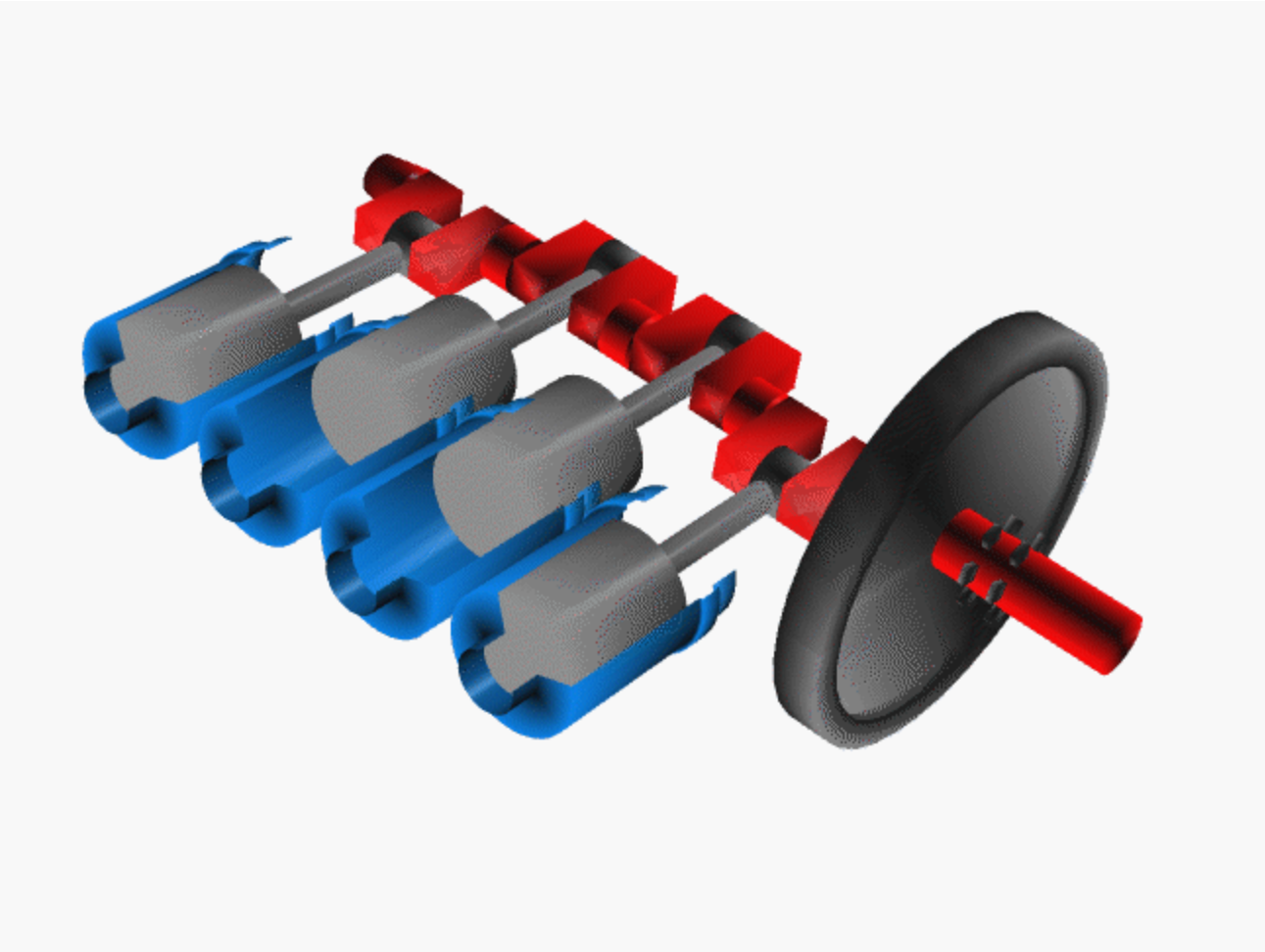
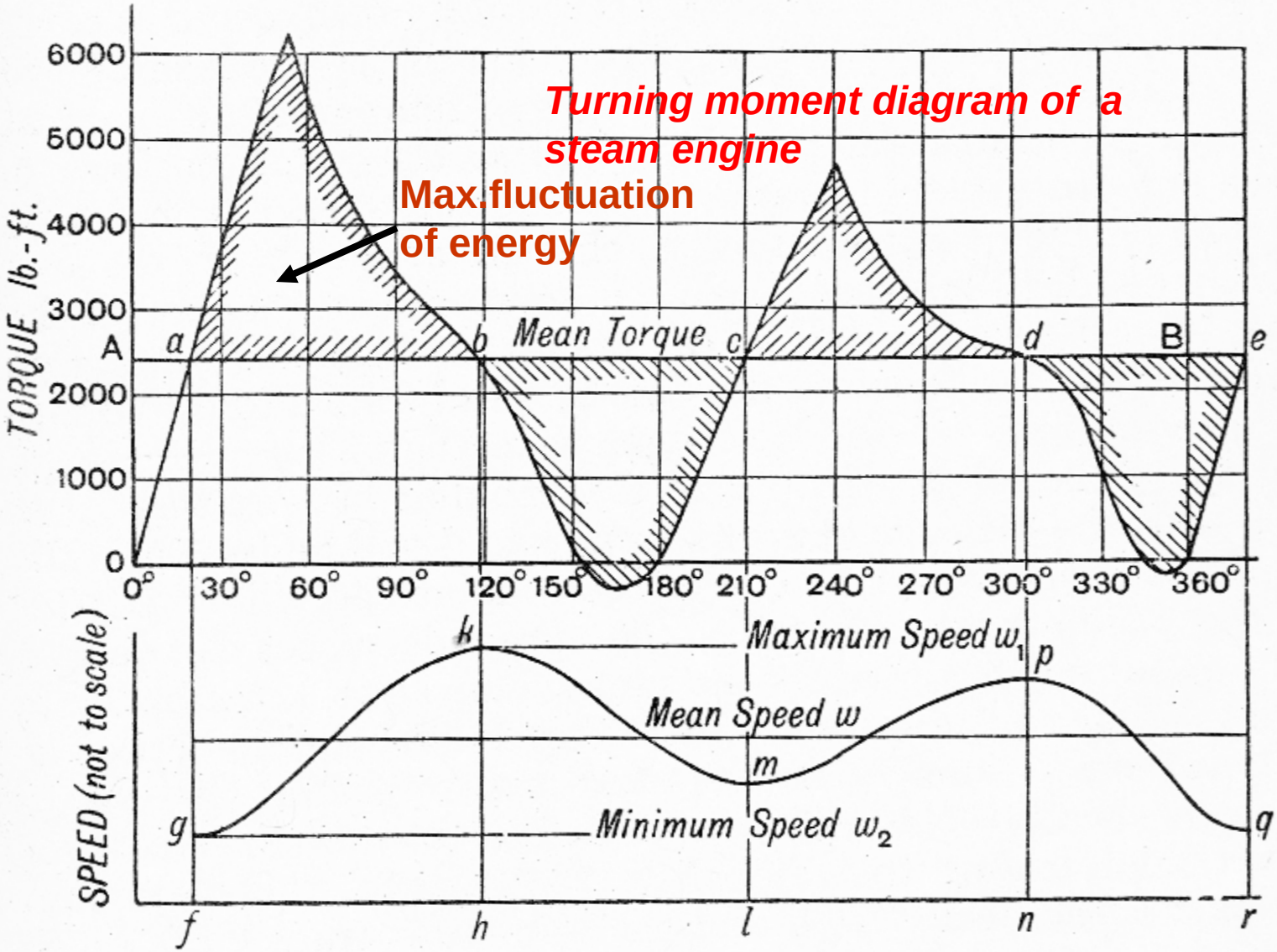
Dynamics of Machinery

Credits: 3

Lecturer: Abbasi




Turning moment diagram of a steam engine
Max.fluctuation of energy
TORQUE lb.-ft.
6000
5000
4000
3000
2000
1000
0
A
B
a
b
c
d
e
Mean Torque
0°
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°
360°
SPEED (not to scale)
Maximum Speed w_1
Mean Speed w
Minimum Speed w_2
k
p
m
g
q
f
h
l
n
r

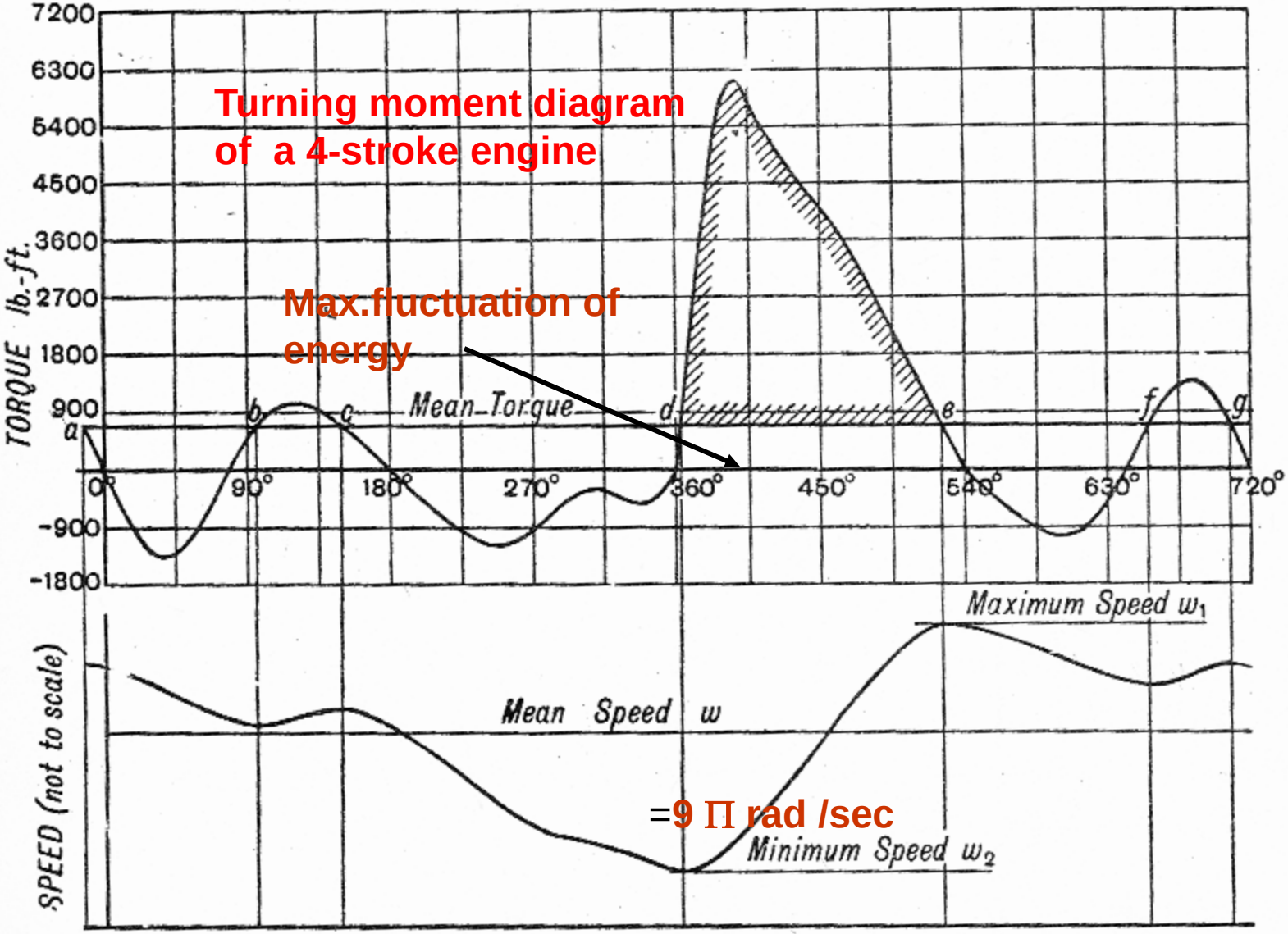

Turning moment diagram
of a 4-stroke engine
Max.fluctuation of
energy
TORQUE lb.-ft.
7200
6300
5400
4500
3600
2700
1800
900
-900
-1800
Mean Torque
a
b
c
d
e
f
g
0°
90°
180°
270°
360°
450°
540°
630°
720°
SPEED (not to scale)
Maximum Speed w_1
Mean Speed w
=9 Π rad /sec
Minimum Speed w_2

FIG. 289

C=Coefficient of speed flucatation
(From Handbooks of machinery)

$$C=\frac{\omega max-\omega min}{\omega ave}$$

$$C=\frac{VMax-Vmin}{Vave}$$

$$E\ flywheel=\frac{1}{2}m(V^2max-V^2min)$$

$$E\ flywheel=\frac{1}{2}I(\omega^2max-\omega^2min)$$

$$m=\frac{E}{CV^2ave}$$

$$I=\frac{E}{C\omega^2ave}$$

Solid-based flywheel

$I = \frac{1}{2}\ mr^2$

Ring-based flywheel

$I = 0.9\ mr^2$; if 90% of mass in ring

Turning moment diagram of a mechanical press

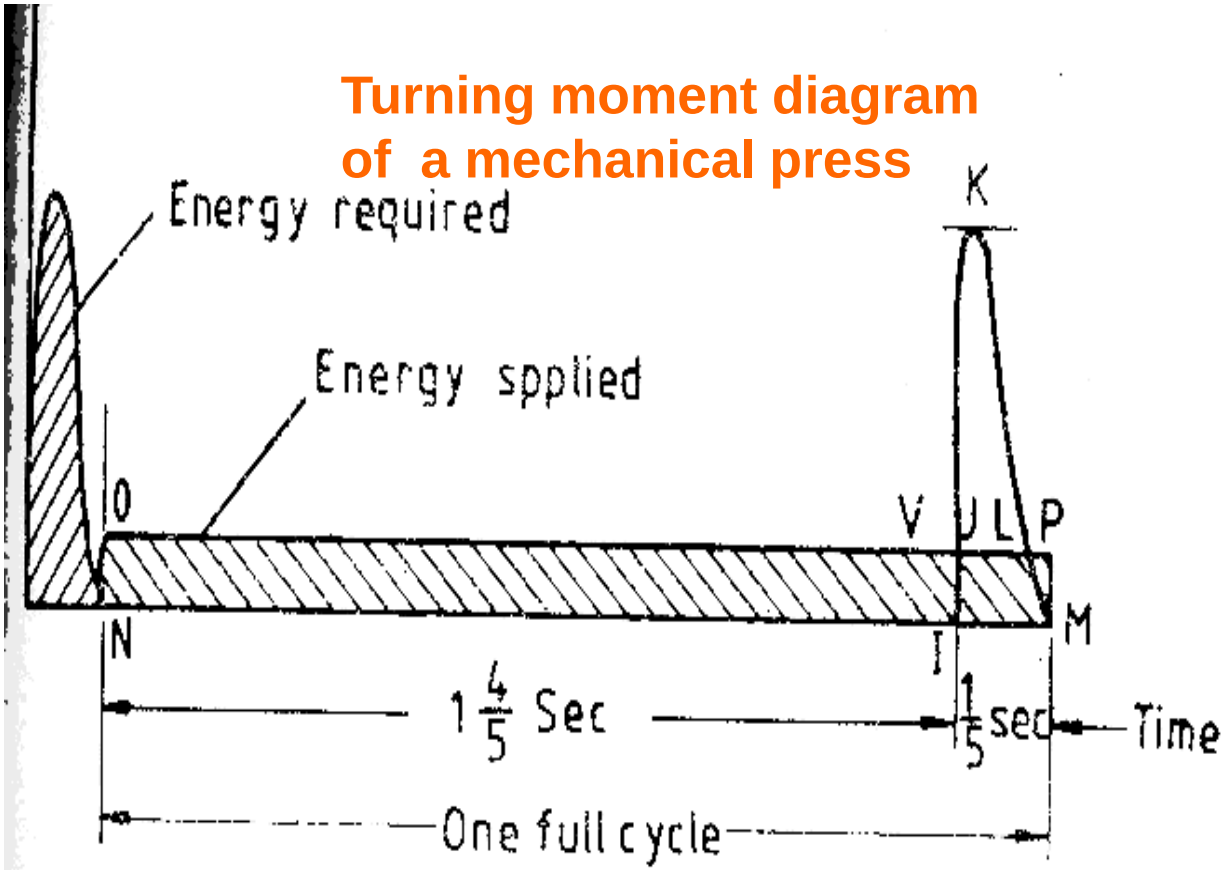


Fig. 14.5 Force-time Relationship for One Full Cycle with a Flywheel

Example : (flywheel calculation)

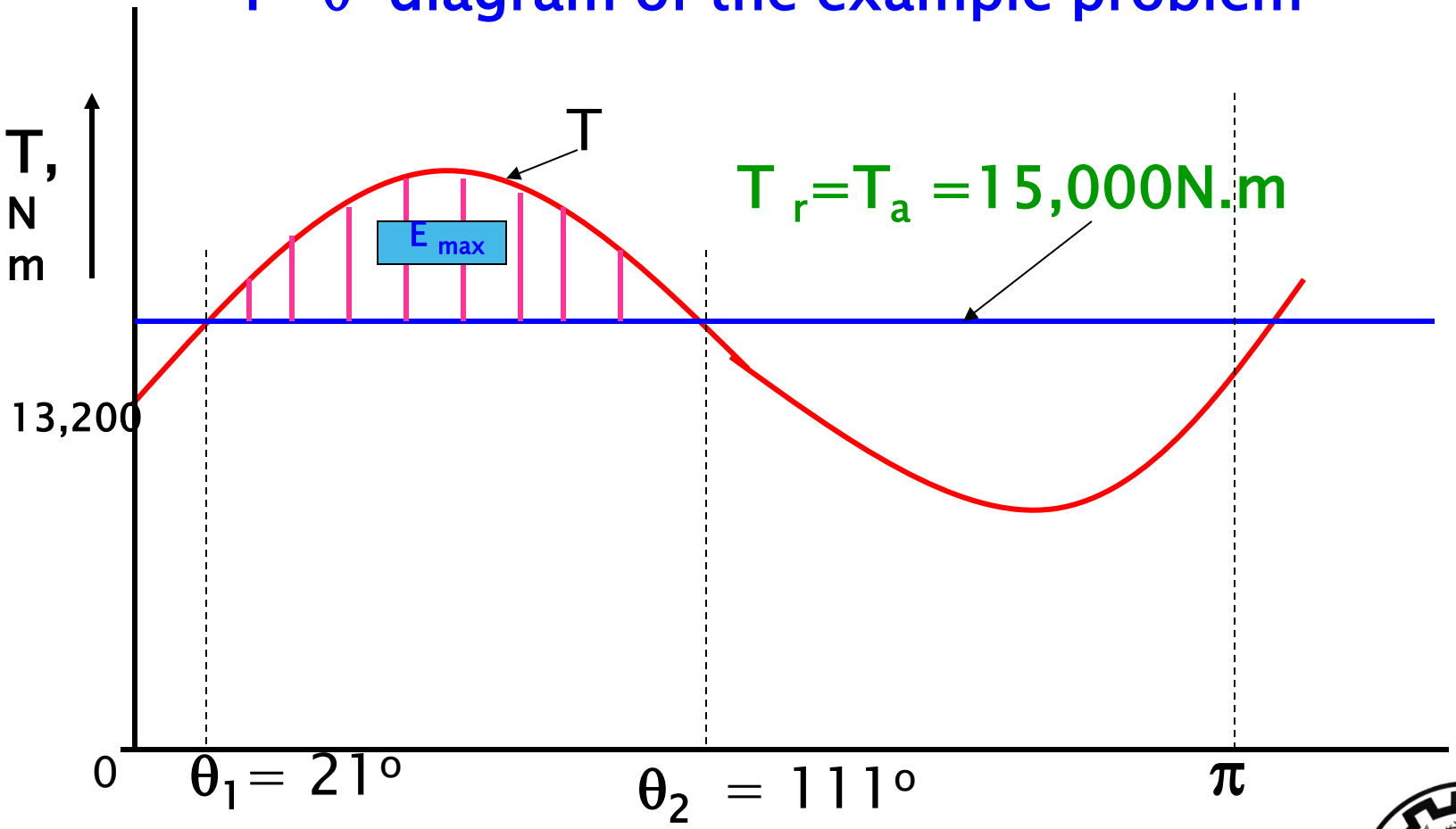
The torque exerted on the crank of a **two stroke** engine Is given as

$$T(\theta) = [15{,}000 + 2{,}000\sin 2(\theta) - 1{,}800\cos 2(\theta)] \ \text{N.m}$$

If the **load torque** on the engine is **constant**,
Determine the following :

1. Power of the engine if the mean speed is 150 rpm.
2. E of the flywheel if **C** =0.01
3. Angular acceleration of the flywheel for $\theta = 30^{o}$

T- θ diagram of the example problem



Solution :

$$T_{av} = 1/2\pi \int_0^{2\pi} M \, d\theta$$

$$= 1/2\pi \int_0^{2\pi} (15000+2000\sin 2(\theta) - 1800\cos 2(\theta)) \, d\theta$$

$$= \mathbf{15{,}000\ N.m}$$

Power of the engine $= 15{,}000 \times 2\pi \ (150/60)$ W

$= \underline{\mathbf{235.5\ k\ W}}$

The value of θ at which T-θ diagram intersect with T_r are given by

$$[15000+2000\sin 2(\theta) - 1800\cos 2(\theta)] = 15{,}000 \text{ N.m}$$

or

$$[2000\sin 2(\theta) - 1800\cos 2(\theta)] = 0,$$

$$\therefore \tan 2\theta = 0.9$$

$$\mathbf{2(\theta)_1 = 42^o \quad \& \quad 2(\theta)_2 = (180+42)^o}$$

$$\mathbf{(\theta)_1 = 21^o \quad \& \quad (\theta)_2 = 111^o}$$

$$\mathbf{E}_{\mathbf{max}} = \int_{(\theta)1}^{(\theta)2} (2000\sin 2(\theta) - 1800\cos 2(\theta))\, d\theta$$

$$= \underline{\mathbf{2690.2\ N.m}}$$

Contd….

We have $C = 0.01$ & $\omega = (150 \times 2\pi)/60$

$= 15.7$ rad /s

Using the relation , $\boxed{E = I\,\omega^2\,C}$

M.I of the flywheel , $I = 2690.2 / (15.7^2 \times 0.01)$

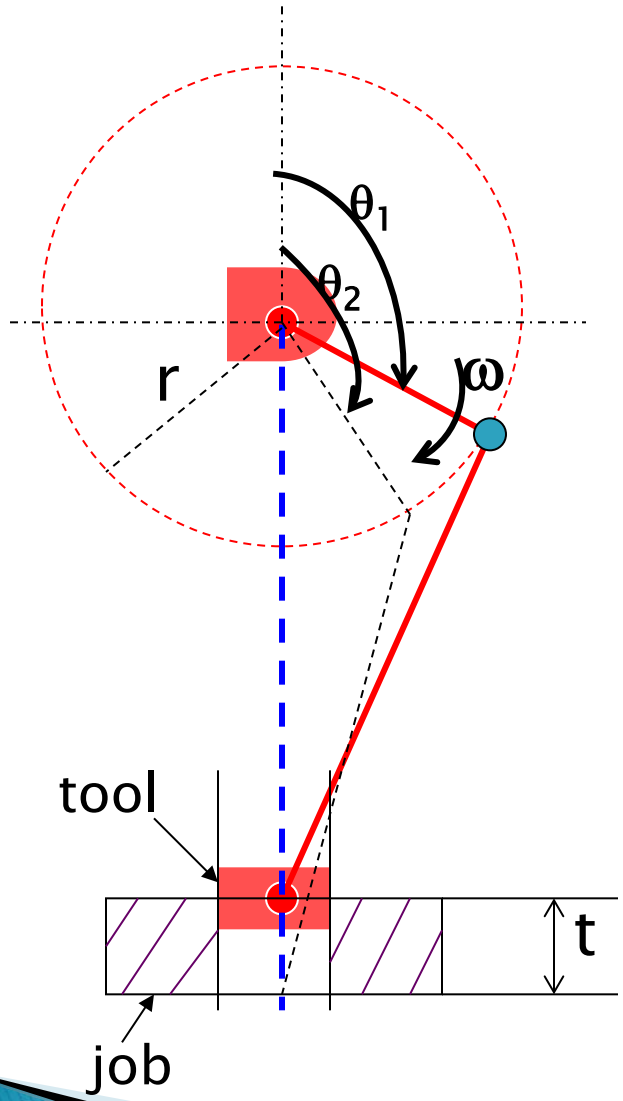
$= \underline{\mathbf{1090\ kg.m^2}}$

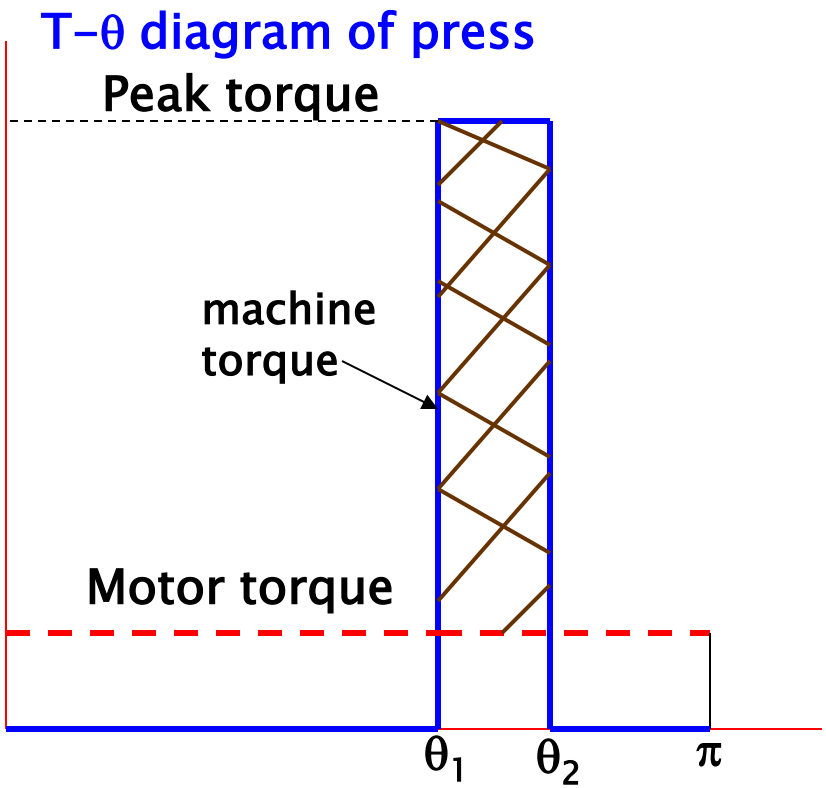
when $\theta = 30^o$, $T = 15{,}000 + 1732 - 900 = \underline{\mathbf{15{,}832\ N.m}}$

Hence ang. acceleration of the flywheel at $\boldsymbol{\theta = 30^o}$

$$\alpha_{30} = (15{,}832 - 15{,}000) / 1090$$

$$= \underline{\mathbf{0.764\ rad/s^2}}$$





Assuming uniform velocity of the tool,

$$(\theta_2 - \theta_1) / 2\pi = t / 2s = t / 4r$$

Example :

Operation :3.8 cm dia hole in 3.2 cm plate

Work done in punching : 600 N.m / cm^2 of sheared area

Stroke of the punch : 10.2 cm

No. of holes punched : 6 holes /min

Max speed of the flywheel at its $\mathbf{r_g}$ = 27.5 m /s

Min speed of the flywheel at its $\mathbf{r_g}$ = 24.5 m /s

To find :

Power of the motor of the machine ?

Mass of the required flywheel ?

Solution :

Sheared area /hole $= \pi\, d\, t = \pi \times 3.8 \times 3.2 = 38.2\ cm^2$

$\therefore$ work done in shearing /hole $= 38.2 \times 600$

$= 22{,}920$ N.m

$\therefore$ work done / minute $= (22{,}920 \times 6)$ N.m

Power of the motor $= (22{,}920 \times 6) / 60$

$=$ **2.292 k W**

We have, $t/2s = 3.2 / (2 \times 10.2)$

$= 3.2 / 20.4 = \mathbf{(\theta_2 - \theta_1) / 2\pi}$

Energy required by the machine / cycle = **22,920 N.m**

Energy delivered by the motor during actual Punching

i.e (when crank rotates from θ_1 to θ_2)

$= 22{,}920 \times (t/2s)$

$= 22{,}920 \times (3.2/20.4)$

$=$ **3,595 N.m**

Contd …

$\therefore$ **Max .fluctuation of energy,**

$$E_{max} = 22{,}920 - 3595 = \underline{19{,}325 \text{ N.m}}$$

$$= I\,(\omega_1^2 - \omega_2^2)\,/\,2$$

$$= m\,r_g^2\,(\omega_1^2 - \omega_2^2)\,/\,2$$

But $r_g^2\ \omega_1^2 = 27.5^2$ & $r_g^2\ \omega_2^2 = 24.5^2$ m/s

$$\therefore\ m\,(27.5^2 - 24.5^2)\,/\,2 = 19{,}325$$

$$m = \underline{244\ \text{kg}}$$