5.5

5.5 Water enters a cylindrical tank through two pipes at rates of 250 and 100 gal/min (see Fig. P5.5). If the level of the water in the tank remains constant, calculate the average velocity of the flow leaving the tank through an 8-in. inside-diameter pipe.

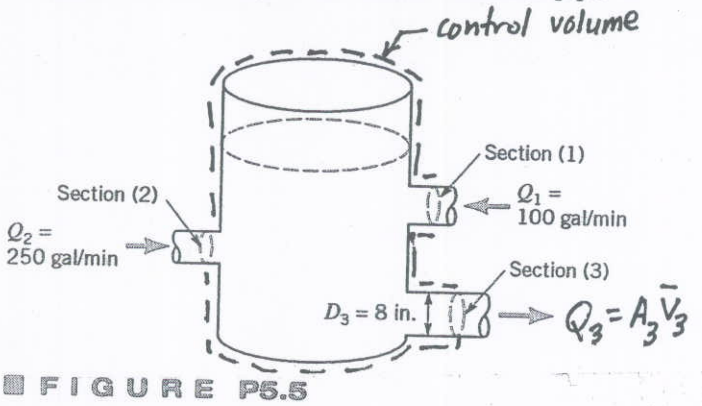


FIGURE P5.5

For steady and incompressible flow through the control volume shown

$$Q_3 = Q_1 + Q_2$$

or

$$\bar{V}_3 = \frac{1}{A_3}(Q_1 + Q_2) = \frac{1}{\frac{\pi d_3^2}{4}}(Q_1 + Q_2)$$

$$\bar{V}_3 = \frac{1}{\frac{\pi (8 \text{ in.})^2}{4}}(100 \text{ gpm} + 250 \text{ gpm})\left(231 \frac{\text{in.}^3}{\text{gal}}\right)\left(\frac{1}{60 \frac{\text{s}}{\text{min}}}\right)\left(\frac{1}{12 \frac{\text{in.}}{\text{ft}}}\right)$$

$$\bar{V}_3 = \underline{\underline{2.23}} \ \frac{\text{ft}}{\text{s}}$$

5.6

5.6 Water flows out through a set of thin, closely spaced blades as shown in Fig. 5.6 with a speed of $V = 10$ ft/s around the entire circumference of the outlet. Determine the mass flowrate through the inlet pipe.

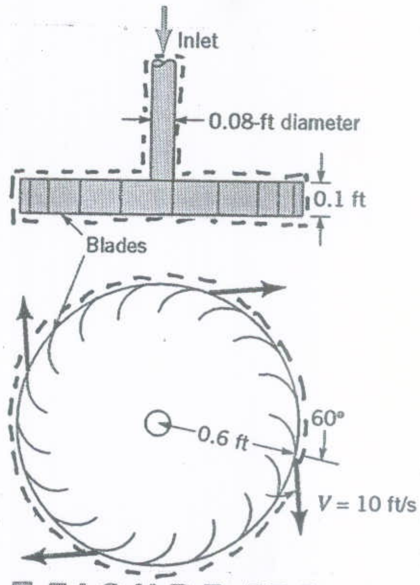


FIGURE P5.6

Use the control volume contained within the broken lines shown in the sketch above.

From the conservation of mass principle

$$\dot{m}_{inlet} = \dot{m}_{outlet}$$

Also

$$\dot{m}_{outlet} = \rho A_{outlet} V_{outlet} \cos 60^\circ$$

$$= \rho\, 2\pi r_{outlet} h V_{outlet} \cos 60^\circ$$

$$= \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) 2\pi (0.6\,\text{ft})(0.1\,\text{ft})\left(10 \frac{\text{ft}}{\text{s}}\right)\cos 60^\circ$$

$$= \underline{\underline{3.66}} \frac{\text{slugs}}{\text{s}}$$

5.7

5.7 The pump shown in Fig. P5.7 produces a steady flow of 10 gal/s through the nozzle. Determine the nozzle exit diameter, D_2, if the exit velocity is to be $V_2 = 100$ ft/s.

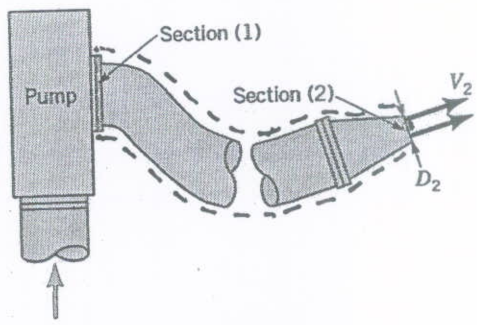


FIGURE P5.7

For steady flow $Q_1 = Q_2$, where $Q_1 = 10\,\frac{gal}{s}\left(231\,\frac{in.^3}{gal}\right)\left(\frac{1\,ft^3}{1728\,in.^3}\right) = 1.337\,\frac{ft^3}{s}$

Thus, with $V_2 = 100\,\frac{ft}{s}$,

$$1.337\,\frac{ft^3}{s} = A_2 V_2 = \frac{\pi}{4} D_2^2 \left(100\,\frac{ft}{s}\right)$$

or

$$D_2 = 0.130\,ft = \underline{\underline{1.57\,in.}}$$

5.8

5.8 Water flows into a sink as shown in Video V5.1 and Fig. P5.8 at a rate of 2 gallons per minute. Determine the average velocity through each of the three 0.4-in.-diameter overflow holes if the drain is closed and the water level in the sink remains constant.

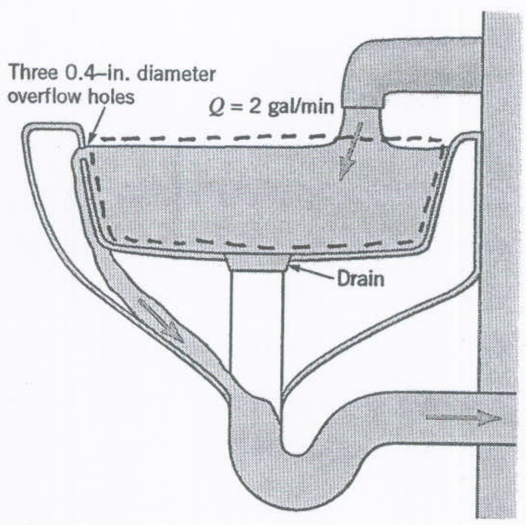


FIGURE P5.8

$Q_1 = Q_2$ for the control volume indicated, where

$$Q_1 = 2\,\frac{gal}{min}\left(\frac{1\ min}{60\ s}\right)\frac{1}{7.48\ \frac{gal}{ft^3}} = 0.00446\ \frac{ft^3}{s}$$

Thus,

$$Q_1 = A_2 V_2 \quad \text{or} \quad V_2 = \frac{Q_1}{A_2} = \frac{0.00446\ \frac{ft^3}{s}}{3\left[\frac{\pi}{4}\left(\frac{0.4}{12}ft\right)^2\right]} = \underline{\underline{1.70\ \frac{ft}{s}}}$$

5.9

5.9 The wind blows through a 7 ft × 10 ft garage door opening with a speed of 5 ft/s as shown in Fig. P5.9. Determine the average speed, V, of the air through the two 3 ft × 4 ft openings in the windows.

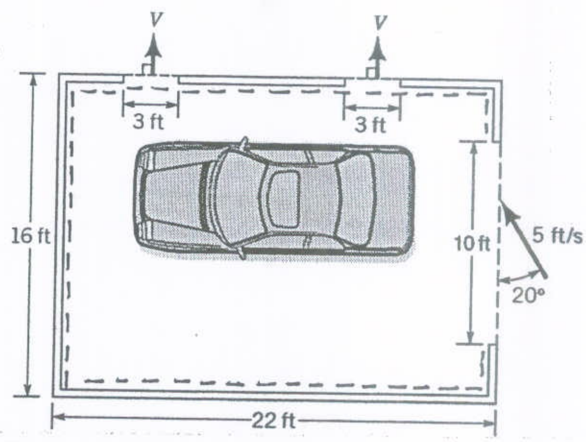


FIGURE P5.9

For steady incompressible flow

$$Q_{\substack{garage\\door}} = Q_{window} + Q_{window}$$

or

$$A_{\substack{garage\\door}} V_{\substack{normal\ to\\garage\ door}} = A_{window} V + A_{window} V$$

so the average speed, V, of the air through the two windows is

$$V = \frac{A_{\substack{garage\\door}} V_{normal\ to\ garage\ door}}{2A_{window}} = \frac{(7ft)(10ft)(5\frac{ft}{s})\sin 20^\circ}{2(3ft)(4ft)} = \underline{\underline{4.99\ \frac{ft}{s}}}$$

5.10

5.10 The human circulatory system consists of a complex branching pipe network ranging in diameter from the aorta (largest) to the capillaries (smallest). The average radii and the number of these vessels is shown in the table below. Does the average blood velocity increase, decrease, or remain constant as it travels from the aorta to the capillaries?

Vessel	Average Radius, mm	Number	r^2N, mm^2
Aorta	12.5	1	156
Arteries	2.0	159	636
Arterioles	0.03	1.4×10^7	12,600
Capillaries	0.006	3.9×10^9	140,400

The average blood velocity, V, is related to the blood mass flow, $\dot{m}$, by

$$V = \frac{\dot{m}}{\rho A N}$$

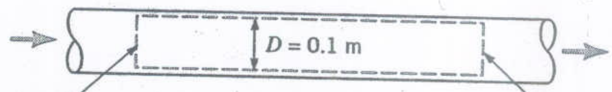
where ρ is blood density, A is vessel cross section area (πr^2) and N is number of vessels. So for constant $\dot{m}$ and ρ,

$$V = \dot{m}/(\rho \pi r^2 N)$$

and since the r^2N product becomes larger, the average velocity <u>becomes smaller.</u>

5.11

5.11 Air flows steadily between two cross sections in a long, straight section of 0.1-m inside diameter pipe. The static temperature and pressure at each section are indicated in Fig. P5.11. If the average air velocity at section (1) is 205 m/s, determine the average air velocity at section (2).



■ FIGURE P5.11

This analysis is similar to the one of Example 5.2.
For steady flow between sections (1) and (2)

$$\dot{m}_2 = \dot{m}_1$$

or

$$\rho_2 A_2 \bar{V}_2 = \rho_1 A_1 \bar{V}_1$$

Thus

$$\bar{V}_2 = \frac{\rho_1}{\rho_2} \frac{A_1}{A_2} \bar{V}_1 \qquad (1)$$

Assuming that under the conditions of this problem, air behaves as an ideal gas we use the ideal gas equation of state (Eq. 1.8) to get

$$\frac{\rho_1}{\rho_2} = \frac{p_1}{p_2} \frac{T_2}{T_1} \qquad (2)$$

Combining Eqs. 1 and 2 and observing that $A_1 = A_2$ we get

$$\bar{V}_2 = \frac{p_1}{p_2} \frac{T_2}{T_1} \bar{V}_1 = \frac{[77\ kPa(abs)](240K)}{[45\ kPa(abs)](268K)} \left(205 \frac{m}{s}\right)$$

$$\bar{V}_2 = \underline{\underline{314 \frac{m}{s}}}$$

5.12

5.12 A hydraulic jump (see Video V10.10) is in place downstream from a spillway as indicated in Fig. P5.12. Upstream of the jump, the depth of the stream is 0.6 ft and the average stream velocity is 18 ft/s. Just downstream of the jump, the average stream velocity is 3.4 ft/s. Calculate the depth of the stream, h, just downstream of the jump.

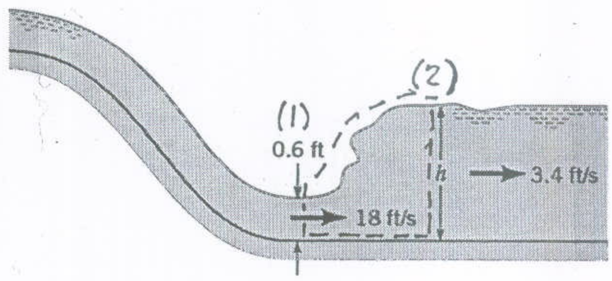


FIGURE P5.12

For steady incompressible flow between sections (1) and (2)

$$Q_1 = Q_2$$

or

$$\bar{V}_1 A_1 = \bar{V}_2 A_2$$

Thus

$$\bar{V}_1 h_1 = \bar{V}_2 h_2$$

and

$$h_2 = \frac{\bar{V}_1 h_1}{\bar{V}_2} = \frac{(18\ \frac{ft}{s})(0.6\ ft)}{(3.4\ \frac{ft}{s})} = \underline{\underline{3.18\ ft}}$$

5.13

5.13 An evaporative cooling tower (see Fig. P5.13) is used to cool water from 110 to 80°F. Water enters the tower at a rate of 250,000 lbm/hr. Dry air (no water vapor) flows into the tower at a rate of 151,000 lbm/hr. If the rate of wet air flow out of the tower is 156,900 lbm/hr, determine the rate of water evaporation in lbm/hr and the rate of cooled water flow in lbm/hr.

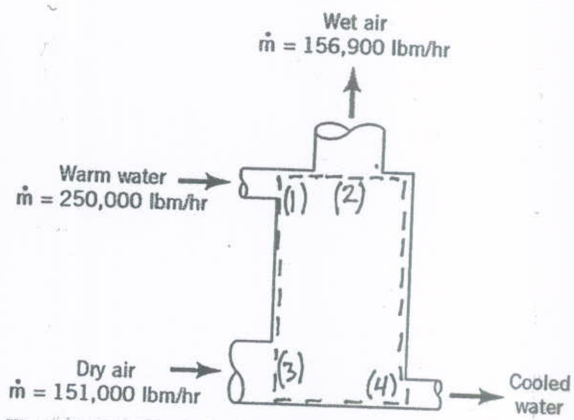


FIGURE P5.13

For steady flow of dry air

$$\dot{m}_3 = \dot{m}_{2,\,dry\ air} \qquad (1)$$

For steady flow of water

$$\dot{m}_1 = \dot{m}_{2,\,water} + \dot{m}_4 \qquad (2)$$

Also

$$\dot{m}_2 = \dot{m}_{2,\,dry\ air} + \dot{m}_{2,\,water} \qquad (3)$$

Combining Eqs. 1 and 3 we get

$$\dot{m}_{2,\,water} = \dot{m}_2 - \dot{m}_3 = \text{rate of water evaporation}$$

So

$$\dot{m}_{2,\,water} = 156{,}900\ \frac{lbm}{hr} - 151{,}000\ \frac{lbm}{hr} = \underline{\underline{5900\ \frac{lbm}{hr}}}$$

From Eq. 2 we get

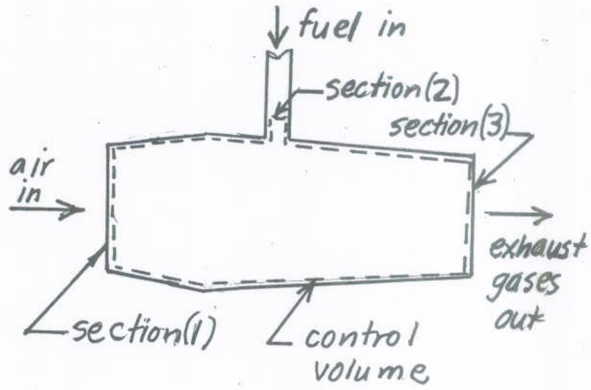
$$\dot{m}_4 = \dot{m}_1 - \dot{m}_{2,\,water} = \text{rate of cooled water flow}$$

or

$$\dot{m}_4 = 250{,}000\ \frac{lbm}{hr} - 5900\ \frac{lbm}{hr} = \underline{\underline{244{,}000\ \frac{lbm}{hr}}}$$

5.14

5.14 At cruise conditions, air flows into a jet engine at a steady rate of 65 lbm/s. Fuel enters the engine at a steady rate of 0.60 lbm/s. The average velocity of the exhaust gases is 1500 ft/s relative to the engine. If the engine exhaust effective cross-sectional area is 3.5 ft^2, estimate the density of the exhaust gases in lbm/ft^3.



For steady flow

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

or

$$\rho_3 A_3 \bar{V}_3 = \dot{m}_1 + \dot{m}_2$$

Thus

$$\rho_3 = \frac{\dot{m}_1 + \dot{m}_2}{A_3 \bar{V}_3} = \frac{65\ \frac{lbm}{s} + 0.60\ \frac{lbm}{s}}{(3.5\ ft^2)(1500\ \frac{ft}{s})}$$

$$\rho_3 = \underline{\underline{0.0125}}\ \frac{lbm}{ft^3}$$

5.15

5.15 Water at 0.1 m^3/s and alcohol (SG=0.8) at 0.3 m^3/s are mixed in a y-duct as shown in Fig. 5.15. What is the average density of the mixture of alcohol and water?

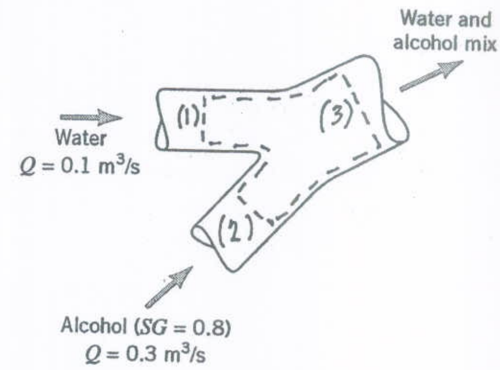


FIGURE P5.15

For steady flow

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

or

$$\rho_1 Q_1 + \rho_2 Q_2 = \rho_3 Q_3 \qquad (1)$$

Also, since the water and alcohol may be considered incompressible

$$Q_1 + Q_2 = Q_3 \qquad (2)$$

Combining Eqs. 1 and 2 we get

$$\rho_1 Q_1 + \rho_2 Q_2 = \rho_3 (Q_1 + Q_2)$$

or

$$\rho_3 = \frac{\rho_1 Q_1 + \rho_2 Q_2}{Q_1 + Q_2}$$

and

$$\rho_3 = \rho_1 \frac{(Q_1 + SG_2 Q_2)}{Q_1 + Q_2}$$

Thus

$$\rho_3 = \frac{\left(999 \frac{kg}{m^3}\right)\left[0.1 \frac{m^3}{s} + (0.8)\left(0.3 \frac{m^3}{s}\right)\right]}{0.1 \frac{m^3}{s} + 0.3 \frac{m^3}{s}} = \underline{\underline{849 \frac{kg}{m^3}}}$$

5.16

5.16 Freshwater flows steadily into an open 55-gal drum initially filled with seawater. The freshwater mixes thoroughly with the seawater and the mixture overflows out of the drum. If the freshwater flowrate is 10 gal/min, estimate the time in seconds required to decrease the difference between the density of the mixture and the density of fresh water by 50%.

A fixed, non-deforming control volume that contains the water mixture in the 55-gal drum is used. Fresh water enters the control volume with density, ρ_1, and volume flowrate, Q_1. The mixture is assumed to be homogeneous throughout the control volume and leaves the control volume with density, ρ_2, and volume flowrate, Q_2. Application of the conservation of mass equation (Eq. 5.5) to the flow through this control volume yields

$$\frac{\partial}{\partial t}\int_{cv} \rho_2 \, d\forall + \rho_2 Q_2 - \rho_1 Q_1 = 0 \qquad (1)$$

Since the fluids involved are incompressible, $Q_1 = Q_2 = Q$. Also, the volume of the control volume is constant. Thus Eq. 1 leads to

$$\forall_{cv} \frac{d(\rho_2/\rho_1)}{dt} + \frac{\rho_2}{\rho_1} Q = Q$$

or

$$\frac{d(\rho_2/\rho_1)}{dt} + \left(\frac{\rho_2}{\rho_1}\right)\frac{Q}{\forall_{cv}} = \frac{Q}{\forall_{cv}} \qquad (2)$$

The solution of Eq. 2 is

$$\frac{\rho_2}{\rho_1} = C e^{-\frac{Q}{\forall_{cv}} t} + 1.0 \qquad (3)$$

At $t = 0$, $\rho_2/\rho_1 = \frac{\rho_{seawater}}{\rho_{fresh\ water}} = \frac{(1.99 \ slugs/ft^3)}{(1.94 \ slugs/ft^3)} = 1.026$

thus

$$C = 0.026$$

(con't)

5.16 (Con't)

Then for

$$\rho_{2,f} - \rho_1 = 0.5(\rho_{2,i} - \rho_1)$$

where

$\rho_{2,f}$ = final mixture density

$\rho_{2,i}$ = initial mixture density

we have

$$\frac{\rho_{2,f}}{\rho_1} = 0.5\left(\frac{\rho_{2,i}}{\rho_1} + 1\right) = 0.5\left(1.026 + 1\right) = 1.013$$

Substituting this value and other givens into Eq. 3 we get

$$1.013 = 0.026\, e^{-\frac{(10\ \text{gal/min})}{(55\ \text{gal})(60\ \frac{\text{min}}{\text{s}})} t} + 1.0$$

and

$$t = \underline{\underline{229\ \text{s}}}$$

5.17

5.17 A water jet pump (see Fig. P5.17) involves a jet cross-sectional area of 0.01 m^2, and a jet velocity of 30 m/s. The jet is surrounded by entrained water. The total cross-sectional area associated with the jet and entrained streams is 0.075 m^2. These two fluid streams leave the pump thoroughly mixed with an average velocity of 6 m/s through a cross-sectional area of 0.075 m^2. Determine the pumping rate (i.e., the entrained fluid flowrate) involved in liters/s.

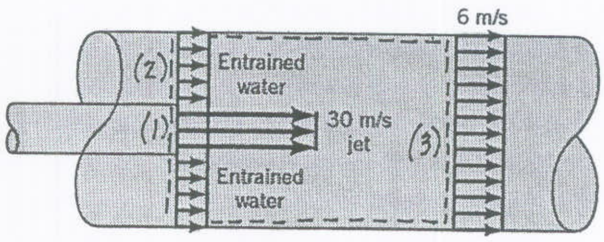


FIGURE P5.17

For steady incompressible flow through the control volume

$$Q_1 + Q_2 = Q_3$$

or

$$\bar{V}_1 A_1 + Q_2 = \bar{V}_3 A_3$$

Thus

$$Q_2 = \bar{V}_3 A_3 - \bar{V}_1 A_1 = \left[\left(6\ \frac{m}{s}\right)\left(0.075\ m^2\right) - \left(30\ \frac{m}{s}\right)\left(0.01\ m^3\right)\right]\left(1000\ \frac{liters}{m^3}\right)$$

$$\underline{\underline{Q_2 = 150\ \frac{liters}{s}}}$$

5.18

5.18 Two rivers merge to form a larger river as shown in Fig. P5.18. At a location downstream from the junction (before the two streams completely merge), the nonuniform velocity profile is as shown and the depth is 6 ft. Determine the value of V.

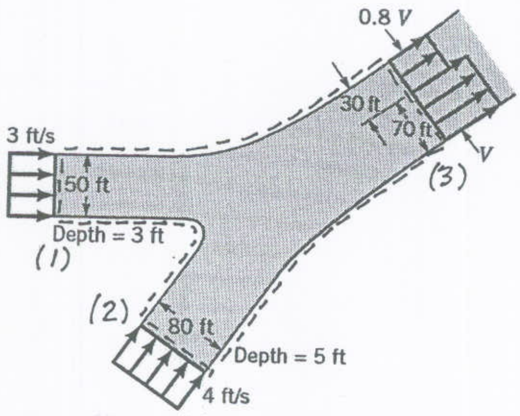


FIGURE P5.18

Use the control volume shown within broken lines in the sketch above. We note that $\dot{m} = \rho A V$ and from the conservation of mass principle we get

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = \dot{m}_{0.8V} + \dot{m}_V$$

Thus

$$\rho A_1 V_1 + \rho A_2 V_2 = \rho A_{0.8V}\, 0.8V + \rho A_V V$$

and

$$V = \frac{A_1 V_1 + A_2 V_2}{A_{0.8V}(0.8) + A_V} = \frac{(50ft)(3ft)(3\frac{ft}{s}) + (80ft)(5ft)(4\frac{ft}{s})}{(30ft)(6ft)(0.8) + (70ft)(6ft)}$$

$$V = \underline{\underline{3.63}}\ \frac{ft}{s}$$

5.19

5.19 Various types of attachments can be used with the shop vac shown in Video V5.2. Two such attachments are shown in Fig. P5.19—a nozzle and a brush. The flowrate is 1 ft³/s. **(a)** Determine the average velocity through the nozzle entrance, V_n. **(b)** Assume the air enters the brush attachment in a radial direction all around the brush with a velocity profile that varies linearly from 0 to V_b along the length of the bristles as shown in the figure. Determine the value of V_b.

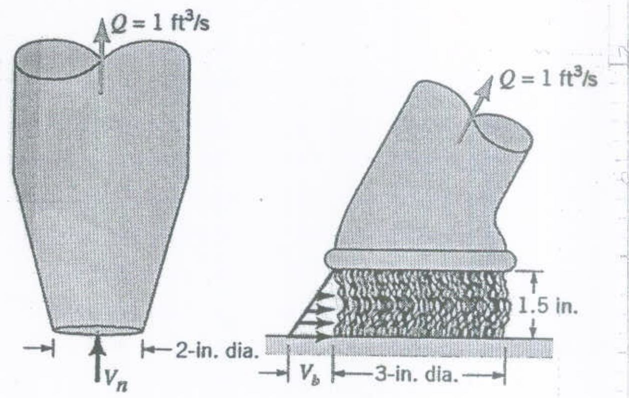


FIGURE P5.19

(a) $Q_1 = Q_2$ where $Q_2 = 1\,\frac{ft^3}{s}$

Thus,

$$A_1V_1 = Q_2 \quad \text{or} \quad V_1 \equiv V_n = \frac{1\,\frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{2}{12}\,ft\right)^2}$$

so

$$V_n = \underline{\underline{45.8\,\frac{ft}{s}}}$$

(b) $Q_3 = Q_4$ where $Q_4 = 1\,\frac{ft^3}{s}$ and $Q_3 = \overline{V}_3 A_3$ where

$\overline{V}_3$ = average velocity at (3) $= \frac{1}{2}V_b$ and

$A_3 = \pi D_3 h_3$

Thus,

$$\frac{1}{2}V_b\left[\pi\left(\frac{3}{12}\,ft\right)\left(\frac{1.5}{12}\,ft\right)\right] = 1\,\frac{ft^3}{s}, \text{ or}$$

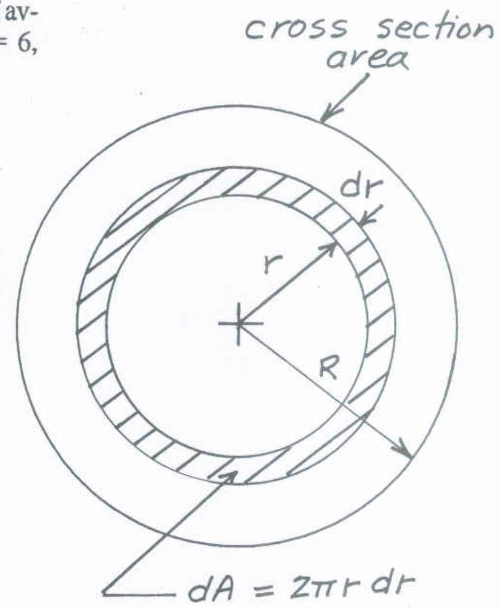
$$V_b = \underline{\underline{20.4\,\frac{ft}{s}}}$$

5.20

5.20 An appropriate turbulent pipe flow velocity profile is

$$\mathbf{V} = u_c\left(\frac{R-r}{R}\right)^{1/n}\hat{\mathbf{i}}$$

where u_c = centerline velocity, r = local radius, R = pipe radius, and $\hat{\mathbf{i}}$ = unit vector along pipe centerline. Determine the ratio of average velocity, $\overline{u}$, to centerline velocity, u_c, for (a) $n = 4$, (b) $n = 6$, (c) $n = 8$, (d) $n = 10$. Compare the different velocity profiles.



For any cross section area

$$\dot{m} = \rho A \overline{u} = \int_A \rho\, \vec{V}\cdot\hat{n}\, dA$$

Also

$$\vec{V}\cdot\hat{n} = \vec{V}\cdot\hat{i} = u_c\left(\frac{R-r}{R}\right)^{\frac{1}{n}}$$

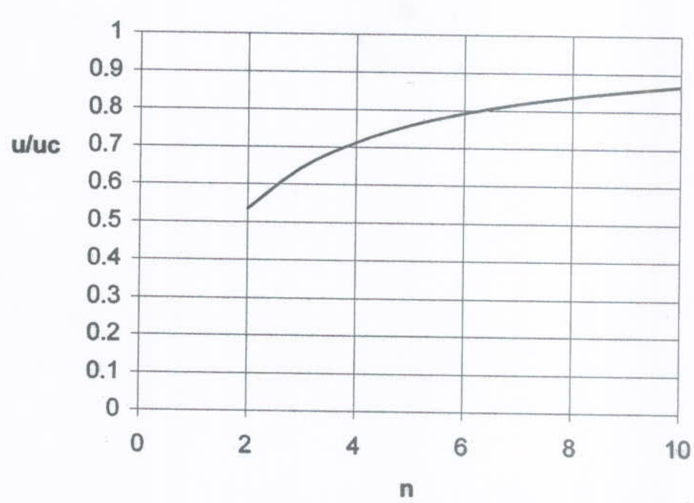
Thus for a uniformly distributed density, ρ, over area A

$$\overline{u} = \frac{\int_0^R u_c\left(\frac{R-r}{R}\right)^{\frac{1}{n}} 2\pi r\, dr}{\pi R^2}$$

and

$$\frac{\overline{u}}{u_c} = 2\int_0^1 \left(1-\frac{r}{R}\right)^{\frac{1}{n}}\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = \frac{2n^2}{2n^2+3n+1}$$

n	$\frac{\overline{u}}{u_c}$
4	0.711
6	0.791
8	0.837
10	0.866



The different velocity profiles (including for laminar flow) are compared in Fig. 8.18. Since the profile for $n = 4$ is not practically significant, it is not shown.

5.21

5.21 As shown in Fig. P5.21, at the entrance to a 3-ft-wide channel the velocity distribution is uniform with a velocity V. Further downstream the velocity profile is given by $u = 4y - 2y^2$, where u is in ft/s and y is in ft. Determine the value of V.

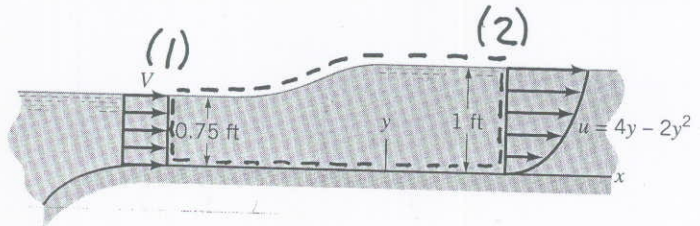


FIGURE P5.21

Use the control volume indicated by the broken lines in the sketch above.

From the conservation of mass principle

$$Q_1 = Q_2$$

$$V_1 A_1 = \int_{A_2} u\, dA \quad \int_0^{1\,ft} (4y - 2y^2)\, b\, dy$$

$$V(0.75\,ft)\,b = 3\left[\frac{4y^2}{2} - \frac{2y^3}{3}\right]_0^{1\,ft} b = \frac{4b}{3}\ \frac{ft^3}{s}$$

$$V = \frac{4}{3(0.75)} = \underline{\underline{1.78}}\ \frac{ft}{s}$$

5.22

5.22 A water flow situation is described by the velocity field equation

$$\mathbf{V} = (3x + 2)\hat{\mathbf{i}} + (2y - 4)\hat{\mathbf{j}} - 5z\hat{\mathbf{k}} \text{ ft/s}$$

where x, y, and z are in feet. **(a)** Determine the mass flow rate through the rectangular area in the plane corresponding to $z = 2$ feet having corners at $(x, y, z) = (0, 0, 2), (5, 0, 2), (5, 5, 2)$, and $(0, 5, 2)$ as shown in Fig. P5.22a. **(b)** Show that mass is conserved in the control volume having corners at $(x, y, z) = (0, 0, 2), (5, 0, 2), (5, 5, 2), (0, 5, 2), (0, 0, 0), (5, 0, 0), (5, 5, 0)$, and $(0, 5, 0)$ as shown in Fig. P5.22b.

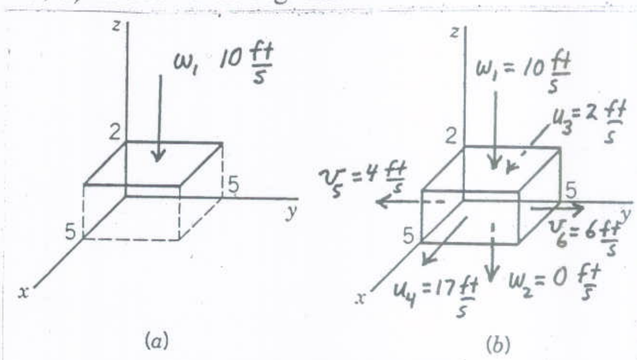


FIGURE P5.22

(a) The general expression for mass flowrate across area A_1 is

$$\dot{m}_1 = \int_{A_1} \rho \vec{V} \cdot \hat{n}\, dA$$

Since the z-direction component of velocity, w_1, is uniformly distributed over A_1, we can use

$$\dot{m}_1 = \rho (w_1 A_1) = \left(1.94 \frac{slugs}{ft^3}\right)\left(10 \frac{ft}{s}\right)\left(25\, ft^2\right)$$

or

$$\dot{m}_1 = 485 \frac{slugs}{s}$$

(b) If $\int_{CS} \rho \vec{V} \cdot \hat{n}\, dA = 0$, then mass is conserved.

However $\int_{CS} \rho \vec{V} \cdot \hat{n}\, dA = \sum \dot{m}$ and since the component of velocity normal to each plane area of the control volume is uniformly distributed over that area we have

$$\sum \dot{m} = \rho\left(-w_1 A_1 + w_2 A_2 - u_3 A_3 + u_4 A_4 + v_5 A_5 + v_6 A_6\right)$$

$$\sum \dot{m} = \rho\left(-250 \frac{ft^3}{s} + 0 \frac{ft^3}{s} - 20 \frac{ft^3}{s} + 170 \frac{ft^3}{s} + 40 \frac{ft^3}{s} + 60 \frac{ft^3}{s}\right)$$

$\sum \dot{m} = 0$ and mass is conserved.

5.23

5.23 An incompressible flow velocity field (water) is given as

$$\mathbf{V} = -\frac{1}{r}\,\hat{\mathbf{e}}_r + \frac{1}{r}\,\hat{\mathbf{e}}_\theta \text{ m/s}$$

where r is in meters. **(a)** Calculate the mass flowrate through the cylindrical surface at $r = 1$ m from $z = 0$ to $z = 1$ m as shown in Fig. P5.23a. **(b)** Show that mass is conserved in the annular control volume from $r = 1$ m to $r = 2$ m and $z = 0$ to $z = 1$ m as shown in Fig. P5.23b.

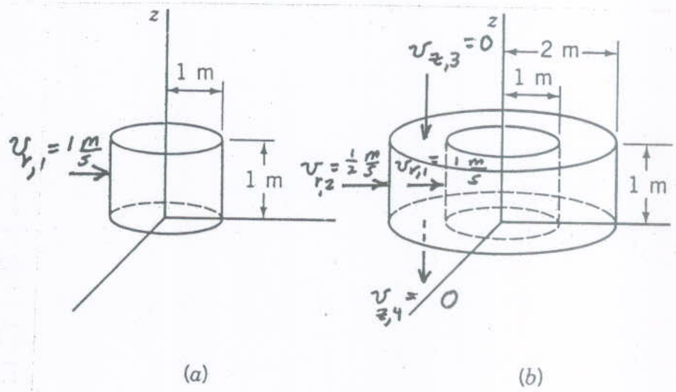


FIGURE P5.23

(a) The general expression for mass flowrate across cylindrical area A_1 is

$$\dot{m}_1 = \int_{A_1} \rho \vec{V} \cdot \hat{n}\, dA$$

Since the radial direction component of velocity, v_r, is uniformly distributed over A_1 we can use

$$\dot{m}_1 = \rho\,(v_{r,1} A_1) = \left(999\ \frac{kg}{m^3}\right)\left(1\ \frac{m}{s}\right)\left(2\pi\ m^2\right)$$

$$\dot{m}_1 = 6280\ \frac{kg}{s}$$

(b) If $\int_{CS} \rho \vec{V} \cdot \hat{n}\, dA = 0$, then mass is conserved.

However $\int_{CS} \rho \vec{V} \cdot \hat{n}\, dA = \Sigma \dot{m}$ and since the component of velocity normal to each cylindrical and plane area of the control volume is uniformly distributed over that area we have

$$\Sigma \dot{m} = \rho\left(v_{r,1} A_1 - v_{r,2} A_2 - v_{z,3} A_3 + v_{z,4} A_4\right)$$

$$\Sigma \dot{m} = \rho\left(2\pi\ \frac{m^3}{s} - 2\pi\ \frac{m^3}{s} - 0\ \frac{m^3}{s} + 0\ \frac{m^3}{s}\right)$$

$\Sigma \dot{m} = 0$ and mass is conserved.

5.24

5.24 Flow of a viscous fluid over a flat plate surface results in the development of a region of reduced velocity adjacent to the wetted surface as depicted in Fig. P5.24. This region of reduced flow is called a boundary layer. At the leading edge of the plate, the velocity profile may be considered uniformly distributed with a value U. All along the outer edge of the boundary layer, the fluid velocity component parallel to the plate surface is also U. If the x direction velocity profile at section (2) is

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$

develop an expression for the volume flowrate through the edge of the boundary layer from the leading edge to a location downstream at x where the boundary layer thickness is δ.

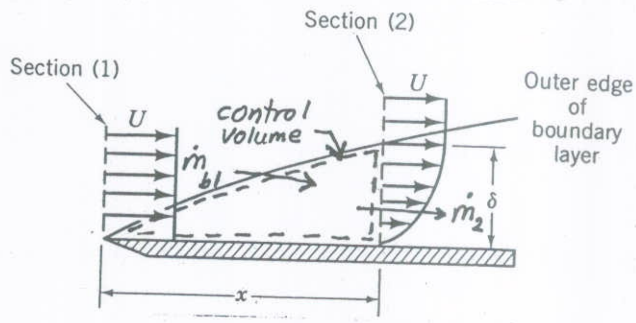


FIGURE P5.24

From the conservation of mass principle applied to the flow through the control volume shown in the figure we have

$$\dot{m}_{bl} = \dot{m}_2 = \int_{A_2} \rho \vec{V} \cdot \hat{n} \, dA$$

For incompressible flow

$$\rho Q_{bl} = \rho U \ell \delta \int_0^1 \left(\frac{y}{\delta}\right)^{\frac{1}{7}} d\left(\frac{y}{\delta}\right)$$

where

$$\ell = \text{width of the plate}$$

and thus

$$Q_{bl} = \frac{7}{8} U \ell \delta$$

5.25

5.25 Air at standard conditions enters the compressor shown in Fig. P5.25 at a rate of 10 ft^3/s. It leaves the tank through a 1.2-in.-diameter pipe with a density of 0.0035 slugs/ft^3 and a uniform speed of 700 ft/s. **(a)** Determine the rate (slugs/s) at which the mass of air in the tank is increasing or decreasing. **(b)** Determine the average time rate of change of air density within the tank.

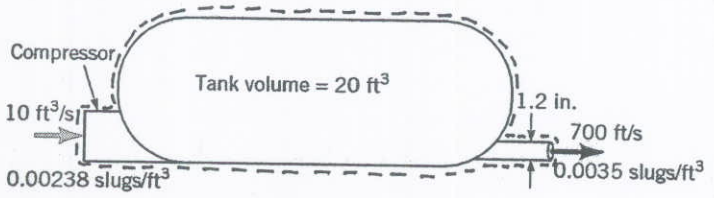


FIGURE P5.25

Use the control volume within the broken lines.

(a) From the conservation of mass principle we get

$$\frac{DM_{sys}}{Dt} = \dot{m}_{in} - \dot{m}_{out} = \rho_{in} Q_{in} - \rho_{out} A_{out} V_{out}$$

$$\frac{DM_{sys}}{Dt} = \left(0.00238 \frac{slug}{ft^3}\right)\left(10 \frac{ft^3}{s}\right) - \left(0.0035 \frac{slug}{ft^3}\right) \frac{\pi (1.2 \text{ in.})^2}{\left(144 \frac{in^2}{ft^2}\right)} \left(700 \frac{ft}{s}\right)$$

$$\frac{DM_{sys}}{Dt} = \underline{\underline{0.00456}} \frac{slug}{s} \quad \text{increasing}$$

(b) $$\frac{DM_{sys}}{Dt} = \frac{D(\rho V_{sys})}{Dt} = V_{sys} \frac{D\rho}{Dt} = 0.00456 \frac{slug}{s}$$

so

$$\frac{D\rho}{Dt} = \frac{0.00456 \frac{slug}{s}}{V_{sys}} = \frac{0.00456 \frac{slug}{s}}{20 ft^3} = \underline{\underline{2.28 \times 10^{-4}}} \frac{slug}{ft^3 \cdot s}$$

5.26

5.26 Estimate the time required to fill with water a cone-shaped container (see Fig. P5.26) 5 ft high and 5 ft across at the top if the filling rate is 20 gal/min.

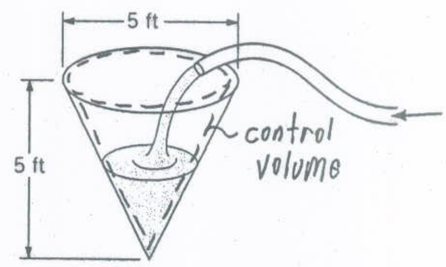


FIGURE P5.26

From application of the conservation of mass principle to the control volume shown in the figure we have

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V} \cdot \hat{n} \, dA = 0$$

For incompressible flow

$$\frac{\partial \forall}{\partial t} - Q = 0$$

or

$$\int_0^{\forall} d\forall = Q \int_0^t dt$$

Thus

$$t = \frac{\forall}{Q} = \frac{\pi D^2 h}{12\, Q} = \frac{\pi (5\,ft)^2 (5\,ft)\left(1728 \frac{in.^3}{ft^3}\right)}{(12)\left(20 \frac{gal}{min}\right)\left(231 \frac{in.^3}{gal}\right)}$$

and

$$t = \underline{\underline{12.2 \ min}}$$

5.29

5.29 A hypodermic syringe (see Fig. P5.29) is used to apply a vaccine. If the plunger is moved forward at the steady rate of 20 mm/s and if vaccine leaks pass the plunger at 0.1 of the volume flowrate out the needle opening, calculate the average velocity of the needle exit flow. The inside diameters of the syringe and the needle are 20 mm and 0.7 mm.

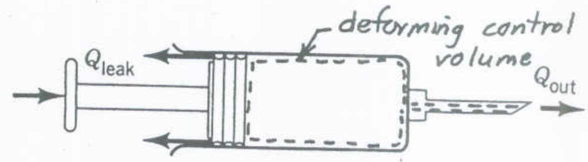


FIGURE P5.29

Using a deforming control volume and the conservation of mass principle (Eq. 5.17) as outlined in Example 5.8, we obtain (see Eq. 8 of Example 5.8)

$$-\rho A_1 V_p + \rho Q_2 + \rho Q_{leak} = 0 \qquad (1)$$

Since $\rho =$ constant, $Q_{leak} = 0.1\, Q_2$ and $Q_2 = A_2 V_2$ we obtain from Eq. 1

$$1.1\, A_2 V_2 = A_1 V_p$$

or

$$V_2 = \left(\frac{A_1}{A_2}\right)\frac{V_p}{1.1} = \left(\frac{d_1^2}{d_2^2}\right)\frac{V_p}{1.1} = \frac{(20\,mm)^2(20\,mm/s)}{(0.7\,mm)^2(1.1)\left(1000\,\frac{mm}{m}\right)}$$

and

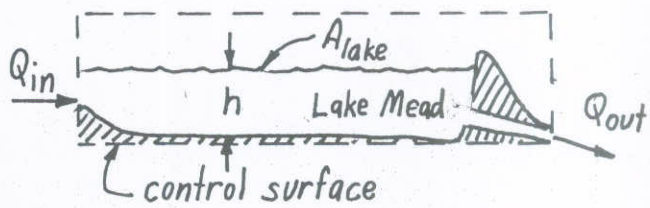
$$V_2 = \underline{\underline{14.8}}\ \frac{m}{s}$$

5.30

5.30 The Hoover Dam (see **Video V2.4**) backs up the Colorado River and creates Lake Mead, which is approximately 115 miles long and has a surface area of approximately 225 square miles. If during flood conditions the Colorado River flows into the lake at a rate of 45,000 cfs and the outflow from the dam is 8000 cfs, how many feet per 24-hour day will the lake level rise?

For the control volume shown:

$$\dot{m}_{in} - \dot{m}_{out} = \frac{\partial}{\partial t}\int_{cv\,water} \rho \, d\forall$$



or since $\dot{m} = \rho Q$,

$$Q_{in} - Q_{out} = \frac{\partial}{\partial t}(A_{lake} h) = A_{lake}\frac{dh}{d}$$

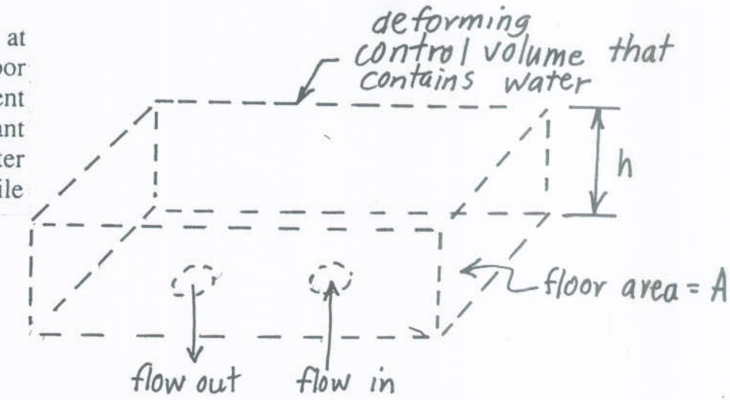
Thus,

$$\frac{dh}{dt} = \frac{Q_{out} - Q_{in}}{A_{lake}} = \frac{(45{,}000 - 8{,}000)\frac{ft^3}{s}}{225\ mi^2\left(5280\frac{ft}{mi}\right)^2} = 5.90\times10^{-6}\ \frac{in.}{s}$$

$$= 5.90\times10^{-6}\ \frac{in.}{s}\left(3{,}600\frac{s}{hr}\right)\left(24\frac{hr}{day}\right) = \underline{\underline{0.510\ \frac{ft}{day}}}$$

5.31

5.31 Storm sewer backup causes your basement to flood at the steady rate of 1 in. of depth per hour. The basement floor area is 1500 ft^2. What capacity (gal/min) pump would you rent to **(a)** keep the water accumulated in your basement at a constant level until the storm sewer is blocked off, **(b)** reduce the water accumulation in your basement at a rate of 3 in./hr even while the backup problem exists?



For a deforming control volume that contains the water over the basement floor (see sketch above), the conservation of mass principle (Eq. 5.17) leads to

$$\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V}_r \cdot \hat{n} \, dA = 0$$

or for constant fluid density and area (A)

$$A\frac{dh}{dt} - Q_{in} + Q_{out} = 0 \qquad (1)$$

(a) For part a, Eq. 1 leads to

$$Q_{out} = Q_{in}$$

To evaluate Q_{in}, we use Eq. 1 with $Q_{out} = 0$. Thus,

$$Q_{in} = A\frac{dh}{dt} = (1500 \text{ ft}^2)\left(1 \frac{\text{in.}}{\text{hr}}\right)\left(\frac{1}{12 \frac{\text{in.}}{\text{ft}}}\right) = 125 \frac{\text{ft}^3}{\text{hr}}$$

and

$$Q_{out} = \left(125 \frac{\text{ft}^3}{\text{hr}}\right)\left(7.48 \frac{\text{gal}}{\text{ft}^3}\right)\left(\frac{1}{60 \frac{\text{min}}{\text{hr}}}\right) = \underline{\underline{15.6}} \frac{\text{gal}}{\text{min}}$$

(b) For part b, Eq. 1 yields

$$Q_{out} = Q_{in} - A\frac{dh}{dt}$$

$$Q_{out} = 15.6 \frac{\text{gal}}{\text{min}} - (1500 \text{ ft}^2)\left(-3 \frac{\text{in.}}{\text{hr}}\right)\left(\frac{1}{12 \frac{\text{in.}}{\text{ft}}}\right)\left(7.48 \frac{\text{gal}}{\text{ft}^3}\right)\left(\frac{1}{60 \frac{\text{min}}{\text{hr}}}\right)$$

$$Q_{out} = \underline{\underline{62.4}} \frac{\text{gal}}{\text{min}}$$

5.32

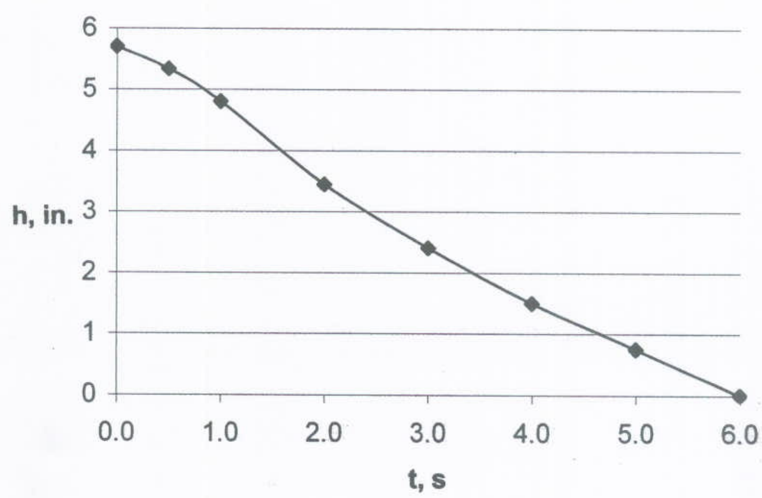
5.32 (See Fluids in the News article "New 1.6 gpf standards," Section 5.1.2.) When a toilet is flushed, the water depth, h, in the tank as a function of time, t, is as given in the table. The size of the rectangular tank is 19 in. by 7.5 in. **(a)** Determine the volume of water used per flush, gpf. **(b)** Plot the flowrate for $0 \le t \le 6$ s.

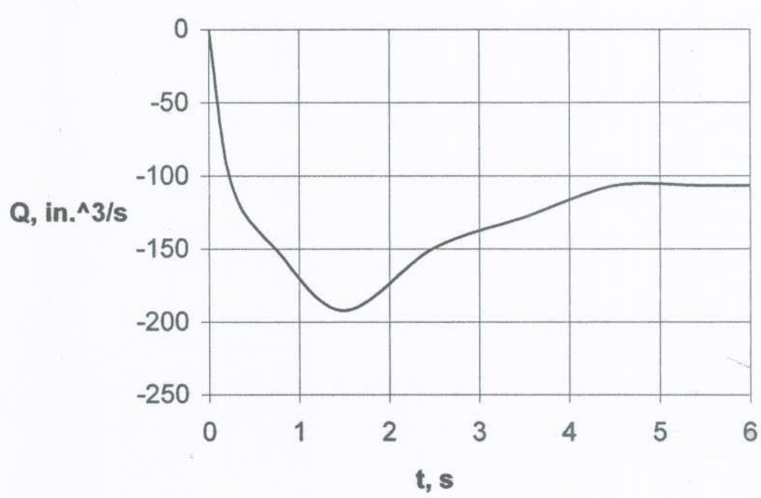
t (s)	h (in.)
0	5.70
0.5	5.33
1.0	4.80
2.0	3.45
3.0	2.40
4.0	1.50
5.0	0.75
6.0	0

(a) Volume of water per flush $= 5.70\,in.\,(19\,in. \times 7.5\,in.) = 812\,in.^3$

$$= 812\,in.^3\left(\frac{1\,gal}{231\,in.^3}\right) = \underline{\underline{3.52\,gal.}}$$

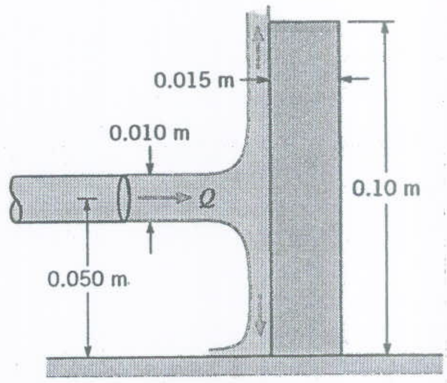
(b) $Q = \dfrac{d(\text{volume in tank})}{dt} = A_{tank}\dfrac{dh}{dt}$, where $\dfrac{dh}{dt}$ is obtained by numerical differentiation of the h vs t data shown below. The resulting Q vs t results are also shown below.





5.38

5.38 A 10-mm diameter jet of water is deflected by a homogeneous rectangular block (15 mm by 200 mm by 100 mm) that weighs 6 N as shown in **Video V5.96** and Fig. P5.38. Determine the minimum volume flowrate needed to tip the block.



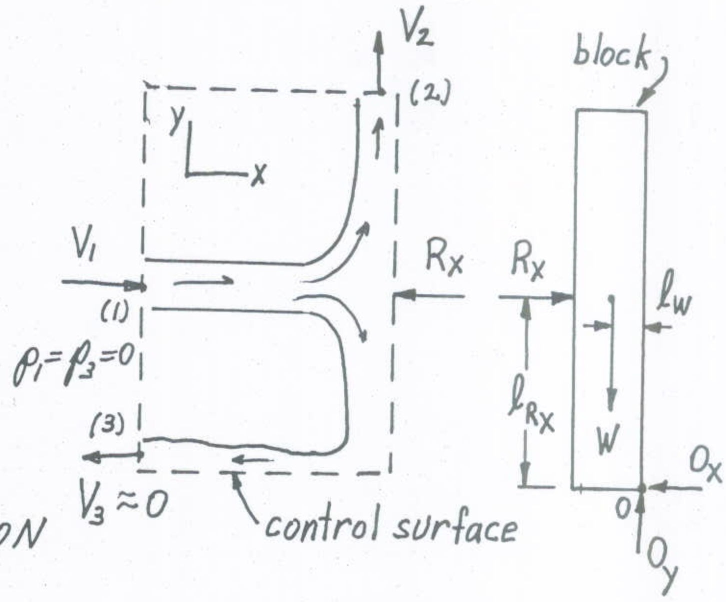
■ FIGURE P5.38

From the free body diagram of the block when it is ready to tip $\Sigma M_0 = 0$, or

$R_x \ell_{R_x} = W \ell_w$ where R_x is the fore that the water puts on the block.

Thus,

$$R_x = \frac{W \ell_w}{\ell_{Rx}} = \frac{6N\left(\frac{0.015}{2}m\right)}{0.050m} = 0.90N$$



For the control volume shown the x-component of the momentum equation

$$\int_{cs} u \rho \vec{V} \cdot \hat{n} \, dA = \Sigma F_x$$

becomes

$$V_1 \rho (-V_1) A_1 = -R_x \quad \text{or} \quad V_1 = \sqrt{\frac{R_x}{\rho A_1}}$$

Thus,

$$V_1 = \sqrt{\frac{0.9N}{\left(999 \frac{kg}{m^3}\right) \frac{\pi}{4} (0.01m)^2}} = 3.39 \frac{m}{s}$$

Hence,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.01m)^2 \left(3.39 \frac{m}{s}\right) = \underline{\underline{2.66 \times 10^{-4} \frac{m^3}{s}}}$$

5.39

5.39 Determine the anchoring force required to hold in place the conical nozzle attached to the end of the laboratory sink faucet shown in Fig. P5.39 when the water flowrate is 10 gal/min. The nozzle weight is 0.2 lb. The nozzle inlet and exit inside diameters are 0.6 and 0.2 in., respectively. The nozzle axis is vertical and the axial distance between sections (1) and (2) is 1.2 in. The pressure at section (1) is 68 psi.

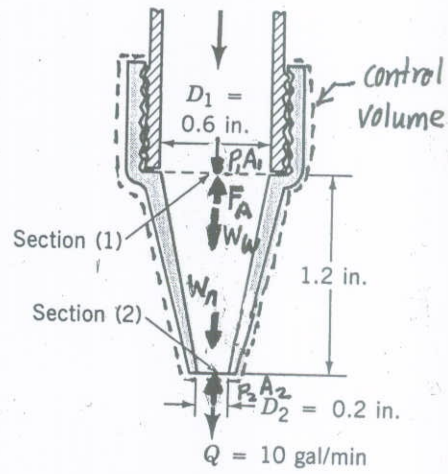


FIGURE P5.39

The analysis leading to the solution of this problem is similar to the one outlined in Example 5.10. Included in the control volume are the nozzle and the water in the nozzle at an instant. Application of the vertical or z-direction component of the linear momentum equation (Eq. 5.22) to the flow through this control volume leads to

$$F_A = \rho w_1^2 A_1 - \rho w_2^2 A_2 + W_n + P_1 A_1 + W_w - P_2 A_2 \qquad (1)$$

which is Eq. 4 of Example 5.10.

The conservation of mass equation yields

$$\dot{m} = \rho w_1 A_1 = \rho w_2 A_2$$

thus Eq. 1 becomes

$$F_A = \dot{m}(w_1 - w_2) + W_n + P_1 A_1 + W_w - P_2 A_2 \qquad (2)$$

The different terms in Eq. 2 are calculated below.

$$\dot{m} = \rho Q = \left(1.94 \frac{slugs}{ft^3}\right)\left(10 \frac{gal}{min}\right)\frac{1}{\left(7.48 \frac{gal}{ft^3}\right)}\frac{1}{\left(60 \frac{s}{min}\right)} = 0.0432 \frac{slug}{s}$$

$$w_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{\left(10 \frac{gal}{min}\right)\left(12 \frac{in.}{ft}\right)^2}{\frac{\pi (0.6 in.)^2}{4}\left(7.48 \frac{gal}{ft^3}\right)\left(60 \frac{s}{min}\right)} = 11.4 \frac{ft}{s}$$

$$w_2 = \frac{Q}{A_2} = \frac{Q}{\frac{\pi D_2^2}{4}} = \frac{\left(10 \frac{gal}{min}\right)\left(12 \frac{in.}{ft}\right)^2}{\frac{\pi (0.2 in.)^2}{4}\left(7.48 \frac{gal}{ft^3}\right)\left(60 \frac{s}{min}\right)} = 102 \frac{ft}{s}$$

$$P_1 A_1 = P_1 \frac{\pi D_1^2}{4} = \left(68 \frac{lb}{in.^2}\right)\frac{\pi (0.6 in.)^2}{4} = 19.2 \text{ lb}$$

(con't)

5.39	(con't)

$$W_w = \rho g \forall_w = \rho g \frac{\pi}{12}\left(D_1^2 + D_2^2 + D_1 D_2\right) h$$

$$W_w = \left(1.94 \frac{slugs}{ft^3}\right)\left(32.2 \frac{ft}{s^2}\right)\left(1 \frac{lb}{slug \frac{ft}{s^2}}\right) \frac{\pi}{12} \frac{\left[(0.6 \text{ in.})^2 + (0.2 \text{ in.})^2 + (0.6 \text{ in.})(0.2 \text{ in.})\right](1.2 \text{ in.})}{\left(1728 \frac{in.^3}{ft^3}\right)}$$

$$W_w = 0.00591 \text{ lb}$$

$$P_2 A_2 = P_2 \pi \frac{D_2^2}{4} = \left(0 \frac{lb}{in.^2}\right) \pi \frac{(0.2 \text{ in.})^2}{4} = 0 \text{ lb}$$

Thus with Eq. 2

$$F_A = \left(0.0432 \frac{slug}{s}\right)\left(11.4 \frac{ft}{s} - 102 \frac{ft}{s}\right)\left(1 \frac{lb}{slug \frac{ft}{s^2}}\right) + 0.2 lb + 19.2 lb + 0.00591 lb - 0 lb$$

$$F_A = \underline{\underline{15.5 \text{ lb}}}$$

5.40

5.40 Water flows through a horizontal, 180° pipe bend as is illustrated in Fig. P5.40. The flow cross section area is constant at a value of 9000 mm^2. The flow velocity everywhere in the bend is 15 m/s. The pressures at the entrance and exit of the bend are 210 and 165 kPa, respectively. Calculate the horizontal (x and y) components of the anchoring force needed to hold the bend in place.

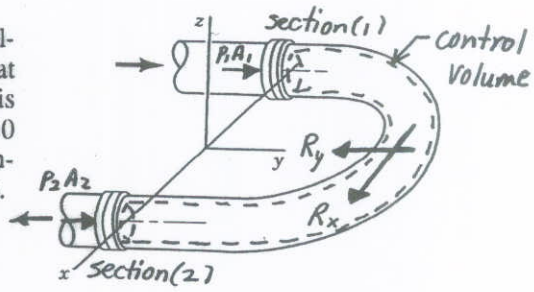


FIGURE P5.40

This analysis is similar to the one of Example 5.11. A fixed, non-deforming control volume that contains the water within the elbow between sections (1) and (2) at an instant is used. The horizontal forces acting on the contents of the control volume in the x and y directions are shown. Application of the x-direction component of the linear momentum equation (Eq. 5.22) leads to

$$\underline{\underline{R_x = 0}}$$

Application of the y-direction component of the linear momentum equation yields

$$-v_1 \rho v_1 A_1 - v_2 \rho v_2 A_2 = p_1 A_1 - R_y + p_2 A_2$$

or

$$R_y = \rho A_1 v_1 (v_1 + v_2) + p_1 A_1 + p_2 A_2$$

Thus

$$R_y = \left(999 \frac{kg}{m^3}\right)\frac{(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2}\left(15 \frac{m}{s}\right)\left(15\frac{m}{s} + 15\frac{m}{s}\right)\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right) + \frac{(210\ kPA)(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2\left(\frac{1}{1000 \frac{N}{m^2 \cdot kPA}}\right)}$$

$$+ \frac{(165\ kPA)(9000\ mm^2)}{\left(1000 \frac{mm}{m}\right)^2\left(\frac{1}{1000 \frac{N}{m^2 \cdot kPA}}\right)}$$

$$R_y = \underline{\underline{7420}}\ N$$

5.41

5.41 Water enters the horizontal, circular cross-sectional, sudden contraction nozzle sketched in Fig. P5.41 at section (1) with a uniformly distributed velocity of 25 ft/s and a pressure of 75 psi. The water exits from the nozzle into the atmosphere at section (2) where the uniformly distributed velocity is 100 ft/s. Determine the axial component of the anchoring force required to hold the contraction in place.

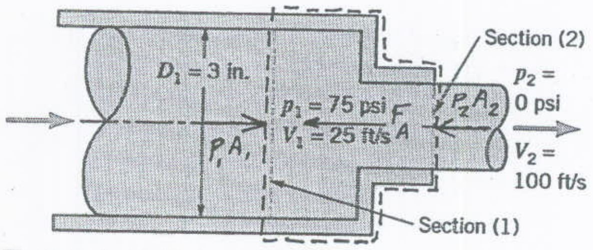


FIGURE P5.41

For this problem we include in the control volume the nozzle as well as the water at an instant between sections (1) and (2) as indicated in the sketch above. The horizontal forces acting on the contents of the control volume are shown in the sketch. Note that the atmospheric forces cancel out and are not shown. Application of the horizontal or x-direction component of the linear momentum equation (Eq. 5.22) to the flow through this control volume yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = P_1 A_1 - F_A - P_2 A_2 \qquad (1)$$

From the conservation of mass equation (Eq. 5.12) we obtain

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2$$

Thus Eq. (1) may be expessed as

$$\dot{m}(u_2 - u_1) = P_1 A_1 - F_A - P_2 A_2$$

or

$$F_A = P_1 A_1 - P_2 A_2 + \dot{m}(u_2 - u_1) = P_1 \frac{\pi D_1^2}{4} - P_2 \frac{\pi D_2^2}{4} - \rho u_1 \frac{\pi D_1^2}{4}(u_2 - u_1)$$

and

$$F_A = \left(75 \frac{lb}{in^2}\right)\frac{\pi (3 in.)^2}{4} - 0\, lb - \left(1.94 \frac{slugs}{ft^3}\right)\left(25 \frac{ft}{s}\right)\frac{\frac{\pi (3 in.)^2}{4}}{\left(144 \frac{in.^2}{ft^2}\right)}\left(100 \frac{ft}{s} - 25 \frac{ft}{s}\right)\left(1 \frac{lb \cdot s^2}{slug \cdot ft}\right)$$

$$F_A = \underline{\underline{352}}\ lb$$

5.42

5.42 The four devices shown in Fig. P5.42 rest on frictionless wheels, are restricted to move in the x direction only and are initially held stationary. The pressure at the inlets and outlets of each is atmospheric, and the flow is incompressible. The contents of each device is not known. When released, which devices will move to the right and which to the left? Explain.

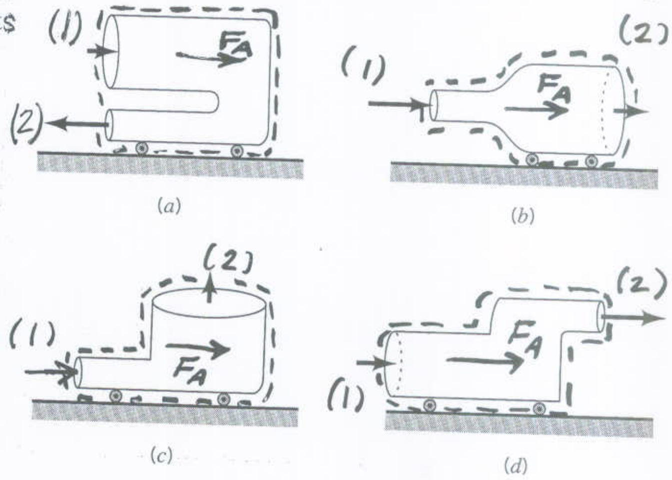


FIGURE P5.42

We apply the horizontal component of the linear momentum equation to the contents of the control volume (broken lines) and determine the sense of the anchoring force F_A. If F_A is in the direction shown in the sketches, motion will be to the left. If F_A is in a direction opposite to that shown, the motion is to the right. If $F_A = 0$, there is no horizontal motion.

For sketch (a)

$$-V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 = F_A$$

Since F_A is to the left, motion is to the right.

For sketch (b)

$$-V_1 \rho V_1 A_1 + V_2 \rho V_2 A_2 = F$$

and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

and since $V_1 > V_2$, then F_A is to the left and motion is to the right.

For sketch (c) (note: flow is into CV at (1))

$$-V_1 \rho V_1 A_1 = F_A$$

and F_A is to the left so motion is to the right.

For sketch (d)

$$-V_1 \rho V_1 A + V_2 \rho V_2 A_2 = F_A$$

and from conservation of mass

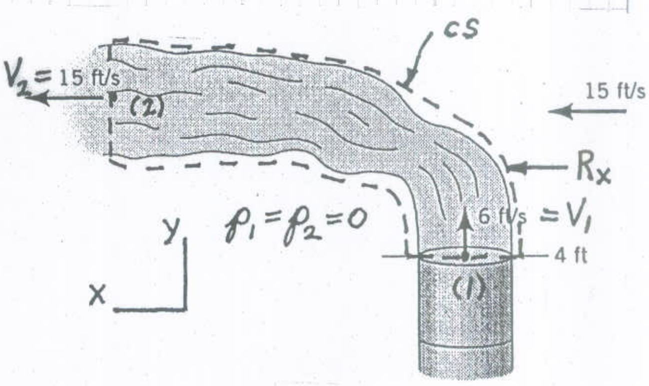
$$\rho V_1 A_1 = \rho V_2 A_2$$

and $V_1 < V_2$

so F_A is to the right and motion is to the left.

5.43

5.43 Exhaust (assumed to have the properties of standard air) leaves the 4-ft-diameter chimney shown in **Video V5.4** and Fig. P5.43 with a speed of 6 ft/s. Because of the wind, after a few diameters downstream the exhaust flows in a horizontal direction with the speed of the wind, 15 ft/s. Determine the horizontal component of the force that the blowing wind puts on the exhaust gases.



■ FIGURE P5.43

For the control volume indicated the x-component of the momentum equation

$$\int_{cs} u \rho \vec{V} \cdot \hat{n} \, dA = \Sigma F_x \text{ becomes}$$

$V_2 \rho V_2 A_2 = R_x$, where R_x is the net horizontal force that the wind puts on the exhaust gases.

Thus,

$$R_x = \dot{m}_2 V_2 \quad \text{where } \dot{m}_2 = \rho A_2 V_2 = \rho A_1 V_1 \quad (\text{i.e. } \dot{m}_1 = \dot{m}_2)$$

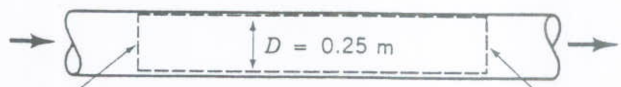
or $\dot{m}_2 = (0.00238 \frac{slugs}{s})\left[\frac{\pi}{4}(4ft)^2\right](6\frac{ft}{s}) = 0.179 \frac{slugs}{s}$

Hence,

$$R_x = 0.179 \frac{slugs}{s}\left(15\frac{ft}{s}\right) = 2.69 \frac{slug \cdot ft}{s^2} = \underline{\underline{2.69 \, lb}}$$

5.44

5.44 Air flows steadily between two cross section in a long, straight section of 12-in.-inside diameter pipe. The static temperature and pressure at each section are indicated in Fig P5.44. If the average air velocity at section (2) is 320 m/s, determine the average air velocity at section (1). Determine the frictional force exerted by the pipe wall on the air flowing between sections (1) and (2). Assume uniform velocity distributions at each section.



■ FIGURE P5.44

This analysis is similar to the one of Example 5.2. For steady flow between sections (1) and (2)

$$\dot{m}_1 = \dot{m}_2$$

or

$$\rho_1 A_1 \overline{V}_1 = \rho_2 A_2 \overline{V}_2$$

Thus

$$\overline{V}_1 = \frac{\rho_2}{\rho_1}\frac{A_2}{A_1}\overline{V}_2 \qquad (1)$$

Assuming that under the conditions of this problem, air behaves as an ideal gas we use the ideal gas equation of state (Eq. 1.8) to get

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}\frac{T_1}{T_2} \qquad (2)$$

Combining Eqs. 1 and 2 and observing that $A_1 = A_2$ we get

$$\overline{V}_1 = \frac{p_2}{p_1}\frac{T_1}{T_2}\overline{V}_2 = \frac{[127\ kPa(abs)](300\,K)}{[690\ kPa(abs)](252\,K)}\left(320\ \frac{m}{s}\right)$$

$$\underline{\underline{\overline{V}_1 = 70.1\ \frac{m}{s}}}$$

(con't)

5.44 (con't)

The analysis for this problem is similar to the one of Example 5.12. For the control volume shown in the sketch above application of the axial component of the linear momentum equation leads to

$$-\bar{V}_1 \rho_1 \bar{V}_1 A_1 + \bar{V}_2 \rho_2 \bar{V}_2 A_2 = p_1 A_1 - R_x - p_2 A_2$$

From the conservation of mass principle

$$\dot{m} = \rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2$$

Also the ideal equation of state is

$$\rho_2 = \frac{p_2}{RT_2}$$

Thus

$$R_x = \frac{p_2}{RT_2} A_2 \bar{V}_2 (\bar{V}_1 - \bar{V}_2) + A(p_1 - p_2) = \frac{\pi D^2}{4}\left[\frac{p_2}{RT_2}\bar{V}_2(\bar{V}_1 - \bar{V}_2) + (p_1 - p_2)\right]$$

$$R_x = \frac{\pi (12\ in.)^2}{4}\left(0.0254\frac{m}{in.}\right)^2 \left[\frac{(127\ kPa)\left(320\frac{m}{s}\right)\left(70\frac{m}{s} - 320\frac{m}{s}\right)\left(1000\frac{N}{kPa\cdot m^2}\right)\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right)}{\left(286.9\frac{J}{kg\cdot K}\right)(252\ K)\left(1\frac{N\cdot m}{J}\right)} + (690\ kPa - 127\ kPa)\left(1000\frac{N}{kPa\cdot m^2}\right)\right]$$

$$R_x = \underline{\underline{30{,}800\ N}}$$

5.45

5.45 Determine the magnitude and direction of the anchoring force needed to hold the horizontal elbow and nozzle combination shown in Fig. P5.45 in place. Atmospheric pressure is 100 kPa. The gage pressure at section (1) is 100 kPa. At section (2), the water exits to the atmosphere.

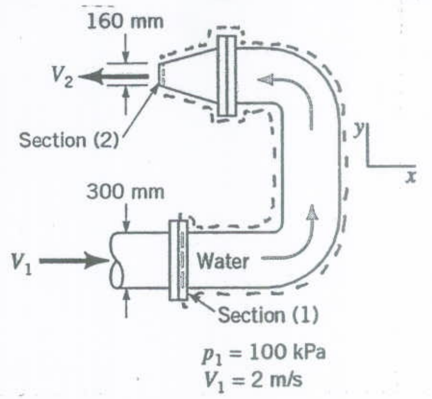


FIGURE P5.45

The control volume shown in the sketch above is used. Application of the y direction component of the linear momentum equation yields

$$R_y = \underline{\underline{0}}$$

Application of the x direction linear momentum equation leads to

$$-u_1 \rho u_1 A_1 - u_2 \rho u_2 A_2 = p_1 A_1 - R_x + p_2 A_2$$

From the conservation of mass equation

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2$$

Thus

$$R_x = \rho u_1 A_1 (u_1 + u_2) + p_1 A_1 + p_2 A_2 = \rho u_1 \frac{\pi D_1^2}{4}\left(u_1 + \frac{D_1^2}{D_2^2} u_1\right) + p_1 \frac{\pi D_1^2}{4} + (0) A_2$$

or

$$R_x = \left(999 \frac{kg}{m^3}\right)\left(2 \frac{m}{s}\right)\frac{\pi}{4}\frac{(300\ mm)^2}{\left(1000 \frac{mm}{m}\right)^2}\left[\left(2 \frac{m}{s}\right) + \frac{(300\ mm)^2}{(160}\left(2 \frac{m}{s}\right)\right]$$

$$+ (100\ kPa)\frac{\pi}{4}\frac{(300\ mm)^2}{\left(1000 \frac{mm}{m}\right)^2}\left(1000 \frac{N}{m^2 \cdot kPa}\right)$$

and

$$R_x = \underline{\underline{8340\ N}}$$

5.46

5.46 Water flows as two free jets from the tee attached to the pipe shown in Fig. P5.46. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the x and y components of the force that the pipe exerts on the tee.

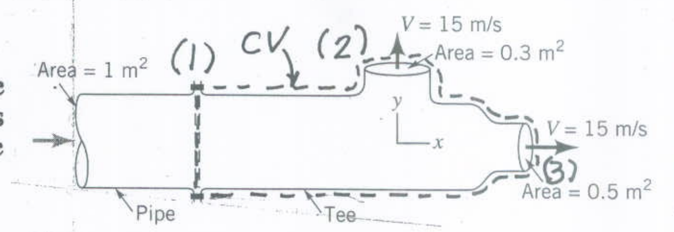


FIGURE P5.46

Use the control volume shown.

For the x-component of the force exerted by the pipe on the tee we use the x-component of the linear momentum equation.

$$-V_1\rho V_1 A_1 + V_3 \rho V_3 A_3 = p_1 A_1 - p_3 A_3 - p_{atm}(A_1 - A_3) + F_x$$

$$= (p_{1\,gage} + p_{atm})A_1 - (p_{3\,gage} + p_{atm})A_3 - p_{atm}(A_1 - A_3) + F_x$$

$$= p_{1\,gage} A_1 + F_x \qquad (1)$$

To get V_1 we use conservation of mass

$$Q_1 = Q_2 + Q_3$$

or $A_1 V_1 = A_2 V_2 + A_3 V_3$

so $$V_1 = \frac{A_2 V_2 + A_3 V_3}{A_1} = \frac{(0.3\,m^2)(15\,\frac{m}{s}) + (0.5\,m^2)(15\,\frac{m}{s})}{1\,m^2} = 12\,\frac{m}{s}$$

To estimate $p_{1\,gage}$ we use Bernoulli's equation for flow between (1) and (2)

$$\frac{p_{1\,gage}}{\rho} + \frac{V_1^2}{2} = \frac{p_{2\,gage}}{\rho} + \frac{V_2^2}{2}$$

$$p_{1\,gage} = \rho\left(\frac{V_2^2 - V_1^2}{2}\right) = \left(999\,\frac{kg}{m^3}\right)\frac{\left[(15\,\frac{m}{s})^2 - (12\,\frac{m}{s})^2\right]}{2}\left(\frac{1\,N \cdot s^2}{kg \cdot m}\right)$$

$$p_{1\,gage} = 40{,}500\,\frac{N}{m^2}$$

Now using Eq. (1) we get:

$$\left[-(12\,\tfrac{m}{s})(999\,\tfrac{kg}{m^3})(12\,\tfrac{m}{s})(1\,m^2) + (15\,\tfrac{m}{s})(999\,\tfrac{kg}{m^3})(15\,\tfrac{m}{s})(0.5\,m^2)\right]\left(\frac{1\,N \cdot s^2}{kg \cdot m}\right) = \left(40{,}500\,\frac{N}{m^2}\right)(1\,m^2) + F_x$$

or $-72{,}000\,N = F_x$

so $F_x = \underline{\underline{72{,}000\,N}} \leftarrow$

For the y component of the force exerted by the pipe on the tee we use the y component of the linear momentum equation to get

$$V_2 \rho V_2 A_2 = F_y$$

$$(15\,\tfrac{m}{s})(999\,\tfrac{kg}{m^3})(15\,\tfrac{m}{s})(0.3\,m^2) = \underline{\underline{67{,}400\,N}} \uparrow = F_y$$

5.47

5.47 A converging elbow (see Fig. P5.47) turns water through an angle of 135° in a vertical plane. The flow cross section diameter is 400 mm at the elbow inlet, section (1), and 200 mm at the elbow outlet, section (2). The elbow flow passage volume is 0.2 m³ between sections (1) and (2). The water volume flowrate is 0.4 m³/s and the elbow inlet and outlet pressures are 150 kPa and 90 kPa. The elbow mass is 12 kg. Calculate the horizontal (x direction) and vertical (z direction) anchoring forces required to hold the elbow in place.

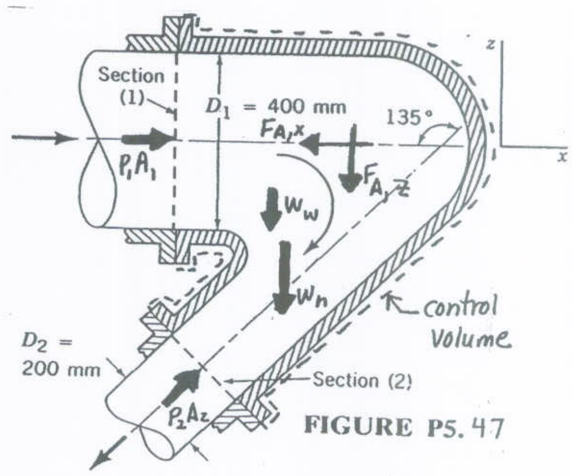


FIGURE P5.47

A control volume that contains the elbow and the water within the elbow between sections (1) and (2) as shown in the sketch above is used. Application of the horizontal or x direction component of the linear momentum equation yields

$$-u_1\rho u_1 A_1 - V_2\cos 45^\circ \rho V_2 A_2 = p_1A_1 - F_{A,x} + p_2A_2\cos 45^\circ$$

From conservation of mass

$$\dot m = \rho u_1 A_1 = \rho V_2 A_2 = \rho Q \qquad (1)$$

Thus

$$F_{A,x} = \frac{\rho Q^2}{A_1} + \frac{\rho Q^2\cos 45^\circ}{A_2} + p_1A_1 + p_2A_2\cos 45^\circ = \frac{\rho Q^2}{\frac{\pi D_1^2}{4}} + \frac{\rho Q^2\cos 45^\circ}{\frac{\pi D_2^2}{4}} + p_1\frac{\pi D_1^2}{4} + p_2\frac{\pi D_2^2}{4}\cos 45^\circ$$

$$F_{A,x} = \left(999\frac{kg}{m^3}\right)\left(0.4\frac{m^3}{s}\right)^2\frac{1}{\left(\frac{\pi}{4}\right)}\left[\frac{\left(1000\frac{mm}{m}\right)^2}{(400\ mm)^2} + \frac{\cos 45^\circ\left(1000\frac{mm}{m}\right)^2}{(200\ mm)^2}\right]\left(\frac{1\ N}{kg\frac{m}{s^2}}\right)$$

$$+\ \frac{\pi}{4}\frac{\left(1000\frac{N}{kPa\ m^2}\right)}{\left(1000\frac{mm}{m}\right)^2}\left[(150\ kPa)(400\ mm)^2 + (90\ kPa)(200\ mm)^2\cos 45^\circ\right]$$

$$\underline{\underline{F_{A,x} = 25{,}700\ N}}$$

Application of the vertical or z direction component of the linear momentum equation leads to

$$-V_2\sin 45^\circ \rho V_2 A_2 = p_2A_2\sin 45^\circ - F_{A,z} - W_w - W_e$$

which when combined with Eq. 1 gives

$$F_{A,z} = \frac{\rho Q^2\sin 45^\circ}{A_2} + p_2A_2\sin 45^\circ - W_w - W_e = \frac{\rho Q^2\sin 45^\circ}{\frac{\pi D_2^2}{4}} + p_2\frac{\pi D_2^2}{4}\sin 45^\circ - \rho g V\!\!\!\!-_w - m_e g$$

(con't)

5.47 (con't)

$$F_{A,z} = \frac{\left(999\,\frac{kg}{m^3}\right)\left(0.4\,\frac{m^3}{s}\right)^2 \sin 45^\circ}{\frac{\pi}{4}\frac{(200\,mm)^2}{\left(1000\,\frac{mm}{m}\right)^2}}\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right) + \frac{(90\,kPa)\,\pi\,(200\,mm)^2 \sin 45^\circ}{4\left(1000\,\frac{mm}{m}\right)^2}$$

$$-\left(999\,\frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)\left(0.2\,m^3\right)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right) - (12\,kg)\left(9.81\,\frac{m}{s^2}\right)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)$$

$$F_{A,z} = \underline{\underline{8920\ N}}$$

5.48

5.48 The hydraulic dredge shown in Fig. P5.48 is used to dredge sand from a river bottom. Estimate the thrust needed from the propeller to hold the boat stationary. Assume the specific gravity of the sand/water mixture is $SG = 1.2$.

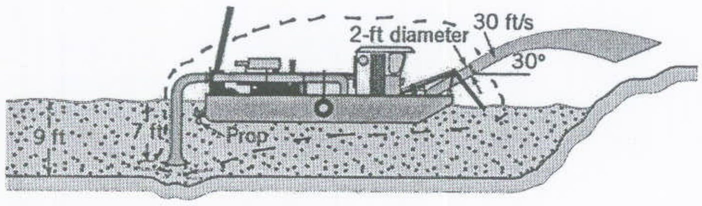


FIGURE P5.48

Using the control volume shown by the broken line in the sketch above we use the horizontal or x component of the linear momentum equation to get

$$F_x = \rho_2 A_2 V_2 V_{2x} = \rho_{H_2O} (SG)_2 \frac{\pi d_2^2}{4} V_2 V_2 \cos 30^\circ$$

where section 1 is where flow enters the control volume vertically and section 2 is where flow leaves the control volume at an angle of 30° from the horizontal direction. Note that there is no horizontal direction linear momentum flow at section 1.

$$F_x = \left(1.94 \frac{slugs}{ft^3}\right)(1.4) \frac{\pi (2\,ft)^2}{4} \left(30 \frac{ft}{s}\right)\left(30 \frac{ft}{s}\right) \cos 30^\circ \left(\frac{1\ lb}{\frac{ft\ slug}{s^2}}\right)$$

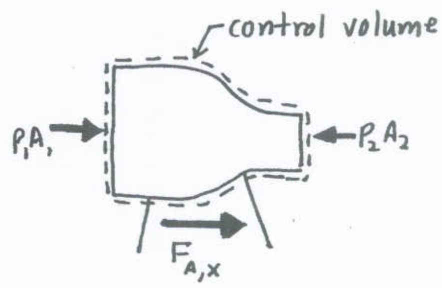
$$F_x = \underline{\underline{6650}}\ lb$$

5.49

5.49 A static thrust stand is to be designed for testing a specific jet engine. Knowing the following conditions for a typical test,

intake air velocity = 700 ft/s
exhaust gas velocity = 1640 ft/s
intake cross section area = 10 ft²
intake static pressure = 11.4 psia
intake static temperature = 480 °R
exhaust gas pressure = 0 psi

estimate a nominal thrust to design for.



The analysis for this problem is similar to the one of Example 5.14. A control volume that contains the entire engine and the fluid in the engine as indicated in the sketch is used. Application of the horizontal or x direction component of the linear momentum equation leads to

$$-u_1\rho_1 u_1 A_1 + u_2 \rho_2 u_2 A_2 = p_1 A_1 + F_{A,x}$$

or

$$F_{A,x} = -u_1 \rho u_1 A_1 + u_2 \rho_2 u_2 A_2 - p_1 A_1$$

The conservation of mass principle yields

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

Thus

$$F_{A,x} = \rho_1 u_1 A_1 (u_2 - u_1) - p_1 A_1$$

or since

$$\rho_1 = \frac{p_1}{RT_1}$$

then

$$F_{A,x} = \frac{p_1}{RT_1} u_1 A_1 (u_1 - u_2) \quad p_1 A_1$$

$$F_{A,x} = \frac{\left(11.4 \frac{lb}{in.^2}\right)\left(700 \frac{ft}{s}\right)\left(10\,ft^2\right)\left(1640\frac{ft}{s} - 700\frac{ft}{s}\right)\left(144 \frac{in.^2}{ft^2}\right)\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)}{\left(1716 \frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)(480\,{}^\circ R)}$$

$$-\left(11.4 \frac{lb}{in.^2} - 14.7\frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)\left(10\,ft^2\right)$$

and

$$F_{A,x} = \underline{\underline{17{,}900\ lb}}$$

5.50

5.50 A horizontal circular cross section jet of air having a diameter of 6 in. strikes a conical deflector as shown in Fig. P5.50. A horizontal anchoring force of 5 lb is required to hold the cone in place. Estimate the nozzle flow rate in ft^3/s. The magnitude of the velocity of the air remains constant.

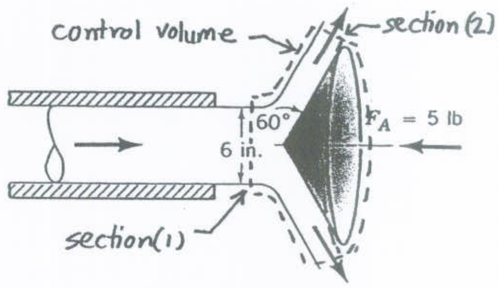


FIGURE P5.50

The control volume shown in the sketch is used. Application of the axial or x-direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = -F_{A,x}$$

With the conservation of mass principle we can conclude for this incompressible flow that

$$u_1 A_1 = u_2 A_2 = Q$$

Also

$$u_2 = V \cos 60°$$

and

$$u_1 = V = \frac{Q}{A_1}$$

Thus

$$-V\rho Q + V\cos 60° \rho Q = -F_{A,x} = -\frac{Q^2}{A_1}\rho + \frac{Q^2}{A_1}\cos 60° \rho$$

or

$$Q = \left[\frac{F_{A,x} A_1}{\rho(1-\cos 60°)}\right]^{\frac{1}{2}} = \left[\frac{F_{A,x}\left(\frac{\pi D_1^2}{4}\right)}{\rho(1-\cos 60°)}\right]^{\frac{1}{2}}$$

Thus

$$Q = \left[\frac{(5\ lb)(\pi)(6\ in.)^2}{\left(0.00238 \frac{slugs}{ft^3}\right)(1-\cos 60°)(4)\left(144 \frac{in.^2}{ft^2}\right)\left(\frac{1\ lb}{slug \frac{ft}{s^2}}\right)}\right]^{\frac{1}{2}}$$

and

$$Q = \underline{\underline{28.7}}\ \frac{ft^3}{s}$$

5.51

5.51 A vertical, circular cross-sectional jet of air strikes a conical deflector as indicated in Fig. P5.51. A vertical anchoring force of 0.1 N is required to hold the deflector in place. Determine the mass (kg) of the deflector. The magnitude of velocity of the air remains constant.

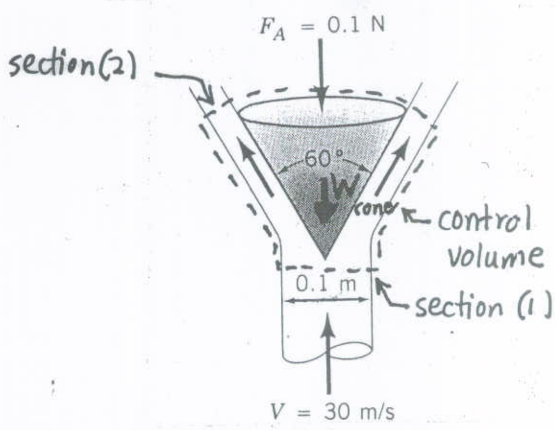


FIGURE P5.51

To determine the mass of the conical deflector we use the stationary, non-deforming control volume shown in the sketch above. Application of the vertical direction component of the linear momentum equation (Eq. 5.22) to the contents of this control volume yields

$$\dot{m}\left(-V_1 + V_2 \cos 30^\circ\right) = -F_A - W_{cone}$$

or

$$W_{cone} = m_{cone}\, g = \dot{m}\left(V_1 - V_2 \cos 30^\circ\right) - F_A = \rho A_1 V_1 \left(V_1 - V_2 \cos 30^\circ\right) - F_A \quad (1)$$

However

$$V_1 = V_2$$

and

$$A_1 = \frac{\pi D_1^2}{4}$$

Thus Eq. 1 can be expressed as

$$m_{cone} = \rho \frac{\pi D_1^2}{4g} V_1 \left(V_1 - V_1 \cos 30^\circ\right) - \frac{F_A}{g}$$

or

$$m_{cone} = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi (0.1\,m)^2 \left(30 \frac{m}{s}\right)\left[30 \frac{m}{s} - \left(30 \frac{m}{s}\right)\cos 30^\circ\right]}{(4)\left(9.81 \frac{m}{s^2}\right)} - \frac{0.1\,N}{\left(9.81 \frac{m}{s^2}\right)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)}$$

and

$$m_{cone} = \underline{\underline{0.108}}\ kg$$

5.52

5.52 Water flows from a large tank into a dish as shown in Fig. P5.52. **(a)** If at the instant shown the tank and the water in it weigh W_1 lb, what is the tension, T_1, in the cable supporting the tank? **(b)** If at the instant shown the dish and the water in it weigh W_2 lb, what is the force, F_2, needed to support the dish?

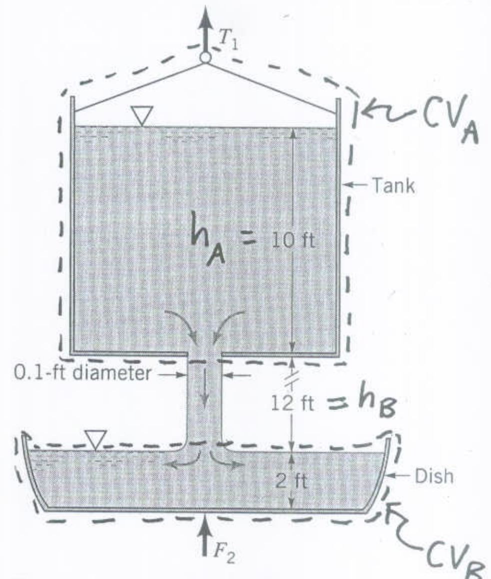


FIGURE P5.52

For part (a) we apply the vertical component of the linear momentum equation to the contents of control volume A, CV_A, to get

$$-V_{out}\,\rho V_{out} A_{out} = T_1 - W_1 \qquad (1)$$

To get value of V_{out} we apply Bernoulli's equation to the flow from the free surface of the water in the tank to the tank outlet to get

$$V_{out} = \sqrt{2gh_A} = \sqrt{(2)(32.2\ \tfrac{ft}{s^2})(10\ ft)} = 25.4\ \tfrac{ft}{s}$$

Then from Eq.(1) we get

$$\frac{-(25.4\ \frac{ft}{s})(1.94\ \frac{slugs}{ft^3})(25.4\ \frac{ft}{s})\ \frac{\pi\,(0.1\ ft)^2}{4}}{1\ \frac{slug\cdot ft}{lb\cdot s^2}} = T_1 - W_1$$

and

$$\underline{\underline{T_1 = W_1 - 9.8\ lb}}$$

For part (b) we apply the vertical component of the linear momentum equation to the contents of CV_B to get

$$V_{\substack{into\\CV_B}}\ \dot m_{\substack{into\\CV_B}} = F_2 - W_2 \qquad (2)$$

To get $V_{\substack{into\\CV_B}}$ we use Bernoulli's equation between free surface of water in tank to free surface of water in dish to get

$$V_{\substack{into\\CV_B}} = \sqrt{2g(h_A+h_B)} = \sqrt{2(32.2\ \tfrac{ft}{s^2})(10\ ft + 12\ ft)} = 37.6\ \tfrac{ft}{s}$$

For $\dot m_{\substack{into\\CV_B}}$ we use from conservation of mass, $\dot m_{\substack{into\\CV_B}} = \dot m_{\substack{out\ of\\tank}} = \rho V_{out} A_{out}$

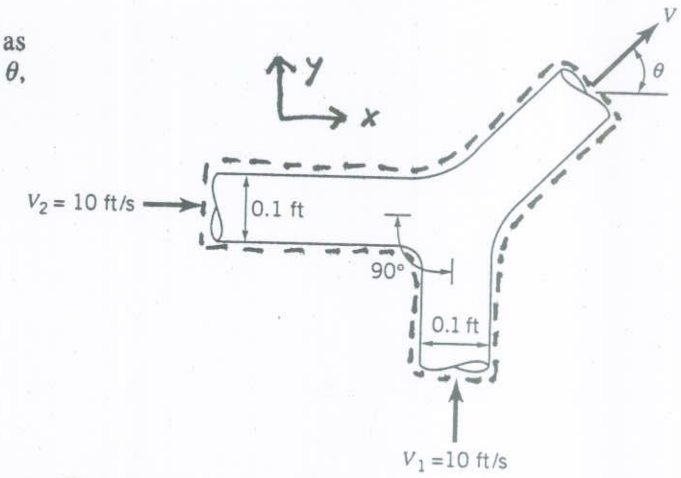
So from Eq.(2) we get

$$(37.6\ \tfrac{ft}{s})(1.94\ \tfrac{slug}{ft^3})(25.4\ \tfrac{ft}{s})\ \frac{\pi\,(0.1\ ft)^2}{4}\left(\frac{1\ lb\cdot s^2}{slug\cdot ft}\right) = F_2 - W_2$$

and $\underline{\underline{F_2 = W_2 + 14.7\ lb}}$

5.53

5.53 Two water jets of equal size and speed strike each other as shown in Fig. P5.53. Determine the speed, V, and direction, θ, of the resulting combined jet. Gravity is negligible.



■ FIGURE P5.53

For the control volume shown in the sketch above the linear momentum equation for the x and y directions are, for the x direction

$$-V_2 \rho V_2 A_2 + (V \cos\theta)\rho V A = 0 \qquad (1)$$

and for the y direction

$$-V_1 \rho V_1 A_1 + (V \sin\theta)\rho V A = 0 \qquad (2)$$

Also for conservation of mass we have

$$\rho_1 V_1 A_1 + \rho V_2 A_2 - \rho V A = 0 \qquad (3)$$

From Eqs. 1 and 2 we get

$$\frac{V_2^2 A_2}{V_1^2 A_1} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

so

$$\theta = \cot^{-1}\frac{V_2^2 A_2}{V_1^2 A_1} = \cot^{-1}\left[\frac{\left(10\,\frac{ft}{s}\right)^2 \pi \frac{(0.1 ft)^2}{4}}{\left(10\,\frac{ft}{s}\right)^2 \pi \frac{(0.1 ft)^2}{4}}\right] = 45^\circ$$

Now, combining Eqs. 2 and 3 we get

$$-V_1^2 A_1 + V \sin\theta\,(V_1 A_1 + V_2 A_2) = 0$$

or

$$V = \frac{V_1^2 A_1}{\sin\theta\,(V_1 A_1 + V_2 A_2)}$$

$$V = \frac{\left(10\,\frac{ft}{s}\right)^2 \pi \frac{(0.1 ft)^2}{4}}{(\sin 45^\circ)\left[\left(10\,\frac{ft}{s}\right)\frac{\pi (0.1 ft)^2}{4} + \left(10\,\frac{ft}{s}\right)\frac{\pi (0.1 ft)^2}{4}\right]}$$

and

$$V = \underline{\underline{7.07}}\ \frac{ft}{s}$$

5.54

5.54 Assuming frictionless, incompressible, one-dimensional flow of water through the horizontal tee connection sketched in Fig. P5.54, estimate values of the x and y components of the force exerted by the tee on the water. Each pipe has an inside diameter of 1 m.

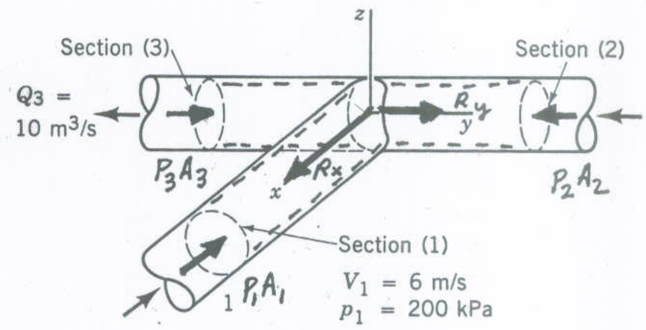


FIGURE P5.54

We can use the x and y components of the linear momentum equation (Eq. 5.22) to determine the x and y components of the reaction force exerted by the water on the tee. For the control volume containing water in the tee, Eq. 5.22 leads to

$$R_x = p_1 A_1 + V_1 \rho Q_1 = p_1 \frac{\pi D_1^2}{4} + V_1 \rho Q_1 \qquad (1)$$

and

$$R_y = p_2 \frac{\pi D_2^2}{4} - p_3 \frac{\pi D_3^2}{4} + V_2 \rho Q_2 - V_3 \rho Q_3 \qquad (2)$$

The reaction forces in Eqs. 1 and 2 are actually exerted by the tee on the water in the control volume. The reaction of the water on the tee is equal in magnitude but opposite in direction.
Conservation of mass (Eq. 5.4) leads to

$$Q_2 = Q_3 - Q_1 = Q_3 - V_1 \frac{\pi D_1^2}{4} = 10 \frac{m^3}{s} - \left(6 \frac{m}{s}\right) \frac{\pi (1m)^2}{4} = 5.288 \frac{m^3}{s}$$

Also

$$Q_1 = V_1 \frac{\pi D_1^2}{4} = \left(6 \frac{m}{s}\right) \frac{\pi (1m)^2}{4} = 4.712 \frac{m^3}{s}$$

Further

$$V_2 = \frac{Q_2}{\frac{\pi D_2^2}{4}} = \frac{\left(5.288 \frac{m^3}{s}\right)}{\frac{\pi (1m)^2}{4}} = 6.733 \frac{m}{s}$$

and

$$V_3 = \frac{Q_3}{\frac{\pi D_3^2}{4}} = \frac{\left(10 \frac{m^3}{s}\right)}{\frac{\pi (1m)^2}{4}} = 12.73 \frac{m}{s}$$

(con't)

5.54 (con't)

Because the flow is incompressible and frictionless we assume that Bernoulli's equation (Eq. 5.74) is valid throughout the control volume. Thus

$$p_3 = p_1 + \frac{\rho}{2}\left(V_1^2 - V_3^2\right) = 200\ kPa + \frac{\left(999\ \frac{kg}{m^3}\right)}{2}\left[\left(6\ \frac{m}{s}\right)^2 - \left(12.73\ \frac{m}{s}\right)^2\right]\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)\left(\frac{10^{-3}\ kPa}{\frac{N}{m^2}}\right)$$

or

$$p_3 = 137\ kPa$$

Also

$$p_2 = p_1 + \frac{\rho}{2}\left(V_1^2 - V_2^2\right) = 200\ kPa + \frac{\left(999\ \frac{kg}{m^3}\right)}{2}\left[\left(6\ \frac{m}{s}\right)^2 - \left(6.733\ \frac{m}{s}\right)^2\right]\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)\left(\frac{10^{-3}\ kPa}{\frac{N}{m^2}}\right)$$

or

$$p_2 = 195.3\ kPa$$

With Eq. 1

$$R_x = \left(200{,}000\ \frac{N}{m^2}\right)\frac{\pi}{4}(1m)^2 + \left(6\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)\left(4.712\ \frac{m^3}{s}\right)\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right) = 185{,}000\ N = 185\ kN$$

and the x-direction component of force exerted by the water on the tee is $\underline{\underline{-185\ kN}}$.

With Eq. 2

$$R_y = \left(195{,}300\ \frac{N}{m^2}\right)\frac{\pi}{4}(1m)^2 - \left(137{,}000\ \frac{N}{m^2}\right)\frac{\pi}{4}(1m)^2 + \left(6.733\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)5.2$$

$$+ \left(6.733\ \frac{m}{s}\right)\left(999\ \frac{kg}{m^3}\right)\left(5.288\ \frac{m^3}{s}\right)\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)$$

or

$$R_y = -45{,}800\ N = -45.8\ kN$$

and the y-direction component of force exerted by the water on the tee is $\underline{\underline{+45.8}}\ kN$.

5.55

5.55 Determine the magnitude of the horizontal component of the anchoring force required to hold in place the sluice gate shown in Fig. 5.55. Compare this result with the size of the horizontal component of the anchoring force required to hold in place the sluice gate when it is closed and the depth of water upstream is 10 ft.

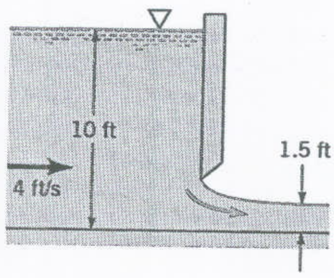


FIGURE P5.55

This analysis is similar to the one of Example 5.15. The control volumes of Fig. E 5.15 are appropriate for use in solving this problem. When the sluice gate is closed (see Figs. E5.15a and E5.15c) application of the x direction component of the linear momentum equation leads to

$$R_x = \frac{1}{2}\gamma H^2 = \frac{1}{2}\left(62.4\,\frac{lb}{ft^3}\right)(10\,ft)^2 = \underline{\underline{3120}}\ \frac{lb}{ft}$$

When the sluice gate is open (see Figs. E5.15b and E5.15d) application of the x direction component of the linear momentum equation leads to

$$R_x = \frac{1}{2}\gamma H^2 - \frac{1}{2}\gamma h^2 - F_f + \rho u_1^2 H - \rho u_2^2 h$$

The exit velocity u_2 may be expressed in terms of the inlet velocity u_1 with the conservation of mass equation as follows

$$u_2 = u_1 \frac{H}{h}$$

Thus

$$R_x = \frac{1}{2}\gamma H^2 - \frac{1}{2}\gamma h^2 - F_f + \rho u_1^2 H - \rho u_1^2 \frac{H^2}{h}$$

Assuming F_f is negligibly small, we obtain

$$R_x = \frac{1}{2}\left(62.4\,\frac{lb}{ft^3}\right)(10\,ft)^2 - \frac{1}{2}\left(62.4\,\frac{lb}{ft^3}\right)(1.5\,ft)^2$$
$$+ \left(1.94\,\frac{slugs}{ft^3}\right)\left(4\,\frac{ft}{s}\right)^2(10\,ft)\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right) - \left(1.94\,\frac{slugs}{ft^3}\right)\left(4\,\frac{ft}{s}\right)^2\frac{(10\,ft)^2}{(1.5\,ft)}\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)$$

$$R_x = \underline{\underline{1290}}\ \frac{lb}{ft}$$

Thus it takes considerably less force to hold in place the sluice gate when it is opened as compared to when it is closed.

5.56

5.56 The rocket shown in Fig. P5.56. is held stationary by the horizontal force, F_x, and the vertical force, F_z. The velocity and pressure of the exhaust gas are 5000 ft/s and 20 psia at the nozzle exit, which has a cross section area of 60 in.^{2}. The exhaust mass flowrate is constant at 21 lbm/s Determine the value of the restraining force F_x. Assume the exhaust flow is essentially horizontal.

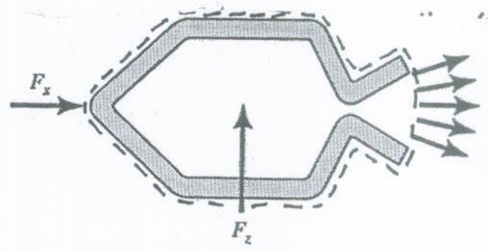


FIGURE P5.56

The control volume contains the rocket and the fluid within the rocket as indicated in the sketch. Application of the x direction component of the linear momentum equation yields

$$\underbrace{\cancel{\frac{\partial}{\partial t}\int_{cv} u \rho \, d\forall}}_{0 \text{ because the rocket is stationary}} + V_1 \rho_1 V_1 A_1 = F_x - p_1 A_1$$

or

$$F_x = p_1 A_1 + V_1 \rho_1 V_1 A_1$$

But

$$\dot{m} = \rho_1 A_1 V_1$$

thus

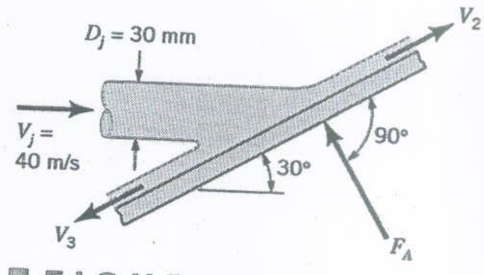
$$F_x = p_1 A_1 + V_1 \dot{m}$$

$$F_x = \left(20 \frac{lb_f}{in.^2} - 14.7 \frac{lb_f}{in.^2}\right)(60 \, in.^2) + \left(5000 \frac{ft}{s}\right)\left(21 \frac{lb_m}{s}\right)\frac{1}{\left(32.2 \frac{lb_m \cdot ft}{lb_f \, s^2}\right)}$$

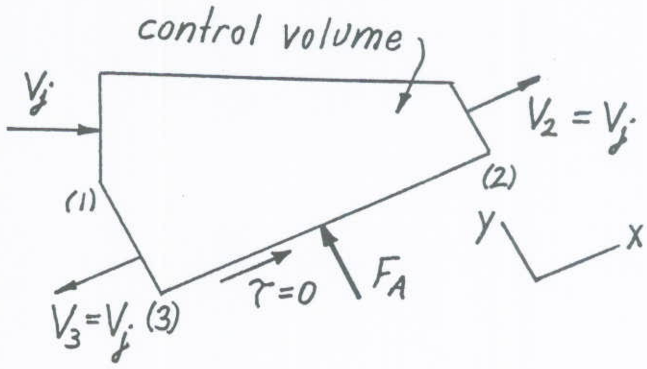
$$F_x = \underline{\underline{3580 \, lb_f}}$$

5.57

5.57 A horizontal circular jet of air strikes a stationary flat plate as indicated in Fig. 5.57. The jet velocity is 40 m/s and the jet diameter is 30 mm. If the air velocity magnitude remains constant as the air flows over the plate surface in the directions shown, determine: **(a)** the magnitude of F_A, the anchoring force required to hold the plate stationary; **(b)** the fraction of mass flow along the plate surface in each of the two directions shown; **(c)** the magnitude of F_A, the anchoring force required to allow the plate to move to the right at a constant speed of 10 m/s.



■ FIGURE P5.57



The non-deforming control volume shown in the sketch above is used.

(a) To determine the magnitude of F_A we apply the component of the linear momentum equation (Eq. 5.22) along the direction of F_A. Thus, $\int_{cs} v \rho \vec{V}\cdot\hat{n}\, dA = \Sigma F_y$, or

$$F_A = \dot{m} V_j \sin 30° = \rho A_j V_j V_j \sin 30° = \frac{\rho \pi D_j^2}{4} V_j^2 \sin 30°$$

or

$$F_A = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi (0.030 m)^2}{(4)} \left(40 \frac{m}{s}\right)^2 (\sin 30°) \left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right) = \underline{\underline{0.696\ N}}$$

(b) To determine the fraction of mass flow along the plate surface in each of the 2 directions shown in the sketch above, we apply the component of the linear momentum equation parallel to the surface of the plate, $\int_{cs} u \rho \vec{V}\cdot\hat{n}\, dA = \Sigma F_x$, to obtain

$$R_{\text{along plate surface}} = \dot{m}_2 V_2 - \dot{m}_3 V_3 - \dot{m}_j V_j \cos 30° \qquad (1)$$

(con't)

5.57 (con't)

Since the air velocity magnitude remains constant, the value of $R_{\text{along plate surface}}$ is zero.* Thus from Eq. 1 we obtain

$$\dot{m}_3 V_3 = \dot{m}_2 V_2 - \dot{m}_j V_j \cos 30° \qquad (2)$$

Since $V_3 = V_2 = V_j$, Eq. 2 becomes

$$\dot{m}_3 = \dot{m}_2 - \dot{m}_j \cos 30° \qquad (3)$$

From conservation of mass we conclude that

$$\dot{m}_j = \dot{m}_2 + \dot{m}_3 \qquad (4)$$

Combining Eqs. 3 and 4 we get

$$\dot{m}_3 = \dot{m}_j - \dot{m}_3 - \dot{m}_j \cos 30°$$

or

$$\dot{m}_3 = \dot{m}_j \frac{(1-\cos 30°)}{2} = \dot{m}_j (0.0670)$$

and

$$\dot{m}_2 = \dot{m}_j (1 - 0.067) = \dot{m}_j (0.933)$$

Thus, $\underline{\underline{\dot{m}_2 \text{ involves } 93.3\% \text{ of } \dot{m}_j}}$ and $\underline{\underline{\dot{m}_3 \text{ involves } 6.7\% \text{ of } \dot{m}_j}}$.

(c) To determine the magnitude of F_A required to allow the plate to move to the right at a constant speed of $10 \frac{m}{s}$, we use a non-deforming control volume like the one in the sketch above that moves to the right with a speed of $10 \frac{m}{s}$. The translating control volume linear momentum equation (Eq. 5.29) leads to

$$F_A = \frac{\rho \pi D_j^2}{4} \left(V_j - 10 \frac{m}{s}\right)^2 \sin 30°$$

or

$$F_A = \left(1.23 \frac{kg}{m^3}\right) \frac{\pi (0.030\,m)^2}{4} \left(40 \frac{m}{s} - 10 \frac{m}{s}\right)^2 (\sin 30°) \left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)$$

and

$$F_A = \underline{\underline{0.391\ N}}$$

* Since $V_1 = V_2 = V_3$ and $\rho_1 = \rho_2 = \rho_3$ and $z_1 = z_2 = z_3$ it follows that the Bernoulli equation is valid from $1 \rightarrow 2$ and $1 \rightarrow 3$.
Thus, there are no viscous effects (Bernoulli equation is valid only for inviscid flow) so that $\tau = 0$. Hence, $R_{\text{along plate}} = 0$.

5.58

5.58 Water is sprayed radially outward over 180° as indicated in Fig. P5.58. The jet sheet is in the horizontal plane. If the jet velocity at the nozzle exit is 20 ft/s, determine the direction and magnitude of the resultant horizontal anchoring force required to hold the nozzle in place.

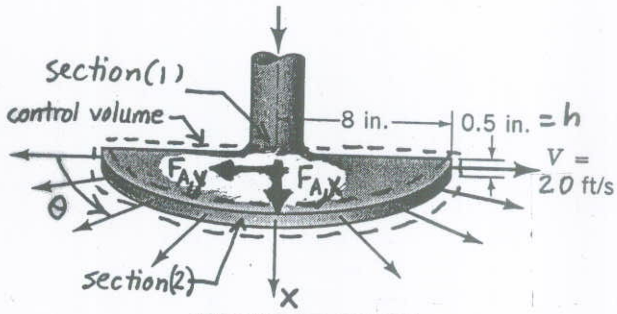


FIGURE P5.58

The control volume includes the nozzle and water between sections (1) and (2) as indicated in the sketch above. Application of the y direction component of the linear momentum equation yields

$$\int_{cs} v \rho \vec{V} \cdot \hat{n}\, dA = -F_{A,y}$$

or

$$F_{A,y} = -\rho \int_0^{\pi} (-V_2 \cos\theta)(V_2)\, h R\, d\theta = \rho h R V_2^2 (\sin\pi - \sin 0)$$

and

$$\underline{\underline{F_{A,y} = 0}}$$

Application of the x direction component of the linear momentum equation leads to

$$\int_{cs} u \rho \vec{V} \cdot \hat{n}\, dA = F_{A,x}$$

or

$$F_{A,x} = \rho \int_0^{\pi} (V_2 \sin\theta)(V_2)\, h R\, d\theta = \rho h R V_2^2 (\cos 0 - \cos\pi)$$

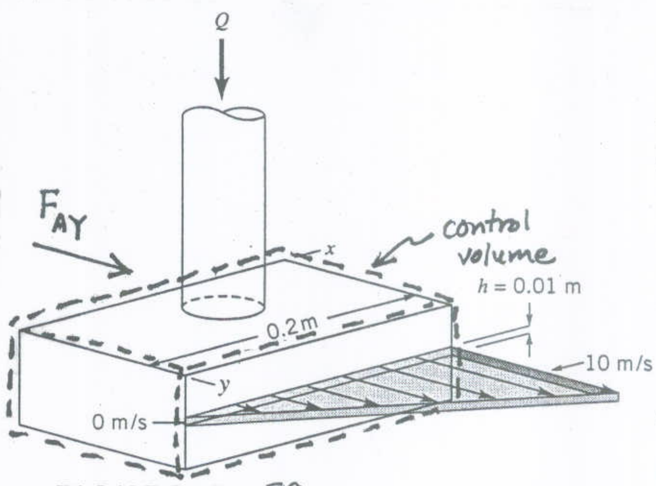
and

$$F_{A,x} = \left(1.94 \frac{slugs}{ft^3}\right) \frac{(0.5\ in.)\ (8\ in.)\ \left(20 \frac{ft}{s}\right)^2 (2)}{\left(12 \frac{in.}{ft}\right)\left(12 \frac{in.}{ft}\right)} \left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$F_{A,x} = \underline{\underline{43\ lb}}$$

5.59

5.59 A sheet of water of uniform thickness ($h = 0.01$ m) flows from the device shown in Fig. P5.59. The water enters vertically through the inlet pipe and exits horizontally with a speed that varies linearly from 0 to 10 m/s along the 0.2-m length of the slit. Determine the y component of anchoring force necessary to hold this device stationary.



■ FIGURE P5.59

A control volume that contains the box portion of the device and the water in the box as shown in the sketch above is used. Application of the y-direction component of the linear momentum equation yields

$$F_{AY} = \int_{A_{slit}} v \rho \vec{V} \cdot \hat{n}\, dA = \rho \int_0^{0.2} v^2 h\, dx$$

The variation of v with x is linear or

$$v = 50x \ \frac{m}{s}$$

Thus

$$F_{AY} = \rho \int_0^{0.2} (50x)^2 h\, dx = \rho (50)^2 h \left.\frac{x^3}{3}\right|_0^{0.2}$$

or

$$F_{AY} = \left(999 \frac{kg}{m^3}\right)\left(50 \frac{1}{s}\right)^2 (0.01 m) \frac{(0.2 m)^3}{3} \left(1 \frac{N \cdot s^2}{kg \cdot m}\right)$$

and

$$F_{AY} = \underline{\underline{66.6 N}}$$

5.60

5.60 A variable mesh screen produces a linear and axisymmetric velocity profile as indicated in Fig. P5.60 in the air flow through a 2-ft-diameter circular cross section duct. The static pressures upstream and downstream of the screen are 0.2 and 0.15 psi and are uniformly distributed over the flow cross section area. Neglecting the force exerted by the duct wall on the flowing air, calculate the screen drag force.

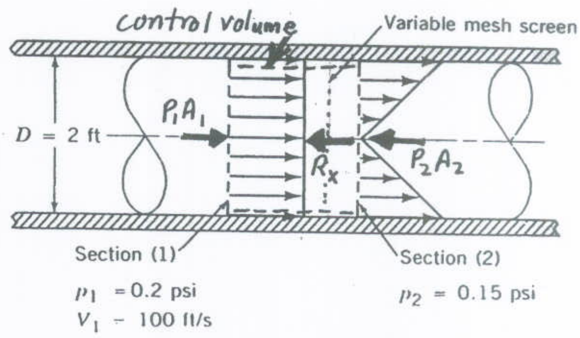


FIGURE P5.60

Application of the axial component of the linear momentum equation to the flow through the control volume shown in the sketch leads to

$$-V_1 \rho V_1 A_1 + \int_0^R u_2 \rho u_2 \, 2\pi r \, dr = P_1 A_1 - R_x - P_2 A_2$$

or

$$R_x = \rho V_1^2 \frac{\pi D_1^2}{4} - 2\pi\rho \int_0^R \left(u_{max} \frac{r}{R}\right)^2 r \, dr + P_1 \frac{\pi D_1^2}{4} - P_2 \frac{\pi D_2^2}{4} \qquad (1)$$

The value of u_{max} may be obtained from conservation of mass as follows

$$\rho V_1 \frac{\pi D_1^2}{4} = \rho \int_0^R \left(u_{max} \frac{r}{R}\right) 2\pi r \, dr$$

Thus

$$u_{max} = \frac{V_1 D_1^2 R}{(2)(4)\int_0^R r^2 \, dr} = \frac{3}{2} V_1 = \frac{3}{2}\left(100 \frac{ft}{s}\right) = 150 \frac{ft}{s}$$

From Eq. 1

$$R_x = \left(0.00238 \frac{slug}{ft^3}\right)\left(100 \frac{ft}{s}\right)^2 \frac{\pi (2\,ft)^2}{4}\left(\frac{1\,lb}{slug \cdot \frac{ft}{s^2}}\right) - 2\pi\left(0.00238 \frac{slug}{ft^3}\right)\left(150 \frac{ft}{s}\right)^2 \frac{(2\,ft)^2}{16}\left(\frac{1\,lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$+ \left(0.2 \frac{lb}{in.^2}\right)\frac{\pi (2\,ft)^2}{4}\left(144 \frac{in.^2}{ft^2}\right) - \left(0.15 \frac{lb}{in.^2}\right)\frac{\pi (2\,ft)^2}{4}\left(144 \frac{in.^2}{ft^2}\right)$$

$$R_x = \underline{\underline{13.3\ lb}}$$

5.61

5.61 Water flows vertically upward in a circular cross section pipe as shown in Fig. P5.61. At section (1), the velocity profile over the cross section area is uniform. At section (2), the velocity profile is

$$\mathbf{V} = w_c \left(\frac{R-r}{R}\right)^{1/7} \hat{\mathbf{k}}$$

where $\mathbf{V}$ = local velocity vector, w_c = centerline velocity in the axial direction, R = pipe radius, and r = radius from pipe axis. Develop an expression for the fluid pressure drop that occurs between sections (1) and (2).

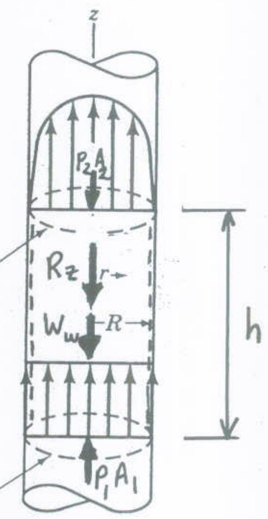


FIGURE P5.61

The analysis for this problem is similar to the one of Example 5.13. The control volume contains the fluid only between sections (1) and (2) as indicated in the sketch. Application of the vertical or z component of the linear momentum equation leads to

$$-w_1 \rho w_1 A_1 + \int_0^R w_2 \rho w_2 \, 2\pi r\,dr = p_1 A_1 - R_z + p_2 A_2 - W_w$$

Thus

$$p_1 - p_2 = \frac{R_z}{A} - \rho w_1^2 + \frac{\rho 2\pi}{A}\int_0^R \left[w_c \left(\frac{R-r}{R}\right)^{\frac{1}{7}}\right]^2 r\,dr + \frac{W_w}{A} \qquad (1)$$

The weight of the water in the control volume may be expressed as

$$W_w = g\rho A h$$

The value of $w_{\mathbb{C}}$ may be obtained from the conservation of mass equation as follows

$$\rho w_1 A_1 = \int_0^R \rho w_c \left(\frac{R-r}{R}\right)^{\frac{1}{7}} 2\pi r\,dr$$

or

$$w_c = \frac{w_1 A_1}{2\pi \int_0^R \left(\frac{R-r}{R}\right)^{\frac{1}{7}} r\,dr} \qquad (2)$$

To evaluate the integral $\int_0^R \left(\frac{R-r}{R}\right)^{\frac{1}{7}} r\,dr$ we substitute

$$\alpha = \frac{R-r}{R} \qquad (3)$$

then

$$d\alpha = -\frac{dr}{R} \qquad (4)$$

(con't)

5.61 (con't)

and
$$\int_0^R \left(\frac{R-r}{R}\right)^{\frac{1}{7}} r\,dr = -\int_1^0 \alpha^{\frac{1}{7}}(1-\alpha)R^2\,d\alpha = \frac{49}{120}R^2 \quad (5)$$
Combining Eqs. 2 and 5 we obtain
$$w_c = \frac{60}{49}w_1$$
Thus from Eq. 1
$$p_1 - p_2 = \frac{R_z}{\pi R^2} - \rho w_1^2 + \frac{\rho(2)(60)^2 w_1^2}{R^2(49)^2}\int_0^R \left(\frac{R-r}{R}\right)^{\frac{2}{7}} r\,dr + g\rho h \quad (6)$$
To evaluate the integral $\int_0^R \left(\frac{R-r}{R}\right)^{\frac{2}{7}} r\,dr$ we use Eqs. 3 and 4.
Thus
$$\int_0^R \left(\frac{R-r}{R}\right)^{\frac{2}{7}} r\,dr = -\int_1^0 \alpha^{2/7}(1-\alpha)R^2\,d\alpha = \frac{49}{144}R^2$$
and Eq. 6 becomes
$$p_1 - p_2 = \frac{R_z}{\pi R^2} - \rho w_1^2 + \rho(1.02)w_1^2 + g\rho h$$
or
$$\underline{\underline{p_1 - p_2 = \frac{R_z}{\pi R^2} + 0.02\rho w_1^2 + g\rho h}}$$
Note that in contrast to the result of Example 5.13, only a very small portion of the pressure drop is due to a change in the momentum flow between sections 1 and 2 in this case.

5.62

5.62 In a laminar pipe flow that is fully developed, the axial velocity profile is parabolic, that is,

$$u = u_c \left[1 - \left(\frac{r}{R}\right)^2\right]$$

as is illustrated in Fig. P5. . Compare the axial direction momentum flowrate calculated with the average velocity, $\bar{u}$, with the axial direction momentum flowrate calculated with the nonuniform velocity distribution taken into account.

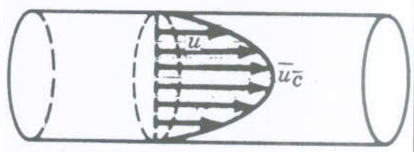


FIGURE P5.62

The axial direction momentum flowrate based on a uniform velocity profile with $u = \bar{u}$ is

$$MF_{x,\,uniform} = \bar{u}\rho\bar{u}A = \rho\bar{u}^2\pi R^2$$

The axial direction momentum flowrate based on the non-uniform parabolic velocity profile is

$$MF_{x,\,non\text{-}uniform} = \int_0^R u\rho u 2\pi r\,dr = \rho u_c^2 2\pi R^2 \int_0^1 \left[1-\left(\frac{r}{R}\right)^2\right]^2 \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

$$MF_{x,\,non\text{-}uniform} = \frac{\rho u_c^2 \pi R^2}{3}$$

To obtain a relationship between $\bar{u}$ and $u_{\mathrm{¢}}$ we use the conservation of mass equation as follows

$$\rho\bar{u}\pi R^2 = \rho 2\pi R^2 u_c \int_0^R \left[1-\left(\frac{r}{R}\right)^2\right]\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

Thus

$$\bar{u} = \frac{u_c}{2}$$

and

$$\underline{MF_{x,\,non\text{-}uniform}} = \frac{4}{3}\rho\bar{u}^2\pi R^2 = \underline{\frac{4}{3} MF_{x,\,uniform}}$$

5.64

5.64 A Pelton wheel vane directs a horizontal, circular cross-sectional jet of water symmetrically as indicated in Fig. P5.64 and Video V5.6. The jet leaves the nozzle with a velocity of 100 ft/s. Determine the x direction component of anchoring force required to (a) hold the vane stationary, (b) confine the speed of the vane to a value of 10 ft/s to the right. The fluid speed magnitude remains constant along the vane surface.

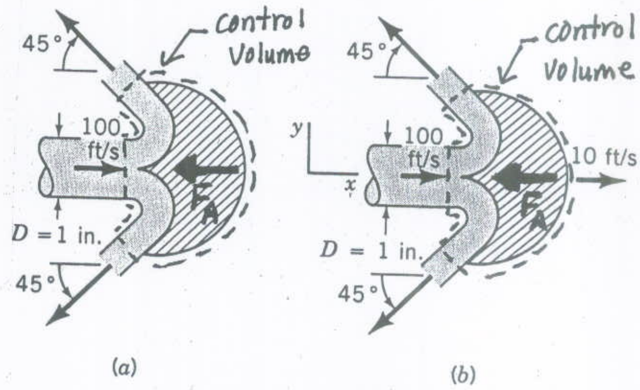


FIGURE P5.64

(a) To determine the x-direction component of anchoring force required to hold the vane stationary we use the stationary control volume shown above and the x-direction component of the linear momentum equation (Eq. 5.22). Thus,

$$F_A = \dot{m}(V_1 + V_2 \cos 45°) = \rho A_1 V_1 (V_1 + V_2 \cos 45°) = \rho \frac{\pi D_1^2}{4} V_1 (V_1 + V_2 \cos 45°)$$

or

$$F_A = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) \frac{\pi (1 \text{ in.})^2 \left(100 \frac{\text{ft}}{\text{s}}\right)}{(4)\left(12 \frac{\text{in.}}{\text{ft}}\right)^2} \left[\left(100 \frac{\text{ft}}{\text{s}}\right) + \left(100 \frac{\text{ft}}{\text{s}}\right) \cos 45°\right] \left(\frac{1 \text{ lb}}{\text{slug} \cdot \frac{\text{ft}}{\text{s}^2}}\right)$$

and

$$F_A = \underline{\underline{181 \text{ lb}}}$$

(b) To determine the x-direction component of anchoring force required to confine the vane to a constant speed of 10 $\frac{ft}{s}$ to the right we use a control volume moving to the right with a speed of 10 $\frac{ft}{s}$ and the x-direction component of the linear momentum equation for a translating control volume (Eq. 5.29). Thus,

$$F_A = \rho A_1 W_1 (W_1 + W_2 \cos 45°) = \rho \frac{\pi D_1^2}{4} W_1 (W_1 + W_2 \cos 45°) \qquad (1)$$

We note that

$$W_1 = V_1 - 10 \frac{\text{ft}}{\text{s}} = 100 \frac{\text{ft}}{\text{s}} - 10 \frac{\text{ft}}{\text{s}} = 90 \frac{\text{ft}}{\text{s}}$$

Thus, Eq. 1 leads to

$$F_A = \left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right) \frac{\pi (1 \text{ in.})^2}{4 \left(12 \frac{\text{in.}}{\text{ft}}\right)^2} \left(90 \frac{\text{ft}}{\text{s}}\right) \left[90 \frac{\text{ft}}{\text{s}} + \left(90 \frac{\text{ft}}{\text{s}}\right) \cos 45°\right] \left(\frac{1 \text{ lb}}{\text{slug} \cdot \frac{\text{ft}}{\text{s}^2}}\right)$$

or

$$F_A = \underline{\underline{146 \text{ lb}}}$$

5.65

5.65 How much power is transferred to the moving vane of Problem 5.65?

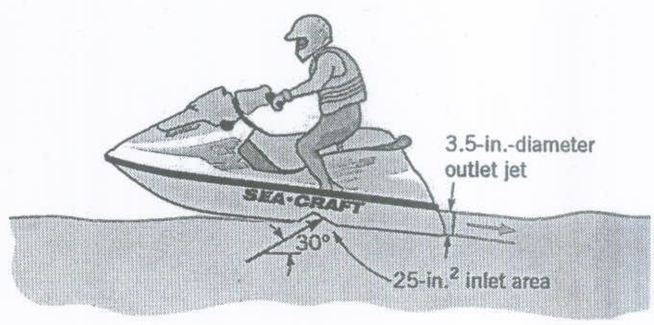
Power $= F_A V$, where from Problem 5.66 $F_A = 146\text{ lb}$

Thus,

$$\text{Power} = \frac{(146\text{ lb})(10\ \frac{\text{ft}}{\text{s}})}{\left(550\ \frac{\text{ft}\cdot\text{lb}}{\text{s}\cdot\text{hp}}\right)} = \underline{\underline{2.65}}\text{ hp}$$

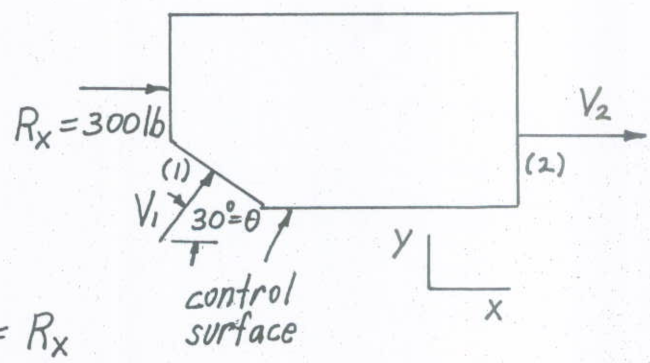
5.66

5.66 The thrust developed to propel the jet ski shown in **Video V9.11** and Fig. P5.66 is a result of water pumped through the vehicle and exiting as a high-speed water jet. For the conditions shown in the figure, what flowrate is needed to produce a 300-lb thrust? Assume the inlet and outlet jets of water are free jets.



■ FIGURE P5.66

For the control volume indicated the x-component of the momentum equation



$$\int_{cs} u\,\rho\,\vec{V}\cdot\hat{n}\,dA = \sum F_x \quad \text{becomes}$$

(1) $$(V_1 \cos 30°)\rho(-V_1)A_1 + V_2\,\rho(+V_2)A_2 = R_x$$

where we have assumed that $p=0$ on the entire control surface and that the exiting water jet is horizontal.

With $\dot{m} = \rho A_1 V_1 = \rho A_2 V_2$ Eq. (1) becomes

$$R_x = \dot{m}(V_2 - V_1 \cos\theta) = \rho V_1 A_1 (V_2 - V_1 \cos 30°) \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$ so that

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{25\,in.^2}{\frac{\pi}{4}(3.5\,in.)^2}\,V_1 = 2.60\,V_1 \qquad (2)$$

By combining Eqs. (1) and (2):

$$R_x = \rho V_1^2 A_1 (2.60 - \cos 30°)$$

or

$$V_1 = \left[\frac{300\,lb}{\left(1.94\,\frac{slugs}{ft^3}\right)\left(\frac{25}{144}\,ft^2\right)(2.60 - \cos 30°)}\right]^{\frac{1}{2}} = 22.7\,\frac{ft}{s}$$

Thus,

$$Q = A_1 V_1 = \left(\frac{25}{144}\,ft^2\right)\left(22.7\,\frac{ft}{s}\right) = \underline{\underline{3.94\,\frac{ft^3}{s}}}$$

5.67

5.67 (See Fluids in the News article titled "Where the plume goes," Section 5.2.2.) Air flows into the jet engine shown in Fig. P5.67 at a rate of 9 slugs/s and a speed of 300 ft/s. Upon landing, the engine exhaust exits through the reverse thrust mechanism with a speed of 900 ft/s in the direction indicated. Determine the reverse thrust applied by the engine to the airplane. Assume the inlet and exit pressures are atmospheric and that the mass flowrate of fuel is negligible compared to the air flowrate through the engine.

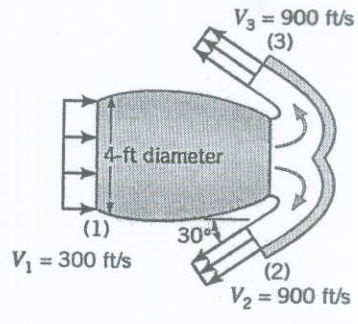
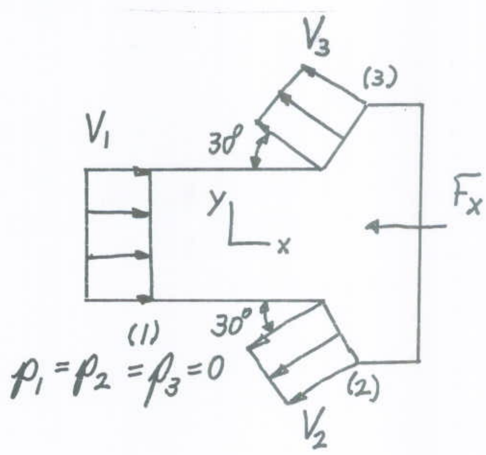


FIGURE P5.67

The momentum equation (x-component),

$\int_{cs} u \rho \vec{V} \cdot \hat{n}\, dA = \Sigma F_x$, for the control volume shown can be written as



$$V_1 \rho(-V_1)A_1 + (-V_2 \cos 30^\circ)\rho V_2 A_2 + (-V_3 \cos 30^\circ)\rho V_3 A_3 = -F_x$$

or

$$F_x = (\rho V_1 A_1)V_1 + (\rho V_2 A_2)V_2 \cos 30^\circ + (\rho V_3 A_3) V_3 \cos 30^\circ \qquad (1)$$

But from conservation of mass,

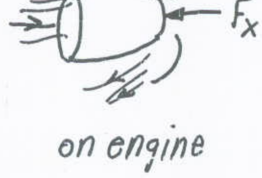
$$\rho V_1 A_1 = \rho V_2 A_2 + \rho V_3 A_3 = \dot{m} = 9\,slugs/s$$

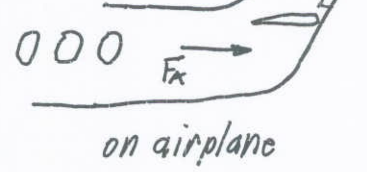
Also, $V_2 = V_3$, so that Eq. (1) becomes

$$F_x = \dot{m}(V_1 + V_2 \cos 30^\circ) = 9\,\frac{slugs}{s}\left(300\,\frac{ft}{s} + 900 \cos 30^\circ\,\frac{ft}{s}\right)$$

$$= 9710\,\frac{slugs \cdot ft}{s^2} = \underline{\underline{9170\ lb}}$$

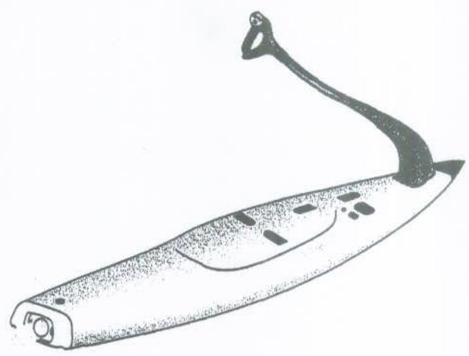
Note direction of F_x on engine and engine on airplane.



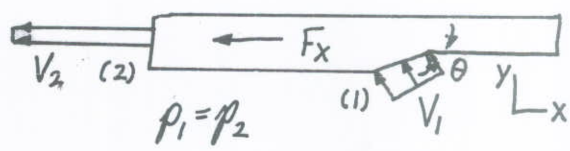


5.68

5.68 (See Fluids in the News article titled "Motorized surfboard," Section 5.2.2.) The thrust to propel the powered surfboard shown in Fig. P5.68 is a result of water pumped through the board that exits as a high-speed 2.75-in.-diameter jet. Determine the flowrate and the velocity of the exiting jet if the thrust is to be 300 lb. Neglect the momentum of the water entering the pump.



■ FIGURE P5.



The x-component of the momentum equation, $\int_{cs} u \rho \vec{V}\cdot\hat{n}\, dA = \sum F_x$, for the control volume shown is

$$(-V_1 \cos\theta)\rho(-V_1)A_1 + (-V_2)\rho V_2 A_2 = -F_x$$

or

$$F_x = \rho V_2^2 A_2 - \rho V_1^2 A_1 \cos\theta \approx \rho V_2^2 A_2$$ if the momentum of the entering water is neglected.

Thus,

$$300\ \text{lb} = \left(1.94\ \frac{\text{slugs}}{\text{ft}^3}\right) V_2^{\,2} \left(\frac{\pi}{4}\left(\frac{2.75}{12}\ \text{ft}^2\right)\right)$$

or

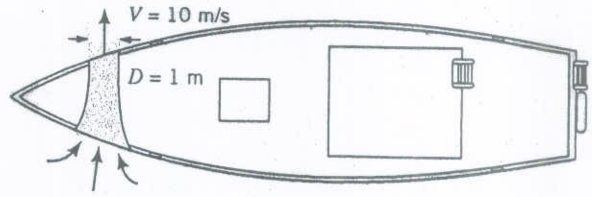
$$\underline{\underline{V_2 = 61.2\ \frac{\text{ft}}{\text{s}}}}$$

and

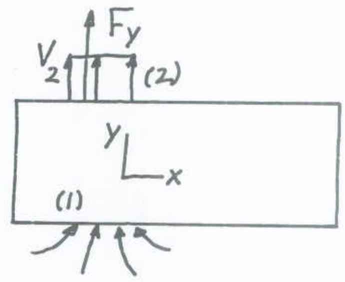
$$Q = A_2 V_2 = \frac{\pi}{4}\left(\frac{2.75}{12}\ \text{ft}\right)^2 \left(61.2\ \frac{\text{ft}}{\text{s}}\right) = \underline{\underline{2.52\ \frac{\text{ft}^3}{\text{s}}}}$$

5.69

5.69 (See Fluids in the News article titled **"Bow thrusters,"** Section 5.2.2) The bow thruster on the boat shown in Fig. P5.69 is used to turn the boat. The thruster produces a 1-m-diameter jet of water with a velocity of 10 m/s. Determine the force produced by the thruster. Assume that the inlet and outlet pressures are zero and that the momentum of the water entering the thruster is negligible.



■ FIGURE P5.69



The y-component of the momentum equation,

$\int_{cs} v \rho \vec{V} \cdot \hat{n}\, dA = \Sigma F_y$, for the control volume shown is,

$$\int_{(1)} v \rho \vec{V} \cdot \hat{n}\, dA + V_2 \rho V_2 A_2 = F_y$$

If the momentum of the entering water is negligible the equation becomes

$$F_y = \rho V_2^2 A_2 = 999 \frac{kg}{m^3} \left(10 \frac{m}{s}\right)^2 \left(\frac{\pi}{4}(1m)^2\right) = 78{,}500 \frac{kg \cdot m}{s^2} = \underline{\underline{78.5\ kN}}$$

5.70

5.70 A snowplow mounted on a truck clears a path 12 *ft* through heavy wet snow, as shown in Figure P5.70. The snow is 8 *in.* deep and its density is 10 lbm/ft^3. The truck travels at 30 *mph*. The snow is discharged from the plow at an angle of 45° from the direction of travel and 45° above the horizontal, as shown in Figure P5.70. Estimate the force required to push the plow.

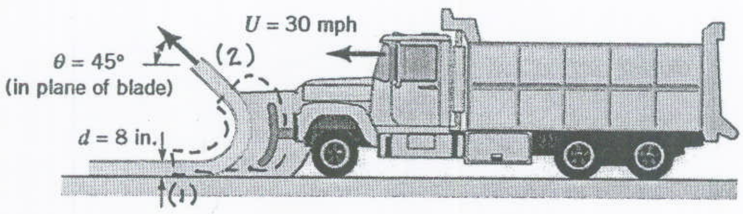


FIGURE P5.70

To estimate the force required to push the snowplow we use the control volume shown in the sketch above and Eq. 5.29. We neglect the friction force between the plow and the road surface. We also neglect any force associated with the plow deflecting air. We only consider how much force is required to turn wet snow 135°.

For the wet snow "flow" we get from Eq. 5.29

$$F_x = \dot{m}\,(W_1 + W_2 \cos 45°)$$

Since

$$\dot{m} = \rho A W_1$$

we assume $W_2 = W_1$ and get

$$F_x = \rho A W_1^2 (1 + \cos 45°)$$

Then

$$F_x = \frac{\left(10\ \frac{lbm}{ft^3}\right)\left(\frac{8\ in.}{12\ in./ft}\right)(12\ ft)\left[\left(30\ \frac{mi}{hr}\right)\frac{\left(5280\ \frac{ft}{mi}\right)}{(3600\ s/hr)}\right]^2 (1 + 0.707)}{\left(32.174\ \frac{lbm \cdot ft}{lb \cdot s^2}\right)}$$

$$F_x = \underline{\underline{8220}}\ lb$$

5.75

5.75 Water enters a rotating lawn sprinkler through its base at the steady rate of 16 gal/min as shown in Fig. P5.75. The exit cross section area of each of the two nozzles is 0.04 in.^{2} and the flow leaving each nozzle is tangential. The radius from the axis of rotation to the centerline of each nozzle is 8 in. **(a)** Determine the resisting torque required to hold the sprinkler head stationary. **(b)** Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min. **(c)** Determine the angular velocity of the sprinkler if no resisting torque is applied.

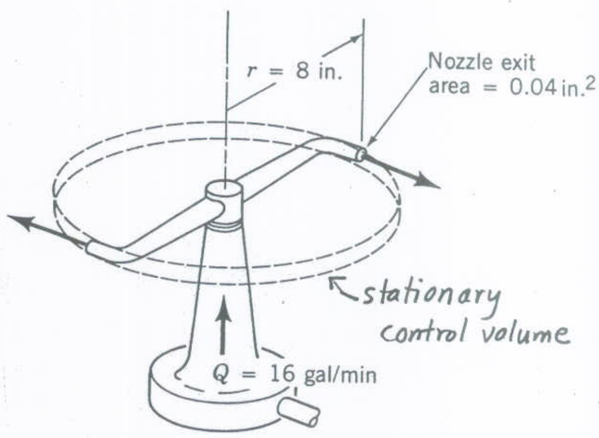


FIGURE P5.75

This is similar to Example 5.17.

(a) To determine the resisting torque required to hold the sprinkler head stationary we use the moment-of-momentum torque equation (Eq. 5.50). Thus,

$$T_{shaft} = \dot{m} r_2 V_{\theta,2} = \rho Q r_2 V_{\theta,2} \qquad (1)$$

For $V_{\theta,2}$ we use

$$V_{\theta,2} = \frac{Q}{2A_{\substack{nozzle\\exit}}} = \frac{\left(16\,\frac{gal}{min}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{2\,(0.04\,in.^2)\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)}$$

or

$$V_{\theta,2} = 64.17\,\frac{ft}{s}$$

With Eq. 1 we obtain

$$T_{shaft} = \frac{\left(1.94\,\frac{slugs}{ft^3}\right)\left(16\,\frac{gal}{min}\right)(8\,in.)\left(64.17\,\frac{ft}{s}\right)\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}{\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)\left(12\,\frac{in.}{ft}\right)}$$

and

$$T_{shaft} = \underline{\underline{2.96\,ft\cdot lb}}$$

(b) To determine the resisting torque associated with a sprinkler speed of 500 $\frac{rev}{min}$ we use Eq. 1 again. However, with rotation we have

$$V_{\theta,2} = W_2 - U_2 \qquad (2)$$

For W_2 we use

$$W_2 = \frac{Q}{2A_{\substack{nozzle\\exit}}} = \frac{\left(16\,\frac{gal}{min}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{(2)(0.04\,in.^2)\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)} = 64.17\,\frac{ft}{s}$$

(con't)

5.75 (con't)

For U_2 we use

$$U_2 = r_2\omega = \frac{(8\text{ in.})\left(500\frac{\text{rev}}{\text{min}}\right)\left(2\pi\frac{\text{rad}}{\text{rev}}\right)}{\left(12\frac{\text{in.}}{\text{ft}}\right)\left(60\frac{\text{s}}{\text{min}}\right)} = 34.91\frac{\text{ft}}{\text{s}}$$

Thus with Eq. 2 we have

$$V_{\theta,2} = 64.17\frac{\text{ft}}{\text{s}} - 34.91\frac{\text{ft}}{\text{s}} = 29.26\frac{\text{ft}}{\text{s}}$$

and with Eq. 1 we obtain

$$T_{\text{shaft}} = \frac{\left(1.94\frac{\text{slugs}}{\text{ft}^3}\right)\left(16\frac{\text{gal}}{\text{min}}\right)(8\text{ in.})\left(29.26\frac{\text{ft}}{\text{s}}\right)}{\left(7.48\frac{\text{gal}}{\text{ft}^3}\right)\left(60\frac{\text{s}}{\text{min}}\right)\left(12\frac{\text{in.}}{\text{ft}}\right)}\left(1\frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)$$

and

$$T_{\text{shaft}} = \underline{\underline{1.35\text{ ft}\cdot\text{lb}}}$$

(c) To determine the angular velocity of the sprinkler if no resisting torque is applied we use the combination of Eqs. 1 and 2 to obtain

$$U_2 = W_2$$

or

$$\omega = \frac{W_2}{r_2} = \frac{\left(64.17\frac{\text{ft}}{\text{s}}\right)\left(12\frac{\text{in.}}{\text{ft}}\right)}{(8\text{ in.})} = 96.3\frac{\text{rad}}{\text{s}}$$

The rotor speed, N, is thus

$$N = \left(96.3\frac{\text{rad}}{\text{s}}\right)\frac{\left(60\frac{\text{s}}{\text{min}}\right)}{\left(2\pi\frac{\text{rad}}{\text{rev}}\right)} = \underline{\underline{920\frac{\text{rev}}{\text{min}}}}$$

5.76

5.76 Five liters/s of water enter the rotor shown in **Video V5.10** and Fig. P5.76 along the axis of rotation. The cross-sectional area of each of the three nozzle exits normal to the relative velocity is 18 mm^2. How large is the resisting torque required to hold the rotor stationary? How fast will the rotor spin steadily if the resisting torque is reduced to zero and **(a)** $\theta = 0°$, **(b)** $\theta = 30°$, **(c)** $\theta = 60°$?

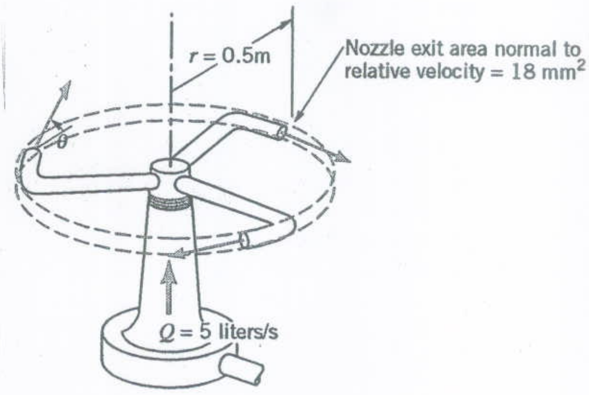


FIGURE P5.76

To determine the torque required to hold the rotor stationary we use the moment-of-momentum torque equation (Eq. 5.50) to obtain

$$T_{shaft} = \dot{m}\, r_{out} V_{out} \cos\theta \qquad (1)$$

We note that

$$\dot{m} = \rho Q \qquad (2)$$

and

$$V_{out} = \frac{Q}{3 A_{\substack{nozzle\\exit}}} \qquad (3)$$

Combining Eqs. 1, 2 and 3 we get

$$T_{shaft} = \frac{\rho Q^2 r_{out} \cos\theta}{3\, A_{\substack{nozzle\\exit}}} \qquad (4)$$

To determine the rotor angular velocity associated with zero shaft torque we again use the moment-of-momentum torque equation (Eq. 5.50) to obtain, this time with rotation,

$$T_{shaft} = \dot{m}\, r_{out} (W_{out} \cos\theta - U_{out}) \qquad (5)$$

We note that

$$U_{out} = r_{out}\,\omega \qquad (6)$$

and

$$W_{out} = \frac{Q}{3 A_{\substack{nozzle\\exit}}} \qquad (7)$$

(con't)

5.76 (con't)

Combining Eqs. 2,5,6 and 7 we get

$$T_{shaft} = \rho Q r_{out}\left(\frac{Q}{3A_{\substack{nozzle\\exit}}}\cos\theta - r_{out}\,\omega\right) \qquad (8)$$

(a) For $\theta = 0°$ we use Eq. 4 to get

$$T_{shaft} = \frac{\left(999\,\frac{kg}{m^3}\right)\left(5\,\frac{liters}{s}\right)^2(0.5\,m)(\cos 0°)\left(1000\,\frac{mm}{m}\right)^2\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)}{\left(1000\,\frac{liters}{m^3}\right)^2(3)(18\,mm^2)}$$

or

$$T_{shaft} = \underline{\underline{231\ N.m}}$$

From Eq. 8 we obtain for $T_{shaft} = 0$

$$\omega = \frac{Q\cos\theta}{3A_{\substack{nozzle\\exit}}\,r_{out}} = \frac{\left(5\,\frac{liters}{s}\right)(\cos 0°)\left(1000\,\frac{mm}{m}\right)^2}{3(18\,mm^2)\left(1000\,\frac{liters}{m^3}\right)(0.5\,m)} = \underline{\underline{185\,\frac{rad}{s}}}$$

(b) For $\theta = 30°$ we use Eq. 4 to get

$$T_{shaft} = \frac{\left(999\,\frac{kg}{m^3}\right)\left(5\,\frac{liters}{s}\right)^2(0.5\,m)(\cos 30°)\left(1000\,\frac{mm}{m}\right)^2\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)}{\left(1000\,\frac{liters}{m^3}\right)^2(3)(18\,mm^2)}$$

or

$$T_{shaft} = \underline{200\ N.m}$$

From Eq. 8 we obtain for $T_{shaft} = 0$

$$\omega = \frac{\left(5\,\frac{liters}{s}\right)(\cos 30°)\left(1000\,\frac{mm}{m}\right)^2}{3(18\,mm^2)\left(1000\,\frac{liters}{m^3}\right)(0.5\,m)} = \underline{\underline{160}}\,\frac{rad}{s}$$

(c) For $\theta = 60°$ we use Eq. 4 to get

$$T_{shaft} = \frac{\left(999\,\frac{kg}{m^3}\right)\left(5\,\frac{liters}{s}\right)^2(0.5\,m)(\cos 60°)\left(1000\,\frac{mm}{m}\right)^2\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)}{\left(1000\,\frac{liters}{m^3}\right)^2(3)(18\,mm^2)}$$

or

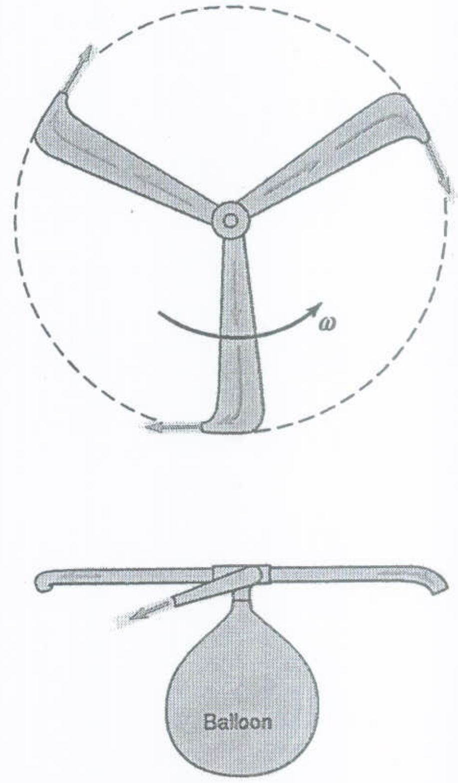
$$T_{shaft} = \underline{\underline{116}}\ N.m$$

From Eq. 8 we obtain for $T_{shaft} = 0$

$$\omega = \frac{\left(5\,\frac{liters}{s}\right)(\cos 60°)\left(1000\,\frac{mm}{m}\right)^2}{(3)(18\,mm^2)\left(1000\,\frac{liters}{m^3}\right)(0.5\,m)} = \underline{\underline{92.5\,\frac{rad}{s}}}$$

5.77

5.77 Shown in Fig. P5.77 is a toy "helicopter" powered by air escaping from a balloon. The air from the balloon flows radially through each of the three propeller blades and out through small nozzles at the tips of the blades. Explain physically how this flow can cause the rotation necessary to rotate the blades to produce the needed lifting force.



■ FIGURE P5.77

As the air flowing radially out through each propeller blade turns to flow out through the nozzle at the blade tip, it exerts a tangential force to the inside surface of the blade. Further, the velocity increase of the air flowing out of each nozzle results in additional force in the opposite direction. These two forces move the blades counter clockwise as shown. The rotating blades experience a lifting force from the air flowing over the blades because of the downward turning of the air.

5.78

5.78 A simplified sketch of a hydraulic turbine runner is shown in Fig. P5.78. Relative to the rotating runner, water enters at section (1) (cylindrical cross section area A_1 at $r_1 = 1.5$ m) at an angle of 100° from the tangential direction and leaves at section (2) (cylindrical cross section area A_2 at $r_2 = 0.85$ m) at an angle of 50° from the tangential direction. The blade height at sections (1) and (2) is 0.45 m and the volume flowrate through the turbine is 30 m³/s. The runner speed is 130 rpm in the direction shown. Determine the shaft power developed.

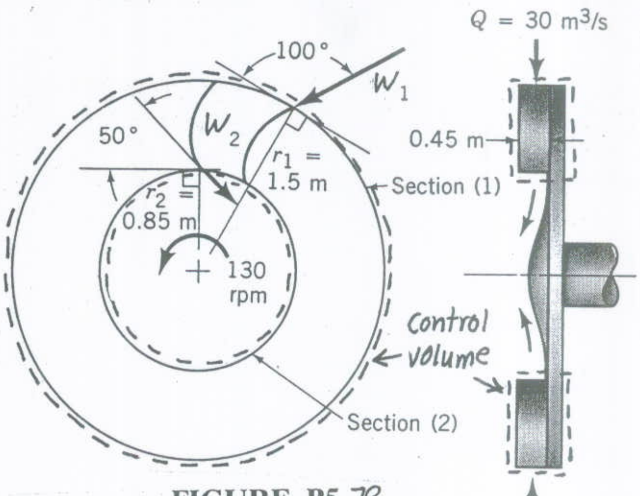


FIGURE P5.78

The stationary and non-deforming control volume shown in the sketch is used. Equation 5.53 can be used to determine to determine the shaft power. Thus

$$\dot{W}_{shaft} = -\dot{m}_1 (U_1 V_{\theta 1}) + \dot{m}_2 (U_2 V_{\theta 2}) \qquad (1)$$

With the conservation of mass equation we can conclude that

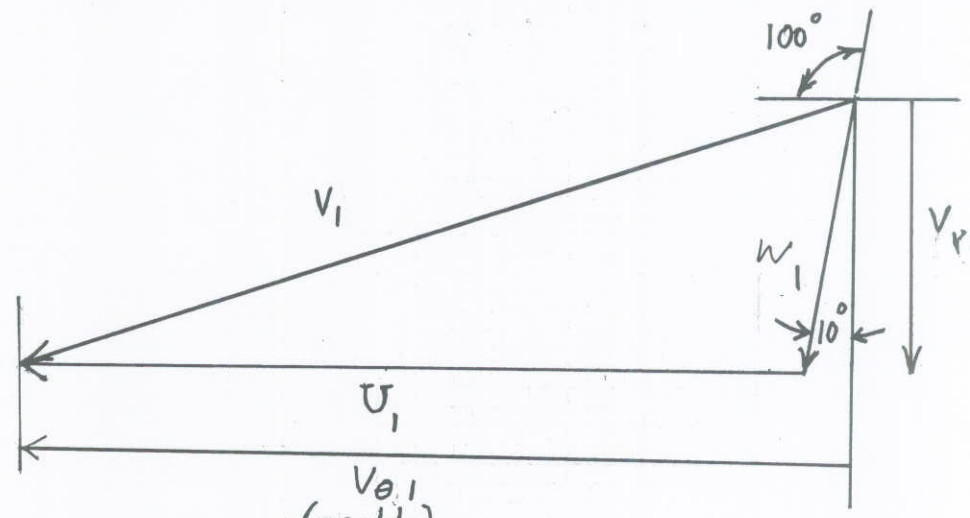
$$\dot{m}_1 = \dot{m}_2 = \rho Q = \left(999 \frac{kg}{m^3}\right)\left(30 \frac{m^3}{s}\right) = 30{,}000 \frac{kg}{s}$$

The blade velocities are easily obtained as follows.

$$U_1 = r_1 \omega = \frac{(1.5\,m)\left(130 \frac{rev}{min}\right)\left(2\pi \frac{rad}{rev}\right)}{\left(60 \frac{s}{min}\right)} = 20.4 \frac{m}{s}$$

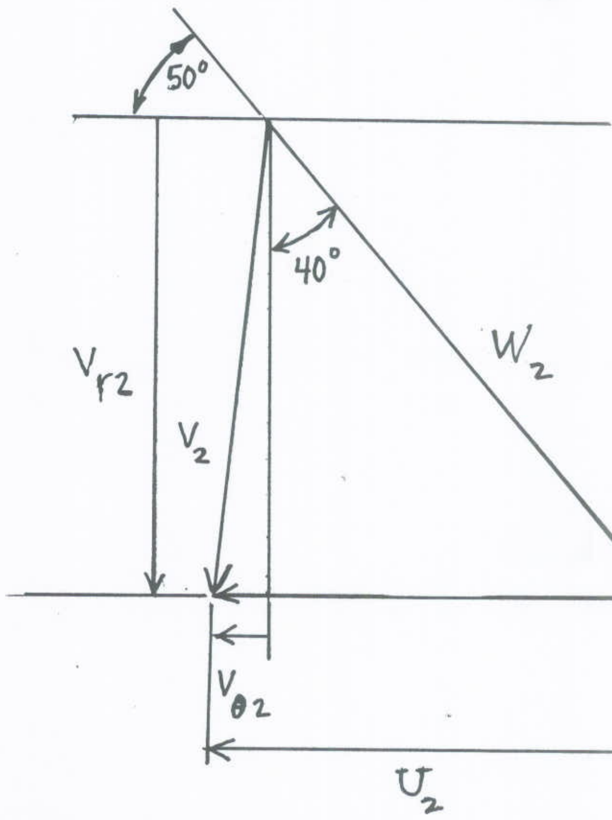
$$U_2 = r_2 \omega = \frac{(0.85\,m)\left(130 \frac{rev}{min}\right)\left(2\pi \frac{rad}{rev}\right)}{\left(60 \frac{s}{min}\right)} = 11.6 \frac{m}{s}$$

The tangential velocities, $V_{\theta,1}$ and $V_{\theta,2}$ may be obtained with the help of the velocity triangles sketched below.



(con't)

5.78 (con't)



With the velocity triangle for section (1) we see that

$$V_{\theta 1} = U_1 + W_1 \sin 10° \qquad (2)$$

Also

$$W_1 \cos 10° = V_{r1}$$

and

$$V_{r1} = \frac{Q}{A_1} = \frac{Q}{2\pi r_1 h_1} = \frac{\left(30\ \frac{m^3}{s}\right)}{2\pi(1.5\,m)(0.45\,m)} = 7.07\ \frac{m}{s}$$

Thus

$$W_1 = \frac{V_{r1}}{\cos 10°} = \frac{\left(7.07\ \frac{m}{s}\right)}{\cos 10°} = 7.18\ \frac{m}{s}$$

and with Eq. 2

$$V_{\theta 1} = 20.4\ \frac{m}{s} + \left(7.18\ \frac{m}{s}\right)\sin 10° = 21.6\ \frac{m}{s}$$

with the velocity triangle for section (2) we conclude that

$$V_{\theta 2} = U_2 - W_2 \sin 40° \qquad (3)$$

(con't)

5.78 (con't)

Also

$$W_2 \cos 40° = V_{r2} = \frac{Q}{A_2} = \frac{Q}{2\pi r_2 h_2} = \frac{\left(30 \frac{m^3}{s}\right)}{2\pi(0.85m)(0.45m)} = 12.5 \frac{m}{s}$$

and

$$W_2 = \frac{V_{r2}}{\cos 40°} = \frac{\left(12.5 \frac{m}{s}\right)}{\cos 40°} = 16.3 \frac{m}{s}$$

Thus from Eq. 3

$$V_{\theta 2} = 11.6 \frac{m}{s} - \left(16.3 \frac{m}{s}\right) \sin 40° = 1.12 \frac{m}{s}$$

Finally, with Eq. 1 we obtain

$$\dot{W}_{shaft} = \left[-\left(30{,}000 \frac{kg}{s}\right)\left(20.4 \frac{m}{s}\right)\left(21.6 \frac{m}{s}\right) + \left(30{,}000 \frac{kg}{s}\right)\left(11.6 \frac{m}{s}\right)\left(1.12 \frac{m}{s}\right)\right]\left(\frac{1 N}{kg \cdot \frac{m}{s^2}}\right)$$

$$\dot{W}_{shaft} = \underline{\underline{-12.8 \times 10^6}} \frac{N \cdot m}{s} = \underline{\underline{-12.8 \times 10^6 W}} = \underline{\underline{-12.8 MW}}$$

The minus sign means power out of the control volume.

5.79

5.79 A water turbine with radial flow has the dimensions shown in Fig. P5.79. The absolute entering velocity is 50 ft/s, and it makes an angle of 30° with the tangent to the rotor. The absolute exit velocity is directed radially inward. The angular speed of the rotor is 120 rpm. Find the power delivered to the shaft of the turbine.

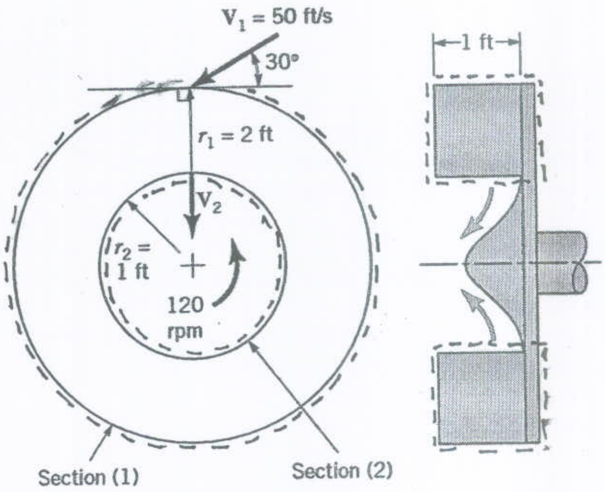


FIGURE P5.79

The stationary and non-deforming control volume shown in the sketch above is used. We use Eq. 5.53 to determine the shaft power involved. Thus

$$\dot{W}_{shaft} = -\dot{m}_1 U_1 V_{\theta 1} \qquad (1)$$

The mass flowrate may be obtained from (2)

$$\dot{m}_1 = \rho V_{r1} A_1 = \rho V_{r1} 2\pi r_1 h_1$$

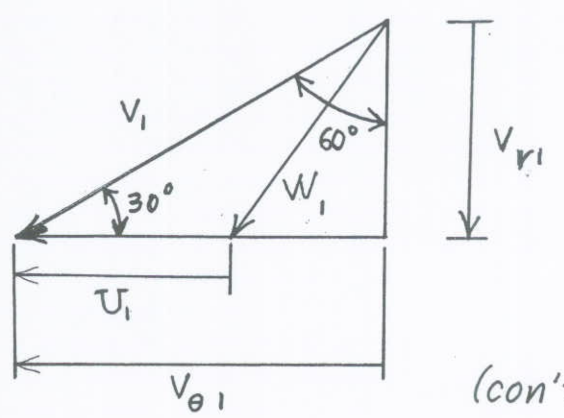
where

V_{r1} = radial component of velocity at section (1)

The blade velocity at section (1) is

$$U_1 = r_1 \omega = (2 \text{ ft})\left(120 \frac{\text{rev}}{\text{min}}\right)\left(2\pi \frac{\text{rad}}{\text{rev}}\right)\frac{1}{\left(60 \frac{\text{s}}{\text{min}}\right)} = 25.1 \frac{\text{ft}}{\text{s}}$$

The values of $V_{\theta 1}$ and V_{r1} may be obtained with the help of a velocity triangle for the flow at section (1) as sketched below.



(con't)

5.79 (con't)

With the velocity triangle we conclude that

$$V_{r1} = V_1 \sin 30° = V_1 \cos 60° = \left(50\ \frac{ft}{s}\right)(\sin 30°) = 25\ \frac{ft}{s}$$

Then from Eq. 2

$$\dot{m}_1 = \left(1.94\ \frac{slugs}{ft^3}\right)\left(25\ \frac{ft}{s}\right) 2\pi (2\ ft)(1\ ft) = 610\ \frac{slugs}{s}$$

Also with the triangle we see that

$$V_{\theta 1} = V_1 \cos 30° = V_1 \sin 60° = \left(50\ \frac{ft}{s}\right) \cos 30° = 43.3\ \frac{ft}{s}$$

Then, with Eq. 1 we obtain

$$\dot{W}_{shaft} = -\left(610\ \frac{slugs}{s}\right)\left(25.1\ \frac{ft}{s}\right)\left(43.3\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

$$\dot{W}_{shaft} = -\underline{\underline{6.63 \times 10^5}}\ \frac{ft \cdot lb}{s}$$

In horsepower we have

$$\dot{W}_{shaft} = -\left(6.63 \times 10^5\ \frac{ft \cdot lb}{s}\right) \frac{1}{\left(550\ \frac{ft \cdot lb}{s \cdot hp}\right)} = \underline{\underline{-1200}}\ hp$$

5.80

5.80 Shown in Fig. P5.80 are front and side views of a centrifugal pump rotor or impeller. If the pump delivers 200 liters/s of water and the blade exit angle is 35° from the tangential direction, determine the power requirement associated with flow leaving at the blade angle. The flow entering the rotor blade row is essentially radial as viewed from a stationary frame.

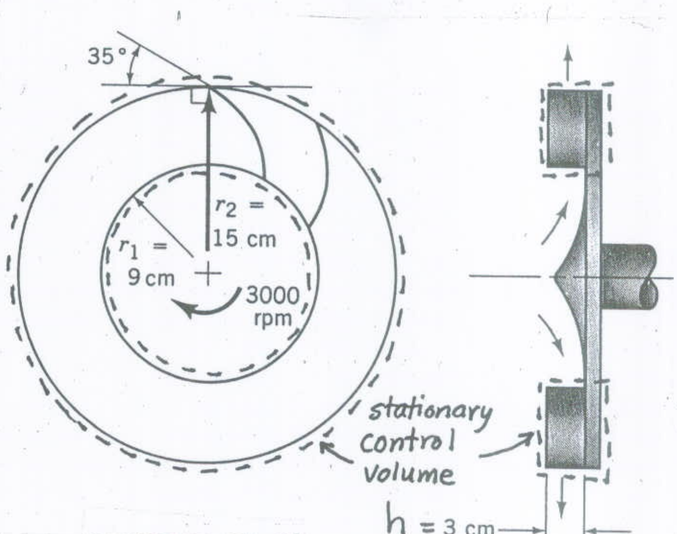


FIGURE P5.80

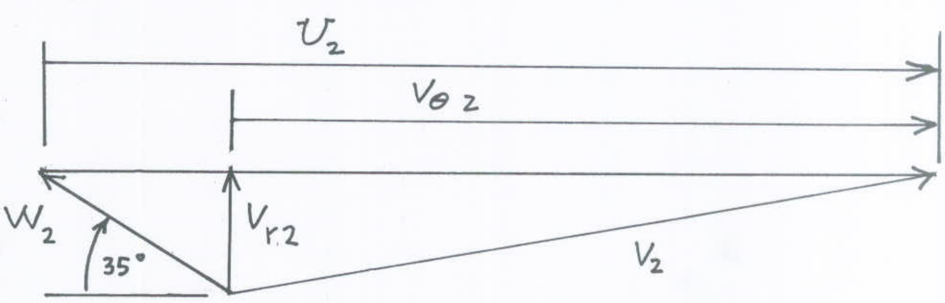
To determine the power, $\dot{W}_{\text{shaft net in}}$, we use the moment-of-momentum power equation (Eq. 5.53) to obtain

$$\dot{W}_{\text{shaft net in}} = \dot{m}\, U_2 V_{\theta 2} = \rho Q U_2 V_{\theta 2} \qquad (1)$$

We obtain U_2 from

$$U_2 = r_2 \omega = \frac{(15\text{ cm})(3000\text{ rpm})\left(2\pi \frac{\text{rad}}{\text{rev}}\right)}{\left(100 \frac{\text{cm}}{\text{m}}\right)\left(60 \frac{\text{s}}{\text{min}}\right)} = 47.12 \frac{\text{m}}{\text{s}}$$

To determine $V_{\theta,2}$ we use the velocity triangle sketched below.



to get

$$V_{\theta 2} = U_2 - \frac{V_{r2}}{\tan 35°} \qquad (3)$$

For V_{r2} we use

$$V_{r2} = \frac{Q}{A_2} = \frac{Q}{2\pi r_2 h} = \frac{\left(200 \frac{\text{liters}}{\text{s}}\right)\left(100 \frac{\text{cm}}{\text{m}}\right)\left(100 \frac{\text{cm}}{\text{m}}\right)}{\left(1000 \frac{\text{liters}}{\text{m}^3}\right) 2\pi (15\text{ cm})(3\text{ cm})} = 7.074 \frac{\text{m}}{\text{s}}$$

(con't)

From Eq. 2 we obtain

$$V_{\theta 2} = 47.12\ \frac{m}{s} - \frac{7.074\ \frac{m}{s}}{\tan 35°} = 37.02\ \frac{m}{s}$$

Thus with Eq. 1 we get

$$\dot{W}_{\substack{shaft \\ net\ in}} = \left(999\ \frac{kg}{m^3}\right)\left(200\ \frac{liters}{s}\right)\frac{\left(47.12\ \frac{m}{s}\right)\left(37.02\ \frac{m}{s}\right)}{\left(1000\ \frac{liters}{m^3}\right)}\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)$$

or

$$\dot{W}_{\substack{shaft \\ net\ in}} = 3.48 \times 10^5\ \frac{N \cdot m}{s}$$

and

$$\dot{W}_{\substack{shaft \\ net\ in}} = \underline{\underline{348}}\ kW$$

5.81

5.81 The velocity triangles for water flow through a radial pump rotor are as indicated in Fig. P5. 1. **(a)** Determine the energy added to each unit mass (kg) of water as it flows through the rotor. **(b)** Sketch an appropriate blade section.

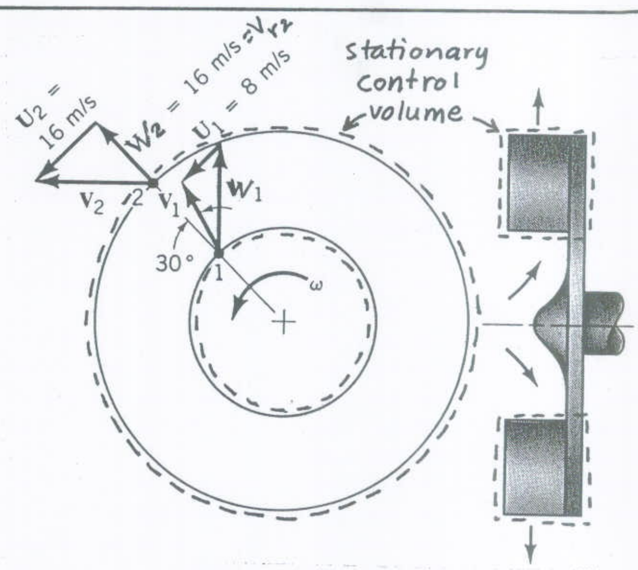


FIGURE P5. 1

(a) To determine the energy per unit mass added to the water flowing through the rotor we use the moment-of-momentum work equation (Eq. 5.54) to get

$$w_{\substack{shaft\\net\ in}} = U_1 V_{\theta 1} + U_2 V_{\theta 2} \qquad (1)$$

We note from the section (2) velocity triangle that

$$V_{\theta 2} = U_2$$

To ascertain $V_{\theta,1}$ we note from the section (1) velocity triangle that

$$V_{\theta 1} = V_{r1} \tan 30° \qquad (2)$$

From conservation of mass between sections (1) and (2) we conclude that

$$V_{r1} A_1 = V_{r2} A_2 = W_2 A_2$$

or

$$V_{r1} = W_2 \frac{A_2}{A_1} = W_2 \frac{r_2}{r_1} = W_2 \frac{U_2}{U_1} = \left(16\ \frac{m}{s}\right)\frac{\left(16\ \frac{m}{s}\right)}{\left(8\ \frac{m}{s}\right)} = 32\ \frac{m}{s}$$

With Eq. 2, $V_{\theta 1} = (32\ \frac{m}{s})(0.577) = 18.48\ m/s$

and with Eq. 1 we obtain

$$w_{\substack{shaft\\net\ in}} = \left[\left(8\ \frac{m}{s}\right)\left(18.48\ \frac{m}{s}\right) + \left(16\ \frac{m}{s}\right)\left(16\ \frac{m}{s}\right)\right]\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right) = \underline{\underline{404}}\ \frac{N \cdot m}{kg}$$

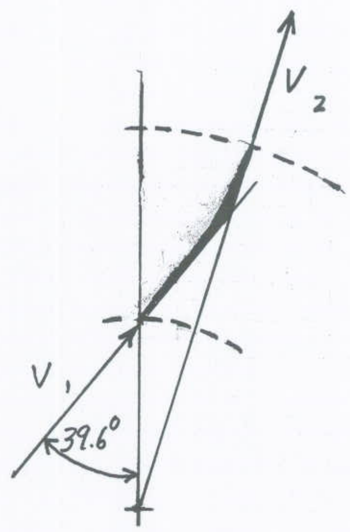
(con't)

5.81 (con't)

(b) An appropriate blade section would be approximately tangent to the section (1) and section (2) relative velocities, W_1 and W_2. The relative flow angle from the radial direction at section (1) is

$$\beta_1 = \tan^{-1}\left[\frac{(U_1 + V_{\theta 1})}{V_{r1}}\right] = \tan^{-1}\left[\frac{(8\,\frac{m}{s} + 18.48\,\frac{m}{s})}{32\,\frac{m}{s}}\right] = 39.6^\circ$$

The relative flow angle from the radial direction at section (2) is 0°. Thus, the blade section is as sketched below.



5.82

5.82 An axial flow turbomachine rotor involves the upstream (1) and downstream (2) velocity triangles shown in Fig.P5.82. Is this turbomachine a turbine or a fan? Sketch an appropriate blade section and determine energy transferred per unit mass of fluid.

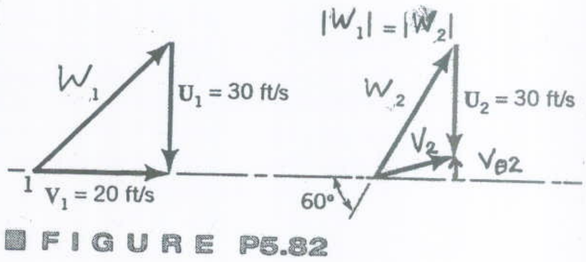


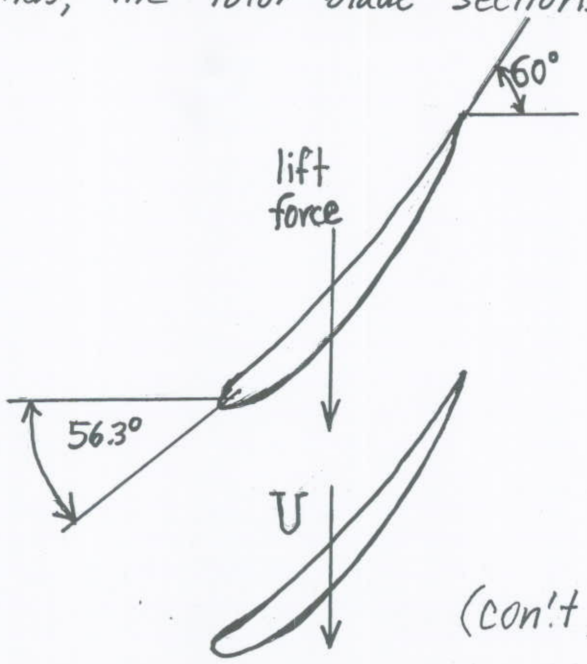
FIGURE P5.82

We can determine whether the axial flow turbomachine involved is a turbine or a fan by comparing the direction of the lift force on the rotor blade section with the direction of the blade velocity, U. If the lift force and the blade velocity are in the same direction a turbine is involved. If the lift force and blade velocity are in opposite directions, a fan is involved. The direction of the lift force can be inferred from the shape of the rotor blade section sketched to be tangent to the relative flows entering and leaving the rotor row.

The entering relative flow angle, β_1, is

$$\beta_1 = \tan^{-1}\frac{U_1}{V_1} = \tan^{-1}\frac{(30\,\frac{ft}{s})}{(20\,\frac{ft}{s})} = 56.3^\circ$$

Thus, the rotor blade sections sketched below are appropriate



(con't)

5.82 (con't)

Since the lift force acting on each rotor blade section is in the same direction as the blade velocity we conclude that this turbomachine is a turbine. The energy transferred per unit mass is the shaft work per unit mass, w_{shaft}, which we can determine with Eq. 5.54. Thus

$$w_{shaft} = -U_2 V_{\theta 2} \qquad (1)$$

From the velocity triangles we obtain

$$V_{\theta 2} = W_2 \sin 60° - U_2$$

and

$$W_2 = W_1 = \sqrt{V_1^2 + U_1^2}$$

Thus

$$w_{shaft} = -U_2\left(\sqrt{V_1^2 + U_1^2}\,\sin 60° - U_2\right)$$

$$w_{shaft} = -\left(30\,\frac{ft}{s}\right)\left[\sqrt{\left(20\,\frac{ft}{s}\right)^2 + \left(30\,\frac{ft}{s}\right)^2}\,\sin 60° - 30\,\frac{ft}{s}\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

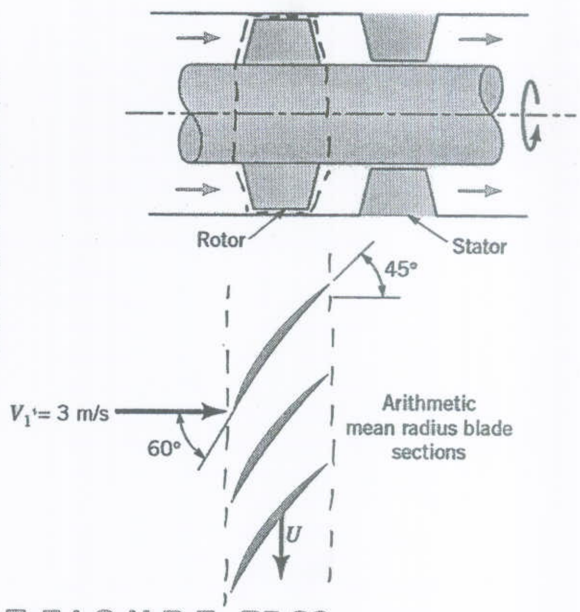
$$w_{shaft} = -\underline{\underline{36.8}}\ \frac{ft \cdot lb}{slug}$$

or

$$w_{shaft} = \frac{-36.8\ \frac{ft \cdot lb}{slug}}{\left(32.2\ \frac{lbm}{slug}\right)} = -\underline{\underline{1.14}}\ \frac{ft \cdot lb}{lbm}$$

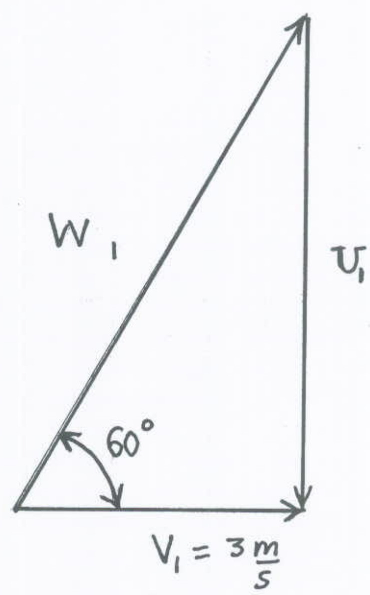
5.83

5.83 An axial flow gasoline pump (see Fig. P5.83) consists of a rotating row of blades (rotor) followed downstream by a stationary row of blades (stator). The gasoline enters the rotor axially (without any angular momentum) with an absolute velocity of 3 m/s. The rotor blade inlet and exit angles are 60° and 45° from the axial direction. The pump annulus passage cross-sectional area is constant. Consider the flow as being tangent to the blades involved. Sketch velocity triangles for flow just upstream and downstream of the rotor and just downstream of the stator where the flow is axial. How much energy is added to each kilogram of gasoline? Is this an actual or ideal amount?



■ FIGURE P5.83

The velocity triangle for flow just upstream of the rotor is sketched below for the arithmetic mean radius.



With the triangle we conclude that

$$W_1 = \frac{V_1}{\cos 60^\circ} = \frac{\left(3\ \frac{m}{s}\right)}{\cos 60^\circ} = 6\ \frac{m}{s}$$

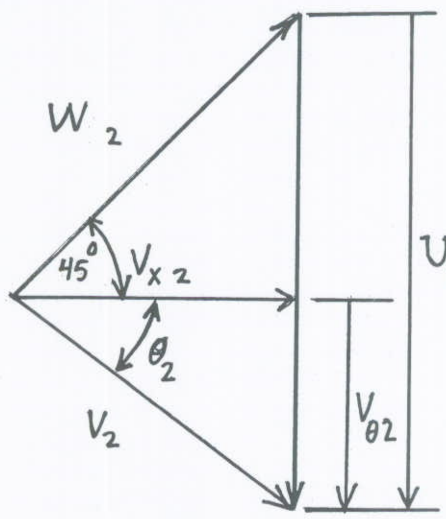
and

$$U_1 = W_1 \sin 60^\circ = \left(6\ \frac{m}{s}\right)\sin 60^\circ = 5.2\ \frac{m}{s}$$

(con't)

5.83 (con't)

The velocity triangle for flow just downstream of the rotor is sketched below for the arithmetic mean radius. For incompressible flow $V_{x2} = V_1$. For mean radius flow $U_2 = U_1$. Thus for relative flow tangent to the blade we obtain the velocity triangle sketched below.



With the triangle we conclude that

$$V_{\theta 2} = U_2 - W_{\theta 2} = U_2 - V_{x2}\tan 45^\circ = 5.2\,\frac{m}{s} - \left(3\,\frac{m}{s}\right)\tan 45^\circ = 2.2\,\frac{m}{s}$$

Also

$$\theta_2 = \tan^{-1}\left(\frac{V_{\theta 2}}{V_{x2}}\right) = \tan^{-1}\left[\frac{\left(2.2\,\frac{m}{s}\right)}{\left(3\,\frac{m}{s}\right)}\right] = 36.2^\circ$$

$$W_2 = \frac{V_{x2}}{\cos 45^\circ} = \frac{\left(3\,\frac{m}{s}\right)}{\cos 45^\circ} = 4.24\,\frac{m}{s}$$

$$V_2 = \frac{V_{x2}}{\cos\theta_2} = \frac{\left(3\,\frac{m}{s}\right)}{\cos 36.2^\circ} = 3.72\,\frac{m}{s}$$

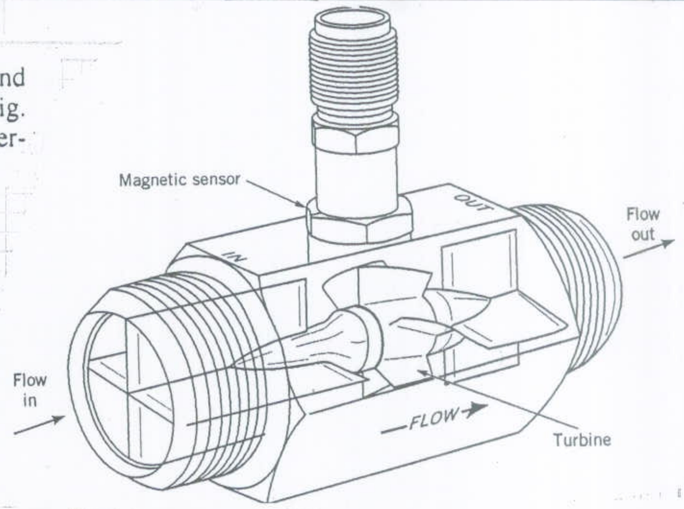
Using the stationary and non-deforming control volume shown above in the first sketch of this solution and Eq. 5.54 we can calculate the energy added to each kg of gasoline.

$$w_{shaft} = U_2 V_{\theta 2} = \left(5.2\,\frac{m}{s}\right)\left(2.2\,\frac{m}{s}\right)\left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right) = \underline{\underline{11.4}}\,\frac{N\cdot m}{kg}$$

This is the actual amount of energy delivered to the gasoline. However, not all of it is absorbed by the gasoline, some is lost.

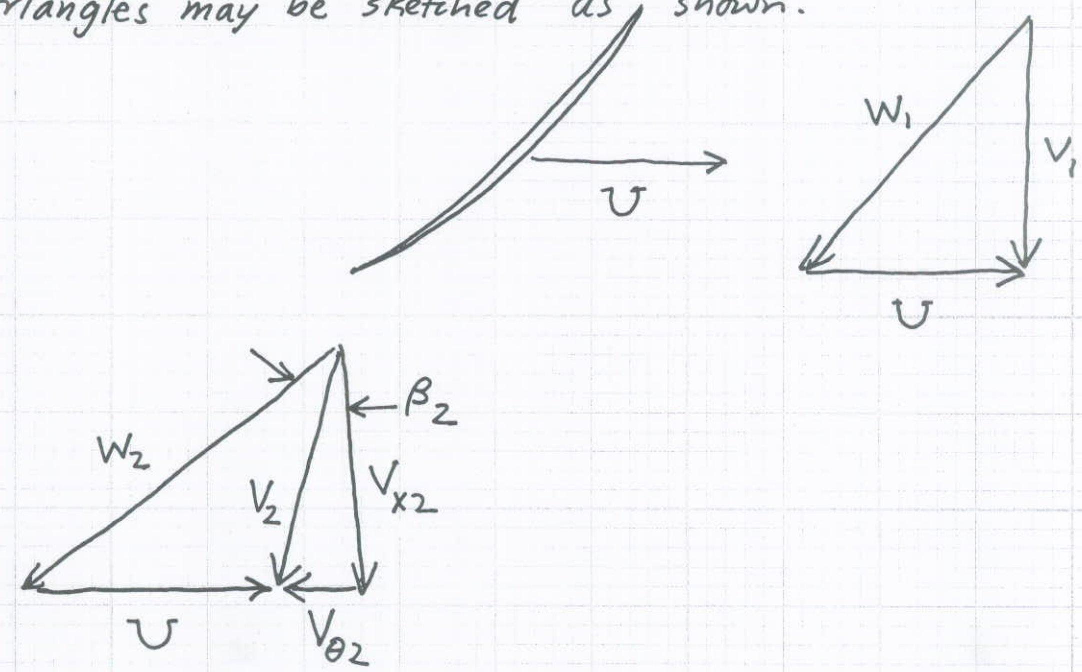
5.84

5.84 Sketch the velocity triangles for the flows entering and leaving the rotor of the turbine-type flow meter shown in Fig. P5.84. Show how rotor angular velocity is proportional to average fluid velocity.



■ FIGURE P5.84 (Courtesy of EG&G Flow Technology, Inc.)

For a section of the turbine blade at radius r, the blade moves tangentially with a velocity $U = r\omega$. The velocity triangles may be sketched as shown.



Using Eq. 5.50 we get

$$T_{shaft} = r_2 V_{\theta 2} \dot{m}_2 = r_2 \left(V_{x2} \tan\beta_2 - U \right) \dot{m}_2$$

For nearly zero T_{shaft}

$$0 = V_{x2} \tan\beta_2 - U = V_{x2} \tan\beta_2 - r\omega$$

So

$$\underline{\underline{\omega = \frac{V_{x2} \tan\beta_2}{r}}}$$

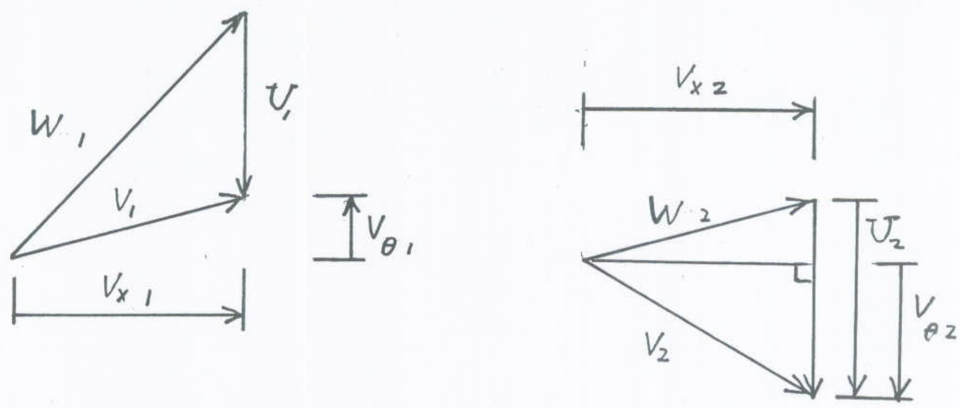
5.85

5.85 By using velocity triangles for flow upstream (1) and downstream (2) of a turbomachine rotor, prove that the shaft work in per unit mass flowing through the rotor is

$$w_{\substack{\text{shaft}\\ \text{net in}}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 + W_1^2 - W_2^2}{2}$$

where V = absolute flow velocity magnitude, W = relative flow velocity magnitude, and U = blade speed.

Any set of velocity triangle for flow through a turbomachine rotor row would give the same result. We use the triangles of Fig. P5.77.



From the inlet flow velocity triangle we get

$$V_{x1}^2 = V_1^2 - V_{\theta 1}^2 \tag{1}$$

and

$$V_{x1}^2 = W_1^2 - (V_{\theta 1} + U_1)^2 = W_1^2 - V_{\theta 1}^2 - 2U_1 V_{\theta 1} - U_1^2 \tag{2}$$

Combining Eqs. 1 and 2 we obtain

$$U_1 V_{\theta 1} = \frac{W_1^2 - V_1^2 - U_1^2}{2} \tag{3}$$

From the outlet flow velocity triangle we get

$$V_{x2}^2 = V_2^2 - V_{\theta 2}^2 \tag{4}$$

and

$$V_{x2}^2 = W_2^2 - (U_2 - V_{\theta 2})^2 = W_2^2 - U_2^2 + 2U_2 V_{\theta 2} - V_{\theta 2}^2 \tag{5}$$

(con't)

5.91

5.91 A 1000-m-high waterfall involves steady flow from one large body to another. Detemine the temperature rise associated with this flow.

This is like Example 5.22.

To determine the temperature change we use the relationship

$$T_2 - T_1 = \frac{\check{u}_2 - \check{u}_1}{c} \qquad (1)$$

where the specific heat, $c = 1 \frac{Btu}{lbm \cdot {}^{\circ}R}$. We use the energy equation (Eq. 5.70) to obtain

$$\check{u}_2 - \check{u}_1 = g(z_1 - z_2) \qquad (2)$$

Combining Eqs. 1 and 2 yields

$$T_2 - T_1 = \frac{g(z_1 - z_2)}{c}$$

or

$$T_2 - T_1 = \frac{\left(9.81 \frac{m}{s^2}\right)(1000\ m)\left(0.4536 \frac{kg}{lbm}\right)\left(0.5556 \frac{K}{{}^{\circ}R}\right)\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)}{\left(1 \frac{Btu}{lbm \cdot {}^{\circ}R}\right)\left(1055 \frac{N \cdot m}{Btu}\right)}$$

and

$$T_2 - T_1 = \underline{\underline{2.34K}}$$

5.85 (con't)

Combining Eqs. 4 and 5 we obtain

$$U_2 V_{\theta 2} = \frac{V_2^2 - W_2^2 + U_2^2}{2} \qquad (6)$$

For the set of velocity triangles

$$w_{\substack{shaft\\ net\ in}} = U_1 V_{\theta 1} + U_2 V_{\theta 2} \qquad (7)$$

Combining Eqs. 3, 6 and 7 we obtain

$$w_{\substack{shaft\\ net\ in}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 + W_1^2 - W_2^2}{2}$$

5.90

5.90 Is a necessary condition associated with application of the Bernoulli equation zero heat transfer? Explain.

From Eq. 5.78, we conclude that for application of the Bernoulli equation (loss = 0)

$$q_{\underset{in}{net}} = \check{u}_{out} - \check{u}_{in}$$

Thus the heat transfer, $q_{\underset{in}{net}}$, with application of the Bernoulli equation is not necessarily zero. No.

5.92

5.92 A 100-ft-wide river with a flowrate of 2400 ft³/s flows over a rock pile as shown in Fig. P5.92. Determine the direction of flow and the head loss associated with the flow across the rock pile.

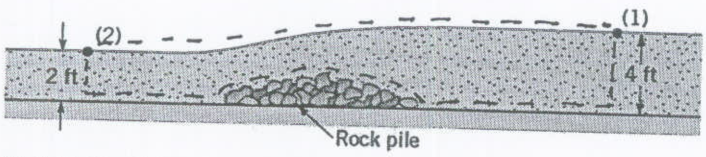


FIGURE P5.92

To determine the direction of flow we will assume a direction, use the energy equation (Eq. 5.84) and calculate the head loss. If the head loss is positive, our assumed direction of flow is correct. If the head loss is negative which is not physically possible, our assumed direction of flow is wrong.

So, assuming the flow is from right to left or from point (1) to point (2) in the sketch above, we get using Eq. 5.84

$$\overbrace{\cancel{\frac{p_2}{\rho}} + \frac{V_2^2}{2g} + z_2 = \cancel{\frac{p_1}{\rho}}}^{\text{same pressure}} + \frac{V_1^2}{2g} + z_1 + \underbrace{\cancel{h_s}}_{0,\ \text{no shaft work}} - h_L$$

Now

$$V_1 = \frac{Q}{A_1} = \frac{\left(2400\ \frac{ft^3}{s}\right)}{(4\ ft)(100\ ft)} = 6\ \frac{ft}{s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{\left(2400\ \frac{ft^3}{s}\right)}{(2\ ft)(100\ ft)} = 12\ \frac{ft}{s}$$

So

$$h_L = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + z_1 - z_2 = \frac{\left(6\ \frac{ft}{s}\right)^2}{2\left(32.2\ \frac{ft}{s^2}\right)} - \frac{\left(12\ \frac{ft}{s}\right)^2}{2\left(32.2\ \frac{ft}{s^2}\right)} + 4\ ft - 2\ ft$$

$h_L = \underline{\underline{0.32\ ft}}$ and since h_L is positive, our assumed <u>right to left</u> flow is correct

5.93

5.93 Air steadily expands adiabatically and without friction from stagnation conditions of 690 kpa (abs) and 290 K to a static pressure of 101 kpa (abs). Determine the velocity of the expanded air assuming: (a) incompressible flow; (b) compressible flow.

This is similar to Example 5.29.

(a) For incompressible flow, the Bernoulli equation (Eq. 5.109) applied to adiabatic and frictionless flow from the stagnation state to the static state leads to

$$V = \sqrt{\frac{2(p_0 - p)}{\rho}} \qquad (1)$$

where the ideal gas equation of state yields

$$\rho_0 = \frac{p_0}{RT_0} \qquad (2)$$

Combining Eqs. 1 and 2 results in

$$V = \sqrt{\frac{2(p_0 - p)RT_0}{p_0}}$$

or

$$V = \sqrt{\frac{2\left[690 \text{ kPa(abs)} - 101 \text{ kPa(abs)}\right]\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)(290 K)}{690 \text{ kPa(abs)}\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}$$

and

$$V = \underline{\underline{377 \frac{m}{s}}}$$

(b) For compressible flow, Eq. 5.113 applied to adiabatic and frictionless flow from the stagnation state to the static state leads to

$$V = \sqrt{\left(\frac{2k}{k-1}\right)\left(\frac{p_0}{\rho_0} - \frac{p}{\rho}\right)} \qquad (3)$$

However for this process

$$\frac{p}{\rho^k} = \text{constant}$$

(con't)

5.93 (con't)

Thus $\rho = \rho_0 \left(\frac{p}{p_0}\right)^{\frac{1}{k}}$ (4)

and combining Eqs. 3 and 4 leads to

$$V = \sqrt{\left(\frac{2k}{k-1}\right)\left[\frac{p_0}{\rho_0} - \frac{p}{\rho_0\left(\frac{p}{p_0}\right)^{\frac{1}{k}}}\right]} \qquad (5)$$

With the ideal equation of state (Eq. 2), Eq. 5 becomes

$$V = \sqrt{\left(\frac{2k}{k-1}\right) RT_0 \left[1 - \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}\right]}$$

or

$$V = \sqrt{\frac{2(1.40)}{(1.40-1)} \frac{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)(290 K)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)} \left\{1 - \left[\frac{101 \text{ kPa(abs)}}{690 \text{ kPa(abs)}}\right]^{\left(\frac{1.40-1}{1.40}\right)}\right\}}$$

and

$$V = \underline{\underline{580}} \frac{m}{s}$$

5.94

5.94 A horizontal Venturi flow meter consists of a converging–diverging conduit as indicated in Fig. P5.94. The diameters of cross sections (1) and (2) are 6 and 4 in. The velocity and static pressure are uniformly distributed at cross sections (1) and (2). Determine the volume flowrate (ft^3/s) through the meter if $p_1 - p_2 = 3$ psi, the flowing fluid is oil ($\rho = 56\ lbm/ft^3$), and the loss per unit mass from (1) to (2) is negligibly small.

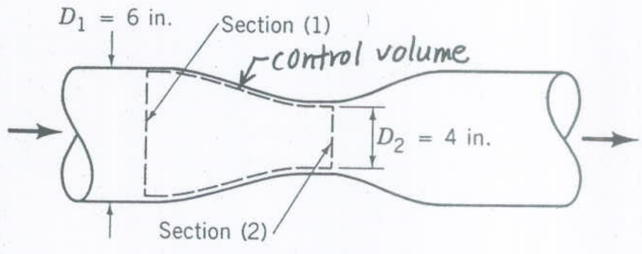


FIGURE P5.94

The control volume shown in the sketch above is used. Application of the conservation of mass equation (Eq. 5.13) to the incompressible steady flow through this control volume leads to

$$Q = A_1 V_1 = A_2 V_2 \qquad (1)$$

Application of the energy equation (Eq. 5.79) to the flow through this control volume yields

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} = \frac{p_1}{\rho} + \frac{V_1^2}{2} \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{p_2}{\rho} + \frac{Q^2}{A_2^2\,2} = \frac{p_1}{\rho} + \frac{Q^2}{A_1^2\,2}$$

or

$$Q = \left\{2\left(\frac{p_1 - p_2}{\rho}\right)\left[\frac{1}{\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)}\right]\right\}^{\frac{1}{2}} = \left\{2\left(\frac{p_1 - p_2}{\rho}\right)\left[\frac{1}{\frac{1}{\left(\frac{\pi D_2^2}{4}\right)^2} - \frac{1}{\left(\frac{\pi D_1^2}{4}\right)^2}}\right]\right\}^{\frac{1}{2}}$$

$$Q = \left\{\frac{2\left(3\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)\left(32.2\ \frac{lbm \cdot ft}{lb\ s^2}\right)}{\left(56\ \frac{lbm}{ft^3}\right)}\left[\frac{1}{\frac{1}{\left(\frac{\pi (4\ in.)^2}{4\left(12\ \frac{in.}{ft}\right)^2}\right)^2} - \frac{1}{\left(\frac{\pi (6\ in.)^2}{4\left(12\ \frac{in.}{ft}\right)^2}\right)^2}}\right]\right\}^{\frac{1}{2}}$$

$$Q = \underline{\underline{2.17}}\ \frac{ft^3}{s}$$

5.95

5.95 Oil ($SG = 0.9$) flows downward through a vertical pipe contraction as shown in Fig. P5.95. If the mercury manometer reading, h, is 100 mm, determine the volume flowrate for frictionless flow. Is the actual flowrate more or less than the frictionless value? Explain.

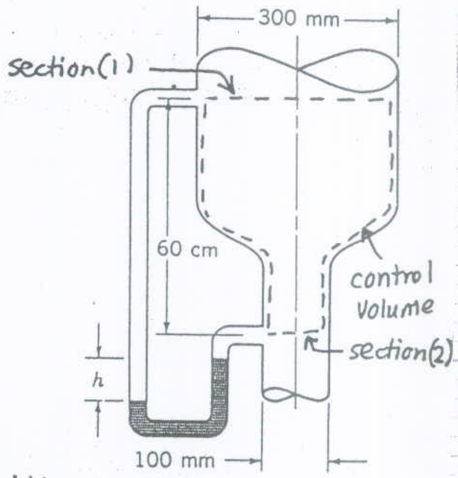


FIGURE P5.95

The volume flowrate may be obtained with

$$Q = A_1 V_1 = A_2 V_2 = \frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2 \qquad (1)$$

To determine either V_1 or V_2 we apply the energy equation (Eq. 5.82) to the flow between sections (1) and (2). Thus,

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 + \cancel{w_{\substack{shaft\\net\ in}}}^{\,0} - \cancel{loss}^{\,neglect} \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{V_2^2}{2}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = \frac{p_1 - p_2}{\rho} + g(z_1 - z_2) \qquad (3)$$

To determine $\frac{p_1 - p_2}{\rho}$ we use the manometer equation from section 2.6 to obtain

$$\frac{p_1 - p_2}{\rho} = gh\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right) - g(z_1 - z_2) \qquad (4)$$

Combining Eqs. 3 and 4 we get

$$V_2 = \sqrt{\frac{2gh\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right)}{1 - \left(\frac{D_2}{D_1}\right)^4}}$$

or

$$V_2 = \sqrt{\frac{(2)(9.81\ \frac{m}{s^2})(0.1\ m)\left(\frac{13.6}{0.9} - 1\right)}{1 - \left(\frac{100\ mm}{300\ mm}\right)^4}} = 5.29\ \frac{m}{s}$$

and from Eq. 1 we have

$$Q = \frac{\pi (0.1 m)^2}{4}\left(5.29\ \frac{m}{s}\right) = \underline{\underline{0.042\ \frac{m^3}{s}}}$$

Actual flowrate would be less than the frictionless value because the loss would be greater than the zero amount used above.

5.96

5.96 An incompressible liquid flows steadily along the pipe shown in Fig. P5.96. Determine the direction of flow and the head loss over the 6-m length of pipe.

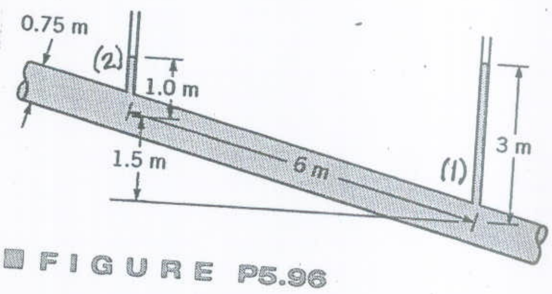


FIGURE P5.96

Assume flow from (1) to (2) and use the energy equation (Eq. 5.84) to get for the contents of the control volume shown:

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + \cancel{h_s}^{\,0} - h_\ell$$

Thus

$$h_\ell = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} + z_1 - z_2 = 3\,m - 1.0\,m - 1.5\,m = \underline{\underline{0.5\,m}}$$

and since $h_\ell > 0$, the assumed direction of flow is correct.

<u>The flow is uphill.</u>

5.97

5.97 Water flows through a vertical pipe, as is indicated in Fig. P5.97. Is the flow up or down in the pipe? Explain.

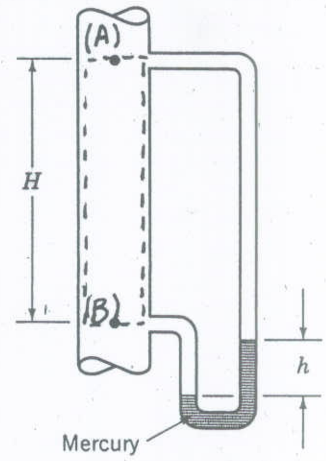


FIGURE P5.97

The control volume shown in the sketch above is used. For steady, incompressible flow downward from (A) to (B) we obtain from Eq. 5.79

$$\frac{p_B}{\rho} + \frac{V_B^2}{2} + gz_B = \frac{p_A}{\rho} + \frac{V_A^2}{2} + gz_A - {}_A\text{loss}_B$$

From conservation of mass we conclude that

$$V_A = V_B$$

Thus from Eq. 1

$$ {}_A\text{loss}_B = gH + \frac{p_A - p_B}{\rho}$$

However the manometer equation (see Section 2.6) yields

$$\frac{p_A - p_B}{\rho} = g\left[h\left(1 - SG_{Hg}\right) - H\right]$$

and

$$ {}_A\text{loss}_B = gh\left(1 - SG_{Hg}\right)$$

which is a negative quantity since $SG_{Hg} = 13.6$. A negative loss is not physically possible so the flow must be upward from B to A. For upward flow the above analysis leads to

$$ {}_B\text{loss}_A = gh\left(SG_{Hg} - 1\right)$$

which is positive and therefore physically reasonable.

5.98

5.98 A circular disk can be lifted up by blowing on it with the device shown in Fig. P5.98. Explain why this happens.

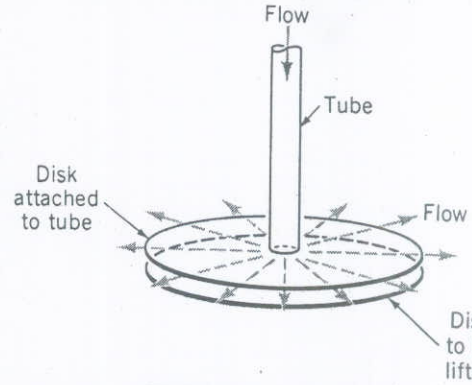


FIGURE P5.98

Applying the energy equation (Eq. 5.82) to the flow from section (1) anywhere within the space between the two circular disks to section (2) at the exit of the flow between the two disks we obtain

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} = \frac{p_1}{\rho} + \frac{V_1^2}{2} - \text{loss}$$

We note that the exit pressure, p_2, is p_{atm}. Thus, Eq. 1 becomes

$$p_1 = p_{atm} + \rho\left(\frac{V_2^2 - V_1^2}{2}\right) + \text{loss} \qquad (1)$$

With conservation of mass we conclude that

$$V_1 = V_2 \frac{A_2}{A_1} = V_2 \frac{D_2}{D_1}$$

which when combined with Eq. 1 yields

$$p_1 = p_{atm} + \rho \frac{V_2^2}{2}\left[1 - \left(\frac{D_2}{D_1}\right)^2\right] + \text{loss} \qquad (2)$$

We conclude with Eq. 2 that the pressures within the flow between the 2 disks are mostly less than $p_2 = p_{atm}$ since $D_1 < D_2$ and loss is small. An exception is the stagnation pressure where the tube flow impacts on the lower disk. The less than atmospheric pressure value of p_1 result in the disk being lifted up.

5.99

5.99 A siphon is used to draw water at 20 °C from a large container as indicated in Fig. P5.99. Does changing the elevation, h, of the siphon centerline above the water level in the tank vary the flowrate through the siphon? Explain. What is the maximum allowable value of h?

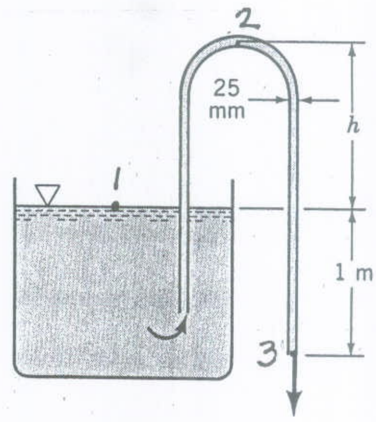


FIGURE P5.99

The volume flowrate through the siphon is related to velocity by the equation

$$Q = VA$$

where A is the constant cross section area of the siphon. Thus V is constant throughout the siphon.
Assuming steady, incompressible flow without friction allows us to use the Bernoulli equation between any two points along a path line. Thus

$$\cancel{\frac{p_1}{\rho}} + \cancel{\frac{V_1^2}{2}} + g z_1 = \cancel{\frac{p_3}{\rho}} + \frac{V_3^2}{2} + g z_3$$

or

$$V_3 = \sqrt{2g\,(1\,m)}$$

and it appears as if V_3 and thus Q is constant and independent of the value of h.
However, if the Bernoulli equation is written for flow between point 2 and 3 we obtain

$$\frac{p_2}{\rho} + \cancel{\frac{V_2^2}{2}} + g z_2 = \frac{p_3}{\rho} + \cancel{\frac{V_3^2}{2}} + g z_3$$

$$h = \frac{p_3 - p_2}{\rho g} - 1$$

and we conclude that since $p_3 = p_{atm}$, as h becomes larger, p_2 becomes smaller.
The maximum value of h is associated with the minimum value of p_2 which is the vapor pressure of water. Thus

$$h_{max} = \frac{p_3 - p_v}{\rho g} - 1 = \frac{\left(101 \times 10^3 \frac{N}{m^2}\right) - \left(2.338 \times 10^3 \frac{N}{m^2}\right)}{\left(998.2 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)\left(1 \frac{N}{kg \cdot m/s^2}\right)} - 1 = \underline{\underline{9.06\,m}}$$

Table B.2 Table B.2

5.100

5.100 A water siphon having a constant inside diameter of 3 in. is arranged as shown in Fig. P5.100. If the friction loss between A and B is $0.8V^2/2$, where V is the velocity of flow in the siphon, determine the flowrate involved.

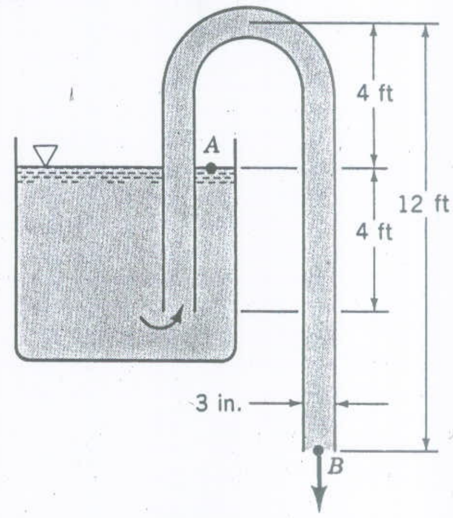


FIGURE P5.100

To determine the flowrate, Q, we use

$$Q = AV = \frac{\pi D^2}{4} V \qquad (1)$$

To obtain V we apply the energy equation (Eq. 5.82) between points A and B in the sketch above. Thus,

$$\cancelto{0}{\frac{p_B}{\rho}} + \frac{V_B^2}{2} + g z_B = \cancelto{0}{\frac{p_A}{\rho}} + \cancelto{0}{\frac{V_A^2}{2}} + g z_A + w_{\substack{shaft \\ net\ in}} - loss$$

or

$$\frac{V^2}{2} + g z_B = g z_A - 0.8\frac{V^2}{2}$$

Thus

$$V = \sqrt{\frac{g(z_A - z_B)}{0.9}} = \sqrt{\frac{(32.2\ \frac{ft}{s^2})(8\ ft)}{0.9}} = 16.9\ \frac{ft}{s}$$

and with Eq. 1

$$Q = \frac{\pi(3 in.)^2 \ (16.9\ \frac{ft}{s})}{4(144\ \frac{in.^2}{ft^2})} = \underline{\underline{0.830\ \frac{ft^3}{s}}}$$

5.101

5.101 Water flows through a valve (see Fig. P5.101) at the rate of 1000 lbm/s. The pressure just upstream of the valve is 90 psi and the pressure drop across the valve is 5 psi. The inside diameters of the valve inlet and exit pipes are 12 and 24 in. If the flow through the valve occurs in a horizontal plane, determine the loss in available energy across the valve.

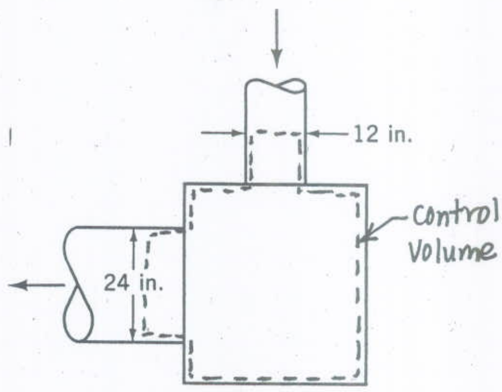


FIGURE P5.101

The control volume shown in the sketch above is used. We can use Eq. 5.79 to determine the loss in available energy associated with the incompressible, steady flow through this control volume. Thus

$$loss = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2}$$

From the conservation of mass principle

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{\dot{m}}{\rho \frac{\pi D_1^2}{4}}$$

and

$$V_2 = \frac{\dot{m}}{\rho \frac{\pi D_2^2}{4}}$$

Thus

$$loss = \frac{P_1 - P_2}{\rho} + \frac{1}{2}\left(\frac{\dot{m}\,4}{\rho\pi}\right)^2\left(\frac{1}{D_1^4} - \frac{1}{D_2^4}\right)$$

$$loss = \frac{\left(50\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{1.94\,\frac{slugs}{ft^3}} + \frac{1}{2}\left[\frac{\left(1000\,\frac{lbm}{s}\right)(4)}{\left(1.94\,\frac{slugs}{ft^3}\right)\left(32.2\,\frac{lbm}{slug}\right)\pi}\right]^2\left[\frac{\left(12\,\frac{in.}{ft}\right)^4}{(12\,in.)^4} - \frac{\left(12\,\frac{in.}{ft}\right)^4}{\left(24\,\frac{in.}{ft}\right)^4}\right]\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)$$

$$loss = \underline{\underline{5660}}\,\frac{ft\cdot lb}{slug}$$

5.102

5.102 Compare the volume flowrates associated with two different vent configurations, a cylindrical hole in the wall having a diameter of 4 in. and the same diameter cylindrical hole in the wall but with a well-rounded entrance (see Fig. P5.102). The room is held at a constant pressure of 1.5 psi above atmospheric. Both vents exhaust into the atmosphere. The loss in available energy associated with flow through the cylindrical vent from the room to the vent exit is $0.5V_2^2/2$, where V_2 is the uniformly distributed exit velocity of air. The loss in available energy associated with flow through the rounded entrance vent from the room to the vent exit is $0.05V_2^2/2$, where V_2 is the uniformly distributed exit velocity of air.

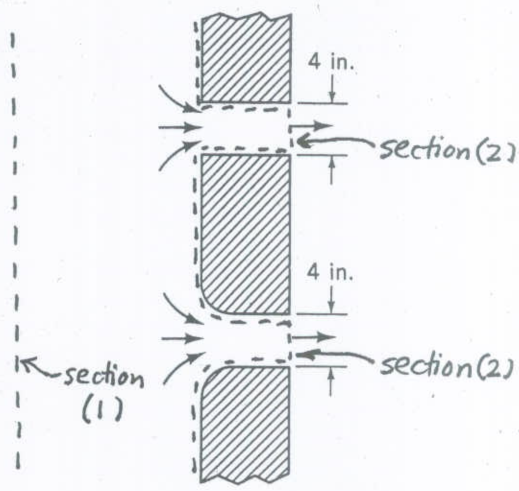


FIGURE P5.102

This is like Example 5.23.

The volume flowrate for each vent configuration is obtained with

$$Q = A_2 V_2 = \frac{\pi D_2^2}{4} V_2 \qquad (1)$$

and the exit velocity of each vent is obtained with the energy equation (Eq. 5.82). Thus,

$$\frac{V_2^2}{2} = \cancel{\frac{V_1^2}{2}}^{0} + \frac{p_1 - p_2}{\rho} + g(\cancel{z_1 - z_2})^{0} - loss$$

or

$$\frac{V_2^2}{2} = \frac{p_1 - p_2}{\rho} - K_L \frac{V_2^2}{2}$$

and

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho(1 + K_L)}} \qquad (2)$$

For the cylindrical vent with an abrupt entrance, Eq. 2 leads to

$$V_2 = \sqrt{\frac{(2)(1.5\,psi)(144\,\frac{in.^2}{ft^2})}{(2.38\times10^{-3}\,\frac{slug}{ft^3})(1+0.5)(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}})}} = 347.9\,\frac{ft}{s}$$

and with Eq. 1 we obtain

$$Q_{\text{abrupt entrance vent}} = \frac{\pi(4\,in.)^2}{4(144\,\frac{in.^2}{ft^2})}\left(347.9\,\frac{ft}{s}\right) = \underline{\underline{30.4\,\frac{ft^3}{s}}}$$

For the cylindrical vent with a rounded entrance, Eq. 2 leads to

$$V_2 = \frac{(2)(1.5\,psi)(144\,\frac{in.^2}{ft^2})}{(2.38\times10^{-3}\,\frac{slug}{ft^3})(1+0.05)(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}})} = 415.8\,\frac{ft}{s}$$

and with Eq. 1 we obtain

$$Q_{\text{rounded entrance vent}} = \frac{\pi(4\,in.)^2}{4(144\,\frac{in.^2}{ft^2})}\left(415.8\,\frac{ft}{s}\right) = \underline{\underline{36.3\,\frac{ft^3}{s}}}$$

5.103

5.103 A gas expands through a nozzle from a pressure of 300 psia to a pressure of 5 psia. The enthalpy change involved, $\check{h}_1 - \check{h}_2$, is 150 Btu/lbm. If the expansion is adiabatic but with frictional effects and the inlet gas speed is negligibly small, determine the exit gas velocity.

Because of the appreciable pressure drop involved in this gas flow we consider this problem to involve compressible flow. From Eq. 5.71 we obtain

$$V_2 = \sqrt{2\left(\check{h}_1 - \check{h}_2\right)}$$

or

$$V_2 = \sqrt{2\left(150\ \frac{\text{Btu}}{\text{lb}_m}\right)\left(778\ \frac{\text{ft}\cdot\text{lb}}{\text{Btu}}\right)\left(32.2\ \frac{\text{lb}_m\cdot\text{ft}}{\text{lb}\cdot\text{s}^2}\right)}$$

$$V_2 = \underline{\underline{2740}}\ \frac{\text{ft}}{\text{s}}$$

5.104

5.104 For the 180° elbow and nozzle flow shown in Fig. P5.104, determine the loss in available energy from section (1) to section (2). How much additional available energy is lost from section (2) to where the water comes to rest?

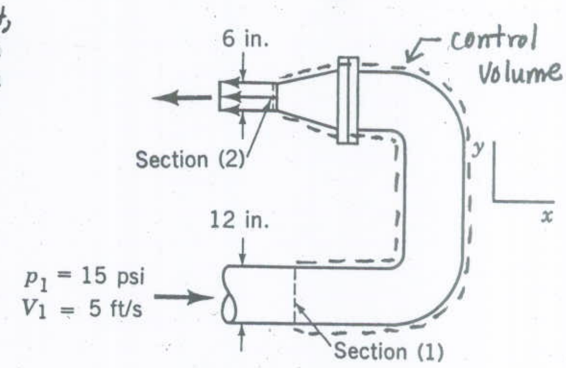


FIGURE P5.104

For solving the first part of this problem, the control volume shown in the sketch above is used. To determine the loss accompanying flow from section 1 to section 2 Eq. 5.79 can be used as follows.

$$_1loss_2 = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)$$

Since x-y coordinates are specified we assume that the flow is horizontal and $z_1 - z_2 = 0$. Also, $p_2 = p_{atm} = 0$ psi.
From the conservation of mass principle we conclude that

$$V_2 = V_1 \frac{A_1}{A_2} = V_1 \left(\frac{D_1^2}{D_2^2}\right)$$

Thus

$$_1loss_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2}\left[1 - \left(\frac{D_1^2}{D_2^2}\right)^2\right] = \frac{p_1}{\rho} + \frac{V_1^2}{2}\left[1 - \left(\frac{D_1}{D_2}\right)^4\right]$$

or

$$_1loss_2 = \frac{\left(15\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1.94\,\frac{slugs}{ft^3}\right)} + \frac{\left(5\,\frac{ft}{s}\right)^2}{2}\left[1 - \left(\frac{12\text{ in.}}{6\text{ in.}}\right)^4\right]\left(\frac{1\text{ lb}}{slug \cdot \frac{ft}{s^2}}\right)$$

$$_1loss_2 = \underline{\underline{926}}\ \frac{ft \cdot lb}{slug}$$

For the second part of this problem we consider the flow of a fluid particle from section 2 to a state of rest, a. Eq. 5.79 leads to

$$_2loss_a = \frac{V_2^2}{2}$$

Note that we have assumed that $p_2 = p_a = p_{atm}$ and $z_2 = z_a$.
Thus

$$_2loss_A = \frac{V_2^2}{2} = \frac{V_1^2}{2}\left(\frac{D_1^2}{D_2^2}\right)^2 = \frac{V_1^2}{2}\left(\frac{D_1}{D_2}\right)^4 = \frac{\left(5\,\frac{ft}{s}\right)^2\left(\frac{12\text{ in.}}{6\text{ in.}}\right)^4\left(1\,\frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}{2}$$

$$_2loss_A = \underline{\underline{200}}\ \frac{ft \cdot lb}{slug}$$

5.105

5.105 An automobile engine will work best when the back pressure at the exhaust manifold, engine block interface is minimized. Show how reduction of losses in the exhaust manifold, piping, and muffler will also reduce the back pressure. How could losses in the exhaust system be reduced? What primarily limits the minimization of exhaust system losses?

We apply the energy equation (Eq. 5.83) to the flow from the engine block, exhaust manifold interface to the exhaust system exit to get

$$p_{in} = p_{out} + \rho\frac{V_{out}^2}{2} - \rho\frac{V_{in}^2}{2} + \rho(\text{loss}) \qquad (1)$$

With Eq. 1 we see that reduction of loss in the exhaust system results in a lower value of p_{in} and thus the engine back pressure. Losses in the exhaust system could be reduced by eliminati g major loss components such as the catalytic converter and the muffler as is often done in race cars. However, noise and emissions legislation limits the extent to which this kind of loss reduction can occur in conventional vehicles. Some loss reduction can also occur by configuring the exhaust system piping with few bends and appropriate area distributions. However, requirements often leads to bends and turns in the piping and costs limit the extend of optimizing area distributions.

5.107

5.107 (See Fluids in the News article titled "Smart shocks," Section 5.3.3.) A 200-lb force applied to the end of the piston of the shock absorber shown in Fig. P5.107 causes the two ends of the shock absorber to move toward each other with a speed of 5 ft/s. Determine the head loss associated with the flow of the oil through the channel. Neglect gravity and any friction force between the piston and cylinder walls.

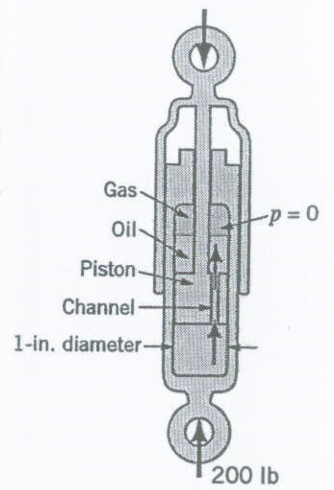


FIGURE P5.107

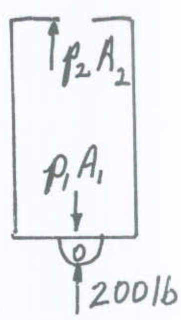
From a force balance on the cylinder

$p_1 A_1 - p_2 A_2 = 200 lb$

or with $p_2 = 0$,

$p_1 = 200 lb / A_1 = 200 lb / \left(\frac{\pi}{4}\left(\frac{1}{12} ft\right)^2\right)$

$= 3.67 \times 10^4 \frac{lb}{ft^2} = 255 psi$



From the energy equation,

$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$, where

$z_1 \approx z_2$, $V_2 = 0$, $V_1 = 5 \frac{ft}{s}$, $p_1 = 255 psi$, and $p_2 = 0$. Assume $\gamma = 50 \frac{lb}{ft^3}$.

Thus,

$$h_L = \frac{p_1}{\gamma} + \frac{V^2}{2g} = \frac{3.67 \times 10^4 \frac{lb}{ft^2}}{\left(50 \frac{lb}{ft^3}\right)} + \frac{\left(5 \frac{ft}{s}\right)^2}{2\left(32.2 \frac{ft}{s^2}\right)} = 734 ft + 0.388 ft = \underline{\underline{734 ft}}$$

5.108

5.108 What is the maximum possible power output of the hydroelectric turbine shown in Fig.P5.108?

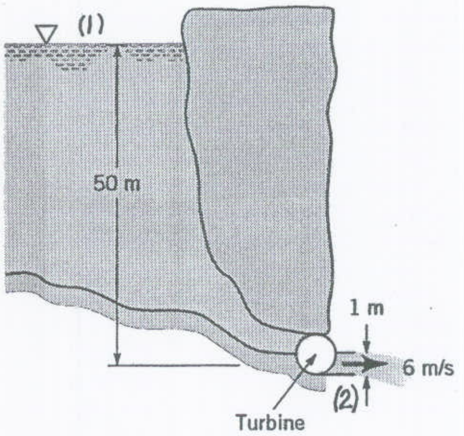


FIGURE P5.108

For flow from section (1) to section (2), Eq. 5.82 yields

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 + w_{\substack{shaft\\net\ in}} - loss \qquad (1)$$

Since $p_1 = p_2 = p_{atm}$, $\quad w_{\substack{shaft\\net\ in}} = -w_{\substack{shaft\\net\ out}}$ Eq. 1 can be expressed as

$$w_{\substack{shaft\\net\ out}} = g(z_1 - z_2) - \frac{V_2^2}{2} - loss$$

The maximum work or power output is achieved when $loss = 0$.

Thus

$$\dot{W}_{\substack{shaft\\net\ out\\maximum}} = \dot{m}\, w_{\substack{shaft\\net\ out\\maximum}} = \dot{m}\left[g(z_1 - z_2) - \frac{V_2^2}{2}\right]$$

Now

$$\dot{m} = \rho V_2 A_2 = \rho V_2 \frac{\pi D_2^2}{4} = \left(999\ \frac{kg}{m^3}\right)\left(6\ \frac{m}{s}\right)\frac{\pi (1\ m)^2}{4} = 4710\ \frac{kg}{s}$$

and

$$\dot{W}_{\substack{shaft\\net\ out\\maximum}} = \left(4710\ \frac{kg}{s}\right)\left[\left(9.81\ \frac{m}{s^2}\right)(50\ m) - \frac{\left(6\ \frac{m}{s}\right)^2}{2}\right]\left(\frac{1\ N}{kg\cdot\frac{m}{s^2}}\right)$$

$$\dot{W}_{\substack{shaft\\net\ out\\maximum}} = \underline{\underline{2.22 \times 10^6}}\ \frac{N\cdot m}{s} = \underline{\underline{2.22 \times 10^6\ W}} = \underline{\underline{2.22}}\ MW$$

5.109

5.109 The pumper truck shown in Fig. P5.109 is to deliver 1.5 ft^3/s to a maximum elevation of 60 ft above the hydrant. The pressure at the 4-in.-diameter outlet of the hydrant is 10 psi. If head losses are negligibly small, determine the power that the pump must add to the water.

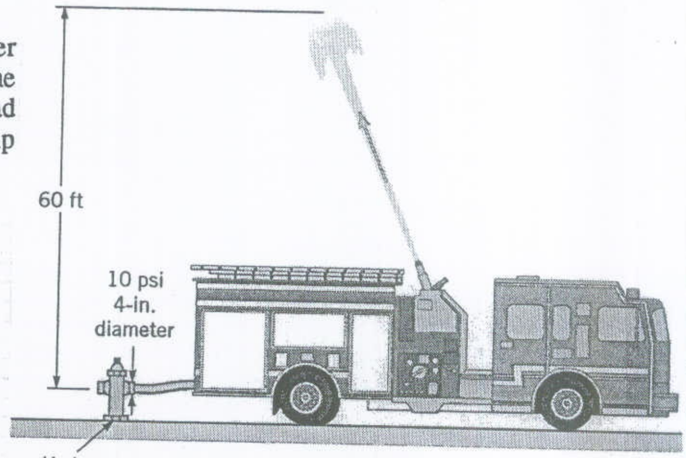


FIGURE P5.109

To solve this problem we first use the energy equation (Eq. 5.84) for flow from the hydrant exit (1) to the maximum desired elevation of 60 ft (2) to get h_s or in this case, the pump head. With the pump head we can get the pump power from Eq. 5.85.

$$\cancel{\frac{p_2}{\rho}}^{\,0} + \cancel{\frac{V_2^2}{2g}}^{\,0} + z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2g} + z_1 + h_s - \cancel{h_L}^{\,0}$$

$$h_s = z_2 - z_1 - \frac{p_1}{\rho} - \frac{V_1^2}{2g}$$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\left(\frac{\pi d_1^2}{4}\right)} = \frac{\left(1.5\,\frac{ft^3}{s}\right)(4)}{\pi\left(\frac{4\text{ in.}}{12\,\frac{\text{in.}}{ft}}\right)^2} = 17.2\ \frac{ft}{s}$$

$$h_s = 60\text{ ft} - \frac{\left(10\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(62.4\,\frac{lb}{ft^3}\right)} - \frac{\left(17.2\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)}$$

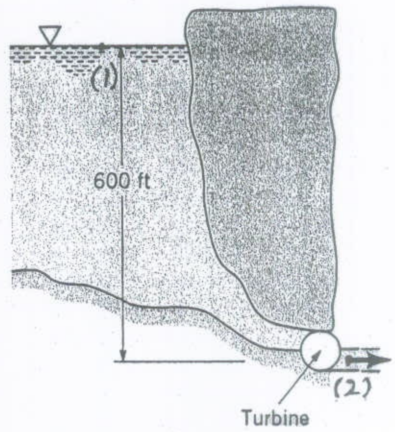
$$h_s = 32.3\text{ ft}$$

$$\dot{W}_{\substack{shaft\\net\ in}} = \gamma Q h_s = \left(62.4\,\frac{lb}{ft^3}\right)\left(1.5\,\frac{ft^3}{s}\right)\frac{(32.2\text{ ft})}{\left(550\,\frac{ft\cdot lb}{s\cdot hp}\right)}$$

$$\dot{W}_{\substack{shaft\\net\ in}} = \underline{\underline{5.48\text{ hp}}}$$

5.110

5.110 The hydroelectric turbine shown in Fig. P5.110 passes 8 million gal/min across a head of 600 ft. What is the maximum amount of power output possible? Why will the actual amount be less?



■ FIGURE P5.110

From the energy equation

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where $p_1 = 0$, $p_2 = 0$, and $V_1 = 0$.

Thus,

$$h_s = (z_2 - z_1) + h_L + \frac{V_2^2}{2g}$$

And, the power is given by

$$\dot{W}_{turb} = \gamma Q h = \gamma Q\left[(z_2 - z_1) + h_L + \frac{V_2^2}{2g}\right]$$

The maximum power would occur if there were no losses ($h_L = 0$) and negligible kinetic energy at the exit ($V_2 \approx 0$; large diameter outlet).

Thus,

$$\dot{W}_{turb\,max} = \gamma Q(z_2 - z_1) = 62.4\frac{lb}{ft^3}\left(8\times10^6\frac{gal}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{1\,ft^3}{7.48\,gal}\right)(-600\,ft)$$

$$-6.67\times10^8\ \frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right) = \underline{\underline{-1.21\times10^6\ hp}}$$

The minus sign is associated with power out.

The actual power will be less by amounts corresponding to loss and exit kinetic energy.

5.111

5.111 A pump is to move water from a lake into a large, pressurized tank as shown in Fig. P5.111 at a rate of 1000 gal in 10 min or less. Will a pump that adds 3 hp to the water work for this purpose? Support your answer with appropriate calculations. Repeat the problem if the tank were pressurized to 3, rather than 2, atmospheres.

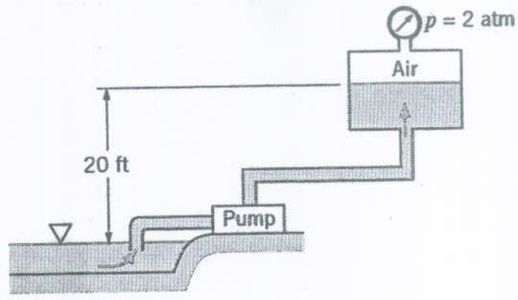


FIGURE P5.111

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}+h_s-h_L=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}, \text{ where } p_1=0,\ z_1=0,\ V_1=0, \text{ and } z_2=20\text{ft}.$$

Thus,

(1) $$h_s=h_L+\frac{p_2}{\gamma}+z_2.$$

Also,

$$Q=[(1000\text{gal})/(10\text{min})]\left(\frac{1\text{ ft}^3}{7.48\text{gal}}\right)\left(\frac{1\text{min}}{60\text{ s}}\right)=0.223\ \frac{\text{ft}^3}{\text{s}}$$

so that

$$h_s=\frac{\dot{W}_s}{\gamma Q}=\frac{(3\text{ hp})(550\ \frac{\text{ft}\cdot\text{lb/s}}{\text{hp}})}{(62.4\frac{\text{lb}}{\text{ft}^3})(0.223\frac{\text{ft}^3}{\text{s}})}=119\text{ ft}$$

(a) If $p_2=2\text{ atm}=2(14.7\frac{\text{lb}}{\text{in.}^2})(144\text{in.}^2/\text{ft}^2)=4{,}230\frac{\text{lb}}{\text{ft}^2}$, then from Eq. (1)

$$h_s=h_L+\frac{4{,}230\frac{\text{lb}}{\text{ft}^2}}{(62.4\frac{\text{lb}}{\text{ft}^3})}+20\text{ft}=h_L+87.8\text{ ft}$$

Thus, if

$h_L\le h_s-87.8\text{ ft}=119\text{ft}-87.8\text{ft}=31.2\text{ ft}$ the given pump will work for $p_2=2\text{atm}$.

(b) If $p_2=3\text{ atm}=6{,}350\frac{\text{lb}}{\text{ft}^2}$, then

$$h_s=h_L+\frac{6{,}350\frac{\text{lb}}{\text{ft}^3}}{(62.4\frac{\text{lb}}{\text{ft}^3})}+20\text{ft}=h_L+122\text{ft}$$

Thus, if this pump is to work

$119\text{ ft}=h_L+122\text{ft}$, or $h_L\le -3\text{ft}$

Since it is not possible to have $h_L<0$, the pump will not work for $p_2=3\text{atm}$.

5.112

5.112 A hydraulic turbine is provided with 4.25 m^3/s of water at 415 kPa. A vacuum gage in the turbine discharge 3 m below the turbine inlet centerline reads 250 mm Hg vacuum. If the turbine shaft output power is 1100 kW, calculate the power loss through the turbine. The supply and discharge pipe inside diameters are identically 80 mm.

We consider the turbine inlet and discharge to be sections (1) and (2). For flow from sections (1) to (2) Eq. 5.82 yields

$$\text{loss} = \frac{P_1 - P_2}{\rho} + g(z_1 - z_2) - w_{\substack{\text{shaft}\\ \text{net out}}} \qquad (1)$$

since

$$V_1 = V_2$$

and

$$w_{\substack{\text{shaft}\\ \text{net out}}} = -w_{\substack{\text{shaft}\\ \text{net in}}}$$

For power loss through the turbine we need to multiply Eq. 1 by the mass flowrate, $\dot{m}$, thus

$$\text{power loss} = \dot{m}\left(\frac{P_1 - P_2}{\rho}\right) + \dot{m}\,g(z_1 - z_2) - \dot{W}_{\substack{\text{shaft}\\ \text{net out}}} \qquad (2)$$

However,

$$\dot{m} = \rho Q = \left(999\ \frac{kg}{m^3}\right)\left(4.25\ \frac{m^3}{s}\right) = 4246\ \frac{kg}{s}$$

Also

$$P_2 = -(0.25\ m\ Hg)(\rho_{Hg})(g) = (0.25\ m)(13.6)\left(999\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)$$

or

$$P_2 = -33{,}300\ \frac{N}{m^2}$$

With Eq. 2

$$\text{power loss} = \left(4246\ \frac{kg}{s}\right)\frac{\left(415{,}000\ \frac{N}{m^2} + 33{,}300\ \frac{N}{m^2}\right)}{\left(999\ \frac{kg}{m^3}\right)} + \left(4246\ \frac{kg}{s}\right)\left(9.81\ \frac{m}{s^2}\right)(3\ m)\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right) - \left(1.1 \times 10^6\ \frac{N \cdot m}{s}\right)$$

or

$$\text{power loss} = 930{,}000\ \frac{N \cdot m}{s} = \underline{\underline{930\ kW}}$$

5.113

5.113 Water is supplied at 150 ft³/s and 60 psi to a hydraulic turbine through a 3-ft inside diameter inlet pipe as indicated in Fig. P5.113. The turbine discharge pipe has a 4-ft inside diameter. The static pressure at section (2), 10 ft below the turbine inlet, is 10-in. Hg vacuum. If the turbine develops 2500 hp, determine the power lost between sections (1) and (2).

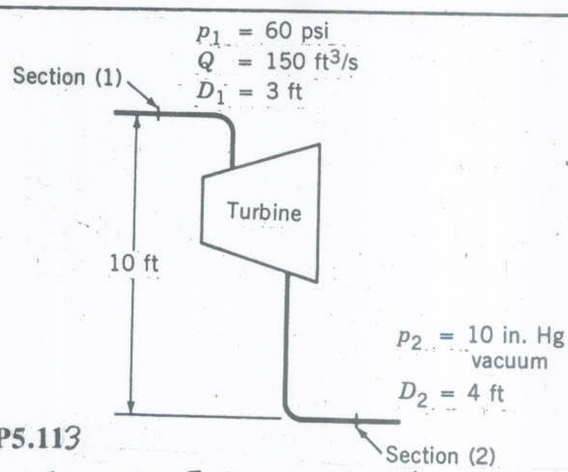


FIGURE P5.113

For flow between sections (1) and (2), Eq. 5.82 leads to

$$\text{power loss} = \rho Q\left[\left(\frac{P_1-P_2}{\rho}\right) + g(z_1-z_2) + \left(\frac{V_1^2-V_2^2}{2}\right)\right] - \dot{W}_{\substack{\text{shaft}\\\text{net out}}} \qquad (1)$$

From given data

$$P_2 = \frac{(-10\text{ in. Hg})(13.6)\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)}{\left(12\,\frac{\text{in.}}{\text{ft}}\right)}\left(\frac{1\text{ lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right) = -708\,\frac{\text{lb}}{\text{ft}^2}$$

Also

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{(4)\left(150\,\frac{\text{ft}^3}{\text{s}}\right)}{\pi\,(3\text{ ft})^2} = 21.22\,\frac{\text{ft}}{\text{s}}$$

From conservation of mass (Eq. 5.13)

$$V_2 = V_1\frac{A_1}{A_2} = V_1\frac{D_1^2}{D_2^2} = \left(21.22\,\frac{\text{ft}}{\text{s}}\right)\frac{(3\text{ ft})^2}{(4\text{ ft})^2} = 11.94\,\frac{\text{ft}}{\text{s}}$$

From Eq. 1

$$\text{power loss} = \frac{\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)\left(150\,\frac{\text{ft}^3}{\text{s}}\right)}{\left(550\,\frac{\text{ft.lb}}{\text{s.hp}}\right)}\left\{\frac{\left(60\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)+\left(708\,\frac{\text{lb}}{\text{ft}^2}\right)}{\left(1.94\,\frac{\text{slugs}}{\text{ft}^3}\right)}\right.$$

$$\left. + \left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)(10\text{ ft})\left(\frac{1\text{ lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right) + \frac{\left[\left(21.22\,\frac{\text{ft}}{\text{s}}\right)^2-\left(11.94\,\frac{\text{ft}}{\text{s}}\right)^2\right]}{2}\left(\frac{1\text{ lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)\right\}$$

$$-2500\text{hp}$$

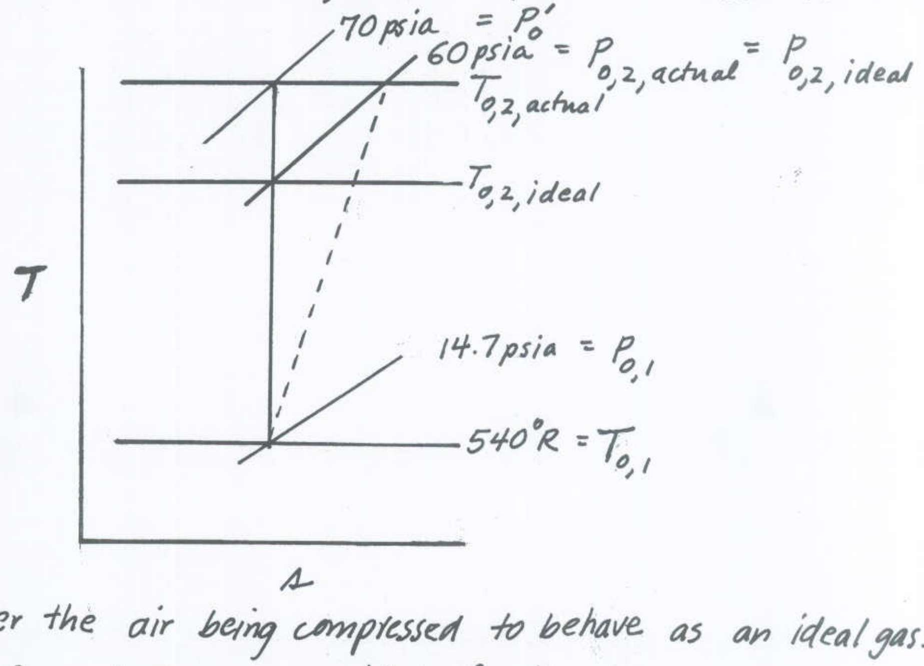
or

$$\text{power loss} = \underline{\underline{301}}\text{ hp}$$

5.114

5.114 A centrifugal air compressor stage operates between an inlet stagnation pressure of 14.7 psia and an exit stagnation pressure of 60 psia. The inlet stagnation temperature is 80 °F. If the loss of total pressure through the compressor stage associated with irreversible flow phenomena is 10 psi, calculate the actual and ideal stagnation temperature rise through the compressor. Calculate the ratio of ideal to actual temperature rise to obtain efficiency.

We assume that the air compressor operates adiabatically. An ideal compression process is frictionless and adiabatic and thus according to Eq. 5.101, it is a constant entropy or isentropic process. With Eq. 5.101 we also conclude that an actual adiabatic compression process with friction must involve an entropy increase. On temperature - entropy coordinates, the ideal and actual compression processes appear as indicated in the sketch below. Also shown is the 10 psi loss in stagnation pressure due to friction.



We consider the air being compressed to behave as an ideal gas. Then from Eqs. 1.8 and 5.111 we obtain for the ideal processes

$$T_{0,2,ideal} = T_{0,1}\left(\frac{P_{0,2,ideal}}{P_{0,1}}\right)^{\frac{k-1}{k}} = (540^\circ R)\left(\frac{60\,psia}{14.7\,psia}\right)^{\frac{1.4-1}{1.4}} = 807\ ^\circ R$$

and

$$T_{0,2,actual} = T_{0,1}\left(\frac{P_0'}{P_{0,1}}\right)^{\frac{1.4-1}{0.4}} = (540^\circ R)\left(\frac{70\,psia}{14.7\,psic}\right)^{\frac{1.4-1}{1.4}} = 843\ ^\circ R$$

(con't)

5.114 (Con't)

Then

actual stagnation temperature rise $= T_{0,2,actual} - T_{0,1} = 843°R - 540°R = \underline{\underline{303°R}}$

and

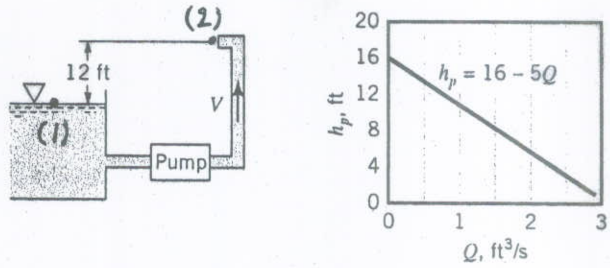
ideal stagnation temperature rise $= T_{0,2,ideal} - T_{0,1} = 807°R - 540°R = \underline{\underline{267°R}}$

Also

$$\text{efficiency} = \frac{T_{0,2,ideal} - T_{0,1}}{T_{0,2,actual} - T_{0,1}} = \frac{267°R}{303°R} = \underline{\underline{0.88}}$$

5.115

5.115 Water is pumped through a 4-in.-diameter pipe as shown in Fig. P5.115*a*. The pump characteristics (pump head versus flowrate) are given in Fig. P5.115*b*. Determine the flowrate if the head loss in the pipe is $h_L = 8V^2/2g$.



(*a*) (*b*)

■ FIGURE P5.115

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}, \text{ where } p_1 = p_2 = 0,\ z_1 = 0,\ z_2 = 12\,ft,\ V_1 = 0, \text{ and } V_2 = Q/A_2$$

Thus,

$$h_s - h_L = z_2 + \frac{V_2^2}{2g}, \text{ with}$$

$$h_s = h_p = 16 - 5Q \text{ and } h_L = 8\frac{V_2^2}{2g} = 8\frac{Q^2}{2gA_2^2}$$

Therefore,

$$16 - 5Q - \frac{4Q^2}{gA_2^2} = 12 + \frac{Q^2}{2gA_2^2}$$

or

(1) $$\left(\frac{9}{2gA_2^2}\right)Q^2 + (5)Q - 4 = 0, \text{ where } g \sim \frac{ft}{s^2},\ A_2 \sim ft^2, \text{ and } Q \sim \frac{ft^3}{s}$$

Using the given data, Eq. (1) becomes

$$\left[\frac{9}{2(32.2)\left(\frac{\pi}{4}\left(\frac{4}{12}\right)^2\right)^2}\right]Q^2 + 5Q - 4 = 0$$

or

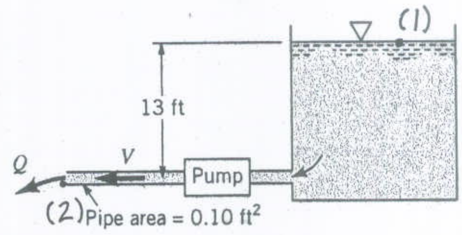
(2) $$18.35\,Q^2 + 5Q - 4 = 0$$

The positive root of Eq. (2) is $Q = \underline{\underline{0.350\,\frac{ft^3}{s}}}$

(The negative root of Eq. (2) has no physical meaning.)

5.116

5.116 Water is pumped from the large tank shown in Fig. P5.116. The head loss is known to be equal to $4V^2/2g$ and the pump head is $h_p = 20 - 4Q^2$, where h_p is in ft when Q is in ft^3/s. Determine the flowrate.



■ FIGURE P5.116

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}, \quad \text{where } p_1 = p_2 = 0,\ z_1 = 13\text{ ft},\ z_2 = 0,\ h_s = h_p \text{ and } V_1 = 0.$$

Thus,

(1) $$z_1 + h_p - h_L = \frac{V_2^2}{2g}$$

Also,

$$h_L = 4\frac{V^2}{2g} = 4\frac{V_2^2}{2g} = 4\frac{(Q/A_2)^2}{2g} \quad \text{since } V_2 = \frac{Q}{A_2}$$

Hence, Eq. (1) becomes

$$z_1 + (20 - 4Q^2) - 4\frac{(Q/A_2)^2}{2g} = \frac{(Q/A_2)^2}{2g}$$

or

$$\left[\left(\frac{5}{2g\,A_2^2}\right) + 4\right]Q^2 = 20 + z_1, \quad \text{where } g \sim \frac{ft}{s^2},\ A_2 \sim ft^2, \text{ and } Q \sim \frac{ft^3}{s}$$

Thus, with the given data

$$\left[\left(\frac{5}{2(32.2\,\frac{ft}{s^2})(0.1\,ft^2)^2}\right) + 4\right]Q^2 = 20 + 13\text{ ft}$$

or

$$\underline{\underline{Q = 1.67\ \frac{ft^3}{s}}}$$

5.117

5.117 When a fan or pump is tested at the factory, head curves (head across the fan or pump versus volume flowrate) are often produced. A generic fan or pump head curve is shown in Fig. P5.117*a*. For any piping system, the drop in pressure or head involved because of loss can be estimated as a function of volume flowrate. A generic piping system loss curve is also shown in Fig. P5.117*b*. When the pump or fan and piping system associated with the two curves of Fig. P5.117 are combined, what will the flowrate be? Why? How can the flowrate through this combined system be varied?

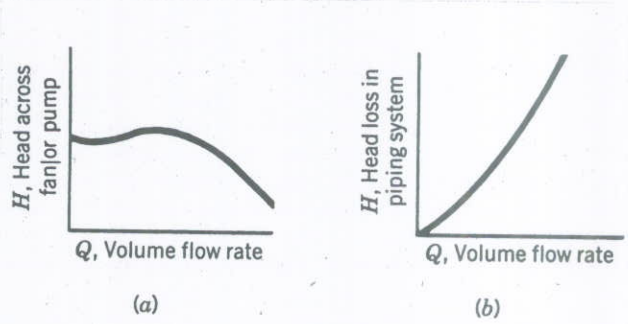
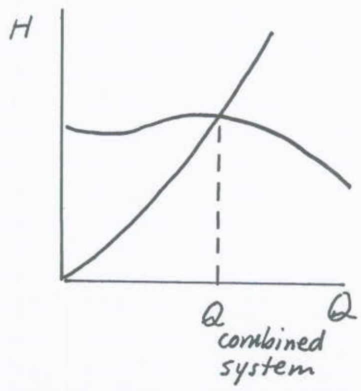
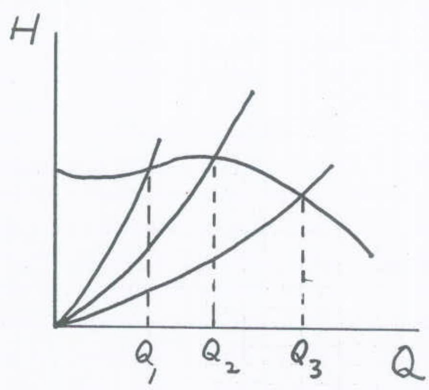


FIGURE P5.117

The flowrate of the combination of the fan or pump and the piping system represented by the two curves sketched above will correspond to the intersection of the two curves as indicated in the sketch below because this conditions satisfies both components in terms of head and flowrate.



To vary the flowrate through the combined system, the piping system curve is normally altered as shown below by changing the resistance to flow of the piping system. This could be accomplished, for example with a variable area valve.



5.118

5.118 Water flows by gravity from one lake to another as sketched in Fig. P5.118 at the steady rate of 80 gpm. What is the loss in available energy associated with this flow? If this same amount of loss is associated with pumping the fluid from the lower lake to the higher one at the same flowrate, estimate the amount of pumping power required.

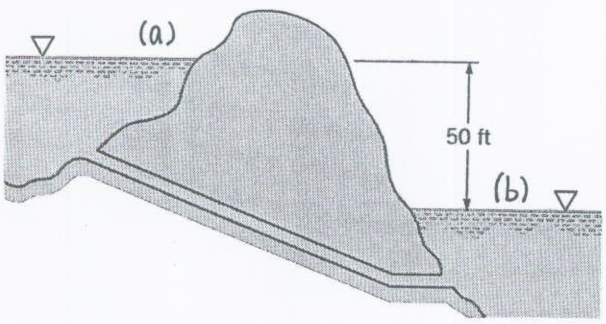


FIGURE P5.118

$$Q = \frac{80\ \frac{gal}{min}}{\left(60\ \frac{s}{min}\right)\left(7.48\ \frac{gal}{ft^3}\right)} = 0.178\ \frac{ft^3}{s}$$

For the flow from section (a) to section (b) Eq. 5.82 leads to

$$loss = g(z_a - z_b) = \left(32.2\ \frac{ft}{s^2}\right)(50\,ft)\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right) = \underline{\underline{1610}}\ \frac{ft\ lb}{slug}$$

For pumped flow from section (b) to section (a) Eq. 5.82 yields

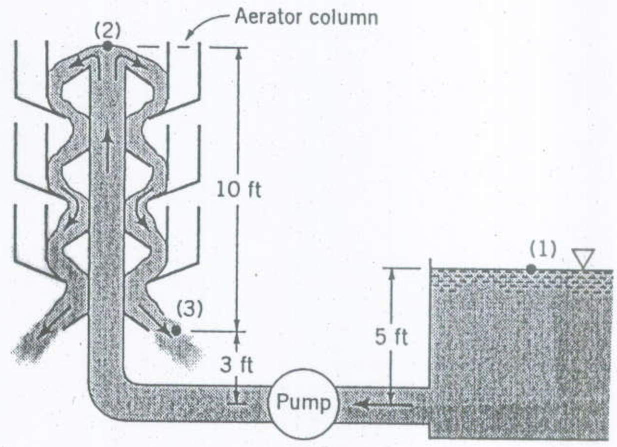
$$\dot{W}_{\substack{shaft\\ net\ in}} = \rho Q\left[g(z_a - z_b) + loss\right] = \left(1.94\ \frac{slugs}{ft^3}\right)\left(0.178\ \frac{ft^3}{s}\right)\left[\left(32.2\ \frac{ft}{s^2}\right)(50\,ft)\left(\frac{1\ lb}{slug\ \frac{ft}{s^2}}\right) + 1610\ \frac{ft \cdot lb}{slug}\right]$$

or

$$\dot{W}_{\substack{shaft\\ net\ in}} = \underline{\underline{1110}}\ \frac{ft \cdot lb}{s} = \underline{\underline{2.02\ hp}}$$

5.119

5.119 Water is pumped from a tank, point (1), to the top of a water plant aerator, point (2), as shown in **Video V5.8** and Fig. P5.119 at a rate of 3.0 ft^3/s. **(a)** Determine the power that the pump adds to the water if the head loss from (1) to (2) where $V_2 = 0$ is 4 ft. **(b)** Determine the head loss from (2) to the bottom of the aerator column, point (3), if the average velocity at (3) is $V_3 = 2$ ft/s.



■ FIGURE P5.119

(a) The energy equation from (1) to (2)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_s - h_L = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

with

$p_1 = p_2 = V_1 = V_2 = 0$ gives

$$h_s = h_L + z_2 - z_1 = 4\,ft + (10+3)\,ft - 5\,ft = 12\,ft$$

Thus, the pump power is

$$\dot{W}_s = \gamma Q h_s = 62.4\,\frac{lb}{ft^3}\left(3\,\frac{ft^3}{s}\right)(12\,ft) = 2246\,\frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right)$$

$$= \underline{\underline{4.08\,hp}}$$

(b) The energy equation from (2) to (3)

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_s - h_L = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

with

$p_2 = p_3 = V_2 = h_s = 0$ gives

$$h_L = z_2 - z_3 - \frac{V_3^2}{2g} = 13\,ft - 3\,ft - \frac{\left(2\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} = 10\,ft - 0.062\,ft$$

or

$$h_L = \underline{\underline{9.94\,ft}}$$

5.120

5.120 A liquid enters a fluid machine at section (1) and leaves at sections (2) and (3) as shown in Fig. P5.120. The density of the fluid is constant at 2 slugs/ft³. All of the flow occurs in a horizontal plane and is frictionless and adiabatic. For the above-mentioned and additional conditions indicated in Fig. P5.120, determine the amount of shaft power involved.

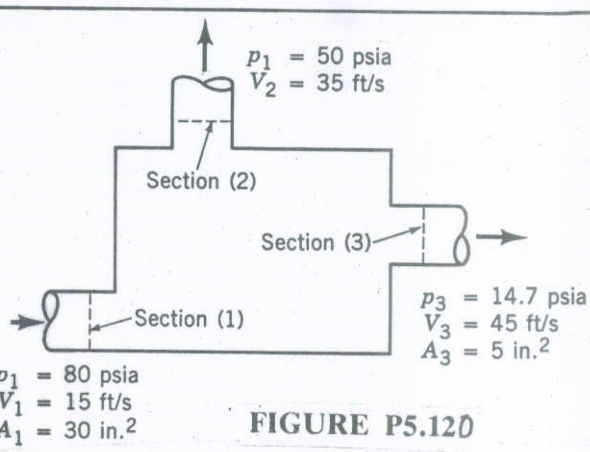


FIGURE P5.120

For the frictionless and adiabatic flow through this fluid machine Eqs. 5.64, 5.65 and 5.76 lead to

$$\dot{W}_{\substack{shaft\\ net\ in}} = \dot{m}_3\left(\frac{p_3}{\rho} + \frac{V_3^2}{2}\right) - \dot{m}_1\left(\frac{p_1}{\rho} + \frac{V_1^2}{2}\right) + \dot{m}_2\left(\frac{p_2}{\rho} + \frac{V_2^2}{2}\right) \quad (1)$$

since

$$\dot{m}_1\check{u}_1 - \dot{m}_2\check{u}_2 - \dot{m}_3\check{u}_3 = (\dot{m}_2 + \dot{m}_3)\check{u}_1 - \dot{m}_2\check{u}_2 - \dot{m}_3\check{u}_3 = \dot{m}_2(\check{u}_1 - \check{u}_2) + \dot{m}_3(\check{u}_1 - \check{u}_2) = 0$$

At section (3)

$$\dot{m}_3 = \rho A_3 V_3 = \left(2\ \frac{slugs}{ft^3}\right)\left(\frac{5\ in.^2}{144\ \frac{in.^2}{ft^2}}\right)\left(45\ \frac{ft}{s}\right) = 3.125\ \frac{slugs}{s}$$

At section (1)

$$\dot{m}_1 = \rho A_1 V_1 = \left(2\ \frac{slugs}{ft^3}\right)\left(\frac{30\ in.^2}{144\ \frac{in.^2}{ft^2}}\right)\left(15\ \frac{ft}{s}\right) = 6.25\ \frac{slugs}{s}$$

From conservation of mass

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 6.25\ \frac{slugs}{s} - 3.125\ \frac{slugs}{s} = 3.125\ \frac{slugs}{s}$$

With Eq. 1 we obtain

$$\dot{W}_{\substack{shaft\\ net\ in}} = \left\{\left(3.125\ \frac{slugs}{s}\right)\left[\frac{\left(14.7\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2\ \frac{slugs}{ft^3}\right)} + \frac{\left(45\ \frac{ft}{s}\right)^2}{2}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\right]\right.$$
$$-\left(6.25\ \frac{slugs}{s}\right)\left[\frac{\left(80\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2\ \frac{slugs}{ft^3}\right)} + \frac{\left(15\ \frac{ft}{s}\right)^2}{2}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\right]$$
$$\left.+\left(3.125\ \frac{slugs}{s}\right)\left[\frac{\left(50\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2\ \frac{slugs}{ft^3}\right)} + \frac{\left(35\ \frac{ft}{s}\right)^2}{2}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)\right]\right\}\left(\frac{1}{550\ \frac{ft\cdot lb}{s\cdot hp}}\right)$$

or

$$\dot{W}_{\substack{shaft\\ net\ in}} = \underline{\underline{-31.1\ hp}}$$, the net shaft power is out (<0)

5.121

5.121 Water is to be moved from one large reservoir to another at a higher elevation as indicated in Fig. P5.121. The loss in available energy associated with 2.5 ft³/s being pumped from sections (1) to (2) is $61\overline{V}^2/2$ where $\overline{V}$ is the average velocity of water in the 8-in.-inside diameter piping involved. Determine the amount of shaft power required.

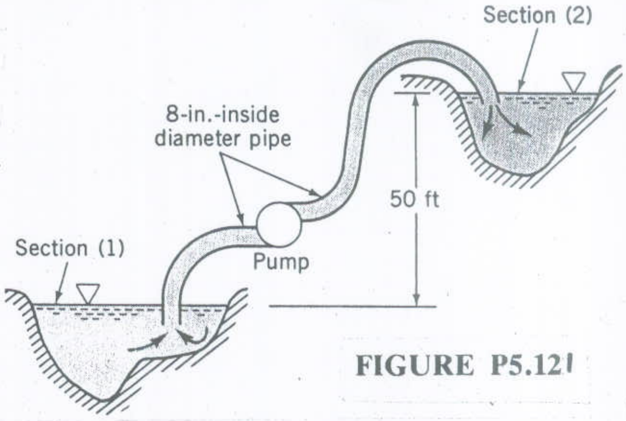


FIGURE P5.121

For the flow from section (1) to section (2) Eq. 5.82 leads to

$$\dot{W}_{\substack{shaft \\ net\ in}} = \rho Q\left[g(z_2 - z_1) + loss\right] = \rho Q\left[g(z_2-z_1) + 61\frac{\overline{V}^2}{2}\right] \quad (1)$$

From the volume flowrate we obtain

$$\overline{V} = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}} = \frac{\left(2.5\ \frac{ft^3}{s}\right)}{\frac{\pi}{4}\left(\frac{8\ in.}{12\ \frac{in.}{ft}}\right)^2} = 7.162\ \frac{ft}{s}$$

Thus, from Eq. 1

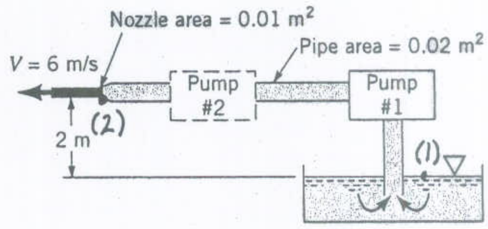
$$\dot{W}_{\substack{shaft \\ net\ in}} = \left(1.94\ \frac{slugs}{ft^3}\right)\left(2.5\ \frac{ft^3}{s}\right)\left[\left(32.2\ \frac{ft}{s^2}\right)(50\ ft) + \frac{(61)\left(7.162\ \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)\left(\frac{1}{550\ \frac{ft \cdot lb}{s \cdot hp}}\right)$$

or

$$\dot{W}_{\substack{shaft \\ net\ in}} = \underline{\underline{28\ hp}}$$

5.122

5.122 Water is to be pumped from the large tank shown in Fig. P5.122 with an exit velocity of 6 m/s. It was determined that the original pump (pump 1) that supplies 1 kW of power to the water did not produce the desired velocity. Hence, it is proposed that an additional pump (pump 2) be installed as indicated to increase the flowrate to the desired value. How much power must pump 2 add to the water? The head loss for this flow is $h_L = 250Q^2$, where h_L is in m when Q is in m^3/s.



■ FIGURE P5.122

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where

$p_1 = p_2 = 0$, $V_1 = 0$, $z_1 = 0$, $z_2 = 2m$.

Thus,

$h_s = h_L + z_2 + \frac{V_2^2}{2g}$, where $V_2 = 6 m/s$ so that $Q = A_2 V_2 = 0.01 m^2 (6 m/s)$
$= 0.06 \, m^3/s$

Note: $h_s = h_{pump1} + h_{pump2}$

Thus, with $h_L = 250Q^2 = 250(0.06)^2 = 0.90 m$ it follows that

$$h_s = 0.90m + 2m + \frac{(6 \, m/s)^2}{2(9.81 \frac{m}{s^2})} = 4.73 m$$

so that

$$\dot{W}_s = \gamma Q h_s = (9.80 \times 10^3 \frac{N}{m^3})(0.06 \frac{m^3}{s})(4.73 m) = 2.78 \times 10^3 \frac{N \cdot m}{s} = 2.78 \, kW$$

Therefore,

$\dot{W}_s = \dot{W}_{pump1} + \dot{W}_{pump2} = 2.78 \, kW$, with $\dot{W}_{pump1} = 1 kW$

Hence,

$$\dot{W}_{pump2} = 2.78 \, kW - 1 \, kW = \underline{\underline{1.78 \, kW}}$$

5.123

5.123 (See Fluids in the News article titled **"Curtain of air,"** Section 5.3.3.) The fan shown in Fig. P5.123 produces an air curtain to separate a loading dock from a cold storage room. The air curtain is a jet of air 10 ft wide, 0.5 ft thick moving with speed $V = 30$ ft/s. The loss associated with this flow is *loss* $= K_L V^2/2$, where $K_L = 5$. How much power must the fan supply to the air to produce this flow?

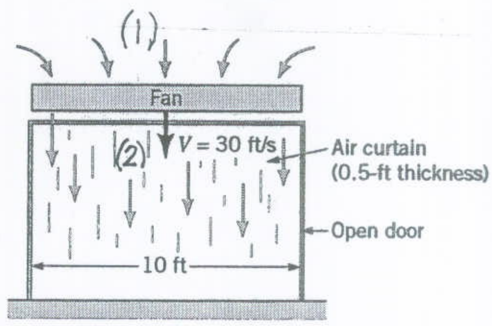


FIGURE P5.123

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g},$$

where

$$p_1 \approx p_2 \approx 0,\ z_1 \approx z_2,\ V_1 \approx 0, \text{ and } h_L = \frac{loss}{g} = 5\frac{V_2^2}{2g}$$

Thus,

$$h_s = h_L + \frac{V_2^2}{2g} = 5\frac{V_2^2}{2g} + \frac{V_2^2}{2g} = \frac{3V_2^2}{g} = \frac{3\left(30\,\frac{ft}{s}\right)^2}{\left(32.2\,\frac{ft}{s^2}\right)} = 83.9\,ft$$

Hence,

$$\dot{W}_s = \gamma Q h_s = \rho g A_2 V_2 h_s = \left(0.00238\,\frac{slugs}{ft^3}\right)\left(32.2\,\frac{ft}{s^2}\right)(10\,ft)(0.5\,ft)\left(30\,\frac{ft}{s}\right)(83.9\,ft)$$

$$= 964\,\frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right)$$

$$= \underline{\underline{1.75\,hp}}$$

5.124

5.124 If a $\frac{3}{4}$-hp motor is required by a ventilating fan to produce a 24-in. stream of air having a velocity of 40 ft/s as shown in Fig. P5.124, estimate (a) the efficiency of the fan and (b) the thrust of the supporting member on the conduit enclosing the fan.

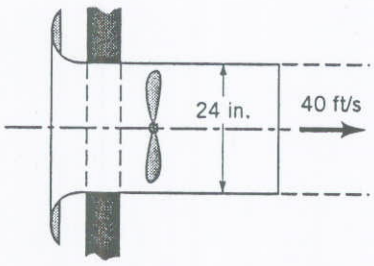


FIGURE P5.124

(a) The solution to this part of the problem is like Example 5.24. We use

$$\eta = \frac{w_{shaft} - loss}{w_{shaft}}$$

to calculate the fan efficiency.

We use the energy equation (Eq. 5.82) for flow through the control volume sketched above to calculate the loss as follows

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 + w_{\substack{shaft \\ net\ in}} - loss$$

But $\quad p_2 = p_1$ and $z_2 = z_1$; $V_1 \simeq 0$; $w_{\substack{shaft \\ net\ in}} = \dfrac{hp}{\dot{m}}$

Also $\quad \dot{m} = \rho A_2 V_2 = \dfrac{p}{RT}\dfrac{\pi d_2^2}{4} V_2$

So

$$loss = w_{\substack{shaft \\ net\ in}} - \frac{V_2^2}{2} = \frac{hp}{\frac{p}{RT}\left(\frac{\pi d_2^2}{4}\right)V_2} - \frac{V_2^2}{2}$$

$$loss = \frac{\left(\frac{3}{4}\,hp\right)\left(550\,\frac{ft \cdot lb}{s \cdot hp}\right)}{\left\{\frac{\left(14.7\,\frac{lb}{in^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)\pi\left[\frac{24\,in.}{\left(12\,\frac{in.}{ft}\right)}\right]^2\left(40\,\frac{ft}{s}\right)}{\left(53.3\,\frac{ft \cdot lb}{lbm \cdot {}^{\circ}R}\right)\left(530\,{}^{\circ}R\right)\,4}\right\}} - \frac{\left(40\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{lbm \cdot ft}{lb \cdot s^2}\right)}$$

(con't)

5.124 (con't)

$$\text{loss} = 44\ \frac{ft \cdot lb}{lbm} - 24.8\ \frac{ft \cdot lb}{lbm} = 19.2\ \frac{ft \cdot lb}{lbm}$$

So

$$\eta = \frac{44\ \frac{ft \cdot lb}{lbm} - 19.2\ \frac{ft \cdot lb}{lbm}}{44\ \frac{ft \cdot lb}{lbm}} = \underline{\underline{0.56}}$$

For

(b) We use the horizontal component of the linear momentum equation to evaluate the anchoring force required to hold the fan in place

$$F_{AX} = V_2\,\dot{m}$$

From part (a)

$$\dot{m} = \frac{P}{RT}\frac{\pi d_2^2}{4} V_2 = \frac{\left(14.7\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)\pi\left(\frac{24\ in.}{12\ \frac{in.}{ft}}\right)^2\left(40\ \frac{ft}{s}\right)}{\left(53.3\ \frac{ft \cdot lb}{lbm\ {}^\circ R}\right)(530\,{}^\circ R)\ 4}$$

$$\dot{m} = 9.41\ \frac{lbm}{s}$$

So

$$F_{AX} = \frac{\left(40\ \frac{ft}{s}\right)\left(9.41\ \frac{lbm}{s}\right)}{\left(32.2\ \frac{lbm \cdot ft}{lb \cdot s^2}\right)} = \underline{\underline{11.7\ lb}}$$

5.125

5.125 Air flows past an object in a pipe of 2-m diameter and exits as a free jet as shown in Fig. P5.125. The velocity and pressure upstream are uniform at 10 m/s and 50 N/m^2, respectively. At the pipe exit the velocity is nonuniform as indicated. The shear stress along the pipe wall is negligible. **(a)** Determine the head loss associated with a particle as it flows from the uniform velocity upstream of the object to a location in the wake at the exit plane of the pipe. **(b)** Determine the force that the air puts on the object.

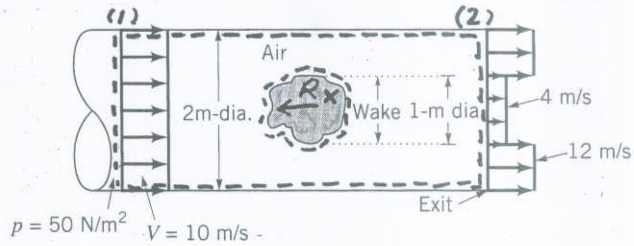


FIGURE P5.125

(a) To determine the loss suffered by a fluid particle as it flows from (1) to a location in the wake at (2) we apply the energy equation (Eq. 5.84) to that particle flow to get:

$$\cancel{\frac{p_2}{\gamma}}^{\,0} + \frac{V_2^2}{2g} + \cancel{z_2} = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} + \cancel{\frac{w_{\text{shaft net in}}}{g}}^{\,0} - h_L \qquad (1)$$

or

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

and

$$h_L = \frac{\left(50 \frac{N}{m^2}\right)}{\left(12 \frac{N}{m^3}\right)} + \frac{\left(10 \frac{m}{s}\right)^2}{2\left(9.81 \frac{m}{s^2}\right)} - \frac{\left(4 \frac{m}{s}\right)^2}{2\left(9.81 \frac{m}{s^2}\right)} = \underline{\underline{8.45\ m}}$$

To determine the head loss associated with the entire flow across the object we use the non-uniform flow energy equation (Eq. 5.89) for flow from (1) to (2) through the control volume shown in the sketch to get:

$$\cancel{\frac{p_2}{\gamma}}^{\,0} + \frac{\alpha_2 \bar{V}_2^2}{2g} + \cancel{z_2} = \frac{p_1}{\gamma} + \frac{\alpha_1 \bar{V}_1^2}{2g} + \cancel{z_1} + \cancel{\frac{w_{\text{shaft net in}}}{g}}^{\,0} - h_L \qquad (2)$$

From Eq. 5.86 we get:

$$\frac{\alpha \bar{V}^2}{2g} = \frac{\int \frac{V^2}{2g} \rho \vec{V} \cdot \hat{n}\, dA}{\dot{m}} = \frac{\int \frac{V^2}{2g} \rho \vec{V} \cdot \hat{n}\, dA}{\rho \bar{V} A}$$

Eq. (2) becomes

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{\int_{A_2} \frac{V^2}{2g} \rho \vec{V} \cdot \hat{n}\, dA}{(\rho V A)_{4\frac{m}{s}} + (\rho V A)_{12\frac{m}{s}}}$$

(con't)

5.125 (con't)

or

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{1}{2g}\left[\frac{V^3_{12\frac{m}{s}} A_{12\frac{m}{s}} + V^3_{3\frac{m}{s}} A_{3\frac{m}{s}}}{V_{4\frac{m}{s}} A_{4\frac{m}{s}} + V_{12\frac{m}{s}} A_{12\frac{m}{s}}}\right]$$

and

$$h_L = \frac{\left(50\,\frac{N}{m^2}\right)}{\left(12\,\frac{N}{m^3}\right)} + \frac{\left(10\,\frac{m}{s}\right)^2}{2\left(9.81\,\frac{m}{s^2}\right)} - \frac{1}{2\left(9.81\,\frac{m}{s^2}\right)}\left\{\frac{\left(12\,\frac{m}{s}\right)^3 \pi\left[\frac{(2m)^2-(1m)^2}{4}\right] + \left(4\,\frac{m}{s}\right)^3 \frac{\pi(1m)^2}{4}}{\left(4\,\frac{m}{s}\right)\frac{\pi(1m)^2}{4} + \left(12\,\frac{m}{s}\right)\pi\left[\frac{(2m)^2-(1m)^2}{4}\right]}\right\}$$

$$h_L = \underline{\underline{2.58\ m}}$$

(b) To determine the force that the air puts on the object, R_x, we use the horizontal component of the linear momentum equation to get:

$$-\rho V_1^2 A_1 + \rho V^2_{12\frac{m}{s}} A_{12\frac{m}{s}} + \rho V^2_{4\frac{m}{s}} A_{4\frac{m}{s}} = p_1 A_1 - R_x$$

and thus

$$R_x = p_1 A_1 + \rho V_1^2 A_1 - \rho\left(V^2_{12\frac{m}{s}} A_{12\frac{m}{s}} + V^2_{4\frac{m}{s}} A_{4\frac{m}{s}}\right)$$

So

$$R_x = \left(50\,\frac{N}{m^2}\right)\frac{\pi(2m)^2}{4} + \left(1.23\,\frac{kg}{m^3}\right)\left(10\,\frac{m}{s}\right)^2 \frac{\pi(2m)^2}{4}\left(1\,\frac{N\cdot s^2}{m\cdot kg}\right)$$
$$-1.23\,\frac{kg}{m^3}\left\{\left(12\,\frac{m}{s}\right)^2 \frac{\pi\left[(2m)^2-(1m)^2\right]}{4} + \left(4\,\frac{m}{s}\right)^2 \frac{\pi(1m)^2}{4}\right\}\left(1\,\frac{N\cdot s^2}{m\cdot kg}\right)$$

and

$$R_x = \underline{\underline{110\ N}}$$

5.126

5.126 Water flows through a 2-ft-diameter pipe arranged horizontally in a circular arc as shown in Fig. P5.126. If the pipe discharges to the atmosphere ($p = 14.7$ psia) determine the x and y components of the resultant force exerted by the water on the piping between sections (1) and (2). The steady flowrate is 3000 ft^3/min. The loss in pressure due to fluid friction between sections (1) and (2) is 60 psi.

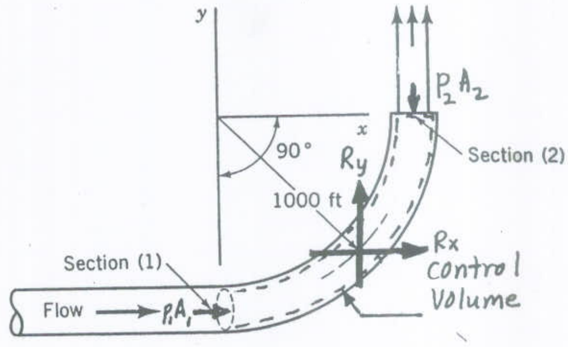


FIGURE P5.126

To determine the x and y components of the resultant force exerted by the water on the piping between section (1) and (2) we use the x and y components of the linear momentum equation (Eq 5.22). For the control volume containing the water in the pipe between section (1) and (2), Eq. 22 leads to

$$R_x = -P_1A_1 - V_1\rho Q = -P_1\frac{\pi D_1^2}{4} - V_1\rho Q \qquad (1)$$

and

$$R_y = P_2A_2 + V_2\rho Q \qquad (2)$$

The resultant force components in Eqs. 1 and 2 are exerted by the pipe on the water. The resultant force of water on pipe is equal in magnitude but opposite in direction.

To determine P_1 we use the energy equation, Eq. 5.83. Thus,

$$P_1 = p(\text{loss}) = 60\,psi = 74.7\,psia \quad (\text{we need to use absolute pressures})$$

Also

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi D_1^2}{4}} = \frac{\left(3000\,\frac{ft^3}{min}\right)}{\frac{\pi(2\,ft)^2}{4}\left(60\,\frac{s}{min}\right)} = 15.92\,\frac{ft}{s}$$

and

$$V_2 = V_1 = 15.92\,\frac{ft}{s}$$

With Eq. 1 we obtain

$$R_x = -(74.7\,psia)\,\pi\frac{(2\,ft)^2}{4}\left(144\,\frac{in.^2}{ft^2}\right) - \left(15.92\,\frac{ft}{s}\right)\frac{\left(1.94\,\frac{slugs}{ft^3}\right)\left(3000\,\frac{ft^3}{min}\right)}{\left(60\,\frac{s}{min}\right)}\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)$$

or

$$R_x = -32{,}200\,lb$$

and the x direction component of the force exerted by the water on the pipe between sections (1) and (2) is $\underline{\underline{+\,32{,}200\,lb}}$.

(con't)

5.126 (con't)

With Eq. 2 we obtain

$$R_y = (14.7\,psia)\frac{\pi(2\,ft)^2}{4}\left(144\,\frac{in^2}{ft^2}\right) + \left(15.92\,\frac{ft}{s}\right)\left(1.94\,\frac{slugs}{ft^3}\right)\left(3000\,\frac{ft^3}{min}\right)\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)\frac{1}{\left(60\,\frac{s}{min}\right)} = 8190\,lb$$

and the y-direction component of the force exerted by the water on the pipe between sections (1) and (2) is $-$ 8190 lb.

5.127

5.127 Water flows steadily down the inclined pipe as indicated in Fig. P5.127. Determine the following: **(a)** The difference in pressure $p_1 - p_2$. **(b)** The loss per unit mass between sections (1) and (2). **(c)** The net axial force exerted by the pipe wall on the flowing water between sections (1) and (2).

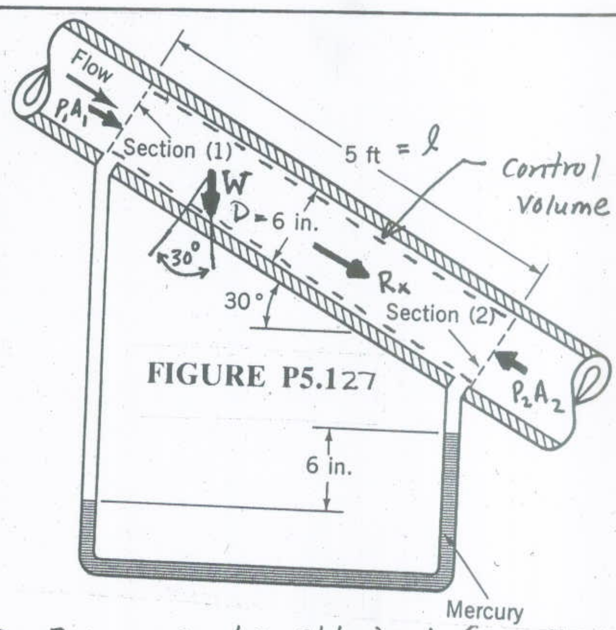


FIGURE P5.127

(a) The difference in pressure, $P_1 - P_2$, may be obtained from the manometer (see Section 2.6) with the fluid statics equation

$$P_1 - P_2 = -\gamma_{H_2O}\left[(5\,ft)\sin 30° + \frac{(6\,in.)}{\left(12\,\frac{in.}{ft}\right)}\right] + \gamma_{Hg}\frac{(6\,in.)}{\left(12\,\frac{in.}{ft}\right)}$$

or

$$P_1 - P_2 = -\left(62.4\,\frac{lb}{ft^3}\right)\left[(5\,ft)\sin 30° + (0.5\,ft)\right] + (13.6)\left(62.4\,\frac{lb}{ft^3}\right)(0.5\,ft) = 237\,\frac{lb}{ft^3}$$

and

$$P_1 - P_2 = 237\,\frac{lb}{ft^3}\,\frac{1}{\left(144\,\frac{in.^2}{ft^2}\right)} = \underline{\underline{1.65\,psi}}$$

(b) The loss per unit mass between sections (1) and (2) may be obtained with Eq. 5.79. Thus

$$loss = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) = \left(237\,\frac{lb}{ft^2}\right)\frac{1}{\left(1.94\,\frac{slugs}{ft^3}\right)} + \left(32.2\,\frac{ft}{s^2}\right)(5\,ft)(\sin 30°)\left(\frac{1\,lb}{slug \cdot \frac{ft}{s^2}}\right)$$

or

$$loss = \underline{\underline{203\,\frac{ft \cdot lb}{slug}}}$$

(c) The net axial force exerted by the pipe wall on the flowing water may be obtained by using the axial component of the linear momentum equation (Eq. 5.22). Thus for the control volume shown above

$$R_x = -\frac{\pi D^2}{4}(P_1 - P_2) - \gamma\frac{\pi D^2}{4}(\ell)\sin 30° = -\frac{\pi D^2}{4}\left[(P_1 - P_2) + \gamma \ell \sin 30°\right]$$

or

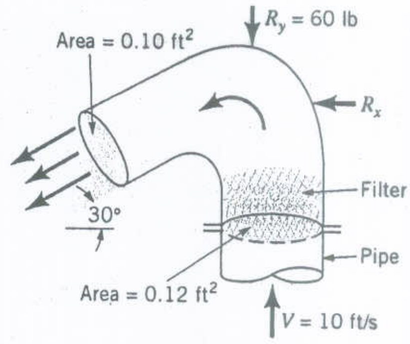
$$R_x = -\frac{\pi}{4}\left(\frac{6\,in.}{12\,\frac{in.}{ft}}\right)^2\left[237\,\frac{lb}{ft^2} + \left(62.4\,\frac{lb}{ft^3}\right)(5\,ft)\sin 30°\right]$$

and

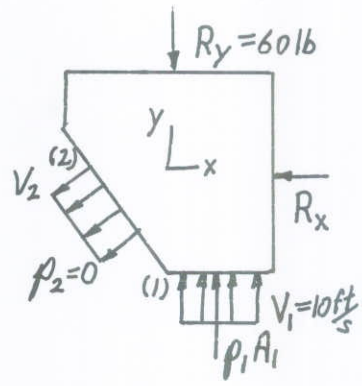
$$R_x = -77.2\,lb = \underline{\underline{77.2\,lb\ \text{opposite to flow direction.}}}$$

5.128

5.128 Water flows steadily in a pipe and exits as a free jet through an end cap that contains a filter as shown in Fig. P5.128. The flow is in a horizontal plane. The axial component, R_y, of the anchoring force needed to keep the end cap stationary is 60 lb. Determine the head loss for the flow through the end cap.



■ FIGURE P5.128



The y-component of the momentum equation,

$\int_{cs} v\, \rho \vec{V}\cdot\hat{n}\, dA = \sum F_y$, for the control volume shown is

(1) $V_1 \rho(-V_1)A_1 + (-V_2 \sin 30°)\rho V_2 A_2 = p_1 A_1 - R_y$

where $V_1 = 10\ ft/s$ and

$$V_2 = \frac{A_1}{A_2} V_1 = \left(\frac{0.12\ ft^2}{0.10\ ft^2}\right)(10\ ft/s) = 12\ ft/s$$

Thus, since $\rho A_1 V_1 = \rho A_2 V_2$, Eq. (1) gives

$$p_1 A_1 = R_y - \rho V_1^2 A_1 - \rho V_2^2 \sin 30° A_2 = R_y - \rho A_1 V_1 [V_1 + V_2 \sin 30°]$$

$$= 60\ lb - \left(1.94\ \frac{slugs}{ft^3}\right)(0.12\ ft^2)\left(10\ \frac{ft}{s}\right)\left[10\ \frac{ft}{s} + 12\ \frac{ft}{s} \sin 30°\right] = 22.8\ lb$$

Hence,

$$p_1 = 22.8\ lb/A_1 = 22.8\ lb/(0.12\ ft^2) = 190\ lb/ft^2$$

From the energy equation for this flow,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} - h_L = \frac{V_2^2}{2g}\ , \text{ or}$$

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2 - V_2^2}{2g} = \frac{190\ lb/ft^2}{62.4\ lb/ft^3} + \frac{(10\ ft/s)^2 - (12\ ft/s)^2}{2\,(32.2\ ft/s^2)} = \underline{\underline{2.36\ ft}}$$

5.129

5.129 When fluid flows through an abrupt expansion as indicated in Fig. P5.1 , the loss in available energy across the expansion, $loss_{ex}$, is often expressed as

$$loss_{ex} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2}$$

where A_1 = cross-sectional area upstream of expansion, A_2 = cross-sectional area downstream of expansion, and V_1 = velocity of flow upstream of expansion. Derive this relationship.

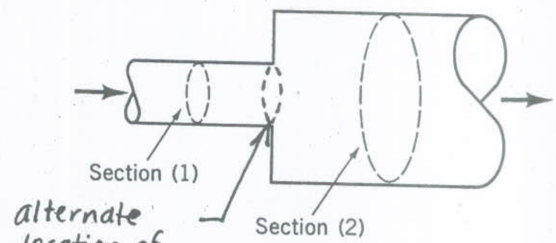


FIGURE P5.129

Applying the energy equation (Eq. 5.82) to the flow from section (1) to section (2) we obtain

$$loss_{ex} = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} \qquad (1)$$

Applying the axial direction component of the linear momentum equation (Eq. 5.22) to the fluid contained in the control volume from section (1) to section (2) we obtain

$$R_x + p_1 A_1 - p_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (2)$$

Now, if we consider section (1) as occurring at the end of the smaller diameter pipe (the beginning of the larger diameter pipe) as indicated in the sketch above, Eq. 1 still yields the expansion loss and Eq. 2 becomes

$$R_x + p_1 A_2 - p_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \qquad (3)$$

Note that with section (1) positioned at the end of the smaller diameter pipe, p_1 acts over area A_2. Also, because of the jet flow from the smaller diameter pipe into the larger diameter pipe, the value of R_x will be small enough compared to the other terms in Eq. 3 that we can drop R_x. From Eq. 3

$$\frac{p_1 - p_2}{\rho} = V_2^2 - V_1^2 \frac{A_1}{A_2} \qquad (4)$$

Combining Eqs. 1 and 4 we obtain

$$loss_{ex} = V_2^2 - V_1^2 \frac{A_1}{A_2} + \frac{V_1^2 - V_2^2}{2}$$

(con't)

5.129 (con't)

From conservation of mass (Eq. 5.13) we have

$$V_2 = V_1 \frac{A_1}{A_2} \qquad (6)$$

Combining Eqs. 5 and 6 we get

$$loss_{ex} = V_1^2\left(\frac{A_1}{A_2}\right)^2 - V_1^2\left(\frac{A_1}{A_2}\right) + \frac{V_1^2 - V_1^2\left(\frac{A_1}{A_2}\right)^2}{2}$$

or

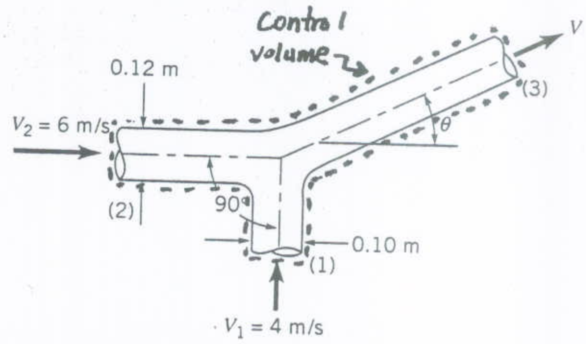
$$loss_{ex} = \frac{V_1^2}{2}\left[2\left(\frac{A_1}{A_2}\right)^2 - 2\frac{A_1}{A_2} + 1 - \left(\frac{A_1}{A_2}\right)^2\right]$$

and

$$loss_{ex} = \frac{V_1^2}{2}\left(1 - \frac{A_1}{A_2}\right)^2$$

5.130

5.130 Two water jets collide and form one homogeneous jet as shown in Fig. P5.130. **(a)** Determine the speed, V, and direction, θ, of the combined jet. **(b)** Determine the loss for a fluid particle flowing from (1) to (3), from (2) to (3). Gravity is negligible.



■ FIGURE P5.130

For the water flowing through the control volume sketched above, the x- and y-direction components of the linear momentum equation are

$$-V_2 \rho V_2 A_2 + V_3 \cos\theta \, \rho V_3 A_3 = 0 \qquad (1)$$

and

$$-V_1 \rho V_1 A_1 + V_3 \sin\theta \, \rho V_3 A_3 = 0 \qquad (2)$$

From the conservation of mass principle we get

$$-\rho V_1 A_1 - \rho V_2 A_2 + \rho V_3 A_3 = 0 \qquad (3)$$

Combining Eqs. 1 and 2 we obtain

$$\tan\theta = \frac{V_1^2 A_1}{V_2^2 A_2} = \frac{V_1 \frac{\pi d_1^2}{4}}{V_2 \frac{\pi d_2^2}{4}} = \frac{\left(4\frac{m}{s}\right)^2 \frac{\pi (0.1m)^2}{4}}{\left(6\frac{m}{s}\right)^2 \frac{\pi (0.12m)^2}{4}} = 0.3086$$

so

$$\theta = \tan^{-1} 0.3086 = \underline{\underline{17.2^\circ}}$$

Now, combining Eqs. 1 and 3 we get

$$-V_2^2 \rho A_2 + V_3 \cos\theta \left(\rho V_1 A_1 + \rho V_2 A_2\right) = 0$$

or

$$V_3 = \frac{V_2^2 A_2}{\cos\theta \left(V_1 A_1 + V_2 A_2\right)} = \frac{V_2^2 d_2^2}{\cos\theta \left(V_1 d_1^2 + V_2 d_2^2\right)}$$

Thus

$$V_3 = \frac{\left(6\frac{m}{s}\right)^2 (0.12m)^2}{(\cos 17.2^\circ)\left[\left(4\frac{m}{s}\right)(0.1m)^2 + \left(6\frac{m}{s}\right)(0.12m)^2\right]}$$

and

$$V_3 = \underline{\underline{4.29 \frac{m}{s}}}$$

(con't)

5.130 (con't)

To determine the loss of available energy associated with the flow through this control volume we obtain by applying the energy equation (Eq. 5.64)

$$-\left(\check{u}_1+\frac{V_1^2}{2}\right)\dot{m}_1-\left(\check{u}_2+\frac{V_2^2}{2}\right)\dot{m}_2+\left(\check{u}_3+\frac{V_3^2}{2}\right)\dot{m}_3=0 \qquad (4)$$

Also, the conservation of mass equation, Eq. 3, can also be written as

$$-\dot{m}_1-\dot{m}_2+\dot{m}_3=0 \qquad (5)$$

Combining Eqs. 4 and 5, we obtain

$$\dot{m}_1(\check{u}_3-\check{u}_1)+\dot{m}_2(\check{u}_3-\check{u}_2)=\dot{m}_1\left(\frac{V_1^2-V_3^2}{2}\right)+\dot{m}_2\left(\frac{V_2^2-V_3^2}{2}\right) \qquad (6)$$

The left hand side of Eq. 6 represents the rate of available energy loss in this fluid flow. Thus rate of available energy loss is

$$\text{rate of loss}=\rho V_1A_1\left(\frac{V_1^2-V_3^2}{2}\right)+\rho V_2A_2\left(\frac{V_2^2-V_3^2}{2}\right)$$

or

$$\text{rate of loss}=\frac{\rho\pi}{4}\left[d_1^2V_1\left(\frac{V_1^2-V_3^2}{2}\right)+d_2^2V_2\left(\frac{V_2^2-V_3^2}{2}\right)\right]$$

Thus

$$\text{rate of loss}=\frac{\left(999\,\frac{kg}{m^3}\right)(3.14)\left(1\,\frac{N\cdot s^2}{kg\cdot m}\right)}{4}\left\{(0.10\,m)^2\left(4\,\frac{m}{s}\right)\left[\frac{\left(4\,\frac{m}{s}\right)^2-\left(4.29\,\frac{m}{s}\right)^2}{2}\right]+(0.12\,m)^2\left(6\,\frac{m}{s}\right)\left[\frac{\left(6\,\frac{m}{s}\right)^2-\left(4.29\,\frac{m}{s}\right)^2}{2}\right]\right\}$$

and

$$\text{rate of loss}=\underline{\underline{558\,\frac{N\cdot m}{s}}}$$

5.131

5.131 Water flows vertically upward in a circular cross section pipe. At section (1), the velocity profile over the cross section area is uniform. At section (2), the velocity profile is

$$\mathbf{V} = w_c \left(\frac{R - r}{R}\right)^{1/7} \hat{\mathbf{k}}$$

where $\mathbf{V}$ = local velocity vector, w_c = centerline velocity in the axial direction, R = pipe inside radius, and, r = radius from pipe axis. Develop an expression for the loss in available energy between sections (1) and (2).

For determining loss we use the energy equation for non-uniform flows, Eq. 5.87. Thus,

$$\text{loss} = \frac{P_1 - P_2}{\rho} + \frac{\alpha_1 \overline{V}_1^2 - \alpha_2 \overline{V}_2^2}{2} + g(z_1 - z_2) \qquad (1)$$

From conservation of mass (Eq. 5.13) we have

$$\overline{V}_1 = \overline{V}_2$$

Also, with Eq. 5.86 for the kinetic energy coefficient, α, we have

$$\alpha_1 = 1.0$$

since the velocity profile at section (1) is uniform. At section (2) we solve Eq. 5.86 (see solution for problem 5.125 (c)) and obtain

$$\alpha_2 = 1.06$$

Thus, Eq. 1 yields

$$\underline{\underline{\text{loss} = \frac{P_1 - P_2}{\rho} - 0.06 \frac{\overline{V}_1^2}{2} + g(z_1 - z_2)}}$$

5.132

5.132 The velocity profile in a turbulent pipe flow may be approximated with the expression

$$\frac{u}{u_c} = \left(\frac{R-r}{R}\right)^{1/n}$$

where u = local velocity in the axial direction, u_c = centerline velocity in the axial direction, R = pipe inner radius from pipe axis, r = local radius from pipe axis, and n = constant. Determine the kinetic energy coefficient, α, for: **(a)** $n = 5$; **(b)** $n = 6$; **(c)** $n = 7$; **(d)** $n = 8$; **(e)** $n = 9$; **(f)** $n = 10$.

For the kinetic energy coefficient, α, we may use Eq. 5.86. Thus,

$$\alpha = \frac{\int_0^R \frac{u^2}{2}\rho u\, 2\pi r\, dr}{\rho \bar{u}\, \pi R^2 \frac{\bar{u}^2}{2}} = \frac{2\int_0^1 u^3\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)}{\bar{u}^3} = \frac{2u_c^3\int_0^1 \left(1-\frac{r}{R}\right)^{\frac{3}{n}}\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)}{\bar{u}^3} \quad (1)$$

For the average velocity, $\bar{u}$, we may use Eq. 5.7. Thus,

$$\bar{u} = \frac{\int_0^R \rho u\, 2\pi r\, dr}{\rho \pi R^2} = 2\int_0^1 u\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) = 2u_c\int_0^1 \left(1-\frac{r}{R}\right)^{\frac{1}{n}}\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right) \quad (2)$$

To facilitate the integrations we make the substitution

$$\beta = 1 - \frac{r}{R} \quad (3)$$

Thus,

$$d\beta = -d\left(\frac{r}{R}\right) \quad (4)$$

and Eq. 2 becomes

$$\bar{u} = -2u_c\int_1^0 \beta^{\frac{1}{n}}(1-\beta)\, d\beta = \frac{2n^2}{(n+1)(2n+1)}\, u_c \quad (5)$$

Combining Eqs. 1, 3, 4 and 5 we obtain

$$\alpha = \frac{-2\int_1^0 \beta^{\frac{3}{n}}(1-\beta)\, d\beta}{\left[\frac{2n^2}{(n+1)(2n+1)}\right]^3} = \left[\frac{2n^2}{(3+n)(3+2n)}\right]\left[\frac{(n+1)(2n+1)}{2n^2}\right]^3 \quad (6)$$

(a) For $n = 5$, Eq. 6 yields

$$\alpha = \left\{\frac{2(5)^2}{(3+5)[3+2(5)]}\right\}\left\{\frac{(5+1)[(2)(5)+1]}{2(5)^2}\right\}^3 = \underline{\underline{1.11}}$$

(b) For $n = 6$

$\alpha = \underline{\underline{1.08}}$

(c) For $n = 7$

$\alpha = \underline{\underline{1.06}}$

(d) For $n = 8$

$\alpha = \underline{\underline{1.05}}$

(e) For $n = 9$

$\alpha = \underline{\underline{1.04}}$

(f) For $n = 10$

$\alpha = \underline{\underline{1.03}}$

Note: Look at Figs. 8.17 and 8.18 for important information about these different velocity profiles.

5.133

5.133 A small fan moves air at a mass flowrate of 0.004 lbm/s. Upstream of the fan, the pipe diameter is 2.5 in., the flow is laminar, the velocity distribution is parabolic, and the kinetic energy coefficient, α_1, is equal to 2.0. Downstream of the fan, the pipe diameter is 1 in., the flow is turbulent, the velocity profile is quite flat, and the kinetic energy coefficient, α_2, is equal to 1.08. If the rise in static pressure across the fan is 0.015 psi and the fan shaft draws 0.00024 hp, compare the value of loss calculated: **(a)** assuming uniform velocity distributions; **(b)** considering actual velocity distributions.

(a) For uniform velocity distributions upstream and downstream of the fan, Eq. 5.82 is applicable. Thus,

$$\text{loss} = \frac{p_{in} - p_{out}}{\rho} + \frac{V_{in}^2 - V_{out}^2}{2} + g\underbrace{(z_{in} - z_{out})}_{0 \text{ for air}} + w_{\substack{shaft\\ net\ in}} \qquad (1)$$

We obtain the shaft work, w_{shaft} from the given shaft power, $\dot{W}_{\substack{shaft\\ net\ in}}$, with

$$w_{\substack{shaft\\ net\ in}} = \frac{\dot{W}_{\substack{shaft\\ net\ in}}}{\dot{m}} = \frac{(0.00024\ hp)\left(550\ \frac{ft \cdot lb}{s \cdot hp}\right)}{0.004\ \frac{lbm}{s}} = 33\ \frac{ft \cdot lb}{lbm}$$

For V_{in} and V_{out} we use Eq. 5.11. Thus,

$$V_{in} = \frac{\dot{m}}{\rho A_{in}} = \frac{\dot{m}}{\rho \frac{\pi D_{in}^2}{4}} = \frac{\left(0.004\ \frac{lbm}{s}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38 \times 10^{-3}\ \frac{slug}{ft^3}\right)\left(32.2\ \frac{lbm}{slug}\right)\frac{\pi (2.5\ in.)^2}{4}} = 1.53\ \frac{ft}{s}$$

and

$$V_{out} = \frac{\dot{m}}{\rho A_{out}} = \frac{\dot{m}}{\rho \frac{\pi D_{out}^2}{4}} = \frac{\left(0.004\ \frac{lbm}{s}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38 \times 10^{-3}\ \frac{slug}{ft^3}\right)\left(32.2\ \frac{lbm}{slug}\right)\frac{\pi (1\ in.)^2}{4}} = 9.57\ \frac{ft}{s}$$

Now from Eq. 1 we obtain

$$\text{loss} = \frac{(-0.015\ psi)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38 \times 10^{-3}\ \frac{slug}{ft^3}\right)\left(32.2\ \frac{lbm}{slug}\right)} + \left[\frac{\left(1.53\ \frac{ft}{s}\right)^2 - \left(9.57\ \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)\frac{1}{\left(32.2\ \frac{lbm}{slug}\right)} + 33\ \frac{ft \cdot lb}{lbm}$$

or

$$\text{loss} = \underline{\underline{3.43\ \frac{ft \cdot lb}{lbm}}}$$

(b) For non-uniform velocity distributions upstream and downstream of the fan Eq. 5.87 is applicable. Thus

$$\text{loss} = \frac{p_{in} - p_{out}}{\rho} + \frac{\alpha_{in} \bar{V}_{in}^2 - \alpha_{out} \bar{V}_{out}^2}{2} + g\underbrace{(z_{in} - z_{out})}_{0 \text{ for air}} + w_{\substack{shaft\\ net\ in}}$$

or

$$\text{loss} = -28.18\ \frac{ft \cdot lb}{lbm} + \left[\frac{(2)(1.53\ \ \ )^2}{2} - \frac{(1.08)\left(9.57\ \frac{ft}{s}\right)^2}{2}\right]\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)\frac{1}{\left(32.2\ \frac{lbm}{slug}\right)} + 33\ \frac{ft \cdot lb}{lbm}$$

and

$$\text{loss} = \underline{\underline{3.36\ \frac{ft \cdot lb}{lbm}}}$$

5.134

5.134 Air enters a radial blower with zero angular momentum. It leaves with an absolute tangential velocity, V_θ, of 200 ft/s. The rotor blade speed at rotor exit is 170 ft/s. If the stagnation pressure rise across the rotor is 0.4 psi, calculate the loss of available energy across the rotor and the rotor efficiency.

To determine the loss of available energy across the rotor we use the energy equation (Eq. 5.82) to obtain

$$loss = \frac{p_{in} - p_{out}}{\rho} + \frac{V_{in}^2 - V_{out}^2}{2} + g(\underbrace{z_{in} - z_{out}}_{0,\ \text{neglect}}) + w_{\substack{shaft \\ net\ in}}$$

or

$$loss = \frac{p_{0,in} - p_{0,out}}{\rho} + w_{\substack{shaft \\ net\ in}} \qquad (1)$$

The shaft work in, $w_{\substack{shaft \\ net\ in}}$ can be obtained with the moment-of-momentum work equation (Eq. 5.54). Thus,

$$w_{\substack{shaft \\ net\ in}} = U_{out} V_{\theta\,out} \qquad (2)$$

Combining Eqs. 1 and 2 leads to

$$loss = \frac{p_{0,in} - p_{0,out}}{\rho} + U_{out} V_{\theta\,out}$$

or

$$loss = -\frac{(0.4\ psi)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(2.38 \times 10^{-3}\ \frac{slug}{ft^3}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)} + \left(170\ \frac{ft}{s}\right)\left(200\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)$$

and

$$loss = \underline{\underline{9800\ \frac{ft \cdot lb}{slug}}} = 9800\ \frac{ft \cdot lb}{slug} \frac{1}{(32.174\ lbm/slug)} = \underline{\underline{305\ \frac{ft \cdot lb}{lbm}}}$$

As was done in Example 5.24, we calculate rotor efficiency from

$$\text{rotor efficiency} = \frac{w_{\substack{shaft \\ net\ in}} - loss}{w_{\substack{shaft \\ net\ in}}} = \frac{U_{out} V_{\theta\,out} - loss}{U_{out} V_{\theta\,o}}$$

$$\text{rotor efficiency} = \frac{\left(170\ \frac{ft}{s}\right)\left(200\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right) - 9800\ \frac{ft \cdot lb}{slug}}{\left(170\ \frac{ft}{s}\right)\left(200\ \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)} = \underline{\underline{0.71}}$$

5.135

5.135 Water enters a pump impeller radially. It leaves the impeller with a tangential component of absolute velocity of 10 m/s. The impeller exit diameter is 60 mm and the impeller speed is 1800 rpm. If the stagnation pressure rise across the impeller is 45 kPa, determine the loss of available energy across the impeller and the hydraulic efficiency of the pump.

The analysis of Example 5.27 is applicable to solving this problem. Using Eq. 6 of Example 5.27 we obtain

$$loss = U_2 V_{\theta 2} - \frac{\text{actual total pressure rise across impeller}}{\rho}$$

However,

$$U_2 = r_2 \omega = \frac{(60\ mm)}{(2)(1000\ \frac{mm}{m})} \frac{(1800\ \frac{rev}{min})(2\pi\ \frac{rad}{rev})}{(60\ \frac{s}{min})} = 5.66\ \frac{m}{s}$$

Thus

$$loss = (5.66\ \tfrac{m}{s})(10\ \tfrac{m}{s})\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right) - \left(45 \times 10^3\ \frac{N}{m^2}\right)\frac{1}{\left(999\ \frac{kg}{m^3}\right)}$$

$$loss = \underline{\underline{11.6}}\ \frac{N \cdot m}{kg}$$

From Eq. 5 of Example 5.27 we obtain

$$\eta = \frac{\frac{\text{actual total pressure rise across impeller}}{\rho}}{U_2 V_{\theta 2}}$$

or

$$\eta = \frac{\left[\frac{(45 \times 10^3\ \frac{N}{m^2})}{(999\ \frac{kg}{m^3})}\right]}{(5.66\ \frac{m}{s})(10\ \frac{m}{s})\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)} = \underline{\underline{0.796}}$$

5.136

5.136 Water enters an axial-flow turbine rotor with an absolute velocity tangential component, V_θ, of 15 ft/s. The corresponding blade velocity, U, is 50 ft/s. The water leaves the rotor blade row with no angular momentum. If the stagnation pressure drop across the turbine is 12 psi, determine the hydraulic efficiency of the turbine.

To determine the efficiency of the turbine we use

$$\eta = \frac{\text{actual work out}}{\text{actual work out} + \text{loss}} \qquad (1)$$

The actual work out, $w_{\substack{shaft \\ net\ out}}$, is obtained with the moment-of-momentum work equation (Eq. 5.54). Thus,

$$w_{\substack{shaft \\ net\ out}} = -w_{\substack{shaft \\ net\ in}} = U_{in} V_{\theta\, in} \qquad (2)$$

To determine the loss of available energy across the rotor we use the energy equation (Eq. 5.82) to obtain

$$\text{loss} = \frac{p_{in} - p_{out}}{\rho} + \frac{V_{in}^2 - V_{out}^2}{2} + \underbrace{\cancel{g(z_{in} - z_{out})}}_{\text{neglect}} + w_{\substack{shaft \\ net\ in}} \qquad (3)$$

Combining Eqs. 2 and 3 we obtain

$$\text{loss} = \frac{p_{0,in} - p_{0,out}}{\rho} - U_{in} V_{\theta\, in} \qquad (4)$$

Combining Eqs. 1, 2 and 4 we obtain

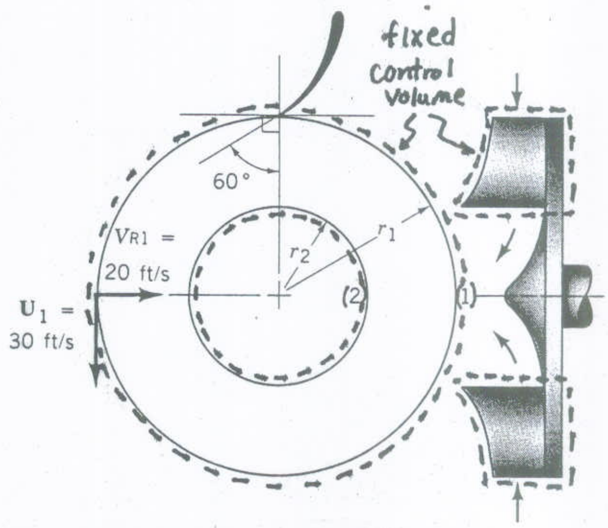
$$\eta = \frac{U_{in} V_{\theta\, in}}{U_{in} V_{\theta\, in} + \text{loss}} = \frac{U_{in} V_{\theta\, in}}{\frac{p_{0,in} - p_{0,out}}{\rho}} = \frac{\left(50 \frac{ft}{s}\right)\left(15 \frac{ft}{s}\right)\left(\frac{1\ lb}{slug \cdot \frac{ft}{s^2}}\right)}{\frac{(12\ psi)\left(144 \frac{in.^2}{ft^2}\right)}{\left(1.94 \frac{slugs}{ft^3}\right)}}$$

and

$$\eta = \underline{\underline{0.842}}$$

5.137

5.137 An inward flow radial turbine (see Fig. P5.137) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed, U_1, of 30 ft/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 20 ft/s through the rotor, and the flow leaving the rotor at section (2) is without angular momentum. If the flowing fluid is water and the stagnation pressure drop across the rotor is 16 psi, determine the loss of available energy across the rotor and the hydraulic efficiency involved.



■ FIGURE P5.137

An analysis like the one of Example 5.28 would be appropriate for solving this problem. Since a turbine is involved in this problem, $w_{\substack{shaft\\net\ in}} = -w_{\substack{shaft\\net\ out}}$ and from Eq. 1 of Example 5.28 we can conclude that

$$loss = \frac{\text{stagnation pressure drop across rotor}}{\rho} - w_{\substack{shaft\\net\ out}}$$

However from Eq. 5.54 we see that

$$w_{shaft} = w_{\substack{shaft\\net\ in}} = -U_1 V_{\theta 1} = -w_{\substack{shaft\\net\ out}}$$

and thus

$$loss = \frac{\text{stagnation pressure drop across rotor}}{\rho} - U_1 V_{\theta 1} \qquad (1)$$

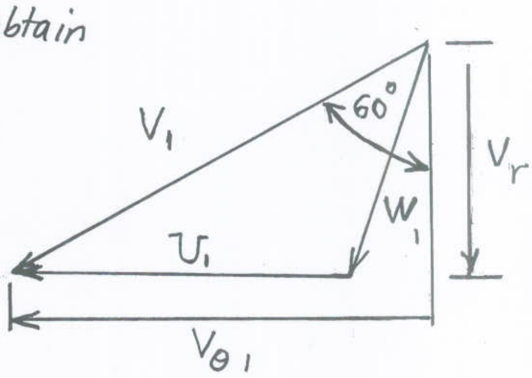
To determine the value of $V_{\theta 1}$ we examine the velocity triangle for the flow entering the rotor that is sketched below.

From the velocity triangle we obtain

$$V_{\theta 1} = V_{r1} \tan 60^\circ$$

or

$$V_{\theta 1} = \left(20 \frac{ft}{s}\right) \tan 60^\circ = 34.64 \frac{ft}{s}$$



(Con't)

5.137 (con't)

From Eq. 1 we obtain

$$loss = \frac{\left(16\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1.94\,\frac{slugs}{ft^3}\right)} - \left(30\,\frac{ft}{s}\right)\left(34.64\,\frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)$$

$$loss = \underline{\underline{148}}\ \frac{ft\cdot lb}{slug}$$

From Eq. 5.82, we can conclude that

$$w_{\substack{shaft\\net\ out}} + loss = \frac{\text{stagnation pressure drop across the rotor}}{\rho}$$

or in other words, the stagnation pressure drop across the rotor results in shaft work and loss of available energy.

Thus a meaningful efficiency is

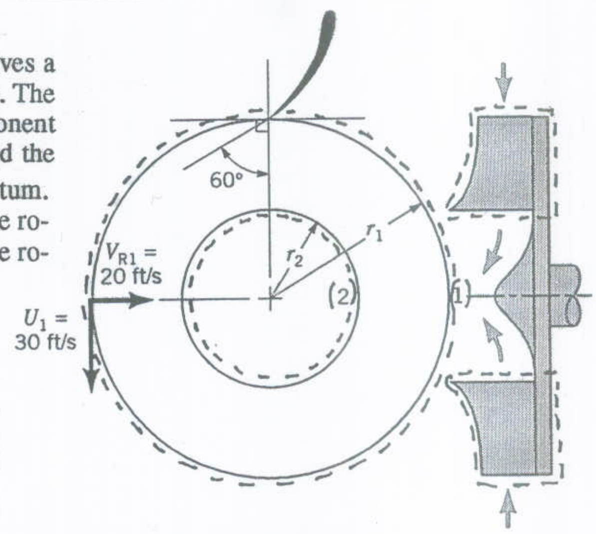
$$\eta = \frac{w_{\substack{shaft\\net\ out}}}{\left(\frac{\text{stagnation pressure drop across the rotor}}{\rho}\right)}$$

or

$$\eta = \frac{\left(30\,\frac{ft}{s}\right)\left(34.64\,\frac{ft}{s}\right)\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right)}{\frac{\left(16\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1.94\,\frac{slugs}{ft^3}\right)}} = \underline{\underline{0.875}}$$

5.138

5.138 An inward flow radial turbine (see Fig. P5.137) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed of 30 ft/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 20 ft/s through the rotor, and the flow leaving the rotor at section (2) is without angular momentum. If the flowing fluid is air and the static pressure drop across the rotor is 0.01 psi, determine the loss of available energy across the rotor and the rotor aerodynamic efficiency.



■ FIGURE P5.137

To determine the loss of available energy across the rotor we use the energy equation (Eq. 5.82). Thus,

$$\text{loss} = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} + \underbrace{g(z_1 - z_2)}_{\text{neglect}} + w_{\substack{\text{shaft}\\ \text{net in}}} \qquad (1)$$

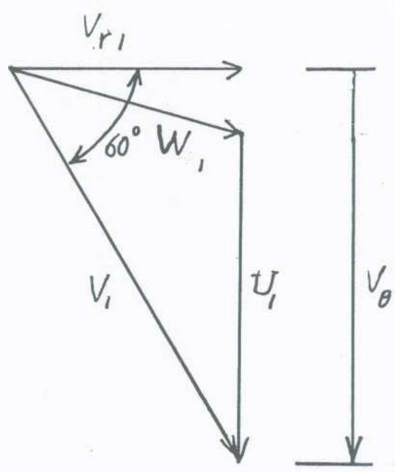
The shaft work, $w_{\substack{\text{shaft}\\ \text{net in}}}$, is obtained with the moment-of-momentum work equation (Eq. 5.54). Thus,

$$w_{\substack{\text{shaft}\\ \text{net in}}} = -U_1 V_{\theta 1} = -w_{\substack{\text{shaft}\\ \text{net out}}} \qquad (2)$$

and combining Eqs. 1 and 2 yields

$$\text{loss} = \frac{P_1 - P_2}{\rho} + \frac{V_1^2 - V_2^2}{2} - U_1 V_{\theta 1} \qquad (3)$$

To determine V_1 and $V_{\theta 1}$, we construct the velocity triangle sketched below.



(con't)

5.138 (con't)

With the velocity triangle we conclude that

$$V_1 = \frac{(20\ \frac{ft}{s})}{\cos 60°} = 40\ \frac{ft}{s}$$

and

$$V_{\theta 1} = V_1 \sin 60° = (40\ \frac{ft}{s}) \sin 60° = 34.64\ \frac{ft}{s}$$

Since the flow leaving the rotor is radial, then

$$V_2 = V_{r2} = 20\ \frac{ft}{s}$$

From Eq. 3 we obtain

$$\text{loss} = \frac{(0.01\ \frac{lb}{in.^2})(144\ \frac{in.^2}{ft^2})}{(2.38\times10^{-3}\ \frac{slug}{ft^3})} + \frac{\left[(40\ \frac{ft}{s})^2 - (20\ \frac{ft}{s})^2\right]}{2}\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right) - (30\ \frac{ft}{s})(34.64\ \frac{ft}{s})\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)$$

or

$$\text{loss} = \underline{\underline{166}}\ \frac{ft\cdot lb}{slug} = \left(166\ \frac{ft\cdot lb}{slug}\right)\frac{1}{(32.174\ \frac{lbm}{slug})} = \underline{\underline{5.16}}\ \frac{ft\cdot lb}{lbm}$$

The efficiency may be obtained with

$$\eta = \frac{\text{actual work out}}{\text{actual work out} + \text{loss}} = \frac{U_1 V_{\theta 1}}{U_1 V_{\theta 1} + \text{loss}}$$

or

$$\eta = \frac{(30\ \frac{ft}{s})(34.64\ \frac{ft}{s})\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}{(30\ \frac{ft}{s})(34.64\ \frac{ft}{s})\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right) + 166\ \frac{ft\cdot lb}{slug}} = \underline{\underline{0.86}}$$

5.140

5.140 Force from a Jet of Air Deflected by a Flat Plate

Objective: A jet of a fluid striking a flat plate as shown in Fig. P5.126 exerts a force on the plate. It is the equal and opposite force of the plate on the fluid that causes the fluid momentum change that accompanies such a flow. The purpose of this experiment is to compare the theoretical force on the plate with the experimentally measured force.

Equipment: Air source with an adjustable flowrate and a flow meter; nozzle to produce a uniform air jet; balance beam with an attached flat plate; weights; barometer; thermometer.

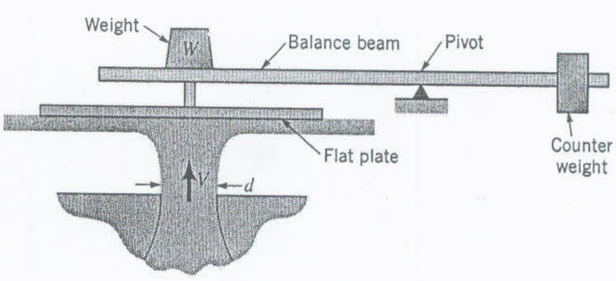
Experimental Procedure: Adjust the counter weight so that the beam is level when there is no mass, m, on the beam and no flow through the nozzle. Measure the diameter, d, of the nozzle outlet. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law. Place a known mass, m, on the flat plate and adjust the fan speed control to produce the necessary flowrate, Q, to make the balance beam level again. The flowrate is related to the flow meter manometer reading, h, by the equation $Q = 0.358\, h^{1/2}$, where Q is in ft^3/s and h is in inches of water. Repeat the measurements for various masses on the plate.

Calculations: For each flowrate, Q, calculate the weight, $W = mg$, needed to balance the beam and use the continuity equation, $Q = VA$, to determine the velocity, V, at the nozzle exit. Use the momentum equation for this problem, $W = \rho V^2 A$, to determine the theoretical relationship between velocity and weight.

Graph: Plot the experimentally measured force on the plate, W, as ordinates and air speed, V, as abscissas.

Results: On the same graph, plot the theoretical force as a function of air speed.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P5.140

(con't)

5.140 (con't)

Solution for Problem 5.140: Force from a Jet of Air Deflected by a Flat Plate

d, in.	H_{atm}, in. Hg	T, deg F	Q = 0.358 h^0.5, with Q in cfs and h in inches of water
1.174	29.25	70	

		Experimental				Theoretical
m, kg	h, in.	Q, ft^3/s	V, ft/s	m, slug	W, lb	W, lb
0.010	0.54	0.263	35.0	0.00069	0.022	0.021
0.020	1.08	0.372	49.5	0.00137	0.044	0.042
0.030	1.52	0.441	58.7	0.00206	0.066	0.059
0.040	2.18	0.529	70.3	0.00274	0.088	0.084
0.050	2.72	0.590	78.5	0.00343	0.110	0.105
0.060	3.25	0.645	85.8	0.00411	0.132	0.126
0.070	3.81	0.699	92.9	0.00480	0.154	0.147
0.080	4.32	0.744	98.9	0.00548	0.177	0.167
0.090	4.92	0.794	105.6	0.00617	0.199	0.190
0.100	5.46	0.837	111.2	0.00685	0.221	0.211
0.150	8.13	1.021	135.7	0.01028	0.331	0.315
0.200	10.85	1.179	156.8	0.01370	0.441	0.420
0.250	13.72	1.326	176.3	0.01713	0.552	0.531

Experimental:

V = Q/A where

$A = \pi d^2/4 = \pi$*(1.174/12 ft)^2/4 = 7.52E-3 ft^2

W = mg

Theoretical:

$W = \rho V^2 A$ where

$\rho = p_{atm}/RT$ with

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.25/12 ft) = 2065 lb/ft^2

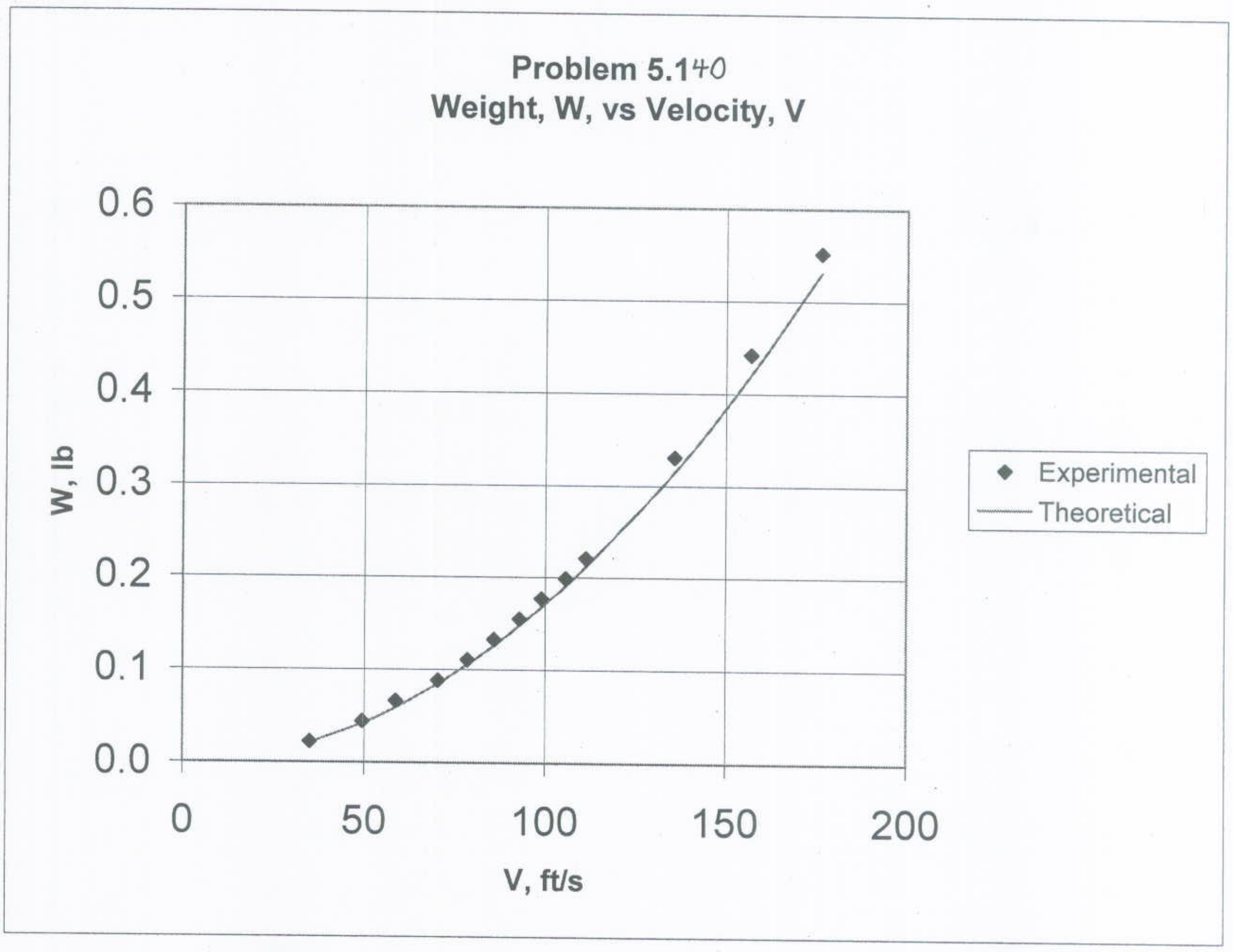
R = 1716 ft lb/slug deg R

T = 70 + 460 = 530 deg R

Thus, ρ = 0.00227 slug/ft^3

(con't)

5.140 (con't)


Problem 5.140
Weight, W, vs Velocity, V
W, lb
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
50
100
150
200
V, ft/s
Experimental
Theoretical

5.141

5.141 Pressure Distribution on a Flat Plate Due to the Deflection of an Air Jet

Objective: In order to deflect a jet of air as shown in Fig. P5.127, the flat plate must push against the air with a sufficient force to change the momentum of the air. This causes an increase in pressure on the plate. The purpose of this experiment is to measure the pressure distribution on the plate and to compare the resultant pressure force to that needed, according to the momentum equation, to deflect the air.

Equipment: Air supply with a flow meter; nozzle to produce a uniform jet of air; circular flat plate with static pressure taps at various radial locations; manometer; barometer; thermometer.

Experimental Procedure: Measure the diameters of the plate, D, and the nozzle exit, d, and the radial locations, r, of the various static pressure taps on the plate. Carefully center the plate over the nozzle exit and adjust the air flowrate, Q, to the desired constant value. Record the static pressure tap manometer readings, h, at various radial locations, r, from the center of the plate. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law.

Calculations: Use the manometer readings, h, to determine the pressure on the plate as a function of location, r. That is, calculate $p = \gamma_m h$, where γ_m is the specific weight of the manometer fluid.

Graph: Plot pressure, p, as ordinates and radial location, r, as abscissas.

Results: Use the experimentally determined pressure distribution to determine the net pressure force, F, that the air jet puts on the plate. That is, numerically or graphically integrate the pressure data to obtain a value for $F = \int p\, dA = \int p\,(2\pi r\, dr)$, where the limits of the integration are over the entire plate, from $r = 0$ to $r = D/2$. Compare this force obtained from the pressure measurements to that obtained from the momentum equation for this flow, $F = \rho V^2 A$, where V and A are the velocity and area of the jet, respectively.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

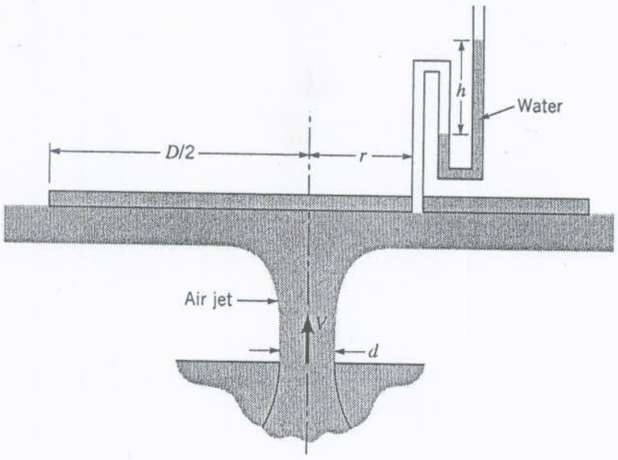


FIGURE P5.141

(con't)

5.141 (con't)

Solution for Problem 5.141: Pressure Distribution on a Flat Plate due to the Deflection of an Air Jet

D, in.	d, in.	H_{atm}, in. Hg	T, deg F	Q. ft^3/s
8.0	1.174	29.25	77	1.41

r, in.	h, in.	p, lb/ft^2	p, lb/in.^2	p*r, lb/in.	i	pr_i+pr_{i+1}	$r_{i+1} - r_i$
0.00	6.62	34.42	0.2391	0.0000	1	0.0834	0.39
0.39	5.92	30.78	0.2138	0.0834	2	0.1701	0.40
0.79	3.04	15.81	0.1098	0.0867	3	0.1114	0.45
1.24	0.55	2.86	0.0199	0.0246	4	0.0355	0.35
1.59	0.19	0.99	0.0069	0.0109	5	0.0205	0.45
2.04	0.13	0.68	0.0047	0.0096	6	0.0174	0.37
2.41	0.09	0.47	0.0033	0.0078	7	0.0130	0.44
2.85	0.05	0.26	0.0018	0.0051	8	0.0086	0.38
3.23	0.03	0.16	0.0011	0.0035	9	0.0035	0.44
3.67	0.00	0.00	0.0000	0.0000			

$p = \gamma_{H2O}*h$

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg}*H_{atm}$ = 847 lb/ft^3*(29.25/12 ft) = 2065 lb/ft^2
R = 1716 ft lb/slug deg R
T = 77 + 460 = 537 deg R

Thus, ρ = 0.00224 slug/ft^3

Using the trapezoidal rule for integration

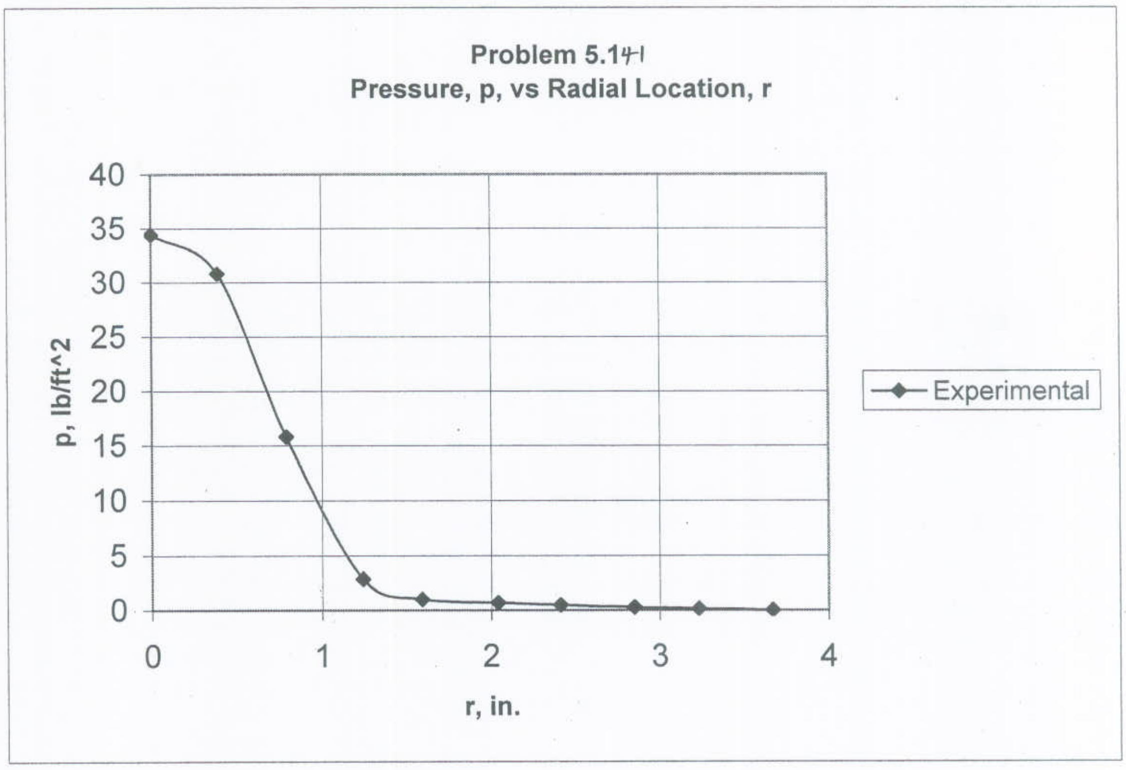
$F_{exp} = 2\pi*0.5*\Sigma_{1\ to\ 9}[(pr_i + pr_{i+1})*(r_{i+1} - r_i)] = 2\pi*0.5*0.189 = \underline{0.594\ lb}$

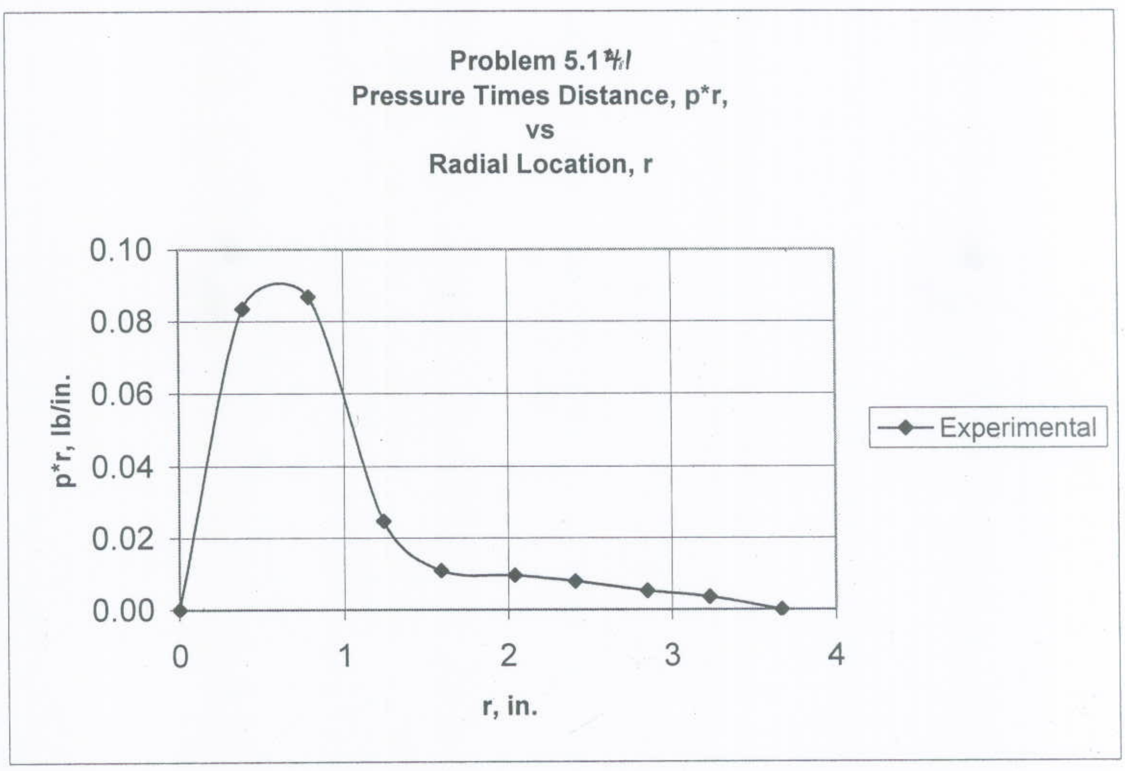
Theory:

$F = \rho V^2 A$ where
$A = \pi d^2/4 = \pi*(1.174/12\ ft)^2/4 = 0.00752\ ft^2$
V = Q/A = (1.41 ft^3/s)/(0.00752 ft^2) = 188 ft/s
Thus,
F_{th} = 0.00224 slug/ft^3*(188 ft/s)^2*(0.00752 ft^2) = <u>0.595 lb</u>

(con't)

5.141 (con't)


Problem 5.141
Pressure, p, vs Radial Location, r
40
35
30
25
20
15
10
5
0
p, lb/ft^2
0
1
2
3
4
r, in.
Experimental


Problem 5.141
Pressure Times Distance, p*r,
vs
Radial Location, r
0.10
0.08
0.06
0.04
0.02
0.00
p*r, lb/in.
0
1
2
3
4
r, in.
Experimental

Master Typing Sheet
10% Reduction
8 1/2 x 11 trim size

5.142

5.142 Force from a Jet of Water Deflected by a Vane

Objective: A jet of a fluid striking a vane as shown in Fig. P5.128 exerts a force on the vane. It is the equal and opposite force of the vane on the fluid that causes the fluid momentum change that accompanies such a flow. The purpose of this experiment is to compare the theoretical force on the vane with the experimentally measured force.

Equipment: Water source; nozzle to produce a uniform jet of water; vanes to deflect the water jet; weigh tank to collect a known amount of water in a measured time period; stop watch; force balance system.

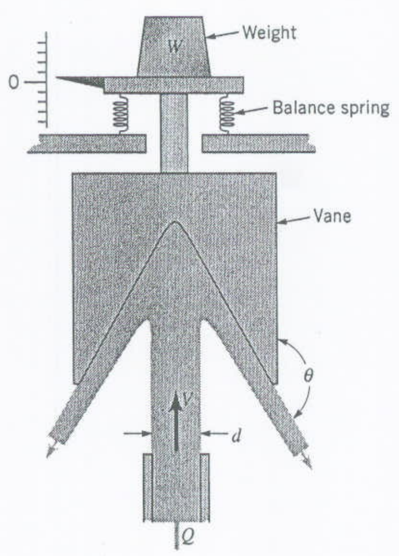
Experimental Procedure: Measure the outlet diameter, d, of the nozzle. Fasten the $\theta = 90$ degree vane to its support and adjust the balance spring to give a zero reading when there is no weight, W, on the platform and no flow through the nozzle. Place a known mass, m, on the platform and adjust the control valve on the pump to provide the necessary flowrate from the nozzle to return the platform to a zero reading. Determine the flowrate by collecting a known weight of water, W_{water}, in the weigh tank during a measured amount of time, t. Repeat the measurements for various masses, m. Repeat the experiment using a $\theta = 180$ degree vane.

Calculations: For each data set, determine the weight, $W = mg$, on the platform and the volume flowrate, $Q = W_{water}/(\gamma t)$, through the nozzle. Determine the exit velocity from the nozzle, V, by using $Q = VA$. Use the momentum equation to determine the theoretical weight that can be supported by the water jet as a function of V and θ.

Graph: For each vane, plot the experimentally determined weight, W, as ordinates and the water velocity, V, as abscissas.

Results: On the same graph plot the theoretical weight as a function of velocity for each vane.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P5.142

(con't)

5.142 (con't)

Solution for Problem 5.142: Force from a Jet of Water Deflected by a Vane

d, in.
0.40

				Experimental			Theoretical
m, kg	W_{water}, lb	t, s	m, slug	W, lb	Q, ft^3/s	V, ft/s	W, lb
Data for θ = 90 deg:							
0.02	7.71	29.8	0.0014	0.044	0.0041	4.7	0.038
0.07	8.66	18.2	0.0048	0.154	0.0076	8.7	0.129
0.17	8.87	10.1	0.0116	0.375	0.0141	16.1	0.440
0.12	8.92	12.6	0.0082	0.265	0.0113	13.0	0.286
0.22	9.66	10.6	0.0151	0.485	0.0146	16.7	0.474
Data for θ = 180 deg:							
0.05	6.81	24.5	0.0034	0.110	0.0045	5.1	0.088
0.10	9.02	20.8	0.0069	0.221	0.0069	8.0	0.215
0.20	8.84	13.2	0.0137	0.441	0.0107	12.3	0.512
0.25	7.88	10.9	0.0171	0.552	0.0116	13.3	0.597
0.30	8.86	11.1	0.0206	0.662	0.0128	14.7	0.727
0.35	7.97	9.5	0.0240	0.772	0.0134	15.4	0.803
0.40	6.37	7.6	0.0274	0.883	0.0134	15.4	0.802

$W = mg$
$Q = W_{water}/(\gamma * t)$
$V = Q/A$ where

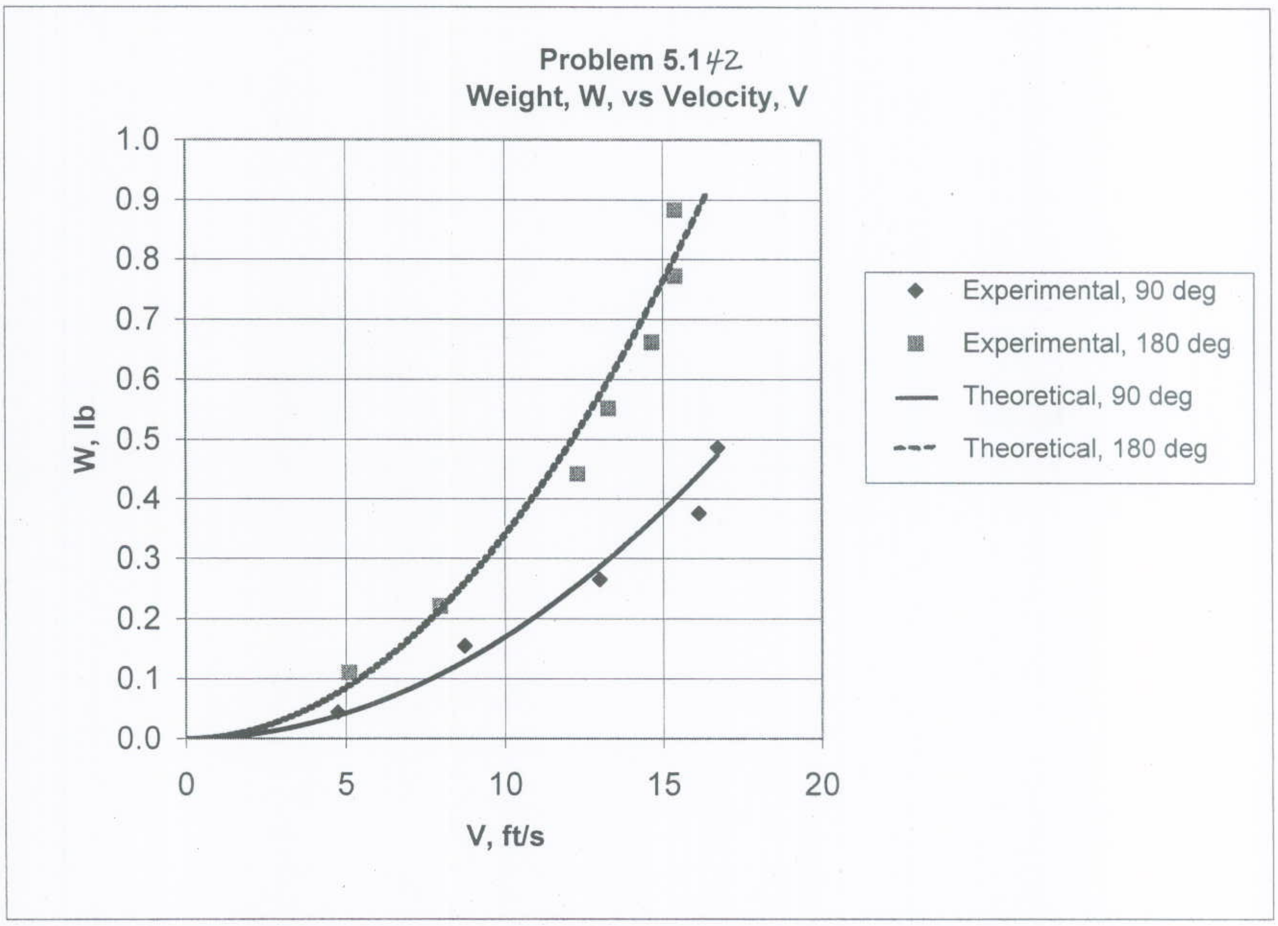
$A = \pi d^2/4 = \pi*(0.40/12 \text{ ft})^2/4 = 0.000873 \text{ ft}^2$

Theoretical:

$W = \rho V^2 A$ for $\theta = 90$ deg
and
$W = 2\rho V^2 A$ for $\theta = 180$ deg

(con't)

5.142 (Con't)


Problem 5.142
Weight, W, vs Velocity, V
W, lb
V, ft/s
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
5
10
15
20
Experimental, 90 deg
Experimental, 180 deg
Theoretical, 90 deg
Theoretical, 180 deg

5.143

5.143 Force of a Flowing Fluid on a Pipe Elbow

Objective: When a fluid flows through an elbow in a pipe system as shown in Fig. P5.129, the fluid's momentum is changed as the fluid changes direction. Thus, the elbow must put a force on the fluid. Similarly, there must be an external force on the elbow to keep it in place. The purpose of this experiment is to compare the theoretical vertical component of force needed to hold an elbow in place with the experimentally measured force.

Equipment: Variable speed fan; Pitot static tube; air speed indicator; air duct and 90-degree elbow; scale; barometer; thermometer.

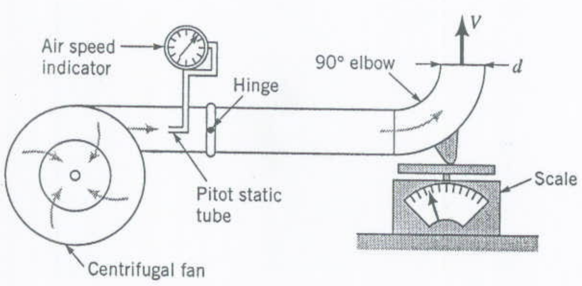
Experimental Procedure: Measure the diameter, d, of the air duct and adjust the scale to read zero when the elbow rests on it and there is no flow through it. Note that the duct is connected to the fan outlet by a pivot mechanism that is essentially friction free. Record the barometer reading, H_{atm}, in inches of mercury and the air temperature, T, so that the air density can be calculated by use of the perfect gas law. Adjust the variable speed fan to give the desired flowrate. Record the velocity, V, in the pipe as given by the Pitot static tube which is connected to an air speed indicator that reads directly in feet per minute. Record the force, F, indicated on the scale at this air speed. Repeat the measurements for various air speeds. Obtain data for two types of elbows: (1) a long radius elbow and (2) a mitered elbow (see Figs. 8.30 and 8.31).

Calculations: For a given air speed, V, use the momentum equation to calculate the theoretical vertical force, $F = \rho V^2 A$, needed to hold the elbow stationary.

Graph: Plot the experimentally measured force, F, as ordinates and the air speed, V, as abscissas.

Results: On the same graph, plot the theoretical force as a function of air speed.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.



■ FIGURE P5.1[illegible]

(con't)

5.143 (con't)

Solution for Problem 5.143: Force of a Flowing Fluid on a Pipe Elbow

d, in.	H_{atm}, in. Hg	T, deg F
8.0	29.07	73

Experiment

V, ft/min	F, lb	V, ft/s
Long Radius Elbow Data		
0	0	0.0
1200	0.38	20.0
1420	0.51	23.7
1800	0.79	30.0
2160	1.05	36.0
2440	1.38	40.7
2700	1.65	45.0
2900	1.91	48.3
3100	2.19	51.7
3520	2.83	58.7
3750	3.12	62.5
3950	3.38	65.8
Mitered Elbow Data		
1400	0.30	23.3
1780	0.55	29.7
2000	0.74	33.3
2300	1.12	38.3
2630	1.44	43.8
2900	1.72	48.3
3150	2.06	52.5
3360	2.38	56.0
3550	2.62	59.2
3620	2.74	60.3

Theory

V, ft/s	F_{th}, lb
0	0
5.0	0.02
10.0	0.08
15.0	0.18
20.0	0.31
25.0	0.49
30.0	0.70
35.0	0.96
40.0	1.25
45.0	1.58
50.0	1.95
55.0	2.36
60.0	2.81
65.0	3.30

$\rho = p_{atm}/RT$ where

$p_{atm} = \gamma_{Hg} * H_{atm}$ = 847 lb/ft^3*(29.07/12ft) = 2052 lb/ft^2

R = 1716 ft lb/slug deg R

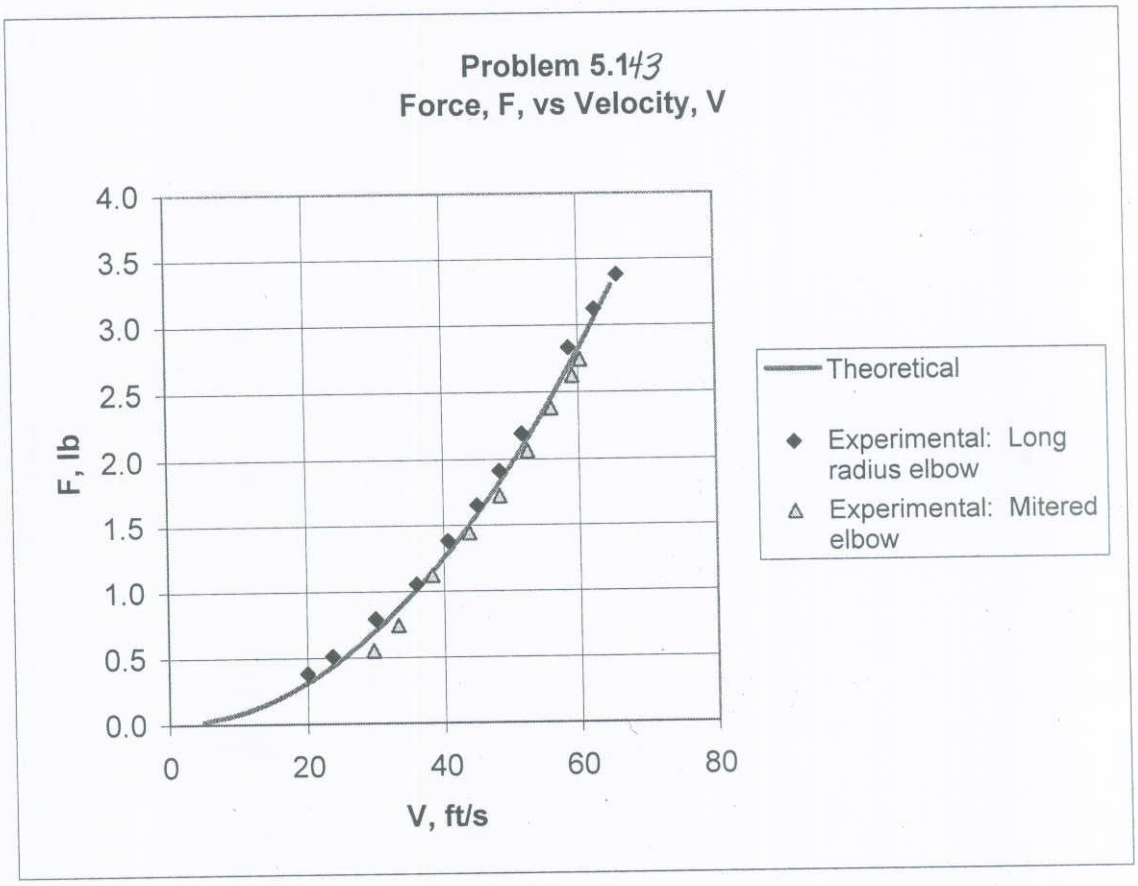
T = 73 + 460 = 533 deg R

Thus, ρ = 0.00224 slug/ft^3

A = πd^2/4 = π*(8/12)^2/4 = 0.349 ft^2

(con't)

5.143 (con't)


Problem 5.143
Force, F, vs Velocity, V
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
F, lb
0
20
40
60
80
V, ft/s
Theoretical
Experimental: Long radius elbow
Experimental: Mitered elbow