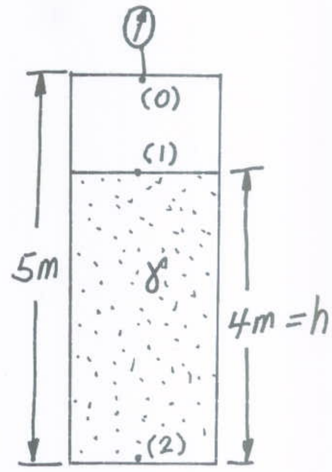
2.2

2.2 A closed, 5-m-tall tank is filled with water to a depth of 4 m. The top portion of the tank is filled with air which, as indicated by a pressure gage at the top of the tank, is at a pressure of 20 kPa. Determine the pressure that the water exerts on the bottom of the tank.



$$p_0 = 20 \times 10^3 \frac{N}{m^2} = p_1$$

$$p_2 = p_1 + \gamma h = 20 \times 10^3 \frac{N}{m^2} + 9.80 \times 10^3 \frac{N}{m^3}(4 m)$$

$$= 59.2 \times 10^3 \frac{N}{m^2} = \underline{\underline{59.2 \text{ kPa}}}$$

2.3

2.3 A closed tank is partially filled with glycerin. If the air pressure in the tank is 6 lb/in.^{2} and the depth of glycerin is 10 ft, what is the pressure in lb/ft^2 at the bottom of the tank?

$$p = \gamma h + p_o = \left(78.6 \frac{lb}{ft^3}\right)(10\ ft) + \left(6 \frac{lb}{in.^2}\right)\left(\frac{144\ in.^2}{ft^2}\right)$$

$$= \underline{\underline{1650 \frac{lb}{ft^2}}}$$

2.4

2.4 Blood pressure is usually given as a ratio of the maximum pressure (systolic pressure) to the minimum pressure (diastolic pressure). As shown in **Video V2.2** such pressures are commonly measured with a mercury manometer. A typical value for this ratio for a human would be 120/70, where the pressures are in mm Hg. **(a)** What would these pressures be in pascals? **(b)** If your car tire was inflated to 120 mm Hg, would it be sufficient for normal driving?

$$p = \gamma h$$

(a) For 120 mm Hg: $p = \left(133 \times 10^3 \frac{N}{m^3}\right)(0.120\ m) = \underline{\underline{16.0\ kPa}}$

For 70 mm Hg: $p = \left(133 \times 10^3 \frac{N}{m^3}\right)(0.070\ m) = \underline{\underline{9.31\ kPa}}$

(b) For 120 mm Hg: $p = \left(16.0 \times 10^3 \frac{N}{m^2}\right)\left(1.450 \times 10^{-4} \frac{lb/in.^2}{N/m^2}\right)$

$$= 2.32\ psi$$

Since a typical tire pressure is 30-35 psi, 120 mm Hg is <u>not sufficient</u> for normal driving.

2.5

2.5 An unknown immiscible liquid seeps into the bottom of an open oil tank. Some measurements indicate that the depth of the unknown liquid is 1.5 m and the depth of the oil (specific weight = 8.5 kN/m^3) floating on top is 5.0 m. A pressure gage connected to the bottom of the tank reads 65 kPa. What is the specific gravity of the unknown liquid?

$$p_{bottom} = (\gamma_{oil})(5\,m) + (\gamma_u)(1.5\,m)$$ where $\gamma_u \sim$ unknown liquid γ

$$\gamma_u = \frac{p_{bottom} - (\gamma_{oil})(5\,m)}{1.5\,m} = \frac{65\times10^3\,\frac{N}{m^2} - \left(8.5\times10^3\,\frac{N}{m^3}\right)(5\,m)}{1.5\,m}$$

$$= 15\times10^3\,\frac{N}{m^3}$$

$$SG = \frac{\gamma_u}{\gamma_{H_2O}\,@\,4^\circ C} = \frac{15\times10^3\,\frac{N}{m^3}}{9.81\times10^3\,\frac{N}{m^3}} = \underline{\underline{1.53}}$$

2.6

2.6 Bathyscaphes are capable of submerging to great depths in the ocean. What is the pressure at a depth of 5 km, assuming that seawater has a constant specific weight of 10.1 kN/m^3? Express your answer in pascals and psi.

$$p = \gamma h + p_0$$

At the surface $p_0 = 0$ so that

$$p = (10.1\times10^3\,\frac{N}{m^3})(5\times10^3\,m) = 50.5\times10^6\,\frac{N}{m^2} = \underline{\underline{50.5\ MPa}}$$

Also,

$$p = \left(50.5\times10^6\,\frac{N}{m^2}\right)\left(1.450\times10^{-4}\,\frac{\frac{lb}{in.^2}}{\frac{N}{m^2}}\right) = \underline{\underline{7320\ psi}}$$

2.7

2.7 For the great depths that may be encountered in the ocean the compressibility of seawater may become an important consideration. **(a)** Assume that the bulk modulus for seawater is constant and derive a relationship between pressure and depth which takes into account the change in fluid density with depth. **(b)** Make use of part (a) to determine the pressure at a depth of 6 km assuming seawater has a bulk modulus of 2.3×10^9 Pa, and a density of 1030 kg/m^3 at the surface. Compare this result with that obtained by assuming a constant density of 1030 kg/m^3.

(a)

$$\frac{dp}{dz} = -\gamma = -\rho g \qquad \text{(Eq. 2.4)}$$

Thus,

$$\frac{dp}{\rho} = -g\,dz \qquad (1)$$

If ρ is a function of p, we must determine $\rho = f(p)$ before integrating Eq.(1). Since,

$$E_v = \frac{dp}{d\rho/\rho} \qquad \text{(Eq. 1.13)}$$

then

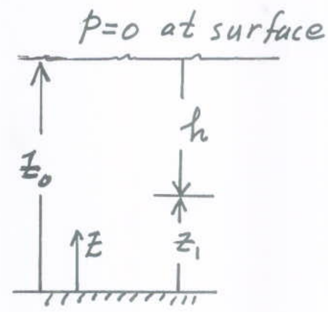
$$\int_0^p dp = E_v \int_{\rho_0}^{\rho} \frac{d\rho}{\rho}$$

so that

$$p = E_v \ln \frac{\rho}{\rho_0}$$

Thus,

$$\rho = \rho_0 e^{\frac{p}{E_v}} \qquad \text{where } \rho = \rho_0 \text{ at } p = 0$$



From Eq.(1)

$$\int_{p_1}^{0} \frac{dp}{\rho_0 e^{\frac{p}{E_v}}} = -g \int_{z_1}^{z_0} dz$$

or

$$\int_{p_1}^{0} e^{-\frac{p}{E_v}}\,dp = -\rho_0 g \int_{z_1}^{z_0} dz$$

so that

$$\underline{\underline{p = -E_v \ln\left(1 - \frac{\rho_0 g h}{E_v}\right)}}$$

where, $h = z_0 - z_1$, the depth below surface

(con't)

2.7 (con't)

(b) From part (a),

$$p = -E_v \ln\left(1 - \frac{\rho_o g h}{E_v}\right)$$

so that at $h = 6\,km$

$$p = -\left(2.3 \times 10^9 \,\frac{N}{m^2}\right) \ln\left[1 - \frac{\left(1.03 \times 10^3 \,\frac{kg}{m^3}\right)\left(9.81 \,\frac{m}{s^2}\right)\left(6 \times 10^3\,m\right)}{2.3 \times 10^9 \,\frac{N}{m^2}}\right]$$

$$= 6.14 \times 10^7 \,\frac{N}{m^2} = \underline{\underline{61.4\ MPa}}$$

(c) For constant density

$$p = \gamma h = \rho g h = \left(1.03 \times 10^3 \,\frac{kg}{m^3}\right)\left(9.81 \,\frac{m}{s^2}\right)\left(6 \times 10^3\,m\right)$$

$$= \underline{\underline{60.6\ MPa}}$$

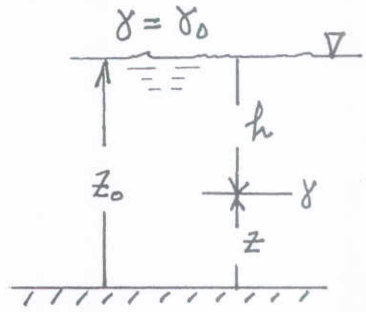
2.8

2.8 Sometimes when riding an elevator or driving up or down a hilly road a person's ears "pop" as the pressure difference between the inside and outside of the ear is equalized. Determine the pressure difference (in psi) associated with this phenomenon if it occurs during a 150 ft elevation change.

$$\Delta p = \gamma \Delta h = 0.0765 \frac{lb}{ft^3}(150 ft)$$
$$= 11.5 \frac{lb}{ft^2}\left(\frac{1 ft^2}{144 in.^2}\right)$$
$$= \underline{\underline{0.0797 psi}}$$

2.9

2.9 Develop an expression for the pressure variation in a liquid in which the specific weight increases with depth, h, as $\gamma = Kh + \gamma_0$, where K is a constant and γ_0 is the specific weight at the free surface.



$$\frac{dp}{dz} = -\gamma \qquad (\text{Eq. } 2.4)$$

Let $h = z_0 - z$

so that $dh = -dz$

Thus,

$$dp = \gamma\, dh$$

and

$$\int_0^p dp = \int_0^h \gamma\, dh$$

For $\gamma = Kh + \gamma_0$,

$$\int_0^p dp = \int_0^h (Kh + \gamma_0)\, dh$$

and

$$\underline{\underline{p = \frac{Kh^2}{2} + \gamma_0 h}}$$

*2.10

*2.10 In a certain liquid at rest, measurements of the specific weight at various depths show the following variation:

h (ft)	γ (lb/ft^3)
0	70
10	76
20	84
30	91
40	97
50	102
60	107
70	110
80	112
90	114
100	115

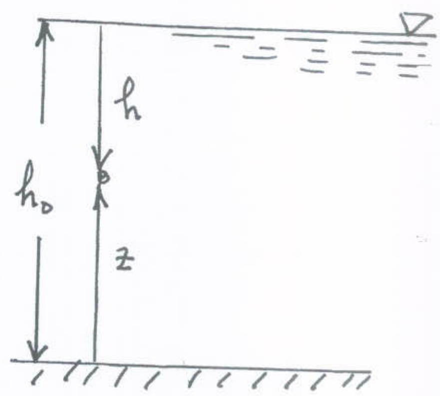
(con't)

The depth $h = 0$ corresponds to a free surface at atmospheric pressure. Determine, through numerical integration of Eq. 2.4, the corresponding variation in pressure and show the results on a plot of pressure (in psf) versus depth (in feet).

$$\frac{dp}{dz} = -\gamma \qquad \text{(Eq. 2.4)}$$

Let $z = h_o - h$ (see figure) so that $dz = -dh$ and therefore

$$dp = -\gamma\, dz = \gamma\, dh$$



Thus,

$$\int_0^{p_i} dp = \int_0^{h_i} \gamma\, dh$$

or

$$p_i = \int_0^{h_i} \gamma\, dh \qquad (1)$$

where p_i is the pressure at depth h_i.

Equation (1) can be integrated numerically using the trapezoidal rule, i.e.,

$$I = \frac{1}{2}\sum_{i=1}^{n-1}(y_i + y_{i+1})(x_{i+1} - x_i)$$

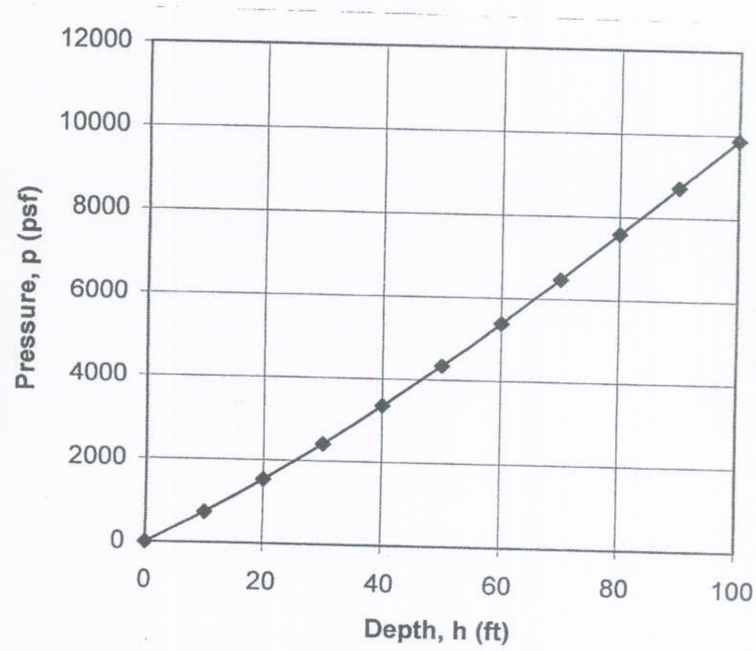
Where $y \sim \gamma$, $x \sim h$, and n = number of data points.

(con't)

*2.10 (Con't)

The tabulated results are given below, along with the corresponding plot of pressure vs. depth.

h(ft)	γ, lb/ft^3	Pressure,psf
0	70	0
10	76	730
20	84	1530
30	91	2405
40	97	3345
50	102	4340
60	107	5385
70	110	6470
80	112	7580
90	114	8710
100	115	9855



*2.12

Elevation (ft)	Temperature (°F)
5000	50.1 (base)
5500	55.2
6000	60.3
6400	62.6
7100	67.0
7400	68.4
8200	70.0
8600	69.5
9200	68.0
9900	67.1 (top)

TABLE P2.12

***2.12** Under normal conditions the temperature of the atmosphere decreases with increasing elevation. In some situations, however, a temperature inversion may exist so that the air temperature increases with elevation. A series of temperature probes on a mountain give the elevation –temperature data shown in Table P2.12. If the barometric pressure at the base of the mountain is 12.1 psia, determine by means of numerical integration the pressure at the top of the mountain.

From Eq. 2.9,

$$\ln \frac{p_2}{p_1} = -\frac{g}{R}\int_{z_1}^{z_2} \frac{dz}{T}$$

In the table below the temperature in °R is given and the integrand $1/T(°R)$ tabulated.

Elevation, ft	T, °F	T, °R	1/ T(°R)
5000	50.1	509.8	0.001962
5500	55.2	514.9	0.001942
6000	60.3	520.0	0.001923
6400	62.6	522.3	0.001915
7100	67.0	526.7	0.001899
7400	68.4	528.1	0.001894
8200	70.0	529.7	0.001888
8600	69.5	529.2	0.00189
9200	68.0	527.7	0.001895
9900	67.1	526.8	0.001898

The approximate value of the integral in Eq. 2.9 is 9.34 obtained using the trapezoidal rule, i.e.,

$$I = \frac{1}{2}\sum_{i=1}^{n-1}(y_i + y_{i+1})(x_{i+1} - x_i) \quad \text{where } y \sim 1/T,\ x \sim \text{elevation},$$

and $n =$ number of data points. Thus,

$$\int_{5000\,ft}^{9900\,ft} \left(\frac{1}{T}\right) dz = 9.34 \ \frac{ft}{°R}$$

so that (with $g = 32.2\ ft/s^2$ and $R = 1716\ ft \cdot lb/slug \cdot °R$

$$\ln \frac{p_2}{p_1} = -\frac{(32.2\ \frac{ft}{s^2})(9.34\ \frac{ft}{°R})}{1716\ ft \cdot lb/slug \cdot °R} = -0.1753 \qquad (1)$$

(con't)

*2.12 (con't)

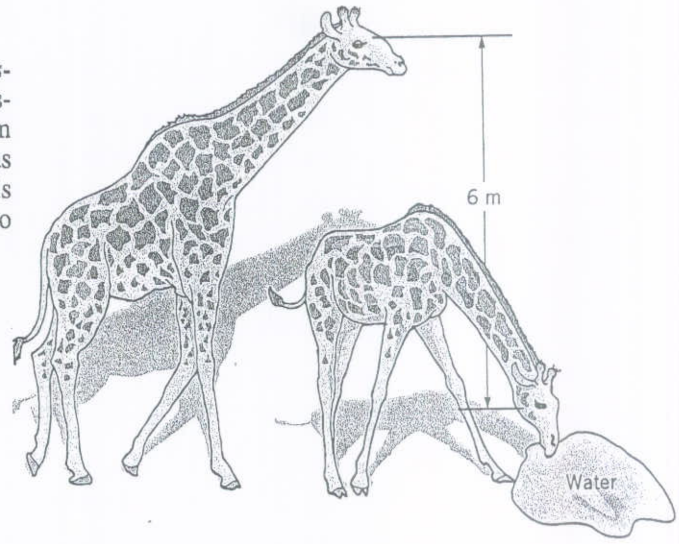
It follows from Eq.(1) with $p_1 = 12.1$ psia that

$$p_2 = (12.1 \text{ psia})\, e^{-0.1753} = \underline{\underline{10.2 \text{ psia}}}$$

(Note: Since the temperature variation is not very large, it would be expected that the assumption of a constant temperature would give good results. If the temperature is assumed to be constant at the base temperature (50.1°F), $p_2 = 10.1$ psia, which is only slightly different from the result given above.)

2.14

2.14 (See Fluids in the News article titled **"Giraffe's blood pressure,"** Section 2.3.1.) **(a)** Determine the change in hydrostatic pressure in a giraffe's head as it lowers its head from eating leaves 6 m above the ground to getting a drink of water at ground level as shown in Fig. P2.14. Assume the specific gravity of blood is $SG = 1$. **(b)** Compare the pressure change calculated in part **(a)** to the normal 120 mm of mercury pressure in a human's heart.



(a) For hydrostatic pressure change,

$$\Delta p = \gamma h = \left(9.80 \frac{kN}{m^2}\right)(6\,m) = 58.8 \frac{kN}{m^2} = \underline{\underline{58.8\,kPa}}$$

(b) To compare with pressure in human heart convert pressure in part (a) to mm Hg:

$$58.8 \frac{kN}{m^2} = \gamma_{Hg}\, h_{Hg} = \left(133 \frac{kN}{m^3}\right) h_{Hg}$$

$$h_{Hg} = (0.442\,m)\left(10^3 \frac{mm}{m}\right) = 442\ mm\ Hg$$

Thus, the pressure change in the giraffe's head is $\underline{\underline{442\ mm\ Hg}}$ compared with $\underline{\underline{120\ mm\ Hg}}$ in the human heart.

2.15

2.15 Assume that a person skiing high in the mountains at an altitude of 15,000 ft takes in the same volume of air with each breath as she does while walking at sea level. Determine the ratio of the mass of oxygen inhaled for each breath at this high altitude compared to that at sea level.

Let $(\ )_0$ denote sea level and $(\ )_{15}$ denote 15,000 ft altitude.
Thus, since m = mass $= \rho \mathcal{V}$, where $\mathcal{V}$ = volume,

$m_0 = \rho_0 \mathcal{V}_0$ and $m_{15} = \rho_{15} \mathcal{V}_{15}$, where $\mathcal{V}_0 = \mathcal{V}_{15}$.

Hence,

$$\frac{m_{15}}{m_0} = \frac{\rho_{15} \mathcal{V}_{15}}{\rho_0 \mathcal{V}_0} = \frac{\rho_{15}}{\rho_0}$$

If it is assumed that the air composition (e.g., % of air that is oxygen) is the same at sea level as it is at 15,000 ft, then we can use the ρ values from Table C.1:

$$\rho_0 = 2.377 \times 10^{-3} \, \frac{\text{slugs}}{\text{ft}^3} \text{ and } \rho_{15} = 1.496 \times 10^{-3} \, \frac{\text{slugs}}{\text{ft}^3} \text{ so that}$$

$$\frac{m_{15}}{m_0} = \frac{1.496 \times 10^{-3} \, \frac{\text{slugs}}{\text{ft}^3}}{2.377 \times 10^{-3} \, \frac{\text{slugs}}{\text{ft}^3}} = 0.629 = \underline{\underline{62.9\,\%}}$$

2.16

2.16 Pikes Peak near Denver, Colorado has an elevation of 14,110 ft. **(a)** Determine the pressure at this elevation, based on Eq. 2.12. **(b)** If the air is assumed to have a constant specific weight of 0.07647 lb/ft³, what would the pressure be at this altitude? **(c)** If the air is assumed to have a constant temperature of 59 °F what would the pressure be at this elevation? For all three cases assume standard atmospheric conditions at sea level (see Table 2.1).

(a)

$$p = p_a \left(1 - \frac{\beta z}{T_a}\right)^{\frac{g}{R\beta}} \qquad \text{(Eq. 2.12)}$$

For $p_a = 2116.2\ \frac{lb}{ft^2}$, $\beta = 0.00357\ \frac{°R}{ft}$, $g = 32.174\ \frac{ft}{s^2}$, $T_a = 518.67°R$, $R = 1716\ \frac{ft \cdot lb}{slug \cdot °R}$, and

$$\frac{g}{R\beta} = \frac{32.174\ \frac{ft}{s^2}}{\left(1716\ \frac{ft \cdot lb}{slug \cdot °R}\right)\left(0.00357\ \frac{°R}{ft}\right)} = 5.252$$

then

$$p = \left(2116.2\ \frac{lb}{ft^2}\right)\left[1 - \frac{\left(0.00357\ \frac{°R}{ft}\right)(14{,}110\ ft)}{518.67\ °R}\right]^{5.252}$$

$$= \underline{\underline{1240\ \frac{lb}{ft^2}\ (abs)}}$$

(b)

$$p = p_a - \gamma h$$

$$= 2116.2\ \frac{lb}{ft^2} - \left(0.07647\ \frac{lb}{ft^3}\right)(14{,}110\ ft)$$

$$= \underline{\underline{1040\ \frac{lb}{ft^2}\ (abs)}}$$

(c)

$$p = p_a\, e^{-\frac{gh}{RT_a}} \qquad \text{(Eq. 2.10)}$$

$$= \left(2116.2\ \frac{lb}{ft^2}\right) e^{-\left[\frac{\left(32.174\ \frac{ft}{s^2}\right)(14{,}110\ ft)}{\left(1716\ \frac{ft \cdot lb}{slug \cdot °R}\right)(518.67°R)}\right]}$$

$$= \underline{\underline{1270\ \frac{lb}{ft^2}\ (abs)}}$$

2.17

2.17 Equation 2.12 provides the relationship between pressure and elevation in the atmosphere for those regions in which the temperature varies linearly with elevation. Derive this equation and verify the value of the pressure given in Table C.2 in Appendix C for an elevation of 5 km.

$$\int_{p_1}^{p_2} \frac{dp}{p} = -\frac{g}{R}\int_{z_1}^{z_2} \frac{dz}{T} \qquad (\text{Eq. } 2.9)$$

Let $p_1 \sim p_a$ for $z_1 = 0$, $p_2 \sim p$ for $z_2 = z$, and $T = T_a - \beta z$.

Thus,

$$\int_{p_a}^{p} \frac{dp}{p} = -\frac{g}{R}\int_{0}^{z} \frac{dz}{T_a - \beta z}$$

or

$$\ln\frac{p}{p_a} = -\frac{g}{R}\left[-\frac{1}{\beta}\ln(T_a - \beta z)\right]_0^z = \frac{g}{R\beta}\left[\ln(T_a - \beta z) - \ln T_a\right]$$

$$= \frac{g}{R\beta}\ln\left(1 - \frac{\beta z}{T_a}\right)$$

and taking logarithm of both sides of equation yields

$$\underline{\underline{p = p_a\left(1 - \frac{\beta z}{T_a}\right)^{\frac{g}{R\beta}}}} \qquad (\text{Eq. } 2.12)$$

For $z = 5\,km$ with $p_a = 101.33\,kPa$, $T_a = 288.15\,K$, $g = 9.807\,\frac{m}{s^2}$, $\beta = 0.00650\,\frac{K}{m}$, $R = 287\,\frac{J}{kg\cdot K}$,

$$p = (101.33\,kPa)\left[1 - \frac{(0.0065\,\frac{K}{m})(5\times10^3\,m)}{288.15\,K}\right]^{\frac{9.807\,\frac{m}{s^2}}{(287\,\frac{J}{kg\cdot K})(0.0065\,\frac{K}{m})}}$$

$$= \underline{\underline{5.40\times10^4\,\frac{N}{m^2}}}$$

(From Table C.2 in Appendix C, $p = 5.405\times10^4\,\frac{N}{m^2}$.)

2.18

2.18 As shown in Fig. 2.6 for the U.S. standard atmosphere, the troposphere extends to an altitude of 11 km where the pressure is 22.6 kPa (abs). In the next layer, called the stratosphere, the temperature remains constant at −56.5 °C. Determine the pressure and density in this layer at an altitude of 15 km. Assume $g = 9.77\ \text{m/s}^2$ in your calculations. Compare your results with those given in Table C.2 in Appendix C.

For isothermal conditions,

$$p_2 = p_1 e^{-\frac{g(z_2 - z_1)}{R T_0}} \qquad \text{(Eq. 2.10)}$$

Let $z_1 = 11\ \text{km}$, $p_1 = 22.6\ \text{kPa}$, $R = 287\ \frac{\text{J}}{\text{kg}\cdot\text{K}}$, $g = 9.77\ \frac{\text{m}}{\text{s}^2}$,

and $T_0 = -56.5\,°\text{C} + 273.15 = 216.65\ \text{K}$.

Thus,

$$p_2 = (22.6\ \text{kPa})\, e^{-\left[\frac{\left(9.77\ \frac{\text{m}}{\text{s}^2}\right)\left(15\times10^3\ \text{m} - 11\times10^3\ \text{m}\right)}{\left(287\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(216.65\ \text{K})}\right]}$$

$$= \underline{\underline{12.1\ \text{kPa}}}$$

Also,

$$\rho_2 = \frac{p}{RT} = \frac{12.1\times10^3\ \frac{\text{N}}{\text{m}^2}}{\left(287\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(216.65\ \text{K})} = \underline{\underline{0.195\ \frac{\text{kg}}{\text{m}^3}}}$$

(From Table C.2 in Appendix C, $p_2 = 12.11\ \text{kPa}$ and $\rho_2 = 0.1948\ \frac{\text{kg}}{\text{m}^3}$.)

2.19

2.19 (See Fluids in the News article titled "Weather, barometers, and bars," Section 2.5.) The record low sea-level barometric pressure ever recorded is 25.8 in. of mercury. At what altitude in the standard atmosphere is the pressure equal to this value?

For record low pressure,

$$p = \gamma_{Hg} h_{Hg} = \left(847 \frac{lb}{ft^3}\right)\left(\frac{25.8 \text{ in.}}{12 \frac{in.}{ft}}\right)\left(\frac{ft^2}{144 \text{ in.}^2}\right) = 12.6 \frac{lb}{in.^2}$$

From Table C.1 in Appendix C

@ 0 ft altitude $p = 14.696 \frac{lb}{in.^2}$

@ 5000 ft altitude $p = 12.228 \frac{lb}{in.^2}$

Assume linear variation change in pressure per foot. Thus,

$$\text{pressure change per foot} = \frac{14.696 \frac{lb}{in.^2} - 12.228 \frac{lb}{in.^2}}{5000 \text{ ft}}$$

$$= 4.936 \times 10^{-4} \frac{lb}{in.^2} \text{ per ft}$$

and

$$14.696 \frac{lb}{in.^2} - d\,(ft)\left[4.936 \times 10^{-4} \frac{\frac{lb}{in.^2}}{ft}\right] = 12.6 \frac{lb}{in.^2}$$

So that $d = \underline{\underline{4,250 \text{ ft}}}$

2.20

2.20 On a given day, a barometer at the base of the Washington Monument reads 29.97 in. of mercury. What would the barometer reading be when you carry it up to the observation deck 500 ft above the base of the monument?

Let $(\ )_b$ and $(\ )_{od}$ correspond to the base and observation deck, respectively.

Thus, with H = height of the monument,

$$p_b - p_{od} = \gamma_{air} H = 7.65 \times 10^{-2} \frac{lb}{ft^3} (500\,ft) = 38.5 \frac{lb}{ft^2}$$

But

$p = \gamma_{Hg} h$, where $\gamma_{Hg} = 847 \frac{lb}{ft^3}$ and h = barometer reading.

Thus,

$$\gamma_{Hg}\left(\frac{29.97}{12} ft\right) - \gamma_{Hg} h_{od} = 38.5 \frac{lb}{ft^2}$$

or

$$h_{od} = \left(\frac{29.97}{12} ft\right) - \frac{38.5 \frac{lb}{ft^2}}{847 \frac{lb}{ft^3}} = \left[\left(\frac{29.97}{12} ft\right) - 0.0455\,ft\right]\left(12 \frac{in.}{ft}\right)$$

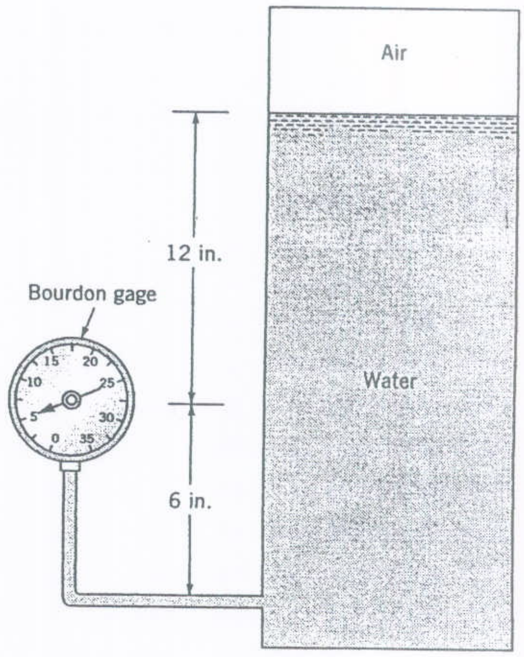
$$= (29.97 - 0.545)\,in.$$

or

$$\underline{\underline{h_{od} = 29.43\,in.}}$$

2.21

2.21 Bourdon gages (see **Video V2.3** and Fig. 2.13) are commonly used to measure pressure. When such a gage is attached to the closed water tank of Fig. P2.21 the gage reads 5 psi. What is the absolute air pressure in the tank? Assume standard atmospheric pressure of 14.7 psi.



■ FIGURE P2.21

$$p = \gamma h + p_o$$

$$p_{gage} - \left(\frac{12}{12}\ ft\right)\gamma_{H_2O} = p_{air}$$

$$p_{air} = \left(5\ \frac{lb}{in.^2} + 14.7\ \frac{lb}{in.^2}\right) - \frac{(1\ ft)\left(62.4\ \frac{lb}{ft^3}\right)}{144\ \frac{in.^2}{ft^2}}$$

$$p_{air} = \underline{\underline{19.3\ psia}}$$

2.22

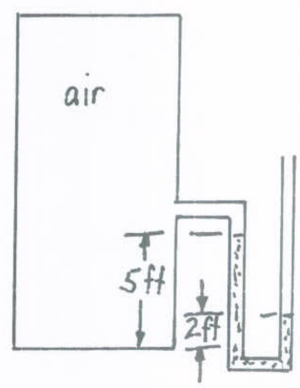
2.22 On the suction side of a pump a Bourdon pressure gage reads 40-kPa vacuum. What is the corresponding absolute pressure if the local atmospheric pressure is 100 kPa (abs)?

$$p(\text{abs}) = p(\text{gage}) + p(\text{atm})$$

$$= -40\ kPa + 100\ kPa = \underline{\underline{60\ kPa}}$$

2.24

2.24 A water-filled U-tube manometer is used to measure the pressure inside a tank that contains air. The water level in the U-tube on the side that connects to the tank is 5 ft above the base of the tank. The water level in the other side of the U-tube (which is open to the atmosphere) is 2 ft above the base. Determine the pressure within the tank.



$$p_{air} + \gamma_{H_2O}(5\text{ft}) - \gamma_{H_2O}(2\text{ft}) = 0$$

or

$$p_{air} = -(3\text{ft})\gamma_{H_2O} = -(3\text{ft})(62.4\ \tfrac{lb}{ft^3})$$

$$= \underline{\underline{-187\ \tfrac{lb}{ft^2}}}$$

2.25

2.25 A barometric pressure of 29.4 in. Hg corresponds to what value of atmospheric pressure in psia, and in pascals?

(In psi) $p = \gamma h = \left(847 \frac{lb}{ft^3}\right)\left(\frac{29.4}{12} ft\right)\left(\frac{1 ft^2}{144 in.^2}\right) = \underline{\underline{14.4 \text{ psia}}}$

(In Pa) $p = \gamma h = \left(133 \times 10^3 \frac{N}{m^3}\right)\left(29.4 in.\right)\left(2.540 \times 10^{-2} \frac{m}{in.}\right) = \underline{\underline{99.3 \text{ kPa (abs)}}}$

2.26

2.26 For an atmospheric pressure of 101 kPa (abs) determine the heights of the fluid columns in barometers containing one of the following liquids: **(a)** mercury, **(b)** water, and **(c)** ethyl alcohol. Calculate the heights including the effect of vapor pressure, and compare the results with those obtained neglecting vapor pressure. Do these results support the widespread use of mercury for barometers? Why?

(Including vapor pressure)

$$p(atm) = \gamma h + p_v$$

where $p_v \sim$ vapor pressure

Thus, $$h = \frac{p(atm) - p_v}{\gamma}$$

(a) For mercury: $$h = \frac{101 \times 10^3 \frac{N}{m^2} - 1.6 \times 10^{-1} \frac{N}{m^2}}{133 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{0.759\ m}}$$

(b) For water: $$h = \frac{101 \times 10^3 \frac{N}{m^2} - 1.77 \times 10^3 \frac{N}{m^2}}{9.80 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{10.1\ m}}$$

(c) For ethyl alcohol: $$h = \frac{101 \times 10^3 \frac{N}{m^2} - 5.9 \times 10^3 \frac{N}{m^2}}{7.74 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{12.3\ m}}$$

(Without vapor pressure)

$$p(atm) = \gamma h$$

$$h = \frac{p(atm)}{\gamma}$$

$$h = \frac{101 \times 10^3 \frac{N}{m^2}}{133 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{0.759\ m}}$$

$$h = \frac{101 \times 10^3 \frac{N}{m^2}}{9.80 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{10.3\ m}}$$

$$h = \frac{101 \times 10^3 \frac{N}{m^2}}{7.74 \times 10^3 \frac{N}{m^3}}$$

$$= \underline{\underline{13.0\ m}}$$

Yes. For mercury barameters the effect of vapor pressure is negligible, and the required height of the mercury column is reasonable.

2.27

2.27 A mercury manometer is connected to a large reservoir of water as shown in Fig. P2.27. Determine the ratio, h_w/h_m, of the distances h_w and h_m indicated in the figure.

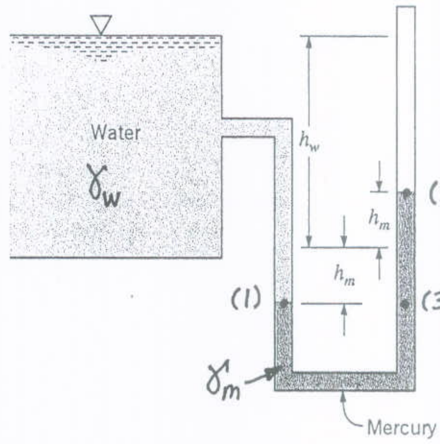


FIGURE P2.27

$p_1 = \gamma_w h_w + \gamma_w h_m$

but $p_1 = p_3 = \gamma_m (2h_m)$

Thus,

$\gamma_w h_w + \gamma_w h_m = 2\gamma_m h_m$

or

$(\gamma_w) h_w = (2\gamma_m - \gamma_w) h_m$

so that

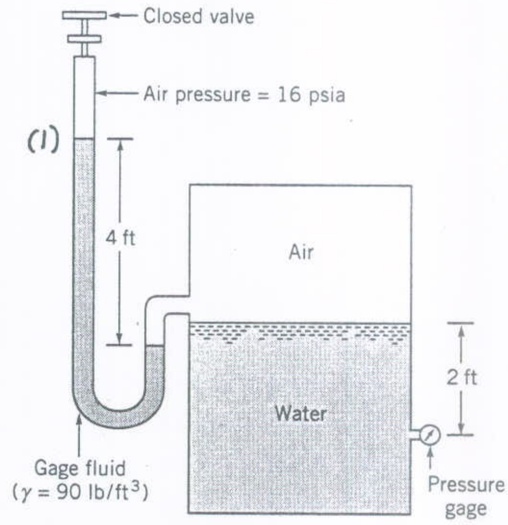
$\dfrac{h_w}{h_m} = \dfrac{(2\gamma_m - \gamma_w)}{\gamma_w} = 2\,SG_m - 1$, where $SG_m = \dfrac{\gamma_m}{\gamma_w} = 13.56$

Thus,

$\dfrac{h_w}{h_m} = 2(13.56) - 1 = \underline{\underline{26.1}}$

2.28

2.28 A U-tube manometer is connected to a closed tank containing air and water as shown in Fig. P2.28. At the closed end of the manometer the air pressure is 16 psia. Determine the reading on the pressure gage for a differential reading of 4 ft on the manometer. Express your answer in psi (gage). Assume standard atmospheric pressure, and neglect the weight of the air columns in the manometer.



■ FIGURE P2.28

$$p_1 + \gamma_{gf}\,(4\,ft) + \gamma_{H_2O}\,(2\,ft) = p_{gage}$$

Thus,

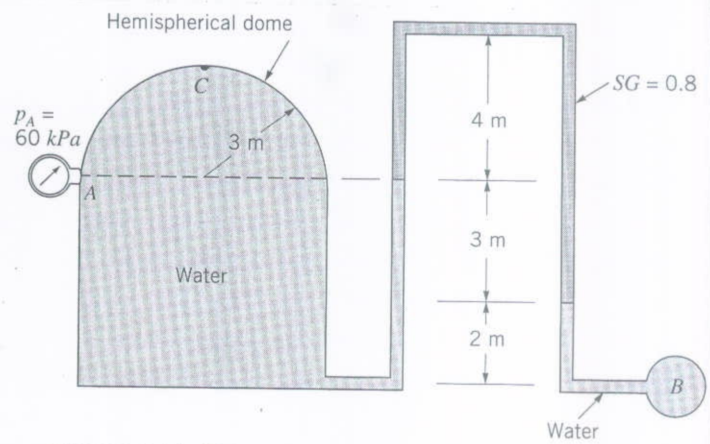
$$p_{gage} = \left(16\,\frac{lb}{in.^2} - 14.7\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right) + \left(90\,\frac{lb}{ft^3}\right)(4\,ft)$$

$$+ \left(62.4\,\frac{lb}{ft^3}\right)(2\,ft)$$

$$= 672\,\frac{lb}{ft^2} = \left(672\,\frac{lb}{ft^2}\right)\left(\frac{1\,ft^2}{144\,in.^2}\right) = \underline{\underline{4.67\ psi}}$$

2.29

2.29 A closed cylindrical tank filled with water has a hemispherical dome and is connected to an inverted piping system as shown in Fig. P2.29. The liquid in the top part of the piping system has a specific gravity of 0.8, and the remaining parts of the system are filled with water. If the pressure gage reading at A is 60 kPa, determine: **(a)** the pressure in pipe B, and **(b)** the pressure head, in millimeters of mercury, at the top of the dome (point C).



■ FIGURE P2.29

(a) $$p_A + (SG)(\gamma_{H_2O})(3\,m) + \gamma_{H_2O}(2\,m) = p_B$$

$$p_B = 60\,kPa + (0.8)\left(9.81\times10^3\,\frac{N}{m^3}\right)(3\,m) + \left(9.80\times10^3\,\frac{N}{m^3}\right)(2\,m)$$

$$= \underline{\underline{103\,kPa}}$$

(b) $$p_C = p_A - \gamma_{H_2O}(3m)$$

$$= 60\,kPa - \left(9.80\times10^3\,\frac{N}{m^3}\right)(3\,m)$$

$$= 30.6\times10^3\,\frac{N}{m^2}$$

$$h = \frac{p_c}{\gamma_{Hg}} = \frac{30.6\times10^3\,\frac{N}{m^2}}{133\times10^3\,\frac{N}{m^3}} = 0.230\,m$$

$$= 0.230\,m\left(\frac{10^3\,mm}{m}\right) = \underline{\underline{230\,mm}}$$

2.30

2.30 Two pipes are connected by a manometer as shown in Fig. P2.30. Determine the pressure difference, $p_A - p_B$, between the pipes.

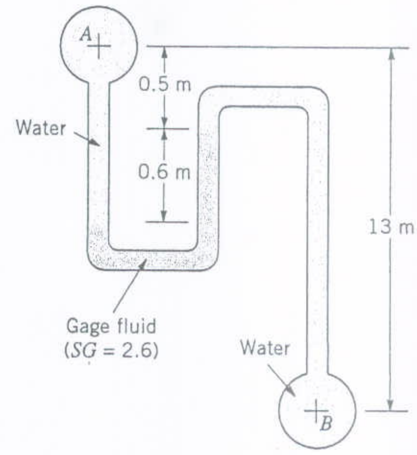


FIGURE P2.30

$$p_A + \gamma_{H_2O}(0.5m + 0.6m) - \gamma_{gf}(0.6m) + \gamma_{H_2O}(1.3m - 0.5m) = p_B$$

Thus,

$$p_A - p_B = \gamma_{gf}(0.6m) - \gamma_{H_2O}(0.5m + 0.6m + 1.3m - 0.5m)$$

$$= (2.6)\left(9.81\,\frac{kN}{m^3}\right)(0.6m) - \left(9.80\,\frac{kN}{m^3}\right)(1.9m)$$

$$= \underline{\underline{-3.32\ kPa}}$$

2.31

2.31 A U-tube manometer is connected to a closed tank as shown in Fig. P2.31. The air pressure in the tank is 0.50 psi and the liquid in the tank is oil ($\gamma = 54.0 \text{ lb/ft}^3$). The pressure at point A is 2.00 psi. Determine: **(a)** the depth of oil, z, and **(b)** the differential reading, h, on the manometer.

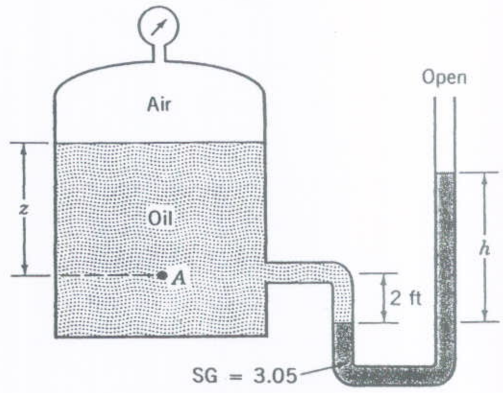


FIGURE P2.31

(a) $p_A = \gamma_{oil}\, z + p_{air}$

Thus,

$$z = \frac{p_A - p_{air}}{\gamma_{oil}} = \frac{\left(2\,\frac{\text{lb}}{\text{in.}^2} - 0.5\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)}{54.0\,\frac{\text{lb}}{\text{ft}^3}} = \underline{\underline{4.00 \text{ ft}}}$$

(b) $p_A + \gamma_{oil}\,(2\text{ ft}) - (SG)(\gamma_{H_2O})\,h = 0$

Thus,

$$h = \frac{p_A + \gamma_{oil}\,(2\text{ ft})}{(SG)(\gamma_{H_2O})}$$

$$= \frac{\left(2\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right) + \left(54.0\,\frac{\text{lb}}{\text{ft}^3}\right)(2\text{ ft})}{(3.05)\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)}$$

$$= \underline{\underline{2.08 \text{ ft}}}$$

2.32

2.32 For the inclined-tube manometer of Fig. P2.32 the pressure in pipe *A* is 0.6 psi. The fluid in both pipes *A* and *B* is water, and the gage fluid in the manometer has a specific gravity of 2.6. What is the pressure in pipe *B* corresponding to the differential reading shown?

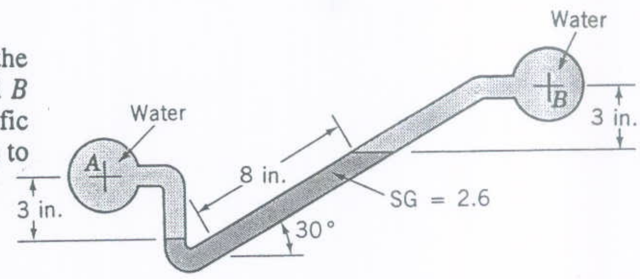


FIGURE P2.32

$$p_A + \gamma_{H_2O}\left(\tfrac{3}{12}\text{ ft}\right) - \gamma_{gf}\left(\tfrac{8}{12}\text{ ft}\right)\sin 30^\circ - \gamma_{H_2O}\left(\tfrac{3}{12}\text{ ft}\right) = p_B$$

(where γ_{gf} is the specific weight of the gage fluid)

Thus,

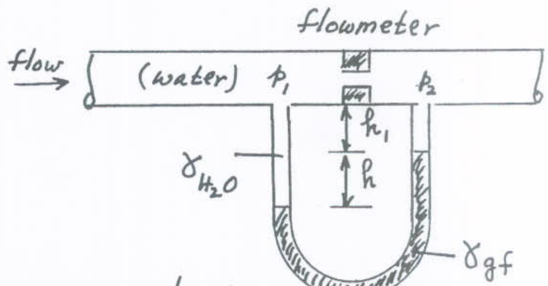
$$p_B = p_A - \gamma_{gf}\left(\tfrac{8}{12}\text{ ft}\right)\sin 30^\circ$$

$$= \left(0.6\ \frac{\text{lb}}{\text{in.}^2}\right)\left(144\ \frac{\text{in.}^2}{\text{ft}^2}\right) - (2.6)\left(62.4\ \frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{8}{12}\text{ ft}\right)(0.5) = 32.3\ \frac{\text{lb}}{\text{ft}^2}$$

$$= 32.3\ \text{lb/ft}^2 / 144\ \text{in.}^2/\text{ft}^2 = \underline{\underline{0.224\ \text{psi}}}$$

2.33

2.33 A flowrate measuring device is installed in a horizontal pipe through which water is flowing. A U-tube manometer is connected to the pipe through pressure taps located 3 in. on either side of the device. The gage fluid in the manometer has a specific weight of 112 lb/ft³. Determine the differential reading of the manometer corresponding to a pressure drop between the taps of 0.5 lb/in.².



Let p_1 and p_2 be pressures at pressure taps.
Write manometer equation between p_1 and p_2. Thus,

$$p_1 + \gamma_{H_2O}(h_1 + h) - \gamma_{gf}\, h - \gamma_{H_2O}\, h_1 = p_2$$

so that

$$h = \frac{p_1 - p_2}{\gamma_{gf} - \gamma_{H_2O}} = \frac{\left(0.5\ \frac{\text{lb}}{\text{in.}^2}\right)\left(144\ \frac{\text{in.}^2}{\text{ft}^2}\right)}{112\ \frac{\text{lb}}{\text{ft}^3} - 62.4\ \frac{\text{lb}}{\text{ft}^3}}$$

$$= \underline{\underline{1.45\ \text{ft}}}$$

2.34

2.34 Small differences in gas pressures are commonly measured with a *micromanometer* of the type illustrated in Fig. P2.34. This device consists of two large reservoirs each having a cross-sectional area, A_r, which are filled with a liquid having a specific weight, γ_1, and connected by a U-tube of cross-sectional area, A_t, containing a liquid of specific weight, γ_2. When a differential gas pressure, $p_1 - p_2$, is applied a differential reading, h, develops. It is desired to have this reading sufficiently large (so that it can be easily read) for small pressure differentials. Determine the relationship between h and $p_1 - p_2$ when the area ratio A_t/A_r is small, and show that the differential reading, h, can be magnified by making the difference in specific weights, $\gamma_2 - \gamma_1$, small. Assume that initially (with $p_1 = p_2$) the fluid levels in the two reservoirs are equal.

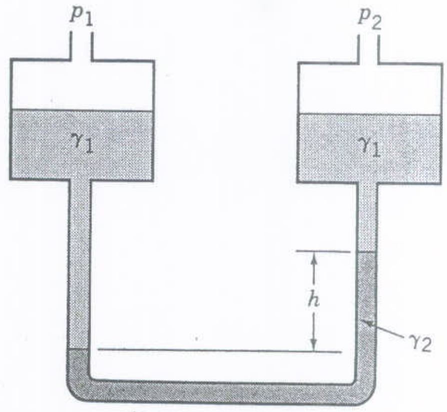
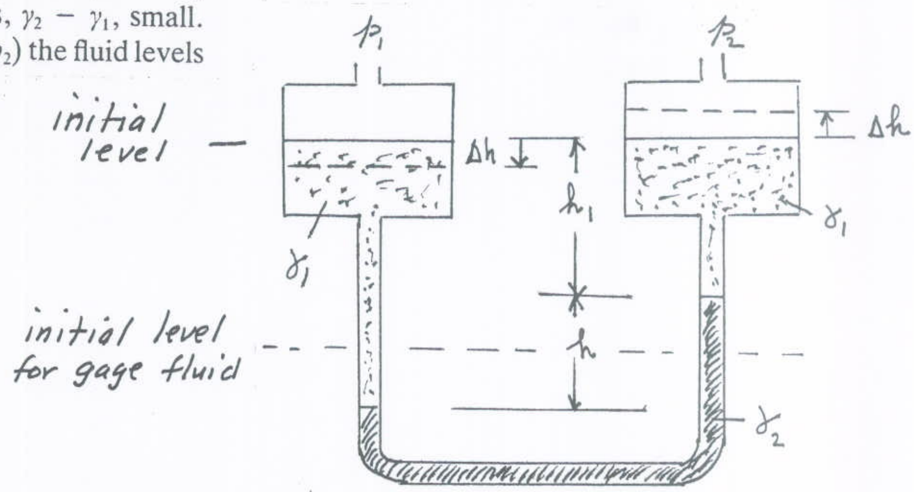


FIGURE P2.34



When a differential pressure, $p_1 - p_2$, is applied we assume that level in left reservoir drops by a distance, Δh, and right level rises by Δh. Thus, the manometer equation becomes

$$p_1 + \gamma_1 (h_1 + h - \Delta h) - \gamma_2 h - \gamma_1 (h_1 + \Delta h) = p_2$$

or

$$p_1 - p_2 = \gamma_2 h - \gamma_1 h + \gamma_1 (2 \Delta h) \qquad (1)$$

Since the liquids in the manometer are incompressible,

$$\Delta h \, A_r = \frac{h}{2} A_t \qquad \text{or} \qquad \frac{2 \Delta h}{h} = \frac{A_t}{A_r}$$

and if $\frac{A_t}{A_r}$ is small then $2\Delta h \ll h$ and last term in Eq.(1) can be neglected. Thus,

$$p_1 - p_2 = (\gamma_2 - \gamma_1) h$$

or

$$\underline{\underline{h = \frac{p_1 - p_2}{\gamma_2 - \gamma_1}}}$$

and large values of h can be obtained for small pressure differentials if $\gamma_2 - \gamma_1$ is small.

2.35

2.35 The cyclindrical tank with hemispherical ends shown in Fig. P2.35 contains a volatile liquid and its vapor. The liquid density is 800 kg/m³, and its vapor density is negligible. The pressure in the vapor is 120 kPa (abs), and the atmospheric pressure is 101 kPa (abs). Determine: **(a)** the gage pressure reading on the pressure gage; and **(b)** the height, h, of the mercury manometer.

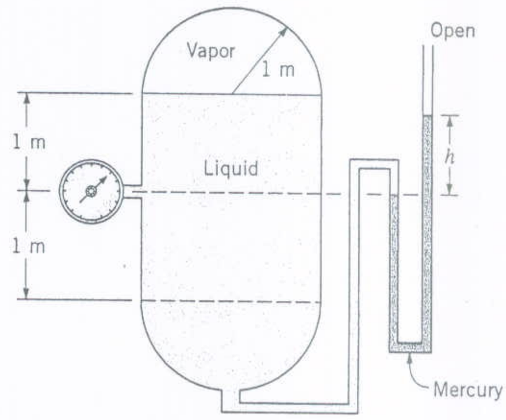


FIGURE P2.35

(a) Let γ_ℓ = sp. wt. of liquid $= \left(800 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right) = 7850 \frac{N}{m^3}$

and

p_{vapor} (gage) = 120 kPa (abs) − 101 kPa (abs) = 19 kPa

Thus,

$$p_{gage} = p_{vapor} + \gamma_\ell\,(1\,m)$$

$$= 19 \times 10^3 \frac{N}{m^2} + \left(7850 \frac{N}{m^3}\right)(1\,m)$$

$$= \underline{\underline{26.9\ kPa}}$$

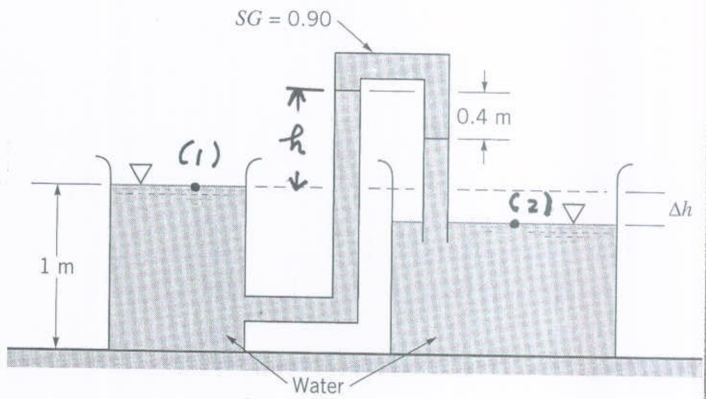
(b) $p_{vapor}\,(gage) + \gamma_\ell\,(1\,m) - \gamma_{Hg}\,(h) = 0$

$$19 \times 10^3 \frac{N}{m^2} + \left(7850 \frac{N}{m^3}\right)(1\,m) - \left(133 \times 10^3 \frac{N}{m^3}\right)(h) = 0$$

$$h = \underline{\underline{0.202\ m}}$$

2.36

2.36 Determine the elevation difference, Δh, between the water levels in the two open tanks shown in Fig. P2.36.



■ FIGURE P2.36

$$p_1 - \gamma_{H_2O} h + (SG)\gamma_{H_2O}(0.4m) + \gamma_{H_2O}(h - 0.4m) + \gamma_{H_2O}(\Delta h) = p_2$$

Since $p_1 = p_2 = 0$

$$\Delta h = 0.4m - (0.9)(0.4m) = \underline{\underline{0.040\,m}}$$

2.37

2.37 For the configuration shown in Fig. P2.37 what must be the value of the specific weight of the unknown fluid? Express your answer in lb/ft³.

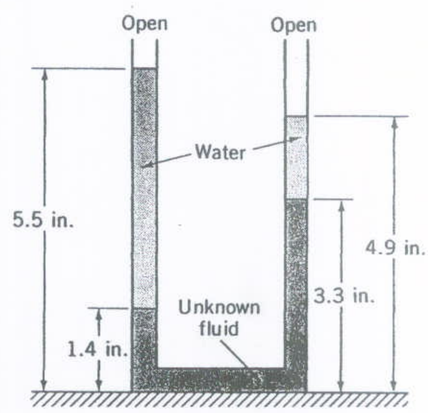


FIGURE P2.37

Let γ be specific weight of unknown fluid. Then,

$$\gamma_{H_2O}\left[\frac{(5.5-1.4)}{12}\,ft\right] - \gamma\left[\frac{(3.3-1.4)}{12}\,ft\right] - \gamma_{H_2O}\left[\frac{(4.9-3.3)}{12}\,ft\right] = 0$$

and

$$\gamma = \frac{\gamma_{H_2O}\left[(5.5-1.4)-(4.9-3.3)\right]in.}{(3.3-1.4)\,in.} = \left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{4.1-1.6}{1.9}\right)$$

$$= \underline{\underline{82.1\,\frac{lb}{ft^3}}}$$

2.38

2.38 An air-filled, hemispherical shell is attached to the ocean floor at a depth of 10 m as shown in Fig. P2.38. A mercury barometer located inside the shell reads 765 mm Hg, and a mercury U-tube manometer designed to give the outside water pressure indicates a differential reading of 735 mm Hg as illustrated. Based on these data what is the atmospheric pressure at the ocean surface?

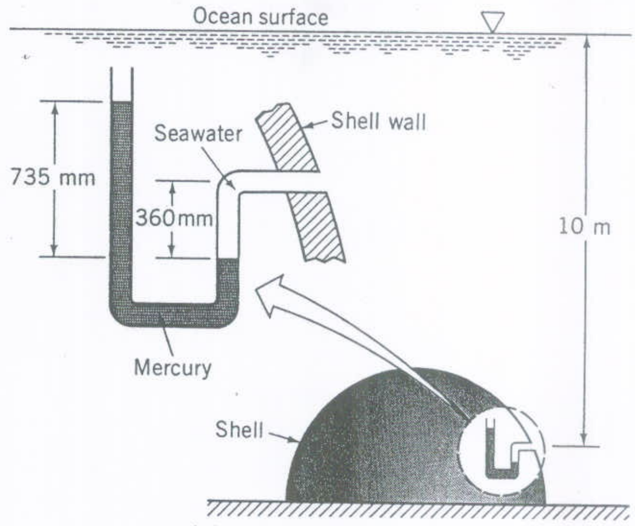


FIGURE P2.38

Let: $p_a \sim$ absolute air pressure inside shell $= \gamma_{Hg}\,(0.765\,m)$

$p_{atm} \sim$ surface atmospheric pressure

$\gamma_{sw} \sim$ specific weight of seawater

Thus, manometer equation can be written as

$$p_{atm} + \gamma_{sw}\,(10\,m) + \gamma_{sw}\,(0.360\,m) - \gamma_{Hg}\,(0.735\,m) = p_a$$

so that

$$p_{atm} = p_a - \gamma_{sw}\,(10.36\,m) + \gamma_{Hg}\,(0.735\,m)$$

$$= \left(133\,\frac{kN}{m^3}\right)(0.765\,m) - \left(10.1\,\frac{kN}{m^3}\right)(10.36\,m) + \left(133\,\frac{kN}{m^3}\right)(0.735\,m)$$

$$= \underline{\underline{94.9\ kPa}}$$

*2.39

*2.39 Both ends of the U-tube mercury manometer of Fig. P2.39 are initially open to the atmosphere and under standard atmospheric pressure. When the valve at the top of the right leg is open the level of mercury below the valve is h_i. After the valve is closed, air pressure is applied to the left leg. Determine the relationship between the differential reading on the manometer and the applied gage pressure, p_g. Show on a plot how the differential reading varies with p_g for h_i = 25, 50, 75, and 100 mm over the range $0 \le p_g \le 300$ kPa. Assume that the temperature of the trapped air remains constant.

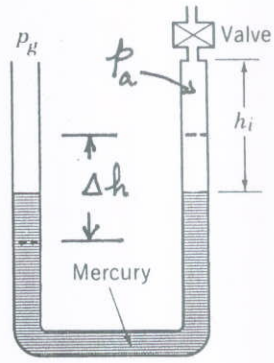


FIGURE P2.39

With the valve closed and a pressure, p_g, applied,

$$p_g - \gamma_{Hg}\,\Delta h = p_a$$

or

$$\Delta h = \frac{p_g - p_a}{\gamma_{Hg}} \qquad (1)$$

where p_g and p_a are gage pressures. For isothermal compression of trapped air

$$\frac{p}{\rho} = \text{constant}$$

so that for constant air mass

$$p_i \forall_i = p_f \forall_f$$

where $\forall$ is air volume, p is absolute pressure, and i and f refer to initial and final states, respectively. Thus,

$$p_{atm} \forall_i = (p_a + p_{atm}) \forall_f \qquad (2)$$

For air trapped in right leg, $\forall_i = h_i$(Area of tube) so that Eq.(2) can be written as

$$p_a = p_{atm}\left[\frac{h_i}{h_i - \frac{\Delta h}{2}} - 1\right] \qquad (3)$$

Substitute Eq.(3) into Eq.(1) to obtain

$$\Delta h = \frac{1}{\gamma_{Hg}}\left[p_g + p_{atm}\left(1 - \frac{h_i}{h_i - \frac{\Delta h}{2}}\right)\right] \qquad (4)$$

(con't)

*2.39 (con't)

Equation (4) can be expressed in the form

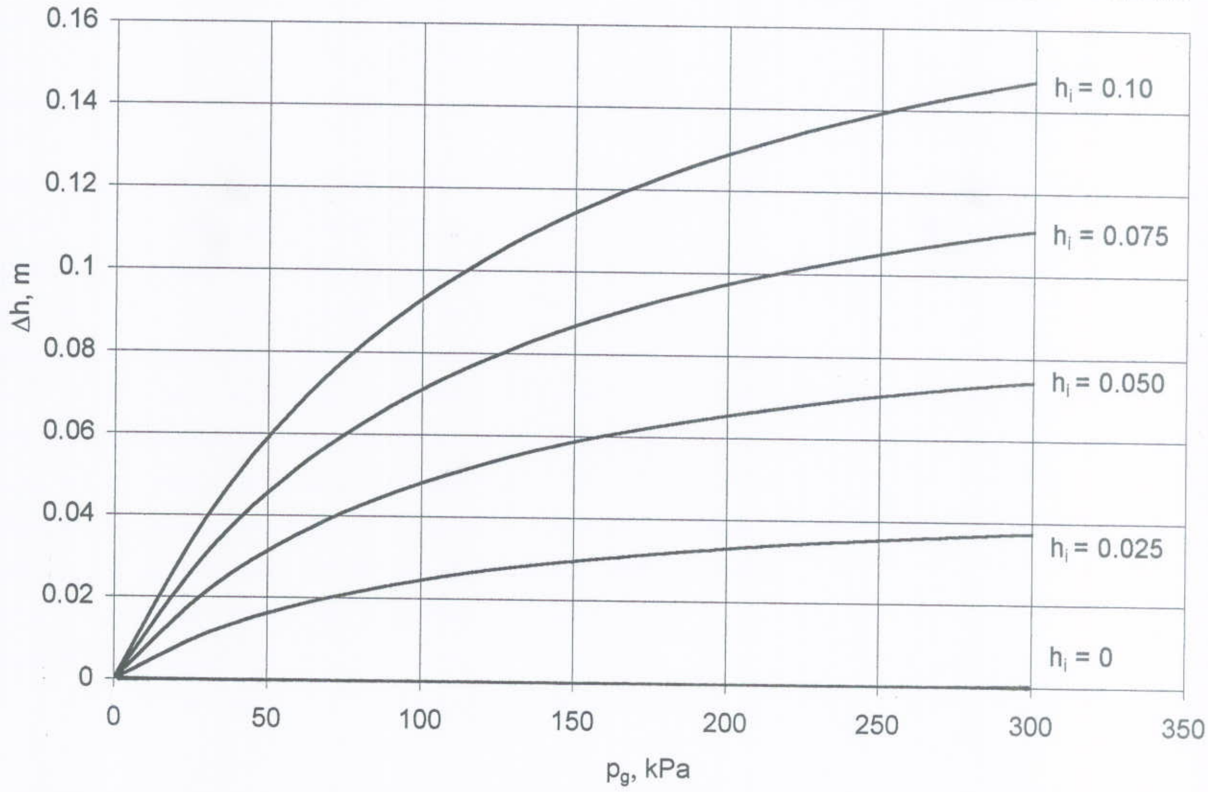
$$(\Delta h)^2 - \left(2h_i + \frac{p_g + p_{atm}}{\gamma_{Hg}}\right)\Delta h + \frac{2 p_g h_i}{\gamma_{Hg}} = 0$$

and the roots of this quadratic equation are

$$\Delta h = \left(h_i + \frac{p_g + p_{atm}}{2\gamma_{Hg}}\right) \pm \sqrt{\left(h_i + \frac{p_g + p_{atm}}{2\gamma_{Hg}}\right)^2 - \frac{2 p_g h_i}{\gamma_{Hg}}} \qquad (5)$$

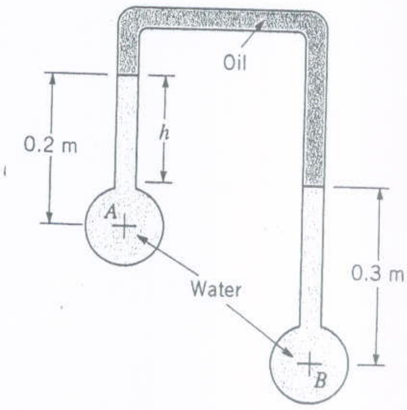
To evaluate Δh the negative sign is used since $\Delta h = 0$ for $p_g = 0$. Tabulated values of Δh for various values of p_g are given in the following table for different values of h_i (with $p_{atm} = 101$ kPa and $\gamma_{Hg} = 133$ kN/m^3). A plot of the data follows.

hi (m)	patm (kPa)	γhg (kN/m3)	p_g (kPa)	$\Delta h(h_i = 0)$ (m)	$\Delta h(h_i=0.025)$ (m)	$\Delta h(h_i=0.05)$ (m)	$\Delta h(h_i=0.075)$ (m)	$\Delta h(h_i=0.1)$ (m)
0.025	101	133	0	0	0	0	0	0
0.05	101	133	30	0	0.0110	0.0212	0.0306	0.0394
0.075	101	133	60	0	0.0182	0.0354	0.0517	0.0672
0.1	101	133	90	0	0.0231	0.0454	0.0668	0.0874
	101	133	120	0	0.0268	0.0528	0.0781	0.1026
	101	133	150	0	0.0296	0.0585	0.0867	0.1143
	101	133	180	0	0.0318	0.0630	0.0936	0.1236
	101	133	210	0	0.0335	0.0666	0.0991	0.1312
	101	133	240	0	0.0350	0.0696	0.1037	0.1374
	101	133	270	0	0.0362	0.0721	0.1075	0.1426
	101	133	300	0	0.0372	0.0742	0.1108	0.1470



2.40

2.40 The inverted U-tube manometer of Fig. P2.40 contains oil ($SG = 0.9$) and water as shown. The pressure differential between pipes A and B, $p_A - p_B$, is -5 kPa. Determine the differential reading, h.



■ FIGURE P2.40

$$p_A - \gamma_{H_2O}(0.2\,m) + \gamma_{oil}(h) + \gamma_{H_2O}(0.3\,m) = p_B$$

Thus,

$$h = \frac{(p_B - p_A) + \gamma_{H_2O}(0.2\,m) - \gamma_{H_2O}(0.3\,m)}{\gamma_{oil}}$$

$$= \frac{5 \times 10^3 \frac{N}{m^2} - (9.80 \times 10^3 \frac{N}{m^3})(0.1\,m)}{8.95 \times 10^3 \frac{N}{m^3}} = \underline{\underline{0.449\,m}}$$

2.41

2.41 An inverted U-tube manometer containing oil ($SG = 0.8$) is located between two reservoirs as shown in Fig. P2.41. The reservoir on the left, which contains carbon tetrachloride, is closed and pressurized to 8 psi. The reservoir on the right contains water and is open to the atmosphere. With the given data, determine the depth of water, h, in the right reservoir.

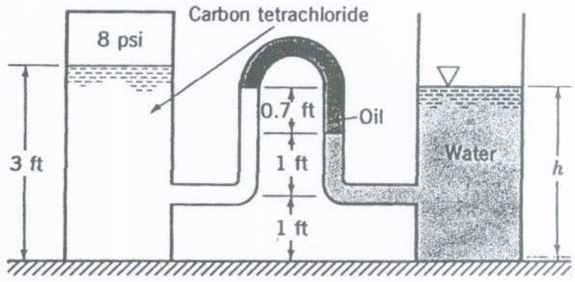


FIGURE P2.41

Let p_A be the air pressure in left reservoir. Manometer equation can be written as

$$p_A + \gamma_{CCl_4}(3\text{ ft} - 1\text{ ft} - 1\text{ ft} - 0.7\text{ ft}) + \gamma_{oil}(0.7\text{ ft}) - \gamma_{H_2O}(h - 1\text{ ft} - 1\text{ ft}) = 0$$

so that

$$h = \frac{p_A + \gamma_{CCl_4}(0.3\text{ ft}) + \gamma_{oil}(0.7\text{ ft})}{\gamma_{H_2O}} + 2\text{ ft}$$

$$= \frac{\left(8\frac{\text{lb}}{\text{in.}^2}\right)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right) + \left(99.5\frac{\text{lb}}{\text{ft}^3}\right)(0.3\text{ ft}) + \left(57.0\frac{\text{lb}}{\text{ft}^3}\right)(0.7\text{ ft})}{62.4\frac{\text{lb}}{\text{ft}^3}} + 2\text{ ft}$$

$$= \underline{\underline{21.6\text{ ft}}}$$

2.42

2.42 Determine the pressure of the water in pipe A shown in Fig. P2.42 if the gage pressure of the air in the tank is 2 psi.

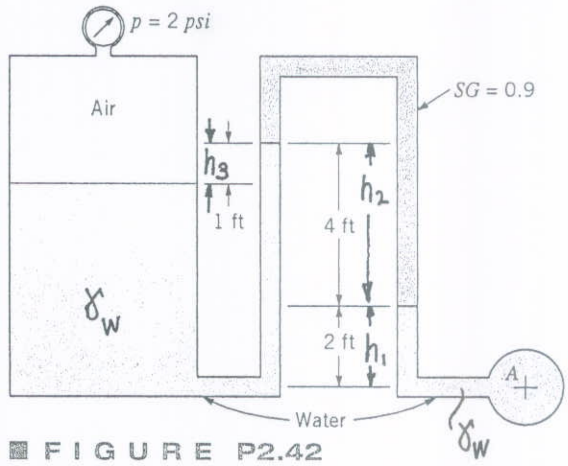


FIGURE P2.42

$$p_A - \gamma_w h_1 - (0.9\gamma_w)h_2 + \gamma_w h_3 = p_{air}$$

or

$$p_A = p_{air} + \gamma_w(h_1 + 0.9h_2 - h_3)$$

$$= 2\,\frac{\text{lb}}{\text{in.}^2}\left(\frac{144\,\text{in.}^2}{\text{ft}^2}\right) + 62.4\,\frac{\text{lb}}{\text{ft}^3}\left(\ \text{ft} + 0.9(4\,\text{ft}) - 1\,\text{ft}\right)$$

$$= \underline{\underline{575\,\frac{\text{lb}}{\text{ft}^2}}}$$

2.43

2.43 In Fig. P2.43 pipe *A* contains gasoline ($SG = 0.7$), pipe *B* contains oil ($SG = 0.9$), and the manometer fluid is mercury. Determine the new differential reading if the pressure in pipe *A* is decreased 25 kPa, and the pressure in pipe *B* remains constant. The initial differential reading is 0.30 m as shown.

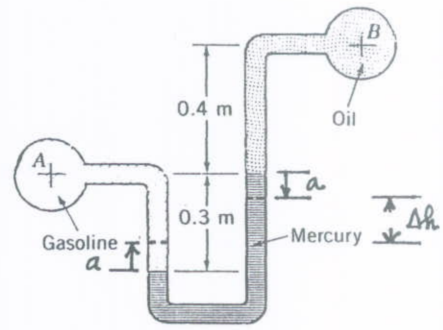


FIGURE P2.43

For the initial configuration:

$$p_A + \gamma_{gas}(0.3\,m) - \gamma_{Hg}(0.3\,m) - \gamma_{oil}(0.4\,m) = p_B \qquad (1)$$

With a decrease in p_A to p_A' gage fluid levels change as shown on figure. Thus, for final configuration:

$$p_A' + \gamma_{gas}(0.3-a) - \gamma_{Hg}(\Delta h) - \gamma_{oil}(0.4+a) = p_B \qquad (2)$$

where all lengths are in m. Subtract Eq.(2) from Eq.(1) to obtain,

$$p_A - p_A' + \gamma_{gas}(a) - \gamma_{Hg}(0.3-\Delta h) + \gamma_{oil}(a) = 0 \qquad (3)$$

Since $2a + \Delta h = 0.3$ (see figure) then

$$a = \frac{0.3-\Delta h}{2}$$

and from Eq.(3)

$$p_A - p_A' + \gamma_{gas}\left(\frac{0.3-\Delta h}{2}\right) - \gamma_{Hg}(0.3-\Delta h) + \gamma_{oil}\left(\frac{0.3-\Delta h}{2}\right) = 0$$

Thus,

$$\Delta h = \frac{p_A - p_A' + \gamma_{gas}(0.15) - \gamma_{Hg}(0.3) + \gamma_{oil}(0.15)}{-\gamma_{Hg} + \frac{\gamma_{gas}}{2} + \frac{\gamma_{oil}}{2}}$$

and with $p_A - p_A' = 25\,kPa$

$$\Delta h = \frac{25\frac{kN}{m^2} + (0.7)(9.81\frac{kN}{m^3})(0.15\,m) - (133\frac{kN}{m^3})(0.3\,m) + (0.9)(9.81\frac{kN}{m^3})(0.15\,m)}{-133\frac{kN}{m^3} + \frac{(0.7)(9.81\frac{kN}{m^3})}{2} + \frac{(0.9)(9.81\frac{kN}{m^3})}{2}}$$

$$= \underline{\underline{0.100\,m}}$$

2.44

2.44 The inclined differential manometer of Fig. P2.44 contains carbon tetrachloride. Initially the pressure differential between pipes A and B, which contain a brine ($SG = 1.1$), is zero as illustrated in the figure. It is desired that the manometer give a differential reading of 12 in. (measured along the inclined tube) for a pressure differential of 0.1 psi. Determine the required angle of inclination, θ.

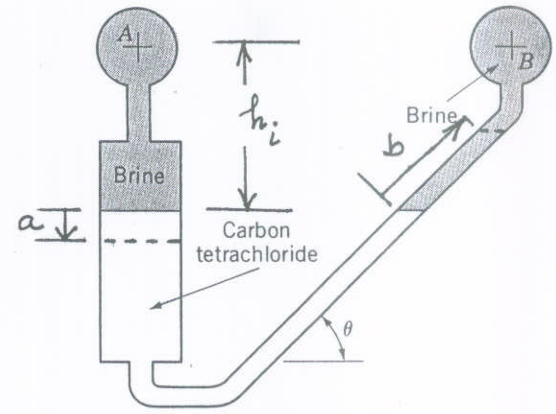


FIGURE P2.44

When $p_A - p_B$ is increased to $p_A' - p_B'$ the left column falls a distance, a, and the right column rises a distance b along the inclined tube as shown in figure. For this final configuration:

$$p_A' + \gamma_{br}(h_i + a) - \gamma_{CCl_4}(a + b\sin\theta) - \gamma_{br}(h_i - b\sin\theta) = p_B'$$

or

$$p_A' - p_B' + (\gamma_{br} - \gamma_{CCl_4})(a + b\sin\theta) = 0 \qquad (1)$$

The differential reading, Δh, along the tube is

$$\Delta h = \frac{a}{\sin\theta} + b$$

Thus, from Eq. (1)

$$p_A' - p_B' + (\gamma_{br} - \gamma_{CCl_4})(\Delta h \sin\theta) = 0$$

or

$$\sin\theta = \frac{-(p_A' - p_B')}{(\gamma_{br} - \gamma_{CCl_4})(\Delta h)}$$

and with $p_A' - p_B' = 0.1\,\text{psi}$

$$\sin\theta = \frac{-\left(0.1\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right)}{\left[(1.1)\left(62.4\,\frac{lb}{ft^3}\right) - 99.5\,\frac{lb}{ft^3}\right]\left(\frac{12}{12}\,ft\right)} = 0.466$$

for $\Delta h = 12$ in.

Thus,

$$\underline{\underline{\theta = 27.8^\circ}}$$

2.45

2.45 Determine the new differential reading along the inclined leg of the mercury manometer of Fig. P2.45, if the pressure in pipe A is decreased 10 kPa and the pressure in pipe B remains unchanged. The fluid in A has a specific gravity of 0.9 and the fluid in B is water.

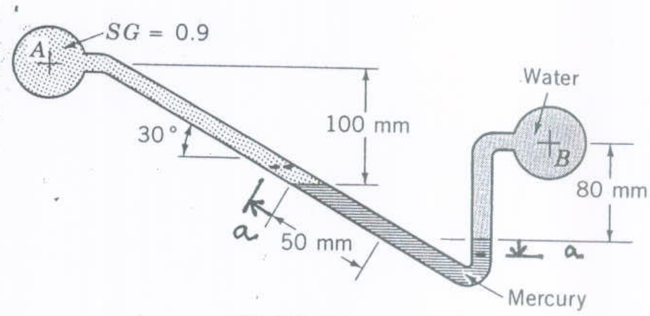


FIGURE P2.45

For the initial configuration:

$$p_A + \gamma_A(0.1) + \gamma_{Hg}(0.05 \sin 30°) - \gamma_{H_2O}(0.08) = p_B \qquad (1)$$

where all lengths are in m. When p_A decreases left column moves up a distance, a, and right column moves down a distance, a, as shown in figure. For the final configuration:

$$p_A' + \gamma_A(0.1 - a \sin 30°) + \gamma_{Hg}(a \sin 30° + 0.05 \sin 30° + a) - \gamma_{H_2O}(0.08 + a) = p_B \qquad (2)$$

where p_A' is the new pressure in pipe A.
Subtract Eq. (2) from Eq. (1) to obtain

$$p_A - p_A' + \gamma_A(a \sin 30°) - \gamma_{Hg}\, a(\sin 30° + 1) + \gamma_{H_2O}(a) = 0$$

Thus,

$$a = \frac{-(p_A - p_A')}{\gamma_A \sin 30° - \gamma_{Hg}(\sin 30° + 1) + \gamma_{H_2O}}$$

For $p_A - p_A' = 10$ kPa

$$a = \frac{-10 \frac{kN}{m^2}}{(0.9)(9.81 \frac{kN}{m^3})(0.5) - (133 \frac{kN}{m^3})(0.5 + 1) + 9.80 \frac{kN}{m^3}}$$

$$= 0.0540 \text{ m}$$

New differential reading, Δh, measured along inclined tube is equal to

$$\Delta h = \frac{a}{\sin 30°} + 0.05 + a$$

$$= \frac{0.0540 \text{ m}}{0.5} + 0.05 \text{ m} + 0.0540 \text{ m} = \underline{\underline{0.212 \text{ m}}}$$

2.46

2.46 Determine the change in the elevation of the mercury in the left leg of the manometer of Fig. P2.46 as a result of an increase in pressure of 5 psi in pipe A while the pressure in pipe B remains constant.

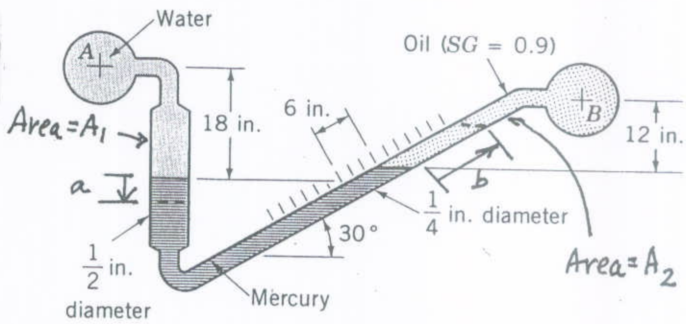


FIGURE P2.46

For the initial configuration:

$$p_A + \gamma_{H_2O}\left(\frac{18}{12}\right) - \gamma_{Hg}\left(\frac{6}{12}\sin 30^\circ\right) - \gamma_{oil}\left(\frac{12}{12}\right) = p_B \qquad (1)$$

where all lengths are in ft. When p_A increases to p_A' the left column falls by the distance, a, and the right column moves up the distance, b, as shown in the figure. For the final configuration:

$$p_A' + \gamma_{H_2O}\left(\frac{18}{12}+a\right) - \gamma_{Hg}\left(a + \frac{6}{12}\sin 30^\circ + b\sin 30^\circ\right) -$$

$$\gamma_{oil}\left(\frac{12}{12} - b\sin 30^\circ\right) = p_B \qquad (2)$$

Subtract Eq. (1) from Eq. (2) to obtain

$$p_A' - p_A + \gamma_{H_2O}(a) - \gamma_{Hg}(a + b\sin 30^\circ) + \gamma_{oil}(b\sin 30^\circ) = 0 \qquad (3)$$

Since the volume of liquid must be constant $A_1 a = A_2 b$, or

$$\left(\tfrac{1}{2}\text{ in.}\right)^2 a = \left(\tfrac{1}{4}\text{ in.}\right)^2 b$$

so that

$$b = 4a$$

Thus, Eq. (3) can be written as

$$p_A' - p_A + \gamma_{H_2O}(a) - \gamma_{Hg}(a + 4a\sin 30^\circ) + \gamma_{oil}(4a\sin 30^\circ) = 0$$

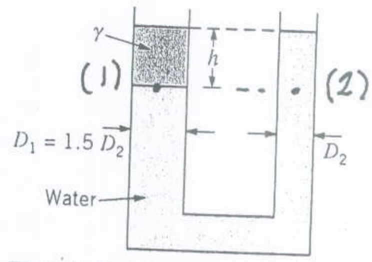
and

$$a = \frac{-(p_A' - p_A)}{\gamma_{H_2O} - \gamma_{Hg}(3) + \gamma_{oil}(2)} = \frac{-\left(5\,\frac{\text{lb}}{\text{in.}^2}\right)\left(144\,\frac{\text{in.}^2}{\text{ft}^2}\right)}{62.4\,\frac{\text{lb}}{\text{ft}^3} - \left(847\,\frac{\text{lb}}{\text{ft}^3}\right)(3) + (0.9)\left(62.4\,\frac{\text{lb}}{\text{ft}^3}\right)(2)}$$

$$= \underline{\underline{0.304\text{ ft (down)}}}$$

2.47

2.47 The U-shaped tube shown in Fig. P2.47 initially contains water only. A second liquid with specific weight, γ, less than water is placed on top of the water with no mixing occurring. Can the height, h, of the second liquid be adjusted so that the left and right levels are at the same height? Provide proof of your answer.



■ FIGURE P2.47

The pressure at point (1) must be equal to the pressure at point (2) since the pressures at equal elevations in a continuous mass of fluid must be the same. Since,

$$p_1 = \gamma h$$

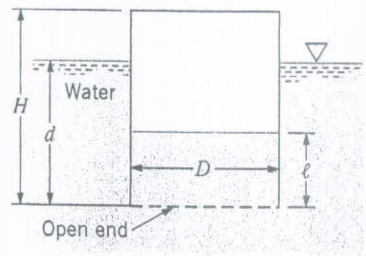
and

$$p_2 = \gamma_{H_2O} h$$

these two pressures can only be equal if $\gamma = \gamma_{H_2O}$. Since $\gamma \neq \gamma_{H_2O}$ the configuration shown in the figure is not possible. No.

*2.48

*2.48 An inverted hollow cylinder is pushed into the water as is shown in Fig. P2.48. Determine the distance, ℓ, that the water rises in the cylinder as a function of the depth, d, of the lower edge of the cylinder. Plot the results for $0 \le d \le H$, when H is equal to 1 m. Assume the temperature of the air within the cylinder remains constant.



■ FIGURE P2.48

For constant temperature compression within the cylinder,

$$p_i \forall_i = p_f \forall_f \qquad (1)$$

where $\forall$ is the air volume, and i and f refer to the initial and final states, respectively. It follows that (see figure)

$$p_i = p_{atm} \qquad p_f = \gamma(d-\ell) + p_{atm}$$

$$\forall_i = \frac{\pi}{4} D^2 H \qquad \forall_f = \frac{\pi}{4} D^2 (H-\ell)$$

Thus, from Eg. (1)

$$p_{atm}\left(\frac{\pi}{4} D^2 H\right) = \left(\gamma(d-\ell) + p_{atm}\right) \frac{\pi}{4} D^2 (H-\ell) \qquad (2)$$

and with

$$p_{atm} = 101\ kPa, \quad \gamma = 9.80 \frac{kN}{m^3}, \text{ and } \quad H = 1\ m$$

Eg. (2) simplifies to

$$\ell^2 - (d+11.31)\ell + d\,(1m) = 0$$

so that (using the quadratic formula)

$$\ell = \frac{(d+11.31) \pm \sqrt{d^2+18.61\,d+128}}{2}$$

Since for $d=0$, $\ell=0$, the negative sign should be used and

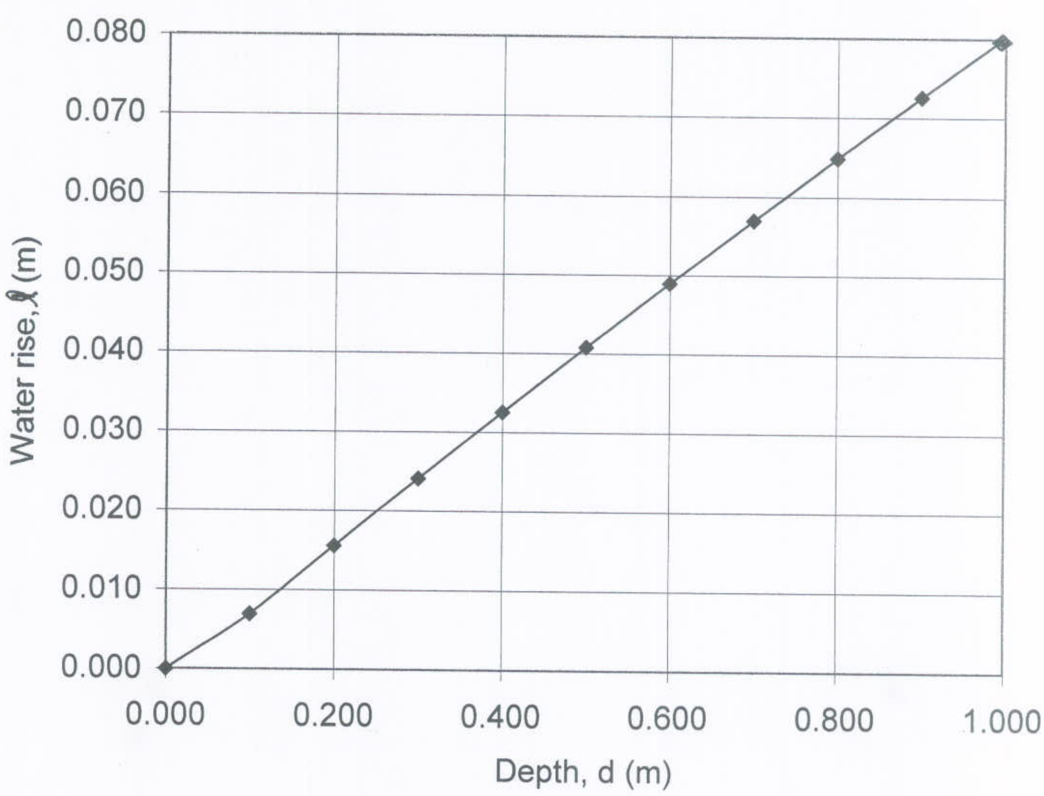
$$\ell = \frac{(d+11.31) - \sqrt{d^2+18.61\,d+128}}{2}$$

Tabulated data with the corresponding plot are shown on the following page.

(con't)

*2.48 (con't)

Depth, d (m)	Water rise, ℓ, (m)
0.000	0.000
0.100	0.007
0.200	0.016
0.300	0.024
0.400	0.033
0.500	0.041
0.600	0.049
0.700	0.057
0.800	0.065
0.900	0.073
1.000	0.080



*2.50

*2.50 A Bourdon gage (see Fig. 2.13 and Video V2.3) is often used to measure pressure. One way to calibrate this type of gage is to use the arrangement shown in Fig. P2.50a. The container is filled with a liquid and a weight, $\mathcal{W}$, placed on one side with the gage on the other side. The weight acting on the liquid through a 0.4-in.-diameter opening creates a pressure that is transmitted to the gage. This arrangement, with a series of weights, can be used to determine what a change in the dial movement, θ, in Fig. P2.50b. corresponds to in terms of a change in pressure. For a particular gage, some data are given below. Based on a plot of these data, determine the relationship between θ and the pressure, p, where p is measured in psi?

$\mathcal{W}$ (lb)	0	1.04	2.00	3.23	4.05	5.24	6.31
θ (deg.)	0	20	40	60	80	100	120

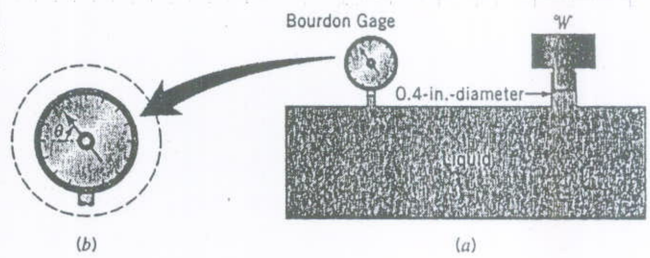


FIGURE P2.50

$$p = \frac{\mathcal{W}}{Area} = \frac{\mathcal{W}\,(lb)}{\frac{\pi}{4}(0.4\text{ in.})^2} = 7.96\,\mathcal{W}(lb) \qquad (1)$$

(where p is in psi)

From graph

$$\mathcal{W} = 0.0522\,\theta$$

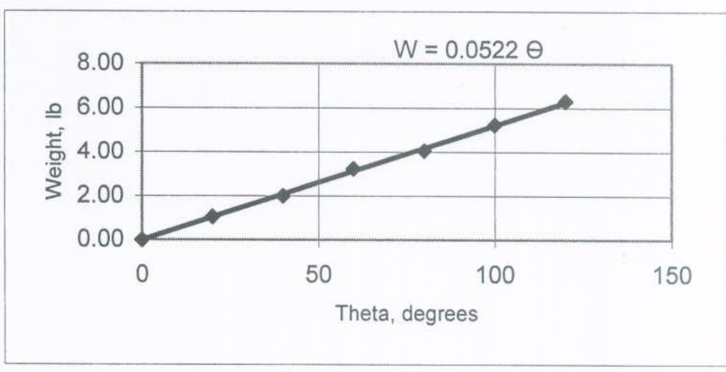
so that from Eq. (1)

$$\frac{p\,(psi)}{7.96} = 0.0522\,\theta$$

and

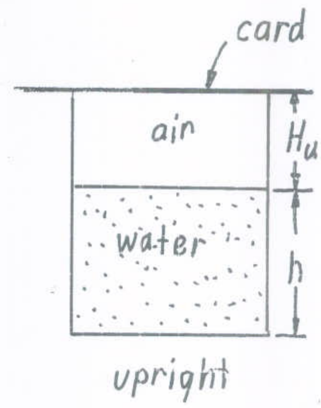
$$\underline{\underline{p\,(psi) = 0.416\,\theta}}$$

Theta, deg.	W, lb
0	0.00
20	1.04
40	2.00
60	3.23
80	4.05
100	5.24
120	6.31

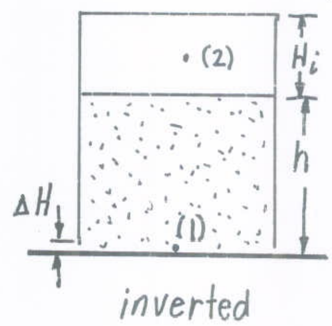


2.51

2.51 You partially fill a glass with water, place an index card on top of the glass, and then turn the glass upside down while holding the card in place. You can then remove your hand from the card and the card remains in place, holding the water in the glass. Explain how this works.



In order to hold the index card in place when the glass is inverted, the pressure at the card-water interface, p_1, must be $p_1 A = -\mathcal{W}$, where A is the area of the glass opening and $\mathcal{W}$ is the card weight. Thus, $p_1 = -\mathcal{W}/A$. Hence, $p_2 = p_1 - \gamma h$, or $p_2 = -\mathcal{W}/A - \gamma h$ (gage).



Since the amount of air in the glass remains the same when it is inverted,

$\rho_u A H_u = \rho_i A H_i$, where u and i subscripts refer to the upright and inverted conditions. Thus,

(1) $H_i = \dfrac{\rho_u}{\rho_i} H_u$ But $p = \rho R T$ so that

(2) $\dfrac{\rho_u}{\rho_i} = \dfrac{(p_u/RT_u)}{(p_i/RT_i)} = \dfrac{p_u}{p_i}$ provided the temperature remains constant: $T_i = T_u$. Note: Since we are using the perfect gas law the pressures must be absolute — $p_u = p_{atm}$, $p_i = p_2 = -\mathcal{W}/A - \gamma h + p_{atm}$

Hence, from Eqs. (1) and (2):

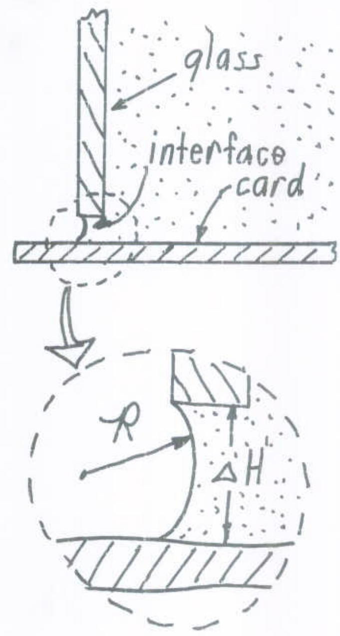
(3) $H_i = \left(\dfrac{p_{atm}}{p_{atm} - \mathcal{W}/A - \gamma h}\right) H_u$ That is, when the glass is inverted the column of air inside expans slightly, causing a small gap of size ΔH between the lip is the glass and the index card. From Eq. (3) this ΔH is

(4) $\Delta H = H_i - H_u = \left(\dfrac{p_{atm}}{p_{atm} - \mathcal{W}/A - \gamma h}\right) H_u - H_u = \left(\dfrac{\mathcal{W}/A + \gamma h}{p_{atm} - \mathcal{W}/A - \gamma h}\right) H_u$

If this gap is "large enough" the water would flow out of the glass and air into it. If it is "small enough," surface tension will allow the slight pressure difference across the air-water interface (i.e., $p_1 = -\mathcal{W}/A$) needed to prevent flow and thus keep the index card in place. Recall from Equation (1.21) in Section 1.9

(con't)

2.51 (con't)



that the pressure diffenence across an interface is proportional to the surface tension of the liquid, σ, and the radius of curvature, $\mathcal{R}$, of the interface.
That is, $p_1 \sim \sigma/\mathcal{R}$
Thus, for small enough gap, ΔH, which gives a small enough interface radius of curvature, $\mathcal{R}$, surface tension is large enough to keep the water from flowing and the index card remains in place.

Consider some typical numbers to obtain an approximation of the gap produced.

Assume $h = 3\,in. = 0.25\,ft$, $H_u = 2\,in. = 0.167\,ft$, $p_{atm} = 14.7\,psia$, and $\mathcal{W}/A \ll \gamma h$. That is, the weight of the card is much less than the weight of the water in the glass (i.e., $\mathcal{W} \ll \gamma A h$).
Hence, from Eq. (4):

$$\Delta H = \left(\frac{\gamma h}{p_{atm} - \gamma h}\right) H_u = \left[\frac{62.4\,\frac{lb}{ft^3}\,(0.25\,ft)}{(14.7\,\frac{lb}{in.^2})(144\,\frac{in.^2}{ft^2}) - 62.4\,\frac{lb}{ft^3}\,(0.25\,ft)}\right](0.167\,ft)$$

or

$$\Delta H = 0.00124\,ft = 0.0149\,in.$$

This is apparently a small enough gap to allow surface tension to keep the water in the glass, air out of it, and the pressure at the water-card interface low enough to keep the card in place.

2.52

2.52 A piston having a cross-sectional area of 0.07 m² is located in a cylinder containing water as shown in Fig. P2.52. An open U-tube manometer is connected to the cylinder as shown. For $h_1 = 60$ mm and $h = 100$ mm, what is the value of the applied force, P, acting on the piston? The weight of the piston is negligible.

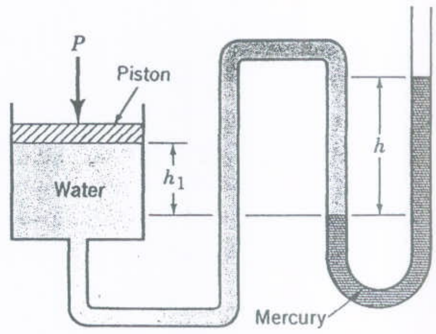


FIGURE P2.52

For equilibrium, $P = p_p A_p$ where p_p is the pressure acting on piston and A_p is the area of the piston. Also,

$$p_p + \gamma_{H_2O} h_1 - \gamma_{Hg} h = 0$$

or

$$p_p = \gamma_{Hg} h - \gamma_{H_2O} h_1$$

$$= \left(133 \frac{kN}{m^3}\right)(0.100 m) - \left(9.80 \frac{kN}{m^3}\right)(0.060 m)$$

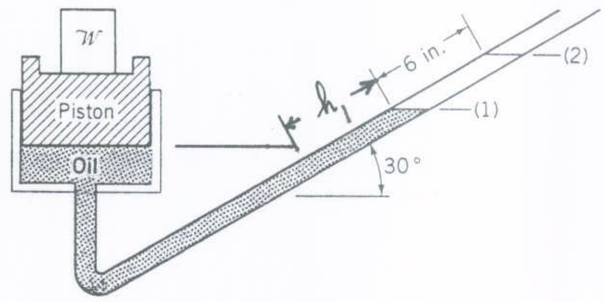
$$= 12.7 \frac{kN}{m^2}$$

Thus,

$$P = \left(12.7 \times 10^3 \frac{N}{m^2}\right)(0.07 m^2) = \underline{\underline{889 N}}$$

2.53

2.53 A 6-in.-diameter piston is located within a cylinder which is connected to a $\frac{1}{2}$-in.-diameter inclined-tube manometer as shown in Fig. P2.53. The fluid in the cylinder and the manometer is oil (specific weight = 59 lb/ft^3). When a weight $\mathcal{W}$ is placed on the top of the cylinder the fluid level in the manometer tube rises from point (1) to (2). How heavy is the weight? Assume that the change in position of the piston is negligible.



■ FIGURE P2.53

With piston alone let pressure on face of piston $= p_p$, and manometer equation becomes

$$p_p - \gamma_{oil}\, h_1 \sin 30° = 0 \qquad (1)$$

With weight added pressure p_p increases to p_p' where

$$p_p' = p_p + \frac{\mathcal{W}}{A_p} \qquad (A_p \sim \text{area of piston})$$

and manometer equation becomes

$$p_p' - \gamma_{oil}\left(h_1 + \tfrac{6}{12}\text{ ft}\right)\sin 30° = 0 \qquad (2)$$

Subtract Eq. (1) from Eq. (2) to obtain

$$p_p' - p_p - \gamma_{oil}\left(\tfrac{6}{12}\text{ ft}\right)\sin 30° = 0$$

or

$$\frac{\mathcal{W}}{A_p} = \gamma_{oil}\left(\tfrac{6}{12}\text{ ft}\right)\sin 30°$$

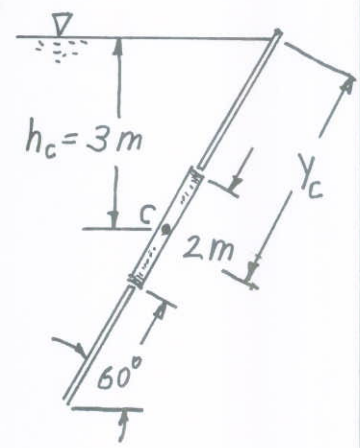
so that

$$\frac{\mathcal{W}}{\frac{\pi}{4}\left(\frac{6}{12}\text{ ft}\right)^2} = \left(59\,\frac{\text{lb}}{\text{ft}^3}\right)\left(\tfrac{6}{12}\text{ ft}\right)(0.5)$$

and

$$\mathcal{W} = \underline{\underline{2.90 \text{ lb}}}$$

2.54



2.54 A circular 2-m-diameter gate is located on the sloping side of a swimming pool. The side of the pool is oriented 60° relative to the horizontal bottom, and the center of the gate is located 3 m below the water surface. Determine the magnitude of the water force acting on the gate and the point through which it acts.

$F_R = p_c A = \gamma h_c A$, where $h_c = 3m$

Thus,

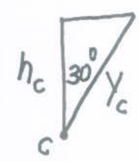
$$F_R = (9.8\,\tfrac{kN}{m^3})(3\,m)\left(\tfrac{\pi}{4}(2m)^2\right) = \underline{\underline{94.2\,kN}}$$

Also,

$$y_R - y_c = \frac{I_{xc}}{y_c A}\,,\quad \text{where for a circle } I_{xc} = \frac{\pi R^4}{4} = \frac{\pi (1m)^4}{4} = \frac{\pi}{4}\,m^4$$

and $\cos 30° = \dfrac{h_c}{y_c}$ so that

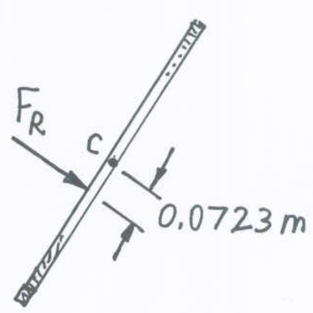
$$y_c = \frac{h_c}{\cos 30°} = \frac{3\,m}{\cos 30°} = 3.46\,m$$



Hence,

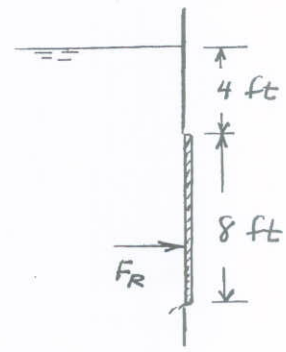
$$y_R - y_c = \frac{I_{xc}}{y_c A} = \frac{\frac{\pi}{4}\,m^4}{(3.46m)\,\frac{\pi}{4}(2m)^2} = \underline{\underline{0.0723\,m}}$$

Thus, the resultant force acts normal to the gate and 0.0723 m from the centroid, along the gate.



2.55

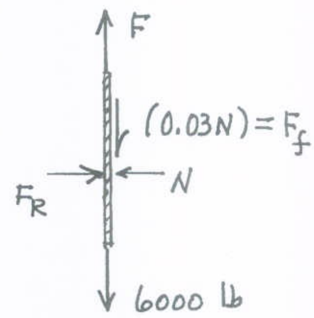
2.55 A vertical rectangular gate is 8 ft wide and 10 ft long and weighs 6000 lb. The gate slides in vertical slots in the side of a reservoir containing water. The coefficient of friction between the slots and the gate is 0.03. Determine the minimum vertical force required to lift the gate when the water level is 4 ft above the top edge of the gate.



$$F_R = \gamma h_c A$$
$$= \left(62.4\,\frac{lb}{ft^3}\right)(8\,ft)(8\,ft \times 10\,ft)$$
$$= 39{,}900\ lb$$

$$\sum F_{horizontal} = 0$$

Thus,
$$N = F_R = 39{,}900\ lb$$



$$\sum F_{vertical} = 0$$

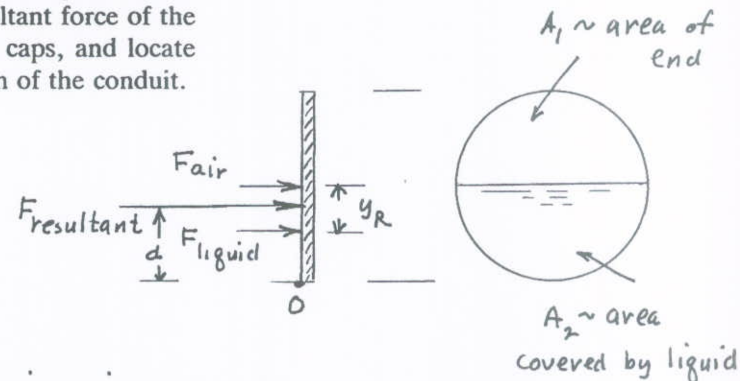
Thus,
$$F = 6000\ lb + (0.03)(39{,}900\ lb)$$
$$= \underline{\underline{7200\ lb}}$$

F_f ~ maximum frictional force

F ~ force to lift gate

2.56

2.56 A horizontal 2-m-diameter conduit is half filled with a liquid ($SG = 1.6$) and is capped at both ends with plane vertical surfaces. The air pressure in the conduit above the liquid surface is 200 kPa. Determine the resultant force of the fluid acting on one of the end caps, and locate this force relative to the bottom of the conduit.



$F_{air} = p A_1$ where p is air pressure

Thus,

$$F_{air} = \left(200 \times 10^3 \frac{N}{m^2}\right)\left(\frac{\pi}{4}\right)(2m)^2 = 200\pi \times 10^3 \text{ N}$$

$$F_{liquid} = \gamma h_c A_2 \quad \text{where} \quad h_c = \frac{4R}{3\pi} \quad \text{(see Fig. 2.18c)}$$

Thus,

$$F_{liquid} = (1.6)\left(9.81 \times 10^3 \frac{N}{m^3}\right)\left[\frac{4(1m)}{3\pi}\right]\left(\frac{1}{2}\right)\left(\frac{\pi}{4}\right)(2m)^2 = 10.5 \times 10^3 \text{ N}$$

For F_{liquid},

$$y_R = \frac{I_{xc}}{y_c A_2} + y_c \qquad \text{where } I_{xc} = 0.1098\, R^4 \quad \text{(see Fig. 2.18c)}$$

$$\text{and } y_c = h_c = \frac{4R}{3\pi}$$

Thus,

$$y_R = \frac{0.1098\,(1m)^4}{\left[\frac{4(1m)}{3\pi}\right]\left(\frac{1}{2}\right)\left(\frac{\pi}{4}\right)(2m)^2} + \frac{4(1m)}{3\pi} = 0.5891\,m$$

Since $F_{resultant} = F_{air} + F_{liquid} = (200\pi + 10.5) \times 10^3 \text{ N} = \underline{\underline{639 \text{ kN}}}$,

we can sum moments about 0 to locate resultant to obtain

$$F_{resultant}(d) = F_{air}(1m) + F_{liquid}(1m - 0.5891m)$$

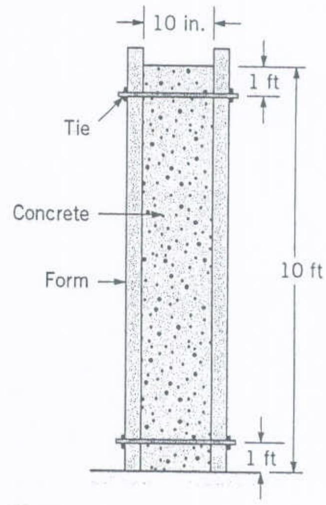
so that

$$d = \frac{(200\pi \times 10^3 N)(1m) + (10.5 \times 10^3 N)(0.4109 m)}{639 \times 10^3 N}$$

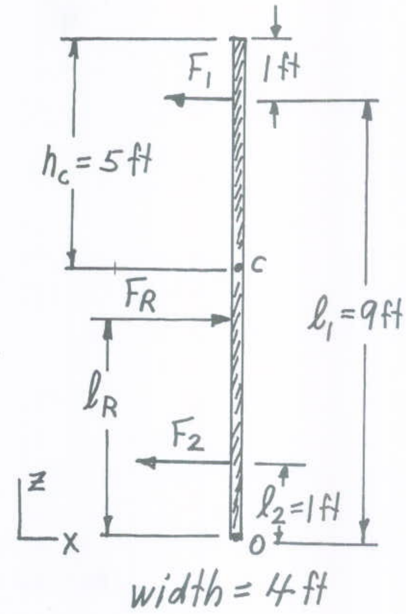
$$= \underline{\underline{0.990 \text{ m above bottom of conduit}}}$$

2.57

2.57 Forms used to make a concrete basement wall are shown in Fig. P2.57. Each 4-ft-long form is held together by four ties—two at the top and two at the bottom as indicated. Determine the tension in the upper and lower ties. Assume concrete acts as a fluid with a weight of 150 lb/ft³.



■ FIGURE P2.57



(1) $\sum F_x = 0$, or $F_1 + F_2 = F_R$

and

(2) $\sum M_0 = 0$, or $\ell_1 F_1 + \ell_2 F_2 = \ell_R F_R$, where $F_R = p_c A = \gamma h_c A$

Thus,

$$F_R = 150 \frac{lb}{ft^3}(5\,ft)(10\,ft)(4\,ft) = 30{,}000\ lb$$

Also,

$$\ell_R = 10ft - y_R = 10ft - y_c - (y_R - y_c) = 10ft - h_c - \frac{I_{xc}}{y_c A}$$

$$= 10ft - 5ft - \frac{\frac{1}{12}(4ft)(10ft)^3}{5\,ft(10ft)(4ft)}$$

$$= 5\,ft - 1.67\,ft$$

or

$$\ell_R = 3.33\ ft$$

Thus, from Eq. (2):

$$(9\,ft)F_1 + (1\,ft)F_2 = (3.33\,ft)(30{,}000\,lb) = 99{,}900\ ft\cdot lb$$

or

(3) $9F_1 + F_2 = 99{,}900$

From Eq. (1), $F_1 + F_2 = 30{,}000\ lb$, or $F_2 = 30{,}000 - F_1$

Thus, from Eq. (3),

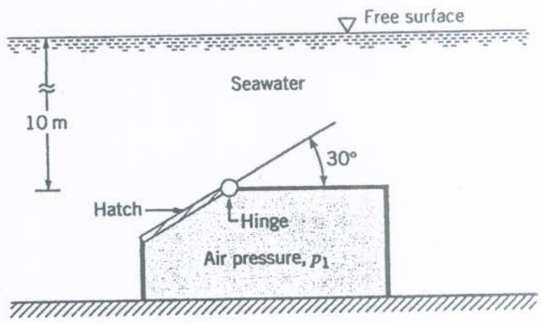
$$9F_1 + 30{,}000 - F_1 = 99{,}900$$

or

$F_1 = \underline{\underline{8{,}740\ lb}}$ so that $F_2 = 30{,}000\,lb - 8{,}740\,lb = \underline{\underline{21{,}260\ lb}}$

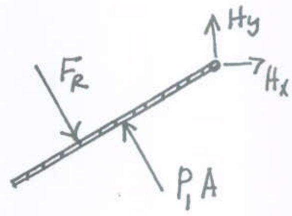
2.58

2.58 A structure is attached to the ocean floor as shown in Fig. P2.58. A 2-m-diameter hatch is located in an inclined wall and hinged on one edge. Determine the minimum air pressure, p_1, within the container to open the hatch. Neglect the weight of the hatch and friction in the hinge.



■ FIGURE P2.58

$$F_R = \gamma h_c A \quad \text{where} \quad h_c = 10\,m + \tfrac{1}{2}(2m)\sin 30°$$
$$= 10.5\,m$$



Thus,

$$F_R = \left(10.1\times10^3 \frac{N}{m^3}\right)(10.5m)\left(\frac{\pi}{4}\right)(2m)^2$$
$$= 3.33\times10^5\,N$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c \quad \text{where} \quad y_c = \frac{10\,m}{\sin 30°} + 1m = 21\,m$$

so that

$$y_R = \frac{\left(\frac{\pi}{4}\right)(1m)^4}{(21m)(\pi)(1m)^2} + 21\,m = 21.012\,m$$

For equilibrium,

$$\Sigma M_H = 0$$

so that

$$F_R\,(21.012m - 20\,m) = p_1(\pi)(1m)^2(1m)$$

and

$$p_1 = \frac{(3.33\times10^5\,N)(1.012\,m)}{\pi\,(1m)^2(1m)} = \underline{\underline{107\,kPa}}$$

2.59

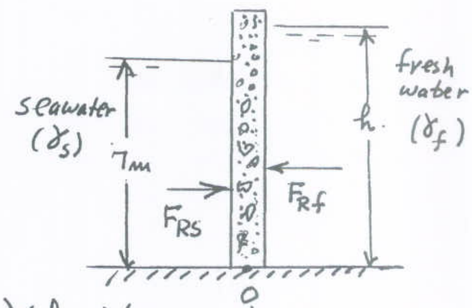
2.59 A long, vertical wall separates seawater from freshwater. If the seawater stands at a depth of 7 m, what depth of freshwater is required to give a zero resultant force on the wall? When the resultant force is zero will the moment due to the fluid forces be zero? Explain.

For a zero resultant force

$$F_{Rs} = F_{Rf}$$

or

$$\gamma_s h_{cs} A_s = \gamma_f h_{cf} A_f$$



Thus, for a unit length of wall

$$\left(10.1 \frac{kN}{m^3}\right)\left(\frac{7m}{2}\right)(7m \times 1m) = \left(9.80 \frac{kN}{m^3}\right)\left(\frac{h}{2} m\right)(h \times 1m)$$

so that

$$\underline{\underline{h = 7.11 \ m}}$$

In order for moment to be zero, F_{Rs} and F_{Rf} must be collinear.

For F_{Rs}:

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(1m)(7m)^3}{\left(\frac{7}{2}m\right)(7m \times 1m)} + \frac{7}{2}m = 4.67m$$

Similarly for F_{Rf}:

$$y_R = \frac{\frac{1}{12}(1m)(7.11m)^3}{\left(\frac{7.11}{2}m\right)(7.11m \times 1m)} + \frac{7.11}{2}m = 4.74m$$

Thus, the distance to F_{Rs} from the bottom (point O) is $7m - 4.67m = 2.33m$. For F_{Rf} this distance is $7.11m - 4.74m = 2.37m$. The forces are not collinear. $\underline{\underline{\text{No.}}}$

2.60

2.60 A pump supplies water under pressure to a large tank as shown in Fig. P2.60. The circular-plate valve fitted in the short discharge pipe on the tank pivots about its diameter A–A and is held shut against the water pressure by a latch at B. Show that the force on the latch is independent of the supply pressure, p, and the height of the tank, h.

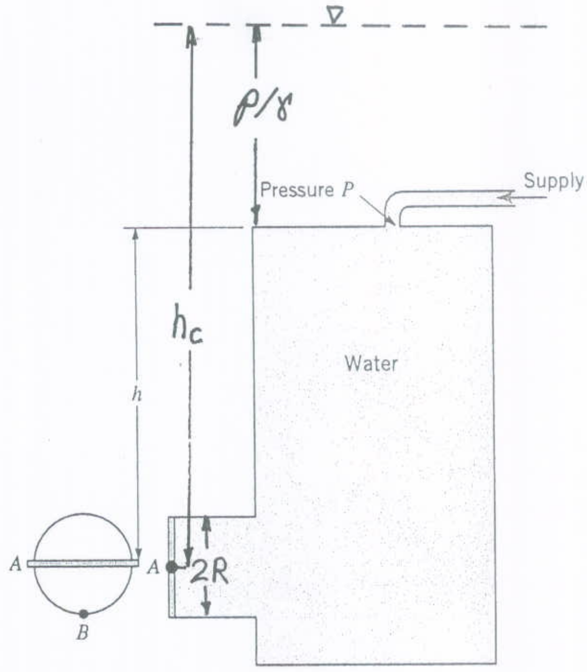
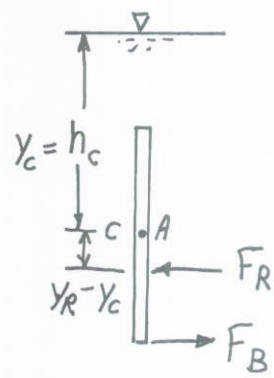


FIGURE P2.60

The pressure on the gate is the same as it would be for an open tank with a depth of

$$h_c = \frac{p + \gamma h}{\gamma}$$

as shown in the figure.



$\Sigma M_A = 0$, or

(1) $(y_R - y_c)\, F_R = R\, F_B$

where

$$F_R = p_c A = \gamma h_c (\pi R^2) = (p + \gamma h)(\pi R^2)$$

and

(2) $$y_R - y_c = \frac{I_{xc}}{y_c A} = \frac{\frac{\pi R^4}{4}}{\left(\frac{p + \gamma h}{\gamma}\right)\pi R^2} = \frac{R^2}{4\left(\frac{p}{\gamma} + h\right)}$$

Thus, from Eqs. (1) and (2)

$$F_B = \frac{(y_R - y_c)}{R} F_R = \frac{R}{4\left(\frac{p}{\gamma} + h\right)} (p + \gamma h)(\pi R^2)$$

or

$$F_B = \underline{\underline{\gamma \frac{\pi}{4} R^3}}$$, which is independent of both p and h.

2.61

2.61 A homogeneous, 4-ft-wide, 8-ft-long rectangular gate weighing 800 lb is held in place by a horizontal flexible cable as shown in Fig. P2.61 Water acts against the gate which is hinged at point A. Friction in the hinge is negligible. Determine the tension in the cable.

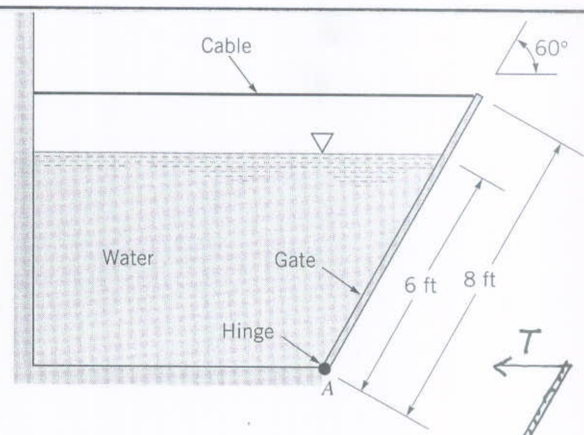
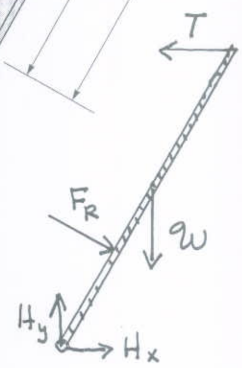


FIGURE P2.61



$F_R = \gamma h_c A$ where $h_c = \left(\frac{6 ft}{2}\right) \sin 60°$

Thus,

$$F_R = \left(62.4 \frac{lb}{ft^3}\right)\left(\frac{6 ft}{2}\right)(\sin 60°)(6 ft \times 4 ft)$$

$$= 3890 \ lb$$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where } y_c = 3 ft$$

so that

$$y_R = \frac{\frac{1}{12}(4 ft)(6 ft)^3}{(3 ft)(6 ft \times 4 ft)} + 3 ft = 4.0 \ ft$$

For equilibrium,

$$\Sigma M_H = 0$$

and

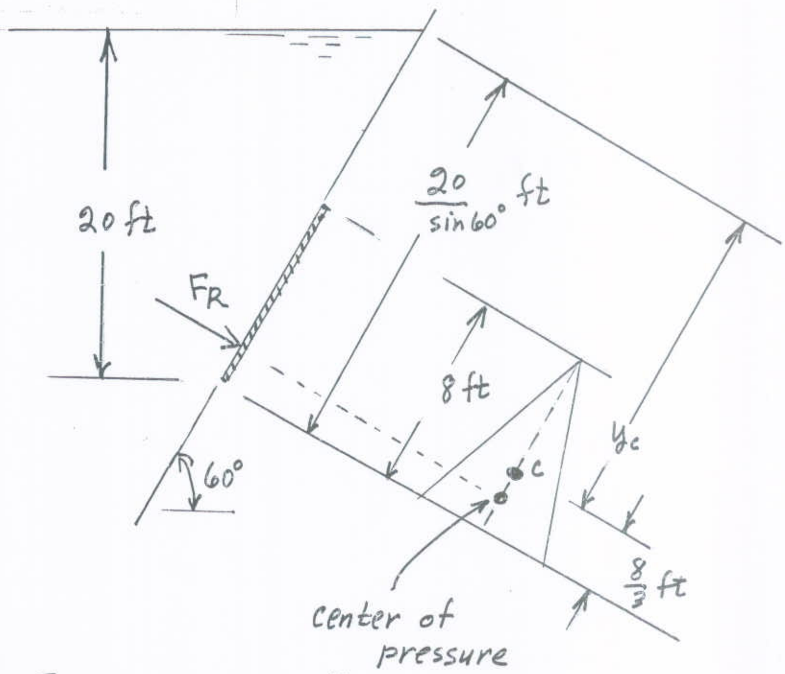
$$T (8 ft)(\sin 60°) = \mathcal{W} (4 ft)(\cos 60°) + F_R \ (2 ft)$$

$$T = \frac{(800 \ lb)(4 ft)(\cos 60°) + (3890 \ lb)(2 ft)}{(8 ft)(\sin 60°)}$$

$$= \underline{\underline{1350 \ lb}}$$

2.63

2.63 An area in the form of an isosceles triangle with a base width of 6 ft and an altitude of 8 ft lies in the plane forming one wall of a tank which contains a liquid having a specific weight of 79.8 lb/ft³. The side slopes upward making an angle of 60° with the horizontal. The base of the triangle is horizontal and the vertex is above the base. Determine the resultant force the fluid exerts on the area when the fluid depth is 20 ft above the base of the triangular area. Show, with the aid of a sketch, where the center of pressure is located.



$$y_c = \left(\frac{20}{\sin 60^\circ}\right) \text{ft} - \left(\frac{8}{3}\right) \text{ft}$$

$$= 20.43 \text{ ft}$$

$$h_c = y_c \sin 60^\circ$$

$$F_R = \gamma h_c A = \left(79.8 \frac{\text{lb}}{\text{ft}^3}\right)\left[(20.43 \text{ ft}) \sin 60^\circ\right]\left(\tfrac{1}{2}\right)(6 \text{ ft} \times 8 \text{ ft})$$

$$= \underline{\underline{33{,}900 \text{ lb}}}$$

$$y_R = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where} \quad I_{xc} = \frac{1}{36}(6 \text{ ft})(8 \text{ ft})^3$$

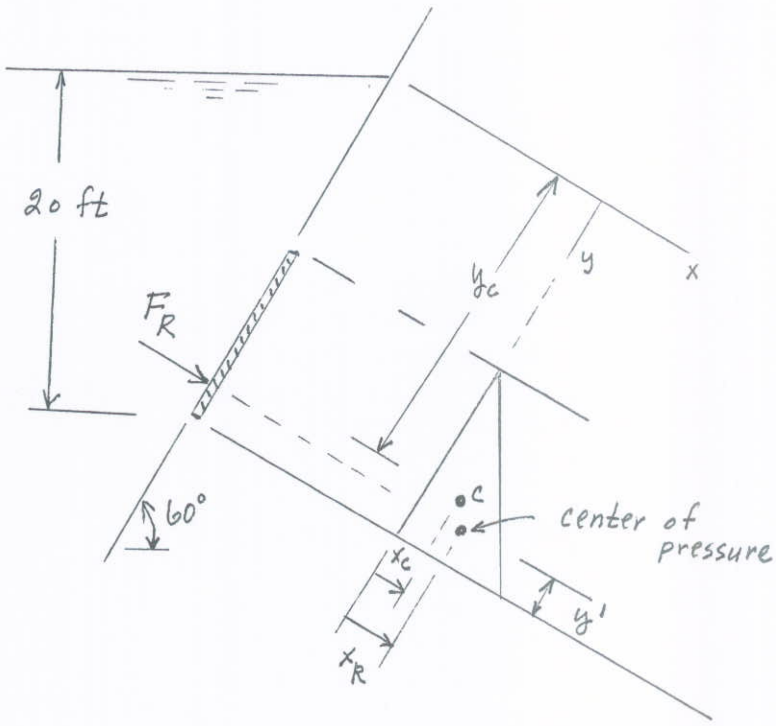
Thus,

$$y_R = \frac{\frac{1}{36}(6 \text{ ft})(8 \text{ ft})^3}{(20.43 \text{ ft})\left(\frac{1}{2}\right)(6 \text{ ft} \times 8 \text{ ft})} + 20.43 \text{ ft} = 20.6 \text{ ft}$$

The force, F_R, acts through the center of pressure which is located a distance of $\frac{20}{\sin 60^\circ}$ ft $-$ 20.6 ft $= \underline{\underline{2.49 \text{ ft}}}$ above the base of the triangle as shown in sketch.

2.64

2.64 Solve Problem **2.63** if the isosceles triangle is replaced with a right triangle having the same base width and altitude as the isosceles triangle.



$F_R = \underline{\underline{33{,}900\ lb}}$

$y' = \underline{\underline{2.49\ ft}}$

(see solution to Problem 2.63)

$$x_R = \frac{I_{xyc}}{y_c A} + x_c \qquad \text{(Eq. 2.20)}$$

where

$$I_{xyc} = \frac{(6\,ft)^2(8\,ft)^2}{72} = 32\ ft^4 \qquad \text{(see Fig. 2.18 d)}$$

and $y_c = 20.43\ ft$ (see solution to Problem 2.63)

Thus,

$$x_R = \frac{32\ ft^4}{(20.43\,ft)(\frac{1}{2})(6\,ft \times 8\,ft)} + \frac{6}{3}\,ft = \underline{\underline{2.07\ ft}}$$

The force, F_R, acts through the center of pressure with coordinates $x_R = 2.07\ ft$ and $y' = 2.49\ ft$ (see sketch).

2.65

2.65 A vertical plane area having the shape shown in Fig. P2.65 is immersed in an oil bath (specific weight = 8.75 kN/m³). Determine the magnitude of the resultant force acting on one side of the area as a result of the oil.

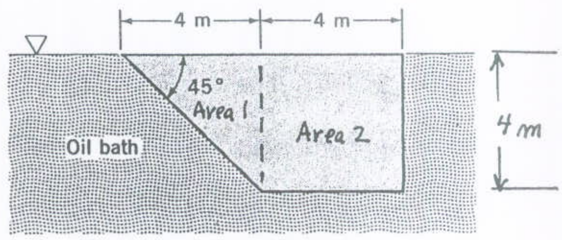


FIGURE P2.65

Break area into two parts as shown in figure.

For area 1:

$$F_{R1} = \gamma h_{c1} A_1$$

$$= \left(8.75 \frac{kN}{m^3}\right)\left(\frac{4\,m}{2}\right)(4\,m \times 4\,m) = 280\ kN$$

For area 2:

$$F_{R2} = \gamma h_{c2} A_2$$

$$= \left(8.75 \frac{kN}{m^3}\right)\left(\frac{4\,m}{3}\right)\left(\frac{1}{2}\right)(4\,m \times 4\,m) = 93.3\ kN$$

Thus,

$$F_R = F_{R1} + F_{R2} = 280\ kN + 93.3\ kN = \underline{\underline{373\ kN}}$$

2.66

2.66 A 3-m-wide, 8-m-high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water as shown in Fig. P2.66. The gate is hinged at its bottom and held closed by a horizontal force, F_H, located at the center of the gate. The maximum value for F_H is 3500 kN. **(a)** Determine the maximum water depth, h, above the center of the gate that can exist without the gate opening. **(b)** Is the answer the same if the gate is hinged at the top? Explain your answer.

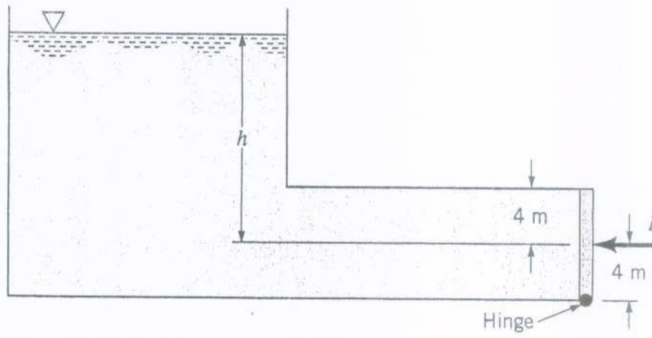


FIGURE P2.66

For gate hinged at bottom

$\sum M_H = 0$

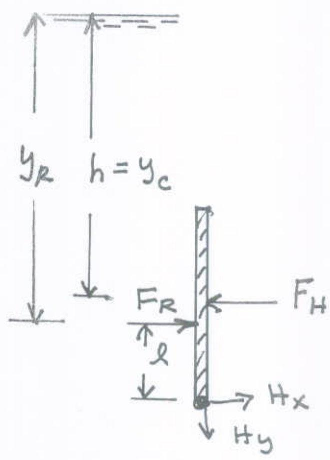
so that

$$(4m)\, F_H = \ell\, F_R \quad \text{(see figure)} \quad (1)$$

and

$$F_R = \gamma h_c A = \left(9.80 \frac{kN}{m^3}\right)(h)(3m \times 8m)$$

$$= (9.80 \times 24\, h)\ kN$$



$\ell = h + 4 - y_R$

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(3m)(8m)^3}{h\,(3m \times 8m)} + h$$

$$= \frac{5.33}{h} + h$$

Thus,

$$\ell(m) = h + 4 - \left(\frac{5.33}{h} + h\right) = 4 - \frac{5.33}{h}$$

and from Eq. (1)

$$(4\ m)(3500\ kN) = \left(4 - \frac{5.33}{h}\right)(9.80 \times 24)(h)\ kN$$

so that

$$\underline{\underline{h = 16.2\ m}}$$

(con't)

2.66 (Con't)

For gate hinged at <u>top</u>

$\sum M_H = 0$

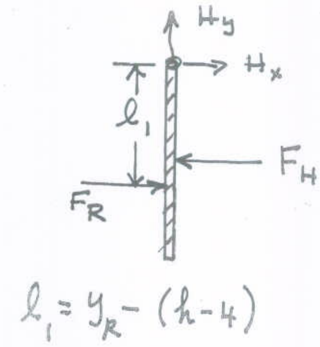
so that

$(4\,m) F_H = \ell_1 F_R$ (see figure) (1)

where

$\ell_1 = y_R - (h-4) = \left(\frac{5.33}{h} + h\right) - (h-4)$

$= \frac{5.33}{h} + 4$



Thus, from Eq. (1)

$(4m)(3500\,kN) = \left(\frac{5.33}{h} + 4\right)(9.80 \times 24)(h)\ kN$

and

$$\underline{\underline{h = 13.5\,m}}$$

Maximum depth for gate hinged at top is less than maximum depth for gate hinged at bottom.

2.67

2.67 A gate having the cross section shown in Fig. P2.67 closes an opening 5 ft wide and 4 ft high in a water reservoir. The gate weighs 500 lb and its center of gravity is 1 ft to the left of AC and 2 ft above BC. Determine the horizontal reaction that is developed on the gate at C.

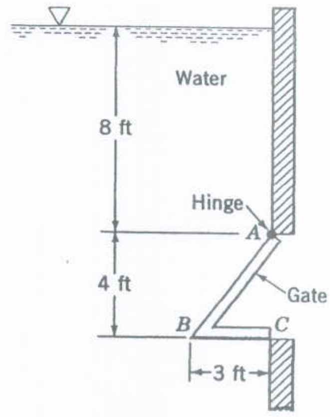


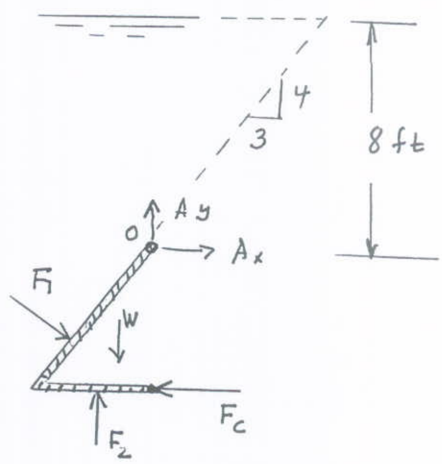
FIGURE P2.67

$F_1 = \gamma h_{c1} A_1$ where $h_{c1} = 8\,ft + 2\,ft$

Thus,

$F_1 = (62.4\,\frac{lb}{ft^3})(10\,ft)(5\,ft \times 5\,ft)$

$= 15,600\ lb$



To locate F_1,

$$y_1 = \frac{I_{xc}}{y_{c1} A_1} + y_{c1}$$

where $y_{c1} = \dfrac{8\,ft}{\frac{4}{5}} + 2.5\,ft = 12.5\,ft$

So that

$$y_1 = \frac{\frac{1}{12}(5\,ft)(5\,ft)^3}{(12.5\,ft)(5\,ft \times 5\,ft)} + 12.5\,ft = 12.67\,ft$$

Also,

$F_2 = p_2 A_2$ where $p_2 = \gamma_{H_2O}\,(8\,ft + 4\,ft)$

so that

$F_2 = \gamma_{H_2O}\,(12\,ft)(A_2) = (62.4\,\frac{lb}{ft^3})(12\,ft)(3\,ft \times 5\,ft) = 11,230\ lb$

For equilibrium,

$$\sum M_O = 0$$

and

$$F_1\left(y_1 - \frac{8\,ft}{\frac{4}{5}}\right) + W(1\,ft) - F_2\left(\tfrac{1}{2}\right)(3\,ft) - F_C\,(4\,ft)$$

so that

$$F_C = \frac{(15,600\,lb)(12.67\,ft - 10\,ft) + (500\,lb)(1\,ft) - (11,230\,lb)(\frac{3}{2}\,ft)}{4\,ft} = \underline{\underline{6330\,lb}}$$

2.68

2.68 The massless, 4-ft-wide gate shown in Fig. P2.68 pivots about the frictionless hinge O. It is held in place by the 2000 lb counterweight, W. Determine the water depth, h.

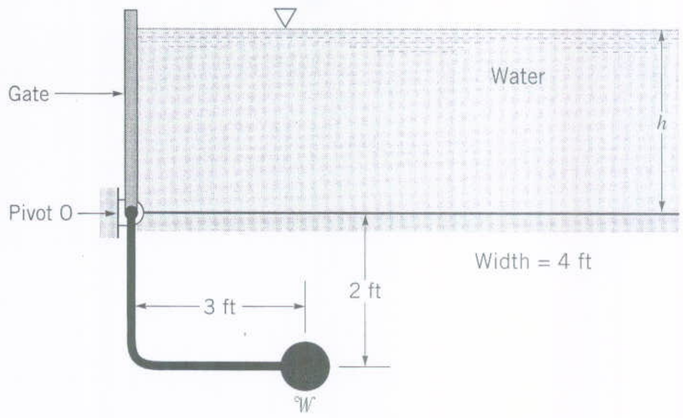


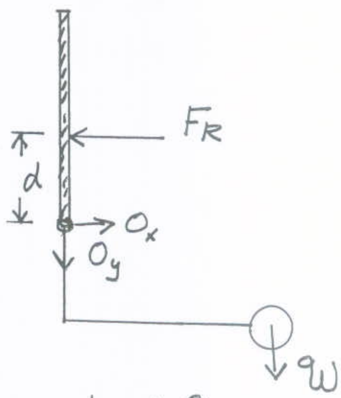
FIGURE P2.68

$$F_R = \gamma h_c A \quad \text{where} \quad h_c = \frac{h}{2}$$

Thus,

$$F_R = \gamma_{H_2O} \frac{h}{2}(h \times b)$$

$$= \gamma_{H_2O} \frac{h^2}{2}(4\,ft)$$



$b = 4\,ft$

To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(4\,ft)(h^3)}{\frac{h}{2}(4\,ft \times h)} + \frac{h}{2}$$

$$= \frac{2}{3}h$$

For equilibrium,

$$\Sigma M_O = 0$$

$$F_R\, d = W(3\,ft) \quad \text{where} \quad d = h - y_R = \frac{h}{3}$$

so that

$$\frac{h}{3} = \frac{(2000\,lb)(3\,ft)}{(\gamma_{H_2O})(\frac{h^2}{2})(4\,ft)}$$

Thus,

$$h^3 = \frac{(3)(2000\,lb)(3\,ft)}{(62.4\,\frac{lb}{ft^3})(\frac{1}{2})(4\,ft)}$$

$$h = \underline{\underline{5.24\,ft}}$$

*2.69

*2.69 A 200-lb homogeneous gate of 10-ft width and 5-ft length is hinged at point A and held in place by a 12-ft-long brace as shown in Fig. P2.69. As the bottom of the brace is moved to the right, the water level remains at the top of the gate. The line of action of the force that the brace exerts on the gate is along the brace. **(a)** Plot the magnitude of the force exerted on the gate by the brace as a function of the angle of the gate, θ, for $0 \leq \theta \leq 90°$. **(b)** Repeat the calculations for the case in which the weight of the gate is negligible. Comment on the results as $\theta \to 0$.

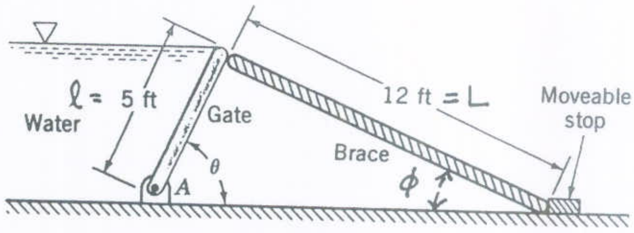
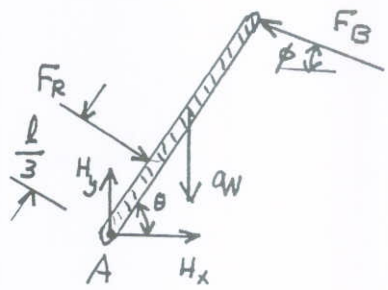


FIGURE P2.69



a) For the free-body-diagram of the gate (see figure),

$$\Sigma F_A = 0$$

so that

$$F_R\left(\frac{\ell}{3}\right) + \mathcal{W}\left(\frac{\ell}{2}\cos\theta\right) = (F_B\cos\phi)(\ell\sin\theta) + (F_B\sin\phi)(\ell\cos\theta) \quad (1)$$

Also,

$$\ell\sin\theta = L\sin\phi \quad \text{(assuming hinge and end of brace at same elevation)}$$

or

$$\sin\phi = \frac{\ell}{L}\sin\theta$$

and

$$F_R = \gamma h_c A = \gamma\left(\frac{\ell\sin\theta}{2}\right)(\ell w)$$

where w is the gate width. Thus, Eq. (1) can be written as

$$\gamma\left(\frac{\ell^3}{6}\right)(\sin\theta)\,w + \frac{\mathcal{W}\ell}{2}\cos\theta = F_B\,\ell\left(\cos\phi\sin\theta + \sin\phi\cos\theta\right)$$

so that

$$F_B = \frac{\left(\frac{\gamma\ell^2 w}{6}\right)\sin\theta + \frac{\mathcal{W}}{2}\cos\theta}{\cos\phi\sin\theta + \sin\phi\cos\theta} = \frac{\left(\frac{\gamma\ell^2 w}{6}\right)\tan\theta + \frac{\mathcal{W}}{2}}{\cos\phi\tan\theta + \sin\phi} \quad (2)$$

For $\gamma = 62.4\ lb/ft^3$, $\ell = 5\,ft$, $w = 10\,ft$, and $\mathcal{W} = 200\,lb$,

$$F_B = \frac{\frac{(62.4\frac{lb}{ft^3})(5\,ft)^2(10\,ft)}{6}\tan\theta + \frac{200\,lb}{2}}{\cos\phi\tan\theta + \sin\phi} = \frac{2600\tan\theta + 100}{\cos\phi\tan\theta + \sin\phi} \quad (3)$$

(con't)

*2.69 (Con't)

Since $\sin\phi = \frac{\ell}{L}\sin\theta$ and $\ell = 5\,ft$, $L = 12\,ft$

$$\sin\phi = \frac{5}{12}\sin\theta$$

and for a given θ, ϕ can be determined. Thus, Eq. (3) can be used to determine F_B for a given θ.

(b) For $\mathcal{W} = 0$, Eq. (3) reduces to

$$F_B = \frac{2600\tan\theta}{\cos\phi\tan\theta + \sin\phi} \qquad (4)$$

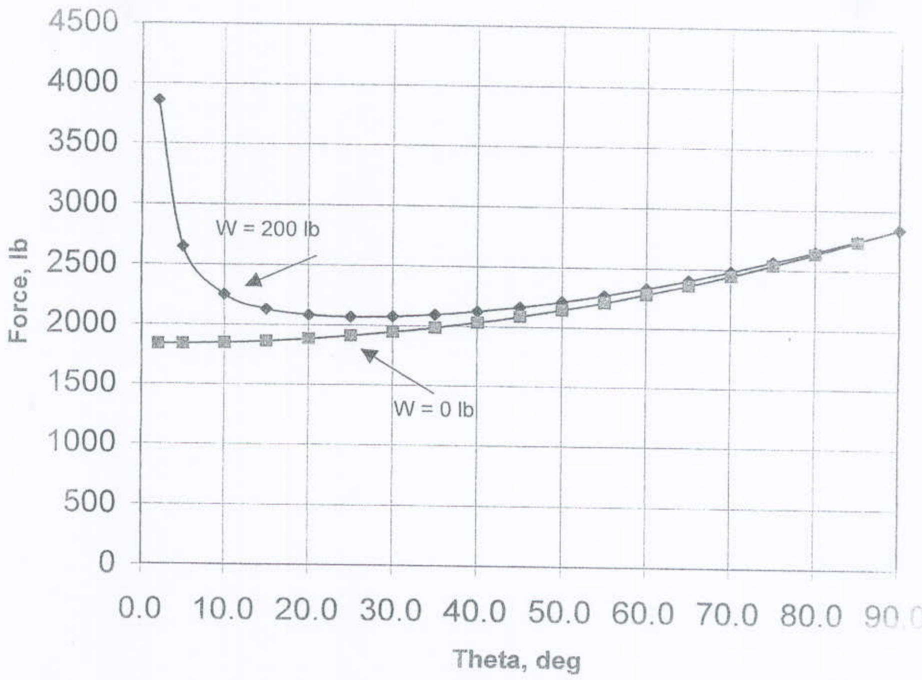
and Eq. (4) can be used to determine F_B for a given θ. Tabulated data of F_B vs. θ for both $\mathcal{W} = 200\,lb$ and $\mathcal{W} = 0\,lb$ are given below.

θ, deg	F_B, lb (W =200 lb)	F_B, lb (W = 0 lb)
90.0	2843	2843
85.0	2745	2736
80.0	2651	2633
75.0	2563	2536
70.0	2480	2445
65.0	2403	2360
60.0	2332	2282
55.0	2269	2210
50.0	2213	2144
45.0	2165	2085
40.0	2125	2032
35.0	2094	1985
30.0	2075	1945
25.0	2069	1911
20.0	2083	1884
15.0	2130	1863
10.0	2250	1847
5.0	2646	1838
2.0	3858	1836

A plot of the data is given on the following page.

(con't)

*2.69 (con't)



(b) (con't)

As $\theta \to 0$ the value of F_B can be determined from Eq.(4),

$$F_B = \frac{2600 \tan\theta}{\cos\phi \tan\theta + \sin\phi}$$

Since

$$\sin\phi = \frac{5}{12}\sin\theta$$

it follows that

$$\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\left(\frac{5}{12}\right)^2\sin^2\theta}$$

and therefore

$$F_B = \frac{2600\tan\theta}{\sqrt{1-\left(\frac{5}{12}\right)^2\sin^2\theta}\,\tan\theta + \frac{5}{12}\sin\theta} = \frac{2600}{\sqrt{1-\left(\frac{5}{12}\right)^2\sin^2\theta} + \frac{5}{12}\cos\theta}$$

Thus, as $\theta \to 0$

$$F_B \to \frac{2600}{1+\frac{5}{12}} = 1840 \text{ lb}$$

Physically this result means that for $\theta \equiv 0$, the value of F_B is indeterminate, but for any "very small" value of θ, F_B will approach 1840 lb.

2.70

2.70 An open tank has a vertical partition and on one side contains gasoline with a density $\rho = 700\ \text{kg/m}^3$ at a depth of 4 m, as shown in Fig. P2.70. A rectangular gate that is 4 m high and 2 m wide and hinged at one end is located in the partition. Water is slowly added to the empty side of the tank. At what depth, h, will the gate start to open?

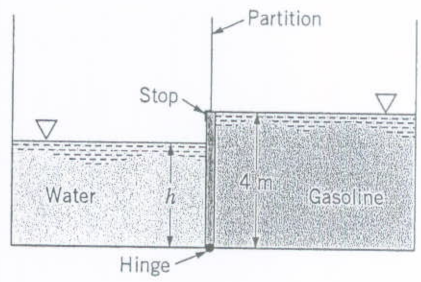
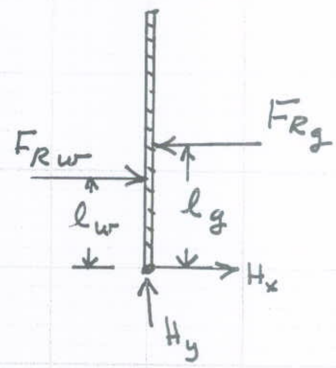


FIGURE P2.70



$F_{R_g} = \gamma_g\, h_{c_g}\, A_g$

where g refers to gasoline.

$F_{R_g} = \left(700\,\frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)(2m)(4m \times 2m)$

$= 110 \times 10^3\ N = 110\,kN$

$F_{R_w} = \gamma_w\, h_{c_w}\, A_w$

where w refers to water.

$F_{R_w} = \left(9.80 \times 10^3\,\frac{N}{m^3}\right)\left(\frac{h}{2}\right)(2m \times h)$

where h is depth of water.

$F_{R_w} = (9.80 \times 10^3)\, h^2$

For equilibrium,

$\sum M_H = 0$

so that

$F_{R_w}\,\ell_w = F_{R_g}\,\ell_g$ with $\ell_w = \frac{h}{3}$ and $\ell_g = \frac{4}{3}\ m$

Thus, $(9.80 \times 10^3)(h^2)\left(\frac{h}{3}\right) = (110 \times 10^3 N)\left(\frac{4}{3}\ m\right)$

and $h = \underline{\underline{3.55\ m}}$

which is the limiting value for h.

2.71

2.71 A 4-ft by 3-ft massless rectangular gate is used to close the end of the water tank shown in Fig. P2.71. A 200 lb weight attached to the arm of the gate at a distance ℓ from the frictionless hinge is just sufficient to keep the gate closed when the water depth is 2 ft, that is, when the water fills the semicircular lower portion of the tank. If the water were deeper the gate would open. Determine the distance ℓ.

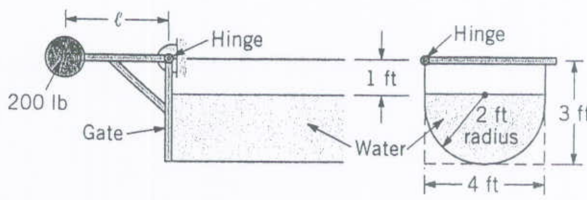


FIGURE P2.71

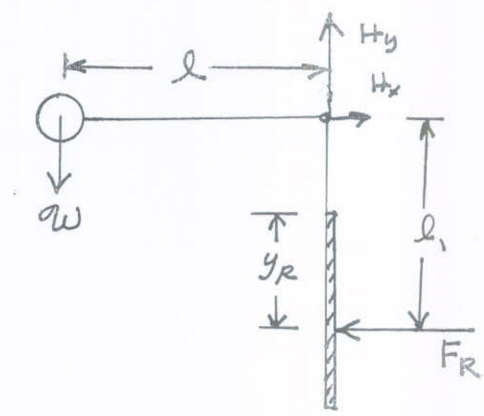
$F_R = \gamma h_c A$ where $h_c = \frac{4R}{3\pi}$ (see Fig. 2.18)

Thus,

$$F_R = \gamma_{H_2O}\left(\frac{4R}{3\pi}\right)\left(\frac{\pi R^2}{2}\right)$$

$$= \left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{4\,(2ft)}{3\pi}\right)\left(\pi\frac{(2ft)^2}{2}\right)$$

$$= 333\ lb$$



To locate F_R,

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$= \frac{0.1098\,R^4}{\left(\frac{4R}{3\pi}\right)\left(\frac{\pi R^2}{2}\right)} + \frac{4R}{3\pi} \quad \text{(see Fig. 2.18)}$$

$\ell_1 = 1\,ft + y_R$

$$= \frac{(0.1098)(2ft)^4}{\left(\frac{4\,(2ft)}{3\pi}\right)\frac{\pi(2ft)^2}{2}} + \frac{4\,(2ft)}{3\pi} = 1.178\ ft$$

For equilibrium,

$$\Sigma M_H = 0$$

so that

$$W\ell = F_R\,(1\,ft + y_R)$$

and

$$\ell = \frac{(333\,lb)(1\,ft + 1.178\,ft)}{200\,lb} = \underline{\underline{3.63\ ft}}$$

2.72

2.72 A rectangular gate that is 2 m wide is located in the vertical wall of a tank containing water as shown in Fig. P2.72. It is desired to have the gate open automatically when the depth of water above the top of the gate reaches 10 m. **(a)** At what distance, d, should the frictionless horizontal shaft be located? **(b)** What is the magnitude of the force on the gate when it opens?

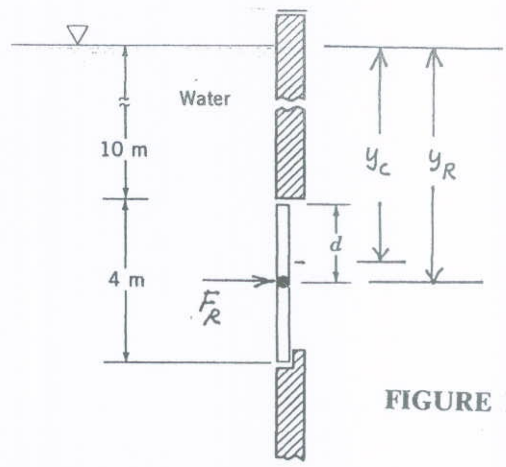


FIGURE P2.72

(a) As depth increases the center of pressure moves toward the centroid of the gate. If we locate hinge at y_R when depth $= 10\,m + d$, the gate will open automatically for any further increase in depth. Since,

$$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(2m)(4m)^3}{(12m)(2m \times 4m)} + 12m = 12.11m$$

then

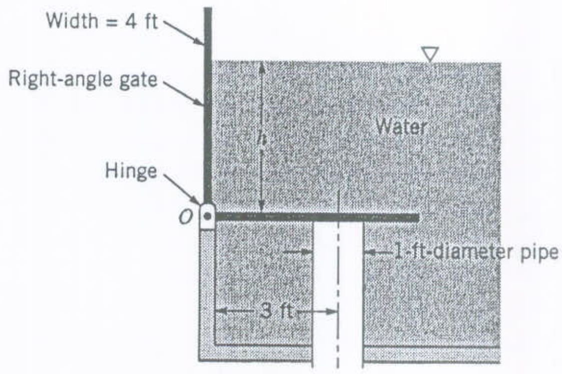
$$d = y_R - 10m = 12.11m - 10m = \underline{\underline{2.11\,m}}$$

(b) For the depth shown,

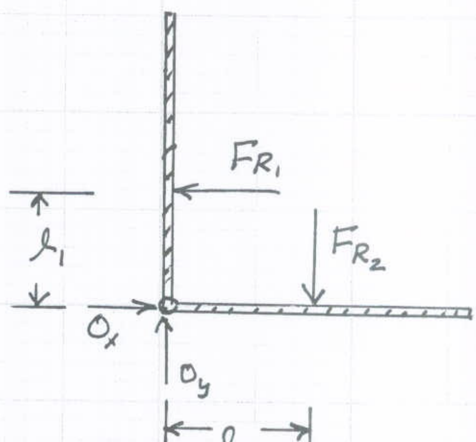
$$F_R = \gamma h_c A = \left(9.80\,\frac{kN}{m^3}\right)(12\,m)(2m \times 4m) = \underline{\underline{941\,kN}}$$

2.73

2.73 A thin 4-ft-wide, right-angle gate with negligible mass is free to pivot about a frictionless hinge at point O, as shown in Fig. P2.73. The horizontal portion of the gate covers a 1-ft-diameter drain pipe which contains air at atmospheric pressure. Determine the minimum water depth, h, at which the gate will pivot to allow water to flow into the pipe.



■ FIGURE P2.73



For equilibrium

$$\sum M_0 = 0$$

$$F_{R_1} \times \ell_1 = F_{R_2} \times \ell_2 \qquad (1)$$

$$F_{R_1} = \gamma h_{c_1} A_1$$

$$= \left(62.4\ \frac{lb}{ft^3}\right)\left(\frac{h}{2}\right)(4ft \times h)$$

$$= 125\ h^2$$

For the force on the horizontal portion of the gate (which is balanced by pressure on both sides except for the area of the pipe)

$$F_{R_2} = \gamma h \left(\frac{\pi}{4}\right)(1ft)^2 = \left(62.4\ \frac{lb}{ft^3}\right)(h)\left(\frac{\pi}{4}\right)(1ft)^2$$

$$= 49.0\ h$$

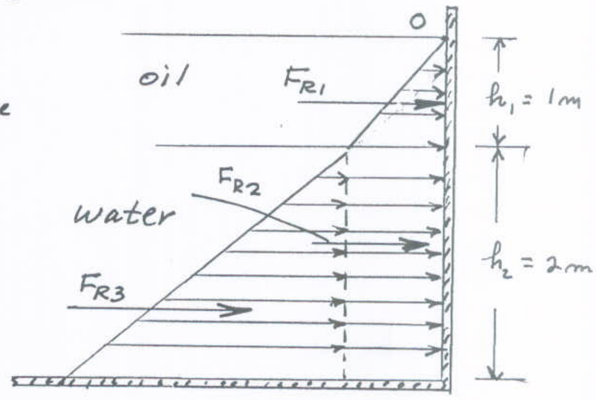
Thus, from Eq. (1) with $\ell_1 = \frac{h}{3}$ and $\ell_2 = 3\ ft$

$$(125 h^2)\left(\frac{h}{3}\right) = (49.0\ h)(3ft)$$

$$h = \underline{\underline{1.88\ ft}}$$

2.74

2.74 An open rectangular tank is 2 m wide and 4 m long. The tank contains water to a depth of 2 m and oil ($SG = 0.8$) on top of the water to a depth of 1 m. Determine the magnitude and location of the resultant fluid force acting on one end of the tank.



Use the concept of the pressure prism (see figure).

$$F_{R1} = \gamma_{oil}\, h_{c_1} A_1$$

so that

$$F_{R1} = (0.8)\left(9.81\frac{kN}{m^3}\right)\left(\frac{1m}{2}\right)(1m \times 2m)$$
$$= 7.85\ kN$$

Let $w \sim$ width $= 2m$

$F_{R2} = p_2 A_2$ where p_2 is pressure at depth h_1. Thus,

$$F_{R2} = (\gamma_{oil}\, h_1)(h_2 \times w) = (0.8)\left(9.81\frac{kN}{m^3}\right)(1m)(2m \times 2m) = 31.4\ kN$$

Also,

$$F_{R3} = \gamma_{H_2O}\, h_{c_3} A_3 \quad \text{so that}$$

$$F_{R3} = \gamma_{H_2O}\left(\frac{h_2}{2}\right)(h_2 \times w) = \left(9.80\frac{kN}{m^3}\right)\left(\frac{2m}{2}\right)(2m \times 2m) = 39.2\ kN$$

Thus,

$$F_R = F_{R1} + F_{R2} + F_{R3} = 7.85kN + 31.4kN + 39.2kN = \underline{\underline{78.5kN}}$$

To locate F_R sum moments around axis through O, so that

$$F_R\, d_R = F_{R1}\, d_1 + F_{R2}\, d_2 + F_{R3}\, d_3 \qquad (1)$$

where d_R is distance to F_R. Since F_{R1}, F_{R2}, and F_{R3} act through the centroids of their respective pressure prisms it follows that

$$d_1 = \frac{2}{3}(1m), \quad d_2 = 1m + 1m = 2m, \quad d_3 = 1m + \frac{2}{3}(2m)$$

and from Eq. (1)

$$d = \frac{(7.85kN)\left(\frac{2}{3}\right)(1m) + (31.4kN)(2m) + (39.2kN)\left(1m + \frac{4m}{3}\right)}{78.5kN}$$

$$= \underline{\underline{2.03\ m \quad (\text{below oil free surface})}}$$

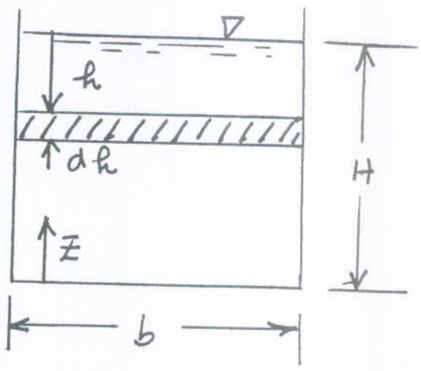
*2.75

***2.75** An open rectangular settling tank contains a liquid suspension that at a given time has a specific weight that varies approximately with depth according to the following data:

h (m)	γ (kN/m^3)
0	10.0
0.4	10.1
0.8	10.2
1.2	10.6
1.6	11.3
2.0	12.3
2.4	12.7
2.8	12.9
3.2	13.0
3.6	13.1

The depth $h = 0$ corresponds to the free surface. By means of numerical integration, determine the magnitude and location of the resultant force that the liquid suspension exerts on a vertical wall of the tank that is 6 m wide. The depth of fluid in the tank is 3.6 m.

The magnitude of the fluid force, F_R, can be found by summing the differential forces acting on the horizontal strip shown in the figure. Thus,



$$F_R = \int_0^H dF_R = b\int_0^H p\,dh \qquad (1)$$

where p is the pressure at depth h.
To find p we use Eq. 2.4

$$\frac{dp}{dz} = -\gamma$$

and with $dz = -dh$

$$p(h) = \int_0^h \gamma\, dh \qquad (2)$$

Equation (2) can be integrated numerically using the trapezoidal rule, i.e., $I = \frac{1}{2}\sum_{i=1}^{n-1}(y_i + y_{i+1})(x_{i+1} - x_i)$ where $y \sim \gamma$, $x \sim h$ and n = number of data points. The pressure distribution is given below.

h, m	γ, kN/m^3	Pressure, kPa
0	10.0	0
0.4	10.1	4.02
0.8	10.2	8.08
1.2	10.6	12.24
1.6	11.3	16.62
2.0	12.3	21.34
2.4	12.7	26.34
2.8	12.9	31.46
3.2	13.0	36.64
3.6	13.1	41.86

(con't)

*2.75 (con't)

Equation (1) can now be integrated numerically using the trapezoidal rule with $y \sim p$ and $x \sim h$. The approximate value of the integral is $71.07 \frac{kN}{m}$.

Thus, with

$$\int_0^H p\,dh = 71.07 \frac{kN}{m}$$

$$F_R = (6\,m)\left(71.07 \frac{kN}{m}\right) = \underline{\underline{426\ kN}}$$

To locate F_R sum moments about axis formed by intersection of vertical wall and fluid surface. Thus,

$$F_R h_R = b \int_0^H h\,p\,dh \qquad (3)$$

The integrand, hp, can now be determined and is tabulated below.

h, m	Pressure, kPa	h *p, kN/m
0	0	0.00
0.4	4.02	1.61
0.8	8.08	6.46
1.2	12.24	14.69
1.6	16.62	26.59
2.0	21.34	42.68
2.4	26.34	63.22
2.8	31.46	88.09
3.2	36.64	117.25
3.6	41.86	150.70

Equation (3) can now be integrated numerically using the trapezoidal rule with $y \sim hp$ and $x \sim h$. The approximate value of the integral is $174.4\ kN$.

Thus, with

$$\int_0^H h\,p\,dh = 174.4\ kN$$

it follows from Eq. (3) that

$$h_R = \frac{b \int_0^H h\,p\,dh}{F_R} = \frac{(6\,m)(174.4\ kN)}{426\ kN} = 2.46\,m$$

The resultant force acts $\underline{\underline{2.46\ m \text{ below fluid surface}}}$.

2.76

2.76 The closed vessel of Fig. P2.76 contains water with an air pressure of 10 psi at the water surface. One side of the vessel contains a spout that is closed by a 6-in.-diameter circular gate that is hinged along one side as illustrated. The horizontal axis of the hinge is located 10 ft below the water surface. Determine the minimum torque that must be applied at the hinge to hold the gate shut. Neglect the weight of the gate and friction at the hinge.

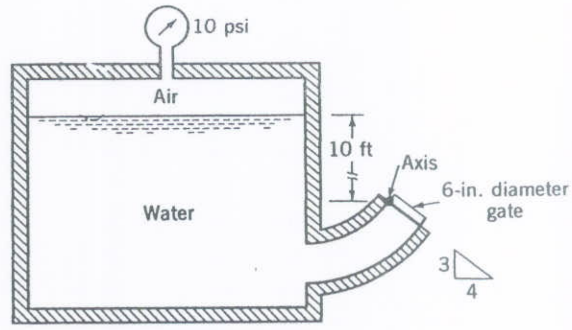
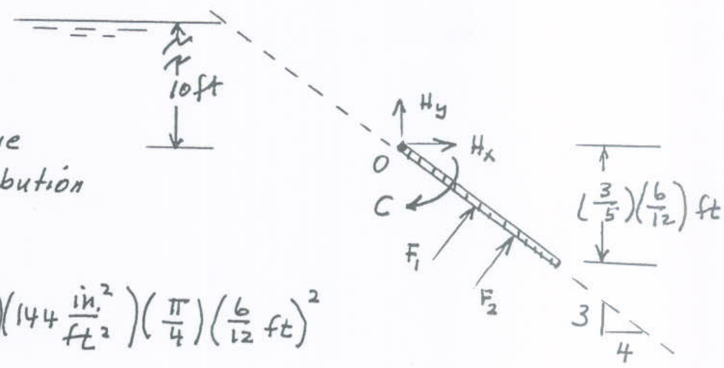


FIGURE P2.76



Let F_1 ~ force due to air pressure, and F_2 ~ force due to hydrostatic pressure distribution of water.

Thus,

$$F_1 = p_{air} A = \left(10 \frac{lb}{in.^2}\right)\left(144 \frac{in.^2}{ft^2}\right)\left(\frac{\pi}{4}\right)\left(\frac{6}{12} ft\right)^2$$

$$= 283 \text{ lb}$$

and

$$F_2 = \gamma h_c A \quad \text{where} \quad h_c = 10 \text{ ft} + \frac{1}{2}\left[\left(\frac{3}{5}\right)\left(\frac{6}{12}\right) ft\right] = 10.15 \text{ ft}$$

so that

$$F_2 = \left(62.4 \frac{lb}{ft^3}\right)(10.15 ft)\left(\frac{\pi}{4}\right)\left(\frac{6}{12} ft\right)^2 = 124 \text{ lb}$$

Also,

$$y_{R2} = \frac{I_{xc}}{y_c A} + y_c \qquad \text{where} \quad y_c = \frac{10 \text{ ft}}{\frac{3}{5}} + \frac{1}{2}\left(\frac{6}{12} ft\right) = 16.92 \text{ ft}$$

so that

$$y_{R2} = \frac{\left(\frac{\pi}{4}\right)\left(\frac{3}{12} ft\right)^4}{(16.92 ft)\left(\frac{\pi}{4}\right)\left(\frac{6}{12} ft\right)^2} + 16.92 \text{ ft} = 16.92 \text{ ft}$$

For equilibrium,

$$\sum M_o = 0$$

and

$$C = F_1\left(\frac{3}{12} ft\right) + F_2\left(y_{R2} - \frac{10 \text{ ft}}{\frac{3}{5}}\right)$$

or

$$C = (283 \text{ lb})\left(\frac{3}{12} ft\right) + (124 \text{ lb})\left(16.92 \text{ ft} - \frac{10 \text{ ft}}{\frac{3}{5}}\right) = \underline{\underline{102 \text{ ft}\cdot\text{lb}}}$$

2.77

2.77 A 4-ft-tall, 8-in.-wide concrete (150 lb/ft^3) retaining wall is built as shown in Fig. P2.77. During a heavy rain, water fills the space between the wall and the earth behind it to a depth h. Determine the maximum depth of water possible without the wall tipping over. The wall simply rests on the ground without being anchored to it.

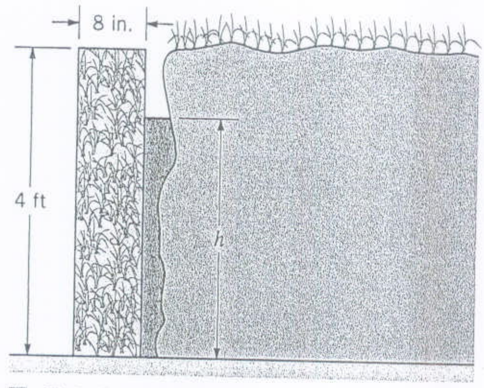
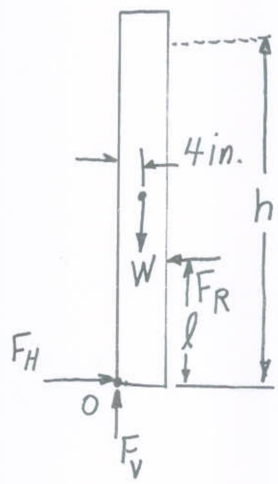


FIGURE P2.77

For equilibrium,

$\sum M_0 = 0$, or

(1) $\ell F_R = (4\text{ in.})\,W$, where with L = wall length,



$$W = \gamma_{concrete}\,\mathcal{V} = \left(150\,\frac{lb}{ft^3}\right)\left(\frac{8}{12}\,ft\right)(4\,ft)\,L = 400L\ lb$$

and

$$F_R = p_c A = \gamma h_c A = \left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{h}{2}\right)L\,h = 31.2\,L\,h^2$$

Also,

$$\ell = \frac{h}{2} - (y_R - y_c) = \frac{h}{2} - \frac{I_{xc}}{y_c A}$$

$$= \frac{h}{2} - \frac{\frac{1}{12}L\,h^3}{\left(\frac{h}{2}\right)L\,h} = \frac{h}{2} - \frac{h}{6} = \frac{h}{3}$$

Thus, Eq (1) becomes

$$\frac{h}{3}\left(31.2\,L\,h^2\right) = \frac{4}{12}\,(400L)$$

or

$$h = \underline{\underline{2.34\ ft}}$$

*2.78

***2.78** Water backs up behind a concrete dam as shown in Fig. P2.78. Leakage under the foundation gives a pressure distribution under the dam as indicated. If the water depth, h, is too great, the dam will topple over about its toe (point A). For the dimensions given, determine the maximum water depth for the following widths of the dam: ℓ = 20, 30, 40, 50, and 60 ft. Base your analysis on a unit length of the dam. The specific weight of the concrete is 150 lb/ft³.

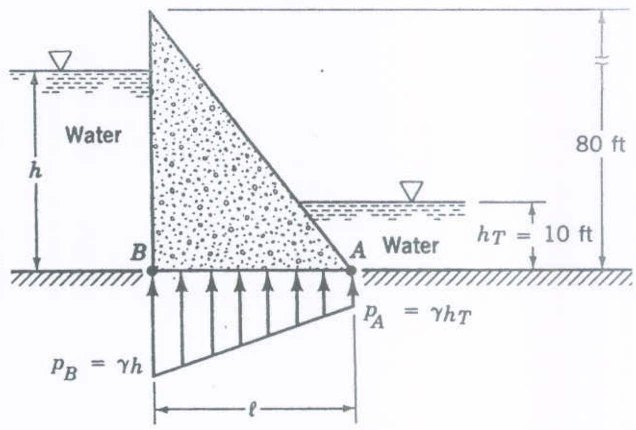
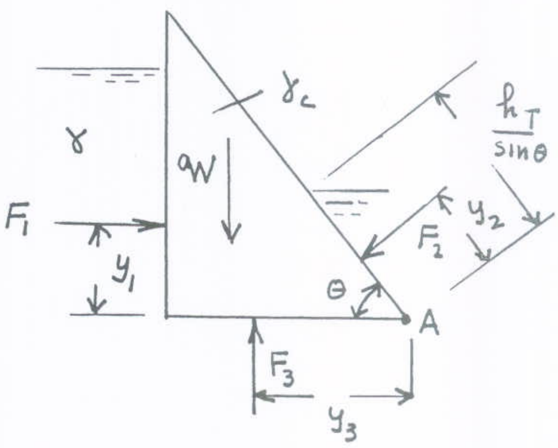


FIGURE P2.78



A free-body-diagram of the dam is shown in the figure at the right, where:

$F_1 = \dfrac{\gamma h^2}{2}$ (for unit length)

$\mathcal{W} = \gamma_c\left(\frac{1}{2}\right)(\ell)(80) = 40\gamma_c\,\ell$

$F_3 = \left(\dfrac{\gamma h + \gamma h_T}{2}\right)\ell$

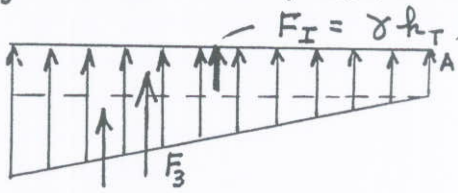
$F_2 = \gamma\left(\dfrac{h_T}{2}\right)\left(\dfrac{h_T}{\sin\theta}\right) = \dfrac{\gamma h_T^2}{2\sin\theta}$

$\tan\theta = \dfrac{80}{\ell}$

$y_1 = \dfrac{h}{3}$ $\qquad y_2 = \dfrac{1}{3}\left(\dfrac{h_T}{\sin\theta}\right)$

To determine y_3 consider the pressure distribution on the bottom:

$F_I = \gamma h_T\,\ell$



$F_{II} = \dfrac{\gamma}{2}(h - h_T)\,\ell$

Summing moments about A,

$$F_3\,y_3 = F_I\left(\frac{\ell}{2}\right) + F_{II}\left(\frac{2}{3}\ell\right)$$

(con't)

*2.78 (con't)

so that

$$y_3 = \frac{F_I(\frac{\ell}{2}) + F_{II}(\frac{2}{3}\ell)}{F_3}$$

where $F_3 = F_I + F_{II}$. Substitution of expressions for F_I and F_{II} yields,

$$y_3 = \frac{\ell\left(\frac{h_T}{3} + \frac{2}{3}h\right)}{h + h_T}$$

For equilibrium of the dam, $\Sigma M_A = 0$, so that

$$F_1\, y_1 - \mathcal{W}\left(\tfrac{2}{3}\ell\right) - F_2\, y_2 + F_3\, y_3 = 0 \qquad (1)$$

and with $\gamma = 62.4\ lb/ft^3$, $\gamma_c = 150\ lb/ft^3$, and $h_T = 10\ ft$, then:

$F_1 = 31.2\, h^2 \qquad \mathcal{W} = 6000\,\ell \qquad F_2 = \dfrac{3120}{\sin\theta} \qquad y_2 = \dfrac{10/3}{\sin\theta}$

$F_3 = 31.2\ (h+10)\,\ell \qquad y_3 = \dfrac{\ell\left(\frac{10}{3} + \frac{3}{3}h\right)}{h + h_T} = \dfrac{(2h+10)\,\ell}{3(h+10)}$

Substitution of these expressions into Eq. (1) yields,

$$(31.2\,h^2)\left(\frac{h}{3}\right) - (6000\,\ell)\left(\frac{2}{3}\ell\right) - \left(\frac{3120}{\sin\theta}\right)\left(\frac{10/3}{\sin\theta}\right) + \left[31.2\ (h+10)\,\ell\right]\left[\frac{(2h+10)\,\ell}{3(h+10)}\right] = 0$$

which can be simplified to

$$\frac{31.2}{3}h^3 + 20.8\,\ell^2 h - 3896\,\ell^2 - \frac{10,400}{\sin^2\theta} = 0 \qquad (2)$$

Thus, for a given ℓ, θ can be determined from the condition $\tan\theta = 80/\ell$, and Eq. (2) solved for h.

For the dam widths specified, the maximum water depths are given below. Note that for the two largest dam widths the water would overflow the dam before it would topple.

Dam width, ℓ, ft	Maximum depth, h, ft
20	48.2
30	61.1
40	71.8
50	81.1
60	89.1

2.79

2.79 (See Fluids in the News article titled "The Three Gorges Dam," Section 2.8.) **(a)** Determine the horizontal hydrostatic force on the 2309-m-long Three Gorges Dam when the average depth of the water against it is 175 m. **(b)** If all of the 6.4 billion people on Earth were to push horizontally against the Three Gorges Dam, could they generate enough force to hold it in place? Support your answer with appropriate calculations.

(a) $F_R = \gamma h_c A = \left(9.80 \times 10^3 \frac{N}{m^3}\right)\left(\frac{175 m}{2}\right)\left(175 m \times 2{,}309 m\right)$

$= \underline{\underline{3.46 \times 10^{11} N}}$

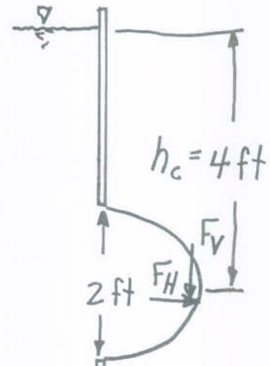
(b) Required average force per person $= \dfrac{3.46 \times 10^{11} N}{6.4 \times 10^9}$

$= \underline{\underline{54.1 \frac{N}{person}}} \left(12.2 \frac{lb}{person}\right)$

Yes. It is likely that enough force could be generated since required average force per person is relatively small.

2.81

2.81 A 2-ft-diameter hemispherical plexiglass "bubble" is to be used as a special window on the side of an above-ground swimming pool. The window is to be bolted onto the vertical wall of the pool and faces outward, covering a 2-ft-diameter opening in the wall. The center of the opening is 4 ft below the surface. Determine the horizontal and vertical components of the force of the water on the hemisphere.



$\sum F_x = 0$, or $F_H = F_R = p_c A$

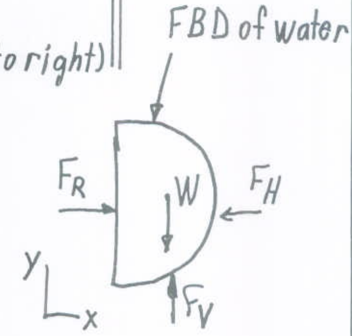
Thus,

$$F_H = \gamma h_c A = 62.4\,\frac{lb}{ft^3}\,(4\,ft)\,\frac{\pi}{4}(2\,ft)^2 = \underline{\underline{784\,lb}} \text{ (to right)}$$

and

$\sum F_y = 0$, or $F_V = W = \gamma \mathcal{V} = \gamma \frac{4}{3}\pi R^3/2$,

where $R = 1\,ft$



Thus,

$$F_V = 62.4\,\frac{lb}{ft^3}\,(4\pi\,(1\,ft)^3/6) = \underline{\underline{131\,lb}} \text{ (down on bubble)}$$

2.82

2.82 Two round, open tanks containing the same type of fluid rest on a table top as shown in Fig. P2.82. They have the same bottom area, A, but different shapes. When the depth, h, of the liquid in the two tanks is the same, the pressure force of the liquids on the bottom of the two tanks is the same. However, the force that the table exerts on the two tanks is different because the weight in each of the tanks is different. How do you account for this apparent paradox?

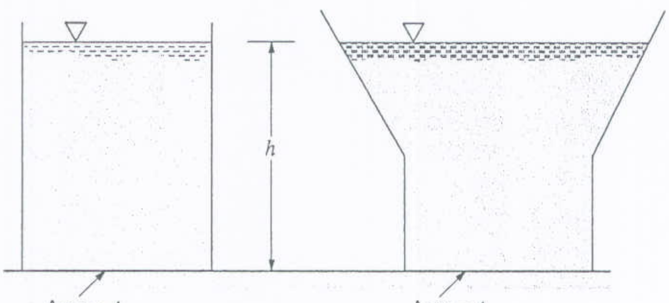
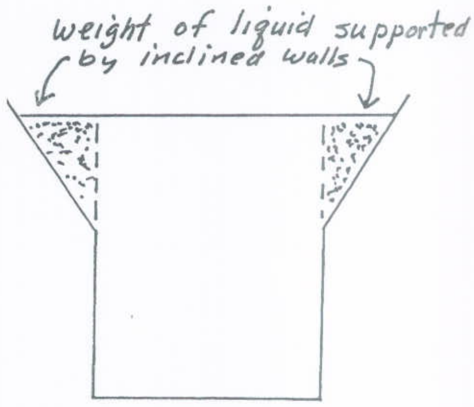


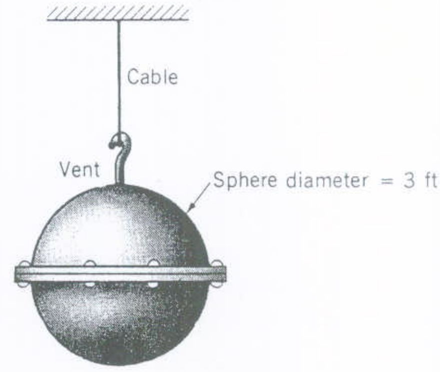
FIGURE P2.82



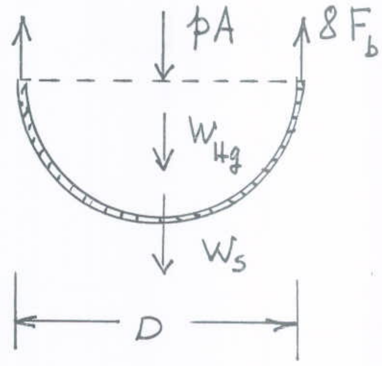
For the tank with the inclined walls, the pressure on the bottom is due to the weight of the liquid in the column directly above the bottom as shown by the dashed lines in the figure. This is the same weight as that for the tank with the straight sides. Thus, the pressure on the bottom of the two tanks is the same. The additional weight in the tank with the inclined walls is supported by the inclined walls, as illustrated in the figure.

2.83

2.83 Two hemispherical shells are bolted together as shown in Fig. P2.83. The resulting spherical container, which weighs 300 lb, is filled with mercury and supported by a cable as shown. The container is vented at the top. If eight bolts are symmetrically located around the circumference, what is the vertical force that each bolt must carry?



■ FIGURE P2.83



F_b ~ force in one bolt

p ~ pressure at mid-plane

A ~ area at mid-plane

W_{Hg} ~ weight of mercury in bottom half of shell

W_s ~ weight of bottom half of shell

For equilibrium,

$$\sum F_{vertical} = 0$$

Thus,

$$8F_b = pA + W_{Hg} + W_s$$

$$= \gamma_{Hg}\left(\frac{D}{2}\right)\left(\frac{\pi}{4}D^2\right) + \gamma_{Hg}\left(\frac{1}{2}\right)\left(\frac{\pi}{6}D^3\right) + \frac{1}{2}\,(300\text{ lb})$$

$$= \left(847\,\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{3\text{ ft}}{2}\right)\left(\frac{\pi}{4}\right)(3\text{ ft})^2 + \left(847\,\frac{\text{lb}}{\text{ft}^3}\right)\left(\frac{1}{2}\right)\left(\frac{\pi}{6}\right)(3\text{ ft})^3 + 150\text{ lb}$$

and

$$F_b = \underline{\underline{1890\text{ lb}}}$$

2.84

2.84 The 18-ft-long gate of Fig. P2.84 is a quarter circle and is hinged at H. Determine the horizontal force, P, required to hold the gate in place. Neglect friction at the hinge and the weight of the gate.

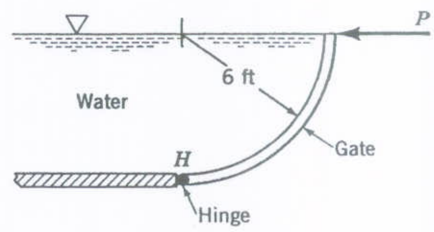
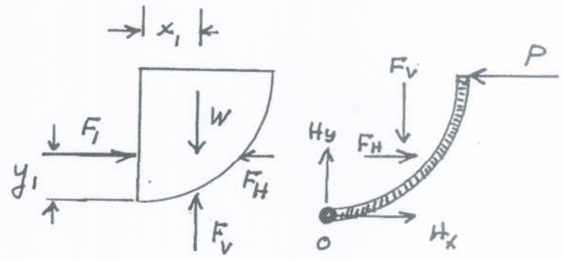


FIGURE P2.84



For equilibrium (from free-body-diagram of fluid mass),

$$\Sigma F_x = 0$$

so that

$$F_H = F_1 = \gamma h_{c1} A_1$$

$$= \left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{6\,ft}{2}\right)(6\,ft \times 18\,ft) = 20{,}200\,lb$$

Similarly,

$$\Sigma F_y = 0$$

so that

$$F_V = W = \gamma_{H_2O} \times (\text{volume of fluid}) = \left(62.4\,\frac{lb}{ft^3}\right)\left[\frac{\pi}{4}(6\,ft)^2 \times 18\,ft\right] = 31{,}800\,lb$$

Also, $$x_1 = \frac{4(6\,ft)}{3\pi} = \frac{8}{\pi}\,ft \qquad (\text{see Fig. } 2.18e)$$

and $$y_1 = \frac{6\,ft}{3} = 2\,ft$$

For equilibrium (from free-body-diagram of gate)

$$\Sigma M_O = 0$$

so that

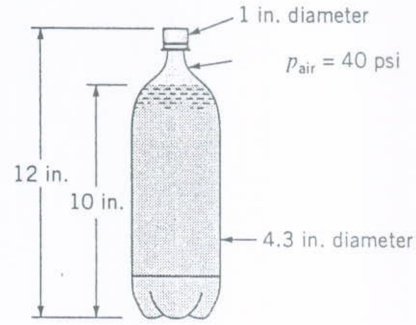
$$P(6\,ft) = F_H(y_1) + F_V(x_1)$$

or

$$P = \frac{(20{,}200\,lb)(2\,ft) + (31{,}800\,lb)\left(\frac{8}{\pi}\,ft\right)}{6\,ft} = \underline{\underline{20{,}200\,lb}}$$

2.85

2.85 The air pressure in the top of the two liter pop bottle shown in Video V2.5 and Fig. P2.85 is 40 psi, and the pop depth is 10 in. The bottom of the bottle has an irregular shape with a diameter of 4.3 in. **(a)** If the bottle cap has a diameter of 1 in. what is magnitude of the axial force required to hold the cap in place? **(b)** Determine the force needed to secure the bottom 2 inches of the bottle to its cylindrical sides. For this calculation assume the effect of the weight of the pop is negligible. **(c)** By how much does the weight of the pop increase the pressure 2 inches above the bottom? Assume the pop has the same specific weight as that of water.



■ FIGURE P2.85

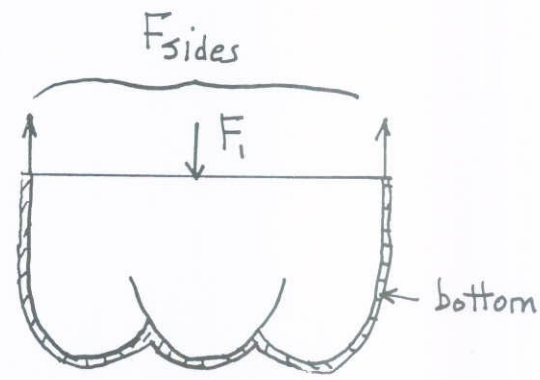
(a) $F_{cap} = p_{air} \times Area_{cap} = \left(40\ \frac{lb}{in.^2}\right)\left(\frac{\pi}{4}\right)(1\ in.)^2 = \underline{\underline{31.4\ lb}}$

(b) $\Sigma F_{vertical} = 0$

$F_{sides} = F_1 = $ (pressure @ 2 in. above bottom) $\times$ (Area)

$= \left(40\ \frac{lb}{in.^2}\right)\left(\frac{\pi}{4}\right)(4.3\ in.)^2$

$= \underline{\underline{581\ lb}}$



(c) $p = p_{air} + \gamma h$

$= 40\ \frac{lb}{in.^2} + \left(62.4\ \frac{lb}{ft^3}\right)\left(\frac{8}{12}\ ft\right)\left(\frac{1}{144\ in.^2/ft^2}\right)$

$= 40\ \frac{lb}{in.^2} + 0.289\ \frac{lb}{in.^2}$

Thus, the increase in pressure due to weight = $\underline{\underline{0.289\ psi}}$ (which is less than 1% of air pressure).

2.86

2.86 Hoover Dam (see **Video 2.4**) is the highest arch-gravity type of dam in the United States. A cross section of the dam is shown in Fig. P2.86(*a*). The walls of the canyon in which the dam is located are sloped, and just upstream of the dam the vertical plane shown in Figure P2.86(*b*) approximately represents the cross section of the water acting on the dam. Use this vertical cross section to estimate the resultant horizontal force of the water on the dam, and show where this force acts.

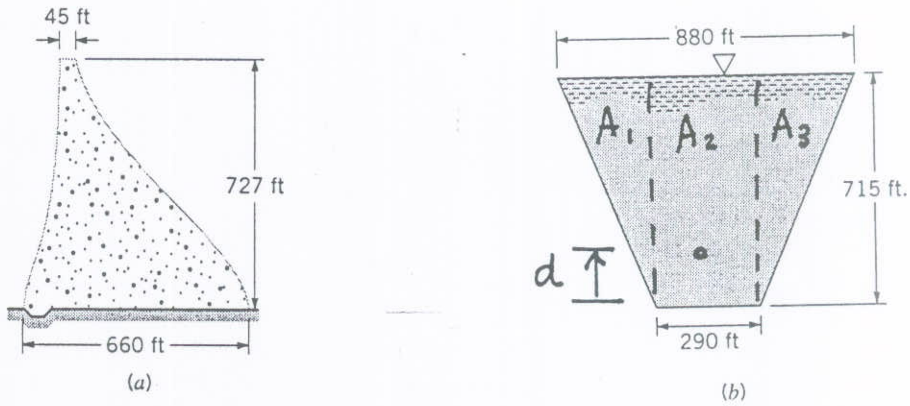


FIGURE P2.86

Break area into 3 parts as shown.

For area 1:

$$F_{R_1} = \gamma h_c A_1 = \left(62.4\ \frac{lb}{ft^3}\right)\left(\frac{1}{3}\right)(715\ ft)\left(\frac{1}{2}\right)(295\ ft)(715\ ft)$$

$$= 1.57 \times 10^9\ lb$$

For area 3: $F_{R_3} = F_{R_1} = 1.57 \times 10^9\ lb$

For area 2:

$$F_{R_2} = \gamma h_c A_2 = \left(62.4\ \frac{lb}{ft^3}\right)\left(\frac{1}{2}\right)(715\ ft)(290\ ft)(715\ ft)$$

$$= 4.63 \times 10^9\ lb$$

Thus,

$$F_R = F_{R_1} + F_{R_2} + F_{R_3} = 1.57 \times 10^9\ lb + 4.63 \times 10^9\ lb + 1.57 \times 10^9\ lb$$

$$= 7.77 \times 10^9\ lb$$

Since the moment of the resultant force about the base of the dam must be equal to the moments due to F_{R_1}, F_{R_2}, and F_{R_3}, it follows that

(con't)

2.86 (con't)

$$F_R \times d = F_{R_1}\left(\frac{2}{3}\right)(715\,ft) + F_{R_2}\left(\frac{1}{2}\right)(715\,ft) + F_{R_3}\left(\frac{2}{3}\right)(715\,ft)$$

and

$$d = \frac{(1.57\times10^9\,lb)\left(\frac{2}{3}\right)(715\,ft) + (4.63\times10^9\,lb)\left(\frac{1}{2}\right)(715\,ft) + (1.57\times10^9\,lb)\left(\frac{2}{3}\right)(715\,ft)}{7.77\times10^9\,lb}$$

$$= 406\,ft$$

Thus, the resultant horizontal force on the dam is $7.77\times10^9\,lb$ acting 406 ft up from the base of the dam along the axis of symmetry of the area.

2.87

2.87 A plug in the bottom of a pressurized tank is conical in shape as shown in Fig. P2.87. The air pressure is 40 kPa and the liquid in the tank has a specific weight of 27 kN/m³. Determine the magnitude, direction, and line of action of the force exerted on the curved surface of the cone within the tank due to the 40 kPa pressure and the liquid.

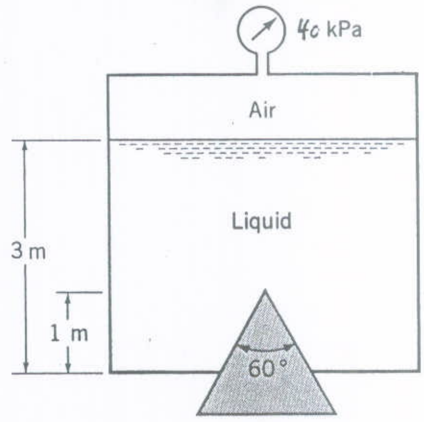


FIGURE P2.87

For equilibrium,

$$\sum F_{vertical} = 0$$

so that

$$F_c = p_{air} A + \mathcal{W}$$

where F_c is the force the cone exerts of the fluid.

Also,

$$p_{air} A = (40\ kPa)\left(\frac{\pi}{4}\right)(d^2)$$

$$= (40\ kPa)\left(\frac{\pi}{4}\right)(1.155 m)^2 = 41.9\ kN$$

$$\tan 30° = \frac{\frac{d}{2}}{1}$$

$$d = 2 \tan 30° = 1.155\ m$$

$$\text{volume of cone} = \frac{\pi}{3}\left(\frac{d}{2}\right)^2 (1)$$

and

$$\mathcal{W} = \gamma \left[\frac{\pi}{4} d^2 (3\ m) - \frac{\pi}{3}\left(\frac{d}{2}\right)^2 (1\ m)\right]$$

$$= \gamma \pi d^2 \left[\frac{3\ m}{4} - \frac{1\ m}{12}\right]$$

$$= \left(27\ \frac{kN}{m^3}\right)(\pi)(1.155\ m)^2\left(\frac{2}{3}\ m\right) = 75.4\ kN$$

Thus,

$$F_c = 41.9\ kN + 75.4\ kN = 117\ kN$$

and the force on the cone has a magnitude of 117 kN and is directed vertically downward along the cone axis.

2.88

2.88 The homogeneous gate shown in Fig. P2.88 consists of one quarter of a circular cylinder and is used to maintain a water depth of 4 m. That is, when the water depth exceeds 4 m, the gate opens slightly and lets the water flow under it. Determine the weight of the gate per meter of length.

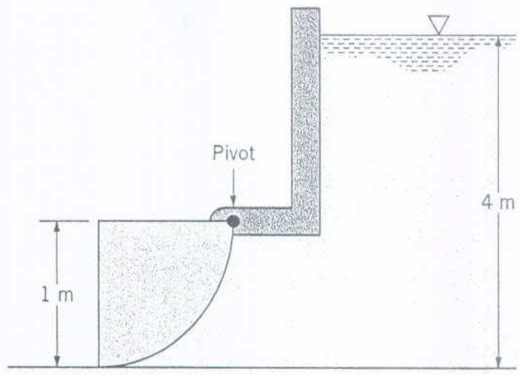
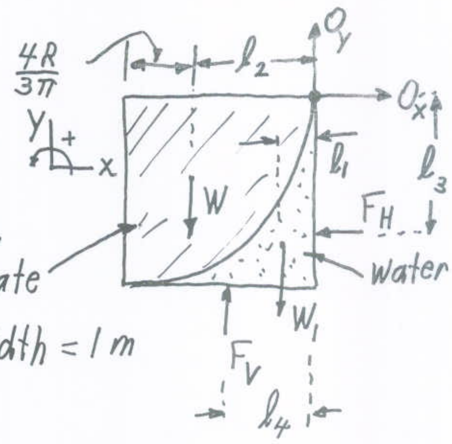


FIGURE P2.88



Consider the free body diagram of the gate and a portion of the water as shown.

width = 1 m

$\sum M_0 = 0$, or

(1) $\ell_2 W + \ell_1 W_1 - F_H \ell_3 - F_V \ell_4 = 0$, where

(2) $F_H = \gamma h_c A = 9.8 \times 10^3 \frac{N}{m^3}(3.5\,m)(1\,m)(1\,m) = 34.3\,kN$

since for the vertical side, $h_c = 4m - 0.5m = 3.5m$

Also,

(3) $F_V = \gamma h_c A = 9.8 \times 10^3 \frac{N}{m^3}(4m)(1m)(1m) = 39.2\,kN$

Also,

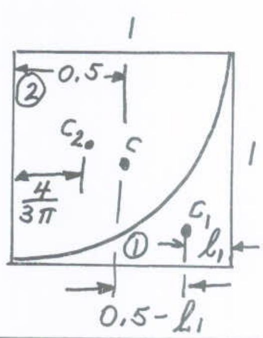
(4) $W_1 = \gamma (1m)^3 - \gamma \left(\frac{\pi}{4}(1m)^2\right)(1m) = 9.8 \times 10^3 \frac{N}{m^3}\left[1 - \frac{\pi}{4}\right] m^3 = 2.10\,kN$

(5) Now, $\ell_4 = 0.5\,m$ and

(6) $\ell_3 = 0.5m + (y_R - y_c) = 0.5m + \frac{I_{xc}}{y_c A} = 0.5m + \frac{\frac{1}{12}(1m)(1m)^3}{3.5m(1m)(1m)} = 0.524m$

(7) and $\ell_2 = 1m - \frac{4R}{3\pi} = 1 - \frac{4(1m)}{3\pi} = 0.576m$

To determine ℓ_1, consider a unit square that consits of a quarter circle and the remainder as shown in the figure. The centroids of areas ① and ② are as indicated.



Thus,

$(0.5 - \frac{4}{3\pi}) A_2 = (0.5 - \ell_1) A_1$

(con't)

2.88	(con't)

so that with $A_2 = \frac{\pi}{4}(1)^2 = \frac{\pi}{4}$ and $A_1 = 1 - \frac{\pi}{4}$ this gives

$$\left(0.5 - \frac{4}{3\pi}\right)\frac{\pi}{4} = (0.5 - \ell_1)\left(1 - \frac{\pi}{4}\right)$$

or

(8) $\ell_1 = 0.223\ m$

Hence, by combining Eqs (1) through (8):

$$(0.576\text{m})W + (0.223\text{m})(2.10\text{ kN}) - (34.3\text{ kN})(0.524\text{m}) - (39.2\text{ kN})(0.5\text{m}) = 0$$

or

$W = \underline{\underline{64.4\text{ kN}}}$

2.89

2.89 The concrete (specific weight = 150 lb/ft³) seawall of Fig. P2.89 has a curved surface and restrains seawater at a depth of 24 ft. The trace of the surface is a parabola as illustrated. Determine the moment of the fluid force (per unit length) with respect to an axis through the toe (point A).

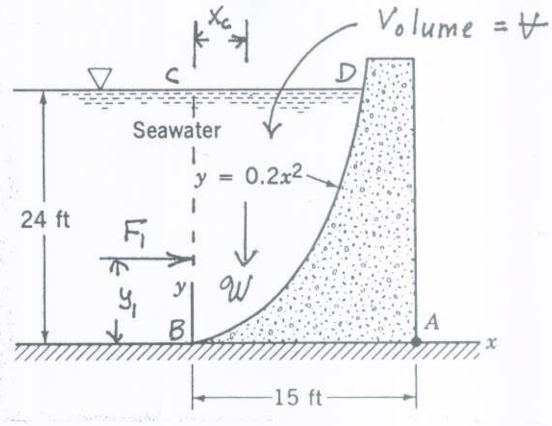


FIGURE P2.89

The components of the fluid force acting on the wall are F_1 and W as shown on the figure where

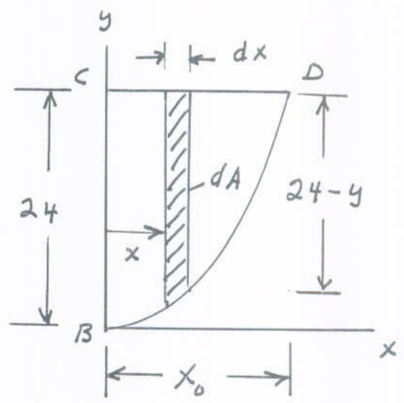
$$F_1 = \gamma h_c A = \left(64.0 \frac{lb}{ft^3}\right)\left(\frac{24\,ft}{2}\right)(24\,ft \times 1\,ft)$$

$$= 18{,}400\ lb \qquad \text{and} \qquad y_1 = \frac{24\,ft}{3} = 8\,ft$$

Also,

$$W = \gamma \forall$$

To determine $\forall$ find area BCD. Thus, (see figure to right)



$x_0 = \sqrt{120}$

(Note: All lengths in ft)

$$A = \int_0^{x_0} (24 - y)\,dx = \int_0^{x_0} (24 - 0.2x^2)\,dx$$

$$= \left[24x - \frac{0.2x^3}{3}\right]_0^{x_0}$$

and with $x_0 = \sqrt{120}$, $A = 175\ ft^2$ so that

$$\forall = A \times 1\,ft = 175\ ft^3$$

Thus,

$$W = \left(64.0 \frac{lb}{ft^3}\right)(175\ ft^3) = 11{,}200\ lb$$

To locate centroid of A:

$$x_c A = \int_0^{x_0} x\,dA = \int_0^{x_0} (24 - y)\,x\,dx = \int_0^{x_0} (24x - 0.2x^3)\,dx = 12x_0^2 - \frac{0.2x_0^4}{4}$$

and

$$x_c = \frac{12(\sqrt{120})^2 - \frac{0.2(\sqrt{120})^4}{4}}{175} = 4.11\ ft$$

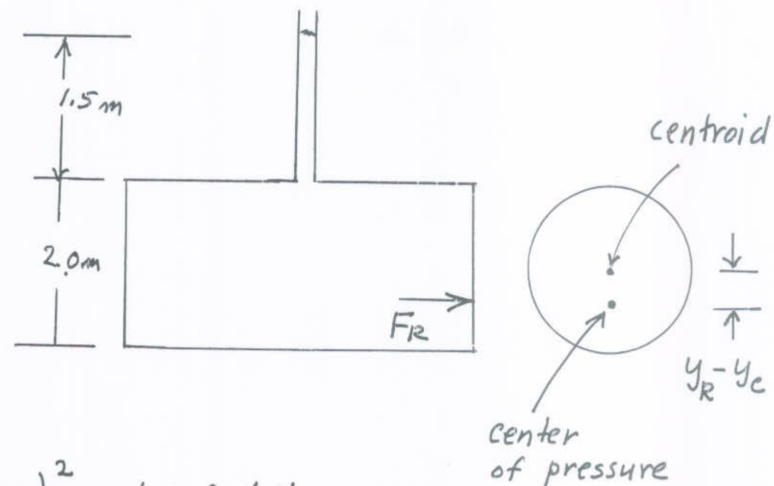
Thus,

$$M_A = F_1 y_1 - W(15 - x_c)$$

$$= (18{,}400\ lb)(8\,ft) - (11{,}200\ lb)(15\,ft - 4.11\,ft) = \underline{\underline{25{,}200\ ft \cdot lb}}$$

2.90

2.90 A cylindrical tank with its axis horizontal has a diameter of 2.0 m and a length of 4.0 m. The ends of the tank are vertical planes. A vertical, 0.1-m-diameter pipe is connected to the top of the tank. The tank and the pipe are filled with ethyl alcohol to a level of 1.5 m above the top of the tank. Determine the resultant force of the alcohol on one end of the tank and show where it acts.



$$F_R = \gamma h_c A$$

where $h_c = 1.5m + 1.0m = 2.5m$

so that

$$F_R = \left(7.74 \frac{kN}{m^3}\right)(2.5\,m)\left(\frac{\pi}{4}\right)(2.0\,m)^2 = 60.8\,kN$$

Also,

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

where $y_c = h_c$ so that

$$y_R = \frac{\frac{\pi (1m)^4}{4}}{(2.5m)\left(\frac{\pi}{4}\right)(2m)^2} + 2.5\,m = 2.60\,m$$

Thus, the resultant force has a magnitude of <u>60.8 kN</u> and acts at a distance of $y_R - y_c = 2.60m - 2.50m = \underline{0.100m}$ <u>below center of tank end wall</u>.

2.91

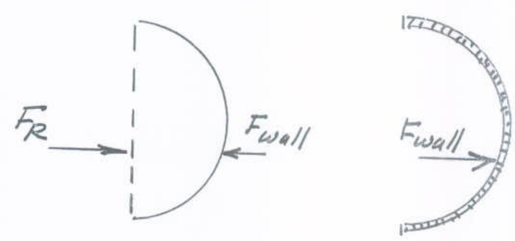
2.91 If the tank ends in Problem 2.90 are hemispherical, what is the magnitude of the resultant horizontal force of the alcohol on one of the curved ends?

For equilibrium,

$F_R = F_{wall}$ (see figure)

$= \underline{\underline{60.8\ kN}}$

(since solution for horizontal force the same as for Problem 2.90).



2.92

2.92 An open tank containing water has a bulge in its vertical side that is semicircular in shape as shown in Fig. P2.92. Determine the horizontal and vertical components of the force that the water exerts on the bulge. Base your analysis on a 1-ft length of the bulge.

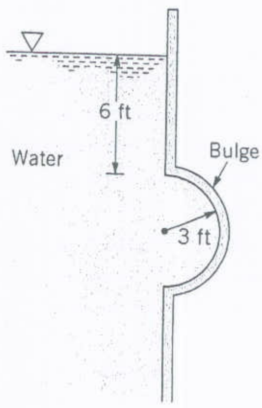
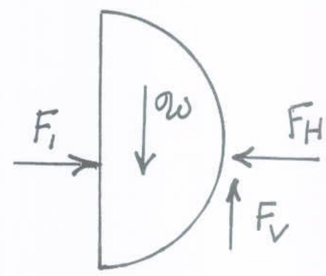


FIGURE P2.92

F_H ~ horizontal force of wall on fluid

F_V ~ vertical force of wall on fluid



$$\mathcal{W} = \gamma_{H_2O}\, \forall_{vol}$$
$$= \left(62.4 \frac{lb}{ft^3}\right)\left(\frac{\pi (3 ft)^2}{2}\right)(1\ ft)$$
$$= 882\ lb$$

$$F_1 = \gamma h_c A = \left(62.4 \frac{lb}{ft^3}\right)(6 ft + 3 ft)(6 ft \times 1 ft)$$
$$= 3370\ lb$$

For equilibrium, $F_V = \mathcal{W} = 882\ lb \uparrow$
and $F_H = F_1 = 3370\ lb \leftarrow$

The force the water exerts on the bulge is equal to, but opposite in direction to F_V and F_H above. Thus,

$$\underline{\underline{(F_H)_{wall} = 3370\ lb \longrightarrow}}$$
$$\underline{\underline{(F_V)_{wall} = 882\ lb \downarrow}}$$

2.93

2.93 A closed tank is filled with water and has a 4-ft-diameter hemispherical dome as shown in Fig. P2.93 A U-tube manometer is connected to the tank. Determine the vertical force of the water on the dome if the differential manometer reading is 7 ft and the air pressure at the upper end of the manometer is 12.6 psi.

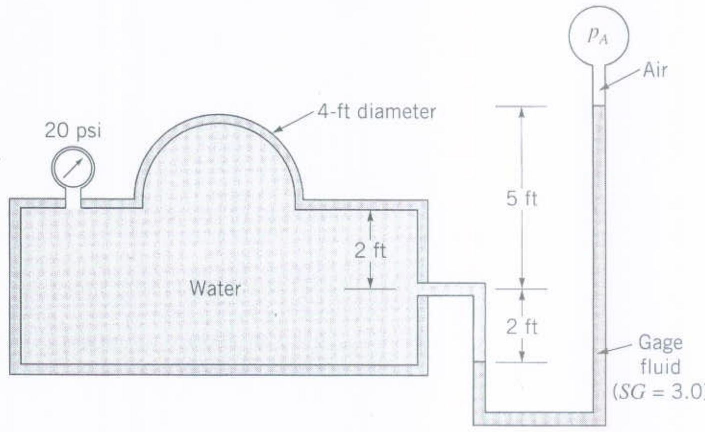
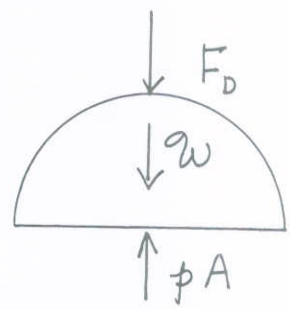


FIGURE P2.93



For equilibrium,

$$\sum F_{vertical} = 0$$

so that

$$F_D = pA - \mathcal{W} \qquad (1)$$

where F_D is the force the dome exerts on the fluid and p is the water pressure at the base of the dome. From the manometer,

$$p_A + \gamma_{gf}(7\,ft) - \gamma_{H_2O}(4\,ft) = p$$

so that

$$p = \left(12.6\,\frac{lb}{in.^2}\right)\left(144\,\frac{in.^2}{ft^2}\right) + (3.0)\left(62.4\,\frac{lb}{ft^3}\right)(7\,ft) - \left(62.4\,\frac{lb}{ft^3}\right)(4\,ft)$$

$$= 2880\,\frac{lb}{ft^2}$$

Thus, from Eq. (1) with volume of sphere $= \frac{\pi}{6}(\text{diameter})^3$

$$F_D = \left(2880\,\frac{lb}{ft^2}\right)\left(\frac{\pi}{4}\right)(4\,ft)^2 - \frac{1}{2}\left[\frac{\pi}{6}(4\,ft)^3\right]\left(62.4\,\frac{lb}{ft^3}\right)$$

$$= 35{,}100\,lb$$

The force that the vertical force that the water exerts on the dome is 35,100 lb ↑.

2.94

2.94 A 3-m-diameter open cylindrical tank contains water and has a hemispherical bottom as shown in Fig. P2.94 Determine the magnitude, line of action, and direction of the force of the water on the curved bottom.

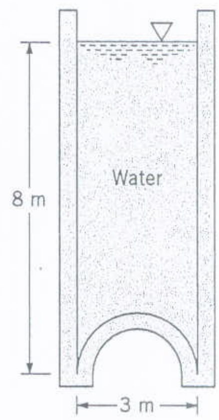


FIGURE P2.94

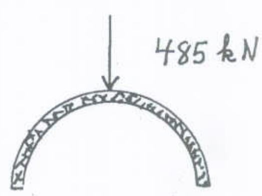
$$\text{Force} = \text{weight of water supported by hemispherical bottom}$$

$$= \gamma_{H_2O}\left[(\text{volume of cylinder}) - (\text{volume of hemisphere})\right]$$

$$= 9.80\,\frac{kN}{m^3}\left[\frac{\pi}{4}(3m)^2(8m) - \frac{\pi}{12}(3m)^3\right]$$

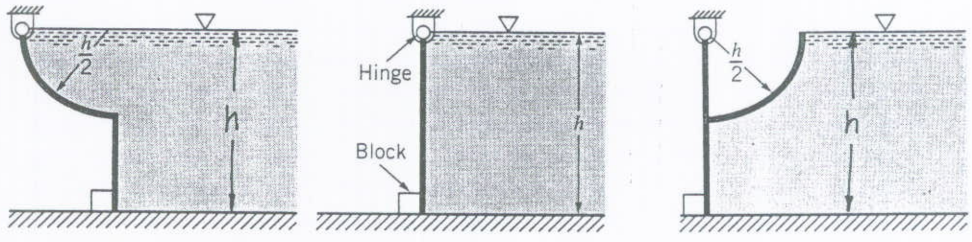
$$= \underline{\underline{485\ kN}}$$

The force is directed vertically downward, and due to symmetry it acts on the hemisphere along the vertical axis of the cylinder.



2.95

2.95 Three gates of negligible weight are used to hold back water in a channel of width b as shown in Fig. P2.95. The force of the gate against the block for gate (**b**) is R. Determine (in terms of R) the force against the blocks for the other two gates.



■ FIGURE P2.95

For case (b)

$$F_R = \gamma h_c A = \gamma\left(\frac{h}{2}\right)(h \times b) = \frac{\gamma h^2 b}{2}$$

and $y_R = \frac{2}{3}h$

Thus,

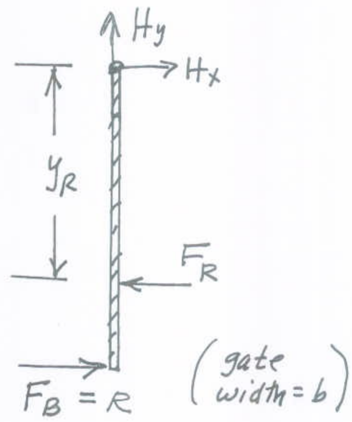
$$\Sigma M_H = 0$$

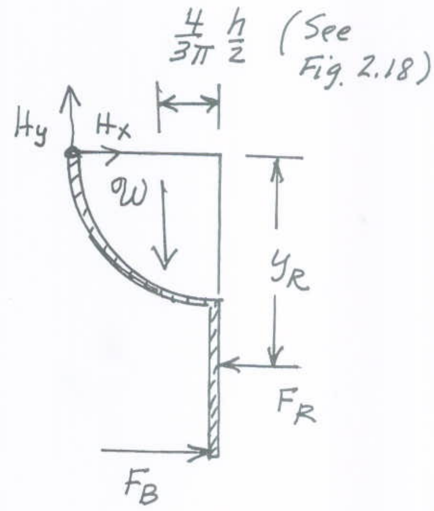
so that

$$hR = \left(\frac{2}{3}h\right)F_R$$

$$hR = \left(\frac{2}{3}h\right)\left(\frac{\gamma h^2 b}{2}\right)$$

$$R = \frac{\gamma h^2 b}{3} \qquad (1)$$





For case (a) on free-body-diagram shown

$F_R = \frac{\gamma h^2 b}{2}$ (from above) and

$y_R = \frac{2}{3}h$

and

$$\mathcal{W} = \gamma \times \text{Vol}$$

$$= \gamma\left[\frac{\pi\left(\frac{h}{2}\right)^2}{4}(b)\right]$$

$$= \frac{\pi\gamma h^2 b}{16}$$

Thus, $\Sigma M_H = 0$

so that

$$\mathcal{W}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + F_R\left(\frac{2}{3}h\right) = F_B\, h$$

and

$$\frac{\pi\gamma h^2 b}{16}\left(\frac{h}{2} - \frac{4h}{6\pi}\right) + \frac{\gamma h^2 b}{2}\left(\frac{2}{3}h\right) = F_B\, h$$

(con't)

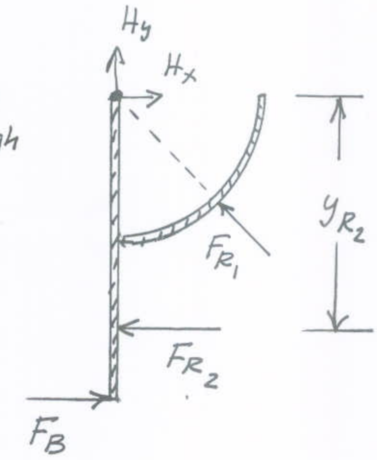
2.95 (Con't)

It follows that

$$F_B = \gamma h^2 b\,(0.390)$$

From Eq. (1) $\gamma h^2 b = 3R$, thus

$$F_B = \underline{\underline{1.17R}}$$



For case (c), for the free-body-diagram shown, the force F_{R_1} on the curved section passes through the hinge and therefore does not contribute to the moment around H. On bottom part of gate

$$F_{R_2} = \gamma h_c A = \gamma\left(\frac{3h}{4}\right)\left(\frac{h}{2}\times b\right) = \frac{3}{8}\gamma h^2 b$$

and

$$y_{R_2} = \frac{I_{xc}}{y_c A} + y_c = \frac{\frac{1}{12}(b)\left(\frac{h}{2}\right)^3}{\left(\frac{3h}{4}\right)\left(\frac{h}{2}\times b\right)} + \frac{3h}{4}$$

$$= \frac{28}{36}h$$

Thus, $\Sigma M_H = 0$

so that

$$F_{R_2}\left(\frac{28}{36}h\right) = F_B\,h$$

or

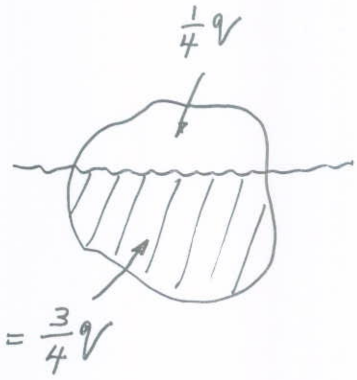
$$F_B = \left(\frac{3}{8}\gamma h^2 b\right)\left(\frac{28}{36}\right) = \frac{7}{24}\gamma h^2 b$$

From Eq. (1) $\gamma h^2 b = 3R$, thus

$$F_B = \frac{7}{8}R = \underline{\underline{0.875R}}$$

2.97

2.97 A freshly cut log floats with one fourth of its volume protruding above the water surface. Determine the specific weight of the log.



$\mathcal{V}$ = log volume

$F_B = W$ or

$\gamma_{H_2O}\mathcal{V}_{H_2O} = \gamma_{log}\mathcal{V}$

Thus,

$$\gamma_{log} = \gamma_{H_2O}\frac{\mathcal{V}_{H_2O}}{\mathcal{V}} = \gamma_{H_2O}\frac{\frac{3}{4}\mathcal{V}}{\mathcal{V}}$$

or

$$\gamma_{log} = \frac{3}{4}\gamma_{H_2O} = \frac{3}{4}\left(62.4\,\frac{lb}{ft^3}\right) = \underline{\underline{46.8\,\frac{lb}{ft^3}}}$$

2.98

2.98 A river barge, whose cross section is approximately rectangular, carries a load of grain. The barge is 28 ft wide and 90 ft long. When unloaded its draft (depth of submergence) is 5 ft, and with the load of grain the draft is 7 ft. Determine: **(a)** the unloaded weight of the barge, and **(b)** the weight of the grain.

(a) For equilibrium,

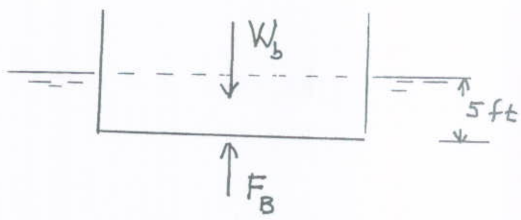
$$\sum F_{vertical} = 0$$

so that

$$W_b = F_B = \gamma_{H_2O} \times (\text{submerged volume})$$

$$= \left(62.4 \frac{lb}{ft^3}\right)(5\,ft \times 28\,ft \times 90\,ft)$$

$$= \underline{\underline{786{,}000\ lb}}$$



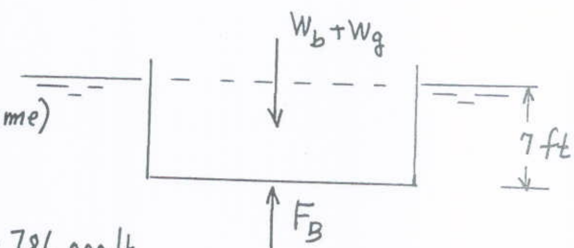
$W_b \sim$ weight of barge (unloaded)

(b) $\sum F_{vertical} = 0$

$$W_b + W_g = F_B = \gamma_{H_2O} \times (\text{submerged volume})$$

$$W_g = \left(62.4 \frac{lb}{ft^3}\right)(7\,ft \times 28\,ft \times 90\,ft) - 786{,}000\ lb$$

$$= \underline{\underline{315{,}000\ lb}}$$



$W_g \sim$ weight of grain

2.99

2.99 A tank of cross-sectional area A is filled with a liquid of specific weight γ_1 as shown in Fig. P2.99*a*. Show that when a cylinder of specific weight γ_2 and volume $\forall$ is floated in the liquid (see Fig. P2.99*b*), the liquid level rises by an amount $\Delta h = (\gamma_2 / \gamma_1)\, \forall / A$.

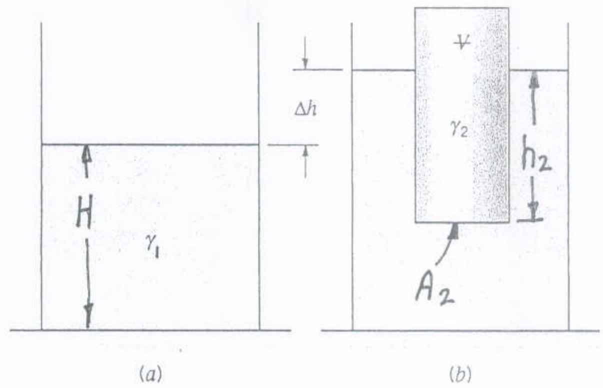


FIGURE P2.99

$\mathcal{W}$ = weight of cylinder $= \gamma_2 \forall$

For equilibrium,

$\mathcal{W}$ = weight of liquid displaced $= \gamma_1 h_2 A_2 = \gamma_1 \forall_2$ where $\forall_2 = h_2 A_2$

Thus,

$\gamma_2 \forall = \gamma_1 \forall_2$, or

$$\forall_2 = \frac{\gamma_2}{\gamma_1} \forall$$

However, the final volume within the tank is equal to the initial volume plus the volume, $\forall_2$, of the cylinder that is submerged. That is,

$$(H + \Delta h) A = HA + \forall_2$$

or

$$\Delta h = \frac{\forall_2}{A} = \underline{\underline{\frac{\gamma_2}{\gamma_1} \frac{\forall}{A}}}$$

2.100

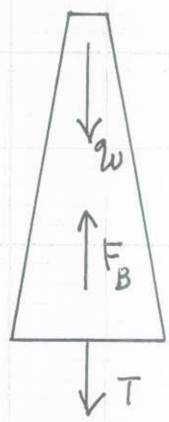
2.100 When the Tucurui dam was constructed in northern Brazil, the lake that was created covered a large forest of valuable hardwood trees. It was found that even after 15 years underwater the trees were perfectly preserved and underwater logging was started. During the logging process a tree is selected, trimmed, and anchored with ropes to prevent it from shooting to the surface like a missile when cut. Assume that a typical large tree can be approximated as a truncated cone with a base diameter of 8 ft, a top diameter of 2 ft, and a height of 100 ft. Determine the resultant vertical force that the ropes must resist when the completely submerged tree is cut. The specific gravity of the wood is approximately 0.6.

For equilibrium,

$$\sum F_{vertical} = 0$$

so that

$$T = F_B - \mathcal{W} \qquad (1)$$



$\mathcal{W} \sim$ weight
$F_B \sim$ buoyant force
$T \sim$ tension in ropes

For a truncated cone,

$$\text{Volume} = \frac{\pi h}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$$

where: r_1 = base radius
r_2 = top radius
h = height

Thus, $$\forall_{tree} = \frac{(\pi)(100\,ft)}{3}\left[(4\,ft)^2 + (4\,ft \times 1\,ft) + (1\,ft)^2\right]$$
$$= 2200\ ft^3$$

For buoyant force,

$$F_B = \gamma_{H_2O} \times \forall_{tree} = \left(62.4\ \frac{lb}{ft^3}\right)\left(2200\,ft^3\right) = 137{,}000\ lb$$

For weight,

$$\mathcal{W} = \gamma_{tree} \times \forall_{tree} = (0.6)\left(62.4\ \frac{lb}{ft^3}\right)\left(2200\,ft^3\right) = 82{,}400\,lb$$

From Eq. (1)

$$T = 137{,}000\ lb - 82{,}400\ lb = \underline{\underline{54{,}600\ lb}}$$

2.102

2.102 An inverted test tube partially filled with air floats in a plastic water-filled soft drink bottle as shown in Video V2.7 and Fig. P2.102. The amount of air in the tube has been adjusted so that it just floats. The bottle cap is securely fastened. A slight squeezing of the plastic bottle will cause the test tube to sink to the bottom of the bottle. Explain this phenomenon.

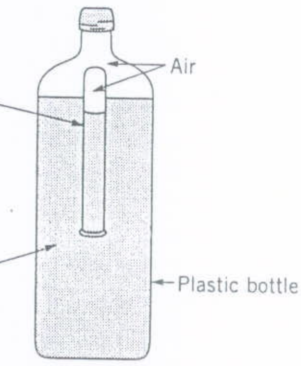
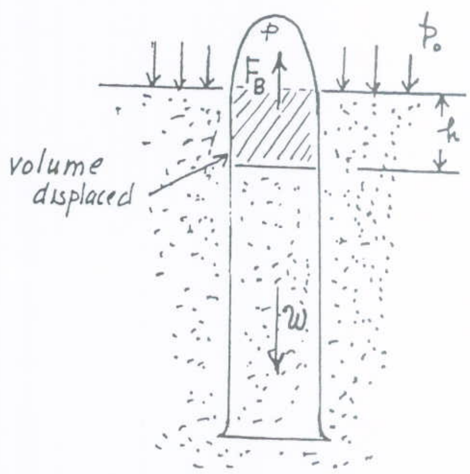


FIGURE P2.102

When the test tube is floating the weight of the tube, $\mathcal{W}$, is balanced by the buoyant force, F_B, as shown in the figure. The buoyant force is due to the displaced volume of water as shown. This displaced volume is due to the air pressure, p, trapped in the tube where $p = p_0 + \gamma_{H_2O} h$. When the bottle is squeezed, the air pressure in the bottle, p_0, is increased slightly and this in turn increases p, the pressure compressing the air in the test tube. Thus, the displaced volume is decreased with a subsequent decrease in F_B. Since $\mathcal{W}$ is constant, a decrease in F_B will cause the test tube to sink.



2.103

2.103 An irregularly shaped piece of a solid material weighs 8.05 lb in air and 5.26 lb when completely submerged in water. Determine the density of the material.

$$W\,(\text{in air}) = \rho g \times (\text{volume}) \qquad \text{where } \rho \sim \text{density of material}$$

$$W\,(\text{in water}) = \rho g \times (\text{volume}) - \text{buoyant force}$$

$$= \rho g \times (\text{volume}) - \rho_{H_2O}\, g \times (\text{volume})$$

Thus,

$$\frac{W\,(\text{in air})}{W\,(\text{in water})} = \frac{\rho}{\rho - \rho_{H_2O}} = \frac{1}{1 - \frac{\rho_{H_2O}}{\rho}}$$

or

$$\rho = \frac{\rho_{H_2O}}{1 - \frac{W\,(\text{in water})}{W\,(\text{in air})}} = \frac{1.94\ \frac{\text{slugs}}{\text{ft}^3}}{1 - \frac{5.26\ \text{lb}}{8.05\ \text{lb}}} = \underline{\underline{5.60\ \frac{\text{slugs}}{\text{ft}^3}}}$$

2.104

2.104 A 1-m-diameter cylindrical mass, M, is connected to a 2-m-wide rectangular gate as shown in Fig. P2.104. The gate is to open when the water level, h, drops below 2.5 m. Determine the required value for M. Neglect friction at the gate hinge and the pulley.

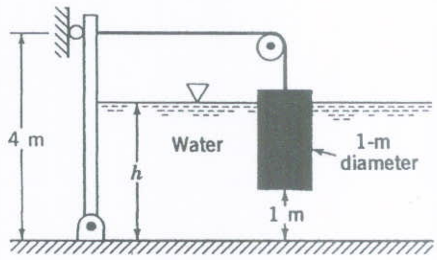
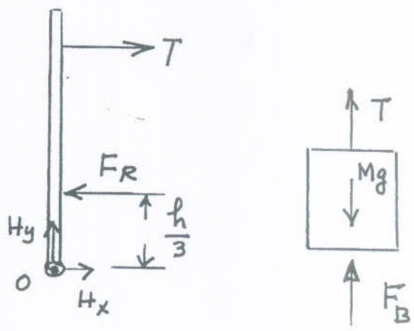


FIGURE P2.104

$$F_R = \gamma h_c A$$
$$= \gamma \left(\frac{h}{2}\right) h\,(2)$$
$$= \gamma h^2$$

where all lengths are in m.



For equilibrium,

$$\sum M_O = 0$$

so that

$$4\,T = \left(\frac{h}{3}\right) F_r = \gamma \frac{h^3}{3}$$

and

$$T = \frac{\gamma h^3}{12}$$

For the cylindrical mass $\sum F_{vertical} = 0$ and

$$T = Mg - F_B = Mg - \gamma \forall_{mass}$$

Thus,

$$M = \frac{T + \gamma \forall_{mass}}{g} = \frac{\frac{\gamma h^3}{12} + \gamma\left(\frac{\pi}{4}\right)(1)^2(h-1)}{g}$$

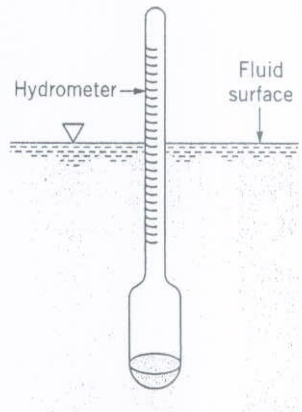
and for $h = 2.5\,m$

$$M = \frac{\left(9.80 \times 10^3 \frac{N}{m^3}\right)\left[\frac{(2.5m)^3}{12} + \frac{\pi}{4}(1m)^2(2.5m - 1.0m)\right]}{9.81 \frac{m}{s^2}}$$

$$= \underline{\underline{2480\ kg}}$$

2.105

2.105 When a hydrometer (see Fig. P2.105 and Video V2.8) having a stem diameter of 0.30 in. is placed in water, the stem protrudes 3.15 in. above the water surface. If the water is replaced with a liquid having a specific gravity of 1.10, how much of the stem would protrude above the liquid surface? The hydrometer weighs 0.042 lb.



■ FIGURE P2.105

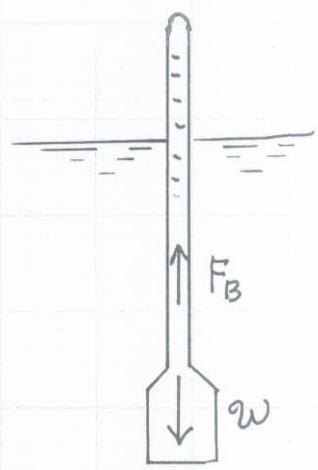
When the hydrometer is floating its weight, $\mathcal{W}$, is balanced by the buoyant force, F_B. For equilibrium,

$$\sum F_{vertical} = 0$$

Thus, for water

$$F_B = \mathcal{W}$$

$$(\gamma_{H_2O}) \forall_1 = \mathcal{W} \qquad (1)$$



where $\forall_1$ is the submerged volume. With the new liquid

$$(SG)(\gamma_{H_2O}) \forall_2 = \mathcal{W} \qquad (2)$$

Combining Eqs. (1) and (2) with $\mathcal{W}$ constant

$$(\gamma_{H_2O}) \forall_1 = (SG)(\gamma_{H_2O}) \forall_2$$

and

$$\forall_2 = \frac{\forall_1}{SG} \qquad (3)$$

(con't)

2.105 (Con't)

From Eq. (1)

$$\forall_1 = \frac{\mathcal{W}}{\gamma_{H_2O}} = \frac{0.042\ lb}{62.4\ \frac{lb}{ft^3}} = 6.73 \times 10^{-4}\ ft^3$$

so that from Eq. (3)

$$\forall_2 = \frac{6.73 \times 10^{-4}\ ft^3}{1.10} = 6.12 \times 10^{-4}\ ft^3$$

Thus, $\forall_1 - \forall_2 = (6.73 - 6.12) \times 10^{-4}\ ft^3 = 0.61 \times 10^{-4}\ ft^3$

To obtain this difference the change in length, Δl, is

$$\left(\frac{\pi}{4}\right)(0.30\ in.)^2\ \Delta l = (0.61 \times 10^{-4}\ ft^3)\left(1728\ \frac{in.^3}{ft^3}\right)$$

$$\Delta l = 1.49\ in.$$

With the new liquid the stem would protrude

3.15 in. + 1.49 in. = $\underline{\underline{4.64\ in.}}$ above the surface

2.106

2.106 A 2-ft-thick block constructed of wood ($SG = 0.6$) is submerged in oil ($SG = 0.8$). and has a 2-ft-thick aluminum (specific weight = 168 lb/ft³) plate attached to the bottom as indicated in Fig. P2.106. Determine completely the force required to hold the block in the position shown. Locate the force with respect to point A.

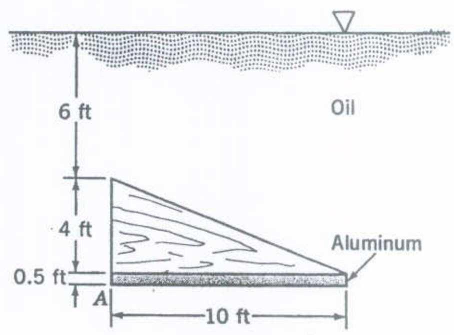
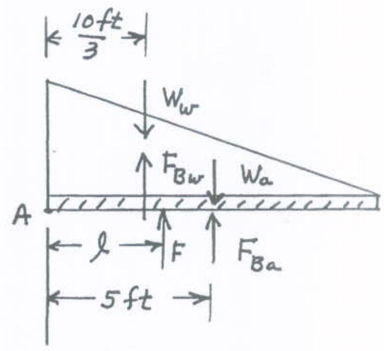


FIGURE P2.106



For equilibrium,

$$\sum F_{vertical} = 0$$

so that

$$F = W_w - F_{Bw} + W_a - F_{Ba}$$

where:

$w \sim$ wood

$a \sim$ aluminum

$F \sim$ force to hold block

$$W_w = (SG_w)(\gamma_{H_2O})\forall_w$$

$$= (0.6)\left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{1}{2}\right)(10ft \times 4ft \times 2ft) = 1500\ lb$$

$$W_a = \left(168\,\frac{lb}{ft^3}\right)(0.5\,ft \times 10ft \times 2ft) = 1680\ lb$$

$$F_{Bw} = (SG_{oil})(\gamma_{H_2O})\forall_w = (0.8)\left(62.4\,\frac{lb}{ft^3}\right)\left(\frac{1}{2}\right)(10ft \times 4ft \times 2\,ft) = 2000\ lb$$

$$F_{Ba} = (SG_{oil})(\gamma_{H_2O})\forall_a = (0.8)\left(62.4\,\frac{lb}{ft^3}\right)(0.5ft \times 10ft \times 2ft) = 499\ lb$$

Thus,

$$F = 1500\ lb - 2000\ lb + 1680\ lb - 499\ lb = \underline{\underline{681\ lb\ \text{upward}}}$$

Also,

$$\sum M_A = 0$$

So that

$$\ell F = \left(\frac{10}{3}\,ft\right)(W_w - F_{Bw}) + (5\,ft)(W_a - F_{Ba})$$

or

$$\ell\,(681\ lb) = \left(\frac{10}{3}\,ft\right)(1500\ lb - 2000\ lb) + (5\ ft)(1680\ lb - 499\ lb)$$

and

$$\underline{\underline{\ell = 6.22\ ft\ \text{to right of point A}}}$$

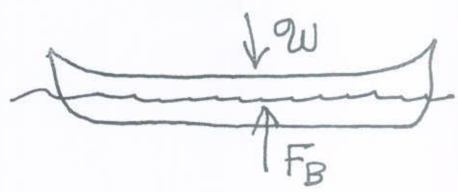
2.107

2.107 (See Fluids in the News article titled "Concrete canoe," Section 2.11.1.) How much extra water does a 147-lb concrete canoe displace compared to an ultralightweight 38-lb Kevlar canoe of the same size carrying the same load?

For equilibrium,

$$\sum F_{vertical} = 0$$

and $\mathcal{W} = F_B = \gamma_{H_2O} \mathcal{V}$ and $\mathcal{V}$ is displaced volume.



For concrete canoe,

$$147 \text{ lb} = \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right) \mathcal{V}_c$$

$$\mathcal{V}_c = 2.36 \text{ ft}^3$$

For Kevlar canoe,

$$38 \text{ lb} = \left(62.4 \frac{\text{lb}}{\text{ft}^3}\right) \mathcal{V}_k$$

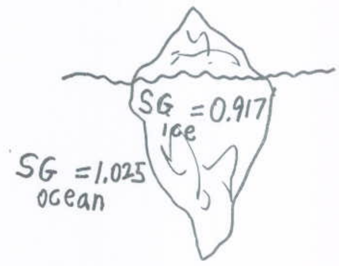
$$\mathcal{V}_k = 0.609 \text{ ft}^3$$

Extra water displacement $= 2.36 \text{ ft}^3 - 0.609 \text{ ft}^3$

$$= \underline{\underline{1.75 \text{ ft}^3}}$$

2.108

2.108 An ice berg (specific gravity 0.917) floats in the ocean (specific gravity 1.025). What percent of the volume of the iceberg is under water?



For equilibrium,

$\mathcal{W}$ = weight of iceberg = F_B = buoyant force

or

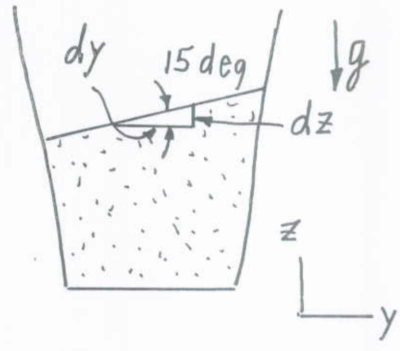
$\mathcal{V}_{ice}\,\gamma_{ice} = \mathcal{V}_{sub}\,\gamma_{ocean}$, where $\mathcal{V}_{sub}$ = volume of ice submerged.

Thus,

$$\frac{\mathcal{V}_{sub}}{\mathcal{V}_{ice}} = \frac{\gamma_{ice}}{\gamma_{ocean}} = \frac{SG_{ice}}{SG_{ocean}} = \frac{0.917}{1.025} = 0.895 = \underline{\underline{89.5\%}}$$

2.110

2.110 It is noted that while stopping, the water surface in a glass of water sitting in the cup holder of a car is slanted at an angle of 15° relative to the horizontal street. Determine the rate at which the car is decelerating.



$$\frac{dz}{dy} = -\frac{a_y}{g + a_z}$$

where $a_z = 0$ and $\frac{dz}{dy} = \tan 15° = 0.268$

Thus,

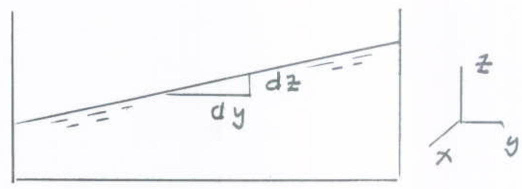
$$0.268 = -\frac{a_y}{g} = -\frac{a_y}{32.2\,ft/s^2}$$

or

$$a_y = -(0.268)(32.2\,\tfrac{ft}{s^2}) = \underline{\underline{-8.63\,\tfrac{ft}{s^2}}}$$

2.111

2.111 An open container of oil rests on the flatbed of a truck that is traveling along a horizontal road at 55 mi/hr. As the truck slows uniformly to a complete stop in 5 s, what will be the slope of the oil surface during the period of constant deceleration?



$$\text{slope} = \frac{dz}{dy} = -\frac{a_y}{g + a_z} \quad (\text{Eq. } 2.28)$$

$$a_y = \frac{\text{final velocity} - \text{initial velocity}}{\text{time interval}}$$

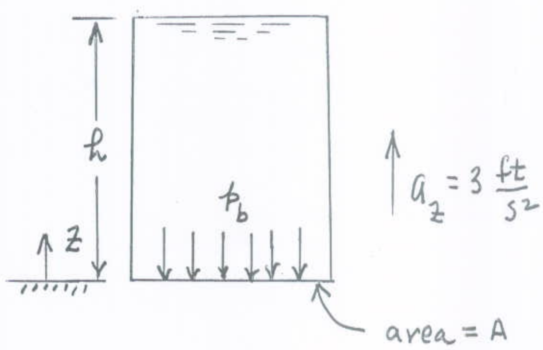
$$= \frac{0 - (55\,\text{mph})\left(0.4470\,\frac{\frac{m}{s}}{\text{mph}}\right)}{5\,s} = -4.92\,\frac{m}{s^2}$$

Thus,

$$\frac{dz}{dy} = -\frac{\left(-4.92\,\frac{m}{s^2}\right)}{9.81\,\frac{m}{s^2} + 0} = \underline{\underline{0.502}}$$

2.112

2.112 A 5-gal, cylindrical open container with a bottom area of 120 in.^{2} is filled with glycerin and rests on the floor of an elevator. **(a)** Determine the fluid pressure at the bottom of the container when the elevator has an upward acceleration of 3 ft/s^2. **(b)** What resultant force does the container exert on the floor of the elevator during this acceleration? The weight of the container is negligible. (Note: 1 gal = 231 in.^{3})



(a) $$\frac{dp}{dz} = -\rho\,(g + a_z) \qquad \text{(Eq. 2.26)}$$

hA = volume

$$h\,(120\ \text{in.}^2) = (5\ \text{gal})\left(\frac{231\ \text{in.}^3}{\text{gal}}\right)$$

$$h = 9.63\ \text{in.}$$

Thus,

$$\int_0^{p_b} dp = -\rho\,(g + a_z)\int_h^0 dz$$

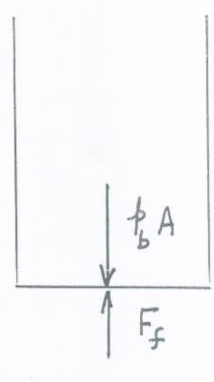
and

$$p_b = \rho\,(g + a_z)\,h$$

$$= \left(2.44\ \frac{\text{slugs}}{\text{ft}^3}\right)\left(32.2\ \frac{\text{ft}}{\text{s}^2} + 3\ \frac{\text{ft}}{\text{s}^2}\right)\left(\frac{9.63}{12}\ \text{ft}\right)$$

$$= \underline{\underline{68.9\ \frac{\text{lb}}{\text{ft}^2}}}$$

(b) From free-body-diagram of container,



$$F_f = p_b A$$

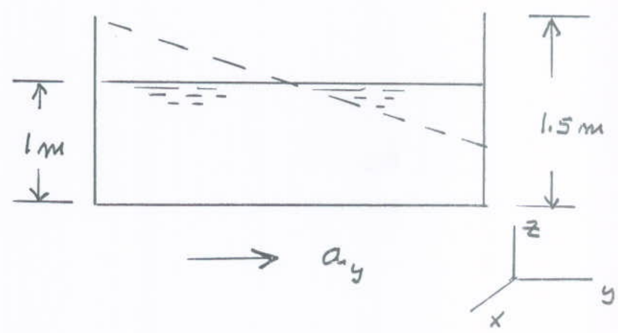
$$= \left(68.9\ \frac{\text{lb}}{\text{ft}^2}\right)(120\ \text{in.}^2)\left(\frac{1\ \text{ft}^2}{144\ \text{in.}^2}\right)$$

$$= 57.4\ \text{lb}$$

Thus, force of container on floor is $\underline{\underline{57.4\ \text{lb downward.}}}$

2.113

2.113 An open rectangular tank 1 m wide and 2 m long contains gasoline to a depth of 1 m. If the height of the tank sides is 1.5 m, what is the maximum horizontal acceleration (along the long axis of the tank) that can develop before the gasoline would begin to spill?



To prevent spilling,

$$\frac{dz}{dy} \geq -\frac{1.5\,m - 1.0\,m}{1\,m} = -0.50$$

(see figure).

Since,

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} \qquad \text{(Eq. 2.28)}$$

or, with $a_z = 0$,

$$a_y = -\left(\frac{dz}{dy}\right) g$$

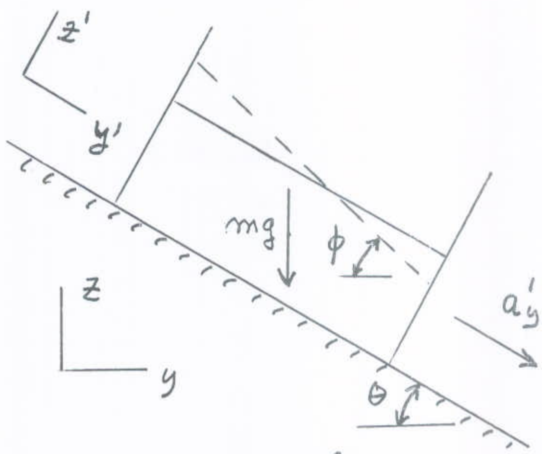
so that

$$(a_y)_{max} = -(-0.50)\left(9.81\,\frac{m}{s^2}\right) = \underline{\underline{4.91\,\frac{m}{s^2}}}$$

(Note: Acceleration could be either to the right or the left.)

2.114

2.114 If the tank of Problem 2.113 slides down a frictionless plane that is inclined at 30° with the horizontal, determine the angle the free surface makes with the horizontal.



$m \sim$ mass of tank and gasoline

From Newton's 2nd law,

$$\sum F_{y'} = m a_y'$$

Since the only force in the y'-direction is the component of weight $(mg)\sin\theta$,

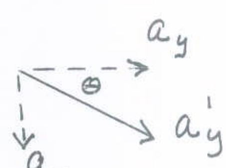
$$(mg)\sin\theta = m a_y'$$

so that

$$a_y' = g\sin\theta$$

and therefore

$$a_y = a_y'\cos\theta \qquad a_z = -a_y'\sin\theta$$



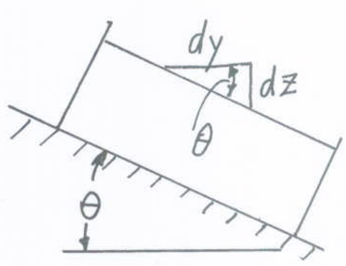
Also,

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} \qquad \text{(Eq. 2.28)}$$

$$= -\frac{a_y'\cos\theta}{g - a_y'\sin\theta} = -\frac{g\sin\theta\cos\theta}{g - g\sin\theta\sin\theta}$$

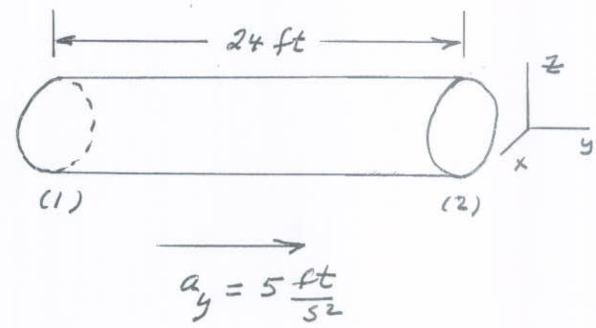
$$= -\frac{\sin\theta\cos\theta}{1-\sin^2\theta} = -\frac{\sin\theta\cos\theta}{\cos^2\theta} = -\tan\theta$$

Hence, $\frac{dz}{dy} = -\tan\theta$, so that the free surface is at <u>the same angle as the plane.</u>



2.115

2.115 A closed cylindrical tank that is 8 ft in diameter and 24 ft long is completely filled with gasoline. The tank, with its long axis horizontal, is pulled by a truck along a horizontal surface. Determine the pressure difference between the ends (along the long axis of the tank) when the truck undergoes an acceleration of 5 ft/s^2.



$$\frac{\partial p}{\partial y} = -\rho a_y \qquad \text{(Eq. 2.25)}$$

Thus,

$$\int_{p_1}^{p_2} dp = -\rho a_y \int_0^{24} dy$$

where $p = p_1$ at $y = 0$ and $p = p_2$ at $y = 24$ ft,

and

$$p_2 - p_1 = -\rho a_y (24\ ft)$$

$$= -\left(1.32\ \frac{slugs}{ft^3}\right)\left(5\ \frac{ft}{s^2}\right)(24\ ft)$$

$$= -158\ \frac{lb}{ft^2}$$

or

$$p_1 - p_2 = \underline{\underline{158\ \frac{lb}{ft^2}}}$$

2.116

2.116 The open U-tube of Fig. P2.116 is partially filled with a liquid. When this device is accelerated with a horizontal acceleration, a, a differential reading, h, develops between the manometer legs which are spaced a distance ℓ apart. Determine the relationship between a, ℓ, and h.

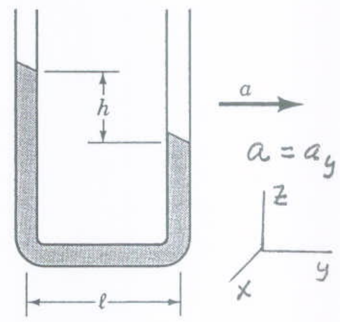


FIGURE P2.116

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} \qquad \text{(Eq. 2.28)}$$

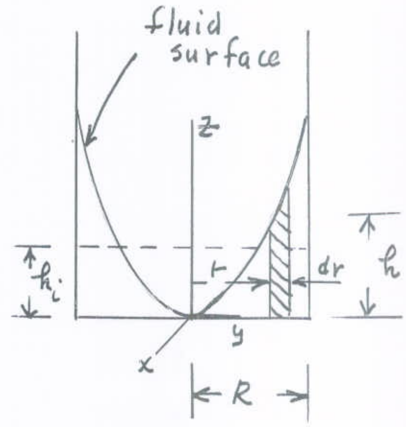
Since, $\frac{dz}{dy} = -\frac{h}{\ell}$ and $a_z = 0$

then
$$-\frac{h}{\ell} = -\frac{a}{g+0}$$

or
$$\underline{\underline{h = \frac{a\ell}{g}}}$$

2.117

2.117 An open 1-m-diameter tank contains water at a depth of 0.7 m when at rest. As the tank is rotated about its vertical axis the center of the fluid surface is depressed. At what angular velocity will the bottom of the tank first be exposed? No water is spilled from the tank.



$h_i \sim$ initial depth

Equation for surfaces of constant pressure (Eq. 2.32):

$$z = \frac{\omega^2 r^2}{2g} + \text{constant}$$

For free surface with $h=0$ at $r=0$,

$$h = \frac{\omega^2 r^2}{2g}$$

The volume of fluid in rotating tank is given by

$$\forall_f = \int_0^R 2\pi r h \, dr = \frac{2\pi \omega^2}{2g}\int_0^R r^3 dr = \frac{\pi \omega^2 R^4}{4g}$$

Since the initial volume, $\forall_i = \pi R^2 h_i$, must equal the final volume,

$$\forall_f = \forall_i$$

so that

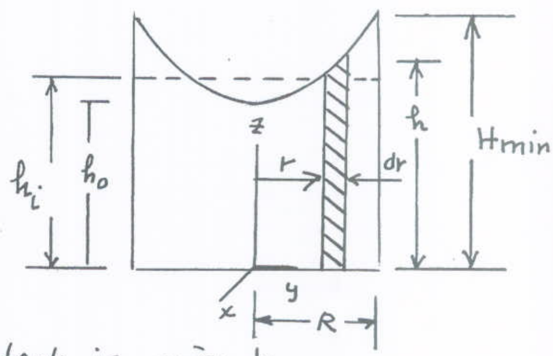
$$\frac{\pi \omega^2 R^4}{4g} = \pi R^2 h_i$$

or

$$\omega = \sqrt{\frac{4 g h_i}{R^2}} = \sqrt{\frac{4\left(9.81 \frac{m}{s^2}\right)(0.7 m)}{(0.5 m)^2}} = \underline{\underline{10.5 \frac{rad}{s}}}$$

2.118

2.118 An open, 2-ft-diameter tank contains water to a depth of 3 ft when at rest. If the tank is rotated about its vertical axis with an angular velocity of 180 rev/min, what is the minimum height of the tank walls to prevent water from spilling over the sides?



For free surface,

$$h = \frac{\omega^2 r^2}{2g} + h_o \qquad (Eq.\ 2.32)$$

The volume of fluid in the rotating tank is given by

$$\forall_f = \int_0^R 2\pi r h \, dr = 2\pi \int_0^R \left(\frac{\omega^2 r^3}{2g} + h_o r \right) dr$$

$$= \frac{\pi \omega^2 R^4}{4g} + \pi h_o R^2$$

$$= \frac{\pi \left(180 \frac{rev}{min} \times 2\pi \frac{rad}{rev} \times \frac{1\,min}{60\,s}\right)^2 (1\,ft)^4}{4\left(32.2 \frac{ft}{s^2}\right)} + \pi h_o (1\,ft)^2$$

$$= \pi\,(2.76 + h_o)\ ft^3 \qquad (\text{with } h_o \text{ in ft})$$

Since the initial volume,

$$\forall_i = \pi R^2 h_i = \pi (1\,ft)^2 (3\,ft) = 3\pi\ ft^3$$

and the final volume must be equal,

$$\forall_f = \forall_i$$

or

$$\pi\,(2.76 + h_o)\ ft^3 = 3\pi\ ft^3$$

and

$$h_o = 0.240\ ft$$

Thus, from the first equation (Eq. 2.32)

$$h = \frac{\omega^2 r^2}{2g} + 0.240\ ft$$

and

$$H_{min} = \frac{\left(180 \frac{rev}{min} \times 2\pi \frac{rad}{rev} \times \frac{1\,min}{60\,s}\right)^2 (1\,ft)^2}{2\left(32.2 \frac{ft}{s^2}\right)} + 0.240\,ft = \underline{\underline{5.76\,ft}}$$

2.119

2.119 A child riding in a car holds a string attached to a floating, helium-filled balloon. As the car decelerates to a stop, the balloon tilts backwards. As the car makes a right-hand turn, the balloon tilts to the right. On the other hand, the child tends to be forced forward as the car decelerates and to the left as the car makes a right-hand turn. Explain these observed effects on the balloon and child.

A floating balloon attached to a string will align itself so that the string it normal to lines of constant pressure. Thus, if the car is not accelerating, the lines of p = constant pressure are horizontal (gravity acts vertically down), and the balloon floats "straight up" (i.e. $\theta = 0$). If forced to the side ($\theta \neq 0$), the balloon will return to the vertical ($\theta = 0$) equilibrium position in which the two forces T and $F_B - W$ line up

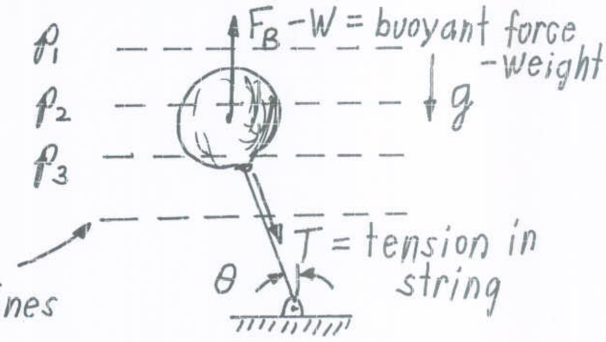


Fig.(1) No acceleration, $\theta = 0$ for equilibrium.

Consider what happens when the car decelerates with an amount $a_y < 0$. As show by Eq. (2.28), the lines of constant pressure are not horizontal, but have a slope of

$$\frac{dz}{dy} = -\frac{a_y}{g + a_z} = -\frac{a_y}{g} > 0 \text{ since } a_z = 0$$

and $a_y < 0$, Again, the balloon's equilibrium position is with the string normal to p = const. lines. That is, the balloon tilts back as the car stops.

When the car turns, $a_y = \frac{V^2}{R}$ (the centrifugal acceleration), the lines of p = const. are as shown, and the balloon tilts to the outside of the curve

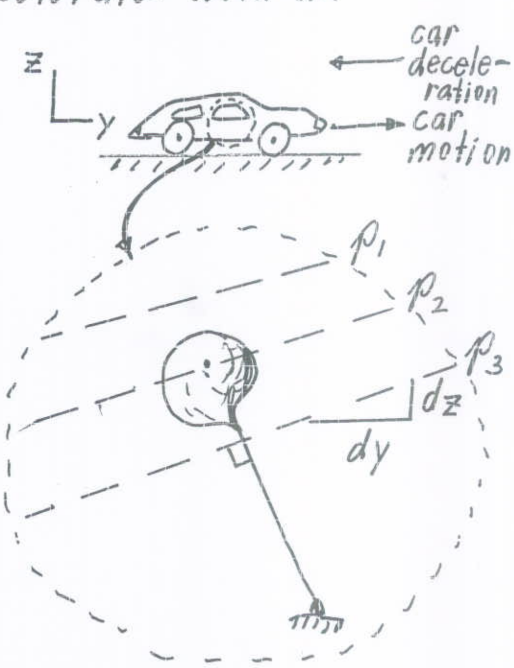
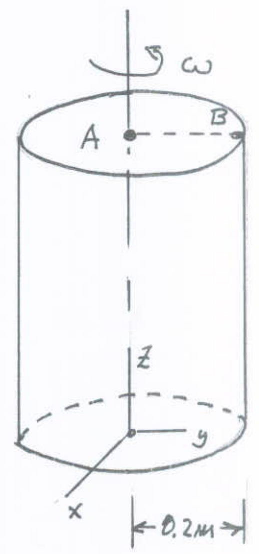


Fig.(2) Balloon aligned so that string is normal to p = constant lines

Fig.(3) Left turn; balloon tilts to right

2.120

2.120 A closed, 0.4-m-diameter cylindrical tank is completely filled with oil ($SG = 0.9$) and rotates about its vertical longitudinal axis with an angular velocity of 40 rad/s. Determine the difference in pressure just under the vessel cover between a point on the circumference and a point on the axis.



Pressure in a rotating fluid varies in accordance with the equation,

$$p = \frac{\rho \omega^2 r^2}{2} - \gamma z + \text{constant} \qquad (\text{Eq. } 2.33)$$

Since $z_A = z_B$,

$$p_B - p_A = \frac{\rho \omega^2}{2}\left(r_B^{\,2} - r_A^{\,2}\right)$$

$$= \frac{(0.9)\left(10^3 \frac{kg}{m^3}\right)\left(40 \frac{rad}{s}\right)^2}{2}\left[(0.2\,m)^2 - 0\right]$$

$$= \underline{\underline{28.8\ kPa}}$$

2.121

2.121 (See Fluids in the News article titled "**Rotating mercury mirror telescope,**" Section 2.12.2.) The largest liquid mirror telescope uses a 6-ft-diameter tank of mercury rotating at 7 rpm to produce its parabolic-shaped mirror as shown in Fig. P2.121. Determine the difference in elevation of the mercury, Δh, between the edge and the center of the mirror.

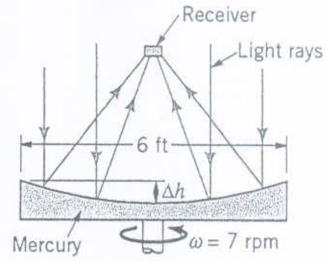


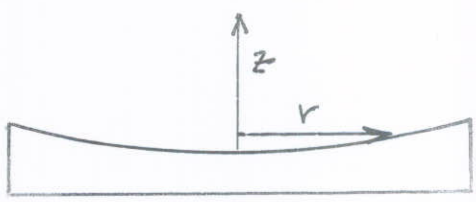
FIGURE P2.121

For free surface of rotating liquid,

$$z = \frac{\omega^2 r^2}{2g} + \text{constant} \qquad \text{(Eq. 2.32)}$$

Let $z=0$ at $r=0$ and therefore constant $=0$. Thus,

$\Delta h = \Delta z$ for $r = 3\text{ ft}$ and with



$$\omega = (7\text{ rpm})\left(2\pi\,\frac{\text{rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)$$

$$= 0.733\ \frac{\text{rad}}{\text{s}}$$

it follows that

$$\Delta h = \frac{\left(0.733\ \frac{\text{rad}}{\text{s}}\right)^2 (3\text{ ft})^2}{2\left(32.2\ \frac{\text{ft}}{\text{s}^2}\right)} = \underline{\underline{0.0751\text{ ft}}}$$

2.122

2.122 Force Needed to Open a Submerged Gate

Objective: A gate, hinged at the top, covers a hole in the side of a water filled tank as shown in Fig. P2.122 and is held against the tank by the water pressure. The purpose of this experiment is to compare the theoretical force needed to open the gate to the experimentally measured force.

Equipment: Rectangular tank with a rectangular hole in its side; gate that covers the hole and is hinged at the top; force transducer to measure the force needed to open the gate; ruler to measure the water depth.

Experimental Procedure: Measure the height, H, and width, b, of the hole in the tank and the distance, L, from the hinge to the point of application of the force, F, that opens the gate. Fill the tank with water to a depth h above the bottom of the gate. Use the force transducer to determine the force, F, needed to slowly open the gate. Repeat the force measurements for various water depths.

Calculations: For arbitrary water depths, h, determine the theoretical force, F, needed to open the gate by equating the moment about the hinge from the water force on the gate to the moment produced by the applied force, F.

Graph: Plot the experimentally determined force, F, needed to open the gate as ordinates and the water depth, h, as abscissas.

Results: On the same graph, plot the theoretical force as a function of water depth.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

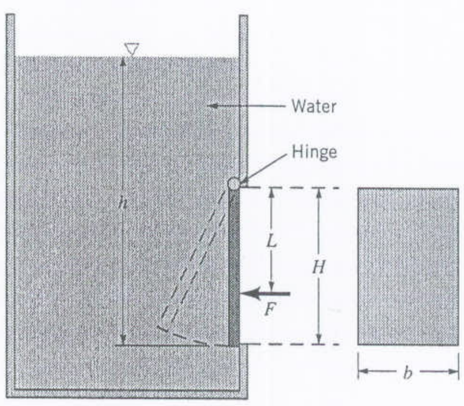


FIGURE P2.122

(con't)

2.122 (Con't)

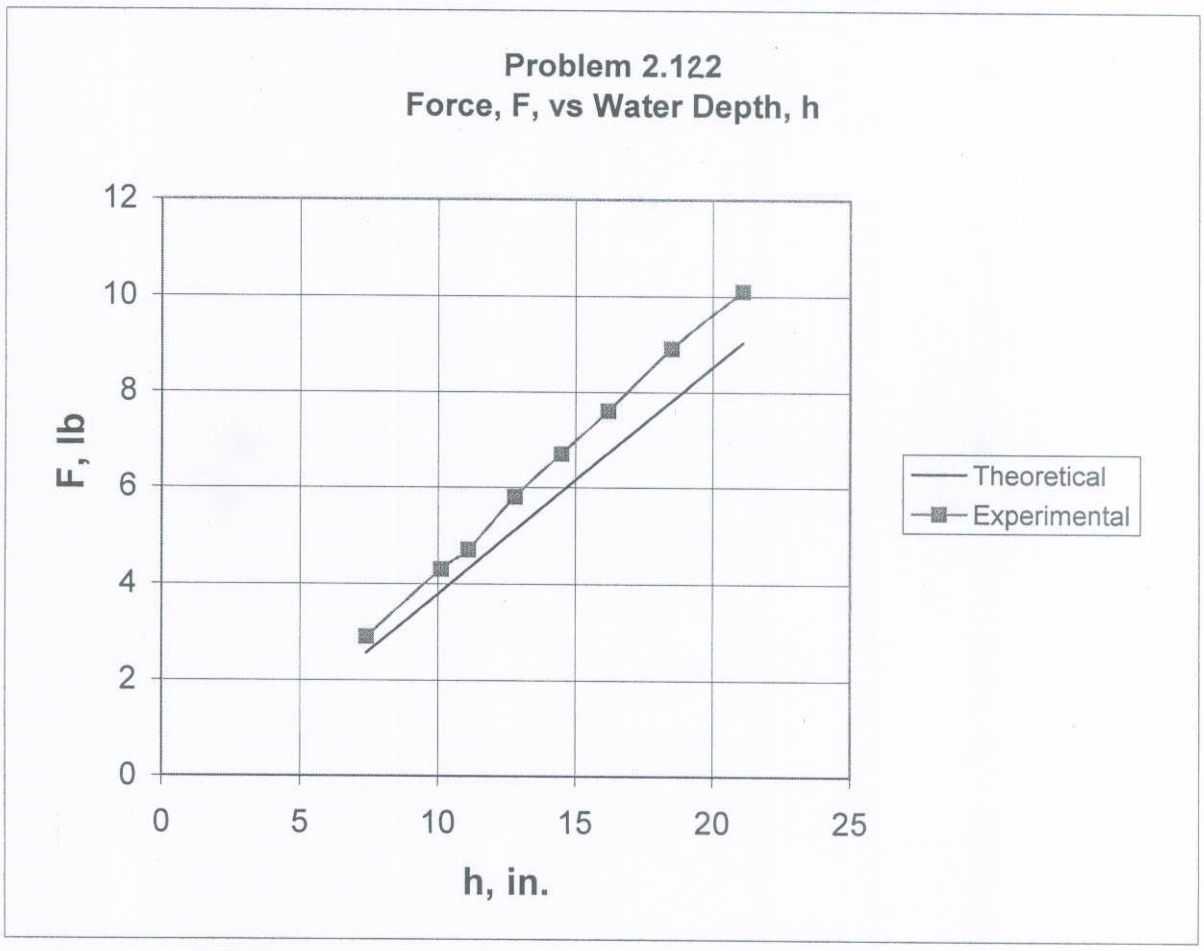
Solution for Problem 2.122: Force Needed to Open a Submerged Gate

L, in.	H, in.	b, in.	γ, lb/ft^3	I_{xc}, ft^4
5.5	6.0	4.0	62.4	0.003472

h, in.	F, lb	F_1, lb	$y_r - y_c$, ft	d, ft	F, lb
21.1	10.1	15.69	0.0138	0.264	9.03
18.5	8.9	13.43	0.0161	0.266	7.80
16.2	7.6	11.44	0.0189	0.269	6.71
14.5	6.7	9.97	0.0217	0.272	5.91
12.8	5.8	8.49	0.0255	0.276	5.11
11.1	4.7	7.02	0.0309	0.281	4.30
10.1	4.3	6.15	0.0352	0.285	3.83
7.4	2.9	3.81	0.0568	0.307	2.55

Since h > H, A = H*b = constant and I_{xc} = b*H^3/12 = constant.

F = F_1*d/L, where $F_1 = \gamma$*(h - H/2)*A, d = H/2 + ($y_r - y_c$), and $y_r - y_c = I_{xc}$/(h - H/2)*A



2.123

2.123 Hydrostatic Force on a Submerged Rectangle

Objective: A quarter-circle block with a vertical rectangular end is attached to a balance beam as shown in Fig. P2.123. Water in the tank puts a hydrostatic pressure force on the block which causes a clockwise moment about the pivot point. This moment is balanced by the counterclockwise moment produced by the weight placed at the end of the balance beam. The purpose of this experiment is to determine the weight, W, needed to balance the beam as a function of the water depth, h.

Equipment: Balance beam with an attached quarter-circle, rectangular cross-section block; pivot point directly above the vertical end of the beam to support the beam; tank; weights; ruler.

Experimental Procedure: Measure the inner radius, R_1, outer radius, R_2, and width, b, of the block. Measure the length, L, of the moment arm between the pivot point and the weight. Adjust the counter weight on the beam so that the beam is level when there is no weight on the beam and no water in the tank. Hang a known mass, m, on the beam and adjust the water level, h, in the tank so that the beam again becomes level. Repeat with different masses and water depths.

Calculations: For a given water depth, h, determine the hydrostatic pressure force, $F_R = \gamma h_c A$, on the vertical end of the block. Also determine the point of action of this force, a distance $y_R - y_c$ below the centroid of the area. Note that the equations for F_R and $y_R - y_c$ are different when the water level is below the end of the block $(h < R_2 - R_1)$ than when it is above the end of the block $(h > R_2 - R_1)$.

For a given water depth, determine the theoretical weight needed to balance the beam by summing moments about the pivot point. Note that both F_R and W produce a moment. However, because the curved sides of the block are circular arcs centered about the pivot point, the pressure forces on the curved sides of the block (which act normal to the sides) do not produce any moment about the pivot point. Thus the forces on the curved sides do not enter into the moment equation.

Graph: Plot the experimentally determined weight, W, as ordinates and the water depth, h, as abscissas.

Result: On the same graph plot the theoretical weight as a function of water depth.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

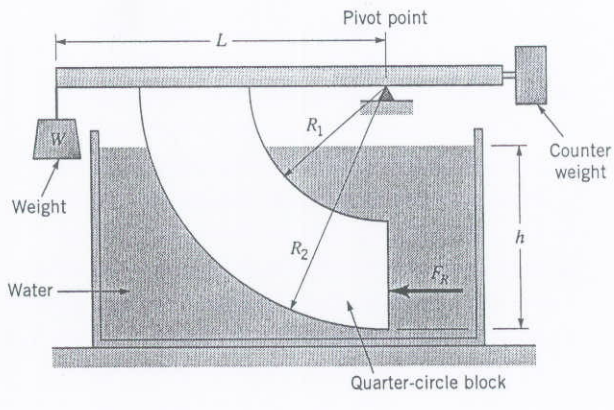


FIGURE P2.123

(con't)

2.123 (Con't)

Solution for Problem 2.123: Hydrostatic Force on a Submerged Rectangle

R_1, in.	R_2, in.	L, in.	b, in.		g, ft/s^2	γ, lb/ft^3
5.0	9.0	12.0	3.0		32.2	62.4

m, kg	h, in.	Experimental W, lb	F_R, lb	y_r -y_c, ft	d, ft	Theoretical W, lb
0.00	0.00	0.00	0.00		0.750	0.000
0.02	1.11	0.04	0.07		0.719	0.048
0.04	1.58	0.09	0.14		0.706	0.095
0.06	1.92	0.13	0.20		0.697	0.139
0.10	2.51	0.22	0.34		0.680	0.232
0.12	2.76	0.26	0.41		0.673	0.278
0.14	2.99	0.31	0.48		0.667	0.323
0.16	3.20	0.35	0.55		0.661	0.367
0.18	3.41	0.40	0.63		0.655	0.413
0.20	3.60	0.44	0.70		0.650	0.456
0.22	3.80	0.48	0.78		0.644	0.504
0.24	3.99	0.53	0.86		0.639	0.551
0.26	4.17	0.57	0.94	0.0512	0.634	0.597
0.28	4.33	0.62	1.01	0.0476	0.631	0.637
0.30	4.50	0.66	1.08	0.0444	0.628	0.680
0.35	4.95	0.77	1.28	0.0376	0.621	0.794
0.40	5.39	0.88	1.47	0.0328	0.616	0.905
0.45	5.83	0.99	1.66	0.0290	0.612	1.016
0.50	6.27	1.10	1.85	0.0260	0.609	1.127
0.55	6.70	1.21	2.04	0.0236	0.607	1.236

W = 32.2 ft/s^2 * (m kg * 6.825E-2 slug/kg)

Sum moments about pivot to give $W*L = F_R*d$

For $h < R_2 - R_1$:

$F_R = \gamma*(h/2)*h*b$

$d = R_2 - (h/3)$

For $h > R_2 - R_1$:

$F_R = \gamma*(h - (R_2 - R_1)/2)*(R_2 - R_1)*b$

$d = R_2 - (R_2 - R_1)/2 + (y_r - y_c)$

$y_r - y_c = I_{xc}/h_c*A$

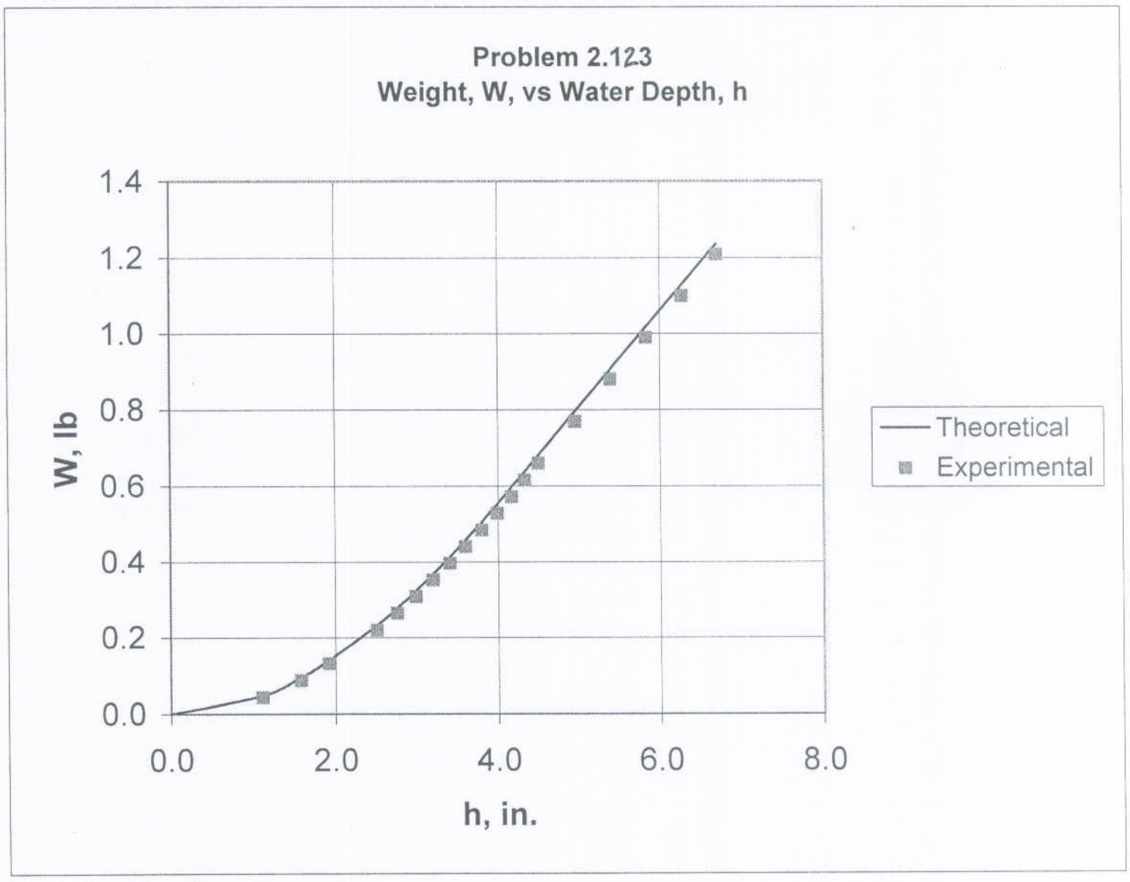
$I_{xc} = b*(R_2 - R_1)^3/12 = 0.000771 \text{ ft}^4$

$h_c = h - (R_2 - R_1)/2$

$A = b*(R_2 - R_1)$

(con't)

2.123 (Con't)



Problem 2.123
Weight, W, vs Water Depth, h
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
W, lb
0.0
2.0
4.0
6.0
8.0
h, in.
Theoretical
Experimental

2.124

2.124 Vertical Uplift Force on an Open-Bottom Box with Slanted Sides

Objective: When a box or form as shown in Fig. P2.124 is filled with a liquid, the vertical force of the liquid on the box tends to lift it off the surface upon which it sits, thus allowing the liquid to drain from the box. The purpose of this experiment is to determine the minimum weight, W, needed to keep the box from lifting off the surface.

Equipment: An open-bottom box that has vertical side walls and slanted end walls; weights; ruler; scale.

Experimental Procedure: Determine the weight, W_{box}, of the empty box and measure its length, L, width, b, wall thickness, t, and the angle of the ends, θ. Set the box on a smooth surface and place a known mass, m, on it. Slowly fill the box with water and note the depth, h, at which the net upward water force is equal to the total weight, $W + W_{box}$, where $W = mg$. This condition will be obvious because the friction force between the box and the surface on which it sits will be zero and the box will "float" effortlessly along the surface. Repeat for various masses and water levels.

Calculations: For an arbitrary water depth, h, determine the theoretical weight, W, needed to maintain equilibrium with no contact force between the box and the surface below it. This can be done by equating the total weight, $W + W_{box}$, to the net vertical hydrostatic pressure force on the box. Calculate this vertical pressure force for two different situations. (1) Assume the vertical pressure force is the vertical component of the pressure forces acting on the slanted ends of the box. (2) Assume the vertical upward force is that from part (1) plus the pressure force acting under the sides and ends of the box because of the finite thickness, t, of the box walls. This additional pressure force is assumed to be due to an average pressure of $p_{avg} = \gamma h/2$ acting on the "foot print" area of the box walls.

Graph: Plot the experimentally determined total weight, $W + W_{box}$, as ordinates and the water depth, h, as abscissas.

Results: On the same graph plot two theoretical total weight verses water depth curves—one involving only the slanted-end pressure force, and the other including the slanted end and the finite-thickness wall pressure forces.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

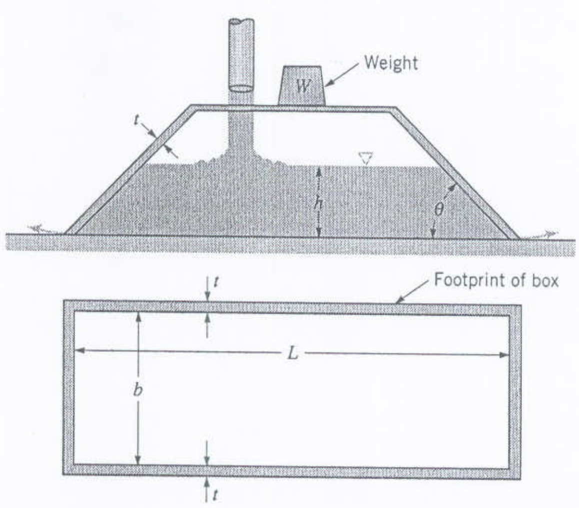


FIGURE P2.124

(con't)

2.124 (Con't)

Solution for Problem 2.124: Vertical Uplift Force on an Open-Bottom Box with Slanted Sides

θ, deg	L, in.	b, in.	t, in.	W_{box}, lb		γ, lb/ft^3
45	10.3	4.0	0.25	0.942		62.4

		Experimental		Theory 1		Theory 2
m, kg	h, in.	$W + W_{box}$, lb	h, in.	$W + W_{box}$, lb	p_{avg}, lb/ft^2	$W + W_{box}$, lb
0.00	2.06	0.942	0.00	0.000	0.00	0.000
0.05	2.23	1.052	0.25	0.009	0.65	0.047
0.10	2.42	1.162	0.50	0.036	1.30	0.111
0.15	2.53	1.272	0.75	0.081	1.95	0.194
0.20	2.67	1.382	1.00	0.144	2.60	0.295
0.25	2.81	1.491	1.25	0.226	3.25	0.414
0.30	2.94	1.601	1.50	0.325	3.90	0.551
0.35	3.06	1.711	1.75	0.442	4.55	0.706
0.40	3.16	1.821	2.00	0.578	5.20	0.879
			2.25	0.731	5.85	1.070
			2.50	0.903	6.50	1.279
			2.75	1.092	7.15	1.506
			3.00	1.300	7.80	1.752
			3.25	1.526	8.45	2.015

W = g*m = 32.2 ft/s^2 * (m kg * 6.825E-2 slug/kg)

Theory 1. Including only the slanted-end pressure force:

$W + W_{box} = \gamma * Vol$

Vol = b*h*h

Theory 2. Including the slanted-end pressure force and the finite-thickness wall pressure force:

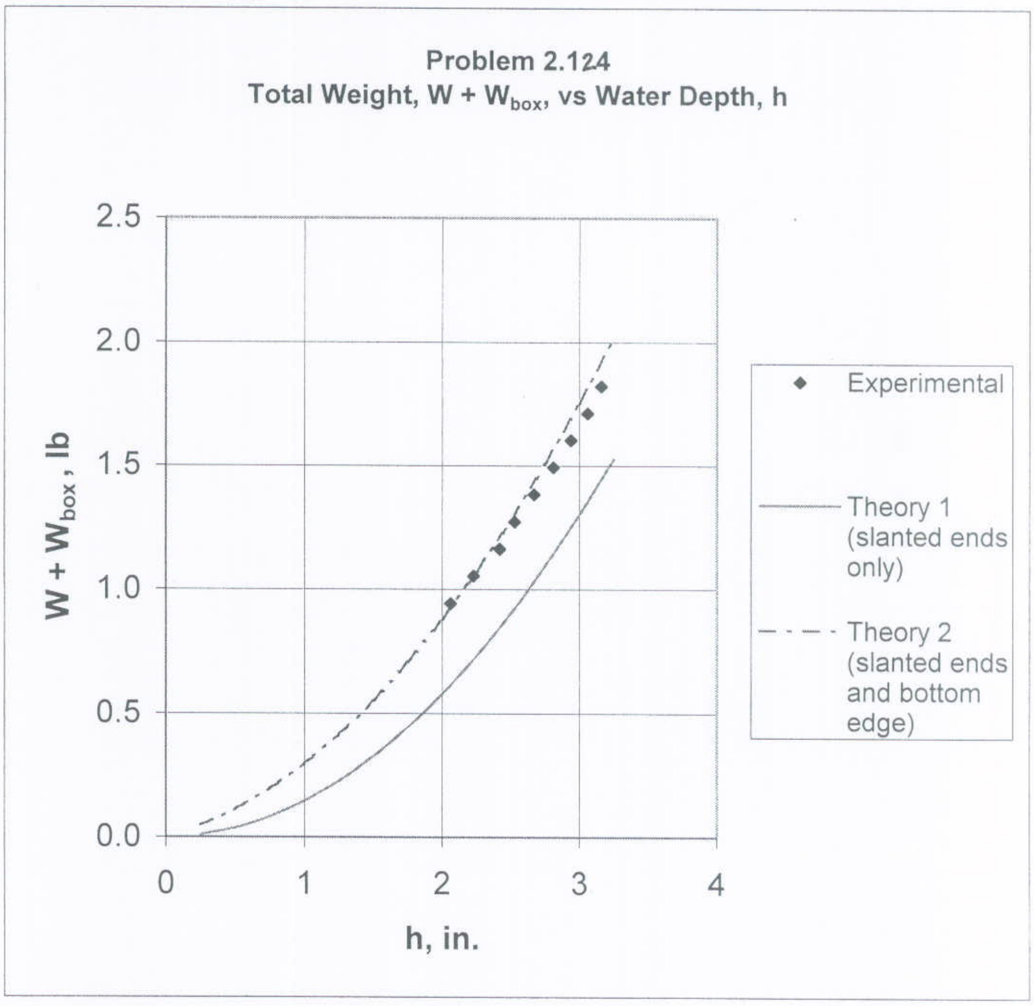
$W + W_{box} = \gamma * Vol + p_{avg} * A$

$p_{avg} = 0.5 * \gamma * h$

A = (b + 2*t)*(L + 2*t/sinθ) - b*L = 8.33 in.^2 = 0.0579 ft^2

(con't)

2.124



Problem 2.124
Total Weight, W + W_box, vs Water Depth, h
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
h, in.
W + W_box , lb
Experimental
Theory 1 (slanted ends only)
Theory 2 (slanted ends and bottom edge)

2.125

2.125 Air Pad Lift Force

Objective: As shown in Fig. P2.125, it is possible to lift objects by use of an air pad consisting of an inverted box that is pressurized by an air supply. If the pressure within the box is large enough, the box will lift slightly off the surface, air will flow under its edges, and there will be very little frictional force between the box and the surface. The purpose of this experiment is to determine the lifting force, W, as a function of pressure, p, within the box.

Equipment: Inverted rectangular box; air supply; weights; manometer.

Experimental Procedure: Connect the air source and the manometer to the inverted square box. Determine the weight, W_{box}, of the square box and measure its length and width, L, and the wall thickness, t. Set the inverted box on a smooth surface and place a known mass, m, on it. Increase the air flowrate until the box lifts off the surface slightly and "floats" with negligible frictional force. Record the manometer reading, h, under these conditions. Repeat the measurements with various masses.

Calculations: Determine the theoretical weight that can be lifted by the air pad by equating the total weight, $W + W_{box}$, to the net vertical pressure force on the box. Here $W = mg$. Calculate this pressure force for two different situations. (1) Assume the pressure force is equal to the area of the box, $A = L^2$, times the pressure, $p = \gamma_m h$, within the box, where γ_m is the specific weight of the manometer fluid. (2) Assume that the net pressure force is that from part (1) plus the pressure force acting under the edges of the box because of the finite thickness, t, of the box walls. This additional pressure force is assumed to be due to an average pressure of $p_{avg} = \gamma_m h/2$ acting on the "foot print" area of the box walls, $4t(L + t)$.

Graph: Plot the experimentally determined total weight, $W + W_{box}$, as ordinates and the pressure within the box, p, as abscissas.

Results: On the same graph, plot two theoretical total weight verses pressure curves—one involving only the pressure times box area pressure force, and the other including the pressure times box area and the finite-thickness wall pressure forces.

Data: To proceed, print this page for reference when you work the problem and *click here* to bring up an EXCEL page with the data for this problem.

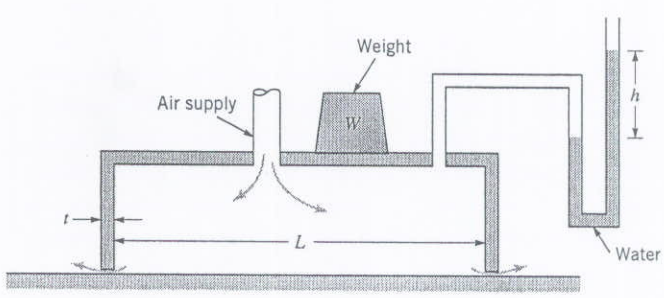


FIGURE P2.125

(Con't)

2.125 (con't)

Solution for Problem 2.125: Air Pad Lift Force

L, in.	t, in.	W_{box}, lb	γ_{H2O}, lb/ft^3
7.5	0.25	1.25	62.4

		Experiment		Theory 1	Theory 2
m, kg	h, in.	$W + W_{box}$, lb	p, lb/ft^2	$W + W_{box}$, lb	$W + W_{box}$, lb
0.0	0.54	1.25	2.81	1.10	1.17
0.1	0.64	1.47	3.33	1.30	1.39
0.2	0.74	1.69	3.85	1.50	1.61
0.3	0.82	1.91	4.26	1.67	1.78
0.4	0.94	2.13	4.89	1.91	2.04
0.5	1.04	2.35	5.41	2.11	2.26
0.6	1.12	2.57	5.82	2.28	2.43
0.7	1.23	2.79	6.40	2.50	2.67
0.8	1.32	3.01	6.86	2.68	2.87
0.9	1.42	3.23	7.38	2.88	3.08
1.0	1.52	3.45	7.90	3.09	3.30
1.1	1.63	3.67	8.48	3.31	3.54
1.2	1.72	3.89	8.94	3.49	3.73
1.3	1.83	4.11	9.52	3.72	3.97
1.4	1.96	4.33	10.19	3.98	4.26
1.5	2.06	4.55	10.71	4.18	4.47
1.6	2.12	4.77	11.02	4.31	4.60
1.7	2.23	4.99	11.60	4.53	4.84
1.8	2.32	5.21	12.06	4.71	5.04

W = g*m = 32.2 ft/s^2 * (m kg * 6.825E-2 slug/kg)

Theory 1. Involving only the pressure times the box area:

$W + W_{box} = p*L^2$

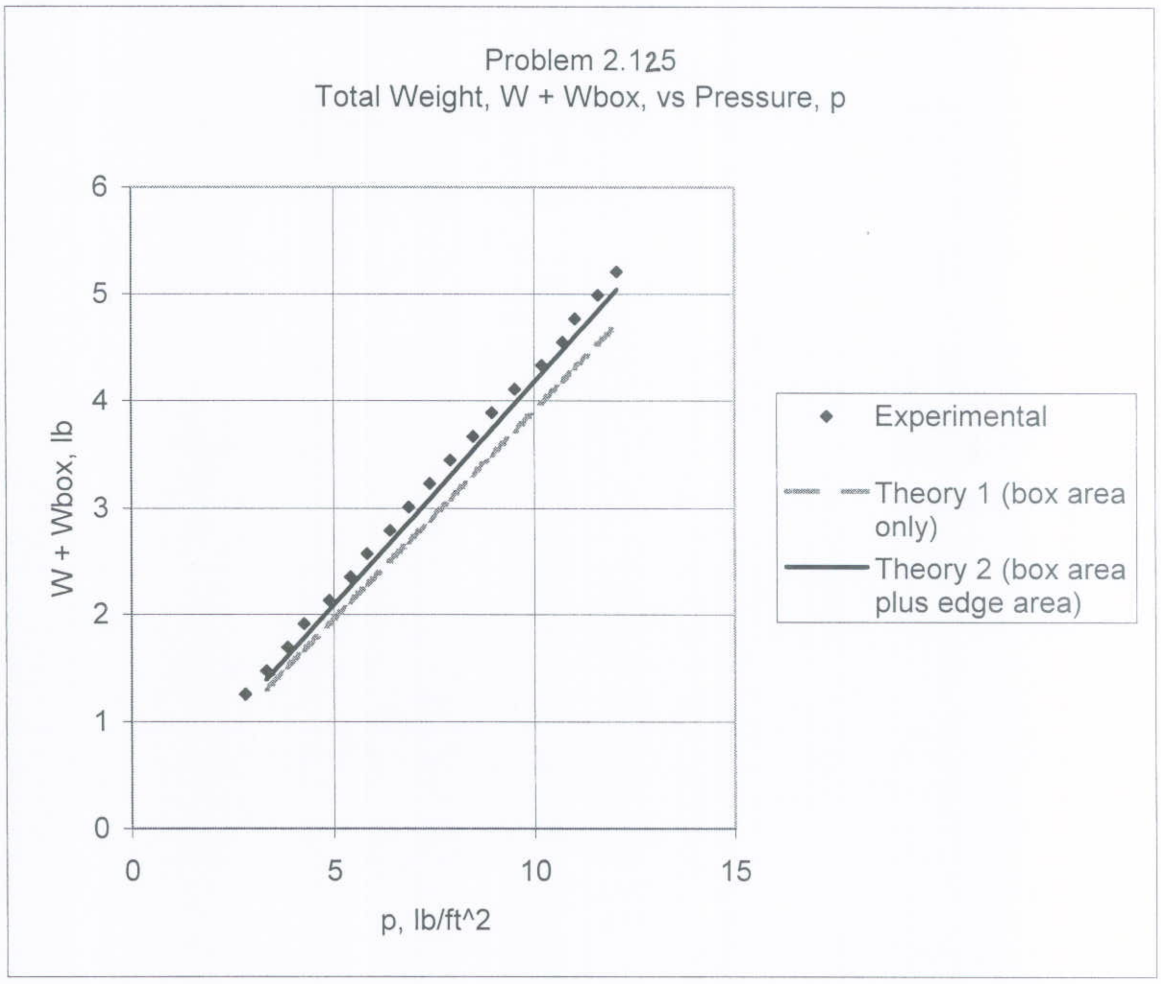
$p = \gamma_{H2O}*h$

Theory 2. Involving the pressure times the box area plus the average pressure times the edge area:

$W + W_{box} = p*L^2 + (p/2)*((L + 2t)^2 - L^2)$

(con't)

2.125 (con't)


Problem 2.125
Total Weight, W + Wbox, vs Pressure, p
W + Wbox, lb
p, lb/ft^2
0
1
2
3
4
5
6
0
5
10
15
Experimental
Theory 1 (box area only)
Theory 2 (box area plus edge area)