

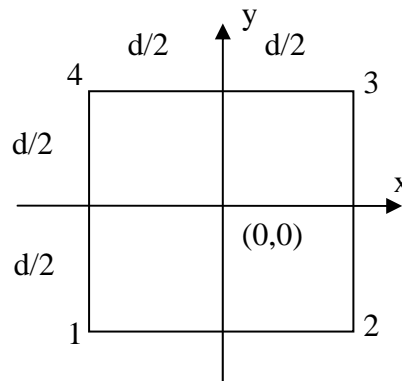
Assignment Number 2

1. Assuming a single degree of freedom at each node and using the following model:

$$u(x, y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 xy$$

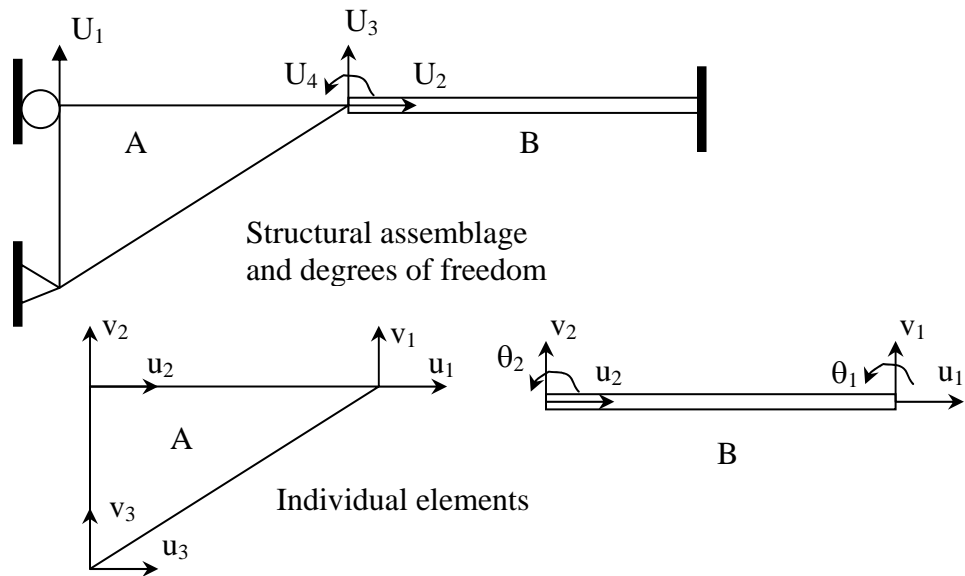
Compute matrix $[A]^{-1}$, displacement transformation matrix.

Compute $[N]$, where $\{u\} = [N]\{q\}$ and $\{q\}$ is vector of nodal degrees of freedom.



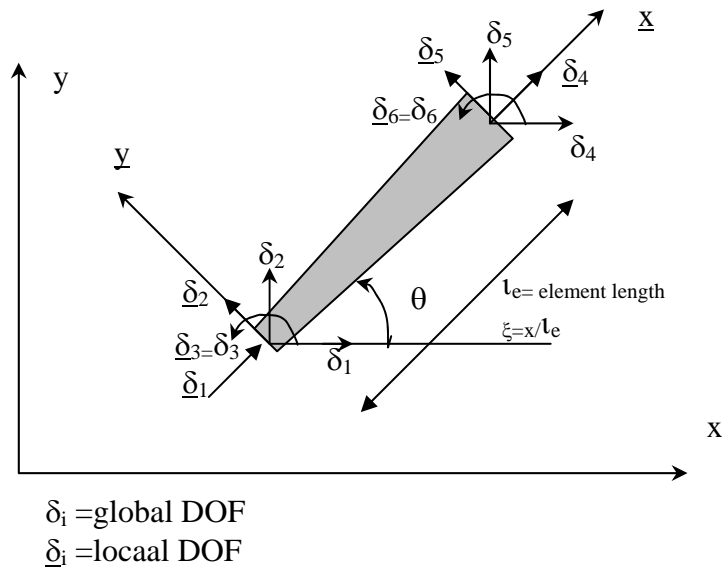
2. Assume that the element stiffness matrices K_A and K_B corresponding to the element displacements shown have been calculated. Assemble these element matrices directly into the global structure stiffness matrix with the displacement boundary condition shown.

$$[K_A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \quad [K_B] = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} & b_{46} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} & b_{56} \\ b_{61} & b_{62} & b_{63} & b_{64} & b_{65} & b_{66} \end{bmatrix}$$



3. Write down an algorithm for obtaining the address of each member stiffness matrix components by skyline method and allocating it in the array of total stiffness matrix. The algorithm must be as exists in the finite element program FEB.For.

4. A tapered beam-column element 1-2 is shown.



There are six degrees of freedom ($\underline{\delta}_1, \underline{\delta}_2, \underline{\delta}_3, \underline{\delta}_4, \underline{\delta}_5, \underline{\delta}_6$) in the local coordinate system \underline{x} and \underline{y} , rotated through an angle θ from x and y (the global) system.

For the tapered element:

$$A(\xi) = (1 - \xi)A_1 + \xi A_2$$

$$I(\xi) = (1 - \xi)I_1 + \xi I_2$$

where A_i 's and I_i 's are the cross sectional areas and the moment of inertias of x -section, respectively, at nodes i ($i=1,2$).

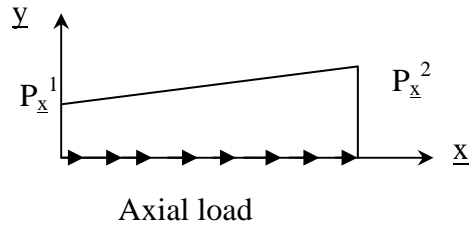
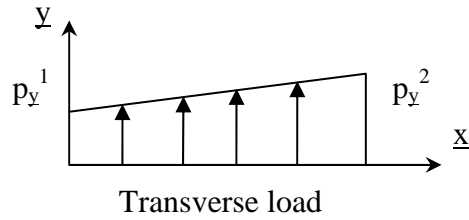
- a) For linear axial displacement and cubic transverse displacement, show that the stiffness matrix in the local \underline{x} and \underline{y} system is given by (E =modulus of elasticity):

$$[K_A] = \begin{bmatrix} \frac{E}{2l_e}(A_1 + A_2) & 0 & 0 & -\frac{E}{2l_e}(A_1 + A_2) & 0 & 0 \\ 0 & \frac{6E}{l_e^3}(I_1 + I_2) & \frac{2E}{l_e^2}(2I_1 + I_2) & 0 & \frac{6E}{l_e^3}(I_1 + I_2) & \frac{2E}{l_e^2}(I_1 + 2I_2) \\ 0 & \frac{2E}{l_e^2}(2I_1 + I_2) & \frac{E}{l_e}(3I_1 + I_2) & 0 & -\frac{2E}{l_e^2}(2I_1 + I_2) & \frac{E}{l_e}(I_1 + I_2) \\ -\frac{E}{2l_e}(A_1 + A_2) & 0 & 0 & \frac{E}{2l_e}(A_1 + A_2) & 0 & 0 \\ 0 & \frac{6E}{l_e^3}(I_1 + I_2) & -\frac{2E}{l_e^2}(2I_1 + I_2) & 0 & \frac{6E}{l_e^3}(I_1 + I_2) & -\frac{2E}{l_e^2}(I_1 + 2I_2) \\ 0 & \frac{2E}{l_e^2}(I_1 + 2I_2) & \frac{E}{l_e}(I_1 + I_2) & 0 & -\frac{2E}{l_e^2}(I_1 + 2I_2) & \frac{E}{l_e}(I_1 + 3I_2) \end{bmatrix}$$

Due to linearly varying transverse and axial loads

$$p_{\underline{y}}(\xi) = (1 - \xi)p_{\underline{y}}^1 + \xi p_{\underline{y}}^2$$

$$p_{\underline{x}}(\xi) = (1 - \xi)p_{\underline{x}}^1 + \xi p_{\underline{x}}^2$$



b) Show that the resulting load vector in the local coordinate is given by the following:

$$\{p\}_{ix6}^T = \left[\frac{l_e}{6} (2p_x^1 + p_x^2) \quad \frac{l_e}{20} (7p_y^1 + p_y^2) \quad \frac{l_e^2}{60} (3p_y^1 + 2p_y^2) \quad \frac{l_e}{6} (p_x^1 + 2p_x^2) \quad \frac{l_e}{20} (3p_y^1 + 7p_y^2) \quad -\frac{l_e^2}{60} (2p_y^1 + 3p_y^2) \right]$$

c) show the transformation matrix for each node as well as [T]. How do you transform the local stiffness matrix and load vector into the global?

Note: Strain energy due to moment and axial force for u as axial displacement and w as transverse displacement are given by:

$$\text{due to moment} = \frac{1}{2} \int_0^l EI w_{xx}^2 dx$$

$$\text{due to axial} = \frac{1}{2} \int_0^l EA u_x^2 dx$$