Chapter 2 • Pressure Distribution in a Fluid

2.1 For the two-dimensional stress field in Fig. P2.1, let

$$\sigma_{xx} = 3000 \text{ psf} \quad \sigma_{yy} = 2000 \text{ psf}$$
$$\sigma_{xy} = 500 \text{ psf}$$

Find the shear and normal stresses on plane AA cutting through at 30°.

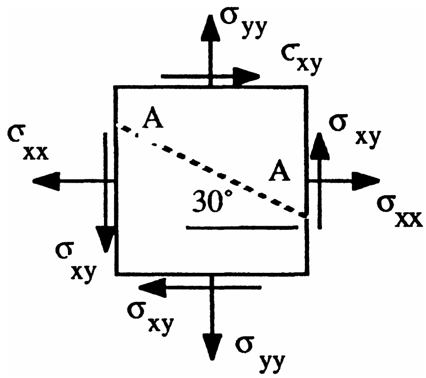
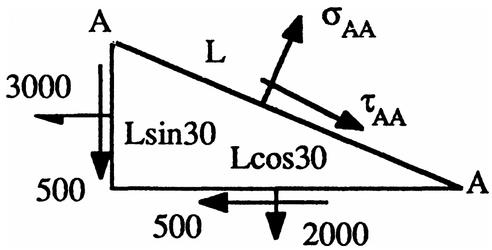


Fig. P2.1

Solution: Make cut "AA" so that it just hits the bottom right corner of the element. This gives the freebody shown at right. Now sum forces normal and tangential to side AA. Denote side length AA as "L."



$$\sum F_{n,AA} = 0 = \sigma_{AA} L - (3000\sin 30 + 500\cos 30)L\sin 30 - (2000\cos 30 + 500\sin 30)L\cos 30$$

Solve for $\sigma_{AA} \approx \mathbf{2683\ lbf/ft^2}$ *Ans.* (a)

$$\sum F_{t,AA} = 0 = \tau_{AA} L - (3000\cos 30 - 500\sin 30)L\sin 30 - (500\cos 30 - 2000\sin 30)L\cos 30$$

Solve for $\tau_{AA} \approx \mathbf{683\ lbf/ft^2}$ *Ans.* (b)

2.2 For the stress field of Fig. P2.1, change the known data to σ_{xx} = 2000 psf, σ_{yy} = 3000 psf, and $\sigma_n(AA)$ = 2500 psf. Compute σ_{xy} and the shear stress on plane AA.

Solution: Sum forces normal to and tangential to AA in the element freebody above, with $\sigma_n(AA)$ known and σ_{xy} unknown:

$$\sum F_{n,AA} = 2500L - (\sigma_{xy}\cos 30° + 2000\sin 30°)L\sin 30° - (\sigma_{xy}\sin 30° + 3000\cos 30°)L\cos 30° = 0$$

$$\text{Solve for} \quad \sigma_{xy} = (2500 - 500 - 2250)/0.866 \approx \mathbf{-289\ lbf/ft^2} \quad \textit{Ans.}\ (a)$$

In like manner, solve for the shear stress on plane AA, using our result for σ_{xy}:

$$\sum F_{t,AA} = \tau_{AA} L - (2000\cos 30° + 289\sin 30°)L\sin 30° + (289\cos 30° + 3000\sin 30°)L\cos 30° = 0$$

$$\text{Solve for} \quad \tau_{AA} = 938 - 1515 \approx \mathbf{-577\ lbf/ft^2} \quad \textit{Ans.}\ (b)$$

This problem and Prob. 2.1 can also be solved using Mohr's circle.

2.3 A vertical clean glass piezometer tube has an inside diameter of 1 mm. When a pressure is applied, water at 20°C rises into the tube to a height of 25 cm. After correcting for surface tension, estimate the applied pressure in Pa.

Solution: For water, let $Y = 0.073$ N/m, contact angle $\theta = 0°$, and $\gamma = 9790\ \text{N/m}^3$. The capillary rise in the tube, from Example 1.9 of the text, is

$$h_{cap} = \frac{2Y\cos\theta}{\gamma R} = \frac{2(0.073\ N/m)\cos(0°)}{(9790\ N/m^3)(0.0005\ m)} = 0.030\ \text{m}$$

Then the rise due to applied pressure is less by that amount: $h_{press} = 0.25\ \text{m} - 0.03\ \text{m} = 0.22$ m. The applied pressure is estimated to be $p = \gamma h_{press} = (9790\ \text{N/m}^3)(0.22\ \text{m}) \approx$ **2160 Pa** *Ans.*

P2.4 For gases over large changes in height, the linear approximation, Eq. (2.14), is inaccurate. Expand the troposphere power-law, Eq. (2.20), into a power series and show that the linear approximation $p \approx p_a - \rho_a g z$ is adequate when

$$\delta z \ll \frac{2T_o}{(n-1)B}, \quad \text{where} \quad n = \frac{g}{RB}$$

Solution: The power-law term in Eq. (2.20) can be expanded into a series:

$$(1 - \frac{Bz}{T_o})^n = 1 - n\frac{Bz}{T_o} + \frac{n(n-1)}{2!}(\frac{Bz}{T_o})^2 - \ldots\ldots \quad \text{where} \quad n = \frac{g}{RB}$$

Multiply by p_a, as in Eq. (2.20), and note that $p_a nB/T_o = (p_a/RT_o)gz = \rho_a gz$. Then the series may be rewritten as follows:

$$p = p_a - \rho_a gz(1 - \frac{n-1}{2}\frac{Bz}{T_o} + \ldots\ldots)$$

For the linear law to be accurate, the 2nd term in parentheses must be much less than unity. If the starting point is not at $z = 0$, then replace z by δz:

$$\frac{n-1}{2}\frac{B\delta z}{T_o} \ll 1 \quad , \qquad \text{or:} \qquad \delta z \ll \frac{2T_o}{(n-1)B} \qquad \textit{Ans.}$$

2.5 Denver, Colorado, has an average altitude of 5300 ft. On a U.S. standard day, pressure gage A reads 83 kPa and gage B reads 105 kPa. Express these readings in gage or vacuum pressure, whichever is appropriate.

Solution: We can find atmospheric pressure by either interpolating in Appendix Table A.6 or, more accurately, evaluate Eq. (2.27) at 5300 ft ≈ 1615 m:

$$p_a = p_o\left(1-\frac{Bz}{T_o}\right)^{g/RB} = (101.35\text{ kPa})\left[1-\frac{(0.0065\text{ K/m})(1615\text{ m})}{288.16\text{ K}}\right]^{5.26} \approx 83.4\text{ kPa}$$

Therefore:

$$\text{Gage A} = 83\text{ kPa} - 83.4\text{ kPa} = -0.4\text{ kPa (gage)} = +0.4\text{ kPa (vacuum)}$$

$$\text{Gage B} = 105\text{ kPa} - 83.4\text{ kPa} = 21.6\text{ kPa (gage)} \quad \textit{Ans.}$$

2.6 Express standard atmospheric pressure as a head, $h = p/\rho g$, in (a) feet of ethylene glycol; (b) inches of mercury; (c) meters of water; and (d) mm of methanol.

Solution: Take the specific weights, $\gamma = \rho g$, from Table A.3, divide p_{atm} by γ:

(a) Ethylene glycol: $h = (2116\text{ lbf/ft}^2)/(69.7\text{ lbf/ft}^3) \approx$ **30.3 ft** *Ans.* (a)

(b) Mercury: $h = (2116\text{ lbf/ft}^2)/(846\text{ lbf/ft}^3) = 2.50\text{ ft} \approx$ **30.0 inches** *Ans.* (b)

(c) Water: $h = (101350\text{ N/m}^2)/(9790\text{ N/m}^3) \approx$ **10.35 m** *Ans.* (c)

(d) Methanol: $h = (101350\text{ N/m}^2)/(7760\text{ N/m}^3) = 13.1\text{ m} \approx$ **13100 mm** *Ans.* (d)

2.7 The deepest point in the ocean is 11034 m in the Mariana Tranch in the Pacific. At this depth $\gamma_{seawater} \approx 10520\text{ N/m}^3$. Estimate the absolute pressure at this depth.

Solution: Seawater specific weight at the surface (Table 2.1) is 10050 N/m^3. It seems quite reasonable to average the surface and bottom weights to predict the bottom pressure:

$$p_{bottom} \approx p_o + \gamma_{abg} h = 101350 + \left(\frac{10050 + 10520}{2}\right)(11034) = 1.136\text{E}8 \text{ Pa} \approx \mathbf{1121\ atm} \quad \textit{Ans.}$$

2.8 A diamond mine is 2 miles below sea level. (a) Estimate the air pressure at this depth. (b) If a barometer, accurate to 1 mm of mercury, is carried into this mine, how accurately can it estimate the depth of the mine?

Solution: (a) Convert 2 miles = 3219 m and use a linear-pressure-variation estimate:

Then $p \approx p_a + \gamma h = 101{,}350\text{ Pa} + (12\text{ N/m}^3)(3219\text{ m}) = 140{,}000\text{ Pa} \approx \mathbf{140\ kPa}$ *Ans.* (a)

Alternately, the troposphere formula, Eq. (2.27), predicts a slightly higher pressure:

$$p \approx p_a(1 - Bz/T_o)^{5.26} = (101.3\text{ kPa})[1 - (0.0065\text{ K/m})(-3219\text{ m})/288.16\text{ K}]^{5.26}$$
$$= \mathbf{147\ kPa} \quad \textit{Ans. (a)}$$

(b) The gage pressure at this depth is approximately 40,000/133,100 ≈ 0.3 m Hg or 300 mm Hg ±1 mm Hg or ±0.3% error. Thus the error in the actual depth is 0.3% of 3220 m or about **±10 m** if all other parameters are accurate. *Ans*. (b)

2.9 Integrate the hydrostatic relation by assuming that the isentropic bulk modulus, $B = \rho(\partial p/\partial \rho)_s$, is constant. Apply your result to the Mariana Trench, Prob. 2.7.

Solution: Begin with Eq. (2.18) written in terms of B:

$$dp = -\rho g\, dz = \frac{B}{\rho}d\rho, \quad \text{or:} \quad \int_{\rho_o}^{\rho} \frac{d\rho}{\rho^2} = -\frac{g}{B}\int_0^z dz = -\frac{1}{\rho} + \frac{1}{\rho_o} = -\frac{gz}{B}, \text{ also integrate:}$$

$$\int_{p_o}^{p} dp = B\int_{\rho_o}^{\rho} \frac{d\rho}{\rho} \quad \text{to obtain } p - p_o = B\ln(\rho/\rho_o)$$

Eliminate ρ between these two formulas to obtain the desired pressure-depth relation:

$$\mathbf{p = p_o - B\ln\left(1 + \frac{g\rho_o z}{B}\right)} \quad \textit{Ans. (a)} \quad \text{With } B_{seawater} \approx 2.33\text{E9 Pa from Table A.3,}$$

$$p_{Trench} = 101350 - (2.33\text{E9})\ln\left[1 + \frac{(9.81)(1025)(-11034)}{2.33\text{E9}}\right]$$
$$= 1.138\text{E8 Pa} \approx \mathbf{1123\ atm} \quad \textit{Ans. (b)}$$

2.10 A closed tank contains 1.5 m of SAE 30 oil, 1 m of water, 20 cm of mercury, and an air space on top, all at 20°C. If p_{bottom} = 60 kPa, what is the pressure in the air space?

Solution: Apply the hydrostatic formula down through the three layers of fluid:

$$p_{bottom} = p_{air} + \gamma_{oil}h_{oil} + \gamma_{water}h_{water} + \gamma_{mercury}h_{mercury}$$

$$\text{or:} \quad 60000\text{ Pa} = p_{air} + (8720\text{ N/m}^3)(1.5\text{ m}) + (9790)(1.0\text{ m}) + (133100)(0.2\text{ m})$$

Solve for the pressure in the air space: $p_{air} \approx \mathbf{10500\ Pa}$ *Ans.*

2.11 In Fig. P2.11, sensor A reads 1.5 kPa (gage). All fluids are at 20°C. Determine the elevations Z in meters of the liquid levels in the open piezometer tubes B and C.

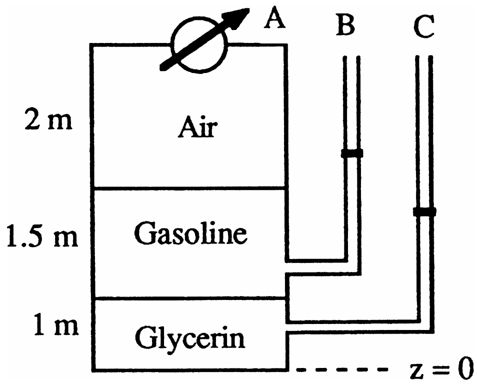


Fig. P2.11

Solution: (B) Let piezometer tube B be an arbitrary distance H above the gasoline-glycerin interface. The specific weights are $\gamma_{air} \approx 12.0\ N/m^3$, $\gamma_{gasoline} = 6670\ N/m^3$, and $\gamma_{glycerin} = 12360\ N/m^3$. Then apply the hydrostatic formula from point A to point B:

$$1500\ N/m^2 + (12.0\ N/m^3)(2.0\ m) + 6670(1.5 - H) - 6670(Z_B - H - 1.0) = p_B = 0\ (gage)$$

Solve for $Z_B = \mathbf{2.73\ m}$ (23 cm above the gasoline-air interface) *Ans.* (b)

Solution (C): Let piezometer tube C be an arbitrary distance Y above the bottom. Then

$$1500 + 12.0(2.0) + 6670(1.5) + 12360(1.0 - Y) - 12360(Z_C - Y) = p_C = 0\ (gage)$$

Solve for $Z_C = \mathbf{1.93\ m}$ (93 cm above the gasoline-glycerin interface) *Ans.* (c)

2.12 In Fig. P2.12 the tank contains water and immiscible oil at 20°C. What is h in centimeters if the density of the oil is 898 kg/m^3?

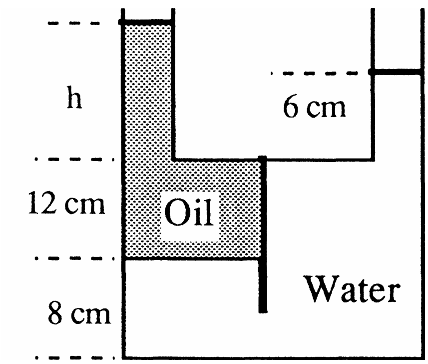


Fig. P2.12

Solution: For water take the density = 998 kg/m^3. Apply the hydrostatic relation from the oil surface to the water surface, skipping the 8-cm part:

$$p_{atm} + (898)(g)(h + 0.12) - (998)(g)(0.06 + 0.12) = p_{atm},$$

Solve for $h \approx 0.08\ m \approx \mathbf{8.0\ cm}$ *Ans.*

2.13 In Fig. P2.13 the 20°C water and gasoline are open to the atmosphere and are at the same elevation. What is the height h in the third liquid?

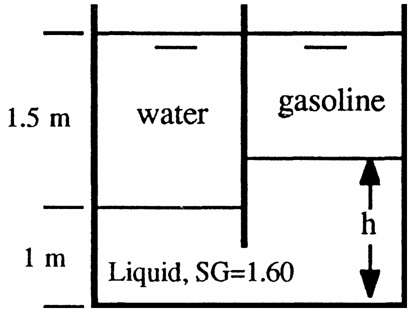


Fig. P2.13

Solution: Take water = 9790 N/m^3 and gasoline = 6670 N/m^3. The bottom pressure must be the same whether we move down through the water or through the gasoline into the third fluid:

$$p_{bottom} = (9790\ \text{N/m}^3)(1.5\ \text{m}) + 1.60(9790)(1.0) = 1.60(9790)h + 6670(2.5 - h)$$

Solve for $h = \mathbf{1.52\ m}$ *Ans.*

P2.14 The symmetric vee-shaped tube in Fig, P2.14 contains static water and air at 20°C. What is the pressure of the air in the closed section at point B?

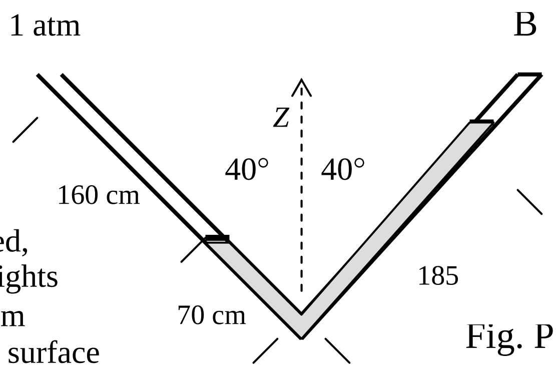


Fig. P2.14

Solution: Naturally the *vertical* depths are needed, not the lengths along the slant. The specific weights are γ_{air} = 11.8 N/m^3 and γ_{water} = 9790 N/m^3, from Table 2.1. From the top left open side to the left surface of the water is a tiny pressure rise, negligible really:

$$p_{left\,surface} = p_a + \gamma_{air}\,\Delta z = 101350 + (11.8)(1.60\cos 40^\circ) = 101350 + 14 = 101364\,Pa$$

Then jump across to the right-hand side and go up to the right-hand surface:

$$p_{right-hand-side} = 101364 - (9790 N/m^3)(1.85 - 0.7m)\cos(40^\circ) = 101364 - 8624 = \mathbf{92{,}700\,Pa}\ \ \textit{Ans.}$$

Certainly close enough – from the right-hand surface to point B is only 4 Pa less. (Some readers write and say the concept of “jumping across” lines of equal pressure is misleading and that I should go down to the bottom of the Vee and then back up. Do you agree?)

2.15 In Fig. P2.15 all fluids are at 20°C. Gage A reads 15 lbf/in^2 absolute and gage B reads 1.25 lbf/in^2 less than gage C. Compute (a) the specific weight of the oil; and (b) the actual reading of gage C in lbf/in^2 absolute.

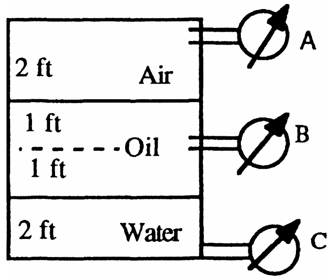


Fig. P2.15

Solution: First evaluate $\gamma_{air} = (p_A/RT)g = [15 \times 144/(1717 \times 528)](32.2) \approx 0.0767$ lbf/ft^3. Take $\gamma_{water} = 62.4$ lbf/ft^3. Then apply the hydrostatic formula from point B to point C:

$$p_B + \gamma_{oil}(1.0\text{ ft}) + (62.4)(2.0\text{ ft}) = p_C = p_B + (1.25)(144)\text{ psf}$$

$$\text{Solve for}\quad \gamma_{oil} \approx \mathbf{55.2\ lbf/ft^3} \qquad \textit{Ans.}\text{ (a)}$$

With the oil weight known, we can now apply hydrostatics from point A to point C:

$$p_C = p_A + \sum \rho gh = (15)(144) + (0.0767)(2.0) + (55.2)(2.0) + (62.4)(2.0)$$

$$\text{or:}\quad p_C = 2395\text{ lbf/ft}^2 = \mathbf{16.6\ psi} \qquad \textit{Ans.}\text{ (b)}$$

P2.16 Suppose that a barometer, using carbon tetrachloride as the working fluid (not recommended), is installed on a standard day in Denver Colorado. (*a*) How high would the fluid rise in the barometer tube? [NOTE: Don't forget the vapor pressure.]
(*b*) Compare this result with a mercury barometer.

Solution: Denver, Colorado is called the "mile-high city" because its average altitude is 5280 ft = 1609 m. By interpolating in Table A.6, we find the standard pressure there is 83,400 Pa. (*a*) From A.4 for carbon tetrachloride, ρ = 1590 kg/m^3. Thus

$$h_{barometer} = \frac{p_{atm} - p_{vapor}}{\rho_{fluid}\, g} = \frac{83400 - 12000\,Pa}{(1590\,kg/m^3)(9.81\,m/s^2)} \approx \mathbf{4.58\,m} \qquad \textit{Ans.}(a)$$

(*b*) For mercury, with ρ = 13550 kg/m^3 and negligible vapor pressure (0.0011 Pa), the same calculation gives $h_{mercury}$ = 83400/[(13550)(9.81)] ≈ **0.627 m** *Ans.*(*b*)

2.17 All fluids in Fig. P2.17 are at 20°C. If p = 1900 psf at point A, determine the pressures at B, C, and D in psf.

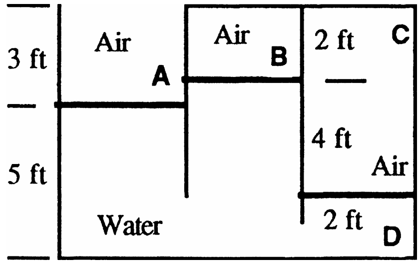


Fig. P2.17

Solution: Using a specific weight of 62.4 lbf/ft^3 for water, we first compute p_B and p_D:

$$p_B = p_A - \gamma_{water}(z_B - z_A) = 1900 - 62.4(1.0\text{ ft}) = \mathbf{1838\ lbf/ft^2} \quad \textit{Ans. (pt. B)}$$

$$p_D = p_A + \gamma_{water}(z_A - z_D) = 1900 + 62.4(5.0\text{ ft}) = \mathbf{2212\ lbf/ft^2} \quad \textit{Ans. (pt. D)}$$

Finally, moving up from D to C, we can neglect the air specific weight to good accuracy:

$$p_C = p_D - \gamma_{water}(z_C - z_D) = 2212 - 62.4(2.0\text{ ft}) = \mathbf{2087\ lbf/ft^2} \quad \textit{Ans. (pt. C)}$$

The air near C has $\gamma \approx 0.074$ lbf/ft^3 times 6 ft yields less than 0.5 psf correction at C.

2.18 All fluids in Fig. P2.18 are at 20°C. If atmospheric pressure = 101.33 kPa and the bottom pressure is 242 kPa absolute, what is the specific gravity of fluid X?

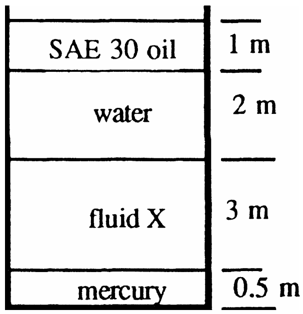


Fig. P2.18

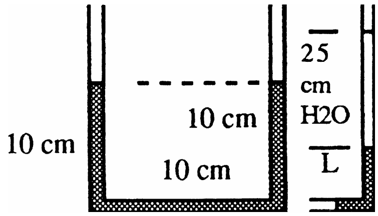
Solution: Simply apply the hydrostatic formula from top to bottom:

$$p_{bottom} = p_{top} + \Sigma\gamma h,$$

or: $$242000 = 101330 + (8720)(1.0) + (9790)(2.0) + \gamma_X(3.0) + (133100)(0.5)$$

Solve for $\gamma_X = 15273\ \text{N/m}^3$, or: $SG_X = \dfrac{15273}{9790} = \mathbf{1.56}$ *Ans.*

2.19 The U-tube at right has a 1-cm ID and contains mercury as shown. If 20 cm^3 of water is poured into the right-hand leg, what will be the free surface height in each leg after the sloshing has died down?



Solution: First figure the height of water added:

$$20\text{ cm}^3 = \frac{\pi}{4}(1\text{ cm})^2 h, \quad \text{or} \quad h = 25.46\text{ cm}$$

Then, at equilibrium, the new system must have 25.46 cm of water on the right, and a 30-cm length of mercury is somewhat displaced so that "L" is on the right, 0.1 m on the bottom, and "0.2 – L" on the left side, as shown at right. The bottom pressure is constant:

$$p_{atm} + 133100(0.2 - L) = p_{atm} + 9790(0.2546) + 133100(L), \quad \text{or:} \quad L \approx 0.0906\text{ m}$$

Thus right-leg-height = 9.06 + 25.46 = **34.52 cm** *Ans.*
left-leg-height = 20.0 – 9.06 = **10.94 cm** *Ans.*

2.20 The hydraulic jack in Fig. P2.20 is filled with oil at 56 lbf/ft^3. Neglecting piston weights, what force F on the handle is required to support the 2000-lbf weight shown?

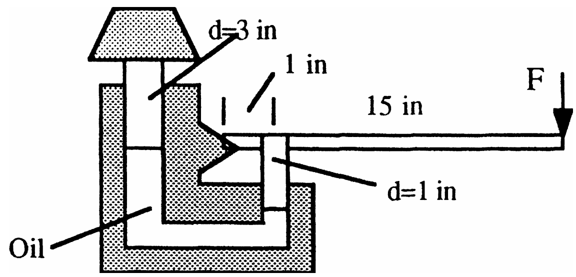


Fig. P2.20

Solution: First sum moments clockwise about the hinge A of the handle:

$$\sum M_A = 0 = F(15+1) - P(1),$$

or: F = P/16, where P is the force in the small (1 in) piston.

Meanwhile figure the pressure in the oil from the weight on the large piston:

$$p_{oil} = \frac{W}{A_{3\text{-in}}} = \frac{2000\text{ lbf}}{(\pi/4)(3/12\text{ ft})^2} = 40744\text{ psf},$$

$$\text{Hence} \quad P = p_{oil}A_{small} = (40744)\frac{\pi}{4}\left(\frac{1}{12}\right)^2 = 222\text{ lbf}$$

Therefore the handle force required is $F = P/16 = 222/16 \approx$ **14 lbf** *Ans.*

2.21 In Fig. P2.21 all fluids are at 20°C. Gage A reads 350 kPa absolute. Determine (a) the height h in cm; and (b) the reading of gage B in kPa absolute.

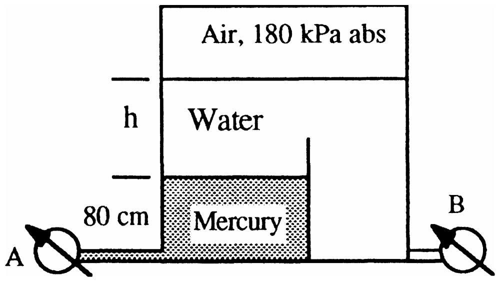


Fig. P2.21

Solution: Apply the hydrostatic formula from the air to gage A:

$$p_A = p_{air} + \sum \gamma h$$

$$= 180000 + (9790)h + 133100(0.8) = 350000 \text{ Pa},$$

Solve for $h \approx$ **6.49 m** *Ans.* (a)

Then, with *h* known, we can evaluate the pressure at gage B:

$$p_B = 180000 + 9790(6.49 + 0.80) = 251000 \text{ Pa} \approx \mathbf{251\ kPa}$$ *Ans.* (b)

2.22 The fuel gage for an auto gas tank reads proportional to the bottom gage pressure as in Fig. P2.22. If the tank accidentally contains 2 cm of water plus gasoline, how many centimeters "h" of air remain when the gage reads "full" in error?

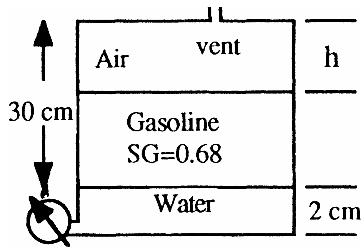


Fig. P2.22

Solution: Given $\gamma_{gasoline} = 0.68(9790) = 6657\ N/m^3$, compute the pressure when "full":

$$p_{full} = \gamma_{gasoline}(\text{full height}) = (6657\ N/m^3)(0.30\ m) = 1997\ Pa$$

Set this pressure equal to 2 cm of water plus "Y" centimeters of gasoline:

$$p_{full} = 1997 = 9790(0.02\ m) + 6657Y, \quad \text{or} \quad Y \approx 0.2706\ m = 27.06\ cm$$

Therefore the air gap $h = 30\ cm - 2\ cm(\text{water}) - 27.06\ cm(\text{gasoline}) \approx \mathbf{0.94\ cm}$ *Ans.*

2.23 In Fig. P2.23 both fluids are at 20°C. If surface tension effects are negligible, what is the density of the oil, in kg/m^3?

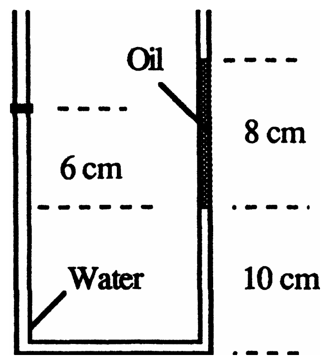


Fig. P2.23

Solution: Move around the U-tube from left atmosphere to right atmosphere:

$$p_a + (9790\ N/m^3)(0.06\ m) - \gamma_{oil}(0.08\ m) = p_a,$$

$$\text{solve for} \quad \gamma_{oil} \approx 7343\ N/m^3,$$

$$\text{or:} \quad \rho_{oil} = 7343/9.81 \approx \mathbf{748\ kg/m^3} \quad \textit{Ans.}$$

2.24 In Prob. 1.2 we made a crude integration of atmospheric density from Table A.6 and found that the atmospheric mass is approximately $m \approx 6.08E18$ kg. Can this result be used to estimate sea-level pressure? Can sea-level pressure be used to estimate m?

Solution: Yes, atmospheric pressure is essentially a result of the weight of the air above. Therefore the air weight divided by the surface area of the earth equals sea-level pressure:

$$p_{sea\text{-}level} = \frac{W_{air}}{A_{earth}} = \frac{m_{air}g}{4\pi R_{earth}^2} \approx \frac{(6.08E18\ kg)(9.81\ m/s^2)}{4\pi(6.377E6\ m)^2} \approx \mathbf{117000\ Pa} \quad \textit{Ans.}$$

This is a little off, thus our mass estimate must have been a little off. If global average sea-level pressure is actually 101350 Pa, then the mass of atmospheric air must be more nearly

$$m_{air} = \frac{A_{earth}p_{sea\text{-}level}}{g} \approx \frac{4\pi(6.377E6\ m)^2(101350\ Pa)}{9.81\ m/s^2} \approx \mathbf{5.28E18\ kg} \quad \textit{Ans.}$$

2.25 Venus has a mass of 4.90E24 kg and a radius of 6050 km. Assume that its atmosphere is 100% CO_2 (actually it is about 96%). Its surface temperature is 730 K, decreasing to 250 K at about z = 70 km. Average surface pressure is 9.1 MPa. Estimate the pressure on Venus at an altitude of 5 km.

Solution: The value of "g" on Venus is estimated from Newton's law of gravitation:

$$g_{Venus} = \frac{Gm_{Venus}}{R^2_{Venus}} = \frac{(6.67E{-}11)(4.90E24\ kg)}{(6.05E6\ m)^2} \approx 8.93\ m/s^2$$

Now, from Table A.4, the gas constant for carbon dioxide is $R_{CO_2} \approx 189\ m^2/(s^2 \cdot K)$. And we may estimate the Venus temperature lapse rate from the given information:

$$B_{Venus} \approx \frac{\Delta T}{\Delta z} \approx \frac{730 - 250\ K}{70000\ m} \approx 0.00686\ K/m$$

Finally the exponent in the p(z) relation, Eq. (2.27), is "n" = g/RB = (8.93)/(189 × 0.00686) ≈ 6.89. Equation (2.27) may then be used to estimate p(z) at z = 10 km on Venus:

$$p_{5\ km} \approx p_o(1 - Bz/T_o)^n \approx (9.1\ MPa)\left[1 - \frac{0.00686\ K/m(5000\ m)}{730\ K}\right]^{6.89} \approx \mathbf{6.5\ MPa} \quad \textit{Ans.}$$

2.26* A *polytropic atmosphere* is defined by the Power-law $p/p_o = (\rho/\rho_o)^m$, where m is an exponent of order 1.3 and p_o and ρ_o are sea-level values of pressure and density. (a) Integrate this expression in the static atmosphere and find a distribution $p(z)$. (b) Assuming an ideal gas, $p = \rho RT$, show that your result in (a) implies a linear temperature distribution as in Eq. (2.25). (c) Show that the standard $B = 0.0065$ K/m is equivalent to $m = 1.235$.

Solution: (a) In the hydrostatic Eq. (2.18) substitute for density in terms of pressure:

$$dp = -\rho g\, dz = -[\rho_o(p/p_o)^{1/m}]g\, dz, \quad \text{or:} \quad \int_{p_o}^{p} \frac{dp}{p^{1/m}} = -\frac{\rho_o g}{p_o^{1/m}} \int_0^z dz$$

Integrate and rearrange to get the result $\displaystyle \boldsymbol{\frac{p}{p_o} = \left[1 - \frac{(m-1)gz}{m(p_o/\rho_o)}\right]^{m/(m-1)}}$ *Ans.* (a)

(b) Use the ideal-gas relation to relate pressure ratio to temperature ratio for this process:

$$\frac{p}{p_o} = \left(\frac{\rho}{\rho_o}\right)^m = \left(\frac{p}{RT}\frac{RT_o}{p_o}\right)^m \quad \text{Solve for} \quad \frac{T}{T_o} = \left(\frac{p}{p_o}\right)^{(m-1)/m}$$

Using p/p_o from *Ans.* (a), we obtain $\dfrac{T}{T_o} = \left[1 - \dfrac{(m-1)gz}{mRT_o}\right]$ *Ans.* (b)

Note that, in using *Ans.* (a) to obtain *Ans.* (b), we have substituted $p_o/\rho_o = RT_o$.

(c) Comparing *Ans.* (b) with the text, Eq. (2.27), we find that lapse rate "*B*" in the text is equal to $(m-1)g/(mR)$. Solve for *m* if $B = 0.0065$ K/m:

$$m = \frac{g}{g - BR} = \frac{9.81\ m/s^2}{9.81\ m/s^2 - (0.0065\ K/m)(287\ m^2/s^2 - R)} = \mathbf{1.235} \quad \textit{Ans. (c)}$$

2.27 This is an *experimental* problem: Put a card or thick sheet over a glass of water, hold it tight, and turn it over without leaking (a glossy postcard works best). Let go of the card. Will the card stay attached when the glass is upside down? **Yes**: This is essentially a *water barometer* and, in principle, could hold a column of water up to 10 ft high!

P2.28 A correlation of numerical results indicates that, all other things being equal, the horizontal distance traveled by a well-hit baseball varies inversely as the cube root of the air density. If a home-run ball hit in New York City travels 400 ft, estimate the distance it would travel in (*a*) Denver, Colorado; and (*b*) La Paz, Bolivia.

Solution: New York City is approximately at sea level, so use the Standard Atmosphere, Table A.6, and take $\rho_{air} = 1.2255$ kg/m^3. Modify Eq. (2.20) for density instead of pressure:

$$\frac{\rho}{\rho_a} = (1 - \frac{Bz}{T_o})^{(g/RB)-1} = (1 - \frac{0.0065\,z}{288.16})^{4.26}$$

Using nominal altitudes from almanacs, apply this formula to Denver and La Paz:

$$(a)\quad \text{Denver, Colorado}: \ z \approx 5280\ ft = 1609\,m\ ; \quad \rho \approx 1.047\ kg/m^3$$

$$(b)\quad \text{La Paz, Bolivia}: \ z \approx 12000\ ft = 3660\,m\ ; \quad \rho \approx 0.849\ kg/m^3$$

Finally apply this to the 400-ft home-run ball:

$$(a)\quad \text{Denver}: \ \text{Distance traveled} = (400\,ft)(\frac{1.2255}{1.047})^{1/3} \approx \mathbf{421\,ft} \quad \textit{Ans.(a)}$$

$$(b)\quad \text{La Paz}: \ \text{Distance traveled} = (400\,ft)(\frac{1.2255}{0.849})^{1/3} \approx \mathbf{452\,ft} \quad \textit{Ans.(b)}$$

In Denver, balls go 5% further, as attested to by many teams visiting Coors Field.

2.29 Show that, for an *adiabatic* atmosphere, $p = C(\rho)^k$, where C is constant, that

$$p/p_o = \left[1 - \frac{(k-1)gz}{kRT_o}\right]^{k/(k-1)}, \quad \text{where } k = c_p/c_v$$

Compare this formula for air at 5 km altitude with the U.S. standard atmosphere.

Solution: Introduce the adiabatic assumption into the basic hydrostatic relation (2.18):

$$\frac{dp}{dz} = -\rho g = \frac{d(C\rho^k)}{dz} = kC\rho^{k-1}\frac{d\rho}{dz}$$

Separate the variables and integrate:

$$\int C\rho^{k-2}d\rho = -\int \frac{g}{k}dz, \quad \text{or:} \quad \frac{C\rho^{k-1}}{k-1} = -\frac{gz}{k} + \text{constant}$$

The constant of integration is related to z = 0, that is, "constant" $= C\rho_o^{k-1}/(k-1)$. Divide this constant out and rewrite the relation above:

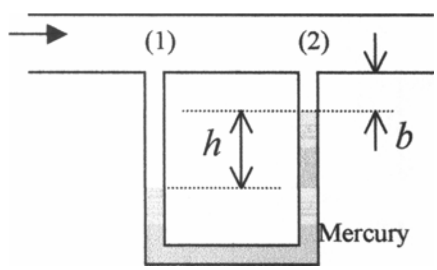
$$\left(\frac{\rho}{\rho_o}\right)^{k-1} = 1 - \frac{(k-1)gz}{kC\rho_o^{k-1}} = (p/p_o)^{(k-1)/k} \quad \text{since } p = C\rho^k$$

Finally, note that $C\rho_o^{k-1} = C\rho_o^k/\rho_o = p_o/\rho_o = RT_o$, where To is the surface temperature. Thus the final desired pressure relation for an adiabatic atmosphere is

$$\frac{p}{p_o} = \left[1 - \frac{(k-1)gz}{kRT_o}\right]^{k/(k-1)} \quad \textit{Ans.}$$

At z = 5,000 m, Table A.6 gives p = 54008 Pa, while the adiabatic formula, with k = 1.40, gives p = **52896 Pa**, or 2.1% lower.

2.30 A mercury manometer is connected at two points to a horizontal 20°C water-pipe flow. If the manometer reading is *h* = 35 cm, what is the pressure drop between the two points?



Solution: This is a classic manometer relation. The two legs of water of height *b* cancel out:

$$p_1 + 9790b + 9790h - 133100h - 9790b = p_2$$
$$p_1 - p_2 = (133{,}100 - 9790 \text{ N/m}^3)(0.35 \text{ m}) \approx \mathbf{43100\ Pa} \quad \textit{Ans.}$$

2.31 In Fig. P2.31 determine Δp between points A and B. All fluids are at 20°C.

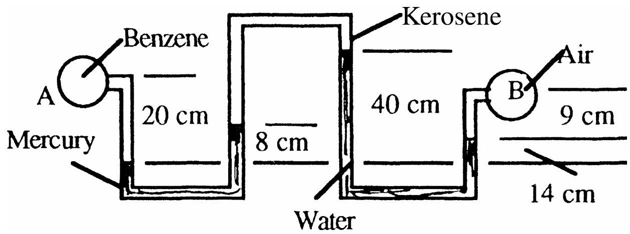


Fig. P2.31

Solution: Take the specific weights to be

Benzene: 8640 N/m^3 Mercury: 133100 N/m^3

Kerosene: 7885 N/m^3 Water: 9790 N/m^3

and γ_{air} will be small, probably around 12 N/m^3. Work your way around from A to B:

$$p_A + (8640)(0.20\text{ m}) - (133100)(0.08) - (7885)(0.32) + (9790)(0.26) - (12)(0.09)$$

$$= p_B, \quad \text{or, after cleaning up,} \quad p_A - p_B \approx \mathbf{8900\ Pa} \quad \textit{Ans.}$$

2.32 For the manometer of Fig. P2.32, all fluids are at 20°C. If $p_B - p_A = 97$ kPa, determine the height H in centimeters.

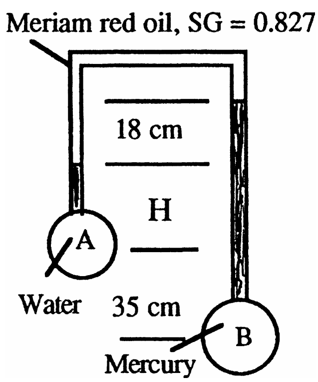


Fig. P2.32

Solution: Gamma = 9790 N/m^3 for water and 133100 N/m^3 for mercury and (0.827)(9790) = 8096 N/m^3 for Meriam red oil. Work your way around from point A to point B:

$$p_A - (9790\text{ N/m}^3)(\text{H meters}) - 8096(0.18)$$

$$+133100(0.18 + H + 0.35) = p_B = p_A + 97000.$$

Solve for $H \approx 0.226\text{ m} = \mathbf{22.6\ cm}$ *Ans.*

2.33 In Fig. P2.33 the pressure at point A is 25 psi. All fluids are at 20°C. What is the air pressure in the closed chamber B?

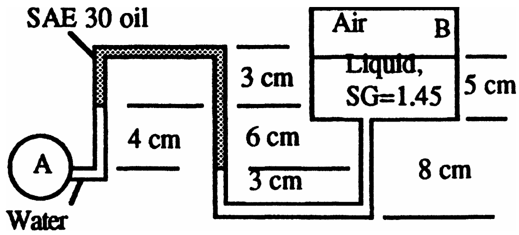


Fig. P2.33

Solution: Take $\gamma = 9790\ N/m^3$ for water, $8720\ N/m^3$ for SAE 30 oil, and (1.45)(9790) = $14196\ N/m^3$ for the third fluid. Convert the pressure at A from 25 lbf/in^2 to 172400 Pa. Compute hydrostatically from point A to point B:

$$p_A + \sum \gamma h = 172400 - (9790\ N/m^3)(0.04\ m) + (8720)(0.06) - (14196)(0.10)$$

$$= p_B = 171100\ Pa \div 47.88 \div 144 = \mathbf{24.8\ psi} \quad \textit{Ans.}$$

2.34 To show the effect of manometer dimensions, consider Fig. P2.34. The containers (a) and (b) are cylindrical and are such that $p_a = p_b$ as shown. Suppose the oil-water interface on the right moves up a distance $\Delta h < h$. Derive a formula for the difference $p_a - p_b$ when (a) $d \ll D$; and (b) $d = 0.15D$. What is the % difference?

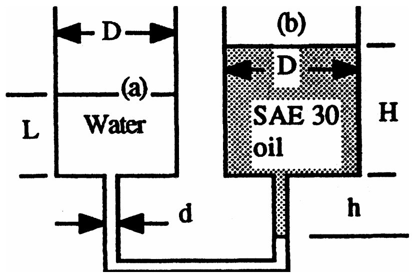


Fig. P2.34

Solution: Take $\gamma = 9790\ \text{N/m}^3$ for water and $8720\ \text{N/m}^3$ for SAE 30 oil. Let "H" be the height of the oil in reservoir (b). For the condition shown, $p_a = p_b$, therefore

$$\gamma_{water}(L + h) = \gamma_{oil}(H + h), \quad \text{or:} \quad H = (\gamma_{water}/\gamma_{oil})(L + h) - h \tag{1}$$

Case (a), $d \ll D$: When the meniscus rises Δh, there will be no significant change in reservoir levels. Therefore we can write a simple hydrostatic relation from (a) to (b):

$$p_a + \gamma_{water}(L + h - \Delta h) - \gamma_{oil}(H + h - \Delta h) = p_b,$$

$$\text{or:} \quad \mathbf{p_a - p_b = \Delta h(\gamma_{water} - \gamma_{oil})} \quad \textit{Ans.}\ \text{(a)}$$

where we have used Eq. (1) above to eliminate H and L. Putting in numbers to compare later with part (b), we have $\Delta p = \Delta h(9790 - 8720) = \mathit{1070}\ \Delta h$, with Δh in meters.

Case (b), $d = 0.15D$. Here we must account for reservoir volume changes. For a rise $\Delta h < h$, a volume $(\pi/4)d^2\Delta h$ of *water* leaves reservoir (a), decreasing "L" by $\Delta h(d/D)^2$, and an identical volume of oil enters reservoir (b), increasing "H" by the same amount $\Delta h(d/D)^2$. The hydrostatic relation between (a) and (b) becomes, for this case,

$$p_a + \gamma_{water}[L - \Delta h(d/D)^2 + h - \Delta h] - \gamma_{oil}[H + \Delta h(d/D)^2 + h - \Delta h] = p_b,$$

$$\text{or:} \quad p_a - p_b = \mathbf{\Delta h[\gamma_{water}(1 + d^2/D^2) - \gamma_{oil}(1 - d^2/D^2)]} \quad \textit{Ans.}\ \text{(b)}$$

where again we have used Eq. (1) to eliminate H and L. If d is not small, this is a *considerable* difference, with surprisingly large error. For the case d = 0.15 D, with water and oil, we obtain $\Delta p = \Delta h[1.0225(9790) - 0.9775(8720)] \approx 1486\ \Delta h$ or **<u>39% more</u>** than (a).

2.35 Water flows upward in a pipe slanted at 30°, as in Fig. P2.35. The mercury manometer reads h = 12 cm. What is the pressure difference between points (1) and (2) in the pipe?

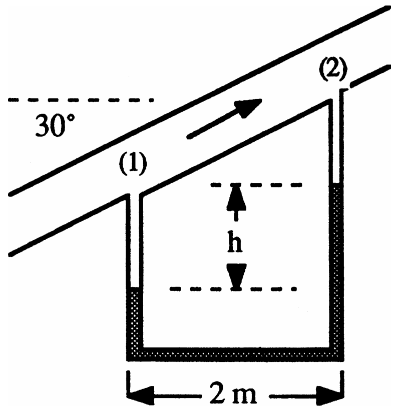


Fig. P2.35

Solution: The vertical distance between points 1 and 2 equals (2.0 m)tan 30° or 1.155 m. Go around the U-tube hydrostatically from point 1 to point 2:

$$p_1 + 9790h - 133100h - 9790(1.155\text{ m}) = p_2,$$

$$\text{or:}\quad p_1 - p_2 = (133100 - 9790)(0.12) + 11300 = \mathbf{26100\ Pa}\quad \textit{Ans.}$$

2.36 In Fig. P2.36 both the tank and the slanted tube are open to the atmosphere. If L = 2.13 m, what is the angle of tilt ϕ of the tube?

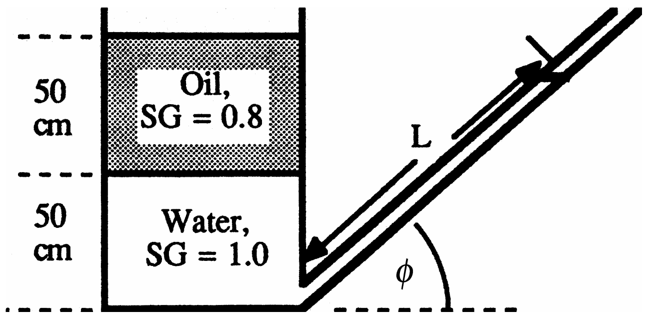


Fig. P2.36

Solution: Proceed hydrostatically from the oil surface to the slanted tube surface:

$$p_a + 0.8(9790)(0.5) + 9790(0.5) - 9790(2.13\sin\phi) = p_a,$$

$$\text{or:}\quad \sin\phi = \frac{8811}{20853} = 0.4225,\quad \text{solve}\quad \phi \approx \mathbf{25°}\quad \textit{Ans.}$$

2.37 The inclined manometer in Fig. P2.37 contains Meriam red oil, SG = 0.827. Assume the reservoir is very large. If the inclined arm has graduations 1 inch apart, what should θ be if each graduation represents 1 psf of the pressure p_A?

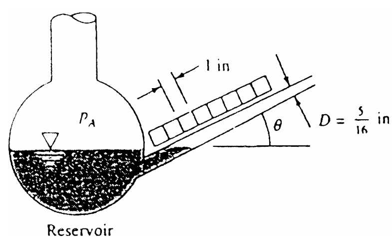


Fig. P2.37

Solution: The specific weight of the oil is (0.827)(62.4) = 51.6 lbf/ft^3. If the reservoir level does not change and ΔL = 1 inch is the scale marking, then

$$p_A(\text{gage}) = 1\,\frac{\text{lbf}}{\text{ft}^2} = \gamma_{\text{oil}}\Delta z = \gamma_{\text{oil}}\Delta L\ \sin\theta = \left(51.6\,\frac{\text{lbf}}{\text{ft}^3}\right)\left(\frac{1}{12}\text{ ft}\right)\sin\theta,$$

$$\text{or:}\quad \sin\theta = 0.2325 \quad \text{or:}\quad \theta = \mathbf{13.45°} \quad \textit{Ans.}$$

2.38 In the figure at right, new tubing contains gas whose density is greater than the outside air. For the dimensions shown, (a) find p1(gage). (b) Find the error caused by assuming ρtube = ρair. (c) Evaluate the error if ρm = 860, ρa = 1.2, and ρt = 1.5 kg/m^3, H = 1.32 m, and h = 0.58 cm.

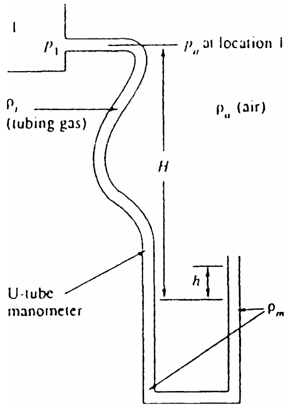


Fig. P2.38

Solution: (a) Work hydrostatically around the manometer:

$$p_1 + \rho_t gH = p_a + \rho_m gh + \rho_a g(H-h),$$

$$\text{or:}\quad \mathbf{p_{1\,gage} = (\rho_m - \rho_a)gh - (\rho_t - \rho_a)gH} \quad \textit{Ans. (a)}$$

(b) From (a), the error is the last term: $\mathbf{Error = -(\rho_t - \rho_a)gH}$ *Ans.* (b)

(c) For the given data, the normal reading is (860 – 1.2)(9.81)(0.0058) = 48.9 Pa, and

$$\textit{Error} = -(1.50-1.20)(9.81)(1.32) = \mathbf{-3.88\ Pa}\ (\textit{about}\ 8\%) \quad \textit{Ans. (c)}$$

2.39 In Fig. P2.39 the right leg of the manometer is open to the atmosphere. Find the gage pressure, in Pa, in the air gap in the tank. Neglect surface tension.

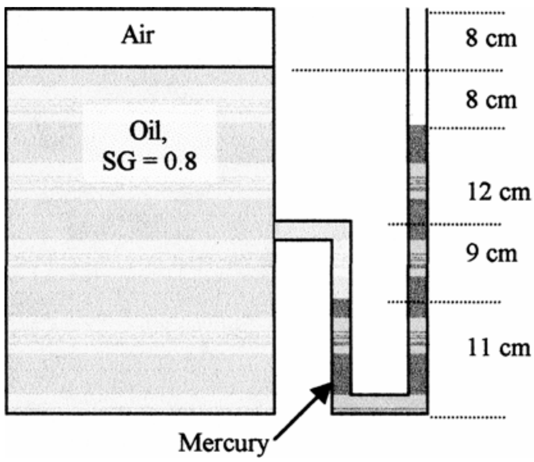


Fig. P2.39

Solution: The two 8-cm legs of air are negligible (only 2 Pa). Begin at the right mercury interface and go to the air gap:

$$0\ \text{Pa-gage} + (133100\ \text{N/m}^3)(0.12+0.09\ \text{m})$$
$$-(0.8\times 9790\ \text{N/m}^3)(0.09-0.12-0.08\ \text{m})$$
$$= p_{\text{airgap}}$$

$$\text{or:}\quad p_{\text{airgap}} = 27951\ \text{Pa} - 2271\ \text{Pa} \approx \mathbf{25700\ Pa\text{-}gage} \quad \textit{Ans.}$$

2.40 In Fig. P2.40 the pressures at A and B are the same, 100 kPa. If water is introduced at A to increase p_A to 130 kPa, find and sketch the new positions of the mercury menisci. The connecting tube is a uniform 1-cm in diameter. Assume no change in the liquid densities.

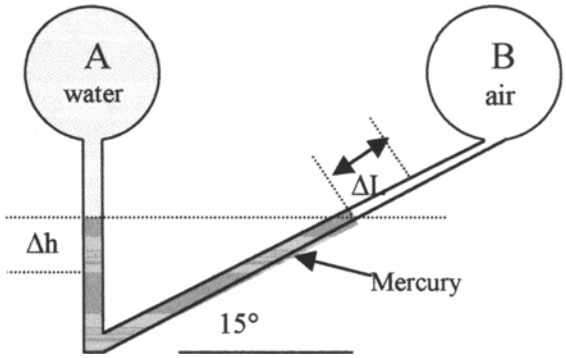


Fig. P2.40

Solution: Since the tube diameter is constant, the volume of mercury will displace a distance Δh down the left side, equal to the volume increase on the right side; Δh = ΔL. Apply the hydrostatic relation to the pressure change, beginning at the right (air/mercury) interface:

$$p_B + \gamma_{Hg}(\Delta L \sin\theta + \Delta h) - \gamma_W(\Delta h + \Delta L \sin\theta) = p_A \quad \text{with} \quad \Delta h = \Delta L$$

$$\text{or:} \quad 100{,}000 + 133100(\Delta h)(1+\sin 15°) - 9790(\Delta h)(1+\sin 15°) = p_A = 130{,}000 \text{ Pa}$$

$$\text{Solve for} \quad \Delta h = (30{,}000 \text{ Pa})/[(133100 - 9790 \text{ N/m}^2)(1+\sin 15°)] = \mathbf{0.193\,m} \quad \textit{Ans.}$$

The mercury in the left (vertical) leg will drop 19.3 cm, the mercury in the right (slanted) leg will rise 19.3 cm along the slant and 5 cm in vertical elevation.

2.41 The system in Fig. P2.41 is at 20°C. Determine the pressure at point A in pounds per square foot.

Solution: Take the specific weights of water and mercury from Table 2.1. Write the hydrostatic formula from point A to the water surface:

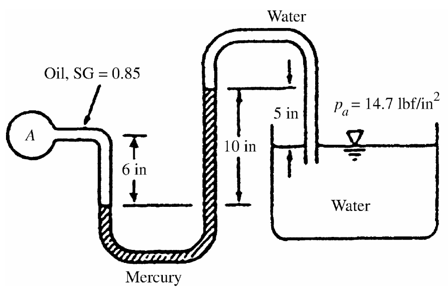


Fig. P2.41

$$p_A + (0.85)(62.4 \text{ lbf/ft}^3)\left(\frac{6}{12}\text{ ft}\right) - (846)\left(\frac{10}{12}\right) + (62.4)\left(\frac{5}{12}\right) = p_{atm} = (14.7)(144)\frac{\text{lbf}}{\text{ft}^2}$$

$$\text{Solve for} \quad p_A = \mathbf{2770\,lbf/ft^2} \quad \textit{Ans.}$$

2.42 Small pressure differences can be measured by the two-fluid manometer in Fig. P2.42, where ρ_2 is only slightly larger than ρ_1. Derive a formula for $p_A - p_B$ if the reservoirs are very large.

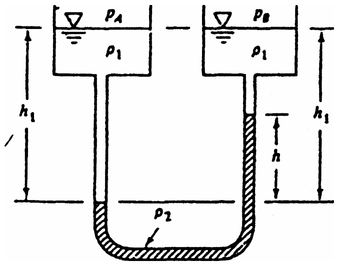


Fig. P2.42

Solution: Apply the hydrostatic formula from A to B:

$$p_A + \rho_1 g h_1 - \rho_2 g h - \rho_1 g(h_1 - h) = p_B$$

Solve for $\mathbf{p_A - p_B = (\rho_2 - \rho_1)gh}$ *Ans.*

If $(\rho_2 - \rho_1)$ is very small, *h* will be very large for a given Δp (a sensitive manometer).

2.43 The traditional method of measuring blood pressure uses a *sphygmomanometer*, first recording the highest (*systolic*) and then the lowest (*diastolic*) pressure from which flowing "Korotkoff" sounds can be heard. Patients with dangerous hypertension can exhibit systolic pressures as high as 5 lbf/in^2. Normal levels, however, are 2.7 and 1.7 lbf/in^2, respectively, for systolic and diastolic pressures. The manometer uses mercury and air as fluids. (a) How high should the manometer tube be? (b) Express normal systolic and diastolic blood pressure in millimeters of mercury.

Solution: (a) The manometer height must be at least large enough to accommodate the largest systolic pressure expected. Thus apply the hydrostatic relation using 5 lbf/in^2 as the pressure,

$$h = p_B/\rho g = (5\ \text{lbf/in}^2)(6895\ \text{Pa/lbf/in}^2)/(133100\ \text{N/m}^3) = 0.26\ \text{m}$$

So make the height about **30 cm**. *Ans.* (a)

(b) Convert the systolic and diastolic pressures by dividing them by mercury's specific weight.

$$h_{systolic} = (2.7\ \text{lbf/in}^2)(144\ \text{in}^2/\text{ft}^2)/(846\ \text{lbf/ft}^3) = 0.46\ \text{ft Hg} = 140\ \text{mm Hg}$$
$$h_{diastolic} = (1.7\ \text{lbf/in}^2)(144\ \text{in}^2/\text{ft}^2)/(846\ \text{lbf/ft}^3) = 0.289\ \text{ft Hg} = 88\ \text{mm Hg}$$

The systolic/diastolic pressures are thus **140/88 mm Hg**. *Ans.* (b)

2.44 Water flows downward in a pipe at 45°, as shown in Fig. P2.44. The mercury manometer reads a 6-in height. The pressure drop $p_2 - p_1$ is partly due to friction and partly due to gravity. Determine the total pressure drop and also the part due to friction only. Which part does the manometer read? Why?

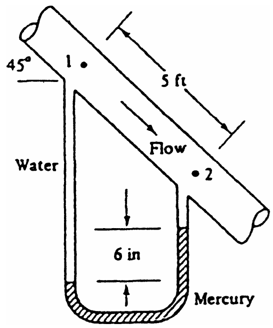


Fig. P2.44

Solution: Let "h" be the distance down from point 2 to the mercury-water interface in the right leg. Write the hydrostatic formula from 1 to 2:

$$p_1 + 62.4\left(5\sin 45° + h + \frac{6}{12}\right) - 846\left(\frac{6}{12}\right) - 62.4h = p_2,$$

$$p_1 - p_2 = \underbrace{(846 - 62.4)(6/12)}_{\text{....friction loss...}} - \underbrace{62.4(5\sin 45°)}_{\text{..gravity head..}} = 392 - 221$$

$$= \mathbf{171\,\frac{lbf}{ft^2}} \quad \textit{Ans.}$$

The manometer reads only the <u>*friction loss*</u> of 392 lbf/ft^2, not the gravity head of 221 psf.

2.45 Determine the gage pressure at point A in Fig. P2.45, in pascals. Is it higher or lower than $P_{atmosphere}$?

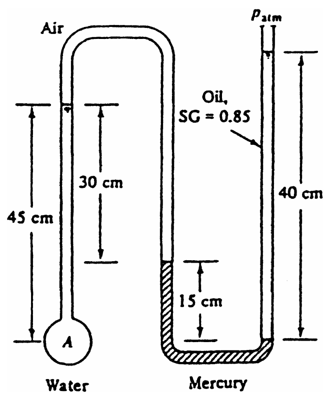


Fig. P2.45

Solution: Take γ = 9790 N/m^3 for water and 133100 N/m^3 for mercury. Write the hydrostatic formula between the atmosphere and point A:

$$p_{atm} + (0.85)(9790)(0.4\text{ m}) - (133100)(0.15\text{ m}) - (12)(0.30\text{ m}) + (9790)(0.45\text{ m}) = p_A,$$

$$\text{or:}\quad p_A = p_{atm} - 12200\text{ Pa} = \mathbf{12200\ Pa\ (vacuum)} \quad \textit{Ans.}$$

2.46 In Fig. P2.46 both ends of the manometer are open to the atmosphere. Estimate the specific gravity of fluid X.

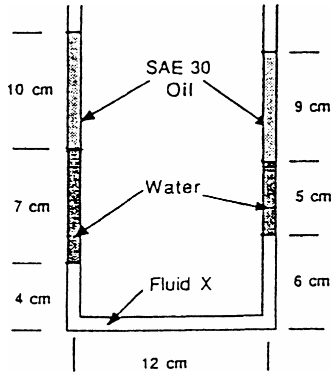


Fig. P2.46

Solution: The pressure at the bottom of the manometer must be the same regardless of which leg we approach through, left or right:

$$p_{atm} + (8720)(0.1) + (9790)(0.07) + \gamma_X(0.04) \quad \text{(left leg)}$$

$$= p_{atm} + (8720)(0.09) + (9790)(0.05) + \gamma_X(0.06) \quad \text{(right leg)}$$

$$\text{or:}\quad \gamma_X = 14150\ \text{N/m}^3, \quad SG_X = \frac{14150}{9790} \approx \mathbf{1.45} \quad \textit{Ans.}$$

2.47 The cylindrical tank in Fig. P2.47 is being filled with 20°C water by a pump developing an exit pressure of 175 kPa. At the instant shown, the air pressure is 110 kPa and H = 35 cm. The pump stops when it can no longer raise the water pressure. Estimate "H" at that time.

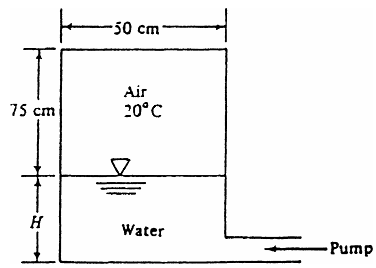


Fig. P2.47

Solution: At the end of pumping, the bottom water pressure must be 175 kPa:

$$p_{air} + 9790H = 175000$$

Meanwhile, assuming isothermal air compression, the final air pressure is such that

$$\frac{p_{air}}{110000} = \frac{Vol_{old}}{Vol_{new}} = \frac{\pi R^2(0.75\ \text{m})}{\pi R^2(1.1\ \text{m} - H)} = \frac{0.75}{1.1 - H}$$

where R is the tank radius. Combining these two gives a quadratic equation for H:

$$\frac{0.75(110000)}{1.1 - H} + 9790H = 175000, \quad \text{or} \quad H^2 - 18.98H + 11.24 = 0$$

The two roots are H = 18.37 m (ridiculous) or, properly, H = **0.614 m** *Ans.*

2.48 Conduct an experiment: Place a thin wooden ruler on a table with a 40% overhang, as shown. Cover it with 2 full-size sheets of newspaper. (a) Estimate the total force on top of the newspaper due to air pressure. (b) With everyone out of the way, perform a karate chop on the outer end of the ruler. (c) Explain the results in b.

Results: (a) Newsprint is about 27 in (0.686 m) by 22.5 in (0.572 m). Thus the force is:

$$F = pA = (101325\ Pa)(0.686\ m)(0.572\ m) = \mathbf{39700\ N!} \quad \textit{Ans.}$$

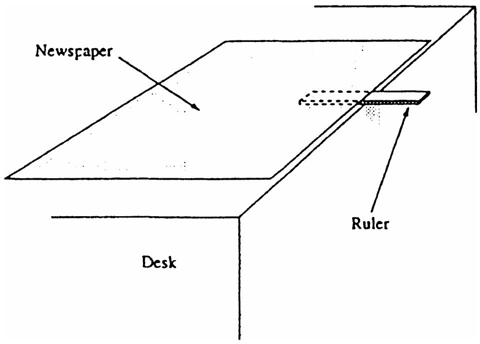


Fig. P2.48

(b) The newspaper will hold the ruler, which will probably *break* due to the chop. *Ans.*
(c) Chop is fast, air does not have time to rush in, partial vacuum under newspaper. *Ans.*

P2.49 The system in Fig. P2.49 is open to 1 atm on the right side.

(*a*) If L = 120 cm, what is the air pressure in container A?

(*b*) Conversely, if p_A = 135 kPa, what is the length L?

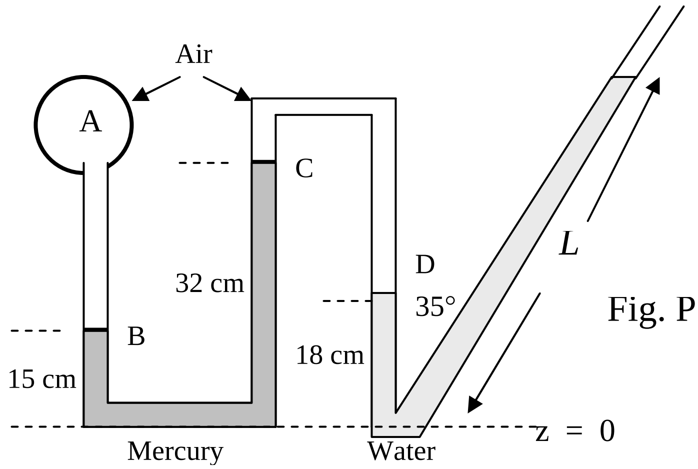


Fig. P2.49

Solution: (*a*) The vertical elevation of the water surface in the slanted tube is (1.2m)(sin55°) = 0.983 m. Then the pressure at the 18-cm level of the water, point D, is

$$p_D = p_{atm} + \gamma_{water}\Delta z = 101350\,Pa + (9790\frac{N}{m^3})(0.983 - 0.18m) = 109200\,Pa$$

Going up from D to C in air is negligible, less than 2 Pa. Thus $p_C \approx p_D$ = 109200 Pa. Going down from point C to the level of point B increases the pressure in mercury:

$$p_B = p_C + \gamma_{mercury}\Delta z_{C-B} = 109200 + (133100\frac{N}{m^3})(0.32 - 0.18m) = \mathbf{131800\,Pa} \quad \textit{Ans.}(a)$$

This is the answer, since again it is negligible to go up to point A in low-density air.
(*b*) Given p_A = 135 kPa, go down from point A to point B with negligible air-pressure change, then jump across the mercury U-tube and go up to point C with a decrease:

$$p_C = p_B - \gamma_{mercury}\Delta z_{B-C} = 135000 - (133100)(0.32 - 0.15) = 112400\,Pa$$

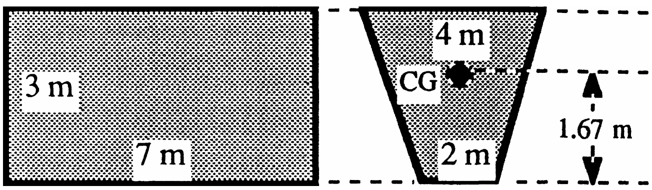
Once again, $p_C \approx p_D \approx$ 112400 Pa, jump across the water and then go up to the surface:

$$p_{atm} = p_D - \gamma_{water}\,\Delta z = 112400 - 9790(z_{surface} - 0.18m) = 101350 Pa$$

$$\text{Solve for} \quad z_{surface} \approx 1.126\,m$$

$$\text{Then the slanted distance } L = 1.126m/\sin 55^\circ = \mathbf{1.375\,m} \quad Ans.(b)$$

2.50 A vat filled with oil (SG = 0.85) is 7 m long and 3 m deep and has a trapezoidal cross-section 2 m wide at the bottom and 4 m wide at the top, as shown in Fig. P2.50. Compute (a) the weight of oil in the vat; (b) the force on the vat bottom; and (c) the force on the trapezoidal end panel.



Solution: (a) The total volume of oil in the vat is (3 m)(7 m)(4 m + 2 m)/2 = 63 m^3. Therefore the weight of oil in the vat is

$$W = \gamma_{oil}(Vol) = (0.85)(9790\ N/m^3)(63\ m^3) = \mathbf{524,000\ N} \quad \textit{Ans. (a)}$$

(b) The force on the horizontal bottom surface of the vat is

$$F_{bottom} = \gamma_{oil} h_{CG} A_{bottom} = (0.85)(9790)(3\ m)(2\ m)(7\ m) = \mathbf{350,000\ N} \quad \textit{Ans. (b)}$$

Note that F is <u>less</u> than the total weight of oil—the student might explain why they differ? (c) I found in my statics book that the centroid of this trapezoid is 1.33 m below the surface, or 1.67 m above the bottom, as shown. Therefore the side-panel force is

$$F_{side} = \gamma_{oil} h_{CG} A_{side} = (0.85)(9790)(1.33\ m)(9\ m^2) = \mathbf{100,000\ N} \quad \textit{Ans. (c)}$$

These are large forces. Big vats have to be strong!

2.51 Gate AB in Fig. P2.51 is 1.2 m long and 0.8 m into the paper. Neglecting atmospheric-pressure effects, compute the force F on the gate and its center of pressure position *X*.

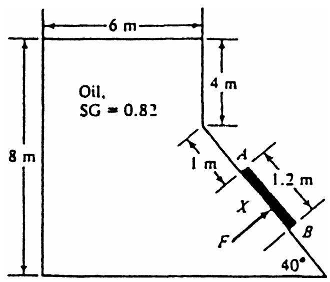


Fig. P2.51

Solution: The centroidal depth of the gate is

$$h_{CG} = 4.0 + (1.0 + 0.6)\sin 40° = 5.028\ m,$$

$$\text{hence} \quad F_{AB} = \gamma_{oil} h_{CG} A_{gate} = (0.82 \times 9790)(5.028)(1.2 \times 0.8) = \mathbf{38750\ N} \quad \textit{Ans.}$$

The line of action of F is slightly below the centroid by the amount

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(1/12)(0.8)(1.2)^3 \sin 40°}{(5.028)(1.2 \times 0.8)} = -0.0153\ m$$

Thus the position of the center of pressure is at $X = 0.6 + 0.0153 \approx \mathbf{0.615\ m}$ *Ans.*

P2.52 Example 2.5 calculated the force on plate AB and its line of action, using the moment-of-inertia approach. Some teachers say it is more instructive to calculate these by *direct integration* of the pressure forces. Using Figs. P2.52 and E2.5*a*, (*a*) find an expression for the pressure variation $p(\zeta)$ along the plate;

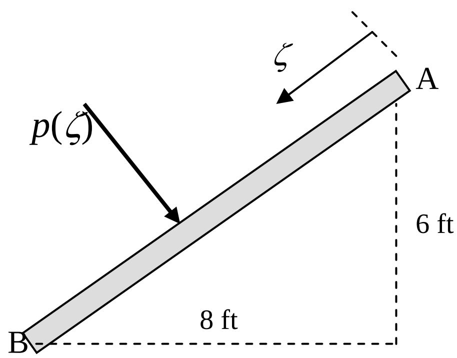


Fig. P2.52

(*b*) integrate this pressure to find the total force *F*;
(*c*) integrate the moments about point A to find the position of the center of pressure.

Solution: (*a*) Point A is 9 ft deep, and point B is 15 ft deep, and $\gamma = 64$ lbf/ft^3. Thus p_A = (64lbf/ft^3)(9ft) = 576 lbf/ft^2 and p_B = (64lbf/ft^3)(15ft) = 960 lbf/ft^2. Along the 10-ft length, pressure increases by (960-576)/10ft = 38.4 lbf/ft^2/ft. Thus the pressure is

(*b*) Given that the plate width b = 5 ft. Integrate for the total force on the plate:

$$p(\zeta) = 576 + 38.4\zeta \quad (lbf/ft^2) \qquad Ans.(a)$$

$$F = \int_{plate} p\,dA = \int p\,b\,d\zeta = \int_0^{10} (576 + 38.4\zeta)(5\,ft)d\zeta =$$

$$= (5)(576\zeta + 38.4\zeta^2/2)\Big|_0^{10} = 28800 + 9600 = \mathbf{38{,}400\ lbf} \qquad Ans.(b)$$

(c) Find the moment of the pressure forces about point A and divide by the force:

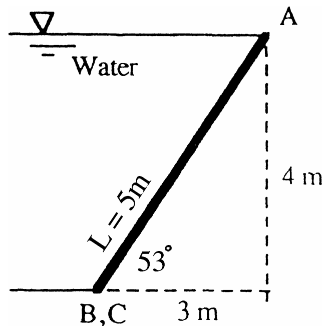
$$M_A = \int_{plate} p\zeta\, b\, dA = \int_0^{10} \zeta(576+38.4\zeta)(5\,ft)d\zeta =$$

$$= (5)(576\zeta^2/2+38.4\zeta^3/3)\Big|_0^{10} = 144000 + 64000 = 208{,}000\,ft-lbf$$

$$Then \quad \zeta_{CP} = \frac{M_A}{F} = \frac{208000\ ft-lbf}{38400\,lbf} = \mathbf{5.42\ ft} \qquad Ans.(c)$$

The center of pressure is 5.417 ft down the plate from Point A.

2.53 Panel ABC in the slanted side of a water tank (shown at right) is an isoceles triangle with vertex at A and base BC = 2 m. Find the water force on the panel and its line of action.



Solution: (a) The centroid of ABC is 2/3 of the depth down, or 8/3 m from the surface. The panel area is (1/2)(2 m)(5 m) = 5 m^2. The water force is

$$F_{ABC} = \gamma h_{CG} A_{panel} = (9790)(2.67\ m)(5\ m^2) = \mathbf{131{,}000\ N} \quad Ans.\ (a)$$

(b) The moment of inertia of ABC is $(1/36)(2\ m)(5\ m)^3 = 6.94\ m^4$. From Eq. (2.44),

$$y_{CP} = -I_{xx}\sin\theta/(h_{CG}A_{panel}) = -6.94\sin(53°)/[2.67(5)] = \mathbf{-0.417\ m} \quad Ans.\ (b)$$

The center of pressure is 3.75 m down from A, or 1.25 m up from BC.

2.54 In Fig. P2.54, the hydrostatic force F is the same on the bottom of all three containers, even though the weights of liquid above are quite different. The three bottom shapes and the fluids are the same. This is called the *hydrostatic paradox*. Explain why it is true and sketch a freebody of each of the liquid columns.

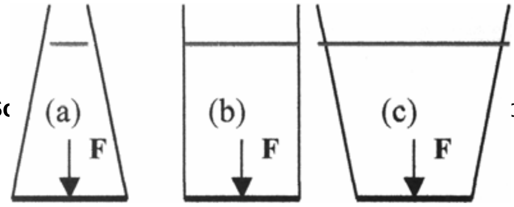
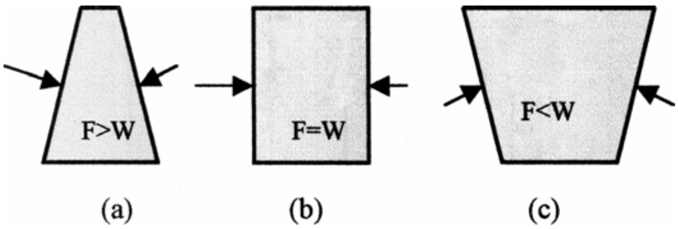


Fig. P2.54

Solution: The three freebodies are shown below. Pressure on the side-walls balances the forces. In (a), downward side-pressure components help add to a light W. In (b) side pressures are horizontal. In (c) upward side pressure helps reduce a heavy W.



2.55 Gate AB in Fig. P2.55 is 5 ft wide into the paper, hinged at A, and restrained by a stop at B. Compute (a) the force on stop B; and (b) the reactions at A if $h = 9.5$ ft.

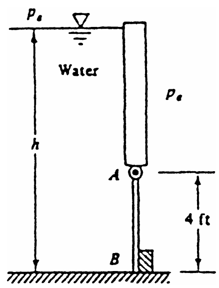


Fig. P2.55

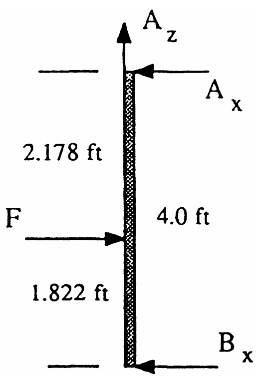
Solution: The centroid of AB is 2.0 ft below A, hence the centroidal depth is $h + 2 - 4 = 7.5$ ft. Then the total hydrostatic force on the gate is

$$F = \gamma h_{CG} A_{gate} = (62.4\ \text{lbf/ft}^3)(7.5\ \text{ft})(20\ \text{ft}^2) = 9360\ \text{lbf}$$

The C.P. is below the centroid by the amount

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{(1/12)(5)(4)^3\sin 90^\circ}{(7.5)(20)}$$
$$= -0.178\ \text{ft}$$

This is shown on the freebody of the gate at right. We find force B_X with moments about A:



$$\sum M_A = B_x(4.0) - (9360)(2.178) = 0,$$

$$\text{or:}\quad B_x = \mathbf{5100\ lbf} \quad \text{(to left)} \quad \textit{Ans. (a)}$$

The reaction forces at A then follow from equilibrium of forces (with *zero* gate weight):

$$\sum F_x = 0 = 9360 - 5100 - A_x, \quad \text{or:} \quad A_x = \mathbf{4260\ lbf} \quad \text{(to left)}$$

$$\sum F_z = 0 = A_z + W_{gate} \approx A_z, \quad \text{or:} \quad A_z = \mathbf{0\ lbf} \quad \textit{Ans. (b)}$$

2.56 For the gate of Prob. 2.55 above, stop "B" breaks if the force on it equals 9200 lbf. For what water depth *h* is this condition reached?

Solution: The formulas must be written in terms of the unknown centroidal depth:

$$h_{CG} = h - 2 \quad F = \gamma h_{CG} A = (62.4)h_{CG}(20) = 1248h_{CG}$$

$$y_{CP} = -\frac{I_{XX}\sin\theta}{h_{CG}A} = -\frac{(1/12)(5)(4)^3\sin 90^\circ}{h_{CG}(20)} = -\frac{1.333}{h_{CG}}$$

Then moments about A for the freebody in Prob. 2.155 above will yield the answer:

$$\sum M_A = 0 = 9200(4) - (1248h_{CG})\left(2 + \frac{1.333}{h_{CG}}\right), \quad \text{or} \quad h_{CG} = 14.08 \text{ ft}, \; h = \mathbf{16.08\ ft} \quad \textit{Ans.}$$

2.57 The tank in Fig. P2.57 is 2 m wide into the paper. Neglecting atmospheric pressure, find the resultant hydrostatic force on panel BC, (a) from a single formula; (b) by computing horizontal and vertical forces separately, in the spirit of curved surfaces.

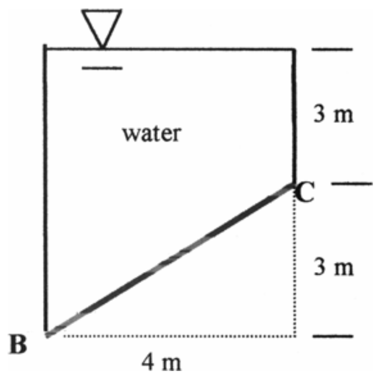


Fig. P2.57

Solution: (a) The resultant force F, may be found by simply applying the hydrostatic relation

$$F = \gamma h_{CG} A = (9790\ \text{N/m}^3)(3+1.5\ \text{m})(5\ \text{m} \times 2\ \text{m}) = 440{,}550\ \text{N} = \mathbf{441\ kN} \quad \textit{Ans.}\ \text{(a)}$$

(b) The horizontal force acts as though BC were vertical, thus h_{CG} is halfway down from C and acts on the projected area of BC.

$$F_H = (9790)(4.5)(3 \times 2) = 264{,}330\ \text{N} = \mathbf{264\ kN} \quad \textit{Ans.}\ \text{(b)}$$

The vertical force is equal to the weight of fluid above BC,

$$F_V = (9790)[(3)(4) + (1/2)(4)(3)](2) = 352{,}440 = \mathbf{352\ kN} \quad \textit{Ans.}\ \text{(b)}$$

The resultant is the same as part (a): $F = [(264)^2 + (352)^2]^{1/2}$ **= 441 kN.**

2.58 In Fig. P2.58, weightless cover gate AB closes a circular opening 80 cm in diameter when weighed down by the 200-kg mass shown. What water level *h* will dislodge the gate?

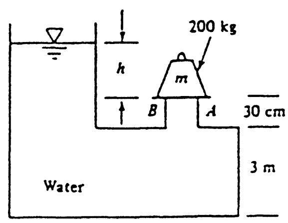


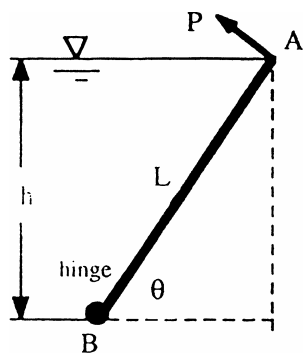
Fig. P2.58

Solution: The centroidal depth is exactly equal to *h* and force F will be upward on the gate. Dislodging occurs when F equals the weight:

$$F = \gamma h_{CG} A_{gate} = (9790\ \text{N/m}^3)\ h \frac{\pi}{4}(0.8\ \text{m})^2 = W = (200)(9.81)\ \text{N}$$

$$\text{Solve for} \quad h = \mathbf{0.40\ m} \quad \textit{Ans.}$$

2.59 Gate AB has length L, width b into the paper, is hinged at B, and has negligible weight. The liquid level h remains at the top of the gate for any angle θ. Find an analytic expression for the force P, perpendicular to AB, required to keep the gate in equilibrium.



Solution: The centroid of the gate remains at distance L/2 from A and depth h/2 below the surface. For any θ, then, the hydrostatic force is F = γ(h/2)Lb. The moment of inertia of the gate is (1/12)bL^3, hence y_{CP} = −(1/12)bL^3sinθ/[(h/2)Lb], and the center of pressure is (L/2 − y_{CP}) from point B. Summing moments about hinge B yields

$$PL = F(L/2 - y_{CP}), \quad \text{or:} \quad \mathbf{P = (\gamma hb/4)(L - L^2\sin\theta/3h)} \quad \textit{Ans.}$$

P2.60 In 1960, Auguste and Jacques Picard's self-propelled bathyscaphe *Trieste* set a record by descending to a depth of 35,800 feet in the Pacific Ocean, near Guam. The passenger sphere was 7 ft in diameter, 6 inches thick, and had a window 16 inches in diameter. (*a*) Estimate the hydrostatic force on the window at that depth. (*b*) If the window is vertical, how far below its center is the center of pressure?

Solution: At the surface, the density of seawater is about 1025 kg/m^3 (1.99 slug/ft^3). Atmospheric pressure is about 2116 lbf/ft^2. We could use these values, or estimate from Eq. (1.19) that the density at depth would be about 4.6% more, or 2.08 slug/ft^3. We could average these two to 2.035 slug/ft^3. The pressure at that depth would thus be approximately

$$p = p_a + \rho_{avg}\, g\, h = 2116 + (2.035\frac{slug}{ft^3})(32.2\frac{ft}{s^2})(35800\ ft) \approx 2{,}350{,}000\ \frac{lbf}{ft^2}$$

(*a*) This pressure, times the area of the 16-inch window, gives the desired force.

$$F_{window} = p_{CG}\, A = (2350000\frac{lbf}{ft^2})[\frac{\pi}{4}(\frac{16}{12}\ ft)^2] = \mathbf{3{,}280{,}000\ \ lbf} \qquad \textit{Ans.}(a)$$

Quite a lot of force, but the bathyscaphe was well designed.
(*b*) The distance down to the center of pressure on the window follows from Eq. (2.27):

$$y_{CP} = -\gamma\sin\theta\frac{I_{xx}}{F} = -[2.035*32.2\frac{lbf}{ft^3}]\sin(90^\circ)\frac{(\pi/4)(8/12\ ft)^4}{3280000\,lbf} = \mathbf{-3.2E-6\ \ ft.} \ \ \textit{Ans.}(b)$$

The center of pressure at this depth is only 38 micro inches below the center of the window.

2.61 Gate AB in Fig. P2.61 is a homogeneous mass of 180 kg, 1.2 m wide into the paper, resting on smooth bottom B. All fluids are at 20°C. For what water depth *h* will the force at point B be zero?

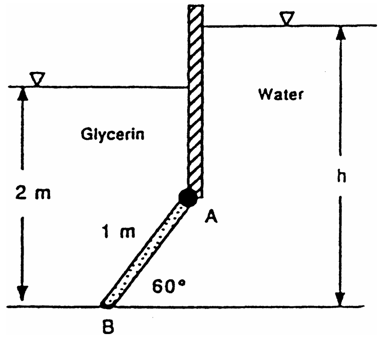


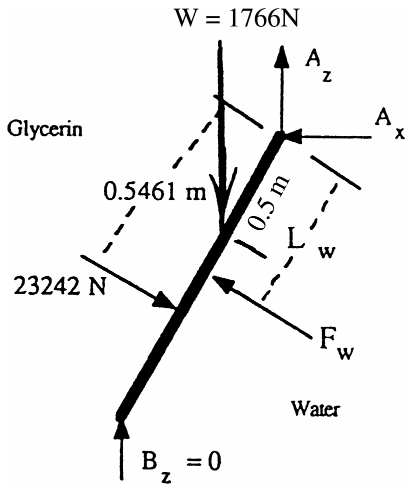
Fig. P2.61

Solution: Let γ = 12360 N/m^3 for glycerin and 9790 N/m^3 for water. The centroid of

AB is 0.433 m vertically below A, so $h_{CG} = 2.0 - 0.433 = 1.567$ m, and we may compute the glycerin force and its line of action:

$$F_g = \gamma\bar{h}A = (12360)(1.567)(1.2) = 23242 \text{ N}$$

$$y_{CP,g} = -\frac{(1/12)(1.2)(1)^3 \sin 60°}{(1.567)(1.2)} = -0.0461 \text{ m}$$



These are shown on the freebody at right. The water force and its line of action are shown without numbers, because they depend upon the centroidal depth on the water side:

$$F_w = (9790)h_{CG}(1.2)$$

$$y_{CP} = -\frac{(1/12)(1.2)(1)^3 \sin 60°}{h_{CG}(1.2)} = -\frac{0.0722}{h_{CG}}$$

The weight of the gate, W = 180(9.81) = 1766 N, acts at the centroid, as shown above. Since the force at B equals zero, we may sum moments counterclockwise about A to find the water depth:

$$\sum M_A = 0 = (23242)(0.5461) + (1766)(0.5\cos 60°) - (9790)h_{CG}(1.2)(0.5 + 0.0722/h_{CG})$$

Solve for $h_{CG,water} = 2.09$ m, or: $h = h_{CG} + 0.433 = \mathbf{2.52\ m}$ *Ans.*

2.62 Gate AB in Fig. P2.62 is 15 ft long and 8 ft wide into the paper, hinged at B with a stop at A. The gate is 1-in-thick steel, SG = 7.85. Compute the 20°C water level *h* for which the gate will start to fall.

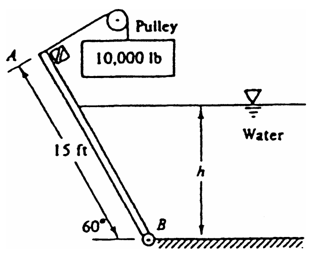
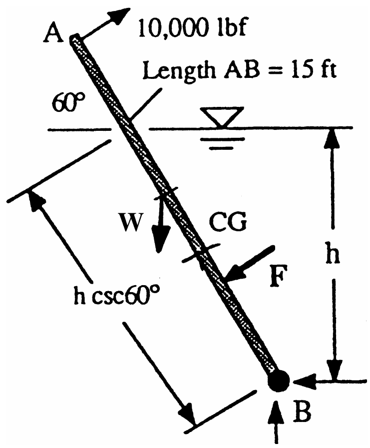


Fig. P2.62

Solution: Only the length (h csc 60°) of the gate lies below the water. Only this part contributes to the hydrostatic force shown in the freebody at right:



$$F = \gamma h_{CG} A = (62.4)\left(\frac{h}{2}\right)(8h\csc 60°)$$
$$= 288.2h^2 \text{ (lbf)}$$
$$y_{CP} = -\frac{(1/12)(8)(h\csc 60°)^3 \sin 60°}{(h/2)(8h\csc 60°)}$$
$$= -\frac{h}{6}\csc 60°$$

The weight of the gate is (7.85)(62.4 lbf/ft^3)(15 ft)(1/12 ft)(8 ft) = 4898 lbf. This weight acts downward at the CG of the *full gate* as shown (not the CG of the submerged portion). Thus, W is 7.5 ft above point B and has moment arm (7.5 cos 60° ft) about B.

We are now in a position to find *h* by summing moments about the hinge line B:

$$\sum M_B = (10000)(15) - (288.2h^2)[(h/2)\csc 60° - (h/6)\csc 60°] - 4898(7.5\cos 60°) = 0,$$

$$\text{or:}\quad 110.9h^3 = 150000 - 18369, \quad h = (131631/110.9)^{1/3} = \mathbf{10.6\ ft} \quad \textit{Ans.}$$

2.63 The tank in Fig. P2.63 has a 4-cm-diameter plug which will pop out if the hydrostatic force on it reaches 25 N. For 20°C fluids, what will be the reading *h* on the manometer when this happens?

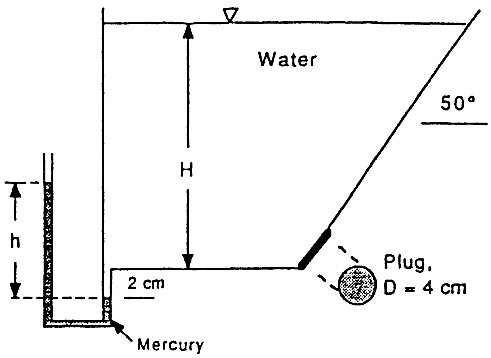


Fig. P2.63

Solution: The water depth when the plug pops out is

$$F = 25\text{ N} = \gamma h_{CG} A = (9790)h_{CG}\frac{\pi(0.04)^2}{4}$$

$$\text{or}\quad h_{CG} = 2.032\text{ m}$$

It makes little numerical difference, but the mercury-water interface is a little deeper than this, by the amount (0.02 sin 50°) of plug-depth, plus 2 cm of tube length. Thus

$$p_{atm} + (9790)(2.032 + 0.02\sin 50° + 0.02) - (133100)h = p_{atm},$$

$$\text{or:}\quad h \approx \mathbf{0.152\ m} \quad \textit{Ans.}$$

2.64 Gate ABC in Fig. P2.64 has a fixed hinge at B and is 2 m wide into the paper. If the water level is high enough, the gate will open. Compute the depth *h* for which this happens.

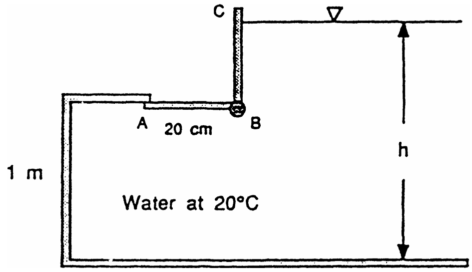
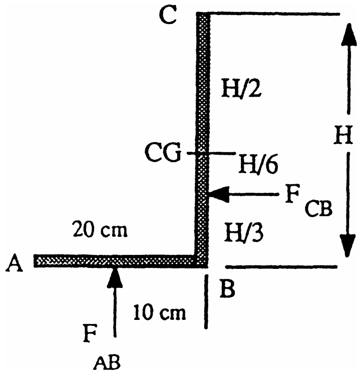


Fig. P2.64

Solution: Let $H = (h - 1\text{ meter})$ be the depth down to the level AB. The forces on AB and BC are shown in the freebody at right. The moments of these forces about B are equal when the gate opens:



$$\sum M_B = 0 = \gamma H(0.2)b(0.1) = \gamma\left(\frac{H}{2}\right)(Hb)\left(\frac{H}{3}\right)$$

$$\text{or:}\quad H = 0.346\text{ m},\qquad h = H + 1 = \mathbf{1.346\ m}\quad \textit{Ans.}$$

This solution is independent of both the water density and the gate width *b* into the paper.

2.65 Gate AB in Fig. P2.65 is semi-circular, hinged at B, and held by a horizontal force P at point A. Determine the required force P for equilibrium.

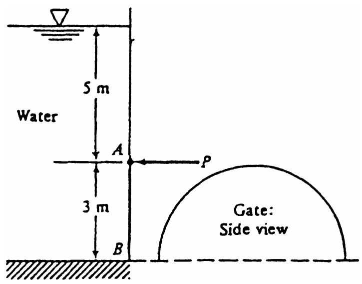
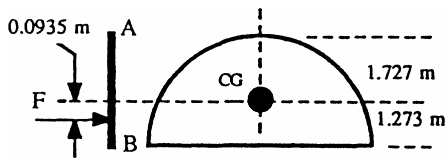


Fig. P2.65

Solution: The centroid of a semi-circle is at $4R/3\pi \approx 1.273$ m off the bottom, as shown in the sketch at right. Thus it is $3.0 - 1.273 = 1.727$ m down from the force P. The water force F is



$$F = \gamma h_{CG}A = (9790)(5.0 + 1.727)\frac{\pi}{2}(3)^2 = 931000\text{ N}$$

The line of action of F lies below the CG:

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(0.10976)(3)^4\sin 90^\circ}{(5 + 1.727)(\pi/2)(3)^2} = -0.0935\text{ m}$$

Then summing moments about B yields the proper support force P:

$$\sum M_B = 0 = (931000)(1.273 - 0.0935) - 3P,\quad \text{or:}\quad P = \mathbf{366000\ N}\quad \textit{Ans.}$$

2.66 Dam ABC in Fig. P2.66 is 30 m wide into the paper and is concrete (SG ≈ 2.40). Find the hydrostatic force on surface AB and its moment about C. Could this force tip the dam over? Would fluid seepage under the dam change your argument?

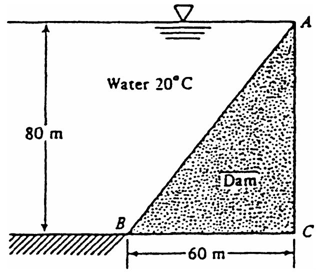
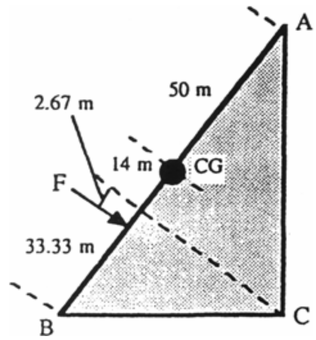


Fig. P2.66

Solution: The centroid of surface AB is 40 m deep, and the total force on AB is

$$F = \gamma h_{CG} A = (9790)(40)(100 \times 30) = 1.175E9\ N$$



The line of action of this force is two-thirds of the way down along AB, or 66.67 m from A. This is seen either by inspection (A is at the surface) or by the usual formula:

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(1/12)(30)(100)^3\sin(53.13°)}{(40)(30\times 100)} = -16.67\ m$$

to be added to the 50-m distance from A to the centroid, or 50 + 16.67 = 66.67 m. As shown in the figure, the line of action of F is 2.67 m to the left of a line up from C normal to AB. The moment of F about C is thus

$$M_C = FL = (1.175E9)(66.67 - 64.0) \approx \mathbf{3.13E9\ N\cdot m} \quad \textit{Ans.}$$

This moment is counterclockwise, hence it cannot tip over the dam. If there were *seepage* under the dam, the main support force at the bottom of the dam would shift to the left of point C and might indeed cause the dam to tip over.

2.67 Generalize Prob. 2.66 with length AB as "H", length BC as "L", and angle ABC as "*q*", with width "b" into the paper. If the dam material has specific gravity "SG", with no seepage, find the critical angle θ_c for which the dam will just tip over to the right. Evaluate this expression for SG = 2.40.

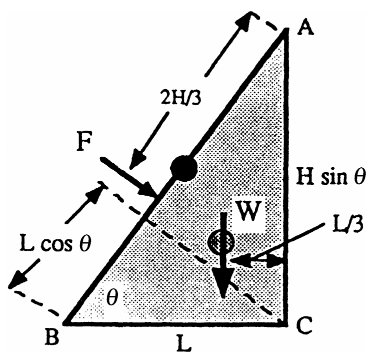


Fig. P2.67

Solution: By geometry, L = Hcosθ and the vertical height of the dam is Hsinq. The force F on surface AB is γ(H/2)(sinθ)Hb, and its position is at 2H/3 down from point A, as shown in the figure. Its moment arm about C is thus (H/3 − Lcosθ). Meanwhile the weight of the dam is W = (SG)γ(L/2)H(sinθ)b, with a moment arm L/3 as shown. Then summation of clockwise moments about C gives, for critical "tip-over" conditions,

$$\sum M_C = 0 = \left(\gamma \frac{H}{2}\sin\theta\, Hb\right)\left[\frac{H}{3} - L\cos\theta\right] - \left[SG(\gamma)\frac{L}{2}H\sin\theta\, b\right]\left(\frac{L}{3}\right) \quad \text{with } L = H\cos\theta.$$

$$\text{Solve for} \quad \cos^2\theta_c = \frac{1}{\mathbf{3+SG}} \quad \textit{Ans.}$$

Any angle greater than θ_c will cause tip-over to the right. For the particular case of concrete, SG ≈ 2.40, cosθ_c ≈ 0.430, or θ_c ≈ **64.5°**, which is greater than the given angle θ = 53.13° in Prob. 2.66, hence there was no tipping in that problem.

2.68 Isosceles triangle gate AB in Fig. P2.68 is hinged at A and weighs 1500 N. What horizontal force P is required at point B for equilibrium?

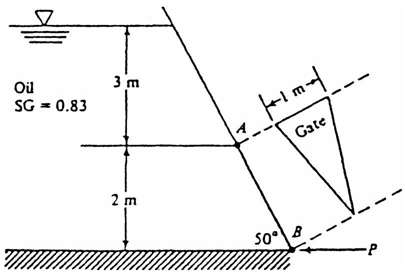
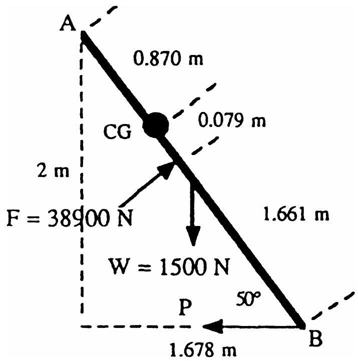


Fig. P2.68



Solution: The gate is 2.0/sin 50° = 2.611 m long from A to B and its area is 1.3054 m^2. Its centroid is 1/3 of the way down from A, so the centroidal depth is 3.0 + 0.667 m. The force on the gate is

$$F = \gamma h_{CG} A = (0.83)(9790)(3.667)(1.3054)$$
$$= 38894 \text{ N}$$

The position of this force is below the centroid:

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A}$$

$$= -\frac{(1/36)(1.0)(2.611)^3\sin 50°}{(3.667)(1.3054)} = -0.0791 \text{ m}$$

The force and its position are shown in the freebody at upper right. The gate weight of 1500 N is assumed at the centroid of the plate, with moment arm 0.559 meters about point A. Summing moments about point A gives the required force P:

$$\sum M_A = 0 = P(2.0) + 1500(0.559) - 38894(0.870 + 0.0791),$$

$$\text{Solve for} \quad P = \mathbf{18040\ N} \quad \textit{Ans.}$$

P2.69 Consider the slanted plate AB of length L in Fig. P2.69. (*a*) Is the hydrostatic force F on the plate equal to the weight of the *missing water* above the plate? If not, correct this hypothesis. Neglect the atmosphere. (*b*) Can a "missing water" approach be generalized to *curved* plates of this type?

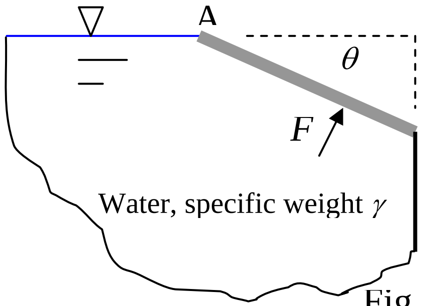


Fig. P2.69

Solution: (*a*) The actual force F equals the pressure at the centroid times the plate area: But the weight of the "missing water" is

$$F = p_{CG} A_{plate} = \gamma h_{CG} L b = \gamma \frac{L\sin\theta}{2} L b = \frac{\gamma}{2} L^2 b \sin\theta$$

$$W_{missing} = \gamma \upsilon_{missing} = \gamma \left[\frac{1}{2}(L\sin\theta)(L\cos\theta) b\right] = \frac{\gamma}{2} L^2 b \sin\theta\cos\theta$$

Why the discrepancy? Because the actual plate force ***is not vertical***. Its vertical component is $F\cos\theta = W_{missing}$. The missing-water weight equals the ***vertical*** component of the force. *Ans*.(*a*) This same approach applies to ***curved*** plates with missing water. *Ans*.(*b*)

P2.70 The swing-check valve in Fig. P2.70 covers a 22.86-cm diameter opening in the slanted wall. The hinge is 15 cm from the centerline, as shown. The valve will open when the hinge moment is 50 N-m. Find the value of

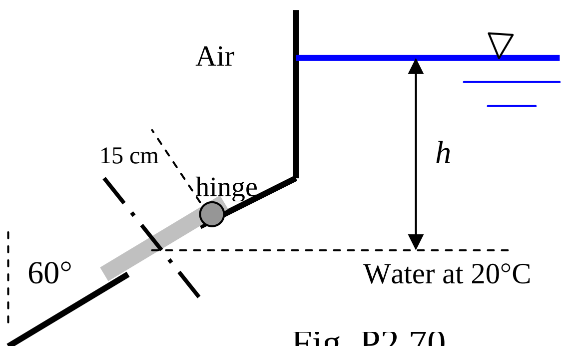


Fig. P2.70

h for the water to cause this condition.

Solution: For water, take γ = 9790 N/m³. The hydrostatic force on the valve is The center of pressure is slightly below the centerline by an amount

$$F = p_{CG} A = \gamma h (\frac{\pi}{4}) R^2 = (9790 \frac{N}{m^3}) h (\frac{\pi}{4})(0.1143 m)^2 = 100.45 h$$

The 60° angle in the figure is a red herring – we need the **30° angle** with the horizontal.

$$y_{CP} = -\frac{\gamma \sin\theta I_{xx}}{F} = \frac{(9790)\sin(30^\circ)(\pi/4)(0.1143)^4}{100.45 h} = \frac{0.00653}{h}$$

Then the moment about the hinge is

$$M_{hinge} = F l = (100.45h)(0.15 + \frac{0.00653}{h}) = 50\, N-m$$

$$\text{Solve for} \quad h = \mathbf{3.28\,m} \qquad \textit{Ans.}$$

Since y_{CP} is so small (2 mm), you don't really need EES. Just iterate once or twice.

2.71 In Fig. P2.71 gate AB is 3 m wide into the paper and is connected by a rod and pulley to a concrete sphere (SG = 2.40). What sphere diameter is just right to close the gate?

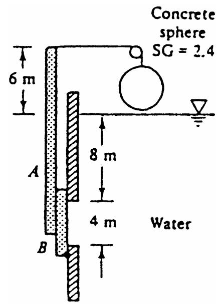


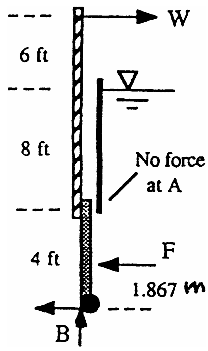
Fig. P2.71

Solution: The centroid of AB is 10 m down from the surface, hence the hydrostatic force is

$$F = \gamma h_{CG} A = (9790)(10)(4 \times 3)$$
$$= 1.175E6 \text{ N}$$

The line of action is slightly below the centroid:

$$y_{CP} = -\frac{(1/12)(3)(4)^3 \sin 90°}{(10)(12)} = -0.133 \text{ m}$$



Sum moments about B in the freebody at right to find the pulley force or weight W:

$$\sum M_B = 0 = W(6+8+4 \text{ m}) - (1.175E6)(2.0 - 0.133 \text{ m}), \quad \text{or} \quad W = 121800 \text{ N}$$

Set this value equal to the weight of a solid concrete sphere:

$$W = 121800 \text{ N} = \gamma_{concrete}\frac{\pi}{6}D^3 = (2.4)(9790)\frac{\pi}{6}D^3, \quad \text{or:} \quad D_{sphere} = \mathbf{2.15\ m} \quad \textit{Ans.}$$

2.72 Gate B is 30 cm high and 60 cm wide into the paper and hinged at the top. What is the water depth *h* which will first cause the gate to open?

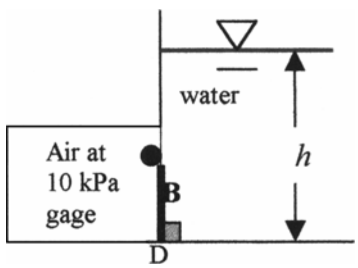


Fig. P2.72

Solution: The minimum height needed to open the gate can be assessed by calculating the hydrostatic force on each side of the gate and equating moments about the hinge. The air pressure causes a force, F_{air}, which acts on the gate at 0.15 m above point D.

$$F_{air} = (10{,}000 \text{ Pa})(0.3 \text{ m})(0.6 \text{ m}) = 1800 \text{ N}$$

Since the air pressure is uniform, F_{air} acts at the centroid of the gate, or 15 cm below the hinge. The force imparted by the water is simply the hydrostatic force,

$$F_w = (\gamma h_{CG} A)_w = (9790\ \text{N/m}^3)(h - 0.15\ \text{m})(0.3\ \text{m})(0.6\ \text{m}) = 1762.2h - 264.3$$

This force has a center of pressure at,

$$y_{CP} = \frac{\frac{1}{12}(0.6)(0.3)^3(\mathbf{sin}90)}{(h-0.15)(0.3)(0.6)} = \frac{0.0075}{h-0.15} \quad \text{with } h \text{ in meters}$$

Sum moments about the hinge and set equal to zero to find the minimum height:

$$\sum M_{hinge} = 0 = (1762.2h - 264.3)[0.15 + (0.0075/(h-0.15))] - (1800)(0.15)$$

This is quadratic in *h*, but let's simply solve by iteration: $h = \mathbf{1.12\ m}$ *Ans.*

2.73 Weightless gate AB is 5 ft wide into the paper and opens to let fresh water out when the ocean tide is falling. The hinge at A is 2 ft above the freshwater level. Find *h* when the gate opens.

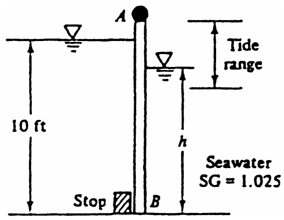


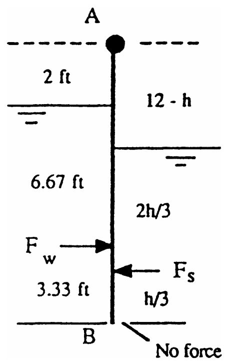
Fig. P2.73

Solution: There are two different hydrostatic forces and two different lines of action. On the water side,

$$F_w = \gamma h_{CG} A = (62.4)(5)(10 \times 5) = 15600\ \text{lbf}$$

positioned at 3.33 ft above point B. In the seawater,

$$F_s = (1.025 \times 62.4)\left(\frac{h}{2}\right)(5h) = 159.9h^2\ \ (\text{lbf})$$



positioned at h/3 above point B. Summing moments about hinge point A gives the desired seawater depth h:

$$\sum M_A = 0 = (159.9h^2)(12 - h/3) - (15600)(12 - 3.33),$$

$$\text{or} \quad 53.3h^3 - 1918.8h^2 + 135200 = 0, \quad \text{solve for} \quad h = \mathbf{9.85\ ft} \quad \textit{Ans.}$$

2.74 Find the height *H* in Fig. P2.74 for which the hydrostatic force on the rectangular panel is the same as the force on the semicircular panel below. Find the force on each panel and set them equal:

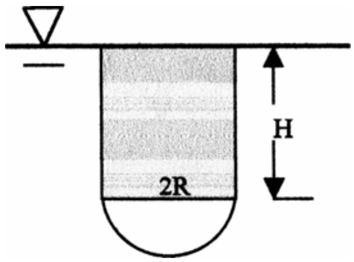


Fig. P2.74

$$F_{rect} = \gamma h_{CG} A_{rect} = \gamma (H/2)[(2R)(H)] = \gamma RH^2$$

$$F_{semi} = \gamma h_{CG} A_{semi} = \gamma (H + 4R/3\pi)[(\pi/2)R^2]$$

Set them equal, cancel γ: $RH^2 = (\pi/2)R^2H + 2R^3/3$, or: $H^2 - (\pi/2)RH - 2R^2/3 = 0$

Solution: $H = \boldsymbol{R[\pi/4 + \{(\pi/4)^2 + 2/3\}^{1/2}] \approx 1.92R}$ *Ans.*

2.75 Gate AB in the figure is hinged at A, has width b into the paper, and makes smooth contact at B. The gate has density ρS and uniform thickness t. For what gate density, expressed as a function of (h, t, ρ, θ), will the gate just begin to lift off the bottom? Why is your answer independent of L and b?

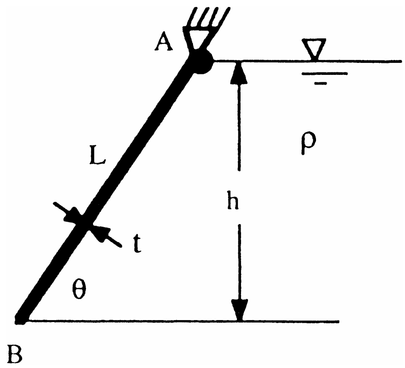


Fig. P2.75

Solution: Gate weight acts down at the center between A and B. The hydrostatic force acts at two-thirds of the way down the gate from A. When "beginning to lift off," there is no force at B. Summing moments about A yields

$$W\frac{L}{2}cos\theta = F\frac{2L}{3}, \quad F = \rho g\frac{h}{2}bL, \quad W = \rho_s gbLt$$

Combine and solve for the density of the gate. *L* and *b* and *g* drop out!

$$\boldsymbol{\rho_s = \frac{2h}{3t\,cos\theta}\rho} \quad \textit{Ans.}$$

2.76 Panel BC in Fig. P2.76 is circular. Compute (a) the hydrostatic force of the water on the panel; (b) its center of pressure; and (c) the moment of this force about point B.

Solution: (a) The hydrostatic force on the gate is:

$$F = \gamma h_{CG} A = (9790\ \text{N/m}^3)(4.5\ \text{m})\sin 50°(\pi)(1.5\ \text{m})^2 = \mathbf{239\ kN} \quad \textit{Ans.}\ \text{(a)}$$

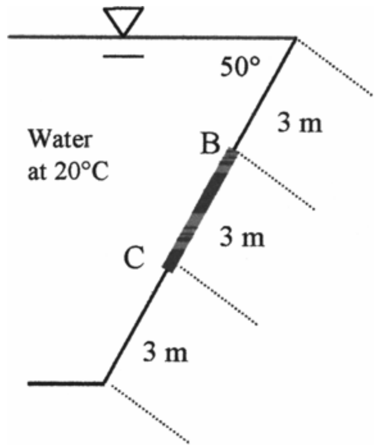


Fig. P2.76

(b) The center of pressure of the force is:

$$y_{CP} = \frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{\frac{\pi}{4} r^4 \sin\theta}{h_{CG}A}$$

$$= \frac{\frac{\pi}{4}(1.5)^4 \sin 50°}{(4.5\sin 50°)(\pi)(1.5^2)} = \mathbf{0.125\ m} \quad \textit{Ans.}\ \text{(b)}$$

Thus y is **1.625 m** down along the panel from B (or 0.125 m down from the center of the circle).

(c) The moment about B due to the hydrostatic force is,

$$M_B = (238550\ \text{N})(1.625\ \text{m}) = 387{,}600\ \text{N}\cdot\text{m} = \mathbf{388\ kN\cdot m} \quad \textit{Ans.}\ \text{(c)}$$

2.77 Circular gate ABC is hinged at B. Compute the force just sufficient to keep the gate from opening when h = 8 m. Neglect atmospheric pressure.

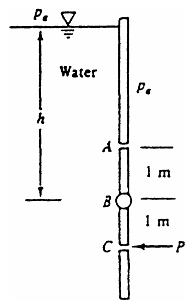


Fig. P2.77

Solution: The hydrostatic force on the gate is

$$F = \gamma h_{CG} A = (9790)(8\ \text{m})(\pi\ \text{m}^2) = 246050\ \text{N}$$

This force acts below point B by the distance

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(\pi/4)(1)^4 \sin 90°}{(8)(\pi)} = -0.03125\ \text{m}$$

Summing moments about B gives P(1 m) = (246050)(0.03125 m), or P ≈ **7690 N** *Ans.*

2.78 Analyze Prob. 2.77 for arbitrary depth *h* and gate radius *R* and derive a formula for the opening force P. Is there anything unusual about your solution?

Solution: Referring to Fig. P2.77, the force F and its line of action are given by

$$F = \gamma h_{CG}A = \gamma h(\pi R^2)$$

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(\pi/4)R^4\sin 90°}{h(\pi R^2)} = -\frac{R^2}{4h}$$

Summing moments about the hinge line B then gives

$$\sum M_B = 0 = (\gamma h\pi R^2)\left(\frac{R^2}{4h}\right) - P(R), \quad \text{or:} \quad P = \frac{\pi}{4}\gamma R^3 \quad \textit{Ans.}$$

What is unusual, at least to non-geniuses, is that the result is independent of depth *h*.

2.79 Gate ABC in Fig. P2.79 is 1-m-square and hinged at B. It opens automatically when the water level is high enough. Neglecting atmospheric pressure, determine the lowest level *h* for which the gate will open. Is your result independent of the liquid density?

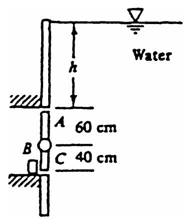
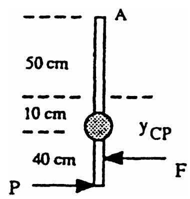


Fig. P2.79

Solution: The gate will open when the hydrostatic force F on the gate is *above B*, that is, when

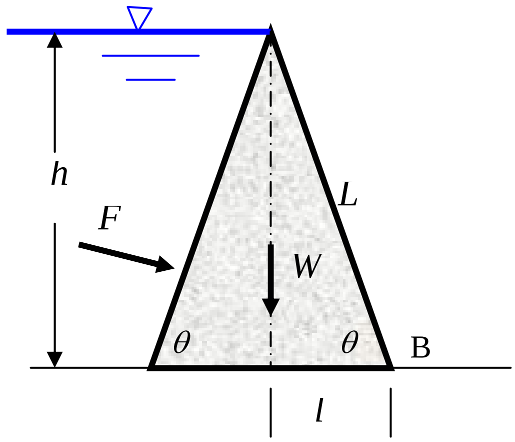


$$|y_{CP}| = \frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{(1/12)(1\text{ m})(1\text{ m})^3\sin 90°}{(h+0.5\text{ m})(1\text{ m}^2)} < 0.1\text{ m},$$

$$\text{or:} \quad h + 0.5 > 0.833\text{ m}, \quad \text{or:} \quad \mathbf{h > 0.333\ m} \quad \textit{Ans.}$$

Indeed, this result is independent of the liquid density.

***P2.80** A concrete dam (SG = 2.5) is made in the shape of an isosceles triangle, as in Fig. P2.80. Analyze this geometry to find the range of angles θ for which the hydrostatic force will tend to tip the dam over at point B. The width into the paper is *b*.



Solution: The critical angle is when the hydrostatic force *F* causes a clockwise moment equal to the counterclockwise moment of the dam weight *W*. The length *L* of the slanted side of the dam is $L = h/\sin\theta$. The force *F* is two-thirds of the way down this face. The moment arm of the weight about point B is $l = h/\tan\theta$. The moment arm of *F* about point B is quite difficult, and you should check this:

$$\text{Moment arm of F about B is } \frac{L}{3} - 2l\cos\theta = \frac{1}{3}\frac{h}{\sin\theta} - \frac{2h}{\tan\theta}\cos\theta$$

Evaluate the two forces and then their moments:

$$F = \gamma\frac{h}{2}\frac{h}{\sin\theta}b \quad ; \quad W = SG\,\gamma\,\upsilon_{dam} = SG\,\gamma\,h\frac{h}{\tan\theta}b$$

$$\Sigma M_B = \frac{\gamma h^2 b}{2\sin\theta}\left(\frac{h}{3\sin\theta} - \frac{2h\cos\theta}{\tan\theta}\right) - \frac{SG\,\gamma\,h^2 b}{\tan\theta}\left(\frac{h}{\tan\theta}\right) \qquad \text{clockwise}$$

When the moment is negative (small θ), the dam is *stable*, it will not tip over. The moment is zero, for SG = 2.5, at $\theta = 77.4°$. Thus tipping is possible in the range **θ > 77.4°**. *Ans*. NOTE: This answer is independent of the numerical values of *h*, *g*, or *b* but requires SG = 2.5.

P2.81 For the semicircular cylinder CDE in Ex. 2.9, find the vertical hydrostatic force by integrating the vertical component of pressure around the surface from $\theta = 0$ to $\theta = \pi$.

Solution: A sketch is repeated here. At any position θ, as in Fig. P2.81, the vertical component of pressure is $p\cos\theta$. The depth down to this point is $h+R(1-\cos\theta)$, and the local pressure is γ times this depth. Thus

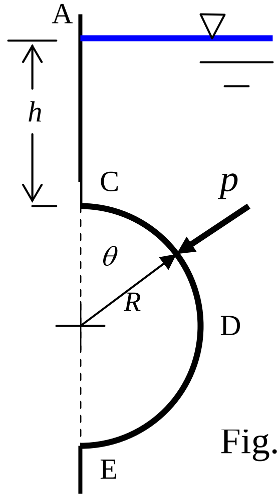


Fig. P2.81

$$F = \int p\cos\theta\, dA = \int_0^{\pi} \gamma[h + R(1-\cos\theta)](\cos\theta)[b\,R\,d\theta]$$

$$= \gamma bR(h+R)\int_0^{\pi}\cos\theta\, d\theta - \gamma bR^2\int_0^{\pi}\cos^2\theta\, d\theta = 0 - \gamma bR^2\frac{\pi}{2}$$

$$\text{Rewrite:} \quad F_{down} = -\gamma\frac{\pi}{2}R^2 b \qquad \textit{Ans.}$$

The negative sign occurs because the sign convention for *dF* was a *downward* force.

2.82 The dam in Fig. P2.82 is a quarter-circle 50 m wide into the paper. Determine the horizontal and vertical components of hydrostatic force against the dam and the point CP where the resultant strikes the dam.

Solution: The horizontal force acts as if the dam were vertical and 20 m high:

$$F_H = \gamma h_{CG} A_{vert}$$
$$= (9790\ \text{N/m}^3)(10\ \text{m})(20\times 50\ \text{m}^2)$$
$$= \mathbf{97.9\ MN} \quad \textit{Ans.}$$

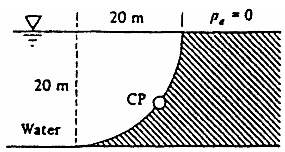
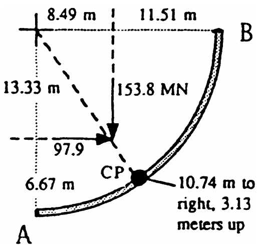


Fig. P2.82



This force acts 2/3 of the way down or 13.33 m from the surface, as in the figure. The vertical force is the weight of the fluid above the dam:

$$F_V = \gamma(\text{Vol})_{dam} = (9790\ \text{N/m}^3)\frac{\pi}{4}(20\ \text{m})^2(50\ \text{m}) = \mathbf{153.8\ MN} \quad \textit{Ans.}$$

This vertical component acts through the centroid of the water above the dam, or $4R/3\pi = 4(20\ \text{m})/3\pi = 8.49$ m to the right of point A, as shown in the figure. The resultant hydrostatic force is $F = [(97.9\ \text{MN})^2 + (153.8\ \text{MN})^2]^{1/2}$ = **182.3 MN** acting down at an angle of **32.5°** from the vertical. The line of action of F strikes the circular-arc dam AB at the center of pressure CP, which is **10.74 m to the right and 3.13 m up from point A**, as shown in the figure. *Ans*.

2.83 Gate AB is a quarter-circle 10 ft wide and hinged at B. Find the force F just sufficient to keep the gate from opening. The gate is uniform and weighs 3000 lbf.

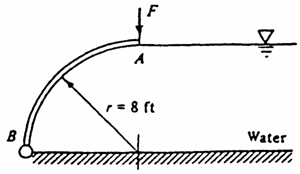
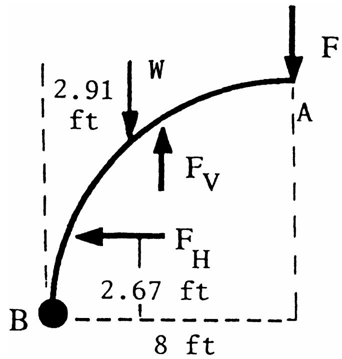


Fig. P2.83

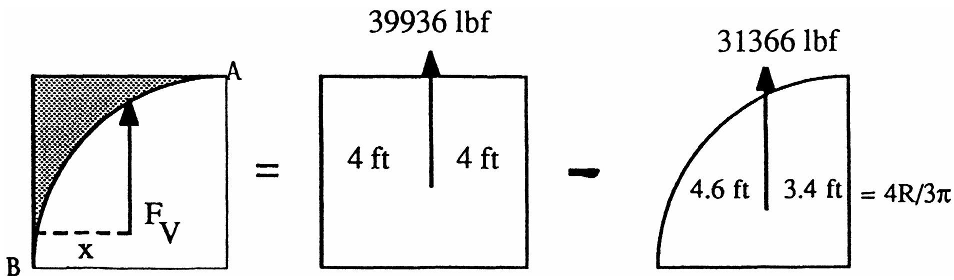
Solution: The horizontal force is computed as if AB were vertical:

$$F_H = \gamma h_{CG} A_{vert} = (62.4)(4\ ft)(8\times 10\ ft^2)$$
$$= 19968\ lbf \quad \text{acting 5.33 ft below A}$$



The vertical force equals the weight of the missing piece of water above the gate, as shown below.

$$F_V = (62.4)(8)(8\times 10) - (62.4)(\pi/4)(8)^2(10)$$
$$= 39936 - 31366 = 8570\ lbf$$



The line of action *x* for this 8570-lbf force is found by summing moments from above:

$$\sum M_B(\text{of } F_V) = 8570x = 39936(4.0) - 31366(4.605), \quad \text{or} \quad x = 1.787\ ft$$

Finally, there is the 3000-lbf gate weight W, whose centroid is $2R/\pi$ = 5.093 ft from force F, or 8.0 – 5.093 = 2.907 ft from point B. Then we may sum moments about hinge B to find the force F, using the freebody of the gate as sketched at the top-right of this page:

$$\sum M_B(\text{clockwise}) = 0 = F(8.0) + (3000)(2.907) - (8570)(1.787) - (19968)(2.667),$$

$$\text{or} \quad F = \frac{59840}{8.0} = \mathbf{7480\ lbf} \quad \textit{Ans.}$$

2.84 Determine (a) the total hydrostatic force on curved surface AB in Fig. P2.84 and (b) its line of action. Neglect atmospheric pressure and assume unit width into the paper.

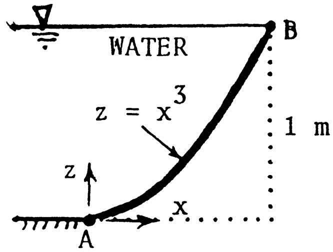
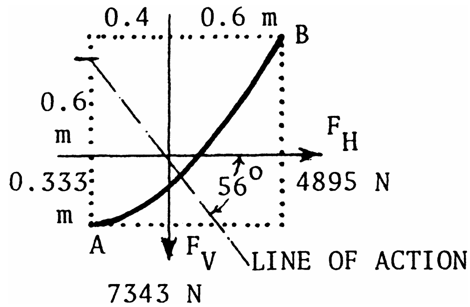


Fig. P2.84

Solution: The horizontal force is

$$F_H = \gamma h_{CG} A_{vert} = (9790\ \text{N/m}^3)(0.5\ \text{m})(1\times 1\ \text{m}^2) = 4895\ \text{N} \ \text{ at } 0.667\ \text{m below B.}$$

For the cubic-shaped surface AB, the weight of water above is computed by integration:



$$F_V = \gamma b \int_0^1 (1 - x^3)\,dx = \frac{3}{4}\gamma b$$

$$= (3/4)(9790)(1.0) = 7343\ \text{N}$$

The line of action (water centroid) of the vertical force also has to be found by integration:

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\int_0^1 x(1-x^3)\,dx}{\int_0^1 (1-x^3)\,dx} = \frac{3/10}{3/4} = 0.4\ \text{m}$$

The vertical force of 7343 N thus acts at 0.4 m to the right of point A, or 0.6 m to the left of B, as shown in the sketch above. The resultant hydrostatic force then is

$$F_{total} = [(4895)^2 + (7343)^2]^{1/2} = \mathbf{8825\ N} \quad \text{acting at } \mathbf{56.31°} \text{ down and to the right.} \quad \textit{Ans.}$$

This result is shown in the sketch at above right. The line of action of F strikes the vertical above point A at 0.933 m above A, or 0.067 m below the water surface.

2.85 Compute the horizontal and vertical components of the hydrostatic force on the quarter-circle panel at the bottom of the water tank in Fig. P2.85.

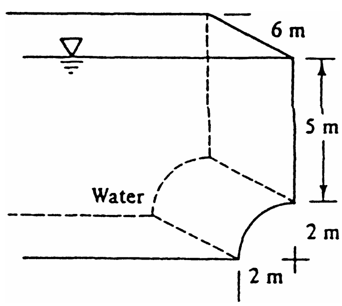


Fig. P2.85

Solution: The horizontal component is

$$F_H = \gamma h_{CG} A_{vert} = (9790)(6)(2\times 6)$$

$$= \mathbf{705000\ N} \quad \textit{Ans.}\ \text{(a)}$$

The vertical component is the weight of the fluid above the quarter-circle panel:

$$\begin{aligned} F_V &= W(\text{2 by 7 rectangle}) - W(\text{quarter-circle}) \\ &= (9790)(2\times 7\times 6) - (9790)(\pi/4)(2)^2(6) \\ &= 822360 - 184537 = \mathbf{638000\ N} \quad \textit{Ans.}\ \text{(b)} \end{aligned}$$

2.86 The quarter circle gate BC in Fig. P2.86 is hinged at C. Find the horizontal force *P* required to hold the gate stationary. The width *b* into the paper is 3 m. Neglect the weight of the gate.

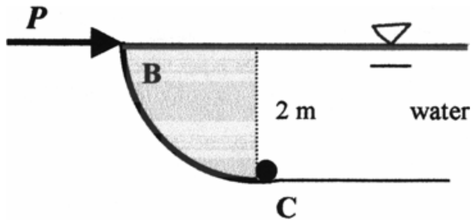


Fig. P2.86

Solution: The horizontal component of water force is

$$F_H = \gamma h_{CG} A = (9790\ \text{N/m}^3)(1\ \text{m})[(2\ \text{m})(3\ \text{m})] = 58{,}740\ \text{N}$$

This force acts 2/3 of the way down or 1.333 m down from the surface (0.667 m up from C). The vertical force is the weight of the quarter-circle of water above gate BC:

$$F_V = \gamma(\text{Vol})_{\text{water}} = (9790\ \text{N/m}^3)[(\pi/4)(2\ \text{m})^2(3\ \text{m})] = 92{,}270\ \text{N}$$

F_V acts down at $(4R/3\pi) = 0.849$ m to the left of C. Sum moments clockwise about point C:

$$\sum M_C = 0 = (2\ \text{m})P - (58740\ \text{N})(0.667\ \text{m}) - (92270\ \text{N})(0.849\ \text{m}) = 2P - 117480$$

$$\text{Solve for} \quad P = 58{,}700\ \text{N} = \mathbf{58.7\ kN} \quad \textit{Ans.}$$

2.87 The bottle of champagne (SG = 0.96) in Fig. P2.87 is under pressure as shown by the mercury manometer reading. Compute the net force on the 2-in-radius hemispherical end cap at the bottom of the bottle.

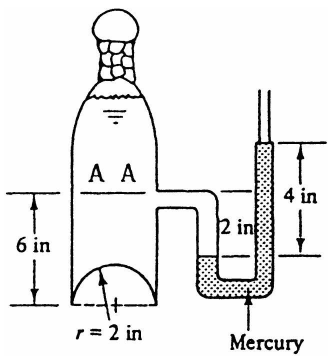


Fig. P2.87

Solution: First, from the manometer, compute the gage pressure at section AA in the champagne 6 inches above the bottom:

$$p_{AA} + (0.96\times 62.4)\left(\frac{2}{12}\text{ ft}\right) - (13.56\times 62.4)\left(\frac{4}{12}\text{ ft}\right) = p_{atmosphere} = 0 \text{ (gage)},$$

$$\text{or:}\quad P_{AA} = 272 \text{ lbf/ft}^2 \text{ (gage)}$$

Then the force on the bottom end cap is vertical only (due to symmetry) and equals the force at section AA plus the weight of the champagne below AA:

$$F = F_V = p_{AA}(\text{Area})_{AA} + W_{6\text{-in cylinder}} - W_{2\text{-in hemisphere}}$$

$$= (272)\frac{\pi}{4}(4/12)^2 + (0.96\times 62.4)\pi(2/12)^2(6/12) - (0.96\times 62.4)(2\pi/3)(2/12)^3$$

$$= 23.74 + 2.61 - 0.58 \approx \mathbf{25.8\ lbf} \quad \textit{Ans.}$$

2.88 Circular-arc *Tainter* gate ABC pivots about point O. For the position shown, determine (a) the hydrostatic force on the gate (per meter of width into the paper); and (b) its line of action. Does the force pass through point O?

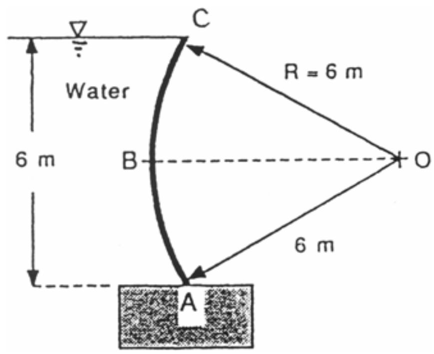
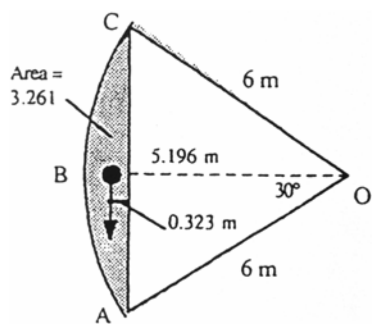


Fig. P2.88

Solution: The horizontal hydrostatic force is based on vertical projection:

$$F_H = \gamma h_{CG} A_{vert} = (9790)(3)(6\times 1) = 176220 \text{ N} \quad \text{at 4 m below C}$$

The vertical force is *upward* and equal to the weight of the missing water in the segment ABC shown shaded below. Reference to a good handbook will give you the geometric properties of a circular segment, and you may compute that the segment area is 3.261 m^2 and its centroid is 5.5196 m from point O, or 0.3235 m from vertical line AC, as shown in the figure. The vertical (upward) hydrostatic force on gate ABC is thus



$$F_V = \gamma A_{ABC}(\text{unit width}) = (9790)(3.2611)$$

$$= 31926 \text{ N} \quad \text{at 0.4804 m from B}$$

The net force is thus $F = [F_H^2 + F_V^2]^{1/2} = \mathbf{179100\ N}$ per meter of width, acting upward to the right at an angle of **10.27°** and passing through a point 1.0 m below and 0.4804 m to the right of point B. This force passes, as expected, *right through point O*.

2.89 The tank in the figure contains benzene and is pressurized to 200 kPa (gage) in the air gap. Determine the vertical hydrostatic force on circular-arc section AB and its line of action.

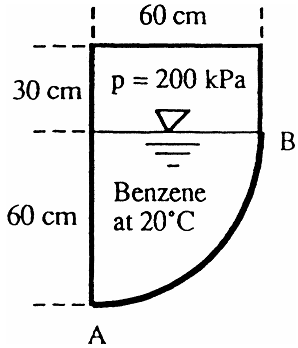


Fig. P2.89

Solution: Assume unit depth into the paper. The vertical force is the weight of benzene plus the force due to the air pressure:

$$F_V = \frac{\pi}{4}(0.6)^2(1.0)(881)(9.81) + (200,000)(0.6)(1.0) = \mathbf{122400\ \frac{N}{m}} \quad \textit{Ans.}$$

Most of this (120,000 N/m) is due to the air pressure, whose line of action is in the middle of the horizontal line through B. The vertical benzene force is 2400 N/m and has a line of action (see Fig. 2.13 of the text) at $4R/(3\pi) = 25.5$ cm to the right or A.

The moment of these two forces about A must equal to moment of the combined (122,400 N/m) force times a distance X to the right of A:

$$(120000)(30\text{ cm}) + (2400)(25.5\text{ cm}) = 122400(X), \quad \textit{solve for}\ \ \mathbf{X = 29.9\ cm} \quad \textit{Ans.}$$

The vertical force is **122400 N/m** (down), acting at **29.9 cm** to the right of A.

P2.90 The tank in Fig. P2.90 is 120 cm long into the paper. Determine the horizontal and vertical hydrostatic forces on the quarter-circle panel AB. The fluid is water at 20°C. Neglect atmospheric pressure.

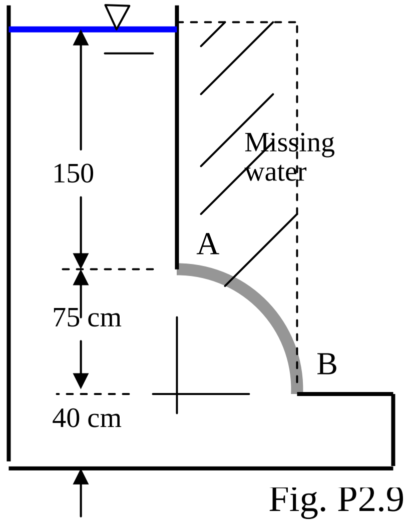


Fig. P2.90

Solution: For water at 20°C, take γ = 9790 N/m^3.

The vertical force on AB is the weight of the missing water above AB – see the dashed lines in Fig. P2.90. Calculate this as a rectangle plus a square-minus-a-quarter-circle:

$$\text{Missing water} = (1.5m)(0.75m)(1.2m) + (1-\pi/4)(0.75m)^2 = 2.16 + 0.145 = 2.305\,m^3$$

$$F_V = \gamma\upsilon = (9790\,N/m^3)(2.305\,m^3) = \mathbf{22{,}600\,N} \quad (\textit{vertical force})$$

The horizontal force is calculated from the vertical projection of panel AB:

$$F_H = p_{CG}\,h\,A_{projection} = (9790\frac{N}{m^3})(1.5 + \frac{0.75}{2}m)(0.75m)(1.2m) = \mathbf{16{,}500\,N} \quad (\textit{horizontal force})$$

2.91 The hemispherical dome in Fig. P2.91 weighs 30 kN and is filled with water and attached to the floor by six equally-spaced bolts. What is the force in each bolt required to hold the dome down?

Solution: Assuming no leakage, the hydrostatic force required equals the *weight of missing water*, that is, the water in a 4-m-diameter cylinder, 6 m high, minus the hemisphere and the small pipe:

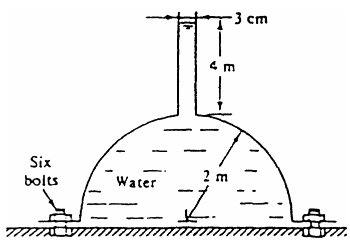


Fig. P2.91

$$F_{total} = W_{2\text{-m-cylinder}} - W_{2\text{-m-hemisphere}} - W_{3\text{-cm-pipe}}$$
$$= (9790)\pi(2)^2(6) - (9790)(2\pi/3)(2)^3 - (9790)(\pi/4)(0.03)^2(4)$$
$$= 738149 - 164033 - 28 = 574088 \text{ N}$$

The dome material helps with 30 kN of weight, thus the bolts must supply 574088–30000 or 544088 N. The force in each of 6 bolts is 544088/6 or $F_{bolt} \approx$ **90700 N** *Ans*.

2.92 A 4-m-diameter water tank consists of two half-cylinders, each weighing 4.5 kN/m, bolted together as in Fig. P2.92. If the end caps are neglected, compute the force in each bolt.

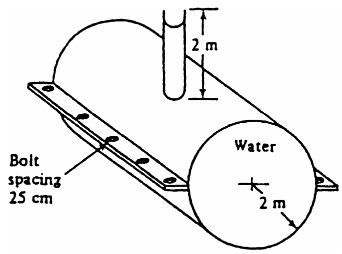


Fig. P2.92

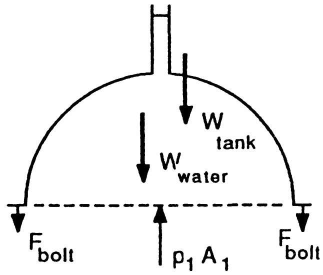
Solution: Consider a 25-cm width of upper cylinder, as at right. The water pressure in the bolt plane is

$$p_1 = \gamma h = (9790)(4) = 39160 \text{ Pa}$$

Then summation of vertical forces on this 25-cm-wide freebody gives

$$\sum F_z = 0 = p_1 A_1 - W_{water} - W_{tank} - 2F_{bolt}$$

$$= (39160)(4\times 0.25) - (9790)(\pi/2)(2)^2(0.25) - (4500)/4 - 2F_{bolt},$$



Solve for $F_{one\ bolt} = \mathbf{11300\ N}$ *Ans.*

2.93 In Fig. P2.93 a one-quadrant spherical shell of radius R is submerged in liquid of specific weight γ and depth h > R. Derive an analytic expression for the hydrodynamic force F on the shell and its line of action.

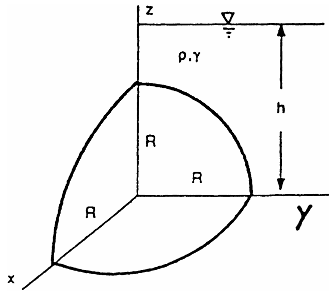


Fig. P2.93

Solution: The two horizontal components are identical in magnitude and equal to the force on the quarter-circle side panels, whose centroids are $(4R/3\pi)$ above the bottom:

$$\text{Horizontal components:}\quad F_x = F_y = \gamma h_{CG} A_{vert} = \gamma\left(h - \frac{4R}{3\pi}\right)\frac{\pi}{4}R^2$$

Similarly, the vertical component is the weight of the fluid above the spherical surface:

$$F_z = W_{cylinder} - W_{sphere} = \gamma\left(\frac{\pi}{4}R^2 h\right) - \gamma\left(\frac{1}{8}\frac{4}{3}\pi R^3\right) = \gamma\frac{\pi}{4}R^2\left(h - \frac{2R}{3}\right)$$

There is no need to find the (complicated) centers of pressure for these three components, for we know that the resultant on a spherical surface <u>*must pass through the center*</u>. Thus

$$F = \left[F_x^2 + F_y^2 + F_z^2\right]^{1/2} = \boldsymbol{\gamma\frac{\pi}{4}R^2\left[(h-2R/3)^2 + 2(h-4R/3\pi)^2\right]^{1/2}} \quad \textit{Ans.}$$

2.94 The 4-ft-diameter log (SG = 0.80) in Fig. P2.94 is 8 ft long into the paper and dams water as shown. Compute the net vertical and horizontal reactions at point C.

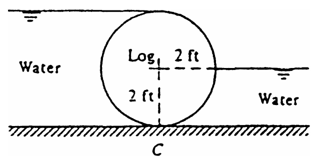
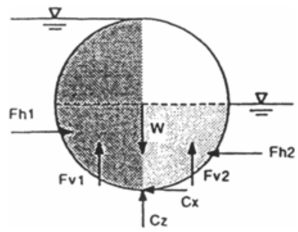


Fig. P2.94

Solution: With respect to the sketch at right, the horizontal components of hydrostatic force are given by



$$F_{h1} = (62.4)(2)(4 \times 8) = 3994 \text{ lbf}$$

$$F_{h2} = (62.4)(1)(2 \times 8) = 998 \text{ lbf}$$

The vertical components of hydrostatic force equal the weight of water in the shaded areas:

$$F_{v1} = (62.4)\frac{\pi}{2}(2)^2(8) = 3137 \text{ lbf}$$

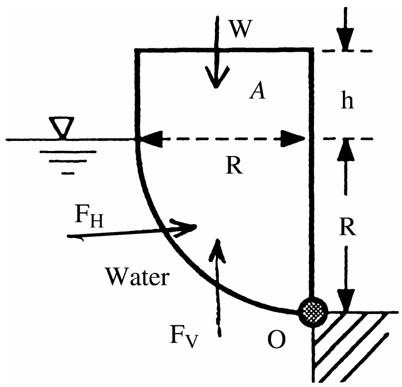
$$F_{v2} = (62.4)\frac{\pi}{4}(2)^2(8) = 1568 \text{ lbf}$$

The weight of the log is $W_{log} = (0.8 \times 62.4)\pi(2)^2(8) = 5018$ lbf. Then the reactions at C are found by summation of forces on the log freebody:

$$\sum F_x = 0 = 3994 - 998 - C_x, \quad \text{or} \quad C_x = \mathbf{2996\ lbf} \quad \textit{Ans.}$$

$$\sum F_z = 0 = C_z - 5018 + 3137 + 1568, \quad \text{or} \quad C_z = \mathbf{313\ lbf} \quad \textit{Ans.}$$

2.95 The uniform body A in the figure has width *b* into the paper and is in static equilibrium when pivoted about hinge O. What is the specific gravity of this body when (a) h = 0; and (b) h = R?



Solution: The water causes a horizontal and a vertical force on the body, as shown:

$$F_H = \gamma \frac{R}{2} Rb \quad at \quad \frac{R}{3} \text{ above } O,$$

$$F_V = \gamma \frac{\pi}{4} R^2 b \quad at \quad \frac{R}{3\pi} \text{ to the left of } O$$

These must balance the moment of the body weight W about O:

$$\sum M_O = \frac{\gamma R^2 b}{2}\left(\frac{R}{3}\right) + \frac{\gamma \pi R^2 b}{4}\left(\frac{4R}{3\pi}\right) - \frac{\gamma_s \pi R^2 b}{4}\left(\frac{4R}{3\pi}\right) - \gamma_s Rhb\left(\frac{R}{2}\right) = 0$$

$$\textit{Solve for:}\quad SG_{body} = \frac{\gamma_s}{\gamma} = \left[\frac{\mathbf{2}}{\mathbf{3}} + \frac{\mathbf{h}}{\mathbf{R}}\right]^{-1} \quad \textit{Ans.}$$

For h = 0, **SG = 3/2** *Ans.* (a). For h = R, **SG = 3/5** *Ans.* (b).

2.96 Curved panel BC is a 60° arc, perpendicular to the bottom at C. If the panel is 4 m wide into the paper, estimate the resultant hydrostatic force of the water on the panel.

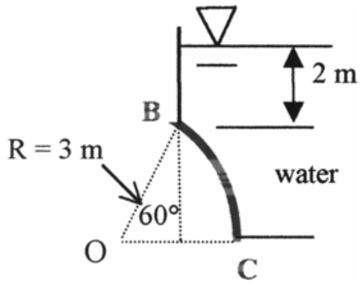
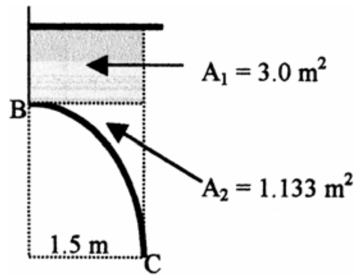


Fig. P2.96

Solution: The horizontal force is,

$$F_H = \gamma h_{CG} A_h$$
$$= (9790\ \text{N/m}^3)[2 + 0.5(3\sin 60°)\ \text{m}]$$
$$\times[(3\sin 60°)\text{m}(4\ \text{m})]$$
$$= 335{,}650\ \text{N}$$



The vertical component equals the weight of water above the gate, which is the sum of the rectangular piece above BC, and the curvy triangular piece of water just above arc BC—see figure at right. (The curvy-triangle calculation is messy and is not shown here.)

$$F_V = \gamma(\text{Vol})_{\text{above BC}} = (9790\ \text{N/m}^3)[(3.0 + 1.133\ \text{m}^2)(4\ \text{m})] = 161{,}860\ \text{N}$$

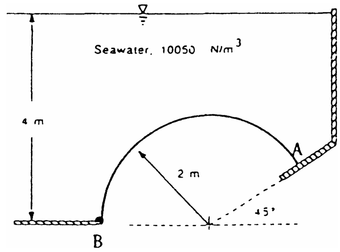
The resultant force is thus,

$$F_R = [(335{,}650)^2 + (161{,}860)^2]^{1/2} = 372{,}635\ \text{N} = \mathbf{373\ kN} \quad \textit{Ans.}$$

This resultant force acts along a line which passes through point O at

$$\theta = \tan^{-1}(161{,}860/335{,}650) = \mathbf{25.7°}$$

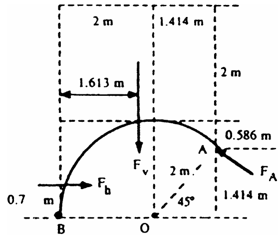
2.97 Gate AB is a 3/8th circle, 3 m wide into the paper, hinged at B and resting on a smooth wall at A. Compute the reaction forces at A and B.



Solution: The two hydrostatic forces are

$$F_h = \gamma h_{CG} A_h = (10050)(4 - 0.707)(1.414 \times 3) = 140 \text{ kN}$$

$$F_v = \text{weight above AB} = 240 \text{ kN}$$



To find the reactions, we need the lines of action of these two forces—a laborious task which is summarized in the figure at right. Then summation of moments on the gate, about B, gives

$$\sum M_{B,clockwise} = 0 = (140)(0.70) + (240)(1.613) - F_A(3.414), \quad \text{or} \quad F_A = \mathbf{142\ kN} \quad \textit{Ans.}$$

Finally, summation of vertical and horizontal forces gives

$$\sum F_z = B_z + 142\sin 45° - 240 = 0, \quad \text{or} \quad B_z = \mathbf{139\ kN}$$

$$\sum F_x = B_x - 142\cos 45° = 0, \quad \text{or} \quad B_x = \mathbf{99\ kN} \quad \textit{Ans.}$$

2.98 Gate *ABC* in Fig. P2.98 is a quarter circle 8 ft wide into the paper. Compute the horizontal and vertical hydrostatic forces on the gate and the line of action of the resultant force.

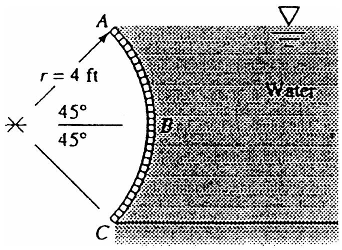


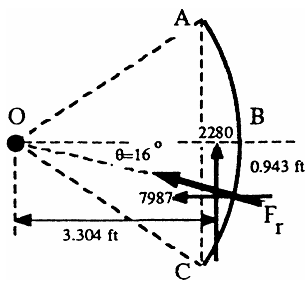
Fig. P2.98

Solution: The horizontal force is

$$F_h = \gamma h_{CG} A_h = (62.4)(2.828)(5.657 \times 8) = \mathbf{7987\ lbf} \leftarrow$$

located at

$$y_{cp} = -\frac{(1/12)(8)(5.657)^3}{(2.828)(5.657 \times 8)} = -0.943 \text{ ft}$$



$$\text{Area ABC} = (\pi/4)(4)^2 - (4\sin 45°)^2 = 4.566 \text{ ft}^2$$

$$\text{Thus} \quad F_v = \gamma \text{Vol}_{ABC} = (62.4)(8)(4.566) = \mathbf{2280\ lbf} \uparrow$$

The resultant is found to be

$$F_R = [(7987)^2 + (2280)^2]^{1/2} = \mathbf{8300\ lbf} \quad \text{acting at } \theta = 15.9° \text{ through the center O.} \quad \textit{Ans.}$$

2.99 A 2-ft-diam sphere weighing 400 kbf closes the 1-ft-diam hole in the tank bottom. Find the force F to dislodge the sphere from the hole.

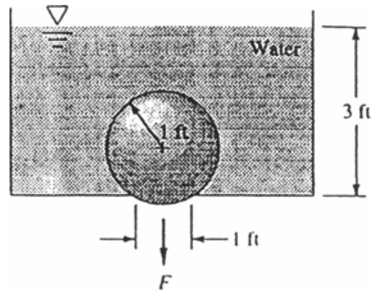
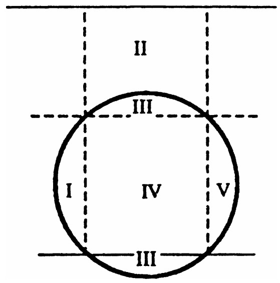


Fig. P2.99



Solution: NOTE: This problem is laborious! Break up the system into regions I,II,III,IV, & V. The respective volumes are:

$$\upsilon_{III} = 0.0539\ \text{ft}^3; \quad \upsilon_{II} = 0.9419\ \text{ft}^3$$
$$\upsilon_{IV} = \upsilon_I = \upsilon_V = 1.3603\ \text{ft}^3$$

Then the hydrostatic forces are:

$$F_{down} = \gamma\upsilon_{II} = (62.4)(0.9419) = 58.8\ \text{lbf}$$
$$F_{up} = \gamma(\upsilon_I + \upsilon_V) = (62.4)(2.7206) = 169.8\ \text{lbf}$$

Then the required force is $F = W + F_{down} - F_{up} = 400 + 59 - 170 = \mathbf{289\ lbf} \uparrow$ *Ans.*

2.100 Pressurized water fills the tank in Fig. P2.100. Compute the hydrostatic force on the conical surface ABC.

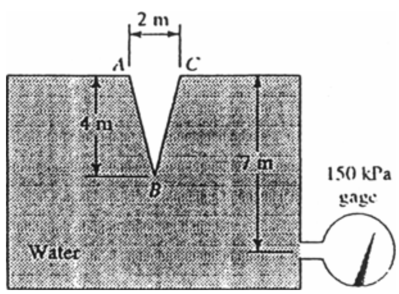
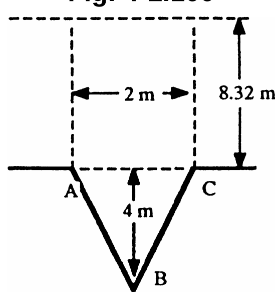


Fig. P2.100



Solution: The gage pressure is equivalent to a fictitious water level $h = p/\gamma = 150000/9790 = 15.32$ m above the gage or 8.32 m above AC. Then the vertical force on the cone equals the weight of fictitious water above ABC:

$$F_V = \gamma \text{Vol}_{above}$$
$$= (9790)\left[\frac{\pi}{4}(2)^2(8.32) + \frac{1}{3}\frac{\pi}{4}(2)^2(4)\right]$$
$$= \mathbf{297,000\ N} \quad \textit{Ans.}$$

P2.101 The closed layered box in Fig. P2.101 has square horizontal cross-sections everywhere. All fluids are at 20°C. Estimate the gage pressure of the air if (*a*) the hydrostatic force on panel AB is 48 kN; or if (*b*) the hydrostatic force on the bottom panel BC is 97 kN.

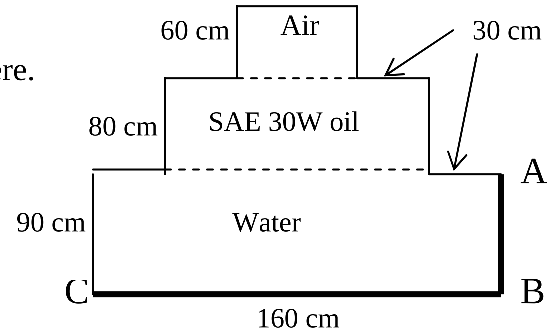


Fig. P2.101

Solution: At 20C, take ρ_{oil} = 891 kg/m^3 and ρ_{water} = 998 kg/m^3. The wedding-cake shape of the box has nothing to do with the problem. (*a*) the force on panel AB equals the pressure at the panel centroid (45 cm down from A) times the panel area:

$$F_{AB} = p_{CG}A_{AB} = (p_{air} + \rho_{oil}gh_{oil} + \rho_{water}gh_{water-CG}), \quad \text{or:}$$
$$48000\,N = [p_{air} + (891)(9.81)(0.8m) + (998)(9.81)(0.45m)][(0.9m)(1.6m)]$$
$$= (p_{air} + 6993 + 4406\,Pa)(1.44\,m^2); \quad \text{Solve} \quad p_{air} = \mathbf{22000\,Pa} \quad Ans.(a)$$

(*b*) The force on the bottom is handled similarly, except we go all the way to the bottom:

$$F_{BC} = p_{BC}A_{AB} = (p_{air} + \rho_{oil}gh_{oil} + \rho_{water}gh_{water}), \quad \text{or:}$$
$$97000\,N = [p_{air} + (891)(9.81)(0.8m) + (998)(9.81)(0.9m)][(1.6m)(1.6m)]$$
$$= (p_{air} + 6993 + 8812\,Pa)(2.56\,m^2); \quad \text{Solve} \quad p_{air} = \mathbf{22000\,Pa} \quad Ans.(b)$$

2.102 A cubical tank is 3 × 3 × 3 m and is layered with 1 meter of fluid of specific gravity 1.0, 1 meter of fluid with SG = 0.9, and 1 meter of fluid with SG = 0.8. Neglect atmospheric pressure. Find (a) the hydrostatic force on the bottom; and (b) the force on a side panel.

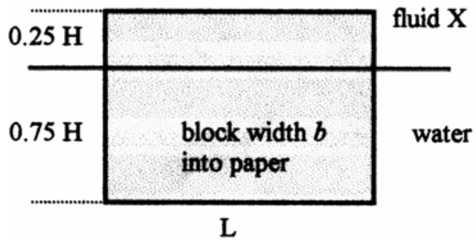
Solution: (a) The force on the bottom is the bottom pressure times the bottom area:

$$F_{bot} = p_{bot}A_{bot} = (9790\ \text{N/m}^3)[(0.8\times 1\ \text{m}) + (0.9\times 1\ \text{m}) + (1.0\times 1\ \text{m})](3\ \text{m})^2$$
$$= \mathbf{238,000\ N} \quad \textit{Ans.}\ \text{(a)}$$

(b) The hydrostatic force on the side panel is the sum of the forces due to each layer:

$$F_{side} = \sum \gamma h_{CG}A_{side} = (0.8\times 9790\ \text{N/m}^3)(0.5\ \text{m})(3\ \text{m}^2) + (0.9\times 9790\ \text{N/m}^3)(1.5\ \text{m})(3\ \text{m}^2)$$
$$+ (9790\ \text{N/m}^3)(2.5\ \text{m})(3\ \text{m}^2) = \mathbf{125,000\ kN} \quad \textit{Ans.}\ \text{(b)}$$

2.103 A solid block, of specific gravity 0.9, floats such that 75% of its volume is in water and 25% of its volume is in fluid X, which is layered above the water. What is the specific gravity of fluid X?



Solution: The block is sketched at right. A force balance is

$$0.9\gamma(\mathrm{HbL}) = \gamma(0.75\mathrm{HbL}) + \mathrm{SG_X}\gamma(0.25\mathrm{HbL})$$

$$0.9 - 0.75 = 0.25\mathrm{SG_X},\quad \boldsymbol{SG_X = 0.6} \quad \textit{Ans.}$$

2.104 The can in Fig. P2.104 floats in the position shown. What is its weight in newtons?

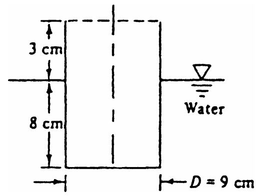


Fig. P2.104

Solution: The can weight simply equals the weight of the displaced water (neglecting the air above):

$$W = \gamma\upsilon_{\text{displaced}} = (9790)\frac{\pi}{4}(0.09\text{ m})^2(0.08\text{ m}) = \mathbf{5.0\ N} \quad \textit{Ans.}$$

2.105 Archimedes, when asked by King Hiero if the new crown was pure gold (SG = 19.3), found the crown weight in air to be 11.8 N and in water to be 10.9 N. Was it gold?

Solution: The buoyancy is the difference between air weight and underwater weight:

$$B = W_{\text{air}} - W_{\text{water}} = 11.8 - 10.9 = 0.9\text{ N} = \gamma_{\text{water}}\upsilon_{\text{crown}}$$

$$\text{But also}\quad W_{\text{air}} = (SG)\gamma_{\text{water}}\upsilon_{\text{crown}},\quad \text{so}\quad W_{\text{in water}} = B(SG - 1)$$

$$\text{Solve for}\quad SG_{\text{crown}} = 1 + W_{\text{in water}}/B = 1 + 10.9/0.9 = \mathbf{13.1\ (not\ pure\ gold)} \quad \textit{Ans.}$$

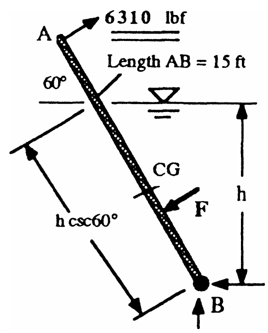
2.106 A spherical helium balloon is 2.5 m in diameter and has a total mass of 6.7 kg. When released into the U. S. Standard Atmosphere, at what altitude will it settle?

Solution: The altitude can be determined by calculating the air density to provide the proper buoyancy and then using Table A.3 to find the altitude associated with this density:

$$\rho_{air} = m_{balloon}/Vol_{sphere} = (6.7\text{ kg})/[\pi(2.5\text{ m}^3)/6] = 0.819\text{ kg/m}^3$$

From Table A.3, atmospheric air has $\rho = 0.819$ kg/m^3 at an altitude of about **4000 m**. *Ans.*

2.107 Repeat Prob. 2.62 assuming that the 10,000 lbf weight is aluminum (SG = 2.71) and is hanging submerged in the water.



Solution: Refer back to Prob. 2.62 for details. The only difference is that the force applied to gate AB by the weight is less due to buoyancy:

$$F_{net} = \frac{(SG-1)}{SG}\gamma\upsilon_{body} = \frac{2.71-1}{2.71}(10000) = 6310\text{ lbf}$$

This force replaces "10000" in the gate moment relation (see Prob. 2.62):

$$\sum M_B = 0 = 6310(15) - (288.2h^2)\left(\frac{h}{2}\csc 60° - \frac{h}{6}\csc 60°\right) - 4898(7.5\cos 60°)$$

$$\text{or: } h^3 = 76280/110.9 = 688, \quad \text{or: } h = \mathbf{8.83\text{ ft}} \quad \textit{Ans.}$$

2.108 A 7-cm-diameter solid aluminum ball (SG = 2.7) and a solid brass ball (SG = 8.5) balance nicely when submerged in a liquid, as in Fig. P2.108. (*a*) If the fluid is water at 20°C, what is the diameter of the brass ball? (*b*) If the brass ball has a diameter of 3.8 cm, what is the density of the fluid?

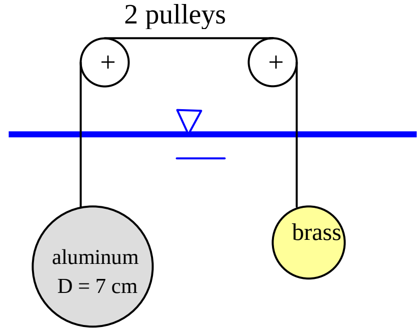


Fig. P2.108

Solution: For water, take $\gamma = 9790\ \text{N/m}^3$. If they balance, net weights are equal:

$$(SG_{alum} - SG_{fluid})\gamma_{water}\frac{\pi}{6}D^3_{alum} = (SG_{brass} - SG_{fluid})\gamma_{water}\frac{\pi}{6}D^3_{brass}$$

We can cancel γ_{water} and (π/6). (a) For water, $SG_{fluid} = 1$, and we obtain

$$(2.7-1)(0.07m)^3 = (8.5-1)D^3_{brass} \quad ; \quad \text{Solve} \quad D_{brass} = \mathbf{0.0427\,m} \qquad Ans.(a)$$

(*b*) For this part, the fluid density (or specific gravity) is unknown:

$$(2.7-SG_{fluid})(0.07\,m)^3 = (8.5-SG_{fluid})(0.038m)^3 \;\; ; \quad \text{Solve} \quad SG_{fluid} = \mathbf{1.595}$$

$$\text{Thus} \quad \rho_{fluid} = 1.595(998) = \mathbf{1592\,kg/m^3} \qquad Ans.(b)$$

According to Table A3, this fluid is probably *carbon tetrachloride*.

2.109 The float level h of a hydrometer is a measure of the specific gravity of the liquid. For stem diameter D and total weight W, if $h = 0$ represents SG = 1.0, derive a formula for h as a function of W, D, SG, and γ_o for water.

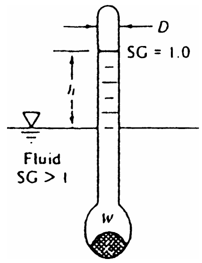
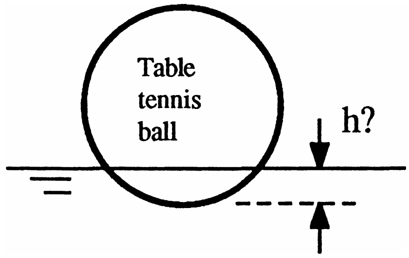


Fig. P2.109

Solution: Let submerged volume be υ_o when SG = 1. Let $A = \pi D^2/4$ be the area of the stem. Then

$$W = \gamma_o \upsilon_o = (SG)\gamma_o(\upsilon_o - Ah), \quad \text{or:} \quad \mathbf{h} = \frac{\mathbf{W(SG-1)}}{\mathbf{SG\gamma_o(\pi D^2/4)}} \quad \textit{Ans.}$$

2.110 An average table tennis ball has a diameter of 3.81 cm and a mass of 2.6 gm. Estimate the (small) depth h at which the ball will float in water at 20°C and sea-level standard air if air buoyancy is (a) neglected; or (b) included.



Solution: For both parts we need the volume of the submerged spherical segment:

$$W = 0.0026(9.81) = 0.0255\ N = \rho_{water} g \frac{\pi h^2}{3}(3R - h), \quad R = 0.01905 \text{ m}, \quad \rho = 998 \frac{kg}{\text{m}^3}$$

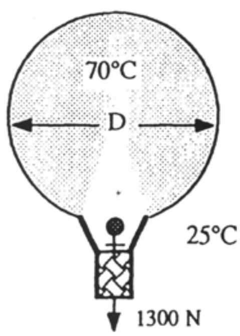
(a) Air buoyancy is neglected. Solve for $h \approx 0.00705 \text{ m} = \mathbf{7.05\ mm}$ *Ans.* (a)

(b) Also include air buoyancy on the exposed sphere volume in the air:

$$0.0255\ N = \rho_w g \upsilon_{seg} + \rho_{air} g \left[\frac{4}{3}\pi R^3 - \upsilon_{seg}\right], \quad \rho_{air} = 1.225 \frac{kg}{\text{m}^3}$$

The air buoyancy is only one-80th of the water. Solve $\boldsymbol{h} = \mathbf{7.00\ mm}$ *Ans.* (b)

2.111 A hot-air balloon must support its own weight plus a person for a total weight of 1300 N. The balloon material has a mass of 60 g/m^2. Ambient air is at 25°C and 1 atm. The hot air inside the balloon is at 70°C and 1 atm. What diameter spherical balloon will just support the weight? Neglect the size of the hot-air inlet vent.



Solution: The buoyancy is due to the difference between hot and cold air density:

$$\rho_{cold} = \frac{p}{RT_{cold}} = \frac{101350}{(287)(273+25)} = 1.185\,\frac{kg}{m^3}; \quad \rho_{hot} = \frac{101350}{287(273+70)} = 1.030\,\frac{kg}{m^3}$$

The buoyant force must balance the known payload of 1300 N:

$$W = 1300\text{ N} = \Delta\rho g\text{ Vol} = (1.185-1.030)(9.81)\frac{\pi}{6}D^3,$$

$$\text{Solve for}\quad D^3 = 1628 \quad\text{or}\quad D_{balloon} \approx \mathbf{11.8\ m} \quad \textit{Ans.}$$

Check to make sure the balloon material is not excessively heavy:

$$W(\text{balloon}) = (0.06\text{ kg/m}^2)(9.81\text{ m/s}^2)(\pi)(11.8\text{ m})^2 \approx 256\text{ N} \quad \text{OK, only 20\% of } W_{total}.$$

2.112 The uniform 5-m-long wooden rod in the figure is tied to the bottom by a string. Determine (a) the string tension; and (b) the specific gravity of the wood. Is it also possible to determine the inclination angle θ?

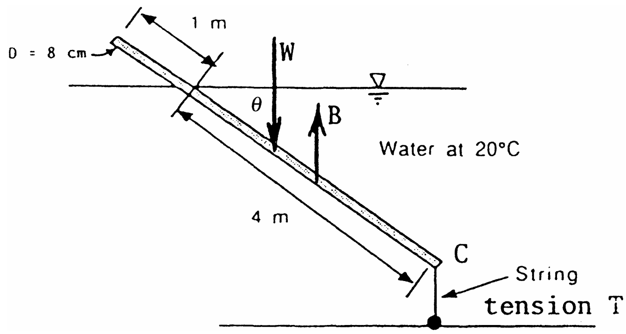


Fig. P2.112

Solution: The rod weight acts at the middle, 2.5 m from point C, while the buoyancy is 2 m from C. Summing moments about C gives

$$\sum M_C = 0 = W(2.5\sin\theta) - B(2.0\sin\theta), \quad\text{or}\quad W = 0.8B$$

$$\text{But}\quad B = (9790)(\pi/4)(0.08\text{ m})^2(4\text{ m}) = 196.8\text{ N}.$$

$$\text{Thus}\quad W = 0.8B = 157.5\text{ N} = SG(9790)(\pi/4)(0.08)^2(5\text{ m}), \quad\text{or:}\quad SG \approx \mathbf{0.64} \quad \textit{Ans.}\text{ (b)}$$

Summation of vertical forces yields

$$\text{String tension}\quad T = B - W = 196.8 - 157.5 \approx \mathbf{39\ N} \quad \textit{Ans.}\text{ (a)}$$

These results are <u>independent of the angle θ</u>, which cancels out of the moment balance.

2.113 A *spar buoy* is a rod weighted to float vertically, as in Fig. P2.113. Let the buoy be maple wood (SG = 0.6), 2 in by 2 in by 10 ft, floating in seawater (SG = 1.025). How many pounds of steel (SG = 7.85) should be added at the bottom so that $h = 18$ in?

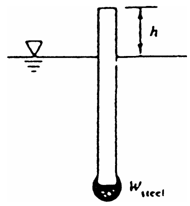


Fig. P2.113

Solution: The relevant volumes needed are

$$\text{Spar volume} = \frac{2}{12}\left(\frac{2}{12}\right)(10) = 0.278\ \text{ft}^3; \quad \text{Steel volume} = \frac{W_{steel}}{7.85(62.4)}$$

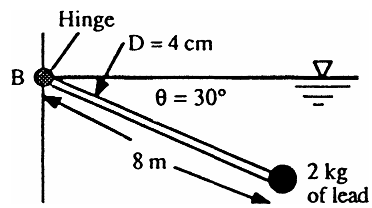
$$\text{Immersed spar volume} = \frac{2}{12}\left(\frac{2}{12}\right)(8.5) = 0.236\ \text{ft}^3$$

The vertical force balance is: buoyancy $B = W_{wood} + W_{steel}$,

$$\text{or:}\quad 1.025(62.4)\left[0.236 + \frac{W_{steel}}{7.85(62.4)}\right] = 0.6(62.4)(0.278) + W_{steel}$$

$$\text{or:}\quad 15.09 + 0.1306W_{steel} = 10.40 + W_{steel}, \quad \text{solve for} \quad W_{steel} \approx \mathbf{5.4\ lbf} \quad \textit{Ans.}$$

2.114 The uniform rod in the figure is hinged at B and in static equilibrium when 2 kg of lead (SG = 11.4) are attached at its end. What is the specific gravity of the rod material? What is peculiar about the rest angle $\theta = 30°$?



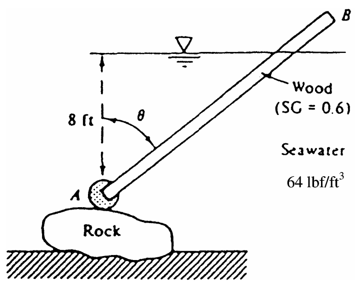
Solution: First compute buoyancies: $B_{rod} = 9790(\pi/4)(0.04)^2(8) = 98.42$ N, and $W_{lead} = 2(9.81) = 19.62$ N, $B_{lead} = 19.62/11.4 = 1.72$ N. Sum moments about B:

$$\sum M_B = 0 = (SG - 1)(98.42)(4\cos 30°) + (19.62 - 1.72)(8\cos 30°) = 0$$

$$\text{Solve for} \quad \mathbf{SG_{rod} = 0.636} \quad \textit{Ans.}\ \text{(a)}$$

The angle θ drops out! The rod is neutrally stable for **any tilt angle!** *Ans.* (b)

2.115 The 2 inch by 2 inch by 12 ft spar buoy from Fig. P2.113 has 5 lbm of steel attached and has gone aground on a rock. If the rock exerts no moments on the spar, compute the angle of inclination θ.



Solution: Let ζ be the submerged length of spar. The relevant forces are:

$$W_{wood} = (0.6)(64.0)\left(\frac{2}{12}\right)\left(\frac{2}{12}\right)(12) = 12.8 \text{ lbf} \quad \textit{at distance } 6\,\mathbf{sin}\,\theta \textit{ to the right of } A\downarrow$$

$$Buoyancy = (64.0)\left(\frac{2}{12}\right)\left(\frac{2}{12}\right)\zeta = 1.778\zeta \quad \textit{at distance } \frac{\zeta}{2}\mathbf{sin}\,\theta \textit{ to the right of } A\uparrow$$

The steel force acts right through A. Take moments about A:

$$\sum M_A = 0 = 12.8(6\,\mathbf{sin}\,\theta) - 1.778\zeta\left(\frac{\zeta}{2}\mathbf{sin}\,\theta\right)$$

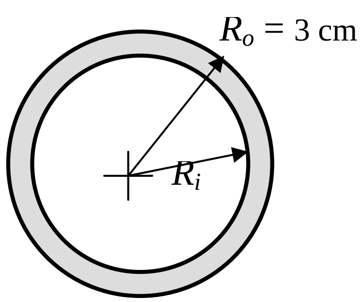
$$\textit{Solve for} \quad \zeta^2 = 86.4, \quad \textit{or} \quad \zeta = 9.295\, ft \quad (\textit{submerged length})$$

Thus the angle of inclination $\theta = \cos^{-1}(8.0/9.295) =$ **30.6°** *Ans.*

P2.116 Ocean currents can be tracked by *Swallow floats* [2], after Dr. John Swallow, of the UK, 1955. There have been many design changes, but the original float was an aluminum tube, of 6 cm outside diameter and about 3 m long, sealed at the ends and slightly pressurized. They had to be etched to obtain the right tube thickness. Estimate the tube thickness to cause neutral buoyancy at a seawater density of 1030 kg/m^3.

Solution: We don't know what the end seals weigh, probably small, neglect them. The density of aluminum is approximately 2700 kg/m^3. Let L = 1 m.

For a neutrally buoyant float, the total weight per meter of aluminum plus air must equal the buoyancy of the seawater:



$$\rho_{alum}\, g\, \pi (R_o^2 - R_i^2) \;+\; \rho_{air}\, g\, \pi R_i^2 \;=\; \rho_{water}\, g\, \pi R_o^2$$

Or:

$$(2700)(9.81)\pi[(0.03)^2 - R_i^2]+1.5(9.81)\pi R_i^2 = 1030(9.81)\pi(0.03)^2$$

$$(2700)(9.81)\pi[(0.03)^2 - R_i^2] + 0.026\,N = 28.57\,N \;; \quad Solve \;\; R_i = 0.0236\,m$$

We guessed at $\rho_{air} \approx 1.5$ kg/m^3. The estimated thickness is R_o–$R_i \approx$ **0.0064 m**. *Ans*.

2.117 The balloon in the figure is filled with helium and pressurized to 135 kPa and 20°C. The balloon material has a mass of 85 g/m^2. Estimate (a) the tension in the mooring line, and (b) the height in the standard atmosphere to which the balloon will rise if the mooring line is cut.

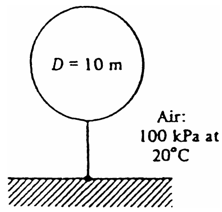


Fig. P2.117

Solution: (a) For helium, from Table A-4, R = 2077 m^2/s^2/K, hence its weight is

$$W_{helium} = \rho_{He} g \upsilon_{balloon} = \left[\frac{135000}{2077(293)}\right](9.81)\left[\frac{\pi}{6}(10)^3\right] = 1139\ N$$

Meanwhile, the total weight of the balloon material is

$$W_{balloon} = \left(0.085\ \frac{kg}{\text{m}^2}\right)\left(9.81\ \frac{\text{m}}{s^2}\right)[\pi(10\ \text{m})^2] = 262\ N$$

Finally, the balloon buoyancy is the weight of displaced air:

$$B_{air} = \rho_{air} g \upsilon_{balloon} = \left[\frac{100000}{287(293)}\right](9.81)\left[\frac{\pi}{6}(10)^3\right] = 6108\ N$$

The difference between these is the tension in the mooring line:

$$T_{line} = B_{air} - W_{helium} - W_{balloon} = 6108 - 1139 - 262 \approx \mathbf{4700\ N} \quad \textit{Ans.}\ \text{(a)}$$

(b) If released, and the balloon remains at 135 kPa and 20°C, equilibrium occurs when the balloon air buoyancy exactly equals the total weight of 1139 + 262 = 1401 N:

$$B_{air} = 1401\ N = \rho_{air}(9.81)\frac{\pi}{6}(10)^3, \quad or: \quad \rho_{air} \approx 0.273\ \frac{kg}{\text{m}^3}$$

From Table A-6, this standard density occurs at approximately

$$\mathbf{Z \approx 12{,}800\ m} \quad \textit{Ans.}\ \text{(b)}$$

P2.118 An intrepid treasure-salvage group has discovered a steel box, containing gold doubloons and other valuables, resting in 80 ft of seawater. They estimate the weight of the box and treasure (in air) at 7000 lbf. Their plan is to attach the box to a sturdy balloon, inflated with air to 3 atm pressure. The empty balloon weighs 250 lbf. The box is 2 ft wide, 5 ft long, and 18 in high. What is the proper diameter of the balloon to ensure an upward lift force on the box that is 20% more than required?

Solution: The specific weight of seawater is approximately 64 lbf/ft^3. The box volume is (2ft)(5ft)(1.5ft) = 12 ft^3, hence the buoyant force on the box is (64)(12) = 768 lbf. Thus the balloon must develop a net upward force of 1.2(7000-768lbf) = 7478 lbf. The air weight in the balloon is negligible, but we can compute it anyway. The air density is:

$$At\ \ p = 3\,atm,\ \ \rho_{air} = \frac{p}{RT} = \frac{3(2116\,lbf/ft^2)}{(1716\,ft^2/s^2-{}^\circ R)(520^\circ R)} = 0.0071\frac{slug}{ft^3}$$

Hence the air specific weight is (0.0071)(32.2) = 0.23 lbf/ft^3, much less than the water. Accounting for balloon weight, the desired net buoyant force on the balloon is

$$F_{net} = (64-0.23\,lbf/ft^3)(\pi/6)D^3_{balloon} - 250\,lbf = 7478\,lbf$$

$$\text{Solve for}\ \ D^3 = 231.4\,lbf^3\ \ ,\quad D_{balloon} \approx \mathbf{6.14\,ft} \quad \textit{Ans.}$$

2.119 With a 5-lbf-weight placed at one end, the uniform wooden beam in the figure floats at an angle θ with its upper right corner at the surface. Determine (a) θ; (b) γwood.

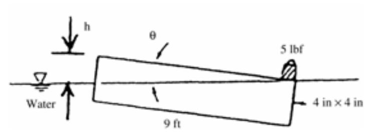


Fig. P2.119

Solution: The total wood volume is (4/12)^2(9) = 1 ft^3. The exposed distance $h = 9\tan\theta$. The vertical forces are

$$\sum F_z = 0 = (62.4)(1.0) - (62.4)(h/2)(9)(4/12) - (SG)(62.4)(1.0) - 5\text{ lbf}$$

The moments of these forces about point C at the right corner are:

$$\sum M_C = 0 = \gamma(1)(4.5) - \gamma(1.5h)(6\text{ ft}) - (SG)(\gamma)(1)(4.5\text{ ft}) + (5\text{ lbf})(0\text{ ft})$$

where γ = 62.4 lbf/ft^3 is the specific weight of water. Clean these two equations up:

$$1.5h = 1 - SG - 5/\gamma \quad \text{(forces)} \qquad 2.0h = 1 - SG \quad \text{(moments)}$$

Solve simultaneously for SG ≈ **0.68** *Ans.* (b); h = 0.16 ft; θ ≈ **1.02°** *Ans.* (a)

2.120 A uniform wooden beam (SG = 0.65) is 10 cm by 10 cm by 3 m and hinged at A. At what angle will the beam float in 20°C water?

Solution: The total beam volume is $3(.1)^2 = 0.03\ m^3$, and therefore its weight is W = (0.65)(9790)(0.03) = 190.9 N, acting at the centroid, 1.5 m down from point A. Meanwhile, if the submerged length is H, the buoyancy is B = $(9790)(0.1)^2H$ = 97.9H newtons, acting at H/2 from the lower end. Sum moments about point A:

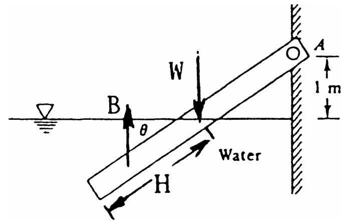


Fig. P2.120

$$\sum M_A = 0 = (97.9H)(3.0 - H/2)\cos\theta - 190.9(1.5\cos\theta),$$

$$\text{or:}\quad H(3 - H/2) = 2.925, \quad \text{solve for} \quad H \approx 1.225\ \text{m}$$

Geometry: 3 – H = 1.775 m is out of the water, or: $\sin\theta = 1.0/1.775$, or $\theta \approx$ **34.3°** *Ans.*

2.121 The uniform beam in the figure is of size L by h by b, with $b,h \ll L$. A uniform heavy sphere tied to the left corner causes the beam to float exactly on its diagonal. Show that this condition requires (a) γb = γ/3; and (b) $D = [Lhb/\{\pi(SG - 1)\}]^{1/3}$.

Solution: The beam weight W = γbLhb and acts in the center, at L/2 from the left corner, while the buoyancy, being a perfect triangle of displaced water, equals B = γLhb/2 and acts at L/3 from the left corner. Sum moments about the left corner, point C:

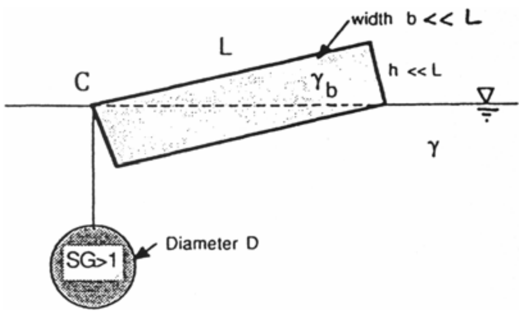


Fig. P2.121

$$\sum M_C = 0 = (\gamma_b Lhb)(L/2) - (\gamma Lhb/2)(L/3), \quad \text{or:} \quad \boldsymbol{\gamma_b = \gamma/3} \quad \textit{Ans. (a)}$$

Then summing vertical forces gives the required string tension T on the left corner:

$$\sum F_z = 0 = \gamma Lbh/2 - \gamma_b Lbh - T, \quad \text{or} \quad T = \gamma Lbh/6 \quad \text{since} \quad \gamma_b = \gamma/3$$

$$\text{But also} \quad T = (W - B)_{sphere} = (SG - 1)\gamma\frac{\pi}{6}D^3, \quad \text{so that} \quad D = \left[\frac{\mathbf{Lhb}}{\boldsymbol{\pi}\mathbf{(SG-1)}}\right]^{1/3} \quad \textit{Ans. (b)}$$

2.122 A uniform block of steel (SG = 7.85) will "float" at a mercury-water interface as in the figure. What is the ratio of the distances *a* and *b* for this condition?

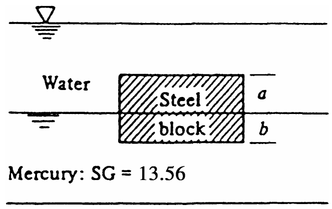


Fig. P2.122

Solution: Let *w* be the block width into the paper and let γ be the water specific weight. Then the vertical force balance on the block is

$$7.85\gamma(a+b)Lw = 1.0\gamma aLw + 13.56\gamma bLw,$$

$$\text{or:}\quad 7.85a + 7.85b = a + 13.56b,\quad \text{solve for } \frac{a}{b} = \frac{13.56-7.85}{7.85-1} = \mathbf{0.834}\quad \textit{Ans.}$$

P2.123 A barge has the trapezoidal shape shown in Fig. P2.123 and is 22 m long into the paper. If the total weight of barge and cargo is 350 tons, what is the draft *H* of the barge when floating in seawater?

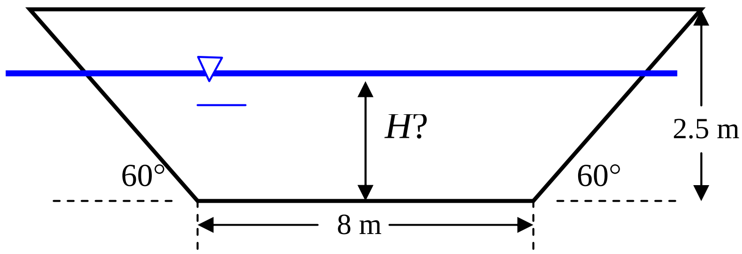


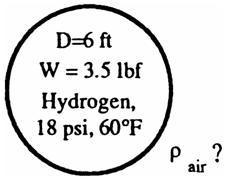
Fig. P2.123

Solution: For seawater, let ρ = 1025 kg/m^3. The top of the barge has length [8m+2(2.5)tan60°] = 8 + 2.89 = 10.89 m. Thus the total volume of the barge is [(8+10.89m)/2](2.5m)(22m) = 519.4 m^3. In terms of seawater, this total volume would be equivalent to (519.4m^3)(1025kg/m^3)(9.81m/s^2) = 5.22E6N ÷ 4.4482lbf/N ÷ 2000lbf/ton = 587 tons. Thus a cargo of 350 tons = 700,000 lbf would fill the barge a bit more than halfway. Thus we solve the following equation for the draft to give W = 350 tons:

$$(22m)(H)\left(8+\frac{H}{\tan 60^\circ}m\right)\left(1025\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right)\left(\frac{1}{4.4482 lbf/N}\right) = 700{,}000\, lbf$$

$$\text{Solve by iteration or EES:}\qquad H \approx \mathbf{1.58\,m}\qquad \textit{Ans.}$$

2.124 A balloon weighing 3.5 lbf is 6 ft in diameter. If filled with hydrogen at 18 psia and 60°F and released, at what U.S. standard altitude will it be neutral?



Solution: Assume that it remains at 18 psia and 60°F. For hydrogen, from Table A-4, $R \approx 24650\ ft^2/(s^2\cdot°R)$. The density of the hydrogen in the balloon is thus

$$\rho_{H_2} = \frac{p}{RT} = \frac{18(144)}{(24650)(460+60)} \approx 0.000202\ slug/ft^3$$

In the vertical force balance for neutral buoyancy, only the outside air density is unknown:

$$\sum F_z = B_{air} - W_{H_2} - W_{balloon} = \rho_{air}(32.2)\frac{\pi}{6}(6)^3 - (0.000202)(32.2)\frac{\pi}{6}(6)^3 - 3.5\ lbf$$

$$\text{Solve for} \quad \rho_{air} \approx 0.00116\ slug/ft^3 \approx 0.599\ kg/m^3$$

From Table A-6, this density occurs at a standard altitude of **6850 m ≈ 22500 ft**. *Ans.*

2.125 Suppose the balloon in Prob. 2.111 is constructed with a diameter of 14 m, is filled at sea level with hot air at 70°C and 1 atm, and released. If the hot air remains at 70°C, at what U.S. standard altitude will the balloon become neutrally buoyant?

Solution: Recall from Prob. 2.111 that the hot air density is $p/RT_{hot} \approx 1.030\ kg/m^3$. Assume that the entire weight of the balloon consists of its material, which from Prob. 2.111 had a density of 60 grams per square meter of surface area. Neglect the vent hole. Then the vertical force balance for neutral buoyancy yields the air density:

$$\sum F_z = B_{air} - W_{hot} - W_{balloon}$$

$$= \rho_{air}(9.81)\frac{\pi}{6}(14)^3 - (1.030)(9.81)\frac{\pi}{6}(14)^3 - (0.06)(9.81)\pi(14)^2$$

$$\text{Solve for} \quad \rho_{air} \approx 1.0557\ kg/m^3.$$

From Table A-6, this air density occurs at a standard altitude of **1500 m**. *Ans.*

2.126 A block of wood (SG = 0.6) floats in fluid X in Fig. P2.126 such that 75% of its volume is submerged in fluid X. Estimate the gage pressure of the air in the tank.

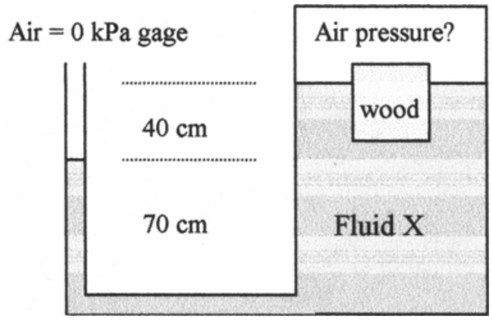


Fig. P2.126

Solution: In order to apply the hydrostatic relation for the air pressure calculation, the density of Fluid X must be found. The buoyancy principle is thus first applied. Let the block have volume V. Neglect the buoyancy of the air on the upper part of the block. Then

$$0.6\gamma_{water}V = \gamma_X(0.75V) + \cancel{\gamma_{air}(0.25V)}; \quad \gamma_X \approx 0.8\gamma_{water} = 7832\ N/m^3$$

The air gage pressure may then be calculated by jumping from the left interface into fluid X:

$$0 \text{ Pa-gage} - (7832 \text{ N/m}^3)(0.4 \text{ m}) = p_{air} = -3130 \text{ Pa-gage} = \mathbf{3130\ Pa\text{-}vacuum} \quad \textit{Ans.}$$

2.127* Consider a cylinder of specific gravity $S < 1$ floating vertically in water ($S = 1$), as in Fig. P2.127. Derive a formula for the stable values of D/L as a function of S and apply it to the case $D/L = 1.2$.

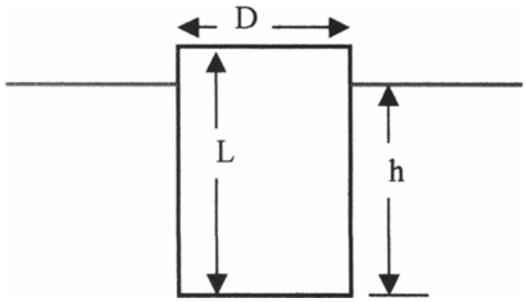


Fig. P2.127

Solution: A vertical force balance provides a relation for h as a function of S and L,

$$\gamma\pi D^2 h/4 = S\gamma\pi D^2 L/4, \quad \text{thus } h = SL$$

To compute stability, we turn Eq. (2.52), centroid G, metacenter M, center of buoyancy B:

$$MB = I_o/v_{sub} = \frac{\frac{\pi}{4}(D/2)^4}{\frac{\pi}{4}Dh} = MG + GB \quad \textit{and substituting } h = SL, \ \frac{D^2}{16SL} = MG + GB$$

where GB = L/2 − h/2 = L/2 − SL/2 = L(1 − S)/2. For neutral stability, MG = 0. Substituting,

$$\frac{D^2}{16SL} = 0 + \frac{L}{2}(1-S) \quad \textit{solving for } D/L, \ \boldsymbol{\frac{D}{L} = \sqrt{8S(1-S)}} \quad \textit{Ans.}$$

For D/L = 1.2, $S^2 - S - 0.18 = 0$ giving **0 ≤ S ≤ 0.235** and **0.765 ≤ S ≤ 1** *Ans.*

2.128 The iceberg of Fig. 2.20 can be idealized as a cube of side length L as shown. If seawater is denoted as S = 1, the iceberg has S = 0.88. Is it stable?

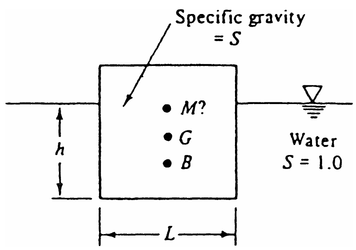


Fig. P2.128

Solution: The distance *h* is determined by

$$\gamma_w hL^2 = S\gamma_w L^3, \quad \text{or:} \quad h = SL$$

The center of gravity is at L/2 above the bottom, and B is at h/2 above the bottom. The metacenter position is determined by Eq. (2.52):

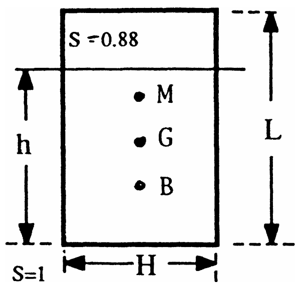
$$MB = I_o/\upsilon_{sub} = \frac{L^4/12}{L^2 h} = \frac{L^2}{12h} = \frac{L}{12S} = MG + GB$$

Noting that GB = L/2 − h/2 = L(1 − S)/2, we may solve for the metacentric height:

$$MG = \frac{L}{12S} - \frac{L}{2}(1-S) = 0 \quad \text{if} \quad S^2 - S + \frac{1}{6} = 0, \ \text{or:} \ S = 0.211 \quad \text{or} \quad 0.789$$

Instability: 0.211 < S < 0.789. Since the iceberg has S = 0.88 > 0.789, **it is stable**. *Ans*.

2.129 The iceberg of Prob. 2.128 may become unstable if its width decreases. Suppose that the height is L and the depth into the paper is L but the width decreases to H < L. Again with S = 0.88 for the iceberg, determine the ratio H/L for which the iceberg becomes unstable.



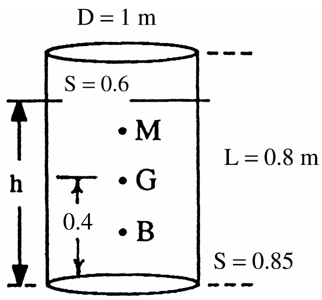
Solution: As in Prob. 2.128, the submerged distance h = SL = 0.88L, with G at L/2 above the bottom and B at h/2 above the bottom. From Eq. (2.52), the distance MB is

$$MB = \frac{I_o}{\upsilon_{sub}} = \frac{LH^3/12}{HL(SL)} = \frac{H^2}{12SL} = MG + GB = MG + \left(\frac{L}{2} - \frac{SL}{2}\right)$$

Then neutral stability occurs when MG = 0, or

$$\frac{H^2}{12SL} = \frac{L}{2}(1-S), \quad \text{or} \quad \frac{H}{L} = [6S(1-S)]^{1/2} = [6(0.88)(1-0.88)]^{1/2} = \mathbf{0.796} \quad \textit{Ans.}$$

2.130 Consider a wooden cylinder (SG = 0.6) 1 m in diameter and 0.8 m long. Would this cylinder be stable if placed to float with its axis vertical in oil (SG = 0.85)?



Solution: A vertical force balance gives

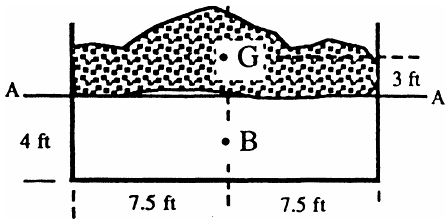
$$0.85\pi R^2 h = 0.6\pi R^2 (0.8 \text{ m}),$$
$$\text{or:} \quad h = 0.565 \text{ m}$$

The point B is at h/2 = 0.282 m above the bottom. Use Eq. (2.52) to predict the metacenter location:

$$MB = I_o/\upsilon_{sub} = [\pi(0.5)^4/4]/[\pi(0.5)^2(0.565)] = 0.111 \text{ m} = MG + GB$$

Now GB = 0.4 m − 0.282 m = 0.118 m, hence **MG** = 0.111 − 0.118 = **−0.007 m**. This float position is thus **slightly unstable**. The cylinder would turn over. *Ans*.

2.131 A barge is 15 ft wide and floats with a draft of 4 ft. It is piled so high with gravel that its center of gravity is 3 ft above the waterline, as shown. Is it stable?



Solution: Example 2.10 applies to this case, with L = 7.5 ft and H = 4 ft:

$$MA = \frac{L^2}{3H} - \frac{H}{2} = \frac{(7.5\text{ ft})^2}{3(4\text{ ft})} - \frac{4\text{ ft}}{2} = 2.69\text{ ft}, \quad \text{where "A" is the waterline}$$

Since G is **3** ft *above* the waterline, MG = 2.69 − 3.0 = **−0.31 ft, unstable**. *Ans.*

2.132 A solid right circular cone has SG = 0.99 and floats vertically as shown. Is this a stable position?

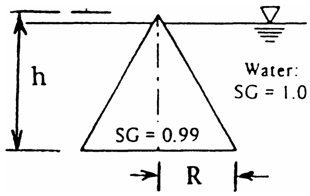


Fig. P2.132

Solution: Let *r* be the radius at the surface and let *z* be the exposed height. Then

$$\sum F_z = 0 = \gamma_w \frac{\pi}{3}(R^2h - r^2z) - 0.99\gamma_w \frac{\pi}{3}R^2h, \quad \text{with } \frac{z}{h} = \frac{r}{R}.$$

$$\text{Thus } \frac{z}{h} = (0.01)^{1/3} = 0.2154$$

The cone floats at a draft $\zeta = h - z = 0.7846h$. The centroid G is at 0.25h above the bottom. The center of buoyancy B is at the centroid of a frustrum of a (submerged) cone:

$$\zeta = \frac{0.7846h}{4}\left(\frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2}\right) = 0.2441h \quad \text{above the bottom}$$

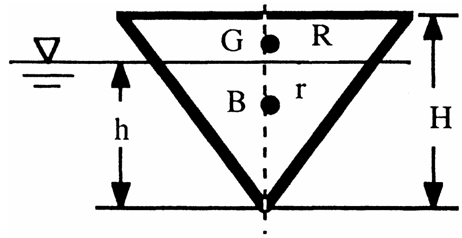
Then Eq. (2.52) predicts the position of the metacenter:

$$MB = \frac{I_o}{\upsilon_{sub}} = \frac{\pi(0.2154R)^4/4}{0.99\pi R^2 h} = 0.000544\frac{R^2}{h} = MG + GB$$

$$= MG + (0.25h - 0.2441h) = MG + 0.0594h$$

Thus MG > 0 (**stability**) if $(R/h)^2 \geq 10.93$ or $\mathbf{R/h \geq 3.31}$ *Ans.*

2.133 Consider a uniform right circular cone of specific gravity S < 1, floating with its vertex down in water, S = 1.0. The base radius is R and the cone height is H, as shown. Calculate and plot the stability parameter MG of this cone, in dimensionless form, versus H/R for a range of cone specific gravities S < 1.



Solution: The cone floats at height *h* and radius *r* such that B = W, or:

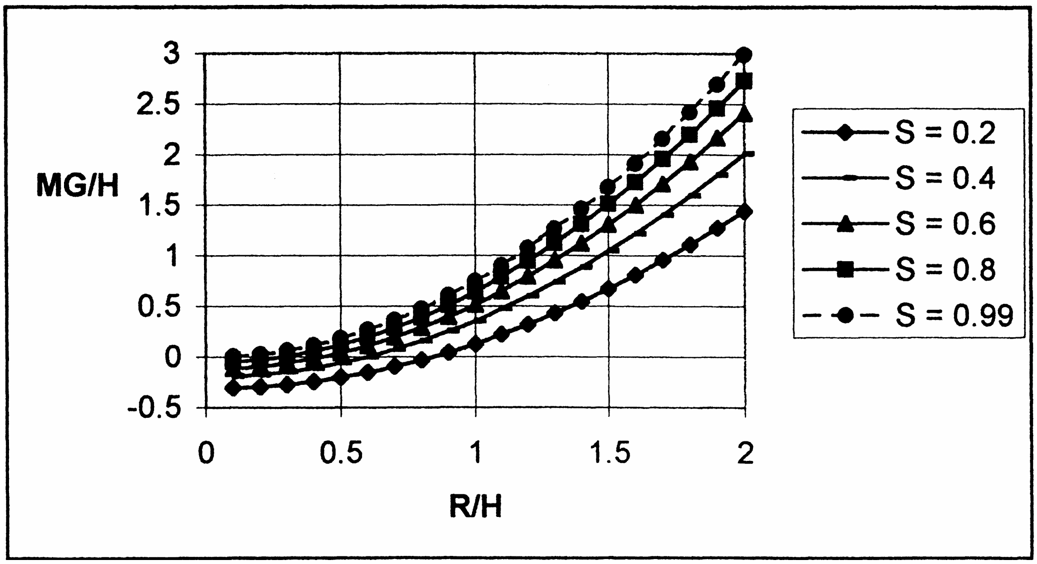
$$\frac{\pi}{3}r^2h(1.0) = \frac{\pi}{3}R^2H(S), \quad or: \quad \frac{h^3}{H^3} = \frac{r^3}{R^3} = S < 1$$

Thus r/R = h/H = $S^{1/3} = \zeta$ for short. Now use the stability relation:

$$MG + GB = MG + \left(\frac{3H}{4} - \frac{3h}{4}\right) = \frac{I_o}{\upsilon_{sub}} = \frac{\pi r^4/4}{\pi r^2 h/3} = \frac{3\zeta R^2}{4H}$$

Non-dimensionalize in the final form: $\frac{\mathbf{MG}}{\mathbf{H}} = \frac{\mathbf{3}}{\mathbf{4}}\left(\zeta\frac{\mathbf{R}^2}{\mathbf{H}^2} - \mathbf{1} + \zeta\right), \ \zeta = S^{1/3}$ *Ans.*

This is plotted below. Floating cones pointing *down* are stable unless slender, $R \ll H$.



2.134 When floating in water (SG = 1), an equilateral triangular body (SG = 0.9) might take *two* positions, as shown at right. Which position is more stable? Assume large body width into the paper.

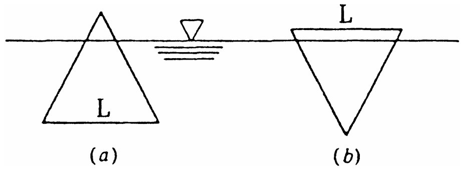


Fig. P2.134

Solution: The calculations are similar to the floating cone of Prob. 2.132. Let the triangle be L by L by L. List the basic results.

(a) Floating with point *up*: Centroid G is 0.289L above the bottom line, center of buoyancy B is 0.245L above the bottom, hence GB = (0.289 − 0.245)L ≈ 0.044L. Equation (2.52) gives

$$MB = I_o/\upsilon_{sub} = 0.0068L = MG + GB = MG + 0.044L$$

$$\text{Hence} \quad MG = \mathbf{-0.037L} \quad \textbf{Unstable} \quad \textit{Ans.} \text{ (a)}$$

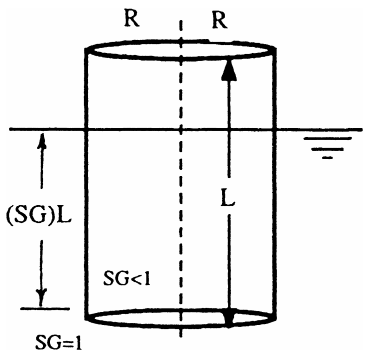
(b) Floating with point *down*: Centroid G is 0.577L above the bottom point, center of buoyancy B is 0.548L above the bottom point, hence GB = (0.577 − 0.548)L ≈ 0.0296L. Equation (2.52) gives

$$MB = I_o/\upsilon_{sub} = 0.1826L = MG + GB = MG + 0.0296L$$

$$\text{Hence} \quad MG = \mathbf{+0.153L} \quad \textbf{Stable} \quad \textit{Ans.} \text{ (b)}$$

2.135 Consider a homogeneous right circular cylinder of length L, radius R, and specific gravity SG, floating in water (SG = 1) with its axis *vertical*. Show that the body is stable if

$$R/L > [2SG(1-SG)]^{1/2}$$



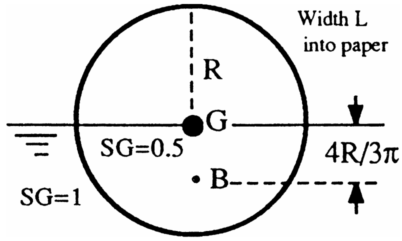
Solution: For a given SG, the body floats with a draft equal to (SG)L, as shown. Its center of gravity G is at L/2 above the bottom. Its center of buoyancy B is at (SG)L/2 above the bottom. Then Eq. (2.52) predicts the metacenter location:

$$MB = I_o/\upsilon_{sub} = \frac{\pi R^4/4}{\pi R^2(SG)L} = \frac{R^2}{4(SG)L} = MG + GB = MG + \frac{L}{2} - SG\frac{L}{2}$$

$$\text{Thus} \quad MG > 0 \text{ (stability)} \quad \text{if} \quad \mathbf{R^2/L^2 > 2SG(1-SG)} \quad \textit{Ans.}$$

For example, if SG = 0.8, stability requires that R/L > 0.566.

2.136 Consider a homogeneous right circular cylinder of length L, radius R, and specific gravity SG = 0.5, floating in water (SG = 1) with its axis *horizontal*. Show that the body is stable if L/R > 2.0.

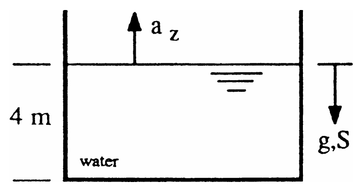


Solution: For the given SG = 0.5, the body floats centrally with a draft equal to R, as shown. Its center of gravity G is exactly at the surface. Its center of buoyancy B is at the centroid of the immersed semicircle: $4R/(3\pi)$ below the surface. Equation (2.52) predicts the metacenter location:

$$MB = I_o/\upsilon_{sub} = \frac{(1/12)(2R)L^3}{\pi(R^2/2)L} = \frac{L^2}{3\pi R} = MG + GB = MG + \frac{4R}{3\pi}$$

$$\text{or:}\quad MG = \frac{L^2}{3\pi R} - \frac{4R}{3\pi} > 0 \text{ (stability)} \quad \text{if } \mathbf{L/R > 2} \quad \textit{Ans.}$$

2.137 A tank of water 4 m deep receives a constant upward acceleration a_z. Determine (a) the gage pressure at the tank bottom if $a_z = 5\ m^2/s$; and (b) the value of a_z which causes the gage pressure at the tank bottom to be 1 atm.



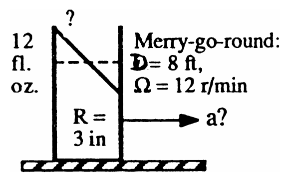
Solution: Equation (2.53) states that $\nabla p = \rho(\mathbf{g} - \mathbf{a}) = \rho(-\mathbf{k}g - \mathbf{k}a_z)$ for this case. Then, for part (a),

$$\Delta p = \rho(g + a_z)\Delta S = (998\ kg/m^3)(9.81 + 5\ m^2/s)(4\ m) = \mathbf{59100\ Pa\ (gage)} \quad \textit{Ans.}\text{ (a)}$$

For part (b), we know Δp = 1 atm but we don't know the acceleration:

$$\Delta p = \rho(g + a_z)\Delta S = (998)(9.81 + a_z)(4.0) = 101350\ Pa \quad \text{if } \mathbf{a_z = 15.6\ \frac{m}{s^2}} \quad \textit{Ans.}\text{ (b)}$$

2.138 A 12 fluid ounce glass, 3 inches in diameter, sits on the edge of a merry-go-round 8 ft in diameter, rotating at 12 r/min. How full can the glass be before it spills?



Solution: First, how high is the container? Well, 1 fluid oz. = 1.805 in^3, hence 12 fl. oz. = 21.66 in^3 = $\pi(1.5\text{ in})^2h$, or $h \approx 3.06$ *in*—It is a fat, nearly square little glass. Second, determine the acceleration toward the center of the merry-go-round, noting that the angular velocity is Ω = (12 rev/min)(1 min/60 s)(2π rad/rev) = 1.26 rad/s. Then, for r = 4 ft,

$$a_x = \Omega^2 r = (1.26\text{ rad/s})^2(4\text{ ft}) = 6.32\text{ ft/s}^2$$

Then, for steady rotation, the water surface in the glass will slope at the angle

$$\tan\theta = \frac{a_x}{g + a_z} = \frac{6.32}{32.2 + 0} = 0.196, \quad \text{or:} \quad \Delta h_{\text{left to center}} = (0.196)(1.5\text{ in}) = 0.294\text{ in}$$

Thus the glass should be filled to no more than 3.06 − 0.294 ≈ 2.77 inches
This amount of liquid is $\upsilon = \pi(1.5\text{ in})^2(2.77\text{ in}) = 19.6\text{ in}^3 \approx$ **10.8 fluid oz.** *Ans.*

2.139 The tank of liquid in the figure P2.139 accelerates to the right with the fluid in rigid-body motion. (a) Compute a_x in m/s^2. (b) Why doesn't the solution to part (a) depend upon fluid density? (c) Compute gage pressure at point A if the fluid is glycerin at 20°C.

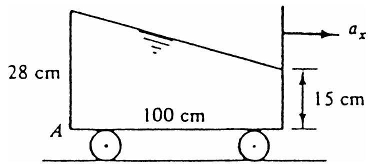


Fig. P2.139

Solution: (a) The slope of the liquid gives us the acceleration:

$$\mathbf{tan}\,\theta = \frac{a_x}{g} = \frac{28 - 15\text{ cm}}{100\text{ cm}} = 0.13, \quad \text{or:} \quad \theta = 7.4°$$

$$\textit{thus} \quad a_x = 0.13g = 0.13(9.81) = \mathbf{1.28\ m/s^2} \quad \textit{Ans.}\text{ (a)}$$

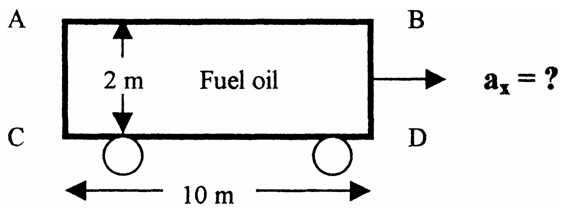
(b) Clearly, the solution to (a) is purely geometric and does not involve fluid density. *Ans.* (b)
(c) From Table A-3 for glycerin, ρ = 1260 kg/m^3. There are many ways to compute p_A. For example, we can go straight down on the left side, using only gravity:

$$p_A = \rho g\Delta z = (1260\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.28\text{ m}) = \mathbf{3460\ Pa\ (gage)} \quad \textit{Ans.}\text{ (c)}$$

Or we can start on the right side, go down 15 cm with g and across 100 cm with a_x:

$$p_A = \rho g\Delta z + \rho a_x\Delta x = (1260)(9.81)(0.15) + (1260)(1.28)(1.00)$$
$$= 1854 + 1607 = \mathbf{3460\ Pa} \quad \textit{Ans.}\text{ (c)}$$

2.140 An elliptical-end fuel tank is 10 m long, with 3-m horizontal and 2-m vertical minor axes, and filled completely with fuel oil (ρ = 890 kg/m^3). Let the tank be pulled along a horizontal road in rigid-body motion. Find the acceleration and direction for which (a) a constant-pressure surface extends from the top of the front end to the bottom of the back end; and (b) the top of the back end is at a pressure 0.5 atm lower than the top of the front end.



Solution: (a) We are given that the isobar or constant-pressure line reaches from point C to point B in the figure above, θ is *negative*, hence the tank is *decelerating*. The elliptical shape is immaterial, only the 2-m height. The isobar slope gives the acceleration:

$$\tan\theta_{C-B} = -\frac{2\text{ m}}{10\text{ m}} = -0.2 = \frac{a_x}{g},\quad \textit{hence}\ \ a_x = -0.2(9.81) = \mathbf{-1.96\ m/s^2}\quad \textit{Ans.}\ \text{(a)}$$

(b) We are now given that pA (back end top) is lower than pB (front end top)—see the figure above. Thus, again, the isobar must slope upward through B but not necessarily pass through point C. The pressure difference along line AB gives the correct *deceleration*:

$$\Delta p_{A-B} = -0.5(101325\ Pa) = \rho_{oil} a_x \Delta x_{A-B} = \left(890\ \frac{kg}{m^3}\right) a_x (10\text{ m})$$

$$\textit{solve for}\quad a_x = \mathbf{-5.69\ m/s^2}\quad \textit{Ans.}\ \text{(b)}$$

This is more than part (a), so the isobar angle must be steeper:

$$\tan\theta = \frac{-5.69}{9.81} = -0.580,\quad \textit{hence}\ \ \theta_{isobar} = -30.1°$$

The isobar in part (a), line CB, has the angle $\theta(\text{a}) = \tan^{-1}(-0.2) = -11.3°$.

2.141 The same tank from Prob. 2.139 is now accelerating while rolling *up* a 30° inclined plane, as shown. Assuming rigid-body motion, compute (a) the acceleration **a**, (b) whether the acceleration is up or down, and (c) the pressure at point A if the fluid is mercury at 20°C.

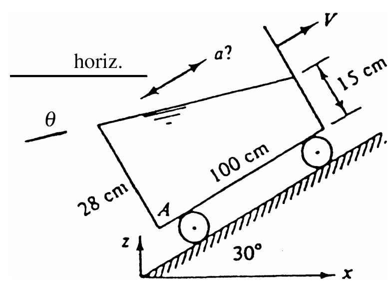


Fig. P2.141

Solution: The free surface is tilted at the angle $\theta = -30° + 7.41° = -22.59°$. This angle must satisfy Eq. (2.55):

$$\tan\theta = \tan(-22.59°) = -0.416 = a_x/(g + a_z)$$

But the 30° incline constrains the acceleration such that $a_x = 0.866a$, $a_z = 0.5a$. Thus

$$\tan\theta = -0.416 = \frac{0.866a}{9.81 + 0.5a}, \quad \text{solve for} \quad \mathbf{a \approx -3.80\ \frac{m}{s^2}\ (down)} \quad \textit{Ans.}\ \text{(a, b)}$$

The cartesian components are $a_x = -3.29\ m/s^2$ and $a_z = -1.90\ m/s^2$.
(c) The distance ΔS normal from the surface down to point A is (28 cosθ) cm. Thus

$$p_A = \rho[a_x^2 + (g + a_z)^2]^{1/2} = (13550)[(-3.29)^2 + (9.81 - 1.90)^2]^{1/2}(0.28\cos 7.41°)$$
$$\approx \mathbf{32200\ Pa\ (gage)} \quad \textit{Ans.}\ \text{(c)}$$

2.142 The tank of water in Fig. P2.142 is 12 cm wide into the paper. If the tank is accelerated to the right in rigid-body motion at 6 m/s^2, compute (a) the water depth at AB, and (b) the water force on panel AB.

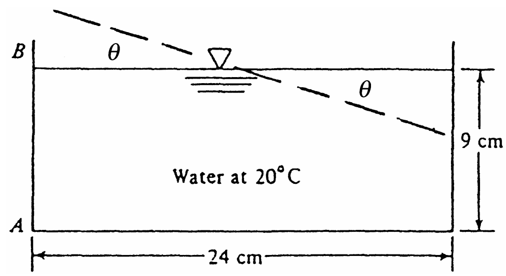


Fig. P2.142

Solution: From Eq. (2.55),

$$\tan\theta = a_x/g = \frac{6.0}{9.81} = 0.612, \quad \text{or} \quad \theta \approx 31.45°$$

Then surface point B on the left rises an additional Δz = 12 tanθ ≈ 7.34 cm,

$$\text{or:} \quad \text{water depth AB} = 9 + 7.34 \approx \mathbf{16.3\ cm} \quad \textit{Ans.}\ \text{(a)}$$

The water pressure on AB varies linearly due to gravity only, thus the water force is

$$F_{AB} = p_{CG}A_{AB} = (9790)\left(\frac{0.163}{2}\ m\right)(0.163\ m)(0.12\ m) \approx \mathbf{15.7\ N} \quad \textit{Ans.}\ \text{(b)}$$

2.143 The tank of water in Fig. P2.143 is full and open to the atmosphere (p_{atm} = 15 psi = 2160 psf) at point A, as shown. For what acceleration a_x, in ft/s^2, will the pressure at point B in the figure be (a) atmospheric; and (b) zero absolute (neglecting cavitation)?

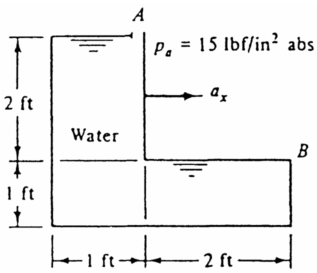


Fig. P2.143

Solution: (a) For $p_A = p_B$, the imaginary 'free surface isobar' should join points A and B:

$$\mathbf{tan}\,\theta_{AB} = \mathbf{tan}\,45° = 1.0 = a_x/g, \quad \text{hence } a_x = g = \mathbf{32.2\ ft/s^2} \quad \textit{Ans.}\ \text{(a)}$$

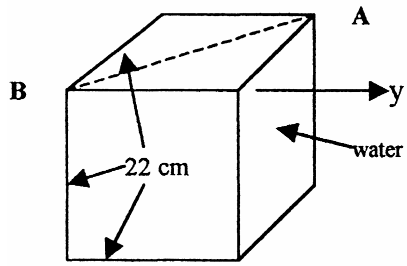
(b) For $p_B = 0$, the free-surface isobar must tilt even more than 45°, so that

$$p_B = 0 = p_A + \rho g\Delta z - \rho a_x \Delta x = 2160 + 1.94(32.2)(2) - 1.94a_x(2),$$

$$\textit{solve} \quad a_x = \mathbf{589\ ft/s^2} \quad \textit{Ans.}\ \text{(b)}$$

This is a very high acceleration (18 g's) and a very steep angle, $\theta = \tan^{-1}(589/32.2) = 87°$.

2.144 Consider a hollow cube of side length 22 cm, full of water at 20°C, and open to p_{atm} = 1 atm at top corner A. The top surface is horizontal. Determine the rigid-body accelerations for which the water at opposite top corner B will *cavitate*, for (a) horizontal, and (b) vertical motion.



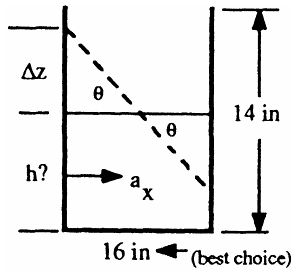
Solution: From Table A-5 the vapor pressure of the water is 2337 Pa. (a) Thus cavitation occurs first when accelerating horizontally along the diagonal AB:

$$p_A - p_B = 101325 - 2337 = \rho a_{x,AB}\Delta L_{AB} = \mathbf{(998)}a_{x,\,AB}\mathbf{(}0.22\sqrt{2}\mathbf{)},$$

$$\textit{solve} \quad a_{x,AB} = \mathbf{319\ m/s^2} \quad \textit{Ans.}\ \text{(a)}$$

If we moved along the *y* axis shown in the figure, we would need $a_y = 319\sqrt{2} = 451$ m/s^2. (b) For *vertical* acceleration, ***nothing would happen***, both points A and B would continue to be atmospheric, although the pressure at deeper points would change. *Ans.*

2.145 A fish tank 16-in by 27-in by 14-inch deep is carried in a car which may experience accelerations as high as 6 m/s^2. Assuming rigid-body motion, estimate the maximum water depth to avoid spilling. Which is the best way to align the tank?



Solution: The best way is to *align the 16-inch width with the car's direction of motion,* to minimize the vertical surface change Δz. From Eq. (2.55) the free surface angle will be

$$\tan\theta_{max} = a_x/g = \frac{6.0}{9.81} = 0.612, \quad \text{thus } \Delta z = \frac{16''}{2}\tan\theta = 4.9 \text{ inches } (\theta = 31.5°)$$

Thus the tank should contain no more than 14 – 4.9 ≈ **9.1 inches of water**. *Ans.*

2.146 The tank in Fig. P2.146 is filled with water and has a vent hole at point A. It is 1 m wide into the paper. Inside is a 10-cm balloon filled with helium at 130 kPa. If the tank accelerates to the right at 5 m/s/s, at what angle will the balloon lean? Will it lean to the left or to the right?

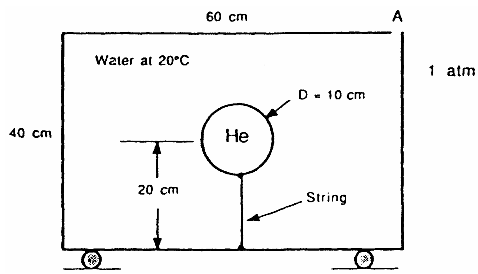
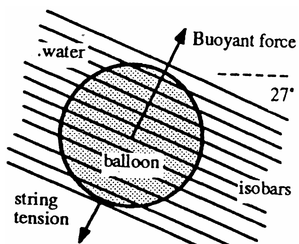


Fig. P2.146

Solution: The acceleration sets up pressure isobars which slant down and to the right, in both the water *and* in the helium. This means there will be a buoyancy force on the balloon up and to the right, as shown at right. It must be balanced by a string tension down and to the left. If we neglect balloon material weight, the balloon leans ***up and to the right*** at angle



$$\theta = \tan^{-1}\left(\frac{a_x}{g}\right) = \tan^{-1}\left(\frac{5.0}{9.81}\right) \approx \mathbf{27°} \quad \textit{Ans.}$$

measured from the vertical. This acceleration-buoyancy effect may seem counter-intuitive.

2.147 The tank of water in Fig. P2.147 accelerates uniformly by rolling without friction down the 30° inclined plane. What is the angle θ of the free surface? Can you explain this interesting result?

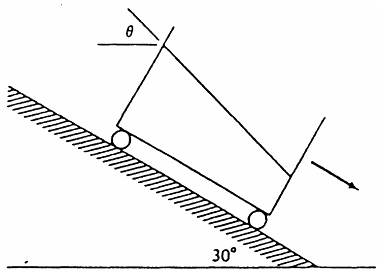


Fig. P2.147

Solution: If frictionless, $\Sigma F = W \sin\theta = ma$ along the incline and thus $a = g \sin 30° = 0.5g$.

$$\text{Thus } \tan\theta = \frac{a_x}{g + a_z} = \frac{0.5g\cos 30°}{g - 0.5g\sin 30°}; \quad \text{solve for } \theta = \mathbf{30°\,!} \quad \textit{Ans.}$$

The free surface aligns itself exactly parallel with the 30° incline.

P2.148 A child is holding a string onto which is attached a helium-filled balloon. (a) The child is standing still and suddenly accelerates forward. In a frame of reference moving with the child, which way will the balloon tilt, forward or backward? Explain. (b) The child is now sitting in a car that is stopped at a red light. The helium-filled balloon is not in contact with any part of the car (seats, ceiling, etc.) but is held in place by the string, which is held by the child. All the windows in the car are closed. When the traffic light turns green, the car accelerates forward. In a frame of reference moving with the car and child, which way will the balloon tilt, forward or backward? Explain. (c) Purchase or borrow a helium-filled balloon. Conduct a scientific experiment to see if your predictions in parts (a) and (b) are correct. If not, explain.

Solution: (a) Only the child and balloon accelerate, not the surrounding air. This is *not* rigid-body fluid motion. **The balloon will tilt backward** due to air drag. *Ans*.(a)

(b) Inside the car, the trapped air will accelerate with the car and the child, etc. This is rigid-body motion. **The balloon will tilt forward**, as in Prob. P2.146. *Ans*.(b)

(c) A student in the writer's class actually tried this experimentally. Our predictions were correct.

2.149 The waterwheel in Fig. P2.149 lifts water with 1-ft-diameter half-cylinder blades. The wheel rotates at 10 r/min. What is the water surface angle θ at pt. A?

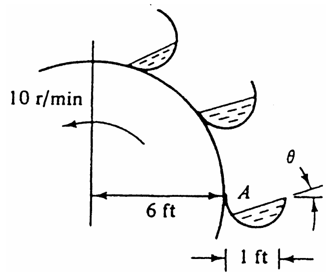


Fig. P2.149

Solution: Convert Ω = 10 r/min = 1.05 rad/s. Use an average radius R = 6.5 ft. Then

$$a_x = \Omega^2 R = (1.05)^2(6.5) \approx 7.13 \text{ ft/s}^2 \quad \text{toward the center}$$

$$\text{Thus} \quad \tan\theta = a_x/g = 7.13/32.2, \quad \text{or:} \quad \theta = \mathbf{12.5°} \quad \textit{Ans.}$$

2.150 A cheap accelerometer can be made from the U-tube at right. If L = 18 cm and D = 5 mm, what will *h* be if $a_x = 6 \text{ m/s}^2$?

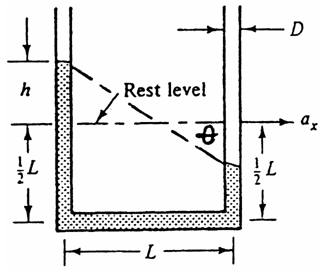


Fig. P2.150

Solution: We assume that the diameter is so small, $D \ll L$, that the free surface is a "point." Then Eq. (2.55) applies, and

$$\tan\theta = a_x/g = \frac{6.0}{9.81} = 0.612, \quad \text{or} \quad \theta = 31.5°$$

$$\text{Then} \quad h = (L/2)\tan\theta = (9 \text{ cm})(0.612) = \mathbf{5.5\ cm} \quad \textit{Ans.}$$

Since $h = (9 \text{ cm})a_x/g$, the scale readings are indeed linear in a_x, but I don't recommend it as an actual accelerometer, there are too many inaccuracies and disadvantages.

2.151 The U-tube in Fig. P2.151 is open at A and closed at D. What uniform acceleration a_x will cause the pressure at point C to be atmospheric? The fluid is water.

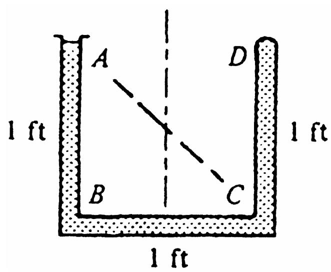
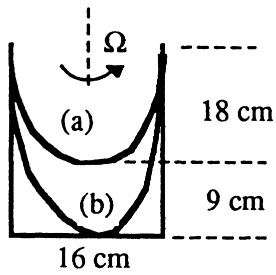


Fig. P2.151

Solution: If pressures at A and C are the same, the "free surface" must join these points:

$$\theta = 45°, \ a_x = g\tan\theta = g = \mathbf{32.2\ ft/s^2} \quad \textit{Ans.}$$

2.152 A 16-cm-diameter open cylinder 27 cm high is full of water. Find the central rigid-body rotation rate for which (a) one-third of the water will spill out; and (b) the bottom center of the can will be exposed.



Solution: (a) One-third will spill out if the resulting paraboloid surface is 18 cm deep:

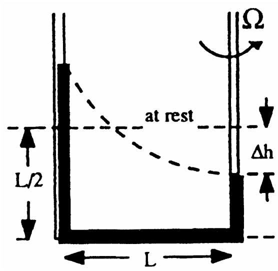
$$h = 0.18\text{ m} = \frac{\Omega^2 R^2}{2g} = \frac{\Omega^2 (0.08\text{ m})^2}{2(9.81)}, \quad \text{solve for} \quad \Omega^2 = 552,$$

$$\Omega = 23.5\text{ rad/s} = \mathbf{224\text{ r/min}} \quad \textit{Ans.}\text{ (a)}$$

(b) The bottom is barely exposed if the paraboloid surface is 27 cm deep:

$$h = 0.27\text{ m} = \frac{\Omega^2 (0.08\text{ m})^2}{2(9.81)}, \quad \text{solve for } \Omega = 28.8\text{ rad/s} = \mathbf{275\text{ r/min}} \quad \textit{Ans.}\text{ (b)}$$

2.153 Suppose the U-tube in Prob. 2.150 is not translated but instead is *rotated about the right leg* at 95 r/min. Find the level *h* in the left leg if L = 18 cm and D = 5 mm.



Solution: Convert Ω = 95 r/min = 9.95 rad/s. Then "R" = L = 18 cm, and, since $D \ll L$,

$$\Delta h = \frac{\Omega^2 R^2}{4g} = \frac{(9.95)^2 (0.18)^2}{4(9.81)} = 0.082\, m$$

Thus $h_{left\,leg} = 9 + 8.2 = \mathbf{17.2\text{ cm}}$ *Ans.*

P2.154 A very tall 10-cm-diameter vase contains 1178 cm^3 of water. When spun steadily to achieve rigid-body rotation, a 4-cm-diameter dry spot appears at the bottom of the vase. What is the rotation rate, r/min, for this condition?

Solution: It is interesting that the answer has nothing to do with the water *density*. The value of 1178 cubic centimeters was chosen to make the rest depth a nice number:

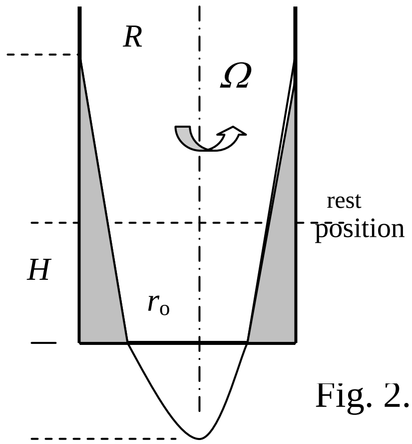


Fig. 2.154

$$\upsilon = 1178\,cm^3 = \pi(5cm)^2 H \text{ , solve } H = 15.0\,cm$$

One way would be to integrate and find the volume of the shaded liquid in Fig. P2.154 in terms of vase radius R and dry-spot radius r_o. That would yield the following formula:

$$d\upsilon = \pi(R^2 - r_o^2)\,dz \text{ , but } z = \Omega^2 r^2/2g \text{ , hence } dz = (\Omega^2 r/g)\,dr$$

$$\text{Thus } \upsilon = \int_{r_o}^{R} \pi(R^2 - r_o^2)(\Omega^2 r/g)\,dr = \frac{\pi\Omega^2}{g}\int_{r_o}^{R}(R^2 r - r^3)\,dr = \frac{\pi\Omega^2}{g}\left(\frac{R^2 r^2}{2} - \frac{r^4}{4}\right)\Big|_{r_o}^{R}$$

$$\text{Finally: } \upsilon = \frac{\pi\Omega^2}{g}\left(\frac{R^4}{4} - \frac{R^2 r_o^2}{2} + \frac{r_o^4}{4}\right) = 0.001178\,m^3$$

$$\text{Solve for } R = 0.05m,\ r_o = 0.02m \ :\ \Omega^2 = 3336,\ \Omega = 57.8\frac{rad}{s} = \mathbf{552\frac{r}{min}} \quad \textit{Ans.}$$

The formulas in the text, concerning the paraboloids of “air”, would, in the writer’s opinion, be difficult to apply because of the free surface extending below the bottom of the vase.

2.155 For what uniform rotation rate in r/min about axis C will the U-tube fluid in Fig. P2.155 take the position shown? The fluid is mercury at 20°C.

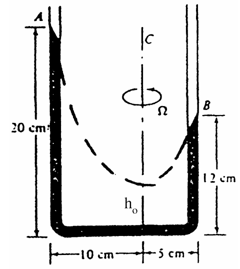


Fig. P2.155

Solution: Let h_o be the height of the free surface at the centerline. Then, from Eq. (2.64),

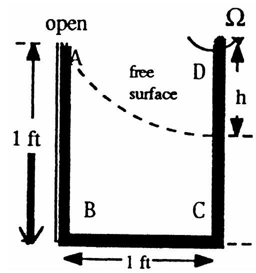
$$z_B = h_o + \frac{\Omega^2 R_B^2}{2g}; \quad z_A = h_o + \frac{\Omega^2 R_A^2}{2g}; \quad R_B = 0.05 \text{ m} \quad \text{and} \quad R_A = 0.1 \text{ m}$$

$$\text{Subtract:} \quad z_A - z_B = 0.08 \text{ m} = \frac{\Omega^2}{2(9.81)}[(0.1)^2 - (0.05)^2],$$

$$\text{solve} \quad \Omega = 14.5 \frac{\text{rad}}{\text{s}} = \mathbf{138} \ \frac{\mathbf{r}}{\mathbf{min}} \quad \textit{Ans.}$$

The fact that the fluid is mercury does not enter into this “kinematic” calculation.

2.156 Suppose the U-tube of Prob. 2.151 is rotated about axis DC. If the fluid is water at 122°F and atmospheric pressure is 2116 psfa, at what rotation rate will the fluid begin to vaporize? At what point in the tube will this happen?



Solution: At 122°F = 50°C, from Tables A-1 and A-5, for water, ρ = 988 kg/m^3 (or 1.917 slug/ft^3) and p_v = 12.34 kPa (or 258 psf). When spinning around DC, the free surface comes down from point A to a position *below* point D, as shown. Therefore the fluid pressure is <u>lowest</u> at point D (*Ans*.). With *h* as shown in the figure,

$$p_D = p_{vap} = 258 = p_{atm} - \rho g h = 2116 - 1.917(32.2)h, \quad h = \Omega^2 R^2/(2g)$$

Solve for h ≈ 30.1 ft (!) Thus the drawing is wildly distorted and the dashed line falls **far below** point C! (The solution is correct, however.)

$$\text{Solve for} \quad \Omega^2 = 2(32.2)(30.1)/(1 \text{ ft})^2 \quad \text{or:} \quad \Omega = 44 \text{ rad/s} = \mathbf{420 \ rev/min}. \quad \textit{Ans.}$$

2.157 The 45° V-tube in Fig. P2.157 contains water and is open at A and closed at C. (a) For what rigid-body rotation rate will the pressure be equal at points B and C? (b) For the condition of part (a), at what point in leg BC will the pressure be a minimum?

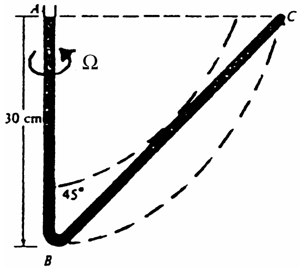


Fig. P2.157

Solution: (a) If pressures are equal at B and C, they must lie on a constant-pressure paraboloid surface as sketched in the figure. Taking $z_B = 0$, we may use Eq. (2.64):

$$z_C = 0.3\text{ m} = \frac{\Omega^2 R^2}{2g} = \frac{\Omega^2 (0.3)^2}{2(9.81)}, \quad \text{solve for} \quad \Omega = 8.09\ \frac{\text{rad}}{\text{s}} = \mathbf{77}\ \frac{\textbf{rev}}{\textbf{min}} \quad \textit{Ans.}\text{ (a)}$$

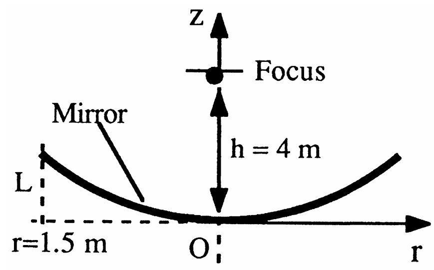
(b) The minimum pressure in leg BC occurs where the highest paraboloid pressure contour is tangent to leg BC, as sketched in the figure. This family of paraboloids has the formula

$$z = z_o + \frac{\Omega^2 r^2}{2g} = r\tan 45°, \quad \text{or:} \quad z_o + 3.333r^2 - r = 0 \quad \text{for a pressure contour}$$

$$\text{The minimum occurs when} \quad dz/dr = 0, \quad \text{or} \quad r \approx \mathbf{0.15\ m} \quad \textit{Ans.}\text{ (b)}$$

The minimum pressure occurs *halfway between points B and C.*

2.158* It is desired to make a 3-m-diameter parabolic telescope mirror by rotating molten glass in rigid-body motion until the desired shape is achieved and then cooling the glass to a solid. The focus of the mirror is to be 4 m from the mirror, measured along the centerline. What is the proper mirror rotation rate, in rev/min?



Solution: We have to review our math book, or Mark's Manual, to recall that the *focus* F of a parabola is the point for which all points on the parabola are equidistant from both the focus and a so-called "directrix" line (which is one focal length below the mirror). For the focal length *h* and the *z-r* axes shown in the figure, the equation of the parabola is given by $\mathbf{r^2 = 4hz}$, with $h = 4$ m for our example.
Meanwhile the equation of the free-surface of the liquid is given by $\mathbf{z = r^2\Omega^2/(2g)}$.
Set these two equal to find the proper rotation rate:

$$z = \frac{r^2\Omega^2}{2g} = \frac{r^2}{4h}, \quad \text{or:} \quad \Omega^2 = \frac{g}{2h} = \frac{9.81}{2(4)} = 1.226$$

$$\textit{Thus} \quad \Omega = 1.107 \ \frac{\text{rad}}{\text{s}} \left(\frac{60}{2\pi}\right) = \mathbf{10.6\ rev/min} \quad \textit{Ans.}$$

The focal point F is far above the mirror itself. If we put in $r = 1.5$ m and calculate the mirror depth "L" shown in the figure, we get L ≈ 14 centimeters.

2.159 The three-legged manometer in Fig. P2.159 is filled with water to a depth of 20 cm. All tubes are long and have equal small diameters. If the system spins at angular velocity Ω about the central tube, (a) derive a formula to find the change of height in the tubes; (b) find the height in cm in each tube if Ω = 120 rev/min. [*HINT*: The central tube must supply water to *both* the outer legs.]

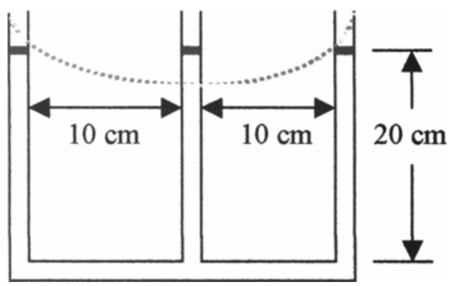


Fig. P2.159

Solution: (a) The free-surface during rotation is visualized as the **red** dashed line in Fig. P2.159. The outer right and left legs experience an increase which is one-half that of the central leg, or $\Delta h_O = \Delta h_C/2$. The total displacement between outer and center menisci is, from Eq. (2.64) and Fig. 2.23, equal to $\Omega^2R^2/(2g)$. The center meniscus falls two-thirds of this amount and feeds the outer tubes, which each rise one-third of this amount above the rest position:

$$\boldsymbol{\Delta h_{outer} = \frac{1}{3}\Delta h_{total} = \frac{\Omega^2 R^2}{6g} \qquad \Delta h_{center} = -\frac{2}{3}\Delta h_{total} = -\frac{\Omega^2 R^2}{3g}} \quad \textit{Ans.}\ \text{(a)}$$

For the particular case $R = 10$ cm and $\Omega = 120$ r/min $= (120)(2\pi/60) = 12.57$ rad/s, we obtain

$$\frac{\Omega^2 R^2}{2g} = \frac{(12.57\ \text{rad/s})^2 (0.1\ \text{m})^2}{2(9.81\ \text{m/s}^2)} = 0.0805\ \text{m};$$

$$\Delta h_O \approx \mathbf{0.027\ m}\ (up) \quad \Delta h_C \approx \mathbf{-0.054\ m}\ (down) \quad \textit{Ans.}\ \text{(b)}$$

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: Answers

FE-2.1 A gage attached to a pressurized nitrogen tank reads a gage pressure of 28 inches of mercury. If atmospheric pressure is 14.4 psia, what is the absolute pressure in the tank?

(a) 95 kPa (b) 99 kPa (c) 101 kPa **(d) 194 kPa** (e) 203 kPa

FE-2.2 On a sea-level standard day, a pressure gage, moored below the surface of the ocean (SG = 1.025), reads an absolute pressure of 1.4 MPa. How deep is the instrument?

(a) 4 m **(b) 129 m** (c) 133 m (d) 140 m (e) 2080 m

FE-2.3 In Fig. FE-2.3, if the oil in region B has SG = 0.8 and the absolute pressure at point A is 1 atmosphere, what is the absolute pressure at point B?

(a) 5.6 kPa (b) 10.9 kPa **(c) 106.9 kPa** (d) 112.2 kPa (e) 157.0 kPa

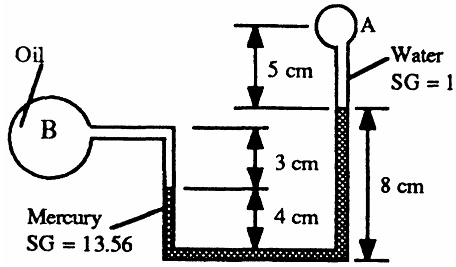


Fig. FE-2.3

FE-2.4 In Fig. FE-2.3, if the oil in region B has SG = 0.8 and the absolute pressure at point B is 14 psia, what is the absolute pressure at point B?

(a) 11 kPa (b) 41 kPa (c) 86 kPa **(d) 91 kPa** (e) 101 kPa

FE-2.5 A tank of water (SG = 1.0) has a gate in its vertical wall 5 m high and 3 m wide. The top edge of the gate is 2 m below the surface. What is the hydrostatic force on the gate?

(a) 147 kN (b) 367 kN (c) 490 kN **(d) 661 kN** (e) 1028 kN

FE-2.6 In Prob. FE-2.5 above, how far below the surface is the center of pressure of the hydrostatic force?

(a) 4.50 m (b) 5.46 m (c) 6.35 m (d) 5.33 m **(e) 4.96 m**

FE-2.7 A solid 1-m-diameter sphere floats at the interface between water (SG = 1.0) and mercury (SG = 13.56) such that 40% is in the water. What is the specific gravity of the sphere?

(a) 6.02 (b) 7.28 (c) 7.78 **(d) 8.54** (e) 12.56

FE-2.8 A 5-m-diameter balloon contains helium at 125 kPa absolute and 15°C, moored in sea-level standard air. If the gas constant of helium is 2077 $m^2/(s^2 \cdot K)$ and balloon material weight is neglected, what is the net lifting force of the balloon?

(a) 67 N (b) 134 N (c) 522 N **(d) 653 N** (e) 787 N

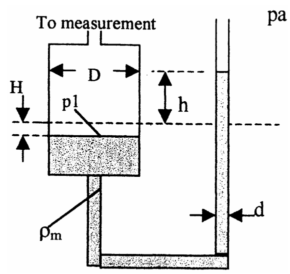
FE-2.9 A square wooden (SG = 0.6) rod, 5 cm by 5 cm by 10 m long, floats vertically in water at 20°C when 6 kg of steel (SG = 7.84) are attached to the lower end. How high above the water surface does the wooden end of the rod protrude?

(a) 0.6 m (b) 1.6 m **(c) 1.9 m** (d) 2.4 m (e) 4.0 m

FE-2.10 A floating body will always be stable when its

(a) CG is above the center of buoyancy (b) center of buoyancy is below the waterline (c) center of buoyancy is above its metacenter (d) metacenter is above the center of buoyancy **(e) metacenter is above the CG**

COMPREHENSIVE PROBLEMS



C2.1 Some manometers are constructed as in the figure at right, with one large reservoir and one small tube open to the atmosphere. We can then neglect movement of the reservoir level. If the reservoir is not large, its level will move, as in the figure. Tube height *h* is measured from the zero-pressure level, as shown.

(a) Let the reservoir pressure be high, as in the Figure, so its level goes down. Write an exact Expression for p_{1gage} as a function of h, d, D, and gravity *g*. (b) Write an approximate expression for p_{1gage}, neglecting the movement of the reservoir. (c) Suppose h = 26 cm, p_a = 101 kPa, and ρ_m = 820 kg/m^3. Estimate the ratio (D/d) required to keep the error in (b) less than 1.0% and also < 0.1%. Neglect surface tension.

Solution: Let *H* be the downward movement of the reservoir. If we neglect air density, the pressure difference is $p_1 - p_a = \rho_m g(h + H)$. But volumes of liquid must balance:

$$\frac{\pi}{4}D^2H = \frac{\pi}{4}d^2h, \quad \text{or:} \quad H = (d/D)^2 h$$

Then the pressure difference (exact except for air density) becomes

$$\boldsymbol{p_1 - p_a = p_{1gage} = \rho_m gh(1 + d^2/D^2)} \quad \textit{Ans.} \text{ (a)}$$

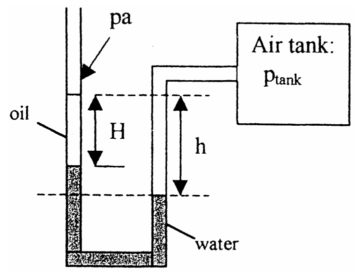
If we ignore the displacement *H*, then $\boldsymbol{p_{1gage} \approx \rho_m gh}$ *Ans.* (b)

(c) For the given numerical values, h = 26 cm and ρ_m = 820 kg/m^3 are irrelevant, all that matters is the ratio d/D. That is,

$$Error\ E = \frac{\Delta p_{exact} - \Delta p_{approx}}{\Delta p_{exact}} = \frac{(d/D)^2}{1+(d/D)^2}, \quad or: \quad D/d = \sqrt{(1-E)/E}$$

For E = 1% or 0.01, D/d = $[(1 - 0.01)/0.01]^{1/2} \geq$ **9.95** *Ans.* (c-1%)
For E = 0.1% or 0.001, D/d = $[(1 - 0.001)/0.001]^{1/2} \geq$ **31.6** *Ans.* (c-0.1%)

C2.2 A prankster has added oil, of specific gravity SG_o, to the left leg of the manometer at right. Nevertheless, the U-tube is still to be used to measure the pressure in the air tank. (a) Find an expression for *h* as a function of *H* and other parameters in the problem.
(b) Find the special case of your result when $p_{tank} = p_a$.
(c) Suppose H = 5 cm, p_a = 101.2 kPa, SG_o = 0.85, and p_{tank} is 1.82 kPa higher than p_a. Calculate *h* in cm, ignoring surface tension and air density effects.



Solution: Equate pressures at level *i* in the tube:

$$p_i = p_a + \rho gH + \rho_w g(h - H) = p_{tank},$$

$$\rho = SG_o\rho_w \quad \text{(ignore the column of air in the right leg)}$$

$$\text{Solve for:} \quad \boldsymbol{h = \frac{p_{tk} - p_a}{\rho_w g} + H(1 - SG_o)} \quad \textit{Ans.} \text{ (a)}$$

If $p_{tank} = p_a$, then

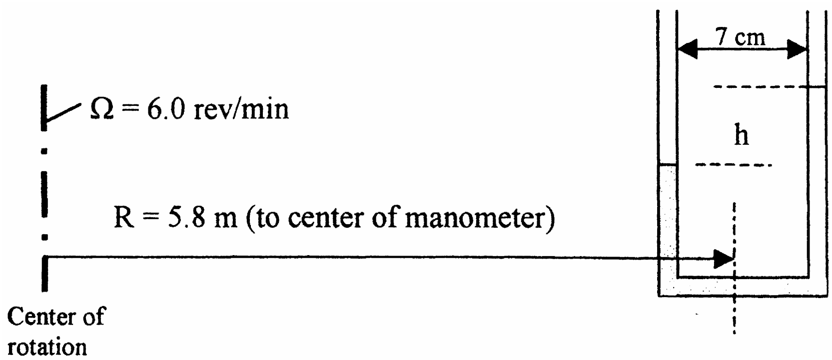
$$\boldsymbol{h = H(1 - SG_o)} \quad \textit{Ans.} \text{ (b)}$$

(c) For the particular numerical values given above, the answer to (a) becomes

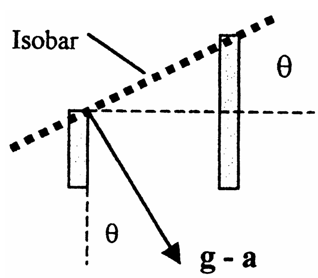
$$h = \frac{1820\ Pa}{998(9.81)} + 0.05(1 - 0.85) = 0.186 + 0.0075 = 0.193\ m = \mathbf{19.3\ cm} \quad \textit{Ans.} \text{ (c)}$$

Note that this result is not affected by the actual value of atmospheric pressure.

C2.3 Professor F. Dynamics, riding the merry-go-round with his son, has brought along his U-tube manometer. (You never know when a manometer might come in handy.) As shown in Fig. C2.2, the merry-go-round spins at constant angular velocity and the manometer legs are 7 cm apart. The manometer center is 5.8 m from the axis of rotation. Determine the height difference *h* in two ways: (a) approximately, by assuming rigid body translation with **a** equal to the average manometer acceleration; and (b) exactly, using rigid-body rotation theory. How good is the approximation?



Solution: (a) Approximate: The average acceleration of the manometer is $R_{avg}\Omega^2 = 5.8[6(2\pi/60)]^2 = 2.29$ rad/s toward the center of rotation, as shown. Then



$$\tan(\theta) = a/g = 2.29/9.81 = h/(7\text{ cm}) = 0.233$$

$$\text{Solve for} \quad h = \mathbf{1.63\ cm} \quad \textit{Ans.} \text{ (a)}$$

(b) Exact: The isobar in the figure at right would be on the parabola $z = C + r^2\Omega^2/(2g)$, where C is a constant. Apply this to the left leg (z_1) and right leg (z_2). As above, the rotation rate is $\Omega = 6.0*(2\pi/60) = 0.6283$ rad/s. Then

$$h = z_2 - z_1 = \frac{\Omega^2}{2g}(r_2^2 - r_1^2) = \frac{(0.6283)^2}{2(9.81)}[(5.8+0.035)^2 - (5.8-0.035)^2]$$

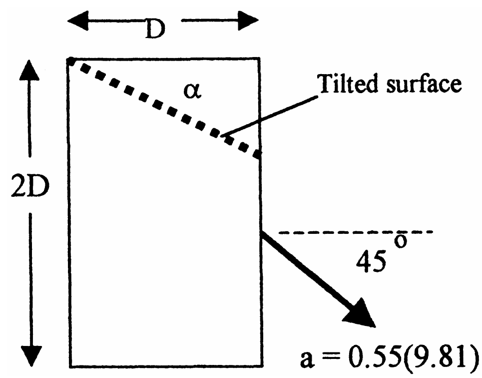
$$= \mathbf{0.0163\ m} \quad \textit{Ans.}\ \text{(b)}$$

This is nearly identical to the approximate answer (a), because R >> Δr.

C2.4 A student sneaks a glass of cola onto a roller coaster ride. The glass is cylindrical, twice as tall as it is wide, and filled to the brim. He wants to know what percent of the cola he should drink before the ride begins, so that none of it spills during the big drop, in which the roller coaster achieves 0.55-g acceleration at a 45° angle below the horizontal. Make the calculation for him, neglecting sloshing and assuming that the glass is vertical at all times.

Solution: We have both horizontal and vertical acceleration. Thus the angle of tilt α is

$$\tan\alpha = \frac{a_x}{g + a_z} = \frac{0.55g\cos 45°}{g - 0.55g\sin 45°} = 0.6364$$



Thus $\alpha = 32.47°$. The tilted surface strikes the centerline at $R\tan\alpha = 0.6364R$ below the top. So the student should drink the cola until its rest position is 0.6364R below the top. The percentage drop in liquid level (and therefore liquid volume) is

$$\%\ removed = \frac{0.6364R}{4R} = 0.159 \quad \textit{or:} \quad \mathbf{15.9\%} \quad \textit{Ans.}$$

C2.5 Dry adiabatic lapse rate is defined as DALR = –dT/dz when T and p vary isentropically. Assuming $T = Cp^a$, where $a = (\gamma - 1)/\gamma$, $\gamma = c_p/c_v$, (a) show that DALR = $g(\gamma - 1)/(\gamma R)$, R = gas constant; and (b) calculate DALR for air in units of °C/km.

Solution: Write T(p) in the form $T/T_o = (p/p_o)^a$ and differentiate:

$$\frac{dT}{dz} = T_o a\left(\frac{p}{p_o}\right)^{a-1}\frac{1}{p_o}\frac{dp}{dz}, \quad \text{But for the hydrostatic condition: } \frac{dp}{dz} = -\rho g$$

Substitute $\rho = p/RT$ for an ideal gas, combine above, and rewrite:

$$\frac{dT}{dz} = -\frac{T_o}{p_o} a\left(\frac{p}{p_o}\right)^{a-1} \frac{p}{RT} g = -\frac{ag}{R}\left(\frac{T_o}{T}\right)\left(\frac{p}{p_o}\right)^a. \quad \text{But: } \frac{T_o}{T}\left(\frac{p}{p_o}\right)^a = 1 \ (\textit{isentropic})$$

Therefore, finally,

$$-\frac{dT}{dz} = DALR = \frac{ag}{R} = \frac{\mathbf{(\gamma-1)g}}{\boldsymbol{\gamma}\mathbf{R}} \quad \textit{Ans.} \text{ (a)}$$

(b) Regardless of the actual air temperature and pressure, the DALR for **air** equals

$$DALR = -\frac{dT}{dz}\Big|_s = \frac{(1.4-1)(9.81\ m/s^2)}{1.4(287\ m^2/s^2/°C)} = 0.00977\ \frac{°C}{m} = \mathbf{9.77}\ \frac{\mathbf{°C}}{\mathbf{km}} \quad \textit{Ans.} \text{ (b)}$$

C2.6 Use the approximate pressure-density relation for a "soft" liquid,

$$dp = a^2 d\rho, \quad \text{or} \quad p = p_o + a^2(\rho - \rho_o)$$

to derive a formula for the density distribution $\rho(z)$ and pressure distribution p(z) in a column of soft liquid. Then find the force F on a vertical wall of width b, extending from z = 0 down to z = –h, and compare with the incompressible result $F = \rho_o g h^2 b/2$.

Solution: Introduce this $p(\rho)$ relation into the hydrostatic relation (2.18) and integrate:

$$dp = a^2 d\rho = -\gamma\, dz = -\rho g\, dz, \quad \text{or:} \quad \int_{\rho_o}^{\rho} \frac{d\rho}{\rho} = -\int_0^z \frac{g\, dz}{a^2}, \quad \text{or:} \quad \boldsymbol{\rho = \rho_o} \mathbf{e}^{\mathbf{-gz/a^2}} \quad \textit{Ans.}$$

assuming constant a^2. Substitute into the $p(\rho)$ relation to obtain the pressure distribution:

$$p \approx p_o + a^2\rho_o[e^{-gz/a^2} - 1] \qquad (1)$$

Since p(z) increases with z at a greater than linear rate, the center of pressure will always be *a little lower than predicted by linear theory* (Eq. 2.44). Integrate Eq. (1) above, neglecting p_o, into the pressure force on a vertical plate extending from z = 0 to z = –h:

$$F = -\int_0^{-h} pb\, dz = \int_{-h}^{0} a^2\rho_o(e^{-gz/a^2} - 1)b\, dz = \mathbf{ba^2}\boldsymbol{\rho_o}\left[\frac{\mathbf{a^2}}{\mathbf{g}}(\mathbf{e^{gh/a^2}} - \mathbf{1}) - \mathbf{h}\right] \quad \textit{Ans.}$$

In the limit of small depth change relative to the "softness" of the liquid, $h \ll a^2/g$, this reduces to the linear formula $F = \rho_o g h^2 b/2$ by expanding the exponential into the first three terms of its series. For "hard" liquids, the difference in the two formulas is negligible. For example, for water (a ≈ 1490 m/s) with h = 10 m and b = 1 m, the linear formula predicts F = 48950**0** N while the exponential formula predicts F = 48950**7** N.

C2.7 Venice, Italy is slowly sinking, so now, especially in winter, plazas and walkways are flooded. The proposed solution is the floating levee of Fig. P2.7. When filled with air, it rises to block off the sea. The levee is 30 m high and 5 m wide. Assume a uniform density of 300 kg/m^3 when floating. For the 1-meter Sea-Lagoon difference shown, estimate the angle at which the levee floats.

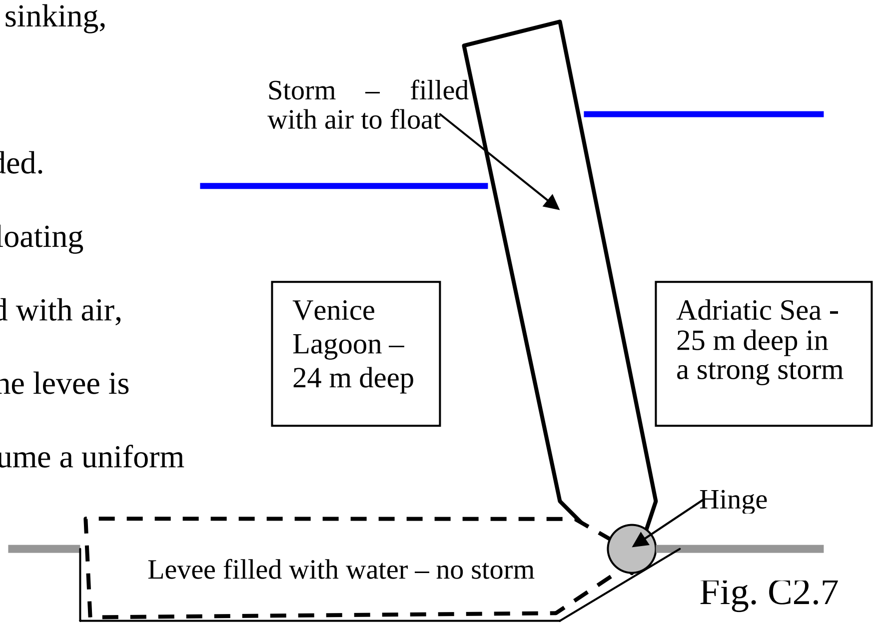
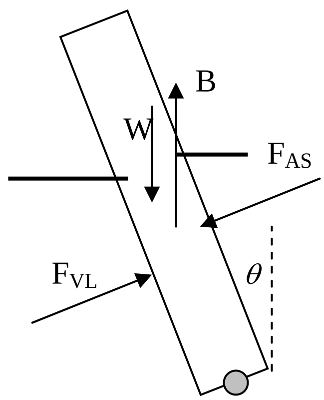


Fig. C2.7

Solution: The writer thinks this problem is rather laborious. Assume $\rho_{seawater}$ = 1025 kg/m^3. There are 4 forces: the hydrostatic force F_{AS} on the Adriatic side, the hydrostatic force F_{VL} on the lagoon side, the weight W of the levee, and the buoyancy B of the submerged part of the levee. On the Adriatic side, 25/cosθ meters are submerged. On the lagoon side, 24/cosθ meters are submerged. For buoyancy, average the two depths, (25+24)/2 = 24.5 m. For weight, the whole length of 30 m is used. Compute the four forces per unit width into the paper (since this width *b* will cancel out of all moments):



$$F_{AS} = \rho g h_{AS} L_{submerged} = (1025)(9.81)(25/2)(25/\cos\theta) = 3.142E6/\cos\theta$$
$$F_{VL} = \rho g h_{VL} L_{submerged} = (1025)(9.81)(24/2)(24/\cos\theta) = 2.896E6/\cos\theta$$
$$W = \rho_{levee} g L(\text{levee width}) = (300)(9.81)(30)(5) = 441500\ N/m$$
$$B = \rho g L_{sub-average}(\text{levee width}) = (1025)(9.81)(24.5)(5) = 1.232E6\ N/m$$

The hydrostatic forces have CP two-thirds of the way down the levee surfaces. The weight CG is in the center of the levee (15 m above the hinge). The buoyancy center is halfway down from the surface, or about (24.5)/2 m. The moments about the hinge are

$$\Sigma M_{hinge} = F_{AS}\left(\frac{25/\cos\theta}{3}m\right) + W(15m)\sin\theta - F_{VL}\left(\frac{24/\cos\theta}{3}m\right) - B\left(\frac{24.5}{2}m\right)\sin\theta = 0$$

where the forces are listed above and are not retyped here. Everything is known except the listing angle θ (measured from the vertical). Some iteration is required, say, on Excel, or, for a good initial guess (about θ = 15-30°), EES converges nicely to

$$\boldsymbol{\theta} \approx \mathbf{23.1°} \quad \textit{Ans.}$$

C2.8 What is the uncertainty is using pressure measurement as an altimeter? A gage on the side of an airplane measures a local pressure of 54 kPa, with an uncertainty of 3 kPa. The estimated lapse rate that day is 0.0070 K/m, with an uncertainty of 0.001 K/m. Effective sea-level temperature is 10°C, with an uncertainty of 4°C. Effective sea-level pressure is 100 kPa, with an uncertainty of 3 kPa. Estimate the airplane's altitude and its uncertainty.

Solution: We are dealing with the troposphere pressure variation formula, Eq. (2.20):

$$\frac{p}{p_o} = \left(1 - \frac{Bz}{T_o}\right)^{g/RB} \quad ; \quad \text{Invert}: \quad z = \frac{T_o}{B}\left[1 - \left(\frac{p}{p_o}\right)^{RB/g}\right]$$

To estimate the plane's altitude, just insert the given data for pressure, temperature, etc.:

$$z = \frac{283K}{0.0070K/m}\left[1 - \left(\frac{54kPa}{100kPa}\right)^{(287)(0.0070)/9.81}\right] \approx \mathbf{4800\,m} \quad \textit{Ans.}$$

To evaluate the overall uncertainty in *z*, we have to compute four derivatives:

$$\delta z = [(\frac{\partial z}{\partial p}\delta p)^2 + (\frac{\partial z}{\partial p_o}\delta p_o)^2 + (\frac{\partial z}{\partial T_o}\delta T_o)^2 + (\frac{\partial z}{\partial B}\delta B)^2]^{1/2}$$

where we are given δp = 3 kPa, δp_o = 3 kPa, δT_o = 4°C, and δB = 0.001. Typing out those four derivatives is a nightmare for the writer, so we will just give the four results:

$$\frac{\partial z}{\partial p}\delta p = -404\,m\ ;\ \frac{\partial z}{\partial p_o}\delta p_o = 218\,m\ ;\ \frac{\partial z}{\partial T_o}\delta T_o = 68\,m\ ;\ \frac{\partial z}{\partial B}\delta B = 42\,m$$

$$\text{whence}\quad \delta z \approx [(404m)^2 + (218m)^2 + (68m)^2 + (42m)^2]^{1/2} = \mathbf{466\,m} \quad \textit{Ans.}$$

The overall uncertainty is about ±10%. The largest effect is the 5.6% uncertainty in pressure, *p*, which has a strong effect on the altitude formula.