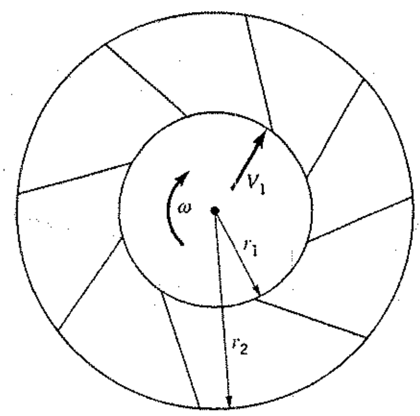
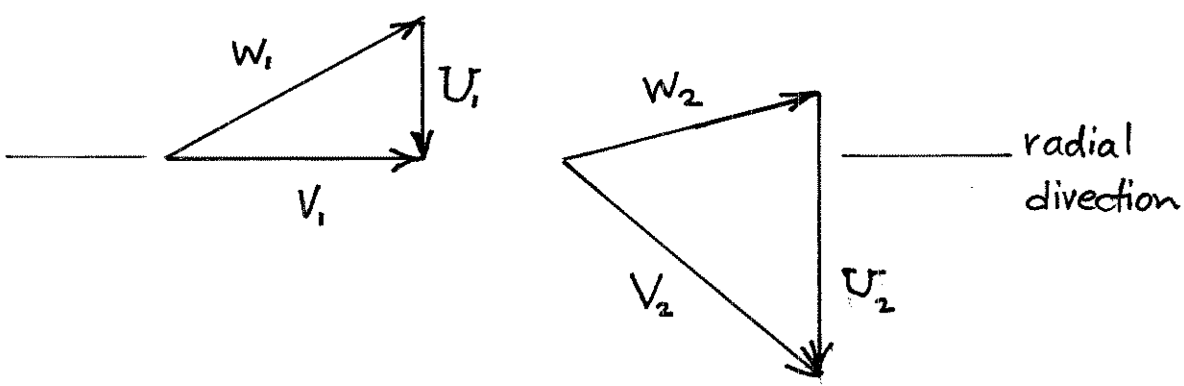
12.4

12.4 The rotor shown in Fig. P12.4 rotates clockwise. Assume that the fluid enters in the radial direction and the relative velocity is tangent to the blades and remains constant across the entire rotor. Is the device a pump or a turbine? Explain.



■ FIGURE P12.4

$W_1 = W_2$ according to the problem statement and $U_2 > U_1$ since $r_2 > r_1$. Thus a reasonable set of velocity triangles for this situation looks like



By comparing the velocity triangles at the rotor inlet (1) and exit (2) we see that the absolute velocity vector, V, has been turned in the direction of blade motion and work has been done on the fluid. This is a pump.

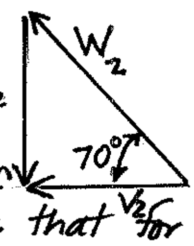
12.10

12.10 Water flows through a rotating sprinkler arm as shown in Fig. P12.10 and **Video V12.2**. Estimate the minimum water pressure necessary for an angular velocity of 150 rpm. Is this a turbine or a pump?

To estimate the minimum water pressure we consider the flow through the sprinkler and into the atmosphere to be without any loss of available energy.

So, using Bernoulli's equation we get

$$P_{min} = P_2 + \rho\frac{V_2^2}{2}$$



where $P_2 = P_{atm}$

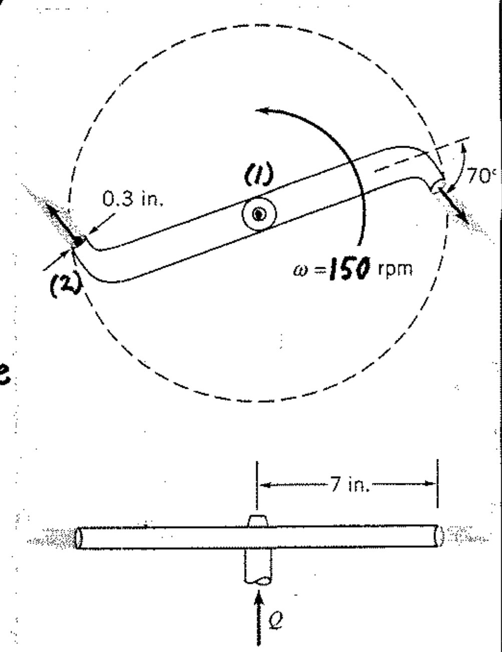
To determine V_2 we recognize that for the minimum pressure condition, the torque resisting sprinkler rotation is zero.

So

$$T = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1}) = 0$$

since $V_{\theta 1} = 0$

then $V_{\theta 2} = 0$



■ FIGURE P12.10

From the exit (2) velocity triangle,

$$V_2 = W_2 \cos 70° \quad \text{and} \quad W_2 \sin 70° = U_2 = r_2\omega$$

So

$$V_2 = r_2\omega\frac{\cos 70°}{\sin 70°} = \frac{(7\text{ in.})\left(150\frac{\text{rev}}{\text{min}}\right)\left(2\pi\frac{\text{rad}}{\text{rev}}\right)}{\left(12\frac{\text{in.}}{\text{ft}}\right)\left(60\frac{\text{s}}{\text{min}}\right)}\frac{\cos 70°}{\sin 70°} = 3.34\frac{\text{ft}}{\text{s}}$$

Then

$$P_{min} = P_{atm} + \frac{1}{2}\frac{\left(62.4\frac{\text{lbm}}{\text{ft}^3}\right)\left(3.34\frac{\text{ft}}{\text{s}}\right)^2}{\left(32.2\frac{\text{lbm}}{\text{lb}}\frac{\text{ft}}{\text{s}^2}\right)} = P_{atm} + 10.8\frac{\text{lb}}{\text{ft}^2}$$

So $\underline{\underline{P_{min} = 10.8\frac{\text{lb}}{\text{ft}^2}}}$ above P_{atm}

The actual pressure needed for sprinkler rotation is larger because of fluid flow losses and finite torque resisting rotation.

This is a turbine, the sprinker moves in the same direction as the fluid force on it.

12.11

12.11 Water is supplied to a dishwasher through the manifold shown in Fig. P12.11. Determine the rotational speed of the manifold if bearing friction and air resistance are neglected. The total flowrate of 2.3 gpm is divided evenly among the six outlets, each of which produces a 5/16-in.-diameter stream.

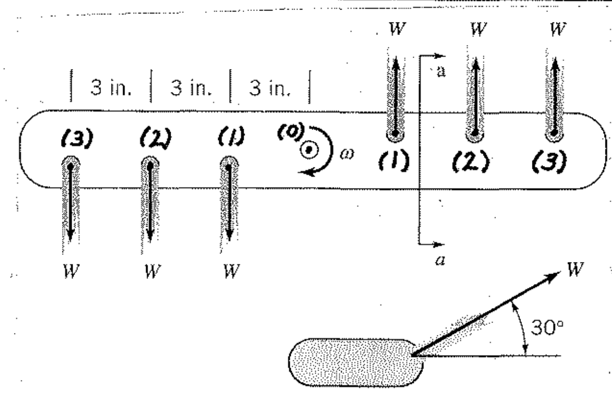


FIGURE P12.11

With points (0), (1), (2), and (3) located as in the diagram above,

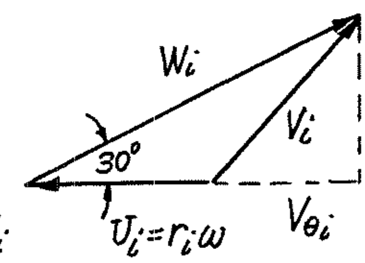
$T = \text{"}\dot{m}(r_{out} V_{\theta out} - r_{in} V_{\theta in})\text{"}$ where $V_{\theta in} = 0$. Thus,

$T = 2\dot{m}_1 r_1 V_{\theta 1} + 2\dot{m}_2 r_2 V_{\theta 2} + 2\dot{m}_3 r_3 V_{\theta 3} = 0$ since there is no friction.

But $\dot{m}_1 = \dot{m}_2 = \dot{m}_3$ so that the above becomes

$$r_1 V_{\theta 1} + r_2 V_{\theta 2} + r_3 V_{\theta 3} = 0 \qquad (1)$$

But $U_i + V_{\theta i} = W_i \cos 30°$, $i = 1, 2, 3 \qquad (2)$



where

$$Q = 6 A_i W_i = 6\left[\frac{\pi}{4}\left(\frac{\left(\frac{5}{16}\right)}{12}\,ft\right)^2\right] W_i = 0.00320\,W_i$$

with

$$Q = \left(2.3\,\frac{gal}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(231\,\frac{in.^3}{gal}\right)\left(\frac{1\,ft^3}{1728\,in.^3}\right) = 0.00512\,\frac{ft^3}{s}$$

Thus, $W_i = 1.60\ \frac{ft}{s}$ so that from Eq. (2) $V_{\theta i} = W_i \cos 30° - U_i$

or $V_{\theta i} = (1.60\ \frac{ft}{s})\cos 30° - r_i \omega$. With $r_1 = \frac{3}{12}\,ft$, $r_2 = \frac{6}{12}\,ft$, and $r_3 = \frac{9}{12}\,ft$

Eq. (1) becomes

$$\frac{3}{12}\left(1.386 - \frac{3}{12}\omega\right) + \frac{6}{12}\left(1.386 - \frac{6}{12}\omega\right) + \frac{9}{12}\left(1.386 - \frac{9}{12}\omega\right) = 0$$

or $24.9 = 10.5\,\omega$

Thus, $\omega = 2.37\,\frac{rad}{s} \times \frac{1\,rev}{2\pi\,rad} = \underline{\underline{0.378\,\frac{rev}{s}}}$

12.12

12.12 Water flows axially up the shaft and out through the two sprinkler arms as sketched in Fig. P12.10 and as shown in **Video V12.2**. With the help of the moment-of-momentum equation explain why only at a threshold amount of water flow, the sprinkler arms begin to rotate. What happens when the flowrate increases above this threshold amount? If the exit nozzle could be varied, what would happen for a set flowrate above the threshold amount, when the angle is increased to 90°? Decreased to 0°?

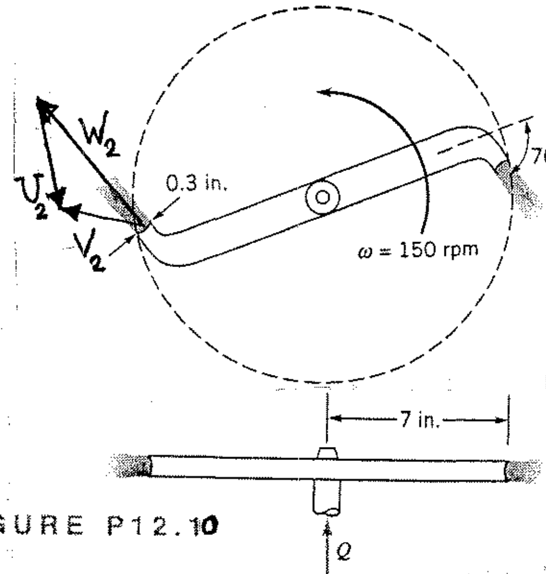


FIGURE P12.10

This sprinkler is similar to the one of Example 5.18.

Thus,

$$T_{shaft} = -r_2 V_{\theta 2} \dot{m}$$

From the velocity triangle shown in the sketch above, we conclude that

$$V_{\theta 2} = \left(W_2 \sin 70^\circ - U_2\right)$$

where

$$U_2 = r_2 \omega$$

Combining, we get

$$T_{shaft} = -r_2 \left(W_2 \sin 70^\circ - r_2 \omega\right) \dot{m} \qquad (1)$$

So, when W_2 and $\dot{m}$ combined is large enough with $\omega = 0$ to overcome T_{shaft}, the sprinkler rotor begins to rotate.

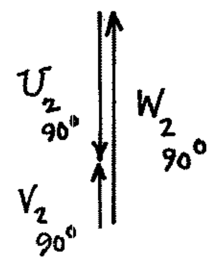
When flowrate increases further, ω is no longer zero but set at a value that satisfies Eq. 1

(con't)

12.12	(con't)

When the nozzle angle is increased from 70° to 90°

$$T_{shaft_{90°}} = (W_{2_{90°}} \sin 90° - r_2 \omega)\, \dot{m}_{90°}$$



For

$$T_{shaft_{90°}} \approx T_{shaft_{70°}}$$

$$W_{2_{90°}} = W_{2_{70°}}$$

$$\dot{m}_{90°} = \dot{m}_{70°}$$

We get

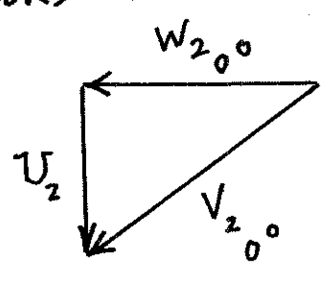
$$W_{2_{90°}} - r_2 \omega_{90°} = W_{2_{70°}} \sin 70° - r_2 \omega_{70°}$$

For this to be true

$$\omega_{90°} > \omega_{70°}$$

or the sprinkler speeds up when the nozzle angle is increased from 70° to 90°.

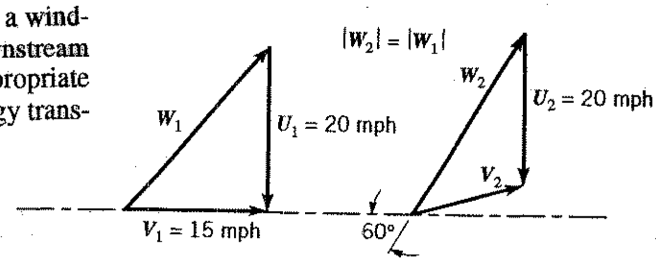
When the nozzle angle is decreased to 0°, the exit velocity triangle now looks like



and the shaft torque associated with this flow opposes and eventually stops sprinkler rotation.

12.13

12.13 At a given radial location, a 15-mph wind against a windmill (see **Video V12.1**) results in the upstream (1) and downstream (2) velocity triangles shown in Fig. P12.13. Sketch an appropriate blade section at that radial location and determine the energy transferred per unit mass of fluid.



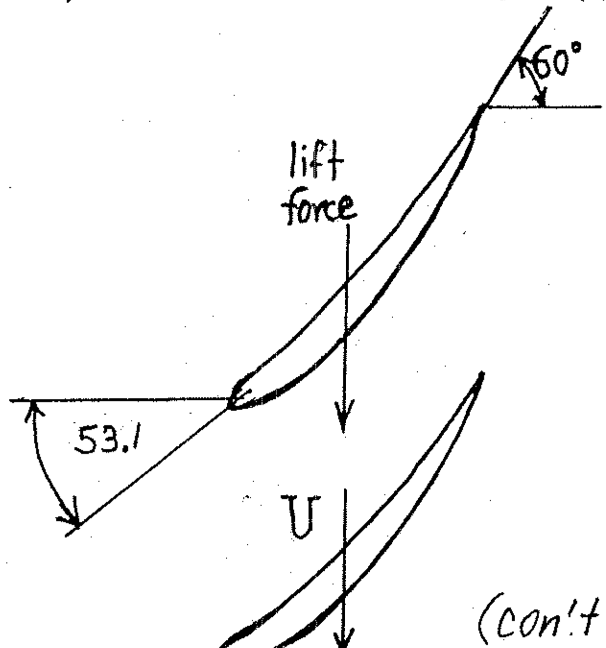
■ **FIGURE P12.13**

We can determine whether the axial flow turbomachine involved is a turbine or a fan by comparing the direction of the lift force on the rotor blade section with the direction of the blade velocity, U. If the lift force and the blade velocity are in the same direction a turbine is involved. If the lift force and blade velocity are in opposite directions, a fan is involved. The direction of the lift force can be inferred from the shape of the rotor blade section sketched to be tangent to the relative flows entering and leaving the rotor row.

The entering relative flow angle, β_1, is

$$\beta_1 = \tan^{-1}\frac{U_1}{V_1} = \tan^{-1}\frac{(20\,\text{mph})}{(15\,\text{mph})} = 53.1°$$

Thus, the rotor blade sections sketched below are appropriate



(con't)

12.13 (con't)

Since the lift force acting on each rotor blade section is in the same direction as the blade velocity we conclude that this turbomachine is a turbine. The energy transferred per unit mass is the shaft work per unit mass, w_{shaft}, which we can determine with Eq. 11.5. Thus

$$w_{shaft} = -U_2 V_{\theta,2} \qquad (1)$$

From the velocity triangles we obtain

$$V_{\theta,2} = W_2 \sin 60° - U_2$$

and

$$W_2 = W_1 = \sqrt{V_1^2 + U_1^2}$$

Thus

$$w_{shaft} = -U_2\left(\sqrt{V_1^2 + U_1^2}\ \sin 60° - U_2\right)$$

$$w_{shaft} = -(20\,mph)\left[\sqrt{(15\,mph)^2 + (20\,mph)^2}\ \sin 60° - 20\,mph\right]\frac{\left(5280\,\frac{ft}{mi}\right)^2}{\left(3600\,\frac{s}{hr}\right)^2}\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)$$

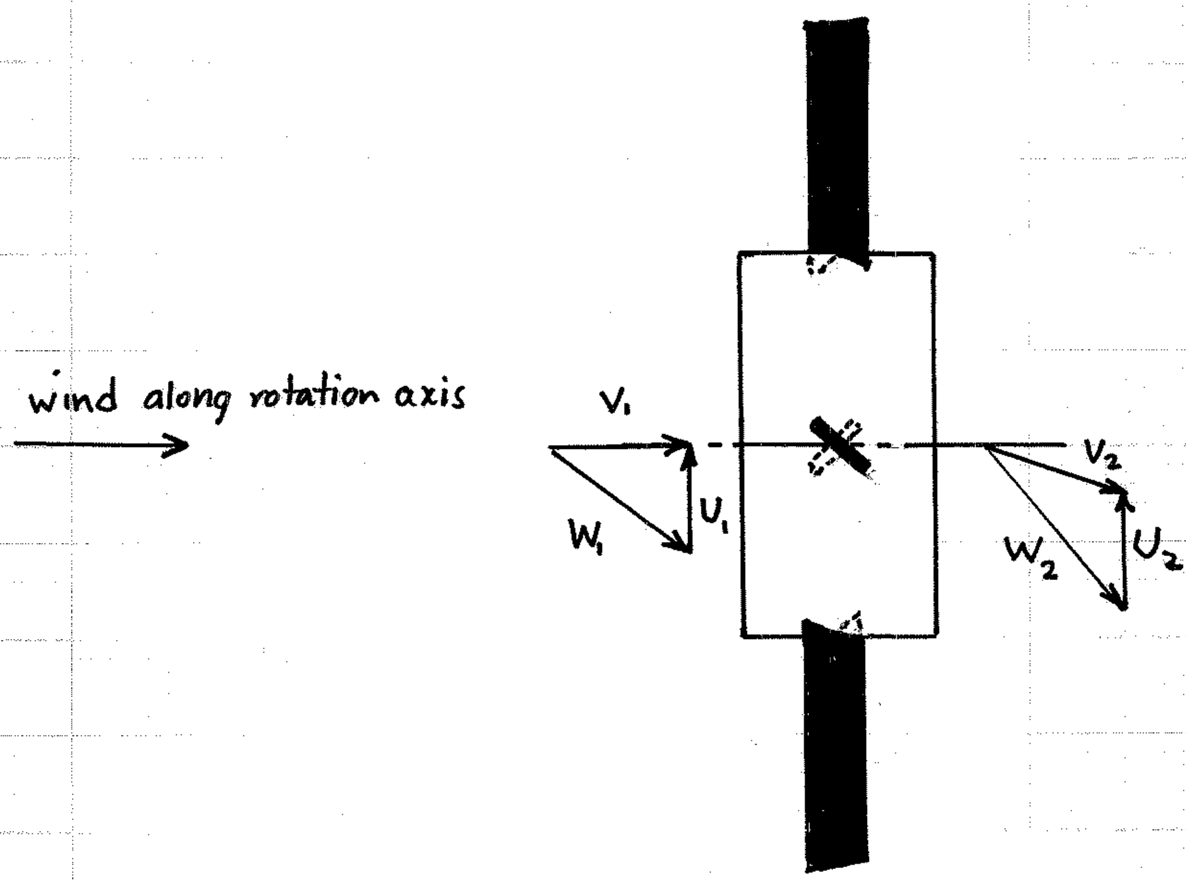
$$w_{shaft} = \underline{\underline{-71}}\ \frac{ft\cdot lb}{slug}$$

or

$$w_{shaft} = -71\ \frac{ft\cdot lb}{slug}\ \frac{1}{\left(32.2\ \frac{lbm}{slug}\right)} = -2.2\ \frac{ft\cdot lb}{lbm}$$

12.14

12.14 Sketch how you would arrange four 3-in.-wide by 12-in.-long thin but rigid strips of sheet metal on a hub to create a windmill like the one shown in **Video V12.1.** Discuss, with the help of velocity triangles, how you would arrange each blade on the hub and how you would orient your windmill in the wind.



12.15

12.15 Sketched in Fig. P12.15 are the upstream [section (1)] and downstream [section (2)] velocity triangles at the arithmetic mean radius for flow through an axial-flow turbomachine rotor. The axial component of velocity is 50 ft/s at sections (1) and (2). (a) Label each velocity vector appropriately. Use **V** for absolute velocity, **W** for relative velocity, and **U** for blade velocity. (b) Are you dealing with a turbine or a fan? (c) Calculate the work per unit mass involved. (d) Sketch a reasonable blade section. Do you think the actual blade exit angle will need to be less or greater than 15°? Why?

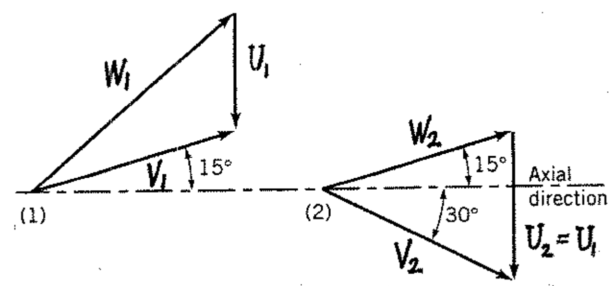


FIGURE P.12.15

(a) See figure above.

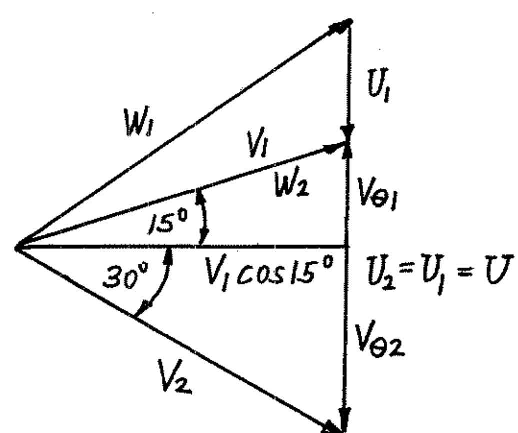
(b) $T = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1}) = \dot{m}\, r_{mean}(V_{\theta 2} - V_{\theta 1})$

where $V_{\theta 2} > 0$ and $V_{\theta 1} < 0$ (see figure above)

Thus, $T > 0$. The machine is a <u>fan.</u>

(c) $w_{shaft} = U_2 V_{\theta 2} - U_1 V_{\theta 1} = U(V_{\theta 2} - V_{\theta 1})$ where $U = U_1 = U_2$

Since $V_{x1} = V_{x2} = 50\,\frac{ft}{s}$, it follows from the figure that:



$V_1 \cos 15° = 50\,\frac{ft}{s}$

or $V_1 = 51.8\,\frac{ft}{s}$

and

$V_2 \cos 30° = 50\,\frac{ft}{s}$ or $V_2 = 57.7\,\frac{ft}{s}$

so that

$V_{\theta 1} = -V_1 \sin 15° = -51.8 \sin 15° = -13.4\,\frac{ft}{s}$

and

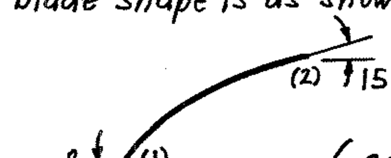
$V_{\theta 2} = V_2 \sin 30° = 28.8\,\frac{ft}{s}$

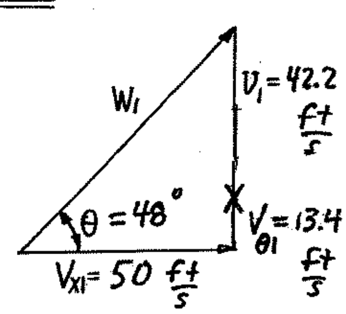
Also, $U = |V_{\theta 1}| + |V_{\theta 2}| = 42.2\,\frac{ft}{s}$

Hence, $w_{shaft} = 42.2\,\frac{ft}{s}\left(28.8\,\frac{ft}{s} - \left(-13.4\,\frac{ft}{s}\right)\right) = \underline{\underline{1780\,\frac{ft^2}{s^2}}}$

(d) From the figure $\tan\theta = \frac{42.2 + 13.4}{50}$, or $\theta = 48°$

Thus, the blade shape is as shown:





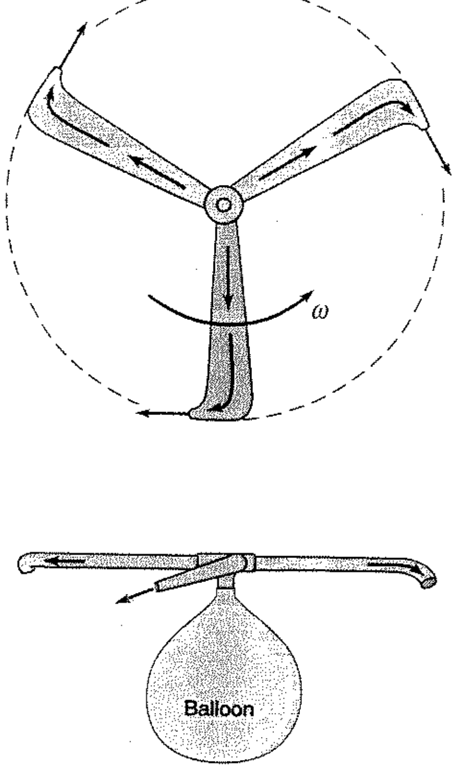
(con't)

12.15 (con't)

The actual blade angle will need to be less than 15°
to achieve a 15° flow angle at the blade exit.
Because of boundary layer development on both surfaces of the blade, the fluid angle will be different from the blade angle. Less turning than expected will be actually achieved.

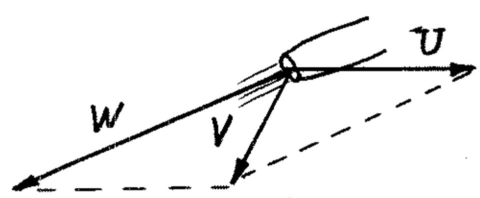
12.16

12.16 Shown in Fig. P12.16 is a toy "helicopter" powered by air escaping from a balloon. The air from the balloon flows radially through each of the three propeller blades and out small nozzles at the tips of the blades. The nozzles (along with the rotating propeller blades) are tilted at a small angle as indicated. Sketch the velocity triangle (i.e., blade, absolute, and relative velocities) for the flow from the nozzles. Explain why this toy tends to move upward. Is this a turbine? Pump?



■ FIGURE P12.16

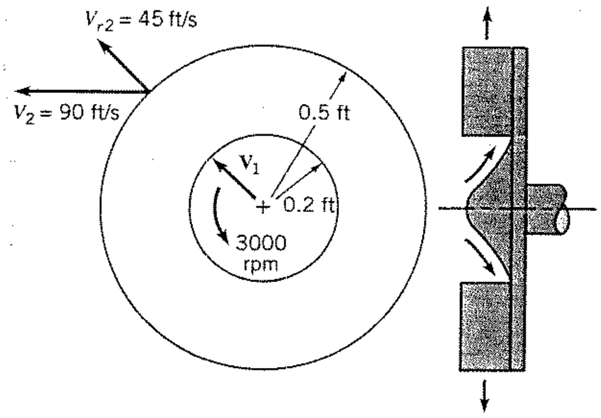
If we assume the helicopter is stationary, then the blade speed is ωR in the horizontal plane as shown in the side view below. The relative velocity, $\vec{W}$, is directed along the nozzle, and the absolute velocity, $\vec{V}=\vec{W}+\vec{U}$, is as indicated.



The toy tends to move upward because the flow over the blades push up on them. The air from the balloon forces the blades to rotate like a turbine. However, the blades act on the ambient air as a pump.

12.18

12.18 The radial component of velocity of water leaving the centrifugal pump sketched in Fig. P12.18 is 45 ft/s. The magnitude of the absolute velocity at the pump exit is 90 ft/s. The fluid enters the pump rotor radially. Calculate the shaft work required per unit mass flowing through the pump.



■ FIGURE P12.18

$$w_{shaft} = U_2 V_{\theta 2} - U_1 V_{\theta 1} \qquad (Eq.\ 12.5)$$

$$w_{shaft} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - (V_{r2}^2 - V_{r1}^2)}{2} \qquad (Eq.\ 12.8)$$

Since the fluid enters radially, $V_{\theta 1} = 0$ so that Eq. 12.5 becomes

$$w_{shaft} = U_2 V_{\theta 2} \qquad (1)$$

with

$$U_2 = r_2 \omega = (0.5\,ft)\left[\frac{\left(3000\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)}{\left(60\,\frac{s}{min}\right)}\right] = 157\,\frac{ft}{s}$$

From Fig. 12.8c

$$V_{\theta 2} = \left(V_2^2 - V_{r2}^2\right)^{1/2}$$

$$= \left[\left(90\,\frac{ft}{s}\right)^2 - \left(45\,\frac{ft}{s}\right)^2\right]^{1/2} = 77.9\,\frac{ft}{s}$$

Thus, from Eq. (1)

$$w_{shaft} = \left(157\,\frac{ft}{s}\right)\left(77.9\,\frac{ft}{s}\right)\left(\frac{1\,lb}{slug\cdot\frac{ft}{s^2}}\right)$$

$$= \underline{\underline{1.22\times 10^4\,\frac{ft\cdot lb}{slug}}} = \frac{1.22\times 10^4\,\frac{ft\cdot lb}{slug}}{32.174\,\frac{lbm}{slug}}$$

$$= \underline{\underline{379}}\,\frac{ft\cdot lb}{lbm}$$

(con't)

12.18 (con't)

We proceed to calculate the component velocities of Eq. 12.8

$$U_1 = r_1\omega = (0.2\ ft)\left[\frac{\left(3000\ \frac{rev}{min}\right)\left(2\pi\ \frac{rad}{rev}\right)}{60\ \frac{s}{min}}\right] = 62.8\ \frac{ft}{s}$$

From conservation of mass

$$V_1 A_1 = V_{r2} A_2$$

or

$$V_1 2\pi r_1 b = V_{r2}\, 2\pi r_2 b$$

and

$$V_1 = V_{r2}\frac{r_2}{r_1} = 45\ \frac{ft}{s}\left(\frac{0.5\ ft}{0.2\ ft}\right) = 112\ \frac{ft}{s}$$

For the entering flow

$$V_{r_1}^2 = U_1^2 + V_1^2 = \left(62.8\ \frac{ft}{s}\right)^2 + \left(112\ \frac{ft}{s}\right)^2$$

so

$$V_{r_1} = 128\ \frac{ft}{s}$$

Then from Eq. 12.8

$$w_{shaft} = \frac{\left(90\ \frac{ft}{s}\right)^2 - \left(112\ \frac{ft}{s}\right)^2 + \left(157\ \frac{ft}{s}\right)^2 - \left(62.8\ \frac{ft}{s}\right)^2 - \left[\left(45\ \frac{ft}{s}\right)^2 - \left(128\ \frac{ft}{s}\right)^2\right]}{2}$$

$$= 1.53 \times 10^4\ \frac{ft^2}{s^2}\left(\frac{1\ lb}{slug\cdot\frac{ft}{s^2}}\right) = \underline{1.53\times10^4}\ \frac{ft\cdot lb}{slug} = \frac{1.53\times10^4\ \frac{ft\cdot lb}{slug}}{32.174\ \frac{lbm}{slug}}$$

$$= \underline{\underline{476}}\ \frac{ft\cdot lb}{lbm}$$

Eq. 12.8 invites round off error because of the differences of velocity squared involved

12.19

12.19 A centrifugal water pump having an impeller diameter of 0.5 m operates at 900 rpm. The water enters the pump parallel to the pump shaft. If the exit blade angle, β_2 (see Fig. 12.8), is 25°, determine the shaft power required to turn the impeller when the flow through the pump is 0.16 m³/s. The uniform blade height is 50 mm.

$$\dot{W}_{shaft} = T_{shaft}\,\omega = T_{shaft}\,2\pi N$$

$$T_{shaft} = \rho Q\,(r_2 V_{\theta 2} - r_1 V_{\theta 1}) \qquad (\text{Eq. } 12.10)$$

With $V_{\theta 1} = 0$

$$T_{shaft} = \rho Q r_2 V_{\theta 2} \qquad (1)$$

From Fig. 12.8c

$$\cot\beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

so that

$$V_{\theta 2} = U_2 - V_{r2}\cot\beta_2 \qquad (2)$$

For $r_2 = \frac{0.5\,m}{2} = 0.25\,m$ with $\omega = \frac{\left(900\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)}{60\,\frac{s}{min}} = 94.2\,\frac{rad}{s}$

then

$$U_2 = r_2\omega = (0.25\,m)\left(94.2\,\frac{rad}{s}\right) = 23.6\,\frac{m}{s}$$

Since the flowrate is given, it follows that

$$Q = 2\pi r_2 b_2 V_{r2}$$

or

$$V_{r2} = \frac{Q}{2\pi r_2 b_2} = \frac{\left(0.16\,\frac{m^3}{s}\right)}{(2\pi)(0.25\,m)(0.05\,m)} = 2.04\,\frac{m}{s}$$

Thus, from Eq. (2)

$$V_{\theta 2} = (23.6 - 2.04\cot 25°)\,\frac{m}{s} = 19.2\,\frac{m}{s}$$

and from Eq. (1)

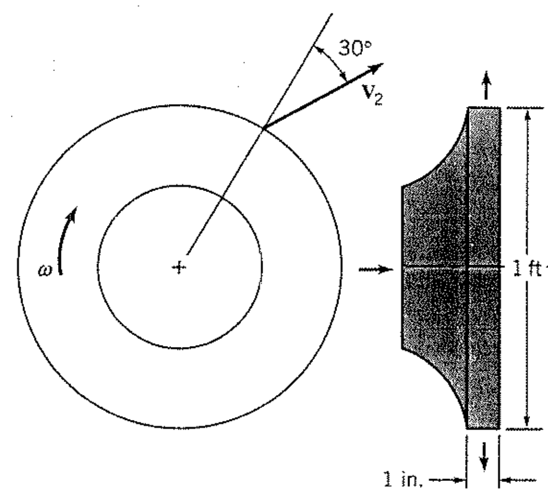
$$T_{shaft} = \left(999\,\frac{kg}{m^3}\right)\left(0.16\,\frac{m^3}{s}\right)(0.25\,m)\left(19.2\,\frac{m}{s}\right) = 768\,N\cdot m$$

so,

$$\dot{W}_{shaft} = (768\,N\cdot m)\left(2\pi\,\frac{rad}{rev}\right)\left(900\,\frac{rev}{min}\right)\frac{1}{\left(60\,\frac{s}{min}\right)}\left(\frac{1}{1000\,\frac{N\cdot m}{s\cdot kW}}\right) = \underline{\underline{0.08\,kW}}$$

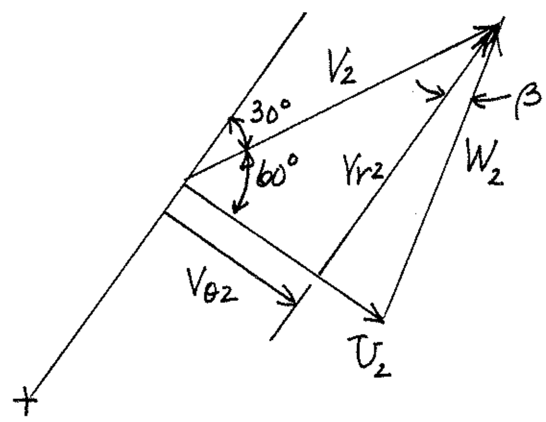
12.20

12.20 A centrifugal pump impeller is rotating at 1200 rpm in the direction shown in Fig. P12.20. The flow enters parallel to the axis of rotation and leaves at an angle of 30° to the radial direction. The absolute exit velocity, V_2, is 90 ft/s. **(a)** Draw the velocity triangle for the impeller exit flow. **(b)** Estimate the torque necessary to turn the impeller if the fluid is water. What will the impeller rotation speed become if the shaft breaks?



■ FIGURE P12.20

(a) The exit flow velocity triangle can be constructed graphically as indicated below,



with $U_2 = r_2 \omega = (0.5\,ft)\dfrac{\left(1200\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)}{\left(60\,\frac{s}{min}\right)} = 62.8\,\frac{ft}{s}$

From the velocity triangle

$$\tan\beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

Since $V_{\theta 2} = V_2 \sin 30°$ and $V_{r2} = V_2 \cos 30°$ it follows that

$$\beta_2 = \tan^{-1}\left[\frac{U_2 - V_2 \sin 30°}{V_2 \cos 30°}\right]$$

$$= \tan^{-1}\left[\frac{62.8\,\frac{ft}{s} - \left(90\,\frac{ft}{s}\right)\sin 30°}{\left(90\,\frac{ft}{s}\right)\cos 30°}\right] = 12.9°$$

(con't)

12.20 (con't)

Thus, from the velocity triangle

$$W_2 = \frac{V_{r2}}{\cos 12.9°} = \frac{V_2 \cos 30°}{\cos 12.9°} = \frac{(90 \frac{ft}{s}) \cos 30°}{\cos 12.9°}$$

$$= 80.0 \ \frac{ft}{s}$$

With β_2 and W known, the velocity triangle is completely specified.

(b) From Eq. 12.9 with $V_{\theta 1} = 0$

$$T_{shaft} = \dot{m} r_2 V_{\theta 2} \qquad (1)$$

Since

$$\dot{m} = \rho 2\pi r_2 b_2 V_{r2}$$ and ρ for water from Table 1.5 is $1.94 \frac{slugs}{ft^3}$

$$= (1.94 \frac{slugs}{ft^3})(2\pi)(0.5 ft)(\frac{1}{12} ft)(90 \frac{ft}{s}) \cos 30°$$

$$= 39.6 \ \frac{slugs}{s}$$

so that from Eq. (1)

$$T_{shaft} = (39.6 \ \frac{slugs}{s})(0.5 ft)(90 \frac{ft}{s}) \sin 30°$$

$$= \underline{\underline{891 \ ft \cdot lb}}$$

A positive torque is in the same direction as the rotation.

When the shaft breaks, the torque becomes zero and the impeller eventually stops because there is no longer a driving torque to force it to rotate. In a pump, the shaft torque drives the impeller and the impeller moves fluid. On the other hand, in a turbine, the moving fluid drives the impeller.

12.21

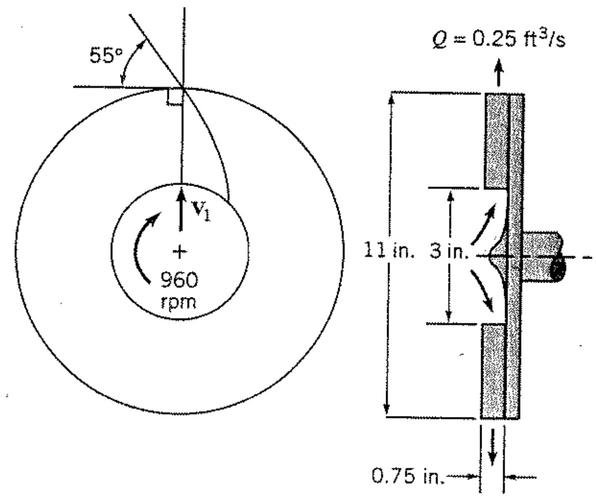
12.21 Discuss the main simplifying assumptions associated with **Eq. 12.13** and explain why actual head rise is always less than ideal head rise. Discuss how ideal head rise is head "added" to the fluid and actual head rise is head "gained" by the fluid. Can Eq. 12.13 be used for a turbine? Explain in terms of actual and ideal changes in head.

Eq. 12.13 is obtained assuming that no loss of available energy occurs in the flow through the pump impeller. The actual head rise across the pump is thus equal to the ideal head rise across the pump minus the loss of available energy suffered by the flowing fluid because of friction. The blades add the ideal head rise amount to the flowing fluid, however, the fluid flow loss results in the actual head rise realized by the flowing fluid being less than the ideal amount by the loss.

Eq. 12.13 may also be used for flow across a turbine rotor, however the change in head will now be negative or in other words the flowing fluid head will drop across the rotor. Further, this head drop across the turbine rotor is the ideal amount, or the amount in the absence of any loss of available energy suffered by the flowing fluid because of viscosity. The actual head drop is larger than the ideal head drop the difference due to losses.

12.22

12.22 A centrifugal radial water pump has the dimensions shown in Fig. P12.22. The volume rate of flow is 0.25 ft³/s, and the absolute inlet velocity is directed radially outward. The angular velocity of the impeller is 960 rpm. The exit velocity as seen from a coordinate system attached to the impeller can be assumed to be tangent to the vane at its trailing edge. Calculate the power required to drive the pump.



■ FIGURE P12.22

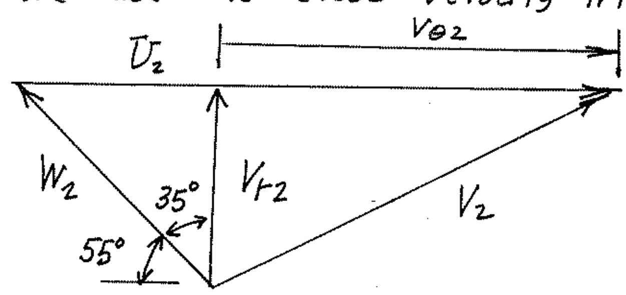
From Eq. 12.11, with $V_{\theta 1} = 0$,

$$\dot{W}_{shaft} = \rho Q U_2 V_{\theta 2} \qquad (1)$$

To determine U_2 we use

$$U_2 = r_2 \omega = \left(\frac{5.5\,in.}{12\,in./ft}\right)\left(960\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)\left(\frac{1}{60\,\frac{s}{min}}\right) = 46.1\,\frac{ft}{s}$$

To obtain $V_{\theta 2}$ we use the exit velocity triangle shown below.



Since $$V_{\theta 2} = U_2 - V_{r2} \tan 35°$$

and $$V_{r2} = \frac{Q}{A_2} = \frac{Q}{2\pi r_2 b_2} = \frac{\left(0.25\,\frac{ft^3}{s}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{(2\pi)(5.5\,in.)(0.75\,in.)} = 1.39\,\frac{ft}{s}$$

it follows that

$$V_{\theta 2} = (46.1 - 1.39 \tan 35°)\,\frac{ft}{s} = 45.1\,\frac{ft}{s}$$

Thus, from Eq. (1)

$$\dot{W}_{shaft} = \left(1.94\,\frac{slugs}{ft^3}\right)\left(0.25\,\frac{ft^3}{s}\right)\left(46.1\,\frac{ft}{s}\right)\left(45.1\,\frac{ft}{s}\right)$$

$$= 1010\,\frac{ft \cdot lb}{s}$$

or

$$\dot{W}_{shaft} = \frac{1010\,\frac{ft \cdot lb}{s}}{550\,\frac{ft \cdot lb}{s \cdot hp}} = \underline{\underline{1.84\,hp}}$$

12.23

12.23 Water is pumped with a centrifugal pump, and measurements made on the pump indicate that for a flowrate of 240 gpm the required input power is 6 hp. For a pump efficiency of 62%, what is the actual head rise of the water being pumped?

From Eq. 12.23 the pump efficiency is given by the equation

$$\eta = \frac{\gamma Q h_a/550}{bhp}$$

so that

$$h_a = \frac{(\eta)(bhp)(550)}{\gamma Q}$$

$$= \frac{(0.62)(6\,hp)\left(550\,\frac{ft\cdot lb}{s\cdot hp}\right)}{\left(62.4\,\frac{lb}{ft^3}\right)\left[\left(240\,\frac{gal}{min}\right)\Big/\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)\right]}$$

$$= \underline{\underline{61.3\,ft}}$$

12.24

12.24 The performance characteristics of a certain centrifugal pump are determined from an experimental setup similar to that shown in Fig. 12.10. When the flowrate of a liquid ($SG = 0.9$) through the pump is 120 gpm, the pressure gage at (1) indicates a vacuum of 95 mm of mercury and the pressure gage at (2) indicates a pressure of 80 kPa. The diameter of the pipe at the inlet is 110 mm and at the exit it is 55 mm. If $z_2 - z_1 = 0.5$ m, what is the actual head rise across the pump? Explain how you would estimate the pump motor power requirement.

From Eq. 12.19

$$h_a = \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2 - V_1^2}{2g} \qquad (1)$$

Since

$$V_1 = \frac{Q}{A_1} = \frac{(120\ gpm)\left(6.309 \times 10^{-5}\ \frac{m^3/s}{gpm}\right)}{\frac{\pi}{4}(0.110\ m)^2} = 0.797\ \frac{m}{s}$$

and

$$V_1 A_1 = V_2 A_2$$

$$V_2 = V_1 \left(\frac{110\ mm}{55\ mm}\right)^2 = \left(0.797\ \frac{m}{s}\right)(2)^2 = 3.19\ \frac{m}{s}$$

Thus, from Eq. (1), with $p_1 = -(h_{Hg})(\gamma_{Hg}) = -(0.095\,m)\left(133 \times 10^3\ \frac{N}{m^3}\right)$

and $p_2 = 80 \times 10^3\ N/m^2$,

$$h_a = \frac{80 \times 10^3\ \frac{N}{m^2} + (0.095)(133 \times 10^3)\ \frac{N}{m^2}}{(0.9)(9.80 \times 10^3)\ \frac{N}{m^3}} + 0.5\,m + \frac{\left(3.19\ \frac{m}{s}\right)^2 - \left(0.797\ \frac{m}{s}\right)^2}{2\left(9.81\ \frac{m}{s^2}\right)}$$

$$h_a = \underline{\underline{11.5\ m}}$$

To estimate the pump motor power requirement use Eq. 12.23

$$\eta = \frac{\gamma Q h_a}{bhp\,(550)}$$

to get

$$bhp = \frac{\gamma Q h_a}{\eta (550)}$$

For differing values of η, a corresponding bhp can be calculated.

12.25

12.25 The performance characteristics of a certain centrifugal pump having a 9-in.-diameter impeller and operating at 1750 rpm are determined using an experimental setup similar to that shown in Fig. 12.10. The following data were obtained during a series of tests in which $z_2 - z_1 = 0$, $V_2 = V_1$, and the fluid was water.

Q (gpm)	20	40	60	80	100	120	140
$p_2 - p_1$ (psi)	40.2	40.1	38.1	36.2	33.5	30.1	25.8
Power input (hp)	1.58	2.27	2.67	2.95	3.19	3.49	4.00

Based on these data, show or plot how the actual head rise, h_a, and the pump efficiency, η, vary with the flowrate. What is the design flowrate for this pump?

From Eq. 12.19 with $z_1 = z_2$ and $V_1 = V_2$

$$h_a = \frac{p_2 - p_1}{\gamma}$$

Thus, for the first set of data in the table

$$h_a = \frac{\left(40.2 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)}{62.4 \frac{lb}{ft^3}} = 92.8\,ft$$

From Eq. 12.23

$$\eta = \frac{\gamma Q h_a / 550}{bhp}$$

and for the first set of data in the table

$$\eta = \frac{\left(62.4 \frac{lb}{ft^3}\right)\left[(20\,gpm)/\left(7.48 \frac{gal}{ft^3}\right)\left(60 \frac{s}{min}\right)\right](92.8\,ft)}{(1.58\,hp)\left(550 \frac{ft \cdot lb}{s \cdot hp}\right)}$$

$$= 0.297$$

or

$$\eta = 29.7\,\%$$

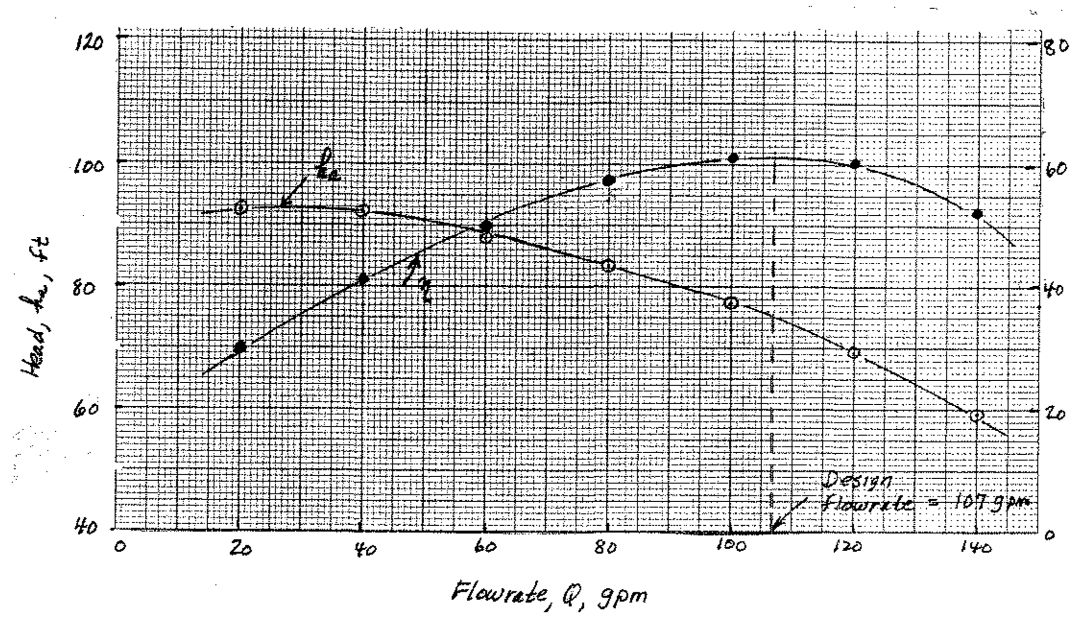
Remaining values for h_a and η can be calculated in a similar manner, and all values are tabulated in the table below.

Q (gpm)	20	40	60	80	100	120	140
h_a (ft)	92.8	92.5	87.9	83.5	77.3	69.5	59.5
η (%)	29.7	41.2	49.9	57.5	61.3	60.4	52.6

(con't)

12.25 (con't)

A plot of the data is shown below. The design flowrate occurs at peak efficiency and is 107 gpm.



12.26

12.26 It is sometimes useful to have $h_a - Q$ pump performance curves expressed in the form of an equation. Fit the $h_a - Q$ data given in Problem 12.25 to an equation of the form $h_a = h_o - kQ^2$ and compare the values of h_a determined from the equation with the experimentally determined values. (*Hint*: Plot h_a versus Q^2 and use the method of least squares to fit the data to the equation.)

Based on the data from Problem 12.15, the following table can be created and from a standard, linear regression curve fitting program the following results are obtained.

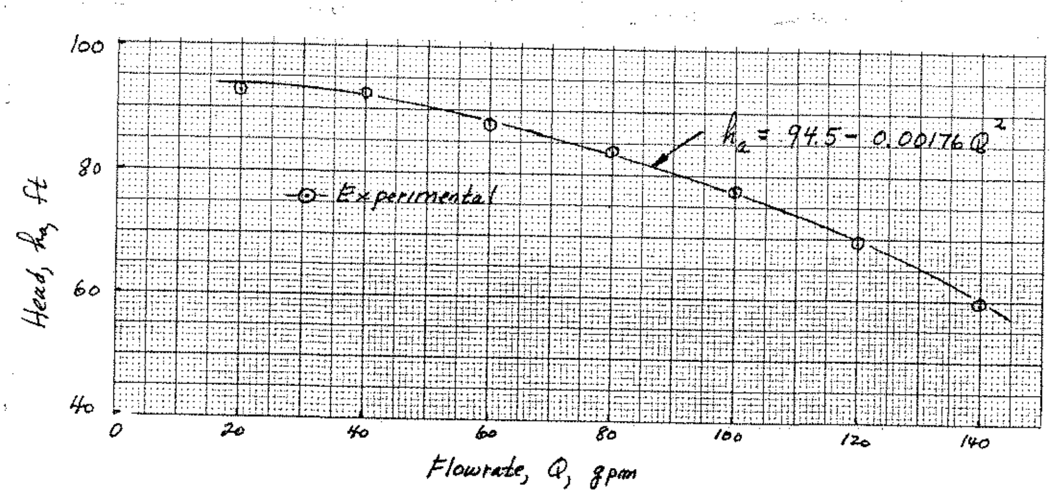
Q (gpm)	20	40	60	80	100	120	140
$[Q\,(\text{gpm})]^2$	4×10^2	16×10^2	36×10^2	64×10^2	100×10^2	144×10^2	196×10^2
h_a (ft)	92.8	92.5	87.9	83.5	77.3	69.5	59.5
Δh_a (ft)*	−1.00	0.81	−0.27	0.26	0.39	0.33	−0.52

$^*\Delta h_a = h_a$ (experimental) $- h_a$ (predicted)

The equation obtained from the data using linear regression is

$$h_a = 94.5 - 0.00176\,Q^2 \qquad (1)$$

where h_a is in ft with Q in gpm. A plot showing the comparison between the experimental data and the predicted results (from Eq. 1) is shown below.



12.28

12.28 In **Example 12.3**, how will the maximum height, z_1, that the pump can be located above the water surface change if the water temperature is decreased to 40 °F?

From Table B.1 for 40°F water, vapor pressure is 0.1217 psia and $\gamma = 62.43\ lb/ft^3$. Thus, with this change in Eq.(2) in Example 12.3

$$(z_1)_{max} = \frac{\left(14.7\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{62.43\ \frac{lb}{ft^3}} - 10.2\ ft - \frac{\left(0.1217\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{62.43\ \frac{lb}{ft^3}} - 15\ ft$$

so that

$$(z_1)_{max} = \underline{\underline{8.43\ ft}}$$

Thus, there is an increase in height from 7.65 ft to 8.43 ft with decrease in water temperature from 80°F to 40°F.

12.29

12.29 A centrifugal pump with a 7-in.-diameter impeller has the performance characteristics shown in Fig. 12.12. The pump is used to pump water at 100 °F, and the pump inlet is located 12 ft above the open water surface. When the flowrate is 200 gpm the head loss between the water surface and the pump inlet is 6 ft of water. Would you expect cavitation in the pump to be a problem? Assume standard atmospheric pressure. Explain how you arrived at your answer.

From Eq. 12.25

$$NPSH_A = \frac{p_{atm}}{\gamma} - z_1 - \Sigma h_L - \frac{p_v}{\gamma} \qquad (1)$$

From Table B.1 the water vapor pressure at 100°F is 0.9493 psia and $\gamma = 62.00\ \frac{lb}{ft^3}$. Thus, with $p_{atm} = 14.7$ psia, $z_1 = 12$ ft, and $\Sigma h_L = 6$ ft, Eq. (1) yields

$$NPSH_A = \frac{\left(14.7\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{62.00\ \frac{lb}{ft^3}} - 12\ ft - 6\ ft - \frac{\left(0.9493\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{62.00\ \frac{lb}{ft^3}}$$

$$= 13.9\ ft$$

From Fig. 12.12 at 200 gpm

$$NPSH_R = \sim 12\ ft$$

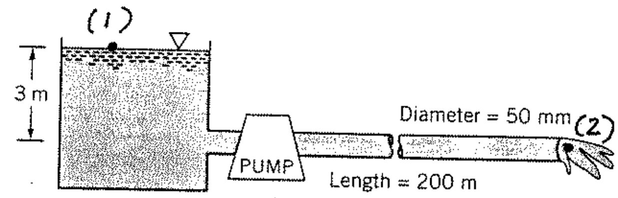
For proper pump operation

$$NPSH_A \geq NPSH_R$$

Since this is true in this case, we expect that cavitation in the pump would not be a problem. No.

12.30

12.30 Water at 40 °C is pumped from an open tank through 200 m of 50-mm-diameter smooth horizontal pipe as shown in Fig. P12.30 and discharges into the atmosphere with a velocity of 3 m/s. Minor losses are negligible. (**a**) If the efficiency of the pump is 70%, how much power is being supplied to the pump? (**b**) What is the $NPSH_A$ at the pump inlet? Neglect losses in the short section of pipe connecting the pump to the tank. Assume standard atmospheric pressure.



■ FIGURE P12.30

(a)
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + f\frac{\ell}{D}\frac{V_2^2}{2g} \quad (1)$$

where $p_1 = p_2 = 0$, $V_1 = 0$, $V_2 = 3\ m/s$, $z_1 = 3m$, and $z_2 = 0$. Thus, Eq. (1) becomes

$$z_1 + h_p = \frac{V_2^2}{2g}\left(1 + f\frac{\ell}{D}\right) \quad (2)$$

Also,

$$Re = \frac{VD}{\nu} = \frac{(3\frac{m}{s})(0.05m)}{(6.580 \times 10^{-7}\frac{m^2}{s})} = 2.28 \times 10^5$$

and from Fig. 8.23 for smooth pipe $f = 0.0152$. Thus, from Eq. (2)

$$h_p = \frac{(3\frac{m}{s})^2}{2(9.81\frac{m}{s^2})}\left[1 + 0.0152\left(\frac{200\ m}{0.05m}\right)\right] - 3m = 25.3\ m$$

Hence,

$$\text{Power gained by fluid} = \gamma Q h_p$$
$$= \left(9.731 \times 10^3 \frac{N}{m^3}\right)\left(\frac{\pi}{4}\right)(0.05m)^2\left(3\frac{m}{s}\right)(25.3m)$$
$$= 1.45 \times 10^3 \frac{N \cdot m}{s} = 1.45\ kW$$

and

$$\text{Power supplied to pump} = \frac{\text{Power gained by fluid}}{\text{Efficiency}}$$
$$= \frac{1.45\ kW}{0.7} = \underline{\underline{2.07\ kW}}$$

(b) From Eq. 12.24

$$NPSH = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} - \frac{p_v}{\gamma} \quad (3)$$

where p_s and V_s refer to the pressure and velocity at the pump inlet, respectively. Also,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} + z_s + h_L$$

so that with $p_1 = p_{atm}$, $V_1 = 0$, $z_s = 0$, and $h_L = 0$

(con't)

12.30 (Con't)

$$\frac{p_{atm}}{\gamma} + z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g}$$

and therefore from Eq.(3) the available NPSH is

$$NPSH_A = \frac{p_{atm}}{\gamma} + z_1 - \frac{p_v}{\gamma} \qquad (4)$$

Note that this result corresponds to Eq. 12.25 with z_1 positive (since pump is below reservoir) and $\Sigma h_L = 0$.

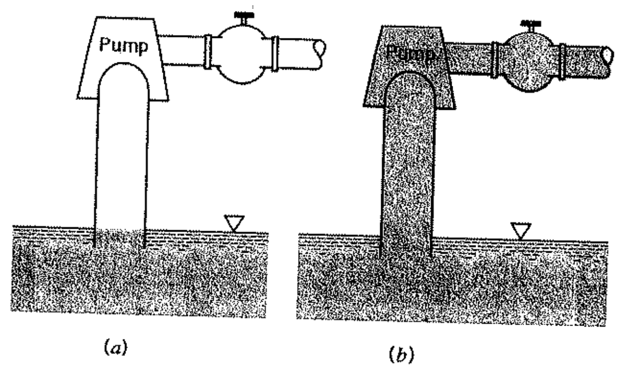
From Table B.2 the water vapor pressure at 40°C is 7.376×10^3 N/m² (abs) and $\gamma = 9.731 \times 10^3$ N/m³. Thus, from Eq.(4) with $p_{atm} = 101$ kPa

$$NPSH_A = \frac{(101 \times 10^3 \frac{N}{m^2})}{(9.731 \times 10^3 \frac{N}{m^3})} + 3m - \frac{(7.376 \times 10^3 \frac{N}{m^2})}{(9.731 \times 10^3 \frac{N}{m^3})}$$

$$= \underline{\underline{12.6\ m}}$$

12.31

12.31 The centrifugal pump shown in Fig. P12.31 is not self-priming. That is, if the water is drained from the pump and pipe as shown in Fig. P12.31(*a*), the pump will not draw the water into the pump and start pumping when the pump is turned on. However, if the pump is primed [i.e., filled with water as in Fig. P12.31(*b*)], the pump does start pumping water when turned on. Explain this behavior.



■ **FIGURE P12.31**

The head-flowrate characteristics for a typical centrifugal pump are shown in Fig. 12.11. The maximum head that the pump can add occurs when $Q \approx 0$ (i.e., at start up for example). This head is in terms of the fluid in the pump. Neglecting losses and the velocity head (and cavitation effects) the pump can lift the fluid a height H equal to the head added by the pump. However, if the fluid in the pump is air (i.e., not primed) the head added is in terms of ft or m of air. For example, if $h_a = 30\,ft$ the pump could raise water that high if it is primed (filled with water). If the pump is not primed (filled with air) then the pump can only raise water up to a distance

$$H = 30\,ft\,\frac{\gamma_{air}}{\gamma_{water}} = 30\,ft\,\frac{\left(0.0765\,\frac{lb}{ft^3}\right)}{\left(62.4\,\frac{lb}{ft^3}\right)} = 0.0368\,ft$$

Hence the water will not get into the pump.

12.33 Owing to fouling of the pipe wall, the friction factor for the pipe of Example 12.4 increases from 0.02 to 0.03. Determine the new flowrate, assuming all other conditions remain the same. What is the pump efficiency at this new flowrate? Explain how a line valve could be used to vary the flowrate through the pipe of Example 12.4. Would it be better to place the valve upstream or downstream of the pump? Why?

With $f = 0.03$, Eq.(2) in Example 12.4 becomes

$$h_p = 10\text{ ft} + \left[0.03\,\frac{(200\text{ ft})}{(\frac{6}{12}\text{ ft})} + (0.5+1.5+1.0)\right]\frac{V^2}{2(32.2\,\frac{\text{ft}}{\text{s}^2})} \qquad (1)$$

Since,

$$V = \frac{Q}{A} = \frac{Q\,(\frac{\text{ft}^3}{\text{s}})}{(\frac{\pi}{4})(\frac{6}{12}\text{ ft})^2}$$

Eq.(1) can be written as

$$h_p = 10 + 6.04\,Q^2$$

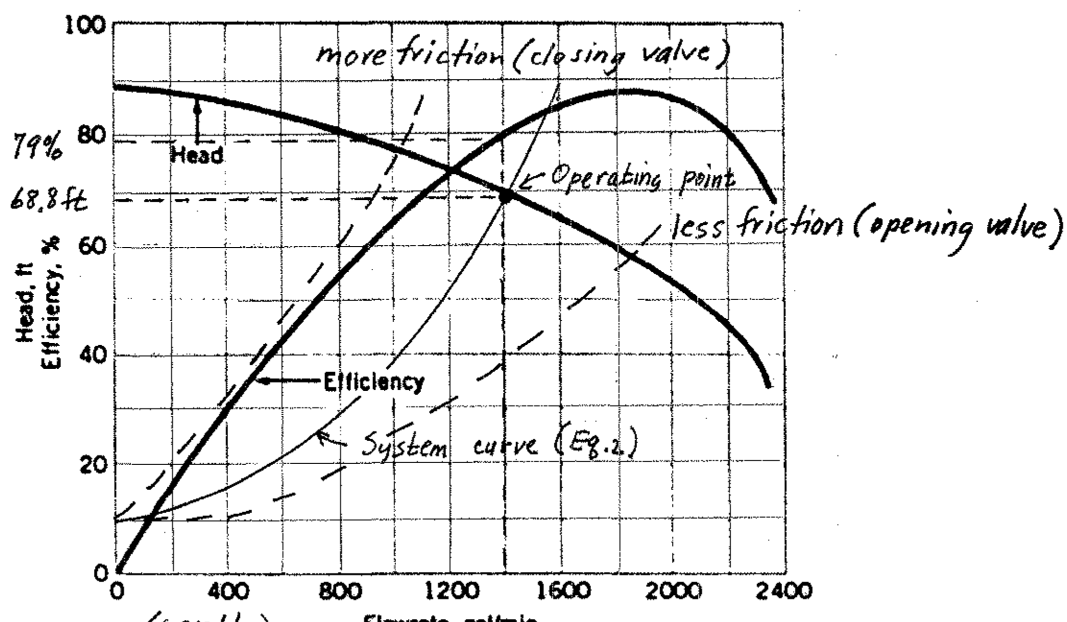
or with Q in gal/min

$$h_p = 10 + 3.00\times10^{-5}\left[Q(\text{gal/min})\right]^2 \qquad (2)$$

The intersection of Eq.(2) (the system equation) with the performance curve for the pump, as shown below, indicates that the new flowrate is

$$Q = 1400\ \frac{\text{gal}}{\text{min}}$$

and the efficiency at this flowrate is approximately $\underline{79.0\%}$.



(con't)

12.33 (Con't)

A line valve acts as a variable frictional resistance to the flow. Closing the valve is equivalent to adding friction and moving the system curve to the left intersecting the head curve at an operational point involving less flowrate than with a more open valve setting. This system curve is sketched in the figure on the previous page and labeled "more friction (closing valve)." Opening the valve is similar to removing friction and moving the system curve to the right intersecting the head curve at an operating point involving more flowrate than with a less open valve setting. This system curve is sketched on the previous page and labeled "less friction (opening valve)."

It would be generally better to place the valve downstream of the pump to avoid the low suction pressure and cavitation possible with upstream placement of the valve.

12.34

12.34 A centrifugal pump having a head-capacity relationship given by the equation $h_a = 180 - 6.10 \times 10^{-4}Q^2$, with h_a in feet when Q is in gpm, is to be used with a system similar to that shown in Fig. 12.14. For $z_2 - z_1 = 50$ ft, what is the expected flowrate if the total length of constant-diameter pipe is 600 ft and the fluid is water? Assume the pipe diameter to be 4 in. and the friction factor to be equal to 0.02. Neglect all minor losses.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} \qquad (1)$$

and with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_2 - z_1 = 50\,ft$, $f = 0.02$, $D = 4/12\,ft$, and $\ell = 600\,ft$, Eq. (1) becomes

$$h_p = 50\,ft + 0.02\,\frac{(600\,ft)}{(\frac{4}{12}\,ft)}\,\frac{V^2}{(2)(32.2\,\frac{ft}{s^2})} \qquad (2)$$

Since

$$V = \frac{Q}{A} = \frac{Q(\frac{ft^3}{s})}{(\frac{\pi}{4})(\frac{4}{12}\,ft)^2}$$

Eq. (2) can be written as

$$h_p = 50 + 73.4\,Q^2$$

or with Q in gal/min

$$h_p = 50 + 3.64 \times 10^{-4}\,[Q\,(gal/min)]^2 \qquad (3)$$

The pump head-capacity relationship is

$$h_a = 180 - 6.10 \times 10^{-4}\,[Q\,(gal/min)]^2 \qquad (4)$$

Thus, the operating point will occur at the flowrate where $h_a = h_p$, or

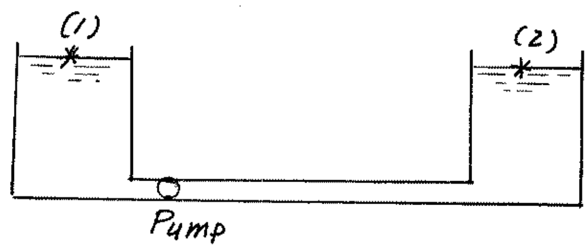
$$180 - 6.10 \times 10^{-4}\,Q^2 = 50 + 3.64 \times 10^{-4}\,Q^2$$

So that

$$Q = \underline{\underline{365\,gpm}}$$

12.35

12.35 A centrifugal pump having a 6-in.-diameter impeller and the characteristics shown in Fig. 12.12 is to be used to pump gasoline through 4000 ft of commercial steel 3-in.-diameter pipe. The pipe connects two reservoirs having open surfaces at the same elevation. Determine the flowrate. Do you think this pump is a good choice? Explain.



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} \qquad (1)$$

and with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $z_1 = z_2 = 0$, $\ell = 4000\ ft$, and $D = 3/12\ ft$ (neglecting minor losses), Eq. (1) becomes

$$h_p = f\frac{(4000\ ft)}{(3/12\ ft)}\frac{V^2}{(2)(32.2\frac{ft}{s^2})} \qquad (2)$$

Since $$V = \frac{Q}{A} = \frac{Q\,(ft^3/s)}{(\frac{\pi}{4})(\frac{3}{12}ft)^2}$$

Eq. (2) can be written as

$$h_p = 1.03 \times 10^5 f\left[Q\,(ft^3/s)\right]^2 \qquad (3)$$

The friction factor depends on $Re = VD/\nu = 4Q/\pi D\nu$ and with $\nu = 4.9 \times 10^{-6}\ ft^2/s$ for gasoline

$$Re = \frac{4\,Q\,(ft^3/s)}{(\pi)(3/12\ ft)(4.9 \times 10^{-6}\frac{ft^2}{s})} = 1.04 \times 10^6\, Q\ (ft^3/s)$$

For commercial steel 3-in. diameter pipe (from Fig. 8.22)

$$\frac{\varepsilon}{D} = 5.8 \times 10^{-4}$$

Thus, for a given Q, f can be obtained from the Moody chart, or the Colebrook equation (Eq. 8.35), and h_p determined from Eq. (3). Tabulated values are given in the following table.

(con't)

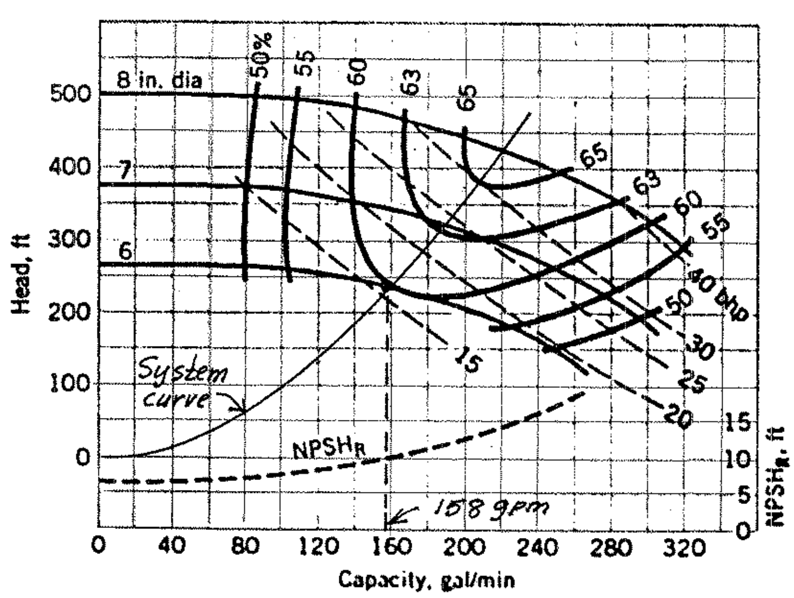
12.35 (Con't)

$Q\left(\frac{gal}{min}\right)$	$Q\left(\frac{ft^3}{s}\right)$	Re	f	$h_p\,(ft)$
40	0.0891	9.27×10^4	0.0208	17.0
80	0.178	1.85×10^5	0.0193	63.0
120	0.267	2.78×10^5	0.0187	137
160	0.357	3.71×10^5	0.0184	242
200	0.446	4.64×10^5	0.0182	373
240	0.535	5.56×10^5	0.0181	534

These data (h_p vs. Q) are plotted on Fig. 12.12 (reproduced below), and the flowrate at the intersection of the system curve and the pump curve is

$$Q = 158\ \frac{gal}{min}$$

Since at this flowrate the pump operates near peak efficiency this type of pump would appear to be a good choice if the 158 gal/min flowrate is at or near the desired flowrate.



12.36

12.36 Determine the new flowrate for the system described in Problem 12.35 if the pipe diameter is increased from 3 in. to 4 in. Is this pump still a good choice? Explain.

Refer to solution to Problem 12.23. With $D = 4/12 \text{ ft}$ Eq.(2) becomes

$$h_p = f \frac{(4000 \text{ ft})}{(\frac{4}{12} \text{ ft})} \frac{V^2}{(2)(32.2 \frac{ft}{s^2})} \qquad (2)$$

and

$$V = \frac{Q}{A} = \frac{Q\,(\frac{ft^3}{s})}{(\frac{\pi}{4})(\frac{4}{12} ft)^2}$$

so that

$$h_p = 2.45 \times 10^4 f \left[Q(\tfrac{ft^3}{s})\right]^2 \qquad (3)$$

The Reynolds number becomes

$$Re = \frac{4Q}{\pi D \nu} = \frac{4\, Q\,(ft^3/s)}{(\pi)(4/12 \text{ ft})(4.9 \times 10^{-6} \frac{ft^2}{s})} = 7.80 \times 10^5 Q\,(\tfrac{ft^3}{s})$$

For commercial steel 4-in. diameter pipe (from Fig. 8.22), $\frac{\varepsilon}{D} = 4.3 \times 10^{-4}$. Thus, for a given Q, f can be obtained from the Moody chart, or the Colebrook equation (Eq. 8.35), and h_p determined from Eq.(3). Tabulated values are given in the following table.

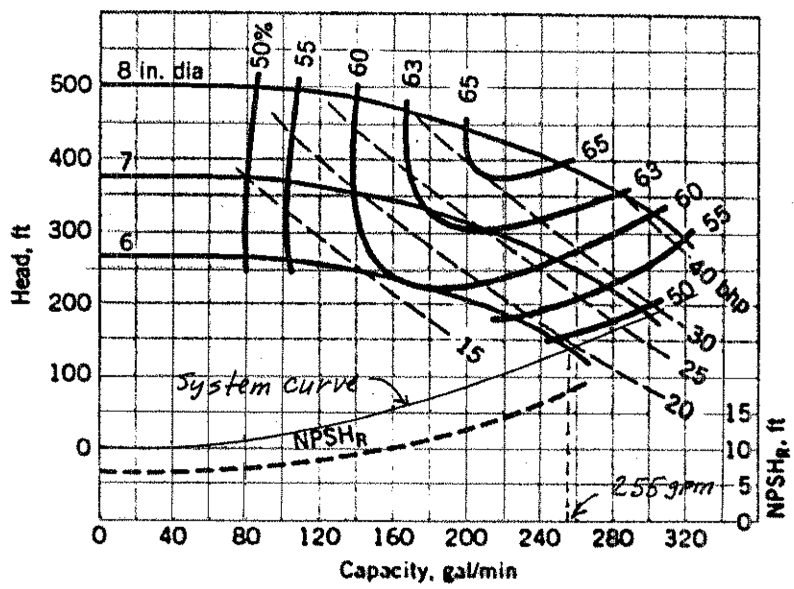
$Q\,(\frac{gal}{min})$	$Q\,(\frac{ft^3}{s})$	Re	f	h_p (ft)
40	0.0891	6.95×10^4	0.0211	4.1
80	0.178	1.39×10^5	0.0192	14.9
120	0.267	2.08×10^5	0.0183	32.0
160	0.357	2.78×10^5	0.0179	55.9
200	0.446	3.48×10^5	0.0176	85.8
240	0.535	4.17×10^5	0.0174	122
280	0.624	4.87×10^5	0.0172	164
320	0.713	5.56×10^5	0.0170	212

These data (h_p vs. Q) are plotted on Fig. 12.12 (reproduced on the following page), and the flowrate at the intersection of the system curve and the pump curve is

$$\underline{\underline{Q = 255 \frac{gal}{min}}}$$

(con't)

12.36 (Con't)



Since at this flowrate the pump efficiency is fairly low (~49%), this pump is no longer a good choice.

12.37

12.37 A centrifugal pump having the characteristics shown in Example 12.4 is used to pump water between two large open tanks through 100 ft of 8-in.-diameter pipe. The pipeline contains 4 regular flanged 90° elbows, a check valve, and a fully open globe valve. Other minor losses are negligible. Assume the friction factor $f = 0.02$ for the 100-ft section of pipe. If the static head (difference in height of fluid surfaces in the two tanks) is 30 ft, what is the expected flowrate? Do you think this pump is a good choice? Explain.

Application of the energy equation between the two free surfaces, points (1) and (2), gives

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \quad (1)$$

and with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, and $z_2 - z_1 = 30\text{ ft}$, Eq. (1) becomes

$$h_p = 30\text{ ft} + \Sigma h_L \quad (2)$$

The head loss term can be expressed as

$$\Sigma h_L = \left[4(0.3) + 10 + 2 + 0.02\frac{(100\text{ ft})}{(\frac{8}{12}\text{ ft})}\right]\frac{V^2}{2(32.2\frac{\text{ft}}{\text{s}^2})}$$

with the minor loss coefficients obtained from Table 8.3. Also,

$$V = \frac{Q}{A} = \frac{Q(\text{ft}^3/\text{s})}{(\frac{\pi}{4})(\frac{8}{12}\text{ ft})^2}$$

and Eq. (2) becomes

$$h_p = 30 + 2.06\left[Q(\text{ft}^3/\text{s})\right]^2$$

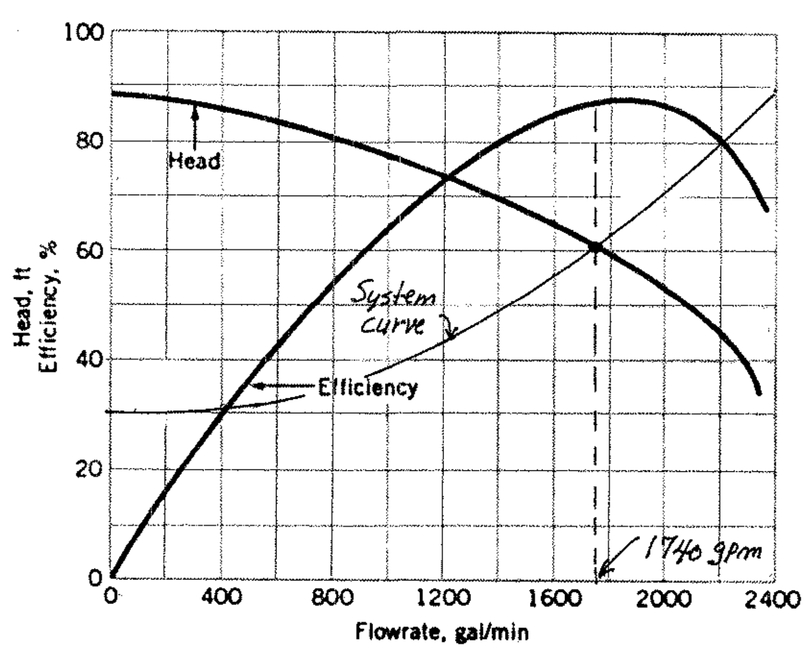
or the system equation can be written as

$$h_p = 30 + 1.02\times10^{-5}\left[Q(\tfrac{\text{gal}}{\text{min}})\right]^2 \quad (3)$$

The intersection of the system curve (Eq. 3) with the pump curve, as shown on the figure, indicates that

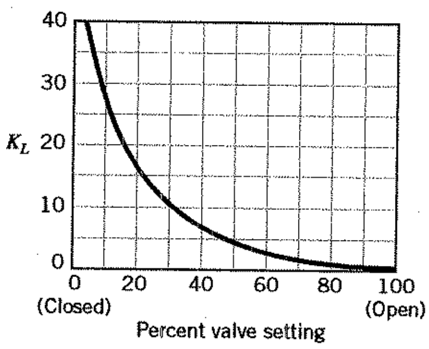
$$\underline{Q = 1740\ \frac{\text{gal}}{\text{min}}}$$

Since the efficiency at this flowrate is near peak efficiency, as shown on the figure, this pump would be satisfactory.

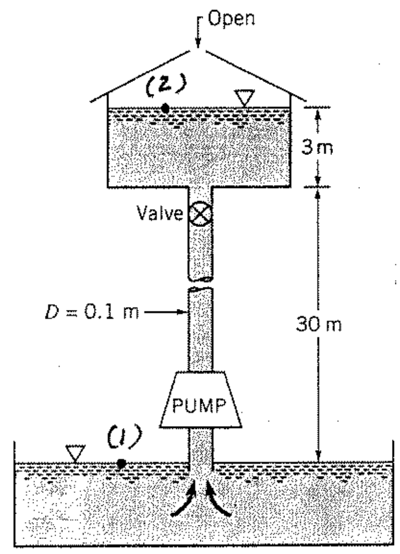


12.38

12.38 In a chemical processing plant a liquid is pumped from an open tank, through a 0.1-m-diameter vertical pipe, and into another open tank as shown in Fig. P12.38(*a*). A valve is located in the pipe, and the minor loss coefficient for the valve as a function of the valve setting is shown in Fig. P12.38(*b*). The pump head-capacity relationship is given by the equation $h_a = 52.0 - 1.01 \times 10^3 Q^2$ with h_a in meters when Q is in m^3/s. Assume the friction



(*b*)



(*a*)

■ **FIGURE P12.38**

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \Sigma h_L \quad (1)$$

and with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, and $z_2 - z_1 = 33\,m$, Eq. (1) becomes

$$h_p = 33\,m + \Sigma h_L \quad (2)$$

The head loss term can be expressed as

$$\Sigma h_L = \left(K_L + f\frac{\ell}{D}\right)\frac{V^2}{2g}$$

(a) With the valve open $K_L \approx 1.0$ (from Fig. P12.29 b) so that with $f = 0.02$, $\ell = 30\,m$, and $D = 0.1\,m$, Eq. (2) can be written as

$$h_p = 33\,m + \left[1.0 + 0.02\frac{(30\,m)}{(0.1\,m)}\right]\frac{V^2}{2\left(9.81\frac{m}{s^2}\right)} \quad (3)$$

and with

$$V = \frac{Q}{A} = \frac{Q\left(\frac{m^3}{s}\right)}{\left(\frac{\pi}{4}\right)(0.1\,m)^2}$$

Eq. (3) becomes

$$h_p = 33\,m + \left[1.0 + 6.0\right](826)\left[Q\left(\tfrac{m^3}{s}\right)\right]^2 \quad (4)$$

or

$$h_p = 33 + 5.78 \times 10^3\left[Q\left(\tfrac{m^3}{s}\right)\right]^2 \quad (5)$$

(con't)

12.38 (con't)

Since the pump equation is

$$h_p = 52.0 - 1.01\times 10^3\left[Q\left(\tfrac{m^3}{s}\right)\right]^2 \qquad (6)$$

Eq. (5) and Eq. (6) can be equated to determine the flowrate. Thus,

$$33 + 5.78\times 10^3 Q^2 = 52.0 - 1.01\times 10^3 Q^2$$

and

$$Q = \underline{\underline{0.0529\ \tfrac{m^3}{s}}}$$

(b) If the flowrate is to be cut in half so that $Q = 0.0529/2 = 0.0265\ m^3/s$, the head added by the pump is

$$h_p = 52.0 - 1.01\times 10^3\left(0.0265\ \tfrac{m^3}{s}\right)^2$$

$$= 50.6\ m$$

From Eq. (4) with K_L unknown

$$50.6\ m = 33\ m + (K_L + 6.0)(826)\left(0.0265\ \tfrac{m^3}{s}\right)^2$$

So that

$$K_L = 24.3$$

From Fig. 12.29 (b) the valve would be $\underline{\underline{13\%\ \text{open}}}$ to obtain this K_L

12.41

12.41 What is the rationale for operating two geometrically similar pumps differing in feature size at the same flow coefficient?

If the pumps are similar in geometry and other important ways, operating both of them at a flow coefficient associated with high efficiency would make sense.

12.42

12.42 A centrifugal pump having an impeller diameter of 1 m is to be constructed so that it will supply a head rise of 200 m at a flowrate of 4.1 m^3/s of water when operating at a speed of 1200 rpm. To study the characteristics of this pump, a 1/5 scale, geometrically similar model operated at the same speed is to be tested in the laboratory. Determine the required model discharge and head rise. Assume both model and prototype operate with the same efficiency (and therefore the same flow coefficient).

For similarity the model pump must operate at the same flow coefficient, Eq. 12.32, so that

$$\left(\frac{Q}{\omega D^3}\right)_m = \left(\frac{Q}{\omega D^3}\right)_p$$

where the subscript (m) refers to the model and (p) to the prototype. Thus,

$$Q_m = \frac{\omega_m}{\omega_p}\left(\frac{D_m}{D_p}\right)^3 Q_p$$

and with $\omega_m = \omega_p$, $D_m/D_p = 1/5$, and $Q_p = 4.1\,m^3/s$, then

$$Q_m = (1)\left(\frac{1}{5}\right)^3\left(4.1\,\frac{m^3}{s}\right) = \underline{\underline{0.0328\,\frac{m^3}{s}}}$$

From Eq. 12.33

$$\left(\frac{g h_a}{\omega^2 D^2}\right)_m = \left(\frac{g h_a}{\omega^2 D^2}\right)_p$$

so that

$$h_{a_m} = \frac{g_p}{g_m}\left(\frac{\omega_m}{\omega_p}\right)^2\left(\frac{D_m}{D_p}\right)^2 h_{a_p}$$

and with $g_p = g_m$, $\omega_m = \omega_p$, $D_m/D_p = 1/5$, and $h_{a_p} = 200\,m$, then

$$h_{a_m} = (1)(1)^2\left(\frac{1}{5}\right)^2(200\,m) = \underline{\underline{8.00\,m}}$$

12.43

12.43 A centrifugal pump with a 12-in.-diameter impeller requires a power input of 60 hp when the flowrate is 3200 gpm against a 60-ft head. The impeller is changed to one with a 10-in. diameter. Determine the expected flowrate, head, and input power if the pump speed remains the same.

For geometrically similar pumps operating at the same speed the effect of a change in impeller diameter is given by Eqs. 12.39, 12.40, 12.41. Thus,

$$\frac{Q_1}{Q_2} = \frac{D_1^3}{D_2^3} \qquad \text{(Eq. 12.39)}$$

and with $Q_1 = 3200\ gpm$, $D_1 = 12\ in.$, and $D_2 = 10\ in.$

$$Q_2 = \left(\frac{D_2}{D_1}\right)^3 Q_1 = \left(\frac{10\ in.}{12\ in.}\right)^3 (3200\ gpm) = \underline{\underline{1850\ gpm}}$$

From Eq. 12.40

$$\frac{h_{a_1}}{h_{a_2}} = \frac{D_1^2}{D_2^2} \qquad \text{(Eq. 12.40)}$$

so that with $h_{a_1} = 60\ ft$

$$h_{a_2} = \left(\frac{D_2}{D_1}\right)^2 h_{a_1} = \left(\frac{10\ in.}{12\ in.}\right)^2 (60\ ft) = \underline{\underline{41.7\ ft}}$$

Similarly from Eq. 12.41

$$\frac{\dot{W}_{shaft_1}}{\dot{W}_{shaft_2}} = \frac{D_1^5}{D_2^5} \qquad \text{(Eq. 12.41)}$$

and with $\dot{W}_{shaft_1} = 60\ hp$

$$\dot{W}_{shaft_2} = \left(\frac{D_2}{D_1}\right)^5 \dot{W}_{shaft_1} = \left(\frac{10\ in.}{12\ in.}\right)^5 (60\ hp) = \underline{\underline{24.1\ hp}}$$

12.44

12.44 Do the head-flowrate data shown in Fig. 12.12 appear to follow the similarity laws as expressed by Eqs. 12.39 and 12.40? Explain.

The data in Fig. 12.12 show the effect of changing impeller diameter on head-flowrate characteristics. According to the similarity laws expressed by Eq. 12.39 and Eq. 12.40

$$\frac{Q_1}{Q_2} = \frac{D_1^3}{D_2^3} \qquad \text{(Eq. 12.39)}$$

$$\frac{h_{a_1}}{h_{a_2}} = \frac{D_1^2}{D_2^2} \qquad \text{(Eq. 12.40)}$$

Thus, as the diameter is increased from 6 in. to 7 in. to 8 in. the flowrate increases according to Eq. 12.39 as

(from 6 in. to 7 in.) $Q_2 = \left(\frac{D_2}{D_1}\right)^3 Q_1 = \left(\frac{7\text{ in.}}{6\text{ in.}}\right)^3 Q_1 = 1.59\, Q_1$

and

(from 6 in. to 8 in.) $Q_2 = \left(\frac{8\text{ in.}}{6\text{ in.}}\right)^3 Q_1 = 2.37\, Q_1$

Similarly, from Eq. 12.40

(from 6 in. to 7 in.) $h_{a_2} = \left(\frac{D_2}{D_1}\right)^2 h_{a_1} = \left(\frac{7\text{ in.}}{6\text{ in.}}\right)^2 h_{a_1} = 1.36\, h_{a_1}$

and

(from 6 in. to 8 in.) $h_{a_2} = \left(\frac{8\text{ in.}}{6\text{ in.}}\right)^2 h_{a_1} = 1.78\, h_{a_1}$

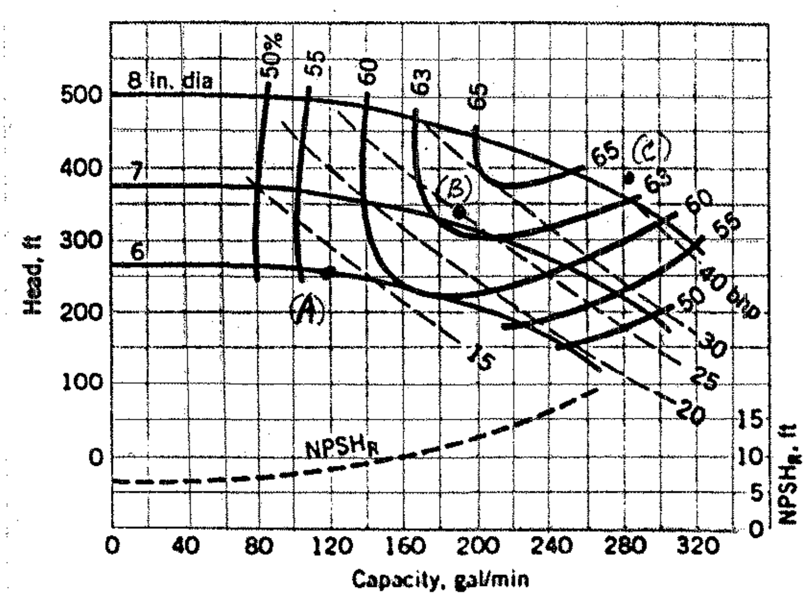
Thus, for any given point, such as (A) where $Q = 120$ gpm and $h_a = 250$ ft (see Fig. 12.12 on following page) for the 6-in. diameter impeller, the corresponding predicted point would be at (B) where

$$Q_2 = (1.59)(120\text{ gpm}) = 191\text{ gpm}$$

$$h_{a_2} = (1.36)(250\text{ ft}) = 340\text{ ft}$$

(con't)

12.44 (Con't)



Similarly, for the 8-in. diameter impeller the predicted point, point (C), would be at

$$Q_2 = (2.37)(120\ gpm) = 284\ gpm$$

and

$$h_{a_2} = (1.78)(250\ ft) = 445\ ft$$

Points (B) and (C) fall near the corresponding curves in Fig. 12.12 thereby demonstrating that they do appear to follow the similarity laws. Yes.

Note that according to the similarity laws the 6-in. diameter curve is simply translated to the right and upward to obtain the corresponding head-flowrate curves for the 7-in. and 8-in. diameter pumps. It is clear from Fig. 12.12 that this is generally how the three curves are related.

12.45

12.45 A centrifugal pump has the performance characteristics of the pump with the 6-in.-diameter impeller described in Fig. 12.12. Note that the pump in this figure is operating at 3500 rpm. What is the expected head gained if the speed of this pump is reduced to 2800 rpm while operating at peak efficiency?

From Fig. 12.12 for the 6-in. diameter impeller operating at 3500 rpm, $Q = 170 \text{ gpm}$ and $h_a = 230 \text{ ft}$ when operating at peak efficiency (see figure below). Thus, if the pump is still operated at peak efficiency with the speed reduced to 2800 rpm then from Eq. 12.36

$$\frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2} \qquad \text{(Eq. 12.36)}$$

so that

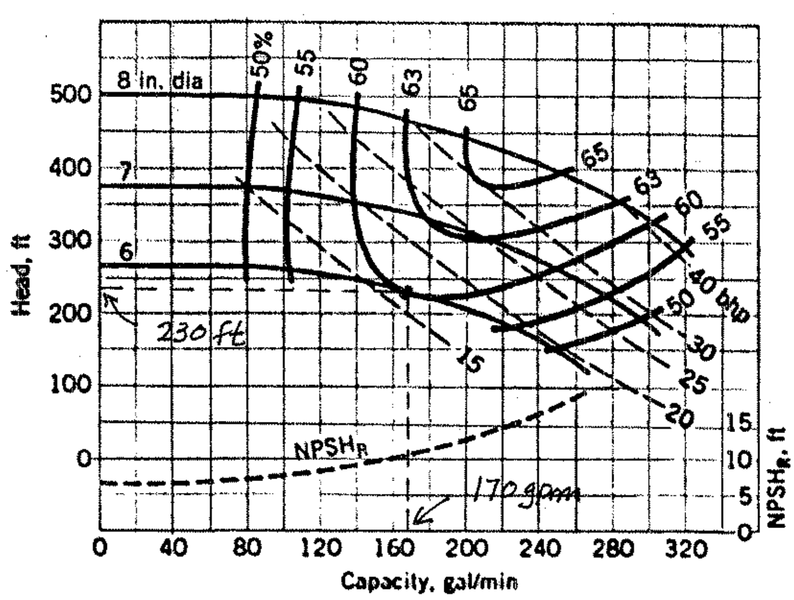
$$Q_2 = \frac{\omega_2}{\omega_1} Q_1 = \left(\frac{2800 \text{ rpm}}{3500 \text{ rpm}}\right)(170 \text{ gpm}) = \underline{\underline{136 \text{ gpm}}}$$

From Eq. 12.37

$$\frac{h_{a1}}{h_{a2}} = \frac{\omega_1^2}{\omega_2^2} \qquad \text{(Eq. 12.37)}$$

so that

$$h_{a_2} = \left(\frac{\omega_2}{\omega_1}\right)^2 h_{a_1} = \left(\frac{2800 \text{ rpm}}{3500 \text{ rpm}}\right)^2 (230 \text{ ft}) = \underline{\underline{147 \text{ ft}}}$$



12.46

12.46 A centrifugal pump provides a flowrate of 500 gpm when operating at 1750 rpm against a 200-ft head. Determine the pump's flowrate and developed head if the pump speed is increased to 3500 rpm.

For a given pump the effect of a change in speed on Q and h_a is given by Eqs. 12.36 and 12.37. Thus,

$$\frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2} \qquad (\text{Eq. } 12.36)$$

and with $Q_1 = 500\,gpm$, $\omega_1 = 1750\,rpm$, and $\omega_2 = 3500\,rpm$, then

$$Q_2 = \frac{\omega_2}{\omega_1} Q_1 = \frac{(3500\,rpm)}{(1750\,rpm)}(500\,gpm)$$

$$= \underline{\underline{1000\,gpm}}$$

Similarly,

$$\frac{h_{a_1}}{h_{a_2}} = \frac{\omega_1^2}{\omega_2^2} \qquad (\text{Eq. } 12.37)$$

so that with $h_{a_1} = 200\,ft$

$$h_{a_2} = \left(\frac{\omega_2}{\omega_1}\right)^2 h_{a_1} = \left(\frac{3500\,rpm}{1750\,rpm}\right)^2 (200\,ft)$$

$$= \underline{\underline{800\,ft}}$$

12.47

12.47 Explain how Fig. 12.18 was constructed from test data. Why is this use of specific speed important? Illustrate with a specific example.

A variety of pump configurations like the ones shown in Fig. 12.18 were tested over a range of flow rates. Performance data like those shown in Fig. 12.17 were acquired. For each pump configuration, the operation at maximum efficiency was noted and the specific speed, N_s, (Eq. 12. 43) was calculated for that condition of flow. These specific speed values calculated at maximum efficiency operation were then used to distribute the different pump configurations as shown in Fig. 12.18.

Specific speed is important because from desired design operational data (ω, Q, and h_a) a specific speed value can be determined. With that value of specific speed and Fig. 12.18 the designer can decide what kind of pump configuration to use for maximum efficiency operation. For example, at lower values of specific speed, a centrifugal pump is generally best. At higher values of specific speed, an axial-flow pump may be best. In between values of specific speed may suggest that a mixed-flow pump would serve most efficiently.

12.48

12.48 Use the data given in Problem 12.25 and plot the dimensionless coefficients C_H, $C_{\mathcal{P}}$, η versus C_Q for this pump. Calculate a meaningful value of specific speed, discuss its usefulness, and compare the result with data of Fig. 12.18.

From Problem 12.25 the following data were obtained:

Q (gpm)	20	40	60	80	100	120	140
h_a (ft)	92.8	92.5	87.9	83.5	77.3	69.5	59.5
η (%)	29.7	41.2	49.9	57.5	61.3	60.4	52.6
Power input (hp)	1.58	2.27	2.67	2.95	3.19	3.49	4.00

For $\omega = \left(1750\,\frac{rev}{min}\right)\left(2\pi\,\frac{rad}{rev}\right)\left(\frac{1}{60\,\frac{s}{min}}\right) = 183.3\,\frac{rad}{s}$ and $D = \frac{9}{12}$ ft, it follows that

$$C_Q = \frac{Q}{\omega D^3} = \frac{Q(gpm)/\left(7.48\,\frac{gal}{ft^3}\right)\left(60\,\frac{s}{min}\right)}{\left(183.3\,\frac{rad}{s}\right)\left(\frac{9}{12}\,ft\right)^3}$$

$$= 2.88 \times 10^{-5}\, Q\,(gpm)$$

$$C_H = \frac{g h_a}{\omega^2 D^2} = \frac{\left(32.2\,\frac{ft}{s^2}\right) h_a(ft)}{\left(183.3\,\frac{rad}{s}\right)^2\left(\frac{9}{12}\,ft\right)^2}$$

$$= 1.70 \times 10^{-3}\, h_a(ft)$$

$$C_{\mathcal{P}} = \frac{\dot{W}_{shaft}}{\rho \omega^3 D^5} = \frac{\dot{W}_{shaft}(hp)\left(550\,\frac{ft\cdot lb}{s\cdot hp}\right)}{\left(1.94\,\frac{slugs}{ft^3}\right)\left(183.3\,\frac{rad}{s}\right)^3\left(\frac{9}{12}\,ft\right)^5}$$

$$= 1.94 \times 10^{-4}\, \dot{W}_{shaft}\,(hp)$$

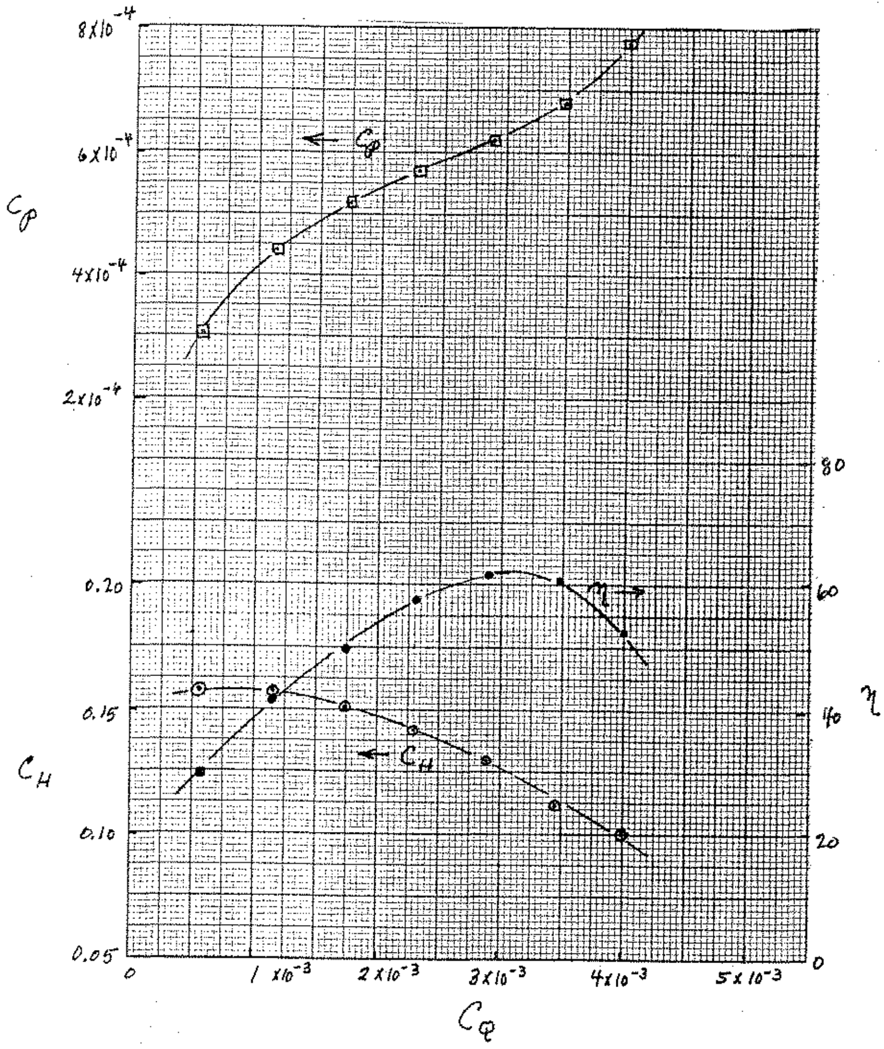
Based on the data above:

Q (gpm)	20	40	60	80	100	120	140
C_Q	5.76×10^{-4}	1.15×10^{-3}	1.73×10^{-3}	2.30×10^{-3}	2.88×10^{-3}	3.46×10^{-3}	4.03×10^{-3}
C_H	0.1581	0.1576	0.1498	0.1423	0.1317	0.1184	0.104
$C_{\mathcal{P}}$	3.07×10^{-4}	4.40×10^{-4}	5.18×10^{-4}	5.72×10^{-4}	6.19×10^{-4}	6.77×10^{-4}	7.76×10^{-4}
η	29.7	41.2	49.9	57.7	61.3	60.4	52.6

(con't)

12.48 (con't)

The plot of C_H, $C_{\mathcal{P}}$, η versus C_Q is shown below.



$$N_{sd} = \frac{\omega(rpm)\sqrt{Q(gpm)}}{[h_a(ft)]^{3/4}}$$

so for $Q = 100\ gpm$ at $\eta_{max} = 61.3\%$

$$N_{sd} = \frac{(1750\ rpm)\sqrt{(100\ gpm)}}{[(77.3\ ft)]^{3/4}} = 671$$

which is within the range of N_{sd} values for radial flow pumps in Fig. 12.18

12.49

12.49 In a certain application a pump is required to deliver 5000 gpm against a 300-ft head when operating at 1200 rpm. What type of pump would you recommend?

For $Q = 5000$ gpm, $h_a = 300$ ft, and $\omega = 1200$ rpm, the specific speed is

$$N_{sd} = \frac{\omega\,(\text{rpm})\sqrt{Q\,(\text{gpm})}}{[h_a\,(\text{ft})]^{3/4}}$$

$$= \frac{(1200\text{ rpm})\sqrt{5000\text{ gpm}}}{(300\text{ ft})^{3/4}}$$

$$= \underline{\underline{1180}}$$

From Fig. 12.18, at this specific speed a radial flow pump (centrifugal pump) would be recommended.

12.53

12.53 A certain axial-flow pump has a specific speed of $N_S = 5.0$. If the pump is expected to deliver 3000 gpm when operating against a 15-ft head, at what speed (rpm) should the pump be run?

Since

$$N_s = \frac{\omega\,(rad/s)\sqrt{Q\,(ft^3/s)}}{\left[g\,(ft/s^2)\;h_a\,(ft)\right]^{3/4}}$$

for $N_s = 5.0$, $g = 32.2\ ft/s^2$, $h_a = 15\ ft$, and with

$$Q = \frac{3000\ \frac{gal}{min}}{\left(7.48\ \frac{gal}{ft^3}\right)\left(60\ \frac{s}{min}\right)} = 6.68\ \frac{ft^3}{s}$$

it follows that

$$\omega\,(rad/s) = \frac{(5.0)\left[\left(32.2\ \frac{ft}{s^2}\right)(15\ ft)\right]^{3/4}}{\sqrt{6.68\ \frac{ft^3}{s}}}$$

$$= 199\ \frac{rad}{s}$$

Hence

$$\omega\,(rpm) = \frac{\left(199\ \frac{rad}{s}\right)\left(60\ \frac{s}{min}\right)}{2\pi\ \frac{rad}{rev}}$$

$$= \underline{\underline{1900\ rpm}}$$

12.54

12.54 A certain pump is known to have a capacity of 3 m^3/s when operating at a speed of 60 rad/s against a head of 20 m. Based on the information in Fig. 12.18, would you recommend a radial-flow, mixed-flow, or axial-flow pump?

Since

$$N_s = \frac{\omega\,(\text{rad/s})\sqrt{Q\,(\text{m}^3/\text{s})}}{\left[g\,(\text{m/s}^2)\;h_a\,(\text{m})\right]^{3/4}}$$

for $\omega = 60\ \text{rad/s}$, $Q = 3\ \text{m}^3/\text{s}$, $g = 9.81\ \text{m/s}^2$, and $h_a = 20\text{m}$

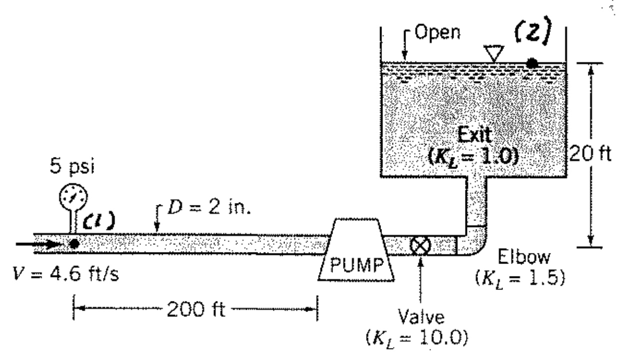
$$N_s = \frac{(60\ \text{rad/s})\sqrt{3\ \text{m}^3/\text{s}}}{\left[(9.81\ \text{m/s}^2)(20\ \text{m})\right]^{3/4}}$$

$$= \underline{\underline{1.98}}$$

From Fig. 12.18 with $N_s = 1.98$ the pump is a <u>mixed-flow pump.</u>

12.55

12.55 Fuel oil (sp. wt = 48.0 lb/ft^3, viscosity = 2.0×10^{-5} lb·s/ft^2) is pumped through the piping system of Fig. P12. with a velocity of 4.6 ft/s. The pressure 200 ft upstream from the pump is 5 psi. Pipe losses downstream from the pump are negligible, but minor losses are not (minor loss coefficients are given on the figure). (a) For a pipe diameter of 2 in. with a relative roughness $\varepsilon/D = 0.001$, determine the head that must be added by the pump. (b) For a pump operating speed of 1750 rpm, what type of pump (radial-flow, mixed-flow, or axial-flow) would you recommend for this application?



■ FIGURE P12.55

(a) $$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \sum h_L \qquad (1)$$

With $\gamma = 48.0\ \text{lb/ft}^3$, $p_1 = 5\text{psi}$, $p_2 = 0$, $V_1 = 4.6\ \text{ft/s}$, $V_2 = 0$, and $z_2 - z_1 = 20\ \text{ft}$, Eq. (1) becomes

$$\frac{\left(5\frac{lb}{in^2}\right)\left(144\frac{in.^2}{ft^2}\right)}{48.0\frac{lb}{ft^3}} + \frac{\left(4.6\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)} + h_p = 20ft + \sum h_L \qquad (2)$$

The head loss term can be expressed as

$$\sum h_L = \left[\underbrace{10.0}_{valve} + \underbrace{1.5}_{elbow} + \underbrace{1.0}_{exit} + f\,\frac{200ft}{2/12\ ft}\right]\frac{\left(4.6\frac{ft}{s}\right)^2}{(2)\left(32.2\frac{ft}{s^2}\right)}$$

The Reynolds number is

$$Re = \frac{\rho V D}{\mu} = \frac{\left(\frac{48.0\frac{lb}{ft^3}}{32.2\frac{ft}{s^2}}\right)\left(4.6\frac{ft}{s}\right)\left(\frac{2}{12}ft\right)}{2.0\times10^{-5}\ \frac{lb\cdot s}{ft^2}} = 5.71\times10^4$$

and with $\varepsilon/D = 0.001$ $\quad f = 0.024$ (from Fig. 8.23).

Thus, $h_L = 13.6\ ft$ and from Eq. (2)

$$h_p = \underline{\underline{18.3\ ft}}$$

(b) Since

$$Q = VA = \left(4.6\frac{ft}{s}\right)\left(\frac{\pi}{4}\right)\left(\frac{2}{12}ft\right)^2 = 0.100\ \frac{ft^3}{s}$$

or

$$Q = \left(0.100\frac{ft^3}{s}\right)\left(7.48\frac{gal}{ft^3}\right)\left(60\frac{s}{min}\right) = 45.0\ gpm$$

the specific speed at 1750 rpm is

$$N_{sd} = \frac{\omega\,(rpm)\sqrt{Q(gpm)}}{[h_a(ft)]^{3/4}} = \frac{(1750\,rpm)\sqrt{45.0\,gpm}}{[18.3\,ft]^{3/4}} = 1330$$

For this specific speed a radial-flow pump would be recommended for this application (see Fig. 12.18).

Master Typ...
10% Rec...
8 1/2 x ...

12.56

12.56 The axial-flow pump shown in Fig. 12.19 is designed to move 5000 gal/min of water over a head rise of 5 ft of water. Estimate the motor power requirement and the $U_2 V_{\theta 2}$ needed to achieve this flowrate on a continuous basis. Comment on any cautions associated with where the pump is placed vertically in the pipe.

From Eq. 12.21 we get the power equivalent to the head rise and flowrate involved. This is the minimum power required to achieve the performance specified.

$$P = \gamma Q h_a$$

$$P = \left(62.4 \frac{lb}{ft^3}\right)\left(5000 \frac{gal}{min}\right) \frac{1}{\left(7.48 \frac{gal}{ft^3}\right)} \frac{1}{\left(60 \frac{s}{min}\right)} (4 \, ft) \frac{1}{\left(550 \frac{ft.lb}{hp.s}\right)}$$

$$P = 5.1 \, hp$$

To estimate the shaft or motor power requirement, we need to assume the efficiency of the conversion of shaft or motor power into the pump performance specified.

$$P_{shaft} = \frac{P}{\eta} \quad \text{or for } 80\% \text{ efficiency}$$

$$P_{shaft} = \frac{5.1 \, hp}{0.8} = \underline{\underline{6.4 \, hp}}$$

$U_2 V_{\theta 2}$ and P_{shaft} are related in Eq. 12.4 through

$$P_{Shaft} = \dot{m} U_2 V_{\theta 2} = \rho A V U_2 V_{\theta 2} = \rho Q U_2 V_{\theta 2}$$

so

$$U_2 V_{\theta 2} = \frac{P_{shaft}}{\rho Q} = \frac{(6.4 \, hp)\left(32.2 \frac{lbm.s^2}{lbf.ft}\right)\left(7.48 \frac{gal}{ft^3}\right)\left(60 \frac{s}{min}\right)\left(550 \frac{ft.lb}{hp.s}\right)}{\left(62.4 \frac{lbm}{ft^3}\right)\left(5000 \frac{gal}{min}\right)}$$

$$U_2 V_{\theta 2} = \underline{\underline{163}} \frac{ft^2}{s^2}$$

(con't)

12.56 (con't)

The main caution in placing the pump vertically in the intake pipe is to do so in a way to avoid cavitation in the pump. The collapse of cavitation bubbles in the pump can erode pump blade and other wetted surfaces. Applying the energy equation, Eq. 5.84, between the free surface (1) and the pump entrance (2) we get

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - h_L$$

So

$$\frac{p_2}{\gamma} = \frac{p_1}{\gamma} + z_1 - z_2 - \frac{V_2^2}{2g} - h_L$$

and to maximize $\frac{p_2}{\gamma}$, we minimize $z_1 - z_2$. To achieve this we place the pump high vertically in the intake pipe. This will tend to keep p_2 high enough to avoid cavitation which occurs when p_2 and/or related pressures in the pump become less than the vapor pressure of the fluid.

Master Type...
10% Reduc...
8 1/2 x ...

12.61

12.61 Consider the Pelton wheel turbine illustrated in Figs. 12.24, 12.25, 12.26, and 12.27. This kind of turbine is used to drive the oscillating sprinkler shown in **Video V12.3** Explain how this kind of sprinkler is started, and subsequently operated at constant oscillating speed. What is the physical significance of the zero torque condition with the Pelton wheel rotating?

As shown on page 795 below Eq. 12.50

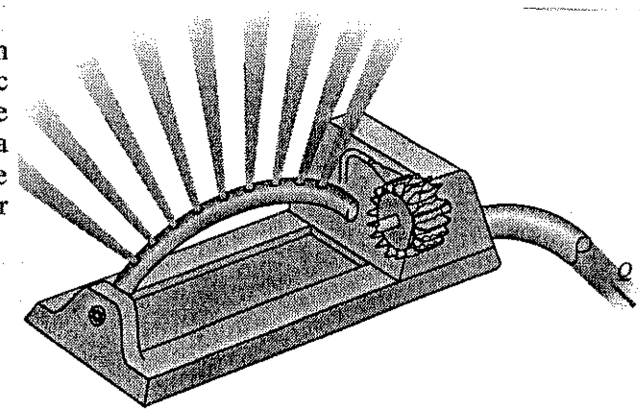
$$T_{shaft} = \dot{m} r_m (U - V_1)(1 - \cos\beta)$$

So for no rotation of the wheel or $U = 0$, the variation of T_{shaft} with changing $\dot{m}$ is linear. When T_{shaft} is just larger than the resisting torque provided by the sprinkler, the Pelton wheel rotates and drives the oscillation of the sprinkler. After wheel rotation and sprinkler oscillation begins, any constant value of $\dot{m}$ and T_{shaft} results in a constant value of U and thus rotation speed and also oscillation period.

If the shaft connecting the oscillating sprinkler to the Pelton wheel breaks during operation, the sprinkler will cease oscillating and the Pelton wheel will run at constant rotation speed corresponding to $U = V_1$.

12.62

12.62 A small Pelton wheel is used to power an oscillating lawn sprinkler as shown in **Video V12.3** and Fig. P12.62. The arithmetic mean radius of the turbine is 1 in., and the exit angle of the blade is 135° relative to the blade motion. Water is supplied through a single 0.20-in.-diameter nozzle at a speed of 50 ft/s. Determine the flowrate, the maximum torque developed, and the maximum power developed by this turbine.



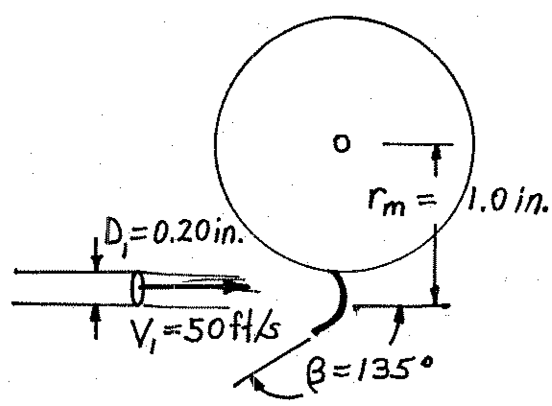
■ FIGURE P12.62

For the Pelton wheel shown

$$Q = A_1 V_1 = \frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4}\left(\frac{0.20}{12}\text{ft}\right)^2 \left(50\,\frac{\text{ft}}{\text{s}}\right)$$

or

$$Q = \underline{\underline{0.0109\,\frac{\text{ft}^3}{\text{s}}}}$$



From Fig. 11.22

$$T_{shaft_{max}} = \dot{m}\, r_m V_1 (1-\cos\beta)$$

and

$$\dot{W}_{shaft_{max}} = 0.25\,\dot{m}\, V_1^2 (1-\cos\beta)$$

where $\dot{m} = \rho Q = 1.94\,\frac{\text{slugs}}{\text{ft}^3}\left(0.0109\,\frac{\text{ft}^3}{\text{s}}\right) = 0.0211\,\frac{\text{slugs}}{\text{s}}$

Thus,

$$T_{shaft_{max}} = 0.0211\,\frac{\text{slugs}}{\text{s}}\left(\frac{1}{12}\text{ft}\right)\left(50\,\frac{\text{ft}}{\text{s}}\right)(1-\cos 135°) = 0.150\,\frac{\text{slugs}\cdot\text{ft}^2}{\text{s}^2}$$

$$= \underline{\underline{0.150\ \text{ft}\cdot\text{lb}}}$$

and

$$\dot{W}_{shaft_{max}} = 0.25\left(0.0211\,\frac{\text{slugs}}{\text{s}}\right)\left(50\,\frac{\text{ft}}{\text{s}}\right)^2(1-\cos 135°) = 22.5\,\frac{\text{slugs}\cdot\text{ft}^2}{\text{s}^3}$$

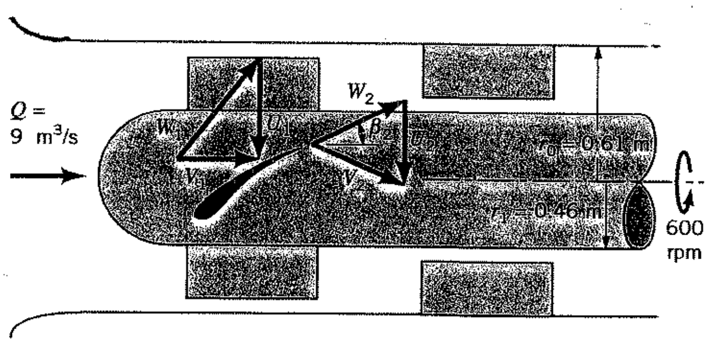
$$= 22.5\,\frac{\text{ft}\cdot\text{lb}}{\text{s}}$$

or

$$\dot{W}_{shaft_{max}} = 22.5\,\frac{\text{ft}\cdot\text{lb}}{\text{s}}\;\frac{1\ \text{hp}}{550\,\frac{\text{ft}\cdot\text{lb}}{\text{s}}} = \underline{\underline{0.0409\ \text{hp}}}$$

12.63

12.63 The single-stage, axial-flow turbomachine shown in Fig. P12.63 involves water flow at a volumetric flowrate of 9 m³/s. The rotor revolves at 600 rpm. The inner and outer radii of the annular flow path through the stage are 0.46 and 0.61 m, and $\beta_2 = 60°$. The flow entering the rotor row and leaving the stator row is axial when viewed from the stationary casing. Is this device a turbine or a pump? Estimate the amount of power transferred to or from the fluid.



■ **FIGURE P12.63**

$$\dot{W}_{shaft} = \dot{m}\left(U_2 V_{\theta 2} - U_1 V_{\theta 1}\right) \quad \text{where } V_{\theta 1} = 0 \qquad (1)$$

and

$U_2 = \omega r_{mean} = \omega \dfrac{(r_i + r_o)}{2}$. Thus, with $\omega = \left(600 \frac{rev}{min}\right)\left(\frac{1 min}{60 s}\right)\left(\frac{2\pi\, rad}{rev}\right) = 62.8 \frac{rad}{s}$

this gives

$$U_2 = \left(62.8 \frac{rad}{s}\right)\left(\frac{0.46 m + 0.61 m}{2}\right) = 33.6 \frac{m}{s}$$

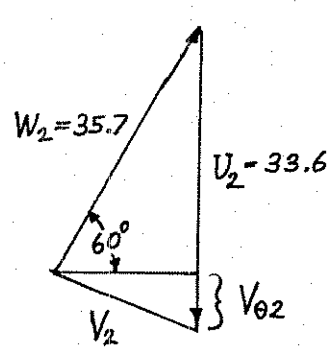
Also,

$$\dot{m} = \rho Q = 999 \frac{kg}{m^3}\left(9 \frac{m^3}{s}\right) = 8991 \frac{kg}{s}$$

But $Q = W_2 \cos 60° A_2$ or since $A_2 = \pi (r_o^2 - r_i^2)$

$$W_2 = \frac{9 \frac{m^3}{s}}{\pi \cos 60° (0.61^2 - 0.46^2) m^2} = 35.7 \frac{m}{s}$$

Hence, from the velocity triangle,



$$V_{\theta 2} = -W_2 \sin 60° + U_2$$

$$= -35.7 \sin 60° + 33.6 = 2.70 \frac{m}{s}$$

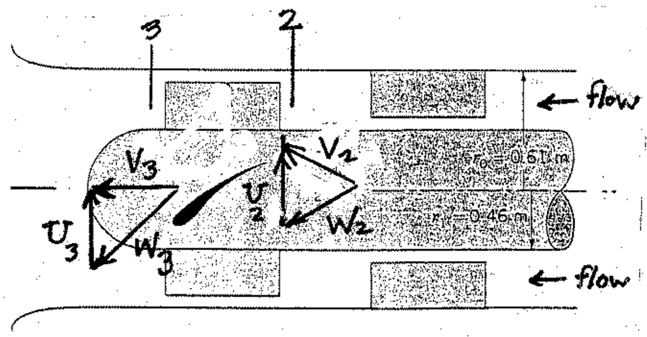
From Eq. (1):

$$\dot{W}_{shaft} = \left(8991 \frac{kg}{s}\right)\left(33.6 \frac{m}{s}\right)\left(2.70 \frac{m}{s}\right) = 8.16 \times 10^5 > 0$$

The device is an 816 kW pump

12.64

12.64 Describes what will happen when the flow through the turbomachine of Fig. P12.63 is in the opposite direction (right to left) and the shaft is freed up to rotate in response to the reversed flow.



When flow is reversed as shown in the sketch above, V_2, the velocity of the flow out of the stationary blade row (now a nozzle) will leave at approximately the blade exit angle. The magnitude of V_2 will depend on the magnitude of the flowrate Q. From the velocity triangles sketched above we conclude that the rotor will now move in a direction opposite to the one of problem 12.44. The rotor speed will depend on values of Q and the restraining shaft torque, T. From the velocity triangles we also conclude that the fluid forces on the moving blade sections are in the same direction as blade motion so the fluid is doing work on the rotor. The device is now acting as a turbine. $\dot{W}_{shaft} = \dot{m}(U_3 V_{\theta 3} - U_2 V_{\theta 2})$ may be used to determine shaft power.

12.65

12.65 For an air turbine of a dentist's drill like the one shown in Fig. E12.8 and **Video V12.4** calculate the average blade speed associated with a rotational speed of 350,000 rpm. Estimate the air pressure needed to run this turbine.

We calculate the average blade speed, U, from

$$U = r_m \omega = \left(\frac{r_i + r_o}{2}\right)\omega = \frac{(0.133+0.168)\,\text{in}}{(2)\left(12\,\frac{\text{in}}{\text{ft}}\right)}\left(350{,}000\,\frac{\text{rev}}{\text{min}}\right)\left(2\pi\,\frac{\text{rad}}{\text{s}}\right)\frac{1}{\left(60\,\frac{\text{s}}{\text{min}}\right)}$$

$$U = 459\,\frac{\text{ft}}{\text{s}}$$

To estimate the air pressure, p_0, needed to run this turbine, we estimate that the nozzle exit velocity is about twice as large as the average blade velocity or

$$V = 2U = 918\ \text{ft/s}$$

So, the corresponding Mach number, M, is approximately

$$M = \frac{V}{c} = \frac{918\ \text{ft/s}}{1100\ \text{ft/s}}$$ with c estimated to be about $1100\,\frac{\text{ft}}{\text{s}}$

$$M = 0.83$$

Then from Fig. D.1 the value of $\frac{p}{p_0}$ corresponding to $M = 0.83$ is

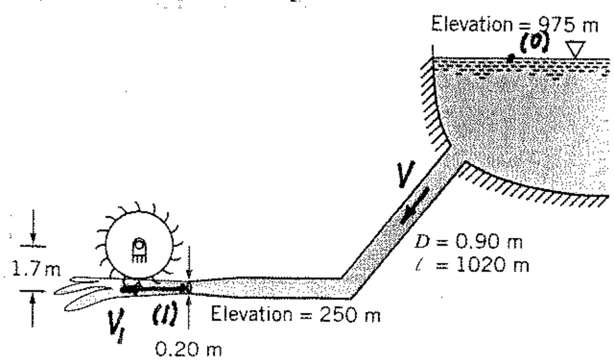
$$\frac{p}{p_0} = 0.1 \quad \text{and} \quad p_0 = \frac{p}{0.1} = 10p$$

So

$$p_0 \approx 10\,(14.7\,\text{psia}) = \underline{\underline{147\ \text{psia}}}$$

12.66

12.66 Water for a Pelton wheel turbine flows from the headwater and through the penstock as shown in Fig. P12.66. The effective friction factor for the penstock, control valves, and the like is 0.032 and the diameter of the jet is 0.20 m. Determine the maximum power output.



■ FIGURE P12.66

$\dot{W}_{shaft} = \rho Q U (U - V_1)(1 - \cos\beta)$ or for maximum power $\beta = 180^\circ$, $U = \frac{V_1}{2}$

Thus,

$$\dot{W}_{shaft\,max} = -\rho Q \frac{V_1^2}{2} \qquad (1)$$

But $\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + f \frac{\ell}{D} \frac{V^2}{2g}$ where $p_0 = p_1 = 0$, $z_0 = 975\,m$, $z_1 = 250\,m$, and $V_0 = 0$

Hence,

$$z_0 = z_1 + \frac{V_1^2}{2g} + f \frac{\ell}{D} \frac{V^2}{2g} \quad \text{where } A_1 V_1 = A V \qquad (2)$$

or $\frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} D^2 V$. That is $V = \left(\frac{d_1}{D}\right)^2 V_1 = \left(\frac{0.2\,m}{0.9\,m}\right)^2 V_1 = 0.0494\, V_1$

so that Eq. (2) becomes:

$$975\,m = 250\,m + \frac{V_1^2}{2(9.81\,\frac{m}{s^2})}\left[1 + 0.032\left(\frac{1020\,m}{0.9\,m}\right)(0.0494)^2\right] \quad \text{where } V_1 \sim \frac{m}{s}$$

or $V_1 = 114.3\,\frac{m}{s}$

Hence,

$$Q = A_1 V_1 = \frac{\pi}{4}(0.2\,m)^2 (114.3\,\frac{m}{s}) = 3.56\,\frac{m^3}{s}$$

Therefore, from Eq. (1):

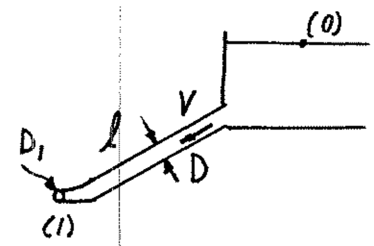
$$\dot{W}_{shaft\,max} = -(999\,\frac{kg}{m^3})(3.56\,\frac{m^3}{s})\frac{(114.3\,\frac{m}{s})^2}{2} = 23.2 \times 10^6\,\frac{N \cdot m}{s} = \underline{\underline{23,200\,kW}}$$

12.67

12.67 Water to run a Pelton wheel is supplied by a penstock of length ℓ and diameter D with a friction factor f. If the only losses associated with the flow in the penstock are due to pipe friction, shown that the maximum power output of the turbine occurs when the nozzle diameter, D_1, is given by $D_1 = D/(2f\ell/D)^{1/4}$.

$\dot{W}_{shaft} = \rho Q U (U - V_1)(1-\cos\beta)$ so the maximum power output occurs with $\beta = 180^\circ$ and $U = \frac{V_1}{2}$. Thus,

$$\dot{W}_{shaft} = \rho Q \frac{V_1^2}{2} \quad \text{where} \qquad (1)$$



$$\frac{p_0}{\gamma} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + f\frac{\ell}{D}\frac{V^2}{2g}$$

But $p_0 = p_1 = 0$, $V_0 = 0$, and $z_0 - z_1 = h$. Thus,

$h = \frac{V_1^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g}$ where since $A_1V_1 = AV$ or $\frac{\pi}{4}D_1^2V_1 = \frac{\pi}{4}D^2V$ we have

$$V_1 = \left(\frac{D}{D_1}\right)^2 V$$

Therefore, $h = \frac{V_1^2}{2g}\left[1 + f\frac{\ell}{D}\frac{D_1^4}{D^4}\right]$ or $\frac{V_1^2}{2g} = \frac{h}{\left[1 + f\ell\frac{D_1^4}{D^5}\right]}$ and Eq. (1) gives

$$\dot{W}_{shaft} = \frac{\rho Q h}{\left(1 + f\frac{\ell}{D^5}D_1^4\right)} = \frac{\rho\frac{\pi}{4}D_1^2 V_1 h}{\left(1 + f\frac{\ell}{D^5}D_1^4\right)}, \text{ but } V_1 = \frac{\sqrt{2gh}}{\left(1 + f\frac{\ell}{D^5}D_1^4\right)^{\frac{1}{2}}} \qquad (2), (3)$$

For this problem f, ℓ, D and h are constants; D_1 is variable.
Thus, from Eqs. (2) and (3):

$$\dot{W}_{shaft} = \frac{K D_1^2}{(1 + cD_1^4)^{3/2}} \text{ where } K = \text{const., and } c = \text{const.} = f\frac{\ell}{D^5}$$

Note: $\dot{W}_{shaft} \to 0$ as $D_1 \to 0$ and as $D_1 \to \infty$. To find the D_1 that gives maximum power over all, set $\frac{d\dot{W}_{shaft}}{dD_1} = 0$

$$\frac{d\dot{W}_{shaft}}{dD} = \frac{2KD_1}{(1+cD_1^4)^{3/2}} + \frac{(-\frac{3}{2})KD_1^2}{(1+cD_1^4)^{5/2}}(c)4D_1^3 = 0$$

or $\frac{2KD_1}{(1+cD_1^4)^{3/2}}\left[1 - \frac{3cD_1^4}{(1+cD_1^4)}\right] = 0$, or $1 + cD_1^4 = 3cD_1^4$, or $D_1^4 = \frac{1}{2c}$

$$\text{Thus, } D_1 = \frac{1}{\left(2f\frac{\ell}{D^5}\right)^{1/4}} = \underline{\underline{\frac{D}{\left(2f\frac{\ell}{D}\right)^{1/4}}}}$$

12.68

12.68 A hydraulic turbine operating at 180 rpm with a head of 100 feet develops 20,000 horsepower. Estimate the power if the same turbine were to operate under a head of 50 ft.

Since hydraulic turbine flow is incompressible, we use the dimensionless parameters developed for hydraulic pumps, namely, flow, head and power coefficients. For this situation we specify operation at the same efficiency and thus flow coefficient with one half the head. Thus, head coefficient remains constant and

$$\left.\frac{g h_T}{\omega^2 D^2}\right)_1 = \left.\frac{g h_T}{\omega^2 D^2}\right)_2$$

so with $D_1 = D_2$ and $g_1 = g_2$:

$$\frac{100}{(180)^2} = \frac{50}{\omega_2^2} \quad \text{or} \quad \omega_2 = \underline{\underline{127 \text{ rpm}}}$$

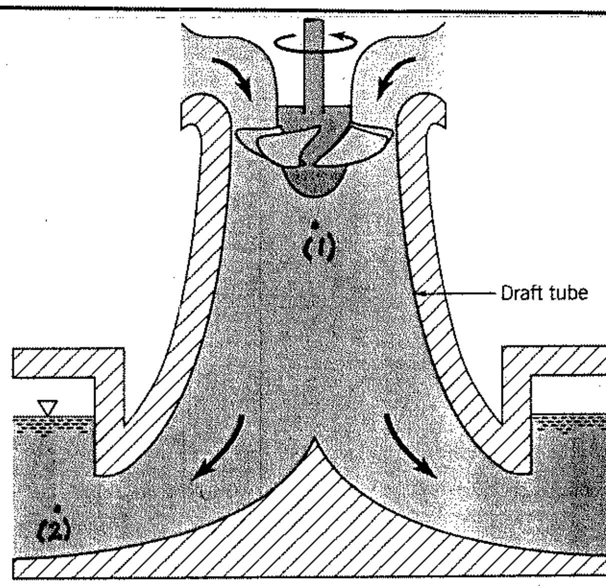
Also power coefficient is the same so $\left.\frac{\dot{W}_{shaft}}{\rho\,\omega^3 D^5}\right)_1 = \left.\frac{\dot{W}_{shaft}}{\rho\,\omega^3 D^5}\right)_2$

so with $D_1 = D_2$ and $\rho_1 = \rho_2$:

$$\frac{20{,}000}{(100)^3} = \frac{\dot{W}_{shaft_2}}{(127)^3} \quad \text{or} \quad \dot{W}_{shaft_2} = \underline{\underline{41{,}000 \text{ hp}}}$$

12.69

12.69 Draft tubes as shown in Fig. P12.69 are often installed at the exit of Kaplan and Francis turbines. Explain why such draft tubes are advantageous.



■ FIGURE P12.69

Without the draft tube there would be a relatively high speed exit jet (speed V_1, pressure $p_1 = 0$). With the draft tube (which acts as a diffuser) the exit speed is much smaller ($V_2 \approx 0$, $p_2 = 0$). From Bernoulli equation it follows that $p_1 < 0$ (with the draft tube). Hence there is a larger head available to the turbine. More energy can be removed from the fluid.

12.70

12.70 Turbines are to be designed to develop 30,000 horsepower while operating under a head of 70 ft and an angular velocity of 60 rpm. What type of turbines is best suited for this purpose? Estimate the flowrate needed.

$\dot{W}_{shaft} = 30{,}000\ hp$; $h_T = 70 ft$; and $\omega = 60\ rpm$ so that

$$N'_{sd} = \frac{\omega\sqrt{\dot{W}_{shaft}}}{(h_T)^{5/4}} = \frac{60\sqrt{3\times10^4}}{(70)^{5/4}} = 51.3$$

For this value a Francis turbine would be appropiate.

Also, since $\dot{W}_{shaft} = \gamma Q h_T$ it follows that

$$Q = \frac{\dot{W}_{shaft}}{\gamma h_T} = \frac{(30{,}000\ hp)(550\ \frac{ft\cdot lb}{s}/hp)}{(62.4\ \frac{lb}{ft^3})(70 ft)} = \underline{\underline{378\ \frac{ft^3}{s}}}$$

12.71

12.71 Show how you would estimate the relationship between feature size and power production for a wind turbine like the one shown in **Video V12.1.**

To estimate the relationship between feature size and power production for a wind turbine we use the dimensionless pi terms of Eqs. 12.29 and 12.30 which are applicable for this incompressible flow. For similar turbines and operating conditions

$$\frac{\dot{W}_{shaft\ 1}}{\rho_1 \omega_1^3 D_1^5} = \frac{\dot{W}_{shaft\ 2}}{\rho_2 \omega_2^3 D_2^5}$$

and

$$\frac{g h_{a1}}{\omega_1^2 D_1^2} = \frac{g h_{a2}}{\omega_2^2 D_2^2}$$

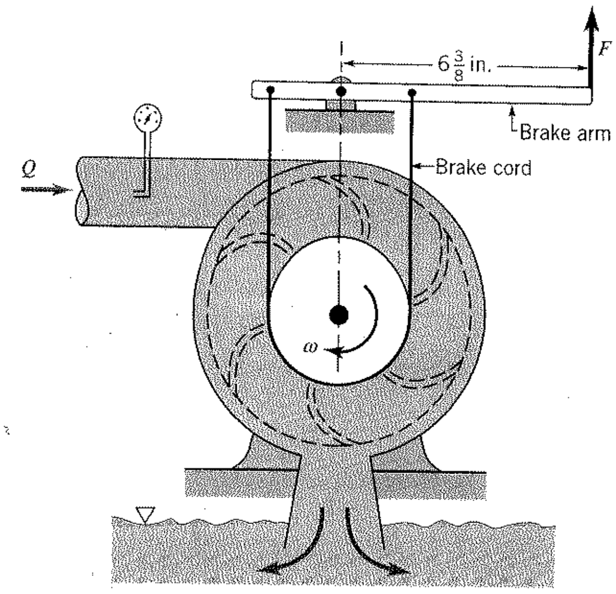
Since $\rho_1 = \rho_2$ and $h_{a1} = h_{a2}$, we combine and get

$$\frac{\dot{W}_{shaft\ 1}}{\dot{W}_{shaft\ 2}} = \frac{D_1^2}{D_2^2}$$

Or power varies with feature size squared.

12.72

12.72 Test data for the small Francis turbine shown in Fig. P12.72 is given in the table below. The test was run at a constant 32.8 ft head just upstream of the turbine. The Prony brake on the turbine output shaft was adjusted to give various angular velocities, and the force on the brake arm, F, was recorded. Use the given data to plot curves of torque as a function of angular velocity and turbine efficiency as a function of angular velocity.

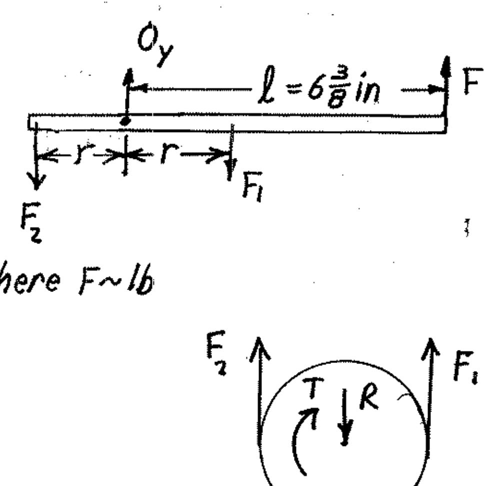


ω (rpm)	Q (ft³/s)	F (lb)
0	0.129	2.63
1000	0.129	2.40
1500	0.129	2.22
1870	0.124	1.91
2170	0.118	1.49
2350	0.0942	0.876
2580	0.0766	0.337
2710	0.068	0.089

■ FIGURE P12.72

Since $\Sigma M_0 = 0$ for the brake arm it follows that $F\ell = F_1 r - F_2 r$

Also, the torque on the turbine is $T = F_1 r - F_2 r$



or $T = F\ell = \left(\frac{6\frac{3}{8}}{12} ft\right) F = 0.531\, F$ ft·lb where F~lb (1)

Also, $\eta = \dfrac{T\omega}{\gamma Q h_T}$ where $h_T = 32.8$ ft

Thus,

$$\eta = \frac{(T\, ft\cdot lb)(\omega \frac{rev}{min})\left(\frac{1 min}{60 s}\right)\left(\frac{2\pi\, rad}{rev}\right)}{(62.4 \frac{lb}{ft^3})(Q \frac{ft^3}{s})(32.8 ft)}$$

or

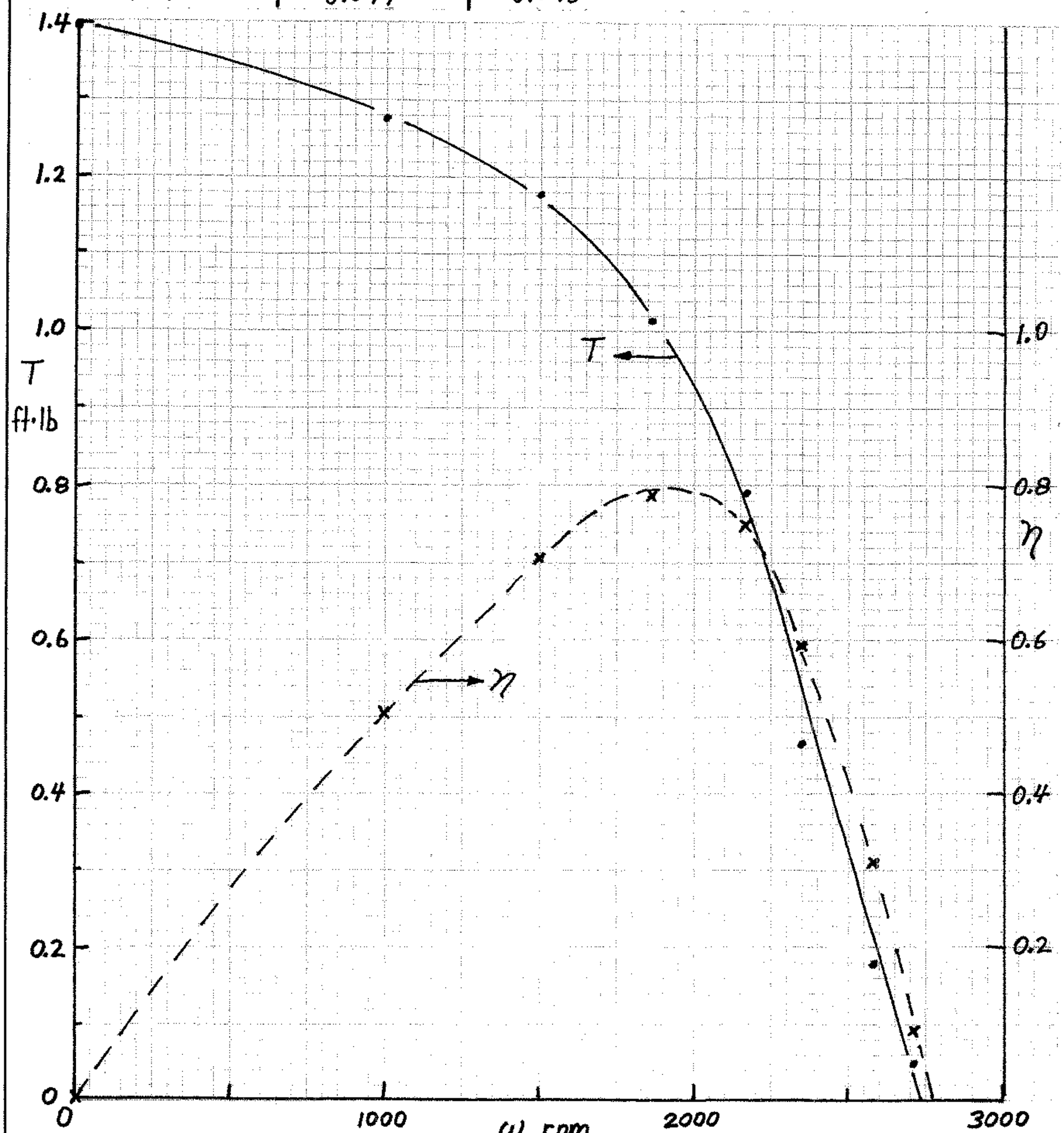
$\eta = 5.116 \times 10^{-5} \dfrac{T\omega}{Q}$ where $T \sim$ ft·lb, $\omega \sim$ rpm, $Q \sim$ cfs (2)

Values of T and η are given in the table below and plotted in the graphs shown.

(con't)

12.72 (con't)

ω, rpm	T, ft·lb	η
0	1.397	0
1000	1.275	0.506
1500	1.179	0.701
1870	1.015	0.783
2170	0.792	0.745
2350	0.465	0.593
2580	0.179	0.308
2710	0.047	0.096

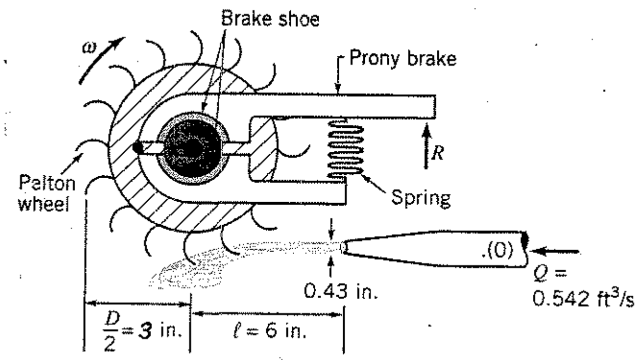


12.74

12.74 The device shown in Fig. P12.74 is used to investigate the power produced by a Pelton wheel turbine. Water supplied at a constant flowrate issues from a nozzle and strikes the turbine buckets as indicated. The angular velocity, ω, of the turbine wheel is varied by adjusting the tension on the Prony brake spring, thereby varying the torque, T_{shaft}, applied to the output shaft. This torque can be determined from the measured force, R, needed to keep the brake arm stationary as $T_{shaft} = F\ell$, where ℓ is the moment arm of the brake force.

Experimentally determined values of ω and R are shown in the following table. Use these results to plot a graph of torque as a function of the angular velocity. On another graph plot the power output, $\dot{W}_{shaft} = T_{shaft}\,\omega$, as a function of the angular velocity. On each of these graphs plot the theoretical curves for this turbine, assuming 100 percent efficiency.

Compare the experimental and theoretical results and discuss some possible reasons for any differences between them.



■ FIGURE P12.74

ω (rpm)	R (lb)
0	2.47
360	1.91
450	1.84
600	1.69
700	1.55
940	1.17
1120	0.89
1480	0.16

(a) Experimental: $T = R\ell = (0.5\text{ ft})R$ or $T = 0.5R$ ft·lb, where $R \sim$ lb (1)

and $\dot{W}_{shaft} = T\omega = T\left(\omega \frac{rev}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(\frac{2\pi\,rad}{rev}\right)$

or

$\dot{W}_{shaft} = 0.1047\,T\omega\ \frac{ft\cdot lb}{s}$, where $T \sim$ ft·lb, $\omega \sim$ rpm (2)

Values of ω, T, and $\dot{W}_{shaft}$ are given in the table and graph below.

(b) Theoretical: $T = \dot{m}r(U - V_1)(1 - \cos\beta)$ where assume $\beta = 180°$,

$$V_1 = \frac{Q}{A_1} = \frac{0.542\ \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{0.43}{12}\,ft\right)^2} = 53.7\ \frac{ft}{s}\text{, and}$$

$$\dot{m} = \rho Q = \left(1.94\ \frac{slugs}{ft^3}\right)\left(0.542\ \frac{ft^3}{s}\right) = 0.105\ \frac{slugs}{s}$$

Hence, with $U = \omega\frac{D}{2} = \left(\frac{3}{12}ft\right)\left(\frac{2\pi\omega}{60}\ \frac{rad}{s}\right) = 0.0262\,\omega\ \frac{ft}{s}$, $\omega \sim$ rpm

$$T = \left(0.105\ \frac{slugs}{s}\right)\left(\frac{3}{12}\,ft\right)\left[0.0262\,\omega - 53.7\right]\frac{ft}{s}$$

or

$T = 1.41\left[4.88\times10^{-4}\omega - 1\right]$ ft·lb, where $\omega \sim$ rpm (3)

(con't)

12.74 (con't)

Also, $\dot{W}_{shaft} = T\omega = T\left(\frac{2\pi}{60}\omega\right) = 0.1047\,T\omega \;\frac{ft\cdot lb}{s}$, where $T \sim ft\cdot lb$, $\omega \sim rpm$ (4)

Values of T and $\dot{W}_{shaft}$ from Eqs. (3) and (4) are plotted in the graph below.

	experiment		theory	
ω, rpm	T, ft·lb	$\dot{W}_{shaft}$, $\frac{ft\cdot lb}{s}$	$-T$, ft·lb	$-\dot{W}_{shaft}$, $\frac{ft\cdot lb}{s}$
0	1.235	0	1.41	0
360	0.955	36.0	1.16	43.8
450	0.920	43.3	1.100	51.8
600	0.845	53.1	0.997	62.6
700	0.775	56.8	0.928	68.0
940	0.585	57.6	0.763	75.1
1120	0.445	52.2	0.639	75.0
1480	0.080	12.4	0.392	60.7