11.1

11.1 Distinguish between flow of an ideal gas and inviscid flow of a fluid.

The flow of an ideal gas involves a gas that obeys the equation of state, Eq. 11.1

$$\rho = \frac{p}{RT}$$

and for which internal energy, $\check{u}$, is a function of temperature only. An ideal gas may have non-zero viscosity.

The inviscid flow of a fluid involves a fluid that has zero viscosity. That fluid may or may not be an ideal gas.

11.3

11.3 Five pounds mass of air are heated in a closed, rigid container from 80 °F, 15 psia to 500 °F. Estimate the final pressure of the air and the entropy rise involved.

To determine the final pressure, p_{final}, we can use the ideal gas equation (Eq. 11.1). Thus, for constant $\frac{mass}{volume}$ = density,

$$p_{final} = \frac{p_{initial}\,T_{final}}{T_{initial}} = \frac{(15\text{ psia})(960\,^\circ R)}{540\,^\circ R} = \underline{\underline{26.7\text{ psia}}}$$

Eq. 11.22 may be used to determine the entropy rise, $s_2 - s_1$. Thus,

$$s_2 - s_1 = c_p \ln \frac{T_{final}}{T_{initial}} - R \ln \frac{p_{final}}{p_{initial}}$$

and

From Table 1.7

$$s_2 - s_1 = \left(6006\,\frac{ft\cdot lb}{slug\cdot{}^\circ R}\right)\ln\left(\frac{960\,^\circ R}{540\,^\circ R}\right) - \left(1716\,\frac{ft\cdot lb}{slug\cdot{}^\circ R}\right)\ln\left(\frac{26.7\text{ psia}}{15\text{ psia}}\right) = \underline{\underline{2466}}\,\frac{ft\cdot lb}{slug\cdot{}^\circ R}$$

11.4

11.4 Air flows steadily between two sections in a duct. At section (1), the temperature and pressure are $T_1 = 80\ °C$, $p_1 = 301$ kPa(abs), and at section (2), the temperature and pressure are $T_2 = 180\ °C$, $p_2 = 181$ kPa(abs). Calculate the **(a)** change in internal energy between sections (1) and (2), **(b)** change in enthalpy between sections (1) and (2), **(c)** change in density between sections (1) and (2), **(d)** change in entropy between sections (1) and (2). How would you estimate the loss of available energy between the two sections of this flow?

(a) Eq. 11.5 may be used to evaluate the change in internal energy, $\check{u}_2 - \check{u}_1$. Thus,

$$\check{u}_2 - \check{u}_1 = c_v (T_2 - T_1) = \left(717.2 \frac{J}{kg \cdot K}\right)(453\,K - 353\,K) = \underline{\underline{71{,}720 \frac{J}{kg}}}$$

(b) Eq. 11.9 may be used to evaluate the change in enthalpy, $\check{h}_2 - \check{h}_1$. Thus,

$$\check{h}_2 - \check{h}_1 = c_p (T_2 - T_1) = \left(1004 \frac{J}{kg \cdot K}\right)(453\,K - 353\,K) = \underline{\underline{100{,}400 \frac{J}{kg}}}$$

(c) The ideal gas equation (Eq. 11.1) may be used to evaluate the density at each section. Thus,

$$\rho_2 - \rho_1 = \frac{p_2}{RT_2} - \frac{p_1}{RT_1} = \frac{1}{R}\left(\frac{p_2}{T_2} - \frac{p_1}{T_1}\right)$$

or

$$\rho_2 - \rho_1 = \frac{1}{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)} \left[\frac{\left(181{,}000 \frac{N}{m^2}\right)}{(453\,K)} - \frac{\left(301{,}000 \frac{N}{m^2}\right)}{(353\,K)}\right] = \underline{\underline{-1.58 \frac{kg}{m^3}}}$$

From Table 1.8 ↑

(d) Eq. 11.22 may be used to evaluate the change in entropy, $s_2 - s_1$. Thus,

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = \left(1004 \frac{J}{kg \cdot K}\right) \ln\left[\frac{(453\,K)}{(353\,K)}\right] - \left(286.9 \frac{N \cdot m}{kg \cdot K}\right) \ln\left[\frac{(181\,kPa)}{(301\,kPa)}\right]$$

or

$$s_2 - s_1 = \underline{\underline{396}} \ \frac{J}{kg \cdot K}$$

(con't)

11.4 (con't)

Since the flow involves a significant change in density, see solution to part (c) above, it is compressible and Eq. 5.108 must be used to evaluation the loss in available energy between sections (1) and (2). So from Eq. 5.108 we get

$$loss = \check{u}_2 - \check{u}_1 + \int_1^2 p\,d\left(\frac{1}{\rho}\right) - q_{\substack{net\\in}}$$

and to complete this solution we need more information so we can evaluate the integral and $q_{\substack{net\\in}}$.

11.5

11.5 Does the entropy change during the process of Example 11.2 indicate a loss of available energy by the flowing fluid?

We combine Eq. 5.106

$$d\check{u} + p\,d\left(\frac{1}{\rho}\right) - \delta q_{\substack{net\\in}} = \delta(loss)$$

with Eq. 5.92

$$T\,ds = d\check{u} + p\,d\left(\frac{1}{\rho}\right)$$

to get

$$T\,ds - \delta q_{\substack{net\\in}} = \delta(loss)$$

and conclude that if this flow is adiabatic ($\delta q_{\substack{net\\in}} = 0$) then entropy change is related to loss.

11.6

11.6 As demonstrated in **Video V11.1**, fluid density differences in a flow may be seen with the help of a schlieren optical system. Discuss what variables affect fluid density and the different ways in which a variable density flow can be achieved.

For an ideal gas:

$$\rho = \frac{p}{RT}$$

so changes in density, ρ, will accompany changes in pressure, p, gas composition, R, and/or temperature, T. Variations in fluid velocity and/or heating and cooling may result in pressure and temperature changes. Changes in gas composition that affect the value of the gas constant, R, will result in changes of density, ρ.

11.7

11.7 Describe briefly how a schlieren optical visualization system (**Videos V11.1** and **V11.4**, also Fig. 11.4) works. How else might density changes in a fluid flow be made visible to the eye?

Density variations in a transparent flowing fluid result in variations in the local speed of light through the fluid. These light speed variations result in changes in light ray direction and phase. Changes in light ray direction result in local variations in perceived light brightness. The shadowgraph and schlieren methods make visible these variations in light brightness. An interferometer makes visible the local variations in light ray phase. A good description of these three flow vizualization methods may be found in The Handbook of Fluid Dynamics edited by Richard W. Johnson and published by the CRC Press (1998).

11.8

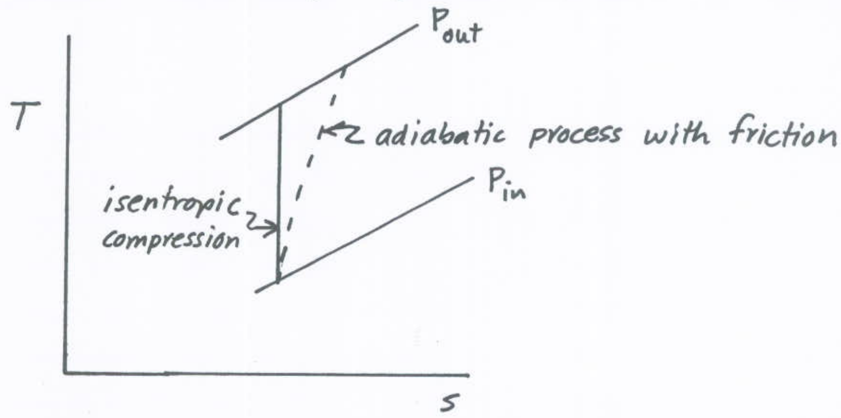
11.8 Explain why the Bernoulli equation (Eq. 3.7) cannot be accurately used for compressible flows.

Refer to Section 3.8.1 Compressibility Effects

11.9

11.9 Air at 14.7 psia and 70 °F is compressed adiabatically by a centrifugal compressor to a pressure of 100 psia. What is the minimum temperature rise possible? Explain.

The minimum temperature rise would occur with an adiabatic and frictionless process which involves a constant entropy or isentropic flow. According to the second law of thermodynamics, Eq. 5.101, the entropy must increase or remain constant during an adiabatic process, it cannot decrease. The T-s diagram sketched below illustrates how the isentropic process results in a minimum temperature rise.



For the isentropic process, Eq. 11.24 is valid. Thus,

$$T_{\substack{out \\ minimum}} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{k-1}{k}} = (530°R)\left(\frac{100\,psia}{14.7\,psia}\right)^{\frac{1.4-1}{1.4}} = 917°R$$

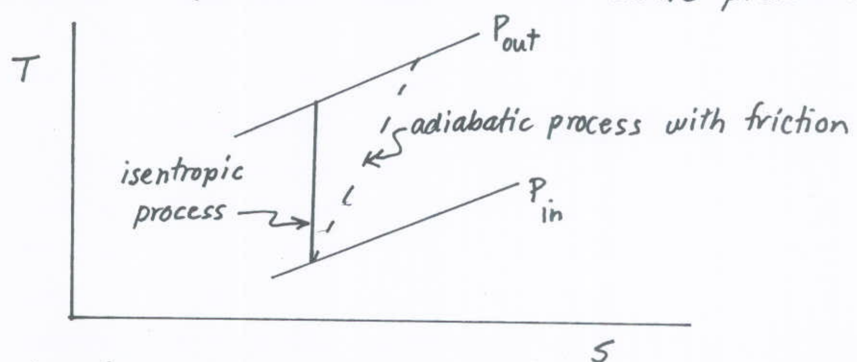
and

$$T_{\substack{out \\ minimum}} - T_{in} = 917°R - 530°R = \underline{\underline{387°R}}$$

11.10

11.10 Methane is compressed adiabatically from 100 kPa(abs) and 25 °C to 200 kPa(abs). What is the minimum compressor exit temperature possible? Explain.

The minimum compressor exit temperature would occur with an adiabatic and frictionless process which involves a constant entropy or isentropic flow. According to the second law of thermodynamics, Eq. 5.101, the entropy must increase or remain constant during an adiabatic process, it cannot decrease. The T-s diagram sketched below illustrates how the isentropic process results in a lower exit temperature than any actual adiabatic process between the same pressures.



For the isentropic compression, we conclude from Eq. 11.24 that

$$T_{out\ minimum} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{k-1}{k}}$$

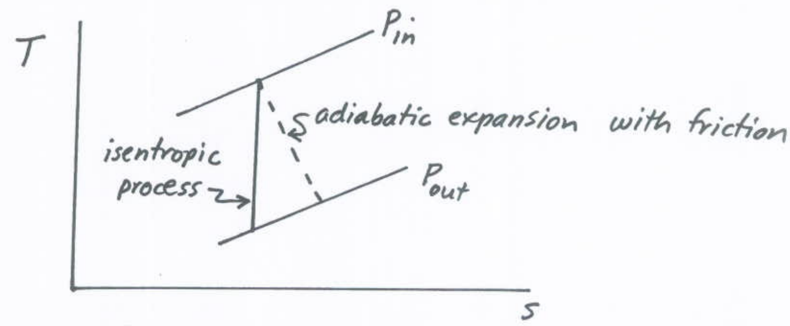
or

$$T_{out\ minimum} = (298\ K)\left(\frac{200\ kPa}{100\ kPa}\right)^{\frac{1.31-1}{1.31}} = \underline{\underline{351\ K}}$$

11.11

11.11 Air expands adiabatically through a turbine from a pressure and temperature of 180 psia, 1600 °R to a pressure of 14.7 psia. If the actual temperature change is 85% of the ideal temperature change, determine the actual temperature of the expanded air and the actual enthalpy and entropy differences across the turbine.

To determine the actual temperature of the expanded air and the actual enthalpy and entropy differences across the turbine we need first to determine the ideal temperature change across the turbine. The ideal temperature change across the turbine is associated with an adiabatic and frictionless and thus isentropic turbine expansion. The actual process involves a smaller temperature change as illustrated with the T-s diagram sketch below.



Eq. 11.24 is valid for the isentropic expansion. Thus,

$$T_{\substack{out\\ideal}} = T_{in}\left(\frac{P_{out}}{P_{in}}\right)^{\frac{k-1}{k}} = (1600\,°R)\left(\frac{14.7\,psia}{180\,psia}\right)^{\frac{1.4-1}{1.4}} = 782\,°R$$

Since

$$\left(T_{\substack{out\\actual}} - T_{in}\right) = 0.85\left(T_{\substack{out\\ideal}} - T_{in}\right)$$

then

$$T_{\substack{out\\actual}} = 0.85\,(782\,°R - 1600\,°R) + 1600\,°R = \underline{\underline{905\,°R}}$$

The actual enthalpy difference, $\check{h}_{\substack{out\\actual}} - \check{h}_{in}$, may be obtained with Eq. 11.9. Thus,

$$\check{h}_{\substack{out\\actual}} - \check{h}_{in} = c_p\left(T_{\substack{out\\actual}} - T_{in}\right) = \left(6006\,\frac{ft\cdot lb}{slug\cdot °R}\right)(905\,°R - 1600\,°R) = \underline{\underline{-4.17\times10^{6}\,\frac{ft\cdot lb}{slug}}}$$

The actual entropy difference, $s_{\substack{out\\actual}} - s_{in}$, may be calculated with Eq. 11.22. Thus,

$$s_{\substack{out\\actual}} - s_{in} = c_p\ln\left(\frac{T_{\substack{out\\actual}}}{T_{in}}\right) - R\ln\left(\frac{P_{out}}{P_{in}}\right) = \left(6006\,\frac{ft\cdot lb}{slug\cdot °R}\right)\ln\left(\frac{905\,°R}{1600\,°R}\right) - \left(1716\,\frac{ft\cdot lb}{slug\cdot °R}\right)\ln\left(\frac{14.7\,psia}{180\,psia}\right)$$

From Table 1.7

or

$$s_{\substack{out\\actual}} - s_{in} = \underline{\underline{877}}\,\frac{ft\cdot lb}{slug\cdot °R}$$

11.12

11.12 An expression for the value of c_p for carbon dioxide as a function of temperature is

$$c_p = 286 - \frac{1.15 \times 10^5}{T} + \frac{2.49 \times 10^6}{T^2}$$

where c_p is in (ft · lb)/(lbm · °R) and T is in °R. Compare the change in enthalpy of carbon dioxide using the constant value of c_p (see Table 1.7) with the change in enthalpy of carbon dioxide using the expression above, for $T_2 - T_1$ equal to **(a)** 10 °R, **(b)** 1000 °R, **(c)** 3000 °R. Set $T_1 = 540$ °R.

For constant c_p, the change in enthalpy, $\check{h}_2 - \check{h}_1$, may be evaluated with Eq. 11.9. Thus,

$$(\check{h}_2 - \check{h}_1)_{\substack{\text{constant}\\ c_p}} = c_p (T_2 - T_1)$$

For varying c_p, the change in enthalpy, $\check{h}_2 - \check{h}_1$, may be evaluated with Eq. 11.8. Thus,

$$\check{h}_2 - \check{h}_1 = \int_{T_1}^{T_2} c_p \, dT = \int_{T_1}^{T_2} \left(286 - \frac{1.15 \times 10^5}{T} + \frac{2.49 \times 10^6}{T^2}\right) dT$$

or

$$(\check{h}_2 - \check{h}_1)_{\substack{\text{varying}\\ c_p}} = 286 \frac{\text{ft.lb}}{\text{lbm.°R}} (T_2 - T_1) - 1.15 \times 10^5 \frac{\text{ft.lb}}{\text{lbm}} \ln\left(\frac{T_2}{T_1}\right) - 2.49 \times 10^6 \frac{\text{ft.lb.°R}}{\text{lbm}} \left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

(a) For $T_1 = 540$ °R and $T_2 = 550$ °R

$$(\check{h}_2 - \check{h}_1)_{\substack{\text{constant}\\ c_p}} = \left(152 \frac{\text{ft.lb}}{\text{lbm.°R}}\right)(550\text{°R} - 540\text{°R}) = \underline{\underline{1520}} \ \frac{\text{ft.lb}}{\text{lbm}}$$

and

$$(\check{h}_2 - \check{h}_1)_{\substack{\text{varying}\\ c_p}} = \left(286 \frac{\text{ft.lb}}{\text{lbm.°R}}\right)(550\text{°R} - 540\text{°R}) - \left(1.15 \times 10^5 \frac{\text{ft.lb}}{\text{lbm}}\right) \ln\left(\frac{550\text{°R}}{540\text{°R}}\right) - \left(2.49 \times 10^6 \frac{\text{ft.lb.°R}}{\text{lbm}}\right)\left(\frac{1}{550\text{°R}} - \frac{1}{540\text{°R}}\right)$$

$$(\check{h}_2 - \check{h}_1)_{\substack{\text{varying}\\ c_p}} = \underline{\underline{1580}} \ \frac{\text{ft.lb}}{\text{lbm}}$$

(con't)

11.12 (con't)

(b) For $T_1 = 540\,°R$ and $T_2 = 1540\,°R$

$$(\check{h}_2 - \check{h}_1)_{\text{constant } c_p} = \left(152 \frac{ft \cdot lb}{lbm \cdot °R}\right)(1540°R - 540°R) = \underline{\underline{1.52 \times 10^5}} \frac{ft \cdot lb}{lbm}$$

and

$$(\check{h}_2 - \check{h}_1)_{\text{varying } c_p} = \left(286 \frac{ft \cdot lb}{lbm \cdot °R}\right)(1540°R - 540°R)$$

$$- \left(1.15 \times 10^5 \frac{ft \cdot lb}{lbm}\right) \ln\left(\frac{1540°R}{540°R}\right)$$

$$- \left(2.49 \times 10^6 \frac{ft \cdot lb \cdot °R}{lbm}\right)\left(\frac{1}{1540°R} - \frac{1}{540°R}\right)$$

$$(\check{h}_2 - \check{h}_1)_{\text{varying } c_p} = \underline{\underline{1.95 \times 10^5}} \frac{ft \cdot lb}{lbm}$$

(c) For $T_1 = 540°R$ and $T_2 = 3540°R$

$$(\check{h}_2 - \check{h}_1)_{\text{constant } c_p} = \left(152 \frac{ft \cdot lb}{lbm \cdot °R}\right)(3540°R - 540°R) = \underline{\underline{4.56 \times 10^5}} \frac{ft \cdot lb}{lbm}$$

$$(\check{h}_2 - \check{h}_1)_{\text{varying } c_p} = \left(286 \frac{ft \cdot lb}{lbm \cdot °R}\right)(3540°R - 540°R) - \left(1.15 \times 10^5 \frac{ft \cdot lb}{lbm}\right) \ln\left(\frac{3540°R}{540°R}\right)$$

$$- \left(2.49 \times 10^6 \frac{ft \cdot lb \cdot °R}{lbm}\right)\left(\frac{1}{3540°R} - \frac{1}{540°R}\right)$$

$$(\check{h}_2 - \check{h}_1)_{\text{varying } c_p} = \underline{\underline{6.80 \times 10^5}} \frac{ft \cdot lb}{lbm}$$

11.14

11.14 Confirm the speed of sound for air at 70 °F listed in Table B.3.

Eq. 11.36 is suitable for calculating the speed of sound in air. Thus,

$$c = \sqrt{RTk} = \sqrt{\left(1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right) \frac{(530\,^\circ R)(1.401)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 1129 \frac{ft}{s}$$

From Table B.3

$c = 1128 \frac{ft}{s}$ for air at $70^\circ F$. The values of c are comparable.

11.15

11.15 From Table B.1 we can conclude that the speed of sound in water at 60 °F is 4814 ft/s. Is this value of c consistent with the value of bulk modulus, E_v, listed in Table 1.5?

The speed of sound in water may be approximated from a nominal value of the bulk modulus, E_v, and density, ρ, with Eq. 11.38. Thus

$$c = \sqrt{\frac{E_v}{\rho}} = \sqrt{\frac{\left(3.12 \times 10^5 \frac{lb}{in.^2}\right)\left(144 \frac{in^2}{ft^2}\right)}{\left(1.94 \frac{slugs}{ft^3}\right)\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{4812 \frac{ft}{s}}}$$

From Table B.1

$$c = 4814 \frac{ft}{s}$$

11.16

11.16 If the observed speed of sound in steel is 5300 m/s, determine the bulk modulus of elasticity of steel in N/m^3. The density of steel is nominally 7790 kg/m^3. How does your value of E_v for steel compare with E_v for water at 15.6 °C? Compare the speeds of sound in steel, water, and air at standard atmospheric pressure and 15 °C and comment on what you observe.

The speed of sound, c, is related to the bulk modulus of elasticity, E_v, and density, ρ, by Eq. 11.38 as follows

$$c = \sqrt{\frac{E_v}{\rho}}$$

Thus

$$E_v = \rho c^2$$ Table 1.7).

and for steel

$$E_{v_{steel}} = \left(7790 \frac{kg}{m^3}\right)\left(5300 \frac{m}{s}\right)^2 \left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)$$

or

$$E_{v_{steel}} = \underline{\underline{2.19 \times 10^{11}}} \frac{N}{m^2}$$

For water at 15.6 °C we get from Table 1.6 m Table 1.7.

$$E_{v_{water}} = 2.15 \times 10^9 \frac{N}{m^2}$$

For water at 15.6 °C

$$c_{water} = \sqrt{\frac{E_v}{\rho}} = \sqrt{\frac{\left(2.15 \times 10^9 \frac{N}{m^2}\right)}{\left(999 \frac{kg}{m^3}\right)\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = \underline{\underline{1470}} \frac{m}{s}$$

For steel

$c_{steel} = \underline{\underline{5300}} \frac{m}{s}$ which is much higher than the speed of sound in water

For air at 15 °C we get from Table B.4

$$c_{air} = 340.4 \text{ m/s}$$

The least compressible material, steel, involves the largest speed of sound. The most compressible material, air, involves the smallest speed of sound. This matches our intuition.

11.17

11.17 Using information provided in Table C.1, develop a table of speed of sound in ft/s as a function of elevation for U.S. standard atmosphere.

We can use Eq. 11.36 to determine the speed of sound in U.S. standard atmosphere at the elevations listed in Table C.1. Thus,

$$c = \sqrt{RTk}$$

We use $R = 1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}$ and $k = 1.40$ from Table 1.7. For absolute temperature we add $460\ {}^\circ R$ to ${}^\circ F$. For altitude $= -5000$ ft

$$c = \sqrt{\frac{\left(1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(538.84\ {}^\circ R)(1.40)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 1136 \frac{ft}{s}$$

For all elevations, the same procedure shown above was used. The results are:

altitude ft	c ft/s
-5000	1136
0	1117
5000	1097
10,000	1078
15,000	1058
20,000	1037
25,000	1016
30,000	995
35,000	973
40,000	968
45,000	968
50,000	968
60,000	968
70,000	971
80,000	978
90,000	984
100,000	991
150,000	1073
200,000	1028
250,000	944

11.18

11.18 Using information provided in Table C.2, develop a table of speed of sound in m/s as a function of elevation for U.S. standard atmosphere.

We can use Eq. 11.36 to determine the speed of sound in U.S. standard atmosphere at the elevations listed in Table C.2. Thus,

$$c = \sqrt{RTk}$$

We use $R = 286.9 \frac{J}{kg \cdot K}$ and $k = 1.40$ from Table 1.8. For absolute temperature we add 273K to °C. For altitude = -1000 m,

$$c = \sqrt{\frac{\left(286.9 \frac{J}{kg \cdot K}\right)(294.5\,K)(1.40)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = 344 \frac{m}{s}$$

For all elevations, the above procedure was used. The results are:

altitude m	c m/s
-1000	344
0	340
1000	336
2000	332
3000	328
4000	324
5000	320
6000	316
7000	312
8000	308
9000	304
10,000	299
15,000	295
20,000	295
25,000	298
30,000	302
40,000	317
50,000	330
60,000	315
70,000	297
80,000	282

11.19

11.19 Determine the Mach number of a car moving in standard air at a speed of **(a)** 25 mph, **(b)** 55 mph, and **(c)** 100 mph.

The Mach number is the ratio of local velocity to speed of sound. Thus

$$Ma = \frac{V}{c}$$

For standard air

$$c = \sqrt{RTk} = \sqrt{\left(1716 \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)(519\ {}^\circ R)(1.4)} = 1117 \frac{ft}{s}$$

or

$$c = \left(1117 \frac{ft}{s}\right)\frac{\left(3600 \frac{s}{hr}\right)}{\left(5280 \frac{ft}{mi}\right)} = 761.6 \text{ mph}$$

(a) For $V = 25$ mph

$$Ma = \frac{25 \text{ mph}}{761.6 \text{ mph}} = \underline{\underline{0.0328}}$$

(b) For $V = 55$ mph

$$Ma = \frac{55 \text{ mph}}{761.6 \text{ mph}} = \underline{\underline{0.0722}}$$

(c) For $V = 100$ mph

$$Ma = \frac{100 \text{ mph}}{761.6 \text{ mph}} = \underline{\underline{0.131}}$$

11.23

11.23 At a given instant of time, two of the pressure waves, each moving at the speed of sound, emitted by a point source moving with constant velocity in a fluid at rest are shown in Fig. P11.23 Determine the Mach number involved and indicate with a sketch the instantaneous location of the point source.

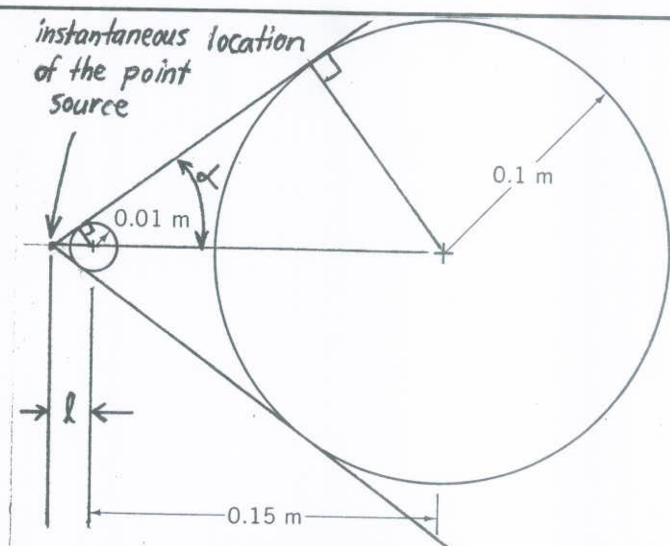


FIGURE P11.23

The Mach number associated with the motion of the point source involved in the sketch above is easily obtained with Eq. 11.39 as shown below.

$$Ma = \frac{1}{\sin\alpha}$$

From the sketch above we note that

$$\sin\alpha = \frac{0.01\,m}{\ell} = \frac{0.1\,m}{0.15\,m + \ell}$$

Thus

$$(0.01\,m)(0.15\,m + \ell) = (0.1\,m)\,\ell$$

or

$$\ell = \frac{(0.01\,m)(0.15\,m)}{(0.09\,m)} = \underline{\underline{0.0167\,m}}$$

and

$$\sin\alpha = \frac{0.01\,m}{0.0167\,m} = 0.599$$

Thus

$$Ma = \frac{1}{\sin\alpha} = \frac{1}{0.599} = \underline{\underline{1.67}}$$

11.24

11.24 At a given instant of time, two of the pressure waves, each moving at the speed of sound, emitted by a point source moving with constant velocity in a fluid at rest are shown in Fig. P11.24. Determine the Mach number involved and indicate with a sketch the instantaneous location of the point source.

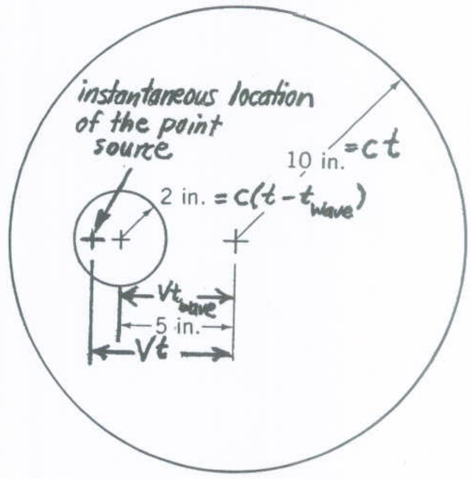


FIGURE P11.24

To determine the Mach number, Ma, we use

$$Ma = \frac{Vt_{wave}}{c\,t_{wave}} \qquad (1)$$

However, from the sketch above we have

$$c(t - t_{wave}) = 2 \text{ in.} = ct - ct_{wave} = 10 \text{ in.} - ct_{wave}$$

Thus,

$$ct_{wave} = 10 \text{ in.} - 2 \text{ in.} = 8 \text{ in.}$$

and with Eq. 1

$$Ma = \frac{5 \text{ in.}}{8 \text{ in.}} = \underline{\underline{0.625}}$$

Also

$$Ma = \frac{Vt}{ct} = \frac{Vt}{10 \text{ in.}} = 0.625$$

Thus,

$$Vt = (0.625)(10 \text{ in.}) = \underline{\underline{6.25 \text{ in.}}}$$

11.25

11.25 Sound waves are very small amplitude pressure pulses that travel at the "speed of sound." Do very large amplitude waves such as a blast wave caused by an explosion (see **Video V11.7**) travel less than, equal to, or greater than the speed of sound? Explain.

The speed of sound is the speed at which an infinitesimal pressure disturbance travels through a fluid and it represents the minimum speed of this disturbance. Finite pressure disturbances travel faster than sound waves because the larger pressure difference acts as a driver of faster movement.

11.26

11.26 How would you estimate the distance between you and an approaching storm front involving lightning and thunder?

One way to estimate the distance between you and approaching storm clouds, x, is to count the number of seconds, t, between seeing the lightning and hearing thunder. Using an approximate value of the speed of sound, $1145\ \frac{ft}{s}$ (see Table B.3) we can approximate distance, x from

$$x = \left(1145\ \frac{ft}{s}\right)(t)$$

11.27

11.27 If a person inhales helium and then talks, his or her voice sounds like "Donald Duck." Explain why this happens.

The speed of sound in helium is nearly three times the speed of sound in air.

11.28

11.28 If a high-performance aircraft is able to cruise at a Mach number of 3.0 at an altitude of 80,000 ft, how fast is this in **(a)** mph, **(b)** ft/s, **(c)** m/s?

(b) With Eq. 11.46

$$V = (Ma)\,c$$

and at 80,000 ft in U.S. standard atmosphere, we have from the solution of problem 11.16

$$c = 978\ \frac{ft}{s}$$

Thus

$$V = (3.0)\left(978\ \frac{ft}{s}\right) = \underline{\underline{2930}}\ \frac{ft}{s}$$

(a) Then

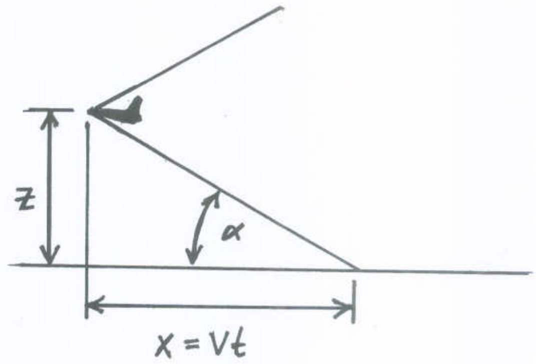
$$V = \left(2930\ \frac{ft}{s}\right)\frac{\left(3600\ \frac{s}{hr}\right)}{\left(5280\ \frac{ft}{mi}\right)} = \underline{\underline{2000}}\ mph$$

(c) Also

$$V = \left(2930\ \frac{ft}{s}\right)\left(0.3048\ \frac{m}{ft}\right) = \underline{\underline{893}}\ \frac{m}{s}$$

11.29

11.29 At the seashore, you observe a high-speed aircraft moving overhead at an elevation of 10,000 ft. You hear the plane 8 s after it passes directly overhead. Using a nominal air temperature of 40 °F, estimate the Mach number and speed of the aircraft.



The Mach number is related to the angle α by Eq. 11.39. Thus

$$Ma = \frac{1}{\sin\alpha} = \frac{V}{c} \qquad (1)$$

Also

$$\tan\alpha = \frac{z}{Vt} \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{\sin\alpha}{\cos\alpha} = \frac{z\sin\alpha}{c\,t}$$

or

$$\alpha = \cos^{-1}\left(\frac{c\,t}{z}\right)$$

Now

$$c = \sqrt{RTk} = \sqrt{\frac{\left(1716\,\frac{ft\cdot lb}{slug\cdot{}^\circ R}\right)(500\,{}^\circ R)(1.4)}{\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = 1096\,\frac{ft}{s}$$

Then

$$\alpha = \cos^{-1}\left[\frac{\left(1096\,\frac{ft}{s}\right)(8\,s)}{(10000\,ft)}\right] = 28.7^\circ$$

and

$$Ma = \frac{1}{\sin 28.7^\circ} = \underline{\underline{2.08}}$$

Further

$$V = (Ma)\,c = (2.08)\left(1096\,\frac{ft}{s}\right) = \underline{\underline{2280}}\,\frac{ft}{s}$$

11-30

11.30 Explain how you could vary the Mach number but not the Reynolds number in air flow past a sphere. For a constant Reynolds number of 300,000, estimate how much the drag coefficient will increase as the Mach number is increased from 0.3 to 1.0.

Considering air as an ideal gas, we can express the Mach number Ma, as

$$Ma = \frac{V}{c} = \frac{V}{\sqrt{RTk}} \qquad (1)$$

The Reynolds number, Re, is

$$Re = \frac{\rho V d}{\mu} = \frac{pVd}{RT\mu} \qquad (2)$$

Looking at equations 1 and 2 we reason that we can vary Ma while holding Re constant by varying V and p only with pV held constant.

From the graph below we conclude that at $Re = 3\times10^5$, the drag coefficient increases from 0.47 to 0.75 at Ma increases from 0.3 to 1.0.

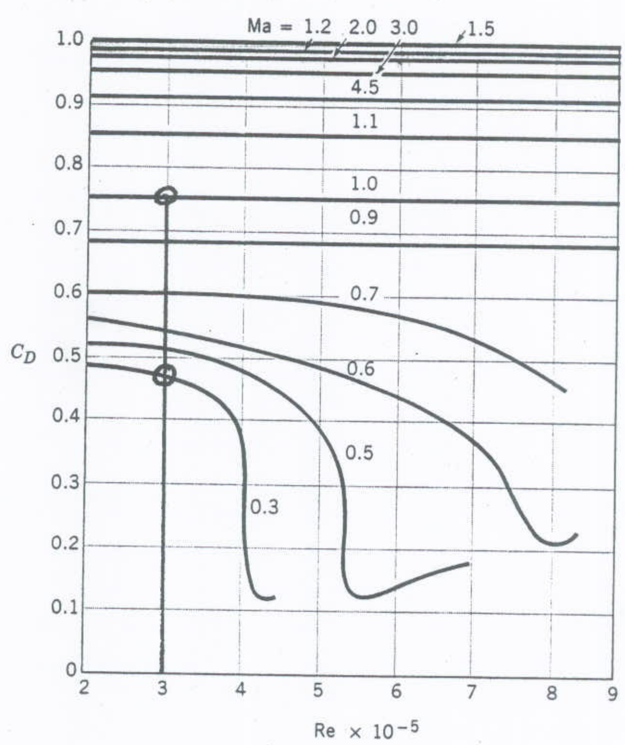


FIGURE 11.2 The variation of the drag coefficient of a sphere with Reynolds number and Mach number. (Adapted from Fig. 1.8 in Ref. 1 of Chapter 9)

Master Tyshin Ease
10% Reduction
8 1/2 x 11

11.33

11.33 Starting with the enthalpy form of the energy equation (Eq. 5.69), show that for isentropic flows, the stagnation temperature remains constant. Why is this important?

starting with Eq. 5.69 we have

$$\dot{m}\left[\check{h}_{out} - \check{h}_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = \dot{Q}_{\substack{net\\in}} + \dot{W}_{\substack{shaft\\net\ in}}$$

For isentropic flow the entropy remains constant and $\dot{Q}_{net} = 0$. Stagnation enthalpy is defined as

$$\check{h}_0 = \check{h} + \frac{V^2}{2}$$

So, for negligible change in elevation (okay for gases) and no shaft work, $\dot{W}_{shaft}$, then

$\check{h}_0$ remains constant.

and since for an ideal gas enthalpy is a function of temperature only, we conclude that constant $\check{h}_0$ means constant stagnation temperature T_0.

This constant stagnation temperature provides us with a convenient reference property at every location in a specific isentropic flow.

11.34

11.34 Explain how fluid pressure varies with cross section area change for the isentropic flow of an ideal gas when the flow is **(a)** subsonic; **(b)** supersonic.

With the help of Eq. 11.47 we can comment on how pressure varies with area change in an isentropic flow. From Eq. 11.47 we obtain

$$dp = \frac{\rho V^2}{(1-Ma^2)}\frac{dA}{A} \qquad (1)$$

(a) For subsonic flow, Eq. 1 suggests that changes of p follow changes of A. If A increases, p increases and vice versa.

(b) For supersonic flow, Eq. 1 suggests that changes of p are opposite to changes of A. If A increases, p decreases and vice versa.

11.35

11.35 For any ideal gas, prove that the slope of constant pressure lines on a temperature–entropy diagram is positive and that higher pressure lines are above lower pressure lines. Why is this important?

From the second Tds equation (Eq. 11.18) we note that for a constant pressure line

$$\frac{d\check{h}}{ds} = T$$

and since for an ideal gas Eq. 11.7 is valid, we have

$$d\check{h} = c_p\, dT$$

and thus

$$\frac{dT}{ds} = \frac{T}{c_p} \qquad (1)$$

With Eq. 1 we conclude that the slope of a constant pressure line on a temperature - entropy diagram is positive.

Further, from Eq. 11.24 we conclude that

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\left(\frac{k}{k-1}\right)}$$

for any isentropic process and thus higher pressure lines are above lower pressure lines in temperature - entropy diagrams. This information is important because it enables us to sketch T-s diagrams correctly.

11.36

11.36 Air flows steadily and isentropically from standard atmospheric conditions to a receiver pipe through a converging duct. The cross-sectional area of the throat of the converging duct is 0.05 ft². Determine the mass flowrate through the duct if the receiver pressure is **(a)** 10 psia, **(b)** 5 psia. Sketch temperature–entropy diagrams for situations **(a)** and **(b)**. Verify results obtained with values from the appropriate graph in Appendix D with calculations involving ideal gas equations. Is condensation of water vapour a concern? Explain.

This problem is similar to Example 11.5

The mass flowrate is obtained at the throat with Eq. 11.40. Thus,

$$\dot{m} = \rho_{th} A_{th} V_{th} \qquad (1)$$

The throat density can be obtained with Eq. 11.60. Thus,

$$\rho_{th} = \rho_0 \left[\frac{1}{1 + \left(\frac{k-1}{2}\right) Ma_{th}^2} \right]^{\frac{1}{k-1}} \qquad (2)$$

To determine the throat Mach number we use Eq. 11.59. Thus,

$$Ma_{th} = \sqrt{\left(\frac{2}{k-1}\right)\left[\left(\frac{p_0}{p_{th}}\right)^{\frac{k-1}{k}} - 1\right]} \qquad (3)$$

The critical throat pressure is obtained with Eq. 11.61. Thus,

$$p_{th}^* = p_0 \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = (14.7\ \text{psia})\left(\frac{2}{1.40+1}\right)^{\frac{1.4}{1.4-1}} = 7.76\ \text{psia}$$

If the receiver pressure, p_{re}, is greater than or equal to p_{th}^*, then $p_{th} = p_{re}$ and the flow is not choked. If $p_{re} < p_{th}^*$, then $p_{th} = p_{th}^*$ and the flow is choked.

The velocity at the throat is obtained with Eqs. 11.36 and 11.46 combined to yield

$$V_{th} = Ma_{th} \sqrt{R T_{th} k} \qquad (4)$$

where T_{th} is obtained with Eq. 11.56. Thus,

$$T_{th} = \frac{T_0}{1 + \left(\frac{k-1}{2}\right) Ma_{th}^2} \qquad (5)$$

(con't)

11.36 (con't)

(a) For $p_{re} = 10$ psia $> p_{th}^* = 7.76$ psia, $p_{th} = 10$ psia and we use Eq. 3 to calculate the throat Mach number. Thus,

$$Ma_{th} = \sqrt{\left(\frac{2}{1.40-1}\right)\left[\left(\frac{14.7\text{ psia}}{10\text{ psia}}\right)^{\frac{1.40-1}{1.40}} - 1\right]} = 0.7628$$

From Eq. 2 we obtain

$$\rho_{th} = \left(2.38\times10^{-3}\ \frac{\text{slug}}{\text{ft}^3}\right)\left[\frac{1}{1+\left(\frac{1.40-1}{2}\right)(0.7628)^2}\right]^{\frac{1}{1.40-1}} = 1.807\times10^{-3}\ \frac{\text{slug}}{\text{ft}^3}$$

From Eq. 5 we get

$$T_{th} = \frac{519\,^\circ\text{R}}{1+\left(\frac{1.40-1}{2}\right)(0.7628)^2} = 464.9\,^\circ\text{R}$$

and with Eq. 4

$$V_{th} = (0.7628)\sqrt{\left(1716\ \frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ\text{R}}\right)\frac{(1.40)(464.9\,^\circ\text{R})}{\left(1\ \frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)}} = 806.2\ \frac{\text{ft}}{\text{s}}$$

With Eq. 1 we obtain

$$\dot{m} = \left(1.807\times10^{-3}\ \frac{\text{slug}}{\text{ft}^3}\right)\left(0.05\ \text{ft}^2\right)\left(806.2\ \frac{\text{ft}}{\text{s}}\right) = \underline{\underline{0.0728}}\ \frac{\text{slug}}{\text{s}}$$

$$\dot{m} = \left(0.0728\ \frac{\text{slug}}{\text{s}}\right)\left(32.174\ \frac{\text{lbm}}{\text{s}}\right)$$

$$\dot{m} = \underline{\underline{2.34}}\ \frac{\text{lbm}}{\text{s}}$$

Alternatively, using Fig. D.1 with

$$\frac{p_{th}}{p_0} = \frac{10\text{ psia}}{14.7\text{ psia}} = 0.68$$

The value of Ma_{th} is

$$Ma_{th} = 0.76$$

For $Ma_{th} = 0.76$, we get from Fig. D.1

$$T_{th} = (0.9\quad)T_0 = (0.9\quad)(519\,^\circ\text{R}) = 467\,^\circ\text{R}$$

Then with Eq. 4

$$V_{th} = 0.76\sqrt{\left(1716\ \frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ\text{R}}\right)\frac{(1.40)(467\,^\circ\text{R})}{\left(1\ \frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)}} = 805\ \frac{\text{ft}}{\text{s}}$$

(con't)

11.36 (con't)

For $Ma_{th} = 0.76$ we get from Fig. D.1

$$\rho_{th} = 0.76086\,\rho_o = (0.76)\left(2.38\times10^{-3}\frac{slug}{ft^3}\right) = 1.8\times10^{-3}\frac{slug}{ft^3}$$

Now, with Eq. 1 we obtain

$$\dot{m} = \left(1.8\times10^{-3}\frac{slug}{ft^3}\right)(0.05\ ft^2)\left(805\frac{ft}{s}\right) = \underline{\underline{0.076}}\ \frac{slug}{ft^3} = \underline{\underline{2.44}}\ \frac{lbm}{s}$$

(b) For $p_{re} = 5\,psia < p^* = 7.76\,psia$, $p_{th} = 7.76\,psia$ and $Ma_{th} = 1.0$. From Eq. 2,

$$\rho_{th} = \left(2.38\times10^{-3}\frac{slug}{ft^3}\right)\left[\frac{1}{1+\left(\frac{1.40-1}{2}\right)}\right]^{\frac{1}{1.40-1}} = 1.509\times10^{-3}\frac{slug}{ft^3}$$

From Eq. 5 we obtain

$$T_{th} = \frac{519°R}{1+\left(\frac{1.40-1}{2}\right)} = 432.5°R$$

and with Eq. 4

$$V_{th} = \sqrt{\frac{\left(1716\frac{ft\cdot lb}{slug\cdot °R}\right)(1.40)(432.5°R)}{\left(1\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = 1019\frac{ft}{s}$$

With Eq. 1 we obtain

$$\dot{m} = \left(1.509\times10^{-3}\frac{slug}{ft^3}\right)(0.05\ ft^2)\left(1019\frac{ft}{s}\right) = \underline{\underline{0.0769}}\ \frac{slug}{s} = \underline{\underline{2.47}}\ \frac{lbm}{s}$$

Alternatively, from Fig. D.1 for $Ma = 1.0$

$$T_{th} = (0.83)(519°R) = 431\ °R$$

and

$$\rho_{th} = (0.64)\left(2.38\times10^{-3}\frac{slug}{ft^3}\right) = 1.52\times10^{-3}\frac{slug}{ft^3}$$

Then with Eq. 4

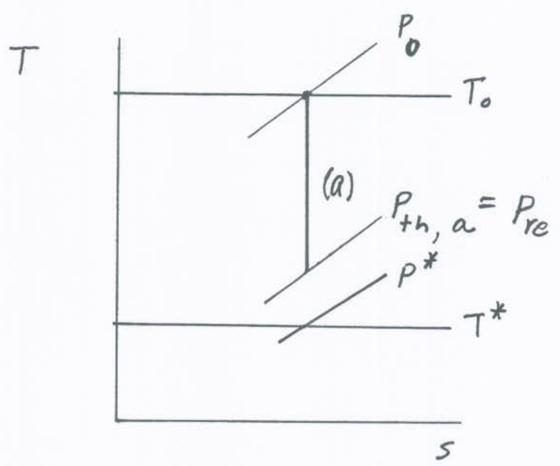
$$V_{th} = \sqrt{\frac{\left(1716\frac{ft\cdot lb}{slug\cdot °R}\right)(1.40)(431\ °R)}{\left(1\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}} = 1020\ ft/s$$

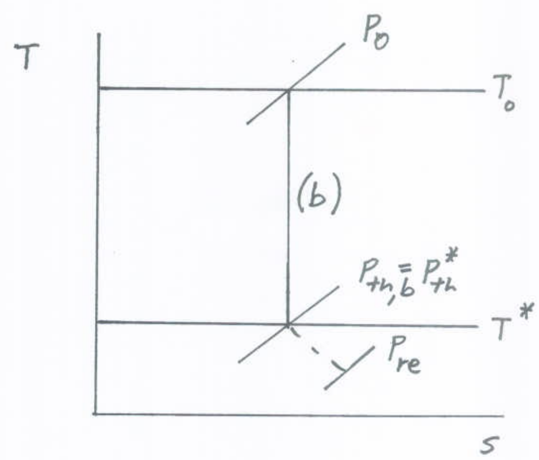
(con't)

11.36 (con't)

and with Eq. 1 we obtain

$$\dot{m} = \left(1.52 \times 10^{-3}\ \frac{slug}{ft^3}\right)\left(0.05\ ft^2\right)\left(1020\ \frac{ft}{s}\right) = \underline{\underline{0.078}}\ \frac{slug}{s} = \underline{\underline{2.51}}\ \frac{lbm}{s}$$





Condensation of water vapour is a topic that deserves further study and discussion.

11.37

11.37 Determine the static pressure to stagnation pressure ratio associated with the following motion in standard air: **(a)** a runner moving at the rate of 10 mph, **(b)** a cyclist moving at the rate of 40 mph, **(c)** a car moving at the rate of 65 mph, **(d)** an airplane moving at the rate of 500 mph.

With a value of Mach number calculated with

$$Ma = \frac{V}{c} \qquad (1)$$

we can calculate $\frac{p}{p_0}$ with

$$\frac{p}{p_0} = \left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma^2}\right]^{\frac{k}{k-1}} \qquad (11.59)$$

For c we use for parts a, b and c

$$c = \sqrt{RTk} = \sqrt{\frac{\left(1716\ \frac{ft \cdot lb}{slug \cdot °R}\right)(519\ °R)(1.40)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 1117\ \frac{ft}{s}$$

or

$$c = \frac{\left(1117\ \frac{ft}{s}\right)\left(3600\ \frac{s}{hr}\right)}{\left(5280\ \frac{ft}{mi}\right)} = 761.6\ mph$$

(a) For $V = 10\ mph$

$$Ma = \frac{10\ mph}{761.6\ mph} = 0.0131$$

and

$$\frac{p}{p_0} = \left[\frac{1}{1+\left(\frac{1.4-1}{2}\right)(0.0131)^2}\right]^{\frac{1.4}{1.4-1}} = \left[\frac{1}{1+(0.2)(0.0131)^2}\right]^{3.5} = 0.99988$$

(b) For $V = 40\ mph$

$$Ma = \frac{40\ mph}{761.6\ mph} = 0.0525$$

and

$$\frac{p}{p_0} = \left[\frac{1}{1+0.2(0.0525)^2}\right]^{3.5} = 0.998$$

(c) For $V = 65\ mph$

$$Ma = \frac{65\ mph}{761.6\ mph} = 0.0854$$

and

$$\frac{p}{p_0} = \left[\frac{1}{1+0.2(0.0854)^2}\right]^{3.5} = 0.9949$$

(con't)

11.37 (con't)

(d) For airplane we assume a nominal altitude of 30,000 ft. From Table C.1 we note a corresponding temperature of $-47.83°F$. Then

$$c = \sqrt{\frac{\left(1716 \frac{ft \cdot lb}{slug \cdot °R}\right)\left[(-47.83+460)°R\right](1.4)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}}$$

$$c = 995 \frac{ft}{s}$$

or

$$c = \left(995 \frac{ft}{s}\right)\frac{\left(3600 \frac{s}{hr}\right)}{\left(5280 \frac{ft}{mi}\right)} = 678 \text{ mph}$$

Then for

$$Ma = \frac{500 \text{ mph}}{678 \text{ mph}} = 0.738$$

$$\frac{p}{p_0} = \left[\frac{1}{1+0.2(0.738)^2}\right]^{3.5} = 0.696$$

11.38

11.38 The static pressure to stagnation pressure ratio at a point in a gas flow field is measured with a Pitot-static probe as being equal to 0.6. The stagnation temperature of the gas is 20 °C. Determine the flow speed in m/s and the Mach number if the gas is air. What error would be associated with assuming that the flow is incompressible?

To determine the flow speed and Mach number having been given the static pressure to stagnation pressure ratio, $\frac{p}{p_0}$, and stagnation temperature, T_0, for air we enter Fig. D.1 with the given value of $\frac{p}{p_0}$ and read the corresponding value of Ma. Thus with $\frac{p}{p_0} = 0.6$, the corresponding value in Fig. D.1 is

$$Ma = \underline{\underline{0.89}}$$

For $Ma = 0.89$, Fig. D.1 gives

$$\frac{T}{T_0} = 0.86$$

and thus

$$T = \left(\frac{T}{T_0}\right) T_0 = (0.86)(293\,K) = 252\ K$$

Then

$$V = (Ma)c = Ma\sqrt{RTk} = 0.89\sqrt{\frac{\left(286.9\ \frac{N \cdot m}{kg \cdot K}\right)(252\ K)(1.4)}{\left(1\ \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}$$

or

$$V = \underline{\underline{283}}\ \frac{m}{s}$$

Inspection of Fig. 3.24 suggests that for this Mach number level, the error associated with assuming that the flow is incompressible would be unacceptably large.

11.39

11.39 The stagnation pressure and temperature of air flowing past a probe are 120 kPa (abs) and 100 °C, respectively. The air pressure is 80 kPa (abs). Determine the air speed and Mach number considering the flow to be **(a)** incompressible; **(b)** compressible.

(a) Assuming incompressible flow we use Bernoulli's equation (Eq. 3.7) to connect the static and stagnation states and get

$$V = \sqrt{\frac{2(P_0 - P)}{\rho_0}} \qquad (1)$$

With the ideal gas equation of state (Eq. 1) we obtain

$$\rho_0 = \frac{P_0}{RT_0} \qquad (2)$$

and combining Eqs. 1 and 2 we obtain

$$V = \sqrt{\frac{2(P_0 - P)RT_0}{P_0}}$$

or

$$V = \sqrt{\frac{2\,[120\text{ kPa(abs)} - 80\text{ kPa(abs)}]\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(373K)}{[120\text{ kPa(abs)}]\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = \underline{\underline{267\,\frac{m}{s}}}$$

For Mach number we need

$$Ma = \frac{V}{c} = \frac{V}{\sqrt{RTk}} \qquad (3)$$

To determine T we use the equation of motion (Eq. 11.54) to obtain

$$T = T_0 - \frac{V^2(k-1)}{2kR} = 373K - \frac{\left(267\,\frac{m}{s}\right)^2(1.4-1)}{2(1.4)\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)}\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)$$

or $T = 337.5\,K$

(con't)

11.39 (con't)

With Eq. 3 we obtain

$$Ma = \frac{267\ \frac{m}{s}}{\sqrt{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(337.5\,K)(1.4)}{\left(1\ \frac{N}{kg\cdot\frac{m}{s^2}}\right)}}} = \underline{\underline{0.725}}$$

(b) For compressible flow

$$\frac{p}{p_0} = \frac{80\ kPa\,(abs)}{120\ kPa\,(abs)} = 0.67$$

and from Fig. D.1 we read

$$Ma = \underline{\underline{0.78}}$$

Also from Fig. D.1 we read

$$\frac{T}{T_0} = 0.89$$

and thus

$$T = (0.89)(373\,K) = 332\ K$$

Thus,

$$V = Ma\sqrt{RTk} = (0.78)\sqrt{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(332\ K)(1.4)}{\left(1\ \frac{N}{kg\cdot\frac{m}{s^2}}\right)}}$$

and

$$V = \underline{\underline{285}}\ \frac{m}{s}$$

11.40

11.40 The stagnation pressure indicated by a Pitot tube mounted on an airplane in flight is 45 kPa (abs). If the aircraft is cruising in standard atmosphere at an altitude of 10,000 m, determine the speed and Mach number involved.

For 10,000 m standard atmosphere we get from Table C.2

$$p = 26.50 \text{ kPa (abs)}$$

and

$$T = 223.1 K$$

Thus

$$\frac{p}{p_0} = \frac{26.50 \text{ kPa (abs)}}{45 \text{ kPa (abs)}} = 0.59$$

and from Fig. D.1 we read

$$Ma = \underline{\underline{0.90}}$$

Thus

$$V = (Ma)c = Ma\sqrt{RTk} = (0.9)\sqrt{\frac{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)(223.1 K)(1.4)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}$$

or

$$V = \underline{\underline{269}} \frac{m}{s}$$

11.42

*11.42 An ideal gas enters subsonically and flows isentropically through a choked converging–diverging duct having a circular cross-sectional area A that varies with axial distance from the throat, x, according to the formula

$$A = 0.1 + x^2$$

where A is in square feet and x is in feet. For this flow situation, sketch the side view of the duct and graph the variation of Mach number, static temperature to stagnation temperature ratio, T/T_0, and static pressure to stagnation pressure ratio, p/p_0, through the duct from $x = -0.6$ ft to $x = +0.6$ ft. Also show the possible fluid states at $x = -0.6$ ft, 0 ft, and $+0.6$ ft using temperature–entropy coordinates. Consider the gas as being helium (use $0.051 \leq \text{Ma} \leq 5.193$). Sketch on your pressure variation graph the nonisentropic paths that would occur with over- and under-expanded duct exit flows (see **Video V11.6**) and explain when they will occur. When will isentropic supersonic duct exit flow occur?

This is like Example 11.8.
Since

$$A = \pi r^2$$

and

$$A = 0.1 + x^2$$

then

$$r = \frac{0.1 + x^2}{\pi} \qquad (1)$$

With Eq. 1 we can determine r values corresponding to values of x. The are summarized in the graph and tables
duct is choked,

$$A^* = 0.1 \text{ ft}^2$$

and

$$\frac{A}{A^*} = 1 + \frac{x^2}{0.1} \qquad (2)$$

With Eq. 2 we can determine $\frac{A}{A^*}$ values corresponding to values of x. These $\frac{A}{A^*}$ values are tabulated

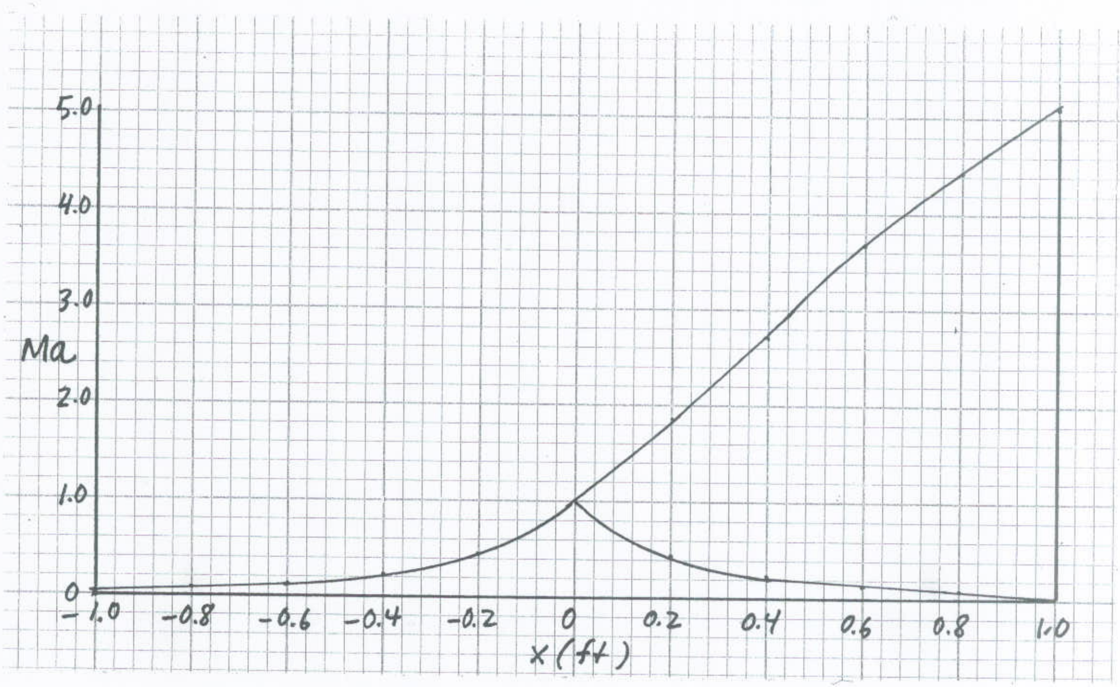
For helium we enter program ISENTROP with $k = 1.66$ and with Ma values within the range specified in the problem statement and obtain values of $\frac{A}{A^*}$ (Eq. 11.71), x (Eq. 2), $\frac{T}{T_0}$ (Eq. 11.56) and $\frac{p}{p_0}$ (Eq. 11.59). These values are tabulated and graphed on pages that follow.

(con't)

11.42 (con't)

From program ISENTROP with $k = 1.66$

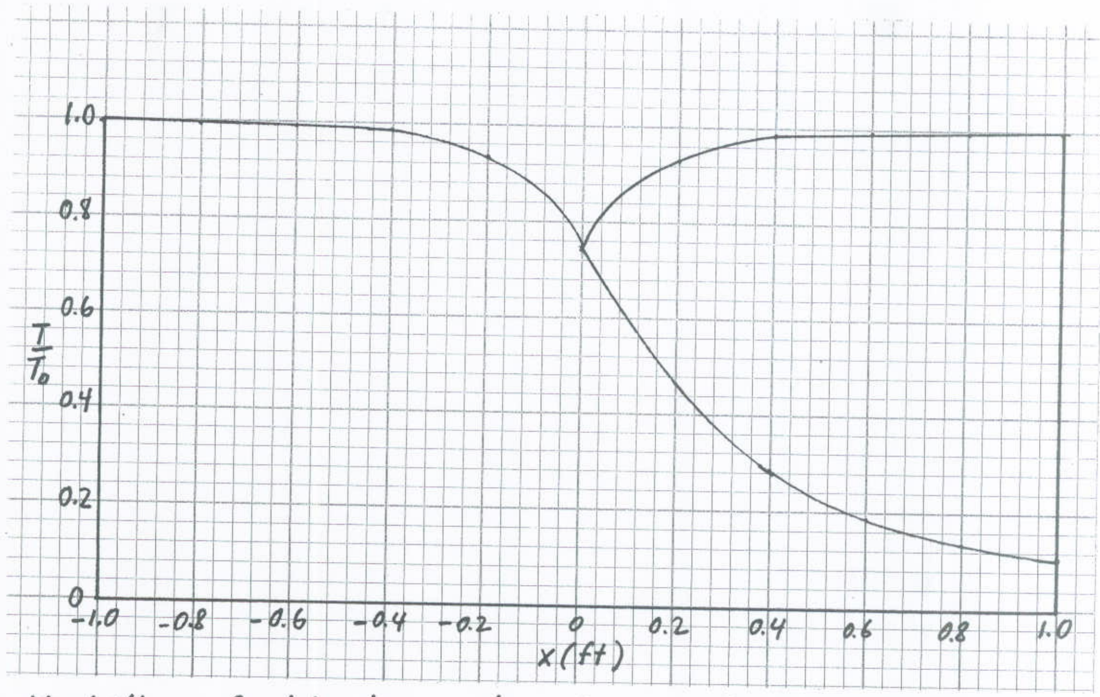
Ma	$\frac{A}{A^*}$	Eq. 2 x(ft)	$\frac{T}{T_0}$	$\frac{p}{p_0}$	state
		subsonic solution			
0.051	11.06	±1.00	0.99914	0.99784	a, c
0.076	7.43	±0.80	0.99809	0.99522	
0.123	4.62	±0.60	0.99503	0.98755	
0.223	2.61	±0.40	0.98385	0.95989	
0.460	1.40	±0.20	0.93473	0.84386	
1.00	1.00	0	0.75188	0.48808	b
		supersonic solution			
1.855	1.40	0.20	0.46827	0.14833	
2.778	2.60	0.40	0.28195	0.04141	
3.647	4.60	0.60	0.18556	0.01446	
4.448	7.40	0.80	0.13282	0.00624	
5.193	11.0	1.00	0.10102	0.00313	d



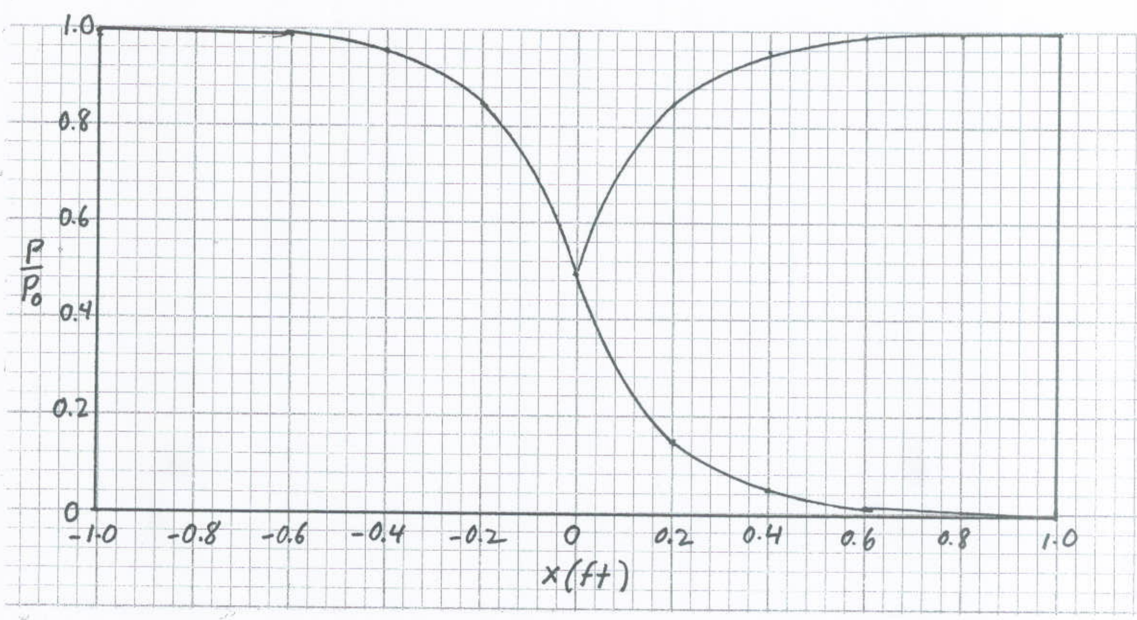
Variation of Mach number for helium

(con't)

11.42 (con't)



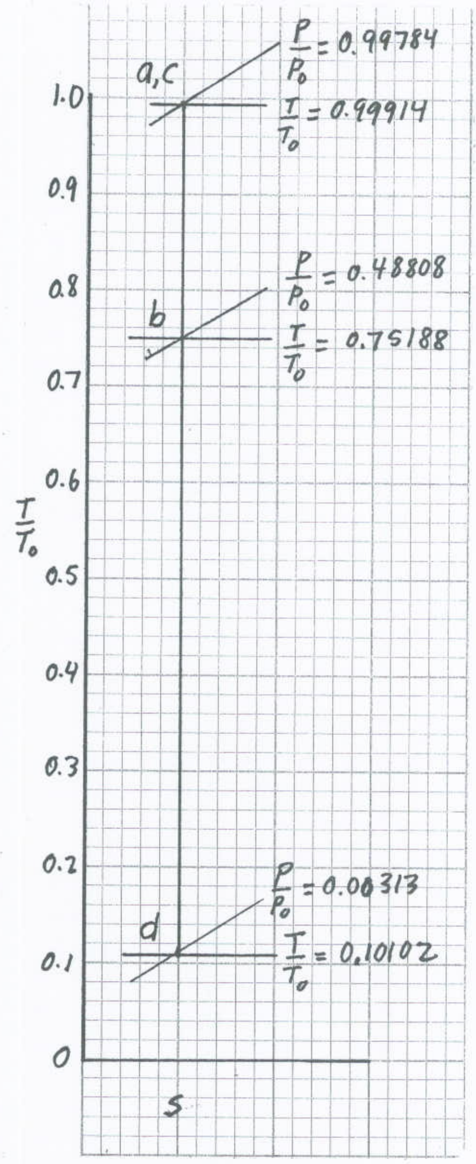
Variation of static temperature to stagnation temperature ratio for helium



Variation of static pressure to stagnation pressure ratio for helium

(con't)

11.42 (con't)

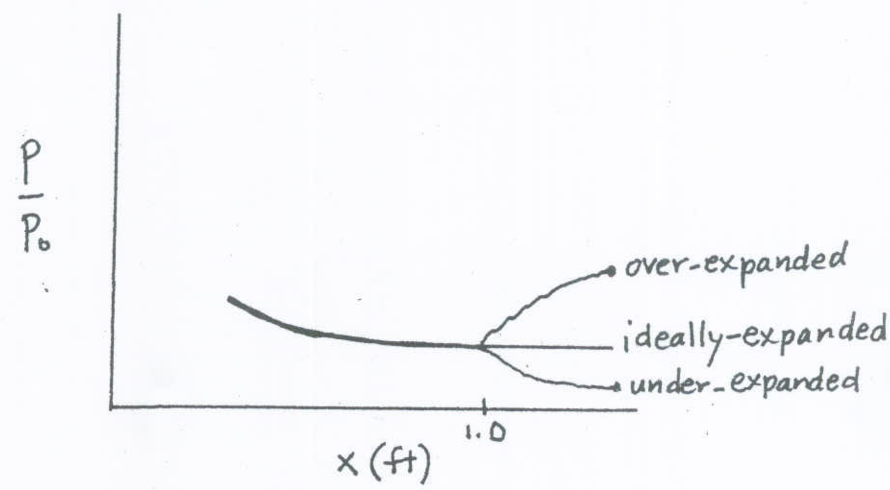


Temperature entropy diagram for helium

(con't)

11.42 (con't)

Over- and under-expanded duct exit flows will occur on approximate paths sketched on the magnified pressure variation graph below when the ambient pressure of the surroundings into which the duct is discharging is respectively greater than and less than the flowing fluid pressure at the duct exit.This illustrates how the flow adjusts to these pressure differences through oblique shock waves that involve irreversible and thus non-isentropic flows. When these two pressures are equal, the flow is "ideally expanded" and the flow into the immediate surroundings is nearly isentropic.



11.43

***11.43** An ideal gas enters supersonically and flows isentropically through the choked converging–diverging duct described in Problem 11.42. Graph the variation of Ma, T/T_0, and p/p_0 from the entrance to the exit sections of the duct for helium (use $0.051 \le \text{Ma} \le 5.193$). Show the possible fluid states at $x = -0.6$ ft, 0 ft, and $+0.6$ ft using temperature–entropy coordinates. Sketch on your pressure variation graph the nonisentropic paths that would occur with over- and underexpanded duct exit flows (see **Video V11.6**) and explain when they will occur. When will isentropic supersonic duct exit flow occur?

This is similar to Example 11.9.
This problem involves the duct of Problem 11· . However the flow enters supersonically. We can use values from the tables of problem 11· with a little rearrangement to account for the supersonic entering flow.

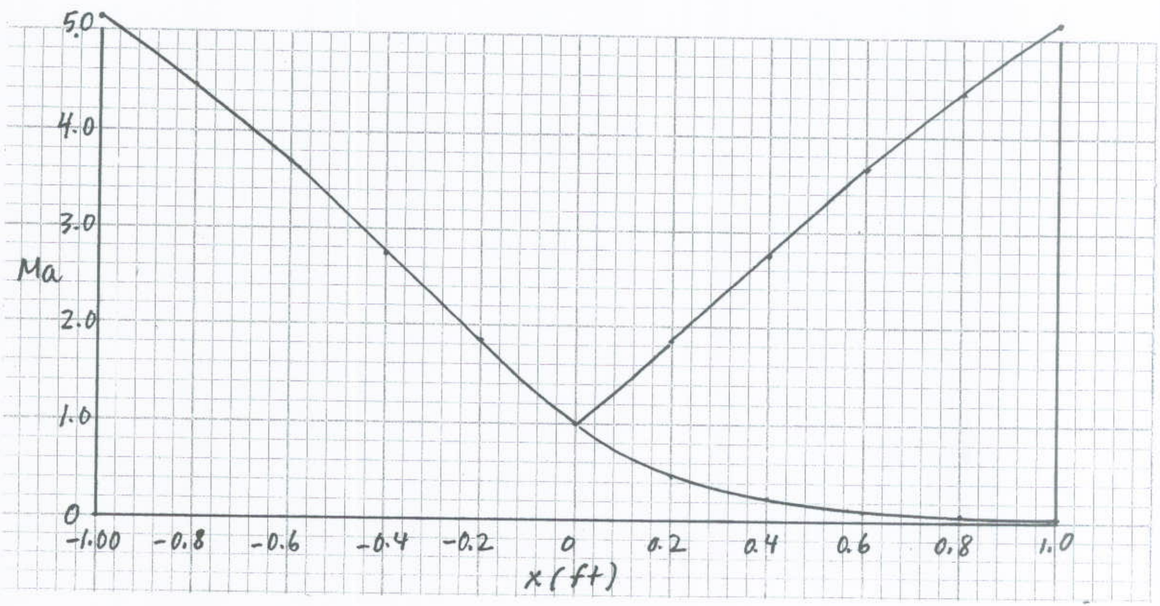
For helium we have

From program ISENTROP with $k = 1.66$

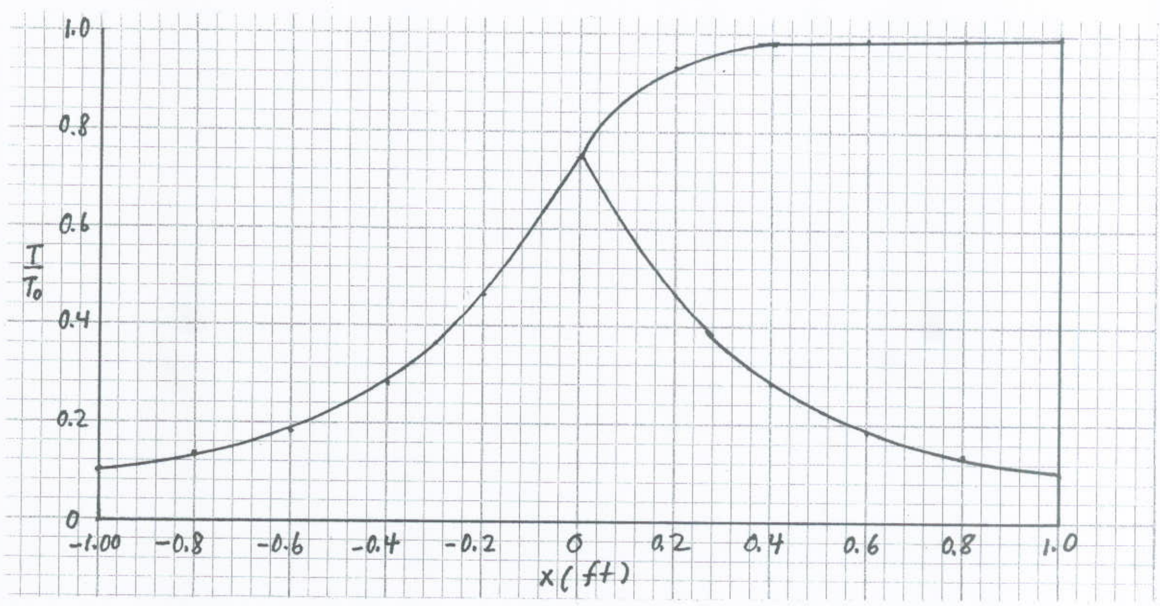
Ma	$\frac{A}{A^*}$	Eq. 2 of 11.39 x (ft)	$\frac{T}{T_0}$	$\frac{P}{P_0}$	state
		supersonic solution			
5.193	11.0	−1.00	0.10102	0.00313	a
4.448	7.4	−0.80	0.13282	0.00624	
3.647	4.6	−0.60	0.18556	0.01446	
2.778	2.6	−0.40	0.28195	0.04141	
1.855	1.4	−0.20	0.46827	0.14833	
1.0	1.0	0	0.75188	0.48808	b
1.855	1.4	0.20	0.46827	0.14833	
2.778	2.6	0.40	0.28195	0.04141	
3.647	4.6	0.60	0.18556	0.01446	
4.448	7.4	0.80	0.13282	0.00624	
5.193	11.0	1.00	0.10102	0.00313	c
		subsonic solution			
0.460	1.40	0.20	0.93473	0.84386	
0.223	2.61	0.40	0.98385	0.95989	
0.123	4.62	0.60	0.99503	0.98755	
0.076	7.43	0.80	0.99809	0.99522	
0.051	11.06	1.00	0.99914	0.99784	d

(Con't)

11.43 (con't)



Variation of Mach number for helium

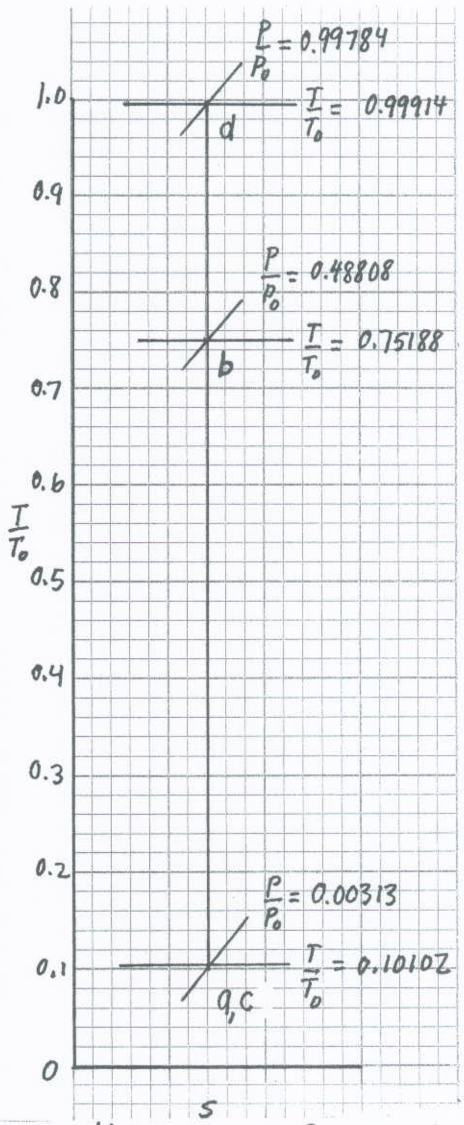


Variation of static temperature to stagnation temperature ratio for helium

(con't)

11.43 (con't)

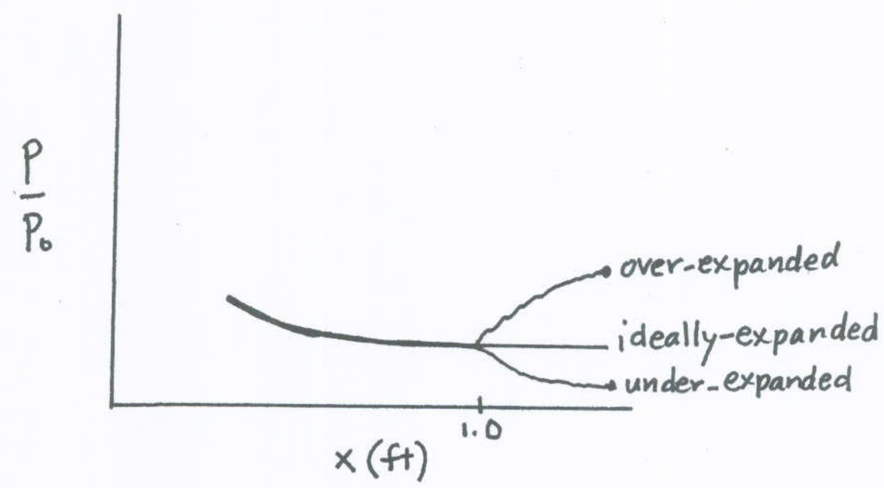
Variation of static pressure to stagnation pressure ratio for helium



(con't) T-s diagram for helium

11.43 (con't)

Over- and under-expanded duct exit flows will occur on approximate paths sketched on the magnified pressure variation graph below when the ambient pressure of the surroundings into which the duct is discharging is respectively greater than and less than the flowing fluid pressure at the duct exit.This illustrates how the flow adjusts to these pressure differences through oblique shock waves that involve irreversible and thus non-isentropic flows. When these two pressures are equal, the flow is "ideally expanded" and the flow into the immediate surroundings is nearly isentropic.



11.44

11.44 An ideal gas flows subsonically and isentropically through the converging–diverging duct described in Problem 11.42. Graph the variation of Ma, T/T_0, and p/p_0 from the entrance to the exit sections of the duct for air. The value of p/p_0 is 0.6708 at $x = 0$ ft. Sketch important states on a T–s diagram.

This is like Example 11.10.
Since $\frac{p}{p_0} = 0.6708$ at $x = 0$ is greater than $\frac{p^*}{p_0} = 0.5283$ for air

the air flow through the converging-diverging duct is not choked. For values of $\frac{A}{A^*}$ at different values of x we obtain corresponding values of Ma, $\frac{T}{T_0}$ and $\frac{p}{p_0}$.

(a) For air we enter Fig. D.1 with values of $\frac{A}{A^*}$ to get Ma, $\frac{T}{T_0}$ and $\frac{p}{p_0}$. For A^* we use

$$A^* = \frac{A}{\left(\frac{A}{A^*}\right)}$$

evaluated at $x = 0$ where $A = 0.1\ ft^2$. We determine $\frac{A}{A^*}$ at $x = 0$ from Fig. D.1 for the subsonic flow value of $\frac{p}{p_0} = 0.6708$, we get

$$\frac{A}{A^*} = 1.05 \quad \text{and thus}$$

$$A^* = \frac{0.1\ ft^2}{1.05} = 0.095\ ft^2$$

We then determine the $\frac{A}{A^*}$ variation through the duct with

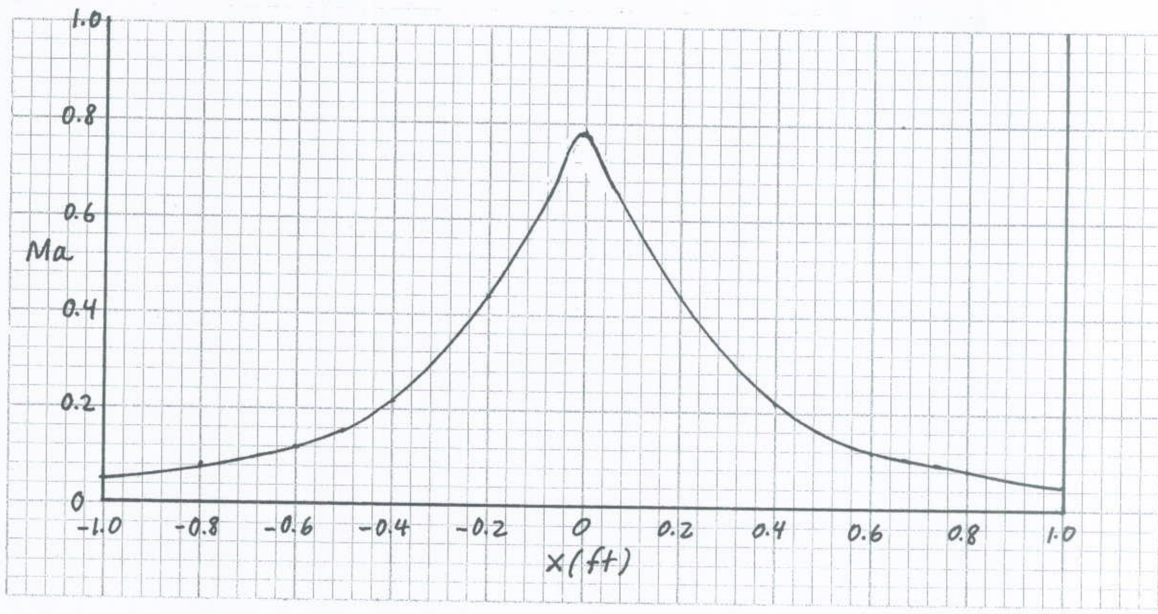
$$\frac{A}{A^*} = \frac{x^2 + 0.1}{A^*} = \frac{x^2+0.1}{0.095} \qquad (1)$$

The corresponding values of $\frac{A}{A^*}$, Ma, $\frac{T}{T_0}$ and $\frac{p}{p_0}$ from Fig. D.1 are also tabulated on the next page.

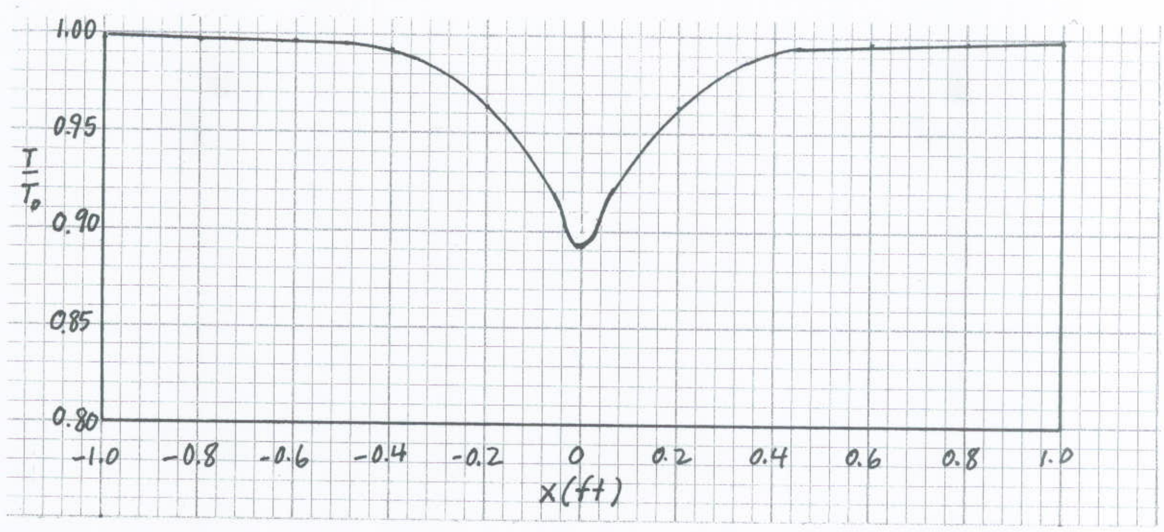
(con't)

11.44 (con't)

x (ft)	With Eq. 1 $\frac{A}{A^*}$	From Fig. D.1 Ma	$\frac{T}{T_0}$	$\frac{P}{P_0}$	state
−1.0	11.6	0.05	0.99	0.99	a
−0.8	7.8	0.08	0.99	0.99	
−0.6	4.8	0.12	0.99	0.98	
−0.4	2.7	0.22	0.99	0.966	
−0.2	1.5	0.44	0.96	0.87	
0	1.0	0.78	0.89	0.66	b
0.2	1.5	0.44	0.96	0.87	
0.4	2.7	0.22	0.99	0.96	
0.6	4.8	0.12	0.99	0.98	
0.8	7.8	0.08	0.99	0.99	
1.0	11.6	0.05	0.99	0.99	c



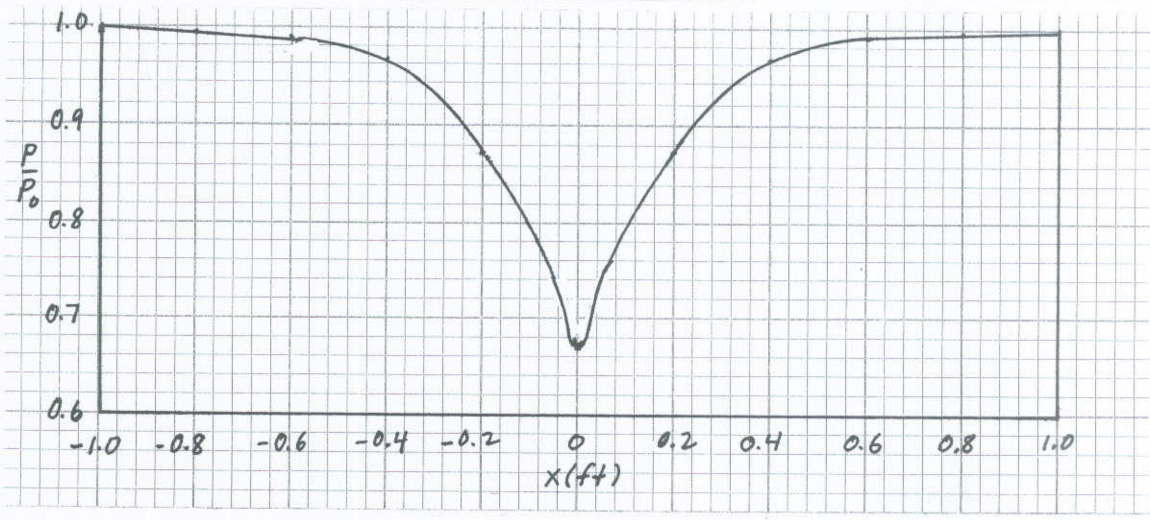
Variation of Mach number for air



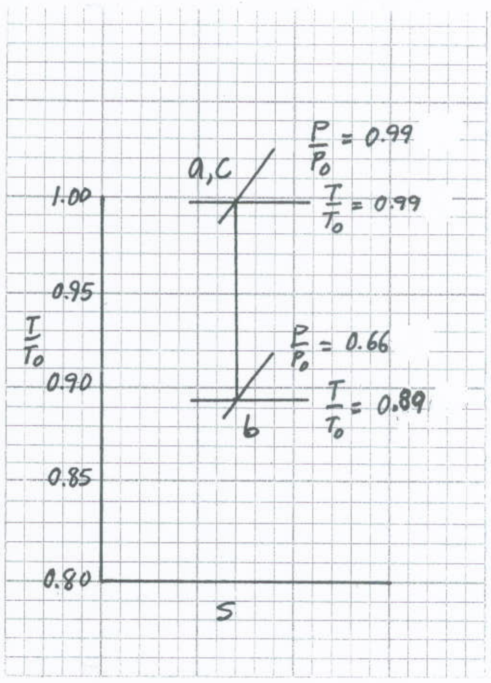
Variation of static temperature to stagnation temperature ratio for air

(con't)

11.44 (con't)



Variation of static pressure to stagnation pressure ratio for air



T-s diagram for air

11.45

11.45 An ideal gas is to flow isentropically from a large tank where the air is maintained at a temperature and pressure of 59 °F and 80 psia to standard atmospheric discharge conditions. Describe in general terms the kind of duct involved and determine the duct exit Mach number and velocity in ft/s if the gas is air.

To determine the duct exit Mach number, Ma_{exit}, we use Eq. 11.59 or for air, Fig. D.1. Thus,

$$Ma_{exit} = \sqrt{\left[\frac{1}{\left(\frac{p_{exit}}{p_0}\right)^{\frac{k-1}{k}}} - 1\right]\left(\frac{2}{k-1}\right)} \qquad (1)$$

or for air

$$Ma_{exit} = \text{Fig. D.1 value as a function of } \frac{p_{exit}}{p_0} \qquad (2)$$

To determine exit velocity, V_{exit}, we use

$$V_{exit} = (Ma_{exit})\,c_{exit} = Ma_{exit}\sqrt{R T_{exit} k} \qquad (3)$$

where

$$T_{exit} = \frac{T_0}{1 + \left(\frac{k-1}{2}\right) Ma_{exit}^2} \qquad (4)$$

or for air

$$T_{exit} = T_0\left(\frac{T_{exit}}{T_0} \text{ value from Fig. D.1 for } Ma_{exit}\right) \qquad (5)$$

$$\frac{p_{exit}}{p_0} = \frac{14.7 \text{ psia}}{80 \text{ psia}} = 0.1838$$

and thus from Fig. D.1, the corresponding values are

$$Ma_{exit} = \underline{\underline{1.8}}$$

and

$$\frac{T_{exit}}{T_0} = 0.62$$

(con't)

11.45 (con't)

Then with Eq. 5 we obtain

$$T_{exit} = (519\,^\circ R)(0.62\quad) = 322\,^\circ R$$

and with Eq. 3 we conclude that

$$V_{exit} = (1.8)\sqrt{\left(1716\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right)\frac{(322\ {}^\circ R)(1.4)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{1580\ \frac{ft}{s}}}$$

A converging-diverging nozzle is required because the exit flow is supersonic.

11.46

11.46 An ideal gas flows isentropically through a converging–diverging nozzle. At a section in the converging portion of the nozzle, $A_1 = 0.1\ \text{m}^2$, $p_1 = 600\ \text{kPa(abs)}$, $T_1 = 20\ °\text{C}$, and $\text{Ma}_1 = 0.6$. For section (2) in the diverging part of the nozzle, determine A_2, p_2, and T_2 if $\text{Ma}_2 = 3.0$ and the gas is air.

To determine A_2 we use Eq. 11.71 or for air, Fig. D.1 Thus,

$$A_2 = A_1 \frac{\left(\frac{A_2}{A^*}\right)}{\left(\frac{A_1}{A^*}\right)} = A_1 \left\{ \frac{\frac{1}{Ma_2}\left[\frac{1+\left(\frac{k-1}{2}\right)Ma_2^2}{1+\left(\frac{k-1}{2}\right)}\right]^{\frac{k+1}{2(k-1)}}}{\frac{1}{Ma_1}\left[\frac{1+\left(\frac{k-1}{2}\right)Ma_1^2}{1+\left(\frac{k-1}{2}\right)}\right]^{\frac{k+1}{2(k-1)}}} \right\} \quad (1)$$

or for air

$$A_2 = A_1 \left[\frac{\left(\text{Fig. D.1 value of } \frac{A_2}{A^*} \text{ for } Ma_2\right)}{\left(\text{Fig. D.1 value of } \frac{A_1}{A^*} \text{ for } Ma_1\right)}\right] \quad (2)$$

To determine P_2 we use Eq. 11.59 or for air, Fig. D.1. Thus,

$$P_2 = P_1 \frac{\left(\frac{P_2}{P_0}\right)}{\left(\frac{P_1}{P_0}\right)} = P_1 \left\{ \frac{\left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_2^2}\right]^{\frac{k}{k-1}}}{\left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_1^2}\right]^{\frac{k}{k-1}}} \right\} \quad (3)$$

or for air,

$$P_2 = P_1 \left[\frac{\left(\text{Fig. D.1 value of } \frac{P_2}{P_0} \text{ for } Ma_2\right)}{\left(\text{Fig D.1 value of } \frac{P_1}{P_0} \text{ for } Ma_1\right)}\right] \quad (4)$$

To determine T_2 we use Eq. 11.56 or for air, Fig. D.1. Thus,

$$T_2 = T_1 \frac{\left(\frac{T_2}{T_0}\right)}{\left(\frac{T_1}{T_0}\right)} = T_1 \left\{ \frac{\left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_2^2}\right]}{\left[\frac{1}{1+\left(\frac{k-1}{2}\right)Ma_1^2}\right]} \right\} \quad (5)$$

or for air

$$T_2 = T_1 \left[\frac{\left(\text{Fig. D.1 value of } \frac{T_2}{T_0} \text{ for } Ma_2\right)}{\left(\text{Fig D.1 value of } \frac{T_1}{T_0} \text{ for } Ma_1\right)}\right] \quad (6)$$

Eq. 2 leads to

$$A_2 = (0.1\,m^2)\frac{(4.3)}{(1.2)} = \underline{\underline{0.36\ m^2}}$$

Eq. 4 leads to

$$P_2 = [600\ kPa(abs)]\frac{(0.03)}{(0.78)} = \underline{\underline{23\ kPa(abs)}}$$

and Eq. 6 gives

$$T_2 = (293\,K)\frac{(0.36)}{(0.93)} = \underline{\underline{113\ K}}$$

11.47

11.47 Upstream of the throat of an isentropic converging–diverging nozzle at section (1), $V_1 = 150$ m/s, $p_1 = 100$ kPa(abs), and $T_1 = 20$ °C. If the discharge flow is supersonic and the throat area is 0.1 m^2, determine the mass flowrate in kg/s for the flow of air.

We determine the Mach number at section (1) with

$$Ma_1 = \frac{V_1}{c_1} = \frac{V_1}{\sqrt{R T_1 k}} \qquad (1)$$

For the gas involved it is likely that Ma_1 is less than 1.0 because V_1 is low. Thus, the flow at the throat is choked since the entering flow is subsonic and the leaving flow is supersonic. For mass flowrate we use Eq. 11.40 to obtain

$$\dot{m} = \rho^* A^* V^* \qquad (2)$$

For throat velocity, V^*, we use

$$V^* = \sqrt{R T^* k} \qquad (3)$$

To obtain T^* we use Eq. 11.63. Thus,

$$T^* = T_0 \left(\frac{2}{k+1}\right) \qquad (4)$$

or for air,

$$T^* = T_0 \left(\text{value of } \frac{T}{T_0} \text{ from Fig. D.1 for } Ma = 1.0\right) \qquad (5)$$

To determine T_0 we use Eq. 11.56. Thus,

$$T_0 = T_1 \left[1 + \left(\frac{k-1}{2}\right) Ma_1^2\right] \qquad (6)$$

or for air,

$$T_0 = \frac{T_1}{\left(\text{Value of } \frac{T_1}{T_0} \text{ from Fig. D.1 for } Ma_1\right)} \qquad (7)$$

(con't)

11.47 (con't)

To determine ρ^* we use the ideal gas equation of state (Eq. 11.1). Thus,

$$\rho^* = \frac{p^*}{RT^*} \qquad (8)$$

For p^* we use Eq. 11.61. Thus

$$p^* = p_0 \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \qquad (9)$$

or for air,

$$p^* = p_0 \left(\text{value of } \frac{p^*}{p_0} \text{ from Fig. D.1 for Ma} = 1.0\right) \qquad (10)$$

For p_0 we use Eq. 11.59. Thus,

$$p_0 = p_1 \left[1 + \left(\frac{k-1}{2}\right) Ma_1^2\right]^{\frac{k}{k-1}} \qquad (11)$$

or for air

$$p_0 = \frac{p_1}{\left(\text{value of } \frac{p_1}{p_0} \text{ from Fig. D.1 for } Ma_1\right)} \qquad (12)$$

(a) For air we use Eq. 1 to obtain

$$Ma_1 = \frac{\left(150 \frac{m}{s}\right)}{\sqrt{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)\frac{(293 K)(1.4)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}} = 0.4372$$

Thus, the flow is choked at the throat. From Eq. 7 we obtain for corresponding value in Fig. D.1 for $Ma_1 = 0.44$

$$T_0 = \frac{293 K}{(0.96)} = 305 K$$

With Eq. 5 we obtain

$$T^* = (305 K)(0.83333) = 254 K$$

Thus

$$V^* = \sqrt{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)\frac{(254 K)(1.4)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = 319 \frac{m}{s}$$

(con't)

11.47 (con't)

From Eq. 12 we obtain with the help of Fig. D.1

$$P_o = \frac{100 \text{ kPa(abs)}}{0.87} = 115 \text{ kPa(abs)}$$

and with Eq. 10

$$p^* = [115 \text{ kPa(abs)}](0.52828) = 60.8 \text{ kPa(abs)}$$

Then with Eq. 8

$$\rho^* = \frac{\left(60.8 \times 10^3 \frac{N}{m^2}\right)}{\left(286.9 \frac{N \cdot m}{kg \cdot K}\right)(254 \text{ K})} = 0.83 \frac{kg}{m^3}$$

Finally, with Eq. 2 we obtain

$$\dot{m} = \left(0.83 \frac{kg}{m^3}\right)(0.1 \, m^2)\left(319 \frac{m}{s}\right) = \underline{\underline{26.5}} \frac{kg}{s}$$

11.48

11.48 The flow blockage associated with the use of an intrusive probe can be important. Determine the percentage increase in section velocity corresponding to a 0.5% reduction in flow area due to probe blockage for air flow if the section area is 1.0 m², T_0 = 20 °C, and the unblocked flow Mach numbers are **(a)** Ma = 0.2; **(b)** Ma = 0.8; **(c)** Ma = 1.5; **(d)** Ma = 3.0.

We want to ascertain

$$\frac{V_{blocked} - V_{unblocked}}{V_{unblocked}} \times 100$$

To determine the unblocked area velocity, $V_{unblocked}$, we use

$$V_{unblocked} = Ma_{unblocked}\sqrt{RT_{unblocked}\,k} \tag{1}$$

For $T_{unblocked}$ we use

$$T_{unblocked} = T_0\left(\frac{T}{T_0} \text{ for } Ma_{unblocked} \text{ from Eq. 11.56 for } Ma_{unblocked}\right) \tag{2}$$

To determine the blocked area velocity, $V_{blocked}$, we use

$$V_{blocked} = Ma_{blocked}\sqrt{RT_{blocked}\,k} \tag{3}$$

For $Ma_{blocked}$ we use $\frac{A_{blocked}}{A^*}$ and determine $Ma_{blocked}$ from Eq. 11.71.

Solution of Eq. 11.71 for $Ma_{blocked}$ from $\frac{A_{blocked}}{A^*}$ requires trial and error.

To determine $\frac{A_{blocked}}{A^*}$ we set

$$\frac{A_{blocked}}{A^*} = 0.995\,\frac{A_{unblocked}}{A^*} \tag{4}$$

We obtain $\frac{A_{unblocked}}{A^*}$ from Eq. 11.71 with the given value of $Ma_{unblocked}$.

To determine $T_{blocked}$ we use Eq. 11.56 to obtain

$$T_{blocked} = \frac{T_0}{1+\left(\frac{k-1}{2}\right)Ma^2_{blocked}} \tag{5}$$

(con't)

11.~~48~~ (con't)

(a) For $Ma_{unblocked} = 0.2$ we obtain with Eqs. 2 and 11.56

$$T_{unblocked} = (293\,K)(0.99206) = 290.7\,K$$

Then with Eq. 1 we have

$$V_{unblocked} = (0.2)\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(290.7\,K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = 68.34\,\frac{ft}{s}$$

we use Eqs. 4 and 11.71 to get

$$\frac{A_{blocked}}{A^*} = (0.995)(2.9635) = 2.949$$

and with Eq. 11.71 we obtain

$$Ma_{blocked} = 0.201$$

With Eq. 5 we get

$$T_{blocked} = \frac{293\,K}{1+\left(\frac{1.4-1}{2}\right)(0.201)^2} = 290.6\,K$$

With Eq. 3 we have

$$V_{blocked} = (0.201)\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(290.6\,K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = 68.67\,\frac{m}{s}$$

and

$$\frac{(V_{blocked} - V_{unblocked})\times 100}{V_{unblocked}} = \frac{\left(68.67\,\frac{m}{s} - 68.34\,\frac{m}{s}\right)(100)}{68.34\,\frac{m}{s}} = \underline{\underline{0.483\,\%}}$$

(b) For $Ma = 0.8$ we obtain with Eqs. 2 and 11.56

$$T_{unblocked} = (293\,K)(0.88652) = 259.8\,K$$

Then with Eq. 1 we get

$$V_{unblocked} = 0.8\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(259.8\,K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = 258.4\,\frac{m}{s}$$

We use Eqs. 4 and 11.71

$$\frac{A_{blocked}}{A^*} = (0.995)(1.03823) = 1.033$$

and with Eq. 11.71 we obtain

$$Ma_{blocked} = 0.813$$

With Eq. 5 we get

$$T_{blocked} = \frac{293\,K}{1+\left(\frac{1.4-1}{2}\right)(0.813)^2} = 258.8\,K$$

(con't)

11.48 (Con't)

With Eq. 3 we have

$$V_{blocked} = (0.813)\sqrt{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(258.8\ K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = 262.1\ \frac{m}{s}$$

and

$$\frac{(V_{blocked} - V_{unblocked})}{V_{unblocked}} \times 100 = \frac{\left(262.1\ \frac{m}{s} - 258.4\ \frac{m}{s}\right)}{\left(258.4\ \frac{m}{s}\right)}(100) = \underline{\underline{1.43\ \%}}$$

(c) For $Ma = 1.5$, we obtain with Eqs. 2 and 11.56

$$T_{unblocked} = (293\ K)(0.68965) = 202.1\ K$$

Then with Eq. 1 we get

$$V_{unblocked} = (1.5)\sqrt{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(202.1\ )(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = 427.4\ \frac{m}{s}$$

We use Eqs. 4 and 11.71 to get

$$\frac{A_{blocked}}{A^*} = (0.995)(1.1762) = 1.17$$

and with Eq. 11.71 we obtain

$$Ma_{blocked} = 1.491$$

With Eq. 5 we get

$$T_{blocked} = \frac{293\ K}{1 + \left(\frac{1.4-1}{2}\right)(1.491)^2} = 202.8\ K$$

With Eq. 3 we have

$$V_{blocked} = (1.491)\sqrt{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(202.8\ K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = 425.5\ \frac{m}{s}$$

and

$$\frac{(V_{blocked} - V_{unblocked})}{V_{unblocked}} \times 100 = \frac{\left(425.5\ \frac{m}{s} - 427.4\ \frac{m}{s}\right)(100)}{427.4\ \frac{m}{s}} = \underline{\underline{-0.445\%}}$$

(d) For $Ma = 3.0$ we obtain with Eqs. 2 and 11.56

$$T_{unblocked} = (293\ K)(0.35714) = 104.6\ K$$

Then with Eq. 1 we get

$$V_{unblocked} = (3.0)\sqrt{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)\frac{(104.6\ K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = 614.9\ \frac{m}{s}$$

(con't)

11.48 (Con't)

We use Eq. 4 and 11.71

$$\frac{A_{blocked}}{A^*} = (0.995)(4.2346) = 4.213$$

and with Eq. 11.71 we obtain

$$Ma_{blocked} = 2.995$$

With Eq. 5 we get

$$T_{blocked} = \frac{293\ K}{1+\left(\frac{1.4-1}{2}\right)(2.995)^2} = 104.9\,K$$

With Eq. 3 we have

$$V_{blocked} = (2.995)\sqrt{\frac{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)(104.9\,K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = 614.8\ \frac{m}{s}$$

and

$$\frac{(V_{blocked} - V_{unblocked})}{V_{unblocked}} \times 100 = \frac{\left(614.8\ \frac{m}{s} - 614.9\ \frac{m}{s}\right)(100)}{\left(614.9\ \frac{m}{s}\right)} = \underline{\underline{-0.0163\%}}$$

11.49

11.49 (See Fluids in the News article titled "Rocket nozzles," Section 11.4.2.) Comment on the practical limits of area ratio for the diverging portion of a converging–diverging nozzle designed to achieve supersonic exit flow.

From Fig. D.1 we see that the A/A^* vs. Ma curve becomes very steep with increasing values of Ma (very large increase in A/A^* needed to achieve even small gains in Ma level) suggesting practical limits to area divergence ratio is actual devices. For example, using Eq. 11.71, the A/A^* divergence ratio needed for $Ma = 5$ is 3450!

11.51

11.51 An ideal gas enters [section (1)] an insulated, constant cross-sectional area duct with the following properties:

$$T_0 = 293 \text{ K}$$
$$p_0 = 101 \text{ kPa(abs)}$$
$$\text{Ma}_1 = 0.2$$

For Fanno flow, determine corresponding values of fluid temperature and entropy change for various levels of pressure and plot the Fanno line if the gas is helium.

This is similar to Example 11.11. For Fanno flow of an ideal gas we use Eqs. 11.75 and 11.76 to establish the Fanno line states. Thus,

$$T + \frac{(\rho V)^2 T^2}{2 c_p \left(\frac{p^2}{R^2}\right)} = T_0 \qquad (1)$$

and

$$s - s_1 = c_p \ln\left(\frac{T}{T_1}\right) - R \ln\left(\frac{p}{p_1}\right) \qquad (2)$$

For helium, $k = 1.66$ and $R = 2077 \frac{N \cdot m}{kg \cdot K}$ (Table 1.8) and $c_p = 5224 \frac{N \cdot m}{kg \cdot K}$ from Problem 11.1 (c). We determine the constant value of ρV by calculating ρ_1 with the ideal gas equation of state (Eq. 3) and V_1 with Eq. 4. For T_1 we use Eq. 11.56 to obtain

$$T_1 = \frac{T_0}{1 + \left(\frac{k-1}{2}\right) Ma_1^2} = \frac{(293 K)}{1 + \left(\frac{1.66 - 1}{2}\right)(0.2)^2} = 289.2 \text{ K}$$

Then, with Eq. 4 we obtain

$$V_1 = 0.2 \sqrt{\frac{\left(2077 \frac{N \cdot m}{kg \cdot K}\right)(289.2 \text{ K})(1.66)}{\left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = 199.7 \frac{m}{s}$$

For p_1 we use Eq. 11.59 to get

$$p_1 = p_0 \left[\frac{1}{1 + \left(\frac{k-1}{2}\right) Ma_1^2}\right]^{\frac{k}{k-1}} = [101 \text{ kPa(abs)}] \left[\frac{1}{1 + \left(\frac{1.66-1}{2}\right)(0.2)^2}\right]^{\frac{1.66}{1.66-1}} = 97.72 \text{ kPa(abs)}$$

and with Eq. 3 we obtain

$$\rho_1 = \frac{\left(97.72 \times 10^3 \frac{N}{m^2}\right)}{\left(2077 \frac{N \cdot m}{kg \cdot K}\right)(289.2 K)} = 0.1627 \frac{kg}{m^3}$$

Thus, the value of ρV is

$$\rho_1 V_1 = \left(0.1627 \frac{kg}{m^3}\right)\left(199.7 \frac{m}{s}\right) = 32.49 \frac{kg}{m^2 \cdot s}$$

(con't)

11.51 (con't)

Eq. 1 becomes for helium

$$T + \frac{\left(32.49\,\frac{kg}{m^2\cdot s}\right)^2 T^2 \left(\frac{1\,N}{kg\cdot\frac{m}{s^2}}\right)}{2\left(5224\,\frac{N\cdot m}{kg\cdot K}\right)\frac{p^2}{\left(2077\,\frac{N\cdot m}{kg\cdot K}\right)^2}} = 293\,K$$

or

$$T + 4.358\times10^5\,\frac{T^2}{p^2} = 293 \qquad (6)$$

where T is in K and p is in $\frac{N}{m^2}$.

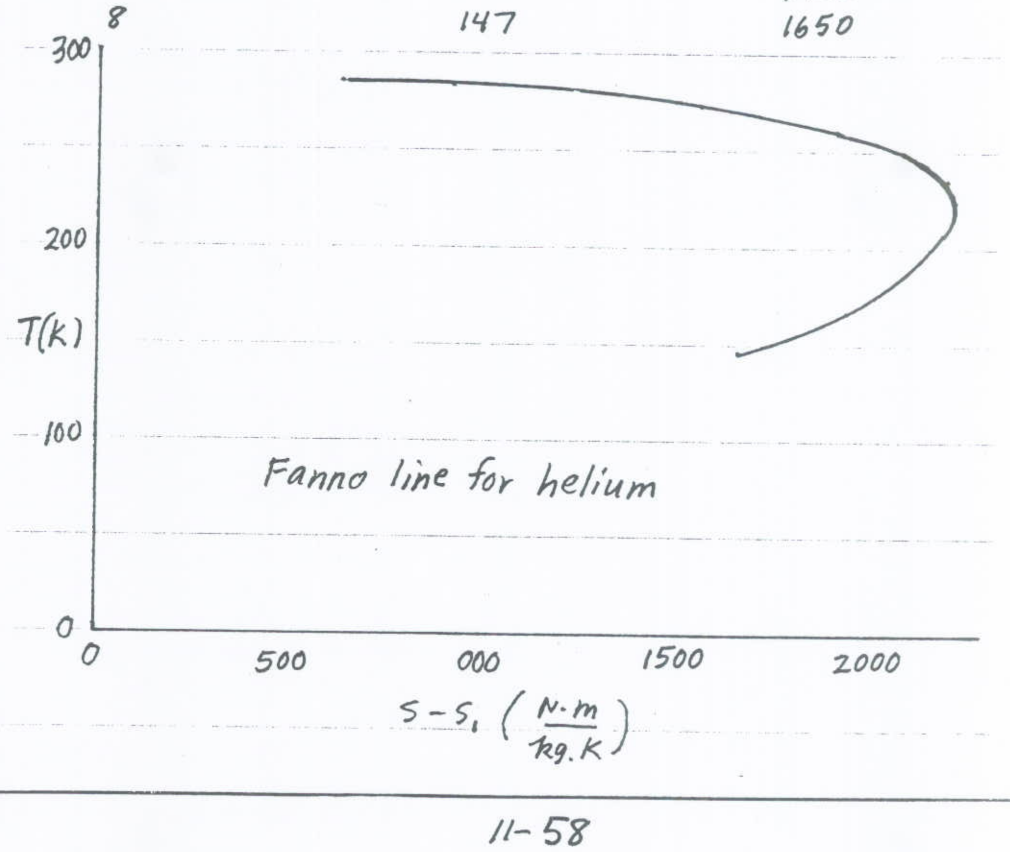
Eq. 2 becomes for helium

$$s - s_1 = \left(5224\,\frac{N\cdot m}{kg\cdot K}\right)\ln\left(\frac{T}{289.2\,K}\right) - \left(2077\,\frac{N\cdot m}{kg\cdot K}\right)\ln\left[\frac{p}{97.72\,kPa(abs)}\right] \qquad (7)$$

where T is in K and p is in kPa(abs).

With Eqs. 6 and 7 we construct the table of values shown below.

p [kPa(abs)]	T (K)	$s-s_1$ $\left(\frac{N\cdot m}{kg\cdot K}\right)$
70	286	630
60	283	905
50	279	1210
40	273	1550
30	260	1900
25	250	2060
20	234	2179
18	225	2200
15	209	2188
10	169	1923
8	147	1650



Fanno line for helium

11.52

11.52 For Fanno flow, prove that

$$\frac{dV}{V} = \frac{fk(\mathrm{Ma}^2/2)(dx/D)}{1 - \mathrm{Ma}^2}$$

and in so doing show that when the flow is subsonic, friction accelerates the fluid, and when the flow is supersonic, friction decelerates the fluid.

Starting with Eq. 11.95 we have

$$\frac{1}{2}\left(1 + k\,Ma^2\right)\frac{d(V^2)}{V^2} - \frac{d(Ma^2)}{Ma^2} + \frac{fk}{2}Ma^2\frac{dx}{D} = 0 \qquad (1)$$

From Eq. 11.93 we have

$$\frac{d(Ma^2)}{Ma^2} = \frac{d(V^2)}{V^2}\left[1 + \left(\frac{k-1}{2}\right)Ma^2\right] \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{1}{2}\left(1 + k\,Ma^2\right)\frac{d(V^2)}{V^2} - \left[1 + \left(\frac{k-1}{2}\right)Ma^2\right]\frac{d(V^2)}{V^2} + \frac{fk}{2}Ma^2\frac{dx}{D} = 0 \qquad (3)$$

or

$$\frac{1}{2}\left(Ma^2 - 1\right)\frac{d(V^2)}{V^2} = -\frac{fk}{2}Ma^2\frac{dx}{D}$$

and

$$\frac{d(V^2)}{V^2} = \frac{Ma^2}{(Ma^2 - 1)}fk\frac{dx}{D} \qquad (4)$$

However

$$d(V^2) = 2V\,dV \qquad (5)$$

Thus combining Eqs. 4 and 5 we get

$$\frac{dV}{V} = \frac{fk\left(\frac{Ma^2}{2}\right)\left(\frac{dx}{D}\right)}{1 - Ma^2} \qquad (6)$$

When the flow is subsonic ($Ma < 1.0$), Eq. 6 leads to $\frac{dV}{V} = +$ and thus friction accelerates the fluid. On the other hand when the flow is supersonic ($Ma > 1.0$), Eq. 6 leads to $\frac{dV}{V} = -$ and in this case friction decelerates the fluid.

11.53

11.53 Standard atmospheric air (T_0 = 59 °F, p_0 = 14.7 psia) is drawn steadily through a frictionless and adiabatic converging nozzle into an adiabatic, constant cross section area duct. The duct is 10 ft long and has an inside diameter of 0.5 ft. The average friction factor for the duct may be estimated as being equal to 0.03. What is the maximum mass flowrate in slugs/s through the duct? For this maximum flowrate determine the values of static temperature, static pressure, stagnation temperature, stagnation pressure, and velocity at the inlet [section (1)] and exit [section (2)] of the constant area duct. Sketch a temperature–entropy diagram for this flow.

This is similar to Example 11.12. As explained in Example 11.12, the maximum flowrate through the duct will occur when the constant area duct chokes and the Mach number at the duct exit [section (2)] is 1.0. The maximum flowrate can be obtained with

$$\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad (1)$$

We note that T_0 is constant throughout the entire flow since the flow is adiabatic. Thus, $T_{0,1} = T_{0,2} = 519\,^\circ R$. Also, p_0 is constant in the converging nozzle but decreases through the constant area duct because of friction. Thus, $p_{0,1} = 14.7\text{ psia}$.

For choked flow

$$\frac{f(\ell_2-\ell_1)}{D} = \frac{(0.03)(10\text{ ft})}{0.5\text{ ft}} = 0.6 = \frac{f(\ell^*-\ell_1)}{D}$$

and from Fig. D.2

we can read values of Ma_1, $\frac{T_1}{T^*}$, $\frac{V_1}{V^*}$, $\frac{p_1}{p^*}$ and $\frac{p_{0,1}}{p^*}$. Then $T^* = T_2$ can be obtained with Eq. 11.63 since T_0 is constant. Thus,

$$T^* = \left(\frac{2}{k+1}\right)T_0 = \left(\frac{2}{1.4+1}\right)(519\,^\circ R) = \underline{\underline{432\,^\circ R}} = T_2$$

and $V^* = V_2$ can be determined with

$$V^* = \sqrt{RT^*k} = \sqrt{\frac{\left(1716\,\frac{\text{ft}\cdot\text{lb}}{\text{slug}\cdot{}^\circ R}\right)(432\,^\circ R)(1.4)}{\left(1\,\frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)}} = \underline{\underline{1020\,\frac{\text{ft}}{\text{s}}}} = V_2$$

(con't)

11.53 (con't)

For $\frac{f(\ell^* - \ell)}{D} = 0.6$, from Fig. D.2 we read

$$Ma_1 = 0.57$$

$$\frac{T_1}{T^*} = 1.13 \qquad (2)$$

$$\frac{V_1}{V^*} = 0.6 \qquad (3)$$

$$\frac{p_1}{p^*} = 1.86 \qquad (4)$$

$$\frac{p_{0,1}}{p_0^*} = 1.22 \qquad (5)$$

From Eq. 2 we get

$$T_1 = (1.13)(432\,°R) = \underline{\underline{488\,°R}}$$

With Eq. 3 we obtain

$$V_1 = (0.6)\left(1020\ \frac{ft}{s}\right) = \underline{\underline{612}}\ \frac{ft}{s}$$

With Eq. 5 we have

$$p_0^* = p_{0,2} = \frac{p_{0,1}}{1.22} = \frac{14.7\ psia}{1.22} = \underline{\underline{12}}\ psia$$

To determine p_1 we enter Fig. D.1 with $Ma_1 = 0.57$ and read

$$\frac{p_1}{p_{0,1}} = 0.8$$

and

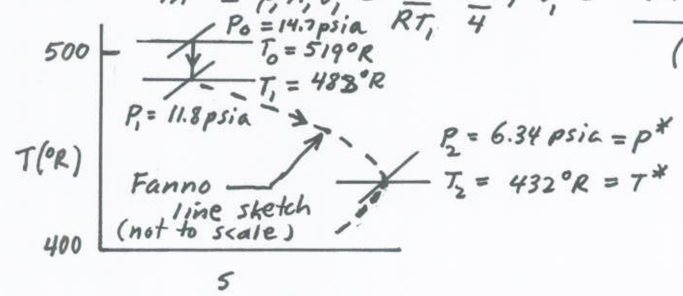
$$p_1 = (0.8)(14.7\ psia) = 11.8\ psia$$

With Eq. 4 we obtain

$$p^* = p_2 = \frac{p_1}{1.86} = \frac{11.8\ psia}{1.86} = 6.34\ psia$$

With Eq. 1 we have

$$\dot{m} = \rho_1 A_1 V_1 = \frac{p_1}{RT_1}\frac{\pi D_1^2}{4} V_1 = \frac{(11.8\ psia)\left(144\ \frac{in.^2}{ft^2}\right)\pi(0.5\ ft)^2\left(612\ \frac{ft}{s}\right)}{\left(1716\ \frac{ft \cdot lb}{slug \cdot °R}\right)(488\,°R)(4)} = \underline{\underline{0.244\ \frac{slug}{s}}}$$



11.54

11.54 The upstream pressure of a Fanno flow venting to the atmosphere is increased until the flow chokes. What will happen to the flowrate when the upstream pressure is further increased?

For a Fanno flow

$$\rho V = \frac{p}{RT}\, Ma \sqrt{RTk} = \text{constant}$$

Also at any one axial location in the flow, from Eq. 11.56

$$\frac{T}{T_0} = \frac{1}{1 + \left(\frac{k-1}{2}\right) Ma^2}$$

Combining we get

$$\rho V = \frac{p}{R\left[\frac{T_0}{1+\left(\frac{k-1}{2}\right)Ma^2}\right]}\, Ma \sqrt{R\left[\frac{T_0}{1+\left(\frac{k-1}{2}\right)Ma^2}\right]k} = \text{constant}$$

So for any one axial location of the flow where the Ma level is the same, T_0 is also the same but p is higher. Thus ρV is also higher and we conclude that increasing the inlet pressure of a choked Fanno flow into the atmosphere results in an increase of flowrate also.

Following the procedure of Example 11.11 one could plot a series of Fanno lines for different values of increased inlet pressure.

11.55

11.55 The duct in Problem 11.53 is shortened by 50%. The duct discharge pressure is maintained at the choked flow value determined in Problem 11.53. Determine the change in mass flowrate through the duct associated with the 50% reduction in length. The average friction factor remains constant at a value of 0.03.

This is like Example 11.13. We guess that the shortened duct will still choke and check our assumption by comparing p_d with p^* If $p_d < p^*$, the flow is choked. If not, another assumption must be made. For choked flow we calculate the mass flowrate as we did in Example 11.12 or in the solution of problem 11.51. For unchoked flow, we must devise another strategy.

For choked flow

$$\frac{f(\ell_2-\ell_1)}{D} = \frac{(0.03)(5\,ft)}{(0.5\,ft)} = 0.3 = \frac{f(\ell^*-\ell)}{D}$$

From Fig. D.2 we read

$$Ma_1 = 0.66$$

$$\frac{T_1}{T^*} = 1.1 \qquad (1)$$

$$\frac{V_1}{V^*} = 0.7 \qquad (2)$$

$$\frac{p_1}{p^*} = 1.6 \qquad (3)$$

With $Ma_1 = 0.66$, we enter Fig. D.1 and read

$$\frac{p_1}{p_{0,1}} = 0.75$$

Thus

$$p_1 = (0.75)(14.7\,psia) = 11 \;psia$$

(con't)

11.55 (con't)

and with Eq. 3 we obtain

$$p^* = p_2 = \frac{p_1}{1.6} = \frac{11 \text{ psia}}{1.6} = 6.88 \text{ psia}$$

Since

$$p_2 = 6.88 \text{ psia} > p_d = 6.34 \text{ psia}$$

the flow is choked as assumed.

$T^* = T_2$ can be obtained with Eq. 11.63 since T_0 is constant. Thus,

$$T^* = \left(\frac{2}{k+1}\right) T_0 = \left(\frac{2}{1.4+1}\right)(519°R) = 432°R = T_2$$

and $V_2 = V^*$ can be determined with

$$V_2 = V^* = \sqrt{RT^*k} = \sqrt{\frac{\left(1716 \frac{ft \cdot lb}{slug \cdot °R}\right)(432°R)(1.4)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = 1020 \frac{ft}{s}$$

With Eq. 1 we have

$$\dot{m} = \rho_2 A_2 V_2 = \frac{p_2}{RT_2} \frac{\pi D_2^2}{4} V_2 = \frac{(6.88 \text{ psia})\left(144 \frac{in.^2}{ft^2}\right)}{\left(1716 \frac{ft \cdot lb}{slug \cdot °R}\right)(432°R)} \frac{\pi (0.5 ft)^2 \left(1020 \frac{ft}{s}\right)}{(4)}$$

or

$$\dot{m} = 0.268 \frac{slug}{s}$$

The change in mass flowrate is

$$\left(\frac{\dot{m}_{5 ft} - \dot{m}_{10 ft}}{\dot{m}_{10 ft}}\right) \times 100 = \left(\frac{0.268 \frac{slug}{s} - 0.244 \frac{slug}{s}}{0.244 \frac{slug}{s}}\right)(100) = \underline{\underline{9.8 \%}}$$

The mass flowrate increased by 9.8% when the tube was shortened by 50%.

11.56

11.56 If the same mass flowrate of air obtained in Problem 11.53 is desired through the shortened duct of Problem 11.55, determine the back pressure, p_2, required. Assume f remains constant at a value of 0.03.

This is similar to Example 11.14. Since the same mass flowrate achieved in Problem 11.51 is desired with the shortened duct of Problem 11.53, we need to achieve the value of Ma_1 obtained in Problem 11.51. Thus, for the same value of Ma_1 as in Problem 11.51 we have

$$\frac{f(\ell^* - \ell_1)}{D} = 0.6$$

However,

$$\frac{f(\ell^* - \ell_2)}{D} = \frac{f(\ell^* - \ell_1)}{D} - \frac{f(\ell_2 - \ell_1)}{D}$$

or

$$\frac{f(\ell^* - \ell_2)}{D} = 0.6 - \frac{(0.03)(5\ ft)}{0.5\ ft} = 0.3$$

With $\frac{f(\ell^* - \ell_2)}{D} = 0.3$ we enter Fig. D.2 and read

$$\frac{p_2}{p^*} = 1.6 \qquad (1)$$

The value of p^* obtained in Problem 11.53 is still valid, so

$$p^* = 6.88\ psia$$

and with Eq. 1 we get

$$p_2 = (1.6)(6.88\ psia) = \underline{\underline{11\ psia}}$$

11.57

11.57 If the average friction factor of the duct of Example 11.12 is changed to **(a)** 0.01 or **(b)** 0.03, determine the maximum mass flowrate of air through the duct associated with each new friction factor and compare with the maximum mass flowrate value of Example 11.12.

(a) For $f = 0.01$ we have

$$\frac{f(\ell^* - \ell_1)}{D} = \frac{(0.01)(2m)}{(0.1m)} = 0.2$$

and on Fig. D.2 we read

$$Ma_1 = 0.7 \qquad (1)$$

$$\frac{T_1}{T^*} = 1.1 \qquad (2)$$

$$\frac{V_1}{V^*} = 0.73$$

From Example 11.12

$$T^* = 240\ K$$

and

$$V^* = 310\ \frac{m}{s}$$

Thus, with Eq. 1 we get

$$T_1 = (1.1)(240K) = 264\ K$$

and with Eq. 2 we obtain

$$V_1 = (0.73)(310\ \frac{m}{s}) = 226\ \frac{m}{s}$$

To determine p_1 we enter Fig. D.1 with $Ma_1 = 0.7$ and read

$$\frac{p_1}{p_{0,1}} = 0.72$$

Thus,

$$p_1 = (0.72)[101\ kPa(abs)] = 72.7\ kPa\,(abs)$$

To determine the mass flowrate we use

$$\dot{m} = \rho_1 A_1 V_1 = \frac{p_1}{RT_1}\frac{\pi D_1^2}{4}V_1 = \frac{(72.7\,10^3\ \frac{N}{m^2})\,\pi\,(0.1m)^2(226\ \frac{m}{s})}{(286.9\ \frac{N\cdot m}{kg\cdot K})(264K)(4)} = \underline{\underline{1.7}}\ \frac{kg}{s}$$

(Con't)

11.57 (con't)

For $f = 0.03$ we have

$$\frac{f(\ell^* - \ell_1)}{D} = \frac{(0.03)(2m)}{(0.1m)} = 0.6$$

and on Fig. D.2 we read

$$Ma_1 = 0.57$$

$$\frac{T_1}{T^*} = 1.13$$

$$\frac{V_1}{V^*} = 0.6$$

Thus,

$$T_1 = (1.13)(240K) = 271K$$

$$V_1 = (0.6)(310 \frac{m}{s}) = 186 \frac{m}{s}$$

From Fig. D.1 we read for $Ma_1 = 0.57$

$$\frac{P_1}{P_{0,1}} = 0.8$$

Thus,

$$P_1 = (0.8)[101 \text{ kPa(abs)}] = 81 \text{ kPa(abs)}$$

To determine $\dot{m}$ we use

$$\dot{m} = \frac{P_1}{RT_1} \frac{\pi D_1^2}{4} V_1 = \frac{(81 \times 10^3 \frac{N}{m^2}) \pi (0.1m)^2 (186 \frac{m}{s})}{(286.9 \frac{N \cdot m}{kg \cdot K})(271K)(4)} = \underline{\underline{1.52 \frac{kg}{s}}}$$

The maximum (choked duct) flowrates for different values of f are

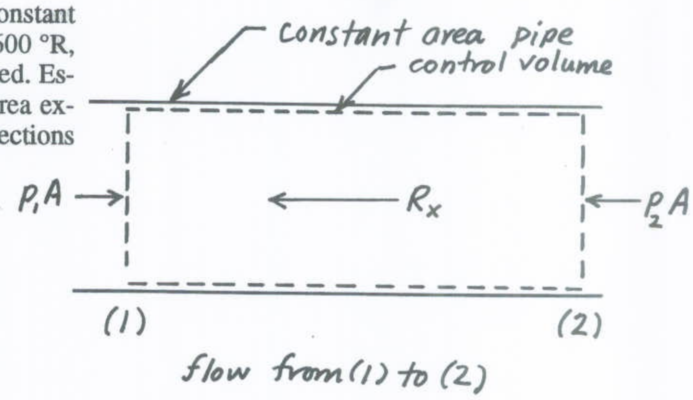
$$\dot{m}_{f=0.01} = 1.70 \frac{kg}{s}$$

$$\dot{m}_{f=0.02} = 1.65 \frac{kg}{s}$$

$$\dot{m}_{f=0.03} = 1.52 \frac{kg}{s}$$

11.58

11.58 Air flows adiabatically between two sections in a constant area pipe. At upstream section (1), $p_{0,1} = 100$ psia, $T_{0,1} = 600$ °R, and $Ma_1 = 0.5$. At downstream section (2), the flow is choked. Estimate the magnitude of the force per unit cross-sectional area exerted by the inside wall of the pipe on the fluid between sections (1) and (2).



The control volume sketched above is used. Applying the axial component of the linear momentum equation (Eq. 5.22) to the contents of this control volume we get for the force exerted by the pipe wall on the fluid, R_x,

$$R_x = p_1 A - p_2 A + \dot{m}(V_1 - V_2)$$

or

$$\frac{R_x}{A} = p_1 - p_2 + \rho_1 V_1 (V_1 - V_2) \qquad (1)$$

Thus we need p_1, p_2, ρ_1, V_1 and V_2.

(a) For air we enter Fig. D.1 with $Ma_1 = 0.5$ and get

$$\frac{T_1}{T_{0,1}} = 0.95$$

and

$$\frac{p_1}{p_{0,1}} = 0.84$$

Thus

$$T_1 = (0.95)(600\,°R) = 570\,°R$$

and

$$p_1 = (0.84)(100 \text{ psia}) = 84 \text{ psia}$$

Then

$$V_1 = Ma_1\sqrt{RT_1 k} = (0.5)\sqrt{\frac{\left(1716 \frac{ft\cdot lb}{slug\cdot °R}\right)(570\,°R)(1.4)}{\left(1 \frac{lb}{slug\cdot \frac{ft}{s^2}}\right)}} = 585 \frac{ft}{s}$$

and

$$\rho_1 = \frac{p_1}{RT_1} = \frac{(84 \text{ psia})\left(144 \frac{in.^2}{ft^2}\right)}{\left(1716 \frac{ft\cdot lb}{slug\cdot °R}\right)(570\,°R)} = 0.0124 \frac{slug}{ft^3}$$

(con't)

11.58

At section (2) the flow is choked. Thus we use the $*$ state of the Fanno flow, Fig. D.2 for section (2). Entering Fig. D.2 with $Ma_1 = 0.5$ we read

$$\frac{p_1}{p^*} = 2.14 = \frac{p_1}{p_2}$$

and

$$\frac{V_1}{V^*} = 0.54 = \frac{V_1}{V_2}$$

Thus

$$p_2 = \frac{p_1}{2.14} = \frac{(84.3\ psia)}{(2.14\ )} = 39.4\ psia$$

and

$$V_2 = \frac{V_1}{0.54} = \frac{\left(586\ \frac{ft}{s}\right)}{(0.54\ \ )} = 1080\ \frac{ft}{s}$$

Now with Eq. 1 we have

$$\frac{R_x}{A} = (84\ psia)\left(144\ \frac{in.^2}{ft^2}\right) - (39.4\ psia)\left(144\ \frac{in.^2}{ft^2}\right)$$

$$+\left(0.0124\ \frac{slug}{ft^3}\right)\left(585\ \frac{ft}{s}\right)\left(585\ \frac{ft}{s} - 1080\ \frac{ft}{s}\right)\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)$$

and

$$\frac{R_x}{A} = \underline{\underline{2830}}\ \frac{lb}{ft^2}$$

11.59

11.59 Cite an example of an actual subsonic flow of practical importance that may be approximated with a Rayleigh flow.

The flow through the combustor of a gas turbine engine is sometimes approximated with a Rayleigh flow.

11.60

11.60 Standard atmospheric air [T_0 = 288 K, p_0 = 101 kPa (abs)] is drawn steadily through an isentropic converging nozzle into a frictionless and diabatic (q = 500 kJ/kg) constant cross section area duct. For maximum flow determine the values of static temperature, static pressure, stagnation temperature, stagnation pressure, and flow velocity at the inlet [section (1)] and exit [section (2)] of the constant area duct. Sketch a temperature–entropy diagram for this flow.

For maximum flow, the Rayleigh flow is choked. For the isentropic nozzle

$$T_{0,1} = T_0 = \underline{\underline{288\,K}}$$

$$p_{0,1} = p_0 = \underline{\underline{101\,kPa\,(abs)}}$$

To determine the static state at the nozzle exit, Rayleigh flow inlet, we need the value of Ma_1. To determine Ma_1 we use

$$h_{0,2} - h_{0,1} = q = c_p\,(T_{0,2} - T_{0,1})$$

or

$$T_{0,2} = \frac{q}{c_p} + T_{0,1} = \frac{\left(500{,}000\ \frac{N.m}{kg}\right)}{\left(1004\ \frac{N.m}{kg.K}\right)} + 288K = \underline{\underline{786\ K}}$$

and noting that for choked flow, $T_{0,2} = T_{0,a}$ we get

$$\frac{T_{0,1}}{T_{0,2}} = \frac{T_{0,1}}{T_{0,a}} = \frac{288\,K}{786\,K} = 0.37$$

With $\frac{T_{0,1}}{T_{0,a}} = 0.37$ we enter Fig. D.3 and read

$$Ma_1 = 0.31$$

$$\frac{p_1}{p_a} = 2.1 \qquad (1)$$

$$\frac{T_1}{T_a} = 0.42 \qquad (2)$$

(con't)

11.60 (con't)

$$\frac{V_1}{V_a} = 0.2 \qquad (3)$$

$$\frac{p_{0,1}}{p_{0,a}} = 1.19 \qquad (4)$$

With Eq. 4 we obtain

$$p_{0,a} = \frac{p_{0,1}}{1.19} = \frac{101\ kPa(abs)}{1.19} = \underline{\underline{84.9\ kPa(abs)}} = p_{0,2}$$

With $Ma_1 = 0.31$ we read from Fig. D.1

$$\frac{p_1}{p_{0,1}} = 0.94 \qquad (5)$$

and

$$\frac{T_1}{T_{0,1}} = 0.98 \qquad (6)$$

With Eqs. 5 and 6 we get

$$p_1 = (0.94)[101\ kPa(abs)] = \underline{\underline{95\ kPa(abs)}} \qquad (7)$$

and

$$T_1 = (0.98114)(288K) = \underline{\underline{282K}}$$

Thus

$$V_1 = Ma_1\sqrt{RT_1k} = (0.31)\sqrt{\frac{\left(286.9\ \frac{N\cdot m}{kg\cdot K}\right)(282K)(1.4)}{\left(1\ \frac{N}{kg\cdot \frac{m}{s^2}}\right)}} = \underline{\underline{104\ \frac{m}{s}}} \qquad (9)$$

Combining Eqs. 1 and 7 we obtain

$$p_a = \frac{p_1}{2.1} = \frac{[95\ kPa(abs)]}{(2.1)} = \underline{\underline{45\ kPa(abs)}} = p_2$$

Combining Eqs. 2 and 8 we have

$$T_a = \frac{T_1}{0.42} = \frac{(283\ K)}{(0.42)} = \underline{\underline{674K}} = T_2$$

Combining Eqs. 3 and 9 we have

$$V_a = \frac{V_1}{0.2} = \frac{104\ \frac{m}{s}}{0.2} = \underline{\underline{520\ \frac{m}{s}}} = V_2$$

(con't)

11.60 (con't)

To sketch a T-s diagram we obtain $s_2 - s_1$ from

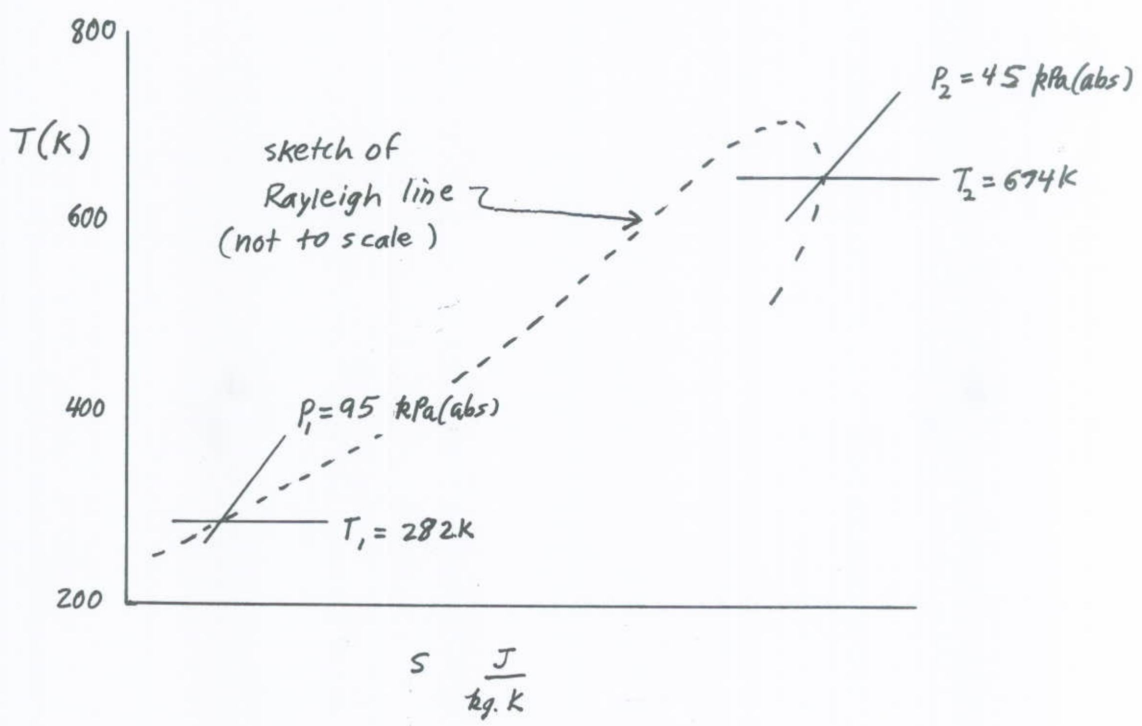
$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

or

$$s_2 - s_1 = \left(1004 \frac{N.m}{kg.K}\right) \ln\left(\frac{674}{282}\right) - \left(286.9 \frac{N.m}{kg.K}\right) \ln\left[\frac{45 \text{ kPa(abs)}}{95 \text{ kPa(abs)}}\right]$$

and

$$s_2 - s_1 = 1090 \frac{N.m}{kg.K}$$



11.61

11.61 Air enters a 0.5-ft inside diameter duct with $p_1 = 20$ psia, $T_1 = 80$ °F, and $V_1 = 200$ ft/s. What frictionless heat addition rate in Btu/s is necessary for an exit gas temperature $T_2 = 1500$ °F? Determine p_2, V_2, and Ma_2 also.

To determine the heat transfer rate we use the energy equation (Eq. 5.69) to get

$$\dot{Q}_{\substack{net\\in}} = \dot{m}\,(h_{0,2} - h_{0,1}) = \dot{m}\,c_p\,(T_{0,2} - T_{0,1}) \qquad (1)$$

For mass flowrate we use

$$\dot{m} = \rho_1 A_1 V_1 = \frac{p_1}{RT_1}\,\frac{\pi D_1^2}{4}\,V_1 \qquad (2)$$

To determine $T_{0,2}$ and $T_{0,1}$ we use Eq. 11.56. Thus,

$$\frac{T}{T_0} = \frac{1}{1 + \left(\frac{k-1}{2}\right) Ma^2} \qquad (3)$$

or for air

$$\frac{T}{T_0} = f(Ma) \text{ in Fig. D.1} \qquad (4)$$

To determine p_2 we use

$$p_2 = p_1 \left(\frac{p_a}{p_1}\right)\left(\frac{p_2}{p_a}\right) \qquad (5)$$

where with Eq. 11.123 for Rayleigh flow

$$\frac{p}{p_a} = \frac{1+k}{1+k\,Ma^2} \qquad (6)$$

or for air

$$\frac{p}{p_a} = f(Ma) \text{ in Fig. D.3} \qquad (7)$$

For exit velocity, V_2, we use

$$V_2 = Ma_2\sqrt{RT_2 k} \qquad (8)$$

We determine Ma_1 with

$$Ma_1 = \frac{V_1}{c_1} = \frac{V_1}{\sqrt{RT_1 k}} \qquad (9)$$

(con't)

11.61 (con't)

and we determine Ma_2 with

$$\frac{T_2}{T_a} = \left(\frac{T_2}{T_1}\right)\left(\frac{T_1}{T_a}\right) \tag{10}$$

and Eq. 11.128 for Rayleigh flow, namely

$$\frac{T}{T_a} = \left[\frac{(1+k)\,Ma}{1+k\,Ma^2}\right]^2 \tag{11}$$

or for air with

$$\frac{T}{T_a} = f(Ma) \text{ on Fig. D.3} \tag{12}$$

For air we determine Ma_1 with Eq. 9. Thus,

$$Ma_1 = \frac{\left(200\ \frac{ft}{s}\right)}{\sqrt{\left(1716\ \frac{ft\cdot lb}{slug\cdot {}^\circ R}\right)\frac{(540\,{}^\circ R)(1.4)}{\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}}} = 0.18$$

For $Ma_1 = 0.18$ we read on Fig. D.1

$$\frac{T_1}{T_{0,1}} = 0.99$$

Thus

$$T_{0,1} = \frac{540\,{}^\circ R}{0.99} = 545\,{}^\circ R$$

With $Ma_1 = 0.18$ we read on Fig. D.3 the values

$$\frac{T_1}{T_a} = 0.17$$

and

$$\frac{P_1}{P_a} = 2.3$$

Thus with Eq. 10 we obtain

$$\frac{T_2}{T_a} = \left(\frac{1960\,{}^\circ R}{540\,{}^\circ R}\right)(0.17) = 0.62$$

(con't)

11.61 (con't)

For $\frac{T_2}{T_a} = 0.62$ we get from Fig. D.3

$$Ma_2 = \underline{\underline{0.40}}$$

and

$$\frac{P_2}{P_a} = 1.96$$

With $Ma_2 = 0.40$ we read on Fig. D.1

$$\frac{T_2}{T_{0,2}} = 0.97$$

Thus,

$$T_{0,2} = \frac{1960}{0.97} = 2020\,°R$$

Then with Eq. 5 we have

$$P_2 = (20\,psia)\left(\frac{1}{2.3}\right)(1.96) = \underline{\underline{17}}\ psia$$

With Eq. 8 we have

$$V_2 = (0.40)\sqrt{\frac{\left(1716\,\frac{ft.lb}{slug.°R}\right)(1960\,°R)(1.4)}{\left(1\,\frac{lb}{slug.\frac{ft}{s^2}}\right)}} = \underline{\underline{868}}\ \frac{ft}{s}$$

With Eq. 2 we get

$$\dot{m} = \frac{(20\,psia)\left(144\,\frac{in.^2}{ft^2}\right)}{\left(1716\,\frac{ft.lb}{slug.°R}\right)(540\,°R)}\,\frac{\pi\,(0.5\,ft)^2\left(200\,\frac{ft}{s}\right)}{(4)} = 0.122\ \frac{slug}{s}$$

and with Eq. 1 we obtain

$$\dot{Q}_{\substack{net\\in}} = \left(0.122\,\frac{slug}{s}\right)\left(6006\,\frac{ft.lb}{slug.°R}\right)\frac{(2020\,°R - 545\,°R)}{\left(778\,\frac{ft.lb}{Btu}\right)} = \underline{\underline{1390}}\ \frac{Btu}{s}$$

11.62

11.6 Air enters a length of constant cross section area pipe with $p_1 = 200$ kPa (abs), $T_1 = 500$ K, and $V_1 = 400$ m/s. If 500 kJ/kg of energy is removed from the air by frictionless heat transfer between sections (1) and (2), determine p_2, T_2, and V_2. Sketch a temperature–entropy diagram for the flow between sections (1) and (2).

To determine the state of the air at section (2) we use the energy equation (Eq. 5.69) to calculate the value of $T_{0,2}$. Thus,

$$q_{\substack{net\\in}} = h_{0,2} - h_{0,1} = c_p (T_{0,2} - T_{0,1})$$

or

$$T_{0,2} = \frac{q_{\substack{net\\in}}}{c_p} + T_{0,1} = -\frac{q_{\substack{net\\out}}}{c_p} + T_{0,1} \qquad (1)$$

We obtain $T_{0,1}$ from $\frac{T_1}{T_{0,1}}$ which we read from Fig. D.1 with a value of Ma_1. We determine Ma_1 with

$$Ma_1 = \frac{V_1}{c_1} = \frac{V_1}{\sqrt{RT_1 k}} \qquad (2)$$

With Ma_1 we also enter Fig. D.3 and read values of $\frac{p_1}{p_a}$, $\frac{T_1}{T_a}$, $\frac{V_1}{V_a}$, and $\frac{T_{0,1}}{T_{0,a}}$. Then we determine $\frac{T_{0,2}}{T_{0,a}}$ with

$$\frac{T_{0,2}}{T_{0,a}} = \left(\frac{T_{0,2}}{T_{0,1}}\right)\left(\frac{T_{0,1}}{T_{0,a}}\right) \qquad (3)$$

With this value of $\frac{T_{0,2}}{T_{0,a}}$ we enter Fig. D.3 and read corresponding values of $\frac{p_2}{p_a}$, $\frac{T_2}{T_a}$, and $\frac{V_2}{V_a}$. Then we determine p_2, T_2 and V_2 with

$$p_2 = \left(\frac{p_2}{p_a}\right)\left(\frac{p_a}{p_1}\right) p_1 \qquad (4)$$

$$T_2 = \left(\frac{T_2}{T_a}\right)\left(\frac{T_a}{T_1}\right) T_1 \qquad (5)$$

and

$$V_2 = \left(\frac{V_2}{V_a}\right)\left(\frac{V_a}{V_1}\right) V_1 \qquad (6)$$

(con't)

11.62 (con't)

We use Eq. 2 to get

$$Ma_1 = \frac{\left(400\ \frac{m}{s}\right)}{\sqrt{\frac{\left(286.9\ \frac{N \cdot m}{kg \cdot K}\right)(500K)(1.4)}{\left(1\ \frac{N}{kg \cdot \frac{m}{s^2}}\right)}}} = 0.89$$

For $Ma_1 = 0.89$ we get from Fig. D.1

$$\frac{T_1}{T_{0,1}} = 0.86$$

Thus,

$$T_{0,1} = \frac{(500\ K)}{(0.86\ \ )} = 580\ K$$

and with Eq. 1 we have

$$T_{0,2} = -\frac{\left(500{,}000\ \frac{J}{kg}\right)}{\left(1004\ \frac{J}{kg \cdot K}\right)} + 580K = 82\ K$$

With $Ma_1 = 0.893$ we enter Fig. D.3 and read

$$\frac{P_1}{P_a} = 1.14$$

$$\frac{T_1}{T_a} = 1.02$$

$$\frac{V_1}{V_a} = 0.9$$

and

$$\frac{T_{0,1}}{T_{0,a}} = 0.99$$

Now with $\frac{T_{0,1}}{T_{0,a}} = 0.99$ and Eq. 3 we obtain

$$\frac{T_{0,2}}{T_{0,a}} = \left(\frac{82K}{579K}\right)(0.99\ \ ) = 0.14$$

(con't)

11.62 (con't)

which has as corresponding values in Fig. D.3 of

$$Ma_2 = 0.18$$

$$\frac{p_2}{p_a} = 2.3$$

$$\frac{T_2}{T_a} = 0.17$$

and

$$\frac{V_2}{V_a} = 0.07$$

With these ratios and those ratios corresponding to $Ma_1 = 0.89$ we use Eqs. 4, 5 and 6 to obtain

$$p_2 = (2.3)\left(\frac{1}{1.14}\right)\left[200\ kPa(abs)\right] = \underline{\underline{404}}\ kPa\,(abs)$$

$$T_2 = (0.17)\left(\frac{1}{1.02}\right)(500K) = \underline{\underline{83}}\ K$$

and

$$V_2 = (0.07)\left(\frac{1}{0.9}\right)\left(400\ \frac{m}{s}\right) = \underline{\underline{31}}\ \frac{m}{s}$$

Note that according to our calculations, $T_2 = 83.2K$ is slightly larger than $T_{0,2} = 82K$. This is not correct and is a result of the inaccuracy associated with using the graphs.

For more precision we ascertain the value of Ma_2 knowing $\frac{T_{0,2}}{T_{0,a}}$ using Eq. 11.131. First however, we determine $\frac{T_{0,1}}{T_{0,a}}$ knowing Ma_1 with Eq. 11.131. Thus,

$$\frac{T_{0,1}}{T_{0,a}} = \frac{2(k+1)\,Ma_1^2\left(1+\frac{k-1}{2}Ma_1^2\right)}{(1+kMa_1^2)^2} = \frac{2(1.4+1)(0.893)^2\left[1+\left(\frac{1.4-1}{2}\right)(0.893)^2\right]}{\left[1+(1.4)(0.893)^2\right]^2}$$

or

$$\frac{T_{0,1}}{T_{0,a}} = 0.9908$$

(con't)

11.62 (con't)

Now we use Eq. 11.56 to determine $\frac{T_1}{T_{0,1}}$. Thus,

$$\frac{T_1}{T_{0,1}} = \frac{1}{1+\left(\frac{k-1}{2}\right)Ma_1^2} = \frac{1}{1+\left(\frac{1.4-1}{2}\right)(0.893)^2} = 0.8624$$

and

$$T_{0,1} = \frac{T_1}{0.8624} = \frac{(500K)}{0.8624} = 579.8K$$

Now with Eq. 1 we have

$$T_{0,2} = \frac{-\left(500{,}000\ \frac{J}{kg}\right)}{\left(1004\ \frac{J}{kg.K}\right)} + 579.8K = 81.79K$$

With Eq. 3 we obtain

$$\frac{T_{0,2}}{T_{0,a}} = \left(\frac{81.79K}{579.8K}\right)(0.9908) = 0.1398$$

With Eq. 11.131 and $\frac{T_{0,2}}{T_{0,a}} = 0.1398$ we get

$$Ma_2 = 0.1776$$

Then with Eq. 11.128 and $Ma_1 = 0.893$ and $Ma_2 = 0.1776$ we get

$$\frac{T_1}{T_a} = \left[\frac{(1+k)Ma_1}{1+kMa_1^2}\right]^2 = \left[\frac{(1+1.4)(0.893)}{1+(1.4)(0.893)^2}\right]^2 = 1.026$$

and

$$\frac{T_2}{T_a} = \left[\frac{(1+1.4)(0.1776)}{1+(1.4)(0.1776)^2}\right]^2 = 0.1666$$

(con't)

11.62 (con't)

Now with Eq. 5 we have

$$T_2 = (0.1666)\left(\frac{1}{1.026}\right)(500\,K) = 81.19\,K$$

and

$$T_2 = 81.19\,K < T_{0,2} = 81.79\,K$$

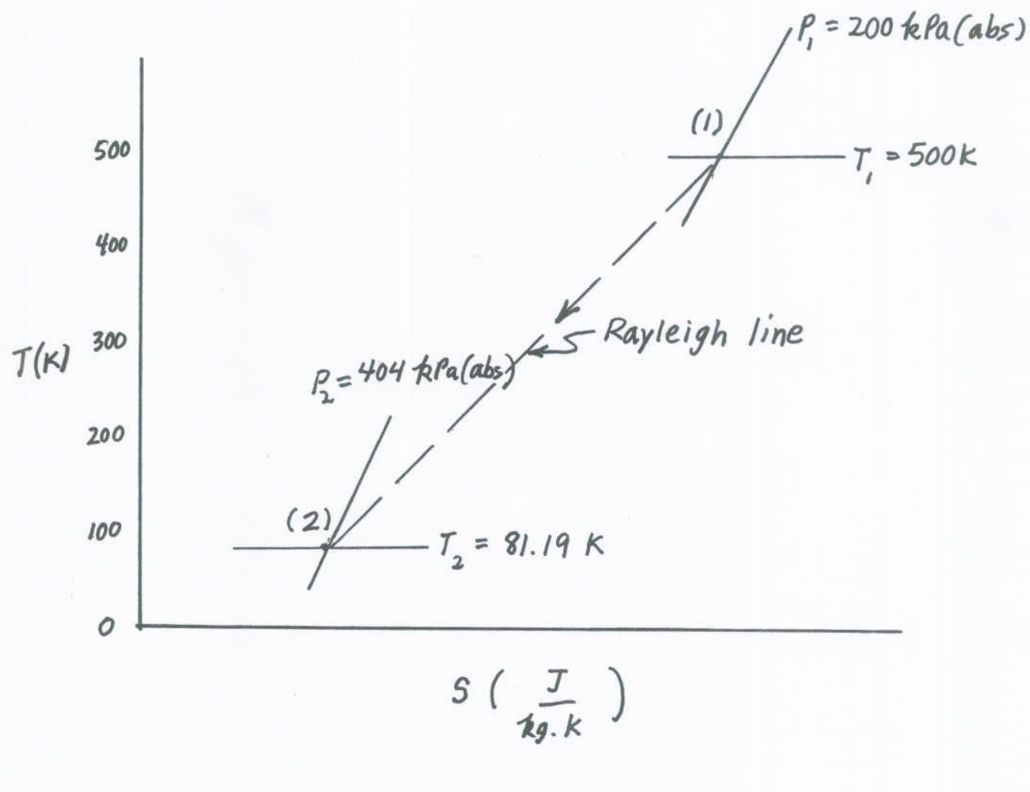
as it should be.

For our T-s sketch we use Eq. 11.76 to calculate $s_2 - s_1$. Thus,

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 1004\,\frac{J}{kg \cdot K} \ln\left(\frac{81.19\,K}{500\,K}\right) - 286.9\,\frac{J}{kg \cdot K} \ln\left[\frac{404\,kPa\,(abs)}{200\,kPa\,(abs)}\right]$$

and

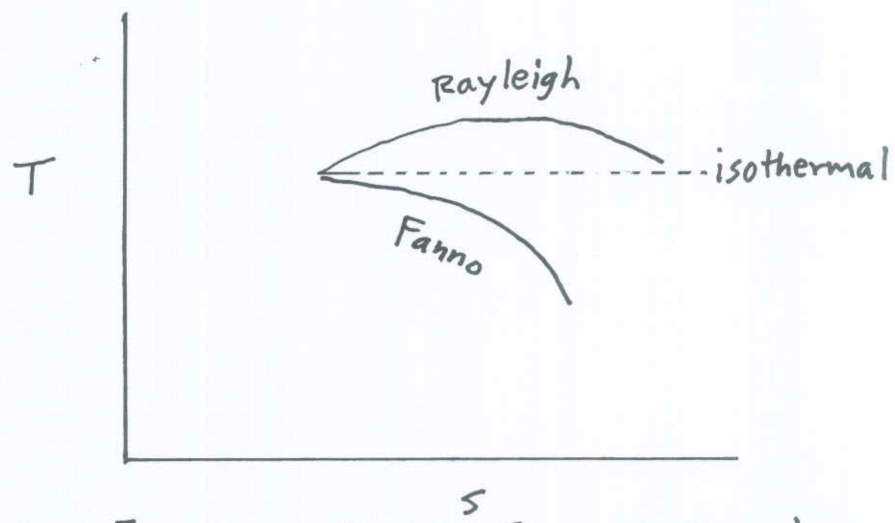
$$s_2 - s_1 = -2030\,\frac{J}{kg \cdot K}$$



11.63

11.63 Describe what happens to a Fanno flow when heat transfer is allowed to occur. Is this the same as a Rayleigh flow with friction considered?

One way to respond to this problem statement is to consider what the path of these flows would look like on temperature-entropy (T-s) coordinates. Starting with the subsonic portions of Fig. 11.25



we can show Fanno and Rayleigh flows. Another classical case described in a number of fluid mechanics texts is isothermal pipe flow (constant temperature pipe flow with friction and heat transfer). This kind of flow approximates what occurs in long underground pipelines. As shown in the sketch above by the broken line the isothermal flow path is generally above the Fanno flow path and below the Rayleigh flow path. We conclude that the path for pipe flow with friction and heating would be above the Fanno flow path and the path for pipe flow with friction and cooling would be below the Fanno flow path with friction and flow rates constant. A Rayleigh flow with friction would track below the Rayleigh flow path shown other things equal. Rayleigh flows approximate flows with heat transfer over short path lengths over which friction can be ignored as an approximation of reality.

11.65

11.65 The Mach number and stagnation pressure of air are 2.0 and 200 kPa(abs) just upstream of a normal shock. Estimate the stagnation pressure loss across the shock.

We want to determine the stagnation pressure loss across a normal shock, or

$$p_{0,x} - p_{0,y} = p_{0,x}\left(1 - \frac{p_{0,y}}{p_{0,x}}\right) \qquad (1)$$

To determine the stagnation pressure ratio we use Eq. 11.156. Thus,

$$\frac{p_{0,y}}{p_{0,x}} = \frac{\left[\left(\frac{k+1}{2}\right)\mathrm{Ma}_x^2\right]^{\frac{k}{k-1}}\left[1+\left(\frac{k-1}{2}\right)\mathrm{Ma}_x^2\right]^{\frac{k}{1-k}}}{\left[\left(\frac{2k}{k+1}\right)\mathrm{Ma}_x^2-\left(\frac{k-1}{k+1}\right)\right]^{\frac{1}{k-1}}} \qquad (2)$$

or for air

$$\frac{p_{0,y}}{p_{0,x}} = f(\mathrm{Ma}_x) \quad \text{in Fig. D.4.}$$

For air ($k=1.4$) we have from Fig. D.4 for $\mathrm{Ma}_x = 2.0$,

$$\frac{p_{0,y}}{p_{0,x}} = 0.72$$

Thus, with Eq. 1 we obtain

$$p_{0,x} - p_{0,y} = [200 \text{ kPa(abs)}](1 - 0.72) = \underline{\underline{56}} \text{ kPa}$$

21

11.66

11.66 The stagnation pressure ratio across a normal shock in an air flow is 0.6. Estimate the Mach number of the flow entering the shock.

To determine the Mach number of the air flow entering a normal shock, Ma_x, given the stagnation pressure ratio, $\frac{p_{0,x}}{p_{0,y}}$, we enter Fig. D.4 with

$$\frac{p_{0,x}}{p_{0,y}} = 0.6$$

and read on Fig. D.4

$$Ma_x = \underline{\underline{2.29}}$$

11.67

11.67 Just upstream of a normal shock in an air flow, Ma = 3.0, T = 600 °R, and p = 30 psia. Estimate values of Ma, T_0, T, p_0, p, and V downstream of the shock.

To determine Ma_y knowing Ma_x we use Eq. 11.149. Thus,

$$Ma_y = \sqrt{\frac{Ma_x^2 + \left(\frac{2}{k-1}\right)}{\left(\frac{2k}{k-1}\right)Ma_x^2 - 1}} \qquad (1)$$

or for air we use Fig. D.4 for Ma_y as a function of Ma_x.
To determine $T_{0,y}$ we use Eq. 11.56. Thus,

$$T_{0,y} = T_y\left[1 + \left(\frac{k-1}{2}\right)Ma_y^2\right] \qquad (2)$$

or for air we use Fig. D.1 for $\frac{T_y}{T_{0,y}}$ as a function of Ma_y.
To obtain T_y we use Eq. 11.151. Thus,

$$T_y = T_x\left\{\frac{\left[1+\left(\frac{k-1}{2}\right)Ma_x^2\right]\left[2\left(\frac{k}{k-1}\right)Ma_x^2 - 1\right]}{\left[\frac{(k+1)^2}{2(k-1)}\right]Ma_x^2}\right\} \qquad (3)$$

or for air we use Fig. D.4 for $\frac{T_y}{T_x}$ as a function of Ma_x.
For $p_{0,y}$ we use Eq. 2 of Example 11.19 to get

$$p_{0,y} = p_x\left\{\frac{\left[\left(\frac{k+1}{2}\right)Ma_x^2\right]^{\frac{k}{k-1}}}{\left[\left(\frac{2k}{k+1}\right)Ma_x^2 - \left(\frac{k-1}{k+1}\right)\right]^{\frac{1}{k-1}}}\right\} \qquad (4)$$

or for air we use Fig. D.4 for $\frac{p_{0,y}}{p_x}$ as a function of Ma_x.
For p_y we use Eq. 11.150 to obtain

$$p_y = p_x\left[\left(\frac{2k}{k+1}\right)Ma_x^2 - \left(\frac{k-1}{k+1}\right)\right] \qquad (5)$$

or for air we use Fig. D.4 for $\frac{p_y}{p_x}$ as a function of Ma_x.
For V_y we use

$$V_y = Ma_y\sqrt{RT_y k} \qquad (6)$$

(con't)

11.67 (con't)

For air we read from Fig. D.4 for $Ma_x = 3.0$

$$Ma_y = \underline{\underline{0.475}}$$

$$\frac{p_y}{p_x} = 10.3 \qquad (7)$$

$$\frac{T_y}{T_x} = 2.7 \qquad (8)$$

$$\frac{p_{0,y}}{p_x} = 12 \qquad (9)$$

and we obtain from Fig. D.1 for $Ma_y = 0.475$

$$\frac{T_y}{T_{0,y}} = 0.96 \qquad (10)$$

From Eq. 8 we get

$$T_y = (2.7)(600°R) = \underline{\underline{1620°R}}$$

and thus with Eq. 10

$$T_{0,y} = \frac{T_y}{0.96} = \frac{1620°R}{0.96} = \underline{\underline{1690°R}}$$

With Eq. 7 we obtain

$$p_y = (10.3)(30\,psia) = \underline{\underline{309}}\ psia$$

and Eq. 9 yields

$$p_{0,y} = (12)(30\,psia) = \underline{\underline{360}}\ psia$$

Then with Eq. 6 we obtain

$$V_y = (0.475)\sqrt{\frac{\left(1716\ \frac{ft \cdot lb}{slug \cdot °R}\right)(1620°R)(1.4)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{937}}\ \frac{ft}{s}$$

11.68

11.68 A total pressure probe like the one shown in **Video V3.8** is inserted into a supersonic air flow. A shock wave forms just upstream of the impact hole. The probe measures a total pressure of 500 kPa(abs). The stagnation temperature at the probe head is 500 K. The static pressure upstream of the shock is measured with a wall tap to be 100 kPa(abs). From these data, estimate the Mach number and velocity of the flow.

This is like Example 11.19.

We enter Fig. D.4 with

$$\frac{p_{0,y}}{p_x} = \frac{500 \text{ kPa(abs)}}{100 \text{ kPa(abs)}} = 5$$

and read

$$Ma_x = \underline{\underline{1.9}}$$

We determine the value of V_x with

$$V_x = Ma_x \sqrt{R T_x k} \tag{1}$$

For T_x we read from Fig. D.1 for $Ma_x = \underline{\underline{1.9}}$

$$\frac{T_x}{T_{0,x}} = 0.58$$

and since

$$T_{0,x} = T_{0,y} = 500 \text{ K}$$

we have

$$T_x = (0.58)\,500 \text{ K} = 290 \text{ K}$$

and with Eq. 1 we obtain

$$V_x = 1.9 \sqrt{\frac{\left(286.9 \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(290 \text{ K})(1.4)}{\left(1 \frac{\text{N}}{\text{kg}\cdot\frac{\text{m}}{\text{s}^2}}\right)}} = \underline{\underline{648 \frac{\text{m}}{\text{s}}}}$$

11.69

11.69 The Pitot tube on a supersonic aircraft (see **Video V3.8**) cruising at an altitude of 30,000 ft senses a stagnation pressure of 12 psia. If the atmosphere is considered standard, determine the airspeed and Mach number of the aircraft. A shock wave is present just upstream of the probe impact hole.

At 30,000 ft, we read from Table C.1 for standard atmosphere

$$T = -47.83°F = 412.2 \ °R$$

and

$$p = 4.373 \ psia$$

Thus,

$$\frac{p_{0,y}}{p_x} = \frac{12 \ psia}{4.373 \ psia} = 2.74$$

and with this value of $\frac{p_{0,y}}{p_x}$ we read from Fig. D.4

$$Ma_y = \underline{\underline{1.25}}$$

Thus,

$$V_x = Ma_x\sqrt{RT_x k} = 1.25\sqrt{\frac{\left(1716 \frac{ft \cdot lb}{slug \cdot °R}\right)(412.2°R)(1.4)}{\left(1 \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}}$$

and

$$V_x = \underline{\underline{1240}} \ \frac{ft}{s}$$

11.7D

11.7D An aircraft cruises at a Mach number of 2.0 at an altitude of 15 km. Inlet air is decelerated to a Mach number of 0.4 at the engine compressor inlet. A normal shock occurs in the inlet diffuser upstream of the compressor inlet at a section where the Mach number is 1.2. For isentropic diffusion, except across the shock, and for standard atmosphere determine the stagnation temperature and pressure of the air entering the engine compressor.

The deceleration process in the inlet diffuser is assumed to be adiabatic since we are considering isentropic diffusion except across the shock. Thus,

$$T_0 = \text{constant}$$

and

$$T_{0,\text{comp inlet}} = T_{0,\text{diffuser inlet}} \qquad (1)$$

To determine the diffuser inlet stagnation temperature we enter Fig. D.1 with $Ma = 2.0$ and read

$$\frac{T}{T_0} = 0.55 \qquad (2)$$

At 15 km elevation in standard atmosphere we read from Table C.2

$$T = -56.5\,^\circ C = 216.5\ K$$

Thus, with Eqs. 1 and 2 we obtain

$$T_{0,\text{comp inlet}} = T_{0,\text{diffuser inlet}} = \frac{(216.5\ K)}{(0.55)} = \underline{\underline{394\ K}}$$

To determine the stagnation pressure at the compressor inlet we use

$$p_{0,\text{comp inlet}} = p_{0,\text{diffuser inlet}} \left(\frac{p_{0,x}}{p_{0,\text{diffuser inlet}}}\right)\left(\frac{p_{0,y}}{p_{0,x}}\right)\left(\frac{p_{0,\text{comp inlet}}}{p_{0,y}}\right) \qquad (3)$$

For $p_{0,\text{diffuser inlet}}$ we use

$$p_{0,\text{diffuser inlet}} = \left(\frac{p_{0,\text{diffuser inlet}}}{p_{\text{diffuser inlet}}}\right) p_{\text{diffuser inlet}} \qquad (4)$$

where $p_{\text{diffuser inlet}} = p_{\text{atm}}$ at 15 km or $p_{\text{diffuser inlet}} = 1.211 \times 10^4\ \frac{N}{m^2}$ (abs) from Table C.2.

(con't)

11.70 (con't)

We obtain $\frac{p_{diffuser\ inlet}}{p_{0,diffuser\ inlet}}$ from Fig. D.1 for $Ma_{diffuser\ inlet} = 2.0$.

Thus from Fig. D.1 we have

$$\frac{p_{diffuser\ inlet}}{p_{0,diffuser\ inlet}} = 0.13 \qquad (5)$$

Combining Eqs. 4 and 5 we obtain

$$p_{0,\ diffuser\ inlet} = \frac{1.211 \times 10^4 \frac{N}{m^2}(abs)}{(0.13)} = 93{,}000\ \frac{N}{m^2}(abs)$$

For $Ma_x = 1.2$, we read from Fig. D.4

$$\frac{p_{0,y}}{p_{0,x}} = 0.99$$

Also, since the flow is isentropic except across the shock,

$$\frac{p_{0,x}}{p_{0,\ diffuser\ inlet}} = 1.0$$

and

$$\frac{p_{0,\ comp\ inlet}}{p_{0,y}} = 1.0$$

Thus, with Eq 3 we obtain

$$p_{0,\ comp\ inlet} = \left[93{,}000\ \frac{N}{m^2}(abs)\right](1.0)(0.9928)(1.0) = \underline{\underline{92{,}000\ \frac{N}{m^2}(abs)}} = \underline{\underline{92\ kPa(abs)}}$$

To determine the static pressure at the compressor inlet we enter Fig. D.1 with $Ma_{comp\ inlet} = 0.4$ and read

$$\frac{p_{comp\ inlet}}{p_{0,\ comp\ inlet}} = 0.89$$

Thus,

$$p_{comp\ inlet} = (0.89)[92\ kPa(abs)] = \underline{\underline{82}}\ kPa(abs)$$

11.71

11.71 Determine, for the air flow through the frictionless and adiabatic converging–diverging duct of Example 11.8, the ratio of duct exit pressure to duct inlet stagnation pressure that will result in a standing normal shock at: **(a)** $x = +0.1$ m; **(b)** $x = +0.2$ m; **(c)** $x = +0.4$ m. How large is the stagnation pressure loss in each case?

This is similar to Example 11.20.

(a) For a standing normal shock at $x = +0.1\,m$ we note from the table of Example 11.8 that

$$Ma_x = 1.37$$

and

$$\frac{p_x}{p_{0,x}} = 0.33 \qquad (1)$$

From Fig. D.4, for $Ma_x = 1.37$ we obtain

$$Ma_y = 0.75$$

and

$$\frac{p_{0,y}}{p_{0,x}} = 0.96 \qquad (2)$$

From Fig. D.1 we find for

$$Ma_y = 0.75$$

$$\frac{A_y}{A^*} = 1.1 \qquad (3)$$

For $x = +0.1\,m$, the ratio of duct exit area to local area (A_2/A_y) is

$$\frac{A_2}{A_y} = \frac{0.1\,m^2 + (0.5\,m)^2}{0.1\,m^2 + (0.1\,m)^2} = 3.18 \qquad (4)$$

and using Eqs. 3 and 4 we get

$$\frac{A_2}{A^*} = \left(\frac{A_y}{A^*}\right)\left(\frac{A_2}{A_y}\right) = (1.1)(3.18) = 3.5$$

(con't)

11.71 (con't)

With $\frac{A_2}{A^*} = 3.5$ we get from Fig. D.1

$$Ma_2 = 0.17$$

and

$$\frac{p_2}{p_{0,2}} = \frac{p_2}{p_{0,y}} = 0.98$$

Thus

$$\frac{p_2}{p_{0,1}} = \frac{p_2}{p_{0,x}} = \left(\frac{p_2}{p_{0,y}}\right)\left(\frac{p_{0,y}}{p_{0,x}}\right) = (0.98)(0.96) = \underline{\underline{0.94}}$$

The loss in stagnation pressure is

$$p_{0,1} - p_{0,2} = p_{0,x} - p_{0,y} = p_{0,x}\left(1 - \frac{p_{0,y}}{p_{0,x}}\right) = [101\ kPa(abs)](1 - 0.96) = \underline{\underline{4}}\ kPa$$

(b) For a standing normal shock at $x = +0.2\,m$ we note from the table of Example 11.8 that

$$Ma_x = 1.76$$

and

$$\frac{p_x}{p_{0,x}} = 0.18$$

From Fig. D.4, for $Ma_x = 1.76$ we obtain

$$Ma_y = 0.62$$

and

$$\frac{p_{0,y}}{p_{0,x}} = 0.83$$

From Fig. D.1 we find for

$$Ma_y = 0.63$$

$$\frac{A_y}{A^*} = 1.16$$

For $x = +0.2m$, the ratio of duct exit area to local area, $\frac{A_2}{A_y}$, is

$$\frac{A_2}{A_y} = \frac{0.1m^2 + (0.5m)^2}{0.1m^2 + (0.2m)^2} = 2.5$$

(con't)

11.71 (con't)

and thus

$$\frac{A_2}{A^*} = \left(\frac{A_2}{A_y}\right)\left(\frac{A_y}{A^*}\right) = (2.5)(1.16) = 2.9$$

With $\frac{A_2}{A^*} = 2.9$ we get from Fig. D.1

$$Ma_2 = 0.20$$

and

$$\frac{p_2}{p_{0,2}} = \frac{p_2}{p_{0,y}} = 0.97$$

Thus

$$\frac{p_2}{p_{0,1}} = \frac{p_2}{p_{0,x}} = \left(\frac{p_2}{p_{0,y}}\right)\left(\frac{p_{0,y}}{p_{0,x}}\right) = (0.97)(0.83) = \underline{\underline{0.8}}$$

The loss in stagnation pressure is

$$p_{0,1} - p_{0,2} = p_{0,x} - p_{0,y} = p_{0,x}\left(1 - \frac{p_{0,y}}{p_{0,x}}\right) = [101\,kPa(abs)](1-0.83) = \underline{\underline{17}}\ kPa$$

(C) For a standing normal shock at $x = +0.4\,m$ we note from the table of Example 11.8 that

$$Ma_x = 2.48$$

and

$$\frac{p_x}{p_{0,x}} = 0.06$$

From Fig. D.4, for $Ma_x = 2.48$ we obtain

$$Ma_y = 0.515$$

and

$$\frac{p_{0,y}}{p_{0,x}} = 0.51$$

From Fig. D.1 we find

$$Ma_y = 0.51$$

$$\frac{A_y}{A^*} = 1.3$$

(con't)

11.71 (con't)

For $x = +0.4\,m$, the ratio of duct exit area to local area, $\frac{A_2}{A_y}$, is

$$\frac{A_2}{A_y} = \frac{0.1\,m^2 + (0.5\,m)^2}{0.1\,m^2 + (0.4\,m)^2} = 1.35$$

and thus

$$\frac{A_2}{A^*} = \left(\frac{A_2}{A_y}\right)\left(\frac{A_y}{A^*}\right) = (1.35)(1.3\quad) = 1.8$$

With $\frac{A_2}{A^*} = 1.8$ we get from Fig. D.1

$$Ma_2 = 0.34$$

and

$$\frac{p_2}{p_{0,2}} = \frac{p_2}{p_{0,y}} = 0.92$$

Thus,

$$\frac{p_2}{p_{0,1}} = \frac{p_2}{p_{0,x}} = \left(\frac{p_2}{p_{0,y}}\right)\left(\frac{p_{0,y}}{p_{0,x}}\right) = (0.92\quad)(0.51) = \underline{\underline{0.47}}$$

The loss in stagnation pressure is

$$p_{0,1} - p_{0,2} = p_{0,x} - p_{0,y} = p_{0,x}\left(1 - \frac{p_{0,y}}{p_{0,x}}\right) = [101\,kPa(abs)](1-0.51\quad) = \underline{\underline{50\,kPa}}$$

11.72

11.72 A normal shock is positioned in the diverging portion of a frictionless, adiabatic, converging–diverging air flow duct where the cross section area is 0.1 ft^2 and the local Mach number is 2.0. Upstream of the shock, $p_0 = 200$ psia and $T_0 = 1200$ °R. If the duct exit area is 0.15 ft^2, determine the exit area temperature and pressure and the duct mass flowrate.

To determine the duct exit temperature, T_2, and pressure, P_2, we need $\frac{T_2}{T_{0,2}}$ and $\frac{P_2}{P_{0,2}}$. We can obtain these ratios from Fig. D.1 knowing the value of Ma_2. The value of Ma_2 we obtain from Fig. D.1 with a known value of $\frac{A_2}{A^*}$ which we get from

$$\frac{A_2}{A^*} = \left(\frac{A_2}{A_y}\right)\left(\frac{A_y}{A^*}\right) \qquad (1)$$

The value of $\left(\frac{A_y}{A^*}\right)$ is obtained from Fig. D.1 with the value of Ma_y obtained from Fig. D.4 with a known value of $Ma_x = 2.0$. Thus from Fig. D.4 for $Ma_x = 2.0$

$$Ma_y = 0.58$$

and from Fig. D.1 we read for $Ma_y = 0.58$

$$\frac{A_y}{A^*} = 1.2$$

From the problem statement

$$\frac{A_2}{A_y} = \frac{0.15 \text{ ft}^2}{0.1 \text{ ft}^2} = 1.5$$

and thus with Eq. 1 we have

$$\frac{A_2}{A^*} = (1.5)(1.2) = 1.8$$

(con't)

11.72 (con't)

With $\frac{A_2}{A^*} = 1.8$ we get from Fig. D.1

$$Ma_2 = 0.34 \qquad (2)$$

$$\frac{T_2}{T_{0,2}} = 0.97 \qquad (3)$$

and

$$\frac{P_2}{P_{0,2}} = 0.92 \qquad (4)$$

The value of $T_{0,2}$ is obtained from

$$T_{0,2} = T_{0,x} = T_{0,y} = T_0 = 1200\,°R \qquad (5)$$

The value of $P_{0,2}$ is obtained from

$$P_{0,2} = P_{0,y} = P_{0,x}\left(\frac{P_{0,y}}{P_{0,x}}\right)$$

where

$$\frac{P_{0,y}}{P_{0,x}} = 0.72$$

from Fig. D.4 for $Ma_x = 2.0$.

Thus

$$P_{0,2} = (200\ psia)(0.72) = 144\ psia \qquad (6)$$

With Eqs 3 and 5 we obtain

$$T_2 = T_{0,2}\left(\frac{T_2}{T_{0,2}}\right) = (1200°R)(0.97) = \underline{\underline{1160°R}}$$

With Eqs. 4 and 6 we have

$$P_2 = P_{0,2}\left(\frac{P_2}{P_{0,2}}\right) = (144\ psia)(0.92) = \underline{\underline{132}}\ psia$$

For mass flowrate we use

$$\dot{m} = \rho_2 A_2 V_2 = \frac{P_2}{RT_2} A_2\, Ma_2\, c_2 = \frac{P_2}{RT_2} A_2\, Ma_2 \sqrt{RT_2 k}$$

and

$$\dot{m} = \frac{(132\,psia)\left(144\,\frac{in.^2}{ft^2}\right)(0.15\,ft^2)(0.34)}{\left(1716\,\frac{ft \cdot lb}{slug \cdot °R}\right)(1160\,°R)} \sqrt{\frac{\left(1716\,\frac{ft\cdot lb}{slug\cdot °R}\right)(1160°R)(1.4)}{\left(1\,\frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}}$$

$$\dot{m} = \underline{\underline{0.81}}\ \frac{slug}{s} = \underline{\underline{26.1}}\ \frac{lbm}{s}$$

11.73

11.73 Supersonic air flow enters an adiabatic, constant cross section area (inside diameter = 1 ft) pipe 30 ft long with $Ma_1 = 3.0$. The pipe friction factor is estimated to be 0.02. What ratio of pipe exit pressure to pipe inlet stagnation pressure would result in a normal shock wave standing at **(a)** $x = 5$ ft, or **(b)** $x = 10$ ft, where x is the distance downstream from the pipe entrance? Determine also the duct exit Mach number and sketch the temperature–entropy diagram for each situation.

This is similar to Example 11.21.

With $Ma_1 = 3.0$ we enter Fig. D.2 and get

$$\frac{f(\ell^* - \ell_1)}{D} = 0.52$$

We note that

$$\frac{f(\ell^* - \ell_1)}{D} = \frac{f(\ell^* - \ell_x)}{D} + \frac{f(\ell_x - \ell_1)}{D} \qquad (1)$$

(a) With Eq. 1 we get for $\ell_x - \ell_1 = 5\text{ ft}$

$$\frac{f(\ell^* - \ell_x)}{D} = \frac{f(\ell^* - \ell_1)}{D} - \frac{f(\ell_x - \ell_1)}{D} = 0.52 - \frac{(0.02)(5\text{ ft})}{(1\text{ ft})}$$

or

$$\frac{f(\ell^* - \ell_x)}{D} = 0.42$$

With $\frac{f(\ell^* - \ell_x)}{D} = 0.42$ we enter Fig. D.2 and find

$$Ma_x = 2.5$$

With $Ma_x = 2.5$ we enter Fig. D.4 and read

$$Ma_y = 0.52$$

Now with $Ma_y = 0.52$ we obtain from Fig. D.2

$$\frac{f(\ell^* - \ell_y)}{D} = 0.9$$

Since

$$\frac{f(\ell^* - \ell_2)}{D} = \frac{f(\ell^* - \ell_y)}{D} - \frac{f(\ell_2 - \ell_y)}{D}$$ (con't)

11.73 (con't)

we get

$$\frac{f(\ell^* - \ell_2)}{D} = 0.9 \qquad \frac{(0.02)(25\,ft)}{(1\,ft)} = 0.4$$

and entering Fig. D.2 with $\frac{f(\ell^* - \ell_2)}{D} = 0.4$ we obtain

$$Ma_2 = \underline{\underline{0.62}} \quad (\text{subsonic flow})$$

Now we note that

$$\frac{p_2}{p_{0,1}} = \left(\frac{p_2}{p^*}\right)\left(\frac{p^*}{p_y}\right)\left(\frac{p_y}{p_x}\right)\left(\frac{p_x}{p^*}\right)\left(\frac{p^*}{p_1}\right)\left(\frac{p_1}{p_{0,1}}\right) \qquad (2)$$

With $Ma_2 = 0.62$ we obtain from Fig. D.2

$$\frac{p_2}{p^*} = 1.7 \qquad (3)$$

With $Ma_y = 0.52$ we obtain from Fig. D.2

$$\frac{p_y}{p^*} = 2.05 \qquad (4)$$

With $Ma_x = 2.5$ we get from Fig. D.4

$$\frac{p_y}{p_x} = 7 \qquad (5)$$

and we obtain from Fig. D.2

$$\frac{p_x}{p^*} = 0.3 \qquad (6)$$

For $Ma_1 = 3.0$ we get from Fig. D.2.

$$\frac{p_1}{p^*} = 0.22 \qquad (7)$$

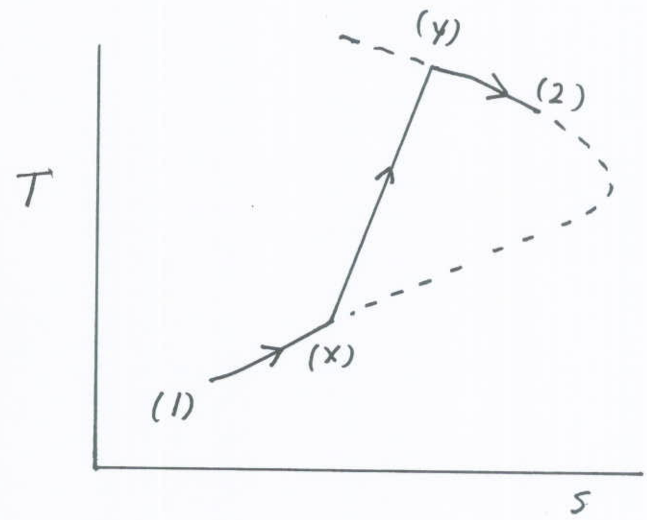
and from Fig. D.1

$$\frac{p_1}{p_{0,1}} = 0.03 \qquad (8)$$

(con't)

Combining Eqs. 2 through 8 we obtain

$$\frac{p_2}{p_{0,1}} = (1.7)\left(\frac{1}{2.05}\right)(7)(0.3)\left(\frac{1}{0.22}\right)(0.03) = \underline{\underline{0.213}}$$



Since we do not have values of temperature or pressure anywhere in the flow, we can only sketch qualitatively what happens on T-s coordinates. The T-s diagram will be similar to the one of Fig. E11.21(b) as indicated above.

(b) With Eq. 1 we get for $\ell_x - \ell_1 = 10\,ft$

$$\frac{f(\ell^* - \ell_x)}{D} = 0.52 - \frac{(0.02)(10\,ft)}{(1\,ft)} = 0.32$$

With $\frac{f(\ell^* - \ell_x)}{D} = 0.32$ we enter Fig. D.2 and find

$$Ma_x = 2$$

With $Ma_x = 2$ we enter Fig. D.4 and read

$$Ma_y = 0.58$$

Now with $Ma_y = 0.58$ we obtain from Fig. D.2

(con't)

11.73 (con't)

$$\frac{f(\ell^* - \ell_y)}{D} = 0.62$$

Since

$$\frac{f(\ell^* - \ell_2)}{D} = \frac{f(\ell^* - \ell_y)}{D} - \frac{f(\ell_2 - \ell_y)}{D}$$

we get

$$\frac{f(\ell^* - \ell_2)}{D} = 0.62 - \frac{(0.02)(20\ ft)}{(1\ ft)} = 0.22$$

and entering Fig. D.2 with $\frac{f(\ell^* - \ell_2)}{D} = 0.22$ we obtain

$$Ma_2 = \underline{\underline{0.89}}$$

With $Ma_2 = 0.89$ we obtain from Fig. D.2

$$\frac{p_2}{p^*} = 1.14 \qquad (9)$$

With $Ma_y = 0.57$ we obtain from Fig. D.2

$$\frac{p_y}{p^*} = 1.86 \qquad (10)$$

With $Ma_x = 2$ we get from Fig. D.4

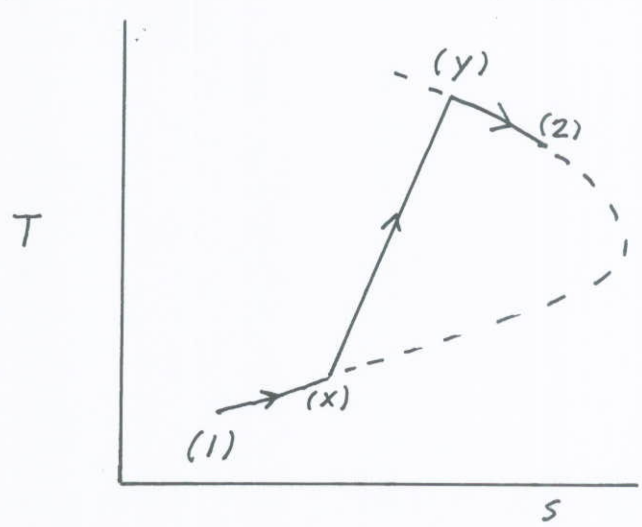
$$\frac{p_y}{p_x} = 4.8 \qquad (11)$$

and we obtain from Fig. D.2

$$\frac{p_x}{p^*} = 0.4 \qquad (12)$$

Combining Eqs. 2, 7, 8, 9, 10, 11 and 12 we obtain

$$\frac{p_2}{p_{0,1}} = (1.14)\left(\frac{1}{1.86}\right)(4.8)(0.4)\left(\frac{1}{0.22}\right)(0.03) = \underline{\underline{0.16}}$$



11.74

11.74 Supersonic air flow enters an adiabatic, constant area pipe (inside diameter = 0.1 m) with $Ma_1 = 2.0$. The pipe friction factor is 0.02. If a standing normal shock is located right at the pipe exit, and the Mach number just upstream of the shock is 1.2, determine the length of the pipe.

We note that

$$\frac{f(\ell_2-\ell_1)}{D} = \frac{f(\ell^*-\ell_1)}{D} - \frac{f(\ell^*-\ell_2)}{D} \qquad (1)$$

where according to Eq. 11.98

$$\frac{f(\ell^*-\ell)}{D} = \frac{1}{k}\frac{(1-Ma^2)}{(Ma^2)} + \left(\frac{k+1}{2k}\right)\ln\left[\frac{\left(\frac{k+1}{2}\right)Ma^2}{1+\left(\frac{k-1}{2}\right)Ma^2}\right] \qquad (2)$$

or for air, $\frac{f(\ell^*-\ell)}{D}$ is graphed as a function of Ma in Fig. D.2.

Thus, knowing Ma_1 and Ma_2 we can determine $\frac{f(\ell^*-\ell_1)}{D}$ and $\frac{f(\ell^*-\ell_2)}{D}$ and with Eq. 1 we obtain $\frac{f(\ell_2-\ell_1)}{D}$. With f and D also known we can determine $\ell_2-\ell_1$.

For air, we find in Fig. D.2 corresponding to $Ma_1 = 2.0$ and $Ma_2 = 1.2$,

$$\frac{f(\ell^*-\ell_1)}{D} = 0.3$$

and

$$\frac{f(\ell^*-\ell_2)}{D} = 0.03$$

Thus, with Eq. 1 we have

$$\frac{f(\ell_2-\ell_1)}{D} = 0.3 - 0.03 = 0.27$$

and

$$\ell_2-\ell_1 = \frac{(0.27)(0.1\,m)}{0.02} = \underline{\underline{1.35\,m}}$$

11.75

11.75 Air enters a frictionless, constant cross section area duct with $Ma_1 = 2.0$, $T_{0,1} = 59\ °F$, and $p_{0,1} = 14.7$ psia. The air is decelerated by heating until a normal shock wave occurs where the local Mach number is 1.5. Downstream of the normal shock, the subsonic flow is accelerated with heating until it chokes at the duct exit. Determine the static temperature and pressure, the stagnation temperature and pressure, and the fluid velocity at the duct entrance, just upstream and downstream of the normal shock and at the duct exit. Sketch the temperature–entropy diagram for this flow.

At the duct entrance, section (1), we have

$$T_{0,1} = \underline{\underline{59°F}} = \underline{\underline{519°R}}$$

and

$$p_{0,1} = \underline{\underline{14.7\ psia}}$$

With $Ma_1 = 2.0$ we enter Fig. D.1 and read

$$\frac{T_1}{T_{0,1}} = 0.56 \qquad (1)$$

and

$$\frac{p_1}{p_{0,1}} = 0.13 \qquad (2)$$

Thus with Eqs. 1 and 2 we obtain

$$T_1 = (0.56)(519°R) = \underline{\underline{291}}\ °R$$

and

$$p_1 = (0.13)(14.7\ psia) = \underline{\underline{1.91\ psia}}$$

Then

$$V_1 = Ma_1\sqrt{RT_1k} = (2.0)\sqrt{\frac{\left(1716\ \frac{ft \cdot lb}{slug \cdot °R}\right)(288°R)(1.4)}{\left(1\ \frac{lb}{slug \cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{1660}}\ \frac{ft}{s}$$

At section (x) just upstream of the shock

$$T_{0,x} = T_{0,1}\left(\frac{T_{0,a}}{T_{0,1}}\right)\left(\frac{T_{0,x}}{T_{0,a}}\right) \qquad (3)$$

(con't)

11.75 (con't)

and

$$P_{0,x} = P_{0,1}\left(\frac{P_{0,a}}{P_{0,1}}\right)\left(\frac{P_{0,x}}{P_{0,a}}\right) \qquad (4)$$

For $Ma_1 = 2.0$ and $Ma_x = 1.5$ we read from Fig. D.3

$$\frac{T_{0,1}}{T_{0,a}} = 0.79 \qquad (5)$$

$$\frac{P_{0,1}}{P_{0,a}} = 1.5 \qquad (6)$$

$$\frac{T_{0,x}}{T_{0,a}} = 0.91$$

$$\frac{P_{0,x}}{P_{0,a}} = 1.12$$

With these ratios and Eqs. 3 and 4 we obtain

$$T_{0,x} = (519\,°R)\left(\frac{1}{0.79}\right)(0.91) = \underline{\underline{598\,°R}}$$

$$P_{0,x} = (14.7\,psia)\left(\frac{1}{1.5}\right)(1.12) = \underline{\underline{11}}\ psia$$

With $Ma_x = 1.5$ we enter Fig. D.1 and read

$$\frac{T_x}{T_{0,x}} = 0.69$$

and

$$\frac{P_x}{P_{0,x}} = 0.27$$

Thus,

$$T_x = (0.69)(595\,°R) = \underline{\underline{411\,°R}}$$

and

$$P_x = (0.27)(11\,psia) = \underline{\underline{3}}\ psia$$

Then

$$V_x = Ma_x\sqrt{RT_x k} = (1.5)\sqrt{\frac{\left(1716\,\frac{ft\cdot lb}{slug\cdot °R}\right)(410\,°R)(1.4)}{\left(1\,\frac{lb}{slug\cdot \frac{ft}{s^2}}\right)}} = \underline{\underline{1490\,\frac{ft}{s}}}$$

(con't)

11.75 (con't)

At section (y) just downstream of the shock we obtain from Fig. D.4 for $Ma_x = 1.5$

$$Ma_y = 0.7$$

$$\frac{p_y}{p_x} = 2.5$$

$$\frac{T_y}{T_x} = 1.3$$

$$\frac{V_x}{V_y} = 1.9$$

$$\frac{p_{0,y}}{p_{0,x}} = 0.93$$

With these ratios and values of properties at section (x) previously determined we have

$$p_y = (2.5)(3.00\ psia) = \underline{\underline{7.5\ psia}}$$

$$T_y = (1.3)(410\,°R) = \underline{\underline{533\,°R}}$$

$$V_y = \frac{\left(1490\ \frac{ft}{s}\right)}{1.9} = \underline{\underline{784\ \frac{ft}{s}}}$$

$$p_{0,y} = (0.93)(11.0\ psia) = \underline{\underline{10.2}}\ psia$$

Also, since the flow across the normal shock is adiabatic,

$$T_{0,y} = T_{0,x} = \underline{\underline{598\,°R}}$$

At the duct exit, section (2) we have the subscript "a" state in Fig. D.3 since the flow is choked there. Thus from Eqs. 5 and 6 we conclude that

$$T_{0,a} = \frac{T_{0,1}}{0.79} = \frac{(519\,°R)}{(0.79)} = \underline{\underline{657}}\,°R = T_{0,2}$$

and

$$p_{0,a} = \frac{p_{0,1}}{1.5} = \frac{(14.7\ psia)}{(1.5)} = \underline{\underline{9.8}}\ psia = p_{0,2}$$

(con't)

11.75 (con't)

With $Ma_1 = 2.0$ we read further from Fig. D.3:

$$\frac{p_1}{p_a} = 0.36$$

$$\frac{T_1}{T_a} = 0.53$$

$$\frac{V_1}{V_a} = 1.45$$

Thus,

$$p_a = \frac{(1.91\ psia)}{(0.36)} = \underline{\underline{5.31\ psia}} = p_2$$

$$T_a = \frac{(291^\circ R)}{(0.53)} = \underline{\underline{549^\circ R}} = T_2$$

and

$$V_a = \frac{\left(1660\ \frac{ft}{s}\right)}{(1.45)} = \underline{\underline{1140\ \frac{ft}{s}}} = V_2$$

To sketch a T-s diagram we need values of $s-s_1$ and we calculate these values with

$$s - s_1 = c_p \ln \frac{T}{T_1} - R \ln \frac{p_x}{p_1}$$

So, for example,

$$s_x - s_1 = \left(6006\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}\right) \ln\left(\frac{411^\circ R}{519^\circ R}\right) - \left(1716\ \frac{ft\ lb}{slug \cdot {}^\circ R}\right) \ln\left(\frac{3\ psia}{14.7\ psia}\right) = 1310\ \frac{ft \cdot lb}{slug\ {}^\circ R}$$

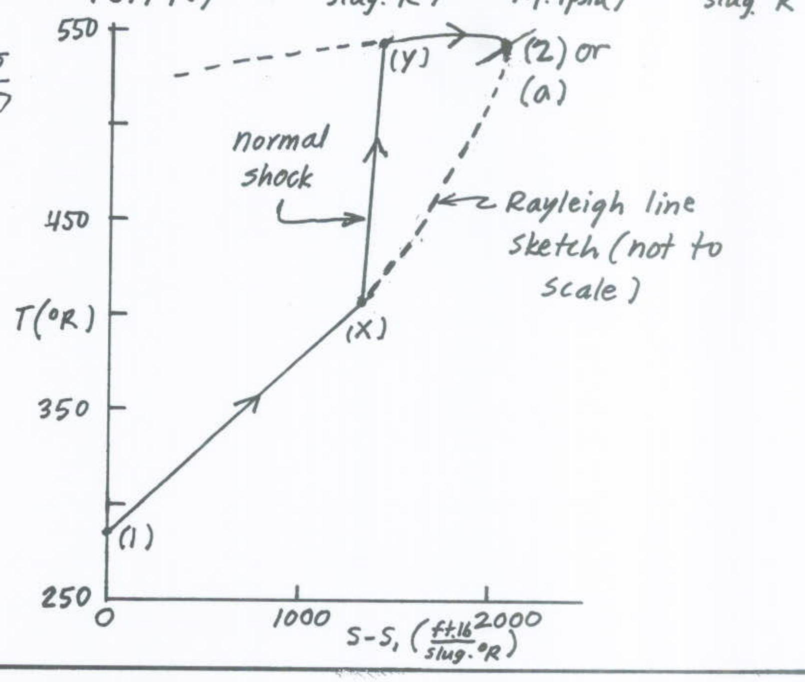
Similarly

$$s_y - s_1 = 6006 \ln \frac{533}{519} - 1716 \ln \frac{7.5}{14.7}$$

$$s_y - s_1 = 1320\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}$$

$$s_2 - s_1 = 6006 \ln \frac{549}{519} - 1716 \ln \frac{5.31}{14.7}$$

$$s_2 - s_1 = 2080\ \frac{ft \cdot lb}{slug \cdot {}^\circ R}$$



11.76

11.76 Air enters a frictionless, constant area duct with Ma = 2.5, $T_0 = 20\ °C$, and $p_0 = 101$ kPa(abs). The gas is decelerated by heating until a normal shock occurs where the local Mach number is 1.3. Downstream of the shock, the subsonic flow is accelerated with heating until it exits with a Mach number of 0.9. Determine the static temperature and pressure, the stagnation temperature and pressure, and the fluid velocity at the duct entrance, just upstream and downstream of the normal shock, and at the duct exit. Sketch the temperature–entropy diagram for this flow.

(a) For air we have at the duct entrance, section (1)

$$Ma_1 = 2.5$$

$$T_{0,1} = 20°C = \underline{\underline{293\ K}}$$

$$p_{0,1} = \underline{\underline{101\ kPa\,(abs)}}$$

With $Ma_1 = 2.5$, we enter Fig. D.1 and read

$$\frac{T_1}{T_{0,1}} = 0.44 \qquad (1)$$

and

$$\frac{p_1}{p_{0,1}} = 0.06 \qquad (2)$$

Thus we have with Eqs. 1 and 2

$$T_1 = (0.44)(293\ K) = \underline{\underline{130\ K}}$$

and

$$p_1 = (0.06)[101\ kPa\,(abs)] = \underline{\underline{6.0\ kPa\,(abs)}}$$

Then,

$$V_1 = Ma_1\sqrt{RT_1 k} = (2.5)\sqrt{\frac{\left(286.9\ \frac{N \cdot m}{kg \cdot K}\right)(130\ K)(1.4)}{\left(1\ \frac{N}{kg \cdot \frac{m}{s^2}}\right)}} = \underline{\underline{571\ \frac{m}{s}}}$$

At section (x) just upstream of the shock,

$$T_{0,x} = T_{0,1}\left(\frac{T_{0,a}}{T_{0,1}}\right)\left(\frac{T_{0,x}}{T_{0,a}}\right) \qquad (3)$$

and

$$p_{0,x} = p_{0,1}\left(\frac{p_{0,a}}{p_{0,1}}\right)\left(\frac{p_{0,x}}{p_{0,a}}\right) \qquad (4)$$

(con't)

11.76 (con't)

For $Ma_1 = 2.5$ and $Ma_x = 1.3$ we read from Fig. D.3

$$\frac{T_{0,1}}{T_{0,a}} = 0.71$$

$$\frac{p_{0,1}}{p_{0,a}} = 2.2$$

$$\frac{T_{0,x}}{T_{0,a}} = 0.95$$

$$\frac{p_{0,x}}{p_{0,a}} = 1.04$$

With these values and Eqs. 3 and 4 we obtain

$$T_{0,x} = (293\,K)\left(\frac{1}{0.71}\right)(0.95) = \underline{\underline{396}}\,K$$

$$p_{0,x} = [101\,kPa(abs)]\left(\frac{1}{2.2}\right)(1.04) = \underline{\underline{47.7}}\,kPa(abs)$$

With $Ma_x = 1.3$ we enter Fig. D.1 and read

$$\frac{T_x}{T_{0,x}} = 0.75$$

and

$$\frac{p_x}{p_{0,x}} = 0.36$$

Thus,

$$T_x = (0.75)(395\,K) = \underline{\underline{296}}\,K$$

and

$$p_x = (0.36)[47.4\,kPa(abs)] = \underline{\underline{17}}\,kPa(abs)$$

Then

$$V_x = Ma_x\sqrt{RT_x k} = (1.3)\sqrt{\frac{\left(286.9\,\frac{N\cdot m}{kg\cdot K}\right)(296\,K)(1.4)}{\left(1\,\frac{N}{kg\cdot\frac{m}{s^2}}\right)}} = \underline{\underline{448}}\,\frac{m}{s}$$

(con't)

11.76 (con't)

At section (y) just downstream of the shock we obtain from Fig. D.4 for $Ma_x = 1.3$

$$Ma_y = 0.79$$

$$\frac{p_y}{p_x} = 1.8$$

$$\frac{T_y}{T_x} = 1.2$$

$$\frac{V_x}{V_y} = 1.5$$

$$\frac{p_{0,y}}{p_{0,x}} = 0.98$$

With these ratios and values of properties at section (x) previously determined we have

$$p_y = (1.8)\,[17.1\ kPa(abs)] = \underline{\underline{30.8}}\ kPa(abs)$$

$$T_y = (1.2)(295\ K) = \underline{\underline{354}}\ K$$

$$V_y = \frac{(448\ \frac{m}{s})}{(1.5)} = \underline{\underline{299}}\ \frac{m}{s}$$

$$p_{0,y} = (0.98)\,[47.4\ kPa(abs)] = \underline{\underline{46.4}}\ kPa(abs)$$

Also, since the flow across the normal shock is adiabatic,

$$T_{0,y} = T_{0,x} = \underline{\underline{396}}\ K$$

At the duct exit, section (2), we have

$$p_2 = p_y\left(\frac{p_a}{p_y}\right)\left(\frac{p_2}{p_a}\right) \quad (5)$$

$$T_2 = T_y\left(\frac{T_a}{T_y}\right)\left(\frac{T_2}{T_a}\right) \quad (6)$$

$$T_{0,2} = T_{0,y}\left(\frac{T_{0,a}}{T_{0,y}}\right)\left(\frac{T_{0,2}}{T_{0,a}}\right) \quad (7)$$

$$p_{0,2} = p_{0,y}\left(\frac{p_{0,a}}{p_{0,y}}\right)\left(\frac{p_{0,2}}{p_{0,a}}\right) \quad (8)$$

$$V_2 = V_y\left(\frac{V_a}{V_y}\right)\left(\frac{V_2}{V_y}\right) \quad (9)$$

(con't)

11.76 (con't)

Appropriate ratios to use in Eqs. 5 through 9 are obtained from Fig. D.3 for $Ma_y = 0.79$ and $Ma_2 = 0.9$.

Thus,

$$\frac{p_y}{p_a} = 1.3$$

$$\frac{p_2}{p_a} = 1.12$$

$$\frac{T_y}{T_a} = 1.02$$

$$\frac{T_2}{T_a} = 1.02$$

$$\frac{T_{0,y}}{T_{0,a}} = 0.96$$

$$\frac{T_{0,2}}{T_{0,a}} = 0.99$$

$$\frac{p_{0,y}}{p_{0,a}} = 1.02$$

$$\frac{p_{0,2}}{p_{0,a}} = 1.01$$

$$\frac{V_y}{V_a} = 0.8$$

$$\frac{V_2}{V_a} = 0.91$$

With these ratios and Eqs. 5 through 9 we obtain

$$p_2 = [30.9\ kPa\ (abs)]\left(\frac{1}{1.3}\right)(1.12) = \underline{\underline{26.6\ kPa\ (abs)}}$$

$$T_2 = (351\ K)\left(\frac{1}{1.02}\right)(1.02) = \underline{\underline{351\ K}}$$

$$T_{0,2} = (395\ K)\left(\frac{1}{0.96}\right)(0.99) = \underline{\underline{407\ K}}$$

(con't)

11.76 (con't)

$$p_{0,2} = [46.4\ kPa(abs)]\left(\frac{1}{1.02}\right)(1.01) = \underline{\underline{45.9\ kPa(abs)}}$$

$$V_2 = \left(296\ \frac{m}{s}\right)\left(\frac{1}{0.8}\right)(0.91) = \underline{\underline{337\ \frac{m}{s}}}$$

For sketching a T-s diagram we need values of $s - s_1$. We use,

$$s - s_1 = c_p \ln \frac{T}{T_1} - R \ln \frac{p}{p_1}$$

Thus, for example,

$$s_x - s_1 = \left(1004\ \frac{J}{kg \cdot K}\right) \ln\left(\frac{296\ K}{130\ K}\right) - \left(286.9\ \frac{J}{kg \cdot K}\right) \ln\left[\frac{17\ kPa(abs)}{6.0\ kPa(abs)}\right]$$

or

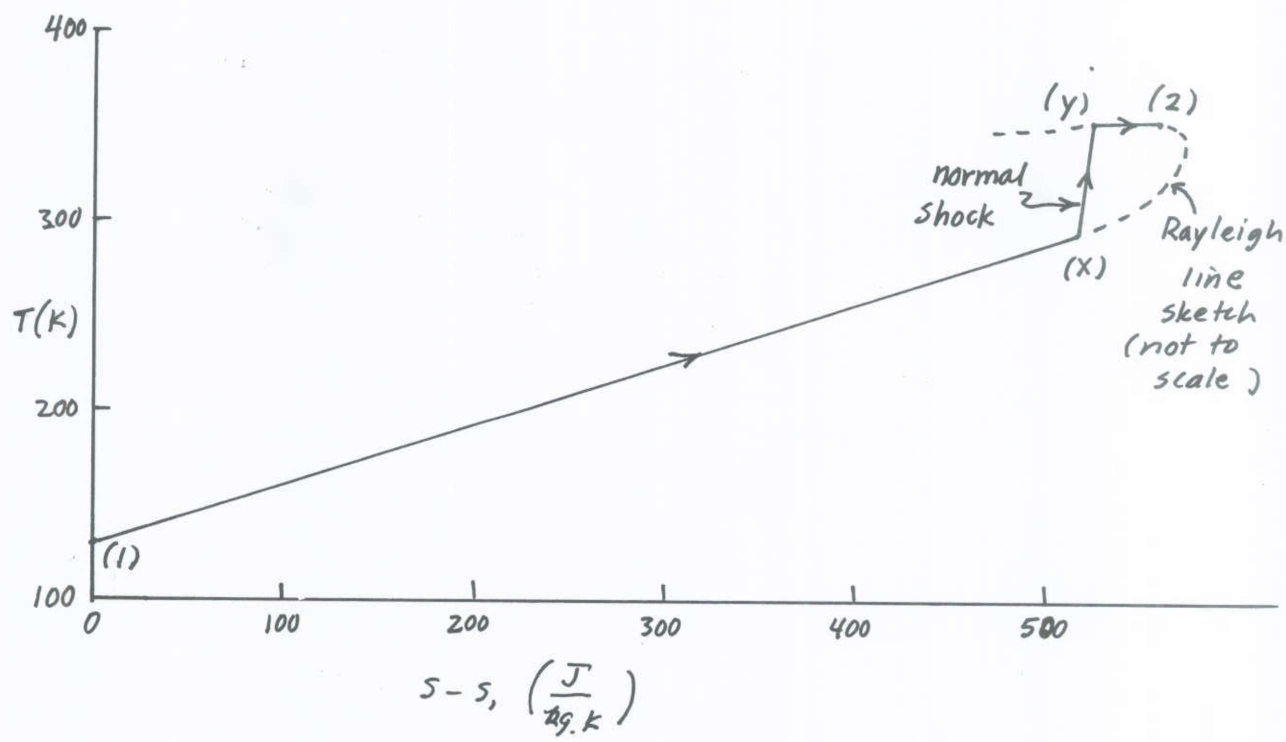
$$s_x - s_1 = 527\ \frac{J}{kg \cdot K}$$

Similarly

$$s_y - s_1 = 536\ \frac{J}{kg \cdot K}$$

and

$$s_2 - s_1 = 570\ \frac{J}{kg \cdot K}$$



11.80

11.80 [See Fluids in the News article titled "Hilsch tube (Ranque vortex tube)," Section 11.1.] Explain why a Hilsch tube works and cite some high and low gas temperatures actually achieved. What is the most important limitation of a Hilsch tube and how can it be overcome?

A Hilsch tube works because the core flow of the associated compressible swirling flow is in solid body rotation (forced vortex). As shown by Eckert and Drake (Eckert, E.R.G. and Drake, Jr., R.M., Analysis of Heat and Mass Transfer, McGraw-Hill, New York, 1972), the difference is total temperature across the radius of this forced vortex can be appreciable, especially when the flow is turbulent. Kurosaka (Kurosaka, M., Acoustic Streaming in Swirling Flow and the Ranque-Hilsch (Vortex Tube) Effect, Journal of Fluid Mechanics 124: 139-172, 1982) concluded that periodic unsteadiness of the swirling flow is the primary cause of the formation of this forced vortex. According to measurements (Ahlborn, B., Keller, J.U., Staudt, R., Trietz, G. and Rebhan, E., Limits of Temperature Separation in a Vortex Tube, J. Phys. D: Appl. Phys. 27: 480-488, 1994) typical hot and cold stream temperatures are 57°C and -13°C.

The most important limitation of the Hilsch tube is the inefficiency of the process, a challenge that remains to be resolved.

11.81

11.81 [See Fluids in the News article titled "Supersonic and compressible flows in gas turbines," Section 11.3.] Using typical physical dimensions and rotation speeds of manufactured gas turbine rotors, consider the possibility that supersonic Fluid velocities relative to blade surfaces are possible. How do designers use this knowledge?

For the fan of a regional turbofan gas turbine engine

tip radius = 19 in.

rotation speed = 8300 rpm

so blade tip speed, U_t, is

$$U_t = r_t \omega = \frac{(19\ \text{in.})\left(8300\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right)}{\left(12\ \frac{\text{in.}}{\text{ft}}\right)\left(60\ \frac{\text{s}}{\text{min}}\right)} = 1376\ \frac{\text{ft}}{\text{s}}$$

For a typical fan inflow velocity triangle (see Fig. 12.3) the velocity relative to the fan blade, W, is much larger than the blade velocity, U. At takeoff and nominal ambient temperatures we see from Table B.3 that the relative velocity of the air flow over the fan blade near its leading edge is most likely supersonic.

For the core compressor of this same engine

tip radius = 10 in.

rotation speed = 16250 rpm

the resultant blade tip speed is $1417\ \frac{\text{ft}}{\text{s}}$

Even at higher temperatures within the core compressor the relative velocity, W, is quite likely to be supersonic.

Designers continue to improve the fan, compressor and turbine components of gas turbines.