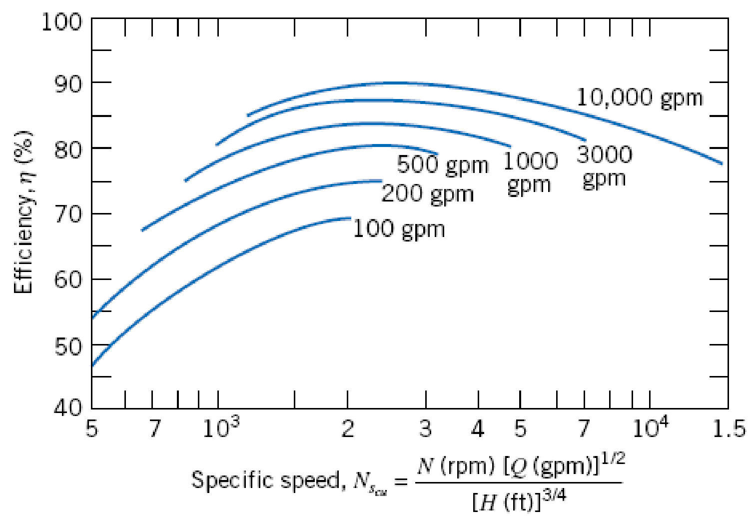
The hydraulic total power is $\quad W_{hTotal} = \frac{W_{mTotal}}{\eta}$ $\quad W_{hTotal} = 304 \cdot kW$

The total flow rate will then be $\quad Q_{Total} = \frac{W_{hTotal}}{\rho \cdot g \cdot H}$ $\quad Q_{Total} = 71.95 \cdot \frac{ft^3}{s}$ $\quad Q_{Total} = 32293 \cdot gpm$

The flow rate per pump is $\quad Q = \frac{Q_{Total}}{6}$ $\quad Q = 12.0 \cdot \frac{ft^3}{s}$ $\quad Q = 5382 \cdot gpm$

From Fig. 10.15 the peak effiiciency is at a specific speed of about

$N_{Scu} = 2500$



Hence $\quad N = N_{Scu} \cdot \frac{H^{\frac{3}{4}}}{Q^{\frac{1}{2}}}$ $\quad N = 641$

The nearest standard speed to $N = 641$ rpm should be used

Problem 10.38

10.38 Appendix D contains area bound curves for pump model selection and performance curves for individual pump models. Use these data and the similarity rules to predict and plot the curves of head H (ft) versus Q (gpm) of a Peerless Type 10AE12 pump, with impeller diameter $D = 12$ in., for nominal speeds of 1000, 1200, 1400, and 1600 rpm.

Given: Data on Peerless Type 10AE12 pump at 1760 rpm

Find: Data at speeds of 1000, 1200, 1400, and 1600 rpm

Solution:

The governing equations are the similarity rules

$$\frac{Q_1}{\omega_1 \cdot D_1^{\,3}} = \frac{Q_2}{\omega_2 \cdot D_2^{\,3}} \qquad (10.19a)$$

$$\frac{h_1}{\omega_1^{\,2} \cdot D_1^{\,2}} = \frac{h2}{\omega_2^{\,2} \cdot D_2^{\,2}} \qquad (10.19b)$$

where $h = g \cdot H$

For scaling from speed ω_1 to speed ω_2, with $D_1 = D_2$ from Eq. 10.19a

$$Q_2 = Q_1 \cdot \frac{\omega_2}{\omega_1}$$

and from Eq. 10.19b $$H_2 = H_1 \cdot \left(\frac{\omega_2}{\omega_1}\right)^2$$

Speed (rpm) = 1760

Q (gal/min)	Q^2	H (ft)	H (fit)
0	0	170	161
500	250000	160	160
1000	1000000	155	157
1500	2250000	148	152
2000	4000000	140	144
2500	6250000	135	135
3000	9000000	123	123
3500	12250000	110	109
4000	16000000	95	93

Speed (rpm) = 1000

Q (gal/min)	H (ft)
0	52.0
284	51.7
568	50.7
852	49.0
1136	46.6
1420	43.5
1705	39.7
1989	35.3
2273	30.2

Speed (rpm) = 1200

Q (gal/min)	H (ft)
0	74.9
341	74.5
682	73.0
1023	70.5
1364	67.1
1705	62.6
2045	57.2
2386	50.8
2727	43.5

Speed (rpm) = 1400

Q (gal/min)	H (ft)
0	102.0
398	101.3
795	99.3
1193	96.0
1591	91.3
1989	85.3
2386	77.9
2784	69.2
3182	59.1

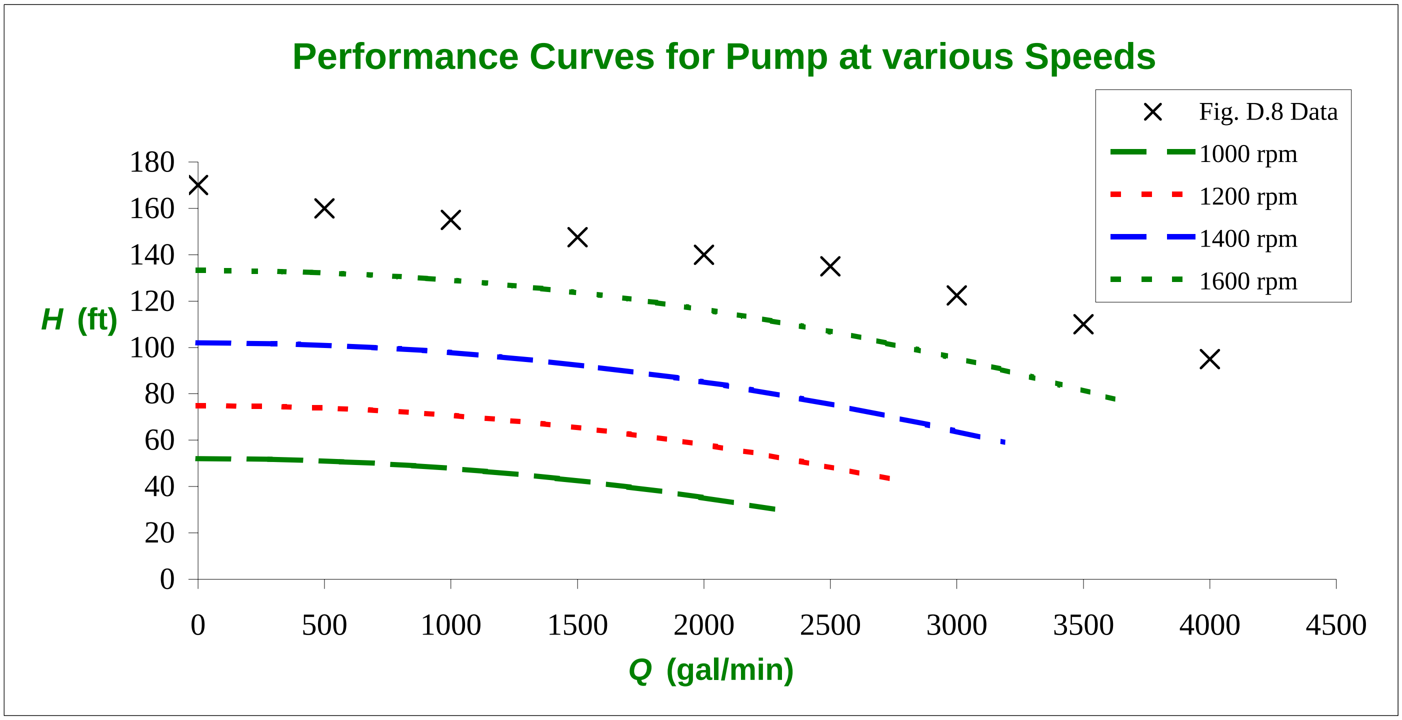
Speed (rpm) = 1600

Q (gal/min)	H (ft)
0	133.2
455	132.4
909	129.7
1364	125.4
1818	119.2
2273	111.4
2727	101.7
3182	90.4
3636	77.2

Data from Fig. D.8 is "eyeballed"

The fit to data is obtained from a least squares fit to $H = H_0 - AQ^2$

H_0 =	161	ft
A =	4.23E-06	ft/(gal/min)



Problem 10.39

Given: Area bound curves for pump model selection and performance curves for individual pump models, Appendix D.

Find: Use these data to verify the similarity rules for a Peerless Type 4AE12 pump operated at 1750 and 3550 nominal rpm, with 11.00 in. impeller.

Solution: From Figs. D.4 and D.5, at the best efficiency point (BEP):

N (rpm)	Q (gpm)	H (ft)	$\dot{W}_m$ (hp)	η (%)
1750	470	104	17	73$^+$
3550	970	430	135	74$^+$

The similarity rules are

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}, \quad \frac{H_1}{\omega_1^2 D_1^2} = \frac{H_2}{\omega_2^2 D_2^2}, \quad \frac{\mathcal{P}_1}{\omega_1^3 D_1^5} = \frac{\mathcal{P}_2}{\omega_2^3 D_2^5}, \text{ and } \eta_1 = \eta_2$$

Evaluating, with $D_1 = D_2$,

$$Q_1 = Q_2 \frac{\omega_1}{\omega_2} = 970 \text{ gpm} \frac{1750 \text{ rpm}}{3550 \text{ rpm}} = 478 \text{ gpm}$$

$$H_1 = H_2\left(\frac{\omega_1}{\omega_2}\right)^2 = 430 \text{ ft}\left(\frac{1750 \text{ rpm}}{3550 \text{ rpm}}\right)^2 = 104 \text{ ft}$$

$$\mathcal{P}_1 = \mathcal{P}_2\left(\frac{\omega_1}{\omega_2}\right)^3 = 135 \text{ hp}\left(\frac{1750 \text{ rpm}}{3550 \text{ rpm}}\right)^3 = 16.2 \text{ hp}$$

$$\eta_1 = \eta_2 = 0.74^+$$

Comparing shows excellent agreement.

Problem 10.40

Given: Data from Appendix D for Peerless Type 4AE12 pump.

Find: Verify the similarity rules for the effects of diameter change at 1750 and 3550 nominal rpm.

Solution: From Figs. D.4 and D.5 at the best efficiency point (BEP):

N (rpm)	D (in.)	Q (gpm)	H (ft)	$\dot{W}_m$ (hp)	η (%)
1750	10.00	455	87	13	73+
	11.00	470	104	17	73+
	12.12	500	123	22	73+
3550	10.00	930	360	115	74+
	11.00	970	430	135	74+
	12.12	1030	500	180	74+

The similarity rules are:

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}; \quad \frac{H_1}{\omega_1^2 D_1^2} = \frac{H_2}{\omega_2^2 D_2^2}, \quad \frac{\mathcal{P}_1}{\omega_1^3 D_1^5} = \frac{\mathcal{P}_2}{\omega_2^3 D_2^5}, \text{ and } \eta_1 = \eta_2.$$

Evaluating, with $\omega_1 = \omega_2 = 1750$ rpm,

$Q_{10} = Q_{11}\left(\frac{10}{11}\right)^3 = 353$; $Q_{12} = Q_{11}\left(\frac{12.12}{11}\right)^3 = 629$; $H_{10} = 86.0$ ft, $H_{12} = 126$ ft

$\mathcal{P}_{10} = 10.6$ hp, $\mathcal{P}_{12} = 27.6$ hp; η = constant

Evaluating, with $\omega_1 = \omega_2 = 3550$ rpm,

$Q_{10} = 729$ gpm, $Q_{12} = 1300$ gpm; $H_{10} = 355$ ft, $H_{12} = 522$ ft; $\mathcal{P}_{10} = 83.8$ hp,

$\mathcal{P}_{12} = 219$ hp; η = constant

Comparing results with data shows:

(1) flow rate is scaled poorly

(2) head is scaled well

(3) power is scaled poorly (because flow rate is scaled poorly)

Better results are obtained using the modified scaling rules (see p. 526); then

$Q \sim D^2$ so $Q_{10} = 388$ gpm and $\mathcal{P} \sim D^4$ so $\mathcal{P}_{10} = 11.6$ hp at 1750 rpm

and $Q_{10} = 802$ gpm and $\mathcal{P}_{10} = 92.2$ hp at 3550 rpm.

Problem 10.41

[3]

Given: Data in Appendix D for Peerless Type 16A18B pump.

Find: Verify the similarity rules for (a) impeller diameter change and (b) speed change.

Solution: From Figs. D.9 and D.10 at the best efficiency point (BEP):

N (rpm)	D (in.)	Q (gpm)	H (ft)	Wm (hp)	η (%)
705	18.0	6250	42	76	86+
	17.0	5850	37	63	86+
	16.0	5600	32	54	86
880	18.0	7900	69	155	87+
	17.0	7400	59	125	87
	16.0	7100	50	105	85

The similarity rules are:

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}, \quad \frac{H_1}{\omega_1^2 D_1^2} = \frac{H_2}{\omega_2^2 D_2^2}, \quad \frac{\mathscr{P}_1}{\omega_1^3 D_1^5} = \frac{\mathscr{P}_2}{\omega_2^3 D_2^5}, \text{ and } \eta_1 = \eta_2$$

Evaluating with $\omega_1 = \omega_2 = 705$ rpm,

$Q_{17} = Q_{18}\left(\frac{17}{18}\right)^3 = 5270$ gpm, $Q_{16} = 4390$ gpm; $H_{17} = H_{18}\left(\frac{17}{18}\right)^2 = 37.5$ ft,

$H_{16} = 33.2$ ft; $\mathscr{P}_{17} = \mathscr{P}_{18}\left(\frac{17}{18}\right)^5 = 57.1$ hp, $\mathscr{P}_{16} = 42.2$ hp; η = constant

At 880 rpm, $Q_{17} = 6660$ gpm, $Q_{16} = 5550$ gpm; $H_{17} = 61.5$ ft, $H_{16} = 54.5$ ft;

$\mathscr{P}_{17} = 116$ hp, $\mathscr{P}_{16} = 86.0$ hp; η = constant

Evaluating with $D_1 = D_2 = 18.0$ in.,

$Q_{705} = Q_{880}\left(\frac{705}{880}\right) = 6330$ gpm; $H_{705} = H_{880}\left(\frac{705}{880}\right)^2 = 44.3$ ft;

$\mathscr{P}_{705} = \mathscr{P}_{880}\left(\frac{705}{880}\right)^3 = 79.7$ hp; η = constant

Comparing results with data shows at <u>constant speed</u>:

(1) flow rate scales poorly, (2) head scales well, (3) power scales poorly with changes in diameter.

Comparing results with data shows at <u>constant diameter</u>:

all quantities scale well with changes in speed.

Flow rate scaling may be improved using the modified procedure discussed on page 526, in which $Q \sim D^2$ and $\mathscr{P} \sim D^4$.

Problem 10.42

Given: Performance curves for Peerless Type 16 A 18B pump, Appendix D. (with D = 18.0 in. impeller).

Find: (a) Develop and plot curve-fits for 705 and 880 nominal rpm.
(b) Verify the effect of pump speed on scaling pump curves using the procedure of Example Problem 10.7.

Solution: Tabulate performance data and curve-fits:

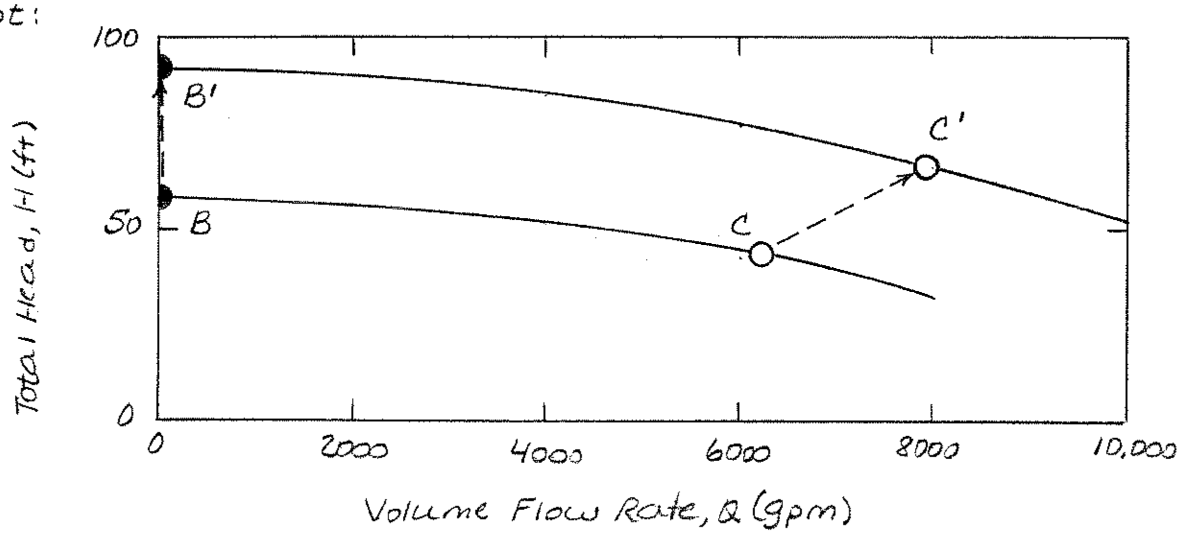
705 rpm:	Q (gpm)	0	2000	4000	6000	8000	BEP:	6250
	H (ft)	59	56	50	43	32		42

Curve-fit: $\hat{H}(ft) = 57.8 - 4.09\times10^{-7}[Q(gpm)]^2$; $r^2 = 0.994$

880 rpm:	Q (gpm)	0	2000	4000	6000	8000	BEP:	7900
	H (ft)	92	89	84	78	68		69

Curve-fit: $\hat{H}(ft) = 91.5 - 4.01\times10^{-7}[Q(gpm)]^2$; $r^2 = 0.992$

Plot:



Using the procedure of Example Problem 10.7:

$$Q_{B'} = Q_B = 0;\ H_{B'} = H_B\left(\frac{\omega'}{\omega}\right)^2 = 59\,ft\left(\frac{880}{705}\right)^2 = 91.9\ ft$$

$$Q_{C'} = Q_C\left(\frac{\omega'}{\omega}\right) = 6250\,gpm\left(\frac{880}{705}\right) = 7800\ gpm$$

$$H_{C'} = H_C\left(\frac{\omega'}{\omega}\right)^2 = 42\,ft\left(\frac{880}{705}\right)^2 = 65.4\ ft$$

Comparing the curve-fit parameters shows good agreement:

$\hat{H} = H_0 - AQ^2$ $\quad H_0 = 91.5$ ft compared to $H_{B'} = 91.9$ ft

$$\left.\begin{aligned} A' &= 4.01\times10^{-7}\ ft/(gpm)^2 \\ A' &= 4.09\times10^{-7}\ ft/(gpm)^2 \end{aligned}\right\}\ \text{within } 2.0\%$$

Problem 10.43

Given: Performance curves for Peerless Type 10AE12 pump, Appendix D.

Find: (a) Develop and plot a curve-fit for 1760 nominal rpm.
(b) Scale the curve-fit to a pump speed of 1150 nominal rpm, using the procedure of Example Problem 10.7.

Solution: Tabulate performance data and curve-fit, for D = 12 in. diameter impeller at 1760 nominal rpm:

Q (gpm)	1500	2000	2500	3000	3500	4000
H (ft)	148	141	133	123	110	95
$\hat{H}$ (ft)	148	141	133	122	110	95.5

Curve-fit: $\hat{H}(\text{ft}) = 157 - 3.83\times10^{-6}[Q(\text{gpm})]^2$; $r^2 = 0.999$

or $\hat{H} = H_0 - AQ^2$

The similarity rules are

$$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3} \; ; \; \frac{H_1}{\omega_1^2 D_1^2} = \frac{H_2}{\omega_2^2 D_2^2}$$

The pump diameter stays constant, so

$$Q_2 = Q_1\left(\frac{\omega_2}{\omega_1}\right) \quad \text{and} \quad H_2 = H_1\left(\frac{\omega_2}{\omega_1}\right)^2 = H_1\left(\frac{1150}{1760}\right)^2 = 0.427\,H_1$$

Following the procedure of Example Problem 10.7, then at 1150 rpm,

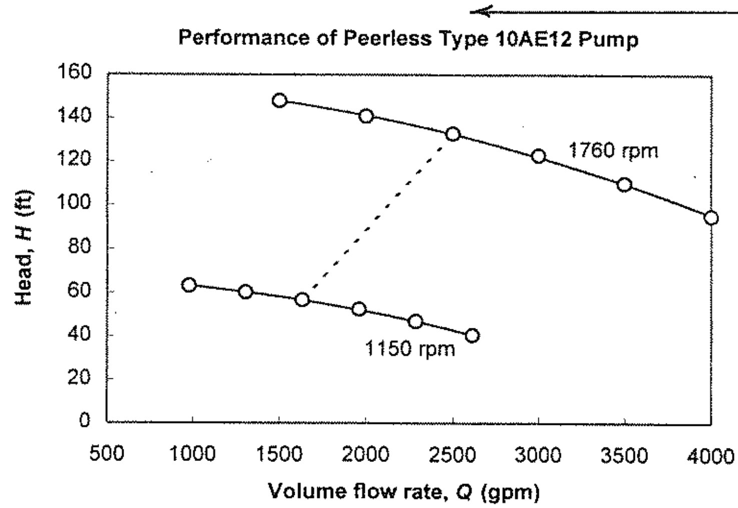
$$\hat{H}(\text{ft}) = 0.427\,H_0 - AQ^2$$

$$= (0.427)\,157\text{ ft} - 3.83\times10^{-6}[Q(\text{gpm})]^2$$

$$\hat{H}(\text{ft}) = 67.0\text{ ft} - 3.83\times10^{-6}[Q(\text{gpm})]^2$$

$\hat{H}$ (ft) (1150 rpm)

The plot is:



Problem 10.44

Open-Ended Problem Statement: Problem 10.20 suggests that pump head at best efficiency is typically about 70 percent of shutoff head. Use pump data from Appendix D to evaluate this suggestion. A further suggestion in Section 10-4 is that the appropriate scaling for tests of a pump casing with different impeller diameters is $Q \propto D^2$. Use pump data to evaluate this suggestion.

Discussion: Data selected from pump performance curves in Appendix D is tabulated and plotted on the next page. Data were selected at the maximum efficiency point for the largest (D_{max}) and smallest (D_{min}) diameter impellers with which each pump was tested.

The head at the best efficiency point with the largest impeller was selected to compare with the shutoff head for the same impeller. These data are shown in the first graph, where they are compared to the average ratio, $H_{BEP} = 0.766\ H_0$. There is some scatter, but the trend of agreement is fairly clear. The actual values suggest a higher ratio than the 0.7 mentioned in Problem 10.20.

The flow rate ratio Q_{max}/Q_{min} was compared with the square of the impeller diameter ratio $(D_{max}/D_{min})^2$. These data ratios are shown in the second graph, where they are compared to the correlation line. Agreement is not perfect, but the trend supports a positive correlation of 0.751. The predicted relationship between diameter and flow rate is $Q_{max}/Q_{min} = 0.751\ (D_{max}/D_{min})^2$.

(Use of three significant figures probably is not justified in this problem. The data are read from small graphs in the Appendix that have already been smoothed by the manufacturer. Also there is some uncertainty in selecting the best efficiency point on each curve.)

Problem 10.44

Sample	Fig.	Model	Speed (rpm)	H_0 (ft)	H_{BEP} (ft)	H_{BEP}/H_0 (---)
1	D.3	4AE11	1750	113	95	0.84
2	D.5	4AE12	3550	636	500	0.79
3	D.6	6AE14	1750	209	160	0.77
4	D.7	8AE20G	1770	430	365	0.85
5	D.8	10AE12	1760	170	112	0.66
6	D.9	16A18B	705	59	42	0.71
7	D.10	16A18B	880	92	69	0.75
					Average:	0.766

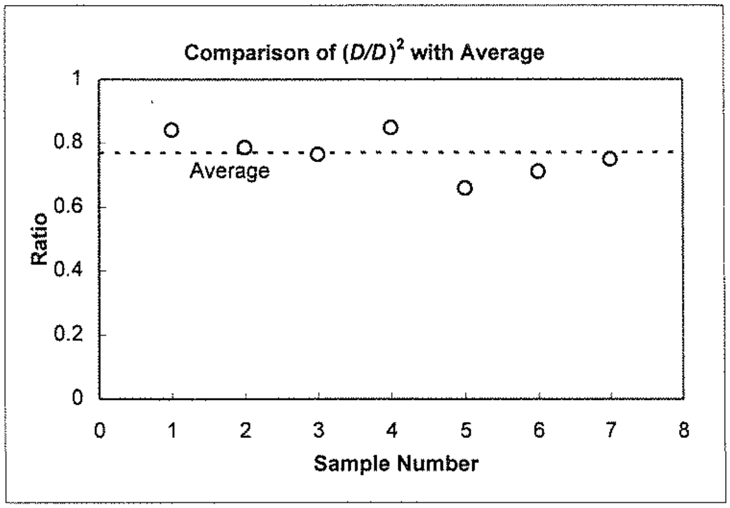
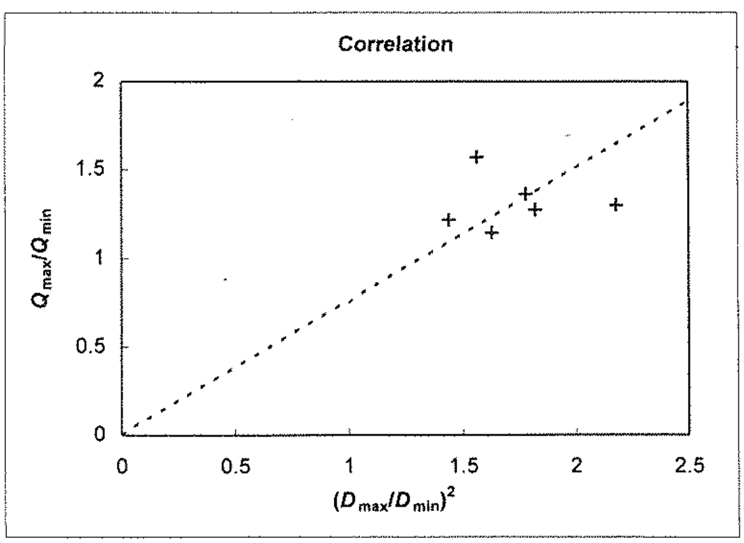


Fig.	Model	Speed (rpm)	D_{min} (in.)	Q_{BEP} (gpm)	D_{max} (in.)	Q_{BEP} (gpm)	$(D_{max}/D_{min})^2$ (---)	Q_{max}/Q_{min} (---)
D.3	4AE11	1750	7.62	740	11.25	960	2.2	1.3
D.5	4AE12	3550	9.5	910	12.12	1040	1.6	1.1
D.6	6AE14	1750	10.38	1375	14.0	1750	1.8	1.3
D.7	8AE20G	1770	16.0	2200	20.0	3450	1.6	1.6
D.8	10AE12	1760	9.0	2500	12.0	3400	1.8	1.4
D.9	16A18B	705	15.0	5100	18.0	6200	1.4	1.2
D.10	16A18B	880	15.0	6500	18.0	7900	1.4	1.2
						Correlation:		0.751



Problem 10.45

Given: Catalog data for centrifugal pump at design conditions:

$Q = 250\text{ gpm} \quad \Delta p = 18.6\text{ psi} \quad N = 1750\text{ rpm}$

Laboratory flume requires $Q_f = 200$ gpm at $H_f = 32$ ft; the only available motor develops 3 hp at 1750 rpm.

Find: (a) Is motor suitable?
(b) How might the pump/motor match be improved?

Solution: To obtain efficiency and pump power requirement, find specific speed.

$$H = \frac{\Delta p}{\rho g} = 18.6\frac{\text{lbf}}{\text{in.}^2}\times\frac{\text{ft}^3}{62.4\text{ lbf}}\times 144\frac{\text{in.}^2}{\text{ft}^2} = 42.9\text{ ft} \quad ; Q = 250\frac{\text{gal}}{\text{min}} = 0.557\text{ cfs}$$

$$N_{S_{cu}} = \frac{N Q^{1/2}}{H^{3/4}} = \frac{1750\text{ rpm }(250\text{ gpm})^{1/2}}{(42.9\text{ ft})^{3/4}} = 1650$$

From Fig. 10.15, $\eta \approx 0.73$. Thus

$$\dot{W}_m = \frac{\dot{W}_h}{\eta} = \frac{\rho Q g H}{\eta} = \frac{1}{0.73}\times 62.4\frac{\text{lbf}}{\text{ft}^3}\times 0.557\frac{\text{ft}^3}{\text{s}}\times 42.9\text{ ft}\times\frac{\text{hp}\cdot\text{s}}{550\text{ ft}\cdot\text{lbf}} = 3.71\text{ hp}$$

The motor is not suitable to drive the pump directly.

The pump at 1750 rpm produces more head and flow than necessary. It may be run at reduced speed, e.g., by using a belt drive.

To produce $Q_f = 200$ gpm, solve $\frac{Q_p}{\omega_p D_p^3} = \frac{Q_f}{\omega_f D_f^3}$; $\omega_f = \frac{200}{250}\times 1750 = 1400$ rpm

To produce $H_f = 32$ ft, solve $\frac{H_p}{\omega_p^2 D_p^2} = \frac{H_f}{\omega_f^2 D_f^2}$; $\omega_f = \sqrt{\frac{H_f}{H_p}}\,\omega_p = \sqrt{\frac{32}{42.9}}\times 1750 = 1510$ rpm

At 1510 rpm the power requirement will be given by $\frac{\mathcal{P}_p}{\omega_p^3 D_p^5} = \frac{\mathcal{P}_f}{\omega_f^3 D_f^5}$, so

$$\mathcal{P}_f = \mathcal{P}_p\left(\frac{\omega_f}{\omega_p}\right)^3 = 3.71\text{ hp}\left(\frac{1510}{1750}\right)^3 = 2.38\text{ hp}$$

This is well within the capability of the 3 hp motor. Therefore run pump at 1510 rpm.

Problem 10.46

10.46 A reaction turbine is designed to produce 17.5 MW at 120 rpm under 45 m of head. Laboratory facilities are available to provide 10 m of head and to absorb 35 kW from the model turbine. Assume comparable efficiencies for the model and prototype turbines. Determine the appropriate model test speed, scale ratio, and volume flow rate.

Given: Data on turbine system

Find: Model test speed; Scale; Volume flow rate

Solution:

Basic equations: $W_h = \rho \cdot Q \cdot g \cdot H$ $\qquad \eta = \frac{W_{mech}}{W_h}$ $\qquad N_S = \frac{\omega \cdot P^{\frac{1}{2}}}{\rho^{\frac{1}{2}} \cdot h^{\frac{5}{4}}}$

The given or available data is

$\rho = 1000 \cdot \frac{kg}{m^3}$ $\quad W_p = 17.5 \cdot MW$ $\quad H_p = 45 \cdot m$ $\quad \omega_p = 120 \cdot rpm$ $\quad H_m = 10 \cdot m$ $\quad W_m = 35 \cdot kW$

where sub p stands for prototype and sub m stands for model

Note that we need h (energy/mass), not H (energy/weight) $\quad h_p = H_p \cdot g$ $\quad h_p = 441 \frac{m^2}{s^2}$ $\quad h_m = H_m \cdot g$ $\quad h_m = 98.1 \frac{m^2}{s^2}$

Hence for the prototype $\quad N_S = \frac{\omega_p \cdot W_p^{\frac{1}{2}}}{\rho^{\frac{1}{2}} \cdot h_p^{\frac{5}{4}}}$ $\quad N_S = 0.822$

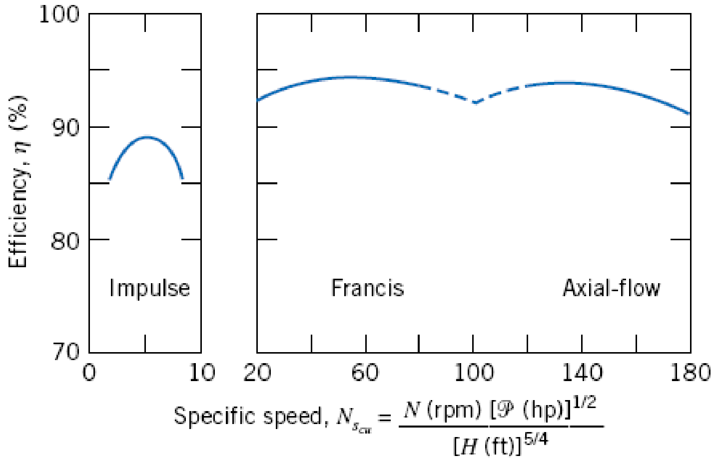
Then for the model $\quad N_S = \frac{\omega_m \cdot W_m^{\frac{1}{2}}}{\rho^{\frac{1}{2}} \cdot h_m^{\frac{5}{4}}}$ $\quad \omega_m = N_S \cdot \frac{\rho^{\frac{1}{2}} \cdot h_m^{\frac{5}{4}}}{W_m^{\frac{1}{2}}}$ $\quad \omega_m = 42.9 \frac{rad}{s}$ $\quad \omega_m = 409 \, rpm$

For dynamically similar conditions $\quad \frac{H_p}{\omega_p^2 \cdot D_p^2} = \frac{H_m}{\omega_m^2 \cdot D_m^2}$ $\quad$ so $\quad \frac{D_m}{D_p} = \frac{\omega_p}{\omega_m} \cdot \sqrt{\frac{H_m}{H_p}} = 0.138$

Also $\quad \frac{Q_p}{\omega_p \cdot D_p^3} = \frac{Q_m}{\omega_m \cdot D_m^3}$ $\quad$ so $\quad Q_m = Q_p \cdot \frac{\omega_m}{\omega_p} \cdot \left(\frac{D_m}{D_p}\right)^3$

To find Q_p we need efficiency. At $W_p = 17.5 \, MW$ or $W_p = 23468 \, hp$ and $H_p = 45 \, m$ or $H_p = 148 \, ft$
from F ig. 10.17 we find (see below), for

$$N_{Scu} = \frac{N(rpm) \cdot P(hp)^{\frac{1}{2}}}{H(ft)^{5.4}} = 35.7 \qquad \eta = 93 \cdot \%$$



Hence from $\eta = \frac{W_{mech}}{W_h} = \frac{W_{mech}}{\rho \cdot Q \cdot g \cdot H}$ $\quad Q_p = \frac{W_p}{\rho \cdot g \cdot H_p \cdot \eta}$ $\quad Q_p = 42.6 \frac{m^3}{s}$

and also $Q_m = \frac{W_m}{\rho \cdot g \cdot H_m \cdot \eta}$ $\quad Q_m = 0.384 \frac{m^3}{s}$

Problem 10.47

10.47 A 1/3 scale model of a centrifugal water pump, when running at $N_m = 100$ rpm, produces a flow rate of $Q_m = 1\ m^3/s$ with a head of $H_m = 4.5$ m. Assuming the model and prototype efficiencies are comparable, estimate the flow rate, head, and power requirement if the design speed is 125 rpm.

Given: Data on a model pump

Find: Prototype flow rate, head, and power at 125 rpm

Solution:

Basic equation: $W_h = \rho \cdot Q \cdot g \cdot H$ and similarity rules

$$\frac{Q_1}{\omega_1 \cdot D_1^3} = \frac{Q_2}{\omega_2 \cdot D_2^3} \quad (10.19a) \qquad \frac{h_1}{\omega_1^2 \cdot D_1^2} = \frac{h2}{\omega_2^2 \cdot D_2^2} \quad (10.19b) \qquad \frac{P_1}{\rho_1 \cdot \omega_1^3 \cdot D_1^5} = \frac{P_2}{\rho_2 \cdot \omega_2^3 \cdot D_2^5} \quad (10.19a)$$

The given or available data is $N_m = 100 \cdot rpm$ $\quad N_p = 125 \cdot rpm$ $\quad \rho = 1000 \cdot \frac{kg}{m^3}$

$Q_m = 1 \cdot \frac{m^3}{s}$ $\quad H_m = 4.5 \cdot m$

From Eq. 10.8a $\quad W_{hm} = \rho \cdot Q_m \cdot g \cdot H_m \qquad W_{hm} = 44.1 \cdot kW$

From Eq. 10.19a (with $D_m/D_p = 1/3$) $\quad \frac{Q_p}{\omega_p \cdot D_p^3} = \frac{Q_m}{\omega_m \cdot D_m^3}$ or $\quad Q_p = Q_m \cdot \frac{\omega_p}{\omega_m} \cdot \left(\frac{D_p}{D_m}\right)^3 = 3^3 \cdot Q_m \cdot \frac{\omega_p}{\omega_m}$

$$Q_p = 27 \cdot Q_m \cdot \frac{N_p}{N_m} \qquad Q_p = 33.8 \frac{m^3}{s}$$

From Eq. 10.19b (with $D_m/D_p = 1/3$) $\quad \frac{h_p}{\omega_p^2 \cdot D_p^2} = \frac{h_m}{\omega_m^2 \cdot D_m^2}$ or $\quad \frac{g \cdot H_p}{\omega_p^2 \cdot D_p^2} = \frac{g \cdot H_m}{\omega_m^2 \cdot D_{pm}^2}$

$$H_p = H_m \cdot \left(\frac{\omega_p}{\omega_m}\right)^2 \cdot \left(\frac{D_p}{D_m}\right)^2 = 3^2 \cdot H_m \cdot \left(\frac{\omega_p}{\omega_m}\right)^2 \qquad H_p = 9 \cdot H_m \cdot \left(\frac{N_p}{N_m}\right)^2 \qquad H_p = 63.3\,m$$

From Eq. 10.19c (with $D_m/D_p = 1/3$) $\quad \frac{P_p}{\rho \cdot \omega_p^3 \cdot D_p^5} = \frac{P_m}{\rho \cdot \omega_m^3 \cdot D_m^5}$ or $\quad W_{hp} = W_{hm} \cdot \left(\frac{\omega_p}{\omega_m}\right)^3 \cdot \left(\frac{D_p}{D_m}\right)^5 = 3^5 \cdot W_{hm} \cdot \left(\frac{\omega_p}{\omega_m}\right)^3$

$$W_{hp} = 243 \cdot W_{hm} \cdot \left(\frac{N_p}{N_m}\right)^3 \qquad W_{hp} = 20.9 \cdot MW$$

Problem 10.48

Given: Pump to operate at $Q = 250$ cfs, $H = 400$ ft, and $N = 870$ rpm.

Model test to be run in facility where $Q \leq 5$ cfs and a 300 hp dynamometer is available. Assume model and prototype efficiencies are comparable.

Find: Appropriate model test speed and scale ratio.

Solution: To obtain homologous operating points, run model test at same specific speed as prototype.

$$Q = 250\,\frac{ft^3}{sec} \times 7.48\,\frac{gal}{ft^3} \times 60\,\frac{sec}{min} = 112{,}000 \text{ gpm}$$

$$N_{s_{cu}} = \frac{NQ^{1/2}}{H^{3/4}} = \frac{870\text{ rpm}(112{,}000\text{ gpm})^{1/2}}{(400\text{ ft})^{3/4}} = 3260$$

(The dimensionless specific speed is $N_{s,nd} = 3260/2733 = 1.19$. Figure 10.14 indicates a mixed-flow geometry.) Figure 10.15 indicates $\eta = 0.92$ at this Ns. Thus

$$\dot{W}_m = \frac{\dot{W}_h}{\eta} = \frac{\rho Q g H}{\eta} = \frac{1}{0.92} \times 62.4\,\frac{lbf}{ft^3} \times 250\,\frac{ft^3}{s} \times 400\,ft \times \frac{hp \cdot s}{550\,ft \cdot lbf} = 12{,}300 \text{ hp}$$

For the model,

$$H_m = \eta \dot{W}_m / \rho Q g = 0.92 \times 300\,hp \times \frac{ft^3}{62.4\,lbf} \times \frac{s}{5\,ft^3} \times 550\,\frac{ft \cdot lbf}{hp \cdot s} = 487\text{ ft}$$

To match specific speeds, then

$$N_{m_{cu}} = N_{s_{cu}} \frac{H_m^{3/4}}{Q_m^{1/2}} = 3260 \frac{(487\text{ ft})^{3/4}}{(2240\text{ gpm})^{1/2}} = 7140\text{ rpm}$$ ← N_m

The scale ratio may be obtained from the scaling laws. For example, since

$$\frac{Q_m}{\omega_m D_m^3} = \frac{Q_p}{\omega_p D_p^3} \qquad \frac{D_m}{D_p} = \left[\frac{Q_m}{Q_p}\,\frac{\omega_p}{\omega_m}\right]^{\frac{1}{3}} = \left[\frac{1}{50} \times \frac{870}{7140}\right]^{\frac{1}{3}} = 0.135$$ ← D_m/D_p

Thus $D_m = 0.135\,D_p$ (scale ratio is $1/0.135 = 7.43$ to 1)

Check using the head ratio, $\frac{H_m}{\omega_m^2 D_m^2} = \frac{H_p}{\omega_p^2 D_p^2}$

$$H_m = H_p \left(\frac{\omega_m}{\omega_p}\right)^2 \left(\frac{D_m}{D_p}\right)^2 = 400\text{ ft}\left(\frac{7140}{870}\right)^2 (0.135)^2 = 491\text{ ft} \approx 487\text{ ft}$$

This is acceptable agreement, considering roundoff error.

{Great care would be needed to avoid cavitation in the model pump at speeds above 7000 rpm.}

Problem 10.49

Given: Model efficiency using curve-fit, $\eta = aQ - bQ^3$, where a and b are constants.

Find: (a) Describe a procedure to evaluate a and b from data.
(b) Evaluate using data for Peerless Type 10AE12 pump, with impeller diameter $D = 12.0$ in., operating at 1760 rpm.

Solution: From Fig. D.8, data are:

η (%)	70	75	80	84	86	86	84
Q (gpm)	1850	2100	2400	2780	3100	3700	4075

Two equations are needed to solve for constants a and b directly. A second equation may be obtained by differentiating. At peak efficiency,

$$\frac{d\eta}{dQ} = a - 3bQ^2 = 0$$

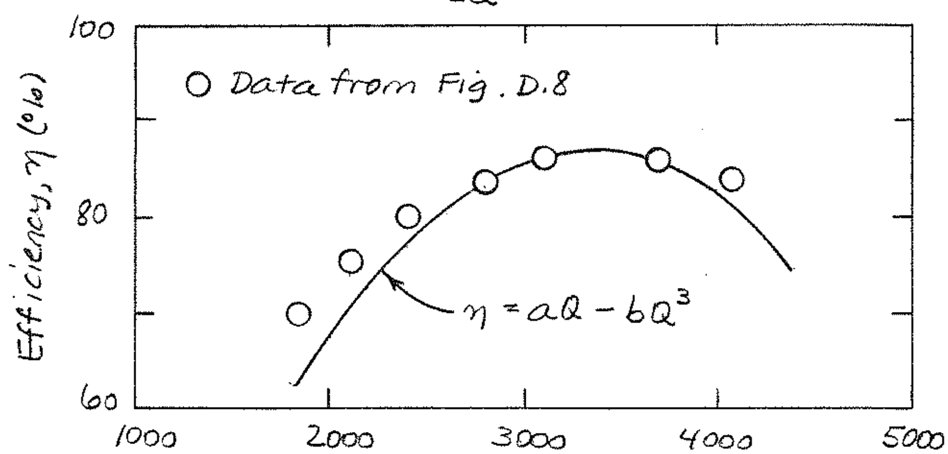
Assume peak efficiency is 87 percent at 3400 gpm. Then

$$\eta_{max} = aQ - bQ^3$$
$$0 = a - 3bQ^2$$

Substituting from the second equation into the first gives

$$\eta_{max} = 3bQ^3 - bQ^3 = 2bQ^3 \; ; \; b = \frac{\eta_{max}}{2Q^3} = 1.11 \times 10^{-9} \; ; \; a = 3bQ^2 = 0.0384$$ a, b

Plotting:



The curve-fit does a good job near peak efficiency, but tends to underestimate the measured data elsewhere.

{An alternative curve-fit procedure is to plot η/Q versus $a - bQ^2$, then do a least-squares fit (using all the data) to obtain a and b. Then $a = 0.0426\,(\text{gpm})^{-1}$, $b = -1.56 \times 10^{-9}\,(\text{gpm})^{-2}$; $r^2 = 0.996$. This underestimates η at $Q > 3500$ gpm.}

Problem 10.50

10.50 Sometimes the variation of water viscosity with temperature can be used to achieve dynamic similarity. A model pump delivers 20 gpm of water at 59°F against a head of 60 ft, when operating at 3500 rpm. Determine the water temperature that must be used to obtain dynamically similar operation at 1750 rpm. Estimate the volume flow rate and head produced by the pump at the lower-speed test condition. Comment on the *NPSH* requirements for the two tests.

Given:

Data on a model pump

Find:

Temperature for dynamically similar operation at 1750 rpm; Flow rate and head; Comment on NPSH

Solution:

Basic equation: $Re_1 = Re_2$ and similarity rules $\dfrac{Q_1}{\omega_1 \cdot D_1^{\,3}} = \dfrac{Q_2}{\omega_2 \cdot D_2^{\,3}}$ $\quad \dfrac{H_1}{\omega_1^{\,2} \cdot D_1^{\,2}} = \dfrac{H2}{\omega_2^{\,2} \cdot D_2^{\,2}}$

The given or available data is $\omega_1 = 3500 \cdot rpm$ $\quad \omega_2 = 1750 \cdot rpm$ $\quad Q_1 = 20 \cdot gpm$ $\quad H_1 = 60 \cdot ft$

From Table A.7 at 59ºF $\quad \nu_1 = 1.23 \times 10^{-5} \cdot \dfrac{ft^2}{s}$

For D = constant $\quad Re_1 = \dfrac{V_1 \cdot D}{\nu_1} = \dfrac{\omega_1 \cdot D \cdot D}{\nu_1} = Re_2 = \dfrac{\omega_2 \cdot D \cdot D}{\nu_2}$ $\quad$ or $\quad \nu_2 = \nu_1 \cdot \dfrac{\omega_2}{\omega_1}$ $\quad \nu_2 = 6.15 \times 10^{-6} \cdot \dfrac{ft^2}{s}$

From Table A.7, at $\nu_2 = 6.15 \times 10^{-6} \cdot \dfrac{ft^2}{s}$, we find, by linear interpolation

$$T_2 = 110 + \frac{(120 - 110)}{(6.05 - 6.68)} \cdot (6.15 - 6.68) \qquad T_2 = 118 \quad \text{degrees F}$$

From similar operation $\quad \dfrac{Q_1}{\omega_1 \cdot D^3} = \dfrac{Q_2}{\omega_2 \cdot D^3}$ $\quad$ or $\quad Q_2 = Q_1 \cdot \dfrac{\omega_2}{\omega_1}$ $\quad Q_2 = 10 \cdot gpm$

and also $\quad \dfrac{H_1}{\omega_1^{\,2} \cdot D^2} = \dfrac{H_2}{\omega_2^{\,2} \cdot D^2}$ $\quad$ or $\quad H_2 = H_1 \cdot \left(\dfrac{\omega_2}{\omega_1}\right)^2$ $\quad H_2 = 15 \cdot ft$

The water at 118ºF is closer to boiling. The inlet pressure would have to be changed to avoid cavitation. The increase between runs 1 and 2 would have to be $\Delta p = p_{v2} - p_{v1}$ where p_{v2} and p_{v1} are the vapor pressures at T_2 and T_1. From the steam tables (find them by Googling!)

$$p_{v1} = 0.247 \cdot psi \qquad p_{v2} = 1.603 \cdot psi \qquad \Delta p = p_{v2} - p_{v1} \qquad \Delta p = 1.36 \cdot psi$$

Problem 10.51

[3]

10.51 A four-stage boiler feed pump has suction and discharge lines of 10 cm and 7.5 cm inside diameter. At 3500 rpm, the pump is rated at 0.025 m^3/s against a head of 125 m while handling water at 115°C. The inlet pressure gage, located 50 cm below the impeller centerline, reads 150 kPa. The pump is to be factory certified by tests at the same flow rate, head rise, and speed, but using water at 27°C. Calculate the *NPSHA* at the pump inlet in the field installation. Evaluate the suction head that must be used in the factory test to duplicate field suction conditions.

Given:

Data on a boiler feed pump

Find:

NPSHA at inlet for field temperature water; Suction head to duplicate field conditions

Solution:

Basic equation: $NPSHA = p_t - p_v = p_g + p_{atm} + \frac{1}{2}\cdot\rho\cdot V^2 - p_v$

Given or available data is

$D_s = 10\cdot cm$ $\quad D_d = 7.5\cdot cm$ $\quad H = 125\cdot m$ $\quad Q = 0.025\cdot\frac{m^3}{s}$

$p_{inlet} = 150\cdot kPa$ $\quad p_{atm} = 101\cdot kPa$ $\quad z_{inlet} = -50\cdot cm$ $\quad \rho = 1000\cdot\frac{kg}{m^3}$ $\quad \omega = 3500\cdot rpm$

For field conditions $\quad p_g = p_{inlet} + \rho\cdot g\cdot z_{inlet}$ $\quad p_g = 145\,kPa$

From continuity $\quad V_s = \frac{4\cdot Q}{\pi\cdot D_s^2}$ $\quad V_s = 3.18\frac{m}{s}$

From steam tables (try Googling!) at 115ºC $\quad p_v = 169\cdot kPa$

Hence $\quad NPSHA = p_g + p_{atm} + \frac{1}{2}\cdot\rho\cdot V_s^2 - p_v$ $\quad NPSHA = 82.2\,kPa$

Expressed in meters or feet of water $\quad \frac{NPSHA}{\rho\cdot g} = 8.38\,m$ $\quad \frac{NPSHA}{\rho\cdot g} = 27.5\,ft$

In the laboratory we must have the same NPSHA. From Table A.8 (or steam tables - try Googling!) at 27ºC $\quad p_v = 3.57\cdot kPa$

Hence $\quad p_g = NPSHA - p_{atm} - \frac{1}{2}\cdot\rho\cdot V_s^2 + p_v$ $\quad p_g = -20.3\,kPa$

The absolute pressure is $\quad p_g + p_{atm} = 80.7\,kPa$

Problem 10.52

10.52 Data from tests of a pump operated at 1500 rpm, with a 30-cm diameter impeller, are

Flow rate, Q ($m^3/s \times 10^3$)	10	20	30	40	50	60	70
Net positive suction head required, $NPSR$ (m)	2.2	2.4	2.6	3.1	3.6	4.1	5.1

Develop and plot a curve-fit equation for $NPSHR$ versus volume flow rate in the form $NPSHR = a + bQ^2$, where a and b are constants. If the $NPSHA = 6$ m, estimate the maximum allowable flow rate of this pump.

Given: Data on a NPSHR for a pump

Find: Curve fit; Maximum allowable flow rate

Solution:

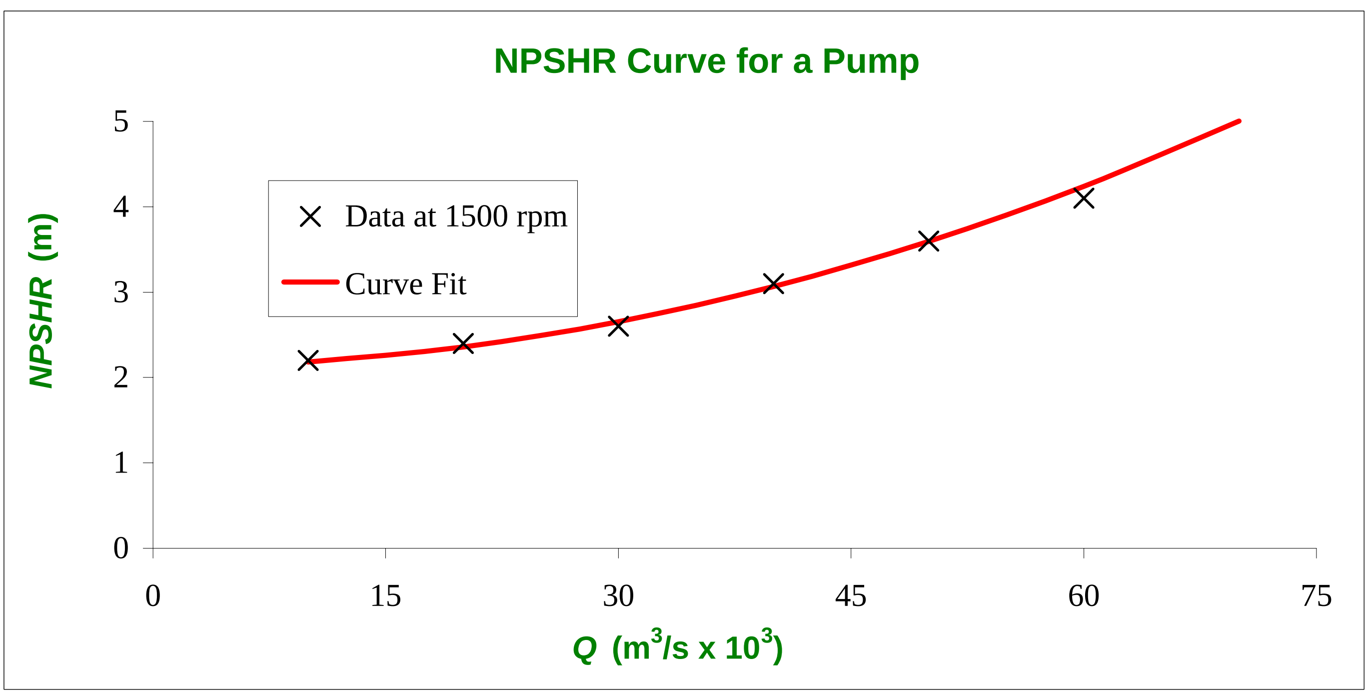
Q ($m^3/s \times 10^3$)	Q^2	$NPSHR$ (m)	$NPSHR$ (fit)
10	1.00E+02	2.2	2.2
20	4.00E+02	2.4	2.4
30	9.00E+02	2.6	2.7
40	1.60E+03	3.1	3.1
50	2.50E+03	3.6	3.6
60	3.60E+03	4.1	4.2
70	4.90E+03	5.1	5.0

The fit to data is obtained from a least squares fit to $NPSHR = a + bQ^2$

a = 2.12 m

b = 5.88E-04 $m/(m^3/s \times 10^3)^2$

Q ($m^3/s \times 10^3$)	$NPSHR$ (m)	
81.2	6.00	Use *Goal Seek* to find Q!



Problem 10.53

Open-Ended Problem Statement: A large deep fryer at a snack-food plant contains hot oil that is circulated through a heat exchanger by pumps. Solid particles and water droplets coming from the food product are observed in the flowing oil. What special factors must be considered in specifying the operating conditions for the pumps?

Discussion: Any solid particles must be able to pass through the pumps without clogging. If the particles are large, this may require larger than normal clearances within the pumps.

If the water droplets flashed to steam, they would form local pockets of water vapor. The pockets of water vapor would disrupt the flow patterns in the pumps in the same way as cavitation in a homogeneous liquid. To prevent this "cavitation" from occurring, static pressure everywhere in the flow circuit must be maintained above the saturation pressure of the water droplets at the temperature of the flowing oil.

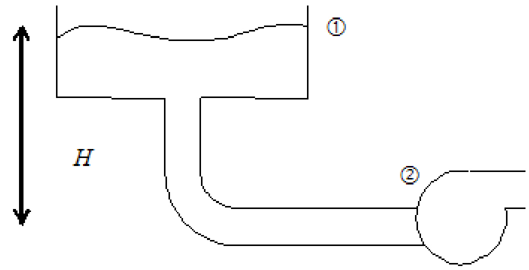
The net positive suction head at the pump inlets must be sufficiently high to prevent any problems from occurring within the pumps themselves.

The solid particles may act as nucleation sites, which would foster the development of vapor pockets in the flow. This might increase the net positive suction head required by the pump above that measured in tests using water. The system must be sized to maintain a large net positive suction head at the design flow rate.

Finally, the viscosity of the oil must be considered. If viscosity is high, pump performance will be degraded compared to pumping water. Then a larger pump must be specified to handle the flow requirement of the hot oil circulation system.

Problem 10.54

10.54 The net positive suction head required (*NPSHR*) by a pump may be expressed approximately as a parabolic function of volume flow rate. The *NPSHR* for a particular pump operating at 1750 rpm is given as $H_r = H_0 + AQ^2$, where $H_0 = 3$ m of water and $A = 3000\ \text{m}/(\text{m}^3/\text{s})^2$. Assume the pipe system supplying the pump suction consists of a reservoir, whose surface is 6 m above the pump centerline, a square entrance, 6 m of 15 cm cast-iron pipe, and a 90° elbow. Calculate the maximum volume flow rate at 20°C for which the suction head is sufficient to operate this pump without cavitation.



Given: Pump and supply pipe system

Find: Maximum operational flow rate

Solution:

Basic equations:

$$\left(\frac{p_1}{\rho} + \alpha_1\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_{lT} \qquad h_{lT} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} + f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} + K\cdot\frac{V^2}{2}$$

L_e for the elbow, and K for the square entrance

$$\text{NPSHA} = \frac{p_t - p_v}{\rho\cdot g} \qquad H_r = H_0 + A\cdot Q^2$$

Assumptions: 1) $p_1 = 0$ 2) $V_1 = 0$ 3) $\alpha_2 = 0$ 4) $z_2 = 0$

We must match the NPSHR (=H_r) and NPSHA

From the energy equation

$$g\cdot H - \left(\frac{p_2}{\rho} + \frac{V^2}{2}\right) = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} + f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} + K\cdot\frac{V^2}{2} \qquad \frac{p_2}{\rho\cdot g} = H - \frac{V^2}{2\cdot g}\cdot\left[1 + f\cdot\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]$$

$$\text{NPSHA} = \frac{p_t - p_v}{\rho\cdot g} = \frac{p_2}{\rho\cdot g} + \frac{p_{atm}}{\rho\cdot g} + \frac{V_2^2}{2\cdot g} - \frac{p_v}{\rho\cdot g} \qquad \text{NPSHA} = H - \frac{V^2}{2\cdot g}\cdot\left[f\cdot\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right] + \frac{(p_{atm} - p_v)}{\rho\cdot g}$$

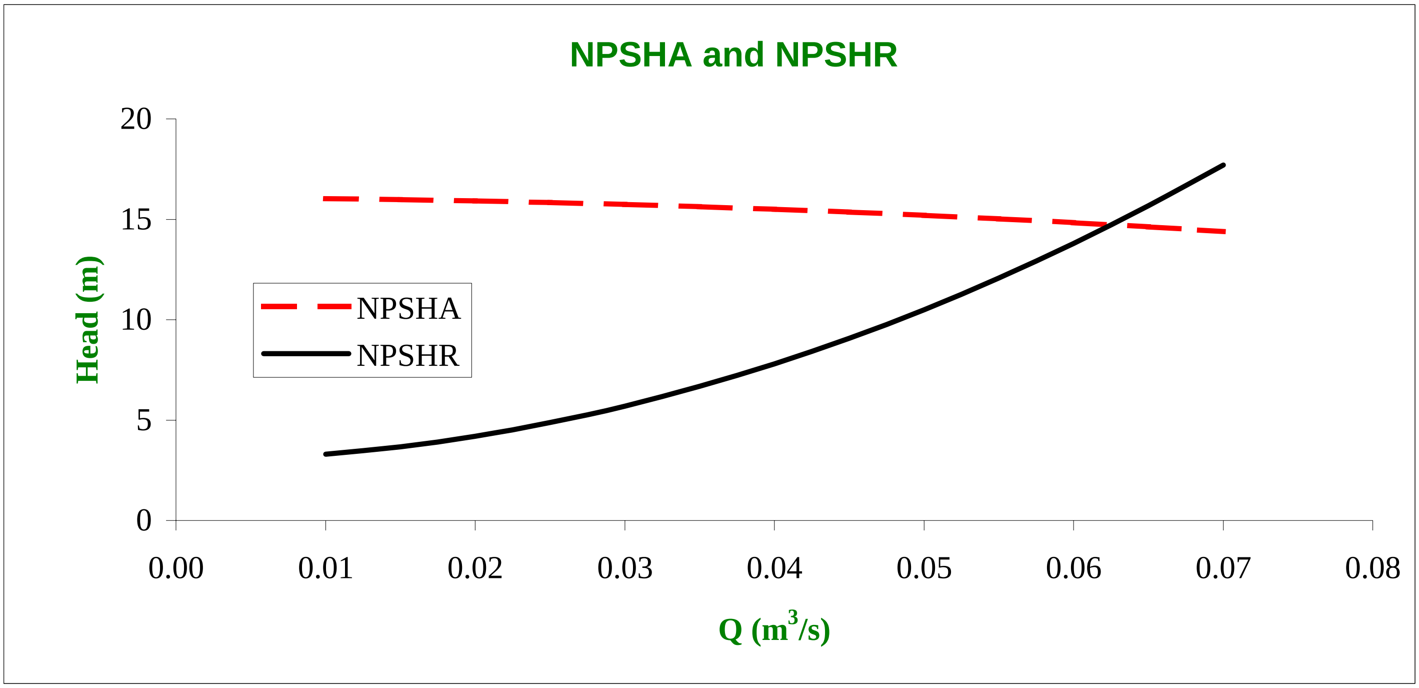
Given data:

L =	6	m
e =	0.26	mm
D =	15	cm
K_{ent} =	0.5	
L_e/D =	30	
H_0 =	3	m
A =	3000	$\text{m}/(\text{m}^3/\text{s})^2$
H =	6	m
p_{atm} =	101	kPa
p_v =	2.34	kPa
ρ =	1000	kg/m^3
ν =	1.01E-06	m^2/s

Computed results:

Q (m^3/s)	V (m/s)	Re	f	*NPSHA* (m)	*NPSHR* (m)
0.010	0.566	8.40E+04	0.0247	16.0	3.30
0.015	0.849	1.26E+05	0.0241	16.0	3.68
0.020	1.13	1.68E+05	0.0237	15.9	4.20
0.025	1.41	2.10E+05	0.0235	15.8	4.88
0.030	1.70	2.52E+05	0.0233	15.7	5.70
0.035	1.98	2.94E+05	0.0232	15.6	6.68
0.040	2.26	3.36E+05	0.0232	15.5	7.80
0.045	2.55	3.78E+05	0.0231	15.4	9.08
0.050	2.83	4.20E+05	0.0230	15.2	10.5
0.055	3.11	4.62E+05	0.0230	15.0	12.1
0.060	3.40	5.04E+05	0.0230	14.8	13.8
0.065	3.68	5.46E+05	0.0229	14.6	15.7
0.070	3.96	5.88E+05	0.0229	14.4	17.7

						Error
0.0625	3.54	5.25E+05	0.0229	14.7	14.7	0.00

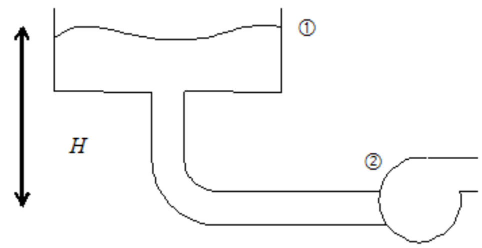


NPSHA and NPSHR
Head (m)
20
15
10
5
0
NPSHA
NPSHR
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
Q (m3/s)

Problem 10.55

[5]

10.55 For the pump and flow system of Problem 10.54, calculate the maximum flow rate for hot water at various temperatures and plot versus water temperature. (Be sure to consider the density variation as water temperature is varied.)



Given: Pump and supply pipe system

Find: Maximum operational flow rate as a function of temperature

Solution:

Basic equations:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^{\,2}}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^{\,2}}{2}+g\cdot z_2\right)=h_{lT} \qquad h_{lT}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2}$$

L_e for the elbow, and K for the square entrance

$$NPSHA=\frac{p_t-p_v}{\rho\cdot g} \qquad H_r=H_0+A\cdot Q^2$$

Assumptions: 1) $p_1 = 0$ 2) $V_1 = 0$ 3) $\alpha_2 = 0$ 4) $z_2 = 0$

We must match the NPSHR ($=H_r$) and NPSHA

From the energy equation

$$g\cdot H-\left(\frac{p_2}{\rho}+\frac{V^2}{2}\right)=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}+f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2}+K\cdot\frac{V^2}{2} \qquad \frac{p_2}{\rho\cdot g}=H-\frac{V^2}{2\cdot g}\left[1+f\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)+K\right]$$

$$NPSHA=\frac{p_t-p_v}{\rho\cdot g}=\frac{p_2}{\rho\cdot g}+\frac{p_{atm}}{\rho\cdot g}+\frac{V_2^{\,2}}{2\cdot g}-\frac{p_v}{\rho\cdot g} \qquad NPSHA=H-\frac{V^2}{2\cdot g}\left[f\cdot\left(\frac{L}{D}+\frac{L_e}{D}\right)+K\right]+\frac{(p_{atm}-p_v)}{\rho\cdot g}$$

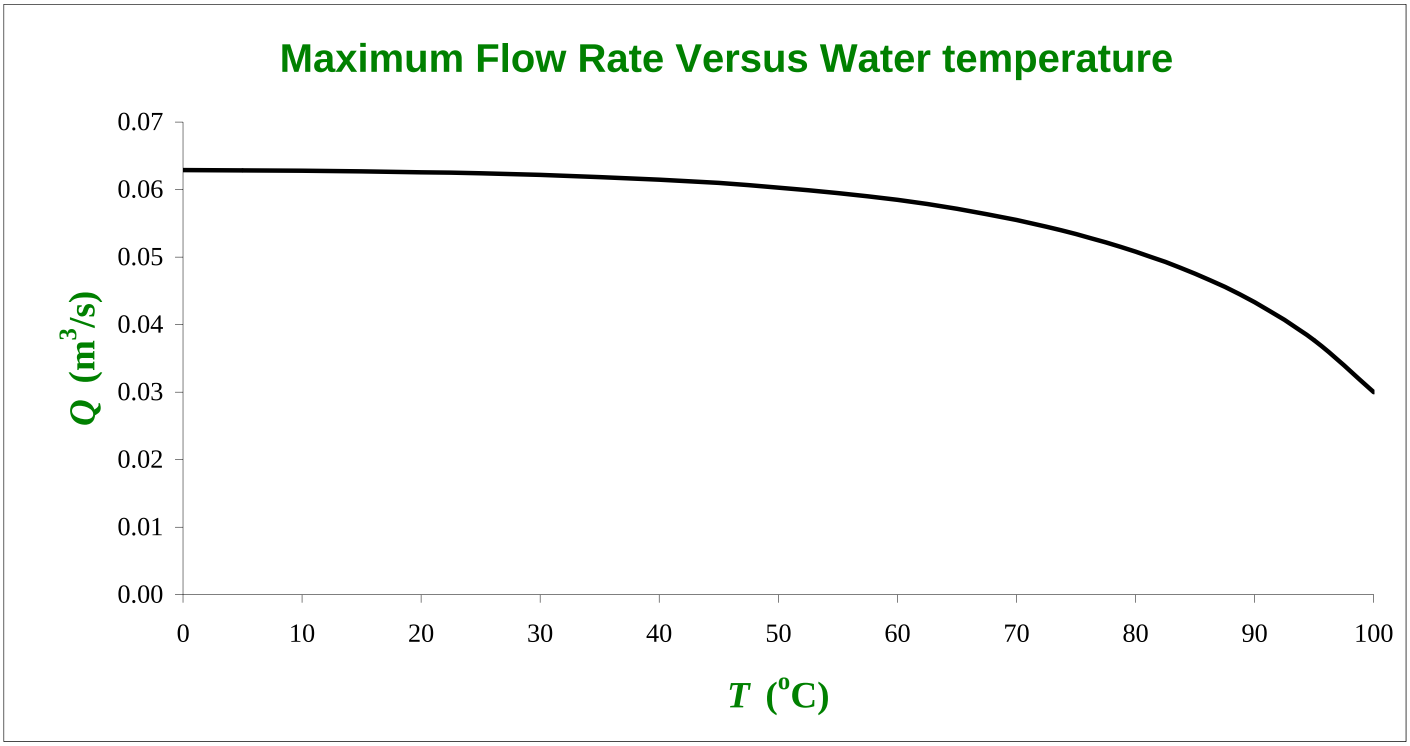
Given data:

L =	6	m
e =	0.26	mm
D =	15	cm
K_{ent} =	0.5	
L_e/D =	30	
H_0 =	3	m
A =	3000	$m/(m^3/s)^2$
H =	6	m
p_{atm} =	101	kPa
ρ =	1000	kg/m^3
ν =	1.01E-06	m^2/s

Computed results:

T (°C)	p_v (kPa)	ρ (kg/m^3)	ν (m^3/s)	*Q* (m^3/s)	*V* (m/s)	*Re*	*f*	*NPSHA* (m)	*NPSHR* (m)	*Error*
0	0.661	1000	1.76E-06	0.06290	3.56	3.03E+05	0.0232	14.87	14.87	0.00
5	0.872	1000	1.51E-06	0.06286	3.56	3.53E+05	0.0231	14.85	14.85	0.00
10	1.23	1000	1.30E-06	0.06278	3.55	4.10E+05	0.0230	14.82	14.82	0.00
15	1.71	999	1.14E-06	0.06269	3.55	4.67E+05	0.0230	14.79	14.79	0.00
20	2.34	998	1.01E-06	0.06257	3.54	5.26E+05	0.0229	14.75	14.75	0.00
25	3.17	997	8.96E-07	0.06240	3.53	5.91E+05	0.0229	14.68	14.68	0.00
30	4.25	996	8.03E-07	0.06216	3.52	6.57E+05	0.0229	14.59	14.59	0.00
35	5.63	994	7.25E-07	0.06187	3.50	7.24E+05	0.0228	14.48	14.48	0.00
40	7.38	992	6.59E-07	0.06148	3.48	7.92E+05	0.0228	14.34	14.34	0.00
45	9.59	990	6.02E-07	0.06097	3.45	8.60E+05	0.0228	14.15	14.15	0.00
50	12.4	988	5.52E-07	0.06031	3.41	9.27E+05	0.0228	13.91	13.91	0.00
55	15.8	986	5.09E-07	0.05948	3.37	9.92E+05	0.0228	13.61	13.61	0.00
60	19.9	983	4.72E-07	0.05846	3.31	1.05E+06	0.0228	13.25	13.25	0.00
65	25.0	980	4.40E-07	0.05716	3.23	1.10E+06	0.0227	12.80	12.80	0.00
70	31.2	978	4.10E-07	0.05548	3.14	1.15E+06	0.0227	12.24	12.24	0.00
75	38.6	975	3.85E-07	0.05342	3.02	1.18E+06	0.0227	11.56	11.56	0.00
80	47.4	972	3.62E-07	0.05082	2.88	1.19E+06	0.0227	10.75	10.75	0.00
85	57.8	969	3.41E-07	0.04754	2.69	1.18E+06	0.0227	9.78	9.78	0.00
90	70.1	965	3.23E-07	0.04332	2.45	1.14E+06	0.0227	8.63	8.63	0.00
95	84.6	962	3.06E-07	0.03767	2.13	1.05E+06	0.0228	7.26	7.26	0.00
100	101	958	2.92E-07	0.02998	1.70	8.71E+05	0.0228	5.70	5.70	0.00
Use *Solver* to make the sum of absolute errors between *NPSHA* and *NPSHR* zero by varying the *Q*'s										0.00

NPSHR increases with temperature because the p_v increases; NPHSA decreases because ρ decreases and p_v increases



Problem 10.56

Given: Centrifugal pump, operating at $N = 2265$ rpm, lifts water between two reservoirs connected by two cast-iron pipes in series.

$L_1 = 300$ ft, $D_1 = 6$ in.; $L_2 = 100$ ft, $D_2 = 3$ in.; $\Delta z = 25$ ft.

Find: (a) head requirement, (b) power need, and (c) hourly cost of electrical energy, for $Q = 200$ gpm if electricity costs 12¢/kw.hr and $\eta_m = 0.85$

Solution:

Apply the energy equation to the total system for steady, incompressible flow using ③ and ④ at reservoir surfaces.

Computing eq.: $\cancel{\frac{p_3}{\rho g}} + \cancel{\alpha_3 \frac{V_3^2}{2g}} + z_3 + H_a = \cancel{\frac{p_4}{\rho g}} + \cancel{\alpha_4 \frac{V_4^2}{2g}} + z_4 + \frac{h_{l_T}}{g}$ (10.24b)

$$h_{l_T} = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2} + f_2 \frac{L_2}{D_2} \frac{V_2^2}{2}$$

Assumptions: (1) $p_3 = p_4 = p_{atm}$, $V_3 = V_4 \approx 0$
(2) neglect minor losses

Then

$$H_a = z_4 - z_3 + f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} \quad \text{------ (1)}$$

$V_1 = \frac{Q}{A_1} = 200 \frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} \times \frac{4}{\pi (0.50)^2 ft^2} = 2.27$ ft/s

$V_2 = 9.08$ ft/s. For water at 59°F, $\nu = 1.23 \times 10^{-5}$ ft²/s (Table A.7)

$Re_1 = \frac{V_1 D_1}{\nu} = 2.27 \frac{ft}{s} \times \frac{6}{12} ft \times \frac{s}{1.23 \times 10^{-5}\,ft^2} = 9.23 \times 10^4$

$Re_2 = 1.85 \times 10^5$. From Table 8.1 for cast iron, $e = 0.00085$ ft

$e/D_1 = 0.0017$, $e/D_2 = 0.0034$

From Eq. 8.37, $f_1 = 0.0244$, $f_2 = 0.0278$. Substituting into (1)

$H_a = 25\,ft + 0.0244 \times \frac{300 \times 12}{6} \times \frac{(2.27)^2}{2} \frac{ft^2}{s^2} \times \frac{s^2}{32.2\,ft} + 0.0278 \times \frac{100 \times 12}{3} \times \frac{(9.08)^2}{2} \frac{ft^2}{s^2} \times \frac{s^2}{32.2\,ft}$

$H_a = 40.4$ ft ← H_a

Specific speed $N_{s_{cu}} = \frac{N Q^{1/2}}{H^{3/4}} = \frac{2265 (200)^{1/2}}{(40.4)^{3/4}} = 2000$

From Fig 10.15, $\eta_p \approx 0.75$. Then $P_m = \frac{\dot{W}_h}{\eta_p} = \frac{\rho Q g H}{\eta_p}$

$P_m = \frac{1}{0.75} \times 200 \frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} \times 62.4 \frac{lbf}{ft^3} \times 40.4\,ft \times \frac{hp \cdot s}{550\,ft \cdot lbf} = 2.72$ hp ← P_m

Cost $= c P_e$. Since $c = \$0.12$/kw.hr & $\eta_m = 0.85$

Then $P_e = \frac{P_m}{\eta_m}$ and

Cost $= c \frac{P_m}{\eta_m} = \frac{\$0.12}{kw \cdot hr} \times \frac{2.72\,hp}{0.85} \times 0.746 \frac{kw}{hp} = 28.7$¢/hr ← C

Problem 10.57

Given: Water supply for Grand Canyon National Park,
$L = 13{,}200$ ft, $Q = 600$ gpm
Location 1: Colorado River, $z_1 = 3734$ ft
Location 2: South Rim, $z_2 = 7022$ ft in storage tank.
Head loss due to friction is $h_{\ell T}/g = 290$ ft

Find: (a) estimate diameter of commercial steel pipe.
(b) pumping power if $\eta_p = 0.61$.

Solution:

Apply the energy equation to the total system for steady incompressible flow using ① and ② at inlet and reservoir surface respectively.

Computing eq.: $\frac{p_1}{\rho g} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + H_a = \frac{p_2}{\rho g} + \alpha_2 \frac{\bar{V}_2^2}{2} + z_2 + \frac{h_{\ell T}}{g}$ (10.24b)

$h_{\ell T} = f \frac{L}{D} \frac{V^2}{2}$

Assumptions: (1) $p_1 = p_2 = p_{atm}$, $\bar{V}_1 \approx \bar{V}_2 \approx 0$
(2) neglect minor losses.

Then $H_a = z_2 - z_1 + \frac{h_{\ell T}}{g}$ ......(1) and $\frac{h_{\ell T}}{g} = f \frac{L}{D} \frac{V^2}{2g} = 290$ ft

Since $f = f(Re, e/D)$ and D is unknown, we must iterate.

For commercial steel, $e = 0.00015$ ft

The procedure is
- assume D, calculate V, Re; determine f (Eqs 8.37a, b); calculate $h_{\ell T}/g$ and compare to value of 290 ft.

D (in.)	V (ft/s)	Re	f_0	$f^{0.5}$	f	h_{lT}/g (ft)
12	1.70	1.46E+06	0.0139	8.517	0.0138	8.2
10	2.45	1.75E+06	0.0141	8.433	0.0141	20.8
8	3.83	2.19E+06	0.0146	8.306	0.0145	65.3
6	6.81	2.92E+06	0.0153	8.111	0.0152	289

Thus $D = 6.0$ in ← D

The total pump head is $H_a = 7022 - 3734 + 290 = 3578$ ft.

The pump power is $P_m = \frac{\dot{W}_h}{\eta_p} = \frac{\rho g Q H}{\eta_p}$

$P_m = \frac{1}{0.61} \times 62.4 \frac{lbf}{ft^3} \times 600 \frac{gal}{min} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} \times 3578\,ft \times \frac{hp \cdot s}{550\,ft \cdot lbf}$

$P_m = 890$ hp ← P_m

Problem 10.58

10.58 A centrifugal pump is installed in a piping system with $L = 300$ m of $D = 40$ cm cast-iron pipe. The downstream reservoir surface is 15 m lower than the upstream reservoir. Determine and plot the system head curve. Find the volume flow rate (magnitude and direction) through the system when the pump is not operating. Estimate the friction loss, power requirement, and hourly energy cost to pump water at 1 m^3/s through this system.

Given: Pump and reservoir system

Find: System head curve; Flow rate when pump off; Loss, Power required and cost for 1 m^3/s flow rate

Solution:

Basic equations: $\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_{lT} - h_p$ $\qquad h_{lT} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} + \Sigma K \cdot \frac{V^2}{2}$ (K for the exit)

where points 1 and 2 are the reservoir free surfaces, and h_p is the pump head

Note also $\quad H = \frac{h}{g} \qquad$ Pump efficiency: $\quad \eta_p = \frac{W_h}{W_m}$

Assumptions: 1) $p_1 = p_2 = p_{atm}$ 2) $V_1 = V_2 = 0$ 3) $\alpha_2 = 0$ 4) $z_1 = 0,\ z_2 = -15 \cdot m$ 4) $K = K_{ent} + K_{ent} = 1.5$

From the energy equation $-g \cdot z_2 = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} - h_p + K \cdot \frac{V^2}{2} \qquad h_p = g \cdot z_2 + f \cdot \frac{L}{D} \cdot \frac{V^2}{2} + K \cdot \frac{V^2}{2} \qquad H_p = z_2 + f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g} + K \cdot \frac{V^2}{2 \cdot g}$

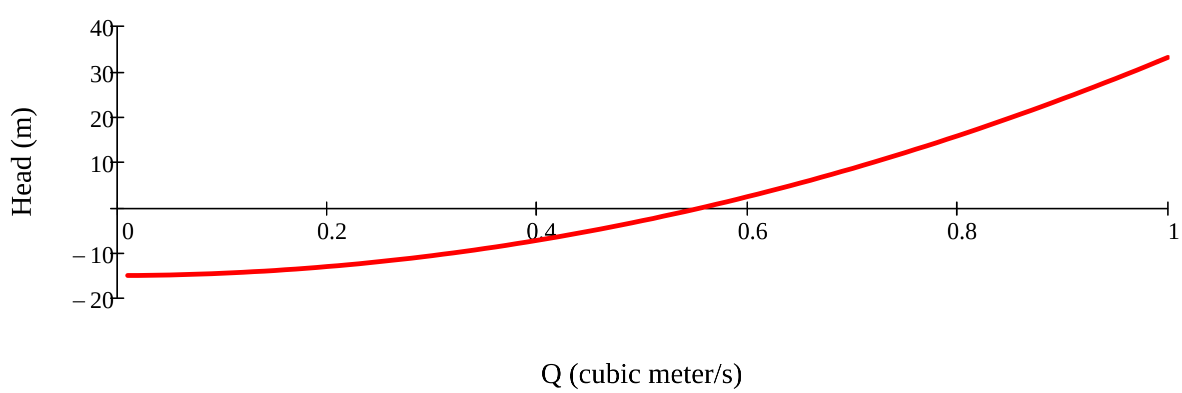
Given or available data $\quad L = 300 \cdot m \qquad D = 40 \cdot cm \qquad e = 0.26 \cdot mm \quad$ (Table 8.1)

$\rho = 1000 \cdot \frac{kg}{m^3} \qquad \nu = 1.01 \times 10^{-6} \cdot \frac{m^2}{s} \quad$ (Table A.8)

The set of equations to solve for each flow rate Q are

$$V = \frac{4 \cdot Q}{\pi \cdot D^2} \qquad Re = \frac{V \cdot D}{\nu} \qquad \frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right) \qquad H_p = z_2 + f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g} + K \cdot \frac{V^2}{2 \cdot g}$$

For example, for $\quad Q = 1 \cdot \frac{m^3}{s} \qquad V = 7.96 \cdot \frac{m}{s} \qquad Re = 3.15 \times 10^6 \qquad f = 0.0179 \qquad H_p = 33.1 \cdot m$



The above graph can be plotted in Excel. In Excel, Solver can be used to find Q for $H_p = 0$ $\quad Q = 0.557\frac{m^3}{s}$ (Zero power rate)

At $\quad Q = 1\cdot\frac{m^3}{s}$ we saw that $\quad H_p = 33.1\cdot m$

Assuming optimum efficiency at $Q = 1.59\times 10^4\cdot gpm$ from Fig. 10.15 $\quad \eta_p = 92\cdot\%$

Then the hydraulic power is $\quad W_h = \rho\cdot g\cdot H_p\cdot Q \quad\quad W_h = 325\cdot kW$

The pump power is then $\quad W_m = \frac{W_h}{\eta_p} \quad\quad W_m\cdot 2 = 706\cdot kW$

If electricity is 10 cents per kW-hr then the hourly cost is about $35

If electricity is 15 cents per kW-hr then the hourly cost is about $53

If electricity is 20 cents per kW-hr then the hourly cost is about $71

Problem 10.59

Given: Peerless horizontal split-case 4AE12 pump with 11 in diameter impeller, operating at 1750 rpm, lifts water between two reservoirs connected by two cast-iron pipes in series.

$L_1 = 200$ ft, $D_1 = 4$ in.; $L_2 = 200$ ft, $D_2 = 3$ in.; $\Delta z = 10$ ft

Plot the system head curve and determine the pump operating point.

Solution:

Apply the energy equation to the total system for steady, incompressible flow using ③ and ④ at reservoir surfaces.

Computing eq.: $$\frac{p_3}{\rho g} + \alpha_3 \frac{\bar{V}_3^2}{2g} + z_3 + H_a = \frac{p_4}{\rho g} + \alpha_4 \frac{\bar{V}_4^2}{2g} + z_4 + \frac{h_{\ell T}}{g} \quad (10.24b)$$

$$h_{\ell T} = f_1 \frac{L_1}{D_1} \frac{\bar{V}_1^2}{2g} + f_2 \frac{L_2}{D_2} \frac{\bar{V}_2^2}{2g}$$

Assumptions: (1) $p_3 = p_4 = p_{atm}$, $V_3 = V_4 \approx 0$
(2) neglect minor losses

then
$$H_a = z_4 - z_3 + f_1 \frac{L_1}{D_1} \frac{\bar{V}_1^2}{2g} + f_2 \frac{L_2}{D_2} \frac{\bar{V}_2^2}{2g} \quad \text{------- (1)}$$

Express V as a function of Q

$$V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4}{\pi} \times \left(\frac{12}{4}\right)^2 \frac{1}{ft^2} Q\left(\frac{gal}{min}\right) \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60s} = 0.0255\, Q(gpm)$$

$V_2 = 0.0454\, Q(gpm)$

The friction factor is determined from the Colebrook eq.

$$\frac{1}{f^{0.5}} = -2.0 \log\left(\frac{e/D}{3.7} + \frac{2.51}{Re\, f^{0.5}}\right) \quad (8.37a)$$

using the equation of Miller for the original estimate

$$f_0 = 0.25\left[\log\left(\frac{e/D}{3.7} + \frac{5.74}{Re^{0.9}}\right)\right]^{-2} \quad (8.37b)$$

Assuming $T = 59°F$, $\nu = 1.23 \times 10^{-5}\ ft^2/s$ (Table A.7)

$Re_1 = \frac{DV_1}{\nu} = \frac{4}{12} \times \frac{10^5}{1.23} V_1 = 2.71 \times 10^4\, V_1$, $Re_2 = 2.03 \times 10^4\, V_2$

For cast iron, $e = 0.00085$ ft (Table 8.1)

$e/D_1 = 0.00255 \qquad e/D_2 = 0.00340$

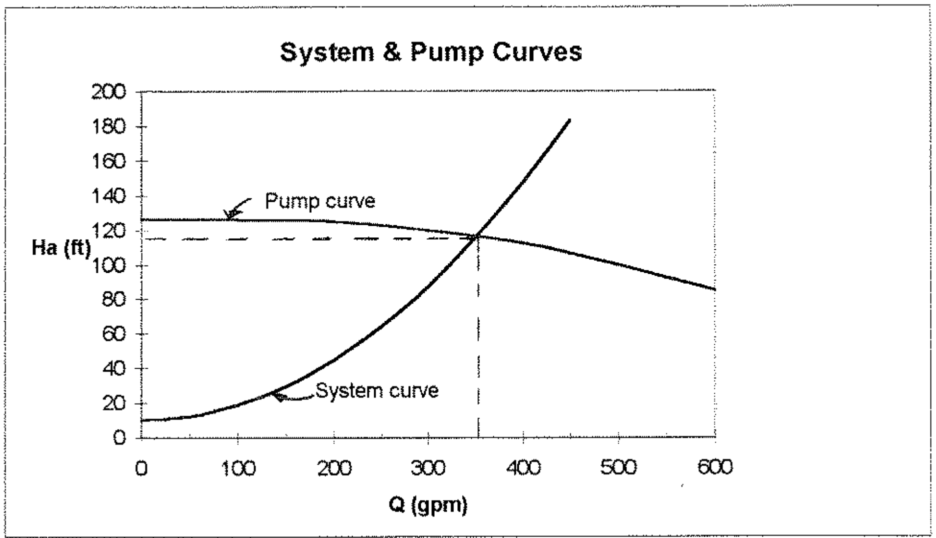
$$H_a = 10\,ft + f_1 \times 9.317\, V_1^2 + f_2 \times 12.42\, V_2^2$$

The pump curve is obtained from Fig. D.4

Problem 10.59

Q (gpm)	V_1 (ft/s)	Re_1	$f_0)_1$	$f^{-0.5})_1$	f_1	V_2 (ft/s)	Re_2	$f_0)_2$	$f^{-0.5})_2$	f_2	H_a (ft)
0	0.0	0.00E+00				0					10.0
50	1.3	3.46E+04	0.0290	5.905	0.0287	2.3	4.61E+04	0.0299	5.817	0.0295	12
100	2.6	6.91E+04	0.0273	6.083	0.0270	4.5	9.22E+04	0.0287	5.933	0.0284	19
150	3.8	1.04E+05	0.0266	6.154	0.0264	6.8	1.38E+05	0.0282	5.977	0.0280	30
200	5.1	1.38E+05	0.0263	6.193	0.0261	9.1	1.84E+05	0.0280	6.000	0.0278	45
250	6.4	1.73E+05	0.0261	6.217	0.0259	11.4	2.30E+05	0.0278	6.014	0.0277	64
300	7.7	2.07E+05	0.0259	6.233	0.0257	13.6	2.76E+05	0.0277	6.023	0.0276	88
350	8.9	2.42E+05	0.0258	6.245	0.0256	15.9	3.23E+05	0.0276	6.030	0.0275	115
400	10.2	2.76E+05	0.0257	6.255	0.0256	18.2	3.69E+05	0.0276	6.036	0.0275	147
450	11.5	3.11E+05	0.0256	6.262	0.0255	20.4	4.15E+05	0.0275	6.040	0.0274	183

Q (gpm)	H_p (ft)
0	126
100	126
200	125
300	120
400	113
500	100
600	85



Pump operates at Q = 350 gpm.
H = 115 ft

Problem 10.60

10.60 A pump transfers water from one reservoir to another through two cast-iron pipes in series. The first is 3000 ft of 9 in. pipe and the second is 1000 ft of 6 in. pipe. A constant flow rate of 75 gpm is tapped off at the junction between the two pipes. Obtain and plot the system head versus flow rate curve. Find the delivery if the system is supplied by the pump of Example 10.7, operating at 1750 rpm.

Given: Data on pump and pipe system

Find: Delivery through system

Solution:

Given or available data:

$L_1 =$	3000	ft	$\nu =$	1.23E-05	ft^2/s (Table A.7)
$D_1 =$	9	in	$K_{ent} =$	0.5	(Fig. 8.14)
$L_2 =$	1000	ft	$K_{exp} =$	1	
$D_2 =$	6	in	$Q_{loss} =$	75	gpm
$e =$	0.00085	ft (Table 8.1)			

Governing Equations:

For the pump and system

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} - \Delta h_{pump} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f \frac{L}{D} \frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = K \frac{\bar{V}^2}{2} \qquad (8.40a)$$

and the pump head (in energy/mass) is given by (from Example 10.7)

$$H_{pump}(ft) = 55.9 - 3.44 \times 10^{-5} \cdot Q(gpm)^2$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 = z_2$) we have

$$0 = h_{lT} - \Delta h_{pump}$$

$$h_{lT} = g \cdot H_{system} = \Delta h_{pump} = g \cdot H_{pump}$$

or

$$H_{lT} = H_{pump} \qquad (1)$$

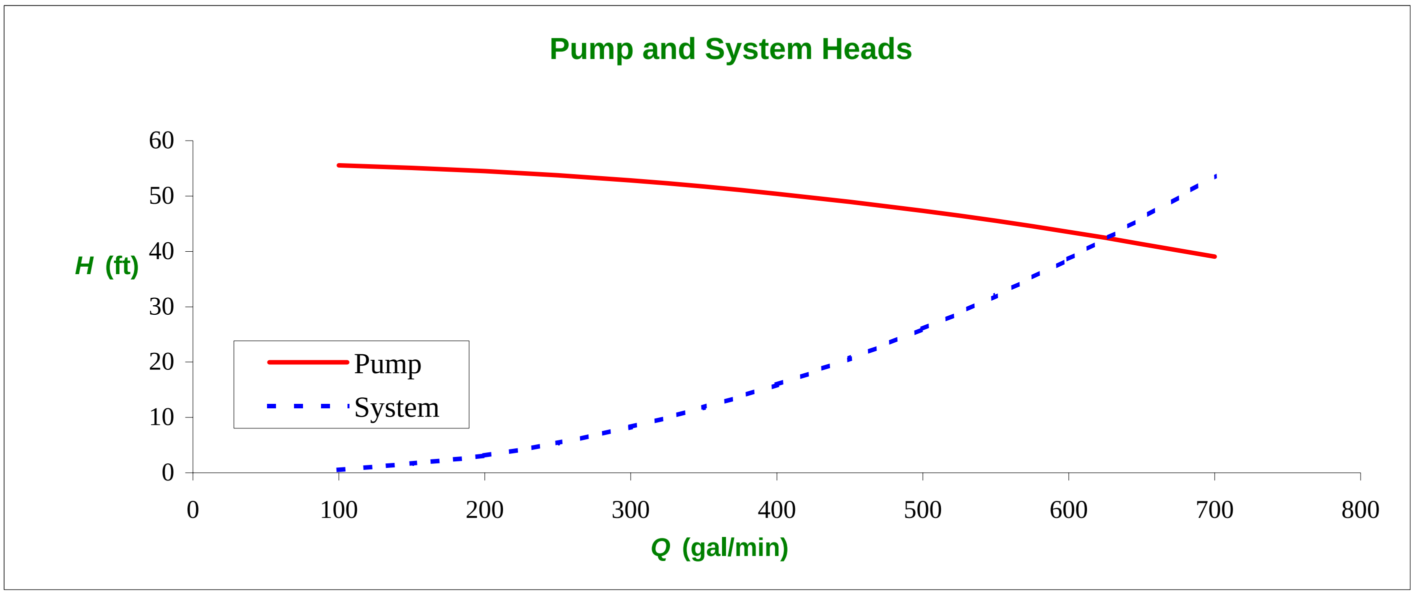
where

$$H_{lT} = \left(f_1 \cdot \frac{L_1}{D_1} + K_{ent}\right) \cdot \frac{V_1^2}{2 \cdot g} + \left(f_2 \cdot \frac{L_2}{D_2} + K_{exit}\right) \cdot \frac{V_2^2}{2}$$

The system and pump heads are computed and plotted below.
To find the operating condition, *Goal Seek* is used to vary Q_1
so that the error between the two heads is zero.

Q_1 (gpm)	Q_2 (gpm)	V_1 (ft/s)	V_2 (ft/s)	Re_1	Re_2	f_1	f_2	H_{IT} (ft)	H_{pump} (ft)
100	25	0.504	0.284	30753	11532	0.0262	0.0324	0.498	55.6
200	125	1.01	1.42	61506	57662	0.0238	0.0254	3.13	54.5
300	225	1.51	2.55	92260	103792	0.0228	0.0242	8.27	52.8
400	325	2.02	3.69	123013	149922	0.0222	0.0237	15.9	50.4
500	425	2.52	4.82	153766	196052	0.0219	0.0234	26.0	47.3
600	525	3.03	5.96	184519	242182	0.0216	0.0233	38.6	43.5
700	625	3.53	7.09	215273	288312	0.0215	0.0231	53.6	39.0

Q_1 (gpm)	Q_2 (gpm)	V_1 (ft/s)	V_2 (ft/s)	Re_1	Re_2	f_1	f_2	H_{IT} (ft)	H_{pump} (ft)	Error)
627	552	3.162	6.263	192785	254580	0.0216	0.0232	42.4	42.4	0%



Problem 10.61

10.61 Performance data for a pump are

H (m)	27.5	27	25	22	18	13	6.5
Q (m^3/s)	0	0.025	0.050	0.075	0.100	0.125	0.150

The pump is to be used to move water between two open reservoirs with an elevation increase of 7.5 m. The connecting pipe system consists of 500 m of commercial steel pipe containing two 90° elbows and an open gate valve. Find the flow rate if we use a) 20 cm, b) 30 cm, and c) 40 cm pipe.

Given: Pump and reservoir/pipe system

Find: Flow rate using different pipe sizes

Solution:

Basic equations:

$$\left(\frac{p_1}{\rho} + \alpha_1 \cdot \frac{V_1^2}{2} + g \cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \cdot \frac{V_2^2}{2} + g \cdot z_2\right) = h_{lT} - h_p$$

$$h_{lT} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} + \Sigma f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2} + \Sigma K \cdot \frac{V^2}{2}$$

L_e for the elbows, and K for the square entrance and exit

and also $H = \frac{h}{g}$

Assumptions: 1) $p_1 = p_2 = p_{atm}$ 2) $V_1 = V_2 = 0$ 3) $\alpha = 0$ 4) $z_1 = 0,\ z_2 = 7.5 \cdot m$ 4) $K = K_{ent} + K_{ent}$ 5) $\frac{L_e}{D}$ is for two elbows

Hence

$$h_{lT} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2} + f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2} + K \cdot \frac{V^2}{2}$$

and also $-z_2 = h_{lT} - h_p$ or $h_{lT} = h_p - z_2$

We want to find a flow that satisfies these equations, rewritten as energy/weight rather than energy/mass

$$H_{lT} = \left[f \cdot \left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right] \cdot \frac{V^2}{2 \cdot g} \qquad H_{lT} + z_2 = H_p$$

Given or available data **(Note: final results will vary depending on fluid data selected)**:

L =	500	m	K_{ent} =	0.5	(Fig. 8.14)
e =	0.046	mm (Table 8.1)	K_{exp} =	1	
D =	20	cm	L_e/D_{elbow} =	60	(Two)
ν =	1.01E-06	m^2/s (Table A.8)	L_e/D_{valve} =	8	(Table 8.4)
z_2 =	7.5	m			

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

Q (m^3/s)	Q^2	H_p (m)
0.000	0.00000	27.5
0.025	0.00063	27.0
0.050	0.00250	25.0
0.075	0.00563	22.0
0.100	0.01000	18.0
0.125	0.01563	13.0
0.150	0.02250	6.5

V (m/s)	Re	f
0.00	0	0.0000
0.80	157579	0.0179
1.59	315158	0.0164
2.39	472737	0.0158
3.18	630317	0.0154
3.98	787896	0.0152
4.77	945475	0.0150

H_p (fit)	$H_{lT} + z_2$ (m)
27	7.5
27	9.0
25	13.1
22	19.7
18	28.7
12.9	40.2
6.5	54.1

$H_0 =$ 27 m

$A =$ 9.30E+02 /(m^3/s)^2

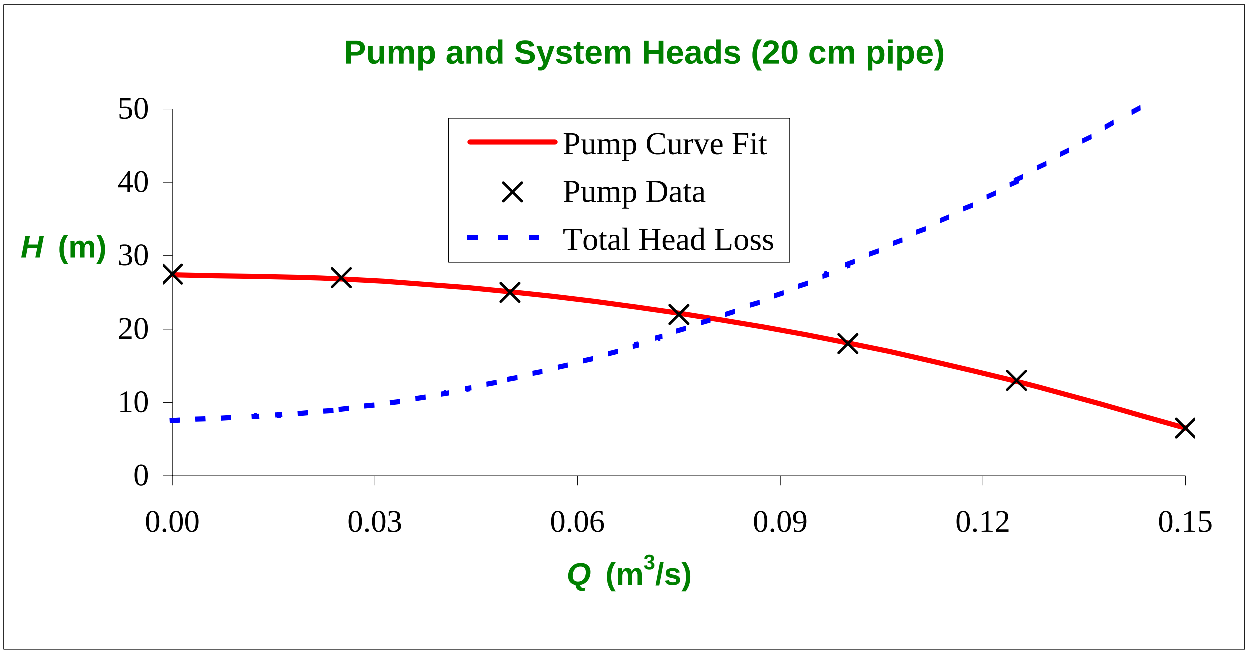
Q (m^3/s)	V (m/s)	Re	f	H_p (fit)	$H_{lT} + z_2$ (m)	Error)
0.0803	2.56	506221	0.0157	21.4	21.4	0.00%

Repeating for: D = 30 cm

Q (m^3/s)	V (m/s)	Re	f	H_p (fit)	$H_{lT} + z_2$ (m)	Error)
0.1284	1.82	539344	0.0149	12.1	12.1	0.00%

Repeating for: D = 40 cm

Q (m^3/s)	V (m/s)	Re	f	H_p (fit)	$H_{lT} + z_2$ (m)	Error)
0.1413	1.12	445179	0.0148	8.9	8.9	0.00%



Problem 10.62

10.62 Performance data for a pump are

H (ft)	179	176	165	145	119	84	43
Q (gpm)	0	500	1000	1500	2000	2500	3000

Estimate the delivery when the pump is used to move water between two open reservoirs, through 1200 ft of 12 in. commercial steel pipe containing two 90° elbows and an open gate valve, if the elevation increase is 50 ft. Determine the gate valve loss coefficient needed to reduce the volume flow rate by half.

Given: Data on pump and pipe system

Find: Delivery through system; valve position to reduce delivery by half

Solution:

Given or available data **(Note: final results will vary depending on fluid data selected)**:

$L =$ 1200 ft

$D =$ 12 in

$e =$ 0.00015 ft (Table 8.1)

$\nu =$ 1.23E-05 ft^2/s (Table A.7)

$\Delta z =$ -50 ft

$K_{ent} =$ 0.5 (Fig. 8.14)

$K_{exp} =$ 1

$L_e/D_{elbow} =$ 30

$L_e/D_{valve} =$ 8 (Table 8.4)

Governing Equations:

For the pump and system

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} - \Delta h_{pump} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2} \qquad (8.40b)$$

$$h_{l_m} = K\frac{\bar{V}^2}{2} \qquad (8.40a)$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0,\ V_1 = V_2 = 0,\ z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} + \Delta z = H_{pump}$$

where

$$H_{lT} = \left[f\cdot\left(\frac{L}{D} + 2\cdot\frac{L_e}{D_{elbow}} + \frac{L_e}{D_{valve}}\right) + K_{ent} + K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

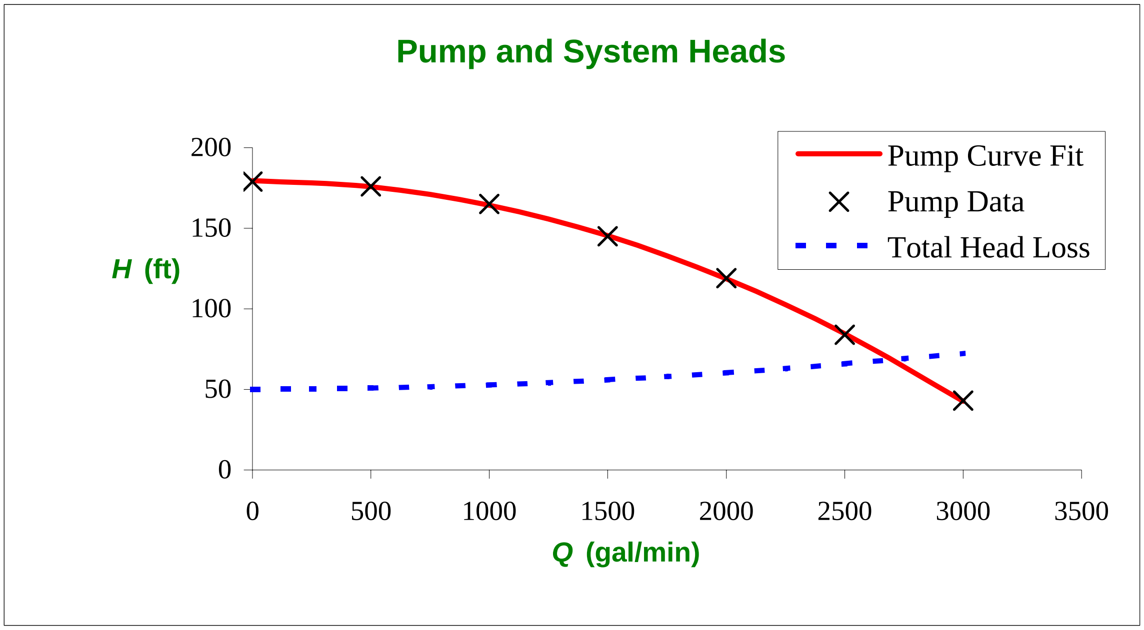
Q (gpm)	Q^2 (gpm)	H_{pump} (ft)
0	0	179
500	250000	176
1000	1000000	165
1500	2250000	145
2000	4000000	119
2500	6250000	84
3000	9000000	43

V (ft/s)	Re	f
0.00	0	0.0000
1.42	115325	0.0183
2.84	230649	0.0164
4.26	345974	0.0156
5.67	461299	0.0151
7.09	576623	0.0147
8.51	691948	0.0145

H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)
180	50.0
176	50.8
164	52.8
145	56.0
119	60.3
84.5	65.8
42.7	72.4

H_0 = 180 ft
A = 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2705	7.67	623829	0.0146	68.3	68.3	0%



For the valve setting to reduce the flow by half, use *Solver* to vary the value below to minimize the error.

L_e/D_{valve} = 26858

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
1352	3.84	311914	0.0158	151.7	151.7	0%

Problem 10.63

10.63 Consider again the pump and piping system of Problem 10.62. Determine the volume flow rate and gate valve loss coefficient for the case of two identical pumps installed in *series*.

Given: Data on pump and pipe system

Find: Delivery through series pump system; valve position to reduce delivery by half

Solution:

Given or available data **(Note: final results will vary depending on fluid data selected)**:

L =	1200	ft	K_{ent} =	0.5	(Fig. 8.14)
D =	12	in	K_{exp} =	1	
e =	0.00015	ft (Table 8.1)	L_e/D_{elbow} =	30	
ν =	1.23E-05	ft^2/s (Table A.7)	L_e/D_{valve} =	8	(Table 8.4)
Δz =	-50	ft			

Governing Equations:

For the pumps and system

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)=h_{l_T}-\Delta h_{pump} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2} \qquad (8.40b)$$

$$h_{l_m} = K\frac{\bar{V}^2}{2} \qquad (8.40a)$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} + \Delta z = H_{pump}$$

where

$$H_{lT} = \left[f\cdot\left(\frac{L}{D}+2\cdot\frac{L_e}{D_{elbow}}+\frac{L_e}{D_{valve}}\right)+K_{ent}+K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

For pumps in series

$$H_{pump} = 2\cdot H_0 - 2\cdot A\cdot Q^2$$

where for a single pump

$$H_{pump} = H_0 - A\cdot Q^2$$

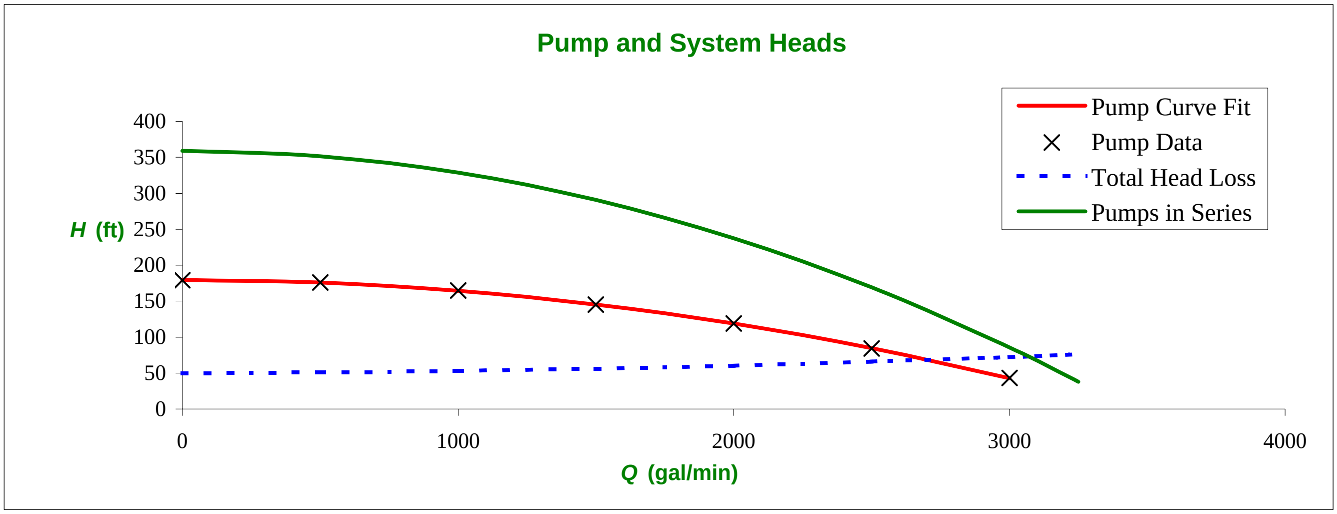
The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

Q (gpm)	Q^2 (gpm)	H_{pump} (ft)	H_{pump} (fit)	V (ft/s)	Re	f
0	0	179	180	0.00	0	0.0000
500	250000	176	176	1.42	115325	0.0183
1000	1000000	165	164	2.84	230649	0.0164
1500	2250000	145	145	4.26	345974	0.0156
2000	4000000	119	119	5.67	461299	0.0151
2500	6250000	84	85	7.09	576623	0.0147
3000	9000000	43	43	8.51	691948	0.0145
3250				9.22	749610	0.0144

H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)
359	50.0
351	50.8
329	52.8
291	56.0
237	60.3
169	65.8
85	72.4
38	76.1

H_0 = 180 ft
A = 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
3066	8.70	707124	0.0145	73.3	73.3	0%



For the valve setting to reduce the flow by half, use *Solver* to vary the value below to minimize the error.

L_e/D_{valve} = 50723

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
1533	4.35	353562	0.0155	287.7	287.7	0%

Problem 10.64

10.64 Consider again the pump and piping system of Problem 10.62. Determine the volume flow rate and gate valve loss coefficient for the case of two identical pumps installed in *parallel*.

Given: Data on pump and pipe system

Find: Delivery through parallel pump system; valve position to reduce delivery by half

Solution:

Given or available data **(Note: final results will vary depending on fluid data selected)**:

L =	1200	ft	K_{ent} =	0.5	(Fig. 8.14)
D =	12	in	K_{exp} =	1	
e =	0.00015	ft (Table 8.1)	L_e/D_{elbow} =	30	
ν =	1.23E-05	ft^2/s (Table A.7)	L_e/D_{valve} =	8	(Table 8.4)
Δz =	-50	ft			

Governing Equations:

For the pumps and system

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} - \Delta h_{pump} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2} \qquad (8.40b)$$

$$h_{l_m} = K\frac{\bar{V}^2}{2} \qquad (8.40a)$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} + \Delta z = H_{pump}$$

where

$$H_{lT} = \left[f\cdot\left(\frac{L}{D} + 2\cdot\frac{L_e}{D_{elbow}} + \frac{L_e}{D_{valve}}\right) + K_{ent} + K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

For pumps in parallel

$$H_{pump} = H_0 - \frac{1}{4}\cdot A\cdot Q^2$$

where for a single pump

$$H_{pump} = H_0 - A\cdot Q^2$$

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

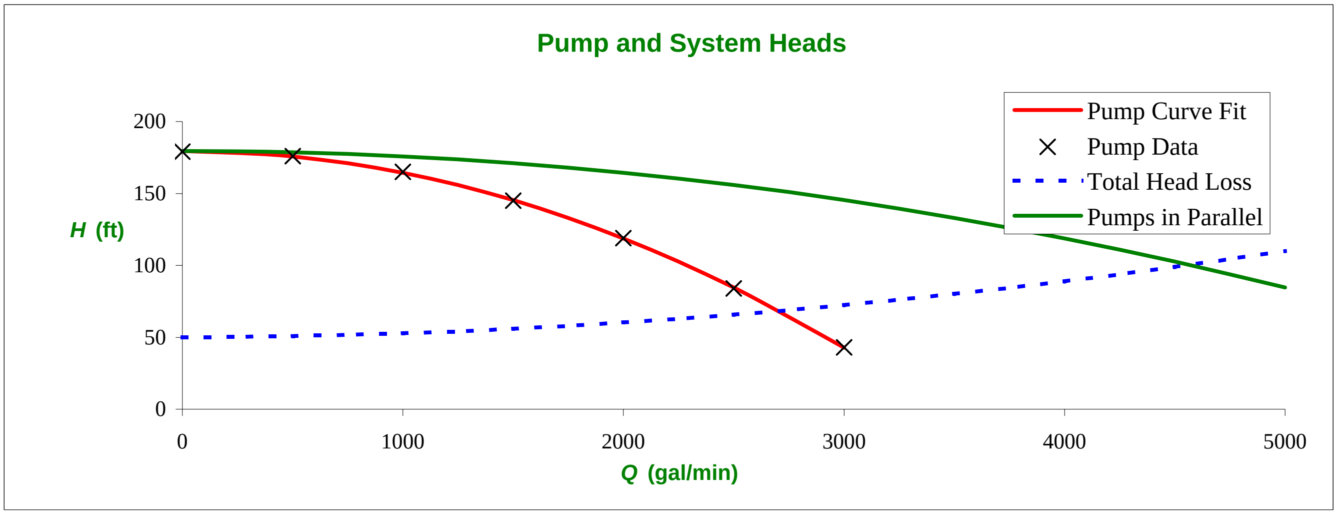
Q (gpm)	Q^2 (gpm)	H_{pump} (ft)	H_{pump} (fit)	V (ft/s)	Re	f
0	0	179	180	0.00	0	0.0000
500	250000	176	176	1.42	115325	0.0183
1000	1000000	165	164	2.84	230649	0.0164
1500	2250000	145	145	4.26	345974	0.0156
2000	4000000	119	119	5.67	461299	0.0151
2500	6250000	84	85	7.09	576623	0.0147
3000	9000000	43	43	8.51	691948	0.0145
3500				9.93	807273	0.0143
4000				11.35	922597	0.0142
4500				12.77	1037922	0.0141
5000				14.18	1153247	0.0140

H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)
180	50.0
179	50.8
176	52.8
171	56.0
164	60.3
156	65.8
145	72.4
133	80.1
119	89.0
103	98.9
85	110.1

H_0 = 180 ft

A = 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
4565	12.95	1053006	0.0141	100.3	100.3	0%



For the valve setting to reduce the flow by half, use *Solver* to vary the value below to minimize the error.

L_e/D_{valve} = 9965

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
2283	6.48	526503	0.0149	159.7	159.7	0%

Problem 10.65

10.65 The resistance of a given pipe increases with age as deposits form, increasing the roughness and reducing the pipe diameter (see Fig. 8.14). Typical multipliers to be applied to the friction factor are given in [16]:

Pipe Age (years)	Small Pipes, 4–10 in.	Large Pipes, 12–60 in.
New	1.00	1.00
10	2.20	1.60
20	5.00	2.00
30	7.25	2.20
40	8.75	2.40
50	9.60	2.86
60	10.0	3.70
70	10.1	4.70

Consider again the pump and piping system of Problem 10.62. Estimate the percentage reductions in volume flow rate that occur after (a) 20 years and (b) 40 years of use, if the pump characteristics remain constant. Repeat the calculation if the pump head is reduced 10 percent after 20 years of use and 25 percent after 40 years.

Given: Data on pump and pipe system, and their aging

Find: Reduction in delivery through system after 20 and 40 years (aging and non-aging pumps)

Solution:

Given or available data **(Note: final results will vary depending on fluid data selected)** :

$L =$	1200	ft	$K_{ent} =$	0.5	(Fig. 8.14)
$D =$	12	in	$K_{exp} =$	1	
$e =$	0.00015	ft (Table 8.1)	$L_e/D_{elbow} =$	30	
$\nu =$	1.23E-05	ft^2/s (Table A.7)	$L_e/D_{valve} =$	8	(Table 8.4)
$\Delta z =$	-50	ft			

Governing Equations:

For the pump and system

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{\bar{V}_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{\bar{V}_2^2}{2}+gz_2\right)=h_{l_T}-\Delta h_{pump} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2} \qquad (8.40b)$$

$$h_{l_m} = K\frac{\bar{V}^2}{2} \qquad (8.40a)$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} + \Delta z = H_{pump}$$

where

$$H_{lT} = \left[f\cdot\left(\frac{L}{D}+2\cdot\frac{L_e}{D_{elbow}}+\frac{L_e}{D_{valve}}\right)+K_{ent}+K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

New System:

Q (gpm)	Q^2 (gpm)	H_{pump} (ft)
0	0	179
500	250000	176
1000	1000000	165
1500	2250000	145
2000	4000000	119
2500	6250000	84
3000	9000000	43

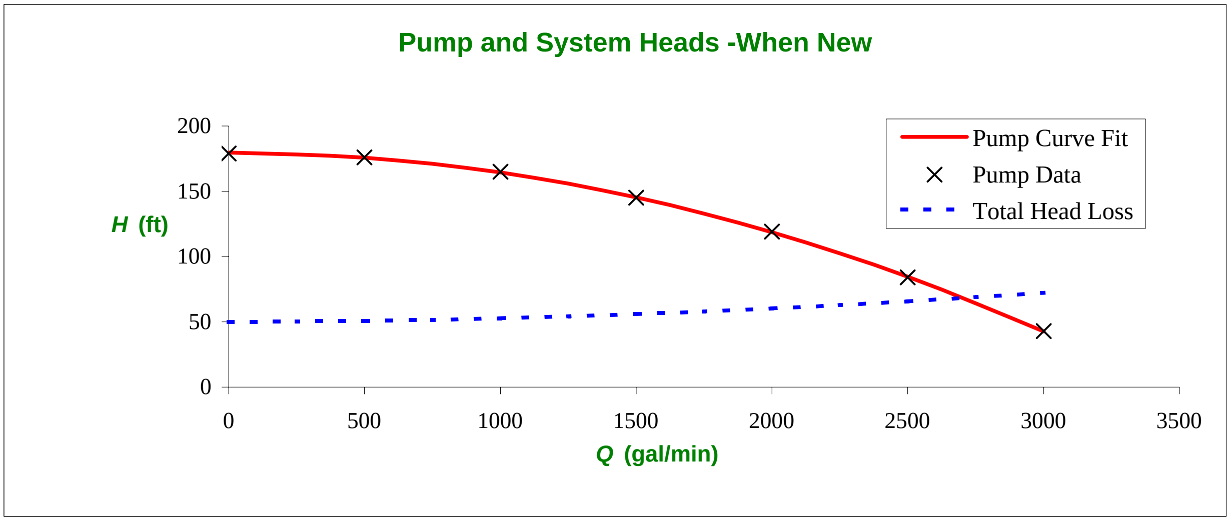
V (ft/s)	Re	f
0.00	0	0.0000
1.42	115325	0.0183
2.84	230649	0.0164
4.26	345974	0.0156
5.67	461299	0.0151
7.09	576623	0.0147
8.51	691948	0.0145

H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)
180	50.0
176	50.8
164	52.8
145	56.0
119	60.3
84.5	65.8
42.7	72.4

$H_0 =$ 180 ft

$A =$ 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2705	7.67	623829	0.0146	68.3	68.3	0%



20-Year Old System:

$f = 2.00\, f_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2541	7.21	586192	0.0295	81.4	81.4	0%

Flow reduction:

163 gpm
6.0% Loss

40-Year Old System:

$f = 2.40\, f_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2484	7.05	572843	0.0354	85.8	85.8	0%

Flow reduction:

221 gpm
8.2% Loss

20-Year Old System and Pump:

$f = 2.00\, f_{new}$ $\quad H_{pump} = 0.90\, H_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2453	6.96	565685	0.0296	79.3	79.3	0%

Flow reduction:

252 gpm
9.3% Loss

40-Year Old System and Pump:

$f = 2.40\, f_{new}$ $\quad H_{pump} = 0.75\, H_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2214	6.28	510754	0.0358	78.8	78.8	0%

Flow reduction:

490 gpm
18.1% Loss

Problem 10.66

10.66 Consider again the pump and piping system of Problem 10.63. Estimate the percentage reductions in volume flow rate that occur after (a) 20 years and (b) 40 years of use, if the pump characteristics remain constant. Repeat the calculation if the pump head is reduced 10 percent after 20 years of use and 25 percent after 40 years. (Use the data of Problem 10.65 for increase in pipe friction factor with age.)

Given: Data on pump and pipe system

Find: Delivery through series pump system; reduction after 20 and 40 years

Solution:

Given or available data **(Note: final results will vary depending on fluid data selected)** :

L = 1200 ft

D = 12 in

e = 0.00015 ft (Table 8.1)

ν = 1.23E-05 ft^2/s (Table A.7)

Δz = -50 ft

K_{ent} = 0.5 (Fig. 8.14)

K_{exp} = 1

L_e/D_{elbow} = 30

L_e/D_{valve} = 8 (Table 8.4)

Governing Equations:

For the pumps and system

$$\left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} - \Delta h_{pump} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2} \qquad (8.40b)$$

$$h_{l_m} = K\frac{\bar{V}^2}{2} \qquad (8.40a)$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} + \Delta z = H_{pump}$$

where

$$H_{lT} = \left[f\cdot\left(\frac{L}{D} + 2\cdot\frac{L_e}{D_{elbow}} + \frac{L_e}{D_{valve}}\right) + K_{ent} + K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

For pumps in series

$$H_{pump} = 2\cdot H_0 - 2\cdot A\cdot Q^2$$

where for a single pump

$$H_{pump} = H_0 - A\cdot Q^2$$

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

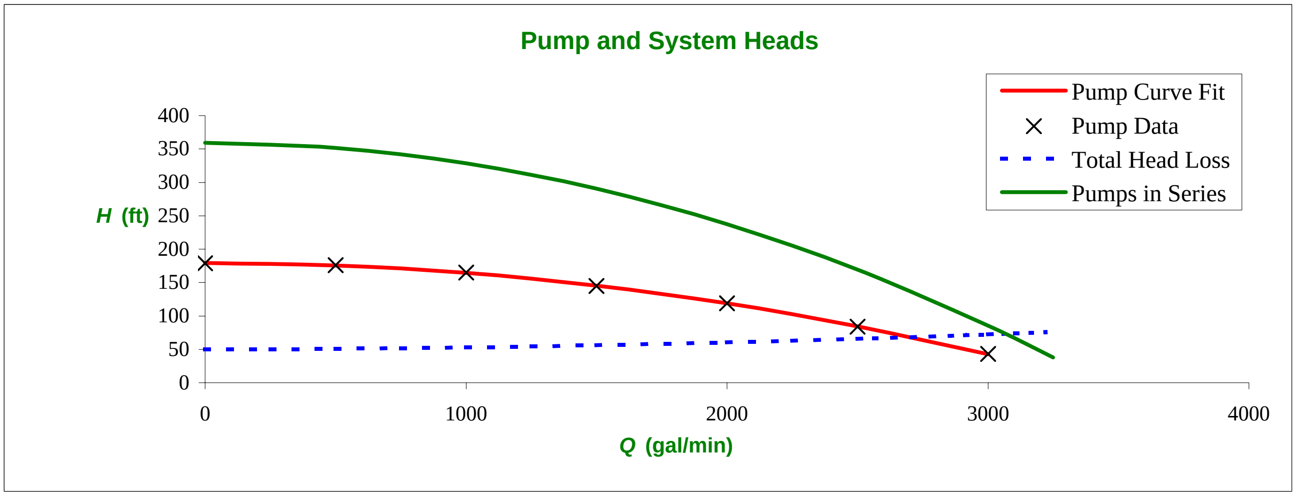
Q (gpm)	Q^2 (gpm)	H_{pump} (ft)	H_{pump} (fit)	V (ft/s)	Re	f
0	0	179	180	0.00	0	0.0000
500	250000	176	176	1.42	115325	0.0183
1000	1000000	165	164	2.84	230649	0.0164
1500	2250000	145	145	4.26	345974	0.0156
2000	4000000	119	119	5.67	461299	0.0151
2500	6250000	84	85	7.09	576623	0.0147
3000	9000000	43	43	8.51	691948	0.0145
3250				9.22	749610	0.0144

H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)
359	50.0
351	50.8
329	52.8
291	56.0
237	60.3
169	65.8
85	72.4
38	76.1

$H_0 =$ 180 ft

$A =$ 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
3066	8.70	707124	0.0145	73.3	73.3	0%



20-Year Old System:

$f = 2.00\, f_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{lT} + \Delta z$ (ft)	Error)
2964	8.41	683540	0.0291	92.1	92.1	0%

Flow reduction:

102 gpm

3.3% Loss

40-Year Old System:

$f = 2.40\, f_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2925	8.30	674713	0.0349	98.9	98.9	0%

Flow reduction:

141 gpm

4.6% Loss

20-Year Old System and Pumps:

$f = 2.00\, f_{new}$ $H_{pump} = 0.90\, H_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2915	8.27	672235	0.0291	90.8	90.8	0%

Flow reduction:

151 gpm

4.9% Loss

40-Year Old System and Pumps:

$f = 2.40\, f_{new}$ $H_{pump} = 0.75\, H_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{lT} + \Delta z$ (ft)	Error)
2772	7.86	639318	0.0351	94.1	94.1	0%

Flow reduction:

294 gpm

9.6% Loss

Problem 10.67

10.67 Consider again the pump and piping system of Problem 10.64. Estimate the percentage reductions in volume flow rate that occur after (a) 20 years and (b) 40 years of use, if the pump characteristics remain constant. Repeat the calculation if the pump head is reduced 10 percent after 20 years of use and 25 percent after 40 years. (Use the data of Problem 10.65 for increase in pipe friction factor with age.)

Given: Data on pump and pipe system

Find: Delivery through parallel pump system; reduction in delivery after 20 and 40 years

Solution:

Given or available data **(Note: final results will vary depending on fluid data selected)** :

L =	1200	ft	K_{ent} =	0.5	(Fig. 8.14)
D =	12	in	K_{exp} =	1	
e =	0.00015	ft (Table 8.1)	L_e/D_{elbow} =	30	
ν =	1.23E-05	ft^2/s (Table A.7)	L_e/D_{valve} =	8	(Table 8.4)
Δz =	-50	ft			

Governing Equations:

For the pumps and system

$$\left(\frac{p_1}{\rho} + \alpha_1\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} - \Delta h_{pump} \qquad (8.49)$$

where the total head loss is comprised of major and minor losses

$$h_l = f\frac{L}{D}\frac{\bar{V}^2}{2} \qquad (8.34)$$

$$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2} \qquad (8.40b)$$

$$h_{l_m} = K\frac{\bar{V}^2}{2} \qquad (8.40a)$$

Hence, applied between the two reservoir free surfaces ($p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 - z_2 = \Delta z$) we have

$$g\cdot\Delta z = h_{lT} - \Delta h_{pump}$$

$$h_{lT} + g\cdot\Delta z = g\cdot H_{system} + g\cdot\Delta z = \Delta h_{pump} = g\cdot H_{pump}$$

or

$$H_{lT} + \Delta z = H_{pump}$$

where

$$H_{lT} = \left[f\cdot\left(\frac{L}{D} + 2\cdot\frac{L_e}{D_{elbow}} + \frac{L_e}{D_{valve}}\right) + K_{ent} + K_{exit}\right]\cdot\frac{V^2}{2\cdot g}$$

For pumps in parallel

$$H_{pump} = H_0 - \frac{1}{4}\cdot A\cdot Q^2$$

where for a single pump

$$H_{pump} = H_0 - A\cdot Q^2$$

The pump data is curve-fitted to $H_{pump} = H_0 - AQ^2$.
The system and pump heads are computed and plotted below.
To find the operating condition, *Solver* is used to vary Q
so that the error between the two heads is minimized.

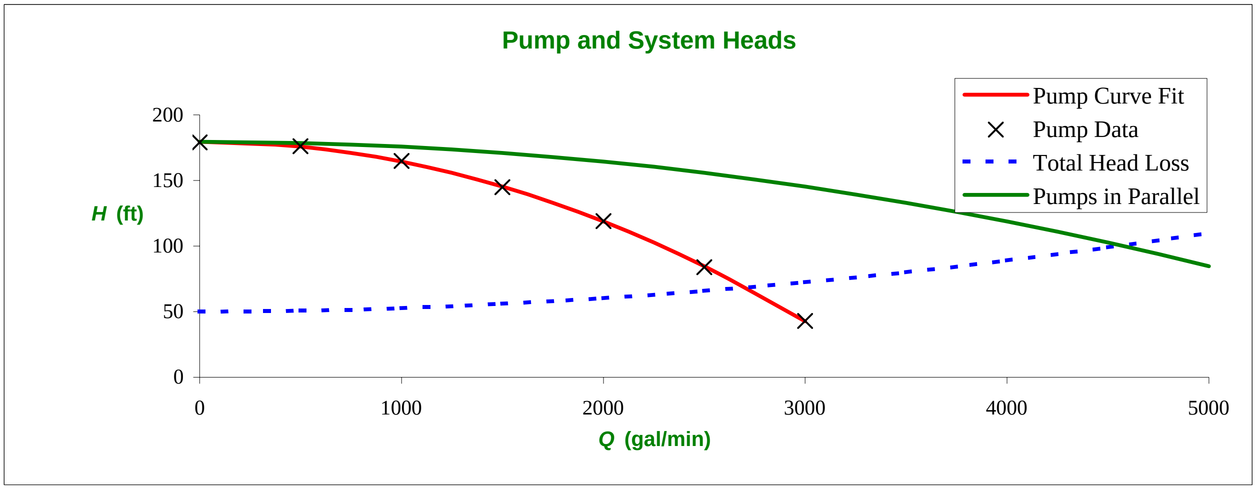
Q (gpm)	Q^2 (gpm)	H_{pump} (ft)	H_{pump} (fit)	V (ft/s)	Re	f
0	0	179	180	0.00	0	0.0000
500	250000	176	176	1.42	115325	0.0183
1000	1000000	165	164	2.84	230649	0.0164
1500	2250000	145	145	4.26	345974	0.0156
2000	4000000	119	119	5.67	461299	0.0151
2500	6250000	84	85	7.09	576623	0.0147
3000	9000000	43	43	8.51	691948	0.0145
3500				9.93	807273	0.0143
4000				11.35	922597	0.0142
4500				12.77	1037922	0.0141
5000				14.18	1153247	0.0140

H_{pumps} (par)	$H_{IT} + \Delta z$ (ft)
180	50.0
179	50.8
176	52.8
171	56.0
164	60.3
156	65.8
145	72.4
133	80.1
119	89.0
103	98.9
85	110.1

$H_0 =$ 180 ft

$A =$ 1.52E-05 ft/(gpm)^2

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{IT} + \Delta z$ (ft)	Error)
4565	12.95	1053006	0.0141	100.3	100.3	0%



20-Year Old System:

$f = 2.00\ f_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pumps} (par)	$H_{IT} + \Delta z$ (ft)	Error)
3906	11.08	900891	0.0284	121.6	121.6	0%

Flow reduction:

660 gpm

14.4% Loss

40-Year Old System:

$f = 2.40\ f_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{IT} + \Delta z$ (ft)	Error)
3710	10.52	855662	0.0342	127.2	127.2	0%

Flow reduction:

856

18.7%

20-Year Old System and Pumps:

$f = 2.00\ f_{new}$ $H_{pump} = 0.90\ H_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{IT} + \Delta z$ (ft)	Error)
3705	10.51	854566	0.0285	114.6	114.6	0%

Flow reduction:

860 gpm

18.8% Loss

40-Year Old System and Pumps:

$f = 2.40\ f_{new}$ $H_{pump} = 0.75\ H_{new}$

Q (gpm)	V (ft/s)	Re	f	H_{pump} (fit)	$H_{IT} + \Delta z$ (ft)	Error)
3150	8.94	726482	0.0347	106.4	106.4	0%

Flow reduction:

1416

31.0%

Problem 10.68

[3]

Given: Water supply for Englewood, Colorado; $L = 5800$ ft, $D = 27$ in., steel.

Section 1: North Platte River at $z_1 = 5280$ ft.

Section 2: Reservoir at $z_2 = 5310$ ft

Design flow rates: $Q = 31$ cfs (initially), 38 cfs (ultimately).

Find: (a) Calculate and plot the system resistance curve.
(b) Specify an appropriate pumping system.
(c) Estimate power required for steady-state operation at both flow rates.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $$\frac{p_1}{\rho g} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + H_p = \frac{p_2}{\rho g} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2 + \frac{h_{l_T}}{g}; \quad h_{l_T} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2g}$$
(with $\bar{V}_1 \approx 0$ (2), $\bar{V}_2 \approx 0$ (2), $L_e/D = 0$ (4))

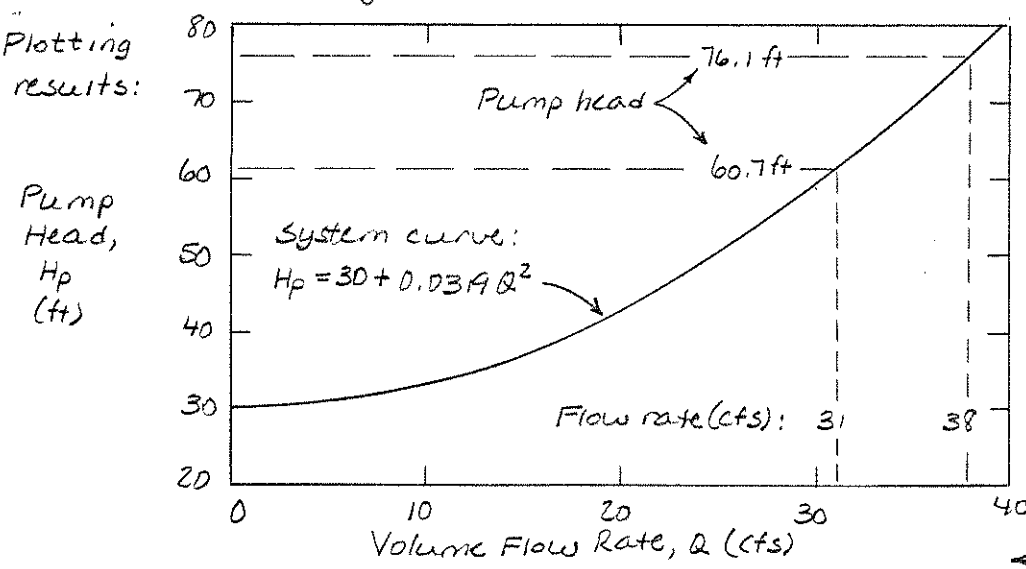
Assumptions: (1) $p_1 = p_2 = p_{atm}$, (2) $\bar{V}_1 \approx \bar{V}_2 \approx 0$, (3) $z_1 = 5280$ ft, $z_2 = 5310$ ft, (4) $K_{ent} = 0$, $K_{exit} = 1$, $\frac{L_e}{D} = 0$

Sample calculation at $Q = 31$ cfs (13,900 gpm), with $T = 70°F$, $e = 0.00015$ ft (Table 8.1)

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 31 \frac{ft^3}{s} \times \left(\frac{12}{27}\right)^2 \frac{1}{ft^2} = 7.80 \text{ ft/s}; \quad Re = \frac{\bar{V}D}{\nu} = 1.64 \times 10^6; \quad \frac{e}{D} = 6.67 \times 10^{-5}; \quad f = 0.0124$$

$$H_p = z_2 - z_1 + \left(f\frac{L}{D} + K\right)\frac{\bar{V}^2}{2g} = (5{,}310 - 5{,}280)\text{ ft} + \left(0.0124 \frac{5800}{(27/12)} + 1\right)\frac{1}{2} \times (7.80)^2 \frac{ft^2}{s^2} \times \frac{s^2}{32.2 \text{ ft}} = 61.1 \text{ ft}$$

Plotting results:



Plot

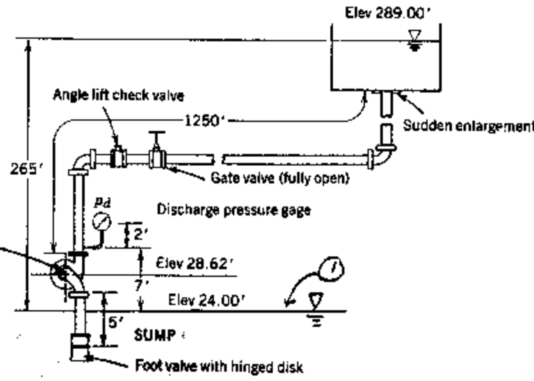
The maximum flow rate is $Q_{max} = 38 \frac{ft^3}{s} \times 7.48 \frac{gal}{ft^3} \times 60 \frac{s}{min} = 17{,}100$ gpm.

Could choose two Peerless 16A 18B pumps at 880⁺ rpm (Fig. D.10) or three 10AE 14(G) pumps at 1750 rpm (Fig. D.2). Efficiency (Fig. 11.15) might be $\eta_p = 0.91$. — n, η

Pump power is $\dot{W}_m = \frac{\dot{W}_h}{\eta_p} = \frac{\rho Q g H}{\eta_p} = 361$ hp (at $Q = 38$ cfs), 235 hp (at 31 cfs) — $\dot{W}_m$

Problem 10.69

Given: Flow system shown.



Design flow rate: $Q = 200\, gpm$

Pipe is commercial steel, $D = 4$ in.

Find: (a) Calculate the NPSHA.
(b) Select a suitable pump.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\frac{p_i}{\rho g} + \alpha_i \frac{\bar{V}_i^2}{2g} + z_i + H_p = \frac{p_j}{\rho g} + \alpha_j \frac{\bar{V}_j^2}{2g} + z_j + \frac{h_{\ell T}}{g}$; $h_{\ell T} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2}$ (both velocity terms marked $\approx 0(2)$)

Assumptions: (1) Reservoirs open to atmosphere, (2) Reservoir velocities negligible, (3) $T = 70°F$ ($p_v = 0.363$ psia, Table A.7)

At $Q = 200\, gpm\, (0.446\, ft^3/s)$, so $\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 0.446 \frac{ft^3}{s} \times \left(\frac{12}{4.026}\right)^2 \frac{1}{ft^2} = 5.04\, ft/s$; $e = 0.00015\, ft$

Thus $Re = \frac{\bar{V}D}{\nu} = 1.58 \times 10^5$; $\frac{e}{D} = 0.000447$; $f = 0.0190$; $\frac{\bar{V}^2}{2g} = \frac{1}{2} \times (5.04)^2 \frac{ft^2}{s^2} \times \frac{s^2}{32.2\, ft} = 0.394\, ft$

For the inlet, $\frac{p_2}{\rho g} + \frac{\alpha_2 \bar{V}_2^2}{2g} = z_1 - z_2 - \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2g} = -5.60\, ft$, since $\bar{V}_1 \approx 0$ (foot valve, 75; entrance, 0.78)

Then $NPSHA = \frac{p_{t,abs} - p_v}{\rho g} = \frac{p_t}{\rho g} + \frac{p_{atm}}{\rho g} - \frac{p_v}{\rho g} = (-5.60 + 33.9 - 0.838)\, ft = 27.5\, ft$ NPSHA

For the complete system, $L/D = (1250 + 5)\, ft \times \left(\frac{12}{4.026}\right)\frac{1}{ft} = 3740$, $\frac{L_e}{D} = 75 + 55 + 8 = 138$ (check valve; gate valve)

$H_p = z_2 - z_1 + \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2g} = (289 - 24)\, ft + [0.019(3740 + 138) + 0.78 + 1]\, 0.394\, ft$

$H_p = 295\, ft$ at $Q = 200\, gpm$ Hp

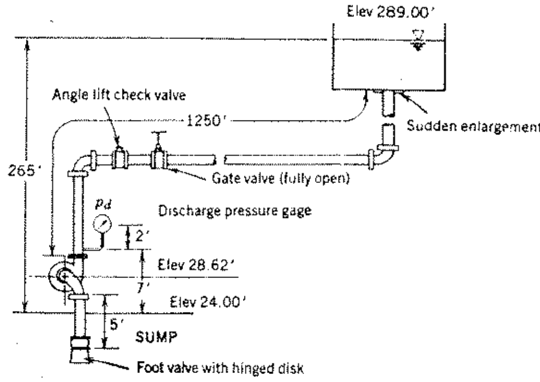
From Fig. D.1, a 4AE12 pump (4 in. discharge line) would fit the application. This pump could produce the required head at a speed between 1750 and 3550 rpm (Figs. D.4 and D.5), but the efficiency may not be acceptable.

{Consult a complete catalog to make a better selection.}

Problem 10.70

Given: Flow system and data of Problem 10.69.

Data for pipe aging from Problem 10.65.



Find: (a) Select pumps that will maintain system flow rate for 10 and 20 years.
(b) Compare delivery to that with pump sized for new pipes only.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\frac{p_1}{\rho g} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + H_p = \frac{p_2}{\rho g} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2 + \frac{h_{lT}}{g}$; $h_{lT} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right] \frac{\bar{V}^2}{2}$ (with $\alpha_1 \frac{\bar{V}_1^2}{2g} \approx 0(2)$, $\alpha_2 \frac{\bar{V}_2^2}{2g} \approx 0(3)$)

Assumptions: (1) $p_1 = p_2 = p_{atm}$, (2) $\bar{V}_1 = \bar{V}_2 \approx 0$, (3) Minor losses as in Problem 10.61

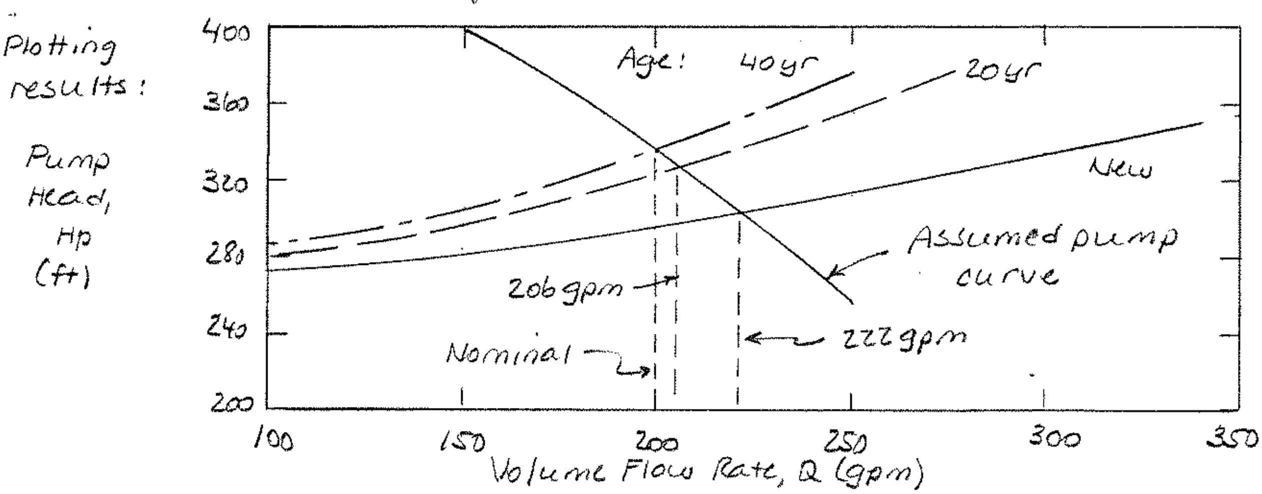
Sample calculations at $Q = 200\,gpm\,(0.446\,ft^3/s)$ for new pipe, $e = 0.00015\,ft$ (Table 8.1):

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 0.446 \frac{ft^3}{s} \times \left(\frac{12}{4.026}\right)^2 \frac{1}{ft^2} = 5.04\,ft/s\,; \quad Re = 1.58 \times 10^5;\ \frac{e}{D} = 0.000447;\ f = 0.0190$$

For the complete system, $\frac{L}{D} = (1250 + 5)\,ft \times \left(\frac{12}{4.026}\right) \frac{1}{ft} = 3740\,;\ \frac{L_e}{D} = 138\,;\ K = 1.28$

$$H_p = z_2 - z_1 + \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right] \frac{\bar{V}^2}{2g} = (289 - 24)\,ft + \left[0.019(3740 + 138) + 1.28\right] 0.394\,ft = 295\,ft$$

Plotting results:



Assume H at 200 gpm is 70% of H_0. Then $H_{200} = 336\,ft$, $H_0 = \frac{1}{0.7} 336\,ft = 480\,ft$ and $H = H_0 - AQ^2$; $A = (H_0 - H)/Q^2 = 0.3 H_0/Q^2 = 0.3 \times 480\,ft/(200\,gpm)^2 = 3.60 \times 10^{-3}\,ft/(gpm)^2$.

Sizing the pump for 200 gpm at 40 years would (assuming no change in pump characteristics) produce 206 gpm at 20 years and 222 gpm in the new system.

The extra head (336 ft, compared to 295 ft) at 200 gpm could be obtained by increasing impeller diameter about 7-10% compared to the pump of Problem 10.69

Problem 10.71

Given: Flow system of Problem 8.155

Location	Elevation	Pressure
Entrance	50.0 ft	20 psig
Discharge	94.0 ft	0 psig

$D = 2.5$ in. (nominal), $L = 290$ ft

2 open gate valves, 1 open angle valve, 7 standard 90° elbows
1 square-edged entrance from a reservoir, 1 free discharge

$Q = 0.439\ \text{ft}^3/\text{s}$ $\quad (Q = 197\ \text{gpm})$ $\quad$ Galvanized pipe

Find: (a) Select an appropriate pump.
(b) Check the NPSHR vs. the NPSHA for this system.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $$\frac{p_1}{\rho g} + \alpha_1\frac{\bar{V}_1^2}{2g} + z_1 + H_p = \frac{p_2}{\rho g} + \alpha_2\frac{\bar{V}_2^2}{2g} + z_2 + \frac{h_{lT}}{g}; \quad h_{lT} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2}$$ ($\bar{V}_1 \approx 0$ (1))

Assumptions: (1) $\bar{V}_1 \approx 0$, (2) $K_{ent} = 0.5$, (3) $\frac{L_e}{D} = 2(8) + 1(150) + 7(30) = 376$; $D = 2.47$ in.

(4) Galvanized pipe, $e = 0.0005$ ft; $\frac{e}{D} = \frac{0.0005\ \text{ft}}{2.47\ \text{in.}} \times 12\frac{\text{in.}}{\text{ft}} = 0.00243$

Then
$$H_p = \frac{p_2 - p_1}{\rho g} + \frac{\alpha_2 \bar{V}_2^2}{2g} + z_2 - z_1 + \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2g}; \quad \bar{V} = \frac{Q}{A} = 13.2\ \text{ft/s}; \quad Re = \frac{\bar{V}D}{\nu} = 2.54\times10^5; f = 0.025$$

$$H_p = (0 - 20)\frac{\text{lbf}}{\text{in.}^2} \times 144\frac{\text{in.}^2}{\text{ft}^2} \times \frac{\text{ft}^3}{62.4\ \text{lbf}} + \frac{1}{2} \times (13.2)^2\frac{\text{ft}^2}{\text{s}^2} \cdot \frac{\text{s}^2}{32.2\ \text{ft}} + (94.0 - 50.0)\ \text{ft}$$
$$+ \left[0.025\left(\frac{290}{2.47/12} + 376\right) + 0.5\right]\frac{1}{2} \times (13.2)^2\frac{\text{ft}^2}{\text{s}^2} \cdot \frac{\text{s}^2}{32.2\ \text{ft}} = 123\ \text{ft}$$ $\quad H_p$

The pump requirement is $Q = 197$ gpm at $H = 123$ ft. This could be supplied by a Peerless Type 4AE12 pump, with impeller $D = 11$ in., operating at 1750 rpm.

(This pump may be slightly too large, since this operating point is at a flow rate below that for best efficiency.)

The NPSHR for this pump at $Q = 197$ gpm is about 5 ft.

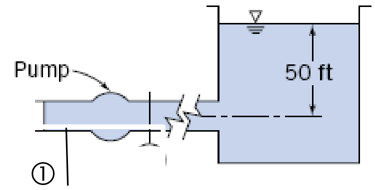
The NPSHA is $\frac{p_1}{\rho g} + \frac{p_{atm}}{\rho g} + \frac{\alpha_1 \bar{V}_1^2}{2g} - p_v = 46.2 + 33.9 + 2.71 - 0.782\ \text{ft} = 82.0\ \text{ft}$

Thus NPSHA >> NPSHR $\quad$ NPSH

Cavitation-free operation is assured.

Problem 10.72

10.72 Consider the flow system shown in Problem 8.110. Assume the minimum *NPSHR* at the pump inlet is 15 ft of water. Select a pump appropriate for this application. Use the data for increase in friction factor with pipe age given in Problem 10.65 to determine and compare the system flow rate after 10 years of operation.



Given: Flow from pump to reservoir

Find: Select a pump to satisfy NPSHR

Solution:

Basic equations
$$\left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) = h_{lT} - h_p \qquad h_{lT} = h_l + h_{lm} = f\cdot\frac{L}{D}\cdot\frac{V_1^2}{2} + K_{exit}\cdot\frac{V_1^2}{2}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 is approximately 1 4) V_2 <<

Note that we compute head per unit weight, H, not head per unit mass, h, so the energy equation between Point 1 and the free surface (Point 2) becomes
$$\left(\frac{p_1}{\rho\cdot g} + \frac{V^2}{2\cdot g}\right) - (z_2) = f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g} + K_{exit}\cdot\frac{V^2}{2\cdot g} - H_p$$

Solving for H_p
$$H_p = z_2 - \frac{p_1}{\rho\cdot g} - \frac{V^2}{2\cdot g} + f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g} + K_{exit}\cdot\frac{V^2}{2\cdot g}$$

From Table A.7 (68°F) $\rho = 1.94\cdot\frac{slug}{ft^3}$ $\quad \nu = 1.08\times10^{-5}\cdot\frac{ft^2}{s}$ $\quad Re = \frac{V\cdot D}{\nu}$ $\quad Re = 6.94\times10^5$

For commercial steel pipe $e = 0.00015\cdot ft$ (Table 8.1) so $\frac{e}{D} = 0.0002$

Flow is turbulent: Given
$$\frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right) \qquad f = 0.0150$$

For the exit $K_{exit} = 1.0$ so we find $H_p = z_2 - \frac{p_1}{\rho\cdot g} + f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}$

Note that for an NPSHR of 15 ft this means $\frac{p_1}{\rho\cdot g} = 15\cdot ft$ $\quad H_p = z_2 - \frac{p_1}{\rho\cdot g} + f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}$ $\quad H_p = 691\,ft$

Note that $Q = \frac{\pi\cdot D^2}{4}\cdot V$ $\quad Q = 4.42\frac{ft^3}{s}$ $\quad Q = 1983\,gpm$

For this combination of Q and Hp, from Fig. D.11 the best pump appears to be a Peerless two-stage 10TU22C operating at 1750 rpm

After 10 years, from Problem 10.65, the friction factor will have increased by a factor of 2.2 $f = 2.2\times0.150$ $\quad f = 0.330$

We now need to solve $H_p = z_2 - \frac{p_1}{\rho\cdot g} + f\cdot\frac{L}{D}\cdot\frac{V^2}{2\cdot g}$ for the new velocity V

$$V = \sqrt{\frac{2\cdot D\cdot g}{f\cdot L}\cdot\left(H_p - z_2 + \frac{p_1}{\rho\cdot g}\right)} \qquad V = 2.13\frac{ft}{s}$$

and f will still be 2.2×0.150

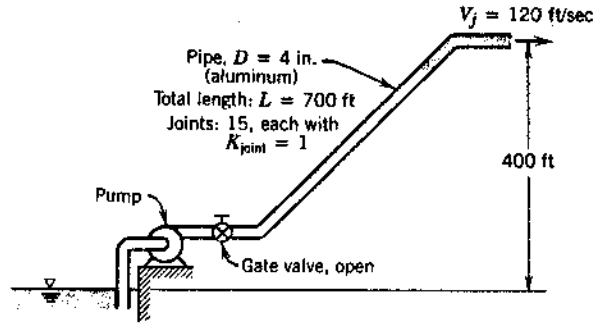
$Q = \frac{\pi\cdot D^2}{4}\cdot V$ $\quad Q = 0.94\frac{ft^3}{s}$ $\quad Q = 423\,gpm$ $\quad$ Much less!

Problem 10.73

Given: Flow system of Problem 8.156:

$Q = 600\ gpm$

$\eta_p \approx 0.70$



Find: (a) select an appropriate pump.
(b) Compare pump efficiency with estimate in problem.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\frac{p_1}{\rho g} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + H_p = \frac{p_2}{\rho g} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2 + \frac{h_{lT}}{g}$; $h_{lT} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2}$ (with $\bar{V}_1 \approx 0$ (2))

Assumptions: (1) $p_1 = p_2 = p_{atm}$, (2) $\bar{V}_1 \approx 0$, (3) Neglect elbow and nozzle losses, (4) $\frac{L_e}{D}$(valve) = 8

$$\bar{V} = \frac{Q}{A} = 600 \frac{gal}{min} \times \frac{4}{\pi}\left(\frac{12}{4}\right)^2 \frac{1}{ft^2} \times \frac{ft^3}{7.48\ gal} \times \frac{min}{60\ s} = 15.3\ \frac{ft}{s} \ ; \ \frac{\bar{V}^2}{2g} = \frac{1}{2} \times \frac{(15.3)^2 ft^2}{s^2} \times \frac{s^2}{32.2\ ft} = 3.63\ ft$$

$$Re = \frac{\bar{V}D}{\nu} = 15.3\frac{ft}{s} \times \left(\frac{4}{12}\right) ft \times \frac{s}{1.08 \times 10^{-5} ft^2} = 4.77 \times 10^5 \ ; \text{ smooth}; \ f = 0.013 \quad (T = 68°F)$$

$$H_p = z_2 - z_1 + \frac{\alpha_2 \bar{V}_2^2}{2g} + \frac{h_{lT}}{g} = z_2 - z_1 + \frac{\bar{V}_2^2}{2g} + \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + 15 K_j\right]\frac{\bar{V}^2}{2g}$$

$$H_p = 400\ ft + \frac{1}{2} \times \frac{(120)^2 ft^2}{s^2} \cdot \frac{s^2}{32.2\ ft} + \left[0.013\left(\frac{700}{4/12} + 8\right) + 15\right] 3.63\ ft = 778\ ft$$

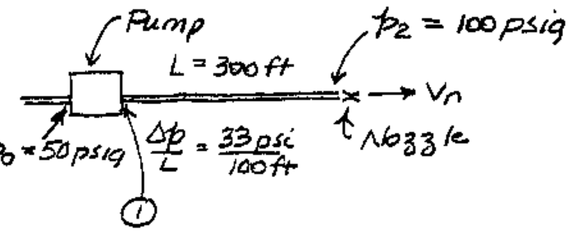
Thus the pump requirement is $H_p = 778$ ft at $Q = 600$ gpm. This head is too great to be developed by a single-stage pump (Fig. D.1). From Fig. D.12, the flow could be supplied by a 5TUT168 3-stage pump, driven at 1750 rpm.

Single-stage pumps have peak efficiencies of 86 percent at 1750 rpm (Fig. D.8). Thus 70 percent efficiency might be reasonable for a 3-stage pump, since $(0.86)^3 = 0.636$.

Problem 10.74

Given: Fire hose and nozzle as shown.

Canvas hose: $D = 3$ in.
$e = 0.001$ ft



Find: (a) Design flow rate
(b) Maximum nozzle exit speed
(c) Pump selection
(d) Efficiency and power

Solution: Apply the energy equation for pipe flow:

Computing equation: $\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) = h_{l_T} = h_l + h_{lm}$

$h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$

Assumptions: (1) $\bar{V}_1 = \bar{V}_2$; (2) $z_1 = z_2$, (3) $h_{lm} = 0$

Then $\frac{p_1}{\rho} - \frac{p_2}{\rho} = \frac{\Delta p}{\rho} = f \frac{L}{D} \frac{\bar{V}^2}{2}$; $\bar{V} = \left[\frac{2 D \Delta p}{f \rho L}\right]^{\frac{1}{2}}$; $\frac{e}{D} = \frac{0.001 \text{ ft}}{3 \text{ in.}} \times 12 \frac{\text{in.}}{\text{ft}} = 0.004$; $f = 0.028$ (fully rough)

$$\bar{V} = \left[\frac{2}{0.028} \times 3 \text{ in.} \times 3(33) \frac{\text{lbf}}{\text{in.}^2} \times \frac{\text{ft}^3}{1.94 \text{ slug}} \times \frac{1}{300 \text{ ft}} \times 12 \frac{\text{in.}}{\text{ft}} \times \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}\right]^{\frac{1}{2}} = 20.9 \text{ ft/s}$$

$Q = \bar{V} A$; $A = \frac{\pi D^2}{4} = \frac{\pi}{4}\left(\frac{3}{12}\right)^2 \text{ft}^2 = 0.0491 \text{ ft}^2$

$$Q = 20.9 \frac{\text{ft}}{\text{s}} \times 0.0491 \text{ ft}^2 \times 7.48 \frac{\text{gal}}{\text{ft}^3} \times \frac{60 \text{ s}}{\text{min}} = 461 \text{ gpm (design flow rate)}$$ Q

Apply Bernoulli to nozzle.

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{p_{atm}}{\rho} + \frac{V_n^2}{2} + g z_n \;;\; V_n = \left[\frac{2 p_2}{\rho} + V_2^2\right]^{\frac{1}{2}}$$

(with $p_{atm} = 0$)

$$V_n = \left[2 \times 100 \frac{\text{lbf}}{\text{in.}^2} \times \frac{\text{ft}^3}{1.94 \text{ slug}} \times 144 \frac{\text{in.}^2}{\text{ft}^2} + (20.9)^2 \frac{\text{ft}^2}{\text{s}^2}\right]^{\frac{1}{2}} = 124 \text{ ft/s}$$ V_n

The pump head requirement (neglecting V and z) will be

$$H_p = \frac{p_1 - p_0}{\rho g} = [100 + 3(33) - 50] \frac{\text{lbf}}{\text{in.}^2} \times \frac{\text{ft}^3}{62.4 \text{ lbf}} \times 144 \frac{\text{in.}}{\text{ft}} = 344 \text{ ft}$$ H_p

From the pump selector chart (Fig. D.1) choose 3AE9S or 4AE10 pump, 3500 rpm.

Based on 4AE12 at 3550 rpm (Fig. D.5), expect $\eta \approx 0.75$ η

$$P = \frac{Q \Delta p}{\eta} = \frac{Q \rho g \Delta h}{\eta} = \frac{1}{0.75} \times 20.9 \frac{\text{ft}}{\text{s}} \times 0.0491 \text{ ft}^2 \times 62.4 \frac{\text{lbf}}{\text{ft}^3} \times 344 \text{ ft} \times \frac{\text{hp} \cdot \text{s}}{550 \text{ ft} \cdot \text{lbf}} = 53.4 \text{ hp}$$ P

Problem 10.75

[3]

10.75 Consider the pipe network of Problem 8.168. Select a pump suitable to deliver a total flow rate of 300 gpm through the pipe network.

Given: Water pipe system

Find: Pump suitable for 300 gpm

Solution:

$$\left(\frac{p_1}{\rho}+\alpha_1\cdot\frac{V_1^2}{2}+g\cdot z_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\cdot\frac{V_2^2}{2}+g\cdot z_2\right)=h_l \qquad h_{lT}=f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$$

$$f=\frac{64}{Re} \quad \text{(Laminar)} \qquad \frac{1}{\sqrt{f}}=-2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7}+\frac{2.51}{Re\cdot\sqrt{f}}\right) \quad \text{(Turbulent)}$$

The energy equation can be simplified to $\Delta p=\rho\cdot f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$

This can be written for each pipe section

Pipe A (first section) $$\Delta p_A=\rho\cdot f_A\cdot\frac{L_A}{D_A}\cdot\frac{V_A^2}{2} \qquad (1)$$

Pipe B (1.5 in branch) $$\Delta p_B=\rho\cdot f_B\cdot\frac{L_B}{D_B}\cdot\frac{V_B^2}{2} \qquad (2)$$

Pipe C (1 in branch) $$\Delta p_C=\rho\cdot f_C\cdot\frac{L_C}{D_C}\cdot\frac{V_C^2}{2} \qquad (3)$$

Pipe D (last section) $$\Delta p_D=\rho\cdot f_D\cdot\frac{L_D}{D_D}\cdot\frac{V_D^2}{2} \qquad (4)$$

In addition we have the following contraints

$$Q_A=Q_D=Q \qquad (5)$$

$$Q=Q_B+Q_C \qquad (6)$$

$$\Delta p=\Delta p_A+\Delta p_B+\Delta p_D \qquad (7)$$

$$\Delta p_B=\Delta p_C \qquad (8)$$

We have 2 unknown flow rates (or, equivalently, velocities); We solve the above eight equations simultaneously

Once we compute the flow rates and pressure drops, we can compute data for the pump

$$\Delta p_{pump}=\Delta p \quad \text{and} \quad Q_{pump}=Q_A \qquad W_{pump}=\Delta p_{pump}\cdot Q_{pump}$$

Pipe Data:

Pipe	*L* (ft)	*D* (in)	*e* (ft)
A	150	1.5	0.00085
B	150	1.5	0.00085
C	150	1	0.00085
D	150	1.5	0.00085

Fluid Properties:

ρ =	1.94	slug/ft^3
μ =	2.10E-05	lbf·s/ft^2

Flow Rate:

Q =	300	gpm
=	0.668	ft^3/s

Flows:

Q_A (ft^3/s)	Q_B (ft^3/s)	Q_C (ft^3/s)	Q_D (ft^3/s)
0.668	0.499	0.169	0.668

V_A (ft/s)	V_B (ft/s)	V_C (ft/s)	V_D (ft/s)
54.47	40.67	31.04	54.47

Re_A	Re_B	Re_C	Re_D
6.29E+05	4.70E+05	2.39E+05	6.29E+05

f_A	f_B	f_C	f_D
0.0335	0.0336	0.0384	0.0335

Heads:

Δp_A (psi)	Δp_B (psi)	Δp_C (psi)	Δp_D (psi)
804.0	448.8	448.8	804.0

Constraints:

(6) $Q = Q_B + Q_C$	(8) $\Delta p_B = \Delta p_C$
0.00%	**0.00%**

Error: **0.00%** Vary Q_B and Q_C using *Solver* to minimize total error

For the pump:

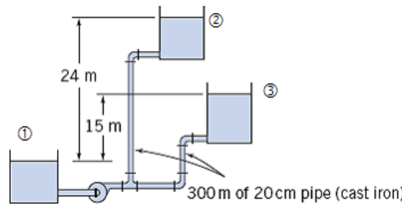
Δp (psi)	Q (gpm)	$\mathscr{P}$ (hp)
2057	300	360

This is a very high pressure; a sequence of pumps would be needed

Problem 10.76

[4]

10.76 A pumping system with two different static lifts is shown. Each reservoir is supplied by a line consisting of 300 m of 20 cm cast-iron pipe. Evaluate and plot the system head versus flow curve. Explain what happens when the pump head is less than the height of the upper reservoir. Calculate the flow rate delivered at a pump head of 26 m.



Given: Pump and supply pipe system

Find: Head versus flow curve; Flow for a head of 26 m

Solution:

Basic equations:
$$\left(\frac{p_1}{\rho} + \alpha_1\cdot\frac{V_1^{\,2}}{2} + g\cdot z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\cdot\frac{V_2^{\,2}}{2} + g\cdot z_2\right) = h_{lT} - h_{pump} \qquad h_{lT} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2} + f\cdot\frac{L_e}{D}\cdot\frac{V^2}{2} + K\cdot\frac{V^2}{2}$$

Applying to the 24 m branch (branch a)
$$-g\cdot H_a = f\cdot\frac{L}{D}\cdot\frac{V_a^{\,2}}{2} + f\cdot\frac{L_e}{D}\cdot\frac{V_a^{\,2}}{2} + K\cdot\frac{V_a^{\,2}}{2} - g\cdot H_{pump}$$

where $H_a = 24\cdot m$ and $\frac{L_{ea}}{D}$ is due to a standard T branch (= 60) and a standard elbow (= 30) from Table 8.4, and $K = K_{ent} + K_{exit} = 1.5$ from Fig. 8.14

$$H_{pump} = H_a + \left[f\cdot\left(\frac{L}{D} + \frac{L_{ea}}{D}\right) + K\right]\cdot\frac{V_a}{2\cdot g} \qquad (1)$$

Applying to the 15 m branch (branch b)
$$H_{pump} = H_b + \left[f\cdot\left(\frac{L}{D} + \frac{L_{eb}}{D}\right) + K\right]\cdot\frac{V_b}{2\cdot g} \qquad (2)$$

where $H_b = 15\cdot m$ and $\frac{L_{eb}}{D}$ is due to a standard T run (= 20) and two standard elbows (= 60), and $K = K_{ent} + K_{exit} = 1.5$

Given data:

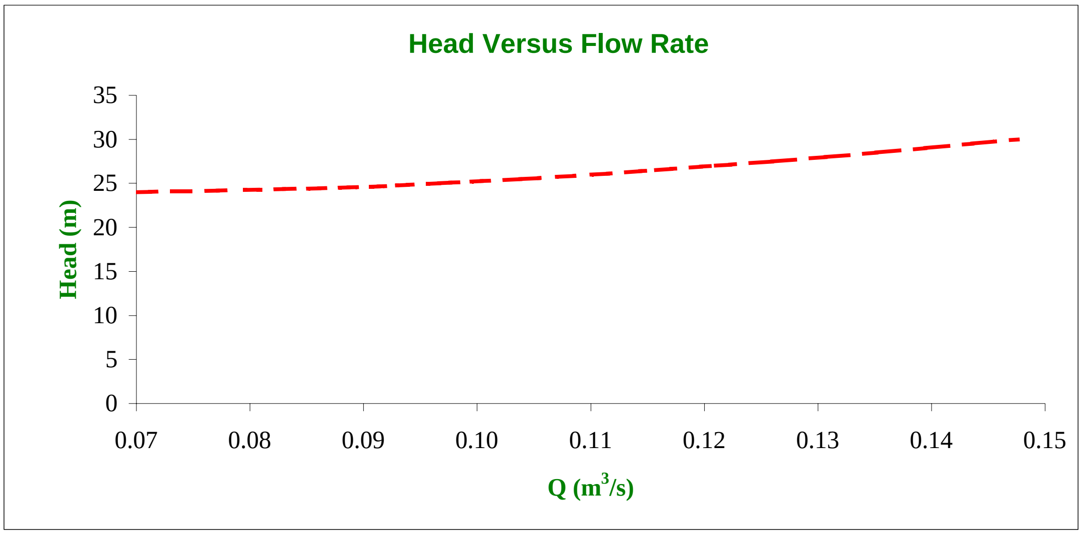
L =	300	m
e =	0.26	mm
D =	20	cm
K =	1.5	
L_{ea}/D =	90	
L_{eb}/D =	80	
H_a =	24	m
H_b =	15	m
ρ =	1000	kg/m^3
ν =	1.01E-06	m^2/s

Computed results: Set up Solver so that it varies all flow rates to make the total head error zero

H_{pump} (m)	Q (m^3/s)	Q_a (m^3/s)	V_a (m/s)	Re_a	f_a	H_{pump} (Eq. 1)	Q_b (m^3/s)	V_b (m/s)	Re_b	f_b	H_{pump} (Eq. 2)	H (Errors)
24.0	0.070	0.000	0.000	8.62E+00	7.4264	24.0	0.070	2.230	4.42E+05	0.0215	24.0	0.00
24.5	0.088	0.016	0.506	1.00E+05	0.0231	24.5	0.072	2.292	4.54E+05	0.0215	24.5	0.00
25.0	0.097	0.023	0.72	1.44E+05	0.0225	25.0	0.074	2.35	4.66E+05	0.0215	25.0	0.00
25.5	0.104	0.028	0.89	1.77E+05	0.0223	25.5	0.076	2.41	4.78E+05	0.0215	25.5	0.00
26.0	0.110	0.033	1.03	2.05E+05	0.0221	26.0	0.078	2.47	4.89E+05	0.0215	26.0	0.00
26.5	0.116	0.036	1.16	2.30E+05	0.0220	26.5	0.079	2.52	5.00E+05	0.0215	26.5	0.00
27.0	0.121	0.040	1.27	2.52E+05	0.0219	27.0	0.081	2.58	5.11E+05	0.0214	27.0	0.00
27.5	0.126	0.043	1.38	2.73E+05	0.0218	27.5	0.083	2.63	5.21E+05	0.0214	27.5	0.00
28.0	0.131	0.046	1.47	2.92E+05	0.0218	28.0	0.084	2.69	5.32E+05	0.0214	28.0	0.00
28.5	0.135	0.049	1.56	3.10E+05	0.0217	28.5	0.086	2.74	5.42E+05	0.0214	28.5	0.00
29.0	0.139	0.052	1.65	3.27E+05	0.0217	29.0	0.088	2.79	5.52E+05	0.0214	29.0	0.00
29.5	0.144	0.054	1.73	3.43E+05	0.0217	29.5	0.089	2.84	5.62E+05	0.0214	29.5	0.00
30.0	0.148	0.057	1.81	3.59E+05	0.0216	30.0	0.091	2.89	5.72E+05	0.0214	30.0	0.00

For the pump head less than the upper reservoir head flow will be out of the reservoir (into the lower one)

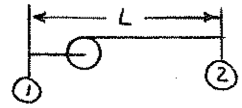
Total Error: 0.00



Problem 10.77

[4]

Given: Chilled water circulation system of Problem 8.158:



$L = 3$ mi (15,800 ft), $D = 2$ ft (steel), $Q = 11{,}200$ gpm, Loop configuration

Find: (a) Select suitable pumps for parallel operation.
(b) Calculate power for 3 pumps in parallel.
(c) Calculate volume flow rate and power if 1 or 2 pumps operate.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\frac{p_1}{\rho g} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 + H_a = \frac{p_2}{\rho g} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2 + \frac{h_{lT}}{g}$; $h_{lT} = \left[f\left(\frac{L_e}{D} + \frac{L}{D}\right) + K\right]\frac{\bar{V}^2}{2}$

Assumptions: (1) $p_1 = p_2$, (2) $\alpha_1 \bar{V}_1^2 = \alpha_2 \bar{V}_2^2$, (3) $z_1 = z_2$, (4) Neglect minor losses, $\frac{L_e}{D} \approx 0$, $K \approx 0$

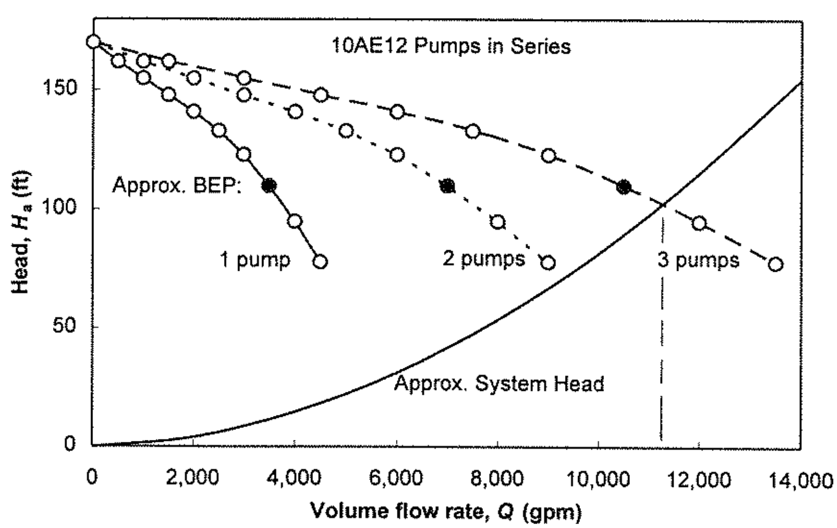
$\bar{V} = \frac{Q}{A} = 11{,}200 \frac{gal}{min} \times \frac{4}{\pi (2)^2 ft^2} \times \frac{ft^3}{7.48\, gal} \times \frac{min}{60\, sec} = 7.94$ ft/s ; $\frac{\bar{V}^2}{2g} = \frac{1}{2} \times (7.94)^2 \frac{ft^2}{s^2} \times \frac{s^2}{32.2\, ft} = 0.979$ ft

Assume $T = 40$ F, so $\nu = 1.64 \times 10^{-5}$ ft²/s ; $Re = \frac{\bar{V}D}{\nu} = 9.68 \times 10^5$; $\frac{e}{D} = 7.5 \times 10^{-5}$; $f = 0.013$

$H_a = f \frac{L}{D} \frac{\bar{V}^2}{2g} = 0.013 \times \frac{3(5280)\, ft}{2\, ft} \times 0.979\, ft = 101$ ft H_p

For three pumps in parallel, each will operate at $Q/3 = 3730$ gpm. The requirement for each pump is $H = 101$ ft at $Q = 3730$ gpm. This can be supplied by Peerless Type 10AE12 pumps with impellers of $D = 12$ in. diameter, operating at $N = 1760$ nominal rpm. The efficiency at this operating point is $\eta \approx 0.85$. 3 pump

Find operating points graphically for 1, 2, and 3 pumps:



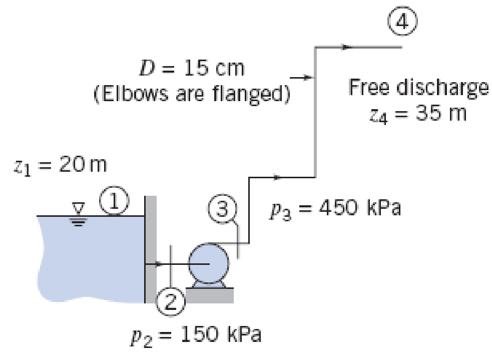
The graphical solution is shown
Q_1 = not satisfactory, $Q_2 = 9400$ gpm (marginal), $Q_3 = 11{,}200$ gpm (OK) Q

Assuming $\eta_p \approx 0.7$, then $\dot{W}_{m_1} = \frac{\rho Q g H}{\eta} \approx 78$ hp, $\dot{W}_{m_2} \approx 241$ hp, and $\dot{W}_{m_3} \approx 409$ hp

Problem 10.78

[4]

10.78 Consider the flow system shown in Problem 8.76. Evaluate the *NPSHA* at the pump inlet. Select a pump appropriate for this application. Use the data on pipe aging from Problem 10.65 to estimate the reduction in flow rate after 10 years of operation.



Given: Data on flow from reservoir/pump

Find: Appropriate pump; Reduction in flow after 10 years

Solution:

Basic equation: $$\left(\frac{p_1}{\rho \cdot g} + \alpha \cdot \frac{V_1^2}{2 \cdot g} + z_1\right) - \left(\frac{p_4}{\rho \cdot g} + \alpha \cdot \frac{V_4^2}{2 \cdot g} + z_4\right) = H_{lT} - H_p$$ for flow from 1 to 4

$$H_{lT} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g} + f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2 \cdot g} + K \cdot \frac{V^2}{2 \cdot g}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) $V_2 = V_3 = V_4$ (constant area pipe)

Given or available data $\rho = 1000 \cdot \frac{kg}{m^3}$ $\nu = 1.01 \times 10^{-6} \cdot \frac{m^2}{s}$ $p_v = 2.34 \cdot kPa$ (Table A.8)

$p_2 = 150 \cdot kPa$ $p_3 = 450 \cdot kPa$ $D = 15 \cdot cm$ $e = 0.046 \cdot mm$ $Q = 0.075 \cdot \frac{m^3}{s}$

$z_1 = 20 \cdot m$ $z_4 = 35 \cdot m$ $V = \frac{4 \cdot Q}{\pi \cdot D^2}$ $V = 4.24 \frac{m}{s}$

For minor losses we have Four elbows: $\frac{L_e}{D} = 4 \times 12 = 48$ (Fig. 8.16) Square inlet: $K_{ent} = 0.5$

At the pump inlet $NPSHA = \dfrac{p_2 + \frac{1}{2} \cdot \rho \cdot V^2 - p_v}{\rho \cdot g}$ $NPSHA = 16.0 m$

The head rise through the pump is $H_p = \frac{p_3 - p_2}{\rho \cdot g}$ $H_p = 30.6 m$

Hence for a flow rate of $Q = 0.075 \frac{m^3}{s}$ or $Q = 1189 gpm$ and $H_p = 30.6 m$ or $H_p = 100 ft$, from Appendix D. Fig. D3 a Peerless4AE11 would suffice

We do not know the pipe length L! Solving the energy equation for it $z_1 - z_4 = H_{lT} - H_p = f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g} + f \cdot \frac{L_e}{D} \cdot \frac{V^2}{2 \cdot g} + K_{ent} \cdot \frac{V^2}{2 \cdot g} - H_p$

For f $Re = \frac{V \cdot D}{\nu}$ $Re = 6.303 \times 10^5$ and $\frac{e}{D} = 3.07 \times 10^{-4}$

Given $\frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re \cdot \sqrt{f}}\right)$ $f = 0.0161$

Hence, substituting values $L = \frac{2 \cdot g \cdot D}{f \cdot V^2} \cdot (z_1 - z_4 + H_p) - D \cdot \left(\frac{L_e}{D}\right) - \frac{K_{ent} \cdot D}{f}$ $L = 146\,m$

From Problem 10.65, for a pipe $D = 0.15\,m$ or $D = 5.91\,in$, the aging over 10 years leads to $f_{worn} = 2.2 \cdot f$

We need to solve the energy equation for a new V

$$V_{worn} = \sqrt{\frac{2 \cdot g \cdot (z_1 - z_4 \cdot}{f_{worn} \cdot \left(\frac{L}{D} + \frac{L_e}{D}\right) + K_{ent}}} \quad V_{worn} = 2.88\,\frac{m}{s}$$

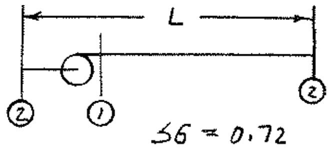
Hence $Q_{worn} = \frac{\pi \cdot D^2}{4} \cdot V_{worn}$ $Q_{worn} = 0.0510\,\frac{m^3}{s}$

$\Delta Q = Q_{worn} - Q$ $\Delta Q = -0.0240\,\frac{m^3}{s}$ $\frac{\Delta Q}{Q} = -32.0\%$

Check f $Re_{worn} = \frac{V_{worn} \cdot D}{\nu}$ Given $\frac{1}{\sqrt{f}} = -2.0 \cdot \log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re_{worn} \cdot \sqrt{f}}\right)$ $f = 0.0165$

Hence using 2.2 x 0.0161 is close enough to using 2.2 x 0.0165

Given: Gasoline pipeline of Problem 8.124:



$L = 13$ km, $D = 0.6$ m, $\Delta p_{12} = 1.4$ MPa $\quad SG = 0.72$

Roughness equivalent to galvanized iron ($e = 0.15$ mm $= 0.00015$ m)

Find: (a) Select suitable pumps for parallel operation.
(b) Calculate the power required for 4 pumps in parallel.
(c) Calculate volume flow rate and power with 1, 2, and 3 pumps.

<u>Solution</u>: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\frac{p_i}{\rho g} + \frac{\alpha_i \bar{V}_i^2}{2g} + z_i + H_p = \frac{p_j}{\rho g} + \frac{\alpha_j \bar{V}_j^2}{2g} + z_j + \frac{h_{lT}}{g}$; $h_{lT} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2}$

Assumptions: (1) $\bar{V}_1 = \bar{V}_2$, $\alpha_1 = \alpha_2$, (2) $z_1 = z_2$, (3) Neglect minor losses, $\frac{L_e}{D} \approx 0$, $K \approx 0$

Find flow rate to size pump. From 1 to 2, $H_p = 0$, so $\quad D = 0.6\,\text{m} \times \frac{\text{ft}}{0.305\,\text{m}} = 1.97$ ft

$\frac{p_1}{\rho g} = \frac{p_2}{\rho g} + \frac{h_{lT}}{g}$; $\Delta p = \rho h_{lT} = \rho f \frac{L}{D}\frac{\bar{V}^2}{2}$; $\bar{V} = \left[\frac{2 \Delta p D}{\rho f L}\right]^{\frac{1}{2}}$ $\quad \frac{e}{D} = \frac{0.00015\,\text{m}}{0.6\,\text{m}} = 2.5 \times 10^{-4}$

But $f = f(Re, e/D)$; Re is not known. Choose f from fully-rough region, $f = 0.014$.

$$\Delta p = 1.4 \times 10^6\,\text{Pa} \times \frac{14.7\,\text{psi}}{101 \times 10^3\,\text{Pa}} = 204\,\text{psi} \; ; \; \rho = 0.72\rho_{H_2O} = 0.72 \times 1.94\,\frac{\text{slug}}{\text{ft}^3} = 1.40\,\text{slug/ft}^3$$

$$\bar{V} = \left[2 \times 204\,\frac{\text{lbf}}{\text{in.}^2} \times 1.97\,\text{ft} \times \frac{\text{ft}^3}{1.40\,\text{slug}} \times \frac{1}{0.014} \times \frac{1}{42{,}600\,\text{ft}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} \times 144\,\frac{\text{in.}^2}{\text{ft}^2}\right]^{\frac{1}{2}} = 11.8\,\text{ft/s}$$

Check: $Re = \frac{\bar{V}D}{\nu} = 11.8\,\frac{\text{ft}}{\text{s}} \times 1.97\,\text{ft} \times \frac{\text{s}}{8.6 \times 10^{-6}\,\text{ft}^2} = 2.70 \times 10^6$; $f = 0.0146$ (See Note below.)

$\bar{V} = \left[\frac{0.014}{0.0146}\right]^{1/2} 11.8\,\frac{\text{ft}}{\text{s}} = 11.6$ ft/s, $\quad Q = \bar{V}A = 35.3\,\frac{\text{ft}^3}{\text{s}} = 15{,}700$ gpm $\quad$ Q

For parallel operation with four pumps, each must supply $\frac{Q}{4} = 3930$ gpm.

The head requirement is $H_p = \frac{p_1 - p_2}{\rho g} = 204\,\frac{\text{lbf}}{\text{in.}^2} \times \frac{\text{ft}^3}{(0.72)62.4\,\text{lbf}} \times 144\,\frac{\text{in.}^2}{\text{ft}^2} = 654$ ft (gasoline)

This combination of head and flow rate cannot be supplied by a single-stage pump. From Fig. D.11, the two-stage Peerless Type 10 TU 22C pump may be chosen. $\quad$ Pump

The input power requirement is $\dot{W}_m = \frac{\rho Q g H}{\eta_p}$. Assuming $\eta_p = 0.65$,

$$\dot{W}_m = \frac{1}{0.65} \times 15{,}700\,\frac{\text{gal}}{\text{min}} \times 204\,\frac{\text{lbf}}{\text{in.}^2} \times \frac{\text{ft}^3}{7.48\,\text{gal}} \times \frac{\text{min}}{60\,\text{s}} \times 144\,\frac{\text{in.}^2}{\text{ft}^2} \times \frac{\text{hp}\cdot\text{s}}{550\,\text{ft}\cdot\text{lbf}} = 2870\,\text{hp (total)}$$ $\quad \dot{W}_m$

Flow rates with fewer pumps operating may be found from a plot. The pump characteristic may be approximated as (assume $H = 0.7\,H_0$):

$$\hat{H}_p \approx 934 - 1.81 \times 10^{-5} Q^2 \quad \left\{H_0 = \frac{654}{0.7} = 934 ; \; B = \frac{H_0 - H}{Q^2} = \frac{(0.3)934}{(3930)^2} = 1.81 \times 10^{-5}\right\}$$

{Note: gasoline is between octane and heptane, Fig. A.3. For $T = 15^\circ$C, $\nu \approx 8 \times 10^{-7}\,\frac{\text{m}^2}{\text{s}} = 8.6 \times 10^{-6}\,\frac{\text{ft}^2}{\text{s}}$.}

Problem 10.79

For two pumps in parallel, $\hat{H} = H_0 - A\left(\frac{Q}{2}\right)^2 \approx 934 - 4.54\times10^{-6}\,Q^2$

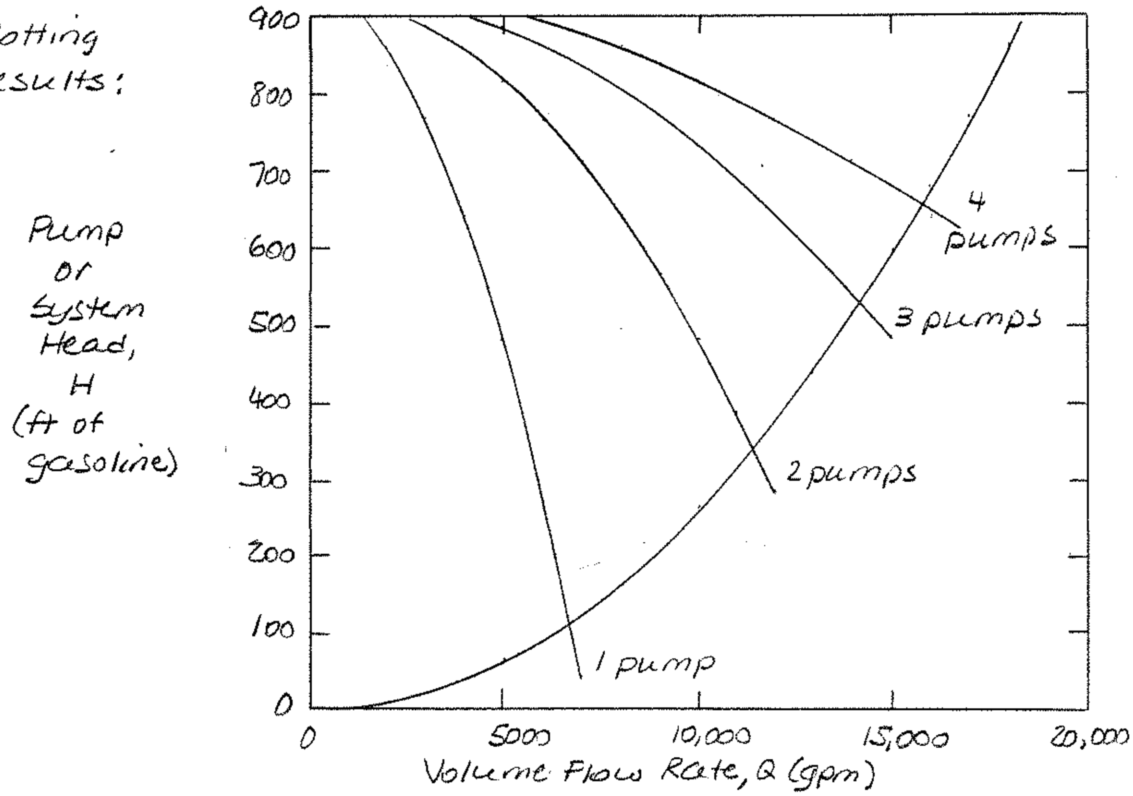
For three pumps in parallel, $\hat{H} = H_0 - A\left(\frac{Q}{3}\right)^2 \approx 934 - 2.02\times10^{-6}\,Q^2$

For four pumps in parallel, $\hat{H} = H_0 - A\left(\frac{Q}{4}\right)^2 \approx 934 - 1.13\times10^{-6}\,Q^2$

The pipe system characteristic is approximately given by

$$\hat{H}_s \approx BQ^2\,;\quad B = \frac{\hat{H}_s}{Q^2} = \frac{654}{(15{,}700)^2} = 2.65\times10^{-6}\ \text{ft/gpm}^2,\ \text{so}\ \hat{H}_s = 2.65\times10^{-6}\,Q^2$$

Plotting results:

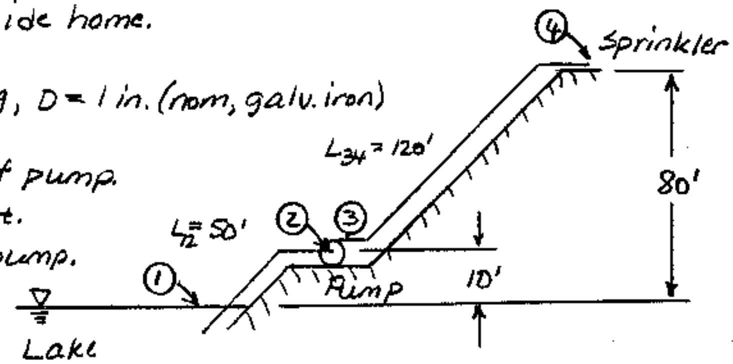


The approximate volume flow rates, heads, and power requirements (assuming $\eta_p = 0.65$) are:

Number of Pumps	1	2	3	4
Flow Rate (gpm)	6710	11,400	14,200	15,700
Head (ft gasoline)	119	345	531	654
Power (hp)	224	1100	2110	2880

Problem 10.80

Given: Sprinkler system for lakeside home.



$Q = 10\,gpm$, $p_4 = 50\,psig$, $D = 1\,in.$ (nom, galv. iron)

Find: (a) Head loss on suction side of pump.
(b) Gage pressure at pump inlet.
(c) Hydralic power req'mt for pump.
(d) Change to $D = 1.5\,in.$?
(e) Pump halfway up hill?

Solution: Assume $T = 68°F$, so $\nu = 1.08\times10^{-5}\,ft^2/s$ (Table A.7).

For 1 in. (nominal) pipe, $D = 1.049\,in.$ (Table 8.5). $\bar{V} = \frac{Q}{A}$; $A = \frac{\pi}{4}D^2 = \frac{\pi}{4}\left(\frac{1.049}{12}\right)^2 ft^2 = 0.00600\,ft^2$

$$\bar{V} = \frac{Q}{A} = 10\,\frac{gal}{min}\times\frac{1}{0.00600\,ft^2}\times\frac{ft^3}{7.48\,gal}\times\frac{min}{60\,s} = 3.71\,ft/s\;;\;\frac{\bar{V}^2}{2} = \frac{1}{2}\times(3.71)^2\frac{ft^2}{s^2} = 6.90\,ft^2/s^2$$

$$Re = \frac{\bar{V}D}{\nu} = 3.71\,\frac{ft}{s}\times1.049\,in.\times\frac{s}{1.08\times10^{-5}\,ft^2}\times\frac{ft}{12\,in.} = 3.00\times10^4\;;\;e = 0.0005\,ft\ \text{(Table 8.1)}$$

Therefore $\frac{e}{D} = 0.0005\,ft\times\frac{1}{1.049\,in.}\times12\,\frac{in.}{ft} = 0.00572$ and $f \approx 0.034$ (Fig. 8.13).

Apply energy equation for steady incompressible pipe flow:

$$\left(\frac{p_1}{\rho g} + \alpha_1\frac{\bar{V}_1^2}{2g} + z_1\right) - \left(\frac{p_2}{\rho g} + \alpha_2\frac{\bar{V}_2^2}{2g} + z_2\right) = \frac{h_{lT_{12}}}{g} = \left(f\frac{L_{12}}{D} + K_{ent} + f\frac{L_e}{D}_{45°} + f\frac{L_e}{D}_{90°}\right)\frac{\bar{V}^2}{2g}$$

(terms marked $\approx 0(1)$, $0(2)$, $=0(3)$; $K_{ent} = 0.78$ (Table 8.2), $\frac{L_e}{D}_{45°} = 16$, $\frac{L_e}{D}_{90°} = 30$ (Table 8.4))

Assumptions: (1) $p_1 = p_{atm} = 0$ (gage)
(2) $V_1 = 0$
(3) $z_1 = 0$
(4) $\alpha_2 \approx 1$

$$\frac{L_{12}}{D} = \frac{50\,ft}{1.049\,in.}\times12\,\frac{in.}{ft} = 572$$

$$H_{lT_{12}} = \left[0.034(572 + 30 + 16) + 0.78\right]6.88\,\frac{ft^2}{s^2}\times\frac{s^2}{32.2\,ft}$$

$$H_{lT_{12}} = 4.67\,ft \quad \leftarrow h_{lT_{12}}$$

$$p_2(gage) = -\rho g\left(\frac{\bar{V}_2^2}{2g} + z_2 + H_{lT}\right)$$

$$p_2(gage) = -62.4\,\frac{lbf}{ft^3}\left(6.88\,\frac{ft^2}{s^2}\times\frac{s^2}{32.2\,ft} + 10\,ft + 4.67\,ft\right)\frac{ft^2}{144\,in.^2} = -6.45\,psig \quad \leftarrow p_2$$

To find hydraulic power, must know p_3. Apply energy equation:

$$\left(\frac{p_3}{\rho g} + \alpha_3\frac{\bar{V}_3^2}{2g} + z_3\right) - \left(\frac{p_4}{\rho g} + \alpha_4\frac{\bar{V}_4^2}{2g} + z_4\right) = h_{lT_{34}} = \left(f\frac{L_{34}}{D} + 2K_{45°}\right)\frac{\bar{V}^2}{2g}$$

Assumptions: (1) $\bar{V}_3 = \bar{V}_4 = \bar{V} = \bar{V}_2$

$$\frac{L_{34}}{D} = \frac{120\,ft}{1.049\,in.}\times12\,\frac{in.}{ft} = 1{,}370$$

$$p_3 = p_4 + \rho g\left[z_4 - z_3 + \left(f\frac{L_{34}}{D} + 2f\frac{L_e}{D}_{45°}\right)\frac{\bar{V}^2}{2g}\right]$$

$$p_3 = 50\,psig + 62.4\,\frac{lbf}{ft^3}\left[(80-10)ft + (0.034\times(1370 + 2\times16))6.90\,\frac{ft^2}{s^2}\times\frac{s^2}{32.2\,ft}\right]\frac{ft^2}{144\,in.^2} = 84.8\,psig$$

Thus the pump head is $H_p = \frac{p_3 - p_2}{\rho g} = [84.8 - (-6.45)]\,\frac{lbf}{in.^2}\times\frac{ft^3}{62.4\,lbf}\times144\,\frac{in.^2}{ft^2} = 211\,ft$

since $\bar{V}_2 = \bar{V}_3$ and the elevations are corrected to the pump centerline ($z_2 = z_3$).

Problem 10.80

From Eq. 10.8a, $\dot{W}_h = \rho Q g H_p$. Thus $\dot{W}_h = \rho g Q H_p$

$$\dot{W}_h = 62.4 \frac{lbf}{ft^3} \times 10 \frac{gal}{min} \times 211\ ft \times \frac{ft^3}{7.48\ gal} \times \frac{hp \cdot min}{33{,}000\ ft \cdot lbf} = 0.533\ hp$$

$\dot{W}_h$

Changing to D = 1.5 in. (nominal) pipe would reduce the mean velocity and hence the head loss and the minor loss. For this pipe D = 1.610 in. (Table 8.4).

$$A = \frac{\pi D^2}{4} = \frac{\pi}{4}\left(\frac{1.610}{12}\right)^2 ft^2 = 0.0141\ ft^2; \text{ so } \bar{V} = \frac{Q}{A} = 10 \frac{gal}{min} \times \frac{ft^3}{7.48\ gal} \times \frac{min}{60\ s} \times \frac{1}{0.0141\ ft^2} = 1.58\ ft/s$$

$$Re = \frac{\bar{V}D}{\nu} = 1.58 \frac{ft}{s} \times \frac{1.61}{12} ft \times \frac{s}{1.08 \times 10^{-5}\ ft^2} = 1.96 \times 10^4;\ \frac{e}{D} = 0.00373;\ f \approx 0.032\ \text{(Fig. 8.13)}$$

Thus $\frac{\bar{V}_2^2}{2g} = \frac{1}{2} \times (1.58)^2 \frac{ft^2}{s^2} \times \frac{s^2}{32.2\ ft} = 0.0388\ ft$

$$H_{\ell T_{12}} = \left(\left(0.031 \times \frac{50}{(1.61/12)} + 30 + 16\right) + 0.78\right) 0.0388\ ft = 0.534\ ft$$

$$H_{\ell T_{34}} = \left(0.031 \times \left(\frac{120}{(1.61/12)} + 2 \times 16\right)\right) 0.0388\ ft = 1.11\ ft$$

$$p_2(gage) = -\rho g \left(\frac{\bar{V}_2^2}{2g} + z_2 + H_{\ell T_{12}}\right)$$

$$p_2(gage) = -62.4 \frac{lbf}{ft^3} (0.0388 + 10 + 0.534)\ ft \times \frac{ft^2}{144\ in.^2} = -4.58\ psig$$

and

$$p_3(gage) = p_4 + \rho g (z_4 - z_3 + H_{\ell T_{34}})$$

$$p_3(gage) = 50\ psig + 62.4 \frac{lbf}{ft^3} (80 - 10 + 1.11)\ ft \times \frac{ft^2}{144\ in.^2} = 80.8\ psig$$

The pump head is $H_p = \frac{p_3 - p_2}{\rho g} = (80.8 - (-4.58)) \frac{lbf}{in.^2} \times \frac{ft^3}{62.4\ lbf} \times \frac{144\ in.^2}{ft^2} = 197\ ft$

Thus the pump power is $\dot{W}_h = \rho g Q H_p = 0.498\ hp$

The power reduction is $\Delta \dot{W}_h = \frac{\dot{W}_h(1.5\ in.) - \dot{W}_h(1\ in.)}{\dot{W}_h(1\ in.)} = -6.7$ percent

$\Delta \dot{W}_h$

The pump should <u>not</u> be moved up the hill. The NPSHA now is

$$NPSHA = \frac{1}{\rho g}\left[p(abs) + \rho \frac{\bar{V}_2^2}{2} - p_v\right] = \frac{1}{\rho g}\left[p_{atm} + p(gage) + \rho \frac{\bar{V}_2^2}{2} - p_v\right]$$

At $T = 68°F$, $p_v = 0.339$ psia (Table A.7).

$$NPSHA = \frac{ft^3}{62.4\ lbf}\left[14.7 \frac{lbf}{in.^2} + (-6.45) \frac{lbf}{in.^2} + 1.94 \frac{slug}{ft^3} \times 6.90 \frac{ft^2}{s^2} \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{ft^2}{144\ in.^2} - 0.339 \frac{lbf}{in.^2}\right] 144 \frac{in.^2}{ft^2}$$

NPSHA = 18.5 ft (1 in. pipe) 22.6 ft (1.5 in. pipe)

NPSHA

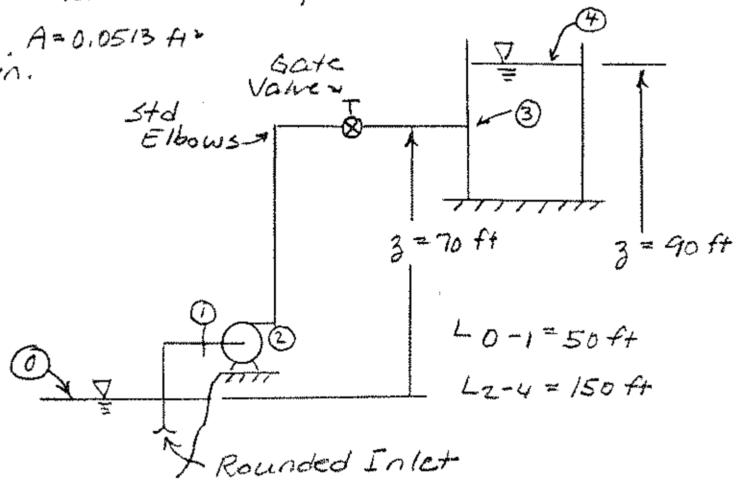
This should not be reduced further by raising the pump. If anything, the pump should be lowered to increase NPSHA.

Problem 10.81

Given: Pump and piping system at lakeside home, as shown.

Galvanized iron pipe. $A = 0.0513\ ft^2$

$D = 3$ in. (nominal) $= 3.068$ in.



Find:
(a) System head-flow curve
(b) System operating point
(c) Power input if $\eta_p = 0.8$
(d) Sketch of system curve when $z_4 = 90$ ft
(e) Sketch of system curve when $z_3 = 75$ ft, valve partclosed, and $Q = 0.1\ ft^3/s$.
(f) Which case has higher pump efficiency?

Solution: Apply the energy equation for pipe flow. The pump must overcome the gravity lift plus the head losses in the pipe and fittings.

Assume: (1) Nominal speed is $\bar{V} = 12$ ft/s, $T = 60°F$, $\nu = 1.21 \times 10^{-5}\ ft^2/s$ (Table A.7)
(2) Flow in fully rough zone ($e = 0.0005$ ft (Table 8.1), $e/D = 0.002$, $f \approx 0.024$)
(3) Cases: (1) Water in tank below ③ } Valve open
(2) Water in tank at $z = 90$ ft }
(3) Valve closed so $Q = 0.1\ ft^3/s$, valve part closed.

Then

$$H_{\ell T} = \frac{h_{\ell T}}{g} = \left[K_{ent} + f\left(\frac{L}{D} + 3\frac{L_e}{D}(\text{elbow}) + \frac{L_e}{D}(\text{gate valve})\right) + K_{exit}\right]\frac{\bar{V}^2}{2g}$$

$$H_{\ell T} = \left[0.04 + 0.024\left(\frac{200\ ft}{3.068\ in.} \times 12\frac{in}{ft} + 3(30) + 8\right) + 1\right]\frac{\bar{V}^2}{2g} = 22.2\ \frac{\bar{V}^2}{2g}$$

and $\quad H_s = z_{end} + 22.2\dfrac{\bar{V}^2}{2g}$ $\qquad$ **H_s**

Assume $\bar{V} = 12$ ft/s, $Q = 276$ gpm, $\dfrac{\bar{V}^2}{2g} = 2.24$ ft, $H_s = 70 + 22.2(2.24) = 120$ ft

Case 1: $z_{end} = 70$ ft $\quad$ Operating point: $Q = 276$ gpm, $H_p = H_s = 120$ ft $\qquad$ **Oper. Pt.**

$$P = \frac{\rho g Q H}{\eta_p} = 62.4\frac{lbf}{ft^3} \times 12.0\frac{ft}{s} \times 0.0513\ ft^2 \times 120\ ft \times \frac{1}{0.8} \times \frac{hp \cdot s}{550\ ft \cdot lbf} = 10.5\ hp$$ **P**

Case 2: $z_{end} = 90$ ft ; $H_s = 90 + 22.2(2.24) = 140$ ft

$$H_s = 90 + 50.0\frac{[Q(gpm)]^2}{(276)^2(gpm)^2} = 90 + 6.56 \times 10^{-4}[Q(gpm)]^2$$ **H_s**

Case 3: $Q = 0.1\ ft^3/s = 44.9$ gpm ; $H_s = H_p$; Assume $H_{BEP} = 0.7\ H_0$

$$H_p = H_0 + \frac{(H_0 - H_{op})}{Q_{op}^2}Q^2 = \frac{120}{0.7} - \frac{(120/0.7 - 120)}{(276)^2}Q^2 = 169 - 6.75 \times 10^{-4} Q^2$$

$H_p = 168$ ft at $Q = 44.9$ gpm $\qquad$ **H_p**

over

Problem 10.81

Water pumped from lake to storage tank on bluff:

Input Data:

Friction factor:	$f =$	0.024	(---)
Pipe diameter:	$D =$	3.068	in.

Calculated Results:

Pipe area:	$A =$	0.0513	ft^2

System Curves for Various Conditions:

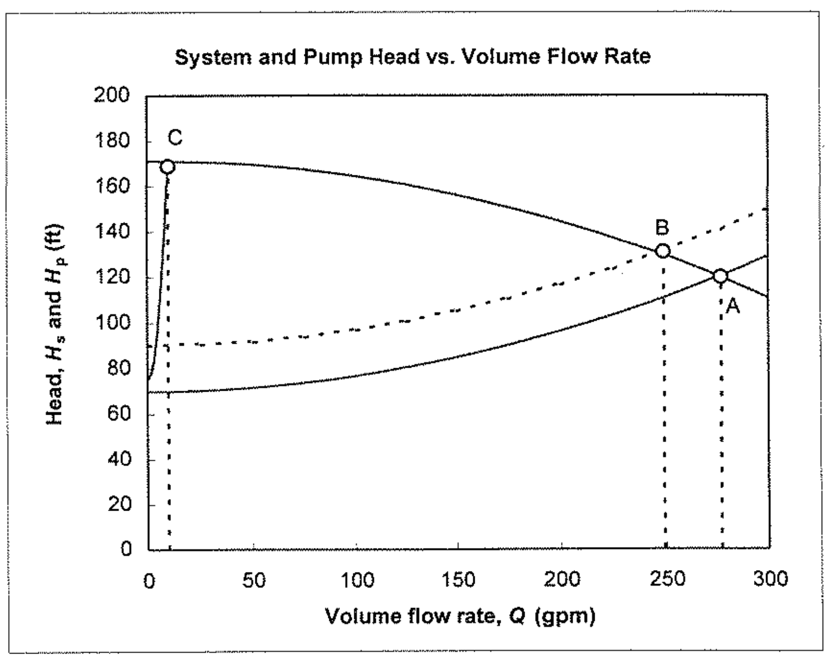
			Case 1:	Case 2:
			H_s	H_s
Q	V	$V^2/2g$	(z_3 = 70 ft)	(z_3 = 90 ft)
(gpm)	(ft/s)	(ft)	(ft)	(ft)
0	0	0.00	70.0	90.0
25	1.09	0.02	70.4	90.4
50	2.17	0.07	71.7	91.7
75	3.26	0.16	73.7	93.7
100	4.34	0.29	76.6	96.6
125	5.43	0.46	80.3	100
150	6.51	0.66	84.9	105
175	7.60	0.90	90.2	110
200	8.68	1.17	96.4	116
225	9.77	1.48	103	123
249.3	10.8	1.82	111	131
277	12.0	2.24	121	141
300	13.0	2.63	129	149

Case 3: Valve partially closed

Q	H_s (z_3 = 75 ft)
(gpm)	(ft)
0	75.0
2	78.8
4	90.0
6	109
8	135
10	169

Pump Head Curve:

Q	H_p
(gpm)	(ft)
0	171
50	170
100	165
150	156
200	145
250	130
277	120
300	111



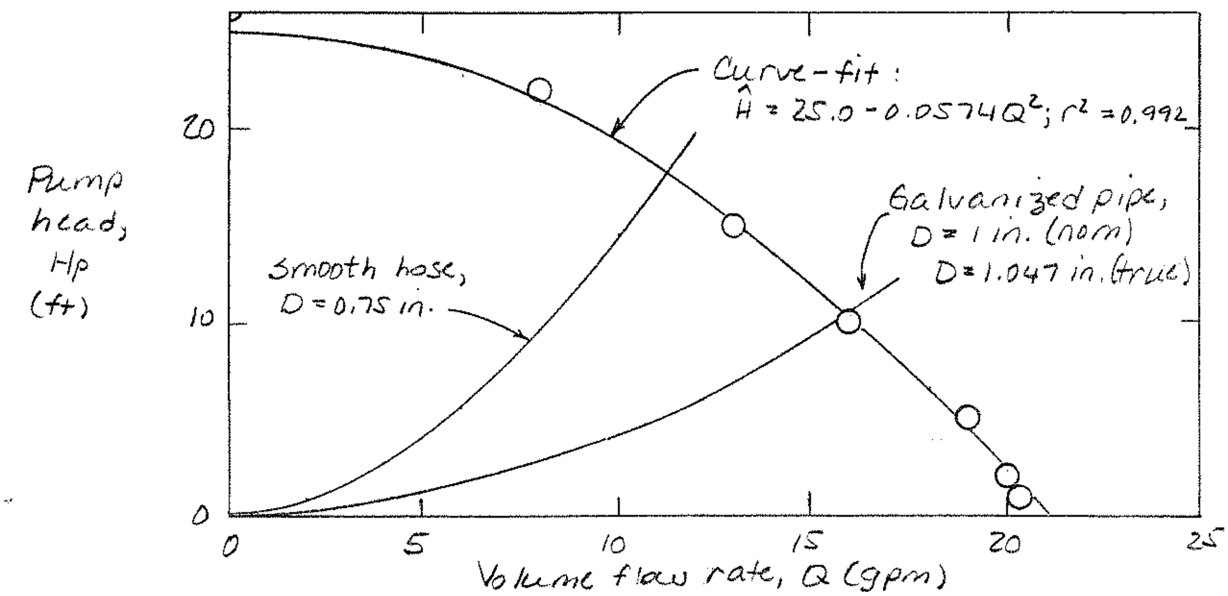
Problem 10.82

Given: Manufacturer data for submersible pump:

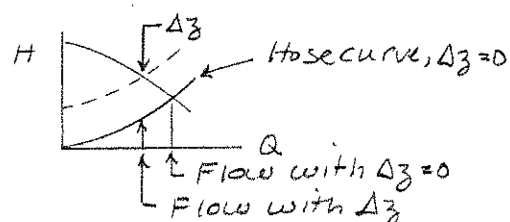
Discharge Height (ft)	1	2	5	10	15	20	26.3
Water Flow Rate (gpm)	20.4	20	19	16	13	8	0

Find: (a) Plot a performance curve for this pump.
(b) Develop and show a curve-fit to the data.
(c) Calculate and plot the pump delivery versus discharge height for (1) 50' of 3/4 in. garden hose, (2) 50' of 1 in. pipe.

Solution: The performance data and curve-fit are plotted below:

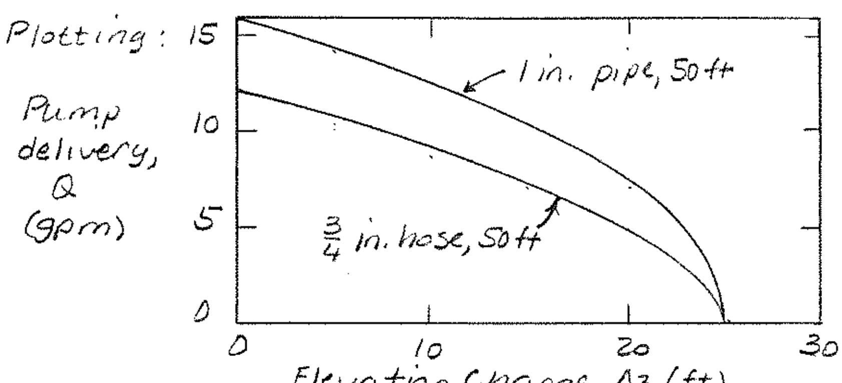


The energy equation for steady, incompressible pipe flow was used to develop the system curves shown above, for $\Delta z = 0$. To obtain delivery versus height, add the elevation change to the head for $\Delta z = 0$ to find the new intersection with the pump curve:



The same result is obtained when the difference between the pump curve and the hose, at a given flow rate, equals Δz.

Plotting:



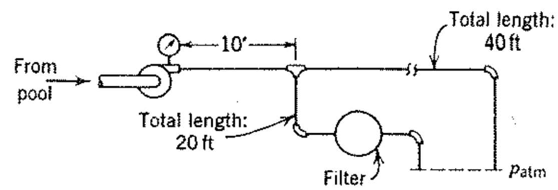
Problem 10.83

Given: Swimming pool filtration system of Problem 8.169.

Assume pipe used is 3/4 in. (nominal) smooth PVC plastic.

Pump delivers $Q = 30$ gpm of water at 75°F.

Filter pressure drop is $\Delta p = 0.6\,Q^2$, where Δp in psi and Q in gpm.



Find: (a) Specify speed and impeller diameter of suitable pump.
(b) Estimate pump efficiency.

Solution: Need head to specify pump. Using FM, with $Q = 30$ gpm (0.0668 cfs) and $D = 0.824$ in. (0.0687 ft), $\Delta p_{12} = 5.56$ psi.

Flow must split to give same pressure drop in each branch. Assuming L_e/D for each elbow is 30, iteration gives:

$Q_{24} = 24.8$ gpm (0.0553 cfs) and $\Delta p = 16.69$ psi

$Q_{23} = 5.2$ gpm (0.0116 cfs) and $\Delta p = 16.2 + 0.6023 = 16.8$ psi

Neglecting any pressure at the pump inlet, the pump must supply

$$\Delta p_{pump} = \Delta p_{12} + \Delta p_{23} = 5.56 + 16.7 = 22.3 \text{ psi}$$

The pump head is $H = \dfrac{\Delta p}{\rho g} = 22.3\,\dfrac{\text{lbf}}{\text{in.}^2} \times \dfrac{\text{ft}^3}{62.4\text{ lbf}} \times 144\,\dfrac{\text{in.}^2}{\text{ft}^2} = 51.5\text{ ft}$ H

This pump is too small to be found in Fig. D.1. Therefore approximate its characteristics using N_s and H_0. Assume $N = 3500$ rpm:

$$N_{s_{cu}} = \frac{N Q^{1/2}}{H^{3/4}} = \frac{3500\,(30)^{1/2}}{(51.5)^{3/4}} = 997$$

From Fig. 10.15, $\eta \approx 0.62$ or less. Assuming $H = 0.7\,H_0$, then

$$H = 0.7\,H_0 = 0.7\,\frac{(\omega R)^2}{g}\,; \quad R = \frac{1}{\omega}\left(\frac{gH}{0.7}\right)^{\frac{1}{2}} = \frac{s}{367\text{ rad}}\left[32.2\,\frac{\text{ft}}{s^2} \times 51.5\text{ ft} \times \frac{1}{0.7}\right]^{\frac{1}{2}} = 0.133\text{ ft}$$

The impeller diameter is approximately

$$D = 2R = 2 \times 0.133\text{ ft} \times 12\,\frac{\text{in.}}{\text{ft}} = 3.18\text{ in.}$$ D

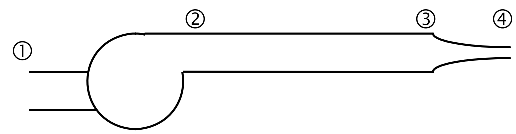
The pump power requirement is

$$\dot{W}_m = \frac{\rho Q g H}{\eta_p} = \frac{1}{0.6} \times 0.0668\,\frac{\text{ft}^3}{s} \times 62.4\,\frac{\text{lbf}}{\text{ft}^3} \times 51.5\text{ ft} \times \frac{\text{hp}\cdot s}{550\text{ ft}\cdot\text{lbf}} = 0.651\text{ hp}$$

A 3/4 horsepower motor should be used. $\dot{W}_m$

Problem 10.84

10.84 Consider the fire hose and nozzle of Problem 8.159. Specify an appropriate pump and impeller diameter to supply four such hoses simultaneously. Calculate the power input to the pump.



Given:
Fire nozzle/pump system

Find:
Appropriate pump; Impeller diameter; Pump power input needed

Solution:

Basic equations $\left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) - \left(\frac{p_3}{\rho} + \alpha\cdot\frac{V_3^2}{2} + g\cdot z_3\right) = h_l$ $\quad h_l = f\cdot\frac{L}{D}\cdot\frac{V_2^2}{2}$ for the hose

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 2 and 3 is approximately 1 4) No minor loss

$$\left(\frac{p_2}{\rho} + \alpha\cdot\frac{V_2^2}{2} + g\cdot z_2\right) - \left(\frac{p_1}{\rho} + \alpha\cdot\frac{V_1^2}{2} + g\cdot z_1\right) = h_{pump} \qquad \text{for the pump}$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) α at 1 and 2 is approximately 1 4) No minor loss

The first thing we need is the flow rate. Below we repeat Problem 8.159 calculations

Hence for the hose $\frac{\Delta p}{\rho} = \frac{p_2 - p_3}{\rho} = f\cdot\frac{L}{D}\cdot\frac{V^2}{2}$ or $V = \sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$

We need to iterate to solve this for V because f is unknown until Re is known. This can be done using *Excel*'s Solver, but here:

$\Delta p = 750\cdot kPa$ $\quad L = 100\cdot m$ $\quad e = 0$ $\quad D = 3.5\cdot cm$ $\quad \rho = 1000\cdot\frac{kg}{m^3}$ $\quad \nu = 1.01\times 10^{-6}\cdot\frac{m^2}{s}$

Make a guess for f $\quad f = 0.01$ $\quad V = \sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$ $\quad V = 7.25\frac{m}{s}$ $\quad Re = \frac{V\cdot D}{\nu}$ $\quad Re = 2.51\times 10^5$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f = 0.0150$

$V = \sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$ $\quad V = 5.92\frac{m}{s}$ $\quad Re = \frac{V\cdot D}{\nu}$ $\quad Re = 2.05\times 10^5$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f = 0.0156$

$V = \sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$ $\quad V = 5.81\frac{m}{s}$ $\quad Re = \frac{V\cdot D}{\nu}$ $\quad Re = 2.01\times 10^5$

Given $\quad \frac{1}{\sqrt{f}} = -2.0\cdot\log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\cdot\sqrt{f}}\right)$ $\quad f = 0.0156$

$V = \sqrt{\frac{2\cdot\Delta p\cdot D}{\rho\cdot f\cdot L}}$ $\quad V = 5.80\frac{m}{s}$ $\quad Re = \frac{V\cdot D}{\nu}$ $\quad Re = 2.01\times 10^5$

$Q = \frac{\pi \cdot D^2}{4} \cdot V$ $\quad Q = 5.578 \times 10^{-3} \frac{m^3}{s}$ $\quad Q = 0.335 \cdot \frac{m^3}{min}$

We have $\quad p_1 = 350 \cdot kPa \quad p_2 = 700 \cdot kPa + 750 \cdot kPa \quad p_2 = 1450\, kPa$

For the pump

$$\left(\frac{p_2}{\rho} + \alpha \cdot \frac{V_2^{\ 2}}{2} + g \cdot z_2\right) - \left(\frac{p_1}{\rho} + \alpha \cdot \frac{V_1^{\ 2}}{2} + g \cdot z_1\right) = h_{pump}$$

so $\quad h_{pump} = \frac{p_2 - p_1}{\rho}$ $\quad$ or $\quad H_{pump} = \frac{p_2 - p_1}{\rho \cdot g}$ $\quad H_{pump} = 112\, m$

We need a pump that can provide a flow of $Q = 0.335 \frac{m^3}{min}$ or $Q = 88.4\, gpm$, with a head of $H_{pump} = 112\, m$ or $H_{pump} = 368\, ft$

From Appendix D, Fig. D.1 we see that a Peerless 2AE11 can provide this kind of flow/head combination; it could also handle four such hoses (the flow rate would be $4 \cdot Q = 354\, gpm$). An impeller diameter could be chosen from proprietary curves.

The required power input is $\quad W_m = \frac{W_h}{\eta_p}$ $\quad$ where we choose $\eta_p = 75 \cdot \%$ from Fig. 10.15

$W_m = \frac{\rho \cdot Q \cdot g \cdot H_{pump}}{\eta_p}$ $\quad W_m = 8.18\, kW$ for one hose or $4 \cdot W_m = 32.7\, kW$ for four

$P_{required} = \frac{P_{pump}}{\eta}$ $\quad P_{required} = \frac{6.14 \cdot kW}{70 \cdot \%}$ $\quad P_{required} = 8.77 \cdot kW$ or $4 \cdot P_{required} = 35.1\, kW$ for four

10.85 Performance data for a centrifugal fan of 1 m diameter, tested at 650 rpm, are

Volume flow rate Q (m^3/s)	3	4	5	6	7	8
Static pressure rise, Δp (mm H_2O)	53	51	45	35	23	11
Power output $\mathscr{P}$ (kW)	2.05	2.37	2.60	2.62	2.61	2.4

Plot the performance data versus volume flow rate. Calculate static efficiency and show the curve on the plot. Find the best efficiency point and specify the fan rating at this point.

Given: Data on centrifugal fan

Find: Plot of performance curves; Best effiiciency point

Solution:

Basic equations: $\eta_p = \frac{W_h}{W_m}$ $\quad W_h = Q\cdot\Delta p$ $\quad \Delta p = \rho_w\cdot g\cdot\Delta h$ (Note: Software cannot render a dot!)

$\rho_w = 1000$ kg/m^3

Q (m^3/s)	Δp (mm)	$\mathscr{P}_m$ (kW)	$\mathscr{P}_h$ (kW)	η (%)
3	53	2.05	1.56	76.1%
4	51	2.37	2.00	84.4%
5	45	2.60	2.21	84.9%
6	35	2.62	2.06	78.6%
7	23	2.61	1.58	60.5%
8	11	2.40	0.86	36.0%

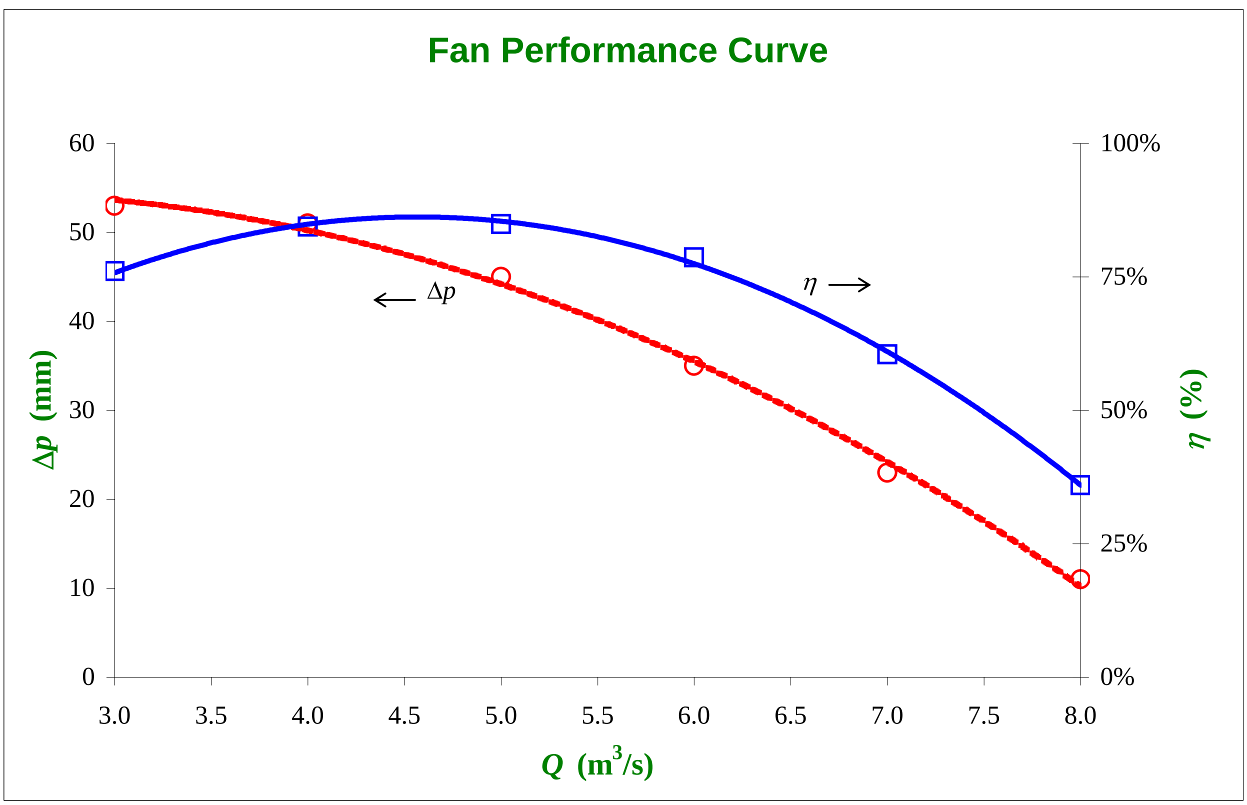
Fitting a 2nd order polynomial to each set of data we find

$\Delta p = -1.32Q^2 + 5.85Q + 48.0$

$\eta = -0.0426Q^2 + 0.389Q - 0.0267$

Finally, we use Solver to maximize η by varying Q:

Q (m^3/s)	Δp (mm)	η (%)
4.57	47.2	86.1%



Problem 10.86

[3]

10.86 Using the fan of Problem 10.85 determine the minimum-size square sheet-metal duct that will carry a flow of 5.75 m^3/s over a distance of 15 m. Estimate the increase in delivery if the fan speed is increased to 800 rpm.

Given: Data on centrifugal fan and square metal duct

Find: Minimum duct geometry for flow required; Increase if fan speed is increased

Solution:

Basic equations: $\eta_p = \dfrac{W_h}{W_m}$ $\quad W_h = Q \cdot \Delta p$ $\quad \Delta p = \rho_w \cdot g \cdot \Delta h$ (Note: Software cannot render a dot!)

and for the duct $\quad \Delta p = \rho_{air} \cdot f \cdot \dfrac{L}{D_h} \cdot \dfrac{V^2}{2}$ $\quad D_h = \dfrac{4 \cdot A}{P} = \dfrac{4 \cdot H^2}{4 \cdot H} = H$

and fan scaling $\quad Q = 5.75 \cdot \dfrac{m^3}{s}$ $\quad \omega = 650 \cdot rpm$ $\quad \omega' = 800 \cdot rpm$ $\quad Q' = \dfrac{\omega'}{\omega} \cdot Q$ $\quad Q' = 7.08 \dfrac{m^3}{s}$

$\rho_w =$ 1000 kg/m^3

$\rho_{air} =$ 1.225 kg/m^3

$\nu_{air} =$ 1.50.E-05 m^2/s

$L =$ 15 m

Assume smooth ducting

Note: Efficiency curve not needed for this problem

Q (m^3/s)	Δp (mm)	$\mathscr{P}_m$ (kW)	$\mathscr{P}_h$ (kW)	η (%)
3	53	2.05	1.56	76.1%
4	51	2.37	2.00	84.4%
5	45	2.60	2.21	84.9%
6	35	2.62	2.06	78.6%
7	23	2.61	1.58	60.5%
8	11	2.40	0.86	36.0%

Fitting a 2nd order polynomial to each set of data we find

$\Delta p = -1.32Q^2 + 5.85Q + 48.0$

User Solver to vary H so the error in Δp is zero

	Fan
Q (m^3/s)	Δp (mm)
7.08	23.3

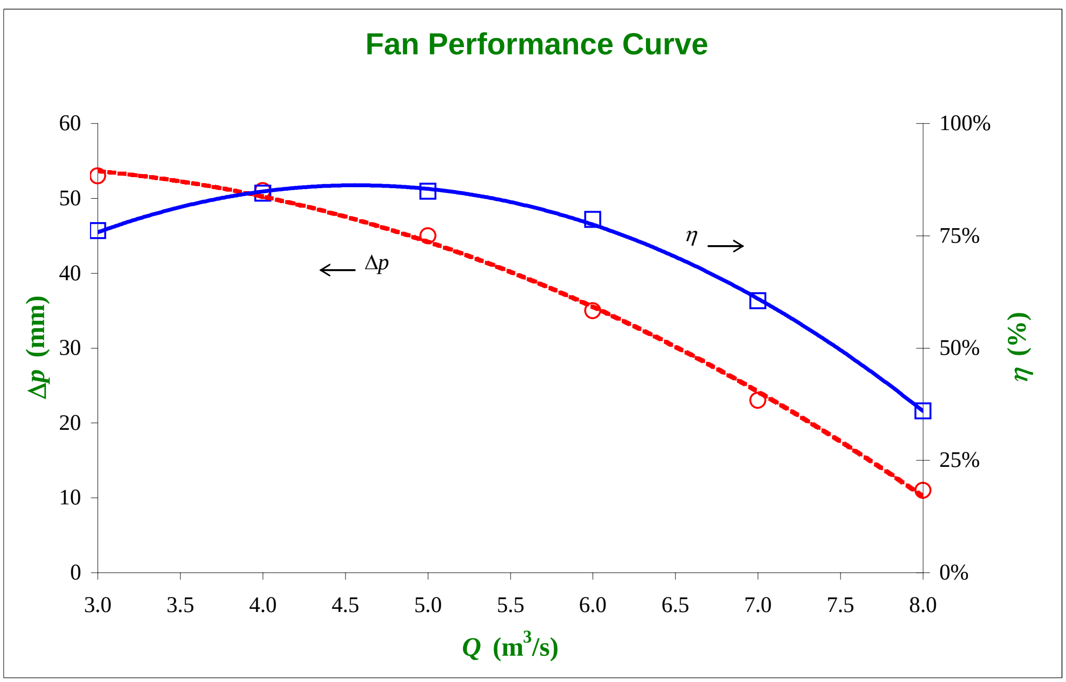
				Duct
H (m)	V (m/s)	Re	f	Δp (mm)
0.472	31.73	9.99.E+05	0.0116	23.3

Error in Δp: 0.00%

Answers:

Q (m^3/s)	H (m)
5.75	0.394

Q (m^3/s)	H (m)
7.08	0.472



Problem 10.87

10.87 Consider the fan and performance data of Problem 10.85. At $Q = 5.75\ m^3/s$, the dynamic pressure is equivalent to 4 mm of water. Evaluate the fan outlet area. Plot total pressure rise and input horsepower for this fan versus volume flow rate. Calculate the fan total efficiency and show the curve on the plot. Find the best efficiency point and specify the fan rating at this point.

Given: Data on centrifugal fan

Find: Fan outlet area; Plot total pressure rise and power; Best efficiency point

Solution:

Basic equations: $\eta_p = \frac{W_h}{W_m}$ $\quad W_h = Q\cdot\Delta p_t$ $\quad \Delta p = \rho_w\cdot g\cdot\Delta h_t$ (Note: Software cannot render a dot!)

$$p_{dyn} = \frac{1}{2}\cdot\rho_{air}\cdot V^2$$

At $Q = 5.75\cdot\frac{m^3}{s}$ we have $h_{dyn} = 4\cdot mm$ $\quad Q = V\cdot A$ and $h_{dyn} = \frac{p_{dyn}}{\rho_w\cdot g} = \frac{\rho_{air}}{\rho_w}\cdot\frac{V^2}{2}$

Hence $V = \sqrt{\frac{\rho_w}{\rho_{air}}\cdot 2\cdot g\cdot h_{dyn}}$ and $A = \frac{Q}{V}$

The velocity V is directly proportional to Q, so the dynamic pressure at any flow rate Q is $h_{dyn} = 4\cdot mm\cdot\left(\frac{Q}{5.75\cdot\frac{m^3}{s}}\right)^2$

The total pressure Δh_t will then be $\Delta h_t = \Delta h + h_{dyn}$ $\quad \Delta h$ is the tabulated static pressure rise

At $Q =$ 5.75 m^3/s

$h_{dyn} =$ 4 mm Hence $V =$ 8.00 m/s

$A =$ 0.71838 m^2

$\rho_w =$ 1000 kg/m^3

$\rho_{air} =$ 1.225 kg/m^3

Fitting a 2nd order polynomial to each set of data we find

$h_t = -0.12Q^2 + 0.585Q + 4.7986$

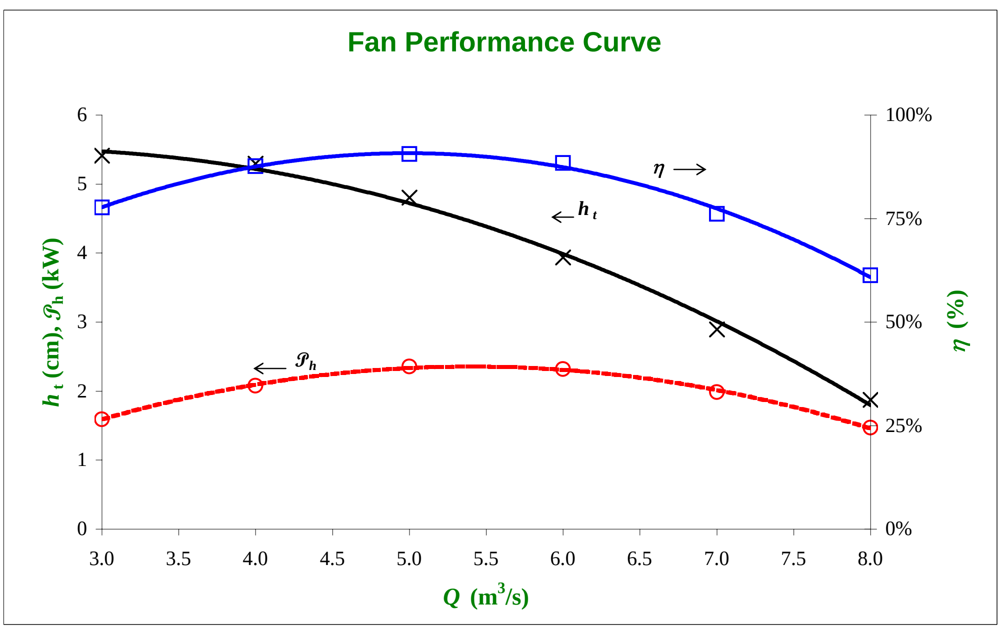
$\mathcal{P}_h = -0.133Q^2 + 1.43Q - 1.5202$

$\eta = -0.0331Q^2 + 0.330Q + 0.0857$

$Q\ (m^3/s)$	Δp (mm)	$\mathcal{P}_m$ (kW)	h_{dyn} (mm)	h_t (cm)	$\mathcal{P}_h$ (kW)	η (%)
3	53	2.05	1.09	5.41	1.59	77.7%
4	51	2.37	1.94	5.29	2.08	87.6%
5	45	2.60	3.02	4.80	2.36	90.6%
6	35	2.62	4.36	3.94	2.32	88.4%
7	23	2.61	5.93	2.89	1.99	76.1%
8	11	2.40	7.74	1.87	1.47	61.3%

Finally, we use Solver to maximize η by varying Q:

$Q\ (m^3/s)$	h_t (cm)	$\mathcal{P}_h$ (kW)	η (%)
4.98	4.73	2.30	90.8%



Problem 10.88

10.88 The performance data of Problem 10.85 are for a 1-m diameter fan wheel. This fan also is manufactured with 1.025, 1.125-, 1.250-, and 1.375-m diameter wheels. Pick a standard fan to deliver 14 m^3/s against a 25-mm H_2O static pressure rise. Assume standard air at the fan inlet. Determine the required fan speed and the input power needed.

Volume flow rate Q (m^3/s)	3	4	5	6	7	8
Static pressure rise, Δp (mm H_2O)	53	51	45	35	23	11
Power output $\mathscr{P}$ (kW)	2.05	2.37	2.60	2.62	2.61	2.4

Given: Data on centrifugal fan and various sizes

Find: Suitable fan; Fan speed and input power

Solution:

Basic equations: $\frac{Q'}{Q} = \left(\frac{\omega'}{\omega}\right)\cdot\left(\frac{D'}{D}\right)^3$ $\quad \frac{h'}{h} = \left(\frac{\omega'}{\omega}\right)^2\cdot\left(\frac{D'}{D}\right)^2$ $\quad \frac{P'}{P} = \left(\frac{\omega'}{\omega}\right)^3\cdot\left(\frac{D'}{D}\right)^5$

We choose data from the middle of the table above as being in the region of the best efficiency

$Q = 5\cdot\frac{m^3}{s}$ $\quad h = 45\cdot mm$ $\quad P = 2.62\cdot kW$ $\quad$ and $\quad \omega = 650\cdot rpm$ $\quad D = 1\cdot m$

The flow and head are $\quad Q' = 14\cdot\frac{m^3}{s}$ $\quad h' = 25\cdot mm$

These equations are the scaling laws for scaling from the table data to the new fan. Solving for scaled fan speed, and diameter using the first two equations

$\omega' = \omega\cdot\left(\frac{Q}{Q'}\right)^{\frac{1}{2}}\cdot\left(\frac{h'}{h}\right)^{\frac{3}{4}}$ $\quad \omega' = 250\,rpm$ $\quad D' = D\cdot\left(\frac{Q'}{Q}\right)^{\frac{1}{2}}\cdot\left(\frac{h}{h'}\right)^{\frac{1}{4}}$ $\quad D' = 1.938\,m$

This size is too large; choose (by trial and error)

$Q = 7\cdot\frac{m^3}{s}$ $\quad h = 23\cdot mm$ $\quad P = 2.61\cdot kW$

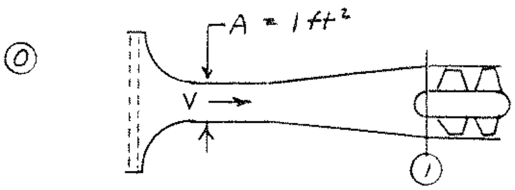
$\omega' = \omega\cdot\left(\frac{Q}{Q'}\right)^{\frac{1}{2}}\cdot\left(\frac{h'}{h}\right)^{\frac{3}{4}}$ $\quad \omega' = 489\,rpm$ $\quad D' = D\cdot\left(\frac{Q'}{Q}\right)^{\frac{1}{2}}\cdot\left(\frac{h}{h'}\right)^{\frac{1}{4}}$ $\quad D' = 1.385\,m$

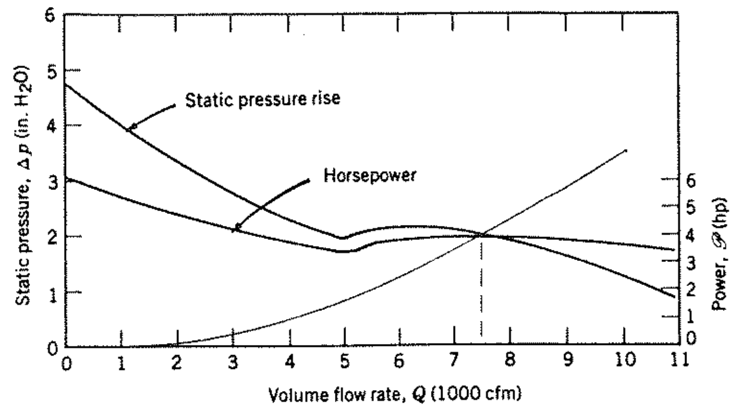
Hence it looks like the largest fan (1.375 m) will be the only fit; it must run at about 500 rpm. Note that it will NOT be running at best efficiency. The power will be

$P' = P\cdot\left(\frac{\omega'}{\omega}\right)^3\cdot\left(\frac{D'}{D}\right)^5$ $\quad P' = 5.67\,kW$

Problem 10.89

Given: Wind tunnel, with 1 foot square test section, powered by fan below. Tunnel contains two screens, each with $K = 0.12$, and a diffuser between the test section and the 24 in. ϕ fan inlet.





Find: (a) Calculate and plot Δp versus Q.
(b) Estimate the maximum air speed.

<u>Solution</u>: Apply energy

Computing equation: $\frac{p_0}{\rho} + \frac{V_0^2}{2} + gz_0 = \frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1 + h_{lT}$; $h_{lT} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{V^2}{2}$ ($V_0 \approx 0$ (2))

Assumptions: (1) $p_0 = p_{atm}$, (2) $V_0 \approx 0$, $\alpha_1 \approx 1$, (3) $z_0 = z_1$, (4) Losses in diffuser, screens

$$\frac{\Delta p_{fan}}{\rho} = \frac{p_{atm} - p_1}{\rho} = \frac{V_1^2}{2} + h_{lT} = \frac{V_1^2}{2} + (2K_{screen} + K_{diffuser})\frac{V^2}{2} = \left[2K_s + K_d + \left(\frac{A}{A_1}\right)^2\right]\frac{Q^2}{2A^2}$$

From continuity, $V_1 A_1 = VA$; $V_1^2 = V^2\left(\frac{A}{A_1}\right)^2$; $V = \frac{Q}{A}$; $V^2 = \frac{Q^2}{A^2}$; $A = 1\text{ ft}^2$; $A_1 = \frac{\pi}{4}D_1^2 = 3.14\text{ ft}^2$

From Fig. 8.19, $K_d = C_{p_i} - C_p = 1 - \left(\frac{1}{AR}\right)^2 - 0.70 = 1 - \left(\frac{1}{3.14}\right)^2 - 0.70 = 0.199$

$$\Delta p_{fan} = [2(0.12) + 0.199 + 0.101]\frac{1}{2} \times Q^2 \frac{ft^6}{min^2} \times \frac{1}{(1)^2 ft^4} \times 0.00238\frac{slug}{ft^3} \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{min^2}{3600\, s^2}$$

$$\Delta h_{fan} = \frac{\Delta p}{\rho g} = 1.79 \times 10^{-7}[Q(cfm)^2]\frac{lbf}{ft^2} \times \frac{ft^3}{62.4\, lbf} \times 12\frac{in.}{ft} = 3.43 \times 10^{-8}[Q(cfm)^2]$$

The resulting curve is plotted above; computed values are tabulated below.

The system will operate where the fan curve and system curve cross. The approximate operating point is $Q = 7400$ cfm at $h = 1.9$ in. H_2O.

The test section speed is

$$V = \frac{Q}{A} = 7400\frac{ft^3}{min} \times \frac{1}{1\, ft^2} \times \frac{min}{60\, s} = 123\text{ ft/s}$$ V

Q (1000 cfm)	1	2	3	4	5	6	7	8	9	10
Δp (in. H_2O)	0.03	0.14	0.31	0.55	0.86	1.24	1.68	2.20	2.78	3.43

Problem 10.90

Given: Axial-flow fan and wind tunnel of Problem 10.89.

Find: (a) Scale performance of fan as it varies with operating speed.
(b) Develop and plot a "calibration curve" showing test section flow speed (m/sec) versus fan speed (rpm).

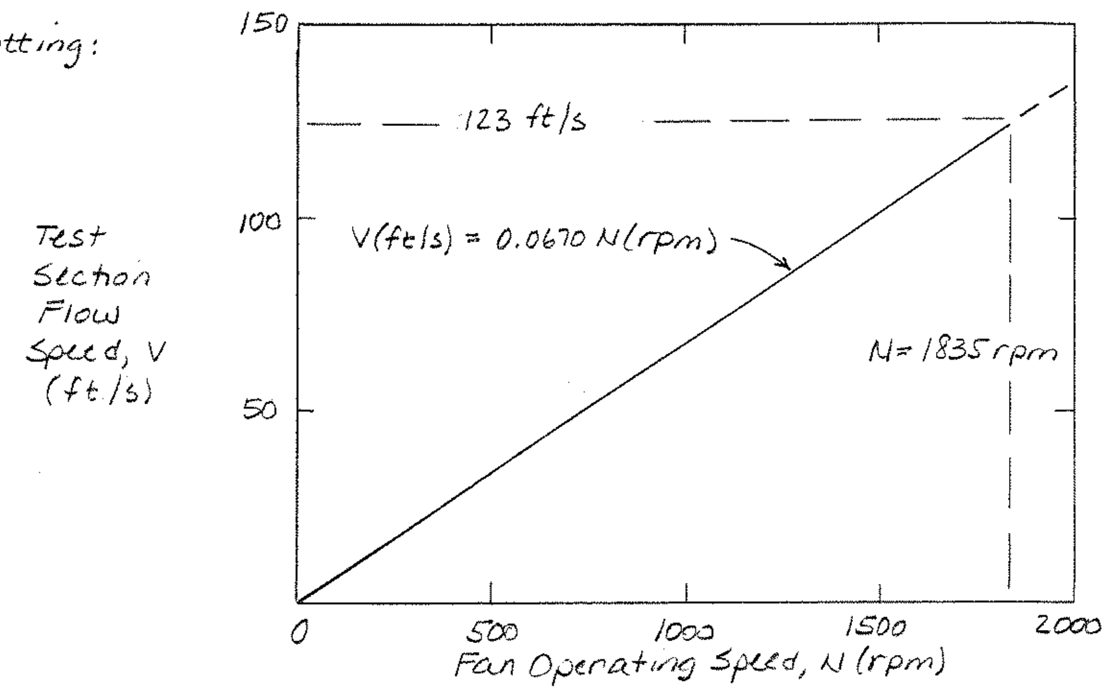
Solution: From the solution to Problem 10.89, $\Delta h_{fan} \propto Q^2 \propto V^2$.

The scaling laws for varying fan speed suggest $Q \propto \omega$ and $p \propto \omega^2$.

Thus $\Delta h_{fan} \propto Q^2 \propto p \propto \omega^2$ or $Q \propto \omega$. The volume flow rate (and the test section flow speed) should vary directly with ω.

From the results of Problem 10.89, $V = 123$ ft/s when $N = 1835$ rpm.

Plotting:



The slope of the linear relationship is $\frac{V}{N} = 123 \frac{ft}{s} \times \frac{1}{1835\ rpm} = 0.0670$ ft/s/rpm.

Thus $V(ft/s) = 0.0670\ N(rpm)$ ← V

Problem 10.91

Given: Experimental test data for aircraft engine fuel pump:

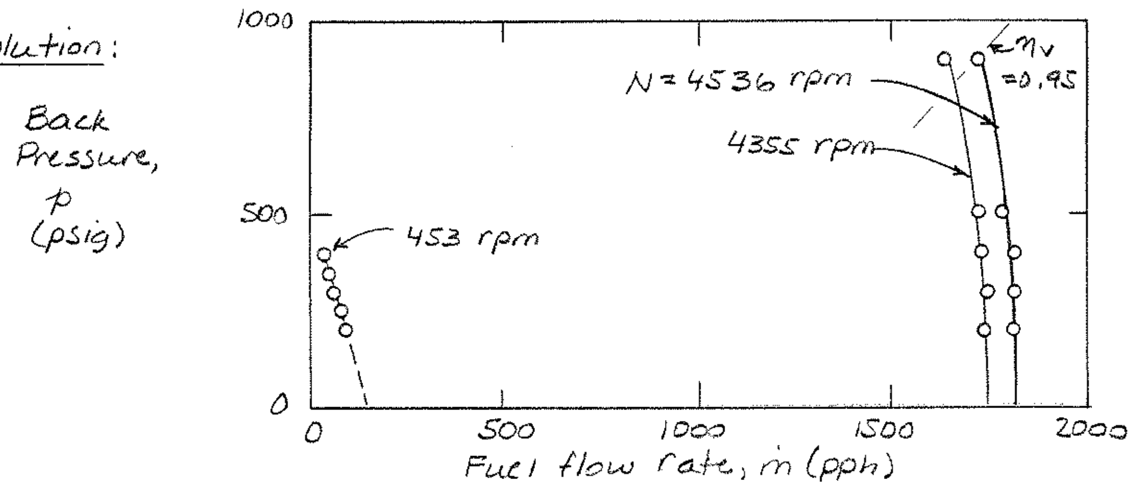
Pump supplies fuel at 450 pph, 150 psig to the controller.

Pump Speed (rpm)	Back Pressure (psig)	Fuel Flow (pph*)	Pump Speed (rpm)	Back Pressure (psig)	Fuel Flow (pph)	Pump Speed (rpm)	Back Pressure (psig)	Fuel Flow (pph)
	200	1810		200	1730		200	89
4536	300	1810	4355	300	1750	453	250	73
(100%)	400	1810	(96%)	400	1735	(10%)	300	58.5
	500	1790		500	1720		350	45
	900	1720		900	1635		400	30

* Fuel flow rate measured in pounds per hour (pph).

Find: (a) Plot curves of fuel pressure versus delivery at the three constant speeds.
(b) Estimate the pump displacement volume per revolution.
(c) Calculate volumetric efficiency at each point, sketch η_v contours.
(d) Evaluate energy loss due to throttling at 100% speed, full delivery.

Solution:



For the pump, $\dot{m} = \rho \forall N$, so $\forall = \frac{\dot{m}}{\rho N}$. Analyzing the 4536 rpm case,

$$\forall \approx 1810 \frac{lbm}{hr} \times \frac{gal}{6.8\ lbm} \times \frac{min}{4536\ rev} \times \frac{ft^3}{7.48\ gal} \times 1728 \frac{in.^3}{ft^3} \times \frac{hr}{60\ min} = 0.226\ in.^3/rev$$ $\forall$

At constant speed, $\eta_v = \frac{\forall_{actual}}{\forall_{geometric}} = \frac{\dot{m}}{\dot{m}(p=0)}$. Calculation shows η_v decreases as speed is reduced, see below.

Energy loss is
$$\dot{W}_L = \left(\frac{\dot{m}_p - \dot{m}_L}{\rho}\right) p_L = (1810-450)\frac{lbm}{hr} \times \frac{gal}{6.8\ lbm} \times 150 \frac{lbf}{in.^2} \times \frac{ft^3}{7.48\ gal} \times 144 \frac{in.^2}{ft^2} \times \frac{hp \cdot s}{550\ ft \cdot lbf} \times \frac{hr}{3600\ s}$$

$$\dot{W}_L = 0.292\ hp$$ $\dot{W}_L$

At 453 rpm, the best volumetric efficiency is

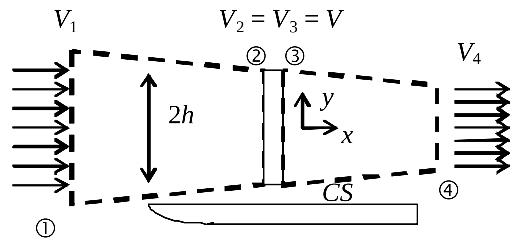
$$\eta_v \approx \frac{\dot{m}}{\dot{m}(p=0)} \times \frac{4536}{453} \approx \frac{89\ pph}{1810\ pph} \times \frac{4536}{453} = 0.0492, \text{ or about } 5\%$$ η_v

At 4355 rpm,

$$\eta_v \approx \frac{1730\ pph}{1810\ pph} \times \frac{4536}{4355} = 0.996, \text{ or more than } 99\% \text{ (this is doubtful).}$$ η_v

Problem 10.92

10.92 The propeller on an airboat used in the Florida Everglades moves air at the rate of 90 lbm/s. When at rest, the speed of the slipstream behind the propeller is 90 mph at a location where the pressure is atmospheric. Calculate (a) the propeller diameter, (b) the thrust produced at rest, and (c) the thrust produced when the airboat is moving ahead at 30 mph if the mass flow rate through the propeller remains constant.



Given: Data on boat and propeller

Find: Propeller diameter; Thrust at rest; Thrust at 30 mph

Solution:

Basic equation:
$$\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz}\,\rho\,dV\!\!\!\!-\; + \int_{CS} \vec{V}_{xyz}\,\rho\vec{V}_{xyz}\cdot d\vec{A} \qquad (4.26)$$

Assumption: 1) Atmospheric pressure on CS 2) Horizontal 3) Steady w.r.t. the CV 4) Use velocities relative to CV

The x-momentum is then $T = u_1\cdot(-m_{rate}) + u_4\cdot(m_{rate}) = (V_4 - V_1)\cdot m_{rate}$ where $m_{rate} = 90\cdot\frac{\text{lbm}}{\text{s}}$ is the mass flow rate

It can be shown (see Example 10.13) that $V = \frac{1}{2}\cdot(V_4 + V_1)$

For the static case $V_1 = 0\cdot\text{mph}$ $V_4 = 90\cdot\text{mph}$ so $V = \frac{1}{2}\cdot(V_4 + V_1)$ $V = 45\,\text{mph}$

From continuity $m_{rate} = \rho\cdot V\cdot A = \rho\cdot V\cdot\frac{\pi\cdot D^2}{4}$ with $\rho = 0.002377\cdot\frac{\text{slug}}{\text{ft}^3}$

Hence $D = \sqrt{\frac{4\cdot m_{rate}}{\rho\cdot\pi\cdot V}}$ $D = 4.76\,\text{ft}$

For $V_1 = 0$ $T = m_{rate}\cdot(V_4 - V_1)$ $T = 369\,\text{lbf}$

When in motion $V_1 = 30\cdot\text{mph}$ and $V = \frac{1}{2}\cdot(V_4 + V_1)$ so $V_4 = 2\cdot V - V_1$ $V_4 = 60\,\text{mph}$

Hence for $V_1 = 30$ mph $T = m_{rate}\cdot(V_4 - V_1)$ $T = 123\,\text{lbf}$

Problem 10.93

10.93 An air boat in the Florida Everglades is powered by a propeller, with $D = 1.5$ m, driven at maximum speed, $N = 1800$ rpm, by a 125 kW engine. Estimate the maximum thrust produced by the propeller at (a) standstill and (b) $V = 12.5$ m/s.

Given: Data on air boat and propeller

Find: Thrust at rest; Thrust at 12.5 m/s

Solution:

Assume the aircraft propeller coefficients in Fi.g 10.40 are applicable to this propeller.

At V = 0, J = 0. Extrapolating from Fig. 10.40b $C_F = 0.16$

We also have $D = 1.5 \cdot m$ $n = 1800 \cdot rpm$ $n = 30 \cdot \frac{rev}{s}$ and $\rho = 1.225 \cdot \frac{kg}{m^3}$

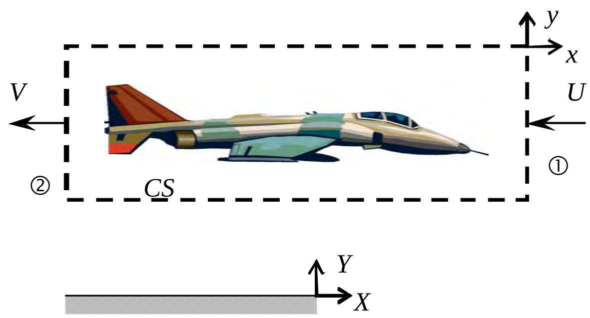
The thrust at standstill (J = 0) is found from $F_T = C_F \cdot \rho \cdot n^2 \cdot D^4$ (Note: n is in rev/s) $F_T = 893\,N$

At a speed $V = 12.5 \cdot \frac{m}{s}$ $J = \frac{V}{n \cdot D}$ $J = 0.278$ and so from Fig. 10.40b $C_P = 0.44$ and $C_F = 0.145$

The thrust and power at this speed can be found $F_T = C_F \cdot \rho \cdot n^2 \cdot D^4$ $F_T = 809\,N$ $P = C_P \cdot \rho \cdot n^3 \cdot D^5$ $P = 111\,kW$

Problem 10.94 [3]

10.94 A jet-propelled aircraft traveling at 450 mph takes in 90 lbm/s of air and discharges it at 1200 mph relative to the aircraft. Determine the propulsive efficiency (defined as the ratio of the useful work output to the mechanical energy input to the fluid) of the aircraft.



Given: Data on jet-propelled aircraft

Find: Propulsive efficiency

Solution:

Basic equation:

$$\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz}\,\rho\, dV\!\!\!- + \int_{CS} \vec{V}_{xyz}\,\rho \vec{V}_{xyz}\cdot d\vec{A} \qquad (4.26)$$

$$\dot{Q} - \dot{W}_s - \dot{W}_{shear} - \dot{W}_{other} = \frac{\partial}{\partial t}\int_{CV} e\,\rho\, dV\!\!\!- + \int_{CS}\left(u + pv + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot d\vec{A} \qquad (4.56)$$

Assumption: 1) Atmospheric pressure on CS 2) Horizontal 3) Steady w.r.t. the CV 4) Use velocities relative to CV

The x-momentum is then $\quad -F_D = u_1\cdot\left(-m_{rate}\right) + u_4\cdot\left(m_{rate}\right) = (-U)\cdot\left(-m_{rate}\right) + (-V)\cdot\left(m_{rate}\right)$

or $\quad F_D = m_{rate}\cdot(V - U) \quad$ where $m_{rate} = 90\cdot\frac{lbm}{s}$ is the mass flow rate

The useful work is then $\quad F_D\cdot U = m_{rate}\cdot(V - U)\cdot U$

The energy equation simplifies to $\quad -W = \left(\frac{U^2}{2}\right)\cdot\left(-m_{rate}\right) + \left(\frac{V^2}{2}\right)\cdot\left(m_{rate}\right) = \frac{m_{rate}}{2}\cdot\left(V^2 - U^2\right)$

Hence $\quad \eta = \dfrac{m_{rate}\cdot(V-U)\cdot U}{\frac{m_{rate}}{2}\cdot\left(V^2 - U^2\right)} = \dfrac{2\cdot(V-U)\cdot U}{\left(V^2 - U^2\right)} = \dfrac{2}{1 + \frac{V}{U}}$

With $\quad U = 450\cdot mph \quad$ and $\quad V = 1200\cdot mph \qquad \eta = \dfrac{2}{1 + \frac{V}{U}} \qquad \eta = 54.5\%$

Problem 10.95

Given: Ship drag data (Figs. 7.2 and 7.3) and dimensions (Problem 9.89).

Performance characteristics of marine propeller (Fig. 10.40). Propeller operates at maximum efficiency when ship steams at maximum speed, $V = 37.6$ kt.

Find: Calculate size, operating speed, and power input for a single propeller to propel this vessel.

Solution: From Problem 9.89, $L = 409$ ft and $A = 19{,}500\ \text{ft}^2$. At maximum speed,

$$V = 37.6\ \frac{\text{nm}}{\text{hr}} \times 6076\ \frac{\text{ft}}{\text{nm}} \times \frac{\text{hr}}{3600\ \text{s}} = 63.5\ \text{ft/s}$$

The Froude number is

$$Fr = \frac{V}{\sqrt{gL}} = 63.5\ \frac{\text{ft}}{\text{s}} \times \left[\frac{\text{s}^2}{32.2\ \text{ft}} \times \frac{1}{409\ \text{ft}}\right]^{\frac{1}{2}} = 0.553$$

From Fig. 7.2, $C_D \approx 0.0054$. The definition is $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$, so

$$F_D = C_D A \tfrac{1}{2}\rho V^2 \;;\; \tfrac{1}{2}\rho V^2 = \tfrac{1}{2} \times (1.025)\, 1.94\ \frac{\text{slug}}{\text{ft}^3} \times (63.5)^2 \frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} = 4010\ \text{lbf/ft}^2$$

$$F_D = 0.0054 \times 19{,}500\ \text{ft}^2 \times 4010\ \frac{\text{lbf}}{\text{ft}^2} = 422{,}000\ \text{lbf}$$

From Fig. 10.40(a), the maximum efficiency is $\eta = 0.67$ at $J = 0.85$. Then

$$nD = \frac{V}{J} = 63.5\ \frac{\text{ft}}{\text{s}} \times \frac{1}{0.85} = 74.7\ \text{ft/s} \qquad (1)$$

Since $C_F = 0.11 = \dfrac{F_D}{\rho n^2 D^4} = \dfrac{F_D}{\rho (n^2 D^2) D^2} = \dfrac{F_D}{\rho (nD)^2 D^2}$, then

$$D = \left[\frac{F_D}{\rho (nD)^2 C_F}\right]^{\frac{1}{2}} = \left[422{,}000\ \text{lbf} \times \frac{\text{ft}^3}{(1.025)\,1.94\ \text{slug}} \times \frac{\text{s}^2}{(74.7)^2\ \text{ft}^2} \times \frac{1}{0.11} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right]^{\frac{1}{2}} = 18.6\ \text{ft}$$

D

From Eq. 1,

$$n = \frac{nD}{D} = 74.7\ \frac{\text{ft}}{\text{s}} \times \frac{1}{18.6\ \text{ft}} = 4.02\ \text{rev/s} \quad (241\ \text{rpm})$$

n

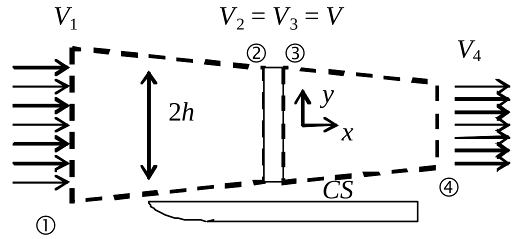
The input power would be

$$\mathcal{P}_{in} = \frac{\mathcal{P}_{out}}{\eta} = \frac{F_D V}{\eta} = 422{,}000\ \text{lbf} \times 63.5\ \frac{\text{ft}}{\text{s}} \times \frac{1}{0.67} \times \frac{\text{hp}\cdot\text{s}}{550\ \text{ft}\cdot\text{lbf}} = 72{,}700\ \text{hp}$$

$\mathcal{P}_{in}$

Problem 10.96

10.96 The propulsive efficiency, η, of a propeller is defined as the ratio of the useful work produced to the mechanical energy input to the fluid. Determine the propulsive efficiency of the moving airboat of Problem 10.92. What would be the efficiency if the boat were not moving?



Given: Definition of propulsion efficiency η

Find: η for moving and stationary boat

Solution:

Assumption: 1) Atmospheric pressure on CS 2) Horizontal 3) Steady w.r.t. the CV 4) Use velocities relative to CV

The x-momentum (Example 10.3): $T = u_1 \cdot (-m_{rate}) + u_4 \cdot (m_{rate}) = m_{rate} \cdot (V_4 - V_1)$

Applying the energy equation to steady, incompressible, uniform flow through the moving CV gives the minimum power input requiremen

$$P_{min} = m_{rate} \cdot \left(\frac{V_4^2}{2} - \frac{V_1^2}{2} \right)$$

On the other hand, useful work is done at the rate of

$$P_{useful} = V_1 \cdot T = V_1 \cdot m_{rate} \cdot (V_4 - V_1)$$

Combining these expressions

$$\eta = \frac{V_1 \cdot m_{rate} \cdot (V_4 - V_1)}{m_{rate} \cdot \left(\frac{V_4^2}{2} - \frac{V_1^2}{2} \right)} = \frac{V_1 \cdot (V_4 - V_1)}{\frac{1}{2} \cdot (V_4 - V_1) \cdot (V_4 + V_1)}$$

or

$$\eta = \frac{2 \cdot V_1}{V_1 + V_4}$$

When in motion $V_1 = 30 \cdot mph$ and $V_4 = 90 \cdot mph$ $\eta = \frac{2 \cdot V_1}{V_1 + V_4}$ $\eta = 50\%$

For the stationary case $V_1 = 0 \cdot mph$ $\eta = \frac{2 \cdot V_1}{V_1 + V_4}$ $\eta = 0\%$

Problem 10.97

Given: Propeller for Gossamer Condor human-powered aircraft has $D = 12$ ft and rotates at $N = 107$ rpm.

Additional information on the aircraft in Problem 9.170:
$W = 200$ lbf, $W/A = 0.4$ lbf/ft², $ar = 17$, $\mathcal{P}_{pilot} = 0.39$ hp, $F_D = 6$ lbf at $V = 12$ mph

Find: Estimate the dimensionless performance characteristics and efficiency of this propeller at cruise conditions.

Solution: From the solution to Problem 9.170, minimum power to propel the aircraft occurs at $V = 10.7$ mph (16.0 ft/s). Assume this is the cruise condition.

From the given data, at 12 mph (17.6 ft/s), $F_D = 6$ lbf

$$\frac{1}{2}\rho V^2 = \frac{1}{2} \times 0.00238 \frac{\text{slug}}{\text{ft}^3} \times (17.6)^2 \frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}} = 0.369 \text{ lbf/ft}^2$$

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} = \frac{W}{qA} = \frac{W/A}{q} = \frac{0.4 \text{ lbf/ft}^2}{0.369 \text{ lbf/ft}^2} = 1.08$$

$$C_D = C_L \frac{F_D}{F_L} = 1.08 \frac{6 \text{ lbf}}{200 \text{ lbf}} = 0.0324$$

$$C_{D,0} = C_D - C_{D,i} = C_D - \frac{C_L^2}{\pi ar} = 0.0324 - \frac{(1.08)^2}{\pi(17)} = 0.0106$$

At $V = 10.7$ mph (16.0 ft/s), $q = 0.305$ lbf/ft²

$$C_L = \frac{W}{qA} = \frac{W/A}{q} = 0.4 \frac{\text{lbf}}{\text{ft}^2} \times \frac{\text{ft}^2}{0.305 \text{ lbf}} = 1.31 \; ; \; C_{D,i} = \frac{C_L^2}{\pi ar} = 0.0321$$

$$C_D = C_{D,0} + C_{D,i} = 0.0106 + 0.0321 = 0.0427 \; ; \; F_D = F_L \frac{C_D}{C_L} = 200 \text{ lbf} \frac{0.0427}{1.31} = 6.52 \text{ lbf}$$

For the propeller,

$$J = \frac{V}{nD} = 16.0 \frac{\text{ft}}{\text{s}} \times \left(\frac{60}{107}\right)\frac{\text{s}}{\text{rev}} \times \frac{1}{12.0 \text{ ft}} = 0.748$$ J

$$C_F = \frac{F_D}{\rho n^2 D^4} = 6.52 \text{ lbf} \times \frac{\text{ft}^3}{0.00238 \text{ slug}} \times \left(\frac{60}{107}\right)^2 \frac{\text{s}^2}{\text{rev}^2} \times \frac{1}{(12.0)^4 \text{ ft}^4} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} = 0.0415$$ C_F

Assume a 30 percent reserve for climbing and maneuvers. Then if $\eta_d = 0.9$,

$$\eta_p = \frac{\mathcal{P}_{out}}{\mathcal{P}_{in}\eta_d} = \frac{F_D V}{(0.7)\,0.39 \text{ hp}\,(0.9)} = \frac{1}{0.246 \text{ hp}} \times 6.52 \text{ lbf} \times 16.0 \frac{\text{ft}}{\text{s}} \times \frac{\text{hp}\cdot\text{s}}{550 \text{ ft}\cdot\text{lbf}} = 0.771$$ η

Finally, $\mathcal{P}_{prop} = \eta_d \mathcal{P}_{in} = 0.246 \text{ hp} = \omega T \; ; \; T = \frac{0.246 \text{ hp}}{\omega} = 12.1 \text{ ft}\cdot\text{lbf}$

$$C_T = \frac{T}{\rho n^2 D^5} = 12.1 \text{ ft}\cdot\text{lbf} \times \frac{\text{ft}^3}{0.00238 \text{ slug}} \times \left(\frac{60}{107}\right)^2 \frac{\text{s}^2}{\text{rev}^2} \times \frac{1}{(12.0)^5 \text{ ft}^5} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} = 0.00642$$ C_T

$$C_P = \frac{C_T}{n} = 0.00642 \times \frac{60}{107} = 0.0036$$ C_P

Problem 10.98

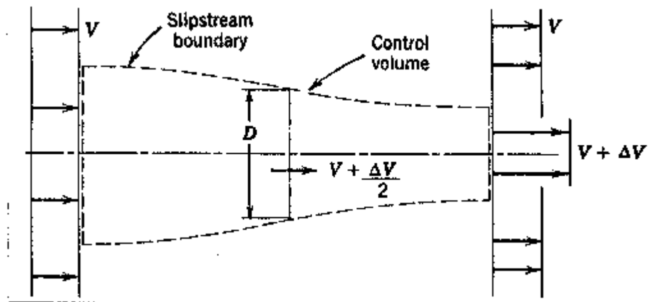
Given: Equations for thrust, power, and efficiency of propulsion devices derived in Section 10-5.

Find: (a) Show that for constant thrust, $\eta = \dfrac{2}{1+\left(1+\dfrac{F_T}{\rho\frac{V^2}{2}\frac{\pi D^2}{2}}\right)^{\frac{1}{2}}}$

(b) Interpret physically.

Solution: Apply 1-D forms of momentum and energy to CV of Fig. 10.38:

Computing equations:



Continuity $\dot{m} = \rho\left(V+\frac{\Delta V}{2}\right)\frac{\pi D^2}{4}$

Momentum $F_T = \dot{m}\,\Delta V$ (10.28)

Energy $\mathcal{P}_{in} = \dot{m}V\Delta V\left(1+\frac{\Delta V}{2V}\right)$ (10.29)

Assumptions: (1) Steady flow, (2) Incompressible flow, (3) Uniform flow, (4) Frictionless flow

Propulsion efficiency is $\eta_p = \dfrac{\mathcal{P}_{out}}{\mathcal{P}_{in}} = \dfrac{F_T V}{\dot{m}V\Delta V\left(1+\frac{\Delta V}{2V}\right)} = \dfrac{\dot{m}V\Delta V}{\dot{m}V\Delta V\left(1+\frac{\Delta V}{2V}\right)} = \dfrac{1}{1+\frac{\Delta V}{2V}}$ (1)

F_T may be written using continuity as

$$F_T = \dot{m}\,\Delta V = \rho\left(V+\frac{\Delta V}{2}\right)\frac{\pi D^2}{4}\Delta V = 2\rho V^2\frac{\pi D^2}{4}\left(1+\frac{\Delta V}{2V}\right)\left(\frac{\Delta V}{2V}\right) = 2\rho V^2\frac{\pi D^2}{4}(1+\lambda)\lambda$$

where $\lambda = \frac{\Delta V}{2V}$. For constant F_T,

$$\lambda^2 + \lambda - \frac{F_T}{2\rho V^2\frac{\pi D^2}{4}} = 0$$

Solving via the quadratic equation, and choosing the positive root

$$\lambda = \frac{-1 \pm \sqrt{1 + 4\dfrac{F_T}{2\rho V^2\frac{\pi D^2}{4}}}}{2} = \frac{1}{2}\left\{-1 + \sqrt{1 + \frac{F_T}{\rho\frac{V^2}{2}\frac{\pi D^2}{4}}}\right\}$$

From Eq. 1, $\eta_p = \dfrac{1}{1+\lambda} = \dfrac{1}{1+\frac{1}{2}\{\ \}} = \dfrac{2}{2+\{\ \}} = \dfrac{2}{1+\left(1+\dfrac{F_T}{\rho\frac{V^2}{2}\frac{\pi D^2}{4}}\right)^{\frac{1}{2}}}$ η_p

The ratio, $F_T/\frac{\pi D^2}{4}$, may be interpreted as the disk loading: the force developed per unit area of the actuator disk. Note $\eta_p \to 1$ as $\frac{\pi D^2}{4}$ increases.

Problem 10.99

Given: Preliminary calculations for a hydroelectric power generation site show a net head, $H = 2350$ ft, is available at water flow rate, $Q = 75$ ft³/s.

Find: Compare the geometry and efficiency of Pelton wheels designed to run at (a) 450 rpm and (b) 600 rpm.

Solution: Apply specific speed equation to classify performance.

Computing equation: $N_{s_{cu}} = \dfrac{N \mathcal{P}^{1/2}}{H^{5/4}}$ (rpm, hp, and ft units)

From Fig. 10.17, $\eta_{max} \approx 0.89$ at $N_{s_{cu}} = 5$. The output power (used to define $N_{s_{cu}}$) is

$$\mathcal{P}_{out} = \eta \rho Q g H = 0.89 \times 62.4 \frac{lbf}{ft^3} \times 75 \frac{ft^3}{s} \times 2350\, ft \times \frac{hp \cdot s}{550\, ft \cdot lbf} = 17{,}800\ hp$$

At $N = 450$ rpm

$$N_{s_{cu}} = \frac{450\, rpm (17{,}800\, hp)^{\frac{1}{2}}}{(2350)^{5/4}} = 3.67, \text{ so } \eta \approx 0.88$$

Neglect nozzle losses and elevation above the tailrace. Then

$$V_j \approx \sqrt{2gH} = \left[2 \times 32.2 \frac{ft}{s^2} \times 2350\, ft\right]^{\frac{1}{2}} = 389\ ft/s$$

From Fig. 10.10, $U = R\omega \approx 0.47\, V_j = 183$ ft/s. Thus

$$D = 2R = \frac{2(0.47\, V_j)}{\omega} = 2 \times 183 \frac{ft}{s} \times \frac{s}{47.1\, rad} = 7.77\ ft$$

The jet diameter is found from $Q = V_j A_j = \pi V_j D_j^2/4$, so

$$D_j = \sqrt{\frac{4Q}{\pi V_j}} = \left[\frac{4}{\pi} \times 75 \frac{ft^3}{s} \times \frac{s}{389\, ft}\right]^{\frac{1}{2}} = 0.495\ ft\ (5.95\ in.)$$

The ratio of jet diameter to wheel diameter is

$$r = \frac{D_j}{D} = \frac{0.495\, ft}{7.77\, ft} = 0.0637 \quad \text{or } 1:15.7 \quad \text{(this is reasonable)}$$

Results from similar computations at $N = 600$ rpm are:

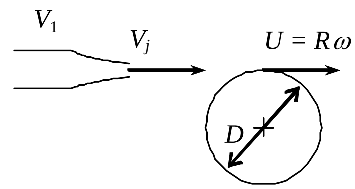
N (rpm)	$N_{s_{cu}}$ (USCS)	D (ft)	D_j/D (---)	η (---)
450	3.67	7.77	1:15.7	0.88
600	4.89	5.83	1:11.8	0.89

The unit operating at 600 rpm is closer to $N_{s_{cu}} = 5$, where peak hydraulic efficiency is expected.

Problem 10.100

[2]

10.100 Conditions at the inlet to the nozzle of a Pelton wheel are $p = 700$ psig and $V = 15$ mph. The jet diameter is $d = 7.5$ in. and the nozzle loss coefficient is $K_{nozzle} = 0.04$. The wheel diameter is $D = 8$ ft. At this operating condition, $\eta = 0.86$. Calculate (a) the power output, (b) the normal operating speed, (c) the approximate runaway speed, (d) the torque at normal operating speed, and (e) the approximate torque at zero speed.



Given:
Pelton turbine

Find:
1) Power 2) Operating speed 3) Runaway speed 4) Torque 5) Torque at zero speed

Solution:

Basic equations
$$\left(\frac{p_1}{\rho \cdot g} + \alpha \cdot \frac{V_1^2}{2 \cdot g} + z_1\right) - \left(\frac{p_j}{\rho \cdot g} + \alpha \cdot \frac{V_j^2}{2 \cdot g} + z_j\right) = \frac{h_{lT}}{g} \qquad h_{lT} = h_l + h_{lm} = K \cdot \frac{V^2}{2}$$

and from Example 10.5 $\quad T_{ideal} = \rho \cdot Q \cdot R \cdot (V_j - U) \cdot (1 - \cos(\theta)) \qquad \theta = 165 \cdot deg$

Assumptions: 1) $p_j = p_{amt}$ 2) Incompressible flow 3) α at 1 and j is approximately 1 4) Only minor loss at nozzle 5) $z_1 = z_j$

Given data $\quad p_{1g} = 700 \cdot psi \qquad V_1 = 15 \cdot mph \qquad V_1 = 22 \frac{ft}{s} \qquad \eta = 86 \cdot \%$

$d = 7.5 \cdot in \qquad D = 8 \cdot ft \qquad R = \frac{D}{2} \qquad K = 0.04 \qquad \rho = 1.94 \cdot \frac{slug}{ft^3}$

Then $\quad \frac{p_{1g}}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} - \frac{V_j^2}{2 \cdot g} = \frac{K}{g} \cdot \frac{V_j^2}{2}$ $\quad$ or $\quad V_j = \sqrt{\frac{2 \cdot \left(\frac{p_{1g}}{\rho} + \frac{V_1^2}{2}\right)}{1 + K}} \qquad V_j = 317 \frac{ft}{s}$

and $\quad Q = V_j \cdot \frac{\pi \cdot d^2}{4} \qquad Q = 97.2 \frac{ft^3}{s} \qquad H = \frac{p_{1g}}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} \qquad H = 1622\,ft$

Hence $\quad P = \eta \cdot \rho \cdot Q \cdot g \cdot H \qquad P = 15392\,hp$

From Fig. 10.10, normal operating speed is around $U = 0.47 \cdot V_j \qquad U = 149 \frac{ft}{s} \qquad \omega = \frac{U}{R} \qquad \omega = 37.2 \frac{rad}{s} \qquad \omega = 356\,rpm$

At runaway $\quad U_{run} = V_j \qquad \omega_{run} = \frac{U_{run}}{\left(\frac{D}{2}\right)} \qquad \omega_{run} = 79.2 \frac{rad}{s} \qquad \omega_{run} = 756\,rpm$

From Example 10.5 $\quad T_{ideal} = \rho \cdot Q \cdot R \cdot (V_j - U) \cdot (1 - \cos(\theta)) \qquad T_{ideal} = 2.49 \times 10^5\,ft \cdot lbf$

Hence $\quad T = \eta \cdot T_{ideal} \qquad T = 2.14 \times 10^5\,ft \cdot lbf$

Stall occurs when $\quad U = 0 \qquad T_{stall} = \eta \cdot \rho \cdot Q \cdot R \cdot V_j \cdot (1 - \cos(\theta)) \qquad T_{stall} = 4.04 \times 10^5\,ft \cdot lbf$

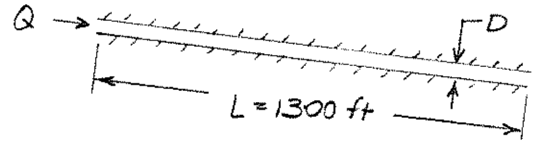
Problem 10.101

Given: Francis (reaction) turbines at Niagara Falls:

$D_0 = 176$ in., $\mathcal{P} = 72{,}500$ hp, $N = 107$ rpm, $\eta = 0.938$, $H = 214$ ft (net)

Penstock has $L = 1300$ ft; $H_{net} = 0.85\, H_{gross}$.

Find: (a) Calculate specific speed.
(b) Evaluate volume flow rate.
(c) Estimate penstock size.



Solution: $N_{s_{cu}} = \dfrac{N \mathcal{P}^{\frac{1}{2}}}{H^{\frac{5}{4}}} = \dfrac{107 \text{ rpm} (72{,}500 \text{ hp})^{\frac{1}{2}}}{(214 \text{ ft})^{\frac{5}{4}}} = 35.1$ — N_s

Efficiency is defined as $\eta = \mathcal{P}/\rho Q g H$, so

$$Q = \frac{\mathcal{P}}{\eta \rho g H} = \frac{1}{0.938} \times 72{,}500 \text{ hp} \times \frac{\text{ft}^3}{62.4 \text{ lbf}} \times \frac{1}{214 \text{ ft}} \times \frac{550 \text{ ft}\cdot\text{lbf}}{\text{hp}\cdot\text{s}} = 31800 \text{ ft}^3/\text{s}$$ — Q

Apply the definition of head loss: $h_{\ell T} = g H_{\ell T} = f \dfrac{L}{D} \dfrac{\bar{V}^2}{2}$

$$H_{\ell T} = 0.15\, H_{gross} = 0.15 \frac{H_{net}}{0.85} = 0.176\, H_{net} = 0.176 \times 214 \text{ ft} = 37.7 \text{ ft}$$

$$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2}\ ; \ \bar{V}^2 = \frac{16 Q^2}{\pi^2 D^4}\ ; \ g\,\Delta H = f \frac{L}{D} \frac{\bar{V}^2}{2} = f \frac{L}{D} \frac{8Q^2}{\pi^2 D^4} = \frac{8 f L Q^2}{\pi^2 D^5}$$

(Assume $T = 50°$F)

Solving,
$$D = \left[\frac{8 f L Q^2}{\pi^2 g \Delta H}\right]^{\frac{1}{5}}$$

This system is very large, so it is difficult to estimate f. Assuming concrete-lined penstocks, $e = 0.01$ ft (Table 8.1). Start with $f = 0.01$ to get by iteration (see below),

$$D = 26.8 \text{ ft}$$

The flow properties are

$$\bar{V} = 56.8 \text{ ft/sec},\ Re = 1.18 \times 10^8, \text{ and } f = 0.0157$$

Flow is in the fully rough zone with $e/D = 0.01/26.8 = 0.00037$.

Using $e = 0.02$ gives $D = 28.1$ ft, so $D \approx 27$–28 ft — D

The iterations are:

Assumed D (ft)	V (ft/s)	Re	f_0	$f^{-0.5}$	Calculated f	Calculated D (ft)
					0.01	24.5
24.5	67.5	1.18E+08	0.0160	7.91	0.0160	26.9
26.9	56.0	1.08E+08	0.0157	7.99	0.0156	26.8
26.80	56.4	1.08E+08	0.0157	7.99	0.0157	26.77
26.77	56.5	1.08E+08	0.0157	7.99	0.0157	26.77
					0.02	28.1

Problem 10.102

Given: Francis turbine Units 19, 20, and 21 at Grand Coulee Dam.

Rated conditions: $\mathcal{P} = 820{,}000$ hp, $N = 72$ rpm, $H = 285$ ft, $\eta = 0.95$

Turbines operate from $220 < H < 355$ ft.

Find: (a) Calculate specific speed at rated conditions.
(b) Estimate maximum water flow rate through each turbine.

Solution: Apply definitions of specific speed and efficiency.

Computing equations: $N_{s_{cu}} = \dfrac{N\mathcal{P}^{1/2}}{H^{5/4}}$ $\quad \eta = \dfrac{\mathcal{P}}{\rho Q g H}$

Thus

$$N_{s_{cu}} = \frac{72 \text{ rpm} \,(820{,}000 \text{ hp})^{\frac{1}{2}}}{(285 \text{ ft})^{5/4}} = 55.7$$

$N_{s_{cu}}$

From η,

$$Q = \frac{\mathcal{P}}{\eta \rho g H}$$

so Q is maximum at minimum head. Assuming $\eta = 0.95$, the

$$Q \approx \frac{1}{0.95} \times 820{,}000 \text{ hp} \times \frac{\text{ft}^3}{62.4 \text{ lbf}} \times \frac{1}{220 \text{ ft}} \times 550 \frac{\text{ft} \cdot \text{lbf}}{\text{hp} \cdot \text{s}} = 34{,}600 \text{ ft}^3/\text{s} \quad (\text{max})$$

Q

{ This is an estimate because η may not be constant, nor may it be possible to develop full power at $H = 220$ ft. }

Problem 10.103

Given: Measured data for reaction turbines at Shasta Dam, Fig. 10.13.

Each turbine is rated at $\mathcal{P} = 103{,}000$ hp at $N = 138.6$ rpm, under a net head of $H = 380$ ft.

Find: (a) Specific speed at rated conditions.
(b) Shaft torque at rated conditions.
(c) Calculate the water flow rate per turbine needed to produce rated output power; plot versus head.

Solution: Apply the definitions of specific speed and efficiency, use data from Fig. 10.13:

Computing equations: $N_{s_{cu}} = \dfrac{N\mathcal{P}^{\frac{1}{2}}}{H^{\frac{5}{4}}}$ $\qquad \eta = \dfrac{\mathcal{P}}{\rho Q g H}$ $\qquad \mathcal{P} = \omega T$

At rated conditions, $N_{s_{cu}} = \dfrac{(138.6\text{ rpm})(103{,}000\text{ hp})^{\frac{1}{2}}}{(380\text{ ft})^{\frac{5}{4}}} = 26.5$ $\qquad N_s$

$$T = \frac{\mathcal{P}}{\omega} = 103{,}000\text{ hp} \times \frac{\text{s}}{14.5\text{ rad}} \times 550\,\frac{\text{ft}\cdot\text{lbf}}{\text{hp}\cdot\text{s}} = 3.91 \times 10^6\text{ ft}\cdot\text{lbf}$$

Find Q from definition of η; at rated conditions, $\eta \approx 0.93$ (Fig. 10.13):

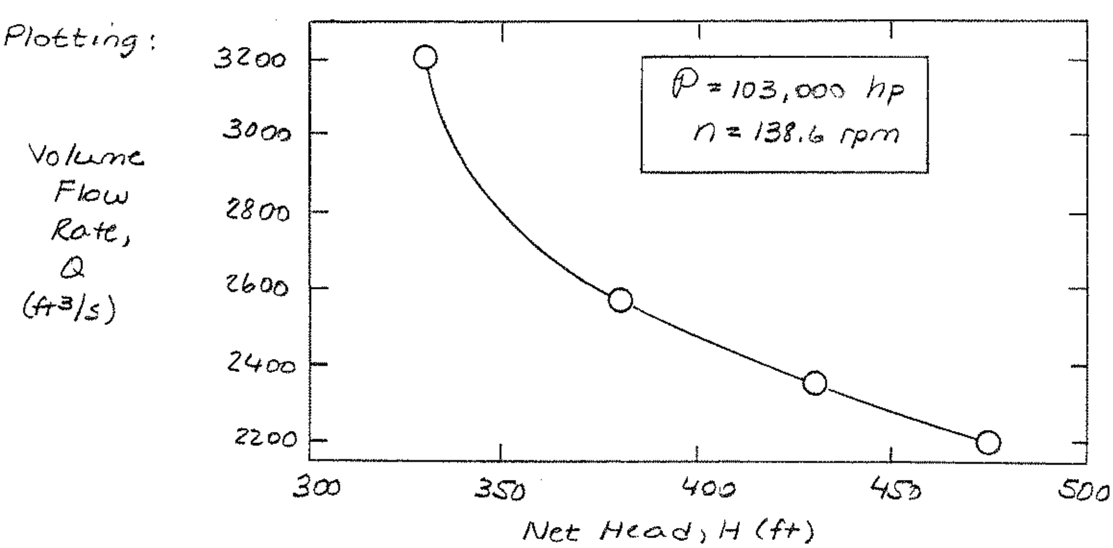
$$Q = \frac{\mathcal{P}}{\eta\rho g H} = \frac{1}{0.93} \times 103{,}000\text{ hp} \times \frac{\text{ft}^3}{62.4\text{ lbf}} \times \frac{1}{380\text{ ft}} \times 550\,\frac{\text{ft}\cdot\text{lbf}}{\text{hp}\cdot\text{s}} = 2570\text{ ft}^3/\text{s}$$

Tabulating similar calculations:

H (ft)	$\mathcal{P}$ (hp)	η (---)	Q (ft^3/s)
238	*	–	–
280	*	–	–
330	103,000	0.86	3200
380	103,000	0.93	2570
430	103,000	0.90	2350
475	103,000	0.87	2200

* cannot produce rated power at this head

Plotting:



Problem 10.104

Given: Data in Fig. 10.11 for Pelton wheel in PG&E Tiger Creek plant.

Rating is $\mathcal{P} = 36{,}000$ hp at $N = 225$ rpm under $H = 1190$ ft (net)

Assume reasonable flow angles and nozzle loss coefficient.

(a) Determine rotor diameter.
(b) Estimate jet diameter.
(c) Compute volume flow rate of water.

Solution: From Bernoulli, the ideal jet velocity would be $V_i = \sqrt{2gH}$. Assuming $C_v = 0.98$ (4 percent loss in nozzle), then

$$V_j = C_v V_i = C_v\sqrt{2gH} = 0.98\left[2 \times 32.2\,\frac{ft}{s^2} \times 1190\,ft\right]^{\frac{1}{2}} = 271\ ft/s$$

From Fig. 10.10, $U = R\omega = 0.47\,V_j$ at optimum conditions. Then

$$D = 2R = \frac{2(0.47)V_j}{\omega} = 0.94 \times 271\,\frac{ft}{s} \times \frac{s}{23.6\ rad} = 10.8\ ft$$ D

From Fig. 11.11, $\eta = 0.86$ at full load. Thus

$$\eta = \frac{\mathcal{P}}{\rho Q g H}\ ;\ Q = \frac{\mathcal{P}}{\eta \rho g H}\ ;\ Q = V_j A_j = V_j\,\frac{\pi D_j^2}{4}$$

$$Q = \frac{1}{0.86} \times 36{,}000\ hp \times \frac{ft^3}{62.4\ lbf} \times \frac{1}{1190\ ft} \times 550\,\frac{ft\cdot lbf}{hp\cdot s} = 310\ ft^3/s$$ Q

$$D_j = \left[\frac{4Q}{\pi V_j}\right]^{\frac{1}{2}} = \left[\frac{4}{\pi} \times 310\,\frac{ft^3}{s} \times \frac{s}{271\ ft}\right]^{\frac{1}{2}} = 1.21\ ft\ (14.5\ in.)$$ D_j

Note $D_j/D = 1.21/10.8 = 0.112$ (1:8.93).

Problem 10.105

10.105 An impulse turbine is to develop 15 MW from a single wheel at a location where the net head is 350 m. Determine the appropriate speed, wheel diameter, and jet diameter for single- and multiple-jet operation. Compare with a double-overhung wheel installation. Estimate the required water consumption.

Given: Impulse turbine requirements

Find: 1) Operating speed 2) Wheel diameter 4) Jet diameter 5) Compare to multiple-jet and double-overhung

Solution:

Basic equations: $V_j = \sqrt{2 \cdot g \cdot H}$ $\quad N_S = \dfrac{\omega \cdot P^{\frac{1}{2}}}{\rho^{\frac{1}{2}} \cdot h^{\frac{5}{4}}}$ $\quad \eta = \dfrac{P}{\rho \cdot Q \cdot g \cdot H}$ $\quad Q = V_j \cdot A_j$

Model as optimum. This means. from Fig. 10.10 $U = 0.47 \cdot V_j$ and from Fig. 10.17 $N_{Scu} = 5$ with $\eta = 89 \cdot \%$

Given or available data $H = 350 \cdot m$ $\quad P = 15 \cdot MW$ $\quad \rho = 1.94 \cdot \dfrac{slug}{ft^3}$

Then $V_j = \sqrt{2 \cdot g \cdot H}$ $\quad V_j = 82.9 \dfrac{m}{s}$ $\quad U = 0.47 \cdot V_j$ $\quad U = 38.9 \dfrac{m}{s}$

We need to convert from N_{Scu} (from Fig. 10.17) to N_S (see discussion after Eq. 10.18b). $N_S = \dfrac{N_{Scu}}{43.46}$ $\quad N_S = 0.115$

The water consumption is $Q = \dfrac{P}{\eta \cdot \rho \cdot g \cdot H}$ $\quad Q = 4.91 \dfrac{m^3}{s}$

For a single jet $\omega = N_S \cdot \dfrac{\rho^{\frac{1}{2}} \cdot (g \cdot H)^{\frac{5}{4}}}{P^{\frac{1}{2}}}$ (1) $\quad \omega = 236\,rpm$ $\quad D_j = \sqrt{\dfrac{4 \cdot Q}{\pi \cdot V_j}}$ (2) $\quad D_j = 0.275\,m$

The wheel radius is $D = \dfrac{2 \cdot U}{\omega}$ (3) $\quad D = 3.16\,m$

For multiple (n) jets, we use the power and flow per jet

From Eq 1 $\omega_n = \omega \cdot \sqrt{n}$ $\quad$ From Eq. 2 $D_{jn} = \dfrac{D_j}{\sqrt{n}}$ and $D_n = \dfrac{D}{\sqrt{n}}$ from Eq. 3

Results:

$n =$	$\omega_n(n) =$ (rpm)	$D_{jn}(n) =$ (m)	$D_n(n) =$ (m)
1	236	0.275	3.16
2	333	0.194	2.23
3	408	0.159	1.82
4	471	0.137	1.58
5	527	0.123	1.41

A double-hung wheel is equivalent to having a single wheel with two jets

Problem 10.106

10.106 Tests of a model impulse turbine under a net head of 20 m produced the following results:

Wheel Speed (rpm)	No-Load Discharge (m^3/hr)	Net Brake Scale Reading (N) (R = 2 m)					
300	10	33	72	107	140	194	233
325	11.4	29	63	96	124	175	213
Discharge (m^3/hr)		44	86	124	157	211	257

Calculate and plot the machine power output and efficiency versus water flow rate.

Given: Data on impulse turbine

Find: Plot of power and efficiency curves

Solution:

Basic equations: $T = F \cdot R$ $\qquad P = \omega \cdot T$ $\qquad \eta = \dfrac{P}{\rho \cdot Q \cdot g \cdot H}$

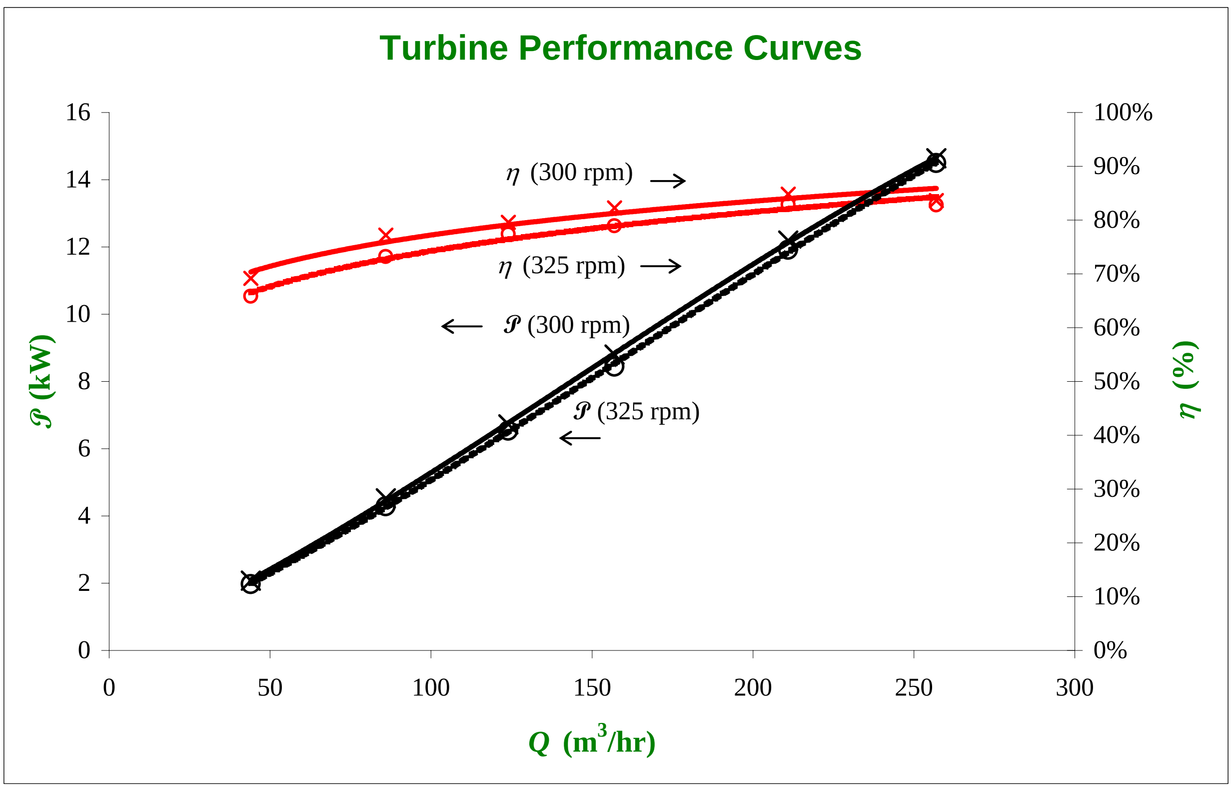
H = 25 m **NOTE: Earlier printings had H incorrectly as 20 m, which gives efficiencies > 100%**

ρ = 1000 kg/m^3

R = 2.00 m

ω = 300 rpm				
Q (m^3/hr)	F (N)	T (N·m)	$\mathscr{P}$ (kW)	η (%)
44	33	66	2.07	69.2%
86	72	144	4.52	77.2%
124	107	214	6.72	79.6%
157	140	280	8.80	82.2%
211	194	388	12.19	84.8%
257	233	466	14.64	83.6%

ω = 325 rpm				
Q (m^3/hr)	F (N)	T (N·m)	$\mathscr{P}$ (kW)	η (%)
44	29	58	1.97	65.9%
86	63	126	4.29	73.2%
124	96	192	6.53	77.4%
157	124	248	8.44	78.9%
211	175	350	11.91	82.9%
257	213	426	14.50	82.8%



Problem 10.107

Given: Definition of specific speed for a hydraulic turbine in U.S. units:

$$N_{s_{cu}} = \frac{N(rpm)[\mathcal{P}(hp)]^{\frac{1}{2}}}{[h(ft)]^{5/4}}$$

Impulse turbine: $N = 400$ rpm, $H = 1190$ ft, $\eta = 0.86$, with $D_j = 6$ in.

Find: (a) Develop a conversion from $N_{s_{cu}}$ to a dimensionless N_s in SI units.
(b) Evaluate N_s of turbine in U.S. and S.I. units.
(c) Estimate the wheel diameter.

Solution: Apply definitions of specific speed and efficiency.

Computing equations: $N_{s_{cu}} = \frac{N\mathcal{P}^{1/2}}{H^{5/4}}$ $\quad \eta \equiv \frac{\mathcal{P}}{\rho Q g H}$ $\quad N_s = \frac{\omega \mathcal{P}^{1/2}}{\rho^{1/2} h^{5/4}}$

From dimensional analysis, recall that for pumps, $N_s = \frac{NQ^{1/2}}{H^{3/4}}$ was dimensionless only when H is expressed as gH, energy per unit mass. Thus the dimensions are

$$[N_s] = \left[\frac{NQ^{1/2}}{(gH)^{3/4}}\right] = \frac{1}{t}\left(\frac{L^3}{t}\right)^{\frac{1}{2}}\left(\frac{t^2}{L^2}\right)^{\frac{3}{4}} = \frac{1}{t}\frac{L^{3/2}}{t^{1/2}}\frac{t^{3/2}}{L^{3/2}} = 1 \quad \checkmark\checkmark$$

To form the N_s for turbines, Q must be multiplied by $\rho g H$ to obtain power. Thus for turbines, the dimensionless specific speed is

$$[N_s] = \left[\frac{NQ^{1/2}}{(gH)^{3/4}} \times \frac{(\rho g H)^{1/2}}{(\rho g H)^{1/2}}\right] = \left[\frac{N\mathcal{P}^{1/2}}{\rho^{1/2}(gH)^{5/4}}\right] = \frac{1}{t}\left(\frac{FL}{t}\times\frac{ML}{Ft^2}\right)^{\frac{1}{2}}\left(\frac{L^3}{M}\right)^{\frac{1}{2}}\left(\frac{t^2}{L^2}\right)^{\frac{5}{4}} = \frac{M^{\frac{1}{2}}L^{\frac{5}{2}}t^{\frac{5}{2}}}{M^{\frac{1}{2}}L^{5/2}t^{5/2}} = 1 \quad \checkmark\checkmark$$

The simplest way to convert is to evaluate each specific speed, then take the ratio.

The jet speed will be approximately $V_j = \sqrt{2gH} = \left[2 \times 32.2\frac{ft}{s^2} \times 1190\,ft\right]^{\frac{1}{2}} = 277$ ft/s

and $Q = V_j\frac{\pi D_j^2}{4} = 277\frac{ft}{s}\times\frac{\pi}{4}\left(\frac{1}{2}\right)^2 ft^2 = 54.4\ ft^3/s$. Thus

$$\mathcal{P} = \eta\rho Q g H = 0.86 \times 62.4\frac{lbf}{ft^3}\times 54.4\frac{ft^3}{s}\times 1190\,ft \times \frac{hp\cdot s}{550\,ft\cdot lbf} = 6320\ hp\ (4.71\ MW)$$

For the wheel, $U = 0.47V_j = R\omega$; $R = 3.10$ ft; $D = 6.20$ ft — D

In U.S. units, $N_{s_{cu}} = \frac{400\,rpm\,(6320\,hp)^{\frac{1}{2}}}{(1190\,ft)^{5/4}} = 4.55$ — N_s

In S.I. units, $\omega = 41.9$ rad/s and $gH = 3560\ m^2/s^2$, so

$$N_s = \frac{\omega\mathcal{P}^{1/2}}{\rho^{1/2}(gH)^{5/4}} = 41.9\frac{rad}{s}\times(4.71\times10^6 W)^{\frac{1}{2}}\times\left(\frac{m^3}{999\,kg}\right)^{\frac{1}{2}}\left(\frac{s^2}{3560\,m^2}\right)^{\frac{5}{4}} = 0.105$$ — N_s

The conversion is $\frac{N_{s_{cu}}}{N_s} = \frac{4.55}{0.105} = 43.5$ — Ratio

Problem 10.108

Given: Published data for PG&E Tiger Creek Power Plant:

$H_{gross} = 1219$ ft, $Q = 750$ ft³/s, $\mathcal{P} = 58$ MW (rating is 60 MW)

Plant is claimed to produce 968 kW·hr/acre·ft of water and 336.4×10^6 kW·hr/yr of operation.

Find: (a) Estimate the net head at the site, the turbine specific speed, and the turbine efficiency.
(b) Comment on the internal consistency of the published data.

Solution: Apply definitions of specific speed and efficiency.

Computing equations: $N_{s_{cu}} = \dfrac{N \mathcal{P}^{1/2}}{H^{5/4}}$ $\qquad \eta = \dfrac{\mathcal{P}}{\rho Q g H_{net}}$

Use the operating point to estimate net head. From Fig. 10.11, assume $\eta = 0.87$: $\qquad \eta$

$$H_{net} = \frac{\mathcal{P}}{\eta \rho Q g} = \frac{1}{0.87} \times 58 \times 10^6\ \text{W} \times \frac{\text{ft}^3}{62.4\ \text{lbf}} \times \frac{\text{s}}{750\ \text{ft}^3} \times \frac{\text{hp}}{746\ \text{W}} \times 550\ \frac{\text{ft}\cdot\text{lbf}}{\text{hp}\cdot\text{s}} = 1050\ \text{ft}$$ H_{net}

Thus

$$H_{net}/H_{gross} = 1050/1219 = 0.861 \text{ or } 86.1 \text{ percent (Reasonable)}$$

The specific speed should be $N_{s_{cu}} \approx 5$. Checking, $\qquad N_s$

$$N = \frac{N_s H^{5/4}}{\mathcal{P}^{1/2}} = \frac{5\,(1050\ \text{ft})^{5/4}}{(77{,}700\ \text{hp})^{1/2}} = 107$$

This is too low, so the plant must have several turbines. Reducing $\mathcal{P}$ to the output per turbine would raise N.

Check data consistency:

$$58 \times 10^6\ \text{W} \times 24\ \frac{\text{hr}}{\text{day}} \times 365\ \frac{\text{day}}{\text{yr}} = 508 \times 10^6\ \text{kW}\cdot\text{hr/yr}$$

$$60 \times 10^6\ \text{W} \qquad = 526 \times 10^6\ \text{kW}\cdot\text{hr/yr}$$

Both values are ~50% higher than quoted.

$$58 \times 10^6\ \text{W} \times \frac{\text{s}}{750\ \text{ft}^3} \times \frac{\text{hr}}{3600\ \text{s}} \times \frac{43{,}600\ \text{ft}^3}{\text{acre}\cdot\text{ft}} = 937\ \text{kW}\cdot\text{hr/acre}\cdot\text{ft}$$

$$60 \times 10^6\ \text{W} \qquad = 969\ \text{kW}\cdot\text{hr/acre}\cdot\text{ft}$$

Excellent agreement

Problem 10.109

10.109 Design the piping system to supply a water turbine from a mountain reservoir. The reservoir surface is 1000 ft above the turbine site. The turbine efficiency is 80 percent, and it must produce 35 hp of mechanical power. Define the minimum standard-size pipe required to supply water to the turbine and the required volume flow rate of water. Discuss the effects of turbine efficiency, pipe roughness, and installing a diffuser at the turbine exit on the performance of the installation.

Given: Hydraulic turbine site

Find: Minimum pipe size; Fow rate; Discuss

Solution:

Basic equations: $H_l = \frac{h_l}{g} = f \cdot \frac{L}{D} \cdot \frac{V^2}{2 \cdot g}$ and also, from Example 10.15 the optimum is when $H_l = \frac{\Delta z}{3}$

As in Fig. 10.41 we assume $L = 2 \cdot \Delta z$ and $f = 0.02$

Then, for a given pipe diameter D $V = \sqrt{\frac{2 \cdot g \cdot D \cdot H_l}{f \cdot L}} = \sqrt{\frac{g \cdot D}{3 \cdot f}}$

Also $Q = V \cdot \frac{\pi \cdot D^2}{4}$ $P_h = \rho \cdot Q \cdot \frac{V^2}{2}$ $P_m = \eta \cdot P_h$

f = 0.02
ρ = 1.94 slug/ft^3
R = 2.00 m
η = 80%

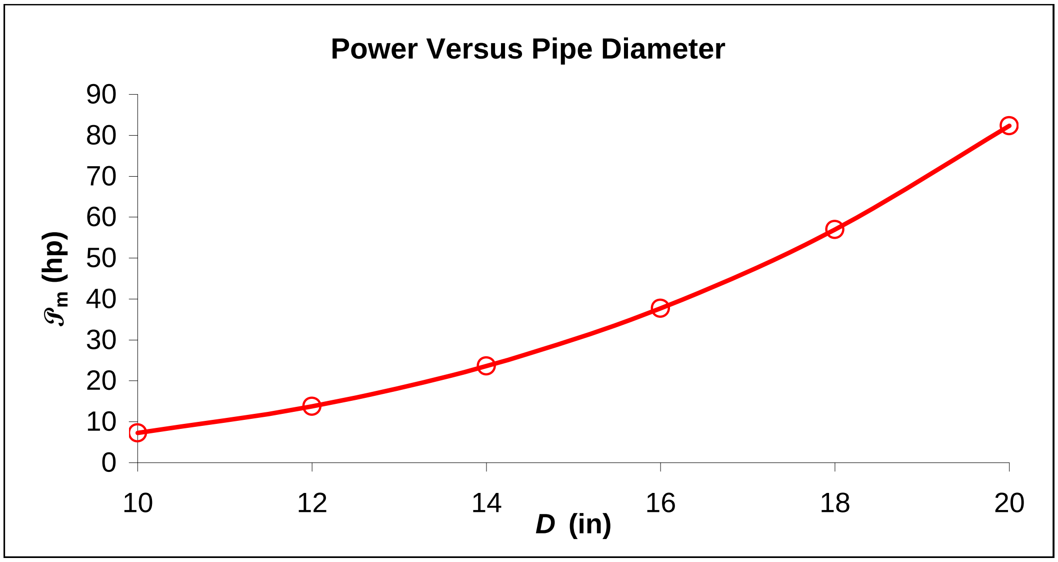
D (in)	V (m/s)	Q (ft^3/s)	$\mathcal{P}_h$ (hp)	$\mathcal{P}_m$ (hp)
10	21.1	11.5	9.10	7.28
12	23.2	18.2	17.22	13.78
14	25.0	26.7	29.54	23.63
16	26.7	37.3	47.13	37.71
18	28.4	50.1	71.18	56.95
20	29.9	65.2	102.93	82.34

Turbine efficiency varies with specific speed (Fig. 10.17).
Pipe roughness appears to the 1/2 power, so has a secondary effect.
A 20% error in f leads to a 10% change in water speed and 30% change in power.
A Pelton wheel is an impulse turbine that does not flow full of water; it directs the stream with open buckets.
A diffuser could not be used with this system.

15.7	26.5	35.4	43.75	35.00

Use Goal Seek or Solver to vary D to make $\mathcal{P}_m$ 35 hp!

The smallest standard size is 16 in.



Problem 10.110

Given: NASA-DOE wind turbine generator at Plum Brook, Ohio.

Two blades, $D = 38$ m; delivers $\mathcal{P} = 100$ kW when $V \geq 29$ km/hr, operating at 40 rpm, with powertrain efficiency, $\eta = 0.75$.

Find: For the maximum power condition, estimate the rotor tip speed and power coefficient.

Solution: Apply definitions

Computing equations: $U = \omega R \quad X = \omega R / V \quad C_P = \dfrac{\mathcal{P}}{\frac{1}{2}\rho V^3 \pi R^2}$

At $N = 40$ rpm,

$$\omega = 40 \frac{\text{rev}}{\text{min}} \times 2\pi \frac{\text{rad}}{\text{rev}} \times \frac{\text{min}}{60\text{ s}} = 4.19 \text{ rad/s}$$

$$U = 4.19 \frac{\text{rad}}{\text{s}} \times \left(\frac{38}{2}\right) \text{m} = 79.6 \text{ m/s}$$

U

$$V = 29 \frac{\text{km}}{\text{hr}} \times 1000 \frac{\text{m}}{\text{km}} \times \frac{\text{hr}}{3600\text{ s}} = 8.06 \text{ m/s}$$

$$X = U/V = 79.6/8.06 = 9.88$$

(Obviously X decreases as wind speed goes up.)

$$\mathcal{P}_m = \frac{\mathcal{P}_e}{0.75} = \frac{1}{0.75} \times 100 \text{ kW} = 133 \text{ kW}$$

$$\frac{1}{2}\rho V^3 \pi R^2 = \frac{\pi}{2} \times 1.23 \frac{\text{kg}}{\text{m}^3} \times (8.06)^3 \frac{\text{m}^3}{\text{s}^3} \times \left(\frac{38}{2}\right)^2 \text{m}^2 \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \times \frac{\text{W} \cdot \text{s}}{\text{N} \cdot \text{m}} = 365 \text{ kW}$$

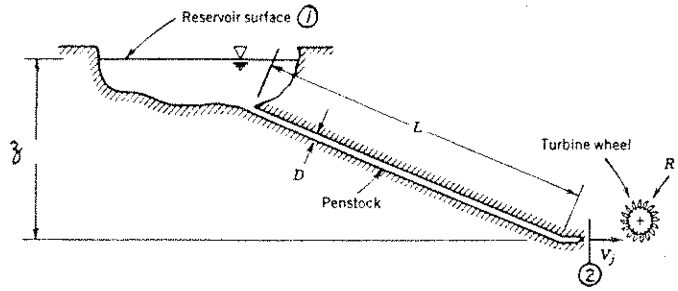
$$C_P = \frac{\mathcal{P}_m}{\frac{1}{2}\rho V^3 \pi R^2} = \frac{133 \text{ kW}}{365 \text{ kW}} = 0.364$$

C_P

Problem 10.111

Given: Small hydraulic impulse turbine installation as shown.

$H = 300$ ft
$L = 1000$ ft (welded steel)
$D = 6$ in., $d_j = 2$ in.
$K_{ent} = 0.5$
$K_{nozzle} = 0.04$



(a) Find V_j, Q, and $\mathcal{P}_h$
(b) Plot $\mathcal{P}_h$ vs. d_j to find optimum d_j.
(c) Then explore effects of varying loss coefficients and pipe roughness.

Solution: Apply the energy equation for steady, incompressible pipe flow.

Computing equation: $\frac{p_1}{\rho g} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\alpha_2 \bar{V}_2^2}{2g} + z_2 + \frac{h_{lT}}{g}$; $h_{lT} = \left[f\left(\frac{L}{D} + \frac{L_e}{D}\right) + K\right]\frac{\bar{V}^2}{2}$ (with ≈ 0 (2) marked over $\alpha_1 \bar{V}_1^2/2g$)

Assumptions: (1) $p_1 = p_2 = p_{atm}$, (2) $\bar{V}_1 \approx 0$, $\alpha_2 = 1$, (3) $L_e/D = 0$, (4) K_{nozzle} based on V_j^2

Then

$$H = \frac{V_j^2}{2g} + \left(f\frac{L}{D} + K_{entrance}\right)\frac{\bar{V}^2}{2g} + K_{nozzle}\frac{V_j^2}{2g}$$

From continuity, $\bar{V}A = V_j A_j$, so $\bar{V} = V_j A_j/A = V_j (d/D)^2$; $\bar{V}^2 = V_j^2 (d/D)^4$, and

$$H = \left[\left(f\frac{L}{D} + K_{ent}\right)\left(\frac{d}{D}\right)^4 + 1 + K_{nozzle}\right]\frac{V_j^2}{2g} \; ; \; V_j = \left[\frac{2gH}{\left(f\frac{L}{D} + K_{ent}\right)\left(\frac{d}{D}\right)^4 + 1 + K_n}\right]^{\frac{1}{2}}$$

Assume $e = 0.00015$ ft (Table 8.1), so $e/D = 0.0003$. From Fig. 8.13, in the fully rough zone, $f = 0.015$. Then for $d_j = 2$ in.

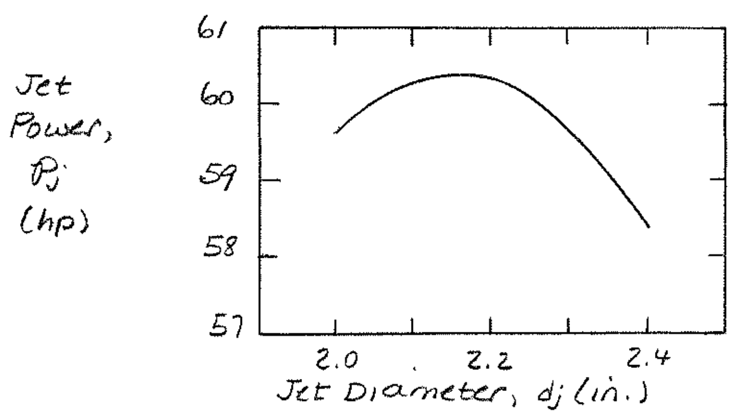
$$V_j = \left[2 \times 32.2\,\frac{ft}{s^2} \times 300\,ft \times \frac{1}{\left(0.015\,\frac{1000\,ft}{0.5\,ft} + 0.5\right)\left(\frac{2}{6}\right)^4 + 1 + 0.04}\right]^{\frac{1}{2}} = 117\ ft/sec$$

($\bar{V} = 13.0$ ft/s, $Re = \bar{V}D/\nu = 6.05 \times 10^5$, so $f = 0.016$, which makes $V_j = 116$ ft/s.)

The jet flow rate is $Q = V_j A_j = 116\,\frac{ft}{s} \times \frac{\pi}{4}\left(\frac{2}{12}\right)^2 ft^2 = 2.53\ ft^3/s$, and the jet power is

$$\mathcal{P}_h = \rho Q \frac{V_j^2}{2} = \frac{1}{2} \times 1.94\,\frac{slug}{ft^3} \times (116)^2\,\frac{ft^2}{s^2} \times 2.53\,\frac{ft^3}{s} \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{hp \cdot s}{550\ ft \cdot lbf} = 60.0\ hp$$

Repeating these calculations using a computer program gives:



Peak power, $\mathcal{P}_j \approx 60.3$ hp, occurs for $2.15 < d < 2.20$ in. ← Opt

Loss coefficients have a minor effect. Making both K_{ent} and K_n zero increases $\mathcal{P}_j$ by 4.8 percent.

Pipe roughness causes larger changes; $\mathcal{P}_j$ increased 12.8 percent with $e = 0$ (smooth).

Problem 10.112

[2]

10.112 A model of an American multiblade farm windmill is to be built for display. The model, with $D = 1$ m, is to develop full power at $V = 10$ m/s wind speed. Calculate the angular speed of the model for optimum power generation. Estimate the power output.

Given: Model of farm windmill

Find: Angular speed for optimum power; Power output

Solution:

Basic equations: $C_P = \dfrac{P}{\frac{1}{2}\cdot\rho\cdot V^3\cdot\pi\cdot R^2}$ $\quad X = \dfrac{\omega\cdot R}{V}$ $\quad$ and we have $\quad \rho = 1.225\cdot\dfrac{kg}{m^3}$

From Fig. 10.45 $\quad C_{Pmax} = 0.3$ $\quad$ at $\quad X = 0.8$ $\quad$ and $\quad D = 1\cdot m$ $\quad R = \dfrac{D}{2}$ $\quad R = 0.5\,m$

Hence, for $\quad V = 10\cdot\dfrac{m}{s}$ $\quad \omega = \dfrac{X\cdot V}{R}$ $\quad \omega = 16\,\dfrac{rad}{s}$ $\quad \omega = 153\,rpm$

Also $\quad P = C_{Pmax}\cdot\dfrac{1}{2}\cdot\rho\cdot V^3\cdot\pi\cdot R^2$ $\quad P = 144\,W$

Problem 10.113

Given: Typical American multiblade farm windmill, $D = 7$ ft, designed to produce maximum power in winds with $V = 15$ mph.

Find: Estimate the rate of water delivery, as a function of the height to which the water is pumped, for this windmill.

Solution: Assume the efficiency trends shown in Fig. 10.45.

Computing equations: $C_P = \dfrac{\mathcal{P}}{\frac{1}{2}\rho V^3 \pi R^2}$ $\qquad X = \omega R/V$

From Fig. 10.45, $C_{P\,max} \approx 0.3$ at $X = 0.8$. $V = 15$ mph (22.0 ft/s). Then the power developed is

$$\mathcal{P} = \frac{\pi}{2} \times 0.3 \times 0.00238 \frac{slug}{ft^3} \times (22.0)^3 \frac{ft^3}{s^3} \times \left(\frac{7}{2}\right)^2 ft^2 \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{hp \cdot s}{550\, ft \cdot lbf} = 0.266 \text{ hp}$$

Converting this mechanical power to pumping gives hydraulic power as

$$\mathcal{P}_h = \rho Q g h = \eta \mathcal{P}_m$$

Thus

$$Qh = \frac{\eta \mathcal{P}_m}{\rho g} = 0.7 \times 0.266\, hp \times \frac{ft^3}{62.4\, lbf} \times 550 \frac{ft \cdot lbf}{hp \cdot s} \times 7.48 \frac{gal}{ft^3} \times 60 \frac{s}{min}$$

$$Qh = 737 \text{ gpm} \cdot \text{ft}$$ $\qquad Qh$

Q varies inversely with the distance lifted, h. The volume flow rate actually delivered would be less, due to suction lift, pipe friction, and minor losses.

Problem 10.114

Given: Largest known Darrieus vertical-axis wind turbine built by DOE near Sandia, New Mexico, is 60 ft tall and 30 ft diameter; the rotor swept area is $A \approx 1200\ ft^2$.

Find: Estimate the maximum power this windmill can produce in a wind with $V = 20$ mph (29.3 ft/s).

Solution: Assume the efficiency trends shown in Fig. 10.45.

Computing equations: $C_P = \dfrac{\mathcal{P}}{\frac{1}{2}\rho V^3 \pi R^2}$ $\quad X = \omega R / V$

From Fig. 10.45, $C_{P\,max} \approx 0.34$ at $X = 5.3$. Use swept area in place of πR^2;

$$\mathcal{P} = \frac{1}{2} \times 0.34 \times 0.00238 \frac{slug}{ft^3} \times (29.3)^3 \frac{ft^3}{s^3} \times 1200\ ft^2 \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{hp \cdot s}{550\ ft \cdot lbf} = 22.2\ hp$$

$\mathcal{P}$

To generate maximum power, the windmill must rotate at

$$\omega = \frac{X V}{R} = 5.3 \times 29.3 \frac{ft}{s} \times \frac{1}{15\ ft} = 10.4\ rad/s \quad (98.9\ rpm)$$

ω

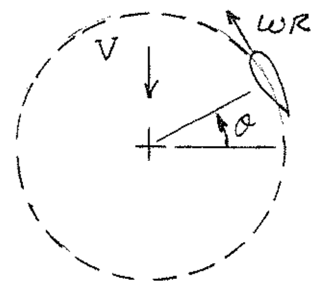
Problem 10.115

Given: Section lift and drag coefficient data for a NACA 0012 section, tested at $Re \approx 6\times10^6$ with standard roughness:

Angle of attack, α (deg)	0	2	4	6	8	10	12
Lift coefficient, C_L (−)	0	0.23	0.45	0.68	0.82	0.94	1.02
Drag coefficient, C_D (−)	0.0098	0.0100	0.0119	0.0147	0.0194	–	–

Find: (a) Analyze air flow relative to a blade element in a Darrieus rotor.
(b) Develop a numerical model for a blade element.
(c) Calculate the power coefficient as a function of tip speed ratio.
(d) Compare with the trend shown in Fig. 10.45.

Solution: Consider plan view of rotor element, absolute velocities:



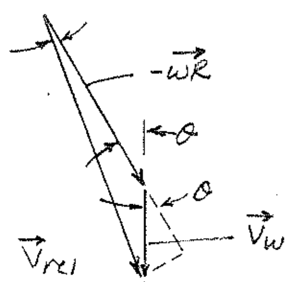
Computing equations: $F_L = C_L \frac{1}{2}\rho V_r^2 A_p$, V_r = relative velocity

$F_D = C_D \frac{1}{2}\rho V_r^2 A_p$ A_p = planform, A_s = swept

$C_p = \dfrac{\mathcal{P}}{\frac{1}{2}\rho V^3 A_s}$ V = wind velocity

Resolve to relative velocity, for position shown:

$\vec{V}_{abs} = \vec{V}_{blade} + \vec{V}_{rel}$; $\vec{V}_{rel} = \vec{V}_{abs} - \vec{V}_{blade}$



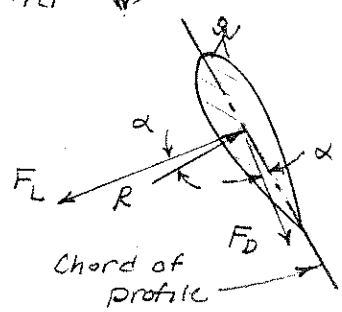
To compute V_{rel}, resolve into components along (a) and transverse (t) to the airfoil chord:

$V_{rel})_a = \omega R + V_w \cos\theta$

$V_{rel})_t = V_w \sin\theta$

$V_{rel} = \left[V_{rel})_a^2 + V_{rel})_t^2 \right]^{\frac{1}{2}}$

$\alpha = \tan^{-1}[V_{rel})_t / V_{rel})_a]$



Lift force (F_L) is normal to $\vec{V}_{rel}$ and drag force (F_D) is parallel to $\vec{V}_{rel}$. Thus

$T = R(F_L \sin\alpha - F_D \cos\alpha)$ (torque, $T > 0$ when $F_L/F_D > \cot\alpha$)

Both C_L and C_D must be modeled as functions of angle of attack, α. From a graph of C_L and C_D versus α, a satisfactory representation is

$C_L = 0.12\,\alpha - 0.0026\,|\alpha|\alpha$, $-12 < \alpha < 12$ degrees ; $C_L = 0$, $|\alpha| > 12$ degrees

$C_D = 0.00952 + 1.52\times10^{-4}\alpha^2$; $-12 < \alpha < 12$ degrees; $C_D = 0.0314$, $|\alpha| > 12$ degrees

(The models in the stalled region, $|\alpha| > 12$ degrees, obviously are crude.)

Sample calculation: Choose $R = 10$ ft, $c = 0.5$ ft, $w = 1$ ft, $X = 5$, $V_w = 20$ mph

At $\theta = 30°$, with $V_w = 20$ mph (29.3 ft/s)

$$X = \omega R/V_w \;;\quad \omega = \frac{X V_w}{R} = 5 \times 29.3\,\frac{ft}{s} \times \frac{1}{10\,ft} = 14.7 \text{ rad/s} \quad (N = 140 \text{ rpm})$$

$$\omega R = 14.7\,\frac{rad}{s} \times 10\,ft = 147\,ft/s$$

$$V_{rel})_a = \omega R + V_w \cos\theta = 147 + 29.3 \cos 30° = 172\,ft/s$$

$$V_{rel})_t = V_w \sin\theta = 29.3 \sin\theta = 14.7\,ft/s$$

$$V_{rel} = \left[V_{rel})_a^2 + V_{rel})_t^2\right]^{\frac{1}{2}} = \left[(172)^2 + (14.7)^2\right]^{\frac{1}{2}} = 173\,ft/s$$

$$\alpha = \tan^{-1}\left[V_{rel})_t / V_{rel})_a\right] = \tan^{-1}(14.7/172) = 4.88 \text{ degrees}$$

$$q = \frac{1}{2}\rho V_{rel}^2 = \frac{1}{2} \times 0.00238\,\frac{slug}{ft^3} \times (173)^2\,\frac{ft^2}{s^2} \times \frac{lbf \cdot s^2}{slug \cdot ft} = 35.6\,lbf/ft^2$$

A_p (projected area of airfoil section) $= cw = 0.5\,ft \times 1\,ft = 0.5\,ft^2$

$$C_L = 0.12\alpha - 0.0026|\alpha|\alpha = 0.12 \times 4.88 - 0.0026|4.88|4.88 = 0.524$$

$$C_D = 0.00952 + 1.52 \times 10^{-4}\alpha^2 = 0.00952 + 1.52 \times 10^{-4}(4.88)^2 = 0.0131$$

$$F_L = C_L q A_p = 0.524 \times 35.6\,\frac{lbf}{ft^2} \times 0.5\,ft^2 = 9.33\,lbf$$

$$F_D = C_D q A_p = 0.0131 \times 35.6\,\frac{lbf}{ft^2} \times 0.5\,ft^2 = 0.233\,lbf$$

$$F_L/F_D = 40.0$$

$$T = R(F_L \sin\alpha - F_D \cos\alpha) = 10\,ft\,(9.33 \sin(4.88°) - 0.233 \cos(4.88°))\,lbf = 5.62\,ft \cdot lbf$$

$$\mathscr{P} = \omega T = 14.7\,\frac{rad}{s} \times 5.62\,ft \cdot lbf = 82.6\,\frac{ft \cdot lbf}{s} \quad (0.150\,hp)$$

$$C_p = \frac{\mathscr{P}}{\frac{1}{2}\rho V_w^3 A_s} \;;\quad A_s = \text{area swept by element} = 2Rw = 2 \times 10\,ft \times 1\,ft = 20\,ft^2$$

$$C_p = 82.6\,\frac{ft \cdot lbf}{s} \times \frac{ft^3}{(\frac{1}{2})\,0.00238\,slug} \times \frac{s^3}{(29.3)^3\,ft^3} \times \frac{1}{20\,ft^2} \times \frac{slug \cdot ft}{lbf \cdot s^2} = 0.138 \quad (\text{at } \theta = 15°)$$

Obtain $\bar{C}_p$ for a complete rotor revolution by integrating numerically. Such results are presented on the next page, and plotted versus tip speed ratio, $X = \omega R/V_w$.

From the plot, $\bar{C}_p$ is small at low X. It increases as X is raised, then peaks and decreases again. Comparison with Fig. 10.45 shows the trends are similar, but the model predicts useful power at larger X than observed experimentally. Blade elements at smaller radii on the rotor would produce less power, since ω = constant along rotor. $\bar{C}_p$ at large X is also sensitive to C_D.

Low $\bar{C}_p$ at small X occurs because the airfoil is stalled.

Problem 10.115

Computed results:

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* *

Airfoil: NACA 0012 Section; Chord, c = 6 in.

Blade element: Span, w = 1 ft; Radius, R = 10 ft

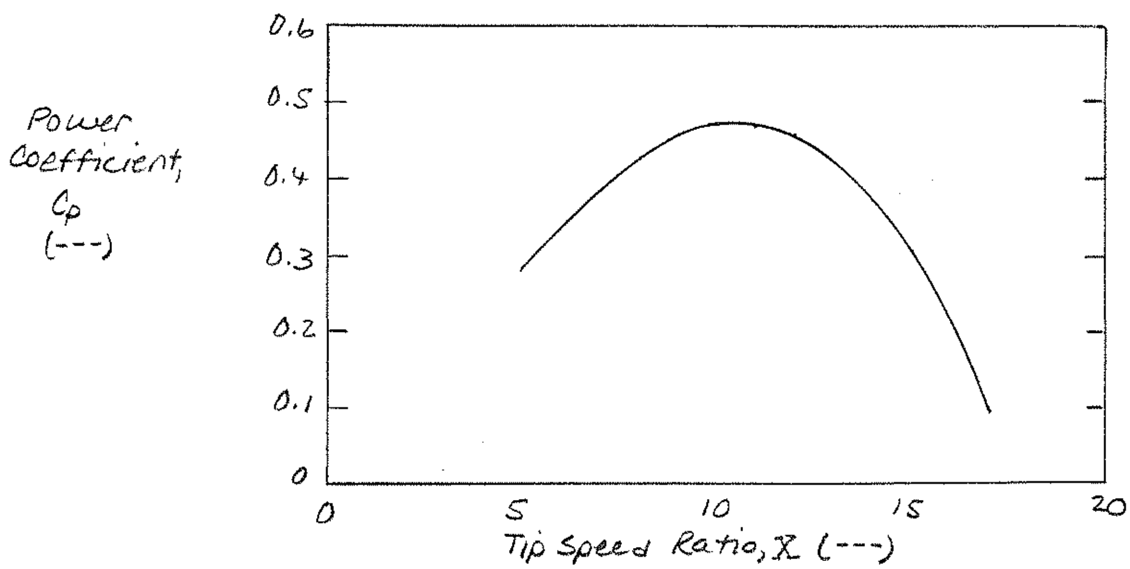
Input data: Tip speed ratio, X = 5.0 (---)
Wind speed, Vw = 20 mph (29.3 ft/sec)

Calculated: Rotor speed, omega = 140.4 rpm

theta (deg)	Vrel (ft/s)	alpha (deg)	Cl (--)	Cd (--)	Fl (lbf)	Fd (lbf)	T (ft-lbf)	Cp (--)
0	176	0.00	0.00	0.010	0.0	0.176	-1.8	-0.043
30	173	4.87	0.52	0.013	9.3	0.233	5.6	0.136
60	164	8.95	0.87	0.022	13.8	0.343	18.1	0.439
90	150	11.31	1.02	0.029	13.7	0.384	23.1	0.562
120	135	10.89	1.00	0.027	10.8	0.295	17.5	0.425
150	122	6.90	0.70	0.017	6.3	0.148	6.1	0.147
180	118	-0.00	-0.00	0.010	-0.0	0.078	-0.8	-0.019
210	122	-6.90	-0.70	0.017	-6.3	0.148	6.1	0.147
240	135	-10.89	-1.00	0.027	-10.8	0.295	17.5	0.425
270	150	-11.31	-1.02	0.029	-13.7	0.384	23.1	0.562
300	164	-8.95	-0.87	0.022	-13.8	0.343	18.1	0.439
330	173	-4.87	-0.52	0.013	-9.3	0.233	5.6	0.136
360	176	0.00	0.00	0.010	0.0	0.176	-1.8	-0.043

Average power coefficient for complete revolution: Cp,bar = 0.280

Plotting results of similar calculations at various tip speed ratios give:



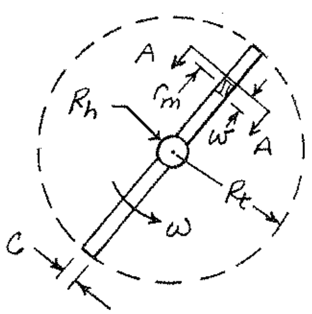
Problem 10.116

Given: Lift and drag data for NACA 23015 airfoil section, Fig. 9.17.

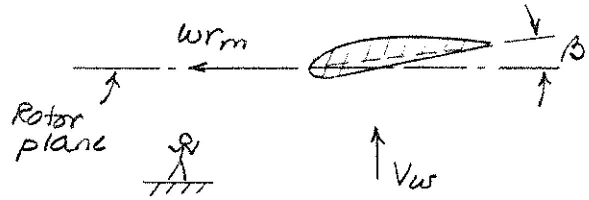
Consider two-blade, horizontal-axis wind turbine with this section.

Find: (a) Analyze air flow relative to a blade element in rotating turbine.
(b) Develop numerical model for blade element.
(c) Calculate power coefficient as a function of tip speed ratio.
(d) Compare with the trend shown in Fig. 10.45.

Solution: Front view of rotor; blade element shown cross-hatched:

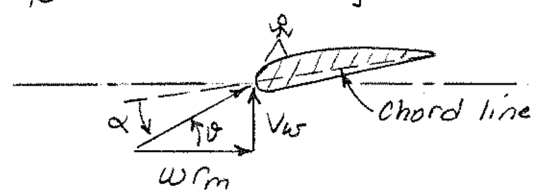


Section A-A: (absolute velocities)

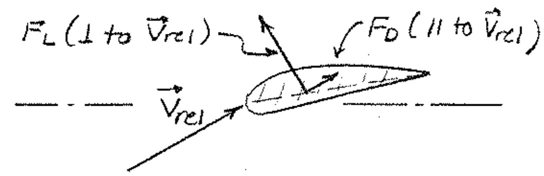


β = blade twist angle

(relative velocities)



α = angle of attack



Resolve to relative velocity:

$$\vec{V}_{abs} = \vec{V}_{blade\ element} + \vec{V}_{rel}$$

$$\vec{V}_{rel} = \vec{V}_w - \vec{V}_{blade} = \vec{V}_w + (-\vec{V}_{blade})$$

Computing equations:

{Note: $\beta > 0$ is required for windmill to self start.}

$F_L = C_L \frac{1}{2}\rho V_r^2 A_p$; V_r = relative velocity

$F_D = C_D \frac{1}{2}\rho V_r^2 A_p$; A_p = planform area

$T = n_b r_m (F_L \sin\theta - F_D \cos\theta)$; n_b = number of blades (2)

$V_{rel} = \left[(\omega r_m)^2 + V_w^2\right]^{\frac{1}{2}}$; $\theta = \tan^{-1}(V_w/\omega r_m)$; $\alpha = \theta - \beta$

Both C_L and C_D must be modeled as functions of angle of attack, α. From Fig. 9.17, satisfactory representations are:

$0 \le \alpha < 12°$	$C_L = 0.12 + 0.107\alpha$	$0 \le \alpha < 4°$	$C_D = 0.0065 + 4.55\times10^{-5}\alpha$
$12° \le \alpha < 18°$	$C_L = 0.12 + 0.107\alpha - 0.00852(\alpha - 12)^2$	$4° \le \alpha < 16°$	C_D = above $+ 7.72\times10^{-5}(\alpha-4)^2$
$18° \le \alpha$	$C_L = 0.2$	$16° \le \alpha$	$C_D = 0.02$

{Obviously both models are crude for $C_L(\alpha > 18°)$, $C_D(\alpha > 16°)$.} Choose $R_t = 10$ ft, $r_m = 5.5$ ft, $c = 6$ in., $w = 1$ ft, $V_w = 20$ mph (29.4 ft/s), $X = 5$, and $\beta = 5°$. Then

$$\omega = \frac{X V_w}{R_t} = 5.0 \times 29.4\,\frac{ft}{s} \times \frac{1}{10\,ft} = 14.7\ rad/s\ \ (140\ rpm);\ \omega r_m = 80.9\ ft/s$$

$V_{rel} = \left[(80.9)^2 + (29.4)^2\right]^{\frac{1}{2}} = 86.1\ ft/s$; $q = \frac{1}{2}\rho V_{rel}^2 = 8.82\ lbf/ft^2$; $A_p = wc = 0.5\ ft^2$

$\theta = \tan^{-1}(29.4/80.9) = 20.0°$; $\alpha = \theta - \beta = 20.0 - 5.0 = 15.0°$; $C_L = 1.65$; $C_D = 0.017$

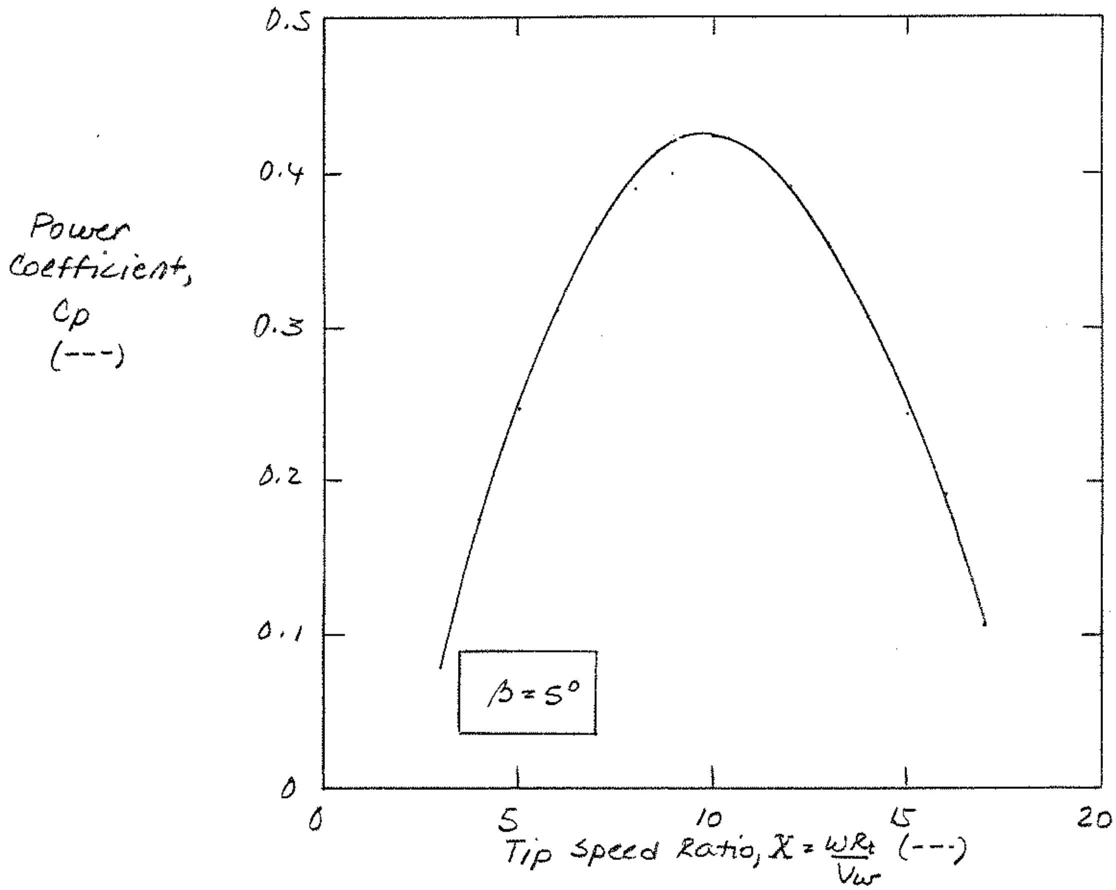
$F_L = 1.65 \times 8.82 \frac{lbf}{ft^2} \times 0.5\ ft^2 = 7.28\ lbf$; $F_D = 0.017 \times 8.82 \frac{lbf}{ft^2} \times 0.5\ ft^2 = 0.075\ lbf$

$T = 2 \times 5.5\ ft\ (7.28 \times \sin 20° - 0.075 \cos 20°)\ lbf = 26.6\ ft \cdot lbf$

Similar calculations for the other blade elements show that torque for the complete propeller is $T_p = 159\ ft \cdot lbf$. The power coefficient is

$$C_P = \frac{\mathcal{P}}{\frac{1}{2}\rho V_w^3 \pi R_t^2} = \frac{\omega T_p}{\frac{1}{2}\rho V_w^3 \pi R_t^2} = \frac{2}{\pi} \times 14.7 \frac{rad}{s} \times 159\ ft \cdot lbf \times \frac{ft^3}{0.00238\ slug} \times \frac{s^3}{(29.4)^3 ft^3} \times \frac{1}{(10)^2 ft^2} \times \frac{slug \cdot ft}{lbf \cdot s^2} = 0.246$$

Calculated results are tabulated on the next page, plotted and discussed below:



Trends shown are similar to Fig. 10.45. At small X, the blade is entirely stalled so useful output is low. At large X, α becomes negative near the tips, reducing output.

This model does not include: (1) axial interference that reduces normal velocity below V_w as loading increases, or (2) swirl introduce by blade drag. Both these effects reduce performance. For more details, see Division L, Section XI of [30].

Computed results:

****

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Airfoil: NACA 23015 Section; Chord, c = 6 in.
Tip radius, Rt = 10 ft
Twist angle, beta = 5 degrees

Blade element: Delta r, dr = 1.00 ft

Input data: Tip speed ratio, X = 5.0 (---)
Wind speed, Vw = 20 mph (29.4 ft/s)

Calculated: Rotor speed, omega = 140.4 rpm

Rm (ft)	Vrel (ft/s)	alpha (deg)	Cl (--)	Cd (--)	Fl (lbf)	Fd (lbf)	T (ft-lbf)
1.50	22	48.13	0.20	0.020	0.06	0.006	0.13
2.50	37	33.66	0.20	0.020	0.16	0.016	0.44
3.50	51	24.74	0.20	0.020	0.32	0.032	0.90
4.50	66	18.96	0.20	0.020	0.52	0.052	1.48
5.50	81	14.98	1.65	0.017	6.41	0.067	23.39
6.50	96	12.10	1.41	0.013	7.69	0.069	28.53
7.50	110	9.93	1.18	0.010	8.55	0.074	31.99
8.50	125	8.24	1.00	0.009	9.31	0.081	34.90
9.50	140	6.89	0.86	0.008	9.95	0.091	37.25

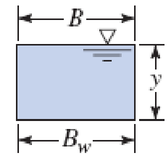
Torque for complete propeller: T = 159.0 ft-lbf

Power coefficient for windmill: Cp = 0.246 (---)

Problem 11.1

[1]

11.1 A 2-m-wide rectangular channel with a bed slope of 0.0005 has a depth of flow of 1.5 m. Manning's roughness coefficient is 0.015. Determine the steady uniform discharge in the channel.



Given: Rectangular channel flow

Find: Discharge

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For a rectangular channel of width $B_w = 2 \cdot m$ and depth $y = 1.5 \cdot m$ we find from Table 11.2

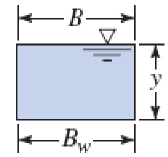
$A = B_w \cdot y$ $\quad A = 3.00 \cdot m^2$ $\quad R = \frac{B_w \cdot y}{B_w + 2 \cdot y}$ $\quad R = 0.600 \cdot m$

Manning's roughness coefficient is $n = 0.015$ and $S_0 = 0.0005$

$Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$ $\quad Q = 3.18 \cdot \frac{m^3}{s}$

Problem 11.2

11.2 Determine the uniform flow depth in a rectangular channel 2.5 m wide with a discharge of 3 m^3/s. The slope is 0.0004 and Manning's roughness factor is 0.015.



Given: Data on rectangular channel

Find: Depth of flow

Solution:

Basic equation: $$Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For a rectangular channel of width $B_w = 2.5\cdot m$ and flow rate $Q = 3\cdot\frac{m^3}{s}$ we find from Table 11.2 $A = B_w\cdot y$ $R = \frac{B_w\cdot y}{B_w + 2\cdot y}$

Manning's roughness coefficient is $n = 0.015$ and $S_0 = 0.0004$

Hence the basic equation becomes $$Q = \frac{1}{n}\cdot B_w\cdot y\cdot\left(\frac{B_w\cdot y}{B_w + 2\cdot y}\right)^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$$

Solving for y $$y\cdot\left(\frac{B_w\cdot y}{B_w + 2\cdot y}\right)^{\frac{2}{3}} = \frac{Q\cdot n}{B_w\cdot S_0^{\frac{1}{2}}}$$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below, to make the left side evaluate to $\frac{Q\cdot n}{B_w\cdot S_0^{\frac{1}{2}}} = 0.900$.

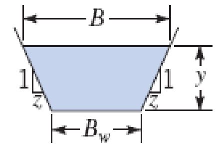
For $y = 1$ (m) $y\cdot\left(\frac{B_w\cdot y}{B_w + 2\cdot y}\right)^{\frac{2}{3}} = 0.676$ For $y = 1.2$ (m) $y\cdot\left(\frac{B_w\cdot y}{B_w + 2\cdot y}\right)^{\frac{2}{3}} = 0.865$

For $y = 1.23$ (m) $y\cdot\left(\frac{B_w\cdot y}{B_w + 2\cdot y}\right)^{\frac{2}{3}} = 0.894$ For $y = 1.24$ (m) $y\cdot\left(\frac{B_w\cdot y}{B_w + 2\cdot y}\right)^{\frac{2}{3}} = 0.904$

The solution to three figures is $y = 1.24$ (m)

Problem 11.3

11.3 Determine the uniform flow depth in a trapezoidal channel with a bottom width of 8 ft and side slopes of 1 vertical to 2 horizontal. The discharge is 100 ft^3/s. Manning's roughness factor is 0.015 and the channel bottom slope is 0.0004.



Given: Data on trapzoidal channel

Find: Depth of flow

Solution:

Basic equation: $Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 8 \cdot ft$ $\quad z = 2$ $\quad Q = 100 \cdot \frac{ft^3}{s}$ $\quad S_0 = 0.0004$

$n = 0.015$

Hence from Table 11.2 $A = (B_w + z \cdot y) \cdot y = (8 + 2 \cdot y) \cdot y$ $\quad R = \frac{(B_w + z \cdot y) \cdot y}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}} = \frac{(8 + 2 \cdot y) \cdot y}{8 + 2 \cdot y \cdot \sqrt{5}}$

Hence $Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1.49}{0.015} \cdot (8 + 2 \cdot y) \cdot y \cdot \left[\frac{(8 + 2 \cdot y) \cdot y}{8 + 2 \cdot y \cdot \sqrt{5}}\right]^{\frac{2}{3}} \cdot 0.0004^{\frac{1}{2}} = 100$ (Note that we don't use units!)

Solving for y $\frac{[(8 + 2 \cdot y) \cdot y]^{\frac{5}{3}}}{(8 + 2 \cdot y \cdot \sqrt{5})^{\frac{2}{3}}} = 50.3$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

For $y = 2$ (ft) $\frac{[(8 + 2 \cdot y) \cdot y]^{\frac{5}{3}}}{(8 + 2 \cdot y \cdot \sqrt{5})^{\frac{2}{3}}} = 30.27$ $\quad$ For $y = 3$ (ft) $\frac{[(8 + 2 \cdot y) \cdot y]^{\frac{5}{3}}}{(8 + 2 \cdot y \cdot \sqrt{5})^{\frac{2}{3}}} = 65.8$

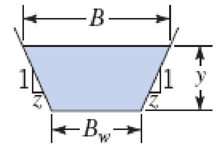
For $y = 2.6$ (ft) $\frac{[(8 + 2 \cdot y) \cdot y]^{\frac{5}{3}}}{(8 + 2 \cdot y \cdot \sqrt{5})^{\frac{2}{3}}} = 49.81$ $\quad$ For $y = 2.61$ (ft) $\frac{[(8 + 2 \cdot y) \cdot y]^{\frac{5}{3}}}{(8 + 2 \cdot y \cdot \sqrt{5})^{\frac{2}{3}}} = 50.18$

The solution to three figures is $y = 2.61$ (ft)

Problem 11.4

[3]

11.4 Determine the uniform flow depth in a trapezoidal channel with a bottom width of 2.5 m and side slopes of 1 vertical to 2 horizontal with a discharge of 3 m^3/s. The slope is 0.0004 and Manning's roughness factor is 0.015.



Given: Data on trapezoidal channel

Find: Depth of flow

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 2.5 \cdot m$ $\quad z = 2$ $\quad Q = 3 \cdot \frac{m^3}{s}$ $\quad S_0 = 0.0004$

$n = 0.015$

Hence from Table 11.2 $A = (B_w + z \cdot y) \cdot y = (8 + 2 \cdot y) \cdot y$ $\quad R = \frac{(B_w + z \cdot y) \cdot y}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}} = \frac{(2.5 + 2 \cdot y) \cdot y}{2.5 + 2 \cdot y \cdot \sqrt{5}}$

Hence $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1}{0.015} \cdot (2.5 + 2 \cdot y) \cdot y \cdot \left[\frac{(2.5 + 2 \cdot y) \cdot y}{2.5 + 2 \cdot y \cdot \sqrt{5}}\right]^{\frac{2}{3}} \cdot 0.0004^{\frac{1}{2}} = 3$ (Note that we don't use units!)

Solving for y $\frac{[(2.5 + 2 \cdot y) \cdot y]^{\frac{5}{3}}}{(2.5 + 2 \cdot y \cdot \sqrt{5})^{\frac{2}{3}}} = 2.25$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

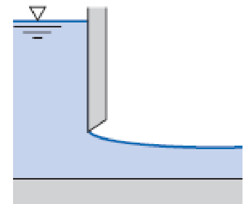
For $y = 1$ (m) $\frac{[(2.5 + 2 \cdot y) \cdot y]^{\frac{5}{3}}}{(2.5 + 2 \cdot y \cdot \sqrt{5})^{\frac{2}{3}}} = 3.36$ For $y = 0.8$ (m) $\frac{[(2.5 + 2 \cdot y) \cdot y]^{\frac{5}{3}}}{(2.5 + 2 \cdot y \cdot \sqrt{5})^{\frac{2}{3}}} = 2.17$

For $y = 0.81$ (m) $\frac{[(2.5 + 2 \cdot y) \cdot y]^{\frac{5}{3}}}{(2.5 + 2 \cdot y \cdot \sqrt{5})^{\frac{2}{3}}} = 2.23$ For $y = 0.815$ (m) $\frac{[(2.5 + 2 \cdot y) \cdot y]^{\frac{5}{3}}}{(2.5 + 2 \cdot y \cdot \sqrt{5})^{\frac{2}{3}}} = 2.25$

The solution to three figures is $y = 0.815$ (m)

Problem 11.5

11.5 A partially open sluice gate in a 3-m-wide rectangular channel carries water at 8.5 m^3/sec. The upstream depth is 2 m. Find the downstream depth and Froude number.



Given: Data on sluice gate

Find: Downstream depth; Froude number

Solution:

Basic equation: $$\frac{p_1}{\rho\cdot g} + \frac{V_1^{\ 2}}{2\cdot g} + y_1 = \frac{p_2}{\rho\cdot g} + \frac{V_2^{\ 2}}{2\cdot g} + y_2 + h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow.

Noting that $p_1 = p_2 = p_{atm}$, (1 = upstream, 2 = downstream) the Bernoulli equation becomes

$$\frac{V_1^{\ 2}}{2\cdot g} + y_1 = \frac{V_2^{\ 2}}{2\cdot g} + y_2$$

The given data is $b = 3\cdot m$ $\quad y_1 = 2\cdot m \quad Q = 8.5\cdot\frac{m^3}{s}$

For mass flow $Q = V\cdot A$ so $V_1 = \frac{Q}{b\cdot y_1}$ and $V_2 = \frac{Q}{b\cdot y_2}$

Using these in the Bernoulli equation $$\frac{\left(\frac{Q}{b\cdot y_1}\right)^2}{2\cdot g} + y_1 = \frac{\left(\frac{Q}{b\cdot y_2}\right)^2}{2\cdot g} + y_2 \qquad (1)$$

The only unknown on the right is y_2. The left side evaluates to $\frac{\left(\frac{Q}{b\cdot y_1}\right)^2}{2\cdot g} + y_1 = 2.10\,m$

To find y_2 we need to solve the non-linear equation. We must do this numerically; we may use the Newton method or similar, or *Excel*'s *Solver* or *Goal Seek*. Here we interate manually, starting with an arbitrary value less than y_1.

For $y_2 = 0.5\cdot m$ $\quad \frac{\left(\frac{Q}{b\cdot y_2}\right)^2}{2\cdot g} + y_2 = 2.14\,m$ $\quad$ For $y_2 = 0.51\cdot m$ $\quad \frac{\left(\frac{Q}{b\cdot y_2}\right)^2}{2\cdot g} + y_2 = 2.08\,m$

For $y_2 = 0.505\cdot m$ $\quad \frac{\left(\frac{Q}{b\cdot y_2}\right)^2}{2\cdot g} + y_2 = 2.11\,m$ $\quad$ For $y_2 = 0.507\cdot m$ $\quad \frac{\left(\frac{Q}{b\cdot y_2}\right)^2}{2\cdot g} + y_2 = 2.10\,m$

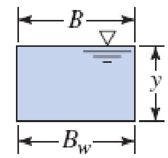
Hence $y_2 = 0.507\,m$

Then $V_2 = \frac{Q}{b\cdot y_2}$ $\quad V_2 = 5.59\frac{m}{s}$ $\quad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}}$ $\quad Fr_2 = 2.51$

Problem 11.6

[1]

11.6 A rectangular flume built of concrete, with 1 ft per 1000 ft slope, is 6 ft wide. Water flows at a normal depth of 3 ft. Compute the discharge.



Given: Data on flume

Find: Discharge

Solution:

Basic equation: $$Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For a rectangular channel of width $B_w = 6 \cdot ft$ and depth $y = 3 \cdot ft$ we find from Table 11.2

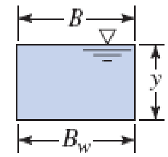
$A = B_w \cdot y$ $\quad A = 18 \cdot ft^2$ $\quad R = \frac{B_w \cdot y}{B_w + 2 \cdot y}$ $\quad R = 1.50 \cdot ft$

For concrete (Table 11.1) $\quad n = 0.013$ and $\quad S_0 = \frac{1 \cdot ft}{1000 \cdot ft}$ $\quad S_0 = 0.001$

$Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$ $\quad Q = 85.5 \cdot \frac{ft^3}{s}$

Problem 11.7

11.7 A rectangular flume built of timber is 3 ft wide. The flume is to handle a flow of 90 ft^3/sec at a normal depth of 6 ft. Determine the slope required.



Given: Data on flume

Find: Slope

Solution:

Basic equation: $Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For a rectangular channel of width $B_w = 3 \cdot ft$ and depth $y = 6 \cdot ft$ we find

$A = B_w \cdot y$ $\quad A = 18 \cdot ft^2$ $\quad R = \frac{B_w \cdot y}{B_w + 2 \cdot y}$ $\quad R = 1.20 \cdot ft$

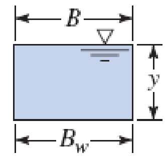
For wood (not in Table 11.1) a Google search finds n = 0.012 to 0.017; we use $n = 0.0145$ with $Q = 90 \cdot \frac{ft^3}{s}$

$$S_0 = \left(\frac{n \cdot Q}{1.49 \cdot A \cdot R^{\frac{2}{3}}} \right)^2 \qquad S_0 = 1.86 \times 10^{-3}$$

Problem 11.8

[2]

11.8 A channel with square cross section is to carry 20 m^3/sec of water at normal depth on a slope of 0.003. Compare the dimensions of the channel required for (a) concrete and (b) soil cement.



Given: Data on square channel

Find: Dimensions for concrete and soil cement

Solution:

Basic equation: $$Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For a square channel of width B_w we find $A = B_w^2$ $$R = \frac{B_w \cdot y}{B_w + 2 \cdot y} = \frac{B_w^2}{B_w + 2 \cdot B_w} = \frac{B_w}{3}$$

Hence $$Q = \frac{1}{n} \cdot B_w^2 \cdot \left(\frac{B_w}{3}\right)^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{S_0^{\frac{1}{2}}}{n \cdot 3^{\frac{2}{3}}} \cdot B_w^{\frac{8}{3}}$$ or $$B_w = \left(\frac{3^{\frac{2}{3}} \cdot Q}{S_0^{\frac{1}{2}}} \cdot n\right)^{\frac{3}{8}}$$

The given data is $Q = 20 \cdot \frac{m^3}{s}$ $S_0 = 0.003$

For concrete, from Table 11.1 (assuming large depth) $n = .013$

$B_w = 2.36\,m$

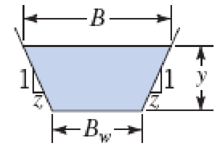
For soil cement from Table 11.1 (assuming large depth) $n = .020$

$B_w = 2.77\,m$

Problem 11.9

[1]

11.9 Water flows in a trapezoidal channel at a normal depth of 1.2 m. The bottom width is 2.4 m and the sides slope at 1:1 (45°). The flow rate is 7.1 m^3/sec. The channel is excavated from bare soil. Find the bed slope.



Given: Data on trapezoidal channel

Find: Bed slope

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 2.4 \cdot m$ $\quad z = 1$ $\quad y = 1.2 \cdot m$ $\quad Q = 7.1 \cdot \frac{m^3}{s}$

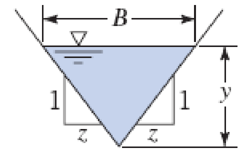
For bare soil (Table 11.1) $n = 0.020$

Hence from Table 11.2 $A = (B_w + z \cdot y) \cdot y$ $\quad A = 4.32\,m^2$ $\quad R = \frac{(B_w + z \cdot y) \cdot y}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}}$ $\quad R = 0.746\,m$

Hence $S_0 = \left(\frac{Q \cdot n}{A \cdot R^{\frac{2}{3}}} \right)^2$ $\quad S_0 = 1.60 \times 10^{-3}$

Problem 11.10

11.10 A triangular channel with side angles of 45° is to carry 10 m^3/sec at a slope of 0.001. The channel is concrete. Find the required dimensions.



Given: Data on triangular channel

Find: Required dimensions

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the triangular channel we have $z = 1$ $\quad S_0 = 0.001$ $\quad Q = 10 \cdot \frac{m^3}{s}$

For concrete (Table 11.1) $n = 0.013$ (assuming y > 60 cm: verify later)

Hence from Table 11.2 $A = z \cdot y^2 = y^2$ $\quad R = \frac{z \cdot y}{2 \cdot \sqrt{1 + z^2}} = \frac{y}{2 \cdot \sqrt{2}}$

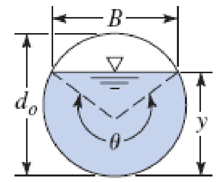
Hence $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1}{n} \cdot y^2 \cdot \left(\frac{y}{2 \cdot \sqrt{2}}\right)^{\frac{2}{3}} \cdot S_0 = \frac{1}{n} \cdot y^{\frac{8}{3}} \cdot \left(\frac{1}{8}\right)^{\frac{1}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1}{2 \cdot n} \cdot y^{\frac{8}{3}} \cdot S_0^{\frac{1}{2}}$

Solving for y $y = \left(\frac{2 \cdot n \cdot Q}{\sqrt{S_0}}\right)^{\frac{3}{8}}$ $\quad y = 2.20\,m$ (The assumption that y > 60 cm is verified)

Problem 11.11

[2]

11.11 A semicircular trough of corrugated steel, with diameter $D = 1$ m, carries water at depth $y_n = 0.25$ m. The slope is 0.01. Find the discharge.



Given: Data on semicircular trough

Find: Discharge

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the semicircular channel $d_0 = 1 \cdot m$ $y = 0.25 \cdot m$ $S_0 = 0.01$

Hence, from geometry $\theta = 2 \cdot \text{asin}\left(\frac{y - \frac{d_0}{2}}{\frac{d_0}{2}}\right) + 180 \cdot \text{deg}$ $\theta = 120 \cdot \text{deg}$

For corrugated steel, a Google search leads to $n = 0.022$

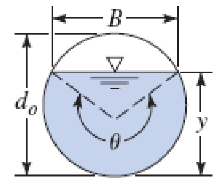
Hence from Table 11.2 $A = \frac{1}{8} \cdot (\theta - \sin(\theta)) \cdot d_0^{2}$ $A = 0.154 \, m^2$

$R = \frac{1}{4} \cdot \left(1 - \frac{\sin(\theta)}{\theta}\right) \cdot d_0$ $R = 0.147 \, m$

Then the discharge is $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} \cdot \frac{m^3}{s}$ $Q = 0.194 \frac{m^3}{s}$

Problem 11.12

11.12 Find the discharge at which the channel of Problem 11.11 flows full.



Given: Data on semicircular trough

Find: Discharge

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the semicircular channel $d_0 = 1 \cdot m$ $\theta = 180 \cdot deg$ $S_0 = 0.01$

For corrugated steel, a Google search leads to (Table 11.1) $n = 0.022$

Hence from Table 11.2 $A = \frac{1}{8} \cdot (\theta - \sin(\theta)) \cdot d_0^{2}$ $A = 0.393\, m^2$

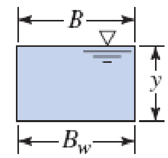
$R = \frac{1}{4} \cdot \left(1 - \frac{\sin(\theta)}{\theta}\right) \cdot d_0$ $R = 0.25\, m$

Then the discharge is $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} \cdot \frac{m^3}{s}$ $Q = 0.708 \frac{m^3}{s}$

Problem 11.13

[3]

11.13 The flume of Problem 11.6 is fitted with a new plastic film liner ($n = 0.010$). Find the new depth of flow if the discharge remains constant at 85.5 ft^3/sec.



Given: Data on flume with plastic liner

Find: Depth of flow

Solution:

Basic equation: $$Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For a rectangular channel of width $B_w = 6 \cdot ft$ and depth y we find from Table 11.2

$$A = B_w \cdot y = 6 \cdot y \qquad R = \frac{B_w \cdot y}{B_w + 2 \cdot y} = \frac{6 \cdot y}{6 + 2 \cdot y}$$

and also $n = 0.010$ and $S_0 = \frac{1 \cdot ft}{1000 \cdot ft}$ $\quad S_0 = 0.001$

Hence $$Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1.49}{0.010} \cdot 6 \cdot y \cdot \left(\frac{6 \cdot y}{6 + 2 \cdot y}\right)^{\frac{2}{3}} \cdot 0.001^{\frac{1}{2}} = 85.5$$ (Note that we don't use units!)

Solving for y $$\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = \frac{85.5 \cdot 0.010}{1.49 \cdot .001^{\frac{1}{2}} \cdot 6 \cdot 6^{\frac{2}{3}}}$$ or $$\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = 0.916$$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with Problem 11.6's depth

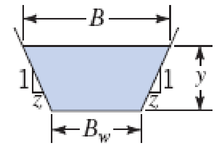
For $y = 3$ (feet) $\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = 1.191$ For $y = 2$ (feet) $\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = 0.684$

For $y = 2.5$ (feet) $\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = 0.931$ For $y = 2.45$ (feet) $\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = 0.906$

For $y = 2.47$ (feet) $\frac{y^{\frac{5}{3}}}{(6 + 2 \cdot y)^{\frac{2}{3}}} = 0.916$ $\qquad y = 2.47$ (feet)

Problem 11.14

11.14 Discharge through the channel of Problem 11.9 is increased to 15 m^3/sec. Find the corresponding normal depth, if the bed slope is 0.00193.



Given: Data on trapzoidal channel

Find: New depth of flow

Solution:

Basic equation: $$Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 2.4\cdot m$ $\quad z = 1$ $\quad Q = 15\cdot\frac{m^3}{s}$ $\quad S_0 = 0.00193$

For bare soil (Table 11.1) $n = 0.020$

Hence from Table 11.2 $A = \left(B_w + z\cdot y\right)\cdot y = (2.4 + y)\cdot y$ $\quad R = \frac{\left(B_w + z\cdot y\right)\cdot y}{B_w + 2\cdot y\cdot\sqrt{1 + z^2}} = \frac{(2.4 + y)\cdot y}{2.4 + 2\cdot y\cdot\sqrt{2}}$

Hence $$Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}} = \frac{1}{0.020}\cdot(2.4 + y)\cdot y\cdot\left[\frac{(2.4 + y)\cdot y}{2.4 + 2\cdot y\cdot\sqrt{2}}\right]^{\frac{2}{3}}\cdot 0.00193^{\frac{1}{2}} = 15$$ (Note that we don't use units!)

Solving for y $$\frac{[(2.4 + y)\cdot y]^{\frac{5}{3}}}{\left(2.4 + 2\cdot y\cdot\sqrt{2}\right)^{\frac{2}{3}}} = 6.83$$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with a larger depth than Problem 11.9's.

For $y = 1.5$ (m) $\quad \frac{[(2.4 + y)\cdot y]^{\frac{5}{3}}}{\left(2.4 + 2\cdot y\cdot\sqrt{2}\right)^{\frac{2}{3}}} = 5.37$ $\qquad$ For $y = 1.75$ (m) $\quad \frac{[(2.4 + y)\cdot y]^{\frac{5}{3}}}{\left(2.4 + 2\cdot y\cdot\sqrt{2}\right)^{\frac{2}{3}}} = 7.2$

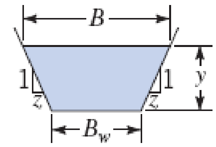
For $y = 1.71$ (m) $\quad \frac{[(2.4 + y)\cdot y]^{\frac{5}{3}}}{\left(2.4 + 2\cdot y\cdot\sqrt{2}\right)^{\frac{2}{3}}} = 6.89$ $\qquad$ For $y = 1.70$ (m) $\quad \frac{[(2.4 + y)\cdot y]^{\frac{5}{3}}}{\left(2.4 + 2\cdot y\cdot\sqrt{2}\right)^{\frac{2}{3}}} = 6.82$

The solution to three figures is $y = 1.70$ (m)

Problem 11.15

[3]

11.15 The channel of Problem 11.9 has 0.00193 bed slope. Find the normal depth for the given discharge after a new plastic liner ($n = 0.010$) is installed.



Given: Data on trapzoidal channel

Find: New depth of flow

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 2.4 \cdot m$ $\quad z = 1$ $\quad Q = 7.1 \cdot \frac{m^3}{s}$ $\quad S_0 = 0.00193$

For bare soil (Table 11.1) $n = 0.010$

Hence from Table 11.2 $A = (B_w + z \cdot y) \cdot y = (2.4 + y) \cdot y$ $\quad R = \frac{(B_w + z \cdot y) \cdot y}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}} = \frac{(2.4 + y) \cdot y}{2.4 + 2 \cdot y \cdot \sqrt{2}}$

Hence $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1}{0.010} \cdot (2.4 + y) \cdot y \cdot \left[\frac{(2.4 + y) \cdot y}{2.4 + 2 \cdot y \cdot \sqrt{2}}\right]^{\frac{2}{3}} \cdot 0.00193^{\frac{1}{2}} = 7.1$ (Note that we don't use units!)

Solving for y $\quad \frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{(2.4 + 2 \cdot y \cdot \sqrt{2})^{\frac{2}{3}}} = 1.62$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with a shallower deptl than that of Problem 11.9.

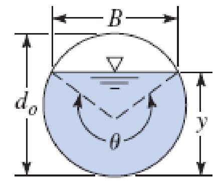
For $y = 1$ (m) $\quad \frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{(2.4 + 2 \cdot y \cdot \sqrt{2})^{\frac{2}{3}}} = 2.55$ $\quad$ For $y = 0.75$ (m) $\quad \frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{(2.4 + 2 \cdot y \cdot \sqrt{2})^{\frac{2}{3}}} = 1.53$

For $y = 0.77$ (m) $\quad \frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{(2.4 + 2 \cdot y \cdot \sqrt{2})^{\frac{2}{3}}} = 1.60$ $\quad$ For $y = 0.775$ (m) $\quad \frac{[(2.4 + y) \cdot y]^{\frac{5}{3}}}{(2.4 + 2 \cdot y \cdot \sqrt{2})^{\frac{2}{3}}} = 1.62$

The solution to three figures is $y = 0.775$ (m)

Problem 11.16

11.16 Consider again the semicircular channel of Problem 11.11. Find the normal depth that corresponds to a discharge of 0.3 m^3/sec.



Given:
Data on semicircular trough

Find:
New depth of flow

Solution:

Basic equation: $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the semicircular channel $d_0 = 1 \cdot m$ $S_0 = 0.01$ $Q = 0.3 \cdot \frac{m^3}{s}$

For corrugated steel, a Google search leads to (Table 11.1) $n = 0.022$

From Table 11.2 $A = \frac{1}{8} \cdot (\theta - \sin(\theta)) \cdot d_0^2 = \frac{1}{8} \cdot (\theta - \sin(\theta))$

$$R = \frac{1}{4} \cdot \left(1 - \frac{\sin(\theta)}{\theta}\right) \cdot d_0 = \frac{1}{4} \cdot \left(1 - \frac{\sin(\theta)}{\theta}\right)$$

Hence $Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1}{0.022} \cdot \left[\frac{1}{8} \cdot (\theta - \sin(\theta))\right] \cdot \left[\frac{1}{4} \cdot \left(1 - \frac{\sin(\theta)}{\theta}\right)\right]^{\frac{2}{3}} \cdot 0.01^{\frac{1}{2}} = 0.3$ (Note that we don't use units!)

Solving for θ $\theta^{-\frac{2}{3}} \cdot (\theta - \sin(\theta))^{\frac{5}{3}} = 1.33$

This is a nonlinear implicit equation for θ and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with a half-full channe

For $\theta = 180 \cdot deg$ $\theta^{-\frac{2}{3}} \cdot (\theta - \sin(\theta))^{\frac{5}{3}} = 3.14$ For $\theta = 140 \cdot deg$ $\theta^{-\frac{2}{3}} \cdot (\theta - \sin(\theta))^{\frac{5}{3}} = 1.47$

For $\theta = 135 \cdot deg$ $\theta^{-\frac{2}{3}} \cdot (\theta - \sin(\theta))^{\frac{5}{3}} = 1.30$ For $\theta = 136 \cdot deg$ $\theta^{-\frac{2}{3}} \cdot (\theta - \sin(\theta))^{\frac{5}{3}} = 1.33$

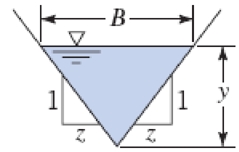
The solution to three figures is $\theta = 136 \cdot deg$

From geometry $y = \frac{d_0}{2} \cdot \left(1 - \cos\left(\frac{\theta}{2}\right)\right)$ $y = 0.313\,m$

Problem 11.17

[2]

11.17 Consider a symmetric open channel of triangular cross section. Show that for a given flow area, the wetted perimeter is minimized when the sides meet at a right angle.



Given: Triangular channel

Find: Proof that wetted perimeter is minimized when sides meet at right angles

Solution:

From Table 11.2 $A = z \cdot y^2$ $P = 2 \cdot y \cdot \sqrt{1 + z^2}$

We need to vary z to minimize P while keeping A constant, which means tha $y = \sqrt{\frac{A}{z}}$ with A = constant

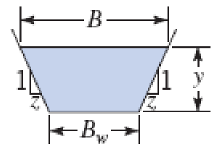
Hence we eliminate y in the expression for P $P = 2 \cdot \sqrt{\frac{A}{z}} \cdot \sqrt{1 + z^2} = 2 \cdot \sqrt{\frac{A \cdot \left(1 + z^2\right)}{z}}$

For optimizing P $\frac{dP}{dz} = \frac{z^2 - 1}{z} \cdot \sqrt{\frac{A}{z \cdot \left(z^2 + 1\right)}} = 0$ or $z = 1$

For z = 1 we find from the figure that we have the case where the sides are inclined at 45º, so meet at 90º. Note that we have only proved that this is a minimum OR maximum of P! It makes sense that it's the minimum, as, for constant A, we get a huge P if we set z to a large number (almost vertical walls); taking the second derivative at z = 1 results in a value of $\sqrt{2 \cdot A}$, which is positive, so we DO have a minimum.

Problem 11.18

11.18 A trapezoidal channel with a bottom width of 20 ft, side slopes of 1 to 2, channel bottom slope of 0.0016, and a Manning's n of 0.025 carries a discharge of 400 cfs. Compute the critical depth and velocity of this channel.



Given:

Data on trapezoidal channel

Find:

Critical depth and velocity

Solution:

Basic equation: $E = y + \frac{V^2}{2\cdot g}$

The given data is: $B_w = 20\cdot ft$ $\quad z = \frac{1}{2}$ $\quad S_0 = 0.0016$ $\quad n = 0.025$ $\quad Q = 400\cdot\frac{ft^3}{s}$

In terms of flow rate $E = y + \frac{Q^2}{2\cdot A^2\cdot g}$ where (Table 11.2) $A = (B_w + z\cdot y)\cdot y$

Hence in terms of y $E = y + \frac{Q^2}{2\cdot(B_w + z\cdot y)^2\cdot y^2\cdot g}$

For critical conditions $\frac{dE}{dy} = 0 = 1 - \frac{Q^2\cdot z}{g\cdot y^2\cdot(B_w + y\cdot z)^3} - \frac{Q^2}{g\cdot y^3\cdot(B_w + y\cdot z)^2} = 1 - \frac{B_w\cdot Q^2}{g\cdot y^3\cdot(B_w + y\cdot z)^3}$

Hence $g\cdot y^3\cdot(B_w + y\cdot z)^3 = B_w\cdot Q^2$

The only unknown on the right is y. The right side evaluates to $B_w\cdot Q^2 = 3.20\times10^6\frac{ft^7}{s^2}$

To find y we need to solve the non-linear equation. We must do this numerically; we may use the Newton method or similar, or *Excel*'s *Solver* or *Goal Seek*. Here we interate manually, starting with an arbitrary value

For $y = 1\cdot ft$ $\quad g\cdot y^3\cdot(B_w + y\cdot z)^3 = 2.77\times10^5\frac{ft^7}{s^2}$ $\qquad$ For $y = 2\cdot ft$ $\quad g\cdot y^3\cdot(B_w + y\cdot z)^3 = 2.38\times10^6\frac{ft^7}{s^2}$

For $y = 2.5\cdot ft$ $\quad g\cdot y^3\cdot(B_w + y\cdot z)^3 = 4.82\times10^6\frac{ft^7}{s^2}$ $\qquad$ For $y = 2.2\cdot ft$ $\quad g\cdot y^3\cdot(B_w + y\cdot z)^3 = 3.22\times10^6\frac{ft^7}{s^2}$

For $y = 2.19\cdot ft$ $\quad g\cdot y^3\cdot(B_w + y\cdot z)^3 = 3.17\times10^6\frac{ft^7}{s^2}$ $\qquad$ For $y = 2.20\cdot ft$ $\quad g\cdot y^3\cdot(B_w + y\cdot z)^3 = 3.22\times10^6\frac{ft^7}{s^2}$

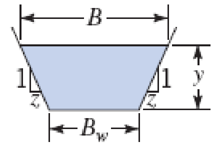
Hence the critical depth is $y = 2.20\,ft$

Also $A = (B_w + z\cdot y)\cdot y$ $\quad A = 46.4\,ft^2$ so critical speed is $V = \frac{Q}{A}$ $\quad V = 8.62\frac{ft}{s}$

Problem 11.19

[3]

11.19 Compute the normal depth and velocity of the channel of Problem 11.18.



Given: Data on trapezoidal channel

Find: Normal depth and velocity

Solution:

Basic equation: $Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 20 \cdot ft$ $\quad z = \frac{1}{2}$ $\quad Q = 400 \cdot \frac{ft^3}{s}$ $\quad S_0 = 0.0016$ $\quad n = 0.025$

Hence from Table 11.2 $\quad A = (B_w + z \cdot y) \cdot y = \left(20 + \frac{1}{2} \cdot y\right) \cdot y$ $\quad R = \frac{(B_w + z \cdot y) \cdot y}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}} = \frac{\left(20 + \frac{1}{2} \cdot y\right) \cdot y}{20 + y \cdot \sqrt{5}}$

Hence $\quad Q = \frac{1}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1}{0.025} \cdot \left(20 + \frac{1}{2} \cdot y\right) \cdot y \cdot \left[\frac{\left(20 + \frac{1}{2} \cdot y\right) \cdot y}{20 + y \cdot \sqrt{5}}\right]^{\frac{2}{3}} \cdot 0.0016^{\frac{1}{2}} = 400$ (Note that we don't use units!)

Solving for y $\quad \frac{\left[\left(20 + \frac{1}{2} \cdot y\right) \cdot y\right]^{\frac{5}{3}}}{\left(20 + y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 250$

This is a nonlinear implicit equation for y and must be solved numerically. We can u one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with an arbitrary depth

For $\quad y = 5$ (ft) $\quad \frac{\left[\left(20 + \frac{1}{2} \cdot y\right) \cdot y\right]^{\frac{5}{3}}}{\left(20 + y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 265$

For $\quad y = 4.9$ (ft) $\quad \frac{\left[\left(20 + \frac{1}{2} \cdot y\right) \cdot y\right]^{\frac{5}{3}}}{\left(20 + y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 256$

For $\quad y = 4.85$ (ft) $\quad \frac{\left[\left(20 + \frac{1}{2} \cdot y\right) \cdot y\right]^{\frac{5}{3}}}{\left(20 + y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 252$

For $\quad y = 4.83$ (ft) $\quad \frac{\left[\left(20 + \frac{1}{2} \cdot y\right) \cdot y\right]^{\frac{5}{3}}}{\left(20 + y \cdot \sqrt{5}\right)^{\frac{2}{3}}} = 250$

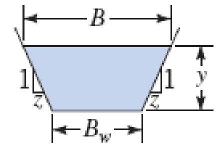
The solution to three figures is $y = 4.83 \cdot ft$ $\quad$ Then $\quad A = (B_w + z \cdot y) \cdot y$ $\quad A = 108 \cdot ft^2$

Finally, the normal velocity is $V = \frac{Q}{A}$ $\quad V = 3.69 \cdot \frac{ft}{s}$

Problem 11.20

[3]

11.20 Derive an expression for the hydraulic radius of a trapezoidal channel with bottom width B_w, liquid depth y, and side slope angle θ. Verify the equation given in Table 11.2. Plot the ratio R/y for $B_w = 2$ m with side slope angles of 30° and 60° for $0.5 < y < 3$ m.



Given: Trapezoidal channel

Find: Derive expression for hydraulic radius; Plot R/y versus y for two different side slopes

Solution:

The area is (from simple geometry or Table 11.2) $$A = B_w \cdot y + 2 \cdot \frac{1}{2} \cdot y \cdot y \cdot z = (B_w + z \cdot y) \cdot y$$

The wetted perimeter is (from simple geometry or Table 11.2) $$P = B_w + 2 \cdot y \cdot \sqrt{1 + z^2}$$

Hence the hydraulic radius is $$R = \frac{A}{P} = \frac{(B_w + z \cdot y) \cdot y}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}}$$ which is the same as that listed in Table 11.2

We are to plot $$\frac{R}{y} = \frac{(B_w + z \cdot y)}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}}$$ with $B_w = 2 \cdot m$ for θ = 30º and 60º, and 0.5 < y < 3 m.

Note: For θ = 30º $$z = \frac{1}{\tan(30 \cdot deg)}$$ $z = 1.73$

Note: For θ = 60º $$z = \frac{1}{\tan(60 \cdot deg)}$$ $z = 0.577$

The graph is plotted in the associated *Excel* workbook

Problem 11.20

11.20 Derive an expression for the hydraulic radius of a trapezoidal channel with bottom width B_w, liquid depth y, and side slope angle θ. Verify the equation given in Table 11.2. Plot the ratio R/y for $B_w = 2$ m with side slope angles of 30° and 60° for $0.5 < y < 3$ m.

Given: Trapezoidal channel

Find: Derive expression for hydraulic radius; Plot R/y versus y for two different side slopes

Solution:

Given data: $B_w = 2$ m

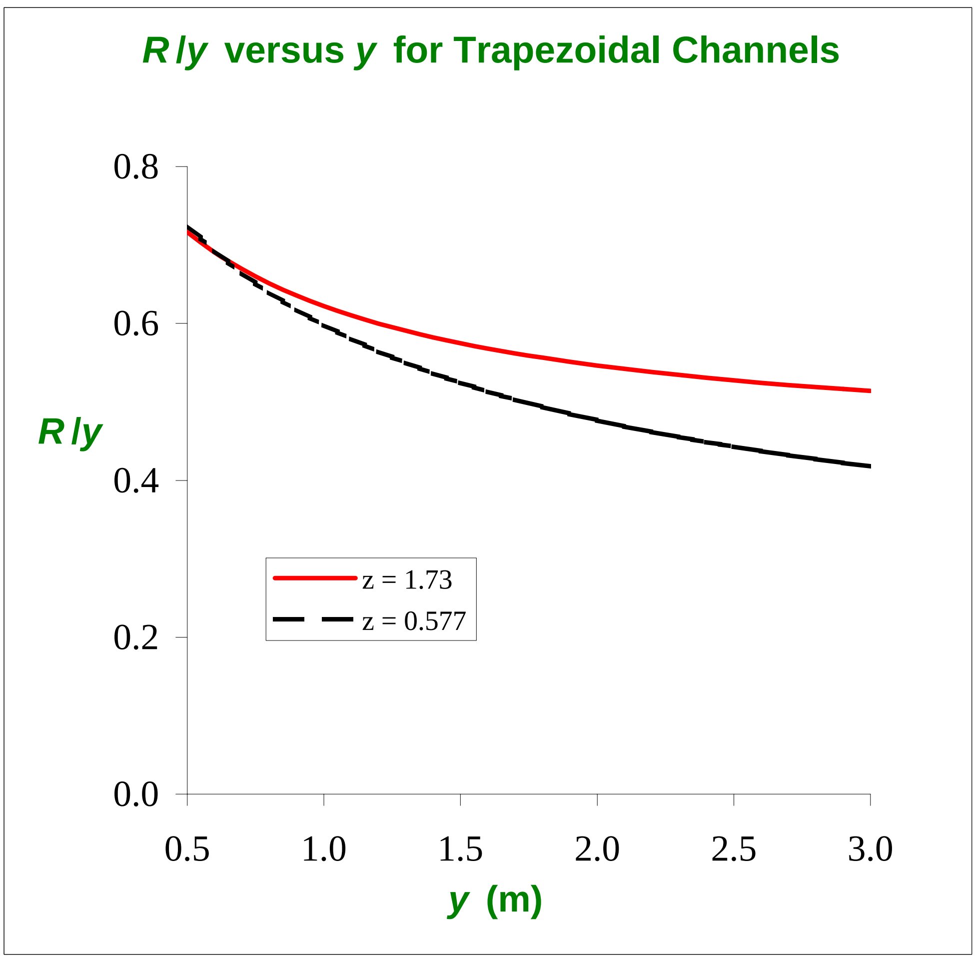
We are to plot $\frac{R}{y} = \frac{(B_w + z \cdot y)}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}}$ with $B_w = 2 \cdot m$ for θ = 30° and 60°, and $0.5 < y < 3$ m.

Note: For θ = 30° $z = \frac{1}{\tan(30 \cdot deg)}$ $z = 1.73$

Note: For θ = 60° $z = \frac{1}{\tan(60 \cdot deg)}$ $z = 0.577$

Computed results:

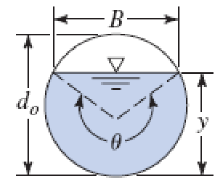
	θ = 30°	θ = 60°
	z = 1.73	*z* = 0.577
y (m)	*R*/*y*	*R*/*y*
0.5	0.717	0.725
0.6	0.691	0.693
0.7	0.669	0.665
0.8	0.651	0.640
0.9	0.636	0.618
1.0	0.622	0.598
1.1	0.610	0.580
1.2	0.600	0.564
1.3	0.591	0.550
1.4	0.582	0.537
1.5	0.575	0.524
1.6	0.568	0.513
1.7	0.562	0.503
1.8	0.556	0.494
1.9	0.551	0.485
2.0	0.546	0.477
2.1	0.542	0.469
2.2	0.538	0.462
2.3	0.534	0.455
2.4	0.531	0.449
2.5	0.527	0.443
2.6	0.524	0.437
2.7	0.522	0.432
2.8	0.519	0.427
2.9	0.516	0.422
3.0	0.514	0.418



Problem 11.21

[3]

11.21 Verify the equation given in Table 11.2 for the hydraulic radius of a circular channel. Evaluate and plot the ratio R/d_o, for liquid depths between 0 and d_o.



Given: Circular channel

Find: Derive expression for hydraulic radius; Plot R/d_0 versus d_0 for a range of depths

Solution:

The area is (from simple geometry or Table 11.2)

$$A = \frac{d_0^2}{8}\cdot\theta + 2\cdot\frac{1}{2}\cdot\frac{d_0}{2}\cdot\sin\left(\pi - \frac{\theta}{2}\right)\cdot\frac{d_0}{2}\cdot\cos\left(\pi - \frac{\theta}{2}\right) = \frac{d_0^2}{8}\cdot\theta + \frac{d_0^2}{4}\cdot\sin\left(\pi - \frac{\theta}{2}\right)\cdot\cos\left(\pi - \frac{\theta}{2}\right)$$

$$A = \frac{d_0^2}{8}\cdot\theta + \frac{d_0^2}{8}\cdot\sin(2\cdot\pi - \theta) = \frac{d_0^2}{8}\cdot\theta - \frac{d_0^2}{8}\cdot\sin(\theta) = \frac{d_0^2}{8}\cdot(\theta - \sin(\theta))$$

The wetted perimeter is (from simple geometry or Table 11.2) $\quad P = \frac{d_0}{2}\cdot\theta$

Hence the hydraulic radius is $\quad R = \frac{A}{P} = \frac{\frac{d_0^2}{8}\cdot(\theta - \sin(\theta))}{\frac{d_0}{2}\cdot\theta} = \frac{1}{4}\cdot\left(1 - \frac{\sin(\theta)}{\theta}\right)\cdot d_0 \quad$ which is the same as that listed in Table 11.2

We are to plot $\quad \frac{R}{d_0} = \frac{1}{4}\cdot\left(1 - \frac{\sin(\theta)}{\theta}\right)$

We will need y as a function of θ: $\quad y = \frac{d_0}{2} + \frac{d_0}{2}\cdot\cos\left(\pi - \frac{\theta}{2}\right) = \frac{d_0}{2}\cdot\left(1 - \cos\left(\frac{\theta}{2}\right)\right) \quad$ or $\quad \frac{y}{d_0} = \frac{1}{2}\cdot\left(1 - \cos\left(\frac{\theta}{2}\right)\right)$

The graph is plotted in the associated *Excel* workbook

Problem 11.21

11.21 Verify the equation given in Table 11.2 for the hydraulic radius of a circular channel. Evaluate and plot the ratio R/d_o, for liquid depths between 0 and d_o.

Given: Circular channel

Find: Derive expression for hydraulic radius; Plot R/d_0 versus d_0

Solution:

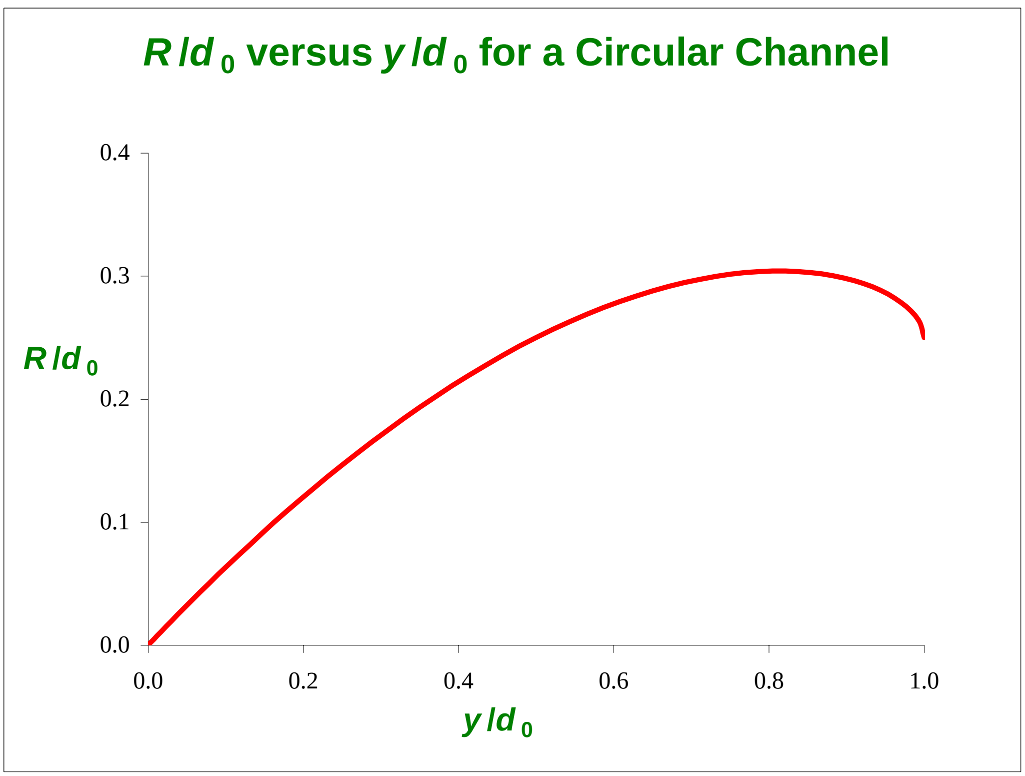
Given data

The hydraulic radius is $$R = \frac{1}{4}\cdot\left(1 - \frac{\sin(\theta)}{\theta}\right)\cdot d_0$$

We are to plot $$\frac{R}{d_0} = \frac{1}{4}\cdot\left(1 - \frac{\sin(\theta)}{\theta}\right)$$

We will need y as a function of θ: $$y = \frac{d_0}{2} + \frac{d_0}{2}\cdot\cos\left(\pi - \frac{\theta}{2}\right) = \frac{d_0}{2}\cdot\left(1 - \cos\left(\frac{\theta}{2}\right)\right)$$ or $$\frac{y}{d_0} = \frac{1}{2}\cdot\left(1 - \cos\left(\frac{\theta}{2}\right)\right)$$

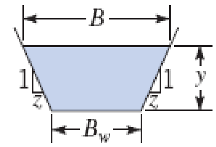
θ (°)	y/d_0	R/d_0
0	0.000	0.000
20	0.008	0.005
40	0.030	0.020
60	0.067	0.043
80	0.117	0.074
100	0.179	0.109
120	0.250	0.147
140	0.329	0.184
160	0.413	0.219
180	0.500	0.250
200	0.587	0.274
220	0.671	0.292
240	0.750	0.302
260	0.821	0.304
280	0.883	0.300
300	0.933	0.291
320	0.970	0.279
340	0.992	0.264
360	1.000	0.250



Problem 11.22

[5]

11.22 Determine the cross-section of the greatest hydraulic efficiency for a trapezoidal channel with side slope of 1 vertical to 2 horizontal if the design discharge is 10 m^3/s. The channel slope is 0.001 and Manning's roughness factor is 0.020.



Given: Data on trapezoidal channel

Find: Geometry for greatest hydraulic efficiency

Solution:

Basic equation: $$Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $z = 2$ $\quad Q = 10\cdot\frac{m^3}{s}$ $\quad S_0 = 0.001$ $\quad n = 0.020$

From Table 11.2 $\quad A = (B_w + z\cdot y)\cdot y \quad P = B_w + 2\cdot y\cdot\sqrt{1+z^2}$

We need to vary B_w and y to obtain optimum conditions. These are when the area and perimeter are optimized. Instead of two independent variables B_w and y, we eliminate B_w by doing the following

$$B_w = \frac{A}{y} - z\cdot y \quad \text{and so} \quad P = \frac{A}{y} - z\cdot y + 2\cdot y\cdot\sqrt{1+z^2}$$

Taking the derivative w.r.t. y $$\frac{\partial}{\partial y}P = \frac{1}{y}\cdot\frac{\partial}{\partial y}A - \frac{A}{y^2} - z + 2\cdot\sqrt{1+z^2}$$

But at optimum conditions $$\frac{\partial}{\partial y}P = 0 \quad \text{and} \quad \frac{\partial}{\partial y}A = 0$$

Hence $$0 = -\frac{A}{y^2} - z + 2\cdot\sqrt{1+z^2} \quad \text{or} \quad A = 2\cdot y^2\cdot\sqrt{1+z^2} - z\cdot y^2$$

Comparing to $$A = (B_w + z\cdot y)\cdot y \quad \text{we find} \quad A = (B_w + z\cdot y)\cdot y = 2\cdot y^2\cdot\sqrt{1+z^2} - z\cdot y^2$$

Hence $$B_w = 2\cdot y\cdot\sqrt{1+z^2} - 2\cdot z\cdot y$$

Then $$A = (B_w + z\cdot y)\cdot y = y^2\cdot\left(2\cdot\sqrt{1+z^2} - z\right)$$

$$P = B_w + 2\cdot y\cdot\sqrt{1+z^2} = 4\cdot y\cdot\sqrt{1+z^2} - 2\cdot z\cdot y$$

and $$R = \frac{A}{P} = \frac{y^2\cdot\left(2\cdot\sqrt{1+z^2}-z\right)}{4\cdot y\cdot\sqrt{1+z^2}-2\cdot z\cdot y} = \frac{\left(2\cdot\sqrt{1+z^2}-z\right)}{4\cdot\sqrt{1+z^2}-2\cdot z}\cdot y$$

Hence $$Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}} = \frac{1}{n}\cdot\left[y^2\cdot\left(2\cdot\sqrt{1+z^2}-z\right)\right]\cdot\left[\frac{\left(2\cdot\sqrt{1+z^2}-z\right)}{4\cdot\sqrt{1+z^2}-2\cdot z}\cdot y\right]^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$$

$$Q = \frac{\left(2\cdot\sqrt{1+z^2}-z\right)^{\frac{5}{3}}\cdot S_0^{\frac{1}{2}}}{n\cdot\left(4\cdot\sqrt{1+z^2}-2\cdot z\right)^{\frac{2}{3}}}\cdot y^{\frac{8}{3}}$$

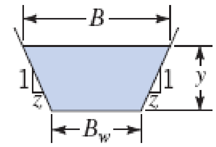
Solving for y $$y = \left[\frac{n\cdot\left(4\cdot\sqrt{1+z^2}-2\cdot z\right)^{\frac{2}{3}}}{\left(2\cdot\sqrt{1+z^2}-z\right)^{\frac{5}{3}}\cdot S_0^{\frac{1}{2}}}\cdot Q\right]^{\frac{3}{8}}$$ $y = 1.69$ (m)

Finally $$B_w = 2\cdot y\cdot\sqrt{1+z^2}-2\cdot z\cdot y$$ $B_w = 0.799$ (m)

Problem 11.23

[3]

11.23 For a trapezoidal shaped channel (n = 0.014 and slope S_o = 0.0002 with a 20-ft bottom width and side slopes of 1 vertical to 1.5 horizontal), determine the normal depth for a discharge of 1000 cfs.



Given: Data on trapezoidal channel

Find: Normal depth

Solution:

Basic equation: $$Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$$

Note that this is an "engineering" equation, to be used without units!

For the trapezoidal channel we have $B_w = 20 \cdot ft$ $\quad z = 1.5 \quad Q = 1000 \cdot \frac{ft^3}{s} \quad S_0 = 0.0002$

$n = 0.014$

Hence from Table 11.2 $\quad A = (B_w + z \cdot y) \cdot y = (20 + 1.5 \cdot y) \cdot y \qquad R = \frac{(B_w + z \cdot y) \cdot y}{B_w + 2 \cdot y \cdot \sqrt{1 + z^2}} = \frac{(20 + 1.5 \cdot y) \cdot y}{20 + 2 \cdot y \cdot \sqrt{3.25}}$

Hence $$Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} = \frac{1.49}{0.014} \cdot (20 + 1.5 \cdot y) \cdot y \cdot \left[\frac{(20 + 1.5 \cdot y) \cdot y}{20 + 2 \cdot y \cdot \sqrt{3.25}}\right]^{\frac{2}{3}} \cdot 0.0002^{\frac{1}{2}} = 1000$$ (Note that we don't use units!)

Solving for y $$\frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{(20 + 2 \cdot y \cdot \sqrt{3.25})^{\frac{2}{3}}} = 664$$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

For $y = 7.5$ (ft) $$\frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{(20 + 2 \cdot y \cdot \sqrt{3.25})^{\frac{2}{3}}} = 684$$ For $y = 7.4$ (ft) $$\frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{(20 + 2 \cdot y \cdot \sqrt{3.25})^{\frac{2}{3}}} = 667$$

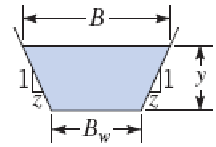
For $y = 7.35$ (ft) $$\frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{(20 + 2 \cdot y \cdot \sqrt{3.25})^{\frac{2}{3}}} = 658$$ For $y = 7.38$ (ft) $$\frac{[(20 + 1.5 \cdot y) \cdot y]^{\frac{5}{3}}}{(20 + 2 \cdot y \cdot \sqrt{3.25})^{\frac{2}{3}}} = 663$$

The solution to three figures is $y = 7.38$ (ft)

Problem 11.24

[5]

11.24 Show that the best hydraulic trapezoidal section is one-half of a hexagon.



Given: Trapezoidal channel

Find: Geometry for greatest hydraulic efficiency

Solution:

From Table 11.2 $\quad A = (B_w + z\cdot y)\cdot y \qquad P = B_w + 2\cdot y\cdot\sqrt{1+z^2}$

We need to vary B_w and y (and then z!) to obtain optimum conditions. These are when the area and perimeter are optimized. Instead of two independent variables B_w and y, we eliminate B_w by doing the following

$$B_w = \frac{A}{y} - z\cdot y \qquad \text{and so} \qquad P = \frac{A}{y} - z\cdot y + 2\cdot y\cdot\sqrt{1+z^2}$$

Taking the derivative w.r.t. y $\quad \frac{\partial}{\partial y}P = \frac{1}{y}\cdot\frac{\partial}{\partial y}A - \frac{A}{y^2} - z + 2\cdot\sqrt{1+z^2}$

But at optimum conditions $\quad \frac{\partial}{\partial y}P = 0 \qquad \text{and} \qquad \frac{\partial}{\partial y}A = 0$

Hence $\quad 0 = -\frac{A}{y^2} - z + 2\cdot\sqrt{1+z^2} \qquad \text{or} \qquad A = 2\cdot y^2\cdot\sqrt{1+z^2} - z\cdot y^2 \qquad (1)$

Now we optimize A w.r.t. z $\quad \frac{\partial}{\partial z}A = \frac{2\cdot y^2\cdot z}{\sqrt{z^2+1}} - y^2 = 0 \qquad \text{or} \qquad 2\cdot z = \sqrt{z^2+1}$

Hence $\quad 4\cdot z^2 = z^2 + 1 \qquad \text{or} \qquad z = \frac{1}{\sqrt{3}}$

We can now evaluate A from Eq 1 $\quad A = 2\cdot y^2\cdot\sqrt{1+z^2} - z\cdot y^2 = 2\cdot y^2\cdot\sqrt{1+\frac{1}{3}} - \frac{1}{3}\cdot y^2 = \left(\frac{4}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)\cdot y^2 = \sqrt{3}\cdot y^2$

But for a trapezoid $\quad A = (B_w + z\cdot y)\cdot y = \left(B_w + \frac{1}{\sqrt{3}}\cdot y\right)\cdot y$

Comparing the two A expressions $\quad A = \left(B_w + \frac{1}{\sqrt{3}}\cdot y\right)\cdot y = \sqrt{3}\cdot y^2 \qquad \text{we find} \qquad B_w = \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)\cdot y = \frac{2}{\sqrt{3}}\cdot y$

But the perimeter is $\quad P = B_w + 2\cdot y\cdot\sqrt{1+z^2} = B_w + 2\cdot y\cdot\sqrt{1+\frac{1}{3}} = B_w + \frac{4}{\sqrt{3}}\cdot y = B_w + 2\cdot B_w = 3\cdot B_w$

In summary we have $\quad z = \frac{1}{\sqrt{3}} \qquad \theta = \operatorname{atan}\left(\frac{1}{z}\right) \qquad \theta = 60\,\text{deg}$ $\quad$ where θ is the angle the sides make with the vertical

and $\quad B_w = \frac{1}{3}\cdot P \quad$ so each of the symmetric sides is $\quad \frac{P - \frac{1}{3}\cdot P}{2} = \frac{1}{3}\cdot P$

We have proved that the optimum shape is equal side and bottom lengths, with 60 angles i.e., half a hexagon!

Problem 11.25

[3]

11.25 Solve Example 11.4 for discharges of 0, 25, 75, 125, and 200 ft^3/s.

Given:

Rectangular channel

Find:

Plot of specific energy curves; Critical depths; Critical specific energy

Solution:

Given data: $B =$ 20 ft

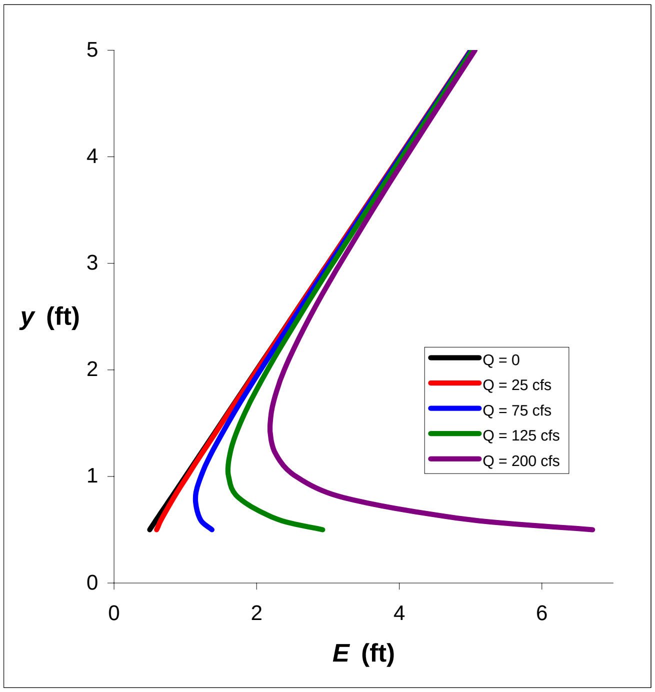
Specific energy: $$E = y + \left(\frac{Q^2}{2gB^2}\right)\frac{1}{y^2}$$

Critical depth: $$y_c = \left(\frac{Q^2}{gB^2}\right)^{\frac{1}{3}}$$

Specific Energy, *E* (ft·lb/lb)

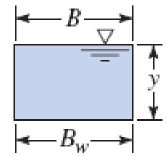
y **(ft)**	***Q*** **= 0**	***Q*** **= 25**	***Q*** **= 75**	***Q*** **= 125**	***Q*** **= 200**
0.5	0.50	0.60	1.37	2.93	6.71
0.6	0.60	0.67	1.21	2.28	4.91
0.8	0.80	0.84	1.14	1.75	3.23
1.0	1.00	1.02	1.22	1.61	2.55
1.2	1.20	1.22	1.35	1.62	2.28
1.4	1.40	1.41	1.51	1.71	2.19
1.6	1.60	1.61	1.69	1.84	2.21
1.8	1.80	1.81	1.87	1.99	2.28
2.0	2.00	2.01	2.05	2.15	2.39
2.2	2.20	2.21	2.25	2.33	2.52
2.4	2.40	2.40	2.44	2.51	2.67
2.6	2.60	2.60	2.63	2.69	2.83
2.8	2.80	2.80	2.83	2.88	3.00
3.0	3.00	3.00	3.02	3.07	3.17
3.5	3.50	3.50	3.52	3.55	3.63
4.0	4.00	4.00	4.01	4.04	4.10
4.5	4.50	4.50	4.51	4.53	4.58
5.0	5.00	5.00	5.01	5.02	5.06

y_c (ft)	**0.365**	**0.759**	**1.067**	**1.46**
E_c (ft)	**0.547**	**1.14**	**1.60**	**2.19**



Problem 11.26

11.26 Rework Example 11.5 for a 30-cm-high hump and a side wall constriction that reduces the channel width to 1.6 m.



Given: Rectangular channel flow with hump and/or side wall restriction

Find: Whether critical flow occurs

Solution:

Basic equations: $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ $E = y + \frac{Q^2}{2 \cdot g \cdot A^2}$ $A = B_w \cdot y$ $E_{min} = \frac{3}{2} \cdot y_c$ (From Example 11.5)

Given data: $B_w = 2 \cdot m$ $y = 1 \cdot m$ $\Delta z = 30 \cdot cm$ $B = 1.6 \cdot m$ $Q = 2.4 \cdot \frac{m^3}{s}$

(a) For a hump with $\Delta z = 30\,cm$ $E = y + \frac{Q^2}{2 \cdot g \cdot B_w^2} \cdot \frac{1}{y^2}$ $E = 1.07\,m$

$$y_c = \left[\frac{\left(\frac{Q}{B_w}\right)^2}{g}\right]^{\frac{1}{3}} \qquad y_c = 0.528\,m \qquad E_{min} = \frac{3}{2} \cdot y_c \qquad E_{min} = 0.791\,m$$

$$\Delta z_{crit} = E - E_{min} \qquad \Delta z_{crit} = 0.282\,m$$

Hence we have $\Delta z = 0.3\,m > \Delta z_{crit} = 0.282\,m$ so the hump IS sufficient for critical flow

(b) For the sidewall restriction with $B = 1.6\,m$

$$y_c = \left[\frac{\left(\frac{Q}{B}\right)^2}{g}\right]^{\frac{1}{3}} \qquad y_c = 0.612\,m \qquad E_{min} = \frac{3}{2} \cdot y_c \qquad E_{min} = 0.918\,m$$

Hence we have $E = 1.073\,m > E_{min} = 0.918\,m$ so the restriction is insufficient for critical flow

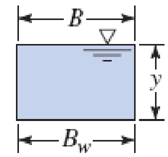
(a) For both, we can use the minimum energy from case (b) $E_{min} = 0.918\,m$

$$\Delta z_{crit} = E - E_{min} \qquad \Delta z_{crit} = 0.155\,m$$

Hence we have $\Delta z = 0.3\,m > \Delta z_{crit} = 0.155\,m$ so in this case the conditions ARE sufficient for critical flow

Problem 11.27

11.27 Compute the critical depth for the channel in Problem 11.1.



Given: Rectangular channel flow

Find: Critical depth

Solution:

Basic equations: $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ $\qquad Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}}$

For a rectangular channel of width $B_w = 2 \cdot m$ and depth $y = 1.5 \cdot m$ we find from Table 11.2

$$A = B_w \cdot y \qquad A = 3.00 \cdot m^2 \qquad R = \frac{B_w \cdot y}{B_w + 2 \cdot y} \qquad R = 0.600 \cdot m$$

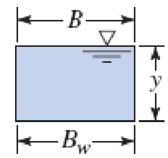
Manning's roughness coefficient is $n = 0.015$ and $S_0 = 0.0005$

$$Q = \frac{1.49}{n} \cdot A \cdot R^{\frac{2}{3}} \cdot S_0^{\frac{1}{2}} \qquad Q = 3.18 \cdot \frac{m^3}{s}$$

Hence $q = \frac{Q}{B_w}$ $\qquad q = 1.59 \frac{m^2}{s}$ $\qquad y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ $\qquad y_c = 0.637\, m$

Problem 11.28

11.28 Compute the critical depth for the channel in Problem 11.2.



Given: Rectangular channel flow

Find: Critical depth

Solution:

Basic equations: $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$

Given data: $B_w = 2.5 \cdot m$ $\quad Q = 3 \cdot \frac{m^3}{s}$

Hence $q = \frac{Q}{B_w}$ $\quad q = 1.2 \frac{m^2}{s}$ $\quad y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ $\quad y_c = 0.528\,m$

Problem 11.29

11.29 Rework Example 11.6 with discharges of 0, 25, 75, 125, and 200 cfs.

Given: Rectangular channel

Find: Plot of specific force curves

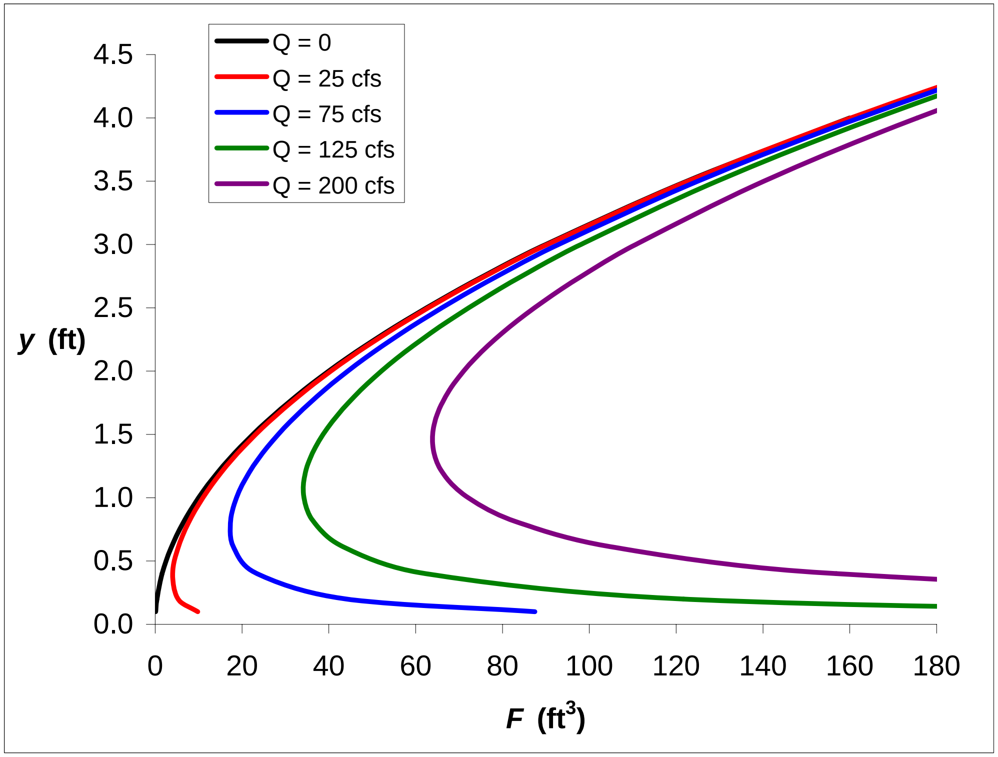
Solution:

Given data: $B =$ 20 ft

Specific force: $$F = \frac{Q^2}{gBy} + \frac{By^2}{2}$$

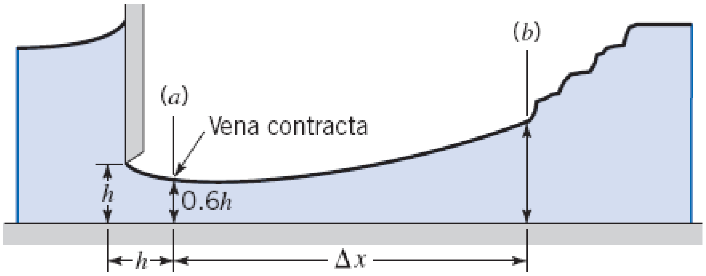
Specific Force, F (ft^3)

y (ft)	*Q* = 0	*Q* = 25	*Q* = 75	*Q* = 125	*Q* = 200
0.1	0.10	9.80	87.44	242.72	621.22
0.2	0.40	5.25	44.07	121.71	310.96
0.4	1.60	4.03	23.44	62.26	156.88
0.6	3.60	5.22	18.16	44.04	107.12
0.8	6.40	7.61	17.32	36.73	84.04
1.0	10.00	10.97	18.73	34.26	72.11
1.2	14.40	15.21	21.68	34.62	66.16
1.4	19.60	20.29	25.84	36.93	63.97
1.6	25.60	26.21	31.06	40.76	64.42
1.8	32.40	32.94	37.25	45.88	66.91
2.0	40.00	40.49	44.37	52.13	71.06
2.2	48.40	48.84	52.37	59.43	76.63
2.4	57.60	58.00	61.24	67.71	83.48
2.6	67.60	67.97	70.96	76.93	91.49
2.8	78.40	78.75	81.52	87.07	100.58
3.0	90.00	90.32	92.91	98.09	110.70
3.5	122.50	122.78	125.00	129.43	140.25
4.0	160.00	160.24	162.18	166.07	175.53
4.5	202.50	202.72	204.44	207.89	216.30
5.0	250.00	250.19	251.75	254.85	262.42



Problem 11.30

11.30 Resolve Example 11.7 for a channel bed slope of 0.003.



Flow downstream of a sluice gate in a wide rectangular channel.

Given: Vena contracta at a sluice gate

Find: Distance from vena contracta at which depth is 0.5 m

Solution:

Basic equations: $E = y + \frac{V^2}{2 \cdot g}$ $\quad R = y$ (Wide channel) $\quad S_f = \left(\frac{V_{ave} \cdot n}{R_{ave}^{\frac{2}{3}}}\right)^2$ $\quad \Delta x = \frac{E_a - E_b}{S_f - S_0}$

(Some equations from Example 11.7)

Given data: $q = 4.646 \cdot \frac{\frac{m^3}{s}}{m}$ $\quad y_a = 0.457 \cdot m$ $\quad y_b = 0.5 \cdot m$ $\quad n = 0.020$ $\quad S_0 = 0.003$

Hence we find $V_a = \frac{q}{y_a}$ $\quad V_a = 10.2 \frac{m}{s}$ $\quad V_b = \frac{q}{y_b}$ $\quad V_b = 9.29 \frac{m}{s}$

Then $E_a = y_a + \frac{V_a^2}{2 \cdot g}$ $\quad E_a = 5.73 m$ $\quad E_b = y_b + \frac{V_b^2}{2 \cdot g}$ $\quad E_b = 4.90 m$

and $V_{ave} = \frac{V_a + V_b}{2}$ $\quad V_{ave} = 9.73 \frac{m}{s}$

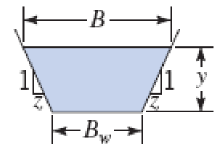
$R_a = y_a$ $\quad R_b = y_b$ $\quad R_{ave} = \frac{R_a + R_b}{2}$ $\quad R_a = 0.457 m$

Then $S_f = \left(\frac{V_{ave} \cdot n}{R_{ave}^{\frac{2}{3}}}\right)^2$ $\quad S_f = 0.101$

Finally $\Delta x = \frac{E_a - E_b}{S_f - S_0}$ $\quad \Delta x = 8.40 m$

Problem 11.31

11.31 Once again consider the trapezoidal channel in Problem 11.8 with a dam placed in the channel so that water backs up to a depth of 5 ft immediately behind the dam. How far upstream would you expect the depth to be 4.80 ft? Consider an energy correction coefficient of 1.1.



Given:
Data on trapezoidal channel and dam

Find:
Location upstream at which depth is 4.80 ft

Solution:

Basic equations:

From Example 11.7
$$\Delta x = \frac{\Delta y + \left(\frac{V_1^2}{2\cdot g} - \frac{V_2^2}{2\cdot g}\right)}{S_0 - S_f} \quad \text{and} \quad S_f = \left(\frac{n\cdot V}{1.49\cdot R^{\frac{2}{3}}}\right)^2$$
(note the factor 1.49 because this is not SI units)

The given data is: $B_w = 20\cdot ft$ $\quad z = \frac{1}{2}$ $\quad S_0 = 0.0016$ $\quad n = 0.025$ $\quad Q = 400\cdot\frac{ft^3}{s}$ $\quad y_1 = 5\cdot ft$ $\quad y_2 = 4.80\cdot ft$

We need to modify the specific energy equation to allow for the emergy correction coefficient (Section 8-6): instead of $\frac{V^2}{2\cdot g}$, the kinetic energy per unit weight is $\alpha\cdot\frac{V^2}{2\cdot g}$ where $\alpha = 1.1$

Hence
$$\Delta x = \frac{\Delta y + \alpha\cdot\left(\frac{V_1^2}{2\cdot g} - \frac{V_2^2}{2\cdot g}\right)}{S_0 - S_f}$$

We need to obtain terms on the right $\quad S_0 = 0.0016 \quad \alpha = 1.1 \quad \Delta y = y_1 - y_2 \quad \Delta y = 0.200\,ft$

We will need (Table 11.2) $\quad A = \left(B_w + z\cdot y\right)\cdot y \qquad R = \frac{\left(B_w + z\cdot y\right)\cdot y}{B_w + 2\cdot y\cdot\sqrt{1+z^2}}$

Then
$$V_1 = \frac{Q}{A_1} \qquad V_1 = \frac{Q}{\left(B_w + z\cdot y_1\right)\cdot y_1} \qquad V_1 = 3.56\,\frac{ft}{s}$$

$$V_2 = \frac{Q}{A_2} \qquad V_2 = \frac{Q}{\left(B_w + z\cdot y_2\right)\cdot y_2} \qquad V_2 = 3.72\,\frac{ft}{s}$$

For S_f we use averages for V and R (as in Example 11.7)

and
$$V_{ave} = \frac{V_1 + V}{2} \qquad V_{ave} = 1.11\,\frac{m}{s}$$

$$R_1 = \frac{\left(B_w + z\cdot y_1\right)\cdot y_1}{B_w + 2\cdot y_1\cdot\sqrt{1+z^2}} \quad R_1 = 3.61\,ft \qquad R_2 = \frac{\left(B_w + z\cdot y_2\right)\cdot y_2}{B_w + 2\cdot y_2\cdot\sqrt{1+z^2}} \quad R_2 = 3.50\,ft$$

$$R_{ave} = \frac{R_1 + R_2}{2} \qquad R_{ave} = 3.55 \cdot ft$$

Then

$$S_f = \left(\frac{V_{ave} \cdot n}{1.49 \cdot R_{ave}^{\frac{2}{3}}} \right)^2 \qquad S_f = 0.000687$$

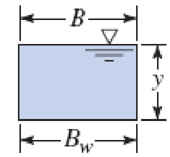
Finally

$$\Delta x = \frac{\Delta y + \alpha \cdot \left(\frac{V_1^2}{2 \cdot g} - \frac{V_2^2}{2 \cdot g} \right)}{S_0 - S_f} \qquad \Delta x = 197\,ft$$

Problem 11.32

[2]

11.32 A rectangular channel carries a discharge of 10 ft³/sec per foot of width. Determine the minimum specific energy possible for this flow. Compute the corresponding flow depth and speed.



Given: Data on rectangular channel

Find: Minimum specific energy; Flow depth; Speed

Solution:

Basic equation: $E = y + \frac{V^2}{2\cdot g}$ (11.14)

In Section 11-2 we prove that the minimum specific energy is when we have critical flow; here we rederive the minimum energy point

For a rectangular channel $Q = V\cdot B_w\cdot y$ or $V = \frac{Q}{B_w\cdot y}$ with $\frac{Q}{B_w} = 10\cdot\frac{\frac{ft^3}{s}}{ft} = \text{constant}$

Hence, using this in Eq. 11.14 $E = y + \left(\frac{Q}{B_w\cdot y}\right)^2\cdot\frac{1}{2\cdot g} = y + \left(\frac{Q^2}{2\cdot B_w^{\;2}\cdot g}\right)\cdot\frac{1}{y^2}$

E is a minimum when $\frac{dE}{dy} = 1 - \left(\frac{Q^2}{B_w^{\;2}\cdot g}\right)\cdot\frac{1}{y^3} = 0$ or $y = \left(\frac{Q^2}{B_w^{\;2}\cdot g}\right)^{\frac{1}{3}}$ $y = 1.46\cdot ft$

Note that from Eq. 11.22 we have $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{Q^2}{B_w^{\;2}\cdot g}\right)^{\frac{1}{3}}$ which is the same result we derived

The speed is then given by $V = \frac{Q}{B_w\cdot y}$ $V = 6.85\cdot\frac{ft}{s}$

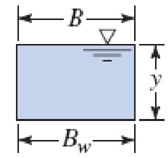
Note that from Eq. 11.20 we also have $V = \sqrt{g\cdot D}$ where D is the hydraulic depth $D = y$ $V = \sqrt{g\cdot D}$ $V = 6.85\cdot\frac{ft}{s}$

The minimum energy is then $E_{min} = y + \frac{V^2}{2\cdot g}$ $E_{min} = 2.19\cdot ft$

Problem 11.33

[3]

11.33 Flow in the channel of Problem 11.32 ($E_{min} = 2.19$ ft) is to be at twice the minimum specific energy. Compute the alternate depths for this E.



Given: Data on rectangular channel

Find: Depths for twice the minimum energy

Solution:

Basic equation: $E = y + \frac{V^2}{2 \cdot g}$ (11.14)

For a rectangular channel $Q = V \cdot B_w \cdot y$ or $V = \frac{Q}{B_w \cdot y}$ with $\frac{Q}{B_w} = 10 \cdot \frac{\frac{ft^3}{s}}{ft}$ = constant

Hence, using this in Eq. 11.14 $E = y + \left(\frac{Q}{B_w \cdot y}\right)^2 \cdot \frac{1}{2 \cdot g} = y + \left(\frac{Q^2}{2 \cdot B_w^{\ 2} \cdot g}\right) \cdot \frac{1}{y^2}$ and $E = 2 \cdot 2.19 \cdot ft$ $E = 4.38 \cdot ft$

We have a nonlinear implicit equation for y $y + \left(\frac{Q^2}{2 \cdot B_w^{\ 2} \cdot g}\right) \cdot \frac{1}{y^2} = E$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with a y larger than the critical, and evaluate the left side of the equation so that it is equal to $E = 4.38\,ft$

For $y = 2 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\ 2} \cdot g}\right) \cdot \frac{1}{y^2} = 2.39 \cdot ft$ For $y = 4 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\ 2} \cdot g}\right) \cdot \frac{1}{y^2} = 4.10 \cdot ft$

For $y = 4.5 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\ 2} \cdot g}\right) \cdot \frac{1}{y^2} = 4.58 \cdot ft$ For $y = 4.30 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\ 2} \cdot g}\right) \cdot \frac{1}{y^2} = 4.38 \cdot ft$

Hence $y = 4.3 \cdot ft$

For the shallow depth

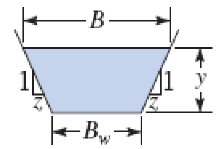
For $y = 1 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\ 2} \cdot g}\right) \cdot \frac{1}{y^2} = 2.55 \cdot ft$ For $y = 0.5 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\ 2} \cdot g}\right) \cdot \frac{1}{y^2} = 6.72 \cdot ft$

For $y = 0.6 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\ 2} \cdot g}\right) \cdot \frac{1}{y^2} = 4.92 \cdot ft$ For $y = 0.65 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\ 2} \cdot g}\right) \cdot \frac{1}{y^2} = 4.33 \cdot ft$

For $y = 0.645 \cdot ft$ $y + \left(\frac{Q^2}{2 \cdot B_w^{\ 2} \cdot g}\right) \cdot \frac{1}{y^2} = 4.38 \cdot ft$ Hence $y = 0.645 \cdot ft$

Problem 11.34

11.34 Water flows at 300 ft^3/sec in a trapezoidal channel with bottom width of 8 ft. The sides are sloped at 2:1. Find the critical depth for this channel.



Given: Data on trapezoidal channel

Find: Critical depth

Solution:

Basic equation: $E = y + \frac{V^2}{2 \cdot g}$ (11.14)

In Section 11-2 we prove that the minimum specific energy is when we have critical flow; here we rederive the minimum energy point

For a trapezoidal channel (Table 11.2) $A = (B_w + z \cdot y) \cdot y$ and $B_w = 8 \cdot ft$ $z = 0.5$

Hence for V $V = \frac{Q}{A} = \frac{Q}{(B_w + z \cdot y) \cdot y}$ and $Q = 300 \cdot \frac{ft^3}{s}$

Using this in Eq. 11.14 $E = y + \left[\frac{Q}{(B_w + z \cdot y) \cdot y}\right]^2 \cdot \frac{1}{2 \cdot g}$

E is a minimum when $\frac{dE}{dy} = 1 - \frac{Q^2 \cdot z}{g \cdot y^2 \cdot (B_w + y \cdot z)^3} - \frac{Q^2}{g \cdot y^3 \cdot (B_w + y \cdot z)^2} = 0$

Hence we obtain for y $\frac{Q^2 \cdot z}{g \cdot y^2 \cdot (B_w + y \cdot z)^3} + \frac{Q^2}{g \cdot y^3 \cdot (B_w + y \cdot z)^2} = 1$ or $\frac{Q^2 \cdot (B_w + 2 \cdot y \cdot z)}{g \cdot y^3 \cdot (B_w + y \cdot z)^3} = 1$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below, to make the left side equal unity

$y = 1 \cdot ft$ $\frac{Q^2 \cdot (B_w + 2 \cdot y \cdot z)}{g \cdot y^3 \cdot (B_w + y \cdot z)^3} = 41$

$y = 5 \cdot ft$ $\frac{Q^2 \cdot (B_w + 2 \cdot y \cdot z)}{g \cdot y^3 \cdot (B_w + y \cdot z)^3} = 0.251$

$y = 3 \cdot ft$ $\frac{Q^2 \cdot (B_w + 2 \cdot y \cdot z)}{g \cdot y^3 \cdot (B_w + y \cdot z)^3} = 1.33$

$y = 3.5 \cdot ft$ $\frac{Q^2 \cdot (B_w + 2 \cdot y \cdot z)}{g \cdot y^3 \cdot (B_w + y \cdot z)^3} = 0.809$

$y = 3.25 \cdot ft$ $\frac{Q^2 \cdot (B_w + 2 \cdot y \cdot z)}{g \cdot y^3 \cdot (B_w + y \cdot z)^3} = 1.03$

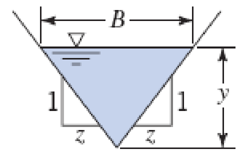
$y = 3.28 \cdot ft$ $\frac{Q^2 \cdot (B_w + 2 \cdot y \cdot z)}{g \cdot y^3 \cdot (B_w + y \cdot z)^3} = 0.998$

The critical depth is $y = 3.28 \cdot ft$

Problem 11.35

[2]

11.35 For a channel of nonrectangular cross section, critical depth occurs at minimum specific energy. Obtain a general equation for critical depth in a triangular channel in terms of Q, g, and z.



Given: Triangular channel

Find: Critcal depth

Solution:

Basic equation: $E = y + \frac{V^2}{2 \cdot g}$ (11.14)

For a triangular channel (Table 11.2) $A = z \cdot y^2$

Hence for V $V = \frac{Q}{A} = \frac{Q}{z \cdot y^2}$

Using this in Eq. 11.14 $E = y + \left(\frac{Q}{z \cdot y^2}\right)^2 \cdot \frac{1}{2 \cdot g}$

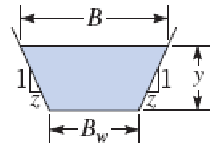
E is a minimum when $\frac{dE}{dy} = 1 - 4 \cdot \frac{Q^2}{z^2 \cdot y^5} \cdot \frac{1}{2 \cdot g} = 0$

Hence we obtain for y $y = \left(\frac{2 \cdot Q^2}{z^2 \cdot g}\right)^{\frac{1}{5}}$

Problem 11.36

[2]

11.36 For a channel of nonrectangular cross section, critical depth occurs at minimum specific energy. Obtain a general equation for critical depth in a channel of trapezoidal section in terms of Q, g, B_w, and z.



Given:

Trapezoidal channel

Find:

Critcal depth

Solution:

Basic equation: $$E = y + \frac{V^2}{2\cdot g} \qquad (11.14)$$

The critical depth occurs when the specific energy is minimized

For a trapezoidal channel (Table 11.2) $A = \left(B_w + z\cdot y\right)\cdot y$ and $B_w = 8\cdot ft$ $z = 0.5$

Hence for V $V = \frac{Q}{A} = \frac{Q}{\left(B_w + z\cdot y\right)\cdot y}$ and $Q = 300\cdot\frac{ft^3}{s}$

Using this in Eq. 11.14 $$E = y + \left[\frac{Q}{\left(B_w + z\cdot y\right)\cdot y}\right]^2\cdot\frac{1}{2\cdot g}$$

E is a minimum when $$\frac{dE}{dy} = 1 - \frac{Q^2\cdot z}{g\cdot y^2\cdot\left(B_w + y\cdot z\right)^3} - \frac{Q^2}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^2} = 0$$

Hence we obtain for y $$\frac{Q^2\cdot z}{g\cdot y^2\cdot\left(B_w + y\cdot z\right)^3} + \frac{Q^2}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^2} = 1$$

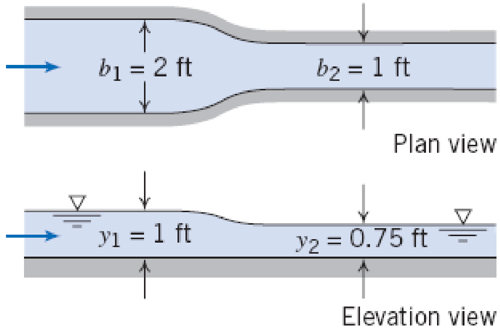
This can be simplified to $$\frac{Q^2\cdot\left(B_w + 2\cdot y\cdot z\right)}{g\cdot y^3\cdot\left(B_w + y\cdot z\right)^3} = 1$$

This expression is the simplest one for y; it is implicit

Problem 11.37

[2]

11.37 Consider the Venturi flume shown. The bed is horizontal and flow may be considered frictionless. The upstream depth is 1 ft and the downstream depth is 0.75 ft. The upstream breadth is 2 ft and the breadth of the throat is 1 ft. Estimate the flow rate through the flume.



Given: Data on venturi flume

Find: Flow rate

Solution:

Basic equation: $\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + y_2$ The Bernoulli equation applies because we have steady, incompressible, frictionless flow

At each section $Q = V \cdot A = V \cdot b \cdot y$ or $V = \frac{Q}{b \cdot y}$

The given data is $b_1 = 2 \cdot ft$ $y_1 = 1 \cdot ft$ $b_2 = 1 \cdot ft$ $y_2 = 0.75 \cdot ft$

Hence the Bernoulli equation becomes (with $p_1 = p_2 = p_{atm}$)

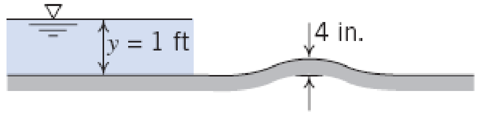
$$\frac{\left(\frac{Q}{b_1 \cdot y_1}\right)^2}{2 \cdot g} + y_1 = \frac{\left(\frac{Q}{b_2 \cdot y_2}\right)^2}{2 \cdot g} + y_2$$

Solving for Q

$$Q = \sqrt{\frac{2 \cdot g \cdot (y_1 - y_2)}{\left(\frac{1}{b_2 \cdot y_2}\right)^2 - \left(\frac{1}{b_1 \cdot y_1}\right)^2}} \qquad Q = 3.24 \cdot \frac{ft^3}{s}$$

Problem 11.38

11.38 A rectangular channel 10 ft wide carries 100 cfs on a horizontal bed at 1.0 ft depth. A smooth bump across the channel rises 4 in. above the channel bottom. Find the elevation of the liquid free surface above the bump.



Given: Data on rectangular channel and a bump

Find: Elevation of free surface above the bump

Solution:

Basic equation: $\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + y_2 + h$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow. Note that at location 2 (the bump), the potential is $y_2 + h$, where h is the bump height

Recalling the specific energy $E = \frac{V^2}{2 \cdot g} + y$ and noting that $p_1 = p_2 = p_{atm}$, the Bernoulli equation becomes $E_1 = E_2 + h$

At each section $Q = V \cdot A = V \cdot b \cdot y$ or $V = \frac{Q}{b \cdot y}$

The given data is $b = 10 \cdot ft$ $y_1 = 1 \cdot ft$ $h = 4 \cdot in$ $Q = 100 \cdot \frac{ft^3}{s}$

Hence we find $V_1 = \frac{Q}{b \cdot y_1}$ $V_1 = 10 \cdot \frac{ft}{s}$

and $E_1 = \frac{V_1^2}{2 \cdot g} + y_1$ $E_1 = 2.554 \cdot ft$

Hence $E_1 = E_2 + h = \frac{V_2^2}{2 \cdot g} + y_2 + h = \frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 + h$ or $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = E_1 - h$

This is a nonlinear implicit equation for y_2 and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We select y_2 so the left side of the equation equals $E_1 - h = 2.22 \cdot ft$

For $y_2 = 1 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 2.55 \cdot ft$ For $y_2 = 1.5 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 2.19 \cdot ft$

For $y_2 = 1.4 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 2.19 \cdot ft$ For $y_2 = 1.3 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 2.22 \cdot ft$

Hence $y_2 = 1.30 \cdot ft$

Note that $V_2 = \frac{Q}{b \cdot y_2}$ $V_2 = 7.69 \cdot \frac{ft}{s}$

so we have $Fr_1 = \frac{V_1}{\sqrt{g \cdot y_1}}$ $Fr_1 = 1.76$ and $Fr_2 = \frac{V_2}{\sqrt{g \cdot y_2}}$ $Fr_2 = 1.19$

Problem 11.39

11.39 A rectangular channel 10 ft wide carries a discharge of 20 ft^3/sec at 0.9 ft depth. A smooth bump 0.2 ft high is placed on the floor of the channel. Estimate the local change in flow depth caused by the bump.

Given: Data on rectangular channel and a bump

Find: Local change in flow depth caused by the bump

Solution:

Basic equation: $$\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + y_2 + h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow. Note that at location 2 (the bump), the potential is $y_2 + h$, where h is the bump height

Recalling the specific energy $E = \frac{V^2}{2 \cdot g} + y$ and noting that $p_1 = p_2 = p_{atm}$, the Bernoulli equation becomes $E_1 = E_2 + h$

At each section $Q = V \cdot A = V \cdot b \cdot y$ or $V = \frac{Q}{b \cdot y}$

The given data is $b = 10 \cdot ft$ $y_1 = 0.9 \cdot ft$ $h = 0.2 \cdot ft$ $Q = 20 \cdot \frac{ft^3}{s}$

Hence we find $V_1 = \frac{Q}{b \cdot y_1}$ $V_1 = 2.22 \cdot \frac{ft}{s}$

and $E_1 = \frac{V_1^2}{2 \cdot g} + y_1$ $E_1 = 0.977 \cdot ft$

Hence $E_1 = E_2 + h = \frac{V_2^2}{2 \cdot g} + y_2 + h = \frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 + h$ or $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = E_1 - h$

This is a nonlinear implicit equation for y_2 and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We select y_2 so the left side of the equation equals $E_1 - h = 0.777 \cdot ft$

For $y_2 = 0.9 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 0.977 \cdot ft$ For $y_2 = 0.5 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 0.749 \cdot ft$

For $y_2 = 0.6 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 0.773 \cdot ft$ For $y_2 = 0.61 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^2} + y_2 = 0.777 \cdot ft$

Hence $y_2 = 0.61 \cdot ft$ and $\frac{y_2 - y_1}{y_1} = -32.2\%$

Note that $V_2 = \frac{Q}{b \cdot y_2}$ $V_2 = 3.28 \cdot \frac{ft}{s}$

so we have $Fr_1 = \frac{V_1}{\sqrt{g \cdot y_1}}$ $Fr_1 = 0.41$ and $Fr_2 = \frac{V_2}{\sqrt{g \cdot y_2}}$ $Fr_2 = 0.74$

Problem 11.40

11.40 At a section of a 10-ft-wide rectangular channel, the depth is 0.3 ft for a discharge of 20 ft^3/sec. A smooth bump 0.1 ft high is placed on the floor of the channel. Determine the local change in flow depth caused by the bump.

Given: Data on rectangular channel and a bump

Find: Local change in flow depth caused by the bump

Solution:

Basic equation: $$\frac{p_1}{\rho \cdot g} + \frac{V_1^{\ 2}}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^{\ 2}}{2 \cdot g} + y_2 + h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow. Note that at location 2 (the bump), the potential is y_2 + h, where h is the bump height

Recalling the specific energy $E = \frac{V^2}{2 \cdot g} + y$ and noting that $p_1 = p_2 = p_{atm}$, the Bernoulli equation becomes $E_1 = E_2 + h$

At each section $Q = V \cdot A = V \cdot b \cdot y$ or $V = \frac{Q}{b \cdot y}$

The given data is $b = 10 \cdot ft$ $y_1 = 0.3 \cdot ft$ $h = 0.1 \cdot ft$ $Q = 20 \cdot \frac{ft^3}{s}$

Hence we find $V_1 = \frac{Q}{b \cdot y_1}$ $V_1 = 6.67 \cdot \frac{ft}{s}$

and $E_1 = \frac{V_1^{\ 2}}{2 \cdot g} + y_1$ $E_1 = 0.991 \cdot ft$

Hence $$E_1 = E_2 + h = \frac{V_2^{\ 2}}{2 \cdot g} + y_2 + h = \frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^{\ 2}} + y_2 + h$$ or $$\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^{\ 2}} + y_2 = E_1 - h$$

This is a nonlinear implicit equation for y_2 and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We select y_2 so the left side of the equation equals $E_1 - h = 0.891 \cdot ft$

For $y_2 = 0.3 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^{\ 2}} + y_2 = 0.991 \cdot ft$ For $y_2 = 0.35 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^{\ 2}} + y_2 = 0.857 \cdot ft$

For $y_2 = 0.33 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^{\ 2}} + y_2 = 0.901 \cdot ft$ For $y_2 = 0.334 \cdot ft$ $\frac{Q^2}{2 \cdot g \cdot b^2 \cdot y_2^{\ 2}} + y_2 = 0.891 \cdot ft$

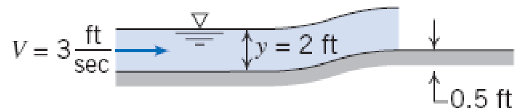
Hence $y_2 = 0.334 \cdot ft$ and $\frac{y_2 - y_1}{y_1} = 11.3\%$

Note that $V_2 = \frac{Q}{b \cdot y_2}$ $V_2 = 5.99 \cdot \frac{ft}{s}$

so we have $Fr_1 = \frac{V_1}{\sqrt{g \cdot y_1}}$ $Fr_1 = 2.15$ and $Fr_2 = \frac{V_2}{\sqrt{g \cdot y_2}}$ $Fr_2 = 1.83$

Problem 11.41

11.41 Water, at 3 ft/sec and 2 ft depth, approaches a smooth rise in a wide channel. Estimate the stream depth after the 0.5 ft rise.



Given: Data on wide channel

Find: Stream depth after rise

Solution:

Basic equation: $$\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + y_2 + h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow. Note that at location 2 (the bump), the potential is $y_2 + h$, where h is the bump height

Recalling the specific energy $E = \frac{V^2}{2 \cdot g} + y$ and noting that $p_1 = p_2 = p_{atm}$, the Bernoulli equation becomes $E_1 = E_2 + h$

At each section $Q = V \cdot A = V_1 \cdot b \cdot y_1 = V_2 \cdot b \cdot y_2$ $\qquad V_2 = V_1 \cdot \frac{y_1}{y_2}$

The given data is $y_1 = 2 \cdot ft$ $\qquad V_1 = 3 \cdot \frac{ft}{s}$ $\qquad h = 0.5 \cdot ft$

Hence $E_1 = \frac{V_1^2}{2 \cdot g} + y_1$ $\qquad E_1 = 2.14 \cdot ft$

Then $$E_1 = E_2 + h = \frac{V_2^2}{2 \cdot g} + y_2 + h = \frac{V_1^2 \cdot y_1^2}{2 \cdot g} \cdot \frac{1}{y_2^2} + y_2 + h$$ or $$\frac{V_1^2 \cdot y_1^2}{2 \cdot g} \cdot \frac{1}{y_2^2} + y_2 = E_1 - h$$

This is a nonlinear implicit equation for y_2 and must be solved numerically. We can use one of a number of numerical root finding techniques, such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We select y_2 so the left side of the equation equals $E_1 - h = 1.64 \cdot ft$

For $y_2 = 2 \cdot ft$ $\qquad \frac{V_1^2 \cdot y_1^2}{2 \cdot g} \cdot \frac{1}{y_2^2} + y_2 = 2.14 \cdot ft$ $\qquad$ For $y_2 = 1.5 \cdot ft$ $\qquad \frac{V_1^2 \cdot y_1^2}{2 \cdot g} \cdot \frac{1}{y_2^2} + y_2 = 1.75 \cdot ft$

For $y_2 = 1.3 \cdot ft$ $\qquad \frac{V_1^2 \cdot y_1^2}{2 \cdot g} \cdot \frac{1}{y_2^2} + y_2 = 1.63 \cdot ft$ $\qquad$ For $y_2 = 1.31 \cdot ft$ $\qquad \frac{V_1^2 \cdot y_1^2}{2 \cdot g} \cdot \frac{1}{y_2^2} + y_2 = 1.64 \cdot ft$

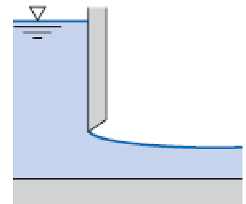
Hence $y_2 = 1.31 \cdot ft$

Note that $V_2 = V_1 \cdot \frac{y_1}{y_2}$ $\qquad V_2 = 4.58 \cdot \frac{ft}{s}$

so we have $Fr_1 = \frac{V_1}{\sqrt{g \cdot y_1}}$ $\qquad Fr_1 = 0.37$ $\qquad$ and $\qquad Fr_2 = \frac{V_2}{\sqrt{g \cdot y_2}}$ $\qquad Fr_2 = 0.71$

Problem 11.42

11.42 Water issues from a sluice gate at 0.6 m depth. The discharge per unit width is 6.0 m^3/sec/m. Estimate the water level far upstream where the flow speed is negligible. Calculate the maximum rate of flow per unit width that could be delivered through the sluice gate.



Given: Data on sluice gate

Find: Water level upstream; Maximum flow rate

Solution:

Basic equation: $$\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + y_2 + h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow.

Noting that $p_1 = p_2 = p_{atm}$, and V_1 is approximately zero (1 = upstream, 2 = downstream) the Bernoulli equation becomes

$$y_1 = \frac{V_2^2}{2 \cdot g} + y_2$$

The given data is $\frac{Q}{b} = 6.0 \frac{m^2}{s}$ $\qquad y_2 = 0.6 \cdot m$

Hence $Q = V_2 \cdot A_2 = V_2 \cdot b \cdot y_2$ $\qquad$ or $\qquad V_2 = \frac{Q}{b \cdot y_2}$ $\qquad V_2 = 10 \frac{m}{s}$

Then upstream $y_1 = \left(\frac{V_2^2}{2 \cdot g} + y_2\right)$ $\qquad y_1 = 5.70\,m$

The maximum flow rate occurs at critical conditions (see Section 11-2), for constant specific energy

In this case $V_2 = V_c = \sqrt{g \cdot y_c}$

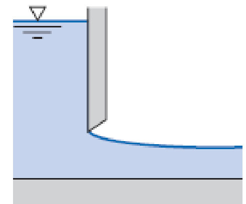
Hence we find $$y_1 = \frac{V_c^2}{2 \cdot g} + y_c = \frac{g \cdot y_c}{2 \cdot g} + y_c = \frac{3}{2} \cdot y_c$$

Hence $y_c = \frac{2}{3} \cdot y_1$ $\qquad y_c = 3.80\,m$ $\qquad V_c = \sqrt{g \cdot y_c}$ $\qquad V_c = 6.10 \frac{m}{s}$

$\frac{Q}{b} = V_c \cdot y_c$ $\qquad \frac{Q}{b} = 23.2 \frac{\frac{m^3}{s}}{m}$ $\qquad$ (Maximum flow rate)

Problem 11.43

11.43 A horizontal rectangular channel 3 ft wide contains a sluice gate. Upstream of the gate the depth is 6 ft; the depth downstream is 0.9 ft. Estimate the volume flow rate in the channel.



Given: Data on sluice gate

Find: Flow rate

Solution:

Basic equation:

$$\frac{p_1}{\rho \cdot g} + \frac{V_1^2}{2 \cdot g} + y_1 = \frac{p_2}{\rho \cdot g} + \frac{V_2^2}{2 \cdot g} + y_2 + h$$

The Bernoulli equation applies because we have steady, incompressible, frictionless flow.

Noting that $p_1 = p_2 = p_{atm}$, (1 = upstream, 2 = downstream) the Bernoulli equation becomes

$$\frac{V_1^2}{2 \cdot g} + y_1 = \frac{V_2^2}{2 \cdot g} + y_2$$

The given data is $b = 3 \cdot ft$ $\quad y_1 = 6 \cdot ft$ $\quad y_2 = 0.9 \cdot ft$

Also $Q = V \cdot A$ so $V_1 = \frac{Q}{b \cdot y_1}$ and $V_2 = \frac{Q}{b \cdot y_2}$

Using these in the Bernoulli equation

$$\frac{\left(\frac{Q}{b \cdot y_1}\right)^2}{2 \cdot g} + y_1 = \frac{\left(\frac{Q}{b \cdot y_2}\right)^2}{2 \cdot g} + y_2$$

Solving for Q $\quad Q = \sqrt{\frac{2 \cdot g \cdot b^2 \cdot y_1^2 \cdot y_2^2}{y_1 + y_2}}$ $\quad Q = 49.5 \frac{ft^3}{s}$

Note that $V_1 = \frac{Q}{b \cdot y_1}$ $\quad V_1 = 2.75 \frac{ft}{s}$ $\quad Fr_1 = \frac{V_1}{\sqrt{g \cdot y_1}}$ $\quad Fr_1 = 0.198$

$V_2 = \frac{Q}{b \cdot y_2}$ $\quad V_2 = 18.3 \frac{ft}{s}$ $\quad Fr_2 = \frac{V_2}{\sqrt{g \cdot y_2}}$ $\quad Fr_2 = 3.41$

Problem 11.44

11.44 Consider a 2.45-m-wide rectangular channel with a bed slope of 0.0004 and a Manning's roughness factor of 0.015. A weir is placed in the channel and the depth upstream of the weir is 1.52 m for a discharge of 5.66 m^3/s. Determine if a hydraulic jump forms upstream of the weir.

Given: Data on rectangular channel and weir

Find: If a hydraulic jump forms upstream of the weir

Solution:

Basic equations: $Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}}$ $\qquad y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$

Note that the Q equation is an "engineering" equation, to be used without units!

For a rectangular channel of width $B_w = 2.45\cdot m$ and depth y we find from Table 11.2

$$A = B_w\cdot y = 2.45\cdot y \qquad R = \frac{B_w\cdot y}{B_w + 2\cdot y} = \frac{2.45\cdot y}{2.45 + 2\cdot y}$$

and also $\quad n = 0.015 \quad$ and $\quad S_0 = 0.0004 \quad Q = 5.66\cdot\frac{m^3}{s}$

Hence $\quad Q = \frac{1}{n}\cdot A\cdot R^{\frac{2}{3}}\cdot S_0^{\frac{1}{2}} = \frac{1}{0.015}\cdot 2.45\cdot y\cdot\left(\frac{2.45\cdot y}{2.45 + 2\cdot y}\right)^{\frac{2}{3}}\cdot 0.0004^{\frac{1}{2}} = 5.66 \quad$ (Note that we don't use units!)

Solving for y $\quad \frac{y^{\frac{5}{3}}}{(2.45 + 2\cdot y)^{\frac{2}{3}}} = \frac{5.66\cdot 0.015}{.0004^{\frac{1}{2}}\cdot 2.54\cdot 2.54^{\frac{2}{3}}} \quad$ or $\quad \frac{y^{\frac{5}{3}}}{(2.54 + 2\cdot y)^{\frac{2}{3}}} = 0.898$

This is a nonlinear implicit equation for y and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below. We start with the given depth

For $\quad y = 1.52 \quad$ (m) $\quad \frac{y^{\frac{5}{3}}}{(2.54 + 2\cdot y)^{\frac{2}{3}}} = 0.639 \qquad$ For $\quad y = 2 \quad$ (m) $\quad \frac{y^{\frac{5}{3}}}{(2.54 + 2\cdot y)^{\frac{2}{3}}} = 0.908$

For $\quad y = 1.95 \quad$ (m) $\quad \frac{y^{\frac{5}{3}}}{(2.54 + 2\cdot y)^{\frac{2}{3}}} = 0.879 \qquad$ For $\quad y = 1.98 \quad$ (m) $\quad \frac{y^{\frac{5}{3}}}{(2.54 + 2\cdot y)^{\frac{2}{3}}} = 0.896$

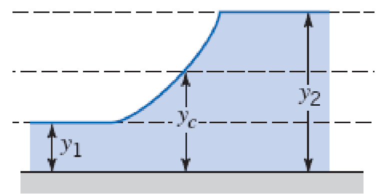
$y = 1.98 \quad$ (m) $\quad$ This is the normal depth.

We also have the critical depth: $q = \frac{Q}{B_w}$ $q = 2.31\frac{m^2}{s}$ $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ $y_c = 0.816\,m$

Hence the given depth is 1.52 m > y_c, but 1.52 m < y_n, the normal depth. This implies the flow is subcritical (far enough upstream it is dep 1.98 m), and that it draws down to 1.52 m as it gets close to the wier. There is no jump.

Problem 11.45

11.45 A hydraulic jump occurs in a rectangular channel 4.0 m wide. The water depth before the jump is 0.4 m and after the jump is 1.7 m. Compute the flow rate in the channel, the critical depth, and the headloss in the jump.



Given:
Data on rectangular channel and hydraulic jump

Find:
Flow rate; Critical depth; Head loss

Solution:

Basic equations: $\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^2}\right)$ $\qquad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^2}{2\cdot g}\right) - \left(y_2 + \frac{V_2^2}{2\cdot g}\right)$ $\qquad y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$

The given data is $b = 4\cdot m$ $\qquad y_1 = 0.4\cdot m$ $\qquad y_2 = 1.7\cdot m$

We can solve for Fr_1 from the basic equation $\sqrt{1 + 8\cdot Fr_1^2} = 1 + 2\cdot\frac{y_2}{y_1}$

$$Fr_1 = \sqrt{\frac{\left(1 + 2\cdot\frac{y_2}{y_1}\right)^2 - 1}{8}} \qquad Fr_1 = 3.34 \qquad \text{and} \qquad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$$

Hence $V_1 = Fr_1\cdot\sqrt{g\cdot y_1}$ $\qquad V_1 = 6.62\frac{m}{s}$

Then $Q = V_1\cdot b\cdot y_1$ $\qquad Q = 10.6\cdot\frac{m^3}{s}$ $\qquad q = \frac{Q}{b}$ $\qquad q = 2.65\frac{m^2}{s}$

The critical depth is $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ $\qquad y_c = 0.894\,m$

Also $V_2 = \frac{Q}{b\cdot y_2}$ $\qquad V_2 = 1.56\frac{m}{s}$ $\qquad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}}$ $\qquad Fr_2 = 0.381$

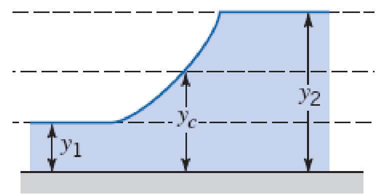
The energy loss is $H_l = \left(y_1 + \frac{V_1^2}{2\cdot g}\right) - \left(y_2 + \frac{V_2^2}{2\cdot g}\right)$ $\qquad H_l = 0.808\,m$

Note that we could use the result of Example 11.9 $\qquad H_l = \frac{(y_2 - y_1)^3}{4\cdot y_1\cdot y_2}$ $\qquad H_l = 0.808\,m$

Problem 11.46

[2]

11.46 A wide channel carries 20 ft^3/sec per foot of width at a depth of 1 ft at the toe of a hydraulic jump. Determine the depth of the jump and the head loss across it.



Given: Data on wide channel and hydraulic jump

Find: Jump depth; Head loss

Solution:

Basic equations: $\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right)$ $\quad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$

The given data is $\frac{Q}{b} = 20\frac{\frac{ft^3}{s}}{ft}$ $\quad y_1 = 1\cdot ft$

Also $Q = V\cdot A = V\cdot b\cdot y$

Hence $V_1 = \frac{Q}{b\cdot y_1}$ $\quad V_1 = 20.0\frac{ft}{s}$ $\quad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$ $\quad Fr_1 = 3.53$

Then $y_2 = \frac{y_1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right)$ $\quad y_2 = 4.51\,ft$

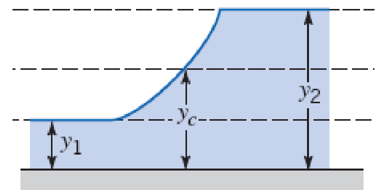
$V_2 = \frac{Q}{b\cdot y_2}$ $\quad V_2 = 4.43\frac{ft}{s}$ $\quad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}}$ $\quad Fr_2 = 0.368$

The energy loss is $H_l = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$ $\quad H_l = 2.40\,ft$

Note that we could use the result of Example 11.9 $\quad H_l = \frac{(y_2 - y_1)^3}{4\cdot y_1\cdot y_2}$ $\quad H_l = 2.40\,ft$

Problem 11.47

11.47 A hydraulic jump occurs in a wide horizontal channel. The discharge is 30 ft^3/sec per foot of width. The upstream depth is 1.3 ft. Determine the depth of the jump.



Given: Data on wide channel and hydraulic jump

Find: Jump depth

Solution:

Basic equations: $$\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\ 2}}\right)$$

The given data is $$\frac{Q}{b} = 30\frac{\frac{ft^3}{s}}{ft} \qquad y_1 = 1.3\cdot ft$$

Also $$Q = V\cdot A = V\cdot b\cdot y$$

Hence $$V_1 = \frac{Q}{b\cdot y_1} \qquad V_1 = 23.1\frac{ft}{s} \qquad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}} \qquad Fr_1 = 3.57$$

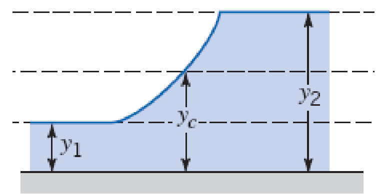
Then $$y_2 = \frac{y_1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\ 2}}\right) \qquad y_2 = 5.94\,ft$$

Note: $$V_2 = \frac{Q}{b\cdot y_2} \qquad V_2 = 5.05\frac{ft}{s} \qquad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}} \qquad Fr_2 = 0.365$$

Problem 11.48

[2]

11.48 A hydraulic jump occurs in a rectangular channel. The flow rate is 200 ft^3/sec, and the depth before the jump is 1.2 ft. Determine the depth behind the jump and the head loss, if the channel is 10 ft wide.



Given: Data on wide channel and hydraulic jump

Find: Jump depth; Head loss

Solution:

Basic equations:
$$\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1+\sqrt{1+8\cdot Fr_1^{\,2}}\right) \qquad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$$

The given data is $Q = 200\cdot\frac{ft^3}{s}$ $\quad b = 10\cdot ft$ $\quad y_1 = 1.2\cdot ft$

Also $Q = V\cdot A = V\cdot b\cdot y$

Hence $V_1 = \frac{Q}{b\cdot y_1}$ $\quad V_1 = 16.7\cdot\frac{ft}{s}$ $\quad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$ $\quad Fr_1 = 2.68$

Then $y_2 = \frac{y_1}{2}\cdot\left(-1+\sqrt{1+8\cdot Fr_1^{\,2}}\right)$ $\quad y_2 = 3.99\cdot ft$

$V_2 = \frac{Q}{b\cdot y_2}$ $\quad V_2 = 5.01\cdot\frac{ft}{s}$ $\quad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}}$ $\quad Fr_2 = 0.442$

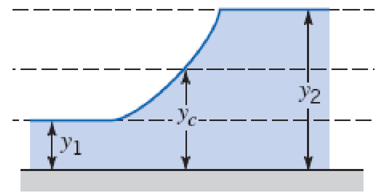
The energy loss is $H_l = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$ $\quad H_l = 1.14\cdot ft$

Note that we could use the result of Example 11.9 $\quad H_l = \frac{(y_2 - y_1)^3}{4\cdot y_1\cdot y_2}$ $\quad H_l = 1.14\cdot ft$

Problem 11.49

[2]

11.49 The hydraulic jump may be used as a crude flow meter. Suppose that in a horizontal rectangular channel 5 ft wide the observed depths before and after a hydraulic jump are 0.66 and 3.0 ft. Find the rate of flow and the head loss.



Given: Data on wide channel and hydraulic jump

Find: Flow rate; Head loss

Solution:

Basic equations: $\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\ 2}}\right)$ $\qquad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^{\ 2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\ 2}}{2\cdot g}\right)$

The given data is $b = 5\cdot ft$ $\qquad y_1 = 0.66\cdot ft$ $\qquad y_2 = 3.0\cdot ft$

We can solve for Fr_1 from the basic equation $\sqrt{1 + 8\cdot Fr_1^{\ 2}} = 1 + 2\cdot\frac{y_2}{y_1}$

$$Fr_1 = \sqrt{\frac{\left(1 + 2\cdot\frac{y_2}{y_1}\right)^2 - 1}{8}} \qquad Fr_1 = 3.55 \qquad \text{and} \qquad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$$

Hence $V_1 = Fr_1\cdot\sqrt{g\cdot y_1}$ $\qquad V_1 = 16.4\cdot\frac{ft}{s}$

Then $Q = V_1\cdot b\cdot y_1$ $\qquad Q = 54.0\,\frac{ft^3}{s}$

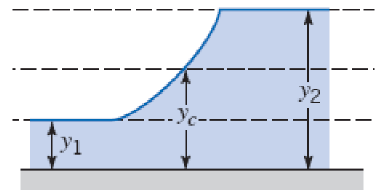
Also $V_2 = \frac{Q}{b\cdot y_2}$ $\qquad V_2 = 3.60\cdot\frac{ft}{s}$ $\qquad Fr_2 = \frac{V_2}{\sqrt{g\cdot y_2}}$ $\qquad Fr_2 = 0.366$

The energy loss is $H_l = \left(y_1 + \frac{V_1^{\ 2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\ 2}}{2\cdot g}\right)$ $\qquad H_l = 1.62\cdot ft$

Note that we could use the result of Example 11.9 $\qquad H_l = \frac{(y_2 - y_1)^3}{4\cdot y_1\cdot y_2}$ $\qquad H_l = 1.62\cdot ft$

Problem 11.50

11.50 A hydraulic jump occurs on a horizontal apron downstream from a wide spillway, at a location where depth is 0.9 m and speed is 25 m/sec. Estimate the depth and speed downstream from the jump. Compare the specific energy downstream of the jump to that upstream.



Given:

Data on wide spillway flow

Find:

Depth after hydraulic jump; Specific energy change

Solution:

Basic equations: $\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right)$ $\qquad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$

The given data is $\quad y_1 = 0.9\cdot m \qquad V_1 = 25\frac{m}{s}$

Then Fr_1 is $\quad Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}} \qquad Fr_1 = 8.42$

Hence $\quad y_2 = \frac{y_1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right) \qquad y_2 = 10.3\,m$

Then $\quad Q = V_1\cdot b\cdot y_1 = V_2\cdot b\cdot y_2 \qquad V_2 = V_1\cdot\frac{y_1}{y_2} \qquad V_2 = 2.19\frac{m}{s}$

For the specific energies $\quad E_1 = y_1 + \frac{V_1^{\,2}}{2\cdot g} \qquad E_1 = 32.8\,m$

$E_2 = y_2 + \frac{V_2^{\,2}}{2\cdot g} \qquad E_2 = 10.5\,m \qquad \frac{E_2}{E_1} = 0.321$

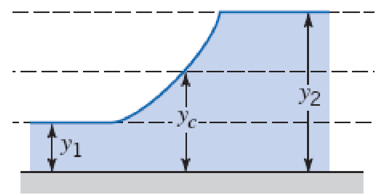
The energy loss is $\quad H_l = E_1 - E_2 \qquad H_l = 22.3\,m$

Note that we could use the result of Example 11.9 $\quad H_l = \frac{\left(y_2 - y_1\right)^3}{4\cdot y_1\cdot y_2} \qquad H_l = 22.3\cdot m$

Problem 11.51

[2]

11.51 A hydraulic jump occurs in a rectangular channel. The flow rate is 6.5 m^3/sec and the depth before the jump is 0.4 m. Determine the depth after the jump and the head loss, if the channel is 1 m wide.



Given: Data on rectangular channel flow

Find: Depth after hydraulic jump; Specific energy change

Solution:

Basic equations:
$$\frac{y_2}{y_1} = \frac{1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right) \qquad H_l = E_1 - E_2 = \left(y_1 + \frac{V_1^{\,2}}{2\cdot g}\right) - \left(y_2 + \frac{V_2^{\,2}}{2\cdot g}\right)$$

The given data is $y_1 = 0.4\cdot m$ $\quad b = 1\cdot m$ $\quad Q = 6.5\frac{m^3}{s}$

Then $Q = V_1\cdot b\cdot y_1 = V_2\cdot b\cdot y_2$ $\quad V_1 = \frac{Q}{b\cdot y_1}$ $\quad V_1 = 16.3\frac{m}{s}$

Then Fr_1 is $Fr_1 = \frac{V_1}{\sqrt{g\cdot y_1}}$ $\quad Fr_1 = 8.20$

Hence $y_2 = \frac{y_1}{2}\cdot\left(-1 + \sqrt{1 + 8\cdot Fr_1^{\,2}}\right)$ $\quad y_2 = 4.45\,m$

and $V_2 = \frac{Q}{b\cdot y_2}$ $\quad V_2 = 1.46\frac{m}{s}$

For the specific energies $E_1 = y_1 + \frac{V_1^{\,2}}{2\cdot g}$ $\quad E_1 = 13.9\,m$

$E_2 = y_2 + \frac{V_2^{\,2}}{2\cdot g}$ $\quad E_2 = 4.55\,m$

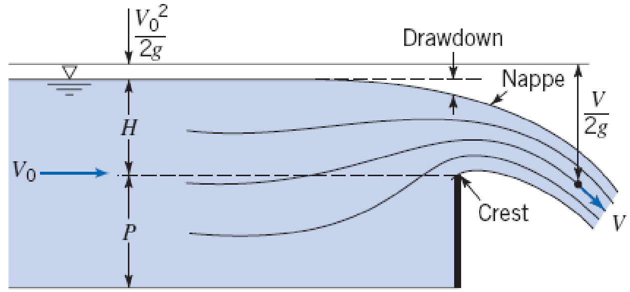
The energy loss is $H_l = E_1 - E_2$ $\quad H_l = 9.31\,m$

Note that we could use the result of Example 11.9 $\quad H_l = \frac{\left(y_2 - y_1\right)^3}{4\cdot y_1\cdot y_2}$ $\quad H_l = 9.31\cdot m$

Problem 11.52

[3]

11.52 A rectangular, sharp-crested weir with end contraction is 1.6 m long. How high should it be placed in a channel to maintain an upstream depth of 2.5 m for 0.5 m^3/s flow rate?



Given: Data on rectangular, sharp-crested weir

Find: Required weir height

Solution:

Basic equations: $Q = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot B' \cdot H^{\frac{3}{2}}$ where $C_d = 0.62$ and $B' = B - 0.1 \cdot n \cdot H$ with $n = 2$

Given data: $B = 1.6 \cdot m$ $\quad Q = 0.5 \cdot \frac{m^3}{s}$

Hence we find

$$Q = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot B' \cdot H^{\frac{3}{2}} = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}}$$

Rearranging $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = \frac{3 \cdot Q}{2 \cdot \sqrt{2 \cdot g} \cdot C_d}$

This is a nonlinear implicit equation for H and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

The right side evaluates to $\frac{3 \cdot Q}{2 \cdot \sqrt{2 \cdot g} \cdot C_d} = 0.273 m^{\frac{5}{2}}$

For $H = 1 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 1.40 m^{\frac{5}{2}}$ $\quad$ For $H = 0.5 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.530 m^{\frac{5}{2}}$

For $H = 0.3 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.253 m^{\frac{5}{2}}$ $\quad$ For $H = 0.35 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.317 m^{\frac{5}{2}}$

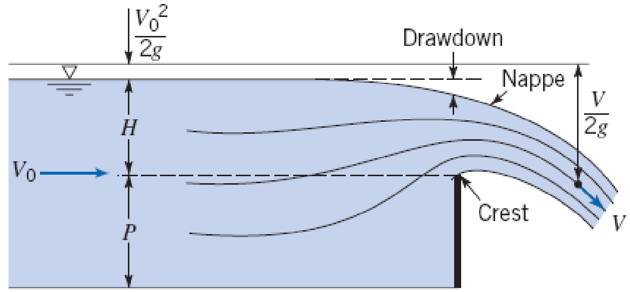
For $H = 0.31 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.265 m^{\frac{5}{2}}$ $\quad$ For $H = 0.315 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.272 m^{\frac{5}{2}}$

For $H = 0.316 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.273 m^{\frac{5}{2}}$ $\quad H = 0.316 m$

But from the figure $\quad H + P = 2.5 \cdot m$ $\quad P = 2.5 \cdot m - H$ $\quad P = 2.18 m$

Problem 11.53

11.53 For a sharp-crested suppressed weir ($C_w = 3.33$) of length $B = 8.0$ ft, $P = 2.0$ ft, and $H = 1.0$ ft, determine the discharge over the weir. Neglect the velocity of approach head.



Given: Data on rectangular, sharp-crested weir

Find: Discharge

Solution:

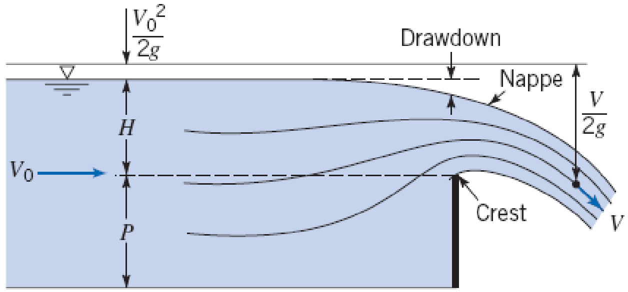
Basic equation: $Q = C_w \cdot B \cdot H^{\frac{3}{2}}$ where $C_w = 3.33$ and $B = 8 \cdot ft$ $P = 2 \cdot ft$ $H = 1 \cdot ft$

Note that this is an "engineering" equation, to be used without units!

$$Q = C_w \cdot B \cdot H^{\frac{3}{2}} \qquad Q = 26.6 \, \frac{ft^3}{s}$$

Problem 11.54

11.54 A rectangular sharp-crested weir with end contractions is 1.5 m long. How high should the weir crest be placed in a channel to maintain an upstream depth of 2.5 m for 0.5 m^3/s flow rate?



Given:
Data on rectangular, sharp-crested weir

Find:
Required weir height

Solution:

Basic equations: $Q = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot B' \cdot H^{\frac{3}{2}}$ where $C_d = 0.62$ and $B' = B - 0.1 \cdot n \cdot H$ with $n = 2$

Given data: $B = 1.5 \cdot m$ $\quad Q = 0.5 \cdot \frac{m^3}{s}$

Hence we find

$$Q = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot B' \cdot H^{\frac{3}{2}} = C_d \cdot \frac{2}{3} \cdot \sqrt{2 \cdot g} \cdot (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}}$$

Rearranging

$$(B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = \frac{3 \cdot Q}{2 \cdot \sqrt{2 \cdot g} \cdot C_d}$$

This is a nonlinear implicit equation for H and must be solved numerically. We can use one of a number of numerical root finding techniqu such as Newton's method, or we can use *Excel*'s *Solver* or *Goal Seek*, or we can manually iterate, as below.

The right side evaluates to $\frac{3 \cdot Q}{2 \cdot \sqrt{2 \cdot g} \cdot C_d} = 0.273 \cdot m^{\frac{5}{2}}$

For $H = 1 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 1.30 \cdot m^{\frac{5}{2}}$ $\quad$ For $H = 0.5 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.495 \cdot m^{\frac{5}{2}}$

For $H = 0.3 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.237 \cdot m^{\frac{5}{2}}$ $\quad$ For $H = 0.35 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.296 \cdot m^{\frac{5}{2}}$

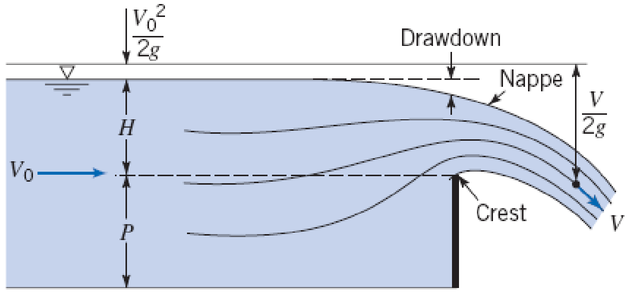
For $H = 0.34 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.284 \cdot m^{\frac{5}{2}}$ $\quad$ For $H = 0.33 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.272 \cdot m^{\frac{5}{2}}$

For $H = 0.331 \cdot m$ $\quad (B - 0.1 \cdot n \cdot H) \cdot H^{\frac{3}{2}} = 0.273 \cdot m^{\frac{5}{2}}$ $\quad H = 0.331\,m$

But from the figure $\quad H + P = 2.5 \cdot m$ $\quad P = 2.5 \cdot m - H$ $\quad P = 2.17\,m$

Problem 11.55

11.55 Determine the head on a 60° V-notch weir for a discharge of 150 l/s. Take $C_d = 0.58$.



Given:

Data on V-notch weir

Find:

Flow head

Solution:

Basic equation:

$$Q = C_d \cdot \frac{8}{15} \cdot \sqrt{2 \cdot g} \cdot \tan\left(\frac{\theta}{2}\right) \cdot H^{\frac{5}{2}}$$

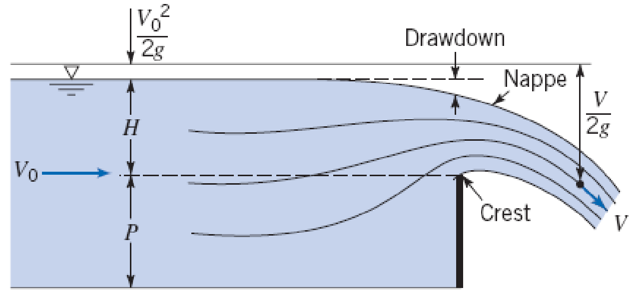
where $C_d = 0.58$ $\quad \theta = 60 \cdot deg$ $\quad Q = 150 \cdot \frac{L}{s}$

$$H = \left(\frac{Q}{C_d \cdot \frac{8}{15} \cdot \sqrt{2 \cdot g} \cdot \tan\left(\frac{\theta}{2}\right)}\right)^{\frac{2}{5}}$$

$H = 0.514\,m$

Problem 11.56

11.56 The head on a 90° V-notch weir is 1.5 ft. Determine the discharge.



Given:
Data on V-notch weir

Find:
Discharge

Solution:

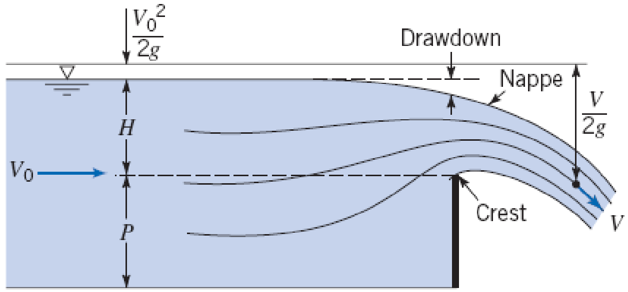
Basic equation: $Q = C_w \cdot H^{\frac{5}{2}}$ where $H = 1.5 \cdot ft$ $C_w = 2.50$ for $\theta = 90 \cdot deg$

Note that this is an "engineering" equation in which we ignore units!

$$Q = C_w \cdot H^{\frac{5}{2}} \qquad Q = 6.89 \frac{ft^3}{s}$$

Problem 11.57

11.57 Determine the weir coefficient of a 90° V-notch weir for a head of 180 mm for a flow rate of 20 l/s.



Given: Data on V-notch weir

Find: Weir coefficient

Solution:

Basic equation: $Q = C_w \cdot H^{\frac{5}{2}}$ where $H = 180 \cdot mm$ $Q = 20 \cdot \frac{L}{s}$

Note that this is an "engineering" equation in which we ignore units!

$$C_w = \frac{Q}{H^{\frac{5}{2}}} \qquad C_w = 1.45$$

Problem 12.1

12.1 An air flow in a duct passes through a thick filter. What happens to the pressure, temperature, and density of the air as it does so? *Hint*: This is a throttling process.

Given: Air flow through a filter

Find: Change in p, T and ρ

Solution:

Basic equations: $h_2 - h_1 = c_p \cdot (T_2 - T_1)$ $\quad p = \rho \cdot R \cdot T$

Assumptions: 1) Ideal gas 2) Throttling process

In a throttling process enthalpy is constant. Hence $h_2 - h_1 = 0$ so $T_2 - T_1 = 0$ or $T = \text{constant}$

The filter acts as a resistance through which there is a pressure drop (otherwise there would be no flow. Hence $p_2 < p_1$

From the ideal gas equation $\frac{p_1}{p_2} = \frac{\rho_1 \cdot T_1}{\rho_2 \cdot T_2}$ so $\rho_2 = \rho_1 \cdot \left(\frac{T_1}{T_2}\right) \cdot \left(\frac{p_2}{p_1}\right) = \rho_1 \cdot \left(\frac{p_2}{p_1}\right)$ Hence $\rho_2 < \rho_1$

The governing equation for entropy is $\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$

Hence $\Delta s = -R \cdot \ln\left(\frac{p_2}{p_1}\right)$ and $\frac{p_2}{p_1} < 1$ so $\Delta s > 0$

Entropy increases because throttling is an irreversible adiabatic process

Problem 12.2

Given: Steady flow through a turbine. Air expands from $T_1 = 1300°C$, $p_1 = 2.0$ MPa (abs) to $T_2 = 500°C$, $p_2 = 101$ kPa.

Find: (a) $u_2 - u_1$ (b) $h_2 - h_1$ (c) $s_2 - s_1$
(d) show process on a Ts diagram

Solution:

$$\Delta u = u_2 - u_1 = c_v(T_2 - T_1) = 717.4\ \frac{J}{kg\cdot K}(500-1300)K = -574\ kJ/kg \quad \leftarrow u_2 - u_1$$

$$\Delta h = h_2 - h_1 = c_p(T_2 - T_1) = 1004\ \frac{J}{kg\cdot K}(-800K) = -803\ kJ/kg \quad \leftarrow h_2 - h_1$$

To calculate the entropy change, we use the Tds equation

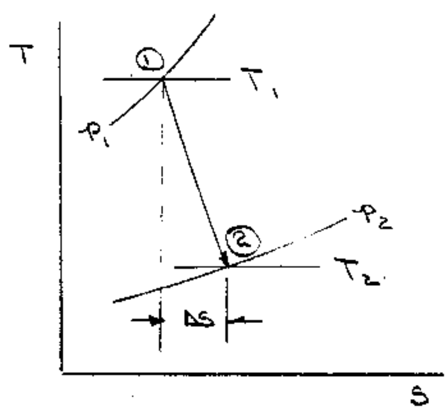
$$T\,ds = dh - v\,dp = c_p\,dT - RT\frac{dp}{p}$$

$$\therefore\ ds = c_p\frac{dT}{T} - R\frac{dp}{p}$$

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

$$= 1004\ \frac{J}{kg\cdot K}\ln\frac{500+273}{1300+273} - 286.9\ \frac{J}{kg\cdot K}\ln\frac{0.101}{2.0}$$

$$s_2 - s_1 = (-713.3 + 856.6)\ J/kg\cdot K = 143\ J/kg\cdot K \quad \leftarrow s_2 - s_1$$



Problem 12.3

12.3 A vendor claims that an adiabatic air compressor takes in air at standard atmosphere conditions and delivers the air at 650 kPa (gage) and 285°C. Is this possible? Justify your answer by calculation. Sketch the process on a Ts diagram.

Given: Data on an air compressor

Find: Whether or not the vendor claim is feasible

Solution:

Basic equation: $$\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$$

The data provided, or available in the Appendices, is:

$p_1 = 101 \cdot kPa$ $\qquad T_1 = (20 + 273) \cdot K$

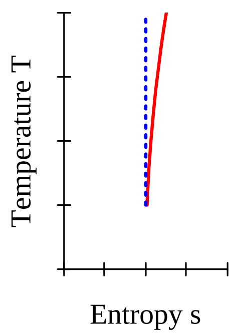
$p_2 = (650 + 101) \cdot kPa$ $\qquad T_2 = (285 + 273) \cdot K$

$c_p = 1004 \cdot \frac{J}{kg \cdot K}$ $\qquad R = 287 \cdot \frac{J}{kg \cdot K}$

Then $$\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right) \qquad \Delta s = 71.0 \frac{J}{kg \cdot K}$$

The second law of thermodynamics states that, for an adiabatic process

$\Delta s \geq 0$ or for all real processes $\Delta s > 0$



Hence the process is feasible!

Problem 12.4

12.4 What is the lowest possible delivery temperature generated by an adiabatic air compressor, starting with standard atmosphere conditions and delivering the air at 100 psig? Sketch the process on a *Ts* diagram.

Given: Adiabatic air compressor

Find: Lowest delivery temperature; Sketch the process on a Ts diagram

Solution:

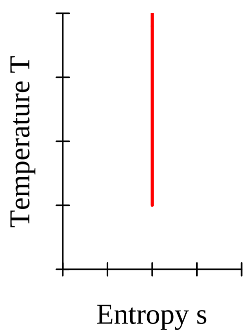
Basic equation: $$\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$$

The lowest temperature implies an ideal (reversible) process; it is also adiabatic, so $\Delta s = 0$, and $T_2 = T_1 \cdot \left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}}$

The data provided, or available in the Appendices, is $p_1 = 14.7 \cdot psi$ $\quad p_2 = (100 + 14.7) \cdot psi$ $\quad T_1 = (68 + 460) \cdot R$ $\quad k = 1.4$

Hence $$T_2 = T_1 \cdot \left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}}$$ $T_2 = 950\,R$ $\quad T_2 = 490\,°F$

The process is



Problem 12.5

[2]

12.5 A test chamber is separated into two equal chambers by a rubber diaphragm. One contains air at 20°C and 200 kPa (absolute), and the other has a vacuum. If the diaphragm is punctured, find the pressure and temperature of the air after it expands to fill the chamber. *Hint*: This is a rapid, violent event, so is irreversible but adiabatic.

Given: Test chamber with two chambers

Find: Pressure and temperature after expansion

Solution:

Basic equation: $p = \rho \cdot R \cdot T$ $\quad \Delta u = q - w$ (First law - closed system) $\quad \Delta u = c_V \cdot \Delta T$

Assumptions: 1) Ideal gas 2) Adiabatic 3) No work

For no work and adiabatic the first law becomes $\Delta u = 0$ or for an Ideal gas $\Delta T = 0$ $\quad T_2 = T_1$

We also have $M = \rho \cdot Vol = const$ and $Vol_2 = 2 \cdot Vol_1$ so $\rho_2 = \frac{1}{2} \cdot \rho_1$

From the ideal gas equation $\frac{p_2}{p_1} = \frac{\rho_2}{\rho_1} \cdot \frac{T_2}{T_1} = \frac{1}{2}$ so $p_2 = \frac{1}{2} \cdot p_1$

Hence $T_2 = 20\,°F$ $\quad p_2 = \frac{200 \cdot kPa}{2}$ $\quad p_2 = 100 \cdot kPa$

Note that $\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right) = -R \cdot \ln\left(\frac{1}{2}\right) = 0.693 \cdot R$ so entropy increases (irreversible adiabatic)

Problem 12.6

[2]

12.6 An automobile supercharger is a device that pressurizes the air that is used by the engine for combustion to increase the engine power (how does it differ from a turbocharger?). A supercharger takes in air at 70°F and atmospheric pressure and boosts it to 200 psig, at an intake rate of 0.5 ft³/s. What are the pressure, temperature, and volume flow rate at the exit? (The relatively high exit temperature is the reason an intercooler is also used.) Assuming a 70% efficiency, what is the power drawn by the supercharger? *Hint*: the efficiency is defined as the ratio of the isentropic power to actual power.

Given: Supercharger

Find: Pressure, temperature and flow rate at exit; power drawn

Solution:

Basic equation: $p = \rho \cdot R_{air} \cdot T$ $\quad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$

$\Delta h = q - w$ (First law - open system) $\quad \Delta h = c_p \cdot \Delta T$

Assumptions: 1) Ideal gas 2) Adiabatic

In an ideal process (reversible and adiabatic) the first law becomes $\Delta h = w$ or for an Ideal gas $w_{ideal} = c_p \cdot \Delta T$

For an isentropic process $\Delta s = 0 = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$ or $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$

The given or available data is $T_1 = (70 + 460) \cdot R$ $\quad p_1 = 14.7 \cdot psi$ $\quad p_2 = (200 + 14.7) \cdot psi$ $\quad \eta = 70 \cdot \%$

$Q_1 = 0.5 \cdot \frac{ft^3}{s}$ $\quad k = 1.4$ $\quad c_p = 0.2399 \cdot \frac{Btu}{lbm \cdot R}$ $\quad R_{air} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$

Hecne $T_2 = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} \cdot T_1$ $\quad T_2 = 1140 \cdot R$ $\quad T_2 = 681 \cdot °F$ $\quad p_2 = 215 \cdot psi$

We also have $m_{rate} = \rho_1 \cdot Q_1 = \rho_2 \cdot Q_2$ $\quad Q_2 = Q_1 \cdot \frac{\rho_1}{\rho_2}$ $\quad Q_2 = Q_1 \cdot \frac{p_1}{p_2} \cdot \frac{T_2}{T_1}$ $\quad Q_2 = 0.0737 \cdot \frac{ft^3}{s}$

For the power we use $P_{ideal} = m_{rate} \cdot w_{ideal} = \rho_1 \cdot Q_1 \cdot c_p \cdot \Delta \cdot T$

From the ideal gas equation $\rho_1 = \frac{p_1}{R_{air} \cdot T_1}$ $\quad \rho_1 = 0.00233 \cdot \frac{slug}{ft^3}$ or $\rho_1 = 0.0749 \cdot \frac{lbm}{ft^3}$

Hence $P_{ideal} = \rho_1 \cdot Q_1 \cdot c_p \cdot (T_2 - T_1)$ $\quad P_{ideal} = 5.78 \cdot kW$

The actual power needed is $P_{actual} = \frac{P_{ideal}}{\eta}$ $\quad P_{actual} = 8.26 \cdot kW$

A supercharger is a pump that forces air into an engine, but generally refers to a pump that is driven directly by the engine, as opposed to a turbocharger that is driven by the pressure of the exhaust gases.

Problem 12.7

12.7 Five kilograms of air is cooled in a closed tank from 250 to 50°C. The initial pressure is 3 MPa. Compute the changes in entropy, internal energy, and enthalpy. Show the process state points on a *Ts* diagram.

Given: Cooling of air in a tank

Find: Change in entropy, internal energy, and enthalpy

Solution:

Basic equation: $p = \rho \cdot R \cdot T$ $\qquad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$

$\Delta u = c_v \cdot \Delta T$ $\qquad \Delta h = c_p \cdot \Delta T$

Assumptions: 1) Ideal gas 2) Constant specific heats

Given or available data $M = 5 \cdot kg$ $\quad T_1 = (250 + 273) \cdot K$ $\quad T_2 = (50 + 273) \cdot K$ $\quad p_1 = 3 \cdot MPa$

$c_p = 1004 \cdot \frac{J}{kg \cdot K}$ $\quad c_v = 717.4 \cdot \frac{J}{kg \cdot K}$ $\quad k = \frac{c_p}{c_v}$ $\quad k = 1.4$ $\quad R = c_p - c_v$ $\quad R = 287 \cdot \frac{J}{kg \cdot K}$

For a constant volume process the ideal gas equation gives $\frac{p_2}{p_1} = \frac{T_2}{T_1}$ $\quad p_2 = \frac{T_2}{T_1} \cdot p_1$ $\quad p_2 = 1.85 \cdot MPa$

Then $\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$ $\qquad \Delta s = -346 \cdot \frac{J}{kg \cdot K}$

$\Delta u = c_v \cdot (T_2 - T_1)$ $\qquad \Delta u = -143 \cdot \frac{kJ}{kg}$

$\Delta h = c_p \cdot (T_2 - T_1)$ $\qquad \Delta h = -201 \cdot \frac{kJ}{kg}$

Total amounts are $\Delta S = M \cdot \Delta s$ $\qquad \Delta S = -1729 \cdot \frac{J}{K}$

$\Delta U = M \cdot \Delta u$ $\qquad \Delta U = -717 \cdot kJ$

$\Delta H = M \cdot \Delta h$ $\qquad \Delta H = -1004 \cdot kJ$

Problem 12.8

12.8 Air is contained in a piston-cylinder device. The temperature of the air is 100°C. Using the fact that for a reversible process the heat transfer $q = \int T ds$, compare the amount of heat (J/kg) required to raise the temperature of the air to 1200°C at (a) constant pressure and (b) constant volume. Verify your results using the first law of thermodynamics. Plot the processes on a Ts diagram.

Given: Air in a piston-cylinder

Find: Heat to raise temperature to 1200ºC at a) constant pressure and b) constant volume

Solution:

The data provided, or available in the Appendices, is:

$T_1 = (100 + 273)\cdot K \qquad T_2 = (1200 + 273)\cdot K \qquad R = 287\cdot\frac{J}{kg\cdot K} \qquad c_p = 1004\cdot\frac{J}{kg\cdot K} \qquad c_v = c_p - R \qquad c_v = 717\cdot\frac{J}{kg\cdot K}$

a) For a constant pressure process we start with $\quad T\cdot ds = dh - v\cdot dp$

Hence, for p = const. $\quad ds = \frac{dh}{T} = c_p\cdot\frac{dT}{T}$

But $\quad \delta q = T\cdot ds$

Hence $\quad \delta q = c_p\cdot dT \qquad q = \int c_p\, dT \qquad q = c_p\cdot(T_2 - T_1) \qquad q = 1104\cdot\frac{kJ}{kg}$

b) For a constant volume process we start $\quad T\cdot ds = du + p\cdot dv$

Hence, for v = const. $\quad ds = \frac{du}{T} = c_v\cdot\frac{dT}{T}$

But $\quad \delta q = T\cdot ds$

Hence $\quad \delta q = c_v\cdot dT \qquad q = \int c_v\, dT \qquad q = c_v\cdot(T_2 - T_1) \qquad q = 789\cdot\frac{kJ}{kg}$

Heating to a higher temperature at constant pressure requires more heat than at constant volume: some of the heat is used to do work in expanding the gas; hence for constant pressure less of the heat is available for raising the temperature.

From the first law: Constant pressure: $q = \Delta u + w$ Constant volume: $q = \Delta u$

The two processes can be plotted using Eqs. 11.11b and 11.11a, simplified for the case of constant pressure and constant volume.

a) For constant pressure $\quad s_2 - s_1 = c_p\cdot\ln\left(\frac{T_2}{T_1}\right) - R\cdot\ln\left(\frac{p_2}{p_1}\right) \quad$ so $\quad \Delta s = c_p\cdot\ln\left(\frac{T_2}{T_1}\right)$

b) For constant volume $\quad s_2 - s_1 = c_v\cdot\ln\left(\frac{T_2}{T_1}\right) + R\cdot\ln\left(\frac{v_2}{v_1}\right) \quad$ so $\quad \Delta s = c_v\cdot\ln\left(\frac{T_2}{T_1}\right)$

The processes are plotted in the associated *Excel* workbook

Problem 12.8 (In Excel)

12.8 Air is contained in a piston-cylinder device. The temperature of the air is 100°C. Using the fact that for a reversible process the heat transfer $q = \int T ds$, compare the amount of heat (J/kg) required to raise the temperature of the air to 1200°C at (a) constant pressure and (b) constant volume. Verify your results using the first law of thermodynamics. Plot the processes on a Ts diagram.

Given: Air in a piston-cylinder

Find: Heat to raise temperature to 1200^{o}C at a) constant pressure and b) constant volume; plot

Solution:

The given or available data is:

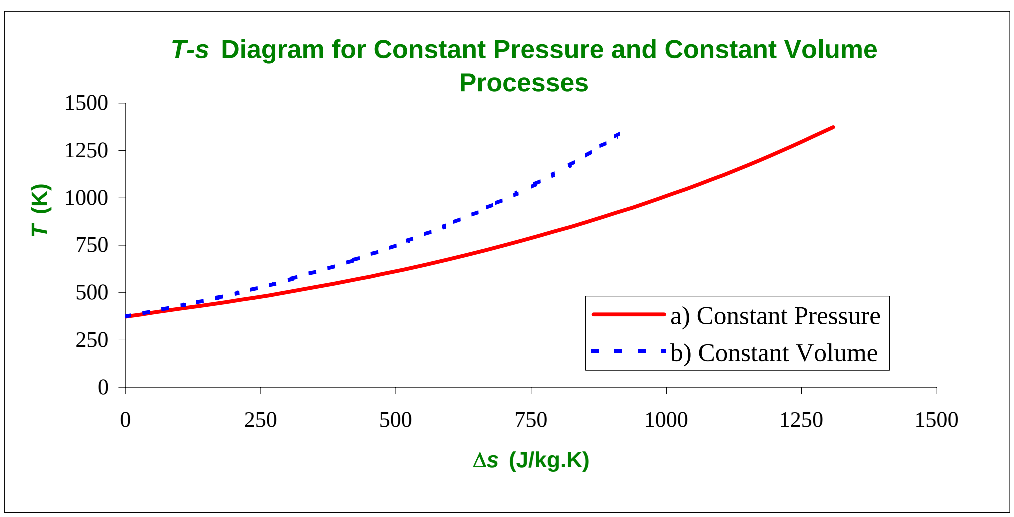
$T_1 =$	100	^{o}C
$T_2 =$	1200	^{o}C
$R =$	287	J/kg.K
$c_p =$	1004	J/kg.K
$c_v =$	717	J/kg.K

The equations to be plotted are:

a) For constant pressure $$s_2 - s_1 = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$$

b) For constant volume $$s_2 - s_1 = c_v \cdot \ln\left(\frac{T_2}{T_1}\right) + R \cdot \ln\left(\frac{v_2}{v_1}\right)$$

T (K)	a) Δs J/kg·K)	b) Δs J/kg·K)
373	0	0
473	238	170
573	431	308
673	593	423
773	732	522
873	854	610
973	963	687
1073	1061	758
1173	1150	821
1273	1232	880
1373	1308	934



Problem 12.9

12.9 The four-stroke Otto cycle of a typical automobile engine is sometimes modeled as an ideal air-standard closed system. In this simplified system the combustion process is modeled as a heating process, and the exhaust-intake process as a cooling process of the working fluid (air). The cycle consists of: isentropic compression from state (1) (p_1 = 100 kPa (abs), T_1 = 20°C, $\forall_1$ = 500 cc) to state (2) ($\forall_2 = \forall_1/8.5$); isometric (constant volume) heat addition to state (3) (T_3 = 2750°C); isentropic expansion to state (4) ($\forall_4 = \forall_1$); and isometric cooling back to state (1). Plot the $p\forall$ and Ts diagrams for this cycle, and find the efficiency, defined as the net work (the cycle area in $p\forall$ space) divided by the heat added.

Given: Data on Otto cycle

Find: Plot of *pV* and *Ts* diagrams; efficiency

Solution:

The data provided, or available in the Appendices, is:

$c_p = 1004\cdot\frac{J}{kg\cdot K}$ $\quad R = 287\cdot\frac{J}{kg\cdot K}$ $\quad c_v = c_p - R$ $\quad c_v = 717\cdot\frac{J}{kg\cdot K}$ $\quad k = \frac{c_p}{c_v}$ $\quad k = 1.4$

$p_1 = 100\cdot kPa$ $\quad T_1 = (20 + 273)\cdot K$ $\quad T_3 = (2750 + 273)\cdot K$ $\quad V_1 = 500\cdot cc$ $\quad V_2 = \frac{V_1}{8.5}$ $\quad V_2 = 58.8\cdot cc$

$V_4 = V_1$

Computed results: $\quad M = \frac{p_1\cdot V_1}{R\cdot T_1}$ $\quad M = 5.95\times 10^{-4} kg$

For process 1-2 we have isentropic behavior $\quad T\cdot v^{k-1} = \text{constant}$ $\quad p\cdot v^k = \text{constant}$ $\quad$ (12.12 a and 12.12b)

Hence $\quad T_2 = T_1\cdot\left(\frac{V_1}{V_2}\right)^{k-1}$ $\quad T_2 = 690\,K$ $\quad p_2 = p_1\cdot\left(\frac{V_1}{V_2}\right)^{k}$ $\quad p_2 = 2002\cdot kPa$

The process from 1 -2 is $\quad p(V) = p_1\cdot\left(\frac{V_1}{V}\right)^k$ $\quad$ and $\quad s = \text{constant}$

The work is $\quad W_{12} = \left(\int_{V_1}^{V_2} p(V)\,dV = \frac{p_1\cdot V_1 - p_2\cdot V_2}{k-1}\right)$ $\quad W_{12} = -169\,J$ $\quad Q_{12} = 0\cdot J$ $\quad$ (Isentropic)

For process 2 - 3 we have constant volume $\quad V_3 = V_2$ $\quad V_3 = 58.8\cdot cc$

Hence $\quad p_3 = p_2\cdot\frac{T_3}{T_2}$ $\quad p_3 = 8770\cdot kPa$

The process from 2 -3 is $V = V_2 = \text{constant}$ and $\Delta s = c_V \cdot \ln\left(\frac{T}{T_2}\right)$ $W_{23} = 0 \cdot J$

(From 12.11a)

$Q_{23} = M \cdot \Delta u = M \cdot \int c_V\, dT$ $Q_{23} = M \cdot c_V \cdot (T_3 - T_2)$ $Q_{23} = 995\, J$

For process 3 - 4 we again have isentropic behavior

Hence $T_4 = T_3 \cdot \left(\frac{V_3}{V_4}\right)^{k-1}$ $T_4 = 1284\, K$ $p_4 = p_3 \cdot \left(\frac{V_3}{V_4}\right)^k$ $p_4 = 438 \cdot kPa$

The process from 3 - 4 is $p(V) = p_3 \cdot \left(\frac{V_3}{V}\right)^k$ and $s = \text{constant}$

The work is $W_{34} = \frac{p_3 \cdot V_3 - p_4 \cdot V_4}{k - 1}$ $W_{34} = 742\, J$ $Q_{34} = 0 \cdot J$

For process 4-1 we again have constant volume

The process from 4 -1 is $V = V_4 = \text{constant}$ and $\Delta s = c_V \cdot \ln\left(\frac{T}{T_4}\right)$ $W_{41} = 0 \cdot J$

(From 12.11a)

$Q_{41} = M \cdot c_V \cdot (T_1 - T_4)$ $Q_{41} = -422\, J$

The net work is $W_{net} = W_{12} + W_{23} + W_{34} + W_{41}$ $W_{net} = 572\, J$

The efficiency is $\eta = \frac{W_{net}}{Q_{23}}$ $\eta = 57.5 \cdot \%$

This is consistent with the expression for the Otto efficiency $\eta_{Otto} = 1 - \frac{1}{r^{k-1}}$

where *r* is the compression ratio $r = \frac{V_1}{V_2}$ $r = 8.5$

$\eta_{Otto} = 57.5 \cdot \%$

Plots of the cycle in *pV* and *Ts* space are shown in the associated *Excel* workbook

Problem 12.9 (In Excel)

12.9 The four-stroke Otto cycle of a typical automobile engine is sometimes modeled as an ideal air-standard closed system. In this simplified system the combustion process is modeled as a heating process, and the exhaust-intake process as a cooling process of the working fluid (air). The cycle consists of: isentropic compression from state ① (p_1 = 100 kPa (abs), T_1 = 20°C, V_1 = 500 cc) to state ② ($V_2 = V_1/8.5$); isometric (constant volume) heat addition to state ③ (T_3 = 2750°C); isentropic expansion to state ④ ($V_4 = V_1$); and isometric cooling back to state ①. Plot the pV and Ts diagrams for this cycle, and find the efficiency, defined as the net work (the cycle area in pV space) divided by the heat added.

Given: Data on Otto cycle

Find: Plot of pV and Ts diagrams; efficiency

Solution:

The given, available, or computed data is:

R =	287	J/kg.K
c_p =	1004	J/kg.K
c_v =	717	J/kg.K
k =	1.4	

T_1 =	293	K	p_1 =	100	kPa	V_1 =	500	cc
T_2 =	690	K	p_2 =	2002	kPa	V_2 =	58.8	cc
T_3 =	3023	K	p_3 =	8770	kPa	V_3 =	58.8	cc
T_4 =	1284	K	p_4 =	438	kPa	V_4 =	500	cc

The process from 1 -2 is $p(V) = p_1 \cdot \left(\frac{V_1}{V}\right)^k$ and $s = \text{constant}$

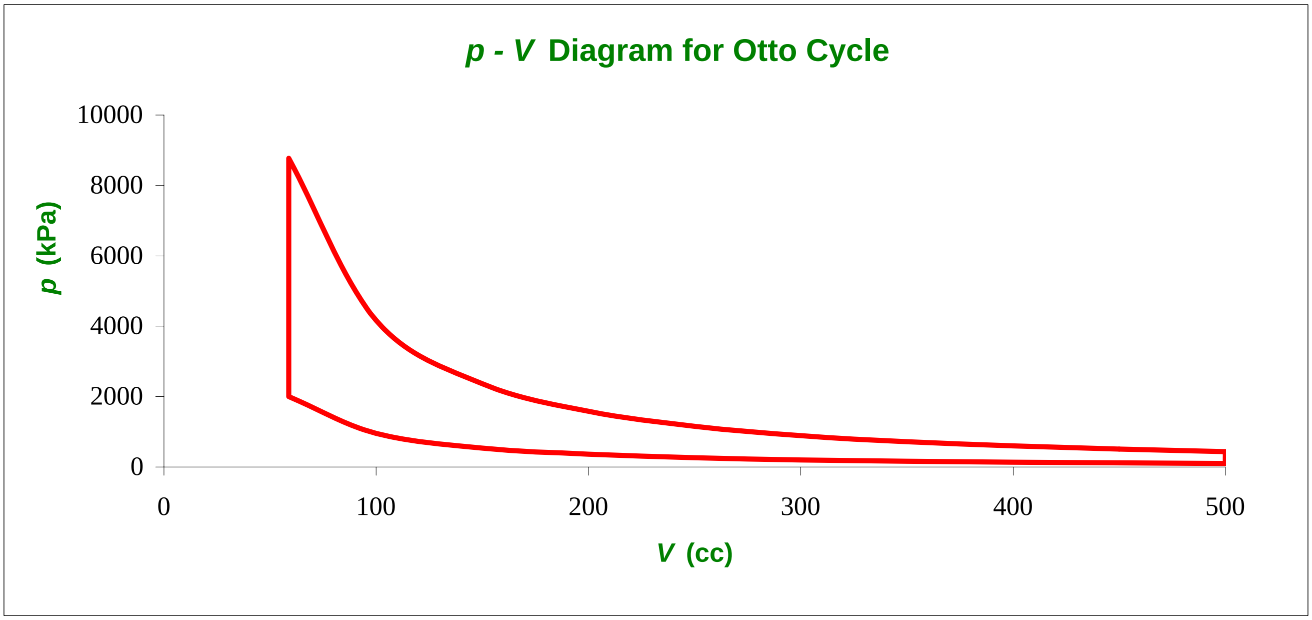
The process from 2 -3 is $V = V_2 = \text{constant}$ and $\Delta s = c_v \cdot \ln\left(\frac{T}{T_2}\right)$

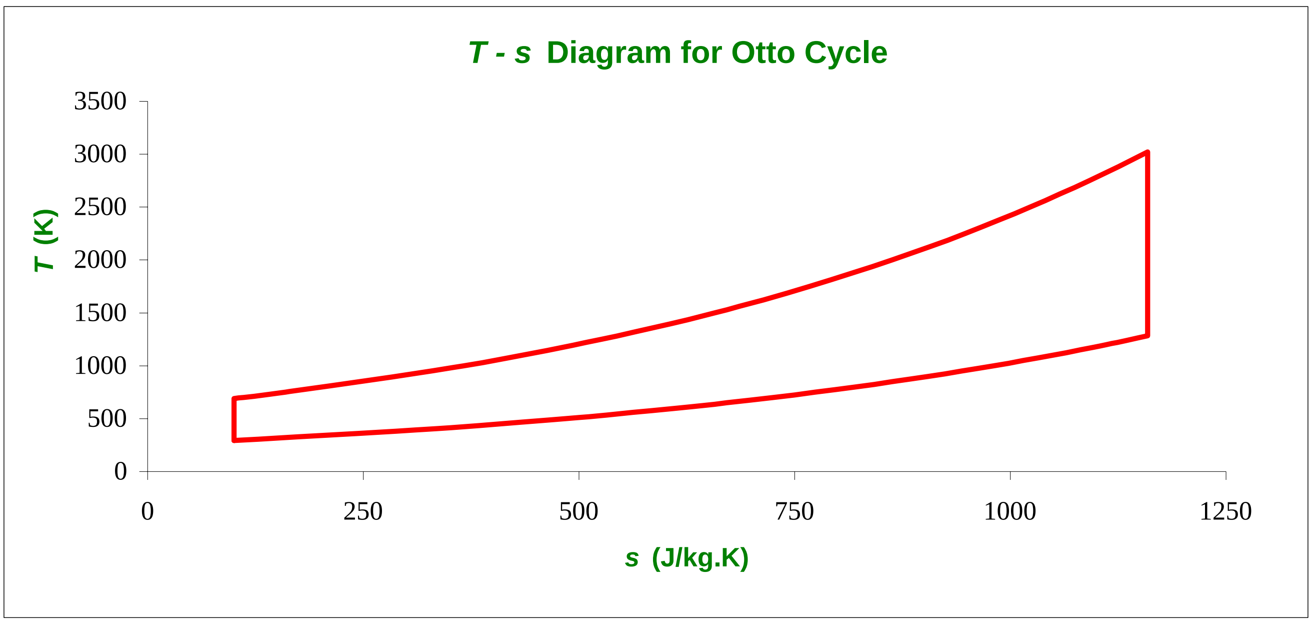
The process from 3 - 4 is $p(V) = p_3 \cdot \left(\frac{V_3}{V}\right)^k$ and $s = \text{constant}$

The process from 4 -1 is $V = V_4 = \text{constant}$ and $\Delta s = c_v \cdot \ln\left(\frac{T}{T_4}\right)$

The computations are:

	V (cc)	*p* (kPa)	*T* (K)	*s* J/kg·K)	
1	500	100	293	100	Initial entropy is arbitrary
	450	116	306	100	Temperatures from Eq. 12.12b
	400	137	320	100	
	350	165	338	100	
	300	204	359	100	
	250	264	387	100	
	200	361	423	100	
	150	540	474	100	
	100	952	558	100	
2	58.8	2002	690	100	Uniform temperature steps
	58.8	2176	750	160	
	58.8	2901	1000	366	
	58.8	3626	1250	526	
	58.8	4352	1500	657	
	58.8	5077	1750	767	
	58.8	5802	2000	863	
	58.8	6527	2250	947	
	58.8	7253	2500	1023	
	58.8	7978	2750	1091	
3	58.8	8770	3023	1159	Temperatures from Eq. 12.12b
	100	4172	2445	1159	
	150	2364	2078	1159	
	200	1580	1852	1159	
	250	1156	1694	1159	
	300	896	1575	1159	
	350	722	1481	1159	
	400	599	1403	1159	
	450	508	1339	1159	
4	500	438	1284	1159	Uniform temperature steps
	500	410	1200	1111	
	500	375	1100	1049	
	500	341	1000	980	
	500	307	900	905	
	500	273	800	820	
	500	239	700	724	
	500	205	600	614	
	500	171	500	483	
	500	137	400	323	
1	500	100	293	100	


p - V Diagram for Otto Cycle
10000
8000
6000
4000
2000
0
p (kPa)
0
100
200
300
400
500
V (cc)


T - s Diagram for Otto Cycle
3500
3000
2500
2000
1500
1000
500
0
T (K)
0
250
500
750
1000
1250
s (J/kg.K)

Problem 12.10

12.10 The four-stroke cycle of a typical diesel engine is sometimes modeled as an ideal air-standard closed system. In this simplified system the combustion process is modeled as a heating process, and the exhaust-intake process as a cooling process of the working fluid (air). The cycle consists of: isentropic compression from state ① ($p_1 = 100$ kPa (abs), $T_1 = 20°\text{C}$, $V_1 = 500$ cc) to state ② ($V_2 = V_1/12.5$); isometric (constant volume) heat addition to state ③ ($T_3 = 3000°\text{C}$); isobaric heat addition to state ④ ($V_4 = 1.75V_3$); isentropic expansion to state ⑤; and isometric cooling back to state ①. Plot the pV and Ts diagrams for this cycle, and find the efficiency, defined as the net work (the cycle area in pV space) divided by the heat added.

Given: Data on diesel cycle

Find: Plot of pV and Ts diagrams; efficiency

Solution:

The data provided, or available in the Appendices, is:

$c_p = 1004 \cdot \frac{J}{kg \cdot K}$ $\quad R = 287 \cdot \frac{J}{kg \cdot K}$ $\quad c_v = c_p - R$ $\quad c_v = 717 \frac{J}{kg \cdot K}$ $\quad k = \frac{c_p}{c_v}$ $\quad k = 1.4$

$p_1 = 100 \cdot kPa$ $\quad T_1 = (20 + 273) \cdot K$ $\quad T_3 = (3000 + 273) \cdot K$ $\quad V_1 = 500 \cdot cc$

$V_2 = \frac{V_1}{12.5}$ $\quad V_2 = 40\,cc$ $\quad V_3 = V_2$ $\quad V_4 = 1.75 \cdot V_3$ $\quad V_4 = 70\,cc$ $\quad V_5 = V_1$

Computed results: $\quad M = \frac{p_1 \cdot V_1}{R \cdot T_1}$ $\quad M = 5.95 \times 10^{-4} kg$

For process 1-2 we have isentropic behavior $\quad T \cdot v^{k-1} = \text{constant}$ (12.12a) $\quad p \cdot v^k = \text{constant}$ (12.12c)

Hence $\quad T_2 = T_1 \cdot \left(\frac{V_1}{V_2}\right)^{k-1}$ $\quad T_2 = 805\,K$ $\quad p_2 = p_1 \cdot \left(\frac{V_1}{V_2}\right)^k$ $\quad p_2 = 3435\,kPa$

The process from 1 -2 is $\quad p(V) = p_1 \cdot \left(\frac{V_1}{V}\right)^k$ $\quad$ and $\quad s = \text{constant}$

The work is $\quad W_{12} = \int_{V_1}^{V_2} p(V)\,dV = \frac{p_1 \cdot V_1 - p_2 \cdot V_2}{k - 1}$ $\quad W_{12} = -218\,J$ $\quad Q_{12} = 0 \cdot J$ (Isentropic)

For process 2 - 3 we have constant volume $\quad V_3 = V_2$ $\quad V_3 = 40\,cc$

Hence $\quad p_3 = p_2 \cdot \frac{T_3}{T_2}$ $\quad p_3 = 13963\,kPa$

The process from 2 -3 is $V = V_2 = \text{constant}$ and $\Delta s = c_v \cdot \ln\left(\frac{T}{T_2}\right)$ $W_{23} = 0 \cdot J$

(From Eq. 12.11a)

$Q_{23} = M \cdot \Delta u = M \cdot \int c_v \, dT$ $Q_{23} = M \cdot c_v \cdot (T_3 - T_2)$ $Q_{23} = 1052\,J$

For process 3 - 4 we have constant pressure $p_4 = p_3$ $p_4 = 13963\,kPa$ $T_4 = T_3 \cdot \left(\frac{V_4}{V_3}\right)$ $T_4 = 5728\,K$

The process from 3 - 4 is $p = p_3 = \text{constant}$ and $\Delta s = c_p \cdot \ln\left(\frac{T}{T_3}\right)$

(From Eq. 12.11b)

$W_{34} = p_3 \cdot (V_4 - V_3)$ $W_{34} = 419\,J$ $Q_{34} = M \cdot c_p \cdot (T_4 - T_3)$ $Q_{34} = 1465\,J$

For process 4 - 5 we again have isentropic behavior $T_5 = T_4 \cdot \left(\frac{V_4}{V_5}\right)^{k-1}$ $T_5 = 2607\,K$

Hence $p_5 = p_4 \cdot \left(\frac{V_4}{V_5}\right)^{k}$ $p_5 = 890\,kPa$

The process from 4 - 5 is $p(V) = p_4 \cdot \left(\frac{V_4}{V}\right)^{k}$ and $s = \text{constant}$

The work is $W_{45} = \frac{p_4 \cdot V_4 - p_5 \cdot V_5}{k - 1}$ $W_{45} = 1330\,J$ $Q_{45} = 0 \cdot J$

For process 5-1 we again have constant volume

The process from 5 -1 is $V = V_5 = \text{constant}$ and $\Delta s = c_v \cdot \ln\left(\frac{T}{T_5}\right)$

(From Eq. 12.11a)

$Q_{51} = M \cdot c_v \cdot (T_1 - T_5)$ $Q_{51} = -987\,J$ $W_{51} = 0 \cdot J$

The net work is $W_{net} = W_{12} + W_{23} + W_{34} + W_{45} + W_{51}$ $W_{net} = 1531\,J$

The heat added is $Q_{added} = Q_{23} + Q_{34}$ $Q_{added} = 2517\,J$

The efficiency is $\eta = \frac{W_{net}}{Q_{added}}$ $\eta = 60.8\,\%$

This is consistent with the expression from thermodynamics for the diesel efficiency

$$\eta_{diesel} = 1 - \frac{1}{r^{k-1}} \cdot \left[\frac{r_c^{\,k} - 1}{k \cdot (r_c - 1)} \right]$$

where r is the compression ratio $\quad r = \frac{V_1}{V_2} \qquad r = 12.5$

and r_c is the cutoff ratio $\quad r_c = \frac{V_4}{V_3} \qquad r_c = 1.75$

$$\eta_{diesel} = 58.8\,\%$$

The plots of the cycle in *pV* and *Ts* space are shown in the associated *Excel* workbook

Problem 12.10 (In Excel)

12.10 The four-stroke cycle of a typical diesel engine is sometimes modeled as an ideal air-standard closed system. In this simplified system the combustion process is modeled as a heating process, and the exhaust-intake process as a cooling process of the working fluid (air). The cycle consists of: isentropic compression from state ① (p_1 = 100 kPa (abs), T_1 = 20°C, $\forall_1$ = 500 cc) to state ② ($\forall_2 = \forall_1/12.5$); isometric (constant volume) heat addition to state ③ (T_3 = 3000°C); isobaric heat addition to state ④ ($\forall_4 = 1.75\forall_3$); isentropic expansion to state ⑤; and isometric cooling back to state ①. Plot the $p\forall$ and Ts diagrams for this cycle, and find the efficiency, defined as the net work (the cycle area in $p\forall$ space) divided by the heat added.

Given: Data on diesel cycle

Find: Plot of pV and Ts diagrams; efficiency

Solution:

The given, available, or computed data is:

R =	287	J/kg.K
c_p =	1004	J/kg.K
c_v =	717	J/kg.K
k =	1.4	

T_1 =	293	K	p_1 =	100	kPa	V_1 =	500	cc
T_2 =	805	K	p_2 =	3435	kPa	V_2 =	40	cc
T_3 =	3273	K	p_3 =	13963	kPa	V_3 =	40	cc
T_4 =	5728	K	p_4 =	13963	kPa	V_4 =	70	cc
T_5 =	2607	K	p_5 =	890	kPa	V_5 =	500	cc

The process from 1 -2 is $p(V) = p_1 \cdot \left(\frac{V_1}{V}\right)^k$ and $s = \text{constant}$

The process from 2 -3 is $V = V_2 = \text{constant}$ and $\Delta s = c_v \cdot \ln\left(\frac{T}{T_2}\right)$

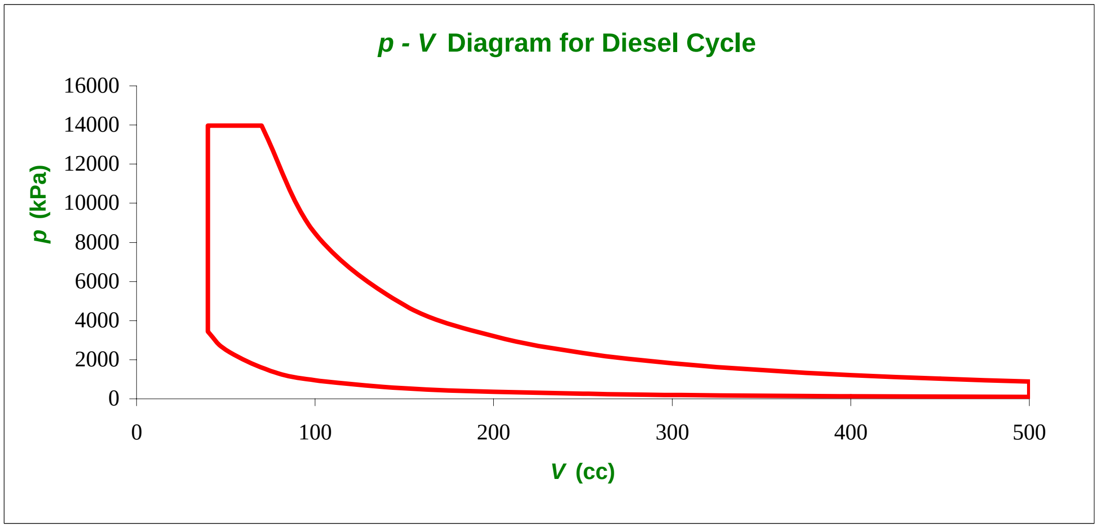
The process from 3 - 4 is $p = p_3 = \text{constant}$ and $\Delta s = c_p \cdot \ln\left(\frac{T}{T_3}\right)$

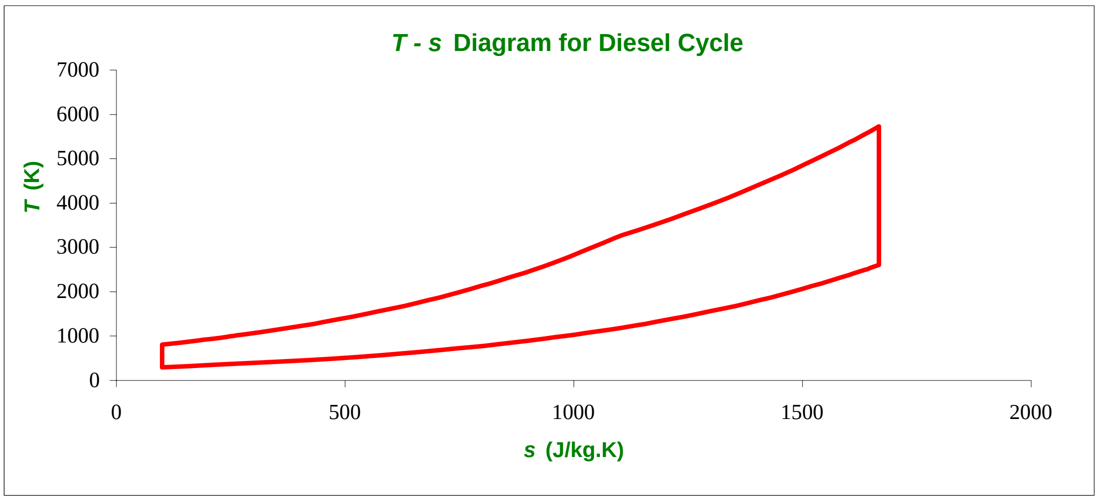
The process from 4 - 5 is $p(V) = p_4 \cdot \left(\frac{V_4}{V}\right)^k$ and $s = \text{constant}$

The process from 5 -1 is $V = V_5 = \text{constant}$ and $\Delta s = c_v \cdot \ln\left(\frac{T}{T_5}\right)$

The computations are:

	V (cc)	*p* (kPa)	*T* (K)	*s* J/kg.K)	
1	500	100	293	100	Initial entropy is arbitrary
	400	137	320	100	Temperatures from Eq. 12.12b
	300	204	359	100	
	250	264	387	100	
	200	361	423	100	
	150	540	474	100	
	100	952	558	100	
	75.0	1425	626	100	
	50.0	2514	736	100	
2	40.0	3435	805	100	Uniform temperature steps
	40.0	3840	900	180	
	40.0	4266	1000	255	
	40.0	5333	1250	415	
	40.0	6399	1500	546	
	40.0	7466	1750	657	
	40.0	8532	2000	752	
	40.0	9599	2250	837	
	40.0	10666	2500	912	
	40.0	11732	2750	981	
3	40.0	13963	3273	1105	Uniform temperature steps
	42.8	13963	3500	1173	
	45.8	13963	3750	1242	
	48.9	13963	4000	1307	
	51.9	13963	4250	1368	
	55.0	13963	4500	1425	
	58.1	13963	4750	1479	
	61.1	13963	5000	1531	
	64.2	13963	5250	1580	
	67.2	13963	5500	1627	
4	70.0	13963	5728	1667	Temperatures from Eq. 12.12b
	100	8474	4966	1667	
	150	4803	4222	1667	
	200	3210	3763	1667	
	250	2349	3441	1667	
	300	1820	3199	1667	
	350	1466	3007	1667	
	400	1216	2851	1667	
	450	1031	2720	1667	
5	500	890	2607	1667	Uniform temperature steps
	500	853	2500	1637	
	500	768	2250	1562	
	500	683	2000	1477	
	500	597	1750	1381	
	500	512	1500	1271	
	500	427	1250	1140	
	500	341	1000	980	
	500	256	750	774	
	500	171	500	483	
1	500	100	293	100	


p - V Diagram for Diesel Cycle
p (kPa)
16000
14000
12000
10000
8000
6000
4000
2000
0
0
100
200
300
400
500
V (cc)


T - s Diagram for Diesel Cycle
T (K)
7000
6000
5000
4000
3000
2000
1000
0
0
500
1000
1500
2000
s (J/kg.K)

Problem 12.11

Given: Air compressed from standard conditions to fill tank with $\forall = 10\ m^3$, $p = 4.5$ MPa (gage). Ideal gas. Reversible.

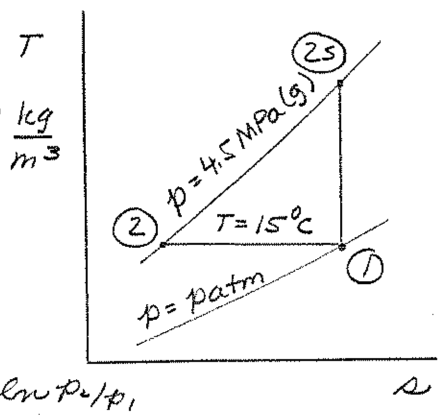
Find: (a) Energy for isothermal compression
(b) Energy for isentropic compression
(c) Energy removed by cooling from (b) to (a)
(d) Sketch Ts & Pv diagrams

Solution: Apply ideal gas, energy, isentropic process equations.

Computing equations: $Q - W = \Delta E = m(u_2 - u_1)$, $W = -\int p d\forall$

$$p/\rho^k = \text{constant} , \quad p = \rho R T$$

The tank contains $m = \rho \forall$



$$\rho = \frac{p}{RT} = (4.5+0.101)10^6 \frac{N}{m^2} \times \frac{kg \cdot K}{287\ N \cdot m} \times \frac{1}{(273+15)\ K} = 55.7\ \frac{kg}{m^3}$$

$$m = \rho \forall = 55.7\ \frac{kg}{m^3} \times 10\ m^3 = 557\ kg$$

For process $1 \to 2$,

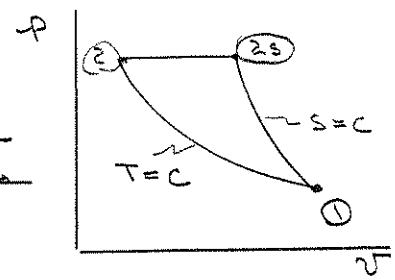
$$-W = -\int p d\forall = -m \int p dv = -m \int RT \frac{dp}{p} = -mRT \ln p_2/p_1$$

$$W_{12} = -557\ kg \times 287 \frac{J}{kg \cdot K} \times (273+15)\ K \times \ln\left(\frac{4.5+0.101}{0.101}\right) = 176\ MJ$$ W_{12}

For process $2 \to 2s$, $\frac{p_{2s}}{p_1} = \left(\frac{\rho_{2s}}{\rho_1}\right)^k \to \frac{T_{2s}}{T_1} = \left(\frac{p_{2s}}{p_1}\right)^{\frac{k-1}{k}} = \left(\frac{4.601}{0.101}\right)^{0.286} = 2.98$

$$T_{2s} = 2.98\ T_1 = 2.98(273+15)K = 858\ K$$ T_{2s}

$$W_{12s} = m(u_{2s} - u_1) - \cancel{Q_{12s}}^{\,0} = m c_v (T_{2s} - T_1)$$



$$W_{12s} = 557\ kg \times 0.717 \frac{kJ}{kg \cdot K} (858-288)K = 228\ MJ$$ W_{12s}

Process $2s \to 2$ is at constant pressure

$$Q_{2s2} = m(u_2 - u_{2s}) - W_{2s2} = m(u_2 - u_{2s}) - mp(v_2 - v_{2s}) = m(h_2 - h_{2s})$$

$$= m c_p (T_2 - T_{2s})$$

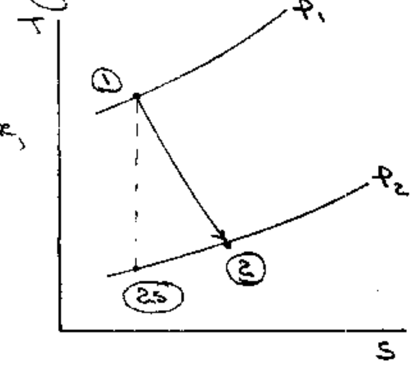
$$Q_{2s2} = 557\ kg \times 1.00 \frac{kJ}{kg \cdot K} \times (288-858)K = -317\ MJ \quad \text{(out of air)}$$ Q_{2s2}

Problem 12.12

Given: Steady flow of air, $\dot{m} = 0.5$ kg/s, through a turbine.
At inlet, $V_1 \approx 0$, $T_1 = 1300°C$, $p_1 = 2.0$ MPa (abs).
At outlet, $V_2 = 200$ m/s, $p_2 = 101$ kPa, $T_2 = 500°C$

Find: (a) power produced by the turbine
(b) label state points on a Ts diagram

Solution:



For an isentropic expansion through the turbine,

$$T_{2s=c} = T_1 \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$$

$$T_{2s=c} = (1300+273)K \left(\frac{0.101\ MPa}{2.0\ MPa}\right)^{\frac{1.4-1}{1.4}}$$

$$T_{2s=c} = 670\ K\ (397°C)$$

Writing the first law of thermodynamics between the turbine inlet and outlet

$$\dot{W} + \dot{Q} = \dot{m}\left[\left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \frac{V_1^2}{2}\right)\right] \quad (V_1 \approx 0)$$

(Assume $\dot{Q} = 0$)

For an ideal gas with constant specific heats, $h_2 - h_1 = c_p(T_2 - T_1)$

$$\therefore \dot{W} = \dot{m}\left[c_p(T_2 - T_1) - \frac{V_2^2}{2}\right]$$

$$= 0.5\ \frac{kg}{s}\left[1004\ \frac{N.m}{kg.K}(773 - 1573)K - \frac{1}{2}(200)^2\ \frac{m^2}{s^2} \times \frac{N.s^2}{kg.m}\right]$$

$$\dot{W} = -392 \times 10^3\ \frac{N.m}{s}$$ (negative sign indicates work out).

$$\therefore \dot{W}_{out} = 392\ kW$$ $\dot{W}_{out}$

Problem 12.13

Given: Natural gas (thermodynamic properties of methane) flows in a pipe of diameter, $D = 0.6\text{m}$. At compressor inlet: $T_1 = 13°C$, $V_1 = 32\ \text{m/s}$, $p_1 = 0.5\ \text{MPa (gage)}$. At compressor outlet, $p_2 = 8.0\ \text{MPa (gage)}$. The compressor efficiency $\eta_c = 0.85$.

Find: (a) $\dot{m}$ (b) T_2, V_2 (c) $\dot{W}_{in}$ d) label state points on Ts diagram

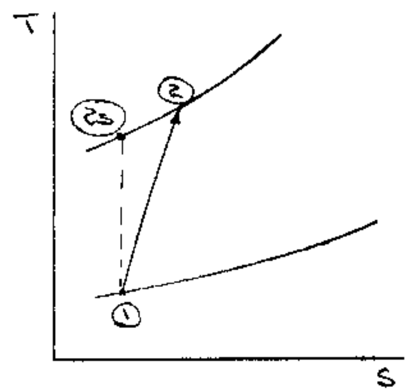
Solution:

The mass flow rate is given by $\dot{m} = \rho V A$ where $\rho = \frac{p}{RT}$

$$\therefore \dot{m} = \frac{p_1}{RT_1} V_1 \frac{\pi D^2}{4} = (500+101)10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{518.3\ N \cdot m} \times \frac{1}{286\ K} \times 32\ \frac{m}{s} \times \frac{\pi}{4}(0.6)^2 m^2$$

$\dot{m} = 36.7\ \text{kg/s}$ ← $\dot{m}$

For an isentropic compression



$$T_{2s} = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = 286\text{K} \left(\frac{8.101\ \text{MPa}}{0.601\ \text{MPa}}\right)^{\frac{1.31-1}{1.31}} = 529\ \text{K}$$

$$\eta_c = \frac{T_{2s} - T_1}{T_2 - T_1} \qquad \therefore T_2 - T_1 = \frac{T_{2s} - T_1}{\eta_c}$$

$$T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_c} = 286\text{K} + \frac{(529-286)\text{K}}{0.85}$$

$T_2 = 572\ \text{K}$ ← T_2

From continuity, $\dot{m} = \rho_1 V_1 A_1 = \rho_2 V_2 A_2$. Assuming $A_1 = A_2$, then

$$V_2 = \frac{\rho_1}{\rho_2} V_1 = \frac{p_1}{p_2}\frac{T_2}{T_1} V_1 = \frac{0.601}{8.101} \times \frac{572}{286} \times 32\ \frac{m}{s} = 4.75\ \text{m/s}$$ ← V_2

Writing the first law of thermodynamics between compressor inlet + outlet

$$\dot{W} + \dot{Q} = \dot{m}\left[\left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \frac{V_1^2}{2}\right)\right] \qquad (\text{Assume } \dot{Q} = 0)$$

$$\dot{W} = \dot{m}\left[(h_2 - h_1) + \tfrac{1}{2}(V_2^2 - V_1^2)\right] = \dot{m}\left[c_p(T_2 - T_1) + \tfrac{1}{2}(V_2^2 - V_1^2)\right]$$

$$\dot{W} = 36.7 \frac{kg}{s}\left[2190 \frac{N \cdot m}{kg \cdot K}(572-286)K + \frac{1}{2}\left\{(4.75)^2 - (32)^2\ \frac{m^2}{s^2} \times \frac{N \cdot s^2}{kg \cdot m}\right\}\right]$$

$$\dot{W} = 36.7\left[626 \times 10^3 - 501\right] \frac{N \cdot m}{s}$$

$\dot{W} = 23\ \text{MW}$ ← $\dot{W}_{in}$

Problem 12.14

12.14 Over time the efficiency of the compressor of Problem 12.13 drops. At what efficiency will the power required to attain 8.0 MPa (gage) exceed 30 MW? Plot the required power and the gas exit temperature as functions of efficiency.

Given: Data on flow through compressor

Find: Efficiency at which power required is 30 MW; plot required efficiency and exit temperature as functions of efficiency

Solution:

The data provided, or available in the Appendices, is:

$$R = 518.3 \cdot \frac{J}{kg \cdot K} \qquad c_p = 2190 \cdot \frac{J}{kg \cdot K} \qquad c_v = c_p - R \qquad c_v = 1672 \frac{J}{kg \cdot K} \qquad k = \frac{c_p}{c_v} \qquad k = 1.31$$

$$T_1 = (13 + 273) \cdot K \qquad p_1 = 0.5 \cdot MPa + 101 \cdot kPa \qquad V_1 = 32 \cdot \frac{m}{s}$$

$$p_2 = 8 \cdot MPa + 101 \cdot kPa \qquad W_{comp} = 30 \cdot MW \qquad D = 0.6 \cdot m$$

The governing equation is the first law of thermodynamics for the compressor

$$M_{flow} \cdot \left[\left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \frac{V_1^2}{2}\right)\right] = W_{comp} \qquad \text{or} \qquad W_{comp} = M_{flow} \cdot \left[c_p \cdot (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}\right]$$

We need to find the mass flow rate and the temperature and velocity at the exit

$$M_{flow} = \rho_1 \cdot A_1 \cdot V_1 = \frac{p_1}{R \cdot T_1} \cdot \frac{\pi}{4} \cdot D^2 \cdot V_1 \qquad M_{flow} = \frac{p_1}{R \cdot T_1} \cdot \frac{\pi}{4} \cdot D^2 \cdot V_1 \qquad M_{flow} = 36.7 \frac{kg}{s}$$

The exit velocity is then given by $M_{flow} = \frac{p_2}{R \cdot T_2} \cdot \frac{\pi}{4} \cdot D^2 \cdot V_2 \qquad V_2 = \frac{4 \cdot M_{flow} \cdot R \cdot T_2}{\pi \cdot p_2 \cdot D^2}$ (1)

The exit velocity cannot be computed until the exit temperature is determined!

Using Eq. 1 in the first law

$$W_{comp} = M_{flow} \cdot \left[c_p \cdot (T_2 - T_1) + \frac{\left(\frac{4 \cdot M_{flow} \cdot R \cdot T_2}{\pi \cdot p_2 \cdot D^2}\right)^2 - V_1^2}{2}\right]$$

In this complicated expression the only unknown is T_2, the exit temperature. The equation is a quadratic, so is solvable explicitly for T_2, but instead we use *Excel*'s *Goal Seek* to find the solution (the second solution is mathematically correct but physically unrealistic - a very large negative absolute temperature). The exit temperature is $T_2 = 660 \cdot K$

If the compressor was ideal (isentropic), the exit temperature would be given by

$$T \cdot p^{\frac{1-k}{k}} = \text{constant} \qquad (12.12b)$$

Hence $T_{2s} = T_1 \cdot \left(\frac{p_1}{p_2}\right)^{\frac{1-k}{k}}$ $T_{2s} = 529\,K$

For a compressor efficiency η, we have $\eta = \frac{h_{2s} - h_1}{h_2 - h_1}$ or $\eta = \frac{T_{2s} - T_1}{T_2 - T_1}$ $\eta = 65.1\,\%$

To plot the exit temperature and power as a function of efficiency we use $T_2 = T_1 + \frac{T_{2s} - T_1}{\eta}$

with $V_2 = \frac{4 \cdot M_{flow} \cdot R \cdot T_2}{\pi \cdot p_2 \cdot D^2}$ and $W_{comp} = M_{flow} \cdot \left[c_p \cdot (T_2 - T_1) + \frac{V_2^{\,2} - V_1^{\,2}}{2}\right]$

The dependencies of T_2 and W_{comp} on efficiency are plotted in the associated *Excel* workbook

Problem 12.14 (In Excel)

[3]

12.14 Over time the efficiency of the compressor of Problem 12.13 drops. At what efficiency will the power required to attain 8.0 MPa (gage) exceed 30 MW? Plot the required power and the gas exit temperature as functions of efficiency.

Given: Data on flow through compressor

Find: Efficiency at which power required is 30 MW; plot required efficiency and exit temperature as functions of efficiency

Solution:

The given or available data is:

R =	518.3	J/kg.K
c_p =	2190	J/kg.K
c_v =	1672	J/kg.K
k =	1.31	
T_1 =	286	K
p_1 =	601	kPa
V_1 =	32	m/s
p_2 =	8101	kPa
D =	0.6	m/s
W_{comp} =	30	MW

Computed results:

$$M_{flow} = \frac{p_1}{R \cdot T_1} \cdot \frac{\pi}{4} \cdot D^2 \cdot V_1$$

M_{flow} = 36.7 kg/s

$$W_{comp} = M_{flow} \cdot \left[c_p \cdot (T_2 - T_1) + \frac{\left(\frac{4 \cdot M_{flow} \cdot R \cdot T_2}{\pi \cdot p_2 \cdot D^2} \right)^2 - V_1^2}{2} \right]$$

Use *Goal Seek* to vary T_2 below so that the error between the left and right sides is zero!

T_2 = 660 K

LHS (MW)	RHS (MW)	Error
30.0	30.0	0.00%

$$T_{2s} = T_1 \cdot \left(\frac{p_1}{p_2} \right)^{\frac{1-k}{k}}$$

T_{2s} = 529 K

$$\eta = \frac{h_{2s} - h_1}{h_2 - h_1}$$

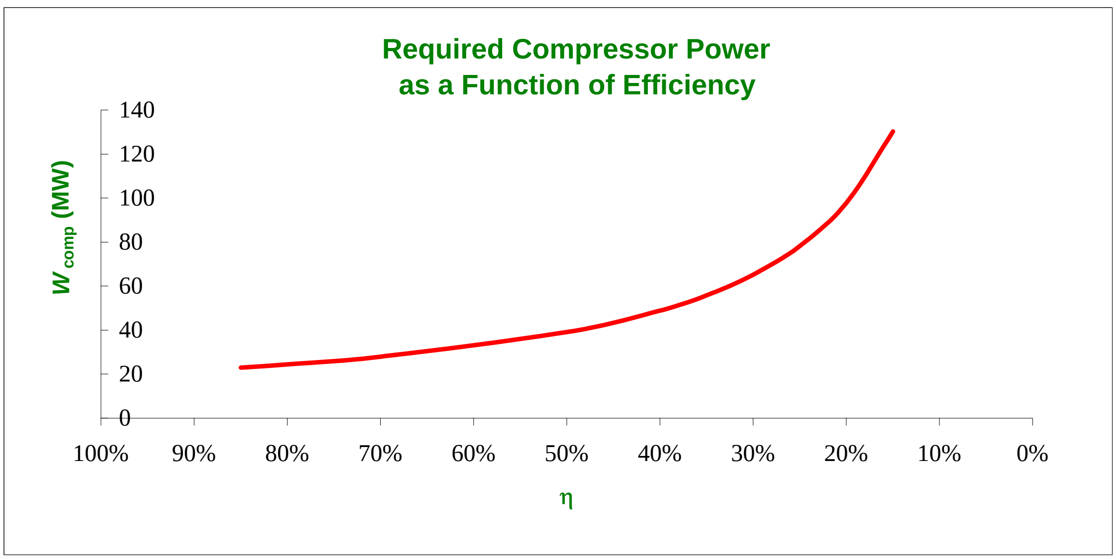
η = 65.1%

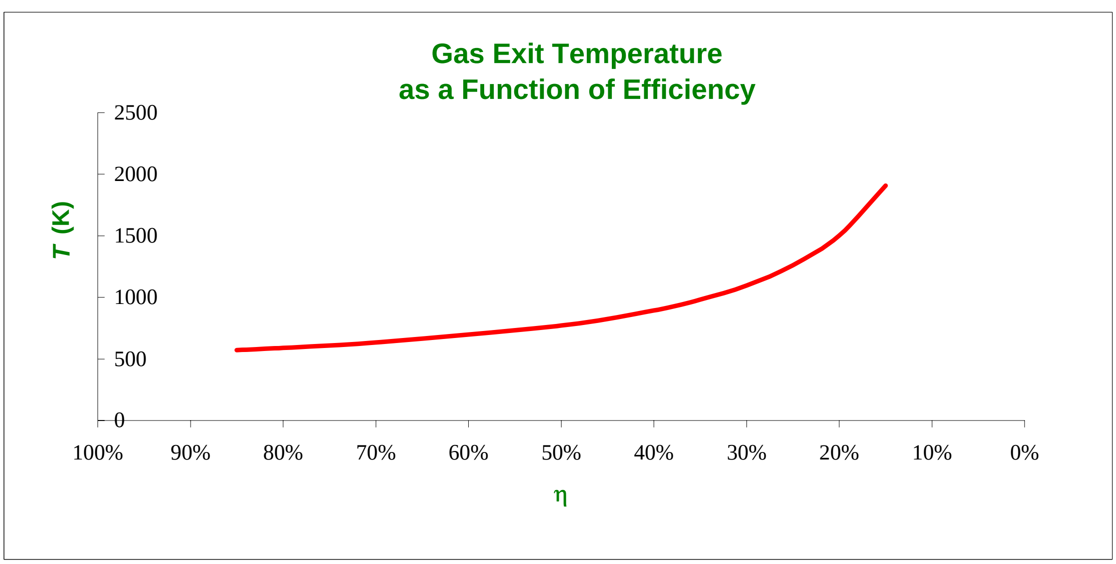
$$\eta = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$V_2 = \frac{4 \cdot M_{flow} \cdot R \cdot T_2}{\pi \cdot p_2 \cdot D^2}$$

$$W_{comp} = M_{flow} \cdot \left[c_p \cdot (T_2 - T_1) + \frac{V_2^{\,2} - V_1^{\,2}}{2} \right]$$

η	T_2 (K)	V_2 (m/s)	W_{comp} (MW)
85%	572	4.75	23
80%	590	4.90	24
70%	634	5.26	28
50%	773	6.41	39
40%	894	7.42	49
35%	981	8.14	56
30%	1097	9.11	65
25%	1259	10.45	78
20%	1503	12.47	98
15%	1908	15.84	130





Problem 12.15

[4]

Given: Balloon inflated isothermally from $r = 5$ to $r = 7$ in.

Flow is $Q = 0.10$ cfm of standard air (59F, 14.7 psia)

Balloon skin tension is $\sigma = kA$, where $k = 200\ \text{lbf/ft}^3$, and A = surface area of balloon.

Find: Time required.

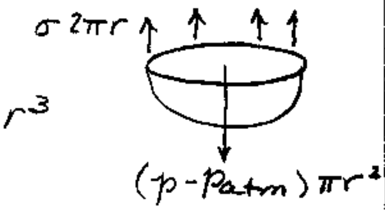
Solution: The mass flow rate is $\dot{m} = \rho_{std} Q$ = constant, so

Computing equation: $\Delta t = \dfrac{\Delta m}{\dot{m}}$ $\qquad p = \rho R T$

Assume: (1) Standard air, $\rho = 0.0765\ \text{lbm/ft}^3$; (2) Ideal gas

Then $\dot{m} = \rho Q = 0.0765 \dfrac{\text{lbm}}{\text{ft}^3} \times 0.10 \dfrac{\text{ft}^3}{\text{min}} \times \dfrac{\text{min}}{60\ \text{s}} = 1.28 \times 10^{-4}\ \text{lbm/s}$

From a force balance on the balloon:



$$(p - p_{atm})\pi r^2 = \sigma 2\pi r = k(4\pi r^2) 2\pi r = 8\pi^2 k r^3$$

or $p = p_{atm} + 8\pi k r$

For $r = 5$ in., $p = 14.7 + 8\pi \times 200 \dfrac{\text{lbf}}{\text{ft}^3} \times 5\ \text{in.} \times \dfrac{\text{ft}^3}{1728\ \text{in.}^3} = 29.2$ psia

$$\rho = \frac{p}{RT} = 29.2 \frac{\text{lbf}}{\text{in.}^2} \times \frac{\text{lbm}\cdot{}^\circ\text{R}}{53.3\ \text{ft}\cdot\text{lbf}} \times \frac{1}{519^\circ\text{R}} \times 144 \frac{\text{in.}^2}{\text{ft}^2} = 0.152\ \text{lbm/ft}^3$$

$$\forall = \frac{4}{3}\pi r^3 = \frac{4\pi}{3} \times (5)^3\ \text{in.}^3 \times \frac{\text{ft}^3}{1728\ \text{in.}^3} = 0.303\ \text{ft}^3$$

$$m = \rho \forall = 0.152 \frac{\text{lbm}}{\text{ft}^3} \times 0.303\ \text{ft}^3 = 0.0461\ \text{lbm}$$

For $r = 7$ in., $p = 14.7 \dfrac{\text{lbf}}{\text{in.}^2} + 8\pi \times 200 \dfrac{\text{lbf}}{\text{ft}^3} \times 7\ \text{in.} \times \dfrac{\text{ft}^3}{1728\ \text{in.}^3} = 35.1$ psia

Tabulating,

r (in.)	p (psia)	ρ (lbm/ft³)	∀ (ft³)	m (lbm)
5	29.2	0.152	0.303	0.0461
7	35.1	0.183	0.831	0.152

Then $\Delta m = m_7 - m_5 = 0.152 - 0.0461\ \text{lbm} = 0.106\ \text{lbm}$

and

$$\Delta t = 0.106\ \text{lbm} \times \frac{\text{s}}{1.28 \times 10^{-4}\ \text{lbm}} = 828\ \text{s} \quad (\simeq 14\ \text{min})$$

Δt

Problem 12.16

12.16 For the balloon process of Problem 12.15 we could define a "volumetric ratio" as the ratio of the volume of standard air supplied to the volume increase of the balloon, per unit time. Plot this ratio over time as the balloon radius is increased from 5 to 7 inches.

Given: Data on flow rate and balloon properties

Find: "Volumetric efficiency" over time

Solution:

The given or available data is:

R =	53.3	ft.lbf/lb°R
T_{atm} =	519	R
p_{atm} =	14.7	psi
k =	200	lbf/ft^3
V_{rate} =	0.1	ft^3/min

Computing equations:

Standard air density $$\rho_{air} = \frac{p_{atm}}{R \cdot T_{atm}}$$

Mass flow rate $$M_{rate} = V_{rate} \cdot \rho_{air}$$

From a force balance on each hemisphere $$\left(p - p_{atm}\right) \cdot \pi \cdot r^2 = \sigma \cdot 2 \cdot \pi \cdot r \quad \text{where} \quad \sigma = k \cdot A = k \cdot 4 \cdot \pi \cdot r^2$$

Hence $$p = p_{atm} + \frac{2 \cdot \sigma}{r} \quad \text{or} \quad p = p_{atm} + 8 \cdot \pi \cdot k \cdot r$$

Density in balloon $$\rho = \frac{p}{R \cdot T_{air}}$$

The instantaneous volume is $$V_{ball} = \frac{4}{3} \cdot \pi \cdot r^3$$

The instantaneous mass is $$M_{ball} = V_{ball} \cdot \rho$$

The time to fill to radius r from r = 5 in is $$t = \frac{M_{ball}(r) - M_{ball}(r = 5in)}{M_{rate}}$$

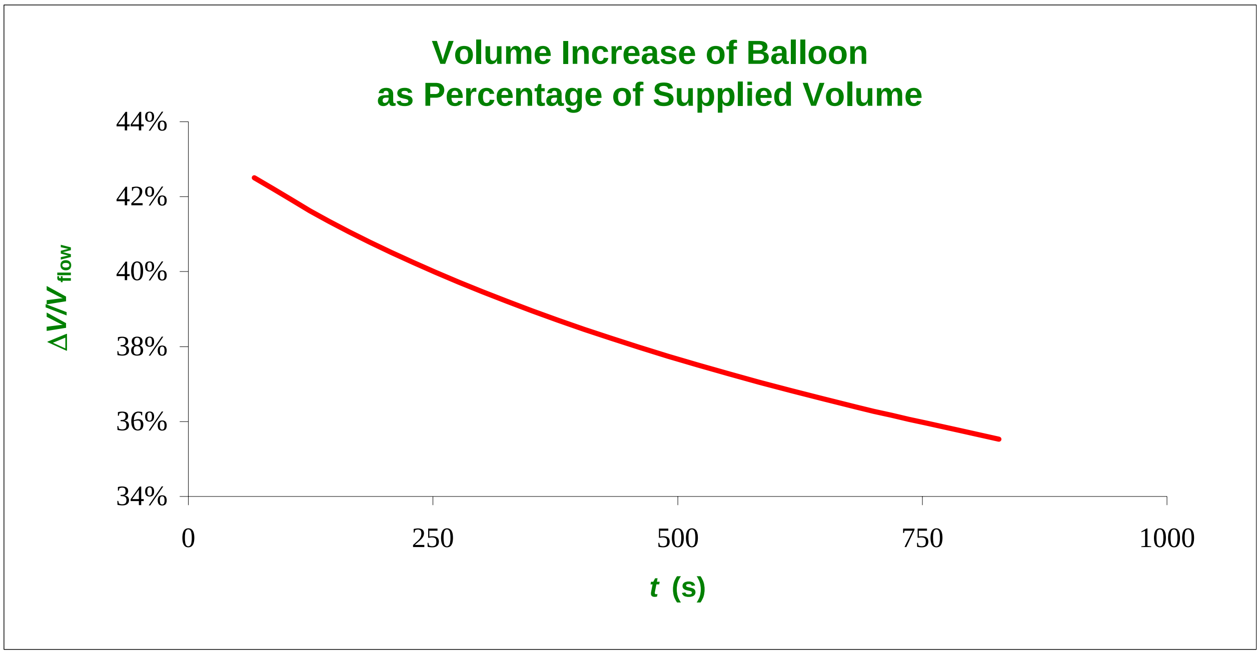
The volume change between time steps Δt is $$\Delta V = V_{ball}(t + \Delta t) - V_{ball}(t)$$

Computed results:

ρ_{air} = 0.0765 lb/ft^3

M_{rate} = 0.000128 lb/s

r (in)	p (psi)	ρ (lb/ft^3)	V_{ball} (ft^3)	M_{ball} (lb)	t (s)	$\Delta V/V_{rate}$
5.00	29.2	0.152	0.303	0.0461	0.00	0.00
5.25	30.0	0.156	0.351	0.0547	67.4	42.5%
5.50	30.7	0.160	0.403	0.0645	144	41.3%
5.75	31.4	0.164	0.461	0.0754	229	40.2%
6.00	32.2	0.167	0.524	0.0876	325	39.2%
6.25	32.9	0.171	0.592	0.101	433	38.2%
6.50	33.6	0.175	0.666	0.116	551	37.3%
6.75	34.3	0.179	0.746	0.133	683	36.4%
7.00	35.1	0.183	0.831	0.152	828	35.5%



Problem 12.17

12.17 A sound pulse level above about 20 Pa can cause permanent hearing damage. Assuming such a sound wave travels through air at 20°C and 100 kPa, estimate the density, temperature, and velocity change immediately after the sound wave passes.

Given: Sound wave

Find: Estimate of change in density, temperature, and velocity after sound wave passes

Solution:

Basic equation: $p = \rho \cdot R \cdot T$ $\quad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$

$du = c_v \cdot dT \quad dh = c_p \cdot dT$

Assumptions: 1) Ideal gas 2) Constant specific heats 3) Isentropic process 4) infinitesimal changes

Given or available data

$T_1 = (20 + 273) \cdot K \quad p_1 = 100 \cdot kPa \quad dp = 20 \cdot Pa \quad k = 1.4 \quad R = 286.9 \frac{J}{kg \cdot K}$

$c = \sqrt{k \cdot R \cdot T_1} \quad c = 343 \frac{m}{s}$

For small changes, from Section 11-2 $\quad dp = c^2 \cdot d\rho \quad$ so $\quad d\rho = \frac{dp}{c^2} \quad d\rho = 1.70 \times 10^{-4} \cdot \frac{kg}{m^3}$ a very small change!

The air density is $\rho_1 = \frac{p_1}{R \cdot T_1} \quad \rho_1 = 1.19 \frac{kg}{m^3}$

Then $\quad dV_x = \frac{1}{\rho_1 \cdot c} \cdot dp \quad dV_x = 0.049 \frac{m}{s}$ This is the velocity of the air after the sound wave!

For the change in temperature we start with the ideal gas equation $p = \rho \cdot R \cdot T$ and differentiate $dp = d\rho \cdot R \cdot T + \rho \cdot R \cdot dT$

Dividing by the ideal gas equation we find $\quad \frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$

Hence $\quad dT = T_1 \cdot \left(\frac{dp}{p_1} - \frac{d\rho}{\rho_1}\right) \quad dT = 0.017 K \quad dT = 0.030 \cdot \Delta°F$ a very small change!

Problem 12.18

[3]

12.18 The bulk modulus E_v of a material indicates how hard it is to compress the material; a large E_v indicates the material requires a large pressure to compress. Is air "stiffer" when suddenly or slowly compressed? To answer this, find expressions in terms of instantaneous pressure p for the bulk modulus of air (kPa) when it is a) rapidly compressed and b) slowly compressed. *Hint*: Rapid compression is approximately isentropic (it is adiabatic because it is too quick for heat transfer to occur), and slow compression is isothermal (there is plenty of time for the air to equilibrate to ambient temperature).

Given: Sound wave

Find: Estimate of change in density, temperature, and velocity after sound wave passes

Solution:

Basic equations: $p = \rho \cdot R \cdot T$ $\qquad$ $E_v = \dfrac{dp}{\frac{d\rho}{\rho}}$

Assumptions: 1) Ideal gas 2) Constant specific heats 3) Infinitesimal changes

To find the bulk modulus we need $\dfrac{dp}{d\rho}$ in $\qquad$ $E_v = \dfrac{dp}{\frac{d\rho}{\rho}} = \rho \cdot \dfrac{dp}{d\rho}$

For rapid compression (isentropic) $\dfrac{p}{\rho^k} = \text{const}$ and so $\dfrac{dp}{d\rho} = k \cdot \dfrac{p}{\rho}$

Hence $\qquad E_v = \rho \cdot \left(k \cdot \dfrac{p}{\rho}\right)$ $\qquad$ $E_v = k \cdot p$

For gradual compression (isothermal) we can use the ideal gas equation $\qquad p = \rho \cdot R \cdot T$ $\qquad$ so $\qquad dp = d\rho \cdot R \cdot T$

Hence $\qquad E_v = \rho \cdot (R \cdot T) = p$ $\qquad$ $E_v = p$

We conclude that the "stiffness" (E_v) of air is equal to kp when rapidly compressed and p when gradually compressed. To give an idea of v

For water $\qquad E_v = 2.24 \cdot \text{GPa}$

For air ($k = 1.4$) at $p = 101 \cdot \text{kPa}$ $\qquad$ Rapid compression $\qquad E_v = k \cdot p$ $\qquad E_v = 141 \cdot \text{kPa}$

Gradual compression $\qquad E_v = p$ $\qquad E_v = 101 \cdot \text{kPa}$

Problem 12.19

[2]

12.19 You have designed a device for determining the bulk modulus, E_v, of a material. It works by measuring the time delay between sending a sound wave into a sample of the material and receiving the wave after it travels through the sample and bounces back. As a test, you use a 1 m rod of steel ($E_v \approx$ 200 GN/m^2). What time delay should your device indicate? You now test a 1 m rod (1 cm diameter) of an unknown material and find a time delay of 0.5 ms. The mass of the rod is measured to be 0.25 kg. What is this material's bulk modulus?

Given: Device for determining bulk modulus

Find: Time delay; Bulk modulus of new material

Solution:

Basic equation: $c = \sqrt{\frac{E_v}{\rho}}$

Hence for given data $E_v = 200 \cdot \frac{GN}{m^2}$ $\quad L = 1 \cdot m$ $\quad$ and for steel $\quad SG = 7.83$ $\quad \rho_w = 1000 \cdot \frac{kg}{m^3}$

For the steel $c = \sqrt{\frac{E_v}{SG \cdot \rho_w}}$ $\quad c = 5054 \frac{m}{s}$

Hence the time to travel distance L is $\Delta t = \frac{L}{c}$ $\quad \Delta t = 1.98 \times 10^{-4} s$ $\quad \Delta t = 0.198 ms$ $\quad \Delta t = 198 \mu s$

For the unknown material $M = 0.25 \cdot kg$ $\quad D = 1 \cdot cm$ $\quad \Delta t = 0.5 \cdot ms$

The density is then $\rho = \frac{M}{L \cdot \frac{\pi \cdot D^2}{4}}$ $\quad \rho = 3183 \frac{kg}{m^3}$

The speed of sound in it is $c = \frac{L}{\Delta t}$ $\quad c = 2000 \frac{m}{s}$

Hence th bulk modulus is $E_v = \rho \cdot c^2$ $\quad E_v = 12.7 \frac{GN}{m^2}$

Problem 12.20

[2]

12.20 Dolphins often hunt by listening for sounds made by their prey. They "hear" with the lower jaw, which conducts the sound vibrations to the middle ear via a fat-filled cavity in the lower jaw bone. If the prey is 1000 m away, how long after a sound is made does a dolphin hear it? Assume the seawater is at 20°C.

Given: Hunting dolphin

Find: Time delay before it hears prey at 1000 m

Solution:

Basic equation: $c = \sqrt{\dfrac{E_v}{\rho}}$

Given (and Table A.2) data $L = 1000\cdot m$ $\quad SG = 1.025 \quad E_v = 2.42\cdot\dfrac{GN}{m^2} \quad \rho_w = 1000\cdot\dfrac{kg}{m^3}$

For the seawater $c = \sqrt{\dfrac{E_v}{SG\cdot\rho_w}} \qquad c = 1537\dfrac{m}{s}$

Hence the time for sound to travel distance L is $\Delta t = \dfrac{L}{c} \qquad \Delta t = 0.651\cdot s \qquad \Delta t = 651\cdot ms$

Problem 12.21

12.21 A submarine sends a sonar signal to detect the enemy. The reflected wave returns after 25 s. Estimate the separation between the submarines. (As an approximation assume the seawater is at 20°C.)

Given: Submarine sonar

Find: Separation between submarines

Solution:

Basic equation: $c = \sqrt{\dfrac{E_v}{\rho}}$

Given (and Table A.2) data $\Delta t = 25 \cdot s$ $\quad SG = 1.025$ $\quad E_v = 2.42 \cdot \dfrac{GN}{m^2}$ $\quad \rho_w = 1000 \cdot \dfrac{kg}{m^3}$

For the seawater $c = \sqrt{\dfrac{E_v}{SG \cdot \rho_w}}$ $\quad c = 1537 \dfrac{m}{s}$

Hence the distance sound travels in time Δt is $L = c \cdot \Delta t$ $\quad L = 38.4\,km$

The distance between submarines is half of this $x = \dfrac{L}{2}$ $\quad x = 19.2\,km$

Problem 12.22

[1]

12.22 An airplane flies at 400 mph at 1600 ft altitude on a standard day. The plane climbs to 50,000 ft and flies at 725 mph. Calculate the Mach number of flight in both cases.

Given: Airplane cruising at two different elevations

Find: Mach numbers

Solution:

Basic equation: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$

Available data $R = 286.9 \frac{J}{kg \cdot K}$ $\quad k = 1.4$

At $z = 1600 \cdot ft$ $\quad z = 488\,m$ interpolating from Table A.3 $T = 288.2 \cdot K + \frac{(284.9 - 288.2) \cdot K}{(500 - 0) \cdot m} \cdot (z - 0m)$

$T = 285\,K$

Hence $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 338 \frac{m}{s}$ $\quad c = 757\,mph$ and we have $V = 400 \cdot mph$

The Mach number is $M = \frac{V}{c}$ $\quad M = 0.529$

Repeating at $z = 50000 \cdot ft$ $\quad z = 15240\,m$ $\quad T = 216.7 \cdot K$

Hence $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 295 \frac{m}{s}$ $\quad c = 660\,mph$ and we have $V = 725 \cdot mph$

The Mach number is $M = \frac{V}{c}$ $\quad M = 1.10$

Problem 12.23

Given: The Lockheed SR-71 aircraft is thought to cruise at $M = 3.3$ at altitude $z = 85{,}000$ ft.

Find: (a) speed of sound and flight speed for these conditions.
(b) Compare speed to muzzle speed (700 m/sec) of a 30-06 rifle bullet.

Solution:

At altitude, $z = 85{,}000 \text{ ft} \times 0.3048 \frac{\text{m}}{\text{ft}} = 25.9 \text{ km}$

From Table A.3, $T = 222 \text{ K}$

$$\therefore c = \sqrt{kRT} = \left[1.4 \times 287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 222\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^{1/2} = 299 \text{ m/s}$$ c

$$V = Mc = 3.3 \times 299 \text{ m/s} = 987 \text{ m/s}$$ V

$$\frac{V}{V_{bullet}} = \frac{987}{700} = 1.41$$

Problem 12.24

Given: Boeing 727 cruises at 520 mi/hr at an altitude of 33,000 ft on a standard day

Find: (a) cruise Mach number of the aircraft
(b) flight speed corresponding to $M_{max} = 0.9$

Solution:

At 33,000 ft, $z = 10.06$ km. From Table A.3, $T = 223$ K.

Then, $c = \sqrt{kRT} = \left[1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 223 K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 299$ m/s

$$V = 520 \frac{mi}{hr} \times 5280 \frac{ft}{mi} \times \frac{hr}{3600 s} \times 0.3048 \frac{m}{ft} = 232 \text{ m/s}$$

$$M = \frac{V}{c} = \frac{232 \text{ m/s}}{299 \text{ m/s}} = 0.776$$ M_{cr}

At $M = 0.90$

$$V = Mc = 0.90 \times 299 \text{ m/s} = 269 \text{ m/s} \quad (603 \text{ mph})$$ V

Problem 12.25

[2]

12.25 Investigate the effect of altitude on Mach number by plotting the Mach number of a 500 mph airplane as it flies at altitudes ranging from sea level to 10 km.

Given: Airplane cruising at 550 mph

Find: Mach number versus altitude

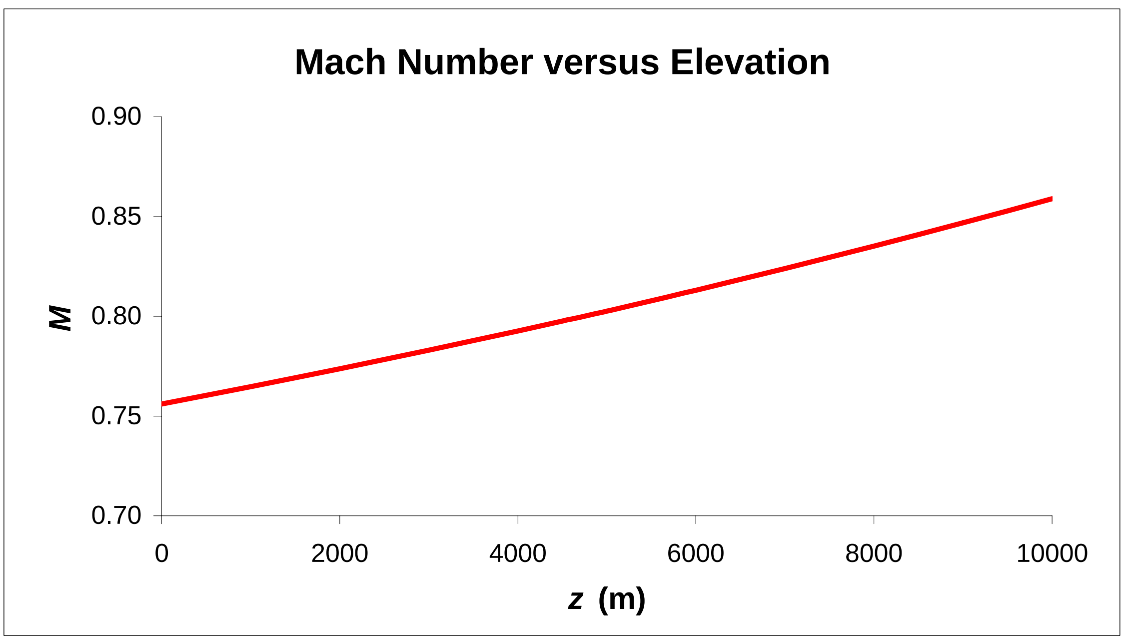
Solution:

Basic equation: $c = \sqrt{k \cdot R \cdot T}$ $\qquad M = \frac{V}{c}$

V = 500 mph
R = 286.90 J/kg·K (Table A.6)
k = 1.40

Data on temperature versus height obtained from Table A.3

z (m)	*T* (K)	*c* (m/s)	*c* (mph)	*M*
0	288.2	340	661	0.756
500	284.9	338	658	0.760
1000	281.7	336	654	0.765
1500	278.4	334	650	0.769
2000	275.2	332	646	0.774
2500	271.9	330	642	0.778
3000	268.7	329	639	0.783
3500	265.4	326	635	0.788
4000	262.2	325	631	0.793
4500	258.9	322	627	0.798
5000	255.7	320	623	0.803
6000	249.2	316	615	0.813
7000	242.7	312	607	0.824
8000	236.2	308	599	0.835
9000	229.7	304	590	0.847
10000	223.3	299	582	0.859



Problem 12.26

12.26 You are watching a July 4th fireworks display from a distance of one mile. How long after you see an explosion do you hear it? You also watch New Year's fireworks (same place and distance). How long after you see an explosion do you hear it? Assume it's 75°F in July and 5°F in January.

Given: Fireworks displays!

Find: How long after seeing them do you hear them?

Solution:

Basic equation: $c = \sqrt{k \cdot R \cdot T}$

Assumption: Speed of light is essentially infinite (compared to speed of sound)

The given or available data is $T_{July} = (75 + 460) \cdot R$ $L = 1 \cdot mi$ $k = 1.4$ $R_{air} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$

Hence $c_{July} = \sqrt{k \cdot R_{air} \cdot T_{July}}$ $c_{July} = 1134 \frac{ft}{s}$

Then the time is $\Delta t_{July} = \frac{L}{c_{July}}$ $\Delta t_{July} = 4.66\,s$

In January $T_{Jan} = (5 + 460) \cdot R$

Hence $c_{Jan} = \sqrt{k \cdot R_{air} \cdot T_{Jan}}$ $c_{Jan} = 1057 \frac{ft}{s}$

Then the time is $\Delta t_{Jan} = \frac{L}{c_{Jan}}$ $\Delta t_{Jan} = 5.00\,s$

Problem 12.27

12.27 Use data for specific volume to calculate and plot the speed of sound in saturated liquid water over the temperature range from 0 to 200°C.

Given: Data on water specific volume

Find: Speed of sound over temperature range

Solution:

Basic equation: $c = \sqrt{\frac{\partial}{\partial\rho} p}$ at isentropic conditions

As an approximation for a liquid $c = \sqrt{\frac{\Delta p}{\Delta\rho}}$ using available data.

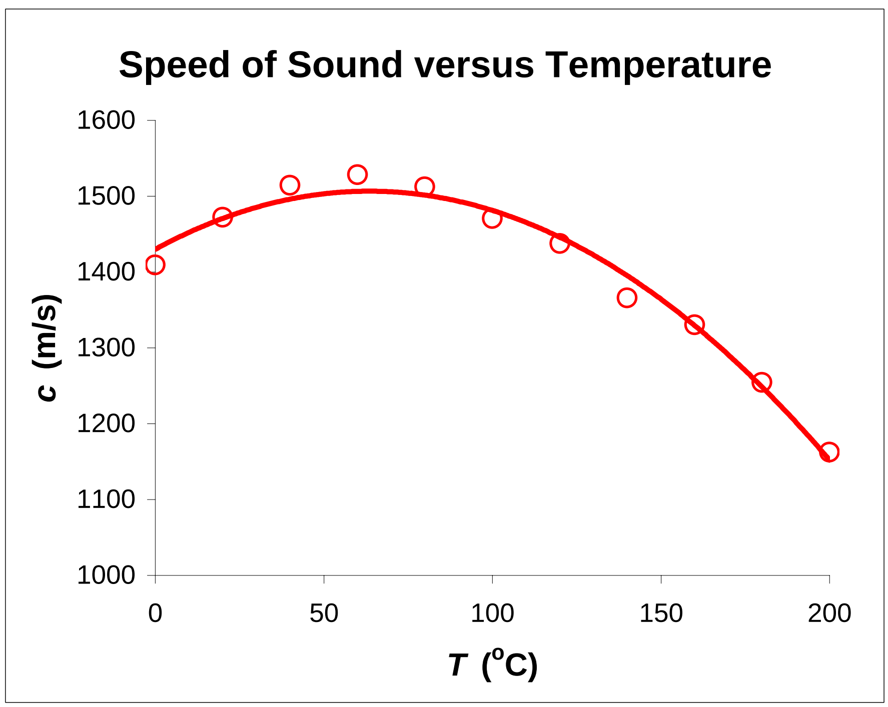
We use compressed liquid data at adjacent pressures of 5 MPa and 10 MPa, and estimate the change in density between these pressures from the corresponding specific volume changes

$$\Delta p = p_2 - p_1 \qquad \Delta\rho = \frac{1}{v_2} - \frac{1}{v_1} \qquad \text{and} \qquad c = \sqrt{\frac{\Delta p}{\Delta\rho}} \quad \text{at each temperature}$$

$p_2 =$ 10 MPa
$p_1 =$ 5 MPa
$\Delta p =$ 5 MPa

Data on specific volume versus temperature can be obtained fro any good thermodynamics text (try the Web!)

	p_1	p_2		
T (°C)	v (m³/kg)	v (m³/kg)	Δρ (kg/m³)	c (m/s)
0	0.0009977	0.0009952	2.52	1409
20	0.0009996	0.0009973	2.31	1472
40	0.0010057	0.0010035	2.18	1514
60	0.0010149	0.0010127	2.14	1528
80	0.0010267	0.0010244	2.19	1512
100	0.0010410	0.0010385	2.31	1470
120	0.0010576	0.0010549	2.42	1437
140	0.0010769	0.0010738	2.68	1366
160	0.0010988	0.0010954	2.82	1330
180	0.0011240	0.0011200	3.18	1254
200	0.0011531	0.0011482	3.70	1162



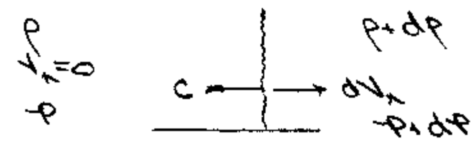
Problem 12.28

[3]

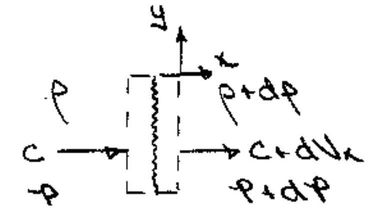
Given: Derivation of sonic speed (Eq. 12.18, Section 12-2)

Rederive assuming direction of fluid motion behind the wave is dV_x to the right.

Solution:



(a) Propagating Wave



(b) Inertial CV moving with wave

Apply continuity to CV (b)

$$0 = \{-|\rho c A|\} + \{|(\rho + d\rho)(c + dV_x)A|\}$$

$$0 = -\rho c A + \rho c A + \rho dV_x A + cA d\rho + d\rho\, dV_x \quad (d\rho\, dV_x \simeq 0)$$

then

$$\rho\, dV_x + c\, d\rho = 0 \qquad \text{or} \qquad dV_x = -\frac{c}{\rho}\, d\rho \quad \text{......(1)}$$

Applying x-momentum equation to same CV gives

$$\sum F_{s_x} = pA - (p + dp)A = \int_{CS} V_x \rho \vec{V}_{xyz} \cdot dA = c\{-|\rho A c|\} + (c + dV_x)\{|\rho A c|\} \quad \{\dot{m}_{in} = \dot{m}_{out}\}$$

$$\therefore\; -A\, dp = \rho A c\, dV_x \;, \text{ and}$$

$$dV_x = -\frac{dp}{\rho c} \quad \text{.......(2)}$$

Combining equations (1) and (2) we obtain

$$-\frac{c}{\rho}\, d\rho = -\frac{dp}{\rho c}$$

or

$$c^2 = \frac{dp}{d\rho}$$

This is the same result as obtained in the derivation of Section 12-2 with the direction of the fluid motion behind the wave to the left

Problem 12.29

[2]

12.29 Compute the speed of sound at sea level in standard air. By scanning data from Table A.3 into your PC (or using Fig. 3.3), evaluate the speed of sound and plot for altitudes to 90 km.

Given: Data on atmospheric temperature variation with altitude

Find: Sound of speed at sea level; plot speed as function of altitude

Solution

The given or available data is:

$R =$ 286.9 J/kg.K
$k =$ 1.4

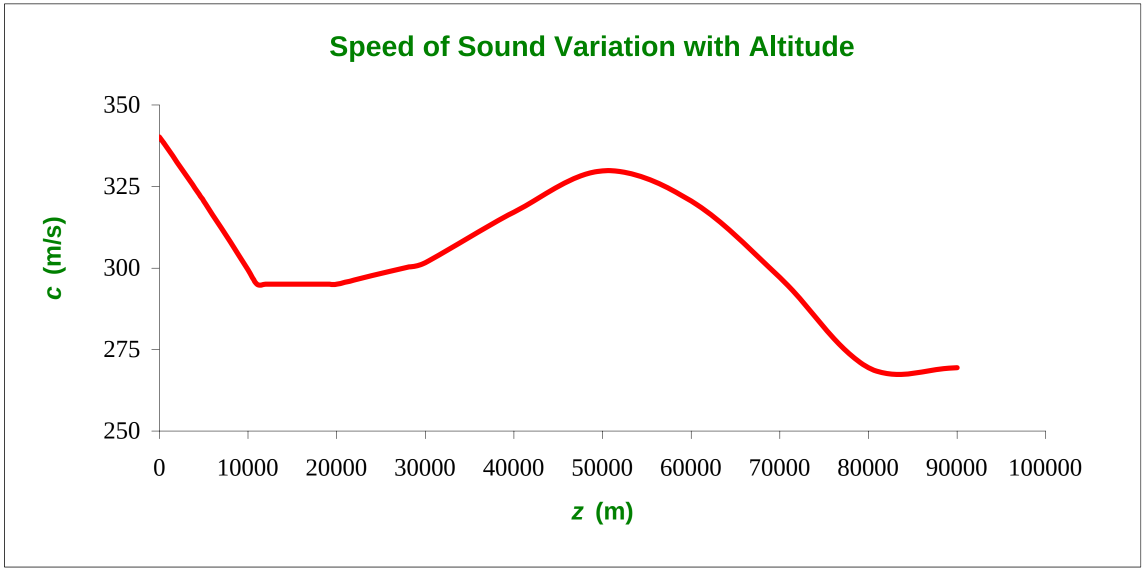
Computing equation:

$$c = \sqrt{kRT}$$

Computed results:

(Only partial data is shown in table)

z (m)	T (K)	c (m/s)
0	288.2	340
500	284.9	338
1000	281.7	336
1500	278.4	334
2000	275.2	332
2500	271.9	330
3000	268.7	329
3500	265.4	326
4000	262.2	325
4500	258.9	322
5000	255.7	320
6000	249.2	316
7000	242.7	312
8000	236.2	308
9000	229.7	304
10000	223.3	299



Problem 12.30

12.30 The temperature varies linearly from sea level to approximately 11 km altitude in the standard atmosphere. Evaluate the *lapse rate*—the rate of decrease of temperature with altitude—in the standard atmosphere. Derive an expression for the rate of change of sonic speed with altitude in an ideal gas under standard atmospheric conditions. Evaluate and plot from sea level to 10 km altitude.

Given: Data on atmospheric temperature variation with altitude

Find: Lapse rate; plot of rate of change of sonic speed with altitude

Solution:

The given or available data is:

R = 286.9 J/kg.K

k = 1.4

T_0 = 288.2 K

T_{10k} = 223.3 K

Computing equations:

For a linear temperature variation $T = T_0 + m \cdot z$

$$\frac{dT}{dz} = m = \frac{T - T_0}{z}$$ which can be evaluated at z = 10 km

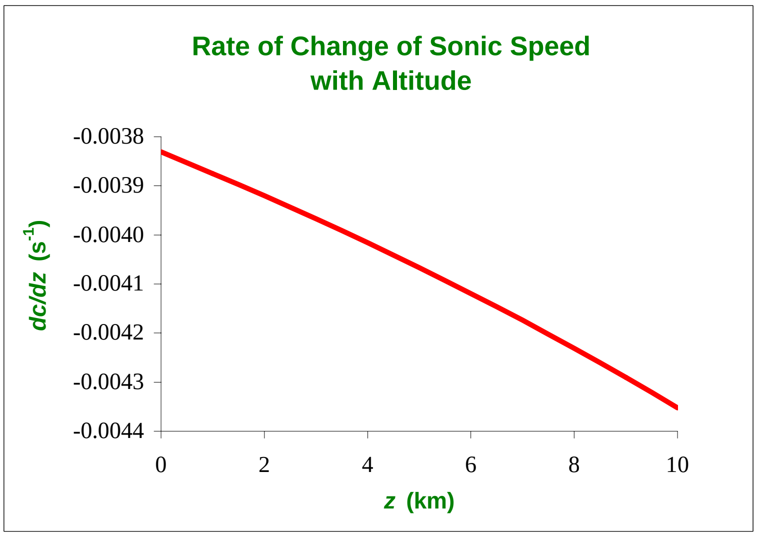
For an ideal gas $c = \sqrt{k \cdot R \cdot T} = \sqrt{k \cdot R \cdot (T_0 + m \cdot z)}$

Hence $$\frac{dc}{dz} = \frac{m \cdot k \cdot R}{2 \cdot c}$$

Computed results:

m = -0.00649 K/m (Using T at z = 10 km)

z (km)	T (K)	dc/dz (s^{-1})
0	288.2	-0.00383
1	281.7	-0.00387
2	275.2	-0.00392
3	268.7	-0.00397
4	262.2	-0.00402
5	255.8	-0.00407
6	249.3	-0.00412
7	242.8	-0.00417
8	236.3	-0.00423
9	229.8	-0.00429
10	223.3	-0.00435



Problem 12.31

12.31 Air at 77°F flows at $M = 1.9$. Determine the air speed and the Mach angle.

Given: Air flow at M = 1.9

Find: Air speed; Mach angle

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \dfrac{V}{c}$ $\quad \alpha = \text{asin}\left(\dfrac{1}{M}\right)$

The given or available data is $T = (77 + 460) \cdot R$ $\quad M = 1.9$ $\quad k = 1.4$ $\quad R_{air} = 53.33 \cdot \dfrac{ft \cdot lbf}{lbm \cdot R}$

Hence $c = \sqrt{k \cdot R_{air} \cdot T}$ $\quad c = 1136 \cdot \dfrac{ft}{s}$

Then the air speed is $V = M \cdot c$ $\quad V = 2158 \dfrac{ft}{s}$ $\quad V = 1471\,mph$

The Mach angle is given by $\alpha = \text{asin}\left(\dfrac{1}{M}\right)$ $\quad \alpha = 31.8\,deg$

Problem 12.32

Given: Gas at $P = 50$ psia $\rho = 0.27$ lbm/ft^3
Projectile fired into gas. Total angle of Mach cone is $20°$

Find: speed of projectile relative to gas

Solution:

Basic equation: $P = \rho R T$

Definitions: $\sin\alpha = \frac{1}{M}$ $M = \frac{V}{c}$

Computing eq: $c = \sqrt{kRT}$

Assumption: ideal gas

$\sin\alpha = \frac{1}{M}$ $M = \frac{1}{\sin\alpha} = \frac{1}{\sin 10°} = 5.76$

$c = \left[kRT\right]^{1/2} = \left[k \frac{P}{\rho}\right]^{1/2}$

$= \left[1.4 \times 50 \frac{\text{lbf}}{\text{in}^2} \times \frac{\text{ft}^3}{0.27\,\text{lbm}} \times 32.2 \frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} \times \frac{144\,\text{in}^2}{\text{ft}^2}\right]^{1/2}$

$c = 1100$ ft/s

$M = \frac{V}{c}$ ∴ $V = Mc = 5.76 \times 1100$ ft/s $= 6320$ ft/s ← V

Problem 12.33

Given : Photo of bullet moving through standard air shows Mach angle, $\alpha = 32°$

Find : Speed of bullet

Solution:

Computing equations, $\sin\alpha = \frac{1}{M}$ $\quad c = \sqrt{kRT}$

Assumptions: (1) air behaves as an ideal gas
(2) constant specific heats.

$M = \frac{1}{\sin\alpha}$, $M = \frac{V}{c}$ $\therefore V = cM = \frac{c}{\sin\alpha}$

Since $c = \sqrt{kRT}$, then

$$V = \frac{1}{\sin\alpha}(kRT)^{1/2} = \frac{1}{\sin 32°}\left(1.4 \times 287\,\frac{N\cdot m}{kg\cdot K} \times 288\,K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2}$$

$V = 642$ m/s (2110 ft/s) ← V

Problem 12.34

Given: A schlieren photograph taken in the NTF shows a Mach angle, $\alpha = 57°$, at a location where $T = -270°F$ and $p = 1.3$ psia.

Find: (a) the local Mach number and flow speed
(b) the unit Reynolds number for the flow

Solution:

$$\sin\alpha = \frac{1}{M} \quad \therefore \quad M = \frac{1}{\sin\alpha} = \frac{1}{\sin 57°} = 1.19$$ M

$$c = \sqrt{kRT} = \left[1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times 32.2 \frac{lbm}{slug} \times 190°R \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{1/2} = 676 \ ft/s$$

$$V = Mc = 1.19\,(676 \ ft/s) = 804 \ ft/s$$ V

$$Re_x = \frac{\rho V x}{\mu}$$

$$\rho = \frac{p}{RT} = 1.3 \frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3 \ ft \cdot lbf} \times \frac{1}{190°R} \times \frac{144 \ in^2}{ft^2} = 0.0185 \ lbm/ft^3$$

From Eq. A.1 (Appendix A)

$$\mu = \frac{b T^{1/2}}{1 + S/T} \qquad b = 1.458 \times 10^{-6} \ kg/m \cdot s \cdot K^{1/2}$$

$S = 110.4 K$

T in K

$T = -270 F = -167 C = 106 K.$

$$\mu = 1.458 \times 10^{-6} \frac{kg}{m \cdot s \cdot K^{1/2}} (106 K)^{1/2} \times \frac{1}{1 + \frac{110.4}{106}} = 7.35 \times 10^{-6} \ kg/m \cdot s$$

$$\mu = 7.35 \times 10^{-6} \frac{kg}{m \cdot s} \times \frac{N \cdot s^2}{kg \cdot m} \times \frac{2.089 \times 10^{-2} \ lbf \cdot s/ft^2}{1 \ N \cdot s}$$

$$\mu = 1.54 \times 10^{-7} \ lbf \cdot s/ft^2$$

$$\therefore \frac{Re}{x} = \frac{\rho V}{\mu} = 0.0185 \frac{lbm}{ft^3} \times 804 \frac{ft}{s} \times \frac{ft^2}{1.54 \times 10^{-7} \ lbf \cdot s} \times \frac{slug}{32.2 \ lbm} \times \frac{lbf \cdot s^2}{slug \cdot ft}$$

$$\frac{Re}{x} = 3.00 \times 10^6 \ ft^{-1} = 9.84 \times 10^6 \ m^{-1}$$ Re/x

Problem 12.35

Given: An aircraft flies at M = 1.4 at elevation z = 200m. The air temperature is 35°C

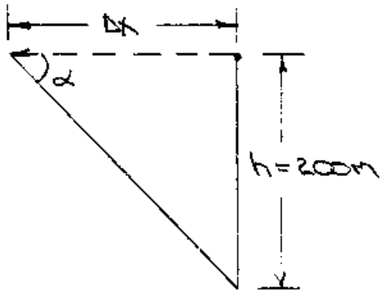
Find: (a) air speed of aircraft
(b) time between instant when aircraft passes directly overhead and instant when Mach cone passes a point on the ground.

Solution: Assume T = constant over 200m elevation.

$T = 35\,C = 308\,K$

$c = \sqrt{kRT} = \left[1.4 \times 287 \frac{N\cdot m}{kg\cdot K} \times 308\,K \times \frac{kg\cdot m}{N\cdot s^2}\right]^{1/2} = 352\ m/s$

$V = Mc = 1.4 \times 352\ m/s = 493\ m/s$ ← V



From the instant the aircraft is directly overhead until the Mach cone reaches the ground, the plane travels a distance Δx at speed V = 493 m/s.

$\sin\alpha = \frac{1}{M} = \frac{1}{1.4} = 0.7143$

$\alpha = 45.6°$

$\frac{h}{\Delta x} = \tan\alpha \quad \therefore \Delta x = \frac{h}{\tan\alpha} = \frac{200\,m}{\tan 45.6°} = 196\ m$

Since the plane moves at constant speed V

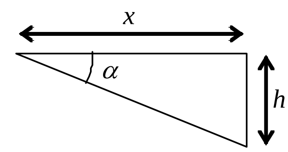
$\Delta x = V\,\Delta t$ and $\Delta t = \frac{\Delta x}{V} = \frac{196\,m}{493\,m/s}$

$\Delta t = 0.398\ s$ ← Δt

Problem 12.36

[2]

12.36 While jogging on the beach (it's a warm summer day, about 30°C) a high-speed jet flies overhead. You guesstimate it's at an altitude of about 3500 m, and count off about 5 s before you hear it. Estimate the speed and Mach number of the jet.



Given: High-speed jet flying overhead

Find: Estimate speed and Mach number of jet

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \alpha = \mathrm{asin}\left(\frac{1}{M}\right)$

Given or available data $T = (30 + 273) \cdot K$ $\quad h = 3500 \cdot m$ $\quad k = 1.4$ $\quad R = 286.9 \frac{J}{kg \cdot K}$

The time it takes to fly from directly overhead to where you hear it is $\Delta t = 5 \cdot s$

The distance traveled, moving at speed V, is $x = V \cdot \Delta t$

The Mach angle is related to height h and distance x by $\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{h}{x} = \frac{h}{V \cdot \Delta t}$ (1)

and also we have $\sin(\alpha) = \frac{1}{M} = \frac{c}{V}$ (2)

Dividing Eq. 2 by Eq 1 $\cos(\alpha) = \frac{c}{V} \cdot \frac{V \cdot \Delta t}{h} = \frac{c \cdot \Delta t}{h}$

Note that we could have written this equation from geometry directly!

We have $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 349 \frac{m}{s}$ so $\alpha = \mathrm{acos}\left(\frac{c \cdot \Delta t}{h}\right)$ $\quad \alpha = 60.1 \cdot deg$

Hence $M = \frac{1}{\sin(\alpha)}$ $\quad M = 1.15$

Then the speed is $V = M \cdot c$ $\quad V = 402 \frac{m}{s}$

Note that we assume the temperature of the air is uniform. In fact the temperature will vary over 3500 m, so the Mach cone will be curved. This speed and Mach number are only rough estimates

Problem 12.37

Given: Aircraft passes overhead at an altitude of 3 km, travelling at M = 1.35. The air temperature is constant at T = 30°C and a head wind blows at $V_{air} = 10$ m/s.

Find: (a) the airspeed of the aircraft
(b) time between instant when aircraft passes directly overhead and instant when sound reaches the ground.

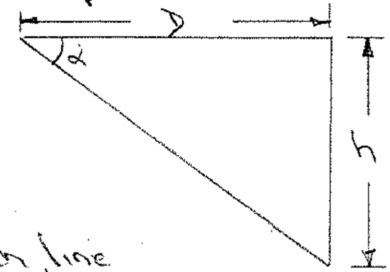
Solution:

$T = \text{constant} = 20°C = 293\ K$

$c = (kRT)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 293\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 343\ m/s$

$V = Mc = 1.5 \times 343 \frac{m}{sec} = 515\ m/s$ ← airspeed

The airspeed is the velocity of the plane relative to the air. The ground speed is then $\vec{V}_p = \vec{V}_{air} + \vec{V}_{pla}$ or $V_p = 485$ m/sec



From the instant the aircraft is directly overhead until the Mach cone reaches the ground, the plane travels a distance, D, at speed $V_p = 485$ m/s.
The value of the time, t, is then $t = D/V_p$
Since the air temperature is constant, the Mach line is straight and $D = h/\tan\alpha$, where $\alpha = \sin^{-1}(1/M)$

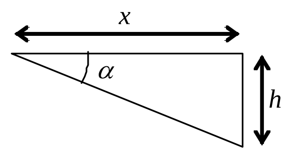
$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8°$$

Then,

$$t = \frac{D}{V_p} = \frac{h}{\tan\alpha}\frac{1}{V_p} = \frac{3000\ m}{\tan 41.8} \times \frac{s}{485\ m} = 6.92\ s$$ ← t

Problem 12.38

12.38 A supersonic aircraft flies at 3 km altitude at a speed of 1000 m/s on a standard day. How long after passing directly above a ground observer is the sound of the aircraft heard by the ground observer?



Given: Supersonic aircraft flying overhead

Find: Time at which airplane heard

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \alpha = \operatorname{asin}\left(\frac{1}{M}\right)$

Given or available data $V = 1000 \cdot \frac{m}{s}$ $\quad h = 3 \cdot km$ $\quad k = 1.4$ $\quad R = 286.9 \frac{J}{kg \cdot K}$

The time it takes to fly from directly overhead to where you hear it is $\Delta t = \frac{x}{V}$

If the temperature is constant then $x = \frac{h}{\tan(\alpha)}$

The temperature is not constant so the Mach line will not be straight. We can find a range of Δt by considering the temperature range

At $h = 3\,km$ we find from Table A.3 that $T = 268.7 \cdot K$

Using this temperature $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 329 \frac{m}{s}$ $\quad$ and $\quad M = \frac{V}{c}$ $\quad M = 3.04$

Hence $\alpha = \operatorname{asin}\left(\frac{1}{M}\right)$ $\quad \alpha = 19.2\,deg$ $\quad x = \frac{h}{\tan(\alpha)}$ $\quad x = 8625\,m$ $\quad \Delta t = \frac{x}{V}$ $\quad \Delta t = 8.62\,s$

At sea level we find from Table A.3 that $T = 288.2 \cdot K$

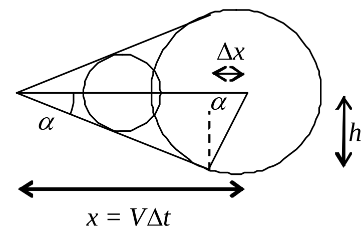
Using this temperature $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 340 \frac{m}{s}$ $\quad$ and $\quad M = \frac{V}{c}$ $\quad M = 2.94$

Hence $\alpha = \operatorname{asin}\left(\frac{1}{M}\right)$ $\quad \alpha = 19.9\,deg$ $\quad x = \frac{h}{\tan(\alpha)}$ $\quad x = 8291\,m$ $\quad \Delta t = \frac{x}{V}$ $\quad \Delta t = 8.29\,s$

Thus we conclude that the time is somwhere between 8.62 and 8.29 s. Taking an average $\Delta t = 8.55 \cdot s$

Problem 12.39

12.39 For the conditions of Problem 12.38, find the location at which the sound wave that first reaches the ground observer was emitted.



Given:
Supersonic aircraft flying overhead

Find:
Location at which first sound wave was emitted

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \alpha = \text{asin}\left(\frac{1}{M}\right)$

Given or available data $V = 1000 \cdot \frac{m}{s}$ $\quad h = 3 \cdot km$ $\quad k = 1.4$ $\quad R = 286.9 \frac{J}{kg \cdot K}$

We need to find Δx as shown in the figure $\quad \Delta x = h \cdot \tan(\alpha)$

The temperature is not constant so the Mach line will not be straight (α is not constant). We can find a range of α and Δx by considering the temperature range

At $h = 3\,km$ we find from Table A.3 that $\quad T = 268.7 \cdot K$

Using this temperature $\quad c = \sqrt{k \cdot R \cdot T}$ $\quad c = 329\frac{m}{s}$ $\quad$ and $\quad M = \frac{V}{c}$ $\quad M = 3.04$

Hence $\quad \alpha = \text{asin}\left(\frac{1}{M}\right)$ $\quad \alpha = 19.2\,deg$ $\quad \Delta x = h \cdot \tan(\alpha)$ $\quad \Delta x = 1043\,m$

At sea level we find from Table A.3 that $\quad T = 288.2 \cdot K$

Using this temperature $\quad c = \sqrt{k \cdot R \cdot T}$ $\quad c = 340\frac{m}{s}$ $\quad$ and $\quad M = \frac{V}{c}$ $\quad M = 2.94$

Hence $\quad \alpha = \text{asin}\left(\frac{1}{M}\right)$ $\quad \alpha = 19.9\,deg$ $\quad \Delta x = h \cdot \tan(\alpha)$ $\quad \Delta x = 1085\,m$

Thus we conclude that the distance is somwhere between 1043 and 1085 m. Taking an average $\quad \Delta x = 1064 \cdot m$

Problem 12.40

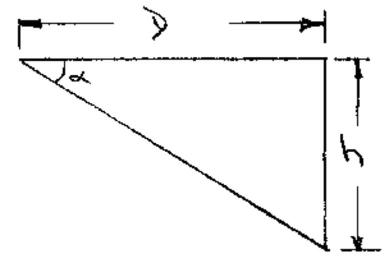
Given: Concorde supersonic transport cruises at $M = 2.2$ at an altitude, $h = 17$ km on a standard day.
At $t = 0$, plane is directly overhead.

Find: value of t when aircraft is first heard.

Solution:

At altitude, $T = 216.7\,K$ $\quad c = (kRT)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 216.7 K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 295\, m/s$

$V = Mc = 2.2 \times 295\, m/s = 649\, m/s.$



If the speed of sound were constant all the way to the ground, the Mach line would remain straight. The Mach angle α, would be constant with

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{2.2}\right) = 27^\circ$$

Then, from the diagram $D = \frac{h}{\tan\alpha}$

and $t = \frac{D}{V} = \frac{h}{\tan\alpha\, V} = \frac{17000\, m}{\tan 27^\circ} \times \frac{1}{649} \frac{sec}{m} = 51.4\, s$

However, the speed of sound varies over the altitude because the temperature varies with altitude.

At sea level, $T = 288.2\, K$

$$c = (kRT)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 288.2 K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 340\, m/s$$

The corresponding value of Mach number for $V = 649\, m/s$ is

$$M = \frac{V}{c} = \frac{649}{340} = 1.91$$

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{1.91}\right) = 31.6^\circ$$

Thus, if the speed of sound were constant (at the sea level) value over the entire altitude, then

$$t = \frac{D}{V} = \frac{h}{\tan\alpha\, V} = \frac{17000\, m}{\tan 31.6} \times \frac{s}{649\, m} = 42.6\, s$$

We can obtain a better approximate by considering the variation of temperature with altitude.
From Table A.3

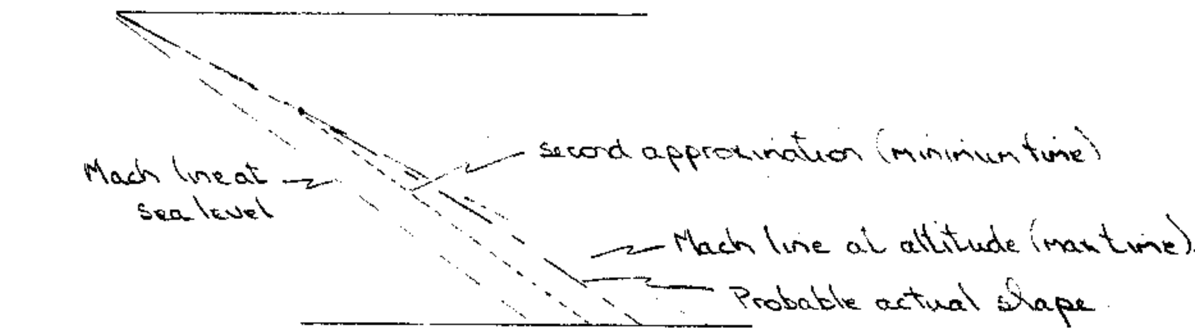
$11\, km \le y < 20\, km \quad T = 216.7\, K$

$0 < y \le 11\, km \quad T$ varies linearly with y

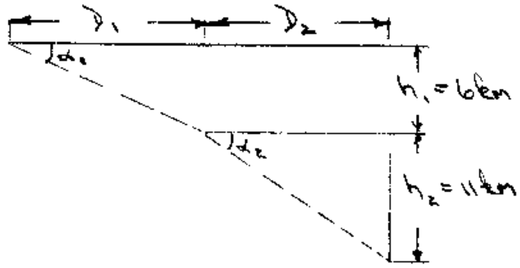
$T = T_0 - by \qquad T_0 = 288.2\, K$

$\qquad\qquad\qquad\qquad b = 6.50\, K/km$

Problem 12.40



Since T is constant for $y > y_0 = 11$ km, the second approximation which assumes the Mach line at sea level for $0 < y \le 11$ km gives the minimum time



$$D_1 = \frac{h_1}{\tan\alpha_1} = \frac{6\text{ km}}{\tan 27^\circ} = 11.77\text{ km}$$

$$D_2 = \frac{h_2}{\tan\alpha_2} = \frac{11\text{ km}}{\tan 31.6^\circ} = 17.88\text{ km}$$

$$D = D_1 + D_2$$

$$t = \frac{D}{V} = \frac{29.65\text{ km}}{649\text{ m/s}} = 45.7\text{ s}$$

Consequently, $45.7 \le t \le 51.4$ s

Since the two values are reasonably close, it is appropriate to take the average value and say $t \doteq 48.5$ s

Problem 12.41

12.41 The airflow around an automobile is assumed to be incompressible. Investigate the validity of this assumption for an automobile traveling at 60 mph. (Relative to the automobile the minimum air velocity is zero, and the maximum is approximately 120 mph.)

Given: Speed of automobile

Find: Whether flow can be considered incompressible

Solution:

Consider the automobile at rest with 60 mph air flowing over it. Let state 1 be upstream, and point 2 the stagnation point on the automobile

The data provided, or available in the Appendices, is:

$R = 287 \cdot \frac{J}{kg \cdot K}$ $\quad k = 1.4$ $\quad V_1 = 60 \cdot mph$ $\quad p_1 = 101 \cdot kPa$ $\quad T_1 = (20 + 273) \cdot K$

The basic equation for the density change is $\frac{\rho_0}{\rho} = \left[1 + \frac{(k-1)}{2} \cdot M^2\right]^{\frac{1}{k-1}}$ (12.20c)

or $\rho_0 = \rho_1 \cdot \left[1 + \frac{(k-1)}{2} \cdot M_1^{\,2}\right]^{\frac{1}{k-1}}$

$\rho_1 = \frac{p_1}{R \cdot T_1}$ $\quad \rho_1 = 1.201 \frac{kg}{m^3}$

For the Mach number we need c $\quad c_1 = \sqrt{k \cdot R \cdot T_1}$ $\quad c_1 = 343 \frac{m}{s}$

$V_1 = 26.8 \frac{m}{s}$ $\quad M_1 = \frac{V_1}{c_1}$ $\quad M_1 = 0.0782$

$\rho_0 = \rho_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^{\,2}\right)^{\frac{1}{k-1}}$ $\quad \rho_0 = 1.205 \frac{kg}{m^3}$ $\quad$ The percentage change in density is $\left|\frac{\rho_0 - \rho_1}{\rho_0}\right| = 0.305\%$

This is an insignificant change, so the flow can be considered incompressible. Note that *M* < 0.3, the usual guideline for incompressibility

For the maximum speed present $\quad V_1 = 120 \cdot mph$ $\quad V_1 = 53.6 \frac{m}{s}$ $\quad M_1 = \frac{V_1}{c_1}$ $\quad M_1 = 0.156$

$\rho_0 = \rho_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^{\,2}\right)^{\frac{1}{k-1}}$ $\quad \rho_0 = 1.216 \frac{kg}{m^3}$ $\quad$ The percentage change in density is $\left|\frac{\rho_0 - \rho_1}{\rho_0}\right| = 1.21\%$

This is still an insignificant change, so the flow can be considered incompressible.

Problem 12.42

12.42 Opponents of supersonic transport aircraft claim that sound waves can be refracted in the upper atmosphere and that, as a result, sonic booms can be heard several hundred miles away from the ground track of the aircraft. Explain the phenomenon of sound wave refraction.

Given: Supersonic transport aircraft

Find: Explanation of sound wave refraction

Solution:

A sound wave is refracted when the speed of sound varies with altitude in the atmosphere. (The variation in sound speed is caused by temperature variations in the atmosphere, as shown in Fig. 3.3)

Imagine a plane wave front that initially is vertical. When the wave encounters a region where the temperature increase with altitude (such as between 20.1 km and 47.3 km altitude in Fig. 3.3), the sound speed increases with elevation. Therefore the upper portion of the wave travels faster than the lower portion. The wave front turns gradually and the sound wave follows a curved path through the atmosphere. Thus a wave that initially is horizontal bends and follows a curved path, tending to reach the ground some distance from the source.

The curvature and the path of the sound could be calculated for any specific temperature variation in the atmosphere. However, the required analysis is beyond the scope of this text.

Problem 12.43

[2]

12.43 Plot the percentage discrepancy between the density at the stagnation point and the density at a location where the Mach number is *M*, of a compressible flow, for Mach numbers ranging from 0.05 to 0.95. Find the Mach numbers at which the discrepancy is 1 percent, 5 percent, and 10 percent.

Given: Mach number range from 0.05 to 0.95

Find: Plot of percentage density change; Mach number for 1%, 5%, and 10% change

Solution:

The given or available data is:

$k = 1.4$

Computing equation:

$$\frac{\rho_0}{\rho} = \left[1 + \frac{(k-1)}{2} \cdot M^2\right]^{\frac{1}{k-1}} \qquad (12.20c)$$

Hence $$\frac{\Delta\rho}{\rho_0} = \frac{\rho_0 - \rho}{\rho_0} = 1 - \frac{\rho}{\rho_0}$$

so $$\frac{\Delta\rho}{\rho_0} = 1 - \left[1 + \frac{(k-1)}{2} \cdot M^2\right]^{\frac{1}{1-k}}$$

Computed results:

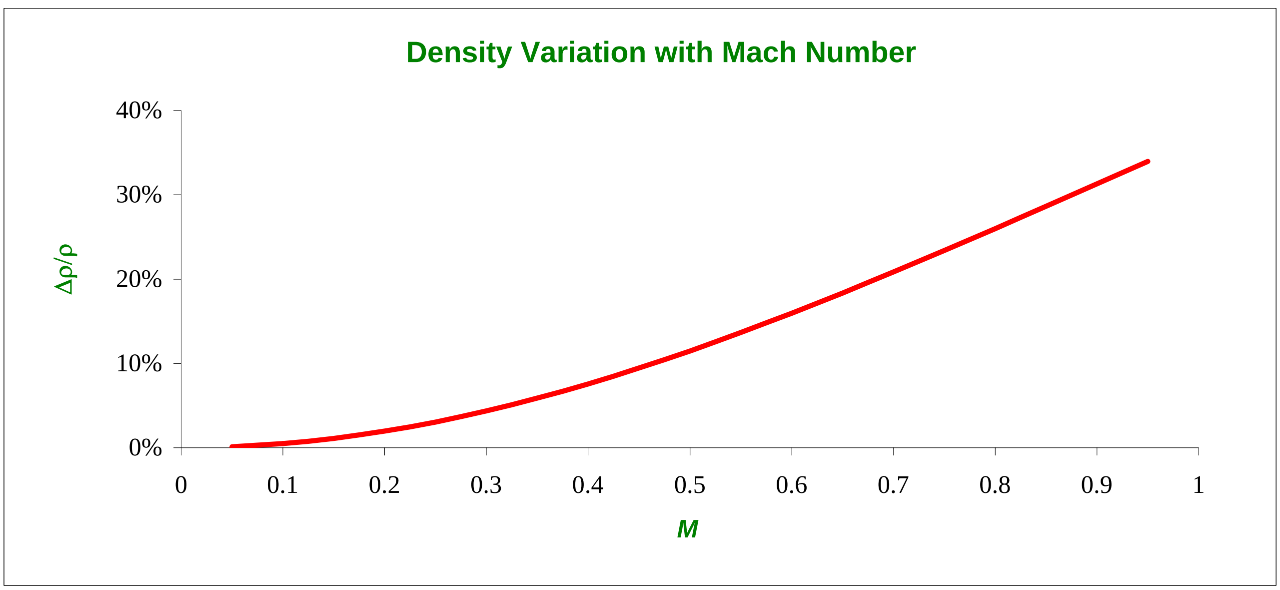
M	$\Delta\rho/\rho_o$
0.05	0.1%
0.10	0.5%
0.15	1.1%
0.20	2.0%
0.25	3.1%
0.30	4.4%
0.35	5.9%
0.40	7.6%
0.45	9.4%
0.50	11%
0.55	14%
0.60	16%
0.65	18%
0.70	21%
0.75	23%
0.80	26%
0.85	29%
0.90	31%
0.95	34%

To find M for specific density changes use *Goal Seek* repeatedly

M	$\Delta\rho/\rho_o$
0.142	1%
0.322	5%
0.464	10%

Note: Based on ρ (not ρ_o) the results are:

0.142	0.314	0.441



Problem 12.44

12.44 An aircraft flies at 250 m/s in air at 28 kPa and −50°C. Find the stagnation pressure at the nose of the aircraft.

Given:
Aircraft flying at 250 m/s

Find:
Stagnation pressure

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$

Given or available data $V = 250 \cdot \frac{m}{s}$ $\quad T = (-50 + 273) \cdot K$ $\quad p = 28 \cdot kPa$ $\quad k = 1.4$ $\quad R = 286.9 \frac{J}{kg \cdot K}$

First we need $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 299 \frac{m}{s}$ then $M = \frac{V}{c}$ $\quad M = 0.835$

Finally we solve for p_0 $\quad p_0 = p \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad p_0 = 44.2\,kPa$

Problem 12.45

[2]

12.45 Compute the air density in the undisturbed air, and at the stagnation point, of Problem 12.44. What is the percentage increase in density? Can we approximate this as an incompressible flow?

Given: Pressure data on aircraft in flight

Find: Change in air density; whether flow can be considered incompressible

Solution:

The data provided, or available in the Appendices, is:

$$k = 1.4 \qquad p_0 = 48 \cdot kPa \qquad p = 27.6 \cdot kPa \qquad T = (-55 + 273) \cdot K$$

Governing equation (assuming isentropic flow):

$$\frac{p}{\rho^k} = \text{constant} \qquad (12.12c)$$

Hence

$$\frac{\rho}{\rho_0} = \left(\frac{p}{p_0}\right)^{\frac{1}{k}}$$

so

$$\frac{\Delta\rho}{\rho} = \frac{\rho_0 - \rho}{\rho} = \frac{\rho_0}{\rho} - 1 = \left(\frac{p_0}{p}\right)^{\frac{1}{k}} - 1 \qquad \frac{\Delta\rho}{\rho} = 48.5 \cdot \%$$

NOT an incompressible flow!

Problem 12.46

[1]

12.46 Find the ratio of static to total pressure for a car moving at 55 mph at sea level and an airplane moving at 550 mph at 30,000 ft.

Given: Car at sea level and aircraft flying at 30,000 ft

Find: Ratio of static to total pressure in each case

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$

Given or available data $\quad V_{car} = 55 \cdot mph \quad V_{car} = 80.7\frac{ft}{s} \quad V_{plane} = 550 \cdot mph \quad V_{plane} = 807\frac{ft}{s}$

$k = 1.4 \quad R_{air} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$

At sea level, from Table A.3 $\quad T = 288.2 \cdot K \quad$ or $\quad T = 519\,R$

Hence $\quad c = \sqrt{k \cdot R_{air} \cdot T} \quad c = 1116\frac{ft}{s} \quad M_{car} = \frac{V_{car}}{c} \quad M_{car} = 0.0723$

The pressure ratio is $\quad \frac{p}{p_0} = \left(1 + \frac{k-1}{2} \cdot M_{car}{}^2\right)^{-\frac{k}{k-1}} = 0.996$

Note that the Bernoulli equation would give the same result!

At $h = 30000 \cdot ft$ or $h = 9144\,m$,interpolating from Table A.3

$$T = 229.7 \cdot K + \frac{(223.3 - 229.7) \cdot K}{(10000 - 9000)} \cdot (9144 - 9000) \qquad T = 229\,K \qquad T = 412\,R$$

Hence $\quad c = \sqrt{k \cdot R_{air} \cdot T} \quad c = 995\frac{ft}{s} \quad M_{plane} = \frac{V_{plane}}{c} \quad M_{plane} = 0.811$

The pressure ratio is $\quad \frac{p}{p_0} = \left(1 + \frac{k-1}{2} \cdot M_{plane}{}^2\right)^{-\frac{k}{k-1}} = 0.649$

Problem 12.47

12.47 For an aircraft traveling at $M = 2$ at an elevation of 12 km, find the dynamic and stagnation pressures.

Given: Aircraft flying at 12 km

Find: Dynamic and stagnation pressures

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad p_{dyn} = \frac{1}{2} \cdot \rho \cdot V^2$

Given or available data $M = 2$ $\quad h = 12 \cdot km$ $\quad k = 1.4$ $\quad R = 286.9 \cdot \frac{J}{kg \cdot K}$

$\rho_{SL} = 1.225 \cdot \frac{kg}{m^3}$ $\quad p_{SL} = 101.3 \cdot kPa$

At $h = 12\,km$,from Table A.3 $\rho = 0.2546 \cdot \rho_{SL}$ $\quad \rho = 0.312 \frac{kg}{m^3}$ $\quad p = 0.1915 \cdot p_{SL}$ $\quad p = 19.4\,kPa$ $\quad T = 216.7 \cdot K$

Hence $p_0 = p \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad p_0 = 152\,kPa$

Also $c = \sqrt{k \cdot R \cdot T}$ $\quad c = 295 \frac{m}{s}$ $\quad V = M \cdot c$ $\quad V = 590 \frac{m}{s}$

Hence $p_{dyn} = \frac{1}{2} \cdot \rho \cdot V^2$ $\quad p_{dyn} = 54.3\,kPa$

Problem 12.48

Given: Body moving through standard air at 200 m/s

Find: Stagnation pressure, assuming
(a) incompressible flow
b) compressible flow

Solution:

For standard air, $P = 101$ kPa, $T = 15°C$

Computing equations $P_o = P + \frac{1}{2}\rho V^2$ (incompressible)

$$\frac{P_o}{P} = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1} \text{ (compressible).}$$

(a) Incompressible flow

$$P_o = P + \frac{1}{2}\rho V^2 = 101 \text{ kPa} + \frac{1}{2} \times 1.225 \frac{\text{kg}}{\text{m}^3} \times (200)^2 \frac{\text{m}^2}{\text{s}^2} \times \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} \times \frac{\text{kPa}}{10^3} \frac{\text{m}^2}{\text{N}}$$

$P_o = 125.5$ kPa ← $P_{o\,inc}$

b. Compressible flow

$$M = \frac{V}{c} \qquad c = (kRT)^{1/2} = \left(1.4 \times 287 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 288\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)^{1/2} = 340 \text{ m/s}$$

$$M = \frac{200}{340} = 0.588$$

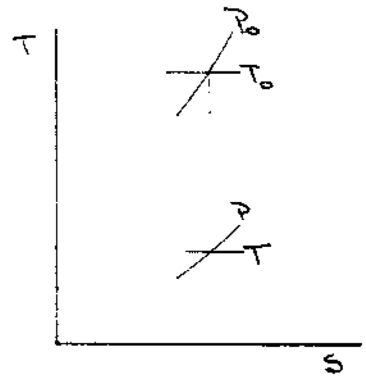
$$P_o = P\left[1 + \frac{k-1}{2} M^2\right]^{k/k-1} = 101 \text{ kPa}\left[1 + 0.2(0.588)^2\right]^{3.5} = 127.6 \text{ kPa}$$ ← $P_{o\,com}$

Problem 12.49

Given: Flow of standard air, $V = 600\ \text{m/s}$

Find: p_0, h_0, T_0

Solution:



Computing equations: $\dfrac{p_0}{p} = \left[1 + \dfrac{k-1}{2}M^2\right]^{k/k-1}$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$$

$$c = \sqrt{kRT}$$

Assumption: air behaves as an ideal gas, $k = 1.4$

$$c = (kRT)^{1/2} = \left(1.4 \times 287\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 288\,\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)^{1/2} = 340\ \text{m/s}$$

$$M = \frac{V}{c} = \frac{600}{340} = 1.76$$

$$p_0 = p\left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} = 101\ \text{kPa}\left[1 + 0.2(1.76)^2\right]^{3.5} = 546\ \text{kPa}$$ ← p_0

$$T_0 = T\left[1 + \frac{k-1}{2}M^2\right] = 288\,\text{K}\left[1 + 0.2(1.76)^2\right] = 466\ \text{K}$$ ← T_0

$dh = c_p\,dT$. For c_p = constant

$$h_0 - h = \int_h^{h_0} dh = \int_T^{T_0} c_p\,dT = c_p(T_0 - T) = 1000\,\frac{\text{J}}{\text{kg}\cdot\text{K}}\,(466 - 288)\,\text{K}$$

$$h_0 - h = 178\ \text{kJ/kg}$$ ← $h_0 - h$

Problem 12.50

Given: DC-10 aircraft cruises at altitude, $z = 12$ km on a standard day.

$p_0 = 29.6$ kPa $\quad p = 19.4$ kPa

Find: (a) M (b) V (c) T_0 (on aircraft).

Solution:

Computing equations: $\frac{p_0}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{k/k-1} \qquad \frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$

Solving the first equation for M

$$M = \left\{\frac{2}{(k-1)}\left[\left(\frac{p_0}{p}\right)^{\frac{k-1}{k}} - 1\right]\right\} = \left\{\frac{2}{1.4-1}\left[\left(\frac{29.6}{19.4}\right)^{\frac{1.4-1}{1.4}} - 1\right]\right\}^{1/2} = 0.801 \quad \leftarrow M$$

At $z = 12$ km, $T = 216.7$ K (Table A.3)

$$c = (kRT)^{1/2} = \left(1.4 \times 286.9\,\frac{N \cdot m}{kg \cdot K} \times 216.7\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 295 \text{ m/s}$$

$$V = Mc = 0.801 \times 295 \text{ m/s} = 236 \text{ m/s} \quad \leftarrow V$$

$$T_0 = T\left(1 + \frac{k-1}{2}M^2\right) = 216.7\,K\left[1 + \frac{(1.4-1)}{2}(0.801)^2\right] = 245\,K \quad \leftarrow T_0$$

Problem 12.51

12.51 An aircraft cruises at $M = 0.65$ at 10 km altitude on a standard day. The aircraft speed is deduced from measurement of the difference between the stagnation and static pressures. What is the value of this difference? Compute the air speed from this actual difference assuming (a) compressibility and (b) incompressibility. Is the discrepancy in air-speed computations significant in this case?

Given: Mach number of aircraft

Find: Pressure difference; air speed based on a) compressible b) incompressible assumptions

Solution:

The data provided, or available in the Appendices, is:

$R = 287 \cdot \frac{J}{kg \cdot K}$ $\quad c_p = 1004 \cdot \frac{J}{kg \cdot K}$ $\quad k = 1.4$ $\quad M = 0.65$

From Table A.3, at 10 km altitude $\quad T = 223.3 \cdot K$ $\quad p = 0.2615 \cdot 101 \cdot kPa$ $\quad p = 26.4\,kPa$

The governing equation for pressure change is: $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad$ (12.20a)

Hence $\quad p_0 = p \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad p_0 = 35.1\,kPa$

The pressure difference is $\quad p_0 - p = 8.67\,kPa$

a) Assuming compressibility $\quad c = \sqrt{k \cdot R \cdot T}$ $\quad c = 300\,\frac{m}{s}$ $\quad V = M \cdot c$ $\quad V = 195\,\frac{m}{s}$

b) Assuming incompressibility

Here the Bernoulli equation applies in the form $\quad \frac{p}{\rho} + \frac{V^2}{2} = \frac{p_0}{\rho}$ $\quad$ so $\quad V = \sqrt{\frac{2 \cdot (p_0 - p)}{\rho}}$

For the density $\quad \rho = \frac{p}{R \cdot T}$ $\quad \rho = 0.412\,\frac{kg}{m^3}$ $\quad V = \sqrt{\frac{2 \cdot (p_0 - p)}{\rho}}$

Hence $\quad V = 205\,\frac{m}{s}$

In this case the error at $M = 0.65$ in computing the speed of the aircraft using Bernoulli equation is $\quad \frac{205 - 195}{195} = 5.13\%$

Problem 12.52

Given: The Anglo-French "Concorde" cruises at $M = 2.2$ at an altitude, $z = 20$ km.

Find: (a) c (b) V (c) α
(d) Maximum T_0 on aircraft

Solution:

At $z = 20$ km, $T = 216.7$ K (Table A.3).

$$c = (kRT)^{1/2} = \left[1.4 \times 286.9 \frac{N \cdot m}{kg \cdot K} \times 216.7 K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 295 \text{ m/s}$$ ← c

$$V = Mc = 2.2 \times 295 \text{ m/s} = 649 \text{ m/s}$$ ← V

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{2.2}\right) = 27.0°$$ ← α

$$T_0 = T\left(1 + \frac{k-1}{2}M^2\right) = 216.7 K\left[1 + \frac{(1.4-1)}{2}(2.2)^2\right] = 426 \text{ K}$$ ← T_0

Problem 12.53

[2]

12.53 Modern high-speed aircraft use "air data computers" to compute air speed from measurement of the difference between the stagnation and static pressures. Plot, as a function of actual Mach number M, for $M = 0.1$ to $M = 0.9$, the percentage error in computing the Mach number assuming incompressibility (i.e., using the Bernoulli equation), from this pressure difference. Plot the percentage error in speed, as a function of speed, of an aircraft cruising at 12 km altitude, for a range of speeds corresponding to the actual Mach number ranging from $M = 0.1$ to $M = 0.9$.

Given: Flight altitude of high-speed aircraft

Find: Mach number and aircraft speed errors assuming incompressible flow; plot

Solution:

The governing equation for pressure change is: $\dfrac{p_0}{p} = \left(1 + \dfrac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ (12.20a)

Hence $\Delta p = p_0 - p = p\cdot\left(\dfrac{p_0}{p} - 1\right)$ $\quad \Delta p = p\cdot\left[\left(1 + \dfrac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} - 1\right]$ (1)

For each Mach number the actual pressure change can be computed from Eq. 1

Assuming incompressibility, the Bernoulli equation applies in the form $\dfrac{p}{\rho} + \dfrac{V^2}{2} = \dfrac{p_0}{\rho}$ so $V = \sqrt{\dfrac{2\cdot(p_0 - p)}{\rho}} = \sqrt{\dfrac{2\cdot\Delta p}{\rho}}$

and the Mach number based on this is $M_{incomp} = \dfrac{V}{c} = \dfrac{\sqrt{\dfrac{2\cdot\Delta p}{\rho}}}{\sqrt{k\cdot R\cdot T}} = \sqrt{\dfrac{2\cdot\Delta p}{k\cdot\rho\cdot R\cdot T}}$

Using Eq. 1 $M_{incomp} = \sqrt{\dfrac{2}{k}\cdot\left[\left(1 + \dfrac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} - 1\right]}$

The error in using Bernoulli to estimate the Mach number is $\dfrac{\Delta M}{M} = \dfrac{M_{incomp} - M}{M}$

For errors in speed:

Actual speed: $V = M\cdot c$ $\quad V = M\cdot\sqrt{k\cdot R\cdot T}$

Speed assuming incompressible flow: $V_{inc} = M_{incomp}\cdot\sqrt{k\cdot R\cdot T}$

The error in using Bernoulli to estimate the speed from the pressure difference is $\dfrac{\Delta V}{V} = \dfrac{V_{incomp} - V}{V}$

The computations and plots are shown in the associated *Excel* workbook

Problem 12.53 (In Excel)

12.53 Modern high-speed aircraft use "air data computers" to compute air speed from measurement of the difference between the stagnation and static pressures. Plot, as a function of actual Mach number M, for $M = 0.1$ to $M = 0.9$, the percentage error in computing the Mach number assuming incompressibility (i.e., using the Bernoulli equation), from this pressure difference. Plot the percentage error in speed, as a function of speed, of an aircraft cruising at 12 km altitude, for a range of speeds corresponding to the actual Mach number ranging from $M = 0.1$ to $M = 0.9$.

Given: Flight altitude of high-speed aircraft

Find: Mach number and aircraft speed errors assuming incompressible flow; plot

Solution:

The given or available data is:

R =	286.9	J/kg.K	
k =	1.4		
T =	216.7	K	(At 12 km, Table A.3)

Computing equations:

$$M_{incomp} = \sqrt{\frac{2}{k}\cdot\left[\left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} - 1\right]}$$

$$\frac{\Delta M}{M} = \frac{M_{incomp} - M}{M}$$

$$V = M\cdot\sqrt{k\cdot R\cdot T}$$

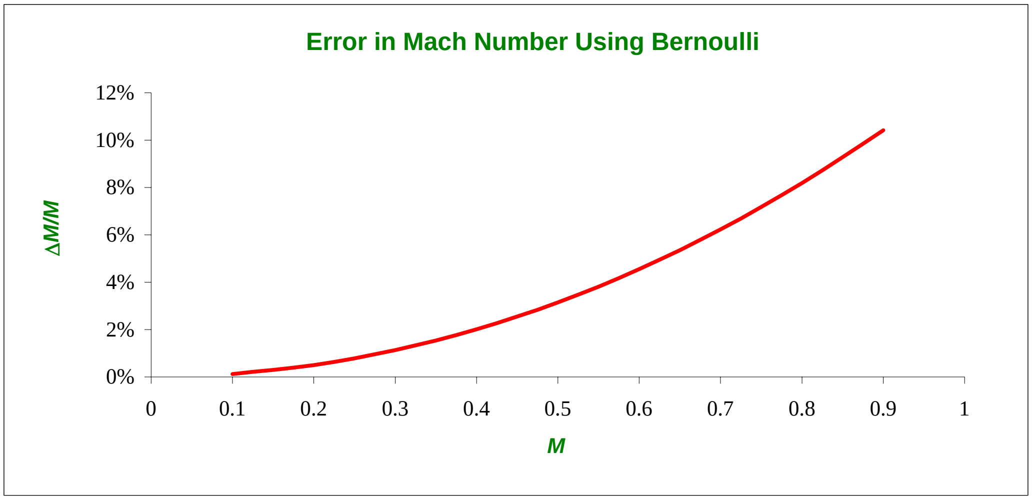
$$V_{inc} = M_{incomp}\cdot\sqrt{k\cdot R\cdot T}$$

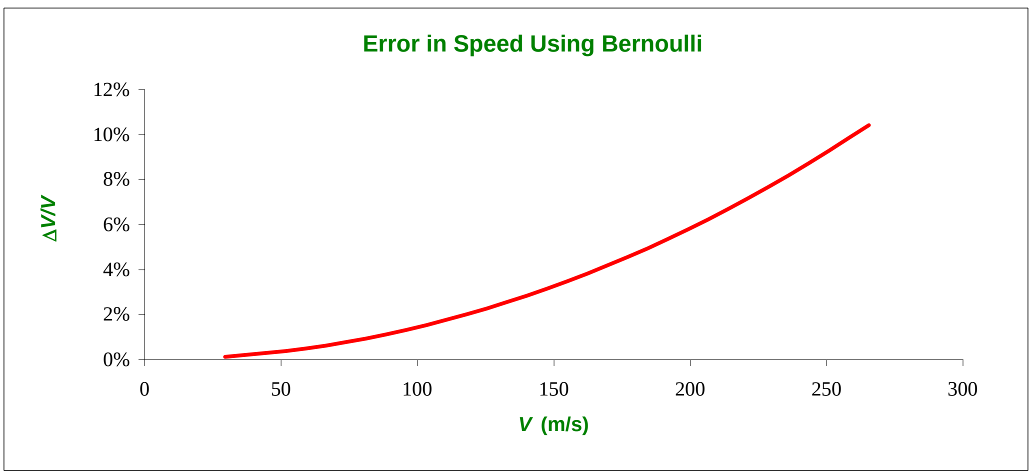
$$\frac{\Delta V}{V} = \frac{V_{incomp} - V}{V}$$

Computed results:

c = 295 m/s

M	M incomp	$\Delta M/M$	V (m/s)	V incomp (m/s)	$\Delta V/V$
0.1	0.100	0.13%	29.5	29.5	0.13%
0.2	0.201	0.50%	59.0	59.3	0.50%
0.3	0.303	1.1%	88.5	89.5	1.1%
0.4	0.408	2.0%	118	120	2.0%
0.5	0.516	3.2%	148	152	3.2%
0.6	0.627	4.6%	177	185	4.6%
0.7	0.744	6.2%	207	219	6.2%
0.8	0.865	8.2%	236	255	8.2%
0.9	0.994	10.4%	266	293	10.4%





Problem 12.54

12.54 A supersonic wind tunnel test section is designed to have $M = 2.5$ at 15°C and 35 kPa (abs). The fluid is air. Determine the required inlet stagnation conditions, T_0 and p_0. Calculate the required mass flow rate for a test section area of 0.175 m^2.

Given: Wind tunnel at M = 2.5

Find: Stagnation conditions; mass flow rate

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad \frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$

Given or available data $\quad M = 2.5 \quad T = (15 + 273) \cdot K \quad p = 35 \cdot kPa \quad A = 0.175 \cdot m^2$

$k = 1.4 \quad R = 286.9 \cdot \frac{J}{kg \cdot K}$

Then $\quad T_0 = T \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right) \quad T_0 = 648\,K \quad T_0 = 375 \cdot °C$

Also $\quad p_0 = p \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}} \quad p_0 = 598 \cdot kPa$

The mass flow rate is given by $\quad m_{rate} = \rho \cdot A \cdot V$

We need $\quad c = \sqrt{k \cdot R \cdot T} \quad c = 340 \frac{m}{s} \quad V = M \cdot c \quad V = 850 \frac{m}{s}$

and also $\quad \rho = \frac{p}{R \cdot T} \quad \rho = 0.424 \frac{kg}{m^3}$

Then $\quad m_{rate} = \rho \cdot A \cdot V \quad m_{rate} = 63.0 \frac{kg}{s}$

Problem 12.55

[2]

Given: Steady air flow through a constant area duct. Properties change due to friction, but flow is adiabatic.

Find: (a) Show that the energy equation reduces to

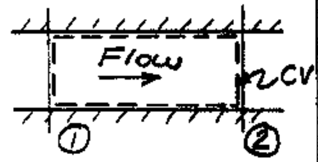
$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = \text{Constant}$$

(b) Show that for adiabatic flow $\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$

(c) Effects on T_0, p_0.

Solution: Apply the energy equation to the CV shown:

$$\text{BE} \quad \dot{Q}^{=0(1)} + \dot{W}_s^{=0(2)} + \dot{W}_{shear}^{=0(3)} = \frac{\partial}{\partial t}\int_{CV}^{=0(4)} e\rho\, d\forall + \int_{CS} (e + pv)\rho \vec{V} \cdot d\vec{A}$$



Assumptions: (1) $\dot{Q} = 0$ (adiabatic)
(2) $\dot{W}_s = 0$
(3) $\dot{W}_{shear} = 0$
(4) Steady flow
(5) Uniform flow at each section
(6) Neglect Δz

Then

$$0 = \left(u_1 + p_1 v_1 + \frac{V_1^2}{2}\right)\{-|\rho_1 V_1 A|\} + \left(u_2 + p_2 v_2 + \frac{V_2^2}{2}\right)\{|\rho_2 V_2 A|\}$$

But $h \equiv u + pv$, and $|\rho_1 V_1 A| = |\rho_2 V_2 A| = |\rho V A| = \dot{m}$, so

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = h + \frac{V^2}{2} = h_0 = \text{Constant}$$ h_0

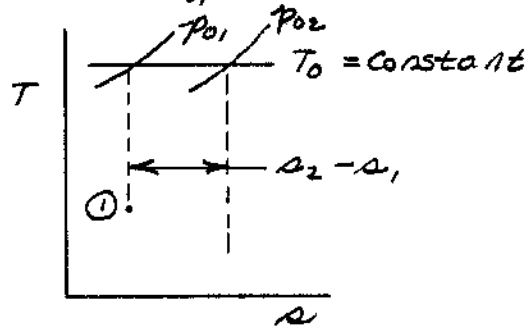
Assumption (7) Ideal gas; $h_0 - h = c_p(T_0 - T)$, $c_p = \frac{kR}{k-1}$, $c^2 = kRT$

Thus $c_p T_0 = c_p T + \frac{V^2}{2}$

$$\frac{T_0}{T} = 1 + \frac{V^2}{2c_p T} = 1 + \frac{(k-1)V^2}{2kRT} = 1 + \frac{k-1}{2}\frac{V^2}{c^2} = 1 + \frac{k-1}{2}M^2$$ $\frac{T_0}{T}$

From the energy equation, $T_{01} = T_{02} = T_0 = \text{constant}$ T_0

The Ts diagram is



Since flow is frictional, $s_2 > s_1$. Therefore $p_{02} < p_{01}$ p_{02}

Problem 12.56

12.56 A new design for a supersonic transport is tested in a wind tunnel at $M = 1.8$. Air is the working fluid. The stagnation temperature and pressure for the wind tunnel are 200 psia and 500°F, respectively. The model wing area is 100 in^2. The measured lift and drag are 12,000 lbf and 1600 lbf, respectively. Find the lift and drag coefficients.

Given: Wind tunnel test of supersonic transport

Find: Lift and drag coefficients

Solution:

Basic equations:

$$c = \sqrt{k \cdot R \cdot T} \qquad M = \frac{V}{c} \qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}} \qquad \frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$$

$$C_L = \frac{F_L}{\frac{1}{2} \cdot \rho \cdot V^2 \cdot A} \qquad C_D = \frac{F_D}{\frac{1}{2} \cdot \rho \cdot V^2 \cdot A}$$

Given or available data

$$M = 1.8 \qquad T_0 = (500 + 460) \cdot R \qquad p_0 = 200 \cdot psi \qquad F_L = 12000 \cdot lbf \qquad F_D = 1600 \cdot lbf$$

$$A = 100 \cdot in^2 \qquad k = 1.4 \qquad R_{air} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$$

We need local conditions

$$p = p_0 \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)^{-\frac{k}{k-1}} \qquad p = 34.8\,psi$$

$$T = \frac{T_0}{1 + \frac{k-1}{2} \cdot M^2} \qquad T = 583\,R \qquad T = 123\,°F$$

Then

$$c = \sqrt{k \cdot R_{air} \cdot T} \qquad c = 1183\,\frac{ft}{s} \qquad c = 807\,mph$$

and

$$V = M \cdot c \qquad V = 2129\,\frac{ft}{s} \qquad V = 1452\,mph$$

We also need

$$\rho = \frac{p}{R_{air} \cdot T} \qquad \rho = 0.00501\,\frac{slug}{ft^3}$$

Finally

$$C_L = \frac{F_L}{\frac{1}{2} \cdot \rho \cdot V^2 \cdot A} \qquad C_L = 1.52$$

$$C_D = \frac{F_D}{\frac{1}{2} \cdot \rho \cdot V^2 \cdot A} \qquad C_D = 0.203$$

Problem 12.57

Given: For aircraft flying at supersonic speeds, the lift and drag coefficients are functions of M only.

Aircraft: $S = 75\ m$, $V = 780\ m/s$, $z = 20\ km$ standard atmosphere

Model: $S = 0.9\ m$, $T = 10°C$, $p = 10\ kPa$ (abs)

Find: (a) air speed for model test
(b) stagnation temperature for model test
(c) stagnation pressure for model test

Solution:

At $z = 20\ km$, $T = 216.7\ K$ (Table A.3).

$$c = (kRT)^{1/2} = \left(1.4 \times 286.9\ \frac{N \cdot m}{kg \cdot K} \times 216.7\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 295\ m/s$$

Thus for aircraft, $M = \frac{V}{c} = \frac{780}{295} = 2.64$

This Mach number must be duplicated in the model test. In the tunnel,

$$c = (kRT)^{1/2} = \left(1.4 \times 287\ \frac{N \cdot m}{kg \cdot K} \times 283\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 337\ m/s$$

$$\therefore V = Mc = 2.64\,(337\ m/s) = 890\ m/s \qquad V$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \qquad \therefore T_0 = T\left(1 + \frac{k-1}{2} M^2\right)$$

$$T_0 = 283\ K\left[1 + \frac{(1.4-1)}{2}(2.64)^2\right] = 677\ K \qquad T_0$$

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1} \qquad \therefore p_0 = p\left[1 + \frac{k-1}{2} M^2\right]^{k/k-1}$$

$$p_0 = 10\ kPa\left[1 + 0.2(2.64)^2\right]^{3.5} = 212\ kPa \qquad p_0$$

Problem 12.58

Given: The Lockheed "Blackbird" aircraft is thought to cruise at $M = 3.3$ at altitude, $z = 26$ km.

Because the speed is supersonic, a normal shock occurs in front of a total-head tube. The stagnation pressure decreases by 74.7 percent across the shock.

Find: (a) V (b) p_o (c) p_o on aircraft (d) T_o on aircraft

Solution:

At altitude $z = 26$ km, $T = 222.5$ K, $p/p_{SL} = 0.0216$ (Table A.3).

$$c = (kRT)^{1/2} = \left[1.4 \times 286.9 \frac{N \cdot m}{kg \cdot K} \times 222.5\,K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 299 \text{ m/s}$$

$$V = Mc = 3.3 \times 299 \text{ m/s} = 987 \text{ m/s} \qquad V$$

The stagnation pressure ahead of the shock (designated p_{o_1}) is given by

$$p_{o_1} = p\left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} = 0.0216\, p_{SL}\left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$$

$$p_{o_1} = 0.0216 \times 101.3 \text{ kPa}\left[1 + \frac{(1.4-1)}{2}(3.3)^2\right]^{\frac{1.4}{0.4}} = 125 \text{ kPa} \qquad p_{o_1}$$

Designating the stagnation pressure behind the shock as p_{o_2}, then

$$\frac{p_{o_1} - p_{o_2}}{p_{o_1}} = 0.747 \quad \text{or} \quad p_{o_2} = p_{o_1} - 0.747\, p_{o_1} = 0.253\, p_{o_1}$$

$$p_{o_2} = 0.253 \times 125 \text{ kPa} = 31.6 \text{ kPa} \qquad p_{o_2}$$

The stagnation temperature does not change across a shock.

$$T_o = T\left[1 + \frac{k-1}{2}M^2\right] = 222.5\,K\left[1 + \frac{(1.4-1)}{2}(3.3)^2\right] = 707 \text{ K} \qquad T_o$$

Problem 12.59

[2]

12.59 Air flows in an insulated duct. At point ① the conditions are $M_1 = 0.1$, $T_1 = 20°\text{C}$, and $p_1 = 1.0$ MPa (abs). Downstream, at point ②, because of friction the conditions are $M_2 = 0.7$, $T_2 = -5.62°\text{C}$, and $p_2 = 136.5$ kPa (abs). (Four significant figures are given to minimize roundoff errors.) Compare the stagnation temperatures at points ① and ②, and explain the result. Compute the stagnation pressures at points ① and ②. Can you explain how it can be that the velocity *increases* for this frictional flow? Should this process be isentropic or not? Justify your answer by computing the change in entropy between points ① and ②. Plot static and stagnation state points on a *Ts* diagram.

Given: Data on air flow in a duct

Find: Stagnation pressures and temperatures; explain velocity increase; isentropic or not?

Solution:

The data provided, or available in the Appendices, is:

$$R = 287 \cdot \frac{\text{J}}{\text{kg}\cdot\text{K}} \qquad c_p = 1004 \cdot \frac{\text{J}}{\text{kg}\cdot\text{K}} \qquad k = 1.4$$

$$M_1 = 0.1 \qquad T_1 = (20 + 273)\cdot\text{K} \qquad p_1 = 1000\cdot\text{kPa} \qquad M_2 = 0.7 \qquad T_2 = (-5.62 + 273)\cdot\text{K} \qquad p_2 = 136.5\cdot\text{kPa}$$

For stagnation temperatures:

$$T_{01} = T_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right) \qquad T_{01} = 293.6\,\text{K} \qquad T_{01} = 20.6\cdot\text{C}$$

$$T_{02} = T_2\cdot\left(1 + \frac{k-1}{2}\cdot M_2^2\right) \qquad T_{02} = 293.6\,\text{K} \qquad T_{02} = 20.6\cdot\text{C}$$

(Because the stagnation temperature is constant, the process is adiabatic)

For stagnation pressures:

$$p_{01} = p_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right)^{\frac{k}{k-1}} \qquad p_{01} = 1.01\cdot\text{MPa}$$

$$p_{02} = p_2\cdot\left(1 + \frac{k-1}{2}\cdot M_2^2\right)^{\frac{k}{k-1}} \qquad p_{02} = 189\cdot\text{kPa}$$

The entropy change is:

$$\Delta s = c_p\cdot\ln\left(\frac{T_2}{T_1}\right) - R\cdot\ln\left(\frac{p_2}{p_1}\right) \qquad \Delta s = 480\cdot\frac{\text{J}}{\text{kg}\cdot\text{K}}$$

Note that

$$V_1 = M_1\cdot\sqrt{k\cdot R\cdot T_1} \qquad V_1 = 34.3\,\frac{\text{m}}{\text{s}} \qquad V_2 = M_2\cdot\sqrt{k\cdot R\cdot T_2} \qquad V_2 = 229\,\frac{\text{m}}{\text{s}}$$

Although there is friction, suggesting the flow should decelerate, because the static pressure drops so much, the net effect is flow acceleration!

The entropy increases because the process is adiabatic but irreversible (friction).

From the second law of thermodynamics $ds \geq \frac{\delta q}{T}$: becomes $ds > 0$

Problem 12.60

12.60 Air is cooled as it flows without friction at a rate of 0.05 kg/s in a duct. At point (1) the conditions are $M_1 = 0.5$, $T_1 = 500°C$, and $p_1 = 500$ kPa (abs). Downstream, at point (2), the conditions are $M_2 = 0.2$, $T_2 = -18.57°C$, and $p_2 = 639.2$ kPa (abs). (Four significant figures are given to minimize roundoff errors.) Compare the stagnation temperatures at points (1) and (2), and explain the result. Compute the rate of cooling. Compute the stagnation pressures at points (1) and (2). Should this process be isentropic or not? Justify your answer by computing the change in entropy between points (1) and (2). Plot static and stagnation state points on a *Ts* diagram.

Given: Data on air flow in a duct

Find: Stagnation temperatures; explain; rate of cooling; stagnation pressures; entropy change

Solution:

The data provided, or available in the Appendices, is: $R = 287 \cdot \frac{J}{kg \cdot K}$ $\quad c_p = 1004 \cdot \frac{J}{kg \cdot K}$ $\quad k = 1.4$

$T_1 = (500 + 273) \cdot K$ $\quad p_1 = 500 \cdot kPa$ $\quad T_2 = (-18.57 + 273) \cdot K$ $\quad p_2 = 639.2 \cdot kPa$

$M_1 = 0.5$ $\quad M_2 = 0.2$ $\quad M_{rate} = 0.05 \cdot \frac{kg}{s}$

For stagnation temperatures:

$$T_{01} = T_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^{\,2}\right) \qquad T_{01} = 811.7\,K \qquad T_{01} = 539 \cdot C$$

$$T_{02} = T_2 \cdot \left(1 + \frac{k-1}{2} \cdot M_2^{\,2}\right) \qquad T_{02} = 256.5\,K \qquad T_{02} = -16.5 \cdot C$$

The fact that the stagnation temperature (a measure of total energy) decreases suggests cooling is taking place.

For the heat transfer:

$$Q = M_{rate} \cdot c_p \cdot \left(T_{02} - T_{01}\right) \qquad Q = -27.9\,kW$$

For stagnation pressures:

$$p_{01} = p_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^{\,2}\right)^{\frac{k}{k-1}} \qquad p_{01} = 593\,kPa$$

$$p_{02} = p_2 \cdot \left(1 + \frac{k-1}{2} \cdot M_2^{\,2}\right)^{\frac{k}{k-1}} \qquad p_{02} = 657\,kPa$$

The entropy change is:

$$\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right) \qquad \Delta s = -1186 \frac{J}{kg \cdot K}$$

The entropy decreases because the process is a cooling process (*Q* is negative).

From the second law of thermodynamics: $ds \geq \frac{\delta q}{T}$ becomes $ds \geq -ve$

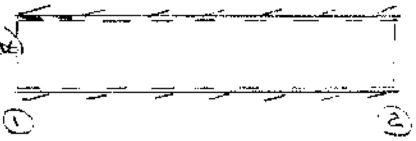
Hence, if the process is reversible, the entropy must decrease; if it is irreversible, it may increase or decrease

Problem 12.61

[3]

Given: Adiabatic flow through long straight pipe of cross-sectional area, $A = 0.05\ m^2$, as shown. Fluid is air

$T_1 = 60°C$, $p_1 = 200\ kPa\ (abs)$, $V_1 = 146\ m/s$



$p_2 = 95.6\ kPa\ (abs)$, $V_2 = 280\ m/s$

Find: $p_{01}, p_{02}, T_{01}, T_{02}, s_2 - s_1$

Solution:

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$ $\qquad \frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Basic equations: $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A}$ (=0(4) on the first term)

$\dot{Q} + \dot{W}_s + \dot{W}_{shear} = \frac{\partial}{\partial t}\int_{CV} e\rho\, d\forall + \int_{CS}\left(e + \frac{p}{\rho}\right)\rho \vec{V} \cdot d\vec{A}$ (=0(1), =0(2), =0(3), =0(4))

Assumptions:
(1) $\dot{Q} = 0$ (adiabatic flow)
(2) $\dot{W}_s = 0$
(3) $\dot{W}_{shear} = 0$
(4) steady flow
(5) uniform flow at each section
(6) $\Delta z = 0$
(7) ideal gas, $k = 1.4$
(8) $A_1 = A_2 = A =$ constant

$M_1 = \frac{V_1}{c_1}$ $\qquad c_1 = (kRT_1)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 333\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 366\ m/s$

$M_1 = \frac{V_1}{c_1} = \frac{146}{366} = 0.399$ $\qquad T_{01} = T_1\left[1 + \frac{k-1}{2}M_1^2\right] = 333\,K\left[1 + 0.2(0.399)^2\right] = 344\,K$ ← T_{01}

$p_{01} = p_1\left[1 + \frac{k-1}{2}M_1^2\right]^{k/k-1} = 200\ kPa\left[1 + 0.2(0.399)^2\right]^{3.5} = 223\ kPa$ ← p_{01}

From the energy equation, $0 = \left(u_1 + p_1 v_1 + \frac{V_1^2}{2}\right)\{-|\rho_1 V_1 A|\} + \left(u_2 + p_2 v_2 + \frac{V_2^2}{2}\right)\{|\rho_2 V_2 A|\}$

From continuity, $0 = -|\rho_1 V_1 A| + |\rho_2 V_2 A|$ or $\rho_1 V_1 = \rho_2 V_2$

Then, using $h = u + pv$,

$h_1 + \frac{V_1^2}{2} = h_{01} = h_2 + \frac{V_2^2}{2} = h_{02}$ or $h_{01} = h_{02}$

For an ideal gas with constant specific heats, $T_{02} = T_{01} = 344\,K$ ← T_{02}

From continuity, $\rho_2 = \rho_1 \frac{V_1}{V_2} = \frac{p_1}{RT_1}\frac{V_1}{V_2} = 200 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\,N \cdot m} \times \frac{1}{333\,K} \times \frac{146}{280} = 1.09\ kg/m^3$

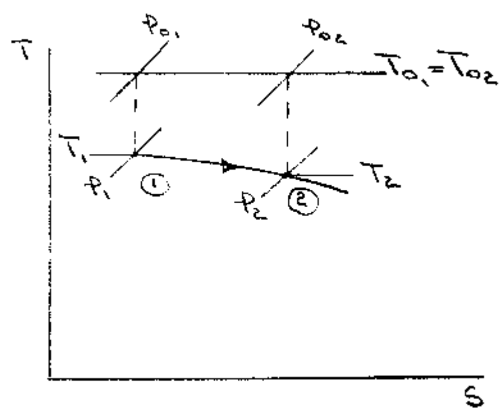
Then,
$T_2 = \frac{p_2}{\rho_2 R} = 95.6 \times 10^3 \frac{N}{m^2} \times \frac{m^3}{1.09\,kg} \times \frac{kg \cdot K}{287\,N \cdot m} = 306\,K \quad (33°C)$

$c_2 = (kRT_2)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 306\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 351\ m/s$

$M_2 = \frac{V_2}{c_2} = \frac{280}{351} = 0.798$

$p_{02} = p_2\left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} = 95.6\,kPa\left[1 + 0.2(0.798)^2\right]^{3.5}$

$p_{02} = 145\ kPa$ ← p_{02}



$T\,ds = dh - v\,dp = c_p\,dT - \frac{RT}{p}\,dp$

$ds = c_p \frac{dT}{T} - R\frac{dp}{p} = ds_0 = -R\frac{dp_0}{p_0}$

$s_{02} - s_{01} = s_2 - s_1 = -R \ln \frac{p_{02}}{p_{01}}$

$s_2 - s_1 = -287 \frac{J}{kg \cdot K} \ln \frac{145}{223} = 0.124 \frac{kJ}{kg \cdot K}$ ← $s_2 - s_1$

Problem 12.62

12.62 Air flows steadily through a constant-area duct. At section (1), the air is at 400 kPa (abs), 325 K, and 150 m/s. As a result of heat transfer and friction, the air at section (2) downstream is at 275 kPa (abs), 450 K. Calculate the heat transfer per kilogram of air between sections (1) and (2), and the stagnation pressure at section (2).

Given: Air flow in duct with heat transfer and friction

Find: Heat transfer; Stagnation pressure at location 2

Solution:

Basic equations: $c = \sqrt{k \cdot R \cdot T}$ $\quad M = \frac{V}{c}$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$

$\rho \cdot V \cdot A = \text{const}$ $\quad h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$

Given or available data $p_1 = 400 \cdot kPa$ $\quad T_1 = 325 \cdot K$ $\quad V_1 = 150 \cdot \frac{m}{s}$

$p_2 = 275 \cdot kPa$ $\quad T_2 = 450 \cdot K$

$c_p = 1004 \cdot \frac{J}{kg \cdot K}$ $\quad k = 1.4$ $\quad R = 286.9 \cdot \frac{J}{kg \cdot K}$

Then $\rho_1 = \frac{p_1}{R \cdot T_1}$ $\quad \rho_1 = 4.29 \frac{kg}{m^3}$ $\quad \rho_2 = \frac{p_2}{R \cdot T_2}$ $\quad \rho_2 = 2.13 \frac{kg}{m^3}$

and from $\rho \cdot V \cdot A = \text{const}$ $\quad V_2 = V_1 \cdot \frac{\rho_1}{\rho_2}$ $\quad V_2 = 302 \frac{m}{s}$

Also $\frac{\delta Q}{dm} = q = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$

$q = c_p \cdot (T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$ $\quad q = 160 \frac{kJ}{kg}$

We also have $c_2 = \sqrt{k \cdot R \cdot T_2}$ $\quad c_2 = 425 \frac{m}{s}$ so $\quad M_2 = \frac{V_2}{c_2}$ $\quad M_2 = 0.711$

Hence $p_{02} = p_2 \cdot \left(1 + \frac{k-1}{2} \cdot M_2^2\right)^{\frac{k}{k-1}}$ $\quad p_{02} = 385 \, kPa$

Problem 12.63

Given: Air passes through a normal shock in a supersonic wind tunnel. Conditions upstream (state ①) and downstream (state ②) of the shock are given below.

①	②
$M_1 = 1.8$	$M_2 = 0.6165$
$T_1 = 270\,K$	$T_2 = 413.6\,K$
$p_1 = 10\,kPa\,(abs)$	$p_2 = 36.13\,kPa\,(abs)$

Find: T_{0_1}, p_{0_1}, T_{0_2}, p_{0_2}, $s_2 - s_1$

Solution:

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \qquad \frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

$T_{0_1} = T_1\left[1 + \frac{k-1}{2}M_1^2\right] = 270\,K\left[1 + 0.2(1.8)^2\right] = 445\,K$ ← T_{0_1}

$p_{0_1} = p_1\left[1 + \frac{k-1}{2}M_1^2\right]^{k/k-1} = 10.0\,kPa\left[1 + 0.2(1.8)^2\right]^{3.5} = 57.5\,kPa\,(abs)$ ← p_{0_1}

$T_{0_2} = T_2\left[1 + \frac{k-1}{2}M_2^2\right] = 413.6\,K\left[1 + 0.2(0.6165)^2\right] = 445\,K$ ← T_{0_2}

(Flow through the shock is adiabatic, $T_{0_2} = T_{0_1}$)

$p_{0_2} = p_2\left[1 + \frac{k-1}{2}M_2^2\right]^{k/k-1} = 36.13\,kPa\left[1 + 0.2(0.6165)^2\right]^{3.5} = 46.7\,kPa\,(abs)$ ← p_{0_2}

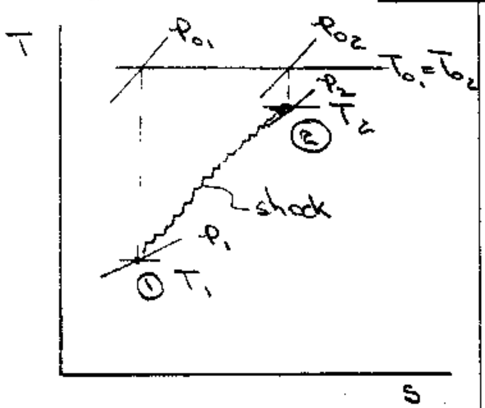
$T\,ds = dh - v\,dp = c_p\,dT - RT\frac{dp}{p}$

$ds = c_p\frac{dT}{T} - R\frac{dp}{p}$

$s_2 - s_1 = s_{0_2} - s_{0_1} = -R\ln\frac{p_{0_2}}{p_{0_1}}$

$= -287\,\frac{J}{kg\cdot K}\ln\frac{46.69}{57.46}$

$s_2 - s_1 = 59.6\,J/kg\cdot K$ ← $s_2 - s_1$



Problem 12.64

12.64 Air enters a turbine at $M_1 = 0.4$, $T_1 = 1250°C$, and $p_1 =$ 625 kPa (abs). Conditions leaving the turbine are $M_2 = 0.8$, $T_2 =$ 650°C, and $p_2 = 20$ kPa (abs). Evaluate local isentropic stagnation conditions (a) at the turbine inlet and (b) at the turbine outlet. Calculate the change in specific entropy across the turbine. Plot static and stagnation state points on a Ts diagram.

Given: Air flow through turbine

Find: Stagnation conditions at inlet and exit; change in specific entropy; Plot on Ts diagram

Solution:

Basic equations:

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} \qquad \frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2 \qquad \Delta s = c_p\cdot\ln\left(\frac{T_2}{T_1}\right) - R\cdot\ln\left(\frac{p_2}{p_1}\right)$$

Given or available data

$M_1 = 0.4$ $\quad p_1 = 625\cdot kPa$ $\quad T_1 = (1250 + 273)\cdot K$

$M_2 = 0.8$ $\quad p_2 = 20\cdot kPa$ $\quad T_2 = (650 + 273)\cdot K$

$c_p = 1004\cdot\frac{J}{kg\cdot K}$ $\quad k = 1.4$ $\quad R = 286.9\cdot\frac{J}{kg\cdot K}$

Then

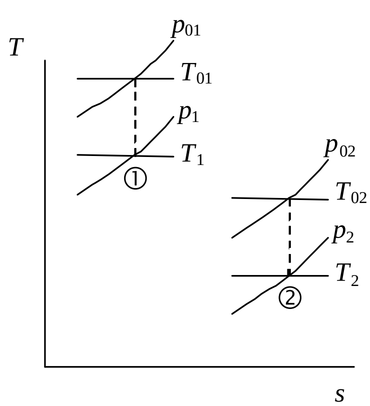
$$T_{01} = T_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right) \qquad T_{01} = 1572\,K \qquad T_{01} = 1299\cdot°C$$

$$p_{01} = p_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right)^{\frac{k}{k-1}} \qquad p_{01} = 698\cdot kPa$$

$$T_{02} = T_2\cdot\left(1 + \frac{k-1}{2}\cdot M_2^2\right) \qquad T_{02} = 1041\,K \qquad T_{02} = 768\cdot°C$$

$$p_{02} = p_2\cdot\left(1 + \frac{k-1}{2}\cdot M_2^2\right)^{\frac{k}{k-1}} \qquad p_{02} = 30\cdot kPa$$

$$\Delta s = c_p\cdot\ln\left(\frac{T_2}{T_1}\right) - R\cdot\ln\left(\frac{p_2}{p_1}\right) \qquad \Delta s = 485\cdot\frac{J}{kg\cdot K}$$



Problem 12.65

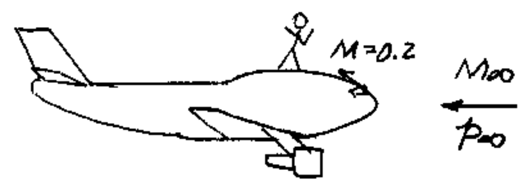
Given: Boeing 747 cruising at $M = 0.87$ at $z = 13$ km, std. day. Window located where $M = 0.2$ relative to surface. Cabin pressurized to equivalent of $z = 2.5$ km, std. day.

Find: Pressure difference across window.

Solution: Apply isentropic stagnation relations.

Computing equation: $p_0 = p\left(1+\frac{k-1}{2}M^2\right)^{k/k-1}$

Assumptions: (1) Ideal gas
(2) Isentropic flow



Consider observer on aircraft: air is decelerated isentropically from $M_\infty = 0.87$ to $M = 0.2$.

From Table A.3:

Calculated: $p = \left(\frac{p}{p_0}\right) p_0$

$p_0 = 101.3$ kPa

Altitude (km)	p/p_0 (---)	p (kPa)
2.5	0.7372	74.7
13.0	0.1636	16.6

For isentropic stagnation:

$$p_0 = p_\infty\left(1+\frac{k-1}{2}M_\infty^2\right)^{k/k-1} = 16.6\text{ kPa}\left(1+\frac{1.4-1}{2}(0.87)^2\right)^{3.5} = 27.2\text{ kPa (abs)}$$

From stagnation to $M = 0.2$:

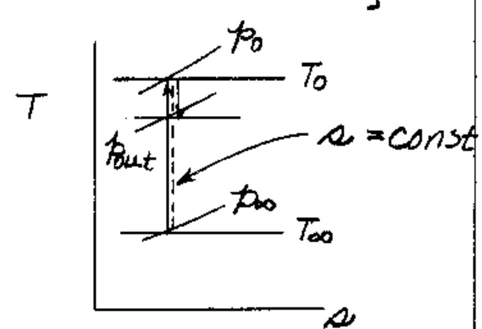
$$p_{out} = \frac{p_0}{\left(1+\frac{k-1}{2}M^2\right)^{k/k-1}} = \frac{27.2\text{ kPa}}{\left(1+0.2(0.2)^2\right)^{3.5}} = 26.5\text{ kPa (abs)}$$

Pressure difference across window is:

$$\Delta p = p_{in} - p_{out} = (74.7 - 26.5)\text{ kPa} = 48.2\text{ kPa}$$ Δp

{Inside pressure is higher; window force is toward outside.}

The corresponding Ts diagram is:



Problem 12.66

[2]

12.66 If a window of the cockpit in Problem 12.65 develops a tiny leak the air will start to rush out at critical speed. Find the mass flow rate if the leak area is 1 mm^2.

Given: Air flow leak in window of airplane

Find: Mass flow rate

Solution:

Basic equations: $m_{rate} = \rho\cdot V\cdot A$ $\quad V_{crit} = \sqrt{\dfrac{2\cdot k}{k+1}\cdot R\cdot T_0}$ $\quad \dfrac{\rho_0}{\rho_{crit}} = \left(\dfrac{k+1}{2}\right)^{\frac{1}{k-1}}$

The interior conditions are the stagnation conditions for the flow

Given or available data $\quad T_0 = 271.9\cdot K$ $\quad \rho_{SL} = 1.225\cdot\dfrac{kg}{m^3}$ $\quad \rho_0 = 0.7812\cdot\rho_{SL}$ $\quad \rho_0 = 0.957\dfrac{kg}{m^3}$

(Above data from Table A.3 at an altitude of 2500 m)

$A = 1\cdot mm^2$ $\quad c_p = 1004\cdot\dfrac{J}{kg\cdot K}$ $\quad k = 1.4$ $\quad R = 286.9\cdot\dfrac{J}{kg\cdot K}$

Then $\quad \rho_{crit} = \dfrac{\rho_0}{\left(\dfrac{k+1}{2}\right)^{\frac{1}{k-1}}}$ $\quad \rho_{crit} = 0.607\dfrac{kg}{m^3}$ $\quad V_{crit} = \sqrt{\dfrac{2\cdot k}{k+1}\cdot R\cdot T_0}$ $\quad V_{crit} = 302\dfrac{m}{s}$

The mass flow rate is $\quad m_{rate} = \rho_{crit}\cdot V_{crit}\cdot A$ $\quad m_{rate} = 1.83\times 10^{-4}\dfrac{kg}{s}$

Problem 12.67

Given: A CO_2 cartridge contains gas at $p_0 = 45$ MPa (gage) and $T_0 = 25°C$

Find: T^*, p^*, V^* that correspond to these stagnation conditions

Solution:

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{k/k-1}$

For CO_2, $k = 1.29$. At critical conditions, $M = 1$

$\frac{T_0^*}{T^*} = 1 + \frac{k-1}{2} = 1.145 \qquad \therefore T^* = \frac{T_0^*}{1.145} = \frac{298\text{K}}{1.145} = 260\text{ K}$ ← T^*

$\frac{p_0^*}{p^*} = \left[1 + \frac{k-1}{2}\right]^{k/k-1} = \left[1.145\right]^{4.448} = 1.826$

$\therefore p^* = \frac{p_0^*}{1.826} = \frac{45.101\text{ MPa}}{1.826} = 24.7\text{ MPa (abs)}$ ← p^*

$V^* = c^* = (kRT^*)^{1/2} = \left(1.29 \times 189\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 260\text{K} \times \frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)^{1/2} = 252\text{ m/s}$ ← V^*

Problem 12.68

[1]

12.68 The gas storage reservoir for a high-speed wind tunnel contains helium at 3600 R and 725 psig. Calculate the critical conditions (temperature, pressure, and flow speed) that correspond to these stagnation conditions.

Given: Data on helium in reservoir

Find: Critical conditions

Solution:

The data provided, or available in the Appendices, is:

$R_{He} = 386.1 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$ $\quad k = 1.66$ $\quad T_0 = 3600 \cdot R$ $\quad p_0 = (725 + 14.7) psi$ $\quad p_0 = 740\, psi$

For critical conditions $\quad \frac{T_0}{T_{crit}} = \frac{k+1}{2}$ $\quad T_{crit} = \frac{T_0}{\frac{k+1}{2}}$ $\quad T_{crit} = 2707\, R$

$$\frac{p_0}{p_{crit}} = \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}} \qquad p_{crit} = \frac{p_0}{\left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}} \qquad p_{crit} = 361\, psi \quad \text{absolute}$$

$$V_{crit} = \sqrt{k \cdot R_{He} \cdot T_{crit}} \qquad V_{crit} = 7471 \frac{ft}{s}$$

Problem 12.69

Given: Stagnation conditions in a solid propellant rocket motor are $T_0 = 3000\,K$ and $p_0 = 45\,MPa$ (gage). Assume ideal gas behavior with $R = 323\,J/kg \cdot K$, $k = 1.2$. The Mach number is unity at the throat of the nozzle.

Find: T_t, p_t, V_t

Solution:

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$ $\qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{k/k-1}$

Assume flow to throat is isentropic.

At throat, $M = 1$

$$\therefore T_{0_t} = T_t\left(1 + \frac{k-1}{2}\right) = 1.1\,T_t \qquad \therefore T_t = \frac{T_0}{1.1} = \frac{3000\,K}{1.10} = 2730\,K \qquad T_t$$

$$\frac{p_{0_t}}{p_t} = \left(1 + \frac{k-1}{2}\right)^{k/k-1} = (1.1)^{\frac{1.2}{0.2}} = 1.7716$$

$$\therefore p_t = \frac{p_0}{1.7716} = \frac{45.101\,MPa}{1.7716} = 25.5\,MPa\,(abs) \qquad p_t$$

$$V_t = (kRT_t)^{1/2} = \left[1.2 \times 323\,\frac{N \cdot m}{kg \cdot K} \times 2730\,K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 1030\,m/s \qquad V_t$$

Problem 12.70

[1]

12.70 The hot gas stream at the turbine inlet of a JT9-D jet engine is at 1500°C, 140 kPa (abs), and $M = 0.32$. Calculate the critical conditions (temperature, pressure, and flow speed) that correspond to these conditions. Assume the fluid properties of pure air.

Given: Data on hot gas stream

Find: Critical conditions

Solution:

The data provided, or available in the Appendices, is:

$R = 287 \cdot \frac{J}{kg \cdot K}$ $\quad k = 1.4$ $\quad T_0 = (1500 + 273) \cdot K$ $\quad T_0 = 1773\,K$ $\quad p_0 = 140 \cdot kPa$

For critical conditions $\quad \frac{T_0}{T_{crit}} = \frac{k+1}{2}$ $\quad T_{crit} = \frac{T_0}{\frac{k+1}{2}}$ $\quad T_{crit} = 1478\,K$

$$\frac{p_0}{p_{crit}} = \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}} \qquad p_{crit} = \frac{p_0}{\left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}} \qquad p_{crit} = 74.0\,kPa \quad \text{absolute}$$

$$V_{crit} = \sqrt{k \cdot R \cdot T_{crit}} \qquad V_{crit} = 770\,\frac{m}{s}$$

Problem 13.1

[2]

13.1 Air is extracted from a large tank in which the temperature and pressure are 70°C and 101 kPa (abs), respectively, through a nozzle. At one location in the nozzle the static pressure is 25 kPa and the diameter is 15 cm. What is the mass flow rate? Assume isentropic flow.

Given: Air extracted from a large tank

Find: Mass flow rate

Solution:

Basic equations: $m_{rate} = \rho \cdot V \cdot A$ $\quad h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$ $\quad \frac{p}{\rho^k} = \text{const}$ $\quad T \cdot p^{\frac{(1-k)}{k}} = \text{const}$

Given or available data $T_0 = (70 + 273) \cdot K$ $\quad p_0 = 101 \cdot kPa$ $\quad p = 25 \cdot kPa$

$D = 15 \cdot cm$ $\quad c_p = 1004 \cdot \frac{J}{kg \cdot K}$ $\quad k = 1.4$ $\quad R = 286.9 \cdot \frac{J}{kg \cdot K}$

The mass flow rate is given by $m_{rate} = \rho \cdot A \cdot V$ $\quad A = \frac{\pi \cdot D^2}{4}$ $\quad A = 0.0177\, m^2$

We need the density and velocity at the nozzle. In the tank $\rho_0 = \frac{p_0}{R \cdot T_0}$ $\quad \rho_0 = 1.026 \frac{kg}{m^3}$

From the isentropic relation $\rho = \rho_0 \cdot \left(\frac{p}{p_0}\right)^{\frac{1}{k}}$ $\quad \rho = 0.379 \frac{kg}{m^3}$

We can apply the energy equation between the tank (stagnation conditions) and the point in the nozzle to find the velocity

$$h_0 = h + \frac{V^2}{2} \qquad V = \sqrt{2 \cdot (h_0 - h)} = \sqrt{2 \cdot c_p \cdot (T_0 - T)}$$

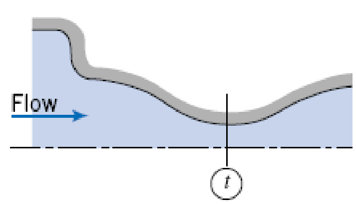
Fot T we again use insentropic relations $T = T_0 \cdot \left(\frac{p_0}{p}\right)^{\frac{(1-k)}{k}}$ $\quad T = 230.167\, K$ $\quad T = -43.0 \cdot °C$

Then $V = \sqrt{2 \cdot c_p \cdot (T_0 - T)}$ $\quad V = 476 \frac{m}{s}$

The mass flow rate is $m_{rate} = \rho \cdot A \cdot V$ $\quad m_{rate} = 3.18 \frac{kg}{s}$

Note that the flow is supersonic at this point $c = \sqrt{k \cdot}$ $c = 304 \frac{m}{s}$ $\quad M = \frac{V}{c}$ $\quad M = 1.57$

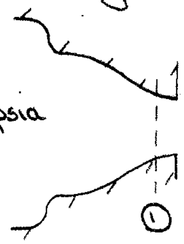
Hence we must have a converging-diverging nozzle



Problem 13.2

Given: Steam flows steadily and isentropically through a nozzle

$T_0 = 900°F$ $\quad$ $P_0 = 900$ psia



$P_1 = 600$ psia $\quad$ $D_1 = 0.188$ in

(nozzle shape unspecified)

Find: V_1, M_1, $\dot{m}$, passage shape

Solution:

Basic equations:
$$h_1 + \frac{V_1^2}{2} = h + \frac{V^2}{2} = h_0 = \text{constant}$$
$$\rho A V = \text{constant} = \dot{m}$$

Assumptions:
(1) steady flow
(2) isentropic flow
(3) uniform flow at a section
(4) superheated steam can be treated as an ideal gas, $R = 85.8$ ft·lbf/lbm·°R, $k = 1.28$
(5) $\Delta z = 0$

From steam tables for superheated vapor with $T_0 = 900°F$, $P_0 = 900$ psia,
$h_0 = 1451.8$ Btu/lbm $\quad$ $s_0 = 1.6257$ Btu/lbm·°R

From steam tables for superheated vapor with $s_1 = 1.6257$ Btu/lbm·°R, and $P_1 = 600$ psia,
$T_1 = 780°F$, $h_1 = 1396.5$ Btu/lbm, $v_1 = 1.167$ ft³/lbm

From the first law,
$$V_1 = [2(h_0 - h_1)]^{1/2} = \left[2(1451.8 - 1396.5)\frac{\text{Btu}}{\text{lbm}} \times 778\frac{\text{ft}\cdot\text{lbf}}{\text{Btu}} \times 32.2\frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right]^{1/2}$$

$V_1 = 1660$ ft/s ⟵ V_1

$$\dot{m} = \rho A V = \frac{1}{v}\frac{\pi D^2}{4}V = \frac{\text{lbm}}{1.167\,\text{ft}^3} \times \frac{\pi}{4} \times \left(\frac{0.188}{12}\right)^2 \text{ft}^2 \times 1660\frac{\text{ft}}{\text{s}} = 0.274 \text{ lbm/s}$$ ⟵ $\dot{m}$

$M_1 = \frac{V_1}{c_1}$ $\quad$ $c_1 = (kRT)^{1/2} = \left(1.30 \times 85.8\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot°\text{R}} \times 1240\,\text{R} \times 32.2\frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right)^{1/2}$

$c_1 = 2110$ ft/s

$$M_1 = \frac{V_1}{c_1} = \frac{1660}{2110} = 0.787$$ ⟵

Since $M_1 < 1.0$, passage converges as shown above

Problem 13.3

13.3 Steam flows steadily and isentropically through a nozzle. At an upstream section where the speed is negligible, the temperature and pressure are 450°C and 6 MPa (abs). At a section where the nozzle diameter is 2 cm, the steam pressure is 2 MPa (abs.). Determine the speed and Mach number at this section and the mass flow rate of steam. Sketch the passage shape.

Given: Steam flow through a nozzle

Find: Speed and Mach number; Mass flow rate; Sketch the shape

Solution:

Basic equations: $m_{rate} = \rho \cdot V \cdot A$ $\qquad h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$

Assumptions: 1) Steady flow 2) Isentropic 3) Uniform flow 4) Superheated steam can be treated as ideal gas

Given or available data $\quad T_0 = (450 + 273) \cdot K \qquad p_0 = 6 \cdot MPa \qquad p = 2 \cdot MPa$

$D = 2 \cdot cm \qquad k = 1.30 \qquad R = 461.4 \cdot \frac{J}{kg \cdot K}$ (Table A.6)

From the steam tables (try finding interactive ones on the Web!), at stagnation conditions

$$s_0 = 6720 \cdot \frac{J}{kg \cdot K} \qquad h_0 = 3.302 \times 10^6 \cdot \frac{J}{kg}$$

Hence at the nozzle section $\quad s = s_0 = 6720 \cdot \frac{J}{kg \cdot K}$ and $p = 2\,MPa$

From these values we find from the steam tables that $\quad T = 289\,°C \qquad h = 2.997 \times 10^6 \cdot \frac{J}{kg} \qquad v = 0.1225 \cdot \frac{m^3}{kg}$

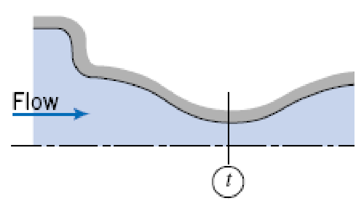
Hence the first law becomes $\quad V = \sqrt{2 \cdot (h_0 - h)} \qquad V = 781 \frac{m}{s}$

The mass flow rate is given by $\quad m_{rate} = \rho \cdot A \cdot V = \frac{A \cdot V}{v} \qquad A = \frac{\pi \cdot D^2}{4} \qquad A = 3.14 \times 10^{-4} m^2$

Hence $\quad m_{rate} = \frac{A \cdot V}{v} \qquad m_{rate} = 2.00 \frac{kg}{s}$

For the Mach number we need $\quad c = \sqrt{k \cdot R \cdot T} \qquad c = 581 \frac{m}{s} \qquad M = \frac{V}{c} \qquad M = 1.35$

The flow is supersonic starting from rest, so must be converging-diverging



Problem 13.4

13.4 At a section in a passage, the pressure is 150 kPa (abs), the temperature is 10°C, and the speed is 120 m/s. For isentropic flow of air, determine the Mach number at the point where the pressure is 50 kPa (abs). Sketch the passage shape.

Given: Air flow in a passage

Find: Mach number; Sketch shape

Solution:

Basic equations: $\frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad c = \sqrt{k \cdot R \cdot T}$

Given or available data $T_1 = (10 + 273) \cdot K$ $\quad p_1 = 150 \cdot kPa$ $\quad V_1 = 120 \cdot \frac{m}{s}$

$p_2 = 50 \cdot kPa$ $\quad k = 1.4$ $\quad R = 286.9 \cdot \frac{J}{kg \cdot K}$

The speed of sound at state 1 is $c_1 = \sqrt{k \cdot R \cdot T_1}$ $\quad c_1 = 337 \frac{m}{s}$

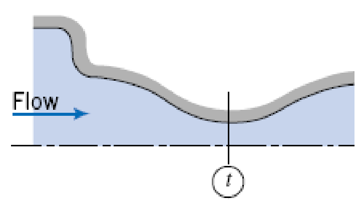
Hence $M_1 = \frac{V_1}{c_1}$ $\quad M_1 = 0.356$

For isentropic flow stagnation pressure is constant. Hence at state 2 $\quad \frac{p_0}{p_2} = \left(1 + \frac{k-1}{2} \cdot M_2^2\right)^{\frac{k}{k-1}}$

Hence $p_0 = p_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^2\right)^{\frac{k}{k-1}}$ $\quad p_0 = 164\,kPa$

Solving for M_2 $\quad M_2 = \sqrt{\frac{2}{k-1} \cdot \left[\left(\frac{p_0}{p_2}\right)^{\frac{k-1}{k}} - 1\right]}$ $\quad M_2 = 1.42$

Hence, as we go from subsonic to supersonic we must have a converging-diverging nozzle



Problem 13.5

[2]

13.5 At a section in a passage, the pressure is 30 psia, the temperature is 100°F, and the speed is 1750 ft/s. At a section downstream the Mach number is 2.5. Determine the pressure at this downstream location for isentropic flow of air. Sketch the passage shape.

Given: Data on flow in a passage

Find: Pressure at downstream location

Solution:

The given or available data is:

R =	53.33	ft·lbf/lbm·°R
k =	1.4	
T_1 =	560	°R
p_1 =	30	psi
V_1 =	1750	ft/s
M_2 =	2.5	

Equations and Computations:

From T_1 and Eq. 12.18

$$c = \sqrt{kRT}$$

c_1 = 1160 ft/s

Then

M_1 = 1.51

From M_1 and p_1, and Eq. 13.7a
(using built-in function *Isenp* (M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)}$$

p_{01} = 111 psi

For isentropic flow ($p_{01} = p_{02}$)

p_{02} = 111 psi

From M_2 and p_{02}, and Eq. 13.7a
(using built-in function *Isenp* (M, k))

p_2 = 6.52 psi

Problem 13.6

[3]

13.6 Air flows isentropically through a converging-diverging nozzle from a large tank containing air at 250°C. At two locations where the area is 1 cm^2, the static pressures are 200 kPa and 50 kPa. Find the mass flow rate, the throat area, and the Mach numbers at the two locations.

Given: Data on flow in a nozzle

Find: Mass flow rate; Throat area; Mach numbers

Solution:

The given or available data is:

$R =$	286.9	J/kg·K			
$k =$	1.4				
$T_0 =$	523	K			
$p_1 =$	200	kPa	$p_2 =$	50	kPa
$A =$	1	cm^2			

Equations and Computations:

We don't know the two Mach numbers. We do know for each that Eq. 13.7a applies:

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)}$$

Hence we can write two equations, but have three unknowns (M_1, M_2, and p_0)!

We also know that states 1 and 2 have the same area. Hence we can write Eq. 13.7d twice:

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)}$$

We now have four equations for four unknowns (A^*, M_1, M_2, and p_0)!

We make guesses (using Solver) for M_1 and M_2, and make the errors in computed A^* and p_0 zero.

For:	$M_1 =$	0.512		$M_2 =$	1.68		**Errors**
from Eq. 13.7a:	$p_0 =$	239	kPa	$p_0 =$	239	kPa	0.00%
and from Eq. 13.7d:	$A^* =$	0.759	cm^2	$A^* =$	0.759	cm^2	0.00%
Note that the throat area is the critical area						**Sum**	0.00%

The stagnation density is then obtained from the ideal gas equation

$\rho_0 =$ 1.59 kg/m^3

The density at critical state is obtained from Eq. 13.7a (or 12.22c)

$\rho^* =$ 1.01 kg/m^3

The velocity at critical state can be obtained from Eq. 12.23)

$$V^* = c^* = \sqrt{\frac{2k}{k+1}RT_0}$$

$V^* =$ 418 m/s

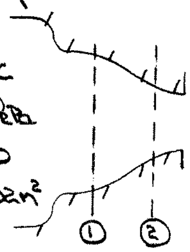
The mass flow rate is $\rho^* V^* A^*$

$m_{rate} =$ 0.0321 kg/s

Problem 13.7

Given: Steady, isentropic flow of air through a passage

$T_1 = 60°C$
$p_1 = 40$ kPa
$M_1 = 2.0$
$A_1 = 0.02\,m^2$



$V_2 = 519$ m/s

(passage shape unspecified)

Find: M_2, shape of passage

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) $\Delta z = 0$ (5) ideal gas

$M_2 = \frac{V_2}{c_2}$ where $c_2 = (kRT_2)^{1/2}$. Hence T_2 must be found

$$h_2 = h_1 + \frac{1}{2}(V_1^2 - V_2^2)$$

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 2.0\left(1.4 \times 287 \frac{N\cdot m}{kg\cdot K} \times 333K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 732 \text{ m/s}$$

$$T_2 = T_1 + \frac{1}{2c_p}(V_1^2 - V_2^2)$$

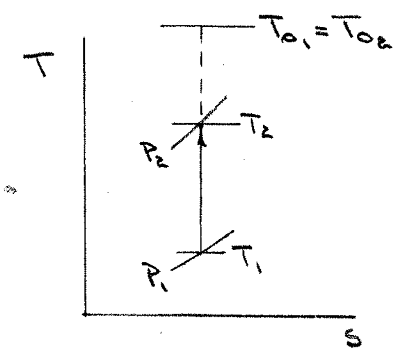
$$= 333K + \left[(732)^2 - (519)^2\right]\frac{m^2}{s^2} \times \frac{1}{2} \times \frac{kg\cdot K}{10^3\,N\cdot m} \times \frac{N\cdot s^2}{kg\cdot m}$$

$$T_2 = 466\text{ K}$$

$$c_2 = (kRT_2)^{1/2} = \left(1.4 \times 287 \frac{N\cdot m}{kg\cdot K} \times 466K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 433 \text{ m/s}$$

$$M_2 = \frac{V_2}{c_2} = \frac{519}{433} = 1.20$$ ← M_2

Since $M_2 < M_1$ and $M_2 > 1.0$, then passage from ① to ② is a supersonic diffuser as shown above



Problem 13.8

13.8 Air flows steadily and isentropically through a passage at 150 lbm/s. At the section where the diameter is $D = 3$ ft, $M = 1.75$, $T = 32°$F, and $p = 25$ psia. Determine the speed and cross-sectional area downstream where $T = 225°$F. Sketch the flow passage.

Given: Air flow in a passage

Find: Speed and area downstream; Sketch flow passage

Solution:

Basic equations:

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2 \qquad c = \sqrt{k\cdot R\cdot T} \qquad \frac{A}{A_{crit}} = \frac{1}{M}\cdot\left(\frac{1+\frac{k-1}{2}\cdot M^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}}$$

Given or available data

$$T_1 = (32+460)\cdot R \qquad p_1 = 25\cdot psi \qquad M_1 = 1.75$$

$$T_2 = (225+460)\cdot R \qquad k = 1.4 \qquad R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}$$

$$D_1 = 3\cdot ft \qquad A_1 = \frac{\pi\cdot D_1^{\,2}}{4} \qquad A_1 = 7.07\,ft^2$$

Hence

$$T_0 = T_1\cdot\left(1+\frac{k-1}{2}\cdot M_1^{\,2}\right) \qquad T_0 = 793\,R \qquad T_0 = 334\,°F$$

For isentropic flow stagnation conditions are constant. Hence

$$M_2 = \sqrt{\frac{2}{k-1}\cdot\left(\frac{T_0}{T_2}-1\right)} \qquad M_2 = 0.889$$

We also have

$$c_2 = \sqrt{k\cdot R_{air}\cdot T_2} \qquad c_2 = 1283\,\frac{ft}{s}$$

Hence

$$V_2 = M_2\cdot c_2 \qquad V_2 = 1141\,\frac{ft}{s}$$

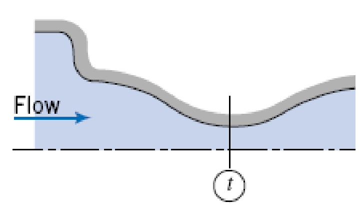
From state 1

$$A_{crit} = \frac{A_1\cdot M_1}{\left(\frac{1+\frac{k-1}{2}\cdot M_1^{\,2}}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}}} \qquad A_{crit} = 5.10\,ft^2$$

Hence at state 2

$$A_2 = \frac{A_{crit}}{M_2}\cdot\left(\frac{1+\frac{k-1}{2}\cdot M_2^{\,2}}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}} \qquad A_2 = 5.15\,ft^2$$

Hence, as we go from supersonic to subsonic we must have a converging-diverging diffuser



Given: Steady, isentropic flow of air through a passage.

$T_1 = 27°C$

$P_1 = 60$ kPa

$V_1 = 486$ m/s

$A_1 = 0.02$ m²

$P_2 = 78.8$ kPa

(passage shape unspecified)

Find: M_2

Solution:

Computing equations: $\frac{P_0}{P} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$ $c = \sqrt{kRT}$

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) ideal gas

For isentropic flow, $P_{0_1} = P_{0_2} = P_0 = \text{constant}$

$M_1 = \frac{V_1}{c_1}$ $c_1 = (kRT_1)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 300K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 347$ m/s

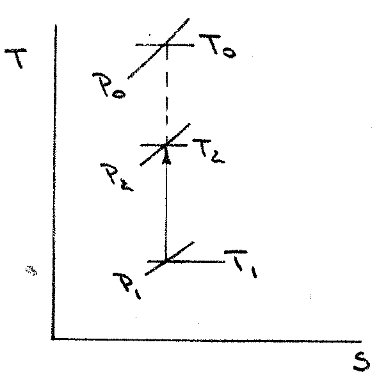
$M_1 = \frac{V_1}{c_1} = \frac{486}{347} = 1.40$ $P_{0_1} = P_1\left[1 + \frac{k-1}{2}M_1^2\right]^{k/k-1} = 60\,kPa\left[1 + 0.2(1.40)^2\right]^{3.5}$

$P_{0_1} = 191$ kPa

$\frac{P_{0_2}}{P_2} = \left[1 + \frac{k-1}{2}M_2^2\right]^{k/k-1}$, $P_{0_2} = P_{0_1}$

$$M_2 = \left\{\frac{2}{k-1}\left[\left(\frac{P_{0_1}}{P_2}\right)^{k-1/k} - 1\right]\right\}^{1/2} = \left\{\frac{2}{0.4}\left[\left(\frac{191}{78.8}\right)^{0.286} - 1\right]\right\}^{1/2} = 1.20 \longleftarrow M_2$$

Since $M_2 < M_1$ and $M_2 > 1.0$, then passage from ① to ② is a supersonic diffuser as shown above



Problem 13.10

[2]

13.10 For isentropic flow of air, at a section in a passage, $A = 0.25\ \text{m}^2$, $p = 15$ kPa (abs), $T = 10°\text{C}$, and $V = 590$ m/s. Find the Mach number and the mass flow rate. At a section downstream the temperature is 137°C and the Mach number is 0.75. Determine the cross-sectional area and pressure at this downstream location. Sketch the passage shape.

Given: Data on flow in a passage

Find: Flow rate; area and pressure at downstream location; sketch passage shape

Solution:

The given or available data is:

$R =$	286.9	J/kg.K
$k =$	1.4	
$A_1 =$	0.25	m^2
$T_1 =$	283	K
$p_1 =$	15	kPa
$V_1 =$	590	m/s
$T_2 =$	410	
$M_2 =$	0.75	

Equations and Computations:

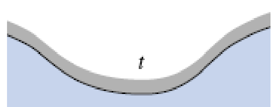
From T_1 and Eq. 12.18

$$c = \sqrt{kRT} \qquad (12.18)$$

$c_1 =$ 337 m/s

Then $M_1 =$ **1.75**

Because the flow decreases isentropically from supersonic to subsonic the passage shape must be convergent-divergent



From p_1 and T_1 and the ideal gas equation

$\rho_1 =$ 0.185 kg/m^3

The mass flow rate is $m_{rate} = \rho_1 A_1 V_1$

$m_{rate} =$ **27.2** kg/s

From M_1 and A_1, and Eq. 13.7d
(using built-in function *IsenA* (M, k))

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)} \qquad (13.7d)$$

$A^* =$ 0.180 m^2

From M_2 and A^*, and Eq. 13.7d
(using built-in function *IsenA* (M, k))

$A_2 =$ 0.192 m^2

From M_1 and p_1, and Eq. 13.7a
(using built-in function *Isenp* (M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$p_{01} =$ 79.9 kPa

For isentropic flow ($p_{01} = p_{02}$)

$p_{02} =$ 79.9 kPa

From M_2 and p_{02}, and Eq. 13.7a
(using built-in function *Isenp* (M, k))

$p_2 =$ 55.0 kPa

Problem 13.11

13.11 Atmospheric air (101 kPa and 20°C) is drawn into a receiving pipe via a converging nozzle. The throat cross-section diameter is 1 cm. Plot the mass flow rate delivered for the receiving pipe pressure ranging from 100 kPa down to 5 kPa.

Given: Flow in a converging nozzle to a pipe

Find: Plot of mass flow rate

Solution:

The given or available data is

$R =$ 287 J/kg·K
$k =$ 1.4
$T_0 =$ 293 K
$p_0 =$ 101 kPa
$D_t =$ 1 cm
$A_t =$ 0.785 cm^2

Equations and Computations:

The critical pressure is given by

$$\frac{p_0}{p^*} = \left[\frac{k+1}{2}\right]^{k/(k-1)} \qquad (12.22a)$$

$p^* =$ 53.4 kPa

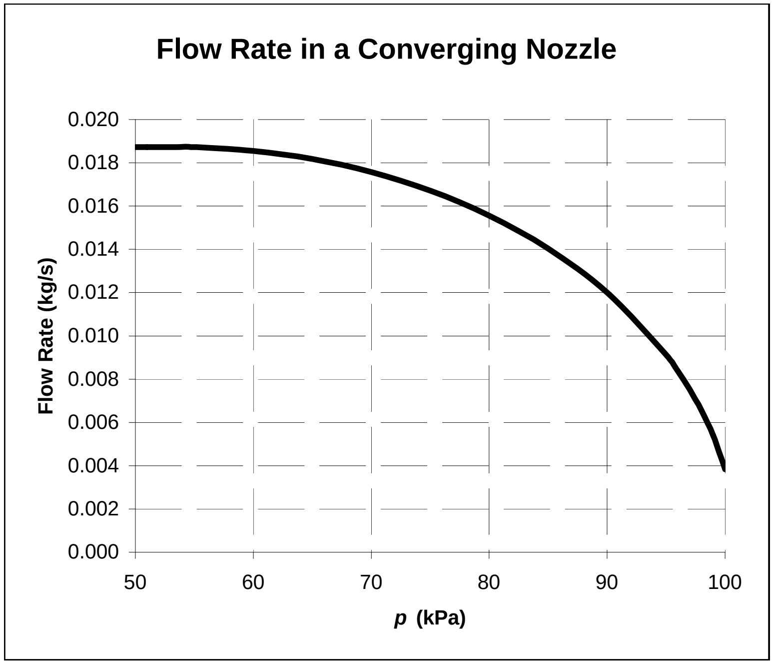
Hence for p = 100 kPa down to this pressure the flow gradually increases; then it is constant

p (kPa)	M (Eq. 13.7a)	T (K) (Eq. 13.7b)	c (m/s)	$V = M \cdot c$ (m/s)	$\rho = p/RT$ (kg/m^3)	Flow (kg/s)
100	0.119	292	343	41	1.19	0.00383
99	0.169	291	342	58	1.18	0.00539
98	0.208	290	342	71	1.18	0.00656
97	0.241	290	341	82	1.17	0.00753
96	0.270	289	341	92	1.16	0.00838
95	0.297	288	340	101	1.15	0.0091
90	0.409	284	337	138	1.11	0.0120
85	0.503	279	335	168	1.06	0.0140
80	0.587	274	332	195	1.02	0.0156
75	0.666	269	329	219	0.971	0.0167
70	0.743	264	326	242	0.925	0.0176
65	0.819	258	322	264	0.877	0.0182
60	0.896	252	318	285	0.828	0.0186
55	0.974	246	315	306	0.778	0.0187
53.4	1.000	244	313	313	0.762	0.0187
53	1.000	244	313	313	0.762	0.0187
52	1.000	244	313	313	0.762	0.0187
51	1.000	244	313	313	0.762	0.0187
50	1.000	244	313	313	0.762	0.0187

Using critical conditions, and Eq. 13.9 for mass flow rate:

53.4	1.000	244	313	313	0.762	0.0185

(Note: discrepancy in mass flow rate is due to round-off error)



Problem 13.12

13.12 Repeat Problem 13.11 if the converging nozzle is replaced with a converging-diverging nozzle with an exit diameter of 2.5 cm (same throat area).

Given: Flow in a converging-diverging nozzle to a pipe

Find: Plot of mass flow rate

Solution:

The given or available data is

R =	286.9	J/kg·K			
k =	1.4				
T_0 =	293	K			
p_0 =	101	kPa			
D_t =	1	cm	De =	2.5	cm
A_t =	0.785	cm^2	A_e =	4.909	cm^2

Equations and Computations:

The critical pressure is given by

$$\frac{p_0}{p^*} = \left[\frac{k+1}{2}\right]^{k/(k-1)} \qquad (12.22a)$$

p^* = 53.4 kPa This is the minimum throat pressure

For the CD nozzle, we can compute the pressure at the exit required for this to happen

A^* =	0.785	cm^2	(= A_t)		
A_e/A^* =	6.25				
M_e =	0.0931	or	3.41	(Eq. 13.7d)	
p_e =	100.4	or	67.2	kPa (Eq. 13.7a)	

Hence we conclude flow occurs in regimes *iii* down to *v* (Fig. 13.8); the flow is ALWAYS choked!

p^* (kPa)	M 13.7a)	T^* (K) (Eq. 13.7b)	c^* (m/s)	$V^* = c^*$ (m/s)	$\rho = p/RT$ (kg/m^3)	Flow (kg/s)
53.4	1.000	244	313	313	0.762	0.0187
(Note: discrepancy in mass flow rate is due to round-off error)						0.0185

(Using Eq. 13.9)

Problem 13.13

[3]

13.13 A passage is designed to expand air isentropically to atmospheric pressure from a large tank in which properties are held constant at 40°F and 45 psia. The desired flow rate is 2.25 lbm/s. Assuming the passage is 20 ft long, and that the Mach number increases linearly with position in the passage, plot the cross-sectional area and pressure as functions of position.

Given: Data on tank conditions; isentropic flow

Find: Plot cross-section area and pressure distributions

Solution:

The given or available data is:

$R =$	53.33	ft·lbf/lbm·°R
$k =$	1.4	
$T_0 =$	500	°R
$p_0 =$	45	psia
$p_e =$	14.7	psia
$m_{rate} =$	2.25	lbm/s

Equations and Computations:

From p_0, p_e and Eq. 13.7a (using built-in function *IsenMfromp* (M,k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$M_e =$ 1.37

Because the exit flow is supersonic, the passage must be a CD nozzle
We need a scale for the area.
From p_0, T_0, m_{flow}, and Eq. 13.10c

$$\dot{m}_{\text{choked}} = 76.6\,\frac{A_t p_0}{\sqrt{T_0}} \qquad (13.10c)$$

Then $A_t = A^* =$ 0.0146 ft^2

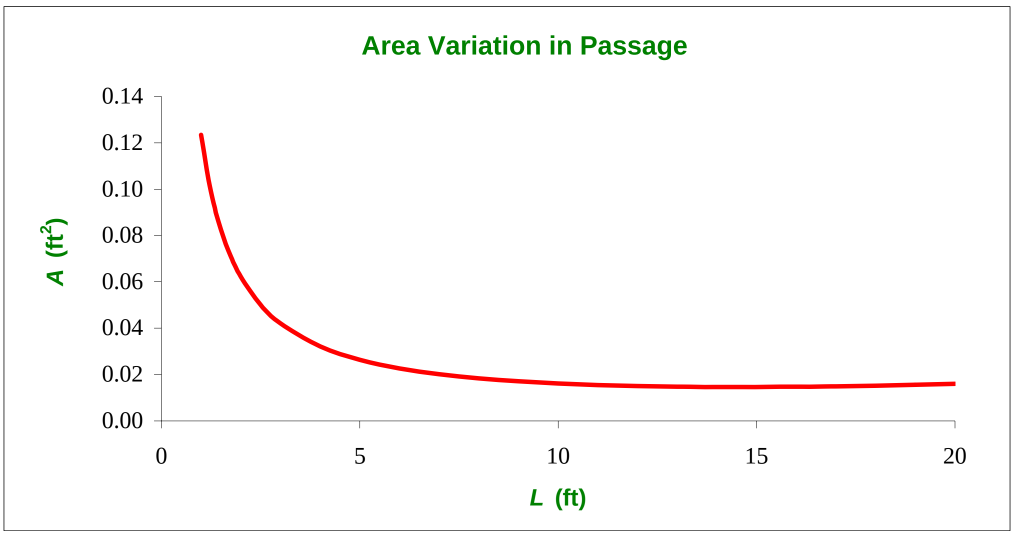
For each M, and A^*, and Eq. 13.7d
(using built-in function *IsenA* (M,k)

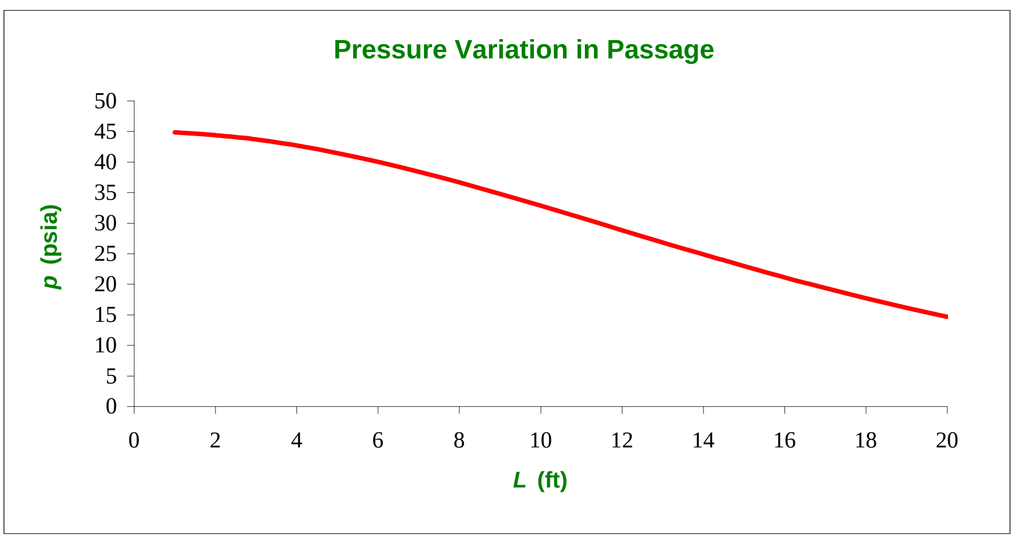
$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)} \qquad (13.7d)$$

we can compute each area A.

From each M, and p_0, and Eq. 13.7a
(using built-in function *Isenp* (M,k)
we can compute each pressure p.

L (ft)	M	A (ft^2)	p (psia)
1.00	0.069	0.1234	44.9
1.25	0.086	0.0989	44.8
1.50	0.103	0.0826	44.7
1.75	0.120	0.0710	44.5
2.00	0.137	0.0622	44.4
2.50	0.172	0.0501	44.1
3.00	0.206	0.0421	43.7
4.00	0.274	0.0322	42.7
5.00	0.343	0.0264	41.5
6.00	0.412	0.0227	40.0
7.00	0.480	0.0201	38.4
8.00	0.549	0.0183	36.7
9.00	0.618	0.0171	34.8
10.00	0.686	0.0161	32.8
11.00	0.755	0.0155	30.8
12.00	0.823	0.0150	28.8
13.00	0.892	0.0147	26.8
14.00	0.961	0.0146	24.9
14.6	**1.000**	**0.0146**	**23.8**
16.00	1.098	0.0147	21.1
17.00	1.166	0.0149	19.4
18.00	1.235	0.0152	17.7
19.00	1.304	0.0156	16.2
20.00	1.372	0.0161	14.7





Problem 13.14

[2]

13.14 Air flows isentropically through a converging nozzle into a receiver in which the absolute pressure is 35 psia. The air enters the nozzle with negligible speed at a pressure of 60 psia and a temperature of 200°F. Determine the mass flow rate through the nozzle for a throat diameter of 4 in.

Given: Air flow in a converging nozzle

Find: Mass flow rate

Solution:

Basic equations: $m_{rate} = \rho \cdot V \cdot A$ $\quad p = \rho \cdot R \cdot T$ $\quad \frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$

Given or available data $p_b = 35 \cdot psi$ $\quad p_0 = 60 \cdot psi$ $\quad T_0 = (200 + 460) \cdot R$ $\quad D_t = 4 \cdot in$

$k = 1.4$ $\quad R_{air} = 53.33 \cdot \frac{ft \cdot lbf}{lbm \cdot R}$ $\quad A_t = \frac{\pi}{4} \cdot D_t^2$ $\quad A_t = 0.0873 \cdot ft^2$

Since $\frac{p_b}{p_0} = 0.583$ is greater than 0.528, the nozzle is not choked and $p_t = p_b$

Hence $M_t = \sqrt{\frac{2}{k-1} \cdot \left[\left(\frac{p_0}{p_t}\right)^{\frac{k-1}{k}} - 1\right]}$ $\quad M_t = 0.912$

and $T_t = \frac{T_0}{1 + \frac{k-1}{2} \cdot M_t^2}$ $\quad T_t = 566 \cdot R$ $\quad T_t = 106 \cdot {}^\circ F$

$c_t = \sqrt{k \cdot R_{air} \cdot T_t}$ $\quad V_t = c_t$ $\quad V_t = 1166 \cdot \frac{ft}{s}$

$\rho_t = \frac{p_t}{R_{air} \cdot T_t}$ $\quad \rho_t = 5.19 \times 10^{-3} \cdot \frac{slug}{ft^3}$

$m_{rate} = \rho_t \cdot A_t \cdot V_t$ $\quad m_{rate} = 0.528 \cdot \frac{slug}{s}$ $\quad m_{rate} = 17.0 \cdot \frac{lbm}{s}$

Problem 13.15

[2]

13.15 Air flows isentropically through a converging nozzle into a receiver where the pressure is 250 kpa (abs). If the pressure is 350 kpa (abs) and the speed is 150 m/s at the nozzle location where the Mach number is 0.5, determine the pressure, speed, and Mach number at the nozzle throat.

Given: Isentropic air flow in converging nozzle

Find: Pressure, speed and Mach number at throat

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$ $\qquad$ $\frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$

Given or available data $\quad p_1 = 350 \cdot kPa \qquad V_1 = 150 \cdot \frac{m}{s} \qquad M_1 = 0.5 \qquad p_b = 250 \cdot kPa$

$k = 1.4 \qquad R = 286.9 \cdot \frac{J}{kg \cdot K}$

The flow will be choked if $p_b/p_0 < 0.528$

$$p_0 = p_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^2\right)^{\frac{k}{k-1}} \qquad p_0 = 415\,kPa \qquad \frac{p_b}{p_0} = 0.602 \qquad \text{(Not choked)}$$

Hence $\quad \frac{p_0}{p_t} = \left(1 + \frac{k-1}{2} \cdot M_t^2\right)^{\frac{k}{k-1}}$ where $p_t = p_b \qquad p_t = 250\,kPa$

so $\quad M_t = \sqrt{\frac{2}{k-1} \cdot \left[\left(\frac{p_0}{p_t}\right)^{\frac{k-1}{k}} - 1\right]} \qquad M_t = 0.883$

Also $\quad V_1 = M_1 \cdot c_1 = M_1 \cdot \sqrt{k \cdot R \cdot T_1}$ or $T_1 = \frac{1}{k \cdot R} \cdot \left(\frac{V_1}{M_1}\right)^2 \qquad T_1 = 224\,K \qquad T_1 = -49.1\,°C$

Then $\quad T_0 = T_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^2\right) \qquad T_0 = 235\,K \qquad T_0 = -37.9\,°C$

Hence $\quad T_t = \frac{T_0}{1 + \frac{k-1}{2} \cdot M_t^2} \qquad T_t = 204\,K \qquad T_t = -69.6\,°C$

Then $\quad c_t = \sqrt{k \cdot R \cdot T_t} \qquad c_t = 286 \frac{m}{s}$

Finally $\quad V_t = M_t \cdot c_t \qquad V_t = 252 \frac{m}{s}$

Problem 13.16

[3]

13.16 Air is flowing steadily through a series of three tanks. The first very large tank contains air at 650 kPa and 35°C. Air flows from it to a second tank through a converging nozzle with exit area 1 cm^2. Finally the air flows from the second tank to a third very large tank through an identical nozzle. The flow rate through the two nozzles is the same, and the flow in them is isentropic. The pressure in the third tank is 65 kPa. Find the mass flow rate, and the pressure in the second tank.

Given: Data on three tanks

Find: Mass flow rate; Pressure in second tank

Solution:

The given or available data is:

	R =	286.9	J/kg.K
	k =	1.4	
	A_t =	1	cm^2

We need to establish whether each nozzle is choked. There is a large total pressure drop so this is likely. However, BOTH cannot be choked and have the same flow rate. This is because Eq. 13.9a, below

$$\dot{m}_{\text{choked}} = 0.04\frac{A_e p_0}{\sqrt{T_0}} \qquad (13.9b)$$

indicates that the choked flow rate depends on stagnation temperature (which is constant) but also stagnation pressure, which drops because of turbulent mixing in the middle chamber. Hence BOTH nozzles cannot be choked. We assume the second one only is choked (why?) and verify later.

Temperature and pressure in tank 1:	T_{01} =	308	K
	p_{01} =	650	kPa
We make a guess at the pressure at the first nozzle exit:	p_{e1} =	527	kPa
NOTE: The value shown is the final answer! It was obtained using *Solver* !			
This will also be tank 2 stagnation pressure:	p_{02} =	527	kPa
Pressure in tank 3:	p_3 =	65	kPa

Equations and Computations:

From the p_{e1} guess and Eq. 13.17a:	M_{e1} =	0.556		
Then at the first throat (Eq.13.7b):	T_{e1} =	290	K	
The density at the first throat (Ideal Gas) is:	ρ_{e1} =	6.33	kg/m^3	
Then c at the first throat (Eq. 12.18) is:	c_{e1} =	341	m/s	
Then V at the first throat is:	V_{e1} =	190	m/s	
Finally the mass flow rate is:	m_{rate} =	0.120	kg/s	First Nozzle!

For the presumed choked flow at the second nozzle we use Eq. 13.9a, with $T_{01} = T_{02}$ and p_{02}:

m_{rate} = 0.120 kg/s Second Nozzle!

For the guess value for p_{e1} we compute the error between the two flow rates:

Δm_{rate} = 0.000 kg/s

Use Solver to vary the guess value for p_{e1} to make this error zero!

Note that this could also be done manually.

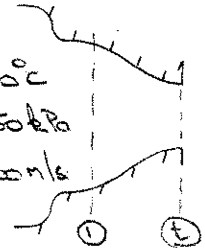
Problem 13.17

Given: Air flows isentropically through a converging nozzle, discharging to the atmosphere

$T_1 = 20°C$

$P_1 = 250\,kPa$

$V_1 = 200\,m/s$



$P_b = P_{atm} = 101\,kPa$

Find: P_t

Solution:

Computing equations: $\frac{P_0}{P} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$ $\qquad c = \sqrt{kRT}$

Assumptions: (1) steady flow
(2) isentropic flow in the nozzle
(3) uniform flow at a section
(4) ideal gas

The nozzle will be choked, i.e. $M_t = 1.0$ if $\frac{P_b}{P_0} \leq 0.528$

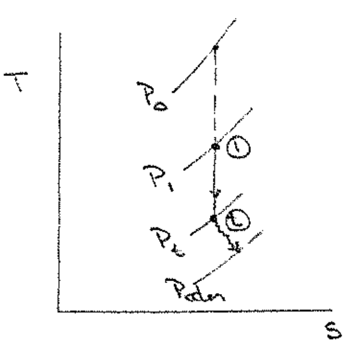
$M_1 = \frac{V_1}{c_1}$ $\quad c_1 = \sqrt{kRT_1} = \left(1.4 \times 287\,\frac{N \cdot m}{kg \cdot K} \times 293\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 343\,m/s$

$M_1 = \frac{V_1}{c_1} = \frac{200}{343} = 0.583$

$\frac{P_0}{P} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$ $\quad P_0 = P_1\left[1 + \frac{k-1}{2}M_1^2\right]^{k/k-1} = 250\,kPa\left[1 + 0.2(0.583)^2\right]^{3.5} = 315\,kPa$

Then, $\frac{P_b}{P_0} = \frac{101}{315} = 0.321 < 0.528$ $\quad \therefore M_t = 1.0$

For $M_t = 1.0$, $\frac{P_t}{P_0} = 0.528$ $\qquad P_t = 0.528\,P_0 = 0.528 \times 315\,kPa = 166\,kPa$ ← P_t



Given: Isentropic flow of air from a large tank through a converging nozzle discharges to atmosphere.

$T_0 = 550°C$

$p_0 = 650$ kPa (abs)

$p_b = p_{atm} = 101$ kPa (abs).

$A_t = 600$ mm²

Find: $\dot{m}$

Solution:

Basic equations: $\dot{m} = \rho V A = \text{const}$ $\quad p = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \quad \frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1} \quad c = \sqrt{kRT}$

Assumptions: (1) steady flow (2) isentropic flow in nozzle (3) uniform flow at a section (4) ideal gas

Since $p_b / p_0 = \frac{101}{650} = 0.155 < 0.528$, the nozzle is choked and $M_t = 1.0$

From the continuity equation $\dot{m} = \rho V A$ and hence we need to determine ρ_t and V_t

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \;; \quad T_t = \frac{T_0}{1 + \frac{k-1}{2} M_t^2} = \frac{823 \text{ K}}{1 + 0.2(1.0)^2} = 686 \text{ K}$$

$$V_t = M_t c_t = M_t (kRT_t)^{1/2} = 1.0\left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 686 K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 525 \text{ m/s}$$

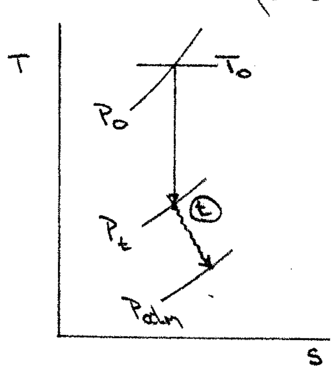
$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1} ; \quad p_t = \frac{p_0}{\left[1 + \frac{k-1}{2} M_t^2\right]^{k/k-1}} = \frac{650 \text{ kPa}}{\left[1 + 0.2(1.0)^2\right]^{3.5}} = 343 \text{ kPa}$$

$$\rho_t = \frac{p_t}{RT_t} = 343 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287 \, N \cdot m} \times \frac{1}{686 K} = 1.74 \text{ kg/m}^3$$

Finally

$$\dot{m} = \rho_t V_t A_t = 1.74 \frac{kg}{m^3} \times 525 \frac{m}{s} \times 6 \times 10^{-4} m^2 = 0.548 \text{ kg/s}$$

$\dot{m}$



Problem 13.19

[2]

13.19 Air flowing isentropically through a converging nozzle discharges to the atmosphere. At a section the area is $A = 0.05\ \text{m}^2$, $T = 3.3°\text{C}$, and $V = 200$ m/s. If the flow is just choked, find the pressure and the Mach number at this location. What is the throat area? What is the mass flow rate?

Given: Data on converging nozzle; isentropic flow

Find: Pressure and Mach number; throat area; mass flow rate

Solution:

The given or available data is:

$R =$	286.9	J/kg.K
$k =$	1.4	
$A_1 =$	0.05	m^2
$T_1 =$	276.3	K
$V_1 =$	200	m/s
$p_{atm} =$	101	kPa

Equations and Computations:

From T_1 and Eq. 12.18

$$c = \sqrt{kRT} \qquad (12.18)$$

$c_1 =$ 333 m/s

Then $M_1 =$ 0.60

To find the pressure, we first need the stagnation pressure.
If the flow is just choked

$p_e =$ $p_{atm} =$ $p^* =$ 101 kPa

From $p_e = p^*$ and Eq. 12.22a

$$\frac{p_0}{p^*} = \left[\frac{k+1}{2}\right]^{k/(k-1)} \qquad (12.22a)$$

$p_0 =$ 191 kPa

From M_1 and p_0, and Eq. 13.7a
(using built-in function *Isenp* (*M* ,*k*)

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

Then $p_1 =$ 150 kPa

The mass flow rate is $m_{rate} = \rho_1 A_1 V_1$

Hence, we need ρ_1 from the ideal gas equation.

$\rho_1 =$ 1.89 kg/m^3

The mass flow rate m_{rate} is then

$m_{rate} =$ 18.9 kg/s

The throat area $A_t = A^*$ because the flow is choked.
From M_1 and A_1, and Eq. 13.7d
(using built-in function *IsenA* (M,k)

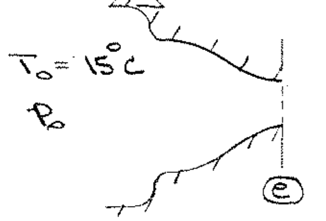
$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)} \quad (13.7d)$$

$A^* =$ 0.0421 m^2

Hence $A_t =$ 0.0421 m^2

Problem 13.20

Given: Flow of air from stagnation state through a converging nozzle discharges to the atmosphere. $p_e = 325$ kPa (gage)



$p_b = 101$ kPa

$A_e = 0.001\ m^2$

$p_e = 325$ kPa (gage) $= 426$ kPa (abs)

Find: p_0, $\dot{m}$

Solution:

Basic equations: $\dot{m} = \rho A V$ $\quad p = \rho R T$

Computing equations: $\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1}$ $\quad \frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) ideal gas behavior

Since $p_e > p_b$, nozzle is choked and $M_e = 1.0$

$p_0 = p_e\left[1 + \frac{k-1}{2} M_e^2\right]^{k/k-1} = 426 \text{ psia} \left[1 + 0.2\right]^{3.5} = 806$ kPa $\longleftarrow$ p_0

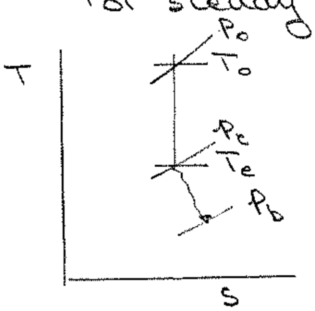
$\frac{T_0}{T_e} = 1 + \frac{k-1}{2} M_e^2 \quad \therefore \quad T_e = \frac{T_0}{1.2} = \frac{288 K}{1.2} = 240 K$

$V_e = c_e = (kRT_e)^{1/2} = \left[1.4 \times 287 \frac{J}{kg \cdot K} \times 240 K \times \frac{N \cdot m}{J} \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 311$ m/s

$\rho_e = \frac{p_e}{RT_e} = 426 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287 J} \times \frac{1}{240 K} \times \frac{J}{N \cdot m} = 6.18$ kg/m^3

Then $\dot{m} = \rho_e V_e A_e = 6.18 \frac{kg}{m^3} \times 311 \frac{m}{s} \times 0.001 m^2 = 1.92$ kg/s

For steady flow, $\dot{m} = 1.92$ kg/s must be supplied to the tank $\longleftarrow$ $\dot{m}$



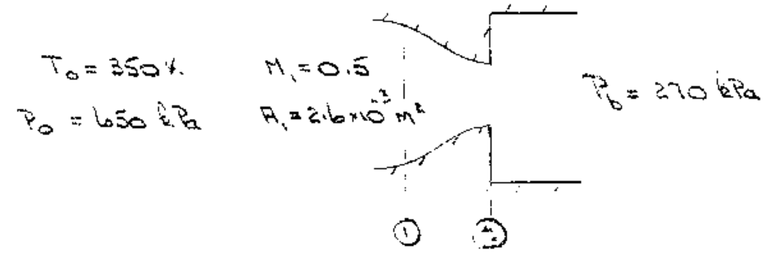
The corresponding volume flow rate of standard air is

$Q = \frac{\dot{m}}{\rho_{stand}} = 1.92 \frac{kg}{s} \times \frac{m^3}{6.18 kg}$

$Q = 0.31$ m^3/s

Problem 13.21

Given: Isentropic flow of air through a converging nozzle discharges to a back pressure, p_b.



Find: A_t

Solution:

Computing equations: $p_0/p = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

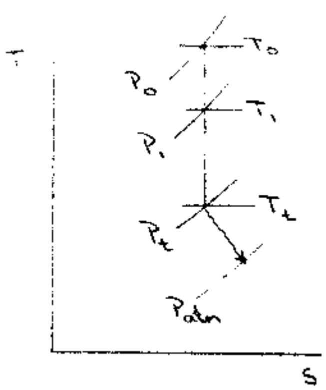
$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{1 + \frac{k-1}{2}}\right]^{(k+1)/2(k-1)} \qquad (12.6)$$

Assumptions:
(1) steady flow
(2) isentropic flow in nozzle
(3) uniform flow at a section
(4) ideal gas

Since $p_b/p_0 = \frac{270 \text{ kPa}}{650 \text{ kPa}} = 0.415 < 0.528$, the nozzle is choked and $M_t = 1.0$

From Eq 12.6 with $M_1 = 0.5$, $A_1/A^* = 1.340$

then $A_t = A^* = A_1/1.340 = \frac{2.6 \times 10^{-3}\ m^2}{1.340} = 1.94 \times 10^{-3}\ m^2$ ← A_t



Problem 13.22

Given: Reversible, adiabatic flow of air from a large tank through a converging nozzle discharges to atmosphere.

$T_o = 600\,K$

$P_o = 600\,kPa$

$P_b = P_{atm} = 101\,kPa$

$A_t = 1.29 \times 10^{-3}\,m^2$

Find: (a) range of tank pressure, P_o, for which $M_t = 1.0$

(b) $\dot{m}$ for conditions given

Solution:

Basic equations: $\dot{m} = \rho V A = \text{const.}$ $\quad P = \rho R T$

Computing equations: $\frac{T_o}{T} = 1 + \frac{k-1}{2} M^2 \qquad \frac{P_o}{P} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow (2) isentropic flow in nozzle (3) uniform flow at a section (4) ideal gas

The nozzle will be choked, ie $M_t = 1.0$ for $P_b/P_o \le 0.528$

Since $P_b = 101\,kPa$, nozzle is choked for

$$P_o \ge \frac{P_b}{0.528} = \frac{101\,kPa}{0.528} = 191\,kPa \qquad P_o$$

Thus for $P_o = 600\,kPa$, $M_t = 1.0$

$$\frac{T_o}{T} = 1 + \frac{k-1}{2}M^2 \; ; \quad T_t = \frac{T_o}{1 + \frac{k-1}{2}M_t^2} = \frac{600\,K}{1 + 0.2(1.0)^2} = 500\,K$$

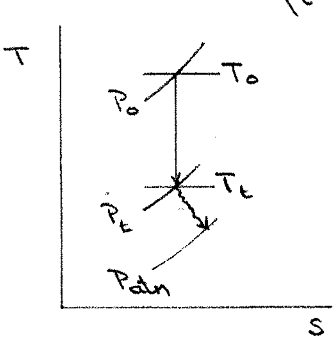
$$V_t = M_t c_t = M_t (kRT_t)^{1/2} = 1.0\left(1.4 \times 287\frac{N\cdot m}{kg\cdot K} \times 500K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 448\,m/s$$

$$\frac{P_o}{P} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} ; \quad P_t = \frac{P_o}{\left[1 + \frac{k-1}{2}M_t^2\right]^{k/k-1}} = \frac{600\,kPa}{\left[1 + 0.2(1.0)^2\right]^{3.5}} = 317\,kPa$$

$$\rho_t = \frac{P_t}{RT_t} = 317 \times 10^3 \frac{N}{m^2} \times \frac{kg\cdot K}{287\,N\cdot m} \times \frac{1}{500K} = 2.21\,kg/m^3$$

Finally,

$$\dot{m} = \rho_t V_t A_t = 2.21\frac{kg}{m^3} \times 448\frac{m}{s} \times 1.29 \times 10^{-3}\,m^2 = 1.28\,kg/s \qquad \dot{m}$$



Problem 13.23

[2]

13.23 Air at 0°C is contained in a large tank on the space shuttle. A converging section with exit area 1×10^{-3} m^2 is attached to the tank, through which the air exits to space at a rate of 2 kg/s. What are the pressure in the tank, and the pressure, temperature, and speed at the exit?

Given: Temperature in and mass flow rate from a tank

Find: Tank pressure; pressure, temperature and speed at exit

Solution:

The given or available data is:

R = 286.9 J/kg.K
k = 1.4
T_0 = 273 K
A_t = 0.001 m^2
m_{rate} = 2 kg/s

Equations and Computations:

Because $p_b = 0$ $\quad p_e = p^*$
Hence the flow is choked!

Hence $\quad T_e = T^*$

From T_0, and Eq. 12.22b

$$\frac{T_0}{T^*} = \frac{k+1}{2} \qquad (12.22b)$$

T^* = 228 K

T_e = 228 K
-45.5 °C

Also $\quad M_e$ = 1
Hence $\quad V_e$ = V^* = c_e

From T_e and Eq. 12.18

$$c = \sqrt{kRT} \qquad (12.18)$$

c_e = 302 m/s

Then $\quad V_e$ = 302 m/s

To find the exit pressure we use the ideal gas equation
after first finding the exit density.
The mass flow rate is $m_{rate} = \rho_e A_e V_e$

Hence $\quad \rho_e$ = 6.62 kg/m^3

From the ideal gas equation $p_e = \rho_e R T_e$

p_e = 432 kPa

From $p_e = p^*$ and Eq. 12.22a

$$\frac{p_0}{p^*} = \left[\frac{k+1}{2}\right]^{k/(k-1)} \qquad (12.22a)$$

p_0 = 817 kPa

We can check our results:
From p_0, T_0, A_t, and Eq. 13.9a

$$\dot{m}_{\text{choked}} = A_e p_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \qquad (13.9a)$$

Then $\quad m_{choked}$ = 2.00 kg/s
m_{choked} = m_{rate} Correct!

Problem 13.24

13.24 A large tank initially is evacuated to −10 kPa (gage). (Ambient conditions are 101 kPa at 20°C.) At $t = 0$, an orifice of 5 mm diameter is opened in the tank wall; the vena contracta area is 65 percent of the geometric area. Calculate the mass flow rate at which air initially enters the tank. Show the process on a Ts diagram. Make a schematic plot of mass flow rate as a function of time. Explain why the plot is nonlinear.

Given: Isentropic air flow into a tank

Find: Initial mass flow rate; Ts process; explain nonlinear mass flow rate

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $\quad m_{rate} = \rho\cdot A\cdot V$

Given or available data $p_0 = 101\cdot kPa$ $\quad p_b = p_0 - 10\cdot kPa$ $\quad p_b = 91\cdot kPa$ $\quad T_0 = (20 + 273)\cdot K$

$k = 1.4$ $\quad R = 286.9\cdot\frac{J}{kg\cdot K}$ $\quad D = 5\cdot mm$

Then $A = \frac{\pi}{4}\cdot D^2$ $\quad A_{vena} = 65\cdot\%\cdot A$ $\quad A_{vena} = 12.8\cdot mm^2$

The flow will be choked if $p_b/p_0 < 0.528$ $\quad \frac{p_b}{p_0} = 0.901$ (Not choked)

Hence $\frac{p_0}{p_{vena}} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ where $p_{vena} = p_b$ $\quad p_{vena} = 91\cdot kPa$

so $M_{vena} = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_{vena}}\right)^{\frac{k-1}{k}} - 1\right]}$ $\quad M_{vena} = 0.389$

Then $T_{vena} = \frac{T_0}{1 + \frac{k-1}{2}\cdot M_{vena}^2}$ $\quad T_{vena} = 284\,K$ $\quad T_{vena} = 11.3\cdot°C$

Then $c_{vena} = \sqrt{k\cdot R\cdot T_{vena}}$ $\quad c_{vena} = 338\frac{m}{s}$

and $V_{vena} = M_{vena}\cdot c_{vena}$ $\quad V_{vena} = 131\frac{m}{s}$

Also $\rho_{vena} = \frac{p_{vena}}{R\cdot T_{vena}}$ $\quad \rho_{vena} = 1.12\frac{kg}{m^3}$

Finally $m_{rate} = \rho_{vena}\cdot A_{vena}\cdot V_{vena}$ $\quad m_{rate} = 1.87\times 10^{-3}\frac{kg}{s}$

The Ts diagram will be a vertical line (T decreases and s = const). After entering the tank there will be turbulent mixing (s increases) and t comes to rest (T increases). The mass flow rate versus time will look like the curved part of Fig. 13.6b; it is nonlinear because V AND ρ va

Problem 13.25

13.25 A-50 cm diameter spherical cavity initially is evacuated. The cavity is to be filled with air for a combustion experiment. The pressure is to be 45 kPa (abs), measured after its temperature reaches T_{atm}. Assume the valve on the cavity is a converging nozzle with throat diameter of 1 mm, and the surrounding air is at standard conditions. For how long should the valve be opened to achieve the desired final pressure in the cavity? Calculate the entropy change for the air in the cavity.

Given: Spherical cavity with valve

Find: Time to reach desired pressure; Entropy change

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$

$p = \rho \cdot R \cdot T$ $\quad c = \sqrt{k \cdot R \cdot T}$ $\quad m_{rate} = \rho \cdot A \cdot V$ $\quad m_{choked} = A_t \cdot p_0 \cdot \sqrt{\frac{k}{R \cdot T_0}} \cdot \left(\frac{2}{k+1}\right)^{\frac{k+1}{2 \cdot (k-1)}}$

Given or available data $\quad p_0 = 101 \cdot kPa$ $\quad T_{atm} = (20 + 273) \cdot K$ $\quad T_0 = T_{atm}$ $\quad d = 1 \cdot mm$ $\quad D = 50 \cdot cm$

$p_f = 45 \cdot kPa$ $\quad T_f = T_{atm}$ $\quad k = 1.4$ $\quad R = 286.9 \cdot \frac{J}{kg \cdot K}$ $\quad c_p = 1004 \cdot \frac{J}{kg \cdot K}$

Then the inlet area is $\quad A_t = \frac{\pi}{4} \cdot d^2$ $\quad A_t = 0.785\, mm^2$ $\quad$ and tank volume is $\quad V = \frac{\pi}{3} \cdot D^3$ $\quad V = 0.131\, m^3$

The flow will be choked if $p_b/p_0 < 0.528$; the MAXIMUM back pressure is $\quad p_b = p_f$ $\quad$ so $\quad \frac{p_b}{p_0} = 0.446$ $\quad$ (Choked)

The final density is $\quad \rho_f = \frac{p_f}{R \cdot T_f}$ $\quad \rho_f = 0.535 \frac{kg}{m^3}$ $\quad$ and final mass is $M = \rho_f \cdot V$ $\quad M = 0.0701\, kg$

Since the mass flow rate is constant (flow is always choked) $\quad M = m_{rate} \cdot \Delta t$ $\quad$ or $\quad \Delta t = \frac{M}{m_{rate}}$

We have choked flow so $\quad m_{rate} = A_t \cdot p_0 \cdot \sqrt{\frac{k}{R \cdot T_0}} \cdot \left(\frac{2}{k+1}\right)^{\frac{k+1}{2 \cdot (k-1)}}$ $\quad m_{rate} = 1.873 \times 10^{-4} \frac{kg}{s}$

Hence $\quad \Delta t = \frac{M}{m_{rate}}$ $\quad \Delta t = 374\, s$ $\quad \Delta t = 6.23\, min$

The air in the tank will be cold when the valve is closed. Because $\rho = M/V$ is constant, $p = \rho RT = \text{const} \times T$, so as the temperature rises to ambient, the pressure will rise too.

For the entropy change during the charging process is given by $\Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$ where $\quad T_1 = T_{atm}$ $\quad T_2 = T_{atm}$

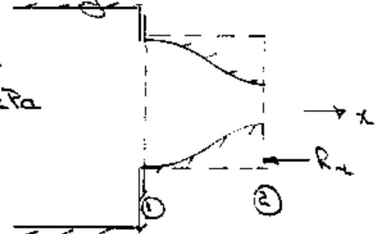
and $\quad p_1 = p_0$ $\quad p_2 = p_f$ $\quad$ Hence $\quad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{p_2}{p_1}\right)$ $\quad \Delta s = 232 \frac{J}{kg \cdot K}$

Problem 13.26

Given: Isentropic flow of air from a large tank through a converging nozzle discharges to the atmosphere

$T_0 = 27°C$

$P_0 = 171$ kPa



$M_1 = 0.2$ $\quad P_b = 101$ kPa

$A_2 = 0.015\ m^2$

Find: Magnitude and direction of force required to keep nozzle in place

Solution:

Basic equations: $F_{S_x} = P_1A_1 - P_2A_2 - P_{atm}(A_1 - A_2) - R_x = \dot{m}(V_2 - V_1)$

$\dot{m} = \rho V A = \text{const} \qquad P = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \qquad \frac{P_0}{P} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow (3) uniform flow at a section
(2) isentropic flow (4) ideal gas

$M_2 = \left\{\frac{2}{k-1}\left[\left(\frac{P_0}{P_2}\right)^{k-1/k} - 1\right]\right\}^{1/2} = \left\{\frac{2}{0.4}\left[\left(\frac{171}{101}\right)^{0.286} - 1\right]\right\}^{1/2} = 0.901$ Hence flow not choked

$T_2 = T_0 / \left[1 + \frac{k-1}{2}M_2^2\right] = 300\ K / [1 + 0.2(0.901)^2] = 258\ K$

$V_2 = M_2 c_2 = M_2 (kRT_2)^{1/2} = 0.901\left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 258\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 290\ m/s$

$\rho_2 = \frac{P_2}{RT_2} = 101 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\ N \cdot m} \times \frac{1}{258\ K} = 1.36\ kg/m^3$

$\dot{m} = \rho_2 V_2 A_2 = 1.36 \frac{kg}{m^3} \times 290 \frac{m}{s} \times 0.015\ m^2 = 5.92\ kg/s$

$T_1 = T_0 / \left[1 + \frac{k-1}{2}M_1^2\right] = 300\ K / [1 + 0.2(0.2)^2] = 298\ K$

$V_1 = M_1 c_1 = M_1 (kRT_1) = 0.2\left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 298\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 69.2\ m/s$

$P_1 = P_0 / \left[1 + \frac{k-1}{2}M_1^2\right]^{k/k-1} = 171\ kPa / [1 + 0.2(0.2)^2]^{3.5} = 166\ kPa$

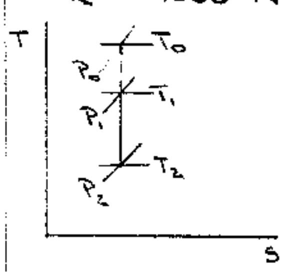
$\rho_1 = \frac{P_1}{RT_1} = 166 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\ N \cdot m} \times \frac{1}{298\ K} = 1.94\ kg/m^3$

$A_1 = \dot{m} / \rho_1 V_1 = 5.92 \frac{kg}{s} \times \frac{m^3}{1.94\ kg} \times \frac{s}{69.2\ m} = 0.0441\ m^2$

$R_x = P_1A_1 - P_2A_2 - P_{atm}(A_1 - A_2) - \dot{m}(V_2 - V_1) = P_{1g}A_1 - P_{2g}A_2 - \dot{m}(V_2 - V_1)$ (with $P_{2g} = 0$)

$= (166 - 101) \times 10^3 \frac{N}{m^2} \times 0.0441\ m^2 - 5.92 \frac{kg}{s}(290 - 69.2)\frac{m}{s} \times \frac{N \cdot s^2}{kg \cdot m}$

$R_x = 1560\ N$ (to the left) ⟵ R_x



Problem 13.27

[3]

Given: "Rocket" cart propelled by compressed air, converging nozzle.

Initially in tank, $p_0 = 1.3$ MPa (abs), $T_0 = 20°C$, $M_0 = 25$ kg

$A_e = 30\ mm^2$; $F_R = 6$ N, aerodynamic drag is negligible.

Find: (a) Pressure in exit plane.
(b) Mass flow rate of air through nozzle.
(c) Acceleration of assembly.
(d) Static and stagnation states on Ts diagram.

Solution: Assume steady, one-dimensional flow of an ideal gas.

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$; $\frac{p_0}{p} = \left(1+\frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}$; $c = \sqrt{kRT}$; $p = \rho RT$

Check for choking: $\frac{p_{atm}}{p_0} = \frac{101 \times 10^3}{1.3 \times 10^6} = 0.0777 < 0.528$, so Choked. $M_e = 1$

Thus

$$T_e = \frac{T_0}{1+\frac{k-1}{2}M_e^2} = \frac{(273+20)K}{1+\frac{k-1}{2}} = 244\ K$$

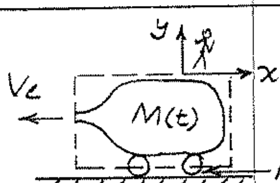
$$p_e = \frac{p_0}{\left(1+\frac{k-1}{2}M_e^2\right)^{k/k-1}} = \frac{1.3 \times 10^6 Pa}{\left(1+\frac{k-1}{2}\right)^{k/k-1}} = \frac{1.3 \times 10^6 Pa}{(1.2)^{3.5}} = 687\ kPa\ (abs)$$ p_e

$$\dot{m} = \rho_e V_e A_e\ ;\ \rho_e = \frac{p_e}{RT_e} = 687 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287 N \cdot m} \times \frac{1}{244 K} = 9.81\ kg/m^3$$

$$V_e = M_e c_e = c_e = \sqrt{kRT_e} = \left[1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 244 K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 313\ m/s$$

$$\dot{m} = 9.81 \frac{kg}{m^3} \times 313 \frac{m}{s} \times 30 \times 10^{-6} m^2 = 0.0921\ kg/s$$ $\dot{m}$

Apply momentum to find acceleration of assembly.



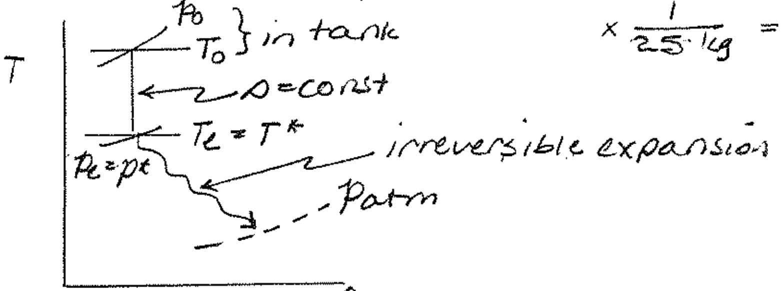
Basic equation: $F_{S_x} + F_{B_x} - \int_{CV} a_{rf_x}\rho\, d\forall = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho \vec{V}\cdot d\vec{A}$ (with $F_{B_x} = 0(4)$, $\frac{\partial}{\partial t}\int_{CV} \approx 0(5)$)

Assume: (4) Horizontal; (5) $u \approx 0$ within CV; (5) Uniform flow at exit.

Then $-F_R + (p_e - p_{atm})A_e - M a_{rf_x} = u_e\{+\dot{m}\} = -V_e \dot{m}$

$u_e = -V_e$

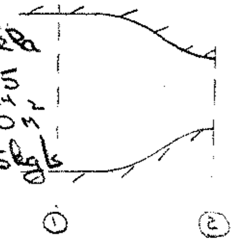
$$a_{rf_x} = \frac{V_e \dot{m} - F_R + (p_e - p_{atm})A_e}{M} = \left[313 \frac{m}{s} \times 0.0921 \frac{kg}{s} - 6\ N + (687-101)10^3 \frac{N}{m^2} \times 30 \times 10^{-6} m^2\right] \times \frac{1}{25\ kg} = 1.62\ m/s^2$$ a_{rf_x}



Problem 13.28

Given: Steady, isentropic flow of air. Duct area to be reduced as much as possible without reducing flowrate.

$p_1 = 300\ \text{kPa}$
$M_1 = 0.5$
$A_1 = 5\times10^{-4}\ \text{m}^2$
$\dot{m} = 0.25\ \text{kg/s}$



Find: (a) T_0 (b) % ΔA possible, (c) V_2 and p_2

Solution:

Basic equations: $\dot{m} = \rho V A$ $\qquad p = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$ $\qquad \frac{p_0}{p} = \left[1+\frac{k-1}{2}M^2\right]^{k/k-1}$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1+\frac{k-1}{2}M^2}{1+\frac{k-1}{2}}\right]^{(k+1)/2(k-1)} \qquad (12.7d)$$

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) ideal gas

To determine T_0, we first need to find T_1

$$\dot{m} = \rho_1 V_1 A_1 = \frac{p_1}{RT_1} M_1 c_1 A_1 = \frac{p_1}{RT_1} M_1 (kRT_1)^{1/2} A_1 = p_1 M_1 A_1 \left(\frac{k}{RT_1}\right)^{1/2}$$

Solving for T_1,

$$T_1 = \frac{k}{R}\left(\frac{p_1 M_1 A_1}{\dot{m}}\right)^2 = 1.4 \times \frac{\text{kg}\cdot\text{K}}{287\ \text{J}}\left[300\times10^3\ \frac{\text{N}}{\text{m}^2}\times 0.5 \times 5\times10^{-4}\ \text{m}^2 \times \frac{\text{s}}{0.25\ \text{kg}}\times\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^2 \frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}$$

$T_1 = 439\ \text{K}$

$$T_{0_1} = T_1\left[1+\frac{k-1}{2}M_1^2\right] = 439\ \text{K}\left[1+0.2(0.5)^2\right] = 461\ \text{K} \qquad \longleftarrow T_0$$

Maximum area reduction occurs where $M_2 = 1$.

For $M_1 = 0.5$, from Eq 12.7d $A/A^* = 1.34$ $\therefore A^* = A_2 = \frac{A_1}{1.34} = 3.73\times10^{-4}\ \text{m}^2$

$$\%\Delta A = \frac{A_2 - A_1}{A_1}\times 100 = \frac{3.73-5}{5}\times100 = -25.4\% \qquad \longleftarrow \%\Delta A$$

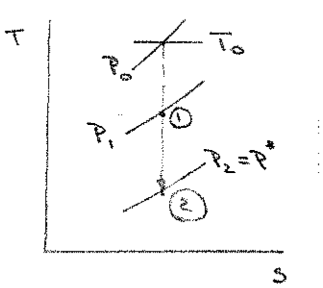
$$T_2 = T_0/\left[1+\frac{k-1}{2}M_2^2\right] = 461\ \text{K}/\left[1+0.2(1)^2\right] = 384\ \text{K}$$

$$V_2 = M_2 c_2 = M_2(kRT_2)^{1/2} = 1.0\left(1.4\times287\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\times384\ \text{K}\times\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)^{1/2}$$

$$V_2 = 393\ \text{m/s} \qquad \longleftarrow V_2$$

$$p_{0_1} = p_{0_2} = p_1\left[1+\frac{k-1}{2}M_1^2\right]^{k/k-1} = 300\ \text{kPa}\left[1+0.2(0.5)^2\right]^{3.5} = 356\ \text{kPa}$$

$$p_2 = p_{0_2}/\left[1+\frac{k-1}{2}M_2^2\right]^{k/k-1} = 356\ \text{kPa}/\left[1+0.2(1)^2\right]^{3.5} = 188\ \text{kPa} \qquad \longleftarrow p_2$$



Problem 13.29

[3]

13.29 An air-jet-driven experimental rocket of 25 kg mass is to be launched from the space shuttle into space. The temperature of the air in the rocket's tank is 125°C. A converging section with exit area 25 mm^2 is attached to the tank, through which the air exits to space at a rate of 0.05 kg/s. What is the pressure in the tank, and the pressure, temperature, and air speed at the exit when the rocket is first released? What is the initial acceleration of the rocket?

Given: Air-driven rocket in space

Find: Tank pressure; pressure, temperature and speed at exit; initial acceleration

Solution:

The given or available data is:

R =	286.9	J/kg.K
k =	1.4	
T_0 =	398	K
A_t =	25	mm^2
M =	25	kg
m_{rate} =	0.05	kg/s

Equations and Computations:

Because $p_b = 0$ $\quad p_e = p^*$

Hence the flow is choked!

Hence $\quad T_e = T^*$

From T_0, and Eq. 12.22b

$$\frac{T_0}{T^*} = \frac{k+1}{2} \qquad (12.22b)$$

$T^* =$ 332 K

$T_e =$ 332 K

58.7 °C

Also $\quad M_e =$ 1

Hence $\quad V_e = \quad V^* = \quad c_e$

From T_e and Eq. 12.18

$$c = \sqrt{kRT} \qquad (12.18)$$

$c_e =$ 365 m/s

Then $\quad V_e =$ 365 m/s

To find the exit pressure we use the ideal gas equation
after first finding the exit density.
The mass flow rate is $m_{rate} = \rho_e A_e V_e$

Hence $\rho_e =$ 0.0548 kg/m^3

From the ideal gas equation $p_e = \rho_e R T_e$

$p_e =$ 5.21 kPa

From $p_e = p^*$ and Eq. 12.22a

$$\frac{p_0}{p^*} = \left[\frac{k+1}{2}\right]^{k/(k-1)} \qquad (12.22a)$$

$p_0 =$ 9.87 kPa

We can check our results:
From p_0, T_0, A_t, and Eq. 13.9a

$$\dot{m}_{\text{choked}} = A_e p_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \qquad (13.9a)$$

Then $m_{choked} =$ 0.050 kg/s
$m_{choked} = m_{rate}$ Correct!

The initial acceleration is given by:

$$\vec{F} - \int_{CV} \vec{a}_{rf}\, \rho\, d\forall = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz}\, \rho\, d\forall + \int_{CS} \vec{V}_{xyz}\, \rho \vec{V}_{xyz} \cdot d\vec{A} \qquad (4.33)$$

which simplifies to: $p_e A_t - M a_x = m_{rate} V$ or: $a_x = \dfrac{m_{rate} V + p_e A_t}{M}$

$a_x =$ 1.25 m/s^2

Problem 13.30

[3]

13.30 A cylinder of gas used for welding contains helium at 20 MPa (gage) and room temperature. The cylinder is knocked over, its valve is broken off, and gas escapes through a converging passage. The minimum flow diameter is 10 mm at the outlet section where the gas flow is uniform. Find (a) the mass flow rate at which gas leaves the cylinder and (b) the instantaneous acceleration of the cylinder (assume the cylinder axis is horizontal and its mass is 65 kg). Show static and stagnation states and the process path on a *Ts* diagram.

Given: Gas cylinder with broken valve

Find: Mass flow rate; acceleration of cylinder

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $\quad p = \rho\cdot R\cdot T$ $\quad c = \sqrt{k\cdot R\cdot T}$ $\quad m_{rate} = \rho\cdot A\cdot V$

$$\vec{F}_S + \vec{F}_B - \int_{CV} \vec{a}_{rf}\,\rho\, dV = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz}\,\rho\, dV + \int_{CS} \vec{V}_{xyz}\,\rho\vec{V}_{xyz}\cdot d\vec{A} \qquad (4.33)$$

Given or available data $p_{atm} = 101\cdot kPa$ $\quad p_0 = 20\cdot MPa$ $\quad T_0 = (20 + 273)\cdot K$ $\quad k = 1.4$ $\quad R = 286.9\cdot\frac{J}{kg\cdot K}$

$d = 10\cdot mm$ so the nozzle area is $A_e = \frac{\pi}{4}\cdot d^2$ $\quad A_e = 78.5\cdot mm^2$ $\quad M_{CV} = 65\cdot kg$

The flow will be choked if $p_b/p_0 < 0.528$: $\quad p_b = p_{atm}$ so $\frac{p_b}{p_0} = 5.05\times 10^{-3}$ (Choked: Critical conditions)

The exit temperature is $T_e = \frac{T_0}{\left(1 + \frac{k-1}{2}\right)}$ $\quad T_e = 244\,K$ $\quad T_e = -29\cdot{}^\circ C$ $\quad c_e = \sqrt{k\cdot R\cdot T_e}$

The exit speed is $V_e = c_e$ $\quad V_e = 313\frac{m}{s}$

The exit pressure is $p_e = \frac{p_0}{\left(1 + \frac{k-1}{2}\right)^{\frac{k}{k-1}}}$ $\quad p_e = 10.6\cdot MPa$ and exit density is $\rho_e = \frac{p_e}{R\cdot T_e}$ $\quad \rho_e = 151\frac{kg}{m^3}$

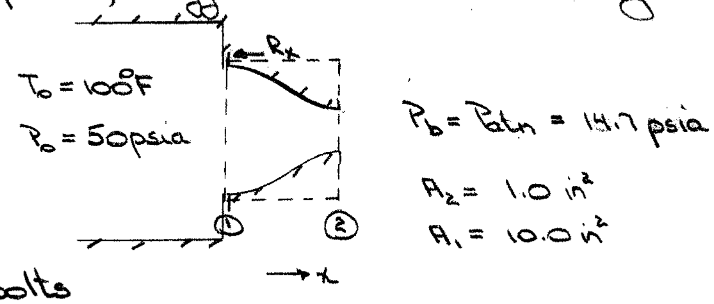
Then $m_{rate} = \rho_e\cdot A_e\cdot V_e$ $\quad m_{rate} = 3.71\frac{kg}{s}$

The momentum equation (Eq. 4.33) simplifies to $\left(p_e - p_{atm}\right)\cdot A_e - M_{CV}\cdot a_x = -V_e\cdot m_{rate}$

Hence $a_x = \frac{\left(p_e - p_{atm}\right)\cdot A_e + V_e\cdot m_{rate}}{M_{CV}}$ $\quad a_x = 30.5\frac{m}{s^2}$

The process is isentropic, followed by nonisentropic expansion to atmospheric pressure

Given: Isentropic flow of air through a converging nozzle discharges to atmosphere; nozzle is bolted to a large tank.



Find: Force in bolts

Solution:

Basic equations: $F_{S_x} = P_1A_1 - P_2A_2 - P_{atm}(A_1 - A_2) - R_x = \dot{m}(V_2 - V_1)$

$\dot{m} = \rho V A = \text{const} \qquad P = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \qquad \frac{P_0}{P} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow
(2) isentropic flow in nozzle
(3) uniform flow at a section
(4) $F_{B_x} = 0$
(5) $V_1 \approx 0$

First check for choking

$\frac{P_b}{P_0} = \frac{14.7}{50} = 0.294 < 0.528$ and hence the nozzle is choked

$M_2 = 1.0$ and $P_2 = 0.528\,P_0 = 0.528(50\,psia) = 26.4\,psia$

$T_0/T = 1 + \frac{k-1}{2}M^2 \; ; \quad T_2 = T_0/\left[1+\frac{k-1}{2}M_2^2\right] = 560°R/\left[1+0.2(1.0)^2\right] = 467°R$

$\rho_2 = \frac{P_2}{RT_2} = 26.4\,\frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3\,ft \cdot lbf} \times \frac{1}{467°R} \times 144\,\frac{in^2}{ft^2} = 0.153\,lbm/ft^3$

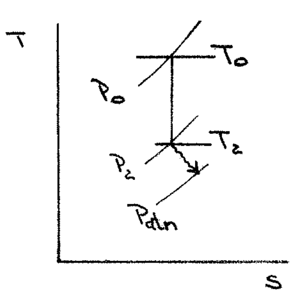
$V_2 = M_2C_2 = M_2(kRT_2)^{1/2} = 1.0\left(1.4 \times 53.3\,\frac{ft \cdot lbf}{lbm \cdot °R} \times 467°R \times 32.2\,\frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{1/2} = 1060\,ft/s$

$\dot{m} = \rho_2 V_2 A_2 = 0.153\,\frac{lbm}{ft^3} \times 1060\,\frac{ft}{s} \times 1.0\,in^2 \times \frac{ft^2}{144\,in^2} = 1.13\,lbm/s$

$R_x = P_1A_1 - P_2A_2 - P_{atm}(A_1 - A_2) - \dot{m}(V_2 - \cancel{V_1}^{\approx 0}) = P_{1g}A_1 - P_{2g}A_2 - \dot{m}V_2$

$R_x = (50 - 14.7)\,\frac{lbf}{in^2} \times 10\,in^2 - (26.4 - 14.7)\,\frac{lbf}{in^2} \times 1.0\,in^2 - 1.13\,\frac{lbm}{s} \times 1060\,\frac{ft}{s} \times \frac{slug}{32.2\,lbm} \times \frac{lbf \cdot s^2}{ft \cdot slug}$

$R_x = 304\,lbf$ Since R_x acts to the left on CV, bolts are in tension R_x



Problem 13.32

13.32 An insulated spherical air tank with diameter $D = 2$ m is used in a blowdown installation. Initially the tank is charged to 2.75 MPa (abs) at 450 K. The mass flow rate of air from the tank is a function of time; during the first 30 s of blowdown 30 kg of air leaves the tank. Determine the air temperature in the tank after 30 s of blowdown. Estimate the nozzle throat area.

Given: Spherical air tank

Find: Air temperature after 30s; estimate throat area

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\qquad \frac{p}{\rho^k} = \text{const}$ $\qquad \frac{\partial}{\partial t}\int \rho\, dV_{CV} + \int \rho\cdot\vec{V}\, d\vec{A}_{CS} = 0$ (4.12)

Assumptions: 1) Large tank (stagnation conditions) 2) isentropic 3) uniform flow

Given or available data $\quad p_{atm} = 101\cdot kPa \quad p_1 = 2.75\cdot MPa \quad T_1 = 450\cdot K \quad D = 2\cdot m \quad V = \frac{\pi}{6}\cdot D^3 \quad V = 4.19\cdot m^3$

$\Delta M = 30\cdot kg \quad \Delta t = 30\cdot s \quad k = 1.4 \quad R = 286.9\cdot\frac{J}{kg\cdot K}$

The flow will be choked if $p_b/p_1 < 0.528$: $\quad p_b = p_{atm}$ $\quad$ so $\quad \frac{p_b}{p_1} = 0.037$ $\quad$ (Initially choked: Critical conditions)

We need to see if the flow is still choked after 30s

The initial (State 1) density and mass are $\quad \rho_1 = \frac{p_1}{R\cdot T_1} \quad \rho_1 = 21.3\frac{kg}{m^3} \quad M_1 = \rho_1\cdot V \quad M_1 = 89.2\,kg$

The final (State 2) mass and density are then $\quad M_2 = M_1 - \Delta M \quad M_2 = 59.2\,kg \quad \rho_2 = \frac{M_2}{V} \quad \rho_2 = 14.1\frac{kg}{m^3}$

For an isentropic process $\quad \frac{p}{\rho^k} = \text{const}$ $\quad$ so $\quad p_2 = p_1\cdot\left(\frac{\rho_2}{\rho_1}\right)^k \quad p_2 = 1.55\cdot MPa \quad \frac{p_b}{p_2} = 0.0652$ $\quad$ (Still choked)

The final temperature is $\quad T_2 = \frac{p_2}{\rho_2\cdot R} \quad T_2 = 382\,K \quad T_2 = 109\cdot °C$

To estimate the throat area we use $\quad \frac{\Delta M}{\Delta t} = m_{tave} = \rho_{tave}\cdot A_t\cdot V_{tave}$ $\quad$ or $\quad A_t = \frac{\Delta M}{\Delta t\cdot\rho_{tave}\cdot V_{tave}}$

where we use average values of density and speed at the throat.

The average stagnation temperature is $\quad T_{0ave} = \frac{T_1 + T_2}{2} \quad T_{0ave} = 416\,K$

The average stagnation pressure is $\quad p_{0ave} = \frac{p_1 + p_2}{2} \quad p_{0ave} = 2.15\cdot MPa$

Hence the average temperature and pressure (critical) at the throat are

$$T_{tave} = \frac{T_{0ave}}{\left(1 + \frac{k-1}{2}\right)} \qquad T_{tave} = 347\,K \qquad \text{and} \qquad p_{tave} = \frac{p_{0ave}}{\left(1 + \frac{k-1}{2}\right)^{\frac{k}{k-1}}} \qquad p_{tave} = 1.14 \cdot MPa$$

Hence

$$V_{tave} = \sqrt{k \cdot R \cdot T_{tave}} \qquad V_{tave} = 373\,\frac{m}{s} \qquad \rho_{tave} = \frac{p_{tave}}{R \cdot T_{tave}} \qquad \rho_{tave} = 11.4\,\frac{kg}{m^3}$$

Finally

$$A_t = \frac{\Delta M}{\Delta t \cdot \rho_{tave} \cdot V_{tave}} \qquad A_t = 2.35 \times 10^{-4}\,m^2 \qquad A_t = 235 \cdot mm^2$$

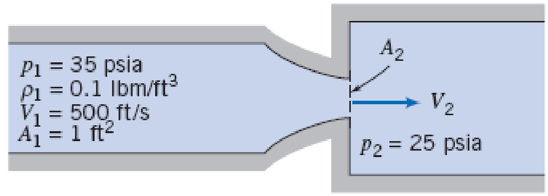
This corresponds to a diameter

$$D_t = \sqrt{\frac{4 \cdot A_t}{\pi}} \qquad D_t = 0.0173\,m \qquad D_t = 17.3 \cdot mm$$

The process is isentropic, followed by nonisentropic expansion to atmospheric pressure

Problem 13.33

13.33 An ideal gas, with $k = 1.25$, flows isentropically through the converging nozzle shown and discharges into a large duct where the pressure is $p_2 = 25$ psia. The gas is *not* air and the gas constant, R, is unknown. Flow is steady and uniform at all cross-sections. Find the exit area of the nozzle, A_2, and the exit speed, V_2.



Given: Ideal gas flow in a converging nozzle

Find: Exit area and speed

Solution:

Basic equations:

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2 \qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}} \qquad \frac{A}{A_{crit}} = \frac{1}{M}\cdot\left(\frac{1 + \frac{k-1}{2}\cdot M^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}}$$

Given or available data $\quad p_1 = 35\cdot psi \quad \rho_1 = 0.1\cdot\frac{lbm}{ft^3} \quad V_1 = 500\cdot\frac{ft}{s} \quad A_1 = 1\cdot ft^2 \quad p_2 = 25\cdot psi \quad k = 1.25$

Check for choking: $\quad c_1 = \sqrt{k\cdot R\cdot T_1}$ or, replacing R using the ideal gas equation $\quad c_1 = \sqrt{k\cdot\frac{p_1}{\rho_1}} \quad c_1 = 1424\frac{ft}{s}$

Hence $\quad M_1 = \frac{V_1}{c_1} \quad M_1 = 0.351$

Then $\quad p_0 = p_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right)^{\frac{k}{k-1}} \quad p_0 = 37.8\,psi$

The critical pressure is then $\quad p_{crit} = \frac{p_0}{\left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}} \quad p_{crit} = 21.0\,psi \quad$ Hence $p_2 > p_{crit}$, so NOT choked

Then we have $\quad M_2 = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_2}\right)^{\frac{k-1}{k}} - 1\right]} \quad M_2 = 0.830$

From M_1 we find $\quad A_{crit} = \frac{M_1\cdot A_1}{\left(\frac{1 + \frac{k-1}{2}\cdot M_1^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}}} \quad A_{crit} = 0.557\,ft^2 \quad A_2 = \frac{A_{crit}}{M_2}\cdot\left(\frac{1 + \frac{k-1}{2}\cdot M_2^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}} \quad A_2 = 0.573\,ft^2$

For isentropic flow $\quad p\cdot\rho^k = const \quad$ so $\quad \rho_2 = \rho_1\cdot\left(\frac{p_1}{p_2}\right)^{\frac{1}{k}} \quad \rho_2 = 0.131\frac{lbm}{ft^3}$

Finally from continuity $\quad \rho\cdot A\cdot V = const \quad$ so $\quad V_2 = V_1\cdot\frac{A_1\cdot\rho_1}{A_2\cdot\rho_2} \quad V_2 = 667\frac{ft}{s}$

Problem 13.34

Given: Jet transport aircraft cruises at 11 km. altitude. Initial cabin conditions are $T_i = 25°C$, P_i = 2.5 km altitude. Cabin volume $\forall = 25 m^3$. Air escapes through a small hole with effective flow area, $A = 0.002 m^2$

Find: Time required for cabin pressure to decrease by 40 percent. Plot cabin pressure as a function of time.

Solution:

Basic equations: $\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \vec{V}\cdot d\vec{A} = 0$

$$\frac{P}{\rho^k} = \text{constant} \qquad P = \rho R T$$

Assumptions: (1) model flow as isentropic flow through a converging nozzle
(2) assume uniform properties within the cabin, isentropic expansion
(3) ideal gas behavior.

Stagnation conditions within the cabin are

$T_i = 298\ K \qquad P_i = P_{atm}$ at 2.5 km $= 74.7$ kPa (Table A.3)

$$P_f = 0.60 P_i = 44.8 \text{ kPa}$$

Back pressure $P_b = P_{atm}$ at 11 km $= 22.7$ kPa

Then $P_b / P_i = 0.304$ and $P_b / P_f = 0.507$. Flow is choked

Note: conditions in cabin are stagnation conditions.

From continuity.

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall = -\int_{cs} \rho \vec{V}\cdot d\vec{A} = -\rho_e V_e A_e$$

$$\forall \frac{d\rho}{dt} = -\rho_e V_e A_e.$$

For choked flow, $M_e = 1.0$

$$\frac{\rho}{\rho_e} = \left[1 + \frac{k-1}{2} M_e^2\right]^{\frac{1}{k-1}} = (1.20)^{2.5} = 1.5774 \qquad \therefore \rho_e = 0.6339\, \rho$$

$$\frac{T}{T_e} = 1 + \frac{k-1}{2} M_e^2 = 1.2 \qquad \therefore T_e = 0.8333\, T$$

$$V_e = (kRT_e)^{1/2} = (kR)^{1/2}(0.8333T)^{1/2} = 0.9129\,(kR)^{1/2}\, T^{1/2}$$

Then

$$\forall \frac{d\rho}{dt} = -\rho_e V_e A_e = -0.6339\rho\,(0.9129)(kR)^{1/2}\, T^{1/2}\, A_e$$

$$\forall \frac{d\rho}{dt} = -0.5787\,(kR)^{1/2} A_e\, \rho T^{1/2}$$

For an isentropic expansion ρ and T can be related

$$\frac{P}{\rho^k} = \text{const} = \frac{\rho R T}{\rho^k} \qquad \therefore \rho^{(1-k)}\, T = \text{constant}$$

Then, $\rho^{(1-k)} T = \rho_i^{(1-k)} T_i$ or $T = T_i \left(\frac{\rho_i}{\rho}\right)^{(1-k)}$ and $T^{1/2} = \frac{T_i^{1/2}}{\rho_i^{(k-1)/2}} \rho^{(k-1)/2}$

Substituting we obtain

$$\forall \frac{d\rho}{dt} = -0.5787 (kR)^{1/2} A_e \rho \frac{T_i^{1/2}}{\rho_i^{(k-1)/2}} \rho^{(k-1)/2}$$

$$\frac{d\rho}{dt} = -0.5787 \frac{(kR)^{1/2}}{\forall} A_e \frac{T_i^{1/2}}{\rho_i^{(k-1)/2}} \rho^{\frac{k+1}{2}} = c_1 \rho^{(k+1)/2}$$

$$\frac{d\rho}{\rho^{(k+1)/2}} = -c_1\, dt \quad \text{where } c_1 = 0.5787 (kR)^{1/2} \frac{A_e}{\forall} \frac{T_i^{1/2}}{\rho_i^{(k-1)/2}}$$

To integrate, we write

$$-c_1 t = \int \rho^{-\frac{(k+1)}{2}} d\rho = \frac{1}{1-\frac{(k+1)}{2}} \rho^{1-\frac{(k+1)}{2}} \Big]_{\rho_i}^{\rho_f} = \frac{2}{(1-k)} \rho^{\frac{(1-k)}{2}} \Big]_{\rho_i}^{\rho_f}$$

$$-c_1 t = \frac{2}{(k-1)} \left[\rho_i^{(1-k)/2} - \rho_f^{(1-k)/2} \right] = \frac{2}{(k-1)} \rho_i^{(1-k)/2} \left[1 - \left(\frac{\rho_f}{\rho_i}\right)^{(1-k)/2} \right]$$

$$c_1 t = \frac{2}{(k-1)} \rho_i^{(1-k)/2} \left[\left(\frac{\rho_i}{\rho_f}\right)^{(k-1)/2} - 1 \right]$$

$$0.5787 (kR)^{1/2} \frac{A_e}{\forall} \frac{T_i^{1/2}}{\rho_i^{(k-1)/2}} t = \frac{2}{(k-1)} \rho_i^{(1-k)/2} \left[\left(\frac{\rho_i}{\rho_f}\right)^{(k-1)/2} - 1 \right]$$

$$0.5787 (kR)^{1/2} \frac{A_e}{\forall} T_i^{1/2} t = \frac{2}{(k-1)} \left[\left(\frac{\rho_i}{\rho_f}\right)^{\frac{(k-1)}{2}} - 1 \right]$$

Since $p/\rho^k = \text{const}$, $\frac{\rho_i}{\rho_f} = \left(\frac{p_i}{p_f}\right)^{1/k}$

and
$$0.5787 (kR)^{1/2} \frac{A_e}{\forall} T_i^{1/2} t = \frac{2}{(k-1)} \left[\left(\frac{p_i}{p_f}\right)^{\frac{(k-1)}{2k}} - 1 \right] \qquad (1)$$

Substituting numerical values

$$0.5787 \left[1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 298 K \times \frac{kg \cdot m}{N \cdot s^2} \right]^{1/2} 0.002 m^2 \times \frac{1}{25 m^3} t = \frac{2}{0.4} \left[\left(\frac{1}{0.6}\right)^{0.1429} - 1 \right]$$

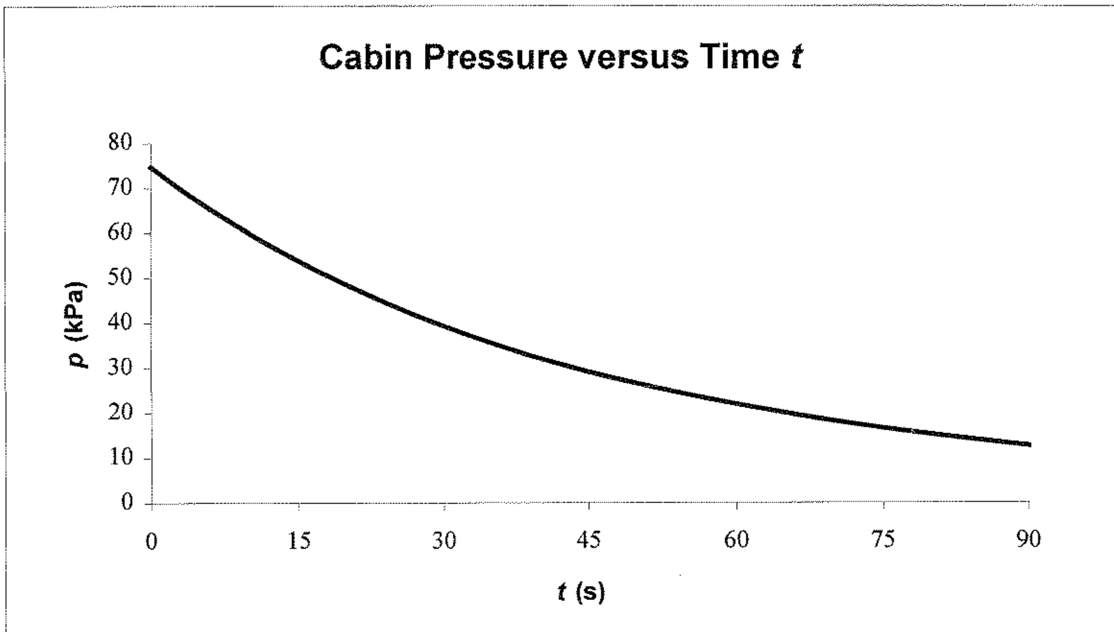
$$0.01602\, t = 0.3786$$

$$t = 23.6 \text{ s.} \longleftarrow t$$

Problem 13.34

Equation 1 is plotted using *Excel*
Note that it's easier to compute *t* from *p* values!

t (s)	*p* (kPa)
0.000	74.7
1.03	73
2.27	71
3.56	69
4.89	67
6.26	65
7.69	63
9.17	61
10.7	59
12.3	57
14.0	55
15.7	53
17.5	51
19.4	49
21.4	47
23.6	**44.8**
25.6	43
27.9	41
30.4	39
33.0	37
35.7	35
38.6	33
41.8	31
45.2	29
48.8	27
52.8	25
57.2	23
62.0	21
67.4	19
73.5	17
80.5	15
90.0	12.7



Problem 13.35

Given: Large insulated tank, pressurized to 620 kPa (gage), supplying air to converging nozzle with discharge to atmosphere. Initial temperature in tank is 127°C.

Find: (a) Initial Mach number at nozzle exit plane.
(b) p in exit plane when flow starts.
(c) How exit plane pressure varies with time.
(d) How flow rate varies with time.
(e) Air temperature in tank when flow rate approaches zero.

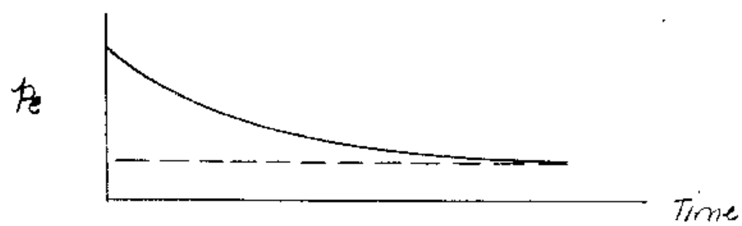
Solution: Assume stagnation conditions in tank, $p_b = p_{atm}$. Then

$$\frac{p_b}{p_0} = \frac{101.3 \text{ kPa}}{(620+101.3)\text{ kPa (abs)}} = 0.140 \ll 0.528 \text{, so flow is choked! } M_e = 1$$ M_e

At exit plane, $M_e = 1$, so

$$p_e = p_0\left(\frac{1}{1+\frac{k-1}{2}M_e^2}\right)^{\frac{k}{k-1}} = (620+101.3)\text{ kPa (abs)}\left(\frac{1}{1.2}\right)^{3.5} = 381 \text{ kPa (abs)}$$ p_e

Exit plane pressure decreases with time, asymptotically approaching p_{atm}.



Flow rate varies similarly.

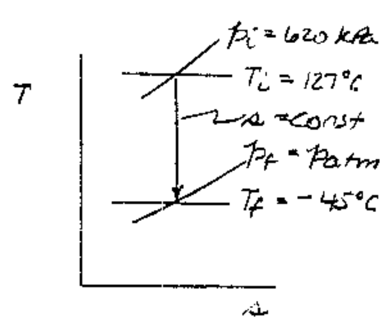
When flow rate approaches zero, $p_0 \rightarrow p_{atm}$. Assume tank behaves as a reversible adiabatic process. Thus

$$\frac{p_f}{p_0} = \left(\frac{T_f}{T_0}\right)^{\frac{k}{k-1}}$$

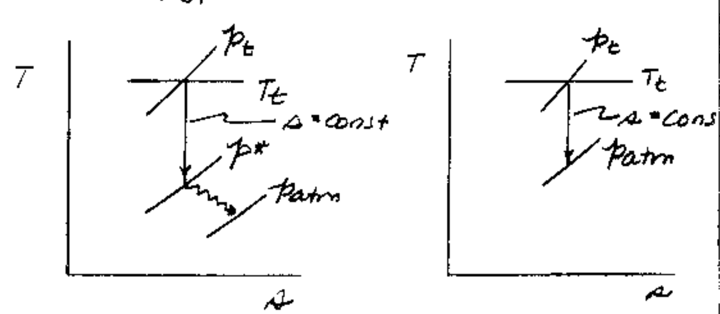
Thus $$T_f = T_0\left(\frac{p_f}{p_0}\right)^{\frac{k-1}{k}} = (273+127)\text{K}\left(\frac{101.3}{620+101.3}\right)^{0.286} = 228 \text{ K}$$

$$T_f = (228-273)°\text{C} = -45°\text{C}$$ T_f

The Ts diagram for air in the tank is:



The Ts diagrams for air flowing from the tank are:



While choked ($p_t > p_{atm}/0.528$)

No longer choked ($p_t \le p_{atm}/0.528$)

Problem 13.36

13.36 A converging-diverging nozzle is attached to a very large tank of air in which the pressure is 150 kPa and the temperature is 35°C. The nozzle exhausts to the atmosphere where the pressure is 101 kPa. The exit diameter of the nozzle is 2.75 cm. What is the flow rate through the nozzle? Assume the flow is isentropic.

Given: CD nozzle attached to large tank

Find: Flow rate

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$ $\quad m_{rate} = \rho \cdot V \cdot A$

Given or available data $\quad p_0 = 150 \cdot kPa \quad T_0 = (35 + 273) \cdot K \quad p_e = 101 \cdot kPa \quad D = 2.75 \cdot cm$

$k = 1.4 \quad R = 286.9 \cdot \frac{J}{kg \cdot K} \quad A_e = \frac{\pi}{4} \cdot D^2 \quad A_e = 5.94\, cm^2$

For isentropic flow $\quad M_e = \sqrt{\frac{2}{k-1} \cdot \left[\left(\frac{p_0}{p_e}\right)^{\frac{k-1}{k}} - 1\right]} \quad M_e = 0.773$

Then $\quad T_e = \frac{T_0}{\left(1 + \frac{k-1}{2} \cdot M_e^2\right)} \quad T_e = 275\, K \quad T_e = 1.94\, °C$

Also $\quad c_e = \sqrt{k \cdot R \cdot T_e} \quad c_e = 332 \frac{m}{s} \quad V_e = M_e \cdot c_e \quad V_e = 257 \frac{m}{s}$

$\rho_e = \frac{p_e}{R \cdot T_e} \quad \rho_e = 1.28 \frac{kg}{m^3}$

Finally $\quad m_{rate} = \rho_e \cdot V_e \cdot A_e \quad m_{rate} = 0.195 \frac{kg}{s}$

Problem 13.37

[2]

13.37 At the design condition of the system of Problem 13.36, the exit Mach number is $M_e = 2.0$. Find the pressure in the tank of Problem 13.36 (keeping the temperature constant) for this condition. What is the flow rate? What is the throat area?

Given: Design condition in a converging-diverging nozzle

Find: Tank pressure; flow rate; throat area

Solution:

The given or available data is:

R =	53.33	ft.lbf/lbm.°R
k =	1.4	
T_0 =	560	°R
A_e =	1	in^2
p_b =	14.7	psia
M_e =	2	

Equations and Computations:

At design condition

$p_e = p_b$

$p_e =$ 14.7 psia

From M_e and p_e, and Eq. 13.7a
(using built-in function *Isenp* (M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$p_0 =$ 115 psia

From M_e and A_e, and Eq. 13.7d
(using built-in function *IsenA* (M, k))

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2} M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)} \qquad (13.7d)$$

$A^* =$ 0.593 in^2

Hence $A_t =$ 0.593 in^2

From p_0, T_0, A_t, and Eq. 13.10a

$$\dot{m}_{\text{choked}} = A_t p_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \qquad (13.10a)$$

$m_{\text{choked}} =$ 1.53 lb/s

Problem 13.38

Given: Air escapes from high-pressure bicycle tire through hole having $D = 0.254\,mm$, $p_1 = 620\,kPa$ (gage), and $T = 27°C$, which remains constant. Internal volume of tire is $\forall = 4.26\times10^{-4}\,m^3$, and also is constant.

Find: (a) Time needed for p in tire to drop to 310 kPa (gage).
(b) Entropy change of air in tire during this process.
(c) Sketch a Ts diagram showing states and process paths.
Plot pressure as a function of time.

Solution: Apply continuity equation, isentropic relationships.

Basic equations: $0 = \frac{\partial}{\partial t}\int_{CV}\rho\, d\forall + \int_{CS}\rho \vec{V}\cdot d\vec{A}$ $\qquad \frac{T_0}{T} = \left(1+\frac{k-1}{2}M^2\right)$; $\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{k-1}}$

Check for choking: $\frac{p_{atm}}{p_{min}} = \frac{101}{310+101} = 0.246 < 0.528$ so always choked.

Thus $\dot{m} = \rho^* V^* A^*$. Assume: (1) Uniform density in tire: $\int_{CV} = \rho\forall$
(2) Uniform flow at throat
(3) Isentropic process to throat.

Then
$$0 = \forall\frac{d\rho}{dt} + \rho^* V^* A_e$$

But $\rho^* = \frac{\rho}{\left(1+\frac{k-1}{2}M_*^2\right)^{1/k-1}} = \frac{\rho}{(1.2)^{2.5}} = 0.634\rho$

so
$$\frac{d\rho}{\rho} = -0.634\frac{V^* A_e}{\forall}dt$$

Integrating, $\ln\frac{\rho_2}{\rho_1} = -0.634\frac{V^* A_e}{\forall}t = \ln\frac{p_2}{p_1}$ since T = constant

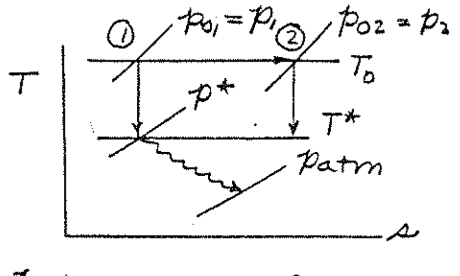
Thus
$$t = -\frac{\forall}{0.634\,V^* A_e}\ln\frac{p_2}{p_1} \qquad (1)$$

$$V^* = c^* = \sqrt{kRT^*} = \left[1.4\times 287\frac{N\cdot m}{kg\cdot K}\times\frac{273+27}{1.2}K\times\frac{kg\cdot m}{N\cdot s^2}\right]^{1/2} = 317\ m/s$$

$$A^* = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.000254)^2 m^2 = 5.07\times10^{-8}\,m^2$$

$$t = -\frac{1}{0.634}\times 4.26\times10^{-4}m^3\times\frac{s}{317\,m}\times\frac{1}{5.07\times10^{-8}m^2}\times\ln\left(\frac{310+101}{620+101}\right) = 23.5\ s$$ $\qquad t$

Ts diagram:



Process ① → ② in tire

Process (isentropic) ① → $*$ (moving to) ② → $*$ in converging passage.

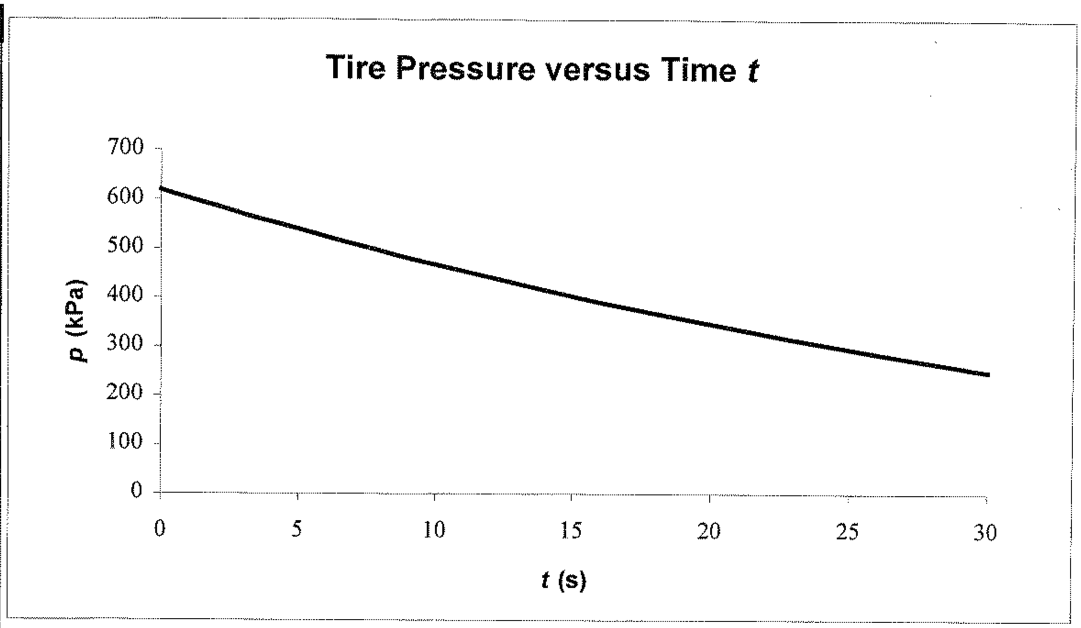
In tire,
$$\Delta s = c_p\ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} = -287\frac{N\cdot m}{kg\cdot K}\times\ln\frac{(310+101)}{(620+101)} = 161\ J/(kg\cdot K)$$ $\qquad \Delta s$

Problem 13.38

[4] Part 2/2

Equation 1 is plotted using *Excel*
Note that it's easier to compute *t* from *p* values!

t (s)	*p* (kPa)
0.000	620
0.584	610
1.18	600
1.78	590
2.39	580
3.00	570
3.63	560
4.27	550
4.92	540
5.57	530
6.24	520
6.92	510
7.61	500
8.31	490
9.03	480
9.75	470
10.5	460
11.2	450
12.0	440
12.8	430
13.6	420
14.4	410
15.2	400
16.1	390
16.9	380
17.8	370
18.7	360
19.6	350
20.6	340
21.5	330
22.5	320
23.5	310
24.5	300
25.6	290
26.7	280
27.8	270
28.9	260
30.0	251



Problem 13.39

Given: Converging-diverging nozzle with $M_{design} = 3.0$ and $A_e = 250\ mm^2$. Nozzle bolted to side of large tank with $p = 4.5$ MPa (gage) and $T = 750$ K. Flow in nozzle is isentropic.

Find: (a) Pressure in nozzle exit plane.
(b) Mass flow rate in nozzle.
(c) Sketch Ts diagram: label tank, throat, exit, and ambient.

Solution: Assume stagnation conditions in tank, isentropic flow in nozzle.

Computing equations: $\frac{p_0}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}$; $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$; $c = \sqrt{kRT}$

For $M_e = 3.0$, $\frac{p_0}{p_e} = \left[1 + \frac{k-1}{2}(3.0)^2\right]^{\frac{k}{k-1}} = 36.7$

$$p_e = \frac{p_0}{36.7} = \frac{(4.5 \times 10^6 + 101 \times 10^3)\ Pa}{36.7} = 125\ kPa\ (abs)\ \text{or}\ 24\ kPa\ (gage)$$ p_e

Thus the nozzle is underexpanded. For steady, 1-D flow, $\dot{m} = \rho_e V_e A_e$.

$$T_e = \frac{T_0}{1 + \frac{k-1}{2}(3.0)^2} = \frac{750\ K}{1 + 0.2(3)^2} = 268\ K$$

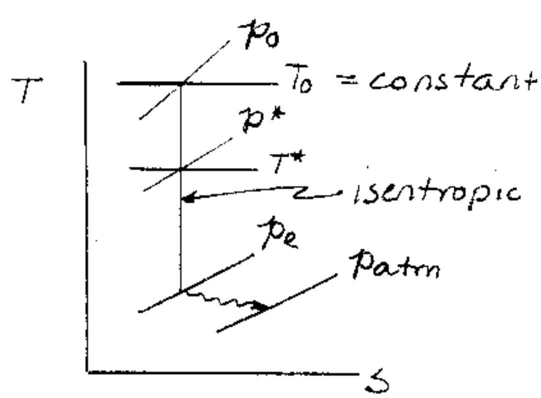
$$c_e = \sqrt{kRT_e} = \left[1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 268\ K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 328\ m/s$$

$$\rho_e = \frac{p_e}{RT_e} = 125 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\ N \cdot m} \times \frac{1}{268\ K} = 1.63\ kg/m^3$$

Finally, $V_e = M_e c_e = 3.0 \times 328\ m/s = 984\ m/s$

$$\dot{m} = \rho_e V_e A_e = 1.63 \frac{kg}{m^3} \times 984 \frac{m}{s} \times 250\ mm^2 \times \frac{m^2}{10^6\ mm^2} = 0.401\ kg/s$$ $\dot{m}$

Ts diagram:

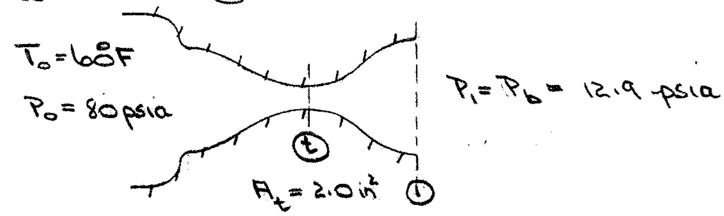


0 = tank conditions

* = throat conditions

e = exit plane conditions

Given: Isentropic flow of air from a large tank through a converging-diverging nozzle discharges to a back pressure, p_b



Find: $\dot{m}$, A_1

Solution

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$, $\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

$$A/A^* = \frac{1}{M}\left[\frac{1+\frac{k-1}{2}M^2}{1+\frac{k-1}{2}}\right]^{(k+1)/2(k-1)} \qquad (12.6)$$

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) ideal gas

$p_0/p = \left[1+\frac{k-1}{2}M^2\right]^{k/k-1}$; $M_1 = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p_1}\right)^{k-1/k} - 1\right]\right\}^{1/2} = \left\{\frac{2}{0.4}\left[\left(\frac{80}{12.9}\right)^{0.286} - 1\right]\right\}^{1/2} = 1.85$

$T_0/T = 1 + \frac{k-1}{2}M^2$; $T_1 = \frac{T_0}{1+\frac{k-1}{2}M_1^2} = \frac{520°R}{1+0.2(1.85)^2} = 309°R$

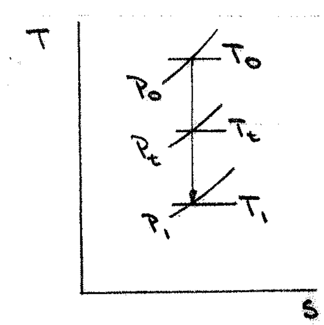
$V_1 = M_1 c_1 = M_1(kRT_1)^{1/2} = 1.85\left(1.4 \times 53.3\,\frac{ft\cdot lbf}{lbm\,°R} \times 309°R \times 32.2\,\frac{lbm}{slug} \times \frac{slug\cdot ft}{lbf\cdot s^2}\right)^{1/2} = 1594\ ft/s$

$\rho_1 = \frac{p_1}{RT_1} = 12.9\,\frac{lbf}{in^2} \times \frac{lbm\,°R}{53.3\,ft\cdot lbf} \times \frac{1}{309°R} \times \frac{144\,in^2}{ft^2} = 0.113\ lbm/ft^3$

Since $M_1 = 1.85$, nozzle must be choked and $M_t = 1.0$; $A_t = A^*$

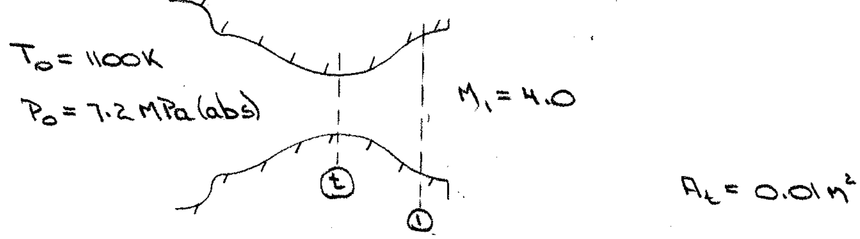
For $M_1 = 1.85$, from Eq. 12.6 (and Fig E.1), $A_1/A^* = 1.496$; $\therefore A_1 = 2.99\ in^2$ ← A_1

$\dot{m} = \rho_1 V_1 A_1 = 0.113\,\frac{lbm}{ft^3} \times 1594\,\frac{ft}{s} \times 2.99\,in^2 \times \frac{ft^2}{144\,in^2} = 3.74\ lbm/s$ ← $\dot{m}$



Problem 13.46

Given: Isentropic flow of air from stagnation state through a converging-diverging nozzle as shown.



Find: V_1, $\dot{m}$

Solution:

Basic equations: $\dot{m} = \rho V A \qquad P = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \qquad \frac{P_0}{P} = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1}$

Assumptions: (1) steady flow (3) uniform flow at a section
(2) isentropic flow (4) ideal gas

$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \quad ; \quad T_1 = \frac{T_0}{1 + \frac{k-1}{2} M^2} = \frac{1100\,K}{1 + 0.2\,(4.0)^2} = 262\,K$

$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 4.0\left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 262\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 1300\,m/s$ ← V_1

Since $M_1 = 4.0$, nozzle must be choked and $M_t = 1.0$

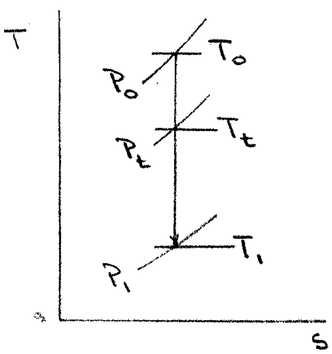
$P_t = \frac{P_0}{\left[1 + \frac{k-1}{2} M_t^2\right]^{k/k-1}} = \frac{7.2 \times 10^6\,Pa}{\left[1 + 0.2(1.0)^2\right]^{3.5}} = 3.80\,MPa$

$T_t = \frac{T_0}{1 + \frac{k-1}{2} M_t^2} = \frac{1100\,K}{1 + 0.2(1.0)^2} = 917\,K$

$\rho_t = \frac{P_t}{RT_t} = 3.80 \times 10^6 \frac{N}{m^2} \times \frac{kg \cdot K}{287\,N \cdot m} \times \frac{1}{917\,K} = 14.4\,kg/m^3$

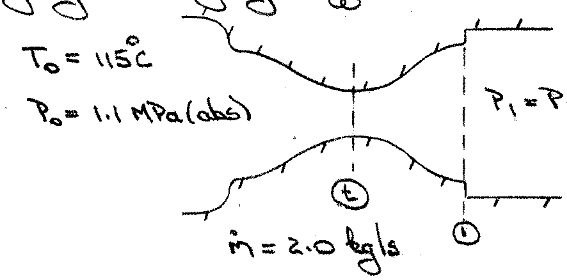
$V_t = M_t c_t = M_t (kRT_t)^{1/2} = 1.0\left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 917\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 607\,m/s$

$\dot{m} = \rho_t V_t A_t = 14.4 \frac{kg}{m^3} \times 607 \frac{m}{s} \times 0.01\,m^2 = 87.4\,kg/s$ ← $\dot{m}$



Problem 13.42

Given: Isentropic flow of air from stagnation state through a converging-diverging nozzle to a back pressure, p_b.



Find: A_t, A_1

Solution:

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$, $\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{1 + \frac{k-1}{2}}\right]^{(k+1)/2(k-1)} \quad (12.6)$$

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) ideal gas

$p_0/p = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$; $M_1 = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p_1}\right)^{k-1/k} - 1\right]\right\}^{1/2} = \left\{\frac{2}{0.4}\left[\left(\frac{1.1}{0.141}\right)^{0.286} - 1\right]\right\}^{1/2} = 2.0$

$T_0/T = 1 + \frac{k-1}{2}M^2$; $T_1 = \frac{T_0}{1 + \frac{k-1}{2}M_1^2} = \frac{388\,K}{1 + 0.2(2.0)^2} = 216\,K$

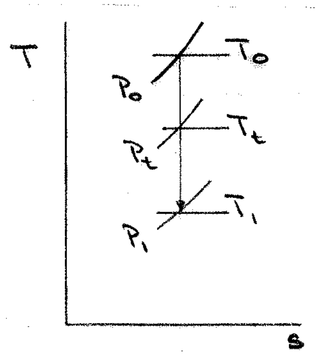
$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 2.0\left(1.4 \times 287 \frac{N\cdot m}{kg\cdot K} \times 216\,K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 589$ m/s

$\rho_1 = \frac{p_1}{RT_1} = 141 \times 10^3 \frac{N}{m^2} \times \frac{kg\cdot K}{287\,N\cdot m} \times \frac{1}{216\,K} = 2.27$ kg/m^3

$A_1 = \frac{\dot{m}}{\rho_1 V_1} = 2.0 \frac{kg}{s} \times \frac{m^3}{2.27\,kg} \times \frac{s}{589\,m} = 1.50 \times 10^{-3}\ m^2$ ← A_1

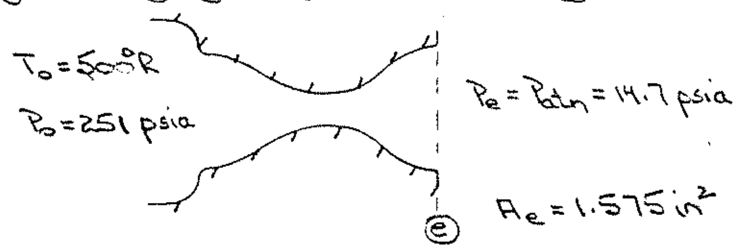
For $M_1 = 2.0$, from Eq. 12.6 and Fig. E.1, $A_1/A^* = 1.688$

Then $A_t = A^* = A_1/1.688 = 8.89 \times 10^{-4}\ m^2$ ← A_t



Problem 13.43

Given: Isentropic flow of air from a large tank through a converging-diverging nozzle discharging to atmosphere



Find: (a) $\dot{m}$
(b) effect on $\dot{m}$ of raising T_0 to $2000°R$
(c) plot $\dot{m}(T)$ for $500°R \le T \le 2000°R$

Solution:

Basic equations: $\dot{m} = \rho V A \qquad p = \rho R T$

Computing equations: $\dfrac{T_0}{T} = 1 + \dfrac{k-1}{2} M^2 \qquad \dfrac{p_0}{p} = \left[1 + \dfrac{k-1}{2} M^2\right]^{k/k-1}$

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) ideal gas

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} \; ; \; M_e = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p_e}\right)^{k-1/k} - 1\right]\right\}^{1/2} = \left\{\frac{2}{0.4}\left[\left(\frac{251}{14.7}\right)^{0.286} - 1\right]\right\}^{1/2} = 2.50$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \; ; \; T_e = \frac{T_0}{1 + \frac{k-1}{2}M_e^2} = \frac{500°R}{1 + 0.2\,(2.50)^2} = 222R$$

$$V_e = M_e c_e = M_e (kRT_e)^{1/2} = 2.5\left(1.4 \times 53.3\,\frac{ft \cdot lbf}{lbm \cdot °R} \times 222°R \times 32.2\,\frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{1/2} = 1826\ ft/s$$

$$\rho_e = \frac{p_e}{RT_e} = 14.7\,\frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3\,ft \cdot lbf} \times \frac{1}{222°R} \times 144\,\frac{in^2}{ft^2} = 0.179\ lbm/ft^3$$

$$\dot{m} = \rho_e V_e A_e = 0.179\,\frac{lbm}{ft^3} \times 1827\,\frac{ft}{s} \times 1.575\,in^2 \times \frac{ft^2}{144\,in^2} = 3.57\ lbm/s \longleftarrow \dot{m}$$

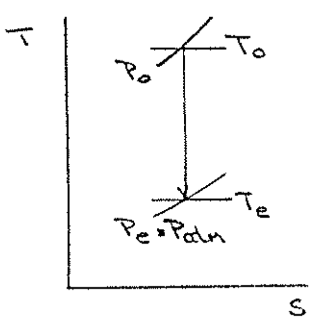
$\dot{m} = \rho_e V_e A_e$ If T_0 is increased by a factor of 4 (holding pressures constant)

(1) T_e will increase by a factor of 4 (since T_0/T = const)

(2) V_e " " " " " 2 (since $V_e \propto T_e^{1/2}$)

(3) ρ_e " decrease " " " " 4 (since $\rho_e \propto 1/T_e$)

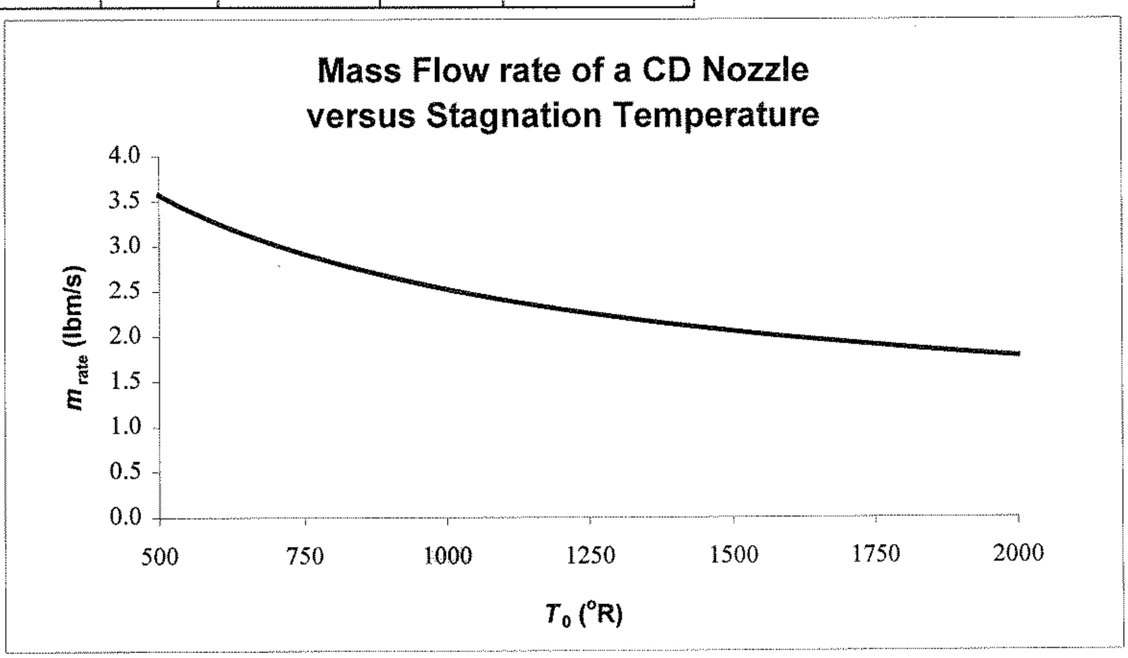
Thus the mass flow rate, $\dot{m}$, will decrease by a factor of 2 $\longleftarrow$



Problem 13.43

The calculations from page 1 are repeated for various T_0 values and plotted using *Excel*

T_0 (°R)	T_e (°R)	ρ_e (lbm/ft^3)	V_e (ft/s)	m_{rate} (lbm/s)
500	222	0.179	1827	3.57
550	244	0.162	1916	3.40
600	267	0.149	2001	3.26
650	289	0.137	2083	3.13
700	311	0.128	2161	3.02
750	333	0.119	2237	2.92
800	356	0.112	2311	2.82
850	378	0.105	2382	2.74
900	400	0.0993	2451	2.66
950	422	0.0941	2518	2.59
1000	444	0.0894	2583	2.52
1050	467	0.0851	2647	2.46
1100	489	0.0812	2710	2.41
1150	511	0.0777	2770	2.35
1200	533	0.0745	2830	2.30
1250	556	0.0715	2888	2.26
1300	578	0.0687	2946	2.21
1350	600	0.0662	3002	2.17
1400	622	0.0638	3057	2.13
1450	644	0.0616	3111	2.10
1500	667	0.0596	3164	2.06
1550	689	0.0577	3216	2.03
1600	711	0.0558	3268	2.00
1650	733	0.0542	3319	1.97
1700	756	0.0526	3368	1.94
1750	778	0.0511	3418	1.91
1800	800	0.0496	3466	1.88
1850	822	0.0483	3514	1.86
1900	844	0.0470	3561	1.83
1950	867	0.0458	3608	1.81
2000	889	0.0447	3654	1.79



Problem 13.44

[3]

13.44 A small, solid fuel rocket motor is tested on a thrust stand. The chamber pressure and temperature are 4 MPa and 3250 K. The propulsion nozzle is designed to expand the exhaust gases isentropically to a pressure of 75 kPa. The nozzle exit diameter is 25 cm. Treat the gas as ideal with $k = 1.25$ and $R = 300\ \mathrm{J/(kg \cdot K)}$. Determine the mass flow rate of propellant gas and the thrust force exerted against the test stand.

Given: Rocket motor on test stand

Find: Mass flow rate; thrust force

Solution:

Basic equations: $\dfrac{T_0}{T} = 1 + \dfrac{k-1}{2}\cdot M^2$ $\qquad \dfrac{p_0}{p} = \left(1 + \dfrac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $\qquad p = \rho\cdot R\cdot T$ $\qquad c = \sqrt{k\cdot R\cdot T}$ $\qquad m_{rate} = \rho\cdot A\cdot V$

$\left(p_{atm} - p_e\right)\cdot A_e + R_x = m_{rate}\cdot V_e$ Momentum for pressure p_e and velocity V_e at exit; R_x is the reaction for

Given or available data $p_e = 75\cdot kPa$ $\quad p_{atm} = 101\cdot kPa$ $\quad p_0 = 4\cdot MPa$ $\quad T_0 = 3250\cdot K$ $\quad k = 1.25$ $\quad R = 300\cdot\dfrac{J}{kg\cdot K}$

$d = 25\cdot cm$ so the nozzle exit area is $A_e = \dfrac{\pi}{4}\cdot d^2$ $\qquad A_e = 491\cdot cm^2$

From the pressures $M_e = \sqrt{\dfrac{2}{k-1}\cdot\left[\left(\dfrac{p_0}{p_e}\right)^{\frac{k-1}{k}} - 1\right]}$ $\qquad M_e = 3.12$

The exit temperature is $T_e = \dfrac{T_0}{\left(1 + \dfrac{k-1}{2}\cdot M_e^2\right)}$ $\qquad T_e = 1467\,K$ $\qquad c_e = \sqrt{k\cdot R\cdot T_e}$ $\qquad c_e = 742\dfrac{m}{s}$

The exit speed is $V_e = M_e\cdot c_e$ $\qquad V_e = 2313\dfrac{m}{s}$ and $\rho_e = \dfrac{p_e}{R\cdot T_e}$ $\qquad \rho_e = 0.170\cdot\dfrac{kg}{m^3}$

Then $m_{rate} = \rho_e\cdot A_e\cdot V_e$ $\qquad m_{rate} = 19.3\dfrac{kg}{s}$

The momentum equation (Eq. 4.33) simplifies to $\left(p_e - p_{atm}\right)\cdot A_e - M_{CV}\cdot a_x = -V_e\cdot m_{rate}$

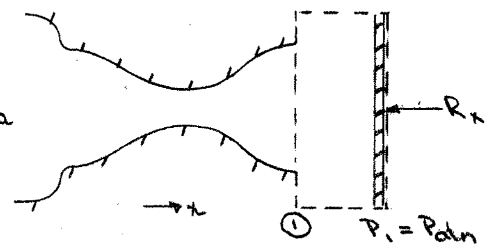
Hence $R_x = \left(p_e - p_{atm}\right)\cdot A_e + V_e\cdot m_{rate}$ $\qquad R_x = 43.5\cdot kN$

Problem 13.45

Given: Flow of nitrogen through a converging-diverging nozzle leaves nozzle at atmospheric pressure, exhaust impinges on vertical flat plate.

$T_0 = 400\,K$

$p_0 = 371\,kPa$



① $p_1 = p_{atm}$ $\quad A_1 = 0.003\,m^2$

Find: Force required to hold the plate.

Solution:

Basic equations: $F_{S_x} = -R_x = -\dot{m}V_1$ (momentum eq. for cv shown)

$\dot{m} = \rho V A = \text{const}$ $\qquad p = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$ $\qquad \frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow
(2) isentropic flow in nozzle
(3) uniform flow at a section
(4) p_{atm} over entire c.s.
(5) $F_{B_x} = 0$
(6) ideal gas, $k = 1.40$

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} ; \quad M_1 = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p_1}\right)^{k-1/k} - 1\right]\right\}^{1/2} = \left\{\frac{2}{0.4}\left[\left(\frac{371}{101}\right)^{0.286} - 1\right]\right\}^{1/2} = 1.50$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M_1^2 ; \quad T_1 = \frac{T_0}{1 + \frac{k-1}{2}M_1^2} = \frac{400K}{1 + 0.2(1.50)^2} = 276K$$

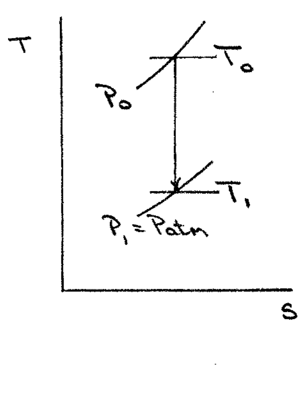
$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 1.5\left(1.4 \times 297\,\frac{N\cdot m}{kg\cdot K} \times 276K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 508\ m/s$$

$$\rho_1 = \frac{p_1}{RT_1} = 101\times10^3\,\frac{N}{m^2} \times \frac{kg\cdot K}{297\,N\cdot m} \times \frac{1}{276K} = 1.23\ kg/m^3$$

$$\dot{m} = \rho_1 V_1 A_1 = 1.23\,\frac{kg}{m^3} \times 508\,\frac{m}{s} \times 0.003\,m^2 = 1.87\ kg/s$$

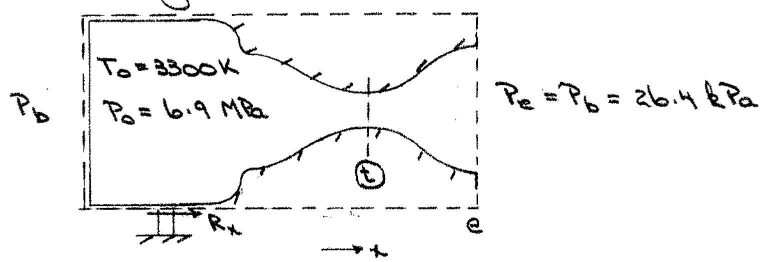
$$R_x = \dot{m}V_1 = 1.87\,\frac{kg}{s} \times 508\,\frac{m}{s} \times \frac{N\cdot s^2}{kg\cdot m} = 950\,N$$

$\leftarrow$ (to the left as shown). $\quad R_x$



Problem 13.46

Given: Liquid rocket motor designed to expand exhaust gases isentropically to design back pressure corresponding to an altitude of 10,000 m. Thrust produced is 100 kN. Exhaust gases treated as water vapor and ideal gas behavior assumed.



Find: (a) $\dot{m}$ (b) A_e (c) A_e/A_t

Solution

Basic equations: $F_{S_x} = R_x = \dot{m} V_e \qquad \dot{m} = \rho V A \qquad p = \rho R T$

Computing equations: $\dfrac{T_0}{T} = 1 + \dfrac{k-1}{2}M^2 \qquad \dfrac{p_0}{p} = \left[1 + \dfrac{k-1}{2}M^2\right]^{k/k-1}$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{1 + \frac{k-1}{2}}\right]^{(k+1)/2(k-1)} \qquad (12.6)$$

Assumptions: (1) steady flow (2) isentropic flow (3) uniform flow at a section (4) thrust specified at altitude (5) $F_{B_x} = 0$ (6) ideal gas, $k = 1.3$, $R = 461 \dfrac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}$

$p_0/p = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$; $M_e = \left\{\dfrac{2}{k-1}\left[\left(\dfrac{p_0}{p}\right)^{\frac{k-1}{k}} - 1\right]\right\}^{1/2} = \left\{\dfrac{2}{0.30}\left[\left(\dfrac{69.0}{0.264}\right)^{0.231} - 1\right]\right\}^{1/2} = 4.18$

$T_0/T = 1 + \frac{k-1}{2}M^2$; $T_e = \dfrac{T_0}{1 + \frac{k-1}{2}M_e^2} = \dfrac{3300\text{ K}}{1 + 0.15(4.18)^2} = 911\text{ K}$

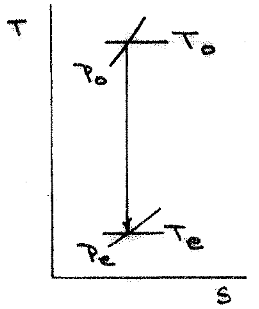
$V_e = M_e c_e = M_e (kRT_e)^{1/2} = 4.18\left[1.3 \times 461 \dfrac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}} \times 911\text{ K} \times \dfrac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right]^{1/2} = 3090\text{ m/s}$

$R_x = \dot{m} V_e$; $\dot{m} = \dfrac{R_x}{V_e} = 10^5\text{ N} \times \dfrac{\text{s}}{3090\text{ m}} \times \dfrac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2} = 32.4\text{ kg/s}$ ← $\dot{m}$

$\rho_e = \dfrac{p_e}{RT_e} = 26.4 \times 10^3 \dfrac{\text{N}}{\text{m}^2} \times \dfrac{\text{kg}\cdot\text{K}}{461\text{ N}\cdot\text{m}} \times \dfrac{1}{911\text{ K}} = 6.29 \times 10^{-2}\text{ kg/m}^3$

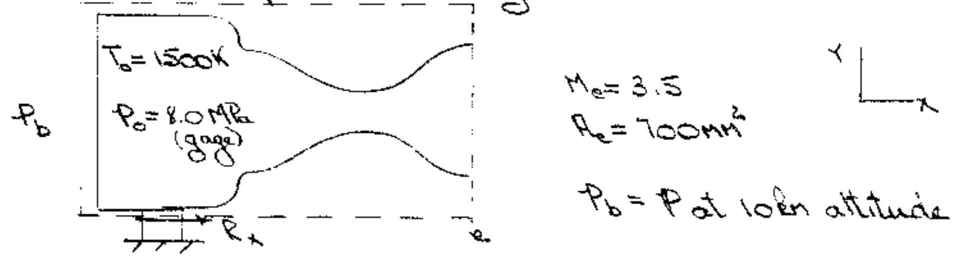
$\dot{m} = \rho_e V_e A_e$; $A_e = \dfrac{\dot{m}}{\rho_e V_e} = 32.4 \dfrac{\text{kg}}{\text{s}} \times \dfrac{\text{m}^3}{6.29 \times 10^{-2}\text{ kg}} \times \dfrac{\text{s}}{3090\text{ m}} = 0.167\text{ m}^2$ ← A_e

For $M_e = 4.18$, from Eq 12.6, $A_e/A^* = A_e/A_t = 19.4$ ← A_e/A_t



Problem 13.47

Given: Small rocket motor, fueled with H_2 and O_2, is tested on thrust stand at a simulated altitude of 10 km. Combustion product is water vapor which may be treated as an ideal gas.



Find: (a) p_e (b) $\dot{m}$ (c) force on test stand

Solution:

Basic equations: $\dot{m} = \rho V A$, $p = \rho R T$

Assumptions: (1) steady flow (2) isentropic flow (3) ideal gas behavior $k = 1.3$, $R = 461\ \text{J/kg·K}$

At 10 km altitude, $p_b = 26.5\text{ kPa}$ (Table A.3)

Evaluate design pressure at exit

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} \quad \therefore p_d = \frac{8.10\times10^6\ \text{Pa}}{\left[1+0.15(3.5)^2\right]^{4.333}} = 88.3\text{ kPa (abs)}$$

Since $p_b < p_d$, $p_e = p_d = 88.3\text{ kPa}$ ——— p_e

$$\frac{T_0}{T_e} = 1 + \frac{k-1}{2}M_e^2 \quad \therefore T_e = \frac{T_0}{1+\frac{k-1}{2}M^2} = \frac{1500\text{K}}{1+0.15(3.5)^2} = 529\text{ K}$$

$$\rho_e = \frac{p_e}{RT_e} = 88.3\times10^3\ \frac{N}{m^2} \times \frac{kg\cdot K}{461\ N\cdot m} \times \frac{1}{529\ K} = 0.362\ kg/m^3$$

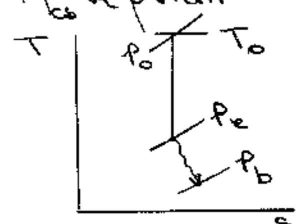
$$V_e = Mc_e = 3.5\left[1.30 \times 461\ \frac{N\cdot m}{kg\cdot K} \times 529K \times \frac{kg\cdot m}{N\cdot s^2}\right]^{1/2} = 1970\ m/s$$

$$\dot{m} = \rho_e V_e A_e = 0.362\ \frac{kg}{m^3} \times 1970\ \frac{m}{s} \times 700\,mm^2 \times \frac{m^2}{10^6\,mm^2} = 0.499\ kg/s$$ ——— $\dot{m}$

To determine force on test stand apply x momentum equation to CV shown

Basic equation: $F_{S_x} + F_{B_x}^{\,\approx 0} = \frac{\partial}{\partial t}\int_{CV} V_x \rho\, d\forall + \int_{CS} V_x \rho \vec{V}\cdot d\vec{A}$

$$R_x + p_b A_e - p_e A_e = \dot{m} V_e$$



Force on test stand is $K_x = -R_x$

$$\therefore K_x = -R_x = -\dot{m}V_e - A_e(p_e - p_b)$$

$$K_x = -0.499\ \frac{kg}{s} \times 1970\ \frac{m}{s} \times \frac{N\cdot s^2}{kg\cdot m} - 700\,mm^2 \times \frac{m^2}{10^6\,mm^2}(88.3-26.5)\times10^3\ \frac{N}{m^2}$$

$K_x = -1{,}026\text{ N}$ (to left) ——— K_x

Problem 13.48

13.48 A CO_2 cartridge is used to propel a small rocket cart. Compressed gas, stored at 35 MPa and 20°C, is expanded through a smoothly contoured converging nozzle with 0.5 mm throat diameter. The back pressure is atmospheric. Calculate the pressure at the nozzle throat. Evaluate the mass flow rate of carbon dioxide through the nozzle. Determine the thrust available to propel the cart. How much would the thrust increase if a diverging section were added to the nozzle to expand the gas to atmospheric pressure? What is the exit area? Show stagnation states, static states, and the processes on a *Ts* diagram.

Given: Compressed CO_2 in a cartridge expanding through a nozzle

Find: Throat pressure; Mass flow rate; Thrust; Thrust increase with diverging section; Exit area

Solution:

Basic equations: $$F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\,dV + \int_{CS} u\,\rho\,\vec{V}\cdot d\vec{A}$$

Assumptions: 1) Isentropic flow 2) Stagnation in cartridge 3) Ideal gas 4) Uniform flow

Given or available data: $k = 1.29$ $\quad R = 188.9\cdot\frac{J}{kg\cdot K}$ $\quad p_{atm} = 101\cdot kPa$

$p_0 = 35\cdot MPa$ $\quad T_0 = (20 + 273)\cdot K$ $\quad d_t = 0.5\cdot mm$

From isentropic relations $$p_{crit} = \frac{p_0}{\left(1 + \frac{k-1}{2}\right)^{\frac{k}{k-1}}} \qquad p_{crit} = 19.2\,MPa$$

Since $p_b << p_{crit}$, then $$p_t = p_{crit} \qquad p_t = 19.2\,MPa$$

Throat is critical so $$m_{rate} = \rho_t\cdot V_t\cdot A_t$$

$$T_t = \frac{T_0}{1 + \frac{k-1}{2}} \qquad T_t = 256\,K$$

$$V_t = \sqrt{k\cdot R\cdot T_t} \qquad V_t = 250\,\frac{m}{s}$$

$$A_t = \frac{\pi\cdot d_t^{\,2}}{4} \qquad A_t = 1.963\times 10^{-7}\,m^2$$

$$\rho_t = \frac{p_t}{R\cdot T_t} \qquad \rho_t = 396\,\frac{kg}{m^3}$$

$$m_{rate} = \rho_t\cdot V_t\cdot A_t \qquad m_{rate} = 0.0194\,\frac{kg}{s}$$

For 1D flow with no body force the momentum equation reduces to $R_x - p_{tgage} \cdot A_t = m_{rate} \cdot V_t$ $\qquad p_{tgage} = p_t - p_{atm}$

$$R_x = m_{rate} \cdot V_t + p_{tgage} \cdot A_t \qquad R_x = 8.60\,N$$

When a diverging section is added the nozzle can exit to atmospheric pressu $p_e = p_{atm}$

Hence the Mach number at exit is $M_e = \left[\frac{2}{k-1} \cdot \left[\left(\frac{p_0}{p_e}\right)^{\frac{k-1}{k}} - 1\right]\right]^{\frac{1}{2}}$ $\qquad M_e = 4.334$

$$T_e = \frac{T_0}{1 + \frac{k-1}{2} \cdot M_e^2} \qquad T_e = 78.7\,K$$

$$c_e = \sqrt{k \cdot R \cdot T_e} \qquad c_e = 138\,\frac{m}{s}$$

$$V_e = M_e \cdot c_e \qquad V_e = 600\,\frac{m}{s}$$

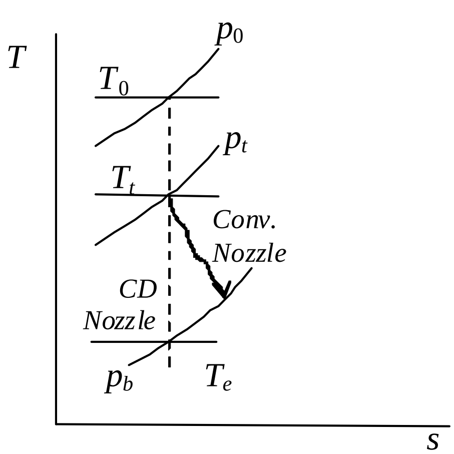
The mass flow rate is unchanged (choked flow)

From the momentum equation $R_x = m_{rate} \cdot V_e$ $\qquad R_x = 11.67\,N$

The percentage increase in thrust is $\frac{11.67 \cdot N - 8.60 \cdot N}{8.60 \cdot N} = 35.7\%$

The exit area is obtained from $m_{rate} = \rho_e \cdot V_e \cdot A_e$ and $\rho_e = \frac{p_e}{R \cdot T_e}$ $\qquad \rho_e = 6.79\,\frac{kg}{m^3}$

$$A_e = \frac{m_{rate}}{\rho_e \cdot V_e} \qquad A_e = 4.77 \times 10^{-6}\,m^2 \qquad A_e = 4.77\,mm^2$$



Problem 13.49

13.49 Consider the converging-diverging option of Problem 13.48. To what pressure would the compressed gas need to be raised (keeping the temperature at 20°C) to develop a thrust of 15N? (Assume isentropic flow.)

Given: CO_2 cartridge and convergent nozzle

Find: Tank pressure to develop thrust of 15 N

Solution:

The given or available data is:

R =	188.9	J/kg·K
k =	1.29	
T_0 =	293	K
p_b =	101	kPa
D_t =	0.5	mm

Equations and Computations:

A_t = 0.196 mm^2

The momentum equation gives

$$R_x = m_{flow} V_e$$

Hence, we need m_{flow} and V_e

For isentropic flow

$p_e = p_b$

p_e = 101 kPa

If we knew p_0 we could use it and p_e, and Eq. 13.7a, to find M_e.

Once M_e is known, the other exit conditions can be found.

Make a guess for p_0, and eventually use *Goal Seek* (see below).

p_0 = 44.6 MPa

From p_0 and p_e, and Eq. 13.7a
(using built-in function *IsenMfromp* (M, k)

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

M_e = 4.5

From M_e and T_0 and Eq. 13.7b
(using built-in function *IsenT*(M, k))

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \qquad (13.7b)$$

$T_e =$ 74.5 K

From T_e and Eq. 12.18

$$c = \sqrt{kRT} \qquad (12.18)$$

$c_e =$ 134.8 m/s

Then

$V_e =$ 606 m/s

The mass flow rate is obtained from p_0, T_0, A_t, and Eq. 13.10a

$$\dot{m}_{\text{choked}} = A_t p_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)} \qquad (13.10a)$$

$m_{\text{choked}} =$ 0.0248 kg/s

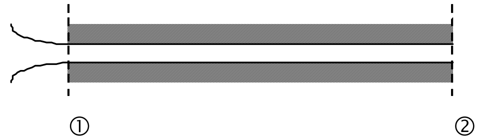
Finally, the momentum equation gives

$R_x = m_{\text{flow}} V_e$
$=$ 15.0 N

We need to set R_x to 15 N. To do this use *Goal Seek*
to vary p_0 to obtain the result!

Problem 13.50

13.50 Room air is drawn into an insulated duct of constant area through a smoothly contoured converging nozzle. Room conditions are $T = 80°\text{F}$ and $p = 14.7$ psia. The duct diameter is $D = 1$ in. The pressure at the duct inlet (nozzle outlet) is $p_1 = 13$ psia. Find (a) the mass flow rate in the duct and (b) the range of exit pressures for which the duct exit flow is choked.



Given:
Air flow in an insulated duct

Find:
Mass flow rate; Range of choked exit pressures

Solution:

Basic equations:
$$\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2 \qquad c = \sqrt{k\cdot R\cdot T} \qquad \frac{A}{A_{crit}} = \frac{1}{M}\cdot\left(\frac{1+\frac{k-1}{2}\cdot M^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}}$$

Given or available data
$$T_0 = (80+460)\cdot R \qquad p_0 = 14.7\cdot psi \qquad p_1 = 13\cdot psi \qquad D = 1\cdot in$$
$$k = 1.4 \qquad R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R} \qquad A = \frac{\pi\cdot D^2}{4} \qquad A = 0.785\cdot in^2$$

Assuming isentropic flow, stagnation conditions are constant. Hence

$$M_1 = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}} - 1\right]} \qquad M_1 = 0.423 \qquad T_1 = \frac{T_0}{1+\frac{k-1}{2}\cdot M_1^2} \qquad T_1 = 521\cdot R \qquad T_1 = 61.7\cdot °F$$

$$c_1 = \sqrt{k\cdot R_{air}\cdot T_1} \qquad c_1 = 341\frac{m}{s} \qquad V_1 = M_1\cdot c_1 \qquad V_1 = 144\frac{m}{s}$$

Also
$$\rho_1 = \frac{p_1}{R_{air}\cdot T_1} \qquad \rho_1 = 0.0673\cdot\frac{lbm}{ft^3}$$

Hence
$$m_{rate} = \rho_1\cdot V_1\cdot A \qquad m_{rate} = 0.174\cdot\frac{lbm}{s}$$

When flow is choked
$$M_2 = 1 \qquad T_2 = \frac{T_0}{1+\frac{k-1}{2}} \qquad T_2 = 450\cdot R \qquad T_2 = -9.7\cdot °F$$

We also have
$$c_2 = \sqrt{k\cdot R_{air}\cdot T_2} \qquad c_2 = 1040\cdot\frac{ft}{s} \qquad V_2 = c_2 \qquad V_2 = 1040\cdot\frac{ft}{s}$$

From continuity
$$\rho_1\cdot V_1 = \rho_2\cdot V_2 \qquad \rho_2 = \rho_1\cdot\frac{V_1}{V_2} \qquad \rho_2 = 0.0306\cdot\frac{lbm}{ft^3}$$

Hence
$$p_2 = \rho_2\cdot R_{air}\cdot T_2 \qquad p_2 = 5.11\cdot psi$$

The flow will therefore choke for any back pressure (pressure at the exit) less than or equal to this pressure

(From Fanno line function $\frac{p_1}{p_{crit}} = 2.545$ at $M_1 = 0.423$ so $p_{crit} = \frac{p_1}{2.545}$ $\qquad p_{crit} = 5.11\,psi$ Check!)

Problem 13.51

[4]

13.51 Air from a large reservoir at 25 psia and 250°F flows isentropically through a converging nozzle into an insulated pipe at 24 psia. The pipe flow experiences friction effects. Obtain a plot of the *Ts* diagram for this flow, until $M = 1$. Also plot the pressure and speed distributions from the entrance to the location at which $M = 1$.

Given: Air flow from converging nozzle into pipe

Find: Plot Ts diagram and pressure and speed curves

Solution:

The given or available data is:

$R =$	53.33	ft·lbf/lbm·°R
$k =$	1.4	
$c_p =$	0.2399	Btu/lbm·°R
	187	ft·lbf/lbm·°R
$T_0 =$	710	°R
$p_0 =$	25	psi
$p_e =$	24	psi

Equations and Computations:

From p_0 and p_e, and Eq. 13.7a
(using built-in function *IsenMfromp* (M, k)) $M_e =$ 0.242

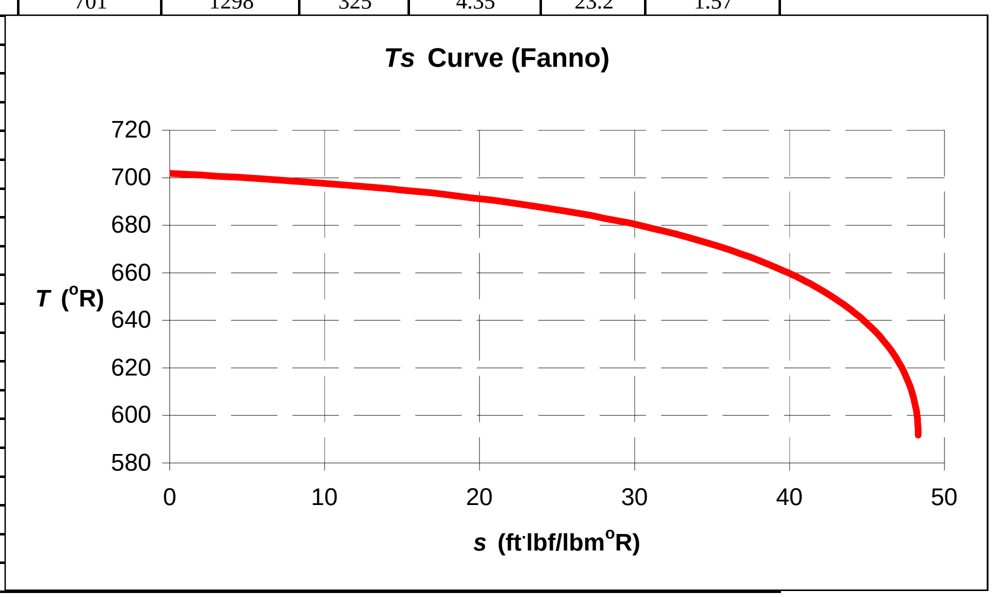
Using built-in function *IsenT* (M, k) $T_e =$ 702 °R

Using p_e, M_e, and function *Fannop* (M, k) $p^* =$ 5.34 psi

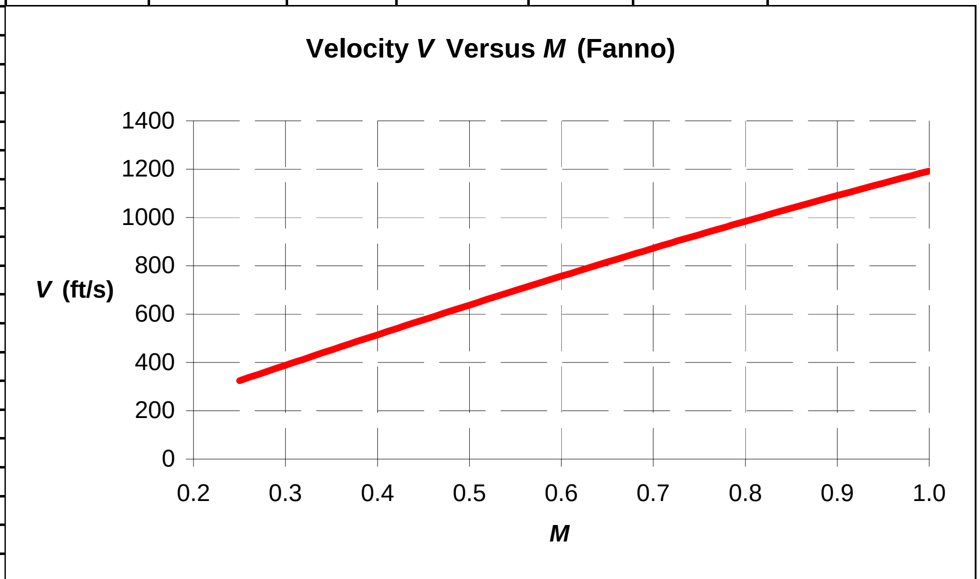
Using T_e, M_e, and function *FannoT* (M, k) $T^* =$ 592 °R

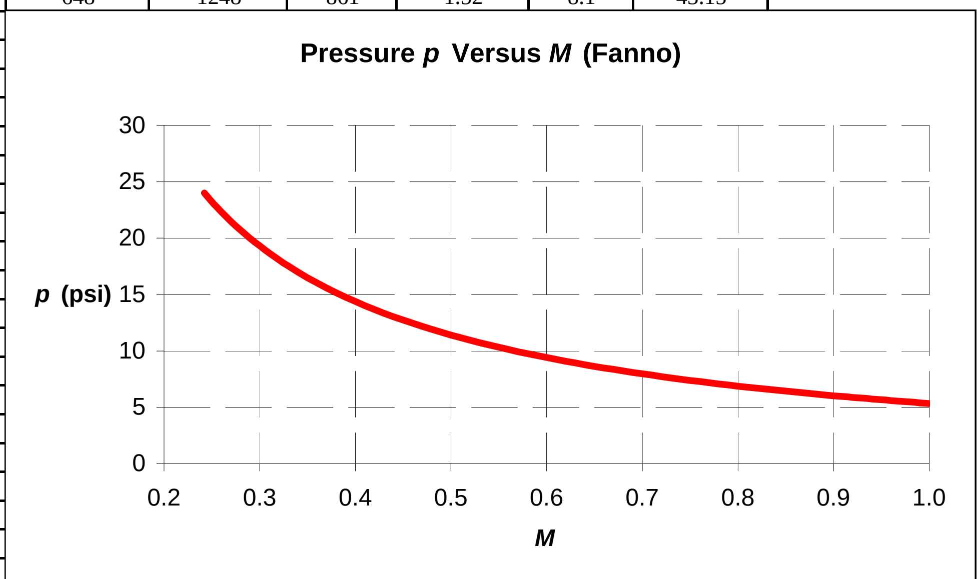
We can now use Fanno-line relations to compute values for a range of Mach numbers:

M	T/T^*	T (°R)	c (ft/s)	V (ft/s)	p/p^*	p (psi)	Δs (ft·lbf/lbm·°R) Eq. (12.11b)
0.242	1.186	702	1299	315	4.50	24.0	0.00
0.25	1.185	701	1298	325	4.35	23.2	1.57
0.26	1.184						
0.27	1.183						
0.28	1.181						
0.29	1.180						
0.3	1.179						
0.31	1.177						
0.32	1.176						
0.33	1.174						
0.34	1.173						
0.35	1.171						
0.36	1.170						
0.37	1.168						
0.38	1.166						
0.39	1.165						
0.4	1.163						
0.41	1.161						
0.42	1.159						
0.43	1.157						
0.44	1.155						
0.45	1.153						



0.46	1.151	681	1280	589	2.33	12.4	29.44
0.47	1.149	680	1279	601	2.28	12.2	30.31
0.48	1.147						
0.49	1.145						
0.5	1.143						
0.51	1.141						
0.52	1.138						
0.53	1.136						
0.54	1.134						
0.55	1.132						
0.56	1.129						
0.57	1.127						
0.58	1.124						
0.59	1.122						
0.6	1.119						
0.61	1.117						
0.62	1.114						
0.63	1.112						
0.64	1.109						
0.65	1.107						
0.66	1.104						
0.67	1.101						
0.68	1.098	650	1250	850	1.54	8.2	42.77
0.69	1.096	648	1248	861	1.52	8.1	43.15
0.7	1.093						
0.71	1.090						
0.72	1.087						
0.73	1.084						
0.74	1.082						
0.75	1.079						
0.76	1.076						
0.77	1.073						
0.78	1.070						
0.79	1.067						
0.8	1.064						
0.81	1.061						
0.82	1.058						
0.83	1.055						
0.84	1.052						
0.85	1.048						
0.86	1.045						
0.87	1.042						
0.88	1.039						
0.89	1.036						
0.9	1.033	611	1212	1091	1.13	6.0	47.84
0.91	1.029	609	1210	1101	1.11	6.0	47.94
0.92	1.026	607	1208	1112	1.10	5.9	48.02
0.93	1.023	605	1206	1122	1.09	5.8	48.09
0.94	1.020	603	1204	1132	1.07	5.7	48.15
0.95	1.017	601	1202	1142	1.06	5.7	48.20
0.96	1.013	600	1201	1153	1.05	5.6	48.24
0.97	1.010	598	1199	1163	1.04	5.5	48.27
0.98	1.007	596	1197	1173	1.02	5.5	48.30
0.99	1.003	594	1195	1183	1.01	5.4	48.31
1	1.000	592	1193	1193	1.00	5.3	48.31





Problem 13.52

13.52 Repeat Problem 13.51 except the nozzle is now a converging-diverging nozzle delivering the air to the pipe at 2.5 psia.

Given: Air flow from converging-diverging nozzle into pipe

Find: Plot Ts diagram and pressure and speed curves

Solution:

The given or available data is:

R =	53.33	ft·lbf/lbm·°R
k =	1.4	
c_p =	0.2399	Btu/lbm·°R
	187	ft·lbf/lbm·°R
T_0 =	710	°R
p_0 =	25	psi
p_e =	2.5	psi

Equations and Computations:

From p_0 and p_e, and Eq. 13.7a
(using built-in function *IsenMfromp* (M,k)) M_e = 2.16

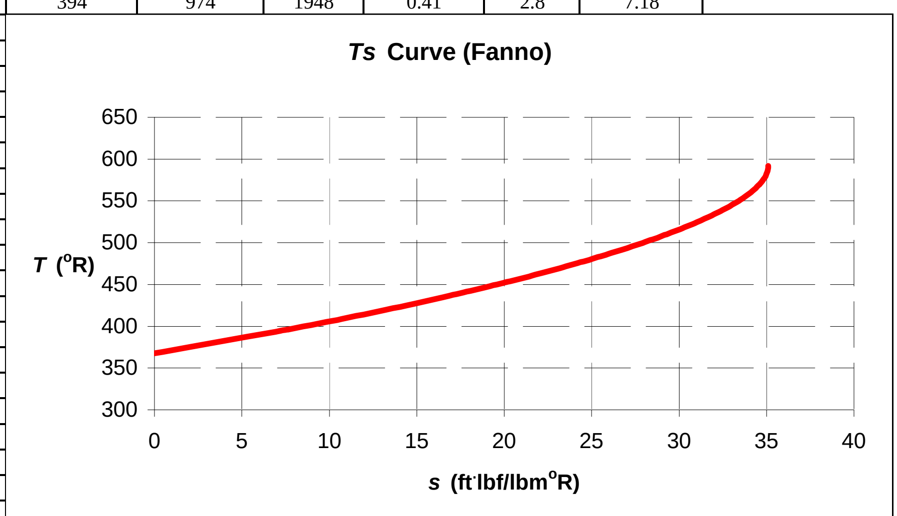
Using built-in function *IsenT* (M,k) T_e = 368 °R

Using p_e, M_e, and function *Fannop* (M,k) p^* = 6.84 psi

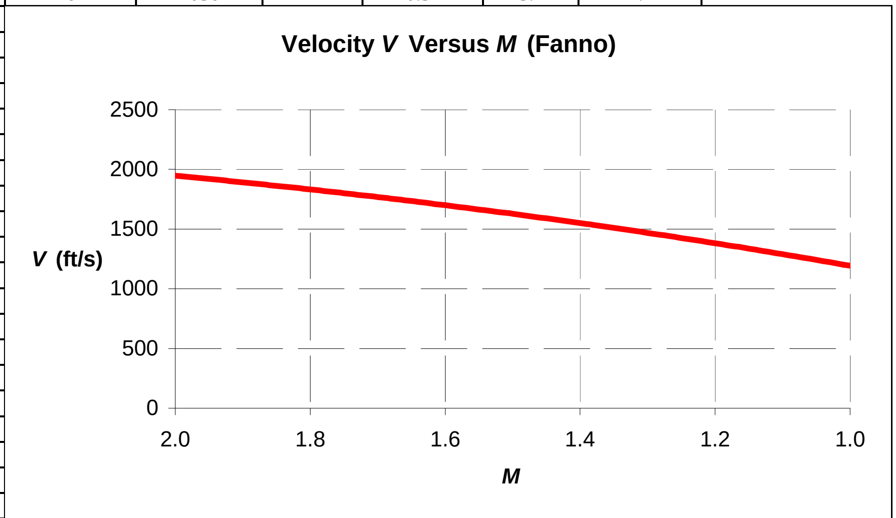
Using T_e, M_e, and function *FannoT* (M,k) T^* = 592 °R

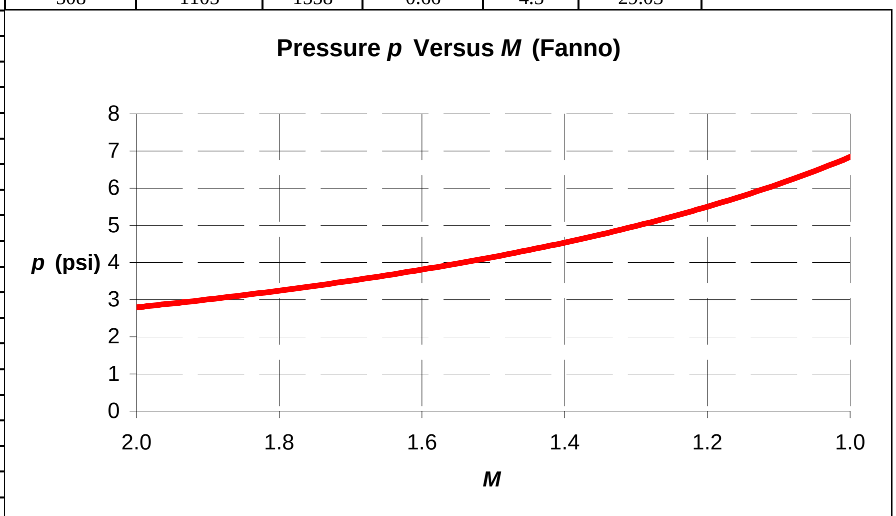
We can now use Fanno-line relations to compute values for a range of Mach numbers:

M	T/T^*	T (°R)	c (ft/s)	V (ft/s)	p/p^*	p (psi)	Δs (ft·lbf/lbm·°R) Eq. (12.11b)
2.157	0.622	368	940	2028	0.37	2.5	0.00
2	0.667	394	974	1948	0.41	2.8	7.18
1.99	0.670						
1.98	0.673						
1.97	0.676						
1.96	0.679						
1.95	0.682						
1.94	0.685						
1.93	0.688						
1.92	0.691						
1.91	0.694						
1.9	0.697						
1.89	0.700						
1.88	0.703						
1.87	0.706						
1.86	0.709						
1.85	0.712						
1.84	0.716						
1.83	0.719						
1.82	0.722						
1.81	0.725						
1.8	0.728						
1.79	0.731	433	1020	1826	0.48	3.3	16.08
1.78	0.735	435	1022	1819	0.48	3.3	16.48
1.77	0.738	436	1024	1813	0.49	3.3	16.88
1.76	0.741	438	1027	1807	0.49	3.3	17.27
1.75	0.744	440	1029	1801	0.49	3.4	17.66
1.74	0.747	442	1031	1794	0.50	3.4	18.05
1.73	0.751	444	1033	1788	0.50	3.4	18.44
1.72	0.754	446	1036	1781	0.50	3.5	18.82
1.71	0.757	448	1038	1775	0.51	3.5	19.20
1.7	0.760	450	1040	1768	0.51	3.5	19.58



1.69	0.764	452	1042	1761	0.52	3.5	19.95
1.68	0.767	454	1045	1755	0.52	3.6	20.32
1.67	0.770	456	1047	1748	0.53	3.6	20.69
1.66	0.774	458	1049	1741	0.53	3.6	21.06
1.65	0.777	460	1051	1735	0.53	3.7	21.42
1.64	0.780	462	1054	1728	0.54	3.7	21.78
1.63	0.784	464	1056	1721	0.54	3.7	22.14
1.62	0.787						
1.61	0.790						
1.6	0.794						
1.59	0.797						
1.58	0.800						
1.57	0.804						
1.56	0.807						
1.55	0.811						
1.54	0.814						
1.53	0.817						
1.52	0.821						
1.51	0.824						
1.5	0.828						
1.49	0.831						
1.48	0.834						
1.47	0.838						
1.46	0.841						
1.45	0.845						
1.44	0.848						
1.43	0.852						
1.42	0.855	506	1103	1566	0.65	4.5	28.76
1.41	0.859	508	1105	1558	0.66	4.5	29.03
1.4	0.862						
1.39	0.866						
1.38	0.869						
1.37	0.872						
1.36	0.876						
1.35	0.879						
1.34	0.883						
1.33	0.886						
1.32	0.890						
1.31	0.893						
1.3	0.897						
1.29	0.900						
1.28	0.904						
1.27	0.907						
1.26	0.911						
1.25	0.914						
1.24	0.918						
1.23	0.921						
1.22	0.925						
1.21	0.928						
1.2	0.932	551	1151	1381	0.80	5.5	33.50
1.19	0.935	553	1153	1372	0.81	5.6	33.65
1.18	0.939	555	1155	1363	0.82	5.6	33.79
1.17	0.942	557	1158	1354	0.83	5.7	33.93
1.16	0.946	559	1160	1345	0.84	5.7	34.05
1.15	0.949	561	1162	1336	0.85	5.8	34.18
1.14	0.952	564	1164	1327	0.86	5.9	34.29
1.13	0.956	566	1166	1318	0.87	5.9	34.40
1.12	0.959	568	1168	1308	0.87	6.0	34.50
1.11	0.963	570	1170	1299	0.88	6.0	34.59
1.1	0.966	572	1172	1290	0.89	6.1	34.68
1.09	0.970	574	1174	1280	0.90	6.2	34.76
1.08	0.973	576	1176	1271	0.91	6.2	34.83
1.07	0.976	578	1179	1261	0.92	6.3	34.89
1.06	0.980	580	1181	1251	0.93	6.4	34.95
1.05	0.983	582	1183	1242	0.94	6.5	34.99
1.04	0.987	584	1185	1232	0.96	6.5	35.03
1.03	0.990	586	1187	1222	0.97	6.6	35.06
1.02	0.993	588	1189	1212	0.98	6.7	35.08
1.01	0.997	590	1191	1203	0.99	6.8	35.10
1	1.000	592	1193	1193	1.00	6.8	35.10





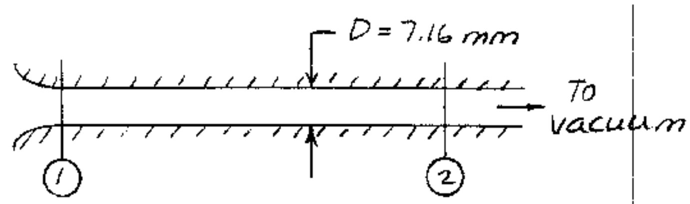
Problem 13.53

[2]

Given: Fanno line flow apparatus in laboratory, smooth brass tube fed by converging nozzle.

$T = 23°C$

$h_{barometer} = 755.1 \text{ mm Hg}$



Find: (a) M_1 $\quad h_1 = -20.8 \text{ mm Hg}$ $\quad M_2 = 0.40$

(b) Mass flow rate in tube

(c) p_2

Solution: Apply equations for steady, 1-D compressible flow:

Computing equations: $T_0 = T\left(1 + \frac{k-1}{2}M^2\right)$ {Entire flow adiabatic}

$$p_0 = p\left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}} \quad \text{\{Isentropic in nozzle\}}$$

Assume: (1) Stagnation conditions in laboratory

(2) Ideal gas

Then $\dfrac{p_0}{p_1} = \left(1 + \frac{k-1}{2}M_1^2\right)^{\frac{k}{k-1}} = \dfrac{755.1 \text{ mm Hg}}{(755.1 - 20.8) \text{ mm Hg}} = 1.03$

$$M_1 = \left\{\frac{2}{k-1}\left[\left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}} - 1\right]\right\}^{1/2} = \left\{5\left[\left(\frac{755.1}{734.3}\right)^{0.286} - 1\right]\right\}^{1/2} = 0.200$$ M_1

From continuity, $\dot{m} = \rho_1 V_1 A_1$; $\rho_1 = \dfrac{p_1}{RT_1}$

$$p_1 = \rho_{Hg} g h_1 = (13.5)1000 \frac{kg}{m^3} \times 9.81 \frac{m}{s^2} \times 0.734 \, m \times \frac{N \cdot s^2}{kg \cdot m} = 97.2 \text{ kPa (abs)}$$

$$T_1 = \frac{T_0}{1 + \frac{k-1}{2}M_1^2} = \frac{(273+23)K}{1 + 0.2(0.200)^2} = 294 \text{ K} \; ; \; c_1 = \sqrt{kRT_1} = 344 \text{ m/s}$$

$$\rho_1 = 97.2 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287 \, N \cdot m} \times \frac{1}{294 \, K} = 1.15 \text{ kg/m}^3$$

$$V_1 = M_1 c_1 = 0.200 \times 344 \text{ m/s} = 68.8 \text{ m/s}$$

$$A_1 = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.00716)^2 m^2 = 4.03 \times 10^{-5} m^2$$

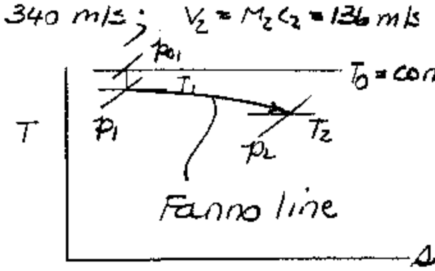
$$\dot{m} = 1.15 \frac{kg}{m^3} \times 68.8 \frac{m}{s} \times 4.03 \times 10^{-5} m^2 = 3.19 \times 10^{-3} \text{ kg/s}$$ $\dot{m}$

Since T_0 = constant, $T_2 = T_0 / \left(1 + \frac{k-1}{2}M_2^2\right) = 287 \text{ K}$; $c_2 = 340 \text{ m/s}$; $V_2 = M_2 c_2 = 136 \text{ m/s}$

$$\rho_2 = \rho_1 \frac{V_1}{V_2} = 1.15 \frac{kg}{m^3} \times \frac{68.8}{136} = 0.582 \text{ kg/m}^3$$

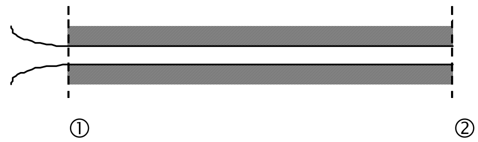
Then $p_2 = \rho_2 R T_2$

$$p_2 = 0.582 \frac{kg}{m^3} \times 287 \frac{N \cdot m}{kg \cdot K} \times 287 \, K = 47.9 \text{ kPa (abs)}$$ p_2



Problem 13.54

13.54 Air flows steadily and adiabatically from a large tank through a converging nozzle connected to an insulated constant-area duct. The nozzle may be considered frictionless. Air in the tank is at $p = 145$ psia and $T = 250°$F. The absolute pressure at the nozzle exit (duct inlet) is 125 psia. Determine the pressure at the end of the duct, if the temperature there is 150°F. Find the entropy increase.



Given: Air flow in a converging nozzle and insulated duct

Find: Pressure at end of duct; Entropy increase

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\quad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $\quad \Delta s = c_p \cdot \ln\left(\frac{T_2}{T_1}\right) - R_{air}\cdot \ln\left(\frac{p_2}{p_1}\right)$ $\quad c = \sqrt{k\cdot R\cdot T}$

Given or available data $T_0 = (250 + 460)\cdot R$ $\quad p_0 = 145\cdot psi$ $\quad p_1 = 125\cdot psi$ $\quad T_2 = (150 + 460)\cdot R$

$k = 1.4$ $\quad c_p = 0.2399\cdot\frac{Btu}{lbm\cdot R}$ $\quad R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}$

Assuming isentropic flow in the nozzle

$M_1 = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}} - 1\right]}$ $\quad M_1 = 0.465$ $\quad T_1 = \frac{T_0}{1 + \frac{k-1}{2}\cdot M_1^2}$ $\quad T_1 = 681\cdot R$ $\quad T_1 = 221\cdot °F$

In the duct T_0 (a measure of total energy) is constant, so $M_2 = \sqrt{\frac{2}{k-1}\cdot\left[\left(\frac{T_0}{T_2}\right) - 1\right]}$ $\quad M_2 = 0.905$

At each location $c_1 = \sqrt{k\cdot R_{air}\cdot T_1}$ $\quad c_1 = 1279\cdot\frac{ft}{s}$ $\quad V_1 = M_1\cdot c_1$ $\quad V_1 = 595\cdot\frac{ft}{s}$

$c_2 = \sqrt{k\cdot R_{air}\cdot T_2}$ $\quad c_2 = 1211\cdot\frac{ft}{s}$ $\quad V_2 = M_2\cdot c_2$ $\quad V_2 = 1096\cdot\frac{ft}{s}$

Also $\rho_1 = \frac{p_1}{R_{air}\cdot T_1}$ $\quad \rho_1 = 0.4960\cdot\frac{lbm}{ft^3}$

Hence $m_{rate} = \rho_1\cdot V_1\cdot A = \rho_2\cdot V_2\cdot A$ so $\rho_2 = \rho_1\cdot\frac{V_1}{V_2}$ $\quad \rho_2 = 0.269\cdot\frac{lbm}{ft^3}$

Then $p_2 = \rho_2\cdot R_{air}\cdot T_2$ $\quad p_2 = 60.8\cdot psi$ Finally $\Delta s = c_p\cdot\ln\left(\frac{T_2}{T_1}\right) - R_{air}\cdot\ln\left(\frac{p_2}{p_1}\right)$ $\quad \Delta s = 0.0231\cdot\frac{Btu}{lbm\cdot R}$

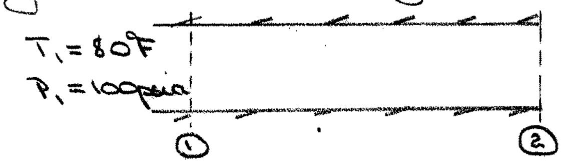
(Note: Using Fanno line relations, at $M_1 = 0.465$ $\quad \frac{T_1}{T_{crit}} = 1.150$ $\quad T_{crit} = \frac{T_1}{1.150}$ $\quad T_{crit} = 329 K$

$\frac{p_1}{p_{crit}} = 2.306$ $\quad p_{crit} = \frac{p_1}{2.3060}$ $\quad p_{crit} = 54.2\cdot psi$

Then $\frac{T_2}{T_{crit}} = 1.031$ so $M_2 = 0.907$ $\quad \frac{p_2}{p_{crit}} = 1.119$ $\quad p_2 = 1.119\cdot p_{crit}$ $\quad p_2 = 60.7\cdot psi$ Check!)

Problem 13.55

Given: Steady flow of air through an insulated pipe



$\dot{m} = 600$ lbm/min
$D_1 = D_2 = D = 4$ in

Find: (a) p_{min} (b) V_{max}, in pipe

Solution:

Basic equations: $\dot{m} = \rho V A$ $\quad p = \rho R T$

Computing equations: $T_0/T = 1 + \frac{k-1}{2} M^2$, $T_0 = \text{constant}$

Assumptions: (1) steady flow (2) adiabatic flow (3) uniform flow at a section (4) ideal gas

$\rho_1 = \frac{p_1}{RT_1} = 100 \frac{\text{lbf}}{\text{in}^2} \times \frac{\text{lbm} \cdot R}{53.3 \text{ ft} \cdot \text{lbf}} \times \frac{1}{540 R} \times \frac{144 \text{ in}^2}{\text{ft}^2} = 0.500 \text{ lbm/ft}^3$

$\dot{m} = \rho V_1 A \quad ; \quad A = \pi D^2/4$

$V_1 = \frac{4\dot{m}}{\pi D^2 \rho_1} = \frac{4}{\pi} \times 600 \frac{\text{lbm}}{\text{min}} \times \left(\frac{12}{4}\right)^2 \frac{1}{\text{ft}^2} \times \frac{\text{ft}^3}{0.500 \text{ lbm}} \times \frac{\text{min}}{60 \text{ s}} = 229 \text{ ft/s}$

$M_1 = \frac{V_1}{c_1} \quad c_1 = (kRT_1)^{1/2} = \left(1.4 \times 53.3 \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot °R} \times 540°R \times \frac{32.2 \text{ lbm}}{\text{slug}} \times \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}\right)^{1/2} = 1140 \text{ ft/s}$

$M_1 = \frac{229}{1140} = 0.201$

Since $M_1 < 1.0$, the minimum pressure and max velocity occur for $M_2 = 1.0$

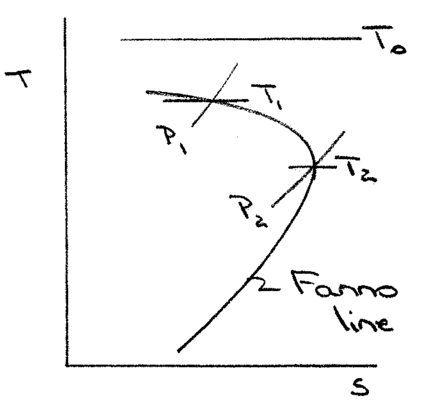
$T_0/T = 1 + \frac{k-1}{2} M^2 \quad ; \quad \frac{T_2}{T_1} = \frac{1 + \frac{k-1}{2} M_1^2}{1 + \frac{k-1}{2} M_2^2} = \frac{1 + 0.2(0.201)^2}{1 + 0.2(1.0)^2} = 0.840$

$T_2 = 0.840(T_1) = 0.840(540°R) = 454°R$

$V_2 = M_2 c_2 = M_2 (kRT_2)^{1/2} = 1.0\left(1.4 \times 53.3 \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot °R} \times 454°R \times 32.2 \frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}\right)^{1/2} = 1040 \text{ ft/s}$ ← V_{max}

$\dot{m} = \rho_1 V_1 A = \rho_2 V_2 A \quad \therefore \quad \rho_2 = \frac{V_1}{V_2} \rho_1 = \frac{229}{1040} \times 0.500 \frac{\text{lbm}}{\text{ft}^3} = 0.110 \text{ lbm/ft}^3$

$p_2 = \rho_2 R T_2 = 0.110 \frac{\text{lbm}}{\text{ft}^3} \times 53.3 \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot °R} \times 454°R \times \frac{\text{ft}^2}{144 \text{ in}^2} = 18.5 \text{ psia}$ ← p_{min}



*Fanno-Line Flow Functions
From Appendix E with $M_1 = 0.20$,

$T_1/T^* = 1.191$ (12.18a) $\quad \therefore T_2 = T^* = 453°R$

$p_1/p^* = 5.456$ (12.18d) $\quad \therefore p_2 = p^* = 18.3$ psia

$V_1/V^* = 0.2182$ (12.18e) $\quad \therefore V_2 = V^* = 1050$ ft/s

Given: Compressible flow through long tube (7.16 mm i.d). Air from atmosphere drawn through by vacuum pump downstream. As back pressure is lowered, pressure distribution along tube changes until p_b = 626 mm Hg (vacuum)

Find: (a) $\dot{m}_{max}$ (b) p_{0e} (c) $s_e - s_i$

Solution:

Basic equations: $\dot{m} = \rho V A$ $\quad p = \rho R T \quad T ds = dh - v\,dp$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \qquad \frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{k/k-1}$

Assumptions: (1) steady flow (2) ideal gas
(3) adiabatic flow through tube
(4) uniform flow at a section

Since pressure distribution does not change when p_b reaches 626 mm Hg (vacuum), flow is choked, $M_e = 1.0$ and $p_e = p_b$

$$p_e = p_{atm} - \rho g \Delta h = 101\times10^3 \frac{N}{m^2} - 13.55\times999 \frac{kg}{m^3}\times 9.81\frac{m}{s^2}\times 0.626\,m \times \frac{N\cdot s^2}{kg\cdot m} = 17.87\times10^3 \frac{N}{m^2}$$

$$T_e = \frac{T_0}{1+\frac{k-1}{2}M_e^2} = \frac{(273+20)K}{1+0.2} = 244\,K$$

$$\rho_e = \frac{p_e}{RT_e} = 17.87\times10^3 \frac{N}{m^2}\times\frac{kg\cdot K}{287\,N\cdot m}\times\frac{1}{244\,K} = 0.255\ kg/m^3$$

$$V_e = (kRT_e)^{1/2} = \left(1.4\times287\frac{N\cdot m}{kg\cdot K}\times 244\,K\times\frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 313\ m/s$$

$$\dot{m} = \rho_e V_e A = 0.255\frac{kg}{m^3}\times 313\frac{m}{s}\times\frac{\pi}{4}(0.00716\,m)^2 = 0.00321\ kg/s \longleftarrow \dot{m}$$

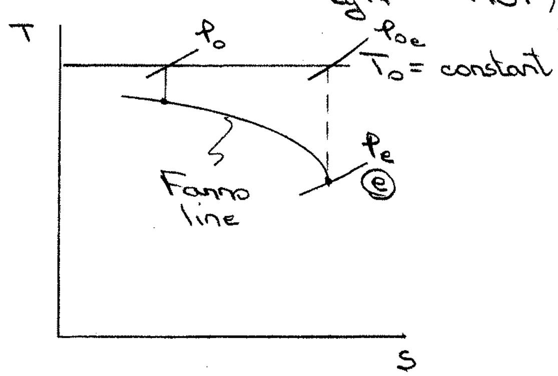
$$p_{0e} = p_e\left(1+\frac{k-1}{2}M_e^2\right)^{k/k-1} = 17.87\,kPa\,(1+0.2)^{3.5} = 33.8\ kPa\ (abs) \longleftarrow p_{0e}$$

For an ideal gas, the Tds equation can be written as

$$T ds = dh - v\,dp = c_p dT - RT\frac{dp}{p} \quad \therefore \quad ds = c_p\frac{dT}{T} - R\frac{dp}{p}$$

Then, $s_e - s_1 = s_{0e} - s_{01} = c_p \ln \frac{T_{02}}{T_{01}} - R \ln\frac{p_{02}}{p_{01}}$ (with the $c_p \ln \frac{T_{02}}{T_{01}}$ term crossed out)

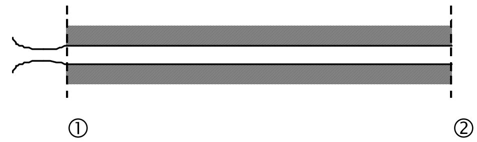
$$s_e - s_1 = -287\frac{J}{kg\cdot K}\ln\left(\frac{33.8}{101}\right) = 314\ J/kg\cdot K \longleftarrow s_e - s_1$$



Problem 13.57

[3]

13.57 A converging-diverging nozzle discharges air into an insulated pipe with area $A = 1\ \text{in}^2$. At the pipe inlet, $p = 18.5$ psia, $T = 100°$F, and $M = 2.0$. For shockless flow to a Mach number of unity at the pipe exit, calculate the exit temperature, the net force of the fluid on the pipe, and the entropy change.



Given: Air flow in a CD nozzle and insulated duct

Find: Temperature at end of duct; Force on duct; Entropy increase

Solution:

Basic equations: $F_s = p_1 \cdot A - p_2 \cdot A + R_x = m_{rate} \cdot (V_2 - V_1)$ $\quad \dfrac{T_0}{T} = 1 + \dfrac{k-1}{2} \cdot M^2$ $\quad \Delta s = c_p \cdot \ln\left(\dfrac{T_2}{T_1}\right) - R_{air} \cdot \ln\left(\dfrac{p_2}{p_1}\right)$

Given or available data $\quad T_1 = (100 + 460) \cdot R \quad p_1 = 18.5 \cdot psi \quad M_1 = 2 \quad M_2 = 1 \quad A = 1 \cdot in^2$

$k = 1.4 \quad c_p = 0.2399 \cdot \dfrac{Btu}{lbm \cdot R} \quad R_{air} = 53.33 \cdot \dfrac{ft \cdot lbf}{lbm \cdot R}$

Assuming isentropic flow in the nozzle

$$\frac{T_0}{T_1} \cdot \frac{T_2}{T_0} = \frac{1 + \frac{k-1}{2} \cdot M_1^2}{1 + \frac{k-1}{2} \cdot M_2^2} \quad \text{so} \quad T_2 = T_1 \cdot \frac{1 + \frac{k-1}{2} \cdot M_1^2}{1 + \frac{k-1}{2} \cdot M_2^2} \quad T_2 = 840 \cdot R \quad T_2 = 380 \cdot °F$$

Also $\quad c_1 = \sqrt{k \cdot R_{air} \cdot T_1} \quad V_1 = M_1 \cdot c_1 \quad V_1 = 2320 \cdot \dfrac{ft}{s} \quad c_2 = \sqrt{k \cdot R_{air} \cdot T_2} \quad V_2 = M_2 \cdot c_2 \quad V_2 = 1421 \cdot \dfrac{ft}{s}$

$\rho_1 = \dfrac{p_1}{R_{air} \cdot T_1} \quad \rho_1 = 0.0892 \cdot \dfrac{lbm}{ft^3} \quad m_{rate} = \rho_1 \cdot V_1 \cdot A = \rho_2 \cdot V_2 \cdot A_2 \quad \text{so} \quad \rho_2 = \rho_1 \cdot \dfrac{V_1}{V_2} \quad \rho_2 = 0.146 \cdot \dfrac{lbm}{ft^3}$

$m_{rate} = \rho_1 \cdot V_1 \cdot A \quad m_{rate} = 1.44 \cdot \dfrac{lbm}{s} \quad p_2 = \rho_2 \cdot R_{air} \cdot T_2 \quad p_2 = 45.3 \cdot psi$

Hence $\quad R_x = (p_2 - p_1) \cdot A + m_{rate} \cdot (V_2 - V_1) \quad R_x = -13.3 \cdot lbf \quad$ (Force is to the right)

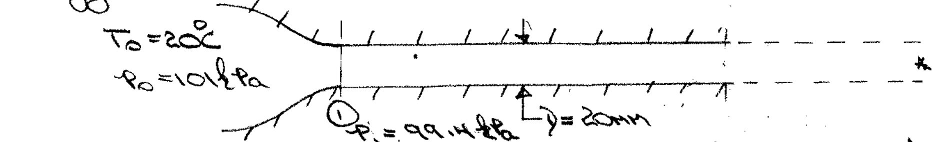
Finally $\quad \Delta s = c_p \cdot \ln\left(\dfrac{T_2}{T_1}\right) - R_{air} \cdot \ln\left(\dfrac{p_2}{p_1}\right) \quad \Delta s = 0.0359 \cdot \dfrac{Btu}{lbm \cdot R}$

(Note: Using Fanno line relations, at $\quad M_1 = 2 \quad \dfrac{T_1}{T_{crit}} = \dfrac{T_1}{T_2} = 0.6667 \quad T_2 = \dfrac{T_1}{0.667} \quad T_2 = 840 \cdot R$

$\dfrac{p_1}{p_{crit}} = \dfrac{p_1}{p_2} = 0.4083 \quad p_2 = \dfrac{p_1}{0.4083} \quad p_2 = 45.3 \cdot psi \quad$ Check!)

Problem 13.58

Given: Air, at 20°C and 101 kPa, is drawn through a converging nozzle into a long, 20 mm diameter, insulated tube. At the nozzle outlet (tube inlet) $p_1 = 99.4$ kPa



Find: (a) $\dot{m}$ (b) T^* and p^* for nozzle (isentropic flow)
(c) T^* and p^* for adiabatic tube flow

Solution:

Basic equations: $p = \rho R T$ $\quad \dot{m} = \rho V A$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \quad \frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{k/k-1}$

Assumptions: (1) steady flow (2) ideal gas
(3) uniform flow at a section
(4) isentropic flow in nozzle, adiabatic flow in tube.

Since flow in nozzle is isentropic, $T_{0_1} = T_0$ and $p_{0_1} = p_0$

$\therefore T_1 = \frac{T_{0_1}}{1+\frac{k-1}{2}M_1^2} = \frac{293\,K}{1+0.2(0.15)^2} = 291.7\,K \;;\; M_1 = \left\{\frac{2}{k-1}\left[\left(\frac{p_{0_1}}{p_1}\right)^{k-1/k} - 1\right]\right\}^{1/2} = 0.151$

$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 0.151\left[1.4 \times 287 \frac{N\cdot m}{kg\cdot K} \times 291.7\,K \times \frac{kg\cdot m}{N\cdot s^2}\right]^{1/2} = 51.7\,m/s$

$\rho_1 = \frac{p_1}{RT_1} = 99.4 \times 10^3 \frac{N}{m^2} \times \frac{kg\cdot K}{287\,N\cdot m} \times \frac{1}{291.7\,K} = 1.187\,kg/m^3$

$\dot{m} = \rho_1 V_1 A_1 = 1.187 \frac{kg}{m^3} \times 51.7 \frac{m}{s} \times \frac{\pi}{4}(0.020\,m)^2 = 0.0193\,kg/s$ ← $\dot{m}$

(b) For isentropic flow, $\frac{T_0}{T^*} = 1 + \frac{k-1}{2} = 1.20 \quad \therefore T^* = \frac{T_0}{1.20} = \frac{293\,K}{1.20} = 244\,K$ ← $T^*_{s=c}$

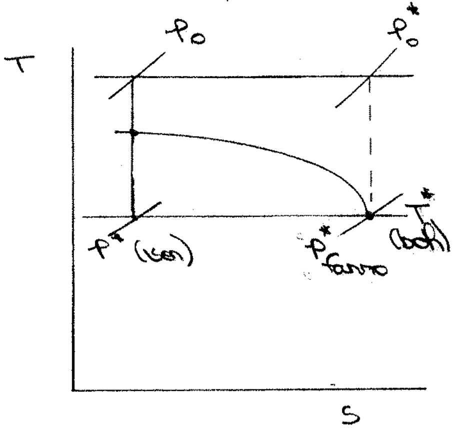
$\frac{p_0}{p^*} = \left(\frac{T_0}{T^*}\right)^{k/k-1} \quad \therefore p^* = \frac{p_0}{(T_0/T^*)^{k/k-1}} = \frac{101\,kPa}{(1.2)^{3.5}} = 53.4\,kPa\,(abs)$ ← $p^*_{s=c}$

(c) For Fanno line flow $T_0^* = T_{0_1} \quad \therefore T^* = T^*_{isen} = 244\,K$ ← T^*

At *, M=1, $\therefore V = c = (kRT^*)^{1/2} = \left[1.4 \times 287 \frac{N\cdot m}{kg\cdot K} \times 244\,K \times \frac{kg\cdot m}{N\cdot s^2}\right]^{1/2} = 313\,m/s$

From continuity $\rho_1 V_1 = \rho^* V^* \quad \therefore \rho^* = \frac{V_1}{V^*}\rho_1 = \frac{51.4}{313} \times 1.187 \frac{kg}{m^3} = 0.1949\,kg/m^3$

$p^* = \rho^* R T^* = 0.1949 \frac{kg}{m^3} \times 287 \frac{N\cdot m}{kg\cdot K} \times 244\,K = 13.6\,kPa\,(abs)$ ← p^*



* Fanno-Line Flow Functions (Appendix E.2)

For $M_1 = 0.151$, Eq 12.18d gives $p_1/p^* = 7.238$

$\therefore p^* = 13.7$ kPa ✓

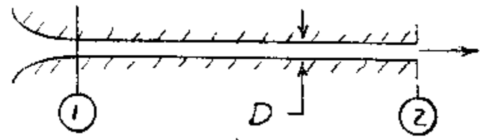
Problem 13.59

[4]

Given: Air flow through nozzle then duct, from tank at 15°C with variable pressure. The duct exhausts to atmosphere.

When $M_2 = 1$: $T_0 = 15°C$

$D = 0.249$ in.



$p_1 = 53.2$ psia $\quad p_2 = 14.7$ psia

$M_1 = 0.30 \quad M_2 = 1.0$

Find: (a) Tank pressure, p_{0_t}

(b) T_2, p_{02}, and $\dot{m}$

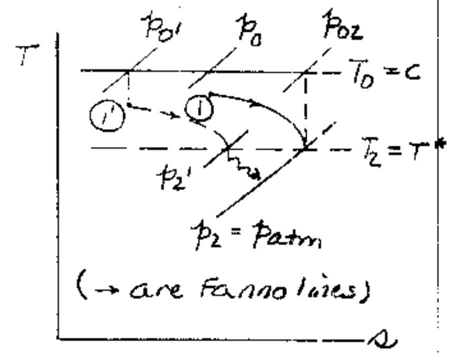
(c) Show on a Ts diagram the effect of raising to $p_0 = 100$ psia.

(d) Plot pressure distribution vs. distance.

Solution: Assume conditions in tank are stagnation properties; flow is isentropic in nozzle, Fanno flow in duct.

Basic equations: $T_0 = T(1 + \frac{k-1}{2}M^2) = \text{const}$ (energy)

$p_0 = p(1 + \frac{k-1}{2}M^2)^{\frac{k}{k-1}}$ (isentropic)



Assume steady, 1-D compressible flow, with constant stagnation temperature, $T_0 = 15°C$ (59°F)

$T_0 = (460 + 59)°R = 519°R$

$p_{0_t} = p_1(1 + \frac{k-1}{2}M_1^2)^{\frac{k}{k-1}} = 53.2 \text{ psia}(1 + 0.2(0.3)^2)^{3.5} = 56.6 \text{ psia}$ $\qquad p_{0_t}$

At exit plane, $M_2 = 1$

$T_2 = \dfrac{T_0}{(1 + \frac{k-1}{2}M_2^2)} = \dfrac{519°R}{1 + 0.2(1)^2} = 433°R$ $\qquad T_2$

$p_{02} = p_2(1 + \frac{k-1}{2}M_2^2)^{\frac{k}{k-1}} = 14.7 \text{ psia}(1 + 0.2(1)^2)^{3.5} = 27.8 \text{ psia}$ $\qquad p_{02}$

At exit plane, $V = V^* = c^* = \sqrt{kRT^*} = 1020$ ft/s

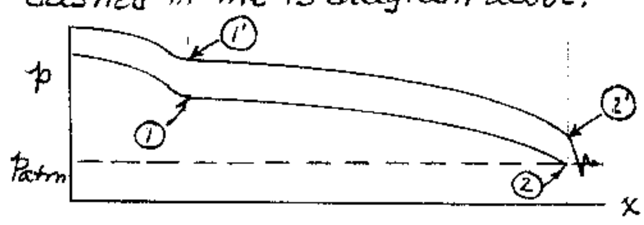
$\rho_2 = \dfrac{p_2}{RT_2} = 14.7 \dfrac{\text{lbf}}{\text{in.}^2} \times \dfrac{\text{lbm} \cdot °R}{53.3 \text{ ft} \cdot \text{lbf}} \times \dfrac{1}{433°R} \times 144 \dfrac{\text{in.}^2}{\text{ft}^2} = 0.0917 \text{ lbm/ft}^3$

$\dot{m} = \rho_2 V_2 A = 0.0917 \dfrac{\text{lbm}}{\text{ft}^3} \times 1020 \dfrac{\text{ft}}{\text{s}} \times \dfrac{\pi}{4}\left(\dfrac{0.249}{12}\right)^2 \text{ft}^2 = 0.0316 \text{ lbm/s}$ $\qquad \dot{m}$

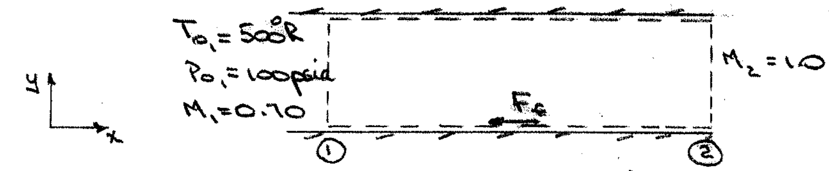
When p_t is increased, density and mass flow rate increase. Flow will shift to a new Fanno line as shown dashed in the Ts diagram above.

T_0 is fixed, so T^* remains fixed.

Since ρ and $\dot{m}$ increase, $p_2' > p_2 = p_{atm}$. The plot of p vs. x is as shown.



Given: Adiabatic flow of air in a constant-area duct with friction



$A = 1.0 \text{ ft}^2$

Find: Friction force exerted on the fluid by the pipe

Solution:

Basic equations: $F_{S_x} = P_1A - P_2A - F_f = \dot{m}(V_2 - V_1)$ $\quad \dot{m} = \rho V A$

Computing equations: $T_0/T = 1 + \frac{k-1}{2}M^2 \qquad P_0/P = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow $\quad$ (4) $F_{B_x} = 0$
(2) adiabatic flow, T_0 = const $\quad$ (5) ideal gas
(3) uniform flow at a section

$P_0/P = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} \qquad P_1 = \frac{P_{0_1}}{\left[1+\frac{k-1}{2}M^2\right]^{k/k-1}} = \frac{100 \text{ psia}}{[1+0.2(0.70)^2]^{3.5}} = 72.1 \text{ psia}$

$T_0/T = 1 + \frac{k-1}{2}M^2 \qquad T_1 = \frac{T_0}{1+\frac{k-1}{2}M_1^2} = \frac{500°R}{1+0.2(0.70)^2} = 455°R$

$\rho_1 = \frac{P_1}{RT_1} = 72.1 \frac{\text{lbf}}{\text{in}^2} \times \frac{\text{lbm}\,°R}{53.3 \text{ ft}\cdot\text{lbf}} \times \frac{1}{455 R} \times \frac{144 \text{ in}^2}{\text{ft}^2} = 0.428 \text{ lbm/ft}^3$

$V_1 = M_1 C_1 = M_1 (kRT_1)^{1/2} = 0.70\left(1.4 \times 53.3 \frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\,°R} \times 455 R \times 32.2 \frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right)^{1/2} = 732 \text{ ft/s}$

$\dot{m} = \rho_1 V_1 A = 0.428 \frac{\text{lbm}}{\text{ft}^3} \times 732 \frac{\text{ft}}{\text{s}} \times 1.0 \text{ ft}^2 = 313 \text{ lbm/s}$

$T_0/T = 1 + \frac{k-1}{2}M^2, \quad T_{0_2} = T_{0_1}, \quad T_2 = \frac{T_0}{1+\frac{k-1}{2}M_2^2} = \frac{500°R}{1+0.2(1.0)^2} = 417°R$

$V_2 = M_2 C_2 = M_2 (kRT_2)^{1/2} = 1.0\left(1.4 \times 53.3 \frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\,°R} \times 417 R \times 32.2 \frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right)^{1/2} = 1000 \text{ ft/s}$

$\dot{m} = \rho_1 V_1 A = \rho_2 V_2 A \qquad \rho_2 = \frac{V_1}{V_2}\rho_1 = \frac{732}{1000} \times 0.428 \text{ lbm/ft}^3 = 0.313 \text{ lbm/ft}^3$

$P_2 = \rho_2 R T_2 = 0.313 \frac{\text{lbm}}{\text{ft}^3} \times 53.3 \frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\,°R} \times 417 R \times \frac{\text{ft}^2}{144 \text{ in}^2} = 48.3 \text{ psia}$

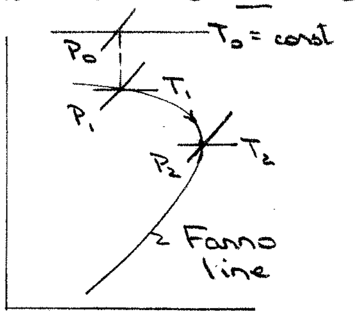
Solving the momentum equation for F_f

$F_f = (P_1 - P_2)A - \dot{m}(V_2 - V_1)$

$= (72.1 - 48.3) \frac{\text{lbf}}{\text{in}^2} \times 1 \text{ ft}^2 \times 144 \frac{\text{in}^2}{\text{ft}^2} - 313 \frac{\text{lbm}}{\text{s}} (1000 - 732) \frac{\text{ft}}{\text{s}} \times \frac{\text{slug}}{32.2 \text{ lbm}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}$

$F_f = 822 \text{ lbf}$ ← F_f

F_f is the force on the control volume from the surroundings. Consequently, F_f is the force on the fluid from the pipe; F_f opposes the motion



* Fanno-Line Flow Functions

From Appendix E.2 with $M_1 = 0.70$,

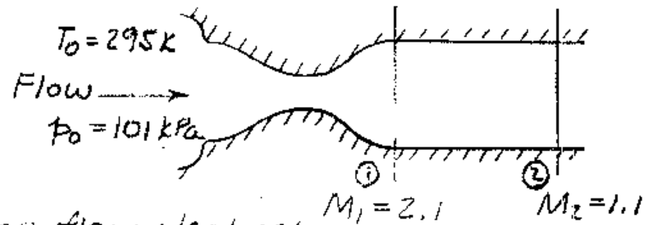
$P_1/P^* = 1.493$ (12.18d) $\quad \therefore P^* = P_2 = 48.3$ psia

$V_1/V^* = 0.732$ (12.18e) $\quad \therefore V^* = V_2 = 1000$ ft/s

Problem 13.61

Given: Supersonic wind tunnel with conditions shown, no shocks.

System is insulated.



Find: (a) T_2, (b) p_2, (c) $s_2 - s_1$, (d) Ts diagram

Solution: Consider steady, 1-D, comp. flow, ideal gas.

Computing equations: $T_0 = T\left(1+\frac{k-1}{2}M^2\right)$; $p_0 = p\left(1+\frac{k-1}{2}M^2\right)^{k/k-1}$ (s = const)

$$T ds = dh - v dp \qquad \rho VA = \text{constant}$$

Assumptions: (1) Isentropic flow in nozzle
(2) Insulated, so T_0 = constant

Then $T_2 = \dfrac{T_0}{1+\frac{k-1}{2}M_2^2} = \dfrac{295\ K}{1+0.2(1.1)^2} = 238\ K$ ⟵ T_2

From continuity, $\rho_1 V_1 = \rho_2 V_2$, since $A_1 = A_2$. But $V = Mc = M\sqrt{kRT}$ and $\rho = p/RT$, so

$$\frac{p_1}{RT_1} M_1\sqrt{kRT_1} = \frac{p_2}{RT_2} M_2 \sqrt{kRT_2} \ ; \ \frac{p_1}{\sqrt{T_1}} M_1 = \frac{p_2}{\sqrt{T_2}} M_2 \ ; \ p_2 = p_1 \frac{M_1}{M_2}\sqrt{\frac{T_2}{T_1}}$$

From isentropic relations,

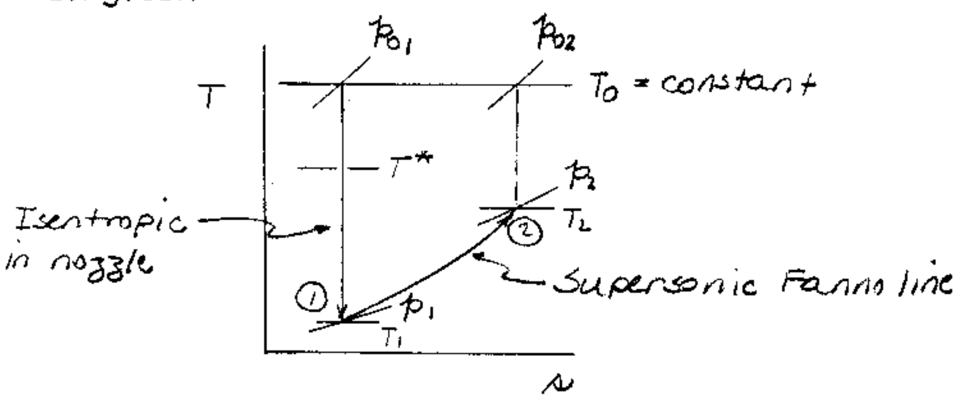
$$T_1 = \frac{T_0}{1+\frac{k-1}{2}M_1^2} = \frac{295\ K}{1+0.2(2.1)^2} = 157\ K \ ; \ p_1 = \frac{p_{01}}{\left(1+\frac{k-1}{2}M_1^2\right)^{k/k-1}} = \frac{101\ kPa}{(1.88)^{3.5}} = 11.1\ kPa$$

Thus $p_2 = \dfrac{2.1}{1.1}\sqrt{\dfrac{238\ K}{157\ K}}\ 11.1\ kPa = 26.1\ kPa\ (abs)$ ⟵ p_2

$$ds = \frac{dh}{T} - v\frac{dp}{T} = c_p \frac{dT}{T} - \frac{RT}{p}\frac{dp}{T} = c_p\frac{dT}{T} - R\frac{dp}{p}$$

$$\Delta s = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} = 1004\frac{J}{kg\cdot K}\ln\left(\frac{238}{157}\right) - 287\frac{J}{kg\cdot K}\ln\left(\frac{26.1}{11.1}\right) = 172\ J/kg\cdot K$$ ⟵ Δs

The Ts diagram is

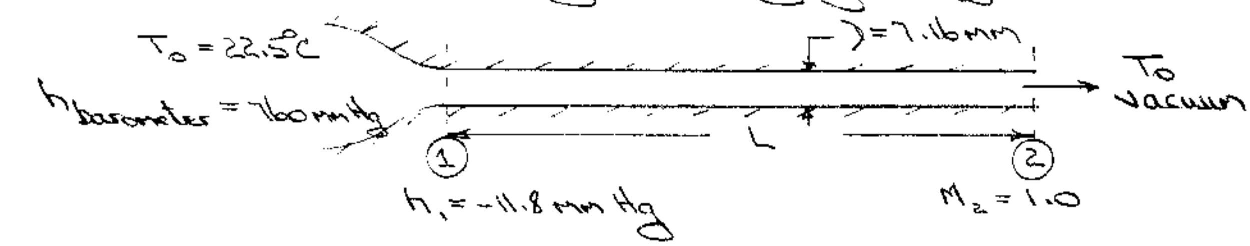


* Fanno-Line Flow Functions (Appendix E.2):

For $M_1 = 2.1$, Eq. 12.18d gives $p_1/p^* = 0.380$, so $p_2 = 26.1\ kPa\ (abs)$
$M_2 = 1.1$ $\qquad p_2/p^* = 0.894$

Problem 13.62

Given: Fanno line flow apparatus in laboratory, smooth brass tube fed by converging nozzle.



Find: (a) M_1 (b) L_2 (c) T_2, p_{02}

Solution: * Compressible flow functions to be used

Assumptions: (1) steady flow (5) $\Delta z = 0$
(2) isentropic flow in nozzle (6) ideal gas, air
(3) adiabatic " in duct (7) uniform flow at a section
(4) $\dot{W}_s = \dot{W}_{shear} = 0$

$$\frac{p_1}{p_0} = \frac{p_1}{p_{atm}} = \frac{\rho g \Delta h_1}{\rho g \Delta h_{atm}} = \frac{(760 - 11.8)\,mm}{760\,mm} = 0.9845$$

From App. E.1 (Eq. 11.17a) $M_1 = 0.150$ ← M_1

$p_1 = \rho_{Hg} g h_1 = (13.5) 1000 \frac{kg}{m^3} \times 9.81 \frac{m}{sec^2} \times 0.748\, m \times \frac{N \cdot sec^2}{kg \cdot m} = 99.1$ kPa (abs)

From App. E.1 (Eq. 11.17b), $T_1/T_0 = 0.9555$ $\therefore T_1 = 294$ K

From App. E.2, with $M_1 = 0.150$

$p_1/p^* = 7.287$, $T/T^* = 1.195$, $\frac{\bar{f} L_{max}}{D} = 27.93$

$p_2 = p^* = 13.60$ and $p_{02} = \frac{p_2}{0.5283} = 25.7$ kPa, $T^* = T_2 = 247$ K ← p_{02}, T_2

$f = f(Re)$, $Re = \frac{\rho V D}{\mu}$

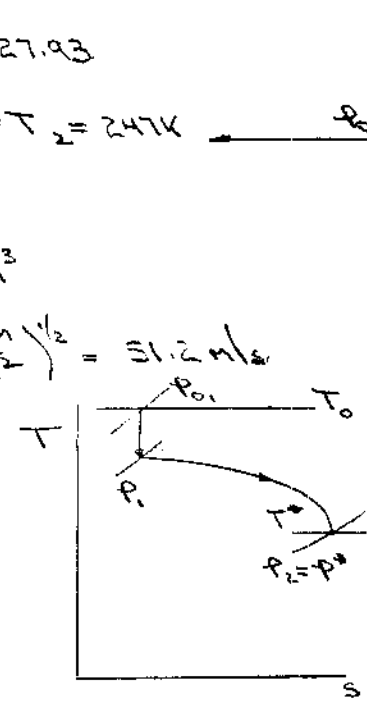
$$\rho_1 = \frac{p_1}{RT_1} = 99.1 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\, N \cdot m} \times \frac{1}{294\, K} = 1.17\, kg/m^3$$

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 0.149 \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 294\, K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 51.2\, m/s$$

From Table A.10 @ $T_1 = 21°C$, $\mu = 1.82 \times 10^{-5}\, N \cdot sec/m^2$

$$Re_1 = \frac{\rho_1 V_1 D}{\mu} = 1.17 \frac{kg}{m^3} \times 51.2 \frac{m}{s} \times 7.16 \times 10^{-3}\, m \times \frac{1}{1.82 \times 10^{-5}} \frac{m^2}{N \cdot s} \times \frac{N \cdot s^2}{kg \cdot m}$$

$Re_1 = 2.36 \times 10^4$



From Fig. 8.13, friction factor $f = 0.0245$

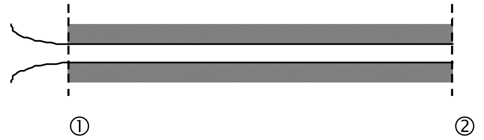
$$\therefore L_{12} = 27.9 \frac{D}{f} = 27.9 \times 7.16 \times 10^{-3}\, m \cdot \frac{1}{0.0245}$$

$L_{12} = 8.2$ m ← L_{12}

Problem 13.63

[2]

13.63 For the conditions of Problem 13.54, find the length, L, of commercial steel pipe of 2 in. diameter between sections ① and ②.



Given: Air flow in a converging nozzle and insulated duct

Find: Length of pipe

Solution:

Basic equations: Fanno-line flow equations, and friction factor

Given or available data

$T_0 = (250 + 460)\cdot R$ $\quad p_0 = 145\cdot psi$ $\quad p_1 = 125\cdot psi$ $\quad T_2 = (150 + 460)\cdot R$

$D = 2\cdot in$ $\quad k = 1.4$ $\quad c_p = 0.2399\cdot\frac{Btu}{lbm\cdot R}$ $\quad R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}$

From isentropic relations

$$M_1 = \left[\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}} - 1\right]\right]^{\frac{1}{2}} \qquad M_1 = 0.465$$

$$\frac{T_0}{T_1} = 1 + \frac{k-1}{2}\cdot M_1^2 \quad \text{so} \quad T_1 = \frac{T_0}{\left(1 + \frac{k-1}{2}\cdot M_1^2\right)} \qquad T_1 = 681\cdot R \qquad T_1 = 221\cdot°F$$

Then for Fanno-line flow

$$\frac{f_{ave}\cdot L_{max1}}{D_h} = \frac{1 - M_1^2}{k\cdot M_1^2} + \frac{k+1}{2\cdot k}\cdot\ln\left[\frac{(k+1)\cdot M_1^2}{2\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right)}\right] = 1.3923$$

$$\frac{p_1}{p_{crit}} = \frac{p_1}{p_2} = \frac{1}{M_1}\cdot\left(\frac{\frac{k+1}{2}}{1 + \frac{k-1}{2}\cdot M_1^2}\right)^{\frac{1}{2}} = 2.3044 \qquad \frac{T_1}{T_{crit}} = \frac{\frac{k+1}{2}}{1 + \frac{k-1}{2}\cdot M_1^2} = 1.150 \qquad T_{crit} = \frac{T_1}{1.150}$$

$$p_{crit} = \frac{p_1}{2.3044} \qquad p_{crit} = 54.2\cdot psi \qquad T_{crit} = 592\cdot R \qquad T_{crit} = 132\cdot°F$$

Also, for $\frac{T_2}{T_{crit}} = 1.031$

$$\frac{T_2}{T_{crit}} = \frac{\frac{k+1}{2}}{1 + \frac{k-1}{2}\cdot M_2^2} \quad \text{leads to} \quad M_2 = \sqrt{\frac{2}{k-1}\cdot\left(\frac{k+1}{2}\cdot\frac{T_{crit}}{T_2} - 1\right)} \qquad M_2 = 0.906$$

Then

$$\frac{f_{ave}\cdot L_{max2}}{D_h} = \frac{1 - M_2^2}{k\cdot M_2^2} + \frac{k+1}{2\cdot k}\cdot\ln\left[\frac{(k+1)\cdot M_2^2}{2\cdot\left(1 + \frac{k-1}{2}\cdot M_2^2\right)}\right] = 0.01271$$

Also

$$\rho_1 = \frac{p_1}{R_{air}\cdot T_1} \qquad \rho_1 = 0.496\frac{lbm}{ft^3} \qquad V_1 = M_1\cdot\sqrt{k\cdot R_{air}\cdot T_1} \qquad V_1 = 595\frac{ft}{s}$$

For air at $T_1 = 221\,°F$, from Table A.9 (approximately) $\mu = 4.48 \times 10^{-7} \cdot \frac{lbf \cdot s}{ft^2}$ so $Re_1 = \frac{\rho_1 \cdot V_1 \cdot D}{\mu}$

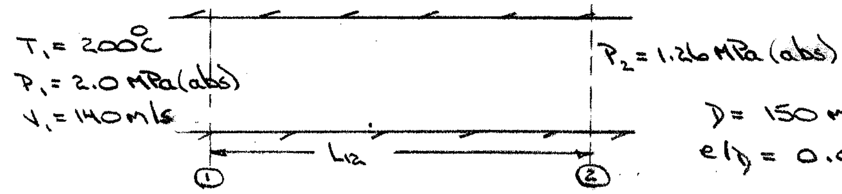
For commercial steel pipe (Table 8.1) $e = 0.00015 \cdot ft$ $\frac{e}{D} = 9 \times 10^{-4}$ and $Re_1 = 3.41 \times 10^{6}$

Hence at this Reynolds number and roughness (Eq. 8.37) $f = 0.01924$

Combining results $L_{12} = \frac{D}{f} \cdot \left(\frac{f_{ave} \cdot L_{max2}}{D_h} - \frac{f_{ave} \cdot L_{max1}}{D_h} \right) = \frac{\frac{2}{12} \cdot ft}{.01924} \cdot (1.3923 - 0.01271)$ $L_{12} = 12.0 \cdot ft$

These calculations are a LOT easier using the *Excel* Add-ins!

Given: Adiabatic flow of air in a constant-area duct



$D = 150$ mm

$e/D = 0.0003$

Find: T_2, L_{12}

Solution:

* Compressible flow functions to be used in the solution

Assumptions: (1) steady flow (2) adiabatic flow (3) uniform flow at a section (4) ideal gas

$M_1 = \frac{V_1}{c_1}$ $\quad c_1 = (kRT_1)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 473 K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 436$ m/s

$M_1 = \frac{140}{436} = 0.321$ From Appendix E.2, with $M_1 = 0.321$,

$T_1/T^* = 1.176 \quad p_1/p^* = 3.378 \quad \bar{f} L_{max}/D]_1 = 4.409$

$\therefore T^* = 402 K$, $p^* = 0.592$ MPa

For $p_2/p^* = \frac{1.26}{0.592} = 2.128$. From App. E.2, $M_2 = 0.502$, $T_2/T^* = 1.142$, $\bar{f}\frac{L_{max}}{D}]_2 = 1.053$

$\therefore T_2 = 1.142 T^* = 1.142(402 K) = 459 K$ ← T_2

$\bar{f}\frac{L_{12}}{D} = \bar{f}\frac{L_{max}}{D}]_1 - \bar{f}\frac{L_{max}}{D}]_2 = 4.409 - 1.053 = 3.356$

$\rho_1 = \frac{p_1}{RT_1} = 2.0 \times 10^6 \frac{N}{m^2} \times \frac{kg \cdot K}{287 N \cdot m} \times \frac{1}{473 K} = 14.7$ kg/m³

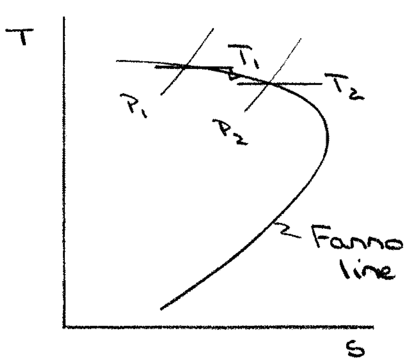
$Re = \frac{\rho_1 V_1 D}{\mu}$. To obtain μ at 200C, use Sutherland correlation (Appendix A)

$$\mu = \frac{bT^{1/2}}{1 + S/T} = 1.458 \times 10^{-6} \frac{kg}{m \cdot s \cdot K^{1/2}} \times (473 K)^{1/2} \times \frac{1}{\left(1 + \frac{110.4}{473}\right)} \times \frac{N \cdot s^2}{kg \cdot m} = 2.57 \times 10^{-5}\ N \cdot s/m^2$$

$$Re = \frac{\rho_1 V_1 D}{\mu} = 14.7 \frac{kg}{m^3} \times 140 \frac{m}{s} \times 1.5 \times 10^{-1} m \times \frac{m^2}{2.57 \times 10^{-5} N \cdot s} = 1.20 \times 10^7$$

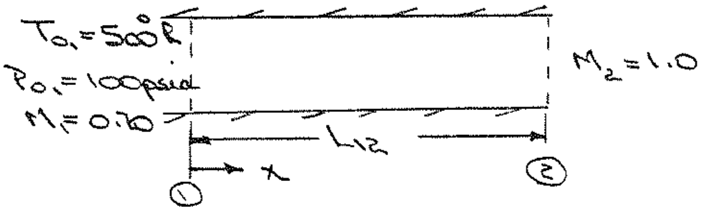
With $e/D = 0.0003$, from Fig. 8.13, friction factor, $f = 0.0147$

$\bar{f}\frac{L_{12}}{D} = 3.356$. $L_{12} = 3.356 \frac{D}{f} = \frac{3.356}{0.0147} \times 0.15 m = 34.2 m$ ← L_{12}



Problem 13.65

Given: Adiabatic flow of air in a constant area duct with friction



circular duct: $A = 1.0\ ft^2$ commercial steel

Find: L_{12}

Plot: $p(x)$, $M(x)$

Solution:

Compressible flow functions to be used in the solution

Assumptions: (1) steady flow (2) adiabatic flow, T_0 = const (3) uniform flow at a section (4) ideal gas

For $M_1 = 0.70$, from Appendix E.1, $T_0/T_1 = 1.098$ (11.17b) $\therefore T_1 = 455°R$

$p_0/p_1 = 1.387$ (11.17a) $\therefore p_1 = 72.1$ psia

For $M_1 = 0.70$, from Appendix E.2, $p_1/p^* = 1.493$ (12.18d) $\therefore p^* = 48.3$ psia

$\bar{f}L_{max}/D_h = 0.2081$ (12.17)

For $M_2 = 1.0$, $\bar{f}L_{max}/D_h = 0$ $\therefore \bar{f}L_{12}/D_h = 0.2081$

$\bar{f} = \bar{f}(Re, e/D)$

$A = \frac{\pi D^2}{4} \quad \therefore D = \left(\frac{4A}{\pi}\right)^{1/2} = \left[\frac{4}{\pi} \times 1.0\ ft^2\right]^{1/2} = 1.13\ ft$

For commercial steel from Table 8.1, $e = 0.00015$ ft $\therefore \frac{e}{D} = 0.00013$

$$\rho_1 = \frac{p_1}{RT_1} = 72.1\frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3\ ft \cdot lbf} \times \frac{1}{455°R} \times 144\frac{in^2}{ft^2} = 0.428\ lbm/ft^3$$

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 0.7\left(1.4 \times 53.3\frac{ft \cdot lbf}{lbm \cdot °R} \times 455°R \times 32.2\frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{1/2} = 732\ ft/s$$

$T_1 = 455°R = -5°F = -20°C = 253\ K$.

From the Sutherland correlation, $\mu = \frac{bT^{1/2}}{1 + S/T}$ (A.1)

$$\mu = 1.458 \times 10^{-6}\frac{kg}{m \cdot s \cdot K^{1/2}} \times \frac{(253\ K)^{1/2}}{1 + 110/253} = 1.62 \times 10^{-5}\frac{kg}{m \cdot s} \times \frac{N \cdot s^2}{kg \cdot m} = 1.62 \times 10^{-5}\frac{N \cdot s}{m^2}$$

$$\mu = 1.62 \times 10^{-5}\frac{N \cdot s}{m^2} \times 0.0209\frac{lbf \cdot s/ft^2}{N \cdot s/m^2} = 3.39 \times 10^{-7}\ lbf \cdot s/ft^2$$

$$Re = \frac{\rho_1 V_1 D}{\mu} = 0.428\frac{lbm}{ft^3} \times 732\frac{ft}{s} \times 1.13\ ft \times \frac{ft^2}{3.39 \times 10^{-7}\ lbf \cdot s} \times \frac{slug}{32.2\ lbm} \times \frac{lbf \cdot s^2}{ft \cdot slug}$$

$Re = 3.24 \times 10^7$.

From Fig 8.13, friction factor $f = 0.0125$

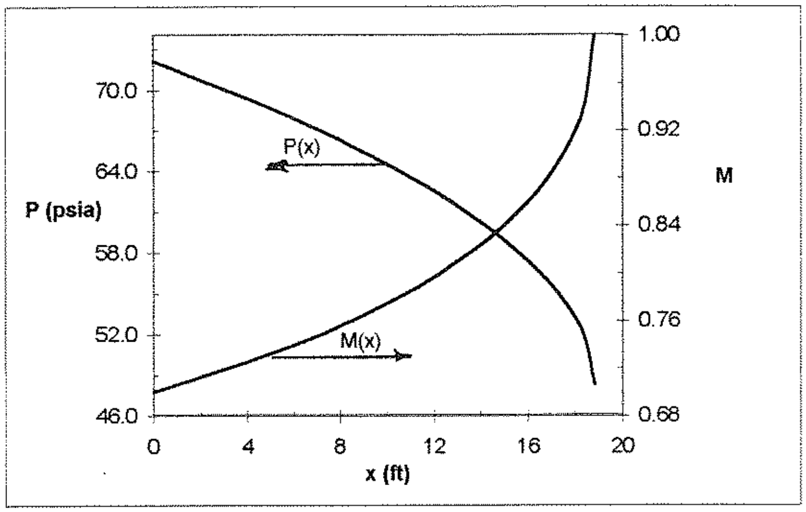
$$\bar{f}\frac{L_{12}}{D} = 0.2081 \quad \therefore L_{12} = \frac{0.2081\ D}{\bar{f}} = \frac{0.2081}{0.0125} \times 1.13\ ft = 18.8\ ft \longleftarrow L_{12}$$

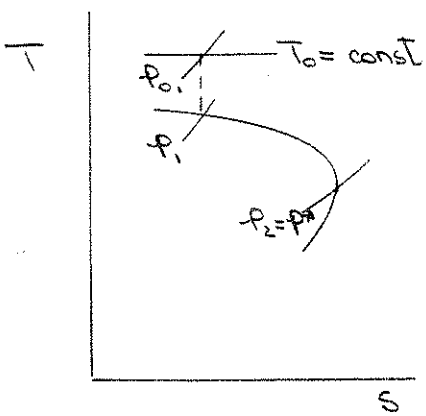
Problem 13.65

To plot $p(x)$, $M(x)$

- assume values of M, $0.70 \le M \le 1.00$
  - calculate corresponding $\bar{f}L/D$ from Eq. 12.17
    - solve for corresponding $\bar{f}\Delta L/D$ where $\Delta L = x$, assuming constant $\bar{f}$
  - calculate corresponding p/p^* from Eq. 12.18d

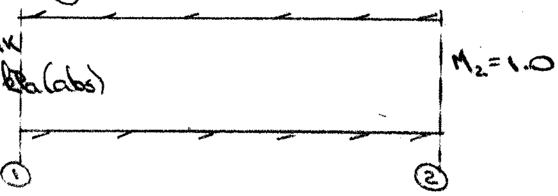
M	$(fL/D)_M$	$\Delta(fL/D)$	x(ft)	P/P*	P(psia)
0.70	0.2081	0	0	1.4935	72.1
0.74	0.1411	0.0670	6.1	1.4054	67.9
0.78	0.0917	0.1165	10.5	1.3261	64.0
0.82	0.0559	0.1522	13.8	1.2542	60.6
0.86	0.0310	0.1772	16.0	1.1889	57.4
0.90	0.0145	0.1936	17.5	1.1291	54.5
0.94	0.0048	0.2033	18.4	1.0743	51.9
1.0	0.0000	0.2081	18.8	1	48.3





Given: Air flow in a smooth, insulated, constant-area tube. Inlet conditions as given in Example Problem 13.8

$T_{0_1} = 296\text{K}$ $T_1 = 294\text{K}$
$P_{0_1} = 101 \text{ kPa(abs)}$ $P_1 = 98.8 \text{ kPa(abs)}$
$M_1 = 0.19$



s^* is the entropy at the condition where $M = 1.0$

Plot: the Fanno line $[T/T_0 \text{ vs } (s-s^*)/c_p]$ for $0.19 \le M \le 1.0$

Solution:

- Compressible flow functions to be used in the solution

Basic equations: $Tds = dh - v\,dP$, $P = \rho RT$

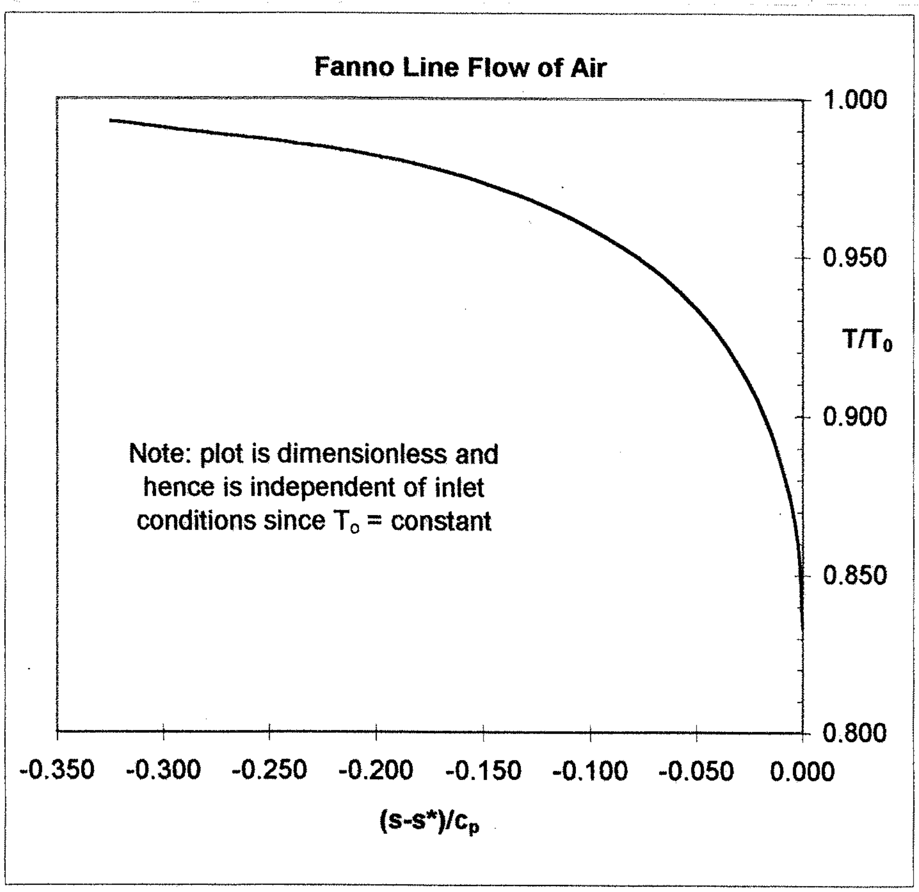
$$Tds = dh - v\,dP = c_p\,dT - \frac{1}{\rho}\,dP \qquad ds = c_p \frac{dT}{T} - \frac{dP}{P}$$

$$s - s^* = s_0 - s_0^* = c_p \int_{T_0^*}^{T_0} \frac{dT}{T} - R \int_{P_0^*}^{P_0} \frac{dP}{P} = \cancel{c_p \ln \frac{T_0}{T_0^*}} - R \ln \frac{P_0}{P_0^*} = -(c_p - c_v) \ln \frac{P_0}{P_0^*}$$

$$\frac{s - s^*}{c_p} = -\frac{(c_p - c_v)}{c_p} \ln \frac{P_0}{P_0^*} = -\frac{(k-1)}{k} \ln \frac{P_0}{P_0^*} = -\frac{0.4}{1.4} \ln \frac{P_0}{P_0^*} = -0.286 \ln \frac{P_0}{P_0^*}$$

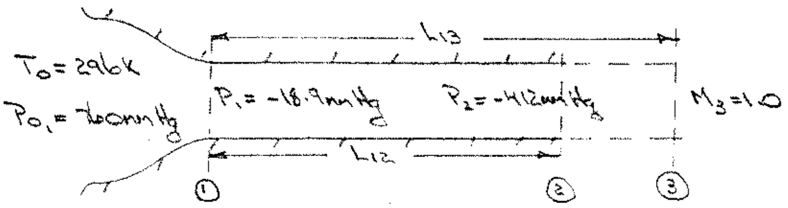
For a given value of M, T/T_0 is found from Appendix E.1 (11.17b)
P_0/P_0^* " " " " E.2 (12.18e)

M	T/T_0	P_0/P_0^*	$(s-s^*)/c_p$
0.19	0.993	3.112	-0.325
0.30	0.982	2.035	-0.203
0.40	0.969	1.590	-0.133
0.50	0.952	1.340	-0.084
0.60	0.933	1.188	-0.049
0.70	0.911	1.094	-0.026
0.80	0.887	1.038	-0.011
0.90	0.861	1.009	-0.003
1.00	0.833	1.000	0.000



Problem 13.67

Given: Air flow in a smooth, insulated, constant-area tube, as given in Example 13.7



$D = 7.16mm$

Plot: $P(x), T(x), M(x)$ vs L_{13} from pipe inlet to choked condition

Solution:

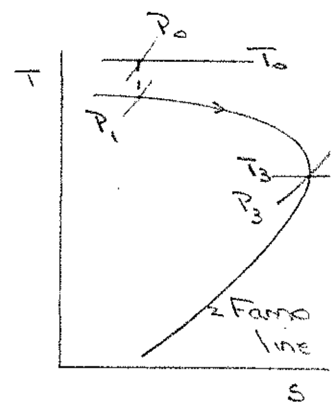
* Compressible flow functions to be used in the solution

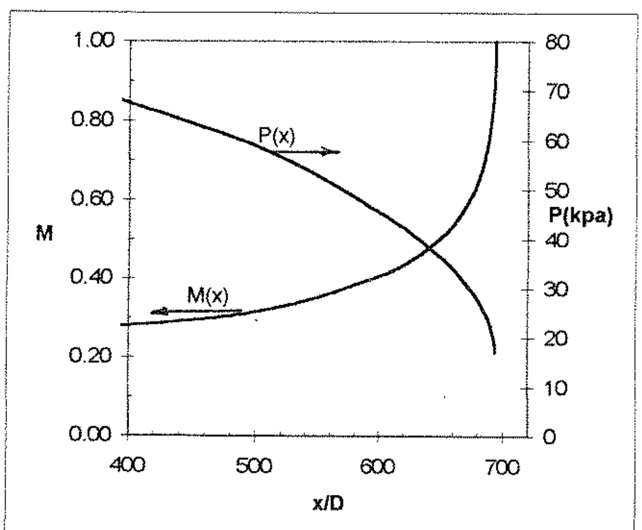
Assumptions: (1) steady flow (4) uniform flow at a section
(2) isentropic flow in nozzle (5) ideal gas
(3) $\dot{Q} = 0$ in duct

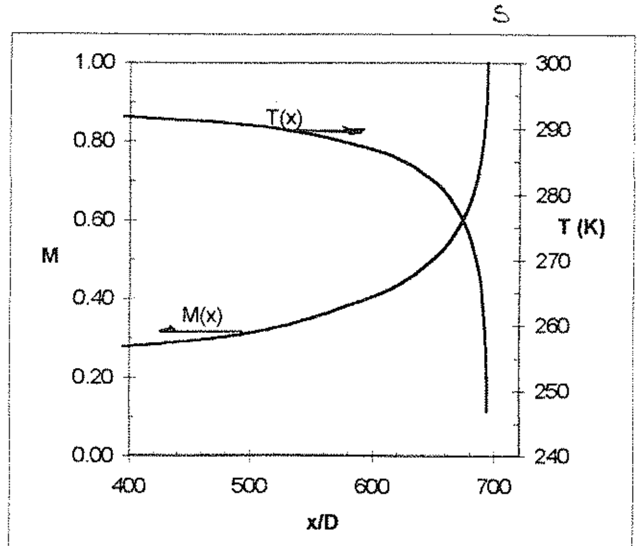
From the solution of Example 13.7 we know that
$M_1 = 0.190$, $T_1 = 294K$, $P_1 = 98.8kPa$, $P_{0_1} = 101kPa$, $\bar{f} = 0.0235$

To determine $P(x), T(x), M(x)$, use Appendix E.2

M	$(fL/D)_M$	$\Delta(fL/D)$	x/D	T/T*	T(K)	P/P*	P(kPa)
0.19	16.38	0	0	1.191	294	5.745	98
0.30	5.30	11.08	469	1.179	291	3.619	62
0.40	2.31	14.07	596	1.163	287	2.696	46
0.50	1.07	15.31	649	1.143	282	2.138	37
0.60	0.49	15.88	673	1.119	276	1.763	30
0.70	0.21	16.17	685	1.093	270	1.493	26
0.80	0.07	16.30	691	1.064	263	1.289	22
0.90	0.01	16.36	693	1.033	255	1.129	19
1.00	0.00	16.38	694	1.000	247	1.000	17







Problem 13.68

Given: Fanno line flow for air ; s^* is the entropy at the condition where M = 1.0.

Plot: Fanno line for Mach numbers in the range $0.1 < M < 3.0$, $\frac{T}{T^*}$ vs $\frac{s-s^*}{c_p}$

Solution:

* Compressible flow functions to be used in the solution

Basic equation: $T\,ds = dh - v\,dp$, $p = \rho R T$

$$T\,ds = dh - v\,dP = c_p\,dT - \frac{1}{\rho}\,dP \quad ; \quad ds = c_p \frac{dT}{T} - \frac{dP}{\rho}$$

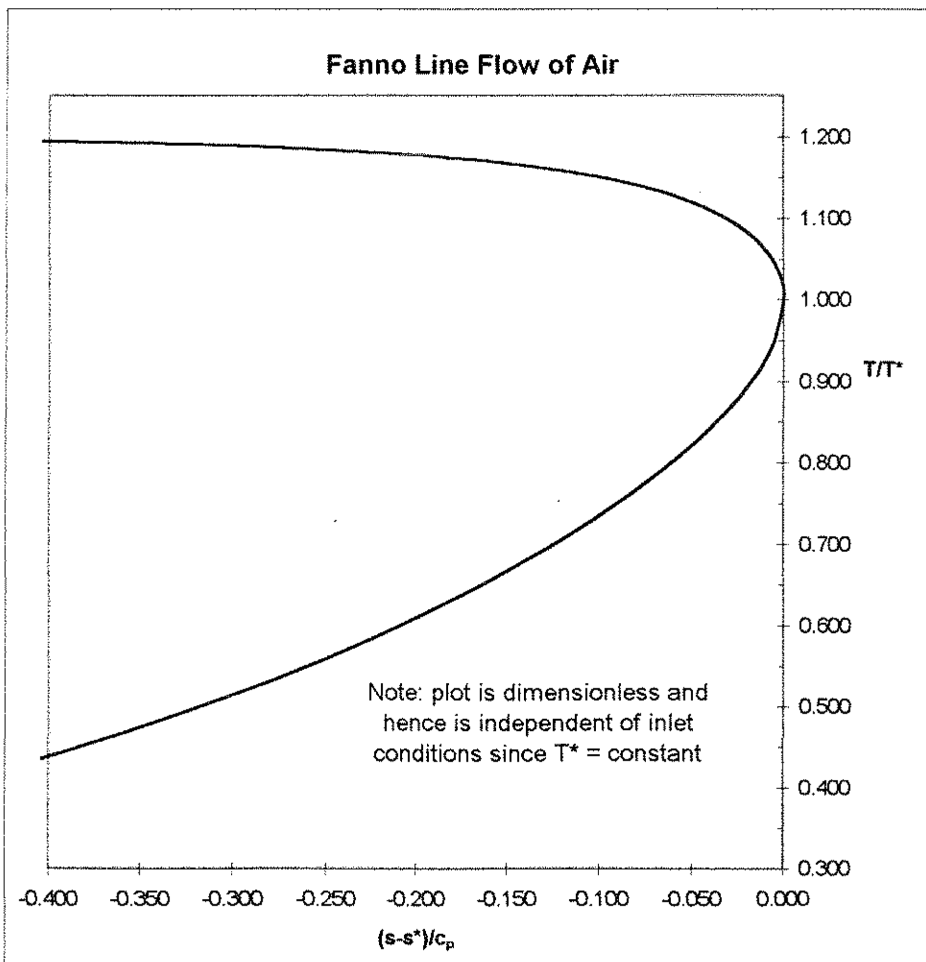
$$s - s^* = s_0 - s_0^* = c_p \int_{T_0^*}^{T_0} \frac{dT}{T} - R \int_{P_0^*}^{P_0} \frac{dP}{P} = c_p \ln \frac{T_0}{T_0^*} - R \ln \frac{P_0}{P_0^*} = -(c_p - c_v) \ln \frac{P_0}{P_0^*}$$

(the term $c_p \ln \frac{T_0}{T_0^*}$ is crossed out, $\to 0$)

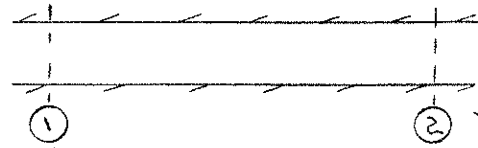
$$\frac{s-s^*}{c_p} = -\frac{(c_p - c_v)}{c_p} \ln \frac{P_0}{P_0^*} = -\frac{(k-1)}{k} \ln \frac{P_0}{P_0^*} = -\frac{(0.4)}{1.4} \ln \frac{P_0}{P_0^*} = -0.286 \ln \frac{P_0}{P_0^*}$$

For a given value of M, use Appendix E.2 to determine T/T^* (12.18a) & P_0/P_0^* (12.18e).

M	T/T^*	P_0/P_0^*	$(s-s^*)/c_p$
0.10	1.198	5.822	-0.504
0.14	1.195	4.182	-0.409
0.20	1.190	2.964	-0.311
0.30	1.179	2.035	-0.203
0.40	1.163	1.590	-0.133
0.50	1.143	1.340	-0.084
0.60	1.119	1.188	-0.049
0.70	1.093	1.094	-0.026
0.80	1.064	1.038	-0.011
0.90	1.033	1.009	-0.003
1.00	1.000	1.000	0.000
1.20	0.932	1.030	-0.009
1.40	0.862	1.115	-0.031
1.60	0.794	1.250	-0.064
1.80	0.728	1.439	-0.104
2.00	0.667	1.688	-0.150
2.20	0.610	2.005	-0.199
2.40	0.558	2.403	-0.251
2.60	0.510	2.896	-0.304
2.80	0.467	3.500	-0.358
3.00	0.429	4.235	-0.413

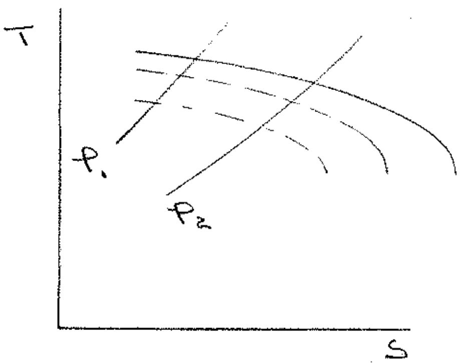


Given: Air flows through an insulated constant-area duct ($D = 2.12$ ft, $e/D = 0.002$, $L_{12} = 40$ ft) with conditions at sections ① & ② as shown.



Find: (a) Is it possible to solve for M_1 and M_2? Prove answer graphically
(b) Find $\dot{m}$ and T_2

Solution:



(a) It is possible to solve for M_1 and M_2. There is a different fanno line for each different flow rate (or M_1)

- need to find the value of M_1 that gives the pressure drop $p_1 - p_2$ over the length L_{12}

(b) Procedure for trial and error solution is to assume M_1 and calculate p_2

Use Fanno-line flow functions of Appendix E-2

Assume M_1

- determine p^*, Appendix E (12.18d)
  $\bar{f}L/D$ " " (12.17)
  $\bar{f} = \bar{f}\,(Re)$
- calculate $\bar{f}L_{12}/D$ and $\bar{f}L/D)_2 = \bar{f}L/D)_1 - \bar{f}L_{12}/D$
- knowing $\bar{f}L/D)_2$, iterate (12.17) to determine M_2
- determine p_2 from M_2 (12.18d) and check vs known p_2

Repeat with another assumed value of M_1

Additional computing equations:

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = M_1 \left(1.4 \times 53.3\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot{}^\circ\text{R}} \times 560^\circ\text{R} \times 32.2\,\frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right)^{1/2} = 1160\,M_1$$

$$\rho_1 = \frac{p_1}{RT_1} = 17\,\frac{\text{lbf}}{\text{in}^2} \times 144\,\frac{\text{in}^2}{\text{ft}^2} \times \frac{\text{lbm}\cdot{}^\circ\text{R}}{53.3\,\text{ft}\cdot\text{lbf}} \times \frac{1}{560^\circ\text{R}} \times \frac{\text{slug}}{32.2\,\text{lbm}} = 2.55\times10^{-3}\,\frac{\text{slug}}{\text{ft}^3}$$

From Table A.9, $\mu = 3.97\times10^{-7}$ lbf·s/ft²

$$Re = \frac{\rho_1 V_1 D}{\mu} = 2.55\times10^{-3}\,\frac{\text{slug}}{\text{ft}^3} \times 1160\,M_1\,\frac{\text{ft}}{\text{s}} \times 2.12\,\text{ft} \times \frac{\text{ft}^2}{3.97\times10^{-7}\,\text{lbf}\cdot\text{s}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{ft}\cdot\text{slug}} = 7.45\times10^{6}\,M_1$$

For $M_1 = 0.4$, $Re = 3.0\times10^6$. With $e/D = 0.002$, Fig. 8.13 gives $f = 0.023$

Assume f is constant

Problem 13.69

[4] Part 2/2

M_1	T_1/T^*	P_1/P^*	P^*(psia)		$(fL_m/D)_1$	V_1 (ft/s)	Re(10^6)	f	fL_{12}/D	$(fL_m/D)_2$	M_2	T_2/T^*	P_2/P^*	P_2(psia)
0.400	1.163	2.6958	6.30606	0.186	2.31	464	2.98	0.0230	0.434	1.875	0.427	1.158	2.520	15.9
0.450	1.153	2.3865	7.12347	0.234	1.57	522	3.35	0.0230	0.434	1.132	0.493	1.144	2.170	15.5
0.500	1.143	2.1381	7.95102	0.286	1.07	580	3.73	0.0230	0.434	0.635	0.568	1.127	1.869	14.9
0.510	1.141	2.0942	8.118	0.297	0.99	592	3.80	0.023	0.434	0.556	0.585	1.123	1.812	14.7

Iterate to determine M_2 for known M_1

M_1	$(fL_m/D)_2$	$(M_2)_{guess}$		$(fL_m/D)_2$
0.400	1.875	0.427	0.211	1.870
0.450	1.132	0.493	0.278	1.128
0.500	0.635	0.568	0.364	0.633
0.510	0.556	0.585	0.384	0.553

$M_1 = 0.510 \quad \frac{T_1}{T^*} = 1.141, \quad \frac{T_2}{T^*} = 1.123, \quad \therefore \frac{T_2}{T_1} = 0.984$

$T_2 = 551°R$ ← T_2

$\dot{m} = \rho V A = 2.55\times10^{-3} \frac{slug}{ft^3} \times 592 \frac{ft}{s} \times \frac{\pi}{4}(2/12)^2 ft^2 = 5.33\ slug/s$ ← $\dot{m}$

Problem 13.70

[2]

13.70 Air brought into a tube through a converging-diverging nozzle initially has stagnation temperature and pressure of 550 K and 1.35 MPa (abs.). Flow in the nozzle is isentropic; flow in the tube is adiabatic. At the junction between the nozzle and tube the pressure is 15 kPa. The tube is 1.5 m long and 2.5 cm in diameter. If the outlet Mach number is unity, find the average friction factor over the tube length. Calculate the change in pressure between the tube inlet and discharge.

Given: Air flow through a CD nozzle and tube.

Find: Average friction factor; Pressure drop in tube

Solution:

Assumptions: 1) Isentropic flow in nozzle 2) Adiabatic flow in tube 3) Ideal gas 4) Uniform flow

Given or available data: $k = 1.40$ $\quad R = 286.9\cdot\frac{J}{kg\cdot K}$ $\quad p_1 = 15\cdot kPa$ where State 1 is the nozzle exit

$p_0 = 1.35\cdot MPa$ $\quad T_0 = 550\cdot K$ $\quad D = 2.5\cdot cm$ $\quad L = 1.5\cdot m$

From isentropic relations $\quad M_1 = \left[\frac{2}{k-1}\cdot\left[\left(\frac{p_0}{p_1}\right)^{\frac{k-1}{k}} - 1\right]\right]^{\frac{1}{2}}$ $\quad M_1 = 3.617$

Then for Fanno-line flow (for choking at the exit)

$$\frac{f_{ave}\cdot L_{max}}{D_h} = \frac{1-M_1^{\,2}}{k\cdot M_1^{\,2}} + \frac{k+1}{2\cdot k}\cdot\ln\left[\frac{(k+1)\cdot M_1^{\,2}}{2\cdot\left(1+\frac{k-1}{2}\cdot M_1^{\,2}\right)}\right] = 0.599$$

Hence $\quad f_{ave} = \frac{D}{L}\cdot\left[\frac{1-M_1^{\,2}}{k\cdot M_1^{\,2}} + \frac{k+1}{2\cdot k}\cdot\ln\left[\frac{(k+1)\cdot M_1^{\,2}}{2\cdot\left(1+\frac{k-1}{2}\cdot M_1^{\,2}\right)}\right]\right]$ $\quad f_{ave} = 0.0100$

$$\frac{p_1}{p_{crit}} = \frac{p_1}{p_2} = \frac{1}{M_1}\cdot\left(\frac{\frac{k+1}{2}}{1+\frac{k-1}{2}\cdot M_1^{\,2}}\right)^{\frac{1}{2}} = 0.159$$

$$p_2 = \frac{p_1}{\left[\frac{1}{M_1}\cdot\left(\frac{\frac{k+1}{2}}{1+\frac{k-1}{2}\cdot M_1^{\,2}}\right)^{\frac{1}{2}}\right]}$$ $\quad p_2 = 94.2\,kPa$

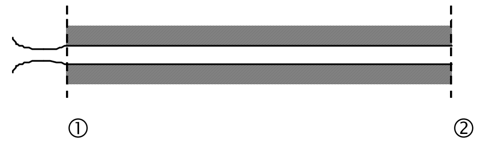
$\Delta p = p_1 - p_2$ $\quad \Delta p = -79.2\,kPa$

These calculations are a LOT easier using the Excel Add-ins!

Problem 13.71

[3]

13.71 For the conditions of Problem 13.57, determine the duct length. Assume the duct is circular and made from commercial steel. Plot the variations of pressure and Mach number versus distance along the duct.



Given:
Air flow in a CD nozzle and insulated duct

Find:
Duct length; Plot of M and p

Solution:

Basic equations: Fanno-line flow equations, and friction factor

Given or available data $T_1 = (100 + 460)\cdot R$ $\quad p_1 = 18.5\cdot psi$ $\quad M_1 = 2$ $\quad M_2 = 1$ $\quad A = 1\cdot in^2$

$k = 1.4$ $\quad c_p = 0.2399\cdot\frac{Btu}{lbm\cdot R}$ $\quad R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}$

Then for Fanno-line flow at $M_1 = 2$

$$\frac{p_1}{p_{crit}} = \frac{p_1}{p_2} = \frac{1}{M_1}\cdot\left(\frac{\frac{k+1}{2}}{1+\frac{k-1}{2}\cdot M_1^2}\right)^{\frac{1}{2}} = 0.4082 \qquad \frac{f_{ave}\cdot L_{max1}}{D_h} = \frac{1-M_1^2}{k\cdot M_1^2} + \frac{k+1}{2\cdot k}\cdot\ln\left[\frac{(k+1)\cdot M_1^2}{2\cdot\left(1+\frac{k-1}{2}\cdot M_1^2\right)}\right] = 0.3050$$

so $\quad p_{crit} = \frac{p_1}{0.4082}$ $\quad p_{crit} = 45.3\cdot psi$

and at $M_2 = 1$

$$\frac{f_{ave}\cdot L_{max2}}{D_h} = \frac{1-M_2^2}{k\cdot M_2^2} + \frac{k+1}{2\cdot k}\cdot\ln\left[\frac{(k+1)\cdot M_2^2}{2\cdot\left(1+\frac{k-1}{2}\cdot M_2^2\right)}\right] = 0$$

Also $\quad \rho_1 = \frac{p_1}{R_{air}\cdot T}$ $\quad \rho_1 = 0.089\cdot\frac{lbm}{ft^3}$ $\quad V_1 = M_1\cdot\sqrt{k\cdot R_{air}\cdot T_1}$ $\quad V_1 = 2320\cdot\frac{ft}{s}$ $\quad D = \sqrt{\frac{4\cdot A}{\pi}}$ $\quad D = 1.13\cdot in$

For air at $T_1 = 100\cdot°F$, from Table A.9 $\quad \mu = 3.96\times 10^{-7}\cdot\frac{lbf\cdot s}{ft^2}$ $\quad$ so $\quad Re_1 = \frac{\rho_1\cdot V_1\cdot D}{\mu}$

For commercial steel pipe (Table 8.1) $\quad e = 0.00015\cdot ft$ $\quad \frac{e}{D} = 1.595\times 10^{-3}$ $\quad$ and $\quad Re_1 = 1.53\times 10^6$

Hence at this Reynolds number and roughness (Eq. 8.37) $\quad f = .02222$

Combining results

$$L_{12} = \frac{D}{f}\cdot\left(\frac{f_{ave}\cdot L_{max2}}{D_h} - \frac{f_{ave}\cdot L_{max1}}{D_h}\right) = \frac{\frac{1.13}{12}\cdot ft}{.02222}\cdot(0.3050 - 0) \qquad L_{12} = 1.29\cdot ft \qquad L_{12} = 15.5\cdot in$$

These calculations are a LOT easier using the *Excel* Add-ins! The *M* and *p* plots are shown in the associated *Excel* workbook

Problem 13.71 (In Excel)

13.71 For the conditions of Problem 13.57, determine the duct length. Assume the duct is circular and made from commercial steel. Plot the variations of pressure and Mach number versus distance along the duct.

Given: Air flow in a CD nozzle and insulated duct

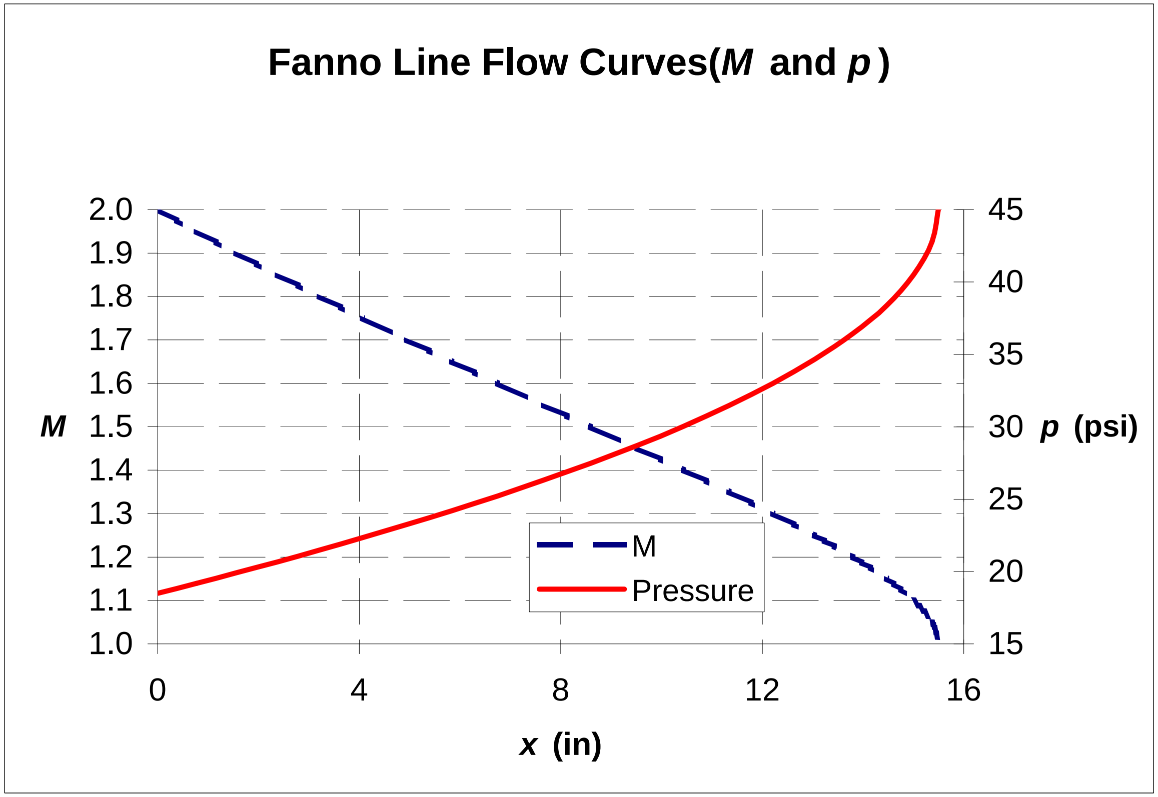
Find: Duct length; Plot of M and p

Solution:

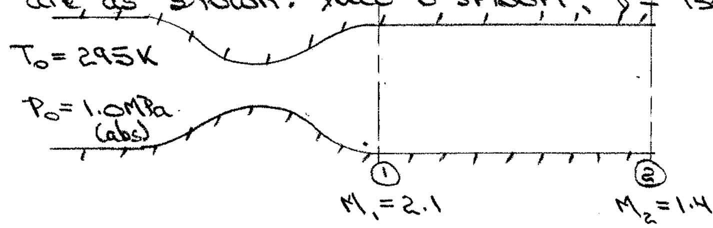
The given or available data is:

f = 0.0222
p^* = 45.3 kPa
D = 1.13 in

M	fL_{max}/D	$\Delta fL_{max}/D$	x (in)	p/p^*	p (psi)
2.00	0.305	0.000	0	0.408	18.49
1.95	0.290	0.015	0.8	0.423	19.18
1.90	0.274	0.031	1.6	0.439	19.90
1.85	0.258	0.047	2.4	0.456	20.67
1.80	0.242	0.063	3.2	0.474	21.48
1.75	0.225	0.080	4.1	0.493	22.33
1.70	0.208	0.097	4.9	0.513	23.24
1.65	0.190	0.115	5.8	0.534	24.20
1.60	0.172	0.133	6.7	0.557	25.22
1.55	0.154	0.151	7.7	0.581	26.31
1.50	0.136	0.169	8.6	0.606	27.47
1.45	0.118	0.187	9.5	0.634	28.71
1.40	0.100	0.205	10.4	0.663	30.04
1.35	0.082	0.223	11.3	0.695	31.47
1.30	0.065	0.240	12.2	0.728	33.00
1.25	0.049	0.256	13.0	0.765	34.65
1.20	0.034	0.271	13.8	0.804	36.44
1.15	0.021	0.284	14.5	0.847	38.37
1.10	0.010	0.295	15.0	0.894	40.48
1.05	0.003	0.302	15.4	0.944	42.78
1.00	0.000	0.305	15.5	1.000	45.30



Given: Flow of air from a large tank ($p_0 = 1.0$ MPa (abs), $T_0 = 295$ K) through a C-D nozzle to a constant area duct. Properties are as shown. Duct is smooth, $D = 150$ mm.



Find: (a) p_2 (b) L_{1-2} (c) $s_2 - s_1$

Solution: * Compressible flow functions to be used in solution.

Assumptions: (1) steady flow (2) uniform flow at a section
(3) isentropic flow in nozzle, adiabatic flow in duct
(4) ideal gas

(a)

From App. E.1 for $M_1 = 2.1$ $T/T_0 = 0.5314$ $\therefore T_1 = 156.7$ K

$p/p_0 = 0.1094$ $\therefore p_1 = 109.4$ kPa.

$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 2.1\left[1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 156.7 K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 527$ m/s

For App. E.2 for $M = 2.1$ $p_0/p_0^* = 1.837$ $\therefore p_0^* = 544.4$ kPa

$p/p^* = 0.3802$ $\therefore p^* = 287.7$ kPa

for $M_2 = 1.4$ $p_2/p^* = 0.6632$ $\therefore p_2 = 190.8$ kPa ← p_2

$p_{0_2}/p_0^* = 1.115$ $\therefore p_{0_2} = 320.8$ kPa

(b) From App. E.2 for $M_1 = 2.1$, $\bar{f} L_{max}/D_h = 0.3339$

$M_2 = 1.4$, $\bar{f} L_{max}/D_h = 0.09974$

$\therefore \bar{f} L_{12}/D_h = 0.3339 - 0.09974 = 0.2342$

To obtain L_{12}, we need f; $f = f(Re)$; $Re = \frac{\rho V D}{\mu}$ {use conditions at ①}

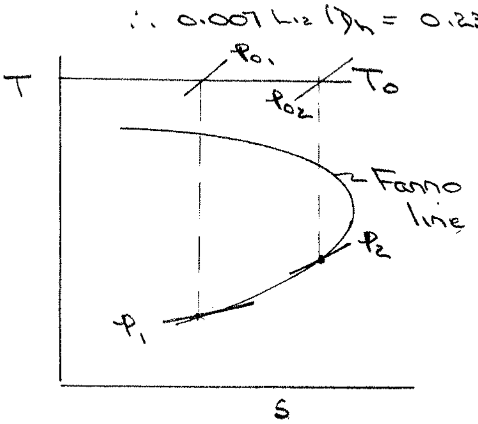
To obtain μ at 156.7K, use Sutherland correlation (Appendix A)

$$\mu = \frac{bT^{1/2}}{1 + S/T} = 1.458 \times 10^{-6} \frac{kg}{m \cdot s \cdot K^{1/2}} \times (156.7 K)^{1/2} \times \frac{1}{1 + \frac{110.4}{156.7}} = 1.071 \times 10^{-5} \frac{N \cdot s}{m^2}$$

$$Re = \frac{\rho_1 V_1 D}{\mu} = \frac{p_1 V_1 D}{RT_1 \mu} = 109.4 \times 10^3 \frac{N}{m^2} \times 527 \frac{m}{s} \times (0.15 m) \times \frac{kg \cdot K}{287 N \cdot m} \times \frac{1}{156.7 K} \times \frac{1}{1.071 \times 10^{-5}} \frac{m^2}{N \cdot s} \times \frac{N \cdot s^2}{kg \cdot m}$$

$Re = 1.80 \times 10^7$. For smooth pipe from Fig 8.13 friction factor, $f \approx 0.007$

$\therefore 0.007 L_{12}/D_h = 0.2342$ $L_{12} = \frac{0.2342}{0.007} D = 5.02$ m ← L_{12}



From the Tds eq.

$$s_2 - s_1 = s_{0_2} - s_{0_1} = c_p \ln \frac{T_{0_2}}{T_{0_1}} - R \ln \frac{p_{0_2}}{p_{0_1}}$$

(the $c_p \ln \frac{T_{0_2}}{T_{0_1}}$ term is struck out)

$$s_2 - s_1 = -287 \frac{N \cdot m}{kg \cdot K} \ln \frac{320.8}{1000} = 0.326 \frac{kJ}{kg \cdot K}$$ ← $s_2 - s_1$

Problem *13.73

*13.73 In long, constant-area pipelines, as used for natural gas, temperature is constant. Assume gas leaves a pumping station at 350 kPa and 20°C at $M = 0.10$. At the section along the pipe where the pressure has dropped to 150 kPa, calculate the Mach number of the flow. Is heat added to or removed from the gas over the length between the pressure taps? Justify your answer: Sketch the process on a Ts diagram. Indicate (qualitatively) T_{0_1}, T_{0_2}, and p_{0_2}.

Given: Isothermal air flow in a duct

Find: Downstream Mach number; Direction of heat transfer; Plot of Ts diagram

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\qquad \frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$ $\qquad m_{rate} = \rho \cdot V \cdot A$

Given or available data $\quad T_1 = (20 + 273) \cdot K \qquad p_1 = 350 \cdot kPa \qquad M_1 = 0.1 \qquad p_2 = 150 \cdot kPa$

From continuity $\quad m_{rate} = \rho_1 \cdot V_1 \cdot A = \rho_2 \cdot V_2 \cdot A$ $\quad$ so $\quad \rho_1 \cdot V_1 = \rho_2 \cdot V_2$

Also $\quad p = \rho \cdot R \cdot T$ $\quad$ and $\quad M = \frac{V}{c}$ $\quad$ or $\quad V = M \cdot c$

Hence continuity becomes $\quad \frac{p_1}{R \cdot T_1} \cdot M_1 \cdot c_1 = \frac{p_2}{R \cdot T_2} \cdot M_2 \cdot c_2$

Since $\quad T_1 = T_2 \qquad c_1 = c_2$ $\quad$ so $\quad p_1 \cdot M_1 = p_2 \cdot M_2$

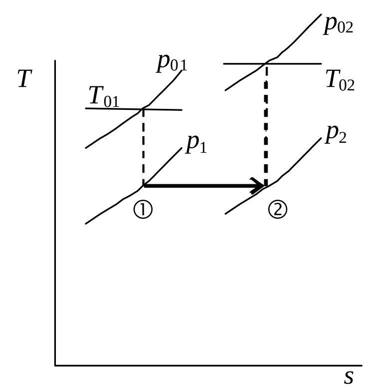
Hence $\quad M_2 = \frac{p_1}{p_2} \cdot M_1 \qquad M_2 = 0.233$

From energy $\quad \frac{\delta Q}{dm} = \left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \frac{V_1^2}{2}\right) = h_{02} - h_{01} = c_p \cdot (T_{02} - T_{01})$

But at each state $\quad \frac{T_0}{T} = 1 + \frac{k-1}{2} \cdot M^2$ $\quad$ or $\quad T_0 = T \cdot \left(1 + \frac{k-1}{2} \cdot M^2\right)$

Since T = const, but $M_2 > M_1$, then $T_{02} > T_{01}$, and

$\frac{\delta Q}{dm} > 0$ $\quad$ so energy is ADDED to the system



Problem *13.74

*13.74 Air enters a 15-cm diameter pipe at 15°C, 1.5 MPa, and 60 m/s. The average friction factor is 0.013. Flow is isothermal. Calculate the local Mach number and the distance from the entrance of the channel, at the point where the pressure reaches 500 kPa.

Given: Isothermal air flow in a pipe

Find: Mach number and location at which pressure is 500 kPa

Solution:

Basic equations: $m_{rate} = \rho \cdot V \cdot A$ $\quad p = \rho \cdot R \cdot T$ $\quad \frac{f \cdot L_{max}}{D} = \frac{1 - k \cdot M^2}{k \cdot M^2} + \ln\left(k \cdot M^2\right)$

Given or available data $T_1 = (15 + 273) \cdot K$ $\quad p_1 = 1.5 \cdot MPa$ $\quad V_1 = 60 \cdot \frac{m}{s}$ $\quad f = 0.013$ $\quad p_2 = 500 \cdot kPa$

$D = 15 \cdot cm$ $\quad k = 1.4$ $\quad R = 286.9 \cdot \frac{J}{kg \cdot K}$

From continuity $\rho_1 \cdot V_1 = \rho_2 \cdot V_2$ or $\frac{p_1}{T_1} \cdot V_1 = \frac{p_2}{T_2} \cdot V_2$

Since $T_1 = T_2$ and $V = M \cdot c = M \cdot \sqrt{k \cdot R \cdot T}$ $\quad M_2 = M_1 \cdot \frac{p_1}{p_2}$

$c_1 = \sqrt{k \cdot R \cdot T_1}$ $\quad c_1 = 340 \frac{m}{s}$ $\quad M_1 = \frac{V_1}{c_1}$ $\quad M_1 = 0.176$

Then $M_2 = M_1 \cdot \frac{p_1}{p_2}$ $\quad M_2 = 0.529$

At $M_1 = 0.176$ $\quad \frac{f \cdot L_{max1}}{D} = \frac{1 - k \cdot M_1^2}{k \cdot M_1^2} + \ln\left(k \cdot M_1^2\right) = 18.819$

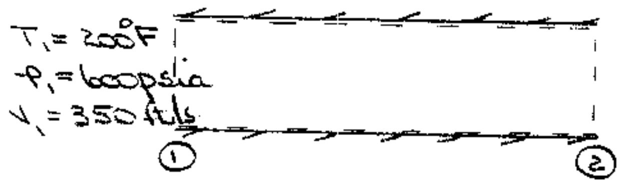
At $M_2 = 0.529$ $\quad \frac{f \cdot L_{max2}}{D} = \frac{1 - k \cdot M_2^2}{k \cdot M_2^2} + \ln\left(k \cdot M_2^2\right) = 0.614$

Hence $\frac{f \cdot L_{12}}{D} = \frac{f \cdot L_{max2}}{D} - \frac{f \cdot L_{max1}}{D} = 18.819 - 0.614 = 18.2$

$L_{12} = 18.2 \cdot \frac{D}{f}$ $\quad L_{12} = 210 m$

Problem *13.75

Given: Air enters a constant area channel at conditions shown and procedes to choking under isothermal flow conditions.



Find: limiting pressure, p_2
Compare with p_2 for frictional adiabatic flow.

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\qquad \dot{m} = \rho V A$

Computing equation: $T_0/T = 1 + \frac{k-1}{2}M^2$

Assumptions: (1) steady flow (2) ideal gas (3) uniform flow at a section (4) $\dot{W}_s = \dot{W}_{shear} = 0$

$$c_1 = (kRT_1)^{1/2} = \left(1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times 660°R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{1/2} = 1260 \text{ ft/s}$$

$$M_1 = \frac{V_1}{c_1} = \frac{350}{1260} = 0.278$$

$$V_2 = M_2 c_2 = M_2 c_1 = \frac{1}{\sqrt{1.4}} \times 1260 \text{ ft/s} = 1060 \text{ ft/s}$$

$$\rho_1 V_1 = \rho_2 V_2 \quad \text{or} \quad \frac{p_1}{RT_1} V_1 = \frac{p_2}{RT_2} V_2 \quad \text{------(1)}$$

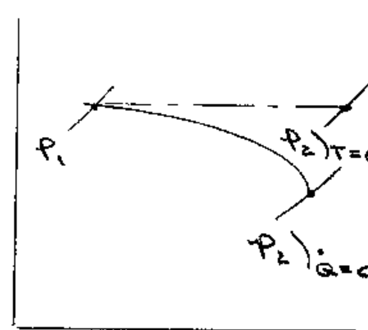
Since $T_1 = T_2$, $p_2 = p_1 \frac{V_1}{V_2} = 600 \text{ psia} \times \frac{350}{1060} = 198$ psia ← $p_2)_{T=C}$

For adiabatic flow, T_0 = constant and $M_2 = 1.0$

$$T_{0_2} = T_{0_1} = T_1\left(1 + \frac{k-1}{2}M_1^2\right) = T_2\left(1 + \frac{k-1}{2}M_2^2\right)$$

$$\frac{T_2}{T_1} = \frac{1 + 0.2(0.278)^2}{1 + 0.2} = 0.846$$

$$T_2 = 558°R$$



$$V_2 = c_2 = (kRT_2)^{1/2} = (1.4 \times 53.3 \times 558 \times 32.2)^{1/2} = 1160 \text{ ft/s}$$

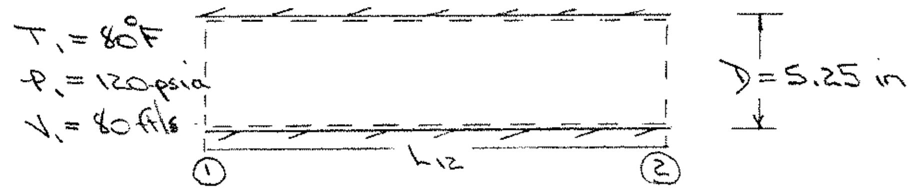
From continuity (Eq. 1)

$$p_2 = p_1 \frac{V_1}{V_2}\frac{T_2}{T_1} = 600 \text{ psia} \times \frac{350}{1160} \times 0.846$$

$p_2 = 153$ psia ← $p_2)_{\dot{Q}=0}$

Problem *13.76

Given: Air enters a clean steel pipe of length $L = 950$ ft and diameter $D = 5.25$ in. at the conditions shown.



Find: pressure drop, $p_1 - p_2$, assuming (a) incompressible (b) isothermal, and (c) adiabatic flow

Solution:

Basic equations: $\dot{m} = \rho V A \qquad p = \rho R T$

Computing equations: $p_1 - p_2 = \rho f \frac{L}{D} \frac{\bar{V}^2}{2}$ (ρ = constant).

$$f \frac{L_{max}}{D} = \frac{1 - kM^2}{kM^2} + \ln kM^2 \quad (T = \text{constant})$$

$$\frac{\bar{f} L_{max}}{D} = \frac{1 - M^2}{kM^2} + \frac{k+1}{2k} \ln\left[\frac{(k+1)M^2}{2\left(1 + \frac{k-1}{2}M^2\right)}\right] \quad (\dot{Q} = 0)$$

$$\rho_1 = \frac{p_1}{RT_1} = 120 \frac{\text{lbf}}{\text{in}^2} \times 144 \frac{\text{in}^2}{\text{ft}^2} \times \frac{\text{lbm} \cdot {}^\circ\text{R}}{53.3 \text{ ft} \cdot \text{lbf}} \times \frac{1}{540^\circ\text{R}} = 0.600 \text{ lbm/ft}^3$$

From Table A.9 {at T = 80°F} $\mu = 3.86 \times 10^{-7}$ lbf·s/ft²

$$Re_1 = \frac{\rho D V}{\mu} = 0.600 \frac{\text{lbm}}{\text{ft}^3} \times \frac{5.25}{12} \text{ft} \times 80 \frac{\text{ft}}{\text{s}} \times \frac{1}{3.86 \times 10^{-7}} \frac{\text{ft}^2}{\text{lbf} \cdot \text{s}} \times \frac{\text{slug}}{32.2 \text{ lbm}} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}}$$

$Re_1 = 1.69 \times 10^6$

For commercial steel (Table 8.1), $e = 0.00015$ ft. $\therefore e/D = 0.00034$

From Fig. 8.13, $f = 0.0155$

(a) For incompressible flow

$$p_1 - p_2 = \rho f \frac{L}{D} \frac{\bar{V}^2}{2} = 0.60 \frac{\text{lbm}}{\text{ft}^3} \times 0.0155 \times \frac{950 \times 12}{5.25} \times \frac{(80)^2}{2} \frac{\text{ft}^2}{\text{s}^2} \times \frac{\text{slug}}{32.2 \text{ lbm}} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \times \frac{\text{ft}^2}{144 \text{ in}^2}$$

$p_1 - p_2 = 13.9$ psia ← $(p_1 - p_2)_{\rho = c}$

(b) For isothermal flow

$$c_1 = (kRT_1)^{1/2} = \left(1.4 \times 53.3 \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot {}^\circ\text{R}} \times 540^\circ\text{R} \times 32.2 \frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}\right)^{1/2} = 1140 \text{ ft/s}$$

$$M_1 = \frac{V_1}{c_1} = \frac{80}{1140} = 0.0702$$

At state ①
$$\left(f \frac{L_{max}}{D}\right)_1 = \frac{1 - kM_1^2}{kM_1^2} + \ln kM_1^2$$

$$= \frac{1 - 1.4(0.0702)^2}{1.4(0.0702)^2} + \ln\left[1.4(0.0702)^2\right]$$

$$\left(f \frac{L_{max}}{D}\right)_1 = 139$$

$$f\frac{L_{12}}{D} = 0.0155 \times \frac{950 \times 12}{5.25} = 33.6$$

$$\therefore \left.f\frac{L_{max}}{D}\right)_2 = 139 - 33.6 = 105 = \frac{1-kM_2^2}{kM_2^2} + \ln kM_2^2$$

Trial and error solution for M_2

M_2	$\left.f\frac{L_{max}}{D}\right)_2$
0.10	66.2
0.08	106
0.081	103
0.0805	105

$V_2 = M_2 c_2 = M_2 c_1 = 0.0805 \times 1140$ ft/s $= 91.8$ ft/s

$\rho_1 V_1 = \rho_2 V_2$ or $\frac{p_1}{T_1} V_1 = \frac{p_2}{T_2} V_2$ ......(1)

Since $T_2 = T_1$, $p_2 = p_1 \frac{V_1}{V_2} = 120 \text{ psia} \times \frac{80}{91.8} = 105$ psia

$p_1 - p_2 = 15.0$ psia $\qquad (p_2 - p_1)_{T=c}$

(c) For adiabatic flow, $M_1 = 0.0702$

$$\left.f\frac{L_{max}}{D}\right)_1 = \frac{1-(0.0702)^2}{1.4(0.0702)^2} + \frac{1.4+1}{2(1.4)} \ln\left[\frac{(2.4)(0.0702)^2}{2(1+0.2(0.0702)^2)}\right] = 139.8$$

$$\therefore \left.f\frac{L_{max}}{D}\right)_2 = 139.8 - 33.6 = 106.2 = \frac{1-M_2^2}{1.4M_2^2} + \frac{2.4}{2.8}\ln\left[\frac{2.4M_2^2}{2(1+0.2M_2^2)}\right]$$

Trial and error solution for M_2

M_2	$\left.f\frac{L_{max}}{D}\right)_2$
0.085	94.1
0.080	106.7
0.0802	106.2 ✓

For adiabatic flow, T_0 = constant

$$\therefore \frac{T_2}{T_1} = \frac{1+\frac{k-1}{2}M_1^2}{1+\frac{k-1}{2}M_2^2} = \frac{1+0.2(0.0702)^2}{1+0.2(0.0802)^2} = 1.00$$

$\therefore V_2 = M_2 c_2 = M_2 c_1 = 0.0802 \times 1140$ ft/sec $= 91.4$ ft/s

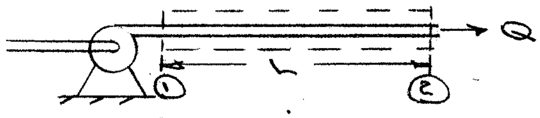
From continuity (Eq. 1)

$$p_2 = p_1 \frac{V_1}{V_2} = 120 \text{ psia} \times \frac{80}{91.4} = 105 \text{ psia}$$

$\therefore p_1 - p_2 = 15.0$ psia $\qquad (p_2 - p_1)_{Q=0}$

Note: p_2 is essentially the same for isothermal and adiabatic flow. The value is higher that for incompressible flow

Given: Natural gas (molecular mass $M_m = 18$, $k = 1.3$) is pumped through a constant area pipe ($D = 36$ in) for a distance $L = 40$ miles



$D = 36$ in
$T = 78°F = \text{const.}$

$p_1 \le 90$ psig $\quad p_2 \ge 10$ psig

Find: Volume flowrate, Q (ft^3 day @ 70°F and 1 atm)

<u>Solution:</u>

Basic equations: $\dot{m} = \rho V A \qquad p = \rho R T$

Computing equation: $$f\frac{L_{max}}{D} = \frac{1-kM^2}{kM^2} + \ln kM^2$$

Then, $$f\frac{L_{12}}{D} = \left(f\frac{L_{max}}{D}\right)_1 - \left(f\frac{L_{max}}{D}\right)_2 = \frac{1-kM_1^2}{kM_1^2} - \frac{1-kM_2^2}{kM_2^2} + \ln\frac{M_1^2}{M_2^2}$$

Since information on pressures is known, we relate p and M

From the ideal gas equation of state, for T = constant $\quad \frac{p_1}{p_2} = \frac{\rho_1}{\rho_2}$

From the continuity equation $\frac{\rho_1}{\rho_2} = \frac{V_2}{V_1}$ and for T = constant, $\frac{V_2}{V_1} = \frac{M_2}{M_1}$

Hence $\frac{p_1}{p_2} = \frac{M_2}{M_1}$ and $M_2 = \frac{p_1}{p_2}M_1$

Substituting for M_2 into the equation for $f\frac{L_{12}}{D_h}$ and rearranging we obtain
$$f\frac{L_{12}}{D_h} = \frac{1-(p_2/p_1)^2}{kM_1^2} - \ln\left(\frac{p_1}{p_2}\right)^2$$

Solving this equation for M_1, then $$M_1 = \left\{\frac{1}{k}\left[\frac{1-\left(\frac{p_2}{p_1}\right)^2}{f\frac{L_{12}}{D_h} + \ln\left(\frac{p_1}{p_2}\right)^2}\right]\right\}^{1/2}$$

$$\frac{L}{D_h} = 40\,mi \times 5280\,\frac{ft}{mi} \times \frac{1}{3\,ft} = 70{,}400$$

Assume pipe is commercial steel. From Table 8.1, $e = 0.00015$ ft and hence $e/D = 0.00005$. Assume $Re > 3.0 \times 10^7$, then $f = 0.0105$. Solving for M_1,

$$M_1 = \left\{\frac{1}{1.3}\left[\frac{1-\left(\frac{24.7}{104.7}\right)^2}{0.0105(70{,}400) + \ln\left(\frac{104.7}{24.7}\right)^2}\right]\right\}^{1/2} = 0.0313$$

$$R_{Nat\,gas} = \frac{R_u}{M} = 1544\,\frac{ft\cdot lbf}{lbmole\cdot°R}\,\frac{lbmole}{18\,lbm} = 85.8\,\frac{ft\cdot lbf}{lbm\cdot°R}$$

$$c_1 = (kRT_1)^{1/2} = \left(1.3 \times 85.8\,\frac{ft\cdot lbf}{lbm\cdot°R} \times 530°R \times \frac{32.2\,lbm}{slug} \times \frac{slug\cdot ft}{lbf\cdot s^2}\right)^{1/2} = 1380\,ft/s$$

$$V_1 = M_1 c_1 = 0.0313 \times 1380\,ft/s = 43.2\,ft/s$$

$$\rho_1 = \frac{p_1}{RT_1} = 104.7\,\frac{lbf}{in^2} \times \frac{lbm\cdot°R}{85.8\,ft\cdot lbf} \times \frac{1}{530°R} \times 144\,\frac{in^2}{ft^2} = 0.332\,lbm/ft^3$$

Check assumption on Re; from Fig A.2 (for methane), $\mu = 1.08 \times 10^{-5} \times 2.09 \times 10^{-2}\,lbf\cdot s/ft^2$

$$Re = \frac{\rho V D}{\mu} = 0.332\,\frac{lbm}{ft^3} \times 43.2\,\frac{ft}{s} \times 3\,ft \times \frac{1}{2.26 \times 10^{-7}}\,\frac{ft^2}{lbf\cdot s} \times \frac{slug}{32.2\,lbm} \times \frac{lbf\cdot s^2}{ft\cdot slug} = 5.91 \times 10^6$$

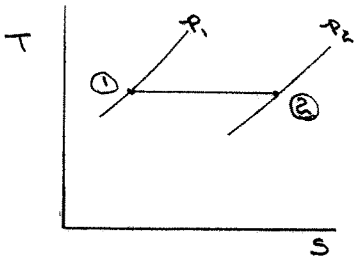
Then $f = 0.011$ and $\frac{f L_{12}}{D} = 774$ $M_1 = 0.0305$ and $V_1 = 42.1$ ft/sec

$$\dot{m} = \rho_1 V_1 A = 0.332 \frac{lbm}{ft^3} \times 42.1 \frac{ft}{sec} \times \frac{\pi}{4}(3)^2 ft^2 = 98.8 \text{ lbm/sec}$$

$$\rho_{atm} = \frac{P_{atm}}{RT_1} = 14.7 \frac{lbf}{in^2} \times \frac{lbm \cdot R}{85.8 \, ft \cdot lbf} \times \frac{1}{530R} \times \frac{144 \, in^2}{ft^2} = 4.65 \times 10^{-2} \text{ lbm/ft}^3$$

Volume flowrate at atmospheric pressure

$$Q = \frac{\dot{m}}{\rho_{atm}} = 98.8 \frac{lbm}{sec} \times \frac{ft^3}{4.65 \times 10^{-2} \, lbm} \times \frac{3600 \, sec}{hr} \times \frac{24 \, hr}{day} = 1.84 \times 10^8 \text{ ft}^3/\text{day}$$ ← Q



Problem 13.78

[4]

13.78 Air from a large reservoir at 25 psia and 250°F flows isentropically through a converging nozzle into a frictionless pipe at 24 psia. The flow is heated as it flows along the pipe. Obtain a plot of the Ts diagram for this flow, until $M = 1$. Also plot the pressure and speed distributions from the entrance to the location at which $M = 1$.

Given: Air flow from converging nozzle into heated pipe

Find: Plot Ts diagram and pressure and speed curves

Solution:

The given or available data is:

$R =$	53.33	ft·lbf/lbm·°R
$k =$	1.4	
$c_p =$	0.2399	Btu/lbm·°R
	187	ft·lbf/lbm·°R
$T_0 =$	710	°R
$p_0 =$	25	psi
$p_e =$	24	psi

Equations and Computations:

From p_0 and p_e, and Eq. 13.7a
(using built-in function *IsenMfromp* (M,k)) $M_e =$ 0.242

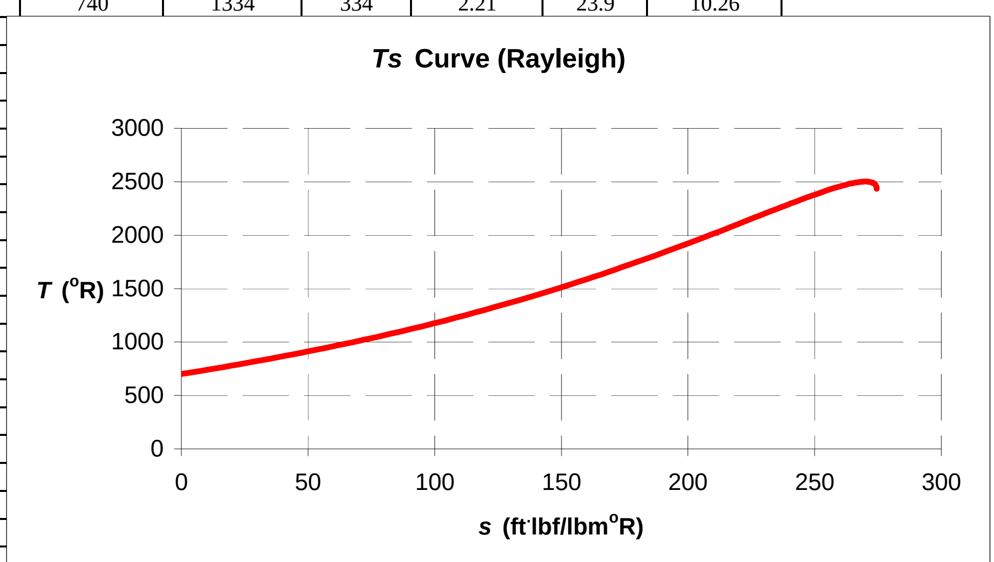
Using built-in function *IsenT* (M,k) $T_e =$ 702 °R

Using p_e, M_e, and function *Rayp* (M,k) $p^* =$ 10.82 psi

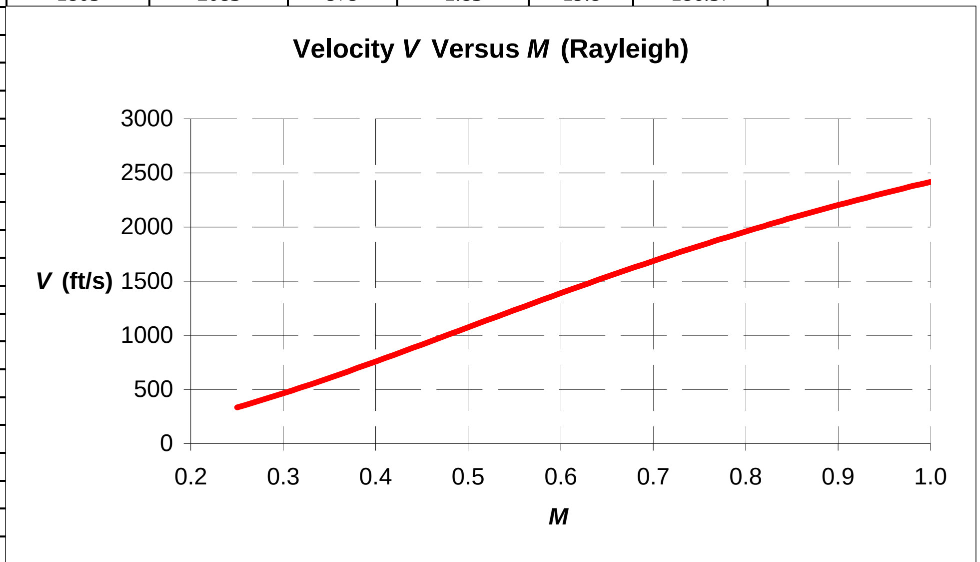
Using T_e, M_e, and function *RayT* (M,k) $T^* =$ 2432 °R

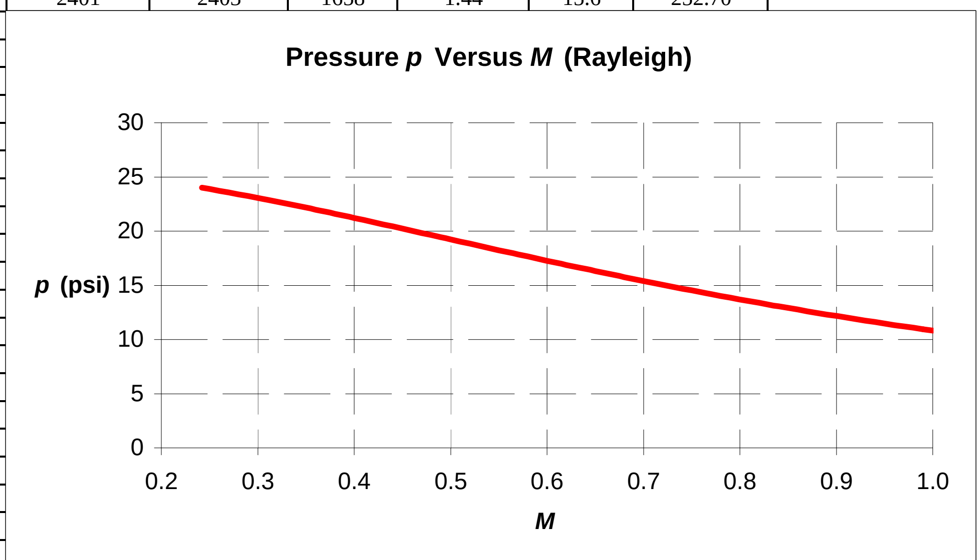
We can now use Rayleigh-line relations to compute values for a range of Mach numbers:

M	T/T^*	T (°R)	c (ft/s)	V (ft/s)	p/p^*	p (psi)	Δs (ft·lbf/lbm·°R) Eq. (12.11b)
0.242	0.289	702	1299	315	2.22	24.0	0.00
0.25	0.304	740	1334	334	2.21	23.9	10.26
0.26	0.325						
0.27	0.346						
0.28	0.367						
0.29	0.388						
0.3	0.409						
0.31	0.430						
0.32	0.451						
0.33	0.472						
0.34	0.493						
0.35	0.514						
0.36	0.535						
0.37	0.555						
0.38	0.576						
0.39	0.595						
0.4	0.615						
0.41	0.634						
0.42	0.653						
0.43	0.672						
0.44	0.690						
0.45	0.708						
0.46	0.725	1764	2059	947	1.85	20.0	181.73



0.47	0.742	1805	2083	979	1.83	19.8	186.57
0.48	0.759						
0.49	0.775						
0.5	0.790						
0.51	0.805						
0.52	0.820						
0.53	0.834						
0.54	0.847						
0.55	0.860						
0.56	0.872						
0.57	0.884						
0.58	0.896						
0.59	0.906						
0.6	0.917						
0.61	0.927						
0.62	0.936						
0.63	0.945						
0.64	0.953						
0.65	0.961						
0.66	0.968						
0.67	0.975						
0.68	0.981	2387	2396	1629	1.46	15.8	250.96
0.69	0.987	2401	2403	1658	1.44	15.6	252.70
0.7	0.993						
0.71	0.998						
0.72	1.003						
0.73	1.007						
0.74	1.011						
0.75	1.014						
0.76	1.017						
0.77	1.020						
0.78	1.022						
0.79	1.024						
0.8	1.025						
0.81	1.027						
0.82	1.028						
0.83	1.028						
0.84	1.029						
0.85	1.029						
0.86	1.028						
0.87	1.028						
0.88	1.027						
0.89	1.026						
0.9	1.025	2492	2448	2203	1.12	12.2	272.78
0.91	1.023	2488	2446	2226	1.11	12.0	273.13
0.92	1.021	2484	2444	2248	1.10	11.9	273.43
0.93	1.019	2479	2441	2270	1.09	11.7	273.70
0.94	1.017	2474	2439	2292	1.07	11.6	273.92
0.95	1.015	2468	2436	2314	1.06	11.5	274.11
0.96	1.012	2461	2433	2335	1.05	11.3	274.26
0.97	1.009	2455	2429	2356	1.04	11.2	274.38
0.98	1.006	2448	2426	2377	1.02	11.1	274.46
0.99	1.003	2440	2422	2398	1.01	10.9	274.51
1	1.000	2432	2418	2418	1.00	10.8	274.52





Problem 13.79

13.79 Repeat Problem 13.78 except the nozzle is now a converging-diverging nozzle delivering the air to the pipe at 2.5 psia.

Given: Air flow from converging-diverging nozzle into heated pipe

Find: Plot Ts diagram and pressure and speed curves

Solution:

The given or available data is:

R =	53.33	ft·lbf/lbm·°R
k =	1.4	
c_p =	0.2399	Btu/lbm·°R
	187	ft·lbf/lbm·°R
T_0 =	710	°R
p_0 =	25	psi
p_e =	2.5	psi

Equations and Computations:

From p_0 and p_e, and Eq. 13.7a
(using built-in function *IsenMfromp* (M,k)) M_e = 2.16

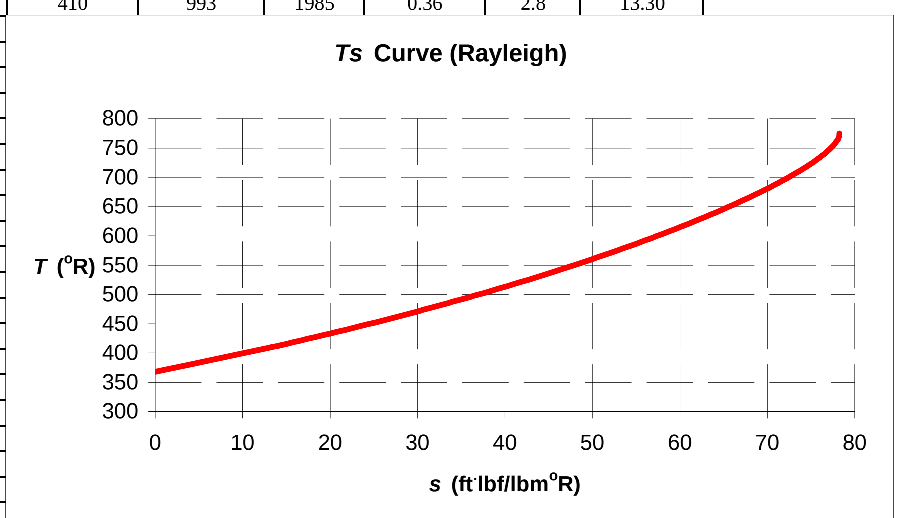
Using built-in function *IsenT* (M,k) T_e = 368 °R

Using p_e, M_e, and function *Rayp* (M,k) p^* = 7.83 psi

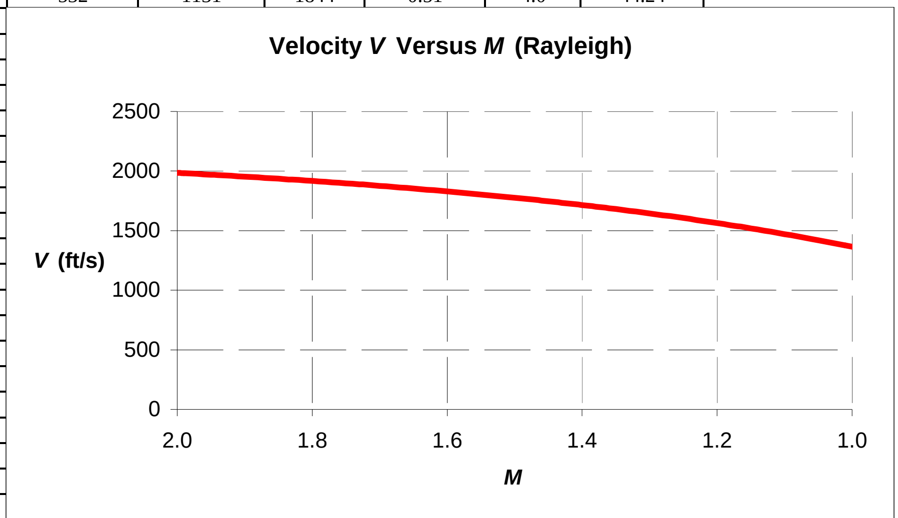
Using T_e, M_e, and function *RayT* (M,k) T^* = 775 °R

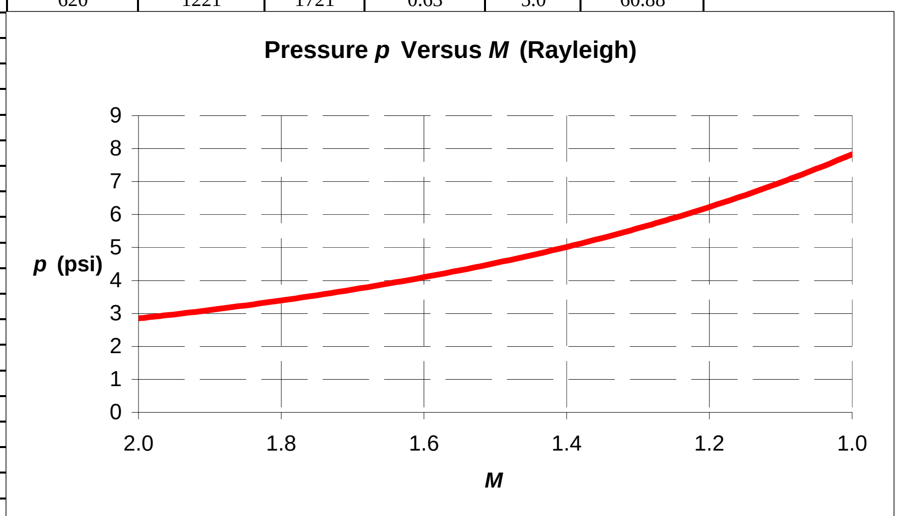
We can now use Rayleigh-line relations to compute values for a range of Mach numbers:

M	T/T^*	T (°R)	c (ft/s)	V (ft/s)	p/p^*	p (psi)	Δs (ft·lbf/lbm·°R) Eq. (12.11b)
2.157	0.475	368	940	2028	0.32	2.5	0.00
2	0.529	410	993	1985	0.36	2.8	13.30
1.99	0.533						
1.98	0.536						
1.97	0.540						
1.96	0.544						
1.95	0.548						
1.94	0.552						
1.93	0.555						
1.92	0.559						
1.91	0.563						
1.9	0.567						
1.89	0.571						
1.88	0.575						
1.87	0.579						
1.86	0.584						
1.85	0.588						
1.84	0.592						
1.83	0.596						
1.82	0.600						
1.81	0.605						
1.8	0.609						
1.79	0.613	475	1069	1913	0.44	3.4	31.06
1.78	0.618	479	1073	1909	0.44	3.5	31.90
1.77	0.622	482	1076	1905	0.45	3.5	32.73
1.76	0.626	485	1080	1901	0.45	3.5	33.57
1.75	0.631	489	1084	1897	0.45	3.6	34.40
1.74	0.635	492	1088	1893	0.46	3.6	35.23
1.73	0.640	496	1092	1889	0.46	3.6	36.06
1.72	0.645	499	1096	1885	0.47	3.7	36.89
1.71	0.649	503	1100	1880	0.47	3.7	37.72
1.7	0.654	507	1104	1876	0.48	3.7	38.54



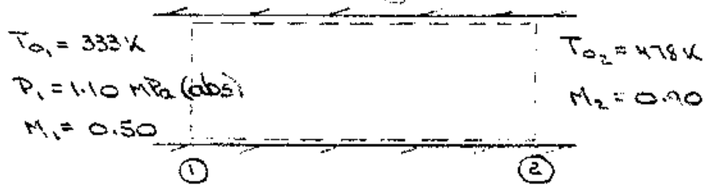
1.69	0.658	510	1107	1872	0.48	3.8	39.36
1.68	0.663	514	1111	1867	0.48	3.8	40.18
1.67	0.668	517	1115	1863	0.49	3.8	41.00
1.66	0.673	521	1119	1858	0.49	3.9	41.81
1.65	0.677	525	1123	1853	0.50	3.9	42.62
1.64	0.682	529	1127	1849	0.50	3.9	43.43
1.63	0.687	532	1131	1844	0.51	4.0	44.24
1.62	0.692						
1.61	0.697						
1.6	0.702						
1.59	0.707						
1.58	0.712						
1.57	0.717						
1.56	0.722						
1.55	0.727						
1.54	0.732						
1.53	0.737						
1.52	0.742						
1.51	0.747						
1.5	0.753						
1.49	0.758						
1.48	0.763						
1.47	0.768						
1.46	0.773						
1.45	0.779						
1.44	0.784						
1.43	0.789						
1.42	0.795	616	1217	1728	0.63	4.9	60.18
1.41	0.800	620	1221	1721	0.63	5.0	60.88
1.4	0.805						
1.39	0.811						
1.38	0.816						
1.37	0.822						
1.36	0.827						
1.35	0.832						
1.34	0.838						
1.33	0.843						
1.32	0.848						
1.31	0.854						
1.3	0.859						
1.29	0.865						
1.28	0.870						
1.27	0.875						
1.26	0.881						
1.25	0.886						
1.24	0.891						
1.23	0.896						
1.22	0.902						
1.21	0.907						
1.2	0.912	706	1303	1564	0.80	6.2	73.21
1.19	0.917	710	1307	1555	0.80	6.3	73.65
1.18	0.922	714	1310	1546	0.81	6.4	74.08
1.17	0.927	718	1314	1537	0.82	6.4	74.50
1.16	0.932	722	1318	1528	0.83	6.5	74.89
1.15	0.937	726	1321	1519	0.84	6.6	75.27
1.14	0.942	730	1324	1510	0.85	6.7	75.63
1.13	0.946	733	1328	1500	0.86	6.7	75.96
1.12	0.951	737	1331	1491	0.87	6.8	76.28
1.11	0.956	741	1334	1481	0.88	6.9	76.58
1.1	0.960	744	1337	1471	0.89	7.0	76.86
1.09	0.965	747	1341	1461	0.90	7.1	77.11
1.08	0.969	751	1344	1451	0.91	7.1	77.34
1.07	0.973	754	1347	1441	0.92	7.2	77.55
1.06	0.978	757	1349	1430	0.93	7.3	77.73
1.05	0.982	761	1352	1420	0.94	7.4	77.88
1.04	0.986	764	1355	1409	0.95	7.5	78.01
1.03	0.989	767	1358	1398	0.97	7.6	78.12
1.02	0.993	769	1360	1387	0.98	7.6	78.19
1.01	0.997	772	1362	1376	0.99	7.7	78.24
1	1.000	775	1365	1365	1.00	7.8	78.25





Problem 13.80

Given: Frictionless flow of air through a constant area duct



Find: $\delta Q/dm$, $p_1 - p_2$

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\qquad p_1A - p_2A = \dot{m}(V_2 - V_1)$

Computing equation: $T_0/T = 1 + \frac{k-1}{2}M^2$

Assumptions: (1) steady flow (2) frictionless flow (3) uniform flow at a section (4) ideal gas (5) $F_{B_x} = 0$ (6) $\dot{W}_s = \dot{W}_{shear} = 0$ (7) $\Delta z = 0$

$h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\qquad \frac{\delta Q}{dm} = h_{02} - h_{01} = c_p(T_{02} - T_{01}) = 10^3 \frac{J}{kg\cdot K}(478 - 333)K = 145\,\frac{kJ}{kg}$ ← $\frac{\delta Q}{dm}$

$p_1A - p_2A = \dot{m}(V_2 - V_1) = \rho_1 V_1 A(V_2 - V_1)$ and $p_1 - p_2 = \rho_1 V_1 (V_2 - V_1)$

$T_0/T = 1 + \frac{k-1}{2}M^2 \qquad T_1 = \frac{T_{01}}{1 + \frac{k-1}{2}M_1^2} = \frac{333\,K}{1 + 0.2(0.50)^2} = 317\,K$

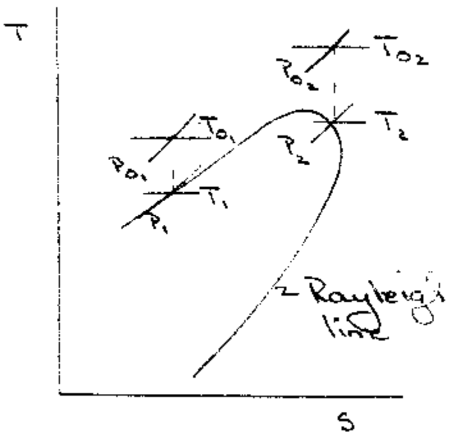
$$T_2 = \frac{T_{02}}{1 + \frac{k-1}{2}M_2^2} = \frac{478\,K}{1 + 0.2(0.90)^2} = 411\,K$$

$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 0.50\left(1.4 \times 287\frac{N\cdot m}{kg\cdot K} \times 317\,K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 178\,m/s$

$V_2 = M_2 c_2 = M_2 (kRT_2)^{1/2} = 0.90\left(1.4 \times 287\frac{N\cdot m}{kg\cdot K} \times 411\,K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 366\,m/s$

$\rho_1 = \frac{p_1}{RT_1} = 1.10 \times 10^6 \frac{N}{m^2} \times \frac{kg\cdot K}{287\,N\cdot m} \times \frac{1}{317\,K} = 12.1\,kg/m^3$

$p_1 - p_2 = \rho_1 V_1 (V_2 - V_1) = 12.1\frac{kg}{m^3} \times 178\frac{m}{s} \times (366 - 178)\frac{m}{s} \times \frac{N\cdot s^2}{kg\cdot m} = 405\,kPa$ ← $p_1 - p_2$



* Rayleigh-Line Flow Functions

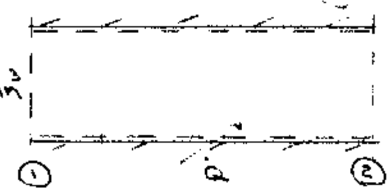
From Appendix E.3

- for $M_1 = 0.5$, $p_1/p^* = 1.778$ (12.30a) $\therefore p^* = 619\,kPa$
- for $M_2 = 0.90$, $p_2/p^* = 1.125$ (12.30a) $\therefore p_2 = 696\,kPa$. $p_1 - p_2 = 404\,kPa$.

Problem 13.81

[2]

Given: Frictionless flow of air through a constant area duct.



Find: $\delta Q/dm$, $s_2 - s_1$

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\qquad$ $T ds = dh - v dp$

Computing equations: $T_0/T = 1 + \frac{k-1}{2} M^2$

Assumptions: (1) Steady flow $\qquad$ (4) ideal gas
(2) frictionless flow $\qquad$ (5) $\dot{W}_s = \dot{W}_{shear} = 0$
(3) uniform flow at a section $\qquad$ (6) $\Delta z = 0$

$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 0.3 (1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 323 K \times \frac{kg \cdot m}{N \cdot s^2})^{1/2} = 108\ m/sec$

$p_1 = \rho_1 R T_1 = 2.16 \frac{kg}{m^3} \times 287 \frac{N \cdot m}{kg \cdot K} \times 323 K = 200\ kPa$

$\rho_1 V_1 = \rho_2 V_2 \qquad \rho_2 = \frac{p_2}{RT_2} \quad ; \quad V_2 = M_2 c_2 = M (kRT_2)^{1/2}$

$\therefore \rho_1 V_1 = \frac{p_2}{RT_2} M (kRT_2)^{1/2} = p_2 M_2 \left(\frac{k}{RT_2}\right)^{1/2}$. Solving for T_2,

$$T_2 = \frac{k}{R}\left(\frac{p_2 M_2}{\rho_1 V_1}\right)^2 = 1.4 \times \frac{kg \cdot K}{287\ N \cdot m}\left[150 \times 10^3 \frac{N}{m^2} \times 0.60 \times \frac{m^3}{2.16\ kg} \times \frac{s}{108\ m} \times \frac{kg \cdot m}{N \cdot s^2}\right]^2 \times \frac{N \cdot s^2}{kg \cdot m}$$

$T_2 = 726\ K$.

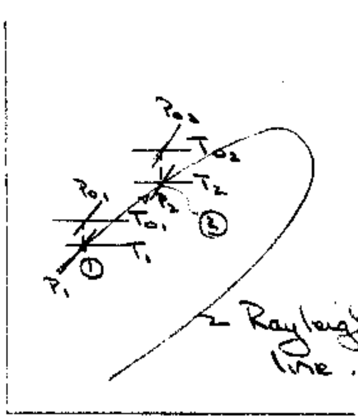
$$s_2 - s_1 = \int_1^2 ds = \int_{T_1}^{T_2} c_p \frac{dT}{T} - \int_{p_1}^{p_2} R \frac{dp}{p} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

$$s_2 - s_1 = 1.0 \frac{kJ}{kg \cdot K} \ln \frac{726}{323} - 0.287 \frac{kJ}{kg \cdot K} \ln \frac{150}{200} = 0.892\ kJ/kg \cdot K$$ $\qquad s_2 - s_1$

$$T_{01} = T_1\left[1 + \frac{k-1}{2} M_1^2\right] = 323 K\left[1 + 0.2 (0.3)^2\right] = 329 K$$

$$T_{02} = T_2\left[1 + \frac{k-1}{2} M_2^2\right] = 726 K\left[1 + 0.2 (0.6)^2\right] = 778 K$$

$$\frac{\delta Q}{dm} = h_{02} - h_{01} = c_p (T_{02} - T_{01}) = 1.0 \frac{kJ}{kg \cdot K} (778 - 329) K = 449\ kJ/kg$$ $\qquad \frac{\delta Q}{dm}$



* Rayleigh-Line Flow Functions (App. E.3)

For $M_1 = 0.30$: $T_{01}/T_0^* = 0.3469 \quad \therefore T_0^* = 948 K$
$T_1/T^* = 0.4089 \quad \therefore T^* = 790 K$
$p_1/p^* = 2.131 \quad \therefore p^* = 93.9\ kPa$

For $M_2 = 0.60$: $T_{02}/T_0^* = 0.8189 \quad \therefore T_{02} = 776 K$
$T_2/T^* = 0.9167 \quad \therefore T_2 = 724 K$
$p_2/p^* = 1.596 \quad \therefore p_2 = 150\ kPa$

Note: In using the flow functions, it is not necessary to know $p_2 = 150\ kPa$.

Problem 13.82

[2]

13.82 Air flows through a 5-cm-inside diameter pipe with negligible friction. Inlet conditions are $T_1 = 15°C$, $p_1 = 1$ MPa (abs), and $M_1 = 0.35$. Determine the heat exchange per pound of air required to produce $M_2 = 1.0$ at the pipe exit, where $p_2 = 500$ kPa.

Given: Frictionless air flow in a pipe

Find: Heat exchange per lb (or kg) at exit, where 500 kPa

Solution:

Basic equations: $m_{rate} = \rho \cdot V \cdot A$ $\quad p = \rho \cdot R \cdot T$ $\quad \frac{\delta Q}{dm} = c_p \cdot (T_{02} - T_{01})$ (Energy) $\quad p_1 - p_2 = \rho_1 \cdot V_1 \cdot (V_2 - V_1)$ (Momentum)

Given or available data $T_1 = (15 + 273) \cdot K$ $\quad p_1 = 1 \cdot MPa$ $\quad M_1 = 0.35$ $\quad p_2 = 500 \cdot kPa$ $\quad M_2 = 1$

$D = 5 \cdot cm$ $\quad k = 1.4$ $\quad c_p = 1004 \cdot \frac{J}{kg \cdot K}$ $\quad R = 286.9 \cdot \frac{J}{kg \cdot K}$

At section 1 $\quad \rho_1 = \frac{p_1}{R \cdot T_1}$ $\quad \rho_1 = 12.1 \frac{kg}{m^3}$ $\quad c_1 = \sqrt{k \cdot R \cdot T_1}$ $\quad c_1 = 340 \frac{m}{s}$

$V_1 = M_1 \cdot c_1$ $\quad V_1 = 119 \frac{m}{s}$

From momentum $\quad V_2 = \frac{p_1 - p_2}{\rho_1 \cdot V_1} + V_1$ $\quad V_2 = 466 \frac{m}{s}$

From continuity $\quad \rho_1 \cdot V_1 = \rho_2 \cdot V_2$ $\quad \rho_2 = \rho_1 \cdot \frac{V_1}{V_2}$ $\quad \rho_2 = 3.09 \frac{kg}{m^3}$

Hence $\quad T_2 = \frac{p_2}{\rho_2 \cdot R}$ $\quad T_2 = 564 K$ $\quad T_2 = 291 °C$

and $\quad T_{02} = T_2 \cdot \left(1 + \frac{k-1}{2} \cdot M_2^2\right)$ $\quad T_{02} = 677 K$ $\quad T_{02} = 403 °C$

with $\quad T_{01} = T_1 \cdot \left(1 + \frac{k-1}{2} \cdot M_1^2\right)$ $\quad T_{01} = 295 K$ $\quad T_{01} = 21.9 °C$

Then $\quad \frac{\delta Q}{dm} = c_p \cdot (T_{02} - T_{01}) = 164 \cdot \frac{Btu}{lbm} = 383 \cdot \frac{kJ}{kg}$

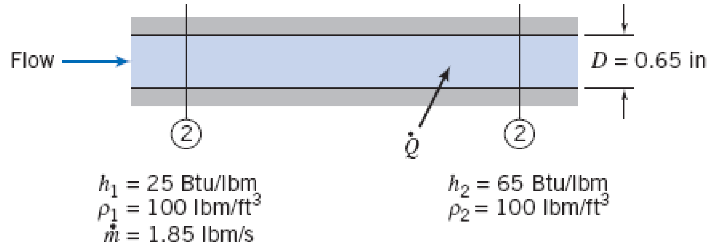
(Note: Using Rayleigh line functions, for $M_1 = 0.35$ $\frac{T_0}{T_{0crit}} = 0.4389$

so $\quad T_{0crit} = \frac{T_{01}}{0.4389}$ $\quad T_{0crit} = 672 K$ close to T_2 ... Check!)

Problem 13.83

[2]

13.83 Liquid Freon, used to cool electronic components, flows steadily into a horizontal tube of constant diameter, $D = 0.65$ in. Heat is transferred to the flow, and the liquid boils and leaves the tube as vapor. The effects of friction are negligible compared with the effects of heat addition. Flow conditions are shown. Find (a) the rate of heat transfer and (b) the pressure difference, $p_1 - p_2$.



Given: Frictionless flow of Freon in a tube

Find: Heat transfer; Pressure drop

NOTE: ρ_2 is NOT as stated; see below

Solution:

Basic equations: $m_{rate} = \rho \cdot V \cdot A$ $\quad p = \rho \cdot R \cdot T$ $\quad Q = m_{rate} \cdot \left(h_{02} - h_{01}\right)$ $\quad h_0 = h + \frac{V^2}{2}$ $\quad p_1 - p_2 = \rho_1 \cdot V_1 \cdot \left(V_2 - V_1\right)$

Given or available data $h_1 = 25 \cdot \frac{\text{Btu}}{\text{lbm}}$ $\quad \rho_1 = 100 \cdot \frac{\text{lbm}}{\text{ft}^3}$ $\quad h_2 = 65 \cdot \frac{\text{Btu}}{\text{lbm}}$ $\quad \rho_2 = 0.850 \cdot \frac{\text{lbm}}{\text{ft}^3}$

$D = 0.65 \cdot \text{in}$ $\quad A = \frac{\pi}{4} \cdot D^2$ $\quad A = 0.332\,\text{in}^2$ $\quad m_{rate} = 1.85 \cdot \frac{\text{lbm}}{\text{s}}$

Then $\quad V_1 = \frac{m_{rate}}{\rho_1 \cdot A}$ $\quad V_1 = 8.03 \frac{\text{ft}}{\text{s}}$ $\quad h_{01} = h_1 + \frac{V_1^{\,2}}{2}$ $\quad h_{01} = 25.0 \frac{\text{Btu}}{\text{lbm}}$

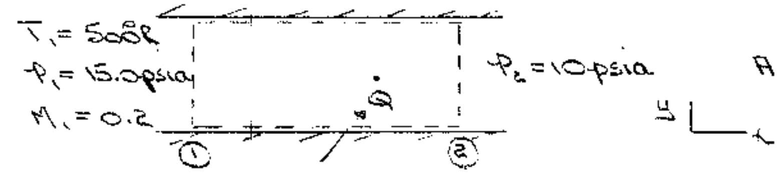
$V_2 = \frac{m_{rate}}{\rho_2 \cdot A}$ $\quad V_2 = 944 \frac{\text{ft}}{\text{s}}$ $\quad h_{02} = h_2 + \frac{V_2^{\,2}}{2}$ $\quad h_{02} = 82.8 \frac{\text{Btu}}{\text{lbm}}$

The heat transfer is $\quad Q = m_{rate} \cdot \left(h_{02} - h_{01}\right)$ $\quad Q = 107 \frac{\text{Btu}}{\text{s}}$ $\quad$ (74 Btu/s with the wrong ρ_2!)

The pressure drop is $\quad \Delta p = \rho_1 \cdot V_1 \cdot \left(V_2 - V_1\right)$ $\quad \Delta p = 162\,\text{psi}$ $\quad$ (-1 psi with the wrong ρ_2!)

Problem 13.84

Given: Frictionless flow of air in a constant-area duct



Find: (a) V_2 and T_2 (b) $\dot{Q}$

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\qquad p_1A - p_2A = \dot{m}(V_2 - V_1)$

Computing equation: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$

Assumptions: (1) steady flow (2) uniform flow at a section (3) ideal gas (4) frictionless flow (5) $F_{B_x} = 0$ (6) $\dot{W}_s = \dot{W}_{shear} = 0$

$$\rho_1 = \frac{p_1}{RT_1} = 15\frac{lbf}{in^2} \times \frac{lbm\cdot°R}{53.3\ ft\cdot lbf} \times \frac{1}{500°R} \times 144\frac{in^2}{ft^2} = 0.0811\ lbm/ft^3$$

$$V_1 = M_1c_1 = M_1(kRT_1)^{1/2} = 0.2\left[1.4 \times 53.3\frac{ft\cdot lbf}{lbm\cdot°R} \times 500°R \times 32.2\frac{lbm}{slug} \times \frac{slug\cdot ft}{lbf\cdot s^2}\right]^{1/2} = 219\ ft/s$$

From the momentum eq. $(p_1 - p_2)A = \dot{m}(V_2 - V_1) = \rho_1V_1A(V_2 - V_1)$

$$\therefore V_2 = \frac{(p_1 - p_2)}{\rho_1V_1} + V_1 = (15-10)\frac{lbf}{in^2} \times \frac{ft^3}{0.0811\ lbm} \times \frac{s}{219\ ft} \times 144\frac{in^2}{ft^2} \times 32.2\frac{lbm}{slug} \times \frac{slug\cdot ft}{lbf\cdot s^2} + 219\ ft/s$$

$V_2 = 1520\ ft/s$ ←—— V_2

From continuity $\rho_2 = \frac{V_1}{V_2}\rho_1 = \frac{219}{1520} \times 0.0811\frac{lbm}{ft^3} = 0.0117\ lbm/ft^3$

$$T_2 = \frac{p_2}{\rho_2R} = 10\frac{lbf}{in^2} \times \frac{ft^3}{0.0117\ lbm} \times \frac{lbm\cdot°R}{53.3\ ft\cdot lbf} \times 144\frac{in^2}{ft^2} = 2310°R$$ ←—— T_2

$T_{0_1} = T_1\left(1 + \frac{k-1}{2}M_1^2\right) = 500\left[1 + 0.2(0.2)^2\right] = 504°R$

$$c_2 = (kRT_2)^{1/2} = \left[1.4 \times 53.3\frac{ft\cdot lbf}{lbm\cdot°R} \times 2310°R \times 32.2\frac{lbm}{slug} \times \frac{slug\cdot ft}{lbf\cdot s^2}\right]^{1/2} = 2360\ ft/s$$

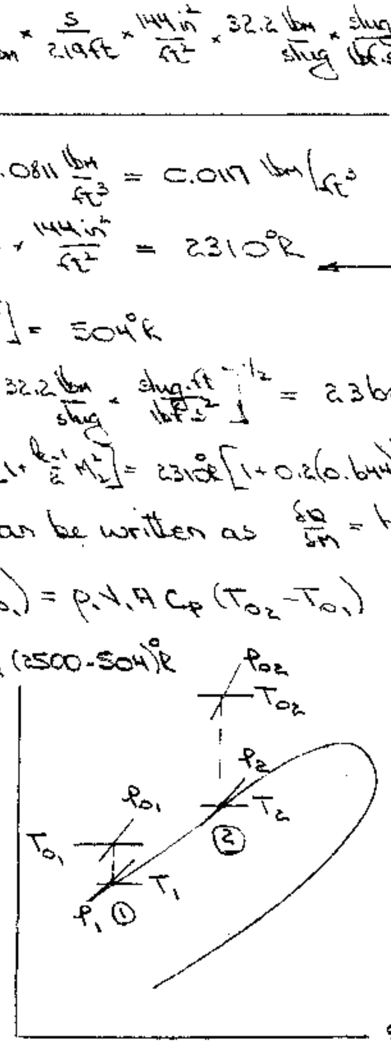
$M_2 = \frac{V_2}{c_2} = \frac{1520}{2360} = 0.644$ $\qquad T_{0_2} = T_2\left[1 + \frac{k-1}{2}M_2^2\right] = 2310°R\left[1 + 0.2(0.644)^2\right] = 2500°R$

Since $h_0 = h + \frac{V^2}{2}$, the energy eq. can be written as $\frac{\delta Q}{dm} = h_{0_2} - h_{0_1}$

$$\therefore \dot{Q} = \dot{m}\frac{\delta Q}{dm} = \dot{m}(h_{0_2} - h_{0_1}) = \dot{m}c_p(T_{0_2} - T_{0_1}) = \rho_1V_1Ac_p(T_{0_2} - T_{0_1})$$

$$\dot{Q} = 0.0811\frac{lbm}{ft^3} \times 219\frac{ft}{s} \times 0.087\ ft^2 \times 0.24\frac{Btu}{lbm\cdot°R} \times (2500 - 504)°R$$

$\dot{Q} = 740\ Btu/s$ ←——



* Rayleigh-Line Flow Functions (App. E.3)

At $M_1 = 0.2$

$p_1/p^* = 2.273$ (12.30a) $\therefore p^* = 6.60$ psia

$T_1/T^* = 0.2066$ (12.30b) $\therefore T^* = 2420°R$

$T_{0_1}/T_0^* = 0.1736$ (12.30d) $\therefore T_0^* = 2900°R$

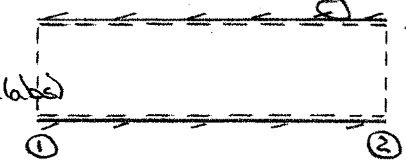
At ② $p_2/p^* = 1.515$ $\therefore M_2 = 0.646$

Also $T_{0_2}/T_0^* = 0.8644$, $\therefore T_{0_2} = 2510°R$ and $T_2/T^* = 0.9408$ $\therefore T_2 = 2280°R$

Problem 13.85

Given: Frictionless flow of air through a constant area duct.

$T_1 = 52°C$ $P_1 = 60$ kPa (abs)



$T_2 = 45°C$ $M_2 = 1.0$ $\dot{m} = 1.42$ kg/s $D = 100$ mm

Find: $\delta Q/dm$, $s_2 - s_1$, $P_{0_1} - P_{0_2}$

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $T\,ds = dh - v\,dP$

Computing equations: $T_0/T = 1 + \frac{k-1}{2}M^2$ $P_0/P = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow   (4) ideal gas
(2) frictionless flow   (5) $\dot{W}_s = \dot{W}_{shear} = 0$
(3) uniform flow at a section   (6) $\Delta z = 0$

$A = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.1)^2 m^2 = 7.85 \times 10^{-3}\ m^2$

$P = \rho RT$ $\rho_1 = \frac{P_1}{RT_1} = 60 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\ N \cdot m} \times \frac{1}{325\ K} = 0.643\ kg/m^3$

$\dot{m} = \rho V A$ $V_1 = \frac{\dot{m}}{\rho_1 A} = 1.42 \frac{kg}{s} \times \frac{m^3}{0.643\ kg} \times \frac{1}{7.85 \times 10^{-3}\ m^2} = 281\ m/s$

$V_2 = M_2 c_2 = M_2 (kRT_2)^{1/2} = 1.0\left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 318\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 357\ m/s$

$\dot{m} = \rho_1 V_1 A = \rho_2 V_2 A$; $\rho_2 = \frac{V_1}{V_2}\rho_1 = \frac{281}{357} \times 0.643\ kg/m^3 = 0.506\ kg/m^3$

$P = \rho RT$ $P_2 = \rho_2 R T_2 = 0.506 \frac{kg}{m^3} \times 287 \frac{N \cdot m}{kg \cdot K} \times 318\ K = 46.2\ kPa$

$T\,ds = dh - v\,dP = c_p dT - \frac{1}{\rho}dP$ $ds = c_p \frac{dT}{T} - R\frac{dP}{P}$

$s_2 - s_1 = \int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} c_p \frac{dT}{T} - \int_{P_1}^{P_2} R\frac{dP}{P} = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = 1.0 \frac{kJ}{kg \cdot K}\ln\frac{318}{325} - 0.287\frac{kJ}{kg \cdot K}\ln\frac{46.2}{60}$

$s_2 - s_1 = 0.0532\ kJ/kg \cdot K$ ← $s_2 - s_1$

$M_1 = \frac{V_1}{c_1}$ $c_1 = (kRT_1)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 325\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 361\ m/s$, $M_1 = \frac{V_1}{c_1} = \frac{281}{361} = 0.778$

$T_0/T = 1 + \frac{k-1}{2}M^2$ $T_{0_1} = T_1\left[1 + \frac{k-1}{2}M_1^2\right] = 325\ K\left[1 + 0.2(0.778)^2\right] = 364\ K$

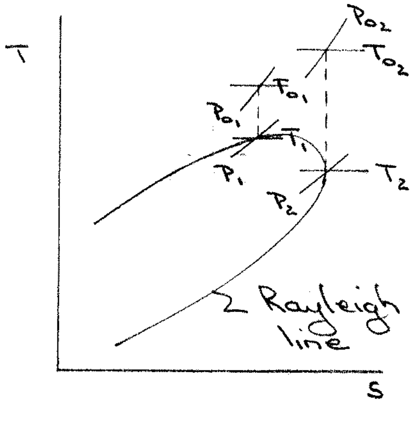
$T_{0_2} = T_2\left[1 + \frac{k-1}{2}M_2^2\right] = 318\ K\left[1 + 0.2(1.0)^2\right] = 382\ K$

$h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\frac{\delta Q}{dm} = h_{0_2} - h_{0_1} = c_p(T_{0_2} - T_{0_1}) = 1.0 \frac{kJ}{kg \cdot K}(382 - 364)K = 18\ kJ/kg$ ← $\delta Q/dm$

$P_0/P = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$ $P_{0_1} = P_1\left[1 + \frac{k-1}{2}M_1^2\right]^{k/k-1} = 60\ kPa\left[1 + 0.2(0.778)^2\right]^{3.5} = 89.5\ kPa$

$P_{0_2} = P_2\left[1 + \frac{k-1}{2}M_2^2\right]^{k/k-1} = 46.2\ kPa\left[1 + 0.2(1.0)^2\right]^{3.5} = 87.5\ kPa$

$P_{0_1} - P_{0_2} = (89.5 - 87.5)\ kPa = 2.0\ kPa$ ← $P_{0_1} - P_{0_2}$



* Rayleigh-Line Flow Functions
From Appendix E.3 for $M_1 = 0.778$

$T_{0_1}/T_0^* = 0.954$ (12.30d) $\therefore T_0^* = T_{0_2} = 381\ K$

$P_{0_1}/P_0^* = 1.024$ (12.30e) $\therefore P_0^* = P_{0_2} = 87.4\ kPa$

$P_1/P^* = 1.299$ (12.30a) $\therefore P^* = P_2 = 46.2\ kPa$

Problem 13.86

[3]

Given: Combustor modeled as frictionless flow through a constant area duct with heat transfer. Air-fuel ratio is large enough so properties are those of air.

$T_1 = 1260°R$ $\quad$ $p_1 = 235$ psia $\quad$ $V_1 = 609$ ft/s $\quad$ $\dot{m}_{air} = 15$ lbm/s $\quad$ $T_2 = 1840°R$ $\quad$ $M_2 = 0.476$

Heating value of the fuel is 18,000 Btu/lbm.

Find: (a) p_2 (b) $\dot{Q}$ (c) $\dot{m}_{fuel}$

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\qquad$ $p_1A - p_2A = \dot{m}(V_2 - V_1)$

Computing equation: $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$

Assumptions: (1) steady flow (2) frictionless flow
(3) ideal gas, properties are those of air
(4) uniform flow at a section
(5) $\dot{W}_s = \dot{W}_{shear} = 0$

$\rho_1 = \frac{p_1}{RT_1} = 235\,\frac{lbf}{in^2} \times \frac{lbm\,°R}{53.3\,ft\,lbf} \times \frac{1}{1260\,°R} \times 144\,\frac{in^2}{ft^2} = 0.504\ lbm/ft^3$

$c_1 = (kRT_1)^{1/2} = \left[1.4 \times 53.3\,\frac{ft\,lbf}{lbm\,R} \times 1260\,R \times 32.2\,\frac{lbm}{slug} \times \frac{slug\,ft}{lbf\,s^2}\right]^{1/2} = 1740\ ft/s$

$M_1 = V_1/c_1 = 609/1740 = 0.350$ $\qquad$ $T_{01} = T_1\left[1 + \frac{k-1}{2}M_1^2\right] = 1260\,°R[1 + 0.2(0.35)^2] = 1290\,°R$

$T_{02} = T_2\left[1 + \frac{k-1}{2}M_2^2\right] = 1840[1 + 0.2(0.476)^2] = 1920\,°R$

From energy eq. $\quad \dot{Q} = \dot{m}(h_{02} - h_{01}) = \dot{m}c_p(T_{02} - T_{01}) = 15\,\frac{lbm}{s} \times 0.24\,\frac{Btu}{lbm\,R} \times (1920 - 1290)\,R$

$\dot{Q} = 2270\ Btu/s$ $\qquad$ ← $\dot{Q}$

But $\dot{Q} = \dot{m}_{fuel} h_{fuel}$ $\quad \therefore \dot{m}_f = \frac{\dot{Q}}{h_f} = 2270\,\frac{Btu}{s} \times \frac{lbm}{18{,}000\,Btu} = 0.126\ lbm/s$ $\qquad$ ← $\dot{m}_f$

$\dot{m}_f/\dot{m}_{air} = 0.126/15 = 0.0084$ $\qquad$ ← $\dot{m}_f/\dot{m}_{air}$

$V_2 = M_2 c_2 = M_2(kRT_2)^{1/2} = 0.476\left[1.4 \times 53.3\,\frac{ft\,lbf}{lbm\,R} \times 1840\,R \times 32.2\,\frac{lbm}{slug} \times \frac{slug\,ft}{lbf\,s^2}\right]^{1/2} = 1000\ ft/s$

From momentum

$p_2 = p_1 - \frac{\dot{m}}{A}(V_2 - V_1) = p_1 - \rho_1 V_1 (V_2 - V_1)$

$p_2 = 235\,\frac{lbf}{in^2} - 0.504\,\frac{lbm}{ft^3} \times 609\,\frac{ft}{s} \times (1000 - 609)\,\frac{ft}{s} \times \frac{slug}{32.2\,lbm} \times \frac{lbf\,s^2}{ft\,slug} \times \frac{ft^2}{144\,in^2} = 209\ psia$ $\qquad$ ← p_2

* Rayleigh-Line Flow Functions (App. E.3)

For $M_1 = 0.35$,

$T_0/T_0^* = 0.4389$ (12.30d) $\quad \therefore T_0^* = 2940\,°R$

$T_1/T^* = 0.5141$ (12.30b) $\quad \therefore T^* = 2450\,°R$

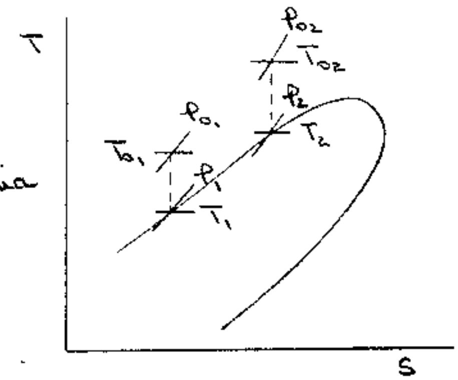
$p_1/p^* = 2.0487$ (12.30a) $\quad \therefore p^* = 114.7$ psia

For $M_2 = 0.476$

$T_{02}/T_0^* = 0.6551 \quad \therefore T_{02} = 1930\,°R$

$T_2/T^* = 0.7522 \quad \therefore T_2 = 1840\,°R$

$p_2/p^* = 1.822 \quad \therefore p_2 = 209$ psia



Problem 13.87

13.87 Consider frictionless flow of air in a duct with $D = 10$ cm. At section ①, the temperature and pressure are 0°C and 70 kPa; the mass flow rate is 0.5 kg/s. How much heat may be added without choking the flow? Evaluate the resulting change in stagnation pressure.

Given: Frictionless flow of air in a duct

Find: Heat transfer without choking flow; change in stagnation pressure

Solution:

Basic equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2}\cdot M^2$ $\qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$ $\qquad p = \rho\cdot R\cdot T$ $\qquad m_{rate} = \rho\cdot A\cdot V$

$p_1 - p_2 = \frac{m_{rate}}{A}\cdot(V_2 - V_1)$ $\qquad \frac{\delta Q}{dm} = c_p\cdot(T_{02} - T_{01})$

Given or available data $T_1 = (0 + 273)\cdot K$ $\qquad p_1 = 70\cdot kPa$ $\qquad m_{rate} = 0.5\cdot\frac{kg}{s}$ $\qquad D = 10\cdot cm$

$A = \frac{\pi}{4}\cdot D^2$ $\qquad A = 78.54\,cm^2$ $\qquad k = 1.4$ $\qquad M_2 = 1$ $\qquad c_p = 1004\cdot\frac{J}{kg\cdot K}$ $\qquad R = 286.9\cdot\frac{J}{kg\cdot K}$

At state 1 $\quad \rho_1 = \frac{p_1}{R\cdot T_1}$ $\qquad \rho_1 = 0.894\,\frac{kg}{m^3}$ $\qquad c_1 = \sqrt{k\cdot R\cdot T_1}$ $\qquad c_1 = 331\,\frac{m}{s}$

From continuity $\quad V_1 = \frac{m_{rate}}{\rho_1\cdot A}$ $\qquad V_1 = 71.2\,\frac{m}{s}$ $\qquad$ then $\quad M_1 = \frac{V_1}{c_1}$ $\qquad M_1 = 0.215$

From momentum $\quad p_1 - p_2 = \frac{m_{rate}}{A}\cdot(V_2 - V_1) = \rho_2\cdot V_2^2 - \rho_1\cdot V_1^2$ $\quad$ but $\quad \rho\cdot V^2 = \rho\cdot c^2\cdot M^2 = \frac{p}{R\cdot T}\cdot k\cdot R\cdot T\cdot M^2 = k\cdot p\cdot M^2$

Hence $\quad p_1 - p_2 = k\cdot p_2\cdot M_2^2 - k\cdot p_1\cdot M_1^2$ $\quad$ or $\quad p_2 = p_1\cdot\left(\frac{1 + k\cdot M_1^2}{1 + k\cdot M_2^2}\right)$ $\qquad p_2 = 31.1\,kPa$

From continuity $\quad \rho_1\cdot V_1 = \frac{p_1}{R\cdot T_1}\cdot M_1\cdot c_1 = \frac{p_1}{R\cdot T_1}\cdot M_1\cdot\sqrt{k\cdot R\cdot T_1} = \sqrt{\frac{k}{R}}\cdot\frac{p_1\cdot M_1}{\sqrt{T_1}} = \rho_2\cdot V_2 = \sqrt{\frac{k}{R}}\cdot\frac{p_2\cdot M_2}{\sqrt{T_2}}$

Hence $\quad \frac{p_1\cdot M_1}{\sqrt{T_1}} = \frac{p_2\cdot M_2}{\sqrt{T_2}}$ $\qquad T_2 = T_1\cdot\left(\frac{p_2}{p_1}\cdot\frac{M_2}{M_1}\right)^2$ $\qquad T_2 = 1161\,K$ $\qquad T_2 = 888\,°C$

Then $\quad T_{02} = T_2\cdot\left(1 + \frac{k-1}{2}\cdot M_2^2\right)$ $\qquad T_{02} = 1394\,K$ $\qquad T_{01} = T_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right)$ $\qquad T_{01} = 276\,K$

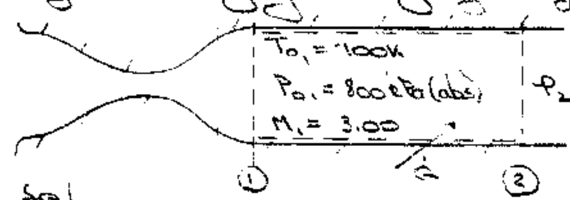
$p_{02} = p_2\cdot\left(1 + \frac{k-1}{2}\cdot M_2^2\right)^{\frac{k}{k-1}}$ $\qquad p_{02} = 58.8\,kPa$ $\qquad p_{01} = p_1\cdot\left(1 + \frac{k-1}{2}\cdot M_1^2\right)^{\frac{k}{k-1}}$ $\qquad p_{01} = 72.3\,kPa$

Finally $\quad \frac{\delta Q}{dm} = c_p\cdot(T_{02} - T_{01}) = 1.12\cdot\frac{MJ}{kg}$ $\qquad \Delta p_0 = p_{02} - p_{01}$ $\qquad \Delta p_0 = -13.5\,kPa$

(Using Rayleigh functions, at $M_1 = 0.215$ $\quad \frac{T_{01}}{T_{0crit}} = \frac{T_{01}}{T_{02}} = 0.1975$ $\quad T_{02} = \frac{T_{01}}{0.1975}$ $\qquad T_{02} = 1395\,K$ $\qquad$ and ditto for p_{02} ...Check!)

Problem 13.88

Given: Frictionless flow of air through a constant-area duct supplied by a converging-diverging nozzle



Find: V_2, M_2, $\delta Q/dm$

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\quad p_1A - p_2A = \dot{m}(V_2 - V_1)$

Computing equations: $T_0/T = 1 + \frac{k-1}{2}M^2$ $\quad p_0/p = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1}$

Assumptions: (1) steady flow (5) $F_{B_x} = 0$
(2) frictionless flow (6) $\dot{W}_s = \dot{W}_{shear} = 0$
(3) uniform flow at a section (7) $\Delta z = 0$
(4) ideal gas

$T_0/T = 1 + \frac{k-1}{2}M^2 \qquad T_1 = \frac{T_{0_1}}{1 + \frac{k-1}{2}M_1^2} = \frac{700\text{ K}}{1 + 0.2(3.0)^2} = 250\text{ K}$

$p_0/p = \left[1 + \frac{k-1}{2}M^2\right]^{k/k-1} \qquad p_1 = \frac{p_{0_1}}{\left[1 + \frac{k-1}{2}M_1^2\right]^{k/k-1}} = \frac{800\text{ kPa}}{\left[1 + 0.2(3.0)^2\right]^{3.5}} = 21.8\text{ kPa}$

$V_1 = M_1c_1 = M_1(kRT_1)^{1/2} = 3.0\left(1.4 \times 287\frac{N\cdot m}{kg\cdot K} \times 250\text{ K} \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 951\text{ m/s}$

$p = \rho RT \qquad \rho_1 = \frac{p_1}{RT_1} = 21.8 \times 10^3\frac{N}{m^2} \times \frac{kg\cdot K}{287\, N\cdot m} \times \frac{1}{250\text{ K}} = 0.304\text{ kg/m}^3$

$(p_1 - p_2)A = \rho_1V_1A(V_2 - V_1) \quad \therefore V_2 = V_1 + \frac{(p_1 - p_2)}{\rho_1V_1}$. Solving for V_2,

$V_2 = 951\text{ m/s} + (21.8 - 46.4)10^3\frac{N}{m^2} \times \frac{m^3}{0.304\text{ kg}} \times \frac{s}{951\text{ m}} \times \frac{kg\cdot m}{N\cdot s^2} = 866\text{ m/s}$ ← V_2

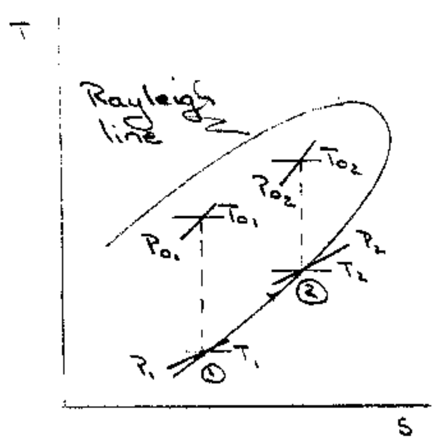
From continuity, $\rho_2 = \rho_1\frac{V_1}{V_2} = 0.304\frac{kg}{m^3} \times \frac{951}{866} = 0.334\text{ kg/m}^3$

$T_2 = \frac{p_2}{\rho_2R} = 46.4 \times 10^3\frac{N}{m^2} \times \frac{m^3}{0.334\text{ kg}} \times \frac{kg\cdot K}{287\, N\cdot m} = 484\text{ K}$

$c_2 = (kRT_2)^{1/2} = \left[1.4 \times 287\frac{N\cdot m}{kg\cdot K} \times 484\text{ K} \times \frac{kg\cdot m}{N\cdot s^2}\right]^{1/2} = 441\text{ m/s}\ ; \quad M_2 = \frac{V_2}{c_2} = \frac{866}{441} = 1.96$ ← M_2

$T_{0_2} = T_2\left[1 + \frac{k-1}{2}M_2^2\right] = 484\left[1 + 0.2(1.96)^2\right] = 856\text{ K}$

$\delta Q/dm = h_{0_2} - h_{0_1} = c_p(T_{0_2} - T_{0_1}) = 1\frac{kJ}{kg\cdot K}(856\text{ K} - 700\text{ K}) = 156\text{ kJ/kg}$ ← $\frac{\delta Q}{dm}$



• Rayleigh-Line Flow Functions (Appendix E.3)

For $M_1 = 3.0$, $T_{0_1}/T_0^* = 0.6540$, $\therefore T_0^* = 1070\text{ K}$

$T_1/T^* = 0.2803 \quad \therefore T^* = 892\text{ K}$

$p_1/p^* = 0.1765 \quad \therefore p^* = 124\text{ kPa}$

$V_1/V^* = 1.588 \quad \therefore V^* = 599\text{ m/s}$

At section ② for $p_2/p^* = 0.3742$

$M_2 = 1.966$ ✓

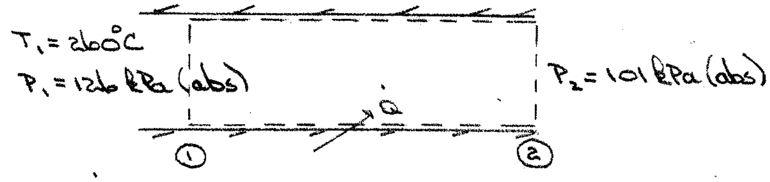
$T_{0_2}/T_0^* = 0.800 \quad \therefore T_{0_2} = 856\text{ K}$ ✓

$T_2/T^* = 0.542 \quad \therefore T_2 = 483\text{ K}$

$V_2/V^* = 1.447 \quad \therefore V_2 = 867\text{ m/s}$

Problem 13.89

Given: Frictionless flow of air through a constant area duct



$A = 0.02\ m^2$

$\dot{m} = 1.83\ kg/s$

Find: M_2, T_2, T_{02}, $\dot{Q}$

Solution:

* Compressible flow functions (Appendix E) to be used in solution

Basic equation: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$

Assumptions:
(1) steady flow
(2) frictionless flow
(3) uniform flow at a section
(4) ideal gas
(5) $F_{Bx} = 0$
(6) $\dot{W}_s = \dot{W}_{shear} = 0$
(7) $\Delta z = 0$

$\rho_1 = \frac{P_1}{RT_1} = 126 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\ N \cdot m} \times \frac{1}{533\ K} = 0.8237\ kg/m^3$

$\dot{m} = \rho_1 V_1 A \qquad V_1 = \frac{\dot{m}}{\rho_1 A} = 1.83 \frac{kg}{s} \times \frac{m^3}{0.8237\ kg} \times \frac{1}{0.02\ m^2} = 111\ m/s$

$M_1 = \frac{V_1}{c_1} \qquad c_1 = (kRT_1)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 533\ K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 463\ m/s$

$M_1 = \frac{111}{463} = 0.240$

From App. E.1, $T_{01}/T_1 = 1.012$ (11.17b) $\therefore T_{01} = 539\ K$

From App E.3, $T_{01}/T_0^* = 0.2395$ (12.30d), $T_1/T^* = 0.2841$ (12.30b), $P_1/P^* = 2.221$ (12.30a)

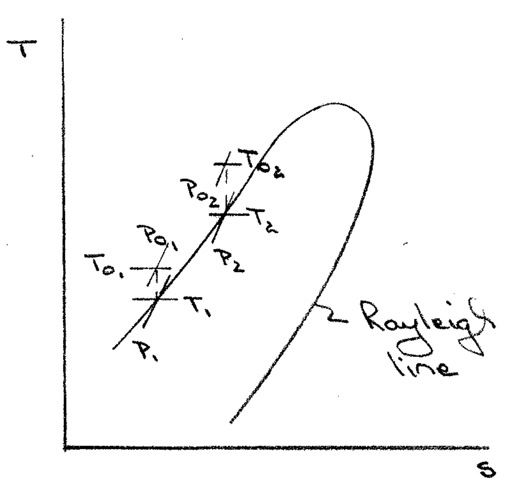
$\therefore T_0^* = 2250\ K \qquad T^* = 1876\ K \qquad P^* = 56.73\ kPa$

At section ② $\frac{P_2}{P^*} = \frac{101}{56.73} = 1.780$ From App E.3 (Eq. 12.30a), $M = 0.50$ ——— M_2

Also, $T_{02}/T_0^* = 0.6914$ (12.30d), $T_2/T^* = 0.7901$. Therefore: $T_{02} = 1556\ K$, $T_2 = 1480\ K$ ——— T_{02}, T_2

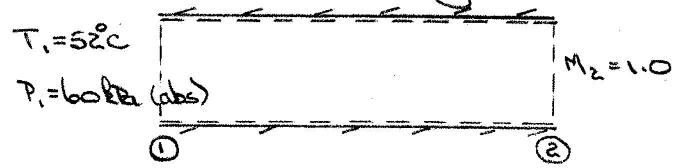
$\dot{Q} = \dot{m} \frac{\delta Q}{dm} = \dot{m}(h_{02} - h_{01}) = \dot{m} c_p (T_{02} - T_{01}) = 1.83 \frac{kg}{s} \times 1.0 \frac{kJ}{kg \cdot K} \times (1556 - 539)\ K$

$\dot{Q} = 1.86\ MJ/s$ ——— $\dot{Q}$



Problem 13.90

Given: Frictionless flow of air through a constant area duct.



$\dot{m} = 1.42\text{ kg/sec}$
$D = 100\text{ mm}$

Find: $\delta Q/dm$, fluid properties at section ②

Solution:

* Compressible flow functions (Appendix E) to be used in solution

Basic equation: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$

Assumptions:
(1) steady flow
(2) frictionless flow
(3) uniform flow at a section
(4) ideal gas
(5) $\dot{W}_s = \dot{W}_{shear} = 0$
(6) $\Delta z = 0$

$A = \frac{\pi D^2}{4} = \frac{\pi}{4}(0.1)^2\text{ m}^2 = 7.85\times10^{-3}\text{ m}^2$

$P = \rho R T \qquad \rho_1 = \frac{P_1}{RT_1} = 60\times10^3\frac{N}{m^2}\times\frac{kg\cdot K}{287\,N\cdot m}\times\frac{1}{325K} = 0.643\text{ kg/m}^3$

$\dot{m} = \rho V A \qquad V_1 = \frac{\dot{m}}{\rho A} = 1.42\frac{kg}{s}\times\frac{m^3}{0.643\,kg}\times\frac{1}{7.85\times10^{-3}\,m^2} = 281\text{ m/s}$

$M_1 = \frac{V_1}{c_1} \qquad c_1 = (kRT_1)^{1/2} = \left(1.4\times287\frac{N\cdot m}{kg\cdot K}\times325K\times\frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 362\text{ m/s}$

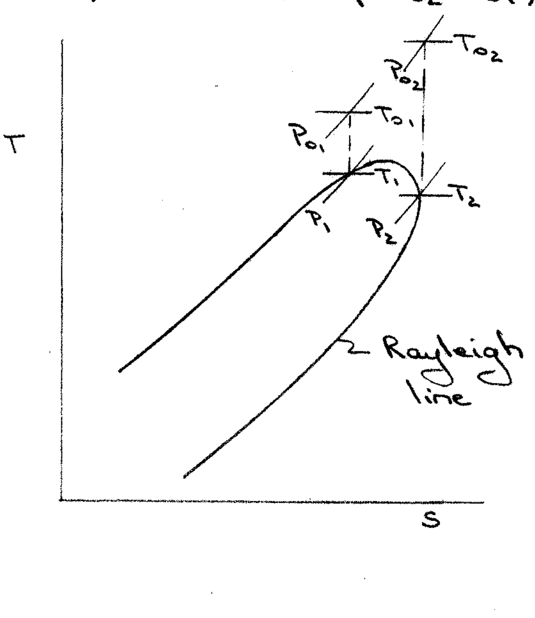
$M_1 = \frac{V_1}{c_1} = \frac{281}{362} = 0.776$

From App E.1, $T_1/T_{o_1} = 0.8925 \quad P_1/P_{o_1} = 0.6717 \quad \therefore T_{o_1} = 364K,\ P_{o_1} = 89.3\text{ kPa}$

From App E.3, $\frac{T_{o_1}}{T_o^*} = 0.9535 \quad \frac{P_{o_1}}{P_o^*} = 1.024 \quad \frac{T_1}{T^*} = 1.022 \quad \frac{P_1}{P^*} = 1.302 \quad \frac{V}{V^*} = \frac{\rho^*}{\rho} = 0.7844$

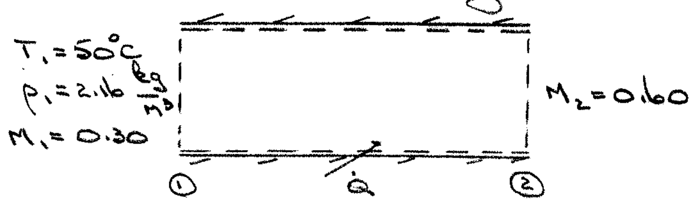
$\therefore T_{o_2} = 382K,\ P_{o_2} = 87.2\text{ kPa},\ T_2 = 318K,\ P_2 = 46.1\text{ kPa},\ \rho_2 = 0.504\frac{kg}{m^3},\ V_2 = 358\frac{m}{s}$ ← $(\ )_2$

$\frac{\delta Q}{dm} = h_{o_2} - h_{o_1} = c_p(T_{o_2} - T_{o_1}) = 1.0\frac{kJ}{kg\cdot K}(382-364)K = 18.0\text{ kJ/kg}$ ← $\delta Q/dm$



Problem 13.91

Given: Frictionless flow of air through a constant area duct



Find: $\delta Q/dm$, $s_2 - s_1$, $p_{0_1} - p_{0_2}$

Solution:

* Compressible flow functions (Appendix E) to be used in solution

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $Tds = dh - vdP$

Assumptions: (1) steady flow (4) ideal gas
(2) frictionless flow (5) $\dot{W}_s = \dot{W}_{shear} = 0$
(3) uniform flow at a section (6) $\Delta z = 0$

$p_1 = \rho_1 R T_1 = 2.16 \frac{kg}{m^3} \times 287 \frac{N \cdot m}{kg \cdot K} \times 323\ K = 200\ kPa$

$M_1 = 0.30$ From App. E.1 $T_1/T_{0_1} = 0.9823$ (11.17b), $p_1/p_{0_1} = 0.9395$ (11.17a)

$\therefore T_{0_1} = 329\ K$ $p_{0_1} = 213\ kPa$

From App. E.3 $T_{0_1}/T_0^* = 0.3469$ (12.30d), $p_{0_1}/p_0^* = 1.199$ (12.30e)

$\therefore T_0^* = 948\ K$ $p_0^* = 178\ kPa$

$M_2 = 0.60$ From App. E.3 $T_{0_2}/T_0^* = 0.8189$ (12.30d), $p_{0_2}/p_0^* = 1.075$ (12.30e)

$\therefore T_{0_2} = 776\ K$ $p_{0_2} = 191\ kPa$

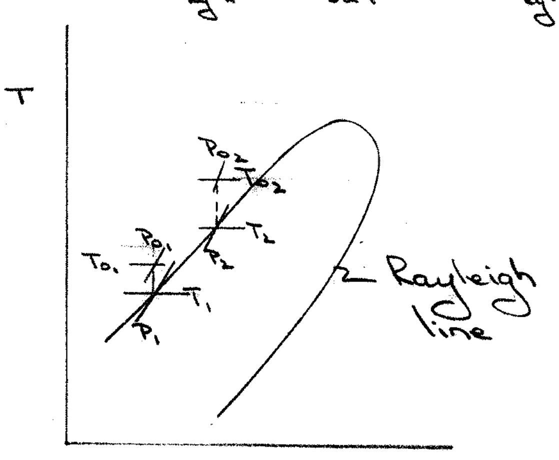
$p_{0_1} - p_{0_2} = (213 - 191)\ kPa = 22\ kPa$ ← $p_{0_1} - p_{0_2}$

$\delta Q/dm = h_{0_2} - h_{0_1} = c_p (T_{0_2} - T_{0_1}) = 1.0 \frac{kJ}{kg \cdot K} (776 - 329) K = 447\ kJ/kg$ ← $\delta Q/dm$

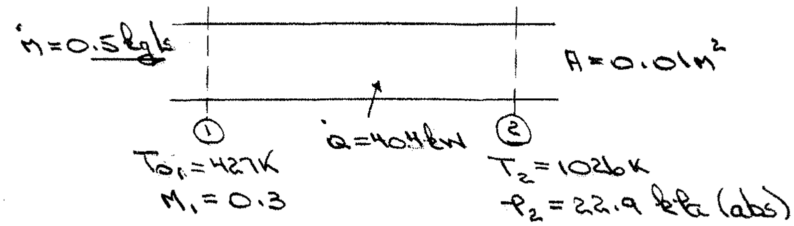
$Tds = dh - vdP = c_p dT - \frac{1}{\rho} dP$ $ds = c_p \frac{dT}{T} - R \frac{dP}{P}$

$s_2 - s_1 = s_{0_2} - s_{0_1} = \int_{s_{0_1}}^{s_{0_2}} ds = \int_{T_{0_1}}^{T_{0_2}} c_p \frac{dT}{T} - \int_{p_{0_1}}^{p_{0_2}} R \frac{dP}{P} = c_p \ln \frac{T_{0_2}}{T_{0_1}} - R \ln \frac{p_{0_2}}{p_{0_1}}$

$s_2 - s_1 = 1.0 \frac{kJ}{kg \cdot K} \ln \frac{776}{329} - 0.287 \frac{kJ}{kg \cdot K} \ln \frac{191}{213} = 0.889\ kJ/kg \cdot K$ ← $s_2 - s_1$



Given: Air enters an engine combustion chamber where heat is added during a frictionless process in a tube with constant area, $A = 0.01\,m^2$. Conditions are as shown.



Find: (a) M_2 (b) p_1 (c) $p_{0_2} - p_{0_1}$

Solution: * Compressible flow functions (Appendix E) to be used in solution

Basic equations: $h_{0_1} + \frac{\delta Q}{dm} = h_{0_2}$ $\quad \dot{m} = \rho V A \quad p = \rho R T$

Assumptions: (1) steady flow (2) frictionless flow
(3) ideal gas (4) uniform flow at a section
(5) $\dot{W}_s = \dot{W}_{shear} = 0$

From the energy equation, $\dot{Q} = \dot{m}(h_{0_2} - h_{0_1}) = \dot{m} c_p (T_{0_2} - T_{0_1})$

$$\therefore T_{0_2} = T_{0_1} + \frac{\dot{Q}}{\dot{m} c_p} = 427\,K + 404 \times 10^3\,W \times \frac{s}{0.5\,kg} \times \frac{kg \cdot K}{1004\,J} \times \frac{J}{W \cdot s} = 1232\,K$$

At $M_1 = 0.3$, from App. E.3, $T_{0_1}/T_0^* = 0.3469 \quad \therefore T_0^* = 1231\,K$

Since $T_{0_2} \approx T_0^*$, then $M_2 = 1.0$ ← M_2

$\therefore p_2 = p^* = 22.9\,kPa$

From Appendix E.3 for $M_1 = 0.3$, $p_1/p^* = 2.131 \quad \therefore p_1 = 48.8\,kPa$ ← p_1

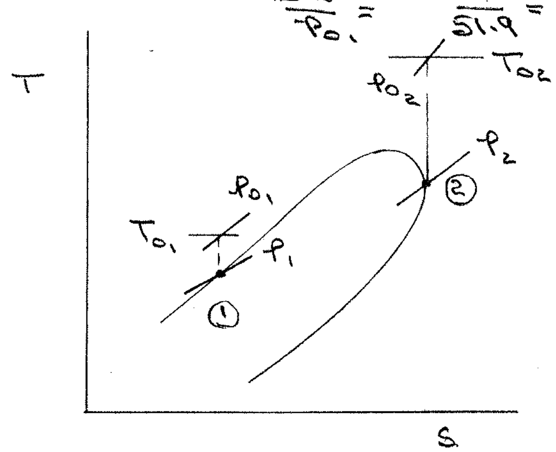
" " E.1 " $M_1 = 0.3$, $p_1/p_{0_1} = 0.9395 \quad \therefore p_{0_1} = 51.9\,kPa$

From Appendix E.1 for $M_2 = 1.0$, $p_2/p_{0_2} = 0.5283 \quad \therefore p_{0_2} = 43.3\,kPa$

$$\therefore p_{0_2} - p_{0_1} = (43.3 - 51.9)\,kPa = -8.6\,kPa$$

$$\frac{\Delta p_0}{p_{0_1}} = \frac{-8.6}{51.9} = -0.166 \quad (\text{or } -16.6\%)$$

← Δp_0



Problem 13.93

13.93 Flow in a gas turbine combustor is modeled as steady, one-dimensional, frictionless heating of air in a channel of constant area. For a certain process, the inlet conditions are 500°C, 1.5 MPa (abs), and $M = 0.5$. Calculate the maximum possible heat addition. Find all fluid properties at the outlet section and the reduction in stagnation pressure. Show the process path on a Ts diagram, indicating all static and stagnation state points.

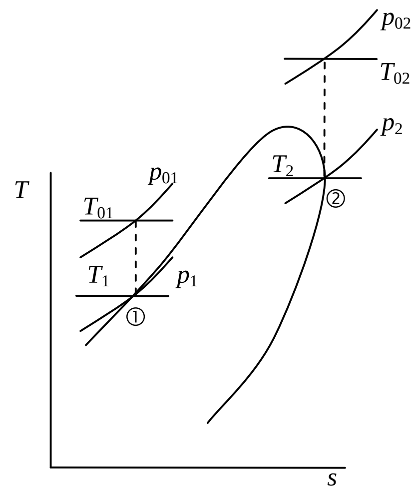
Given: Data on flow through gas turbine combustor

Find: Maximum heat addition; Outlet conditions; Reduction in stagnation pressure; Plot of process

Solution:

The given or available data is:

$R =$	286.9	J/kg·K
$k =$	1.4	
$c_p =$	1004	J/kg·K
$T_1 =$	773	K
$p_1 =$	1.5	MPa
$M_1 =$	0.5	



Equations and Computations:

From $p_1 = \rho_1 R T_1$ $\quad \rho_1 =$ 6.76 kg/m^3

From $V_1 = M_1\sqrt{kRT_1}$ $\quad V_1 =$ 279 m/s

Using built-in function *IsenT* (M,k):

$T_{01}/T_1 =$ 1.05 $\quad T_{01} =$ 812 K

Using built-in function *Isenp* (M,k):

$p_{01}/p_1 =$ 1.19 $\quad p_{01} =$ 1.78 MPa

For maximum heat transfer: $M_2 =$ 1

Using built-in function *rayT0* (M,k), *rayp0* (M,k), *rayT* (M,k), *rayp* (M,k), *rayV* (M,k):

$T_{01}/T_0^* =$	0.691	$T_0^* =$	1174	K	$(= T_{02})$
$p_{01}/p_0^* =$	1.114	$p_0^* =$	1.60	MPa	$(= p_{02})$
$T/T^* =$	0.790	$T^* =$	978	K	$(= T_{02})$
$p/p^* =$	1.778	$p^* =$	0.844	MPa	$(= p_2)$
$\rho^*/\rho =$	0.444	$\rho^* =$	3.01	kg/m^3	$(= \rho_2)$

Note that at state 2 we have critical conditions!

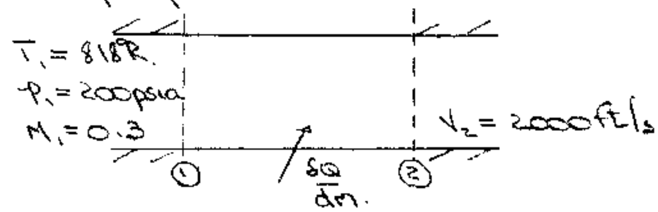
Hence: $p_{012} - p_{01} =$ -0.182 MPa $\quad$ -182 kPa

From the energy equation: $$\frac{\delta Q}{dm} = c_p\left(T_{02} - T_{01}\right)$$

$\delta Q/dm =$ 364 kJ/kg

Problem 13.94

Given: Combustor modeled as frictionless flow through a constant-area duct with heat addition. Air fuel ratio is large enough so properties are those of air



$A = 0.5\ ft^2$

Find: (a) T_2, p_2, ρ_2, M_2 (b) $\dot{Q}$

Solution: (using compressible flow functions - Appendix E)

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\quad p = \rho RT \quad \dot{m} = \rho V A$

Assumptions: (1) steady flow (2) frictionless flow
(3) ideal gas, properties are those of air
(4) uniform flow at a section
(5) $\dot{W}_s = \dot{W}_{shear} = 0$

$\rho_1 = \frac{p_1}{RT_1} = 200 \frac{lbf}{in^2} \times \frac{lbm\,°R}{53.3\,ft\,lbf} \times \frac{1}{818°R} \times 144 \frac{in^2}{ft^2} = 0.6606\ lbm/ft^3$

$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 0.3\left[1.4 \times 53.3 \frac{ft\,lbf}{lbm\,°R} \times 818°R \times 32.2 \frac{lbm}{slug} \times \frac{slug\,ft}{lbf\,s^2}\right]^{1/2} = 421\ ft/s$

From App. E.1 $\quad M_1 = 0.3 \quad T/T_0 = 0.9823 \quad \therefore T_{0_1} = 832.7°R$
$p/p_0 = 0.9395 \quad \therefore p_{0_1} = 212.9$ psia

From continuity, $\rho_1 V_1 = \rho_2 V_2$
$\therefore \rho_2 = \frac{V_1}{V_2}\rho_1 = \frac{421}{2000} \times 0.6606\ lbm/ft^3 = 0.139\ lbm/ft^3$ ← ρ_2

From App E.3
$M_1 = 0.3 \quad \frac{T_{0_1}}{T_0^*} = 0.3469 \quad \frac{p_{0_1}}{p_0^*} = 1.199 \quad \frac{T_1}{T^*} = 0.4089 \quad \frac{p_1}{p^*} = 2.131 \quad \frac{V_1}{V^*} = 0.1918$

$\therefore T_0^* = 2400R \quad p_0^* = 177.6$ psia $\quad T^* = 2000R \quad p^* = 93.85$ psia $\quad V^* = 2193\ \frac{ft}{sec}$

At section ② $\quad V_2/V^* = 2000/2193 = 0.9120$

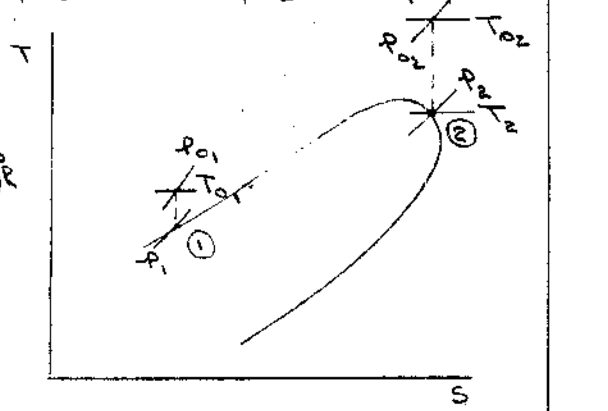
From App. E.3 $\quad M_2 = 0.90$ ← M_2

$T_{0_2}/T_0^* = 0.9921$, $p_{0_2}/p_0^* = 1.005$, $T_2/T^* = 1.025$, $p_2/p^* = 1.125$
$\therefore T_{0_2} = 2380R$, $p_{0_2} = 178$ psia, $T_2 = 2050R$, $p_2 = 106$ psia ← $T_{0_2}, p_{0_2}, T_2, p_2$

From the energy equation,
$\dot{Q} = \dot{m}\frac{\delta Q}{dm} = \rho_1 V_1 A (h_{0_2} - h_{0_1}) = \rho_1 V_1 A c_p (T_{0_2} - T_{0_1})$

$\dot{Q} = 0.6606 \frac{lbm}{ft^3} \times 420.6 \frac{ft}{s} \times 0.5\,ft^2 \times 0.24 \frac{BTU}{lbm\,°R} \times (2380 - 833)°R$

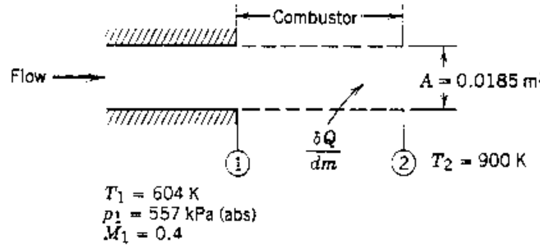
$\dot{Q} = 5.16 \times 10^4$ Btu/s ← $\dot{Q}$



Problem 13.95

Given: Steady flow combustor operating under conditions shown. Assume thermodynamic properties are those of pure air.

Find: (a) T_{o2}
(b) M_2
(c) $\delta Q/dm$, $\dot{Q}$
(d) $\dot{Q}/\dot{Q}_{max}$



Solution:
(using compressible flow functions - Appendix E)

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\dot{m} = \rho V A$ $p = \rho R T$

Assumptions: (1) steady flow (2) ideal gas
(3) uniform flow at a section
(4) $\dot{W}_s = \dot{W}_{shear} = 0$

$\rho_1 = \frac{p_1}{RT_1} = 557\times10^3 \frac{N}{m^2} \times \frac{kg\cdot K}{287 N\cdot m} \times \frac{1}{604 K} = 3.21\ kg/m^3$

$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 0.4\left(1.4\times287 \frac{N\cdot m}{kg\cdot K}\times 604K \times \frac{kg\cdot m}{N\cdot s^2}\right)^{1/2} = 197\ m/s$

$\dot{m} = \rho_1 V_1 A = 3.21 \frac{kg}{m^3}\times 197 \frac{m}{s} \times 0.0185\ m^2 = 11.7\ kg/s$

From App. E.1 at $M_1 = 0.4$, $T_1/T_{o1} = 0.969$ $\therefore T_{o1} = 623\ K$

From App. E.3 at $M_1 = 0.4$, $T/T^* = 0.6152$, $T_o/T_o^* = 0.5290$

$\therefore T^* = 982$ and $T_o^* = 1180\ K$

At section ② $T_2 = 900K$ $\therefore (T/T^*)_2 = 0.916$

From App. E.3, $M_2 = 0.60$, $(T_o/T_o^*)_2 = 0.8189$

$\therefore T_{o2} = 966\ K$, $M_2 = 0.60$ ← M_2, T_{o2}

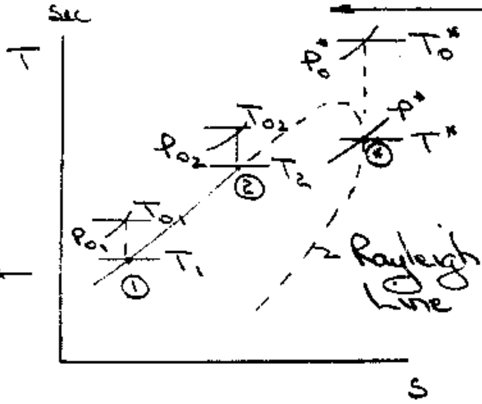
From the energy equation,

$\frac{\delta Q}{dm} = h_{o2} - h_{o1} = c_p(T_{o2} - T_{o1}) = 1.00 \frac{kJ}{kg\cdot K}(966-623)K = 343 \frac{kJ}{kg}$ ← $\delta Q/dm$

$\dot{Q} = \dot{m}\frac{\delta Q}{dm} = 11.7 \frac{kg}{sec}\times 343 \frac{kJ}{kg} = 4010 \frac{kJ}{sec} = 4010\ kW$ ← $\dot{Q}$

The heat transfer may be expressed as a fraction of the maximum possible heat addition

$\frac{\delta Q/dm}{(\delta Q/dm)_{max}} = \frac{T_{o2} - T_{o1}}{T_o^* - T_{o1}} = \frac{966-623}{1180-623} = 0.616$ ←

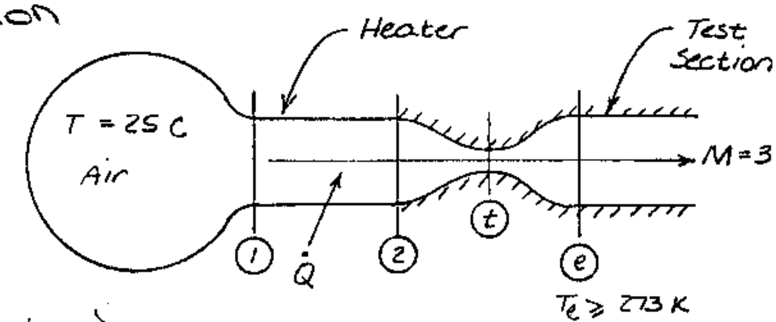


Problem 13.96

[3]

Given: Supersonic wind tunnel, with test section Mach number $M = 3$, is supplied from a high-pressure tank of air at 25°C. Air from the tank is heated in a short, constant area section upstream of the converging-diverging nozzle which feeds the test section. The heat addition, $\dot{Q} = 10$ kW is sufficient to ensure $T > 0$°C at the entrance to the test section

Find: (a) T_{0_2}
(b) $\dot{m}_{max}$
(c) A_e/A_t



Solution:

(using compressible flow functions)

Basic equations: $h_1 + \frac{V_1^2}{2} + \frac{\delta Q}{dm} = h_2 + \frac{V_2^2}{2}$ $\quad \dot{m} = \rho V A \quad p = \rho R T$

Assumptions: (1) steady flow (2) uniform flow at a section
(3) frictionless flow in the heater
(4) isentropic flow through the nozzle
(5) ideal gas (6) $\dot{W}_s = \dot{W}_{shear} = 0$

$T_{0_1} = T = 298$ K

From Appendix E.1 at $M = 3$, $T/T_0 = 0.3571$, $A_e/A^* = A_e/A_t = 4.23$ ← $\frac{A}{A_t}$

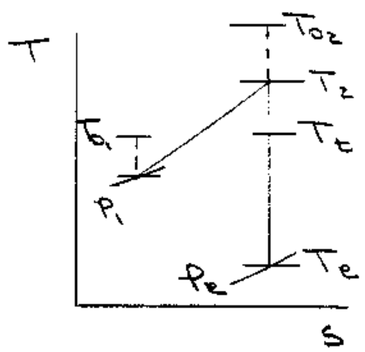
With $T_e = 273$K, $T_{0_e} = T_{0_2} = 273\text{K}/0.3571 = 764$ K ← T_{0_2}

From the energy equation $\frac{\delta Q}{dm} = h_{0_2} - h_{0_1} = c_p(T_{0_2} - T_{0_1})$

Since $\dot{Q} = \dot{m}\frac{\delta Q}{dm}$, then $\dot{m} = \frac{\dot{Q}}{c_p(T_{0_2} - T_{0_1})}$

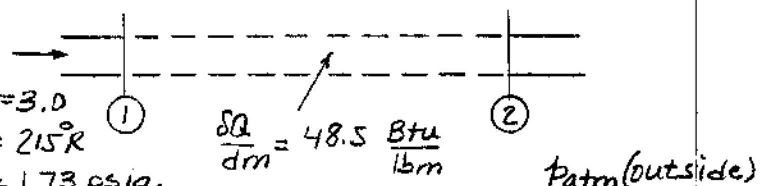
and

$$\dot{m} = 10\,\text{kW} \times \frac{10^3\,\text{N}\cdot\text{m}}{\text{s}\cdot\text{kW}} \times \frac{\text{kg}\cdot\text{K}}{1000\,\text{N}\cdot\text{m}} \times \frac{1}{(764-298)\text{K}} = 0.0215\ \text{kg/s}$$ ← $\dot{m}$



Problem 13.97

Given: Frictionless flow of air in a constant-area duct, discharging to atmospheric pressure, with the flow conditions shown.



$M_1 = 3.0$
$T_1 = 215°R$
$p_1 = 1.73$ psia

$\frac{\delta Q}{dm} = 48.5 \frac{Btu}{lbm}$

Find: (a) Compare p_2 with atmospheric pressure.
(b) Is M_2 greater than, equal to, or less than unity?
(c) Sketch p vs. x along the channel.

Solution: Apply equations for Rayleigh line flow of an ideal gas.

Basic equations: $c_p T_{01} + \frac{\delta Q}{dm} = c_p T_{02}$ $\quad p_1 A - p_2 A = \dot{m}(V_2 - V_1)$

Computing equation: $T_0 = T\left(1 + \frac{k-1}{2} M^2\right)$

Assumptions: (1) Steady flow
(2) Frictionless flow
(3) Uniform flow at each section
(4) Ideal gas
(5) $F_{B_x} = 0$
(6) $\dot{W}_s = \dot{W}_{shear} = 0$
(7) $\Delta z = 0$

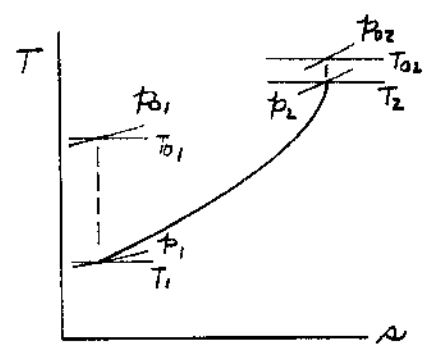
The minimum possible Mach number for supersonic flow with heating is $M_2 = 1$.

$T_{01} = T_1\left(1 + \frac{k-1}{2} M^2\right) = 215°R\left(1 + 0.2(3.0)^2\right) = 613°R$

$T_{02} = T_{01} + \frac{1}{c_p}\frac{\delta Q}{dm} = 613°R + \frac{lbm \cdot °R}{0.240\ Btu} \times 48.5 \frac{Btu}{lbm} = 815°R$

Check for $M_2 = 1.0$: $T_2 = \frac{T_{02}}{1 + 0.2(1)^2} = \frac{815°R}{1.2} = 679°R$

$V_2 = c_2 = \sqrt{kRT_2} = 1{,}280$ ft/s



Thus $p_2 = p_1 + \frac{\dot{m}}{A}(V_1 - V_2) = p_1 + \rho_1 V_1 (V_1 - V_2)$

$V_1 = M_1 c_1 = M_1 \sqrt{kRT_1} = 3.0 \times 719 \frac{ft}{s} = 2{,}160$ ft/s ; $\rho_1 = \frac{p_1}{RT_1} = 0.0217$ lbm/ft³

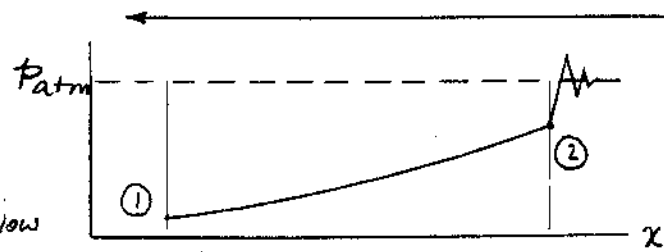
$p_2 = 1.73 \frac{lbf}{in.^2} + 0.0217 \frac{lbm}{ft^3} \times 2160 \frac{ft}{s} (2160 - 1280) \frac{ft}{s} \times \frac{slug}{32.2\ lbm} \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{ft^2}{144\ in.^2} = 10.6$ psia

Thus if $M_2 = 1.0$, the $p_2 < p_{atm}$, which is not possible for sonic flow. Therefore

$p_2 < p_{atm}$ and $M_2 > 1.0$ for this flow.

p_2
M_2

The pressure vs. distance plot is:



{This problem could be solved quantitatively, but only by either iteration or by use of compressible flow functions; that solution is presented on the next page.}

Problem 13.97

For $M_1 = 3.0$, from App. E.3: $\frac{T_0}{T_0^*} = 0.6540$, and $\frac{p}{p^*} = 0.1765$

Thus $T_0^* = \frac{T_{01}}{(T_0/T_0^*)_1} = \frac{613°R}{0.6540} = 937°R$; $p^* = \frac{p_1}{(p/p^*)_1} = \frac{1.73\ psia}{0.1765} = 9.80\ psia$

At section 2, $T_{02} = 815\ R$ and $\frac{T_{02}}{T_0^*} = \frac{815°R}{937°R} = 0.870$

From App. E.3 this corresponds to $M_2 = 1.74$. At this Mach number, $\frac{p}{p^*} = 0.4581$.

Thus $p_2 = p^* \left(\frac{p}{p^*}\right)_2 = 9.80\ psia \times 0.4581 = 4.49\ psia$ ← p_2

{These calculations confirm that $M_2 > 1$ and $p_2 < p_{atm}$.}

Given: Aircraft cabin pressurization system, $\dot{m} = 0.75$ lbm/s

Cruise is $M = 0.85$ at $z = 40{,}000$ ft (State 1)

Air slowed isentropically to 100 ft/s w.r.to plane (State 2)

Air compressed adiabatically to pressure equivalent to 8,000 ft altitude; $\Delta T = 170°F$ (State 3); $V_3 \simeq V_2$

Air is cooled at constant pressure, with negligible friction to 70°F (State 4); $V_4 \simeq V_3$

Find: (a) Sketch a system diagram, labeling all components.
(b) Determine static and stagnation temperatures at each cross-section.
(c) Evaluate compressor work added and heat rejected in cooling process.

Solution: From the standard atmosphere, at $z_1 = 40{,}000$ ft, $T_1 = -70°F$ and $p_1 = 2.73$ psia

$$c_1 = \sqrt{kRT_1} = \left[1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times (460-70)°R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{\frac{1}{2}} = 968 \frac{ft}{s}$$

$$V_1 = M_1 c_1 = 0.85 \times 968 \frac{ft}{s} = 823 \text{ ft/s}$$

For isentropic deceleration $T_{01} = T_1 + \frac{V_1^2}{2c_p} = T_{02} = T_2 + \frac{V_2^2}{2c_p}$

$$T_2 = T_1 + \frac{1}{2c_p}(V_1^2 - V_2^2)$$

$$T_2 = 390°R + \frac{1}{2} \times \frac{lbm \cdot °R}{0.240\ Btu}\left[(823)^2 - (100)^2\right]\frac{ft^2}{s^2} \times \frac{Btu}{778\ ft \cdot lbf} \times \frac{lbf \cdot s^2}{slug \cdot ft} \times \frac{slug}{32.2\ lbm} = 445°R$$

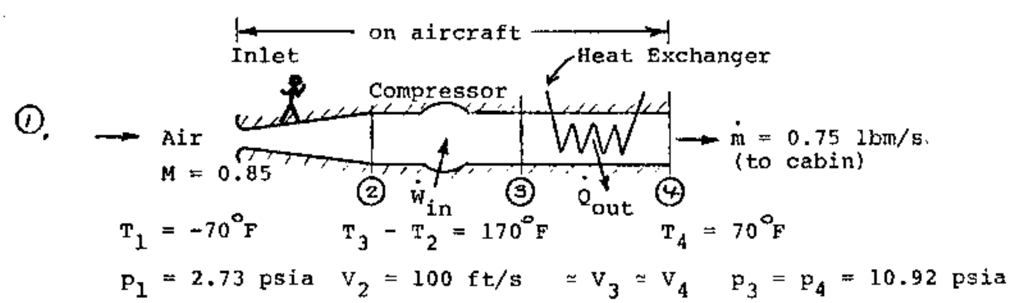
For isentropic deceleration,

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{k/k-1} = 2.73\text{ psia}\left(\frac{445}{390}\right)^{3.5} = 4.33\text{ psia}$$

The compressor raises the air the equivalent of $z_3 = z_4 = 8{,}000$ ft; from the Standard Atmosphere

$$p_3 = p_4 = 10.92\text{ psia};\ p_{03} = p_3\left(1 + \frac{k-1}{2}M_3^2\right)^{\frac{k}{k-1}} = 10.92\left(1 + 0.2(0.0823)^2\right)^{3.5} = 10.97\text{ psia}$$

Assume $V_2 \simeq V_3 \simeq V_4$ since no other data are known. The system is



Evaluating properties: $T_{01} = T_1\left(1+\frac{k-1}{2}M_1^2\right) = 390°R\,(1+0.2(0.85)^2) = 446°R = T_{02}$

$p_{01} = p_1\left(1+\frac{k-1}{2}M_1^2\right)^{\frac{k}{k-1}} = 2.73\ \text{psia}\,(1+0.2(0.85)^2)^{3.5} = 4.38\ \text{psia} = p_{02}$

$T_3 = T_2 + 170°R = 445 + 170°R = 615°R$; $T_{03} = T_{02} + 170°R = 616°R$ (since $V_2 \simeq$ const)

From the energy equation

$\dot{W}_{in} = \dot{m}(h_3 - h_2) = \dot{m}c_p(T_3 - T_2)$

$\dot{W}_{in} = 0.75\ \frac{\text{lbm}}{\text{s}} \cdot 0.240\ \frac{\text{Btu}}{\text{lbm}\cdot°R}\,(170)°R \times 778\ \frac{\text{ft}\cdot\text{lbf}}{\text{Btu}} \times \frac{\text{hp}\cdot\text{s}}{550\ \text{ft}\cdot\text{lbf}} = 43.3\ \text{hp}$ $\dot{W}_{in}$

$\dot{Q}_{in} = \dot{m}(h_4 - h_3) = \dot{m}c_p(T_4 - T_3)$

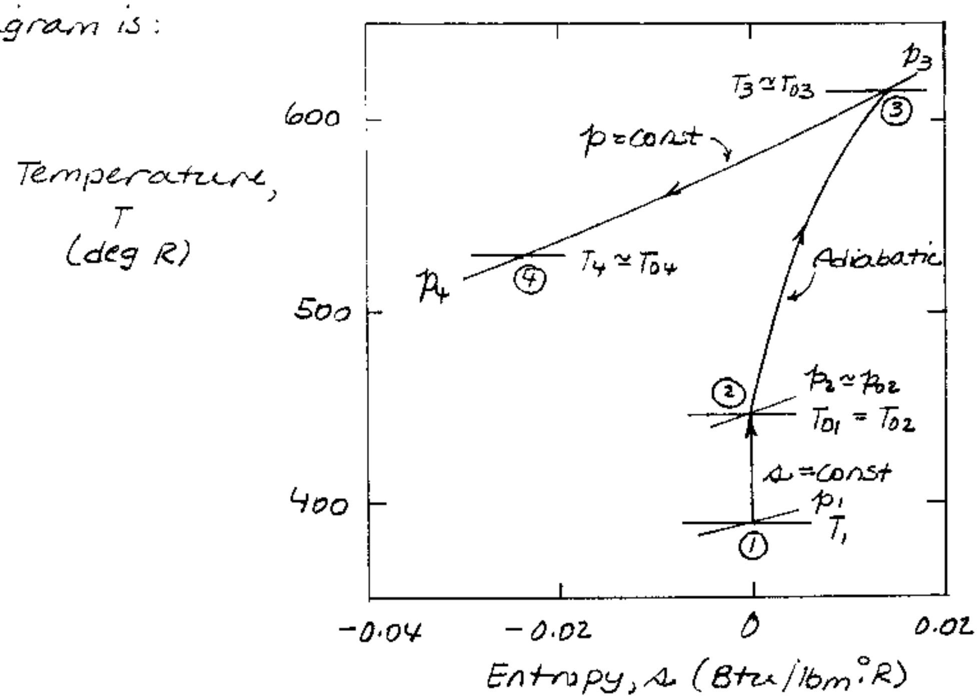
$\dot{Q}_{in} = 0.75\ \frac{\text{lbm}}{\text{s}} \cdot 0.240\ \frac{\text{Btu}}{\text{lbm}\cdot°R}\,(530 - 615)°R = -15.3\ \text{Btu/s}$ (out) $\dot{Q}_{in}$

The entropy changes are computed using $Tds = dh - vdp$, as

$\Delta s_{23} = c_p \ln\frac{T_{03}}{T_{02}} - R\ln\frac{p_{03}}{p_{02}} = 0.240\ \frac{\text{Btu}}{\text{lbm}\cdot°R}\ln\left(\frac{616}{446}\right) - 53.3\ \frac{\text{ft}\cdot\text{lbf}}{\text{lbm}\cdot°R}\ln\left(\frac{10.97}{4.38}\right)\frac{\text{Btu}}{778\ \text{ft}\cdot\text{lbf}} = 0.0146\ \frac{\text{Btu}}{\text{lbm}\cdot°R}$

$\Delta s_{34} = c_p \ln\frac{T_{04}}{T_{03}} - R\ln\frac{p_{04}}{p_{03}} = 0.240\ \frac{\text{Btu}}{\text{lbm}\cdot°R}\ln\left(\frac{530}{616}\right) - \frac{53.3}{778}\ \frac{\text{Btu}}{\text{lbm}\cdot°R}\ln\left(\frac{10.97}{10.97}\right) = -0.0361\ \text{Btu/lbm}\cdot°R$

The Ts diagram is:

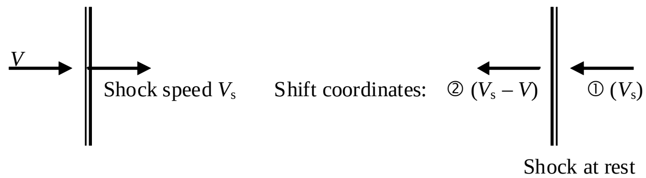


Problem 13.99

13.99 Testing of a demolition explosion is to be evaluated. Sensors indicate that the shock wave generated at the instant of explosion is 30 MPa (abs). If the explosion occurs in air at 20°C and 101 kPa, find the speed of the shock wave, and the temperature and speed of the air just after the shock passes. As an approximation assume $k = 1.4$. (Why is this an approximation?)

Given: Normal shock due to explosion

Find: Shock speed; temperature and speed after shock



Solution:

Basic equations:

$$M_2^2 = \frac{M_1^2 + \frac{2}{k-1}}{\left(\frac{2 \cdot k}{k-1}\right) \cdot M_1^2 - 1} \qquad V = M \cdot c = M \cdot \sqrt{k \cdot R \cdot T}$$

$$\frac{p_2}{p_1} = \frac{2 \cdot k}{k+1} \cdot M_1^2 - \frac{k-1}{k+1} \qquad \frac{T_2}{T_1} = \frac{\left(1 + \frac{k-1}{2} \cdot M_1^2\right) \cdot \left(k \cdot M_1^2 - \frac{k-1}{2}\right)}{\left(\frac{k+1}{2}\right)^2 \cdot M_1^2}$$

Given or available data $\quad k = 1.4 \quad R = 286.9 \cdot \frac{J}{kg \cdot K} \quad p_2 = 30 \cdot MPa \quad p_1 = 101 \cdot kPa \quad T_1 = (20 + 273) \cdot K$

From the pressure ratio $\quad M_1 = \sqrt{\left(\frac{k+1}{2 \cdot k}\right) \cdot \left(\frac{p_2}{p_1} + \frac{k-1}{k+1}\right)} \qquad M_1 = 16.0$

Then we have

$$T_2 = T_1 \cdot \frac{\left(1 + \frac{k-1}{2} \cdot M_1^2\right) \cdot \left(k \cdot M_1^2 - \frac{k-1}{2}\right)}{\left(\frac{k+1}{2}\right)^2 \cdot M_1^2} \qquad T_2 = 14790\,K \qquad T_2 = 14517 \cdot °C$$

$$M_2 = \sqrt{\frac{M_1^2 + \frac{2}{k-1}}{\left(\frac{2 \cdot k}{k-1}\right) \cdot M_1^2 - 1}} \qquad M_2 = 0.382$$

Then the speed of the shock ($V_s = V_1$) is $\quad V_1 = M_1 \cdot \sqrt{k \cdot R \cdot T_1} \quad V_1 = 5475 \frac{m}{s} \quad V_s = V_1 \quad V_s = 5475 \frac{m}{s}$

After the shock (V_2) the speed is $\quad V_2 = M_2 \cdot \sqrt{k \cdot R \cdot T_2} \quad V_2 = 930 \frac{m}{s}$

But we have $\quad V_2 = V_s - V \quad V = V_s - V_2 \quad V = 4545 \frac{m}{s}$

These results are unrealistic because at the very high post-shock temperatures experienced, the specific heat ratio will NOT be constant! The extremely high initial air velocity and temperature will rapidly decrease as the shock wave expands in a spherical manner and thus weakens.

Problem 13.100

13.100 A large tank containing air at 125 psia and 175°F is attached to a converging-diverging nozzle that has a throat area of 1.5 in^2 through which the air is exiting. A normal shock sits at a point in the nozzle where the area is 2.5 in^2. The nozzle exit area is 3.5 in^2. What are the Mach numbers just after the shock and at the exit? What are the stagnation and static pressures before and after the shock?

Given: C-D nozzle with normal shock

Find: Mach numbers at the shock and at exit; Stagnation and static pressures before and after the shock

Solution:

Basic equations: Isentropic flow

$$\frac{A}{A_{crit}} = \frac{1}{M}\cdot\left(\frac{1+\frac{k-1}{2}\cdot M^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}} \qquad \frac{p_0}{p} = \left(1+\frac{k-1}{2}\cdot M^2\right)^{\frac{k}{k-1}}$$

Normal shock

$$M_2^{\,2} = \frac{M_1^{\,2}+\frac{2}{k-1}}{\left(\frac{2\cdot k}{k-1}\right)\cdot M_1^{\,2}-1} \qquad \frac{p_2}{p_1} = \frac{2\cdot k}{k+1}\cdot M_1^{\,2}-\frac{k-1}{k+1} \qquad \frac{p_{02}}{p_{01}} = \frac{\left(\frac{\frac{k+1}{2}\cdot M_1^{\,2}}{1+\frac{k-1}{2}\cdot M_1^{\,2}}\right)^{\frac{k}{k-1}}}{\left(\frac{2\cdot k}{k+1}\cdot M_1^{\,2}-\frac{k-1}{k+1}\right)^{\frac{1}{k-1}}}$$

Given or available data $k = 1.4$ $\quad R_{air} = 53.33\cdot\frac{ft\cdot lbf}{lbm\cdot R}$ $\quad p_{01} = 125\cdot psi$ $\quad T_0 = (175+460)\cdot R$

$A_t = 1.5\cdot in^2$ $\quad A_s = 2.5\cdot in^2$ $\quad$ (Shock area) $\quad A_e = 3.5\cdot in^2$

Because we have a normal shock the CD must be accelerating the flow to supersonic so the throat is at critical state.

$$A_{crit} = A_t$$

At the shock we have $\frac{A_s}{A_{crit}} = 1.667$

At this area ratio we can find the Mach number before the shock from the isentropic relation $\frac{A_s}{A_{crit}} = \frac{1}{M_1}\cdot\left(\frac{1+\frac{k-1}{2}\cdot M_1^{\,2}}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}}$

Solving iteratively (or using *Excel*'s *Solver*, or even better the function *isenMsupfromA* from the Web site!) $\quad M_1 = 1.985$

The stagnation pressure before the shock was given: $\quad p_{01} = 125\,psi$

The static pressure is then

$$p_1 = \frac{p_{01}}{\left(1+\frac{k-1}{2}\cdot M_1^{\,2}\right)^{\frac{k}{k-1}}} \qquad p_1 = 16.4\,psi$$

After the shock we have $$M_2 = \sqrt{\frac{M_1^2 + \frac{2}{k-1}}{\left(\frac{2\cdot k}{k-1}\right)\cdot M_1^2 - 1}}$$ $M_2 = 0.580$

Also $$p_{02} = p_{01}\cdot\frac{\left(\frac{\frac{k+1}{2}\cdot M_1^2}{1 + \frac{k-1}{2}\cdot M_1^2}\right)^{\frac{k}{k-1}}}{\left(\frac{2\cdot k}{k+1}\cdot M_1^2 - \frac{k-1}{k+1}\right)^{\frac{1}{k-1}}}$$ $p_{02} = 91.0\,\text{psi}$

and $$p_2 = p_1\cdot\left(\frac{2\cdot k}{k+1}\cdot M_1^2 - \frac{k-1}{k+1}\right)$$ $p_2 = 72.4\,\text{psi}$

Finally, for the Mach number at the exit, we could find the critical area change across the shock; instead we find the new critical area from isentropic conditions at state 2.

$$A_{crit2} = A_s\cdot M_2\cdot\left(\frac{1 + \frac{k-1}{2}\cdot M_2^2}{\frac{k+1}{2}}\right)^{-\frac{k+1}{2\cdot(k-1)}}$$ $A_{crit2} = 2.06\,\text{in}^2$

At the exit we have $$\frac{A_e}{A_{crit2}} = 1.698$$

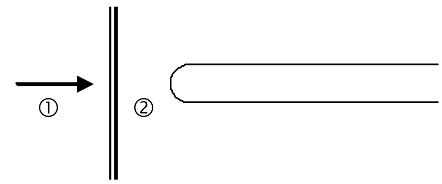
At this area ratio we can find the Mach number before the shock from the isentropic relation $$\frac{A_e}{A_{crit2}} = \frac{1}{M_e}\cdot\left(\frac{1 + \frac{k-1}{2}\cdot M_e^2}{\frac{k+1}{2}}\right)^{\frac{k+1}{2\cdot(k-1)}}$$

Solving iteratively (or using *Excel*'s *Solver*, or even better the function *isenMsubfromA* from the Web site!) $M_e = 0.369$

These calculations are obviously a LOT easier using the *Excel* functions available on the Web site!

Problem 13.101

13.101 A normal shock occurs when a pitot-static tube is inserted into a supersonic wind tunnel. Pressures measured by the tube are $p_{0_2} = 10$ psia and $p_2 = 8$ psia. Before the shock, $T_1 = 285°$R and $p_1 = 1.5$ psia. Calculate the air speed in the wind tunnel.



Given:
Normal shock near pitot tube

Find:
Air speed

Solution:

Basic equations: $p_1 - p_2 = \rho_1 \cdot V_1 \cdot (V_2 - V_1)$ (Momentum) $\quad \dfrac{p_0}{p} = \left(1 + \dfrac{k-1}{2} \cdot M^2\right)^{\frac{k}{k-1}}$

Given or available data $\quad T_1 = 285 \cdot R \quad p_1 = 1.75 \cdot psi \quad p_{02} = 10 \cdot psi \quad p_2 = 8 \cdot psi$

$k = 1.4 \quad R_{air} = 53.33 \cdot \dfrac{ft \cdot lbf}{lbm \cdot R}$

At state 2 $\quad M_2 = \sqrt{\dfrac{2}{k-1} \cdot \left[\left(\dfrac{p_{02}}{p_2}\right)^{\frac{k-1}{k}} - 1\right]} \quad M_2 = 0.574$

From momentum $\quad p_1 - p_2 = \rho_2 \cdot V_2^2 - \rho_1 \cdot V_1^2$ but $\rho \cdot V^2 = \rho \cdot c^2 \cdot M^2 = \dfrac{p}{R \cdot T} \cdot k \cdot R \cdot T \cdot M^2 = k \cdot p \cdot M^2$

$p_1 - p_2 = k \cdot p_2 \cdot M_2^2 - k \cdot p_1 \cdot M_1^2$ or $p_1 \cdot \left(1 + k \cdot M_1^2\right) = p_2 \cdot \left(1 + k \cdot M_2^2\right)$

Hence $\quad M_1 = \sqrt{\dfrac{1}{k} \cdot \left[\dfrac{p_2}{p_1} \cdot \left(1 + k \cdot M_2^2\right) - 1\right]} \quad M_1 = 2.01$

Also $\quad c_1 = \sqrt{k \cdot R_{air} \cdot T_1} \quad c_1 = 827 \dfrac{ft}{s}$

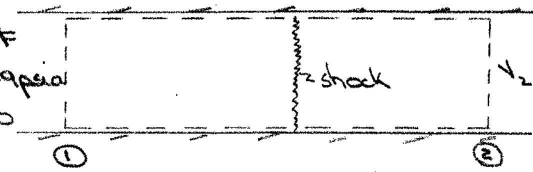
Then $\quad V_1 = M_1 \cdot c_1 \quad V_1 = 1666 \dfrac{ft}{s}$

Note: With $p_1 = 1.5$ psi we obtain $\quad V_1 = 1822 \cdot \dfrac{ft}{s}$

(Using normal shock functions, for $\dfrac{p_2}{p_1} = 4.571$ we find $\quad M_1 = 2.02 \quad M_2 = 0.573$ Check!)

Problem 13.102

Given: Steady flow of air through a constant area duct



Find: ρ_2, M_2, sketch pressure distribution

Solution:

Basic equation: $h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$

Assumptions:
(1) steady flow
(2) uniform flow at a section
(3) $\dot{Q} = \dot{W}_s = \dot{W}_{shear} = 0$
(4) $\Delta z = 0$
(5) ideal gas
(6) $A_1 = A_2 = A$

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 2.0\left(1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times 600°R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{1/2} = 2400 \text{ ft/s}$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \qquad T_2 = T_1 + \frac{1}{2c_p}(V_1^2 - V_2^2)$$

$$T_2 = 600°R + \frac{1}{2}\left[(2.4)^2 - (1.08)^2\right] \times 10^6 \frac{ft^2}{s^2} \times \frac{lbm \cdot °R}{0.24 \, Btu} \times \frac{Btu}{778 \, ft \cdot lbf} \times \frac{slug}{32.2 \, lbm} \times \frac{lbf \cdot s^2}{slug \cdot ft}$$

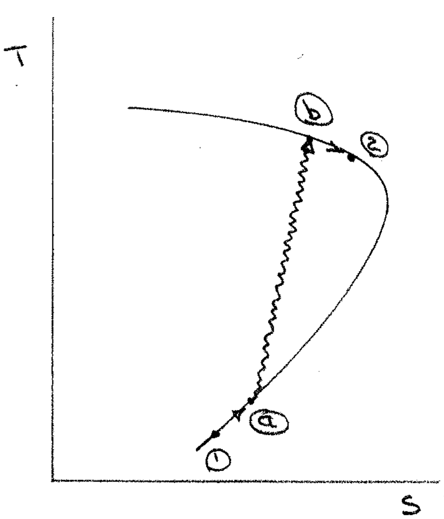
$$T_2 = 982°R$$

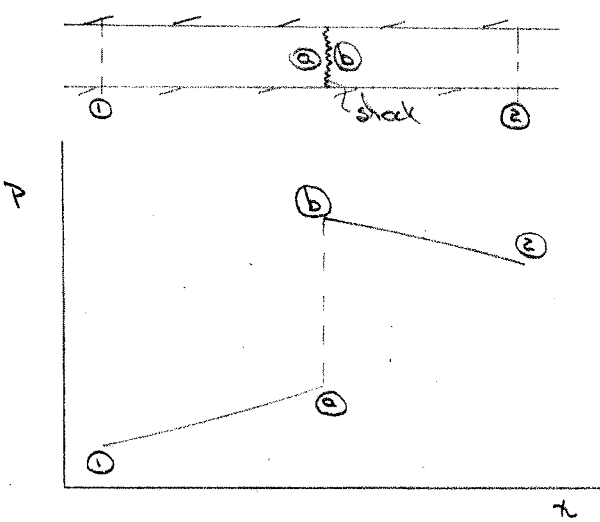
$$M_2 = \frac{V_2}{c_2} \qquad c_2 = (kRT_2)^{1/2} = \left(1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times 982°R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right)^{1/2} = 1540 \text{ ft/s}$$

$$M_2 = \frac{1080}{1540} = 0.701 \longleftarrow M_2$$

From continuity $\rho_1 V_1 = \rho_2 V_2 \qquad \rho_2 = \frac{V_1}{V_2}\rho_1 = \frac{V_1}{V_2}\frac{p_1}{RT_1}$

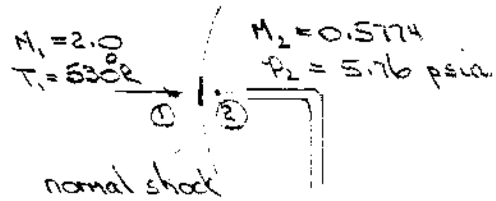
$$\rho_2 = \frac{2400}{1080} \times 35.9 \frac{lbf}{in^2} \times \frac{lbm \cdot °R}{53.3 \, ft \cdot lbf} \times \frac{1}{600°R} \times \frac{144 \, in^2}{ft^2} = 0.359 \, lbm/ft^3 \longleftarrow \rho_2$$





Problem 13.103

Given: A total-pressure probe is placed in a supersonic flow, $M_1 = 2.0$. Behind the shock $M_2 = 0.5774$, $p_2 = 5.76$ psia



Find: a) T_{0_2}, p_{0_2} (b) $p_{0_1}, T_{0_1}, p_1, \rho_1, V_1$

Solution:

Computing equations: $p(1+kM^2) = \text{const.}$ (across shock)

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \qquad \frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{k/k-1}$$

Assumptions: (1) steady flow (3) uniform flow at a section (2) ideal gas

Across the shock $p(1+kM^2) = \text{const}$

$$\therefore p_1 = p_2\frac{(1+kM_2^2)}{(1+kM_1^2)} = 5.76\text{ psia}\frac{[1+1.4(0.5774)^2]}{[1+1.4(2)^2]} = 1.28\text{ psia} \quad \longleftarrow p_1$$

$$\rho_1 = \frac{p_1}{RT_1} = 1.28\frac{lbf}{in^2} \times \frac{lbm\,R}{53.3\,ft\,lbf} \times \frac{1}{530\,R} \times 144\frac{in^2}{ft^2} = 0.00653\text{ lbm/ft}^3 \quad \longleftarrow \rho_1$$

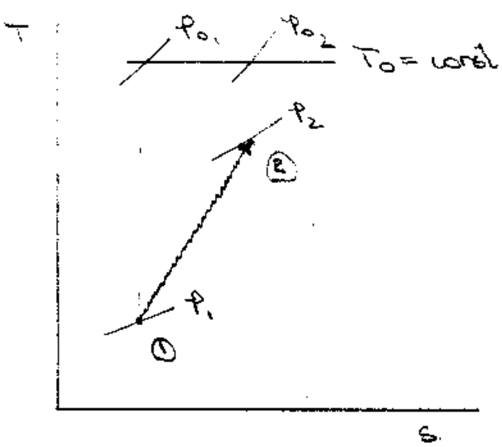
$$V_1 = M_1c_1 = M_1(kRT_1)^{1/2} = 2.0\left[1.4 \times 53.3\frac{ft\,lbf}{lbm\,R} \times 530\,R \times 32.2\frac{lbm}{slug} \times \frac{slug\,ft}{lbf\,s^2}\right]^{1/2} = 2260\text{ ft/s} \quad \longleftarrow V_1$$

$$T_{0_1} = T_1\left(1+\frac{k-1}{2}M_1^2\right) = 530\,R[1+0.2(2)^2] = 954\,R \quad \longleftarrow T_{0_1}$$

$$p_{0_1} = p_1\left(1+\frac{k-1}{2}M_1^2\right)^{k/k-1} = 1.28\text{ psia}(1.8)^{3.5} = 10.0\text{ psia} \quad \longleftarrow p_{0_1}$$

$$p_{0_2} = p_2\left(1+\frac{k-1}{2}M_2^2\right)^{k/k-1} = 5.76\text{ psia}[1+0.2(0.5774)^2]^{3.5} = 7.22\text{ psia} \quad \longleftarrow p_{0_2}$$

$$T_{0_2} = T_{0_1} = 954\,R \quad \longleftarrow$$



Compressible-Flow Functions (Appendix E)

For $M_1 = 2.0$, from App E.4

$M_2 = 0.577$ (12.34b)

$p_2/p_1 = 4.50$ (12.34a) $\therefore p_1 = 1.28$ psia

$p_{0_2}/p_{0_1} = 0.721$ (12.37)

For $M_1 = 2.0$, from App. E.1

$p_1/p_{0_1} = 0.1278$ (11.17a) $\therefore p_{0_1} = 10.0$ psia

and $p_{0_2} = 0.721\,p_{0_1} = 7.21$ psia

$T_1/T_0 = 0.556$ (11.17b) $\therefore T_{0_1} = 954\,°R$

Note: In using the tables it is not necessary to know the downstream Mach number.

Problem 13.104

13.104 Air approaches a normal shock at $M_1 = 2.5$, with $T_{0_1} = 1250°\text{R}$ and $p_1 = 20$ psia. Determine the speed and temperature of the air leaving the shock and the entropy change across the shock.

Given: Normal shock

Find: Speed and temperature after shock; Entropy change

Solution:

The given or available data is:

$R =$	53.33	ft·lbf/lbm·R	0.0685	Btu/lbm·R
$k =$	1.4			
$c_p =$	0.2399	Btu/lbm·R		
$T_{01} =$	1250	$^{\text{o}}$R		
$p_1 =$	20	psi		
$M_1 =$	2.5			

Equations and Computations:

From $p_1 = \rho_1 R T_1$ $\qquad \rho_1 =$ 300.02 kg/m^3

$V_1 =$ 764 m/s

Using built-in function *IsenT* (M,k):

$T_{01}/T_1 =$ 2.25 $\qquad T_1 =$ 556 $^{\text{o}}$R

96 $^{\text{o}}$F

Using built-in function *NormM2fromM* (M,k):

$M_2 =$ 0.513

Using built-in function *NormTfromM* (M,k):

$T_2/T_1 =$ 2.14 $\qquad T_2 =$ 1188 $^{\text{o}}$R

728 $^{\text{o}}$F

Using built-in function *NormpfromM* (M,k):

$p_2/p_1 =$ 7.13 $\qquad p_2 =$ 143 psi

From $V_2 = M_2\sqrt{kRT_2}$ $\qquad V_2 =$ 867 ft/s

From $$\Delta s = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$

$\Delta s =$ 0.0476 Btu/lbm·R

37.1 ft·lbf/lbm·R

Given: Air flow through a normal shock as shown:

$T_1 = 35°C$

$P_1 = 229$ kPa (abs)

$V_1 = 704$ m/sec

Find: T_2, P_{o_2}

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions:
(1) steady flow
(2) uniform flow at a section
(3) $\dot{Q} = \dot{W}_s = \dot{W}_{shear} = 0$
(4) $\Delta z = 0$
(5) $F_{B_x} = 0$
(6) no friction forces
(7) ideal gas
(8) $A_1 = A_2 = A$

$M_1 = \frac{V_1}{c_1}$ $\quad c_1 = (kRT_1)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 308\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 352$ m/s

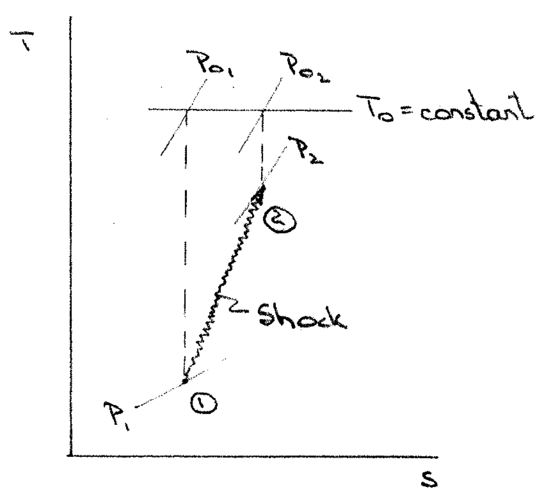
$M_1 = \frac{704}{352} = 2.00$

From App. E.1 $\quad P_1/P_{o_1} = 0.1278$

From App. E.4 $\quad P_{o_2}/P_{o_1} = 0.7209 \quad T_2/T_1 = 1.687$

$T_2 = \frac{T_2}{T_1} \times T_1 = 1.687 \times 308\,K = 520\,K$ ——— T_2

$P_{o_2} = \frac{P_{o_2}}{P_{o_1}} \times \frac{P_{o_1}}{P_1} \times P_1 = 0.7209 \times \frac{1}{0.1278} \times 229\ kPa = 1.29$ MPa (abs) ——— P_{o_2}



Problem 13.106

13.106 A normal shock stands in a constant-area duct. Air approaches the shock with $T_{0_1} = 550\,\text{K}$, $p_{0_1} = 650\,\text{kPa (abs)}$, and $M_1 = 2.5$. Determine the static pressure downstream from the shock. Compare the downstream pressure with that reached by decelerating isentropically to the same subsonic Mach number.

Given: Normal shock

Find: Pressure after shock; Compare to isentropic deceleration

Solution:

The given or available data is:

$R =$	286.9	J/kg·K
$k =$	1.4	
$T_{01} =$	550	K
$p_{01} =$	650	kPa
$M_1 =$	2.5	

Equations and Computations:

Using built-in function *Isenp* (M,k):

$p_{01}/p_1 =$ 17.09 $\qquad p_1 =$ 38 kPa

Using built-in function *NormM2fromM* (M,k):

$M_2 =$ 0.513

Using built-in function *NormpfromM* (M,k):

$p_2/p_1 =$ 7.13 $\qquad p_2 =$ 271 kPa

Using built-in function *Isenp* (M,k) at M_2:

$p_{02}/p_2 =$ 1.20

But for the isentropic case: $p_{02} = p_{01}$

Hence for isentropic deceleration: $p_2 =$ 543 kPa

Problem 13.107

13.107 A normal shock occurs in air at a section where $V_1 =$ 2000 mph, $T_1 = -15°F$, and $p_1 = 5$ psia. Determine the speed and Mach number downstream from the shock, and the change in stagnation pressure across the shock.

Given: Normal shock

Find: Speed and Mach number after shock; Change in stagnation pressure

Solution:

The given or available data is:

$R =$	53.33	ft·lbf/lbm·R	0.0685	Btu/lbm·R
$k =$	1.4			
$T_1 =$	445	^oR		
$p_1 =$	5	psi		
$V_1 =$	2000	mph	2933	ft/s

Equations and Computations:

From $c_1 = \sqrt{kRT_1}$ $\quad c_1 =$ 1034 ft/s

Then $\quad M_1 =$ 2.84

Using built-in function *NormM2fromM* (M,k):

$M_2 =$ 0.486

Using built-in function *NormdfromM* (M,k):

$\rho_2/\rho_1 =$ 3.70

Using built-in function *Normp0fromM* (M,k):

$p_{02}/p_{01} =$ 0.378

Then $V_2 = \dfrac{\rho_1}{\rho_2} V_1$ $\quad V_2 =$ 541 mph $\quad$ 793 ft/s

Using built-in function *Isenp* (M,k) at M_1:

$p_{01}/p_1 =$ 28.7

From the above ratios and given p_1:

$p_{01} =$ 143 psi

$p_{02} =$ 54.2 psi

$p_{01} - p_{02} =$ 89.2 psi

Problem 13.108

[2]

13.108 Air approaches a normal shock with $T_1 = -7.5°F$, $p_1 = 14.7$ psia, and $V_1 = 1750$ mph. Determine the speed immediately downstream from the shock and the pressure change across the shock. Calculate the corresponding pressure change for a frictionless, shockless deceleration between the same speeds.

Given: Normal shock

Find: Speed; Change in pressure; Compare to shockless deceleration

Solution:

The given or available data is:

$R =$	53.33	ft·lbf/lbm·R	0.0685	Btu/lbm·R
$k =$	1.4			
$T_1 =$	452.5	^oR		
$p_1 =$	14.7	psi		
$V_1 =$	1750	mph	2567	ft/s

Equations and Computations:

From $c_1 = \sqrt{kRT_1}$ $\quad c_1 =$ 1043 ft/s

Then $\quad M_1 =$ 2.46

Using built-in function *NormM2fromM* (M,k):

$M_2 =$ 0.517

Using built-in function *NormdfromM* (M,k):

$\rho_2/\rho_1 =$ 3.29

Using built-in function *NormpfromM* (M,k):

$p_2/p_1 =$ 6.90 $\quad p_2 =$ 101 psi

$p_2 - p_1 =$ 86.7 psi

Then $V_2 = \dfrac{\rho_1}{\rho_2}V_1$ $\quad V_2 =$ 532 mph $\quad$ 781 ft/s

Using built-in function *Isenp* (M,k) at M_1:

$p_{01}/p_1 =$ 16.1

Using built-in function *Isenp* (M,k) at M_2:

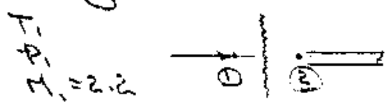
$p_{02}/p_2 =$ 1.20

From above ratios and p_1, for isentropic flow (p_0 = const): $\quad p_2 =$ 197 psi

$p_2 - p_1 =$ 182 psi

Problem 13.109

Given: Supersonic aircraft cruises at M = 2.2 at 12 km altitude. Normal shock stands in front of a pitot tube which senses a stagnation pressure, p_{02}.



Find: (a) T_{01}, p_{01} (b) p_{02}

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) uniform flow at a section
(3) thin shock
(4) ideal gas

Use table A.3 to determine properties at state ① At 12 km altitude,

$T_1 = 216.7$ K $\quad p_1 = 19.4$ kPa

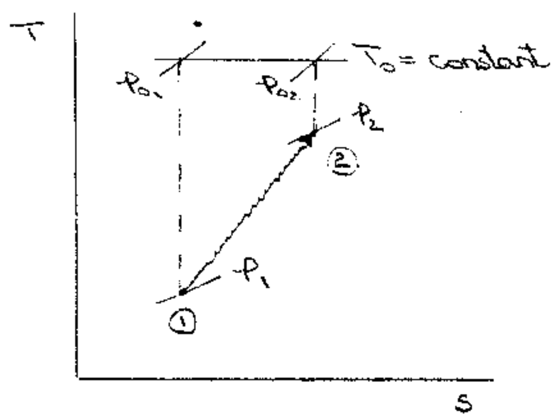
From App. E.1, for $M_1 = 2.2$,

$T_1/T_{01} = 0.5081 \quad \therefore T_{01} = 426$ K ——— T_{01}

$p_1/p_{01} = 0.09352 \quad \therefore p_{01} = 207$ kPa (abs) ——— p_{01}

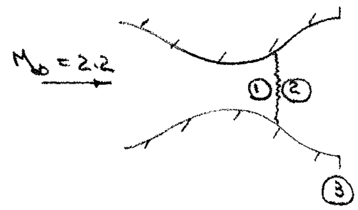
From App. E.4, for $M_1 = 2.2$, $M_2 = 0.5471$

$p_{02}/p_{01} = 0.6281 \quad \therefore p_{02} = 130$ kPa (abs) ——— p_{02}



Problem 13.110

Given: Concorde flew at $M = 2.2$ at an altitude of 20 km. In the engine inlet system the air is decelerated isentropically to a local Mach number of 1.3; air undergoes a normal shock and then decelerates isentropically to $M = 0.4$.



$M_1 = 1.3$

$M_3 = 0.4$

Find: T_3, P_3, P_{o3}

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow
(2) ideal gas
(3) flow is isentropic except across the shock.

At 20 km altitude $T_\infty = 217\text{ K}$, $P_\infty = 5.53\text{ kPa}$. (Table A.3)

$M_\infty = 2.2$ From App. E.1, $T_\infty / T_o = 0.5081$ $P_\infty / P_o = 0.09352$

$\therefore T_o = 427\text{ K}$ $P_o = 59.1\text{ kPa}$

$M_1 = 1.3$ From App. E.1, $T_1/T_o = 0.7474$ $P_1/P_o = 0.3609$

$\therefore T_1 = 319\text{ K}$ $P_1 = 21.3\text{ kPa}$

From App. E.4, $M_2 = 0.786$ $P_{o2}/P_{o1} = 0.9794$ $\frac{T_2}{T_1} = 1.191$ $\frac{P_2}{P_1} = 1.805$

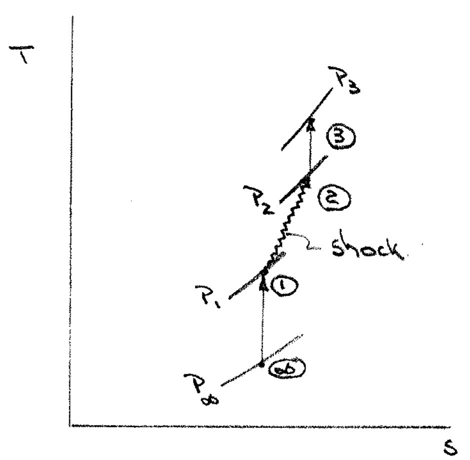
$\therefore P_{o2} = 57.9\text{ kPa}$, $T_2 = 380\text{ K}$, $P_2 = 38.4\text{ kPa}$.

$P_{o3} = P_{o2} = 57.9\text{ kPa (abs)}$ ← P_{o3}

$M_3 = 0.4$ From App. E.1, $T_3/T_o = 0.9690$ $P_3/P_{o3} = 0.8956$.

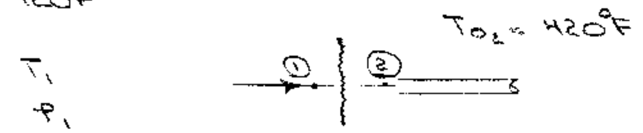
$\therefore T_3 = 414\text{ K}.$ ← T_3

$P_3 = 0.8956\, P_{o3} = 0.8956\, P_{o2} = 51.9\text{ kPa (abs)}$ ← P_3



Problem 13.111

Given: Supersonic aircraft flies at 35000 ft. Normal shock stands in front of stagnation temperature probe. Probe reads $T_0 = 420°F$



Find: (a) M_1, V_1 (b) p_2, p_{0_2}

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) uniform flow at a section
(3) thin shock, $R_x = 0$
(4) ideal gas

Use table A.3 to determine properties at state ①.

Altitude $= 35000\,ft \times 0.3048 \frac{m}{ft} = 10{,}670\ m$

From Table A.3 $\quad T_1 = 219\,K = -54°C = -65°F$

$$p_1 = 23.9\times10^3 \frac{N}{m^2} \times \frac{lbf}{4.448\,N} \times \left(\frac{0.3048\,m}{ft}\right)^2 = 499\ psfa$$

At state ① , $T_1/T_{0_1} = 395/880 = 0.4489$

From App. E.1, for $T_1/T_{0_1} = 0.4489$, $M_1 = 2.48$ ← M_1

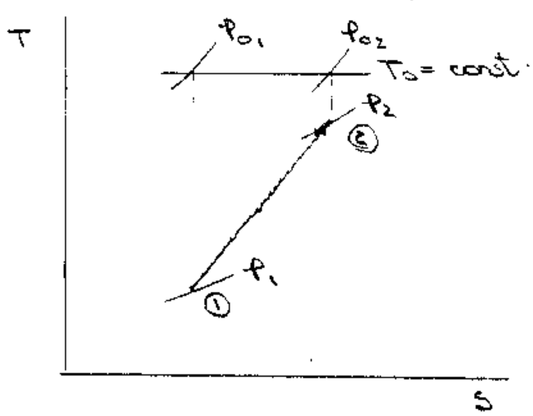
$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 2.48\left[1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times 395°R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{1/2} = 2420\ ft/s$ ← V_1

From App. E.1, for $M_1 = 2.48$, $p_1/p_{0_1} = 0.06038$ $\therefore p_{0_1} = 57.4$ psia

From App. E.4, for $M_1 = 2.48$, $M_2 = 0.5149$

$p_{0_2}/p_{0_1} = 0.5071 \quad \therefore p_{0_2} = 29.1$ psia ← p_{0_2}

$p_2/p_1 = 7.009 \quad \therefore p_2 = 24.3$ psia ← p_2



Problem 13.112

13.112 Equations 13.41 are a useful set of equations for analyzing flow through a normal shock. Derive another useful equation, the Rankine-Hugoniot relation,

$$\frac{p_2}{p_1} = \frac{(k+1)\frac{\rho_2}{\rho_1} - (k-1)}{(k+1) - (k-1)\frac{\rho_2}{\rho_1}}$$

and use it to find the density ratio for air as $p_2/p_1 \to \infty$.

Given: Normal shock

Find: Rankine-Hugoniot relation

Solution:

Basic equations: Momentum: $p_1 + \rho_1 \cdot V_1^2 = p_2 + \rho_2 \cdot V_2^2$ Mass: $\rho_1 \cdot V_1 = \rho_2 \cdot V_2$

Energy: $h_1 + \frac{1}{2} \cdot V_1^2 = h_2 + \frac{1}{2} \cdot V_2^2$ Ideal Gas: $p = \rho \cdot R \cdot T$

From the energy equation $2 \cdot (h_2 - h_1) = 2 \cdot c_p \cdot (T_2 - T_1) = V_1^2 - V_2^2 = (V_1 - V_1) \cdot (V_1 + V_2)$ (1)

From the momentum equation $p_2 - p_1 = \rho_1 \cdot V_1^2 - \rho_2 \cdot V_2^2 = \rho_1 \cdot V_1 \cdot (V_1 - V_2)$ where we have used the mass equation

Hence $V_1 - V_2 = \frac{p_2 - p_1}{\rho_1 \cdot V_1}$

Using this in Eq 1 $2 \cdot c_p \cdot (T_2 - T_1) = \frac{p_2 - p_1}{\rho_1 \cdot V_1} \cdot (V_1 + V_2) = \frac{p_2 - p_1}{\rho_1} \cdot \left(1 + \frac{V_2}{V_1}\right) = \frac{p_2 - p_1}{\rho_1} \cdot \left(1 + \frac{\rho_1}{\rho_2}\right) = (p_2 - p_1) \cdot \left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$

where we again used the mass equation

Using the idea gas equation $2 \cdot c_p \cdot \left(\frac{p_2}{\rho_2 \cdot R} - \frac{p_1}{\rho_1 \cdot R}\right) = (p_2 - p_1) \cdot \left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$

Dividing by p_1 and multiplying by ρ_2, and using $R = c_p - c_v$, $k = c_p/c_v$

$$2 \cdot \frac{c_p}{R} \cdot \left(\frac{p_2}{p_1} - \frac{\rho_2}{\rho_1}\right) = 2 \cdot \frac{k}{k-1} \cdot \left(\frac{p_2}{p_1} - \frac{\rho_2}{\rho_1}\right) = \left(\frac{p_2}{p_1} - 1\right) \cdot \left(\frac{\rho_2}{\rho_1} + 1\right)$$

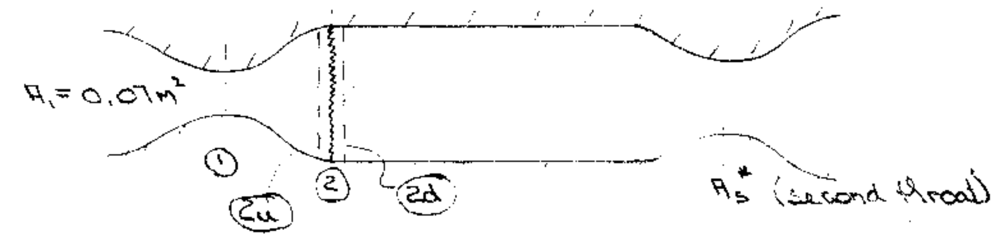
Collecting terms $\frac{p_2}{p_1} \cdot \left(\frac{2 \cdot k}{k-1} - 1 - \frac{\rho_2}{\rho_1}\right) = \frac{2 \cdot k}{k-1} \cdot \frac{\rho_2}{\rho_1} - \frac{\rho_2}{\rho_1} - 1$

$$\frac{p_2}{p_1} = \frac{\frac{2 \cdot k}{k-1} \cdot \frac{\rho_2}{\rho_1} - \frac{\rho_2}{\rho_1} - 1}{\left(\frac{2 \cdot k}{k-1} - 1 - \frac{\rho_2}{\rho_1}\right)} = \frac{\frac{(k+1)}{(k-1)} \cdot \frac{\rho_2}{\rho_1} - 1}{\frac{(k+1)}{(k-1)} - \frac{\rho_2}{\rho_1}} \quad \text{or} \quad \frac{p_2}{p_1} = \frac{(k+1) \cdot \frac{\rho_2}{\rho_1} - (k-1)}{(k+1) - (k-1) \cdot \frac{\rho_2}{\rho_1}}$$

For an infinite pressure ratio $(k+1) - (k-1) \cdot \frac{\rho_2}{\rho_1} = 0$ or $\frac{\rho_2}{\rho_1} = \frac{k+1}{k-1}$ (= 6 for air)

Problem 13.113

Given: Supersonic wind tunnel, supplied with air at $T_0 = 500K$ and $p_0 = 1.0$ MPa(abs), is to operate at a test section mach number, $M = 2.2$. Normal shock stands in exit plane of inlet nozzle.



Find: a) M_{2d} (b) p_{2d} (c) p_{o2d} (d) A_s

Solution:

Compressible flow functions (Appendix E) to be used in solution.

Assumptions: (1) steady flow (2) uniform flow at a section
(3) isentropic flow in nozzles, adiabatic flow across shock
(4) ideal gas.

At $M_{2u} = 2.2$, from App E.1, $p_{2u}/p_0 = 0.09352$ $\therefore p_{2u} = 93.5$ kPa

At $M_{2u} = 2.2$, from App E.4, $M_{2d} = 0.547$ (12.34b) ← M_{2d}

$p_2/p_1 = 5.480$ $\therefore p_{2d} = 512$ kPa ← p_{2d}

$p_{o2}/p_{o1} = 0.6281$ $\therefore p_{o2d} = 628$ kPa ← p_{o2d}

At $M_{2u} = 2.2$, from App E.1, $A_2/A^* = A_2/A_1 = 2.005$ $\therefore A_2 = 0.1404\ m^2$

At $M_{2d} = 0.547$, from App E.1, $A_2/A^* = A_2/A_s^* = 1.259$ $\therefore A_s^* = 0.111\ m^2$ ← A_s^*

Problem 13.114

[3]

Given: Supersonic wind tunnel starting as shown.

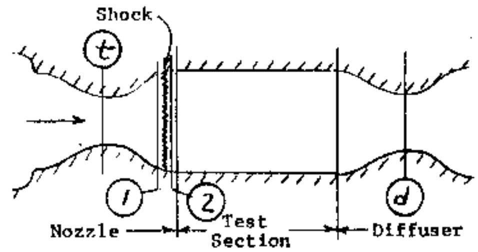
$A_t = 1.25\ ft^2$

$A_1 = A_2 = 3.05\ ft^2$

$M_{design} = 2.50$

$T_0 = 1080\ R$

$p_0 = 115\ psia$



Find: (a) Minimum possible A_d at this condition.
(b) Entropy increase across the shock.

Solution: Use functions for steady, one-dimensional compressible flow.

Computing equations: A/A^* vs. M from isentropic flow functions (App. E.1)

p_{02}/p_{01} vs. M from shock flow functions (App. E.4)

Assumptions: (1) Steady flow
(2) Uniform flow at each section
(3) Ideal gas
(4) Isentropic except across shock
(5) Adiabatic flow
(6) $F_{Bx} = 0$
(7) $\Delta z = 0$

Then from App. E.1, $M_1 = 2.416$ at $\frac{A_1}{A^*} = \frac{A_1}{A_t} = \frac{3.05\ ft^2}{1.25\ ft^2} = 2.44$.

From App. E.4, at $M_1 = 2.416$, $\frac{p_{02}}{p_{01}} = 0.5395$. Thus $p_{0d} = 0.5395\ p_{01} = 62.0$ psia.

For adiabatic flow, $T_0 =$ constant and $T^* = \frac{T_0}{1.2} = \frac{1080\ R}{1.2} = 900\ R =$ constant.

From continuity, $\dot{m} = \rho_t A_t V_t = \rho_d V_d A_d$. Substituting $\rho = \frac{p}{RT}$ and $V = M\sqrt{kRT}$,

$$\frac{p_t}{RT_t}\sqrt{kRT_t}\,A_t = \frac{p_d}{RT_d}\sqrt{kRT_d}\,A_d\ ;\quad A_d = A_t\frac{p_t}{p_d} = A_t\frac{p_{0t}}{p_{0d}} = \frac{A_t}{0.5395}$$

$$A_d = \frac{1.25\ ft^2}{0.5395} = 2.32\ ft^2$$

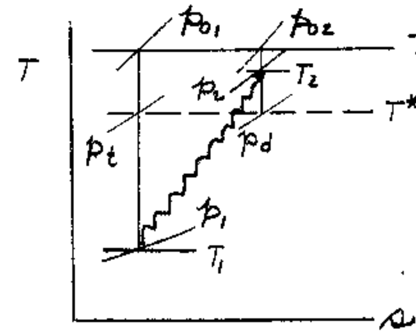
A_d

From the Gibbs equation,

$$T\,ds = dh - v\,dp\ ;\quad ds = c_p\frac{dT}{T} - R\frac{dp}{p}$$

$$\Delta s = c_p \ln\frac{T_{02}}{T_{01}} - R\ln\frac{p_{02}}{p_{01}}$$

Since $T_0 =$ constant, $\ln(T_{02}/T_{01}) = 0$, and



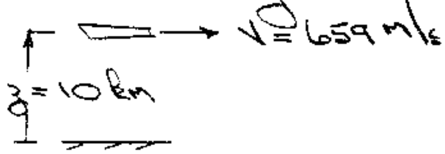
$$\Delta s = -53.3\ \frac{ft\cdot lbf}{lbm\cdot {}^\circ R} \times \ln(0.5395) \times \frac{Btu}{778\ ft\cdot lbf} = 0.0423\ Btu/lbm\cdot{}^\circ R$$

Δs

{ Note a slightly stronger shock could occur at $M = 2.50$; a larger diffuser throat ($A_d \simeq 2.51\ ft^2$) would be needed to start this tunnel. }

Problem 13.115

Given: Aircraft in supersonic flight on standard day
Total-head tube senses stagnation pressure.
Mach number computed assuming $s = c$, i.e. ignoring shock in front of tube.



Find: (a) flight Mach number
(b) pressure sensed by total-head tube
(c) air speed computed assuming s = constant

<u>Solution:</u> (using compressible flow functions - Appendix E)

From Table A.3 at $z = 10$ km, $T = 223.3$ K, $p/p_{SL} = 0.2615$

Thus $T = 223$ K, $p = 0.2615\, p_{SL} = 0.2615 \times 101.3$ kPa $= 26.5$ kPa

$c_1 = (kRT_1)^{1/2} = \left[1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 223\, K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 299$ m/s

$\overleftarrow{M_2}$ | shock | $\overleftarrow{M_1}$ $\qquad M_1 = \frac{V_1}{c_1} = \frac{659}{299} = 2.20$ ← M_1

From App. E.1 at $M_1 = 2.20$, $p/p_0 = 0.09352$ $\therefore p_{0_1} = 283$ kPa

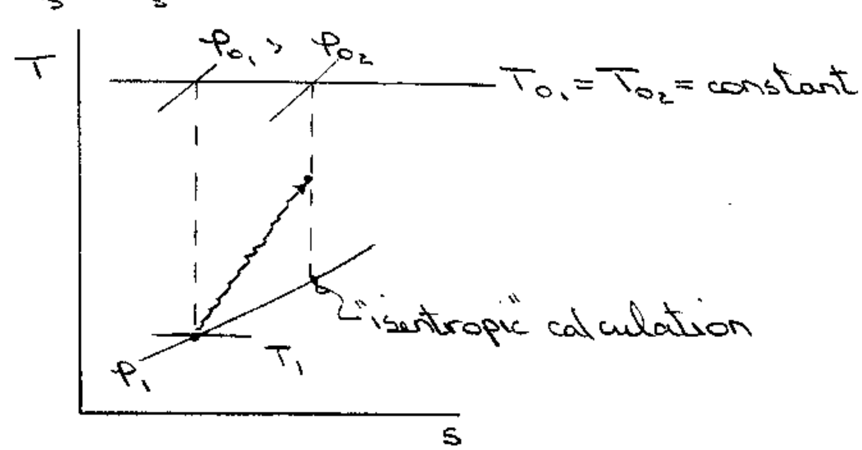
From App E.4 at $M_1 = 2.20$, $p_{0_2}/p_{0_1} = 0.6281$, $p_2/p_1 = 5.480$

$\therefore p_{0_2} = 0.6281\, p_{0_1} = 0.6281 \times 283$ kPa $= 178$ kPa (abs) ← p_{0_2}

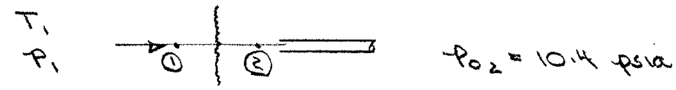
If Mach number is calculated neglecting the shock then M is calculated with $p = 26.5$ kPa and $p_0 = 178$ kPa.

For $p/p_0 = 26.5/178 = 0.149$, from App. E.1, $M_s = 1.90$

$V_{1_s} = M_s c_1 = 1.90 \times 299$ m/s $= 568$ m/s ← V_{1s}



Given: Supersonic aircraft cruises at $M = 2.7$ at 60,000 ft altitude. Normal shock stands in front of a pitot tube which senses a stagnation pressure of 10.4 psia.



Find: (a) T_2, p_2 (b) $p_{0_2} - p_{0_1}$ (c) $s_2 - s_1$

Solution.

Compressible flow functions (Appendix E) to be used in solution

Basic equation: $Tds = dh - v dp$

Assumptions: (1) steady flow (2) uniform flow at a section
(3) thin shock, $R_x = 0$
(4) ideal gas

Use table A.3 to determine properties at state ①

Altitude $= 60{,}000\,ft \times 0.3048\,\frac{m}{ft} = 18{,}290\,m$

From table A.3 $\quad T_1 = 216.7\,K = -56.3°C = -69°F$

$$p_1 = 7.25 \times 10^3\,\frac{N}{m^2} \times \frac{lbf}{4.448\,N} \times \left(\frac{0.3048\,m}{ft}\right)^2 = 151\,lbf/ft^2$$

From App. E.1 for $M_1 = 2.7$ $\quad p_1/p_{0_1} = 0.04295$ $\quad \therefore p_{0_1} = 24.4\,lbf/in^2$

From App. E.4 for $M_1 = 2.7$, $\quad M_2 = 0.4956$

$p_{0_2}/p_{0_1} = 0.4236$ $\quad \therefore p_{0_2} = 10.3$ psia

$T_2/T_1 = 2.343$ $\quad \therefore T_2 = 916\,R$ ——— T_2

$p_2/p_1 = 8.338$ $\quad \therefore p_2 = 8.74$ psia ——— p_2

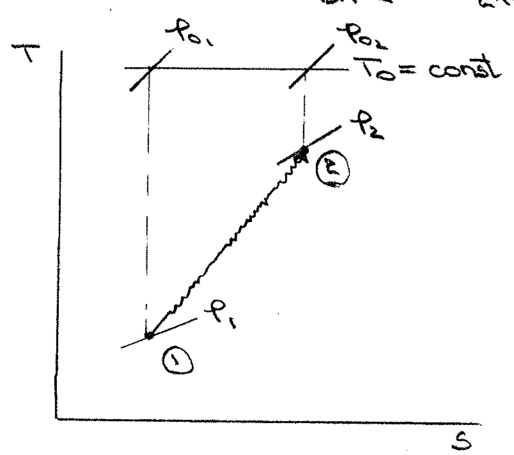
$p_{0_2} - p_{0_1} = 10.3\,psi - 24.4\,psi = -14.1\,psi$ ——— $p_{0_2} - p_{0_1}$

From the Tds equation, $Tds = dh - v dp = c_p dT - RT\frac{dp}{p}$

$$\therefore ds = c_p \frac{dT}{T} - R\frac{dp}{p}$$

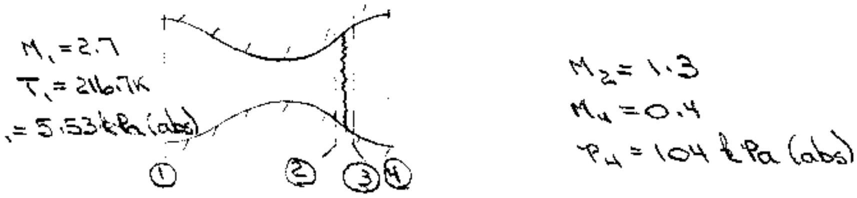
Then $s_2 - s_1 = s_{0_2} - s_{0_1} = c_p \ln \frac{T_{0_2}}{T_{0_1}} - R \ln \frac{p_{0_2}}{p_{0_1}}$ (first term crossed out, $=0$)

$$s_2 - s_1 = -53.3\,\frac{ft\cdot lbf}{lbm\cdot°R} \ln\frac{10.3}{24.4} \times \frac{Btu}{778\,ft\cdot lbf} = 0.0591\,\frac{Btu}{lbm\cdot°R}$$ ——— $s_2 - s_1$



Problem 13.117

Given: Supersonic aircraft flies at $M = 2.7$ at 20 km altitude. Air is slowed isentropically in inlet system to $M = 1.3$. A normal shock occurs at that location. Following the shock, the flow is decelerated adiabatically, but not isentropically, to $p = 104$ kPa and $M = 0.4$



Find: (a) T_0 (b) $p_3 - p_2$ (c) $s_4 - s_1$ d) p_{0_4}

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) uniform flow at a section
(3) thin shock, $A_2 = A_3$
(4) ideal gas

For $M_1 = 2.7$, from App. E.1, $T_{0_1}/T_1 = 2.458$ $\therefore T_{0_1} = 533$ K — T_0
$p_{0_1}/p_1 = 23.283$ $\therefore p_{0_1} = 128.8$ kPa
Note: T_0 = constant, $p_{0_2} = p_{0_1}$

For $M_2 = 1.3$, from App. E.1, $T_{0_2}/T_2 = 1.338$ $\therefore T_2 = 398$ K
$p_{0_2}/p_2 = 2.771$ $p_2 = 46.48$ kPa

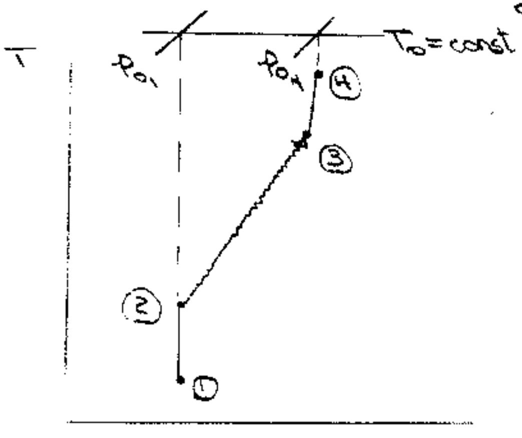
For $M_2 = 1.3$, from App E.4, $M_3 = 0.786$
$p_3/p_2 = 1.805$ $\therefore p_3 = 83.9$ kPa
$p_{0_3}/p_{0_2} = 0.9794$ $p_{0_3} = 126$ kPa
$T_3/T_2 = 1.191$ $T_3 = 474$ K

$p_3 - p_2 = (83.9 - 46.5)$ kPa $= 37.4$ kPa — $p_3 - p_2$

For $M_4 = 0.4$, from App. E.1, $p_{0_4}/p_4 = 1.117$ $\therefore p_{0_4} = 116$ kPa

$$s_4 - s_1 = s_{0_4} - s_{0_1} = c_p \ln \frac{T_{0_4}}{T_{0_1}} - R \ln \frac{p_{0_4}}{p_{0_1}} = -287 \frac{\text{J}}{\text{kg}\cdot\text{K}} \ln \frac{116}{129}$$

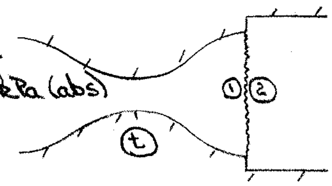
$s_4 - s_1 = -30.5$ J/kg·K — $s_4 - s_1$



Problem 13.118

Given: Steady, adiabatic flow of air through a converging-diverging nozzle with a normal shock in the exit plane

$T_0 = 15°C$
$P_0 = 101$ kPa (abs)



$A_1/A_t = 3.5$

Find: P_2, V_2

Solution:

* Compressible flow tables to be used in the solution

Assumptions: (1) steady flow (2) uniform flow at a section (3) ideal gas (4) isentropic flow except across the shock

$A_1/A^* = 3.5$

From Fig. E.1 and Eq. 12.6, $M_1 = 2.80$; then $T_1/T_0 = 0.3894$ $\quad P_1/P_{0_1} = 0.03665$

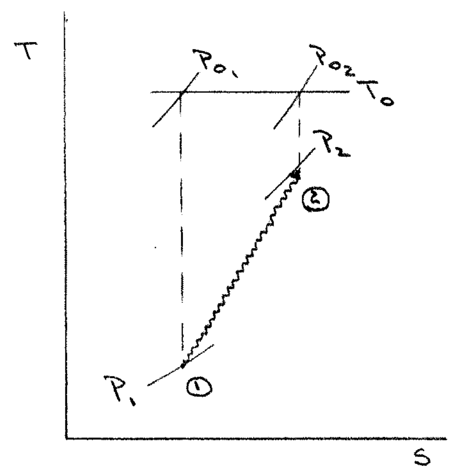
$\therefore T_1 = 112$ K $\quad P_1 = 3.72$ kPa (abs)

For $M_1 = 2.80$, from App. E.4, $M_2 = 0.488$ $\quad T_2/T_1 = 2.451$ $\quad P_2/P_1 = 8.980$

$\therefore P_2 = 8.98\,P_1 = 8.98 \times 3.72 \text{ kPa} = 33.4 \text{ kPa (abs)}$ ——— P_2

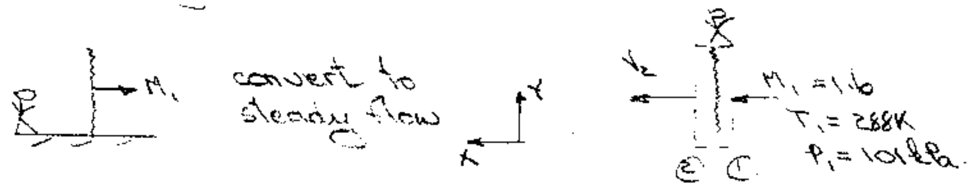
$T_2 = 2.451\,T_1 = 275$ K

$V_2 = M_2 c_2 = M_2 (kRT_2)^{1/2} = 0.488 \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 275 K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 162$ m/s ← V_2



Problem 13.119

Given: Blast wave propagates outward from an explosion. Model the wave as strong normal shock. Wave front travels at M=1.6 through undisturbed air



Find: (a) V_2 relative to wave (b) V_2 relative to ground.

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow as seen by an observer on the wave
(2) uniform flow at a section
(3) thin shock, $R_x = 0$
(4) ideal gas

At state ①

$$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 1.6 \left[1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 288 K \times \frac{kg \cdot m}{N \cdot s^2}\right]^{1/2} = 544 \text{ m/s}$$

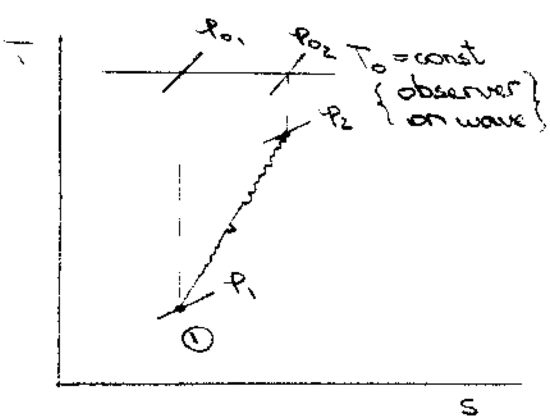
For $M_1 = 1.6$, from Appendix E.4, $V_1 / V_2 = \rho_2/\rho_1 = 2.032$

$$\therefore \vec{V}_2 = 268\hat{i} \text{ m/s} \quad \underline{\vec{V}_{2 \text{ rel wave}}}$$

The wave moves to the right at $V_1 = 544$ m/s. Air moves to the left with respect to the wave at $V_{2rel} = 268$ m/s.

$$\vec{V}_{2abs} = \vec{V}_{2wave} + \vec{V}_{2rel} = -544\hat{i} + 268\hat{i} = -276\hat{i} \text{ m/s} \quad \underline{\vec{V}_{2abs}}$$

{relative to ground, air behind wave moves to right}.



Problem 13.120

Given: Steady, adiabatic air flow from a reservoir through a converging-diverging nozzle with a shock present.

$T_0 = 300\,K$

$p_0 = 250\,kPa\,(abs)$

$p_2 = 132\,kPa\,(abs)$

$p_1 = 68.1\,kPa$

$p_3 = 180\,kPa$

Find: (a) M_2 (b) p_2 (c) $s_2 - s_1$ (d) Sketch Ts diagram

Solution: Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (3) uniform flow at each section
(2) ideal gas (4) isentropic flow, except across shock

At section ① $p_0 = 250\,kPa\,(abs)$, $p_1 = 68.1\,kPa\,(abs)$ $\therefore p/p_0 = 0.2724$

From App. E.1 $M_1 = 1.50$

From App. E.4 with $M_1 = 1.50$, $M_2 = 0.701$ —— M_2

$p_2/p_1 = 2.458$ $\therefore p_2 = 167\,kPa\,(abs)$ —— p_2

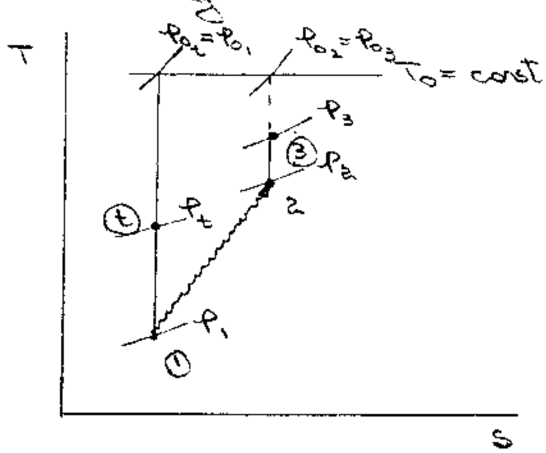
$p_{02}/p_{01} = 0.9298$

From the Tds equation, $T\,ds = dh - v\,dp = c_p\,dT - RT\,\frac{dp}{p}$

$\therefore ds = c_p\,dT - R\,\frac{dp}{p}$

$$s_2 - s_1 = s_{02} - s_{01} = c_p \ln \frac{T_{02}}{T_{01}} - R \ln \frac{p_{02}}{p_{01}}$$

$$s_2 - s_1 = -287\,\frac{J}{kg \cdot K} \ln 0.9298 = 20.9\,J/kg \cdot K$$ ←—— $s_2 - s_1$



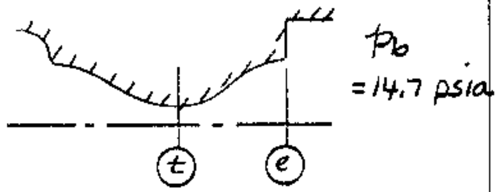
Problem 13.121

[2]

Given: C-D nozzle expanding air as shown.

$T_0 = 250°F$

$p_0 = 50.5$ psia



Find: (a) Exit Mach number.
(b) Mass flow rate.

Solution: Use functions for steady, one-dimensional compressible flow.

Computing equations: A/A^*, p/p_0, and T/T_0 from isentropic (Appendix E.1)

Assumptions: (1) Steady flow
(2) Uniform flow at each section
(3) Isentropic if no shock
(4) Ideal gas
(5) $F_{Bx} = 0$
(6) $\Delta z = 0$

Check the exit condition: $\frac{A_e}{A^*} = \frac{A_e}{A_t} = \frac{0.917}{0.801} = 1.145 \rightarrow M_e = 1.452$ (App. E.1, Eq. 12.6)

Also from App. E.1, at $M = 1.452$, $\frac{p}{p_0} = 0.2919$; $p_e = 0.2919\, p_0 = 14.74$ psia.

Thus p_e is just slightly above $p_{atm} = 14.7$ psia; flow at exit is supersonic, so $M_t = 1.0$. From continuity

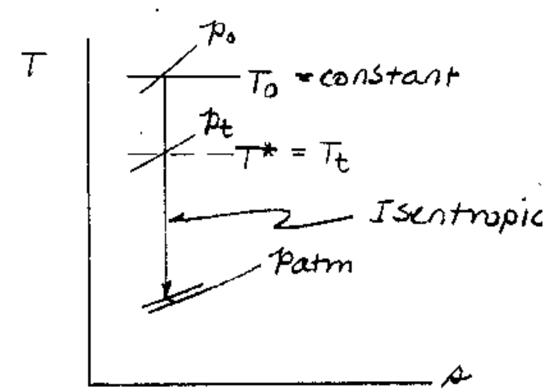
$$\dot{m} = \rho_t V_t A_t = \frac{p_t}{RT_t} M_t \sqrt{kRT_t}\, A_t = p_t \sqrt{\frac{k}{RT_t}}\, A_t$$

From App. E.1, at $M = 1$, $T/T_0 = 0.8333$ and $p/p_0 = 0.5283$, so

$$T_t = 0.8333 \times (460 + 250)°R = 592°R \quad \text{and} \quad p_t = 0.5283 \times 50.5\ \text{psia} = 26.7\ \text{psia}$$

$$\dot{m} = 26.7\ \frac{\text{lbf}}{\text{in.}^2}\left[1.4 \times \frac{\text{lbm}\cdot°R}{53.3\ \text{ft}\cdot\text{lbf}} \times \frac{1}{592°R} \times \frac{\text{slug}}{32.2\ \text{lbm}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{slug}\cdot\text{ft}}\right]^{1/2} 0.801\ \text{in.}^2 \times \frac{\text{slug}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2} \times 32.2\ \frac{\text{lbm}}{\text{slug}}$$

$\dot{m} = 0.808$ lbm/s $\qquad \dot{m}$



{ Flow in the nozzle is just slightly underexpanded, since $p_{exit} > p_{back}$. }

Problem * 13.122

Given: Steady, adiabatic air flow from a reservoir through a converging-diverging nozzle.

$A_t = 1.0 \text{ in}^2$

$A_e = 1.58 \text{ in}^2$

$T_0 = 600°R$

$P_0 = 100$ psia

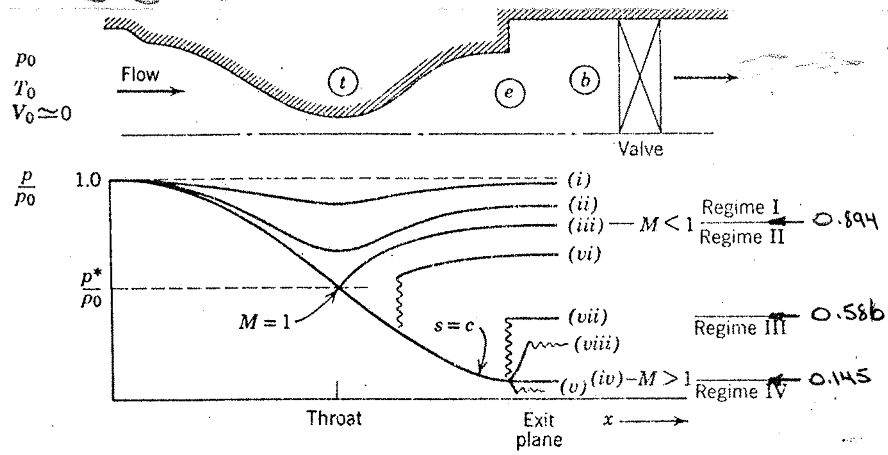


Fig. 13.20 Pressure distributions for flow in a converging-diverging nozzle as a function of back pressure.

Find: M_e at design conditions; P_b corresponding to regime boundaries of Fig 12.20, sketch $P(x)$

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) ideal gas (3) uniform flow at a section (4) isentropic flow except across a shock

For $A_e/A^* = 1.58$, from Fig. E.1 and Eq. 12.6, $M_e = 0.403$ $\quad P_e/P_0 = 0.8940$

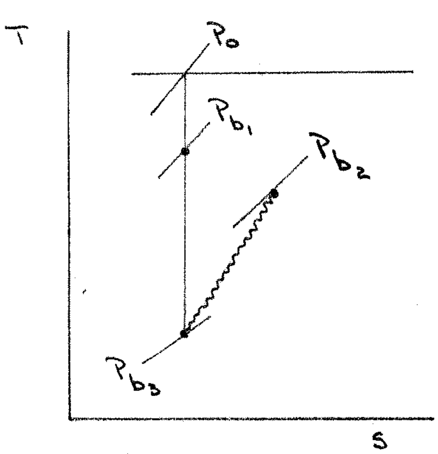
or $M_e = 1.92$ $\quad P_e/P_0 = 0.1447$ ← M_e

Then $P_{b_1} = \frac{P_{e_1}}{P_0} \cdot P_0 = 0.8940 \times 100 \text{ psia} = 89.4 \text{ psia}$ ← P_{b_1}

$P_{b_3} = \frac{P_{e_3}}{P_0} \times P_0 = 0.1447 \times 100 \text{ psia} = 14.5 \text{ psia}$ ← P_{b_3}

For $M_e = 1.92$, from App. E.4, $\frac{P_2}{P_1} = 4.045$

$P_{b_2} = \frac{P_2}{P_1} \times P_{e_3} = 4.045 \times 14.5 \text{ psia} = 58.6 \text{ psia}$ ← P_{b_2}



Given: Steady, adiabatic air flow from a reservoir through a converging-diverging nozzle; nozzle is designed to discharge to atmosphere

$A_e / A_t = 4.0$

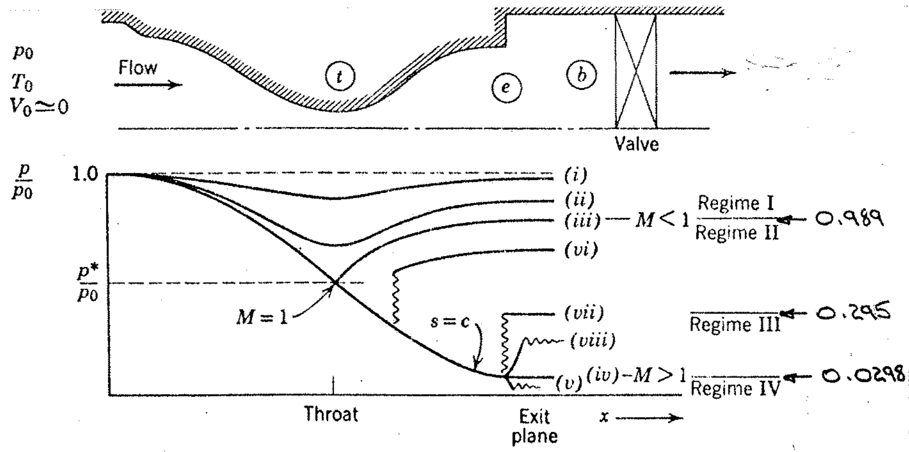


Fig. 13.20 *Pressure distributions for flow in a converging-diverging nozzle as a function of back pressure.*

Find: M_e at design conditions, P_0; P_b corresponding to regime boundaries of Fig 12.20, sketch $P(x)$

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) ideal gas
(3) uniform flow at a section
(4) isentropic flow except across a shock

For $A_e / A^* = 4.00$,
from Fig. E.1 and Eq. 12.6b, $M_e = 2.94$ $\quad P/P_0 = 0.0298$
or $M_e = 0.147$ $\quad P/P_0 = 0.9887$

Then

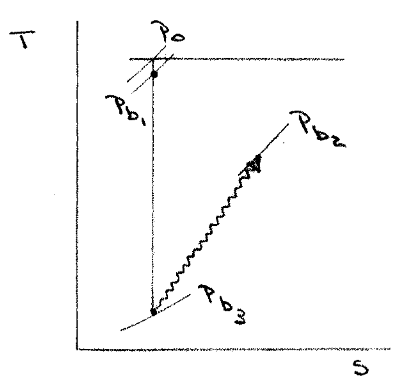
$M_{ed} = 2.94 \qquad P_{0d} = \frac{P_e}{0.0298} = \frac{101 \text{ kPa}}{0.0298} = 3.39 \text{ MPa (abs)}$ ← M_e, P_0

$P_{b_1} = \frac{P_{e_1}}{P_0} \times P_0 = 0.9887 \times 3.39 \text{ MPa} = 3.35 \text{ MPa (abs)}$ ← P_{b_1}

$P_{b_3} = 101 \text{ kPa (abs)}$ ← P_{b_3}

For $M_e = 2.94$ from App. E.4, $P_2/P_1 = 9.918$

$P_{b_2} = \frac{P_2}{P_1} \times P_{e_3} = 9.918 \times 101 \text{ kPa} = 1.00 \text{ MPa (abs)}$ ← P_{b_2}



Problem 13.124

[3]

13.124 A normal shock occurs in the diverging section of a converging-diverging nozzle where $A = 25\text{ cm}^2$ and $M = 2.75$. Upstream, $T_0 = 550$ K and $p_0 = 700$ kPa (abs). The nozzle exit area is 40 cm^2. Assume the flow is isentropic except across the shock. Determine the nozzle exit pressure, throat area, and mass flow rate.

Given: Normal shock in CD nozzle

Find: Exit pressure; Throat area; Mass flow rate

Solution:

The given or available data is:

R =	286.9	J/kg·K
k =	1.4	
T_{01} =	550	K
p_{01} =	700	kPa
M_1 =	2.75	
A_1 =	25	cm^2
A_e =	40	cm^2

Equations and Computations (assuming State 1 and 2 before and after the shock):

Using built-in function *Isenp* (M,k):

$p_{01}/p_1 =$ 25.14 $\quad p_1 =$ 28 kPa

Using built-in function *IsenT* (M,k):

$T_{01}/T_1 =$ 2.51 $\quad T_1 =$ 219 K

Using built-in function *IsenA* (M,k):

$A_1/A_1^* =$ 3.34 $\quad A_1^* = A_t =$ 7.49 cm^2

Then from the Ideal Gas equation:

$\rho_1 =$ 0.4433 kg/m^3

Also: $c_1 =$ 297 m/s

So: $V_1 =$ 815 m/s

Then the mass flow rate is: $m_{rate} = \rho_1 V_1 A_1$

$m_{rate} =$ 0.904 kg/s

For the normal shock:

Using built-in function *NormM2fromM* (M,k):

$M_2 =$ 0.492

Using built-in function *Normp0fromM* (M,k) at M_1:

$p_{02}/p_{01} =$ 0.41 $\quad p_{02} =$ 284 kPa

For isentropic flow after the shock:

Using built-in function *IsenA* (M,k):

$A_2/A_2^* =$ 1.356

But: $A_2 = A_1$

Hence: $A_2^* =$ 18.44 cm^2

Using built-in function *IsenAMsubfromA* (Aratio,k):

For: $A_e/A_2^* =$ 2.17 $\quad M_e =$ 0.279

Using built-in function *Isenp* (M,k):

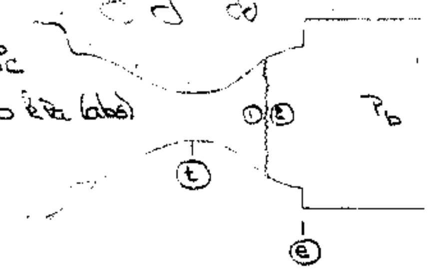
$p_{02}/p_e =$ 1.06 $\quad p_e =$ 269 kPa

Problem 13.125

Given: Steady, adiabatic air flow from a reservoir through a converging-diverging nozzle with a shock present

$T_0 = 60°C$

$p_0 = 600$ kPa (abs)



$M_1 = 2.42$

$A_e/A_t = 4.0$

Find: p_b, sketch the pressure distribution.

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions (1) steady flow
(2) uniform flow at a section
(3) ideal gas
(4) isentropic flow except across the shock

$M_1 = 2.42$ From App. E.1, $p/p_0 = 0.06630$ $\quad A_1/A_1^* = A_1/A_t = 2.448$

$\therefore p_1 = 39.8$ kPa(abs)

From App. E.4, $M_2 = 0.521$ $\quad p_2/p_1 = 6.666$ $\quad p_{02}/p_{01} = 0.5318$

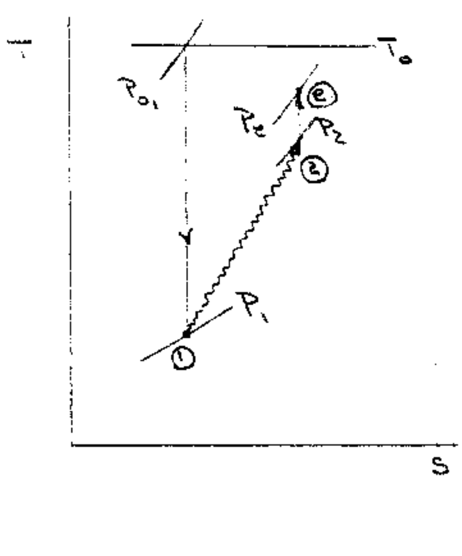
$\therefore p_2 = 265$ kPa(abs) $\quad p_{02} = 319$ kPa (abs)

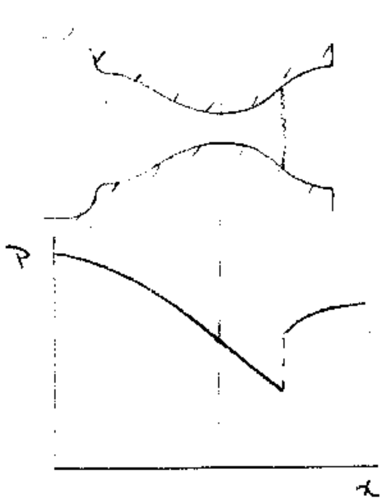
$M_2 = 0.521$ From App. E.1, $A_2/A_2^* = 1.301 = A_1/A_2^*$

Then $\dfrac{A_e}{A_2^*} = \dfrac{A_e}{A_1^*} \times \dfrac{A_1^*}{A_1} \times \dfrac{A_1}{A_2^*} = 4.0 \times \dfrac{1}{2.448} \times 1.301 = 2.126$

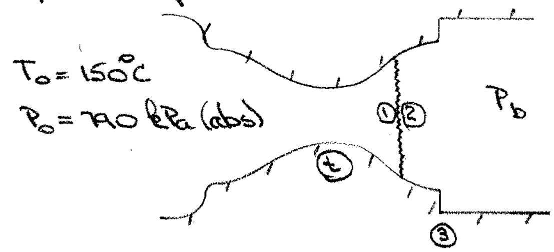
For $A_e/A^* = 2.126$, from Fig. E.1 and Eq. 12.6, $M_e = 0.286$, $p_e/p_0 = 0.9447$

Then $p_b = p_e = 0.9447\, p_0 = 0.9447 \times 319$ kPa $= 301$ kPa $\longleftarrow$ p_e





Given: Steady, adiabatic air flow from a reservoir through a converging-diverging nozzle; nozzle is designed to discharge to atmospheric pressure.



$P_1 = 160$ kPa (abs)

$A_1 = A_2 = 600\ \text{mm}^2$

Find: P_b, A_3, A_t

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) uniform flow at a section (3) ideal gas (4) isentropic flow except across the shock.

At design conditions,

$P_3/P_0 = 101/790 = 0.1278$. From App. E.1, $M_{3\,design} = 2.00$ $\quad \frac{A_3}{A_t} = 1.688$

At section ①, $\frac{P_1}{P_0} = \frac{160}{790} = 0.2025$. From App. E.1, $M_1 = 1.70$ $\quad A_1/A_1^* = 1.338$

$$A_t = A_1^* = \frac{A_1}{1.338} = \frac{600\ \text{mm}^2}{1.338} = 448\ \text{mm}^2 \quad \text{———} \quad A_t$$

Then $A_3 = 1.688\, A_t = 1.688\,(448\ \text{mm}^2) = 756\ \text{mm}^2$ ——— A_3

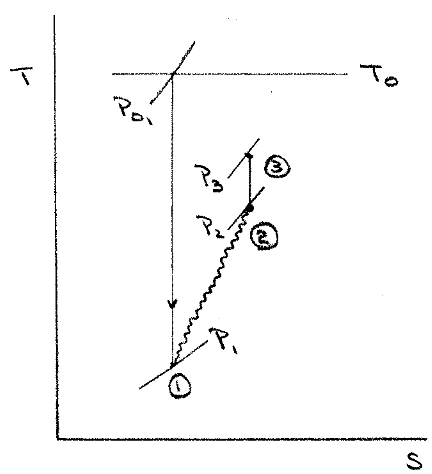
$M_1 = 1.70$. From App. E.4, $M_2 = 0.641$ $\quad P_{0_2}/P_{0_1} = 0.8557$ $\quad P_2/P_1 = 3.205$

$\therefore P_{0_2} = 676$ kPa (abs) $\quad P_2 = 513$ kPa (abs)

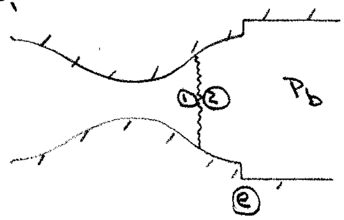
$M_2 = 0.641$. From App. E.1, $A_2/A_2^* = 1.145$ $\quad \therefore A_2^* = \frac{A_2}{1.145} = 524\ \text{mm}^2$

$A_3/A_2^* = \frac{756}{524} = 1.443$. From Fig. E.1 and Eq. 12.6, $M_3 = 0.453$; $\frac{P_3}{P_{0_3}} = 0.8688$

$\therefore P_b = P_3 = 0.8688\, P_{0_3} = 0.8688\, P_{0_2} = 587$ kPa (abs) ——— P_b



Given: Steady, adiabatic flow of air through a converging-diverging nozzle with a shock present; $p_b / p_{0_1} = 0.830$. At design conditions $p_e / p_{0_1} = 0.1278$



$p_b/p_{0_1} = p_e/p_{0_1} = 0.8300$

Find: M_1

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) uniform flow at a section (3) ideal gas (4) isentropic flow except across the shock

At design conditions, $p_e/p_{0_1} = 0.1278$

From App. E.1, $M_d = 2.0$ and $A_e/A_1^* = 1.688$

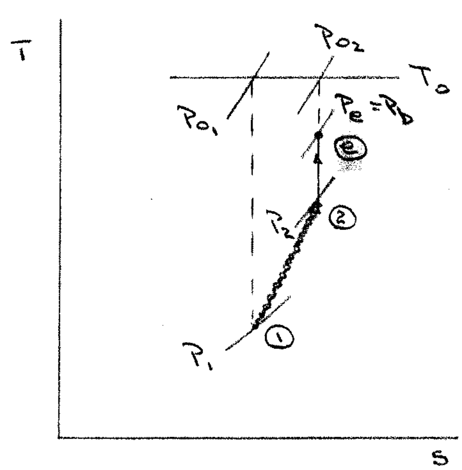
Now $\frac{p_e}{p_{0_1}} = \frac{p_e}{p_{0_2}} \times \frac{p_{0_2}}{p_{0_1}}$ and $\frac{A_e}{A_1^*} = \frac{A_e}{A_2^*} \times \frac{A_2^*}{A_1^*}$ $\qquad \frac{A_2^*}{A_1^*} = \frac{A_2^*}{A_2} \times \frac{A_1}{A_1^*}$

(p_e/p_{0_2} and A_e/A_2^*: $f(M_e)$; p_{0_2}/p_{0_1} and A_2^*/A_1^*: $f(M_1)$)

We thus have a trial and error solution to determine M_1

M_1	$\frac{A_1}{A_1^*}$ (a)	M_2 (b)	$\frac{p_{0_2}}{p_{0_1}}$ (c)	$\frac{A_2}{A_2^*}$ (d)	$\frac{A_2^*}{A_1^*}$	$\frac{A_e}{A_2^*}$	M_e (e)	$\frac{p_e}{p_{0_2}}$ (f)	$\frac{p_e}{p_{0_1}} \overset{?}{=} 0.8300$
1.60	1.25	0.67	0.8952	1.119	1.117	1.511	0.42	0.8857	0.7929
1.40	1.150	0.74	0.9582	1.068	1.077	1.567	0.41	0.8907	0.8534
1.50	1.176	0.70	0.9298	1.094	1.075	1.570	0.403	0.8923	0.830 ✓

$\therefore M_1 = 1.50 \longleftarrow M_1$



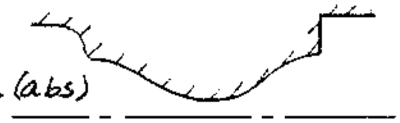
a. App. E.1 (12.6)
b. App E.4 (12.34b)
c. App E.4 (12.37)
d. App. E.1 (12.6)
e. App. E.1 Fig. E.1 & Eq. 12.6
f. App. E.1 (11.17a)

Problem 13.128

[3]

Given: Air flow through a converging-diverging nozzle. $A_e/A_t = 3.5$.

$T_0 = 15°C$
$p_0 = 101$ kPa (abs)



$A_t = 500\ mm^2$

Find: Range of back pressure for which a normal shock will occur in the nozzle, and the corresponding mass flowrate.

Solution: Use compressible flow functions in solution.

Computing equation: $\dot{m} = \rho V A$

Assumptions: (1) Steady flow (2) Ideal gas (3) Uniform flow at a section (4) Isentropic, except across shock

A normal shock will occur within the nozzle for back pressure conditions in Regime II of Fig. 12.20. For isentropic flow, with $A_e/A_t = 3.5$, from Fig. E.1 and Eq. 12.6,

M	p/p_0	p_b
0.169	0.9858	99.6 kPa
2.80	0.03685	

From Appendix E.4,

M_1	M_2	p_2/p_1
2.80	0.4882	8.980

Thus $p_b = p_2 = p_0 \dfrac{p_1}{p_0}\dfrac{p_2}{p_1} = 101\ kPa\ (0.03685)(8.980) = 33.4\ kPa$

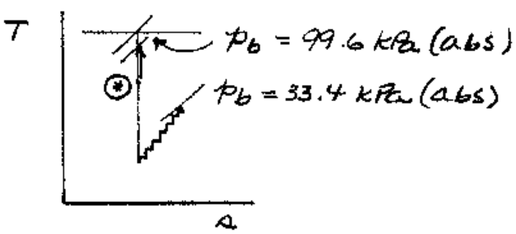
$33.4\ kPa < p_b < 99.6\ kPa$ (abs) (for normal shock in nozzle) p_b

Flow is choked throughout this regime. Thus

$$\dot{m} = \rho_t V_t A_t \qquad \rho_t = \frac{p_t}{RT_t} = (0.5283)\,1.01\times10^5\frac{N}{m^2}\times\frac{kg\cdot K}{287\ N\cdot m}\times\frac{1}{(0.8333)\,288\ K} = 0.775\ \frac{kg}{m^3}$$

$$V_t = c_t = \sqrt{kRT_t} = \left[1.4\times287\frac{N\cdot m}{kg\cdot K}\times(0.8333)\,288\ K\times\frac{kg\cdot m}{N\cdot s^2}\right]^{\frac{1}{2}} = 311\ m/s$$

$$\dot{m} = 0.775\ \frac{kg}{m^3}\times311\ \frac{m}{s}\times500\ mm^2\times\frac{m^2}{10^6\ mm^2} = 0.121\ \frac{kg}{s}$$ $\dot{m}$



Problem 13.129

Given: Converging-diverging nozzle with $A_e/A_t = 1.633$, designed to operate at atmospheric pressure at the exit plane.

Find: Ranges of stagnation pressure for which nozzle will be free from shocks.

Solution: Use compressible flow tables in solution.

Assume flow in nozzle is isentropic when shock-free. From Appendix E.1 and Eq. 12.6,

M	p/p_0	A/A^*
1.96	0.1360	1.633
0.38	0.9052	1.659
0.40	0.8956	1.590

At design conditions, $M_e = 1.96$, and

$$p_0 \geq \frac{p_e}{(p/p_0)_e} = \frac{101\text{ kPa}}{0.1360} = 743\text{ kPa (abs)}$$

By iteration, then the given area ratio corresponds to isentropic choked flow with $M_e = 0.388$ and $(p/p_0)_e = 0.9014$. The corresponding stagnation pressure is

$$p_0 = \frac{p_e}{(p/p_0)_e} = \frac{101\text{ kPa}}{0.9014} = 112\text{ kPa (abs)}$$

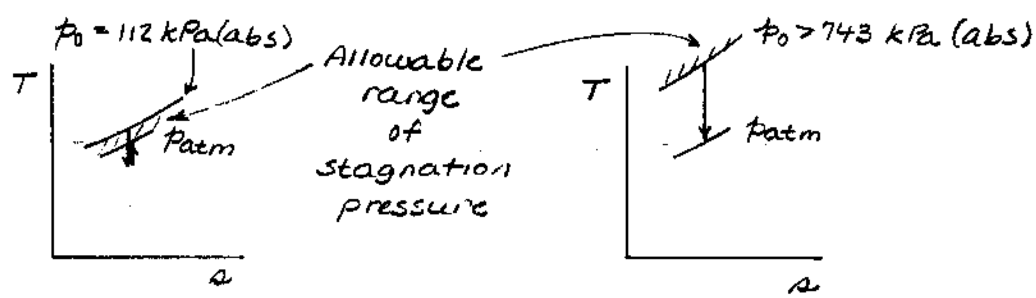
Flow will be isentropic and shock-free for

(a) $p_{atm} < p_0 < 112$ kPa (abs) $(0 < M_e < 0.388)$

(b) $p_0 > 743$ kPa (abs) $(M_e = 1.96)$

p_0

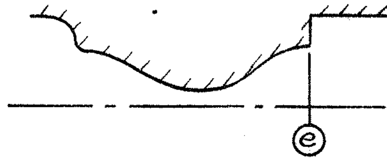
The corresponding Ts diagrams are:



Given: Air flow through a converging-diverging nozzle. $A_e/A_t = 1.87$.

$T_0 = 240°F$

$p_0 = 100$ psia



Find: Mach number and flow velocity in exit plane.

Solution: Use compressible flow functions in solution. Assume ideal gas.

For isentropic flow through the nozzle to $A_e/A_t = 1.87$, from Fig. E.1 and Eq. 12.6,

M	T/T_0	p/p_0	p_d (psia)
2.12	0.5266	0.1060	10.6
~0.33	~0.98	~0.92	~92.0

Neither of these conditions matches the back pressure. Check the case of a shock (at $M = 2.12$) in the exit plane. From Appendix E.4,

M_1	M_2	p_2/p_1
2.12	0.5583	5.077

Then $p_e = 5.077\, p_d = 5.077(10.6 \text{ psia}) = 53.8$ psia.

The back pressure of 40 psia is therefore between the design pressure and the pressure that would exist downstream from a normal shock in the exit plane. The flow is in regime III of Fig. 12.20: supersonic in the exit plane with external compression. Thus

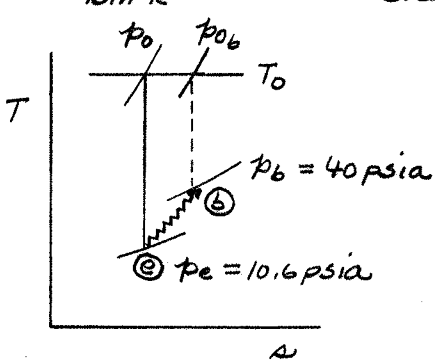
$$M_e = M_d = 2.12$$ Me

$$V_e = M_e c_e = M_e\sqrt{kRT_e}$$

$$T_e = \frac{T}{T_0}T_0 = 0.5266(460+240)°R = 369°R$$

$$V_e = 2.12\left[1.4 \times 53.3\,\frac{ft\cdot lbf}{lbm\cdot °R} \times 369°R \times 32.2\,\frac{lbm}{slug} \times \frac{slug\cdot ft}{lbf\cdot s^2}\right]^{\frac{1}{2}} = 2000\,\frac{ft}{s}$$ Ve

The Ts diagram is

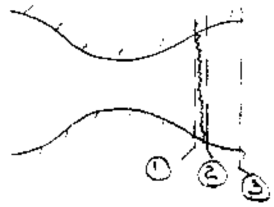


Problem 13.131

[3]

Given: Steady, adiabatic flow of air through a converging-diverging nozzle with shock in diverging section under conditions shown.

$T_0 = 1000°R$
$p_0 = 100$ psia



$A_1 = A_2 = 4.0\ in^2$
$M_1 = 2.0$
$A_3 = 6.0\ in^2$

Find: p_3

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) ideal gas (3) uniform flow at each section (4) isentropic flow, except across shock.

For $M_1 = 2.0$, from App. E.1, $p_1/p_{0_1} = 0.1278$ $\therefore p_1 = 12.78$ psia.

For $M_1 = 2.0$, from App. E.4, $M_2 = 0.5774$

$p_2/p_1 = 4.50$ $\therefore p_2 = 57.5$ psia

$p_{0_2}/p_{0_1} = 0.7209$ $\therefore p_{0_2} = 72.1$ psia

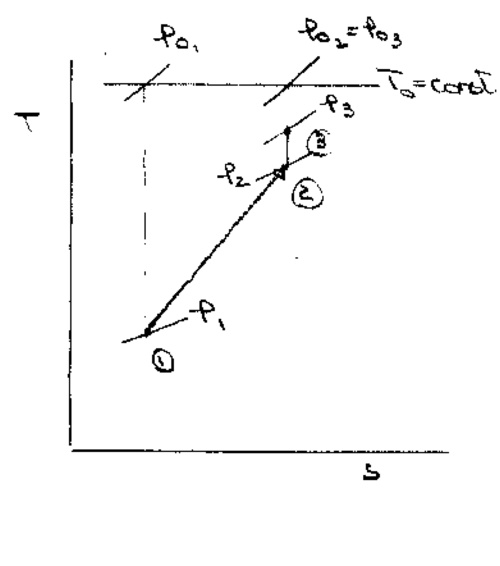
For $M_2 = 0.5774$, from App. E.1, $A_2/A_2^* = 1.216$ $\therefore A_2^* = 3.288\ in^2$

Then $A_3/A_2^* = 6.0/3.288 = 1.825$

With $A_3/A^* = 1.825$, from App. E.1 (Fig. E.1 and Eq. 12.6) $M_3 = 0.340$

With $M_3 = 0.340$, from App. E.1, $p_{0_3}/p_3 = 1.083$

$$p_3 = \frac{p_{0_3}}{1.083} = \frac{p_{0_2}}{1.083} = \frac{72.1\ \text{psia}}{1.083} = 66.6\ \text{psia} \qquad p_3$$

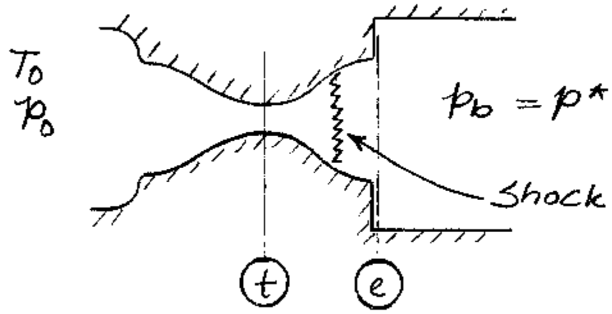


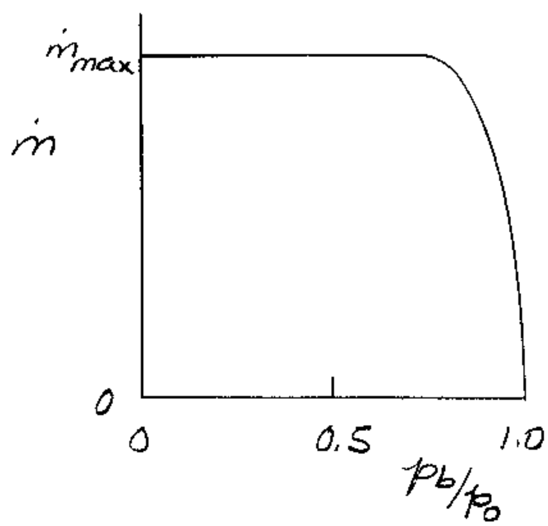
Problem 13.132

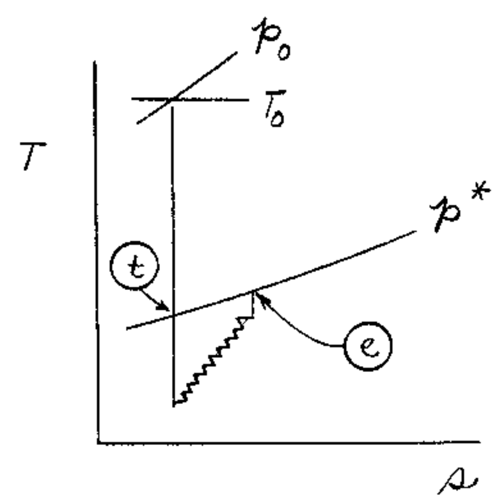
Given: Flow through a converging-diverging nozzle, $p_b = p^*$

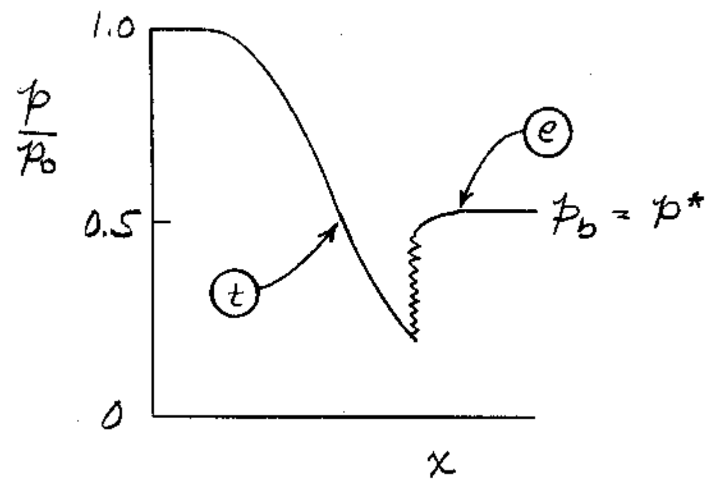
Find: Sketch (a) mass flow rate vs. pressure ratio
(b) pressure vs. distance along nozzle
(c) Ts diagram

Solution: When $p_b = p^*$, flow in the nozzle will be choked and a shock will stand in the diverging section.



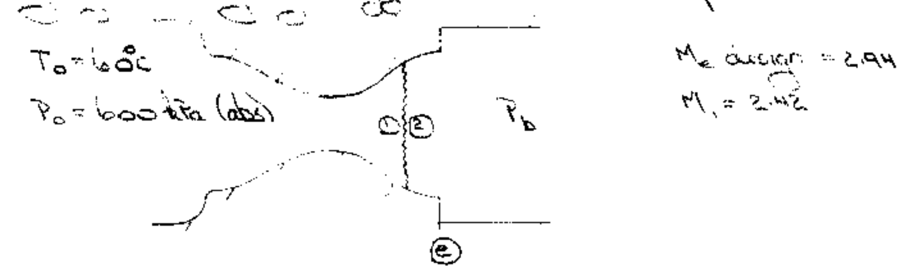






Problem 13.133

Given: Steady, adiabatic air flow from a reservoir through a converging - diverging nozzle with a shock present



Find: p_b, sketch the pressure distribution

Solution:

Compressible flow tables (Appendix E) to be used in solution.

Assumptions: (1) steady flow (3) uniform flow at each section
(2) ideal gas (4) isentropic flow, except across shock.

At design, $M_e = 2.94$, from Appendix E.1, $A_e/A_1^* = 3.999$

For $M_1 = 2.42$, from Appendix E.1, $p_1/p_{0_1} = 0.06630$
$A_1/A_1^* = 2.448$

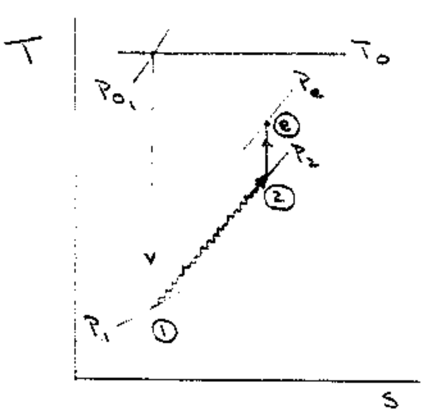
For $M_1 = 2.42$, from Appendix E.4, $M_2 = 0.521$
$p_{0_2}/p_{0_1} = 0.5318$
$p_2/p_1 = 6.666$

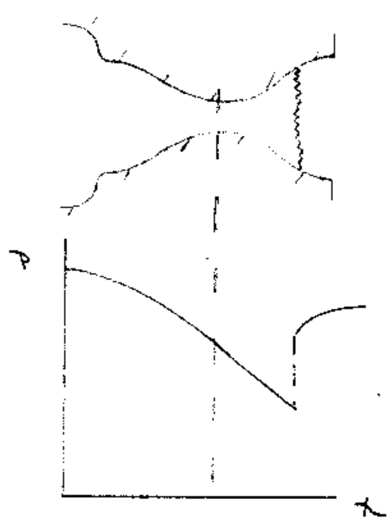
For $M_2 = 0.521$, from Appendix E.1, $A_2/A_2^* = A_1/A_2^* = 1.302$

Then, $\frac{A_e}{A_2^*} = \frac{A_e}{A_1^*} \times \frac{A_1^*}{A_1} \times \frac{A_1}{A_2^*} = 3.999 \times \frac{1}{2.448} \times 1.302 = 2.127$

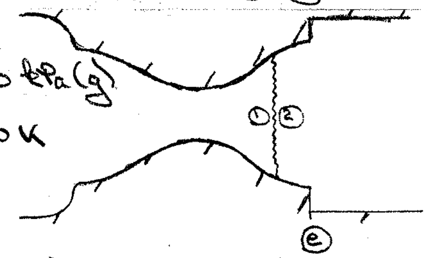
For $A_e/A_2^* = 2.127$, from App. E.1 (Fig E.1 & Eq 12.6), $M_e = 0.286$, $\frac{p_e}{p_{0_2}} = 0.9447$

$\therefore p_b = p_e = \frac{p_e}{p_{0_2}} \times \frac{p_{0_2}}{p_{0_1}} \times p_{0_1} = 0.9447 \times 0.5318 \times 600 \text{ kPa} = 301 \text{ kPa}$ ← p_b





Given: Steady, adiabatic flow of air through a converging-diverging nozzle with shock in diverging section under conditions shown.



$p_0 = 1000\ kPa(g)$

$T_0 = 400\ K$

$M_1 = 3.0$

$A_1 = A_t = 500\ mm^2$

$A_e = 600\ mm^2$

Find: M_2, p_{0_2}, p_2, A_t, $s_2 - s_1$, M_e

Solution:

Compressible flow functions (Appendix E) to be used in solution

Assumptions: (1) steady flow (2) ideal gas (3) uniform flow at each section (4) isentropic flow, except across shock.

For $M_1 = 3.0$, from Appendix E.1, $p_1/p_0 = 0.02722 \quad \therefore p_1 = 29.97\ kPa$

$A_1/A^* = 4.235 \quad \therefore A^* = A_t = 118\ mm^2$ ← A_t

For $M_1 = 3.0$, from Appendix E.4, $M_2 = 0.475$ ← M_2

$p_{0_2}/p_{0_1} = 0.3283 \quad \therefore p_{0_2} = 361\ kPa\ (abs)$ ← p_{0_2}

$p_2/p_1 = 10.33 \quad \therefore p_2 = 310\ kPa\ (abs)$ ← p_2

Since $T_{0_2} = T_{0_1}$, then

$$s_2 - s_1 = s_{0_2} - s_{0_1} = c_p \ln \frac{T_{0_2}}{T_{0_1}} - R \ln \frac{p_{0_2}}{p_{0_1}} = -287 \frac{N\cdot m}{kg\cdot K} \ln 0.3283$$

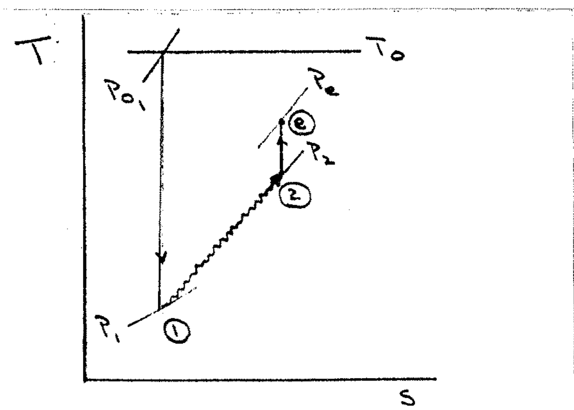
(the $c_p \ln \frac{T_{0_2}}{T_{0_1}}$ term is struck out)

$$s_2 - s_1 = -0.320\ kJ/kg\cdot K$$ ← $s_2 - s_1$

At $M_2 = 0.475$, from App. E.1 $A_2/A_2^* = 1.391 \quad \therefore A_2^* = 359.5\ mm^2$

At exit $A_e/A_2^* = \frac{600}{359.5} = 1.669$. From App E.1 (Fig E.1) & Eq 12.6

$$M_e = 0.377$$ ← M_e



The actual exit Mach number would be higher than the estimate based on isentropic flow downstream from the shock.

Flow downstream from the shock is subsonic. Flow slows in the diverging passage, which acts as a subsonic diffuser, causing pressure to increase in the direction of flow.

The result will be rapid growth of boundary layers on the channel walls. The boundary layers reduce the effective flow area of the passage. Because the boundary layers thicken rapidly, the area ratio for slowing the flow will be less than for isentropic flow. Therefore the actual flow will not slow as much as the isentropic model predicts.

The actual exit Mach number will be higher than the estimate based on isentropic flow.

Problem *13.135

Open-Ended Problem Statement: A supersonic wind tunnel must have two throats, with the second throat larger than the first. Explain why this must be so.

Discussion: The first throat is located in the supersonic nozzle from which flow enters the test section. The second throat is located in the supersonic diffuser that slows flow leaving the test section to subsonic speed for re-compression and re-circulation.

The second throat must be larger than the first for two reasons. First, it is impossible to slow a real flow to a Mach number of exactly one in a supersonic diffuser. The minimum Mach number that can be achieved with stable flow is about $M = 1.3$. Therefore even if the flow were isentropic everywhere the second throat would have to be larger than the first by the area ratio A/A^* corresponding to $M = 1.3$ at the throat.

The second reason is that flow is not isentropic through the tunnel. Some friction exists, which must reduce the stagnation pressure of the flow stream. This also reduces the stagnation density. Therefore a larger area is needed to carry the mass flow at any given flow speed.

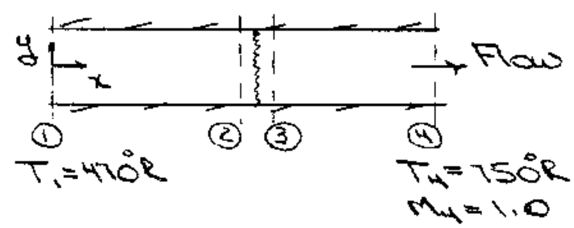
For these reasons the second throat area must be larger than the first throat area.

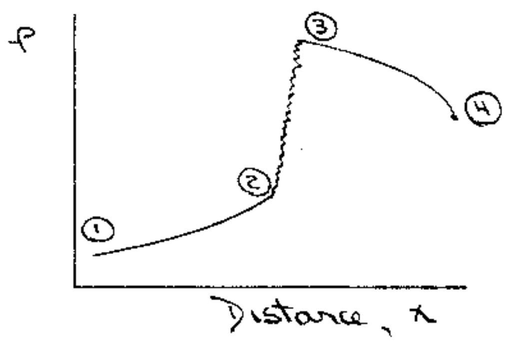
Problem *13.136

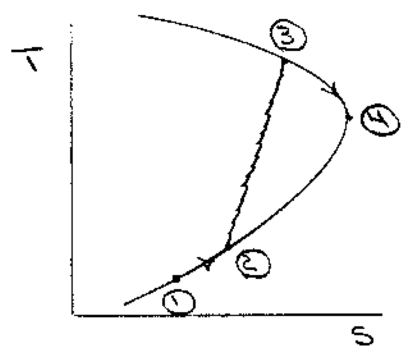
Given: A normal shock stands in a section of insulated constant-area duct; conditions immediately upstream and downstream of the shock are denoted by subscripts 2 and 3, respectively.
Flow in the duct is frictional.
Conditions at ① (some distance upstream) are $T_1 = 470°R$ and at ④ (some distance downstream) are $T_4 = 750°R$ and $M_4 = 1.0$

Find: (a) Sketch pressure distribution all the duct
(b) Sketch a Ts diagram
(c) Determine M_1

Solution:







For adiabatic flow, T_0 = constant (from energy eq.).

Thus, $T_{0_1} = T_{0_4} = T_4\left[1 + \frac{k-1}{2}M_4^2\right] = 750°R\,[1.2] = 900°R$

Since $T_{0_1} = T_1\left[1 + \frac{k-1}{2}M_1^2\right]$, then

$$M_1^2 = \frac{2}{k-1}\left[\frac{T_{0_1}}{T_1} - 1\right] = \frac{2}{0.4}\left[\frac{900R}{470R} - 1\right] = 4.57$$

$M_1 = \sqrt{4.57} = 2.14$ ——— M_1

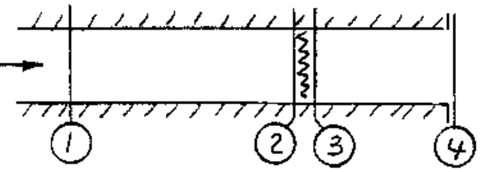
Problem *13.137

Given: Flow with shock in insulated constant-area duct, as shown.

$T_1 = 668°R$

$p_{01} = 78.2$ psia

$M_1 = 2.05$



$T_2 = 388$ F $\qquad M_4 = 1.0$

Find: (a) Speed before shock, V_2.
(b) Entropy change, $s_4 - s_1$.

Solution: Use functions for steady, one-dimensional compressible flow.

Computing equations: $\frac{T}{T_0}$ from isentropic functions (Appendix E.1)

$\frac{p}{p^*}$ from Fanno-line functions (Appendix E.2)

Assumptions: (1) Steady flow
(2) Uniform flow at each cross-section
(3) Ideal gas
(4) Fanno line flow
(5) $F_{B_x} = 0$
(6) $\Delta z = 0$

For adiabatic flow on Fanno line and across shock, T_0 = Constant. At $M_1 = 2.05$, $T/T_0 = 0.5433$ (Eq. 11.17b). Thus

$$T_0 = T_{01} = \frac{T_1}{(T/T_0)_1} = \frac{668°R}{0.5433} = 1230°R$$

Using $T_2 = 388$ F $(848°R)$, $\frac{T}{T_0} = \frac{848°R}{1230°R} = 0.6894$ and $M_2 = 1.50$ (Table E.1).

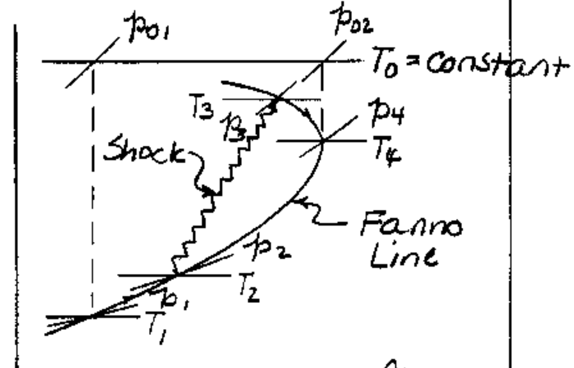
$$V_2 = M_2 c_2 = M_2\sqrt{kRT_2} = 1.50\left[1.4 \times 53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times 848°R \times 32.2 \frac{lbm}{slug} \times \frac{slug \cdot ft}{lbf \cdot s^2}\right]^{\frac{1}{2}}$$

$$V_2 = 2140 \text{ ft/s}$$

V_2

Flow must stay on the same Fanno line. Thus $(p_0/p_0^*)_1 = 1.760$ at $M_1 = 2.05$ (Eq. 12.18e). Thus

$$p_0^* = \frac{p_{01}}{(p_0/p_0^*)_1} = \frac{78.2 \text{ psia}}{1.760} = 44.4 \text{ psia}$$



From the Gibbs equation

$$T\,ds = dh - v\,dp \;;\; ds = c_p \frac{dT}{T} - R\frac{dp}{p}$$

$$\Delta s = c_p \ln \frac{T_{04}}{T_{01}} - R \ln \frac{p_{04}}{p_{01}}$$

(the c_p term is struck out, = 0)

$$\Delta s = -53.3 \frac{ft \cdot lbf}{lbm \cdot °R} \times \ln\left(\frac{44.4}{78.2}\right) \times \frac{Btu}{778\ ft \cdot lbf} = 0.0388 \text{ Btu/lbm} \cdot °R$$

Δs

{This problem could be solved without using functions, but the solution would require more calculations.}

Problem 13.138

13.138 Show that as the upstream Mach number approaches infinity, the Mach number after an oblique shock becomes

$$M_2 \approx \sqrt{\frac{k-1}{2k\sin^2(\beta-\theta)}}$$

Given: Normal shock

Find: Approximation for downstream Mach number as upstream one approaches infinity

Solution:

Basic equations:

$$M_{2n}^2 = \frac{M_{1n}^2 + \frac{2}{k-1}}{\left(\frac{2 \cdot k}{k-1}\right) \cdot M_{1n}^2 - 1} \qquad (13.48a) \qquad M_{2n} = M_2 \cdot \sin(\beta - \theta) \qquad (13.47b)$$

Combining the two equations

$$M_2 = \frac{M_{2n}}{\sin(\beta-\theta)} = \frac{\sqrt{\frac{M_{1n}^2 + \frac{2}{k-1}}{\left(\frac{2 \cdot k}{k-1}\right) \cdot M_{1n}^2 - 1}}}{\sin(\beta-\theta)} = \sqrt{\frac{M_{1n}^2 + \frac{2}{k-1}}{\left[\left(\frac{2 \cdot k}{k-1}\right) \cdot M_{1n}^2 - 1\right] \cdot \sin(\beta-\theta)^2}}$$

$$M_2 = \sqrt{\frac{1 + \frac{2}{(k-1) \cdot M_{1n}^2}}{\left[\left(\frac{2 \cdot k}{k-1}\right) - \frac{1}{M_{1n}^2}\right] \cdot \sin(\beta-\theta)^2}}$$

As M_1 goes to infinity, so does M_{1n}, so

$$M_2 = \sqrt{\frac{1}{\left(\frac{2 \cdot k}{k-1}\right) \cdot \sin(\beta-\theta)^2}} \qquad M_2 = \sqrt{\frac{k-1}{2 \cdot k \cdot \sin(\beta-\theta)^2}}$$

Problem 13.139

[3]

13.139 Supersonic air flow at $M_1 = 2.5$ and 80 kPa (abs) is deflected by an oblique shock with angle $\beta = 35°$. Find the Mach number and pressure after the shock, and the deflection angle. Compare these results to those obtained if instead the flow had experienced a normal shock. What is the smallest possible value of angle β for this upstream Mach number?

Given: Data on an oblique shock

Find: Mach number and pressure downstream; compare to normal shock

Solution:

The given or available data is:

	$R =$	286.9	J/kg.K
	$k =$	1.4	
	$p_1 =$	80	kPa
	$M_1 =$	2.5	
	$\beta =$	35	°

Equations and Computations:

From M_1 and β

$M_{1n} =$ 1.43

$M_{1t} =$ 2.05

From M_{1n} and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (M, k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

$p_2 =$ 178.6 kPa

The tangential velocity is unchanged

$$V_{t1} = V_{t2}$$

Hence

$$c_{t1} M_{t1} = c_{t2} M_{t2}$$

$$(T_1)^{1/2} M_{t1} = (T_2)^{1/2} M_{t2}$$

$$M_{2t} = (T_1/T_2)^{1/2} M_{t1}$$

From M_{1n}, and Eq. 13.48c
(using built-in function *NormTfromM* (M, k))

$T_2/T_1 =$ 1.28

Hence

$M_{2t} =$ 1.81

Also, from M_{1n}, and Eq. 13.48a
(using built-in function *NormM2fromM* (*M* ,*k*))

$$M_{2_n}^2 = \frac{M_{1_n}^2 + \frac{2}{k-1}}{\frac{2k}{k-1} M_{1_n}^2 - 1} \qquad (13.48a)$$

$M_{2n} =$ 0.726

The downstream Mach number is then

$$M_2 = (M_{2t}^{\ 2} + M_{2n}^{\ 2})^{1/2}$$

$M_2 =$ 1.95

Finally, from geometry

$$V_{2n} = V_2 \sin(\beta - \theta)$$

Hence $\theta = \beta - \sin^{-1}(V_{2n}/V_2)$

or $\theta = \beta - \sin^{-1}(M_{2n}/M_2)$

$\theta =$ 13.2 ^{o}

For the normal shock:
From M_1 and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (*M* ,*k*))

$p_2 =$ 570 kPa

Also, from M_1, and Eq. 13.48a
(using built-in function *NormM2fromM* (*M* ,*k*))

$M_2 =$ 0.513

For the minimum β:
The smallest value of β is when the shock is a Mach wave (no deflection)

$$\beta = \sin^{-1}(1/M_1)$$

$\beta =$ 23.6 ^{o}

Problem 13.140

[3]

13.140 Consider supersonic flow of air at $M_1 = 3.0$. What is the range of possible values of the oblique shock angle β? For this range of β, plot the pressure ratio across the shock.

Given: Oblique shock in flow at $M = 3$

Find: Minimum and maximum β, plot of pressure rise across shock

Solution:

The given or available data is:

$R =$	286.9	J/kg.K
$k =$	1.4	
$M_1 =$	3	

Equations and Computations:

The smallest value of β is when the shock is a Mach wave (no deflection)

$$\beta = \sin^{-1}(1/M_1)$$

$$\beta = 19.5^{\text{o}}$$

The largest value is $\beta = 90.0^{\text{o}}$

The normal component of Mach number is

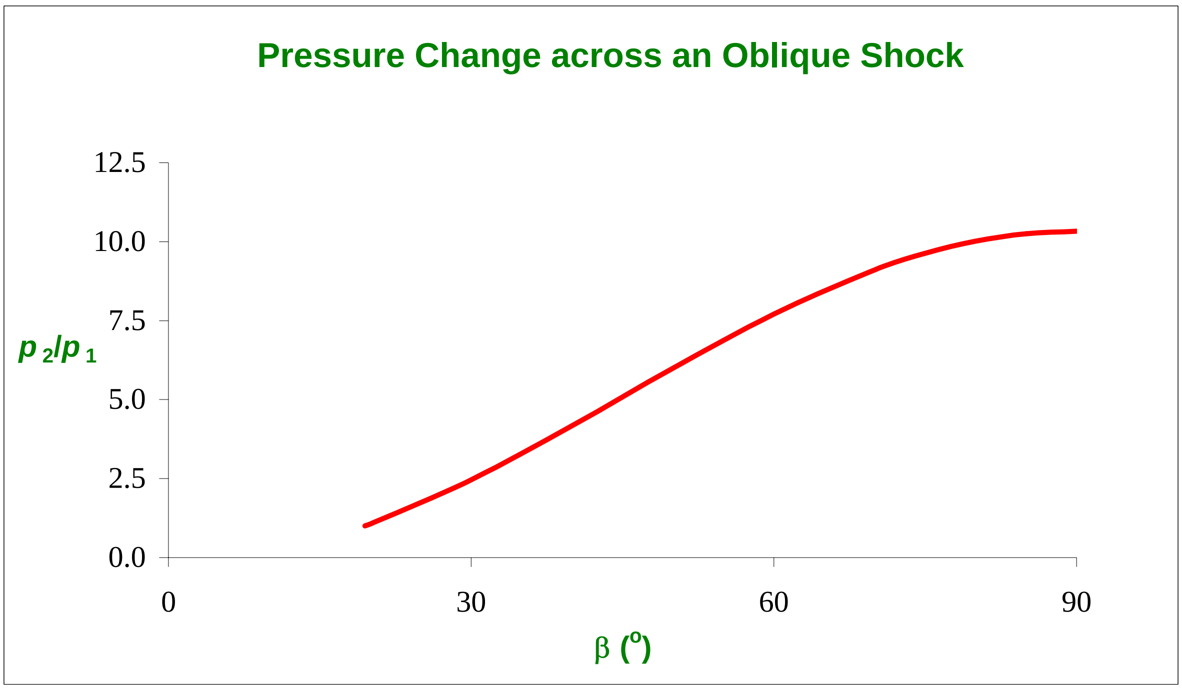
$$M_{1n} = M_1 \sin(\beta) \qquad (13.47a)$$

For each β, p_2/p_1 is obtained from M1n, and Eq. 13.48d
(using built-in function *NormpfromM*(M,k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

Computed results:

β (°)	M_{1n}	p_2/p_1
19.5	1.00	1.00
20	1.03	1.06
30	1.50	2.46
40	1.93	4.17
50	2.30	5.99
60	2.60	7.71
70	2.82	9.11
75	2.90	9.63
80	2.95	10.0
85	2.99	10.3
90	3.00	10.3



Problem 13.141

[3]

13.141 The air velocities before and after an oblique shock are 1250 m/s and 650 m/s, respectively, and the deflection angle is $\theta = 35°$. Find the oblique shock angle β, and the pressure ratio across the shock.

Given: Velocities and deflection angle of an oblique shock

Find: Shock angle β; pressure ratio across shock

Solution:

The given or available data is:

$R =$	286.9	J/kg.K
$k =$	1.4	
$V_1 =$	1250	m/s
$V_2 =$	650	m/s
$\theta =$	35	o

Equations and Computations:

From geometry we can write two equations for tangential velocity:

For V_{1t} $$V_{1t} = V_1\cos(\beta) \quad (1)$$

For V_{2t} $$V_{2t} = V_2\cos(\beta - \theta) \quad (2)$$

For an oblique shock $V_{2t} = V_{1t}$, so Eqs. 1 and 2 give

$$V_1\cos(\beta) = V_2\cos(\beta - \theta) \quad (3)$$

Solving for β $$\beta = \tan^{-1}((V_1 - V_2\cos(\theta))/(V_2\sin(\theta)))$$

$$\beta = 62.5^{o}$$

(Alternatively, solve Eq. 3 using *Goal Seek* !)

For p_2/p_1, we need M_{1n} for use in Eq. 13.48d

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

We can compute M_1 from θ and β, and Eq. 13.49
(using built-in function *Theta* (M, β, k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

For

$\theta =$ 35.0 °
$\beta =$ 62.5 °

$M_1 =$ 3.19

This value of M_1 was obtained by using *Goal Seek* :
Vary M_1 so that θ becomes the required value.
(Alternatively, find M_1 from Eq. 13.49 by explicitly solving for it!)

We can now find M_{1n} from M_1. From M_1 and Eq. 13.47a

$$M_{1n} = M_1\sin(\beta) \qquad (13.47a)$$

Hence $M_{1n} =$ 2.83

Finally, for p_2/p_1, we use M_{1n} in Eq. 13.48d
(using built-in function *NormpfromM* (M, k))

$p_2/p_1 =$ 9.15

Problem 13.142

13.142 The temperature and Mach number before an oblique shock are $T_1 = 10°\text{C}$ and $M_1 = 3.25$, respectively, and the pressure ratio across the shock is 5. Find the deflection angle, θ, the shock angle, β, and the Mach number after the shock, M_2.

Given: Data on an oblique shock

Find: Deflection angle θ; shock angle β; Mach number after shock

Solution:

The given or available data is:

R =	286.9	J/kg.K
k =	1.4	
M_1 =	3.25	
T_1 =	283	K
p_2/p_1 =	5	

Equations and Computations:

From p_2/p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (M,k)
and *Goal Seek* or *Solver*)

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

For p_2/p_1 = 5.00

M_{1n} = 2.10

From M_1 and M_{1n}, and Eq 13.47a

$$M_{1n} = M_1 \sin(\beta) \qquad (13.47a)$$

β = 40.4 °

From M_1 and β, and Eq. 13.49
(using built-in function *Theta* (M,β, k)

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k+\cos 2\beta)+2} \qquad (13.49)$$

θ = 23.6 °

To find M_2 we need M_{2n}. From M_{1n}, and Eq. 13.48a
(using built-in function *NormM2fromM* (M,k))

$$M_{2_n}^2 = \frac{M_{1_n}^2 + \frac{2}{k-1}}{\frac{2k}{k-1}M_{1_n}^2 - 1} \qquad (13.48a)$$

M_{2n} = 0.561

The downstream Mach number is then obtained from
from M_{2n}, θ and β, and Eq. 13.47b

$$M_{2n} = M_2 \sin(\beta - \theta) \qquad (13.47b)$$

Hence M_2 = 1.94

Problem 13.143

13.143 An airfoil at zero angle of attack has a sharp leading edge with an included angle of 20°. It is being tested over a range of speeds in a wind tunnel. The air temperature upstream is maintained at 15°C. Determine the Mach number and corresponding air speed at which a detached normal shock first attaches to the leading edge, and the angle of the resulting oblique shock. Plot the oblique shock angle β as a function of upstream Mach number M_1, from the minimum attached-shock value through $M_1 = 7$.

Given: Airfoil with included angle of 20°

Find: Mach number and speed at which oblique shock forms

Solution:

The given or available data is:

R =	286.9	J/kg.K
k =	1.4	
T_1 =	288	K
θ =	10	o

Equations and Computations:

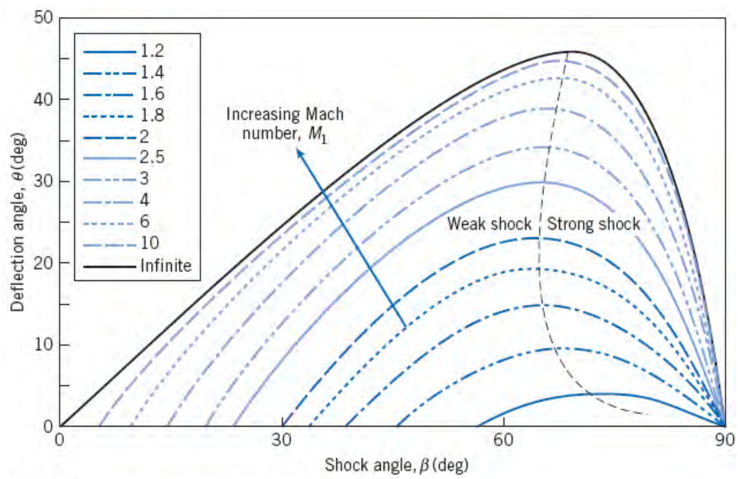


Fig. 13.29 Oblique shock deflection angle.

From Fig. 13.29 the smallest Mach number for which an oblique shock exists at a deflection $\theta = 10^{\circ}$ is approximately $M_1 = 1.4$.

By trial and error, a more precise answer is
(using built-in function *Theta* (M, β, k)

M_1 =	1.42	
β =	67.4	o
θ =	10.00	o
c_1 =	340	m/s
V_1 =	483	m/s

A suggested procedure is:

1) Type in a guess value for M_1
2) Type in a guess value for β

3) Compute θ from Eq. 13.49
   (using built-in function *Theta* (*M* ,β, *k*))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

4) Use *Solver* to maximize θ by varying β
5) If θ is not 10°, make a new guess for M_1
6) Repeat steps 1 - 5 until θ = 10°

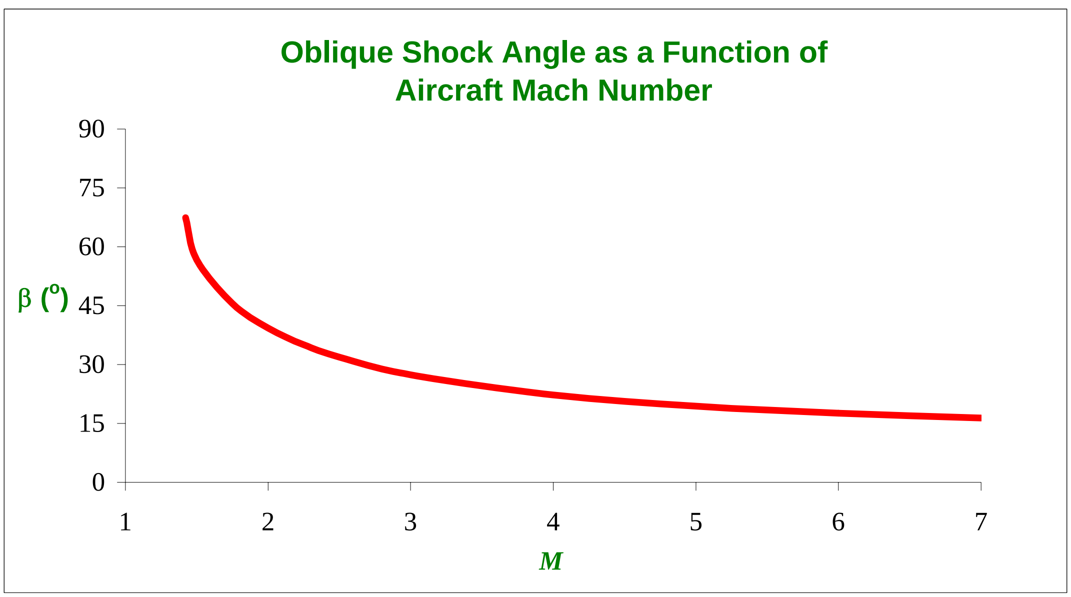
Computed results:

M_1	β (°)	θ (°)	Error
1.42	67.4	10.0	0.0%
1.50	56.7	10.0	0.0%
1.75	45.5	10.0	0.0%
2.00	39.3	10.0	0.0%
2.25	35.0	10.0	0.0%
2.50	31.9	10.0	0.0%
3.00	27.4	10.0	0.0%
4.00	22.2	10.0	0.0%
5.00	19.4	10.0	0.0%
6.00	17.6	10.0	0.0%
7.00	16.4	10.0	0.0%

Sum: 0.0%

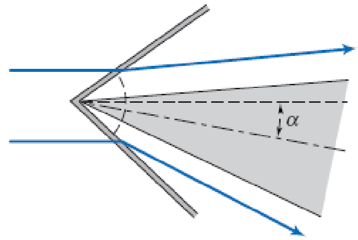
To compute this table:

1) Type the range of M_1
2) Type in guess values for β
3) Compute θ from Eq. 13.49
   (using built-in function *Theta* (*M* ,β, *k*)
4) Compute the absolute error between each θ and θ = 10°
5) Compute the sum of the errors
6) Use *Solver* to minimize the sum by varying the β values
   (Note: You may need to interactively type in new β values
   if *Solver* generates β values that lead to no θ, or to
   β values that correspond to a strong rather than weak shock)



Problem 13.144

13.144 An airfoil has a sharp leading edge with an included angle of $\delta = 60°$. It is being tested in a wind tunnel running at 1200 m/s (the air pressure and temperature upstream are 75 kPa and 3.5°C). Plot the pressure and temperature in the region adjacent to the upper surface as functions of angle of attack, α, ranging from $\alpha = 0°$ to 30°. What are the maximum pressure and temperature? (Ignore the possibility of a detached shock developing if α is too large; see Problem 13.145.)



Given: Airfoil with included angle of 60°

Find: Plot of temperature and pressure as functions of angle of attack

Solution:

The given or available data is:

R = 286.9 J/kg.K
k = 1.4
T_1 = 276.5 K
p_1 = 75 kPa
V_1 = 1200 m/s
δ = 60 °

Equations and Computations:

From T_1 c_1 = 333 m/s

Then M_1 = 3.60

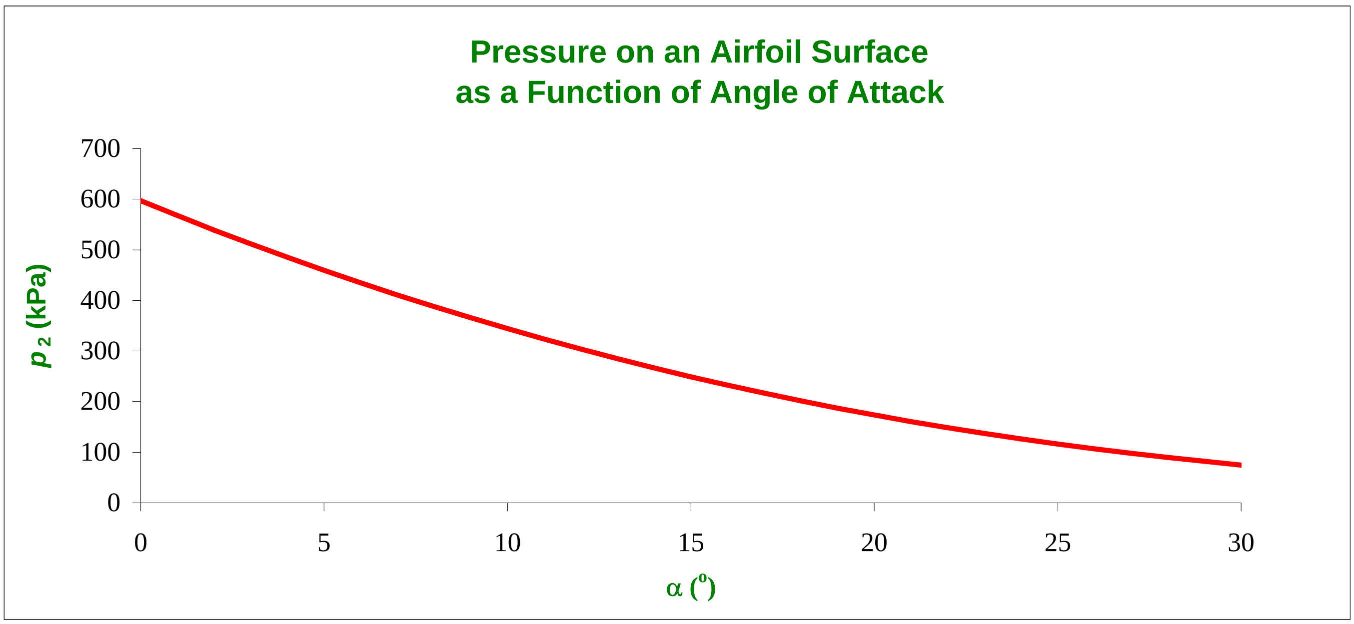
Computed results:

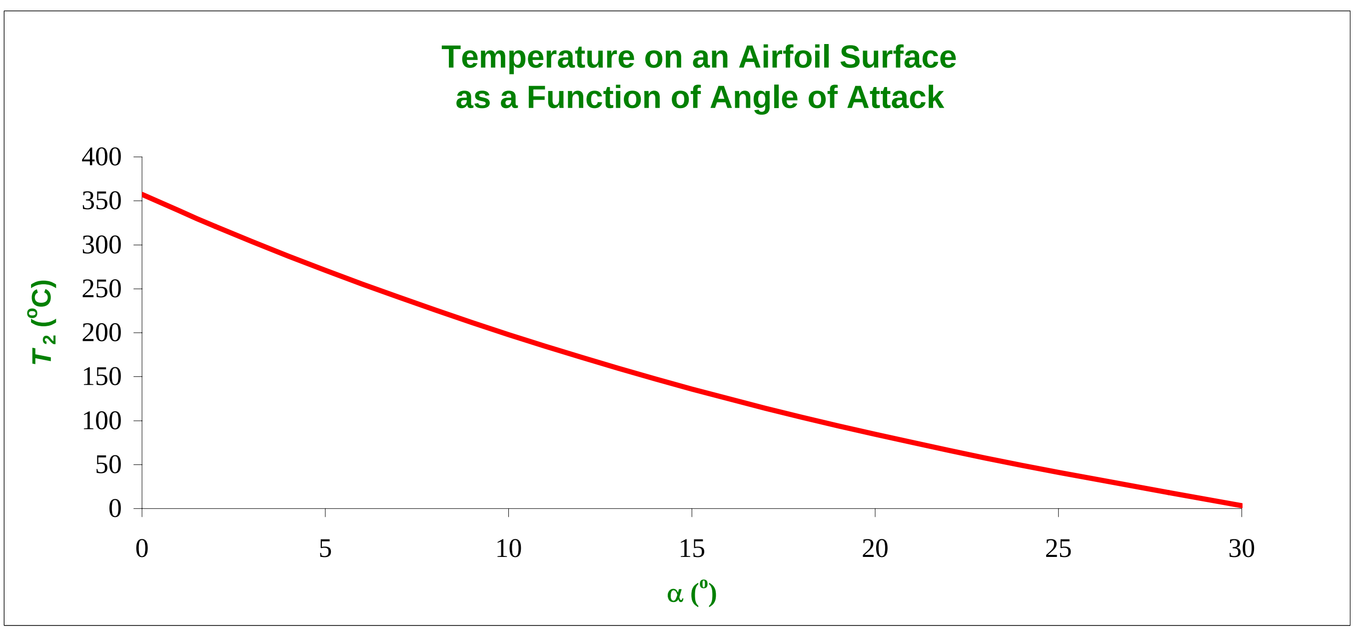
α (°)	β (°)	θ (°) Needed	θ (°)	Error	M_{1n}	p_2 (kPa)	T_2 (°C)
0.00	47.1	30.0	30.0	0.0%	2.64	597	357
2.00	44.2	28.0	28.0	0.0%	2.51	539	321
4.00	41.5	26.0	26.0	0.0%	2.38	485	287
6.00	38.9	24.0	24.0	0.0%	2.26	435	255
8.00	36.4	22.0	22.0	0.0%	2.14	388	226
10.00	34.1	20.0	20.0	0.0%	2.02	344	198
12.00	31.9	18.0	18.0	0.0%	1.90	304	172
14.00	29.7	16.0	16.0	0.0%	1.79	267	148
16.00	27.7	14.0	14.0	0.0%	1.67	233	125
18.00	25.7	12.0	12.0	0.0%	1.56	202	104
20.00	23.9	10.0	10.0	0.0%	1.46	174	84
22.00	22.1	8.0	8.0	0.0%	1.36	149	66
24.00	20.5	6.0	6.0	0.0%	1.26	126	49
26.00	18.9	4.0	4.0	0.0%	1.17	107	33
28.00	17.5	2.0	2.0	0.0%	1.08	90	18
30.00	16.1	0.0	0.0	0.0%	1.00	75	3

Sum: 0.0% Max: 597 357

To compute this table:

1) Type the range of α
2) Type in guess values for β
3) Compute θ_{Needed} from $\theta = \delta/2 - \alpha$
4) Compute θ from Eq. 13.49
   (using built-in function *Theta* (M ,β, k)
5) Compute the absolute error between each θ and θ_{Needed}
6) Compute the sum of the errors
7) Use *Solver* to minimize the sum by varying the β values
   (Note: You may need to interactively type in new β values
   if *Solver* generates β values that lead to no θ)
8) For each α, M_{1n} is obtained from M_1, and Eq. 13.47a
9) For each α, p_2 is obtained from p_1, M_{1n}, and Eq. 13.48d
   (using built-in function *NormpfromM* (M ,k))
10) For each α, T_2 is obtained from T_1, M_{1n}, and Eq. 13.48c
   (using built-in function *NormTfromM* (M ,k))

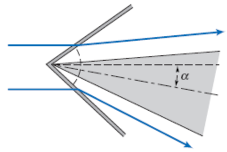




Problem 13.145

[4]

13.145 The airfoil of Problem 13.144 will develop a detached shock on the lower surface if the angle of attack, α, exceeds a certain value. What is this angle of attack? Plot the pressure and temperature in the region adjacent to the lower surface as functions of angle of attack, α, ranging from $\alpha = 0°$ to the angle at which the shock becomes detached. What are the maximum pressure and temperature?



Given: Airfoil with included angle of 60^o

Find: Angle of attack at which oblique shock becomes detached

Solution:

The given or available data is:

R =	286.9	J/kg.K
k =	1.4	
T_1 =	276.5	K
p_1 =	75	kPa
V_1 =	1200	m/s
δ =	60	o

Equations and Computations:

From T_1 $\quad c_1$ = 333 m/s

Then $\quad M_1$ = 3.60

From Fig. 13.29, at this Mach number the smallest deflection angle for which an oblique shock exists is approximately $\theta = 35^o$.

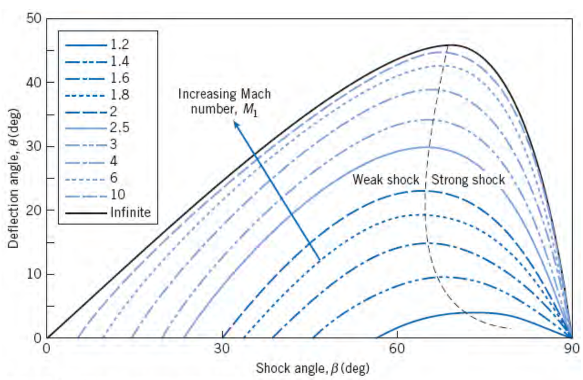


Fig. 13.29 Oblique shock deflection angle.

By using *Solver* , a more precise answer is
(using built-in function *Theta* (M,β, k)

M_1 =	3.60	
β =	65.8	o
θ =	37.3	o

A suggested procedure is:

1) Type in a guess value for β
2) Compute θ from Eq. 13.49
   (using built-in function *Theta* (M,β, k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

3) Use *Solver* to maximize θ by varying β

For a deflection angle θ the angle of attack α is

$\alpha = \theta - \delta/2$

α = 7.31 o

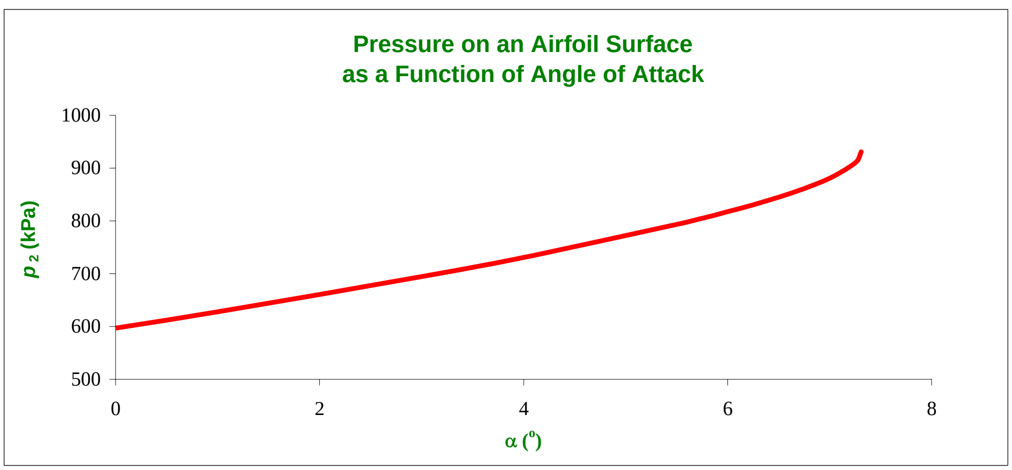
Computed results:

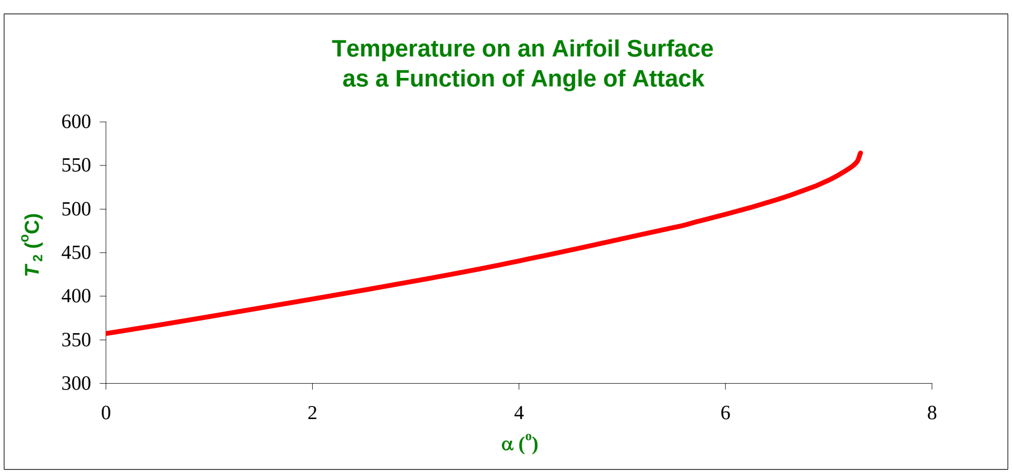
α (°)	β (°)	θ (°) Needed	θ (°)	Error	M_{1n}	p_2 (kPa)	T_2 (°C)
0.00	47.1	30.0	30.0	0.0%	2.64	597	357
1.00	48.7	31.0	31.0	0.0%	2.71	628	377
2.00	50.4	32.0	32.0	0.0%	2.77	660	397
3.00	52.1	33.0	33.0	0.0%	2.84	695	418
4.00	54.1	34.0	34.0	0.0%	2.92	731	441
5.50	57.4	35.5	35.5	0.0%	3.03	793	479
5.75	58.1	35.8	35.7	0.0%	3.06	805	486
6.00	58.8	36.0	36.0	0.0%	3.08	817	494
6.25	59.5	36.3	36.2	0.0%	3.10	831	502
6.50	60.4	36.5	36.5	0.0%	3.13	845	511
6.75	61.3	36.8	36.7	0.0%	3.16	861	521
7.00	62.5	37.0	37.0	0.0%	3.19	881	533
7.25	64.4	37.3	37.2	0.0%	3.25	910	551
7.31	65.8	37.3	37.3	0.0%	3.28	931	564

Sum:	0.0%		Max:	931	564

To compute this table:

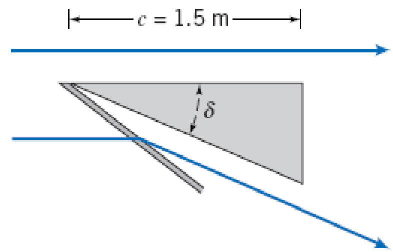
1) Type the range of α
2) Type in guess values for β
3) Compute θ_{Needed} from $\theta = \alpha + \delta/2$
4) Compute θ from Eq. 13.49
   (using built-in function *Theta* (M,β, k)
5) Compute the absolute error between each θ and θ_{Needed}
6) Compute the sum of the errors
7) Use *Solver* to minimize the sum by varying the β values
   (Note: You may need to interactively type in new β values
   if *Solver* generates β values that lead to no θ)
8) For each α, M_{1n} is obtained from M_1, and Eq. 13.47a
9) For each α, p_2 is obtained from p_1, M_{1n}, and Eq. 13.48d
   (using built-in function *NormpfromM* (M,k))
10) For each α, T_2 is obtained from T_1, M_{1n}, and Eq. 13.48c
   (using built-in function *NormTfromM* (M,k))





Problem 13.146

13.146 The wedge-shaped airfoil shown has chord $c = 1.5$ m and included angle $\delta = 7°$. Find the lift per unit span at a Mach number of 2.75 in air for which the static pressure is 70 kPa.



Given: Data on airfoil flight

Find: Lift per unit span

Solution:

The given or available data is:

R =	286.9	J/kg.K
k =	1.4	
p_1 =	70	kPa
M_1 =	2.75	
δ =	7	o
c =	1.5	m

Equations and Computations:

The lift per unit span is

$$L = (p_L - p_U)c \qquad (1)$$

(Note that p_L acts on area $c/\cos(\delta)$, but its normal component is multiplied by $\cos(\delta)$)

For the upper surface:

$$p_U = p_1$$

p_U = 70.0 kPa

For the lower surface:

We need to find M_{1n}

The deflection angle is $\theta = \delta$

$\theta =$ 7 °

From M_1 and θ, and Eq. 13.49
(using built-in function *Theta* (M, β, k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

For $\theta =$ 7.0 °
$\beta =$ 26.7 °

(Use *Goal Seek* to vary β so that $\theta = \delta$)

From M_1 and β $M_{1n} =$ 1.24

From M_{1n} and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (M, k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

$p_2 =$ 113 kPa

$p_L = p_2$

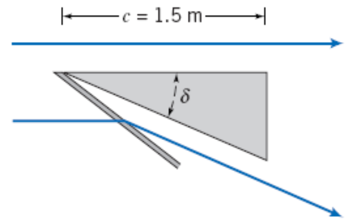
$p_L =$ 113 kPa

From Eq 1 $L =$ 64.7 kN/m

Problem 13.147

[3]

13.147 The wedge-shaped airfoil shown has chord $c = 2$ m and angles $\delta_{lower} = 15°$ and $\delta_{upper} = 5°$. Find the lift per unit span at a Mach number of 2.75 in air at a static pressure of 75 kPa.



Given: Data on airfoil flight

Find: Lift per unit span

Solution:

The given or available data is:

R =	286.9	J/kg.K
k =	1.4	
p_1 =	75	kPa
M_1 =	2.75	
δ_U =	5	o
δ_L =	15	o
c =	2	m

Equations and Computations:

The lift per unit span is

$$L = (p_L - p_U)c \qquad (1)$$

(Note that each p acts on area $c/\cos(\delta)$, but its normal component is multiplied by $\cos(\delta)$)

For the upper surface:

We need to find $M_{1n(U)}$

The deflection angle is $\theta_U = \delta_U$

$\theta_U = 5$ o

From M_1 and θ_U, and Eq. 13.49
(using built-in function *Theta* (M, β, k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

For θ_U = 5.00 o

β_U = 25.1 o

(Use *Goal Seek* to vary β_U so that $\theta_U = \delta_U$)

From M_1 and β_U $M_{1n(U)}$ = 1.16

From $M_{1n(U)}$ and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (M, k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

p_2 = 106 kPa

p_U = p_2

p_U = 106 kPa

For the lower surface:

We need to find $M_{1n(L)}$

The deflection angle is θ_L = δ_L

θ_L = 15 °

From M_1 and θ_L, and Eq. 13.49
(using built-in function *Theta* (M, β, k))

For θ_L = 15.00 °
β_L = 34.3 °

(Use *Goal Seek* to vary β_L so that $\theta_L = \delta_L$)

From M_1 and β_L $M_{1n(L)}$ = 1.55

From $M_{1n(L)}$ and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (M, k))

p_2 = 198 kPa

p_L = p_2

p_L = 198 kPa

From Eq 1 L = 183 kN/m

Problem 13.148

13.148 An oblique shock causes a flow that was at $M = 4$ and a static pressure of 75 kPa to slow down to $M = 2.5$. Find the deflection angle and the static pressure after the shock.

Given: Oblique shock Mach numbers

Find: Deflection angle; Pressure after shock

Solution:

The given or available data is:

$k =$	1.4	
$p_1 =$	75	kPa
$M_1 =$	4	
$M_2 =$	2.5	

Equations and Computations:

We make a guess for β: $\beta =$ 33.6 °

From M_1 and β, and Eq. 13.49 (using built-in function *Theta* (M, β,k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

$\theta =$ 21.0 °

From M_1 and β — $M_{1n} =$ 2.211

From M_2, θ, and β — $M_{2n} =$ 0.546 (1)

We can also obtain M_{2n} from Eq. 13.48a (using built-in function *normM2fromM* (M,k))

$$M_{2_n}^2 = \frac{M_{1_n}^2 + \frac{2}{k-1}}{\frac{2k}{k-1}M_{1_n}^2 - 1} \qquad (13.48a)$$

$M_{2n} =$ 0.546 (2)

We need to manually change β so that Eqs. 1 and 2 give the same answer.

Alternatively, we can compute the difference between 1 and 2, and use *Solver* to vary β to make the difference zero

Error in $M_{2n} =$ 0.00%

Then p_2 is obtained from Eq. 13.48d (using built-in function *normpfromm* (M,k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

$p_2 =$ 415 kPa

Problem 13.149

[4]

13.149 The geometry of the fuselage and engine cowling near the inlet to the engine of a supersonic fighter aircraft is designed so that the incoming air at $M = 3$ is deflected 7.5 degrees, and then experiences a normal shock at the engine entrance. If the incoming air is at 50 kPa, what is the pressure of the air entering the engine? What would be the pressure if the incoming air was slowed down by only a normal shock?

Given: Air flow into engine

Find: Pressure of air in engine; Compare to normal shock

Solution:

The given or available data is:

$k =$	1.4	
$p_1 =$	50	kPa
$M_1 =$	3	
$\theta =$	7.5	°

Equations and Computations:

Assuming isentropic flow deflection

$$p_0 = \text{constant}$$

$$p_{02} = p_{01}$$

For p_{01} we use Eq. 13.7a (using built-in function *Isenp* (M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$p_{01} =$	1837	kPa
$p_{02} =$	1837	kPa

For the deflection $\theta =$ 7.5 °

From M_1 and Eq. 13.55 (using built-in function *Omega* (M, k))

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \qquad (13.55)$$

$\omega_1 =$ 49.8 °

Deflection = $\omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$ (1)

Applying Eq. 1 $\omega_2 = \omega_1 - \theta$ (Compression!)

$\omega_2 =$ 42.3 °

From ω_2, and Eq. 13.55 (using built-in function *Omega* (M, k))

For	ω_2 =	42.3	°
	M_2 =	2.64	

(Use *Goal Seek* to vary M_2 so that ω_2 is correct)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (M, k))

$$p_2 = p_{02}/(p_{02}/p_2)$$

p_2 = 86.8 kPa

For the normal shock (2 to 3) M_2 = 2.64

From M_2 and p_2, and Eq. 13.41d (using built-in function *NormpfromM* (M, k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1} M_1^2 - \frac{k-1}{k+1} \qquad (13.41\text{d})$$

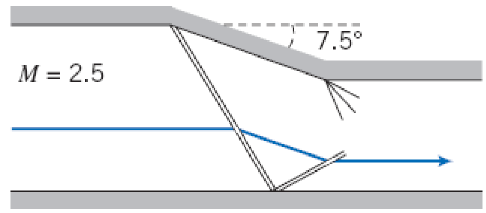
p_3 = 690 kPa

For slowing the flow down from M_1 with only a normal shock, using Eq. 13.41d

p = 517 kPa

Problem 13.150 [3]

13.150 Air flows isentropically at $M = 2.5$ in a duct. There is a 7.5° contraction that triggers an oblique shock, which in turn reflects off a wall generating a second oblique shock. This second shock is necessary so the flow ends up flowing parallel to the channel walls after the two shocks. Find the Mach number and pressure in the contraction and downstream of the contraction. (Note that the convex corner will have expansion waves to redirect the flow along the upper wall.)



Given: Air flow in a duct

Find: Mach number and pressure at contraction and downstream;

Solution:

The given or available data is:

k =	1.4	
M_1 =	2.5	
θ =	7.5	°
p_1 =	50	kPa

Equations and Computations:

For the first oblique shock (1 to 2) we need to find β from Eq. 13.49

$$\tan\theta = \frac{2\cot\beta(M_1^2\sin^2\beta - 1)}{M_1^2(k+\cos 2\beta)+2} \qquad (13.49)$$

We choose β by iterating or by using *Goal Seek* to target θ (below) to equal the given θ
Using built-in function *theta* (*M*, β,*k*)

θ =	7.50	°
β =	29.6	°

Then M_{1n} can be found from geometry (Eq. 13.47a)

M_{1n} = 1.233

Then M_{2n} can be found from Eq. 13.48a)
Using built-in function *NormM2fromM* (*M*,*k*)

$$M_{2_n} = f(M_{1_n}) \qquad (13.48a)$$

M_{2n} = 0.822

Then, from M_{2n} and geometry (Eq. 13.47b)

M_2 = 2.19

From M_{1n} and Eq. 13.48d (using built-in function *NormpfromM* (M ,k))

$$\frac{p_2}{p_1} = f(M_{1_n}) \qquad (13.48d)$$

p_2/p_1 =	1.61	Pressure ratio
p_2 =	80.40	

We repeat the analysis of states 1 to 2 for states 2 to 3, to analyze the second oblique shock

We choose β for M_2 by iterating or by using *Goal Seek* to target θ (below) to equal the given θ
Using built-in function *theta* (M, β,k)

θ =	7.50	o
β =	33.5	o

Then M_{2n} (normal to second shock!) can be found from geometry (Eq. 13.47a)

M_{2n} = 1.209

Then M_{3n} can be found from Eq. 13.48a)
Using built-in function *NormM2fromM* (M,k)

M_{3n} = 0.837

Then, from M_{3n} and geometry (Eq. 13.47b)

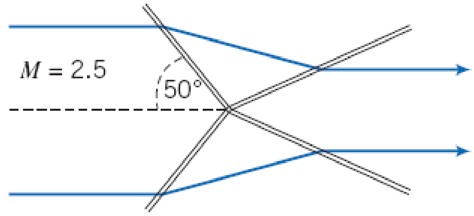
M_3 = 1.91

From M_{2n} and Eq. 13.48d (using built-in function *NormpfromM* (M ,k))

p_3/p_2 =	1.54	Pressure ratio
p_3 =	124	

Problem 13.151

13.151 A flow at $M = 2.5$ is deflected by a combination of interacting oblique shocks as shown. The first shock pair is aligned at 50° to the flow. A second oblique shock pair deflects the flow again so it ends up parallel to the original flow. If the pressure before any deflections is 50 kPa, find the pressure after two deflections.



NOTE: Angle is 30° not 50°!

Given: Air flow in a duct

Find: Mach number and pressure at contraction and downstream;

Solution:

The given or available data is:

k = 1.4

M_1 = 2.5

β = 30 °

p_1 = 50 kPa

Equations and Computations:

For the first oblique shock (1 to 2) we find θ from Eq. 13.49

$$\tan\theta = \frac{2\cot\beta(M_1^2\sin^2\beta - 1)}{M_1^2(k+\cos 2\beta)+2} \qquad (13.49)$$

Using built-in function *theta* (M, β, k)

θ = 7.99 °

Also, M_{1n} can be found from geometry (Eq. 13.47a)

M_{1n} = 1.250

Then M_{2n} can be found from Eq. 13.48a)

Using built-in function *NormM2fromM* (M,k)

$$M_{2_n} = f(M_{1_n}) \qquad (13.48a)$$

M_{2n} = 0.813

Then, from M_{2n} and geometry (Eq. 13.47b)

M_2 = 2.17

From M_{1n} and Eq. 13.48d (using built-in function *NormpfromM* (M ,k))

$$\frac{p_2}{p_1} = f(M_{1_n}) \qquad (13.48d)$$

p_2/p_1 =	1.66	Pressure ratio
p_2 =	82.8	

We repeat the analysis for states 1 to 2 for 2 to 3, for the second oblique shock

We choose β for M_2 by iterating or by using *Goal Seek* to target θ (below) to equal the previous θ
Using built-in function *theta* (M, β,k)

θ =	7.99	o
β =	34.3	o

Then M_{2n} (normal to second shock!) can be found from geometry (Eq. 13.47a)

M_{2n} = 1.22

Then M_{3n} can be found from Eq. 13.48a)
Using built-in function *NormM2fromM* (M,k)

M_{3n} = 0.829

Then, from M_{3n} and geometry (Eq. 13.47b)

M_3 = 1.87

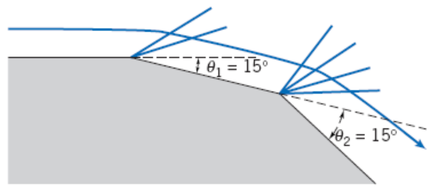
From M_{2n} and Eq. 13.48d (using built-in function *NormpfromM* (M ,k))

p_3/p_2 =	1.58	Pressure ratio
p_3 =	130	

Problem 13.152

[3]

13.152 Air flows at Mach number of 1.5, static pressure 95 kPa, and is expanded by angles $\theta_1 = 15°$ and $\theta_2 = 15°$, as shown. Find the pressure changes.



Given:
Deflection of air flow

Find:
Pressure changes

Solution:

The given or available data is:

R =	286.9	J/kg.K
k =	1.4	
p =	95	kPa
M =	1.5	
θ_1 =	15	°
θ_2 =	15	°

Equations and Computations:

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \qquad (13.55)$$

and

$$\text{Deflection} = \omega_a - \omega_b = \omega(M_a) - \omega(M_b) \qquad (1)$$

From M and Eq. 13.55 (using built-in function *Omega* (M, k))

ω = 11.9 °

For the first deflection:

Applying Eq. 1

$\theta_1 = \omega_1 - \omega$

$\omega_1 = \theta_1 + \omega$

ω_1 = 26.9 °

From ω_1, and Eq. 13.55

(using built-in function *Omega* (M, k))

For $\omega_1 =$ 26.9 ^{o}

$M_1 =$ 2.02

(Use *Goal Seek* to vary M_1 so that ω_1 is correct)

Hence for p_1 we use Eq. 13.7a

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/(k-1)} \qquad (13.7a)$$

The approach is to apply Eq. 13.7a twice, so that
(using built-in function *Isenp* (M, k))

$$p_1 = p\,(p_0/p)/(p_0/p_1)$$

$p_1 =$ 43.3 kPa

For the second deflection:

We repeat the analysis of the first deflection

Applying Eq. 1

$$\theta_2 + \theta_1 = \omega_2 - \omega$$

$$\omega_2 = \theta_2 + \theta_1 + \omega$$

$\omega_2 =$ 41.9 ^{o}

(Note that instead of working from the initial state to state 2 we could have worked from state 1 to state 2 because the entire flow is isentropic)

From ω_2, and Eq. 13.55
(using built-in function *Omega* (M, k))

For $\omega_2 =$ 41.9 ^{o}

$M_2 =$ 2.62

(Use *Goal Seek* to vary M_2 so that ω_2 is correct)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (M, k))

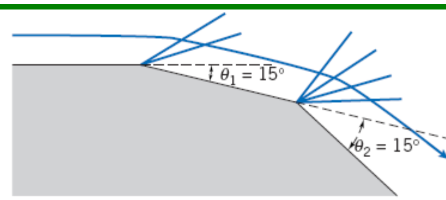
$$p_2 = p\,(p_0/p)/(p_0/p_2)$$

$p_2 =$ 16.9 kPa

Problem 13.153

[3]

13.153 Find the incoming and intermediate Mach numbers and static pressures if, after two expansions of $\theta_1 = 15°$ and $\theta_2 = 15°$, the Mach number is 4 and static pressure is 10 kPa.



Given: Deflection of air flow

Find: Mach numbers and pressures

Solution

The given or available data is:

R =	286.9	J/kg.K
k =	1.4	
p_2 =	10	kPa
M_2 =	4	
θ_1 =	15	o
θ_2 =	15	o

Equations and Computations:

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \qquad (13.55)$$

and

$$\text{Deflection} = \omega_a - \omega_b = \omega(M_a) - \omega(M_b) \qquad (1)$$

From M and Eq. 13.55 (using built-in function *Omega* (M , k))

ω_2 = 65.8 o

For the second deflection:

Applying Eq. 1

$\omega_1 = \omega_2 - \theta_2$

ω_1 = 50.8 o

From ω_1, and Eq. 13.55
(using built-in function *Omega* (M , k))

For ω_1 = 50.8 o
M_1 = 3.05

(Use *Goal Seek* to vary M_1 so that ω_1 is correct)

Hence for p_1 we use Eq. 13.7a

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/(k-1)} \qquad (13.7a)$$

The approach is to apply Eq. 13.7a twice, so that
(using built-in function *Isenp* (M, k))

$$p_1 = p_2(p_0/p_2)/(p_0/p_1)$$

$p_1 =$ 38.1 kPa

For the first deflection:

We repeat the analysis of the second deflection

Applying Eq. 1

$$\theta_2 + \theta_1 = \omega_2 - \omega$$

$$\omega = \omega_2 - (\theta_2 + \theta_1)$$

$\omega =$ 35.8 °

(Note that instead of working from state 2 to the initial state we could have worked from state 1 to the initial state because the entire flow is isentropic)

From ω, and Eq. 13.55
(using built-in function *Omega* (M, k))

For $\omega =$ 35.8 °
$M =$ 2.36

(Use *Goal Seek* to vary M so that ω is correct)

Hence for p we use Eq. 13.7a
(using built-in function *Isenp* (M, k))

$$p = p_2(p_0/p_2)/(p_0/p)$$

$p =$ 110 kPa

Problem 13.154

[4]

13.154 Compare the static and stagnation pressures produced by (a) an oblique shock and (b) isentropic *compression* waves as they each deflect a flow at a Mach number of 3.5 through a deflection angle of 35° in air for which the static pressure is 50 kPa.

Given: Mach number and deflection angle

Find: Static and stagnation pressures due to: oblique shock; compression wave

Solution:

The given or available data is:

R =	286.9	J/kg.K
k =	1.4	
p_1 =	50	kPa
M_1 =	3.5	
θ =	35	°

Equations and Computations:

For the oblique shock:

We need to find M_{1n}

The deflection angle is θ = 35 °

From M_1 and θ, and Eq. 13.49

(using built-in function *Theta* (M, β, k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k+\cos 2\beta)+2} \qquad (13.49)$$

For

θ =	35.0	°
β =	57.2	°

(Use *Goal Seek* to vary β so that $\theta = 35^\circ$)

From M_1 and β $\quad M_{1n}$ = 2.94

From M_{1n} and p_1, and Eq. 13.48d

(using built-in function *NormpfromM* (M, k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

p_2 = 496 kPa

To find M_2 we need M_{2n}. From M_{1n}, and Eq. 13.48a
(using built-in function *NormM2fromM* (M, k))

$$M_{2_n}^2 = \frac{M_{1_n}^2 + \frac{2}{k-1}}{\frac{2k}{k-1} M_{1_n}^2 - 1} \qquad (13.48a)$$

$M_{2n} =$ 0.479

The downstream Mach number is then obtained from
from M_{2n}, θ and β, and Eq. 13.47b

$$M_{2n} = M_2 \sin(\beta - \theta) \qquad (13.47b)$$

Hence $M_2 =$ 1.27

For p_{02} we use Eq. 12.7a
(using built-in function *Isenp* (M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$p_{02} = p_2/(p_{02}/p_2)$

$p_{02} =$ 1316 kPa

For the isentropic compression wave:

For isentropic flow $p_0 =$ constant

$p_{02} = p_{01}$

For p_{01} we use Eq. 13.7a
(using built-in function *Isenp* (M, k))

$p_{01} =$ 3814 kPa

$p_{02} =$ 3814 kPa

(Note that for the oblique shock, as required by Eq. 13.48b

$$\frac{p_{0_2}}{p_{0_1}} = \frac{\left[\frac{\frac{k+1}{2} M_{1_n}^2}{1 + \frac{k-1}{2} M_{1_n}^2}\right]^{k/(k-1)}}{\left[\frac{2k}{k+1} M_{1_n}^2 - \frac{k-1}{k+1}\right]^{1/(k-1)}} \qquad (13.48b)$$

$p_{02}/p_{01} =$ 0.345
(using built-in function *Normp0fromM* (M, k)

$p_{02}/p_{01} =$ 0.345
(using p_{02} from the shock and p_{01})

For the deflection $\theta = -\theta$ (*Compression*)

$\theta =$ -35.0 °

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \quad (13.55)$$

and

Deflection = $\omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$ (1)

From M_1 and Eq. 13.55 (using built-in function *Omega* (M, k))

$\omega_1 =$ 58.5 °

Applying Eq. 1 $\omega_2 = \omega_1 + \theta$

$\omega_2 =$ 23.5 °

From ω_2, and Eq. 13.55
(using built-in function *Omega* (M, k))

For $\omega_2 =$ 23.5 °
$M_2 =$ 1.90

(Use *Goal Seek* to vary M_2 so that $\omega_2 = 23.5^o$)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (M, k))

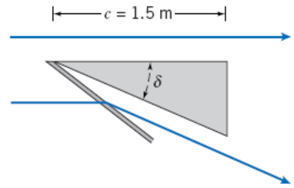
$p_2 = p_{02}/(p_{02}/p_2)$

$p_2 =$ 572 kPa

Problem 13.155

[3]

13.155 Consider the wedge-shaped airfoil of Problem 13.146. Suppose the oblique shock could be replaced by isentropic *compression* waves. Find the lift per unit span at the Mach number of 2.75 in air for which the static pressure is 70 kPa.



Given: Wedge-shaped airfoil

Find: Lift per unit span assuming isentropic flow

Solution:

The given or available data is:

R =	286.9	J/kg.K
k =	1.4	
p =	70	kPa
M =	2.75	
δ =	7	o
c =	1.5	m

Equations and Computations:

The lift per unit span is

$$L = (p_L - p_U)c \qquad (1)$$

(Note that p_L acts on area $c/\cos(\delta)$, but its normal component is multiplied by $\cos(\delta)$)

For the upper surface:

$p_U = p$

$p_U =$ 70 kPa

For the lower surface:

$\theta = -\delta$

$\theta =$ -7.0 o

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \qquad (13.55)$$

and

$$\text{Deflection} = \omega_L - \omega = \omega(M_L) - \omega(M) \qquad (2)$$

From M and Eq. 13.55 (using built-in function *Omega* (M , k))

$\omega =$ 44.7 ^{o}

Applying Eq. 2

$\theta = \omega_L - \omega$

$\omega_L = \theta + \omega$

$\omega_L =$ 37.7 ^{o}

From ω_L, and Eq. 13.55
(using built-in function *Omega* (M , k))

For $\omega_L =$ 37.7 ^{o}

$M_L =$ 2.44

(Use *Goal Seek* to vary M_L so that ω_L is correct)

Hence for p_L we use Eq. 13.7a

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/(k-1)} \qquad (13.7a)$$

The approach is to apply Eq. 13.7a twice, so that
(using built-in function *Isenp* (M , k))

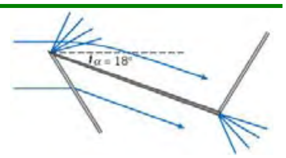
$p_L = p\,(p_0/p)/(p_0/p_L)$

$p_L =$ 113 kPa

From Eq 1 $L =$ 64.7 kN/m

Problem 13.156

13.156 Find the lift and drag per unit span on the airfoil shown for flight at a Mach number of 1.75 in air for which the static pressure is 50 kPa. The chord length is 1 m.



Given: Mach number and airfoil geometry

Find: Lift and drag per unit span

Solution:

The given or available data is:

R = 286.9 J/kg.K

k = 1.4

p_1 = 50 kPa

M_1 = 1.75

α = 18 ^{o}

c = 1 m

Equations and Computations:

The net force per unit span is $F = (p_L - p_U)c$

Hence, the lift force per unit span is

$$L = (p_L - p_U)c\cos(\alpha) \qquad (1)$$

The drag force per unit span is

$$D = (p_L - p_U)c\sin(\alpha) \qquad (2)$$

For the lower surface (oblique shock):

We need to find M_{1n}

The deflection angle is $\theta = \alpha$

θ = 18 ^{o}

From M_1 and θ, and Eq. 13.49

(using built-in function *Theta* (M, β, k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

For θ = 18.0 ^{o}

β = 62.9 ^{o}

(Use *Goal Seek* to vary β so that θ is correct)

From M_1 and β $\quad M_{1n}$ = 1.56

From M_{1n} and p_1, and Eq. 13.48d

(using built-in function *NormpfromM* (M, k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

p_2 = 133.2 kPa

$p_L = p_2$

p_L = 133.2 kPa

For the upper surface (isentropic expansion wave):

For isentropic flow p_0 = constant

p_{02} = p_{01}

For p_{01} we use Eq. 13.7a
(using built-in function *Isenp* (*M* , *k*))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \quad (13.7a)$$

p_{01} = 266 kPa

p_{02} = 266 kPa

For the deflection $\theta = \alpha$ (*Compression*)

θ = 18.0 °

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \quad (13.55)$$

and

Deflection = $\omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$ (3)

From M_1 and Eq. 13.55 (using built-in function *Omega* (*M* , *k*))

ω_1 = 19.3 °

Applying Eq. 3 $\omega_2 = \omega_1 + \theta$

ω_2 = 37.3 °

From ω_2, and Eq. 13.55 (using built-in function *Omega* (*M* , *k*))

For ω_2 = 37.3 °
M_2 = 2.42

(Use *Goal Seek* to vary M_2 so that ω_2 is correct)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (*M* , *k*))

$p_2 = p_{02}/(p_{02}/p_2)$

p_2 = 17.6 kPa

$p_U = p_2$

p_U = 17.6 kPa

From Eq. 1 L = 110.0 kN/m

From Eq. 2 D = 35.7 kN/m

Problem 13.157

13.157 Plot the lift and drag per unit span, and the lift/drag ratio, as functions of angle of attack for $\alpha = 0°$ to $18°$, for the airfoil shown, for flight at a Mach number of 1.75 in air for which the static pressure is 50 kPa. The chord length is 1 m.

Given:
Mach number and airfoil geometry

Find:
Plot of lift and drag and lift/drag versus angle of attack

Solution:

The given or available data is:

k =	1.4	
p_1 =	50	kPa
M_1 =	1.75	
α =	12	o
c =	1	m

Equations and Computations:

The net force per unit span is

$$F = (p_L - p_U)c$$

Hence, the lift force per unit span is

$$L = (p_L - p_U)c\cos(\alpha) \qquad (1)$$

The drag force per unit span is

$$D = (p_L - p_U)c\sin(\alpha) \qquad (2)$$

For each angle of attack the following needs to be computed:

For the lower surface (oblique shock):

We need to find M_{1n}

Deflection $\theta = \alpha$

From M_1 and θ, and Eq. 13.49
(using built-in function *Theta* (M, β, k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

find β

(Use *Goal Seek* to vary β so that θ is the correct value)

From M_1 and β find M_{1n}

From M_{1n} and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (M, k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

find p_2

and $p_L = p_2$

For the upper surface (isentropic expansion wave):

For isentropic flow p_0 = constant

p_{02} = p_{01}

For p_{01} we use Eq. 13.7a
(using built-in function *Isenp* (*M* , *k*))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/(k-1)} \qquad (13.7a)$$

find p_{02} = 266 kPa

Deflection θ = α

we use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}} \tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \qquad (13.55)$$

and

Deflection = $\omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$ (3)

From M_1 and Eq. 13.55 (using built-in function *Omega* (*M* , *k*))

find ω_1 = 19.3 º

Applying Eq. 3 $\omega_2 = \omega_1 + \theta$ (4)

From ω_2, and Eq. 12.55 (using built-in function *Omega* (*M* , *k*))

From ω_2 find M_2

(Use *Goal Seek* to vary M_2 so that ω_2 is the correct value)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (*M* , *k*))

$p_2 = p_{02}/(p_{02}/p_2)$

$p_U = p_2$

Finally, from Eqs. 1 and 2, compute *L* and *D*

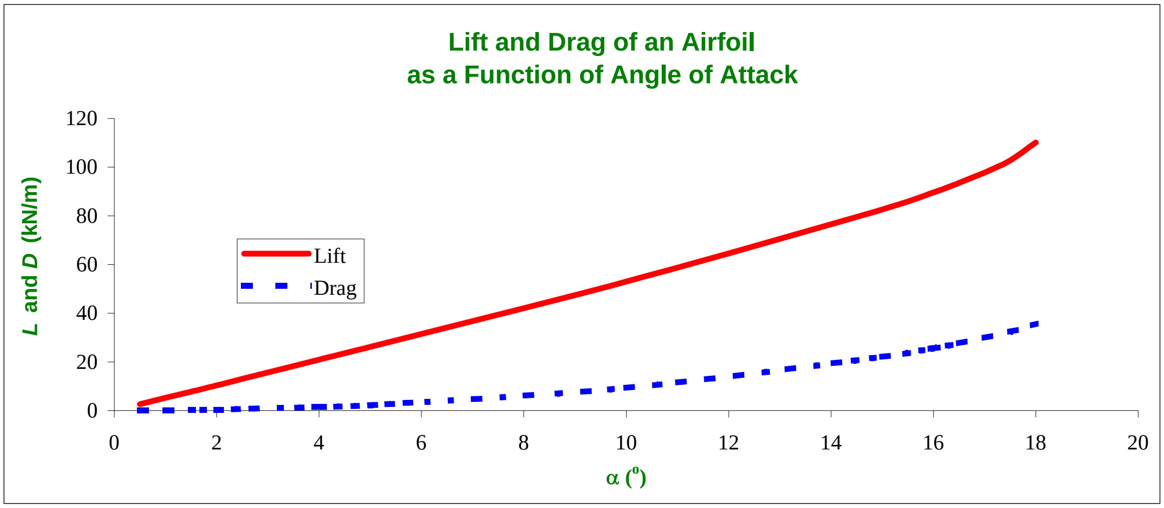
Computed results:

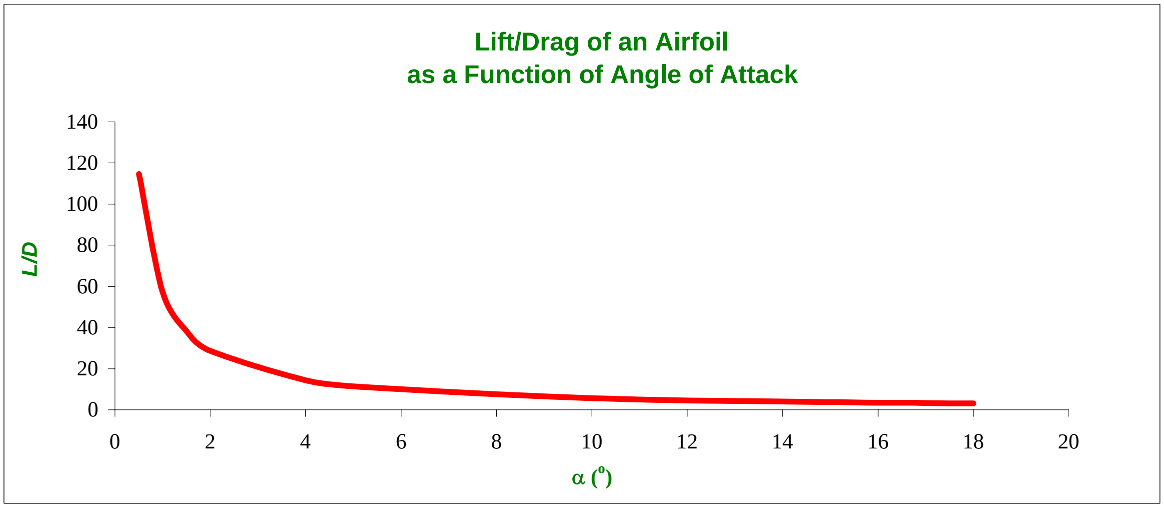
α (º)	β (º)	θ (º)	Error	M_{1n}	p_L (kPa)	ω_2 (º)	ω_2 from M_2 (º)	Error	M_2	p_U (kPa)	*L* (kN/m)	*D* (kN/m)	*L/D*
0.50	35.3	0.50	0.0%	1.01	51.3	19.8	19.8	0.0%	1.77	48.7	2.61	0.0227	115
1.00	35.8	1.00	0.0%	1.02	52.7	20.3	20.3	0.0%	1.78	47.4	5.21	0.091	57.3
1.50	36.2	1.50	0.0%	1.03	54.0	20.8	20.8	0.0%	1.80	46.2	7.82	0.205	38.2
2.00	36.7	2.00	0.0%	1.05	55.4	21.3	21.3	0.0%	1.82	45.0	10.4	0.364	28.6
4.00	38.7	4.00	0.0%	1.09	61.4	23.3	23.3	0.0%	1.89	40.4	20.9	1.46	14.3
5.00	39.7	5.00	0.0%	1.12	64.5	24.3	24.3	0.0%	1.92	38.3	26.1	2.29	11.4
10.00	45.5	10.0	0.0%	1.25	82.6	29.3	29.3	0.0%	2.11	28.8	53.0	9.35	5.67
15.00	53.4	15.0	0.0%	1.41	106.9	34.3	34.3	0.0%	2.30	21.3	82.7	22.1	3.73
16.00	55.6	16.0	0.0%	1.44	113.3	35.3	35.3	0.0%	2.34	20.0	89.6	25.7	3.49
16.50	56.8	16.5	0.0%	1.47	116.9	35.8	35.8	0.0%	2.36	19.4	93.5	27.7	3.38
17.00	58.3	17.0	0.0%	1.49	121.0	36.3	36.3	0.0%	2.38	18.8	97.7	29.9	3.27
17.50	60.1	17.5	0.0%	1.52	125.9	36.8	36.8	0.0%	2.40	18.2	102.7	32.4	3.17
18.00	62.9	18.0	0.0%	1.56	133.4	37.3	37.3	0.0%	2.42	17.6	110	35.8	3.08

Sum: 0.0% Sum: 0.0%

To compute this table:

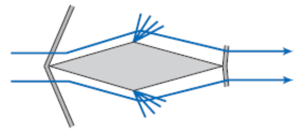
1) Type the range of α
2) Type in guess values for β
3) Compute θ from Eq. 13.49
   (using built-in function *Theta* (M, β, k)
4) Compute the absolute error between each θ and α
5) Compute the sum of the errors
6) Use *Solver* to minimize the sum by varying the β values
   (Note: You may need to interactively type in new β values
   if *Solver* generates β values that lead to no θ)
7) For each α, M_{1n} is obtained from M_1, and Eq. 13.47a
8) For each α, p_L is obtained from p_1, M_{1n}, and Eq. 13.48d
   (using built-in function *NormpfromM* (M, k))
9) For each α, compute ω_2 from Eq. 4
10) For each α, compute ω_2 from M_2, and Eq. 13.55
    (using built-in function *Omega* (M, k))
11) Compute the absolute error between the two values of ω_2
12) Compute the sum of the errors
13) Use *Solver* to minimize the sum by varying the M_2 values
    (Note: You may need to interactively type in new M_2 values)
    if *Solver* generates β values that lead to no θ)
14) For each α, p_U is obtained from p_{02}, M_2, and Eq. 13.47a
    (using built-in function *Isenp* (M, k))
15) Compute L and D from Eqs. 1 and 2





Problem 13.158

13.158 Find the drag coefficient of the symmetric, zero angle of attack airfoil shown for a Mach number of 2.0 in air for which the static pressure is 95 kPa and temperature is 0°C. The included angles at the nose and tail are each 10°.



Given: Mach number and airfoil geometry

Find: Drag coefficient

Solution:

The given or available data is:

R =	286.9	J/kg.K
k =	1.4	
p_1 =	95	kPa
M_1 =	2	
α =	0	o
δ =	10	o

Equations and Computations:

The drag force is

$$D = (p_F - p_R) cs \tan(\delta/2) \qquad (1)$$

(s and c are the span and chord)

This is obtained from the following analysis

Airfoil thickness (frontal area) = $2s(c/2\tan(\delta/2))$

Pressure difference acting on frontal area = $(p_F - p_R)$

(p_F and p_R are the pressures on the front and rear surfaces)

The drag coefficient is $C_D = D/(1/2\rho V^2 A)$ (2)

But it can easily be shown that

$$\rho V^2 = pkM^2$$

Hence, from Eqs. 1 and 2

$$C_D = (p_F - p_R)\tan(\delta/2)/(1/2pkM^2) \qquad (3)$$

For the frontal surfaces (oblique shocks):

We need to find M_{1n}

The deflection angle is $\theta = \delta/2$

θ = 5 o

From M_1 and θ, and Eq. 13.49
(using built-in function *Theta*(M, β,k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k+\cos 2\beta)+2} \qquad (13.49)$$

For	θ =	5.0	^{o}
	β =	34.3	^{o}

(Use *Goal Seek* to vary β so that $\theta = 5^o$)

From M_1 and β $\qquad M_{1n}$ = 1.13

From M_{1n} and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (*M* ,*k*))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

p_2 = 125.0 kPa

p_F = p_2

p_F = 125.0 kPa

To find M_2 we need M_{2n}. From M_{1n}, and Eq. 13.48a
(using built-in function *NormM2fromM* (*M* ,*k*))

$$M_{2_n}^2 = \frac{M_{1_n}^2 + \frac{2}{k-1}}{\frac{2k}{k-1}M_{1_n}^2 - 1} \qquad (13.48a)$$

M_{2n} = 0.891

The downstream Mach number is then obtained from
from M_{2n}, θ and β, and Eq. 13.47b

$$M_{2n} = M_2\sin(\beta - \theta) \qquad (13.47b)$$

Hence $\qquad M_2$ = 1.82

For p_{02} we use Eq. 13.7a
(using built-in function *Isenp* (*M* , *k*))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

p_{02} = 742 kPa

For the rear surfaces (isentropic expansion waves):

Treating as a new problem

Here: M_1 is the Mach number after the shock
and M_2 is the Mach number after the expansion wave
p_{01} is the stagnation pressure after the shock
and p_{02} is the stagnation pressure after the expansion wave

$M_1 = M_2$ (shock)

$M_1 =$ 1.82

$p_{01} = p_{02}$ (shock)

$p_{01} =$ 742 kPa

For isentropic flow $p_0 =$ constant

$p_{02} = p_{01}$

$p_{02} =$ 742 kPa

For the deflection $\theta = \delta$

$\theta =$ 10.0 ^{o}

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \quad (13.55)$$

and

Deflection = $\omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$ (3)

From M_1 and Eq. 13.55 (using built-in function *Omega* (M , k))

$\omega_1 =$ 21.3 ^{o}

Applying Eq. 3 $\omega_2 = \omega_1 + \theta$

$\omega_2 =$ 31.3 ^{o}

From ω_2, and Eq. 13.55 (using built-in function Omega(M, k))

For $\omega_2 =$ 31.3 ^{o}

$M_2 =$ 2.18

(Use *Goal Seek* to vary M_2 so that $\omega_2 = 31.3^o$)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (M , k))

$p_2 = p_{02}/(p_{02}/p_2)$

$p_2 =$ 71.2 kPa

$p_R = p_2$

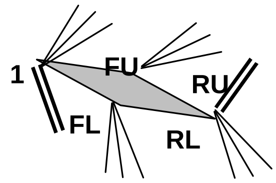
$p_R =$ 71.2 kPa

Finally, from Eq. 1 $C_D =$ 0.0177

Problem 13.159

[4]

13.159 Find the lift and drag coefficients of the airfoil of Problem 13.158 if the airfoil now has an angle of attack of 12°.



Given: Mach number and airfoil geometry

Find: Lift and Drag coefficients

Solution:

The given or available data is:

R =	286.9	J/kg.K
k =	1.4	
p_1 =	95	kPa
M_1 =	2	
α =	12	o
δ =	10	o

Equations and Computations:

Following the analysis of Example 13.14
the force component perpendicular to the major axis, per area, is

$$F_V/sc = 1/2\{(p_{FL} + p_{RL}) - (p_{FU} + p_{RU})\} \quad (1)$$

and the force component parallel to the major axis, per area, is

$$F_H/sc = 1/2\tan(\delta/2)\{(p_{FU} + p_{FL}) - (p_{RU} + p_{RL})\} \quad (2)$$

using the notation of the figure above.
(s and c are the span and chord)

The lift force per area is

$$F_L/sc = (F_V\cos(\alpha) - F_H\sin(\alpha))/sc \quad (3)$$

The drag force per area is

$$F_D/sc = (F_V\sin(\alpha) + F_H\cos(\alpha))/sc \quad (4)$$

The lift coefficient is $C_L = F_L/(1/2\rho V^2 A)$ (5)

But it can be shown that

$$\rho V^2 = pkM^2 \quad (6)$$

Hence, combining Eqs. 3, 4, 5 and 6

$$C_L = (F_V/sc\cos(\alpha) - F_H/sc\sin(\alpha))/(1/2pkM^2) \quad (7)$$

Similarly, for the drag coefficient

$$C_D = (F_V/sc\sin(\alpha) + F_H/sc\cos(\alpha))/(1/2pkM^2) \quad (8)$$

For surface FL (oblique shock):

We need to find M_{1n}

The deflection angle is $\theta = \alpha + \delta/2$

$\theta =$ 17 ^{o}

From M_1 and θ, and Eq. 13.49
(using built-in function *Theta* (M, β, k))

$$\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2(k + \cos 2\beta) + 2} \qquad (13.49)$$

For $\theta =$ 17.0 ^{o}

$\beta =$ 48.2 ^{o}

(Use *Goal Seek* to vary β so that $\theta = 17^o$)

From M_1 and β $\quad M_{1n} =$ 1.49

From M_{1n} and p_1, and Eq. 13.48d
(using built-in function *NormpfromM* (M, k))

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_{1_n}^2 - \frac{k-1}{k+1} \qquad (13.48d)$$

$p_2 =$ 230.6 kPa

$p_{FL} = p_2$

$p_{FL} =$ 230.6 kPa

To find M_2 we need M_{2n}. From M_{1n}, and Eq. 13.48a
(using built-in function *NormM2fromM* (M, k))

$$M_{2_n}^2 = \frac{M_{1_n}^2 + \frac{2}{k-1}}{\frac{2k}{k-1}M_{1_n}^2 - 1} \qquad (13.48a)$$

$M_{2n} =$ 0.704

The downstream Mach number is then obtained from
from M_{2n}, θ and β, and Eq. 13.47b

$$M_{2n} = M_2\sin(\beta - \theta) \qquad (13.47b)$$

Hence $\quad M_2 =$ 1.36

For p_{02} we use Eq. 13.7a
(using built-in function *Isenp* (M, k))

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (13.7a)$$

$p_{02} =$ 693 kPa

For surface RL (isentropic expansion wave):

Treating as a new problem

Here: M_1 is the Mach number after the shock
and M_2 is the Mach number after the expansion wave
p_{01} is the stagnation pressure after the shock
and p_{02} is the stagnation pressure after the expansion wave

$M_1 = M_2$ (shock)

$M_1 =$ 1.36

$p_{01} = p_{02}$ (shock)

$p_{01} =$ 693 kPa

For isentropic flow $p_0 =$ constant

$p_{02} = p_{01}$

$p_{02} =$ 693 kPa

For the deflection $\theta = \delta$

$\theta =$ 10.0 °

We use Eq. 13.55

$$\omega = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\left(\sqrt{\frac{k-1}{k+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \quad (13.55)$$

and

Deflection = $\omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$ (3)

From M_1 and Eq. 13.55 (using built-in function *Omega* (M, k))

$\omega_1 =$ 7.8 °

Applying Eq. 3 $\omega_2 = \omega_1 + \theta$

$\omega_2 =$ 17.8 °

From ω_2, and Eq. 13.55 (using built-in function *Omega* (M, k))

For $\omega_2 =$ 17.8 °
$M_2 =$ 1.70

(Use *Goal Seek* to vary M_2 so that $\omega_2 = 17.8^o$)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (M, k))

$p_2 = p_{02}/(p_{02}/p_2)$

$p_2 =$ 141 kPa

$p_{RL} = p_2$

$p_{RL} =$ 141 kPa

For surface FU (isentropic expansion wave):

$M_1 =$ 2.0

For isentropic flow $p_0 =$ constant

$p_{02} = p_{01}$

For p_{01} we use Eq. 13.7a
(using built-in function *Isenp* (M , k))

$p_{01} =$ 743
$p_{02} =$ 743 kPa

For the deflection $\theta = \alpha - \delta/2$

$\theta =$ 7.0 °

We use Eq. 13.55

and

Deflection = $\omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$ (3)

From M_1 and Eq. 13.55 (using built-in function *Omega* (M , k))

$\omega_1 =$ 26.4 °

Applying Eq. 3 $\omega_2 = \omega_1 + \theta$

$\omega_2 =$ 33.4 °

From ω_2, and Eq. 13.55 (using built-in function Omega(M, k))

For $\omega_2 =$ 33.4 °
$M_2 =$ 2.27

(Use *Goal Seek* to vary M_2 so that $\omega_2 = 33.4^o$)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (M , k))

$p_2 = p_{02}/(p_{02}/p_2)$

$p_2 =$ 62.8 kPa

$p_{FU} = p_2$

$p_{FU} =$ 62.8 kPa

For surface RU (isentropic expansion wave):

Treat as a new problem.

Flow is isentropic so we could analyse from region FU to RU
but instead analyse from region 1 to region RU.

$M_1 =$ 2.0

For isentropic flow $p_0 =$ constant

$p_{02} = p_{01}$

	p_{01} =	743	kPa
	p_{02} =	743	kPa

TOTAL deflection $\theta = \alpha + \delta/2$

θ = 17.0 ^{o}

We use Eq. 13.55

and

Deflection = $\omega_2 - \omega_1 = \omega(M_2) - \omega(M_1)$ (3)

From M_1 and Eq. 13.55 (using built-in function *Omega* (*M* , *k*))

ω_1 = 26.4 ^{o}

Applying Eq. 3 $\omega_2 = \omega_1 + \theta$

ω_2 = 43.4 ^{o}

From ω_2, and Eq. 13.55 (using built-in function Omega(M, k))

For	ω_2 =	43.4	^{o}
	M_2 =	2.69	

(Use *Goal Seek* to vary M_2 so that $\omega_2 = 43.4^o$)

Hence for p_2 we use Eq. 13.7a
(using built-in function *Isenp* (*M* , *k*))

$p_2 = p_{02}/(p_{02}/p_2)$

p_2 = 32.4 kPa

$p_{RU} = p_2$

p_{RU} = 32.4 kPa

The four pressures are:

	p_{FL} =	230.6	kPa
	p_{RL} =	140.5	kPa
	p_{FU} =	62.8	kPa
	p_{RU} =	32.4	kPa
From Eq 1	F_V/sc =	138	kPa
From Eq 2	F_H/sc =	5.3	kPa
From Eq 7	C_L =	0.503	
From Eq 8	C_D =	0.127	