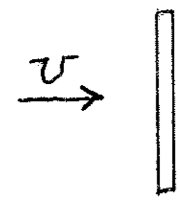
9.2

9.2 A thin square is oriented perpendicular to the upstream velocity in a uniform flow. The average pressure on the front side of the square is 0.7 times the stagnation pressure and the average pressure on the back side is a vacuum (i.e., less than the free stream pressure) with a magnitude 0.4 times the stagnation pressure. Determine the drag coefficient for this square.

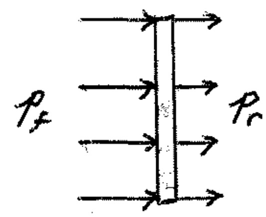


The drag can be determined by summing the pressure forces.

$$\mathcal{D} = p_f A - p_r A$$

$$= 0.7(\tfrac{1}{2}\rho V^2)A - (-0.4)(\tfrac{1}{2}\rho V^2)A$$

The pressure on the rear is in vacuum so is negative.



$$\mathcal{D} = 1.1(\tfrac{1}{2}\rho V^2)A$$

So,

$$C_D = \frac{\mathcal{D}}{\frac{1}{2}\rho V^2 A} = \frac{1.1(\frac{1}{2}\rho V^2)A}{\frac{1}{2}\rho V^2 A}$$

$$\underline{\underline{C_D = 1.1}}$$

9.3

9.3 A small 15-mm-long fish swims with a speed of 20 mm/s. Would a boundary layer type flow be developed along the sides of the fish? Explain.

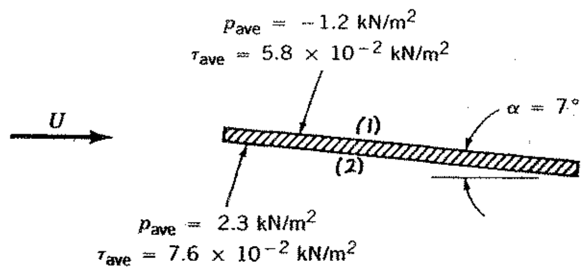
$Re = \frac{U\ell}{\nu}$, or with $\ell = 15 \times 10^{-3}\, m$, $U = 20 \times 10^{-3}\, \frac{m}{s}$ and $\nu = 1.12 \times 10^{-6}\, \frac{m^2}{s}$ (i.e., $15.5\,°C$ water)

$$Re = \frac{(20 \times 10^{-3}\, \frac{m}{s})(15 \times 10^{-3}\, m)}{1.12 \times 10^{-6}\, \frac{m^2}{s}} = 268$$

<u>This Reynolds number is not large enough to have true boundary layer type flow.</u> ($Re \approx 1000$ is often assumed to be the lower limit.)

9.4

9.4 The average pressure and shear stress acting on the surface of the 1-m-square flat plate are as indicated in Fig. P9.4 Determine the lift and drag generated. Determine the lift and drag if the shear stress is neglected. Compare these two sets of results.



■ FIGURE P9.4

Since $\int p\, dA = p_{ave} A$ and $\int \tau_w\, dA = \tau_{ave} A$ it follows that

$\mathcal{D} = -p_1 A_1 \sin\alpha + p_2 A_2 \sin\alpha + \tau_1 A_1 \cos\alpha + \tau_2 A_2 \cos\alpha$

or with $A_1 = A_2 = 1\,m^2$ and $\alpha = 7°$,

$\mathcal{D} = A_1 \sin\alpha\,(p_2 - p_1) + A_1 \cos\alpha\,(\tau_1 + \tau_2)$

$= (1\,m^2)\sin 7°\,(2.3 - (-1.2))\,\frac{kN}{m^2} + (1\,m^2)\cos 7°\,(5.8\times10^{-2} + 7.6\times10^{-2})\,\frac{kN}{m^2}$

$= 0.427\ kN + 0.133\ kN = \underline{0.560\ kN}$

Note, if shear stress is neglected $\mathcal{D} = \underline{0.427\ kN}$ (ie., $\tau_1 = \tau_2 = 0$)

Also, $\mathcal{L} = -p_1 A_1 \cos\alpha + p_2 A_2 \cos\alpha - \tau_1 A_1 \sin\alpha - \tau_2 A_2 \sin\alpha$

or

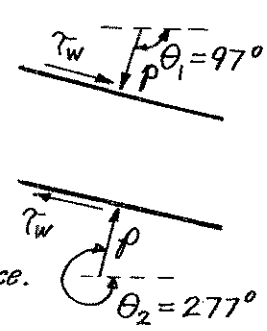
$\mathcal{L} = A_1 \cos\alpha\,(p_2 - p_1) - A_1 \sin\alpha\,(\tau_1 + \tau_2)$

$= (1\,m^2)\cos 7°\,(2.3 - (-1.2))\,\frac{kN}{m^2} - (1\,m^2)\sin 7°\,(5.8\times10^{-2} + 7.6\times10^{-2})\,\frac{kN}{m^2}$

$= 3.47\ kN - 0.0163\ kN = \underline{3.45\ kN}$

Note, if shear stress is neglected $\mathcal{L} = \underline{3.47\ kN}$

Note: If the general expressions $\mathcal{D} = \int p \cos\theta\, dA + \int \tau_w \sin\theta\, dA$ and $\mathcal{L} = -\int p \sin\theta\, dA + \int \tau_w \cos\theta\, dA$ are use, be careful about the signs involved. On the upper surface $\theta_1 = 97°$ and p and τ_w are positive as indicated in the figure. On the lower surface $\theta_2 = 277°$ and p and τ_w are positive as indicated in the lower figure.



For example, with this notation $\tau_w < 0$ on the lower surface.

$\mathcal{L} = -(-1.2\,\frac{kN}{m^2})\sin 97°(1\,m^2) - (2.3\,\frac{kN}{m^2})\sin 277°(1\,m^2)$

$+ (5.8\times10^{-2}\,\frac{kN}{m^2})\cos 97°(1\,m^2) + (-7.6\times10^{-2}\,\frac{kN}{m^2})\cos 277°(1\,m^2)$

$= 3.45\ kN$, as obtained above.

*9.5

*9.5 The pressure distribution on the 1-m-diameter circular disk in Fig. P9.5 is given in the table. Determine the drag on the disk.

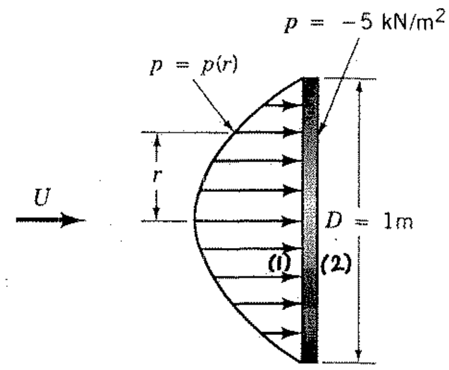


FIGURE P9.5

$$\mathcal{D} = \int_1 p\,dA - \int_2 p\,dA = \int_{r=0}^{r=\frac{D}{2}} p\,(2\pi r\,dr) - p_2\,\frac{\pi}{4}D^2, \text{ since } dA = 2\pi r\,dr$$

Thus,

$$\mathcal{D} = 2\pi \int_0^{0.5\,m} p\,r\,dr - \left(-5\,\frac{kN}{m^2}\right)\frac{\pi}{4}(1\,m^2) = 2\pi\int_0^{0.5} p\,r\,dr + 3.93\,kN$$

where $p \sim \frac{kN}{m^2}$, $r \sim m$

Evaluate the integral numerically using the following integrand:

r, m	pr, kN/m
0	0
0.05	0.214
0.10	0.406
0.15	0.558
0.20	0.620
0.25	0.695
0.30	0.711
0.35	0.662
0.40	0.564
0.45	0.333
0.50	0.000

r (m)	p (kN/m²)
0	4.34
0.05	4.28
0.10	4.06
0.15	3.72
0.20	3.10
0.25	2.78
0.30	2.37
0.35	1.89
0.40	1.41
0.45	0.74
0.50	0.0

Using a standard numerical integration technique with the above integrand gives $\mathcal{D} = \underline{\underline{5.43\,kN}}$

9.6

9.6 When you walk through still air at a rate of 1 m/s, would you expect the character of the air flow around you to be most like that depicted in Fig. 9.6a, b, or c? Explain.

$Re = \frac{U\ell}{\nu}$, where $\nu = 1.46 \times 10^{-5} \frac{m^2}{s}$ and $U = 1 \frac{m}{s}$. Assume $\ell = 1\,m$.

Thus,

$$Re = \frac{(1 \frac{m}{s})(1m)}{1.46 \times 10^{-5} \frac{m^2}{s}} = 6.85 \times 10^4$$

This flow has a large enough Reynolds number to develop a boundary layer. Thus, viscous effects would not be important far from your body, except in the wake region behind you.

Note: The above conclusion is true whether we assume $\ell = 1\,m$, $\ell = 2\,m$, $\ell = 0.1\,m$, or some other reasonable characteristic length of our body.

The flow would be most like that in Fig. 9.6c.

9.7

9.7 A 0.10 m-diameter circular cylinder moves through air with a speed U. The pressure distribution on the cylinder's surface is approximated by the three straight line segments shown in Fig. P9.7 Determine the drag coefficient on the cylinder. Neglect shear forces.

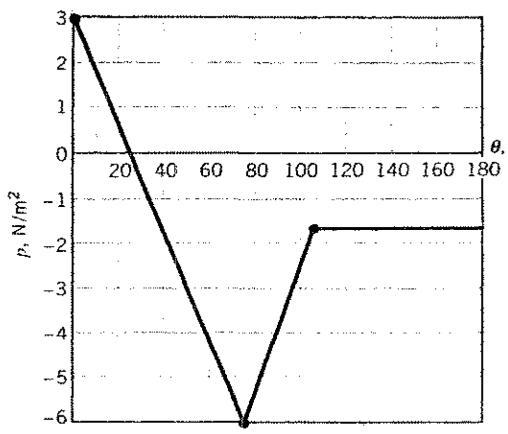
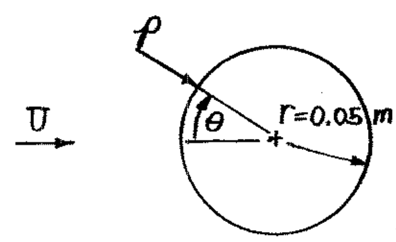


FIGURE P9.7

$$\mathcal{D}_p = \int p\, b r \cos\theta \, d\theta = br \int p \cos\theta \, d\theta$$

or $$\mathcal{D}_p = 2br\int_{\theta=0}^{\pi} p \cos\theta \, d\theta$$



$dA = b\,r d\theta$
b = length

Break up the integration into the following three segments:

1) $0 \le \theta \le 70^\circ = 1.222\ rad$ where

$p = -7.39\,\theta + 3\ \frac{N}{m^2}$, where $\theta \sim rad.$

i.e. $p\big|_{\theta=0} = 3$ and $p\big|_{\theta=1.222} = -6$

2) $70^\circ \le p \le 100^\circ$ or $1.222 \le \theta \le 1.745$ rad where

$p = 8.59\,\theta - 16.5\ \frac{N}{m^2}$, where $\theta \sim rad$

i.e. $p\big|_{\theta=1.222} = -6$ and $p\big|_{\theta=1.745} = -1.5$

and

3) $100^\circ \le p \le 180^\circ$ or $1.745 \le \theta \le 3.14$ rad where

$p = -1.5\ \frac{N}{m^2}$

Thus,

$$\mathcal{D}_p = 2br\left[\int_0^{70^\circ} p\cos\theta\, d\theta + \int_{70^\circ}^{100^\circ} p \cos\theta \, d\theta + \int_{100^\circ}^{180^\circ} p\cos\theta\, d\theta\right] = 2br\left[I_1 + I_2 + I_3\right] \quad (1)$$

where

(con't)

9.7	(con't)

$$I_1 = \int_0^{1.222} (-7.39\,\theta + 3)\cos\theta \, d\theta = \left[-7.39(\cos\theta + \theta\sin\theta) + 3\sin\theta\right|_0^{1.222} = -0.791$$

$$I_2 = \int_{1.222}^{1.745} (8.59\,\theta - 16.5)\cos\theta \, d\theta = \left[8.59(\cos\theta + \theta\sin\theta - 16.5\sin\theta\right|_{1.222}^{1.745} = -0.260$$

and

$$I_3 = \int_{1.745}^{3.14} (-1.5)\cos\theta \, d\theta = -1.5\sin\theta\Big|_{1.745}^{3.14} = 1.477$$

Hence,

$$\mathcal{D}_p = 2br\left[0.791 - 0.260 + 1.477\right] = 0.852\,br$$

or with

$$C_D = \frac{\mathcal{D}_p}{\frac{1}{2}\rho U^2 A} = \frac{0.852\,br}{\frac{1}{2}\rho U^2(2br)} = \frac{0.426}{\frac{1}{2}\rho U^2}$$

But the pressure at $\theta = 0$, the stagnation point, is $3\,\frac{N}{m^2}$.
Thus, $\frac{1}{2}\rho U^2 = 3\,\frac{N}{m^2}$ so that

$$C_D = \frac{0.426}{3} = \underline{\underline{0.142}}$$

9.8

9.8 Typical values of the Reynolds number for various animals moving through air or water are listed below. For which cases is inertia of the fluid important? For which cases do viscous effects dominate? For which cases would the flow be laminar; turbulent? Explain.

Animal	Speed	Re
(a) large whale	10 m/s	300,000,000
(b) flying duck	20 m/s	300,000
(c) large dragonfly	7 m/s	30,000
(d) invertebrate larva	1 mm/s	0.3
(e) bacterium	0.01 mm/s	0.00003

Inertia important if $Re \gtrsim 1$ (i.e whale, duck, dragonfly)

Viscous effects dominate if $Re \lesssim 1$ (i.e larva, bacterium)

Boundary layer flow becomes turbulent for Re on the order of 10^5 to 10^6. (i.e. whale and perhaps the duck)
The flow would be laminar for the dragonfly, larva, and bacterium and perhaps the duck.

9.12

9.12 Water flows past a flat plate that is oriented parallel to the flow with an upstream velocity of 0.5 m/s. Determine the approximate location downstream from the leading edge where the boundary layer becomes turbulent. What is the boundary layer thickness at this location?

$$Re_{cr} = 5\times10^5 = \frac{U x_{cr}}{\nu}$$

$$x_{cr} = \frac{5\times10^5\,\nu}{U} = \frac{5\times10^5\,(1.12\times10^{-6}\ \mathrm{m^2/s})}{0.5\ \mathrm{m/s}} = \underline{\underline{1.12\ \mathrm{m}}}$$

$$\delta = 5\sqrt{\frac{\nu x}{U}} = 5\sqrt{\frac{(1.12\times10^{-6}\ \mathrm{m^2/s})\,1.12\ \mathrm{m}}{0.5\ \mathrm{m/s}}} = \underline{\underline{7.92\times10^{-3}\ \mathrm{m}}}$$

9.13

9.13 A viscous fluid flows past a flat plate such that the boundary layer thickness at a distance 1.3 m from the leading edge is 12 mm. Determine the boundary layer thickness at distances of 0.20, 2.0, and 20 m from the leading edge. Assume laminar flow.

For laminar flow $\delta = C\sqrt{x}$, where C is a constant.

Thus,

$$C = \frac{\delta}{\sqrt{x}} = \frac{12 \times 10^{-3}\,m}{\sqrt{1.3\,m}} = 0.0105 \quad \text{or} \quad \delta = 0.0105\sqrt{x} \quad \text{where } x \sim m,\ \delta \sim m$$

x, m	δ, m	δ, mm
0.2	0.00470	4.70
2.0	0.0148	14.8
20.0	0.0470	47.0

9.14

9.14 If the upstream velocity of the flow in Problem 9.13 is $U = 1.5$ m/s, determine the kinematic viscosity of the fluid.

For laminar flow $\delta = 5\sqrt{\frac{\nu x}{U}}$, or $\nu = \frac{U\delta^2}{25\,x}$

Thus,

$$\nu = \frac{(1.5\,\frac{m}{s})(12 \times 10^{-3}\,m)^2}{25\,(1.3\,m)} = \underline{\underline{6.65 \times 10^{-6}\,\frac{m^2}{s}}}$$

9.15

9.15 Water flows past a flat plate with an upstream velocity of $U = 0.02$ m/s. Determine the water velocity a distance of 10 mm from the plate at distances of $x = 1.5$ m and $x = 15$ m from the leading edge.

From the Blasius solution for boundary layer flow on a flat plate, $u = U f'(\eta)$, where η, the similarity variable, is $\eta = y\sqrt{\frac{U}{\nu x}}$. Values of $f'(\eta)$ are given in Table 9.1.

Since $Re_x = \frac{Ux}{\nu} = \frac{(0.02\frac{m}{s})(15m)}{1.12\times10^{-6}\frac{m^2}{s}} = 2.68\times10^5$ is less than the critical $Re_{x_{cr}} = 5\times10^5$, it follows that the boundary layer flow is laminar.

At $x_1 = 1.5\,m$ and $y = 10\times10^{-3}\,m$ we obtain:

$$\eta_1 = (10\times10^{-3}m)\sqrt{\frac{0.02\frac{m}{s}}{(1.12\times10^{-6}\frac{m^2}{s})(1.5m)}} = 1.091$$

Linear interpolation from Table 9.1 gives:

$$f' = 0.2647 + \frac{(0.3938 - 0.2647)}{(1.2 - 0.8)}(1.091 - 0.8) = 0.359$$

Hence,

$$u_1 = U f'(\eta_1) = (0.02\frac{m}{s})(0.359) = \underline{\underline{0.00718\frac{m}{s}}}$$

Similarly, at $x_2 = 15m$ and $y = 10\times10^{-3}m$ we obtain:

$$\eta_2 = (10\times10^{-3}m)\sqrt{\frac{0.02\frac{m}{s}}{(1.12\times10^{-6}\frac{m^2}{s})(15m)}} = 0.345$$

Linear interpolation from Table 9.1 gives:

$$f' = 0.0 + \frac{(0.1328 - 0.0)}{(0.8 - 0.4)}(0.345 - 0.0) = 0.1145$$

Hence,

$$u_2 = U f'(\eta_2) = (0.02\frac{m}{s})(0.1145) = \underline{\underline{0.00229\frac{m}{s}}}$$

9.16

9.16 Approximately how fast can the wind blow past a 0.25-in.-diameter twig if viscous effects are to be of importance throughout the entire flow field (i.e., Re < 1)? Explain. Repeat for a 0.004-in.-diameter hair and a 6-ft-diameter smokestack.

$Re = \frac{UD}{\nu} < 1$ or $U < \frac{\nu}{D}$ if viscous effects are to be important throughout the flow.

For standard air $\nu = 1.57 \times 10^{-4}\ \frac{ft^2}{s}$

Thus,

$U < \frac{1.57 \times 10^{-4}}{D}$, where D is the diameter in feet.

object	D, ft	U, $\frac{ft}{s}$
twig	2.08×10^{-2}	7.54×10^{-3}
hair	3.33×10^{-4}	0.471
smokestack	6	2.62×10^{-5}

9.17

9.17 As is indicated in Table 9.2, the laminar boundary layer results obtained from the momentum integral equation are relatively insensitive to the shape of the assumed velocity profile. Consider the profile given by $u = U$ for $y > \delta$, and $u = U\{1 - [(y - \delta)/\delta]^2\}^{1/2}$ for $y \leq \delta$ as shown in Fig. P9.17. Note that this satisfies the conditions $u = 0$ at $y = 0$ and $u = U$ at $y = \delta$. However, show that such a profile produces meaningless results when used with the momentum integral equation. Explain.

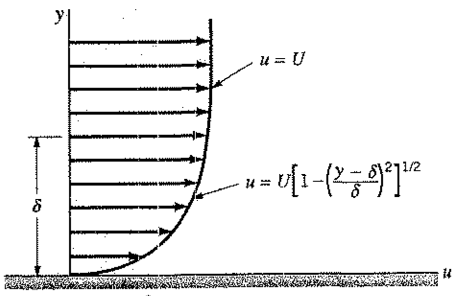


FIGURE P9.17

From the momentum integral equation

$$\delta = \sqrt{\frac{2C_2 \nu x}{U C_1}}, \quad \text{where } \frac{u}{U} = g(Y) = \left[1 - (Y-1)^2\right]^{\frac{1}{2}} \qquad (1)$$

Note: $\frac{u}{U} = 0$ at $Y=0$ and $\frac{u}{U} = 1$ and $Y=1$, as required.

Also, $C_1 = \int_0^1 g(1-g)\,dY$ which can be evaluated for the given $g(Y)$.

However,

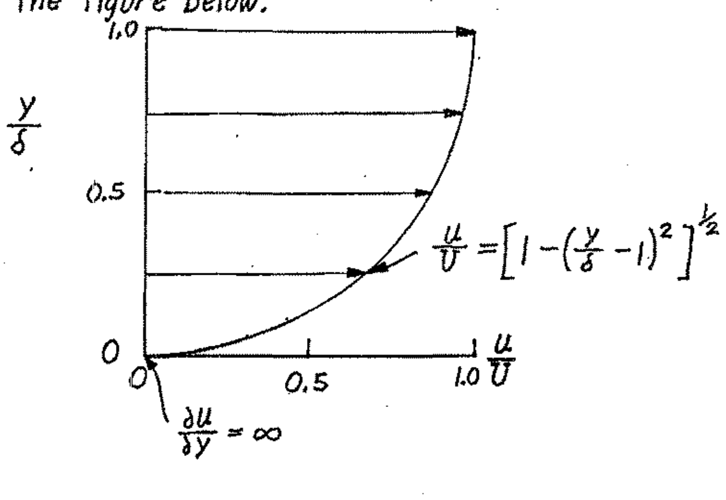
$$C_2 = \left.\frac{dg}{dY}\right|_{Y=0}, \text{ or since } \frac{dg}{dY} = \frac{1}{2}\left[1-(Y-1)^2\right]^{-\frac{1}{2}}(-2)(Y-1) = \frac{(1-Y)}{\left[1-(Y-1)^2\right]^{\frac{1}{2}}}$$

Thus,

$C_2 = \infty$, which from Eq.(1) gives $\delta = \infty$

This profile cannot be used since it gives $\delta = \infty$ due to the physically unrealistic $\frac{\partial u}{\partial y} = \infty$ at the surface $(y=0)$.

See the figure below.



9.19

9.19 Because of the velocity deficit, $U - u$, in the boundary layer, the streamlines for flow past a flat plate are not exactly parallel to the plate. This deviation can be determined by use of the displacement thickness, δ^*. For air blowing past the flat plate shown in Fig. P9.19, plot the streamline A–B that passes through the edge of the boundary layer ($y = \delta_B$ at $x = \ell$) at point B. That is, plot $y = y(x)$ for streamline A–B. Assume laminar boundary layer flow.

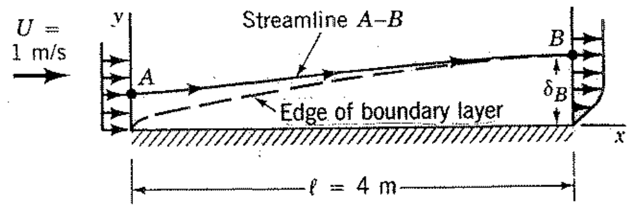
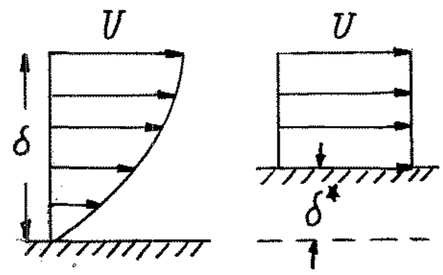


FIGURE P9.19

Since $Re_\ell = \dfrac{U\ell}{\nu} = \dfrac{(1\frac{m}{s})(4m)}{1.46\times10^{-5}\frac{m^2}{s}} = 2.74\times10^5 < 5\times10^5$, the boundary layer flow remains laminar along the entire plate. Hence,

$$\delta = 5\sqrt{\frac{\nu x}{U}} \quad \text{or} \quad \delta_B = 5\left[\frac{(1.46\times10^{-5}\frac{m^2}{s})(4m)}{1\frac{m}{s}}\right]^{1/2} = 0.0382\,m$$

The flowrate carried by the actual boundary layer is by definition equal to that carried by a uniform velocity with the plate displaced by an amount δ^*. Since there is no flow through the plate or streamline A-B,



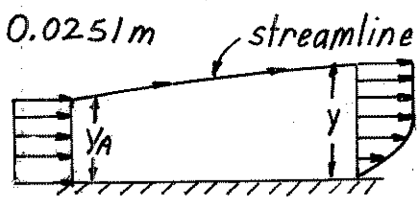
$$Q_A = Q_B, \text{ or } U y_A = (\delta_B - \delta_B^*)U \qquad (1)$$

where $\delta^* = 1.721\sqrt{\dfrac{\nu x}{U}}$

or $\delta_B^* = 1.721\left[\dfrac{(1.46\times10^{-5}\frac{m^2}{s})(4m)}{1\frac{m}{s}}\right]^{1/2} = 0.01315\,m$

Thus,

$$y_A = \delta_B - \delta_B^* = 0.0382\,m - 0.01315\,m = 0.0251\,m$$



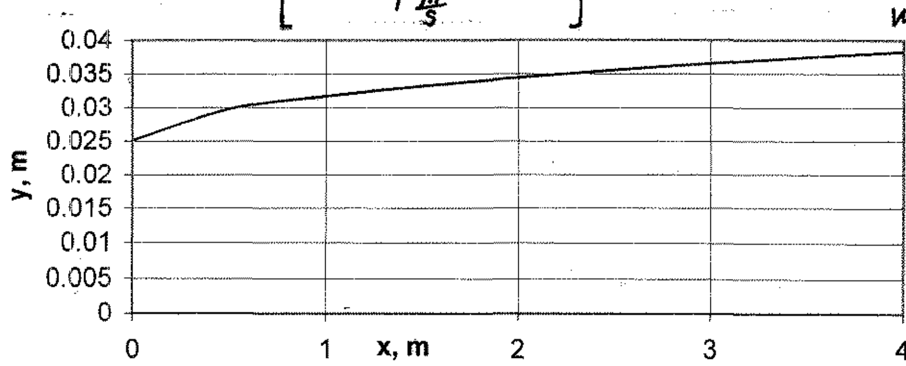
Hence, for any x-location

$Q_A = Q$ or $U y_A = U(y - \delta^*)$

or $y = y_A + \delta^* = y_A + 1.721\sqrt{\dfrac{\nu x}{U}}$

$$= 0.0251\,m + 1.721\left[\frac{(1.46\times10^{-5}\frac{m^2}{s})\,x\,m}{1\frac{m}{s}}\right]^{\frac{1}{2}} = \underline{\underline{0.0251 + 6.58\times10^{-3}\sqrt{x}\ m,}}$$

where $x \sim m$



9.20

9.20 Air enters a square duct through a 1-ft opening as is shown in Fig. P9.20. Because the boundary layer displacement thickness increases in the direction of flow, it is necessary to increase the cross-sectional size of the duct if a constant $U = 2$ ft/s velocity is to be maintained outside the boundary layer. Plot a graph of the duct size, d, as a function of x for $0 \leq x \leq 10$ ft if U is to remain constant. Assume laminar flow.

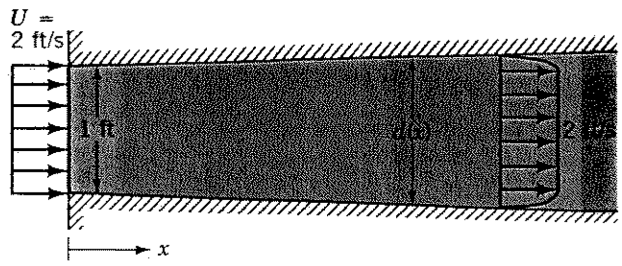


FIGURE P9.20

For incompressible flow $Q_0 = Q(x)$ where Q_0 = flowrate into the duct $= UA_0 = (2\,\frac{ft}{s})(1\,ft^2) = 2\,\frac{ft^3}{s}$

and

$Q(x) = UA$, where $A = (d - 2\delta^*)^2$ is the effective area of the duct (allowing for the decreased flowrate in the boundary layer).

Thus,

$$Q_0 = U(d - 2\delta^*)^2 \quad \text{or} \quad d = 1\,ft + 2\delta^*, \tag{1}$$

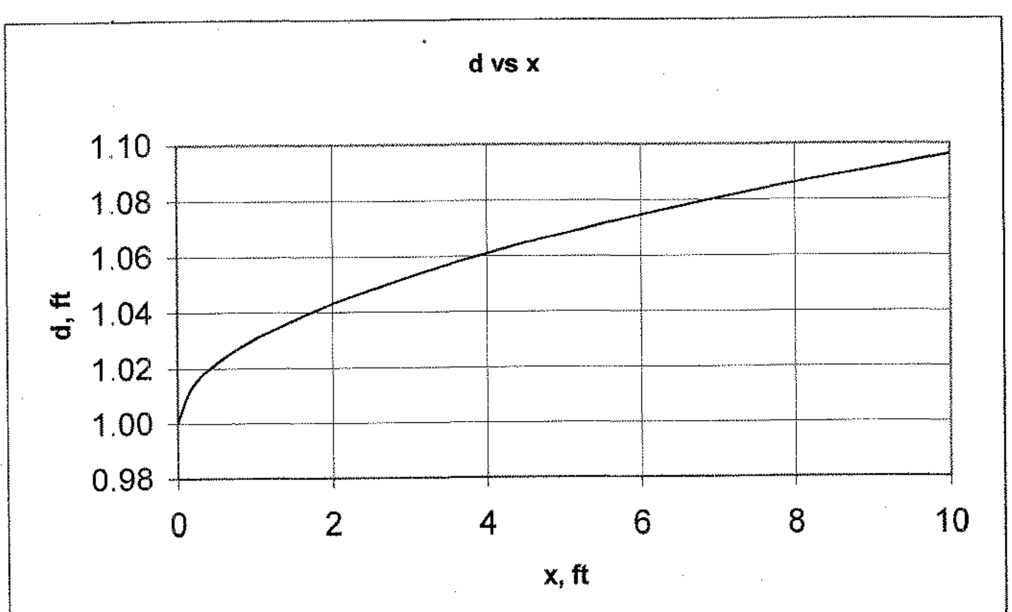
where

$$\delta^* = 1.721\sqrt{\frac{\nu x}{U}} = 1.721\left[\frac{(1.57\times10^{-4}\,\frac{ft^2}{s})\,x}{2\,\frac{ft}{s}}\right]^{1/2} = 0.0152\sqrt{x}\ ft, \text{ where } x \sim ft$$

Hence, from Eq. (1)

$$d = \underline{\underline{1 + 0.0304\sqrt{x}\ ft}}$$

For example, $d = 1$ ft at $x = 0$ and $d = 1.096$ ft at $x = 10$ ft.



9.21

9.21 A smooth, flat plate of length $\ell = 6$ m and width $b = 4$ m is placed in water with an upstream velocity of $U = 0.5$ m/s. Determine the boundary layer thickness and the wall shear stress at the center and the trailing edge of the plate. Assume a laminar boundary layer.

$$\delta = 5\sqrt{\frac{\nu X}{U}} = 5\sqrt{\frac{(1.12\times10^{-6}\,\frac{m^2}{s})\,X}{0.5\,\frac{m}{s}}} = 7.48\times10^{-3}\sqrt{X}\ m, \text{ where } X \sim m$$

and

$$\tau_w = 0.332\,U^{3/2}\sqrt{\frac{\rho\mu}{X}} = 0.332\,(0.5\tfrac{m}{s})^{3/2}\sqrt{\frac{(999\,\frac{kg}{m^3})(1.12\times10^{-3}\,\frac{N\cdot s}{m^2})}{X}}$$

$$= \frac{0.124}{\sqrt{X}}\ \frac{N}{m^2}, \text{ where } X \sim m$$

Thus, at $X = 3$ m $\quad \delta = 7.48\times10^{-3}\sqrt{3} = \underline{\underline{0.0130\ m}}$

$$\tau_w = \frac{0.124}{\sqrt{3}} = \underline{\underline{0.0716\ \frac{N}{m^2}}}$$

while at $X = 6$ m $\quad \delta = 7.48\times10^{-3}\sqrt{6} = \underline{\underline{0.0183\ m}}$

$$\tau_w = \frac{0.124}{\sqrt{6}} = \underline{\underline{0.0506\ \frac{N}{m^2}}}$$

9.22

9.22 An atmospheric boundary layer is formed when the wind blows over the earth's surface. Typically, such velocity profiles can be written as a power law: $u = ay^n$, where the constants a and n depend on the roughness of the terrain. As is indicated in Fig. P9. , typical values are $n = 0.40$ for urban areas, $n = 0.28$ for woodland or suburban areas, and $n = 0.16$ for flat open country (Ref. 23). **(a)** If the velocity is 20 ft/s at the bottom of the sail on your boat ($y = 4$ ft), what is the velocity at the top of the mast ($y = 30$ ft)? **(b)** If the average velocity is 10 mph on the tenth floor of an urban building, what is the average velocity on the sixtieth floor?

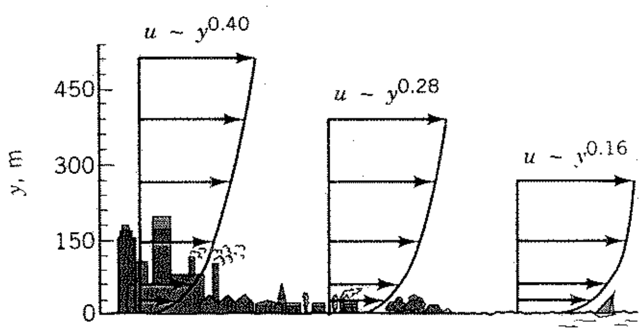


FIGURE P9.22

(a) $u = C\,y^{0.16}$, where C is a constant

Thus, $\dfrac{u_2}{u_1} = \left(\dfrac{y_2}{y_1}\right)^{0.16}$ or $u_2 = 20\,\frac{ft}{s}\left(\dfrac{30\,ft}{4\,ft}\right)^{0.16} = \underline{\underline{27.6\,\frac{ft}{s}}}$

(b) $u = \tilde{C}\,y^{0.4}$, where $\tilde{C}$ is a constant

Thus, $\dfrac{u_2}{u_1} = \left(\dfrac{y_2}{y_1}\right)^{0.40}$ or $u_2 = 10\,mph\left(\dfrac{60}{10}\right)^{0.4} = \underline{\underline{20.5\,mph}}$

9.24

9.24 A 30-story office building (each story is 12 ft tall) is built in a suburban industrial park. Plot the dynamic pressure, $\rho u^2/2$, as a function of elevation if the wind blows at hurricane strength (75 mph) at the top of the building. Use the atmospheric boundary layer information of Problem 9.22

From Fig. P9.22 the boundary layer velocity profile is given by $u \sim y^{0.28}$, or $u = C\,y^{0.28}$, where C is a constant.

Thus, $\dfrac{u}{u_1} = \left(\dfrac{y}{y_1}\right)^{0.28}$

or

$$u = 110\left(\frac{y}{360}\right)^{0.28} \frac{ft}{s} \text{ where } y \sim ft$$

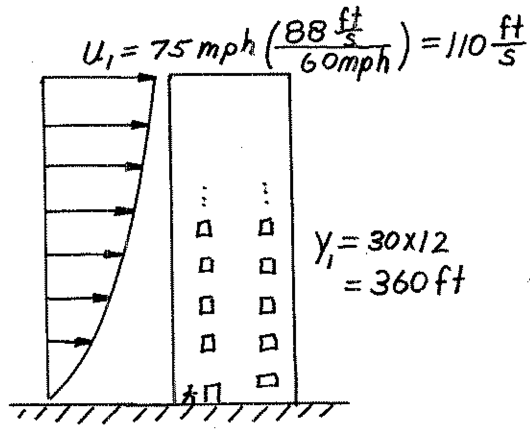
Hence,

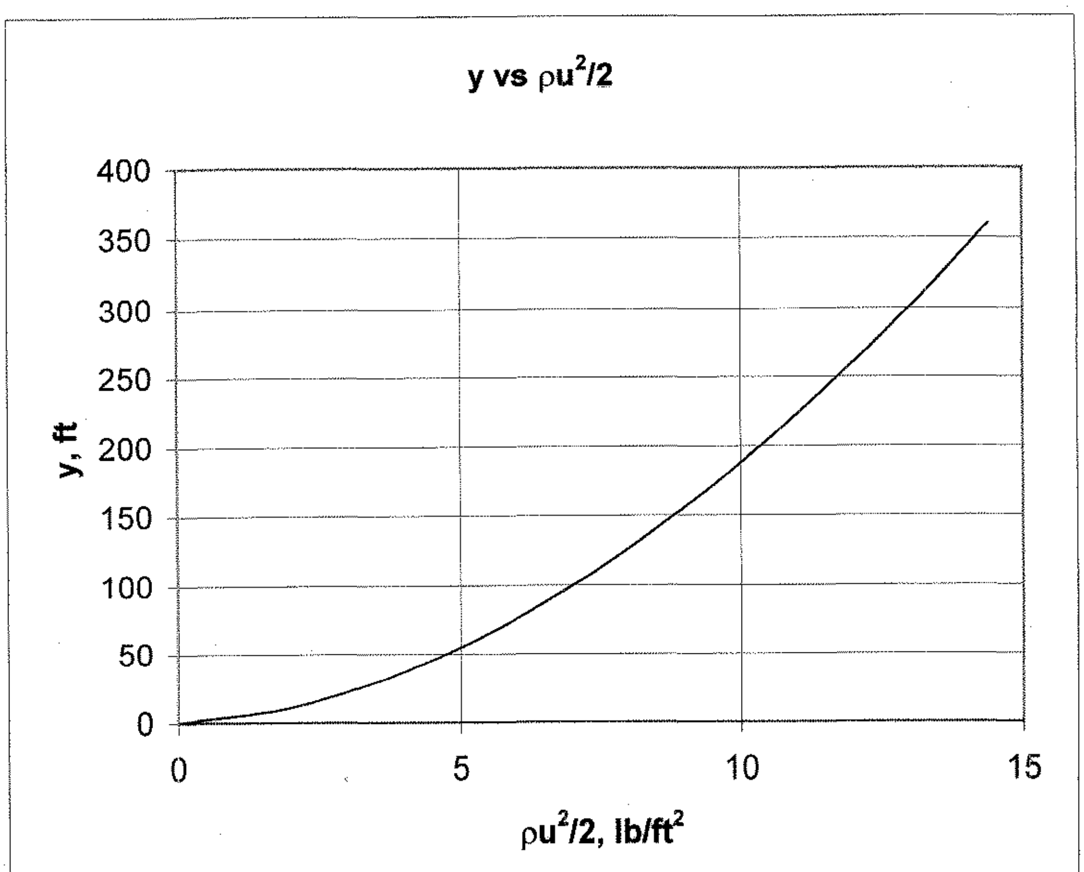
$$\frac{1}{2}\rho u^2 = \frac{1}{2}\left(2.38 \times 10^{-3} \frac{slug}{ft^3}\right)\left[110\left(\frac{y}{360}\right)^{0.28} \frac{ft}{s}\right]^2$$

or

$$\frac{1}{2}\rho u^2 = 14.4\left(\frac{y}{360}\right)^{0.56} \frac{lb}{ft^2}, \text{ where } y \sim ft$$

This is plotted in the figure below.





9.25

9.25 Show that for any function $f = f(\eta)$ the velocity components u and v determined by Eqs. 9.12 and 9.13 satisfy the incompressible continuity equation, Eq. 9.8.

Given $u = U f'(\eta)$, $v = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}(\eta f'(\eta) - f(\eta))$

where $\eta = \left(\frac{U}{\nu x}\right)^{\frac{1}{2}} y$ and $(\ )' \equiv \frac{d}{d\eta}$

Show that $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ for any $f(\eta)$.

$\frac{\partial u}{\partial x} = U \frac{\partial f'}{\partial x} = U \frac{df'}{d\eta}\frac{\partial \eta}{\partial x}$, where $\frac{\partial \eta}{\partial x} = -\frac{U^{\frac{1}{2}} y}{2\nu^{\frac{1}{2}}} x^{-\frac{3}{2}}$

Thus,

$$\frac{\partial u}{\partial x} = -U f''\left[\frac{U^{\frac{1}{2}} y}{2\nu^{\frac{1}{2}} x^{\frac{3}{2}}}\right] = -\frac{U^{\frac{3}{2}} y f''}{2 x^{\frac{3}{2}} \nu^{\frac{1}{2}}} \qquad (1)$$

and

$$\frac{\partial v}{\partial y} = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\left[\frac{\partial \eta}{\partial y} f' + \eta \frac{\partial f'}{\partial \eta}\frac{\partial \eta}{\partial y} - \frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial y}\right]$$

$$= \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\left[\frac{\partial \eta}{\partial y} f' + \eta f'' \frac{\partial \eta}{\partial y} - \frac{\partial \eta}{\partial y} f'\right]$$

$$= \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\left[\eta f'' \frac{\partial \eta}{\partial y}\right], \text{ where } \frac{\partial \eta}{\partial y} = \left(\frac{U}{\nu x}\right)^{\frac{1}{2}}$$

Hence,

$$\frac{\partial v}{\partial y} = \left(\frac{\nu U}{4x}\right)^{\frac{1}{2}}\left(\frac{U}{\nu x}\right)^{\frac{1}{2}} y f'' \left(\frac{U}{\nu x}\right)^{\frac{1}{2}} = \frac{U^{\frac{3}{2}} y f''}{2 x^{\frac{3}{2}} \nu^{\frac{1}{2}}} \qquad (2)$$

By combining Eqs. (1) and (2) we see that

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ for any function $f(\eta)$.

9.26

***9.26 Integrate the Blasius equation (Eq. 9.14) numerically to determine the boundary layer profile for laminar flow past a flat plate. Compare your results with those of Table 9.1.**

9.26* Integrate the Blasius equation (Eq. 9.14) numerically to determine the boundary layer profile for laminar flow past a flat plate. Compare your results with those of Table 9.1.

Solve the following third order differential equation by a numerical integration technique:

$2f''' + ff'' = 0$ with boundary conditions

$f = f' = 0$ at $\eta = 0$ and $f' \to 1$ as $\eta \to \infty$ $\quad \left(( )' \equiv \frac{d}{d\eta} \right)$

Write this third order equation as 3 first order equations and use a Runga-Kutta numerical technique to integrate them. Thus, let

$y_1 \equiv f$, $y_1' = f' \equiv y_2$, $y_2' = f'' \equiv y_3$, and $y_3' = f''' = -\frac{1}{2}ff'' = -\frac{1}{2}y_1 y_3$

That is:

$y_1' = y_2$

$y_2' = y_3$ and

$y_3' = -y_1 y_3/2$

These can be approximated as

$\Delta y_1 = y_2 \Delta\eta$, $\Delta y_2 = y_3 \Delta\eta$, and $\Delta y_3 = (-y_1 y_3/2)\Delta\eta$

Start with $y_1 = y_2 = 0$ at $\eta = 0$. Assume $y_3 = C$ at $\eta = 0$ (where C is some given constant) and "integrate to $\eta = \infty$" by $y_i = y_i(0) + \sum_j \Delta y_{ij} \Delta\eta$

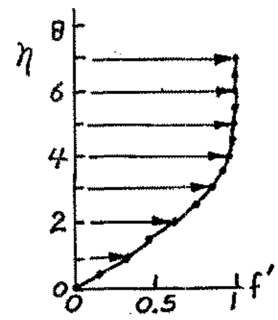
If $y_2(\infty) \neq 1$ (i.e., $f'(\infty) \neq 1$) adjust the value of C (i.e., $f''(0)$) and try again. The two-point boundary value problem (i.e., $f(0) = f'(0) = 0$ and $f'(\infty) = 1$) is solved by iteration as an initial value problem (i.e., $f(0) = f'(0) = 0$, $f''(0) = C$).

A step size of $\Delta\eta = 0.01$ was used, with $0 < \eta < 7$. That is, 700 steps were used. A value of $C = 0.332$ was found to give $f'(\infty) = 1$, or actually $f'(7) = 1$. This value of C and the corresponding velocity profile, $u = f'(\eta)$, shown on the next page agree very well with the standard values given in Table 9.1.

(con't)

9.26 (con't)

eta	f	f'	f''
0.5000	+4.07E-02	+1.66E-01	+3.31E-01
1.0000	+1.64E-01	+3.30E-01	+3.23E-01
1.5000	+3.68E-01	+4.87E-01	+3.03E-01
2.0000	+6.47E-01	+6.30E-01	+2.67E-01
2.5000	+9.93E-01	+7.52E-01	+2.17E-01
3.0000	+1.39E+00	+8.47E-01	+1.61E-01
3.5000	+1.83E+00	+9.14E-01	+1.07E-01
4.0000	+2.30E+00	+9.56E-01	+6.38E-02
4.5000	+2.79E+00	+9.80E-01	+3.36E-02
5.0000	+3.28E+00	+9.92E-01	+1.56E-02
5.5000	+3.78E+00	+9.97E-01	+6.41E-03
6.0000	+4.28E+00	+9.99E-01	+2.32E-03
6.5001	+4.78E+00	+1.00E+00	+7.36E-04
7.0001	+5.28E+00	+1.00E+00	+2.06E-04



9.27

9.27 An airplane flies at a speed of 400 mph at an altitude of 10,000 ft. If the boundary layers on the wing surfaces behave as those on a flat plate, estimate the extent of laminar boundary layer flow along the wing. Assume a transitional Reynolds number of $Re_{xcr} = 5 \times 10^5$. If the airplane maintains its 400-mph speed but descends to sea level elevation, will the portion of the wing covered by a laminar boundary layer increase or decrease compared with its value at 10,000 ft? Explain.

At 10,000 ft:

(a) $Re_{x_{cr}} = \dfrac{U x_{cr}}{\nu}$, where $U = 400\,mph\left(\dfrac{1\,hr}{3600\,s}\right)\left(\dfrac{5280\,ft}{mi}\right) = 587\,\dfrac{ft}{s}$

and from Table C.1, $\nu = \dfrac{\mu}{\rho} = \dfrac{3.534 \times 10^{-7}\,\frac{lb \cdot s}{ft^2}}{1.756 \times 10^{-3}\,\frac{slug}{ft^3}}$

$= 2.01 \times 10^{-4}\,\dfrac{ft^2}{s}$

Hence, with $Re_{x_{cr}} = 5 \times 10^5$,

$$x_{cr} = \frac{\nu\, Re_{x_{cr}}}{U} = \frac{\left(2.01 \times 10^{-4}\,\frac{ft^2}{s}\right)(5 \times 10^5)}{587\,\frac{ft}{s}} = \underline{\underline{0.171\,ft}}$$

At sea-level:

(b) $Re_{x_{cr}} = \dfrac{U x_{cr}}{\nu}$, where $U = 400\,mph\left(\dfrac{1\,hr}{3600\,s}\right)\left(\dfrac{5280\,ft}{mi}\right) = 587\,\dfrac{ft}{s}$

and $\nu = 1.57 \times 10^{-4}\,\dfrac{ft^2}{s}$

Hence,

$$x_{cr} = \frac{\nu\, Re_{cr}}{U} = \frac{\left(1.57 \times 10^{-4}\,\frac{ft^2}{s}\right)(5 \times 10^5)}{587\,\frac{ft}{s}} = \underline{\underline{0.134\,ft}}$$

The laminar boundary layer occupies the first 0.134 ft of the wing at sea level and (from part (a) above) the first 0.171 ft at an altitude of 10,000 ft. This is due mainly to the lower density (larger kinematic viscosity). The dynamic viscosities are approximately the same.

9.29

9.29 A laminar boundary layer velocity profile is approximated by $u/U = [2 - (y/\delta)](y/\delta)$ for $y \le \delta$, and $u = U$ for $y > \delta$. **(a)** Show that this profile satisfies the appropriate boundary conditions. **(b)** Use the momentum integral equation to determine the boundary layer thickness, $\delta = \delta(x)$.

(a) $\frac{u}{U} = g(Y) = 2Y - Y^2$ where $Y = y/\delta$

Thus, $\left.\frac{u}{U}\right|_{y=0} = 0$ as it must, $\left.\frac{u}{U}\right|_{y=\delta} = 2-1=1$ or $u=U$ at $y=\delta$ as it must.

Also, $\frac{du}{dy} = U\left[\frac{2}{\delta} - \frac{2y}{\delta^2}\right]$ so that $\left.\frac{du}{dy}\right|_{y=\delta} = U\left[\frac{2}{\delta} - \frac{2}{\delta}\right] = 0$

(b) From the momentum integral equation,

$$\delta = \sqrt{\frac{2C_2 \nu x}{U C_1}}, \text{ where } C_1 = \int_0^1 g(1-g)\,dY \text{ and } C_2 = \left.\frac{dg}{dY}\right|_{Y=0}$$

Thus,

$$C_1 = \int_0^1 (2Y - Y^2)(1-2Y+Y^2)\,dY = \int_0^1 (2Y - 5Y^2 + 4Y^3 - Y^4)\,dY$$

$$= 1 - \frac{5}{3} + 1 - \frac{1}{5} = \frac{2}{15}$$

and

$$C_2 = (2 - 2Y)\Big|_{Y=0} = 2$$

so that

$$\delta = \sqrt{\frac{2(2)\nu x}{\frac{2}{15} U}} = \sqrt{\frac{30 \nu x}{U}}$$

Hence, with $Re_x = \frac{Ux}{\nu}$,

$$\frac{\delta}{x} = \frac{\sqrt{30}}{\sqrt{Re_x}} = \underline{\underline{\frac{5.48}{\sqrt{Re_x}}}}$$

9.30

9.30 A laminar boundary layer velocity profile is approximated by the two straight-line segments indicated in Fig. P9.30 Use the momentum integral equation to determine the boundary layer thickness, $\delta = \delta(x)$, and wall shear stress, $\tau_w = \tau_w(x)$. Compare these results with those in Table 9.2.

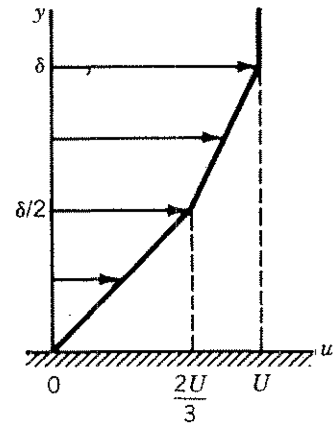


FIGURE P9.30

From the momentum integral equation

$$\delta = \sqrt{\frac{2C_2 \nu x}{U C_1}}, \text{ where } C_1 = \int_0^1 g(1-g)\,dY \text{ and } C_2 = \left.\frac{dg}{dY}\right|_{Y=0} \qquad (1)$$

and $\frac{u}{U} = g(Y)$ with $Y = \frac{y}{\delta}$.

For $0 \le Y < \frac{1}{2}$, $g = a_1 + b_1 Y$ with the constants a_1 and b_1 obtained from $g = \frac{2}{3}$ at $Y = \frac{1}{2}$ and $g = 0$ at $Y = 0$. Thus, $a_1 = 0$, $b_1 = \frac{4}{3}$

or $g = \frac{4}{3}Y$ for $0 \le Y < \frac{1}{2}$

Hence, $C_2 = \frac{4}{3}$ (2)

Similarly, for $\frac{1}{2} \le Y \le 1$, $g = a_2 + b_2 Y$ with $g = \frac{2}{3}$ at $Y = \frac{1}{2}$ and $g = 1$ at $Y = 1$

Thus, $\frac{2}{3} = a_2 + \frac{1}{2}b_2$ and $1 = a_2 + b_2$ which give $a_2 = \frac{1}{3}$, $b_2 = \frac{2}{3}$

or $g = \frac{1}{3} + \frac{2}{3}Y$, for $\frac{1}{2} \le Y < 1$

Hence, $C_1 = \int_0^1 g(1-g)\,dY = \int_0^{1/2} \frac{4}{3}Y\left(1 - \frac{4}{3}Y\right)dY + \int_{1/2}^1 \left(\frac{1}{3} + \frac{2}{3}Y\right)\left(1 - \frac{1}{3} - \frac{2}{3}Y\right)dY$

$= \frac{4}{9}\int_0^{1/2} (3Y - 4Y^2)\,dY + \frac{2}{9}\int_{1/2}^1 (1+2Y)(1-Y)\,dY$ which upon integration gives

$C_1 = 0.1574$ (3)

By combining Eqs. (1), (2), and (3) we obtain

$$\delta = \left[\frac{2\left(\frac{4}{3}\right)\nu x}{0.1574\, U}\right]^{1/2} = \underline{\underline{4.12\sqrt{\frac{\nu x}{U}}}} \text{ or } \frac{\delta\, Re_x^{1/2}}{x} = 4.12$$

Also,

$$\tau_w = \frac{\mu U}{\delta} C_2 = \underline{\underline{\frac{4\mu U}{3\delta}}} \text{ or } c_f = \frac{\sqrt{2C_1C_2}}{\sqrt{Re_x}} = \frac{\sqrt{2(0.1574)\left(\frac{4}{3}\right)}}{\sqrt{Re_x}} = \frac{0.648}{\sqrt{Re_x}}$$

Compare these results to those in Table 9.2.

9.31*

9.31* For a fluid of specific gravity $SG = 0.86$ flowing past a flat plate with an upstream velocity of $U = 5$ m/s, the wall shear stress on a flat plate was determined to be as indicated in the table below. Use the momentum integral equation to determine the boundary layer momentum thickness, $\Theta = \Theta(x)$. Assume $\Theta = 0$ at the leading edge, $x = 0$.

Since $\tau_w = \rho U^2 \frac{d\Theta}{dx}$ it follows that $d\Theta = \frac{\tau_w}{\rho U^2}dx$

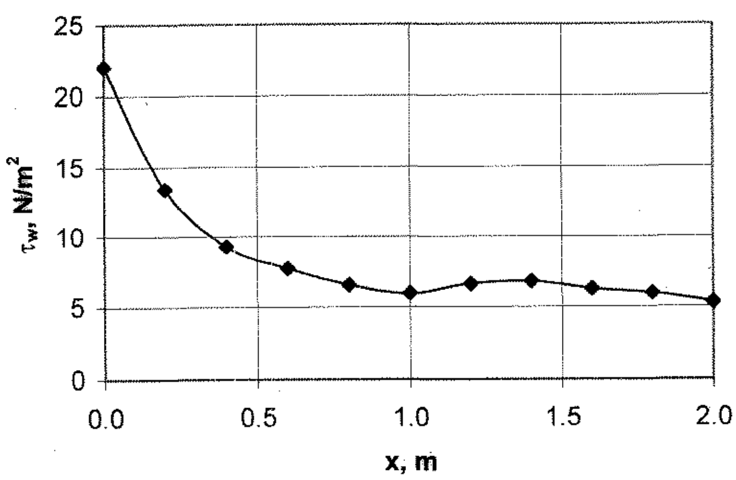
which can be integrated to give (using $\Theta = 0$ at $x = 0$)

$$\Theta = \frac{1}{\rho U^2}\int_0^x \tau_w\, dx = \frac{1}{(0.86)(1000\,\frac{kg}{m^3})(5\,\frac{m}{s})^2}\int_0^x \tau_w\, dx$$

or

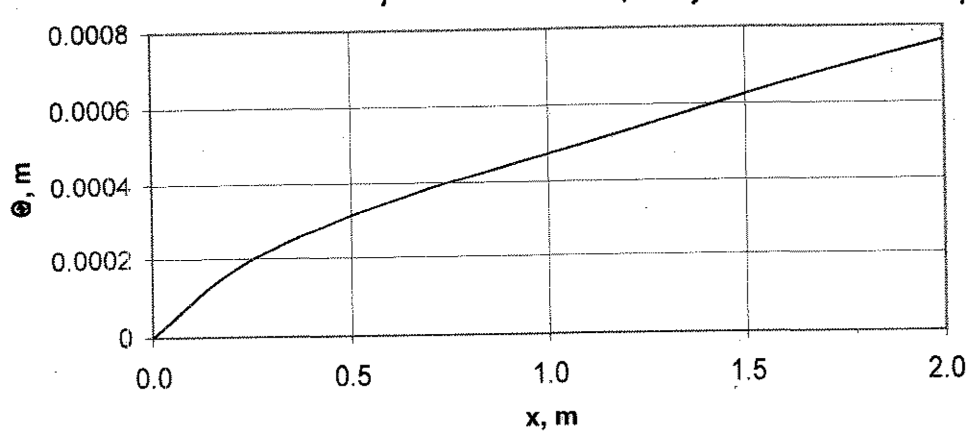
$$\Theta = 4.65\times10^{-5}\int_0^x \tau_w\, dx \;, \text{ where } \Theta \sim m,\; x \sim m, \text{ and } \tau_w \sim \frac{N}{m^2} \qquad (1)$$

For $0 \le x \le 2.0$ m, integrate Eq. (1) to determine Θ as a function of x. To do so, we need the value of τ_w at $x = 0$, which is not given in the table. Theoretically, $\tau_w = \infty$ at the leading. For our purposes, based on the extrapolated curve below, assume $\tau_w = 22\,\frac{N}{m^2}$ at $x = 0$



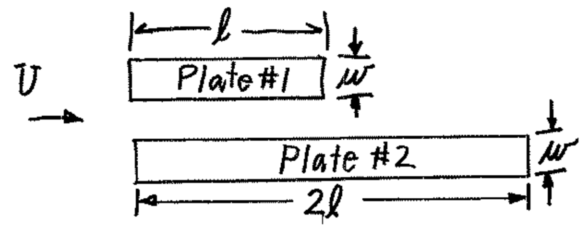
x (m)	τ_w (N/m²)
0	—
0.2	13.4
0.4	9.25
0.6	7.68
0.8	6.51
1.0	5.89
1.2	6.57
1.4	6.75
1.6	6.23
1.8	5.92
2.0	5.26

A standard numerical integration technique gives the following results.



9.35

9.35 Water flows over two flat plates with the same laminar free-stream velocity. Both plates have the same width, but Plate #2 is twice as long as Plate #1. What is the relationship between the drag force for these two plates?



$$\mathscr{D} = C_D \tfrac{1}{2}\rho U^2 A$$

Thus,

$$\mathscr{D}_1 = C_{D_1} \tfrac{1}{2}\rho U^2 \ell w$$

and

$$\mathscr{D}_2 = C_{D_2} \tfrac{1}{2}\rho U^2 (2\ell w) \text{ or}$$

$$\frac{\mathscr{D}_2}{\mathscr{D}_1} = \frac{C_{D_2}}{C_{D_1}} \frac{(2\ell w)}{\ell w} = 2\frac{C_{D_2}}{C_{D_1}} \qquad (1)$$

For laminar flow on a flat plate

$$C_D = \frac{1.328}{\sqrt{Re_\ell}}, \text{ where } Re_\ell = \frac{U\ell}{\nu}, \text{ so that } C_D = \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}$$

Thus,

$$\frac{C_{D_2}}{C_{D_1}} = \left(\frac{1.328\sqrt{\nu}}{\sqrt{U(2\ell)}}\right) \Big/ \left(\frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}\right) = \frac{1}{\sqrt{2}} \qquad (2)$$

Hence, from Eqs. (1) and (2),

$$\frac{\mathscr{D}_2}{\mathscr{D}_1} = 2/\sqrt{2} = \underline{\underline{1.414}}$$

9.36

9.36 Fluid flows past a flat plate with a drag force $\mathcal{D}_1$. If the freestream velocity is doubled, will the new drag force, $\mathcal{D}_2$, be larger or smaller than $\mathcal{D}_1$ and by what amount?

$$\mathcal{D} = C_D \tfrac{1}{2}\rho U^2 A$$

If you assume that the doubling of U, which will change Re, does not significantly change C_D (see Fig. 9.22), then

$$\frac{\mathcal{D}_1}{\mathcal{D}_2} = \frac{C_D \frac{1}{2}\rho U_1^2 A}{C_D \frac{1}{2}\rho U_2^2 A} = \frac{U_1^2}{U_2^2} \quad \text{where } U_2 = 2U_1$$

$$= \frac{U_1^2}{(2U_1)^2} = \frac{1}{4}$$

So,

$$\underline{\underline{\mathcal{D}_2 = 4\mathcal{D}_1}}$$

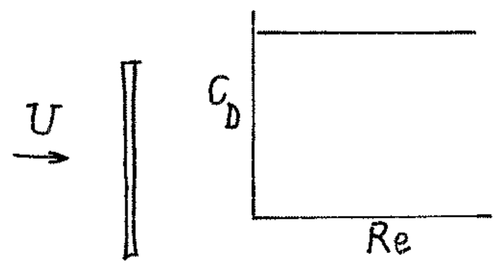


plate normal to flow

Note:

If the plate is parallel to the flow, then C_D changes with Re. See Fig. 9.22.

Thus,

$$\frac{D_1}{D_2} = \frac{C_{D_1} U_1^2}{C_{D_2} U_2^2}$$

so that a numerical answer could not be obtained without additional data about the value of Re.

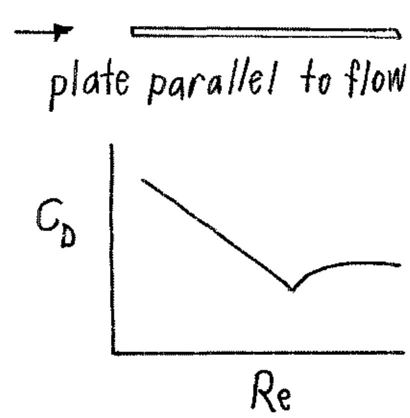


plate parallel to flow

9.37

9.37 A model is placed in an air flow with a given velocity and then placed in water flow with the same velocity. If the drag coefficients are the same between these two cases, how do the drag forces compare between the two fluids?

$$\left(\frac{\mathcal{D}}{\frac{1}{2}\rho U^2 A}\right)_w = \left(\frac{\mathcal{D}}{\frac{1}{2}\rho U^2 A}\right)_a$$

$$\frac{\mathcal{D}_w}{\rho_w} = \frac{\mathcal{D}_a}{\rho_a}$$

$$\frac{\mathcal{D}_w}{\mathcal{D}_a} = \frac{\rho_w}{\rho_a} \quad \text{where } \rho_w \gg \rho_a$$

$$\underline{\underline{\mathcal{D}_w \gg \mathcal{D}_a}}$$

It should be noted that since $Re = \frac{U\ell}{\nu}$, matching ν_w and ν_a would be difficult. Therefore, depending on shape and velocity, the C_D values may not actually be the same. However, this difference would be small compared to the density difference.

Note: At standard conditions,

$$\frac{\mathcal{D}_w}{\mathcal{D}_a} = \frac{\rho_w}{\rho_a} = \frac{1.94\ slugs/ft^3}{2.38\times10^{-3}\ slugs/ft^3} = 815$$

9.38

9.38 The drag coefficient for a newly designed hybrid car is predicted to be 0.21. The cross-sectional area of the car is 30 ft^2. Determine the aerodynamic drag on the car when it is driven through still air at 55 mph.

$$\mathcal{D} = C_D \tfrac{1}{2}\rho V^2 A$$

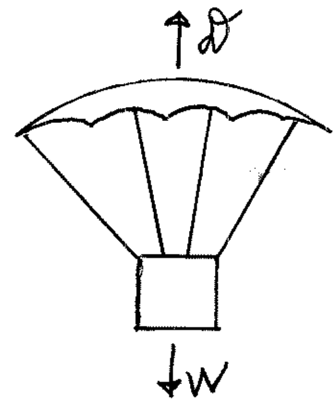
$$V = 55\,\text{mph} \times \frac{88\ \text{ft/s}}{60\,\text{mph}} = 80.7\ \text{ft/s}$$

$$\mathcal{D} = 0.21\left(\tfrac{1}{2}\right)(0.00238\ \text{slugs/ft}^3)(80.7\ \text{ft/s})^2(30\,\text{ft}^2)$$

$$\underline{\underline{\mathcal{D} = 48.8\ \text{lb}}}$$

9.39

9.39 A 5-m-diameter parachute of a new design is to be used to transport a load from flight altitude to the ground with an average vertical speed of 3 m/s. The total weight of the load and parachute is 200 N. Determine the approximate drag coefficient for the parachute.



$$C_D = \frac{\mathcal{D}}{\frac{1}{2}\rho V^2 A}$$

If in equilibrium, at constant velocity, then

$$W = \mathcal{D}$$

$$C_D = \frac{W}{\frac{1}{2}\rho V^2 A}$$

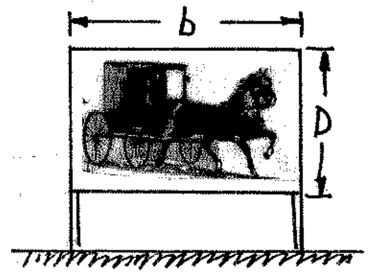
$$= \frac{200\,N}{\frac{1}{2}(1.23\,\frac{kg}{m^3})(3\,\frac{m}{s})^2 \frac{\pi}{4}(5\,m)^2}$$

$$\underline{\underline{C_D = 1.84}}$$

The sea-level density was used to solve this problem. Clearly during the drop, ρ will be changing, but the changes are relatively small.

9.40

9.40 A 50-mph wind blows against an outdoor movie screen that is 70 ft wide and 20 ft tall. Estimate the wind force on the screen.



$\mathcal{D} = C_D \frac{1}{2} \rho V^2 A$, where from Fig. 9.22 with $\frac{b}{D} = \frac{70\,ft}{20\,ft} = 3.5$ we obtain $C_D = 1.15$

Hence,

$$\mathcal{D} = 1.15\left(\frac{1}{2}\right)\left(0.00238\,\frac{slugs}{ft^3}\right)\left[\left(50\,\frac{mi}{hr}\right)\frac{\left(5280\,\frac{ft}{mi}\right)}{\left(3600\,\frac{s}{hr}\right)}\right]^2 (70\,ft)(20\,ft)$$

or

$$\underline{\mathcal{D} = 10{,}300\ lb}$$

9.41

9.41 The aerodynamic drag on a car depends on the "shape" of the car. For example, the car shown in Fig. P9.41 has a drag coefficient of 0.36 with the windows and roof closed. With the windows and roof open, the drag coefficient increases to 0.45. With the windows and roof open, at what speed is the amount of power needed to overcome aerodynamic drag the same as it is at 65 mph with the windows and roof closed? Assume the frontal area remains the same. Recall that power is force times velocity.

Windows and roof closed: $C_D = 0.35$ Windows open; roof open: $C_D = 0.45$

■ FIGURE P9.41

Power $= \mathcal{P} = F \cdot V$

The force is the drag force. Let $(\ )_c$ and $(\ )_o$ denote closed and open.

$$\mathcal{D} = C_D \tfrac{1}{2}\rho U^2 A$$

We want to find U_o when $\mathcal{P}_o = \mathcal{P}_c$

$$\mathcal{P}_o = U_o \mathcal{D}_o = \tfrac{1}{2}\rho U_o^3 A_o C_{Do} = \mathcal{P}_c = U_c \mathcal{D}_c = \tfrac{1}{2}\rho U_c^3 A_c C_{Dc}$$

The frontal areas are the same, so $A_o = A_c$

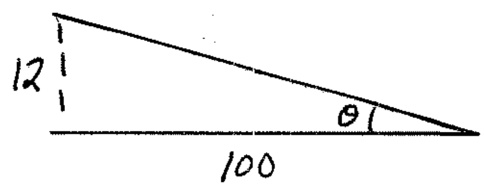
$$U_o^3 C_{Do} = U_c^3 C_{Dc}$$

$$U_o = U_c \left(\frac{C_{Dc}}{C_{Do}}\right)^{1/3} = (65\,\text{mph})\left(\frac{0.36}{0.45}\right)^{1/3}$$

$$\underline{\underline{U_o = 60.3\ \text{mph}}}$$

9.42

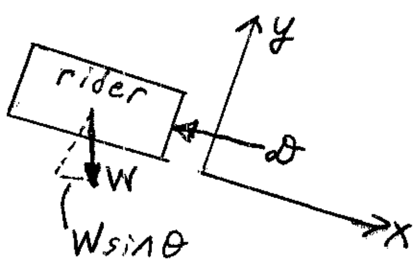
9.42 A rider on a bike with the combined mass of 100 kg attains a terminal speed of 15 m/s on a 12% slope. Assuming that the only forces affecting the speed are the weight and the drag, calculate the drag coefficient. The frontal area is 0.9 m^2. Speculate whether the rider is in the upright or racing position.



$\tan\theta = 12/100 = 0.12$

$\theta = 6.84°$

$\sin\theta = 0.119$



In equilibrium, $\Sigma F = 0$

$$\Sigma F_x = 0$$

$$W\sin\theta = D = C_D \tfrac{1}{2}\rho V^2 A, \text{ where } W = mg = (100\,kg)(9.81\,m/s^2) = 981\,N$$

$$C_D = \frac{W\sin\theta}{\tfrac{1}{2}\rho V^2 A}$$

$$= \frac{(981N)(0.119)}{\tfrac{1}{2}(1.23\,kg/m^3)(15\,m/s)^2(0.6\,m^2)}$$

$$\underline{\underline{C_D = 1.4}}$$

Looking at Fig. 9.30, given A and C_D, the rider is upright.

9.43

9.43 A baseball is thrown by a pitcher at 95 mph through standard air. The diameter of the baseball is 2.82 in. Estimate the drag force on the baseball.

$$\mathcal{D} = C_D \frac{1}{2} \rho U^2 A$$

$$U = 95\,mph \times \frac{88\,ft/s}{60\,mph} = 139.3\,ft/s$$

$$Re = \frac{UD}{\nu} = \frac{(139.3\,ft/s)\left(\frac{2.82}{12}\,ft\right)}{1.57 \times 10^{-4}\,ft^2/s} = 2.09 \times 10^5$$

From Fig. 9.25, and assuming a smooth sphere,

$$C_D \approx 0.5$$

$$\mathcal{D} = 0.5\left(\frac{1}{2}\right)\left(0.00238\,\frac{slugs}{ft^3}\right)(139.3\,ft/s)^2\left(\frac{\pi}{4}\left(\frac{2.82}{12}\right)^2\right)$$

$$\underline{\underline{\mathcal{D} = 0.502\,lb}}$$

9.44

9.44 A logging boat is towing a log that is 2 m in diameter and 8 m long at 4 m/s through water. Estimate the power required if the axis of the log is parallel to the tow direction.

For power, $\mathcal{P} = F \cdot V$

$$F = \mathcal{D} = C_D \tfrac{1}{2} \rho V^2 A$$

For the aspect ratio, $D = 2m$ and $l = 8m$

From Fig. 9.29,

$$\frac{l}{D} = \frac{8}{2} = 4, \text{ so } C_D = 0.85$$

$$\mathcal{D} = 0.85\left(\tfrac{1}{2}\right)\left(999\,\mathrm{kg/m^3}\right)\left(4\,\mathrm{m/s}\right)^2\left(\tfrac{\pi}{4}(2m)^2\right)$$

$$\mathcal{D} = 21{,}341\,N$$

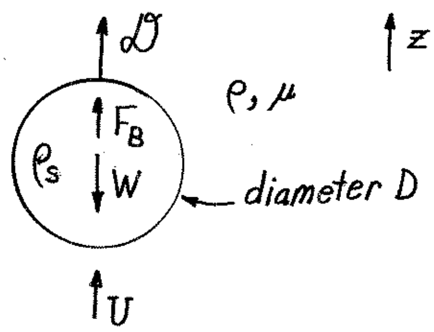
$$\mathcal{P} = \mathcal{D}V = (21{,}341\,N)(4\,\mathrm{m/s}) = 85{,}400\,W$$

$$= \underline{\underline{85.4\,kW}}$$

Note: The above $C_D = 0.85$ assumes that the log is essentially submerged and wave making is not an important contribution to the drag.

9.45

9.45 A sphere of diameter D and density ρ_s falls at a steady rate through a liquid of density ρ and viscosity μ. If the Reynolds number, $Re = \rho DU/\mu$, is less than 1, show that the viscosity can be determined from $\mu = gD^2(\rho_s - \rho)/18\,U$.



For steady flow $\sum F_z = 0$

or $\mathcal{D} + F_B = W$, where F_B = buoyant force $= \rho g \forall = \rho g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$

W = weight $= \rho_s g \forall = \rho_s g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$

and $\mathcal{D}$ = drag $= C_D \frac{1}{2}\rho \frac{\pi}{4} D^2$, or since $Re < 1$

$\mathcal{D} = 3\pi D U \mu$

Thus,

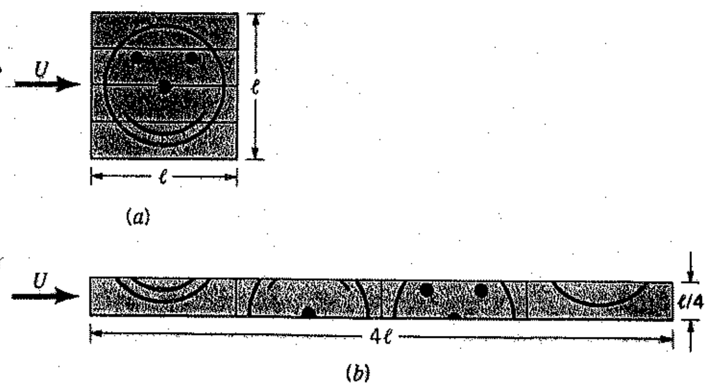
$$3\pi D U \mu + \rho g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3 = \rho_s g\left(\frac{4}{3}\right)\pi\left(\frac{D}{2}\right)^3$$

which can be rearranged to give

$$\mu = \frac{g D^2(\rho_s - \rho)}{18\,U}$$

9.46

9.46 The square flat plate shown in Fig. P9.46a is cut into four equal-sized prices and arranged as shown in Fig. P9.46b. Determine the ratio of the drag on the original plate [case (a)] to the drag on the plates in the configuration shown in (b). Assume laminar boundary flow. Explain your answer physically.



■ FIGURE P9.46

For case (a):

$$\mathscr{D}_{f_a} = \frac{1}{2}\rho U^2 C_{Df} A \quad \text{where } C_{Df} = \frac{1.328}{\sqrt{Re_\ell}} = \frac{1.328}{\sqrt{\frac{U\ell}{\nu}}} \text{ and } A = \ell^2$$

Thus,

$$\mathscr{D}_{f_a} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}\ell^2 = 0.664\,\rho U^{3/2}\sqrt{\nu}\,\ell^{3/2} \qquad (1)$$

For case (b):

$$\mathscr{D}_{f_b} = \frac{1}{2}\rho U^2 C_{Df} A \quad \text{where } C_{Df} = \frac{1.328}{\sqrt{\frac{U(4\ell)}{\nu}}} \quad \text{and } A = (4\ell)\left(\frac{\ell}{4}\right) = \ell^2$$

Thus,

$$\mathscr{D}_{f_b} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{4U\ell}}\ell^2 = \frac{1}{2}\left(0.664\,\rho U^{\frac{3}{2}}\sqrt{\nu}\,\ell^{\frac{3}{2}}\right) \qquad (2)$$

By comparing Eqs. (1) and (2) we see that

$$\underline{\underline{\mathscr{D}_{f_a} = 2.0\,\mathscr{D}_{f_b}}}$$

In case (b) the boundary layer on the rear plate is thicker than on the front plate. Hence the shear stress is less on the rear plate than it is on that plate in configuration (a), giving less drag for case (b) than for case (a), even though the total areas are the same.

9.47

9.47 If the drag on one side of a flat plate parallel to the upstream flow is $\mathcal{D}$ when the upstream velocity is U, what will the drag be when the upstream velocity is $2U$; or $U/2$? Assume laminar flow.

For laminar flow $\mathcal{D} = \frac{1}{2}\rho U^2 C_{Df} A$, where $C_{Df} = \dfrac{1.328}{\sqrt{\frac{U\ell}{\nu}}}$

Thus,

$$\mathcal{D} = \frac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}} A = 0.664\,\rho A \frac{\sqrt{\nu}}{\sqrt{\ell}} U^{3/2} \sim U^{3/2}$$

Hence,

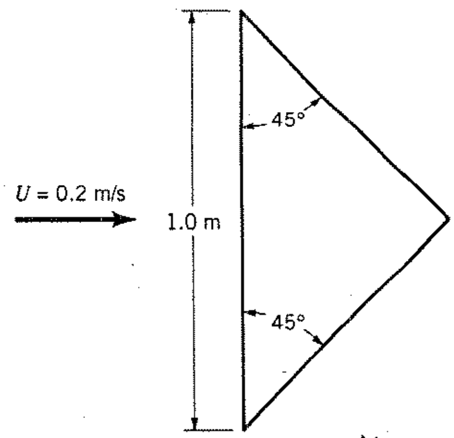
$$\frac{\mathcal{D}_U}{\mathcal{D}_{2U}} = \frac{U^{3/2}}{(2U)^{3/2}} \text{ or } \underline{\underline{\mathcal{D}_{2U} = 2.83\,\mathcal{D}_U}}$$

and

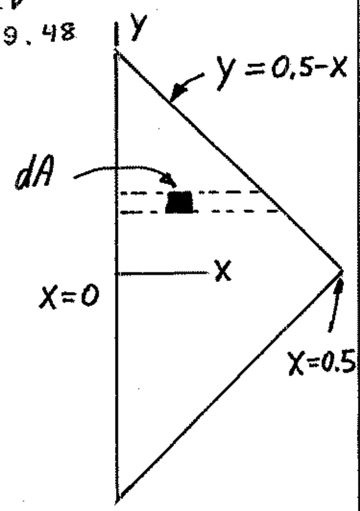
$$\frac{\mathcal{D}_U}{\mathcal{D}_{U/2}} \quad \frac{U^{3/2}}{\left(\frac{U}{2}\right)^{3/2}} \text{ or } \underline{\underline{\mathcal{D}_{U/2} = 0.354\,\mathcal{D}_U}}$$

9.48

9.48 Water flows past a triangular flat plate oriented parallel to the free stream as shown in Fig. P9.48 Integrate the wall shear stress over the plate to determine the friction drag on one side of the plate. Assume laminar boundary layer flow.



■ FIGURE P9.48



$$\mathcal{D} = \int \tau_w \, dA \quad \text{where} \quad \tau_w = 0.332\, U^{3/2} \sqrt{\frac{\rho \mu}{x}}$$

Thus,

$$\mathcal{D} = 0.332\, U^{3/2} \sqrt{\rho\mu} \int \frac{1}{\sqrt{x}}\, dA$$

$$= 0.332\, U^{3/2} \sqrt{\rho\mu}\,(2) \int_{x=0}^{x=0.5} \int_{y=0}^{y=0.5-x} \frac{dy\,dx}{\sqrt{x}}$$

$$= 0.332\, U^{3/2} \sqrt{\rho\mu}\,(2) \int_{x=0}^{0.5} \frac{0.5-x}{\sqrt{x}}\, dx$$

$$= 0.332\, U^{3/2} \sqrt{\rho\mu}\,(2) \left[0.5(2) x^{\frac{1}{2}} - \frac{2}{3} x^{\frac{3}{2}} \right]_0^{0.5}$$

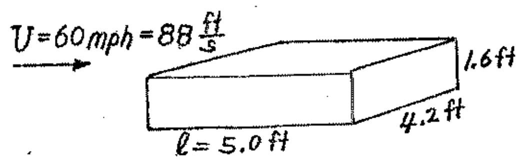
$$= 0.664 \left(0.2 \frac{m}{s}\right)^{3/2} \sqrt{999 \frac{kg}{m^3} \left(1.12 \times 10^{-3} \frac{N \cdot s}{m^2}\right)} \left[\sqrt{0.5} - \frac{2}{3}(0.5)^{3/2} \right]$$

or

$$\mathcal{D} = \underline{\underline{0.0296\, N}}$$

9.50

9.50 A rectangular car-top carrier of 1.6-ft height, 5.0-ft length (front to back), and 4.2-ft width is attached to the top of a car. Estimate the additional power required to drive the car with the carrier at 60 mph through still air compared with the power required to driving only the car at 60 mph.



$\mathcal{D} = C_D \frac{1}{2} \rho V^2 A$ and $\mathcal{P} = V\mathcal{D}$ = power (1)

From Fig. 9.31 with $\frac{\ell}{D} = \frac{5\ ft}{1.6\ ft} = 3.13$ we obtain $C_D = 1.3$

Hence,

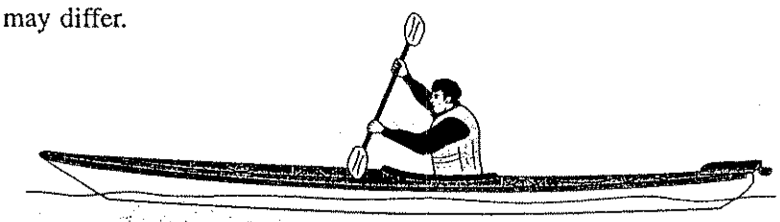
$$\mathcal{D} = 1.3\left(\tfrac{1}{2}\right)\left(0.00238\ \tfrac{slugs}{ft^3}\right)(1.6\,ft)(4.2\,ft)\left(88\ \tfrac{ft}{s}\right)^2 = 80.5\ lb$$

Thus, from Eq. (1),

$$\mathcal{P} = \left(88\ \tfrac{ft}{s}\right)(80.5\ lb)\frac{1\ hp}{550\ \frac{ft\cdot lb}{s}} = \underline{\underline{12.9\ hp}}$$

9.51

9.51 As shown in Video V9.2 and Fig. P9.51a a kayak is a relatively streamlined object. As a first approximation in calculating the drag on a kayak, assume that the kayak acts as if it were a smooth flat plate 17 ft long and 2 ft wide. Determine the drag as a function of speed and compare your results with the measured values given in Fig. P9.51b Comment on reasons why the two sets of values may differ.



(a)

For a flat plate $\mathcal{D} = \frac{1}{2}\rho U^2 C_{Df} A$ where (1)

$A = 17\,ft(2\,ft) = 34\,ft^2$ and C_{Df} is a function of $Re_\ell = \frac{U\ell}{\nu}$

Thus,

$$Re_\ell = \frac{17\,ft\; U}{1.21\times10^{-5}\,ft^2/s} = 1.40\times10^6\, U \qquad (2)$$

Consider $1 \le U \le 8\,\frac{ft}{s}$, or $1.40\times10^6 \le Re_\ell \le 1.12\times10^7$

From Fig. 9.15 we see that in this Re_ℓ range the boundary layer flow is in the transitional range. Thus, from Table 9.3

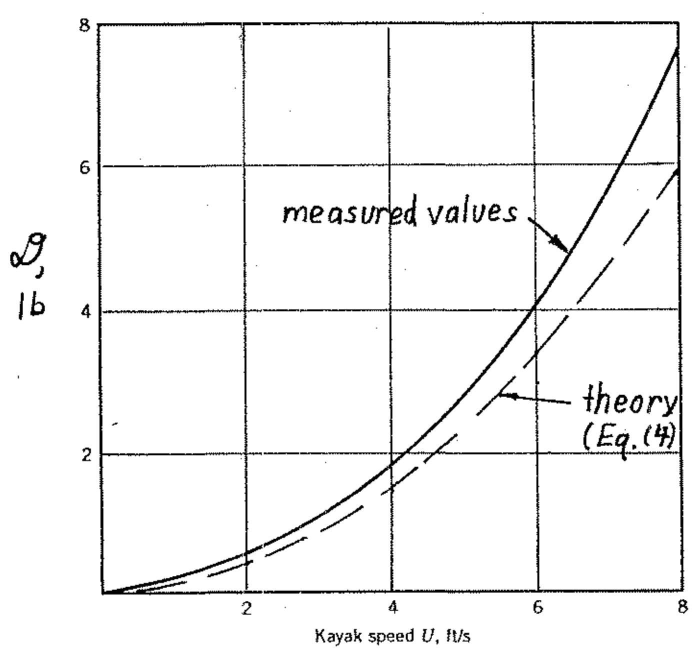
$$C_{Df} = 0.455/(\log Re_\ell)^{2.58} - 1700/Re_\ell \qquad (3)$$

By combining Eqs. (1), (2), and (3):

$$\mathcal{D} = \frac{1}{2}\left(1.94\,\frac{slugs}{ft^3}\right)U^2 C_{Df}\,(34\,ft^2) \quad \text{or}$$

$$\mathcal{D} = 33.0\,U^2\left[0.455/(\log(1.40\times10^6 U))^{2.58} - 1700/(1.40\times10^6\, U)\right] \qquad (4)$$

The results from this equation are plotted below.

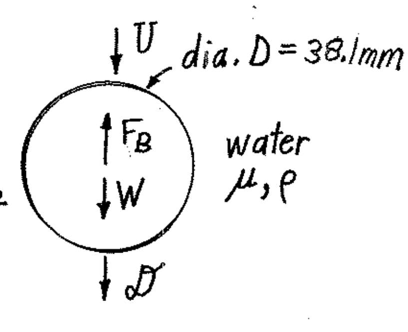


U, ft/s	$\mathcal{D}$, lb
1	0.0986
2	0.410
3	0.909
4	1.58
5	2.42
6	3.43
7	4.59
8	5.90

■ FIGURE P9.51(b)

9.52

9.52 A 38.1-mm-diameter, 0.0245-N table tennis ball is released from the bottom of a swimming pool. With what velocity does it rise to the surface? Assume it has reached its terminal velocity.



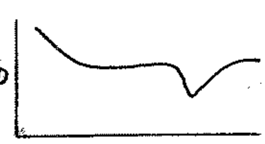
For steady rise $\sum F_z = 0$
or
$F_B = W + \mathcal{D}$, where $\mathcal{D} = \text{drag} = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$

$W = \text{weight} = 0.0245\ N$

$F_B = \text{buoyant force} = \gamma \forall = \gamma \frac{4\pi}{3}\left(\frac{D}{2}\right)^3$

Thus,
$$\gamma \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = W + C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$$
or
$$\left(9.80\times10^3 \frac{N}{m^3}\right)\frac{4\pi}{3}\left(\frac{0.0381}{2}\right)^3 m = 0.0245\,N + \frac{1}{2}C_D\left(999\ \frac{kg}{m^3}\right)U^2 \frac{\pi}{4}(0.0381\,m)^3$$

or
$$C_D U^2 = 0.455\,, \text{ where } U \sim \frac{m}{s} \qquad (1)$$
Also, $Re = \frac{UD}{\nu}$
or
$$Re = \frac{U\,(0.0381\,m)}{1.12\times10^{-6}\,\frac{m^2}{s}} = 3.40\times10^4\,U, \text{ where } U \sim \frac{m}{s} \qquad (2)$$

Finally, from Fig. 9.21: C_D  (3)

Re

Trial and error solution: Assume C_D; obtain U from Eq.(1), Re from Eq.(2); check C_D from Eq.(3), the graph.

Assume $C_D = 0.5 \longrightarrow U = 0.954\,\frac{m}{s} \longrightarrow Re = 3.24\times10^4 \longrightarrow C_D = 0.4 \neq 0.5$

Assume $C_D = 0.4 \longrightarrow U = 1.06\,\frac{m}{s} \longrightarrow Re = 3.62\times10^4 \longrightarrow C_D = 0.4$ (checks)

Thus, $U = \underline{\underline{1.06\,\frac{m}{s}}}$

Note: Because of the graph (Fig. 9.21) the answers are not accurate to three significant figures.

9.53

9.53 To reduce aerodynamic drag on a bicycle, it is proposed that the cross-sectional shape of the handlebar tubes be made "tear-drop" shape rather than circular. Make a rough estimate of the reduction in aerodynamic drag for a bike with this type of handlebars compared with the standard handlebars. List all assumptions.

For a standard racing bike $\mathcal{D}_s = C_{D_s} \frac{1}{2}\rho U^2 A$, where from Fig. 9.33 $C_{D_s} = 0.88$, $A = 3.9\, ft^2$

Thus, $\mathcal{D}_s = 1.716\, \rho U^2$

For the modified bike assume $\mathcal{D}_m = \mathcal{D}_s - \mathcal{D}_c + \mathcal{D}_t$, where (1)

$\mathcal{D}_c$ = drag from standard circular cross section handle bars

and

$\mathcal{D}_t$ = drag from tear-drop shaped handle bars.

That is,

$\mathcal{D}_c = C_{D_c} \frac{1}{2}\rho U^2 A_H$ and $\mathcal{D}_t = C_{Dt} \frac{1}{2}\rho U^2 A_H$ where the handle bars are assumed to be 1 ft long and 1 in. in diameter. (i.e., $A_H = \frac{1}{12}\, ft^2$)

Typical C_D values are $C_{D_c} = 1$ (Fig. 9.23) and $C_{Dt} = 0.12$ (Fig. 9.21)

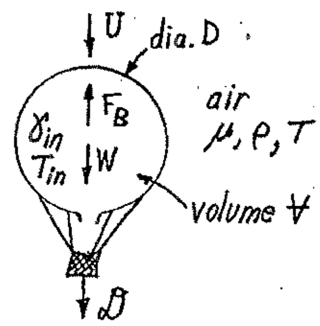
Thus, Eq. (1) gives $\mathcal{D}_m = 1.716\rho U^2 - 1(\frac{1}{2})\rho U^2(\frac{1}{12}) + 0.12(\frac{1}{2})\rho U^2(\frac{1}{12})$
$= (1.716 - 0.0367)\rho U^2$

or $$\frac{\mathcal{D}_s - \mathcal{D}_m}{\mathcal{D}_s} = \frac{1.716\,\rho U^2 - (1.716 - 0.0367)\rho U^2}{1.716\,\rho U^2} = 0.0214$$

i.e., a reduction in drag of approximately 2 percent

9.54

9.54 A hot air balloon roughly spherical in shape has a volume of 70,000 ft^3 and a weight of 500 lb (including passengers, basket, balloon fabric, etc.). If the outside air temperature is 80 °F and the temperature within the balloon is 165 °F, estimate the rate at which it will rise under steady state conditions if the atmospheric pressure is 14.7 psi.



For steady rise $\sum F_z = 0$, or $F_B = W + \mathcal{D}$
where

$\mathcal{D} = \text{drag} = C_D \frac{1}{2} \rho U^2 \frac{\pi}{4} D^2$

$F_B = \text{buoyant force} = \gamma \forall$

and

$W = \text{total weight} = 500 \text{ lb} + \gamma_{in} \forall$

Now $\rho = \dfrac{p}{RT} = \dfrac{(14.7 \frac{lb}{in^2})(12 \frac{in}{ft})^2}{(1715 \frac{ft \cdot lb}{slug \cdot °R})(460+80)°R} = 0.00229 \frac{slugs}{ft^3}$

$\gamma = \rho g = (0.00229 \frac{slugs}{ft^3})(32.2 \frac{ft}{s^2}) = 0.0736 \frac{lb}{ft^3}$

and

$\gamma_{in} = \dfrac{p g}{R\, T_{in}} = \dfrac{(14.7 \frac{lb}{in^2})(12 \frac{in}{ft})^2 (32.2 \frac{ft}{s^2})}{(1715 \frac{ft \cdot lb}{slug \cdot °R})(460+165)°R} = 0.0636 \frac{lb}{ft^3}$

Note: Since the balloon is open at the bottom, the pressure within the balloon is nearly the same as it is outside.

Thus, with $\forall = 7 \times 10^4 \, ft^3 = \frac{4\pi}{3}\left(\frac{D}{2}\right)^3$
or $D = 51.1$ ft we obtain

$\mathcal{D} = C_D \frac{1}{2}(0.00229) U^2 \frac{\pi}{4}(51.1)^2$

$= 2.36\, C_D U^2$ lb, where $U \sim \frac{ft}{s}$

Also,

$W = 500 \text{ lb} + (0.0636 \frac{lb}{ft^3})(70{,}000 \, ft^3) = 4952 \text{ lb}$

and

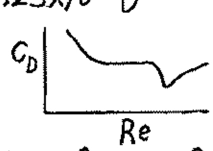
$F_B = (0.0736 \frac{lb}{ft^3})(70{,}000 \, ft^3) = 5152 \text{ lb}$ Thus, $F_B = W + \mathcal{D}$ gives

$5152 \text{ lb} = 4952 \text{ lb} + 2.36\, C_D U^2$ or $C_D U^2 = 84.7$ (1)

Also, $Re = \frac{UD}{\nu}$

or $Re = \dfrac{51.1 \text{ ft } U}{1.57 \times 10^{-4} \frac{ft^2}{s}} = 3.25 \times 10^5 \, U$ (2)

and from Fig. 9.23 (3)



Trial and error solution: Assume C_D; obtain U from Eq.(1), Re from Eq.(2); check C_D from Eq.(3), the graph.

Assume $C_D = 0.5 \rightarrow U = 13.0 \frac{ft}{s} \rightarrow Re = 4.23 \times 10^6 \rightarrow C_D = 0.24 \neq 0.5$

Assume $C_D = 0.24 \rightarrow U = 18.8 \frac{ft}{s} \rightarrow Re = 6.11 \times 10^6 \rightarrow C_D = 0.30 \neq 0.24$

Assume $C_D = 0.30 \rightarrow \underline{\underline{U = 16.8 \frac{ft}{s}}} \rightarrow Re = 5.46 \times 10^6 \rightarrow C_D = 0.30$ (checks)

9.55

9.55 It is often assumed that "sharp objects can cut through the air better than blunt ones." Based on this assumption, the drag on the object shown in Fig. P9.55 should be less when the wind blows from right to left than when it blows from left to right. Experiments show that the opposite is true. Explain.

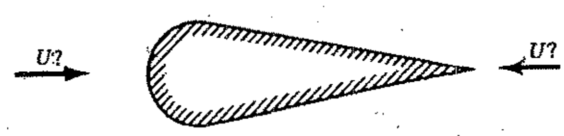
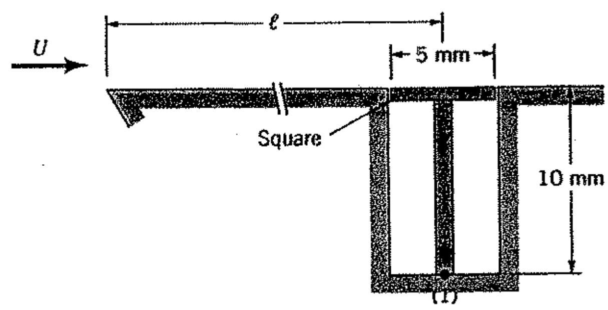


FIGURE P9.55

A significant portion of the drag on an object can be from the relatively low pressure developed in the wake region behind the object. By making the object streamlined (i.e., flow from left to right, not right to left in the above figure) boundary layer separation is avoided and a relatively thin wake with low drag is obtained. Whether the front of the object is "sharp" or "blunt" does not affect the contribution to the drag from the front part of the body—at least not as much a the width of the wake affects the drag.

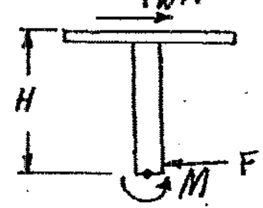
9.56

*9.56 The device shown in Fig. P9.56 is to be designed to measure the wall shear stress as air flows over the smooth surface with an upstream velocity U. It is proposed that τ_w can be obtained by measuring the bending moment, M, at the base [point (1)] of the support that holds the small surface element which is free from contact with the surrounding surface. Plot a graph of M as a function of U for $5 \le U \le 50$ m/s, with $\ell = 2, 3, 4,$ and 5 m.



■ FIGURE P9.56

Since the length of the measuring surface is much less than its distance from the leading edge (i.e., 5mm<<ℓ) we can assume that the shear stress is essentially constant on that surface.



Thus, $M = \tau_w A H$

or

$$M = (5\times10^{-3}m)^2(10\times10^{-3}m)\tau_w = 2.5\times10^{-7}\tau_w \ \ Nm \qquad (1)$$

where $\tau_w \sim \frac{N}{m^2}$

The flow will be laminar or turbulent depending whether $Re_\ell < 5\times10^5$ or $Re_\ell > 5\times10^5$, where $Re_\ell = \frac{U\ell}{\nu}$ and $\nu = 1.46\times10^{-5}\frac{m^2}{s}$.

Since $Re_{\ell_{min}} = \frac{(5\frac{m}{s})(2m)}{1.46\times10^{-5}\frac{m^2}{s}} = 6.84\times10^5$ the flow is always turbulent.

Also, since

$Re_{\ell_{max}} = \frac{(50\frac{m}{s})(5m)}{1.46\times10^{-5}\frac{m^2}{s}} = 1.71\times10^7$ it follows from Table 9.3 that*

$$\tau_w = 0.0225\,\rho U^2\left(\frac{\nu}{U\delta}\right)^{1/4} \quad \text{where } \delta = \frac{0.370x}{Re_x^{1/5}} = \frac{0.370\,\ell\,\nu^{1/5}}{U^{1/5}\ell^{1/5}}$$

That is,

$$\tau_w = 0.0225\,\rho U^2\left[\frac{\nu\,U^{\frac{1}{5}}\ell^{1/5}}{U(0.370)\ell\,\nu^{1/5}}\right]^{1/4} = 0.0225\,\rho\,U^{9/5}\nu^{1/5}\ell^{-1/5}(0.370)^{-1/4}$$

or

$$\tau_w = 0.0225\left(1.23\frac{kg}{m^3}\right)U^{9/5}\left(1.46\times10^{-5}\frac{m^2}{s}\right)^{\frac{1}{5}}\ell^{-\frac{1}{5}}(0.370)^{-\frac{1}{4}}$$

$$= 3.83\times10^{-3}\,U^{9/5}\ell^{-1/5}\ \frac{N}{m^2}, \text{ where } U\sim\frac{m}{s} \text{ and } \ell\sim m$$

Thus, from Eq.(1),

$$M = (2.5\times10^{-7})(4.34\times10^{-3})U^{9/5}\ell^{-1/5} = 9.57\times10^{-10}U^{9/5}\ell^{-1/5}\ N\cdot m$$

The values of M are calculated and plotted for $5 \le U \le 50\ \frac{m}{s}$, with $\ell = 2, 3, 4,$ and 5 m.

(con't)

9.56 (con't)

For ℓ = 2.00 m

U, m/s	M, N.m
5.00	+1.510E-08
10.00	+5.257E-08
15.00	+1.091E-07
20.00	+1.830E-07
25.00	+2.735E-07
30.00	+3.798E-07
35.00	+5.012E-07
40.00	+6.374E-07
45.00	+7.879E-07
50.00	+9.525E-07

For ℓ = 4.00 m

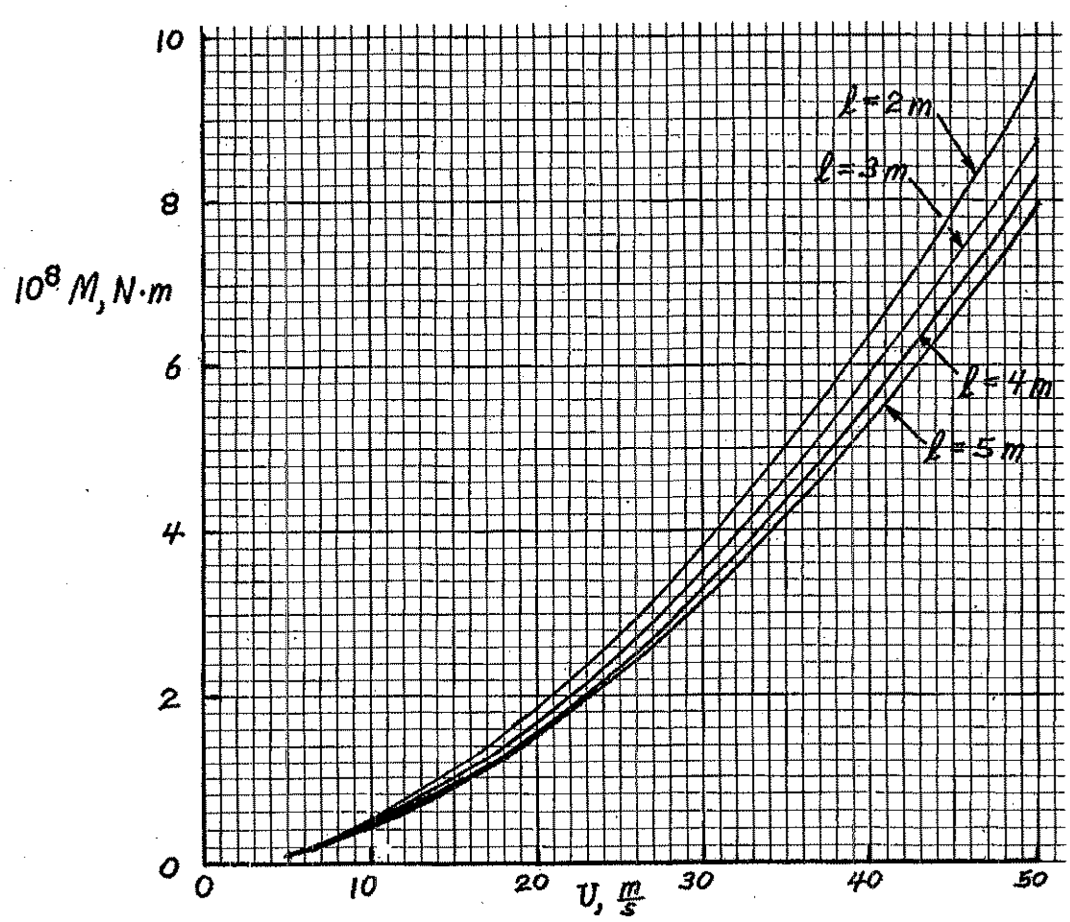
U, m/s	M, N.m
5.00	+1.314E-08
10.00	+4.576E-08
15.00	+9.494E-08
20.00	+1.594E-07
25.00	+2.381E-07
30.00	+3.306E-07
35.00	+4.363E-07
40.00	+5.549E-07
45.00	+6.859E-07
50.00	+8.292E-07

For ℓ = 3.00 m

U, m/s	M, N.m
5.00	+1.392E-08
10.00	+4.847E-08
15.00	+1.006E-07
20.00	+1.688E-07
25.00	+2.522E-07
30.00	+3.502E-07
35.00	+4.622E-07
40.00	+5.878E-07
45.00	+7.266E-07
50.00	+8.783E-07

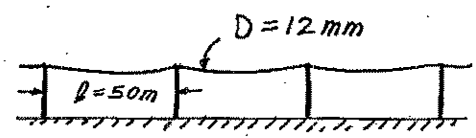
For ℓ = 5.00 m

U, m/s	M, N.m
5.00	+1.257E-08
10.00	+4.376E-08
15.00	+9.080E-08
20.00	+1.524E-07
25.00	+2.277E-07
30.00	+3.162E-07
35.00	+4.173E-07
40.00	+5.307E-07
45.00	+6.560E-07
50.00	+7.930E-07



9.57

9.57 A 12-mm-diameter cable is strung between a series of poles that are 50 m apart. Determine the horizontal force this cable puts on each pole if the wind velocity is 30 m/s.



F_P = force on one pole = $\mathscr{D}$

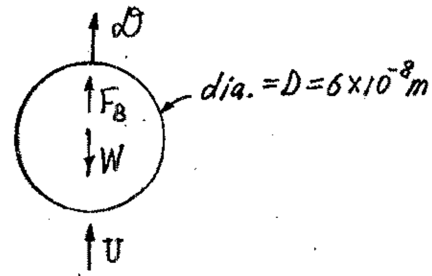
where $\mathscr{D} = C_D \frac{1}{2}\rho U^2 A$

Since $Re = \frac{UD}{\nu} = \frac{(30\frac{m}{s})(0.012m)}{1.46\times10^{-5}\frac{m^2}{s}} = 2.47\times10^4$ it follows from Fig. 9.23 that

$C_D = 0.4$. Hence, $F_P = 0.4\left(\frac{1}{2}\right)(1.23\frac{kg}{m^3})(30\frac{m}{s})^2(50m)(0.012m) = \underline{\underline{133\ N}}$

9.58

9.58 How fast do small water droplets of 0.06-μm (6×10^{-8} m) diameter fall through the air under standard sea-level conditions? Assume the drops do not evaporate. Repeat the problem for standard conditions at 5000-m altitude.



For steady conditions, $\mathscr{D} + F_B = W$,
where if $Re = \frac{UD}{\nu} < 1$

$\mathscr{D} = \text{drag} = 3\pi D U \mu$ Also, $W = \gamma_{H_2O} \forall = \gamma_{H_2O} \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \text{weight}$
and $F_B = \gamma_{air} \forall = \gamma_{air} \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \text{buoyant force}$

Since $\gamma_{air} \ll \gamma_{H_2O}$, we can neglect the buoyant force.
That is, $\mathscr{D} = W$, or

$$3\pi D U \mu = \gamma_{H_2O} \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 \quad \text{or} \quad U = \frac{\gamma_{H_2O} D^2}{18\mu} \qquad (1)$$

At sea level $\mu = 1.789 \times 10^{-5} \frac{N \cdot s}{m^2}$ so that

$$U = \frac{(9.80 \times 10^3 \frac{N}{m^3})(6 \times 10^{-8} m)^2}{18\,(1.789 \times 10^{-5} \frac{N \cdot s}{m^2})} = \underline{\underline{1.10 \times 10^{-7} \frac{m}{s}}}$$

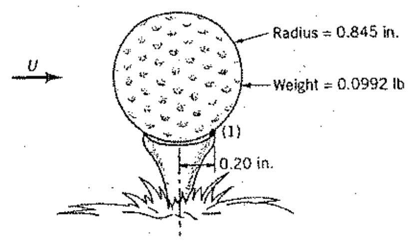
Note that $Re = \frac{(1.10 \times 10^{-7} \frac{m}{s})(6 \times 10^{-8} m)}{1.46 \times 10^{-5} \frac{m^2}{s}} = 4.52 \times 10^{-10} \ll 1$ so the use of the low Re drag equation is valid.

At an altitude of 5000 m, $\mu = 1.628 \times 10^{-5} \frac{N \cdot s}{m^2}$ and from Eq. (1)

$$U = \frac{(9.80 \times 10^3 \frac{N}{m^3})(6 \times 10^{-8} m)^2}{18\,(1.628 \times 10^{-5} \frac{N \cdot s}{m^2})} = \underline{\underline{1.20 \times 10^{-7} \frac{m}{s}}}$$

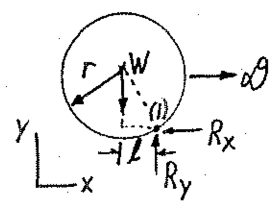
9.59

9.59 A strong wind can blow a golf ball off the tee by pivoting it about point 1 as shown in Fig. P9.59. Determine the wind speed necessary to do this.



■ FIGURE P9.59

When the ball is about to be blown from the tee the free body diagram is as shown. Hence, by summing moments about (1):



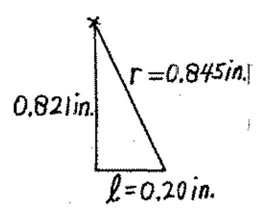
$\Sigma M_1 = 0$, or $W\ell = \mathcal{D} r$

Thus,

$(0.0992\ lb)(0.20 in.) = \mathcal{D}(0.821\ in.)$

or

$\mathcal{D} = 0.0242\ lb$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 \pi r^2$



Thus,

$0.0242\ lb = C_D \frac{1}{2}(0.00238 \frac{slugs}{ft^3}) U^2 \pi \left(\frac{0.845 in.}{12 \frac{in.}{ft}}\right)^2$

or

$C_D U^2 = 1305$, where $U \sim \frac{ft}{s}$ (1)

For a sphere* $C_D = C_D(Re)$ (see Fig. 9.18) where (2)

$Re = \frac{\rho U D}{\mu} = \frac{(0.00238\ slugs/ft^3)\, U\, (2(0.845)/12\ ft)}{3.47 \times 10^{-7} (lb \cdot s/ft^2)}$

or

$Re = 966\, U$, where $U \sim \frac{ft}{s}$ (3)

Trial and error solution:

Assume $C_D = 0.4$ so that from Eq.(1), $U = 57.1 \frac{ft}{s}$ and from Eq.(3), $Re = 966(57.1) = 5.52 \times 10^4$. Thus, from Fig. 9.18, $C_D \approx 0.25 \neq 0.40$ Try again.

Assume $C_D = 0.22$ so that $U = 77.0 \frac{ft}{s}$ and $Re = 7.44 \times 10^4$

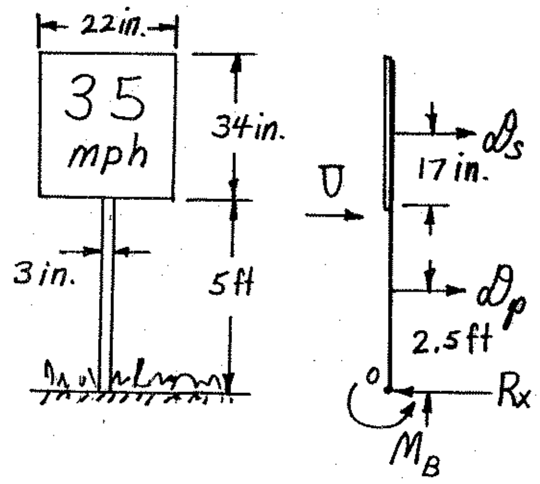
Thus, from Fig. 9.18, $C_D \approx 0.22$ Checks.

Hence, $\underline{\underline{U \approx 77.0 \frac{ft}{s}}}$

* golf ball (i.e. with dimples)

9.60

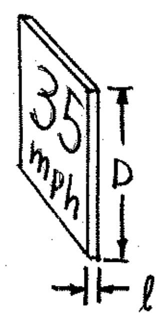
9.60 A 22 in. by 34 in. speed limit sign is supported on a 3-in. wide, 5-ft-long pole. Estimate the bending moment in the pole at ground level when a 30-mph wind blows against the sign. (See Video V9.9.) List any assumptions used in your calculations.



For equalibrium, $\Sigma M_0 = 0$ or

$$M_B = 2.5\,ft\,\mathcal{D}_p + \left(5+\tfrac{17}{12}\right)ft\,\mathcal{D}_s \text{, where} \qquad (1)$$

$\mathcal{D}_p$ = drag on the pole and $\mathcal{D}_s$ = drag on the sign



From Fig. 9.28 with $\ell/D < 0.1$ for the sign,
$C_{D_s} = 1.9$

From Fig. 9.19 if the post acts as a square rod with sharp corners $C_{D_p} = 2.2$ Thus, with $V = 30\,mph = 44\,\frac{ft}{s}$,

$$\mathcal{D}_s = \tfrac{1}{2}\rho V^2 C_{D_s} A_s = \tfrac{1}{2}\left(0.00238\,\tfrac{slugs}{ft^3}\right)\left(44\,\tfrac{ft}{s}\right)^2(1.9)\left(\tfrac{22\,(34)}{144}\,ft^2\right) = 22.7\,lb$$

and

$$\mathcal{D}_p = \tfrac{1}{2}\rho V^2 C_{D_p} A_p = \tfrac{1}{2}\left(0.00238\,\tfrac{slugs}{ft^3}\right)\left(44\,\tfrac{ft}{s}\right)^2(2.2)\left(\tfrac{3}{12}\,(5)\,ft^2\right) = 6.34\,lb$$

Thus, from Eq. (1):

$$M_B = 2.5\,ft\,(6.34\,lb) + \left(5+\tfrac{17}{12}\right)ft\,(22.7\,lb) = \underline{\underline{162\,ft\cdot lb}}$$

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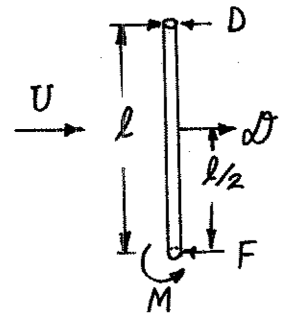
10% Reduction
8 1/2 x 11 trim size

9.61

9.61 Determine the moment needed at the base of 20-m-tall, 0.12-m-diameter flag pole to keep it in place in a 20 m/s wind.



For equilibrium, $M = \frac{\ell}{2}\mathcal{D}$ where (1)

$\mathcal{D} = C_D \frac{1}{2} \rho U^2 \ell D$

Since $Re = \frac{UD}{\nu} = \frac{(20\frac{m}{s})(0.12\,m)}{1.46\times10^{-5}\frac{m^2}{s}} = 1.64\times10^5$, it follows from Fig. 9.21

that $C_D = 1.2$

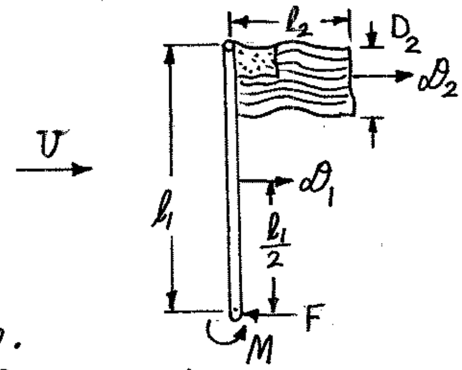
Thus, $\mathcal{D} = 1.2\left(\frac{1}{2}\right)(1.23\,\frac{kg}{m^3})(20\frac{m}{s})^2(20m)(0.12m) = 708N$

Hence, from Eq. (1)

$M = \frac{20m}{2}(708N) = \underline{\underline{7{,}080\,N{\cdot}m}}$

9.62

9.62 Repeat Problem 9.61 if a 2-m by 2.5-m flag is attached to the top of the pole. See Fig. 9.30 for drag coefficient data for flags.



For equilibrium, $M = \frac{\ell_1}{2}\mathcal{D}_1 + \left(\ell_1 - \frac{D_2}{2}\right)\mathcal{D}_2$ (1)

where $\ell_1 = 20\,m$, $\ell_2 = 2.5\,m$, and $D_2 = 2\,m$.

From the solution to Problem 9.48, $\frac{\ell_1}{2}\mathcal{D}_1 = 7{,}080\ N\cdot m$ (2)

Also,

$\mathcal{D}_2 = C_D \frac{1}{2}\rho U^2 \ell_2 D_2$, where from Fig. 9.30 with $\frac{\ell_2}{D_2} = \frac{2.5}{2} = 1.25$

we obtain $C_D = 0.08$.

Thus,

$\mathcal{D}_2 = 0.08\left(\frac{1}{2}\right)\left(1.23\frac{kg}{m^3}\right)\left(20\frac{m}{s}\right)^2(2.5\,m)(2\,m) = 98.4\ N$ (3)

By combining Eqs. (1), (2), and (3) we obtain

$M = 7{,}080\ N\cdot m + (20\,m - 1\,m)(98.4\ N) = \underline{\underline{8{,}950\ N\cdot m}}$

9.64

9.64 How much more power is required to peddle a bicycle at 15 mph into a 20-mph headwind than at 15 mph through still air? Assume a frontal area of 3.9 ft² and a drag coefficient of $C_D = 0.88$.

$\mathcal{P}$ = power = $U_B \mathcal{D}$ and $\mathcal{D} = C_D \frac{1}{2} \rho U^2 A$, where U_B = speed of the bike $= 15 \frac{mi}{hr}\left(\frac{88 \frac{ft}{s}}{60 \frac{mi}{hr}}\right) = 22 \frac{ft}{s}$

and U = wind speed relative to bike.

Thus,

$$\mathcal{P} = \left(22 \tfrac{ft}{s}\right)(0.88)\left(\tfrac{1}{2}\right)\left(0.00238 \tfrac{slugs}{ft^3}\right) U^2 (3.9\, ft^2) = 0.0898\, U^2 \ \tfrac{ft \cdot lb}{s} \quad (1)$$

with $U \sim \frac{ft}{s}$

a) With a 20 mph headwind, $U = (15 + 20)\frac{mi}{hr}\left(\frac{88 \frac{ft}{s}}{60 \frac{mi}{hr}}\right) = 51.3 \frac{ft}{s}$

Thus,

$$\mathcal{P}_a = 0.0898\,(51.3)^2 = 236 \ \tfrac{ft \cdot lb}{s}$$

b) With still air, $U = 15\, mph = 22 \frac{ft}{s}$

Thus,

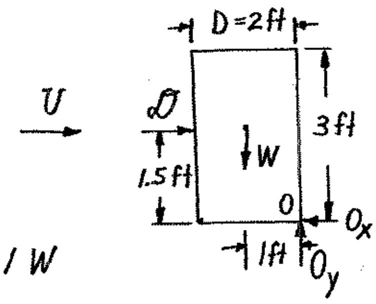
$$\mathcal{P}_b = 0.0898\,(22)^2 = 43.5 \ \tfrac{ft \cdot lb}{s}$$

Hence, need an additional power of $\mathcal{P}_a - \mathcal{P}_b = (236 - 43.5)\frac{ft \cdot lb}{s}\left(\frac{1\, hp}{550 \frac{ft \cdot lb}{s}}\right)$

$$= \underline{\underline{0.350\, hp}}$$

9.65

9.65 Estimate the wind velocity necessary to knock over a 10-lb garbage can that is 3 ft tall and 2 ft in diameter. List your assumptions.



If the can is about to tip around corner O, then $\Sigma M_0 = 0$, or $1.5\mathscr{D} = 1\,W$

or $1.5\, C_D \frac{1}{2}\rho U^2 A = W$ A typical value of C_D for a cylinder is $C_D \approx 1$ (see Fig. 9.21)

Thus,

$(1.5\,ft)(1)(\tfrac{1}{2})(0.00238\,\frac{slugs}{ft^3})\,U^2(2\,ft)(3\,ft) = 10\,ft\cdot lb$, where $U \sim \frac{ft}{s}$

or $U = \underline{\underline{30.6\,\frac{ft}{s}}}$

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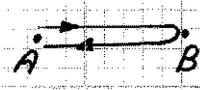
9.66

9.66 On a day without any wind, your car consumes x gallons of gasoline when you drive at a constant speed, U, from point A to point B and back to point A. Assume that you repeat the journey, driving at the same speed, on another day when there is a steady wind blowing from B to A. Would you expect your fuel consumption to be less than, equal to, or greater than x gallons for this windy round-trip? Support your answer with appropriate analysis.

Trip with the larger power lost due to aerodynamic drag will use the most gas. Let $(\ )_1$ mean "no wind" and $(\ )_2$ mean "wind".

(1) No wind:

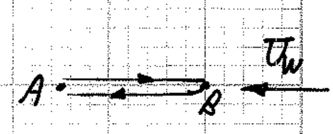
$\mathcal{D}_1 = C_D \frac{1}{2}\rho U^2 A$ for both $A \rightarrow B$ and $B \rightarrow A$



Thus,

$\mathcal{P}_1 = \text{power} = U\mathcal{D}_1 = \frac{1}{2}\rho U^3 C_D A$

(2) Wind (U_w = wind speed; assume $U_w < U$):



$\mathcal{D}_2 = C_D \frac{1}{2}\rho (U+U_w)^2 A$ for $A \rightarrow B$

$\mathcal{D}_2 = C_D \frac{1}{2}\rho (U-U_w)^2 A$ for $B \rightarrow A$

Thus,

$\mathcal{P}_2 = \frac{1}{2}\rho (U+U_w)^2 U C_D A$ for $A \rightarrow B$

$\mathcal{P}_2 = \frac{1}{2}\rho (U-U_w)^2 U C_D A$ for $B \rightarrow A$

Energy used $= \mathcal{P}t$, where t = time to go from $A \rightarrow B$ or $B \rightarrow A$

Thus,

$E_1 = 2\left(\frac{1}{2}\rho U^3 C_D A\right)t$ (Note: Factor of 2 for $A \rightarrow B + B \rightarrow A$)

and

$E_2 = \frac{1}{2}\rho (U+U_w)^2 U C_D A t + \frac{1}{2}\rho (U-U_w)^2 U C_D A t$

Thus,

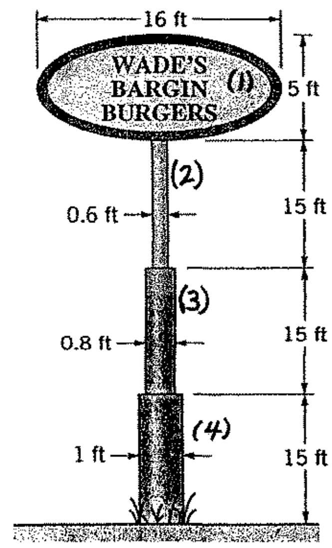
$$\frac{E_1}{E_2} = \frac{2U^3}{(U+U_w)^2 U + (U-U_w)^2 U} = \frac{2U^3}{2U^3 + 2U_w^2 U} = \frac{1}{1+(U_w/U)^2} < 1$$

Hence,

$\frac{E_1}{E_2} < 1$, i.e. more fuel needed when windy

9.67

9.67 The structure shown in Fig. P9.67 consists of three cylindrical support posts to which an elliptical flat-plate sign is attached. Estimate the drag on the structure when a 50-mph wind blows against it.



■ FIGURE P9.67

For the composite body:

(1) $\mathscr{D} = \sum_{i=1}^{4} \mathscr{D}_i = \frac{1}{2}\rho U^2 \left[C_{D_1} A_1 + C_{D_2} A_2 + C_{D_3} A_3 + C_{D_4} A_4 \right]$

where if we assume the sign is an ellipse,

$A_1 = \frac{\pi}{4}(10 ft)(5 ft) = 39.3 ft^2$, and the projected areas of the cylinders are

$A_2 = 0.6 ft (15 ft) = 9.00 ft^2$

$A_3 = 0.8 ft (15 ft) = 12.0 ft^2$ and

$A_4 = 1 ft (15 ft) = 15.0 ft^2$

From Fig. 9.20, for a thin disc $C_{D_1} = 1.1$

For the cylindrical post, obtain C_D from Fig. 9.15 as: ($U = 50 mph = 73.3 \frac{ft}{s}$)

$Re_2 = \frac{U D_2}{\nu} = \frac{73.3 \frac{ft}{s}(0.6 ft)}{1.57 \times 10^{-4} \frac{ft^2}{s}} = 2.8 \times 10^5 \longrightarrow C_{D_2} = 0.6$

Similarly,

$Re_3 = 3.7 \times 10^5 \longrightarrow C_{D_3} = 0.5$

$Re_4 = 4.7 \times 10^5 \longrightarrow C_{D_4} = 0.25$

Thus, from Eq. (1):

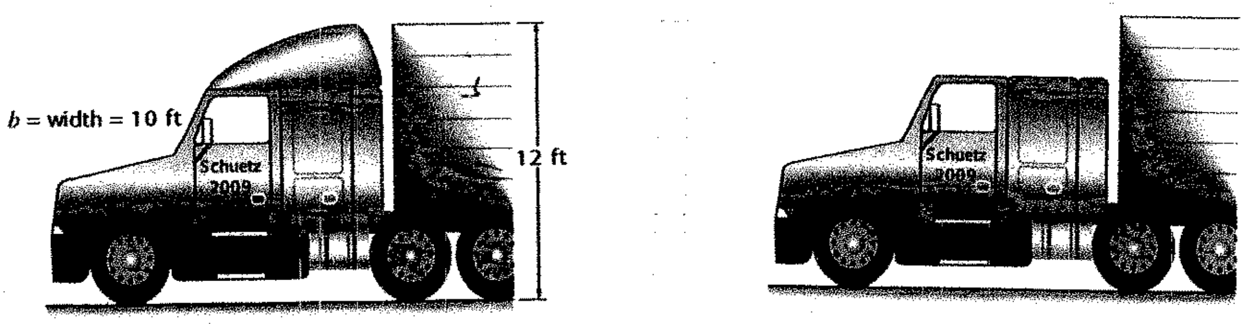
$\mathscr{D} = \frac{1}{2}(0.00238 \frac{slugs}{ft^3})(73.3 \frac{ft}{s})^2 \left[1.1(39.3 ft^2) + 0.6(9.0 ft^2) + 0.5(12 ft^2) + 0.25(15 ft^2) \right]$

$= \underline{\underline{378 \, lb}}$

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

9.68

9.68 As shown in Video V9.13 and Fig. P9.68, the aerodynamic drag on a truck can be reduced by the use of appropriate air deflectors. A reduction in drag coefficient from $C_D = 0.96$ to $C_D = 0.70$ corresponds to a reduction of how many horsepoer needed at a highway speed of 65 mph?



(a) $C_D = 0.70$ (b) $C_D = 0.96$

■ FIGURE P9.68

$\mathcal{P}$ = power = $\mathcal{D}\,U$ where

$$\mathcal{D} = \frac{1}{2}\rho U^2 C_D A$$

Thus, $\Delta\mathcal{P}$ = reduction in power

$$= \mathcal{P}_b - \mathcal{P}_a$$

$$= \frac{1}{2}\rho U^3 A\left[C_{D_b} - C_{Da}\right]$$

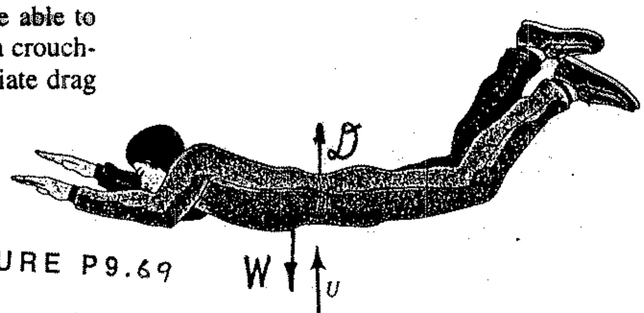
With $U = 65\ mph = 95.3\ fps$,

$$\Delta\mathcal{P} = \frac{1}{2}\left(0.00238\,\frac{slugs}{ft^3}\right)\left(95.3\,\frac{ft}{s}\right)^3(10ft)(12ft)\left[0.96 - 0.70\right]$$

$$= 32{,}100\,\frac{ft\cdot lb}{s}\left(\frac{1\ hp}{550\,\frac{ft\cdot lb}{s}}\right) = \underline{\underline{58.4\ hp}}$$

9.69

9.69 As shown in Video V9.7 and Fig. P9.69 a vertical wind tunnel can be used for skydiving practice. Estimate the vertical wind speed needed if a 150-lb person is to be able to "float" motionless when the person (a) curls up as in a crouching position or (b) lies flat. See Fig. 9.30 for appropriate drag coefficient data.



■ FIGURE P9.69

For equilibrium conditions

$$W = \mathcal{D} = C_D \tfrac{1}{2} \rho U^2 A$$

Assume $W = 160\ lb$ and $C_D A = 9\ ft^2$ (see Fig. 9.30)

Thus,

$$160\ lb = (\tfrac{1}{2})(0.00238 \tfrac{slugs}{ft^3})\, U^2 (9\ ft^2) \quad \text{where } U \sim \tfrac{ft}{s}$$

or

$$U = (122 \tfrac{ft}{s})(\tfrac{1\ mi}{5280\ ft})(\tfrac{3600\ s}{1\ hr}) = \underline{\underline{83.2\ mph}}$$

Note: If the skydiver "curled up into a ball", then $C_D A \approx 2.5\ ft$ (see Fig. 9.30) and $U = 158\ mph$

9.70*

9.70 The helium-filled balloon shown in Fig. P9.70 is to be used as a wind speed indicator. The specific weight of the helium is $\gamma = 0.011\ \text{lb/ft}^3$, the weight of the balloon material is 0.20 lb, and the weight of the anchoring cable is negligible. Plot a graph of θ as a function of U for $1 \le U \le 50$ mph. Would this be an effective device over the range of U indicated? Explain.

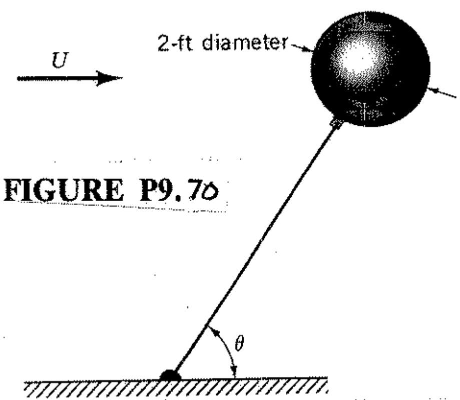
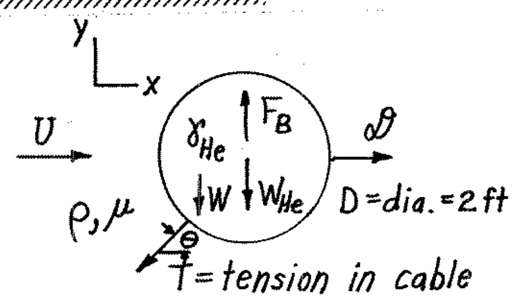


FIGURE P9.70



For the balloon to remain stationary

$\Sigma F_x = 0$ and $\Sigma F_y = 0$

Thus, $\mathscr{D} = T\cos\theta$ or $T = \dfrac{\mathscr{D}}{\cos\theta}$

and $F_B = W + T\sin\theta + W_{He}$

which combine to give

$$F_B = W + \mathscr{D}\tan\theta + W_{He} \tag{1}$$

But $W = 0.2\ lb$, $F_B = \rho g \forall = (7.65\times10^{-2}\ \frac{lb}{ft^3})\ \frac{4\pi}{3}\left(\frac{2}{2}\ ft\right)^3 = 0.3204\ lb$

and $W_{He} = \gamma_{He}\forall = (0.011\ \frac{lb}{ft^3})\ \frac{4\pi}{3}\left(\frac{2}{2}\ ft\right)^3 = 0.0461\ lb$

Thus, Eq. (1) becomes

$0.3204\ lb = 0.2\ lb + \mathscr{D}\tan\theta + 0.0461\ lb$

or

$\mathscr{D}\tan\theta = 0.0743\ lb$ Also, $\mathscr{D} = C_D \frac{1}{2}\rho U^2 \frac{\pi}{4} D^2$

$= C_D U^2 (0.00238\ \frac{slugs}{ft^3})\frac{\pi}{8}(2\ ft)^2$

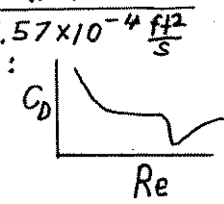
$= 0.00374\, C_D U^2\ lb$, where $U \sim \frac{ft}{s}$

Hence,

$$0.00374\, C_D U^2 \tan\theta = 0.0743 \quad \text{or} \quad \tan\theta = \frac{19.9}{C_D U^2} \tag{2}$$

$$\text{Also, } Re = \frac{UD}{\nu} = \frac{(2\ ft)\ U}{1.57\times10^{-4}\ \frac{ft^2}{s}} \quad \text{or} \quad Re = 1.27\times10^4\ U \tag{3}$$

and from Fig. 9.21: (4)

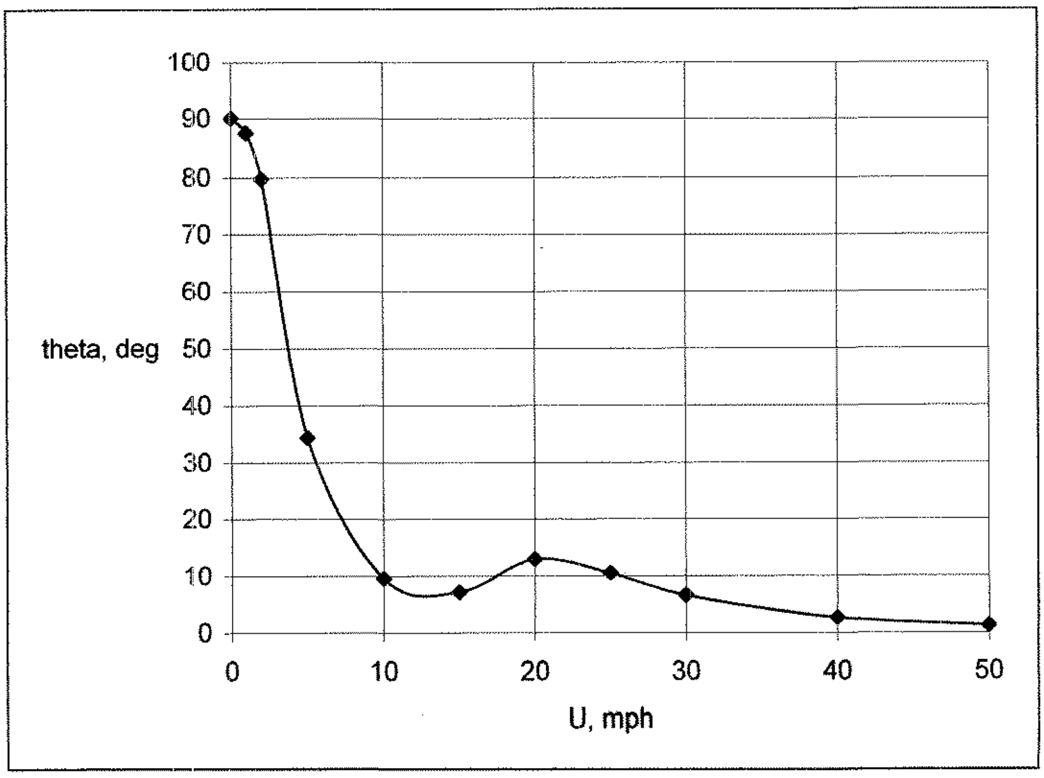


Thus, select various 1 mph $\le U \le$ 50 mph (i.e., $1.47\ \frac{ft}{s} \le U \le 73.3\ \frac{ft}{s}$) and use Eqs. (2), (3), (4) to obtain θ. Plotted results are shown below.

(con't)

9.70* (con't)

U, mph	Re	CD	Θ, deg
0	0	---	90
1	12700	0.40	87.52
2	25400	0.42	79.71
5	63500	0.54	34.42
10	127000	0.55	9.55
15	190500	0.33	7.10
20	254000	0.10	13.02
25	317500	0.08	10.48
30	381000	0.09	6.52
40	508000	0.12	2.76
50	635000	0.16	1.32



Note: Because of the sudden change in C_D when the boundary layer becomes turbulent (at about 15 mph), the θ vs U curve is highly non-linear. In fact, for some values of θ there is more than one possible value of U. It would not work well as a wind speed indicator in this range.

9.71

9.71 A 0.30-m-diameter cork ball ($SG = 0.21$) is tied to an object on the bottom of a river as is shown in Fig. P9.61. Estimate the speed of the river current. Neglect the weight of the cable and the drag on it.

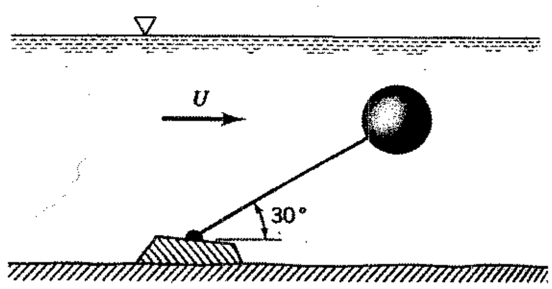
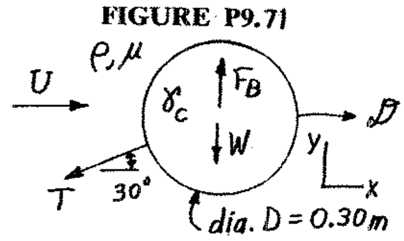


FIGURE P9.71



For the ball to remain stationary

$\Sigma F_x = 0$ and $\Sigma F_y = 0$

Thus, $\mathscr{D} = T\cos 30°$ or $T = \dfrac{\mathscr{D}}{\cos 30°}$

and

$F_B = W + T\sin 30°$

Hence, $F_B = W + \mathscr{D}\tan 30°$, where $F_B = \rho g \forall = \left(9.80\,\frac{kN}{m^3}\right)\left(\frac{4\pi}{3}\left(\frac{0.30}{2}m\right)^3\right)$

$= 0.1385\ kN$

and

$W = \gamma_c \forall = \left(\frac{\gamma_c}{\gamma}\right)\gamma\forall = (SG)\,F_B$

$= 0.21\,(0.1385\ kN)$

$= 0.0291\ kN$

Thus,

$0.1385\ kN = 0.0291\ kN + \mathscr{D}\tan 30°$

or

$\mathscr{D} = 0.189\ kN$, where $\mathscr{D} = C_D \frac{1}{2}\rho U^2 A = C_D U^2\left(\frac{1}{2}\right)\left(999\,\frac{kg}{m^3}\right)\left(\frac{\pi}{4}(0.3m)^2\right)$

$= 35.3\ C_D U^2\ N$, where $U \sim \frac{m}{s}$

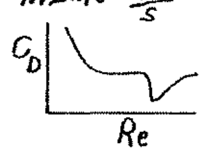
Hence

$35.3\,C_D U^2 = 189$ or $C_D U^2 = 5.35$ (1)

Also, $Re = \dfrac{UD}{\nu} = \dfrac{(0.3m)\,U}{1.12\times10^{-6}\,\frac{m^2}{s}} = 2.68\times10^5\,U$ (2)

and

from Fig. 9.21 (3)



Trial and error solution for U: Assume C_D; calculate U from Eq.(1) and Re from Eq.(2); check C_D from Eq.(3), the graph.

Assume $C_D = 0.5 \rightarrow U = 3.27\,\frac{m}{s} \rightarrow Re = 8.76\times10^5 \rightarrow C_D = 0.15 \neq 0.5$

Assume $C_D = 0.15 \rightarrow U = 5.97\,\frac{m}{s} \rightarrow Re = 1.60\times10^6 \rightarrow C_D = 0.20 \neq 0.15$

Assume $C_D = 0.19 \rightarrow U = 5.31\,\frac{m}{s} \rightarrow Re = 1.42\times10^6 \rightarrow C_D = 0.19$ (checks)

Thus, $\underline{\underline{U = 5.31\,\frac{m}{s}}}$

9.72

9.72 A shortwave radio antenna is constructed from circular tubing, as is illustrated in Fig. P9.72. Estimate the wind force on the antenna in a 100 km/hr wind.

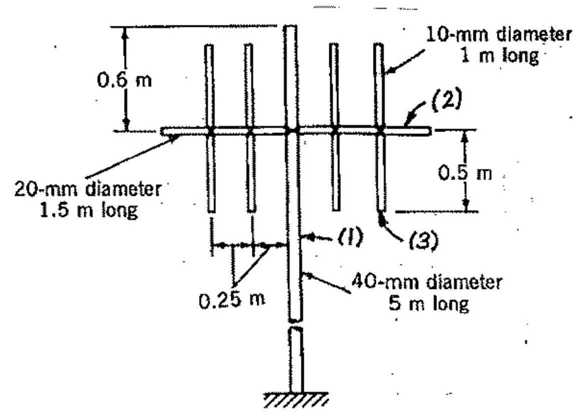


FIGURE P9.72

$$\mathcal{D} = \mathcal{D}_1 + \mathcal{D}_2 + 4\mathcal{D}_3$$

$$= \frac{1}{2}\rho U^2 \left[C_{D_1} A_1 + C_{D_2} A_2 + C_{D_3} A_3 \right]$$

where $U = 100 \frac{km}{hr} \left(\frac{10^3 m}{km}\right)\left(\frac{1h}{3600s}\right) = 27.8 \frac{m}{s}$

Obtain C_{D_i} from Fig. 9.23 for the given $Re_i = \frac{UD_i}{\nu}$.

Thus, $Re_1 = \dfrac{(27.8 \frac{m}{s})(0.04m)}{1.46 \times 10^{-5} \frac{m^2}{s}} = 7.62 \times 10^4 \longrightarrow C_{D_1} = 1.4$

$Re_2 = \dfrac{(27.8 \frac{m}{s})(0.02m)}{1.46 \times 10^{-5} \frac{m^2}{s}} = 3.81 \times 10^4 \longrightarrow C_{D_2} = 1.4$

and

$Re_3 = \dfrac{(27.8 \frac{m}{s})(0.01m)}{1.46 \times 10^{-5} \frac{m^2}{s}} = 1.90 \times 10^4 \longrightarrow C_{D_3} = 1.4 = C_{D_2} = C_{D_1}$

so that

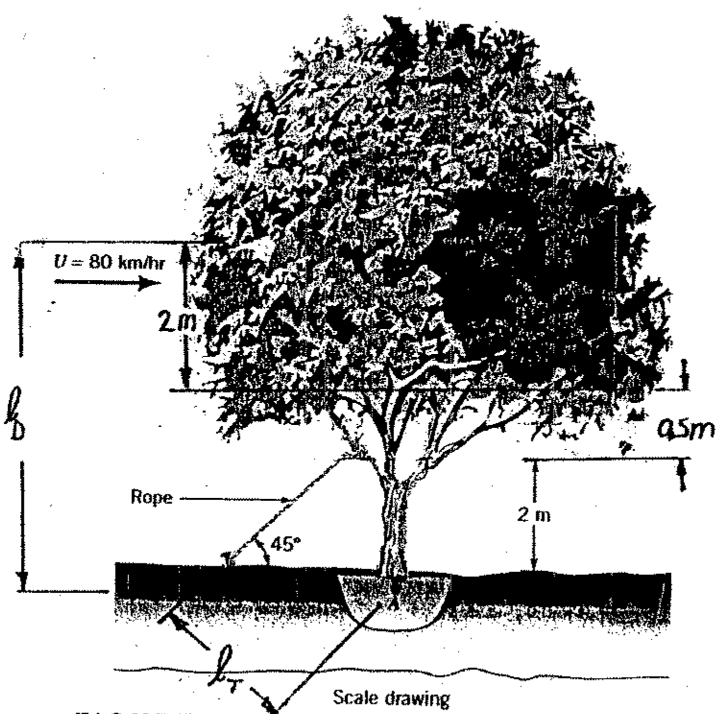
$$\mathcal{D} = \frac{1}{2}(1.23 \frac{kg}{m^3})(27.8 \frac{m}{s})^2 (1.4) \left[(5m)(0.04m) + (1.5m)(0.02m) + 4(1m)(0.01m) \right]$$

or

$$\mathcal{D} = \underline{\underline{180\ N}}$$

9.73

9.73 The large, newly planted tree shown in Fig. P9.73 is kept from tipping over in a wind by use of a rope as shown. It is assumed that the sandy soil cannot support any moment about the center of the soil ball, point A. Estimate the tension in the rope if the wind is 80 km/hr. See Fig. 9.30 for drag coefficient data.



■ FIGURE P9.73

$\Sigma M_A = 0$ where the moments are due to the drag, $\mathcal{D}$, and the tension in the rope, T.

Thus,

$\ell_D \mathcal{D} = \ell_T T$, where from the figure $\ell_D \approx (2 + 2.5 + 0.5)m = 5.0m$

and $\ell_T = \frac{3\,m}{\sqrt{2}} = 2.12\,m$

Hence,

$$T = \frac{\ell_D \mathcal{D}}{\ell_T} = \frac{\ell_D \frac{1}{2}\rho U^2 A C_D}{\ell_T} \quad \text{where from the figure } A \approx \frac{\pi}{4}(5m)^2$$

Thus, with $U = \left(80\frac{km}{hr}\right)\left(\frac{1hr}{3600s}\right)\left(1000\frac{m}{km}\right) = 22.2\frac{m}{s}$

and $C_D = 0.26$ (see Fig. 9.21) we obtain

$$T = \frac{5.0m}{2.12\,m}\left(\frac{1}{2}\right)\left(1.23\frac{kg}{m^3}\right)\left(22.2\frac{m}{s}\right)^2 \frac{\pi}{4}(5m)^2(0.26) = 3650\ N = \underline{\underline{3.65kN}}$$

9.74

9.74 Estimate the wind force on your hand when you hold it out of your car window while driving 55 mph. Repeat your calculations if you were to hold your hand out of the window of an airplane flying 550 mph.

$\mathscr{D} = C_D \frac{1}{2} \rho U^2 A$, where $U = (55\,mph)\left(\frac{88\,\frac{ft}{s}}{60\,mph}\right) = 80.7\,\frac{ft}{s}$

Assume your hand is 4 in. by 6 in. in size and acts like a thin disc with $C_D \approx 1.1$ (see Fig. 9.29).

Thus,

$\mathscr{D} = (1.1)\left(\frac{1}{2}\right)(0.00238)\left(80.7\,\frac{ft}{s}\right)^2\left(\frac{4}{12}\,ft\right)\left(\frac{6}{12}\,ft\right) = \underline{\underline{1.42\,lb}}$

If your hand is normal to the the lift force is zero.

For $U = 550\,mph = 807\,\frac{ft}{s}$ (i.e., a 10 fold increase in U) the drag will increase by a factor of 100 (i.e., $\mathscr{D} \sim U^2$), or $\mathscr{D} = \underline{\underline{142\,lb}}$

Note: We have assumed that C_D is not a function of U. That is, it is not a function of either $Re = \frac{UD}{\nu}$ or $Ma = \frac{U}{c}$.

9.76

9.76 A 2-mm-diameter meteor of specific gravity 2.9 has a speed of 6 km/s at an altitude of 50,000 m where the air density is 1.03×10^{-3} kg/m³. If the drag coefficient at this large Mach number condition is 1.5, determine the deceleration of the meteor.

$\mathcal{D} = ma$ where $m = \rho \forall = \rho \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 = (2.9)(999 \frac{kg}{m^3}) \frac{4\pi}{3}\left(\frac{2\times10^{-3}\ m}{2}\right)^3$

$= 1.21 \times 10^{-5}\ kg$

Also, $\mathcal{D} = C_D \frac{1}{2} \rho U^2 A$

$= 1.5\left(\frac{1}{2}\right)\left(1.03\times10^{-3} \frac{kg}{m^3}\right)\left(6\times10^3 \frac{m}{s}\right)^2 \frac{\pi}{4}\left(2\times10^{-3} m\right)^2 = 8.74\times10^{-2}\ N$

Thus,

$a = \frac{\mathcal{D}}{m} = \frac{8.74\times10^{-2}\ N}{1.21\times10^{-5}\ kg} = \underline{\underline{7220 \frac{m}{s^2}}}$

9.77

9.77 Air flows past two equal sized spheres (one rough, one smooth) that are attached to the arm of a balance as is indicated in Fig. P9.77. With $U = 0$ the beam is balanced. What is the minimum air velocity for which the balance arm will rotate clockwise?

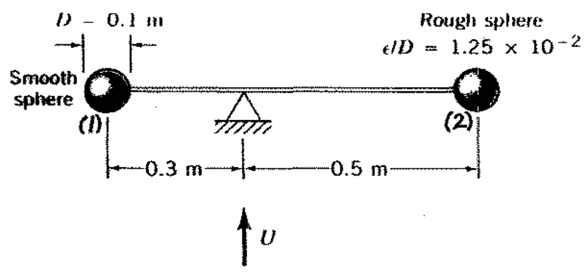


FIGURE P9.77

For clockwise rotation to start, $\Sigma M_o < 0$

That is $0.3\,\mathscr{D}_1 \geq 0.5\,\mathscr{D}_2$, where $\mathscr{D}_1 = C_{D_1}\frac{1}{2}\rho U_1^2 A_1$ and

$$\mathscr{D}_2 = C_{D_2}\frac{1}{2}\rho U_2^2 A_2$$

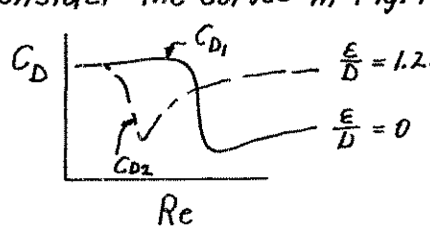
Thus,

$0.3\,C_{D_1}\frac{1}{2}\rho U_1^2 A_1 = 0.5\,C_{D_2}\frac{1}{2}\rho U_2^2 A_2$, or since $U_1 = U_2$ and $A_1 = A_2$

this gives

$$C_{D_2} = 0.6\,C_{D_1} \qquad (1)$$

Consider the curves in Fig. 9.25 with $\frac{\varepsilon}{D} = 0$ and $\frac{\varepsilon}{D} = 1.25 \times 10^{-2}$



Trial and error solution to find Re so that Eq. (1) is satisfied.

Assume $Re = 6 \times 10^4 \longrightarrow C_{D_1} = 0.5,\ C_{D_2} = 0.46$ or $\frac{C_{D2}}{C_{D1}} = 0.92 \neq 0.6$

Assume $Re = 8 \times 10^4 \longrightarrow C_{D_1} = 0.5,\ C_{D_2} = 0.21$ or $\frac{C_{D2}}{C_{D1}} = 0.42 \neq 0.6$

Assume $Re = 7 \times 10^4 \longrightarrow C_{D_1} = 0.5,\ C_{D_2} = 0.33$ or $\frac{C_{D2}}{C_{D1}} = 0.66 \approx 0.6$

Thus, $Re \approx 7.1 \times 10^4 = \frac{UD}{\nu} = \frac{(0.1\,m)\,U}{1.46 \times 10^{-5}\,\frac{m^2}{s}}$ or $U \approx \underline{\underline{10.4\,\frac{m}{s}}}$

9.78

9.78 A 2-in.-diameter sphere weighing 0.14 lb is suspended by the jet of air shown in Fig. P9.78 and Video V3.2. The drag coefficient for the sphere is 0.5. Determine the reading on the pressure gage if friction and gravity effects can be neglected for the flow between the pressure gage and the nozzle exit.

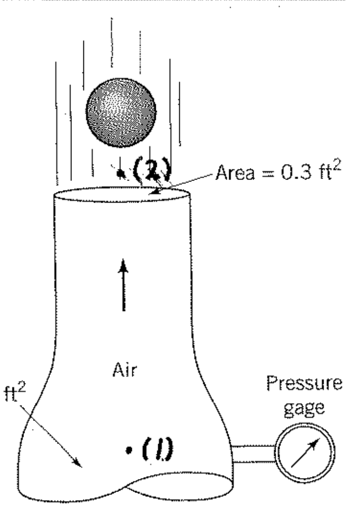
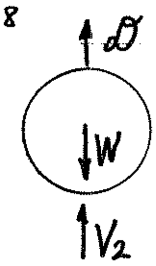


FIGURE P9.78



For equilibrium, $\mathscr{D} = W$ or

$C_D \frac{1}{2}\rho V_2^2 A = W$, where $A = \frac{\pi}{4}D^2$

Thus,

$$V_2 = \left[\frac{2W}{C_D \rho \pi D^2/4}\right]^{1/2}$$

$$= \left[\frac{8\,(0.14\,lb)}{0.5(0.00238\,\frac{slugs}{ft^3})\pi(\frac{2}{12}\,ft)^2}\right]^{1/2} = 104\,\frac{ft}{s}$$

Also,

$V_1 A_1 = V_2 A_2$ or $V_1 = V_2\,\frac{A_2}{A_1} = (104\,\frac{ft}{s})\frac{0.3\,ft^2}{0.6\,ft^2} = 52.0\,\frac{ft}{s}$

and

$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$ where $p_2 = 0$

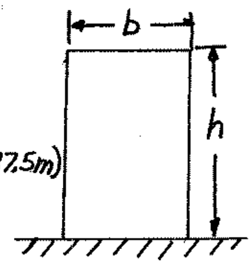
Thus,

$$p_1 = \frac{1}{2}\rho[V_2^2 - V_1^2] = \frac{1}{2}(0.00238\,\frac{slugs}{ft^3})\left[(104\,\frac{ft}{s})^2 - (52.0\,\frac{ft}{s})^2\right]$$

$$= \underline{\underline{9.65\,\frac{lb}{ft^2}}}$$

9.79

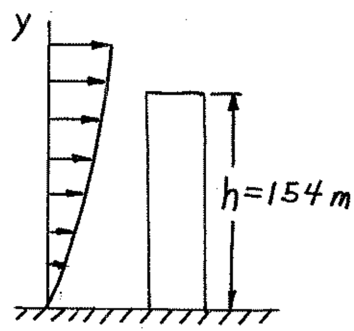
9.79 The United Nations Building in New York is approximately 87.5-m wide and 154-m tall. **(a)** Determine the drag on this building if the drag coefficient is 1.3 and the wind speed is a uniform 20 m/s. **(b)** Repeat your calculations if the velocity profile against the building is a typical profile for an urban area (see Problem 9.22) and the wind speed halfway up the building is 20 m/s.



(a) $\mathscr{D} = C_D \frac{1}{2}\rho U^2 A = 1.3\left(\frac{1}{2}\right)(1.23\ \frac{kg}{m^3})(20\ \frac{m}{s})^2(154m)(87.5m)$

or

$\mathscr{D} = 4.31 \times 10^6\ N = \underline{\underline{4.31\ MN}}$



(b) For an urban area, $u = C\,y^{0.4}$

Thus, with $u = 20\ \frac{m}{s}$ at $y = \frac{h}{2} = 77m$

we obtain

$C = \frac{20}{77^{0.4}} = 3.52$, or $u = 3.52\ y^{0.4}$ with $u \sim \frac{m}{s}$, $y \sim m$

The total drag is

$$\mathscr{D} = \int d\mathscr{D} = \int C_D \frac{1}{2}\rho u^2\, dA = \frac{1}{2}\rho C_D \int_{y=0}^{y=154} (3.52\ y^{0.4})^2 (87.5)\, dy$$

or

$$\mathscr{D} = \frac{1}{2}(1.23)(1.3)(3.52)^2(87.5)\int_0^{154} y^{0.8}\,dy = 867\left(\frac{1}{1.8}\right)(154)^{1.8} = 4.17 \times 10^6\ N$$

Thus,

$\mathscr{D} = \underline{\underline{4.17\ MN}}$

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

9.80

9.80 A regulation football is 6.78 in. in diameter and weighs 0.91 lb. If its drag coefficient is $C_D = 0.2$, determine its deceleration if it has a speed of 20 ft/s at the top of its trajectory.

$$\mathcal{D} = ma \quad , \quad \text{where} \quad m = \frac{W}{g} = \frac{0.91\ lb}{32.2\ \frac{ft}{s^2}} = 0.0283\ slugs$$

and

$$\mathcal{D} = C_D \frac{1}{2} \rho U^2 A = 0.2\left(\tfrac{1}{2}\right)\left(0.00238\ \frac{slugs}{ft^3}\right)\left(20\ \frac{ft}{s}\right)^2\left(\frac{\pi}{4}\left(\frac{6.78}{12}\ ft\right)^2\right) = 0.0239\ lb$$

Thus,

$$a = \frac{\mathcal{D}}{m} = \frac{0.0238\ lb}{0.0283\ slugs} = \underline{\underline{0.841\ \frac{ft}{s^2}}}$$

9.81

9.81 An airplane tows a banner that is b = 0.8 m tall and ℓ = 25 m long at a speed of 150 km/hr. If the drag coefficient based on the area $b\ell$ is C_D = 0.06, estimate the power required to tow the banner. Compare the drag force on the banner with that on a rigid flat plate of the same size. Which has the larger drag force and why?

$\mathcal{P} = \mathcal{D}U$, where $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$ with $A = b\ell$.

Thus, with $C_D = 0.06$ and $U = (150\,\frac{km}{hr})(\frac{1\,hr}{3600\,s})(\frac{1000\,m}{1\,km}) = 41.7\,\frac{m}{s}$ this gives

$\mathcal{P} = (0.06)(\frac{1}{2})(1.23\,\frac{kg}{m^3})(41.7\,\frac{m}{s})^3(0.8\,m)(25\,m) = 53.5 \times 10^3\,W = \underline{\underline{53.5\,kW}}$

For a rigid flat plate

$\mathcal{P} = \mathcal{D}U = 2C_D \frac{1}{2}\rho U^3 b\ell$ (the factor of two is needed because the drag coefficient is based on the drag on one side of the plate)

With $Re_\ell = \frac{U\ell}{\nu} = \frac{(41.7\frac{m}{s})(25\,m)}{1.46 \times 10^{-5}\,\frac{m^2}{s}} = 7.14 \times 10^7$ we obtain from Fig. 9.15 a value of $C_D \approx 0.0025$ for a smooth plate.

Thus,

$\mathcal{P} = 2(0.0025)(\frac{1}{2})(1.23\,\frac{kg}{m^3})(41.7\,\frac{m}{s})^3(0.8\,m)(25\,m) = 4.46 \times 10^3\,W = \underline{\underline{4.46\,kW}}$

For the flat plate case the drag is relatively small because it is due entirely to shear (viscous) forces. Due to the "fluttering" of the banner, a good portion of its drag (and hence power) is a result of pressure forces. It is not as streamlined as a rigid flat plate.

9.83

9.83 The paint stirrer shown in Fig. P9.83 consists of two circular disks attached to the end of a thin rod that rotates at 80 rpm. The specific gravity of the paint is $SG = 1.1$ and its viscosity is $\mu = 2 \times 10^{-2}\ \text{lb}\cdot\text{s/ft}^2$. Estimate the power required to drive the mixer if the induced motion of the liquid is neglected.

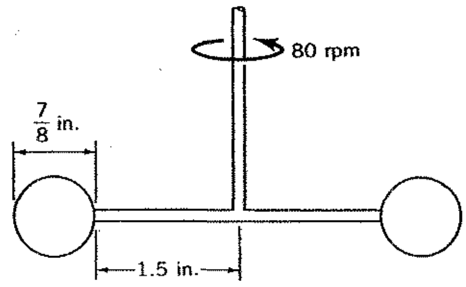


FIGURE P9.83

If we neglect the effects of the shaft and rod and consider the paint to be stationary, then

$\mathcal{M} = 2\mathcal{D}R$, where $\mathcal{M}$ = torque to rotate shaft
and $\mathcal{D}$ = drag on one disk $= C_D \frac{1}{2}\rho U^2 A$

Also, $U = \omega R$ and $\mathcal{P}$ = power to rotate shaft $= \mathcal{M}\omega$

Thus,

$$\mathcal{P} = 2\mathcal{D}R\,\omega = 2C_D\tfrac{1}{2}\rho(\omega R)^2\tfrac{\pi}{4}D^2 R\,\omega$$

or

$$\mathcal{P} = \tfrac{\pi}{4}C_D\,\rho\omega^3R^3D^2 = \tfrac{\pi}{4}C_D\,\rho U^3D^2 \quad \text{where } \rho = SG\,\rho_{H_2O} \qquad (1)$$

With $Re = \dfrac{\rho U D}{\mu} = \dfrac{SG\,\rho_{H_2O}UD}{\mu}$

where

$$U = \omega R = \left(80\tfrac{\text{rev}}{\text{min}}\right)\left(\tfrac{1\ \text{min}}{60\ \text{s}}\right)\left(\tfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\tfrac{1.5+\frac{7}{16}}{12}\ \text{ft}\right) = 1.353\ \tfrac{\text{ft}}{\text{s}}$$

we have

$$Re = \frac{(1.1)(1.94\ \frac{\text{slugs}}{\text{ft}^3})(1.353\ \frac{\text{ft}}{\text{s}})\left(\frac{7}{8(12)}\ \text{ft}\right)}{2\times10^{-2}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}} = 10.5$$

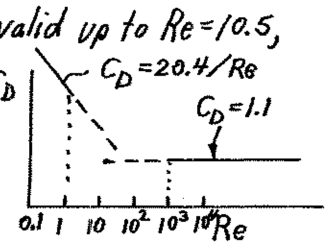
For a circular disk, $C_D = 1.1$ if $Re > 10^3$ (see Fig. 9.29)

while $\qquad C_D = \dfrac{20.4}{Re}$ if $Re < 1$ (see Table 9.4) $\qquad (2)$

For this particular problem $1 < Re = 10.5 < 10^3$

Note: If the low Reynolds number result (Eq. (2)) is valid up to $Re = 10.5$,

then $C_D = \dfrac{20.4}{10.5} = 1.94$



To be on the conservative side (i.e., maximum power)

use the larger C_D — $C_D = 1.94$ From Eq. (1)

$$\mathcal{P} = \tfrac{\pi}{4}(1.94)(1.1)(1.94\ \tfrac{\text{slugs}}{\text{ft}^3})(1.353\ \tfrac{\text{ft}}{\text{s}})^3\left(\tfrac{7}{8(12)}\ \text{ft}\right)^2$$

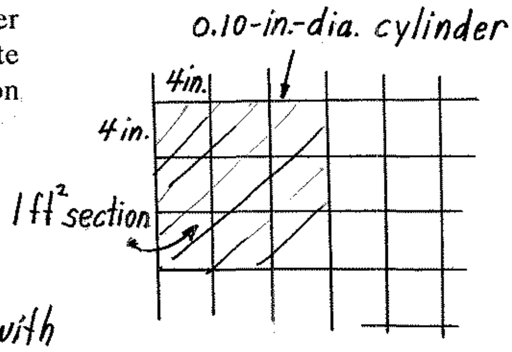
$$= 0.0428\ \tfrac{\text{ft}\cdot\text{lb}}{\text{s}}$$

or

$$\mathcal{P} = \left(0.0428\ \tfrac{\text{ft}\cdot\text{lb}}{\text{s}}\right)\left(\frac{1\ \text{hp}}{550\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}}\right) = \underline{\underline{7.78\times10^{-5}\ \text{hp}}}$$

9.85

9.85 A fishnet consists of 0.10-in.-diameter strings tied into squares 4 in. per side. Estimate the force needed to tow a 15 ft by 30 ft section of this net through seawater at 5 ft/s.



The net can be treated as one long 0.10-in.-diameter circular cylinder with

$\mathscr{D} = C_D \frac{1}{2} \rho U^2 A$, where $U = 5\,\frac{ft}{s}$. Each $1\ ft^2$ section of the net contains 6 feet of string (donot count the edges twice). Thus, the total string length is approximately $\ell = (6\,\frac{ft}{ft^2})(15\,ft)(30\,ft) = 2700\,ft$

Also, since $\rho = 1.99\,\frac{slugs}{ft^3}$ and $\nu = 1.26 \times 10^{-5}\,\frac{ft^2}{s}$ (see Table 1.5)

$$Re = \frac{UD}{\nu} = \frac{(5\,\frac{ft}{s})(\frac{0.10}{12}\,ft)}{1.26 \times 10^{-5}\,\frac{ft^2}{s}} = 3310$$

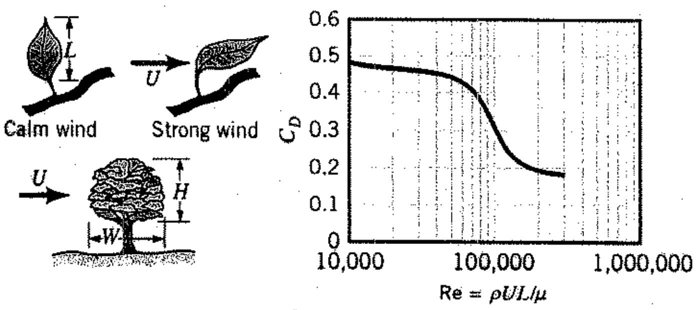
Hence, from Fig. 9.21 that $C_D = 1.1$

Thus,

$$\mathscr{D} = (1.1)(\tfrac{1}{2})(1.99\,\tfrac{slugs}{ft^3})(5\,\tfrac{ft}{s})^2(\tfrac{0.1}{12}\,ft)(2700\,ft) = \underline{\underline{616\ lb}}$$

9.86

9.86 As indicated in Fig. P9.86, the orientation of leaves on a tree is a function of the wind speed, with the tree becoming "more streamlined" as the wind increases. The resulting drag coefficient for the tree (based on the frontal area of the tree, HW) as a function of Reynolds number (based on the leaf length, L) is approximated as shown. Consider a tree with leaves of length $L = 0.3$ ft. What wind speed will produce a drag on the tree that is 6 times greater than the drag on the tree in a 15 ft/s wind?



■ FIGURE P9.86

$\mathscr{D} = C_D \frac{1}{2}\rho U^2 A$ and $Re = \frac{\rho U L}{\mu}$

or

$\mathscr{D} = C_D \frac{1}{2}(0.00238) U^2 HW = 0.00119\, HW C_D U^2$ (1)

and

$Re = \frac{0.00238 \frac{slugs}{ft^3} U (0.3\,ft)}{3.74 \times 10^{-7} lb \cdot s/ft^2} = 1909\, U$, where $U \sim ft/s$ (2)

Thus, with $U = 15$ ft/s, $Re = 1909(15) = 28,600$ so that from Fig. P9.84,

$C_D = 0.46$ so

$\mathscr{D}_{15} = 0.00119\, HW (0.46)(15)^2 = 0.123\, HW$

For the drag 6 time as great, $\mathscr{D} = 6\mathscr{D}_{15} = 6(0.123\, HW) = 0.738\, HW$ (3)

Thus, from Eqs. (1) and (3):

$0.738\, HW = 0.00119\, HW C_D U^2$

or

$C_D U^2 = 621$ (4)

Trial and error solution:

Assume $C_D = 0.3$ so that from Eq. (4), $U = \sqrt{\frac{621}{0.3}} = 45.5$ ft/s and from Eq. (2)

$Re = 1909(45.5) = 86,900$. Thus, from Fig. P9.84, $C_D = 0.33 \neq 0.3$, the assumed value.

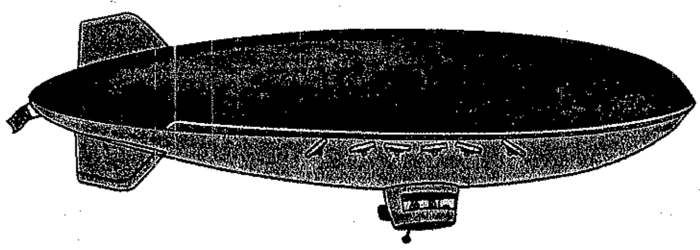
Try again. Assume $C_D = 0.33 \rightarrow U = 43.4$ ft/s $\rightarrow Re = 82,900 \rightarrow C_D = 0.36 \neq 0.33$

Try $C_D = 0.36 \rightarrow U = 41.6$ ft/s $\rightarrow Re = 79,300 \rightarrow C_D = 0.36$

Thus, $\underline{\underline{U = 41.6 \text{ ft/s}}}$

9.87

9.87 The blimp shown in Fig. P9.87 is used at various athletic events. It is 128 ft long and has a maximum diameter of 33 ft. If its drag coefficient (based on the frontal area) is 0.060, estimate the power required to propel it **(a)** at its 35-mph cruising speed, or **(b)** at its maximum 55-mph speed.



■ FIGURE P9.87

$\mathcal{P} = \mathcal{D} U$ where $\mathcal{D} = C_D \frac{1}{2} \rho U^2 A$

Thus, with

$\mathcal{D} = 0.060 \left(\frac{1}{2}\right)\left(0.00238 \frac{slugs}{ft^3}\right) U^2 \frac{\pi}{4} (33\ ft)^2$

$= 0.0611\, U^2\ lb$, where $U \sim ft/s$

(a) Thus, with $U = 35 \frac{mi}{hr} \left(\frac{5280\ ft/mi}{3600 s/hr}\right) = 51.3\ ft/s$,

$\mathcal{D} = 0.0611 (51.3)^2 = 161\ lb$

so that

$\mathcal{P} = \mathcal{D} U = 161 lb\ (51.3\ ft/s)\left(\frac{1\ hp}{550\ ft \cdot lb/s}\right) = \underline{\underline{15.0\ hp}}$

(b) Similarly, with $U = 55\ mph = 80.7\ ft/s$,

$\mathcal{D} = 0.0611 (80.7)^2 = 398\ lb$

so that

$\mathcal{P} = \mathcal{D} U = 398\ lb\ (80.7 \frac{ft}{s})\left(\frac{1\ hp}{550\ ft \cdot lb/s}\right) = \underline{\underline{58.4\ hp}}$

9.88

9.88 Show that for level flight at a given speed, the power required to overcome aerodynamic drag decreases as the altitude increases. Assume that the drag coefficient remains constant. This is one reason why airlines fly at high altitudes.

For level flight $\mathscr{L} = W$, where W = airplane weight = constant
and $\mathscr{L} = C_L \frac{1}{2} \rho V^2 A$

If V is to remain constant, then C_L must increase as ρ decreases (i.e., altitude increases).

Also, $\mathcal{P} = \mathscr{D} V$, where $\mathscr{D} = C_D \frac{1}{2} \rho V^2 A$
or

$\mathcal{P} = C_D \frac{1}{2} \rho V^3 A$. For constant V, C_D, and A, the power decreases as altitude increases (ρ decreases).

9.89

9.89 (See Fluids in the News article "Dimpled baseball bats," Section 9.3.3.) How fast must a 3.5-in.-diameter, dimpled baseball bat move through the air in order to take advantage of drag reduction produced by the dimples on the bat. Although there are differences, assume the bat (a cylinder) acts the same as a golf ball in terms of how the dimples affect the transition from a laminar to a turbulent boundary layer.

From Fig. 9.25, for a golf ball the dimples reduce drag for $Re = \frac{\rho U D}{\mu} \gtrsim 4 \times 10^4$
Thus, assume $Re = 4 \times 10^4$ for the bat so that

$$\frac{\rho U D}{\mu} = 4 \times 10^4$$

or

$$\frac{\left(0.00238 \frac{slugs}{ft^3}\right) U \left(\frac{3.5}{12} ft\right)}{\left(3.74 \times 10^{-7} \frac{lb \cdot s}{ft^2}\right)} = 4 \times 10^4$$

Thus,

$$\underline{\underline{U = 21.6 \frac{ft}{s}}}$$

9.90

9.90 (See Fluids in the News article "At 10,240 mpg it doesn't cost much to 'fill 'er up,'" Section 9.3.3.) (a) Determine the power it takes to overcome aerodynamic drag on a small (6 ft^2 cross section), streamlined ($C_D = 0.12$) vehicle traveling 15 mph. (b) Compare the power calculated in part (a) with that for a large (36 ft^2 cross-sectional area), nonstreamlined ($C_D = 0.48$) SUV traveling 65 mph on the interstate.

$\mathcal{P}$ = power $= U\mathscr{D}$, where $\mathscr{D} = C_D \frac{1}{2}\rho U^2 A$

so that

$$\mathcal{P} = C_D \frac{1}{2}\rho U^3 A$$

$$\text{(a)}\quad \mathcal{P} = 0.12\left(\frac{1}{2}\right)\left(0.00238\,\frac{slugs}{ft^3}\right)\left[\left(15\,\frac{mi}{hr}\right)\left(\frac{5280\ ft/mi}{3600\ s/hr}\right)\right]^3 (6\ ft^2)$$

$$= 9.12\,\frac{ft\cdot lb}{s}\left(\frac{1\ hp}{550\ ft\cdot lb/s}\right) = \underline{\underline{0.0166\ hp}}$$

$$\text{(b)}\quad \mathcal{P} = 0.48\left(\frac{1}{2}\right)\left(0.00238\,\frac{slugs}{ft^3}\right)\left[\left(65\,\frac{mi}{hr}\right)\left(\frac{5280\ ft/mi}{3600\ s/hr}\right)\right]^3 (36\ ft^2)$$

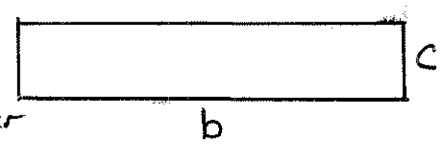
$$= 17{,}800\,\frac{ft\cdot lb}{s}\left(\frac{1\ hp}{550\ ft\cdot lb/s}\right) = \underline{\underline{32.4\ hp}}$$

9.92

9.92 A rectangular wing with an aspect ratio of 6 is to generate 1000 lb of lift when it flies at a speed of 200 ft/s. Determine the length of the wing if its lift coefficient is 1.0.

Aspect ratio, $\mathcal{A} = b^2/A = 6$

$= b/c$ for rectangular wing



The lift coefficient is given by,

$$C_L = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 A}$$

$$\mathcal{L} = C_L \tfrac{1}{2}\rho U^2 A \quad \text{where } A = bc = 6c^2$$

$$\mathcal{L} = C_L \tfrac{1}{2}\rho U^2 (6c^2)$$

$$1000\ lb = 1.0\left(\tfrac{1}{2}\right)(0.00238\ slugs/ft^3)(200\ ft/s)^2(6c^2)$$

$$6c^2 = 21.0$$

$$c = 1.87\ ft$$

$$b = 6(c) = 6(1.87\ ft)$$

$$\underline{\underline{b = 11.2\ ft}}$$

9.94

9.94 A Piper Cub airplane has a gross weight of 1750 lb, a cruising speed of 115 mph, and a wing area of 179 ft^2. Determine the lift coefficient of this airplane for these conditions.

For equilibrium $\mathcal{L} = W = 1750\ lb$, where $\mathcal{L} = C_L \frac{1}{2}\rho U^2 A$

Thus, with $U = (115\ mph)\dfrac{(88\ \frac{ft}{s})}{(60\ mph)} = 169\ \frac{ft}{s}$

$$C_L = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 A} = \frac{1750\ lb}{\frac{1}{2}(0.00238\ \frac{slugs}{ft^3})(169\ \frac{ft}{s})^2(179\ ft^2)} = \underline{\underline{0.288}}$$

9.95

9.95 A light aircraft with a wing area of 200 ft^2 and a weight of 2000 lb has a lift coefficient of 0.40 and a drag coefficient of 0.05. Determine the power required to maintain level flight.

For equilibrium $\mathcal{L} = W = 2000\ lb = C_L \frac{1}{2}\rho U^2 A$

or
$2000\ lb = (0.40)\frac{1}{2}(0.00238\ \frac{slugs}{ft^3})U^2(200\ ft^2)$

Hence,
$U = 145\ \frac{ft}{s}$

Also, $\mathcal{P} = \text{power} = \mathcal{D}U$, where

$\mathcal{D} = C_D \frac{1}{2}\rho U^2 A = (0.05)\frac{1}{2}(0.00238\ \frac{slugs}{ft^3})(145\ \frac{ft}{s})^2(200\ ft^2) = 250\ lb$

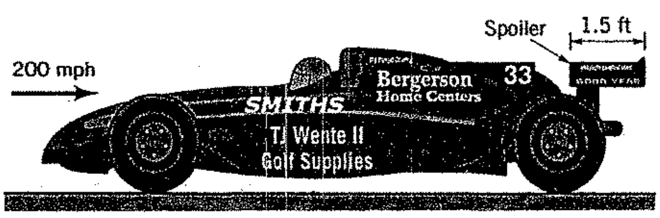
Note: This value of $\mathcal{D}$ could be obtained from

$$\frac{W}{\mathcal{D}} = \frac{\mathcal{L}}{\mathcal{D}} = \frac{C_L}{C_D} = \frac{0.40}{0.05} = 8, \text{ or } \mathcal{D} = \frac{W}{8} = \frac{2000\ lb}{8} = 250\ lb$$

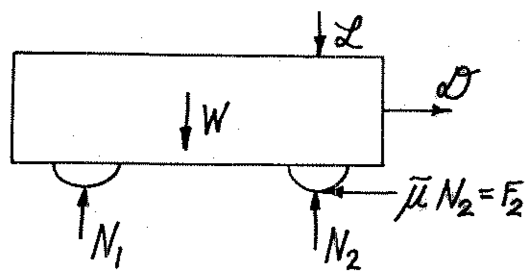
Thus,
$$\mathcal{P} = 250\ lb\,(145\ \tfrac{ft}{s}) = 3.63 \times 10^4\ \frac{ft \cdot lb}{s}\left(\frac{1\ hp}{550\ \frac{ft \cdot lb}{s}}\right) = \underline{\underline{65.9\ hp}}$$

9.96

9.96 As shown in Video V9.19 and Fig. P9.96, a spoiler is used on race cars to produce a negative lift, thereby giving a better tractive force. The lift coefficient for the airfoil shown is $C_L = 1.1$, and the coefficient of friction between the wheels and the pavement is 0.6. At a speed of 200 mph, by how much would use of the spoiler increase the maximum tractive force that could be generated between the wheels and ground? Assume the air speed past the spoiler equals the car speed and that the airfoil acts directly over the drive wheels.



■ FIGURE P9.96



Tractive force $= F_2 = \tilde{\mu} N_2$

where $\tilde{\mu}$ = coefficient of friction $= 0.6$

Thus,

$\Delta F_2 = \tilde{\mu} \Delta N_2 = \tilde{\mu} \mathcal{L}$, where ΔF_2 is the increase in tractive force due to the (downward) lift.

Hence, with $U = 200\,mph = 293\,ft/s$,

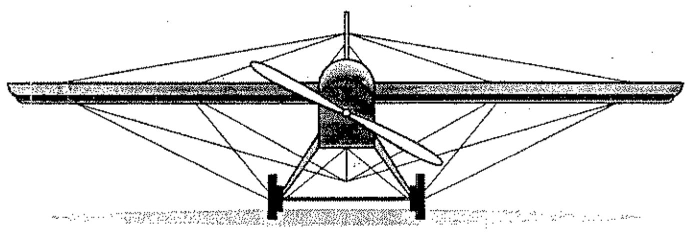
$$\mathcal{L} = \frac{1}{2}\rho U^2 C_L A = \frac{1}{2}\left(0.00238\,\frac{slugs}{ft^3}\right)\left(293\,\frac{ft}{s}\right)^2 (1.1)(1.5ft)(4ft) = 674\,lb,$$

and

$\Delta F_2 = 0.6\,(674\,lb) = \underline{\underline{405\,lb}}$

9.97

9.97 The wings of old airplanes are often strengthened by the use of wires that provided cross-bracing as shown in Fig. P9.97. If the drag coefficient for the wings was 0.020 (based on the planform area), determine the ratio of the drag from the wire bracing to that from the wings.



Speed: 70 mph
Wing area: 148 ft^2
Wire: length = 160 ft
diameter = 0.05 in.

■ FIGURE P9.97

$\mathcal{D}_{wing} = \frac{1}{2}\rho U^2 C_{D_{wing}} A_{wing}$

and

$\mathcal{D}_{wire} = \frac{1}{2}\rho U^2 C_{D_{wire}} A_{wire}$ so that

$\dfrac{\mathcal{D}_{wire}}{\mathcal{D}_{wing}} = \dfrac{C_{D_{wire}} A_{wire}}{C_{D_{wing}} A_{wing}}$, where $A_{wing} = 148\ ft^2$, $C_{D_{wing}} = 0.02$

Also, $A_{wire} = \ell D = (160\ ft)\left(\frac{0.05}{12}\ ft\right) = 0.667\ ft^2$

and since

$$Re = \frac{UD}{\nu} = \frac{(70\ mph)\left(\frac{88\ \frac{ft}{s}}{60\ mph}\right)\left(\frac{0.05}{12}\ ft\right)}{1.57 \times 10^{-4}\ \frac{ft^2}{s}} = 2720.$$

From Fig. 9.21, with $Re = 2720$ we obtain $C_D = 1.0$

Hence,

$$\frac{\mathcal{D}_{wire}}{\mathcal{D}_{wing}} = \frac{(1.0)(0.667\ ft^2)}{(0.02)(148\ ft^2)} = 0.225, \text{ or } \underline{\underline{22.5\%}}$$

9.98

9.98 A wing generates a lift $\mathscr{L}$ when moving through sea-level air with a velocity U. How fast must the wing move through the air at an altitude of 10,000 m with the same lift coefficient if it is to generate the same lift?

$\mathscr{L} = C_L \frac{1}{2} \rho U^2 A$ so with $\mathscr{L}$, C_L, and A constant

$$(\rho U^2)_{sea\ level} = (\rho U^2)_{10{,}000\,m}$$

Hence,

$$U_{10{,}000\,m} = \left(\frac{\rho_{sea\ level}}{\rho_{10{,}000\,m}}\right)^{1/2} U_{sea\ level} = \left(\frac{1.23\,\frac{kg}{m^3}}{0.414\,\frac{kg}{m^3}}\right)^{1/2} U_{sea\ level}$$

or

$$\underline{\underline{U_{10{,}000\,m} = 1.72\,U_{sea\ level}}}$$

*9.99

9.99 Air blows over the flat-bottomed, two-dimensional object shown in Fig. P9.91. The shape of the object, $y = y(x)$, and the fluid speed along the surface, $u = u(x)$, are given in the table. Determine the lift coefficient for this object.

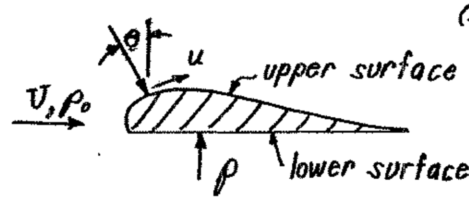
x (% c)	y (% c)	u/U
0	0	0
2.5	3.72	0.971
5.0	5.30	1.232
7.5	6.48	1.273
10	7.43	1.271
20	9.92	1.276
30	11.14	1.295
40	11.49	1.307
50	10.45	1.308
60	9.11	1.195
70	6.46	1.065
80	3.62	0.945
90	1.26	0.856
100	0	0.807

If viscous effects are negligible, then

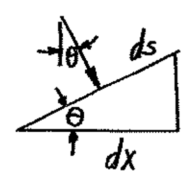
$$\mathcal{L} = \int_{lower} p\cos\theta\, dA - \int_{upper} p\cos\theta\, dA \qquad (1)$$

where from the Bernoulli equation

$$p + \tfrac{1}{2}\rho u^2 = p_0 + \tfrac{1}{2}\rho U^2 \qquad (2)$$



The effect of atmospheric pressure, p_0, drops out when the integration over the entire surface is performed.
With $\theta = 0$ on the lower surface and with $\cos\theta\, dA = \cos\theta\,(\ell\, ds) = \ell\, dx$, where ℓ = wing span, Eqs.(1) and (2) give



$$\mathcal{L} = \int_{lower}\left[p_0 + \tfrac{1}{2}\rho(U^2 - u^2)\right]\ell\, dx - \int_{upper}\left[p_0 + \tfrac{1}{2}\rho(U^2 - u^2)\right]\ell\, dx$$

or, since $u = U$ on the lower surface

$$\mathcal{L} = -\tfrac{1}{2}\rho\ell\int_{x=0}^{x=c}(U^2 - u^2)\,dx = \tfrac{1}{2}\rho U^2 \ell c\int_{x'=0}^{x'=1}\left[\left(\frac{u}{U}\right)^2 - 1\right]dx' \text{ , where } x' = \frac{x}{c} \qquad (3)$$

Thus, since

$$C_L = \frac{\mathcal{L}}{\tfrac{1}{2}\rho U^2 A} = \frac{\mathcal{L}}{\tfrac{1}{2}\rho U^2 \ell c}$$

it follows from Eq.(3) that

$$C_L = \int_{x'=0}^{x'=1}\left[\left(\frac{u}{U}\right)^2 - 1\right]dx'$$

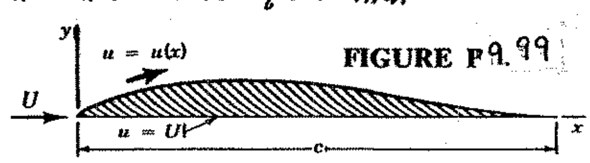


FIGURE P9.99

By using a standard numerical integration routine with the data given we obtain

$C_L = 0.327$

x'	$\left(\frac{u}{U}\right)^2 - 1$
0	-1.00
0.025	-0.0572
0.050	0.518
0.075	0.621
0.100	0.615
0.200	0.628
0.300	0.677
0.400	0.708
0.500	0.711
0.600	0.428
0.700	0.134
0.800	-0.107
0.900	-0.267
1.000	-0.349

9.101

9.101 A Boeing 747 aircraft weighing 580,000 lb when loaded with fuel and 100 passengers takes off with an airspeed of 140 mph. With the same configuration (i.e., angle of attack, flap settings, etc.) what is its takeoff speed if it is loaded with 372 passengers. Assume each passenger with luggage weighs 200 lb.

For steady flight $\mathcal{L} = C_L \frac{1}{2}\rho V^2 A = W$ (1)

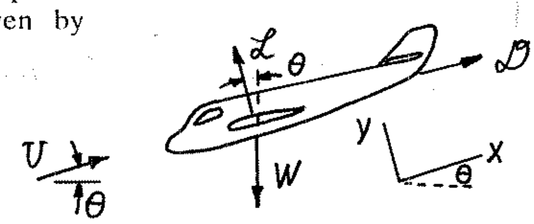
Let $(\ )_{100}$ denote conditions with 100 passengers and $(\ )_{372}$ with 372 passengers. Thus, with $C_{L\,100} = C_{L\,372}$, $A_{100} = A_{372}$, and $\rho_{100} = \rho_{372}$ Eq. (1) gives

$$\frac{\mathcal{L}_{100}}{\mathcal{L}_{372}} = \frac{U_{100}^2}{U_{372}^2} \quad \text{or} \quad U_{372} = U_{100}\left\{\frac{[580{,}000 + (372-100)(200)]\,\text{lb}}{580{,}000\ \text{lb}}\right\}^{1/2}, \text{ with } U_{100} = 140\,\text{mph}$$

Thus, $U_{372} = \underline{\underline{146\ \text{mph}}}$

9.102

9.102 Show that for unpowered flight (for which the lift, drag, and weight forces are in equilibrium) the glide slope angle, θ, is given by $\tan\theta = C_D/C_L$.



For steady unpowered flight

$\Sigma F_x = 0$ gives $\mathcal{D} = W\sin\theta$

and

$\Sigma F_y = 0$ gives $\mathcal{L} = W\cos\theta$

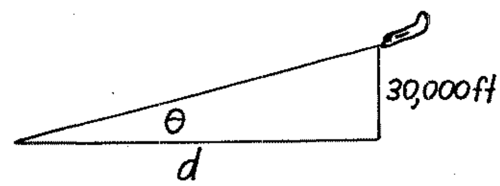
Thus,

$$\frac{\mathcal{D}}{\mathcal{L}} = \frac{W\sin\theta}{W\cos\theta} = \tan\theta, \text{ where } \frac{\mathcal{D}}{\mathcal{L}} = \frac{\frac{1}{2}\rho U^2 A C_D}{\frac{1}{2}\rho U^2 A C_L} = \frac{C_D}{C_L}$$

Hence, $\underline{\underline{\tan\theta = \frac{C_D}{C_L}}}$

9.103

9.103 If the lift coefficient for a Boeing 777 aircraft is 15 times greater than its drag coefficient, can it glide from an altitude of 30,000 ft to an airport 80 mi away if it loses power from its engines? Explain. (See Problem 9.102)



From Problem 9.102, $\tan\theta = \frac{C_D}{C_L} = \frac{1}{15}$

Hence,

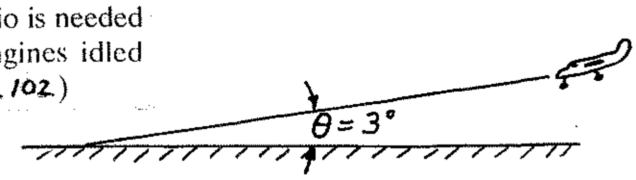
$\frac{30{,}000}{d} = \frac{1}{15}$, or $d = 4.5\times10^5\,ft$

$= 85.2\,mi$

Hence, the plane can glide 80mi.

9.104

9.104 On its final approach to the airport an airplane flies on a flight path that is 3.0° relative to the horizontal. What lift-to-drag ratio is needed if the airplane is to land with its engines idled back to zero power? (See Problem 9.102.)



From Problem 9.102,

$\tan\theta = \frac{C_D}{C_L}$

or

$\frac{C_D}{C_L} = \tan 3° = 0.0524$ $\qquad \frac{C_L}{C_D} = 19.1$

9.105

9.105 Over the years there has been a dramatic increase in the flight speed (U) and altitude (h), weight ($\mathcal{W}$), and wing loading ($\mathcal{W}/A$ = weight divided by wing area) of aircraft. Use the data given in the table below to determine the lift coefficient for each of the aircraft listed.

Aircraft	Year	$\mathcal{W}$, lb	U, mph	$\mathcal{W}/A$, lb/ft^2	h, ft
Wright Flyer	1903	750	35	1.5	0
Douglas DC-3	1935	25,000	180	25.0	10,000
Douglas DC-6	1947	105,000	315	72.0	15,000
Boeing 747	1970	800,000	570	150.0	30,000

$$C_L = \frac{\mathcal{L}}{\frac{1}{2}\rho U^2 A} = \frac{W}{\frac{1}{2}\rho U^2 A} = \frac{2}{\rho U^2}\left(\frac{W}{A}\right)$$

Thus,

	ρ, $slugs/ft^3$	U, ft/s	W/A, lb/ft^2	C_L
Wright Flyer	2.38×10^{-3}	51.3	1.5	0.480
DC-3	1.76×10^{-3}	264	25.0	0.409
DC-6	1.50×10^{-3}	462	72.0	0.451
747	8.91×10^{-4}	836	150	0.482

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

9.106

9.106 The landing speed of an airplane such as the Space Shuttle is dependent on the air density. (See Video V9.1.) By what percent must the landing speed be increased on a day when the temperature is 110 deg F compared to a day when it is 50 deg F? Assume the atmospheric pressure remains constant.

For equalibrium, lift = weight, or

$$\frac{1}{2}\rho U^2 C_L A = W$$

Thus, with constant W, C_L, and A,

$$(\rho U^2)_{T=110^\circ} = (\rho U^2)_{T=50^\circ} \quad \text{or}$$

$$U_{110^\circ} = \left(\frac{\rho_{50}}{\rho_{110}}\right)^{\frac{1}{2}} U_{50^\circ}$$

But $p = \rho R T$ so that $\dfrac{\rho_{50}}{\rho_{110}} = \dfrac{(p_{50}/RT_{50})}{(p_{110}/RT_{110})} = \dfrac{(460+110)}{(460+50)} = 1.1176$

Thus,

$$U_{110^\circ} = \sqrt{1.1176}\, U_{50^\circ} = 1.0572\, U_{50^\circ} \quad \text{or a } \underline{5.72\% \text{ increase}}$$

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

9.107

9.107 Commercial airliners normally cruise at relatively high altitudes (30,000 to 35,000 ft). Discuss how flying at this high altitude (rather than 10,000 ft, for example) can save fuel costs.

For level flight W = aircraft weight $= \mathcal{L} = C_L \frac{1}{2}\rho U^2 A$

Thus, for given W, C_L, and A the dynamic pressure is constant, independent of altitude. That is

$$\left.\tfrac{1}{2}\rho U^2\right)_{10{,}000\,ft} = \left.\tfrac{1}{2}\rho U^2\right)_{30{,}000\,ft}, \text{ or } U_{30{,}000} = \left(\frac{\rho_{10{,}000}}{\rho_{30{,}000}}\right)^{1/2} U_{10{,}000}$$

Hence, $U_{30{,}000} > U_{10{,}000}$

Also, since the drag is $\mathcal{D} = C_D \frac{1}{2}\rho U^2 A$ it follows that

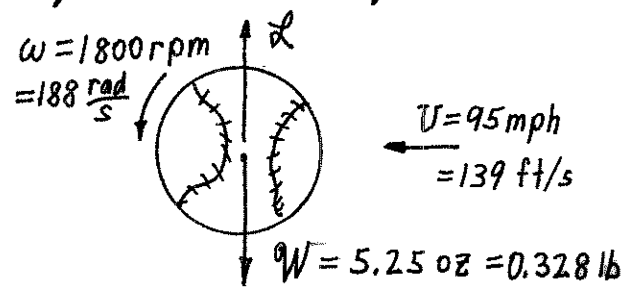
$$\mathcal{D}_{30{,}000} = \left.C_D \tfrac{1}{2}\rho U^2 A\right)_{30{,}000} = \left.C_D \tfrac{1}{2}\rho U^2 A\right)_{10{,}000} \text{ since } \tfrac{1}{2}\rho U_{30{,}000}^2 = \tfrac{1}{2}\rho U_{10{,}000}^2$$

Hence, the aircrart can fly faster at high altituded with the same amount of drag ($\mathcal{D}_{30{,}000} = \mathcal{D}_{10{,}000}$)

9.109

9.109 For many years, hitters have claimed that some baseball pitchers have the ability to actually throw a rising fastball. Assuming that a top major leaguer pitcher can throw a 95-mph pitch and impart a 1800-rpm spin to the ball, is it possible for the ball to actually rise? Assume the baseball diameter is 2.9 in. and its weight is 5.25 oz.

If the lift produced on the spinning ball is greater than its weight the ball will rise.



$$\mathscr{L} = C_L \tfrac{1}{2}\rho U^2 A$$

where C_L is a function of $\frac{\omega D}{2U}$ as shown in Fig. 9.39.

Thus, with

$$\frac{\omega D}{2U} = \frac{(188\,\frac{\text{rad}}{\text{s}})(\frac{2.9}{12}\,\text{ft})}{2\,(139\,\text{ft/s})} = 0.163$$

$C_L = 0.04$

Hence, for the given conditions

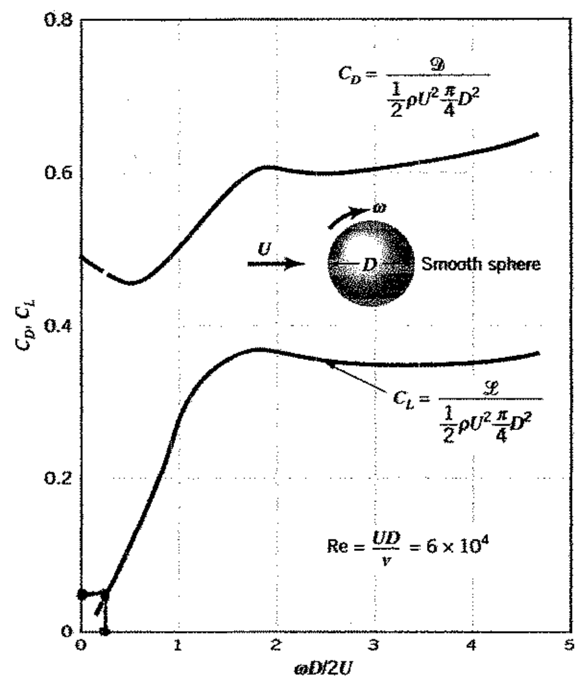
$$\mathscr{L} = 0.04\left(\tfrac{1}{2}\right)(0.00238\,\tfrac{\text{slugs}}{\text{ft}^3})(139\,\tfrac{\text{ft}}{\text{s}})^2 \times \tfrac{\pi}{4}\left(\tfrac{2.9}{12}\,\text{ft}\right)^2 = 0.0422\,\text{lb}$$

so that

$\mathscr{L} = 0.0422\,\text{lb} < \mathscr{W} = 0.328\,\text{lb}$

The ball will not rise.

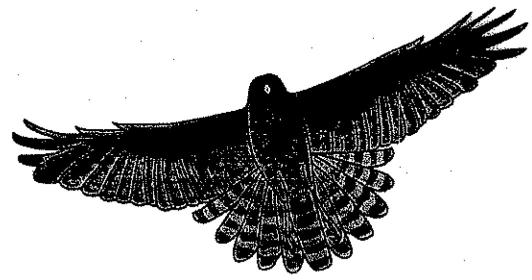
Note: The above result is based on smooth-sphere data. The results for a baseball (with its rough surface containing seams) will probably give a somewhat larger lift because for a given angular velocity it can "drag" more air along as it spins.



■ **FIGURE 9.39** **Lift and drag coefficients for a spinning smooth sphere (Ref. 23).**

9.110

9.110 (See "**Learning from nature,**" Section 9.4.1.) As indicated in Fig. P9.110, birds can significantly alter their body shape and increase their planform area, A, by spreading their wing and tail feathers, thereby reducing their flight speed. If during landing the planform area is increased by 50% and the lift coefficient increased by 30% while all other parameters are held constant, by what percent is the flight speed reduced?



■ FIGURE P9.110

$$\mathcal{L} = C_L \tfrac{1}{2}\rho U^2 A$$

Let $(\ )_2$ denote landing conditions and $(\ )_1$ denote normal flight conditions. Thus, with $\mathcal{L}_1 = \mathcal{L}_2$,

$$C_{L1} \tfrac{1}{2}\rho U_1^2 A_1 = C_{L2} \tfrac{1}{2}\rho U_2^2 A_2$$

or

$$U_2 = U_1 \sqrt{\frac{A_1}{A_2}} \sqrt{\frac{C_{L1}}{C_{L2}}} = U_1 \sqrt{\frac{A_1}{1.5A_1}} \sqrt{\frac{C_{L1}}{1.3C_{L1}}}$$

or

$$U_2 = 0.716\, U_1$$

Hence,

$$\frac{U_2 - U_1}{U_1} = 0.716 - 1 = -0.284$$

i.e., a $\underline{\underline{28.4\% \text{ reduction}}}$ in flight speed

9.111

9.111 (See Fluids in the News article "Why winglets?," Section 9.4.2.) It is estimated that by installing appropriately designed winglets on a certain airplane the drag coefficient will be reduced by 5%. For the same engine thrust, by what percent will the aircraft speed be increased by use of the winglets?

Let $(\ )_1$ denote without winglets and $(\ )_2$ with winglets.
Thus, since drag equals thrust and $\text{thrust}_1 = \text{thrust}_2$, it follows that

$$\mathscr{D}_1 = \mathscr{D}_2$$

or

$$C_{D_1} \tfrac{1}{2}\rho U_1^2 A_1 = C_2 \tfrac{1}{2}\rho U_2^2 A_2$$

so that with $A_1 = A_2$,

$$U_2 = U_1 \sqrt{\frac{C_{D1}}{C_{D_2}}} = U_1 \sqrt{\frac{C_{D1}}{0.95\, C_{D_1}}} = 1.0260\, U_1$$

Thus, a 2.60% increase in speed is realized.

9.112

9.112 Boundary Layer on a Flat Plate

Objective: A boundary layer is formed on a flat plate when air blows past the plate. The thickness, δ, of the boundary layer increases with distance, x, from the leading edge of the plate. The purpose of this experiment is to use an apparatus, as shown in Fig. P9.112, to measure the boundary layer thickness.

Equipment: Wind tunnel; flat plate; boundary layer mouse consisting of ten Pitot tubes positioned at various heights, y, above the flat plate; inclined multiple manometer; measuring calipers; barometer, thermometer.

Experimental Procedure: Position the tips of the Pitot tubes of the boundary layer mouse a known distance, x, downstream from the leading edge of the plate. Use calipers to determine the distance, y, between each Pitot tube and the plate. Fasten the tubing from each Pitot tube to the inclined multiple manometer and determine the angle of inclination, θ, of the manometer board. Adjust the wind tunnel speed, U, to the desired value and record the manometer readings, L. Move the boundary layer mouse to a new distance, x, downstream from the leading edge of the plate and repeat the measurements. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: For each distance, x, from the leading edge, use the manometer data to determine the air speed, u, as a function of distance, y, above the plate (see Eq. 3.13). That is, obtain $u = u(y)$ at various x locations. Note that both the wind tunnel test section and the open end of the manometer tubes are at atmospheric pressure.

Graph: Plot speed, u, as ordinates and distance from the plate, y, as abscissas for each location, x, tested.

Results: Use the $u = u(y)$ results to determine the approximate boundary layer thickness as a function of distance, $\delta = \delta(x)$. Plot a graph of boundary layer thickness as a function of distance from the leading edge. Note that the air flow within the wind tunnel is quite turbulent so that the measured boundary layer thickness is not expected to match the theoretical laminar boundary layer thickness given by the Blassius solution (see Eq. 9.15).

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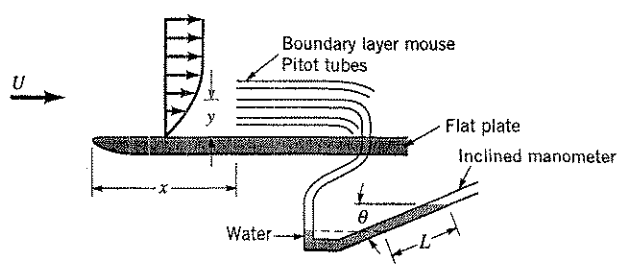


■ FIGURE P9.112

(con't)

9.112 (con't)

Solution for Problem 9.112: Boundary Layer on a Flat Plate

θ, deg	H_{atm}, in. Hg	T, deg F	γ_{H2O}, lb/ft^3
25	29.09	80	62.4

y, in.	L, in.	u, ft/s	y, in.	L, in.	u, ft/s
Data for x = 7.75 in.			Data for x = 3.75 in.		
0.020	0.20	19.9	0.020	0.15	17.2
0.035	0.35	26.3	0.035	0.35	26.3
0.044	0.48	30.8	0.044	0.45	29.8
0.060	0.70	37.2	0.060	0.71	37.5
0.096	0.95	43.4	0.096	1.20	48.7
0.110	1.06	45.8	0.110	1.30	50.7
0.138	1.21	48.9	0.138	1.56	55.6
0.178	1.44	53.4	0.178	1.77	59.2
0.230	1.70	58.0	0.230	1.95	62.1
0.270	1.85	60.5	0.270	2.00	62.9
Data for x = 5.75 in.			Data for x = 1.75 in.		
0.020	0.20	19.9	0.020	0.20	19.9
0.035	0.42	28.8	0.035	0.50	31.5
0.044	0.50	31.5	0.044	0.68	36.7
0.060	0.71	37.5	0.060	0.90	42.2
0.096	0.98	44.0	0.096	1.51	54.7
0.110	1.06	45.8	0.110	1.70	58.0
0.138	1.30	50.7	0.138	1.90	61.3
0.178	1.54	55.2	0.178	1.95	62.1
0.230	1.76	59.0	0.230	2.00	62.9
0.270	1.88	61.0	0.270	2.00	62.9

$\rho u^2/2 = \gamma_{H2O}$*L $\sin\theta$
where
$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{H2O}$*H_{atm} = 847 lb/ft^3*(29.09/12 ft) = 2053 lb/ft^2
R = 1716 ft lb/slug deg R
T = 80 + 460 = 540 deg R

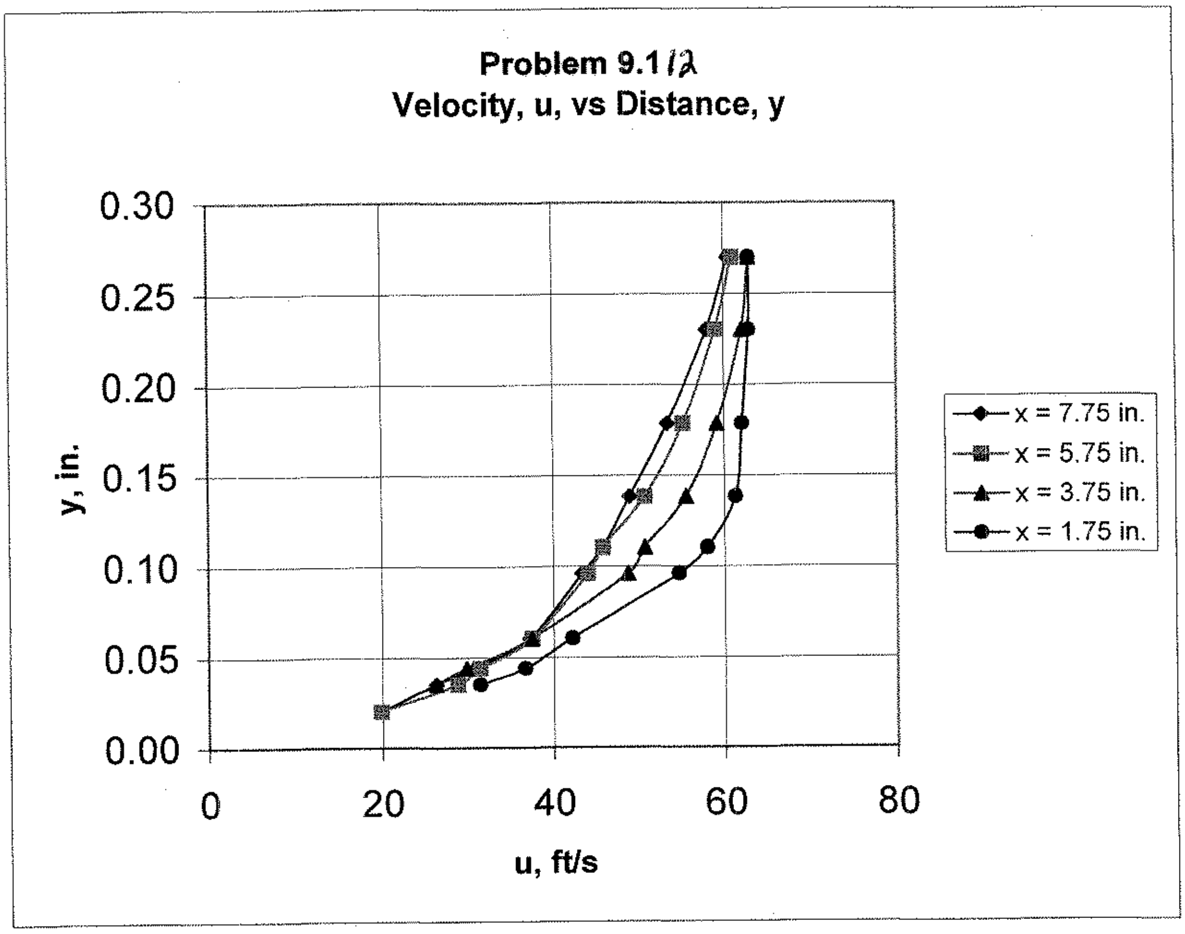
Thus, ρ = 0.00222 slug/ft^3

Approximate boundary layer thickness as obtained from the graph:

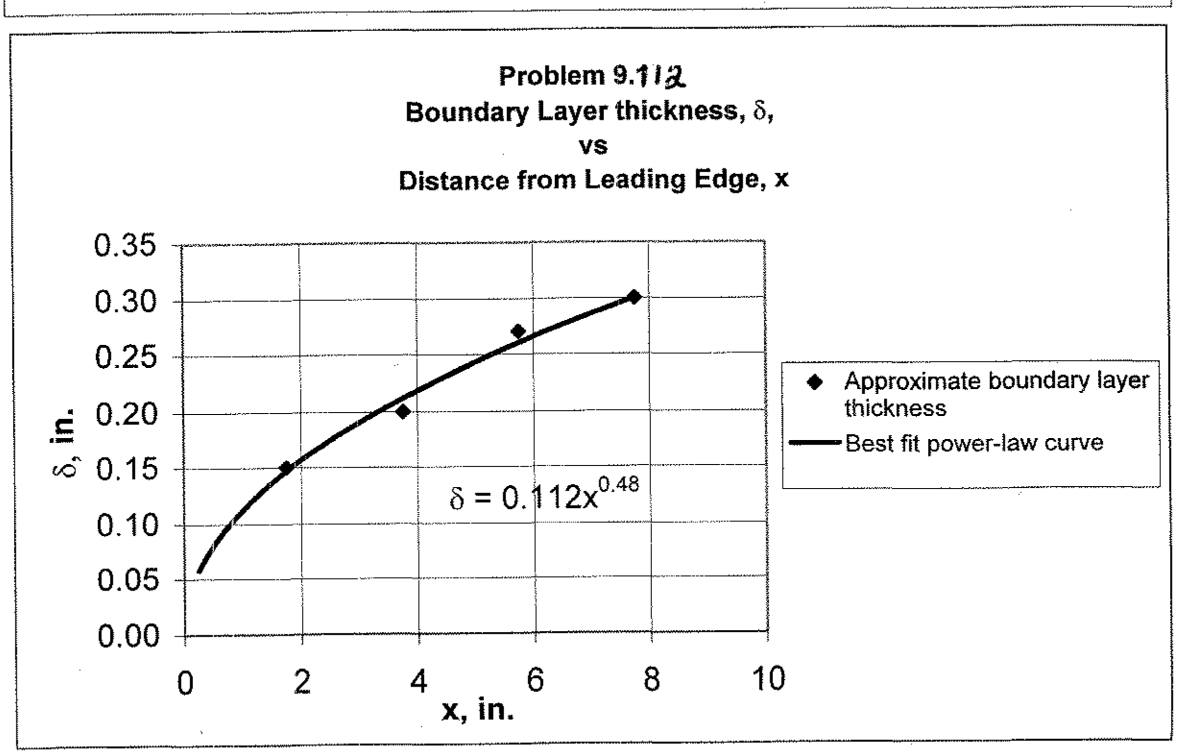
x, in.	δ, in.
1.75	0.15
3.75	0.20
5.75	0.27
7.75	0.30

(con't)

9.112 (con't)



Problem 9.112
Velocity, u, vs Distance, y
y, in.
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
20
40
60
80
u, ft/s
x = 7.75 in.
x = 5.75 in.
x = 3.75 in.
x = 1.75 in.



Problem 9.112
Boundary Layer thickness, δ,
vs
Distance from Leading Edge, x
δ, in.
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
2
4
6
8
10
x, in.
δ = 0.112x^0.48
Approximate boundary layer thickness
Best fit power-law curve

9.113

9.113 Pressure Distribution on a Circular Cylinder

Objective: Viscous effect within the boundary layer on a circular cylinder cause boundary layer separation, thereby causing the pressure distribution on the rear half of the cylinder to be different than that on the front half. The purpose of this experiment is to use an apparatus, as shown in Fig. P9.113, to determine the pressure distribution on a circular cylinder.

Equipment: Wind tunnel; circular cylinder with 18 static pressure taps arranged equally from the front to the back of the cylinder; inclined multiple manometer; barometer; thermometer.

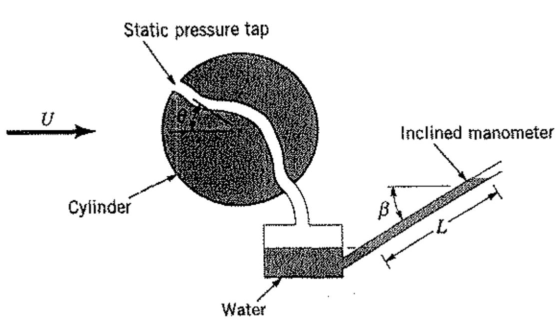
Experimental Procedure: Mount the circular cylinder in the wind tunnel so that a static pressure tap points directly upstream. Measure the angle, β, of the inclined manometer. Adjust the wind tunnel fan speed to give the desired free stream speed, U, in the test section. Attach the tubes from the static pressure taps to the multiple manometer and record the manometer readings, L, as a function of angular position, θ. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the data to determine the pressure coefficient, $C_p = (p - p_0)/(\rho U^2/2)$, as a function of position, θ. Here $p_0 = 0$ is the static pressure upstream of the cylinder in the free stream of the wind tunnel, and $p = \gamma_m L \sin\beta$ is the pressure on the surface of the cylinder.

Graph: Plot the pressure coefficient, C_p, as ordinates and the angular location, θ, as abscissas.

Results: On the same graph, plot the theoretical pressure coefficient, $C_p = 1 - 4 \sin^2\theta$, obtained from ideal (inviscid) theory (see Section 6.6.3).

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P9.113

(con't)

9.113 (con't)

Solution for Problem 9.113: Pressure Distribution on a Circular Cylinder

β, deg	H_{atm}, in. Hg	T, deg F	U, ft/s
25	29.97	75	47.9

		Experiment		Theory
θ, deg	L, in.	p, lb/ft^2	C_p	C_p
0	1.2	2.64	1.00	1.00
10	1.1	2.42	0.92	0.88
20	0.7	1.54	0.58	0.53
30	0.1	0.22	0.08	0.00
40	-0.6	-1.32	-0.50	-0.65
50	-1.6	-3.52	-1.33	-1.35
60	-2.4	-5.27	-2.00	-2.00
70	-3.1	-6.81	-2.58	-2.53
80	-3.0	-6.59	-2.50	-2.88
90	-2.7	-5.93	-2.25	-3.00
100	-2.7	-5.93	-2.25	-2.88
110	-2.6	-5.71	-2.17	-2.53
120	-2.6	-5.71	-2.17	-2.00
130	-2.6	-5.71	-2.17	-1.35
140	-2.6	-5.71	-2.17	-0.65
150	-2.6	-5.71	-2.17	0.00
160	-2.7	-5.93	-2.25	0.53
170	-2.7	-5.93	-2.25	0.88
180	-2.8	-6.15	-2.33	1.00

$p = \gamma_{H20}$*L $\sin\beta$

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg}$*H_{atm} = 847 lb/ft^3*(29.97/12 ft) = 2115 lb/ft^2

R = 1716 ft lb/slug deg R

T = 75 + 460 = 535 deg R

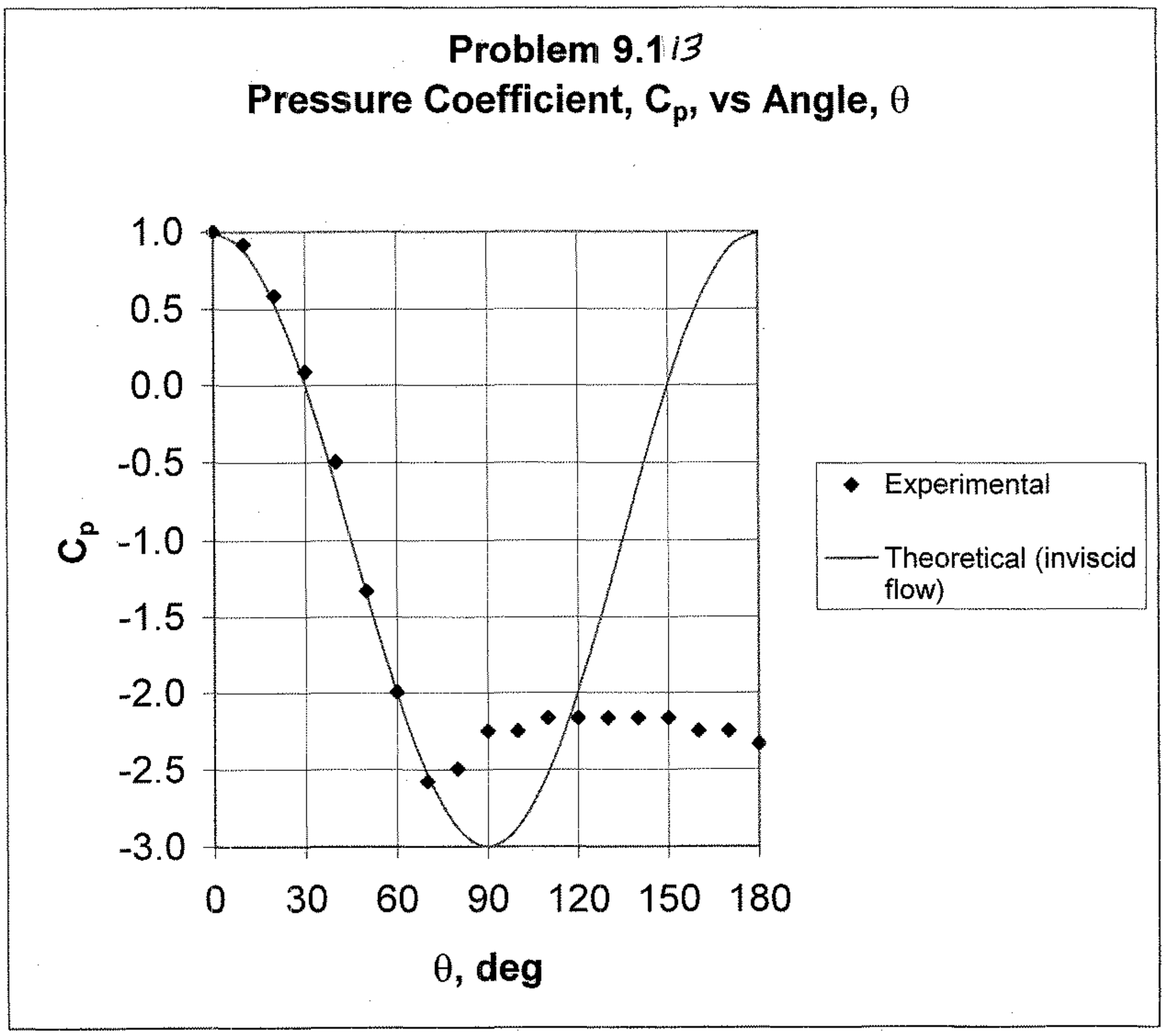
Thus, ρ = 0.00230 slug/ft^3

$C_p = p/(\rho U^2/2)$

Theory: $C_p = 1 - 4\sin^2\theta$

(con't)

9.113 (con't)


Problem 9.113
Pressure Coefficient, C_p, vs Angle, θ
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
C_p
0
30
60
90
120
150
180
θ, deg
Experimental
Theoretical (inviscid flow)