8.2

8.2 Water flows through a 50-ft pipe with a 0.5-in. diameter at 5 gal/min. What fraction of this pipe can be considered an entrance region?

Based on Tables 1.3 & 1.4

$$5\ \text{gal/min} = 1.11 \times 10^{-2}\ \text{ft}^3/\text{s}$$

Determine Re

$$V = Q/A = \frac{1.11\times10^{-2}}{\frac{\pi}{4}\left(\frac{0.5}{12}\right)^2} = 8.17\ \text{ft/s}$$

$$Re = \frac{VD}{\nu} = \frac{(8.17)\,(0.5/12)}{1.21\times10^{-5}} = 2.81\times10^{4}$$

For turbulent flow

$$\frac{l_e}{D} = 4.4(Re)^{1/6}$$

$$l_e = (0.5/12)\,4.4\,(2.81\times10^4)^{1/6}$$

$$l_e = \underline{\underline{1.0\ \text{ft}}}$$

8.3

8.3 Rainwater runoff from a parking lot flows through a 3-ft-diameter pipe, completely filling it. Whether flow in a pipe is laminar or turbulent depends on the value of the Reynolds number. (See Video V8.2) Would you expect the flow to be laminar or turbulent? Support your answer with appropriate calculations.

$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$ If $Re > 4000$ the flow is turbulent. The corresponding velocity is

$$V = \frac{Re\,\nu}{D} = \frac{(4000)(1.21 \times 10^{-5}\,\frac{ft^2}{s})}{3\,ft} = 0.0161\,\frac{ft}{s}$$

Most likely the velocity will be greater than this, i.e., turbulent flow.

8.4

8.4 Blue and yellow streams of paint at 60 °F (each with a density of 1.6 slugs/ft^3 and a viscosity 1000 times greater than water) enter a pipe with an average velocity of 4 ft/s as shown in Fig. P8.4. Would you expect the paint to exit the pipe as green paint or separate streams of blue and yellow paint? Explain. Repeat the problem if the paint were "thinned" so that it is only 10 times more viscous than water. Assume the density remains the same.

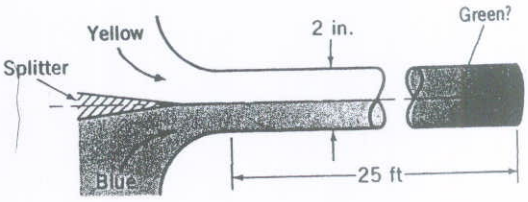


FIGURE P8.4

If the flow is laminar the paint would exit as separate blue and yellow streams.

$$Re = \frac{\rho V D}{\mu} = \frac{\rho V D}{1000\,\mu_{H_2O}} = \frac{1.6\,\frac{slugs}{ft^3}\left(4\,\frac{ft}{s}\right)\left(\frac{2}{12}\,ft\right)}{1000\left(2.34\times10^{-5}\,\frac{lb\cdot s}{ft^2}\right)} = 45.6 < 2100$$

Thus, laminar flow so blue and yellow streams.

If use $\mu = 10\mu_{H_2O}$ obtain

$Re = 4560 > 4000$ so have turbulent flow with natural mixing and green paint.

Note: Check to determine if the 25 ft length is greater than the entrance length, ℓ_e.

For laminar flow $\frac{\ell_e}{D} = 0.06\,Re$, or $\ell_e = 0.06(45.6)\left(\frac{2}{12}\,ft\right) = 0.456\,ft < 25\,ft$

For turbulent flow $\frac{\ell_e}{D} = 4.4\,Re^{1/6}$, or $\ell_e = 4.4(4560)^{\frac{1}{6}}\left(\frac{2}{12}\,ft\right) = 2.99\,ft < 25\,ft$

8.5

8.5 Air at 200 °F flows at standard atmospheric pressure in a pipe at a rate of 0.08 lb/s. Determine the minimum diameter allowed if the flow is to be laminar.

Maximum $Re = \frac{\rho V D}{\mu}$ for laminar flow is $Re = 2100$.
or with

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2} \quad , \quad Re = \frac{\rho\left(\frac{4Q}{\pi D^2}\right)D}{\mu} = \frac{4\rho Q}{\pi \mu D} = 2100$$

Hence,

$$Q = \frac{2100 \pi \mu D}{4 \rho} \qquad (1)$$

Given $\gamma Q = 0.08 \frac{lb}{s}$, where $\gamma = g\rho$ and $\rho = \frac{p}{RT}$
Thus,

$$\rho = \frac{(14.7 \times 144 \frac{lb}{ft^2})}{(1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R})(460+200)^\circ R} = 0.00187 \frac{slugs}{ft^3}$$

so that

$$Q = \frac{0.08 \frac{lb}{s}}{(32.2 \frac{ft}{s^2})(0.00187 \frac{slugs}{ft^3})} = 1.33 \frac{ft^3}{s}$$

Hence, with $\mu = 4.49 \times 10^{-7} \frac{lb \cdot s}{ft^2}$ (see Table B.3), Eq. (1) gives

$$D = \frac{4 \rho Q}{2100 \pi \mu} = \frac{4(0.00187 \frac{slugs}{ft^3})(1.33 \frac{ft^3}{s})}{2100 \pi (4.49 \times 10^{-7} \frac{lb \cdot s}{ft^2})} = \underline{\underline{3.36 \, ft}}$$

8.6

8.6 To cool a given room it is necessary to supply 4 ft^3/s of air through an 8-in.-diameter pipe. Approximately how long is the entrance length in this pipe?

$V = \frac{Q}{A} = \frac{4 \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{8}{12} ft\right)^2} = 11.5 \frac{ft}{s}$ Thus, with $\nu = 1.57 \times 10^{-4} \frac{ft^2}{s}$ (see Table 1.6)

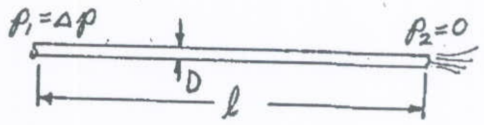
$Re = \frac{VD}{\nu} = \frac{11.5 \frac{ft}{s}\left(\frac{8}{12} ft\right)}{1.57 \times 10^{-4} \frac{ft^2}{s}} = 48{,}800 > 4000$ so the flow is turbulent.

Hence,

$\frac{\ell_e}{D} = 4.4\, Re^{1/6}$, or $\ell_e = 4.4(48{,}800)^{1/6}\left(\frac{8}{12}\right) = \underline{\underline{17.7\ ft}}$

8.7

8.7 A long small-diameter tube is to be used as a viscometer by measuring the flowrate through the tube as a function of the pressure drop along the tube. The calibration constant, $K = Q/\Delta p$, is calculated by assuming the flow is laminar. For tubes of diameter 0.5, 1.0, and 2.0 mm, determine the maximum flowrate allowed (in cm^3/s) if the fluid is **(a)** 20 °C water, or **(b)** standard air.



$Re = \frac{VD}{\nu}$ where $Q = VA = \frac{\pi}{4}D^2 V$

Thus,

$$Re = \frac{4QD}{\pi D^2 \nu} = \frac{4Q}{\pi D \nu}, \text{ or } Q = \frac{\pi D \nu Re}{4}$$

Maximum Q occurs with maximum Re for laminar flow: $Re \approx 2100$

Thus, $Q_{max} = 1650\, \nu D$

a) For 20°C water $\nu = 1.004 \times 10^{-6} \frac{m^2}{s}$

Hence, $Q_{max} = 1650\,(1.004 \times 10^{-6} \frac{m^2}{s})D = 1.66 \times 10^{-3} D \frac{m^3}{s}$ with $D \sim m$

b) For standard air $\nu = 1.46 \times 10^{-5} \frac{m^2}{s}$

Hence, $Q_{max} = 1650\,(1.46 \times 10^{-5} \frac{m^2}{s})D = 2.41 \times 10^{-2} D \frac{m^3}{s}$ with $D \sim m$

Thus, the following values are obtained:

	D, m	Q_{max}, $\frac{m^3}{s}$	Q_{max}, $\frac{cm^3}{s}$
(a) water	0.0005	8.30×10^{-7}	0.83
	0.0010	1.66×10^{-6}	1.66
	0.0020	3.32×10^{-6}	3.32
(b) air	0.0005	1.21×10^{-5}	12.1
	0.001	2.41×10^{-5}	24.1
	0.002	4.82×10^{-5}	48.2

Note: $1\, cm^3 = 10^{-6} m^3$

8.8

8.8 Carbon dioxide at 20 °C and a pressure of 550 kPa (abs) flows in a pipe at a rate of 0.04 N/s. Determine the maximum diameter allowed if the flow is to be turbulent.

For turbulent flow, $Re = \frac{\rho V D}{\mu} > 4000$, where $Q = VA = \frac{\pi}{4} D^2 V$

or $Re = \frac{4 \rho Q D}{\pi \mu D^2} = \frac{4 \rho Q}{\pi \mu D} = 4000$

Thus, $D = \frac{4 \rho Q}{4000 \pi \mu}$, where $g \rho Q = 0.04 \frac{N}{s}$ and $\mu = 1.4 \times 10^{-5} \frac{N \cdot s}{m^2}$ (Table 1.8)

Hence,

$$D = \frac{4 \left(0.04 \frac{N}{s}\right)\left(\frac{1}{9.81 \frac{m}{s^2}}\right)}{4000 \pi \left(1.47 \times 10^{-5} \frac{N \cdot s}{m^2}\right)} = \underline{\underline{0.0883 \ m}}$$

8.9

8.9 The pressure distribution measured along a straight, horizontal portion of a 50-mm-diameter pipe attached to a tank is shown in the table below. Approximately how long is the entrance length? In the fully developed portion of the flow, what is the value of the wall shear stress?

x (m) (± 0.01 m)	p (mm H_2O) (± 5 mm)
0 (tank exit)	520
0.5	427
1.0	351
1.5	288
2.0	236
2.5	188
3.0	145
3.5	109
4.0	73
4.5	36
5.0 (pipe exit)	0

The entrance length extends to the fully developed portion in which $\frac{\partial p}{\partial x} = \text{constant}$. Approximate $\frac{\partial p}{\partial x} \approx \frac{\delta p}{\delta x}$ to obtain the following:

From $X=$	to $X=(\ )m$	δp, mm H_2O	δx	$\frac{\partial p}{\partial x}$, $\frac{mm\,H_2O}{m}$
0	0.5	−93	0.5	−186
0.5	1.0	−76	0.5	−152
1.0	1.5	−63	0.5	−126
1.5	2.0	−52	0.5	−104
2.0	2.5	−48	0.5	−96
2.5	3.0	−43	0.5	−86
3.0	3.5	−36	0.5	−72
3.5	4.0	−36	0.5	−72
4.0	4.5	−37	0.5	−74
4.5	5.0	−36	0.5	−72

Within the error on δp, the pressure gradient is constant for $X \gtrsim 3, m$ Thus, $\underline{\underline{\ell_e \approx 3m}}$.

For $X > 3\,m$, $\frac{\Delta p}{\ell} = 72 \frac{mm\,H_2O}{m}$ Since $1\,mm\,H_2O \times \gamma_{H_2O} = (1 \times 10^{-3} m)(9800 \frac{N}{m^3})$ $= 9.80 \frac{N}{m^2}$, then

$$\frac{\Delta p}{\ell} = 72 \frac{mm\,H_2O}{m} \left(\frac{9.80 \frac{N}{m^2}}{mm\,H_2O} \right) = 706 \frac{N}{m^3}$$

Since $\Delta p = \frac{4 \tau_w \ell}{D}$ it follows that

$$\tau_w = \frac{D}{4} \frac{\Delta p}{\ell} = \frac{0.050m}{4} (706 \frac{N}{m^3}) = \underline{\underline{8.83 \frac{N}{m^2}}}$$

8.10

8.10 (See Fluids in the News article titled "Nanoscale flows," Section 8.1.1.) **(a)** Water flows in a tube that has a diameter of $D = 0.1$ m. Determine the Reynolds number if the average velocity is 10 diameters per second. **(b)** Repeat the calculations if the tube is a nanoscale tube with a diameter of $D = 100$ nm.

(a) $Re = \frac{VD}{\nu}$, where $D = 0.1m$, $V = 10(0.1m)/s = 1\frac{m}{s}$, and $\nu = 1.12 \times 10^{-6}\frac{m^2}{s}$

Thus,

$$Re = \frac{(1\frac{m}{s})(0.1m)}{1.12\times10^{-6}\frac{m^2}{s}} = \underline{\underline{89,300}}$$

(b) $Re = \frac{VD}{\nu}$, where $D = 100\,nm\left(\frac{1\,m}{10^9 nm}\right) = 10^{-7}m$, $V = 10(10^{-7}m)/s = 10^{-6}\frac{m}{s}$, and $\nu = 1.12\times10^{-6}\frac{m^2}{s}$

Thus,

$$Re = \frac{(10^{-6}\frac{m}{s})(10^{-7}m)}{1.12\times10^{-6}\frac{m^2}{s}} = \underline{\underline{8.93\times10^{-8}}}$$

8.12

8.12 For fully developed laminar pipe flow in a circular pipe, the velocity profile is given by $u(r) = 2\,(1 - r^2/R^2)$ in m/s, where R is the inner radius of the pipe. Assuming that the pipe diameter is 4 cm, find the maximum and average velocities in the pipe as well as the volume flow rate.

$$u(r) = 2(1 - r^2/R^2)$$

Based on Eq. (8.7),

Maximum velocity, $\underline{\underline{V_c = 2\ m/s}}$

We could also use the fact that the maximum velocity occurs at the centerline of the pipe, $r=0$

$$u(0) = 2(1 - 0/R^2) = \underline{\underline{2\ m/s}}$$

Average velocity, V

$$V = V_c/2 = 2/2 = \underline{\underline{1\ m/s}}$$

Volume flowrate, Q

$$Q = VA = (1)\,\frac{\pi}{4}(0.04)^2 = \underline{\underline{1.26 \times 10^{-3}\ m^3/s}}$$

8.13

8.13 The wall shear stress in a fully developed flow portion of a 12-in.-diameter pipe carrying water is 1.85 lb/ft². Determine the pressure gradient, $\partial p/\partial x$, where x is in the flow direction, if the pipe is **(a)** horizontal, **(b)** vertical with flow up, or **(c)** vertical with flow down.

In general, $\dfrac{\Delta p - \gamma \ell \sin\theta}{\ell} = \dfrac{2\tau}{r}$

Thus, with $\tau = \tau_w$ at $r = \frac{D}{2}$ and $\frac{\partial p}{\partial x} = -\frac{\Delta p}{\ell}$ this becomes

$$\frac{\partial p}{\partial x} = -\frac{4\tau_w}{D} - \gamma \sin\theta$$

a) For a horizontal pipe $\theta = 0$

$$\frac{\partial p}{\partial x} = -\frac{4\tau_w}{D} = -\frac{4\left(1.85\,\frac{lb}{ft^2}\right)}{1\,ft} = \underline{\underline{-7.40\,\frac{lb}{ft^3}}}$$

b) For vertical flow up $\theta = 90°$

$$\frac{\partial p}{\partial x} = -\frac{4\tau_w}{D} - \gamma = -\frac{4\left(1.85\,\frac{lb}{ft^2}\right)}{1\,ft} - 62.4\,\frac{lb}{ft^3} = \underline{\underline{-69.8\,\frac{lb}{ft^3}}}$$

and

c) For vertical flow down $\theta = -90°$

$$\frac{\partial p}{\partial x} = -\frac{4\tau_w}{D} + \gamma = -\frac{4\left(1.85\,\frac{lb}{ft^2}\right)}{1\,ft} + 62.4\,\frac{lb}{ft^3} = \underline{\underline{55.0\,\frac{lb}{ft^3}}}$$

8.14

8.14 The pressure drop needed to force water through a horizontal 1-in.-diameter pipe is 0.60 psi for every 12-ft length of pipe. Determine the shear stress on the pipe wall. Determine the shear stress at distances 0.3 and 0.5 in. away from the pipe wall.

For a horizontal pipe $\frac{\Delta p}{\ell} = \frac{2\tau}{r}$ or $\tau = \frac{r}{2}\frac{\Delta p}{\ell}$

Thus,

$$\tau = r\,\frac{(0.6 \times 144\,\frac{lb}{ft^2})}{2(12\ ft)} = 3.6\,r\,\frac{lb}{ft^2}\text{, where } r \sim ft$$

Hence,

$$\tau_w = 3.6\left(\frac{0.5}{12}\right) = \underline{\underline{0.15\,\frac{lb}{ft^2}}}$$

and with $r = (0.5-0.3)\,in. = 0.2\,in.$,

$$\tau = 3.6\left(\frac{0.2}{12}\right) = \underline{\underline{0.06\,\frac{lb}{ft^2}}}$$

Finally, with $r = (0.5-0.5)\,in. = 0\,in.$ $\quad \underline{\underline{\tau = 0}}$

8.15

8.15 Repeat Problem 8.14 if the pipe is on a 20° hill. Is the flow up or down the hill? Explain.

For a pipe on a hill $\frac{\Delta p}{\ell} = \frac{2\tau}{r} + \gamma \sin\theta$, where $\theta = \pm 20°$

Assume the flow is uphill : $\theta = +20°$

Thus, $\tau = \frac{r}{2}\left[\frac{\Delta p}{\ell} - \gamma \sin\theta\right]$ or $\tau_w = \frac{1}{2}\left(\frac{0.5}{12}\,ft\right)\left[\frac{0.6 \times 144\,\frac{lb}{ft^2}}{12\,ft} - 62.4\,\frac{lb}{ft^3}\sin 20°\right]$

or $\tau_w = -0.295\,\frac{lb}{ft^2}$ Since we must have $\tau_w > 0$, the flow must not be uphill.

Assume the flow is downhill: $\theta = -20°$

Thus, $\tau = \frac{r}{2}\left[\frac{\Delta p}{\ell} - \gamma \sin\theta\right]$ or $\tau = \frac{r}{2}\left[\frac{0.6 \times 144\,\frac{lb}{ft^2}}{12\,ft} + 62.4\,\frac{lb}{ft^3}\sin 20°\right]$

$= 14.3\,r\,\frac{lb}{ft^2}$, where $r \sim ft$. The $\underline{\underline{\text{flow is downhill}}}$

Hence, with $r = \frac{D}{2}$

$$\tau_w = 14.3\left(\frac{0.5}{12}\right) = \underline{\underline{0.596\,\frac{lb}{ft^2}}}$$

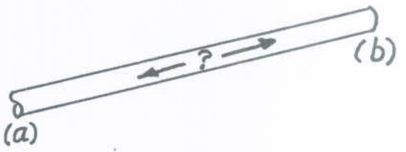
With $r = (0.5-0.3)\,in. = 0.2\,in.$,

$$\tau = 14.3\left(\frac{0.2}{12}\right) = \underline{\underline{0.238\,\frac{lb}{ft^2}}}$$

With $r = (0.5-0.5)\,in. = 0$, $\underline{\underline{\tau = 0}}$

8.16

8.16 Water flows in a constant diameter pipe with the following conditions measured: At section (a) $p_a = 32.4$ psi and $z_a = 56.8$ ft; at section (b) $p_b = 29.7$ psi and $z_b = 68.2$ ft. Is the flow from (a) to (b) or from (b) to (a)? Explain.



Assume the flow is uphill. Thus, $\frac{p_a}{\gamma} + \frac{V_a^2}{2g} + z_a = \frac{p_b}{\gamma} + \frac{V_b^2}{2g} + z_b + h_L$

or with $V_a = V_b$,

$$h_L = \frac{p_a}{\gamma} + z_a - \frac{p_b}{\gamma} - z_b = \frac{(32.4\,psi - 29.7\,psi)(144\,\frac{in^2}{ft^2})}{62.4\,\frac{lb}{ft^3}} + 56.8\,ft - 68.2\,ft$$

or

$h_L = -5.17\,ft < 0$, which is impossible. Thus, the flow is downhill, from (b) to (a).

*** 8.17**

***8.17** Some fluids behave as a non-Newtonian power-law fluid characterized by $\tau = -C(du/dr)^n$, where $n = 1, 3, 5$, and so on, and C is a constant. (If $n = 1$, the fluid is the customary Newtonian fluid.) **(a)** For flow in a round pipe of a diameter D, integrate the force balance equation (**Eq. 8.3**) to obtain the velocity profile

$$u(r) = \frac{-n}{(n+1)}\left(\frac{\Delta p}{2\ell C}\right)^{1/n}\left[r^{(n+1)/n} - \left(\frac{D}{2}\right)^{(n+1)/n}\right]$$

(b) Plot the dimensionless velocity profile u/V_c, where V_c is the centerline velocity (at $r = 0$), as a function of the dimensionless radial coordinate $r/(D/2)$, where D is the pipe diameter. Consider values of $n = 1, 3, 5$, and 7.

(a) For any fluid $\frac{\Delta p}{\ell} = \frac{2\tau}{r}$ so that with $\tau = -C\left(\frac{du}{dr}\right)^n$ we obtain

$\frac{\Delta p}{\ell} = -\frac{2C}{r}\left(\frac{du}{dr}\right)^n$ or $\frac{du}{dr} = -\left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}} r^{\frac{1}{n}}$ *

or

$-\int du = \left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}}\int r^{\frac{1}{n}}dr$ which integrates to give

$u = -\left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}}\frac{n}{(n+1)}\, r^{\left(\frac{n+1}{n}\right)} + C_1$, where C_1 is a constant. (1)

The fluid sticks to the pipe so that $u = 0$ at $r = \frac{D}{2}$.

Hence, from Eq. (1)

$C_1 = \left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}}\frac{n}{(n+1)}\left(\frac{D}{2}\right)^{\frac{(n+1)}{n}}$

so that

$$\underline{\underline{u = \frac{n}{(n+1)}\left(\frac{\Delta p}{2C\ell}\right)^{\frac{1}{n}}\left[-r^{\frac{(n+1)}{n}} + \left(\frac{D}{2}\right)^{\frac{(n+1)}{n}}\right]}}$$

*Note: Since we are considering only odd integer values for n we can use the fact that if

$\left(\frac{du}{dr}\right)^n = -K$, where $K > 0$, then $\frac{du}{dr} = -K^{\frac{1}{n}}$

so that $\frac{du}{dr} < 0$.

(b) From part (a):

$u(r) = \frac{n}{(n+1)}\left(\frac{\Delta p}{2\ell C}\right)^{\frac{1}{n}}\left[-r^{\left(\frac{n+1}{n}\right)} + \left(\frac{D}{2}\right)^{\left(\frac{n+1}{n}\right)}\right]$ (2)

Let $V_c = u(r=0)$, or $V_c = \frac{-n}{(n+1)}\left(\frac{\Delta p}{2\ell C}\right)^{\frac{1}{n}}\left(\frac{D}{2}\right)^{\left(\frac{n+1}{n}\right)}$ (3)

Note: For $\tau = C\left(\frac{du}{dr}\right)^n$ with $\frac{du}{dr} < 0$ and n an odd integer, to have $\tau > 0$, we must have $C < 0$. Thus, from Eq. (2), $V_c > 0$ as it must.

By dividing Eq. (2) by Eq. (3) we obtain

(con't)

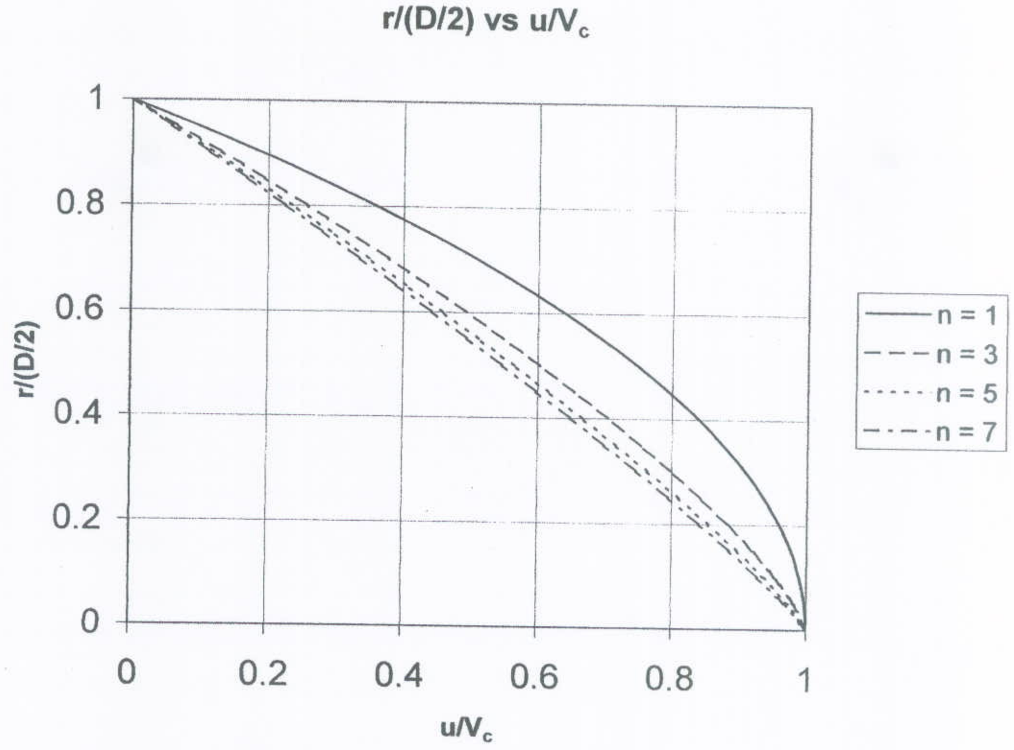
*8.17 (con't)

$$\frac{u}{V_c} = 1 - \left[\frac{r}{\left(\frac{D}{2}\right)}\right]^{\left(\frac{n+1}{n}\right)}$$

This result is plotted below for $n = 1, 3, 5,$ and 7, with $0 \le \frac{r}{\left(\frac{D}{2}\right)} \le 1$.

An EXCEL program was used to do the calculations and plotting.

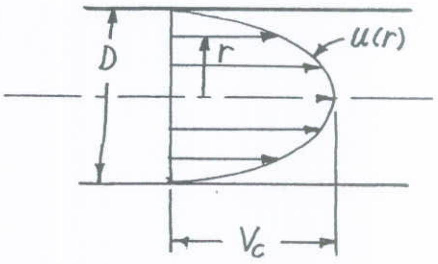
	n = 1	n = 3	n = 5	n = 7
r/(D/2)	u/V$_c$	u/V$_c$	u/V$_c$	u/V$_c$
0	1	1	1	1
0.05	0.998	0.982	0.973	0.967
0.1	0.990	0.954	0.937	0.928
0.15	0.978	0.920	0.897	0.886
0.2	0.960	0.883	0.855	0.841
0.25	0.938	0.843	0.811	0.795
0.3	0.910	0.799	0.764	0.747
0.35	0.878	0.753	0.716	0.699
0.4	0.840	0.705	0.667	0.649
0.45	0.798	0.655	0.616	0.599
0.5	0.750	0.603	0.565	0.547
0.55	0.698	0.549	0.512	0.495
0.6	0.640	0.494	0.458	0.442
0.65	0.578	0.437	0.404	0.389
0.7	0.510	0.378	0.348	0.335
0.75	0.438	0.319	0.292	0.280
0.8	0.360	0.257	0.235	0.225
0.85	0.278	0.195	0.177	0.170
0.9	0.190	0.131	0.119	0.113
0.95	0.097	0.066	0.060	0.057
1	0.000	0.000	0.000	0.000



r/(D/2) vs u/V$_c$

8.18

8.18 For laminar flow in a round pipe of diameter D, at what distance from the centerline is the actual velocity equal to the average velocity?



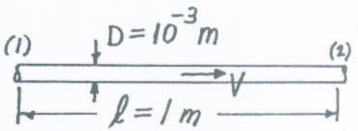
For laminar flow

$$u = V_c\left[1-\left(\frac{2r}{D}\right)^2\right]$$

Thus, if $u = \frac{V_c}{2} = V_c\left[1-\left(\frac{2r}{D}\right)^2\right]$, $r = \frac{D}{2\sqrt{2}} = \underline{\underline{0.354D}}$

8.19

8.19 Water at 20 °C flows through a horizontal 1-mm-diameter tube to which are attached two pressure taps a distance 1 m apart. **(a)** What is the maximum pressure drop allowed if the flow is to be laminar? **(b)** Assume the manufacturing tolerance on the tube diameter is $D = 1.0 \pm 0.1$ mm. Given this uncertainty in the tube diameter, what is the maximum pressure drop allowed if it must be assured that the flow is laminar?



From Table B.2 $\nu = 1.00\times10^{-6}\,\frac{m^2}{s}$
$\mu = 1.00\times10^{-3}\,\frac{N\cdot s}{m^2}$

a) Maximum Δp corresponds to maximum V, or

$$Re = \frac{VD}{\nu} = 2100$$

Thus, $V = \frac{2100\,\nu}{D} = \frac{2100(1\times10^{-6}\frac{m^2}{s})}{10^{-3}\,m} = 2.10\,\frac{m}{s}$

For laminar flow

$$V = \frac{\Delta p\, D^2}{32\,\mu\,\ell}, \text{ or } \Delta p = \frac{32\,\mu\,\ell\, V}{D^2} = \frac{32\,(1\times10^{-3}\frac{N\cdot s}{m^2})(1m)(2.10\frac{m}{s})}{(10^{-3}m)^2}$$

Thus,

$$\underline{\underline{\Delta p = 6.72\times10^{4}\,\frac{N}{m^2}}}$$

b) Since $V = \frac{2100\,\nu}{D}$ and $\Delta p = \frac{32\,\mu\,\ell\, V}{D^2}$ it follows that

$\Delta p = \frac{32\mu\ell(2100\nu)}{D^3}$ Thus, the larger the diameter, the smaller the Δp allowed to maintain laminar flow.

Thus, consider $D = 1.1\,mm = 1.1\times10^{-3}\,m$, or

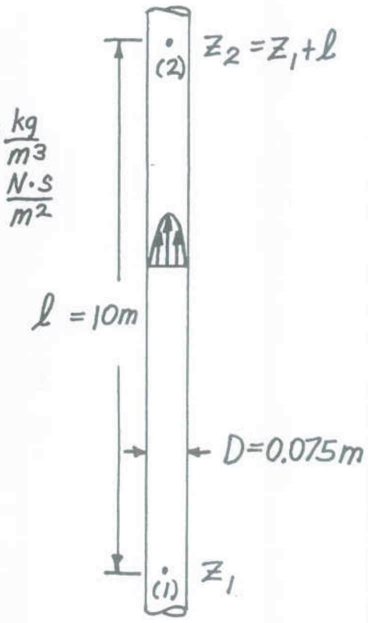
$$\Delta p = \frac{32\,(1\times10^{-3}\frac{N\cdot s}{m^2})(1m)(2100)(1\times10^{-6}\frac{m^2}{s})}{(1.1\times10^{-3}m)^3} = \underline{\underline{5.05\times10^{4}\,\frac{N}{m^2}}}$$

8.20

8.20 Glycerin at 20 °C flows upward in a vertical 75-mm-diameter pipe with a centerline velocity of 1.0 m/s. Determine the head loss and pressure drop in a 10-m length of the pipe.

$\rho = 1260 \frac{kg}{m^3}$

$\mu = 1.50 \frac{N \cdot s}{m^2}$



For laminar flow in a pipe,

$V = \text{average velocity} = \frac{1}{2} V_{max} = \frac{1}{2}(1\frac{m}{s}) = 0.5\frac{m}{s}$

Thus,

$$Re = \frac{\rho V D}{\mu} = \frac{(1260\frac{kg}{m^3})(0.5\frac{m}{s})(0.075\,m)}{1.50\,\frac{N \cdot s}{m^2}} = 31.5 < 2100$$

The flow is laminar so that

$$V = \frac{(\Delta p - \gamma \ell \sin\theta)D^2}{32\mu\ell}, \text{ where } \theta = 90^\circ$$

Thus,

$$\Delta p = \frac{32\mu\ell V}{D^2} + \gamma\ell = \frac{32\,(1.50\frac{N \cdot s}{m^2})(10m)(0.5\frac{m}{s})}{(0.075\,m)^2} + (9.81\frac{m}{s^2})(1260\frac{kg}{m^3})(10m)$$

$$= 1.66 \times 10^5 \frac{N}{m^2}, \text{ or } \underline{\underline{\Delta p = 166\ kPa}}$$

Also,

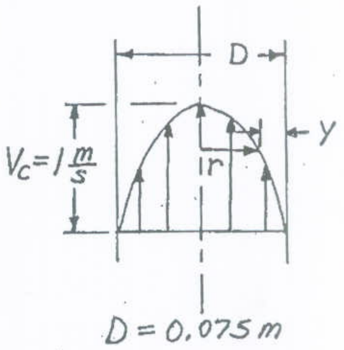
$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_L, \text{ or with } V_1 = V_2,\ z_2 - z_1 = \ell, \text{ and}$$

$p_1 = p_2 + \Delta p$ this gives

$$h_L = \frac{\Delta p}{\gamma} - \ell = \frac{1.66 \times 10^5 \frac{N}{m^2}}{(9.81\frac{m}{s^2})(1260\frac{kg}{m^3})} - 10m = \underline{\underline{3.43\,m}}$$

8.21

8.21 Determine the magnitude of the velocity gradient at points 10, 20, and 30 mm from the pipe wall for the flow in Problem 8.20.*



For laminar flow in a round pipe

$$u(r) = V_c\left[1-\left(\frac{2r}{D}\right)^2\right]$$

or

$$\frac{du}{dr} = -2V_c\left(\frac{2r}{D}\right)\left(\frac{2}{D}\right) = -\frac{8V_c}{D^2}r$$

Thus,

$$\frac{du}{dr} = \frac{-8(1\frac{m}{s})r}{(0.075\,m)^2} = -1422r\ \frac{1}{s},\ \text{where } r \sim m$$

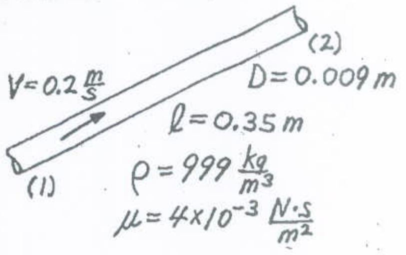
Also, $y = \text{distance from wall} = \frac{D}{2} - r = 0.0375 - r$

y, m	r, m	$\frac{du}{dr}$, $\frac{1}{s}$
0.01	0.0275	-39.1
0.02	0.0175	-24.9
0.03	0.0075	$\underline{\underline{-10.7}}$

* 8.20 Glycerin at 20 °C flows upward in a vertical 75-mm-diameter pipe with a centerline velocity of 1.0 m/s. Determine the head loss and pressure drop in a 10-m length of the pipe.

8.22

8.22 A large artery in a person's body can be approximated by a tube of diameter 9 mm and length 0.35 m. Also assume that blood has a viscosity of approximately $4 \times 10^{-3}\ \mathrm{N \cdot s/m^2}$, a specific gravity of 1.0, and that the pressure at the beginning of the artery is equivalent to 120 mm Hg. If the flow were steady (it is not) with $V =$ 0.2 m/s, determine the pressure at the end of the artery if it is oriented (a) vertically up (flow up) or (b) horizontal.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } V_1=V_2=V \qquad (1)$$

and

$$p_1=\gamma_{Hg}h=133\frac{kN}{m^3}(0.120m)=15.96\frac{kN}{m^2}$$

Also, $Re=\frac{\rho VD}{\mu}=\frac{(999\frac{kg}{m^3})(0.2\frac{m}{s})(0.009m)}{4\times10^{-3}\frac{N\cdot s}{m^2}}=450<2100$ Thus the flow is laminar so that

$$f=\frac{64}{Re}=\frac{64}{450}=0.142$$

Hence, from Eq.(1), $p_2=p_1-\gamma(z_2-z_1)-f\frac{\ell}{D}\frac{1}{2}\rho V^2$

a) For flow vertically up, $z_2-z_1=\ell$ so that

$$p_2=p_1-\gamma\ell-f\frac{\ell}{D}\frac{1}{2}\rho V^2=15.96\frac{kN}{m^2}-(9.81\times10^3\frac{N}{m^3})(0.35m)$$
$$-0.142\frac{0.35m}{0.009m}(\tfrac{1}{2})(999\frac{kg}{m^3})(0.2\frac{m}{s})^2$$

or

$$p_2=15.96\frac{kN}{m^2}-3.43\frac{kN}{m^2}-0.110\frac{kN}{m^2}=\underline{\underline{12.42\ kPa}}$$

b) For horizontal flow $z_1=z_2$ so that

$$p_2-p_1=15.96\frac{kN}{m^2}-0.142\frac{0.35m}{0.009m}(\tfrac{1}{2})(999\frac{kg}{m^3})(0.2\frac{m}{s})^2$$
$$=15.96\frac{kN}{m^2}-0.110\frac{kN}{m^2}=\underline{\underline{15.85\ kPa}}$$

Note the gravitational effects are considerably more important than viscous effects (3.43 kPa compared to 0.110kPa).

8.23

8.23 At time $t = 0$ the level of water in tank *A* shown in Fig. P8.23 is 2 ft above that in tank *B*. Plot the elevation of the water in tank *A* as a function of time until the free surfaces in both tanks are at the same elevation. Assume quasisteady conditions—that is, the steady pipe flow equations are assumed valid at any time, even though the flowrate does change (slowly) in time. Neglect minor losses. *Note:* Verify and use the fact that the flow is laminar.

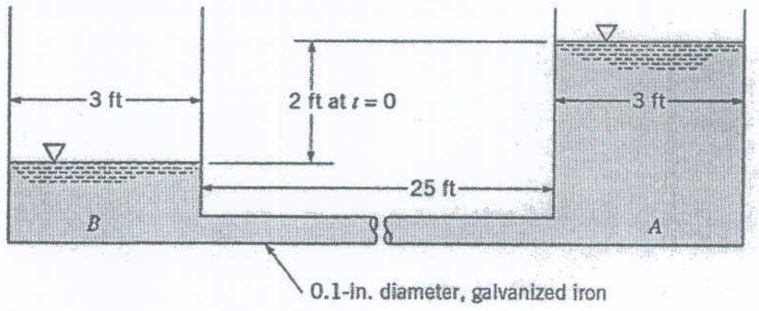
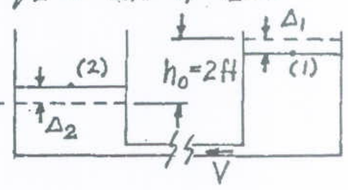


FIGURE P8.23

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } p_1 = p_2 = 0 \text{ and } V_1 = V_2 \approx 0 \qquad (1)$$



At $t = 0$, $z_2 = 0$ and $z_1 = h_0 = 2$ ft

Because the tanks are the same diameter

$\Delta_1 = \Delta_2$ and with $z_2 = \Delta_2$, $z_1 = h_0 - \Delta_2$

we obtain $z_1 = h_0 - z_2$ Thus, Eq.(1) becomes

$$z_1 = z_2 + f\frac{\ell}{D}\frac{V^2}{2g} \text{ or } 2z_1 - h_0 = f\frac{\ell}{D}\frac{V^2}{2g} \qquad (2)$$

Also, $A_1\left(-\frac{dz_1}{dt}\right) = Q = \frac{\pi}{4}D^2V$, where $A_1 = \frac{\pi}{4}D_T^2$ with $D_T = 3\text{ft} = $ tank diameter

Thus, $V = -\left(\frac{D_T}{D}\right)^2 \frac{dz_1}{dt}$ (3)

The maximum $Re = \frac{\rho V D}{\mu}$ occurs when the head, $z_1 - z_2$, is greatest.

From Eq.(2) (with $z_1 = h_0$), $h_0 = f\frac{\ell}{D}\frac{V_{max}^2}{2g}$

Assume laminar flow so that $f = \frac{64}{Re}$ or $f = \frac{64\mu}{\rho V D}$ (4)

Thus, from Eq.(4)

$$h_0 = \frac{64\mu}{\rho V_{max} D}\frac{\ell}{D}\frac{V_{max}^2}{2g} = \frac{32\mu\ell V_{max}}{\gamma D^2}, \text{ or } V_{max} = \frac{\gamma D^2 h_0}{32\mu\ell} = \frac{(62.4\frac{lb}{ft^3})(\frac{0.1}{12}ft)^2(2ft)}{32(2.34\times10^{-5}\frac{lb\cdot s}{ft^2})(25ft)}$$

$$= 0.462\frac{ft}{s}$$

or

$$Re_{max} = \frac{(1.94\frac{slugs}{ft^3})(0.462\frac{ft}{s})(\frac{0.1}{12}ft)}{2.34\times10^{-5}\frac{lb\cdot s}{ft^2}} = 319 < 2100$$ The flow remains laminar.

Thus, Eqs.(2) and (4) give

$$2z_1 - h_0 = \frac{64\mu}{\rho V D}\frac{\ell}{D}\frac{V^2}{2g} = \frac{32\mu\ell V}{\gamma D^2}, \text{ or by using Eq.(3)}$$

$$2z_1 - h_0 = -\left(\frac{D_T}{D}\right)^2 \frac{32\mu\ell}{\gamma D^2}\frac{dz_1}{dt} \qquad (5)$$

Let $F \equiv z_1 - \frac{h_0}{2}$ so that $\frac{dF}{dt} = \frac{dz_1}{dt}$ and Eq.(5) becomes

$$2F = -\left(\frac{D_T}{D}\right)^2 \frac{32\mu\ell}{\gamma D^2}\frac{dF}{dt}$$

(con't)

8.23

(con't)

or $\alpha \frac{dF}{dt} + F = 0$, where $\alpha = \frac{16 \mu \ell}{\gamma D^2} \left(\frac{D_T}{D}\right)^2$

Thus,

$$\alpha \int \frac{dF}{F} = -\int dt \quad \text{or} \quad \alpha \ln F = -t + \tilde{C}, \text{ where } \tilde{C} = \text{constant}$$

Hence,

$F = C e^{-(t/\alpha)}$ That is, $z_1 - \frac{h_0}{2} = C e^{-(t/\alpha)}$ with the initial condition $z_1 = h_0$ when $t = 0$, or $C = \frac{h_0}{2}$

Thus, $z_1 - \frac{h_0}{2} = \frac{h_0}{2} e^{-(t/\alpha)}$

or

$z_1 = \frac{h_0}{2}\left[1 + e^{-(t/\alpha)}\right]$ Note: As $t \to \infty$, $z_1 \to \frac{h_0}{2}$

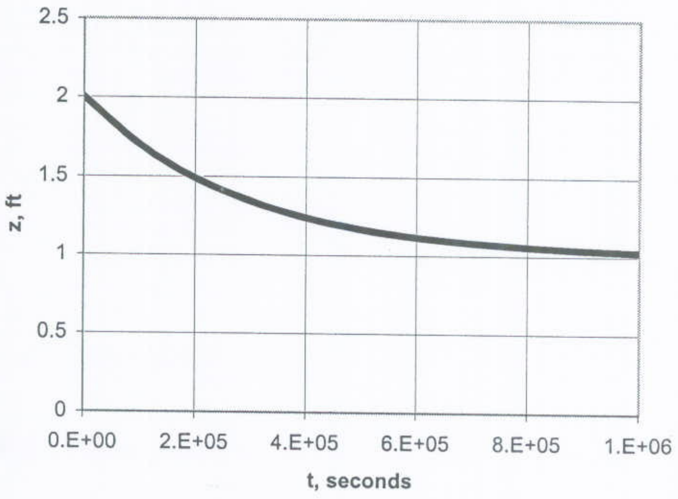
For the conditions given, $h_0 = 2$ ft and

$$\alpha = \frac{16\left(2.34 \times 10^{-5} \frac{lb \cdot s}{ft^2}\right)(25 ft)}{\left(62.4 \frac{lb}{ft^3}\right)\left(\frac{0.1}{12} ft\right)^2} \left(\frac{3 ft}{\left(\frac{0.1}{12} ft\right)}\right)^2 = 2.80 \times 10^5 s$$

Hence,

$\underline{\underline{z_1 = 1 + e^{-\left(\frac{t}{2.8 \times 10^5 s}\right)}}}$, where $z_1 \sim$ ft and $t \sim$ s

This result is plotted below. (Note: $\lim_{t \to \infty} z_1 = 1$ ft)



8.24

8.24 A fluid flows through a horizontal 0.1-in.-diameter pipe. When the Reynolds number is 1500, the head loss over a 20-ft length of the pipe is 6.4 ft. Determine the fluid velocity.

$h_L = f \frac{\ell}{D} \frac{V^2}{2g}$, where since $Re = 1500 < 2100$ the flow is laminar.

Thus, $f = 64/Re = 64/1500 = 0.0427$ so that

$$6.4\,ft = 0.0427 \; \frac{20\,ft}{(0.1/12\,ft)} \; \frac{V^2}{2(32.2\,ft/s^2)}$$

or

$$V = \underline{\underline{2.01 \frac{ft}{s}}}$$

8.25

8.25 A viscous fluid flows in a 0.10-m-diameter pipe such that its velocity measured 0.012 m away from the pipe wall is 0.8 m/s. If the flow is laminar, determine the centerline velocity and the flowrate.

For laminar flow in a pipe

$u(r) = V_c\left[1 - \left(\frac{2r}{D}\right)^2\right]$, where $D = 0.1\,m$ and $u = 0.8 \frac{m}{s}$ at $r = \frac{0.1m}{2} - 0.012\,m = 0.038\,m$

Thus,

$$0.8 \frac{m}{s} = V_c\left[1 - \left(\frac{2(0.038m)}{0.10\,m}\right)^2\right] \quad \text{or} \quad V_c = \underline{\underline{1.89 \frac{m}{s}}}$$

so that

$$Q = \frac{\pi}{4}D^2 V = \frac{\pi}{4}D^2(0.5V_c) = \frac{\pi}{4}(0.1m)^2(0.5)(1.89\frac{m}{s}) = \underline{\underline{7.42 \times 10^{-3} \frac{m^3}{s}}}$$

8.26

8.26 Oil flows through the horizontal pipe shown in Fig. P8.20 under laminar conditions. All sections are the same diameter except one. Which section of the pipe (A, B, C, D, or E) is slightly smaller diameter than the others? Explain.

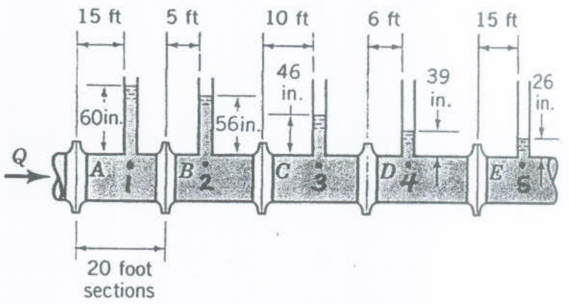
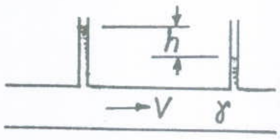


FIGURE P8.26

For laminar flow in a horizontal pipe $Q = \frac{\pi D^4}{128\mu} \frac{\Delta p}{\ell}$, where $Q_A = Q_B = Q_C = Q_D = Q_E$. Thus $\frac{\Delta p}{\ell} \sim \frac{1}{D^4}$ The smallest diameter pipe has the largest $\frac{\Delta p}{\ell}$, where $\Delta p = \gamma h$

Let $a = \left.\frac{\Delta p}{\ell}\right)_{pipe\ A}$, $b = \left.\frac{\Delta p}{\ell}\right)_{pipe\ B}$, etc.



Hence, from the data in the figure for the section between (1) and (2):

$$5a + 5b = \gamma \frac{(60-56)}{12}, \text{ where } a \text{ and } b \sim \frac{lb}{ft^3} \text{ and } \gamma \sim \frac{lb}{ft^3}. \quad (1)$$

Similarly, from (2) to (3)

$$15b + 10c = \gamma \frac{(56-46)}{12}, \quad (2)$$

from (3) to (4)

$$10c + 6d = \gamma \frac{(46-39)}{12}, \quad (3)$$

and from (4) to (5)

$$14d + 15e = \gamma \frac{(39-26)}{12} \quad (4)$$

Eqs. (1) through (4) can be written as

(5) $a + b = 0.0667\gamma$

(6) $1.5b + c = 0.0833\gamma$

(7) $c + 0.6d = 0.0583\gamma$

(8) $d + 1.071e = 0.0774\gamma$

From the problem statement, 4 pipes are the same diameter, one is smaller diameter. Thus, 4 of the 5 variables (a, b, c, d, e) should be equal, one larger than the others.

Assume $a > b = c = d = e$ From Eq. (6), $1.5b + b = 0.0833\gamma$ or $b = 0.0333\gamma$ but from Eq. (7), $b + 0.6b = 0.0583\gamma$ or $b = 0.0364\gamma$ which is not the same as that from Eq. (6).

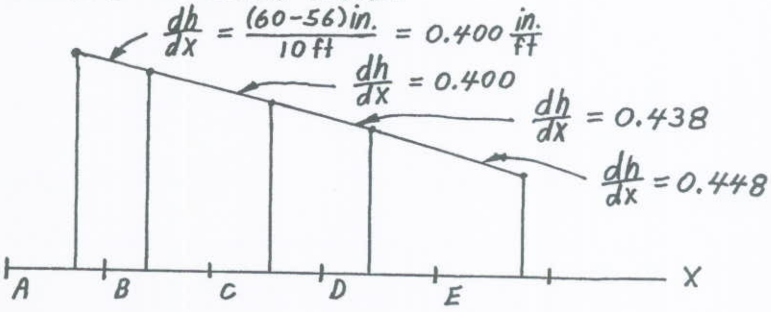
Assuming $b > a = c = d = e$, or $c > a = b = d = e$, or $e > a = b = c = d$ lead to similar inconsistencies. However, if we assume $d > a = b = c = e$ we obtain from Eq (5): $a = 0.0333\gamma$; from Eq. (6): the same value of a; from Eq. (7): $d = 0.0417\gamma$; the same value of d from Eq. (8).

(con't)

8.26 (con't)

Thus, $a = b = c = e$ and $d > a$. That is, the small pipe is pipe D.

Note: This result can also be obtained as follows. From the given data the pressure gradient (average) between piezometer tube locations is as shown below.



Given that all section have the same diameter except for one, it follows (based on the different $\frac{dh}{dx}$ values) that the diameter of section D is less than that of the others.

8.27

8.27 Asphalt at 120 °F, considered to be a Newtonian fluid with a viscosity 80,000 times that of water and a specific gravity of 1.09, flows through a pipe of diameter 2.0 in. If the pressure gradient is 1.6 psi/ft determine the flowrate assuming the pipe is **(a)** horizontal; **(b)** vertical with flow up.

If the flow is laminar, then $Q = \frac{\pi(\Delta p - \gamma \ell \sin\theta)D^4}{128 \mu \ell}$ (1)

where $\gamma = SG\, \gamma_{H_2O} = 1.09\,(62.4 \frac{lb}{ft^3}) = 68.0 \frac{lb}{ft^3}$

and

$$\mu = 80{,}000\, \mu_{H_2O} = 8 \times 10^4 \left(1.164 \times 10^{-5} \frac{lb \cdot s}{ft^2}\right) = 0.931 \frac{lb \cdot s}{ft^2}$$

a) For horizontal flow, $\theta = 0$

Thus, from Eq. (1)

$$Q = \frac{\pi\,(1.6 \times 144 \frac{lb}{ft^2})(\frac{2}{12} ft)^4}{128\,(0.931 \frac{lb \cdot s}{ft^2})(1 ft)} = \underline{\underline{4.69 \times 10^{-3} \frac{ft^3}{s}}}$$

b) For vertical flow up, $\theta = 90$

Thus, from Eq. (1)

$$Q = \frac{\pi\,(1.6 \times 144 \frac{lb}{ft^2} - 68 \frac{lb}{ft^3}(1 ft))(\frac{2}{12} ft)^4}{128\,(0.931 \frac{lb \cdot s}{ft^2})(1 ft)} = \underline{\underline{3.30 \times 10^{-3} \frac{ft^3}{s}}}$$

Note: We must check to see if our assumption of laminar flow is correct.

Since $V = \frac{Q}{A} = \frac{4.69 \times 10^{-3} \frac{ft^3}{s}}{\frac{\pi}{4}(\frac{2}{12})^2} = 0.215 \frac{ft}{s}$ it follows that

$$Re = \frac{\rho V D}{\mu} = \frac{1.09(1.94 \frac{slug}{ft^3})(0.215)(\frac{2}{12} ft)}{0.931 \frac{lb \cdot s}{ft^2}} = 0.0814 < 2100$$

The flow is laminar.

8.28

8.28 Oil of $SG = 0.87$ and a kinematic viscosity $\nu = 2.2 \times 10^{-4}\ \text{m}^2/\text{s}$ flows through the vertical pipe shown in Fig. P8.28 at a rate of $4 \times 10^{-4}\ \text{m}^3/\text{s}$. Determine the manometer reading, h.

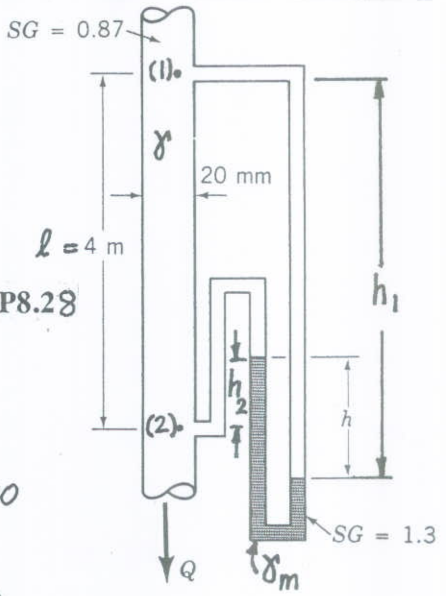


FIGURE P8.28

$$V = \frac{Q}{A} = \frac{4\times10^{-4}\,\frac{m^3}{s}}{\frac{\pi}{4}(0.02m)^2} = 1.27\,\frac{m}{s} \text{ so that}$$

$$Re = \frac{\rho VD}{\mu} = \frac{VD}{\nu} = \frac{(1.27\frac{m}{s})(0.02m)}{2.2\times10^{-4}\frac{m^2}{s}} = 115 < 2100$$

The flow is laminar with

$$Q = \frac{\pi(\Delta p + \gamma\ell)D^4}{128\mu\ell}, \text{ or } \Delta p = p_1 - p_2 = \frac{128\mu\ell Q}{\pi D^4} - \gamma\ell \qquad (1)$$

Hence, with $\gamma = SG\,\gamma_{H_2O} = 0.87(9.81\,\frac{kN}{m^3}) = 8.53\,\frac{kN}{m^3}$ and

$$\mu = \nu\rho = \nu\, SG\, \rho_{H_2O} = (2.2\times10^{-4}\frac{m^2}{s})(0.87)(1000\frac{kg}{m^3}) = 0.191\ \frac{N\cdot s}{m^2}$$

Eq. (1) gives

$$\Delta p = \frac{128(0.191\,\frac{N\cdot s}{m^2})(4m)(4\times10^{-4}\frac{m^3}{s})}{\pi\,(0.020m)^4} - (8.53\,\frac{kN}{m^3})(4m)(10^3\frac{N}{kN})$$

or $\Delta p = 4.37\times10^4\frac{N}{m^2} = 43.7\frac{kN}{m^2}$ (2)

From manometer considerations

$p_1 + \gamma h_1 - \gamma_m h + \gamma h_2 = p_2$, where $\gamma_m = SG_m\gamma_{H_2O} = 1.3(9.81\,\frac{kN}{m^3}) = 12.74\,\frac{kN}{m^3}$

and $h_1 = h - h_2 + \ell$, or $h_2 + h_1 = h + \ell$

Thus,

$$p_1 - p_2 = \Delta p = -\gamma(h_2+h_1) + \gamma_m h = (\gamma_m - \gamma)h - \gamma\ell \qquad (3)$$

Combine Eqs. (2) and (3) to give

$$43.7\,\frac{kN}{m^2} = (12.74-8.53)\frac{kN}{m^3}\,h - (8.53\,\frac{kN}{m^3})(4\,m)$$

or

$$h = \underline{\underline{18.5\,m}}$$

8.29

8.29 Determine the manometer reading, h, for Problem 8.28 if the flow is up rather than down the pipe. Note: The manometer reading will be reversed.

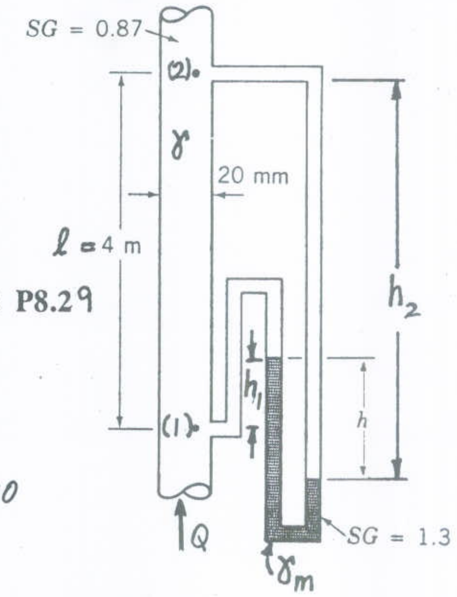


FIGURE P8.29

$$V = \frac{Q}{A} = \frac{4 \times 10^{-4} \frac{m^3}{s}}{\frac{\pi}{4}(0.02m)^2} = 1.27 \frac{m}{s} \text{ so that}$$

$$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{(1.27\frac{m}{s})(0.02m)}{2.2 \times 10^{-4} \frac{m^2}{s}} = 115 < 2100$$

The flow is laminar with

$$Q = \frac{\pi(\Delta p - \gamma \ell)D^4}{128 \mu \ell}, \text{ or } \Delta p = p_1 - p_2 = \frac{128 \mu \ell Q}{\pi D^4} + \gamma \ell \qquad (1)$$

Hence, with $\gamma = SG\, \gamma_{H_2O} = 0.87(9.81 \frac{kN}{m^3}) = 8.53 \frac{kN}{m^3}$ and

$$\mu = \nu \rho = \nu\, SG\, \rho_{H_2O} = (2.2 \times 10^{-4} \frac{m^2}{s})(0.87)(1000 \frac{kg}{m^3}) = 0.191 \frac{N \cdot s}{m^2}$$

Eq. (1) gives

$$\Delta p = \frac{128(0.191 \frac{N \cdot s}{m^2})(4m)(4 \times 10^{-4} \frac{m^3}{s})}{\pi (0.020m)^4} + (8.53 \frac{kN}{m^3})(4m)(10^3 \frac{N}{kN})$$

or

$$\Delta p = 1.119 \times 10^5 \frac{N}{m^2} = 111.9 \frac{kN}{m^2} \qquad (2)$$

From manometer considerations

$$p_1 - \gamma h_1 + \gamma_m h - \gamma h_2 = p_2, \text{ where } \gamma_m = SG_m \gamma_{H_2O} = 1.3(9.81 \frac{kN}{m^3}) = 12.74 \frac{kN}{m^3}$$

and $h_2 = \ell + h - h_1$ or $h_2 + h_1 = \ell + h$

Thus,

$$p_1 - p_2 = \Delta p = \gamma(h_2 + h_1) - \gamma_m h = -(\gamma_m - \gamma)h + \gamma \ell \qquad (3)$$

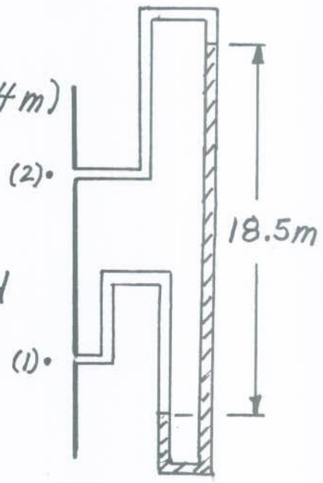
Combine Eqs. (2) and (3) to give

$$111.9 \frac{kN}{m^2} = -(12.74 - 8.53)\frac{kN}{m^3} h + 8.53 \frac{kN}{m^3}(4m)$$

or

$$h = \underline{\underline{-18.5 m}}$$

Note: Since $h < 0$ the manometer is displaced in the direction opposite that shown in the original figure.



8.30

8.30 A liquid with $SG = 0.96$, $\mu = 9.2 \times 10^{-4}\ \mathrm{N \cdot s/m^2}$, and vapor pressure $p_v = 1.2 \times 10^4\ \mathrm{N/m^2(abs)}$ is drawn into the syringe as is indicated in Fig. P8.30. What is the maximum flowrate if cavitation is not to occur in the syringe?

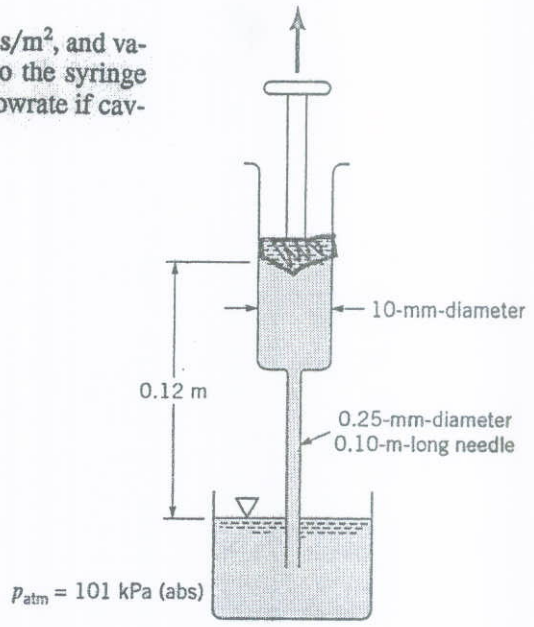


FIGURE P8.30

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(f\frac{\ell}{D} + \Sigma K_L\right)\frac{V^2}{2g}$, where $p_1 = 101\ kPa$, $z_1 = 0$, $V_1 = 0$, $z_2 = 0.12\,m$. The maximum flowrate will occur when p_2 is the minimum allowed: $p_2 = p_v = 1.2 \times 10^4 \frac{N}{m^2}$.

Thus, $\frac{p_1}{\gamma} = \frac{p_v}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(f\frac{\ell}{D} + K_{L_{entrance}} + K_{L_{exit}}\right)\frac{V^2}{2g}$, (1)

where $V_2 = \frac{VA}{A_2} = V\left(\frac{D}{D_2}\right)^2 = V\left(\frac{0.25mm}{10\,mm}\right)^2 = 0.000625\,V$ Thus, $V_2 \approx 0$ and Eq. (1) becomes

$$\frac{(101 \times 10^3 - 1.2 \times 10^4)\frac{N}{m^2}}{0.96\,(9.80 \times 10^3 \frac{N}{m^3})} = 0.12\,m + \left(f\left(\frac{0.1\,m}{0.25 \times 10^{-3}\,m}\right) + 0.5 + 1\right)\frac{V^2}{2(9.81\frac{m}{s^2})}$$

or

$$122 = (267f + 1)V^2 \qquad (2)$$

Assume (because of the small diameter) that the flow is laminar.

Thus, $f = \frac{64}{Re} = \frac{64\mu}{\rho V D}$

or

$$f = \frac{64\,(9.2 \times 10^{-4} \frac{N \cdot s}{m^2})}{0.96(999\frac{kg}{m^3})\,V\,(0.25 \times 10^{-3} m)} = \frac{0.246}{V}$$

Hence, from Eq. (2)

$$122 = \left(267\,\frac{0.246}{V} + 1\right)V^2 \quad or \quad 122\,V = (65.7 + V)V^2$$

Thus,

$V^2 + 65.7\,V - 122 = 0$, which has solutions

$V = \frac{-65.7 \pm \sqrt{65.7^2 + 4(122)}}{2} = 1.81\frac{m}{s}$, or $-67.5\frac{m}{s}$ (neglect the $V<0$ root)

Hence,

$$Q = AV = \frac{\pi}{4}(0.25 \times 10^{-3} m)^2 (1.81\frac{m}{s}) = \underline{\underline{8.88 \times 10^{-8}\frac{m^3}{s}}}$$

Check if laminar flow:

$$Re = \frac{\rho V D}{\mu} = \frac{0.96(999\frac{kg}{m^3})(1.81\frac{m}{s})(0.25 \times 10^{-3} m)}{9.2 \times 10^{-4}\frac{N \cdot s}{m^2}} = 472 < 2100 \text{ (laminar)}$$

8.32

8.32 For oil ($SG = 0.86$, $\mu = 0.025\ \text{Ns/m}^2$) flow of $0.3\ \text{m}^3/\text{s}$ through a round pipe with diameter of 500 mm, determine the Reynolds number. Is the flow laminar or turbulent?

$$SG = \rho/\rho_{H_2O} = 0.86$$

$$\rho_{oil} = 0.86(\rho_{H_2O}) = 0.86\,(999) = 859\ \text{kg/m}^3$$

$$V = Q/A = 0.3/\tfrac{\pi}{4}(0.5)^2 = 1.53\ \text{m/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{(859)(1.53)(0.5)}{0.025} = 2.63 \times 10^4$$

Based on the criterion that $Re < 2100$ represents laminar flow, this flow is <u>turbulent</u>.

8.33

8.33 For air at a pressure of 200 kPa (abs) and temperature of 15 °C, determine the maximum laminar volume flowrate for flow through a 2.0-cm-diameter tube.

For laminar flow, the maximum Re value is 2100

$$Re = \frac{\rho V D}{\mu} = 2100$$

$$V = \frac{2100\,\mu}{\rho D}$$

To determine air density, make use of ideal gas law

$$P = \rho R T \quad \text{or} \quad \rho = P/RT$$

$$\rho = \frac{200 \times 10^3}{(286.9)(273+15)} = 2.42 \ \mathrm{kg/m^3}$$

Viscosity has little variation with pressure, so it is reasonable to assume the use of the standard value for air, $\mu = 1.79 \times 10^{-5}$

$$V = \frac{2100\,(1.79 \times 10^{-5})}{(2.42)\,(0.02)} = 0.78 \ \mathrm{m/s}$$

Maximum laminar volume flowrate

$$Q = VA = (0.78)\left(\frac{\pi}{4}(0.02)^2\right)$$

$$\underline{\underline{Q = 2.4 \times 10^{-4} \ \mathrm{m^3/s}}}$$

8.34

8.34 Show that the power law approximation for the velocity profile in turbulent pipe flow (Eq. 8.31) cannot be accurate at the centerline or at the pipe wall because the velocity gradients at these locations are not correct. Explain.

If $\bar{u} = V_0\left[1-\frac{r}{R}\right]^{\frac{1}{n}}$, then $\frac{d\bar{u}}{dr} = \frac{V_0}{n}\left[1-\frac{r}{R}\right]^{\left(\frac{1}{n}-1\right)}\left(-\frac{1}{R}\right)$

or

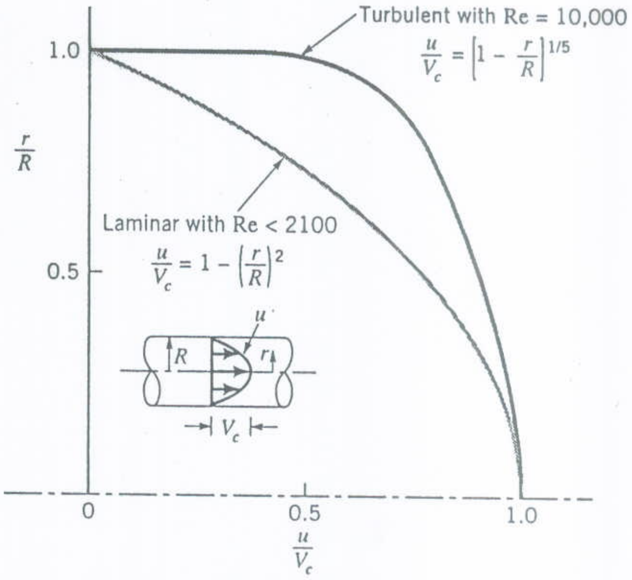
$\frac{d\bar{u}}{dr} = -\frac{V_0}{nR}\left[1-\frac{r}{R}\right]^{\left(\frac{1-n}{n}\right)}$ Thus, $\left.\frac{d\bar{u}}{dr}\right|_{r=0} = \underline{\underline{-\frac{V_0}{nR}}}$, but by symmetry it must be zero.

Also, $\left.\frac{d\bar{u}}{dr}\right|_{r=R} = -\frac{V_0}{nR}\left[1-1\right]^{\left(\frac{1-n}{n}\right)} = \underline{\underline{-\infty}}$ since $\left(\frac{1-n}{n}\right) < 0$ for $n > 1$

Physically, the velocity gradient must be finite.

8.35

8.35 As shown in Video V8.9 and Fig. P8.35 the velocity profile for laminar flow in a pipe is quite different from that for turbulent flow. With laminar flow the velocity profile is parabolic; with turbulent flow at Re = 10,000 the velocity profile can be approximated by the power-law profile shown in the figure. (a) For laminar flow, determine at what radial loaction you would place a Pitot tube if it is to measure the average velocity in the pipe. (b) Repeat part (a) for turbulent flow with Re = 10,000.



■ FIGURE P8.35

For laminar or turbulent flow,

$$Q = AV = \pi R^2 V = \int u\,dA = \int u(2\pi r\,dr) = 2\pi \int_{r=0}^{R} u\,r\,dr$$

a) Laminar flow:

$$\pi R^2 V = 2\pi V_c \int_0^R r\left[1-\left(\frac{r}{R}\right)^2\right]dr = 2\pi V_c\left[\frac{R^2}{2} - \frac{R^2}{4}\right] = \pi \frac{R^2}{2} V_c$$

Thus, $V = \frac{1}{2}V_c$ For $u = V = \frac{V_c}{2}$ the equation for $\frac{u}{V_c}$ gives

$$\frac{u}{V_c} = \frac{1}{2} = 1 - \left(\frac{r}{R}\right)^2, \text{ or } \left(\frac{r}{R}\right)^2 = \frac{1}{2} \text{ Thus, } r = \frac{1}{\sqrt{2}}R = \underline{\underline{0.707R}}$$

b) Turbulent flow

$$\pi R^2 V = 2\pi V_c \int_0^R r\left[1 - \frac{r}{R}\right]^{1/5} dr = 2\pi R^2 V_c \int_0^1 \left(\frac{r}{R}\right)\left[1-\left(\frac{r}{R}\right)\right]^{1/5} d\left(\frac{r}{R}\right)$$

Let $y \equiv 1 - \left(\frac{r}{R}\right)$ so that $\left(\frac{r}{R}\right) = 1-y$ and $d\left(\frac{r}{R}\right) = -dy$

Thus,

$$\pi R^2 V = 2\pi R^2 V_c \int_{y=1}^{y=0} (1-y)\,y^{1/5}(-dy) = 2\pi R^2 V_c \int_0^1 \left(y^{1/5} - y^{6/5}\right)dy$$

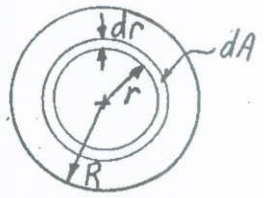
$$= 2\pi R^2 V_c\left[\frac{5}{6} - \frac{5}{11}\right] = 2\pi R^2 V_c\left(\frac{25}{66}\right)$$

or $V = \frac{50}{66}V_c$ For $u = V = \frac{50}{60}$ the equation for $\frac{u}{V_c}$ gives

$$\frac{u}{V_c} = \frac{50}{66} = \left[1 - \frac{r}{R}\right]^{1/5} \text{ or } \frac{r}{R} = 0.750 \text{ so that } r = \underline{\underline{0.750\,R}}$$

8.36

8.36 The kinetic energy coefficient, α, is defined in Eq. 5.86. Show that its value for a power-law turbulent velocity profile (Eq. 8.31) is given by $\alpha = (n+1)^3(2n+1)^3/[4n^4(n+3)(2n+3)]$.



From Eq. 5.86, $\alpha = \dfrac{\rho \int \bar{u}^3 dA}{\rho A V V^2}$ where V = average velocity, $A = \pi R^2$, and $\bar{u} = V_c\left[1 - \frac{r}{R}\right]^{\frac{1}{n}}$. From Example 8.4, $V = \dfrac{2n^2 V_c}{(n+1)(2n+1)}$ (0)

Thus, with $dA = 2\pi r\, dr$

$$\alpha = \frac{\int \bar{u}^3 dA}{AV^3}, \text{ where } \int \bar{u}^3 dA = 2\pi \int_{r=0}^{R} V_c^3\left[1 - \frac{r}{R}\right]^{\frac{3}{n}} r\, dr = 2\pi R^2 V_c^3 \int_{y=0}^{1} [1-y]^{\frac{3}{n}} y\, dy \tag{1}$$

where $y = \frac{r}{R}$.

Let $x = 1 - y$ so that $y = 1 - x$ and $dy = -dx$,

Hence,

$$\int_{y=0}^{1} [1-y]^{\frac{3}{n}} y\, dy = -\int_{x=1}^{0} x^{3/n}(1-x)\, dx = \int_{0}^{1} \left(x^{\frac{3}{n}} - x^{\frac{3}{n}+1}\right) dx$$

$$= \frac{n}{n+3} x^{\frac{n+3}{n}} - \frac{n}{2n+3} x^{\frac{2n+3}{n}} \Bigg|_{x=0}^{x=1}$$

Thus,

$$\int_{y=0}^{1} [1-y]^{\frac{3}{n}} y\, dy = \frac{n^2}{(n+3)(2n+3)} \tag{2}$$

From Eqs. (0), (1), and (2):

$$\alpha = \frac{2\pi R^2 V_c^3 \dfrac{n^2}{(n+3)(2n+3)}}{\pi R^2 \left[\dfrac{2n^2 V_c}{(n+1)(2n+1)}\right]^3} = \underline{\underline{\frac{(n+1)^3(2n+1)^3}{4n^4(n+3)(2n+3)}}}$$

8.38

8.38 Determine the thickness of the viscous sublayer in a smooth 8-in.-diameter pipe if the Reynolds number is 25,000.

$\delta_s = \frac{5\nu}{u^*}$, where $u^* = \left(\frac{\tau_w}{\rho}\right)^{\frac{1}{2}}$ and $\tau_w = \frac{D\Delta p}{4\ell}$. Since $\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2$
we obtain $\tau_w = \frac{\rho f V^2}{8}$ and $u^* = \sqrt{\frac{f}{8}}\,V$
Thus,

$$\delta_s = \frac{5\nu}{\sqrt{\frac{f}{8}}\,V} = \frac{5\nu D}{\sqrt{\frac{f}{8}}\,VD}, \text{ or } \delta_s = \frac{5D}{Re\sqrt{\frac{f}{8}}} \qquad (1)$$

From Fig. 8.20, for a smooth pipe with $Re = 2.5\times10^4$, $f = 0.024$
Thus, from Eq. (1)

$$\delta_s = \frac{5\sqrt{8}\left(\frac{8}{12}\text{ft}\right)}{2.5\times10^4\sqrt{0.024}} = \underline{\underline{0.00243\text{ ft}}}$$

8.39

8.39 Water at 60 °F flows through a 6-in.-diameter pipe with an average velocity of 15 ft/s. Approximately what is the height of the largest roughness element allowed if this pipe is to be classified as smooth?

Let h = roughness height. Thus, $h = \delta_s$, where $\delta_s = \frac{5\nu}{u^*}$ with $u^* = \left(\frac{\tau_w}{\rho}\right)^{1/2}$ and $\tau_w = \frac{D\Delta p}{4\ell}$. Since $\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2$ we obtain

$\tau_w = \frac{\rho f V^2}{8}$ or $u^* = \sqrt{\frac{f}{8}}\,V$

For a smooth pipe with $Re = \frac{VD}{\nu} = \frac{(15\frac{ft}{s})(\frac{6}{12}ft)}{1.21\times10^{-5}\frac{ft^2}{s}} = 6.19\times10^5$ we obtain from Fig. 8.20 $f = 0.0125$

Thus, $u^* = \left(\frac{0.0125}{8}\right)^{1/2}(15\frac{ft}{s}) = 0.593\frac{ft}{s}$

or

$$\delta_s = \frac{5\nu}{u^*} = \frac{5(1.21\times10^{-5}\frac{ft^2}{s})}{0.593\frac{ft}{s}} = \underline{\underline{1.02\times10^{-4}\,ft}}$$

8.41

8.41 A person with no experience in fluid mechanics wants to estimate the friction factor for 1-in.-diameter galvanized iron pipe at a Reynolds number of 8,000. They stumble across the simple equation of $f = 64/\mathrm{Re}$ and use this to calculate the friction factor. Explain the problem with this approach and estimate their error.

For $Re = 8000$ under standard conditions, the pipe flow will be turbulent.

f - laminar

$$f = 64/Re = 64/8000 = 8 \times 10^{-3}$$

f - turbulent

for galvinized iron pipe, $\varepsilon = 0.0005$ ft

so, $\varepsilon/D = 0.0005/(1/12) = 6 \times 10^{-3}$

Making use of the Moody Chart

$f \simeq 0.04$

The error is in using the laminar equation to calculate the friction factor when the flow is turbulent.

$$\frac{f_{actual}}{f_{laminar}} = \frac{f_{turbulent}}{f_{laminar}} = \frac{0.04}{0.008} \simeq 5$$

That is, the friction factor is 5 time greater than if the flow were laminar.

8.42

8.42 Water flows through a horizontal plastic pipe with a diameter of 0.2 m at a velocity of 10 cm/s. Determine the pressure drop per meter of pipe using the Moody chart.

The pressure drop in the pipe can be found from

$$\Delta P = f \frac{l}{D} \frac{\rho V^2}{2}$$

The friction factor is determined from the Moody chart.

$$Re = \frac{\rho V D}{\mu} = \frac{(999)(0.1)(0.2)}{1.12 \times 10^{-3}} = 1.8 \times 10^4$$

For plastic pipe, $\varepsilon = 0.0$ mm

$\varepsilon/D = 0.0/0.2 = 0.0$

From the Moody Chart

$f = 0.026$

So ΔP per meter ($l = 1m$)

$$\Delta P = (0.026)\left(\frac{1}{0.2}\right)\left[\frac{999(0.1)^2}{2}\right]$$

$\underline{\Delta P = 0.649}$ Pa per meter

8.43

8.43 For Problem 8.42, calculate the power lost to the friction per meter of pipe.

$\Delta P = 0.649\,\text{Pa}$ per meter of pipe, $V = 0.1\,\text{m/s}$, $D = 0.2\,\text{m}$

Based on equations in Ch.5, power can be found from

$\mathcal{P} = (\Delta P)\,Q$

$Q = VA = (0.1)\left(\frac{\pi}{4}(0.2)^2\right) = 3.14\times10^{-3}\ \text{m}^3/\text{s}$

$\mathcal{P} = (0.649)\left(3.14\times10^{-3}\right) = 2.04\times10^{-3}\ \text{N}\cdot\text{m/s} = \underline{\underline{2.04\times10^{-3}\ \text{W}}}$

8.44

8.44 Oil ($SG = 0.9$), with a kinematic viscosity of 0.007 ft^2/s, flows in a 3-in.-diameter pipe at 0.01 ft^3/s. Determine the head loss per unit length of this flow.

$h_L = f \frac{\ell}{D} \frac{V^2}{2g}$ where $\ell = 1\ ft$ for "per unit length of pipe".

Determine friction factor based on Re & ε/D

$Q = 0.01\ ft^3/s = VA$

$V = \frac{0.01}{\frac{\pi}{4}\left(\frac{3}{12}\right)^2} = 0.20\ ft/s$

$Re = \frac{VD}{\nu} = \frac{0.20(3/12)}{0.007} = 7.14$

Since Re is below 2100, the flow is laminar.

The friction factor can be determined from

$f = 64/Re = 64/7.14 = 8.96$

$h_L = (8.96)\frac{1}{(3/12)}\frac{(0.2)^2}{2(32.2)} = \underline{0.022\ ft}$ per ft of pipe

8.45

8.45 Water flows through a 6-in.-diameter horizontal pipe at a rate of 2.0 cfs and a pressure drop of 4.2 psi per 100 ft of pipe. Determine the friction factor.

For a horizontal pipe $\Delta p = f \frac{\ell}{D} \frac{1}{2} \rho V^2,$

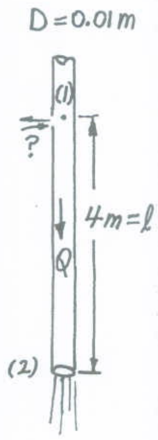
where $V = \frac{Q}{A} = \frac{2.0 \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{6}{12} ft\right)^2} = 10.2 \frac{ft}{s}$

Thus,

$$f = \frac{2 D \Delta p}{\rho \ell V^2} = \frac{2\left(\frac{6}{12} ft\right)\left(4.2 \times 144 \frac{lb}{ft^2}\right)}{\left(1.94 \frac{slugs}{ft^3}\right)(100 ft)\left(10.2 \frac{ft}{s}\right)^2} = \underline{\underline{0.0300}}$$

8.46

8.46 Water flows downward through a vertical 10-mm-diameter galvanized iron pipe with an average velocity of 5.0 m/s and exits as a free jet. There is a small hole in the pipe 4 m above the outlet. Will water leak out of the pipe through this hole, or will air enter into the pipe through the hole? Repeat the problem if the average velocity is 0.5 m/s.



$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}$, where $p_2=0$, $z_2=0$,

$z_1=4m$, $V_1=V_2=V$. Thus,

$\frac{p_1}{\gamma}=f\frac{\ell}{D}\frac{V^2}{2g}-z_1$, or $p_1=f\frac{\ell}{D}\frac{1}{2}\rho V^2-\gamma\ell$ (1) With ε from Table 8.1,

$\frac{\varepsilon}{D}=\frac{0.15\,mm}{10\,mm}=0.015$ so that with $Re=\frac{VD}{\nu}=\frac{(5\frac{m}{s})(0.01m)}{1.12\times10^{-6}\frac{m^2}{s}}=4.46\times10^4$

we obtain $f=0.045$ (see Fig. 8.20).

Thus, from Eq. (1)

$$p_1=0.045\left(\frac{4m}{0.01m}\right)\frac{1}{2}(999\,\frac{kg}{m^3})(5\,\frac{m}{s})^2-9800\,\frac{N}{m^3}\,(4m)=1.86\times10^5\,\frac{N}{m^2}$$

Since $p_1>0$, water will leak out of the pipe when $V=5\frac{m}{s}$

If $V=0.5\frac{m}{s}$, then $Re=4.46\times10^3$ and $f=0.052$

Thus, from Eq. (1)

$$p_1=0.052\left(\frac{4m}{0.01m}\right)\frac{1}{2}(999\,\frac{kg}{m^3})(0.5\,\frac{m}{s})^2-9800\,\frac{N}{m^3}\,(4m)=-3.66\times10^4\,\frac{N}{m^2}$$

Since $p_1<0$, air will enter the pipe when $V=0.5\,\frac{m}{s}$

Note: The above conclusion is valid regardless of the length, ℓ.

8.47

8.47 Air at standard conditions flows through an 8-in.-diameter, 14.6 ft-long, straight duct with the velocity versus pressure drop data indicated in the following table. Determine the average friction factor over this range of data.

V (ft/min)	Δ*p* (in. water)
3950	0.35
3730	0.32
3610	0.30
3430	0.27
3280	0.24
3000	0.20
2700	0.16

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g} \quad \text{where } V_1=V_2=V,\ \Delta p=p_1-p_2,\ z_1=z_2$$

Thus, $\Delta p = f\frac{\ell}{D}\frac{1}{2}\rho V^2$ of $f=\frac{2\Delta p D}{\rho \ell V^2}$ where $\Delta p=\gamma_{H_2O}h$

or

$$f=\frac{2\left(\frac{h}{12}\text{ft}\right)\left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{8}{12}\text{ft}\right)}{\left(0.00238\frac{\text{slugs}}{\text{ft}^3}\right)(14.6\text{ft})\left(\frac{V}{60}\frac{\text{ft}}{\text{s}}\right)^2}=7.18\times10^5\frac{h}{V^2}, \text{ where } h\sim\text{in. of water},\ V\sim\frac{\text{ft}}{\text{min}}$$

Calculated values are given below:

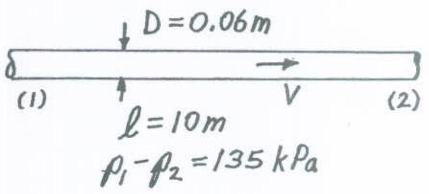
V, ft/min	h, in. water	f
3950	0.35	0.0161
3730	0.32	0.0165
3610	0.30	0.0165
3430	0.27	0.0165
3280	0.24	0.0160
3000	0.20	0.0160
2700	0.16	0.0158
	Average f =	0.0162

The average value of f is

$$f_{ave} = \underline{\underline{0.0162}}$$

8.48

8.48 Water flows through a horizontal 60-mm-diameter galvanized iron pipe at a rate of 0.02 m^3/s. If the pressure drop is 135 kPa per 10 m of pipe, do you think this pipe is **(a)** a new pipe, **(b)** an old pipe with a somewhat increased roughness due to aging, or **(c)** a very old pipe that is partially clogged by deposits? Justify your answer.



For the horizontal pipe $(z_1 = z_2)$ with $V_1 = V_2$ the energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} \text{ reduces to } p_1 - p_2 = f\frac{\ell}{D}\frac{1}{2}\rho V^2$$

or

$$135 \times 10^3 \frac{N}{m^2} = f \frac{10m}{0.06m} \frac{1}{2}(999\frac{kg}{m^3})(7.07\frac{m}{s})^2, \text{ or } f = 0.0324$$

where we have used $V = \frac{Q}{A} = \frac{0.02\frac{m^3}{s}}{\frac{\pi}{4}(0.06m)^2} = 7.07\frac{m}{s}$

With $Re = \frac{VD}{\nu} = \frac{(7.07\frac{m}{s})(0.06m)}{1.12 \times 10^{-6}\frac{m^2}{s}} = 3.79 \times 10^5$ and $\frac{\varepsilon}{D} = \frac{0.15\ mm}{60\ mm} = 2.5 \times 10^{-3}$ for a new galvanized iron pipe (see Table 8.1), the friction factor should be (see Fig. 8.20) $f = 0.0255$. Since this is less than the actual value $f = 0.0324$, the pipe is not a new pipe.

With $Re = 3.79 \times 10^5$ and $f = 0.0324$ we obtain from Fig. 8.20 a relative roughness of $\frac{\varepsilon}{D} = 0.006$. This is approximately twice the roughness of a new pipe — certainly quite possible. A very old partially clogged pipe would have considerably greater head loss. Thus, the pipe is an old pipe with somewhat increased roughness.

8.49

8.49 Water flows at a rate of 10 gallons per minute in a new horizontal 0.75-in.-diameter galvanized iron pipe. Determine the pressure gradient, $\Delta p/\ell$, along the pipe.

$$Q = 10\,\frac{gal}{min}\left(\frac{1\,min}{60\,s}\right)\left(\frac{231\,in.^3}{1\,gal}\right)\left(\frac{1\,gal}{1728\,in.^3}\right) = 0.0223\,\frac{ft^3}{s}$$

Thus,

$$V = \frac{Q}{A} = \frac{0.0223\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{0.75}{12}\,ft\right)^2} = 7.27\,\frac{ft}{s}$$

Now, for a horizontal pipe

$$\Delta p = f\,\frac{\ell}{D}\,\frac{1}{2}\rho V^2$$ where since

$$Re = \frac{VD}{\nu} = \frac{7.27\,\frac{ft}{s}\left(\frac{0.75}{12}\,ft\right)}{1.21\times10^{-5}\,\frac{ft^2}{s}} = 3.76\times10^4$$

and

$$\frac{\varepsilon}{D} = \frac{0.0005\,ft}{\left(\frac{0.75}{12}\,ft\right)} = 0.008$$

it follows from Fig. 8.20 that $f = 0.037$

Thus,

$$\frac{\Delta p}{\ell} = \frac{0.037\,(1.94\,slugs/ft^3)\,(7.27\,ft/s)^2}{\left(\frac{0.75}{12}\,ft\right)(2)} = 30.4\,\frac{lb}{ft^3}\left(\frac{1\,ft^2}{144\,in.^2}\right)$$

$$= \underline{\underline{0.211\,psi/ft}}$$

8.50

8.50 Two equal length, horizontal pipes, one with a diameter of 1 in., the other with a diameter of 2 in., are made of the same material and carry the same fluid at the same flow rate. Which pipe produces the larger head loss? Justify your answer.

For either pipe $h_L = f \frac{\ell}{D} \frac{V^2}{2g}$, where $V = Q/A = Q/(\frac{\pi}{4} D^2)$.

Thus,

$$h_L = f \frac{\ell}{D} \left[4Q/(\pi D^2) \right]^2 /2g = \frac{8}{\pi^2} f \frac{\ell}{D^5} Q^2/g$$

or

$$h_L = \left[\frac{8}{\pi^2} \frac{\ell Q^2}{g} \right] \frac{f}{D^5} \qquad (1)$$

Let $(\ )_1$ and $(\ )_2$ denote the 1 in. and 2 in. diameter pipes, respectively. Thus, with $Q_1 = Q_2$ and $\ell_1 = \ell_2$, Eq. (1) gives

$$\frac{h_{L1}}{h_{L2}} = \frac{(f_1/D_1^5)}{(f_2/D_2^5)} = \left(\frac{f_1}{f_2}\right)\left(\frac{D_2}{D_1}\right)^5 = \left(\frac{f_1}{f_2}\right)\left(\frac{2\text{ in.}}{1\text{ in.}}\right)^5$$

or

$$\frac{h_{L1}}{h_{L2}} = 32\left(\frac{f_1}{f_2}\right) \qquad (2)$$

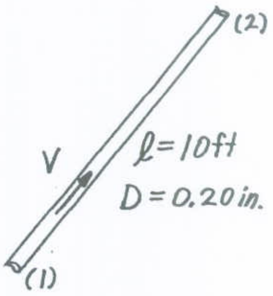
Although $f_1 \neq f_2$ (because $Re_1 \neq Re_2$ and $\varepsilon/D_1 \neq \varepsilon/D_2$) the ratio f_1/f_2 would not be significantly different than 1, especially compared to the factor of 32 in Eq. (2). For example, assume $Re_1 = 10{,}000$ and $\varepsilon/D_1 = 0.001$ so that $f_1 = 0.033$ (see Fig. 8.20). Thus, since

$Re = VD/\nu = (Q/\frac{\pi}{4}D^2)D/\nu = \frac{4Q}{\pi \nu}/D$ it follows that if $Re_1 = 10{,}000$, then $Re_2 = 5{,}000$ and $\varepsilon/D_2 = 0.0005$ if $\varepsilon/D_1 = 0.001$. Hence, $f_2 = 0.037$ so that $h_{L_1}/h_{L_2} = 32\,(0.033/0.037) = 28.5 >> 1$. Similar results would be true for other Re, ε/D values.

Thus, $h_{L_1}/h_{L_2} = 32\,(f_1/f_2) > 1$, The smaller pipe has the larger head loss.

8.52

8.52 Blood (assume $\mu = 4.5 \times 10^{-5}$ lb·s/ft², $SG = 1.0$) flows through an artery in the neck of a giraffe from its heart to its head at a rate of 2.5×10^{-4} ft³/s. Assume the length is 10 ft and the diameter is 0.20 in. If the pressure at the beginning of the artery (outlet of the heart) is equivalent to 0.70 ft Hg, determine the pressure at the end of the artery when the head is **(a)** 8 ft above the heart, or **(b)** 6 ft below the heart. Assume steady flow. How much of this pressure difference is due to elevation effects, and how much is due to frictional effects?



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } V_1 = V_2 = V \qquad (1)$$

and

$$V = \frac{Q}{A} = \frac{2.5 \times 10^{-4} \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{0.2}{12}ft\right)^2} = 1.146 \frac{ft}{s} \qquad \text{Thus, } Re = \frac{\rho V D}{\mu}, \text{ or}$$

$$Re = \frac{(1.94 \frac{slugs}{s})(1.146\frac{ft}{s})(\frac{0.2}{12}ft)}{4.5 \times 10^{-5} \frac{lb \cdot s}{ft^2}} = 823 \quad \text{Hence, the flow is laminar with}$$

$$f = \frac{64}{Re} = \frac{64}{823} = 0.0778$$

Also, $p_1 = \gamma_{Hg} h = (847 \frac{lb}{ft^3})(0.70ft) = 593 \frac{lb}{ft^2}$

Hence, from Eq. (1)

$$p_2 = p_1 - \gamma(z_2 - z_1) - f\frac{\ell}{D}\frac{1}{2}\rho V^2$$

a) With $z_2 - z_1 = 8ft$,

$$p_2 = 593 \frac{lb}{ft^2} - (62.4 \frac{lb}{ft^3})(8ft) - 0.0778 \frac{10ft}{(\frac{0.2}{12}ft)}(\tfrac{1}{2})(1.94 \frac{slugs}{ft^3})(1.146\frac{ft}{s})^2$$

$$= 593 \frac{lb}{ft^2} - 499 \frac{lb}{ft^2} - 59.5 \frac{lb}{ft^2} = \underline{\underline{34.5 \frac{lb}{ft^2}}}$$

Note: $-499 \frac{lb}{ft^2}$ is due to elevation, -59.5 is due to friction.

b) With $z_2 - z_1 = -6ft$,

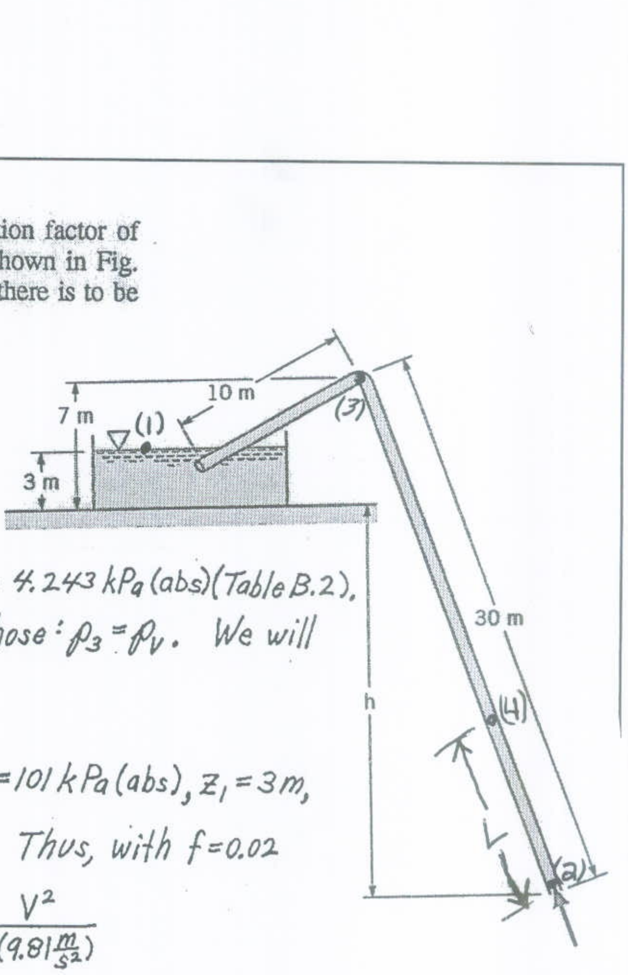
$$p_2 = 593 \frac{lb}{ft^2} - (62.4 \frac{lb}{ft^3})(-6ft) - 0.0778 \frac{10ft}{(\frac{0.2}{12}ft)}(\tfrac{1}{2})(1.94 \frac{slugs}{ft^3})(1.146\frac{ft}{s})^2$$

$$= 593 \frac{lb}{ft^2} + 374 \frac{lb}{ft^2} - 59.5 \frac{lb}{ft^2} = \underline{\underline{908 \frac{lb}{ft^2}}}$$

Note: $374 \frac{lb}{ft^2}$ is due to elevation, $-59.5 \frac{lb}{ft}$ is due to friction.

8.53

8.53 A 40-m-long, 12-mm-diameter pipe with a friction factor of 0.020 is used to siphon 30 °C water from a tank as shown in Fig. P8.53. Determine the maximum value of h allowed if there is to be no cavitation within the hose. Neglect minor losses.



The minimum pressure is the vapor pressure $p_v = 4.243$ kPa (abs)(Table B.2). Assume the minimum pressure is at the top of the hose: $p_3 = p_v$. We will check this assumption after we obtain h.

Thus,

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 + f\frac{\ell}{D}\frac{V^2}{2g}$, where $p_1 = 101$ kPa (abs), $z_1 = 3m$, $z_3 = 7m$, $V_1 = 0$, $V_3 = V$, and $p_3 = 4.243$ kPa (abs). Thus, with $f = 0.02$

$$\frac{(101-4.243)\frac{kN}{m^2}}{9.77\frac{kN}{m^3}} + 3m = 7m + \left(1+0.02\left(\frac{10m}{0.012m}\right)\right)\frac{V^2}{2(9.81\frac{m}{s^2})}$$

or

$V = 2.56\frac{m}{s}$.

Obtain h from

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}$, where $p_2 = 0$, $V_2 = V = 2.56\frac{m}{s}$, $z_2 = -h$, and $\ell = 40m$. That is, with $p_1 = p_2 = 0$

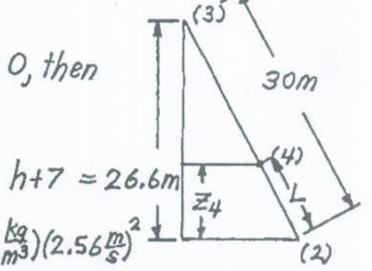
$$3\,m = -h + \left(1+0.02\left(\frac{40m}{0.012m}\right)\right)\frac{(2.56\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}, \text{ or } h = \underline{\underline{19.6\,m}}$$

Check if minimum pressure occurs at (3). Consider point (4).

From $\frac{p_4}{\gamma} + \frac{V_4^2}{2g} + z_4 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{L}{D}\frac{V^2}{2g}$ with $p_2 = 0$, $V_2 = V_4 = V$ we obtain

$p_4 = \gamma(z_2 - z_4) + f\frac{L}{D}\frac{1}{2}\rho V^2$ If we use $z_2 = 0$, then from the figure: $\frac{L}{z_4} = \frac{30}{26.6}$, or $L = 1.128 z_4$

Thus,



$$p_4 = 9.80\frac{kN}{m^3}(-z_4) + (0.02)\left(\frac{1.128z_4}{0.012}\right)\left(\frac{1}{2}\right)\left(999\frac{kg}{m^3}\right)\left(2.56\frac{m}{s}\right)^2$$

or

$p_4 = (-9.80\times10^3 + 6.15\times10^3)z_4 = -3650\,z_4$

Thus, p_4 decreases as z_4 increases. That is, the minimum pressure occurs at section (3) as assumed.

8.54

8.54 Gasoline flows in a smooth pipe of 40-mm diameter at a rate of 0.001 m³/s. If it were possible to prevent turbulence from occurring, what would be the ratio of the head loss for the actual turbulent flow compared to that if it were laminar flow?

Let $(\ )_t$ denote the turbulent flow and $(\ )_\ell$ the laminar flow.

Thus, $h_{L_t} = f_t \frac{\ell}{D} \frac{V^2}{2g}$ and $h_{L_\ell} = f_\ell \frac{\ell}{D} \frac{V^2}{2g}$ (1)

where

$$V = V_t = V_\ell = \frac{Q}{A} = \frac{0.001 \frac{m^3}{s}}{\frac{\pi}{4}(0.04m)^2} = 0.796 \frac{m}{s}$$

From Table 1.6 $\rho = 680 \frac{kg}{m^3}$ and $\mu = 3.1 \times 10^{-4} \frac{N \cdot s}{m^2}$ so that

$$Re = \frac{\rho V D}{\mu} = \frac{(680 \frac{kg}{m^3})(0.796 \frac{m}{s})(0.04m)}{3.1 \times 10^{-4} \frac{N \cdot s}{m^2}} = 6.98 \times 10^4$$

Hence, from Fig. 8.20, for a smooth pipe $f_t = 0.0192$

while for laminar flow $f_\ell = \frac{64}{Re} = \frac{64}{6.98 \times 10^4} = 9.16 \times 10^{-4}$

Thus, from Eq. (1)

$$\frac{h_{Lt}}{h_{L\ell}} = \frac{f_t}{f_\ell} = \frac{0.0192}{9.16 \times 10^{-4}} = \underline{\underline{21.0}}$$

8.55

8.55 A 3-ft-diameter duct is used to carry ventilating air into a vehicular tunnel at a rate of 9000 ft³/min. Tests show that the pressure drop is 1.5 in. of water per 1500 ft of duct. What is the approximate size of the equivalent roughness of the surface of the duct?

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } z_1=z_2,\ V_1=V_2, \text{ and} \qquad (1)$$

$$p_1-p_2=\gamma_{H_2O}h=\left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{1.5}{12}\,ft\right)=7.80\,\frac{lb}{ft^2}$$

Also, $V=\frac{Q}{A}=\frac{\left(9000\,\frac{ft^3}{min}\right)\left(\frac{1\,min}{60\,s}\right)}{\frac{\pi}{4}(3\,ft)^2}=21.2\,\frac{ft}{s}$

Thus, from Eq.(1) $\quad p_1-p_2=f\frac{\ell}{D}\frac{1}{2}\rho V^2$ or

$$f=\frac{2D(p_1-p_2)}{\rho\ell V^2}=\frac{2(3\,ft)\left(7.80\,\frac{lb}{ft^2}\right)}{\left(2.38\times10^{-3}\,\frac{slugs}{ft^3}\right)(1500\,ft)\left(21.2\,\frac{ft}{s}\right)^2}=\underline{\underline{0.0292}}$$

From Fig. 8.20 with $f=0.0292$ and $Re=\frac{VD}{\nu}=\frac{\left(21.2\,\frac{ft}{s}\right)(3\,ft)}{1.57\times10^{-4}\,\frac{ft^2}{s}}=4.05\times10^5$

we obtain $\frac{\varepsilon}{D}=0.0044$ Thus, $\varepsilon=0.0044\,(3\,ft)=\underline{\underline{0.0132\,ft}}$

8.57

8.57 An optional method of stating minor losses from pipe components is to express the loss in terms of equivalent length; the head loss from the component is quoted as the length of straight pipe with the same diameter that would generate an equivalent loss. Develop an equation for the equivalent length, ℓ_{eq}.

$$h_{L\,minor} = K_L\, V^2/2g$$

$$h_{L\,major} = f\,\frac{\ell}{D}\, V^2/2g$$

The pipe length from the major loss can be used to represent the equivalent length, leq.

$$f\,\frac{\ell_{eq}}{D}\, V^2/2g = K_L\, V^2/2g$$

$$f\,\frac{\ell_{eq}}{D} = K_L$$

$$\underline{\underline{\ell_{eq} = \frac{K_L D}{f}}}$$

8.58

8.58 Given 90° threaded elbows used in conjunction with copper pipe (drawn tubing) of 0.75-in. diameter, convert the loss for a single elbow to equivalent length of copper pipe for wholly turbulent flow.

$$\ell_{eq} = \frac{K_L D}{f}$$

For 90° threaded elbow, $K_L = 1.5$

For copper pipe (drawn tubing), $\varepsilon = 0.000005 \text{ ft}$

So

$$\frac{\varepsilon}{D} = \frac{0.000005}{(0.75/12)} = 8 \times 10^{-5}$$

From Moody chart (wholly turbulent flow)

$$f \cong 0.0115$$

$$\ell_{eq} = \frac{(1.5)(0.75/12)}{0.0115} = \underline{\underline{8.15 \text{ ft}}}$$

8.59

8.59 Based on Problem 8.57, develop a graph to predict equivalent length, ℓ_{eq}, as a function of pipe diameter for a 45° threaded elbow connecting copper piping (drawn tubing) for wholly turbulent flow.

$$\ell_{eq} = \frac{K_L D}{f}$$

For 45° threaded elbow, $K_L = 0.4$

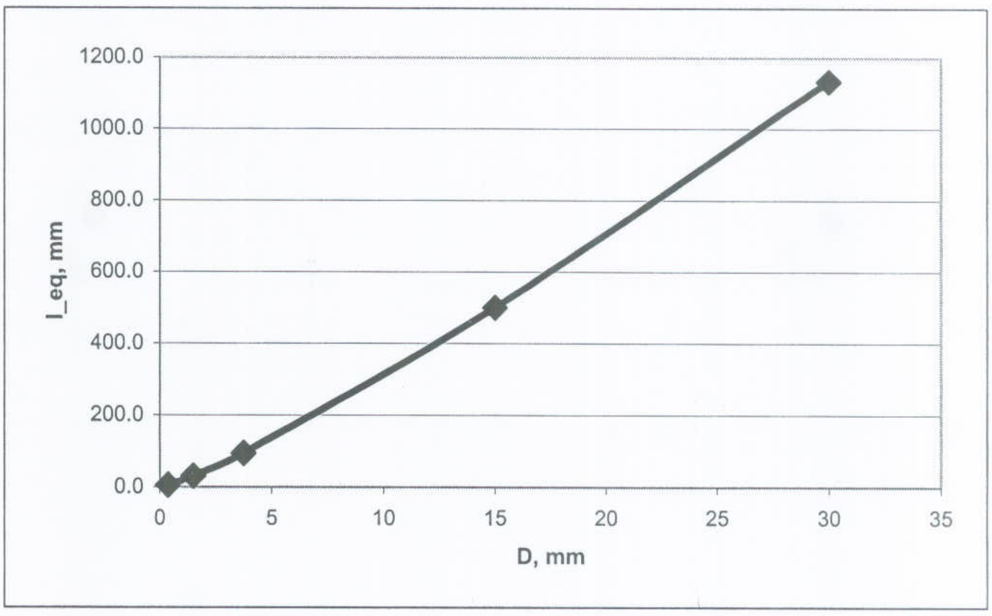
For copper tubing (drawn tubing), $\varepsilon = 0.0015$ mm

To calculate f, use alternate form

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$$

For wholly turbulent flow, assume $Re = 8 \times 10^7$
This is large enough Re to make f essentially independent of Re (see Moody chart, Fig. 8.20).

D(mm)	e/D	f	l_eq (mm)
30	0.00005	0.010602	1131.8
15	0.0001	0.012025	499.0
3.75	0.0004	0.015932	94.2
1.5	0.001	0.019678	30.5
0.375	0.004	0.028474	5.3



8.60

8.60 A regular 90° threaded elbow is used to connect two straight portions of 4-in.-diameter galvanized iron pipe. **(a)** If the flow is assumed to be wholly turbulent, determine the equivalent length of straight pipe for this elbow. **(b)** Does a pipe fitting such as this elbow have a significant or negligible effect on the flow? Explain.

(a) $h_L = K_L \frac{V^2}{2g}$, where from Table 8.2 $K_L = 1.5$ for a 90° threaded elbow.

Also,

$\ell_{eq} = \frac{K_L D}{f}$, where from Table 8.1 $\varepsilon = 0.0005\ \text{ft}$ for a galvanized iron pipe. Thus, with $\varepsilon/D = 0.0005\ \text{ft}/(4/12)\text{ft} = 0.0015$ and a very large Reynolds number (i.e. wholly turbulent flow) it follows from Fig. 8.20 that $f = 0.021$.

Thus,

$$\ell_{eq} = \frac{1.5(4/12)\text{ft}}{0.021} = \underline{\underline{23.8\ \text{ft}}}$$

(b) In general $h_L = K_L \frac{V^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g} = (K_L + f\frac{\ell}{D})\frac{V^2}{2g}$

or

$h_L = f\frac{(\ell + \ell_{eq})}{D}\frac{V^2}{2g}$ since $K_L = f\frac{\ell_{eq}}{D}$

Thus, whether or not a pipe filling such as this elbow has a significant effect on the flow depends on the relative size of ℓ_{eq} (= 23.8ft for this case) and the pipe length, ℓ. If $\ell_{eq} \ll \ell$, then the fitting is negligible.

8.61

8.61 To conserve water and energy, a "flow reducer" is installed in the shower head as shown in Fig. P8.61 If the pressure at point (1) remains constant and all losses except for that in the "flow reducer" are neglected, determine the value of the loss coefficient (based on the velocity in the pipe) of the "flow reducer" if its presence is to reduce the flowrate by a factor of 2. Neglect gravity.

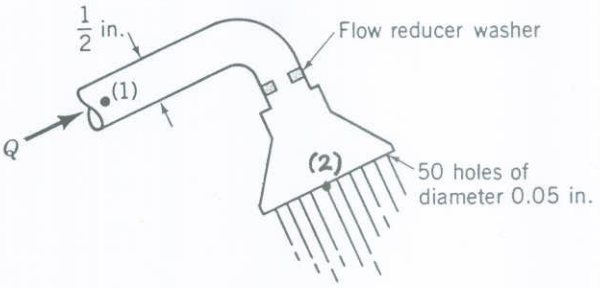


FIGURE P8.61

Without the reducer $\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$, where $p_2=0$, $z_1=z_2$
and

$V_1=\frac{Q}{A_1}=\frac{4Q}{\pi D_1^2}=\frac{4Q}{\pi(\frac{0.5}{12}ft)^2}=733Q$

$V_2=\frac{Q}{A_2}=\frac{4Q}{50\pi(\frac{0.05}{12}ft)^2}=1467Q$ (V_1 and $V_2\sim\frac{ft}{s}$ and $Q\sim\frac{ft^3}{s}$)

Thus, $p_1=\frac{1}{2}\rho(V_2^2-V_1^2)=\frac{1}{2}\rho(1467^2Q^2-733Q^2)=8.07\times10^5\rho Q^2\ \frac{lb}{ft^2}$, (1)

where $\rho\sim\frac{slug}{ft^3}$, $Q\sim\frac{ft^3}{s}$

With the flow reducer the flowrate is reduced by a factor of two.

Thus, $V_1=\frac{1}{2}(733Q)$ and $V_2=\frac{1}{2}(1467Q)$ with (2)

$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}=\frac{V_2^2}{2g}+K_L\frac{V_1^2}{2g}$ or $p_1=\frac{1}{2}\rho(V_2^2+(K_L-1)V_1^2)$ (3)

Hence, by combing Eqs. (1), (2), and (3) we obtain

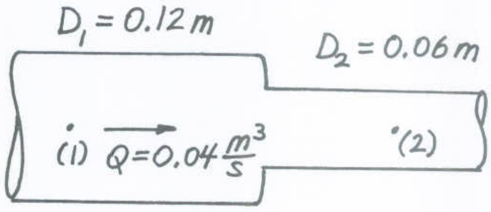
$$8.07\times10^5\rho Q^2=\frac{1}{2}\rho\left[\left(\frac{1467}{2}Q\right)^2+(K_L-1)\left(\frac{733}{2}Q\right)^2\right]$$

or

$\underline{\underline{K_L=9.00}}$

8.62

8.62 Water flows at a rate of 0.040 m^3/s in a 0.12-m-diameter pipe that contains a sudden contraction to a 0.06-m-diameter pipe. Determine the pressure drop across the contraction section. How much of this pressure difference is due to losses and how much is due to kinetic energy changes?



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + K_L\frac{V_2^2}{2g}, \text{ where } z_1 = z_2 \qquad (1)$$

and

$$V_1 = \frac{Q}{A_1} = \frac{0.04\frac{m^3}{s}}{\frac{\pi}{4}(0.12m)^2} = 3.54\,\frac{m}{s}, \quad V_2 = \frac{Q}{A_2} = \frac{0.04\frac{m^3}{s}}{\frac{\pi}{4}(0.06m)^2} = 14.1\,\frac{m}{s}$$

Thus, with $\frac{A_2}{A_1} = \left(\frac{D_2}{D_1}\right)^2 = \left(\frac{0.06m}{0.12m}\right)^2 = 0.25$ we obtain from Fig. 8.30

$K_L = 0.40$

Hence, from Eq. (1)

$$p_1 - p_2 = \frac{1}{2}\rho\left[K_L V_2^2 + V_2^2 - V_1^2\right] = \frac{1}{2}\left(999\,\frac{kg}{m^3}\right)\left[0.40\left(14.1\,\frac{m}{s}\right)^2 + \left(14.1\,\frac{m}{s}\right)^2 - \left(3.54\,\frac{m}{s}\right)^2\right]$$

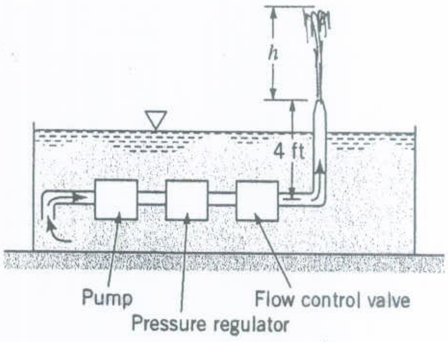
or

$$p_1 - p_2 = 39.7\times10^3\,\frac{N}{m^2} + 93.0\times10^3\,\frac{N}{m^2} = \underline{\underline{133\ kPa}}$$

This represents a $\underline{\underline{39.7\ kPa}}$ drop from losses and a $\underline{\underline{93.0\ kPa}}$ drop due to an increase in kinetic energy.

8.64

8.64 (See "New hi-tech fountains," Section 8.5.) The fountain shown in Fig. P8.64 is designed to provide a stream of water that rises $h = 10$ ft to $h = 20$ ft above the nozzle exit in a periodic fashion. To do this the water from the pool enters a pump, passes through a pressure regulator that maintains a constant pressure ahead of the flow control valve. The valve is electronically adjusted to provide the desired water height. With $h = 10$ ft the loss coefficient for the valve is $K_L = 50$. Determine the valve loss coefficient needed for $h = 20$ ft. All losses except for the flow control valve are negligible. The area of the pipe is 5 times the area of the exit nozzle.



■ FIGURE P8.64

For any height h,

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}-h_L=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}\text{, where } z_1=0,\ z_2=h+4\text{ft},\ p_2=0,\ V_2=0,$$

and $h_L=K_L V_1^2/2$

Thus,

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}-K_L\frac{V_1^2}{2g}=z_2 \qquad (1)$$

For $h=10\text{ft}$: $(K_L=50)$

$$\frac{p_1}{\gamma}=z_2-\frac{V_1^2}{2g}+K_L\frac{V_1^2}{2g}=(10\text{ft}+4\text{ft})+(50-1)\frac{V_1^2}{2g} \qquad (2)$$

Also, from (3) to (2):

$$\frac{p_3}{\gamma}+z_3+\frac{V_3^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}\text{, where } p_2=p_3=0,\ z_2-z_3=h\text{, and } V_2=0$$

Thus,

$$\frac{V_3^2}{2g}=h \quad\text{or}\quad V_3=\sqrt{2gh} \qquad (3)$$

so for $h=10\text{ft}$, $V_3=\sqrt{2(32.2\frac{\text{ft}}{\text{s}^2})(10\text{ft})}=25.4\frac{\text{ft}}{\text{s}}$

Also, $V_1A_1=V_3A_3$ so that $V_1=\frac{A_3}{A_1}V_3=(\frac{1}{5})(25.4\frac{\text{ft}}{\text{s}})=5.08\frac{\text{ft}}{\text{s}}$

Hence, Eq. (2) gives

$$\frac{p_1}{\gamma}=14\text{ft}+49(5.08\tfrac{\text{ft}}{\text{s}})^2/(2(32.2\tfrac{\text{ft}}{\text{s}^2}))=33.6\text{ft}$$

For $h=20\text{ft}$, from Eq. (3): $V_3=\sqrt{(2)(32.2\frac{\text{ft}}{\text{s}^2})(20\text{ft})}=35.9\frac{\text{ft}}{\text{s}}$

Hence, $V_1=\frac{A_3}{A_1}V_3=(\frac{1}{5})(35.9\frac{\text{ft}}{\text{s}})=7.18\frac{\text{ft}}{\text{s}}$

Since p_1 is constant (independent of h), the value $\frac{p_1}{\gamma}=33.6\text{ ft}$ obtained above for $h=10\text{ft}$ is also valid for $h=20\text{ft}$. Thus, with $z_2=h+4\text{ft}=20\text{ft}+4\text{ft}=24\text{ft}$, Eq. (1) is:

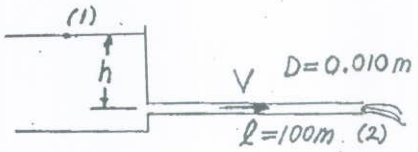
$$33.6\text{ft}+(7.18\tfrac{\text{ft}}{\text{s}})^2/(2(32.2\text{ ft/s}^2))-K_L(7.18\tfrac{\text{ft}}{\text{s}})^2/(2(32.2\text{ ft/s}^2))=24\text{ ft}$$

or

$$K_L=\underline{\underline{13.0}}$$

8.65

***8.65** Water flows from a large open tank through a sharp-edged entrance and into a galvanized iron pipe of length 100 m and diameter 10 mm. The water exits the pipe as a free jet at a distance h below the free surface of the tank. Plot a log–log graph of the flowrate, Q, as a function of h for $0.1 \le h \le 10$ m.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+K_L\right)\frac{V^2}{2g},$$ where $p_1=0$, $V_1=0$, $z_1=h$,

$p_2=0$, $V_2=V$, and $z_2=0$ Thus,

$$h=\left(1+f\frac{\ell}{D}+K_L\right)\frac{V^2}{2g}$$ where from Fig. 8.25 $K_L=0.5$

Hence,

$$h=\left(1+f\left(\frac{100\,m}{0.01\,m}\right)+0.5\right)\frac{V^2}{2(9.81\frac{m}{s^2})} \quad \text{or} \quad 19.6h=(1.5+10{,}000f)V^2, \text{ with} \qquad (1)$$

$h \sim m$, $V \sim \frac{m}{s}$

Also, $\frac{\varepsilon}{D}=\frac{0.15\,mm}{10\,mm}=0.015$ (see Table 8.2)

and

$$Re=\frac{VD}{\nu}=\frac{(0.01m)V}{1.12\times10^{-6}\frac{m^2}{s}} \quad \text{or} \quad Re=8930V \qquad (2)$$

If the flow is laminar, then $f=\frac{64}{Re}$ (i.e. $Re<2100$) (3)

If the flow is turbulent, then from Eq. 8.35b

$$\frac{1}{\sqrt{f}}=-1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}+\frac{6.9}{Re}\right]$$

That is

$$\frac{1}{\sqrt{f}}=-1.8\log\left[4.05\times10^{-3}+\frac{6.9}{Re}\right] \qquad (4)$$

The maximum h for laminar flow occurs when $Re=2100$, or from Eq. (2)

$V=\frac{2100}{8930}=0.235\frac{m}{s}$ and $f=\frac{64}{2100}=0.0304$. Thus, from Eq. (1)

$$h=\frac{(1.5+10{,}000(0.0304))(0.235\frac{m}{s})^2}{19.6\frac{m}{s^2}}=0.861\,m$$

Thus, for $h<0.861\,m$ the flow is laminar. For $h>0.861$ assume flow is turbulent.

For $0.1\,m \le h \le 10\,m$ solve Eqs. (1), (2), and (3) or (4) depending if $h<0.861\,m$ or $h>0.861\,m$ to obtain V. Then

$$Q=\frac{\pi}{4}D^2V=\frac{\pi}{4}(0.01m)^2V=7.85\times10^{-5}V\ \frac{m^3}{s}, \text{ where } V\sim\frac{m}{s} \qquad (5)$$

For laminar flow (i.e., $h \le 0.861m$) Eqs. (1), (2), and (3) give

$$19.6h=\left[1.5+10^4\frac{64}{8930V}\right]V^2$$

or

$V^2+47.8V-13.1h=0$, which can be solved using the quadratic equation to give

$V=-23.9\pm[571+13.1h]^{1/2}$ Since $V>0$ we can disregard the "−" root.

(con't)

8.65 (con't)

Thus, using Eq. (5)

$$Q = 7.85 \times 10^{-5}\left[-23.9 + (571 + 13.1h)^{1/2}\right] \text{ for } 0 \le h \le 0.861 \text{ m} \quad (6)$$

This equation was used in a MS Excel spreadsheet to find Q as a function of h for laminar flow.

NOTE: The coefficients of Eq. (6) must be very precisely given because for small values of h, $(571+13.1h)^{1/2} \simeq -23.9$

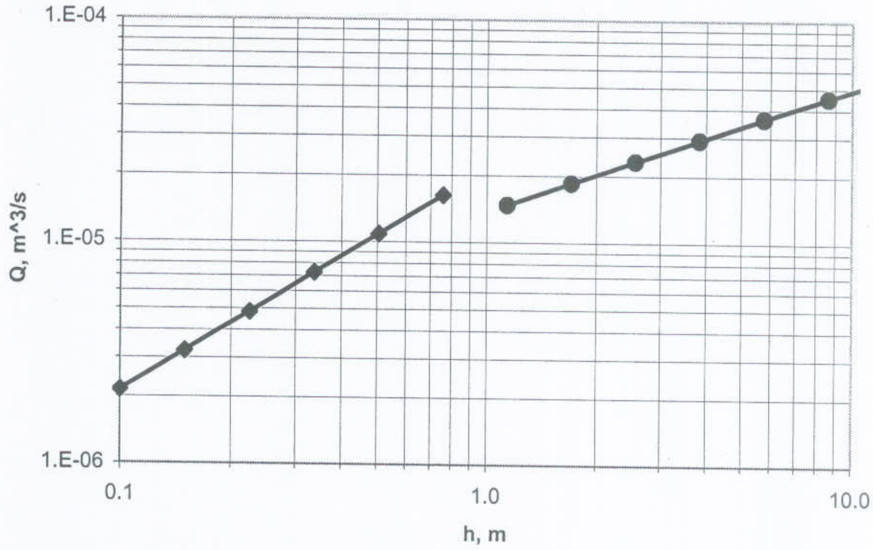
So in the spreadsheet

$$Q = 7.85 \times 10^{-5}\left[-23.88951 + (570.7087 + 13.066667h)^{1/2}\right]$$

For $h > 0.861$ m, Eqns. (1), (2), and (4) were used in the spreadsheet to manually iterate on f. Eqn (5) was used to find Q(h).

Insert a guess value (e.g. f = 0.02) in the f(guess) cell. A new f value will be calculated in the f(new) cell. Use this new value as the updated f(guess). Continue until f(guess) = f(new).

h, m	f(guess)	V, m/s	Re	f(new)	Q, m^3/s
0.100					2.15E-06
0.150					3.22E-06
0.225					4.82E-06
0.338					7.23E-06
0.506					1.08E-05
0.759					1.62E-05
1.139	0.0639	1.87E-01	1667	0.0639	1.47E-05
1.709	0.0604	2.35E-01	2100	0.0604	1.85E-05
2.563	0.0575	2.95E-01	2636	0.0575	2.32E-05
3.844	0.0551	3.69E-01	3298	0.0551	2.90E-05
5.767	0.0531	4.61E-01	4114	0.0531	3.62E-05
8.650	0.0515	5.73E-01	5116	0.0515	4.50E-05
12.975	0.0501	7.11E-01	6353	0.0501	5.58E-05



8,66

8.66 Air flows through the mitered bend shown in Fig. P8.66 at a rate of 5.0 cfs. To help straighten the flow after the bend, a set of 0.25-in.-diameter drinking straws is placed in the pipe as shown. Estimate the extra pressure drop between points (1) and (2) caused by these straws.

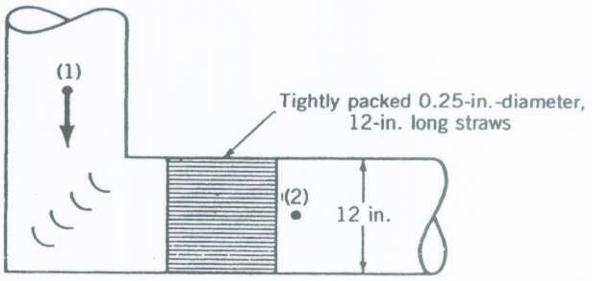


FIGURE P8.66

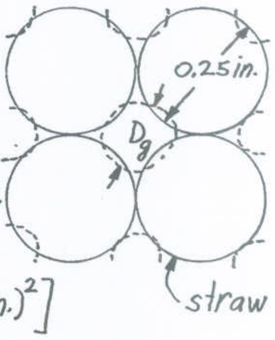
The extra pressure drop, Δp, is equal to the pressure drop through the length of the straws minus the pressure drop in that 12 in. length of the pipe without the straws. That is

$$\Delta p = \Delta p_s - \Delta p_{ns}, \text{ where } \Delta p_{ns} = f\frac{\ell}{D}\frac{1}{2}\rho V^2 \text{ with } V = \frac{Q}{A} = \frac{5\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{12}{12}ft)^2} = 6.37\frac{ft}{s}$$

Also, $Re = \frac{VD}{\nu} = \frac{(6.37\frac{ft}{s})(1ft)}{1.57\times10^{-4}\frac{ft^2}{s}} = 4.06\times10^4$. If we assume the pipe is smooth, it follows from Fig. 8.20 that $f = 0.0215$. Thus,

$$\Delta p_{ns} = 0.0215\left(\frac{12\text{ in.}}{12\text{ in.}}\right)\left(\frac{1}{2}\right)\left(2.38\times10^{-3}\frac{slug}{ft^3}\right)\left(6.37\frac{ft}{s}\right)^2 = 1.04\times10^{-3}\frac{lb}{ft^2} \qquad (1)$$

With the straws in place, $\Delta p_s = f\frac{\ell}{D}\frac{1}{2}\rho V^2$ where the values of f, D, and V and different than those used above. In general the flow geometry is quite complex — flow through the straws and flow in the gaps between the straws. For simplicity, assume the gaps act as a circular flow area of diameter $D_g = \frac{3}{8}D = \frac{3}{8}(0.25 in.) = 0.0938 in.$ Thus, in each 0.5 in. by 0.5 in. cross section there are 4 straws, or a total of $N = 4\frac{straws}{(0.25in^2)}\left[\frac{\pi}{4}(12 in.)^2\right]$ i.e. $N = 1810$ straws.



If the flow is laminar, then $Q \sim D^4$ so that $\frac{Q_{gap}}{Q_{straw}} = \left(\frac{0.0938 in.}{0.25 in.}\right)^4 = 0.0198$

That is, only about 2% of the flow is in the gap region — neglect this amount.

Thus, $V = \frac{Q}{NA} = \frac{5\frac{ft^3}{s}}{1810\left(\frac{\pi}{4}\left(\frac{0.25}{12}ft\right)^2\right)} = 8.10\frac{ft}{s}$

Hence,

$$Re = \frac{VD}{\nu} = \frac{(8.10\frac{ft}{s})(\frac{0.25}{12}ft)}{1.57\times10^{-4}\frac{ft^2}{s}} = 1070 < 2100, \text{ the flow is laminar with}$$

$f = \frac{64}{Re} = \frac{64}{1070} = 0.0598$, or $\Delta p_s = 0.0598\left(\frac{12 in.}{0.25in.}\right)\left(\frac{1}{2}\right)\left(2.38\times10^{-3}\frac{slug}{ft^3}\right)\left(8.10\frac{ft}{s}\right)^2$

or $\Delta p_s = 0.224\frac{lb}{ft^2}$ Hence, when combined with result (1)

$$\Delta p = \Delta p_s - \Delta p_{ns} = (0.224 - 0.00104)\frac{lb}{ft^2} = \underline{\underline{0.223\frac{lb}{ft^2}}}$$

8.67

8.67 Repeat Problem 8.66 if the straws are replaced by a piece of porous foam rubber that has a loss coefficient equal to 5.4.

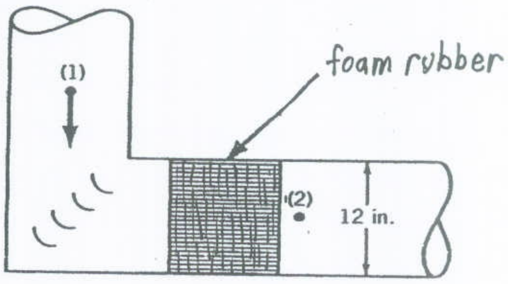


FIGURE P8.67

The extra pressure drop, Δp, is equal to the pressure drop through the length of foam rubber minus the pressure drop in that 12 in. length of the pipe without the foam. That is,

$\Delta p = \Delta p_f - \Delta p_{nf}$, where $\Delta p_{nf} = f \frac{\ell}{D} \frac{1}{2} \rho V^2$ with $V = \frac{Q}{A} = \frac{5 \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{12}{12} ft\right)^2} = 6.37 \frac{ft}{s}$

Also, $Re = \frac{VD}{\nu} = \frac{(6.37 \frac{ft}{s})(1 ft)}{1.57 \times 10^{-4} \frac{ft^2}{s}} = 4.06 \times 10^4$. If we assume the pipe is smooth, it follows from Fig. 8.20 that $f = 0.0215$. Thus,

$$\Delta p_{nf} = 0.0215 \left(\frac{12 \text{ in.}}{12 \text{ in.}}\right)\left(\frac{1}{2}\right)\left(2.38 \times 10^{-3} \frac{slug}{ft^3}\right)\left(6.37 \frac{ft}{s}\right)^2 = 1.04 \times 10^{-3} \frac{lb}{ft^2} \qquad (1)$$

The pressure drop due to the foam is

$$\Delta p_f = K_L \frac{1}{2} \rho V^2$$

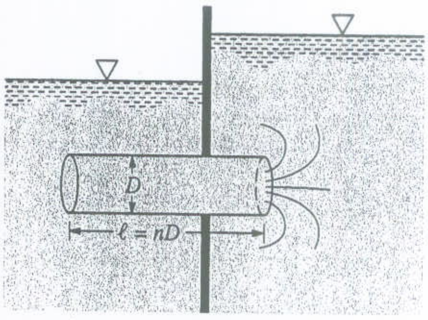
$$= 5.4 \left(\frac{1}{2}\right)\left(0.00238 \frac{slugs}{ft^3}\right)\left(6.37 \frac{ft}{s}\right)^2 = 0.261 \frac{lb}{ft^2}$$

Thus,

$$\Delta p = \Delta p_f - \Delta p_{nf} = 0.261 \frac{lb}{ft^2} - 0.00104 \frac{lb}{ft^2} = \underline{\underline{0.260 \frac{lb}{ft^2}}}$$

8.68

8.68 As shown in Fig. P8.68, water flows from one tank to another through a short pipe whose length is n times the pipe diameter. Head losses occur in the pipe and at the entrance and exit. (See Video V8.10) Determine the maximum value of n if the major loss is to be no more than 10% of the minor loss and the friction factor is 0.02.



■ FIGURE P8.68

If $h_{L_{major}} = 10\% h_{L_{minor}}$, then

$$10 f \frac{\ell}{D} \frac{V^2}{2g} = \Sigma K_L \frac{V^2}{2g} \quad \text{or} \quad \frac{\ell}{D} = \frac{\Sigma K_L}{10 f} \qquad (1)$$

where $\Sigma K_L = K_{L_{entrance}} + K_{L_{exit}} = 0.8 + 1 = 1.8$

Thus, with $f = 0.02$ and $\ell = nD$ Eq. (1) becomes

$$\frac{nD}{D} = \frac{1.8}{10(0.02)}$$

or

$$n = \underline{\underline{9}}$$

8.69

8.69 Air flows through the fine mesh gauze shown in Fig. P8.69 with an average velocity of 1.50 m/s in the pipe. Determine the loss coefficient for the gauze.

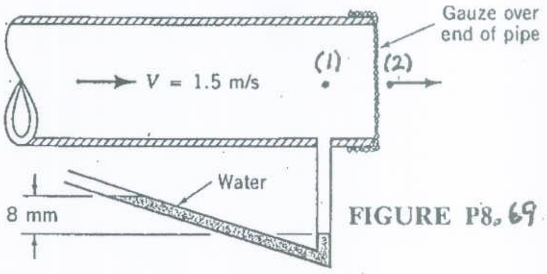


FIGURE P8.69

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+K_L\frac{V^2}{2g}, \text{ where } z_1=z_2,\; V_1=V_2=V=1.5\,\frac{m}{s}$$

Thus, $K_L=\frac{2(p_1-p_2)}{\rho V^2}$ where $p_2=0$ and $p_1=8\,mm$ water

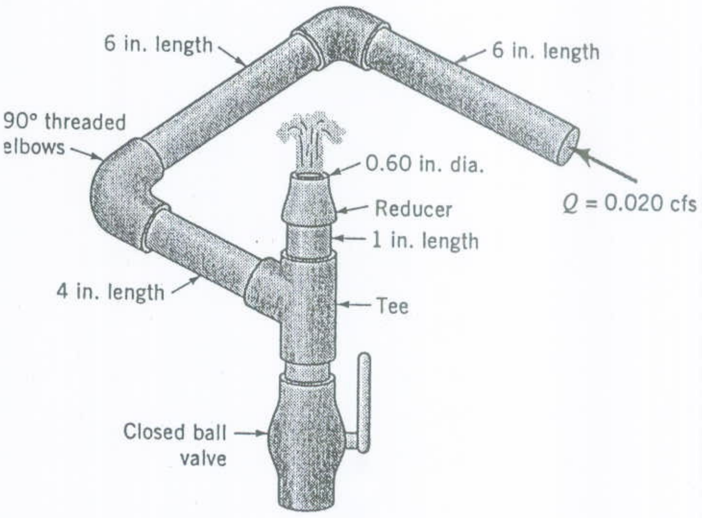
or $p_1=(8\times10^{-3}\,m)(9.80\times10^{3}\,\frac{N}{m^3})=78.4\,\frac{N}{m^2}$

Hence,

$$K_L=\frac{2\,(78.4\,\frac{N}{m^2})}{(1.23\,\frac{kg}{m^3})(1.5\,\frac{m}{s})^2}=\underline{\underline{56.7}}$$

8.70

8.70 Water flows steadily through the 0.75-in. diameter galvanized iron pipe system shown in Video V8.14 and Fig. P8.70 at a rate of 0.020 cfs. Your boss suggests that friction losses in the straight pipe sections are negligible compared to losses in the threaded elbows and fittings of the system. Do you agree or disagree with your boss? Support your answer with appropriate calculations.



■ FIGURE P8.70

Major loss $= f\frac{\ell}{D}\frac{V^2}{2g}$ where

$\ell = (6+6+4+1)\,in. = 17\,in.,\ D = 0.75\,in.$

and

$$V = \frac{Q}{A} = \frac{0.02\,\frac{ft^3}{s}}{\frac{\pi}{4}(0.75/12)^2\,ft^2} = 6.52\,\frac{ft}{s}$$

Thus, with

$$Re = \frac{VD}{\nu} = \frac{6.52\,\frac{ft}{s}\left(\frac{0.75}{12}\,ft\right)}{1.21\times10^{-5}\,\frac{ft^2}{s}} = 3.37\times10^{4} \text{ and}$$

$$\frac{\varepsilon}{D} = \frac{0.0005\,ft}{\left(\frac{0.75}{12}\,ft\right)} = 8\times10^{-3}$$ (see Table 8.1) we obtain (see Fig. 8.20)

$$f = 0.038 \text{ so that } f\frac{\ell}{D}\frac{V^2}{2g} = 0.038\,\frac{17\,in.}{0.75\,in.}\frac{V^2}{2g} = 0.861\,\frac{V^2}{2g} \qquad (1)$$

Also,

$$\text{Minor loss} = \sum K_L\frac{V^2}{2g} = \left[\underset{90^\circ\text{ elbow}}{2(1.5)} + \underset{\text{tee}}{2} + \underset{\text{reducer}}{0.15}\right]\frac{V^2}{2g} = 5.15\,\frac{V^2}{2g} \qquad (2)$$

reducer with $\frac{A_2}{A_1} = \left(\frac{0.6\,in.}{0.75\,in.}\right)^2 = 0.64$
(see Fig. 8.26)

Thus, from Eqs. (1) and (2):

$$\frac{\text{major loss}}{\text{minor loss}} = \frac{0.861\,\frac{V^2}{2g}}{5.15\,\frac{V^2}{2g}} = 0.167 = 16.7\%$$

Probably <u>disagree</u> with boss because pipe friction is about 17% of other losses.

8.72

8.72 Given two rectangular ducts with equal cross-sectional area, but different aspect ratios (width/height) of 2 and 4, which will have the greater frictional losses? Explain your answer.

The duct with the greater losses is the one with the largest headloss per length, h_L/ℓ, where $h_L = f\frac{\ell}{D_h}\frac{V^2}{2g}$. If the areas are equal, then the velocities are equal since $V = Q/A$.

Let $(\;)_2$ and $(\;)_4$ denote ducts with aspect ratios of 2 and 4, respectively. Thus,

$$(h_L/\ell)_4 = \frac{f_4}{D_{h4}}\frac{V_4^2}{2g} \text{ and } (h_L/\ell)_2 = \frac{f_2}{D_{h2}}\frac{V_2^2}{2g}, \text{ where } V_2 = V_4.$$

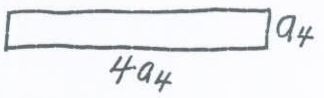
Hence,

$$(h_L/\ell)_4/(h_L/\ell)_2 = \frac{f_4}{D_{h4}} \Big/ \frac{f_2}{D_{h2}} = \frac{f_4}{f_2}\frac{D_{h2}}{D_{h4}} \qquad (1)$$

Let $A_2 = (2a)a$

and

$A_4 = (4a_4)a_4$



Thus, since $A_2 = A_4$,

$$2a^2 = 4a_4^2, \text{ or } a_4 = \frac{1}{\sqrt{2}}a$$

and

$$D_{h_2} = 4A_2/P_2 = 4(2a^2)/[4a+2a] = \frac{4}{3}a = 1.33a \qquad (2)$$

and

$$D_{h4} = 4A_4/P_4 = 4(2a^2)/[\tfrac{2}{\sqrt2}a + \tfrac{8}{\sqrt2}a] = \frac{4\sqrt2}{5}a = 1.13a \qquad (3)$$

so that

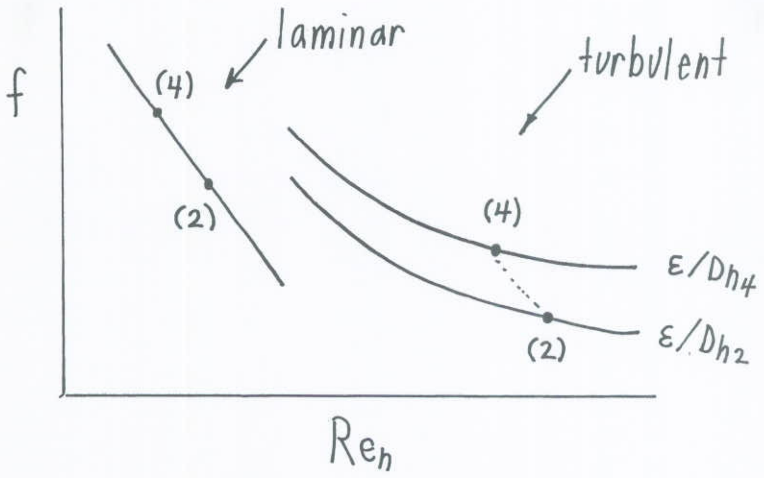
$$\frac{D_{h2}}{D_{h4}} = \frac{\frac{4}{3}a}{\frac{4\sqrt2}{5}a} = \frac{5}{3\sqrt2} = 1.179 \text{ so that Eq. (1) becomes}$$

$$(h_L/\ell)_4/(h_L/\ell)_2 = 1.179\frac{f_4}{f_2} \qquad (4)$$

In general, $f = f(Re, \frac{\varepsilon}{D})$ in such a way that if $\frac{\varepsilon}{D}$ increases, f increases and if Re decreases, f increases. This is seen from the Moody chart as indicated below.

(con't)

8.72 (con't)



For a given ε, $\left(\frac{\varepsilon}{D_h}\right)_4 > \left(\frac{\varepsilon}{D_h}\right)_2$ since $D_{h_4} < D_{h_2}$ (see Eqs. (2) and (3)).
Also, since $Re_h = VD_h/\nu$ it follows that

$Re_{h_4} < Re_{h_2}$ since $D_{h_4} < D_{h_2}$ and $V_2 = V_4$.

Thus, whether the flow is laminar or turbulent it follows that $f_4 > f_2$. It follows from Eq. (4) that

$(h_L/\ell)_4 / (h_L/\ell_2) > 1$

That is, the duct with the $\underline{\underline{\text{aspect ratio of 4}}}$ has the greater headloss.

8.73

8.73 Air at standard temperature and pressure flows at a rate of 7.0 cfs through a horizontal, galvanized iron duct that has a rectangular cross-sectional shape of 12 in. by 6 in. Estimate the pressure drop per 200 ft of duct.

For a horizontal duct $\Delta p = \gamma h_L = f \frac{\ell}{D_h} \frac{1}{2} \rho V^2$, where $V = \frac{Q}{A}$ (1)

or

$$V = \frac{7 \frac{ft^3}{s}}{(12\,in.)(6\,in.)\left(\frac{1\,ft^2}{144\,in.^2}\right)} = 14.0 \frac{ft}{s} \quad \text{and} \quad Re_h = \frac{V D_h}{\nu}$$

with

$$D_h = \frac{4A}{P} = \frac{4(0.5\,ft^2)}{(2+1)ft} = 0.667\,ft$$

Thus,

$$Re_h = \frac{(14.0 \frac{ft}{s})(0.667 ft)}{1.57 \times 10^{-4} \frac{ft^2}{s}} = 5.95 \times 10^4$$

Also, for galvanized iron $\varepsilon = 0.0005\,ft$, or $\frac{\varepsilon}{D_h} = \frac{0.0005\,ft}{0.667\,ft} = 0.000750$

From Fig. 8.20 we obtain $f = 0.0227$

Thus, from Eq. (1) with $\ell = 200\,ft$,

$$\Delta p = (0.0227)\frac{200\,ft}{0.667\,ft} \frac{1}{2}\left(2.38 \times 10^{-3} \frac{slug}{ft^3}\right)\left(14.0 \frac{ft}{s}\right)^2 = 1.59 \frac{lb}{ft^2} = \underline{\underline{0.0110\ psi}}$$

8.74

8.74 Air flows through a rectangular galvanized iron duct of size 0.30 m by 0.15 m at a rate of 0.068 m³/s. Determine the head loss in 12 m of this duct.

$h_L = f\frac{\ell}{D_h}\frac{V^2}{2g}$, where $D_h = \frac{4A}{P} = \frac{4(0.3m)(0.15m)}{2[0.3m+0.15m]} = 0.2\,m$

and

$V = \frac{Q}{A} = \frac{0.068\frac{m^3}{s}}{(0.3m)(0.15m)} = 1.51\frac{m}{s}$ Also, $Re_h = \frac{VD_h}{\nu} = \frac{(1.51\frac{m}{s})(0.2m)}{1.46\times10^{-5}\frac{m^2}{s}} = 20{,}700$

and from Table 8.1,

$\frac{\varepsilon}{D_h} = \frac{0.15\times10^{-3}m}{0.2\,m} = 7.5\times10^{-4}$ Hence, from Fig. 8.20 $f=0.027$

so that

$$h_L = (0.027)\left(\frac{12\,m}{0.2m}\right)\frac{(1.51\frac{m}{s})^2}{2(9.81\frac{m}{s^2})} = \underline{\underline{0.188m}}$$

8.75

8.75 Air at standard conditions flows through a horizontal 1 ft by 1.5 ft rectangular wooden duct at a rate of 5000 ft³/min. Determine the head loss, pressure drop, and power supplied by the fan to overcome the flow resistance in 500 ft of the duct.

$h_L = f\frac{\ell}{D_h}\frac{V^2}{2g}$, where $V = \frac{Q}{A} = \frac{(5000\frac{ft^3}{min})(\frac{1\,min}{60s})}{(1ft)(1.5ft)} = 55.6\frac{ft}{s}$

and $D_h = \frac{4A}{P} = \frac{4(1ft)(1.5ft)}{2[1ft+1.5ft]} = 1.2\,ft$

Also, $Re_h = \frac{VD_h}{\nu} = \frac{(55.6\frac{ft}{s})(1.2ft)}{1.57\times10^{-4}\frac{ft^2}{s}} = 4.25\times10^5$ and from Table 8.1

$\varepsilon \approx 0.0006\,ft$ to $0.003\,ft$. Use an "average" $\varepsilon = 0.0018$ ft so that

$\frac{\varepsilon}{D_h} = \frac{0.0018\ ft}{1.2ft} = 0.0015$ Thus, from Fig. 8.20 $f = 0.022$, or

$$h_L = (0.022)\left(\frac{500\ ft}{1.2ft}\right)\frac{(55.6\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} = \underline{\underline{440\,ft}}$$

For this horizontal pipe $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$, where $z_1 = z_2$ and $V_1 = V_2$.

Thus, $p_1 - p_2 = \gamma h_L = (7.65\times10^{-2}\frac{lb}{ft^3})(440ft) = 33.7\frac{lb}{ft^2} = \underline{\underline{0.234psi}}$

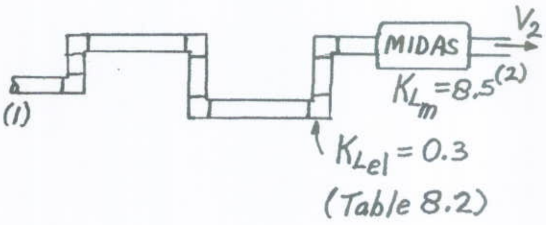
$\mathcal{P} = \gamma Q h_L = Q(p_1 - p_2) = (5000\frac{ft^3}{min})(\frac{1\,min}{60s})(33.7\frac{lb}{ft^2}) = (2810\frac{ft\cdot lb}{s})\left[\frac{1\,hp}{(550\frac{ft\cdot lb}{s})}\right]$

or

$\mathcal{P} = \underline{\underline{5.11\,hp}}$

8.76

8.76 Assume a car's exhaust system can be approximated as 14 ft of 0.125-ft-diameter cast-iron pipe with the equivalent of six 90° flanged elbows and a muffler. (See Video V8.12) The muffler acts as a resistor with a loss coefficient of $K_L = 8.5$. Determine the pressure at the beginning of the exhaust system if the flowrate is 0.10 cfs, the temperature is 250 °F, and the exhaust has the same properties as air.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(f\frac{\ell}{D} + \Sigma K_L\right)\frac{V^2}{2g}$$, where $z_1 = z_2$, $p_2 = 0$,

and

$$V = V_1 = V_2 = \frac{Q}{A} = \frac{0.1\,\frac{ft^3}{s}}{\frac{\pi}{4}(0.125 ft)^2} = 8.15\,\frac{ft}{s}$$

Thus,

$$p_1 = \left(f\frac{\ell}{D} + \Sigma K_L\right)\frac{1}{2}\rho V^2,$$ where $$\rho = \frac{p}{RT} = \frac{(14.7\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2})}{(1716\,\frac{ft\cdot lb}{slug\cdot {}^\circ R})(460+250)^\circ R} = 1.74 \times 10^{-3}\,\frac{slug}{ft^3}$$

Also, $\frac{\varepsilon}{D} = \frac{0.00085 ft}{0.125 ft} = 0.0068$ (Table 8.1)

so that with $$Re = \frac{\rho V D}{\mu} = \frac{(1.74\times10^{-3}\,\frac{slug}{ft^3})(8.15\,\frac{ft}{s})(0.125 ft)}{4.7\times10^{-7}\,\frac{lb\cdot s}{ft^2}} = 3770$$ we

obtain from Fig. 8.20, $f = 0.047$

Hence,

$$p_1 = \left(0.047\left(\frac{14 ft}{0.125 ft}\right) + 6(0.3) + 8.5\right)\left(\frac{1}{2}\right)\left(1.74\times10^{-3}\,\frac{slug}{ft^3}\right)\left(8.15\,\frac{ft}{s}\right)^2$$

$$= \underline{\underline{0.899\,\frac{lb}{ft^2}}}$$

8.77

8.77 The pressure at section (2) shown in Fig. P8.77 is not to fall below 60 psi when the flowrate from the tank varies from 0 to 1.0 cfs and the branch line is shut off. Determine the minimum height, *h*, of the water tank under the assumption that (**a**) minor losses are negligible, (**b**) minor losses are not negligible.

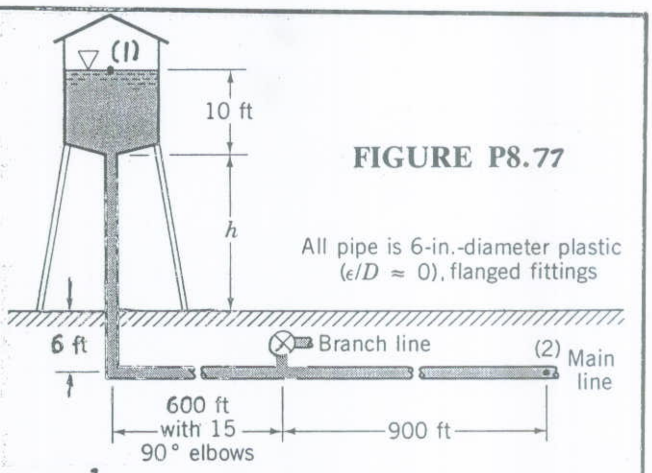


FIGURE P8.77

All pipe is 6-in.-diameter plastic ($\epsilon/D \approx 0$), flanged fittings

$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+(f\frac{\ell}{D}+\Sigma K_L)\frac{V^2}{2g}$, where $p_1=0$, $V_1=0$, $z_1=16\,ft+h$,

and $z_2=0$ Thus, with $V=V_2$

$16+h=\frac{p_2}{\gamma}+\frac{V^2}{2g}+(f\frac{\ell}{D}+\Sigma K_L)\frac{V^2}{2g}$. Note: h must be no less than that with $p_{2\,min}=60\,psi$ and $Q_{max}=1\,cfs$, or

$$V_2=V=\frac{Q}{A_2}=\frac{1\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{6}{12}ft)^2}=5.09\frac{ft}{s}$$

Hence,

$$h=-16ft+\frac{(60\frac{lb}{in^2})(144\frac{in^2}{ft^2})}{62.4\frac{lb}{ft^3}}+\left(1+f\left(\frac{h+6+600+900}{\frac{6}{12}}\right)+\Sigma K_L\right)\frac{(5.09\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$

or

$$h=122.5+\left(1+f\left(\frac{1506+h}{0.5}\right)+\Sigma K_L\right)(0.402)\ ft, \text{ where } h\sim ft \qquad (1)$$

With $\frac{\varepsilon}{D}=0$ and $Re=\frac{VD}{\nu}=\frac{(5.09\frac{ft}{s})(\frac{6}{12}ft)}{1.21\times10^{-5}\frac{ft^2}{s}}=2.10\times10^5$ we obtain

$f=0.0155$ (see Fig. 8.20)

a) Neglect minor losses ($\Sigma K_L=0$):

From Eq. (1)

$$h=122.5+\left(1+(0.0155)\left(\frac{1506+h}{0.5}\right)\right)(0.402)$$

or $h=\underline{\underline{143\ ft}}$

b) Include minor losses:

$\Sigma K_L=K_{L\,entrance}+15K_{L\,elbow}+K_{L\,tee}=0.5+15(0.3)+0.2=5.2$

(see Table 8.2, assume flanged fittings)

Thus, from Eq. (1)

$$h=122.5+\left(1+(0.0155)\left(\frac{1506+h}{0.5}\right)+5.2\right)(0.402)$$

or

$h=\underline{\underline{146\,ft}}$

Note: For this case minor losses are not very important.

8.78

8.78 Repeat Problem 8.77 with the assumption that the branch line is open so that half of the flow from the tank goes into the branch, and half continues in the main line.

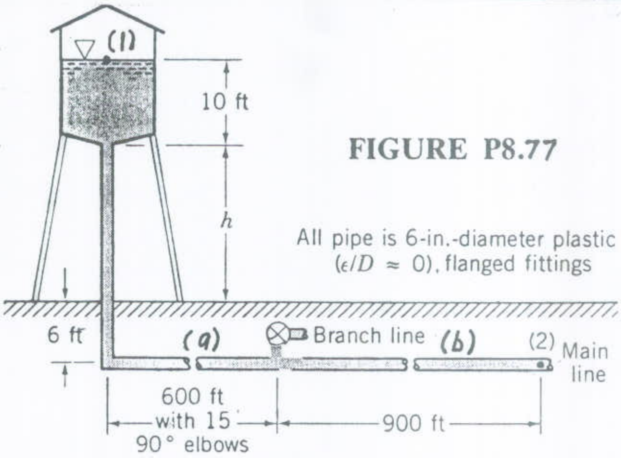


FIGURE P8.77

All pipe is 6-in.-diameter plastic ($\varepsilon/D \approx 0$), flanged fittings

For the flow from (1) to (2):

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f_a\frac{\ell_a}{D_a}+\Sigma K_{L_a}\right)\frac{V_a^2}{2g}+\left(f_b\frac{\ell_b}{D_b}+\Sigma K_{L_b}\right)\frac{V_b^2}{2g} \quad (1)$$

where $(\ )_a$ and $(\ )_b$ denote pipes "a" and "b" as indicated in the figure. Thus, with $p_1=0$, $V_1=0$, $z_1=16\text{ ft}+h$, $z_2=0$, and $p_2=60\text{ psi}$. Also,

$$V_a=\frac{Q_a}{A_a}=\frac{1\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{6}{12}ft)^2}=5.09\frac{ft}{s}\ ,\ V_b=\frac{Q_b}{A_b}=\frac{0.5\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{6}{12}ft)^2}=2.55\frac{ft}{s},$$ Eq.(1) becomes

$$16+h=\frac{(60\frac{lb}{in^2})(144\frac{in^2}{ft^2})}{62.4\frac{lb}{ft^3}}+\left(1+f_a\left(\frac{h+6+600}{\frac{6}{12}}\right)+\Sigma K_{L_a}\right)\frac{(5.09\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$
$$+\left(f_b\left(\frac{900}{\frac{6}{12}}\right)+\Sigma K_{L_b}\right)\frac{(2.55\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}$$

or

$$h=122.5+\left(1+f_a\left(\frac{606+h}{0.5}\right)+\Sigma K_{L_a}\right)(0.402)+(1800f_b+\Sigma K_{L_b})(0.101),\ \text{where } h\sim ft \quad (2)$$

With $\frac{\varepsilon}{D}=0$, $Re_a=\frac{V_aD_a}{\nu}=\frac{(5.09\frac{ft}{s})(\frac{6}{12}ft)}{1.21\times10^{-5}\frac{ft^2}{s}}=2.10\times10^5$, and

$Re_b=\frac{V_bD_b}{\nu}=\frac{1}{2}Re_a=1.05\times10^5$ we obtain $f_a=0.0155$ and $f_b=0.0175$ (Fig. 8.20)

a) Neglect minor losses ($\Sigma K_{L_a}=\Sigma K_{L_b}=0$):

From Eq.(2)

$$h=122.5+\left(1+(0.0155)\left(\frac{606+h}{0.5}\right)\right)(0.402)+(1800(0.0175)(0.101)$$

or

$$h=\underline{\underline{135\text{ ft}}}$$

b) Include minor losses:

$\Sigma K_{L_a}=K_{L_{entrance}}+15K_{L_{elbow}}=0.5+15(0.3)=5.0$ (see Table 8.2; assume flanged fittings)

and

$\Sigma K_{L_b}=K_{L_{tee}}=0.2$

From Eq.(2)

$$h=122.5+\left(1+(0.0155)\left(\frac{606+h}{0.5}\right)+5.0\right)(0.402)+(1800(0.0175)+0.2)(0.101)$$

or

$$h=\underline{\underline{137\text{ ft}}}$$

Note: For this case minor losses are not very important.

8.79

8.79 The exhaust from your car's engine flows through a complex pipe system as shown in Fig. P8.79 and **Video V8.5.** Assume that the pressure drop through this system is Δp_1 when the engine is idling at 1000 rpm at a stop sign. Estimate the pressure drop (in terms of Δp_1) with the engine at 3000 rpm when you are driving on the highway. List all the assumptions that you made to arrive at your answer.

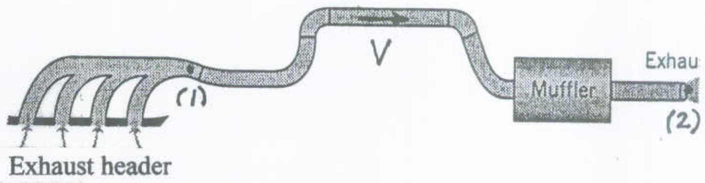


FIGURE P8.79

For steady flow,

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

Assume $z_1 = z_2$ and $V_1 = V_2$ so that with $h_L = \left[f\frac{\ell}{D} + K_L\right]\frac{V^2}{2g}$ and $\Delta p \equiv p_1 - p_2$ we obtain

$$\Delta p = \gamma h_L = \gamma\left(f\frac{\ell}{D} + K_L\right)\frac{V^2}{2g} = \frac{1}{2}\rho V^2\left(f\frac{\ell}{D} + K_L\right)$$

Hence,

$$\frac{\Delta p_{3000}}{\Delta p_{1000}} = \frac{\frac{1}{2}\rho_{3000} V_{3000}^2\left(f_{3000}\frac{\ell}{D} + K_L\right)}{\frac{1}{2}\rho_{1000} V_{1000}^2\left(f_{1000}\frac{\ell}{D} + K_L\right)}$$

Assume $\rho_{1000} = \rho_{3000}$ and $f_{1000} = f_{3000}$ (i.e. f independent of Re)

Thus,

$$\frac{\Delta p_{3000}}{\Delta p_{1000}} = \left(\frac{V_{3000}}{V_{1000}}\right)^2$$

But $V = \frac{Q}{A}$ where Q is assumed proportional to engine rpm.

That is $V_{3000} = 3\,V_{1000}$ so that

$$\frac{\Delta p_{3000}}{\Delta p_{1000}} = (3)^2 = \underline{\underline{9}}$$

8.80

8.80 According to fire regulations in a town, the pressure drop in a commercial steel horizontal pipe must not exceed 1.0 psi per 150 ft of pipe for flowrates up to 500 gal/min. If the water temperature is above 50° F, can a 6-in-diameter pipe be used?

Determine the pressure drop in a 6in. diameter pipe.

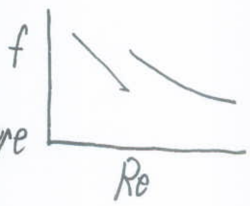
$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } V_1=V_2 \text{ and } z_1=z_2.$$

Thus

$$\frac{p_1-p_2}{\gamma}=f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } f=f(Re,\tfrac{\varepsilon}{D}). \qquad (1)$$

From Table 8.1, $\varepsilon = 0.00015 ft$ so that $\frac{\varepsilon}{D}=\frac{1.5\times10^{-4}}{(6/12 ft)}=3\times10^{-4}$

The largest p_1-p_2 will occur with the largest f, which occurs with the smallest Re, or largest ν. Since the viscosity of water increases a the temperature decreases, we consider the coldest case — $T=50°F$.



From Table B.1, at 50°F, $\gamma=62.41 lb/ft^3$ and $\nu=1.407\times10^{-5}\frac{ft^2}{s}$

Also,

$$V=\frac{Q}{A}=\frac{(500\frac{gal}{min})(\frac{1min}{60s})(231\frac{in.^3}{gal})(\frac{1ft^3}{1728in^3})}{\frac{\pi}{4}(\frac{6}{12}ft)^2}=5.67\frac{ft}{s}$$

Thus,

$$Re=\frac{VD}{\nu}=\frac{(5.67\frac{ft}{s})(6/12 ft)}{1.407\times10^{-5}\frac{ft^2}{s}}=2.01\times10^5$$

Hence, with $Re=2.01\times10^5$ and $\frac{\varepsilon}{D}=3\times10^{-4}$ we obtain from Fig. 8.20,

$f=0.018$

Therefore, from Eq. (1),

$$\frac{p_1-p_2}{\gamma}=0.018\frac{(150ft)}{(6/12ft)}\frac{(5.67\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})}=2.70 ft$$

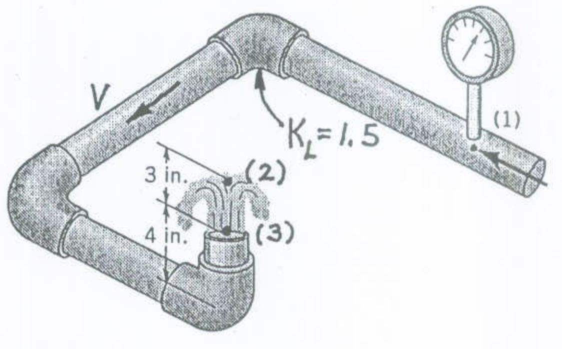
so that

$$p_1-p_2=(2.70ft)(62.4\frac{lb}{ft^3})=168\frac{lb}{ft^2}(\frac{1ft^2}{144in.^2})=1.17 psi > 1.0 psi$$

A 6in. diameter pipe requires slightly more than the allowed 1.0psi per 150ft. Thus, ~~no~~, a 6in. pipe cannot be used. The minimum diameter can be shown to be $D=0.513 ft = 6.37 in.$

8.81

8.81 As shown in Video V8.14 and Fig. P8.81 water "bubbles up" 3 in. above the exit of the vertical pipe attached to three horizontal pipe segments. The total length of the 0.75-in.-diameter galvanized iron pipe between point (1) and the exit is 21 in. Determine the pressure needed at point (1) to produce this flow.



■ FIGURE P8.81

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - h_L = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where $z_1 = 0$, $p_2 = 0$, $V_2 = 0$ Thus,

(1) $$\frac{p_1}{\gamma} = z_2 + h_L - \frac{V_1^2}{2g} \quad \text{where } V_1 = V_3 = V$$

With no head loss from (3) to (2) and $p_2 = p_3 = V_2 = 0$ we obtain

$$\frac{V_3^2}{2g} + z_3 = z_2, \text{ or } V_3 = \sqrt{2g(z_2 - z_3)} = \sqrt{2\left(32.2\frac{ft}{s^2}\right)\left(\frac{3}{12}ft\right)} = 4.01\frac{ft}{s}$$

Thus,

$$Re = \frac{VD}{\nu} = \frac{V_3 D}{\nu} = \frac{4.01\frac{ft}{s}\left(\frac{0.75}{12}ft\right)}{1.21\times10^{-5}\frac{ft^2}{s}} = 2.07\times10^4$$

and

$$\frac{\varepsilon}{D} = \frac{0.0005\,ft}{\left(\frac{0.75}{12}\right)ft} = 0.008$$ (see Table 8.1), so that (see Fig. 8.20)

$f = 0.039$

Also, $h_L = f\frac{\ell}{D}\frac{V^2}{2g} + \sum K_L \frac{V^2}{2g}$ where $\sum K_L = 3(1.5) = 4.5$

Hence, Eq. (1) becomes

$$\frac{p_1}{\gamma} = z_2 + \left[f\frac{\ell}{D} + \sum K_L\right]\frac{V^2}{2g} - \frac{V_1^2}{2g} \quad \text{where } V_1 = V$$

or

$$\frac{p_1}{\gamma} = \frac{7}{12}ft + \left[0.039\,\frac{21\,in.}{0.75\,in.} + 4.5 - 1\right]\frac{\left(4.01\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)} = (0.583 + 1.147)ft$$

$$= 1.73\,ft$$

Thus,

$$p_1 = \left(62.4\frac{lb}{ft^3}\right)(1.73\,ft) = 108\,\frac{lb}{ft^2} = \underline{\underline{0.750\,psi}}$$

8.82

8.82 Water at 10 °C is pumped from a lake as shown in Fig. P8.82. If the flowrate is 0.011 m³/s, what is the maximum length inlet pipe, ℓ, that can be used without cavitation occurring?

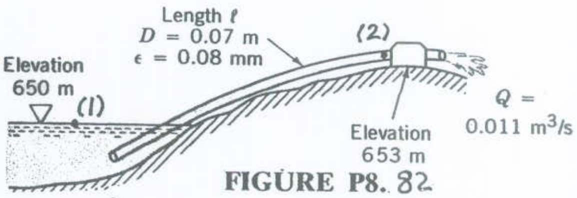


FIGURE P8.82

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}\text{, where } p_1=101\text{ kPa}, z_1=650\text{m} \qquad (1)$$

$V_1=0$, $V_2=V$, $z_2=653\,m$, and from Table B.2 $p_2=p_v=1.228\text{ kPa}$

Also, $V=\frac{Q}{A}=\frac{0.011\frac{m^3}{s}}{\frac{\pi}{4}(0.07m)^2}=2.86\frac{m}{s}$ so that

$Re=\frac{VD}{\nu}=\frac{(2.86\frac{m}{s})(0.07m)}{1.307\times10^{-6}\frac{m^2}{s}}=1.53\times10^5$. With this Re and from Table 8.1 with

$\frac{\varepsilon}{D}=\frac{0.08\,mm}{70\,mm}=0.00114$ we obtain $f=0.0216$ (see Fig. 8.20)

Hence, with $\Sigma K_L=0.8$ for the entrance, Eq.(1) becomes

$$\frac{(101-1.228)\times10^3\frac{N}{m^3}}{9.80\times10^3\frac{N}{m^2}}+650m=653\,m+\left(1+(0.0216)\left(\frac{\ell}{0.07m}\right)+0.8\right)\frac{(2.86\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}$$

or $\ell=\underline{\underline{50.0\,m}}$

8.83

8.83 Water flows through the pipe system shown in Fig. P8.83 at a rate of 0.30 ft³/s. The pipe diameter is 2 in., and its roughness is 0.002 in. The loss coefficient for each of the five filters is 6.0, and all other minor losses are negligible. Determine the power added to the water by the pump if the pressure immediately before the pump is to be the same as that immediately after the last filter. The length of the pipe between these two locations is 80 ft.

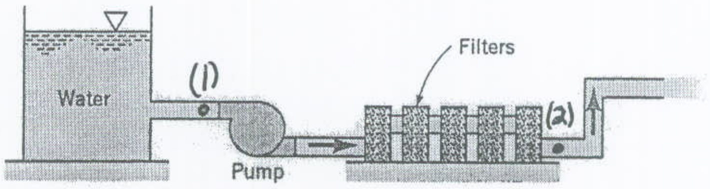


FIGURE P8.83

From the energy equation

$$h_p = h_L = \left(f\frac{l}{D} + K_L\right)V^2/2g$$

$$Q = VA, \quad V = 0.30/\left(\frac{\pi}{4}(2/12)^2\right) = 13.75 \text{ ft/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{(1.94)(13.75)(2/12)}{2.34\times10^{-5}} = 1.9\times10^5$$

$$\varepsilon/D = \frac{0.002}{2} = 1\times10^{-3}$$

From the Moody chart, $f \hat{=} 0.0215$

So

$$h_p = \left[0.0215\frac{80}{2/12} + (5)(6)\right]\frac{(13.75)^2}{2(32.2)}$$

$$= [10.32 + 30](2.94)$$

(Note: the filters produce ~3x the pipe loss)

$$h_p = 118.54 \text{ ft}$$

Calculate the power

$$\dot{W} = \gamma Q h_p = (62.4)(0.3)(118.54)$$

$$= 2219.1\,\frac{\text{ft lb}}{\text{s}} \times \frac{1\text{ hp}}{550\,\frac{\text{ft lb}}{\text{s}}} = \underline{4.03\text{ hp}}$$

8.84

8.84 Water at 40 °F flows through the coils of the heat exchanger as shown in Fig. P8.84 at a rate of 0.9 gal/min. Determine the pressure drop between the inlet and outlet of the horizontal device.

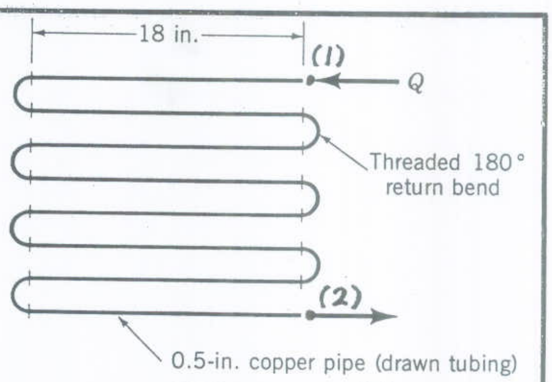


FIGURE P8.84

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(f\frac{\ell}{D} + \Sigma K_L\right)\frac{V^2}{2g}, \text{ where } z_1 = z_2,$$

$$V = V_1 = V_2 = \frac{Q}{A} = \frac{\left(0.9\,\frac{gal}{min}\right)\left(231\,\frac{in.^3}{gal}\right)\left(\frac{1\,ft^3}{1728\,in.^3}\right)\left(\frac{1\,min}{60\,s}\right)}{\frac{\pi}{4}\left(\frac{0.5}{12}\,ft\right)^2} = 1.47\,\frac{ft}{s}$$

Thus,

$$p_1 - p_2 = \left(f\frac{\ell}{D} + \Sigma K_L\right)\frac{1}{2}\rho V^2, \text{ with } \ell = 8\left(\frac{18}{12}\,ft\right) = 12\,ft \qquad (1)$$

and $\Sigma K_L = 7(1.5) = 10.5$ (see Table 8.2)

Also, from Table 8.1 $\frac{\varepsilon}{D} = (0.000005\,ft/(0.5/12\,ft)) = 1.2\times10^{-4}$

and $Re = \frac{VD}{\nu} = \frac{\left(1.47\,\frac{ft}{s}\right)\left(\frac{0.5}{12}\,ft\right)}{1.66\times10^{-5}\,\frac{ft^2}{s}} = 3690$ (see Table 8.1 for ν)

Hence, from Fig. 8.20

$f = 0.041$

and from Eq. (1)

$$p_1 - p_2 = \left(0.041\left(\frac{12\,ft}{\frac{0.5}{12}\,ft}\right) + 10.5\right)\left(\frac{1}{2}\right)\left(1.94\,\frac{slugs}{ft^3}\right)\left(1.47\,\frac{ft}{s}\right)^2$$

or

$$p_1 - p_2 = 46.8\,\frac{lb}{ft^2} = \underline{\underline{0.325\,psi}}$$

8.85

8.85 For the flow in Problem 8.84, ethylene glycol is added to the water for freeze protection if the temperature drops below the freezing point. The density is unchanged, and all flow conditions are the same except that the viscosity of the mixture has changed to 0.01 Ns/m² at the given temperature. Recalculate the pressure drop between inlet and outlet. Discuss how this loss will change if the fluid temperature does drop below freezing.

First, convert the viscosity to BG units Using Table 1.4

$$\mu = 0.01 \frac{N\cdot s}{m^2}\left(2.089\times10^{-2}\right) = 2.09\times10^{-4}\ \frac{lb\, s}{ft^2}$$

From Table B.1, $\rho = 1.94\ slugs/ft^3$

So,

$$\nu = \frac{\mu}{\rho} = \frac{2.09\times10^{-4}\ \frac{lb\cdot s}{ft^2}}{1.94\ slugs/ft^3} = 1.077\times10^{-4}\ ft^2/s$$

Calculate an updated Reynolds number with $V = 1.47\ ft/s$ (see Prob. 8.85)

$$Re = \frac{VD}{\nu} = \frac{(1.47\ ft/s)\,(0.5/12\ ft)}{1.077\times10^{-4}\ ft^2/s} = 569$$

Therefore, the new flow is laminar

$$f = 64/Re = 64/569 = 0.112$$

From Problem 8.84

$$P_1 - P_2 = \left(f\frac{\ell}{D} + \sum K_L\right)\frac{1}{2}\rho V^2$$

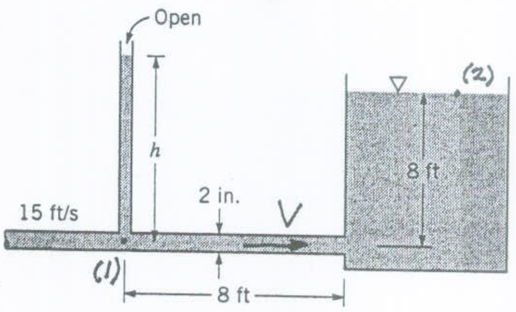
$$= \left(0.112\left(\frac{12\ ft}{0.5/12\ ft}\right) + 10.5\right)\left(\frac{1}{2}\right)\left(1.94\ \frac{slugs}{ft^3}\right)\left(1.47\frac{ft}{s}\right)^2$$

$$P_1 - P_2 = 89.6\ lb/ft^2 = \underline{\underline{0.622\ psi}}$$

This addition approximately doubles the pressure drop. If the fluid temperature does drop below freezing, there will be a further increase in viscosity and the pressure drop.

8.86

8.86 Water flows through a 2-in.-diameter pipe with a velocity of 15 ft/s as shown in Fig. P8.86. The relative roughness of the pipe is 0.004, and the loss coefficient for the exit is 1.0. Determine the height, h, to which the water rises in the piezometer tube.



■ FIGURE P8.86

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$\frac{p_1}{\gamma} = h$, $z_1 = 0$, $p_2 = 0$, $z_2 = 8\,ft$, $V_2 = 0$ and

$h_L = (f\frac{\ell}{D} + K_L)\frac{V^2}{2g}$ with $V = V_1$ and $K_L = 1$

Thus,

(1) $$h + \frac{V^2}{2g} - (f\frac{\ell}{D} + K_L)\frac{V^2}{2g} = z_2$$

But $Re = \frac{\rho VD}{\mu} = \frac{1.94\frac{slug}{ft^3}(15\frac{ft}{s})(\frac{2}{12}ft)}{2.34\times10^{-5}\frac{lb\cdot s}{ft^2}} = 2.07\times10^{5}$

Hence from Fig. 8.20 with $\varepsilon/D = 0.004$ we obtain $f = 0.029$ so that Eq. (1) becomes

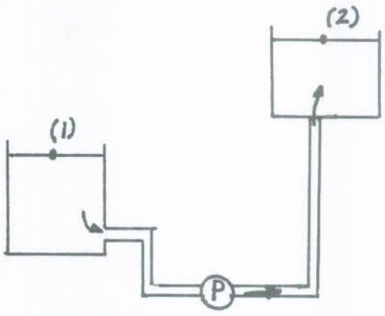
$$h + \left[1 - 0.029\frac{14\,ft}{(\frac{2}{12}ft)} - 1\right]\frac{(15\frac{ft}{s})^2}{2(32.2\frac{ft}{s^2})} = 8\,ft$$

or

$$h = \underline{\underline{16.5\,ft}}$$

8.87

8.87 Water is pumped through a 60-m-long, 0.3-m-diameter pipe from a lower reservoir to a higher reservoir whose surface is 10 m above the lower one. The sum of the minor loss coefficients for the system is $K_L = 14.5$. When the pump adds 40 kW to the water the flowrate is 0.20 m^3/s. Determine the pipe roughness.



$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p - h_L = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$, where $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 = 0$, and $z_2 = 10m$

Thus, $h_p - h_L = z_2$, where

$$h_p = \frac{\dot{W}_P}{\gamma Q} = \frac{40 \times 10^3 N \cdot m/s}{(9.80 \times 10^3 N/m^3)(0.2 m^3/s)} = 20.4 m$$

Hence,

$$20.4m - \left[f \frac{\ell}{D} + \Sigma K_L\right] \frac{V^2}{2g} = 10 m \qquad (1)$$

with

$$V = \frac{Q}{A} = (0.2\ m^3/s) / \left(\frac{\pi}{4}(0.3m)^2\right) = 2.82\ m/s$$

Thus, from Eq. (1)

$$20.4m - \left[f\left(\frac{60\ m}{0.3m}\right) + 14.5\right] \frac{(2.82\ m/s)^2}{2(9.81\ m/s^2)}$$

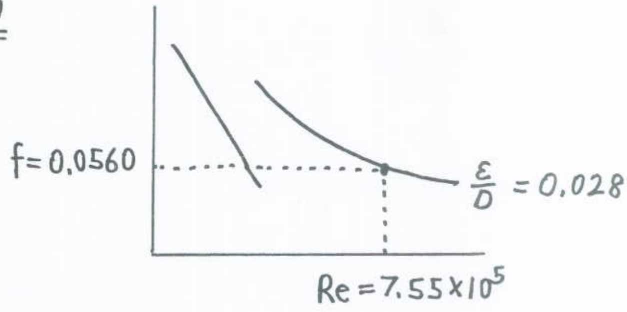
or

$f = 0.0560$

Also, $Re = \frac{\rho V D}{\mu} = \frac{999 kg/m^3 (2.82 m/s)(0.3m)}{1.12 \times 10^{-3} N \cdot s/m^2} = 7.55 \times 10^5$

Thus, from the Moody chart (Fig. 8.35), with $Re = 7.55 \times 10^5$ and $f = 0.0560$ it follows that $\varepsilon/D = 0.028$, or

$\varepsilon = 0.028\ (0.3m) = \underline{\underline{0.0084\ m}}$



8.89

8.89 As shown in Fig. P8.89, a standard household water meter is incorporated into a lawn irrigation system to measure the volume of water applied to the lawn. Note that these meters measure volume, not volume flowrate. (See Video V8.15.) With an upstream pressure of $p_1 = 50$ psi the meter registered that 120 ft³ of water was delivered to the lawn during an "on" cycle. Estimate the upstream pressure, p_1, needed if it is desired to have 150 ft³ delivered during an "on" cycle. List any assumptions needed to arrive at your answer.

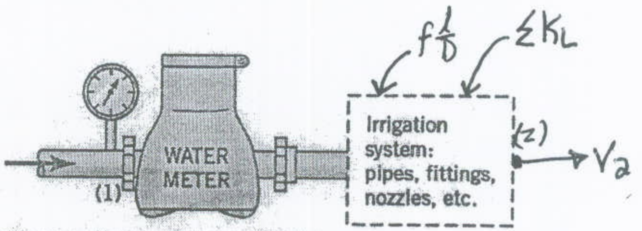


FIGURE P8.89

The energy equation for this flow is

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - \left[f\frac{\ell}{D} + \sum K_L\right]\frac{V^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (1)$$

where $z_1 = z_2$, $p_2 = 0$, $V_1 = V$, and $V_2 = \frac{A_1}{A_2}V_1$

Thus, from Eq. (1)

$$p_1 = \frac{1}{2}\rho V_1^2\left[f\frac{\ell}{D} + \sum K_L + \left(\frac{A_1}{A_2}\right)^2 - 1\right] \qquad (2)$$

But $Q = A_1V_1 = \frac{\forall}{t}$, where $\forall$ is the volume of water supplied during an "on" cycle and t is the length of the cycle.

For a given system $\sum K_L$ is independent of Q. Similarly, for large Re pipe flow, f is independent of Re (or Q). Thus,

$\left[f\frac{\ell}{D} + \sum K_L + \left(\frac{A_1}{A_2}\right)^2 - 1\right]$ is constant, independent of Q.

Hence, from Eq. (2), if the length of the cycle is constant,

$$\frac{p_1)_{150\,ft^3}}{p_1)_{120\,ft^3}} = \frac{\frac{1}{2}\rho V_1^2)_{150}}{\frac{1}{2}\rho V_1^2)_{120}} = \left[\frac{V_1)_{150}}{V_1)_{120}}\right]^2 = \left(\frac{\forall_{150}}{\forall_{120}}\right)^2 = \left(\frac{150}{120}\right)^2 = 1.563$$

or

$$p_1)_{150} = 1.563\,p_1)_{120} = 1.563(50\,psi) = \underline{\underline{78.1\,psi}}$$

8.90

8.90 A fan is to produce a constant air speed of 40 m/s throughout the pipe loop shown in Fig. P8.90. The 3-m-diameter pipes are smooth, and each of the four 90-degree elbows has a loss coefficient of 0.30. Determine the power that the fan adds to the air.

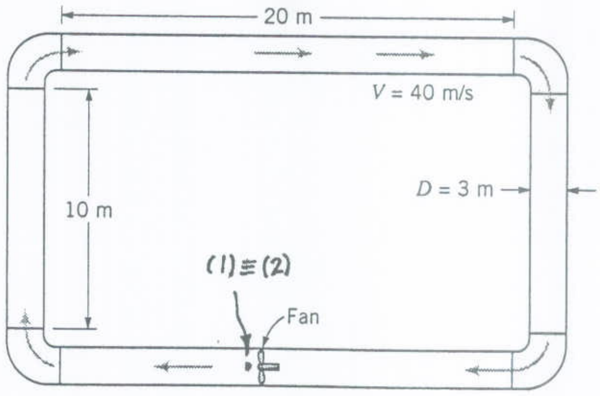


FIGURE P8.90

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}-h_L+h_s=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}$$

If we locate (1) and (2) at the same place it follows that

$p_1=p_2$, $V_1=V_2$, and $z_1=z_2$.

Thus,

$$h_s=h_L=\left(f\frac{\ell}{D}+\sum_i K_{L_i}\right)\frac{V^2}{2g} \quad \text{where } \sum_i K_{L_i}=4(0.30)=1.2$$

Also, $$Re=\frac{\rho VD}{\mu}=\frac{1.23\frac{kg}{m^3}(40\frac{m}{s})(3m)}{1.79\times10^{-5}\frac{N\cdot s}{m^2}}=8.25\times10^6$$

and $\frac{\varepsilon}{D}=0$ so that from Fig. 8.20, $f=0.0083$

Hence,

$$h_s=\left(0.0083\,\frac{(20+20+10+10)m}{3\,m}+1.2\right)\frac{(40\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}=111\,m$$

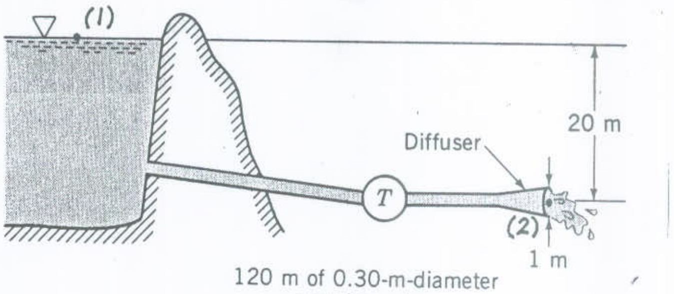
so that

$$\dot{W}_s=\gamma Q h_s=\rho g Q h_s=(1.23\tfrac{kg}{m^3})(9.81\tfrac{m}{s^2})\left[\tfrac{\pi}{4}(3m)^2(40\tfrac{m}{s})\right]111\,m$$

$$=3.79\times10^5\,\frac{N\cdot m}{s}=\underline{\underline{379\,kW}}$$

8.91

8.91 The turbine shown in Fig. P8.91 develops 400 kW. Determine the flowrate if **(a)** head losses are negligible or **(b)** head loss due to friction in the pipe is considered. Assume $f = 0.02$. *Note*: There may be more than one solution or there may be no solution to this problem.



120 m of 0.30-m-diameter cast iron pipe

FIGURE P8.91

$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} + h_T$, where $p_1 = p_2 = 0$, $z_1 = 20m$,

$z_2 = 0$ Thus, $z_1 = \frac{V_2^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g} + h_T$ (1)

a) Neglect head losses $(f=0)$:

$z_1 = \frac{V_2^2}{2g} + h_T$, where $h_T = \frac{P}{\gamma Q} = \frac{400 \times 10^3 \frac{N \cdot m}{s}}{(9.80 \times 10^3 \frac{N}{m^3})\frac{\pi}{4}(1m)^2 V_2} = \frac{52.0}{V_2}$ m

Thus,

$20m = \frac{V_2^2}{2(9.81\frac{m}{s^2})} + \frac{52.0}{V_2}$ or $V_2^3 - 392V_2 + 1020 = 0$ (2)

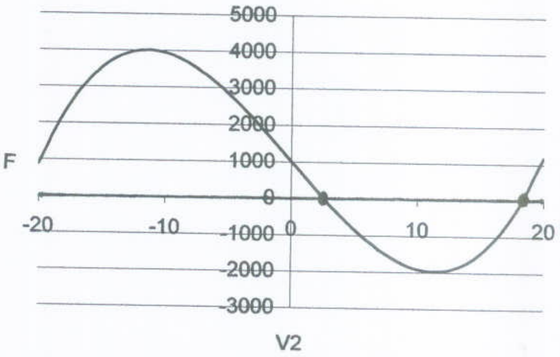
Determine the roots of this cubic equation. Let $V_2^3 - 392V_2 + 1020 \equiv F$

As indicated by the graph, there at two real, positive roots to $F=0$:

$V_2 = 2.65\frac{m}{s}$ or $V_2 = 18.3\frac{m}{s}$ Thus,

$Q = A_2 V_2 = \frac{\pi}{4}(1m)^2 V_2$, or

$Q = 2.08\frac{m^3}{s}$ or $Q = 14.4\frac{m^3}{s}$



The negative root $(V_2 < 0)$ has no physical meaning.

b) Include head loss $(f=0.02)$: From Eq. (1) $V = \frac{V_2 A_2}{A} = V_2\left(\frac{D_2}{D}\right)^2 = V_2\left(\frac{1m}{0.3m}\right)^2$

$= 11.1V_2$

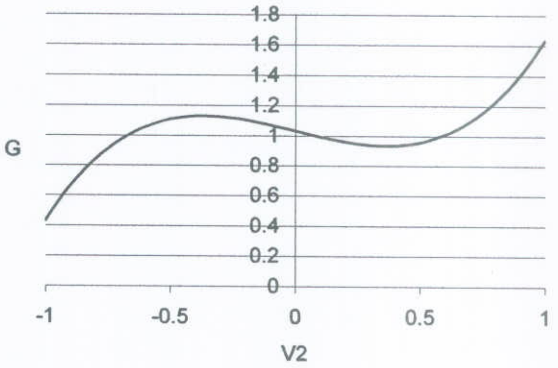
or

$20m = \left(1 + 0.02\left(\frac{120m}{0.3m}\right)(11.1)^2\right)\frac{V_2^2}{2(9.81\frac{m}{s^2})} + \frac{52.0}{V_2}$ m

Thus,

$V_2^3 - 0.398V_2 + 1.034 = 0$ Let $G \equiv V_2^3 - 0.398V_2 + 1.034$; determine V_2 that gives $G=0$.

As indicated by the graph, there is no positive real root. Hence, the flow cannot occur with $\dot{W}_s = 400kW$.



8.92

*8.92 In some locations with very "hard" water, a scale can build up on the walls of pipes to such an extent that not only does the roughness increases with time, but the diameter significantly decreases with time. Consider a case for which the roughness and diameter vary as $\varepsilon = 0.02 + 0.01t$ mm, $D = 50\ (1 - 0.02t)$ mm, where t is in years. Plot the flowrate as a function of time for $t = 0$ to $t = 10$ years if the pressure drop per 12 m of horizontal pipe remains constant at $\Delta p = 1.3$ kPa.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } z_1 = z_2\ ,\ V_1 = V_2 = V, \text{ and}$$
$$\Delta p = p_1 - p_2 = 1.3\ kPa$$

Thus,

$$\Delta p = f\frac{\ell}{D}\frac{V^2}{2g}\gamma, \text{ or } 1.3\times10^3\frac{N}{m^2} = f\left(\frac{12m}{0.05(1-0.02t)}\right)\frac{V^2}{2(9.81\frac{m}{s^2})}(9.80\times10^3\frac{N}{m^3})$$

or

$$fV^2 = 0.0108(1-0.02t), \text{ where } t\sim yr,\ V\sim\frac{m}{s} \qquad (1)$$

Also, $$Re = \frac{VD}{\nu} = \frac{V[0.05(1-0.02t)]}{1.12\times10^{-6}\frac{m^2}{s}}, \text{ or } Re = 4.46\times10^4(1-0.02t)V \qquad (2)$$

and

$$\frac{\varepsilon}{D} = \frac{(0.02+0.01t)}{50(1-0.02t)} \qquad (3)$$

Finally, from the alternate formula, Eq. 8.35b,

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right] \qquad (4)$$

For $0 \le t \le 10$ yr , obtain $\frac{\varepsilon}{D}$ from Eq.(3) and solve Eqs. (1), (2), and (4) for f, V, and Re. Then $Q = VA = V\frac{\pi}{4}(0.05(1-0.02t))^2$

or

$$Q = 1.96\times10^{-3}(1-0.02t)^2V \text{ where } Q\sim\frac{m^3}{s},\ V\sim\frac{m}{s},\ t\sim yr \qquad (5)$$

Eqns (1)-(5) were used in a MS Excel spreadsheet to manually iterate on the solution.

$$f_{guess} \xrightarrow{(1)} V \xrightarrow{(2)} Re \xrightarrow{(4)} f_{new} \overset{?}{=} f_{guess}$$

No

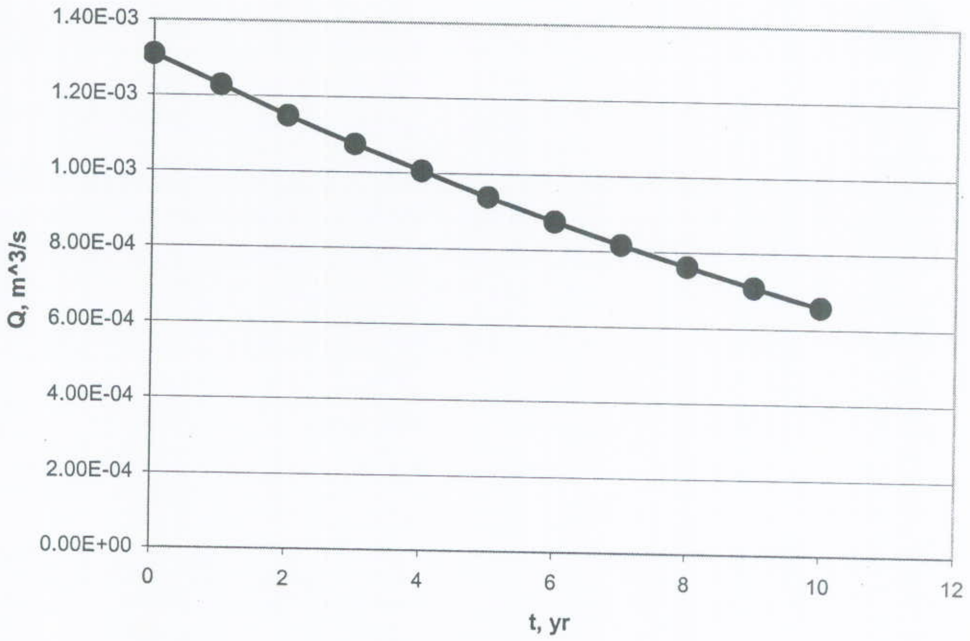
The spreadsheet results are shown below along with a plot of the data.

(cont)

8.92 (cont)

Insert a guess value (e.g. f = 0.02) in the f(guess) cell. A new f value will be calculated in the f(new) cell. Use this new value as the updated f(guess). Continue until f(guess) = f(new).

t, yr	e/D	f(guess)	V, m/s	Re	f(new)	Q, m^3/s
0	4.00E-04	0.0243	6.67E-01	29758	0.0243	1.31E-03
1	6.12E-04	0.0249	6.52E-01	28496	0.0250	1.23E-03
2	8.33E-04	0.0257	6.35E-01	27195	0.0257	1.15E-03
3	1.06E-03	0.0264	6.20E-01	25998	0.0264	1.07E-03
4	1.30E-03	0.0271	6.06E-01	24845	0.0271	1.00E-03
5	1.56E-03	0.0279	5.90E-01	23692	0.0279	9.37E-04
6	1.82E-03	0.0286	5.76E-01	22625	0.0286	8.75E-04
7	2.09E-03	0.0293	5.63E-01	21595	0.0293	8.16E-04
8	2.38E-03	0.0300	5.50E-01	20602	0.0300	7.61E-04
9	2.68E-03	0.0307	5.37E-01	19643	0.0307	7.08E-04
10	3.00E-03	0.0315	5.24E-01	18686	0.0315	6.57E-04



8.93

8.93 Water flows from the nozzle attached to the spray tank shown in Fig. P8.93. Determine the flowrate if the loss coefficient for the nozzle (based on upstream conditions) is 0.75 and the friction factor for the rough hose is 0.11.

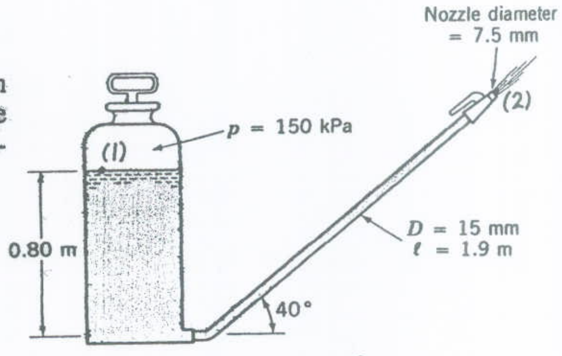


FIGURE P8.93

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+K_L\right)\frac{V^2}{2g}, \text{ where } p_1=150\,kPa,\ p_2=0, \quad (1)$$

$z_1=0.8\,m$, $z_2=\ell \sin 40^\circ=(1.9\,m)\sin 40^\circ=1.22\,m$, $V_1=0$,

$V=\frac{Q}{A}$, and $V_2=\frac{Q}{A_2}=\left(\frac{A}{A_2}\right)V=\left(\frac{D}{D_2}\right)^2 V=\left(\frac{15\,mm}{7.5\,mm}\right)^2 V=4V$

Thus, with $f=0.11$ and $K_L=0.75$ Eq.(1) gives

$$\frac{150\times10^3\frac{N}{m^2}}{9.80\times10^3\frac{N}{m^3}}+0.8\,m=1.22\,m+\left(4^2+0.11\left(\frac{1.9\,m}{0.015\,m}\right)+0.75\right)\frac{V^2}{2(9.81\frac{m}{s^2})}$$

or

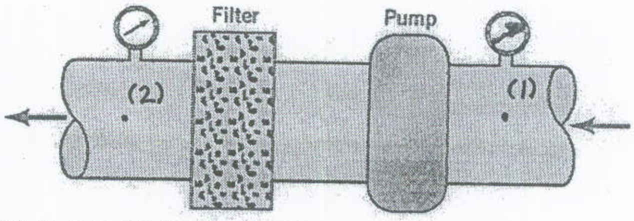
$V=3.09\frac{m}{s}$

Thus, $Q=AV=\frac{\pi}{4}(0.015\,m)^2(3.09\frac{m}{s})=\underline{\underline{5.46\times10^{-4}\frac{m^3}{s}}}$

8.94

8.94 When the pump shown in Fig. P8.94 adds 0.2 horsepower to the flowing water, the pressures indicated by the two gages are equal. Determine the flowrate.

Length of pipe between gages = 60 ft
Pipe diameter = 0.1 ft
Pipe friction factor = 0.03
Filter loss coefficient = 12



■ FIGURE P8.94

$$\cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} + h_p = \cancel{\frac{p_2}{\gamma}} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2} + h_L$$

$$p_1 = p_2\,,\quad z_1 = z_2\,,\quad V_1 = V_2$$

So, $h_p = h_L$ (1)

The pump adds 0.2 hp of power.

$$\dot{W} = 0.2\ hp \times \frac{550\ \frac{ft \cdot lb}{s}}{1\ hp} = 110\ \frac{ft \cdot lb}{s}$$

Convert to head by:

$$h_p = \frac{\dot{W}}{\gamma Q} = \frac{110\ \frac{ft\,lb}{s}}{62.4\frac{lb}{ft^3}\ Q} = \frac{1.76}{Q}$$

Sub into (1)

$$\frac{1.76}{Q} = \left(f\frac{l}{D} + \Sigma K_L\right)\frac{V^2}{2g} = \left(f\frac{l}{D} + \Sigma K_L\right)\frac{(Q/A)^2}{2g}$$

or $Q^3 = \dfrac{1.76(2)(g)A^2}{\left(f\frac{l}{D} + \Sigma K_L\right)}$ where $A = \frac{\pi}{4}(0.1\,ft)^2 = 7.85 \times 10^{-3} ft^2$

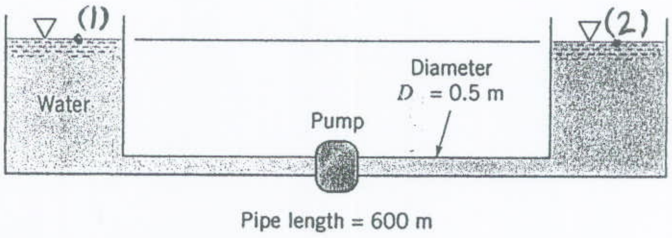
$$= \frac{1.76(64.4)(7.85 \times 10^{-3})^2}{\left(0.03\,\frac{60}{0.1} + 12\right)}$$

$$Q^3 = 2.328 \times 10^{-4}$$

$$\underline{\underline{Q = 0.0615\ ft^3/s}}$$

8.95

8.95 Water is pumped between two large open tanks as shown in Fig. P8.95. If the pump adds 50 kW of power to the fluid, what is the flowrate passing between the tanks? Assume the friction factor to be equal to 0.02 and minor losses to be negligible.



■ FIGURE P8.95

With $P_1 = P_2 = 0$, $V_1 = V_2 = 0$, and $z_1 = z_2$

$$h_p = h_L = f \frac{\ell}{D} \frac{V^2}{2g} \qquad (1)$$

With the pump adding 50 kW of power

$$\dot{W} = 50 \times 10^3 \, W = h_p Q \gamma$$

$$h_p = \frac{50 \times 10^3}{Q(9.8 \times 10^3)} = \frac{5.10}{Q}$$

Sub into (1)

$$\frac{5.10}{Q} = f \frac{\ell}{D} \frac{V^2}{2g} = f \frac{\ell}{D} \frac{Q^2}{2gA^2}$$

$$Q^3 = \frac{5.10(2)(D g A^2)}{f \ell} \quad \text{where } A = \frac{\pi}{4}(0.5)^2 = 0.196 \, m^2$$

Thus,

$$Q^3 = \frac{5.10(2)(0.5)(9.81)(0.196)^2}{(0.02)(600)}$$

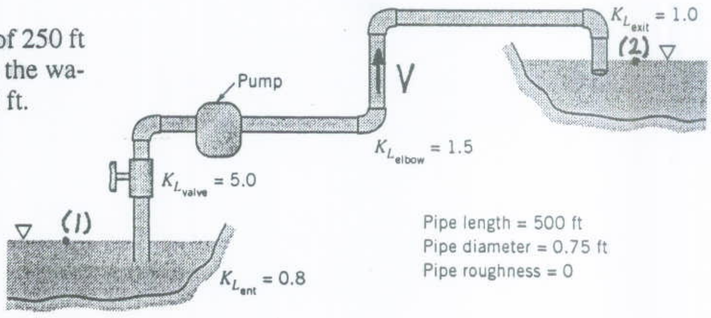
$$= 0.1602$$

or

$$\underline{\underline{Q = 0.543 \, m^3/s}}$$

8.97

8.97 The pump shown in Fig. P8.97 delivers a head of 250 ft to the water. Determine the power that the pump adds to the water. The difference in elevation of the two ponds is 200 ft.



■ FIGURE P8.97

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 = 0$, $z_2 = 200\ ft$, $h_p = 250\ ft$

Thus,

$$-f\frac{\ell}{D}\frac{V^2}{2g} - \sum_i K_{L_i}\frac{V^2}{2g} + h_p = z_2 \quad \text{so that with } \sum_i K_{L_i}\frac{V^2}{2g} = (0.8 + 4(1.5) + 5.0 + 1)\frac{V^2}{2g}$$

$$= 12.8\,\frac{V^2}{2g}$$

$$\left[-f\left(\frac{500}{0.75}\right) - 12.8\right]\frac{V^2}{2(32.2)} + 250 = 200$$

or

(1) $(667f + 12.8)V^2 = 3220$

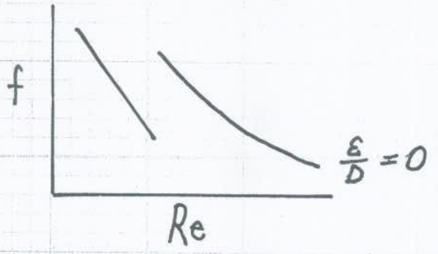
Also, $Re = \frac{\rho V D}{\mu} = \frac{(1.94\ \frac{slugs}{ft^3})\,V\,(0.75\ ft)}{2.34\times10^{-5}\ \frac{lb\cdot s}{ft^2}}$

or

(2) $Re = 6.22\times10^4\,V$

and from Fig. 8.20:

(3)



Trial and error solution. Assume $f = 0.02 \xrightarrow{(1)} V = 11.1\ \frac{ft}{s} \xrightarrow{(2)} Re = 6.9\times10^5$

$\xrightarrow{(3)} f = 0.012 \neq 0.02$

Assume $f = 0.012 \xrightarrow{(1)} V = 12.4\ \frac{ft}{s} \xrightarrow{(2)} Re = 7.7\times10^5 \xrightarrow{(3)} f = 0.0121 \approx 0.012$

Thus, $V = 12.4\ \frac{ft}{s}$ and

$$\dot{W}_s = \gamma Q h_p = \left(62.4\ \frac{lb}{ft^3}\right)\frac{\pi}{4}(0.75\ ft)^2\left(12.4\ \frac{ft}{s}\right)(250\ ft) = 8.55\times10^4\ \frac{ft\cdot lb}{s}$$

$$= 8.55\times10^4\ \frac{ft\cdot lb}{s} \times 1\,\frac{hp}{550\ \frac{ft\cdot lb}{s}} = \underline{\underline{155\ hp}}$$

Alternatively, we could replace Eq. (3) (the Moody chart) by Eq 8.35

(con't)

8.97 (con't)

(the Colebrook equation) and obtain V as follows.

From Eq. (1),

$V = \left[3220/(667f + 12.8) \right]^{1/2}$, which when combined with Eq. (2) gives

(4) $Re = 6.22 \times 10^4 \left[3220/(667f+12.8) \right]^{\frac{1}{2}} = 3.53 \times 10^6 / (667f + 12.8)^{\frac{1}{2}}$

Also, the Colebrook equation with $\varepsilon/D = 0$ is

(5) $\frac{1}{\sqrt{f}} = -2.0 \log \left(\frac{2.51}{Re\sqrt{f}} \right)$

By combining Eqs (4) and (5) we obtain a single equation involving only f:

$$\frac{1}{\sqrt{f}} = -2.0 \log \left[\frac{2.51(667f + 12.8)^{1/2}}{3.53 \times 10^6 \sqrt{f}} \right]$$

Using a compute root-finding program to solve Eq (6) gives $f = 0.0123$, consistent with the above trial and error method.

8.98

8.98 Water flows through two sections of the vertical pipe shown in Fig. P8.98. The bellows connection cannot support any force in the vertical direction. The 0.4-ft-diameter pipe weighs 0.2 lb/ft and the friction factor is assumed to be 0.02. At what velocity will the force, F, required to hold the pipe be zero?

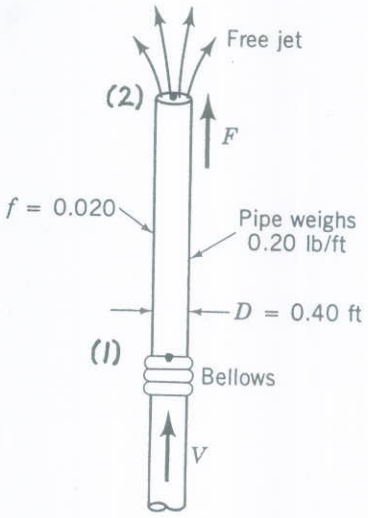
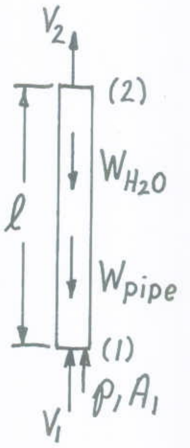


FIGURE P8.98

From the momentum equation applied to the control volume indicated



$$p_1A_1 - W_{H_2O} - W_{pipe} = \dot{m}(V_2 - V_1) = 0 \text{ since } V_1 = V_2$$

Thus, $p_1 = \dfrac{W_{H_2O} + W_{pipe}}{A_1} = \dfrac{\gamma \ell A_1 + \ell\left(\frac{W_{pipe}}{\ell}\right)}{A_1}$

or

$$p_1 = \gamma\ell + \frac{(0.20\frac{lb}{ft})\ell}{\frac{\pi}{4}(0.4ft)^2} = \gamma\ell + 1.59\ell \text{, where } p_1 \sim \frac{lb}{ft^2},\ \ell \sim ft$$

Also,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} \text{, where } p_2 = 0,$$

$V_1 = V_2 = V$, $z_1 = 0$, and $z_2 = \ell$

Thus, $p_1 = \gamma z_2 + f\frac{\ell}{D}\frac{1}{2}\rho V^2$,

or when combined with the above force balance result

$$p_1 = \gamma\ell + f\frac{\ell}{D}\frac{1}{2}\rho V^2 = \gamma\ell + 1.59\ell$$

That is, $\dfrac{f\rho V^2}{2D} = 1.59$ or $V = \sqrt{\dfrac{2D(1.59)}{\rho f}} = \sqrt{\dfrac{2(0.4)(1.59)}{(1.94)(0.02)}} = \underline{\underline{5.73\frac{ft}{s}}}$

Note: This answer is independent of the pipe length, ℓ.

8.99

8.99 Water is circulated from a large tank, through a filter, and back to the tank as shown in Fig. P8.99. The power added to the water by the pump is 200 ft · lb/s. Determine the flowrate through the filter.

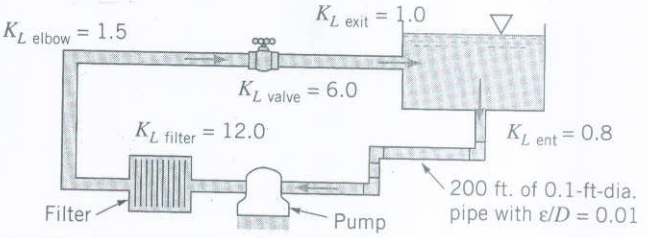


FIGURE P8.99

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + \left(f\frac{\ell}{D} + \sum_i K_{L_i}\right)\frac{V^2}{2g} \qquad (1)$$

where

$p_1 = p_2$, $V_1 = V_2 = 0$, and $z_1 = z_2$

Also, $\dot{W}_p = \gamma Q h_p$ or

$$h_p = \frac{200\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}}{62.4\frac{\text{lb}}{\text{ft}^3}\left(\frac{\pi}{4}(0.1\text{ft})^2\right)V} = \frac{408}{V}$$

Thus, Eq. (1) becomes

$$\frac{408}{V} = \left(\frac{200\text{ft}}{0.1\text{ft}}f + (0.8 + 5(1.5) + 12 + 6 + 1)\right)\frac{V^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$$

or

$$V^3 = \frac{13.13}{(f + 0.01365)} \qquad (2)$$

Also,

$$Re = \frac{\rho V D}{\mu} = \frac{1.94\frac{\text{slugs}}{\text{ft}^3}(V\frac{\text{ft}}{\text{s}})(0.1\text{ft})}{2.34\times10^{-5}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}} \text{ or } Re = 8290V \qquad (3)$$

Trial and error solution:

Assume $f = 0.04$. From Eq. (2), $V = 6.26\frac{\text{ft}}{\text{s}}$; from Eq. (3), $Re = 5.20\times10^4$. Thus, from Fig. 8.20, $f = 0.039 \neq 0.04$

Assume $f = 0.039$, or $V = 6.29\frac{\text{ft}}{\text{s}}$ and $Re = 5.21\times10^4$ and $f = 0.039$ (Checks)

Thus, $Q = AV = \frac{\pi}{4}(0.1\text{ft})^2(6.29\frac{\text{ft}}{\text{s}}) = \underline{\underline{0.0494\frac{\text{ft}^3}{\text{s}}}}$

Alternatively, the Colebrook equation (Eq. 8.35) could be used rather than the Moody chart. Thus,

(con't)

8.99 (con't)

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right), \text{ where from Eq.(2),} \qquad (4)$$

$$f = (13.13/V^3) - 0.01365 \qquad (5)$$

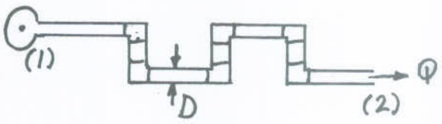
Thus, by combining Eqs. (3), (4), and (5) we obtain the following equation for V:

$$1/[(13.13/V^3) - 0.01365]^{\frac{1}{2}} = -2.0 \log\{[\frac{0.01}{3.7} + 2.51/[(8290V)((13.13/V^3) - 0.01365)^{\frac{1}{2}}]\} \qquad (6)$$

Using a computer root-finding program gives the solution to Eq.(6) as $V = 6.29 \frac{ft}{s}$, the same as obtained by the above trial and error method.

8.100

8.100 A certain process requires 2.3 cfs of water to be delivered at a pressure of 30 psi. This water comes from a large-diameter supply main in which the pressure remains at 60 psi. If the galvanized iron pipe connecting the two locations is 200 ft long and contains six threaded 90° elbows, determine the pipe diameter. Elevation differences are negligible.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}$$, where $p_2=30\,psi$, $p_1=60\,psi$,

$z_1=z_2$, $V_1=0$, $V_2=V=\frac{Q}{A}=\frac{2.3\,\frac{ft^3}{s}}{\frac{\pi}{4}D^2}=\frac{2.93}{D^2}\,\frac{ft}{s}$, with $D\sim ft$

Thus,

$$p_1-p_2=\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{1}{2}\rho V^2$$

or

$$(60-30)\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right)=\left(1+f\left(\frac{200ft}{D}\right)+6(1.5)+0.5\right)\left(\frac{2.93}{D^2}\frac{ft}{s}\right)^2\left(\frac{1}{2}\right)\left(1.94\frac{slugs}{ft^3}\right)$$

where we have used

$$\Sigma K_L=6K_{L_{elbow}}+K_{L_{entrance}}=6(1.5)+0.5$$

Thus,

$$49.4=\left(1+\frac{19.0f}{D}\right)\frac{1}{D^4} \qquad (1)$$

Also,

$$Re=\frac{VD}{\nu}=\frac{\left(\frac{2.93}{D^2}\right)D}{\nu}=\frac{2.93\,\frac{ft^2}{s}}{1.21\times10^{-5}\frac{ft^2}{s}D}\ \text{ or }\ Re=2.42\times10^5\,\frac{1}{D} \qquad (2)$$

and from Table 8.1

$$\frac{\varepsilon}{D}=\frac{0.0005\,ft}{D} \qquad (3)$$

Finally, from Fig. 8.20:

(4)

Trial and error solution of Eqs. (1), (2), (3), and (4) for f, D, $\frac{\varepsilon}{D}$, and Re.

Normally it is easiest to guess a value of f, calculate D, etc. In this case (because of minor losse), Eq.(1) is not easy to use in this fashion. Thus, assume D, calculate f (Eq.(1)), Re (Eq.(2)), and $\frac{\varepsilon}{D}$ (Eq.(3)). Look up f in Fig. 8.20 (Eq.(4)) and compare with that from Eq.(1).

Assume $D=0.4\,ft$. Thus, $f=0.00557$, $Re=6.05\times10^5$, $\frac{\varepsilon}{D}=0.00125$
or from Fig. 8.20 $f=0.021\neq0.00557$

Assume $D=0.5\,ft$; $f=0.0551$, $Re=4.84\times10^5$, $\frac{\varepsilon}{D}=0.001$ or $f=0.0203\neq0.0551$

Assume $D=0.45\,ft$; $f=0.0243$, $Re=5.38\times10^5$, $\frac{\varepsilon}{D}=0.00111$ or $f=0.0205\neq0.0243$

Assume $D=0.44\,ft$; $f=0.0197$, $Re=5.50\times10^5$, $\frac{\varepsilon}{D}=0.00114$ or $f=0.0205\neq0.0197$

After enough trials obtain $D=\underline{\underline{0.442\,ft}}$

Note: If Fig. 8.20 (Eq.(4)) is replaced by the Colebrook equation,

(con't)

8.100 (con't)

this problem can be solved as follows.

Thus, from Eq. (1),

$f = (49.4D^5 - D)/19$ so that with the Colebrook equation (Eq. 8.35), when combined with Eqs. (2) and (3), gives

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

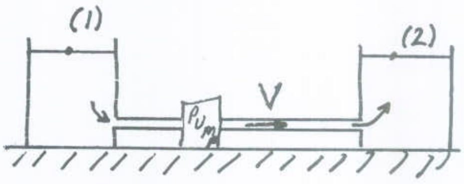
or

$$\left[\frac{19}{(49.4D^5 - D)}\right]^{1/2} = -2.0 \log\left[\frac{0.0005}{3.7\,D} + \frac{2.51\,D\,\sqrt{19}}{2.42 \times 10^5\,(49.4\,D^5 - D)^{1/2}}\right] \qquad (5)$$

Using a computer root-finding routine gives the solution to Eq. (5) as $D = 0.442$ ft which is the same as that obtained by the trial and error method above.

8.101

8.101 Water is pumped between two large open reservoirs through 1.5 km of smooth pipe. The water surfaces in the two reservoirs are at the same elevation. When the pump adds 20 kW to the water the flowrate is 1 m³/s. If minor losses are negligible, determine the pipe diameter.



$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}, \text{ where } p_1 = p_2 = 0,\ V_1 = V_2 = 0,\ z_1 = z_2$$

Thus,

(1) $$h_s = h_L \text{ where } h_s = \frac{\dot{W}_s}{\gamma Q} = \frac{20 \times 10^3 \, N \cdot m/s}{(9.80 \times 10^3 \frac{N}{m^3})(1 \frac{m^3}{s})} = 2.04 m$$

and

$$h_L = f \frac{\ell}{D} \frac{V^2}{2g} \text{ with } V = \frac{Q}{A} = \frac{1 m^3/s}{\frac{\pi}{4} D^2} = \frac{1.273}{D^2} \text{ m/s with } D \sim m$$

Hence,

(2) $$h_L = f \frac{1.5 \times 10^3 m}{D} \frac{(1.23/D^2)^2 m^2/s^2}{2(9.81 m/s^2)} = 123.9 f/D^5 \text{ m}$$

From Eqs (1) and (2), $2.04 = 123.9 f/D^5$ or $f = 0.0165 D^5$

(3) $D = 2.27 f^{1/5}$

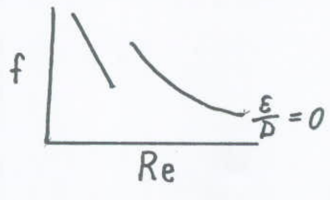
Also,

$$Re = \frac{\rho V D}{\mu} = \frac{999 \frac{kg}{m^3} (1.273/D^2) m \, D m}{1.12 \times 10^{-3} \frac{N \cdot s}{m^2}} \text{ or}$$

(4) $Re = 1.14 \times 10^6 / D$

Finally, with $\varepsilon/D = 0$ the Moody chart (Fig. 8.20) is the final equation.

(5)



Trial and error solution of Eqs. (3), (4), and (5) for f, Re, and D:
Assume $f = 0.02$ so Eq (3) gives $D = 2.27 (0.02)^{1/5} = 1.04 m$ and Eq (4) gives $Re = 1.14 \times 10^6 / 1.04 = 1.10 \times 10^6$. Thus, from Eq (5), $f = 0.0115$ which is not equal to the assumed $f = 0.02$. Try again with $f = 0.0115$ which gives $D = 0.931 m$, $Re = 1.22 \times 10^4$, and $f = 0.0113 \neq 0.0115$. One final try with $f = 0.0113$ gives $D = 0.927 m$, $Re = 1.23 \times 10^4$, and $f = 0.0113$ as assumed. Thus, $\underline{\underline{D = 0.927 m.}}$

An alternate method is to use the Colebrook formula (Eq (8.35)) rather than the Moody chart (Eq (5)). Thus, with $\varepsilon/D = 0$,

(con't)

8.101 (con't)

Eq(8.35) is

$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{2.51}{Re\sqrt{f}}\right)$ which, when combined with Eqs. (3) and (4), gives

$$(6) \quad \frac{1}{(0.0165 D^5)^{1/2}} = -2.0 \log\left[\frac{2.51\, D}{1.14\times10^6 (0.0165 D^5)^{1/2}}\right]$$

Using a computer root-finding program to solve Eq. (6) gives D = 0.926, which is consistent with the trial and error solution given above.

8.102

8.102 Determine the diameter of a steel pipe that is to carry 2,000 gal/min of gasoline with a pressure drop of 5 psi per 100 ft of horizontal pipe.

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } z_1=z_2 \text{ and } V_1=V_2$$

Thus,

$$p_1-p_2=f\frac{\ell}{D}\frac{1}{2}\rho V^2 \text{ with } p_1-p_2=5\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right),\ \ell=100\,ft, \qquad (1)$$

$$\rho=1.32\frac{slugs}{ft^3},\ \mu=6.5\times10^{-6}\frac{lb\cdot s}{ft^2}, \text{ and}$$

$$V=\frac{Q}{A}=\frac{\left(2000\frac{gal}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(231\frac{in.^3}{gal}\right)\left(\frac{1}{1728}\frac{ft^3}{in.^3}\right)}{\frac{\pi}{4}D^2}, \text{ or } V=\frac{5.67}{D^2}\frac{ft}{s} \text{ with } D\sim ft$$

Hence, Eq. (1) gives:

$$5(144)\frac{lb}{ft^2}=f\left(\frac{100\,ft}{D\,ft}\right)\frac{1}{2}\left(1.32\frac{slugs}{ft^3}\right)\left(\frac{5.67}{D^2}\frac{ft}{s}\right)^2$$

or

$$D=1.24\,f^{1/5} \qquad (2)$$

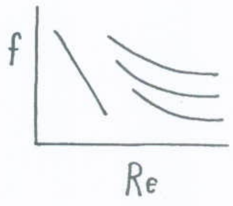
Also,

$$Re=\frac{\rho VD}{\mu}=\frac{\left(1.32\frac{slugs}{ft^3}\right)\left(\frac{5.67}{D^2}\frac{ft}{s}\right)D\,ft}{6.5\times10^{-6}\frac{lb\cdot s}{ft^2}}, \text{ or } Re=\frac{1.15\times10^6}{D} \qquad (3)$$

and

$$\frac{\varepsilon}{D}=\frac{0.00015}{D}, \text{ where } D\sim ft \qquad (4)$$

Finally, the fourth equation is the Moody chart (or the Colebrook equation) (5)



Note: 4 equations ((2), (3), (4), and (5)) and
4 unknowns (f, $\frac{\varepsilon}{D}$, D, Re)

Trial and error solution:

$$\text{Guess } f=0.02 \xrightarrow{(2)} D=0.567\,ft \quad \begin{array}{l}\xrightarrow{(3)} Re=2.03\times10^6 \\ \xrightarrow{(4)} \frac{\varepsilon}{D}=0.000265\end{array}\Bigg\}\overset{(5)}{} f=0.0148\neq0.02$$

Thus, the guessed value is not correct.

$$\text{Guess } f=0.0148 \xrightarrow{(2)} D=0.534\,ft \quad \begin{array}{l}\xrightarrow{(3)} Re=2.15\times10^6 \\ \xrightarrow{(4)} \frac{\varepsilon}{D}=0.000281\end{array}\Bigg\}\overset{(5)}{} f=0.0150\approx0.0148$$

Thus, $D=1.24(0.0150)^{1/5}=\underline{\underline{0.535\,ft}}$

By using the Colebrook equation, Eq. 8.35, rather than the Moody chart, Eq. (5), we have

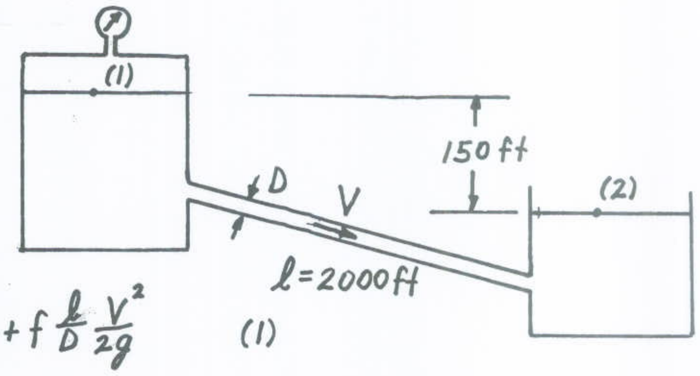
$$\frac{1}{\sqrt{f}}=-2.0\log\left[\frac{\varepsilon/D}{3.7}+\frac{2.51}{Re\sqrt{f}}\right]$$ which, using Eqs (2), (3), and (4) is,

$$\frac{1}{(D/1.24)^{5/2}}=-2.0\log\left[\frac{0.00015}{3.7D}+\frac{2.51\,D}{1.15\times10^6(D/1.24)^{5/2}}\right]$$

Using a computer root-finding program to solve Eq. (6) gives $D=0.536\,ft$ which is consistent with the above trial and error solution.

8.103

8.103 Water is to be moved from a large, closed tank in which the air pressure is 20 psi into a large, open tank through 2000 ft of smooth pipe at the rate of 3 ft^3/s. The fluid level in the open tank is 150 ft below that in the closed tank. Determine the required diameter of the pipe. Neglect minor losses.



$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}+f\frac{\ell}{D}\frac{V^2}{2g} \qquad (1)$$

where

$V_1 = V_2 = 0$, $z_1 - z_2 = 150\ ft$, and $p_1 = 20\ psi$, $p_2 = 0$

Also,

$$V=\frac{Q}{A}=\frac{3\frac{ft^3}{s}}{\frac{\pi}{4}D^2}=\frac{3.82}{D^2}, \text{ where } V\sim\frac{ft}{s},\ D\sim ft$$

Thus, Eq. (1) becomes

$$\frac{(20\frac{lb}{in.^2})(144\frac{in.^2}{ft^2})}{62.4\frac{lb}{ft^3}}+150ft=f\,\frac{2000ft}{D}\,\frac{\left(\frac{3.82}{D^2}\right)^2}{2(32.2\frac{ft}{s^2})}$$

or

$$D = 1.18 f^{1/5} \qquad (2)$$

Also,

$$Re=\frac{\rho VD}{\mu}=\frac{\rho\left(\frac{3.82}{D^2}\right)D}{\mu}=\frac{1.94\ (3.82)}{2.34\times10^{-5}\ D}, \text{ or } Re=\frac{3.17\times10^5}{D} \qquad (3)$$

Trial and error solution:

Assume $f = 0.02$ so from Eq. (2), $D = 0.540\ ft$ and from Eq. (3) $Re = 5.87\times10^5$. Thus, from Fig. 8.20 (with $\frac{\varepsilon}{D}=0$) $f = 0.013 \neq 0.02$

Assume $f = 0.013$ which gives $D = 0.495\ ft$, $Re = 6.40\ 10^5$, and $f = 0.0125$

Assume $f = 0.0125$, so $D = 0.491 ft$, $Re = 6.46\times10^5$, $f = 0.0125$ (Checks)

Thus, $D = \underline{\underline{0.491\ ft}}$

Alternately, the Colebrook equation, Eq. 8.35, rather than the Moody chart, Fig. 8.20, could be used as follows:

(con't)

8.103 (con't)

With $\varepsilon/D = 0$, Eq. 8.35 is

$\frac{1}{\sqrt{f}} = -2.0 \log(2.51/(Re\sqrt{f}))$ where (4)

from Eq. (2), $f = (D/1.18)^5$ (5)

Thus, combining Eqs. (3), (4), and (5) gives

$1/(D/1.18)^{5/2} = -2.0 \log[2.51/((3.17\times10^5/D)(D/1.18)^{5/2})]$ (6)

Using a computer root-finding technique gives the solution to Eq. (6) as $D = 0.492\,ft$, which is consistent with the above trial and error solution.

8.104

8.104 Rainwater flows through the galvanized iron downspout shown in Fig. P8.104 at a rate of 0.006 m³/s. Determine the size of the downspout cross section if it is a rectangle with an aspect ratio of 1.7 to 1 and it is completely filled with water. Neglect the velocity of the water in the gutter at the free surface and the head loss associated with the elbow.

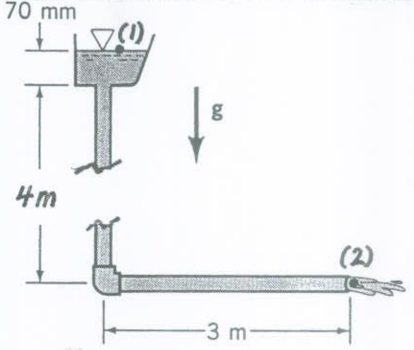
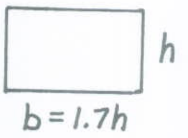


FIGURE P8.104

$$\frac{\cancel{p_1}}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{\cancel{p_2}}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D_h}\frac{V^2}{2g}, \text{ where } p_1=p_2=0,\ V_1=0,\ V_2=V, \quad (1)$$

$z_1=4.07m$, and $z_2=0$

Also, $D_h=\frac{4A}{P}=\frac{4(1.7h^2)}{2(1.7h+h)}=1.26h$



and $V=\frac{Q}{A}=\frac{0.006\frac{m^3}{s}}{1.7h^2}=0.00353h^{-2}\ \frac{m}{s}$, where $h\sim m$

Thus, from Eq.(1)

$$4.07m=\left(1+f\left(\frac{7m}{1.26h\,m}\right)\right)\left(\frac{3.53\times10^{-3}}{h^2}\right)^2\frac{m^2}{s^2}\left(\frac{1}{2(9.81\frac{m}{s^2})}\right)$$

or

$$6.41\times10^6h^4=1+5.55\frac{f}{h} \quad (2)$$

From Table 8.1 $\frac{\varepsilon}{D_h}=\frac{0.15\times10^{-3}m}{1.26h\ m}=\frac{1.19\times10^{-4}}{h}$, where $h\sim m$ (3)

and $Re_h=\frac{VD_h}{\nu}=\frac{(0.00353h^{-2}\frac{m}{s})(1.26h\,m)}{1.12\times10^{-6}\frac{m^2}{s}}$ or $Re_h=\frac{3970}{h}$ (4)

Finally, from Fig. 8.20:

(5)

Trial and error solution of Eqs. (2),(3),(4), and (5) for $f, h, Re_h, \frac{\varepsilon}{D_h}$.

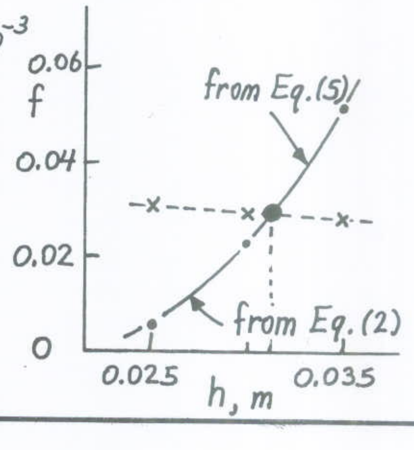
Assume $h=0.04m$; from (2) $f=0.111$, from (3) $\frac{\varepsilon}{D_h}=1.07\times10^{-3}$, and from (4) $Re_h=9.93\times10^4$. Hence, from (4) $f=0.0223\neq0.111$

Assume $h=0.03m$; from (2) $f=0.0227$, $\frac{\varepsilon}{D_h}=4.0\times10^{-3}$ and $Re_h=1.32\times10^5$ Hence, from (5) $f=0.0290\neq0.0227$

Assume $h=0.025m$; or $f=0.00677$, $\frac{\varepsilon}{D_h}=4.76\times10^{-3}$ and $Re_h=1.59\times10^5$ Hence, from (5) $f=0.0303\neq0.00677$

Assume $h=0.035m$; or $f=0.0544$; $\frac{\varepsilon}{D_h}=3.40\times10^{-3}$; $Re_h=1.13\times10^5$. Hence from (5) $f=0.0280$

Plot f from Eq.(2) and f from Eq.(5) as a function of h. Solution is where the two curves intersect.



Thus $h=0.031m$ and $b=1.7(0.031m)$ or <u>0.031 m by 0.053 m</u>

(con't)

8.104 (con't)

This problem can be solved using the Colebrook equation, Eq. 8.35, rather than the Moody chart, Fig. 8.20, as follows:

From Eq. 8.35,

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{Re_h\sqrt{f}}\right) \qquad (6)$$

where, from Eq. (2),

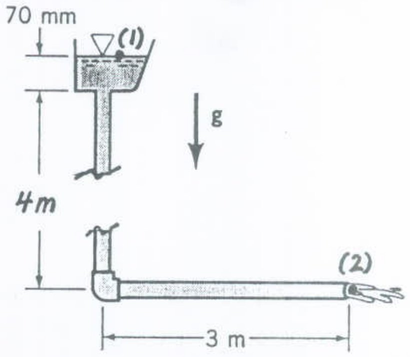
$$f = (6.41\times10^{6} h^{5} - h)/5.55 \qquad (7)$$

Combining Eqs. (3), (4), (6), and (7) gives a single equation for h :

$$\frac{1}{[(6.41\times10^{6}h^{5}-h)/5.55]} = -2.0\log\left\{\frac{1.19\times10^{-4}}{3.7h} + \frac{2.51h}{3970[(6.41\times10^{6}h^{5}-h)/5.55]}\right\} \qquad (8)$$

Using a computer root-finding program gives $h = 0.0313$ m, which is the same as that obtained by the above trial and error method.

*8.105



*8.105 Repeat Problem 8.104 if the downspout is circular.

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+f\frac{\ell}{D}\frac{V^2}{2g}\text{, where } p_1=p_2=0,\ V_1=0,\ V_2=V,$$

$z_1 = 4.07\ m$, and $z_2=0$ Thus, $z_1=(1+f\frac{\ell}{D})\frac{V^2}{2g}$ or

$$(4.07\ m)(2)(9.81\tfrac{m}{s^2})=(1+f(\tfrac{7\ m}{D}))V^2 \qquad (1)$$

Hence, with $V=\frac{Q}{\frac{\pi}{4}D^2}$ or $V=\frac{4(0.006\frac{m^3}{s})}{\pi D^2}=\frac{0.00764}{D^2}$, Eq. (1) becomes

$$79.9=(1+\frac{7f}{D})\left(\frac{0.00764}{D^2}\right)^2$$

or $f=1.956\times10^5D^5-0.1429D$, where $D\sim m$ (2)

Also, $Re=\frac{VD}{\nu}=\frac{(\frac{0.00764}{D^2})D}{\nu}=\frac{0.00764\frac{m}{s}}{(1.12\times10^{-6}\frac{m^2}{s})D}$

or $Re=\frac{6.82\times10^3}{D}$ (3)

From Table 8.1 $\frac{\varepsilon}{D}=\frac{0.15\times10^{-3}}{D}$ so that Eq. 8.35 becomes (4)

$$\frac{1}{\sqrt{f}}=-2.0\log\left[\frac{\frac{\varepsilon}{D}}{3.7}+\frac{2.51}{Re\sqrt{f}}\right]$$ or when combined with Eqs. (3) and (4)

$$\frac{1}{\sqrt{f}}=-2.0\log\left[\frac{4.05\times10^{-5}}{D}+\frac{3.68\times10^{-4}D}{\sqrt{f}}\right] \qquad (5)$$

Solve Eqs. (2) and (5) for f and D as follows: Substitute f from Eq. (2) into Eq. (5) to obtain a single equation for D:

$$\frac{1}{(1.956\times10^5D^5-0.1429D)^{1/2}}=-2.0\log\left[\frac{4.05\times10^{-5}}{D}+\frac{3.68\times10^{-4}D}{(1.956\times10^5D^5-0.1429D)^{1/2}}\right] \qquad (6)$$

Using a computer root-finding technique gives

$\underline{\underline{D=0.0445\ m}}$

8.107

8.107 Air, assumed incompressible, flows through the two pipes shown in Fig. P8.107. Determine the flowrate if minor losses are neglected and the friction factor in each pipe is 0.015. Determine the flowrate if the 0.5-in.-diameter pipe were replaced by a 1-in.-diameter pipe. Comment on the assumption of incompressibility.

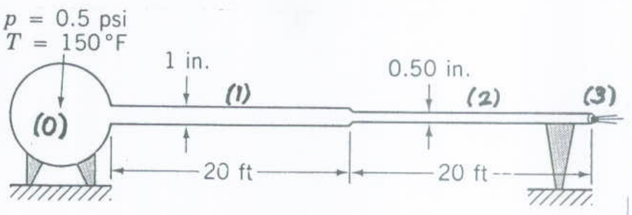


FIGURE P8.107

$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=h_{L_1}+h_{L_2}+\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3 \text{, where } V_0=0,\ z_0=z_2,\ p_3=0, \qquad (1)$$

$$V_2=V_3,\ h_{L_1}=f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g},\ h_{L_2}=f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}, \text{ and } V_1=V_2\frac{A_2}{A_1}=V_2\left(\frac{D_2}{D_1}\right)^2=\left(\frac{0.5\text{ in.}}{1.0\text{ in.}}\right)^2V_2$$

$$=0.25V_2$$

Thus, Eq. (1) becomes

$$\frac{p_0}{\gamma}=f_1\frac{\ell_1}{D_1}\frac{(0.25V_2)^2}{2g}+f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}+\frac{V_2^2}{2g}$$

or

$$p_0=\frac{1}{2}\rho V_2^2\left[f_1\frac{\ell_1}{D_1}(0.25)^2+f_2\frac{\ell_2}{D_2}+1\right] \qquad (2)$$

With $p_0=\rho_0RT_0$ or $\rho_0=\dfrac{p_0}{RT_0}=\dfrac{\left(0.5\frac{lb}{in.^2}+14.7\frac{lb}{in.^2}\right)\left(144\frac{in.^2}{ft^2}\right)}{\left(1716\frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)(150+460)^\circ R}=0.00209\ \frac{slug}{ft^3}$

and $f_1=f_2=0.015$ Eq. (2) gives

$$\left(0.5\frac{lb}{in.^2}\right)\left(144\frac{in.^2}{ft^2}\right)=\frac{1}{2}\left(0.00209\frac{slug}{ft^3}\right)V_2^2\left[(0.015)\left(\left(\frac{20\,ft}{\frac{1}{12}ft}\right)(0.25)^2+\left(\frac{20\,ft}{\frac{1}{24}ft}\right)\right)+1\right]$$

or

$V_2=90.4\ \frac{ft}{s}$ Thus, $Q=A_2V_2=\frac{\pi}{4}\left(\frac{1}{24}ft\right)^2\left(90.4\frac{ft}{s}\right)=\underline{\underline{0.123\ \frac{ft^3}{s}}}$

If both pipes were 1 in. diameter, then $V_1=V_2$ and Eq. (1) becomes

$p_0=\frac{1}{2}\rho V_2^2\left[f_1\frac{\ell_1}{D_1}+f\frac{\ell_2}{D_2}+1\right]$ or with $f_1=f_2$, $\ell_1=\ell_2$, and $D_1=D_2$

$$p_0=\frac{1}{2}\rho V_2^2\left[f_2\frac{(2\ell_2)}{D_2}+1\right]$$

Hence,

$$\left(0.5\frac{lb}{in.^2}\right)\left(144\frac{in.^2}{ft^2}\right)=\frac{1}{2}\left(0.00209\frac{slug}{ft^3}\right)V_2^2\left[0.015\left(\frac{40\,ft}{\frac{1}{12}ft}\right)+1\right]$$

or

$V_2=91.7\ \frac{ft}{s}$ Thus, $Q=A_2V_2=\frac{\pi}{4}\left(\frac{1}{12}ft\right)^2\left(91.7\frac{ft}{s}\right)=\underline{\underline{0.500\frac{ft^3}{s}}}$

Since $p=\rho RT$ it follows that

$\dfrac{\rho_3}{\rho_0}=\dfrac{\left(\frac{p_3}{RT_3}\right)}{\left(\frac{p_0}{RT_0}\right)}=\dfrac{p_3}{p_0}\dfrac{T_0}{T_3}$ If we assume $T_3=T_0$ (it probably will not be, but it should be a reasonable approximation) then

$\dfrac{\rho_3}{\rho_0}\approx\dfrac{p_3}{p_0}=\dfrac{14.7\,psi}{(0.5+14.7)\,psi}=0.967$ The flow is nearly incompressible.

***8.108**

*8.108 Repeat Problem 8.107 if the pipes are galvanized iron and the friction factors are not known a priori.

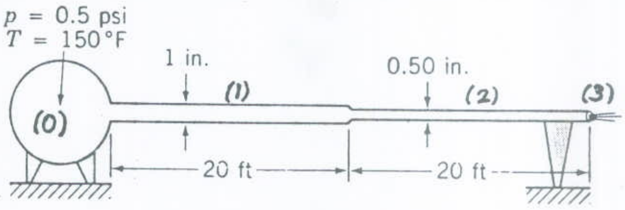


FIGURE P8.107

$$\frac{p_0}{\gamma}+\frac{V_0^2}{2g}+z_0=h_{L_1}+h_{L_2}+\frac{p_3}{\gamma}+\frac{V_3^2}{2g}+z_3\text{, where } V_0=0,\ z_0=z_2,\ p_3=0,\ V_2=V_3, \qquad (1)$$

$$h_{L_1}=f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g},\ h_{L_2}=f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g},\text{ and } V_1=\frac{V_2A_2}{A_1}=V_2\left(\frac{D_2}{D_1}\right)^2=\left(\frac{0.5\text{ in.}}{1.0\text{ in.}}\right)^2V_2=0.25\,V_2$$

Thus, Eq. (1) becomes

$$\frac{p_0}{\gamma}=f_1\frac{\ell_1}{D_1}\frac{(0.25V_2)^2}{2g}+f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}+\frac{V_2^2}{2g}$$

or

$$p_0=\frac{1}{2}\rho V_2^2\left[f_1\frac{\ell_1}{D_1}(0.25)^2+f_2\frac{\ell_2}{D_2}+1\right] \qquad (2)$$

With $p_0=\rho_0RT_0$ or $\rho_0=\frac{p_0}{RT_0}=\frac{\left(0.5\frac{\text{lb}}{\text{in.}^2}+14.7\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right)}{\left(1716\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ\text{R}}\right)(150+460)^\circ\text{R}}=0.00209\ \frac{\text{slug}}{\text{ft}^3}$

Eq. (2) becomes

$$\left(0.5\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right)=\frac{1}{2}\left(0.00209\frac{\text{slug}}{\text{ft}^3}\right)V_2^{\,2}\left[(0.25)^2f_1\left(\frac{20\text{ ft}}{\frac{1}{12}\text{ ft}}\right)+f_2\left(\frac{20\text{ ft}}{\frac{1}{24}\text{ ft}}\right)+1\right]$$

or

$$6.89\times10^4=V_2^{\,2}\left(15f_1+480f_2+1\right) \qquad (3)$$

Also from Table 8.1, $\frac{\varepsilon}{D_1}=\frac{0.0005\text{ ft}}{D_1}=\frac{0.0005\text{ ft}}{\frac{1}{12}\text{ ft}}=0.006$ (4)

and $\frac{\varepsilon}{D_2}=\frac{0.0005\text{ ft}}{\frac{1}{24}\text{ ft}}=0.012$ (5)

and

$Re_1=\frac{V_1D_1}{\nu}$, $Re_2=\frac{V_2D_2}{\nu}$, where from Table B.3

$$\nu=\frac{\mu}{\rho_0}=\frac{4.18\times10^{-7}\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}{0.00209\frac{\text{slug}}{\text{ft}^3}}=2.00\times10^{-4}\frac{\text{ft}^2}{\text{s}}$$

Hence,

$$Re_1=\frac{(0.25V_2)\left(\frac{1}{12}\text{ ft}\right)}{2.00\times10^{-4}\frac{\text{ft}^2}{\text{s}}}=104\,V_2 \qquad (6)$$

and

$$Re_2=\frac{V_2\left(\frac{1}{24}\text{ ft}\right)}{2.00\times10^{-4}\frac{\text{ft}^2}{\text{s}}}=208\,V_2 \qquad (7)$$

For turbulent flow Eq. 8.35 gives $\frac{1}{\sqrt{f}}=-2.0\log\left[\frac{\frac{\varepsilon}{D}}{3.7}+\frac{2.51}{Re\sqrt{f}}\right]$ (8)

By combining Eqs. (4) through (8) we obtain

$$\frac{1}{\sqrt{f_1}}=-2.0\log\left[1.62\times10^{-3}+\frac{2.41\times10^{-2}}{V_2\sqrt{f_1}}\right] \qquad (9)$$

and

$$\frac{1}{\sqrt{f_2}}=-2.0\log\left[3.24\times10^{-3}+\frac{1.21\times10^{-2}}{V_2\sqrt{f_2}}\right] \qquad (10)$$

(con't)

*8.108 (con't)

A computer trial and error solution method gives the solution to Eqs. (3), (9), and (10) as:

$$f_1 = 0.0425\,,\ f_2 = 0.0446\,,\ \text{and}\ V_2 = 5.47\,\frac{ft}{s}$$

Thus,

$$Q = A_2 V_2 = \frac{\pi}{4} D_2^2 V_2 = \frac{\pi}{4}\left(\frac{0.50}{12}\,ft\right)^2 \left(5.47\,\frac{ft}{s}\right) = \underline{\underline{7.46\times10^{-2}\,\frac{ft^3}{s}}}$$

If $D_1 = D_2$, then $V_1 = V_2$, $f_1 = f_2$ since $\frac{\varepsilon}{D_1} = \frac{\varepsilon}{D_2} = 0.006$, and

$$Re_1 = Re_2 = \frac{V_2 D_2}{\nu} = \frac{V_2\left(\frac{1}{12}\,ft\right)}{2.00\times10^{-4}\,\frac{ft^2}{s}} = 416\,V_2$$

Thus, Eq. (1) becomes

$$p_0 = \frac{1}{2}\rho V_2^2\left[f_2\left(\frac{\ell_1+\ell_2}{D_2}\right)+1\right]$$

or

$$\left(0.5\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right) = \frac{1}{2}\left(0.00209\,\frac{slug}{ft^3}\right) V_2^2\left[f_2\left(\frac{40\,ft}{\frac{1}{12}\,ft}\right)+1\right]$$

Hence,

$$6.89\times10^4 = V_2^2\left[480\,f_2 + 1\right] \qquad (11)$$

Also, from Eq. (8)

$$\frac{1}{\sqrt{f_2}} = -2.0\,\log\left[1.62\times10^{-3} + \frac{6.03\times10^{-3}}{V_2\sqrt{f_2}}\right] \qquad (12)$$

A computer solution of Eqs. (11) and (12) gives

$$f_2 = 0.0351 \ \text{and}\ V_2 = 62.2\,\frac{ft}{s}$$

Thus,

$$Q = A_2 V_2 = \frac{\pi}{4} D_2^2 V_2 = \frac{\pi}{4}\left(\frac{1}{12}\,ft\right)^2\left(62.2\,\frac{ft}{s}\right) = \underline{\underline{0.339\,\frac{ft^3}{s}}}$$

Note: Since $p = \rho R T$ it follows that

$$\frac{\rho_3}{\rho_0} = \frac{\left(\frac{p_3}{RT_3}\right)}{\left(\frac{p_0}{RT_0}\right)} = \frac{p_3}{p_0}\,\frac{T_0}{T_3}$$ If we assume $T_3 = T_0$ (it probably will not be, but it should be a reasonable approximation) then

$$\frac{\rho_3}{\rho_0} = \frac{p_3}{p_0} = \frac{14.7\,psi}{(0.5+14.7)\,psi} = 0.967$$ The flow is nearly incompressible.

8.110

8.110 The flowrate between tank A and tank B shown in Fig. P8.110 is to be increased by 30% (i.e., from Q to $1.30Q$) by the addition of a second pipe (indicated by the dotted lines) running from node C to tank B. If the elevation of the free surface in tank A is 25 ft above that in tank B, determine the diameter, D, of this new pipe. Neglect minor losses and assume that the friction factor for each pipe is 0.02.

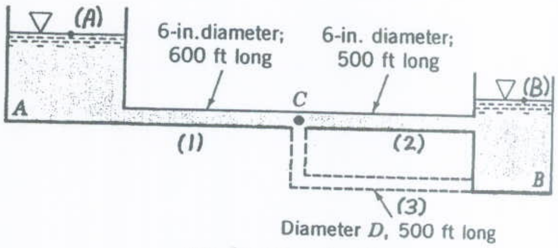


FIGURE P8.110

With the single pipe: $\frac{p_A}{\gamma}+\frac{V_A^2}{2g}+z_A=\frac{p_B}{\gamma}+\frac{V_B^2}{2g}+z_B+f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g}+f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}$ (1)

where $p_A=p_B=0$, $V_A=V_B=0$, $z_A=25\text{ ft}$, $z_B=0$,

and $V_1=V_2$ (since $D_1=D_2$).

Thus, $z_A=f_1\frac{(\ell_1+\ell_2)}{D_1}\frac{V_1^2}{2g}$, or $25\text{ft}=(0.02)\frac{(600+500)\text{ft}}{(\frac{6}{12}\text{ft})}\frac{V_1^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$

or $V_1=6.05\frac{\text{ft}}{\text{s}}$ Hence, $Q=A_1V_1=\frac{\pi}{4}(\frac{6}{12}\text{ft})^2(6.05\frac{\text{ft}}{\text{s}})=1.188\frac{\text{ft}^3}{\text{s}}$

With the second pipe $Q=1.30(1.188\frac{\text{ft}^3}{\text{s}})=1.54\frac{\text{ft}^3}{\text{s}}$

Thus, $Q_1=1.54\frac{\text{ft}^3}{\text{s}}=Q_2+Q_3$ or $V_1=\frac{Q_1}{A_1}=\frac{1.54\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}(\frac{6}{12}\text{ft})^2}=7.84\frac{\text{ft}}{\text{s}}$

For fluid flowing from A to B through pipes 1 and 2,

$z_A=h_{L_1}+h_{L_2}=f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g}+f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g}$ (see Eq. (1))

or

$$25\text{ft}=(0.02)\frac{600\text{ft}}{(\frac{6}{12}\text{ft})}\frac{(7.84\frac{\text{ft}}{\text{s}})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}+(0.02)\frac{500\text{ft}}{(\frac{6}{12}\text{ft})}\frac{V_2^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$$

Hence, $V_2=2.60\frac{\text{ft}}{\text{s}}$

and

$$Q_2=A_2V_2=\frac{\pi}{4}(\frac{6}{12}\text{ft})^2(2.60\frac{\text{ft}}{\text{s}})=0.511\frac{\text{ft}^3}{\text{s}}$$

Thus, $Q_3=Q_1-Q_2=1.54\frac{\text{ft}^3}{\text{s}}-0.511\frac{\text{ft}^3}{\text{s}}=1.03\frac{\text{ft}^3}{\text{s}}$

For fluid flowing from A to B through pipes 1 and 3,

$z_A=h_{L_1}+h_{L_3}=f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g}+f_3\frac{\ell_3}{D_3}\frac{V_3^2}{2g}$, where $V_3=\frac{Q_3}{A_3}=\frac{1.03\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}D_3^2}=\frac{1.31}{D_3^2}$

Thus,

$$25\text{ft}=(0.02)\frac{600\text{ ft}}{(\frac{6}{12}\text{ft})}\frac{(7.84\frac{\text{ft}}{\text{s}})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}+(0.02)\frac{500\text{ft}}{D_3}\frac{(\frac{1.31}{D_3^2})^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$$

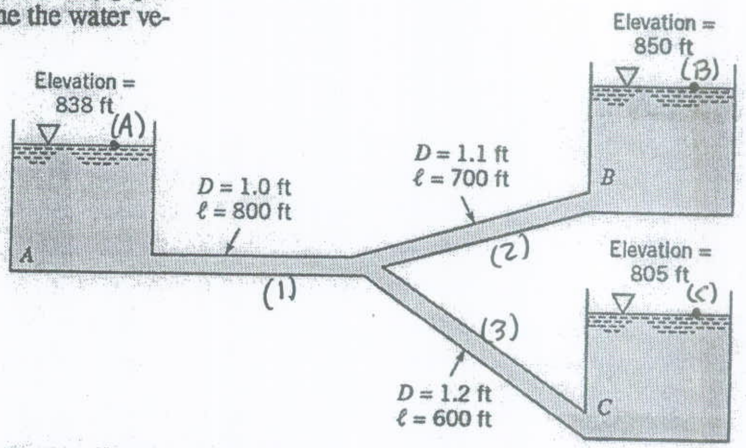
or

$$\underline{\underline{D_3=0.662\text{ft}}}$$

Note: With the parameters given, the solution is quite sensitive to round off errors in the calculations

8.111

8.111 The three tanks shown in Fig. P8.111 are connected by pipes with friction factors of 0.03 for each pipe. Determine the water velocity in each pipe. Neglect minor losses.



■ FIGURE P8.111

Assume the flow from both tanks A and B is into tank C, or $Q_3 = Q_1 + Q_2$

Thus, $\frac{\pi}{4}D_3^2 V_3 = \frac{\pi}{4}D_1^2 V_1 + \frac{\pi}{4}D_2^2 V_2$, or $1.2^2 V_3 = 1.0^2 V_1 + 1.1^2 V_2$

Hence, $V_3 = 0.694\,V_1 + 0.840\,V_2$ (1)

For the flow from A to C, with $p_A = p_C = 0$, $V_A = V_C = 0$, we obtain

$$z_A = z_C + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}, \text{ or } 838\,ft = 805\,ft + \frac{0.03}{2(32.2\,\frac{ft}{s^2})}\left[\frac{800\,ft}{1\,ft}V_1^2 + \frac{600\,ft}{1.2\,ft}V_3^2\right]$$

or

$$33 = 0.373\,V_1^2 + 0.233\,V_3^2 \qquad (2)$$

Similarly for the flow from B to C, with $p_B = p_C = 0$, $V_B = V_C = 0$, we obtain

$$z_B = z_C + f_2 \frac{\ell_2}{D_2}\frac{V_2^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}, \text{ or } 850\,ft = 805\,ft + \frac{0.03}{2(32.2\,\frac{ft}{s^2})}\left[\frac{700\,ft}{1.1\,ft}V_2^2 + \frac{600\,ft}{1.2\,ft}V_3^2\right]$$

or

$$45 = 0.296\,V_2^2 + 0.233\,V_3^2 \qquad (3)$$

Thus, 3 equations ((1), (2), and (3)) for V_1, V_2, and V_3. Solve as follows:

Subtract (2) from (3) to obtain

$$12 = 0.296\,V_2^2 - 0.373\,V_1^2 \qquad (4)$$

From (2): $V_3 = \sqrt{141.6 - 1.6\,V_1^2}$, or when combined with (1):

$$\sqrt{141.6 - 1.6V_1^2} = 0.694\,V_1 + 0.840\,V_2, \text{ or } V_2 = \sqrt{200 - 2.27V_1^2} - 0.826\,V_1 \qquad (5)$$

Combine Eqs. (4) and (5) to obtain:

$$\frac{12}{0.296} = \left[\sqrt{200 - 2.27V_1^2} - 0.826\,V_1\right]^2 - \frac{0.373}{0.296}$$, which can be simplified to

$$V_1\sqrt{200 - 2.27V_1^2} = 96.5 - 1.725\,V_1^2 \qquad (6)$$

By squaring this equation we obtain (after simplification):

$$V_1^4 - 101.5\,V_1^2 + 1774 = 0 \quad \text{Hence: } V_1^2 = \frac{101.5 \pm \sqrt{101.5^2 - 4(1774)}}{2} = \begin{matrix} 79.1 \\ \text{or} \\ 22.4 \end{matrix}$$

Thus, $V_1 = 8.89\,\frac{ft}{s}$ or $V_1 = 4.73\,\frac{ft}{s}$

Note: The $V_1 = 8.89$ solution is an extra root introduced by squaring Eq. (6). It is not a solution of the original Eqs. (1), (2), (3). For this value, Eq. (6) becomes $889\sqrt{200 - 2.27(8.89^2)} \overset{?}{=} 96.5 - 1.725(8.89)^2$, or "40 = −40"

Thus $V_1 = \underline{\underline{4.73\,\frac{ft}{s}}}$, from Eq. (2) $V_3 = \left[\frac{33 - 0.373(4.73)^2}{0.233}\right]^{1/2} = \underline{\underline{10.3\,\frac{ft}{s}}}$,

and from Eq. (1) $V_2 = \frac{10.3 - 0.694(4.73)}{0.840} = \underline{\underline{8.35\,\frac{ft}{s}}}$

8.112

8.112 The three water-filled tanks shown in Fig. P8.112 are connected by pipes as indicated. If minor losses are neglected, determine the flowrate in each pipe.

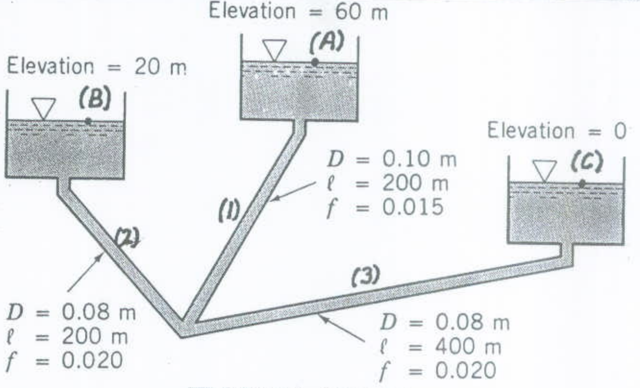


FIGURE P8.112

Assume the fluid flows from A to B and A to C. Thus, $Q_1 = Q_2 + Q_3$

or $\frac{\pi}{4}(0.1m)^2 V_1 = \frac{\pi}{4}(0.08m)^2 V_2 + \frac{\pi}{4}(0.08m)^2 V_3$

Thus, $V_1 = 0.64 V_2 + 0.64 V_3$ (1)

For fluid flowing from A to B with $p_A = p_B = 0$ and $V_A = V_B = 0$,

$z_A = z_B + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{\ell_2}{D_2}\frac{V_2^2}{2g}$

or

$60m - 20m = (0.015)\left(\frac{200m}{0.1m}\right)\frac{V_1^2}{2(9.81\frac{m}{s^2})} + (0.020)\left(\frac{200m}{0.08m}\right)\frac{V_2^2}{2(9.81\frac{m}{s^2})}$

Hence,

$40 = 1.529 V_1^2 + 2.55 V_2^2$ (2)

Similarly, for fluid flowing from A to C with $p_A = p_C = 0$ and $V_A = V_C = 0$,

$z_A = z_C + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$

or

$60m = (0.015)\left(\frac{200m}{0.1m}\right)\frac{V_1^2}{2(9.81\frac{m}{s^2})} + (0.020)\left(\frac{400m}{0.08m}\right)\frac{V_3^2}{2(9.81\frac{m}{s^2})}$

Hence,

$60 = 1.529 V_1^2 + 5.10 V_3^2$ (3)

Solve Eqs. (1), (2), and (3) for V_1, V_2, and V_3. From Eqs. (1) and (3):

$60 = 1.529(0.64)^2(V_2 + V_3)^2 + 5.10 V_3^2$, or $95.8 = (V_2 + V_3)^2 + 8.14 V_3^2$ (4)

Subtract Eq. (2) from Eq. (3):

$60 - 40 = 5.10 V_3^2 + 2.55 V_2^2$ or $V_2 = \sqrt{2V_3^2 - 7.84}$ (5)

Thus, from Eqs. (4) and (5): $8.14 V_3^2 + \left(\sqrt{2V_3^2 - 7.84} + V_3\right)^2 - 95.8 = 0$

This can be simplified to

$2V_3\sqrt{2V_3^2 - 7.84} = 103.6 - 11.14 V_3^2$ Square both sides and (6)
rearrange to give $V_3^4 - 19.63 V_3^2 + 92.5 = 0$ which can be solved by the quadratic formula to give

$V_3^2 = \frac{19.63 \pm \sqrt{19.63^2 - 4(92.5)}}{2} = 11.77$ or 7.86 Thus $V_3 = 3.43 \frac{m}{s}$
or $V_3 = 2.80 \frac{m}{s}$

(con't)

8.112 (con't)

Note: The value $V_3 = 3.43\,\frac{m}{s}$ is not a solution of the original equations, Eqs. (1), (2), and (3). With this value the right hand side of Eq. (6) is negative (i.e. $103.6 - 11.14V_3^2 = 103.6 - 11.14(3.43)^2 = -24.5$). As seen from the left hand side of Eq. (6), this cannot be. This extra root was introduced by squaring Eq. (6).

Thus, $Q_3 = A_3 V_3 = \frac{\pi}{4}(0.08m)^2(2.80\,\frac{m}{s}) = \underline{\underline{0.0141\,\frac{m^3}{s}}}$

Also, from Eq. (3):

$$60 = 1.529\,V_1^2 + 5.10\,(2.80)^2 \quad \text{or} \quad V_1 = 3.62\,\frac{m}{s}$$

or

$$Q_1 = A_1 V_1 = \frac{\pi}{4}(0.10m)^2(3.62\,\frac{m}{s}) = \underline{\underline{0.0284\,\frac{m^3}{s}}}$$

and from Eq. (1):

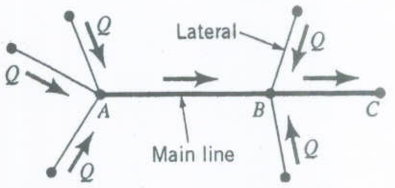
$$3.62 = 0.64\,V_2 + 0.64(2.80) \quad \text{or} \quad V_2 = 2.86\,\frac{m}{s}$$

or

$$Q_2 = A_2 V_2 = \frac{\pi}{4}(0.08m)^2(2.86\,\frac{m}{s}) = \underline{\underline{0.0143\,\frac{m^3}{s}}}$$

8.113

8.113 (See **"Deepwater pipeline,"** Section 8.5.2.) Five oil fields, each producing an output of Q barrels per day, are connected to the 28-in.-diameter "main line pipe" ($A-B-C$) by 16-in.-diameter "lateral pipes" as shown in Fig. P8.113. The friction factor is the same for each of the pipes and elevation effects are negligible. **(a)** For section $A-B$ determine the ratio of the pressure drop per mile in the main line pipe to that in the lateral pipes. **(b)** Repeat the calculations for section $B-C$.



■ FIGURE P8.113

For any of the pipe sections $\frac{\Delta p}{\gamma} = f\frac{\ell}{D}\frac{V^2}{2g}$, or $\Delta p = \frac{1}{2}\rho V^2 f \frac{\ell}{D}$

(a) Thus, $\Delta p_{AB} = \frac{1}{2}\rho V_{AB}^2 f_{AB}\frac{\ell_{AB}}{D_{AB}}$ and $\Delta p_{lat} = \frac{1}{2}\rho V_{lat}^2 f_{lat}\frac{\ell_{lat}}{D_{lat}}$, where $f_{AB} = f_{lat}$

Hence,

$$\frac{\Delta p_{AB}/\ell_{AB}}{\Delta p_{lat}/\ell_{lat}} = \frac{(V_{AB}^2/D_{AB})}{(V_{lat}^2/D_{lat})} \qquad (1)$$

Also,

$Q_{AB} = 3Q$ so that $\frac{\pi}{4}D_{AB}^2 V_{AB} = 3\frac{\pi}{4}D_{lat}^2 V_{lat}$ or $V_{AB}/V_{lat} = 3(D_{lat}/D_{AB})^2$

Thus, Eq. (1) becomes

$$\frac{\Delta p_{AB}/\ell_{AB}}{\Delta p_{lat}/\ell_{lat}} = \left[3\left(\frac{D_{lat}}{D_{AB}}\right)^2\right]^2\left(\frac{D_{lat}}{D_{AB}}\right) = 9\left(\frac{D_{lat}}{D_{AB}}\right)^5 = 9\left(\frac{16\,in.}{28\,in.}\right)^5 = 0.548$$

(b) Similarly, for section BC:

$\Delta p_{BC}/\ell_{BC} = \frac{1}{2}\rho V_{BC}^2 f/D_{BC}$ so that

$$\frac{\Delta p_{BC}/\ell_{BC}}{\Delta p_{lat}/\ell_{lat}} = \frac{(V_{BC}^2/D_{BC})}{(V_{lat}^2/D_{lat})} \qquad (2)$$

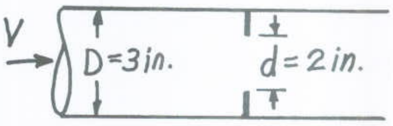
Also,

$Q_{BC} = 5Q_{lat}$, or $V_{BC}/V_{lat} = 5(D_{lat}/D_{BC})^2$ so that Eq. (2) becomes

$$\frac{\Delta p_{BC}/\ell_{BC}}{\Delta p_{lat}/\ell_{lat}} = \left[5\left(\frac{D_{lat}}{D_{BC}}\right)^2\right]^2\left(\frac{D_{lat}}{D_{BC}}\right) = 25\left(\frac{D_{lat}}{D_{BC}}\right)^5 = 25\left(\frac{16\,in.}{28\,in.}\right)^5 = \underline{\underline{1.52}}$$

8.116

8.116 A 2-in.-diameter orifice plate is inserted in a 3-in.-diameter pipe. If the water flowrate through the pipe is 0.90 cfs, determine the pressure difference indicated by a manometer attached to the flow meter.



$$Q = C_o A_o \sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}} \text{ , where } \beta = \frac{d}{D} = \frac{2\,in.}{3\,in.} = \frac{2}{3}\text{ , } Q = 0.70\,\frac{ft^3}{s}\text{ , and}$$

$$A_o = \frac{\pi}{4}d$$

Also,

$$Re = \frac{VD}{\nu}\text{ , where } V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.7\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{3}{12}ft\right)^2} = 14.26\,\frac{ft}{s}$$

Thus,

$$Re = \frac{\left(14.26\frac{ft}{s}\right)\left(\frac{3}{12}ft\right)}{1.21\times10^{-5}\frac{ft^2}{s}} = 2.95\times10^5 \quad \text{Hence, from Fig.8.41 : } C_o = 0.608$$

so that,

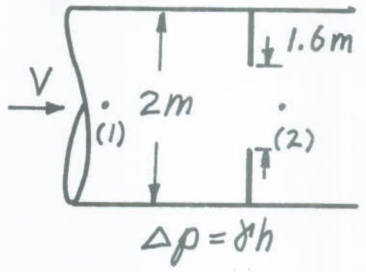
$$0.9\,\frac{ft^3}{s} = (0.608)\frac{\pi}{4}\left(\frac{2}{12}ft\right)^2\sqrt{\frac{2(p_1-p_2)}{\left(1.94\frac{slugs}{ft^3}\right)\left(1-\left(\frac{2}{3}\right)^4\right)}}$$

or

$$p_1 - p_2 = 3590\,\frac{lb}{ft^2} = \underline{\underline{24.9\,\frac{lb}{in.^2}}}$$

8.117

8.117 Air to ventilate an underground mine flows through a large 2-m-diameter pipe. A crude flowrate meter is constructed by placing a sheet metal "washer" between two sections of the pipe. Estimate the flowrate if the hole in the sheet metal has a diameter of 1.6 m and the pressure difference across the sheet metal is 8.0 mm of water.



$$Q = C_o A_o \sqrt{\frac{2(p_1 - p_2)}{\rho(1-\beta^4)}} = C_o \frac{\pi}{4}(1.6m)^2 \sqrt{\frac{2(0.008\,m)(9.80\times10^3 \frac{N}{m^3})}{(1.23\frac{kg}{m^3})\left[1-\left(\frac{1.6\,m}{2.0\,m}\right)^4\right]}}$$

or

$$Q = 29.5\, C_o \ \frac{m^3}{s} \qquad (1)$$

Also, $Re = \frac{DV}{\nu} = \frac{(2m)\,V}{1.46\times10^{-5}\frac{m^2}{s}}$ or $Re = 1.37\times10^5 V$ where $V \sim \frac{m}{s}$ (2)

and

$$\beta = \frac{d}{D} = \frac{1.6\,m}{2.0\,m} = 0.8$$

Trial and error solution:

Assume $C_o = 0.61$ so that from Eq.(1), $Q = 29.5\,(0.61) = 18.0\,\frac{m^3}{s}$

Hence, $V = \frac{Q}{A} = \frac{18.0\frac{m^3}{s}}{\frac{\pi}{4}(2.0m)^2} = 5.73\,\frac{m}{s}$

From Eq.(2), $Re = 1.37\times10^5\,(5.73) = 7.85\times10^5$

This Re and β give $C_o = 0.61$ (see Fig. 8.41) which agrees with the assumed value.

Thus, $Q = \underline{\underline{18.0\,\frac{m^3}{s}}}$

8.118

8.118 Water flows through a 40-mm-diameter nozzle meter in a 75-mm-diameter pipe at a rate of 0.015 m³/s. Determine the pressure difference across the nozzle if the temperature is **(a)** 10 °C, or **(b)** 80 °C.

$Q=0.015\,\frac{m^3}{s}$

$$Q=C_n A_n\sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}}, \text{ where } \beta=\frac{d}{D}=\frac{40\,mm}{75\,mm}=0.533$$

Thus,

$$0.015\,\frac{m^3}{s}=C_n\frac{\pi}{4}(0.04m)^2\sqrt{\frac{2(p_1-p_2)}{\rho(1-0.533^4)}}$$

or

$$C_n\sqrt{p_1-p_2}=8.09\rho^{\frac{1}{2}}, \text{where } \rho\sim\frac{kg}{m^3},\ p_1-p_2\sim\frac{N}{m^2} \qquad (1)$$

Also, $Re=\frac{VD}{\nu}=\frac{V(0.075m)}{\nu}$, with $V=\frac{Q}{\frac{\pi}{4}D^2}=\frac{0.015\frac{m^3}{s}}{\frac{\pi}{4}(0.075m)^2}=3.40\,\frac{m}{s}$

a) Assume $T=10°C$, or from Table B.2: $\rho=999.7\,\frac{kg}{m^3}$, $\nu=1.307\times10^{-6}\,\frac{m^2}{s}$

Thus,

$$Re=\frac{(3.40\frac{m}{s})(0.075m)}{1.307\times10^{-6}\frac{m^2}{s}}=1.95\times10^5 \text{ so that from Fig. 8.43:}$$

$$C_n=0.986$$

From Eq. (1): $0.986\sqrt{p_1-p_2}=8.09(999.7)^{\frac{1}{2}}$ or $p_1-p_2=6.73\times10^4\,\frac{N}{m^2}$

Thus,

$$p_1-p_2=\underline{\underline{67.3\text{ kPa}}}$$

b) Assume $T=80°C$, of from Table B.2: $\rho=971.8\,\frac{kg}{m^3}$, $\nu=3.65\times10^{-7}\,\frac{m^2}{s}$

Thus,

$$Re=\frac{(3.40\frac{m}{s})(0.075m)}{3.65\times10^{-7}\frac{m}{s}}=6.99\times10^5 \text{ so that from Fig. 8.43:}$$

$$C_n=0.991$$

From Eq. (1): $0.991\sqrt{p_1-p_2}=8.09(971.8)^{\frac{1}{2}}$ or $p_1-p_2=6.48\times10^4\,\frac{N}{m^2}$

Thus,

$$p_1-p_2=\underline{\underline{64.8\text{ kPa}}}$$

8.119.

8.119 Air at 200 °F and 60 psia flows in a 4-in.-diameter pipe at a rate of 0.52 lb/s. Determine the pressure at the 2-in-diameter throat of a Venturi meter placed in the pipe.

$$Q = C_v A_T \sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}}, \text{ where } \beta = \frac{d}{D} = \frac{2\text{ in.}}{4\text{ in.}} = 0.5 \text{ and } \gamma Q = 0.52\,\frac{\text{lb}}{\text{s}} \quad (1)$$

Also, $\rho = \frac{p}{RT} = \frac{(60\,\frac{\text{lb}}{\text{in.}^2})(144\,\frac{\text{in.}^2}{\text{ft}^2})}{(1716\,\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ\text{R}})(200+460)^\circ\text{R}} = 7.63\times10^{-3}\,\frac{\text{slug}}{\text{ft}^3}$

so that

$$\gamma = \rho g = (7.63\times10^{-3}\,\frac{\text{slug}}{\text{ft}^3})(32.2\,\frac{\text{ft}}{\text{s}}) = 0.246\,\frac{\text{lb}}{\text{ft}^3}$$

Thus, $Q = \frac{0.52\,\frac{\text{lb}}{\text{s}}}{0.246\,\frac{\text{lb}}{\text{ft}^3}} = 2.11\,\frac{\text{ft}^3}{\text{s}}$ and $V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{2.11\,\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}(\frac{4}{12}\,\text{ft})^2} = 24.2\,\frac{\text{ft}}{\text{s}}$

Also, from Table B.3, $\mu = 4.49\times10^{-7}\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}$ so that

$$Re = \frac{\rho V D}{\mu} = \frac{(7.63\times10^{-3}\,\frac{\text{slug}}{\text{ft}^3})(24.2\,\frac{\text{ft}}{\text{s}})(\frac{4}{12}\,\text{ft})}{4.49\times10^{-7}\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}} = 1.37\times10^5$$

Hence, from Fig. 8.45,

$C_v \approx 0.98$

From Eq.(1): $2.11\,\frac{\text{ft}^3}{\text{s}} = (0.98)\frac{\pi}{4}(\frac{2}{12}\,\text{ft})^2\sqrt{\frac{2(p_1-p_2)}{(7.63\times10^{-3}\,\frac{\text{slug}}{\text{ft}^3})(1-0.5^4)}}$

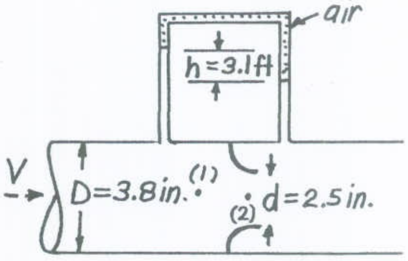
or

$p_1 - p_2 = 34.8\,\frac{\text{lb}}{\text{ft}^2}\left(\frac{1\,\text{ft}^2}{144\,\text{in.}^2}\right) = 0.242\,\frac{\text{lb}}{\text{in.}^2}$

Thus, $p_2 = (60-0.242)\,\text{psia} = \underline{\underline{59.76\,\text{psia}}}$

8.120

8.120 A 2.5-in.-diameter flow nozzle is installed in a 3.8-in.-diameter pipe that carries water at 160 °F. If the air–water manometer used to measure the pressure difference across the meter indicates a reading of 3.1 ft, determine the flowrate.



$$Q = C_n A_n \sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}}\text{, where } \beta = \frac{d}{D} = \frac{2.5\text{ in.}}{3.8\text{ in.}} = 0.658 \qquad (1)$$

From Table B.1: $\rho = 1.896\ \frac{\text{slugs}}{\text{ft}^3}$, $\mu = 8.32\times10^{-6}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}$ so that

$$Re = \frac{\rho V D}{\mu} = \frac{(1.896\ \frac{\text{slugs}}{\text{ft}^3})V(\frac{3.8}{12}\text{ft})}{8.32\times10^{-6}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}$$

or

$$Re = 7.22\times10^4 V\text{, where } V\sim\frac{\text{ft}}{\text{s}} \qquad (2)$$

Also, with $Q = \frac{\pi}{4}D^2V$ Eq.(1) becomes (using $p_1 - p_2 = \gamma h$):

$$\frac{\pi}{4}\left(\frac{3.8}{12}\text{ft}\right)^2 V = C_n\frac{\pi}{4}\left(\frac{2.5}{12}\text{ft}\right)^2\left[\frac{2(32.2\frac{\text{ft}}{\text{s}^2})(1.896\ \frac{\text{slugs}}{\text{ft}^3})(3.1\text{ft})}{(1.896\ \frac{\text{slugs}}{\text{ft}^3})(1-0.658^4)}\right]^{1/2}$$

or

$$V = 6.78\ C_n \qquad (3)$$

Trial and error solution using Fig. 8.43 for $C_n = C_n(Re, \beta=0.658)$:

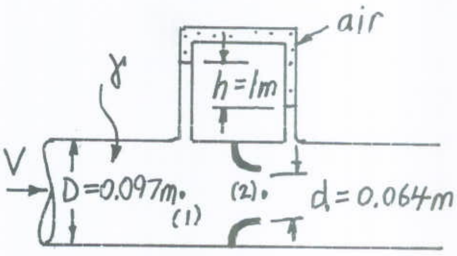
Assume $C_n = 0.99$ From Eq.(3) $V = 6.78\,(0.99) = 6.71\ \frac{\text{ft}}{\text{s}}$

From Eq. (2) $Re = 7.22\times10^4(6.71\frac{\text{ft}}{\text{s}}) = 4.84\times10^5$ which from Fig. 8.43 gives $C_n = 0.99$ (checks with assumed value)

Thus, $V = 6.71\ \frac{\text{ft}}{\text{s}}$ and $Q = \frac{\pi}{4}D^2V = \frac{\pi}{4}(\frac{3.8}{12}\text{ft})^2(6.71\ \frac{\text{ft}}{\text{s}}) = \underline{\underline{0.528\ \frac{\text{ft}^3}{\text{s}}}}$

8.121

8.121 A 0.064 m-diameter nozzle meter is installed in a 0.097 m-diameter pipe that carries water at 60 °C. If the inverted air–water U-tube manometer used to measure the pressure difference across the meter indicates a reading of 1 m, determine the flowrate.



(1) $Q = C_n A_n \sqrt{\dfrac{2(p_1 - p_2)}{\rho(1-\beta^4)}}$, where $\beta = \dfrac{d}{D} = \dfrac{0.064\ m}{0.097\ m} = 0.660$

From Table B.2: $\rho = 983.2\ \dfrac{kg}{m^3}$, $\mu = 4.665 \times 10^{-4}\ \dfrac{N \cdot s}{m^2}$ so that

$$Re = \frac{\rho V D}{\mu} = \frac{(983.2\ \frac{kg}{m^3})\ V\ (0.097m)}{4.665 \times 10^{-4}\ \frac{N \cdot s}{m^2}}$$

or

(2) $Re = 2.04 \times 10^5\ V$, where $V \sim \frac{m}{s}$

Also, with $Q = \frac{\pi}{4} D^2 V$ and $p_1 - p_2 = \gamma h = \rho g h = 983.2\ \frac{kg}{m^3}(9.81\ \frac{m}{s^2})(1m)$
$= 9.65 \times 10^3\ \frac{N}{m^2}$

equation (1) becomes

$$\frac{\pi}{4}(0.097\ m)^2 V = C_n \frac{\pi}{4}(0.064m)^2 \left[\frac{2\ (9.65 \times 10^3 \frac{N}{m^2})}{(983.2\ \frac{kg}{m^3})(1 - 0.660^4)}\right]^{\frac{1}{2}}$$

or

(3) $V = 2.14\ C_n$

Trial and error solution using Fig. 8.43 for $C_n = C_n(Re, \beta = 0.660)$

Assume $C_n = 0.99$ From Eq. (3), $V = 2.14(0.99) = 2.12\ \frac{m}{s}$

From Eq. (2), $Re = 2.04 \times 10^5 (2.12) = 4.32 \times 10^5$ which from Fig. 8.43 gives $C_D = 0.99$ which checks with the assumed value.

Thus, $V = 2.12\ \frac{m}{s}$ and $Q = \frac{\pi}{4} D^2 V = \frac{\pi}{4}(0.097m)^2 (2.12 \frac{m}{s}) = \underline{\underline{0.0157\ \frac{m^3}{s}}}$

8.122

8.122 Water flows through the Venturi meter shown in Fig. P8.122. The specific gravity of the manometer fluid is 1.52. Determine the flowrate.

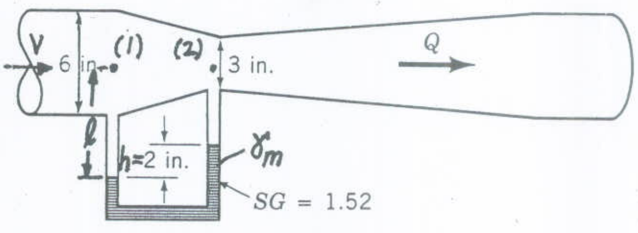


FIGURE P8.122

$$Q = C_V A_T \sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}}, \text{ where } \beta = \frac{d}{D} = \frac{3\text{ in.}}{6\text{ in.}} = 0.5$$

Also,

$$p_1 + \gamma\ell = p_2 + \gamma(\ell - h) + \gamma(SG)h \quad \text{or } p_1 - p_2 = \gamma(SG-1)h = \rho g(SG-1)h$$

Hence,

$$Q = C_V A_T \sqrt{\frac{2\rho g(SG-1)h}{\rho(1-\beta^4)}} \quad \text{or}$$

$$Q = C_V \frac{\pi}{4}\left(\frac{3}{12}\text{ft}\right)^2 \left[\frac{2(32.2\frac{\text{ft}}{\text{s}^2})(1.52-1)(\frac{2}{12}\text{ft})}{(1-0.5^4)}\right]^{1/2}$$

Thus,

$$Q = 0.1198\, C_V \qquad \text{Assume } C_V \approx 0.98 \text{ so that } Q = 0.1198(0.98) = 0.117\frac{\text{ft}^3}{\text{s}}$$

Hence,

$$V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.117\frac{\text{ft}^3}{\text{s}}}{\frac{\pi}{4}(\frac{6}{12}\text{ft})^2} = 0.596\frac{\text{ft}}{\text{s}} \text{ so that}$$

$$Re = \frac{VD}{\nu} = \frac{(0.596\frac{\text{ft}}{\text{s}})(\frac{6}{12}\text{ft})}{1.21\times10^{-5}\frac{\text{ft}^2}{\text{s}}} = 2.46\times10^4$$

From Fig. 8.45 at this Re, $C_V \approx 0.96 \neq 0.98$, the assumed value.
Hence, assume $C_V = 0.96$, or

$$Q \approx 0.1198(0.96) = 0.115\frac{\text{ft}^3}{\text{s}} \text{ and } V = \frac{0.115}{\frac{\pi}{4}(\frac{6}{12})^2} = 0.586\frac{\text{ft}}{\text{s}}$$

Therefore, $Re = \frac{0.586(\frac{6}{12})}{1.21\times10^{-5}} = 2.42\times10^4$ so that from Fig. 8.45,

$C_V \approx 0.96$ Checks with assumed value.

Hence, $\underline{\underline{Q = 0.115\frac{\text{ft}^3}{\text{s}}}}$

8.123

8.123 Water flows through the orifice meter shown in Fig. P8.123 at a rate of 0.10 cfs. If $d = 0.1$ ft, determine the value of h.

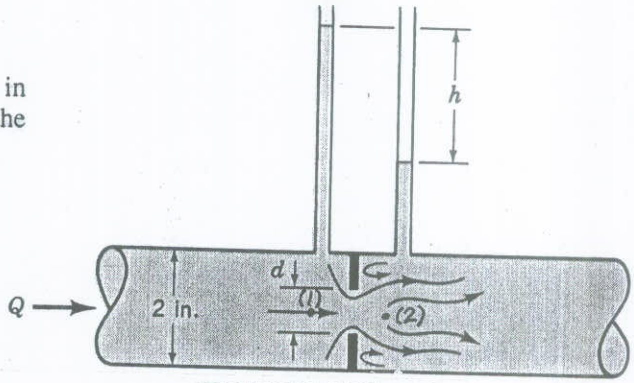


FIGURE P8.123

$$Q = C_o A_o \sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}}, \text{ where } \beta = \frac{d}{D} = \frac{0.1\,ft}{\frac{2}{12}\,ft} = 0.6,\ p_1 - p_2 = \gamma h = \rho g h \qquad (1)$$

Also, $V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{0.10\,\frac{ft^3}{s}}{\frac{\pi}{4}(\frac{2}{12}\,ft)^2} = 4.58\,\frac{ft}{s}$ so that

$$Re = \frac{VD}{\nu} = \frac{(4.58\,\frac{ft}{s})(\frac{2}{12}\,ft)}{1.21 \times 10^{-5}\,\frac{ft^2}{s}} = 6.31 \times 10^4$$ Hence, from Fig. 8.41, $C_o = 0.616$

Therefore, from Eq. (1):

$$0.10\,\frac{ft^3}{s} = (0.616)\,\frac{\pi}{4}(0.1\,ft)^2 \sqrt{\frac{2\rho(32.2\,\frac{ft}{s^2})h}{\rho(1-0.6^4)}} \quad \text{or} \quad h = \underline{\underline{5.77\,ft}}$$

8.124

8.124 Water flows through the orifice meter shown in Fig. P8.123 such that $h = 1.6$ ft with $d = 1.5$ in. Determine the flowrate.

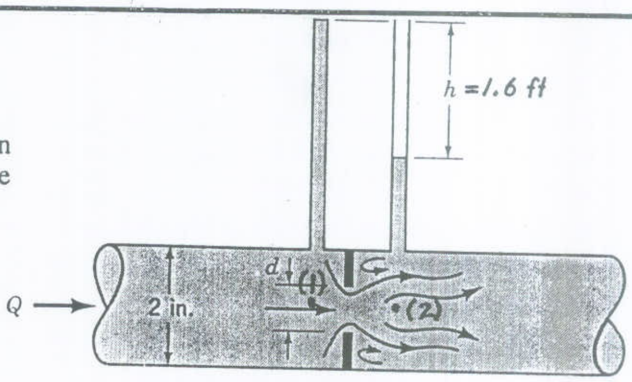


FIGURE P8.123

$Q = C_o A_o \sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}}$, where $\beta = \frac{d}{D} = \frac{1.5 \text{in.}}{2 \text{in.}} = 0.75$ and $p_1 - p_2 = \gamma h = \rho g h$

Thus,

$$Q = C_o \frac{\pi}{4}\left(\frac{1.5}{12} ft\right)^2 \left[\frac{2\rho(32.2 \frac{ft}{s})(1.6 ft)}{\rho(1-0.75^4)}\right]^{\frac{1}{2}}$$

or

$Q = 0.151 C_o$ (1)

Also, $Re = \frac{VD}{\nu} = \frac{V\left(\frac{2}{12} ft\right)}{1.21 \times 10^{-5} \frac{ft^2}{s}} = 1.38 \times 10^4 V$, where $V = \frac{Q}{\frac{\pi}{4}D^2} = 45.8Q$ (2)

Trial and error solution:

Assume $C_o = 0.6$; or from Eq.(1), $Q = 0.151(0.6) = 0.0906 \frac{ft^3}{s}$

Hence, from Eq.(2), $V = 45.8(0.0906) = 4.15$ and $Re = 5.73 \times 10^4$

From Fig. 8.41 with this Re and β, $C_o = 0.62 \neq 0.6$ (the assumed value)

Assume $C_o = 0.62$ or $Q = 0.151(0.62) = 0.0936 \frac{ft^3}{s}$, Thus $V = 45.8(0.0936)$ or $V = 4.29 \frac{ft}{s}$ and $Re = 5.92 \times 10^4$, From Fig. 8.41, $C_o = 0.62$, the assumed value.

Hence, $Q = \underline{\underline{0.0936 \frac{ft^3}{s}}}$

8.125

8.125 The scale reading on the rotameter shown in Fig. P8.125 and Video V8.14 (also see Fig. 8.46) is directly proportional to the volumetric flowrate. With a scale reading of 2.6 the water bubbles up approximately 3 in. How far will it bubble up if the scale reading is 5.0?

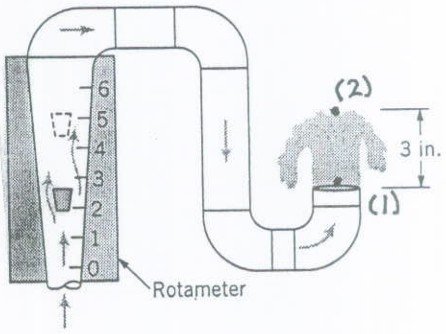


FIGURE P8.125

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$p_1 = p_2 = 0$, $z_1 = 0$, $V_2 = 0$, so that with no losses ($h_L = 0$),

(1) $$\frac{V_1^2}{2g} = z_2$$

For the rotameter $Q = K \times SR$ where SR = scale reading and K is a constant.

Thus,

$$V_1 = \frac{Q_1}{A_1} = \frac{K \times SR}{A_1}$$ so that when combined with Eq.(1),

$$\frac{K^2 (SR)^2}{A_1^2 (2g)} = z_2 \quad \text{or} \quad \frac{K^2 (2.6)^2}{A_1 (2g)} = \left(\tfrac{3}{12}\,\text{ft}\right) \quad \text{and} \quad \frac{K^2 (5.0)^2}{A_1 (2g)} = h$$

By dividing these two equations,

$$\frac{(5.0)^2}{(2.6)^2} = \frac{h}{\left(\frac{3}{12}\,\text{ft}\right)} \quad \text{or} \quad h = 0.925\ \text{ft} = \underline{\underline{11.1\ \text{in.}}}$$

8.126

8.126 Friction Factor for Laminar and Transitional Pipe Flow

Objective: Theoretically, the friction factor, f, for laminar pipe flow is given by $f = 64/\text{Re}$, where the Reynolds number, $\text{Re} = \rho V D/\mu$, is based on the average velocity, V, within the pipe and the pipe diameter, D. Also, the flow is normally laminar for $\text{Re} < 2100$. The purpose of this experiment is to use the device shown in Fig. P8.126 to investigate these two properties.

Equipment: Small diameter metal tubes (pipes), air supply with flow regulator, rotameter flow meter, manometer.

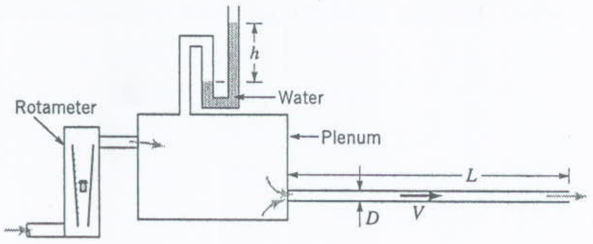
Experimental Procedure: Attach a tube of length L and diameter D to the plenum. Adjust the flow regulator to obtain the desired flowrate as measured by the rotameter. Record the manometer reading, h, so that the pressure difference between the plenum (tank) and the free jet at the end of the tube can be determined. Repeat for several different flowrates and tube diameters. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: For each of the data sets determine the pressure difference, $\Delta p = \gamma_m h$, between the plenum pressure and the free jet pressure. Here γ_m is the specific weight of the manometer fluid. Use the energy equation, Eq. 5.84, to determine the friction factor, f. Assume the loss coefficient for the pipe entrance is $K_L = 0.8$. Also calculate the Reynolds number, Re, for each data set.

Graph: On a log-log graph, plot the experimentally determined friction factor, f, as ordinates and the Reynolds number, Re, as abscissas.

Results: On the same graph, plot the theoretical friction factor for laminar flow, $f = 64/\text{Re}$, as a function of the Reynolds number. Based on the experimental data, determine the maximum value of the Reynolds number for which the flow in these pipes is laminar.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.126

(con't)

8.126 (con't)

Solution for Problem 8.126: Friction Factor for Laminar and Transitional Pipe Flow

L, in.	H_{atm}, in. Hg	T, deg F
24	28.9	73

h, in.	Q, ml/min	Q, cfs	V, fps	Re	f	Theoretical Re	 f
D = 0.108 in. Data						100	0.6400
7.5	6600	0.003887	61.11	3202	0.0341	2100	0.0305
6.75	6200	0.003652	57.40	3008	0.0349		
6.26	6000	0.003534	55.55	2911	0.0345		
5.54	5650	0.003328	52.31	2741	0.0344		
4.66	5150	0.003033	47.68	2499	0.0349		
4.29	5000	0.002945	46.29	2426	0.0339		
3.92	4860	0.002863	45.00	2358	0.0325		
3.48	4600	0.002709	42.59	2232	0.0322		
3.21	4500	0.002651	41.66	2183	0.0307		
2.34	3700	0.002179	34.26	1795	0.0338		
1.86	2900	0.001708	26.85	1407	0.0461		
1.11	1800	0.001060	16.67	873	0.0758		
0.63	1100	0.000648	10.18	534	0.1194		
D = 0.046 in. Data							
9.52	560	0.000330	28.58	638	0.1007		
7.68	475	0.000280	24.24	541	0.1134		
7.08	425	0.000250	21.69	484	0.1311		
5.26	315	0.000186	16.08	359	0.1785		
3.39	221	0.000130	11.28	252	0.2348		
2.61	165	0.000097	8.42	188	0.3256		
D = 0.063 in. Data							
4.58	925	0.000545	25.17	770	0.0838		
3.32	680	0.000401	18.50	566	0.1140		
2.51	530	0.000312	14.42	441	0.1431		
1.48	325	0.000191	8.84	270	0.2270		
0.86	190	0.000112	5.17	158	0.3893		

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{H2O} * H_{atm}$ = 847 lb/ft^3*(28.9/12 ft) = 2040 lb/ft^2
R = 1716 ft lb/slug deg R
T = 73 + 460 = 533 deg R

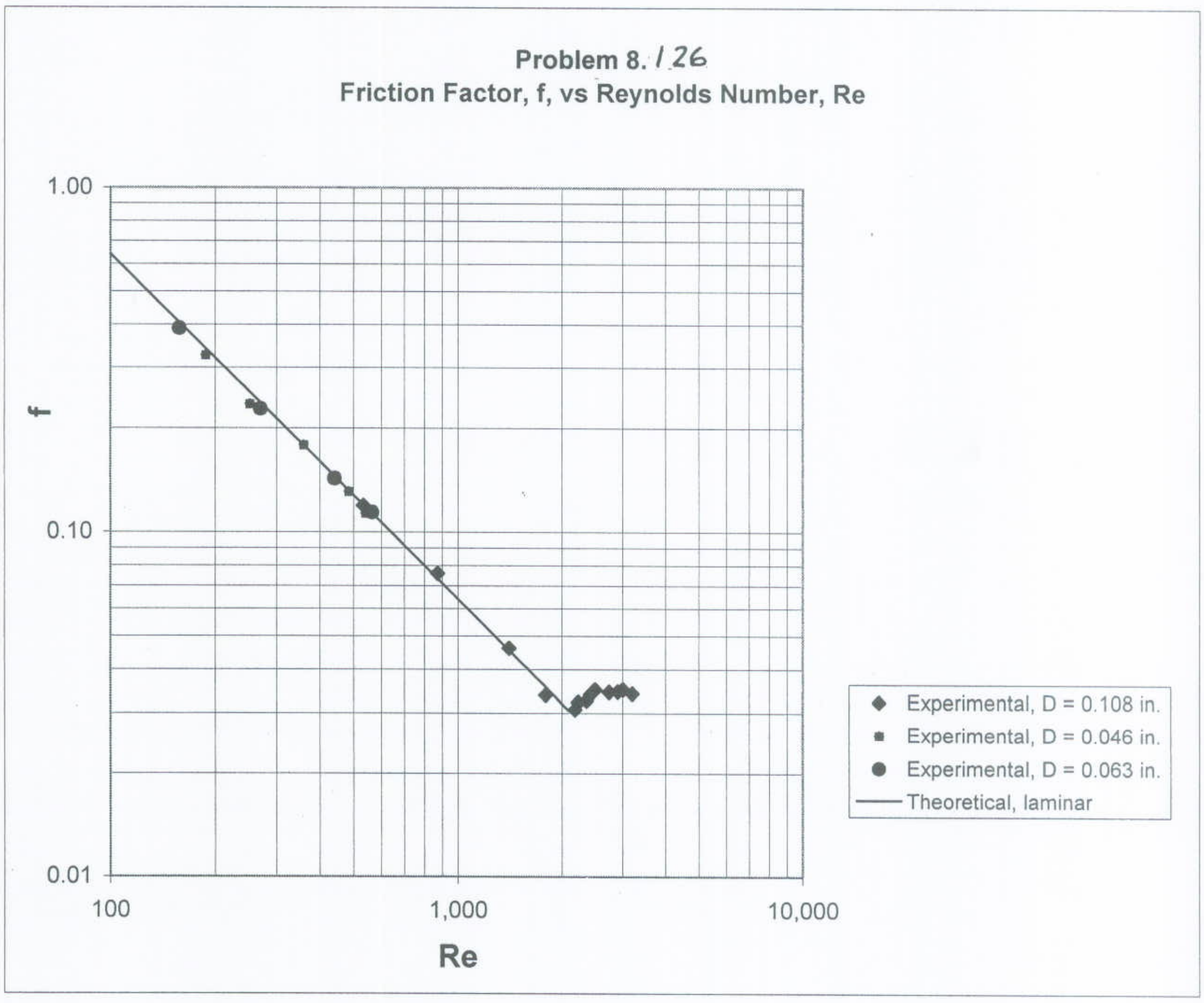
Thus, ρ = 0.00223 slug/ft^3 and $\gamma = \rho$*g = 0.0718 lb/ft^3
Also, μ = 3.83E-7 lb s/ft^2

Theoretical for laminar flow: f = 64/Re = 64/(ρDV/μ)

$\Delta p/\gamma$ = (fL/D + K_L + 1)(V^2/2g) where K_L = entrance loss coefficient = 0.8 and V = Q/(πD^2/4)

(con't)

8.126 (con't)


Problem 8.126
Friction Factor, f, vs Reynolds Number, Re
1.00
0.10
0.01
f
100
1,000
10,000
Re
◆ Experimental, D = 0.108 in.
■ Experimental, D = 0.046 in.
● Experimental, D = 0.063 in.
Theoretical, laminar

8.127

8.127 Calibration of an Orifice Meter and a Venturi Meter

Objective: Because of various real-world, nonideal conditions, neither orifice meters nor Venturi meters operate exactly as predicted by a simple theoretical analysis. The purpose of this experiment is to use the device shown in Fig. P8.127 to calibrate an orifice meter and a Venturi meter.

Equipment: Water tank with sight gage, pump, Venturi meter, orifice meter, manometers.

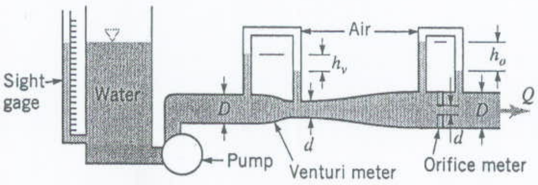
Experimental Procedure: Determine the pipe diameter, D, and the throat diameter, d, for the flow meters. Note that each meter has the same values of D and d. Make sure that the tubes connecting the manometers to the flow meters do not contain any unwanted air bubbles. This can be verified by noting that the manometer readings, h_v, and h_o, are zero when the system is full of water and the flowrate, Q, is zero. Turn on the pump and adjust the valve to give the desired flowrate. Record the time, t, it takes for a given volume, V, of water to be pumped from the tank. The volume can be determined from using the sight gage on the tank. At this flowrate record the manometer readings. Repeat for several different flowrates.

Calculations: For each data set determine the volumetric flowrate, $Q = V/t$, and the pressure differences across each meter, $\Delta p = \gamma_m h$, where γ_m is the specific weight of the manometer fluid. Use the flow meter equations (see Section 8.6.1) to determine the orifice discharge coefficient, C_o, and the Venturi discharge coefficient, C_v, for these meters.

Graph: On a log-log graph, plot flowrate, Q, as ordinates and pressure difference, Δp, as abscissas.

Result: On the same graph, plot the ideal flowrate, Q_{ideal} (see Eq. 8.37), as a function of pressure difference.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.127

(con't)

8.127 (con't)

Solution for Problem 8.127: Calibration of an Orifice Meter and a Venturi Meter

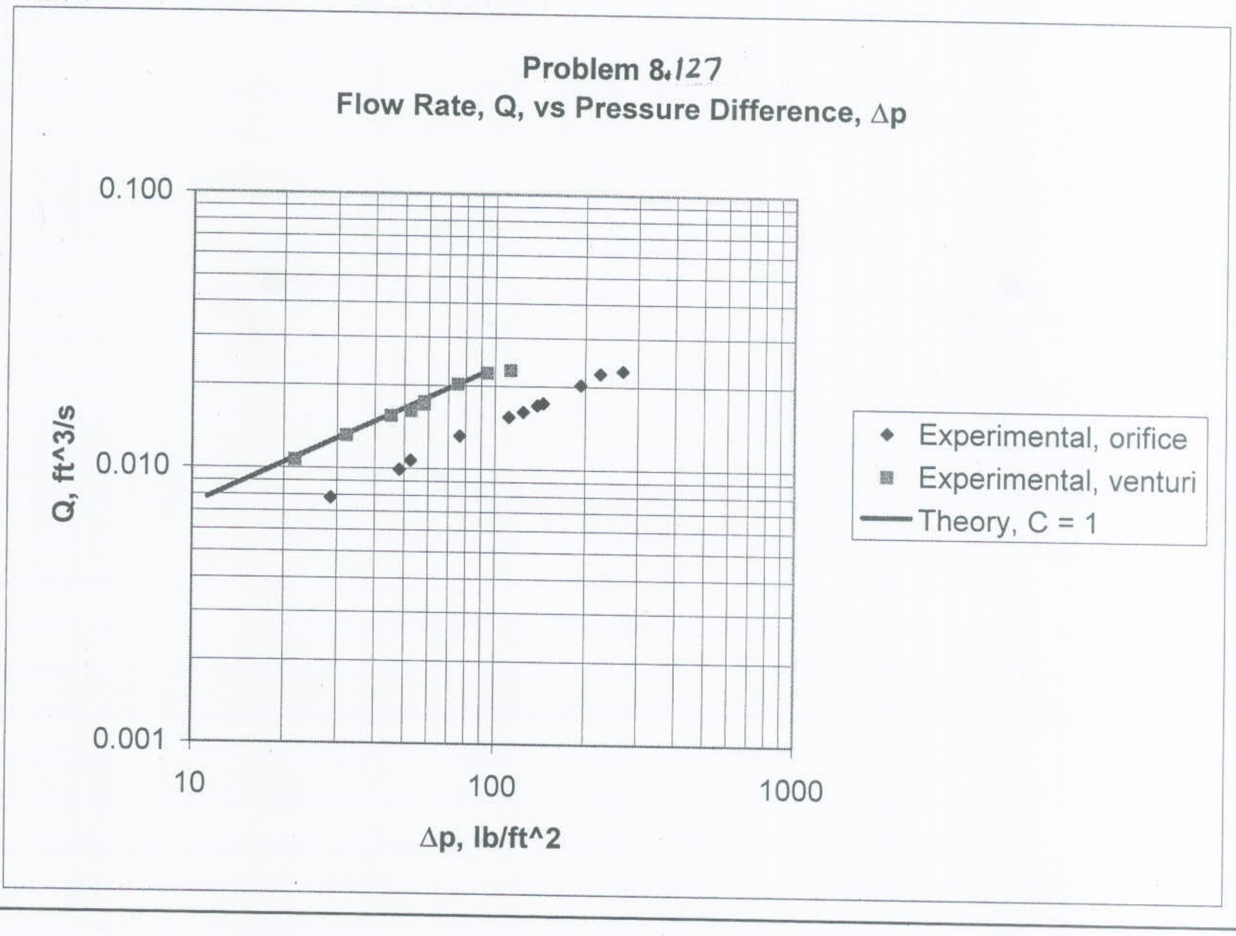
d, in.	D, in.	**V**, gallons
0.625	1.025	2.00

t, s	h_o, in.	h_v, in.	Δp_o, lb/ft^2	Δp_v, lb/ft^2	Q, ft^3/s	C_o	C_v	Ideal C = 1 Δp, lb/ft^2
27.0	9.3	3.8	48.4	19.8	0.0099	0.611	0.956	18.0
13.2	37.1	14.5	192.9	75.4	0.0203	0.626	1.001	75.5
34.2	5.5	1.9	28.6	9.9	0.0078	0.627	1.067	11.2
16.6	23.9	10.1	124.3	52.5	0.0161	0.620	0.953	47.7
12.0	43.2	18.1	224.6	94.1	0.0223	0.638	0.985	91.4
11.7	51.3	21.7	266.8	112.8	0.0229	0.600	0.923	96.1
15.4	27.9	11.2	145.1	58.2	0.0174	0.618	0.976	55.5
25.1	10.1	4.2	52.5	21.8	0.0107	0.631	0.978	20.9
20.4	14.7	6.2	76.4	32.2	0.0131	0.643	0.990	31.6
17.3	21.4	8.7	111.3	45.2	0.0155	0.629	0.986	44.0
15.7	26.7	11.2	138.8	58.2	0.0170	0.620	0.957	53.4
			Average discharge coefficient:			0.624 orifice	0.979 venturi	

Q = **V** gal/t s x (231 in.^3/gal)x(1 ft^3/1728 in.^3)

$\Delta p = \gamma_{H2O}$*h = 62.4 lb/ft^3 *h ft

Q_v =A_2/[1 - (A_2/A_1)^2]^0.5*C_v*(2*g*$\Delta p_v/\gamma_{H2O}$)^0.5
and
Q_o =A_2/[1 - (A_2/A_1)^2]^0.5*C_o*(2*g*$\Delta p_o/\gamma_{H2O}$)^0.5
where
A_1 = π D^2/4 = π (1.025/12 ft)^2/4 = 0.00573 ft^2
and
A_2 = π d^2/4 = π (0.625/12 ft)^2/4 = 0.00213 ft^2



8.128

8.128 Flow from a Tank through a Pipe System

Objective: The rate of flow of water from a tank is a function of the pipe system used to drain the tank. The purpose of this experiment is to use a pipe system as shown in Fig. P8.128 to investigate the importance of major and minor head losses in a typical pipe flow situation.

Equipment: Water tank; various lengths of galvanized iron pipe; various threaded pipe fittings (valves, elbows, etc.); pipe wrenches; stop watch; thermometer.

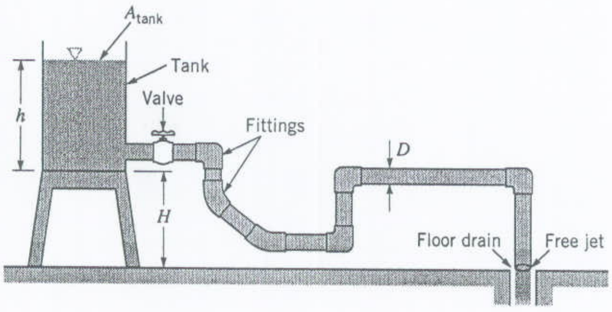
Experimental Procedure: Use the pipe segments and pipe fittings to construct a suitable pipeline through which the tank water may flow into a floor drain. Measure the pipe diameter, D, and the various pipe lengths and note the various valves and fittings used. Measure the elevation difference, H, between the bottom of the tank and the outlet of the pipe. Also determine the cross-sectional area of the tank, A_{tank}. Fill the tank with water and record the water temperature, T. With the pipeline valve wide open, measure the water depth, h, in the tank as a function of time, t, as the tank drains.

Calculations: Calculate the experimentally determined flowrate, Q_{ex}, from the tank as $Q_{\text{ex}} = -A_{\text{tank}}\, dh/dt$, where the time rate of change of water depth, dh/dt, is obtained from the slope of the h versus t graph. Select a typical water depth, h_1, for this calculation.

Graph: Plot the water depth, h, in the tank as ordinates and time, t, as abscissas.

Results: For the pipe system used in this experiment, use the energy equation to calculate the theoretical flowrate, Q_{th}, based on three different assumptions. Use the same typical water depth, h_1, for the theoretical calculations as was used in determining Q_{ex}. First, calculate Q_{th} under the assumption that all losses are negligible. Second, calculate Q_{th} if only major losses (pipe friction) are important. Third, calculate Q_{th} if both major and minor losses are important.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.128

(con't)

8.128 (con't)

Solution for Problem 8.128: Flow from a Tank Through a Pipe System

The pipe is galvanized iron with threaded fittings.
The system contains:

- one sharp edged entrance
- one fully open globe valve
- two 45-deg elbows
- four 90-deg elbows

D, in.	A_{tank}, ft^2	H, ft	Total pipe length, in.	T, deg F
0.595	0.654	1.00	135	71

h, ft	t, s
1.00	0
0.90	13
0.80	26
0.70	40
0.60	54
0.50	67
0.40	81

Experimental: Q_{ex} = -(dh/dt)*A_{tank} = -(0.0074 ft/s)*(0.654 ft^2) = <u>0.00484 ft^3/s</u>

Theoretical with no losses: $Q_{th} = V_2 * A_2$, where when h = 0.90 ft

V_2 = (2g*(h + H))^0.5 = (2*32.2*(0.9 +1.0))^0.5 = 11.06 ft/s

and with $A_2 = \pi D^2/4 = \pi*(0.595/12\text{ ft})^2/4 = 0.00193\text{ ft}^2$

Q_{th} = 0.00193 ft^2*(11.06 ft/s) = <u>0.0213 ft^3/s</u>

Theoretical with major losses: $Q_{th} = V_2 * A_2$, where the energy equation gives

$h + H = V_2^2/2g(1 + fL/D)$, where again use h = 0.90 ft and f is a function of Re and ε/D

Thus, with h = 0.90 ft,

$1.9 = (V_2^2/64.4)*(1 + f*135/0.595)$, or

$122.4 = V_2^2*(1 + 227f)$

$Re = V_2D/\nu = V_2*(0.595/12\text{ ft})/(1.04E-5\text{ ft}^2/s) = 4768*V_2$

and

ε/D =0.0005 ft/(0.595/12 ft) = 0.0101

Trial and error solution*: Guess f, solve for V_2, calculate Re, obtain new f from Moody chart

The solution is: f = 0.041, V_2 = 3.44 ft/s, Re = 16,430

Q_{th} = 0.00193 ft^2*(3.44 ft/s) = <u>0.00664 ft^3/s</u>

Theoretical with major and minor losses: The energy equation gives

$h + H = (1 + fL/D + \Sigma K_L)V_2^2/2g$

where $\Sigma K_L = 0.5 + 10 + 2*0.4 + 4*1.5 = 17.3$

Thus, with h = 0.9 ft

$1.9 = (V_2^2/64.4)*(17.3 + f*135/0.595)$, or

$122.4 = V_2^2*(17.3 + 227f)$

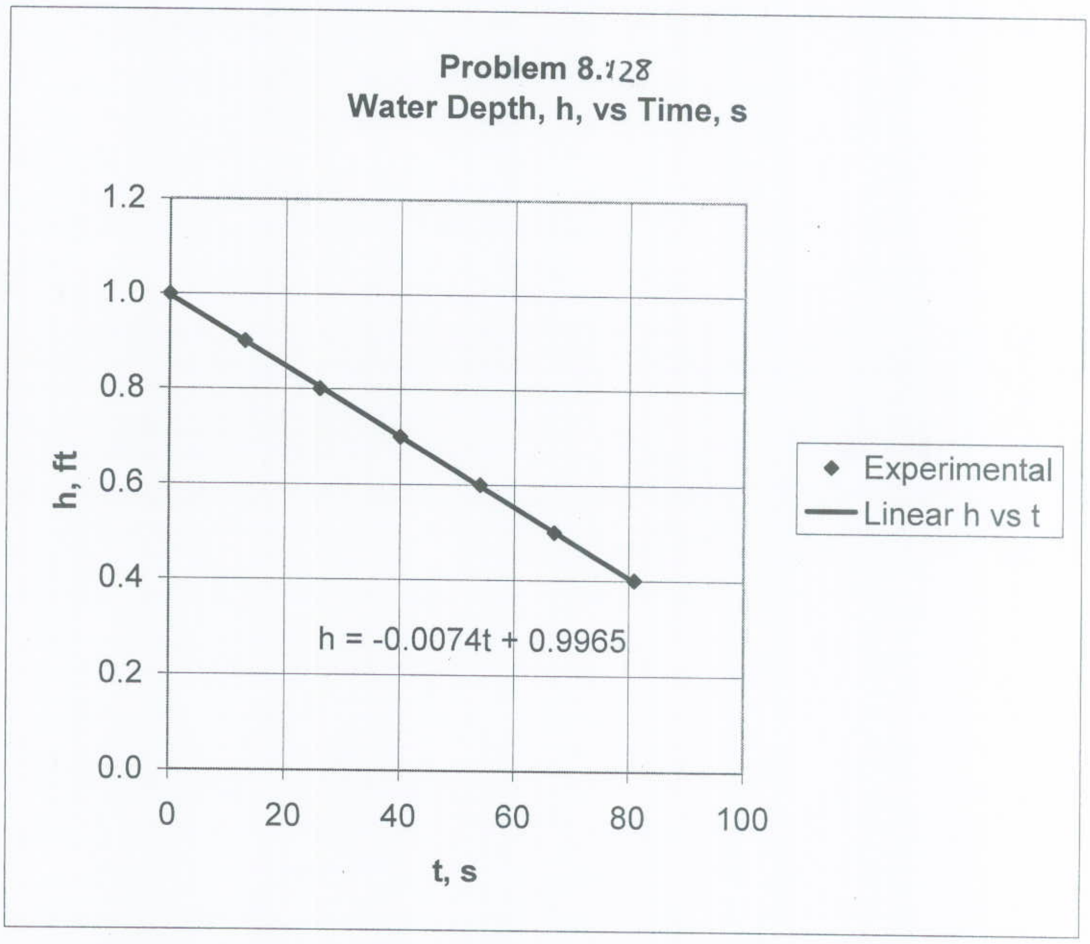
Trial and error solution gives*: f = 0.42, V_2 = 2.14 ft/s, Re = 10,200

Q_{th} = 0.00193 ft^2*(2.14 ft/s) = <u>0.00413 ft^3/s</u>

* As an alternate solution method, use the Colebrook equation (Eq. 8.35) rather than the Moody chart (Fig. 8.20) and use a computer root-finding technique to solve the equation.

(con't)

8.128 (con't)


Problem 8.128
Water Depth, h, vs Time, s
1.2
1.0
0.8
0.6
0.4
0.2
0.0
h, ft
0
20
40
60
80
100
t, s
h = -0.0074t + 0.9965
◆ Experimental
Linear h vs t

8.129

8.129 Flow of Water Pumped from a Tank and through a Pipe System

Objective: The rate of flow of water pumped from a tank is a function of the pump properties and of the pipe system used. The purpose of this experiment is to use a pump and pipe system as shown schematically in Fig. P8.129 to investigate the rate at which the water is pumped from the tank.

Equipment: Water tank; centrifugal pump; various lengths of galvanized iron pipe; various threaded pipe fittings (valves, elbows, unions, etc.); pipe wrenches; stop watch; thermometer.

Experimental Procedure: Use the pipe segments and pipe fittings to construct a suitable pipeline through which the tank water may be pumped into a sink. Measure the pipe diameter, *D*, and the various pipe lengths and note the various valves and fittings used. Measure the elevation difference, *H*, between the bottom of the tank and the outlet of the pipe. Also determine the cross-sectional area of the tank, A_{tank}. Fill the tank with water and record the water temperature, *T*. With the pipeline valves wide open, measure the water depth, *h*, in the tank as a function of time, *t*, as water is pumped from the tank.

Calculations: Calculate the experimentally determined flowrate, Q_{ex}, from the tank as $Q_{\text{ex}} = -A_{\text{tank}}\, dh/dt$, where the time rate of change of water depth, dh/dt, is obtained from the slope of the *h* versus *t* graph.

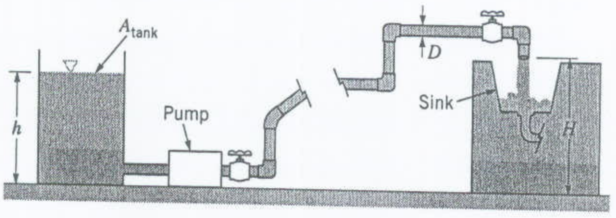
Graph: Plot the water depth, *h*, in the tank as ordinates and time, *t*, as abscissas.

Results: For the pipe system used in this experiment, use the energy equation to calculate the pump head, h_p, needed to in order to produce a given flowrate, *Q*. For these calculations include all major and minor losses in the pipe system. Plot the system curve (i.e., pump head as ordinates and flowrate as abscissas) based on the results of these calculations. On the same graph, plot the pump curve (i.e., h_p as a function of *Q*) as supplied by the pump manufacturer. For the pump used this curve is given by

$$h_p = -2.44 \times 10^5\, Q^2 + 51.0\, Q - 12.5$$

where *Q* is in ft³/s and h_p is in ft. From the intersection of the system curve and the pump curve, determine the theoretical flowrate that the pump should provide for the pipe system used.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.129

(con't)

8.129 (con't)

Solution for Problem 8.129: Flowrate of Water Pumped from a Tank and Through a Pipe System

The pipe is galvanized iron with threaded fittings.
The system contains:

- one sharp entrance
- eight 90-deg elbows
- two 45-deg elbows
- two globe valves
- one union

D, in.	A_{tank}, ft^2	H, ft	Total pipe length, in.	T, deg F
0.625	0.647	3.50	242	62

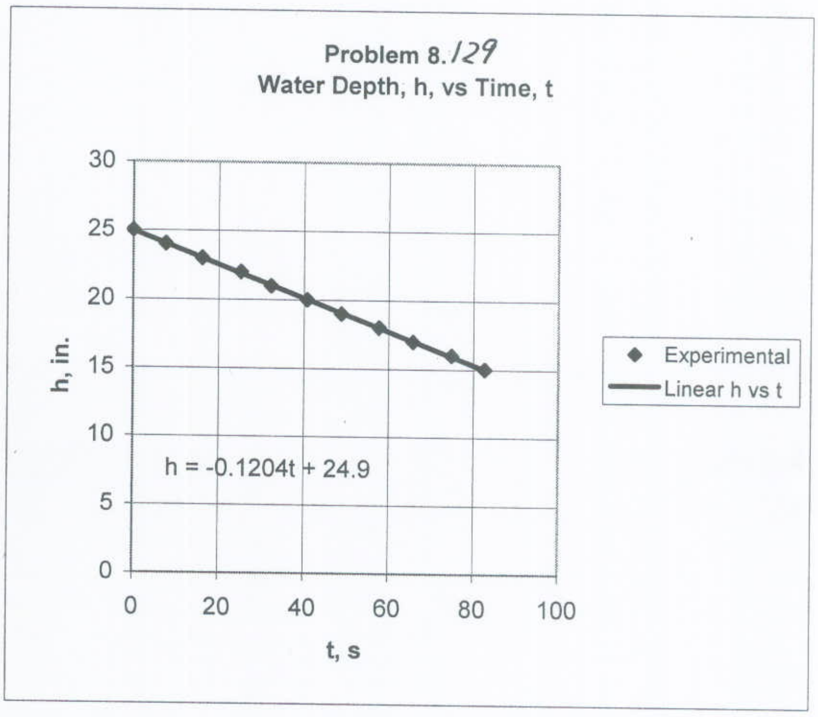
		Pump equation			System equation		
h, in.	t, s	h_p, ft	Q, ft^3/s	V, ft/s	Re	f	h_p, ft
25	0	12.50	0.000	0.00	0		2
24	7.6	12.31	0.001	0.47	2070	0.0309	2.16
23	16.1	11.63	0.002	0.94	4140	0.0490	2.73
22	25.2	10.46	0.003	1.41	6210	0.0470	3.62
21	32.3	8.80	0.004	1.88	8281	0.0450	4.84
20	40.8	6.66	0.005	2.35	10351	0.0430	6.37
19	48.9	4.02	0.006	2.81	12421	0.0425	8.27
18	57.7	0.90	0.007	3.28	14491	0.0420	10.50
17	65.7						
16	74.9						
15	82.7						

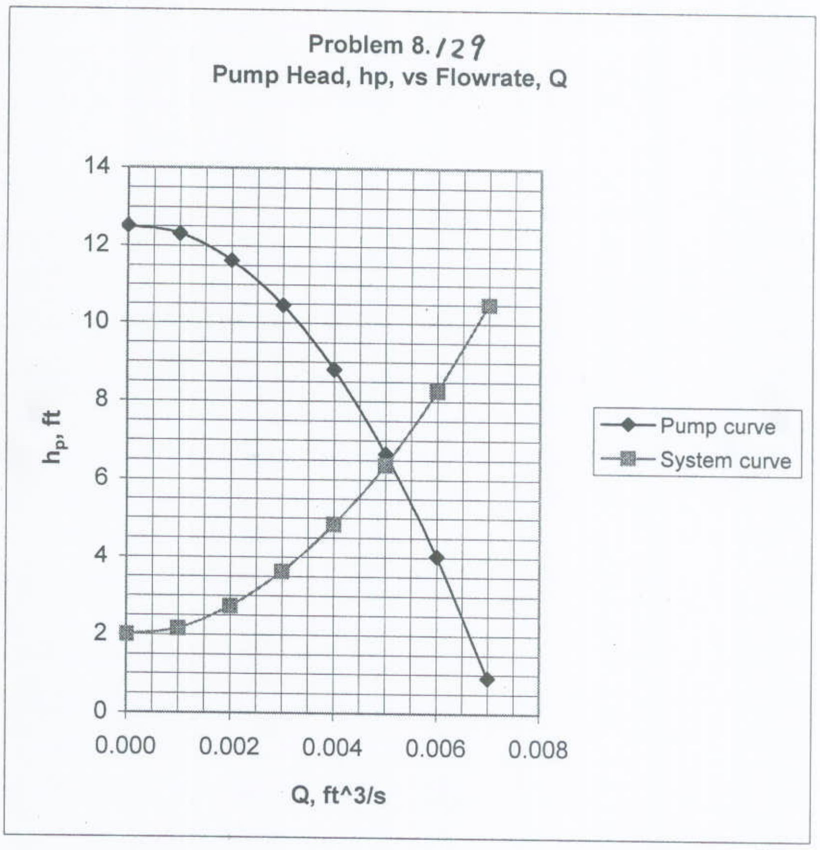
Experimental:
$Q_{ex} = -A_{tank}$*(dh/dt) where from the graph, dh/dt = -0.1204 in./s
Thus,
Q_{ex} = -(0.647 ft^2)*(-0.1204/12 ft/s) = <u>0.00669 ft^3/s</u>

Theoretical:
The energy equation gives
$h + h_p - h_L = H + V^2/2g$, where
$h_L = (fL/D + \Sigma K_L)*V^2/2g = (f*(242\text{ in.}/0.625\text{ in.}) + 0.5 + 8*1.5 + 2*0.4 + 2*10 + 0.08)*V^2/2g$
$= (387*f + 33.4)*V^2/(2*32.2) = (6.01*f + 0.519)*V^2$
Thus, with h = 18 in. = 1.5 ft,
$h_p = H - h + h_L + V^2/2g = 3.5 - 1.5 + (6.01*f + 0.519)*V^2 + V^2/(64.4)$
or
$h_p = 2.0 + (6.01*f + 0.535)*V^2$
But $V = Q/A = Q/(\pi D^2/4) = Q/(\pi*(0.625/12\text{ ft})^2/4) = 469*Q$
Thus, the system equation is
$h_p = 2.0 + (6.01*f + 0.535)*(469*Q)^2 = 2.0 + (1.32\text{E}+6*f + 1.18\text{E}+5)*Q^2$
Also, obtain f from the Moody chart with
$Re = VD/\nu = V*(0.625/12\text{ ft})/(1.18\text{E-}5\text{ ft}^2/\text{s}) = 4414*V$
$\varepsilon/D = 0.0005\text{ ft}/(0.625/12\text{ ft}) = 0.0096$
From the graph, the pump and system equations intersect at Q_{th} = <u>0.0051 ft^3/s</u>

(con't)

8.129 (con't)


Problem 8.129
Water Depth, h, vs Time, t
30
25
20
15
10
5
0
h, in.
0
20
40
60
80
100
t, s
h = -0.1204t + 24.9
◆ Experimental
Linear h vs t


Problem 8.129
Pump Head, hp, vs Flowrate, Q
14
12
10
8
6
4
2
0
h_p, ft
0.000
0.002
0.004
0.006
0.008
Q, ft^3/s
Pump curve
System curve

8.130

8.130 Pressure Distribution in the Entrance Region of a Pipe

Objective: The pressure distribution in the entrance region of a pipe is different than that in the fully developed portion of the pipe. The purpose of this experiment is to use an apparatus, as shown in Fig. P8.130, to determine the pressure distribution and the head loss in the pipe entrance region.

Equipment: Air supply with flow meter, pipe with static pressure taps, manometer, ruler, barometer, thermometer.

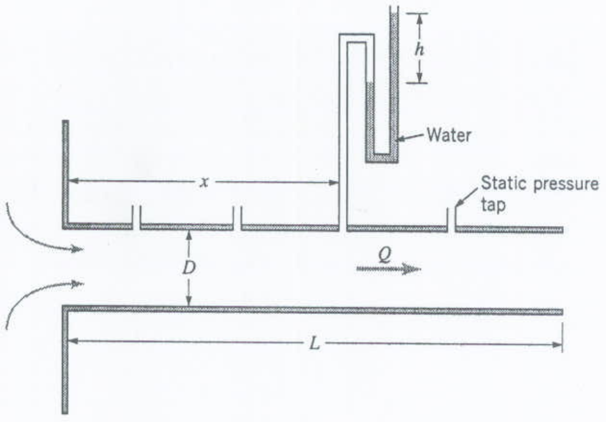
Experimental Procedure: Measure the diameter, D, and length, L, of the pipe and the distance, x, from the pipe inlet to the various static pressure taps. Adjust the flowrate, Q, to the desired value. Record the manometer readings, h, at the various distances from the pipe entrance. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Determine the average velocity, $V = Q/A$, in the pipe and the pressure $p = \gamma_m h$ at the various locations, x, along the pipe. Here γ_m is the specific weight of the manometer fluid.

Graph: Plot the pressure, p, within the pipe as ordinates and the axial location, x, as abscissas.

RESULT: Use the graph to determine the entrance length, L_e, for the pipe. This can be done by noting the approximate location at which the pressure distribution becomes linear with distance along the pipe (i.e., where dp/dx becomes constant). Use the experimental data to determine the friction factor for fully developed flow in this pipe. Also determine the entrance loss coefficient, $K_{L\,\text{ent}}$.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.130

(con't)

8.130 (con't)

Solution for Problem 8.130: Pressure Distribution in the Entrance Region of a Pipe

D, in.	L, in.	Q, ft^3/s	H_{atm}, in. Hg	T, deg F
0.74	50	0.481	29.7	75

x, in.	h, in.	p, lb/ft^2
0	9.98	51.9
1	7.21	37.5
2	6.61	34.4
4	6.19	32.2
6	5.82	30.3
10	5.15	26.8
15	4.23	22.0
20	3.64	18.9
30	2.28	11.9
40	1.09	5.7
50	0	0.0

$\rho = p_{atm}$/RT where

$p_{atm} = \gamma_{Hg}{}^*H_{atm}$ = 847 lb/ft^3*(29.7/12 ft) = 2096 lb/ft^2
R = 1716 ft lb/slug deg R
T = 75 + 460 = 535 deg R

Thus, ρ = 0.00228 slug/ft^3

V = Q/A =(0.481 ft^3/s)/(π*(0.74/12 ft)^2/4) = 161 ft/s

p = γ_{H2O}*h

From the graph, the p vs x results are linear after (approximately) x = 15 in. Thus, L_e = 15 in.

For the fully developed flow portion, dp/dx = -fρV^2/2D and from the graph dp/dx = -0.635 (lb/ft^2)/in. Thus,

f = 0.635 (lb/ft^2)/in.*2*0.74 in./(0.00228 slugs/ft^3*(161 ft/s)^2) = 0.0159

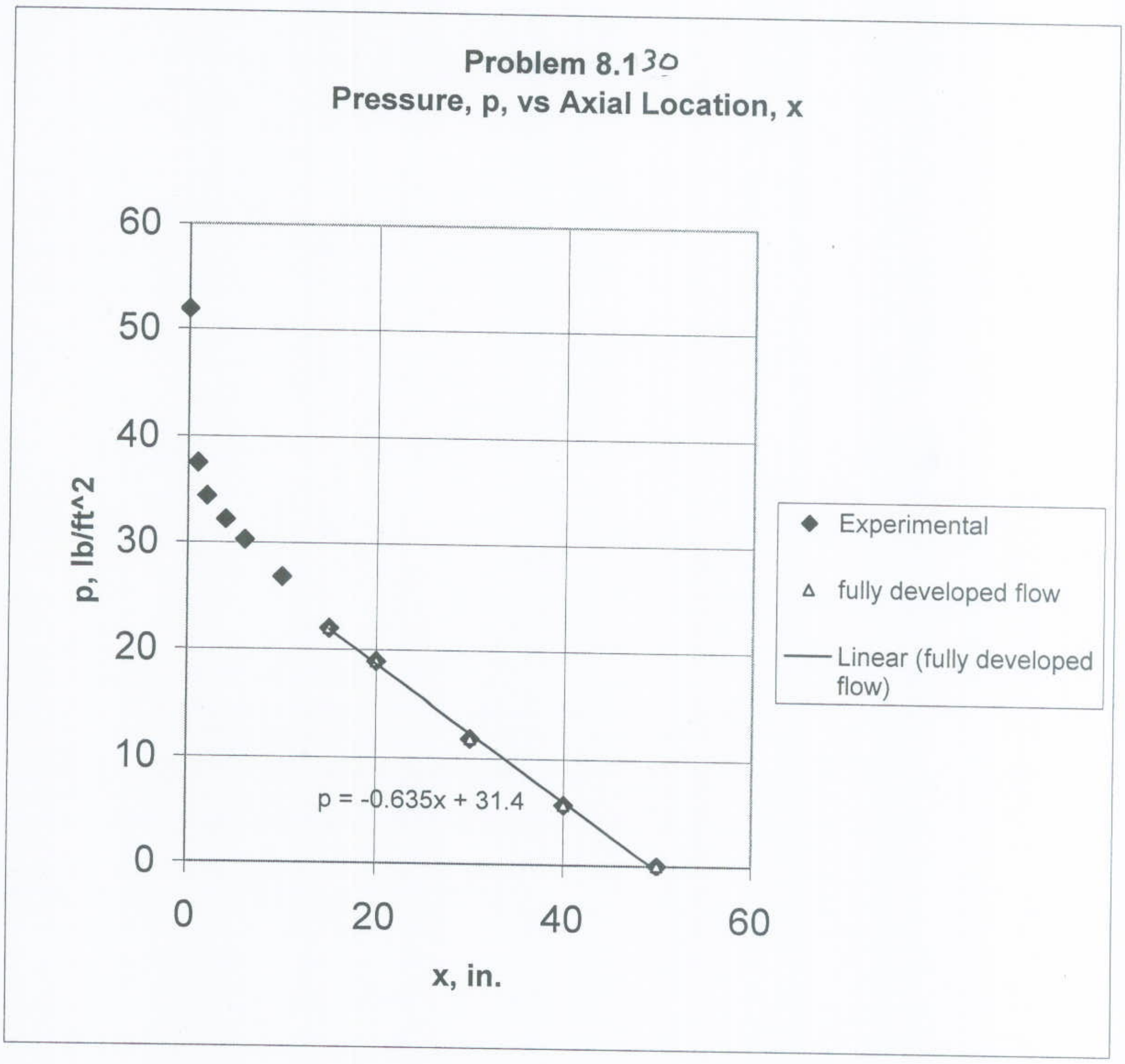
From the entrance to the exit of the pipe p_{ent} = (K_L + fL/D)ρV^2/2
Thus,

K_L = 2p_{ent}/(ρV^2) -fL/D= 2*51.9 lb/ft^2/(0.00228 slugs/ft^3*(161 ft/s)^2) - 0.0159*50in./0.74 in.
=0.682

Results: L_e = 15 in.; f = 0.0159, and K_L = 0.682.

(con't)

8.130 (con't)


Problem 8.130
Pressure, p, vs Axial Location, x
p, lb/ft^2
x, in.
0
10
20
30
40
50
60
0
20
40
60
p = -0.635x + 31.4
◆ Experimental
△ fully developed flow
Linear (fully developed flow)

8.131

8.131 Power Loss in a Coiled Pipe

Objective: The amount of power, P, dissipated in a pipe depends on the head loss, h_L, and the flowrate, Q. The purpose of this experiment is to use an apparatus as shown in Fig. P8.131 to determine the power loss in a coiled pipe and to determine how the coiling of the pipe affects the power loss.

Equipment: Air supply with a flow meter; flexible pipe that can be used either as a straight pipe or formed into a coil; manometer; barometer; thermometer.

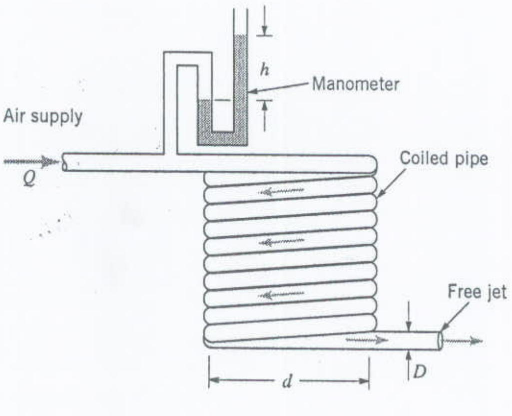
Experimental Procedure: Straighten the pipe and fasten it to the air supply exit. Measure the diameter, D, and length, L, of the pipe. Adjust the flowrate, Q, to the desired value and determine the manometer reading, h. Repeat the measurements for various flowrates. Form the pipe into a coil of diameter d and repeat the flowrate-pressure measurements. Record the barometer reading, H_{bar}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the manometer data to determine the pressure drop, $\Delta p = \gamma_m h$, and head loss, $h_L = \Delta p/\gamma$, as a function of flowrate, Q, for both the straight and coiled pipes. Here γ_m is the specific weight of the manometer fluid and γ is the specific weight of the flowing air. Also calculate the power loss, $P = \gamma Q h_L$, for both the straight and coiled pipes.

Graph: Plot head loss, h_L, as ordinates and flowrate, Q, as abscissas.

Results: On a log-log graph, plot the power loss, P, as a function of flowrate for both the straight and coiled pipes. Determine the best-fit straight lines through the data.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P8.131

(con't)

8.131 (con't)

Solution for Problem 8.131: Power Loss in a Coiled Pipe

D, in.	L, ft	H_{atm}, in. Hg	T, deg F
1.44	18	29.9	80

h, in.	Q, ft^3/s	Δp, lb/ft^2	h_L, ft	P, hp
Straight Pipe Data (d = infinity)				
10	1.19	52.0	709	0.1125
8	1.06	41.6	568	0.0802
6	0.913	31.2	426	0.0518
4	0.731	20.8	284	0.0276
2	0.505	10.4	142	0.0095
Coiled Pipe Data (d = 8 in.)				
10	0.835	52.0	709	0.0789
8	0.745	41.6	568	0.0563
6	0.641	31.2	426	0.0364
4	0.517	20.8	284	0.0196
2	0.357	10.4	142	0.0068

$\Delta p = \gamma_{H2O}h$ where γ_{H2O} = 62.4 lb/ft^3

$h_L = \Delta p/\gamma$ where $\gamma = g\rho$

$\rho = p_{atm}/RT$ where

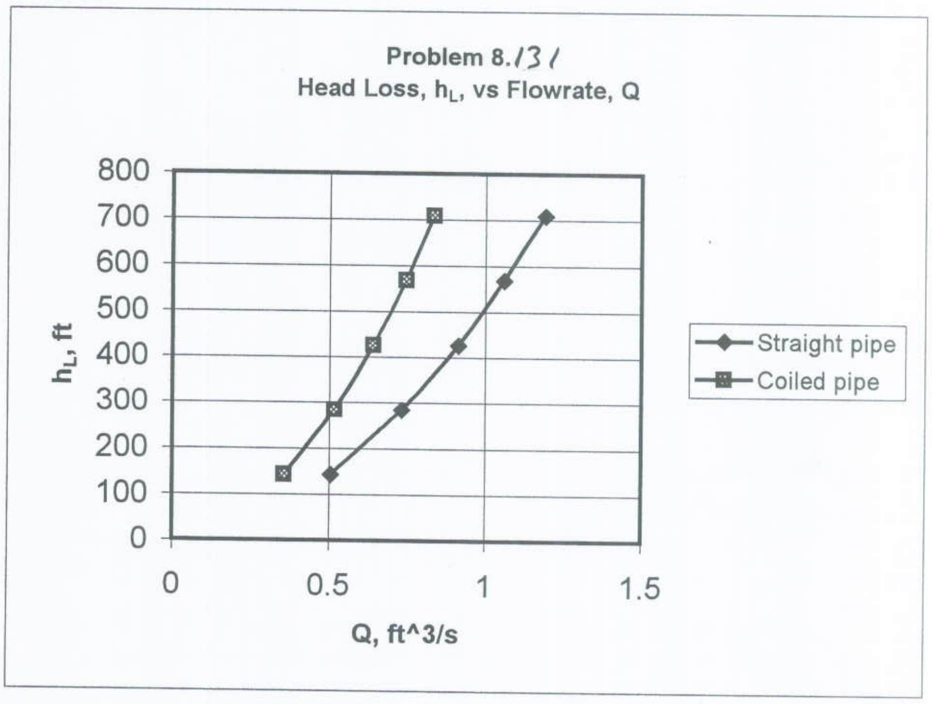
$p_{atm} = \gamma_{Hg}*H_{atm}$ = 847 lb/ft^3*(29.9/12 ft) = 2110 lb/ft^2
R = 1716 ft lb/slug deg R
T = 80 +460 =540 deg R

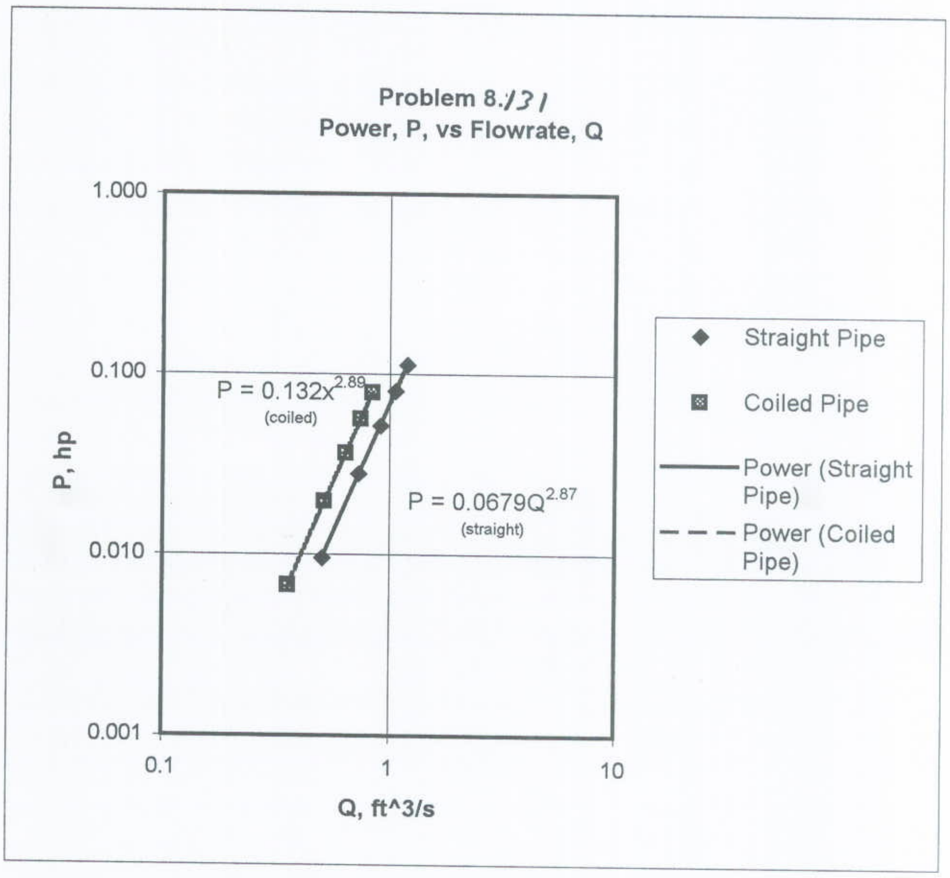
Thus, ρ = 0.00228 slug/ft^3 and γ = 0.0733 lb/ft^3

P = (γQh_L)ft lb/s*(1hp/550 ft lb/s)

(con't)

8.131 (con't)


Problem 8.131
Head Loss, h_L, vs Flowrate, Q
h_L, ft
800
700
600
500
400
300
200
100
0
0
0.5
1
1.5
Q, ft^3/s
Straight pipe
Coiled pipe


Problem 8.131
Power, P, vs Flowrate, Q
P, hp
1.000
0.100
0.010
0.001
0.1
1
10
Q, ft^3/s
P = 0.132x^2.89
(coiled)
P = 0.0679Q^2.87
(straight)
Straight Pipe
Coiled Pipe
Power (Straight Pipe)
Power (Coiled Pipe)