Chapter 5 • Dimensional Analysis and Similarity

5.1 For axial flow through a circular tube, the Reynolds number for transition to turbulence is approximately 2300 [see Eq. (6.2)], based upon the diameter and average velocity. If $d = 5$ cm and the fluid is kerosene at 20°C, find the volume flow rate in m^3/h which causes transition.

Solution: For kerosene at 20°C, take ρ = 804 kg/m^3 and μ = 0.00192 kg/m·s. The only unknown in the transition Reynolds number is the fluid velocity:

$$\mathrm{Re}_{tr} \approx 2300 = \frac{\rho V d}{\mu} = \frac{(804)V(0.05)}{0.00192}, \quad \text{solve for } V = 0.11\ \mathrm{m/s}$$

$$\text{Then} \quad Q = VA = (0.11)\frac{\pi}{4}(0.05)^2 = 2.16E\text{–}4\ \frac{m^3}{s} \times 3600 \approx \mathbf{0.78\ \frac{m^3}{hr}} \quad \textit{Ans.}$$

P5.2 A prototype automobile is designed for cold weather in Denver, CO (-10°C, 83 kPa). Its drag force is to be tested in on a one-seventh-scale model in a wind tunnel at 20°C and 1 atm. If model and prototype satisfy dynamic similarity, what prototype velocity, in mi/h, is matched? Comment on your result.

Solution: First assemble the necessary air density and viscosity data:

$$\text{Denver: } T = 263K \ ; \quad \rho_p = \frac{p}{RT} = \frac{83000}{287(263)} = 1.10\ \frac{kg}{m^3} \ ; \quad \mu_p = 1.75E-5\frac{kg}{m-s}$$

$$\text{Wind tunnel: } T = 293K \ ; \rho_m = \frac{p}{RT} = \frac{101350}{287(293)} = 1.205\ \frac{kg}{m^3} \ ; \quad \mu_m = 1.80E-5\frac{kg}{m-s}$$

Convert 150 mi/h = 67.1 m/s. For dynamic similarity, equate the Reynolds numbers:

$$\mathrm{Re}_p = \frac{\rho VL}{\mu}|_p = \frac{(1.10)V_p(7L_m)}{1.75E-5} = \mathrm{Re}_m = \frac{\rho VL}{\mu}|_m = \frac{(1.205)(67.1)(L_m)}{1.80E-5}$$

$$\text{Solve for} \quad V_{prototype} = 10.2\, m/s = 22.8\ mi/h \qquad \textit{Ans.}$$

This is too *slow*, hardly fast enough to turn into a driveway. Since the tunnel can go no faster, the model drag must be corrected for Reynolds number effects. Note that we did not need to know the actual *length* of the prototype auto, only that it is 7 times larger than the model length.

5.3 An airplane has a chord length $L = 1.2$ m and flies at a Mach number of 0.7 in the standard atmosphere. If its Reynolds number, based on chord length, is 7E6, how high is it flying?

Solution: This is harder than Prob. 5.2 above, for we have to search in the U.S. Stan-dard Atmosphere (Table A-6) to find the altitude with the right density and viscosity

and speed of sound. We can make a first guess of T ≈ 230 K, a ≈ √(kRT) ≈ 304 m/s, U = 0.7a ≈ 213 m/s, and $\mu \approx$ 1.51E−5 kg/m·s. Then our first estimate for density is

$$\mathrm{Re_C} = 7\mathrm{E}6 = \frac{\rho \mathrm{UC}}{\mu} \approx \frac{\rho(213)(1.2)}{1.51\mathrm{E}{-5}}, \quad \text{or} \quad \rho \approx 0.44\ \mathrm{kg/m^3} \quad \text{and} \quad \mathrm{Z} \approx 9500\ \mathrm{m\ (Table\ A\text{-}6)}$$

Repeat and the process converges to $\rho \approx 0.41\ \mathrm{kg/m^3}$ or **Z ≈ 10100 m** *Ans.*

5.4 When tested in water at 20°C flowing at 2 m/s, an 8-cm-diameter sphere has a measured drag of 5 N. What will be the velocity and drag force on a 1.5-m-diameter weather balloon moored in sea-level standard air under dynamically similar conditions?

Solution: For water at 20°C take $\rho \approx 998\ \mathrm{kg/m^3}$ and $\mu \approx 0.001$ kg/m·s. For sea-level standard air take $\rho \approx 1.2255\ \mathrm{kg/m^3}$ and $\mu \approx$ 1.78E–5 kg/m·s. The balloon velocity follows from *dynamic similarity*, which requires identical Reynolds numbers:

$$\mathrm{Re_{model}} = \left.\frac{\rho \mathrm{VD}}{\mu}\right|_{\mathrm{model}} = \frac{998(2.0)(0.08)}{0.001} = 1.6\mathrm{E}5 = \mathrm{Re_{proto}} = \frac{1.2255\mathrm{V_{balloon}}(1.5)}{1.78\mathrm{E}{-5}}$$

or $V_{balloon} \approx 1.55$ m/s. Then the two spheres will have identical drag coefficients:

$$\mathrm{C_{D,model}} = \frac{\mathrm{F}}{\rho \mathrm{V^2 D^2}} = \frac{5\ \mathrm{N}}{998(2.0)^2(0.08)^2} = 0.196 = \mathrm{C_{D,proto}} = \frac{\mathrm{F_{balloon}}}{1.2255(1.55)^2(1.5)^2}$$

Solve for $\mathbf{F_{balloon} \approx 1.3\ N}$ *Ans.*

5.5 An automobile has a characteristic length and area of 8 ft and 60 ft^2, respectively. When tested in sea-level standard air, it has the following measured drag force versus speed:

V, mi/h:	20	40
Drag, lbf:	31	11

The same car travels in Colorado at 65 mi/h at an altitude of 3500 m. Using dimensional analysis, estimate (a) its drag force and (b) the horsepower required to overcome air drag.

Solution: For sea-level air in BG units, take $\rho \approx 0.00238$ slug/ft^3 and $\mu \approx$ 3.72E–7 slug/ft·s. Convert the raw drag and velocity data into dimensionless form:

V (mi/hr):	20	40	60
$C_D = F/(\rho V^2 L^2)$:	0.237	0.220	0.211
$Re_L = \rho VL/\mu$:	1.50E6	3.00E6	4.50E6

Drag coefficient plots versus Reynolds number in a very smooth fashion and is well fit (to ±1%) by the Power-law formula $\mathbf{C_D \approx 1.07 Re_L^{-0.106}}$.

(a) The new velocity is V = 65 mi/hr = 95.3 ft/s, and for air at 3500-m Standard Altitude (Table A-6) take $\rho = 0.001675$ slug/ft^3 and μ = 3.50E–7 slug/ft·s. Then compute the new Reynolds number and use our Power-law above to estimate drag coefficient:

$$\mathbf{Re}_{Colorado} = \frac{\rho VL}{\mu} = \frac{(0.001675)(95.3)(8.0)}{3.50E{-}7} = 3.65E6, \quad hence$$

$$C_D \approx \frac{1.07}{(3.65E6)^{0.106}} = 0.2157, \quad \therefore\ \mathbf{F} = 0.2157(0.001675)(95.3)^2(8.0)^2 = \mathbf{210\ lbf} \quad Ans.\ \text{(a)}$$

(b) The horsepower required to overcome drag is

$$\text{Power} = FV = (210)(95.3) = 20030\ \text{ft·lbf/s} \div 550 = \mathbf{36.4\ hp} \quad Ans.\ \text{(b)}$$

5.6 SAE 10 oil at 20°C flows past an 8-cm-diameter sphere. At flow velocities of 1, 2, and 3 m/s, the measured sphere drag forces are 1.5, 5.3, and 11.2 N, respectively. Estimate the drag force if the same sphere is tested at a velocity of 15 m/s in glycerin at 20°C.

Solution: For SAE 10 oil at 20°C, take $\rho \approx 870$ kg/m^3 and $\mu \approx 0.104$ kg/m·s. Convert the raw drag and velocity data into dimensionless form:

V (m/s):	1	2	3
F (newtons):	1.5	5.3	11.2
$C_D = F/(\rho V^2 D^2)$:	0.269	0.238	0.224
$Re_L = \rho VD/\mu$:	669	1338	2008

Drag coefficient plots versus Reynolds number in a very smooth fashion and is well fit (to ±1%) by the power-law formula $\mathbf{C_D \approx 0.81Re_L^{-0.17}}$.

The new velocity is V = 15 m/s, and for glycerin at 20°C (Table A-3), take $\rho \approx 1260$ kg/m^3 and $\mu \approx 1.49$ kg/m·s. Then compute the new Reynolds number and use our experimental correlation to estimate the drag coefficient:

$$\mathrm{Re}_{glycerin} = \frac{\rho VD}{\mu} = \frac{(1260)(15)(0.08)}{1.49} = 1015 \ (\textit{within the range}), \quad \textit{hence}$$

$$C_D = 0.81/(1015)^{0.17} \approx 0.250, \quad \textit{or}: \quad \mathbf{F}_{glycerin} = 0.250(1260)(15)^2(0.08)^2 = \mathbf{453\ N} \quad \textit{Ans.}$$

5.7 A body is dropped on the moon (g = 1.62 m/s^2) with an initial velocity of 12 m/s. By using option 2 variables, Eq. (5.11), the ground impact occurs at $t^{**} = 0.34$ and $S^{**} = 0.84$. Estimate (a) the initial displacement, (b) the final displacement, and (c) the time of impact.

Solution: (a) The initial displacement follows from the "option 2" formula, Eq. (5.12):

$$\mathrm{S^{**} = gS_o/V_o^2 + t^{**} + \frac{1}{2}t^{**2} = 0.84 = \frac{(1.62)S_o}{(12)^2} + 0.34 + \frac{1}{2}(0.34)^2}$$

$$\text{Solve for} \quad \mathrm{S_o} \approx \mathbf{39\ m} \quad \textit{Ans.}\text{ (a)}$$

(b, c) The final time and displacement follow from the given dimensionless results:

$$\mathrm{S^{**} = gS/V_o^2 = 0.84 = (1.62)S/(12)^2}, \quad \text{solve for } \mathrm{S_{final}} \approx \mathbf{75\ m} \quad \textit{Ans.}\text{ (b)}$$

$$\mathrm{t^{**} = gt/V_o = 0.34 = (1.62)t/(12)}, \quad \text{solve for } \mathrm{t_{impact}} \approx \mathbf{2.52\ s} \quad \textit{Ans.}\text{ (c)}$$

5.8 The *Morton number* Mo, used to correlate bubble-dynamics studies, is a dimensionless combination of acceleration of gravity g, viscosity μ, density ρ, and surface tension coefficient Y. If Mo is proportional to g, find its form.

Solution: The relevant dimensions are $\{g\} = \{LT^{-2}\}$, $\{\mu\} = \{ML^{-1}T^{-1}\}$, $\{\rho\} = \{ML^{-3}\}$, and $\{Y\} = \{MT^{-2}\}$. To have g in the numerator, we need the combination:

$$\{Mo\} = \{g\}\{\mu\}^a\{\rho\}^b\{Y\}^c = \left\{\frac{L}{T^2}\right\}\left\{\frac{M}{LT}\right\}^a\left\{\frac{M}{L^3}\right\}^b\left\{\frac{M}{T^2}\right\}^c = M^0L^0T^0$$

$$\text{Solve for} \quad a = 4,\ b = -1,\ c = -3, \quad \text{or:} \quad \mathbf{Mo} = \frac{\boldsymbol{g\mu^4}}{\boldsymbol{\rho Y^3}} \quad \textit{Ans.}$$

P5.9 The *Richardson number*, Ri, which correlates the production of turbulence by buoyancy,

is a dimensionless combination of the acceleration of gravity g, the fluid temperature T_o, the local temperature gradient $\partial T/\partial z$, and the local velocity gradient $\partial u/\partial z$. Determine the form of the Richardson number if it is proportional to g.

Solution: In the {MLTΘ} system, these variables have the dimensions $\{g\} = \{L/T^2\}$, $\{T_o\} = \{\Theta\}$, $\{\partial T/\partial z\} = \{\Theta/L\}$, and $\{\partial u/\partial z\} = \{T^{-1}\}$. The ratio $g/(\partial u/\partial z)^2$ will cancel time, leaving {L} in the numerator, and the ratio $\{\partial T/\partial z\}/T_o$ will cancel {Θ}, leaving {L} in the numerator. Multiply them together and we have the standard form of the dimensionless Richardson number:

$$\mathrm{Ri} = \frac{g\left(\dfrac{\partial T}{\partial z}\right)}{T_o\left(\dfrac{\partial u}{\partial z}\right)^2} \qquad \textit{Ans.}$$

5.10 Determine the dimension $\{MLT\Theta\}$ of the following quantities:

$$\text{(a) } \rho u\frac{\partial u}{\partial x} \quad \text{(b) } \int_1^2 (p-p_0)\,dA \quad \text{(c) } \rho c_p \frac{\partial^2 T}{\partial x\,\partial y} \quad \text{(d) } \iiint \rho\frac{\partial u}{\partial t}\,dx\,dy\,dz$$

All quantities have their standard meanings; for example, ρ is density, etc.

Solution: Note that $\{\partial u/\partial x\} = \{U/L\}, \{\int p\,dA\} = \{pA\},$ etc. The results are:

$$\text{(a) } \left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\}; \quad \text{(b) } \left\{\frac{\mathbf{ML}}{\mathbf{T^2}}\right\}; \quad \text{(c) } \left\{\frac{\mathbf{M}}{\mathbf{L^3T^2}}\right\}; \quad \text{(d) } \left\{\frac{\mathbf{ML}}{\mathbf{T^2}}\right\} \quad \textit{Ans.}$$

P5.11 During World War II, Sir Geoffrey Taylor, a British fluid dynamicist, used dimensional analysis to estimate the wave speed of an atomic bomb explosion. He assumed that the blast wave radius R was a function of energy released E, air density ρ, and time t. Use dimensional analysis to show how wave radius must vary with time.

Solution: The proposed function is $R = f(E, \rho, t)$. There are four variables ($n = 4$) and three primary dimensions (MLT, or $j = 3$), thus we expect $n\text{-}j = 4\text{-}3 = 1$ pi group. List the dimensions:

$$\{R\} = \{\mathrm{L}\}\ ;\ \{E\} = \{\mathrm{ML}^2/\mathrm{T}^2\}\ ;\quad \{\rho\} = \{\mathrm{M/L}^3\}\ ;\quad \{t\} = \{\mathrm{T}\}$$

Assume arbitrary exponents and make the group dimensionless:

$$R^1 E^a \rho^b t^c = (\mathrm{L})^1 (\mathrm{ML}^2/\mathrm{T}^2)^a (\mathrm{M/L}^3)^b (\mathrm{T})^c = \mathrm{M}^0 \mathrm{L}^0 \mathrm{T}^0\ ,$$

$$\text{whence } a+b=0\ ;\ 1+2a-3b=0\ ;\ -2a+c=0\ ;\ \text{Solve } a=-\frac{1}{5}\ ;\ b=+\frac{1}{5}\ ;\ c=-\frac{2}{5}$$

The single pi group is

$$\Pi_1 = \frac{R\,\rho^{1/5}}{E^{1/5}\,t^{2/5}} = \text{constant}, \quad \text{thus} \quad R_{wave} \propto t^{2/5} \qquad \textit{Ans.}$$

5.12 The *Stokes number*, St, used in particle-dynamics studies, is a dimensionless combination of *five* variables: acceleration of gravity g, viscosity μ, density ρ, particle velocity U, and particle diameter D. (a) If St is proportional to μ and inversely proportional to g, find its form. (b) Show that St is actually the quotient of two more traditional dimensionless groups.

Solution: (a) The relevant dimensions are $\{g\} = \{\mathrm{LT}^{-2}\}$, $\{\mu\} = \{\mathrm{ML}^{-1}\mathrm{T}^{-1}\}$, $\{\rho\} = \{\mathrm{ML}^{-3}\}$, $\{U\} = \{\mathrm{LT}^{-1}\}$, and $\{D\} = \{\mathrm{L}\}$. To have μ in the numerator and g in the denominator, we need the combination:

$$\{St\} = \{\mu\}\{g\}^{-1}\{\rho\}^a\{U\}^b\{D\}^c = \left\{\frac{M}{LT}\right\}\left\{\frac{T^2}{L}\right\}\left\{\frac{M}{L^3}\right\}^a\left\{\frac{L}{T}\right\}^b\{L\}^c = M^0L^0T^0$$

$$\textbf{\textit{Solve for}}\quad a=-1,\ b=1,\ c=-2,\quad \textbf{\textit{or}}:\quad St=\frac{\mu U}{\rho g D^2}\quad \textit{Ans.}\ \text{(a)}$$

$$\textbf{\textit{This has the ratio form:}}\ \ \textbf{\textit{St}}=\frac{U^2/(gD)}{\rho UD/\mu}=\frac{\textit{Froude number}}{\textit{Reynolds number}}\quad \textit{Ans.}\ \text{(b)}$$

5.13 The speed of propagation C of a capillary wave in deep water is known to be a function only of density ρ, wavelength λ, and surface tension Y. Find the proper functional relationship, completing it with a dimensionless constant. For a given density and wavelength, how does the propagation speed change if the surface tension is doubled?

Solution: The "function" of ρ, λ, and Y must have velocity units. Thus

$$\{C\}=\{f(\rho,\lambda,Y)\},\quad \text{or}\quad C=\text{const}\,\rho^a\lambda^b Y^c,\quad \text{or:}\quad \left\{\frac{L}{T}\right\}=\left\{\frac{M}{L^3}\right\}^a\{L\}^b\left\{\frac{M}{T^2}\right\}^c$$

$$\text{Solve for}\quad a=b=-1/2\ \text{and}\ c=+1/2,\quad \text{or:}\quad \mathbf{C=const}\sqrt{\frac{\mathbf{Y}}{\rho\lambda}}\quad \textit{Ans.}$$

Thus, for constant ρ and λ, if Y is doubled, C increases as $\sqrt{2}$, or **+41%.** *Ans.*

P5.14 Consider flow in a pipe of diameter D through a pipe bend of radius R_b. The pressure loss Δp through the bend is a function of these two length scales, plus density ρ, viscosity μ, and average flow velocity V. (a) Use dimensional analysis to rewrite this function in terms of dimensionless pi groups. (b) In analyzing data for such pipe-bend losses (Chap. 6), the dimensionless loss is often correlated with the *Dean number*, De:

$$\text{De} = \text{Re}_D\sqrt{\frac{D}{2R_b}}$$

Can your dimensional analysis produce a similar group? If not, explain why not.

Solution: (*a*) The proposed function is $\Delta p = f(\rho, \mu, V, D, R_b)$. There are six variables (n = 6) and three primary dimensions (j = 3), thus we expect n-j = 6-3 = 3 pi groups. Selecting, for example, (ρ, μ, V) as repeating variables, we would obtain the dimensionless function

$$\frac{\Delta p}{\rho V^2} = fcn\left(\frac{\rho V D}{\mu}, \frac{D}{R_b}\right) \qquad Ans.(a)$$

(b) The Dean number *combines* the two pi's on the right hand side, using fluid flow theory as a guide. This reduction from 3 to 2 pi groups cannot be predicted by pure dimensional analysis.

5.15 The wall shear stress τ_w in a boundary layer is assumed to be a function of stream velocity U, boundary layer thickness δ, local turbulence velocity u', density ρ, and local pressure gradient dp/dx. Using (ρ, U, δ) as repeating variables, rewrite this relationship as a dimensionless function.

Solution: The relevant dimensions are $\{\tau_w\} = \{ML^{-1}T^{-2}\}$, $\{U\} = \{LT^{-1}\}$, $\{\delta\} = \{L\}$, $\{u'\} = \{LT^{-1}\}$, $\{\rho\} = \{ML^{-3}\}$, and $\{dp/dx\} = \{ML^{-2}T^{-2}\}$. With n = 6 and j = 3, we expect $n - j = k = 3$ pi groups:

$$\Pi_1 = \rho^a U^b \delta^c \tau_w = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{L}{T}\right\}^b \{L\}^c \left\{\frac{M}{LT^2}\right\} = M^0 L^0 T^0, \quad solve\ \ a = -1,\ b = -2,\ c = 0$$

$$\Pi_2 = \rho^a U^b \delta^c u' = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{L}{T}\right\}^b \{L\}^c \left\{\frac{L}{T}\right\} = M^0 L^0 T^0, \quad solve\ \ a = 0,\ b = -1,\ c = 0$$

$$\Pi_3 = \rho^a U^b \delta^c \frac{dp}{dx} = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{L}{T}\right\}^b \{L\}^c \left\{\frac{M}{L^2T^2}\right\} = M^0 L^0 T^0, \quad solve\ \ a = -1,\ b = -2,\ c = 1$$

The final dimensionless function then is given by:

$$\Pi_1 = fcn(\Pi_2, \Pi_3), \quad \text{or:} \quad \boldsymbol{\frac{\tau_w}{\rho U^2} = fcn\left(\frac{u'}{U}, \frac{dp}{dx}\frac{\delta}{\rho U^2}\right)} \quad Ans.$$

5.16 Convection heat-transfer data are often reported as a *heat-transfer coefficient h,* defined by

$$\dot{Q} = hA\Delta T$$

where $\dot{Q}$ = heat flow, J/s
A = surface area, m^2
ΔT = temperature difference, K

The dimensionless form of *h*, called the *Stanton number*, is a combination of *h*, fluid density ρ, specific heat c_p, and flow velocity *V*. Derive the Stanton number if it is proportional to *h*.

Solution: If $\{\dot{Q}\} = \{hA\Delta T\}$, then $\left\{\dfrac{ML^2}{T^3}\right\} = \{h\}\{L^2\}\{\Theta\}$, or: $\{h\} = \left\{\dfrac{M}{\Theta T^3}\right\}$

$$\text{Then } \{\text{Stanton No.}\} = \{h^1\rho^b c_p{}^c V^d\} = \left\{\frac{M}{\Theta T^3}\right\}\left\{\frac{M}{L^3}\right\}^b\left\{\frac{L^2}{T^2\Theta}\right\}^c\left\{\frac{L}{T}\right\}^d = M^0L^0T^0\Theta^0$$

$$\text{Solve for} \quad b = -1,\ c = -1, \text{ and } d = -1.$$

$$\text{Thus, finally, Stanton Number} = h\rho^{-1}c_p{}^{-1}V^{-1} = \frac{\mathbf{h}}{\boldsymbol{\rho}\mathbf{Vc_p}} \quad \textit{Ans.}$$

5.17 The pressure drop per unit length $\Delta p/L$ in a porous, rotating duct (Really! See Ref. 35) depends upon average velocity *V*, density ρ, viscosity μ, duct height *h*, wall injection velocity v_w, and rotation rate Ω. Using (ρ, V, h) as repeating variables, rewrite this relationship in dimensionless form.

Solution: The relevant dimensions are $\{\Delta p/L\} = \{ML^{-2}T^{-2}\}$, $\{V\} = \{LT^{-1}\}$, $\{\rho\} = \{ML^{-3}\}$, $\{\mu\} = \{ML^{-1}T^{-1}\}$, $\{h\} = \{L\}$, $\{v_w\} = \{LT^{-1}\}$, and $\{\Omega\} = \{T^{-1}\}$. With $n = 7$ and $j = 3$, we expect $n - j = k = 4$ pi groups: They are found, as specified, using (ρ, V, h) as repeating variables:

$$\Pi_1 = \rho^a V^b h^c \frac{\Delta p}{L} = \left\{\frac{M}{L^3}\right\}^a\left\{\frac{L}{T}\right\}^b\{L\}^c\left\{\frac{M}{L^2T^2}\right\} = M^0L^0T^0, \quad \textit{solve}\ \ a = -1,\ b = -2,\ c = 1$$

$$\Pi_2 = \rho^a V^b h^c \mu^{-1} = \left\{\left\{\frac{M}{L^3}\right\}^a\left\{\frac{L}{T}\right\}^b\{L\}^c\left\{\frac{M}{LT}\right\}\right\}^{-1} = M^0L^0T^0, \quad \textit{solve}\ \ a = 1,\ b = 1,\ c = 1$$

$$\Pi_3 = \rho^a V^b h^c \Omega = \left\{\frac{M}{L^3}\right\}^a\left\{\frac{L}{T}\right\}^b\{L\}^c\left\{\frac{1}{T}\right\} = M^0L^0T^0, \quad \textit{solve}\ \ a = 0,\ b = -1,\ c = 1$$

$$\Pi_4 = \rho^a V^b h^c v_w = \left\{\frac{M}{L^3}\right\}^a\left\{\frac{L}{T}\right\}^b\{L\}^c\left\{\frac{L}{T}\right\} = M^0L^0T^0, \quad \textit{solve}\ \ a = 0,\ b = -1,\ c = 0$$

The final dimensionless function then is given by:

$$\Pi_1 = fcn(\Pi_2, \Pi_3, \Pi_4), \quad \text{or:} \quad \frac{\Delta p}{L}\frac{h}{\rho V^2} = fcn\left(\frac{\rho V h}{\mu}, \frac{\Omega h}{V}, \frac{v_w}{V}\right) \quad \textit{Ans.}$$

5.18 Under laminar conditions, the volume flow Q through a small triangular-section pore of side length b and length L is a function of viscosity μ, pressure drop per unit length $\Delta p/L$, and b. Using the pi theorem, rewrite this relation in dimensionless form. How does the volume flow change if the pore size b is doubled?

Solution: Establish the variables and their dimensions:

$$\begin{array}{ccccc} Q & = fcn(\Delta p/L, & \mu, & b) \\ \{L^3/T\} & \{M/L^2T^2\} & \{M/LT\} & \{L\} \end{array}$$

Then $n = 4$ and $j = 3$, hence we expect n – j = 4 – 3 = **1** Pi group, found as follows:

$$\Pi_1 = (\Delta p/L)^a(\mu)^b(b)^c Q^1 = \{M/L^2T^2\}^a\{M/LT\}^b\{L\}^c\{L^3/T\}^1 = M^0L^0T^0$$

$$M\text{:}\ a + b = 0; \quad L\text{:}\ -2a - b + c + 3 = 0; \quad T\text{:}\ -2a - b - 1 = 0,$$

$$\text{solve} \quad a = -1,\ b = +1,\ c = -4$$

$$\Pi_1 = \frac{\mathbf{Q\mu}}{\mathbf{(\Delta p/L)b^4}} = \textbf{constant} \quad \textit{Ans.}$$

Clearly, if b is doubled, the flow rate Q increases by a factor of **16**. *Ans.*

5.19 The period of oscillation T of a water surface wave is assumed to be a function of density ρ, wavelength λ, depth h, gravity g, and surface tension Y. Rewrite this relationship in dimensionless form. What results if Y is negligible?

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccc} T & = fcn(\ \rho, & \lambda, & h, & g, & Y\ ) \\ \{T\} & \{M/L^3\} & \{L\} & \{L\} & \{L/T^2\} & \{M/T^2\} \end{array}$$

Then $n = 6$ and $j = 3$, hence we expect n – j = 6 – 3 = **3** Pi groups, capable of various arrangements and selected by myself as follows:

$$\text{Typical final result:} \quad \mathbf{T(g/\lambda)^{1/2} = fcn}\left(\frac{\mathbf{h}}{\lambda}, \frac{\mathbf{Y}}{\rho \mathbf{g}\lambda^2}\right) \quad \textit{Ans.}$$

If Y is negligible, ρ drops out also, leaving: $\mathbf{T(g/\lambda)^{1/2} = fcn\left(\frac{h}{\lambda}\right)}$ *Ans.*

5.20 We can extend Prob. 5.18 to the case of laminar duct flow of a non-newtonian fluid, for which the simplest relation for stress versus strain-rate is the *power-law* approximation:

$$\tau = C\left(\frac{d\theta}{dt}\right)^n$$

This is the analog of Eq. (1.23). The constant C takes the place of viscosity. If the exponent n is less than (greater than) unity, the material simulates a pseudoplastic (dilatant) fluid, as illustrated in Fig. 1.7. (a) Using the {MLT} system, determine the dimensions of C. (b) The analog of Prob. 5.18 for Power-law laminar triangular-duct flow is $Q = \text{fcn}(C, \Delta p/L, b)$. Rewrite this function in the form of dimensionless Pi groups.

Solution: The shear stress and strain rate have the dimensions $\{\tau\} = \{ML^{-1}T^{-2}\}$, and $\{d\theta/dt\} = \{T^{-1}\}$.

(a) Using these in the equation enables us to find the dimensions of C:

$$\left\{\frac{M}{LT^2}\right\} = \{C\}\left\{\frac{1}{T}\right\}^n, \quad \textit{hence} \;\; \mathbf{\{C\} = \left\{\frac{M}{LT^{2-n}}\right\}} \quad \textit{Ans.}\text{ (a)}$$

Now that we know {C}, combine it with $\{Q\} = \{L^3T^{-1}\}$, $\{\Delta p/L\} = \{ML^{-2}T^{-2}\}$, and $\{b\} = \{L\}$. Note that there are 4 variables and j = 3 {MLT}, hence we expect 4 − 3 = only ***one*** pi group:

$$\{Q\}^a\left\{\frac{\Delta p}{L}\right\}^b\{L\}^c\{C\} = \left\{\frac{L^3}{T}\right\}^a\left\{\frac{M}{L^2T^2}\right\}^b\{L\}^c\left\{\frac{M}{LT^{2-n}}\right\} = M^0L^0T^0,$$

$$\textit{solve} \quad a = n,\; b = -1,\; c = -3n-1$$

The one and only dimensionless pi group is thus:

$$\mathbf{\Pi_1 = \frac{Q^n C}{(\Delta p/L)b^{3n+1}} = constant} \quad \textit{Ans.}\text{ (b)}$$

5.21 In Example 5.1 we used the pi theorem to develop Eq. (5.2) from Eq. (5.1). Instead of merely listing the primary dimensions of each variable, some workers list the *powers* of each primary dimension for each variable in an array:

$$\begin{array}{c} \\ M \\ L \\ T \end{array} \begin{array}{c} \begin{array}{ccccc} F & L & U & \rho & \mu \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & -3 & -1 \\ -2 & 0 & -1 & 0 & -1 \end{bmatrix} \end{array}$$

This array of exponents is called the *dimensional matrix* for the given function. Show that the *rank* of this matrix (the size of the largest nonzero determinant) is equal to $j = n - k$, the desired reduction between original variables and the pi groups. This is a general property of dimensional matrices, as noted by Buckingham [29].

Solution: The **rank** of a matrix is the size of the largest submatrix within which has a *non-zero determinant*. This means that the constants in that submatrix, when considered as coefficients of algebraic equations, are *linearly independent*. Thus we establish the number of *independent* parameters—adding one more forms a dimensionless group. For the example shown, the rank is **three** (note the very first 3 × 3 determinant on the left has a non-zero determinant). Thus "j" = 3 for the drag force system of variables.

5.22 The angular velocity Ω of a windmill is a function of windmill diameter *D*, wind velocity *V*, air density ρ, windmill height H as compared to atmospheric boundary layer height *L*, and the number of blades *N*: $\Omega = \text{fcn}(D, V, \rho, H/L, N)$. Viscosity effects are negligible. Rewrite this function in terms of dimensionless Pi groups.

Solution: We have $n = 6$ variables, $j = 3$ dimensions (M, L, T), thus expect $n - j = 3$ Pi groups. Since only ρ has *mass* dimensions, it drops out. After some thought, we realize that *H/L* and *N* are already dimensionless! The desired dimensionless function becomes:

$$\frac{\mathbf{\Omega D}}{\mathbf{V}} = \boldsymbol{fcn}\left(\frac{\mathbf{H}}{\mathbf{L}}, \mathbf{N}\right) \quad \textit{Ans.}$$

5.23 The period *T* of vibration of a beam is a function of its length *L*, area moment of inertia *I*, modulus of elasticity *E*, density ρ, and Poisson's ratio σ. Rewrite this relation in dimensionless form. What further reduction can we make if *E* and *I* can occur only in the product form *EI*?

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccc} T = \text{fcn}(& L, & I, & E, & \rho, & \sigma\ ) \\ \{T\} & \{L\} & \{L^4\} & \{M/LT^2\} & \{M/L^3\} & \{\text{none}\} \end{array}$$

Then $n = 6$ and $j = 3$, hence we expect $n - j = 6 - 3 =$ **3** Pi groups, capable of various arrangements and selected by myself as follows: [Note that σ must be a Pi group.]

Typical final result: $$\mathbf{\frac{T}{L}\sqrt{\frac{E}{\rho}} = fcn\left(\frac{L^4}{I}, \sigma\right)}$$ *Ans.*

If E and I can only appear together as EI, then $$\mathbf{\frac{T}{L^3}\sqrt{\frac{EI}{\rho}} = fcn(\sigma)}$$ *Ans.*

5.24 The lift force *F* on a missile is a function of its length *L*, velocity *V*, diameter *D*, angle of attack α, density ρ, viscosity μ, and speed of sound *a* of the air. Write out the dimensional matrix of this function and determine its rank. (See Prob. 5.21 for an explanation of this concept.) Rewrite the function in terms of pi groups.

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccccccc} F & = \text{fcn}(& L, & V, & D, & \alpha, & \rho, & \mu, & a &) \\ \{ML/T^2\} & & \{L\} & \{L/T\} & \{L\} & \{1\} & \{M/L^3\} & \{M/LT\} & \{L/T\} & \end{array}$$

Then $n = 8$ and $j = 3$, hence we expect n – j = 8 – 3 = **5** Pi groups. The matrix is

M:

L:

T:

The rank of this matrix is indeed **three**, hence there are exactly 5 Pi groups, as follows:

Typical final result: $$\mathbf{\frac{F}{\rho V^2 L^2} = fcn\left(\alpha, \frac{\rho VL}{\mu}, \frac{L}{D}, \frac{V}{a}\right)}$$ *Ans.*

P5.25 The thrust *F* of a propeller is generally thought to be a function of its diameter *D* and angular velocity Ω, the forward speed *V*, and the density ρ and viscosity μ of the fluid. Rewrite this relationship as a dimensionless function.

Solution: Write out the function with the various dimensions underneath:

$$\begin{array}{cccccccc} F & = & fcn(& D, & \Omega, & V, & \rho, & \mu\ ) \\ \{ML/T^2\} & & & \{L\} & \{1/T\} & \{L/T\} & \{M/L^3\} & \{M/LT\} \end{array}$$

There are 6 variables and 3 primary dimensions (MLT), and we quickly see that $j = 3$, because (ρ, V, D) cannot form a pi group among themselves. Use the pi theorem to find the three pi's:

$$\Pi_1 = \rho^a V^b D^c F\ ;\ \text{Solve for } a = -1,\ b = -2,\ c = -2. \quad \text{Thus} \quad \Pi_1 = \frac{F}{\rho V^2 D^2}$$

$$\Pi_2 = \rho^a V^b D^c \Omega\ ;\ \text{Solve for } a = 0,\ b = -1,\ c = 1. \quad \text{Thus} \quad \Pi_2 = \frac{\Omega D}{V}$$

$$\Pi_3 = \rho^a V^b D^c \mu\ ;\ \text{Solve for } a = -1,\ b = -1,\ c = -1. \quad \text{Thus} \quad \Pi_3 = \frac{\mu}{\rho V D}$$

One of many forms of the final desired dimensionless function is

$$\frac{F}{\rho V^2 D^2} = fcn\left(\frac{\Omega D}{V}, \frac{\mu}{\rho V D}\right) \qquad \textit{Ans.}$$

5.26 A pendulum has an oscillation period T which is assumed to depend upon its length L, bob mass m, angle of swing θ, and the acceleration of gravity. A pendulum 1 m long, with a bob mass of 200 g, is tested on earth and found to have a period of 2.04 s when swinging at 20°. (a) What is its period when it swings at 45°? A similarly constructed pendulum, with L = 30 cm and m = 100 g, is to swing on the moon (g = 1.62 m/s^2) at θ = 20°. (b) What will be its period?

Solution: First establish the variables and their dimensions so that we can do the numbers:

$$\begin{array}{cccccc} T & = \text{fcn}(& L, & m, & g, & \theta\ ) \\ \{T\} & & \{L\} & \{M\} & \{L/T^2\} & \{1\} \end{array}$$

Then $n = 5$ and $j = 3$, hence we expect n – j = 5 – 3 = **2** Pi groups. They are unique:

$$T\sqrt{\frac{g}{L}} = \text{fcn}(\theta) \quad \text{(mass drops out for dimensional reasons)}$$

(a) If we change the angle to 45°, this changes Π_2, hence we lose dynamic similarity and do not know the new period. More testing is required. *Ans.* (a)

(b) If we swing the pendulum on the moon at the same 20°, we may use similarity:

$$T_1\left(\frac{g_1}{L_1}\right)^{1/2} = (2.04\text{ s})\left(\frac{9.81\text{ m/s}^2}{1.0\text{ m}}\right)^{1/2} = 6.39 = T_2\left(\frac{1.62\text{ m/s}^2}{0.3\text{ m}}\right)^{1/2},$$

$$\text{or:}\quad \mathbf{T_2 = 2.75\ s}\quad \textit{Ans.}\text{(b)}$$

5.27 In studying sand transport by ocean waves, A. Shields in 1936 postulated that the bottom shear stress τ required to move particles depends upon gravity g, particle size d and density ρ_p, and water density ρ and viscosity μ. Rewrite this in terms of dimensionless groups (which led to the *Shields Diagram* in 1936).

Solution: There are six variables (τ, g, d, ρ_p, ρ, μ) and three dimensions (M, L, T), hence we expect $n - j = 6 - 3 =$ **3** Pi groups. The author used (ρ, g, d) as repeating variables:

$$\frac{\tau}{\rho g d} = fcn\left(\frac{\rho g^{1/2} d^{3/2}}{\mu}, \frac{\rho_p}{\rho}\right)\quad \textit{Ans.}$$

The shear parameter used by Shields himself was based on *net* weight: $\boldsymbol{\tau/[(\rho_p - \rho)gd]}$.

5.28 A simply supported beam of diameter D, length L, and modulus of elasticity E is subjected to a fluid crossflow of velocity V, density ρ, and viscosity μ. Its center deflection δ is assumed to be a function of all these variables. (a) Rewrite this proposed function in dimensionless form. (b) Suppose it is known that δ is independent of μ, inversely proportional to E, and dependent only upon ρV^2, not ρ and V separately. Simplify the dimensionless function accordingly.

Solution: Establish the variables and their dimensions:

$$\begin{array}{llllllll} \delta & = \text{fcn}(& \rho, & D, & L, & E, & V, & \mu) \\ \{L\} & & \{M/L^3\} & \{L\} & \{L\} & \{M/LT^2\} & \{L/T\} & \{M/LT\} \end{array}$$

Then $n = 7$ and $j = 3$, hence we expect $n - j = 7 - 3 =$ **4** Pi groups, capable of various arrangements and selected by myself, as follows (a):

$$\text{Well-posed final result:}\quad \frac{\delta}{L} = \mathbf{fcn}\left(\frac{L}{D}, \frac{\rho VD}{\mu}, \frac{E}{\rho V^2}\right)\quad \textit{Ans.}\text{ (a)}$$

(b) If μ is unimportant, then the Reynolds number ($\rho VD/\mu$) drops out, and we have already cleverly combined E with ρV^2, which we can now slip out upside down:

If μ drops out and $\delta \propto \frac{1}{E}$, then $\frac{\delta}{L} = \frac{\rho V^2}{E}\,\mathrm{fcn}\left(\frac{L}{D}\right)$,

$$\text{or:}\quad \boldsymbol{\frac{\delta E}{\rho V^2 L} = \mathrm{fcn}\left(\frac{L}{D}\right)} \quad \textit{Ans.}\text{ (b)}$$

5.29 When fluid in a pipe is accelerated linearly from rest, it begins as laminar flow and then undergoes transition to turbulence at a time t_{tr} which depends upon the pipe diameter D, fluid acceleration a, density ρ, and viscosity μ. Arrange this into a dimensionless relation between t_{tr} and D.

Solution: Establish the variables and their dimensions:

$$\begin{array}{llllll} t_{tr} & = \mathrm{fcn}(& \rho & , D, & a, & \mu\ ) \\ \{T\} & & \{M/L^3\} & \{L\} & \{L/T^2\} & \{M/LT\} \end{array}$$

Then $n = 5$ and $j = 3$, hence we expect $n - j = 5 - 3 = \mathbf{2}$ Pi groups, capable of various arrangements and selected by myself, as required, to isolate t_{tr} versus D:

$$\boldsymbol{t_{tr}\left(\frac{\rho a^2}{\mu}\right)^{1/3} = \mathrm{fcn}\left[D\left(\frac{\rho^2 a}{\mu^2}\right)^{1/3}\right]} \quad \textit{Ans.}$$

5.30 The wall shear stress τ_w for flow in a narrow annular gap between a fixed and a rotating cylinder is a function of density ρ, viscosity μ, angular velocity Ω, outer radius R, and gap width Δr. Using (ρ, Ω, R) as repeating variables, rewrite this relation in dimensionless form.

Solution: The relevant dimensions are $\{\tau_w\} = \{ML^{-1}T^{-2}\}$, $\{\rho\} = \{ML^{-3}\}$, $\{\mu\} = \{ML^{-1}T^{-1}\}$, $\{\Omega\} = \{T^{-1}\}$, $\{R\} = \{L\}$, and $\{\Delta r\} = \{L\}$. With $n = 6$ and $j = 3$, we expect $n - j = k = 3$ pi groups. They are found, as specified, using (ρ, Ω, R) as repeating variables:

$$\Pi_1 = \rho^a \Omega^b R^c \tau_w = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{1}{T}\right\}^b \{L\}^c \left\{\frac{M}{LT^2}\right\} = M^0 L^0 T^0, \quad solve\ \ a = -1, b = -2, c = -2$$

$$\Pi_2 = \rho^a \Omega^b R^c \mu^{-1} = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{1}{T}\right\}^b \{L\}^c \left\{\frac{M}{LT}\right\}^{-1} = M^0 L^0 T^0, \quad solve\ \ a = 1, b = 1, c = 2$$

$$\Pi = \rho^a \Omega^b R^c \Delta r = \left\{\frac{M}{L^3}\right\}^a \left\{\frac{1}{T}\right\}^b \{L\}^c \{L\} = M^0 L^0 T^0, \quad solve\ \ a = 0, b = 0, c = -1$$

The final dimensionless function has the form:

$$\Pi_1 = fcn(\Pi_2, \Pi_3), \quad or: \quad \boldsymbol{\frac{\tau_{wall}}{\rho\Omega^2 R^2} = fcn\left(\frac{\rho\Omega R^2}{\mu}, \frac{\Delta r}{R}\right)} \quad \textit{Ans.}$$

5.31 The heat-transfer rate per unit area *q* to a body from a fluid in natural or gravitational convection is a function of the temperature difference ΔT, gravity *g*, body length *L*, and three fluid properties: kinematic viscosity ν, conductivity *k*, and thermal expansion coefficient β. Rewrite in dimensionless form if it is known that *g* and β appear only as the product $g\beta$.

Solution: Establish the variables and their dimensions:

$$\underset{\{M/T^3\}}{q} = \text{fcn}\Big(\underset{\{\Theta\}}{\Delta T},\ \underset{\{L/T^2\}}{g},\ \underset{\{L\}}{L},\ \underset{\{L^2/T\}}{\nu},\ \underset{\{1/\Theta\}}{\beta},\ \underset{\{ML/\Theta T^3\}}{k}\Big)$$

Then $n = 7$ and $j = 4$, hence we expect $n - j = 7 - 4 = \mathbf{3}$ Pi groups, capable of various arrangements and selected by myself, as follows:

If β and ΔT kept separate, then $\dfrac{qL}{k\Delta T} = \text{fcn}\left(\beta\Delta T, \dfrac{gL^3}{\nu^2}\right)$

If, in fact, β and ΔT must appear <u>together</u>, then Π_2 and Π_3 above <u>combine</u> and we get

$$\underbrace{\mathbf{\frac{qL}{k\Delta T}}}_{\text{Nusselt No.}} \mathbf{= fcn}\underbrace{\left(\mathbf{\frac{\beta\Delta T g L^3}{\nu^2}}\right)}_{\text{Grashof Number}} \quad \textit{Ans.}$$

5.32 A *weir* is an obstruction in a channel flow which can be calibrated to measure the flow rate, as in Fig. P5.32. The volume flow Q varies with gravity g, weir width b into the paper, and upstream water height H above the weir crest. If it is known that Q is proportional to b, use the pi theorem to find a unique functional relationship $Q(g, b, H)$.

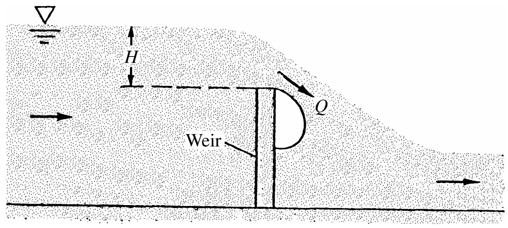


Fig. P5.32

Solution: Establish the variables and their dimensions:

$$\underset{\{L^3/T\}}{Q} = \text{fcn}(\underset{\{L/T^2\}}{g}, \underset{\{L\}}{b}, \underset{\{L\}}{H})$$

Then $n = 4$ and $j = 2$, hence we expect n – j = 4 – 2 = **2** Pi groups, capable of various arrangements and selected by myself, as follows:

$$\frac{Q}{g^{1/2}H^{5/2}} = \text{fcn}\left(\frac{b}{H}\right); \quad \text{but if } Q \propto b, \text{ then we reduce to } \mathbf{\frac{Q}{bg^{1/2}H^{3/2}} = constant} \quad \textit{Ans.}$$

5.33 A spar buoy (see Prob. 2.113) has a period T of vertical (heave) oscillation which depends upon the waterline cross-sectional area A, buoy mass m, and fluid specific weight γ. How does the period change due to doubling of (a) the mass and (b) the area? Instrument buoys should have long periods to avoid wave resonance. Sketch a possible long-period buoy design.

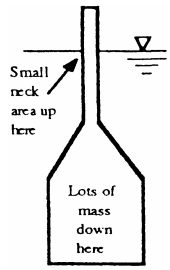


Fig. P5.33

Solution: Establish the variables and their dimensions:

$$\underset{\{T\}}{T} = \text{fcn}(\underset{\{L^2\}}{A}, \underset{\{M\}}{m}, \underset{\{M/L^2T^2\}}{\gamma})$$

Then $n = 4$ and $j = 3$, hence we expect n – j = 4 – 3 = **1** single Pi group, as follows:

$$\mathbf{T\sqrt{\frac{A\gamma}{m}} = dimensionless\ constant} \quad \textit{Ans.}$$

Since we can't do anything about γ, the specific weight of water, we *can* increase period T by increasing buoy mass m and decreasing waterline area A. See the illustrative long-period buoy in Figure P5.33.

5.34 To good approximation, the thermal conductivity k of a gas (see Ref. 8 of Chap. 1) depends only on the density ρ, mean free path ℓ, gas constant R, and absolute temperature T. For air at 20°C and 1 atm, $k \approx 0.026$ W/m·K and $\ell \approx 6.5\text{E}{-8}$ m. Use this information to determine k for hydrogen at 20°C and 1 atm if $\ell \approx 1.2\text{E}{-7}$ m.

Solution: First establish the variables and their dimensions and then form a pi group:

$$\begin{array}{ccccccc} k & = \text{fcn}(& \rho & , \ \ell \ , & R & , & T\ ) \\ \{ML/\Theta T^3\} & & \{M/L^3\} & \{L\} & \{L^2/T^2\Theta\} & & \{\Theta\} \end{array}$$

Thus $n = 5$ and $j = 4$, and we expect $n - j = 5 - 4 = \mathbf{1}$ single pi group, and the result is

$$k/(\rho R^{3/2} T^{1/2} \ell) = \text{a dimensionless constant} = \Pi_1$$

The value of Π_1 is found from the air data, where $\rho = 1.205$ kg/m^3 and R = 287 m^2/s^2·K:

$$\Pi_{1,air} = \frac{0.026}{(1.205)(287)^{3/2}(293)(6.5E{-8})} = 3.99 = \Pi_{1,hydrogen}$$

For hydrogen at 20°C and 1 atm, calculate $\rho = 0.0839$ kg/m^3 with $R = 4124$ m^2/s^2·K. Then

$$\Pi_1 = 3.99 = \frac{k_{hydrogen}}{(0.0839)(4124)^{3/2}(293)^{1/2}(1.2E{-7})}, \quad \textit{solve for } k_{hydrogen} = \mathbf{0.182\ \frac{W}{m{\cdot}K}} \quad \textit{Ans.}$$

This is slightly larger than the accepted value for hydrogen of $k \approx 0.178$ W/m·K.

5.35 The torque M required to turn the cone-plate viscometer in Fig. P5.35 depends upon the radius R, rotation rate Ω, fluid viscosity μ, and cone angle θ. Rewrite this relation in dimensionless form. How does the relation simplify if it is known that M is proportional to θ?

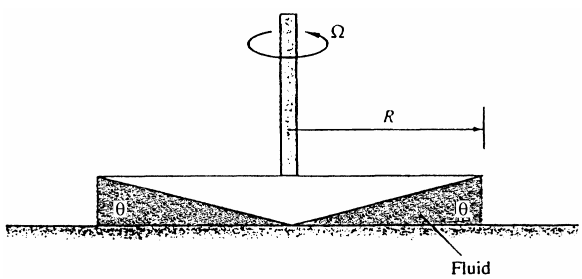


Fig. P5.35

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccc} M & = \text{fcn}(& R, & \Omega, & \mu, & \theta\ ) \\ \{ML^2/T^2\} & & \{L\} & \{1/T\} & \{M/LT\} & \{1\} \end{array}$$

Then $n = 5$ and $j = 3$, hence we expect n – j = 5 – 3 = **2** Pi groups, capable of only one reasonable arrangement, as follows:

$$\mathbf{\frac{M}{\mu\Omega R^3} = fcn(\theta)};\quad \text{if}\ \ M \propto \theta,\quad \text{then}\ \ \mathbf{\frac{M}{\mu\Omega\theta R^3} = constant} \quad \textit{Ans.}$$

See Prob. 1.56 of this Manual, for an analytical solution.

5.36 The rate of heat loss, Q_{loss} through a window is a function of the temperature difference ΔT, the surface area *A*, and the *R* resistance value of the window (in units of ft²·hr·°F/Btu): Q_{loss} = fcn(ΔT, *A*, *R*). (a) Rewrite in dimensionless form. (b) If the temperature difference doubles, how does the heat loss change?

Solution: First figure out the dimensions of *R*: {R} = {$T^3\Theta/M$}. Then note that $n = 4$ variables and $j = 3$ dimensions, hence we expect only 4 – 3 = *one* Pi group, and it is:

$$\Pi_1 = \frac{Q_{loss}R}{A\,\Delta T} = Const, \quad or: \quad \boldsymbol{Q_{loss} = Const\,\frac{A\,\Delta T}{R}} \quad \textit{Ans. (a)}$$

(b) Clearly (to me), Q ∝ ΔT: **if Δt doubles, Q_{loss} also doubles**. *Ans.* (b)

P5.37 The volume flow *Q* through an orifice plate is a function of pipe diameter *D*, pressure drop Δ*p* across the orifice, fluid density *ρ* and viscosity *μ*, and orifice diameter *d*. Using *D*, *ρ*, and Δ*p* as repeating variables, express this relationship in dimensionless form.

Solution: There are 6 variables and 3 primary dimensions (MLT), and we already know that

j = 3, because the problem thoughtfully gave the repeating variables. Use the pi theorem to find the three pi's:

$$\Pi_1 = D^a\rho^b\Delta p^c Q\ ;\ \text{Solve for } a=-2,\ b=1/2,\ c=-1/2. \quad \text{Thus} \quad \Pi_1 = \frac{Q\,\rho^{1/2}}{D^2\,\Delta p^{1/2}}$$

$$\Pi_2 = D^a\rho^b\Delta p^c d\ ;\ \text{Solve for } a=-1\ \ b=0\ \ c=\ 0. \quad \text{Thus} \quad \Pi_1 = \frac{d}{D}$$

$$\Pi_3 = D^a\rho^b\Delta p^c \mu\ ;\ \text{Solve for } a=-1,\ b=-1/2,\ c=-1/2. \quad \text{Thus} \quad \Pi_1 = \frac{\mu}{D\,\rho^{1/2}\Delta p^{1/2}}$$

The final requested orifice-flow function (see Sec. 6.12 later for a different form) is:

$$\frac{Q\rho^{1/2}}{D^2\,\Delta p^{1/2}} = fcn\left(\frac{d}{D},\ \frac{\mu}{D\rho^{1/2}\Delta p^{1/2}}\right) \qquad \textit{Ans.}$$

5.38 The size d of droplets produced by a liquid spray nozzle is thought to depend upon the nozzle diameter D, jet velocity U, and the properties of the liquid ρ, μ, and Y. Rewrite this relation in dimensionless form. *Hint*: Take D, ρ, and U as repeating variables.

Solution: Establish the variables and their dimensions:

$$\begin{array}{ccccccc} d & = \text{fcn}(& D, & U, & \rho, & \mu, & Y\ ) \\ \{L\} & & \{L\} & \{L/T\} & \{M/L^3\} & \{M/LT\} & \{M/T^2\} \end{array}$$

Then $n = 6$ and $j = 3$, hence we expect $n - j = 6 - 3 = \mathbf{3}$ Pi groups, capable of various arrangements and selected by me, as follows:

$$\text{Typical final result:}\quad \mathbf{\frac{d}{D} = fcn\left(\frac{\rho UD}{\mu},\ \frac{\rho U^2 D}{Y}\right)} \qquad \textit{Ans.}$$

5.39 In turbulent flow past a flat surface, the velocity u near the wall varies approximately logarithmically with distance y from the wall and also depends upon viscosity μ, density ρ, and wall shear stress τ_w. For a certain airflow at 20°C and 1 atm, $\tau_w = 0.8$ Pa and $u = 15$ m/s at $y = 3.6$ mm. Use this information to estimate the velocity u at $y = 6$ mm.

Solution: Establish the variables and their dimensions:

$$\begin{array}{cccccc} u & = \text{fcn}(& y, & \rho, & \mu, & \tau_w\ ) \\ \{L/T\} & & \{L\} & \{M/L^3\} & \{M/LT\} & \{M/LT^2\} \end{array}$$

Then n = 5 and j = 3, hence we expect n − j = 5 − 3 = **2** Pi groups, capable of various arrangements and selected by me to form the traditional "u" versus "y," as follows:

$$\text{Ideal non-dimensionalization:} \quad \frac{u}{\sqrt{(\tau_w/\rho)}} \approx \text{const}\ \ln\left(\frac{y\sqrt{(\tau_w\rho)}}{\mu}\right)$$

The logarithmic relation is assumed in the problem, and the "constant" can be evaluated from the given data. For air at 20°C, take $\rho = 1.20$ kg/m^3, and $\mu = 1.8\text{E}{-5}$ kg/m·s. Then

$$\frac{15}{\sqrt{(0.8/1.20)}} \approx C\ \ln\left[\frac{(0.0036)(0.8)^{1/2}(1.20)^{1/2}}{1.8\text{E}{-5}}\right], \quad \text{or:} \quad C \approx 3.48 \quad \text{for this data}$$

Further out, at y = 6 mm, we assume that the same logarithmic relation holds. Thus

$$\text{At } y = 6\text{ mm:}\quad \frac{u}{\sqrt{(0.8/1.20)}} \approx 3.48\ln\left[\frac{(0.006)(0.8)^{1/2}(1.2)^{1/2}}{1.8\text{E–}5}\right],\quad \text{or}\quad \mathbf{u \approx 16.4\ \frac{m}{s}}\quad \textit{Ans.}$$

P5.40 The time t_d to drain a liquid from a hole in the bottom of a tank is a function of the hole diameter d, the initial fluid volume υ_o, the initial liquid depth h_o, and the density ρ and viscosity μ of the fluid. Rewrite this relation as a dimensionless function, using Ipsen's method.

Solution: As asked, use *Ipsen's method*. Write out the function with the dimensions beneath:

$$\begin{array}{ccccccccccccc} t_d & = & fcn(& d & , & \upsilon_o & , & h_o & , & \rho & , & \mu &) \\ \{T\} & & & \{L\} & & \{L^3\} & & \{L\} & & \{M/L^3\} & & \{M/LT\} & \end{array}$$

Eliminate the dimensions by multiplication or division. Divide by μ to eliminate {M}:

$$\begin{array}{ccccccccccccc} t_d & = & fcn(& d & , & \upsilon_o & , & h_o & , & \dfrac{\rho}{\mu} & , & \cancel{\mu} &) \\ \{T\} & & & \{L\} & & \{L^3\} & & \{L\} & & \{T/L^2\} & & & \end{array}$$

Recall Ipsen's rules: Only divide into variables containing mass, in this case only ρ. Now eliminate {T}. Again only one division is necessary:

$$\begin{array}{ccccccccccc} \dfrac{t_d\,\mu}{\rho} & = & fcn(& d & , & \upsilon_o & , & h_o & , & \cancel{\dfrac{\rho}{\mu}} &) \\ \{L^2\} & & & \{L\} & & \{L^3\} & & \{L\} & & & \end{array}$$

Finally, eliminate {L} by dividing by d. This completes our task when we discard d itself:

$$\underset{\{1\}}{\frac{t_d \mu}{\rho d^2}} = fcn\left(\underset{\{1\}}{\frac{\upsilon_o}{d^3}}, \underset{\{1\}}{\frac{h_o}{d}}\right) \qquad \textit{Ans.}$$

Just divide out the dimensions, don't worry about j or selecting repeating variables. Of course, the Pi Theorem would give the same, or comparable, results.

5.41 A certain axial-flow turbine has an output torque M which is proportional to the volume flow rate Q and also depends upon the density ρ, rotor diameter D, and rotation rate Ω. How does the torque change due to a doubling of (a) D and (b) Ω?

Solution: List the variables and their dimensions, one of which can be **M/Q**, since M is stated to be proportional to Q:

$$\underset{\{M/LT\}}{M/Q} = \text{fcn}(\underset{\{L\}}{D}, \underset{\{M/L^3\}}{\rho}, \underset{\{1/T\}}{\Omega})$$

Then $n = 4$ and $j = 3$, hence we expect n – j = 4 – 3 = **1** single Pi group:

$$\frac{M/Q}{\rho \Omega D^2} = \text{dimensionless constant}$$

(a) If turbine diameter D is doubled, the torque M increases by a factor of **4**. *Ans.* (a)
(b) If turbine speed Ω is doubled, the torque M increases by a factor of **2**. *Ans.* (b)

P5.42 When disturbed, a floating buoy will bob up and down at frequency f. Assume that this frequency varies with buoy mass m and waterline diameter d and with the specific weight γ of the liquid. (a) Express this as a dimensionless function. (b) If d and γ are constant and the buoy mass is halved, how will the frequency change?

Solution: The proposed function is $f = fcn(m, d, \gamma)$. Write out their dimensions:

$$\{f\} = \{T^{-1}\} \quad ; \quad \{m\} = \{M\} \quad ; \quad \{d\} = \{L\} \quad ; \quad \{\gamma\} = \{ML^{-2}T^{-2}\}$$

There are four variables and $j = 3$. Hence we expect only *one* Pi group. We find that

$$\Pi_1 = \frac{f}{d}\sqrt{\frac{m}{\gamma}} = \text{constant} \qquad Ans.(a)$$

Hence, for these simplifying assumptions, f is proportional to $m^{-1/2}$. If m halves, f rises by a factor $(0.5)^{-1/2} = 1.414$. In other words, halving m increases f by about 41%. *Ans.*(*b*)

5.43 Non-dimensionalize the thermal energy partial differential equation (4.75) and its boundary conditions (4.62), (4.63), and (4.70) by defining dimensionless temperature $T^* = T/T_o$, where To is the fluid inlet temperature, assumed constant. Use other dimensionless variables as needed from Eqs. (5.23). Isolate all dimensionless parameters which you find, and relate them to the list given in Table 5.2.

Solution: Recall the previously defined variables in addition to T^*:

$$u^* = \frac{u}{U}; \quad x^* = \frac{x}{L}; \quad t^* = \frac{Ut}{L}; \quad \text{similarly,} \quad v^* \text{ or } w^* = \frac{v \text{ or } w}{U}; \quad y^* \text{ or } z^* = \frac{y \text{ or } z}{L}$$

Then the dimensionless versions of Eqs. (4.75, 62, 63, 70) result as follows:

$$\textbf{(4.75):} \quad \frac{dT^*}{dt^*} = \underbrace{\left(\frac{k}{\rho c_p UL}\right)}_{\text{1/Peclet Number}} \nabla^{*2}T^* + \underbrace{\left(\frac{\mu U}{\rho c_p T_o L}\right)}_{\text{Eckert Number divided by Reynolds Number}} \Phi^*$$

5.44 The differential energy equation for incompressible two-dimensional flow through a "Darcy-type" porous medium is approximately

$$\rho c_p \frac{\sigma}{\mu}\frac{\partial p}{\partial x}\frac{\partial T}{\partial x} + \rho c_p \frac{\sigma}{\mu}\frac{\partial p}{\partial y}\frac{\partial T}{\partial y} + k\frac{\partial^2 T}{\partial y^2} = 0$$

where σ is the *permeability* of the porous medium. All other symbols have their usual meanings. (a) What are the appropriate dimensions for σ? (b) Nondimensionalize this equation, using (L, U, ρ, T_o) as scaling constants, and discuss any dimensionless parameters which arise.

Solution: (a) The only way to establish $\{\sigma\}$ is by comparing two terms in the PDE:

$$\left\{\rho c_p \frac{\sigma}{\mu}\frac{\partial p}{\partial x}\frac{\partial T}{\partial x}\right\} = \left\{k\frac{\partial^2 T}{\partial x^2}\right\}, \quad \text{or:} \quad \left\{\frac{M}{L^3T^3}\right\}\{\sigma\} \overset{?}{=} \left\{\frac{M}{LT^3}\right\},$$

$$\text{Thus} \quad \{\sigma\} = \{\mathbf{L^2}\} \quad \textit{Ans.}\text{ (a)}$$

(b) Define dimensionless variables using the stated list of (L, U, ρ, T_o) for scaling:

$$x^* = \frac{x}{L}; \quad y^* = \frac{y}{L}; \quad p^* = \frac{p}{\rho U^2}; \quad T^* = \frac{T}{T_o}$$

Substitution into the basic PDE above yields only a *single* dimensionless parameter:

$$\zeta\left(\frac{\partial p^*}{\partial x^*}\frac{\partial T^*}{\partial x^*} + \frac{\partial p^*}{\partial y^*}\frac{\partial T^*}{\partial y^*}\right) + \frac{\partial^2 T^*}{\partial y^{*2}} = 0, \quad \text{where } \zeta = \frac{\boldsymbol{\rho^2 c_p U^2 \sigma}}{\boldsymbol{\mu k}} \quad \textit{Ans.}\text{ (b)}$$

I don't know the name of this parameter. It is related to the "Darcy-Rayleigh" number.

5.45 A model differential equation, for chemical reaction dynamics in a plug reactor, is as follows:

$$u\frac{\partial C}{\partial x} = \mathcal{D}\frac{\partial^2 C}{\partial x^2} - kC - \frac{\partial C}{\partial t}$$

where u is the velocity, $\mathcal{D}$ is a diffusion coefficient, k is a reaction rate, x is distance along the reactor, and C is the (dimensionless) concentration of a given chemical in the reactor. (a) Determine the appropriate dimensions of $\mathcal{D}$ and k. (b) Using a characteristic length scale L and average velocity V as parameters, rewrite this equation in dimensionless form and comment on any Pi groups appearing.

Solution: (a) Since all terms in the equation contain C, we establish the dimensions of k and $\mathcal{D}$ by comparing $\{k\}$ and $\{\mathcal{D}\partial^2/\partial x^2\}$ to $\{u\partial/\partial x\}$:

$$\{k\} = \{\mathcal{D}\}\left\{\frac{\partial^2}{\partial x^2}\right\} = \{\mathcal{D}\}\left\{\frac{1}{L^2}\right\} = \{u\}\left\{\frac{\partial}{\partial x}\right\} = \left\{\frac{L}{T}\right\}\left\{\frac{1}{L}\right\},$$

$$\text{hence} \quad \mathbf{\{k\} = \left\{\frac{1}{T}\right\}} \quad \text{and} \quad \mathbf{\{\mathcal{D}\} = \left\{\frac{L^2}{T}\right\}} \quad \textit{Ans. (a)}$$

(b) To non-dimensionalize the equation, define $u^* = u/V$, $t^* = Vt/L$, and $x^* = x/L$ and sub-stitute into the basic partial differential equation. The dimensionless result is

$$\boldsymbol{u^*\frac{\partial C}{\partial x^*} = \left(\frac{\mathcal{D}}{VL}\right)\frac{\partial^2 C}{\partial x^{*2}} - \left(\frac{kL}{V}\right)C - \frac{\partial C}{\partial t^*}}, \text{ where } \frac{VL}{\mathcal{D}} = \text{mass-transfer Peclet number} \quad \textit{Ans. (b)}$$

5.46 The differential equation for compressible inviscid flow of a gas in the *xy* plane is

$$\frac{\partial^2\phi}{\partial t^2} + \frac{\partial}{\partial t}(u^2+\upsilon^2) + (u^2-a^2)\frac{\partial^2\phi}{\partial x^2} + (\upsilon^2-a^2)\frac{\partial^2\phi}{\partial y^2} + 2u\upsilon\frac{\partial^2\phi}{\partial x\,\partial y} = 0$$

where ϕ is the velocity potential and a is the (variable) speed of sound of the gas. Nondimensionalize this relation, using a reference length L and the inlet speed of sound a_o as parameters for defining dimensionless variables.

Solution: The appropriate dimensionless variables are $u^* = u/a_o$, $t^* = a_o t/L$, $x^* = x/L$, $a^* = a/a_o$, and $\phi^* = \phi/(a_o L)$. Substitution into the PDE for ϕ as above yields

$$\mathbf{\frac{\partial^2\phi^*}{\partial t^{*2}} + \frac{\partial}{\partial t^*}(u^{*2}+v^{*2}) + (u^{*2}-a^{*2})\frac{\partial^2\phi^*}{\partial x^{*2}} + (v^{*2}-a^{*2})\frac{\partial^2\phi^*}{\partial y^{*2}} + 2u^*v^*\frac{\partial^2\phi^*}{\partial x^*\partial y^*} = 0} \quad \textit{Ans.}$$

The PDE comes clean and there are no dimensionless parameters. *Ans.*

5.47 The differential equation for small-amplitude vibrations $y(x, t)$ of a simple beam is given by

$$\rho A\frac{\partial^2 y}{\partial t^2} + EI\frac{\partial^4 y}{\partial x^4} = 0$$

where ρ = beam material density
A = cross-sectional area
I = area moment of inertia
E = Young's modulus

Use only the quantities ρ, E, and A to nondimensionalize y, x, and t, and rewrite the differential equation in dimensionless form. Do any parameters remain? Could they be removed by further manipulation of the variables?

Solution: The appropriate dimensionless variables are

$$y^* = \frac{y}{\sqrt{A}}; \quad t^* = t\sqrt{\frac{E}{\rho A}}; \quad x^* = \frac{x}{\sqrt{A}}$$

Substitution into the PDE above yields a dimensionless equation with *one* parameter:

$$\mathbf{\frac{\partial^2 y^*}{\partial t^{*2}} + \left(\frac{I}{A^2}\right)\frac{\partial^4 y^*}{\partial x^{*4}} = 0;} \quad \text{One geometric parameter: } \mathbf{\frac{I}{A^2}} \quad \textit{Ans.}$$

We could *remove* (I/A^2) completely by redefining $\mathbf{x^* = x/I^{1/4}}$. *Ans.*

5.48 A smooth steel (SG = 7.86) sphere is immersed in a stream of ethanol at 20°C moving at 1.5 m/s. Estimate its drag in N from Fig. 5.3*a*. What stream velocity would quadruple its drag? Take D = 2.5 cm.

Solution: For ethanol at 20°C, take $\rho \approx 789$ kg/m^3 and $\mu \approx 0.0012$ kg/m·s. Then

$$Re_D = \frac{\rho UD}{\mu} = \frac{789(1.5)(0.025)}{0.0012} \approx 24700; \quad \text{Read Fig. 5.3(a):} \quad C_{D,sphere} \approx 0.4$$

$$\text{Compute drag} \quad F = C_D\left(\frac{1}{2}\right)\rho U^2\frac{\pi}{4}D^2 = (0.4)\left(\frac{1}{2}\right)(789)(1.5)^2\left(\frac{\pi}{4}\right)(0.025)^2$$

$$\approx \mathbf{0.17\ N} \quad \textit{Ans.}$$

Since $C_D \approx$ constant in this range of Re_D, **doubling U quadruples** the drag. *Ans.*

5.49 The sphere in Prob. 5.48 is dropped in gasoline at 20°C. Ignoring its acceleration phase, what will its terminal (constant) fall velocity be, from Fig. 5.3*a*?

Solution: For gasoline at 20°C, take $\rho \approx 680$ kg/m^3 and $\mu \approx 2.92\text{E}-4$ kg/m·s. For steel take $\rho \approx 7800$ kg/m^3. Then, in "terminal" velocity, the net weight equals the drag force:

$$\text{Net weight} = (\rho_{steel} - \rho_{gasoline})g\frac{\pi}{6}D^3 = \text{ Drag force,}$$

$$\text{or:} \quad (7800 - 680)(9.81)\frac{\pi}{6}(0.025)^3 = 0.571\text{ N} = C_D\left(\frac{1}{2}\right)(680)U^2\frac{\pi}{4}(0.025)^2$$

$$\text{Guess} \quad C_D \approx 0.4 \quad \text{and compute} \quad \mathbf{U \approx 2.9\ \frac{m}{s}} \quad \textit{Ans.}$$

Now check $Re_D = \rho UD/\mu = 680(2.9)(0.025)/(2.92E{-}4) \approx 170000$. **Yes**, $C_D \approx 0.4$, OK.

5.50 When a microorganism moves in a viscous fluid, inertia (fluid density) has a negligible influence on the organism's drag force. These are called *creeping flows*. The only important parameters are velocity *U*, viscosity μ, and body length scale *L*. (a) Write this relationship in dimensionless form. (b) The drag coefficient $C_D = F/(1/2\ \rho U^2 A)$ is not appropriate for such flows. Define a more appropriate drag coefficient and call is C_c (for creeping flow). (c) For a spherical organism, the drag force can be calculated exactly from creeping-flow theory: $F = 3\pi\mu U d$. Evaluate both forms of the drag coefficient for creeping flow past a sphere.

Solution: (a) If $F = \text{fcn}(U, \mu, L)$, then $n = 4$ and $j = 3$ (MLT), whence we expect $n - j = 4 - 3$ = only *one* pi group, which must therefore be a constant:

$$\Pi_1 = \frac{F}{\mu UL} = Const, \quad or: \quad \boldsymbol{F_{creeping\ flow} = Const\ \mu UL} \quad \textit{Ans.}\text{ (a)}$$

(b) Clearly, the best 'creeping' coefficient is Π_1 itself: $\mathbf{C_c = F/(\mu UL)}$. *Ans.* (b)
(c) If $F_{sphere} = 3\pi\mu Ud$, then $\mathbf{C_c = F/(\mu Ud) = 3\pi}$. *Ans.* (c—*creeping coeff.*)
The standard (inappropriate) form of drag coefficient would be

$$\mathbf{C_D} = (3\pi\mu Ud)/(1/2\ \rho U^2\pi d^2/4) = 24\mu/(\rho Ud) = \mathbf{24/Re_d}. \quad \text{Ans. (c—}\textit{standard}\text{)}$$

5.51 A ship is towing a sonar array which approximates a submerged cylinder 1 ft in diameter and 30 ft long with its axis normal to the direction of tow. If the tow speed is 12 kn (1 kn = 1.69 ft/s), estimate the horsepower required to tow this cylinder. What will be the frequency of vortices shed from the cylinder? Use Figs. 5.2 and 5.3.

Solution: For seawater at 20°C, take $\rho \approx 1.99$ slug/ft^3 and $\mu \approx 2.23E{-}5$ slug/ft·s. Convert V = 12 knots ≈ 20.3 ft/s. Then the Reynolds number and drag of the towed cylinder is

$$Re_D = \frac{\rho UD}{\mu} = \frac{1.99(20.3)(1.0)}{2.23E{-}5} \approx 1.8E6. \quad \text{Fig. 5.3(a) cylinder:} \quad \text{Read } C_D \approx 0.3$$

$$\text{Then} \quad F = C_D\left(\frac{1}{2}\right)\rho U^2 DL = (0.3)\left(\frac{1}{2}\right)(1.99)(20.3)^2(1)(30) \approx 3700 \text{ lbf}$$

$$\text{Power} \quad P = FU = (3700)(20.3) \div 550 \approx \mathbf{140\ hp} \quad \textit{Ans.}\text{ (a)}$$

Data for cylinder vortex shedding is found from Fig. 5.2*b*. At a Reynolds number $Re_D \approx$ 1.8E6, read fD/U ≈ 0.24. Then

$$f_{shedding} = \frac{StU}{D} = \frac{(0.24)(20.3\ ft/s)}{1.0\ ft} \approx \mathbf{5\ Hz} \quad Ans.\ (b)$$

P5.52 A standard table tennis ball is smooth, weighs 2.6 g, and has a diameter of 1.5 in. If struck with an initial velocity of 85 mi/h, in air at 20°C and 1 atm, (*a*) what is the initial deceleration rate? (*b*) What is the estimated uncertainty of your result in past (*a*)?

Solution: Convert everything to SI units: m = 0.0026 kg, D = 0.0381 m, V = 38 m/s. For air at 20°C, take ρ = 1.205 kg/m^3 and μ = 1.8E-5 kg/m-s. (*a*) Calculate the Reynolds number and then read the drag coefficient from Fig. 5.3*a*.

$$\mathrm{Re}_D = \frac{\rho VD}{\mu} = \frac{(1.205 kg/m^3)(38 m/s)(0.0381 m)}{1.8E-5\, kg/m-s} = 97{,}000 \ ; \ \text{Fig.}5.3a: C_D \approx 0.47$$

$$F_{drag} = C_D \frac{\rho}{2} V^2 \frac{\pi}{4} D^2 = (0.47)(\frac{1.205}{2})(38)^2 \frac{\pi}{4}(0.0381)^2 \approx 0.466\, N$$

$$\text{Then} \quad a = \frac{F}{m} = \frac{0.466\, N}{0.0026\, kg} \approx 180 \frac{m}{s^2} \qquad Ans.(a)$$

(*b*) This estimate is quite uncertain, about ±20%, because Fig. 5.3*a* is too small to read accurately. The writer cheated and took a better value from Chap. 7. Figure Prob. D5.2 is better, also.

5.53 Vortex shedding can be used to design a *vortex flowmeter* (Fig. 6.34). A blunt rod stretched across the pipe sheds vortices whose frequency is read by the sensor downstream. Suppose the pipe diameter is 5 cm and the rod is a cylinder of diameter 8 mm. If the sensor reads 5400 counts per minute, estimate the volume flow rate of water in m^3/h. How might the meter react to other liquids?

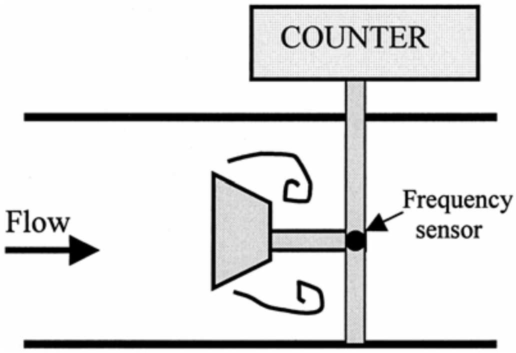


Fig. 6.34

Solution: 5400 counts/min = 90 Hz = f.

$$\text{Guess}\quad \frac{fD}{U} \approx 0.2 = \frac{90(0.008)}{U},\quad \text{or}\quad U \approx 3.6\ \frac{m}{s}$$

$$\text{Check}\quad Re_{D,water} = \frac{998(3.6)(0.008)}{0.001} \approx 29000;\quad \text{Fig. 5.2: Read St} \approx 0.2,\quad \text{OK.}$$

If the centerline velocity is 3.6 m/s and the flow is turbulent, then $V_{avg} \approx 0.82V_{center}$ (see Ex. 3.4 of the text). Then the pipe volume flow is approximately:

$$Q = V_{avg}A_{pipe} = (0.82\times 3.6)\frac{\pi}{4}(0.05\ m)^2 \approx 0.0058\ \frac{m^3}{s} \approx \mathbf{21}\ \frac{\mathbf{m^3}}{\mathbf{hr}}\quad \textit{Ans.}$$

5.54 A fishnet is made of 1-mm-diameter strings knotted into 2 × 2 cm squares. Estimate the horsepower required to tow 300 ft^2 of this netting at 3 kn in seawater at 20°C. The net plane is normal to the flow direction.

Solution: For seawater at 20°C, take $\rho \approx 1025\ kg/m^3$ and $\mu \approx 0.00107$ kg/m·s. Convert V = 3 knots = 1.54 m/s. Then, considering the strings as "cylinders in crossflow," the Reynolds number is Re

$$Re_D = \frac{\rho VD}{\mu} = \frac{(1025)(1.54)(0.001)}{0.00107} \approx 1500;\quad \text{Fig. 5.3(a): } C_{D,cyl} \approx 1.0$$

Drag of one 2-cm strand:

$$F = C_D\frac{\rho}{2}V^2DL = (1.0)\left(\frac{1025}{2}\right)(1.54)^2(0.001)(0.02) \approx 0.0243\ N$$

Now 1 m^2 of net contains *5000* of these 2-cm strands, and 300 ft^2 = 27.9 m^2 of net contains (5000)(27.9) = *139400* strands total, for a total net force F = 139400(0.0243) ≈ 3390 N ÷ 4.4482 = **762 lbf** on the net. Then the horsepower required to tow the net is

$$\text{Power} = FV = (3390\text{ N})(1.54\text{ m/s}) = 5220\text{ W} \div 746 \approx \mathbf{7.0\ hp} \quad \textit{Ans.}$$

5.55 The radio antenna on a car begins to vibrate wildly at **8 Hz** when the car is driven at 45 mi/h over a rutted road which approximates a sine wave of amplitude 2 cm and wavelength λ = 2.5 m. The antenna diameter is 4 mm. Is the vibration due to the road or to vortex shedding?

Solution: Convert U = 45 mi/h = 20.1 m/s. Assume sea level air, ρ = 1.2 kg/m^3, μ = 1.8E–5 kg/m·s. Check the Reynolds number based on antenna diameter: Re_d = (1.2)(20.1)(0.004)/(1.8E–5) = 5400. From Fig. 5.2b, read St ≈ 0.21 = $(\omega/2\pi)d/U$ = (f_{shed})(0.004 m)/(20.1 m/s), or f_{shed} ≈ 1060 Hz ≠ 8 Hz, so **rule out** vortex shedding. Meanwhile, the rutted road introduces a forcing frequency $f_{road} = U/\lambda$ = (20.1 m/s)/(2.5 m) ≈ **8.05 Hz**. We conclude that this resonance is due to ***road roughness***.

5.56 Flow past a long cylinder of square cross-section results in more drag than the comparable round cylinder. Here are data taken in a water tunnel for a square cylinder of side length b = 2 cm:

V, m/s:	1.0	2.0	3.0	4.0
Drag, N/(m of depth):	21	85	191	335

(a) Use this data to predict the drag force per unit depth of wind blowing at 6 m/s, in air at 20°C, over a tall square chimney of side length b = 55 cm. (b) Is there any uncertainty in your estimate?

Solution: Convert the data to the dimensionless form $F/(\rho V^2 bL)$ = fcn($\rho Vb/\mu$), like Eq. (5.2). For air, take ρ = 1.2 kg/m^3 and μ = 1.8E–5 kg/m·s. For water, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. Make a new table using the water data, with L = 1 m:

$F/(\rho V^2 bL)$:	1.05	1.06	1.06	1.05
$\rho Vb/\mu$:	19960	39920	59880	79840

In this Reynolds number range, the force coefficient is approximately constant at about 1.055. Use this value to estimate the air drag on the large chimney:

$$F_{air} = C_F \rho_{air} V_{air}^2 (bL)_{chimney} = (1.055)\left(1.2\ \frac{kg}{m^3}\right)\left(6\ \frac{m}{s}\right)^2 (0.55\ m)(1\ m) \approx \mathbf{25\ N/m} \quad \textit{Ans.}\ \text{(a)}$$

(b) Yes, there **is** uncertainty, because $Re_{chimney} = 220{,}000 > Re_{model} = 80{,}000$ or less.

5.57 The simply supported 1040 carbon-steel rod of Fig. P5.57 is subjected to a crossflow stream of air at 20°C and 1 atm. For what stream velocity *U* will the rod center deflection be approximately 1 cm?

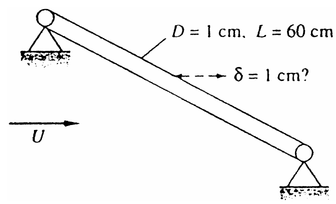


Fig. P5.57

Solution: For air at 20°C, take $\rho \approx 1.2$ kg/m^3 and $\mu \approx 1.8\text{E}{-5}$ kg/m·s. For carbon steel take Young's modulus E ≈ 29E6 psi ≈ 2.0E11 Pa.

This is not an elasticity course, so just use the formula for center deflection of a simply-supported beam:

$$\delta_{center} = \frac{FL^3}{48EI} = 0.01\text{ m} = \frac{F(0.6)^3}{48(2.0E11)[(\pi/4)(0.005)^4]}, \quad \text{solve for } F \approx 218\text{ N}$$

$$\text{Guess} \quad C_D \approx 1.2, \quad \text{then } F = 218\text{ N} = C_D\frac{\rho}{2}V^2DL = (1.2)\left(\frac{1.2}{2}\right)V^2(0.01)(0.6)$$

Solve for V ≈ 225 m/s, check $Re_D = \rho VD/\mu \approx 150{,}000$: OK, $C_D \approx 1.2$ from Fig. 5.3a.

Then **V ≈ 225 m/s**, which is quite high subsonic speed, Mach number ≈ 0.66. *Ans.*

5.58 For the steel rod of Prob. 5.57, at what airstream velocity *U* will the rod begin to vibrate laterally in resonance in its first mode (a half sine wave)? (*Hint:* Consult a vibration text under "lateral beam vibration.")

Solution: From a vibrations book, the first mode frequency for a simply-supported slender beam is given by

$$\omega_n = \pi^2\sqrt{\frac{EI}{mL^4}} \quad \text{where } m = \rho_{steel}\pi R^2 = \text{beam mass per unit length}$$

$$\text{Thus} \quad f_n = \frac{\omega_n}{2\pi} = \frac{\pi}{2}\left[\frac{2.0E11(\pi/4)(0.005)^4}{(7840)\pi(0.005)^2(0.6)^4}\right]^{1/2} \approx 55.1\text{ Hz}$$

The beam will resonate if its vortex shedding frequency is the same. Guess fD/U ≈ 0.2:

$$St = \frac{fD}{U} \approx 0.2 = \frac{55.1(0.01)}{U}, \quad \text{or} \quad U \approx 2.8\ \frac{m}{s}$$

Check $Re_D = \rho VD/\mu \approx 1800$. Fig. 5.2, OK, St ≈ 0.2. Then $\mathbf{V \approx 2.8\ \frac{m}{s}}$ *Ans.*

P5.59 A long, slender, 3-cm-diameter smooth flagpole bends alarmingly in 20 mi/h sea-level winds, causing patriotic citizens to gasp. An engineer claims that the pole will bend less if its surface is deliberately roughened. Is she correct, at least qualitatively?

Solution: For sea-level air, take ρ = 1.2255 kg/m^3 and μ = 1.78E-5 kg/m-s. Convert 20 mi/h = 8.94 m/s. Calculate the Reynolds number of the pole as a "cylinder in crossflow":

$$Re_D = \frac{\rho VD}{\mu} = \frac{(1.2255 kg/m^3)(8.94 m/s)(0.03 m)}{1.78E-5\, kg/m-s} = 18{,}500$$

From Fig. 5.3*b*, we see that this Reynolds number is *below* the region where roughness is effective in reducing cylinder drag. Therefore we think the engineer is ***incorrect***. *Ans.*

[It is more likely that the drag of the *flag* is causing the problem.]

***P5.60** The thrust *F* of a free propeller, either aircraft or marine, depends upon density ρ, the rotation rate *n* in r/s, the diameter *D*, and the forward velocity *V*. Viscous effects are slight and neglected here. Tests of a 25-cm-diameter model aircraft propeller, in a sea-level wind tunnel, yield the following thrust data at a velocity of 20 m/s:

Rotation rate, r/mi	4800	6000	8000
Measured thrust, N	6.1	19	47

(*a*) Use this data to make a crude but effective dimensionless plot. (b) Use the dimensionless data to predict the thrust, in newtons, of a similar 1.6-m-diameter prototype propeller when rotating at 3800 r/min and flying at 225 mi/h at 4000 m standard altitude.

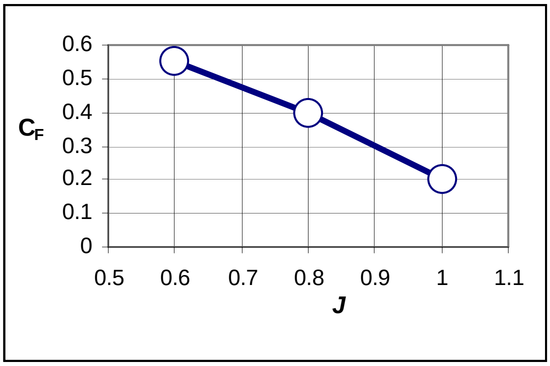
Solution: The given function is $F = \text{fcn}(\rho, n, D, V)$, and we note that $j = 3$. Hence we expect 2 pi groups. The writer chose (ρ, n, D) as repeating variables and found this:

$$C_F = fcn(J), \quad where \quad C_F = \frac{F}{\rho n^2 D^4} \quad and \quad J = \frac{V}{nD}$$

The quantity C_F is called the *thrust coefficient*, while J is called the *advance ratio*. Now use the data (at $\rho = 1.2255\ \text{kg/m}^3$) to fill out a new table showing the two pi groups:

n, r/s	133.3	100.0	80.0
C_F	0.55	0.40	0.20
J	0.60	0.80	1.00

A crude but effective plot of this data is as follows. *Ans*.(*a*)



(*b*) At 4000 m altitude, from Table A.6, ρ = 0.8191 kg/m^3. Convert 225 mi/h = 101.6 m/s. Convert 3800 r/min = 63.3 r/s. Then find the prototype advance ratio:

$$J = (101.6\text{ m/s})/[(63.3\text{ r/s})(1.6\text{ m})] = 1.00$$

Well, lucky us, that's our third data point! Therefore $C_{F,\text{prototype}} \approx 0.20$. And the thrust is

$$F_{prototype} = C_F \rho n^2 D^4 = (0.20)(0.8191\frac{kg}{m^3})(63.3\frac{r}{s})^2(1.6m)^4 \approx \mathbf{4300\ N} \quad \textit{Ans.(b)}$$

5.61 If viscosity is neglected, typical pump-flow results are shown in Fig. P5.61 for a model pump tested in water. The pressure rise decreases and the power required increases with the dimensionless flow coef-ficient. Curve-fit expressions are given for the data. Suppose a similar pump of 12-cm diameter is built to move gasoline at 20°C and a flow rate of 25 m^3/h. If the pump rotation speed is 30 r/s, find (a) the pressure rise and (b) the power required.

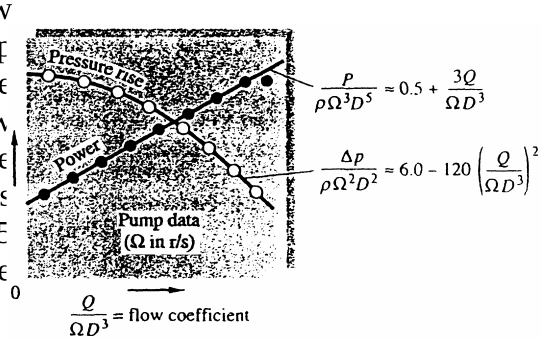


Fig. P5.61

Solution: For gasoline at 20°C, take $\rho \approx 680$ kg/m^3 and $\mu \approx 2.92E{-}4$ kg/m·s. Convert Q = 25 m^3/hr = 0.00694 m^3/s. Then we can evaluate the "flow coefficient":

$$\frac{Q}{\Omega D^3} = \frac{0.00694}{(30)(0.12)^3} \approx 0.134, \quad \text{whence} \quad \frac{\Delta p}{\rho\Omega^2 D^2} \approx 6 - 120(0.134)^2 \approx 3.85$$

$$\text{and} \quad \frac{P}{\rho\Omega^3 D^5} \approx 0.5 + 3(0.134) \approx 0.902$$

With the dimensionless pressure rise and dimensionless power known, we thus find

$$\Delta p = (3.85)(680)(30)^2(0.12)^2 \approx \mathbf{34000\ Pa} \quad \textit{Ans.}\ \text{(a)}$$

$$P = (0.902)(680)(30)^3(0.12)^5 \approx \mathbf{410\ W} \quad \textit{Ans.}\ \text{(b)}$$

5.62 A prototype water pump has an impeller diameter of 2 ft and is designed to pump 12 ft^3/s at 750 r/min. A 1-ft-diameter model pump is tested in 20°C air at 1800 r/min, and Reynolds-number effects are found to be negligible. For similar conditions, what will the volume flow of the model be in ft^3/s? If the model pump requires 0.082 hp to drive it, what horsepower is required for the prototype?

Solution: For air at 20°C, take $\rho \approx 0.00234$ slug/ft^3. For water at 20°C, take $\rho \approx 1.94$ slug/ft^3. The proper Pi groups for this problem are $P/\rho\Omega^3 D^5 = \text{fcn}(Q/\Omega D^3, \rho\Omega D^2/\mu)$. Neglecting μ:

$$\frac{P}{\rho\Omega^3 D^5} = \text{fcn}\left(\frac{Q}{\Omega D^3}\right) \quad \text{if Reynolds number is unimportant}$$

$$\text{Then} \quad Q_{model} = Q_p(\Omega_m/\Omega_p)(D_m/D_p)^3 = 12\left(\frac{1800}{750}\right)\left(\frac{1.0}{2.0}\right)^3 \approx \mathbf{3.6\ \frac{ft^3}{s}} \quad \textit{Ans.}$$

$$\text{Similarly,} \quad P_p = P_m(\rho_p/\rho_m)(\Omega_p/\Omega_m)^3(D_p/D_m)^5 = 0.082\left(\frac{1.94}{0.00234}\right)\left(\frac{750}{1800}\right)^3\left(\frac{2.0}{1.0}\right)^5$$

$$\text{or} \quad P_{proto} \approx \mathbf{157\ hp} \quad \textit{Ans.}$$

5.63 The pressure drop per unit length $\Delta p/L$ in smooth pipe flow is known to be a function only of the average velocity V, diameter D, and fluid properties ρ and μ. The following data were obtained for flow of water at 20°C in an 8-cm-diameter pipe 50 m long:

Q, m^3/s	0.005	0.01	0.015	0.020
Δp, Pa	5800	20,300	42,100	70,800

Verify that these data are slightly outside the range of Fig. 5.10. What is a suitable power-law curve fit for the present data? Use these data to estimate the pressure drop for flow of kerosene at 20°C in a smooth pipe of diameter 5 cm and length 200 m if the flow rate is 50 m^3/h.

Solution: For water at 20°C, take $\rho \approx 998$ kg/m^3 and $\mu \approx 0.001$ kg/m·s. In the spirit of Fig. 5.10 and Example 5.7 in the text, we generate dimensionless Δp and V:

Q, m^3/s:	0.005	0.010	0.015	0.020
$V = Q/A$, m/s:	0.995	1.99	2.98	3.98
$Re = \rho VD/\mu$:	79400	158900	238300	317700
$\rho D^3\Delta p/(L\mu^2)$:	5.93E7	2.07E8	4.30E8	7.24E8

These data, except for the first point, exceed Re = 1E6 and are thus off to the right of the plot in Fig. 5.10. They could fit a "1.75" Power-law, as in *Ans.* (c) as in Ex. 5.7 of the text, but only to ±4%. They fit a "1.80" power-law much more accurately:

$$\frac{\rho \Delta p D^3}{L\mu^2} \approx 0.0901\left(\frac{\rho VD}{\mu}\right)^{1.80} \pm 1\%$$

For kerosene at 20°C, take $\rho \approx 804$ kg/m^3 and $\mu \approx 1.92E{-}3$ kg/m·s. The new length is 200 m, the new diameter is 5 cm, and the new flow rate is 50 m^3/hr. Then evaluate Re:

$$V = \frac{50/3600}{(\pi/4)(0.05)^2} \approx 7.07\ \frac{m}{s}, \quad \text{and} \quad Re_D = \frac{\rho VD}{\mu} = \frac{804(7.07)(0.05)}{1.92E{-}3} \approx 148100$$

$$\text{Then} \quad \rho\Delta pD^3/(L\mu^2) \approx 0.0901(148100)^{1.80} \approx 1.83E8 = \frac{(804)\Delta p(0.05)^3}{(200)(1.92E{-}3)^2}$$

Solve for $\mathbf{\Delta p \approx 1.34E6\ Pa}$ *Ans.*

5.64 The natural frequency ω of vibra-tion of a mass *M* attached to a rod, as in Fig. P5.64, depends only upon *M* and the stiffness *EI* and length *L* of the rod. Tests with a 2-kg mass attached to a 1040 carbon-steel rod of diameter 12 mm and length 40 cm reveal a natural frequency of 0.9 Hz. Use these data to predict the natural frequency of a 1-kg mass attached to a 2024 aluminum-alloy rod of the same size.

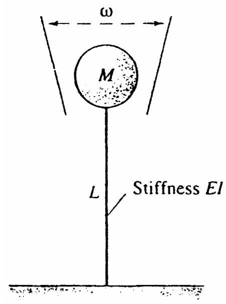


Fig. P5.64

Solution: For steel, E ≈ 29E6 psi ≈ 2.03E11 Pa. If $\omega = f(M, EI, L)$, then $n = 4$ and $j = 3$ (MLT), hence we get only **1** pi group, which we can evaluate from the steel data:

$$\frac{\omega(ML^3)^{1/2}}{(EI)^{1/2}} = \text{constant} = \frac{0.9[(2.0)(0.4)^3]^{1/2}}{[(2.03E11)(\pi/4)(0.006)^4\,]^{1/2}} \approx 0.0224$$

For 2024 aluminum, E ≈ 10.6E6 psi ≈ 7.4E10 Pa. Then re-evaluate the same pi group:

$$\text{New}\quad \frac{\omega(ML^3)^{1/2}}{(EI)^{1/2}} = 0.0224 = \frac{\omega\,[(1.0)(0.4)^3]^{1/2}}{[(7.4E10)(\pi/4)(0.006)^4\,]^{1/2}},\quad \text{or}\quad \boldsymbol{\omega_{alum} \approx 0.77\ \text{Hz}}\quad \textit{Ans.}$$

5.65 In turbulent flow near a flat wall, the local velocity *u* varies only with distance *y* from the wall, wall shear stress τw, and fluid properties ρ and μ. The following data were taken in the University of Rhode Island wind tunnel for airflow, $\rho = 0.0023$ slug/ft^3, $\mu = 3.81E{-}7$ slug/(ft·s), and $\tau w = 0.029$ lbf/ft^2:

y, in	0.021	0.035	0.055	0.080	0.12	0.16
u, ft/s	50.6	54.2	57.6	59.7	63.5	65.9

(a) Plot these data in the form of dimensionless *u* versus dimensionless *y*, and suggest a suitable power-law curve fit. (b) Suppose that the tunnel speed is increased until *u* = 90 ft/s at *y* = 0.11 in. Estimate the new wall shear stress, in lbf/ft^2.

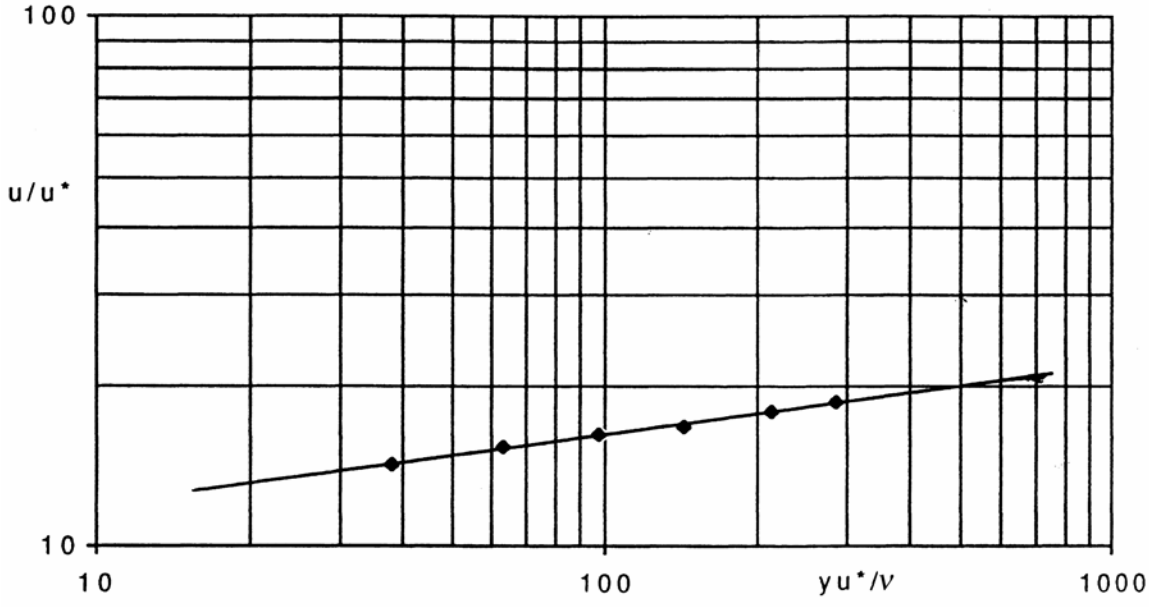
Solution: Given that u = fcn(y, τw, ρ, μ), then *n* = 5 and *j* = 3 (MLT), so we expect n – j = 5 – 3 = **2** pi groups, and they are traditionally chosen as follows (Chap. 6, Section 6.5):

$$\frac{u}{u^*} = \text{fcn}\left(\frac{\rho u^* y}{\mu}\right),\quad \text{where}\ \ u^* = (\tau_w/\rho)^{1/2} = \text{the 'friction velocity'}$$

We may compute $u^* = (\tau_w/\rho)^{1/2} = (0.029/0.0023)^{1/2} = 3.55$ ft/s and then modify the given data into dimensionless parameters:

y, in:	0.021	0.035	0.055	0.080	0.12	0.16
$\rho u^* y/\mu$:	38	63	98	143	214	286
u/u^*:	14.3	15.3	16.2	16.8	17.9	18.6

When plotted on log-log paper as follows, they form nearly a straight line:



The slope of the line is 0.13 and its intercept (at $yu^*/\nu = 1$) is 8.9. Hence the formula:

$$u/u^* \approx 8.9(yu^*/\nu)^{0.13} \pm 1\% \quad \textit{Ans.}\ (a)$$

Now if the tunnel speed is increased until u = 90 ft/s at y = 0.11 in, we may substitute in:

$$\frac{90}{u^*} \approx 8.9\left[\frac{0.0023(0.11/12)u^*}{3.87E{-}7}\right]^{0.13}, \quad \text{solve for } u^* \approx 4.89 \text{ ft/s}$$

$$\text{Solve for} \quad \tau_w = \rho u^{*2} = (0.0023)(4.89)^2 \approx \mathbf{0.055\ lbf/ft^2} \quad \textit{Ans.}\ (b)$$

5.66 A torpedo 8 m below the surface in 20°C seawater cavitates at a speed of 21 m/s when atmospheric pressure is 101 kPa. If Reynolds-number and Froude-number effects are negligible, at what speed will it cavitate when running at a depth of 20 m? At what depth should it be to avoid cavitation at 30 m/s?

Solution: For seawater at 20°C, take ρ = 1025 kg/m^3 and p_v = 2337 Pa. With Reynolds and Froude numbers neglected, the cavitation numbers must simply be the same:

$$Ca = \frac{p_a + \rho gz - p_v}{\rho V^2} \quad \text{for Flow } 1 = \frac{101000 + (1025)(9.81)(8) - 2337}{(1025)(21)^2} \approx 0.396$$

$$\text{(a) At z = 20 m:}\quad Ca = 0.396 = \frac{101000 + 1025(9.81)(20) - 2337}{1025V_a^2},$$

$$\text{or}\quad V_a \approx \mathbf{27.2\ \frac{m}{s}}\quad \textit{Ans.}\text{ (a)}$$

$$\text{(b) At } V_b = 30\ \frac{m}{s}:\quad Ca = 0.396 = \frac{101000 + 1025(9.81)z_b - 2337}{1025(30)^2},$$

$$\text{or}\quad z_b \approx \mathbf{26.5\ m}\quad \textit{Ans.}\text{ (b)}$$

5.67 A student needs to measure the drag on a prototype of characteristic length d_p moving at velocity U_p in air at sea-level conditions. He constructs a model of characteristic length d_m, such that the ratio d_p/d_m = a factor f. He then measures the model drag under dynamically similar conditions, in sea-level air. The student claims that the drag force on the prototype will be identical to that of the model. Is this claim correct? Explain.

Solution: Assuming no compressibility effects, dynamic similarity requires that

$$Re_m = Re_p,\quad or:\quad \frac{\rho_m U_m d_m}{\mu_m} = \frac{\rho_p U_p d_p}{\mu_p},\quad whence\ \ \frac{U_m}{U_p} = \frac{d_p}{d_m} = f$$

Run the tunnel at “f” times the prototype speed, then drag coefficients match:

$$\frac{F_m}{\rho_m U_m^2 d_m^2} = \frac{F_p}{\rho_p U_p^2 d_p^2},\quad or:\quad \frac{F_m}{F_p} = \left(\frac{U_m d_m}{U_p d_p}\right)^2 = \left(\frac{f}{f}\right)^2 = \mathbf{1}\quad \textbf{Yes,}\ \textit{drags are the same}!$$

5.68 Consider viscous flow over a very small object. Analysis of the equations of motion shows that the inertial terms are much smaller than viscous and pressure terms. Fluid density drops out, and these are called *creeping flows*. The only important parameters are velocity U, viscosity μ, and body length scale d. For three-dimensional bodies, like spheres, creeping-flow analysis yields very good results. It is uncertain, however, if creeping flow applies to two-dimensional bodies, such as cylinders, since even though the diameter may be very small, the length of the cylinder is infinite. Let us see if dimensional analysis can help. (a) Apply the Pi theorem to two-dimensional drag force $F_{2\text{-D}}$ as a function of the other parameters. Be careful: two-dimensional drag has dimensions of *force per unit length*, not simply force. (b) Is your analysis in part (a) physically plausible? If not, explain why not. (c) It turns out that fluid density ρ cannot be neglected in analysis of creeping flow over two-dimensional bodies. Repeat the dimensional analysis, this time including ρ as a variable, and find the resulting nondimensional relation between the parameters in this problem.

Solution: If we assume, as given, that $F_{2\text{-D}} = \text{fcn}(\mu,U,d)$, then the dimensions are

$$\{F_{2\text{-}D}\} = \{M/T^2\}; \quad \{\mu\} = \{M/LT\}; \quad \{U\} = \{L/T\}; \quad \{d\} = \{L\}$$

Thus $n = 4$ and $j = 3$ (MLT), hence we expect $n - j$ = only *one* Pi group, which is

$$\Pi_1 = \frac{F_{2\text{-}D}}{\mu U} = Const, \quad or: \quad \mathbf{F_{2\text{-}D} = Const\ \mu U} \quad \textit{Ans.}\ \text{(a)}$$

(b) Is this physically plausible? **No**, because it states that *the body drag is independent of its size L*. Therefore something has been left out of the analysis: ρ. *Ans.* (b)
(c) If density is added, we have $F_{2\text{-D}} = \text{fcn}(\rho,\mu,U,d)$, and a *second* Pi group appears:

$$\Pi_2 = \frac{\rho U d}{\mu} = Re_d; \quad Thus,\ realistically, \quad \frac{\mathbf{F_{2\text{-}D}}}{\mu \mathbf{U}} = \mathbf{fcn}\left(\frac{\rho \mathbf{Ud}}{\mu}\right) \quad \textit{Ans.}\ \text{(c)}$$

Experimental data and theory for *two*-dimensional bodies agree with part (c).

5.69 A simple flow-measurement device for streams and channels is a notch, of angle α, cut into the side of a dam, as shown in Fig. P5.69. The volume flow Q depends only on α, the acceleration of gravity g, and the height δ of the upstream water surface above the notch vertex. Tests of a model notch, of angle α = 55°, yield the following flow rate data:

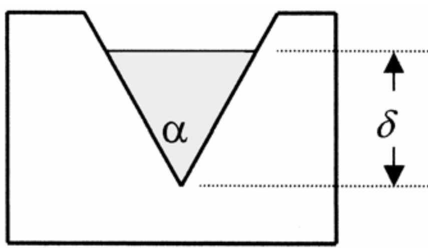


Fig. P5.69

δ, cm:	10	20	30	40
Q, m^3/h:	8	47	126	263

(a) Find a dimensionless correlation for the data. (b) Use the model data to predict the flow rate of a prototype notch, also of angle α = 55°, when the upstream height δ is 3.2 m.

Solution: (a) The appropriate functional relation is $Q = \text{fcn}(\alpha, g, \delta)$ and its dimensionless form is $Q/(g^{1/2}\delta^{5/2}) = \text{fcn}(\alpha)$. Recalculate the data in this dimensionless form, with α constant:

$$Q/(g^{1/2}\delta^{5/2}) = \mathbf{0.224} \quad \mathbf{0.233} \quad \mathbf{0.227} \quad \mathbf{0.230} \quad \text{respectively} \quad \textit{Ans.}\ \text{(a)}$$

(b) The average coefficient in the data is about 0.23. Since the notch angle is still 55°, we may use the formula to predict the larger flow rate:

$$Q_{prototype} = 0.23 g^{1/2}\delta^{5/2} = 0.23\left(9.81\ \frac{m}{s^2}\right)^{1/2}(3.2\ m)^{5/2} \approx \mathbf{13.2\ m^3/s} \quad \textit{Ans.}\ \text{(b)}$$

5.70 A diamond-shaped body, of characteristic length 9 in, has the following measured drag forces when placed in a wind tunnel at sea-level standard conditions:

V, ft/s:	30	38	48	56	61
F, lbf:	1.25	1.95	3.02	4.05	4.81

Use these data to predict the drag force of a similar 15-in diamond placed at similar orientation in 20°C water flowing at 2.2 m/s.

Solution: For sea-level air, take $\rho = 0.00237$ slug/ft^3, $\mu = 3.72\text{E}{-7}$ slug/ft·s. For water at 20°C, take ρ = 1.94 kg/m^3, μ = 2.09E–5 slug/ft·s. Convert the model data into drag coefficient and Reynolds number, taking L_m = 9 in = 0.75 ft:

V_m, ft/s:	30	38	48	56	61
$F/(\rho V^2 L^2)$:	0.667	0.649	0.630	0.621	0.621
$\rho VL/\mu$:	143000	182000	229000	268000	291000

An excellent curve-fit to this data is the power-law

$$C_F \approx 2.5\text{Re}_L^{-0.111} \pm 1\%$$

Now introduce the new case, V_{proto} = 2.2 m/s = 7.22 ft/s, L_{proto} = 15 in = 1.25 ft. Then $\text{Re}_{L,proto} = \dfrac{1.94(7.22)(1.25)}{2.09\text{E}{-5}} \approx 837000$, which is outside the range of the model data. Strictly speaking, we **cannot** use the model data to predict this new case. *Ans.*

If we wish to *extrapolate* to get an estimate, we obtain

$$C_{F,proto} \approx \frac{2.5}{(837000)^{0.111}} \approx 0.550 \approx \frac{F_{proto}}{1.94(7.22)^2(1.25)^2},$$

$$\text{or:} \quad F_{proto} \approx \mathbf{87\ lbf} \quad \textit{Approximately}$$

5.71 The pressure drop in a venturi meter (Fig. P3.165) varies only with the fluid density, pipe approach velocity, and diameter ratio of the meter. A model venturi meter tested in water at 20°C shows a 5-kPa drop when the approach velocity is 4 m/s. A geometrically similar prototype meter is used to measure gasoline at 20°C and a flow rate of 9 m^3/min. If the prototype pressure gage is most accurate at 15 kPa, what should the upstream pipe diameter be?

Solution: Given Δp = fcn(ρ, V, d/D), then by dimensional analysis $\Delta p/(\rho V^2)$ = fcn(d/D). For water at 20°C, take ρ = 998 kg/m^3. For gasoline at 20°C, take ρ = 680 kg/m^3. Then, using the water 'model' data to obtain the function "fcn(d/D)", we calculate

$$\frac{\Delta p_m}{\rho_m V_m^2} = \frac{5000}{(998)(4.0)^2} = 0.313 = \frac{\Delta p_p}{\rho_p V_p^2} = \frac{15000}{(680)V_p^2}, \quad \text{solve for } V_p \approx 8.39\ \frac{m}{s}$$

$$\text{Given}\quad Q = \frac{9}{60}\ \frac{m^3}{s} = V_p A_p = (8.39)\frac{\pi}{4}D_p^2, \quad \text{solve for best } \mathbf{D_p \approx 0.151\ m} \quad \textit{Ans.}$$

5.72 A one-fifteenth-scale model of a parachute has a drag of 450 lbf when tested at 20 ft/s in a water tunnel. If Reynolds-number effects are negligible, estimate the terminal fall velocity at 5000-ft standard altitude of a parachutist using the prototype if chute and chutist together weigh 160 lbf. Neglect the drag coefficient of the woman.

Solution: For water at 20°C, take ρ = 1.94 kg/m^3. For air at 5000-ft standard altitude (Table A-6) take ρ = 0.00205 kg/m^3. If Reynolds number is unimportant, then the two cases have the same drag-force coefficient:

$$C_{Dm} = \frac{F_m}{\rho_m V_m^2 D_m^2} = \frac{450}{1.94(20)^2(D_p/15)^2} = C_{Dp} = \frac{160}{0.00205 V_p^2 D_p^2},$$

$$\text{solve}\quad \mathbf{V_p \approx 24.5\ \frac{ft}{s}} \quad \textit{Ans.}$$

5.73 The power *P* generated by a certain windmill design depends upon its diameter *D*, the air density ρ, the wind velocity *V*, the rotation rate Ω, and the number of blades *n*. (a) Write this relationship in dimensionless form. A model windmill, of diameter 50 cm, develops 2.7 kW at sea level when *V* = 40 m/s and when rotating at 4800 rev/min. (b) What power will be developed by a geometrically and dynamically similar prototype, of diameter 5 m, in winds of 12 m/s at 2000 m standard altitude? (c) What is the appropriate rotation rate of the prototype?

Solution: (a) For the function *P* = fcn(*D*, ρ, *V*, Ω, *n*) the appropriate dimensions are {*P*} = {ML^{2}T^{-3}}, {*D*} = {L}, {ρ} = {ML^{-3}}, {*V*} = {L/T}, {Ω} = {T^{-1}}, and {*n*} = {1}. Using (*D*, ρ, *V*) as repeating variables, we obtain the desired dimensionless function:

$$\boldsymbol{\frac{P}{\rho D^2 V^3} = fcn\left(\frac{\Omega D}{V},\ n\right)} \quad \textit{Ans.}\ \text{(a)}$$

(c) "Geometrically similar" means that *n* is the same for both windmills. For "dynamic similarity," the advance ratio ($\Omega D/V$) must be the same:

$$\left(\frac{\Omega D}{V}\right)_{model} = \frac{(4800\ r/\text{min})(0.5\ m)}{(40\ m/s)} = 1.0 = \left(\frac{\Omega D}{V}\right)_{proto} = \frac{\Omega_{proto}(5\ m)}{12\ m/s},$$

$$\text{or:}\quad \Omega_{proto} = \mathbf{144\ \frac{rev}{min}} \quad \textit{Ans. (c)}$$

(b) At 2000 m altitude, $\rho = 1.0067$ kg/m^3. At sea level, $\rho = 1.2255$ kg/m^3. Since $\Omega D/V$ and n are the same, it follows that the power coefficients equal for model and prototype:

$$\frac{P}{\rho D^2 V^3} = \frac{2700W}{(1.2255)(0.5)^2(40)^3} = \frac{P_{proto}}{(1.0067)(5)^2(12)^3},$$

$$\textit{solve}\quad \mathbf{P_{proto}} = 5990\ W \approx \mathbf{6\ kW} \quad \textit{Ans. (b)}$$

5.74 A one-tenth-scale model of a supersonic wing tested at 700 m/s in air at 20°C and 1 atm shows a pitching moment of 0.25 kN·m. If Reynolds-number effects are negligible, what will the pitching moment of the prototype wing be flying at the same Mach number at 8-km standard altitude?

Solution: If Reynolds number is unimportant, then the dimensionless moment coefficient $M/(\rho V^2 L^3)$ must be a function only of the Mach number, Ma = V/a. For sea-level air, take $\rho = 1.225$ kg/m^3 and sound speed $a = 340$ m/s. For air at 8000-m standard altitude (Table A-6), take $\rho = 0.525$ kg/m^3 and sound speed $a = 308$ m/s. Then

$$Ma_m = \frac{V_m}{a_m} = \frac{700}{340} = 2.06 = Ma_p = \frac{V_p}{308}, \quad \text{solve for}\ V_p \approx 634\ \frac{m}{s}$$

$$\text{Then}\quad M_p = M_m\left(\frac{\rho_p V_p^2 L_p^3}{\rho_m V_m^2 L_m^3}\right) = 0.25\left(\frac{0.525}{1.225}\right)\left(\frac{634}{700}\right)^2\left(\frac{10}{1}\right)^3 \approx \mathbf{88\ kN{\cdot}m} \quad \textit{Ans.}$$

P5.75 According to the web site *USGS Daily Water Data for the Nation*, the mean flow rate in the New River near Hinton, WV is 10,100 ft^3/s. If the hydraulic model in Fig. 5.9 is to match this condition with Froude number scaling, what is the proper model flow rate?

Solution: For Froude scaling, the volume flow rate is a blend of velocity and length terms:

$$\frac{Q_m}{Q_p} = \frac{V_m}{V_p}\frac{A_m}{A_p} = \sqrt{\frac{L_m}{L_p}}\left(\frac{L_m}{L_p}\right)^2 = \left(\frac{L_m}{L_p}\right)^{5/2} \quad \text{or} \quad \alpha^{5/2}$$

$$Fig.5/9: \alpha = 1:65\,; \quad \therefore\ Q_{model} = \left(10100\frac{ft^3}{s}\right)\left(\frac{1}{65}\right)^{5/2} = 0.30\frac{ft^3}{s} \quad Ans.$$

5.76 A 2-ft-long model of a ship is tested in a freshwater tow tank. The measured drag may be split into "friction" drag (Reynolds scaling) and "wave" drag (Froude scaling). The model data are as follows:

Tow speed, ft/s:	0.8	1.6	2.4	3.2	4.0	4.8
Friction drag, lbf:	0.016	0.057	0.122	0.208	0.315	0.441
Wave drag, lbf:	0.002	0.021	0.083	0.253	0.509	0.697

The prototype ship is 150 ft long. Estimate its total drag when cruising at 15 kn in seawater at 20°C.

Solution: For fresh water at 20°C, take ρ = 1.94 slug/ft^3, μ = 2.09E−5 slug/ft·s. Then evaluate the Reynolds numbers and the Froude numbers and respective force coefficients:

V_m, ft/s:	0.8	1.6	2.4	3.2	4.0	4.8
$Re_m = V_m L_m/\nu$:	143000	297000	446000	594000	743000	892000
$C_{F,friction}$:	0.00322	0.00287	0.00273	0.00261	0.00254	0.00247
$Fr_m = V_m/\sqrt{(gL_m)}$:	0.099	0.199	0.299	0.399	0.498	0.598
C_F, wave:	0.00040	0.00106	0.00186	0.00318	0.00410	0.00390

For seawater, take ρ = 1.99 slug/ft^3, μ = 2.23E–5 slug/ft·s. With L_p = 150 ft and V_p = 15 knots = 25.3 ft/s, evaluate

$$Re_{proto} = \frac{\rho_p V_p L_p}{\mu_p} = \frac{1.99(25.3)(150)}{2.23E\text{–}5} \approx 3.39E8; \quad Fr_p = \frac{25.3}{[32.2(150)]^{1/2}} \approx 0.364$$

$$\text{For} \quad Fr \approx 0.364, \quad \text{interpolate to } C_{F,wave} \approx 0.0027$$

Thus we can immediately estimate $F_{wave} \approx 0.0027(1.99)(25.3)^2(150)^2 \approx \underline{77000}$ lbf. However, as mentioned in Fig. 5.8 of the text, Re_p is far outside the range of the friction force data, therefore we must *extrapolate* as best we can. A power-law curve-fit is

$$C_{F,friction} \approx \frac{0.0178}{Re^{0.144}}, \quad \text{hence } C_{F,proto} \approx \frac{0.0178}{(3.39E8)^{0.144}} \approx 0.00105$$

$$\text{Thus} \quad F_{friction} \approx 0.00105(1.99)(25.3)^2(150)^2 \approx \underline{30000}\text{ lbf. } \mathbf{F_{total} \approx 107000\ lbf}. \quad \textit{Ans.}$$

5.77 A dam spillway is to be tested by using Froude scaling with a one-thirtieth-scale model. The model flow has an average velocity of 0.6 m/s and a volume flow of 0.05 m^3/s. What will the velocity and flow of the prototype be? If the measured force on a certain part of the model is 1.5 N, what will the corresponding force on the prototype be?

Solution: Given $\alpha = L_m/L_p$ = 1/30, Froude scaling requires that

$$V_p = \frac{V_m}{\sqrt{\alpha}} = \frac{0.6}{(1/30)^{1/2}} \approx \mathbf{3.3\ \frac{m}{s}}; \quad Q_p = \frac{Q_m}{\alpha^{5/2}} = \frac{0.05}{(1/30)^{5/2}} \approx \mathbf{246\ \frac{m^3}{s}} \quad \textit{Ans.}\text{ (a)}$$

The force scales in similar manner, assuming that the density remains constant (water):

$$F_p = F_m\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2 = F_m(1)\left(\frac{1}{\sqrt{\alpha}}\right)^2\left(\frac{1}{\alpha}\right)^2 = (1.5)(30)^3 \approx \mathbf{40500\ N} \quad \textit{Ans.}\text{ (b)}$$

5.78 A prototype spillway has a characteristic velocity of 3 m/s and a characteristic length of 10 m. A small model is constructed by using Froude scaling. What is the minimum scale ratio of the model which will ensure that its minimum Weber number is 100? Both flows use water at 20°C.

Solution: For water at 20°C, ρ = 998 kg/m^3 and Y = 0.073 N/m, for both model and prototype. Evaluate the Weber number of the prototype:

$$We_p = \frac{\rho_p V_p^2 L_p}{Y_p} = \frac{998(3.0)^2(10.0)}{0.073} \approx 1.23E6; \quad \text{for Froude scaling,}$$

$$\frac{We_m}{We_p} = \frac{\rho_m}{\rho_p}\left(\frac{V_m}{V_p}\right)^2\left(\frac{L_m}{L_p}\right)\left(\frac{Y_p}{Y_m}\right) = (1)(\sqrt{\alpha})^2(\alpha)(1) = \alpha^2 = \frac{100}{1.23E6} \quad \text{if} \quad \alpha = 0.0090$$

Thus the model Weber number will be ≥100 if $\alpha = L_m/L_p \geq 0.0090 = \mathbf{1/111}$. *Ans.*

5.79 An East Coast estuary has a tidal period of 12.42 h (the semidiurnal lunar tide) and tidal currents of approximately 80 cm/s. If a one-five-hundredth-scale model is constructed with tides driven by a pump and storage apparatus, what should the period of the model tides be and what model current speeds are expected?

Solution: Given T_p = 12.42 hr, V_p = 80 cm/s, and $\alpha = L_m/L_p = 1/500$. Then:

$$\text{Froude scaling:} \quad T_m = T_p\sqrt{\alpha} = \frac{12.42}{\sqrt{500}} = 0.555\,\text{hr} \approx \mathbf{33\ min} \quad \textit{Ans.}\text{(a)}$$

$$V_m = V_p\sqrt{\alpha} = 80\Big/\sqrt{(500)} \approx \mathbf{3.6\ cm/s} \quad \textit{Ans.}\text{ (b)}$$

5.80 A prototype ship is 35 m long and designed to cruise at 11 m/s (about 21 kn). Its drag is to be simulated by a 1-m-long model pulled in a tow tank. For Froude scaling find (a) the tow speed, (b) the ratio of prototype to model drag, and (c) the ratio of prototype to model power.

Solution: Given $\alpha = 1/35$, then Froude scaling determines everything:

$$V_{tow} = V_m = V_p\sqrt{\alpha} = 11/\sqrt{(35)} \approx \mathbf{1.86\ m/s}$$

$$F_m/F_p = (V_m/V_p)^2(L_m/L_p)^2 = (\sqrt{\alpha})^2(\alpha)^2 = \alpha^3 = (1/35)^3 \approx \frac{\mathbf{1}}{\mathbf{42900}} \quad \textit{Ans.}$$

$$P_m/P_p = (F_m/F_p)(V_m/V_p) = \alpha^3(\sqrt{\alpha}) = \alpha^{3.5} = 1/35^{3.5} \approx \frac{\mathbf{1}}{\mathbf{254000}}$$

5.81 An airplane, of overall length 55 ft, is designed to fly at 680 m/s at 8000-m standard altitude. A one-thirtieth-scale model is to be tested in a pressurized helium wind tunnel at 20°C. What is the appropriate tunnel pressure in atm? Even at this (high) pressure, exact dynamic similarity is not achieved. Why?

Solution: For air at 8000-m standard altitude (Table A-6), take ρ = 0.525 kg/m^3, μ = 1.53E–5 kg/m·s, and sound speed a = 308 m/s. For helium at 20°C (Table A-4), take gas constant R = 2077 J/(kg·°K), μ = 1.97E–5 kg/m·s, and a = 1005 m/s. For similarity at this supersonic speed, we must match both the Mach and Reynolds numbers. First convert L_p = 55 ft = 16.8 m. Then

$$Ma_p = \frac{680}{308} = 2.21 = Ma_m = \frac{V_m}{1005}, \quad \text{solve for } V_{model} \approx 2219\ \frac{m}{s}$$

$$Re_p = \frac{\rho VL}{\mu}\Big|_p = \frac{0.525(680)(16.8)}{1.53E\text{–}5} = 3.91E8 = Re_m = \frac{\rho_{He}(2219)(16.8/30)}{1.97E\text{–}5}$$

$$\text{Solve for} \quad \rho_{He} \approx 6.21\ kg/m^3 = \frac{p}{RT} = \frac{p_{He}}{(2077)(293)},$$

$$\text{or} \quad \mathbf{p_{He} \approx 3.78\ MPa = 37.3\ atm} \quad \textit{Ans.}$$

Even with Ma and Re matched, true dynamic similarity is not achieved, because the specific heat ratio of helium, $k \approx 1.66$, is not equal to $k_{air} \approx 1.40$.

5.82 A prototype ship is 400 ft long and has a wetted area of 30,000 ft^2. A one-eightieth-scale model is tested in a tow tank according to Froude scaling at speeds of 1.3, 2.0, and 2.7 kn (1 kn = 1.689 ft/s). The measured friction drag of the model at these speeds is 0.11, 0.24, and 0.41 lbf, respectively. What are the three prototype speeds? What is the estimated prototype friction drag at these speeds if we correct for Reynolds-number discrepancy by extrapolation?

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3, μ = 2.09E–5 slug/ft·s. Convert the velocities to ft/sec. Calculate the Reynolds numbers for the model data:

V_m, ft/s:	2.19	3.38	4.56
$Re_m = \rho VL/\mu$:	1.02E6	1.57E6	2.12E6
$C_{Fm} = F/\rho V^2L^2$:	0.000473	0.000433	0.000407

The data may be fit to the Power-law expression $C_{Fm} \approx$ **$0.00805/Re^{0.205}$**. The related *prototype* speeds are given by Froude scaling, $V_p = V_m/\sqrt{\alpha}$, where $\alpha = 1/80$:

V_m, ft/s:	2.19	3.38	4.56	
V_p, ft/s:	**19.6**	**30.2**	**40.8**	*Ans.* (a)

Then we may compute the prototype Reynolds numbers and friction drag coefficients:

$Re_p = \rho VL/\mu$:	7.27E8	1.12E9	1.51E9

Estimate the friction-drag coefficients by extrapolating the Power-law fit listed previously:

$C_{Fp} = F/\rho V^2L^2$:	0.000123	0.000112	0.000106	
$F_p = C_{Fp}\rho V_p^2 L_p^2$:	**14600 lbf**	**31800 lbf**	**54600 lbf**	*Ans.* (b)

Among other approximations, this extrapolation assumes very smooth surfaces.

5.83 A one-fortieth-scale model of a ship's propeller is tested in a tow tank at 1200 r/min and exhibits a power output of 1.4 ft·lbf/s. According to Froude scaling laws, what should the revolutions per minute and horsepower output of the prototype propeller be under dynamically similar conditions?

Solution: Given $\alpha = 1/40$, use Froude scaling laws:

$$\Omega_p/\Omega_m = T_m/T_p = \sqrt{\alpha}, \quad \text{thus } \Omega_p = \frac{1200}{(40)^{1/2}} \approx \mathbf{190\ \frac{rev}{min}} \quad \textit{Ans. (a)}$$

$$P_p = P_m\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{\Omega_p}{\Omega_m}\right)^3\left(\frac{D_p}{D_m}\right)^5 = (1.4)(1)\left(\frac{1}{\sqrt{40}}\right)^3(40)^5$$

$$= 567000 \div 550 = \mathbf{1030\ hp} \quad \textit{Ans. (b)}$$

5.84 A prototype ocean-platform piling is expected to encounter currents of 150 cm/s and waves of 12-s period and 3-m height. If a one-fifteenth-scale model is tested in a wave channel, what current speed, wave period, and wave height should be encountered by the model?

Solution: Given $\alpha = 1/15$, apply straight Froude scaling (Fig. 5.6*b*) to these results:

$$\text{Velocity:} \quad V_m = V_p\sqrt{\alpha} = \frac{150}{\sqrt{15}} = \mathbf{39\ \frac{cm}{s}}$$

$$\text{Period:} \quad T_m = T_p\sqrt{\alpha} = \frac{12}{\sqrt{15}} = \mathbf{3.1\ s}; \quad \text{Height:} \quad H_m = \alpha H_p = \frac{3}{15} = \mathbf{0.20\ m} \quad \textit{Ans.}$$

***P5.85** As shown in Ex. 5.3, pump performance data can be non-dimensionalized. Problem P5.62 gave typical dimensionless data for centrifugal pump "head", $H = \Delta p/\rho g$, as follows:

$$\frac{gH}{n^2D^2} \approx 6.0 - 120\left(\frac{Q}{nD^3}\right)^2$$

where Q is the volume flow rate, n the rotation rate in r/s, and D the impeller diameter. This type of correlation allows one to compute H when (ρ, Q, D) are known. (*a*) Show how to rearrange these Pi groups so that one can *size* the pump, that is, compute D directly when (Q, H, n) are known. (*b*) Make a crude but effective plot of your new function. (*c*) Apply part (*b*) to the following example: When H = 37 m, Q = 0.14 m^3/s, and n = 35 r/s, find the pump diameter for this condition.

Solution: (*a*) We have to eliminate D from one or the other of the two parameters. The writer chose to remove D from the left side. The new parameter will be

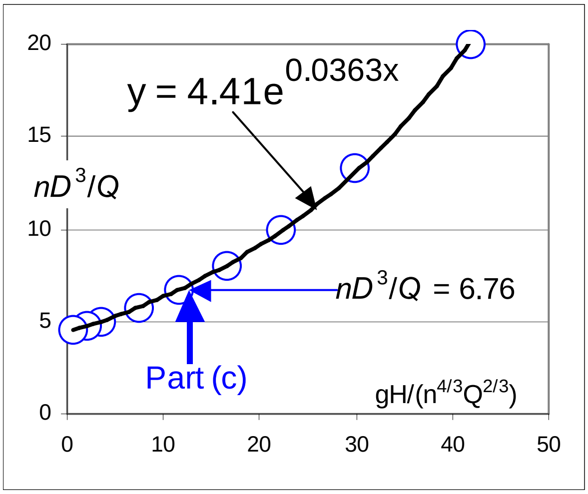
$$\Pi_3 = \frac{gH}{n^2D^2}\left(\frac{nD^3}{Q}\right)^{2/3} = \frac{gH}{n^{4/3}Q^{2/3}}$$

$$\frac{nD^3}{Q} = fcn\left(\frac{gH}{n^{4/3}Q^{2/3}}\right)$$

For convenience, we inverted the right-hand parameter to feature D. Thus the function

will enable one to input (Q, H, n) and immediately solve for the impeller diameter. *Ans*.(*a*)

(*b*) The new variable hopelessly complicates the algebra of the original parabolic formula. However, with a little (well, maybe a *lot of*) work, one can compute and plot a few values:



It fits a least-squared exponential curve quite well, as you see. *Ans.*(*b*)

$$\frac{gH}{n^{4/3}Q^{2/3}} = \frac{(9.81m/s^2)(37\,m)}{(35r/s)^{4/3}(0.14m^3/s)^{2/3}} = 11.76 \quad \textit{Hence}$$

$$\frac{nD^3}{Q} \approx 4.41\exp[0.0363(11.76)] = 6.76 = \frac{35\,D^3}{0.14}, \quad \textit{Solve} \;\; D \approx 0.30\,m \quad \textit{Ans.(c)}$$

(*c*) For the given data, H = 37 m, Q = 0.14 m^3/s, and n = 35 r/s, calculate Π_3:

A 30-cm pump fits these conditions. These Π value solutions are shown on the crude plot above. [NOTE: This problem was set up from the original parabolic function by using D = 30 cm, so the curve-fit is quite accurate.]

5.86 Solve Prob. 5.49 for glycerin, using the modified sphere-drag plot of Fig. 5.11.

Solution: Recall this problem is identical to Prob. 5.85 above except that the fluid is **glycerin**, with ρ = 1260 kg/m^3 and μ = 1.49 kg/m·s. Evaluate the net weight:

$$W = (7800 - 1260)(9.81)\frac{\pi}{6}(0.025)^3 \approx 0.525\text{ N}, \quad \text{whence } \frac{\rho F}{\mu^2} = \frac{1260(0.525)}{(1.49)^2} \approx 298$$

From Fig. 5.11 read Re ≈ 15, or V = 15(1.49)/[1260(0.025)] ≈ **0.7 m/s**. *Ans.*

5.87 In Prob. 5.62 it was difficult to solve for Ω because it appeared in both power and flow coefficients. Rescale the problem, using the data of Fig. P5.61, to make a plot of dimensionless power versus dimension-less rotation speed. Enter this plot directly to solve Prob. 5.62 for Ω.

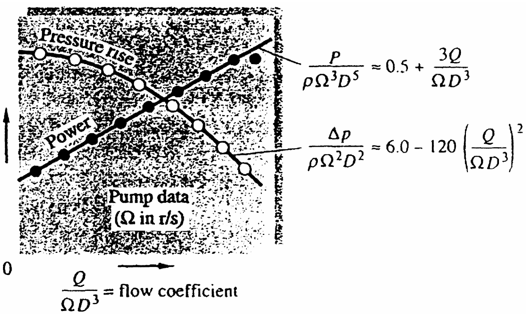


Fig. P5.61

Solution: Recall that the problem was to find the speed Ω for this pump family if D = 12 cm, Q = 25 m^3/hr, and P = 300 W, in gasoline, ρ = 680 kg/m^3 and μ = 2.92E–4 kg/m·s. We can eliminate Ω from the power coefficient for a new type of coefficient:

$$\Pi_3 = \frac{P}{\rho\Omega^3 D^5}\cdot\frac{\Omega^3 D^9}{Q^3} = \frac{\mathbf{PD^4}}{\boldsymbol{\rho}\mathbf{Q^3}}, \quad \text{to be plotted versus } \frac{Q}{\Omega D^3}$$

The plot is shown below, as computed from the expressions in Fig. P5.61.

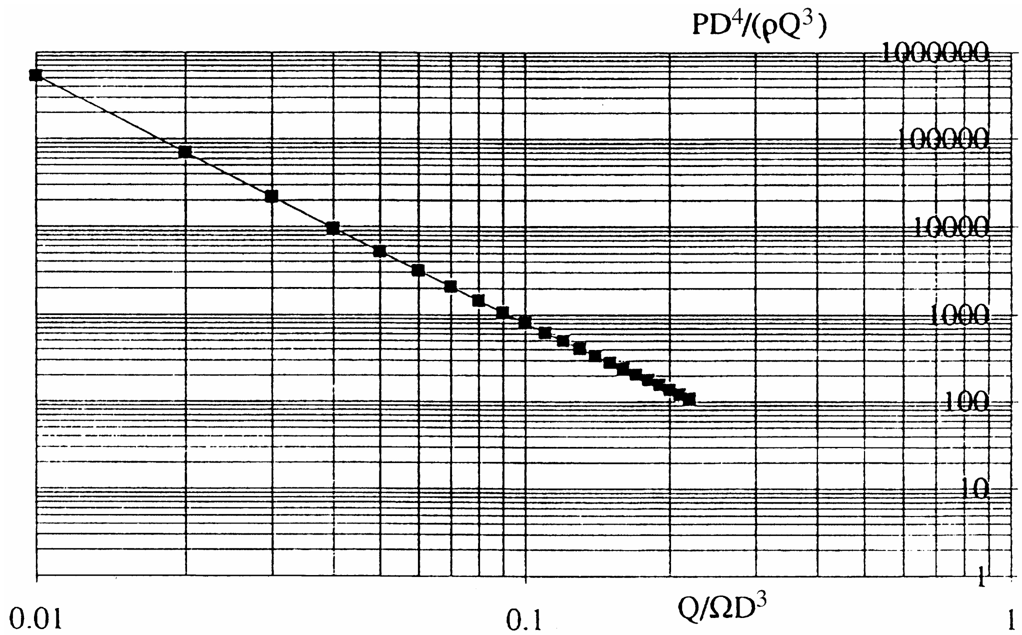


Fig. P5.87

Below $\Pi_3 < 10{,}000$, an excellent Power-law curve-fit is $(Q/\Omega D^3) \approx 1.43/\Pi_3^{0.4} \pm 1\%$.

We use the given data to evaluate Π3 and hence compute $Q/\Omega D^3$:

$$\Pi_3 = \frac{(300)(0.12)^4}{(680)(25/3600)^3} = 273, \quad \text{whence } \frac{Q}{\Omega D^3} \approx \frac{1.43}{(273)^{0.4}} \approx 0.152 = \frac{25/3600}{\Omega(0.12)^3}$$

$$\text{Solve for} \quad \Omega \approx \mathbf{26.5\ rev/s} \quad \textit{Ans.}$$

5.88 Modify Prob. 5.61 as follows: Let Ω = 32 r/s and *Q* = 24 m^3/h for a geometrically similar pump. What is the maximum diameter if the power is not to exceed 340 W? Solve this problem by rescaling the data of Fig. P5.61 to make a plot of dimensionless power versus dimensionless diameter. Enter this plot directly to find the desired diameter.

Solution: We can eliminate D from the power coefficient for an alternate coefficient:

$$\Pi_4 = \frac{P}{\rho\Omega^3 D^5}\cdot\left(\frac{\Omega D^3}{Q}\right)^{5/3} = \frac{\mathbf{P}}{\boldsymbol{\rho\Omega^{4/3}Q^{5/3}}}, \quad \text{to be plotted versus } \frac{Q}{\Omega D^3}$$

The plot is shown below, as computed from the expressions in Fig. P5.61.

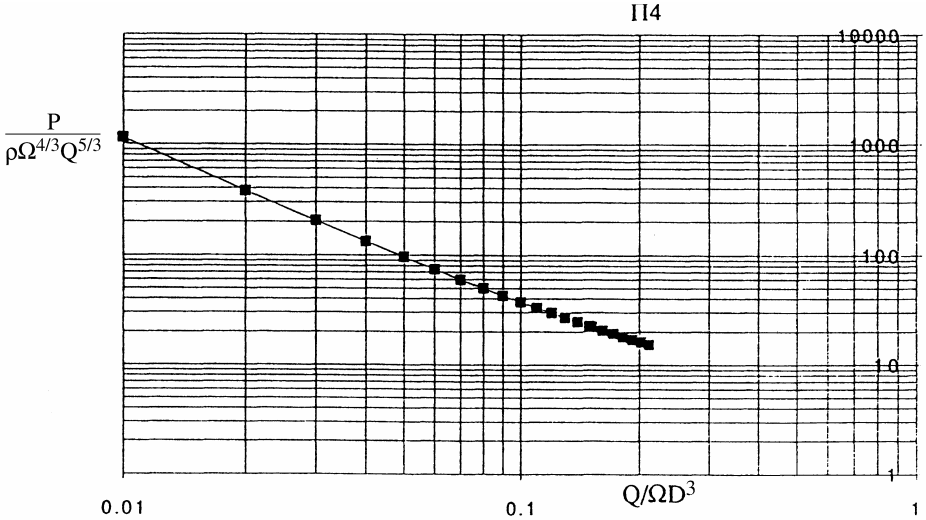


Fig. P5.88

Below Π4 < 1,000, an excellent Power-law curve-fit is $(Q/\Omega D^3) \approx 2.12/\Pi_4^{0.85} \pm 1\%$.
We use the given data to evaluate Π4 and hence compute $Q/\Omega D^3$:

$$\Pi_4 = \frac{340}{680(32)^{4/3}(24/3600)^{5/3}} = 20.8, \quad \text{whence} \quad \frac{Q}{\Omega D^3} = \frac{2.12}{(20.8)^{0.85}} \approx 0.161 = \frac{24/3600}{32D^3}$$

Solve for **D ≈ 0.109 m** *Ans.*

5.89 Knowing that Δp is proportional to L, rescale the data of Example 5.7 to plot dimensionless Δp versus dimensionless *diameter*. Use this plot to find the diameter required in the first row of data in Example 5.7 if the pressure drop is increased to 10 kPa for the same flow rate, length, and fluid.

Solution: Recall that Example 5.7, where $\Delta p/L = \text{fcn}(\rho, V, \mu, D)$, led to the correlation

$$\frac{\rho D^3 \Delta p}{L\mu^2} \approx 0.155\left(\frac{\rho VD}{\mu}\right)^{1.75}$$, which is awkward because D occurs on both sides.

Further, we need $Q = (\pi/4)D^2V$, not V, for the desired correlation, because Q is known. We form this by multiplying the equation by $(\rho Q/D\mu)$ to get a new "Δp vs. D" correlation:

$$\Pi_3 = \frac{\rho D^3 \Delta p}{L\mu^2}\left(\frac{\rho Q}{D\mu}\right)^3 = \frac{\rho^4 Q^3 \Delta p}{L\mu^5} \approx 0.155\left(\frac{4\rho Q}{\pi\mu D}\right)^{1.75}\left(\frac{\rho Q}{D\mu}\right)^3 \approx 0.236\left(\frac{\rho Q}{D\mu}\right)^{4.75} \qquad (2)$$

Correlation "2" can now be used to solve for an unknown diameter. The data are the first row of Example 5.7, with diameter unknown and a new pressure drop listed:

$$L = 5\text{ m}; \quad Q = 0.3\text{ m}^3/\text{hr}; \quad \Delta p = 10,000\text{ Pa}; \quad \rho = 680\text{ kg/m}^3; \quad \mu = 2.92\text{E–}4\text{ kg/m}\cdot\text{s}$$

$$\text{Evaluate}\quad \Pi_3 = \frac{(680)^4(0.3/3600)^3(10000)}{(5)(2.92\text{E}{-}4)^5} \approx 1.17\text{E}20 \approx 0.236(\rho Q/D\mu)^{4.75}$$

$$\text{Solve for}\quad \frac{\rho Q}{D\mu} \approx 22700 = \frac{(680)(0.3/3600)}{D(2.92\text{E}-4)} \quad \text{or} \quad \mathbf{D \approx 0.0085\ m}$$

This solution is restricted to <u>smooth walls</u>, for which the data in Ex. 5.7 was taken.

5.90 Knowing that Δp is proportional to L, rescale the data of Example 5.7 to plot dimensionless Δp versus dimensionless *viscosity*. Use this plot to find the viscosity required in the first row of data in Example 5.7 if the pressure drop is increased to 10 kPa for the same flow rate, length, and density.

Solution: Recall that Example 5.7, where $\Delta p/L = \text{fcn}(\rho, V, \mu, D)$, led to the correlation

$$\frac{\rho D^3\Delta p}{L\mu^2} \approx 0.155\left(\frac{\rho VD}{\mu}\right)^{1.75},\quad \text{which is awkward because } \mu \text{ occurs on both sides.}$$

We can form a "μ-free" parameter by dividing the left side by Reynolds-number-squared:

$$\Pi_4 = \frac{\rho D^3\Delta p/L\mu^2}{(\rho VD/\mu)^2} = \frac{\Delta pD}{\rho V^2 L} \approx \frac{0.155}{(\rho VD/\mu)^{0.25}} \qquad (3)$$

Correlation "3" can now be used to solve for an unknown viscosity. The data are the first row of Example 5.7, with viscosity unknown and a new pressure drop listed:

$$L = 5\text{ m};\quad D = 1\text{ cm};\quad Q = 0.3\text{ m}^3/\text{hr};\quad \Delta p = 10{,}000\text{ Pa};\quad \rho = 680\ \frac{\text{kg}}{\text{m}^3};\quad \cancel{V = 1.06\ \frac{\text{m}}{\text{s}}}$$

$$\text{Evaluate}\quad \Pi_4 = \frac{(10000)(0.01)}{(680)(1.06)^2(5.0)} = 0.0262 \stackrel{?}{=} \frac{0.155}{\text{Re}^{0.25}},\quad \text{or}\quad \text{Re} \approx 1230\ ???$$

This is a trap for the unwary: Re = 1230 is <u>**far below the range of the data**</u> in Ex. 5.7, for which 15000 < Re < 95000. The solution cannot be trusted and in fact is quite <u>incorrect</u>, for the flow would be <u>laminar</u> and follow an entirely different correlation. *Ans.*

***P5.91** The traditional "Moody-type" pipe friction correlation in Chap. 6 is of the form

$$f = \frac{2\Delta p\, D}{\rho V^2 L} = fcn\left(\frac{\rho VD}{\mu}, \frac{\varepsilon}{D}\right)$$

where D is the pipe diameter, L the pipe length, and ε the wall roughness. Note that fluid average velocity V is used on both sides. This form is meant to find Δp when V is known.

(*a*) Suppose that Δp is known and we wish to find V. Rearrange the above function so that V is isolated on the left-hand side. Use the following data, for ε/D = 0.005, to make a plot of your new function, with your velocity parameter as the ordinate of the plot.

f	0.0356	0.0316	0.0308	0.0305	0.0304
$\rho VD/\mu$	15,000	75,000	250,000	900,000	3,330,000

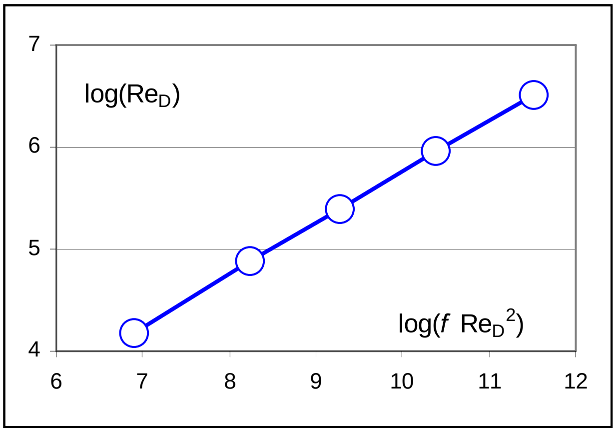
(*b*) Use your plot to determine V, in m/s, for the following pipe flow: D = 5 cm, ε = 0.025 cm, L = 10 m, for water flow at 20°C and 1 atm. The pressure drop Δp is 110 kPa.

Solution: We can eliminate V from the left side by multiplying by Re^2. Then rearrange:

$$\mathrm{Re}_D = fcn\left(f\,\mathrm{Re}_D^2,\ \frac{\varepsilon}{D}\right),\quad or:\quad \frac{\rho VD}{\mu} = fcn\left(\frac{2\rho D^3\,\Delta p}{L\,\mu^2},\ \frac{\varepsilon}{D}\right)$$

We can add a third row to the data above and make a log-log plot:

$f\,\mathrm{Re}_D^2$	8.01E6	1.78E8	1.92E9	2.47E10	3.31E11



It is a pretty good straight line, which means a power-law. A good fit is

$$\frac{\rho VD}{\mu} \approx 4.85\left(\frac{2\rho D^3 \Delta p}{L\mu^2}\right)^{0.507} \quad for \quad \frac{\varepsilon}{D} = 0.005$$

Different power-law constants would be needed for other roughness ratios.

(*b*) Given pipe pressure drop data. For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m-s. Calculate the value of ($f\,Re_D^2$) for this data:

$$f\,\mathrm{Re}_D^2 = \frac{2\rho D^3 \Delta p}{L\mu^2} = \frac{2(998kg/m^3)(0.05m)^3(110000Pa)}{(10\,m)(0.001kg/m-s)^2} = 2.75E9$$

$$\text{Power-law:}\ \frac{\rho VD}{\mu} = 4.85(2.75E9)^{0.507} \approx 296{,}000 = \frac{(998)V(0.05)}{0.001}$$

$$Solve\ \ for \quad V \approx 5.93\ m/s \qquad Ans.(b)$$

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

FE5.1 Given the parameters (U, L, g, ρ, μ) which affect a certain liquid flow problem. The ratio $V^2/(Lg)$ is usually known as the

(a) velocity head (b) Bernoulli head **(c) Froude No.** (d) kinetic energy (e) impact energy

FE5.2 A ship 150 m long, designed to cruise at 18 knots, is to be tested in a tow tank with a model 3 m long. The appropriate tow velocity is

(a) 0.19 m/s (b) 0.35 m/s **(c) 1.31 m/s** (d) 2.55 m/s (e) 8.35 m/s

FE5.3 A ship 150 m long, designed to cruise at 18 knots, is to be tested in a tow tank with a model 3 m long. If the model wave drag is 2.2 N, the estimated full-size ship wave drag is

(a) 5500 N (b) 8700 N (c) 38900 N (d) 61800 N **(e) 275000 N**

FE5.4 A tidal estuary is dominated by the semi-diurnal lunar tide, with a period of 12.42 hours. If a 1:500 model of the estuary is tested, what should be the model tidal period?

(a) 4.0 s (b) 1.5 min (c) 17 min **(d) 33 min** (e) 64 min

FE5.5 A football, meant to be thrown at 60 mi/h in sea-level air (ρ = 1.22 kg/m^3, μ = 1.78E–5 N·s/m^2) is to be tested using a one-quarter scale model in a water tunnel (ρ = 998 kg/m^3, μ = 0.0010 N·s/m^2). For dynamic similarity, what is the proper model water velocity?

(a) 7.5 mi/hr (b) 15.0 mi/hr (c) 15.6 mi/hr **(d) 16.5 mi/hr** (e) 30 mi/hr

FE5.6 A football, meant to be thrown at 60 mi/h in sea-level air (ρ = 1.22 kg/m^3, μ = 1.78E–5 N·s/m^2) is to be tested using a one-quarter scale model in a water tunnel (ρ = 998 kg/m^3, μ = 0.0010 N·s/m^2). For dynamic similarity, what is the ratio of model force to prototype force?

(a) 3.86:1 (b) 16:1 (c) 32:1 (d) 56.2:1 (e) 64:1

FE5.7 Consider liquid flow of density ρ, viscosity μ, and velocity U over a very small model spillway of length scale L, such that the liquid surface tension coefficient Y is important. The quantity $\rho U^2L/Y$ in this case is important and is called the

(a) capillary rise (b) Froude No. (c) Prandtl No. **(d) Weber No.** (e) Bond No.

FE5.8 If a stream flowing at velocity U past a body of length L causes a force F on the body which depends only upon U, L and fluid viscosity μ, then F must be proportional to

(a) $\rho UL/\mu$ (b) ρU^2L^2 (c) $\mu U/L$ **(d) μUL** (e) UL/μ

FE5.9 In supersonic wind tunnel testing, if different gases are used, dynamic similarity requires that the model and prototype have the same Mach number and the same

(a) Euler number (b) speed of sound (c) stagnation enthalpy
(d) Froude number **(e) specific heat ratio**

FE5.10 The Reynolds number for a 1-ft-diameter sphere moving at 2.3 mi/hr through seawater (specific gravity 1.027, viscosity 1.07E–3 N·s/m^2) is approximately

(a) 300 (b) 3000 (c) 30,000 **(d) 300,000** (e) 3,000,000

COMPREHENSIVE PROBLEMS

C5.1 Estimating pipe wall friction is one of the most common tasks in fluids engineering. For long circular, rough pipes in turbulent flow, wall shear τ_W is a function of density ρ, viscosity μ, average velocity V, pipe diameter d, and wall roughness height ε. Thus, functionally, we can write $\tau_W = \text{fcn}(\rho, \mu, V, d, \varepsilon)$. (a) Using dimensional analysis, rewrite this function in dimensionless form. (b) A certain pipe has d = 5 cm and ε = 0.25 mm. For flow of water at 20°C, measurements show the following values of wall shear stress:

Q (in gal/min)	~	1.5	3.0	6.0	9.0	12.0	14.0
τ_W (in Pa)	~	0.05	0.18	0.37	0.64	0.86	1.25

Plot this data in the dimensionless form suggested by your part (a) and suggest a curve-fit formula. Does your plot reveal the entire functional relation suggested in your part (a)?

Solution: (a) There are 6 variables and 3 primary dimensions, therefore we expect 3 Pi groups. The traditional choices are:

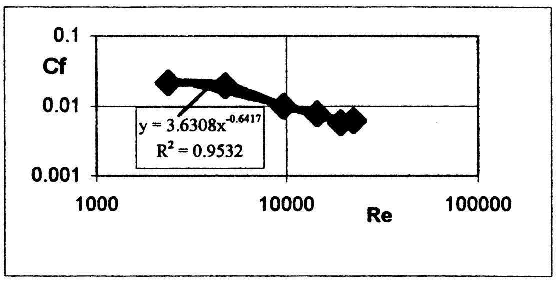
$$\frac{\tau_w}{\rho V^2} = fcn\left(\frac{\rho V d}{\mu}, \frac{\varepsilon}{d}\right) \quad or: \quad \boldsymbol{C_f = fcn\left(\mathrm{Re}, \frac{\varepsilon}{d}\right)} \quad \textit{Ans.}\text{ (a)}$$

(b) In nondimensionalizing and plotting the above data, we find that ε/d = 0.25 mm/50 mm = 0.005 *for all the data*. Therefore we only plot dimensionless shear versus Reynolds number, using ρ = 998 kg/m^3 and μ = 0.001 kg/m·s for water. The results are tabulated as follows:

V, m/s	Re	Cf
0.0481972	2405	0.021567
0.0963944	4810	0.019411
0.1927888	9620	0.009975
0.2891832	14430	0.007668
0.3855776	19240	0.005796
0.4498406	22447	0.00619

When plotted on log-log paper, C_f versus Re makes a slightly curved line.

A reasonable power-law curve-fit is shown o the chart: $\mathbf{C_f \approx 3.63Re^{-0.642}}$ with 95% correlatior *Ans*. (b)
This curve is *only* for the narrow Reynolds numbe range 2000–22000 and a *single* ε/d.



C5.2 When the fluid exiting a nozzle, as in Fig. P3.49, is a *gas*, instead of water, compressibility may be important, especially if upstream pressure p_1 is large and exit diameter d_2 is small. In this case, the difference $(p_1 - p_2)$ is no longer controlling, and the gas mass flow, $\dot{m}$, reaches a maximum value which depends upon p_1 and d_2 and also upon the absolute upstream temperature, T_1, and the gas constant, R. Thus, functionally, $\dot{m} = \text{fcn}(p_1, d_2, T_1, R)$. (a) Using dimensional analysis, rewrite this function in dimensionless form. (b) A certain pipe has d_2 = 1 mm. For flow of air, measurements show the following values of mass flow through the nozzle:

T_1 (in °K)

p_1 (in kPa)

$\dot{m}$ (in kg/s)

Plot this data in the dimensionless form suggested by your part (a). Does your plot reveal the entire functional relation suggested in your part (a)?

Solution: (a) There are $n = 5$ variables and $j = 4$ dimensions (M, L, T, Θ), hence we expect only $n - j = 5 - 4 =$ **1** Pi group, which turns out to be

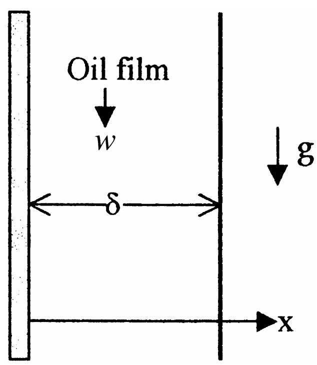
$$\Pi_1 = \frac{\dot{\mathbf{m}}\sqrt{\mathbf{RT_1}}}{\mathbf{p_1 d_2^2}} = \textbf{Constant} \quad \textit{Ans.} \text{ (a)}$$

(b) The data should yield a *single* measured value of Π_1 for all five points:

T_1 (in °K)	~	300	300	300	500	800
$\dot{m}\sqrt{(RT_1)}/(p_1 d_2{}^2)$:		*54.3*	*54.0*	*53.8*	*54.3*	*54.3*

Thus the measured value of Π_1 is about **54.3 ± 0.5** (dimensionless). The problem asks you to *plot* this function, but since it is a *constant*, we shall not bother. *Ans.* (a, b)
PS: The correct value of Π_1 (see Chap. 9) should be about **0.54**, not 54. Sorry: The nozzle diameter d_2 was supposed to be **1 cm**, not 1 mm.

C5.3 Reconsider the fully-developed drain-ing vertical oil-film problem (see Fig. P4.80) as an exercise in dimensional analysis. Let the vertical velocity be a function only of distance from the plate, fluid properties, gravity, and film thickness. That is, $w = \text{fcn}(x, \rho, \mu, g, \delta)$. (a) Use the Pi theorem to rewrite this function in terms of dimensionless parameters. (b) Verify that the exact solution from Prob. 4.80 is consistent with your result in part (a).



Solution: There are $n = 6$ variables and $j = 3$ dimensions (M, L, T), hence we expect only $n - j = 6 - 3 = \mathbf{3}$ Pi groups. The author selects (ρ, g, δ) as repeating variables, whence

$$\Pi_1 = \frac{w}{\sqrt{g\delta}}; \quad \Pi_2 = \frac{\mu}{\rho\sqrt{g\delta^3}}; \quad \Pi_3 = \frac{x}{\delta}$$

Thus the expected function is

$$\boldsymbol{\frac{w}{\sqrt{g\delta}} = fcn\left(\frac{\mu}{\rho\sqrt{g\delta^3}}, \frac{x}{\delta}\right)} \quad \textit{Ans.}\ \text{(a)}$$

(b) The exact solution from Problem 4.80 can be written in just this form:

$$w = \frac{\rho g x}{2\mu}(x - 2\delta), \quad \textit{or:} \quad \underbrace{\frac{w}{\sqrt{g\delta}}}_{\Pi_1}\ \underbrace{\frac{\mu}{\rho\sqrt{g\delta^3}}}_{\Pi_2} = \frac{1}{2}\underbrace{\frac{x}{\delta}}_{\Pi_3}\left(\frac{x}{\delta} - 2\right)$$

Yes, the two forms of dimensionless function are the same. *Ans.* (b)

C5.4 The Taco Inc. Model 4013 centrifugal pump has an impeller of diameter D = 12.95 in. When pumping 20°C water at Ω = 1160 rev/min, the measured flow rate Q and pressure rise Δp are given by the manufacturer as follows:

Q (gal/min)	~	200	300	400	500	600	700
Δp (psi)	~	36	35	34	32	29	23

(a) Assuming that Δp = fcn(ρ, Q, D, Ω), use the Pi theorem to rewrite this function in terms of dimensionless parameters and then plot the given data in dimensionless form. (b) It is desired to use the same pump, running at 900 rev/min, to pump 20°C gasoline at 400 gal/min. According to your dimensionless correlation, what pressure rise Δp is expected, in lbf/in^2?

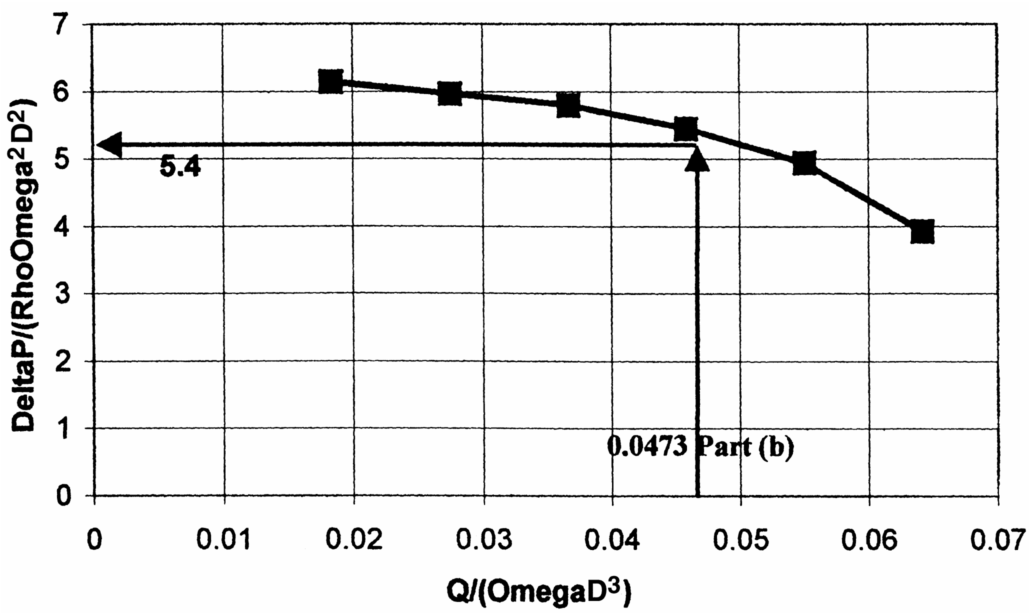
Solution: There are $n = 5$ variables and $j = 3$ dimensions (M, L, T), hence we expect $n - j = 5 - 3 = \mathbf{2}$ Pi groups. The author selects (ρ, D, Ω) as repeating variables, whence

$$\Pi_1 = \frac{\Delta p}{\rho\Omega^2 D^2}; \quad \Pi_2 = \frac{Q}{\Omega D^3}, \quad \textit{or:} \quad \boldsymbol{\frac{\Delta p}{\rho\Omega^2 D^2} = fcn\left(\frac{Q}{\Omega D^3}\right)} \quad \textit{Ans.}\ \text{(a)}$$

Convert the data to this form, using Ω = 19.33 rev/s, D = 1.079 ft, ρ = 1.94 slug/ft^3, and use Δp in lbf/ft^2, not psi, and Q in ft^3/s, not gal/min:

Q (gal/min)	~	200	300	400	500	600	700
$\Delta p/(\rho\Omega^2 D^2)$:		*6.14*	*5.97*	*5.80*	*5.46*	*4.95*	*3.92*
$Q/(\Omega D^3)$:		*0.0183*	*0.0275*	*0.0367*	*0.0458*	*0.0550*	*0.0642*

The dimensionless plot of Π1 versus Π2 is shown below.



(b) The dimensionless chart above is valid for the new conditions, also. Convert 400 gal/min to 0.891 ft^3/s and $\Omega = 900$ rev/min to 15 rev/s. Then evaluate Π2:

$$\Pi_2 = \frac{Q}{\Omega D^3} = \frac{0.891}{15(1.079)^3} = \mathbf{0.0473}$$

This value is entered in the chart above, from which we see that the corresponding value of Π1 is about **5.4**. For gasoline (Table A-3), ρ = 1.32 slug/ft^3. Then this new running condition with gasoline corresponds to

$$\Pi_2 = 5.4 = \frac{\Delta p}{\rho\Omega^2 D^2} = \frac{\Delta p}{1.32(15)^2(1.079)^2}, \quad \textit{solve for} \;\; \Delta p = 1870\ \frac{lbf}{ft^2} = \mathbf{13}\ \frac{\mathbf{lbf}}{\mathbf{in^2}} \quad \textit{Ans.}\text{ (b)}$$

C5.5 Does an automobile radio antenna vibrate in resonance due to vortex shedding? Consider an antenna of length L and diameter D. According to beam-vibration theory [e.g. Kelly [34], p. 401], the first mode natural frequency of a solid circular cantilever beam is $\omega_n = 3.516[EI/(\rho AL^4)]^{1/2}$, where E is the modulus of elasticity, I is the area moment of inertia, ρ is the beam material density, and A is the beam cross-section area. (a) Show that ω_n is proportional to the antenna radius R. (b) If the antenna is steel, with $L = 60$ cm and $D = 4$ mm, estimate the natural vibration frequency, in Hz. (c) Compare with the shedding frequency if the car moves at 65 mi/h.

Solution: (a) From Fig. 2.13 for a circular cross-section, $A = \pi R^2$ and $I = \pi R^4/4$. Then the natural frequency is predicted to be:

$$\omega_n = 3.516\sqrt{\frac{E\pi R^4/4}{\rho\pi R^2 L^4}} = \mathbf{1.758}\sqrt{\frac{\boldsymbol{E}}{\boldsymbol{\rho}}}\,\frac{\boldsymbol{R}}{\boldsymbol{L^2}} = Const \times RP \quad \textit{Ans.}\text{ (a)}$$

(b) For steel, $E = 2.1\text{E}11$ Pa and $\rho = 7840$ kg/m^3. If L = 60 cm and D = 4 mm, then

$$\omega_n = 1.758\sqrt{\frac{2.1E11}{7840}}\,\frac{0.002}{0.6^2} \approx 51\ \frac{rad}{s} \approx \mathbf{8\ Hz} \quad \textit{Ans.}\text{ (b)}$$

(c) For U = 65 mi/h = 29.1 m/s and sea-level air, check $Re_D = \rho UD/\mu = 1.2(29.1)(0.004)/(0.000018) \approx 7800$. From Fig. 5.2*b*, read Strouhal number St ≈ 0.21. Then,

$$\frac{\omega_{shed}D}{2\pi U} = \frac{\omega_{shed}(0.004)}{2\pi(29.1)} \approx 0.21, \quad or: \quad \omega_{shed} \approx 9600\ \frac{rad}{s} \approx \mathbf{1500\ Hz} \quad \textit{Ans.}\text{ (c)}$$

Thus, for a typical antenna, the shedding frequency is far higher than the natural vibration frequency.