Chapter 6 • Viscous Flow in Ducts

P6.1 An engineer claims that flow of SAE 30W oil, at 20°C, through a 5-cm-diameter smooth pipe at 1 million N/h, is laminar. Do you agree? A million newtons is a lot, so this sounds like an awfully high flow rate.

Solution: For SAE 30W oil at 20°C (Table A.3), take ρ = 891 kg/m^3 and μ = 0.29 kg/m-s. Convert the weight flow rate to volume flow rate in SI units:

$$Q = \frac{\dot{w}}{\rho g} = \frac{(1E6\,N/h)(1/3600\,h/s)}{(891 kg/m^3)(9.81 m/s^2)} = 0.0318\frac{m^3}{s} = \frac{\pi}{4}(0.05m)^2 V\ , \ solve\ V = 16.2\frac{m}{s}$$

$$Calculate\quad \mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{(891 kg/m^3)(16.2 m/s)(0.05 m)}{0.29\,kg/m-s} \approx \mathbf{2500}\quad (transitional)$$

This is not high, but **not laminar**. *Ans*. With careful inlet design, low disturbances, and a very smooth wall, it might still be laminar, but **No**, this is *transitional*, not definitely laminar.

6.2 Air at approximately 1 atm flows through a horizontal 4-cm-diameter pipe. (a) Find a formula for Q_{max}, the maximum volume flow for which the flow remains laminar, and plot Q_{max} versus temperature in the range 0°C ≤ T ≤ 500°C. (b) Is your plot linear? If not, explain.

Solution: (a) First convert the Reynolds number from a velocity form to a volume flow form:

$$V = \frac{Q}{(\pi/4)d^2},\quad \text{therefore}\quad \mathrm{Re}_d = \frac{\rho V d}{\mu} = \frac{4\rho Q}{\pi\mu d} \le 2300\quad \text{for laminar flow}$$

Maximum laminar volume flow is given by $\mathbf{Q_{max}} = \dfrac{\mathbf{2300\pi d\mu}}{\mathbf{4\rho}}$ *Ans.* (a)

With d = 0.04 m = constant, get μ and ρ for air from Table A-2 and plot Q_{max} versus T °C:

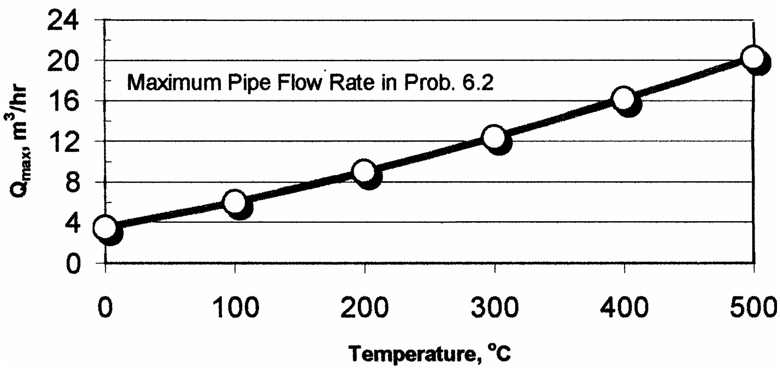


Fig. P6.2

The curve is not quite linear because $\nu = \mu/\rho$ is not quite linear with T for air in this range. *Ans.* (b)

6.3 For a thin wing moving parallel to its chord line, transition to a turbulent boundary layer occurs at a "local" Reynolds number Re_x, where x is the distance from the leading edge of the wing. The critical Reynolds number depends upon the intensity of turbulent fluctuations in the stream and equals 2.8E6 if the stream is very quiet. A semiempirical correlation for this case [Ref. 3 of Ch. 6] is

$$Re_{x_{crit}}^{1/2} \approx \frac{-1+(1+13.25\zeta^2)^{1/2}}{0.00392\zeta^2}$$

where ζ is the tunnel-turbulence intensity in percent. If $V = 20$ m/s in air at 20°C, use this formula to plot the transition position on the wing versus stream turbulence for ζ between 0 and 2 percent. At what value of ζ is x_{crit} decreased 50 percent from its value at $\zeta = 0$?

Solution: This problem is merely to illustrate the *strong* effect of stream turbulence on the transition point. For air at 20°C, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. Compute $Re_{x,crit}$ from the correlation and plot $x_{tr} = \mu Re_x/[\rho(20\text{ m/s})]$ versus percent turbulence:

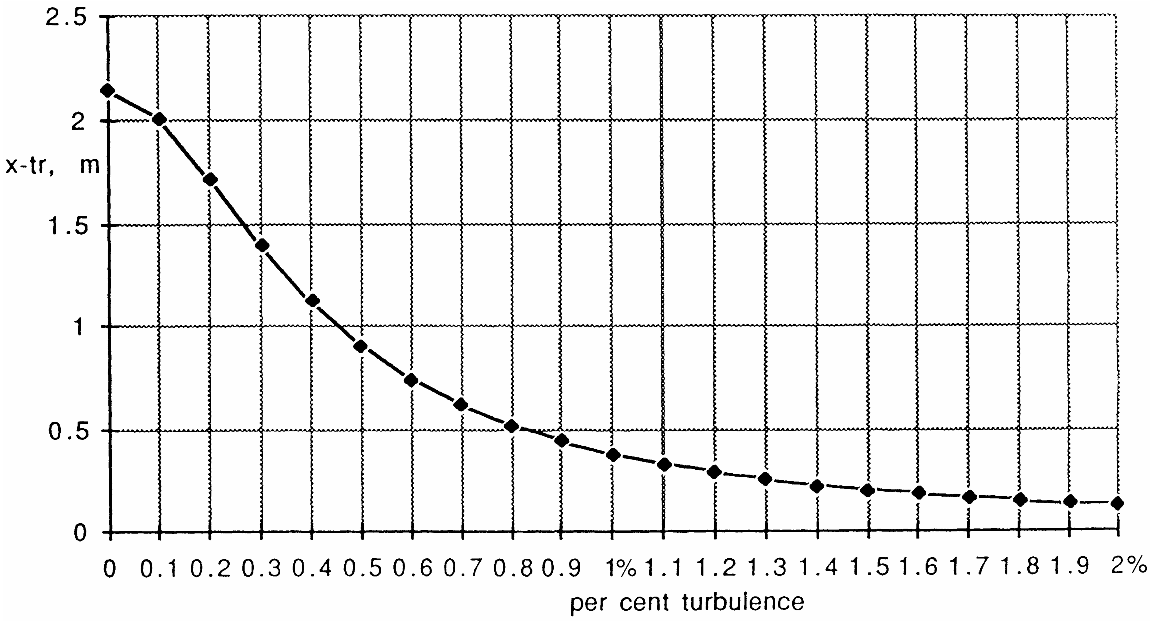


Fig. P6.3

The value of x_{crit} decreases by half (to 1.07 meters) at $\zeta \approx$ **0.42%**. *Ans.*

6.4 For flow of SAE 30 oil through a 5-cm-diameter pipe, from Fig. A.1, for what flow rate in m^3/h would we expect transition to turbulence at (a) 20°C and (b) 100°C?

Solution: For SAE 30 oil take $\rho = 891$ kg/m^3 and take $\mu = 0.29$ kg/m·s at 20°C (Table A.3) and 0.01 kg/m-s at 100°C (Fig A.1). Write the critical Reynolds number in terms of flow rate Q:

$$\text{(a)}\ \ \text{Re}_{crit} = 2300 = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu D} = \frac{4(891\ kg/m^3)Q}{\pi(0.29\ kg/m{\cdot}s)(0.05\ m)},$$

$$\text{solve}\quad Q = 0.0293\ \frac{m^3}{s} = \mathbf{106}\ \frac{\mathbf{m^3}}{\mathbf{h}}\quad \textit{Ans.}\ \text{(a)}$$

$$\text{(b)}\ \ \text{Re}_{crit} = 2300 = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu D} = \frac{4(891\ kg/m^3)Q}{\pi(0.010\ kg/m{\cdot}s)(0.05\ m)},$$

$$\text{solve}\quad Q = 0.00101\ \frac{m^3}{s} = \mathbf{3.6}\ \frac{\mathbf{m^3}}{\mathbf{h}}\quad \textit{Ans.}\ \text{(b)}$$

6.5 In flow past a body or wall, early transition to turbulence can be induced by placing a trip wire on the wall across the flow, as in Fig. P6.5. If the trip wire in Fig. P6.5 is placed where the local velocity is U, it will trigger turbulence if $Ud/\nu = 850$, where d is the wire diameter [Ref. 3 of Ch. 6]. If the sphere diameter is 20 cm and transition is observed at Re_D = 90,000, what is the diameter of the trip wire in mm?

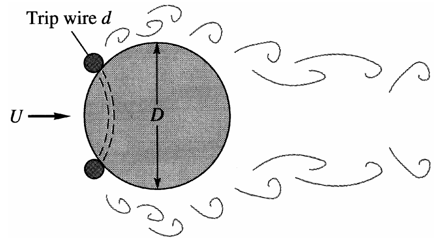


Fig. P6.5

Solution: For the same U and ν,

$$Re_d = \frac{Ud}{\nu} = 850; \quad Re_D = \frac{UD}{\nu} = 90000,$$

$$\text{or} \quad d = D\frac{Re_d}{Re_D} = (200\text{ mm})\left(\frac{850}{90000}\right) \approx \mathbf{1.9\ mm}$$

P6.6 For flow of a uniform stream parallel to a sharp flat plate, transition to a turbulent boundary layer on the plate may occur at $Re_x = \rho Ux/\mu \approx$ 1E6, where U is the approach velocity and x is distance along the plate. If U = 2.5 m/s, determine the distance x for the following fluids at 20°C and 1 atm: (*a*) hydrogen; (*b*) air; (*c*) gasoline; (*d*) water; (*e*) mercury; and (*f*) glycerin.

Solution: We are to calculate $x = (Re_x)(\mu)/(\rho U) = (1E6)(\mu)/[\rho\,(2.5\text{m/s})]$. Make a table:

FLUID	ρ – kg/m^3	μ - kg/m-s	x - meters
Hydrogen	0.00839	9.05E-5	43.
Air	1.205	1.80E-5	6.0
Gasoline	680	2.92E-4	0.17
Water	998	0.0010	0.40

Mercury	13,550	1.56E-3	0.046
Glycerin	1260	1.49	470.

Clearly there are vast differences between fluid properties and their effects on flows.

6.7 Cola, approximated as pure water at 20°C, is to fill an 8-oz container (1 U.S. gal = 128 fl oz) through a 5-mm-diameter tube. Estimate the minimum filling time if the tube flow is to remain laminar. For what cola (water) temperature would this minimum time be 1 min?

Solution: For cola "water", take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. Convert 8 fluid ounces = (8/128)(231 in^3) ≈ 2.37E–4 m^3. Then, if we assume transition at Re = 2300,

$$\mathrm{Re}_{crit} = 2300 = \frac{\rho V D}{\mu} = \frac{4\rho Q}{\pi\mu D}, \quad \text{or:} \quad Q_{crit} = \frac{2300\pi(0.001)(0.005)}{4(998)} \approx 9.05\text{E–}6\ \frac{\mathrm{m}^3}{\mathrm{s}}$$

Then $\Delta t_{fill} = \upsilon/Q = 2.37\text{E–}4/9.05\text{E–}6 \approx$ **26 s** *Ans.* (a)

(b) We fill in exactly one minute if $Q_{crit} = 2.37\text{E–}4/60 = 3.94\text{E–}6$ m^3/s. Then

$$Q_{crit} = 3.94\text{E–}6\ \frac{\mathrm{m}^3}{\mathrm{s}} = \frac{2300\pi\nu D}{4} \quad \text{if} \quad \nu_{water} \approx 4.36\text{E–}7\ \mathrm{m}^2/\mathrm{s}$$

From Table A-1, this kinematic viscosity occurs at **T ≈ 66°C** *Ans.* (b)

6.8 When water at 20°C ($\rho = 998$ kg/m^3, $\mu = 0.001$ kg/m·s) flows through an 8-cm-diameter pipe, the wall shear stress is 72 Pa. What is the axial pressure gradient ($\partial p/\partial x$) if the pipe is (a) horizontal; and (b) vertical with the flow *up*?

Solution: Equation (6.9b) applies in both cases, noting that τ_w is negative:

$$\text{(a)} \quad \textit{Horizontal}: \quad \frac{dp}{dx} = \frac{2\tau_w}{R} = \frac{2(-72\ Pa)}{0.04\ m} = \mathbf{-3600\ \frac{Pa}{m}} \quad \textit{Ans. (a)}$$

$$\text{(b)} \quad \textit{Vertical, up}: \quad \frac{dp}{dx} = \frac{2\tau_w}{R} - \rho g \frac{dz}{dx}^{\nearrow 1} = -3600 - 998(9.81) = \mathbf{-13,400\ \frac{Pa}{m}} \quad \textit{Ans. (b)}$$

6.9 A light liquid (ρ = 950 kg/m^3) flows at an average velocity of 10 m/s through a horizontal smooth tube of diameter 5 cm. The fluid pressure is measured at 1-m intervals along the pipe, as follows:

x, m:	0	1	2	3	4	5	6
p, kPa:	304	273	255	240	226	213	200

Estimate (a) the total head loss, in meters; (b) the wall shear stress in the fully developed section of the pipe; and (c) the overall friction factor.

Solution: As sketched in Fig. 6.6 of the text, the pressure drops fast in the entrance region (31 kPa in the first meter) and levels off to a linear decrease in the "fully developed" region (13 kPa/m for this data).
(a) The overall head loss, for $\Delta z = 0$, is defined by Eq. (6.8) of the text:

$$h_f = \frac{\Delta p}{\rho g} = \frac{304{,}000 - 200{,}000\ Pa}{(950\ kg/m^3)(9.81\ m/s^2)} = \mathbf{11.2\ m} \quad \textit{Ans.}\ \text{(a)}$$

(b) The wall shear stress in the fully-developed region is defined by Eq. (6.9*b*):

$$\left.\frac{\Delta p}{\Delta L}\right|_{fully\ developed} = \frac{13000\ Pa}{1\ m} = \frac{4\tau_w}{d} = \frac{4\tau_w}{0.05\ m}, \quad \text{solve for} \quad \tau_w = \mathbf{163\ Pa} \quad \textit{Ans.}\ \text{(b)}$$

(c) The overall friction factor is defined by Eq. (6.10) of the text:

$$f_{overall} = h_{f,overall}\frac{d}{L}\frac{2g}{V^2} = (11.2\ m)\left(\frac{0.05\ m}{6\ m}\right)\frac{2(9.81\ m/s^2)}{(10\ m/s)^2} = \mathbf{0.0182} \quad \textit{Ans.}\ \text{(c)}$$

NOTE: The fully-developed friction factor is only 0.0137.

6.10 Water at 20°C (ρ = 998 kg/m^3) flows through an inclined 8-cm-diameter pipe. At sections A and B, p_A = 186 kPa, V_A = 3.2 m/s, z_A = 24.5 m, while p_B = 260 kPa, V_B = 3.2 m/s, and z_B = 9.1 m. Which way is the flow going? What is the head loss?

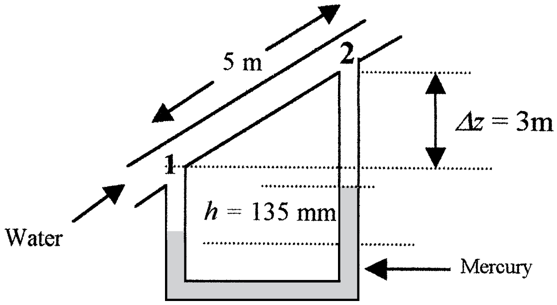
Solution: Guess that the flow is from A to B and write the steady flow energy equation:

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B + h_f, \quad \text{or:} \quad \frac{186000}{9790} + 24.5 = \frac{260000}{9790} + 9.1 + h_f,$$

$$\textit{or}: \quad 43.50 = 35.66 + h_f, \quad \text{solve: } \mathbf{h_f = +7.84\ m} \quad \textit{Yes, flow is from A to B.} \quad \textit{Ans.}\ \text{(a, b)}$$

6.11 Water at 20°C flows upward at 4 m/s in a 6-cm-diameter pipe. The pipe length between points 1 and 2 is 5 m, and point 2 is 3 m higher. A mercury manometer, connected between 1 and 2, has a reading h = 135 mm, with p_1 higher. (a) What is the pressure change ($p_1 - p_2$)? (b) What is the head loss, in meters? (c) Is the manometer reading proportional to head loss? Explain. (d) What is the friction factor of the flow?

Solution: A sketch of this situation is shown at right. By moving through the manometer, we obtain the pressure change between points 1 and 2, which we compare with Eq. (6.9*b*):



$$p_1 + \gamma_w h - \gamma_m h - \gamma_w \Delta z = p_2,$$

$$\textbf{or:}\quad p_1 - p_2 = \left(133100 - 9790\ \frac{N}{m^3}\right)(0.135\ m) + \left(9790\ \frac{N}{m^3}\right)(3\ m)$$

$$= 16650 + 29370 = \mathbf{46{,}000\ Pa} \quad \textit{Ans.}\,\text{(a)}$$

$$\textit{From Eq. (6.9b)},\quad h_f = \frac{\Delta p}{\gamma_w} - \Delta z = \frac{46000\ Pa}{9790\ N/m^3} - 3\ m = 4.7 - 3.0 = \mathbf{1.7\ m} \quad \textit{Ans.}\ \text{(b)}$$

$$\textit{The friction factor is}\quad f = h_f \frac{d}{L}\frac{2g}{V^2} = (1.7\ m)\left(\frac{0.06\ m}{5\ m}\right)\frac{2(9.81\ m/s^2)}{(4\ m/s)^2} = \mathbf{0.025} \quad \textit{Ans.}\,\text{(d)}$$

By comparing the manometer relation to the head-loss relation above, we find that:

$$h_f = \frac{(\gamma_m - \gamma_w)}{\gamma_w} h$$ ***and thus head loss*** **is** ***proportional to manometer reading.*** *Ans.* (c)

NOTE: IN PROBLEMS 6.12 TO 6.99, MINOR LOSSES ARE NEGLECTED.

6.12 A 5-mm-diameter capillary tube is used as a viscometer for oils. When the flow rate is 0.071 m^3/h, the measured pressure drop per unit length is 375 kPa/m. Estimate the viscosity of the fluid. Is the flow laminar? Can you also estimate the density of the fluid?

Solution: Assume laminar flow and use the pressure drop formula (6.12):

$$\frac{\Delta p}{L} \overset{?}{=} \frac{8Q\mu}{\pi R^4},\quad \text{or:}\quad 375000\ \frac{Pa}{m} \overset{?}{=} \frac{8(0.071/3600)\mu}{\pi(0.0025)^4},\quad \text{solve}\quad \mu \approx \mathbf{0.292\ \frac{kg}{m{\cdot}s}} \quad \textit{Ans.}$$

$$\text{Guessing}\quad \rho_{oil} \approx 900\ \frac{kg}{m^3},$$

$$\text{check}\quad Re = \frac{4\rho Q}{\pi\mu d} = \frac{4(900)(0.071/3600)}{\pi(0.292)(0.005)} \approx \mathbf{16}\quad \textbf{OK, laminar}\quad \textit{Ans.}$$

It is not possible to find density from this data, laminar pipe flow is independent of density.

6.13 A soda straw is 20 cm long and 2 mm in diameter. It delivers cold cola, approximated as water at 10°C, at a rate of 3 cm^3/s. (a) What is the head loss through the straw? What is the axial pressure gradient $\partial p/\partial x$ if the flow is (b) vertically up or (c) horizontal? Can the human lung deliver this much flow?

Solution: For water at 10°C, take $\rho = 1000\ kg/m^3$ and $\mu = 1.307E{-}3$ kg/m·s. Check Re:

$$Re = \frac{4\rho Q}{\pi\mu d} = \frac{4(1000)(3E{-}6\ m^3/s)}{\pi(1.307E{-}3)(0.002)} = 1460\quad \text{(OK, laminar flow)}$$

$$\text{Then, from Eq. (6.12),}\quad h_f = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(1.307E{-}3)(0.2)(3E{-}6)}{\pi(1000)(9.81)(0.002)^4} \approx \mathbf{0.204\ m}\quad \textit{Ans. (a)}$$

If the straw is *horizontal*, then the pressure gradient is simply due to the head loss:

$$\frac{\Delta p}{L}\bigg|_{horiz} = \frac{\rho g h_f}{L} = \frac{1000(9.81)(0.204\ m)}{0.2\ m} \approx \mathbf{9980\ \frac{Pa}{m}}\quad \textit{Ans. (c)}$$

If the straw is *vertical*, with flow *up*, the head loss and elevation change add together:

$$\frac{\Delta p}{L}\bigg|_{vertical} = \frac{\rho g(h_f + \Delta z)}{L} = \frac{1000(9.81)(0.204 + 0.2)}{0.2} \approx \mathbf{19800\ \frac{Pa}{m}}\quad \textit{Ans. (b)}$$

The human lung can certainly deliver case (c) and strong lungs can develop case (b) also.

6.14 Water at 20°C is to be siphoned through a tube 1 m long and 2 mm in diameter, as in Fig. P6.14. Is there any height H for which the flow might not be laminar? What is the flow rate if $H = 50$ cm? Neglect the tube curvature.

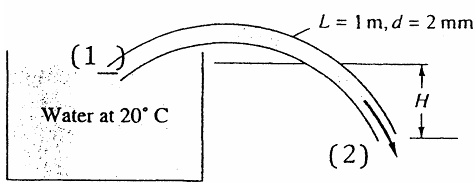


Fig. P6.14

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. Write the steady flow energy equation between points 1 and 2 above:

$$\frac{p_{atm}}{\rho g}+\frac{0^2}{2g}+z_1=\frac{p_{atm}}{\rho g}+\frac{V_{tube}^2}{2g}+z_2+h_f,\quad \text{or:}\quad H-\frac{V^2}{2g}=h_f=\frac{32\mu L}{\rho g d^2}V \qquad (1)$$

Enter data in Eq. (1): $0.5-\dfrac{V^2}{2(9.81)}=\dfrac{32(0.001)(1.0)V}{(998)(9.81)(0.002)^2}$, solve $V \approx 0.590\ \dfrac{m}{s}$

Equation (1) is quadratic in V and has only one positive root. The siphon flow rate is

$$Q_{H=50\text{ cm}}=\frac{\pi}{4}(0.002)^2(0.590)=1.85E{-}6\ \frac{m^3}{s}\approx \mathbf{0.0067\ \frac{m^3}{h}}\quad \textbf{if } \boldsymbol{H} \textbf{ = 50 cm}\quad \textit{Ans.}$$

$$\text{Check}\quad Re=(998)(0.590)(0.002)/(0.001)\approx 1180\ \text{(OK, laminar flow)}$$

It is possible to approach Re ≈ 2000 (possible transition to turbulent flow) for H < 1 m, for the case of the siphon bent over nearly vertical. We obtain **Re = 2000 at H ≈ 0.87 m**.

6.15 Professor Gordon Holloway and his students at the University of New Brunswick went to a fast-food emporium and tried to drink chocolate shakes ($\rho \approx 1200$ kg/m^3, $\mu \approx 6$ kg/m·s) through fat straws 8 mm in diameter and 30 cm long. (a) Verify that their human lungs, which can develop approximately 3000 Pa of vacuum pressure, would be unable to drink the milkshake through the vertical straw. (b) A student cut 15 cm from his straw and proceeded to drink happily. What rate of milkshake flow was produced by this strategy?

Solution: (a) Assume the straw is barely inserted into the milkshake. Then the energy equation predicts

$$\frac{p_1}{\rho g}=\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}=\frac{V_2^2}{2g}+z_2+h_f$$

$$=0+0+0=\frac{(-3000\ Pa)}{(1200\ kg/m^3)(9.81\ m/s^2)}+\frac{V_{tube}^2}{2g}+0.3\ m+h_f$$

$$\textit{Solve for}\quad \boldsymbol{h_f=0.255\ m-0.3\ m-\frac{V_{tube}^2}{2g}<0}\quad \textbf{\textit{which is impossible}}\quad \textit{Ans.}\text{ (a)}$$

(b) By cutting off 15 cm of vertical length and assuming laminar flow, we obtain a new energy equation

$$h_f = 0.255 - 0.15 - \frac{V^2}{2g} = \frac{32\mu LV}{\rho g d^2} = 0.105\ m - \frac{V^2}{2(9.81)} = \frac{32(6.0)(0.15)V}{(1200)(9.81)(0.008)^2} = 38.23V$$

Solve for $V = 0.00275\ m/s, \quad Q = AV = (\pi/4)(0.008)^2(0.00275)$

$$Q = 1.4E\text{–}7\ \frac{m^3}{s} = \mathbf{0.14\ \frac{cm^3}{s}} \quad \textit{Ans.}\ \text{(b)}$$

Check the Reynolds number: $\text{Red} = \rho Vd/\mu = (1200)(0.00275)(0.008)/(6) = 0.0044$ (Laminar).

6.16 Glycerin at 20°C is to be pumped through a horizontal smooth pipe at 3.1 m^3/s. It is desired that (1) the flow be laminar and (2) the pressure drop be no more than 100 Pa/m. What is the minimum pipe diameter allowable?

Solution: For glycerin at 20°C, take $\rho = 1260$ kg/m^3 and $\mu = 1.49$ kg/m·s. We have two different constraints to satisfy, a pressure drop and a Reynolds number:

$$\frac{\Delta p}{L} = \frac{128\mu Q}{\pi d^4} \le 100\ \frac{Pa}{m} \quad (1); \quad \frac{128(1.49)(3.1)}{\pi d^4} \le 100, \quad \mathbf{d \ge 1.17\ m},$$

$$\text{or:} \quad Re = \frac{4\rho Q}{\pi \mu d} \le 2000 \quad (2); \quad \frac{4(1260)(3.1)}{\pi(1.49)d} \le 2000, \quad \mathbf{d \ge 1.67\ m}$$

The second of these is more restrictive. Thus the proper diameter is **d ≥ 1.67 m**. *Ans.*

6.17 A *capillary viscometer* measures the time required for a specified volume υ of liquid to flow through a small-bore glass tube, as in Fig. P6.17. This transit time is then correlated with fluid viscosity. For the system shown, (a) derive an approximate formula for the time required, assuming laminar flow with no entrance and exit losses. (b) If $L = 12$ cm, $l = 2$ cm, $\upsilon = 8$ cm^3, and the fluid is water at 20°C, what capillary diameter D will result in a transit time t of 6 seconds?

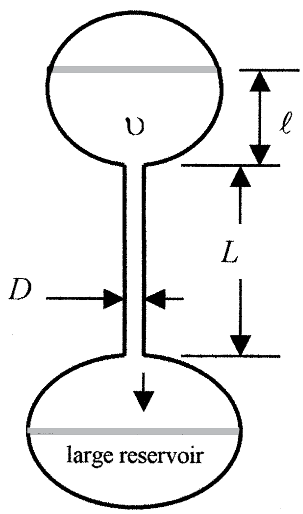


Fig. P6.17

Solution: (a) Assume no pressure drop and neglect velocity heads. The energy equation reduces to:

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=0+0+(L+l)=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f=0+0+0+h_f, \quad \textbf{or}: \quad h_f \approx L+l$$

$$\textit{For laminar flow,} \quad h_f=\frac{128\mu LQ}{\pi\rho g d^4} \quad \textit{and, for uniform draining,} \ Q=\frac{\upsilon}{\Delta t}$$

$$\textit{Solve for} \quad \mathbf{\Delta t=\frac{128\mu L\upsilon}{\pi\rho g d^4(L+l)}} \quad \textit{Ans.} \text{ (a)}$$

(b) Apply to $\Delta t = 6$ s. For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. Formula (a) predicts:

$$\Delta t = 6\ s = \frac{128(0.001\ kg/m{\cdot}s)(0.12\ m)(8E\text{–}6\ m^3)}{\pi(998\ kg/m^3)(9.81\ m/s^2)d^4(0.12+0.02\ m)},$$

Solve for **d ≈ 0.0015 m** *Ans.* (b)

6.18 To determine the viscosity of a liquid of specific gravity 0.95, you fill, to a depth of 12 cm, a large container which drains through a 30-cm-long vertical tube attached to the bottom. The tube diameter is 2 mm, and the rate of draining is found to be 1.9 cm^3/s. What is your estimate of the fluid viscosity? Is the tube flow laminar?

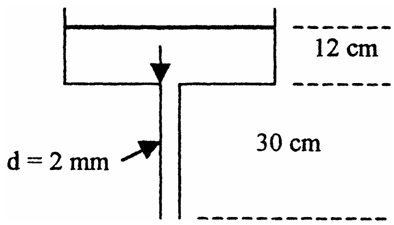


Fig. P6.18

Solution: The known flow rate and diameter enable us to find the velocity in the tube:

$$V=\frac{Q}{A}=\frac{1.9E\text{–}6\ m^3/s}{(\pi/4)(0.002\ m)^2}=0.605\ \frac{m}{s}$$

Evaluate $\rho_{\text{liquid}} = 0.95(998) = 948$ kg/m^3. Write the energy equation between the top surface and the tube exit:

$$\cancel{\frac{p_a}{\rho g}}=\cancel{\frac{V_{top}^2}{2g}}+z_{top}=\cancel{\frac{p_a}{\rho g}}+\frac{V^2}{2g}+0+h_f,$$

$$or: \quad 0.42=\frac{V^2}{2g}+\frac{32\mu LV}{\rho g d^2}=\frac{(0.605)^2}{2(9.81)}+\frac{32\mu(0.3)(0.605)}{948(9.81)(0.002)^2}$$

Note that "L" in this expression is the tube length only (L = 30 cm).

$$\text{Solve for } \mu = \mathbf{0.00257\ \frac{kg}{m{\cdot}s}} \ (\textit{laminar flow}) \quad \textit{Ans.}$$

$$Re_d = \frac{\rho V d}{\mu} = \frac{948(0.605)(0.002)}{0.00257} = 446\ (\textit{laminar})$$

6.19 An oil (SG = 0.9) issues from the pipe in Fig. P6.19 at $Q = 35\ \text{ft}^3/\text{h}$. What is the kinematic viscosity of the oil in ft^3/s? Is the flow laminar?

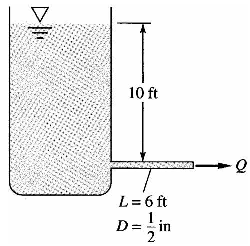


Fig. P6.19

Solution: Apply steady-flow energy:

$$\frac{p_{atm}}{\rho g} + \frac{0^2}{2g} + z_1 = \frac{p_{atm}}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f,$$

$$\text{where} \quad V_2 = \frac{Q}{A} = \frac{35/3600}{\pi(0.25/12)^2} \approx 7.13\ \frac{\text{ft}}{\text{s}}$$

$$\text{Solve} \quad h_f = z_1 - z_2 - \frac{V_2^2}{2g} = 10 - \frac{(7.13)^2}{2(32.2)} = 9.21\ \text{ft}$$

Assuming laminar pipe flow, use Eq. (6.12) to relate head loss to viscosity:

$$h_f = 9.21\ \text{ft} = \frac{128\nu LQ}{\pi g d^4} = \frac{128(6)(35/3600)\nu}{\pi(32.2)(0.5/12)^4}, \quad \text{solve } \nu = \frac{\mu}{\rho} \approx \mathbf{3.76E{-}4\ \frac{ft^2}{s}} \quad \textit{Ans.}$$

Check $Re = 4Q/(\pi\nu d) = 4(35/3600)/[\pi(3.76\text{E}{-}4)(0.5/12)] \approx 790$ (OK, laminar)

P6.20 The oil tanks in Tinyland are only 160 cm high, and they discharge to the Tinyland oil truck through a smooth tube 4 mm in diameter and 55 cm long. The tube exit is open to the atmosphere and 145 cm below the tank surface. The fluid is medium fuel oil, $\rho = 850\ \text{kg/m}^3$ and $\mu = 0.11$ kg/m-s. Estimate the oil flow rate in cm^3/h.

Solution: The steady flow energy equation, with 1 at the tank surface and 2 the exit, gives

$$z_1 = z_2 + \frac{\alpha V^2}{2g} + f\frac{L}{d}\frac{V^2}{2g}, \; or: \; \Delta z = 1.45m = \frac{V^2}{2g}(2.0 + \frac{64}{\mathrm{Re}_d}\frac{0.55m}{0.004m}), \; \mathrm{Re}_d = \frac{850V(0.004)}{0.11}$$

We have taken the energy correction factor $\alpha = 2.0$ for laminar pipe flow.

Solve for $V = 0.10$ m/s, $\mathrm{Re}_d = 3.1$ (laminar), $Q = 1.26\text{E-}6\ \mathrm{m^3/s} \approx$ **4500 $\mathbf{cm^3/h}$**. *Ans.*

The exit jet energy $\alpha V^2/2g$ is properly included but is very small (0.001 m).

6.21 In Tinyland, houses are less than a foot high! The rainfall is laminar! The drainpipe in Fig. P6.21 is only 2 mm in diameter. (a) When the gutter is full, what is the rate of draining? (b) The gutter is designed for a sudden rainstorm of up to 5 mm per hour. For this condition, what is the maximum roof area that can be drained successfully? (c) What is Red?

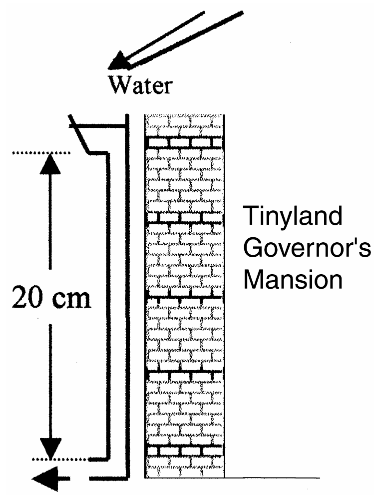


Fig. P6.21

Solution: If the velocity at the gutter surface is neglected, the energy equation reduces to

$$\Delta z = \frac{V^2}{2g} + h_f, \quad \text{where } h_{f,\text{laminar}} = \frac{32\mu LV}{\rho g d^2}$$

For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. (a) With Δz known, this is a quadratic equation for the pipe velocity V:

$$0.2\ m = \frac{V^2}{2(9.81\ m/s^2)} + \frac{32(0.001\ kg/m{\cdot}s)(0.2\ m)V}{(998\ kg/m^3)(9.81\ m/s^2)(0.002\ m)^2},$$

$$\text{or:} \quad 0.051V^2 + 0.1634V - 0.2 = 0, \quad \textit{Solve for } V = 0.945\ \frac{m}{s},$$

$$Q = \frac{\pi}{4}(0.002\ m)^2\left(0.945\ \frac{m}{s}\right) = 2.97E\text{–}6\ \frac{m^3}{s} = \mathbf{0.0107\ \frac{m^3}{h}} \quad \textit{Ans.}\text{ (a)}$$

(b) The roof area needed for maximum rainfall is 0.0107 $m^3/h \div 0.005$ m/h = **2.14 m^2**. *Ans.* (b)
(c) The Reynolds number of the gutter is Re_d = (998)(0.945)(0.002)/(0.001) = **1890** *laminar*. *Ans.* (c)

6.22 A steady push on the piston in Fig. P6.22 causes a flow rate Q = 0.15 cm^3/s through the needle. The fluid has $\rho = 900\ kg/m^3$ and μ = 0.002 kg/(m·s). What force F is required to maintain the flow?

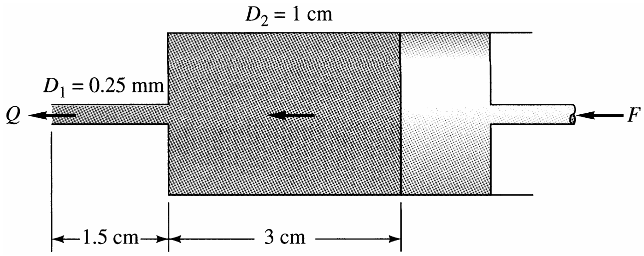


Fig. P6.22

Solution: Determine the velocity of exit from the needle and then apply the steady-flow energy equation:

$$V_1 = \frac{Q}{A} = \frac{0.15}{(\pi/4)(0.025)^2} = 306 \text{ cm/s}$$

$$\text{Energy: } \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_{f1} + h_{f2}, \quad \text{with } z_1 = z_2,\ V_2 \approx 0,\ h_{f2} \approx 0$$

Assume laminar flow for the head loss and compute the pressure difference on the piston:

$$\frac{p_2 - p_1}{\rho g} = h_{f1} + \frac{V_1^2}{2g} = \frac{32(0.002)(0.015)(3.06)}{(900)(9.81)(0.00025)^2} + \frac{(3.06)^2}{2(9.81)} \approx 5.79 \text{ m}$$

$$\text{Then} \quad F = \Delta p A_{piston} = (900)(9.81)(5.79)\frac{\pi}{4}(0.01)^2 \approx \mathbf{4.0\ N} \quad \textit{Ans.}$$

6.23 SAE 10 oil at 20°C flows in a vertical pipe of diameter 2.5 cm. It is found that the pressure is constant throughout the fluid. What is the oil flow rate in m^3/h? Is the flow up or down?

Solution: For SAE 10 oil, take $\rho = 870\ kg/m^3$ and μ = 0.104 kg/m·s. Write the energy equation between point 1 upstream and point 2 downstream:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f, \quad \text{with } p_1 = p_2 \text{ and } V_1 = V_2$$

Thus $h_f = z_1 - z_2 > 0$ by definition. Therefore, **flow is down**. *Ans.*

While flowing down, the pressure drop due to friction exactly balances the pressure rise due to gravity. Assuming laminar flow and noting that Δz = L, the pipe length, we get

$$h_f = \frac{128\mu LQ}{\pi\rho g d^4} = \Delta z = L,$$

$$\text{or:}\quad Q = \frac{\pi(8.70)(9.81)(0.025)^4}{128(0.104)} = 7.87E\text{–}4\ \frac{m^3}{s} = \mathbf{2.83\ \frac{m^3}{h}}\quad \textit{Ans.}$$

6.24 Two tanks of water at 20°C are connected by a capillary tube 4 mm in diameter and 3.5 m long. The surface of tank 1 is 30 cm higher than the surface of tank 2. (a) Estimate the flow rate in m^3/h. Is the flow laminar? (b) For what tube diameter will Re_d be 500?

Solution: For water, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. (a) Both tank surfaces are at atmospheric pressure and have negligible velocity. The energy equation, when neglecting minor losses, reduces to:

$$\Delta z = 0.3\ m = h_f = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(0.001\ kg/m\cdot s)(3.5\ m)Q}{\pi(998\ kg/m^3)(9.81\ m/s^2)(0.004\ m)^4}$$

$$\text{Solve for}\quad Q = 5.3E\text{–}6\ \frac{m^3}{s} = \mathbf{0.019\ \frac{m^3}{h}}\quad \textit{Ans.}\text{ (a)}$$

$$\text{Check}\quad Re_d = 4\rho Q/(\pi\mu d) = 4(998)(5.3E\text{–}6)/[\pi(0.001)(0.004)]$$

$$\mathbf{Re_d = 1675}\ \ \textbf{\textit{laminar.}}\quad \textit{Ans.}\text{ (a)}$$

(b) If $Re_d = 500 = 4\rho Q/(\pi\mu d)$ and $\Delta z = h_f$, we can solve for both Q and d:

$$Re_d = 500 = \frac{4(998\ kg/m^3)Q}{\pi(0.001\ kg/m\cdot s)d},\quad \textbf{\textit{or}}\quad Q = 0.000394d$$

$$h_f = 0.3\ m = \frac{128(0.001\ kg/m\cdot s)(3.5\ m)Q}{\pi(998\ kg/m^3)(9.81\ m/s^2)d^4},\quad \textit{or}\quad Q = 20600d^4$$

Combine these two to solve for $Q = 1.05E\text{–}6\ m^3/s$ *and* **d = 2.67 mm** *Ans.* (b)

6.25 For the configuration shown in Fig. P6.25, the fluid is ethyl alcohol at 20°C, and the tanks are very wide. Find the flow rate which occurs in m^3/h. Is the flow laminar?

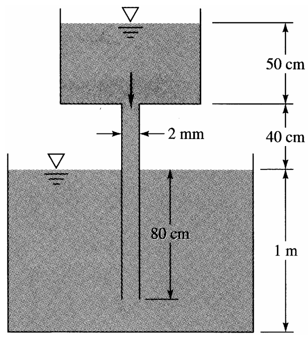


Fig. P6.25

Solution: For ethanol, take ρ = 789 kg/m^3 and μ = 0.0012 kg/m·s. Write the energy equation from upper free surface (1) to lower free surface (2):

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f, \quad \text{with } p_1=p_2 \text{ and } V_1\approx V_2\approx 0$$

$$\text{Then}\quad h_f=z_1-z_2=0.9\text{ m}=\frac{128\mu LQ}{\pi\rho g d^4}=\frac{128(0.0012)(1.2\text{ m})Q}{\pi(789)(9.81)(0.002)^4}$$

Solve for $Q\approx 1.90\text{E–}6\ m^3/s$ = $\mathbf{0.00684\ m^3/h.}$ *Ans.*

Check the Reynolds number $Re = 4\rho Q/(\pi\mu d) \approx 795$ – **OK, laminar flow.**

P6.26 Two oil tanks are connected by two 9-m-long pipes, as in Fig. P6.26. Pipe 1 is 5 cm in diameter and is 6 m higher than pipe 2. It is found that the flow rate in pipe 2 is twice as large as the flow in pipe 1. (*a*) What is the diameter of pipe 2? (*b*) Are both pipe flows laminar? (*c*) What is the flow rate in pipe 2 (m^3/s)?

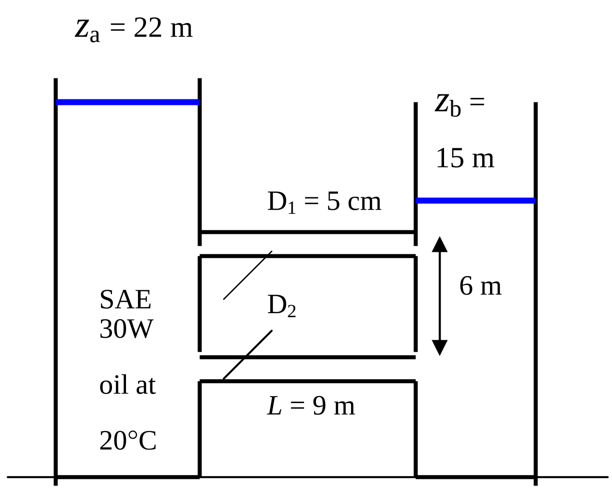


Fig. P6.26

Neglect minor losses.

Solution: (*a*) If we *know* the flows are laminar, and (L, ρ, μ) are constant, then $Q \propto D^4$:

$$\text{From Eq.(6.12), } \frac{Q_2}{Q_1} = 2.0 = \left(\frac{D_2}{D_1}\right)^4, \text{ hence } D_2 = (5\,cm)(2.0)^{1/4} = \mathbf{5.95\,cm} \quad Ans.(a)$$

We will check later in part (*b*) to be sure the flows are laminar. [Placing pipe 1 six meters higher was meant to be a confusing trick, since both pipes have exactly the same head loss and Δz.] (*c*) Find the flow rate first and then backtrack to the Reynolds numbers. For SAE 30W oil at 20°C (Table A.3), take ρ = 891 kg/m^3 and μ = 0.29 kg/m-s. From the energy equation, with $V_1 = V_2 = 0$, and Eq. (6.12) for the laminar head loss,

$$\Delta z = 22 - 15 = 7\,m = h_f = \frac{128\mu LQ}{\pi\rho g D_2^4} = \frac{128(0.29\,kg/m-s)(9m)\,Q_2}{\pi(891\,kg/m^3)(9.81\,m/s^2)(0.0595\,m)^4}$$

$$\text{Solve for} \quad Q_2 = \mathbf{0.0072\,m^3/s} \quad Ans.(c)$$

In a similar manner, insert D_1 = 0.05m and compute Q_1 = **0.0036** m^3/s = (1/2)Q_1.

(*b*) Now go back and compute the Reynolds numbers:

$$\text{Re}_1 = \frac{4\rho Q_1}{\pi\mu D_1} = \frac{4(891)(0.0036)}{\pi(0.29)(0.050)} = \mathbf{281} \;;\; \text{Re}_2 = \frac{4\rho Q_2}{\pi\mu D_2} = \frac{4(891)(0.0072)}{\pi(0.29)(0.0595)} = \mathbf{473} \quad Ans.(b)$$

Both flows are **laminar**, which verifies our flashy calculation in part (*a*).

6.27 Let us attack Prob. 6.25 in symbolic fashion, using Fig. P6.27. All parameters are constant except the upper tank depth *Z*(*t*). Find an expression for the flow rate *Q*(*t*) as a function of *Z*(*t*). Set up a differential equation, and solve for the time *t*0 to drain the upper tank completely. Assume quasi-steady laminar flow.

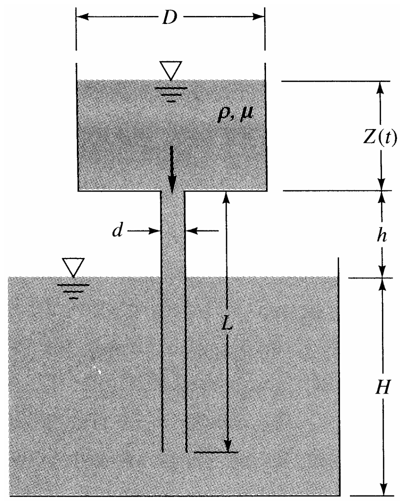


Fig. P6.27

Solution: The energy equation of Prob. 6.25, using symbols only, is combined with a control-volume mass balance for the tank to give the basic differential equation for Z(t):

$$\text{energy: } h_f = \frac{32\mu LV}{\rho g d^2} = h + Z; \quad \text{mass balance: } \frac{d}{dt}\left[\frac{\pi}{4}D^2 Z + \frac{\pi}{4}d^2 L\right] = -Q = -\frac{\pi}{4}d^2 V,$$

$$\text{or: } \frac{\pi}{4}D^2\frac{dZ}{dt} = -\frac{\pi}{4}d^2 V, \quad \text{where } V = \frac{\rho g d^2}{32\mu L}(h+Z)$$

Separate the variables and integrate, combining all the constants into a single "C":

$$\int_{Z_o}^{Z}\frac{dZ}{h+Z} = -C\int_0^t dt, \quad \text{or: } \mathbf{Z = (h + Z_o)e^{-Ct} - h}, \quad \text{where } \mathbf{C = \frac{\rho g d^4}{32\mu L D^2}} \quad \textit{Ans.}$$

$$\text{Tank drains completely when } Z = 0, \text{ at } \mathbf{t_0 = \frac{1}{C}\ln\left(1 + \frac{Z_o}{h}\right)} \quad \textit{Ans.}$$

6.28 For straightening and smoothing an airflow in a 50-cm-diameter duct, the duct is packed with a "honeycomb" of thin straws of length 30 cm and diameter 4 mm, as in Fig. P6.28. The inlet flow is air at 110 kPa and 20°C, moving at an average velocity of 6 m/s. Estimate the pressure drop across the honeycomb.

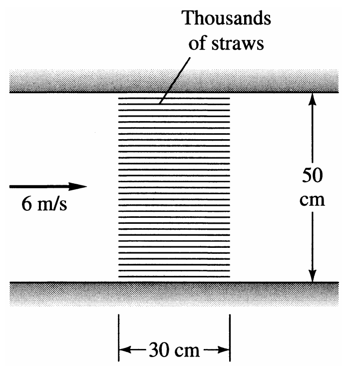


Fig. P6.28

Solution: For air at 20°C, take $\mu \approx$ 1.8E–5 kg/m·s and ρ = 1.31 kg/m^3. There would be approximately 12000 straws, but each one would see the average velocity of 6 m/s. Thus

$$\Delta p_{laminar} = \frac{32\mu LV}{d^2} = \frac{32(1.8E{-}5)(0.3)(6.0)}{(0.004)^2} \approx \mathbf{65\ Pa} \quad \textit{Ans.}$$

Check Re = ρVd/μ = (1.31)(6.0)(0.004)/(1.8E–5) ≈ 1750 OK, laminar flow.

6.29 Oil, with ρ = 890 kg/m^3 and μ = 0.07 kg/m·s, flows through a horizontal pipe 15 m long. The power delivered to the flow is 1 hp. (a) What is the appropriate pipe diameter if the flow is at the laminar transition point? For this condition, what are (b) Q in m^3/h; and (c) τ_w in kPa?

Solution: (a, b) Set the Reynolds number equal to 2300 and the (laminar) power equal to 1 hp:

$$\mathrm{Re}_d = 2300 = \frac{(890\ kg/m^3)Vd}{0.07\ kg/m{\cdot}s} \quad \text{or} \quad Vd = 0.181\ m^2/s$$

$$Power = 1\ hp = 745.7\ W = Q\Delta p_{laminar} = \left(\frac{\pi}{4}d^2V\right)\left(\frac{32\mu LV}{d^2}\right) = \left(\frac{\pi}{4}\right)32(0.07)(15)V^2$$

$$\text{Solve for } V = 5.32\ \frac{m}{s} \quad \text{and} \quad \mathbf{d = 0.034\ m} \quad \textit{Ans.}\ \text{(a)}$$

It follows that $Q = (\pi/4)d^2V = (\pi/4)(0.034\ m)^2(5.32\ m/s) = 0.00484\ m^3/s$ = **17.4 m³/h** *Ans.* (b)
(c) From Eq. (6.12), the wall shear stress is

$$\tau_w = \frac{8\mu V}{d} = \frac{8(0.07\ kg/m{\cdot}s)(5.32\ m/s)}{(0.034\ m)} = 88\ Pa = \mathbf{0.088\ kPa} \quad \textit{Ans.}\ \text{(c)}$$

6.30 SAE 10 oil at 20°C flows through the 4-cm-diameter vertical pipe of Fig. P6.30. For the mercury manometer reading h = 42 cm shown, (a) calculate the volume flow rate in m^3/h, and (b) state the direction of flow.

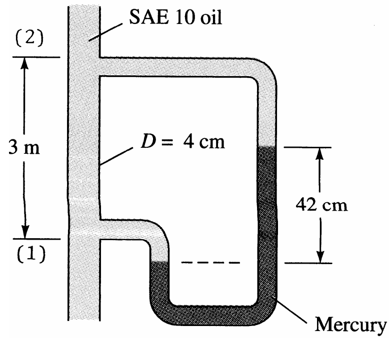


Fig. P6.30

Solution: For SAE 10 oil, take ρ = 870 kg/m^3 and μ = 0.104 kg/m·s. The pressure at the lower point (1) is considerably higher than p2 according to the manometer reading:

$$p_1 - p_2 = (\rho_{Hg} - \rho_{oil})g\Delta h = (13550 - 870)(9.81)(0.42) \approx 52200\text{ Pa}$$

$$\Delta p/(\rho_{oil}g) = 52200/[870(9.81)] \approx 6.12\text{ m}$$

This is more than 3 m of oil, therefore it must include a friction loss: **flow is *up***. *Ans.* (b)
The energy equation between (1) and (2), with $V_1 = V_2$, gives

$$\frac{p_1 - p_2}{\rho g} = z_2 - z_1 + h_f, \quad \text{or} \quad 6.12\text{ m} = 3\text{ m} + h_f, \quad \text{or:} \quad h_f \approx 3.12\text{ m} = \frac{128\mu LQ}{\pi\rho g d^4}$$

$$\text{Compute} \quad Q = \frac{(6.12 - 3)\pi(870)(9.81)(0.04)^4}{128(0.104)(3.0)} = 0.00536\ \frac{m^3}{s} \approx \mathbf{19.3}\ \frac{\mathbf{m^3}}{\mathbf{h}} \quad \textit{Ans. (a)}$$

Check $Re = 4\rho Q/(\pi\mu d) = 4(870)(0.00536)/[\pi(0.104)(0.04)] \approx 1430$ (OK, laminar flow).

P6.31 A *laminar flow element* or LFE (Meriam Instrument Co.) measures low gas-flow rates with a bundle of capillary tubes packed inside a large outer tube. Consider oxygen at 20°C and

1 atm flowing at 84 ft^3/min in a 4-in-diameter pipe. (*a*) Is the flow approaching the element turbulent? (*b*) If there are 1000 capillary tubes, L = 4 in, select a tube diameter to keep Re_d below 1500 and also to keep the tube pressure drop no greater than 0.5 lbf/in^2. (*c*) Do the tubes selected in part (*b*) fit nicely within the approach pipe?

Solution: For oxygen at 20°C and 1 atm (Table A.4), take R = 260 $m^2/(s^2K)$, hence $\rho = p/RT$ = (101350Pa)/[260(293K)] = 1.33 kg/m^3 = 0.00258 slug/ft^3. Also read μ = 2.0E-5 kg/m-s = 4.18E-7 slug/ft-s. Convert Q = 84 ft^3/min = 1.4 ft^3/s. Then the entry pipe Reynolds number is

$$\mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{4\rho Q}{\pi\mu D} = \frac{4(0.00258\,slug/ft^3)(1.4\,ft^3/s)}{\pi(4.18E-7\,slug/ft-s)(4/12\,ft)} = 33{,}000 \quad \text{(turbulent)} \quad Ans.(a)$$

(*b*) To keep Re_d below 1500 and keep the (laminar) pressure drop no more than 72 psf (0.5 psi),

$$\mathrm{Re}_d = \frac{\rho Vd}{\mu} \le 1500 \quad \text{and} \quad \Delta p = \frac{32\mu LV}{d^2} \le 72\frac{lbf}{ft^2}, \quad \text{where } V = \frac{Q/1000}{(\pi/4)d^2}$$

Select values of *d* and iterate, or use EES. The upper limit on Reynolds number gives

$$\mathrm{Re}_d = 1500 \quad \text{if} \quad d = 0.00734\,ft = \mathbf{0.088\,in}\;; \quad \Delta p = 2.74\,lbf/ft^2 \quad Ans.(b)$$

This is a satisfactory answer, since the pressure drop is no problem, quite small. One thousand of these tubes would have an area about one-half of the pipe area, so would fit nicely. *Ans.*(*c*)

Increasing the tube diameter would lower Re_d and have even smaller pressure drop. Example: d = 0.01 ft, Re_d = 1100, Δp = 0.8 psf. These 0.01-ft-diameter tubes would just barely fit into the larger pipe. One disadvantage, however, is that these tubes are *short*: the entrance length is *longer* than the tube length, and thus Δp will be larger than calculated by "fully-developed" formulas.

6.32 SAE 30 oil at 20°C flows in the 3-cm-diameter pipe in Fig. P6.32, which slopes at 37°. For the pressure measurements shown, determine (a) whether the flow is up or down and (b) the flow rate in m^3/h.

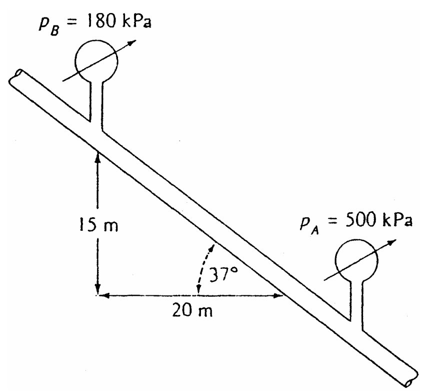


Fig. P6.32

Solution: For SAE 30 oil, take ρ = 891 kg/m^3 and μ = 0.29 kg/m·s. Evaluate the hydraulic grade lines:

$$\mathrm{HGL_B} = \frac{p_B}{\rho g} + z_B = \frac{180000}{891(9.81)} + 15 = 35.6\ \mathrm{m};\quad \mathrm{HGL_A} = \frac{500000}{891(9.81)} + 0 = 57.2\ \mathrm{m}$$

Since $\mathrm{HGL_A} > \mathrm{HGL_B}$ the **flow is *up*** *Ans.* (a)

The head loss is the difference between hydraulic grade levels:

$$h_f = 57.2 - 35.6 = 21.6\ \mathrm{m} = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(0.29)(25)Q}{\pi(891)(9.81)(0.03)^4}$$

Solve for $Q = 0.000518\ \mathrm{m^3/s} \approx \mathbf{1.86\ m^3/h}$ *Ans.* (b)

Finally, check $\mathrm{Re} = 4\rho Q/(\pi\mu d) \approx 68$ (OK, laminar flow).

6.33 In Problem 6.32, suppose it is desired to add a pump between A and B to drive the oil *upward* from A to B at a rate of 3 kg/s. At 100% efficiency, what pump power is required?

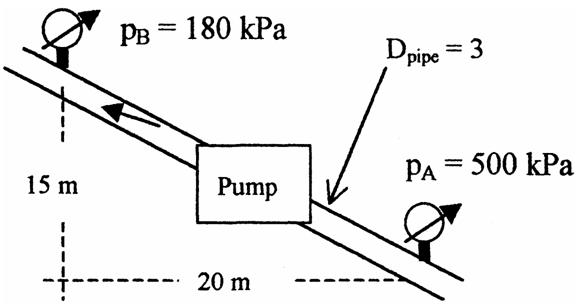


Fig. P6.33

Solution: For SAE 30 oil at 20°C, $\rho = 891\ \mathrm{kg/m^3}$ and $\mu = 0.29$ kg/m·s. With mass flow known, we can evaluate the pipe velocity:

$$V = \frac{\dot{m}}{\rho A} = \frac{3\ kg/s}{891\pi(0.015)^2} = 4.76\ \frac{m}{s},$$

$$\text{Check } Re_d = \frac{891(4.76)(0.03)}{0.29} = \mathbf{439}\ (OK,\ laminar)$$

Apply the steady flow energy equation between A and B:

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B + h_f - h_p,\quad or:\ \frac{500000}{891(9.81)} = \frac{180000}{891(9.81)} + 15 + h_f - h_p$$

$$\text{where}\quad h_f = \frac{32\mu LV}{\rho g d^2} = \frac{32(0.29)(25)(4.76)}{891(9.81)(0.03)^2} = 140.5\ m, \quad \text{Solve for } h_{pump} = 118.9\ m$$

The pump power is then given by

$$\textbf{Power} = \rho g Q h_p = \dot{m} g h_p = \left(3\ \frac{kg}{s}\right)\left(9.81\ \frac{m}{s^2}\right)(118.9\ m) = \textbf{3500 watts} \quad \textit{Ans.}$$

6.34 Derive the time-averaged *x*-momentum equation (6.21) by direct substitution of Eqs. (6.19) into the momentum equation (6.14). It is convenient to write the convective acceleration as

$$\frac{du}{dt} = \frac{\partial}{\partial x}(u^2) + \frac{\partial}{\partial y}(uv) + \frac{\partial}{\partial z}(uw)$$

which is valid because of the continuity relation, Eq. (6.14).

Solution: Into the *x*-momentum eqn. substitute u = u + u', v = v + v', etc., to obtain

$$\rho\left[\frac{\partial}{\partial x}(\bar{u}^2 + 2\bar{u}u' + u'^2) + \frac{\partial}{\partial y}(\bar{v}\,\bar{u} + \bar{v}u' + v'\bar{u} + v'u') + \frac{\partial}{\partial z}(\bar{w}\bar{u} + \bar{w}u' + w'\bar{u} + w'u')\right]$$

$$= -\frac{\partial}{\partial x}(\bar{p} + p') + \rho g_x + \mu[\nabla^2(\bar{u} + u')]$$

Now take the time-average of the entire equation to obtain Eq. (6.21) of the text:

$$\rho\left[\frac{\mathbf{d\bar{u}}}{\mathbf{dt}} + \frac{\partial}{\partial \mathbf{x}}(\overline{\mathbf{u'^2}}) + \frac{\partial}{\partial \mathbf{y}}(\overline{\mathbf{u'v'}}) + \frac{\partial}{\partial \mathbf{z}}(\overline{\mathbf{u'w'}})\right] = -\frac{\partial \mathbf{\bar{p}}}{\partial \mathbf{x}} + \rho \mathbf{g_x} + \mu\nabla^2(\mathbf{\bar{u}}) \quad \textit{Ans.}$$

6.35 By analogy with Eq. (6.21) write the turbulent mean-momentum differential equation for (a) the *y* direction and (b) the *z* direction. How many turbulent stress terms appear in each equation? How many unique turbulent stresses are there for the total of three directions?

Solution: You can re-derive, as in Prob. 6.34, or just permute the axes:

$$\text{(a)}\quad \mathbf{y}:\ \rho\frac{d\bar{v}}{dt} = -\frac{\partial \bar{p}}{\partial y} + \rho g_y + \frac{\partial}{\partial x}\left(\mu\frac{\partial \bar{v}}{\partial x} - \rho u'v'\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial \bar{v}}{\partial y} - \rho v'v'\right)$$

$$+ \frac{\partial}{\partial z}\left(\mu\frac{\partial \bar{v}}{\partial z} - \rho v'w'\right)$$

(b) **z**: $$\rho\frac{d\bar{w}}{dt} = -\frac{\partial\bar{p}}{\partial z} + \rho g_z + \frac{\partial}{\partial x}\left(\mu\frac{\partial\bar{w}}{\partial x} - \rho u'w'\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial\bar{w}}{\partial y} - \rho v'w'\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial\bar{w}}{\partial z} - \rho w'w'\right)$$

6.36 The following turbulent-flow velocity data *u*(*y*), for air at 75°F and 1 atm near a smooth flat wall, were taken in the University of Rhode Island wind tunnel:

y, in:	0.025	0.035	0.047	0.055	0.065
u, ft/s:	51.2	54.2	56.8	57.6	59.1

Estimate (a) the wall shear stress and (b) the velocity *u* at *y* = 0.22 in.

Solution: For air at 75°F and 1 atm, take $\rho = 0.00230$ slug/ft^3 and $\mu = 3.80E-7$ slug/ft·s. We fit each data point to the logarithmic-overlap law, Eq. (6.28):

$$\frac{u}{u^*} \approx \frac{1}{\kappa}\ln\frac{\rho u^* y}{\mu} + B \approx \frac{1}{0.41}\ln\left[\frac{0.0023u^*y}{3.80E-7}\right] + 5.0, \quad u^* = \sqrt{\tau_w/\rho}$$

Enter each value of u and y from the data and estimate the friction velocity u*:

y, in:	0.025	0.035	0.047	0.055	0.065
u*, ft/s:	**3.58**	**3.58**	**3.59**	**3.56**	**3.56**
yu*/ν (approx):	45	63	85	99	117

Each point gives a good estimate of u*, because each point is within the logarithmic layer in Fig. 6.10 of the text. The overall average friction velocity is

$$u^*_{avg} \approx 3.57\ \frac{ft}{s} \pm 1\%, \quad \tau_{w,avg} = \rho u^{*2} = (0.0023)(3.57)^2 \approx \mathbf{0.0293\ \frac{lbf}{ft^2}} \quad \textit{Ans. (a)}$$

Out at y = 0.22 inches, we may estimate that the log-law still holds:

$$\frac{\rho u^* y}{\mu} = \frac{0.0023(3.57)(0.22/12)}{3.80E-7} \approx 396, \quad u \approx u^*\left[\frac{1}{0.41}\ln(396) + 5.0\right]$$

$$\text{or:} \quad u \approx (3.57)(19.59) \approx \mathbf{70\ \frac{ft}{s}} \quad \textit{Ans. (b)}$$

Figure 6.10 shows that this point ($y^+ \approx 396$) seems also to be within the logarithmic layer.

6.37 Two infinite plates a distance h apart are parallel to the xz plane with the upper plate moving at speed V, as in Fig. P6.37. There is a fluid of viscosity μ and constant pressure between the plates. Neglecting gravity and assuming incompressible turbulent flow $u(y)$ between the plates, use the logarithmic law and appropriate boundary conditions to derive a formula for dimensionless wall shear stress versus dimensionless plate velocity. Sketch a typical shape of the profile $u(y)$.

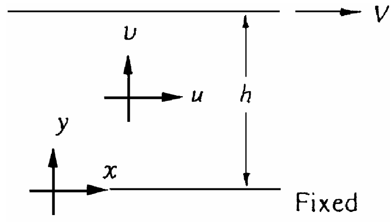
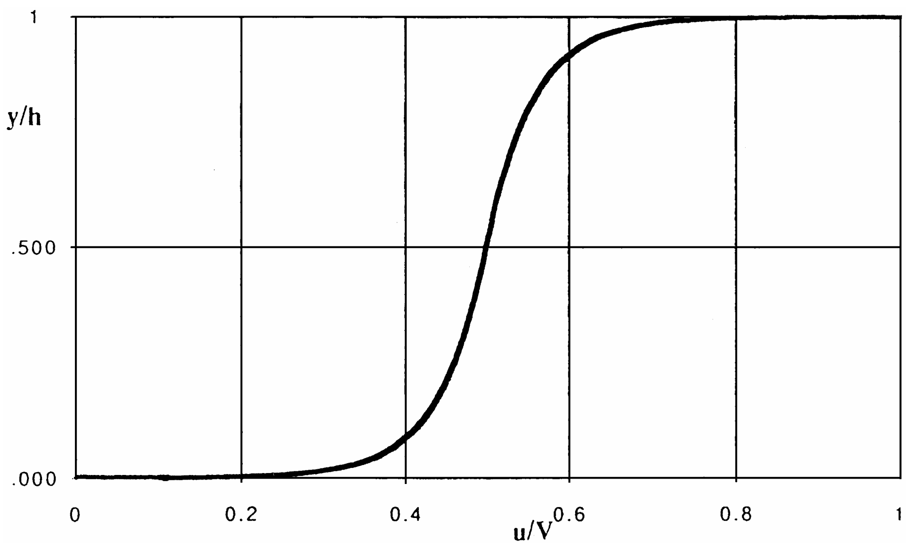


Fig. P6.37

Solution: The shear stress between parallel plates is *constant*, so the centerline velocity must be exactly $u = V/2$ at $y = h/2$. Anti-symmetric log-laws form, one with increasing velocity for $0 < y < h/2$, and one with decreasing velocity for $h/2 < y < h$, as shown below:



The match-point at the center gives us a log-law estimate of the shear stress:

$$\frac{V}{2u^*} \approx \frac{1}{\kappa}\ln\left(\frac{hu^*}{2\nu}\right) + \mathrm{B}, \quad \kappa \approx 0.41,\ \mathrm{B} \approx 5.0,\ \boldsymbol{u^* = (\tau_w/\rho)^{1/2}} \quad \textit{Ans.}$$

This is one form of "dimensionless shear stress." The more normal form is friction coefficient versus Reynolds number. Calculations from the log-law fit a Power-law curve-fit expression in the range $2000 < Re_h < 1E5$:

$$\mathbf{C_f} = \frac{\tau_w}{(1/2)\rho V^2} \approx \frac{0.018}{(\rho V h/\nu)^{1/4}} = \frac{\mathbf{0.018}}{\mathbf{Re_h^{1/4}}} \quad \textit{Ans.}$$

6.38 Suppose in Fig. P6.37 that h = 3 cm, the fluid is water at 20°C (ρ = 998 kg/m^3, μ = 0.001 kg/m·s), and the flow is turbulent, so that the logarithmic law is valid. If the shear stress in the fluid is 15 Pa, estimate V in m/s.

Solution: Just as in Prob. 6.37, apply the log-law at the center between the wall, that is, $y = h/2$, $u = V/2$. With τ_w known, we can evaluate u^* immediately:

$$u^* = \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\frac{15}{998}} = 0.123\ \frac{m}{s}, \quad \frac{V/2}{u^*} \approx \frac{1}{\kappa}\ln\left(\frac{u^* h/2}{\nu}\right) + B,$$

$$\text{or:} \quad \frac{V/2}{0.123} = \frac{1}{0.41}\ln\left[\frac{0.123(0.03/2)}{0.001/998}\right] + 5.0 = 23.3, \quad \textit{Solve for}\ \ \mathbf{V \approx 5.72\ \frac{m}{s}} \quad \textit{Ans.}$$

6.39 By analogy with laminar shear, $\tau = \mu\, du/dy$. T. V. Boussinesq in 1877 postulated that turbulent shear could also be related to the mean-velocity gradient $\tau_{turb} = \varepsilon\, du/dy$, where ε is called the *eddy viscosity* and is much larger than μ. If the logarithmic-overlap law, Eq. (6.28), is valid with $\tau \approx \tau_w$, show that $\varepsilon \approx \kappa\rho u^* y$.

Solution: Differentiate the log-law, Eq. (6.28), to find du/dy, then introduce the eddy viscosity into the turbulent stress relation:

$$\textit{If} \quad \frac{u}{u^*} = \frac{1}{\kappa}\ln\left(\frac{yu^*}{\nu}\right) + B, \quad \textit{then} \quad \frac{du}{dy} = \frac{u^*}{\kappa y}$$

$$\textit{Then, if} \quad \tau \approx \tau_w \equiv \rho u^{*2} = \varepsilon\frac{du}{dy} = \varepsilon\frac{u^*}{\kappa y}, \quad \textit{solve for}\ \ \boldsymbol{\varepsilon = \kappa\rho u^* y} \quad \textit{Ans.}$$

6.40 Theodore von Kármán in 1930 theorized that turbulent shear could be represented by $\tau_{turb} = \varepsilon\, du/dy$ where $\varepsilon = \rho\kappa^2 y^2 |du/dy|$ is called the *mixing-length eddy viscosity* and $\kappa \approx 0.41$ is Kármán's dimensionless *mixing-length constant* [2,3]. Assuming that $\tau_{turb} \approx \tau_w$ near the wall, show that this expression can be integrated to yield the logarithmic-overlap law, Eq. (6.28).

Solution: This is accomplished by straight substitution:

$$\tau_{turb} \approx \tau_w = \rho u^{*2} = \varepsilon\frac{du}{dy} = \left[\rho\kappa^2 y^2\left|\frac{du}{dy}\right|\right]\frac{du}{dy}, \quad \text{solve for } \frac{du}{dy} = \frac{u^*}{\kappa y}$$

$$\text{Integrate: } \int du = \frac{u^*}{\kappa}\int\frac{dy}{y}, \quad \text{or: } \mathbf{u = \frac{u^*}{\kappa}\ln(y) + constant} \quad \textit{Ans.}$$

To convert this to the exact form of Eq. (6.28) requires fitting to experimental data.

P6.41 Two reservoirs, which differ in surface elevation by 40 m, are connected by 350 m of new pipe of diameter 8 cm. If the desired flow rate is at least 130 N/s of water at 20°C, may the pipe material be (a) galvanized iron, (b) commercial steel, or (c) cast iron? Neglect minor losses.

Solution: Applying the extended Bernoulli equation between reservoir surfaces yields

$$\Delta z = 40\,m = f\frac{L}{D}\frac{V^2}{2g} = f\left(\frac{350\,m}{0.08\,m}\right)\frac{V^2}{2(9.81\,m/s^2)}$$

where f and V are related by the friction factor relation:

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re_D\sqrt{f}}\right) \quad \text{where} \quad Re_D = \frac{\rho VD}{\mu}$$

When V is found, the weight flow rate is given by $w = \rho g Q$ where $Q = AV = (\pi D^2/4)V$. For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m-s. Given the desired $w = 130$ N/s, solve this system of equations by EES to yield the wall roughness. The results are:

$$f = 0.0257 \;;\; V = 2.64 \text{ m/s} \;;\; Re_D = 211{,}000 \;;\; \varepsilon_{max} = 0.000203 \text{ m} = \mathbf{0.203\ mm}$$

Any less roughness is OK. From Table 6-1, the three pipe materials have
(a) galvanized: $\varepsilon = 0.15$ mm ; (b) commercial steel: $\varepsilon = 0.046$ mm ; cast iron: $\varepsilon = 0.26$ mm

Galvanized and steel are fine, but cast iron is too rough.. *Ans*. Actual flow rates are

(a) galvanized: 135 N/s; (b) steel: 152 N/s; (c) cast iron: 126 N/s (*not enough*)

6.42 It is clear by comparing Figs. 6.12*b* and 6.13 that the effects of sand roughness and commercial (manufactured) roughness are not quite the same. Take the special case of commercial roughness ratio $\varepsilon/d = 0.001$ in Fig. 6.13, and replot in the form of the wall-law shift ΔB (Fig. 6.12*a*) versus the logarithm of $\varepsilon^+ = \varepsilon u^*/\nu$. Compare your plot with Eq. (6.45).

Solution: To make this plot we must relate ΔB to the Moody-chart friction factor. We use Eq. (6.33) of the text, which is valid for any B, in this case, $B = B_o - \Delta B$, where $B_o \approx 5.0$:

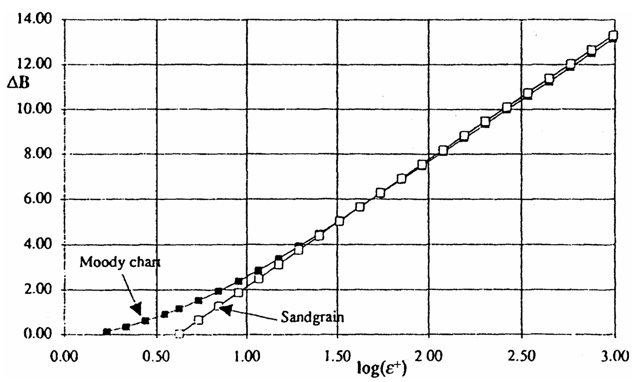
$$\frac{V}{u^*} \approx \frac{1}{\kappa}\ln\left(\frac{Ru^*}{\nu}\right) + B_o - \Delta B - \frac{3}{2\kappa}, \quad \text{where } \frac{V}{u^*} = \sqrt{\frac{8}{f}} \quad \text{and} \quad \frac{Ru^*}{\nu} = \frac{1}{2}Re_d\sqrt{\frac{f}{8}} \qquad (1)$$

Combine Eq. (1) with the Colebrook friction formula (6.48) and the definition of ε^+:

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/d}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \qquad (2)$$

$$\text{and} \quad \varepsilon^+ = \frac{\varepsilon u^*}{\nu} = \frac{\varepsilon}{d}d^+ = \frac{\varepsilon}{d}Re\sqrt{\frac{f}{8}} \qquad (3)$$

Equations (1, 2, 3) enable us to make the plot below of "commercial" log-shift ΔB, which is similar to the 'sand-grain' shift predicted by Eq. (6.45): $\Delta B_{sand} \approx (1/\kappa)\ln(\varepsilon^+) - 3.5$.



Ans.

Fig. P6.42

P6.43 A reservoir supplies water through 100 m of 30-cm-diameter cast iron pipe to a turbine that extracts 80 hp from the flow. The water then exhausts to the atmosphere. Neglect minor losses. (*a*) Assuming that $f \approx 0.019$, find the flow rate (there is a cubic polynomial). Explain why there are two solutions. (*b*) For extra credit, solve for the flow rate using the actual friction factors.

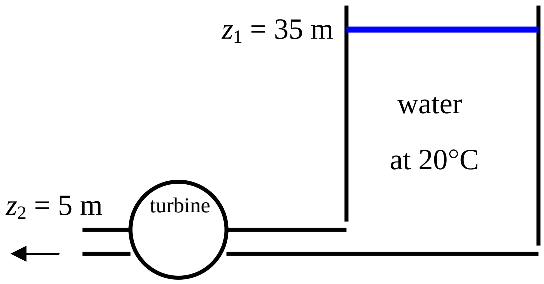


Fig. P6.43

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m-s. The energy equation yields a relation between elevation, friction, and turbine power:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{turb} + h_f$$

$$z_1 - z_2 = 35 - 5m = 30m = h_{turb} + h_f = \frac{Power}{\rho g Q} + (1 + f\frac{L}{D})\frac{V_2^2}{2g}, \quad Q = \frac{\pi}{4}D^2 V_2$$

$$30\,m = \frac{(80\,hp)(745.7W/hp)}{(9790\,N/m^3)(\pi/4)(0.3m)^2 V_2} + [1 + (0.019)\frac{100m}{0.3m}]\frac{V_2^2}{2(9.81m/s^2)}]$$

Clean this up into a cubic polynomial:

$$30 = \frac{86.2}{V} + 0.373V^2, \quad \text{or: } V^3 - 80.3V + 231 = 0$$

$$\textit{Three roots}: \quad V = 3.34\,m/s \;;\; 6.81\,m/s \;;\; -10.15\,m/s$$

The third (negative) root is meaningless. The other two are correct. Either

$$\mathbf{Q} = \mathbf{0.481\ m^3/s}\ ,\quad h_{\text{turbine}} = 12.7\text{ m}\ ,\quad h_f = 17.3\text{ m}$$

$$\mathbf{Q} = \mathbf{0.236\ m^3/s}\ ,\quad h_{\text{turbine}} = 25.8\text{ m}\ ,\quad h_f = 4.2\text{ m}\qquad \textit{Ans.}(a)$$

Both solutions are valid. The higher flow rate wastes a lot of water and creates 17 meters of friction loss. The lower rate uses 51% less water and has proportionately much less friction.

(*b*) The *actual* friction factors are very close to the problem's "Guess". Thus we obtain

Re = 2.04E6, f = 0.0191; $\mathbf{Q} = \mathbf{0.479\ m^3/s}$, $h_{\text{turbine}} = 12.7$ m , $h_f = 17.3$ m

Re = 1.01E6, f = 0.0193 ; $\mathbf{Q} = \mathbf{0.237\ m^3/s}$, $h_{\text{turbine}} = 25.7$ m , $h_f = 4.3$ m *Ans*.(b)

The same remarks apply: The lower flow rate is better, less friction, less water used.

6.44 Mercury at 20°C flows through 4 meters of 7-mm-diameter glass tubing at an average velocity of 5 m/s. Estimate the head loss in meters and the pressure drop in kPa.

Solution: For mercury at 20°C, take ρ = 13550 kg/m^3 and μ = 0.00156 kg/m·s. Glass tubing is considered hydraulically "smooth," ε/d = 0. Compute the Reynolds number:

$$Re_d = \frac{\rho V d}{\mu} = \frac{13550(5)(0.007)}{0.00156} = 304{,}000;\quad \text{Moody chart smooth:}\quad f \approx 0.0143$$

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = 0.0143\left(\frac{4.0}{0.007}\right)\frac{5^2}{2(9.81)} = \mathbf{10.4\ m}\quad \textit{Ans.}\text{ (a)}$$

$$\Delta p = \rho g h_f = (13550)(9.81)(10.4) = 1{,}380{,}000\ Pa = \mathbf{1380\ kPa}\quad \textit{Ans.}\text{ (b)}$$

6.45 Oil, SG = 0.88 and ν = 4E–5 m^2/s, flows at 400 gal/min through a 6-inch asphalted cast-iron pipe. The pipe is 0.5 miles long (2640 ft) and slopes upward at 8° in the flow direction. Compute the head loss in feet and the pressure change.

Solution: First convert 400 gal/min = 0.891 ft^3/s and ν = 0.000431 ft^2/s. For asphalted cast-iron, ε = 0.0004 ft, hence ε/d = 0.0004/0.5 = 0.0008. Compute *V*, *Red*, and *f*:

$$V = \frac{0.891}{\pi(0.25)^2} = 4.54\ \frac{ft}{s};\quad Re_d = \frac{4.54(0.5)}{0.000431} = 5271;\quad \text{calculate}\quad f_{Moody} = 0.0377$$

$$\text{then}\quad h_f = f\frac{L}{d}\frac{V^2}{2g} = 0.0377\left(\frac{2640}{0.5}\right)\frac{(4.54)^2}{2(32.2)} = \mathbf{63.8\ ft}\quad \textit{Ans.}\ \text{(a)}$$

If the pipe slopes upward at 8°, the pressure drop must balance both friction and gravity:

$$\Delta p = \rho g(h_f + \Delta z) = 0.88(62.4)[63.8 + 2640\sin 8°] = \mathbf{23700\ \frac{lbf}{ft^2}}\quad \textit{Ans.}\ \text{(b)}$$

6.46 Kerosene at 20°C is pumped at 0.15 m^3/s through 20 km of 16-cm-diameter cast-iron horizontal pipe. Compute the input power in kW required if the pumps are 85 percent efficient.

Solution: For kerosene at 20°C, take ρ = 804 kg/m^3 and μ = 1.92E−3 kg/m·s. For cast iron take $\varepsilon \approx$ 0.26 mm, hence ε/d = 0.26/160 ≈ 0.001625. Compute V, Re, and f:

$$V = \frac{0.15}{(\pi/4)(0.16)^2} = 7.46\ \frac{m}{s};\quad Re = \frac{4\rho Q}{\pi\mu d} = \frac{4(804)(0.15)}{\pi(0.00192)(0.16)} \approx 500{,}000$$

$$\varepsilon/d \approx 0.001625:\quad \text{Moody chart: } f \approx 0.0226$$

$$\text{Then}\quad h_f = f\frac{L}{d}\frac{V^2}{2g} = (0.0226)\left(\frac{20000}{0.16}\right)\frac{(7.46)^2}{2(9.81)} \approx 8020\ m$$

At 85% efficiency, the pumping power required is:

$$P = \frac{\rho g Q h_f}{\eta} = \frac{804(9.81)(0.15)(8020)}{0.85} \approx 11.2E{+}6\ W = \mathbf{11.2\ \ MW}\quad \textit{Ans.}$$

6.47 The gutter and smooth drainpipe in Fig. P6.47 remove rainwater from the roof of a building. The smooth drainpipe is 7 cm in diameter. (a) When the gutter is full, estimate the rate of draining. (b) The gutter is designed for a sudden rainstorm of up to 5 inches per hour. For this condition, what is the maximum roof area that can be drained successfully?

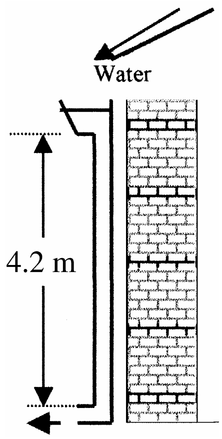


Fig. P6.47

Solution: If the velocity at the gutter surface is neglected, the energy equation reduces to

$$\Delta z = \frac{V^2}{2g} + h_f, \quad h_f = f\frac{L}{d}\frac{V^2}{2g}, \quad \text{solve } V^2 = \frac{2g\Delta z}{1+fL/d} = \frac{2(9.81)(4.2)}{1+f(4.2/0.07)}$$

For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. Guess $f \approx 0.02$ to obtain the velocity estimate V ≈ 6 m/s above. Then $Re_d \approx \rho Vd/\mu \approx (998)(6)(0.07)/(0.001) \approx 428{,}000$ (turbulent). Then, for a smooth pipe, $f \approx 0.0135$, and V is changed slightly to 6.74 m/s. After convergence, we obtain

$$V = 6.77 \text{ m/s}, \quad Q = V(\pi/4)(0.07)^2 = \mathbf{0.026\ m^3/s} \quad \textit{Ans. (a)}$$

A rainfall of 5 in/h = (5/12 ft/h)(0.3048 m/ft)/(3600 s/h) = 0.0000353 m/s. The required roof area is

$$A_{roof} = Q_{drain}/V_{rain} = (0.026\ \text{m}^3/\text{s})/0.0000353\ \text{m/s} \approx \mathbf{740\ m^2} \quad \textit{Ans. (b)}$$

6.48 Show that if Eq. (6.33) is accurate, the position in a turbulent pipe flow where local velocity u equals average velocity V occurs exactly at $r = 0.777R$, independent of the Reynolds number.

Solution: Simply find the log-law position y^+ where u^+ exactly equals V/u*:

$$V = u^*\left[\frac{1}{\kappa}\ln\frac{Ru^*}{\nu} + B - \frac{3}{2\kappa}\right] \stackrel{?}{=} u^*\left[\frac{1}{\kappa}\ln\frac{yu^*}{\nu} + B\right] \text{ if } \frac{1}{\kappa}\ln\frac{y}{R} = -\frac{3}{2\kappa}$$

$$\text{Since } y = R - r, \text{ this is equivalent to } \frac{r}{R} = 1 - e^{-3/2} = 1 - 0.223 \approx \mathbf{0.777} \quad \textit{Ans.}$$

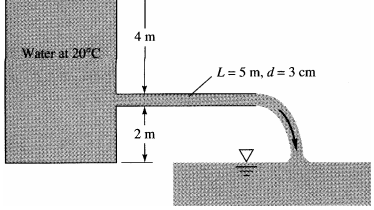


Fig. P6.49

6.49 The tank-pipe system of Fig. P6.49 is to deliver at least 11 m^3/h of water at 20°C to the reservoir. What is the maximum roughness height ε allowable for the pipe?

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. Evaluate V and Re for the expected flow rate:

$$V = \frac{Q}{A} = \frac{11/3600}{(\pi/4)(0.03)^2} = 4.32\ \frac{m}{s};\quad Re = \frac{\rho V d}{\mu} = \frac{998(4.32)(0.03)}{0.001} = 129000$$

The energy equation yields the value of the head loss:

$$\frac{p_{atm}}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_{atm}}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f \quad \text{or} \quad h_f = 4 - \frac{(4.32)^2}{2(9.81)} = 3.05\ m$$

$$\text{But also}\quad h_f = f\frac{L}{d}\frac{V^2}{2g},\quad \text{or:}\quad 3.05 = f\left(\frac{5.0}{0.03}\right)\frac{(4.32)^2}{2(9.81)},\quad \text{solve for } f \approx 0.0192$$

With f and Re known, we can find ε/d from the Moody chart or from Eq. (6.48):

$$\frac{1}{(0.0192)^{1/2}} = -2.0\log_{10}\left[\frac{\varepsilon/d}{3.7} + \frac{2.51}{129000(0.0192)^{1/2}}\right],\quad \text{solve for } \frac{\varepsilon}{d} \approx 0.000394$$

$$\text{Then}\quad \varepsilon = 0.000394(0.03) \approx 1.2\text{E–}5\ m \approx \mathbf{0.012\ mm}\ \text{(very smooth)}\quad \textit{Ans.}$$

6.50 Ethanol at 20°C flows at 125 U.S. gal/min through a horizontal cast-iron pipe with L = 12 m and d = 5 cm. Neglecting entrance effects, estimate (a) the pressure gradient, dp/dx; (b) the wall shear stress, τ_w; and (c) the percent reduction in friction factor if the pipe walls are polished to a smooth surface.

Solution: For ethanol (Table A-3) take ρ = 789 kg/m^3 and μ = 0.0012 kg/m·s. Convert 125 gal/min to 0.00789 m^3/s. Evaluate V = Q/A = 0.00789/[$\pi(0.05)^2$/4] = 4.02 m/s.

$$Re_d = \frac{\rho V d}{\mu} = \frac{789(4.02)(0.05)}{0.0012} = 132{,}000,\quad \frac{\varepsilon}{d} = \frac{0.26\ mm}{50\ mm} = 0.0052\ \text{ Then } f_{Moody} \approx 0.0314$$

$$\text{(b)}\quad \tau_w = \frac{f}{8}\rho V^2 = \frac{0.0314}{8}(789)(4.02)^2 = \mathbf{50\ Pa}\quad \textit{Ans.}\text{ (b)}$$

$$\text{(a)}\quad \frac{dp}{dx} = -\frac{4\tau_w}{d} = \frac{-4(50)}{0.05} = \mathbf{-4000\ \frac{Pa}{m}}\quad \textit{Ans.}\text{ (a)}$$

(c) $Re = 132000, \quad f_{smooth} = 0.0170,$ hence the reduction in f is

$$\left(1 - \frac{0.0170}{0.0314}\right) = \mathbf{46\%} \quad \textit{Ans. (c)}$$

6.51 The viscous sublayer (Fig. 6.10) is normally less than 1 percent of the pipe diameter and therefore very difficult to probe with a finite-sized instrument. In an effort to generate a thick sublayer for probing, Pennsylvania State University in 1964 built a pipe with a flow of glycerin. Assume a smooth 12-in-diameter pipe with $V = 60$ ft/s and glycerin at 20°C. Compute the sublayer thickness in inches and the pumping horsepower required at 75 percent efficiency if $L = 40$ ft.

Solution: For glycerin at 20°C, take $\rho = 2.44$ slug/ft^3 and $\mu = 0.0311$ slug/ft·s. Then

$$\text{Re} = \frac{\rho V d}{\mu} = \frac{2.44(60)(1\text{ ft})}{0.0311} = 4710 \ \text{(barely turbulent!)} \quad \text{Smooth: } f_{\text{Moody}} \approx 0.0380$$

$$\text{Then} \quad u^* = V(f/8)^{1/2} = 60\left(\frac{0.0380}{8}\right)^{1/2} \approx 4.13 \ \frac{\text{ft}}{\text{s}}$$

The sublayer thickness is defined by $y^+ \approx 5.0 = \rho y u^*/\mu$. Thus

$$y_{\text{sublayer}} \approx \frac{5\mu}{\rho u^*} = \frac{5(0.0311)}{(2.44)(4.13)} = 0.0154\text{ ft} \approx \mathbf{0.185\ inches} \quad \textit{Ans.}$$

With f known, the head loss and the power required can be computed:

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = (0.0380)\left(\frac{40}{1}\right)\frac{(60)^2}{2(32.2)} \approx 85\text{ ft}$$

$$P = \frac{\rho g Q h_f}{\eta} = \frac{1}{0.75}(2.44)(32.2)\left[\frac{\pi}{4}(1)^2(60)\right](85) = 419000 \div 550 \approx \mathbf{760\ hp} \quad \textit{Ans.}$$

6.52 The pipe flow in Fig. P6.52 is driven by pressurized air in the tank. What gage pressure p_1 is needed to provide a 20°C water flow rate $Q = 60$ m^3/h?

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. Get V, Re, f:

$$V = \frac{60/3600}{(\pi/4)(0.05)^2} = 8.49 \ \frac{\text{m}}{\text{s}};$$

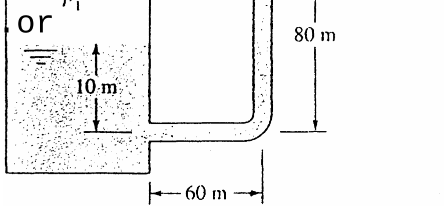


Fig. P6.52

$$\mathrm{Re} = \frac{998(8.49)(0.05)}{0.001} \approx 424000; \quad f_{smooth} \approx \mathbf{0.0136}$$

Write the energy equation between points (1) (the tank) and (2) (the open jet):

$$\frac{p_1}{\rho g} + \frac{0^2}{2g} + 10 = \frac{0}{\rho g} + \frac{V_{pipe}^2}{2g} + 80 + h_f, \quad \text{where } h_f = f\frac{L}{d}\frac{V^2}{2g} \text{ and } V_{pipe} = 8.49\ \frac{m}{s}$$

$$\text{Solve} \quad p_1 = (998)(9.81)\left[80 - 10 + \frac{(8.49)^2}{2(9.81)}\left\{1 + 0.0136\left(\frac{170}{0.05}\right)\right\}\right]$$

$$\approx \mathbf{2.38E6\ \ Pa} \quad \textit{Ans.}$$

[This is a *gage* pressure (relative to the pressure surrounding the open jet.)]

6.53 In Fig. P6.52 suppose p_1 = 700 kPa and the fluid specific gravity is 0.68. If the flow rate is 27 m^3/h, estimate the viscosity of the fluid. What fluid in Table A-5 is the likely suspect?

Solution: Evaluate $\rho = 0.68(998) = 679\ kg/m^3$. Evaluate $V = Q/A = (27/3600)/[\pi(0.025)^2] = 3.82$ m/s. The energy analysis of the previous problem now has f as the unknown:

$$\frac{p_1}{\rho g} = \frac{700000}{679(9.81)} = \Delta z + \frac{V^2}{2g} + f\frac{L}{d}\frac{V^2}{2g} = 70 + \frac{(3.82)^2}{2(9.81)}\left[1 + f\frac{170}{0.05}\right], \quad \text{solve} \quad f = 0.0136$$

$$\text{Smooth pipe: } f = 0.0136,\ Re_d = 416000 = \frac{679(3.82)(0.05)}{\mu},$$

$$\text{Solve} \quad \mu = \mathbf{0.00031}\ \mathbf{\frac{kg}{m\cdot s}} \quad \textit{Ans.}$$

The density and viscosity are close to the likely suspect, **gasoline**. *Ans.*

6.54* A swimming pool W by Y by h deep is to be emptied by gravity through the long pipe shown in Fig. P6.54. Assuming an average pipe friction factor f_{av} and neglecting minor losses, derive a formula for the time to empty the tank from an initial level h_o.

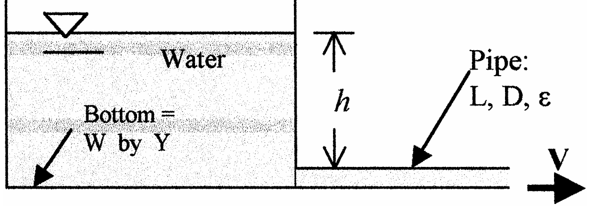


Fig. P6.54

Solution: With no driving pressure and negligible tank surface velocity, the energy equation can be combined with a control-volume mass conservation:

$$h(t) = \frac{V^2}{2g} + f_{av}\frac{L}{D}\frac{V^2}{2g}, \quad or: \quad Q_{out} = A_{pipe}V = \frac{\pi}{4}D^2\sqrt{\frac{2gh}{1+f_{av}L/D}} = -WY\frac{dh}{dt}$$

We can separate the variables and integrate for time to drain:

$$\frac{\pi}{4}D^2\sqrt{\frac{2g}{1+f_{av}L/D}}\int_0^t dt = -WY\int_{h_o}^0 \frac{dh}{\sqrt{h}} = -WY\left(0-2\sqrt{h_o}\right)$$

$$\textit{Clean this up to obtain:} \quad \boldsymbol{t_{drain} \approx \frac{4WY}{\pi D^2}\sqrt{\frac{2h_o(1+f_{av}L/D)}{g}}^{1/2}} \quad \textit{Ans.}$$

6.55 The reservoirs in Fig. P6.55 contain water at 20°C. If the pipe is smooth with L = 4500 m and d = 4 cm, what will the flow rate in m³/h be for Δz = 100 m?

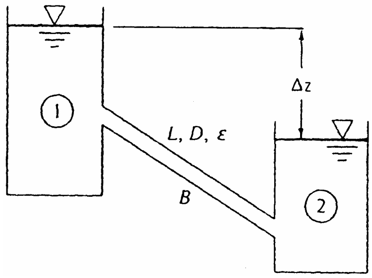


Fig. P6.55

Solution: For water at 20°C, take ρ = 998 kg/m³ and μ = 0.001 kg/m·s. The energy equation from surface 1 to surface 2 gives

$$p_1 = p_2 \quad \text{and} \quad V_1 = V_2,$$

$$\text{thus} \quad h_f = z_1 - z_2 = 100 \text{ m}$$

$$\text{Then } 100 \text{ m} = f\left(\frac{4500}{0.04}\right)\frac{V^2}{2(9.81)}, \quad \text{or} \quad fV^2 \approx 0.01744$$

Iterate with an initial guess of f ≈ 0.02, calculating V and Re and improving the guess:

$$V \approx \left(\frac{0.01744}{0.02}\right)^{1/2} \approx 0.934 \frac{m}{s}, \quad Re \approx \frac{998(0.934)(0.04)}{0.001} \approx 37300, \quad f_{smooth} \approx 0.0224$$

$$V_{better} \approx \left(\frac{0.01744}{0.0224}\right)^{1/2} \approx 0.883 \frac{m}{s}, \quad Re_{better} \approx 35300, \quad f_{better} \approx 0.0226, \text{ etc......}$$

This process converges to

$$f = 0.0227, \ Re = 35000, \ V = 0.877 \text{ m/s}, \ Q \approx 0.0011 \text{ m}^3/\text{s} \approx \mathbf{4.0\ m^3/h}. \quad \textit{Ans.}$$

6.56 Consider a horizontal 4-ft-diameter galvanized-iron pipe simulating the Alaska Pipeline. The oil flow is 70 million U.S. gallons per day, at a density of 910 kg/m³ and viscosity of 0.01 kg/m·s (see Fig. A.1 for SAE 30 oil at 100°C). Each pump along the line raises the oil pressure to 8 MPa, which then drops, due to head loss, to 400 kPa at the entrance to the next pump. Estimate (a) the appropriate distance between pumping stations; and (b) the power required if the pumps are 88% efficient.

Solution: For galvanized iron take ε = 0.15 mm. Convert d = 4 ft = 1.22 m. Convert Q = 7E7 gal/day = 3.07 m³/s. The flow rate gives the velocity and Reynolds number:

$$V = \frac{Q}{A} = \frac{3.07}{\pi(1.22)^2/4} = 2.63 \frac{m}{s}; \quad Re_d = \frac{\rho V d}{\mu} = \frac{910(2.63)(1.22)}{0.01} = 292{,}500$$

$$\frac{\varepsilon}{d} = \frac{0.15\ mm}{1220\ mm} = 0.000123, \quad f_{Moody} \approx 0.0157$$

Relating the known pressure drop to friction factor yields the unknown pipe length:

$$\Delta p = 8{,}000{,}00 - 400{,}000\ Pa = f\frac{L}{d}\frac{\rho}{2}V^2 = 0.0157\frac{L}{1.22}\left(\frac{910}{2}\right)(2.63)^2,$$

$$\text{Solve}\quad L = \mathbf{188,000\ m} = 117\ \text{miles}\quad \textit{Ans.}\ \text{(a)}$$

The pumping power required follows from the pressure drop and flow rate:

$$Power = \frac{Q\,\Delta p}{Efficiency} = \frac{3.07(8E6 - 4E5)}{0.88} = 2.65E7\ \text{watts}$$

$$= \mathbf{26.5\ MW}\ (35{,}500\ \text{hp})\quad \textit{Ans.}\ \text{(b)}$$

6.57 Apply the analysis of Prob. 6.54 to the following data. Let $W = 5$ m, $Y = 8$ m, $h_o = 2$ m, $L = 15$ m, $D = 5$ cm, and $\varepsilon = 0$. (a) By letting $h = 1.5$ m and 0.5 m as representative depths, estimate the average friction factor. Then (b) estimate the time to drain the pool.

Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. The velocity in Prob. 6.54 is calculated from the energy equation:

$$V = \sqrt{\frac{2gh}{1 + fL/D}}\quad \text{with}\quad f = fcn(\text{Re}_D)_{smooth\ pipe}\quad \text{and}\quad \text{Re}_D = \frac{\rho VD}{\mu},\ L/D = 300$$

(a) With a bit of iteration for the Moody chart, we obtain $\text{Re}_D = 108{,}000$ and $f \approx 0.0177$ at $h = 1.5$ m, and $\text{Re}_D = 59{,}000$ and $f \approx .0202$ at $h = 0.5$ m; thus the average value $\boldsymbol{f_{av} \approx 0.019}$. *Ans.* (a)

The drain formula from Prob. 6.54 then predicts:

$$t_{drain} \approx \frac{4WY}{\pi D^2}\sqrt{\frac{2h_o(1 + f_{av}L/D)}{g}} \approx \frac{4(5)(8)}{\pi(0.05)^2}\sqrt{\frac{2(2)[1 + 0.019(300)]}{9.81}}$$

$$= 33700\ s = \mathbf{9.4\ h}\quad \textit{Ans.}\ \text{(b)}$$

6.58 In Fig. P6.55 assume that the pipe is cast iron with $L = 550$ m, $d = 7$ cm, and $\Delta z = 100$ m. If an 80 percent efficient pump is placed at point B, what input power is required to deliver 160 m^3/h of water upward from reservoir 2 to 1?

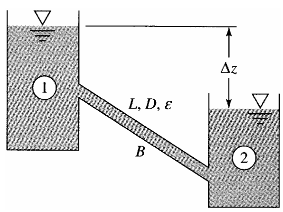


Fig. P6.55

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. Compute V, Re:

$$V = \frac{Q}{A} = \frac{160/3600}{(\pi/4)(0.07)^2} \approx 11.55\ \frac{m}{s}; \quad Re = \frac{998(11.55)(0.07)}{0.001} \approx 807000$$

$$\frac{\varepsilon}{d}\Big|_{\text{cast iron}} = \frac{0.26\text{ mm}}{70\text{ mm}} \approx 0.00371; \quad \text{Moody chart: } f \approx 0.00280$$

The energy equation from surface 1 to surface 2, with a pump at B, gives

$$h_{pump} = \Delta z + h_f = 100 + (0.0280)\left(\frac{550}{0.07}\right)\frac{(11.55)^2}{2(9.81)} = 100 + 1494 \approx 1594 \text{ m}$$

$$\text{Power} = \frac{\rho g Q h_p}{\eta} = \frac{(998)(9.81)(160/3600)(1594)}{0.80} = 8.67\text{E5 W} \approx \mathbf{867\ kW} \quad \textit{Ans.}$$

6.59 The following data were obtained for flow of 20°C water at 20 m^3/hr through a badly corroded 5-cm-diameter pipe which slopes downward at an angle of 8°: p_1 = 420 kPa, z_1 = 12 m, p_2 = 250 kPa, z_2 = 3 m. Estimate (a) the roughness ratio of the pipe; and (b) the percent change in head loss if the pipe were smooth and the flow rate the same.

Solution: The pipe length is given indirectly as $L = \Delta z/\sin\theta = (9\text{ m})/\sin 8° = 64.7$ m. The steady flow energy equation then gives the head loss:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f, \quad or: \quad \frac{420000}{9790} + 12 = \frac{250000}{9790} + 3 + h_f,$$

$$\text{Solve} \quad h_f = 26.4\ m$$

Now relate the head loss to the Moody friction factor:

$$h_f = 26.4 = f\frac{L}{d}\frac{V^2}{2g} = f\frac{64.7}{0.05}\frac{(2.83)^2}{2(9.81)}, \quad \text{Solve } f = 0.050,\ Re = 141000,\ \text{Read } \frac{\varepsilon}{d} \approx 0.0211$$

The estimated (and uncertain) pipe roughness is thus $\varepsilon = 0.0211d \approx$ **1.06 mm** *Ans.* (a)
(b) At the same Re_d = 141000, f_{smooth} = 0.0168, or **66% less head loss**. *Ans.* (b)

P6.60 In the spirit of Haaland's explicit pipe friction factor approximation, Eq. (6.49), Jeppson [20] proposed the following explicit formula:

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/d}{3.7} + \frac{5.74}{Re_d^{0.9}}\right)$$

(a) Is this identical to Haaland's formula and just a simple rearrangement? Explain.

(b) Compare Jeppson to Haaland for a few representative values of (turbulent) Re_d and ε/d and their deviations compared to the Colebrook formula (6.48).

Solution: (a) No, it *looks* like a rearrangement of Haaland's formula, but it is not. Haaland started with Colebrook's smooth-wall formula and added just enough ε/d effect for accuracy. Jeppson started with the rough-wall formula and added just enough Re_d effect for accuracy. Both are excellent approximations over the full (turbulent) range of Re_d and ε/d. Their predicted values of f are nearly the same and very close to the implicit Colebrook formula. Here is a table of their standard deviations of their values when subtracted from Colebrook:

1E4 < Red < 1e8	$\varepsilon/d = 0.03$	0.01	0.001	0.0001	0.00001
Jeppson rms error	0.000398	0.000328	0.000195	0.000067	0.000088
Haaland rms error	0.000034	0.000043	0.000129	0.000113	0.000083

As expected, Jeppson is slightly better for smooth walls, Haaland for rough walls. Both are within ±2% of the Colebrook formula over the entire range of Re_d and ε/d.

6.61 What level h must be maintained in Fig. P6.61 to deliver a flow rate of 0.015 ft^3/s through the $\frac{1}{2}$-in commercial-steel pipe?

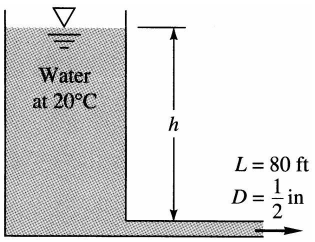


Fig. P6.61

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. For commercial steel, take $\varepsilon \approx$ 0.00015 ft, or ε/d = 0.00015/(0.5/12) ≈ 0.0036. Compute

$$V = \frac{Q}{A} = \frac{0.015}{(\pi/4)(0.5/12)^2} = 11.0\ \frac{ft}{s};$$

$$Re = \frac{\rho V d}{\mu} = \frac{1.94(11.0)(0.5/12)}{2.09E{-}5} \approx 42500 \quad \varepsilon/d = 0.0036, \quad f_{Moody} \approx 0.0301$$

The energy equation, with $p_1 = p_2$ and $V_1 \approx 0$, yields an expression for surface elevation:

$$h = h_f + \frac{V^2}{2g} = \frac{V^2}{2g}\left(1 + f\frac{L}{d}\right) = \frac{(11.0)^2}{2(32.2)}\left[1 + 0.0301\left(\frac{80}{0.5/12}\right)\right] \approx \mathbf{111\ ft} \quad \textit{Ans.}$$

6.62 Water at 20°C is to be pumped through 2000 ft of pipe from reservoir 1 to 2 at a rate of 3 ft^3/s, as shown in Fig. P6.62. If the pipe is cast iron of diameter 6 in and the pump is 75 percent efficient, what horsepower pump is needed?

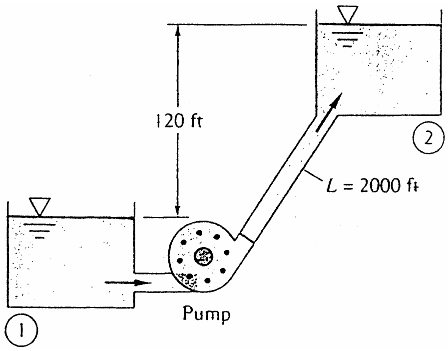


Fig. P6.62

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. For cast iron, take $\varepsilon \approx$ 0.00085 ft, or ε/d = 0.00085/(6/12) ≈ 0.0017. Compute V, Re, and *f*:

$$V = \frac{Q}{A} = \frac{3}{(\pi/4)(6/12)^2} = 15.3\ \frac{ft}{s};$$

$$Re = \frac{\rho V d}{\mu} = \frac{1.94(15.3)(6/12)}{2.09E{-}5} \approx 709000 \quad \varepsilon/d = 0.0017, \quad f_{Moody} \approx 0.0227$$

The energy equation, with $p_1 = p_2$ and $V_1 \approx V_2 \approx 0$, yields an expression for pump head:

$$h_{pump} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 120\ ft + 0.0227\left(\frac{2000}{6/12}\right)\frac{(15.3)^2}{2(32.2)} = 120 + 330 \approx 450\ ft$$

$$\text{Power:} \quad P = \frac{\rho g Q h_p}{\eta} = \frac{1.94(32.2)(3.0)(450)}{0.75} = 112200 \div 550 \approx \mathbf{204\ hp} \quad \textit{Ans.}$$

6.63 A tank contains 1 m^3 of water at 20°C and has a drawn-capillary outlet tube at the bottom, as in Fig. P6.63. Find the outlet volume flux Q in m^3/h at this instant.

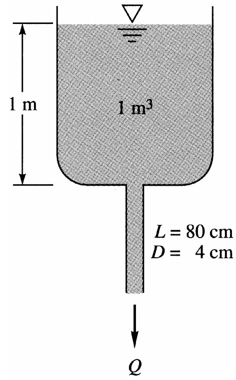


Fig. P6.63

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For drawn tubing, take $\varepsilon \approx 0.0015$ mm, or ε/d = 0.0015/40 ≈ 0.0000375. The steady-flow energy equation, with $p_1 = p_2$ and $V_1 \approx 0$, gives

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = \Delta z - \frac{V^2}{2g}, \quad \text{or:} \quad \frac{V^2}{2g}\left(1+\frac{0.8}{0.04}f\right) \approx 1.8 \text{ m}, \; V^2 \approx \frac{35.32}{1+20f}$$

$$\text{Guess} \quad f \approx 0.015, \quad V = \left[\frac{35.32}{1+20(0.015)}\right]^{1/2} \approx 5.21\ \frac{m}{s}, \quad Re = \frac{998(5.21)(0.04)}{0.001} \approx 208000$$

$$f_{better} \approx 0.0158, \quad V_{better} \approx 5.18 \text{ m/s}, \quad Re \approx 207000 \text{ (converged)}$$

$$\text{Thus} \quad V \approx 5.18 \text{ m/s}, \quad Q = (\pi/4)(0.04)^2(5.18) = 0.00651 \text{ m}^3/\text{s} \approx \mathbf{23.4\ m^3/h}. \quad \textit{Ans.}$$

6.64 Repeat Prob. 6.63 to find the flow rate if the fluid is SAE 10 oil. Is the flow laminar or turbulent?

Solution: For SAE 10 oil at 20°C, take $\rho = 870$ kg/m^3 and $\mu = 0.104$ kg/m·s. For drawn tubing, take $\varepsilon \approx 0.0015$ mm, or ε/d = 0.0015/40 ≈ 0.0000375. Guess laminar flow:

$$h_f = 1.8 \text{ m} - \frac{V^2}{2g} \overset{?}{=} \frac{32\mu LV}{\rho g d^2}, \quad \text{or:} \quad 1.8 - \frac{V^2}{2(9.81)} = \frac{32(0.104)(0.8)V}{870(9.81)(0.04)^2} = 0.195V$$

$$\text{Quadratic equation:} \quad V^2 + 3.83V - 35.32 = 0, \quad \text{solve } V = 4.33 \text{ m/s}$$

$$\text{Check} \quad Re = (870)(4.33)(0.04)/(0.104) \approx \mathbf{1450 \ (OK, laminar)}$$

So it <u>is</u> laminar flow, and Q = $(\pi/4)(0.04)^2(4.33)$ = 0.00544 m^3/s = **19.6 m^3/h**. *Ans.*

6.65 In Prob. 6.63 the initial flow is turbulent. As the water drains out of the tank, will the flow revert to laminar motion as the tank becomes nearly empty? If so, at what tank depth? Estimate the time, in h, to drain the tank completely.

Solution: Recall that $\rho = 998$ kg/m^3, $\mu = 0.001$ kg/m·s, and $\varepsilon/d \approx 0.0000375$. Let Z be the depth of water in the tank (Z = 1 m in Fig. P6.63). When Z = 0, find the flow rate:

$$Z = 0,\ h_f = 0.8\text{ m},\quad V^2 \approx \frac{2(9.81)(0.8)}{1+20f}\quad \text{converges to } f = 0.0171,\ Re = 136000$$

$$V \approx 3.42\text{ m/s},\quad Q \approx 12.2\text{ m}^3/\text{h } (Z=0)$$

So even when the tank is empty, the flow is still turbulent. *Ans.*

$$\text{The time to drain the tank is}\quad \frac{d}{dt}(\upsilon_{tank}) = -Q = \frac{d}{dt}(A_{tank}Z) = (1\text{ m}^2)\frac{dZ}{dt} = -Q,$$

$$\text{or}\quad t_{drain} = -\int_{1m}^{0m}\frac{dZ}{Q} = \left(\frac{1}{Q}\right)_{avg}(1\text{ m})$$

So all we need is the average value of (1/Q) during the draining period. We know Q at Z = 0 and Z = 1 m, let's check it also at Z = 0.5 m: Calculate $Q_{midway} \approx 19.8$ m^3/h. Then

$$\left.\frac{1}{Q}\right|_{avg} \approx \frac{1}{6}\left[\frac{1}{23.4}+\frac{4}{19.8}+\frac{1}{12.2}\right] \approx 0.0544\ \frac{h}{m^3},\quad t_{drain} = \mathbf{0.0544\ h \approx 3.3\ min}\quad \textit{Ans.}$$

6.66 Ethyl alcohol at 20°C flows through a 10-cm horizontal drawn tube 100 m long. The fully developed wall shear stress is 14 Pa. Estimate (a) the pressure drop, (b) the volume flow rate, and (c) the velocity u at $r = 1$ cm.

Solution: For ethyl alcohol at 20°C, $\rho = 789$ kg/m^3, $\mu = 0.0012$ kg/m·s. For drawn tubing, take $\varepsilon \approx 0.0015$ mm, or $\varepsilon/d = 0.0015/100 \approx 0.000015$. From Eq. (6.12),

$$\Delta p = 4\tau_w\frac{L}{d} = 4(14)\left(\frac{100}{0.1}\right) \approx \mathbf{56000\ Pa}\quad \textit{Ans.}\text{ (a)}$$

The wall shear is directly related to *f*, and we may iterate to find V and Q:

$$\tau_w = \frac{f}{8}\rho V^2,\quad \text{or:}\quad fV^2 = \frac{8(14)}{789} = 0.142\quad \text{with}\ \frac{\varepsilon}{d} = 0.000015$$

$$\text{Guess } f \approx 0.015,\quad V = \left[\frac{0.142}{0.015}\right]^{1/2} \approx 3.08\ \frac{m}{s},\quad Re = \frac{789(3.08)(0.1)}{0.0012} \approx 202000$$

$$f_{better} \approx 0.0158,\quad V_{better} \approx 3.00\text{ m/s},\quad Re_{better} \approx 197000\text{ (converged)}$$

Then V ≈ **3.00 m/s**, and Q = (π/4)$(0.1)^2$(3.00) = 0.0236 m^3/s = **85 m^3/h**. *Ans.* (b)
Finally, the log-law Eq. (6.28) can estimate the velocity at r = 1 cm, "y" = R − r = 4 cm:

$$u^* = \left(\frac{\tau_w}{\rho}\right)^{1/2} = \left(\frac{14}{789}\right)^{1/2} = 0.133\ \frac{m}{s};$$

$$\frac{u}{u^*} \approx \frac{1}{\kappa}\ln\left[\frac{\rho u^* y}{\mu}\right] + B = \frac{1}{0.41}\ln\left[\frac{789(0.133)(0.04)}{0.0012}\right] + 5.0 = 24.9$$

$$\text{Then}\quad u \approx 24.9(0.133) \approx \mathbf{3.3\ m/s}\quad \text{at}\ \ r = 1\ \text{cm}.\quad \textit{Ans. (c)}$$

6.67 A straight 10-cm commercial-steel pipe is 1 km long and is laid on a constant slope of 5°. Water at 20°C flows downward, due to gravity only. Estimate the flow rate in m^3/h. What happens if the pipe length is 2 km?

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. If the flow is due to gravity only, then the head loss exactly balances the elevation change:

$$h_f = \Delta z = L\sin\theta = f\frac{L}{d}\frac{V^2}{2g},\quad \text{or}\quad fV^2 = 2gd\sin\theta = 2(9.81)(0.1)\sin 5° \approx 0.171$$

Thus the flow rate is independent of the pipe length L if laid on a constant slope. *Ans.*
For commercial steel, take ε ≈ 0.046 mm, or ε/d ≈ 0.00046. Begin by guessing fully-rough flow for the friction factor, and iterate V and Re and f:

$$f \approx 0.0164,\quad V \approx \left(\frac{0.171}{0.0164}\right)^{1/2} \approx 3.23\ \frac{m}{s},\quad Re = \frac{998(3.23)(0.1)}{0.001} \approx 322000$$

$$f_{better} \approx 0.0179,\quad V_{better} \approx 3.09\ m/s,\quad Re \approx 308000\ \text{(converged)}$$

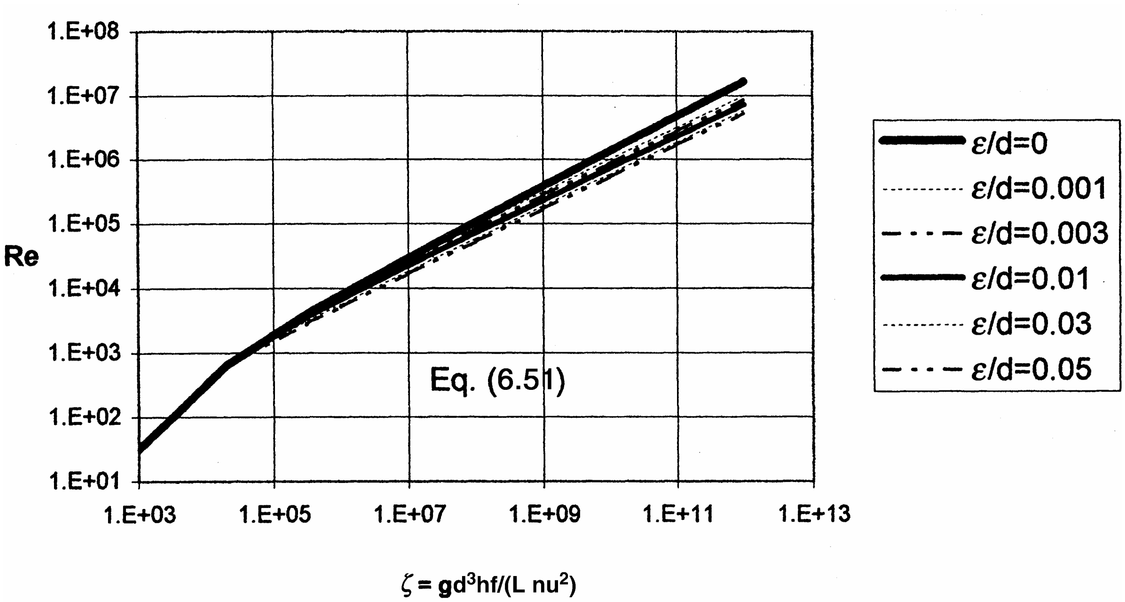
$$\text{Then}\quad Q \approx (\pi/4)(0.1)^2(3.09) \approx 0.0243\ m^3/s \approx \mathbf{87\ m^3/h}.\quad \textit{Ans.}$$

6.68 The Moody chart, Fig. 6.13, is best for finding head loss (or Δp) when Q, V, d, and L are known. It is awkward for the "2nd" type of problem, finding Q when h_f or Δp are known (see Ex. 6.9). Prepare a modified Moody chart whose abscissa is independent of Q and V, using ε/d as a parameter, from which one can immediately read the ordinate to find (dimensionless) Q or V. Use your chart to solve Example 6.9.

Solution: This problem was mentioned *analytically* in the text as Eq. (6.51). The proper parameter which contains head loss only, and *not* flow rate, is ζ:

$$\zeta = \frac{gd^3 h_f}{L\nu^2}\qquad Re_d = -(8\zeta)^{1/2}\log\left(\frac{\varepsilon/d}{3.7} + \frac{1.775}{\sqrt{\zeta}}\right)\qquad \text{Eq. (6.51)}$$

We simply plot Reynolds number versus ζ for various ε/d, as shown below:



To solve Example 6.9, a 100-m-long, 30-cm-diameter pipe with a head loss of 8 m and ε/d = 0.0002, we use that data to compute ζ = **5.3E7**. The oil properties are ρ = 950 kg/m^3 and ν = 2E–5 m^2/s. Enter the chart above: let's face it, the scale is very hard to read, but we estimate, at ζ = 5.3E7, that 6E4 < Re_d < 9E4, which translates to a flow rate of

0.28 < Q < 0.42 m^3/s. *Ans.* (Example 6.9 gave Q = 0.342 m^3/s.)

6.69 For Prob. 6.62 suppose the only pump available can deliver only 80 hp to the fluid. What is the proper pipe size in inches to maintain the 3 ft^3/s flow rate?

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. For cast iron, take $\varepsilon \approx$ 0.00085 ft. We can't specify ε/d because we don't know d. The energy analysis above is correct and should be modified to replace V by Q:

$$h_p = 120 + f\frac{L}{d}\frac{(4Q/\pi d^2)^2}{2g} = 120 + f\frac{2000}{d}\frac{[4(3.0)/\pi d^2]^2}{2(32.2)} = 120 + 453\frac{f}{d^5}$$

$$\text{But also}\quad h_p = \frac{\text{Power}}{\rho g Q} = \frac{80(550)}{62.4(3.0)} = 235 = 120 + \frac{453\,f}{d^5},\quad \text{or:}\quad \mathbf{d^5 \approx 3.94f}$$

Guess f ≈ 0.02, calculate d, ε/d and Re and get a better f and iterate:

$$f \approx 0.020, \quad d \approx [3.94(0.02)]^{1/5} \approx 0.602 \text{ ft}, \quad \text{Re} = \frac{4\rho Q}{\pi\mu d} = \frac{4(1.94)(3.0)}{\pi(2.09E{-}5)(0.602)},$$

$$\text{or} \quad \text{Re} \approx 589000, \quad \frac{\varepsilon}{d} = \frac{0.00085}{0.602} \approx 0.00141, \quad \text{Moody chart:} \quad f_{better} \approx 0.0218 \text{ (repeat)}$$

We are nearly converged. The final solution is f ≈ 0.0217, d ≈ 0.612 ft ≈ **7.3 in** *Ans*.

P6.70 Water at 68°F flows through 200 ft of a horizontal 6-in-diameter asphalted cast iron pipe. (*a*) If the head loss is 4.5 ft, find the average velocity and the flow rate, using the rescaled variable ζ discussed as a "Type 2" problem. (*b*) Does this input data seem familiar to you?

Solution: For water in BG units, take ν = 1.1E-5 ft²/s. For asphalted cast iron, ε = 0.0004 ft, hence ε/d = 0.0004ft/0.5ft = 0.0008. Calculate the velocity-free group ζ:

$$\zeta = \frac{g\,d^3 h_f}{L\nu^2} = \frac{(32.2\,ft/s^2)(0.5\,ft)^3(4.5\,ft)}{(200\,ft)(1.1E-5\,ft^2/s)} = 7.485E8$$

Now get the Reynolds number from the modified Colebrook formula, Eq. (6.51):

$$\text{Re}_d = -(8\zeta)^{1/2}\log_{10}\left(\frac{\varepsilon/d}{3.7} + \frac{1.775}{\sqrt{\zeta}}\right) = -[8\sqrt{7.485E8}]\log_{10}\left[\frac{0.0008}{3.7} + \frac{1.775}{\sqrt{7.485E8}}\right] = 274{,}800$$

$$\text{Then} \quad V = \frac{\nu\,\text{Re}_d}{d} = \frac{(1.1E-5)(274800)}{0.5} = 6.05\ ft/s \qquad Ans.(a)$$

$$Q = \frac{\pi}{4}d^2 V = \frac{\pi}{4}(0.5\,ft)^2(6.05\,ft/s) = 1.19\ ft^3/s \qquad Ans.(a)$$

(*b*) These are the numbers for L. F. Moody's classic example, which was introduced in Ex. 6.6 of the text. We did not get V = 6.00 ft/s because h_f was rounded off from 4.47 ft.

6.71 It is desired to solve Prob. 6.62 for the most economical pump and cast-iron pipe system. If the pump costs \$125 per horsepower delivered to the fluid and the pipe costs \$7000 per inch of diameter, what are the minimum cost and the pipe and pump size to maintain the 3 ft^3/s flow rate? Make some simplifying assumptions.

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For cast iron, take $\varepsilon \approx 0.00085$ ft. Write the energy equation (from Prob. 6.62) in terms of Q and d:

$$P_{in\ hp} = \frac{\rho g Q}{550}(\Delta z + h_f) = \frac{62.4(3.0)}{550}\left\{120 + f\left(\frac{2000}{d}\right)\frac{[4(3.0)/\pi d^2]^2}{2(32.2)}\right\} = 40.84 + \frac{154.2f}{d^5}$$

$$\text{Cost} = \$125P_{hp} + \$7000d_{inches} = 125(40.84 + 154.2f/d^5) + 7000(12d), \quad \text{with d in ft.}$$

$$\text{Clean up:} \quad \text{Cost} \approx \$5105 + 19278f/d^5 + 84000d$$

Regardless of the (unknown) value of f, this Cost relation does show a minimum. If we assume for simplicity that *f* is constant, we may use the differential calculus:

$$\left.\frac{d(\text{Cost})}{d(d)}\right|_{f\approx const} = \frac{-5(19278)f}{d^6} + 84000, \quad \text{or} \quad d_{best} \approx (1.148\ f)^{1/6}$$

$$\text{Guess } f \approx 0.02, \quad d \approx [1.148(0.02)]^{1/6} \approx 0.533\text{ ft}, \quad Re = \frac{4\rho Q}{\pi\mu d} \approx 665000, \quad \frac{\varepsilon}{d} \approx 0.00159$$

$$\text{Then} \quad f_{better} \approx 0.0224, \quad d_{better} \approx 0.543\text{ ft (converged)}$$

Result: $d_{best} \approx 0.543$ ft ≈ **6.5 in,** $Cost_{min} \approx \$14300_{pump} + \$45600_{pipe} \approx$ **\$60000**. *Ans.*

6.72 Modify Prob. P6.57 by letting the diameter be unknown. Find the proper pipe diameter for which the pool will drain in about 2 hours flat.

Solution: Recall the data: Let $W = 5$ m, $Y = 8$ m, $h_o = 2$ m, $L = 15$ m, and $\varepsilon = 0$, with water, $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. We apply the same theory as Prob. 6.57:

$$V = \sqrt{\frac{2gh}{1 + fL/D}}, \quad t_{drain} \approx \frac{4WY}{\pi D^2}\sqrt{\frac{2h_o(1 + f_{av}L/D)}{g}}, \quad f_{av} = fcn(Re_D) \quad \text{for a smooth pipe.}$$

For the present problem, t_{drain} = 2 hours and *D* is the unknown. Use an average value h = 1 m to find f_{av}. Enter these equations on EES (or you can iterate by hand) and the final results are

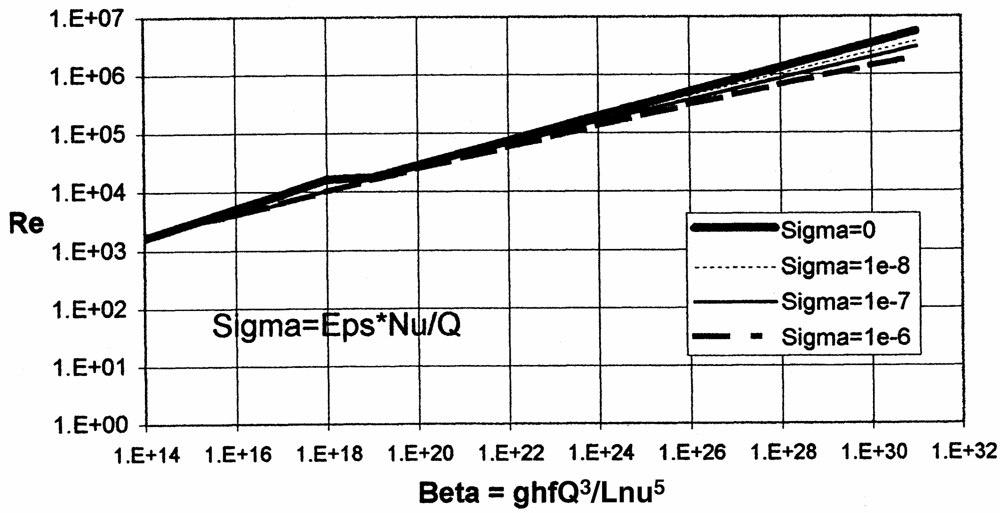
$$V = 2.36\text{ m/s}; \quad Re_D = 217{,}000; \quad f_{av} \approx 0.0154; \quad D = 0.092\text{ m} \approx \mathbf{9.2\ cm} \quad \textit{Ans.}$$

6.73 The Moody chart, Fig. 6.13, is best for finding head loss (or Δp) when *Q*, *V*, *d*, and *L* are known. It is awkward for the "3rd" type of problem, finding *d* when h_f (or Δ*p*) and *Q* are known (see Ex. 6.11). Prepare a modified Moody chart whose abscissa is independent of *d*, using as a parameter ε non-dimensionalized without *d*, from which one can immediately read the (dimensionless) ordinate to find *d*. Use your chart to solve Ex. 6.11.

Solution: An appropriate Pi group which does not contain *d* is $\beta = (gh_fQ^3)/(L\nu^5)$. Similarly, an appropriate roughness parameter without *d* is $\sigma = (\varepsilon\nu/Q)$. After a lot of algebra, the Colebrook friction factor formula (6.48) becomes

$$Re_d^{5/2} = -2.0\left(\frac{128\beta}{\pi^3}\right)^{1/2}\log_{10}\left[\frac{\pi\sigma Re_d}{14.8} + \frac{2.51Re_d^{3/2}}{(128\beta/\pi^3)^{1/2}}\right]$$

A plot of this messy relation is given below.



To solve Example 6.11, a 100-m-long, unknown-diameter pipe with a head loss of 8 m, flow rate of 0.342 m^3/s, and ε = 0.06 mm, we use that data to compute β = **9.8E21** and $\boldsymbol{\sigma}$ **= 3.5E–6**. The oil properties are ρ = 950 kg/m^3 and ν = 2E–5 m^2/s. Enter the chart above: let's face it, the scale is very hard to read, but we estimate, at β = 9.8E21 and σ = 3.5E–6, that 6E4 < Re_d < 8E4, which translates to a diameter of **0.27 < *d* < 0.36 m**. *Ans.* (Example 6.11 gave *d* = 0.3 m.)

P6.74 Two reservoirs, which differ in surface elevation by 40 m, are connected by a new commercial steel pipe of diameter 8 cm. If the desired weight flow rate is 200 N/s of water at 20°C, what is the proper length of the pipe? Neglect minor losses.

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m-s. For commercial steel, ε = 0.046 mm, thus ε/d = 0.046mm/80mm = 0.000575. Find the velocity and the friction factor:

$$V = \frac{\dot{w}/(\rho g)}{(\pi/4)D^2} = \frac{200/[998(9.81)]}{(\pi/4)(0.08)^2} = 4.06\frac{m}{s}, \quad \mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{998(4.06)(0.08)}{0.001} = 324{,}000$$

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}_D\sqrt{f}}\right) \quad \text{yields} \quad f = 0.0185$$

Then we find the pipe length from the energy equation, which is simple in this case:

$$\Delta z = 40\,m = f\frac{L}{D}\frac{V^2}{2g} = (0.0185)\frac{L}{(0.08m)}\frac{(4.06)^2}{2(9.81)}, \quad \textit{Solve} \quad L \approx 205\,m \quad \textit{Ans.}$$

6.75 You wish to water your garden with 100 ft of $\frac{5}{8}$-in-diameter hose whose roughness is 0.011 in. What will be the delivery, in ft^3/s, if the gage pressure at the faucet is 60 lbf/in^2? If there is no nozzle (just an open hose exit), what is the maximum horizontal distance the exit jet will carry?

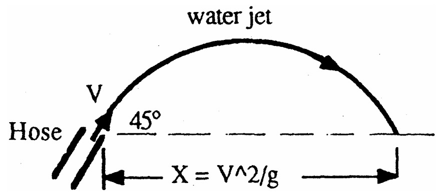


Fig. P6.75

Solution: For water, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09\text{E}{-5}$ slug/ft·s. We are given $\varepsilon/d = 0.011/(5/8) \approx 0.0176$. For constant area hose, $V_1 = V_2$ and energy yields

$$\frac{p_{faucet}}{\rho g} = h_f, \quad \text{or:} \quad \frac{60 \times 144 \text{ psf}}{1.94(32.2)} = 138 \text{ ft} = f\frac{L}{d}\frac{V^2}{2g} = f\frac{100}{(5/8)/12}\frac{V^2}{2(32.2)},$$

$$\text{or} \quad fV^2 \approx 4.64. \quad \text{Guess} \quad f \approx f_{\text{fully rough}} = 0.0463, \quad V \approx 10.0\ \frac{\text{ft}}{\text{s}}, \quad Re \approx 48400$$

$$\text{then} \quad f_{\text{better}} \approx 0.0472, \quad V_{\text{final}} \approx \mathbf{9.91\ ft/s} \text{ (converged)}$$

The hose delivery then is $Q = (\pi/4)(5/8/12)^2(9.91) = \mathbf{0.0211\ ft^3/s}$. *Ans.* (a)

From elementary particle-trajectory theory, the maximum horizontal distance X travelled by the jet occurs at $\theta = 45°$ (see figure) and is $\mathbf{X} = V^2/g = (9.91)^2/(32.2) \approx \mathbf{3.05\ ft}$ *Ans.* (b), which is pitiful. You need a *nozzle* on the hose to increase the exit velocity.

6.76 The small turbine in Fig. P6.76 extracts 400 W of power from the water flow. Both pipes are wrought iron. Compute the flow rate Q m^3/h. Sketch the EGL and HGL accurately.

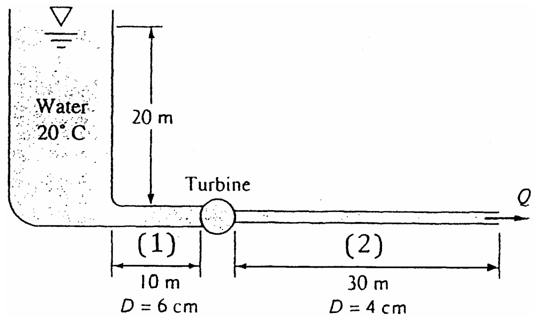


Fig. P6.76

Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For wrought iron, take $\varepsilon \approx 0.046$ mm, hence $\varepsilon/d_1 = 0.046/60 \approx 0.000767$ and $\varepsilon/d_2 = 0.046/40 \approx 0.00115$. The energy equation, with $V_1 \approx 0$ and $p_1 = p_2$, gives

$$z_1 - z_2 = 20 \text{ m} = \frac{V_2^2}{2g} + h_{f2} + h_{f1} + h_{\text{turbine}}, \quad h_{f1} = f_1\frac{L_1}{d_1}\frac{V_1^2}{2g} \quad \text{and} \quad h_{f2} = f_2\frac{L_2}{d_2}\frac{V_2^2}{2g}$$

$$\text{Also,} \quad h_{\text{turbine}} = \frac{P}{\rho g Q} = \frac{400 \text{ W}}{998(9.81)Q} \quad \text{and} \quad Q = \frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_2^2V_2$$

The only unknown is Q, which we may determine by iteration after an initial guess:

$$h_{turb} = \frac{400}{998(9.81)Q} = 20 - \frac{8f_1L_1Q^2}{\pi^2 g d_1^5} - \frac{8f_2L_2Q^2}{\pi^2 g d_2^5} - \frac{8Q^2}{\pi^2 g d_2^4}$$

$$\text{Guess} \quad Q = 0.003\ \frac{m^3}{s}, \quad \text{then}\ Re_1 = \frac{4\rho Q}{\pi\mu d_1} = 63500, \quad f_{1,Moody} \approx 0.0226,$$

$$Re_2 = 95300, \quad f_2 \approx 0.0228.$$

But, for this guess, h_{turb}(left hand side) ≈ 13.62 m, h_{turb}(right hand side) ≈ 14.53 m (wrong). Other guesses converge to h_{turb} ≈ 9.9 meters. For Q ≈ 0.00413 m^3/s ≈ **15 m^3/h**. *Ans.*

6.77 Modify Prob. 6.76 into an economic analysis, as follows. Let the 40 m of wrought-iron pipe have a uniform diameter *d*. Let the steady water flow available be *Q* = 30 m^3/h. The cost of the turbine is $4 per watt developed, and the cost of the piping is $75 per centimeter of diameter. The power generated may be sold for $0.08 per kilowatt hour. Find the proper pipe diameter for minimum *payback time*, i.e., minimum time for which the power sales will equal the initial cost of the system.

Solution: With flow rate known, we need only guess a diameter and compute power from the energy equation similar to Prob. 6.76:

$$P = \rho g Q h_t, \quad \text{where}\ h_t = 20\ m - \frac{V^2}{2g}\left(1 + f\frac{L}{d}\right) = 20 - \frac{8Q^2}{\pi^2 g d^4}\left(1 + f\frac{L}{d}\right)$$

$$\text{Then} \quad \text{Cost} = \$4 * P + \$75(100d) \quad \text{and} \quad \text{Annual income} = \$0.08\left(\frac{P}{1000}\right)(24)(365)$$

The Moody friction factor is computed from Re = 4ρQ/(πμd) and ε/*d* = 0.066/*d*(mm). The payback time, in years, is then the cost divided by the annual income. For example,

$$\text{If}\ d = 0.1\ m, \quad Re \approx 106000, \quad f \approx 0.0200, \quad h_t \approx 19.48\ m, \quad P = 1589.3\ W$$

Cost ≈ $7107 Income = $1,114/year Payback ≈ **6.38 years**

Since the piping cost is very small (<$1000), both cost and income are nearly proportional to power, hence the payback will be nearly the same (6.38 years) regardless of diameter. There is an almost invisible minimum at **d ≈ 7 cm**, Re ≈ 151000, f ≈ 0.0201, h_t ≈ 17.0 m, Cost ≈ $6078, Income ≈ $973, Payback ≈ 6.25 years. However, as diameter d decreases, we generate less power and gain little in payback time.

6.78 In Fig. P6.78 the connecting pipe is commercial steel 6 cm in diameter. Estimate the flow rate, in m^3/h, if the fluid is water at 20°C. Which way is the flow?

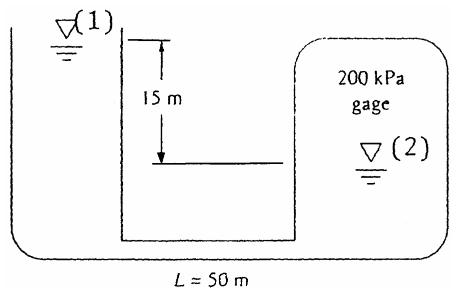


Fig. P6.78

Solution: For water, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For commercial steel, take $\varepsilon \approx$ 0.046 mm, hence ε/d = 0.046/60 ≈ 0.000767. With p_1, V_1, and V_2 all ≈ 0, the energy equation between surfaces (1) and (2) yields

$$0+0+z_1 \approx \frac{p_2}{\rho g}+0+z_2+h_f, \quad \text{or} \quad h_f = 15-\frac{200000}{998(9.81)} \approx -5.43 \text{ m } \textbf{(flow to left)} \leftarrow$$

$$\text{Guess turbulent flow:} \quad h_f = f\frac{L}{d}\frac{V^2}{2g} = f\frac{50}{0.06}\frac{V^2}{2(9.81)} = 5.43, \quad \text{or:} \quad fV^2 \approx 0.1278$$

$$\frac{\varepsilon}{d} = 0.00767, \quad \text{guess } f_{\text{fully rough}} \approx 0.0184, \quad V \approx \left(\frac{0.1278}{0.0184}\right)^{1/2} \approx 2.64\ \frac{m}{s}, \quad Re = 158000$$

$$f_{\text{better}} \approx 0.0204, \quad V_{\text{better}} = 2.50\ \frac{m}{s}, \quad Re_{\text{better}} \approx 149700, \quad f_{\text{3rd iteration}} \approx 0.0205 \text{ (converged)}$$

The iteration converges to

$$f \approx 0.0205,\ V \approx 2.49 \text{ m/s},\ Q = (\pi/4)(0.06)^2(2.49) = 0.00705 \text{ m}^3/\text{s} = \mathbf{25\ m^3/h} \leftarrow \quad \textit{Ans.}$$

6.79 A garden hose is used as the return line in a waterfall display at the mall. In order to select the proper pump, you need to know the hose wall roughness, which is not supplied by the manufacturer. You devise a simple experiment: attach the hose to the drain of an above-ground pool whose surface is 3 m above the hose outlet. You estimate the minor loss coefficient in the entrance region as 0.5, and the drain valve has a minor-loss equivalent length of 200 diameters when fully open. Using a bucket and stopwatch, you open the valve and measure a flow rate of 2.0E–4 m^3/s for a hose of inside diameter 1.5 cm and length 10 m. Estimate the roughness height of the hose inside surface.

Solution: First evaluate the average velocity in the hose and its Reynolds number:

$$V = \frac{Q}{A} = \frac{2.0E\text{–}4}{(\pi/4)(0.015)^2} = 1.13\ \frac{m}{s}, \quad Re_d = \frac{\rho V d}{\mu} = \frac{998(1.13)(0.015)}{0.001} = 16940\ (\textit{turbulent})$$

Write the energy equation from surface (point 1) to outlet (point 2), assuming an energy correction factor $\alpha = 1.05$:

$$\frac{p_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_f + \sum h_{loss}, \quad where\ \sum h_{loss} = \left(K_e + f\frac{L_{eq}}{d}\right)\frac{\alpha_2 V_2^2}{2g}$$

The unknown is the friction factor:

$$f = \frac{\dfrac{z_1 - z_2}{V^2/2g} - \alpha_2 - K_e}{(L + L_{eq})/d} = \frac{\dfrac{3m}{(1.13)^2/2(9.81)} - 1.05 - 0.5}{(10/0.015 + 200)} = 0.0514$$

For $f = 0.0514$ and $\mathrm{Re_d} = 16940$, the Moody chart (Eq. 6.48) predicts $\varepsilon/d \approx 0.0206$. Therefore the estimated hose-wall roughness is $\varepsilon = 0.0206(1.5\text{ cm}) =$ **0.031 cm** *Ans.*

6.80 The head-versus-flow-rate characteristics of a centrifugal pump are shown in Fig. P6.80. If this pump drives water at 20°C through 120 m of 30-cm-diameter cast-iron pipe, what will be the resulting flow rate, in m³/s?

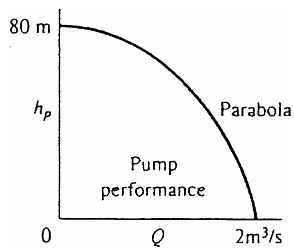


Fig. P6.80

Solution: For water, take $\rho = 998$ kg/m³ and $\mu = 0.001$ kg/m·s. For cast iron, take $\varepsilon \approx 0.26$ mm, hence $\varepsilon/d = 0.26/300 \approx 0.000867$. The head loss must match the pump head:

$$h_f = f\frac{L}{d}\frac{V^2}{2g} = \frac{8fLQ^2}{\pi^2 g d^5} = h_{pump} \approx 80 - 20Q^2, \quad \text{with Q in m}^3/\text{s}$$

$$\text{Evaluate} \quad h_f = \frac{8f(120)Q^2}{\pi^2(9.81)(0.3)^5} = 80 - 20Q^2, \quad \text{or:} \quad Q^2 \approx \frac{80}{20 + 4080f}$$

$$\text{Guess} \quad f \approx 0.02, \quad Q = \left[\frac{80}{20 + 4080(0.02)}\right]^{1/2} \approx 0.887\ \frac{m^3}{s}, \quad Re = \frac{4\rho Q}{\pi\mu d} \approx 3.76E6$$

$$\frac{\varepsilon}{d} = 0.000867, \quad f_{better} \approx 0.0191, \quad Re_{better} \approx 3.83E6, \quad \text{converges to } \mathbf{Q \approx 0.905\ \frac{m^3}{s}} \quad Ans.$$

6.81 The pump in Fig. P6.80 is used to deliver gasoline at 20°C through 350 m of 30-cm-diameter galvanized iron pipe. Estimate the resulting flow rate, in m^3/s. (Note that the pump head is now in meters of gasoline.)

Solution: For gasoline, take ρ = 680 kg/m^3 and μ = 2.92E–4 kg/m·s. For galvanized iron, take $\varepsilon \approx$ 0.15 mm, hence ε/d = 0.15/300 ≈ 0.0005. Head loss matches pump head:

$$h_f = \frac{8fLQ^2}{\pi^2 g d^5} = \frac{8f(350)Q^2}{\pi^2(9.81)(0.3)^5} = 11901fQ^2 = h_{pump} \approx 80 - 20Q^2, \quad Q^2 = \frac{80}{20 + 11901f}$$

$$\text{Guess} \quad f_{rough} \approx 0.017, \quad Q \approx 0.600\ \frac{m^3}{s},$$

$$Re_{better} \approx 5.93E6, \quad \frac{\varepsilon}{d} = 0.0005, \quad f_{better} \approx 0.0168$$

This converges to $f \approx 0.0168$, $Re \approx 5.96E6$, $\mathbf{Q \approx 0.603\ m^3/s}$. *Ans.*

6.82 The pump in Fig. P6.80 has its maximum efficiency at a head of 45 m. If it is used to pump ethanol at 20°C through 200 m of commercial-steel pipe, what is the proper pipe diameter for maximum pump efficiency?

Solution: For ethanol, take ρ = 789 kg/m^3 and μ = 1.2E–3 kg/m·s. For commercial steel, take $\varepsilon \approx$ 0.046 mm, hence ε/d = 0.046/(1000d). We know the head and flow rate:

$$h_{pump} = 45\ m \approx 80 - 20Q^2, \quad \text{solve for } Q \approx 1.323\ m^3/s.$$

$$\text{Then} \quad h_p = h_f = \frac{8fLQ^2}{\pi^2 g d^5} = \frac{8f(200)(1.323)^2}{\pi^2(9.81)d^5} = \frac{28.92f}{d^5} = 45\ m, \quad \text{or:} \quad d \approx 0.915f^{1/5}$$

$$\text{Guess} \quad f \approx 0.02, \quad d \approx 0.915(0.02)^{1/5} \approx 0.419\ m,$$

$$Re = \frac{4\rho Q}{\pi\mu d} \approx 2.6E6, \quad \frac{\varepsilon}{d} \approx 0.000110$$

$$\text{Then} \quad f_{better} \approx 0.0130, \quad d_{better} \approx 0.384\ m, \quad Re_{better} \approx 2.89E6, \quad \frac{\varepsilon}{d}\Big|_{better} \approx 0.000120$$

This converges to $f \approx 0.0129$, $Re \approx 2.89E6$, $\mathbf{d \approx 0.384\ m}$. *Ans.*

6.83 For the system of Fig. P6.55, let Δz = 80 m and L = 185 m of cast-iron pipe. What is the pipe diameter for which the flow rate will be 7 m^3/h?

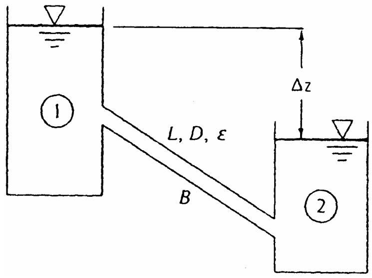


Fig. P6.55

Solution: For water, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For cast iron, take $\varepsilon \approx$ 0.26 mm, but d is unknown. The energy equation is simply

$$\Delta z = 80\text{ m} = h_f = \frac{8fLQ^2}{\pi^2 g d^5} = \frac{8f(185)(7/3600)^2}{\pi^2(9.81)d^5} = \frac{5.78E{-}5 f}{d^5}, \quad \text{or} \quad d \approx 0.0591 f^{1/5}$$

$$\text{Guess} \quad f \approx 0.03, \quad d = 0.0591(0.03)^{1/5} \approx 0.0293\text{ m}, \quad Re = \frac{4\rho Q}{\pi\mu d} \approx 84300, \quad \frac{\varepsilon}{d} \approx 0.00887$$

Iterate: $f_{better} \approx 0.0372$, $d_{better} \approx 0.0306$ m, $Re_{better} \approx 80700$, $\varepsilon/d|_{better} \approx 0.00850$, etc. The process converges to f ≈ 0.0367, **d ≈ 0.0305 m**. *Ans.*

6.84 It is desired to deliver 60 m^3/h of water (ρ = 998 kg/m^3, μ = 0.001 kg/m·s) at 20°C through a horizontal asphalted cast-iron pipe. Estimate the pipe diameter which will cause the pressure drop to be exactly 40 kPa per 100 meters of pipe length.

Solution: Write out the relation between Δp and friction factor, taking "L" = 100 m:

$$\Delta p = f\frac{L}{d}\frac{\rho}{2}V^2 = f\frac{100}{d}\frac{(998)}{2}\left[\frac{60/3600}{(\pi/4)d^2}\right]^2 = 40{,}000 = 22.48\frac{f}{d^5}, \quad or: \quad d^5 = 0.00562f$$

Knowing ε = 0.12 mm, then ε/d = 0.00012/d and $Re_d = 4\rho Q/(\pi\mu d)$ = 21178/d. Use EES, or guess $f \approx$ 0.02 and iterate until the proper diameter and friction factor are found.
Final convergence: $f \approx$ 0.0216; $Re_d \approx$ 204,000; ***d* = 0.104 m**. *Ans.*

P6.85 Repeat Prob. 6.26 if the fluid is *water* at 20°C. This is more laborious than the earlier problem, but the basic concepts are just the same.

The system sketch is repeated here for convenience.

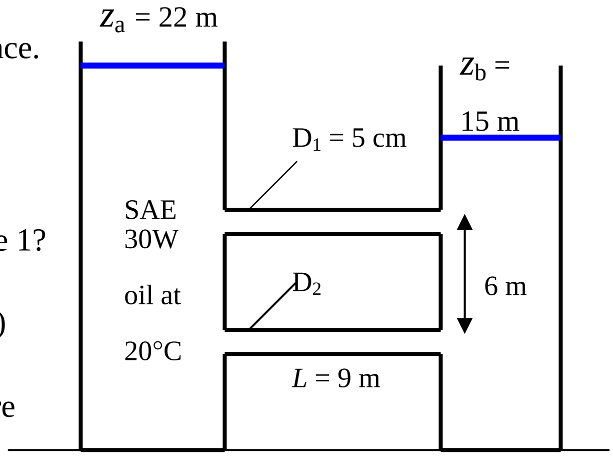


Fig. P6.26

Solution: The problem has three parts.

(*a*) What is D_2 if its flow rate is twice that of pipe 1?

If Re_D is not too large, 10^4-10^5, we use Eq. (6.41) for a quick estimate. If head loss and (L, ρ, μ) are constant, then $Q^{1.75}$ is proportional to $D^{4.75}$, or $Q_{\text{turbulent}} \propto D^{19/7}$. If the flow rate doubles, then

$$\frac{Q_2}{Q_1} = 2.0 \approx \left(\frac{D_2}{D_1}\right)^{19/7}, \quad \text{hence } D_2 \approx (5\,cm)(2.0)^{7/19} \approx 0.0645\,cm \quad \textit{Ans.}(a)$$

$$\text{More accurate result from part(c): } D_2 = \mathbf{0.06495\,m}$$

(*b*) Are both pipe flows turbulent? Well, we think so and used Eq. (6.41). Check this later.

(*c*) What is the flow rate in pipe 2 (m^3/s)? Neglect minor losses. For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m-s. The energy equation yields the same for both pipes:

$$\Delta z = 22m - 15m = 7m = f_2 \frac{L}{D_2}\frac{V_2^2}{2g} = f_1 \frac{L}{D_1}\frac{V_1^2}{2g}, \quad \text{smooth tube}$$

Since we don't know Re_D, or even know the exact D_2, this requires either iteration or EES. The results (obtained by EES) are:

$$Re_1 = 370{,}000 \;;\; f_1 = 0.01391 \;;\; D_1(given) = 0.05\,m \;;\; Q_1 = \mathbf{0.01454\,m^3/s}$$
$$Re_2 = 569{,}000 \;;\; f_2 = 0.01285 \;;\; D_2 = \mathbf{0.06495\,m} \;;\; Q_2 = 0.02908\,m^3/s = 2Q_1$$

The Reynolds numbers are too high for the power-law, Eq. (6.41), but the error is only 0.7%.

6.86 SAE 10 oil at 20°C flows at an average velocity of 2 m/s between two smooth parallel horizontal plates 3 cm apart. Estimate (a) the centerline velocity, (b) the head loss per meter, and (c) the pressure drop per meter.

Solution: For SAE 10 oil, take $\rho = 870$ kg/m^3 and $\mu = 0.104$ kg/m·s. The half-distance between plates is called "*h*" (see Fig. 6.37). Check D_h and Re:

$$D_h = \frac{4A}{P} = 4h = 6\text{ cm},\quad Re_{D_h} = \frac{\rho V D_h}{\mu} = \frac{870(2.0)(0.06)}{0.104} \approx 1004\text{ (laminar)}$$

$$\text{Then}\quad u_{CL} = u_{max} = \frac{3}{2}V = \frac{3}{2}(2.0) \approx \mathbf{3.0\ m/s}\quad \textit{Ans.}\text{ (a)}$$

The head loss and pressure drop per meter folow from laminar theory, Eq. (6.63):

$$\Delta p = \frac{3\mu VL}{h^2} = \frac{3(0.104)(2.0)(1.0)}{(0.015\text{ m})^2} \approx \mathbf{2770\ Pa/m}\quad \textit{Ans.}\text{ (c)}$$

$$h_f = \frac{\Delta p}{\rho g} = \frac{2770}{870(9.81)} \approx \mathbf{0.325\ m/m}\quad \textit{Ans.}\text{ (b)}$$

6.87 A commercial-steel annulus 40 ft long, with a = 1 in and b = $\frac{1}{2}$ in, connects two reservoirs which differ in surface height by 20 ft. Compute the flow rate in ft^3/s through the annulus if the fluid is water at 20°C.

Solution: For water, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For commercial steel, take $\varepsilon \approx 0.00015$ ft. Compute the hydraulic diameter of the annulus:

$$D_h = \frac{4A}{P} = 2(a - b) = 1 \text{ inch};$$

$$h_f = 20 \text{ ft} = f\frac{L}{D_h}\frac{V^2}{2g} = f\left(\frac{40}{1/12}\right)\frac{V^2}{2(32.2)}, \quad \text{or:} \quad fV^2 \approx 2.683$$

We can make a reasonable estimate by simply relating the Moody chart to *Dh*, rather than the more complicated "effective diameter" method of Eq. (6.77). Thus

$$\frac{\varepsilon}{D_h} = \frac{0.00015}{1/12} \approx 0.0018, \quad \text{Guess } f_{rough} \approx 0.023, \quad V = (2.683/0.023)^{1/2} \approx 10.8 \frac{ft}{s}$$

$$Re = \frac{\rho V D_h}{\mu} = \frac{1.94(10.8)(1/12)}{2.09E{-}5} \approx 83550, \quad f_{better} \approx 0.0249, \quad V_{better} \approx 10.4 \frac{ft}{s}$$

This converges to $f \approx 0.0250$, $V \approx 10.37$ ft/s, $Q = \pi(a^2 - b^2)V = \mathbf{0.17\ ft^3/s}$. *Ans.*

6.88 An oil cooler consists of multiple parallel-plate passages, as shown in Fig. P6.88. The available pressure drop is 6 kPa, and the fluid is SAE 10W oil at 20°C. If the desired total flow rate is 900 m^3/h, estimate the appropriate number of passages. The plate walls are hydraulically smooth.

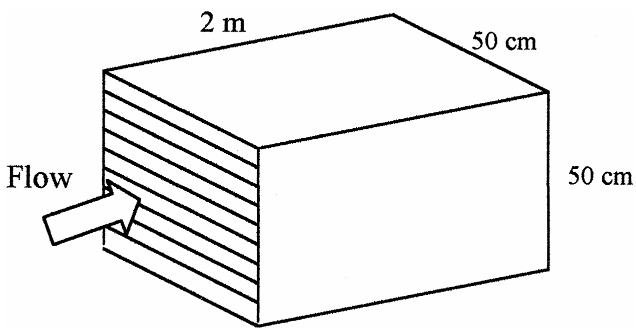


Fig. P6.88

Solution: For SAE 10W oil, $\rho = 870$ kg/m^3 and $\mu = 0.104$ kg/m·s. The pressure drop remains 6 kPa no matter how many passages there are (ducts in parallel). Guess laminar flow, Eq. (6.63),

$$Q_{one\ passage} = \frac{bh^3}{3\mu}\frac{\Delta p}{L}$$

where h is the half-thickness between plates. If there are N passages, then b = 50 cm for all and h = 0.5 m/(2N). We find h and N such that NQ = 900 m^3/h for the full set of passages. The problem is ideal for EES, but one can iterate with a calculator also. We find that 18 passages are one too many—Q only equals 835 m^3/h. The better solution is:

$$\mathbf{N = 17\ passages},\quad Q_N = 935\ \text{m}^3/\text{h},\quad h = 1.47\ \text{cm},\quad \text{Re}_{D_h} = 512\ \text{(laminar flow)}$$

6.89 An annulus of narrow clearance causes a very large pressure drop and is useful as an accurate measurement of viscosity. If a smooth annulus 1 m long with a = 50 mm and b = 49 mm carries an oil flow at 0.001 m^3/s, what is the oil viscosity if the pressure drop is 250 kPa?

Solution: Assuming laminar flow, use Eq. (6.73) for the pressure drop and flow rate:

$$Q = \frac{\pi}{8\mu}\frac{\Delta p}{L}\left[a^4 - b^4 - \frac{(a^2-b^2)^2}{\ln(a/b)}\right],\quad \text{or, for the given data:}$$

$$0.001\ \text{m}^3/\text{s} = \frac{\pi}{8\mu}\left(\frac{250000}{1\ \text{m}}\right)\left[(0.05)^4 - (0.049)^4 - \frac{\{(0.05)^2-(0.049)^2\}^2}{\ln(0.05/0.049)}\right]$$

$$\text{Solve for}\quad \mu \approx \mathbf{0.0065\ kg/m{\cdot}s}\quad \textit{Ans.}$$

6.90 A 90-ft-long sheet-steel duct carries air at approximately 20°C and 1 atm. The duct cross section is an equilateral triangle whose side measures 9 in. If a blower can supply 1 hp to the flow, what flow rate, in ft^3/s, will result?

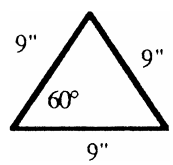


Fig. P6.90

Solution: For air at 20°C and 1 atm, take $\rho \approx 0.00234$ slug/ft^3 and $\mu = 3.76E{-}7$ slug/ft·s. Compute the hydraulic diameter, and express the head loss in terms of Q:

$$D_h = \frac{4A}{P} = \frac{4(1/2)(9)(9\sin 60°)}{3(9)} = 5.2'' = 0.433\ \text{ft}$$

$$h_f = f\frac{L}{D_h}\frac{(Q/A)^2}{2g} = f\left(\frac{90}{0.433}\right)\frac{\{Q/[0.5(9/12)^2\sin 60°]\}^2}{2(32.2)} \approx 54.4fQ^2$$

For sheet steel, take $\varepsilon \approx 0.00015$ ft, hence $\varepsilon/D_h \approx 0.000346$. Now relate everything to the input power:

$$\text{Power} = 1\text{ hp} = 550\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}} = \rho g Q h_f = (0.00234)(32.2)Q[54.4fQ^2],$$

$$\text{or:}\quad fQ^3 \approx 134 \quad \text{with Q in ft}^3/\text{s}$$

$$\text{Guess}\quad f \approx 0.02,\quad Q = (134/0.02)^{1/3} \approx 18.9\ \frac{\text{ft}^3}{\text{s}},\quad Re = \frac{\rho(Q/A)D_h}{\mu} \approx 209000$$

Iterate: $f_{better} \approx 0.0179$, $Q_{better} \approx 19.6$ ft^3/s, $Re_{better} \approx 216500$. The process converges to

$$f \approx 0.01784,\ V \approx 80.4\text{ ft/s},\ \mathbf{Q \approx 19.6\ ft^3/s}.\quad \textit{Ans.}$$

6.91 Heat exchangers often consist of many triangular passages. Typical is Fig. P6.91, with L = 60 cm and an isosceles-triangle cross section of side length a = 2 cm and included angle β = 80°. If the average velocity is V = 2 m/s and the fluid is SAE 10 oil at 20°C, estimate the pressure drop.

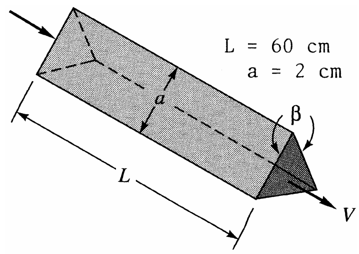


Fig. P6.91

Solution: For SAE 10 oil, take ρ = 870 kg/m^3 and μ = 0.104 kg/m·s. The Reynolds number based on side length a is Re = $\rho Va/\mu$ ≈ **335**, so the flow is *laminar*. The bottom side of the triangle is 2(2 cm)sin40° ≈ *2.57* cm. Calculate hydraulic diameter:

$$A = \frac{1}{2}(2.57)(2\cos 40°) \approx 1.97\text{ cm}^2;\quad P = 6.57\text{ cm};\quad D_h = \frac{4A}{P} \approx 1.20\text{ cm}$$

$$Re_{D_h} = \frac{\rho V D_h}{\mu} = \frac{870(2.0)(0.0120)}{0.104} \approx 201;\quad \text{from Table 6.4},\ \theta = 40°,\quad fRe \approx 52.9$$

$$\text{Then}\quad f = \frac{52.9}{201} \approx 0.263,\quad \Delta p = f\frac{L}{D_h}\frac{\rho}{2}V^2 = (0.263)\left(\frac{0.6}{0.012}\right)\left(\frac{870}{2}\right)(2)^2$$

$$\approx \mathbf{23000\ Pa}\quad \textit{Ans.}$$

6.92 A large room uses a fan to draw in atmospheric air at 20°C through a 30 cm by 30 cm commercial-steel duct 12 m long, as in Fig. P6.92. Estimate (a) the air flow rate in m^3/hr if the room pressure is 10 Pa vacuum; and (b) the room pressure if the flow rate is 1200 m^3/hr. Neglect minor losses.

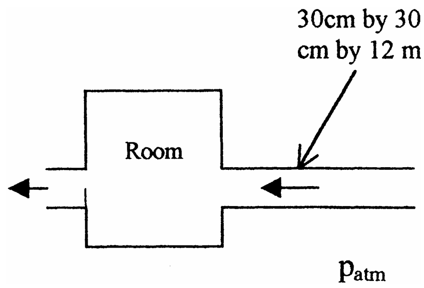


Fig. P6.92

Solution: For air, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. For commercial steel, $\varepsilon = 0.046$ mm. For a square duct, D_h = side-length = 30 cm, hence $\varepsilon/d = 0.046/300 = 0.000153$. The (b) part is easier, with flow rate known we can evaluate velocity, Reynolds number, and friction factor:

$$V = \frac{Q}{A} = \frac{1200/3600}{(0.3)(0.3)} = 3.70\ \frac{m}{s},\quad Re_{D_h} = \frac{1.2(3.70)(0.3)}{1.8E{-}5} = 74100,\quad \text{thus}\quad f_{Moody} \approx 0.0198$$

Then the pressure drop follows immediately:

$$\Delta p = f\frac{L}{D_h}\frac{\rho}{2}V^2 = 0.0198\left(\frac{12}{0.3}\right)\left(\frac{1.2}{2}\right)(3.70)^2 = 6.53\ Pa,$$

or: $\mathbf{p_{room} = 6.5\ Pa}$ (*vacuum*) *Ans.* (b)

(a) If $\Delta p = 10$ Pa (vacuum) is known, we must iterate to find friction factor:

$$\Delta p = 10\ \ Pa = f\left(\frac{12}{0.3}\right)\left(\frac{1.2}{2}\right)V^2,\quad V = \frac{Q}{(0.3)^2},\quad f = fcn\left(\frac{1.2V(0.3)}{1.8E{-}5}, \frac{\varepsilon}{D_h} = 0.000153\right)$$

After iteration, the results converge to:

$V = 4.69$ m/s; $Re_d = 93800$; $f = 0.0190$; $\boldsymbol{Q} = 0.422$ m^3/s = **1520 m^3/h** *Ans.* (a)

P6.93 In Moody's Example 6.6, the 6-inch diameter, 200-ft-long asphalted cast iron pipe has a pressure drop of about 280 lbf/ft^2 when the average water velocity is 6 ft/s. Compare this to an *annular* cast iron pipe with an inner diameter of 6 in and the same annular average velocity of 6 ft/s. (*a*) What outer diameter would cause the flow to have the same pressure drop of 280 lbf/ft^2? (*b*) How do the cross-section areas compare, and why? Use the hydraulic diameter approximation.

Solution: Recall the Ex. 6.6 data, $\varepsilon = 0.0004$ ft. For water at 68°F, take $\rho = 1.94$ slug/ft^3 and $\mu =$ 2.09E-5 slug/ft-sec. The hydraulic diameter of an annulus is $D_h = 2(R_o - R_i)$, where $R_i = 0.25$ ft. We know the pressure drop, hence the head loss is

$$h_f = f\frac{L}{D_h}\frac{V^2}{2g} = f\frac{200\,ft}{2(R_o - 0.25\,ft)}\frac{(6\,ft/s)^2}{32.2\,ft/s^2} = \frac{\Delta p}{\rho g} = \frac{280\,lbf/ft^2}{62.4\,lbf/ft^3} = 4.49\,ft$$

We do not know f or R_o. The additional relation is the Moody friction factor correlation:

$$\frac{1}{\sqrt{f}} \approx -2.0\log_{10}\left(\frac{\varepsilon/D_h}{3.7} + \frac{2.51}{\mathrm{Re}_{D_h}\sqrt{f}}\right) \quad \text{where} \quad \mathrm{Re}_{D_h} = \frac{\rho V D_h}{\mu} = \frac{(1.94)(6.0)[2(R_o - 0.25)]}{2.09E-5}$$

(*a*) For $\varepsilon = 0.0004$ ft, solve these two simultaneously, using EES or Excel, to obtain

$$f = 0.0199\ ;\ \mathrm{Re}_{D_{ho}} = 276{,}000\ ;\ R_o = \mathbf{0.498\,ft} \qquad Ans.(a)$$

(*b*) The annular gap is 0.498 – 0.25 = 0.248 ft, just about equal to the inner radius. However, the annular area is **three times the area of Moody's pipe**! *Ans* .(*b*) The annular pipe has much more wall area than a hollow pipe, more friction, so more area is needed to match the pressure drop.

6.94 As shown in Fig. P6.94, a multiduct cross section consists of seven 2-cm-diameter smooth thin tubes packed tightly in a hexagonal "bundle" within a single 6-cm-diameter tube. Air, at about 20°C and 1 atm, flows through this system at 150 m^3/h. Estimate the pressure drop per meter.

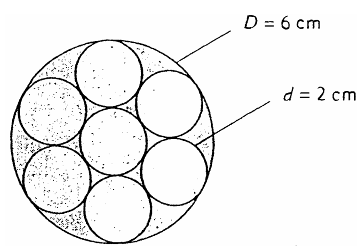


Fig. P6.94

Solution: For air, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. A separate analysis would show that the small triangular cusped passages have *fifty times more resistance* to the flow than the 2-cm-diameter tubes. Therefore we assume all the flow goes through the seven 2-cm tubes. Thus each tube takes one-seventh of the flow rate:

$$V = \frac{Q}{A_{7\text{ tubes}}} = \frac{150/3600}{7\pi(0.01)^2} \approx 18.95\ \frac{m}{s}, \quad Re = \frac{\rho V d}{\mu} = \frac{1.2(18.95)(0.02)}{1.8E{-}5} \approx 25300$$

$$\text{Turbulent:} \quad f_{smooth} \approx 0.0245, \quad \Delta p = f\frac{L}{d}\frac{\rho}{2}V^2 = 0.0245\left(\frac{1.0}{0.02}\right)\frac{1.2}{2}(18.95)^2$$

$$\Delta p \approx \mathbf{260\ Pa} \quad \textit{Ans.}$$

6.95 A wind tunnel is made of wood and is 28 m long, with a rectangular section 50 cm by 80 cm. It draws in sea-level standard air with a fan. If the fan delivers 7 kW of power to the air, estimate (a) the average velocity; and (b) the pressure drop in the wind tunnel.

Solution: For sea-level air, $\rho = 1.22$ kg/m^3 and $\mu = 1.81E{-}5$ kg/m·s. The hydraulic diameter is:

$$D_h = \frac{4A}{P} = \frac{4(50\ cm)(80\ cm)}{2(50+80\ cm)} = 61.54\ cm = 0.6154\ m$$

(a, b) The known power is related to both the flow rate and the pressure drop:

$$Power = Q\Delta p = [HWV]\left[f\frac{L}{D_h}\frac{\rho}{2}V^2\right]$$

$$= [(0.5\ m)(0.8\ m)V]\left[f\frac{28\ m}{0.6154\ m}\frac{1.22\ kg/m^3}{2}V^2\right] = 11.1 fV^3 = 7000\ W$$

Thus we need to find V such that $fV^3 = 631$ m^3/s^3. For wood, take roughness $\varepsilon = 0.5$ mm. Then $\varepsilon/D_h = 0.0005$ m/0.6154 m = 0.000813. Use the Moody chart to find V and the Reynolds number. Guess $f \approx 0.02$ to start, or use EES. The iteration converges to:

$$f = 0.0189, \quad Re_{D_h} = 1.33E6, \quad \mathbf{V = 32\ m/s}, \quad \mathbf{\Delta p = 540\ Pa} \quad \textit{Ans.}\ \text{(a, b)}$$

6.96 Water at 20°C is flowing through a 20-cm-square smooth duct at a (turbulent) Reynolds number of 100,000. For a "laminar flow element" measurement, it is desired to pack the pipe with a honeycomb array of small square passages (see Fig. P6.28 for

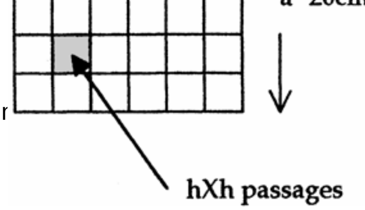


Fig. P6.96

an example). What passage width h will ensure that the flow in each tube will be laminar (Reynolds number less than 2000)?

Solution: The hydraulic diameter of a square is the side length h (or a). For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. The Reynolds number establishes flow velocity:

$$Re_{D_h} = 100,000 = \frac{\rho V d}{\mu} = \frac{998V(0.2)}{0.001}, \quad \text{Solve for } V = 0.501\ \frac{m}{s}$$

This velocity is the same when we introduce small passages, if we neglect the blockage of the thin passage walls. Thus we merely set the passage Reynolds number = 2000:

$$Re_h \frac{\rho V h}{\mu} = \frac{998(0.501)h}{0.001} \le 2000 \quad if\ h \le \mathbf{0.004\ m\ = 4\ mm} \quad Ans.$$

6.97 A heat exchanger consists of multiple parallel-plate passages, as shown in Fig. P6.97. The available pressure drop is 2 kPa, and the fluid is water at 20°C. If the desired total flow rate is 900 m^3/h, estimate the appropriate number of passages. The plate walls are hydraulically smooth.

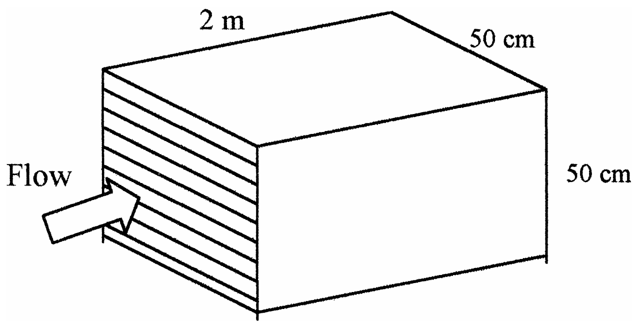


Fig. P6.97

Solution: For water, $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. Unlike Prob. 6.88, here we expect turbulent flow. If there are N passages, then b = 50 cm for all N and the passage thickness is $H = 0.5$ m/N. The hydraulic diameter is $D_h = 2H$. The velocity in each passage is related to the pressure drop by Eq. (6.58):

$$\Delta p = f\frac{L}{D_h}\frac{\rho}{2}V^2 \quad \text{where} \quad f = f_{smooth} = fcn\left(\frac{\rho V D_h}{\mu}\right)$$

$$\text{For the given data,} \quad 2000\ Pa = f\,\frac{2.0\ m}{2(0.5\ m/N)}\frac{998\ kg/m^3}{2}V^2$$

Select N, find H and V and $Q_{total} = AV = b^2V$ and compare to the desired flow of 900 m^3/h. For example, guess N = 20, calculate f = 0.0173 and Q_{total} = 2165 m^3/h. The converged result is

$$Q_{total} = 908\ \text{m}^3/\text{h}, \quad f = 0.028,$$

$$\text{Re}_{D_h} = 14400, \quad H = 7.14\ \text{mm}, \quad \boldsymbol{N = 70\ \textbf{passages}} \quad \textit{Ans.}$$

6.98 A rectangular heat exchanger is to be divided into smaller sections using sheets of commercial steel 0.4 mm thick, as sketched in Fig. P6.98. The flow rate is 20 kg/s of water at 20°C. Basic dimensions are *L* = 1 m, *W* = 20 cm, and *H* = 10 cm. What is the proper number of *square* sections if the overall pressure drop is to be no more than 1600 Pa?

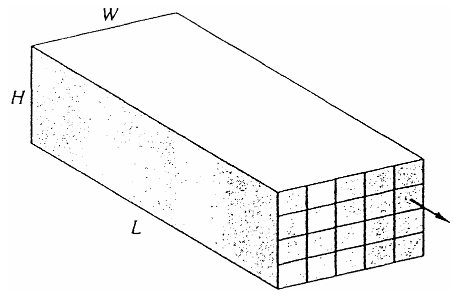


Fig. P6.98

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For commercial steel, $\varepsilon \approx 0.046$ mm. Let the short side (10 cm) be divided into "J" squares. Then the long (20 cm) side divides into "2J" squares and altogether there are N = 2J^2 squares. Denote the side length of the square as "a," which equals (10 cm)/J minus the wall thickness. The hydraulic diameter of a square exactly equals its side length, $D_h = a$. Then the pressure drop relation becomes

$$\Delta p = f\frac{L}{D_h}\frac{\rho}{2}V^2 = f\frac{1.0}{a}\left(\frac{998}{2}\right)\left(\frac{Q}{Na^2}\right)^2 \leq 1600 \text{ Pa}, \quad \text{where} \quad N = 2J^2 \text{ and } a = \frac{0.1}{J} - 0.0004$$

As a first estimate, neglect the 0.4-mm wall thickness, so $a \approx 0.1/J$. Then the relation for Δp above reduces to $fJ \approx 0.32$. Since $f \approx 0.036$ for this turbulent Reynolds number (Re ≈ 1E4) we estimate that J ≈ 9 and in fact this is not bad even including wall thickness:

$$J = 9, \quad N = 2(9)^2 = 162, \quad a = \frac{0.1}{9} - 0.0004 = 0.0107 \text{ m}, \quad V = \frac{20/998}{162(0.0107)^2} \approx 1.078 \ \frac{m}{s}$$

$$\text{Re} = \frac{\rho Va}{\mu} = \frac{998(1.078)(0.0107)}{0.001} \approx 11526, \quad \frac{\varepsilon}{a} = \frac{0.046}{10.7} \approx 0.00429, \quad f_{Moody} \approx 0.0360$$

$$\text{Then} \quad \Delta p = (0.036)\left(\frac{1.0}{0.0107}\right)\left(\frac{998}{2}\right)(1.078)^2 \approx 1950 \text{ Pa}$$

So the wall thickness increases *V* and decreases *a* so Δp is too large. Try J = 8:

$$J = 8, \quad N = 128, \quad a = 0.0121 \text{ m}, \quad V = 1.069 \ \frac{m}{s},$$

$$\text{Re} = 12913, \quad \frac{\varepsilon}{a} = 0.0038, \quad f \approx 0.0347$$

Then $\Delta p = f(L/a)(\rho/2)V^2 \approx$ **1636 Pa**. Close enough, J = 8, **N = 128** *Ans.*

[I suppose a practical person would specify J = 7, N = 98, to keep $\Delta p < 1600$ Pa.]

P6.99 In Sec. 6.11 it was mentioned that Roman aqueduct customers obtained extra water by attaching a diffuser to their pipe exits. Fig. P6.99 shows a simulation: a smooth inlet pipe, with and without a 15° diffuser expanding to a 5-cm-diameter exit. The pipe entrance is sharp-edged.

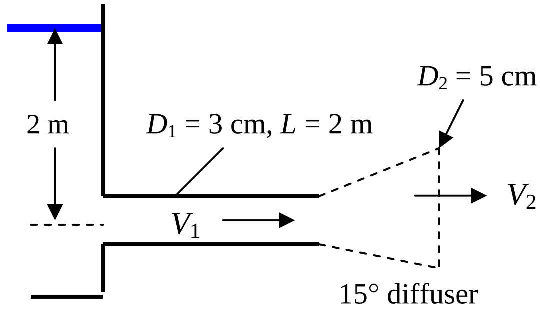


Fig. P6.99

Calculate the flow rate (*a*) without, and (*b*) with the diffuser.

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m-s. The energy equation between the aqueduct surface and the pipe exit yields

$$z_{surf} = z_2 + \frac{V_2^2}{2g} + h_f + \Sigma h_m = z_2 + \frac{V_2^2}{2g} + \frac{V_1^2}{2g}\left(f\frac{L}{D_1} + K_{entrance} + K_{diffuser}\right)$$

(*a*) Without the diffuser, K_{diff} = 0, and $V_1 = V_2$. For a sharp edge, take K_{ent} = 0.5. We obtain

$$2m = \frac{V_1^2}{2g}\left(1 + f\frac{2m}{0.03m} + 0.5\right), \quad with \quad f = fcn(\mathrm{Re} = \rho V_1 D_1/\mu)$$

$$Solve: \ \mathrm{Re} = 115{,}000 \ ; \ f = 0.0175 \ ; V_1 = 4.48\,m/s \ ; \ Q_{without} = 0.00271\,m^3/s \quad Ans.(a)$$

(*b*) *With* the diffuser, from Fig. 6.23, for $D_1/D_2 = 3/5 = 0.6$ and $2\theta = 15°$, read $K_{diffuser} \approx 0.2$. From one-dimensional continuity, $V_2 = V_1(3/5)^2 = 0.36V_1$. The energy equation becomes

$$2m = \frac{(0.36V_1)^2}{2g} + \frac{V_1^2}{2g}\left(1 + f\frac{2m}{0.03m} + 0.5 + 0.2\right)$$

$$Solve: \quad \text{Re} = 134{,}000\ ;\ f = 0.0169\ ;\ V_1 = 3.84\, m/s\ ;\ Q_{with} = 0.00316\, m^3/s \quad Ans.(b)$$

Adding the diffuser increases the flow rate by **17%**. [NOTE: Don't know if the Romans did this, but a well-rounded entrance, $K_{ent} = 0.05$, would increase the flow rate by another 15%.]

6.100 Repeat Prob. 6.92 by including minor losses due to a sharp-edged entrance, the exit into the room, and an open gate valve. If the room pressure is 10 Pa (vacuum), by what percentage is the flow rate decreased from part (a) of Prob. 6.92?

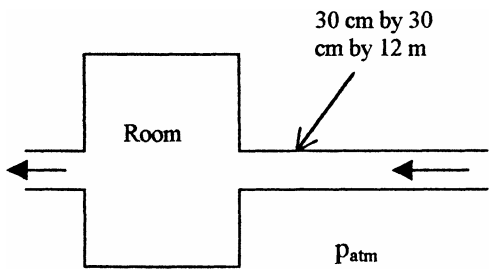


Fig. P6.100

Solution: For air, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8$E–5 kg/m·s. For commercial steel, $\varepsilon = 0.046$ mm. For a square duct, D_h = side-length = 30 cm, hence $\varepsilon/d = 0.046/300 = 0.000153$. Now add $K_{entrance} = 0.5$, $K_{exit} = 1.0$, and $K_{valve} = 0.03$ to the energy equation:

$$\Delta p = 10\ Pa = \frac{\rho}{2}V^2\left(f\frac{L}{D_h} + \sum K_{minor}\right) = \left(\frac{1.2}{2}\right)\left[\frac{Q}{(0.3)^2}\right]^2\left(f\frac{12}{0.3} + 0.5 + 1.0 + 0.03\right)$$

where we compute f based on $\text{Re}_{D_h} = (1.2)V(0.3)/1.8\text{E–}5$ and $\varepsilon/D_h = 0.000153$. The iteration converges to

$$V = 2.65 \text{ m/s}; \quad \text{Re}_d = 53000; \quad f = 0.0212; \quad \boldsymbol{Q} = 0.238 \text{ m}^3/\text{s} = \mathbf{860\ m^3/h}$$

Moral: Don't forget minor losses! The flow rate is **43%** less than Prob. 6.92! *Ans.*

NOTE: IN PROBLEMS 6.100–6.110, MINOR LOSSES ARE *INCLUDED*.

6.101 In Fig. P6.101 a thick filter is being tested for losses. The flow rate in the pipe is 7 m^3/min, and the upstream pressure is 120 kPa. The fluid is air at 20°C. Using the water-manometer reading, estimate the loss coefficient *K* of the filter.

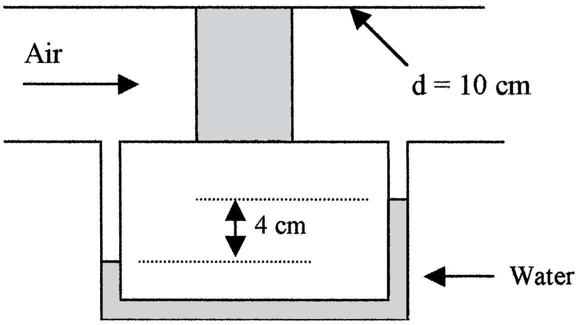


Fig. P6.101

Solution: The upstream density is $\rho_{air} = p/(RT) = 120000/[287(293)] = 1.43\ kg/m^3$. The average velocity *V* (which is used to correlate loss coefficient) follows from the flow rate:

$$V = \frac{Q}{A_{pipe}} = \frac{7/60\ m^3/s}{(\pi/4)(0.1\ m)^2} = 14.85\ m/s$$

The manometer measures the pressure drop across the filter:

$$\Delta p_{mano} = (\rho_w - \rho_a)gh_{mano} = (998 - 1.43\ kg/m^3)(9.81\ m/s^2)(0.04\ m) = 391\ Pa$$

This pressure is correlated as a loss coefficient using Eq. (6.78):

$$K_{filter} = \frac{\Delta p_{filter}}{(1/2)\rho V^2} = \frac{391\ Pa}{(1/2)(1.43\ kg/m^3)(14.85\ m/s)^2} \approx \mathbf{2.5} \quad Ans.$$

6.102 A 70 percent efficient pump delivers water at 20°C from one reservoir to another 20 ft higher, as in Fig. P6.102. The piping system consists of 60 ft of galvanized-iron 2-in pipe, a reentrant entrance, two screwed 90° long-radius elbows, a screwed-open gate valve, and a sharp exit. What is the input power required in horsepower with and without a 6° well-designed conical expansion added to the exit? The flow rate is 0.4 ft^3/s.

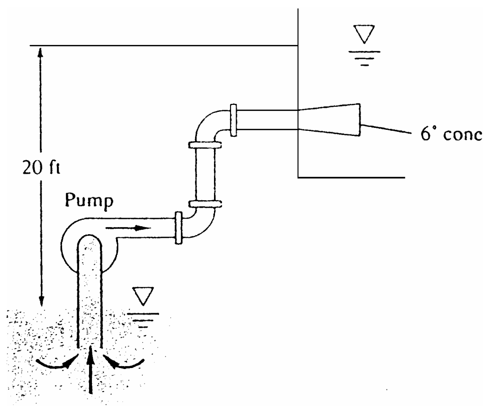


Fig. P6.102

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09\text{E}{-5}$ slug/ft·s. For galvanized iron, $\varepsilon \approx 0.0005$ ft, whence $\varepsilon/d = 0.0005/(2/12 \text{ ft}) \approx 0.003$. Without the 6° cone, the minor losses are:

$$K_{\text{reentrant}} \approx 1.0; \quad K_{\text{elbows}} \approx 2(0.41); \quad K_{\text{gate valve}} \approx 0.16; \quad K_{\text{sharp exit}} \approx 1.0$$

$$\text{Evaluate} \quad V = \frac{Q}{A} = \frac{0.4}{\pi(2/12)^2/4} = 18.3 \frac{\text{ft}}{\text{s}}; \quad \text{Re} = \frac{\rho V d}{\mu} = \frac{1.94(18.3)(2/12)}{2.09\text{E}{-5}} \approx 284000$$

At this Re and roughness ratio, we find from the Moody chart that f ≈ 0.0266. Then

$$\text{(a)} \quad h_{\text{pump}} = \Delta z + \frac{V^2}{2g}\left(f\frac{L}{d} + \sum K\right) = 20 + \frac{(18.3)^2}{2(32.2)}\left[0.0266\left(\frac{60}{2/12}\right) + 1.0 + 0.82 + 0.16 + 1.0\right]$$

$$\text{or} \quad h_{\text{pump}} \approx 85.6 \text{ ft}, \quad \text{Power} = \frac{\rho g Q h_p}{\eta} = \frac{(62.4)(0.4)(85.6)}{0.70}$$

$$= 3052 \div 550 \approx \mathbf{5.55\ hp} \quad \textit{Ans.}\ \text{(a)}$$

(b) If we replace the sharp exit by a 6° conical diffuser, from Fig. 6.23, $K_{\text{exit}} \approx 0.3$. Then

$$h_p = 20 + \frac{(18.3)^2}{2(32.2)}\left[0.0266\left(\frac{60}{2/12}\right) + 1.0 + .82 + .16 + 0.3\right] = 81.95 \text{ ft}$$

$$\text{then} \quad \text{Power} = (62.4)(0.4)(81.95)/0.7 \div 550 \approx \mathbf{5.31\ hp} \text{ (4\% less)} \quad \textit{Ans.}\ \text{(b)}$$

6.103 The reservoirs in Fig. P6.103 are connected by cast-iron pipes joined abruptly, with sharp-edged entrance and exit. Including minor losses, estimate the flow of water at 20°C if the surface of reservoir 1 is 45 ft higher than that of reservoir 2.

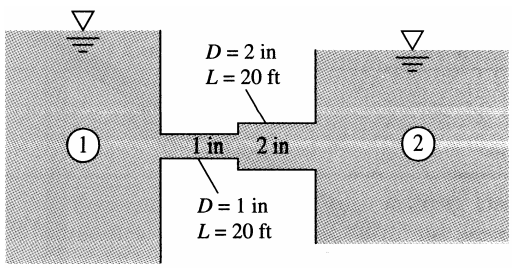


Fig. P6.103

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/ft·s. Let "a" be the small pipe and "b" the larger. For wrought iron, $\varepsilon \approx$ 0.00015 ft, whence ε/d_a = 0.0018 and ε/d_b = 0.0009. From the continuity relation,

$$Q = V_a \frac{\pi}{4} d_a^2 = V_b \frac{\pi}{4} d_b^2 \quad \text{or, since } d_b = 2d_a, \quad \text{we obtain } V_b = \frac{1}{4} V_a$$

For pipe "a" there are two minor losses: a sharp entrance, K_1 = 0.5, and a sudden expansion, Fig. 6.22, Eq. (6.101), $K_2 = [1 - (1/2)^2]^2 \approx 0.56$. For pipe "b" there is one minor loss, the submerged exit, $K_3 \approx 1.0$. The energy equation, with equal pressures at (1) and (2) and near zero velocities at (1) and (2), yields

$$\Delta z = h_{f\text{-}a} + \sum h_{m\text{-}a} + h_{f\text{-}b} + \sum h_{m\text{-}b} = \frac{V_a^2}{2g}\left(f_a \frac{L_a}{d_a} + 0.5 + 0.56\right) + \frac{V_b^2}{2g}\left(f_b \frac{L_b}{d_b} + 1.0\right),$$

$$\text{or, since} \quad V_b = V_a/4, \quad \Delta z = 45 \text{ ft} = \frac{V_a^2}{2(32.2)}\left[240 f_a + 1.06 + \frac{120}{16} f_b + \frac{1.0}{16}\right]$$

where f_a and f_b are separately related to different values of Re and ε/d. Guess to start:

$$f_a \approx f_b \approx 0.02: \quad \text{then } V_a = 21.85 \text{ ft/s}, \quad Re_a \approx 169000, \quad \varepsilon/d_a = 0.0018, \quad f_{a\text{-}2} \approx 0.0239$$

$$V_b = 5.46 \text{ ft/s}, \quad Re_b \approx 84500, \quad \varepsilon/d_b = 0.0009, \quad f_{b\text{-}2} \approx 0.0222$$

$$\text{Converges to:} \quad f_a = 0.024, \quad f_b = 0.0224, \quad V_a \approx 20.3 \text{ ft/s},$$

$$Q = V_a A_a \approx \mathbf{0.111\ ft^3/s}. \quad \textit{Ans.}$$

6.104 Reconsider the air hockey table of Problem 3.162, but with inclusion of minor losses. The table is 3 ft by 6 ft in area, with 1/16-in-diameter holes spaced every inch in a rectangular grid (2592 holes total). The required jet speed from each hole is 50 ft/s. Your job is to select an appropriate blower to meet the requirements.

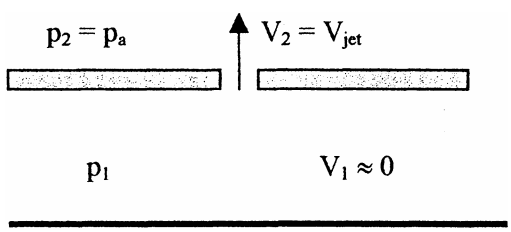


Fig. P3.162

Hint: Assume that the air is stagnant in the manifold under the table surface, and assume sharp-edge inlets at each hole. (a) Estimate the pressure rise (in lbf/in^2) required of the blower. (b) Compare your answer to the previous calculation in Prob. 3.162, where minor losses were ignored. Are minor losses significant?

Solution: Write the energy equation between manifold and atmosphere:

$$\frac{p_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{losses},\quad \text{where } h_{losses}\approx K_{inlet}\frac{V_{jet}^2}{2g}$$

Neglect $V_1 \approx 0$ and $z_1 \approx z_2$, assume $\alpha_{1,2} = 1.0$, and solve for

$$\Delta p = p_1 - p_2 = \frac{\rho}{2}V_{jet}^2(1+K_{inlet}),\quad \text{where } K_{sharp\text{-}edge\text{-}inlet}\approx 0.5$$

Clearly, the pressure drop is about **50% greater** due to the minor loss. *Ans.* (b)
Work out Δp, assuming $\rho_{air} \approx 0.00234$ slug/ft^3:

$$\Delta p = \frac{0.00234}{2}(50)^2(1+0.5) = 4.39\ \frac{lbf}{ft^2}\div 144 = \mathbf{0.0305\ \frac{lbf}{in^2}}\quad \textit{Ans.}\text{ (a)}$$

(Again, this is 50% higher than Prob. 3.162.)

6.105 The system in Fig. P6.105 consists of 1200 m of 5 cm cast-iron pipe, two 45° and four 90° flanged long-radius elbows, a fully open flanged globe valve, and a sharp exit into a reservoir. If the elevation at point 1 is 400 m, what gage pressure is required at point 1 to deliver 0.005 m^3/s of water at 20°C into the reservoir?

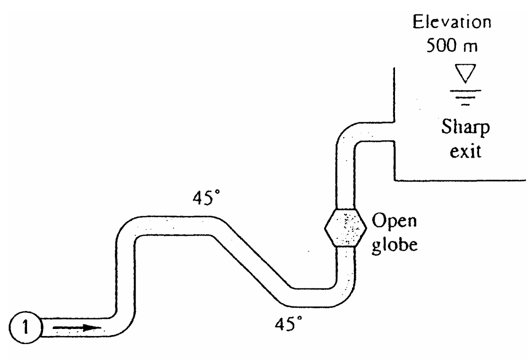


Fig. P6.105

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For cast iron, take $\varepsilon \approx 0.26$ mm, hence $\varepsilon/d = 0.0052$. With the flow rate known, we can compute V, Re:

$$V=\frac{Q}{A}=\frac{0.005}{(\pi/4)(0.05)^2}=2.55\ \frac{m}{s};\quad Re=\frac{998(2.55)(0.05)}{0.001}\approx 127000,\quad f_{Moody}\approx 0.0315$$

The minor losses may be listed as follows:

45° long-radius elbow: K ≈ 0.2; 90° long-radius elbow: K ≈ 0.3

Open flanged globe valve: K ≈ 8.5; submerged exit: K ≈ 1.0

Then the energy equation between (1) and (2—the reservoir surface) yields

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = 0 + 0 + z_2 + h_f + \sum h_m,$$

$$\text{or:}\quad p_1/(\rho g) = 500 - 400 + \frac{(2.55)^2}{2(9.81)}\left[0.0315\left(\frac{1200}{0.05}\right) + 0.5 + 2(0.2) + 4(0.3) + 8.5 + 1 - 1\right]$$

$$= 100 + 253 = 353\text{ m},\quad \text{or:}\quad \mathbf{p_1} = (998)(9.81)(353) \approx \mathbf{3.46\ MPa}\quad \textit{Ans.}$$

6.106 The water pipe in Fig. 6.106 slopes upward at 30°. The pipe is 1-inch diameter and *smooth*. The flanged globe valve is fully open. If the mercury manometer shows a 7-inch deflection, what is the flow rate in cubic feet per sec?

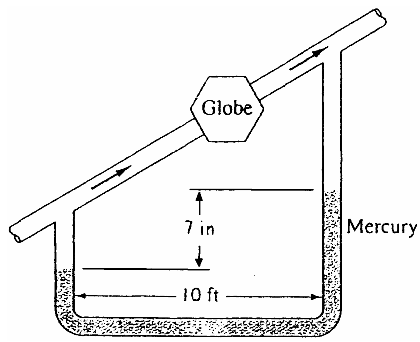


Fig. P6.106

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. The pipe length and elevation change are

$$L = \frac{10\text{ ft}}{\cos 30°} = 11.55\text{ ft};\quad z_2 - z_1 = 10\tan 30° = 5.77\text{ ft},\quad \text{Open 1}''\text{ globe valve: } K \approx 13$$

The manometer indicates the total pressure change between (1) and (2):

$$p_1 - p_2 = (\rho_{Merc} - \rho_w)gh + \rho_w g\Delta z = (13.6-1)(62.4)\left(\frac{7}{12}\right) + 62.4(5.77) \approx 819\text{ psf}$$

The energy equation yields

$$\frac{p_1 - p_2}{\rho g} = \Delta z + h_f + h_m = 5.77 + \frac{V^2}{2(32.2)}\left[f\frac{11.55}{1/12} + 13\right] \approx \frac{819\text{ lbf/ft}^2}{62.4\text{ lbf/ft}^3}$$

$$\text{or:}\quad V^2 \approx \frac{2(32.2)(7.35)}{(139f + 13)}.\quad \text{Guess } f \approx 0.02,\ V \approx 5.48\ \frac{ft}{s},\ Re \approx 42400,\ f_{new} \approx 0.0217$$

Rapid convergence to f ≈ 0.0217, V ≈ 5.44 ft/s, Q = V(π/4)(1/12)^2 ≈ **0.0296 ft^3/s.** *Ans.* [NOTE that the manometer reading of 7 inches exactly balances the friction losses, and the hydrostatic pressure change $\rho g\Delta z$ cancels out of the energy equation.]

6.107 In Fig. P6.107 the pipe is galvanized iron. Estimate the percentage increase in the flow rate (a) if the pipe entrance is cut off flush with the wall and (b) if the butterfly valve is opened wide.

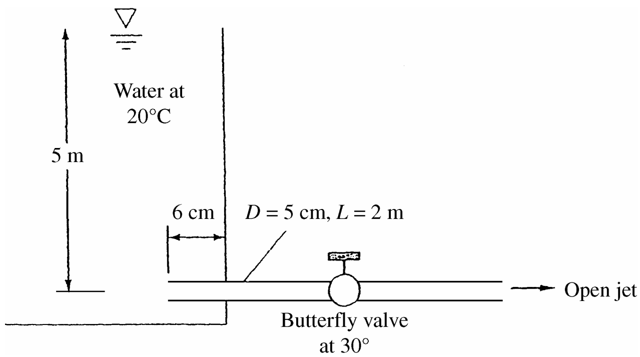


Fig. P6.107

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For galvanized iron, take $\varepsilon \approx 0.15$ mm, hence $\varepsilon/d = 0.003$. First establish minor losses as shown:

$$\text{Protruding entrance (Fig. 6.21a),} \quad \frac{L}{d} \approx 1.2, \ K \approx 1;$$

$$\text{Butterfly @ 30° (Fig 6.19)} \quad K \approx 80 \pm 20$$

The energy equation, with $p_1 = p_2$, yields:

$$\Delta z = \frac{V^2}{2g} + h_f + \sum h_m = \frac{V^2}{2g}\left[1 + f\frac{L}{d} + \sum K\right] = \frac{V^2}{2(9.81)}\left[1 + f\left(\frac{2}{0.05}\right) + 1.0 + 80 \pm 20\right] = 5 \text{ m}$$

$$\text{Guess} \quad f \approx 0.02, \quad V \approx 1.09 \ \frac{m}{s}, \quad Re \approx 54300, \quad \frac{\varepsilon}{d} = 0.003,$$

$$f_{new} \approx 0.0284, \quad V_{new} \approx 1.086 \ \frac{m}{s}$$

Thus the "base" flow, for our comparison, is $V_o \approx 1.086$ m/s, $Q_o \approx 0.00213$ m^3/s.
If we cut off the entrance flush, we reduce K_{ent} from 1.0 to **0.5**; hardly a significant reduction in view of the huge butterfly valve loss $K_{valve} \approx 80$. The energy equation is

$$5 \text{ m} = \frac{V^2}{2(9.81)}[1 + 40f + 0.5 + 80 \pm 20], \quad \text{solve } V \approx 1.090 \ \frac{m}{s},$$

$$Q = \mathbf{0.00214} \ \frac{\mathbf{m^3}}{\mathbf{s}} \ (0.3\% \text{ more}) \quad \textit{Ans. (a)}$$

If we open the butterfly wide, K_{valve} decreases from 80 to only **0.3**, a *huge* reduction:

$$5 \text{ m} = \frac{V^2}{2(9.81)}[1 + 40f + 1.0 + 0.3], \quad \text{solve } V \approx 5.4 \ \frac{m}{s},$$

$$Q = \mathbf{0.0106} \ \frac{\mathbf{m^3}}{\mathbf{s}} \ (5 \text{ times more}) \quad \textit{Ans. (b)}$$

Obviously opening the valve has a dominant effect for this system.

6.108 The water pump in Fig. P6.108 maintains a pressure of 6.5 psig at point 1. There is a filter, a half-open disk valve, and two regular screwed elbows. There are 80 ft of 4-inch diameter commercial steel pipe. (a) If the flow rate is 0.4 ft^3/s, what is the loss coefficient of the filter? (b) If the disk valve is wide open and $K_{filter} = 7$, what is the resulting flow rate?

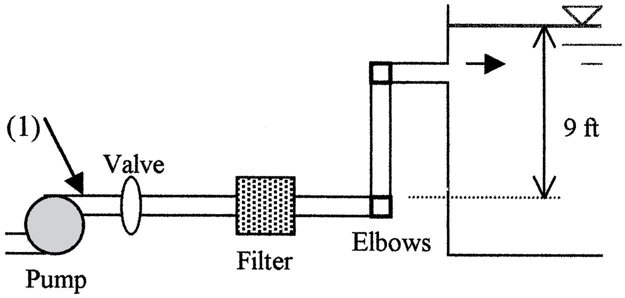


Fig. P6.108

Solution: For water, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. The energy equation is written from point 1 to the surface of the tank:

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f+K_{valve}+K_{filter}+2K_{elbow}+K_{exit}$$

(a) From the flow rate, V_1 = Q/A = (0.4 ft^3/s)/[(π/4)(4/12 ft)^2] = 4.58 ft/s. Look up minor losses and enter into the energy equation:

$$\frac{(6.5)(144)\text{ lbf/ft}^2}{62.4\text{ lbf/ft}^3}+\frac{(4.58\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)}+0$$

$$=0+0+9\text{ ft}+\frac{(4.58)^2}{2(32.2)}\left[f\frac{80\text{ ft}}{(4/12\text{ ft})}+2.8+K_{filter}+2(0.64)+1\right]$$

We can solve for K_{filter} if we evaluate f. Compute Re_D = (1.94)(4.58)(4/12)/(2.09E–5) = 141,700. For commercial steel, ε/D = 0.00015 ft/0.333 ft = 0.00045. From the Moody chart, $f \approx 0.0193$, and fL/D = 4.62. The energy equation above becomes:

$$15.0\text{ ft}+0.326\text{ ft}=9\text{ ft}+0.326(4.62+2.8+K_{\text{filter}}+1.28+1)\text{ ft},$$

$$\text{Solve}\quad \mathbf{K_{filter} \approx 9.7}\quad \textit{Ans. (a)}$$

(b) If K_{filter} = 7.0 and V is unknown, we must iterate for the velocity and flow rate. The energy equation becomes, with the disk valve wide open ($K_{Valve} \approx 0$):

$$15.0\text{ ft}+\frac{V^2}{2(32.2)}=9\text{ ft}+\frac{V^2}{2(32.2)}\left(f\frac{80}{1/3}+0+7.0+1.28+1\right)$$

$$\textit{Iterate to find}\quad f\approx 0.0189,\quad Re_D=169{,}000,\quad V=5.49\text{ ft/s},$$

$$\mathbf{Q = AV = 0.48\ ft^3/s}\quad \textit{Ans. (b)}$$

6.109 In Fig. P6.109 there are 125 ft of 2-in pipe, 75 ft of 6-in pipe, and 150 ft of 3-in pipe, all cast iron. There are three 90° elbows and an open globe valve, all flanged. If the exit elevation is zero, what horsepower is extracted by the turbine when the flow rate is 0.16 ft^3/s of water at 20°C?

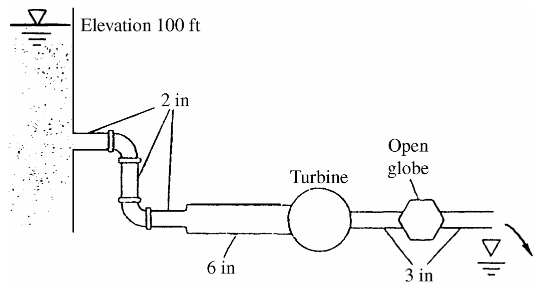


Fig. P6.109

Solution: For water at 20°C, take ρ = 1.94 $slug/ft^3$ and μ = 2.09E–5 slug/ft·s. For cast iron, $\varepsilon \approx$ 0.00085 ft. The 2″, 6″, and 3″ pipes have, respectively,

$$\text{(a)}\quad L/d = 750,\ \varepsilon/d = 0.0051;\quad \text{(b)}\quad L/d = 150,\ \varepsilon/d = 0.0017;$$

$$\text{(c)}\quad L/d = 600,\ \varepsilon/d = 0.0034$$

The flow rate is known, so each velocity, Reynolds number, and *f* can be calculated:

$$V_a = \frac{0.16}{\pi(2/12)^2/4} = 7.33\ \frac{ft}{s};\quad Re_a = \frac{1.94(7.33)(2/12)}{2.09E\text{–}5} = 113500,\quad f_a \approx 0.0314$$

$$\text{Also, } V_b = 0.82\ ft/s,\ Re_b = 37800,\ f_c \approx 0.0266;\ V_c = 3.26,\ Re_c = 75600,\ f_c \approx 0.0287$$

Finally, the minor loss coefficients may be tabulated:

sharp 2″ entrance: K = 0.5; three 2″ 90° elbows: K = 3(0.95)

2″ sudden expansion: K ≈ 0.79; 3″ open globe valve: K ≈ 6.3

The turbine head equals the elevation difference minus losses and the exit velocity head:

$$h_t = \Delta z - \sum h_f - \sum h_m - V_c^2/(2g)$$

$$= 100 - \frac{(7.33)^2}{2(32.2)}[0.0314(750) + 0.5 + 3(0.95) + 0.79]$$

$$- \frac{(0.82)^2}{2(32.2)}(0.0266)(150) - \frac{(3.26)^2}{2(32.2)}[0.0287(600) + 6.3 + 1] \approx \mathbf{72.8\ ft}$$

The resulting turbine power = $\rho g Q h_t$ = (62.4)(0.16)(72.8) ÷ 550 ≈ **1.32 hp**. *Ans.*

6.110 In Fig. P6.110 the pipe entrance is sharp-edged. If the flow rate is 0.004 m^3/s, what power, in W, is extracted by the turbine?

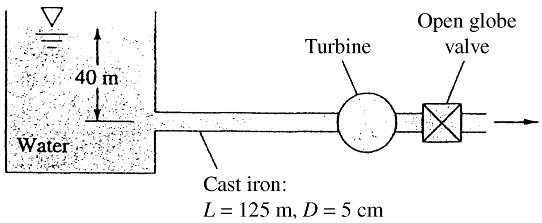


Fig. P6.110

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For cast iron, $\varepsilon \approx 0.26$ mm, hence $\varepsilon/d = 0.26/50 \approx 0.0052$. The minor loss coefficients are Entrance: K ≈ 0.5; 5-cm(≈2″) open globe valve: K ≈ 6.9.

The flow rate is known, hence we can compute V, Re, and f:

$$V = \frac{Q}{A} = \frac{0.004}{(\pi/4)(0.05)^2} = 2.04\ \frac{m}{s},\quad Re = \frac{998(2.04)(0.05)}{0.001} \approx 102000,\quad f \approx 0.0316$$

The turbine head equals the elevation difference minus losses and exit velocity head:

$$h_t = \Delta z - h_f - \sum h_m - \frac{V^2}{2g} = 40 - \frac{(2.04)^2}{2(9.81)}\left[(0.0316)\left(\frac{125}{0.05}\right) + 0.5 + 6.9 + 1\right] \approx 21.5\ m$$

$$\text{Power} = \rho g Q h_t = (998)(9.81)(0.004)(21.5) \approx \mathbf{840\ W} \quad \textit{Ans.}$$

6.111 For the parallel-pipe system of Fig. P6.111, each pipe is cast iron, and the pressure drop $p_1 - p_2$ = 3 lbf/in^2. Compute the total flow rate between 1 and 2 if the fluid is SAE 10 oil at 20°C.

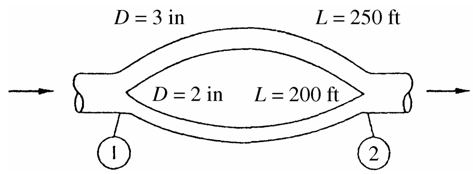


Fig. P6.111

Solution: For SAE 10 oil at 20°C, take ρ = 1.69 $slug/ft^3$ and μ = 0.00217 slug/ft·s. For cast iron, $\varepsilon \approx$ 0.00085 ft. Convert Δp = 3 psi = 432 psf and guess laminar flow in each:

$$\Delta p_a \stackrel{?}{=} \frac{128\mu L_a Q_a}{\pi d_a^4} = 432 = \frac{128(0.00217)(250)Q_a}{\pi(3/12)^4},$$

$$Q_a \approx 0.0763\ \frac{ft^3}{s}.\quad \text{Check } Re \approx 300 \text{ (OK)}$$

$$\Delta p_b \stackrel{?}{=} \frac{128\mu L_b Q_b}{\pi d_b^4} = 432 = \frac{128(0.00217)(200)Q_b}{\pi(2/12)^4},$$

$$Q_b \approx 0.0188\ \frac{ft^3}{s}.\quad \text{Check } Re \approx 112 \text{ (OK)}$$

The total flow rate is $\mathbf{Q} = Q_a + Q_b = 0.0763 + 0.0188 \approx \mathbf{0.095\ ft^3/s}$. *Ans.*

6.112 If the two pipes in Fig. P6.111 are instead laid in **series** with the same total pressure drop of 3 psi, what will the flow rate be? The fluid is SAE 10 oil at 20°C.

Solution: For SAE 10 oil at 20°C, take ρ = 1.69 slug/ft^3 and μ = 0.00217 slug/ft·s. Again guess laminar flow. Now, instead of Δp being the same, $Q_a = Q_b = Q$:

$$\Delta p_a + \Delta p_b = 432 \text{ psf} = \frac{128\mu L_a Q}{\pi d_a^4} + \frac{128\mu L_b Q}{\pi d_b^4} = \frac{128(0.00217)}{\pi} Q\left[\frac{250}{(3/12)^4} + \frac{200}{(2/12)^4}\right]$$

$$\text{Solve for } \quad \mathbf{Q \approx 0.0151\ ft^3/s} \quad \textit{Ans.} \quad \text{Check } Re_a \approx 60 \text{ (OK) and } Re_b \approx 90 \text{ (OK)}$$

In series, the flow rate is six times less than when the pipes are in parallel.

6.113 The parallel galvanized-iron pipe system of Fig. P6.113 delivers water at 20°C with a total flow rate of 0.036 m^3/s. If the pump is wide open and not running, with a loss coefficient K = 1.5, determine (a) the flow rate in each pipe and (b) the overall pressure drop.

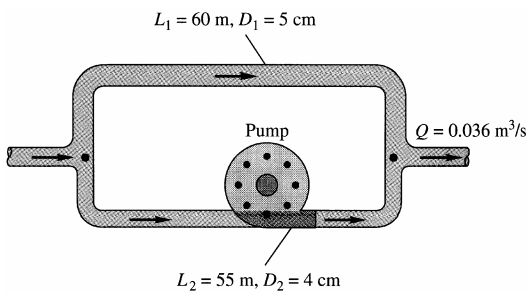


Fig. P6.113

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For galvanized iron, ε = 0.15 mm. Assume turbulent flow, with Δp the same for each leg:

$$h_{f1} = f_1 \frac{L_1}{d_1}\frac{V_1^2}{2g} = h_{f2} + h_{m2} = \frac{V_2^2}{2g}\left(f_2 \frac{L_2}{d_2} + 1.5\right),$$

$$\text{and} \quad Q_1 + Q_2 = (\pi/4)d_1^2 V_1 + (\pi/4)d_2^2 V_2 = Q_{total} = 0.036 \text{ m}^3/\text{s}$$

When the friction factors are correctly found from the Moody chart, these two equations may be solved for the two velocities (or flow rates). Begin by guessing f ≈ 0.020:

$$(0.02)\left(\frac{60}{0.05}\right)\frac{V_1^2}{2(9.81)} = \frac{V_2^2}{2(9.81)}\left[(0.02)\left(\frac{55}{0.04}\right) + 1.5\right], \quad \text{solve for } V_1 \approx 1.10 V_2$$

$$\text{then} \quad \frac{\pi}{4}(0.05)^2(1.10V_2) + \frac{\pi}{4}(0.04)^2 V_2 = 0.036. \quad \text{Solve } V_2 \approx 10.54 \frac{m}{s},\ V_1 \approx 11.59 \frac{m}{s}$$

$$\text{Correct } Re_1 \approx 578000, \quad f_1 \approx 0.0264, \quad Re_2 \approx 421000, \quad f_2 \approx 0.0282, \text{ repeat.}$$

The 2nd iteration converges: $f_1 \approx 0.0264$, $V_1 = 11.69$ m/s, $f_2 \approx 0.0282$, $V_2 = 10.37$ m/s,

$$Q_1 = A_1V_1 = \mathbf{0.023\ m^3/s}, \quad Q_2 = A_2V_2 = \mathbf{0.013\ m^3/s}. \quad \textit{Ans. (a)}$$

The pressure drop is the same in either leg:

$$\Delta p = f_1 \frac{L_1}{d_1}\frac{\rho V_1^2}{2} = \left(f_2 \frac{L_2}{d_2} + 1.5\right)\frac{\rho V_2^2}{2} \approx \mathbf{2.16E6\ Pa} \quad \textit{Ans. (b)}$$

***P6.114** A blower supplies standard air to a plenum that feeds two horizontal square sheet-metal ducts with sharp-edged entrances. One duct is 100 ft long, with a cross-section 6 in by 6 in. The second duct is 200 ft long. Each duct exhausts to the atmosphere. When the plenum pressure is 5.0 lbf/ft^2 gage, the volume flow in the longer duct is three times the flow in the shorter duct. Estimate both volume flows and the cross-section size of the longer duct.

Solution: For standard air, in BG units, take ρ = 0.00238 slug/ft^3 and μ = 3.78E-7 slug/ft-sec. For sheet-metal, take ε = 0.00016 ft. The energy equation for this case is

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f + h_{entrance}\ , \quad or:$$

$$\Delta p = \frac{1}{2}\rho V^2 (1 + f\frac{L}{D_h} + K_{ent}) \quad where\ K_{sharp-edged} \approx 0.5$$

We have abbreviated the duct velocity to *V*, without a subscript. For a square duct, the hydraulic diameter is the side length of the square. First compute the flow rate in the short duct:

$$5.0\frac{lbf}{ft^2} = \frac{0.00238 slug/ft^3}{2}\{1 + f\frac{100\,ft}{0.5\,ft} + 0.5)\ , \quad f = fcn(\mathrm{Re}_{D_h}, \frac{\varepsilon}{D_h})$$

The Reynolds number for the short duct is Re = (0.00238)V(0.5)/(3.78E-7) = 3148*V*, and ε/D_h = 0.00016ft/0.5ft = 0.00032. The solution is

$$L = 100\ ft:\ \mathrm{Re}_{D_h} = 87{,}000\ ;\ f = 0.0200\ ;\ V = 27.7\ ft/s\ ;\ Q_{short} = \mathbf{6.92\,ft^3/s}$$

For the longer duct, Re = (0.00238)VD_h /(3.78E-7), and ε/D_h = 0.00016ft/D_h. We don't know D_h and must solve to make Q_{long} = 3Q_{short}. The solution is

$$L = 200\ ft:\ \mathrm{Re}_{D_h} = 150{,}000\ ;\ f = 0.0177\ ;\ V = 27.5\ ft/s\ ;\ Q_{long} = 20.75\ ft^3/s$$

$$\text{Solve for}\quad D_{h,long} = \mathbf{0.87\,ft} \qquad \textit{Ans.}$$

NOTE: It is an interesting numerical quirk that, for these duct parameters, the velocities in each duct are almost identical, regardless of the magnitude of the pressure drop.

6.115 In Fig. P6.115 all pipes are 8-cm-diameter cast iron. Determine the flow rate from reservoir (1) if valve C is (a) closed; and (b) open, with K_{valve} = 0.5.

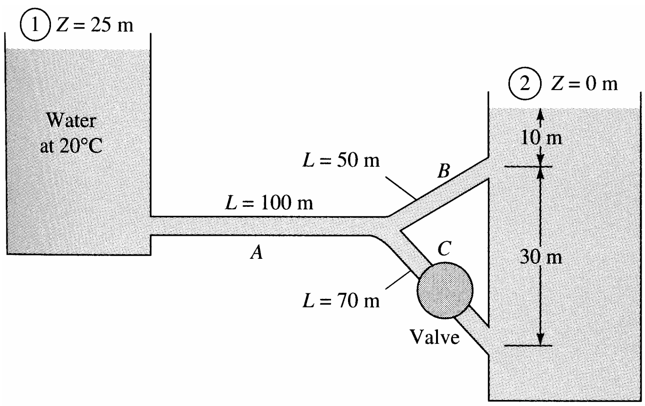


Fig. P6.115

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. For cast iron, $\varepsilon \approx$ 0.26 mm, hence ε/d = 0.26/80 ≈ 0.00325 for all three pipes. Note $p_1 = p_2$, $V_1 = V_2 \approx 0$. These are long pipes, but we might wish to account for minor losses anyway:

sharp entrance at A: $K_1 \approx 0.5$; line junction from A to B: $K_2 \approx 0.9$ (Table 6.5)

branch junction from A to C: $K_3 \approx 1.3$; two submerged exits: $K_B = K_C \approx 1.0$

If valve C is closed, we have a straight *series* path through A and B, with the same flow rate Q, velocity V, and friction factor *f* in each. The energy equation yields

$$z_1 - z_2 = h_{fA} + \sum h_{mA} + h_{fB} + \sum h_{mB},$$

$$\text{or:}\quad 25\text{ m} = \frac{V^2}{2(9.81)}\left[f\,\frac{100}{0.08} + 0.5 + 0.9 + f\,\frac{50}{0.08} + 1.0\right],\quad \text{where } f = \text{fcn}\left(\text{Re}, \frac{\varepsilon}{d}\right)$$

Guess f ≈ $f_{\text{fully rough}}$ ≈ 0.027, then V ≈ 3.04 m/s, Re ≈ 998(3.04)(0.08)/(0.001) ≈ 243000, ε/d = 0.00325, then f ≈ 0.0273 (converged). Then the velocity through A and B is V = 3.03 m/s, and Q = (π/4)$(0.08)^2$(3.03) ≈ **0.0152 m³/s**. *Ans.* (a).

If valve C is open, we have parallel flow through B and C, with $Q_A = Q_B + Q_C$ and, with *d* constant, $V_A = V_B + V_C$. The total head loss is the same for paths A-B and A-C:

$$z_1 - z_2 = h_{fA} + \sum h_{mA\text{-}B} + h_{fB} + \sum h_{mB} = h_{fA} + \sum h_{mA\text{-}C} + h_{fC} + \sum h_{mC},$$

$$\text{or:}\quad 25 = \frac{V_A^2}{2(9.81)}\left[f_A\,\frac{100}{0.08} + 0.5 + 0.9\right] + \frac{V_B^2}{2(9.81)}\left[f_B\,\frac{50}{0.08} + 1.0\right]$$

$$= \frac{V_A^2}{2(9.81)}\left[f_A\,\frac{100}{0.08} + 0.5 + 1.3\right] + \frac{V_C^2}{2(9.81)}\left[f_C\,\frac{70}{0.08} + 1.0\right]$$

plus the additional relation $V_A = V_B + V_C$. Guess f ≈ $f_{\text{fully rough}}$ ≈ 0.027 for all three pipes and begin. The initial numbers work out to

$$2g(25) = 490.5 = V_A^2(1250f_A + 1.4) + V_B^2(625f_B + 1) = V_A^2(1250f_A + 1.8) + V_C^2(875f_C + 1)$$

$$\text{If } f \approx 0.027,\ \text{solve (laboriously)}\ V_A \approx 3.48\text{ m/s},\ V_B \approx 1.91\text{ m/s},\ V_C \approx 1.57\text{ m/s}.$$

$$\text{Compute } Re_A = 278000,\quad f_A \approx 0.0272,\quad Re_B = 153000,\quad f_B = 0.0276,$$
$$Re_C = 125000,\quad f_C = 0.0278$$

Repeat once for convergence: V_A ≈ 3.46 m/s, V_B ≈ 1.90 m/s, V_C ≈ 1.56 m/s. The flow rate from reservoir (1) is $\mathbf{Q_A}$ = (π/4)$(0.08)^2$(3.46) ≈ **0.0174 m³/s**. (14% more) *Ans.* (b)

6.116 For the series-parallel system of Fig. P6.116, all pipes are 8-cm-diameter asphalted cast iron. If the total pressure drop $p_1 - p_2$ = 750 kPa, find the resulting flow rate *Q* m³/h for water at 20°C. Neglect minor losses.

Solution: For water at 20°C, take ρ = 998 kg/m³ and μ = 0.001 kg/m·s. For asphalted cast iron, $\varepsilon \approx$ 0.12 mm, hence ε/d = 0.12/80 ≈ 0.0015 for all three pipes. The head loss is the same through AC and BC:

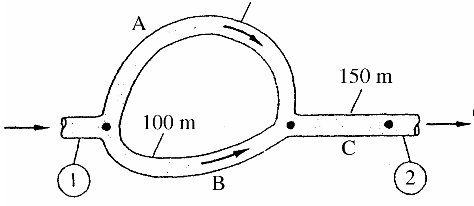


Fig. P6.116

$$\frac{\Delta p}{\rho g} = h_{fA} + h_{fC} = h_{fB} + h_{fC} = \left(f\frac{L}{d}\frac{V^2}{2g}\right)_A + \left(f\frac{L}{d}\frac{V^2}{2g}\right)_C = \left(f\frac{L}{d}\frac{V^2}{2g}\right)_B + \left(f\frac{L}{d}\frac{V^2}{2g}\right)_C$$

Since d is the same, $V_A + V_B = V_C$ and f_A, f_B, f_C are found from the Moody chart. Cancel g and introduce the given data:

$$\frac{750000}{998} = f_A\frac{250}{0.08}\frac{V_A^2}{2} + f_C\frac{150}{0.08}\frac{V_C^2}{2} = f_B\frac{100}{0.08}\frac{V_B^2}{2} + f_C\frac{150}{0.08}\frac{V_C^2}{2}, \quad V_A + V_B = V_C$$

Guess $f_{rough} \approx 0.022$ and solve laboriously: $V_A \approx 2.09\ \frac{m}{s}$, $V_B \approx 3.31\ \frac{m}{s}$, $V_C \approx 5.40\ \frac{m}{s}$

Now compute $Re_A \approx 167000$, $f_A \approx 0.0230$, $Re_B \approx 264000$, $f_B \approx 0.0226$, $Re_C \approx 431000$, and $f_C \approx 0.0222$. Repeat the head loss iteration and we converge: $V_A \approx 2.06$ m/s, $V_B \approx 3.29$ m/s, $V_C \approx 5.35$ m/s, $Q = (\pi/4)(0.08)^2(5.35) \approx \mathbf{0.0269\ m^3/s}$. *Ans.*

6.117 A blower delivers air at 3000 m^3/h to the duct circuit in Fig. P6.117. Each duct is commercial steel and of square cross-section, with side lengths $a_1 = a_3 = 20$ cm and $a_2 = a_4 = 12$ cm. Assuming sea-level air conditions, estimate the power required if the blower has an efficiency of 75%. Neglect minor losses.

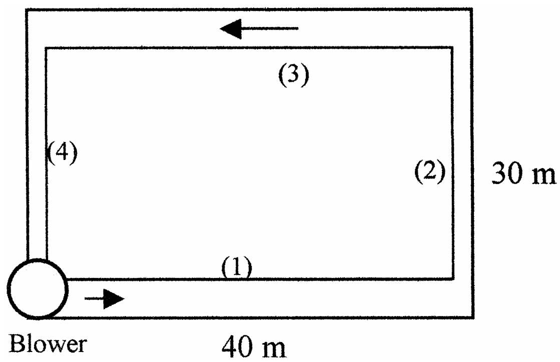


Fig. P6.117

Solution: For air take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. Establish conditions in each duct:

$$Q = \frac{3000}{3600} = 0.833\ \frac{m^3}{s};\ V_{1\&3} = \frac{0.833\ m^3/s}{(0.2\ m)^2} = 20.8\ m/s;\ Re_{1\&3} = \frac{1.2(20.8)(0.2)}{1.8E{-}5} = 278{,}000$$

$$V_{2\&4} = \frac{0.833\ m^3/s}{(0.12\ m)^2} = 57.8\ m/s;\quad Re_{2\&4} = \frac{1.2(57.8)(0.12)}{1.8E{-}5} = 463{,}000$$

For commercial steel (Table 6.1) $\varepsilon = 0.046$ mm. Then we can find the two friction factors:

$$\left.\frac{\varepsilon}{D}\right|_{1\&3} = \frac{0.046}{200} = 0.00023;\quad Re_{1\&3} = 278000;\quad \text{Moody chart: } f_{1\&3} \approx 0.0166$$

$$\left.\frac{\varepsilon}{D}\right|_{2\&4} = \frac{0.046}{120} = 0.000383; \quad Re_{2\&4} = 463000; \quad \text{Moody chart: } f_{1\&3} \approx 0.0170$$

$$\text{Then} \quad \Delta p_{1\&3} = \left(f\frac{L}{D}\frac{\rho V^2}{2}\right)_{1\&3} = (0.0166)\left(\frac{80}{0.2}\right)\frac{(1.2)(20.8)^2}{2} = 1730\ Pa$$

$$\text{and} \quad \Delta p_{2\&4} = \left(f\frac{L}{D}\frac{\rho V^2}{2}\right)_{1\&3} = (0.0170)\left(\frac{60}{0.12}\right)\frac{(1.2)(57.8)^2}{2} = 17050\ Pa$$

The total power required, at 75% efficiency, is thus:

$$Power = \frac{Q\Delta p}{\eta} = \frac{(0.833\ \text{m}^3/\text{s})(1730 + 17050\ \text{Pa})}{0.75} = \mathbf{20900\ W} \quad Ans.$$

6.118 For the piping system of Fig. P6.118, all pipes are concrete with a roughness of 0.04 inch. Neglecting minor losses, compute the overall pressure drop p1 − p2 in lbf/in^2. The flow rate is 20 ft^3/s of water at 20°C.

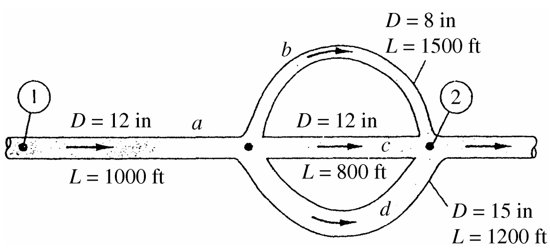


Fig. P6.118

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. Since the pipes are all different make a little table of their respective L/d and ε/d:

(a)	L = 1000 ft,	d = 12 in,	L/d = *1000*,	ε/d = **0.00333**
(b)	1500 ft	8 in	*2250*	**0.00500**
(c)	800 ft	12 in	*800*	**0.00333**
(d)	1200 ft	15 in	*960*	**0.00267**

With the flow rate known, we can find everything in pipe (a):

$$V_a = \frac{Q_a}{A_a} = \frac{20}{(\pi/4)(1\ \text{ft})^2} = 25.5\ \frac{\text{ft}}{\text{s}}, \quad Re_a = \frac{1.94(25.5)(1)}{2.09E{-}5} = 2.36E6, \quad f_a \approx \mathit{0.0270}$$

Then pipes (b,c,d) are in parallel, each having the same head loss and with flow rates which must add up to the total of 20 ft^3/s:

$$h_{fb} = \frac{8f_bL_bQ_b^2}{\pi^2 g\, d_b^5} = h_{fc} = \frac{8f_cL_cQ_c^2}{\pi^2 g\, d_c^5} = h_{fd} = \frac{8f_dL_dQ_d^2}{\pi^2 g\, d_d^5}, \quad \text{and} \quad Q_b + Q_c + Q_d = 20\ \frac{\text{ft}^3}{\text{s}}$$

Introduce L_b, d_b, etc. to find that $Q_c = 3.77Q_b(f_b/f_c)^{1/2}$ and $Q_d = 5.38Q_b(f_b/f_d)^{1/2}$

Then the flow rates are iterated from the relation

$$\sum Q = 20\ \frac{ft^3}{s} = Q_b[1 + 3.77(f_b/f_c)^{1/2} + 5.38(f_b/f_d)^{1/2}]$$

$$\text{First guess:}\quad f_b = f_c = f_d:\quad Q_b \approx 1.97\ ft^3/s;\quad Q_c \approx 7.43\ ft^3/s;\quad Q_d \approx 10.6\ ft^3/s$$

Improve by computing $Re_b \approx 349000$, $f_b \approx 0.0306$, $Re_c \approx 878000$, $f_c \approx 0.0271$, $Re_d \approx 1002000$, $f_d \approx 0.0255$. Repeat to find $Q_b \approx 1.835\ ft^3/s$, $Q_c \approx 7.351\ ft^3/s$, $Q_d \approx 10.814\ ft^3/s$. Repeat once more and quit: $Q_b \approx 1.833\ ft^3/s$, $Q_c \approx 7.349\ ft^3/s$, $Q_d \approx 10.819\ ft^3/s$, from which $V_b \approx 5.25$ ft/s, $V_c \approx 9.36$ ft/s, $V_d \approx 8.82$ ft/s. The pressure drop is

$$p_1 - p_2 = \Delta p_a + \Delta p_b = f_a \frac{L_a}{d_a}\frac{\rho V_a^2}{2} + f_b \frac{L_b}{d_b}\frac{\rho V_b^2}{2}$$

$$= 17000 + 1800 \approx 18800\ \text{psf} \approx \mathbf{131\ \frac{lbf}{in^2}} \quad \textit{Ans.}$$

6.119 Modify Prob. 6.118 as follows. Let the pressure drop ($p_1 - p_2$) be 98 lbf/in^2. Neglecting minor losses, determine the flow rate in ft^3/s.

Solution: From the solution just above for $\Delta p \approx 131$ psi, we can see that Δp_a is about 90.2% of the total drop. Therefore our first guess can be that

$$\Delta p_a \approx 0.902\Delta p = 0.902(98 \times 144) \approx 12729\ \text{psf} = f_a \frac{L_a}{d_a}\frac{\rho V_a^2}{2} \approx (0.027)(1000)\frac{1.94V_a^2}{2}$$

$$\text{Solve for}\quad V_a \approx 22.05\ \frac{ft}{s}$$

$$\text{and}\quad Q_a = A_aV_a \approx 17.3\ \frac{ft^3}{s} \overset{?}{=} Q_b[1 + 3.77(f_b/f_c)^{1/2} + 5.38(f_b/f_d)^{1/2}]$$

The last relation is still valid just as it was in Prob. 6.188. We can iterate to find

$$f_a \approx 0.0270,\quad Q_b \approx 1.585\ \frac{ft^3}{s},\quad Q_c \approx 6.357\ \frac{ft^3}{s},\quad Q_d \approx 9.358\ \frac{ft^2}{s},\quad f_b \approx 0.0307$$

$$p_1 - p_2 = f_a(L_a/d_a)(\rho V_a^2/2) + f_b(L_b/d_b)(\rho V_b^2/2) \approx 12717 + 1382 = 14099\ \text{psf} \approx 97.9\ \text{psi}$$

This is certainly close enough. We conclude the flow rate is $\mathbf{Q \approx 17.3\ ft^3/s}$. *Ans.*

6.120 Three cast-iron pipes are laid in parallel with these dimensions:

Pipe 1:	$L_1 = 800$ m	$d_1 = 12$ cm
Pipe 2:	$L_2 = 600$ m	$d_2 = 8$ cm
Pipe 3:	$L_3 = 900$ m	$d_3 = 10$ cm

The total flow rate is 200 m^3/h of water at 20°C. Determine (a) the flow rate in each pipe; and (b) the pressure drop across the system.

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For cast iron, $\varepsilon = 0.26$ mm. Then, $\varepsilon/d_1 = 0.00217$, $\varepsilon/d_2 = 0.00325$, and $\varepsilon/d_3 = 0.0026$. The head losses are the same for each pipe, and the flow rates add:

$$h_f = \frac{8 f_1 L_1 Q_1^2}{\pi^2 g d_1^5} = \frac{8 f_2 L_2 Q_2^2}{\pi^2 g d_2^5} = \frac{8 f_3 L_3 Q_3^2}{\pi^2 g d_3^5}; \quad \text{and} \quad Q_1 + Q_2 + Q_3 = \frac{200}{3600}\frac{m^3}{s}$$

$$\text{Substitute and combine:} \quad Q_1[1 + 0.418(f_1/f_2)^{1/2} + 0.599(f_1/f_3)^{1/2}] = 0.0556\ m^3/s$$

We could either go directly to EES or begin by guessing $f_1 = f_2 = f_3$, which gives $Q_1 = 0.0275$ m^3/s, $Q_2 = 0.0115$ m^3/s, and $Q_3 = 0.0165$ m^3/s. This is *very* close! Further iteration gives

$$\text{Re}_1 = 298000,\ f_1 = 0.0245;\ \text{Re}_2 = 177000,\ f_2 = 0.0275;\ \text{Re}_3 = 208000,\ f_3 = 0.0259$$

$$Q_1 = \mathbf{0.0281\ m^3/s}, \quad Q_2 = \mathbf{0.0111\ m^3/s}, \quad \text{and} \quad Q_3 = \mathbf{0.0163\ m^3/s} \quad \textit{Ans.}\ \text{(a)}$$

$$h_f = 51.4\ \text{m}, \quad \Delta p = \rho g h_f = (998\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(51.4\ \text{m}) = \mathbf{503{,}000\ Pa} \quad \textit{Ans.}\ \text{(b)}$$

6.121 Consider the three-reservoir system of Fig. P6.121 with the following data:

$L_1 = 95$ m	$L_2 = 125$ m	$L_3 = 160$ m
$z_1 = 25$ m	$z_2 = 115$ m	$z_3 = 85$ m

All pipes are 28-cm-diameter unfinished concrete ($\varepsilon = 1$ mm). Compute the steady flow rate in all pipes for water at 20°C.

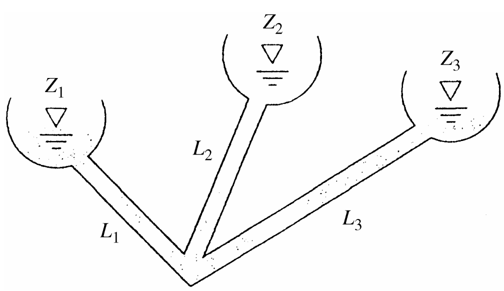


Fig. P6.121

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. All pipes have $\varepsilon/d = 1/280 = 0.00357$. Let the intersection be "a." The head loss at "a" is desired:

$$z_1 - h_a = f_1 \frac{L_1}{d_1}\frac{V_1^2}{2g}; \quad z_2 - h_a = f_2 \frac{L_2}{d_2}\frac{V_2^2}{2g}; \quad z_3 - h_a = f_3 \frac{L_3}{d_3}\frac{V_3^2}{2g}$$

plus the requirement that $Q_1 + Q_2 + Q_3 = 0$ or, for same d, $V_1 + V_2 + V_3 = 0$

We guess h_a then iterate each friction factor to find V and Q and then check if $\sum Q = 0$.

$$h_a = 75\text{ m:}\quad 25 - 75 = (-)50 = f_1\left(\frac{95}{0.28}\right)\frac{V_1^2}{2(9.81)},\quad \text{solve } f_1 \approx 0.02754,\ V_1 \approx -10.25\ \frac{m}{s}$$

$$\text{Similarly,}\quad 115 - 75 = f_2(125/0.28)\left[V_2^2/2(9.81)\right]\quad \text{gives } f_2 \approx 0.02755.\quad V_2 \approx +7.99$$

$$\text{and}\quad 85 - 75 = f_3(160/0.28)\left[V_3^2/2(9.81)\right]$$

$$\text{gives}\quad f_3 \approx 0.02762,\ V_3 \approx +3.53\ \frac{m}{s},\quad \sum V = \mathbf{+1.27}$$

Repeating for $h_a = 80$ m gives $V_1 = -10.75$, $V_2 = +7.47$, $V_3 = +2.49$ m/s, $\sum V = -0.79$.
Interpolate to $h_a \approx 78$ m, gives $V_1 = -10.55$ m/s, $V_2 = +7.68$ m/s, $V_3 = +2.95$ m/s, or:

$$\mathbf{Q_1 = -0.65\ m^3/s,\quad Q_2 = +0.47\ m^3/s,\quad Q_3 = +0.18\ m^3/s}.\quad \textit{Ans.}$$

6.122 Modify Prob. 6.121 by reducing the diameter to 15 cm, with $\varepsilon = 1$ mm. Compute the flow rate in each pipe. They all reduce, compared to Prob. 6.121, by a factor of about *5.2.* Can you explain this?

Solution: The roughness ratio increases to $\varepsilon/d = 1/150 = 0.00667$, and all L/d's increase. Guess $h_a = 75$ m: converges to $f_1 = 0.0333$, $f_2 = 0.0333$, $f_4 = 0.0334$

and $V_1 \approx -6.82$ m/s, $V_2 \approx +5.32$ m/s, $V_3 \approx +2.34$ m/s, $\sum V \approx \mathbf{+0.85}$

We finally obtain $h_a \approx 78.2$ m, giving $V_1 = -7.04$ m/s, $V_2 = +5.10$ m/s, $V_3 = +1.94$ m/s,

or: $\mathbf{Q_1 = -0.124\ m^3/s}$, $\mathbf{Q_2 = +0.090\ m^3/s}$, $\mathbf{Q_3 = +0.034\ m^3/s}$. *Ans.*

6.123 Modify Prob. 6.121 on the previous page as follows. Let z_3 be unknown and find its value such that the flow rate in pipe 3 is 0.2 m^3/s toward the junction. (This problem is best suited for computer iteration.)

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. All pipes have $\varepsilon/d = 1/280 = 0.00357$. Let the intersection be "a." The head loss at "a" is desired for each in order to check the flow rate in pipe 3.

In Prob. 6.121, with $z_3 = 85$ m, we found Q_3 to be 0.18 m^3/s toward the junction, pretty close. We repeat the procedure with a few new values of z_3, closing to $\sum Q = 0$ each time:

Guess $z_3 = 85$ m: $h_a = 78.19$ m, $Q_1 = -0.6508$, $Q_2 = +0.4718$, $Q_3 = +0.1790\ m^3/s$

90 m: 80.65 m, -0.6657, $+0.6657$, $+0.2099\ m^3/s$

Interpolate: $h_a \approx 79.89$, $Q_1 \approx -0.6611$,

$Q_2 \approx +0.4608$, $\mathbf{Q_3 \approx +0.200\ m^3/s}$, $\mathbf{z_3 \approx 88.4\ m}$ *Ans.*

6.124 The three-reservoir system in Fig. P6.124 delivers water at 20°C. The system data are as follows:

$D1 = 8$ in	$D2 = 6$ in	$D3 = 9$ in
$L1 = 1800$ ft	$L2 = 1200$ ft	$L3 = 1600$ ft

All pipes are galvanized iron. Compute the flow rate in all pipes.

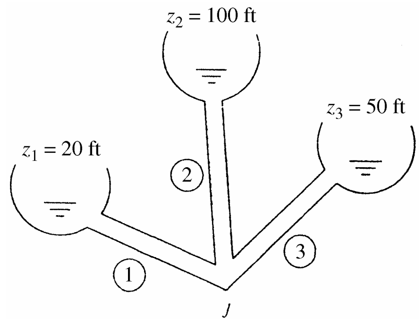


Fig. P6.124

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09$E–5 slug/ft·s. For galvanized iron, take $\varepsilon = 0.0005$ ft. Then the roughness ratios are

$$\varepsilon/d_1 = 0.00075 \quad \varepsilon/d_2 = 0.0010 \quad \varepsilon/d_3 = 0.00667$$

Let the intersection be "a." The head loss at "a" is desired:

$$z_1 - h_a = \frac{f_1 L_1}{d_1}\frac{V_1^2}{2g}; \quad z_2 - h_a = \frac{f_2 L_2}{d_2}\frac{V_2^2}{2g}; \quad z_3 - h_a = \frac{f_3 L_3}{d_3}\frac{V_3^2}{2g}; \quad \text{plus } Q_1 + Q_2 + Q_3 = 0$$

We guess h_a then iterate each friction factor to find V and Q and then check if $\sum Q = 0$.

$$\text{Guess} \quad h_a = 50 \text{ ft:} \quad 20 - 50 = (-)30 \text{ ft} = \frac{f_1(1800)V_1^2}{(8/12)2(32.2)},$$

$$\text{solve} \quad f_1 = 0.0194, \quad V_1 = -6.09 \ \frac{\text{ft}}{\text{s}}$$

$$\text{Similarly, } f_2 = 0.0204,\ V_2 \approx +8.11 \text{ ft/s and of course } V_3 = 0. \quad \text{Get } \sum Q = -0.54 \text{ ft}^3/\text{s}$$

Try again with a slightly lower h_a to reduce Q_1 and increase Q_2 and Q_3:

$$h_a = 48 \text{ ft:} \quad \text{converges to } Q_1 = -2.05 \ \frac{\text{ft}^3}{\text{s}}, \quad Q_2 = +1.62 \ \frac{\text{ft}^3}{\text{s}},$$

$$Q_3 = +0.76 \ \frac{\text{ft}^3}{\text{s}}, \quad \sum Q = +0.33$$

Interpolate to

$$h_a = 49.12 \text{ ft:} \quad \mathbf{Q_1 = -2.09\ ft^3/s}, \quad \mathbf{Q_2 = +1.61\ ft^3/s}, \quad \mathbf{Q_3 = +0.49\ ft^3/s} \quad \textit{Ans.}$$

6.125 Suppose that the three cast-iron pipes in Prob. 6.120 are instead connected to meet smoothly at a point B, as shown in Fig. P6.125. The inlet pressures in each pipe are: p_1 = 200 kPa; p_2 = 160 kPa; p_3 = 100 kPa. The fluid is water at 20°C. Neglect minor losses. Estimate the flow rate in each pipe and whether it is toward or away from point B.

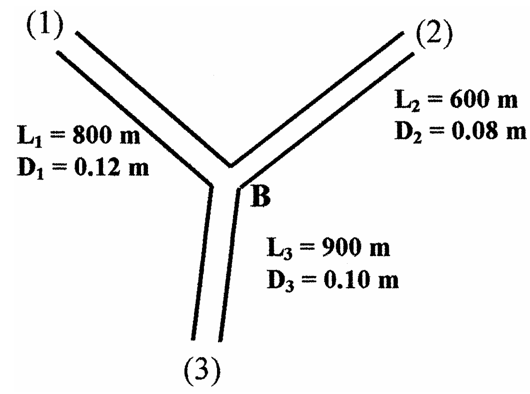


Fig. P6.125

Solution: For water take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. The pressure at point B must be a known (constant) value which makes the net flow rate equal to zero at junction B. The flow clearly goes from (1) to B, and from B to (3), but we are not sure about pipe (2). For cast iron (Table 6.1), ε = 0.26 mm. Each pipe has a flow rate based upon its pressure drop:

$$p_1 - p_B = f_1\frac{L_1}{D_1}\frac{\rho V_1^2}{2}; \quad |p_2 - p_B| = f_2\frac{L_2}{D_2}\frac{\rho V_2^2}{2}; \quad p_B - p_3 = f_3\frac{L_3}{D_3}\frac{\rho V_3^2}{2}$$

where the f's are determined from the Moody chart for each pipe's ε/D and Re_D. The correct value of p_B makes the flow rates $Q_i = (\pi/4)D_i^2 V_i$ balance at junction B. EES is excellent for this type of iteration, and the final results balance for p_B = 166.7 kPa:

$f_1 = 0.0260;\ \ Re_1 = 74300;\ \ \varepsilon/D_1 = 0.00217;\ \ \mathbf{Q_1 = +0.00701\ m^3/s}$ (toward B)

$f_2 = 0.0321;\ \ Re_2 = 18900;\ \ \varepsilon/D_2 = 0.00325;\ \ \mathbf{Q_2 = -0.00119\ m^3/s}$ (away from B) *Ans.*

$f_3 = 0.0270;\ \ Re_3 = 74000;\ \ \varepsilon/D_3 = 0.00260;\ \ \mathbf{Q_3 = -0.00582\ m^3/s}$ (away from B)

6.126 Modify Prob. 6.124 as follows. Let all data be the same except that pipe 1 is fitted with a butterfly valve (Fig. 6.19*b*). Estimate the proper valve opening angle (in degrees) for the flow rate through pipe 1 to be reduced to 1.5 ft^3/s toward reservoir 1. (This problem requires iteration and is best suited to a digital computer.)

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–4 slug/ft·s. For galvanized iron, take ε = 0.0005 ft. Then the roughness ratios are

$$\varepsilon/d_1 = 0.00075 \quad \varepsilon/d_2 = 0.0010 \quad \varepsilon/d_3 = 0.00667$$

For a butterfly valve loss coefficient "K" (to be found). Let the junction be "J." The head loss at "J" is desired and then to be iterated to give the proper flow rate in pipe (1):

$$z_1 - h_J = \frac{V_1^2}{2g}\left(f\frac{L}{d}+K\right)_1; \quad z_2 - h_J = \frac{V_2^2}{2g}\left(f\frac{L}{d}\right)_2;$$

$$z_3 - h_J = \frac{V_3^2}{2g}\left(f\frac{L}{d}\right)_3; \quad \text{and} \quad Q_1 + Q_2 + Q_3 = 0$$

We know z_1 = 20 ft, z_2 = 100 ft, and z_3 = 50 ft. From Prob. 6.124, where **K = 0**, the flow rate was 2.09 ft^3/s toward reservoir 1. Now guess a finite value of K and repeat:

$$K = 40: \quad \text{converges to } h_J = 50.0, \quad Q_1 = -1.59\ ft^3/s, \quad Q_2 = +1.59\ ft^3/s; \quad Q_3 \approx 0$$

$$K = 50: \quad \text{converges to } h_J = 50.03\ ft, \quad Q_1 = -1.513 \quad Q_2 = +1.591 \quad Q_3 = -0.078$$

$$\mathbf{K = 52}: \quad \text{gives } h_J = 50.04\ ft, \quad \mathbf{Q_1 = -1.500\ ft^3/s} \quad Q_2 = 1.591 \quad Q_3 = -0.091 \quad \textit{Ans.}$$

From Fig. 6.19b, a butterfly valve coefficient K ≈ 52 occurs at $\theta_{\mathbf{opening}} \approx \mathbf{35°}$. *Ans.*

6.127 In the five-pipe horizontal network of Fig. P6.127, assume that all pipes have a friction factor *f* = 0.025. For the given inlet and exit flow rate of 2 ft^3/s of water at 20°C, determine the flow rate and direction in all pipes. If p_A = 120 lbf/in^2 gage, determine the pressures at points *B*, *C*, and *D*.

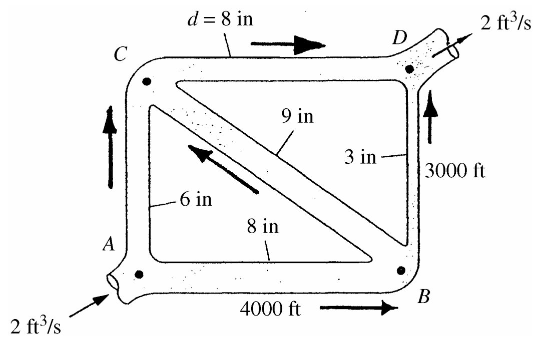


Fig. P6.127

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. Each pipe has a head loss which is known except for the square of the flow rate:

$$\text{Pipe AC:} \quad h_f = \frac{8fLQ^2}{\pi^2 g d^5}\Big|_{AC} = \frac{8(0.025)(3000)Q_{AC}^2}{\pi^2(32.2)(6/12)^5} = K_{AC}Q_{AC}^2, \quad \text{where } K_{AC} \approx 60.42$$

$$\text{Similarly,} \quad K_{AB} = 19.12, \quad K_{BC} = 13.26, \quad K_{CD} = 19.12, \quad K_{BD} = 1933. \quad \left(Q \text{ in } \frac{ft^3}{s}\right)$$

There are two triangular closed loops, and the total head loss must be zero for each. Using the flow directions assumed on the figure P6.127 above, we have

$$\text{Loop A-B-C:} \quad 19.12Q_{AB}^2 + 13.26Q_{BC}^2 - 60.42Q_{AC}^2 = 0$$

$$\text{Loop B-C-D:} \quad 13.26Q_{BC}^2 + 19.12Q_{CD}^2 - 1933.0Q_{BD}^2 = 0$$

And there are three independent junctions which have zero net flow rate:

$$\text{Junction A:}\quad Q_{AB} + Q_{AC} = 2.0;\quad \text{B:}\ Q_{AB} = Q_{BC} + Q_{BD};\quad \text{C:}\ Q_{AC} + Q_{BC} = Q_{CD}$$

These are five algebraic equations to be solved for the five flow rates. The answers are:

$$Q_{AB} = \mathbf{1.19},\quad Q_{AC} = \mathbf{0.81},\quad Q_{BC} = \mathbf{0.99},\quad Q_{CD} = \mathbf{1.80},\quad Q_{BD} = \mathbf{0.20}\ \mathbf{\frac{ft^3}{s}}\quad \textit{Ans.}\ \text{(a)}$$

The pressures follow by starting at A (120 psi) and subtracting off the friction losses:

$$p_B = p_A - \rho g K_{AB} Q_{AB}^2 = 120 \times 144 - 62.4(19.12)(1.19)^2$$

$$p_B = 15590\ \text{psf} \div 144 = \mathbf{108}\ \mathbf{\frac{lbf}{in^2}}$$

$$\text{Similarly,}\quad p_C \approx \mathbf{103\ psi}\quad \text{and}\quad p_D \approx \mathbf{76\ psi}\quad \textit{Ans.}\ \text{(b)}$$

6.128 Modify Prob. 6.127 above as follows: Let the inlet flow at A and the exit flow at D be unknown. Let $p_A - p_B = 100$ psi. Compute the flow rate in all five pipes.

Solution: Our head loss coefficients "K" from above are all the same. Head loss AB is known, plus we have two "loop" equations and two "junction" equations:

$$\frac{p_A - p_B}{\rho g} = \frac{100 \times 144}{62.4} = 231\ \text{ft} = K_{AB} Q_{AB}^2 = 19.12 Q_{AB}^2,\quad \text{or}\quad Q_{AB} = \mathbf{3.47\ ft^3/s}$$

$$\text{Two loops:}\quad 231 + 13.26 Q_{BC}^2 - 60.42 Q_{AC}^2 = 0$$

$$13.26 Q_{BC}^2 + 19.12 Q_{CD}^2 - 1933.0 Q_{BD}^2 = 0$$

$$\text{Two junctions:}\quad Q_{AB} = 3.47 = Q_{BC} + Q_{BD};\quad Q_{AC} + Q_{BC} = Q_{CD}$$

The solutions are in exactly the same ratio as the lower flow rates in Prob. 6.127:

$$Q_{AB} = \mathbf{3.47}\ \mathbf{\frac{ft^3}{s}},\quad Q_{BC} = \mathbf{2.90}\ \mathbf{\frac{ft^3}{s}},\quad Q_{BD} = \mathbf{0.58}\ \mathbf{\frac{ft^3}{s}},$$

$$Q_{CD} = \mathbf{5.28}\ \mathbf{\frac{ft^3}{s}},\quad Q_{AC} = \mathbf{2.38}\ \mathbf{\frac{ft^3}{s}}\quad \textit{Ans.}$$

6.129 In Fig. P6.129 all four horizontal cast-iron pipes are 45 m long and 8 cm in diameter and meet at junction *a*, delivering water at 20°C. The pressures are known at four points as shown:

$p1 = 950$ kPa $p2 = 350$ kPa
$p3 = 675$ kPa $p4 = 100$ kPa

Neglecting minor losses, determine the flow rate in each pipe.

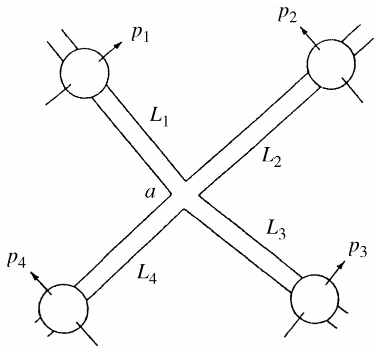


Fig. P6.129

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. All pipes are cast iron, with $\varepsilon/d = 0.26/80 = 0.00325$. All pipes have L/d = 45/0.08 = 562.5. One solution method is to guess the junction pressure pa, iterate to calculate the friction factors and flow rates, and check to see if the net junction flow is zero:

$$\text{Guess } p_a = 500 \text{ kPa:}\quad h_{f1} = \frac{950000 - 500000}{998(9.81)} = 45.96 \text{ m} = \frac{8f_1L_1Q_1^2}{\pi^2 g d_1^5} = 1.135E6 f_1 Q_1^2$$

$$\text{then guess } f_1 \approx 0.02,\quad Q_1 = 0.045 \text{ m}^3/\text{s},\quad Re_1 = 4\rho Q_1/(\pi\mu d_1) = 715000,\quad f_{1\text{-new}} \approx 0.0269$$

$$\text{converges to}\quad f_1 \approx 0.0270,\quad Q_1 \approx 0.0388 \text{ m}^3/\text{s}$$

$$\text{Iterate also to}\quad Q_2 = -0.0223\ \frac{m^3}{s}(\text{away from } a),\quad Q_3 = 0.0241,\quad Q_4 = -0.0365$$

$$\sum Q = +0.00403,\quad \text{so we have guessed } p_a \text{ a little low.}$$

Trying pa = 530 kPa gives $\sum Q = -0.00296$, hence iterate to pa ≈ 517 kPa:

$$Q_1 = \mathbf{+0.0380}\ \frac{\mathbf{m^3}}{\mathbf{s}}\ (\text{toward a}),\quad Q_2 = \mathbf{-0.0236}\ \frac{\mathbf{m^3}}{\mathbf{s}},$$

$$Q_3 = \mathbf{+0.0229}\ \frac{\mathbf{m^3}}{\mathbf{s}},\quad Q_4 = \mathbf{-0.0373}\ \frac{\mathbf{m^3}}{\mathbf{s}}\quad \textit{Ans.}$$

6.130 In Fig. P6.130 lengths *AB* and *BD* are 2000 and 1500 ft, respectively. The friction factor is 0.022 everywhere, and p_A = 90 lbf/in^2 gage. All pipes have a diameter of 6 in. For water at 20°C, determine the flow rate in all pipes and the pressures at points *B*, *C*, and *D*.

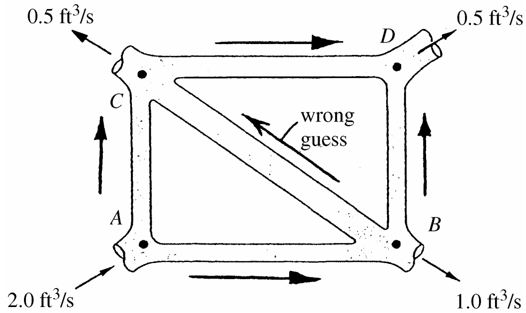


Fig. P6.130

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09E-5$ slug/ft·s. Each pipe has a head loss which is known except for the square of the flow rate:

$$\text{Pipe AC:} \quad h_f = \frac{8fLQ^2}{\pi^2 g d^5} = \frac{8(0.022)(1500)Q_{AC}^2}{\pi^2(32.2)(6/12)^5} = K_{AC}Q_{AC}^2, \quad \text{where } K_{AC} \approx \mathbf{26.58}$$

Similarly, $K_{AB} = K_{CD} = 35.44$, $K_{BD} = 26.58$, and $K_{BC} = 44.30$.

The solution is similar to Prob. 6.127, except that (1) the K's are different; and (2) junctions B and C have additional flow leaving the network. The basic flow relations are:

$$\text{Loop ABC:} \quad 35.44Q_{AB}^2 + 44.3Q_{BC}^2 - 26.58Q_{AC}^2 = 0$$

$$\text{Loop BCD:} \quad 44.3Q_{BC}^2 + 35.44Q_{CD}^2 - 26.58Q_{BD}^2 = 0$$

$$\text{Junctions A,B,C:} \quad Q_{AB} + Q_{AC} = 2.0;$$

$$Q_{AB} = Q_{BC} + Q_{BD} + 1.0; \quad Q_{AC} + Q_{BC} = Q_{CD} + 0.5$$

In this era of PC "equation solvers" such as MathCAD, etc., it is probably not necessary to dwell upon any solution methods. For hand work, one might guess Q_{AB}, then the other four are obtained in sequence from the above relations, plus a check on the original guess for Q_{AB}. The assumed arrows are shown above. It turns out that we have guessed the direction incorrectly on Q_{BC} above, but the others are OK. The final results are:

$$Q_{AB} = \mathbf{0.949\ ft^3/s} \text{ (toward B)}; \quad Q_{AC} = \mathbf{1.051\ ft^3/s} \text{ (toward C)}$$

$$Q_{BC} = \mathbf{0.239} \text{ (toward B)}; \quad Q_{CD} = \mathbf{0.312} \text{ (toward D)}; \quad Q_{BD} = \mathbf{0.188} \text{ (to D)} \quad \textit{Ans. (a)}$$

The pressures start at A, from which we subtract the friction losses in each pipe:

$$p_B = p_A - \rho g K_{AB} Q_{AB}^2 = 90 \times 144 - 62.4(35.44)(0.949)^2 = 10969 \text{ psf} \div 144 = \mathbf{76\ psi}$$

$$\text{Similarly, we obtain} \quad p_C = 11127 \text{ psf} = \mathbf{77\ psi}; \quad p_D = 10911 \text{ psf} \approx \mathbf{76\ psi} \quad \textit{Ans. (b)}$$

6.131 A water-tunnel test section has a 1-m diameter and flow properties $V = 20$ m/s, $p = 100$ kPa, and $T = 20$°C. The boundary-layer blockage at the end of the section is 9 percent. If a conical diffuser is to be added at the end of the section to achieve maximum pressure recovery, what should its angle, length, exit diameter, and exit pressure be?

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. The Reynolds number is very high, Re = $\rho Vd/\mu$ = (998)(20)(1)/(0.001) ≈ 2.0E7; much higher than the diffuser data in Fig. 6.28b (Re ≈ 1.2E5). But what can we do (?) Let's use it anyway:

$$B_t = 0.09, \quad \text{read } C_{p,max} \approx 0.71 \quad \text{at } L/d \approx 25,\ 2\theta \approx 4°,\ AR \approx 8:$$

$$\text{Then} \quad \theta_{cone} \approx 2°, \quad L \approx 25d \approx \mathbf{25\ m}, \quad D_{exit} = d(8)^{1/2} \approx \mathbf{2.8\ m} \quad \textit{Ans.}\ (a)$$

$$C_p \approx 0.71 = \frac{p_e - p_t}{(1/2)\rho V_t^2} = \frac{p_e - 100000}{(1/2)(998)(20)^2}, \quad \text{or:} \quad p_{exit} \approx \mathbf{242000\ Pa} \quad \textit{Ans.}\ (b)$$

6.132 For Prob. 6.131, suppose we are limited by space to a total diffuser length of 10 meters. What should be the diffuser angle, exit diameter, and exit pressure for maximum recovery?

Solution: We are limited to L/D = 10.0. From Fig. 6.28b, read $C_{p,max} \approx 0.62$ at AR ≈ 4 and $2\theta \approx 6°$. *Ans.* The exit diameter and pressure are

$$D_e = d\sqrt{(AR)} = (1.0)(4.0)^{1/2} \approx \mathbf{2.0\ m} \quad \textit{Ans.}$$

$$C_{p,max} \approx 0.62 = (p_e - 100000)/[(1/2)(998)(20)^2], \quad \text{or:} \quad p_{exit} \approx \mathbf{224000\ Pa} \quad \textit{Ans.}$$

6.133 A wind-tunnel test section is 3 ft square with flow properties *V* = 150 ft/s, *p* = 15 lbf/in^2 absolute, and *T* = 68°F. Boundary-layer blockage at the end of the test section is 8 percent. Find the angle, length, exit height, and exit pressure of a flat-walled diffuser added onto the section to achieve maximum pressure recovery.

Solution: For air at 20°C and 15 psi, take $\rho = 0.00238$ slug/ft^3 and $\mu = 3.76\text{E}{-7}$ slug/ft·s. The Reynolds number is rather high, Re = $\rho Vd/\mu$ = (0.00238)(150)(3)/(3.76E–7) ≈ 2.9E6; much higher than the diffuser data in Fig. 6.28a (Re ≈ 2.8E5). But what can we do (?) Let's use it anyway:

$$B_t = 0.08, \quad \text{read } C_{p,max} \approx 0.70 \quad \text{at} \quad L/W_1 \approx 17,\ 2\theta \approx 9.5°,\ AR \approx 3.75:$$

$$\text{Then} \quad \theta_{best} \approx \mathbf{4.75°}, \quad L \approx 17W_1 \approx \mathbf{51\ ft}, \quad W_2 \approx (AR)W_1 = 3.75(3) \approx \mathbf{11\ ft} \quad \textit{Ans.}$$

$$C_p \approx 0.70 = \frac{p_e - p_t}{(1/2)\rho V_1^2} = \frac{p_e - 15 \times 144}{(1/2)(0.00238)(150)^2}, \quad \text{or:} \quad p_{exit} \approx \mathbf{2180\ \frac{lbf}{ft^2}} \quad \textit{Ans.}$$

6.134 For Prob. 6.133 above, suppose we are limited by space to a total diffuser length of 30 ft. What should the diffuser angle, exit height, and exit pressure be for maximum recovery?

Solution: We are limited to L/W1 = 10.0. From Fig. 6.28a, read $C_{p,max} \approx 0.645$ at AR ≈ 2.8 and $2\theta \approx 10°$. *Ans.* The exit height and pressure are

$$W_{1,e} = (AR)W_1 = (2.8)(3.0) \approx \mathbf{8.4\ ft} \quad \textit{Ans.}$$

$$C_{p,max} \approx 0.645 = [p_e - (15)144)]/[(1/2)(0.00238)(150)^2], \quad \text{or} \quad p_e = \mathbf{2180\ \frac{lbf}{ft^2}} \quad \textit{Ans.}$$

6.135 An airplane uses a pitot-static tube as a velocimeter. The measurements, with their uncertainties, are a static temperature of (–11 ± 3)°C, a static pressure of 60 ± 2 kPa, and a pressure difference $(p_0 - p_s)$ = 3200 ± 60 Pa. (a) Estimate the airplane's velocity and its uncertainty. (b) Is a compressibility correction needed?

Solution: The air density is $\rho = p/(RT) = (60000\text{ Pa})/[(287\text{ m}^2/\text{s}^2\cdot\text{K})(262\text{ K})] = 0.798\text{ kg/m}^3$. (a) Estimate the velocity from the incompressible Pitot formula, Eq. (6.97):

$$V = \sqrt{\frac{2\Delta p}{\rho}} = \sqrt{\frac{2\Delta p}{p/(RT)}} = \sqrt{\frac{2(3200\ Pa)}{0.798\text{ kg/m}^3}} = 90\ \frac{\text{m}}{\text{s}}$$

The overall uncertainty involves pressure difference, absolute pressure, and absolute temperature:

$$\frac{\delta V}{V} = \left[\left(\frac{1}{2}\frac{\delta\Delta p}{\Delta p}\right)^2 + \left(\frac{1}{2}\frac{\delta p}{p}\right)^2 + \left(\frac{1}{2}\frac{\delta T}{T}\right)^2\right]^{1/2} = \frac{1}{2}\left[\left(\frac{60}{3200}\right)^2 + \left(\frac{2}{60}\right)^2 + \left(\frac{3}{262}\right)^2\right]^{1/2} = 0.020$$

The uncertainty in velocity is 2%, therefore our final estimate is $\mathbf{V \approx 90 \pm 2\ m/s}$ *Ans.* (a) Check the Mach number. The speed of sound is $a = (kRT)^{1/2} = [1.4(287)(262)]^{1/2} = 324$ m/s. Therefore

$Ma = V/a = 90/324 = 0.28 < 0.3$. **No compressibility correction is needed.** *Ans.* (b)

6.136 For the pitot-static pressure arrangement of Fig. P6.136, the manometer fluid is (colored) water at 20°C. Estimate (a) the centerline velocity, (b) the pipe volume flow, and (c) the (smooth) wall shear stress.

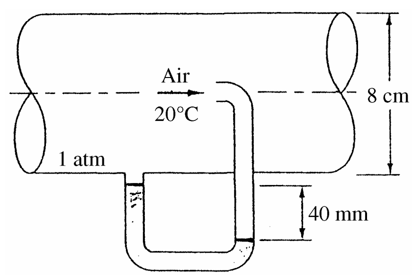


Fig. P6.136

Solution: For air at 20°C and 1 atm, take $\rho = 1.2$ kg/m^3 and $\mu = 1.8E{-}5$ kg/m·s. For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. The manometer reads

$$p_o - p = (\rho_{water} - \rho_{air})gh = (998 - 1.2)(9.81)(0.040) \approx 391\ \text{Pa}$$

$$\text{Therefore}\quad V_{CL} = [2\Delta p/\rho]^{1/2} = [2(391)/1.2]^{1/2} \approx \mathbf{25.5\ m/s}\quad \textit{Ans.}\ \text{(a)}$$

We can estimate the friction factor and then compute average velocity from Eq. (6.43):

$$\text{Guess}\quad V_{avg} \approx 0.85V_{CL} \approx 21.7\ \frac{m}{s},\quad \text{then}\ Re_d = \frac{\rho Vd}{\mu} = \frac{1.2(21.7)(0.08)}{1.8E{-}5} \approx 115{,}700$$

$$\text{Then}\quad f_{smooth} \approx 0.0175,\quad V_{better} = \frac{25.5}{[1 + 1.33\sqrt{0.0175}]} \approx 21.69\ \frac{m}{s}\ \text{(converged)}$$

$$\text{Thus the volume flow is}\quad Q = (\pi/4)(0.08)^2(21.69) \approx \mathbf{0.109\ m^3/s}.\quad \textit{Ans.}\ \text{(b)}$$

$$\text{Finally,}\quad \tau_w = \frac{f}{8}\rho V^2 = \frac{0.0175}{8}(1.2)(21.69)^2 \approx \mathbf{1.23\ Pa}\quad \textit{Ans.}\ \text{(c)}$$

6.137 For the 20°C water flow of Fig. P6.137, use the pitot-static arrangement to estimate (a) the centerline velocity and (b) the volume flow in the 5-in-diameter smooth pipe. (c) What error in flow rate is caused by neglecting the 1-ft elevation difference?

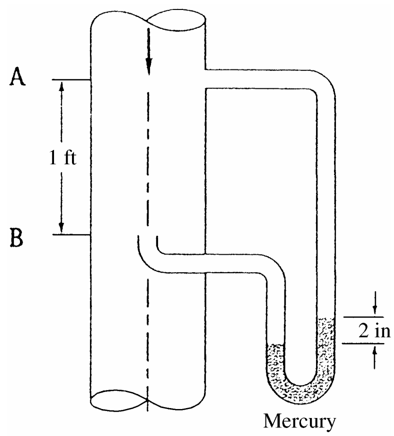


Fig. P6.137

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and $\mu = 2.09E{-}5$ slug/ft·s. For the manometer reading h = 2 inches,

$$p_{oB} - p_A = (SG_{merc} - 1)(\rho g)_{water}h + \rho_{water}g(1\ \text{ft})\quad \text{but from the energy equation,}$$

$$p_A - p_B = \rho_{water}gh_{f\text{-}AB} - \rho_{water}g(1\ \text{ft})\quad \text{Therefore } p_{oB} - p_B = (SG - 1)\rho gh_{mano} + \rho gh_{f\text{-}AB}$$

$$\text{where friction loss}\ h_{f\text{-}AB} \approx f(\Delta L/d)(V^2/2g)$$

Thus the pitot tube reading equals the manometer reading (of about 130 psf) plus the friction loss between A and B (which is only about 3 psf), so there is only a small error:

$$(SG - 1)\rho gh = (13.56 - 1)(62.4)(2/12) \approx 130.6\ \text{psf},\quad V_{CL} \approx \left[\frac{2\Delta p}{\rho}\right]^{1/2} = \left[\frac{2(130.6)}{1.94}\right]^{1/2}$$

$$\text{or}\quad V_{CL} \approx 11.6\ \frac{ft}{s},\quad \text{so}\ V_{avg} \approx 0.85V_{CL} \approx 9.9\ \frac{ft}{s},\quad Re = \frac{1.94(9.9)(5/12)}{2.09E\text{–}5} \approx 381500,$$

$$\text{so}\quad f_{smooth} \approx 0.0138,\quad \text{or}\quad \Delta p_{friction} = f(L/d)\rho V^2/2 \approx 3.2\ \text{lbf/ft}^2$$

If we now correct the pitot tube reading to $\Delta p_{pitot} \approx 130.6 + 3.2 = 133.8$ psf, we may iterate and converge rapidly to the final estimate:

$$f \approx 0.01375,\ V_{CL} \approx \mathbf{11.75\ \frac{ft}{s}};\quad Q \approx \mathbf{1.39\ \frac{ft^3}{s}};\quad V_{avg} \approx \mathbf{10.17\ \frac{ft}{s}}\quad \textit{Ans.}\ \text{(a, b)}$$

The error compared to our earlier estimate V ≈ 9.91 ft/s is about **2.6%** *Ans.* (c)

6.138 An engineer who took college fluid mechanics on a pass-fail basis has placed the static pressure hole far upstream of the stagnation probe, as in Fig. P6.138, thus contaminating the pitot measurement ridiculously with pipe friction losses. If the pipe flow is air at 20°C and 1 atm and the manometer fluid is Meriam red oil (SG = 0.827), estimate the air centerline velocity for the given manometer reading of 16 cm. Assume a smooth-walled tube.

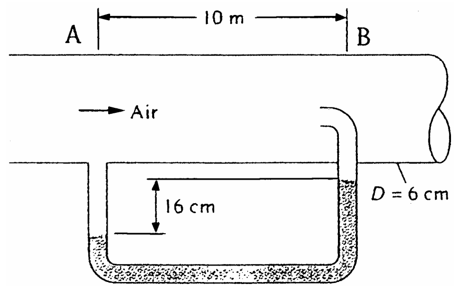


Fig. P6.138

Solution: For air at 20°C and 1 atm, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. Because of the high friction loss over 10 meters of length, the manometer actually shows p_{oB} *less than* p_A, which is a bit weird but correct:

$$p_A - p_{oB} = (\rho_{mano} - \rho_{air})gh = [0.827(998) - 1.2](9.81)(0.16) \approx 1294\ \text{Pa}$$

$$\text{Meanwhile,}\quad p_A - p_B = \rho g h_f = f\frac{L}{d}\frac{\rho V^2}{2},\quad \text{or}\quad p_{oB} - p_B = \frac{fL}{d}\frac{\rho V^2}{2} - 1294 = \frac{\rho}{2}V_{CL}^2$$

$$\text{Guess}\quad f \approx 0.02,\quad V \approx 0.85V_{CL},\quad \text{whence}\ 0.02\left(\frac{10}{0.06}\right)\left(\frac{1.2}{2}\right)V^2 - 1294 = \frac{1.2}{2}\left(\frac{V}{0.85}\right)^2$$

$$\text{Solve for}\quad V \approx 33.3\ \frac{m}{s},\quad Re_d = \frac{1.2(33.3)(0.06)}{1.8E\text{–}5} \approx 133000,\quad f_{better} \approx 0.0170,$$

$$V \approx V_{CL}/[1 + 1.33\sqrt{f}] \approx 0.852V_{CL},\quad \text{repeat to convergence}$$

Finally converges, $f \approx 0.0164$, $V \approx 39.87$ m/s, $V_{CL} = V/0.8546 \approx$ **46.65 m/s**. *Ans.*

6.139 Professor Walter Tunnel must measure velocity in a water tunnel. Due to budgetary restrictions, he cannot afford a pitot-static tube, so he inserts a total-head probe and a static-head probe, as shown, both in the mainstream away from the wall boundary layers. The two probes are connected to a manometer. (a) Write an expression for tunnel velocity V in terms of the parameters in the figure. (b) Is it critical that h1 be measured accurately? (c) How does part (a) differ from a pitot-static tube formula?

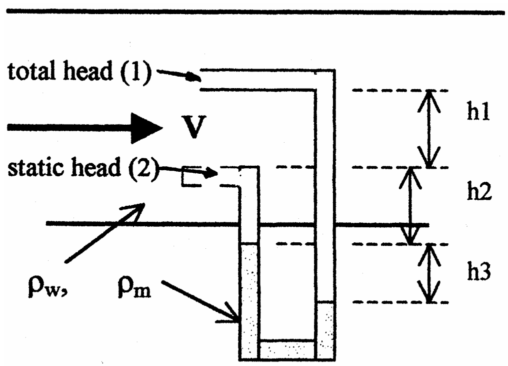


Fig. P6.139

Solution: Write Bernoulli from total-head inlet (1) to static-head inlet (2):

$$p_o + \rho_w g z_1 = p_s + \frac{\rho_w}{2}V^2 + \rho_w g z_2, \quad \text{Solve } V = \sqrt{\frac{2(p_o - p_s + \rho_w g h_1)}{\rho_w}}$$

Combine this with hydrostatics through the manometer:

$$p_s + \rho_w g h_2 + \rho_m g h_3 = p_o + \rho_w g h_1 + \rho_w g h_2 + \rho_w g h_3, \quad \text{cancel out } \rho_w g h_2$$

$$or: \quad p_o - p_s + \rho_w g h_1 = (\rho_m - \rho_w) g h_3$$

Introduce this into the expression for V above, for the final result:

$$V_{tunnel} = \sqrt{\frac{2(\rho_m - \rho_w) g h_3}{\rho_w}} \quad \textit{Ans. (a)}$$

This is exactly the same as a pitot-static tube—*h*1 is not important. *Ans.* (b, c)

6.140 Kerosene at 20°C flows at 18 m^3/h in a 5-cm-diameter pipe. If a 2-cm-diameter thin-plate orifice with corner taps is installed, what will the measured pressure drop be, in Pa?

Solution: For kerosene at 20°C, take ρ = 804 kg/m^3 and μ = 1.92E–3 kg/m·s. The orifice beta ratio is β = 2/5 = 0.4. The pipe velocity and Reynolds number are:

$$V = \frac{Q}{A} = \frac{18/3600}{(\pi/4)(0.05)^2} = 2.55 \frac{m}{s}, \quad Re = \frac{804(2.55)(0.05)}{1.92E\text{–}3} = 53300$$

From Eqs. (6.112) and (6.113a) [corner taps], estimate $C_d \approx 0.6030$. Then the orifice pressure-drop formula predicts

$$Q = \frac{18}{3600} = 0.6030\frac{\pi}{4}(0.02)^2\sqrt{\frac{2\Delta p}{804[1-(0.4)^4]}},\quad \text{solve } \Delta p \approx \mathbf{273\ kPa} \quad \textit{Ans.}$$

6.141 Gasoline at 20°C flows at 105 m^3/h in a 10-cm-diameter pipe. We wish to meter the flow with a thin-plate orifice and a differential pressure transducer which reads best at about 55 kPa. What is the proper β ratio for the orifice?

Solution: For gasoline at 20°C, take ρ = 680 kg/m^3 and μ = 2.92E−4 kg/m·s. This problem is similar to Example 6.21 in the text, but we don't have to be so precise because we don't know the exact geometry: corner taps, $D:\frac{1}{2}D$ taps, etc. The pipe velocity is

$$V_1 = \frac{Q}{A_1} = \frac{105/3600}{(\pi/4)(0.1)^2} = 3.71\ \frac{m}{s},\quad Re_D = \frac{680(3.71)(0.1)}{2.92E\text{–}4} \approx 865000$$

From Fig. 6.41, which is reasonable for all orifice geometries, read $C_d \approx 0.61$. Then

$$V_{throat} = \frac{3.71\ m/s}{\beta^2} = C_d\sqrt{\frac{2(55000)}{680(1-\beta^4)}},\quad \text{or}\quad \frac{\beta^2}{(1-\beta^4)^{1/2}} \approx 0.478$$

$$\text{Solve for}\quad \beta \approx \mathbf{0.66}\quad \textit{Ans.}$$

Checking back with Fig. 6.41, we see that this is about right, so no further iteration is needed for this level of accuracy.

6.142 The shower head in Fig. P6.142 delivers water at 50°C. An orifice-type flow reducer is to be installed. The up-stream pressure is constant at 400 kPa. What flow rate, in gal/min, results without the reducer? What reducer orifice diameter would decrease the flow by 40 percent?

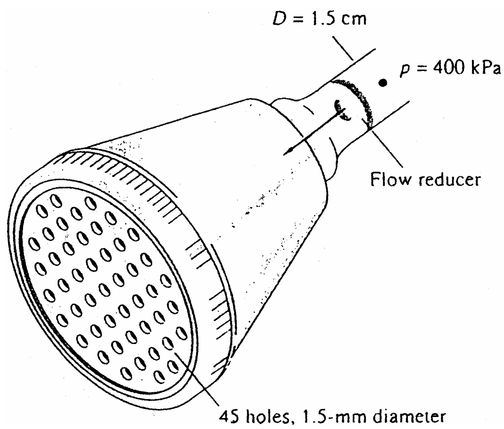


Fig. P6.142

Solution: For water at 50°C, take ρ = 988 kg/m^3 and μ = 0.548E−3 kg/m·s. Further assume that the shower head is a *poor diffuser*, so the pressure in the head is also about 400 kPa. Assume the outside pressure is sea-level standard, 101 kPa. From Fig. 6.41 for a 'typical' orifice, estimate $C_d \approx 0.61$. Then, with $\beta \approx 0$ for the small holes, each hole delivers a flow rate of

$$Q_{1\text{ hole}} \approx C_d A_{hole}\sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} \approx 0.61\left(\frac{\pi}{4}\right)(0.0015)^2\sqrt{\frac{2(400000-101000)}{988(1-0^4)}},$$

$$\text{or}\quad Q_{1\text{ hole}} \approx 2.65\text{E–}5\ \text{m}^3/\text{s} \quad \text{and} \quad Q_{total} = 45Q_{1\text{ hole}} \approx 0.00119\ \frac{\text{m}^3}{\text{s}}\left(\approx 19\ \frac{\text{gal}}{\text{min}}\right)$$

This is a *large* flow rate—a lot of expensive hot water. Checking back, the inlet pipe for this flow rate has $Re_D \approx 183000$, so $C_d \approx 0.60$ would be slightly better and a repeat of the calculation would give $\mathbf{Q_{no\ reducer}} \approx 0.00117\ \text{m}^3/\text{s} \approx$ **18.6 gal/min**. *Ans.*

A 40% reduction would give Q = 0.6(0.00117) = 7.04E–4 m^3/s ÷ 45 = 1.57E–5 m^3/s for each hole, which corresponds to a pressure drop

$$Q_{1\text{ hole}} = 1.57\text{E–}5 = 0.60\left(\frac{\pi}{4}\right)(0.0015)^2\sqrt{\frac{2\Delta p}{988}}, \quad \text{or} \quad \Delta p \approx 108000\ \text{Pa}$$

$$\text{or}\quad p_{\text{inside head}} \approx 101 + 108 \approx 209\ \text{kPa, the reducer must drop the inlet pressure to this.}$$

$$Q = 7.04\text{E–}4 \approx 0.61\left(\frac{\pi}{4}\right)(0.015\beta)^2\left[\frac{2(400000-209000)}{988(1-\beta^4)}\right]^{1/2}, \quad \text{or} \quad \frac{\beta^2}{(1-\beta^4)^{1/2}} \approx 0.332$$

$$\text{Solve for}\quad \beta \approx 0.56, \quad d_{reducer} \approx 0.56(1.5) \approx \mathbf{0.84\ cm} \quad \textit{Ans.}$$

6.143 A 10-cm-diameter smooth pipe contains an orifice plate with D: $\frac{1}{2}D$ taps and β = 0.5. The measured orifice pressure drop is 75 kPa for water flow at 20°C. Estimate the flow rate, in m^3/h. What is the nonrecoverable head loss?

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. We know everything in the orifice relation, Eq. (6.104), except C_d, which we can estimate (as 0.61):

$$Q = C_d A_t\sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} = C_d\,\frac{\pi}{4}(0.05)^2\sqrt{\frac{2(75000)}{998[1-(0.5)^4]}} = 0.0249C_d$$

$$\text{Guess}\quad C_d \approx 0.61, \quad Q \approx 0.0152\ \frac{\text{m}^3}{\text{s}}, \quad Re_D = \frac{4\rho Q}{\pi\mu D} \approx 193000, \quad C_d(\text{Eq. 6.112}) \approx 0.605$$

This is converged: $\mathbf{Q} = 0.0249(0.605) = 0.0150\ m^3/s \approx \mathbf{54\ m^3/h}$. *Ans.* (a)

(b) From Fig. 6.44, the non-recoverable head loss coefficient is K ≈ 1.8, based on V_t:

$$V_t = \frac{Q}{A_t} = \frac{0.0150}{\pi(0.025)^2} \approx 7.66\ \frac{m}{s},$$

$$\Delta p_{loss} = K\frac{\rho}{2}V_t^2 = 1.8\left(\frac{998}{2}\right)(7.66)^2 \approx \mathbf{53000\ Pa} \quad \textit{Ans. (b)}$$

6.144 Accurate solution of Prob. 6.143, using Fig. 6.41, requires iteration because both the ordinate and the abscissa of this figure contain the unknown flow rate *Q*. In the spirit of Example 5.8, rescale the variables and construct a new plot in which *Q* may be read directly from the ordinate. Solve Prob. 6.143 with your new chart.

Solution: Figure 6.41 has C_d versus Re_D, both of which contain Q:

$$C_d = \frac{Q}{A_t[2\Delta p/\rho(1-\beta^4)]^{1/2}}; \quad Re_D = \frac{4\rho Q}{\pi\mu D}, \quad \text{then } \zeta = C_d^{-1}Re_D = \frac{\beta\rho d}{\mu}\left[\frac{2\Delta p}{\rho(1-\beta^4)}\right]^{1/2}$$

The quantity ζ is independent of Q—sort of a Q-less Reynolds number. If we plot C_d versus ζ, we should solve the problem of finding an unknown Q when Δp is known. The plot is shown below. For the data of Prob. 6.143, we compute

$$\left[\frac{2\Delta p}{\rho(1-\beta^4)}\right]^{1/2} = \left[\frac{2(75000)}{998(1-.5^4)}\right]^{1/2} = 12.7\ \frac{m}{s}, \quad \zeta = \frac{0.5(998)(0.05)(12.7)}{0.001} \approx 316000$$

From the figure below, read $\mathbf{C_d \approx 0.605}$ (!) hence $Q = C_dA_t[2\Delta p/\rho(1-\beta^4)]^{1/2} = \mathbf{54\ m^3/h}$.

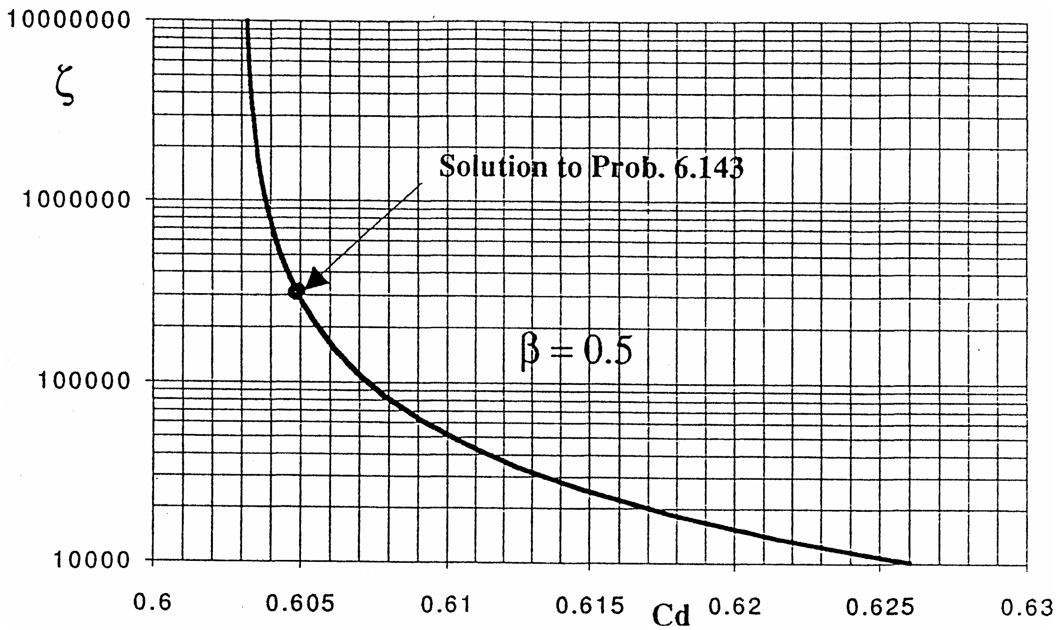


Fig. P6.144

6.145 The 1-m-diameter tank in Fig. P6.145 is initially filled with gasoline at 20°C. There is a 2-cm-diameter orifice in the bottom. If the orifice is suddenly opened, estimate the time for the fluid level *h*(t) to drop from 2.0 to 1.6 meters.

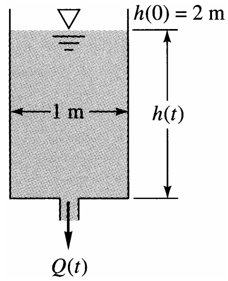


Fig. P6.145

Solution: For gasoline at 20°C, take ρ = 680 kg/m^3 and μ = 2.92E−4 kg/m·s. The orifice simulates "corner taps" with $\beta \approx 0$, so, from Eq. (6.112), $C_d \approx 0.596$. From the energy equation, the pressure drop across the orifice is $\Delta p = \rho g h(t)$, or

$$Q = C_d A_t \sqrt{\frac{2\rho g h}{\rho(1-\beta^4)}} \approx 0.596\left(\frac{\pi}{4}\right)(0.02)^2\sqrt{2(9.81)h} \approx 0.000829\sqrt{h}$$

$$\text{But also}\quad Q = -\frac{d}{dt}(\upsilon_{tank}) = -A_{tank}\frac{dh}{dt} = -\frac{\pi}{4}(1.0\text{ m})^2\frac{dh}{dt}$$

Set the *Q*'s equal, separate the variables, and integrate to find the draining time:

$$-\int_{2.0}^{1.6}\frac{dh}{\sqrt{h}} = 0.001056\int_0^{t_{final}} dt,\quad \text{or}\quad t_{final} = \frac{2[\sqrt{2}-\sqrt{1.6}]}{0.001056} = 283\text{ s} \approx \mathbf{4.7\ min}\quad \textit{Ans.}$$

6.146 A pipe connecting two reservoirs, as in Fig. P6.146, contains a thin-plate orifice. For water flow at 20°C, estimate (a) the volume flow through the pipe and (b) the pressure drop across the orifice plate.

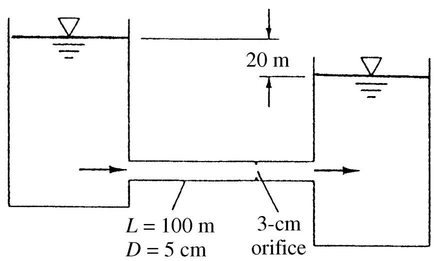


Fig. P6.146

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. The energy equation should include the orifice head loss and the entrance and exit losses:

$$\Delta z = 20\text{ m} = \frac{V^2}{2g}\left(f\frac{L}{d} + \sum K\right),\quad \text{where } K_{entr} \approx 0.5,\ K_{exit} \approx 1.0,\ K_{orifice}^{\beta=0.6} \approx 1.5 \text{ (Fig. 6.44)}$$

$$V^2 = \frac{2(9.81)(20)}{[f(100/0.05)+0.5+1.0+1.5]} = \frac{392.4}{2000f+3.0}; \quad \text{guess } f \approx 0.02, V \approx 3.02 \text{ m/s}$$

$$\text{Iterate to} \quad f_{smooth} \approx 0.0162, \quad V \approx 3.33 \text{ m/s}$$

The final $Re = \rho VD/\mu \approx 166000$, and $Q = (\pi/4)(0.05)^2(3.33) \approx$ **0.00653 m³/s** *Ans.* (a)

(b) The pressure drop across the orifice is given by the orifice formula:

$$Re_D = 166000, \quad \beta = 0.6, \quad C_d \approx 0.609 \text{ (Fig. 6.41)}:$$

$$Q = 0.00653 = C_d A_t\left[\frac{2\Delta p}{\rho(1-\beta^4)}\right]^{1/2} = 0.609\left(\frac{\pi}{4}\right)(0.03)^2\left[\frac{2\Delta p}{998(1-0.6^4)}\right]^{1/2},$$

$$\Delta p = \mathbf{100\ kPa} \quad \textit{Ans.}$$

6.147 Air flows through a 6-cm-diameter smooth pipe which has a 2 m-long perforated section containing 500 holes (diameter 1 mm), as in Fig. P6.147. Pressure outside the pipe is sea-level standard air. If p_1 = 105 kPa and Q_1 = 110 m³/h, estimate p_2 and Q_2, assuming that the holes are approximated by thin-plate orifices. *Hint:* A momentum control volume may be very useful.

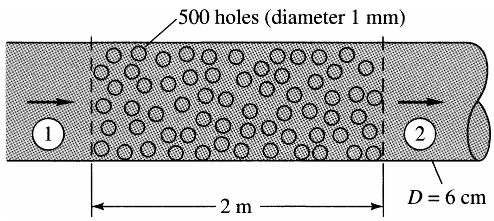


Fig. P6.147

Solution: For air at 20°C and 105 kPa, take ρ = 1.25 kg/m³ and μ = 1.8E–5 kg/m·s. Use the entrance flow rate to estimate the wall shear stress from the Moody chart:

$$V_1 = \frac{Q_1}{A} = \frac{110/3600}{(\pi/4)(0.06)^2} = 10.8\ \frac{m}{s}, \quad Re_1 = \frac{1.25(10.8)(0.06)}{1.8E{-}5} \approx 45000, \quad f_{smooth} \approx 0.0214$$

$$\text{then} \quad \tau_{wall} = \frac{f}{8}\rho V^2 = \frac{0.0214}{8}(1.25)(10.8)^2 \approx 0.390 \text{ Pa}$$

Further assume that the pressure does not change too much, so $\Delta p_{orifice} \approx 105000 - 101350 \approx$ 3650 Pa. Then the flow rate from the orifices is, approximately,

$$\beta \approx 0,\ C_d \approx 0.61: \quad Q \approx 500 C_d A_t(2\Delta p/\rho)^{1/2} = 500(0.61)\left(\frac{\pi}{4}\right)(0.001)^2\left[\frac{2(3650)}{1.25}\right]^{1/2}$$

$$\text{or:}\quad Q \approx \mathbf{0.0183}\ \text{m}^3/\text{s},\quad \text{so}\ \ Q_2 = \frac{110}{3600} - 0.0183 \approx 0.01225\ \text{m}^3/\text{s}$$

Then $V_2 = Q_2/A_2 = 0.01225/[(\pi/4)(0.06)^2] \approx 4.33$ m/s. A control volume enclosing the pipe walls and sections (1) and (2) yields the x-momentum equation:

$$\sum F_x = p_1A - p_2A - \tau_w\pi DL = \dot{m}_2V_2 - \dot{m}_1V_1 = \rho AV_2^2 - \rho AV_1^2,\quad \text{divide by A:}$$

$$p_1 - p_2 = 0.390\left[\frac{\pi(0.06)(2.0)}{(\pi/4)(0.06)^2}\right] + 1.25(4.33)^2 - 1.25(10.8)^2 = 52 + 23 - 146 \approx -71\ \text{Pa}$$

Thus $p_2 = 105000 + 71 \approx$ **105 kPa** also and above is correct: $Q_2 = \mathbf{0.0123\ m^3/s}$. *Ans.*

6.148 A smooth pipe containing ethanol at 20°C flows at 7 m^3/h through a Bernoulli obstruction, as in Fig. P6.148. Three piezometer tubes are installed, as shown. If the obstruction is a thin-plate orifice, estimate the piezometer levels (a) h_2 and (b) h_3.

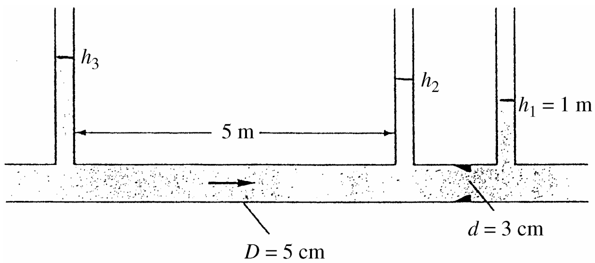


Fig. P6.148

Solution: For ethanol at 20°C, take ρ = 789 kg/m^3 and μ = 0.0012 kg/m·s. With the flow rate known, we can compute Reynolds number and friction factor, etc.:

$$V = \frac{Q}{A} = \frac{7/3600}{(\pi/4)(0.05)^2} = 0.99\ \frac{\text{m}}{\text{s}};\quad Re_D = \frac{789(0.99)(0.05)}{0.0012} = 32600,\quad f_{smooth} \approx 0.0230$$

From Fig. 6.44, at $\beta = 0.6$, K ≈ 1.5. Then the head loss across the orifice is

$$\Delta h = h_2 - h_1 = K\frac{V_t^2}{2g} = (1.5)\left[\frac{\{0.99/(0.6)^2\}^2}{2(9.81)}\right] \approx 0.58\ \text{m},\quad \text{hence}\ \ \mathbf{h_2 \approx 1.58\ m}\quad \textit{Ans.}\ \text{(a)}$$

Then the piezometer change between (2) and (3) is due to Moody friction loss:

$$h_3 - h_2 = h_f = f\frac{L}{d}\frac{V^2}{2g} = (0.023)\left(\frac{5}{0.05}\right)\frac{(0.99)^2}{2(9.81)} = 0.12\ \text{m},$$

$$\text{or} \quad h_3 = 1.58 + 0.12 \approx \mathbf{1.7\ m} \quad \textit{Ans.}\ \text{(b)}$$

6.149 In a laboratory experiment, air at 20°C flows from a large tank through a 2-cm-diameter smooth pipe into a sea-level atmosphere, as in Fig. P6.149. The flow is metered by a long-radius nozzle of 1-cm diameter, using a manometer with Meriam red oil (SG = 0.827). The pipe is 8 m long. The measurements of tank pressure and manometer height are as follows:

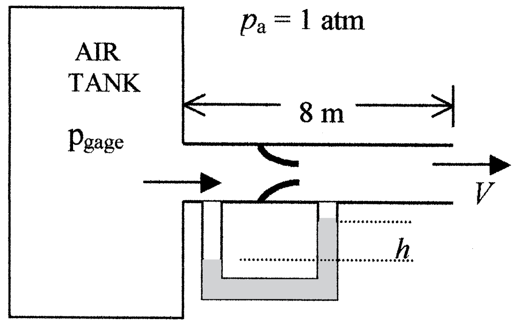


Fig. P6.149

p_{tank}, (gage):	Pa	60	320	1200	2050	2470	3500	4900
h_{mano}, mm:		6	38	160	295	380	575	820

Use this data to calculate the flow rates Q and Reynolds numbers Re_D and make a plot of measured flow rate versus tank pressure. Is the flow laminar or turbulent? Compare the data with theoretical results obtained from the Moody chart, including minor losses. Discuss.

Solution: For air take ρ=1.2 kg/m^3 and μ = 0.000015 kg/m·s. With no elevation change and negligible tank velocity, the energy equation would yield

$$p_{tank} - p_{atm} = \frac{\rho V^2}{2}\left(1 + f\frac{L}{D} + K_{entrance} + K_{nozzle}\right), \quad K_{ent} \approx 0.5 \text{ and } K_{noz} \approx 0.7$$

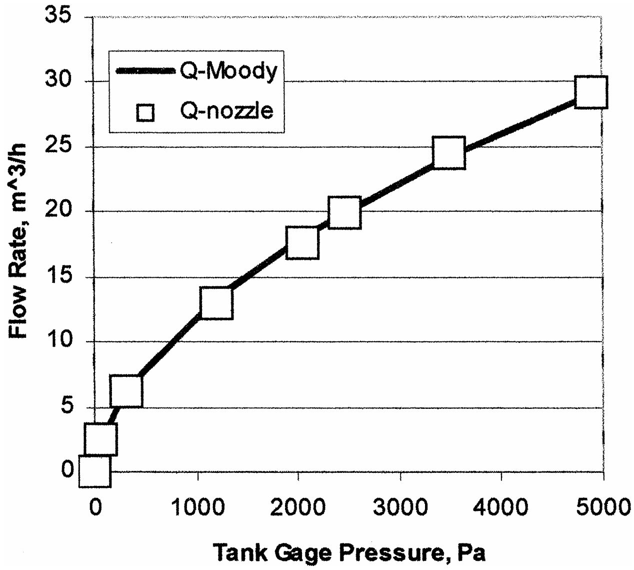
Since Δp is given, we can use this expression plus the Moody chart to predict V and Q = AV and compare with the flow-nozzle measurements. The flow nozzle formula is:

$$V_{throat} = C_d\sqrt{\frac{2\Delta p_{mano}}{\rho(1-\beta^4)}} \quad \text{where } \Delta p = (\rho_{oil} - \rho_{air})gh,\ C_d \text{ from Fig. 6.42 and } \beta = 0.5$$

The friction factor is given by the smooth-pipe Moody formula, Eq. (6.48) for $\varepsilon = 0$. The results may be tabulated as follows, and the plot on the next page shows excellent (too good?) agreement with theory.

p_{tank}, Pa:	60	320	1200	2050	2470	3500	4900
V, m/s (nozzle data):	2.32	5.82	11.9	16.1	18.2	22.3	26.4
Q, m^3/h (nozzle data):	2.39	6.22	12.9	17.6	19.9	24.5	29.1
Q, m^3/h (theory):	2.31	6.25	13.3	18.0	20.0	24.2	28.9

f_{Moody}:	0.0444	0.0331	0.0271	0.0252	0.0245	0.0234	0.0225



6.150 Gasoline at 20°C flows at 0.06 m^3/s through a 15-cm pipe and is metered by a 9-cm-diameter long-radius flow nozzle (Fig. 6.40a). What is the expected pressure drop across the nozzle?

Solution: For gasoline at 20°C, take $\rho = 680$ kg/m and $\mu = 2.92\text{E}{-4}$ kg/m·s. Calculate the pipe velocity and Reynolds number:

$$V = \frac{Q}{A} = \frac{0.06}{(\pi/4)(0.15)^2} = 3.40\ \frac{m}{s},\quad Re_D = \frac{680(3.40)(0.15)}{2.92\text{E}{-4}} \approx 1.19\text{E}6$$

The ISO correlation for discharge (Eq. 6.114) is used to estimate the pressure drop:

$$C_d \approx 0.9965 - 0.00653\left(\frac{10^6\beta}{Re_D}\right)^{1/2} = 0.9965 - 0.00653\left[\frac{10^6(0.6)}{1.19\text{E}6}\right]^{1/2} \approx \mathit{0.9919}$$

$$\text{Then}\quad Q = 0.06 = (0.9919)\left(\frac{\pi}{4}\right)(0.09)^2\sqrt{\frac{2\Delta p}{680(1-0.6^4)}},$$

Solve **$\Delta p \approx 27000$ Pa** *Ans.*

P6.151 An engineer needs to monitor a flow of 20°C gasoline at about 250±25 gal/min through a 4-in-diameter smooth pipe. She can use an orifice plate, a long-radius flow nozzle, or a venturi nozzle, all with 2-in-diameter throats. The only differential pressure gage available is accurate in the range 6 to 10 lbf/in^2. Disregarding flow losses, which device is best?

Solution: For gasoline at 20°C, take $\rho = 680\ kg/m^3$ and $\mu = 2.92E\text{-}4$ kg/m-s. We are given

$\beta = 2/4 = 0.5$. The flow rate is in the range $0.0142 < Q < 0.0174\ m^3/s$. The pipe Reynolds number is in the range $Re_D = 460{,}000 \pm 10\%$. In SI units, the throat diameter is 0.0508 m, and its area is $(\pi/4)(0.0508m)^2 = 0.00203\ m^2$. Our basic "obstruction" formula is Eq. (6.104):

$$Q = C_d A_t \sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} = C_d(0.00203m^2)\sqrt{\frac{2\Delta p}{(680kg/m^3)\{1-(0.5)^4\}}} = 0.0158 \pm 0.0016\frac{m^3}{s}$$

It remains only to determine C_d for the three devices and then calculate Δp. The results are:

Orifice plate, D:1/2D taps: $C_d \approx 0.605$, Δp = **6.2 to 9.3** lbf/in^2 *Ans.*

Long-radius flow nozzle: $C_d \approx 0.99$, Δp = 2.3 to 3.5 lbf/in^2

Venturi nozzle: $C_d \approx 0.977$, Δp = 2.4 to 3.6 lbf/in^2

Only the orifice plate, with its high losses, is compatible with the available pressure gage.

6.152 Kerosene at 20°C flows at 20 m^3/h in an 8-cm-diameter pipe. The flow is to be metered by an ISA 1932 flow nozzle so that the pressure drop is 7 kPa. What is the proper nozzle diameter?

Solution: For kerosene at 20°C, take ρ = 804 kg/m^3 and μ =1.92E−3 kg/m·s. We cannot calculate the discharge coefficient exactly because we don't know β, so just estimate C_d:

$$\text{Guess}\quad C_d \approx 0.99, \quad \text{then } Q \approx 0.99\left(\frac{\pi}{4}\right)(0.08\,\beta)^2\sqrt{\frac{2(7000)}{804(1-\beta^4)}} = \frac{20}{3600}\,\frac{m^3}{s}$$

$$\text{or:}\quad \frac{\beta^2}{(1-\beta^4)^{1/2}} \approx 0.268, \quad \text{solve } \beta \approx 0.508,$$

$$Re_D = \frac{4(804)(20/3600)}{\pi(1.92E\text{–}3)(0.08)} \approx 37000$$

Now compute a better C_d from the ISA nozzle correlation, Eq. (6.115):

$$C_d \approx 0.99 - 0.2262\beta^{4.1} + (0.000215 - 0.001125\beta + 0.00249\beta^{4.7})\left(\frac{10^6}{Re_D}\right)^{1.15} \approx 0.9647$$

Iterate once to obtain a better $\beta \approx 0.515$, d = 0.515(8 cm) ≈ **4.12 cm** *Ans.*

6.153 Two water tanks, each with base area of 1 ft^2, are connected by a 0.5-in-diameter long-radius nozzle as in Fig. P6.153. If h = 1 ft as shown for *t* = 0, estimate the time for h(t) to drop to 0.25 ft.

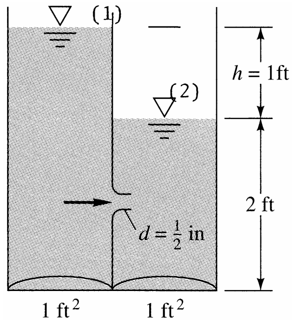


Fig. P6.153

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E–5 slug/ft·s. For a long-radius nozzle with $\beta \approx 0$, guess $C_d \approx$ 0.98 and $K_{loss} \approx$ 0.9 from Fig. 6.44. The elevation difference *h* must balance the head losses in the nozzle and submerged exit:

$$\Delta z = \sum h_{loss} = \frac{V_t^2}{2g}\sum K = \frac{V_t^2}{2(32.2)}(0.9_{nozzle} + 1.0_{exit}) = h, \quad \text{solve } V_t = 5.82\sqrt{h}$$

$$\text{hence} \quad Q = V_t\left(\frac{\pi}{4}\right)\left(\frac{1/2}{12}\right)^2 \approx 0.00794\sqrt{h} = -\frac{\mathbf{1}}{\mathbf{2}}A_{tank}\frac{dh}{dt} = -0.5\frac{dh}{dt}$$

The boldface factor **1/2** accounts for the fact that, as the left tank falls by dh, the right tank rises by the same amount, hence dh/dt changes twice as fast as for one tank alone. We can separate and integrate and find the time for h to drop from 1 ft to 0.25 ft:

$$\int_{0.25}^{1.0}\frac{dh}{\sqrt{h}} = 0.0159\int_0^{t_{final}} dt, \quad \text{or:} \quad t_{final} = \frac{2\left(\sqrt{1}-\sqrt{0.25}\right)}{0.0159} \approx \mathbf{63\ s} \quad \textit{Ans.}$$

6.154 Water at 20°C flows through the orifice in the figure, which is monitored by a mercury manometer. If d = 3 cm, (a) what is *h* when the flow is 20 m^3/h; and (b) what is *Q* when *h* = 58 cm?

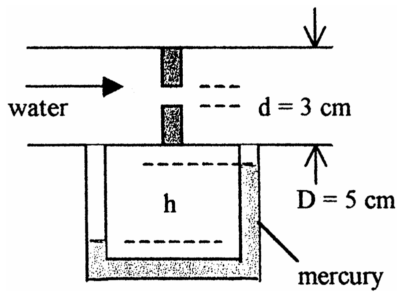


Fig. P6.154

Solution: (a) Evaluate V = Q/A = 2.83 m/s and $Re_D = \rho VD/\mu$ = 141,000, β = 0.6, thus $C_d \approx$ 0.613.

$$Q = \frac{20}{3600} = C_d\frac{\pi}{4}d^2\sqrt{\frac{2\Delta p}{\rho(1-\beta^4)}} = (0.613)\frac{\pi}{4}(0.03)^2\sqrt{\frac{2(13550-998)(9.81)h}{998(1-0.6^4)}}$$

where we have introduced the manometer formula $\Delta p = (\rho_{mercury} - \rho_{water})gh$.

Solve for: $\boldsymbol{h} \approx 0.58$ m = **58 cm** *Ans.* (a)

Solve this problem when h = 58 cm is known and Q is the unknown. Well, we can see that the numbers are the same as part (a), and the solution is

$$\text{Solve for:}\quad \boldsymbol{Q} \approx 0.00556\ \text{m}^3/\text{s} = \mathbf{20\ m^3/h}\quad \textit{Ans.}\ \text{(b)}$$

6.155 It is desired to meter a flow of 20°C gasoline in a 12-cm-diameter pipe, using a modern venturi nozzle. In order for international standards to be valid (Fig. 6.40), what is the permissible range of (a) flow rates, (b) nozzle diameters, and (c) pressure drops? (d) For the highest pressure-drop condition, would compressibility be a problem?

Solution: For gasoline at 20°C, take $\rho = 680\ \text{kg/m}^3$ and $\mu = 2.92\text{E}{-4}$ kg/m·s. Examine the possible range of Reynolds number and beta ratio:

$$1.5\text{E}5 < \text{Re}_D = \frac{4\rho Q}{\pi\mu D} = \frac{4(680)Q}{\pi(2.92\text{E}{-4})(0.12)} < 2.0\text{E}5,$$

$$\text{or}\quad \mathbf{0.0061 < Q < 0.0081\ \frac{m^3}{s}}\quad \textit{Ans.}\ \text{(a)}$$

$$0.316 < \beta = d/D < 0.775,\quad \text{or:}\quad \mathbf{3.8 < d < 9.3\ cm}\quad \textit{Ans.}\ \text{(b)}$$

For estimating pressure drop, first compute $C_d(\beta)$ from Eq. (6.116): $0.924 < C_d < 0.985$:

$$Q = C_d\frac{\pi}{4}(0.12\,\beta)^2\sqrt{\frac{2\Delta p}{680(1-\beta^4)}},\quad \text{or:}\quad \Delta p = 2.66\text{E}6(1-\beta^4)\left[\frac{Q}{C_d\beta^2}\right]^2$$

put in large Q, small β, etc. to obtain the range **200 < Δp < 18000 Pa** *Ans.* (c)

6.156 Ethanol at 20°C flows down through a modern venturi nozzle as in Fig. P6.156. If the mercury manometer reading is 4 in, as shown, estimate the flow rate, in gal/min.

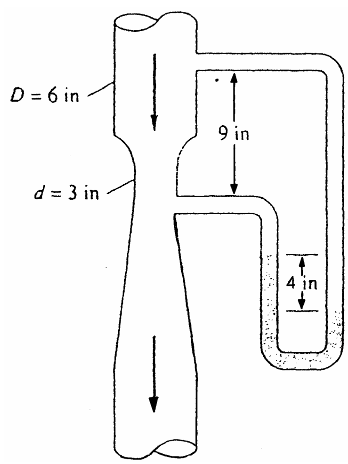


Fig. P6.156

Solution: For ethanol at 20°C, take ρ = 1.53 slug/ft^3 and μ = 2.51E–5 slug/ft·s. Given $\beta = 0.5$, the discharge coefficient is

$$C_d = 0.9858 - 0.196(0.5)^{4.5} \approx 0.9771$$

The 9-inch displacement of manometer taps does not affect the pressure drop reading, because both legs are filled with ethanol. Therefore we proceed directly to Δp and Q:

$$\Delta p_{nozzle} = (\rho_{merc} - \rho_{eth})gh = (26.3 - 1.53)(32.2)(4/12) \approx 266 \text{ lbf/ft}^2$$

$$\text{Hence} \quad Q = C_d A_t\left[\frac{2\Delta p}{\rho(1-\beta^4)}\right]^{1/2} = 0.9771\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\right)^2\sqrt{\frac{2(266)}{1.53(1-0.5^4)}} \approx 0.924\ \frac{\text{ft}^3}{\text{s}} \quad \textit{Ans.}$$

6.157 Modify Prob. 6.156 if the fluid is *air* at 20°C, entering the venturi at a pressure of 18 psia. Should a compressibility correction be used?

Solution: For air at 20°C and 18 psi, take $\rho = 0.00286$ slug/ft^3 and $\mu = 3.76E{-}7$ slug/ft·s. With β still equal to 0.5, C_d still equals 0.9771 as previous page. The manometer reading is

$$\Delta p_{nozzle} = (26.3 - 0.00286)(32.2)(4/12) \approx 282 \text{ lbf/ft}^2,$$

$$\text{whence} \quad Q = 0.9771\left(\frac{\pi}{4}\right)\left(\frac{3}{12}\right)^2\sqrt{\frac{2(282)}{0.00286(1-0.5^4)}} \approx \mathbf{22.0}\ \frac{\mathbf{ft^3}}{\mathbf{s}} \quad \textit{Ans.}$$

From this result, the throat velocity $V_t = Q/A_t \approx 448$ ft/s, quite high, the Mach number in the throat is approximately Ma = 0.4, a **(slight) compressibility correction might be expected.** [Making a one-dimensional subsonic-flow correction, using the methods of Chap. 9, results in a throat volume flow estimate of Q ≈ 22.8 ft^3/s, about 4% higher.]

6.158 Water at 20°C flows in a long horizontal commercial-steel 6-cm-diameter pipe which contains a classical Herschel venturi with a 4-cm throat. The venturi is connected to a mercury manometer whose reading is h = 40 cm. Estimate (a) the flow rate, in m^3/h, and (b) the total pressure difference between points 50 cm upstream and 50 cm downstream of the venturi.

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$kg/m·s. For commercial steel, $\varepsilon \approx 0.046$ mm, hence $\varepsilon/d = 0.046/60 = 0.000767$. First *estimate* the flow rate:

$$\Delta p = (\rho_m - \rho_w)gh = (13560 - 998)(9.81)(0.40) \approx 49293 \text{ Pa}$$

$$\text{Guess} \quad C_d \approx 0.985, \quad Q = (0.985)\left(\frac{\pi}{4}\right)(0.04)^2\sqrt{\frac{2(49293)}{998[1-(4/6)^4]}} \approx 0.0137\ \frac{\text{m}^3}{\text{s}}$$

$$\text{Check} \quad Re_D = \frac{4\rho Q}{\pi\mu D} \approx 291000$$

At this Reynolds number, we see from Fig. 6.42 that C_d does indeed ≈ 0.985 for the Herschel venturi. Therefore, indeed, $Q = 0.0137\ m^3/s \approx$ **49 m^3/h**. *Ans.* (a)

(b) 50 cm upstream and 50 cm downstream are far enough that the pressure recovers from its throat value, and the total Δp is the sum of Moody pipe loss and venturi head loss. First work out the pipe velocity, $V = Q/A = (0.0137)/[(\pi/4)(0.06)^2] \approx 4.85$ m/s. Then

$$Re_D = 291000,\quad \frac{\varepsilon}{d} = 0.000767,\quad \text{then } f_{Moody} \approx 0.0196;\quad \text{Fig. 6.44: } K_{venturi} \approx 0.2$$

$$\text{Then}\quad \Delta p = \Delta p_{Moody} + \Delta p_{venturi} = \frac{\rho V^2}{2}\left(f\frac{L}{d} + K\right)$$

$$= \frac{998(4.85)^2}{2}\left[0.0196\left(\frac{1.0}{0.06}\right) + 0.2\right] \approx \mathbf{6200\ Pa}\quad \textit{Ans. (b)}$$

6.159 A modern venturi nozzle is tested in a laboratory flow with water at 20°C. The pipe diameter is 5.5 cm, and the venturi throat diameter is 3.5 cm. The flow rate is measured by a weigh tank and the pressure drop by a water-mercury manometer. The mass flow rate and manometer readings are as follows:

$\dot{m}$, kg/s:	0.95	1.98	2.99	5.06	8.15
h, mm:	3.7	15.9	36.2	102.4	264.4

Use these data to plot a calibration curve of venturi discharge coefficient versus Reynolds number. Compare with the accepted correlation, Eq. (6.116).

Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. The given data of mass flow and manometer height can readily be converted to discharge coefficient and Reynolds number:

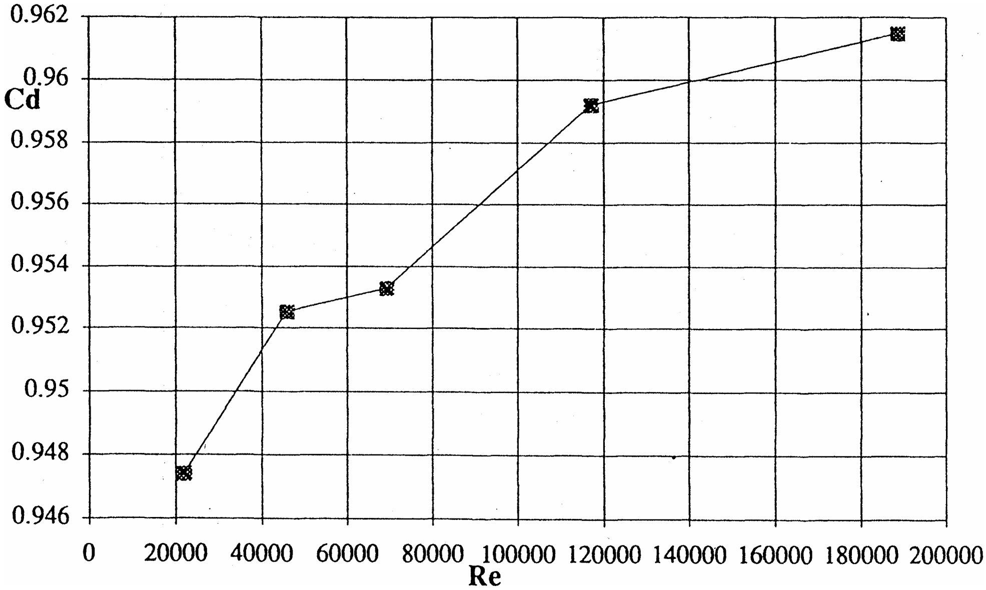
$$Q = \frac{\dot{m}}{998} = C_d\left(\frac{\pi}{4}\right)(0.035)^2\sqrt{\frac{2(13.56-1)\rho_w(9.81)h}{\rho_w[1-(3.5/5.5)^4]}},\quad \text{or:}\quad C_d \approx \frac{\dot{m}\ (kg/s)}{16.485\sqrt{h_{meters}}}$$

$$Re_D = \frac{4\dot{m}}{\pi\mu D} = \frac{4\dot{m}}{\pi(0.001)(0.055)} \approx 23150\ \dot{m}\ (kg/s)$$

The data can then be converted and tabulated as follows:

h, m:	0.037	0.0159	0.0362	0.1024	0.2644
C_d:	0.947	0.953	0.953	0.959	0.962
Re_D:	22000	46000	69000	117000	189000

These data are plotted in the graph below, similar to Fig. 6.42 of the text:



They closely resemble the "classical Herschel venturi," but this data is actually for a *modern* venturi, for which we only know the value of C_d for $1.5E5 < Re_D \le 2E5$:

$$\text{Eq. (6.116)} \quad C_d \approx 0.9858 - 0.196\left(\frac{3.5}{5.5}\right)^{4.5} \approx \mathbf{0.960}$$

The two data points near this Reynolds number range are quite close to 0.960 ± 0.002.

6.160 The butterfly-valve losses in Fig. 6.19*b* may be viewed as a type of Bernoulli obstruction device, as in Fig. 6.39. The "throat area" A_t in Eq. (6.104) can be interpreted as the two slivers of opening around the butterfly disk when viewed from upstream. First fit the average loss K_{mean} versus the opening angle in Fig. 6.19*b* to an exponential curve. Then use your curve fit to compute the "discharge coefficient" of a butterfly valve as a function of the opening angle. Plot the results and compare them to those for a typical flow-meter.

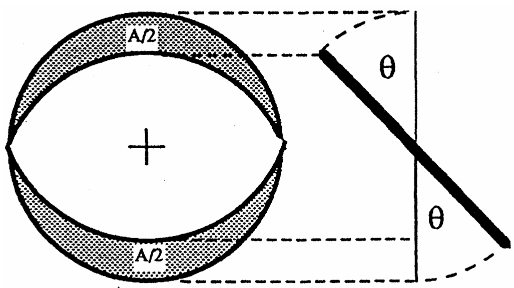


Fig. P6.160

Solution: The two "slivers" referred to are shown. The total sliver area equals *total* area reduced by a cosine factor:

$$A_{sliver} = A_{total}(1-\cos\theta), \quad \text{where } A_{total} = \pi R^2, \ R = \text{valve and pipe radius}$$

The "effective" velocity passing through the slivers may be computed from continuity:

$$V_{eff} = \frac{Q}{A_{slivers}} = \frac{Q}{A_{total}}\frac{A_{total}}{A_{slivers}} = \frac{V_{pipe}}{A_s/A_t} = \frac{V_{pipe}}{(1-\cos\theta)}$$

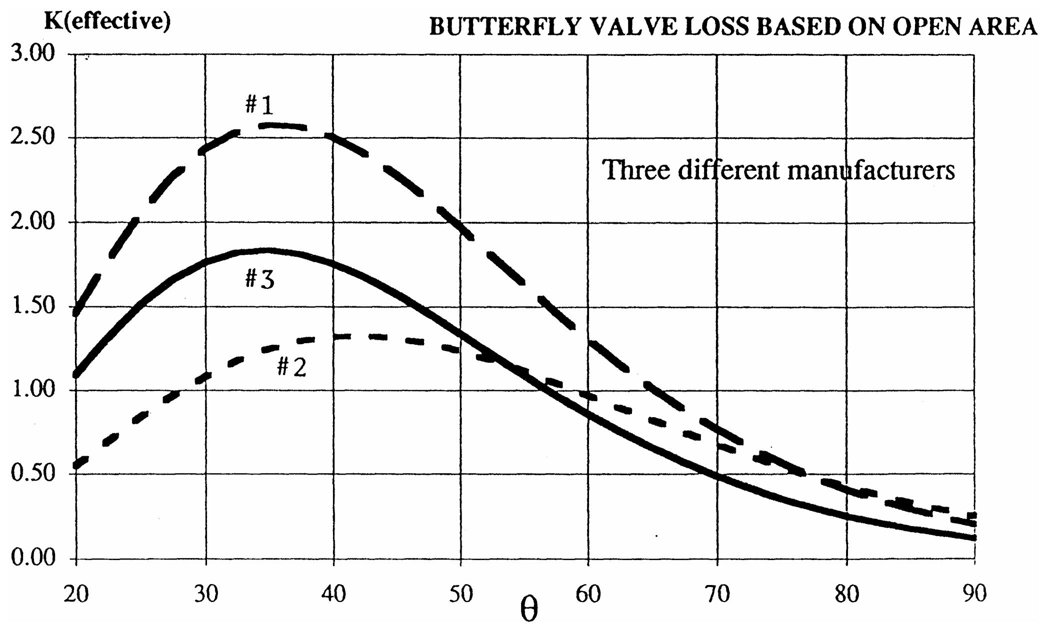
Then the problem suggests that the loss coefficients might correlate better (and not vary so much or be so large as in Fig. 6.19b) if the loss is based on effective velocity:

$$K_{better} = \frac{h_{loss}}{V_{eff}^2/2g} = \frac{h_{loss}}{V_{pipe}^2/2g}\left(\frac{V_{pipe}}{V_{eff}}\right)^2 = K_{Fig.6.19}(1-\cos\theta)^2$$

So we take the data for traditional "K" in Fig. 6.19b, multiply it by $(1-\cos\theta)^2$, and replot it below. Actually, we have taken three exponential curve-fits, one for each manufacturer's data shown in Fig. 6.19, to give an idea of the data uncertainty:

$$\#1: K_1 \approx 3500\ e^{-0.109\theta}; \quad 2: K_2 \approx 930\ e^{-0.091\theta}; \quad \#3: K_3 \approx 2800\ e^{-0.112\theta}, \quad \theta \text{ in degrees}$$

The calculations are made and are shown plotted below. This idea works fairly well, but the K's still vary a bit over the range of θ. However, all K's are now of order unity, which is a better correlation than the huge variations shown in Fig. 6.19b.



6.161 Air flows at high speed through a Herschel venturi monitored by a mercury manometer, as shown in Fig. P6.161. The upstream conditions are 150 kPa and 80°C. If h = 37 cm, estimate the mass flow in kg/s. [HINT: The flow is compressible.]

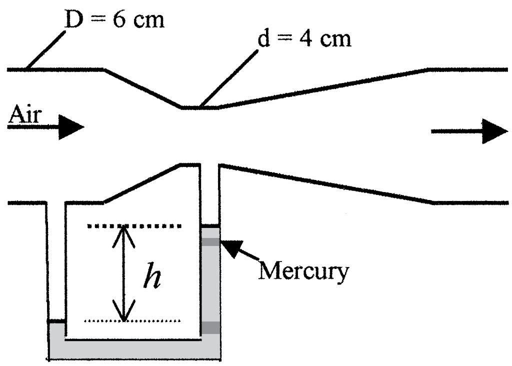


Fig. P6.161

Solution: The upstream density is $\rho_1 = p_1/(RT) = (150000)/[287(273 + 80)] = 1.48$ kg/m^3. The clue "high speed" means that we had better use the *compressible* venturi formula, Eq. (6.117):

$$\dot{m} = C_d Y A_t \sqrt{\frac{2\rho_1(p_1 - p_2)}{1-\beta^4}} \quad \text{where } \beta = 4/6 \text{ for this nozzle.}$$

The pressure difference is measured by the mercury manometer:

$$p_1 - p_2 = (\rho_{merc} - \rho_{air})gh = (13550 - 1.48\ kg/m^3)(9.81\ m/s^2)(0.37\ m) = 49200\ Pa$$

The pressure ratio is thus (150 − 49.2)/150 = 0.67 and, for $\beta = 2/3$, we read $Y \approx 0.76$ from Fig. 6.45. From Fig. 6.43 estimate $C_d \approx 0.985$. The (compressible) venturi formula thus predicts:

$$\dot{m} = 0.985(0.76)\left[\frac{\pi}{4}(0.04\ m)^2\right]\sqrt{\frac{2(1.48)(49200)}{1-(2/3)^4}} = \mathbf{0.40\ \frac{kg}{s}} \quad Ans.$$

6.162 Modify Prob. 6.161 as follows. Find the manometer reading h for which the mass flow through the venturi is approximately 0.4 kg/s. [HINT: The flow is compressible.]

Solution: This is, in fact, the answer to Prob. 6.161, but who knew? The present problem is intended as an iteration exercise, preferably with EES. We know the upstream pressure and density and the discharge coefficient, but we must iterate for Y and p_2 in the basic formula:

$$\dot{m} = C_d Y A_t \sqrt{\frac{2\rho_1(p_1 - p_2)}{1-\beta^4}} = 0.40\ kg/s$$

The answer should be h = 0.37 m, as in Prob. 6.161, but the problem is extremely sensitive to the value of h. A 10% change in h causes only a 2% change in mass flow.

The actual answer to Prob. 6.161 was a mass flow of 0.402 kg/s. EES reports that, for mass flow *exactly* equal to 0.400 kg/s, the required manometer height is ***h* = 0.361 m**. *Ans.*

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

FE 6.1 In flow through a straight, smooth pipe, the diameter Reynolds number for transition to turbulence is generally taken to be

(a) 1500 **(b) 2300** (c) 4000 (d) 250,000 (e) 500,000

FE 6.2 For flow of water at 20°C through a straight, smooth pipe at 0.06 m^3/h, the pipe diameter for which transition to turbulence occurs is approximately

(a) 1.0 cm (b) 1.5 cm (c) 2.0 cm (d) 2.5 cm (e) 3.0 cm

FE 6.3 For flow of oil (μ = 0.1 kg/(m·s), SG = 0.9) through a long, straight, smooth 5-cm-diameter pipe at 14 m^3/h, the pressure drop per meter is approximately

(a) 2200 Pa **(b) 2500 Pa** (c) 10,000 Pa (d) 160 Pa (e) 2800 Pa

FE 6.4 For flow of water at a Reynolds number of 1.03E6 through a 5-cm-diameter pipe of roughness height 0.5 mm, the approximate Moody friction factor is

(a) 0.012 **(b)** 0.018 **(c) 0.038** (d) 0.049 (e) 0.102

FE 6.5 Minor losses through valves, fittings, bends, contractions etc. are commonly modeled as proportional to

(a) total head (b) static head **(c) velocity head** (d) pressure drop (e) velocity

FE 6.6 A smooth 8-cm-diameter pipe, 200 m long, connects two reservoirs, containing water at 20°C, one of which has a surface elevation of 700 m and the other with its surface elevation at 560 m. If minor losses are neglected, the expected flow rate through the pipe is

(a) 0.048 m^3/h (b) 2.87 m^3/h (c) 134 m^3/h **(d) 172 m^3/h** (e) 385 m^3/h

FE 6.7 If, in Prob. FE 6.6 the pipe is rough and the actual flow rate is 90 m^3/hr, then the expected average roughness height of the pipe is approximately

(a) 1.0 mm **(b) 1.25 mm** (c) 1.5 mm (d) 1.75 mm (e) 2.0 mm

FE 6.8 Suppose in Prob. FE 6.6 the two reservoirs are connected, not by a pipe, but by a sharp-edged orifice of diameter 8 cm. Then the expected flow rate is approximately

(a) 90 m^3/h **(b) 579 m^3/h** (c) 748 m^3/h (d) 949 m^3/h (e) 1048 m^3/h

FE 6.9 Oil (μ = 0.1 kg/(m·s), SG = 0.9) flows through a 50-m-long smooth 8-cm-diameter pipe. The maximum pressure drop for which laminar flow is expected is approximately

(a) 30 kPa (b) 40 kPa (c) 50 kPa (d) 60 kPa **(e) 70 kPa**

FE 6.10 Air at 20°C and approximately 1 atm flows through a smooth 30-cm-square duct at 1500 cubic feet per minute. The expected pressure drop per meter of duct length is

(a) 1.0 Pa **(b) 2.0 Pa** (c) 3.0 Pa (d) 4.0 Pa (e) 5.0 Pa

FE 6.11 Water at 20°C flows at 3 cubic meters per hour through a sharp-edged 3-cm-diameter orifice in a 6-cm-diameter pipe. Estimate the expected pressure drop across the orifice.

(a) 440 Pa (b) 680 Pa (c) 875 Pa **(d) 1750 Pa** (e) 1870 Pa

FE 6.12 Water flows through a straight 10-cm-diameter pipe at a diameter Reynolds number of 250,000. If the pipe roughness is 0.06 mm, what is the approximate Moody friction factor?

(a) 0.015 (b) 0.017 **(c) 0.019** (d) 0.026 (e) 0.032

FE 6.13 What is the hydraulic diameter of a rectangular air-ventilation duct whose cross-section is 1 meter by 25 cm?

(a) 25 cm **(b) 40 cm** (c) 50 cm (d) 75 cm (e) 100 cm

FE 6.14 Water at 20°C flows through a pipe at 300 gal/min with a friction head loss of 45 ft. What is the power required to drive this flow?

(a) 0.16 kW (b) 1.88 kW **(c) 2.54 kW** (d) 3.41 kW (e) 4.24 kW

FE 6.15 Water at 20°C flows at 200 gal/min through a pipe 150 m long and 8 cm in diameter. If the friction head loss is 12 m, what is the Moody friction factor?

(a) 0.010 (b) 0.015 **(c) 0.020** (d) 0.025 (e) 0.030

COMPREHENSIVE PROBLEMS

C6.1 A pitot-static probe will be used to measure the velocity distribution in a water tunnel at 20°C. The two pressure lines from the probe will be connected to a U-tube manometer which uses a liquid of specific gravity 1.7. The maximum velocity expected in the water tunnel is 2.3 m/s. Your job is to select an appropriate U-tube from a manufacturer which supplies manometers of heights 8, 12, 16, 24 and 36 inches. The cost increases significantly with manometer height. Which of these should you purchase?

Solution: The pitot-static tube formula relates velocity to the difference between stagnation pressure p_o and static pressure p_s in the water flow:

$$p_o - p_s = \frac{1}{2}\rho_w V^2, \quad \text{where } \rho_w = 998\ \frac{\text{kg}}{\text{m}^3} \quad \text{and} \quad V_{max} = 2.3\ \frac{\text{m}}{\text{s}}$$

Meanwhile, the manometer reading *h* relates this pressure difference to the two fluids:

$$p_o - p_s = (\rho_{mano} - \rho_w)gh = \rho_w(SG_{mano} - 1)gh$$

$$\text{Solve for} \quad h_{max} = \frac{V_{max}^2}{2g(SG_{mano}-1)} = \frac{(2.3)^2}{2(9.81)(1.7-1)} = 0.385\ m = \mathbf{15.2\ in}$$

It would therefore be most economical to ***buy the 16-inch manometer***. But be careful when you use it: a bit of overpressure will pop the manometer fluid out of the tube!

C6.2 A pump delivers a steady flow of water (ρ,μ) from a large tank to two other higher-elevation tanks, as shown. The same pipe of diameter *d* and roughness ε is used throughout. All minor losses *except through the valve* are neglected, and the partially-closed valve has a loss coefficient K_{valve}. Turbulent flow may be assumed with all kinetic energy flux correction coefficients equal to 1.06. The pump net head *H* is a known function of Q_A and hence also of $V_A = Q_A/A_{pipe}$, for example, $H = a - bV_A^2$, where *a* and *b* are constants. Subscript J refers to the junction point at the tee where branch A splits into B and C. Pipe length L_C is much longer than L_B. It is desired to predict the pressure at J, the three pipe velocities and friction factors, and the pump head. Thus there are 8 variables: H, V_A, V_B, V_C, f_A, f_B, f_C, p_J. Write down the eight equations needed to resolve this problem, but *do not solve*, since an elaborate iteration procedure, or an equation solver such as EES, would be required.

Solution: First, equation (1) is clearly the pump performance:

$$H = a - bV_A^2 \qquad (1)$$

$$3\ \textit{Moody factors:}\quad f_A = fcn\left(V_A, \frac{\varepsilon}{d}\right) \tag{2}$$

$$f_B = fcn\left(V_B, \frac{\varepsilon}{d}\right) \tag{3}$$

$$f_C = fcn\left(V_C, \frac{\varepsilon}{d}\right) \tag{4}$$

Conservation of mass (constant area) at the junction J: $V_A = V_B + V_C$ (5)

Finally, there are three independent steady-flow energy equations:

$$(1)\ to\ (2):\quad z_1 = z_2 - H + f_A \frac{L_A}{d}\frac{V_A^2}{2g} + f_B \frac{L_B}{d}\frac{V_B^2}{2g} \tag{6}$$

$$(1)\ to\ (3):\quad z_1 = z_3 - H + f_A \frac{L_A}{d}\frac{V_A^2}{2g} + f_C \frac{L_C}{d}\frac{V_C^2}{2g} + K_{valve}\frac{V_C^2}{2g} \tag{7}$$

$$(J)\ to\ (2):\quad \frac{p_J}{\rho g} + z_J = \frac{p_{atm}}{\rho g} + z_2 + f_B \frac{L_B}{d}\frac{V_B^2}{2g} \tag{8}$$

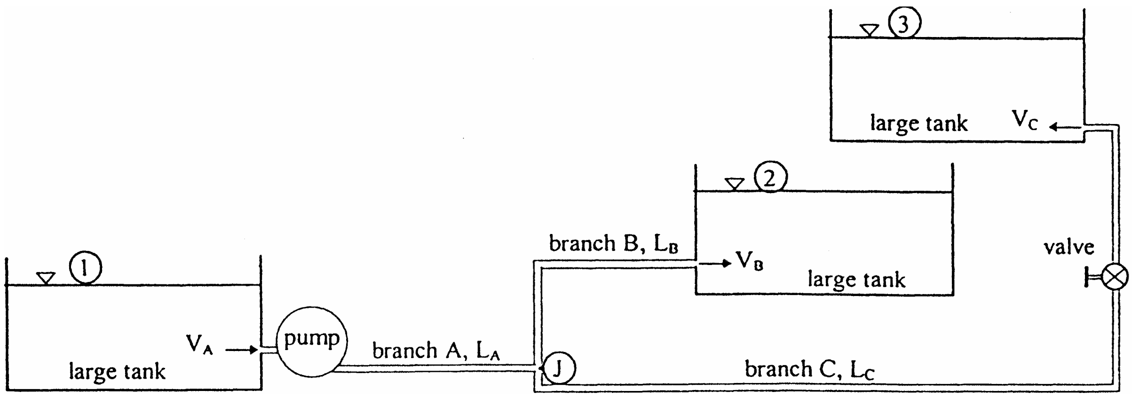


Fig. PC6.2

C6.3 The water slide in the figure is to be installed in a swimming pool. The manufacturer recommends a continuous water flow of 1.39E–3 m^3/s (about 22 gal/min) down the slide to ensure that customers do not burn their bottoms. An 80%-efficient pump under the slide, submerged 1 m below the water surface, feeds a 5-m-long, 4-cm-diameter hose, of roughness 0.008 cm, to the slide. The hose discharges the water at the top of the slide, 4 m above the water surface, as a free jet. Ignore minor losses and assume $\alpha = 1.06$. Find the brake horsepower needed to drive the pump.

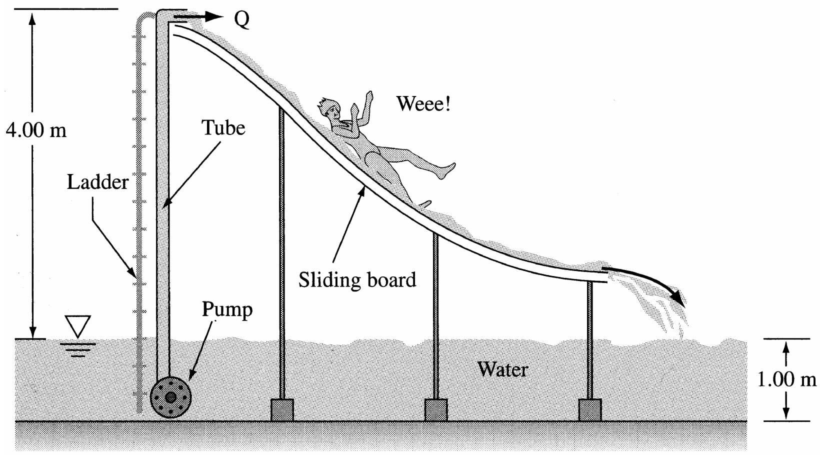


Fig. PC6.3

Solution: For water take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. Write the steady-flow energy equation from the water surface (1) to the outlet (2) at the top of the slide:

$$\frac{p_a}{\rho g}+\frac{\alpha_1 V_1^2}{2g}+z_1=\frac{p_a}{\rho g}+\frac{\alpha_2 V_2^2}{2g}+z_2+h_f-h_{pump},\quad \text{where } V_2=\frac{1.39E\text{–}3}{\pi(0.02)^2}=1.106\ \frac{m}{s}$$

$$\textit{Solve for}\quad h_{pump}=(z_2-z_1)+\frac{V_2^2}{2g}\left(\alpha_2+f\frac{L}{d}\right)$$

Work out Red = ρVd/μ = (998)(1.106)(0.04)/0.001 = 44200, ε/d = 0.008/4 = 0.002, whence f_{Moody} = 0.0268. Use these numbers to evaluate the pump head above:

$$h_{pump}=(5.0-1.0)+\frac{(1.106)^2}{2(9.81)}\left[1.06+0.0268\left(\frac{5.0}{0.04}\right)\right]=4.27\text{ m},$$

$$\text{whence}\quad \mathbf{BHP_{required}}=\frac{\rho g Q h_{pump}}{\eta}=\frac{998(9.81)(1.39E\text{–}3)(4.27)}{0.8}=\mathbf{73\ watts}\quad \textit{Ans.}$$

C6.4 Suppose you build a house out in the ‘boonies,’ where you need to run a pipe to the nearest water supply, which fortunately is about 1 km above the elevation of your house. The gage pressure at the water supply is 1 MPa. You require a minimum of 3 gal/min when your end of the pipe is open to the atmosphere. To minimize cost, you want to buy the smallest possible diameter pipe with an extremely smooth surface.

(a) Find the total head loss from pipe inlet to exit, neglecting minor losses.

(b) Which is more important to this problem, the head loss due to elevation difference, or the head loss due to pressure drop in the pipe?
(c) Find the minimum required pipe diameter.

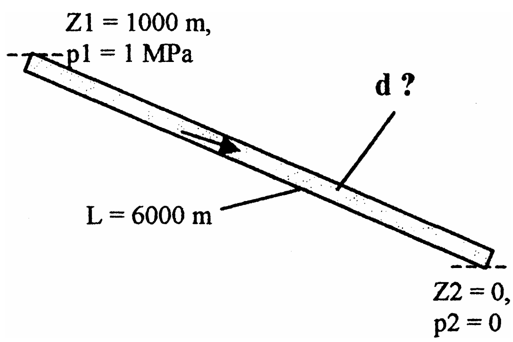


Fig. C6.4

Solution: Convert 3.0 gal/min to 1.89E–4 m^3/s. Let 1 be the inlet and 2 be the outlet and write the steady-flow energy equation:

$$\frac{p_{1gage}}{\rho g}+\frac{\alpha_1 V_1^2}{2g}+z_1=\frac{p_{2gage}}{\rho g}+\frac{\alpha_2 V_2^2}{2g}+z_2+h_f$$

$$\text{or:}\quad h_f=z_1-z_2+\frac{p_{1gage}}{\rho g}=1000\text{ m}+\frac{1E6\text{ kPa}}{998(9.81)}=1000+102=1102\text{ m}\quad \textit{Ans.}\text{ (a)}$$

(b) Thus, *elevation drop* of 1000 m is more important to head loss than $\Delta p/\rho g$ = 102 m.
(c) To find the minimum diameter, iterate between flow rate and the Moody chart:

$$h_f=f\frac{L}{d}\frac{V^2}{2g},\quad L=6000\text{ m},\quad \frac{1}{\sqrt{f}}=-2\log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right),\quad V=\frac{Q}{\pi d^2/4},$$

$$Q=1.89\text{E–}4\ \frac{\text{m}^3}{\text{s}},\quad \text{Re}=\frac{Vd}{\nu}$$

We are given h_f = 1102 m and ν_{water} = 1.005E–6 m^2/s. We can iterate, if necessary, or use **EES**, which can swiftly arrive at the final result:

$$f_{smooth}=0.0266;\quad \text{Re}=17924;\quad V=1.346\text{ m/s};\quad \mathbf{d_{min}=0.0134\text{ m}}\quad \textit{Ans.}\text{(c)}$$

C6.5 Water at 20°C flows, at the same flow rate Q = 9.4E–4 m^3/s, through two ducts, one a round pipe, and one an annulus, as shown. The cross-section area A of each duct is identical, and each has walls of commercial steel. Both are the same length. In the cross-sections shown, R = 15 mm and a = 25 mm.
(a) Calculate the correct radius b for the annulus.
(b) Compare head loss per unit length for the two ducts, first using the hydraulic diameter and second using the 'effective diameter' concept.
(c) If the losses are different, why? Which duct is more 'efficient'? Why?

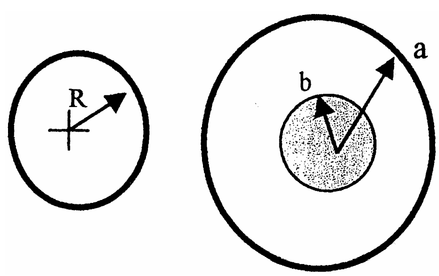


Fig. C6.5

Solution: (a) Set the areas equal:

$$A = \pi R^2 = \pi(a^2 - b^2), \quad or: \quad b = \sqrt{a^2 - R^2} = \sqrt{(25)^2 - (15)^2} = 20 \text{ mm} \quad Ans. \text{ (a)}$$

(b) Find the round-pipe head loss, assuming $\nu = 1.005E{-6}\ \text{m}^2/\text{s}$:

$$V = \frac{Q}{A} = \frac{9.4E{-4}\ \text{m}^3/\text{s}}{\pi(0.015\ \text{m})^2} = 1.33\ \frac{\text{m}}{\text{s}}; \quad \text{Re} = \frac{(1.33)(0.030)}{1.005E{-6}} = 39700;$$

$$\frac{\varepsilon}{d} = 0.00153,\ f_{Moody} = 0.0261$$

Thus $h_f/L = (f/d)(V^2/2g) = (0.0261/0.03)(1.33^2)/2/9.81 = \mathbf{0.0785}$ (*round*) *Ans.* (b)

Annulus: $D_h = 4A/P = 2(a-b) = 20$ mm, same V = 1.33 m/s:

$$\text{Re}_{Dh} = \frac{VD_h}{\nu} = 26500,\ \frac{\varepsilon}{D_h} = 0.0023,\ f_{Moody} = 0.0291,$$

$$h_f/L \approx \left(\frac{f}{D_h}\frac{V^2}{2g}\right) \approx \mathbf{0.131} \quad (annulus) \quad Ans. \text{ (b)}$$

Effective-diameter concept: b/a = 0.8, Table 6.3: $D_{eff} = 0.667D_h = 13.3$ mm. Then

$$\text{Re}_{Deff} = 17700,\ \frac{\varepsilon}{D_{eff}} = 0.00345,\ f_{Moody} = 0.0327,$$

$$\frac{h_f}{L} = \frac{f}{D_h}\frac{V^2}{2g} = \mathbf{0.147} \quad (annulus\text{—}D_{eff}) \quad Ans. \text{ (b)}$$

NOTE: Everything here uses D_{eff} except h_f, which by definition uses D_h!

We see that the annulus has about 85% more head loss than the round pipe, for the same area and flow rate! This is because the annulus has more wall area, thus more friction. *Ans.* (c)

C6.6 John Laufer (*NACA Tech. Rep.* 1174, 1954) gave velocity data for 20°C airflow in a smooth 24.7-cm-diameter pipe at Re ≈ 5 E5:

u/u_{CL}:	1.0	0.997	0.988	0.959	0.908	0.847	0.818	0.771	0.690
r/R:	0.0	0.102	0.206	0.412	0.617	0.784	0.846	0.907	0.963

The centerline velocity u_{CL} was 30.5 m/s. Determine (a) the average velocity by numerical integration and (b) the wall shear stress from the log-law approximation. Compare with the Moody chart and with Eq. (6.43).

Solution: For air at 20°C, take ρ = 1.2 kg/m^3 and μ = 0.00018 kg/m·s. The average velocity is defined by the (dimensionless) integral

$$V = \frac{1}{\pi R^2}\int_0^R u(2\pi r)dr, \quad \text{or:} \quad \frac{V}{u_{CL}} = \int_0^1 \frac{u}{u_{CL}} 2\eta d\eta, \quad \text{where} \quad \eta = \frac{r}{R}$$

Prepare a spreadsheet with the data and carry out the integration by the trapezoidal rule:

$$\int_0^1 \frac{u}{u_c} 2\eta d\eta \approx [(u/u_c)_2\eta_2 + (u/u_c)_1\eta_1](\eta_2 - \eta_1) + [(u/u_c)_3\eta_3 + (u/u_c)_2\eta_2](\eta_3 - \eta_2) + \cdots$$

The integral is evaluated on the spreadsheet below. The result is $V/u_{CL} \approx 0.8356$,

or $V \approx (0.8356)(30.5) \approx$ **25.5 m/s**. *Ans.* (a)

The wall shear stress is estimated by fitting the log-law (6.28) to each data point:

$$\text{For each (u,y)}, \quad \frac{u}{u^*} \approx \frac{1}{\kappa}\ln\left(\frac{yu^*}{\nu}\right) + B, \quad \kappa \approx 0.41 \quad \text{and} \quad B \approx 5.0$$

We know ν for air and are given *u* and *y* from the data, hence we can solve for u*. The spreadsheet gives u* ≈ 1.1 m/s ± 1%, or $\tau_w = \rho u^{*2} = (1.2)(1.1)^2 \approx$ **1.45 Pa**. *Ans.* (b)

y/R	r/R	u/uCL	∫u/uCL 2πr/R dr/R	u*
1.000	0.000	1.000	.0000	—
0.898	0.102	0.997	.0104	1.126
0.794	0.206	0.988	.0421	1.128
0.588	0.412	0.959	.1654	1.126
0.383	0.617	0.908	.3613	1.112
0.216	0.784	0.847	.5657	1.099
0.154	0.846	0.818	.6498	1.101
0.093	0.907	0.771	.7347	1.098
0.037	0.963	0.690	.8111	1.097
0.000	1.000	0.000	**.8356**	—

We make similar estimates from the Moody chart by evaluating Re and *f* and iterating:

$$\text{Guess } V \approx 25 \text{ m/s}, \quad \text{then } Re = \frac{1.2(25)(0.247)}{0.00018} \approx 412000,\ f_{smooth} \approx 0.0136$$

$$V_{better} = u_{CL}/[1 + 1.3\sqrt{f}] \approx 26.5, \quad \text{whence } Re \approx 436000,\ f_{better} \approx 0.0135$$

This converges to **V ≈ 26.5 m/s** *Ans.* and $\tau_w = (f/8)\rho V^2 \approx$ **1.42 Pa**. *Ans.*

C6.7 Consider energy exchange in fully-developed laminar flow between parallel plates, as in Eq. (6.63). Let the pressure drop over a length L be Δp. Calculate the rate of work done by this pressure drop on the fluid in the region ($0 < x < L$, $-h < y < +h$) and compare with the integrated energy dissipated due to the viscous function Φ from Eq. (4.50) over this same region. The two should be equal. Explain why this is so. Can you relate the viscous drag force and the wall shear stress to this energy result?

Solution: From Eq. (6.63), the velocity profile between the plates is parabolic:

$$u = \frac{3}{2}V\left(1 - \frac{y^2}{h^2}\right) \quad \text{where } V = \frac{h^2}{3\mu}\frac{\Delta p}{L} \text{ is the average velocity}$$

Let the width of the flow be denoted by b. The work done by pressure drop Δp is:

$$\dot{W}_{pressure} = \Delta p V A = \left(\frac{3\mu L V}{h^2}\right)(V)(2hb) = \frac{6\mu L b V^2}{h}$$

Meanwhile, from Eq. (4.50), the viscous dissipation function for this fully-developed flow is:

$$\Phi = \mu\left(\frac{\partial u}{\partial y}\right)^2 = \mu\left(\frac{3Vy}{h^2}\right)^2 = \frac{9\mu V^2 y^2}{h^4}$$

Integrate this to get the total dissipated energy over the entire flow region of dimensions L by b by $2h$:

$$\dot{E}_{dissipated} = Lb\int_{-h}^{+h}\left(\frac{9\mu V^2 y^2}{h^4}\right)dy = \frac{6\mu L b V^2}{h} = \dot{W}_{pressure}! \quad \textit{Ans.}$$

The two energy terms are equal. There is no work done by the wall shear stresses (where $u = 0$), so the pressure work is entirely absorbed by viscous dissipation within the flow field. *Ans.*

C6.8 This text has presented the traditional correlations for turbulent smooth-wall friction factor, Eq. (6.38), and the law-of-the-wall, Eq. (6.28). Recently, groups at Princeton and Oregon [56] have made new friction measurements and suggest the following smooth-wall friction law:

$$\frac{1}{\sqrt{f}} = 1.930\log_{10}(\mathrm{Re}_D\sqrt{f}) - 0.537$$

In earlier work, they also report that better values for the constants κ and B in the log-law,

Eq. (6.28), are $\kappa \approx 0.421 \pm 0.002$ and $B \approx 5.62 \pm 0.08$. (*a*) Calculate a few values of f in the range $1E4 \le \mathrm{Re}_D \le 1E8$ and see how the two formulas differ. (*b*) Read Ref. 56 and briefly check the five papers in its bibliography. Report to the class on the general results of this work.

Solution: The two formulas are practically identical except as the Reynolds number is very high or very low. The new formula was fit to new, and extensive, friction data in Ref. 56 and can thus be said to be slightly more accurate. Here is a table of calculations.

Re_D	f_{Prandtl}	$f_{\text{Ref.56}}$	Difference
3000	0.04353	0.04251	-2.41%
10000	0.03089	0.0305	-1.10%
30000	0.02349	0.02344	-0.18%
100000	0.01799	0.01811	0.62%
300000	0.01447	0.01464	1.22%
1000000	0.01165	0.01186	1.76%
3.0E+06	0.009722	0.009938	2.17%
1.0E+07	0.008104	0.008316	2.56%
3.0E+07	0.006949	0.007153	2.86%
1.0E+08	0.005941	0.006134	3.15%

They differ by no more than 3%.