

10.2

10.2 On a distant planet small amplitude waves travel across a 1-m-deep pond with a speed of 5 m/s. Determine the acceleration of gravity on the surface of that planet.

$$c = \sqrt{gy}, \text{ where } c = 5 \frac{m}{s} \text{ and } y = 1m$$

Thus,

$$g = \frac{c^2}{y} = \frac{(5 \frac{m}{s})^2}{1m} = \underline{\underline{25 \frac{m}{s^2}}}$$

10.3

10.3 The flowrate in a 50-ft-wide, 2-ft-deep river is $Q = 190$ cfs. Is the flow subcritical or supercritical?

$$Fr = \frac{V}{\sqrt{gy}}, \text{ where } V = \frac{Q}{A} = \frac{190 \frac{ft^3}{s}}{(2ft)(50ft)} = 1.90 \frac{ft}{s}$$

Thus,

$$Fr = \frac{1.90 \frac{ft}{s}}{\sqrt{(32.2 \frac{ft}{s^2})(2ft)}} = 0.237 < 1 \quad \underline{\underline{\text{The flow is subcritical.}}}$$

10.4 The flowrate per unit width in a wide channel is $q = 2.3 \text{ m}^2/\text{s}$. Is the flow subcritical or supercritical if the depth is (a) 0.2 m, (b) 0.8m, or (c) 2.5 m?

$$V = \frac{Q}{A} = \frac{qb}{yb} = \frac{q}{y} \text{ so that } Fr = \frac{V}{\sqrt{gy}} = \frac{q}{y\sqrt{gy}} = \frac{q}{\sqrt{g} y^{3/2}}$$

or

$$Fr = \frac{2.3 \frac{\text{m}^2}{\text{s}}}{\sqrt{9.81 \frac{\text{m}}{\text{s}^2}} y^{3/2}} = \frac{0.734}{y^{3/2}}, \text{ where } y \sim \text{m}$$

	$y, \text{ m}$	Fr	flow type
a)	0.2	8.21	supercritical
b)	0.8	1.03	supercritical
c)	2.5	0.186	<u>subcritical</u>

10.5

10.5 A rectangular channel 3 m wide carries $10 \text{ m}^3/\text{s}$ at a depth of 2 m. Is the flow subcritical or supercritical? For the same flowrate, what depth will give critical flow?

$$Q = AV \text{ or } V = \frac{Q}{bY} = \frac{10 \frac{\text{m}^3}{\text{s}}}{(3\text{m})(2\text{m})} = 1.667 \frac{\text{m}}{\text{s}}$$

$$\text{Thus, } Fr = \frac{V}{\sqrt{gY}} = \frac{1.667 \frac{\text{m}}{\text{s}}}{[(9.81 \frac{\text{m}}{\text{s}^2})(2\text{m})]^{1/2}} = 0.376 < 1 \text{ The flow is } \underline{\text{subcritical}}.$$

$$\text{Also, } Y_c = \left(\frac{q^2}{g} \right)^{1/3}, \text{ where } q = \frac{Q}{b} = \frac{10 \frac{\text{m}^3}{\text{s}}}{3\text{m}} = 3.33 \frac{\text{m}^2}{\text{s}} \text{ so that}$$

$$Y_c = \left(\frac{(3.33 \frac{\text{m}^2}{\text{s}})^2}{9.81 \frac{\text{m}}{\text{s}^2}} \right)^{1/3} = \underline{\underline{1.04 \text{ m}}}$$

10.6

10.6 Consider waves made by dropping objects (one after another from a fixed location) into a stream of depth y that is moving with speed V as shown in Fig. P10.6 (see Video V10.5). The circular wave crests that are produced travel with speed $c = (gy)^{1/2}$ relative to the moving water. Thus, as the circular waves are washed downstream, their diameters increase and the center of each circle is fixed relative to the moving water. (a) Show that if the flow is supercritical, lines tangent to the waves generate a wedge of half-angle $\alpha/2 = \arcsin(1/Fr)$, where $Fr = V/(gy)^{1/2}$ is the Froude number. (b) Discuss what happens to the wave pattern when the flow is subcritical, $Fr < 1$.

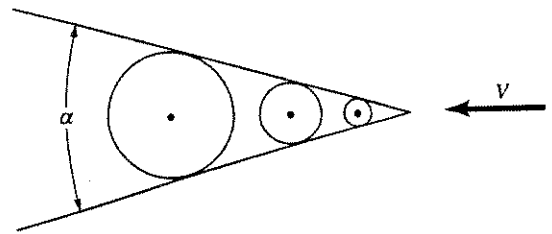
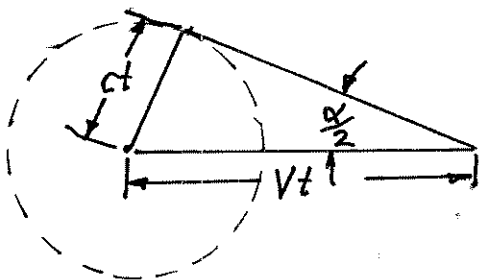


FIGURE P10.6

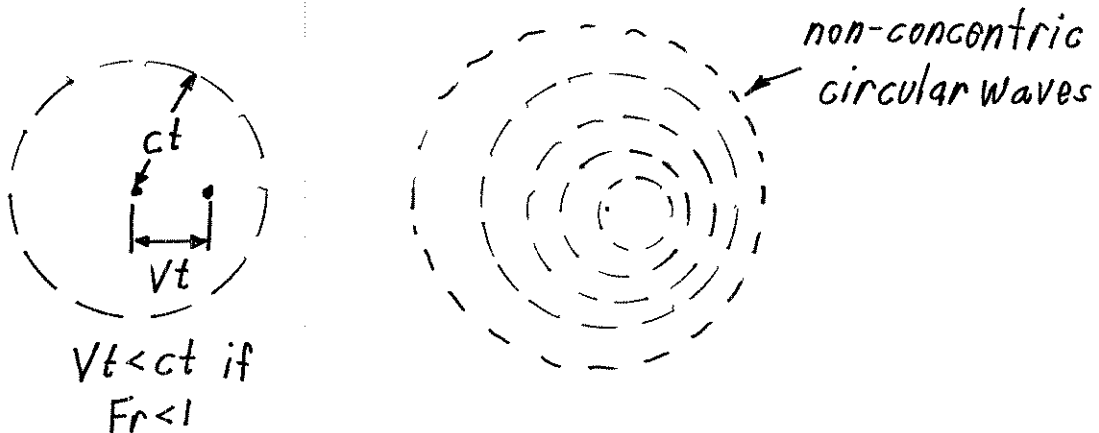
(a) In a time interval of t since the object hit the water (and initiated the wave), the center of the wave has been swept downstream a distance Vt and the wave has expanded to be a distance ct from its center. This is shown in the figure below. Note that $Vt > ct$ if $V > c$ (i.e. $Fr > 1$).



Thus, from the figure,
$$\sin \frac{\alpha}{2} = \frac{ct}{Vt} = \frac{c}{V} = \frac{\sqrt{gy}}{V} = \frac{1}{Fr}$$

or
$$\frac{\alpha}{2} = \arcsin(1/Fr)$$

(b) If $Fr < 1$ the above result gives $\sin \frac{\alpha}{2} > 1$, which is impossible. For $Fr < 1$ the following wave pattern would result. There is no "wedge" produced.



10.7

10.7 Waves on the surface of a tank are observed to travel at a speed of 2 m/s. How fast would these waves travel if (a) the tank were in an elevator accelerating downward at a rate of 4 m/s^2 , (b) the tank accelerates horizontally at a rate of 9.81 m/s^2 , (c) the tank were aboard the orbiting Space Shuttle? Explain.

Since $c = \sqrt{gy}$ it follows that the tank depth is

$$y = \frac{c^2}{g} = \frac{(2 \frac{\text{m}}{\text{s}})^2}{9.81 \frac{\text{m}}{\text{s}^2}} = 0.408 \text{ m}$$

(a) If the tank accelerates down with acceleration a , the effective acceleration of gravity is $g_{\text{eff}} = g - a = (9.81 - 4) \frac{\text{m}}{\text{s}^2} = 5.81 \frac{\text{m}}{\text{s}^2}$

Thus,

$$c = \sqrt{g_{\text{eff}} y} = \sqrt{(5.81 \frac{\text{m}}{\text{s}^2})(0.408 \text{ m})} = \underline{\underline{1.54 \frac{\text{m}}{\text{s}}}}$$

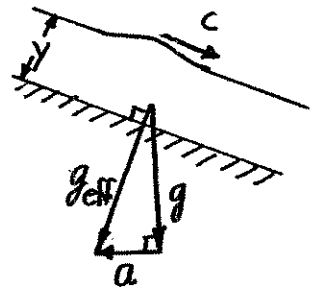
(b) If the tank accelerates horizontally with acceleration a , the effective acceleration is

$$g_{\text{eff}} = \sqrt{g^2 + a^2} = \sqrt{9.81^2 + 9.81^2} = 13.87 \frac{\text{m}}{\text{s}^2}$$

Thus,

$$c = \sqrt{(13.87 \frac{\text{m}}{\text{s}^2})(0.408 \text{ m})} = \underline{\underline{2.38 \frac{\text{m}}{\text{s}}}}$$

(c) In orbit $g_{\text{eff}} = 0$ (weightless) so $c = \underline{\underline{0}}$



10.8 In flowing from section (1) to section (2) along an open channel, the water depth decreases by a factor of two and the Froude number changes from a subcritical value of 0.5 to a supercritical value of 3.0. Determine the channel width at (2) if it is 12 ft wide at (1).

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = 0.5, \text{ or } \sqrt{gy_1} = 2.0 V_1 \quad (1)$$

and

$$Fr_2 = \frac{V_2}{\sqrt{gy_2}} = 3.0 \text{ where } y_2 = 0.5 y_1$$

$$\text{Thus, } \frac{V_2}{\sqrt{0.5gy_1}} = 3.0, \text{ or } \sqrt{gy_1} = V_2 / (3\sqrt{0.5}) \quad (2)$$

By equating Eq. (1) and (2): $2.0 V_1 = V_2 / (3\sqrt{0.5})$
or

$$V_2 = 4.24 V_1$$

However, $Q_1 = Q_2$ or $b_1 y_1 V_1 = b_2 y_2 V_2$, where b = channel width.

Thus, with $b_1 = 12$ ft:

$$(12 \text{ ft}) y_1 (V_1) = b_2 (0.5 y_1) (4.24 V_1), \text{ or } b_2 = \frac{12 \text{ ft}}{0.5 (4.24)} = \underline{\underline{5.66 \text{ ft}}}$$

10.9

10.9 Observations at a shallow sandy beach show that even though the waves several hundred yards out from the shore are not parallel to the beach, the waves often "break" on the beach nearly parallel to the shore as is indicated in Fig. P10.9. Explain this behavior based on the wave speed $c = (gy)^{1/2}$.

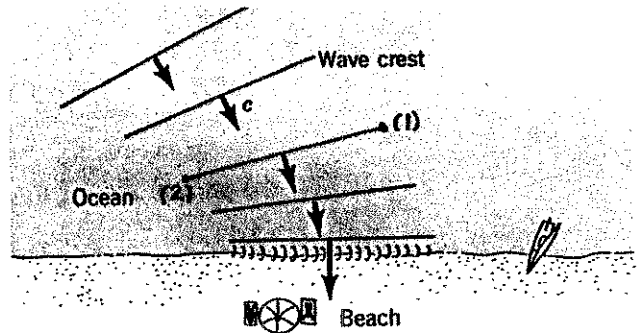


FIGURE P10.9

Since $c = \sqrt{gy}$ it follows that $c_1 > c_2$ because of the fact that $y_1 > y_2$. Therefore, as the waves move, that portion in the deeper water tends to "catch up" with that portion closer to shore in the shallower water. The wave crest tends to become more nearly parallel to the shore line. The waves "break" on the shore as if the wind were blowing normal to the shore.

10.11 Often when an earthquake shifts a segment of the ocean floor, a relatively small amplitude wave of very long wavelength is produced. Such waves go unnoticed as they move across the open ocean; only when they approach the shore do they become dangerous (a tsunami or "tidal wave"). Determine the wave speed if the wavelength, λ , is 6000 ft and the ocean depth is 15,000 ft.

$$\text{From Eq. 10.4: } c = \left[\frac{g\lambda}{2\pi} \tanh\left(\frac{2\pi y}{\lambda}\right) \right]^{\frac{1}{2}}$$

or

$$c = \left[\frac{(32.2 \frac{\text{ft}}{\text{s}^2})(6000 \text{ ft})}{2\pi} \tanh\left(\frac{2\pi(15,000 \text{ ft})}{6000 \text{ ft}}\right) \right]^{\frac{1}{2}} = \underline{\underline{175 \frac{\text{ft}}{\text{s}}}}$$

10.12

10.12 A bicyclist rides through a 3-in. deep puddle of water as shown in Video V10.5 and Fig. P10.12. If the angle made by the V-shaped wave pattern produced by the front wheel is observed to be 40 deg, estimate the speed of the bike through the puddle. *Hint:* Make a sketch of the current location of the bike wheel relative to where it was Δt seconds ago. Also indicate on this sketch the current location of the wave that the wheel made Δt seconds ago. Recall that the wave moves radially outward in all directions with speed c relative to the stationary water.

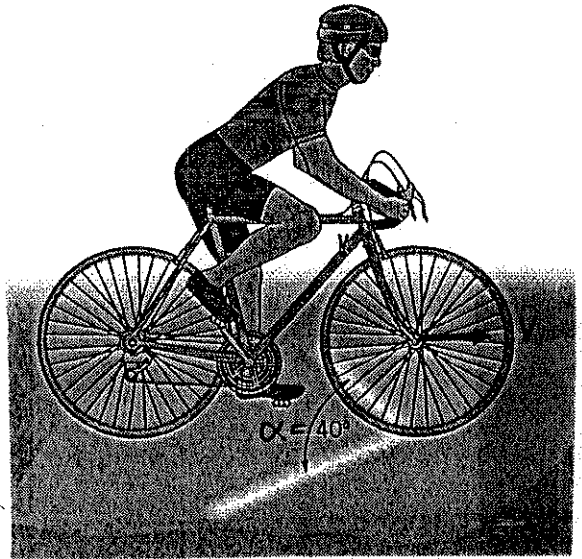
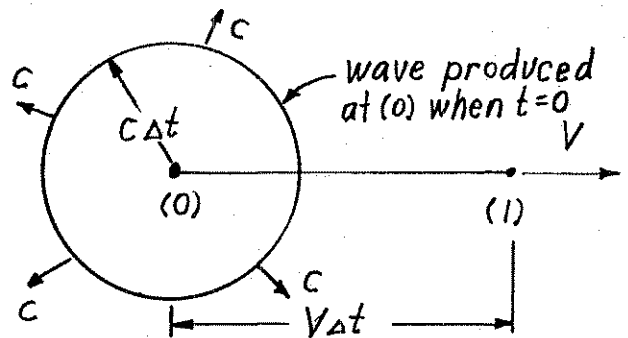
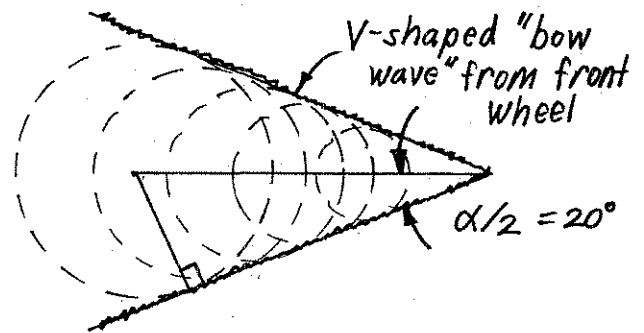


FIGURE P10.12

At time $t=0$ the front wheel was at point (0). At the current time, $t=\Delta t$, the wheel has traveled a distance $d=V\Delta t$ and is at point (1). At time $t=\Delta t$, a wave produced by the wheel when it was at (0) will be a distance $c\Delta t$ from (0) as indicated in the figure.



Waves produced at various times (from $t=0$ to $t=\Delta t$) by the front wheel will form a V-shaped wave as shown in the second figure (provided $V > c$; supercritical bike speed).



From the geometry of the figure

$$\sin \frac{\alpha}{2} = \frac{c\Delta t}{V\Delta t}$$

or

$$V = \frac{c}{\sin \frac{\alpha}{2}}$$

where $c = \sqrt{gy} = \left[32.2 \frac{\text{ft}}{\text{s}^2} \left(\frac{3}{12} \text{ft} \right) \right]^{\frac{1}{2}} = 2.84 \frac{\text{ft}}{\text{s}}$

Thus,

$$V = \frac{2.84 \frac{\text{ft}}{\text{s}}}{\sin 20^\circ} = \underline{\underline{8.30 \frac{\text{ft}}{\text{s}}}}$$

10.13

10.13 Determine the minimum depth in a 3-m-wide rectangular channel if the flow is to be subcritical with a flowrate of $Q = 60 \text{ m}^3/\text{s}$.

$$V = \frac{Q}{A} = \frac{60 \frac{\text{m}^3}{\text{s}}}{(3\text{m})y} = \frac{20}{y}, \text{ where } V \sim \frac{\text{m}}{\text{s}} \text{ when } y = \text{depth} \sim \text{m}$$

Also,

$$Fr = \frac{V}{\sqrt{gy}} = \frac{\left(\frac{20}{y} \frac{\text{m}}{\text{s}}\right)}{\left[\left(9.81 \frac{\text{m}}{\text{s}^2}\right)y\right]^{1/2}} = \frac{6.39}{y^{3/2}}$$

Note: As y decreases, Fr increases

Thus, to have $Fr < 1$ we must have $\frac{6.39}{y^{3/2}} < 1$, or

$$y > (6.39)^{2/3} = \underline{\underline{3.44\text{m}}}$$

10.14

10.14 (See Fluids in the News article titled "Tsunami, the nonstorm wave," Section 10.2.1.) An earthquake causes a shift in the ocean floor that produces a tsunami with a wavelength of 100 km. How fast will this wave travel across the ocean surface if the ocean depth is 3000 m?

$$c = \left[\frac{g\lambda}{2\pi} \tanh\left(\frac{2\pi y}{\lambda}\right) \right]^{1/2}, \text{ where } \lambda = 100 \text{ km} = 10^5 \text{ m and } y = 3000 \text{ m.}$$

Thus,

$$c = \left[\frac{9.81 \frac{\text{m}}{\text{s}^2} (10^5 \text{ m})}{2\pi} \tanh\left(\frac{2\pi (3000 \text{ m})}{10^5 \text{ m}}\right) \right]^{1/2} = 171 \frac{\text{m}}{\text{s}}$$

or

$$c = 171 \frac{\text{m}}{\text{s}} \left(\frac{3600 \text{ s}}{\text{hr}} \right) \left(\frac{1 \text{ km}}{10^3 \text{ m}} \right) = \underline{\underline{616 \frac{\text{km}}{\text{hr}}}}$$

10.15

10.15 Water flows in a 10-m-wide open channel with a flowrate of $5 \text{ m}^3/\text{s}$. Determine the two possible depths if the specific energy of the flow is $E = 0.6 \text{ m}$.

$$(1) \quad E = y + \frac{q^2}{2gy^2}, \quad \text{where } E = 0.6 \text{ m and}$$

$$q = \frac{Q}{b} = \frac{5 \frac{\text{m}^3}{\text{s}}}{10 \text{ m}} = 0.5 \frac{\text{m}^2}{\text{s}}$$

Thus, Eq. (1) becomes

$$0.6 \text{ m} = y + \frac{(0.5 \frac{\text{m}^2}{\text{s}})^2}{(2)(9.80 \frac{\text{m}}{\text{s}^2}) y^2 \text{ m}^2}$$

or

$$0.6 = y + \frac{0.0128}{y^2}, \quad \text{where } y \sim \text{m}$$

Solution to this equation give

$$\underline{\underline{y = 0.560 \text{ m} \quad \text{and} \quad y = 0.173 \text{ m}}}$$

10.16

10.16 Water flows in a rectangular channel with a flowrate per unit width of $q = 2.5 \text{ m}^2/\text{s}$. Plot the specific energy diagram for this flow. Determine the two possible depths of flow if $E = 2.5 \text{ m}$.

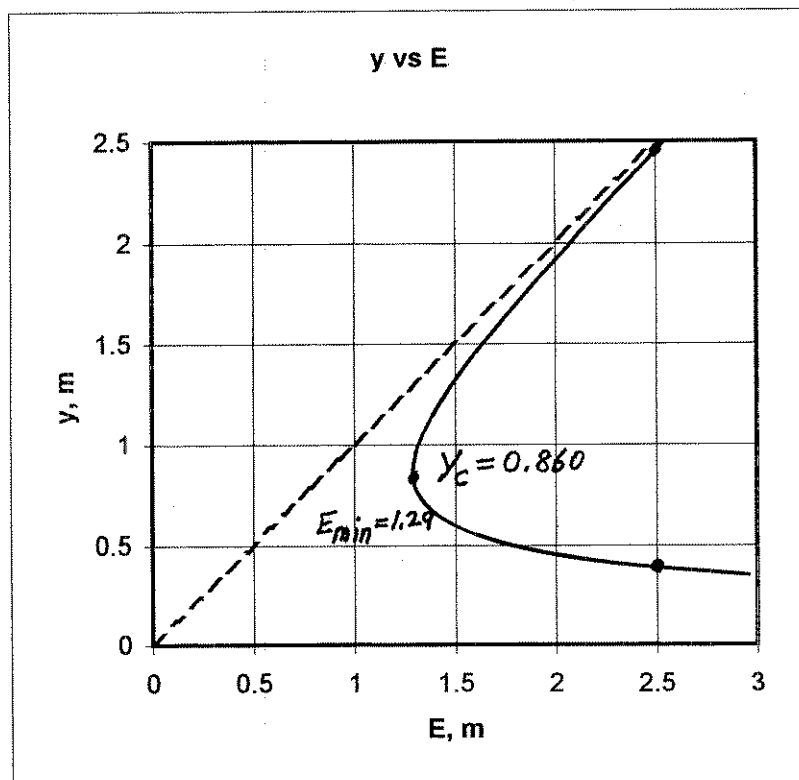
$$E = y + \frac{q^2}{2gy^2} = y + \frac{(2.5 \frac{\text{m}^2}{\text{s}})^2}{2(9.81 \frac{\text{m}}{\text{s}^2})y^2} = y + \frac{0.319}{y^2}$$

Thus, plot

$$E = y + \frac{0.319}{y^2}, \text{ where } E \sim \text{m}, y \sim \text{m}.$$

$$\text{Note: } y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(2.5 \frac{\text{m}^2}{\text{s}})^2}{9.81 \frac{\text{m}}{\text{s}^2}}\right)^{\frac{1}{3}} = 0.860 \text{ m}$$

$$\text{and } E_{\min} = \frac{3}{2} y_c = \frac{3}{2} (0.860 \text{ m}) = 1.29 \text{ m}$$



$$\text{For } E = 2.5 \text{ m, Eq. (1) is } 2.5 = y + \frac{0.319}{y^2}$$

$$\text{or } y^3 - 2.5y^2 + 0.319 = 0$$

The roots to this equation are $y = 2.45, 0.338, \text{ and } -0.335$

Thus, $y = 2.45 \text{ m}$ or $y = 0.338 \text{ m}$

10.17

10.17 Water flows radially outward on a horizontal round disk as is shown in Video V10.12 and Fig. P10.17. (a) Show that the specific energy can be written in terms of the flowrate, Q , the radial distance from the axis of symmetry, r , and the fluid depth, y , as

$$E = y + \left(\frac{Q}{2\pi r}\right)^2 \frac{1}{2gy^2}$$

(b) For a constant flowrate, sketch the specific energy diagram. Recall Fig. 10.7, but note that for the present case r is a variable. Explain the important characteristics of your sketch. (c) Based on the results of Part (b), show that the water depth increases in the flow direction if the flow is subcritical, but that it decreases in the flow direction if the flow is supercritical.

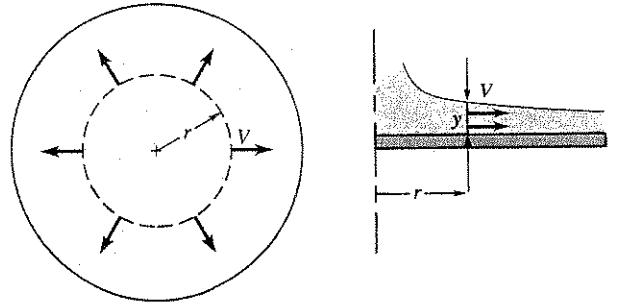


FIGURE P10.17

(a) The specific energy is $E = y + \frac{V^2}{2g}$, where $V = \frac{Q}{A} = \frac{Q}{2\pi r y}$

Thus,

$$E = y + \left(\frac{Q}{2\pi r}\right)^2 \frac{1}{2gy^2}$$

(b) Let $\tilde{q} \equiv \frac{Q}{2\pi r}$ so that $E = y + \frac{\tilde{q}^2}{2gy^2}$ which is the same as for two dimensional flow with $q = \frac{Q}{b}$ being replaced by \tilde{q} . However, for two dimensional flow q is constant; for radial flow \tilde{q} is a variable since r varies. But E vs y curves for constant \tilde{q} would look as shown below (Fig. 10.7).

(c) From the Bernoulli equation

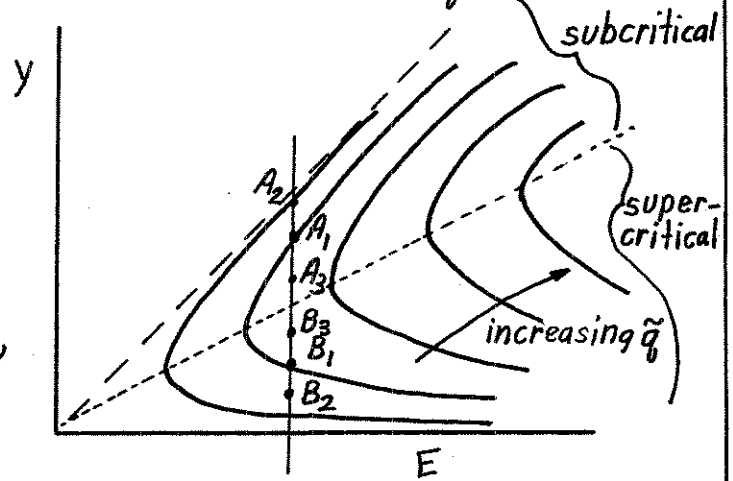
$$E_1 = E_2 \text{ or } E = \text{constant for this flow.}$$

Consider subcritical flow — point A. For outflow r increases so that \tilde{q} decreases. Thus since $E = \text{const.}$, the flow goes from state A_1 to A_2 ; the depth increases. For sub-

critical inflow r decreases, \tilde{q} increases, the flow goes from A_1 to A_3 , and the depth decreases.

For supercritical flow \dagger is true. Thus, outflow increases r , decreases \tilde{q} ; or from B_1 to B_2 — decreasing depth.

Supercritical inflow from B_1 to B_3 — increasing depth.



	subcritical	supercritical
inflow	depth decreases	depth increases
outflow	depth increases	depth decreases

10.18

10.18 Water flows in a 10-ft-wide rectangular channel with a flowrate of 200 ft³/s. Plot the specific energy diagram for this flow. Determine the two possible flowrates when the specific energy is 6 ft.

$$E = y + \frac{q^2}{2gy^2}, \text{ where } q = \frac{Q}{b} = \frac{200 \frac{\text{ft}^3}{\text{s}}}{10 \text{ft}} = 20 \frac{\text{ft}^2}{\text{s}}$$

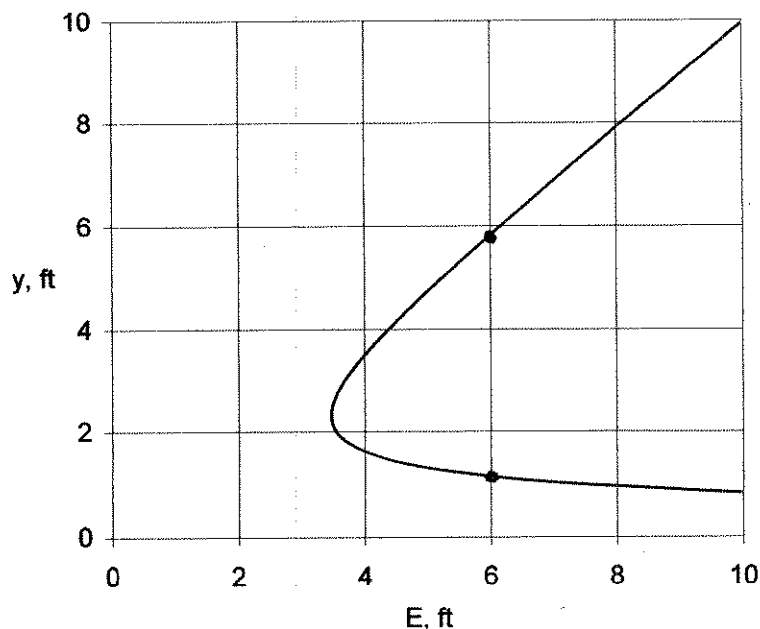
Thus,

$$E = y + \frac{(20 \text{ft}^2/\text{s})^2}{2(32.2 \text{ft}/\text{s}^2) y^2}$$

or

$$(1) \quad E = y + \frac{6.21}{y^2}, \text{ where } E \text{ and } y \sim \text{ft}$$

Eq. (1) is plotted below.



From Eq. (1), when $E = 6 \text{ ft}$,

$$6 = y + \frac{6.21}{y^2}$$

or

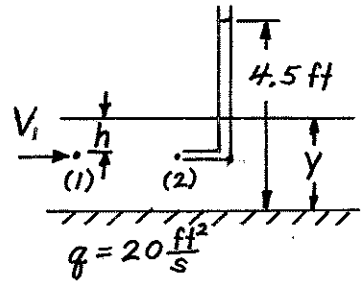
$$y^3 - 6y^2 + 6.21 = 0 \text{ which has solutions}$$

$$\underline{\underline{y = 5.82 \text{ ft} \text{ or } y = 1.129 \text{ ft}}}$$

These values are shown in the above figure.

10.19

10.19 Water flows in a rectangular channel at a rate of $q = 20$ cfs/ft. When a Pitot tube is placed in the stream, water in the tube rises to a level of 4.5 ft above the channel bottom. Determine the two possible flow depths in the channel. Illustrate this flow on a specific energy diagram.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2, \text{ where } z_1 = z_2,$$

$$V_2 = 0, \frac{p_1}{\gamma} = h, \text{ and } \frac{p_2}{\gamma} = 4.5 - (y - h)$$

Thus,

$$h + \frac{V_1^2}{2g} = 4.5 \text{ ft} - y + h, \text{ or } \frac{V_1^2}{2g} = 4.5 - y$$

$$\text{but, } V_1 = \frac{q}{y} = \frac{20 \frac{\text{ft}^2}{\text{s}}}{y}$$

Hence,

$$\frac{\left(\frac{20}{y}\right)^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})} = 4.5 - y \text{ or } y^3 - 4.5y^2 + 6.21 = 0, \text{ where } y \sim \text{ft}$$

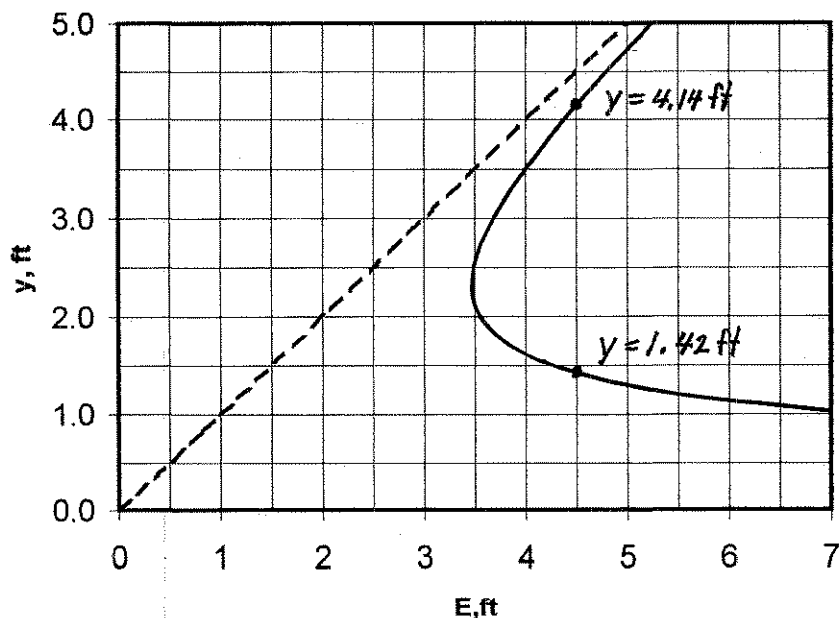
The roots of this equation are $y = 4.14, 1.42, \text{ and } -1.06$

Thus,

$$\underline{\underline{y = 4.14 \text{ ft or } y = 1.42 \text{ ft}}}$$

$$E = y + \frac{q^2}{2gy^2} = y + \frac{(20 \frac{\text{ft}^2}{\text{s}})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})y^2} \text{ or } E = y + \frac{6.21}{y^2} \quad (1)$$

The specific energy diagram (plot of Eq. (1)) is shown below.



10.20

10.20 Water flows in a 5-ft-wide rectangular channel with a flowrate of $Q = 30 \text{ ft}^3/\text{s}$ and an upstream depth of $y_1 = 2.5 \text{ ft}$ as is shown in Fig. P10.20. Determine the flow depth and the surface elevation at section (2).

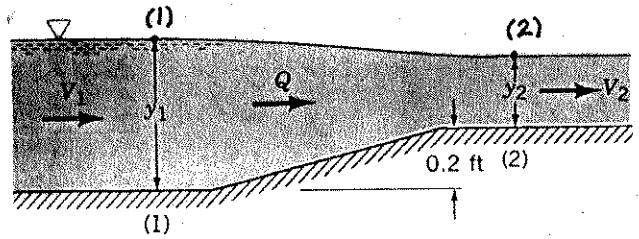


FIGURE P10.20

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0, z_1 = y_1 = 2.5 \text{ ft}, z_2 = 0.2 \text{ ft} + y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{(30 \frac{\text{ft}^3}{\text{s}})}{(2.5 \text{ ft})(5 \text{ ft})} = 2.4 \frac{\text{ft}}{\text{s}}, \text{ and } V_2 = \frac{Q}{A_2} = \frac{30 \frac{\text{ft}^3}{\text{s}}}{(5 \text{ ft})y_2} = \frac{6}{y_2}$$

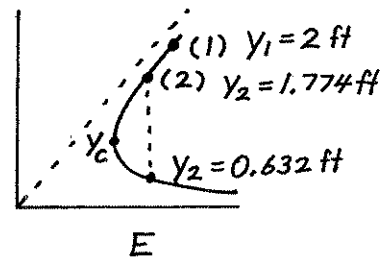
Thus,

$$\frac{(2.4 \frac{\text{ft}}{\text{s}})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})} + 2.5 \text{ ft} = \frac{(\frac{6}{y_2} \frac{\text{ft}}{\text{s}})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})} + 0.2 \text{ ft} + y_2$$

or $y_2^3 - 1.94 y_2^2 + 0.559 = 0$ which has roots $y_2 = 1.774, 0.632, \text{ and } -0.632$

Note: $Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{2.4 \frac{\text{ft}}{\text{s}}}{[(32.2 \frac{\text{ft}}{\text{s}^2})(2.5 \text{ ft})]^{1/2}} = 0.374 < 1$

If $y_2 = 0.632$, then $Fr_2 > 1$. This cannot be since there is no "bump" between (1) and (2) at which critical conditions can occur.



Thus, $y_2 = 1.774 \text{ ft}$ and $z_2 = 1.974 \text{ ft}$

10.21

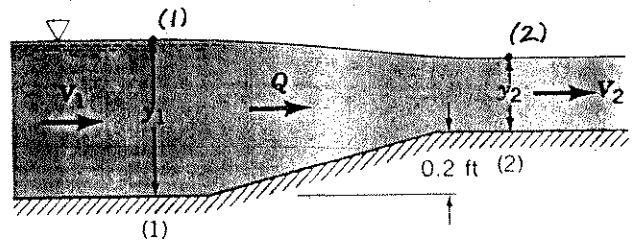
10.21 Repeat Problem 10.20 if the upstream depth is $y_1 = 0.5$ ft.

FIGURE P10.20

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0, z_1 = y_1 = 0.5 \text{ ft}, z_2 = 0.2 \text{ ft} + y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{30 \frac{\text{ft}^3}{\text{s}}}{(0.5 \text{ ft})(5 \text{ ft})} = 12 \frac{\text{ft}}{\text{s}}, \text{ and } V_2 = \frac{Q}{A_2} = \frac{30 \frac{\text{ft}^3}{\text{s}}}{(5 \text{ ft})y_2} = \frac{6}{y_2}$$

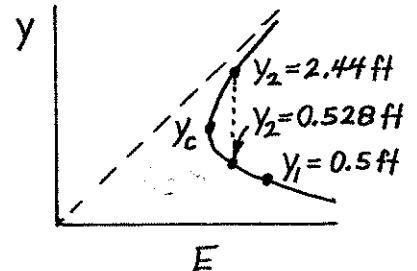
Thus,

$$\frac{(12 \frac{\text{ft}}{\text{s}})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})} + 0.5 \text{ ft} = \frac{(\frac{6}{y_2})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})} + 0.2 \text{ ft} + y_2$$

or $y_2^3 - 2.53 y_2^2 + 0.559 = 0$ which has roots $y_2 = 2.44, 0.528, \text{ and } -0.434$

$$\text{Note: } Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{12 \frac{\text{ft}}{\text{s}}}{[(32.2 \frac{\text{ft}}{\text{s}^2})(0.5 \text{ ft})]^{1/2}} = 2.99 > 1$$

If $y_2 = 2.44$ ft, then $Fr_2 < 1$. This cannot be since there is no "bump" between (1) and (2) at which critical conditions can occur.



Thus, $y_2 = 0.528$ ft and $z_2 = 0.728$ ft

*10.22

10.22* Water flows over the bump in the bottom of the rectangular channel shown in Fig. P10.22 with a flowrate per unit width of $q = 4 \text{ m}^2/\text{s}$. The channel bottom contour is given by $z_B = 0.2e^{-x^2}$, where z_B and x are in meters. The water depth far upstream of the bump is $y_1 = 2 \text{ m}$. Plot a graph of the water depth, $y = y(x)$, and the surface elevation, $z = z(x)$, for $-4 \text{ m} \leq x \leq 4 \text{ m}$. Assume one-dimensional flow.

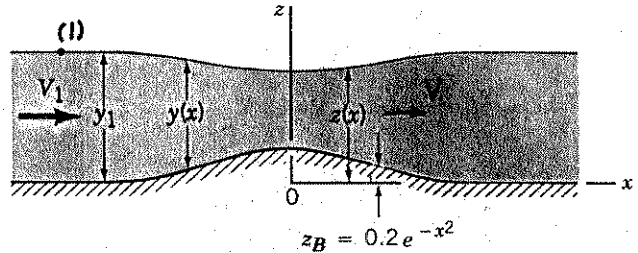


FIGURE P10.22

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{p}{\rho} + \frac{V^2}{2g} + z, \text{ where } p_1 = p = 0, z_1 = y_1 = 2 \text{ m}, z_2 = y + z_B$$

$$\text{or } z = y + 0.2e^{-x^2}, V_1 = \frac{q}{y_1} = \frac{4 \frac{\text{m}^2}{\text{s}}}{2 \text{ m}} = 2 \frac{\text{m}}{\text{s}}, \text{ and } V = \frac{q}{y} = \frac{4}{y}$$

$$\text{Thus, } \frac{(2 \frac{\text{m}}{\text{s}})^2}{2(9.81 \frac{\text{m}}{\text{s}^2})} + 2 \text{ m} = \frac{(\frac{4}{y} \frac{\text{m}}{\text{s}})^2}{2(9.81 \frac{\text{m}}{\text{s}^2})} + y + 0.2e^{-x^2}$$

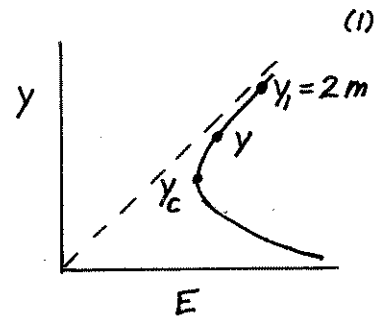
or

$$y^3 - (2.20 - 0.2e^{-x^2})y^2 + 0.815 = 0 \text{ where } y \sim \text{m}$$

Solve for y with $-4 \leq x \leq 4 \text{ m}$

$$\text{Note: } Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{2 \frac{\text{m}}{\text{s}}}{[(9.81 \frac{\text{m}}{\text{s}^2})(2 \text{ m})]^{1/2}} = 0.452 < 1$$

Thus, the flow will remain subcritical throughout — the largest root of Eq. (1) will be the correct one.



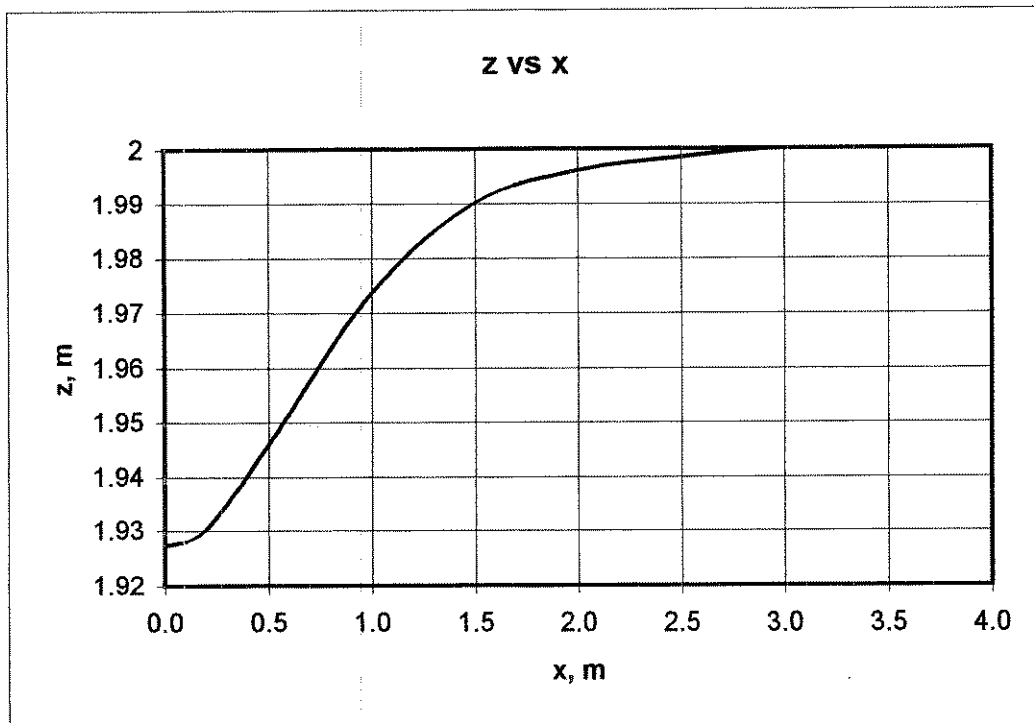
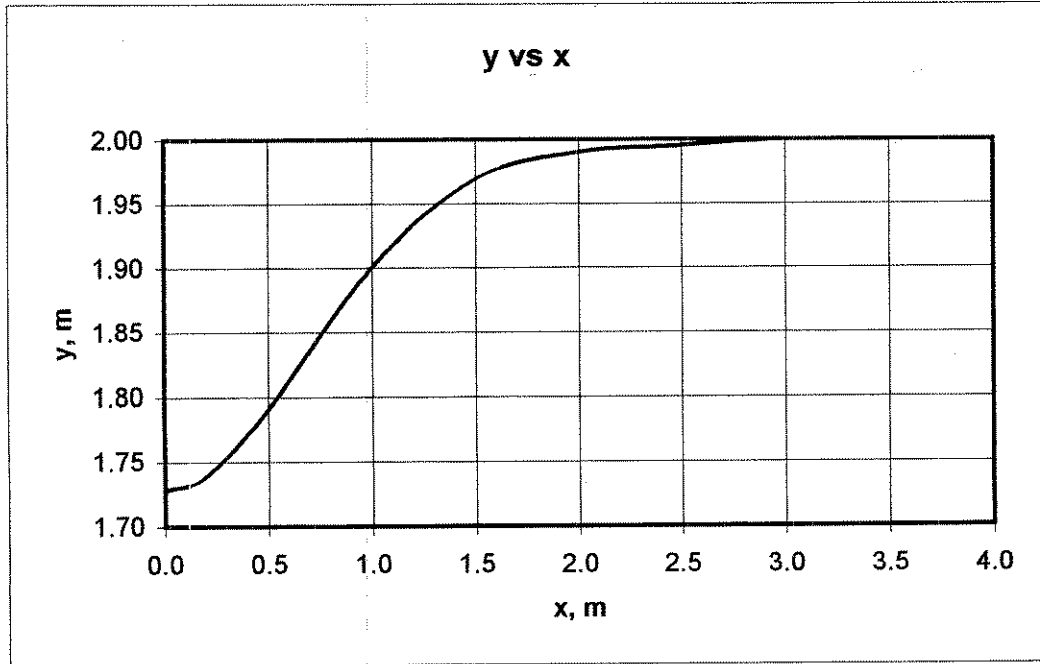
The following results are obtained by solving Eq. (1) for y and then $z = y + 0.2e^{-x^2}$ for $-4 \text{ m} \leq x \leq 4 \text{ m}$.

$\pm x, \text{ m}$	$y, \text{ m}$	$z, \text{ m}$
0.0	1.727	1.927
0.5	1.790	1.946
1.0	1.901	1.974
1.5	1.969	1.990
2.0	1.991	1.994
2.5	1.995	1.995
3.0	1.995	1.995
3.5	1.995	1.995
4.0	1.995	1.995

(con't)

10.22^A (con't)

The above results are plotted in the graph below.



*10.23

*10.23 Repeat Problem 10.22 if the upstream depth is 0.4 m.

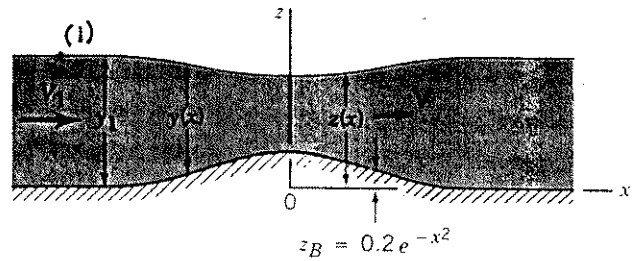


FIGURE P10.22

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{p}{\rho} + \frac{V^2}{2g} + z, \text{ where } p_1 = p = 0, z_1 = y_1 = 0.4 \text{ m}, z_2 = y + z_B$$

$$\text{or } z_2 = y + 0.2e^{-x^2}, V_1 = \frac{Q}{y_1} = \frac{4 \frac{\text{m}^3}{\text{s}}}{0.4 \text{ m}} = 10 \frac{\text{m}}{\text{s}}, \text{ and } V = \frac{Q}{y} = \frac{4}{y}$$

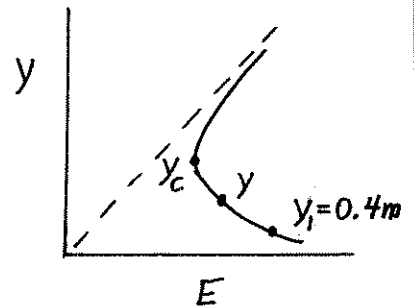
$$\text{Thus, } \frac{(10 \frac{\text{m}}{\text{s}})^2}{2(9.81 \frac{\text{m}}{\text{s}^2})^2} + 0.4 \text{ m} = \frac{(\frac{4}{y} \frac{\text{m}}{\text{s}})^2}{2(9.81 \frac{\text{m}}{\text{s}^2})^2} + y + 0.2e^{-x^2}$$

$$\text{or } y^3 - (5.50 - 0.2e^{-x^2})y^2 + 0.815 = 0 \text{ where } y \sim \text{m} \quad (1)$$

Solve for y with $-4 \leq x \leq 4 \text{ m}$

$$\text{Note: } Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{10 \frac{\text{m}}{\text{s}}}{[(9.81 \frac{\text{m}}{\text{s}^2})(0.4 \text{ m})]^{1/2}} = 5.05 > 1$$

Thus, the flow will remain supercritical throughout — the smallest positive root of Eq.(1) will be the correct one.



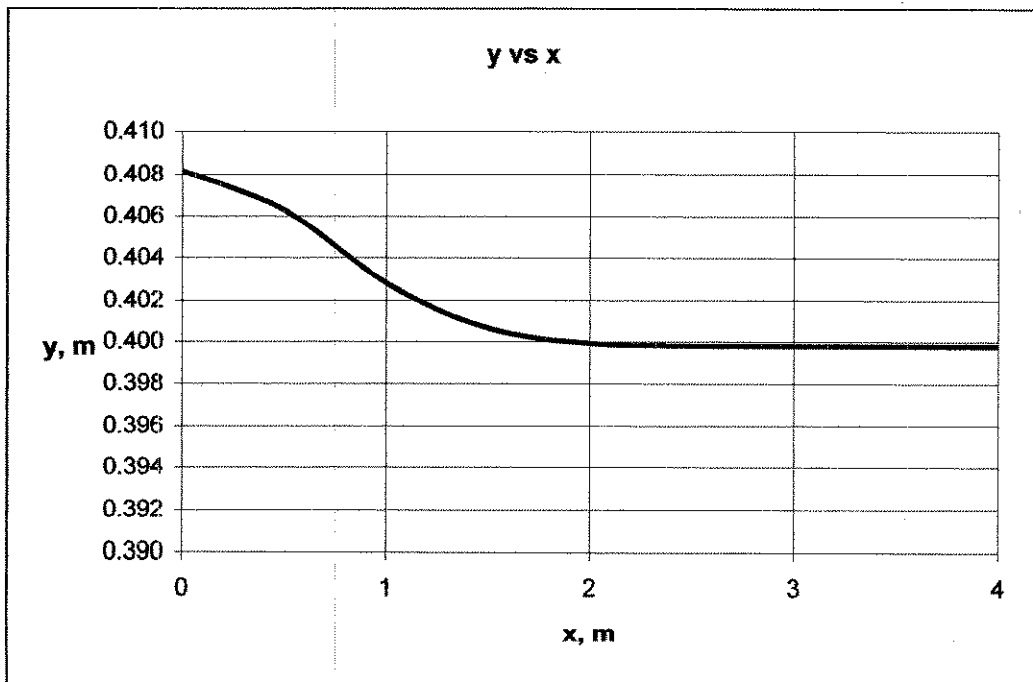
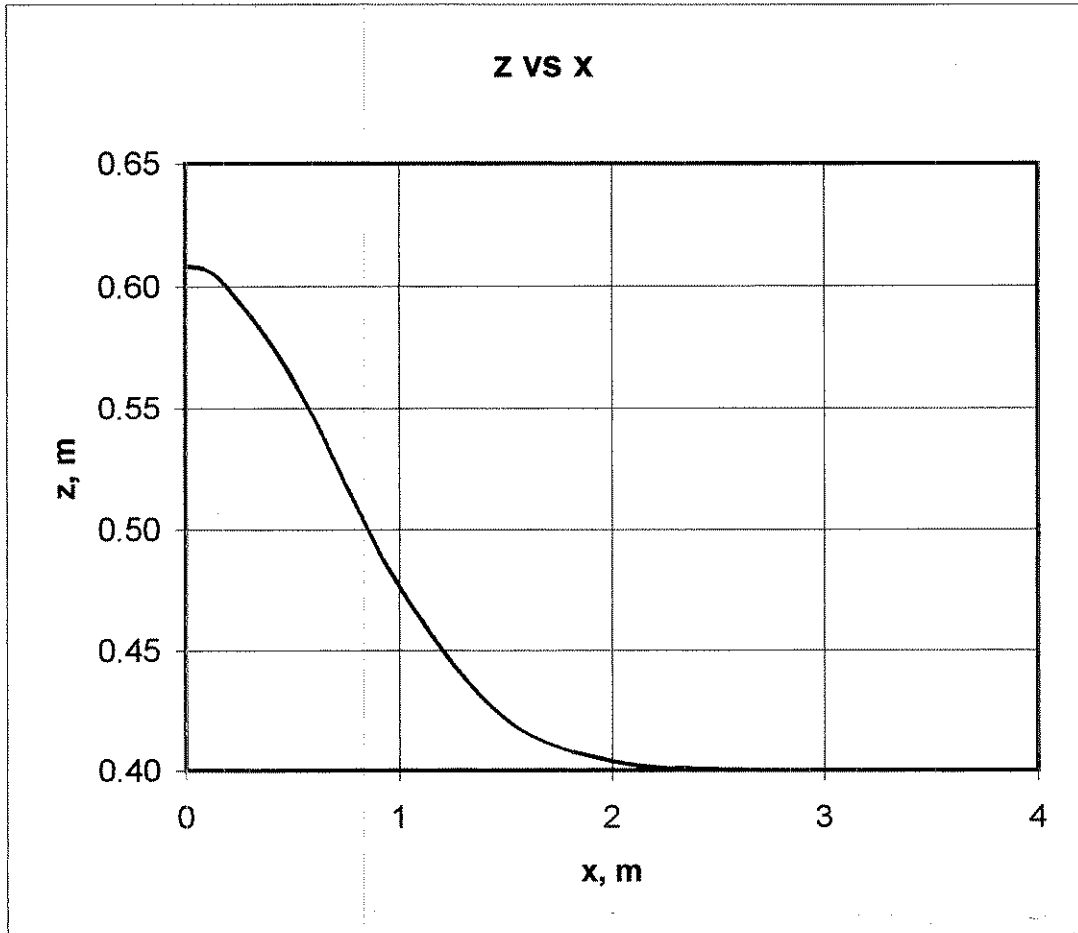
The following results are obtained by solving Eq.(1) for y and then $z = y + 0.2e^{-x^2}$ for $-4 \text{ m} \leq x \leq 4 \text{ m}$.

$\pm x, \text{ m}$	$y, \text{ m}$	$z, \text{ m}$
0.0	0.408	0.608
0.5	0.406	0.562
1.0	0.403	0.476
1.5	0.401	0.422
2.0	0.400	0.404
2.5	0.400	0.400
3.0	0.400	0.400
3.5	0.400	0.400
4.0	0.400	0.400

(cont)

10.23* (con't)

The above results are plotted on the graph below.



10.24

10.24 Water in a rectangular channel flows into a gradual contraction section as is indicated in Fig. P10.24. If the flowrate is $Q = 25 \text{ ft}^3/\text{s}$ and the upstream depth is $y_1 = 2 \text{ ft}$, determine the downstream depth, y_2 .

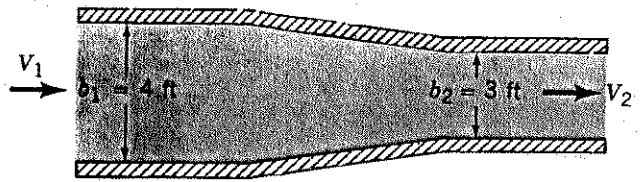
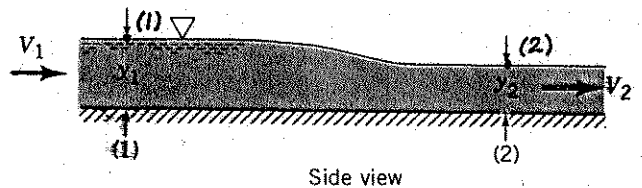


FIGURE P10.24 Top view



Side view

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0, z_1 = y_1 = 2 \text{ ft}, z_2 = y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{25 \frac{\text{ft}^3}{\text{s}}}{(4 \text{ ft})(2 \text{ ft})} = 3.13 \frac{\text{ft}}{\text{s}}, \text{ and } V_2 = \frac{Q}{A_2} = \frac{25 \frac{\text{ft}^3}{\text{s}}}{(3 \text{ ft}) y_2} = \frac{8.33}{y_2}$$

$$\text{Thus, } \frac{(3.13 \frac{\text{ft}}{\text{s}})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})} + 2 \text{ ft} = \frac{(\frac{8.33}{y_2})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})} + y_2$$

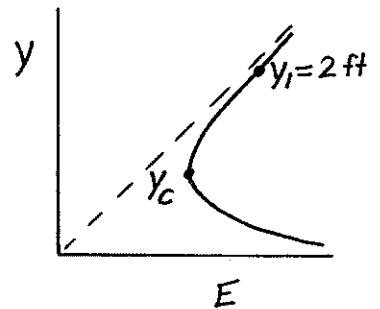
or

$$y_2^3 - 2.15 y_2^2 + 1.077 = 0 \text{ which has roots } y_2 = 1.828, 0.946, \text{ and } -0.623 \quad (1)$$

$$\text{Note: } Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{3.13 \frac{\text{ft}}{\text{s}}}{[(32.2 \frac{\text{ft}}{\text{s}^2})(2 \text{ ft})]^{\frac{1}{2}}} = 0.390 < 1$$

Since there is no relative minimum area between (1) and (2) where critical flow can occur it follows that $Fr_2 < 1$ also. Thus, it is not possible to have $y_2 = 0.946$

$$\text{Thus, } \underline{\underline{y_2 = 1.828 \text{ ft}}}$$



10.25

10.25 Sketch the specific energy diagram for the flow of Problem 10.24 and indicate its important characteristics. Note that $q_1 \neq q_2$.

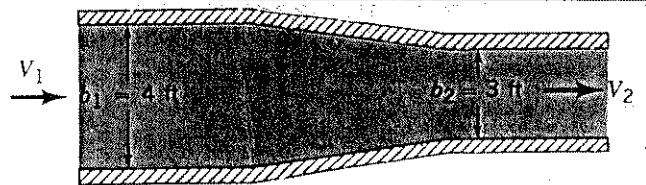
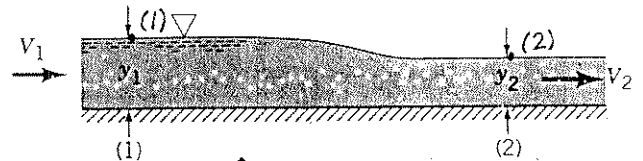


FIGURE P10.24



$$E = y + \frac{q^2}{2gy^2}$$

Thus, for the $b_1 = 4$ ft channel, $q_1 = \frac{Q}{b_1} = \frac{25 \frac{\text{ft}^3}{\text{s}}}{4 \text{ ft}} = 6.25 \frac{\text{ft}^2}{\text{s}}$

or

$$E = y + \frac{(6.25 \frac{\text{ft}^2}{\text{s}})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})y^2} \quad \text{or} \quad E = y + \frac{0.607}{y^2} \tag{1}$$

For the $b_2 = 3$ ft channel, $q_2 = \frac{Q}{b_2} = \frac{25 \frac{\text{ft}^3}{\text{s}}}{3 \text{ ft}} = 8.33 \frac{\text{ft}^2}{\text{s}}$

or

$$E = y + \frac{(8.33 \frac{\text{ft}^2}{\text{s}})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})y^2} \quad \text{or} \quad E = y + \frac{1.077}{y^2} \tag{2}$$

Note: $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ so that $y_{c1} = \left(\frac{q_1^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(6.25 \frac{\text{ft}^2}{\text{s}})^2}{32.2 \frac{\text{ft}}{\text{s}^2}}\right)^{\frac{1}{3}} = 1.067 \text{ ft}$

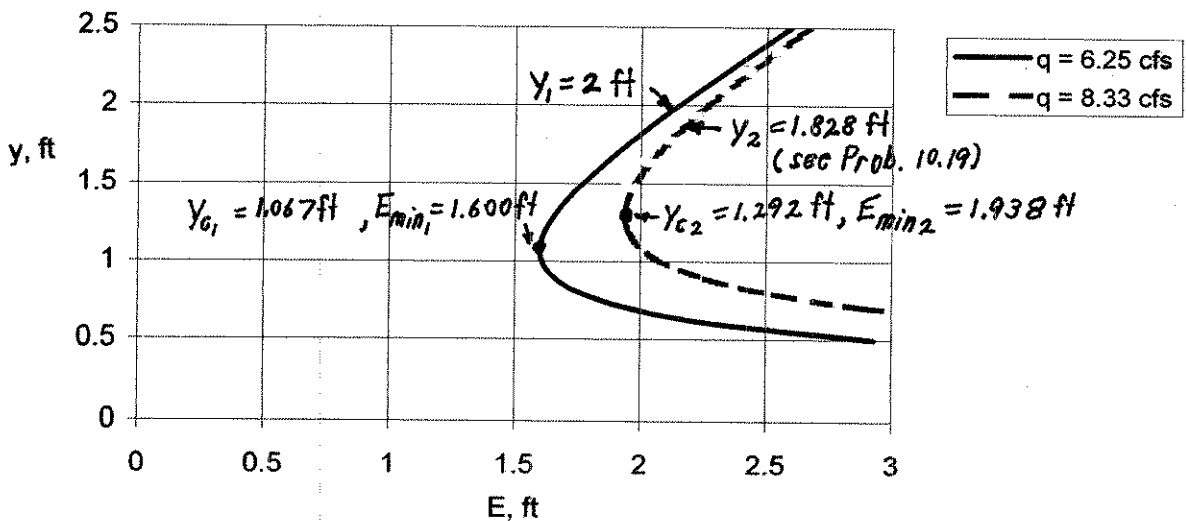
and

$$y_{c2} = \left(\frac{q_2^2}{g}\right)^{\frac{1}{3}} = \left(\frac{(8.33 \frac{\text{ft}^2}{\text{s}})^2}{32.2 \frac{\text{ft}}{\text{s}^2}}\right)^{\frac{1}{3}} = 1.292 \text{ ft}$$

Also, $E_{min} = \frac{3}{2} y_c$, or $E_{min1} = \frac{3}{2} (1.067 \text{ ft}) = 1.600 \text{ ft}$

$$E_{min2} = \frac{3}{2} (1.292 \text{ ft}) = 1.938 \text{ ft}$$

The specific energy diagrams (Eqs. (1) and (2)) are plotted below:



Note: $E_1 = y_1 + \frac{V_1^2}{2g} = E_2 = y_2 + \frac{V_2^2}{2g} = 2 \text{ ft} + \frac{(3.13 \frac{\text{ft}}{\text{s}})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})} = 2.15 \text{ ft}$

10.26

10.26 Repeat Problem 10.24 if the upstream depth is $y_1 = 0.5$ ft. Assume that there are no losses between sections (1) and (2).

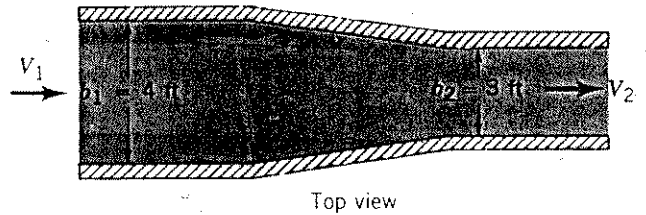
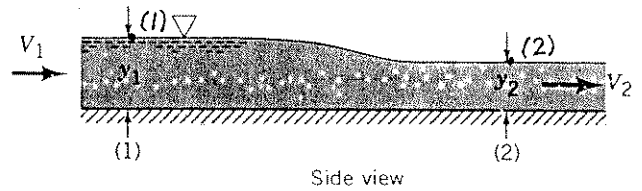


FIGURE P10.24



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0, z_1 = y_1 = 0.5 \text{ ft}, z_2 = y_2,$$

$$V_1 = \frac{Q}{A_1} = \frac{25 \frac{\text{ft}^3}{\text{s}}}{(4 \text{ ft})(0.5 \text{ ft})} = 12.5 \frac{\text{ft}}{\text{s}}, \text{ and } V_2 = \frac{Q}{A_2} = \frac{25 \frac{\text{ft}^3}{\text{s}}}{(3 \text{ ft})y_2} = \frac{8.33}{y_2}$$

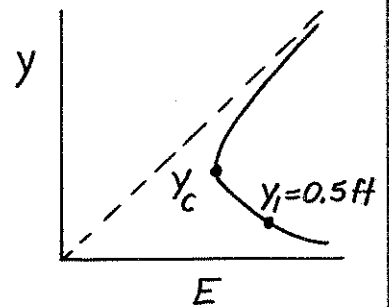
$$\text{Thus, } \frac{(12.5 \frac{\text{ft}}{\text{s}})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})} + 0.5 \text{ ft} = \frac{(\frac{8.33}{y_2})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})} + y_2$$

$$\text{or } y_2^3 - 2.93 y_2^2 + 1.077 = 0 \text{ which has roots } y_2 = 2.79, 0.694, \text{ and } -0.555$$

$$\text{Note: } Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{12.5 \frac{\text{ft}}{\text{s}}}{[(32.2 \frac{\text{ft}}{\text{s}^2})(0.5 \text{ ft})]^{1/2}} = 3.12 > 1$$

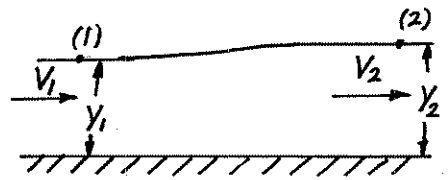
Since there is no relative minimum area between (1) and (2) where critical flow can occur it follows that $Fr_2 > 1$ also. Thus, it is not possible to have $y_2 = 2.79$ (the subcritical root).

$$\text{Thus, } y_2 = \underline{\underline{0.694 \text{ ft}}}$$



10.27

10.27 Water flows in a rectangular channel with a flowrate per unit width of $q = 1.5 \text{ m}^2/\text{s}$ and a depth of 0.5 m at section (1). The head loss between sections (1) and (2) is 0.03 m . Plot the specific energy diagram for this flow and locate states (1) and (2) on this diagram. Is it possible to have a head loss of 0.06 m ? Explain.



$$\frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2g} + z_2 + h_L, \text{ where } p_1 = p_2 = 0, z_1 = 0.5 \text{ m}, z_2 = y_2, \quad (1)$$

$$V_1 = \frac{q}{y_1} = \frac{1.5 \frac{\text{m}^2}{\text{s}}}{0.5 \text{ m}} = 3 \frac{\text{m}}{\text{s}}, \text{ and } V_2 = \frac{q}{y_2} = \frac{1.5 \frac{\text{m}^2}{\text{s}}}{y_2}$$

Thus, with $E = y + \frac{V^2}{2g}$ and $h_L = 0.03 \text{ m}$ Eq.(1) is

$$E_1 = E_2 + 0.03$$

$$\text{Also, } E = y + \frac{q^2}{2gy^2} = y + \frac{(1.5 \frac{\text{m}^2}{\text{s}})^2}{2(9.81 \frac{\text{m}}{\text{s}^2})y^2}$$

$$\text{or } E = y + \frac{0.1146}{y^2} \quad (2)$$

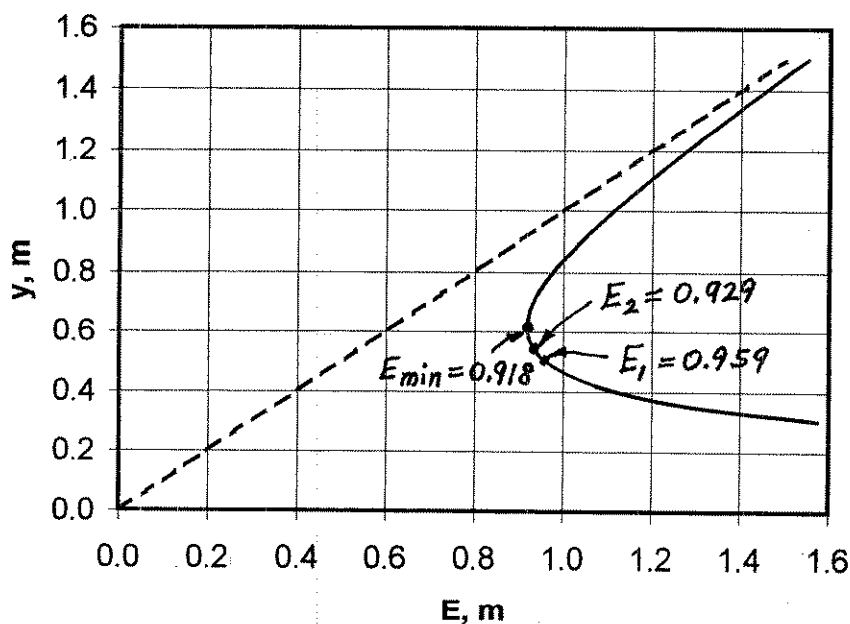
Eq.(2) is plotted below.

$$\text{Note: } y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(1.5 \frac{\text{m}^2}{\text{s}})^2}{9.81 \frac{\text{m}}{\text{s}^2}}\right)^{1/3} = 0.612 \text{ m}$$

$$\text{and } E_{\min} = \frac{3}{2} y_c = \frac{3}{2} (0.612 \text{ m}) = 0.918 \text{ m}$$

$$\text{Also, } E_1 = y_1 + \frac{q^2}{2gy_1^2} = 0.5 + \frac{(1.5 \frac{\text{m}^2}{\text{s}})^2}{2(9.81 \frac{\text{m}}{\text{s}^2})(0.5 \text{ m})^2} = 0.959 \text{ m}$$

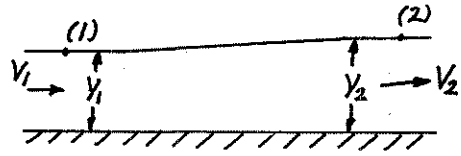
$$\text{and } E_2 = E_1 - 0.03 = 0.929 \text{ m}$$



Note: If $h_L = 0.06 \text{ m}$ with $E_1 = 0.959 \text{ m}$ so that $E_2 = E_1 - 0.06$, then $E_2 = 0.899 \text{ m} < E_{\min}$. Thus, it is not possible to have $h_L = 0.06$ with the given q and y_1 .

10.28

10.28 Water flows in a horizontal rectangular channel with a flowrate per unit width of $q = 10 \text{ ft}^2/\text{s}$ and a depth of 1.0 ft at the downstream section (2). The head loss between section (1) upstream and section (2) is 0.2 ft. Plot the specific energy diagram for this flow and locate states (1) and (2) on this diagram.



$$p_1/\gamma + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L, \text{ where } p_1 = p_2 = 0, V_2 = \frac{q}{y_2} = \frac{10 \frac{\text{ft}^2}{\text{s}}}{1 \text{ ft}} = 10 \frac{\text{ft}}{\text{s}}, \quad (1)$$

$$\text{and } y_2 = 1 \text{ ft}$$

Thus, with $E = y + \frac{V^2}{2g}$

$$E = y + \frac{q^2}{2gy^2} = y + \frac{(10 \frac{\text{ft}^2}{\text{s}})^2}{2(32.2 \frac{\text{ft}}{\text{s}^2})y^2}$$

or

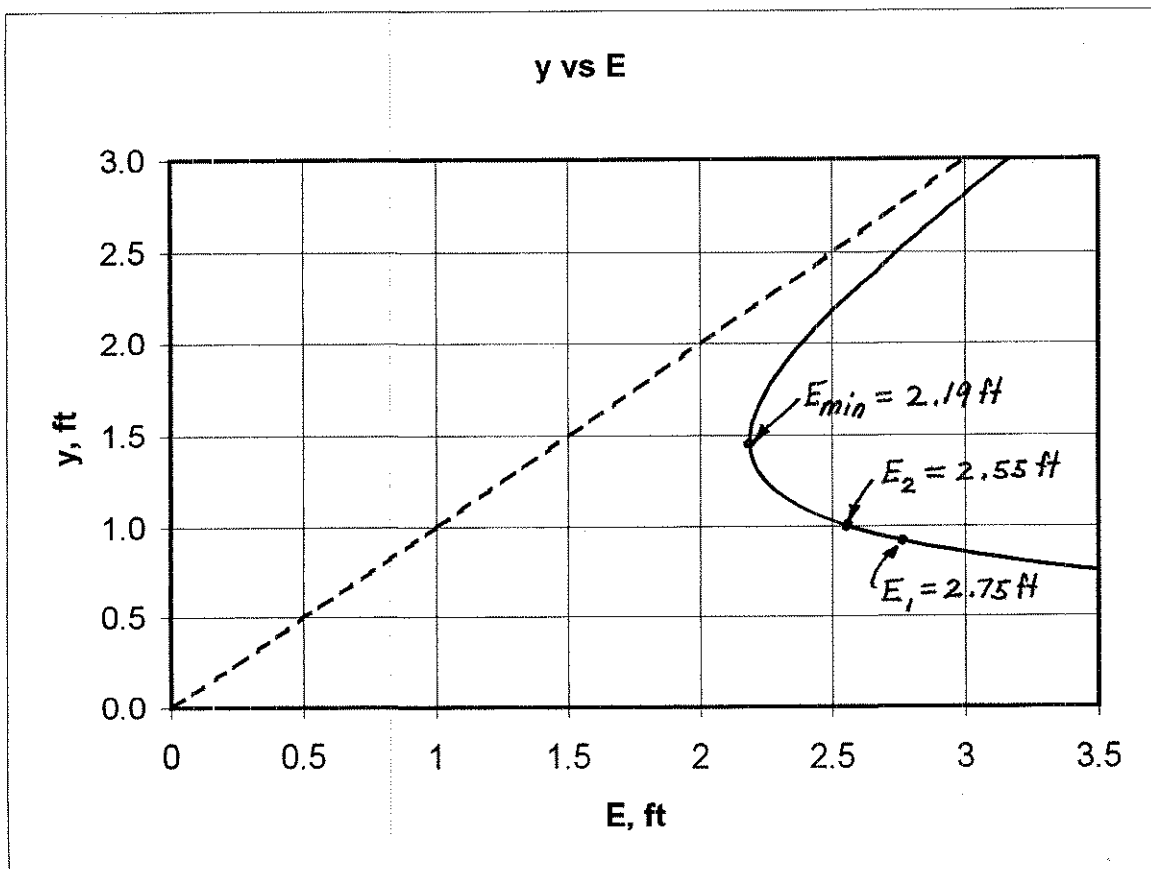
$$E = y + \frac{1.553}{y^2} \text{ where } E \sim \text{ft}, y \sim \text{ft}. \quad (2)$$

and Eq.(1) gives $E_1 = E_2 + h_L = E_2 + 0.2 \text{ ft}$

Eq.(2) is plotted below.

$$\text{Note: } y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(10 \frac{\text{ft}^2}{\text{s}})^2}{32.2 \frac{\text{ft}}{\text{s}^2}}\right)^{1/3} = 1.459 \text{ ft}, E_{\min} = \frac{3}{2} y_c = \frac{3}{2} (1.459 \text{ ft}) = 2.19 \text{ ft},$$

$$E_2 = y_2 + \frac{1.553}{y_2^2} = 1 + \frac{1.553}{1^2} = 2.55 \text{ ft}, \text{ and } E_1 = E_2 + h_L = 2.75 \text{ ft}$$



10.29

10.29 Water flows in a horizontal, rectangular channel with an initial depth of 1 m and an initial velocity of 4 m/s. Determine the depth downstream if losses are negligible. Note that there may be more than one solution.

$$E = y + \frac{q^2}{2gy^2}, \text{ where from the initial conditions,}$$

$$y_1 = 1 \text{ m and } q_1 = \frac{Q}{b} = \frac{V_1 y_1 b}{b} = (4 \text{ m/s})(1 \text{ m}) = 4 \frac{\text{m}^2}{\text{s}}$$

Thus,

$$E_1 = y_1 + \frac{q_1^2}{2gy_1} = 1 \text{ m} + \frac{(4 \frac{\text{m}^2}{\text{s}})^2}{2(9.81 \frac{\text{m}}{\text{s}^2})(1 \text{ m})^2} = 1.815 \text{ m}$$

(Note: Since $q = Vy$, $E = y + \frac{(Vy)^2}{2gy^2} = y + \frac{V^2}{2g}$)

With no losses, $E_2 = E_1$, so that $q_1 = q_2$ (i.e., $Q_1 = q_1 b_1 = Q_2 = q_2 b_2$,

Hence,

$$E_2 = 1.815 \text{ m} = y_2 + \frac{(4 \frac{\text{m}^2}{\text{s}})^2}{2(9.81 \frac{\text{m}}{\text{s}^2})y_2^2}$$

with $b_1 = b_2$)

or

$$1.815 = y_2 + \frac{0.815}{y_2^2}$$

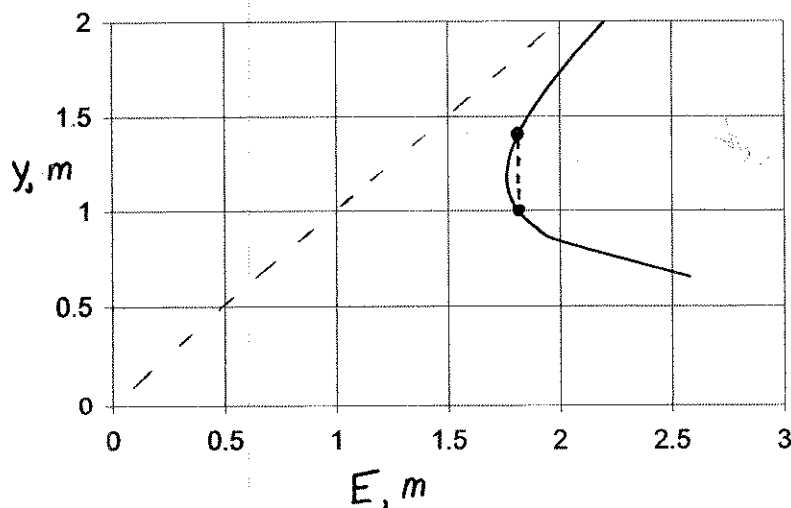
which has solutions

$$y_2 = \underline{\underline{1 \text{ m}}} \text{ (same as the initial depth)}$$

or

$$y_2 = \underline{\underline{1.40 \text{ m}}}$$

The energy diagram and these two depths are shown below.



10.30

10.30 A smooth transition section connects two rectangular channels as shown in Fig. P10.30. The channel width increases from 6.0 to 7.0 ft and the water surface elevation is the same in each channel. If the upstream depth of flow is 3.0 ft, determine h , the amount the channel bed needs to be raised across the transition section to maintain the same surface elevation.

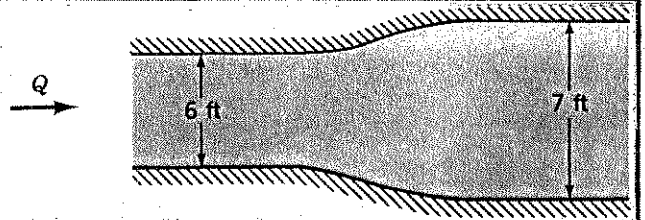
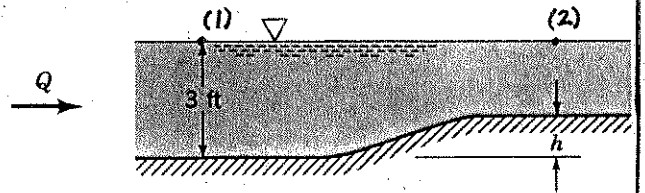


FIGURE P10.30 Top view



Side view

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2g} + z_2, \text{ where } p_1 = p_2 = 0 \text{ and } z_1 = z_2$$

Thus, $V_1 = V_2$ or

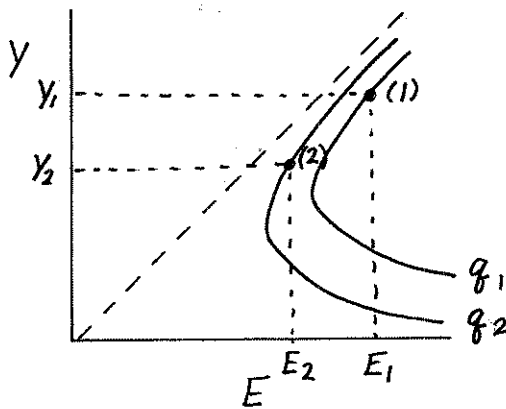
$$\frac{Q}{A_1} = \frac{Q}{A_2} \text{ Hence, } A_1 = A_2 \text{ or } (6\text{ft})(3\text{ft}) = (7\text{ft})(3\text{ft} - h)$$

$$\text{or } h = \underline{0.429\text{ ft}}$$

$$\text{Note: } q_1 = \frac{Q}{b_1} = \frac{Q}{6} \text{ and } q_2 = \frac{Q}{b_2} = \frac{Q}{7} < q_1$$

and $E_1 = y_1 + \frac{V_1^2}{2g}$ and $E_2 = y_2 + \frac{V_2^2}{2g}$ Thus, since $V_1 = V_2$ it follows that $E_1 - E_2 = y_1 - y_2$

The corresponding specific energy diagram is as indicated below:



10.32 Integrate the differential equation obtained in Problem 10.31 to determine the "draw-down" distance, $\ell = \ell(x)$, indicated in Fig. P10.31. Comment on your results.

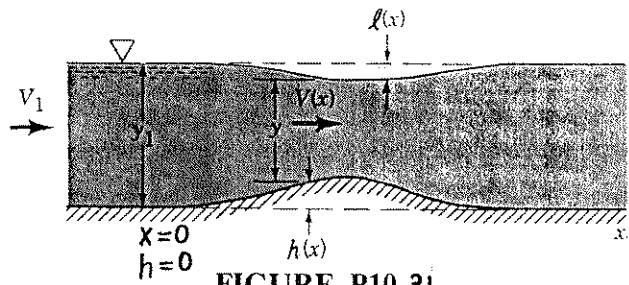


FIGURE P10.31

From Problem 10.31:

$$\frac{dy}{dx} = \frac{-(\frac{dh}{dx})}{(1 - (\frac{V^2}{gy})^2)}, \text{ where } V_1 y_1 = Vy, \text{ or } V = \frac{V_1 y_1}{y}$$

Thus, $\frac{V^2}{gy} = \frac{(\frac{V_1 y_1}{y})^2}{gy} = \frac{V_1^2 y_1^2}{g y^3}$ so that

$$\frac{dy}{dx} = \frac{-(\frac{dh}{dx})}{[1 - (\frac{V_1^2 y_1^2}{g y^3})]}, \text{ or } [1 - (\frac{V_1^2 y_1^2}{g y^3})] dy = -(\frac{dh}{dx}) dx$$

Integrate from $y = y_1$ and $x = 0$, with $\frac{dh}{dx}$ a given function of x .

$$\int_{y=y_1}^y [1 - (\frac{V_1^2 y_1^2}{g y^3})] dy = - \int_{x=0}^x (\frac{dh}{dx}) dx = - \int_{h=0}^h dh = -h$$

or

$$\left[y - (\frac{V_1^2 y_1^2}{g}) (-\frac{1}{2}) y^{-2} \right]_{y_1}^y = -h \quad \text{Thus, } y + (\frac{V_1^2 y_1^2}{2g}) y^{-2} - y_1 - \frac{V_1^2}{2g} = -h$$

or

$$y^3 - (y_1 + \frac{V_1^2}{2g} - h) y^2 + (\frac{V_1^2 y_1^2}{2g}) = 0 \quad (1)$$

Obtain $y = y(x)$ from Eq. (1) and then $\ell = \ell(x)$ from $y_1 = h + y + \ell$
or $\ell = y_1 - h - y$

Note: Eq. (1) is nothing more than the Bernoulli equation:

$$\frac{V_1^2}{2g} + y_1 = \frac{V^2}{2g} + y + h \quad \text{with } V = \frac{V_1 y_1}{y} \text{ so that}$$

$$\frac{V_1^2}{2g} + y_1 = \frac{(\frac{V_1 y_1}{y})^2}{2g} + y + h \quad \text{which simplifies to Eq. (1).}$$

10.33

10.33 Water flows in the river shown in Fig. P10.33 with a uniform bottom slope. The total head at each section is measured by using Pitot tubes as indicated. Determine the value of dy/dx at a location where the Froude number is 0.357.

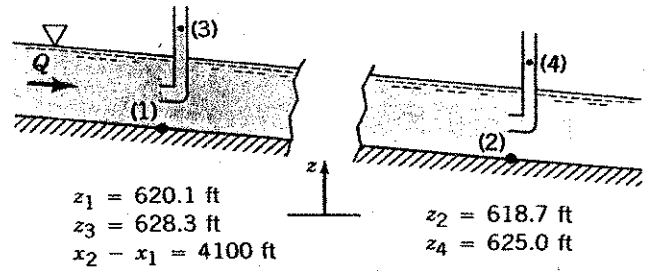


FIGURE P10.33

$$\frac{dy}{dx} = \frac{S_f - S_0}{1 - Fr^2}, \text{ where from the figure } S_f = \frac{h_L}{l} = \frac{z_3 - z_4}{x_1 - x_2} = \frac{(628.3 - 625.0)\text{ft}}{4100\text{ft}}$$

$$\text{or } S_f = 8.05 \times 10^{-4} \text{ and } S_0 = \frac{z_1 - z_2}{l} = \frac{(620.1 - 618.7)\text{ft}}{4100\text{ft}} = 3.41 \times 10^{-4}$$

Thus,

$$\frac{dy}{dx} = \frac{8.05 \times 10^{-4} - 3.41 \times 10^{-4}}{1 - (0.357)^2} = \underline{\underline{0.000532}} \quad (\text{i.e., } 2.81 \frac{\text{ft}}{\text{mi}})$$

10.34

10.34 Repeat Problem 10.33 if the Froude number is 2.75.

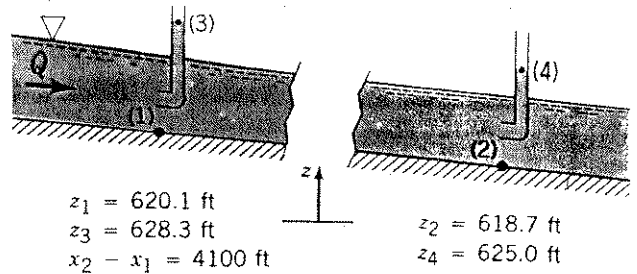


FIGURE P10.33

$$\frac{dy}{dx} = \frac{S_f - S_o}{1 - Fr^2}, \text{ where from the figure } S_f = \frac{h_L}{l} = \frac{z_3 - z_4}{x_1 - x_2} = \frac{(628.3 - 625.0) \text{ ft}}{4100 \text{ ft}}$$

$$\text{or } S_f = 8.05 \times 10^{-4} \text{ and } S_o = \frac{z_1 - z_2}{l} = \frac{(620.1 - 618.7) \text{ ft}}{4100 \text{ ft}} = 3.41 \times 10^{-4}$$

Thus,

$$\frac{dy}{dx} = \frac{8.05 \times 10^{-4} - 3.41 \times 10^{-4}}{1 - (2.75)^2} = \underline{\underline{-7.07 \times 10^{-5}}} \quad (\text{i.e., } -0.373 \frac{\text{ft}}{\text{mi}})$$

10.35

10.35 Water flows in a horizontal rectangular channel at a depth of 0.5 ft and a velocity of 8 ft/s. Determine the two possible depths at a location slightly downstream. Viscous effects between the water and the channel surface are negligible.

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{8 \text{ ft/s}}{\sqrt{32.2 \text{ ft/s}^2 (0.5 \text{ ft})}} = 1.99$$

Thus, with $Fr_1 > 1$ there could be a hydraulic jump with $y_2 > y_1 = 0.5 \text{ ft}$.

If so, then

$$\frac{y_2}{y_1} = \frac{1}{2} [-1 + \sqrt{1 + 8 Fr_1^2}] = \frac{1}{2} [-1 + \sqrt{1 + 8 (1.99)^2}] = 2.36$$

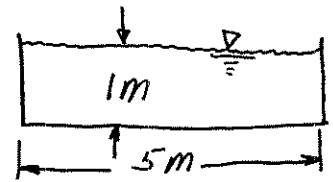
So that

$$y_2 = 2.36 y_1 = 2.36 (0.5 \text{ ft}) = 1.18 \text{ ft}$$

Hence, either $y_2 = 0.5 \text{ ft}$ (no jump) or $y_2 = 1.18 \text{ ft}$ (with jump)

10.36

10.36 Water flows in a 5-m-wide channel with a speed of 2 m/s and a depth of 1 m. The channel bottom slopes at a rate of 1 m per 1000 m. Determine the Manning coefficient for this channel.



$$(1) \quad V = \frac{K}{n} R_h^{2/3} \sqrt{S_o}, \text{ where}$$

$$S_o = \frac{1\text{m}}{1000\text{m}} = 0.001, \quad K=1, \text{ and}$$

$$R_h = \frac{A}{P} = \frac{1\text{m}(5\text{m})}{(5\text{m}+1\text{m}+1\text{m})} = \frac{5}{7}\text{m}$$

Thus, with $V = 2 \frac{\text{m}}{\text{s}}$, from Eq. (1)

$$2 = \frac{1}{n} \left(\frac{5}{7}\right)^{2/3} \sqrt{0.001}$$

or

$$n = \underline{\underline{0.0126}}$$

10.37

10.37 Fluid properties such as viscosity or density do not appear in the Manning equation (Eq. 10.20). Does this mean that this equation is valid for any open-channel flow such as that involving mercury, water, oil, or molasses? Explain.

The Manning equation, $Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, was "derived" specifically for water. It is not in dimensionless form and cannot be use without alteration (i.e. different n values; different dependance on R_h ; etc) for other fluids.

10.38

10.38 The following data are taken from measurements on Indian Fork Creek: $A = 26 \text{ m}^2$, $P = 16 \text{ m}$, and $S_0 = 0.02 \text{ m}/62 \text{ m}$. Determine the average shear stress on the wetted perimeter of this channel.

$$\tau_w = \gamma R_h S_0, \text{ where } R_h = \frac{A}{P}$$

$$\text{Thus, } \tau_w = \left(9800 \frac{\text{N}}{\text{m}^3}\right) \left(\frac{26 \text{ m}^2}{16 \text{ m}}\right) \left(\frac{0.02 \text{ m}}{62 \text{ m}}\right) = \underline{\underline{5.14 \frac{\text{N}}{\text{m}^2}}}$$

10.39

10.39 The following data are obtained for a particular reach of the Provo River in Utah: $A = 183 \text{ ft}^2$, free-surface width = 55 ft, average depth = 3.3 ft, $R_h = 3.22 \text{ ft}$, $V = 6.56 \text{ ft/s}$, length of reach = 116 ft, and elevation drop of reach = 1.04 ft. Determine the (a) average shear stress on the wetted perimeter, (b) the Manning coefficient, n , and (c) the Froude number of the flow.

$$a) \tau_w = \gamma R_h S_o, \text{ where } S_o = \frac{1.04 \text{ ft}}{116 \text{ ft}} = 0.00897$$

$$\text{Thus, } \tau_w = (62.4 \frac{\text{lb}}{\text{ft}^3})(3.22 \text{ ft})(0.00897) = \underline{\underline{1.80 \frac{\text{lb}}{\text{ft}^2}}}$$

$$b) Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2} = AV, \text{ where } K=1.49$$

$$\text{Thus, } n = \frac{1.49 R_h^{2/3} S_o^{1/2}}{V} = \frac{(1.49)(3.22)^{2/3}(0.00897)^{1/2}}{6.56} = \underline{\underline{0.0469}}$$

$$c) Fr = \frac{V}{\sqrt{gY}} = \frac{6.56 \frac{\text{ft}}{\text{s}}}{[(32.2 \frac{\text{ft}}{\text{s}^2})(3.3 \text{ ft})]^{1/2}} = \underline{\underline{0.636}} < 1 \text{ (subcritical)}$$

10.40

10.40 At a particular location the cross section of the Columbia River is as indicated in Fig. P10.40. If on a day without wind it takes 5 min to float 0.5 mi along the river, which drops 0.46 ft in that distance, determine the value of the Manning coefficient, n .

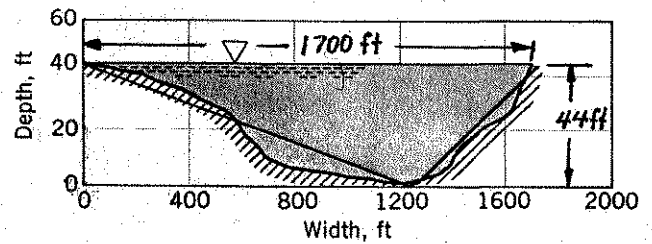


FIGURE P10.40

$$\text{From the given data, } V = \frac{(0.5 \text{ mi})(5280 \frac{\text{ft}}{\text{mi}})}{(5 \text{ min})(60 \frac{\text{s}}{\text{min}})} = 8.8 \frac{\text{ft}}{\text{s}}.$$

From the Manning equation,

$$V = \frac{K}{n} R_h^{2/3} S_0^{1/2}, \text{ where } K=1.49, S_0 = \frac{0.46 \text{ ft}}{(0.5 \text{ mi})(5280 \frac{\text{ft}}{\text{mi}})} = 0.000174, \quad (1)$$

$$\text{and } R_h = \frac{A}{P}.$$

Approximate A and P from the figure as

$$A \approx \frac{1}{2} b y = \frac{1}{2} (1700 \text{ ft})(44 \text{ ft}) = 37,400 \text{ ft}^2$$

and

$$P \approx 1800 \text{ ft} \quad \text{Thus, } R_h \approx \frac{37,400 \text{ ft}^2}{1800 \text{ ft}} = 20.8 \text{ ft}$$

Hence, from Eq. (1):

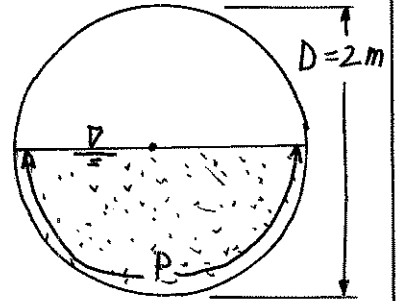
$$8.8 = \frac{1.49}{n} (20.8)^{2/3} (0.000174)^{1/2}$$

$$\text{or}$$

$$n = \underline{\underline{0.0169}}$$

10.41

10.41 A 2-m-diameter pipe made of finished concrete lies on a slope of 1 m elevation change per 1000 m horizontal distance. Determine the flowrate when the pipe is half full.



$$(1) \quad Q = AV = \frac{K}{n} A R_h^{2/3} \sqrt{S_o}, \text{ where } K=1 \text{ and } S_o = \frac{1\text{m}}{1000\text{m}} = 0.001$$

$$\text{Also, } A = \frac{1}{2} \left(\frac{\pi}{4} D^2 \right) = \frac{\pi}{8} (2\text{m})^2 = 1.57 \text{ m}^2$$

$$\text{and } R_h = \frac{A}{P} = \frac{\frac{1}{2} \left(\frac{\pi}{4} D^2 \right)}{\frac{\pi}{2} D} = \frac{D}{4} = \frac{2\text{m}}{4} = 0.5 \text{ m}$$

From Table 10.1, for finished concrete, $n = 0.012$

Thus, from Eq. (1),

$$Q = \frac{1}{0.012} (1.57) (0.5)^{2/3} \sqrt{0.001} = \underline{\underline{2.61 \frac{\text{m}^3}{\text{s}}}}$$

10.42

10.42 Rainwater flows down a street whose cross section is shown in Fig. P10.42. The street is on a hill at an angle of 2° . Determine the maximum flowrate possible if the water is not to overflow onto the sidewalk.

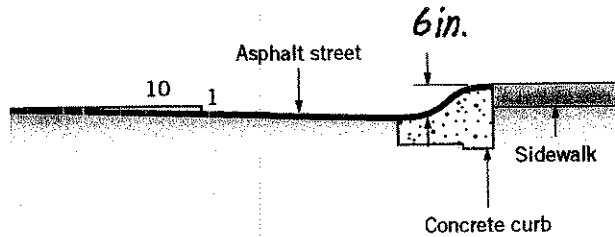
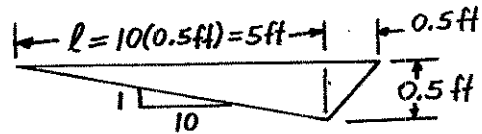


FIGURE P10.42

$$Q = \frac{K}{n} A R_h^{\frac{2}{3}} \sqrt{S_0}, \text{ where } K=1.49 \text{ and } S_0 = \tan 2^\circ = 0.0349 \quad (1)$$

Although part of the channel is asphalt and part is concrete, since the value of n is similar for each, use $n=0.016$, the value for asphalt. Also, approximate the cross section as a triangle as indicated.



Thus,

$$A \approx \frac{1}{2} (5 + 10) (0.5) \text{ ft}^2 = 1.375 \text{ ft}^2$$

and

$$P \approx \sqrt{2} (0.5 \text{ ft}) + \sqrt{(5 \text{ ft})^2 + (0.5 \text{ ft})^2} = 5.73 \text{ ft}$$

so that

$$R_h = \frac{A}{P} = \frac{1.375 \text{ ft}^2}{5.73 \text{ ft}} = 0.240 \text{ ft}$$

Hence, from Eq. (1):

$$Q = \frac{1.49}{0.016} (1.375) (0.240)^{\frac{2}{3}} \sqrt{0.0349} = \underline{\underline{9.25 \frac{\text{ft}^3}{\text{s}}}}$$

10.43

10.43 By what percent is the flowrate reduced in the rectangular channel shown in Fig. P10.43 because of the addition of the thin center board? All surfaces are of the same material.

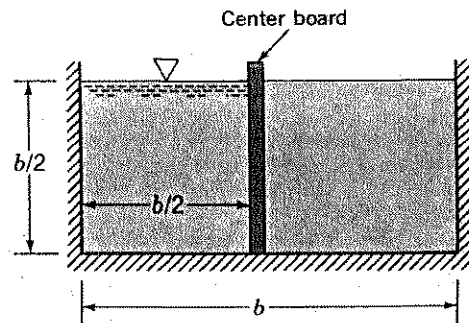


FIGURE P10.43

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$$

Without the centerboard $A = b(\frac{b}{2}) = \frac{b^2}{2}$, $R_h = \frac{A}{P} = \frac{\frac{b^2}{2}}{2b} = \frac{b}{4}$

or

$$Q_{\text{without}} = \frac{K}{n} \left(\frac{b^2}{2}\right) \left(\frac{b}{4}\right)^{2/3} S_o^{1/2} \quad (1)$$

With the centerboard $Q_{\text{with}} = 2Q_2$, where $A_2 = \left(\frac{b}{2}\right)^2$,

$$R_{h2} = \frac{A_2}{P_2} = \frac{\left(\frac{b}{2}\right)^2}{3\left(\frac{b}{2}\right)} = \frac{b}{6}$$

or

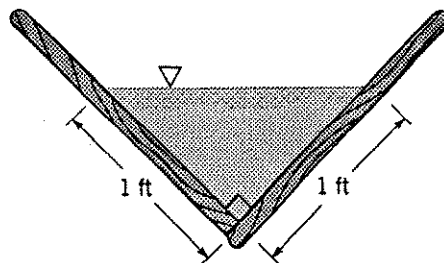
$$Q_{\text{with}} = 2 \frac{K}{n} \left(\frac{b}{2}\right)^2 \left(\frac{b}{6}\right)^{2/3} S_o^{1/2} \quad (2)$$

Divide Eq. (2) by Eq. (1) to obtain $\frac{Q_{\text{with}}}{Q_{\text{without}}} = \frac{2\left(\frac{b}{2}\right)^2 \left(\frac{b}{6}\right)^{2/3}}{\left(\frac{b^2}{2}\right) \left(\frac{b}{4}\right)^{2/3}} = 0.763$

a $100 - 76.3 = \underline{\underline{23.7\% \text{ reduction}}}$

10.44

10.44 The great Kings River flume in Fresno County, California, was used from 1890 to 1923 to carry logs from an elevation of 4500 ft where trees were cut to an elevation of 300 ft at the railhead. The flume was 54 miles long, constructed of wood, and had a V-cross section as indicated in Fig. P10.44. It is claimed that logs would travel the length of the flume in 15 hours. Do you agree with this claim? Provide appropriate calculations to support your answer.



■ FIGURE P10.44

$l = \text{distance traveled} = V_{\log} t$. Thus,

$$V_{\log} = \frac{l}{t} = \frac{(54 \text{ mi})(5280 \text{ ft/mi})}{(15 \text{ hr})(3600 \text{ s/hr})} = 5.28 \frac{\text{ft}}{\text{s}}$$

Determine the average water velocity, V , and compare it with this V_{\log} .

$$V = \frac{K}{n} R_h^{2/3} \sqrt{S_0}, \text{ where } K=1.49, A = \frac{1}{2}(1 \text{ ft}^2) = 0.5 \text{ ft}^2, P = 2 \text{ ft}$$

$$\text{so that } R_h = \frac{A}{P} = \frac{0.5 \text{ ft}^2}{2 \text{ ft}} = 0.25 \text{ ft}$$

Also,

$$S_0 = \frac{\Delta z}{l} = \frac{(4500 - 300) \text{ ft}}{(54 \text{ mi})(5280 \text{ ft/mi})} = 0.0147$$

Thus, with $n = 0.012$ (see Table 10.1, planed wood),

$$V = \frac{1.49}{0.012} (0.25)^{2/3} \sqrt{0.0147} = 5.97 \frac{\text{ft}}{\text{s}}$$

Note: V is slightly larger than V_{\log} . Thus, the claim appears to be correct. Yes.

10.45

10.45 Water flows in a channel as shown in Fig. P10.45. The velocity is 4.0 ft/s when the channel is half full with depth d . Determine the velocity when the channel is completely full, depth $2d$.

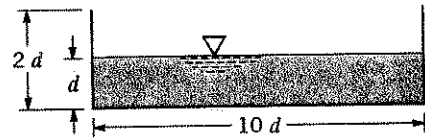


FIGURE P10.45

$$V = \frac{K}{n} R_h^{2/3} \sqrt{S_0} \quad \text{so that}$$

$$(1) \quad \frac{V_{\text{full}}}{V_{\frac{1}{2}\text{full}}} = \frac{\left(\frac{K}{n} R_h^{2/3} \sqrt{S_0}\right)_{\text{full}}}{\left(\frac{K}{n} R_h^{2/3} \sqrt{S_0}\right)_{\frac{1}{2}\text{full}}} = \left(\frac{R_{h\text{full}}}{R_{h\frac{1}{2}\text{full}}}\right)^{2/3} \quad \text{since } K, n, \text{ and } S_0 \text{ are the same for both the full and half-full conditions.}$$

Also,

$$R_{h\text{full}} = \left(\frac{A}{P}\right)_{\text{full}} = \frac{(2d)(10d)}{(10d+2d+2d)} = \frac{10}{7}d$$

and

$$R_{h\frac{1}{2}\text{full}} = \left(\frac{A}{P}\right)_{\frac{1}{2}\text{full}} = \frac{(d)(10d)}{(10d+d+d)} = \frac{5}{6}d$$

Thus, from Eq. (1)

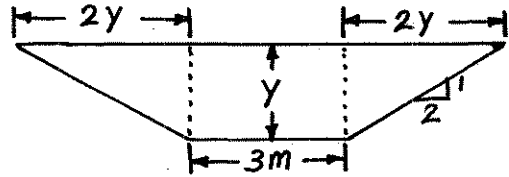
$$\frac{V_{\text{full}}}{V_{\frac{1}{2}\text{full}}} = \left(\frac{\frac{10}{7}d}{\frac{5}{6}d}\right)^{2/3} = 1.432$$

or

$$V_{\text{full}} = 1.432 V_{\frac{1}{2}\text{full}} = 1.432 \left(4 \frac{\text{ft}}{\text{s}}\right) = \underline{\underline{5.73 \frac{\text{ft}}{\text{s}}}}$$

10.46

10.46 A trapezoidal channel with a bottom width of 3.0 m and sides with a slope of 2:1 (horizontal:vertical) is lined with fine gravel ($n = 0.020$) and is to carry $10 \text{ m}^3/\text{s}$. Can this channel be built with a slope of $S_0 = 0.00010$ if it is necessary to keep the velocity below 0.75 m/s to prevent scouring of the bottom? Explain.



Determine V with $Q = 10 \frac{\text{m}^3}{\text{s}}$ and $S_0 = 0.00010$.

$$Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_0}, \text{ where } A = \frac{1}{2} y [3 + (3 + 4y)] = 2y^2 + 3y \quad (0)$$

$$\text{and } R_h = \frac{A}{P}, \text{ with } P = 3 + 2(\sqrt{5}y)$$

Thus,

$$10 = \frac{1}{0.02} (2y^2 + 3y) \left[\frac{2y^2 + 3y}{3 + 2\sqrt{5}y} \right]^{2/3} (0.0001)^{1/2}$$

or

$$20 = \frac{(2y^2 + 3y)^{5/3}}{(3 + 2\sqrt{5}y)^{2/3}} \text{ which can be written as}$$

$$2y^2 + 3y - 6.03 (3 + 2\sqrt{5}y)^{0.4} = 0 \quad (1)$$

A standard root-finding computer program gives the solution to Eq. (1) as $y = 2.25 \text{ m}$

$$\text{Hence, from Eq. (0) } A = 2(2.25)^2 + 3(2.25) = 16.9 \text{ m}^2$$

so that

$$V = \frac{Q}{A} = \frac{10 \frac{\text{m}^3}{\text{s}}}{16.9 \text{ m}^2} = 0.592 \frac{\text{m}}{\text{s}}$$

Thus, $V < 0.75 \frac{\text{m}}{\text{s}}$ so that scouring will not occur.

10.47

10.47 Water flows in a 2-m-diameter finished concrete pipe so that it is completely full and the pressure is constant all along the pipe. If the slope is $S_0 = 0.005$, determine the flowrate by using open-channel flow methods. Compare this result with that obtained by using pipe flow methods of Chapter 8.

For open channel flow $Q = \frac{K}{n} A R_h^{2/3} S_0^{1/2}$, where $K = 1$

Also, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (2\text{ m})^2 = 3.14\text{ m}^2$ and $P = \pi D = 6.28\text{ m}$ so that

$$R_h = \frac{A}{P} = \frac{3.14\text{ m}^2}{6.28\text{ m}} = 0.5\text{ m}$$

Hence, with $n = 0.012$ for finished concrete (see Table 10.1)

$$Q = \frac{1}{0.012} (3.14) (0.5)^{2/3} (0.005)^{1/2} = \underline{\underline{11.7 \frac{\text{m}^3}{\text{s}}}} \text{ (open channel)}$$

For pipe flow with constant pressure:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2g} + z_2 + f \frac{l}{D} \frac{V^2}{2g}$$

where $p_1 = p_2$ and $V_1 = V_2$

Thus, with $z_1 - z_2 = l S_0$,

$$l S_0 = f \frac{l}{D} \frac{V^2}{2g}$$

or

$$f V^2 = 2g D S_0 = 2(9.81 \frac{\text{m}}{\text{s}^2})(2\text{ m})(0.005) \text{ Thus, } f V^2 = 0.196 \quad (1)$$

From Table 8.1, for smooth concrete $\frac{\epsilon}{D} = 0.3 \times 10^{-3} \text{ m} / 2\text{ m} = 1.5 \times 10^{-4}$

$$\text{Also, } Re = \frac{VD}{\nu} = \frac{V(2\text{ m})}{1.12 \times 10^{-6} \frac{\text{m}^2}{\text{s}}} = 1.79 \times 10^6 V \quad (2)$$

and from the Moody chart (Fig. 8.20):

Solve Eqs. (1), (2), and (3) for f, V, Re :

Assume $f = 0.015$ so that from Eq. (1)

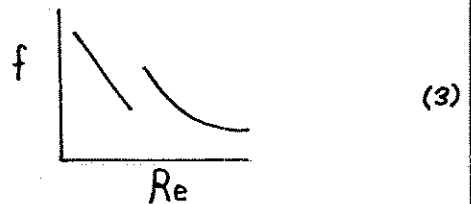
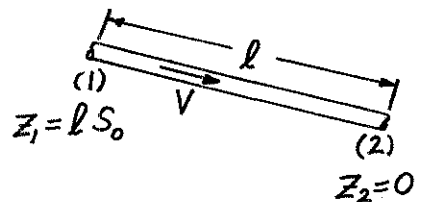
$$V = \left[\frac{0.196}{0.015} \right]^{1/2} = 3.61 \frac{\text{m}}{\text{s}}$$

or $Re = 1.79 \times 10^6 (3.61) = 6.46 \times 10^6$ Thus, from Eq. (3) (Moody chart)

$f = 0.013 \neq 0.015$. Assume $f = 0.013$, or $V = \left[\frac{0.196}{0.013} \right]^{1/2} = 3.88 \frac{\text{m}}{\text{s}}$

so that $Re = 1.79 \times 10^6 (3.88) = 6.95 \times 10^6$ Thus, from Eq. (3) $f = 0.013$ (checks with the assumed value) Hence, $V = 3.88 \frac{\text{m}}{\text{s}}$ or

$$Q = AV = \frac{\pi}{4} (2\text{ m})^2 (3.88 \frac{\text{m}}{\text{s}}) = \underline{\underline{12.2 \frac{\text{m}^3}{\text{s}}}} \text{ (pipe flow)} \approx 11.7 \frac{\text{m}^3}{\text{s}} \text{ (open channel flow)}$$



10.48

10.48 Water flows in a weedy earthen channel at a rate of $30 \text{ m}^3/\text{s}$. What flowrate can be expected if the weeds are removed and the depth remains constant?

$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$ Let $()_{nw}$ denote no weeds; $()_w$ denote with weeds.
Thus, since $A_w = A_{nw}$, $R_{hw} = R_{hnw}$ and $S_{ow} = S_{onw}$ it follows that

$$\frac{Q_w}{Q_{nw}} = \frac{\frac{K}{n_w} A_w R_{hw}^{2/3} S_{ow}^{1/2}}{\frac{K}{n_{nw}} A_{nw} R_{hnw}^{2/3} S_{onw}^{1/2}} = \frac{n_{nw}}{n_w}$$

From Table 10.1 $n_w = 0.030$, $n_{nw} = 0.022$

or

$$Q_{nw} = \frac{n_w}{n_{nw}} Q_w = \frac{0.030}{0.022} (30 \frac{\text{m}^3}{\text{s}}) = \underline{\underline{40.9 \frac{\text{m}^3}{\text{s}}}}$$

10.49

10.49 A round concrete storm sewer pipe used to carry rainfall runoff from a parking lot is designed to be half full when the rainfall rate is a steady 1 in./hr. Will this pipe be able to handle the flow from a 2-in./hr rainfall without water backing up into the parking lot? Support your answer with appropriate calculations.

$$Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_0}$$

Let ()₁ denote conditions when the pipe is half full and ()₂ when the pipe is full.

$$\text{Thus, } A_1 = \frac{\pi}{8} D^2, R_{h1} = A_1 / P_1 = (\frac{\pi}{8} D^2) / (\frac{\pi}{2} D) = D/4$$

$$\text{and } A_2 = \frac{\pi}{4} D^2, R_{h2} = A_2 / P_2 = (\frac{\pi}{4} D^2) / (\pi D) = D/4$$

$$\text{Also, } S_{01} = S_{02} \text{ and } n_1 = n_2$$

Therefore,

$$\frac{Q_1}{Q_2} = \frac{\frac{K}{n_1} A_1 R_{h1}^{2/3} \sqrt{S_{01}}}{\frac{K}{n_2} A_2 R_{h2}^{2/3} \sqrt{S_{02}}} = \frac{A_1 R_{h1}^{2/3}}{A_2 R_{h2}^{2/3}} = \frac{(\frac{\pi}{8} D^2) (\frac{D}{4})^{2/3}}{(\frac{\pi}{4} D^2) (\frac{D}{4})^{2/3}} = \frac{1}{2}$$

That is, $Q_2 = 2Q_1$. The full pipe can carry twice that of the half-full pipe. It can carry the 2 in./hr rainfall.

10.50

10.50 A 10-ft-wide rectangular channel is built to bypass a dam so that fish can swim upstream during their migration. During normal conditions when the water depth is 4 ft, the water velocity is 5 ft/s. Determine the velocity during a flood when the water depth is 8 ft.

Let $()_n$ and $()_f$ denote normal and flood conditions, respectively.

Thus,

$$(1) \quad V_n = \frac{K}{n_n} R_{h_n}^{2/3} \sqrt{S_{0n}} \quad \text{and}$$

$$(2) \quad V_f = \frac{K}{n_f} R_{h_f}^{2/3} \sqrt{S_{0f}}$$

where $n_n = n_f$, $S_{0n} = S_{0f}$ and

$$A_n = 10\text{ft}(4\text{ft}) = 40\text{ft}^2, \quad A_f = 10\text{ft}(8\text{ft}) = 80\text{ft}^2$$

$$P_n = 10\text{ft} + 2(4\text{ft}) = 18\text{ft}, \quad P_f = 10\text{ft} + 2(8\text{ft}) = 26\text{ft}$$

$$\text{Thus, } R_{h_n} = \frac{A_n}{P_n} = \frac{40\text{ft}^2}{18\text{ft}} = 2.22\text{ft}$$

$$\text{and } R_{h_f} = \frac{A_f}{P_f} = \frac{80\text{ft}^2}{26\text{ft}} = 3.08\text{ft}$$

Hence, divide Eq(2) by Eq(1) to obtain:

$$\frac{V_f}{V_n} = \left(\frac{R_{h_f}}{R_{h_n}} \right)^{2/3} = \left(\frac{3.08\text{ft}}{2.22\text{ft}} \right)^{2/3} = 1.24$$

so that

$$V_f = 1.24 V_n = 1.24 \left(5 \frac{\text{ft}}{\text{s}} \right) = \underline{\underline{6.22 \frac{\text{ft}}{\text{s}}}}$$

10.52*

*10.52 Water flows in the painted steel rectangular channel with rounded corners shown in Fig. P10.52. The bottom slope is 1 ft/200 ft. Plot a graph of flowrate as a function of water depth for $0 \leq y \leq 1$ ft with corner radii of $r = 0, 0.2, 0.4, 0.6, 0.8,$ and 1.0 ft.

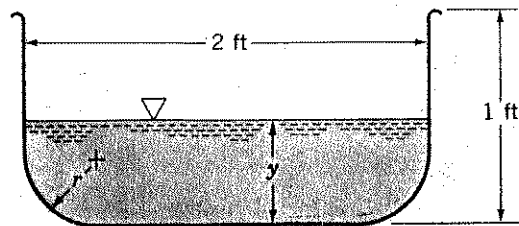


FIGURE P10.52

$$Q = \frac{K}{n} A R_h S_o^{1/2}, \text{ where } K=1.49, \text{ from Table 10.1 } n=0.014, \text{ and} \quad (1)$$

$$S_o = \frac{1 \text{ ft}}{200 \text{ ft}} = 0.005$$

(a) Assume $y \geq r$:

$$\text{Thus, } A = 2(y-r) + r(2-2r) + \frac{1}{2} \pi r^2$$

$$\text{or } A = 2y - (2 - \frac{\pi}{2})r^2$$

$$\text{and } P = 2(y-r) + (2-2r) + \pi r$$

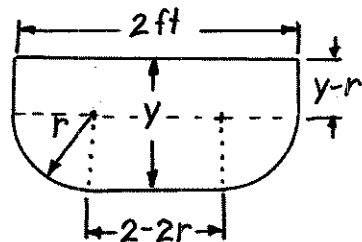
$$\text{or } P = 2y - (4 - \pi)r + 2$$

Hence, with $R_h = \frac{A}{P}$ Eqs. (1), (2), and (3) give

$$Q = \frac{1.49}{0.014} A^{5/3} \frac{1}{P^{2/3}} (0.005)^{1/2}$$

$$\text{or } Q = 7.53 \frac{[2y - (2 - \frac{\pi}{2})r^2]^{5/3}}{[2y - (4 - \pi)r + 2]^{2/3}}$$

$$\text{for } r \leq y \leq 1, \text{ where } r \sim \text{ft}, y \sim \text{ft}, Q \sim \frac{\text{ft}^3}{\text{s}} \quad (4)$$



(2)

(3)

(b) Assume $y \leq r$:

$$\text{Thus, } A = A_1 + A_2 + A_3$$

From Example 10.5, with $D=2r$

$$A_1 + A_3 = \frac{(2r)^2}{8} (\theta - \sin \theta) \text{ where } \theta \sim \text{rad and}$$

$$\cos \frac{\theta}{2} = \frac{r-y}{r}$$

$$\text{Hence, } A = \frac{r^2}{2} (\theta - \sin \theta) + (2-2r)y$$

$$\text{Also, } P = 2-2r + P_1 + P_3, \text{ where}$$

$$\text{from Example 10.5, } P_1 + P_3 = \frac{(2r)\theta}{2} = r\theta$$

$$\text{Thus, } P = 2-2r + r\theta = 2 + (\theta-2)r$$

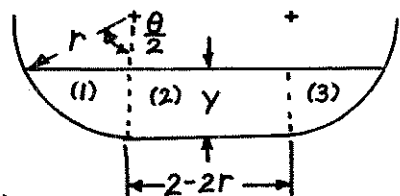
By combining Eqs. (1), (5), and (6) we obtain:

$$Q = \frac{1.49}{0.014} A^{5/3} \frac{1}{P^{2/3}} (0.005)^{1/2}$$

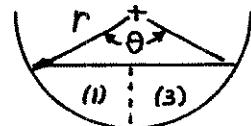
or

$$Q = 7.53 \frac{[\frac{r^2}{2} (\theta - \sin \theta) + (2-2r)y]^{5/3}}{[2 + (\theta-2)r]^{2/3}}$$

$$\text{for } 0 \leq y \leq r, \text{ where } r \sim \text{ft}, y \sim \text{ft}, Q \sim \frac{\text{ft}^3}{\text{s}}, \text{ and } \theta = 2 \cos^{-1}(\frac{r-y}{r}) \sim \text{rad} \quad (7)$$



(5)

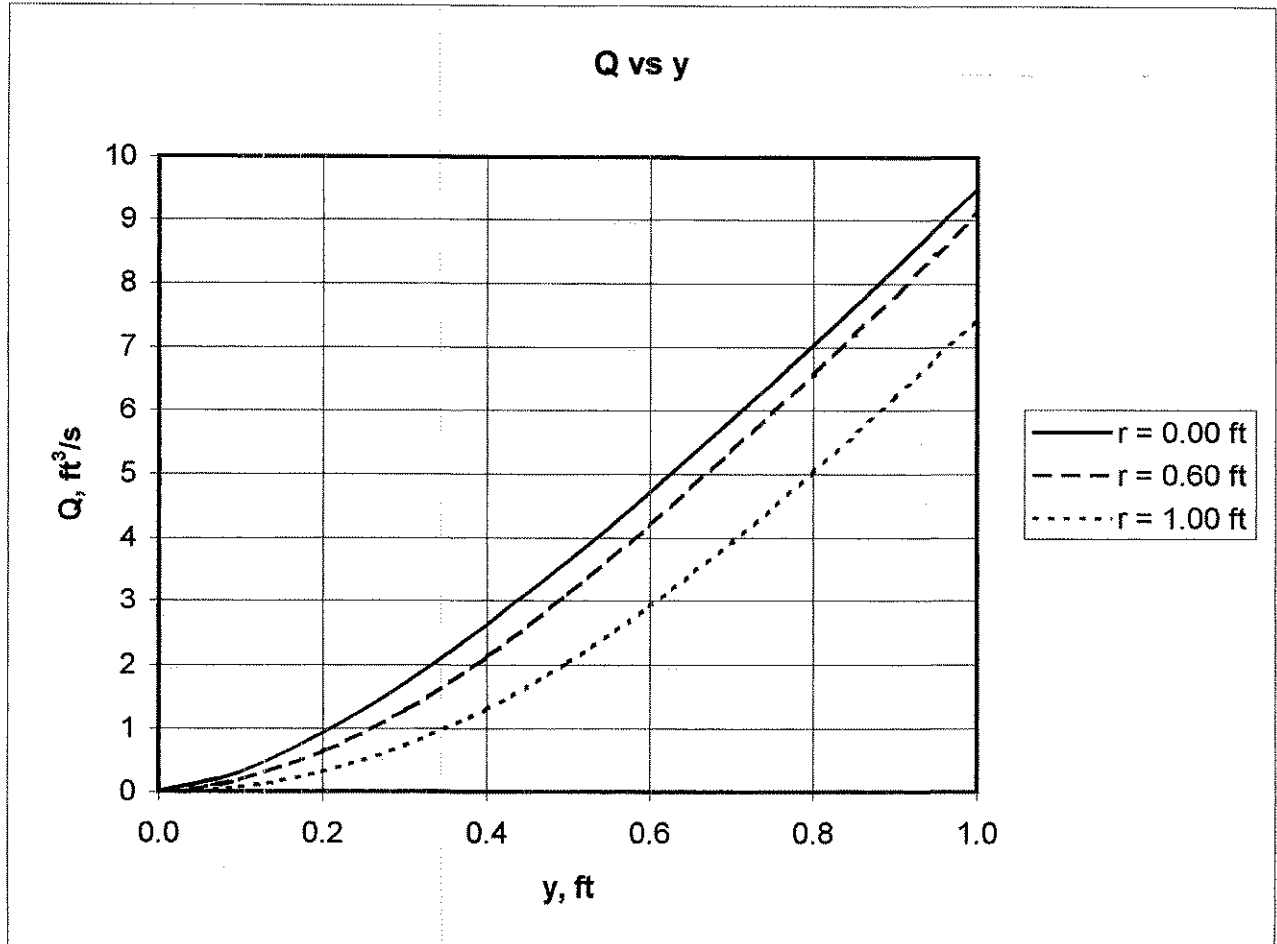


(6)

(cont)

10.52* (con't)

The results, $Q=Q(y)$, are plotted below for $r=0, 0.6$, and $1 ft$.



10.53*

10.53* The cross section of a long tunnel carrying water through a mountain is as indicated in Fig. P10.53. Plot a graph of flowrate as a function of water depth, y , for $0 \leq y \leq 18$ ft. The slope is 2 ft/mi and the surface of the tunnel is rough rock (equivalent to rubble masonry). At what depth is the flowrate maximum? Explain.

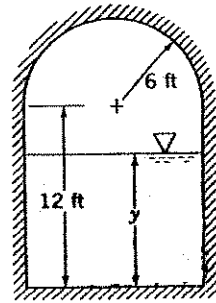


FIGURE P10.53

$$Q = \frac{K}{n} A R_h^{2/3} S_0^{1/2}, \text{ where } K = 1.49, S_0 = \frac{2 \text{ ft}}{5280 \text{ ft}} = 0.000379, \quad (1)$$

and from Table 10.5 $n = 0.025$

(a) Assume $y \leq 12$ ft: Thus, $A = 12y$ and $P = 2y + 12$
so that $R_h = \frac{A}{P} = \frac{12y}{2y + 12} = \frac{6y}{y + 6}$

Hence,

$$Q = \frac{1.49}{0.025} (12y) \left[\frac{6y}{y+6} \right]^{2/3} (0.000379)^{1/2}$$

or

$$Q = 46.0 \frac{y^{5/3}}{(y+6)^{2/3}}, \text{ for } y \leq 12 \text{ where } y \sim \text{ft}, Q \sim \frac{\text{ft}^3}{\text{s}} \quad (2)$$

(b) Assume $12 \leq y \leq 18$ ft:

$$\text{Thus, } A = (12 \text{ ft})^2 + \frac{\pi}{2} (6 \text{ ft})^2 - A_1,$$

where from Example 10.5

$$A_1 = \frac{D^2}{8} (\theta - \sin \theta), \text{ with } \cos \frac{\theta}{2} = \frac{y-12}{6}$$

Hence, from Eq. (3)

$$A = 201 \text{ ft}^2 - \frac{(12 \text{ ft})^2}{8} (\theta - \sin \theta)$$

or

$$A = 201 - 18(\theta - \sin \theta) \text{ ft}, \text{ where } \theta \sim \text{rad}$$

Also,

$$P = 3(12 \text{ ft}) + (\pi - \theta)(6 \text{ ft}) = 36 + 6(\pi - \theta) \text{ ft} \quad (5)$$

Thus, with $R_h = \frac{A}{P}$ Eqs. (1), (4), and (5) give

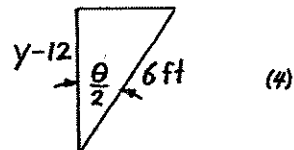
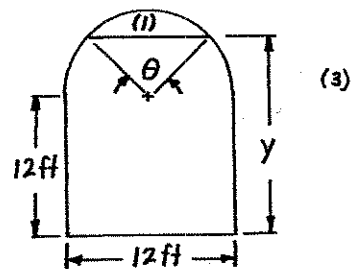
$$Q = \frac{1.49}{0.025} \frac{[201 - 18(\theta - \sin \theta)]^{5/3}}{[36 + 6(\pi - \theta)]^{2/3}} (0.000379)^{1/2}$$

or

$$Q = 43.4 \frac{(11.15 - \theta + \sin \theta)^{5/3}}{(9.14 - \theta)^{2/3}}, \text{ for } 12 \leq y \leq 18 \text{ ft, where } \theta \sim \text{rad}, \quad (6)$$

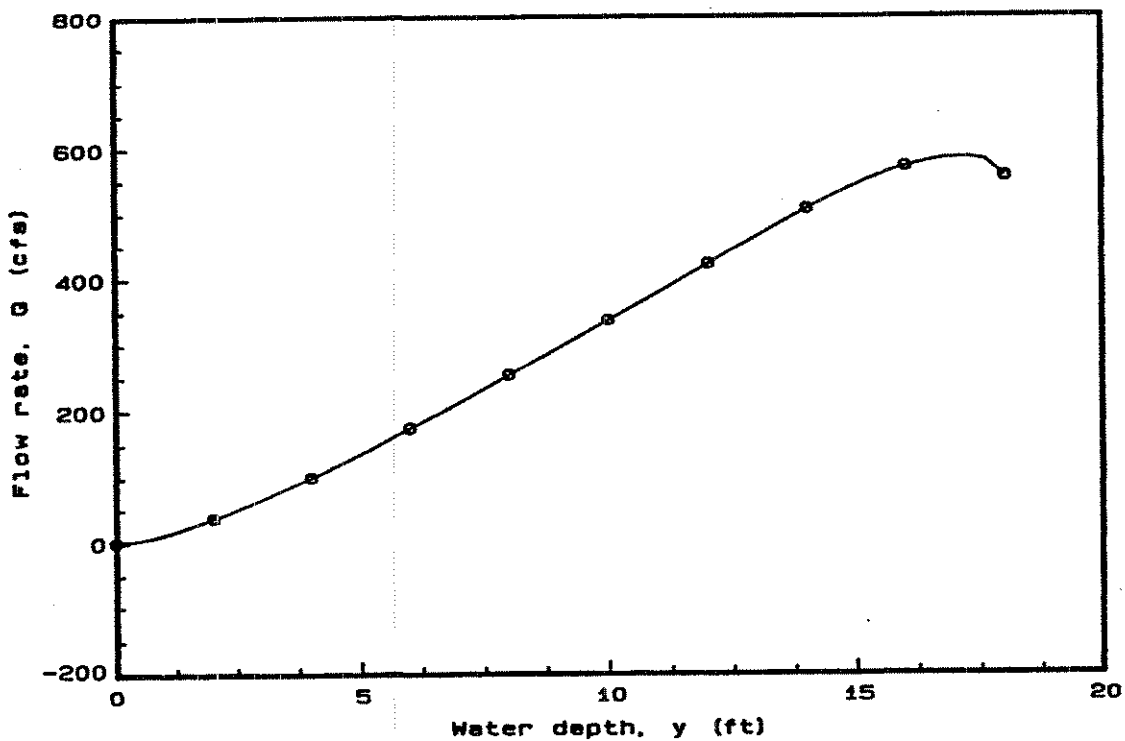
$$Q \sim \frac{\text{ft}^3}{\text{s}}, \text{ and } \theta = 2 \cos^{-1} \left(\frac{y-12}{6} \right)$$

For $0 \leq y \leq 18$ ft calculate $Q = Q(y)$ from either Eq. (2) and Eq. (6),
(cont)



10.53* (cont)

depending on the value of y . The results from this calculation are given below. The maximum flowrate, $Q_{max} = 583 \frac{ft^3}{s}$, occurs at $y = \underline{17.1 ft}$. For $17.1 ft \leq y \leq 18 ft$, an increase in depth adds only little to the flow area, A , but greatly increases the wetted perimeter, P . Thus, the retarding force is increased considerably.



10.54

10.54 The smooth concrete-lined channel shown in Fig. P10.54 is built on a slope of 2 m/km. Determine the flowrate if the depth is $y = 1.5$ m.

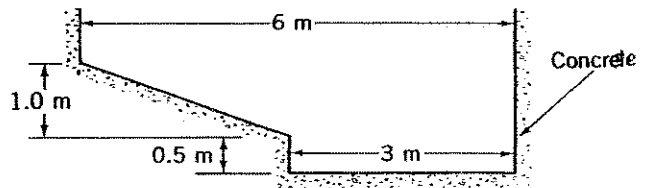


FIGURE P10.54

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K=1, S_o = \frac{2 \text{ m}}{1000 \text{ m}} = 0.002, \text{ and from Table 10.1 (1)}$$

$$n = 0.012$$

$$\text{With } y = 1.5 \text{ m, } A = (3 \text{ m})(0.5 \text{ m}) + \frac{1}{2}(3 \text{ m} + 6 \text{ m})(1.0 \text{ m}) = 6 \text{ m}^2$$

and

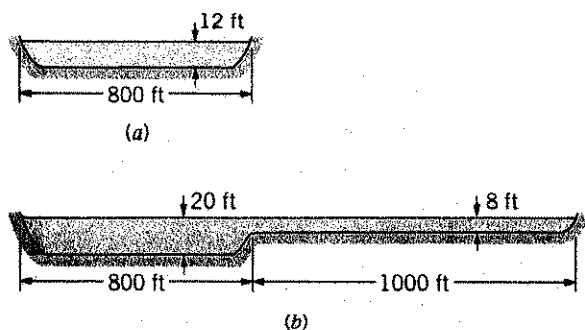
$$P = 1.5 \text{ m} + 3 \text{ m} + 0.5 \text{ m} + (1^2 + 3^2)^{1/2} \text{ m} = 8.16 \text{ m}$$

$$\text{Thus, } R_h = \frac{A}{P} = \frac{6 \text{ m}^2}{8.16 \text{ m}} = 0.735 \text{ m, and Eq. (1) gives}$$

$$Q = \frac{1}{0.012} (6)(0.735)^{2/3} (0.002)^{1/2} = \underline{\underline{18.2 \frac{\text{m}^3}{\text{s}}}}$$

10.55

*10.55 At a given location, under normal conditions a river flows with a Manning coefficient of 0.030 and a cross section as indicated in Fig. P10.55a. During flood conditions at this location, the river has a Manning coefficient of 0.040 (because of trees and brush in the floodplain) and a cross section as shown in Fig. P10.55b. Determine the ratio of the flowrate during flood conditions to that during normal conditions.



■ FIGURE P10.55

$$(1) \quad Q_a = \frac{K}{n_a} A_a R_{ha}^{2/3} \sqrt{S_{0a}}, \quad \text{where } A_a = 12 \text{ ft}(800 \text{ ft}) = 9600 \text{ ft}^2, \quad P_a = 2(12 \text{ ft}) + 800 \text{ ft} = 824 \text{ ft},$$

$$\text{so that } R_{ha} = A_a / P_a = 9600 \text{ ft}^2 / (824 \text{ ft}) = 11.65 \text{ ft}$$

Similarly,

$$(2) \quad Q_b = \frac{K}{n_b} A_b R_{hb}^{2/3} \sqrt{S_{0b}}, \quad \text{where } A_b = 20 \text{ ft}(800 \text{ ft}) + 8 \text{ ft}(1000 \text{ ft}) = 24,000 \text{ ft}^2,$$

$$P_b = 800 \text{ ft} + 1000 \text{ ft} + 2(20 \text{ ft}) = 1840 \text{ ft} \quad \text{so that}$$

$$R_{hb} = A_b / P_b = 24,000 \text{ ft}^2 / (1840 \text{ ft}) = 13.04 \text{ ft}$$

Thus, from Eqs. (1) and (2), with $S_{0a} = S_{0b}$,

$$(3) \quad \frac{Q_b}{Q_a} = \frac{\frac{K}{n_b} A_b R_{hb}^{2/3} \sqrt{S_{0b}}}{\frac{K}{n_a} A_a R_{ha}^{2/3} \sqrt{S_{0a}}} = \frac{n_a A_b R_{hb}^{2/3}}{n_b A_a R_{ha}^{2/3}}$$

By using the given and calculated data,

$$\frac{Q_b}{Q_a} = \left(\frac{0.03}{0.04} \right) \left(\frac{24,000 \text{ ft}^2}{9,600 \text{ ft}^2} \right) \left(\frac{13.04 \text{ ft}}{11.65 \text{ ft}} \right)^{2/3} = \underline{\underline{2.02}}$$

10.56

10.56 Repeat Problem 10.54 if the surfaces are smooth concrete as is indicated, except for the diagonal surface, which is gravelly with $n = 0.025$.

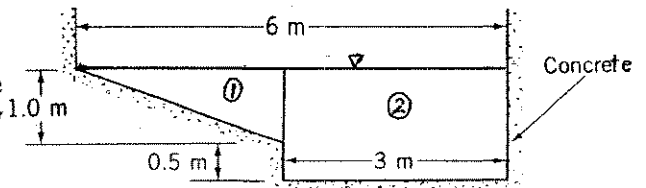


FIGURE P10.54

$$Q = Q_1 + Q_2 = \frac{K}{n_1} A_1 R_{h1}^{2/3} S_0^{1/2} + \frac{K}{n_2} A_2 R_{h2}^{2/3} S_0^{1/2}, \text{ where } K=1, S_0=0.002, \quad (1)$$

$n_1=0.025$, and from Table 10.1 $n_2=0.012$

Also, $A_1 = \frac{1}{2}(1.0\text{ m})(3\text{ m}) = 1.50\text{ m}^2$, $P_1 = (1.0^2 + 3.0^2)^{1/2} = 3.16\text{ m}$

or $R_{h1} = \frac{A_1}{P_1} = \frac{1.50\text{ m}^2}{3.16\text{ m}} = 0.475\text{ m}$

and $A_2 = (3\text{ m})(1.5\text{ m}) = 4.5\text{ m}^2$, $P_2 = 0.5\text{ m} + 3\text{ m} + 1.5\text{ m} = 5\text{ m}$

or $R_{h2} = \frac{A_2}{P_2} = \frac{4.5\text{ m}^2}{5\text{ m}} = 0.90\text{ m}$

Hence, from Eq. (1):

$$Q = \frac{1}{0.025} (1.50) (0.475)^{2/3} (0.002)^{1/2} + \frac{1}{0.012} (4.5) (0.90)^{2/3} (0.002)^{1/2}$$

or

$$Q = \underline{\underline{17.3 \frac{\text{m}^3}{\text{s}}}}$$

Note: With all surfaces concrete, $Q = 18.2 \frac{\text{m}^3}{\text{s}}$ (see Problem 10.64).

10.57*

10.57* Water flows through the storm sewer shown in Fig. P10.57. The slope of the bottom is 2 m/400 m. Plot a graph of the flowrate as a function of depth for $0 \leq y \leq 1.7$ m. On the same graph plot the flowrate expected if the entire surface were lined with material similar to that of a clay tile.

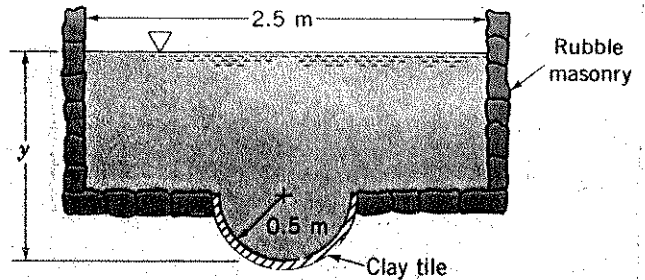


FIGURE P10.57

(a) For $0 \leq y \leq 0.5$ m: The flow is the same as that in a circular pipe.

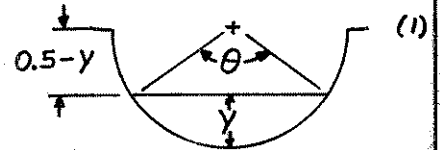
Thus, from Example 10.5 with $D=1$ m, $K=1$, and $n=0.014$ (Table 10.1):

$$Q = \frac{K}{n} S_0^{1/2} \frac{D^{8/3}}{8(4)^{2/3}} \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}} = \frac{1}{0.014} \left(\frac{2}{400}\right)^{1/2} \frac{(1)^{8/3}}{8(4)^{2/3}} \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}}$$

or

$$Q = 0.251 \frac{(\theta - \sin\theta)^{5/3}}{\theta^{2/3}} \frac{m^3}{s}, \text{ where } \theta \sim \text{rad}$$

$$\text{and } \theta = 2 \cos^{-1}\left(\frac{0.5-y}{0.5}\right)$$



(b) For $y \geq 0.5$ m:

$$Q = Q_1 + Q_2, \text{ where}$$

$$Q_1 = \frac{K}{n_1} A_1 R_{h1}^{2/3} S_0^{1/2}, \text{ with } n_1 = 0.014,$$

$$A_1 = \frac{\pi}{2} (0.5)^2 = 0.393 \text{ m}^2, P_1 = \pi(0.5) = 1.57 \text{ m so that}$$

$$R_{h1} = \frac{A_1}{P_1} = \frac{0.393 \text{ m}^2}{1.57 \text{ m}} = 0.250 \text{ m}$$

Thus,

$$Q_1 = \frac{1}{0.014} (0.393)(0.250)^{2/3} \left(\frac{2}{400}\right)^{1/2} = 0.787 \frac{m^3}{s}$$

Also,

$$Q_2 = \frac{K}{n_2} A_2 R_{h2}^{2/3} S_0^{1/2}, \text{ with } n_2 = 0.025 \text{ (see Table 10.1)} \quad (2)$$

$$A_2 = (2.5 \text{ m})(y - 0.5) = 2.5y - 1.25 \text{ and } P_2 = 2(y - 0.5) + 2\left(\frac{3}{4}\right) = 2y + 0.5$$

Hence, with $R_{h2} = \frac{A_2}{P_2}$, Eq. (2) becomes

$$Q_2 = \frac{1}{0.025} (2.5y - 1.25)^{5/3} \frac{1}{(2y + 0.5)^{2/3}} \left(\frac{2}{400}\right)^{1/2} = 13.0 \frac{(y - 0.5)^{5/3}}{(2y + 0.5)^{2/3}}$$

Therefore,

$$Q = 0.787 + 13.0 \frac{(y - 0.5)^{5/3}}{(2y + 0.5)^{2/3}} \frac{m^3}{s} \text{ for } y \geq 0.5 \text{ m.} \quad (3)$$

Plot $Q = Q(y)$ for $0 \leq y \leq 1.7$ m using Eqs. (1) and (3).

(cont)

10.57* (cont)

If the entire surface were lined with material with $n_1 = n_2 = 0.014$, Eqn. (1) would remain valid. The coefficient "13.0" in Eq. (3) would become $13.0 \left(\frac{0.025}{0.014} \right) = 23.2$. For this case,

$$Q = 0.787 + 23.2 \frac{(y-0.5)^{5/3}}{(2y+0.5)^{2/3}} \frac{m^3}{s} \text{ for } y \geq 0.5m \quad (4)$$

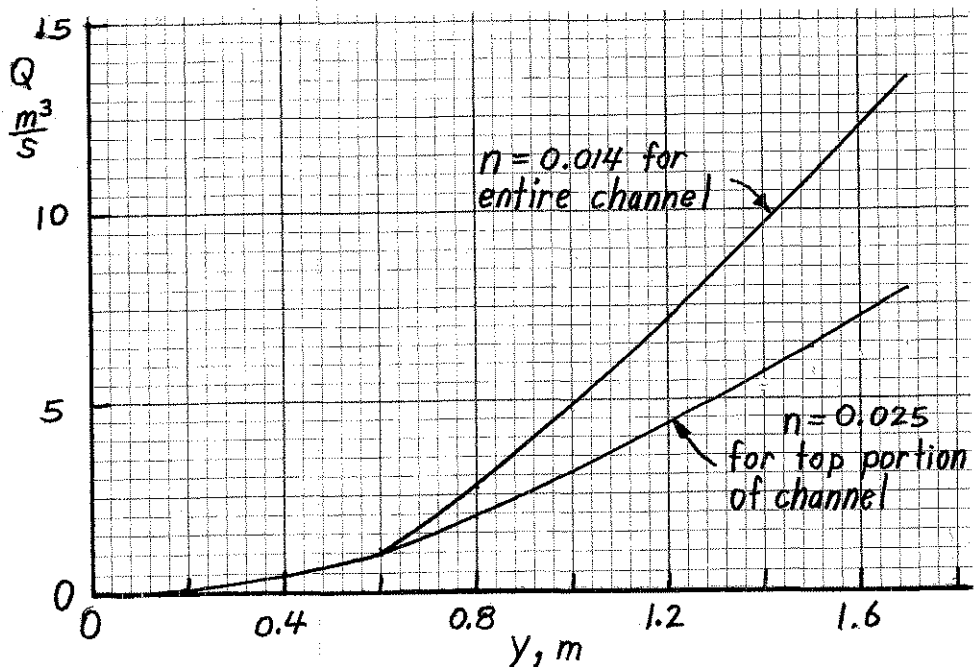
This result is also plotted (i.e. Q from Eq. (1) for $0 \leq y \leq 0.5$, and Q from Eq. (4) for $0.5 < y \leq 1.7m$).

With $n = 0.025$ for part of the channel

y, m	Q, m ³ /s	y, m	Q, m ³ /s
0.0	7.552E-11	0.9	2.407E+00
0.1	3.293E-02	1.0	3.010E+00
0.2	1.381E-01	1.1	3.649E+00
0.3	3.089E-01	1.2	4.315E+00
0.4	5.315E-01	1.3	5.003E+00
0.5	7.870E-01	1.4	5.708E+00
0.6	9.837E-01	1.5	6.426E+00
0.7	1.367E+00	1.6	7.157E+00
0.8	1.853E+00	1.7	7.897E+00

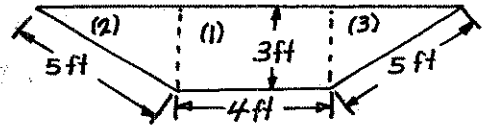
With $n = 0.014$ for the entire channel

y, m	Q, m ³ /s	y, m	Q, m ³ /s
0.0	7.552E-11	0.9	3.678E+00
0.1	3.293E-02	1.0	4.754E+00
0.2	1.381E-01	1.1	5.894E+00
0.3	3.089E-01	1.2	7.083E+00
0.4	5.315E-01	1.3	8.310E+00
0.5	7.870E-01	1.4	9.568E+00
0.6	1.138E+00	1.5	1.085E+01
0.7	1.822E+00	1.6	1.215E+01
0.8	2.689E+00	1.7	1.348E+01



10.58

10.58 Determine the flowrate for the symmetrical channel shown in Fig. P10.80 if the bottom is smooth concrete and the sides are weedy. The bottom slope is $S_0 = 0.001$.



$$Q = Q_1 + Q_2 + Q_3 = Q_1 + 2Q_2, \text{ where } Q_i = \frac{K}{n_i} A_i R_{hi}^{2/3} S_0^{1/2} \text{ with } K = 1.49$$

Also, $A_1 = (3\text{ ft})(4\text{ ft}) = 12\text{ ft}^2$, $A_2 = \frac{1}{2}(3\text{ ft})(4\text{ ft}) = 6\text{ ft}^2$, $P_1 = 4\text{ ft}$, and $P_2 = 5\text{ ft}$,
 so that $R_{h1} = \frac{A_1}{P_1} = \frac{12\text{ ft}^2}{4\text{ ft}} = 3\text{ ft}$ and $R_{h2} = \frac{A_2}{P_2} = \frac{6\text{ ft}^2}{5\text{ ft}} = 1.2\text{ ft}$

Hence, with $n_1 = 0.012$ and $n_2 = 0.030$ (see Table 10.1) we obtain:

$$Q = \frac{1.49}{0.012} (12)(3)^{2/3} (0.001)^{1/2} + 2 \frac{1.49}{0.030} (6)(1.2)^{2/3} (0.001)^{1/2} = \underline{\underline{119 \frac{\text{ft}^3}{\text{s}}}}$$

10.59

10.59 (See Fluids in the News article titled "Done without a GPS or lasers," Section 10.4.3.) Determine the number of gallons of water delivered per day by a rubble masonry, 1.2-m-wide aqueduct laid on an average slope of 14.6 m per 50 km if the water depth is 1.8 m.

$$Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_0},$$

where $A = 1.2\text{ m}(1.8\text{ m}) = 2.16\text{ m}^2$ and

$P = 1.2\text{ m} + 2(1.8\text{ m}) = 4.8\text{ m}$ so that

$$R_h = A/P = (2.16\text{ m}^2)/(4.8\text{ m}) = 0.450\text{ m}$$

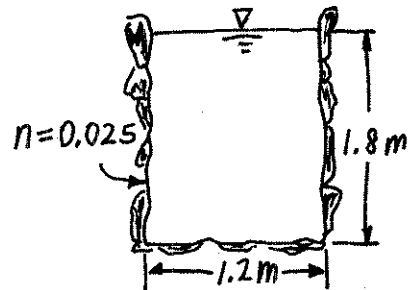
Thus, with $K=1$,

$$Q = \frac{1}{0.025} (2.16\text{ m}^2) (0.450\text{ m})^{2/3} \left(\frac{14.6\text{ m}}{50 \times 10^3\text{ m}} \right)^{1/2} = 0.867\text{ m}^3/\text{s}$$

or

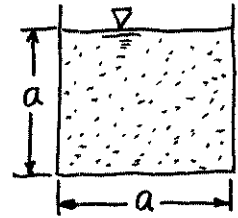
$$Q = 0.867 \frac{\text{m}^3}{\text{s}} \left(\frac{3600\text{ s}}{1\text{ hr}} \right) \left(\frac{24\text{ hr}}{1\text{ day}} \right) \left(\frac{1\text{ ft}^3}{0.0283\text{ m}^3} \right) \left(\frac{1728\text{ in}^3}{1\text{ ft}^3} \right) \left(\frac{1\text{ gal}}{231\text{ in}^3} \right)$$

$$= \underline{\underline{19.8 \times 10^6\text{ gal/day}}}$$



10.60

10.60 Water flows in a rectangular, finished concrete channel at a rate of $2 \text{ m}^3/\text{s}$. The bottom slope is 0.001. Determine the channel width if the water depth is to be equal to its width.



$$(1) \quad Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_0}, \quad \text{where } S_0 = 0.001 \text{ and } Q = 2 \frac{\text{m}^3}{\text{s}}$$

Also, $K=1$ and from Table 10.1, $n=0.012$

For the square channel

$$A = a^2 \text{ and } R_h = \frac{A}{P} = \frac{a^2}{3a} = \frac{a}{3}$$

Thus, from Eq. (1)

$$2 \frac{\text{m}^3}{\text{s}} = \frac{1}{0.012} a^2 \left(\frac{a}{3}\right)^{2/3} \sqrt{0.001}$$

or

$$a^{8/3} = 1.58$$

Hence,

$$a = \underline{\underline{1.19 \text{ m}}}$$

10.61

10.61 An old, rough-surfaced, 2-m-diameter concrete pipe with a Manning coefficient of 0.025 carries water at a rate of $5.0 \text{ m}^3/\text{s}$ when it is half full. It is to be replaced by a new pipe with a Manning coefficient of 0.012 that is also to flow half full at the same flowrate. Determine the diameter of the new pipe.

$$Q_{\text{old}} = \frac{K}{n_{\text{old}}} A_{\text{old}} R_{h_{\text{old}}}^{2/3} \sqrt{S_{o_{\text{old}}}} \quad (1)$$

and

$$Q_{\text{new}} = \frac{K}{n_{\text{new}}} A_{\text{new}} R_{h_{\text{new}}}^{2/3} \sqrt{S_{o_{\text{new}}}} \quad (2)$$

where $Q_{\text{old}} = Q_{\text{new}}$ and $S_{o_{\text{old}}} = S_{o_{\text{new}}}$.

Thus, by equating Eqs. (1) and (2),

$$\frac{A_{\text{old}} R_{h_{\text{old}}}^{2/3}}{n_{\text{old}}} = \frac{A_{\text{new}} R_{h_{\text{new}}}^{2/3}}{n_{\text{new}}} \quad (3)$$

But for a half full pipe, $A = \frac{\pi}{8} D^2$ and $R_h = \frac{A}{P} = \frac{\frac{\pi}{8} D^2}{\frac{\pi}{2} D} = \frac{D}{4}$

Thus, $A R_h^{2/3} = \frac{\pi}{8} D^2 \left(\frac{D}{4}\right)^{2/3}$ so that Eq. (3) becomes

$$\frac{\frac{\pi}{8} D_{\text{old}}^2 \left(\frac{D_{\text{old}}}{4}\right)^{2/3}}{n_{\text{old}}} = \frac{\frac{\pi}{8} D_{\text{new}}^2 \left(\frac{D_{\text{new}}}{4}\right)^{2/3}}{n_{\text{new}}}$$

$$\text{or } \frac{D_{\text{old}}^{8/3}}{n_{\text{old}}} = \frac{D_{\text{new}}^{8/3}}{n_{\text{new}}}$$

$$\text{Thus, } D_{\text{new}} = \left(\frac{n_{\text{new}}}{n_{\text{old}}}\right)^{3/8} D_{\text{old}} = \left(\frac{0.012}{0.025}\right)^{3/8} (2 \text{ m}) = \underline{\underline{1.52 \text{ m}}}$$

10.62

10.62 Four sewer pipes of 0.5-m diameter join to form one pipe of diameter D . If the Manning coefficient, n , and the slope are the same for all of the pipes, and if each pipe flows half-full, determine D .

$$Q_1 = 4Q_0, \text{ where } Q_1 = \frac{K}{n_1} A_1 R_{h_1}^{2/3} S_{o_1}^{1/2} \text{ and } Q_0 = \frac{K}{n_0} A_0 R_{h_0}^{2/3} S_{o_0}^{1/2}, \text{ with } \quad (1)$$

$$n_1 = n_0, S_{o_1} = S_{o_0}, A_1 = \frac{\pi}{8} D^2, R_{h_1} = \frac{A_1}{P_1} = \frac{\frac{\pi}{8} D^2}{\frac{\pi}{2} D} = \frac{D}{4},$$

$$A_0 = \frac{\pi}{8} (0.5)^2, \text{ and } R_{h_0} = \frac{A_0}{P_0} = \frac{0.5}{4}$$

Thus, from Eq. (1)

$$A_1 R_{h_1}^{2/3} = 4 A_0 R_{h_0}^{2/3}, \text{ or}$$

$$\frac{\pi}{8} D^2 \left(\frac{D}{4}\right)^{2/3} = 4 \frac{\pi}{8} (0.5)^2 \left(\frac{0.5}{4}\right)^{2/3}$$

Hence,

$$D^{8/3} = 4 \left(\frac{1}{2}\right)^{8/3}, \text{ or } D = \underline{\underline{0.841 \text{ m}}}$$

10.63

10.63 The flowrate in the clay-lined channel ($n = 0.025$) shown in Fig. P10.63 is to be $300 \text{ ft}^3/\text{s}$. To prevent erosion of the sides, the velocity must not exceed 5 ft/s . For this maximum velocity, determine the width of the bottom, b , and the slope, S_0 .

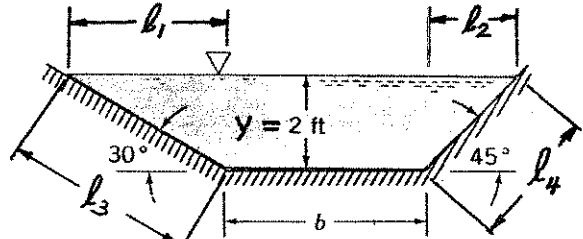


FIGURE P10.63

$$V = \frac{Q}{A}, \text{ where } A = \frac{1}{2}[b + (b + l_1 + l_2)]y \text{ with } l_1 = \frac{2 \text{ ft}}{\tan 30^\circ} = 3.46 \text{ ft} \quad (1)$$

$$\text{and } l_2 = \frac{2 \text{ ft}}{\tan 45^\circ} = 2 \text{ ft}$$

$$\text{Thus, } 5 \frac{\text{ft}}{\text{s}} = \frac{300 \frac{\text{ft}^3}{\text{s}}}{\frac{1}{2}[b + (b + 3.46 \text{ ft} + 2 \text{ ft})](2 \text{ ft})}, \text{ or } b = \underline{\underline{27.3 \text{ ft}}}$$

$$\text{Also, } V = \frac{K}{n} R_h^{2/3} S_0^{1/2}, \text{ where } K = 1.49 \text{ and from Table 10.1, } n = 0.025 \quad (2)$$

$$\text{From Eq. (1), } A = \frac{1}{2}[2(27.3 \text{ ft}) + 3.46 \text{ ft} + 2 \text{ ft}](2 \text{ ft}) = 60.0 \text{ ft}^2$$

$$\text{Also, } P = b + l_3 + l_4 = 27.3 \text{ ft} + \frac{2 \text{ ft}}{\sin 30^\circ} + \frac{2 \text{ ft}}{\sin 45^\circ} = 34.1 \text{ ft}$$

$$\text{Thus, } R_h = \frac{A}{P} = \frac{60.0 \text{ ft}^2}{34.1 \text{ ft}} = 1.76 \text{ ft} \text{ so that Eq. (2) becomes}$$

$$5 = \frac{1.49}{0.025} (1.76)^{2/3} S_0^{1/2}, \text{ or } S_0 = \underline{\underline{0.00331}}$$

10.64 Overnight a thin layer of ice forms on the surface of a 40-ft-wide river that is essentially of rectangular cross-sectional shape. Under these conditions the flow depth is 3 ft. During the following day the sun melts the ice cover. Determine the new depth if the flowrate remains the same and the surface roughness of the ice is essentially the same as that for the bottom and sides of the river.

$$Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_0}$$

Let $()_i$ denote conditions with the ice cover and $()_n$ with no ice cover.

Thus,

$$A_i = (40 \text{ ft})(3 \text{ ft}) = 120 \text{ ft}^2, \quad P_i = 2(40 \text{ ft}) + 2(3 \text{ ft}) = 86 \text{ ft},$$

$$\text{and } R_{h_i} = A_i / P_i = 120 \text{ ft}^2 / 86 \text{ ft} = 1.395 \text{ ft}$$

Also,

$$A_n = 40y, \quad P_n = 40 + 2y, \quad \text{and } R_{h_n} = A_n / P_n = 40y / (40 + 2y)$$

Hence, since $Q_i = Q_n$ it follows that

$$\frac{K}{n_i} A_i R_{h_i}^{2/3} \sqrt{S_{0_i}} = \frac{K}{n_n} A_n R_{h_n}^{2/3} \sqrt{S_{0_n}}$$

so that with $n_i = n_n$ and $S_{0_i} = S_{0_n}$ this becomes

$$A_i R_{h_i}^{2/3} = A_n R_{h_n}^{2/3}$$

Hence,

$$120 (1.395)^{2/3} = 40y \left(\frac{40y}{40 + 2y} \right)^{2/3}$$

or

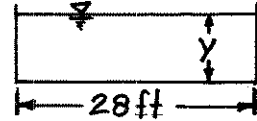
$$3.75 = y \left(\frac{40y}{40 + 2y} \right)^{2/3} \quad (1)$$

A standard root-finding computer program gives the solution to Eq. (1) as

$$y = \underline{\underline{2.31 \text{ ft}}}$$

10.65

10.65 A rectangular unfinished concrete channel of 28-ft-width is laid on a slope of 8 ft/mi. Determine the flow depth and Froude number of the flow if the flowrate is 400 ft³/s.



$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}$, where $K = 1.49$, $S_o = \frac{8 \text{ ft}}{5280 \text{ ft}} = 0.001515$, and from Table 10.1 $n = 0.014$

Also, $A = 28y$ and $P = 2y + 28$ so that $R_h = \frac{A}{P} = \frac{28y}{2y + 28}$

Thus, $400 = \frac{1.49}{0.014} \left(\frac{28y}{2y + 28} \right)^{2/3} (28y) (0.001515)^{1/2}$

or

$$0.594 = \frac{y^{5/3}}{(y + 14)^{2/3}}$$

Hence, $0.458(y + 14) - y^{5/2} = 0$ (1)

The solution to Eq. (1) is $y = \underline{\underline{2.23 \text{ ft}}}$

Thus,

$$V = \frac{Q}{A} = \frac{400 \frac{\text{ft}^3}{\text{s}}}{(28 \text{ ft})(2.23 \text{ ft})} = 6.41 \frac{\text{ft}}{\text{s}}$$

so that

$$Fr = \frac{V}{\sqrt{gy}} = \frac{6.41 \frac{\text{ft}}{\text{s}}}{\left[(32.2 \frac{\text{ft}}{\text{s}^2})(2.23 \text{ ft}) \right]^{1/2}} = \underline{\underline{0.756}}$$

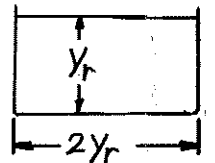
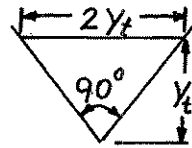
10.66

10.66 An engineer is to design a channel lined with planed wood to carry water at a flowrate of $2 \text{ m}^3/\text{s}$ on a slope of $10 \text{ m}/800 \text{ m}$. The channel cross section can be either a 90° triangle or a rectangle with a cross section twice as wide as its depth. Which would require less wood and by what percent?

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2} \quad (1)$$

Let $()_t$ denote the triangular cross-section and $()_r$ denote the rectangular cross-section

Thus, $Q_r = Q_t = 2 \frac{\text{m}^3}{\text{s}}$, $S_{or} = S_{ot} = \frac{10}{800}$
and $n_r = n_t$ so that Eq. (1) gives



$$A_r R_{hr}^{2/3} = A_t R_{ht}^{2/3}, \text{ where } R_h = \frac{A}{P} \quad (2)$$

Hence,

$$A_r = 2y_r^2, P_r = 4y_r \text{ so that } R_{hr} = \frac{2y_r^2}{4y_r} = \frac{1}{2} y_r$$

Also,

$$A_t = \frac{1}{2} (2y_t) y_t = y_t^2, P_t = 2(\sqrt{2} y_t) \text{ so that } R_{ht} = \frac{y_t}{2\sqrt{2}}$$

Thus, from Eq. (2):

$$2y_r^2 \left(\frac{1}{2} y_r\right)^{2/3} = y_t^2 \left(\frac{1}{2\sqrt{2}} y_t\right)^{2/3}, \text{ or } y_r = 0.707 y_t$$

The amount of wood is proportional to the wetted perimeter, P .

Since

$$\frac{P_t}{P_r} = \frac{2\sqrt{2} y_t}{4y_r} = \frac{2\sqrt{2} y_t}{4(0.707) y_t} = 1.00$$

the triangle requires the same amount of wood as the rectangle

10.67

10.67 A circular finished concrete culvert is to carry a discharge of 50 ft³/s on a slope of 0.0010. It is to flow not more than half-full. The culvert pipes are available from the manufacture with diameters that are multiples of 1 ft. Determine the smallest suitable culvert diameter.

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49, S_o = 0.001, \text{ and (from Table 10.1)}$$

$$n = 0.012$$

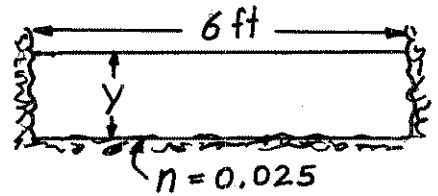
For a circular pipe half full $A = \frac{\pi}{8} D^2$, $P = \frac{\pi}{2} D$ so that $R_h = \frac{A}{P} = \frac{D}{4}$

$$\text{Thus, } 50.0 = \frac{1.49}{0.012} \left(\frac{\pi}{8} D^2 \right) \left(\frac{D}{4} \right)^{2/3} (0.001)^{1/2}, \text{ or } D = 5.12 \text{ ft}$$

To make sure it is not more than half full use the 6 ft diameter pipe.

10.68

10.68 At what depth will 50 ft³/s of water flow in a 6-ft-wide rectangular channel lined with rubble masonry set on a slope of 1 ft in 500 ft? Is a hydraulic jump possible under these conditions? Explain.



$$Q = \frac{1.49}{n} A R_h^{2/3} \sqrt{S_0} \quad \text{where}$$

$$A = 6y, \quad R_h = \frac{A}{P} = \frac{6y}{2y+6}, \quad S_0 = \frac{1 \text{ ft}}{500 \text{ ft}}$$

$$\text{and } n = 0.025 \quad (\text{see Table 10.1})$$

Thus,

$$50 = \frac{1.49}{0.025} (6y) \left[\frac{6y}{2y+6} \right]^{2/3} (0.002)^{1/2}$$

which becomes

$$y^{5/3} = (2y+6)^{2/3} (0.948)$$

By use of a root-finding computer program, the solution is

$$y = \underline{\underline{2.53 \text{ ft}}}$$

$$\text{Thus, } V = \frac{Q}{A} = \frac{50 \text{ ft}^3/\text{s}}{6(2.53) \text{ ft}^2} = 3.29 \text{ ft/s}$$

so that

$$Fr = \frac{V}{\sqrt{g y}} = \frac{3.29 \text{ ft/s}}{\left[(32.2 \text{ ft/s}^2)(2.53 \text{ ft}) \right]^{1/2}} = 0.365$$

Since $Fr < 1$ it is not possible to have a hydraulic jump.

10.69 The rectangular canal shown in Fig. P10.69 changes to a round pipe of diameter D as it passes through a tunnel in a mountain. Determine D if the surface material and slope remain the same and the round pipe is to flow completely full.

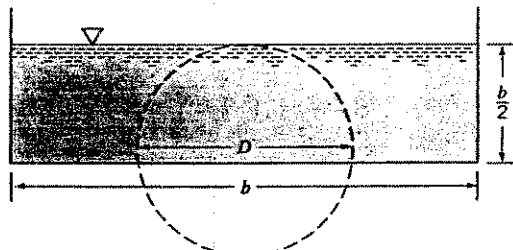


FIGURE P10.69

$$Q = \frac{K}{n} A R_h^{\frac{2}{3}} \sqrt{S_0} \quad \text{Let } ()_r \text{ denote the rectangular channel and } ()_c \text{ denote the circular pipe} \quad (1)$$

Thus, since $Q_r = Q_c$, $n_r = n_c$, $K_r = K_c$, $S_{or} = S_{oc}$ it follows from Eq. (1) that

$$A_r R_{hr}^{\frac{2}{3}} = A_c R_{hc}^{\frac{2}{3}} \quad \text{where } A_r = \frac{b^2}{2} \text{ and } P_r = b + 2\left(\frac{b}{2}\right) = 2b$$

$$\text{so that } R_{hr} = \frac{A_r}{P_r} = \frac{b}{4}$$

$$\text{and } A_c = \frac{\pi}{4} D^2 \text{ and } P_c = \pi D$$

$$\text{so that } R_{hc} = \frac{A_c}{P_c} = \frac{D}{4}$$

Thus,

$$\left(\frac{b}{2}\right)\left(\frac{b}{4}\right)^{\frac{2}{3}} = \left(\frac{\pi}{4} D^2\right)\left(\frac{D}{4}\right)^{\frac{2}{3}} \quad \text{or } D = \left(\frac{2}{\pi}\right)^{\frac{3}{8}} b = \underline{\underline{0.844b}}$$

10.70 The flowrate through the trapezoidal canal shown in Fig. P10.70 is Q . If it is desired to double the flowrate to $2Q$ without changing the depth, determine the additional width, L , needed. The bottom slope, surface material, and the slope of the walls are to remain the same.

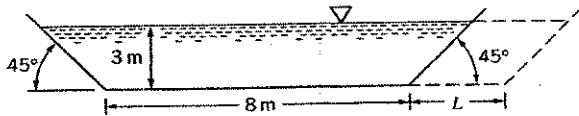


FIGURE P10.70

$$\text{Initially: } Q_i = \frac{K}{n} A_i R_{hi}^{\frac{2}{3}} \sqrt{S_0}$$

$$\text{where } A_i = (3\text{ m})(8\text{ m}) + 2 \left[\frac{1}{2} (3\text{ m})(3\text{ m}) \right] = 33\text{ m}^2$$

$$\text{and } P_i = 8\text{ m} + 2 \left[\sqrt{2} (3\text{ m}) \right] = 16.49\text{ m}$$

$$\text{Thus, } R_{hi} = \frac{A_i}{P_i} = \frac{33\text{ m}^2}{16.49\text{ m}} = 2.00\text{ m}$$

so that

$$Q_i = \frac{K \sqrt{S_0}}{n} (33)(2.00)^{\frac{2}{3}} = 52.4 \frac{K \sqrt{S_0}}{n} \quad (1)$$

$$\text{Finally: } Q_f = \frac{K}{n} A_f R_{hf}^{\frac{2}{3}} \sqrt{S_0} \quad (K, n, S_0 \text{ are constant})$$

$$\text{where } A_f = (3\text{ m})(8+L)\text{ m} + 2 \left[\frac{1}{2} (3\text{ m})(3\text{ m}) \right] = 33+3L\text{ m}^2, \text{ where } L \sim \text{m}$$

$$\text{and } P_f = P_i + L = 16.49 + L\text{ m}$$

$$\text{Thus, } R_{hf} = \frac{A_f}{P_f} = \frac{33+3L}{16.49+L}\text{ m}$$

so that

$$Q_f = \frac{K \sqrt{S_0}}{n} (33+3L) \left(\frac{33+3L}{16.49+L} \right)^{\frac{2}{3}} \quad (2)$$

But $Q_f = 2Q_i$, so that from Eqs. (1) and (2):

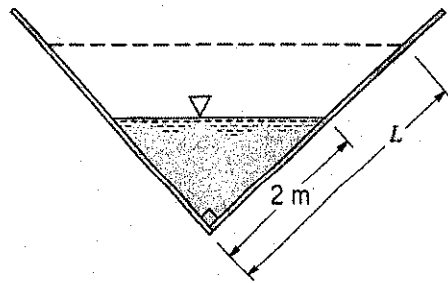
$$(33+3L) \left(\frac{33+3L}{16.49+L} \right)^{\frac{2}{3}} = 2(52.4) = 104.8$$

A standard computer root-finding program gives the solution to this equation as

$$L = \underline{\underline{8.77\text{ m}}}$$

10.71

10.71 When the channel of triangular cross section shown in Fig. P10.71 was new, a flowrate of Q caused the water to reach $L = 2$ m up the side as indicated. After considerable use, the walls of the channel became rougher and the Manning coefficient, n , doubled. Determine the new value of L if the flowrate stayed the same.



■ FIGURE P10.71

$$Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_0} \quad (1)$$

Let $()_o$ and $()_n$ represent the old and new conditions.

Thus, $n_o = 2n_n$, $A_n = \frac{1}{2}(2\text{ m})^2 = 2\text{ m}^2$, $P_n = 4\text{ m}$, so that

$$R_{hn} = A_n / P_n = (2\text{ m}^2) / (4\text{ m}) = \frac{1}{2}\text{ m}$$

Also, $A_o = \frac{1}{2}L^2$, $P_o = 2L$, so that $R_{ho} = A_o / P_o = (\frac{1}{2}L^2) / (2L) = L/4$

Therefore, using Eq. (1) with $Q_o = Q_n$ gives

$$\frac{K}{n_o} A_o R_{ho}^{2/3} \sqrt{S_{o0}} = \frac{K}{n_n} A_n R_{hn}^{2/3} \sqrt{S_{on}}$$

or since $S_{on} = S_{o0}$,

$$\frac{1}{n_o} A_o R_{ho}^{2/3} = \frac{1}{n_n} A_n R_{hn}^{2/3}$$

By using the above data this becomes

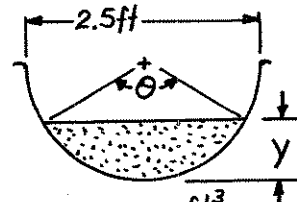
$$\frac{1}{2n_n} \left(\frac{1}{2}L^2\right) \left(\frac{L}{4}\right)^{2/3} = \frac{1}{n_n} (2\text{ m}^2) \left(\frac{1}{2}\text{ m}\right)^{2/3} \quad \text{or } L^{8/3} = 8(2)^{2/3}$$

or

$$L = \underline{\underline{2.59\text{ m}}}$$

10.72

10.72 A smooth steel water slide at an amusement park is of semicircular cross section with a diameter of 2.5 ft. The slide descends a vertical distance of 35 ft in its 420 ft length. If pumps supply water to the slide at a rate of 6 cfs, determine the depth of flow. Neglect the effects of the curves and bends of the slide.



$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K=1.49, S_o = \frac{35 \text{ ft}}{420 \text{ ft}} = 0.0833, Q = 6.0 \frac{\text{ft}^3}{\text{s}}$$

and from Table 10.1 $n = 0.012$

Also (see Example 10.5), $A = \frac{D^2}{8} (\theta - \sin \theta)$ and

$$R_h = \frac{D(\theta - \sin \theta)}{4\theta}, \text{ where } D = 2.5 \text{ ft}$$

Thus,

$$Q = \frac{K}{n} S_o^{1/2} \frac{D^{8/3}}{8(4)^{2/3}} \left[\frac{(\theta - \sin \theta)^{5/3}}{\theta^{2/3}} \right], \text{ where } \theta \sim \text{rad},$$

or

$$6.0 = \frac{1.49}{0.012} (0.0833)^{1/2} \frac{(2.5)^{8/3}}{8(4)^{2/3}} \left[\frac{(\theta - \sin \theta)^{5/3}}{\theta^{2/3}} \right]$$

Hence,

$$0.293 \theta^{2/3} = (\theta - \sin \theta)^{5/3} \quad 0.0252 \theta^2 - (\theta - \sin \theta)^5 = 0 \quad (1)$$

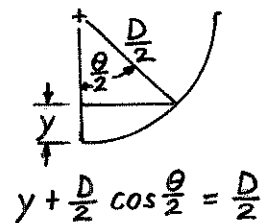
Using a standard root-finding technique gives the solution to Eq. (1) as $\theta = 1.574 \text{ rad}$.

$$\text{Thus, } \theta = (1.574 \text{ rad}) \left(\frac{180 \text{ deg}}{\pi \text{ rad}} \right) = 90.2^\circ$$

or since

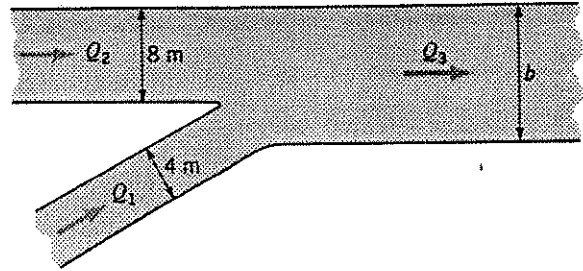
$$y = \frac{D}{2} (1 - \cos(\frac{\theta}{2})) \text{ it follows that}$$

$$y = \left(\frac{2.5 \text{ ft}}{2} \right) (1 - \cos(\frac{90.2^\circ}{2})) = \underline{\underline{0.368 \text{ ft}}}$$



10.73

10.73 Two canals join to form a larger canal as shown in Video V10.6 and Fig. P10.73. Each of the three rectangular canals is lined with the same material and has the same bottom slope. The water depth in each is to be 2 m. Determine the width of the merged canal, b . Explain physically (i.e., without using any equations) why it is expected that the width of the merged canal is less than the combined widths of the two original canals (i.e., $b < 4\text{ m} + 8\text{ m} = 12\text{ m}$).



■ FIGURE P10.73

$$Q_3 = Q_1 + Q_2 \quad \text{where for } i=1,2,3$$

$$Q_i = \frac{K}{n_i} A_i R_{h_i}^{2/3} \sqrt{S_{0i}}$$

Thus,

$$\frac{K}{n_3} A_3 R_{h_3}^{2/3} \sqrt{S_{03}} = \frac{K}{n_2} A_2 R_{h_2}^{2/3} \sqrt{S_{02}} + \frac{K}{n_1} A_1 R_{h_1}^{2/3} \sqrt{S_{01}} \quad (1)$$

But $n_1 = n_2 = n_3$ and $S_{01} = S_{02} = S_{03}$ so that Eq.(1) becomes

$$A_3 R_{h_3}^{2/3} = A_2 R_{h_2}^{2/3} + A_1 R_{h_1}^{2/3} \quad (2)$$

where

$$A_1 = 2\text{ m}(4\text{ m}) = 8\text{ m}^2, \quad P_1 = (2+2+4) = 8\text{ m} \text{ so that } R_{h_1} = \frac{A_1}{P_1} = \frac{8\text{ m}^2}{8\text{ m}} = 1\text{ m}$$

$$A_2 = 2\text{ m}(8\text{ m}) = 16\text{ m}^2, \quad P_2 = (2+2+8) = 12\text{ m} \text{ so that } R_{h_2} = \frac{A_2}{P_2} = \frac{16\text{ m}^2}{12\text{ m}} = 1.333\text{ m}$$

and

$$A_3 = 2b\text{ m}^2, \quad P_3 = (2+2+b) = (b+4)\text{ m} \text{ so that } R_{h_3} = \frac{A_3}{P_3} = \frac{2b}{(b+4)}$$

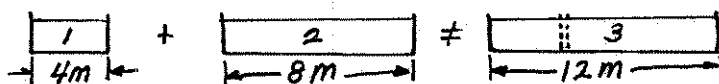
Thus, Eq.(2) becomes

$$(2b) \left[\frac{2b}{(b+4)} \right]^{2/3} = 16 (1.333)^{2/3} + 8 (1)^{2/3} = 27.4$$

$$\text{or } b^{5/3} = 8.63 (b+4)^{2/3} \quad (3)$$

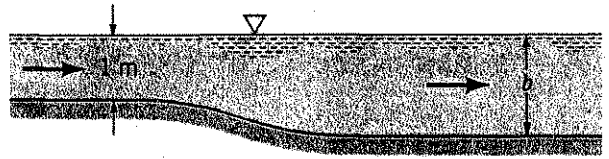
Using a standard root-finding technique gives the solution to Eq.(3):
 $b = \underline{10.66\text{ m}}$

If the two original canals merged to form a 12 m wide canal, the water depth would be less than 2 m because without the two walls there would be less friction force hold the water back. Thus, to maintain the 2 m depth we must have $b < 12\text{ m}$.



10.74

10.74 Water flows uniformly at a depth of 1 m in a channel that is 5 m wide as shown in Fig. P10.74. Further downstream the channel cross section changes to that of a square of width and height b . Determine the value of b if the two portions of this channel are made of the same material and are constructed with the same bottom slope.



Width = 5 m

■ FIGURE P10.74

$Q_U = Q_D$, where $()_U$ and $()_D$ denote upstream and downstream conditions.

Thus, since $Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_0}$, it follows that

$$\frac{K}{n_U} A_U R_{h_U}^{2/3} \sqrt{S_{0U}} = \frac{K}{n_D} A_D R_{h_D}^{2/3} \sqrt{S_{0D}}$$

Also, $S_{0U} = S_{0D}$ and $n_U = n_D$

Hence,

$$(1) \quad A_U R_{h_U}^{2/3} = A_D R_{h_D}^{2/3}, \text{ where } A_U = (1\text{ m})(5\text{ m}) = 5\text{ m}^2, P_U = 2(1\text{ m}) + 5\text{ m} = 7\text{ m},$$

so that $R_{h_U} = A_U / P_U = 5\text{ m}^2 / 7\text{ m} = 0.714\text{ m}$.

Also, $A_D = b^2$, $P_D = 3b$, so that $R_{h_D} = A_D / P_D = b^2 / (3b) = \frac{1}{3}b$

Thus, from Eq. (1):

$$(5\text{ m}^2)(0.714\text{ m})^{2/3} = b^2 \left(\frac{1}{3}b\right)^{2/3}$$

or

$$b = \underline{\underline{2.21\text{ m}}}$$

10.75

10.75 Determine the flow depth for the channel shown in Fig. P10.54 if the flowrate is $15 \text{ m}^3/\text{s}$.

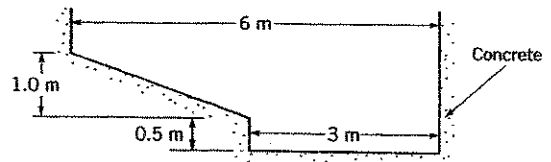


FIGURE P10.54

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K=1, S_o = \frac{3\text{ m}}{3000\text{ m}} = 0.003, \text{ and from Table 10.1 } n=0.012$$

$$\text{Also, } A = 3y + \frac{1}{2} [3(y-0.5)](y-0.5) = \frac{3}{2}y^2 + \frac{3}{2}y + \frac{3}{8}$$

$$\text{and } P = y + 3 + 0.5 + \left[(y - \frac{1}{2})^2 + 9(y - \frac{1}{2})^2 \right]^{1/2}$$

$$= y + 3.5 + \sqrt{10}(y - 0.5) = 4.16y + 1.92$$

Hence, with $R_h = \frac{A}{P}$ and $Q = 15 \frac{\text{m}^3}{\text{s}}$ we obtain

$$15 = \frac{1}{0.012} (1.5y^2 + 1.5y + 0.375)^{5/3} \frac{1}{(4.16y + 1.92)^{2/3}} (0.003)^{1/2}$$

$$\text{or } 2.04 (4.16y + 1.92)^{0.4} - 1.5y^2 - 1.5y - 0.375 = 0 \quad (1)$$

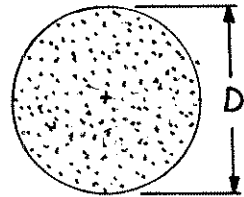
Using a standard root-finding technique, the solution to Eq. (1) is found to be

$$y = \underline{\underline{1.22\text{ m}}}$$

Note: Since $y < 1.5\text{ m}$ the water does not contact the left vertical wall.

10.76

10.76 Rainwater runoff from a 200-ft by 500-ft parking lot is to drain through a circular concrete pipe that is laid on a slope of 3 ft/mi. Determine the pipe diameter if it is to be full with a steady rainfall of 1.5 in./hr.



$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49, S_o = \frac{3 \text{ ft}}{5280 \text{ ft}} = 0.000568, \quad (1)$$

$$A = \frac{\pi}{4} D^2 \quad \text{and} \quad R_h = \frac{A}{P} = \frac{\frac{\pi}{4} D^2}{\pi D} = \frac{D}{4}$$

From Table 10.1, $n = 0.012$

Also, $Q = A_{\text{lot}} r$, where $r = \text{rainfall rate} = 1.5 \frac{\text{in.}}{\text{hr}}$

$$\text{Thus, } Q = (200 \text{ ft})(500 \text{ ft})(1.5 \frac{\text{in.}}{\text{hr}}) \left(\frac{1}{12} \frac{\text{ft}}{\text{in.}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right) = 3.47 \frac{\text{ft}^3}{\text{s}}$$

Hence, from Eq. (1):

$$3.47 = \frac{1.49}{0.012} \left(\frac{\pi}{4} D^2 \right) \left(\frac{D}{4} \right)^{2/3} (0.000568)^{1/2}$$

$$\text{or } D = \underline{\underline{1.64 \text{ ft}}}$$

10.77 (See Fluids in the News article titled "Plumbing the Everglades," Section 10.4.1.) The canal shown in Fig. P10.77 is to be widened so that it can carry twice the amount of water. Determine the additional width, L , required if all other parameters (i.e., flow depth, bottom slope, surface material, side slope) are to remain the same.

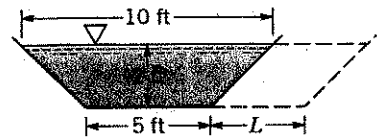


FIGURE P10.77

Let $()_0$ denote the original canal and $()_w$ the widened canal.

Thus,

$$(1) \quad Q_0 = \frac{K}{n_0} A_0 R_{h_0}^{2/3} \sqrt{S_{0_0}} \quad \text{and}$$

$$(2) \quad Q_w = \frac{K}{n_w} A_w R_{h_w}^{2/3} \sqrt{S_{0_w}}, \quad \text{where } n_0 = n_w \text{ and } S_{0_0} = S_{0_w}$$

Hence, from Eqs. (1) and (2)

$$(3) \quad \frac{Q_w}{Q_0} = \frac{\frac{K}{n_w} A_w R_{h_w}^{2/3} \sqrt{S_{0_w}}}{\frac{K}{n_0} A_0 R_{h_0}^{2/3} \sqrt{S_{0_0}}} = \frac{A_w}{A_0} \left(\frac{R_{h_w}}{R_{h_0}} \right)^{2/3}, \quad \text{where } Q_w = 2Q_0$$

$$\text{Also, } A_0 = \frac{1}{2} (5 \text{ ft} + 10 \text{ ft})(2 \text{ ft}) = 15 \text{ ft}^2$$

$$P_0 = 5 \text{ ft} + 2(3.20 \text{ ft}) = 11.4 \text{ ft}, \quad \text{so that}$$

$$R_{h_0} = A_0 / P_0 = 15 \text{ ft}^2 / 11.4 \text{ ft} = 1.316 \text{ ft}$$

and

$$A_w = \frac{1}{2} [(5 \text{ ft} + L) + (10 \text{ ft} + L)](2 \text{ ft}) = (15 + 2L) \text{ ft}^2$$

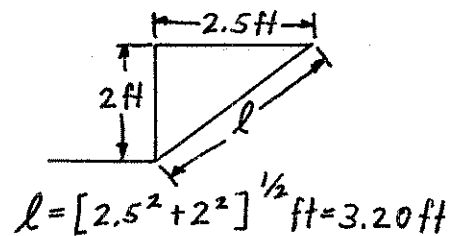
$$P_w = 5 \text{ ft} + L + 2(3.20 \text{ ft}) = (11.4 + L) \text{ ft}, \quad \text{so that}$$

$$R_{h_w} = A_w / P_w = (15 + 2L) / (11.4 + L)$$

Hence, from Eq. (3) with $\frac{Q_w}{Q_0} = 2$,

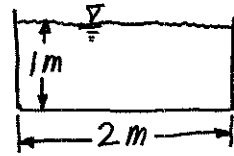
$$(4) \quad 2 = \frac{(15 + 2L)}{15} \left[\frac{(15 + 2L) / (11.4 + L)}{1.316} \right]^{2/3}, \quad \text{where } L \sim \text{ft}$$

By using a standard root-finding program, the solution to Eq. (4) is determined to be $L = \underline{\underline{5.94 \text{ ft}}}$



10.78

10.78 Water flows 1 m deep in a 2-m-wide finished concrete channel. Determine the slope if the flowrate is $3 \text{ m}^3/\text{s}$.



$$(1) \quad Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_0}, \text{ where } K=1, \text{ and from Table 10.1, } n=0.012$$

Also,

$$A = (1\text{m})(2\text{m}) = 2\text{m}^2$$

and

$$R_h = \frac{A}{P} = \frac{2\text{m}^2}{(2\text{m}+1\text{m}+1\text{m})} = 0.5\text{m}$$

Hence, with $Q = 3 \frac{\text{m}^3}{\text{s}}$ Eq.(1) becomes

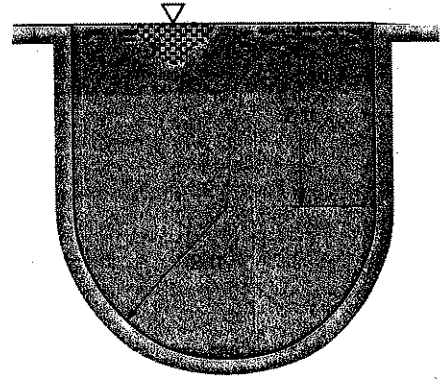
$$3 \frac{\text{m}^3}{\text{s}} = \frac{1}{0.012} (2\text{m}^2) (0.5\text{m})^{2/3} \sqrt{S_0}$$

or

$$S_0 = \underline{\underline{0.000816}}$$

10.79

10.79 Water flows in the channel shown in Fig. P10.79 at a rate of $90 \text{ ft}^3/\text{s}$. Determine the minimum slope that this channel can have so that the water does not overflow the sides. The Manning coefficient for this channel is $n = 0.014$.



■ FIGURE P10.79

$$(1) \quad Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_0}, \quad \text{where } K = 1.49, \quad n = 0.014, \quad \text{and } Q = 90 \frac{\text{ft}^3}{\text{s}}$$

Also,

$$A = (2 \text{ ft})(4 \text{ ft}) + \frac{1}{2} \pi (2 \text{ ft})^2 = 14.28 \text{ ft}^2$$

and

$$R_h = \frac{A}{P} = \frac{14.28 \text{ ft}^2}{(2 \text{ ft} + 2 \text{ ft} + \pi(2 \text{ ft}))} = 1.389 \text{ ft}$$

Thus, from Eq. (1)

$$90 \frac{\text{ft}^3}{\text{s}} = \frac{1.49}{0.014} (14.28 \text{ ft}^2) (1.389 \text{ ft})^{2/3} \sqrt{S_0}$$

or

$$S_0 = \underline{\underline{0.00226}}$$

10.80

10.80 To prevent weeds from growing in a clean earthen-lined canal, it is recommended that the velocity be no less than 2.5 ft/s. For the symmetrical canal shown in Fig. P10.80, determine the minimum slope needed.

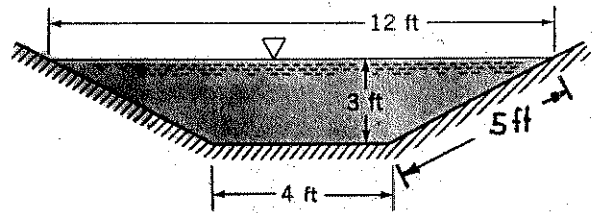


FIGURE P10.80

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K=1.49 \text{ and } R_h = \frac{A}{P} \quad (1)$$

$$A = \frac{1}{2}(4\text{ ft} + 12\text{ ft})(3\text{ ft}) = 24\text{ ft}^2 \text{ and } P = 4\text{ ft} + 2(5\text{ ft}) = 14\text{ ft}$$

$$\text{Thus, } R_h = \frac{24\text{ ft}^2}{14\text{ ft}} = 1.714\text{ ft}$$

From Table 10.1, $n = 0.022$ so that Eq.(1) gives (with $V = 2.5 \frac{\text{ft}}{\text{s}}$)

$$2.5 = \frac{1.49}{0.022} (1.714)^{2/3} S_o^{1/2} \text{ or } S_o = \underline{\underline{0.000664}}$$

10.81

10.81 The smooth, concrete-lined, symmetrical channel shown in Video V10.7 and Fig. P10.80 carries water from the silt-laden Colorado River. If the velocity must be 4.0 ft/s to prevent the silt from settling out (and eventually clogging the channel), determine the minimum slope needed.

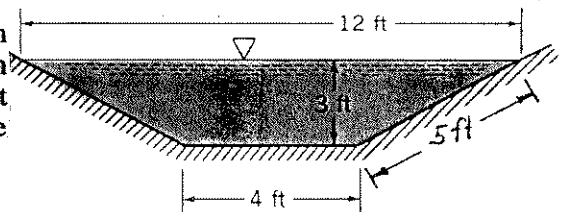


FIGURE P10.80

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K=1.49 \text{ and } R_h = \frac{A}{P} \quad (1)$$

$$A = \frac{1}{2}(4\text{ ft} + 12\text{ ft})(3\text{ ft}) = 24\text{ ft}^2 \text{ and } P = 4\text{ ft} + 2(5\text{ ft}) = 14\text{ ft}$$

$$\text{Thus, } R_h = \frac{24\text{ ft}^2}{14\text{ ft}} = 1.714\text{ ft}$$

From Table 10.1, $n = 0.012$ so that Eq.(1) gives (with $V = 4 \frac{\text{ft}}{\text{s}}$)

$$4.0 = \frac{1.49}{0.012} (1.714)^{2/3} S_o^{1/2} \text{ or } S_o = \underline{\underline{0.000505}}$$

10.82

10.82 The symmetrical channel shown in Fig. P10.80 is dug in sandy loam soil with $n = 0.020$. For such surface material it is recommended that to prevent scouring of the surface the average velocity be no more than 1.75 ft/s. Determine the maximum slope allowed.

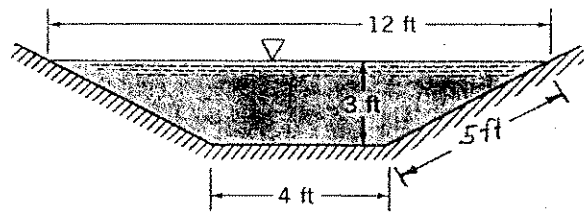


FIGURE P10.80

$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K=1.49 \text{ and } R_h = \frac{A}{P} \quad (1)$$

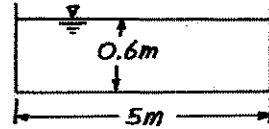
$$A = \frac{1}{2}(4\text{ ft} + 12\text{ ft})(3\text{ ft}) = 24\text{ ft}^2 \text{ and } P = 4\text{ ft} + 2(5\text{ ft}) = 14\text{ ft}$$

$$\text{Thus, } R_h = \frac{24\text{ ft}^2}{14\text{ ft}} = 1.714\text{ ft}$$

With $n = 0.020$ and $V = 1.75 \frac{\text{ft}}{\text{s}}$ Eq.(1) gives

$$1.75 = \frac{1.49}{0.020} (1.714)^{2/3} S_o^{1/2} \text{ or } S_o = \underline{\underline{0.000269}}$$

10.83 The depth downstream of a sluice gate in a rectangular wooden channel of width 5 m is 0.60 m. If the flowrate is 18 m³/s, determine the channel slope needed to maintain this depth. Will the depth increase or decrease in the flow direction if the slope is (a) 0.02; (b) 0.01?



$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K=1 \text{ and from Table 10.1, } n=0.012 \quad (1)$$

$$\text{Also } A = (5\text{ m})(0.6\text{ m}) = 3\text{ m}^2, \quad P = 5\text{ m} + 2(0.6\text{ m}) = 6.2\text{ m}$$

$$\text{so that } R_h = \frac{A}{P} = \frac{3\text{ m}^2}{6.2\text{ m}} = 0.484\text{ m}$$

Hence, from Eq. (1):

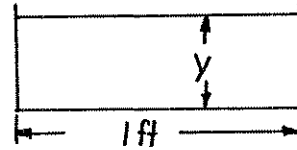
$$18 = \frac{1}{0.012} (3)(0.484)^{2/3} S_o^{1/2} \quad \text{or} \quad S_o = \underline{\underline{0.0136}}$$

With $S_o = 0.02 > 0.0136$ the velocity will increase and the water will become less than 0.6 m deep.

With $S_o = 0.01 < 0.0136$ the velocity will decrease and the water will become greater than 0.6 m deep.

10.84

10.84 Water in a rectangular painted steel channel of width $b = 1$ ft and depth y is to flow at critical conditions, $Fr = 1$. Plot a graph of the critical slope, S_{oc} , as a function of y for $0.05 \text{ ft} \leq y \leq 5 \text{ ft}$. What is the maximum slope allowed if critical flow is not to occur regardless of the depth?



$$V = \frac{K}{n} R_h^{2/3} S_o^{1/2}, \text{ where } K=1.49 \text{ and from Table 10.1 } n=0.014$$

$$\text{Also, } R_h = \frac{A}{P} = \frac{y}{2y+1} \text{ and with } Fr = \frac{V}{\sqrt{gy}} = 1, \quad V = \sqrt{gy}$$

Thus,

$$\sqrt{32.2 y} = \frac{1.49}{0.014} \left(\frac{y}{2y+1} \right)^{2/3} S_{oc}^{1/2} \text{ or } S_{oc} = 0.00284 \left[\frac{(2y+1)^4}{y} \right]^{1/3} \quad (1)$$

Equation (1) is plotted below. To determine the minimum critical slope set $\frac{dS_{oc}}{dy} = 0$. That is:

$$\frac{dS_{oc}}{dy} = \left(\frac{1}{3} \right) (0.00284) \left[\frac{(2y+1)^4}{y} \right]^{-2/3} \left[\frac{4(2y+1)^3(2)y - (2y+1)^4}{y^2} \right] = 0$$

Thus, $y = \frac{1}{6}$ so that from Eq.(1)

$$S_{oc_{min}} = 0.00284 \left[\frac{\left(\frac{2}{6} + 1 \right)^4}{\frac{1}{6}} \right]^{1/3} = \underline{\underline{0.00757}}$$

If $S_o < 0.00757$ critical flow cannot occur at any depth.

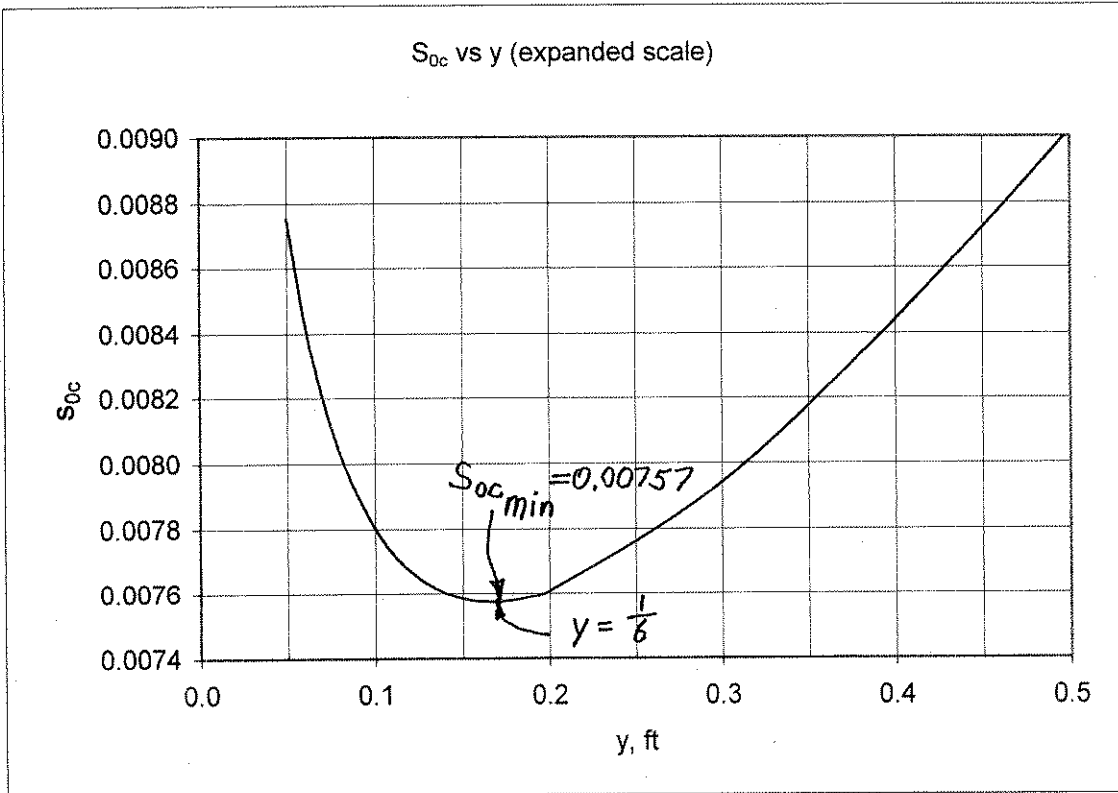
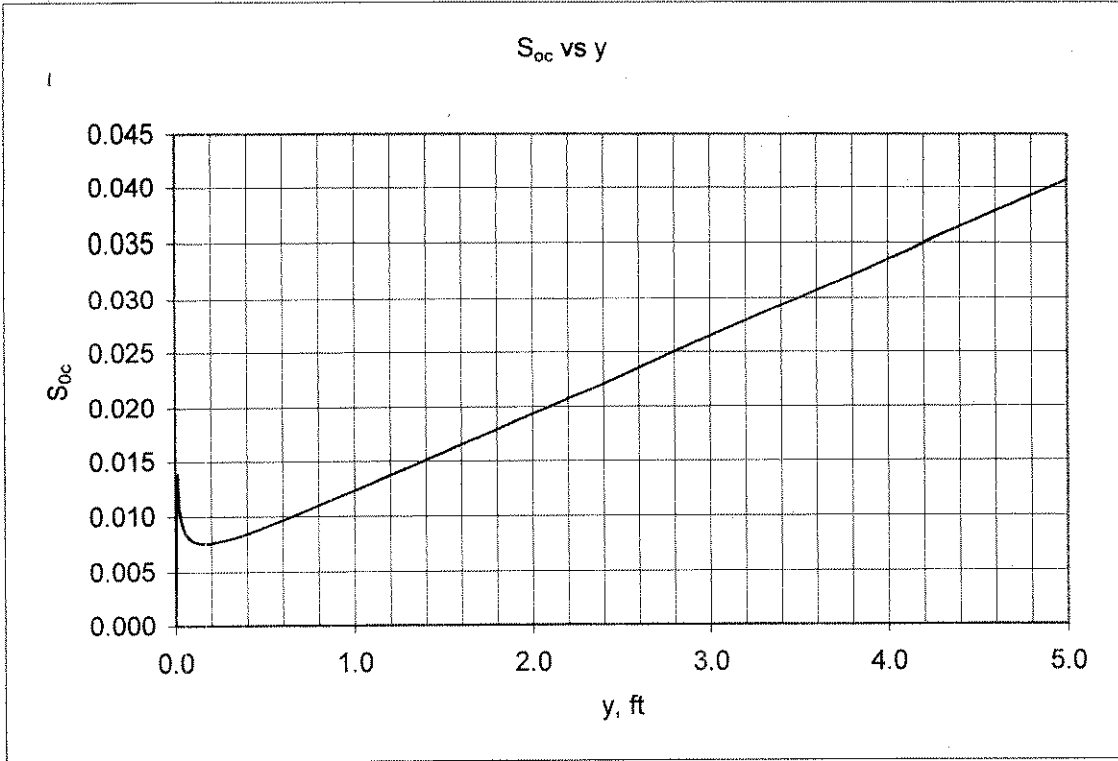
The following values are obtained from Eq.(1). Note that

$$\lim_{y \rightarrow 0} S_{oc} = 0.00284 \lim_{y \rightarrow 0} \left[\frac{(2y+1)^4}{y} \right]^{1/3} = \infty \text{ and } \lim_{y \rightarrow \infty} S_{oc} = \infty$$

See next page for graphs.

(cont)

10.84 (con't)



10.85

10.85 A 50-ft-long aluminum gutter (Manning coefficient $n = 0.011$) on a section of a roof is to handle a flowrate of $0.15 \text{ ft}^3/\text{s}$ during a heavy rain storm. The cross section of the gutter is shown in Fig. P10.85. Determine the vertical distance that this gutter must be pitched (i.e., the difference in elevation between the two ends of the gutter) so that the water does not overflow the gutter. Assume uniform depth channel flow.

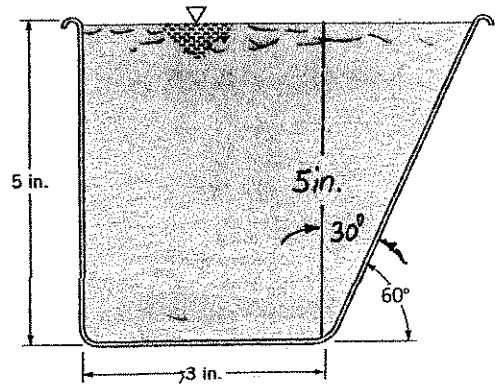


FIGURE P10.85

$$(1) \quad Q = \frac{K}{n} A R_h^{2/3} \sqrt{S_0}, \quad \text{where } K = 1.49, \quad n = 0.011, \quad \text{and}$$

$$A = (5 \text{ in.})(3 \text{ in.}) + \frac{1}{2} (5 \text{ in.})(5 \text{ in.} \tan 30^\circ) = 22.2 \text{ in.}^2 \left(\frac{1 \text{ ft}^2}{144 \text{ in.}^2} \right) = 0.154 \text{ ft}^2$$

Also,

$$R_h = \frac{A}{P}, \quad \text{where } P = (5 \text{ in.}) + (3 \text{ in.}) + \left(\frac{5 \text{ in.}}{\cos 30^\circ} \right) = 13.77 \text{ in.} \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right) = 1.148 \text{ ft}$$

Hence,

$$R_h = \frac{0.154 \text{ ft}^2}{1.148 \text{ ft}} = 0.134 \text{ ft}$$

Thus, from Eq. (1)

$$0.15 \frac{\text{ft}^3}{\text{s}} = \frac{1.49}{0.011} (0.154 \text{ ft}^2) (0.134 \text{ ft})^{2/3} \sqrt{S_0}$$

so that

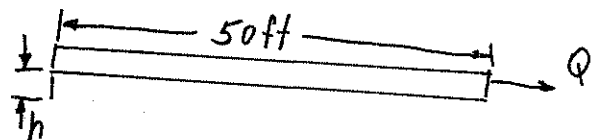
$$S_0 = 7.54 \times 10^{-4}$$

But

$$S_0 = \frac{h}{50 \text{ ft}}$$

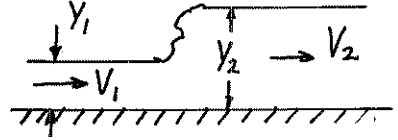
so that

$$h = (50 \text{ ft}) S_0 = (50 \text{ ft}) (7.54 \times 10^{-4}) = 0.0377 \text{ ft} = \underline{\underline{0.452 \text{ in.}}}$$



10.87

10.87 Water flows upstream of a hydraulic jump with a depth of 0.5 m and a velocity of 6 m/s. Determine the depth of the water downstream of the jump.



$$\frac{y_2}{y_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8Fr_1^2} \right], \text{ where}$$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{6 \text{ m/s}}{\sqrt{(9.81 \text{ m/s}^2)(0.5 \text{ m})}} = 2.71$$

Thus,

$$\frac{y_2}{y_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8(2.71)^2} \right] = 3.36$$

so that

$$y_2 = 3.36(0.5 \text{ m}) = \underline{\underline{1.68 \text{ m}}}$$

10.88

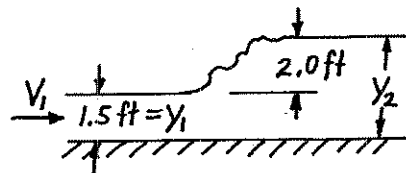
10.88 A 2.0-ft standing wave is produced at the bottom of the rectangular channel in an amusement park water ride. If the water depth upstream of the wave is estimated to be 1.5 ft, determine how fast the boat is traveling when it passes through this standing wave (hydraulic jump) for its final "splash."

$$\frac{y_2}{y_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8Fr_1^2} \right]$$

$$\text{or } \left(\frac{2.0 \text{ ft} + 1.5 \text{ ft}}{1.5 \text{ ft}} \right) = \frac{1}{2} \left[-1 + \sqrt{1 + 8Fr_1^2} \right]$$

$$\text{Thus, } Fr_1 = 1.97, \text{ or since } Fr_1 = \frac{V_1}{\sqrt{g y_1}}$$

$$V_1 = Fr_1 \sqrt{g y_1} = 1.97 \sqrt{(32.2 \frac{\text{ft}}{\text{s}^2})(1.5 \text{ ft})} = \underline{\underline{13.7 \frac{\text{ft}}{\text{s}}}}$$



10.89

10.89 The water depths upstream and downstream of a hydraulic jump are 0.3 and 1.2 m, respectively. Determine the upstream velocity and the power dissipated if the channel is 50 m wide.

$$\frac{y_2}{y_1} = \frac{1.2 \text{ m}}{0.3 \text{ m}} = \frac{1}{2} \left[-1 + \sqrt{1 + 8Fr_1^2} \right] \text{ or } Fr_1 = 3.16 \text{ Thus, since } Fr_1 = \frac{V_1}{(g y_1)^{1/2}}$$

$$\text{it follows that } V_1 = (3.16) \left[(9.81 \frac{\text{m}}{\text{s}^2})(0.3 \text{ m}) \right]^{1/2} = \underline{\underline{5.42 \frac{\text{m}}{\text{s}}}}$$

The power dissipated is given by

$$\mathcal{P} = \gamma Q h_L, \text{ where } \frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{Fr_1^2}{2} \left(1 - \left(\frac{y_1}{y_2} \right)^2 \right)$$

$$\text{or } h_L = (0.3 \text{ m}) \left[1 - \frac{1.2 \text{ m}}{0.3 \text{ m}} + \frac{(3.16)^2}{2} \left(1 - \left(\frac{0.3 \text{ m}}{1.2 \text{ m}} \right)^2 \right) \right] = 0.504 \text{ m}$$

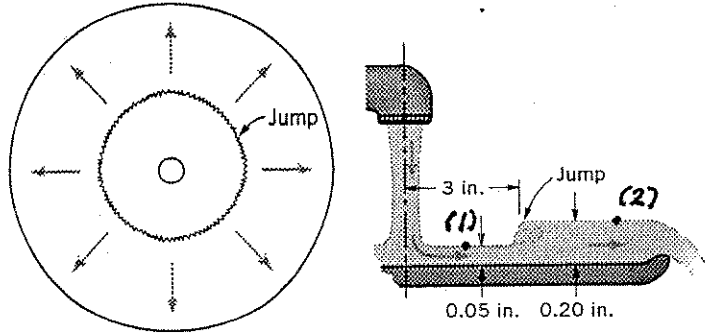
$$\text{Also, } Q = A_1 V_1 = y_1 b V_1 = (0.3 \text{ m})(50 \text{ m})(5.42 \frac{\text{m}}{\text{s}}) = 81.3 \frac{\text{m}^3}{\text{s}}$$

Thus,

$$\mathcal{P} = (9.8 \frac{\text{kN}}{\text{m}^3})(81.3 \frac{\text{m}^3}{\text{s}})(0.504 \text{ m}) = 401 \frac{\text{kN} \cdot \text{m}}{\text{s}} = \underline{\underline{401 \text{ kW}}}$$

10.90

10.90 Under appropriate conditions, water flowing from a faucet, onto a flat plate, and over the edge of the plate can produce a circular hydraulic jump as shown in Fig. P10.90 and Video V10.J2. Consider a situation where a jump forms 3.0 in. from the center of the plate with depths upstream and downstream of the jump of 0.05 in. and 0.20 in., respectively. Determine the flowrate from the faucet.



■ FIGURE P10.90

For a hydraulic jump:

$$\frac{y_2}{y_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8Fr_1^2} \right] \quad \text{or}$$

$$\frac{0.20 \text{ in.}}{0.05 \text{ in.}} = \frac{1}{2} \left[-1 + \sqrt{1 + 8Fr_1^2} \right] \quad \text{so that} \quad Fr_1 = 3.16 = \frac{V_1}{\sqrt{gy_1}}$$

Thus,

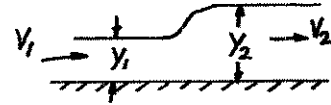
$$V_1 = 3.16 \sqrt{32.2 \frac{\text{ft}}{\text{s}^2} (0.05/12) \text{ ft}} = 1.16 \frac{\text{ft}}{\text{s}}$$

and

$$Q = A_1 V_1 = 2\pi R_1 y_1 V_1 = 2\pi \left(\frac{3}{12} \text{ ft} \right) \left(\frac{0.05}{12} \text{ ft} \right) \left(1.16 \frac{\text{ft}}{\text{s}} \right) = \underline{\underline{0.00759 \frac{\text{ft}^3}{\text{s}}}}$$

10.91

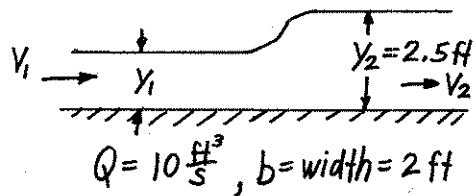
10.91 Show that the Froude number downstream of a hydraulic jump in a rectangular channel is $(y_1/y_2)^{3/2}$ times the Froude number upstream of the jump, where (1) and (2) denote the upstream and downstream conditions, respectively.



$$Fr_2 = \frac{V_2}{(g y_2)^{1/2}}, \text{ where } V_1 A_1 = V_2 A_2, \text{ or } V_2 = \frac{A_1}{A_2} V_1 = \frac{y_1}{y_2} V_1$$

$$\text{Thus, } \frac{(y_1}{y_2}) V_1}{(g y_2)^{1/2}} = \left(\frac{y_1}{y_2}\right)^{3/2} \frac{V_1}{(g y_1)^{1/2}} \quad \text{Hence, } \underline{\underline{Fr_2 = \left(\frac{y_1}{y_2}\right)^{3/2} Fr_1}}$$

10.92 Water flows in a 2-ft-wide rectangular channel at a rate of $10 \text{ ft}^3/\text{s}$. If the water depth downstream of a hydraulic jump is 2.5 ft, determine (a) the water depth upstream of the jump, (b) the upstream and downstream Froude numbers, and (c) the head loss across the jump.



(a) Use $\frac{y_2}{y_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8 Fr_1^2} \right]$ with $y_2 = 2.5 \text{ ft}$ so that
 $5 + y_1 = y_1 \sqrt{1 + 8 Fr_1^2}$ Now, with $Fr_1^2 = \frac{V_1^2}{g y_1} = \frac{(Q/(b y_1))^2}{g y_1} = \frac{(10/(2 y_1))^2}{32.2 y_1}$,
 or $Fr_1^2 = \frac{0.776}{y_1^3}$, we obtain

$5 + y_1 = y_1 \left[1 + 8 \left(\frac{0.776}{y_1^3} \right) \right]^{\frac{1}{2}}$ By squaring both sides and simplifying we
 obtain $y_1^2 + 2.5 y_1 - 0.621 = 0$ which gives $y_1 = \underline{\underline{0.228 \text{ ft}}}$.

(b) From the above results

$$Fr_1^2 = \frac{0.776}{(0.228)^3} \quad \text{or} \quad Fr_1 = \underline{\underline{8.09}}$$

Also,

$$V_2 = \frac{Q}{A_2} = \frac{10 \frac{\text{ft}^3}{\text{s}}}{(2.5 \text{ ft})(2 \text{ ft})} = 2.0 \frac{\text{ft}}{\text{s}} \quad \text{so that} \quad Fr_2 = \frac{V_2}{(g y_2)^{\frac{1}{2}}} = \frac{2 \frac{\text{ft}}{\text{s}}}{\left[(32.2 \frac{\text{ft}}{\text{s}^2})(2.5 \text{ ft}) \right]^{\frac{1}{2}}}$$

$$\text{or} \quad Fr_2 = \underline{\underline{0.223}}$$

(c) Also,

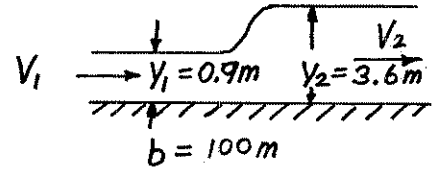
$$h_L = y_1 \left[1 - \frac{y_2}{y_1} + \frac{Fr_1^2}{2} \left(1 - \left(\frac{y_1}{y_2} \right)^2 \right) \right] = 0.228 \text{ ft} \left[1 - \frac{2.5}{0.228} + \frac{(8.10)^2}{2} \left(1 - \left(\frac{0.228}{2.5} \right)^2 \right) \right]$$

or

$$h_L = \underline{\underline{5.15 \text{ ft}}}$$

10.93

10.93 A hydraulic jump at the base of a spillway of a dam is such that the depths upstream and downstream of the jump are 0.90 and 3.6 m, respectively (see Video V10.11). If the spillway is 10 m wide, what is the flowrate over the spillway?



$$\frac{y_2}{y_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8Fr_1^2} \right], \text{ or } \frac{3.6 \text{ m}}{0.9 \text{ m}} = \frac{1}{2} \left[-1 + \sqrt{1 + 8Fr_1^2} \right]$$

Hence, $Fr_1 = 3.16$, but $Fr_1 = \frac{V_1}{(gy_1)^{1/2}}$ so that

$$V_1 = 3.16 \left[(9.81 \frac{\text{m}}{\text{s}^2})(0.9 \text{ m}) \right]^{1/2} = 9.39 \frac{\text{m}}{\text{s}}$$

Thus,

$$Q = A_1 V_1 = by_1 V_1 = (10.0 \text{ m})(0.9 \text{ m})(9.39 \frac{\text{m}}{\text{s}}) = \underline{\underline{84.5 \frac{\text{m}^3}{\text{s}}}}$$

10.94

10.94 Determine the head loss and power dissipated by the hydraulic jump of Problem 10.93.

$$h_L = y_1 \left[1 - \frac{y_2}{y_1} + \frac{Fr_1^2}{2} \left(1 - \left(\frac{y_1}{y_2} \right)^2 \right) \right], \text{ where from } \frac{y_2}{y_1} = \frac{3.6 \text{ m}}{0.9 \text{ m}} = \frac{1}{2} \left[-1 + \sqrt{1 + 8Fr_1^2} \right]$$

Hence, $Fr_1 = 3.16$ so that

$$h_L = (0.9 \text{ m}) \left[1 - \frac{3.6 \text{ m}}{0.9 \text{ m}} + \frac{(3.16)^2}{2} \left(1 - \left(\frac{0.9 \text{ m}}{3.6 \text{ m}} \right)^2 \right) \right] = \underline{\underline{1.51 \text{ m}}}$$

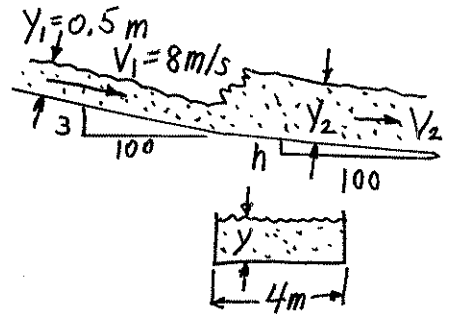
Also, $\mathcal{P} = \gamma Q h_L$, where $V_1 = (gy_1)^{1/2} Fr_1 = \left[(9.81 \frac{\text{m}}{\text{s}^2})(0.9 \text{ m}) \right]^{1/2} (3.16) = 9.39 \frac{\text{m}}{\text{s}}$

Hence,

$$\mathcal{P} = (9.80 \frac{\text{kN}}{\text{m}^3}) \left[(0.9 \text{ m})(100 \text{ m})(9.39 \frac{\text{m}}{\text{s}}) \right] (1.51 \text{ m}) = 12,500 \frac{\text{kN} \cdot \text{m}}{\text{s}} = \underline{\underline{12,500 \text{ kW}}}$$

10.95

10.95 A hydraulic jump occurs in a 4-m-wide rectangular channel at a point where the slope changes from 3 m per 100 m upstream of the jump to h m per 100 m downstream of the jump. The depth and velocity of the uniform flow upstream of the jump are 0.5 m and 8 m/s, respectively. Determine the value of h if the flow downstream of the jump is to be uniform flow.



Upstream of the jump

$$V_1 = \frac{K}{n} R_{h_1}^{2/3} \sqrt{S_{0_1}}, \text{ where } S_{0_1} = \frac{3 \text{ m}}{100 \text{ m}} = 0.03$$

$$\text{and } R_{h_1} = \frac{A_1}{P_1} = \frac{(4 \text{ m})(0.5 \text{ m})}{(4 \text{ m} + 0.5 \text{ m} + 0.5 \text{ m})} = 0.4 \text{ m}$$

Thus,

$$Q_1 = A_1 V_1 = (4 \text{ m})(0.5 \text{ m}) \frac{K}{n} (0.4 \text{ m})^{2/3} \sqrt{0.03}$$

or

$$(1) \quad Q_1 = 0.188 \frac{K}{n} \frac{\text{m}^3}{\text{s}}$$

Also,

$$\frac{y_2}{y_1} = \frac{1}{2} \left[-1 + (1 + 8 Fr_1^2)^{1/2} \right], \text{ where } Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{8 \frac{\text{m}}{\text{s}}}{\sqrt{(9.81 \frac{\text{m}}{\text{s}^2})(0.5 \text{ m})}} = 3.61$$

Thus,

$$\frac{y_2}{y_1} = \frac{1}{2} \left[-1 + (1 + 8(3.61)^2)^{1/2} \right] = 4.63$$

so that

$$y_2 = 4.63 y_1 = 4.63 (0.5 \text{ m}) = 2.31 \text{ m}$$

Therefore,

$$A_2 = (4 \text{ m})(2.31 \text{ m}) = 9.24 \text{ m}^2 \text{ and } R_{h_2} = \frac{A_2}{P_2} = \frac{9.24 \text{ m}^2}{(4 \text{ m} + 2.31 \text{ m} + 2.31 \text{ m})} = 1.07 \text{ m}$$

so that

$$Q_2 = \frac{K}{n} A_2 R_{h_2}^{2/3} \sqrt{S_{0_2}} = \frac{K}{n} (9.24 \text{ m}^2) (1.07 \text{ m})^{2/3} \sqrt{S_{0_2}}$$

or

$$(2) \quad Q_2 = 9.67 \frac{K}{n} \sqrt{S_{0_2}} \frac{\text{m}^3}{\text{s}}$$

But, $Q_1 = Q_2$, so that from Eqs. (1) and (2)

$$0.188 \frac{K}{n} = 9.67 \frac{K}{n} \sqrt{S_{0_2}}$$

or

$$S_{0_2} = 0.000378 = \frac{h}{100 \text{ m}} \quad \text{Hence, } h = \underline{\underline{0.0378 \text{ m}}}$$

10.96

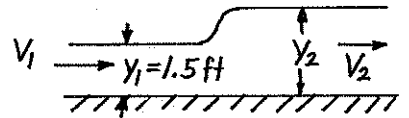
10.96 At a given location in a 12-ft-wide rectangular channel the flowrate is 900 ft³/s and the depth is 4 ft. Is this location upstream or downstream of the hydraulic jump that occurs in this channel? Explain.

$$V = \frac{Q}{A} = \frac{900 \frac{\text{ft}^3}{\text{s}}}{(12 \text{ ft})(4 \text{ ft})} = 18.75 \frac{\text{ft}}{\text{s}} \quad \text{so that} \quad Fr = \frac{V}{(gy)^{1/2}} = \frac{18.75 \frac{\text{ft}}{\text{s}}}{\left[(32.2 \frac{\text{ft}}{\text{s}^2})(4 \text{ ft}) \right]^{1/2}} = 1.65$$

Since $Fr > 1$, the location is upstream of the jump.

*10.97

*10.97 A rectangular channel of width b is to carry water at flowrates from $30 \leq Q \leq 600$ cfs. The water depth upstream of the hydraulic jump that occurs (if one does occur) is to remain 1.5 ft for all cases. Plot the power dissipated in the jump as a function of flowrate for channels of width $b = 10, 20, 30,$ and 40 ft.



$$\mathcal{P} = \gamma Q h_L, \text{ where } h_L = y_1 \left[1 - \left(\frac{y_2}{y_1} \right) + \frac{Fr_1^2}{2} \left(1 - \left(\frac{y_1}{y_2} \right)^2 \right) \right] \quad (1)$$

$$\text{and } \frac{y_2}{y_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8 Fr_1^2} \right], \text{ provided } Fr_1 \geq 0 \quad (2)$$

Also, $Fr_1 = \frac{V_1}{(g y_1)^{1/2}}$, where $V_1 = \frac{Q}{A_1} = \frac{Q}{1.5 b}$ so that

$$Fr_1 = \frac{\left(\frac{Q}{1.5 b} \right)}{\left[(32.2 \frac{ft}{s^2}) (1.5 ft) \right]^{1/2}} = 0.0959 \frac{Q}{b} \text{ Hence, from Eq. (1)}$$

$$h_L = (1.5) \left[1 - \left(\frac{y_2}{y_1} \right) + (0.00460) \left(\frac{Q}{b} \right)^2 \left(1 - \left(\frac{y_1}{y_2} \right)^2 \right) \right] \text{ ft, where } b \sim \text{ft}, Q \sim \frac{ft^3}{s} \quad (3)$$

and from Eq. (2)

$$\frac{y_2}{y_1} = \frac{1}{2} \left[-1 + \left(1 + 0.0736 \left(\frac{Q}{b} \right)^2 \right)^{1/2} \right] \quad (4)$$

For the given values of plot \mathcal{P} from

$$\mathcal{P} = 62.4 Q h_L \frac{ft \cdot lb}{s} \text{ for } 30 \leq Q \leq 600 \frac{ft^3}{s} \quad (5)$$

Note: If $Fr_1 < 1$ there is no jump and $\mathcal{P} = 0$. From above, $Fr_1 = 1$

$$\text{when } Q = \frac{b}{0.0959} = 10.4 b \quad (6)$$

Let $Q_1 =$ flowrate when $Fr_1 = 1$. From Eq. (6) we obtain

$b, \text{ ft}$	$Q_1, \frac{ft^3}{s}$
10	104
20	208
30	312
40	416

With $b = 10, 20, 30,$ or 40 ft calculate and plot \mathcal{P} from:

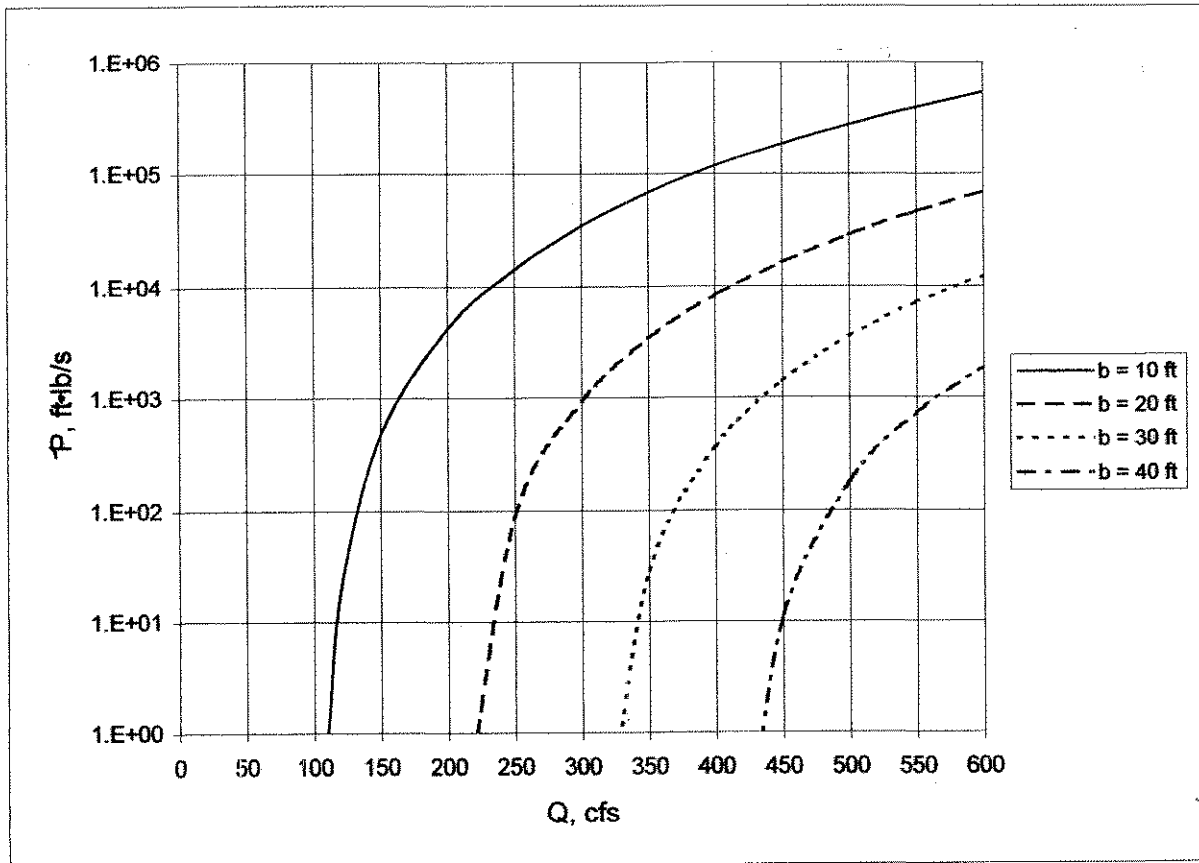
a) $\mathcal{P} = 0$ if $Q < Q_1$

b) $\mathcal{P} = 62.4 Q h_L \frac{ft \cdot lb}{s}$, where obtain h_L from Eq. (3) with $\frac{y_2}{y_1}$ from Eq. (4) if $Q_1 \leq Q \leq 600 \frac{ft^3}{s}$

(cont)

10.97* (con't)

The results of the above calculations are plotted below.



10.98

10.98 Water flows in a rectangular channel at a depth of $y = 1$ ft and a velocity of $V = 20$ ft/s. When a gate is suddenly placed across the end of the channel, a wave (a moving hydraulic jump) travels upstream with velocity V_w as is indicated in Fig. P10.98. Determine V_w . Note that this is an unsteady problem for a stationary observer. However, for an observer moving to the left with velocity V_w , the flow appears as a steady hydraulic jump.

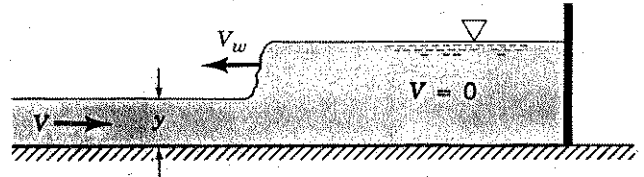


FIGURE P10.98

For an observer moving to the left with speed V_w the flow appears as shown below.

Thus, treat the flow as a jump with

$$Fr_1 = \frac{V_1}{(gy_1)^{1/2}} = \frac{(20 + V_w)}{[(32.2 \frac{ft}{s^2})(1 ft)]^{1/2}}$$

or

$$Fr_1 = 0.176(20 + V_w)$$

Also, $A_1 V_1 = A_2 V_2$, or $\frac{y_2}{y_1} = \frac{V_1}{V_2} = \frac{20 + V_w}{V_w}$

and

$$\frac{y_2}{y_1} = \frac{1}{2}[-1 + \sqrt{1 + 8Fr_1^2}] \text{ which when combined with Eqs. (1) and (2) becomes}$$

$$\frac{20 + V_w}{V_w} = \frac{1}{2}[-1 + \sqrt{1 + 8(0.176)^2(20 + V_w)^2}]$$

or

$$2(20 + V_w) + V_w = V_w(1 + (0.248)(20 + V_w)^2)^{1/2}$$

or

$$(40 + 3V_w)^2 = V_w^2[1 + (0.248)(20 + V_w)^2], \text{ which can be written as}$$

$$0.248 V_w^4 + 9.92 V_w^3 + 91.2 V_w^2 - 240 V_w - 1600 = 0 \quad (3)$$

By using a standard root-finding program, the solution to Eq. (3) is determined to be $V_w = \underline{\underline{4.36 \text{ ft/s}}}$.

10.99

10.99 Water flows in a rectangular channel with velocity $V = 6$ m/s. A gate at the end of the channel is suddenly closed so that a wave (a moving hydraulic jump) travels upstream with velocity $V_w = 2$ m/s as is indicated in Fig. P10.98. Determine the depths ahead of and behind the wave. Note that this is an unsteady problem for a stationary observer. However, for an observer moving to the left with velocity V_w , the flow appears as a steady hydraulic jump.

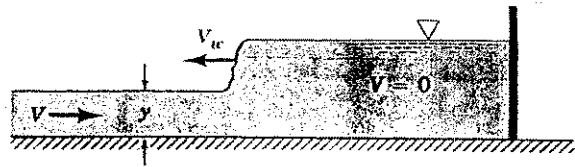


FIGURE P10.98

For an observer moving to the left with speed $V_w = 2 \frac{m}{s}$ the flow appears as shown below.

Thus, treat as a jump with

$$V_1 = 8 \frac{m}{s}, \quad V_2 = 2 \frac{m}{s}$$

Since $A_1 V_1 = A_2 V_2$ or $\frac{y_2}{y_1} = \frac{V_1}{V_2} = \frac{8 \frac{m}{s}}{2 \frac{m}{s}} = 4$ it follows that

$$\frac{y_2}{y_1} = \frac{1}{2} \left[-1 + \sqrt{1 + 8 Fr_1^2} \right] = 4 \quad \text{Hence, } Fr_1 = 3.16$$

However, $Fr_1 = \frac{V_1}{(g y_1)^{1/2}}$ so that

$$y_1 = \frac{V_1^2}{g Fr_1^2} = \frac{(8 \frac{m}{s})^2}{(9.81 \frac{m}{s^2})(3.16)^2} = \underline{\underline{0.652 \text{ m}}}$$

and

$$y_2 = 4 y_1 = 4(0.652 \text{ m}) = \underline{\underline{2.61 \text{ m}}}$$

10.100

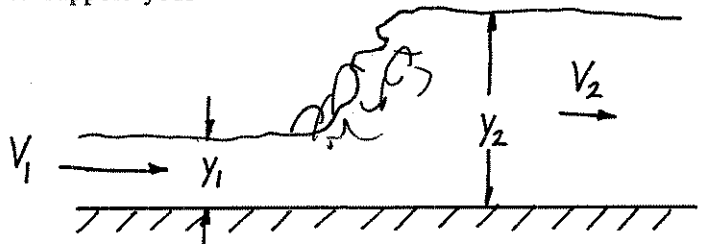
10.100 (See Fluids in the News article titled "Grand Canyon rapids building," Section 10.6.1.) During the flood of 1983, a large hydraulic jump formed at "Crystal Hole" rapid on the Colorado River. People rafting the river at that time report "entering the rapid at almost 30 mph, hitting a 20-ft-tall wall of water, and exiting at about 10 mph." Is this information (i.e., upstream and downstream velocities and change in depth) consistent with the principles of a hydraulic jump? Show calculations to support your answer.

Is the given data consistent with a hydraulic jump?

$$V_1 = 30 \text{ mph} = 44 \text{ ft/s}$$

$$V_2 = 10 \text{ mph} = 14.7 \text{ ft/s}$$

$$y_2 - y_1 = 20 \text{ ft}$$



From conservation of mass: $A_1 V_1 = A_2 V_2$
or $y_1 V_1 = y_2 V_2$ since $b_1 = \text{width} = b_2$

Thus,

$$\frac{y_2}{y_1} = \frac{V_1}{V_2} = \frac{44 \text{ ft/s}}{14.7 \text{ ft/s}} = 2.99 \quad (1)$$

Also, for a hydraulic jump

$$\frac{y_2}{y_1} = \frac{1}{2}(-1 + \sqrt{1 + 8 Fr_1^2}) \text{ so that } 2.99 = \frac{1}{2}(-1 + \sqrt{1 + 8 Fr_1^2})$$

or

$$Fr_1 = 2.44$$

Thus, since $Fr_1 = \frac{V_1}{\sqrt{g y_1}}$ it follows that

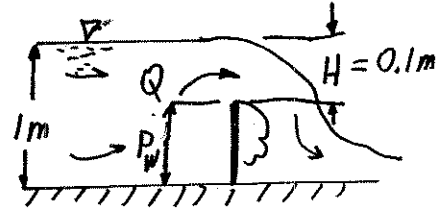
$$2.44 = \frac{44 \text{ ft/s}}{(32.2 \text{ ft/s}^2 y_1)^{1/2}} \text{ or } y_1 = 10.1 \text{ ft. so that from Eq. (1),}$$

$$y_2 = 2.99 y_1 = 2.99 (10.1 \text{ ft}) = 30.2 \text{ ft}$$

Hence, the given data gives $y_2 - y_1 = 30.2 \text{ ft} - 10.1 \text{ ft} = 20.1 \text{ ft}$, which is surprisingly close to the reported depth. Yes, the data is consistent with the principles of a hydraulic jump.

10.102

10.102 Water flows over a 2-m-wide rectangular sharp-crested weir. Determine the flowrate if the weir head is 0.1 m and the channel depth is 1 m.



$$(1) \quad Q = C_{wr} \frac{2}{3} \sqrt{2g} b H^{3/2}, \text{ where } b = 2 \text{ m}, H = 0.1 \text{ m}$$

Also,

$$C_{wr} = 0.611 + 0.075 \left(\frac{H}{P_w} \right), \text{ where } P_w = 1 \text{ m} - H = 1 \text{ m} - 0.1 \text{ m} = 0.9 \text{ m}$$

Thus,

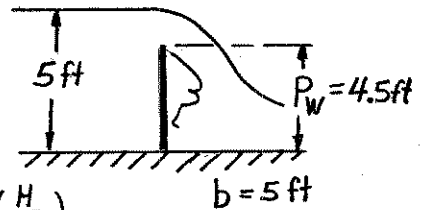
$$C_{wr} = 0.611 + 0.075 \left(\frac{0.1 \text{ m}}{0.9 \text{ m}} \right) = 0.619$$

so that from Eq. (1),

$$Q = 0.619 \left(\frac{2}{3} \right) \sqrt{2 \left(9.81 \frac{\text{m}}{\text{s}^2} \right)} (2 \text{ m}) (0.1 \text{ m})^{3/2} = \underline{\underline{0.116 \frac{\text{m}^3}{\text{s}}}}$$

10.103

10.103 Water flows over a 5-ft-wide, rectangular sharp-crested weir that is $P_w = 4.5$ ft tall. If the depth upstream is 5 ft, determine the flowrate.



$$Q = C_{wr} \frac{2}{3} \sqrt{2g} b H^{3/2}, \text{ where } C_{wr} = 0.611 + 0.075 \left(\frac{H}{P_w} \right)$$

with

$$H = 5 \text{ ft} - 4.5 \text{ ft} = 0.5 \text{ ft}$$

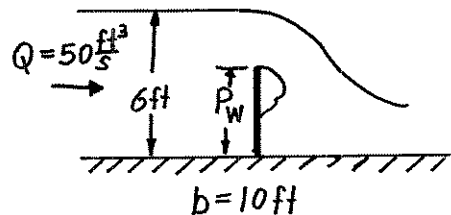
$$\text{Hence, } C_{wr} = 0.611 + 0.075 \left(\frac{0.5 \text{ ft}}{4.5 \text{ ft}} \right) = 0.619$$

and

$$Q = (0.619) \left(\frac{2}{3} \right) (2 (32.2 \frac{\text{ft}}{\text{s}^2}))^{1/2} (5 \text{ ft}) (0.5)^{3/2} = \underline{\underline{5.85 \frac{\text{ft}^3}{\text{s}}}}$$

10.104

10.104 A rectangular sharp crested weir is used to measure the flowrate in a channel of width 10 ft. It is desired to have the channel flow depth be 6 ft when the flowrate is 50 cfs. Determine the height, P_w , of the weir plate.



$$Q = C_{wr} \frac{2}{3} \sqrt{2g} b H^{3/2}, \text{ where } H = 6 \text{ ft} - P_w \text{ and}$$

$$C_{wr} = 0.611 + 0.075 \frac{H}{P_w}$$

Thus,

$$Q = \left(0.611 + 0.075 \left(\frac{6 - P_w}{P_w} \right) \right) \left(\frac{2}{3} \right) (2g)^{1/2} b (6 - P_w)$$

or

$$50 \frac{\text{ft}^3}{\text{s}} = \left(0.611 + 0.075 \left(\frac{6 - P_w}{P_w} \right) \right) \left(\frac{2}{3} \right) (64.4 \frac{\text{ft}}{\text{s}^2})^{1/2} (10 \text{ ft}) (6 - P_w), \text{ where } P_w \sim \text{ft}$$

Hence,

$$\left[8.15 + \frac{(6 - P_w)}{P_w} \right] (6 - P_w)^{3/2} - 12.5 = 0 \quad (1)$$

By using a standard root-finding program, the solution to Eq. (1) is found to be $P_w = \underline{\underline{4.70 \text{ ft}}}$.

10.105

10.105 Water flows from a storage tank, over two triangular weirs, and into two irrigation channels as shown in Video V10.13 and Fig. P10.105. The head for each weir is 0.4 ft, and the flowrate in the channel fed by the 90°-V-notch weir is to be twice the flowrate in the other channel. Determine the angle θ for the second weir.

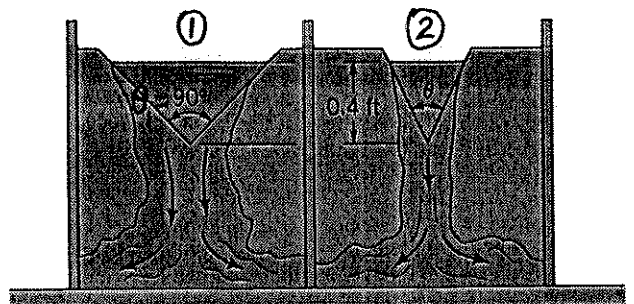


FIGURE P10.105

$$Q = C_{wt} \frac{8}{15} \tan\left(\frac{\theta}{2}\right) \sqrt{2g} H^{5/2} \quad (1)$$

where

$$\theta_1 = 90^\circ, H_1 = H_2 = 0.4 \text{ ft}, \text{ and } Q_1 = 2Q_2 \quad (2)$$

Thus, from Fig. 10.21,

$$C_{wt1} = 0.590$$

From Eqs. (1) and (2),

$$C_{wt1} \frac{8}{15} \tan\left(\frac{\theta_1}{2}\right) \sqrt{2g} H_1^{5/2} = C_{wt2} \frac{8}{15} \tan\left(\frac{\theta_2}{2}\right) \sqrt{2g} H_2^{5/2} \times 2$$

or

$$0.590 \tan 45^\circ = C_{wt2} \tan\left(\frac{\theta_2}{2}\right) \times 2$$

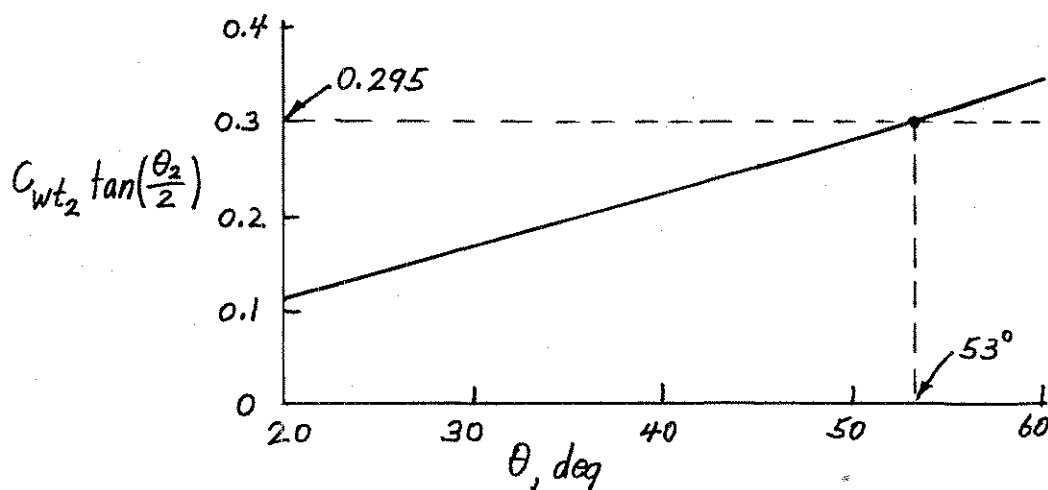
or

$$C_{wt2} \tan\left(\frac{\theta_2}{2}\right) = 0.295 \quad (3)$$

Trial and error solution: Assume $\theta_2 = 20^\circ$. From Fig. 10.21, $C_{wt2} = 0.626$

Thus, $C_{wt2} \tan\left(\frac{\theta_2}{2}\right) = 0.626 \tan(10^\circ) = 0.110 \neq 0.295$. Thus, $\theta_2 \neq 20^\circ$

Repeated tries result in the graph below from which we conclude that $\theta_2 = \underline{\underline{53^\circ}}$



10.106

10.106 Rain water from a parking lot flows into a 2-acre ($8.71 \times 10^4 \text{ ft}^2$) retention pond. After a heavy rain when there is no more inflow into the pond, the rectangular weir shown in Fig. P10.106 at the outlet of the pond has a head of $H = 0.6 \text{ ft}$. (a) Determine the rate at which the level of the water in the pond decreases, dH/dt , at this condition. (b) Determine how long it will take to reduce the pond level by half a foot; that is, to $H = 0.1 \text{ ft}$.

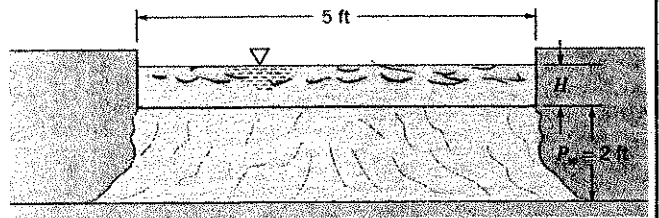


FIGURE P10.106

For a rectangular weir,

$$Q = C_{wr} \frac{2}{3} \sqrt{2g} b H^{3/2}, \text{ where } C_{wr} = 0.611 + 0.075 \frac{H}{P_w}$$

Thus, with $b = 5 \text{ ft}$, and $P_w = 2 \text{ ft}$

$$Q = (0.611 + 0.075 \frac{H}{2}) \frac{2}{3} \sqrt{2(32.2 \text{ ft/s}^2)} (5 \text{ ft}) H^{3/2}$$

$$(1) \quad Q = 26.7 (0.611 + 0.0375 H) H^{3/2} \frac{\text{ft}^3}{\text{s}}, \text{ where } H \sim \text{ft}$$

$$(2) \quad \text{Also, } Q = -A_{\text{pond}} \frac{dH}{dt} = -8.71 \times 10^4 \text{ ft}^2 \frac{dH}{dt}$$

Thus, from Eqs. (1) and (2),

$$-8.71 \times 10^4 \frac{dH}{dt} = 26.7 (0.611 + 0.0375 H) H^{3/2}$$

$$(3) \quad \frac{dH}{dt} = -3.07 \times 10^{-4} (0.611 + 0.0375 H) H^{3/2} = -1.87 \times 10^{-4} H^{3/2} - 1.15 \times 10^{-5} H^{5/2}$$

(a) When $H = 0.6 \text{ ft}$,

$$\frac{dH}{dt} = -1.87 \times 10^{-4} (0.6)^{3/2} - 1.15 \times 10^{-5} (0.6)^{5/2} = -9.01 \times 10^{-5} \frac{\text{ft}}{\text{s}} \left(\frac{3600 \text{ s}}{\text{hr}} \right) = -0.324 \frac{\text{ft}}{\text{hr}}$$

(b) Integrate Eq(3) from $H = 0.6 \text{ ft}$ when $t = 0$ to $H = 0.1 \text{ ft}$ when $t = T$

Thus, from Eq(3)

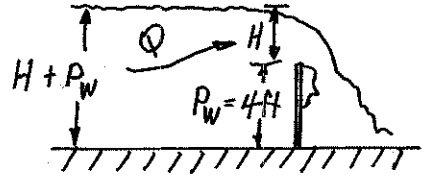
$$\int_{t=0}^{t=T} dt = \int_{H=0.6}^{H=0.1} \frac{-dH}{(1.87 \times 10^{-4} H^{3/2} + 1.15 \times 10^{-5} H^{5/2})}$$

$$(4) \quad T = \int_{0.1}^{0.6} \frac{dH}{(1.87 \times 10^{-4} H^{3/2} + 1.15 \times 10^{-5} H^{5/2})}$$

Numerical integration of Eq(4) gives $T = 19,900 \text{ s} \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right) = \underline{\underline{5.53 \text{ hr}}}$

10.107

10.107 A basin at a water treatment plant is 60 ft long, 10 ft wide, and 5 ft deep. Water flows from the basin over a 3-ft-long, rectangular weir whose crest is 4 ft above the bottom of the basin. Estimate how long it will take for the depth of the water in the basin to change from 4.5 ft to 4.4 ft if there is no flow into the basin.



$$Q = C_{wr} \frac{2}{3} \sqrt{2g} b H^{3/2}, \text{ where } b = 3 \text{ ft and } C_{wr} = (0.611 + 0.075 \left(\frac{H}{P_w}\right))$$

Thus,

$$Q = (0.611 + 0.075 \left(\frac{H}{4}\right)) \frac{2}{3} \sqrt{2(32.2 \text{ ft/s}^2)} (3 \text{ ft}) H^{3/2}$$

or

$$(1) \quad Q = 9.81 H^{3/2} + 0.301 H^{5/2} \text{ ft}^3/\text{s}, \text{ where } H \sim \text{ft}$$

Also,

$$Q = -A_{\text{tank}} \frac{dH}{dt} = -(60 \text{ ft})(10 \text{ ft}) \frac{dH}{dt}$$

or

$$(2) \quad Q = -600 \frac{dH}{dt}$$

Thus, from Eqs (1) and (2),

$$-600 \frac{dH}{dt} = 9.81 H^{3/2} + 0.301 H^{5/2}$$

$$(3) \quad \frac{dH}{dt} = -0.0164 H^{3/2} - 0.000502 H^{5/2}, \text{ where } H = \text{water depth in channel} - 4 \text{ ft.}$$

Thus, with $H = 0.5 \text{ ft}$ at $t = 0$ and $H = 0.4 \text{ ft}$ at $t = T$ it follows that

$$\int_{t=0}^{t=T} dt = - \int_{H=0.5}^{H=0.4} \frac{dH}{(0.0164 H^{3/2} + 0.000502 H^{5/2})}$$

or

$$(4) \quad T = \int_{0.4}^{0.5} \frac{dH}{(0.0164 H^{3/2} + 0.000502 H^{5/2})}$$

A standard numerical integration of Eq. (4) gives

$$T = \underline{\underline{20.1 \text{ s}}}$$

Note: From Eq. (3), $\frac{dH}{dt} = -0.00589 \frac{\text{ft}}{\text{s}}$ when $H = 0.5 \text{ ft}$ and $-0.00420 \frac{\text{ft}}{\text{s}}$ when $H = 0.4 \text{ ft}$. At these rates with $\frac{dH}{dt} = \frac{\Delta H}{T} = \frac{-0.1 \text{ ft}}{T}$ we would obtain $T = -0.1 \text{ ft} / \left(\frac{dH}{dt}\right) = -0.1 \text{ ft} / (-0.00589 \frac{\text{ft}}{\text{s}}) = 17.0 \text{ s}$ or $T = -0.1 \text{ ft} / (-0.00420 \frac{\text{ft}}{\text{s}}) = 23.8 \text{ s}$, which brackets the actual $T = 20.1 \text{ s}$.

10.108

10.108 Water flows over a sharp crested triangular weir with $\theta = 90^\circ$. The head range covered is $0.2 \leq H \leq 1.0$ ft and the accuracy in the measurement of the head, H , is $\delta H = \pm 0.01$ ft. Plot a graph of the percent error expected in Q as a function of Q .

$$Q = C_{wt} \frac{\theta}{15} \tan\left(\frac{\theta}{2}\right) \sqrt{2g} H^{5/2}, \text{ where } \theta = 90^\circ$$

Thus,

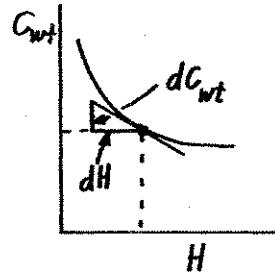
$$\frac{dQ}{dH} = \frac{\theta}{15} \tan\left(\frac{\theta}{2}\right) \sqrt{2g} \left[C_{wt} \left(\frac{5}{2}\right) H^{3/2} + H^{5/2} \frac{dC_{wt}}{dH} \right]$$

$$\text{or } \frac{dQ}{Q} = \frac{\left[C_{wt} \left(\frac{5}{2}\right) H^{3/2} + H^{5/2} \frac{dC_{wt}}{dH} \right] dH}{C_{wt} H^{5/2}}$$

Hence,

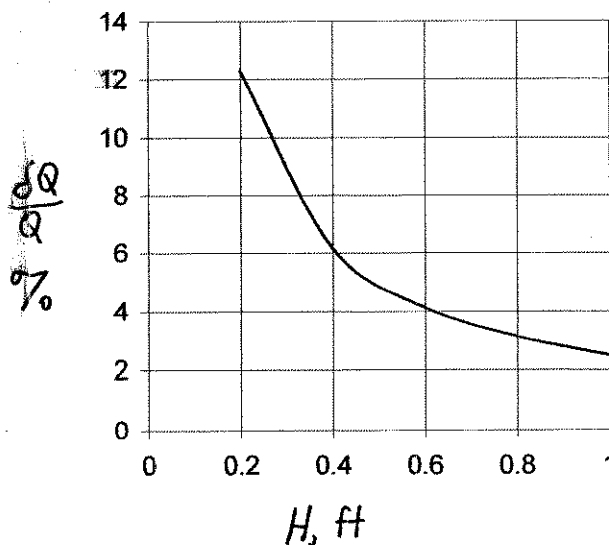
$$\frac{\delta Q}{Q} = \frac{5}{2} \frac{\delta H}{H} + \frac{1}{C_{wt}} \left(\frac{dC_{wt}}{dH} \right) \delta H \quad (1)$$

With $0.2 \leq H \leq 1.0$ ft and $\delta H = 0.01$ ft calculate $\frac{\delta Q}{Q}$ after obtaining C_{wt} and $\frac{dC_{wt}}{dH}$ from Fig. 10.21.



H, ft	C_{wt}	$\frac{dC_{wt}}{dH}$	$\frac{\delta Q}{Q}$
0.2	0.60	-0.100	0.123
0.4	0.588	-0.042	0.0618
0.6	0.582	-0.018	0.0414
0.8	0.581	-0.005	0.0312
1.0	0.581	0	0.0250

The above results are plotted below:



10.109 (a) The rectangular sharp-crested weir shown in Fig. P10.109a is used to maintain a relatively constant depth in the channel upstream of the weir. How much deeper will the water be upstream of the weir during a flood when the flowrate is $45 \text{ ft}^3/\text{s}$ compared to normal conditions when the flowrate is $30 \text{ ft}^3/\text{s}$? Assume the weir coefficient remains constant at $C_{wr} = 0.62$. (b) Repeat the calculations if the weir of part (a) is replaced by a rectangular sharp-crested "duck bill" weir which is oriented at an angle of 30° relative to the channel centerline as shown in Fig. P10.109b. The weir coefficient remains the same.

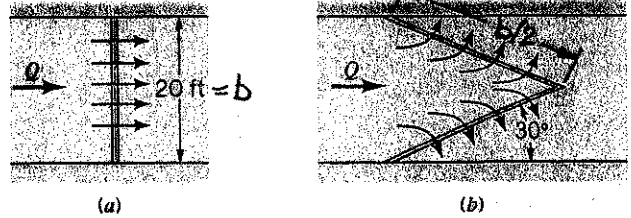


FIGURE P10.109

In either case

$$Q = C_{wr} \frac{2}{3} \sqrt{2g} b H^{3/2} = 0.62 \left(\frac{2}{3}\right) \sqrt{2(32.2 \text{ ft/s}^2)} b H^{3/2},$$

or

$$Q = 3.32 b H^{3/2}, \text{ where } Q \sim \text{ft}^3/\text{s} \text{ when } b \sim \text{ft} \text{ and } H \sim \text{ft} \quad (1)$$

(a) From Eq. (1) with $b = 20 \text{ ft}$, if $Q = 30 \text{ ft}^3/\text{s}$ then

$$30 = 3.32 (20) H_{30}^{3/2}, \text{ or } H_{30} = 0.589 \text{ ft}$$

If $Q = 45 \text{ ft}^3/\text{s}$, then

$$45 = 3.32 (20) H_{45}^{3/2}, \text{ or } H_{45} = 0.772 \text{ ft}$$

$$\text{Thus, } \Delta H_a = H_{45} - H_{30} = 0.772 \text{ ft} - 0.589 \text{ ft} = \underline{\underline{0.183 \text{ ft}}}$$

(b) From Eq. (1) with $b = 2(10 \text{ ft})/\sin 30^\circ = 40 \text{ ft}$, if $Q = 30 \text{ ft}^3/\text{s}$ then

$$30 = 3.32 (40) H_{30}^{3/2}, \text{ or } H_{30} = 0.371 \text{ ft}$$

If $Q = 45 \text{ ft}^3/\text{s}$, then

$$45 = 3.32 (40) H_{45}^{3/2}, \text{ or } H_{45} = \underline{\underline{0.486 \text{ ft}}}$$

$$\text{Thus, } \Delta H_b = H_{45} - H_{30} = 0.486 \text{ ft} - 0.371 \text{ ft} = \underline{\underline{0.115 \text{ ft}}}$$

Note that the "duck bill" weir gives a smaller change in the head than does the "regular" weir.

10.110 Water flows in a rectangular channel of width $b = 20$ ft at a rate of 100 ft³/s. The flowrate is to be measured by using either a rectangular weir of height $P_w = 4$ ft or a triangular ($\theta = 90^\circ$) sharp crested weir. Determine the head, H , necessary. If measurement of the head is accurate to only ± 0.04 ft, determine the accuracy of the measured flowrate expected for each of the weirs. Which weir would be the most accurate? Explain.

(a) Rectangular weir:

$$Q = (0.611 + 0.075 \left(\frac{H}{P_w}\right)) \left(\frac{2}{3}\right) \sqrt{2g} b H^{3/2}, \text{ where } P_w = 4 \text{ ft}$$

Thus,

$$Q = \left[0.611 + 0.075 \left(\frac{H}{4}\right)\right] \left(\frac{2}{3}\right) [2(32.2 \frac{\text{ft}}{\text{s}^2})]^{1/2} (20 \text{ ft}) H^{3/2}$$

or

$$Q = 107 (0.611 + 0.0188H) H^{3/2}, \text{ where } Q \sim \frac{\text{ft}^3}{\text{s}} \text{ and } H \sim \text{ft} \quad (1)$$

$$\text{With } Q = 100 \frac{\text{ft}^3}{\text{s}} \text{ this gives } 0.935 = (0.611 + 0.0188H) H^{3/2}$$

$$\text{or } (32.5 + H) H^{3/2} - 49.7 = 0 \quad (2)$$

By using a standard root-finding program, the solution to Eq. (2) is determined to be

$$H = \underline{\underline{1.294 \text{ ft}}}$$

(b) Triangular weir:

$$Q = C_{wt} \frac{8}{15} \tan\left(\frac{\theta}{2}\right) \sqrt{2g} H^{5/2} = C_{wt} \left(\frac{8}{15}\right) (\tan 45^\circ) [2(32.2 \frac{\text{ft}}{\text{s}^2})]^{1/2} H^{5/2}$$

$$\text{or } Q = 4.28 C_{wt} H^{5/2} \frac{\text{ft}^3}{\text{s}}, \text{ where } H \sim \text{ft} \text{ and } C_{wt} \text{ is from Fig. 10.21.} \quad (2)$$

For $Q = 100 \frac{\text{ft}^3}{\text{s}}$, assume $C_{wt} = 0.58$ so that

$$4.28 (0.58) H^{5/2}, \text{ or } H = \underline{\underline{4.39 \text{ ft}}} \quad \text{Note: The assumed } C_{wt} = 0.58 \text{ checks (see Fig. 10.21)}$$

Calculate Q for $H = H_{100}$, $H_{100} + 0.04$, and $H_{100} - 0.04$ from Eqs. (1) and (2):

(Rectangular) H , ft	Q , cfs	(Triangular) H , ft	Q , cfs
1.254	95.3	4.35	98.0
$H_{100} = 1.294$	100	$H_{100} = 4.39$	100
1.334	104.9	4.43	102.5

With $H \pm 0.04$ ft it is seen that triangular weir is more accurate (i.e. smaller variation in Q).

10.111 Water flows under a sluice gate in a 60-ft-wide finished concrete channel as is shown in Fig. P10.111. Determine the flowrate. If the slope of the channel is 2.5 ft/200 ft, will the water depth increase or decrease downstream of the gate? Assume $C_c = y_2/a = 0.65$. Explain.

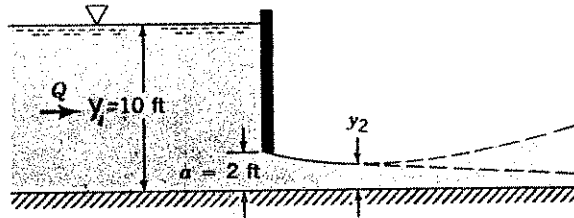


FIGURE P10.111

$$Q = bq = b C_d a \sqrt{2gy_1}, \text{ where } b = 60 \text{ ft}, a = 2 \text{ ft}, \text{ and from Fig. 10.24}$$

$$\text{since } \frac{y_1}{a} = \frac{10 \text{ ft}}{2 \text{ ft}} = 5 \text{ it follows that } C_d = 0.55$$

Hence,

$$Q = (60 \text{ ft})(0.55)(2 \text{ ft}) \left[2 \left(32.2 \frac{\text{ft}}{\text{s}^2} \right) (10 \text{ ft}) \right]^{1/2} = \underline{\underline{1670 \frac{\text{ft}^3}{\text{s}}}}$$

Determine the slope needed to maintain uniform flow downstream of the gate:

$$Q = \frac{K}{n} A R_h^{2/3} S_o^{1/2}, \text{ where } K = 1.49 \text{ and from Table 10.1 } n = 0.012 \quad (1)$$

$$\text{Also, } y_2 = C_c a = 0.65 (2 \text{ ft}) = 1.3 \text{ ft}$$

so that

$$A = (1.3 \text{ ft})(60 \text{ ft}) = 78 \text{ ft}^2, \quad P = (60 + 2(1.3)) \text{ ft} = 62.6 \text{ ft}$$

$$\text{and } R_h = \frac{A}{P} = \frac{78 \text{ ft}^2}{62.6 \text{ ft}} = 1.245 \text{ ft}$$

Thus, from Eq. (1):

$$1670 = \frac{1.49}{0.012} (78) (1.245)^{2/3} S_o^{1/2}, \text{ or } S_o = 0.0222$$

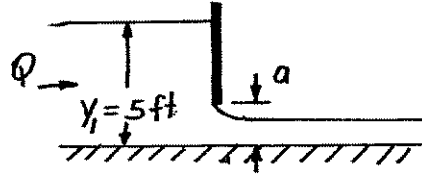
Hence, the required slope for uniform flow is $S_o = 0.0222$

but the actual slope is $S_o = \frac{2.5 \text{ ft}}{200 \text{ ft}} = 0.0125$, less than required.

The fluid will slow down and the depth increase.

10.112

10.112 Water flows under a sluice gate in a channel of 10-ft width. If the upstream depth remains constant at 5 ft, plot a graph of flowrate as a function of the distance between the gate and channel bottom as the gate is slowly opened. Assume free outflow.

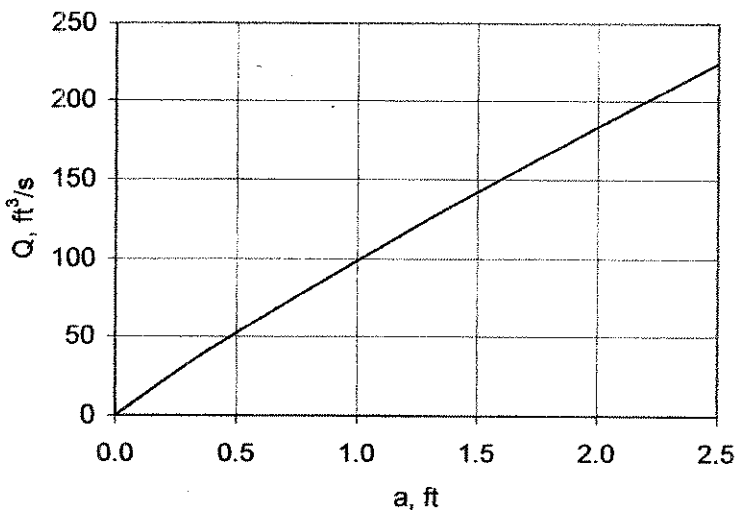
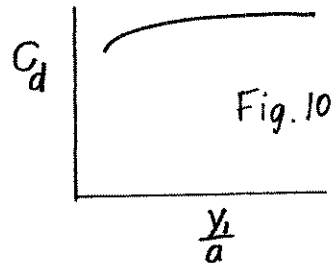


$$Q = qb = b C_d a \sqrt{2gy_1}, \text{ where } y_1 = 5 \text{ ft, } b = 10 \text{ ft, and } C_d \text{ is from Fig. 10.25.}$$

Thus,

$$Q = C_d (10 \text{ ft}) a [2(32.2 \frac{\text{ft}}{\text{s}^2})(5 \text{ ft})]^{1/2} = 179 C_d a \frac{\text{ft}^3}{\text{s}}, \text{ where } a \sim \text{ft}$$

$a, \text{ ft}$	$\frac{y_1}{a}$	C_d	$Q, \frac{\text{ft}^3}{\text{s}}$
0	∞	0.6	0
0.5	10	0.58	51.9
1.0	5	0.55	98.5
1.5	3.33	0.53	142
2.0	2.5	0.51	183
2.5	2	0.50	224



10.113

10.113 A water-level regulator (not shown) maintains a depth of 2.0 m downstream from a 10-m-wide drum gate as shown in Fig. P10.113. Plot a graph of flowrate, Q , as a function of water depth upstream of the gate, y_1 , for $2.0 \leq y_1 \leq 5.0$ m.

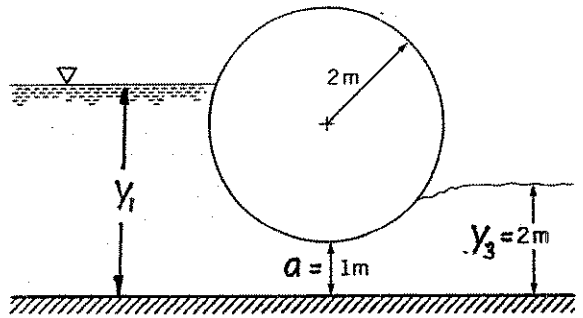


FIGURE P10.113

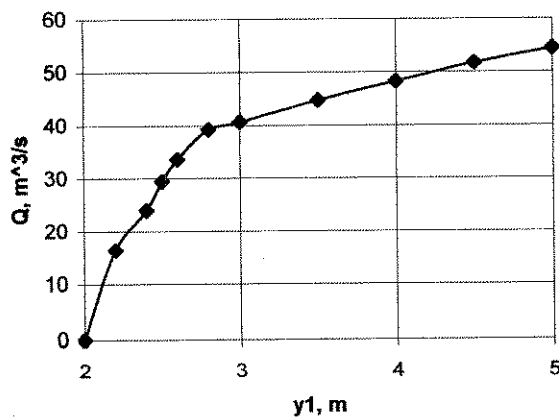
$$Q = bq = b C_d a \sqrt{2g y_1}, \text{ where } a = 1 \text{ m and } b = 10 \text{ m.}$$

Thus,

$$Q = (10 \text{ m}) C_d (1 \text{ m}) \sqrt{2(9.81 \frac{\text{m}}{\text{s}^2})(y_1, \text{m})} = 44.3 C_d \sqrt{y_1}, \text{ where } Q \sim \frac{\text{m}^3}{\text{s}} \text{ when } y_1 \sim \text{m}$$

Obtain C_d from Fig. 10.25 with $\frac{y_3}{a} = 2$.

y_1, m	y_1/a	C_d	$Q, \text{m}^3/\text{s}$
2.00	2.00	0.00	0.00
2.20	2.20	0.25	16.43
2.40	2.40	0.35	24.02
2.50	2.50	0.42	29.42
2.60	2.60	0.47	33.57
2.80	2.80	0.53	39.29
3.00	3.00	0.53	40.67
3.50	3.50	0.54	44.75
4.00	4.00	0.55	48.29
4.50	4.50	0.55	51.69
5.00	5.00	0.55	54.48



10.114 Calibration of a Triangular Weir

Objective: The flowrate over a weir is a function of the weir head. The purpose of this experiment is to use a device as shown in Fig. P10.114 to calibrate a triangular weir and determine the relationship between flowrate, Q , and weir head, H .

Equipment: Water channel (flume) with a pump and a flow control valve; triangular weir; float; point gage; stop watch.

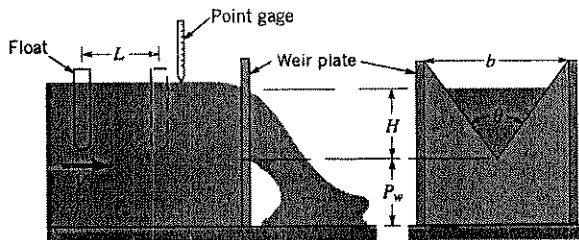
Experimental Procedure: Measure the width, b , of the channel, the distance, P_w , between the channel bottom and the bottom of the V-notch in the weir plate, and the angle, θ , of the V-notch. Fasten the weir plate to the channel bottom, turn on the pump, and adjust the control valve to produce the desired flowrate, Q , over the weir. Use the point gage to measure the weir head, H . Insert the float into the water well upstream from the weir and measure the time, t , it takes for the float to travel a known distance, L . Repeat the measurements for various flowrates (i.e., various weir heads).

Calculations: For each set of data, determine the experimental flowrate as $Q = VA$, where $V = L/t$ is the velocity of the float (assumed to be equal to the average velocity of the water upstream of the weir) and $A = b(P_w + H)$ is the flow area upstream of the weir.

Graph: On log-log graph paper, plot flowrate, Q , as ordinates and weir head, H , as abscissas. Draw the best-fit line with a slope of $5/2$ through the data.

Results: Use the flowrate-weir head data to determine the triangular weir coefficient, C_{wt} , for this weir (see Eq. 10.32). For this experiment, assume that the weir coefficient is a constant, independent of weir head.

Data: To proceed, print this page for reference when you work the problem and [click here](#) to bring up an EXCEL page with the data for this problem.



■ FIGURE P10.114

(cont.)

10.114 (con't)

Solution for Problem 10.114 Calibration of a Triangular Weir

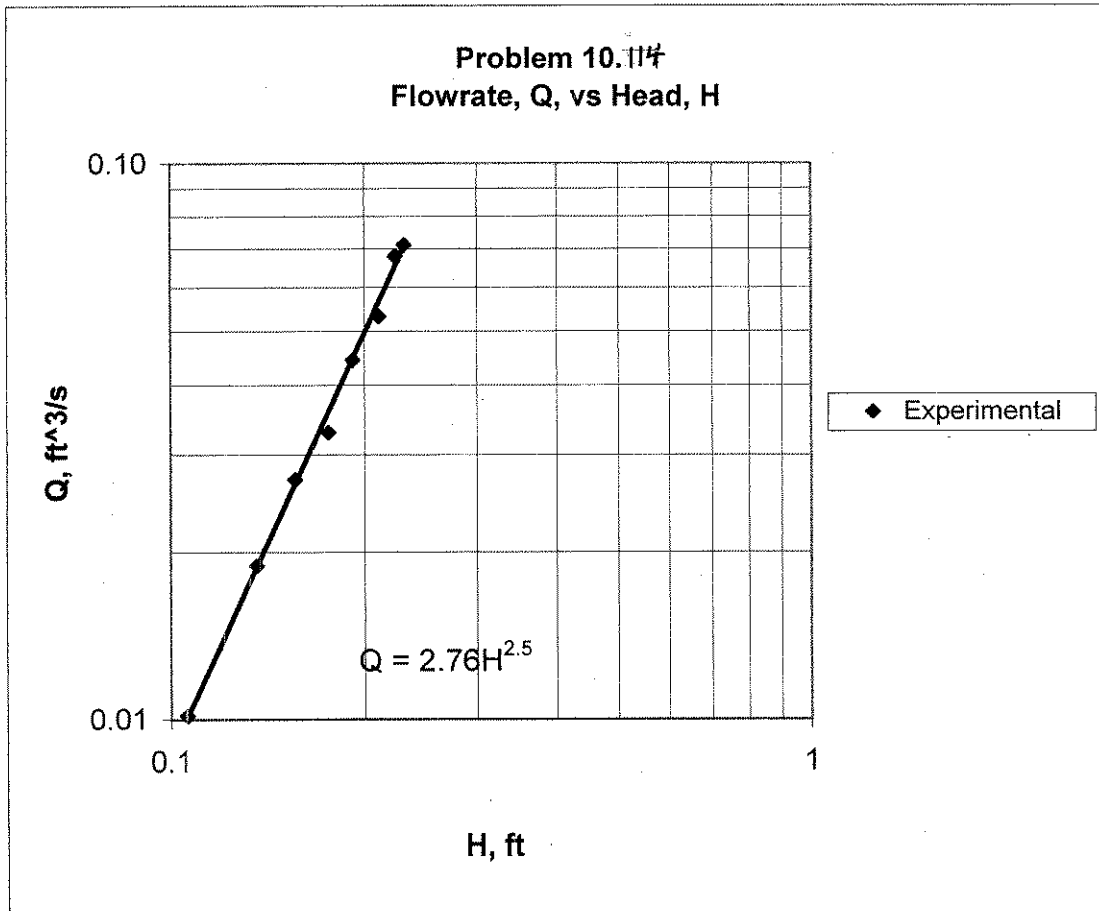
θ , deg	b, in.	P_w , in.	L, ft	H, ft	t, s	V, ft/s	Q, ft ³ /s
90	6.00	6.55	1.50	0.231	8.2	0.183	0.0711
				0.224	8.5	0.176	0.0679
				0.211	10.7	0.140	0.0530
				0.192	12.5	0.120	0.0443
				0.176	16.5	0.091	0.0328
				0.156	19.5	0.077	0.0270
				0.136	27.1	0.055	0.0189
				0.106	48.2	0.031	0.0101
				0.091	62.9	0.024	0.0076
				0.088	68.1	0.022	0.0070

$Q = VA = V \cdot b(P_w + H)$ where $V = L/t$

$Q = C_{wt} (8/15) \tan(\theta/2) (2g)^{1/2} H^{5/2}$ where from the graph

$Q = 2.76 H^{2.5}$

Thus, $C_{wt} = (15/8) \cdot 2.76 / (2 \cdot 32.2)^{1/2} = \underline{0.645}$



10.115

10.115 Calibration of a Rectangular Weir

Objective: The flowrate over a weir is a function of the weir head. The purpose of this experiment is to use a device as shown in Fig. P10.115 to calibrate a rectangular weir and determine the relationship between flowrate, Q , and weir head, H .

Equipment: Water channel (flume) with a pump and a flow control valve; rectangular weir; float; point gage; stop watch.

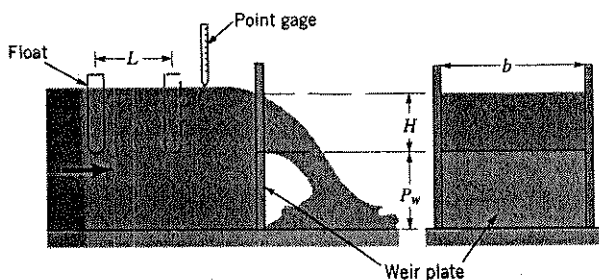
Experimental Procedure: Measure the width, b , of the channel and the distance, P_w , between the channel bottom and the top of the weir plate. Fasten the weir plate to the channel bottom, turn on the pump, and adjust the control valve to produce the desired flowrate, Q , over the weir. Use the point gage to measure the weir head, H . Insert the float into the water well upstream from the weir and measure the time, t , it takes for the float to travel a known distance, L . Repeat the measurements for various flowrates (i.e., various weir heads).

Calculations: For each set of data, determine the experimental flowrate as $Q = VA$, where $V = L/t$ is the velocity of the float (assumed to be equal to the average velocity of the water upstream of the weir) and $A = b(P_w + H)$ is the flow area upstream of the weir.

Graph: On log-log graph paper, plot flowrate, Q , as ordinates and weir head, H , as abscissas. Draw the best-fit line with a slope of $3/2$ through the data.

Results: Use the flowrate-weir head data to determine the rectangular weir coefficient, C_{wr} , for this weir (see Eq. 10.30). For this experiment, assume that the weir coefficient is a constant, independent of weir head.

Data: To proceed, print this page for reference when you work the problem and [click here](#) to bring up an EXCEL page with the data for this problem.



■ FIGURE P10.115

(con't)

10.115 (con't)

Solution for Problem 10.115 Calibration of a Rectangular Weir

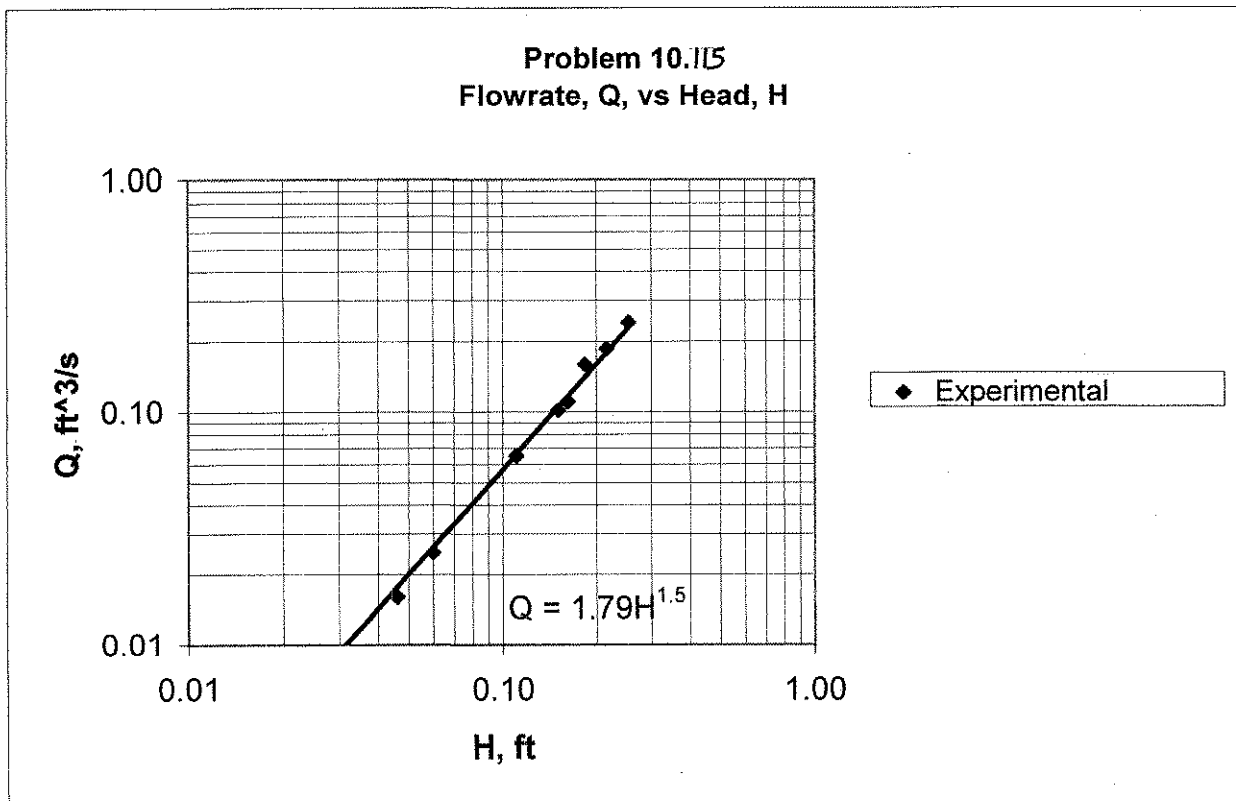
b, in.	P _w , in.	L, ft	H, ft	t, s	V, ft/s	Q, ft ³ /s
6.00	6.00	1.40	0.254	2.2	0.636	0.240
			0.216	2.7	0.519	0.186
			0.184	3.0	0.467	0.160
			0.162	4.2	0.333	0.110
			0.151	4.5	0.311	0.101
			0.111	6.6	0.212	0.065
			0.060	15.8	0.089	0.025
			0.046	23.8	0.059	0.016
			0.031	38.4	0.036	0.010

$Q = VA = V \cdot b(P_w + H)$ where $V = L/t$

$Q = C_{wr} (2/3) (2g)^{1/2} H^{3/2} b$ where from the graph

$Q = 1.79 H^{1.5}$

Thus, $C_{wr} = (3/2) \cdot 1.79 / (0.5 \cdot (2 \cdot 32.2)^{1/2}) = 0.669$



10.116 Hydraulic Jump Depth Ratio

Objective: Under certain conditions, if the flow in a channel is supercritical a hydraulic jump will form. The purpose of this experiment is to use an apparatus as shown in Fig. P10.116 to determine the depth ratio, y_2/y_1 , across the hydraulic jump as a function of the Froude number upstream of the jump, Fr_1 .

Equipment: Water channel (flume) with a pump and a flow control valve; sluice gate; point gage; adjustable tail gate.

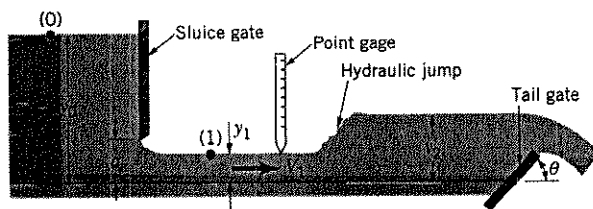
Experimental Procedure: Position the sluice gate so that the distance, a , between the bottom of the gate and the bottom of the channel is approximately 1 inch. Adjust the flow control valve to produce a flowrate that causes the water to back up to the desired depth, y_0 , upstream of the sluice gate. Carefully adjust the angle, θ , of the tail gate so that a hydraulic jump forms at the desired location downstream from the sluice gate. Note that if θ is too small, the jump will be washed downstream and disappear. If θ is too large, the jump will migrate upstream and be swallowed by the sluice gate. With the jump in place, use the point gage to determine the depth upstream from the sluice gate, y_0 , the depth just upstream from the jump, y_1 , and the depth downstream from the jump, y_2 . Repeat the measurements for various flowrates (i.e., various y_0 values).

Calculations: For each data set, use the Bernoulli and continuity equations between points (0) and (1) to determine the velocity, V_1 , and Froude number, $Fr_1 = V_1/(gy_1)^{1/2}$, just upstream from the jump (see Eq. 3.21). Also use the measured depths to determine the depth ratio, y_2/y_1 , across the jump.

Graph: Plot the depth ratio, y_2/y_1 , as ordinates and Froude number, Fr_1 , as abscissas.

Results: On the same graph, plot the theoretical depth ratio as a function of Froude number (see Eq. 10.24).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P10.116.

(cont)

10.116 (con't)

Solution for Problem 10.116 Hydraulic Jump Depth Ratio

y_0 , ft	y_1 , ft	y_2 , ft.	Experimental			Theoretical	
			V_1 , ft/s	Fr_1	y_2/y_1	Fr_1	y_2/y_1
0.855	0.055	0.404	7.19	5.40	7.35	1	1.00
0.759	0.055	0.386	6.75	5.07	7.02	2	2.37
0.691	0.055	0.367	6.42	4.82	6.67	3	3.77
0.578	0.055	0.337	5.83	4.38	6.13	4	5.18
0.492	0.055	0.308	5.34	4.01	5.60	5	6.59
0.414	0.055	0.280	4.85	3.65	5.09	6	8.00
0.289	0.055	0.233	3.95	2.97	4.24		
0.248	0.055	0.211	3.62	2.72	3.84		

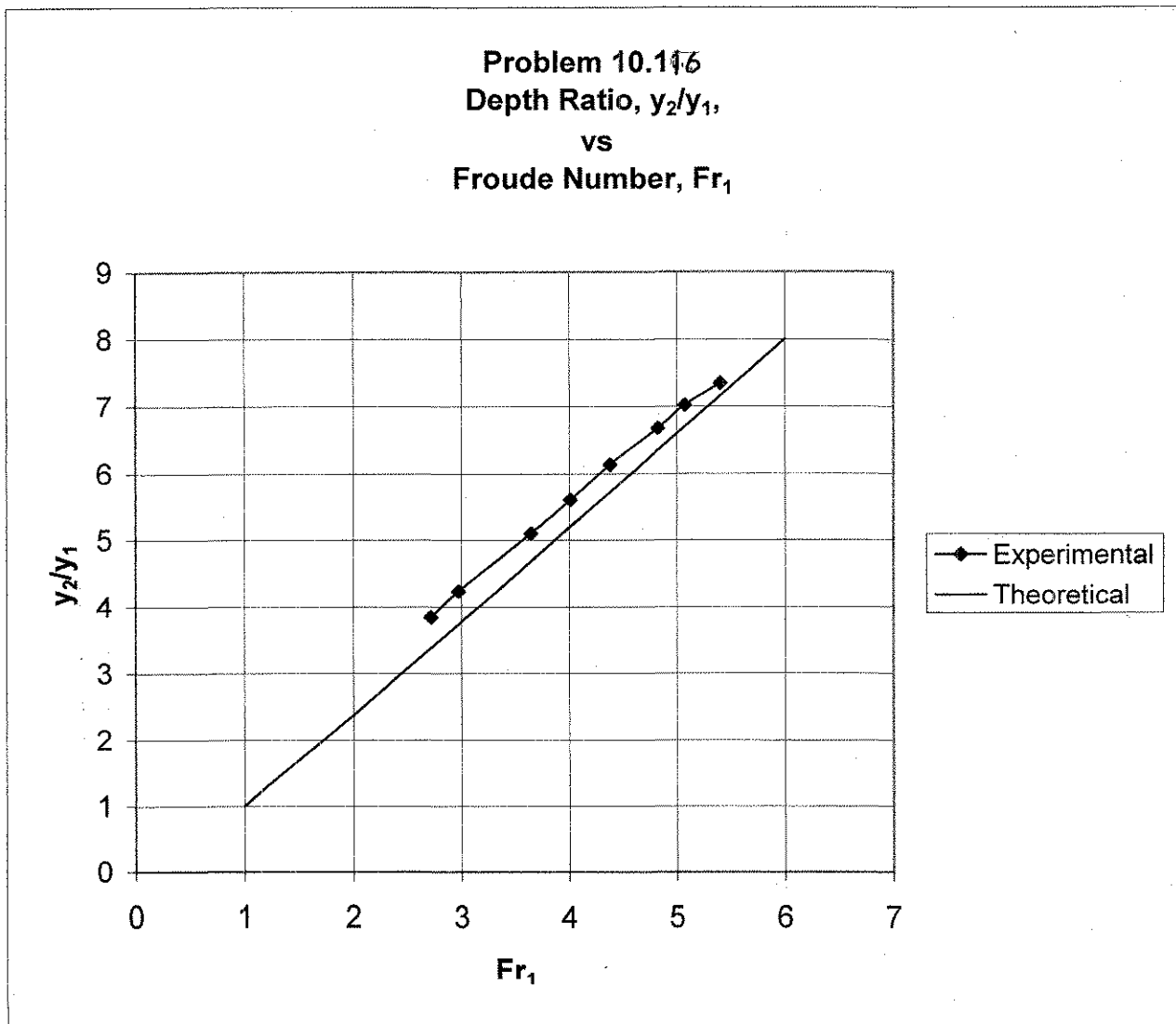
For flow under a sluice gate:

$$V_1 = [2g(y_0 - y_1)/(1 - (y_1/y_0)^2)]^{1/2}$$

Theory:

$$y_2/y_1 = [-1 + (1 + 8Fr_1^2)^{1/2}]/2$$

$$Fr_1 = V_1/(gy_1)^{1/2}$$



10.117 Hydraulic Jump Head Loss

Objective: Under certain conditions, if the flow in a channel is supercritical a hydraulic jump will form. The purpose of this experiment is to use an apparatus as shown in Fig. P10.117 to determine the head loss ratio, h_L/y_1 , across the hydraulic jump as a function of the Froude number upstream of the jump, Fr_1 .

Equipment: Water channel (flume) with a pump and a flow control valve; sluice gate; point gage; Pitot tubes; adjustable tail gate.

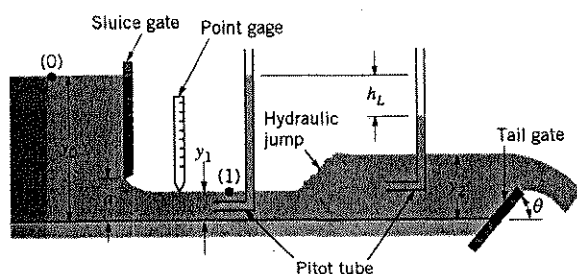
Experimental Procedure: Position the sluice gate so that the distance, a , between the bottom of the gate and the bottom of the channel is approximately 1 inch. Adjust the flow control valve to produce a flowrate that causes the water to back up to the desired depth, y_0 , upstream of the sluice gate. Carefully adjust the angle, θ , of the tail gate so that a hydraulic jump forms at the desired location downstream from the sluice gate. Note that if θ is too small, the jump will be washed downstream and disappear. If θ is too large, the jump will migrate upstream and be swallowed by the sluice gate. With the jump in place, use the point gage to determine the depth upstream from the sluice gate, y_0 , and the depth just upstream from the jump, y_1 . Also measure the head loss, h_L , as the difference in the water elevations in the piezometer tubes connected to the two Pitot tubes located upstream and downstream of the jump. Repeat the measurements for various flowrates (i.e., various y_0 values).

Calculations: For each data set, use the Bernoulli and continuity equations between points (0) and (1) to determine the velocity, V_1 , and the Froude number, $Fr_1 = V_1/(gy_1)^{1/2}$, just upstream from the jump. Also calculate the dimensionless head loss, h_L/y_1 , for each data set.

Graph: Plot the dimensionless head loss across the jump, h_L/y_1 , as ordinates and the Froude number, Fr_1 , as abscissas.

Results: On the same graph, plot the theoretical dimensionless head loss as a function of Froude number (see Eqs. 10.24 and 10.25).

Data: To proceed, print this page for reference when you work the problem and [click here](#) to bring up an EXCEL page with the data for this problem.



■ FIGURE P10.117

(cont)

10.117 (con't)

Solution for Problem 10.117 Hydraulic Jump Head Loss

y_0 , ft	y_1 , ft	y_2 , ft	h_L , ft	Experimental			Theoretical		
				V_1 , ft/s	Fr_1	h_L/y_1	Fr_1	y_2/y_1	h_L/y_1
0.855	0.055	0.404	0.364	7.19	5.40	6.62	1	1.00	0.00
0.759	0.055	0.386	0.313	6.75	5.07	5.69	2	2.37	0.27
0.691	0.055	0.367	0.271	6.42	4.82	4.93	3	3.77	1.41
0.578	0.055	0.337	0.201	5.83	4.38	3.65	4	5.18	3.52
0.492	0.055	0.308	0.152	5.34	4.01	2.76	5	6.59	6.62
0.414	0.055	0.280	0.117	4.85	3.65	2.13	6	8.00	10.72
0.289	0.055	0.233	0.058	3.95	2.97	1.05			
0.248	0.055	0.211	0.042	3.62	2.72	0.76			

For flow under a sluice gate:

$$V_1 = [2g(y_0 - y_1)/(1 - (y_1/y_0)^2)]^{1/2}$$

Theory:

$$h_L/y_1 = 1 - (y_2/y_1) + Fr_1^2 [1 - (y_1/y_2)^2]/2$$

where

$$y_2/y_1 = [-1 + (1 + 8Fr_1^2)^{1/2}]/2$$

