Dynamics of Machinery

Credits: 3

Lecturer: Abbasi

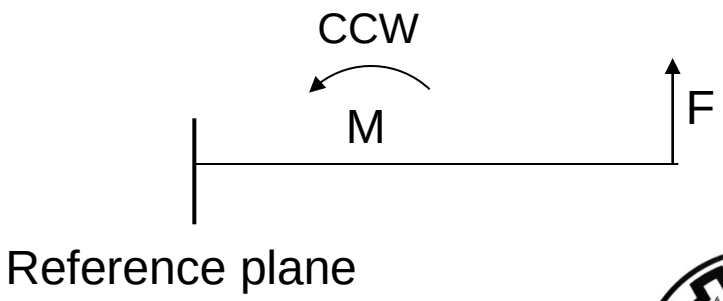
BALANCING

- The technique of correcting or eliminating unwanted inertia forces and moments in rotating or reciprocating masses.
- The two equations to determine the amount and location of the correction

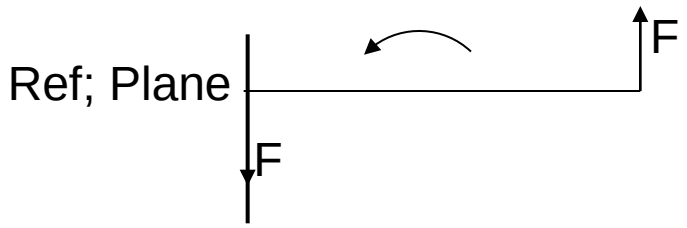
$$\Sigma F = 0 \quad \text{and} \quad \Sigma M = 0$$

Assumptions

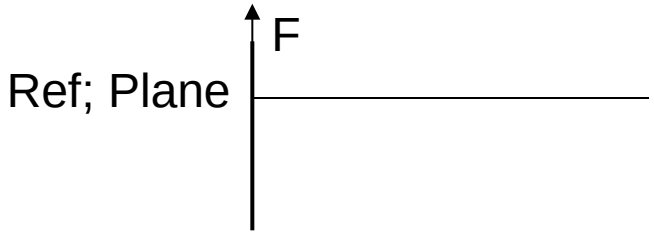
- Upwards force is positive &
- Counter clockwise couple is positive.



- 1. $\Sigma F = 0$ & $\Sigma M = 0$
  - Complete balanced
- 2. $\Sigma F = 0$ & $\Sigma M \neq 0$
  - Unbalanced being due to a couple.

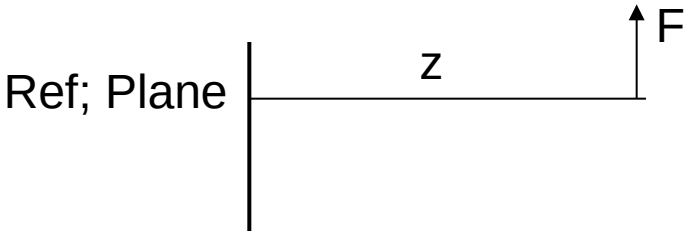


- 3. $\Sigma F \neq 0$ & $\Sigma M = 0$
  - Unbalanced being due to a single resultant force in the reference plane.



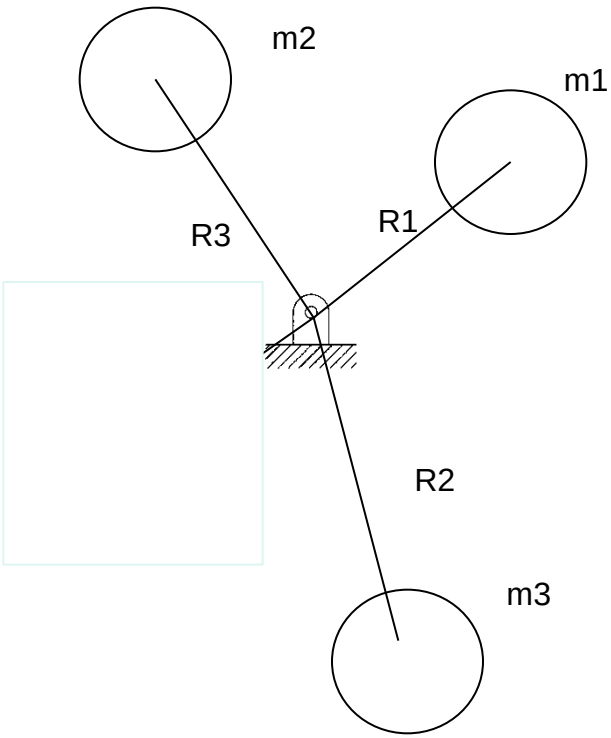
- 4. Σ F ≠ 0 & Σ M ≠ 0
  - Unbalanced being due to a single resultant force which locates at a distance z from the reference plane and

$$z = \frac{M}{F}$$



Inertia Forces and D'Alembert's Principle

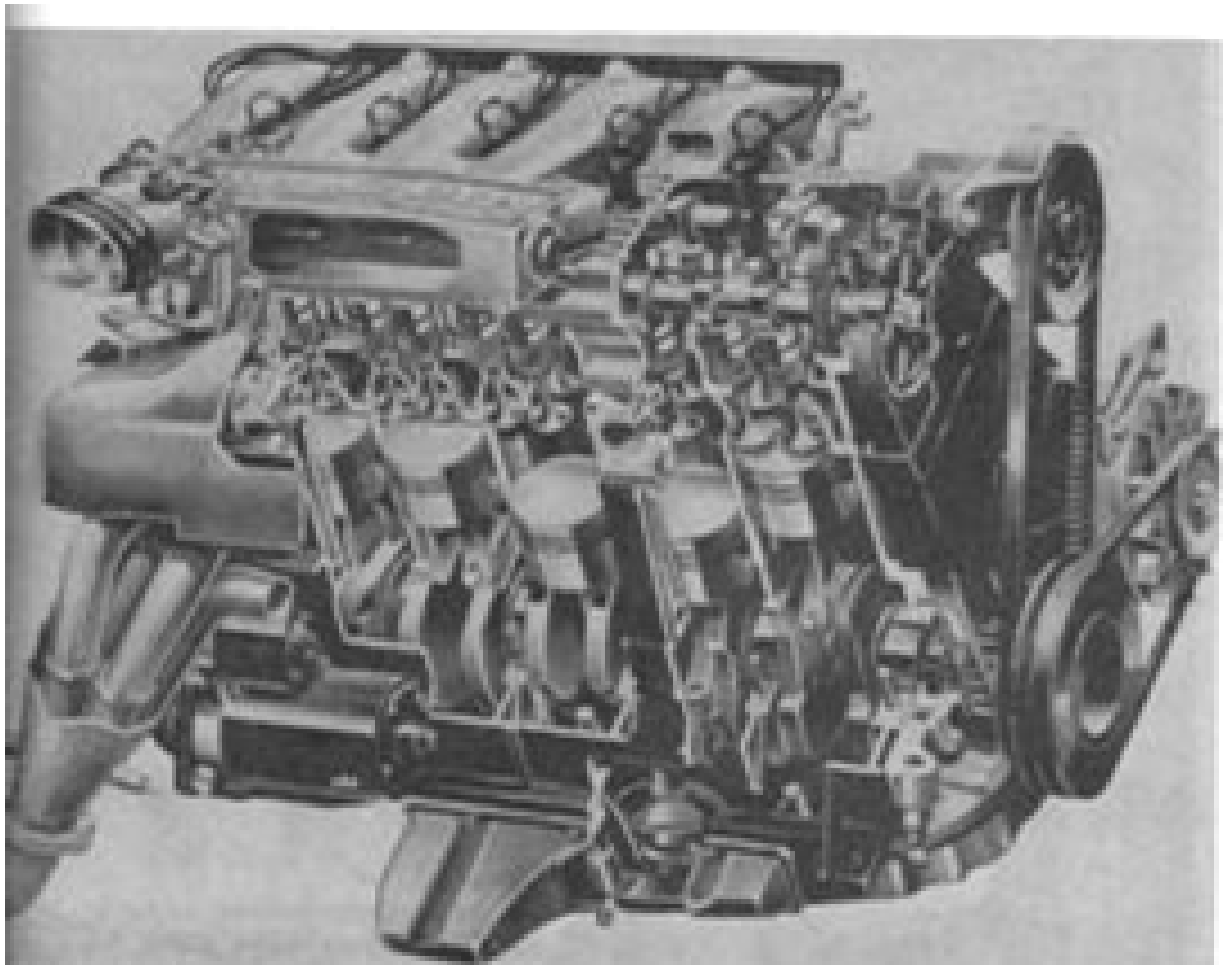
- $\Sigma F = F_1 + F_2 + F_3$, the resultant force will not be through the mass centre, and results the unbalanced force system.
- The effect of this unbalanced system is to produce an acceleration, A_G, of the centre of mass of the body. $\Sigma F = mA_G$ (1)
- Taking moment about centre of mass of the body results the unbalanced moment system. It causes angular acceleration, α, of the body. $\Sigma M = I_G \alpha$ (2)

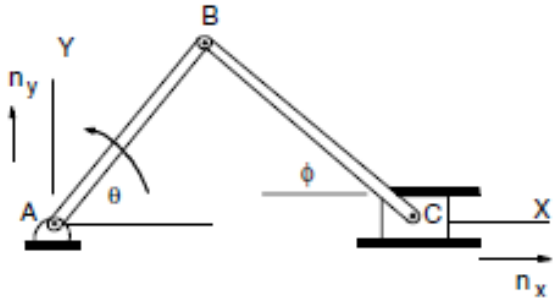


$$\sum miRi = meRe$$

$$\sum miRiai = \Sigma meReae$$

Balancing of Multi-cylinder In-line Engines (Analytical Method)





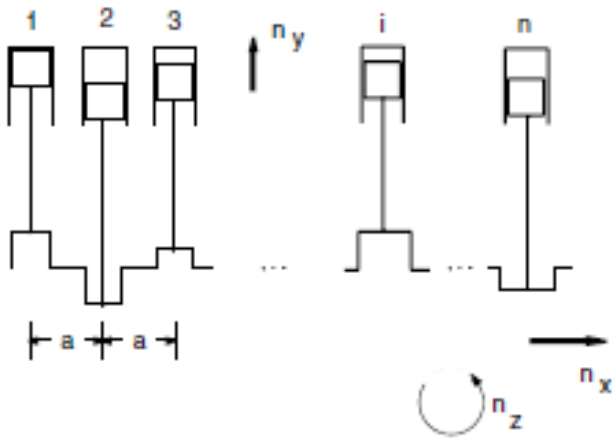
$$x = r\cos\theta + \ell - (1/4)(r/\ell)^2 + (\ell/4)(r/\ell)^2\cos 2\theta$$

$$\ddot{x} = -r\omega^2\left[\cos\theta + (r/\ell)\cos 2\theta\right]$$

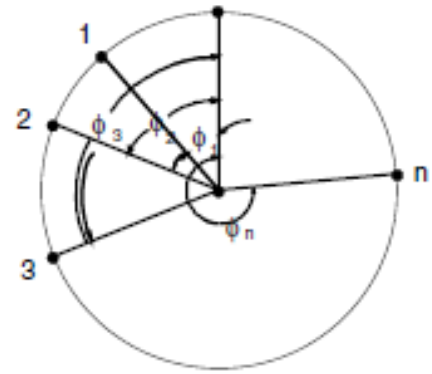
$$\mathbf{F}_C^* = m_C r\omega^2\left[\cos\theta + (r/\ell)\cos 2\theta\right]$$

primary unbalancing force

secondary unbalancing force.



a = x



$$\mathbf{F}^* = \sum_{i=1}^{m} \mathbf{F}_i^* = mr\omega^2 \left[\sum_{i=1}^{n} \cos(\theta + \phi_i) + (r/\ell) \sum_{i=1}^{n} \cos 2(\theta + \phi_i) \right]$$

$$\sum_{i=1}^{n} \cos\phi_i = 0 \quad \text{and} \quad \sum_{i=1}^{n} \sin\phi_i = 0$$

$$\sum_{i=1}^{n} \cos 2\phi_i = 0 \quad \text{and} \quad \sum_{i=1}^{n} \sin 2\phi_i = 0$$

$$\cos(\theta + \phi_i) \equiv \cos\theta \cos\phi_i - \sin\theta \sin\phi_i$$

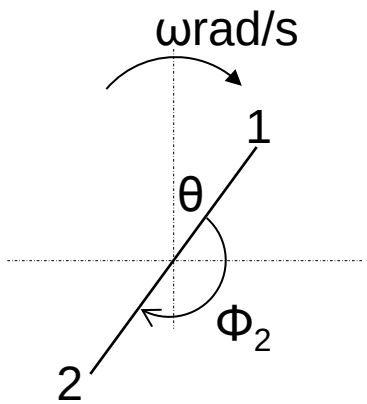
$$\sum_{i=1}^{n} (i-1)\cos\phi_i = 0 \quad \text{and} \quad \sum_{i=1}^{n} (i-1)\sin\phi_i = 0$$

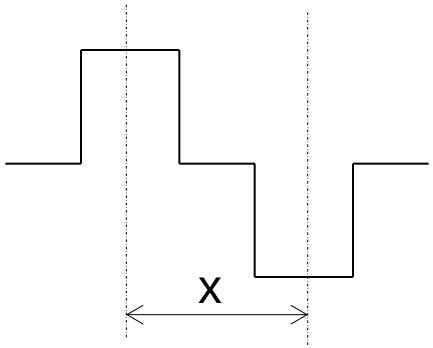
$$\mathbf{M}^* = mr\omega^2 a \left[\sum_{i=1}^{n} (i-1)\cos(\theta + \phi_i) + (r/\ell) \sum_{i=1}^{n} (i-1)\cos 2(\theta + \phi_i) \right]$$

$$\sum_{i=1}^{n} (i-1)\cos 2\phi_i = 0 \quad \text{and} \quad \sum_{i=1}^{n} (i-1)\sin 2\phi_i = 0$$

Example (1)

- 2 stroke, 2 cylinder, In-line Engine
  - Firing interval = $\frac{360}{2} = 180^{\circ}$





Continued

Φ	cosΦ	sinΦ	2Φ	cos2Φ	sin2Φ	x	xcosΦ	xsinΦ	xcos2Φ	xsin2Φ
$\Phi_1 = 0^\circ$	1	0	0°	1	0	0	0	0	x	0
$\Phi_2 = 180^\circ$	-1	0	360°	1	0	x	-x	0	x	0
	0 0 Primary Forces Balanced			2 0 Secondary Forces Unbalanced		i-1	-x 0 Primary Moments Unbalanced		x 0 Secondary Moments Unbalanced	

Cylinder Strokes for Various Crankshaft Angles for 180° Connecting Rod Spacing

Cylinder	Connecting Rod Crank Angle (°)	Crankshaft Rotation/Stroke Type			
		0 to 180°	180 to 360°	360 to 540°	540 to 720°
1	0	Power	Exhaust	Intake	Compression
2	180	Exhaust	Intake	Compression	Power
3	360	Intake	Compression	Power	Exhaust
4	540	Compression	Power	Exhaust	Intake

i	ϕ_i (°)	$\cos\phi_i$	$\sin\phi_i$	$\cos 2\phi_i$	$\sin 2\phi_i$	$(i-1)\cos\phi_i$	$(i-1)\sin\phi_i$	$(i-1)\cos 2\phi_i$	$(i-1)\sin 2\phi_i$
1	0	1	0	1	0	0	0	0	0
2	180	–1	0	1	0	–1	0	1	0
3	360	1	0	1	0	2	0	2	0
4	540	–1	0	1	0	–3	0	3	0
Totals		0	0	4	0	–2	0	6	0

Cylinder Strokes for Various Crankshaft Angles for 90° Connecting Rod Spacing

Cylinder	Connecting Rod Crank Angle (°)	Crankshaft Rotation/Stroke Type			
		0 to 180°	180 to 360°	360 to 540°	540 to 720°
1	0	Power	Exhaust	Intake	Compression
2	90	Power	Exhaust	Intake	Compression
3	180	Exhaust	Intake	Compression	Power
4	270	Exhaust	Intake	Compression	Power

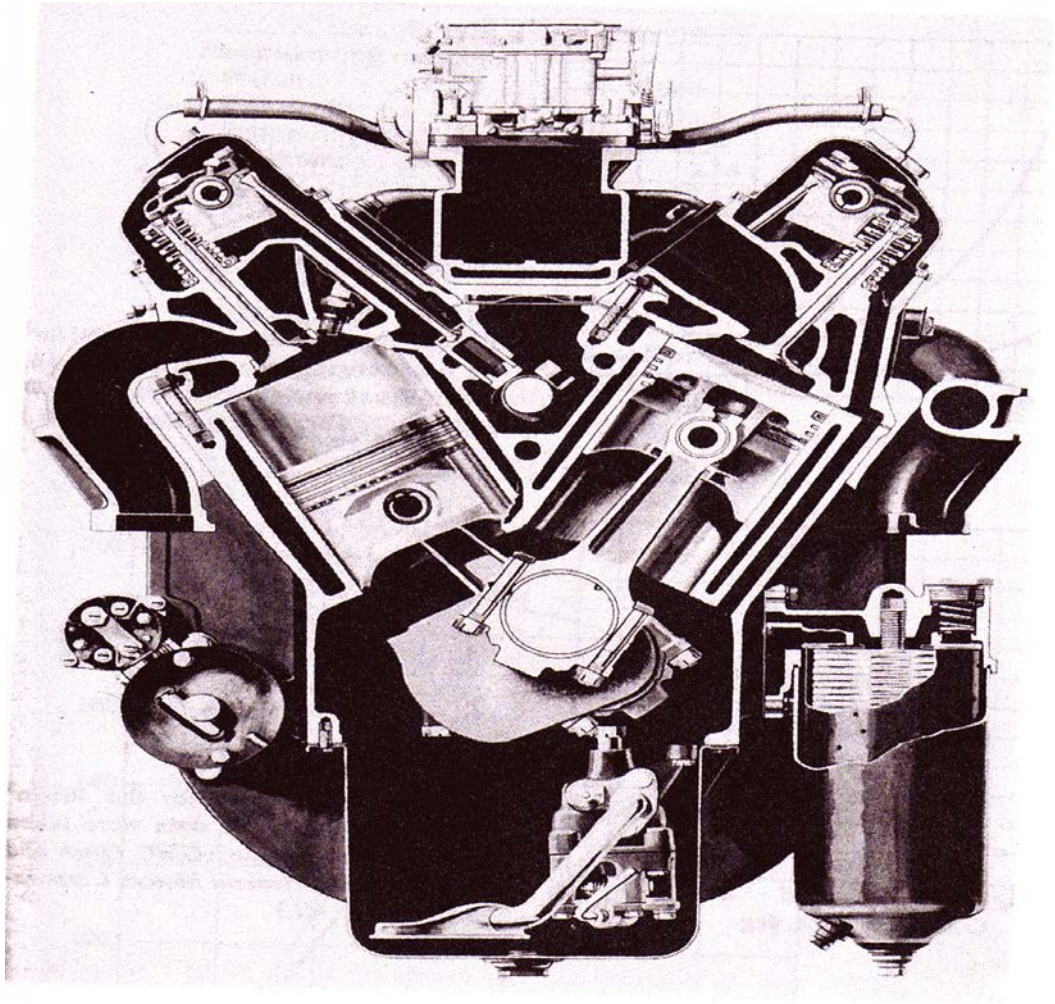
i	ϕ_i (°)	$\cos\phi_i$	$\sin\phi_i$	$\cos 2\phi_i$	$\sin 2\phi_i$	$(i-1)\cos\phi_i$	$(i-1)\sin\phi_i$	$(i-1)\cos 2\phi_i$	$(i-1)\sin 2\phi_i$
1	0	1	0	1	0	0	0	0	0
2	90	0	1	–1	0	0	1	–1	0
3	180	–1	0	1	0	–2	0	2	0
4	270	0	–1	–1	0	0	–3	–3	0
Totals		0	0	0	0	–2	–2	–2	0

Cylinder Strokes for Various Crankshaft Angles for 0, 90, 270, and 180° Connecting Rod Spacing

Cylinder	Connecting Rod Crank Angle (°)	Crankshaft Rotation/Stroke Type			
		0 to 180°	180 to 360°	360 to 540°	540 to 720°
1	0	Power	Exhaust	Intake	Compression
2	90	Intake	Compression	Power	Exhaust
3	270	Exhaust	Intake	Compression	Power
4	180	Compression	Power	Exhaust	Intake

i	ϕ_i (°)	$\cos\phi_i$	$\sin\phi_i$	$\cos 2\phi_i$	$\sin 2\phi_i$	$(i-1)\cos\phi_i$	$(i-1)\sin\phi_i$	$(i-1)\cos 2\phi_i$	$(i-1)\sin 2\phi_i$
1	0	1	0	1	0	0	0	0	0
2	90	0	1	–1	0	0	1	–1	0
3	270	0	–1	–1	0	0	–2	–2	0
4	180	–1	0	1	0	–3	0	3	0
Totals		0	0	0	0	–3	–1	0	0

i	ϕ_i (°)	$\cos\phi_i$	$\sin\phi_i$	$\cos 2\phi_i$	$\sin 2\phi_i$	$(i-1)\cos\phi_i$	$(i-1)\sin\phi_i$	$(i-1)\cos 2\phi_i$	$(i-1)\sin 2\phi_i$
1	0	1	0	1	0	0	0	0	0
2	90	0	1	–1	0	0	1	–1	0
3	270	0	–1	–1	0	0	–2	–2	0
4	180	–1	0	1	0	–3	0	3	0
Totals		0	0	0	0	–3	–1	0	0

i	ϕ_i (°)	$\cos\phi_i$	$\sin\phi_i$	$\cos 2\phi_i$	$\sin 2\phi_i$	$(i-1)\cos\phi_i$	$(i-1)\sin\phi_i$	$(i-1)\cos 2\phi_i$	$(i-1)\sin 2\phi_i$
1	0	1	0	1	0	0	0	0	0
2	90	0	1	–1	0	0	1	–1	0
3	270	0	–1	–1	0	0	–2	–2	0
4	180	–1	0	1	0	–3	0	3	0
5	180	–1	0	1	0	–4	0	4	0
6	270	0	–1	–1	0	0	–5	–5	0
7	90	0	1	–1	0	0	6	–6	0
8	0	1	0	1	0	7	0	7	0
Totals		0	0	0	0	0	0	0	0

Balancing of multi-cylinder V-Engine