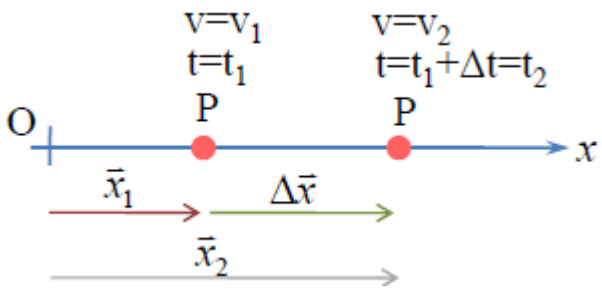
Dynamics of Machinery

Credits: 3

Lecturer: Abbasi



$$|\bar{x}_1| = s_1, \quad |\bar{x}_2| = s_2, \quad |\Delta\bar{x}| = \Delta s$$

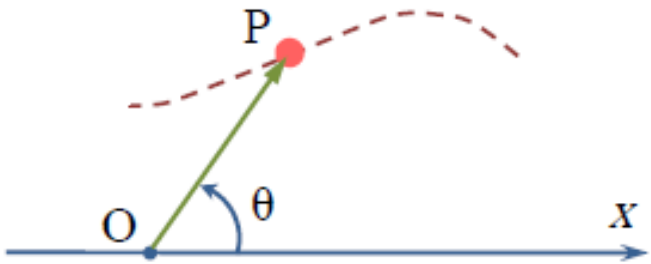
$$\bar{a}_{av.} = \frac{\Delta\bar{v}}{\Delta t} = \frac{\bar{v}_2 - \bar{v}_1}{t_2 - t_1} \quad \text{or} \quad a = \frac{v_2 - v_1}{t_2 - t_1} \quad (\text{m/s}^2), (\text{cm/s}^2), (\text{in/s}^2), \ldots$$

$$\bar{a} = \lim_{\Delta t \to 0} \frac{\Delta\bar{v}}{\Delta t} = \frac{d\bar{v}}{dt} = \dot{\bar{v}} \quad \Rightarrow \quad \bar{a} = \frac{d^2\bar{x}}{dt^2} = \ddot{\bar{x}} \quad \text{or} \quad a = \ddot{x}$$

If a= constant then:

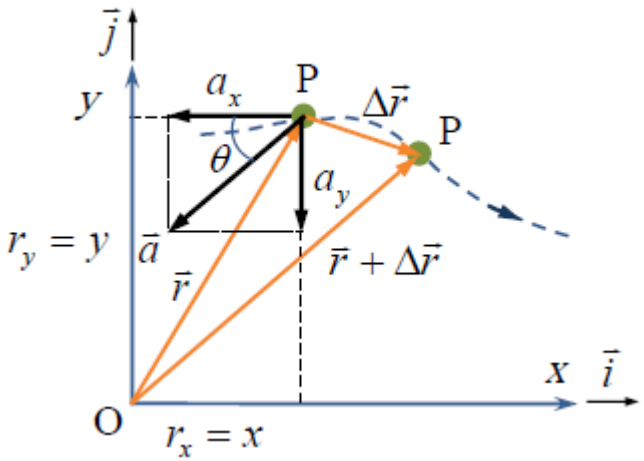
$$\int_{v_0}^{v} dv = \int_0^t a\,dt \quad \Rightarrow \quad \boxed{v = at + v_0}$$

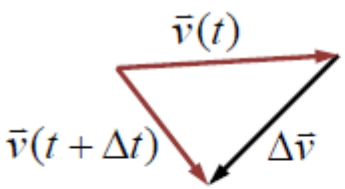
$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a\,ds \quad \Rightarrow \quad \boxed{v^2 = 2a(s - s_0) + v_0^2}$$



$$\bar{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta \bar{\omega}}{\Delta t} = \frac{d\bar{\omega}}{dt} = \dot{\bar{\omega}} = \frac{d^2\bar{\theta}}{dt^2} = \ddot{\bar{\theta}} \quad (\mathrm{rad/s^2})$$

$$dt = \frac{d\theta}{\omega} = \frac{d\omega}{\alpha} \quad \Rightarrow \quad \boxed{\alpha\, d\theta = \omega\, d\omega}$$



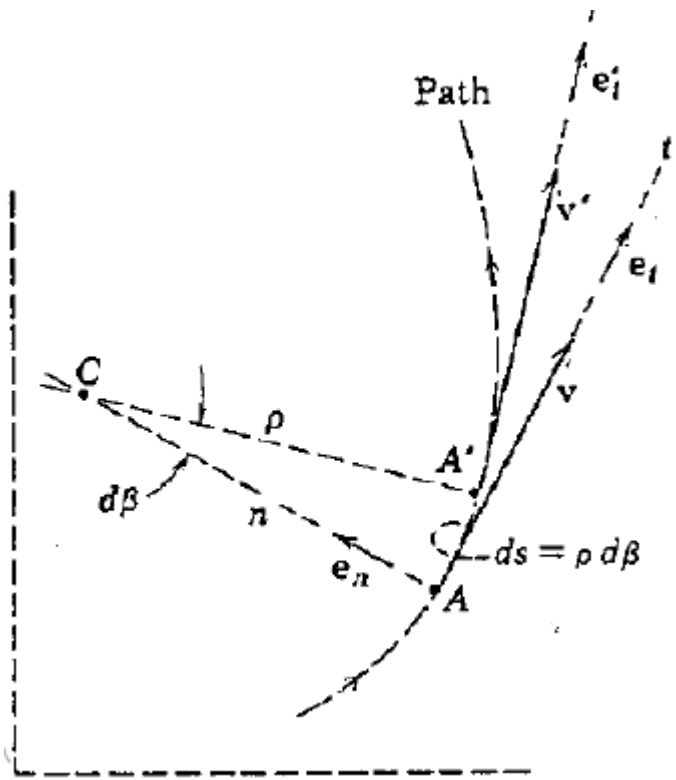


$$\boxed{\vec{a} = \ddot{x}\vec{i} + \ddot{y}\vec{j} = a_x\vec{i} + a_y\vec{j}}$$

$$\vec{r} = x\vec{i} + y\vec{j}$$

$$a = |\vec{a}| = \sqrt{(a_x)^2 + (a_y)^2}, \quad \tan\theta = \frac{a_y}{a_x}$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \dot{\vec{v}} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}}$$



$$\vec{v} = \dot{s}\vec{e}_t = \rho\dot{\beta}\vec{e}_t = v\vec{e}_t$$

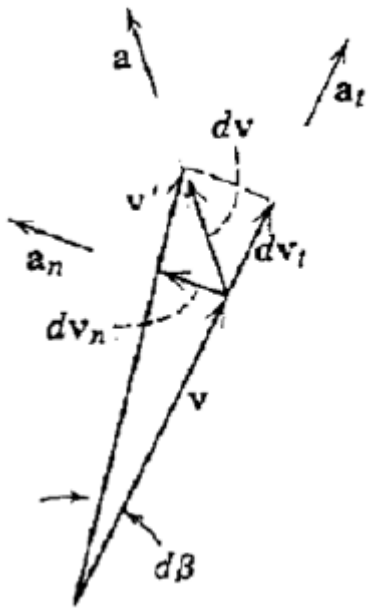
$$\vec{a} = \frac{d\vec{v}}{dt}_t = \frac{d}{dt}(\dot{s}\vec{e}_t) \Rightarrow \vec{a} = \ddot{s}\,\vec{e}_t + \dot{s}\,\dot{\vec{e}}_t$$

$$|\vec{e}_t| = \text{const.} \Rightarrow$$

$$\dot{\vec{e}}_t = \vec{\omega}\times\vec{e}_t = (\dot{\beta}\vec{e}_k)\times\vec{e}_t = \dot{\beta}(\vec{e}_k\times\vec{e}_t) = \dot{\beta}\vec{e}_n$$

$$\xrightarrow{(a)} \quad \vec{a} = \ddot{s}\,\vec{e}_t + \dot{s}\dot{\beta}\,\vec{e}_n$$

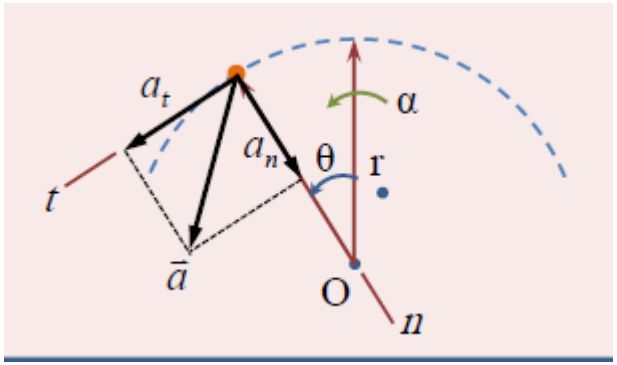
$$\Rightarrow \quad \boxed{\vec{a} = \dot{v}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n = a_t\vec{e}_t + a_n\vec{e}_n}$$



$$a_t = \ddot{s} = \dot{v},$$

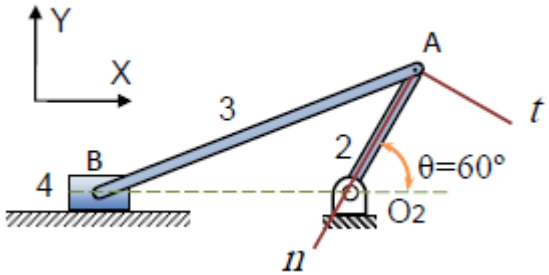
$$a_n = \rho\dot{\beta}^2 = \frac{v^2}{\rho},$$

$$a = \sqrt{(a_t)^2 + (a_n)^2}$$



$$a_t = \dot{v} = \frac{d}{dt}(r\dot{\theta}) = r\ddot{\theta} = r\alpha$$

$$a_n = \frac{v^2}{r} = r\dot{\theta}^2$$

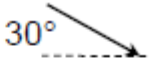


$$\bar{\omega}_2 = -2\bar{k} \text{ rad/s, CW}$$

$$\bar{\alpha}_2 = 1\bar{k} \text{ rad/s}^2$$

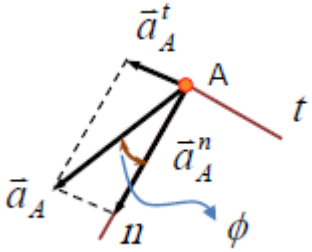
O2A= 20 cm

$$\bar{v}_A = 2 \times 0.2 = 0.4 \text{ m/s}$$



$$\bar{v}_A = 0.4\bar{e}_t \quad \text{m/s}$$

$$\bar{v}_A = 0.4(\cos 30^\circ \bar{i} - \sin 30^\circ \bar{j}) \quad \text{m/s}$$



$$\bar{a}_A = a_A^t \bar{e}_t + a_A^n \bar{e}_n$$

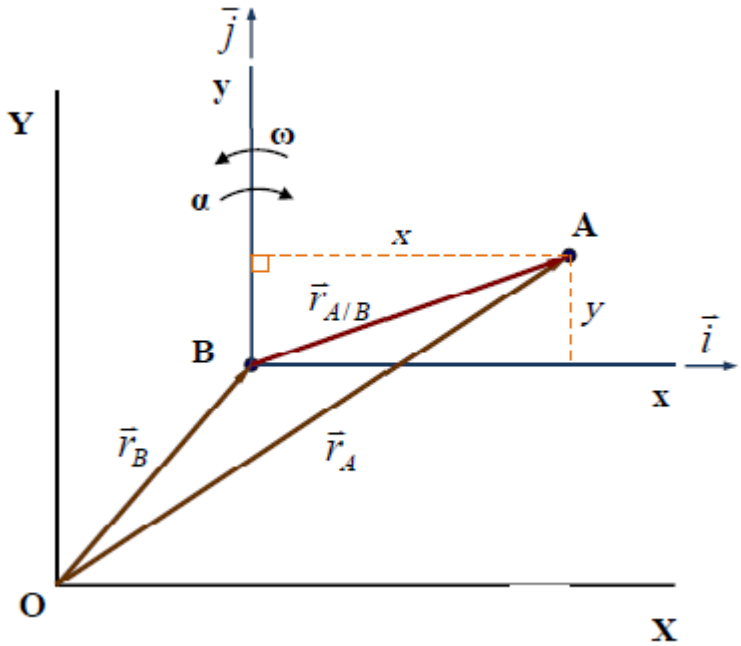
$$a_A^t = r\alpha = -(0.2 \times 1) = -0.2 \text{ m/s}^2$$

$$a_A^n = r\dot{\theta}^2 = 0.2 \times 2^2 = 0.8 \text{ m/s}^2$$

$$\bar{a}_A = -0.2\bar{e}_t + 0.8\bar{e}_n \quad \text{m/s}^2$$

$$a_A = \sqrt{(a_A^t)^2 + (a_A^n)^2} = \text{ 0.825 m/s2}$$

$$\phi = \tan^{-1}\left(\frac{a_t}{a_n}\right) = -14.04^\circ$$



$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$$

$$\vec{r}_{A/B} = x\vec{i} + y\vec{j}$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{rel} + \vec{\omega} \times \vec{r}_{A/B}$$

$$\vec{a}_A = \dot{\vec{v}}_A$$

$$\vec{a}_A = \vec{a}_B + \dot{\vec{v}}_{rel} + \dot{\vec{\omega}} \times \vec{r}_{A/B} + \dot{\vec{\omega}} \times \dot{\vec{r}}_{A/B}$$

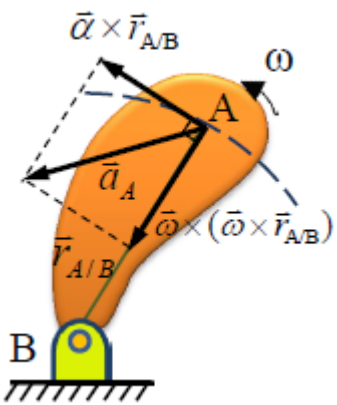
$$\vec{v}_{rel} = \dot{x}\vec{i} + \dot{y}\vec{j}$$

$$\dot{\vec{v}}_{rel} = \ddot{x}\vec{i} + \ddot{y}\vec{j} + \dot{x}\dot{\vec{i}} + \dot{y}\dot{\vec{j}}$$

$$\boxed{\vec{a}_{\mathrm{A}} = \vec{a}_{\mathrm{B}} + \vec{a}_{\mathrm{rel}} + \vec{\alpha} \times \vec{r}_{\mathrm{A/B}} + 2\vec{\omega} \times \vec{v}_{\mathrm{rel}} + \overbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r}_{\mathrm{A/B}})}^{\boxed{-\omega^2 \vec{r}_{\mathrm{A/B}}}}}$$

$(\omega = \alpha = 0)$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{rel}$$

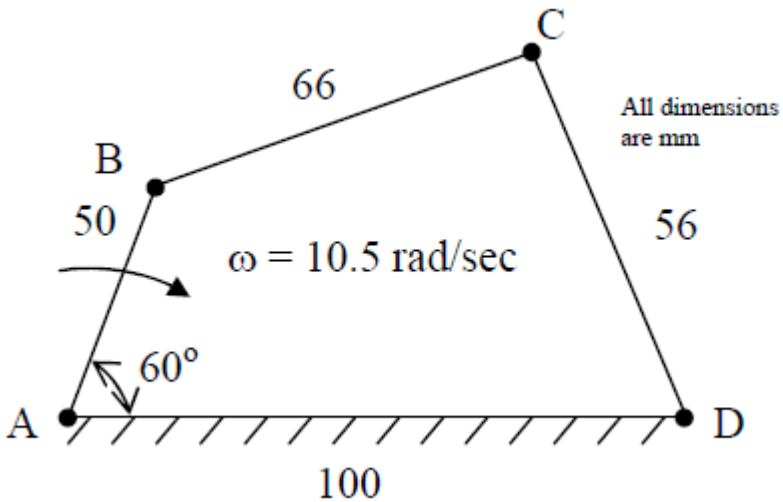


$(a_B = 0)$

$$a_A = \sqrt{(\overline{AB}\alpha)^2 + (\overline{AB}\omega^2)^2}$$

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/B})$$

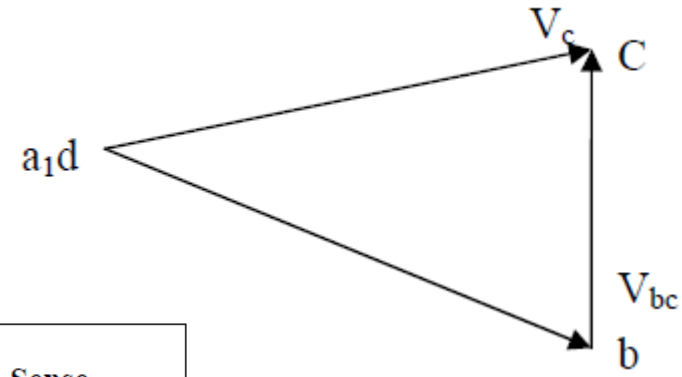
$$= \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B}$$



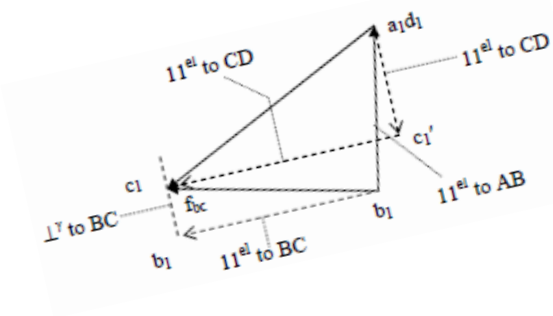
$V_b = \omega_2 \times AB$

$V_b = 10.5 \times 0.05$

$V_b = 0.525$ m/s

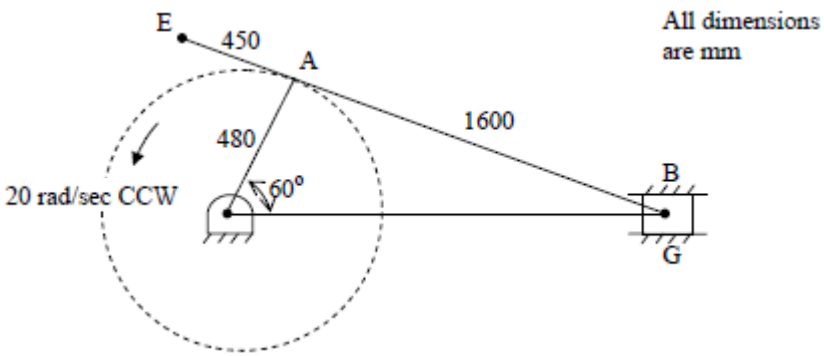


Sl. No.	Link	Magnitude	Direction	Sense
1.	AB	$f^c = \omega^2_{AB} r$ $f^c = (10.5)^2/0.525$ $f^c = 5.51$ m/s^2	Parallel to AB	→ A
2.	BC	$f^c = \omega^2_{BC} r$ $f^c = 1.75$ $f^t = \alpha r$	Parallel to BC $\perp^r$ to BC	→ B –
3.	CD	$f^c = \omega^2_{CD} r$ $f^c = 2.75$ $f^t = ?$	Parallel to DC $\perp^r$ to DC	→ D –



$$\alpha_{BC} = \frac{f^t_{bc}}{BC} = \frac{\overrightarrow{c_1b_1^1}}{BC} = 34.09\,\text{rad/sec (CCW)}$$

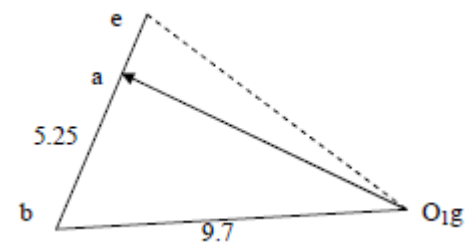
$$\alpha_{CD} = \frac{f^t_{cd}}{CD} = \frac{\overrightarrow{c_1c_1^1}}{CD} = 79.11\,\text{rad/sec (CCW)}$$



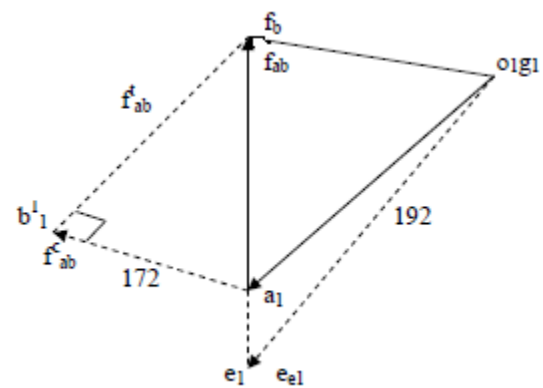
$V_a = \omega_{OA} \times OA$

$V_a = 20 \times 0.48$

$V_a = 9.6$ m/s



Sl. No.	Link	Magnitude	Direction	Sense
1.	OA	$f^c_{aO} = \omega^2_{OA} r = 192$	Parallel to OA	→ O
2.	AB	$f^c_{ab} = \omega^2_{ab} r = 17.2$ f^t_{ab} –	Parallel to AB $\perp^r$ to AB	→ A –
3.	Slider B	–	Parallel to Slider	–

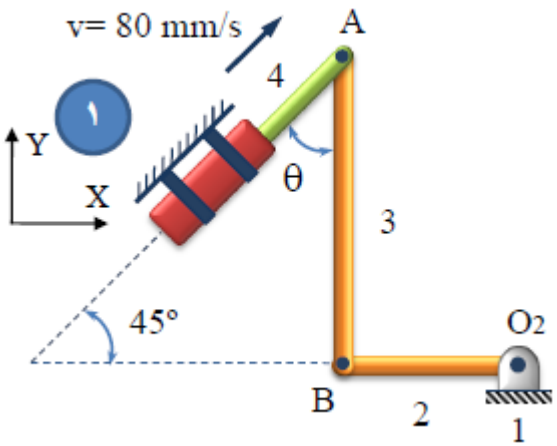


$f_b = 72$ m/sec^2

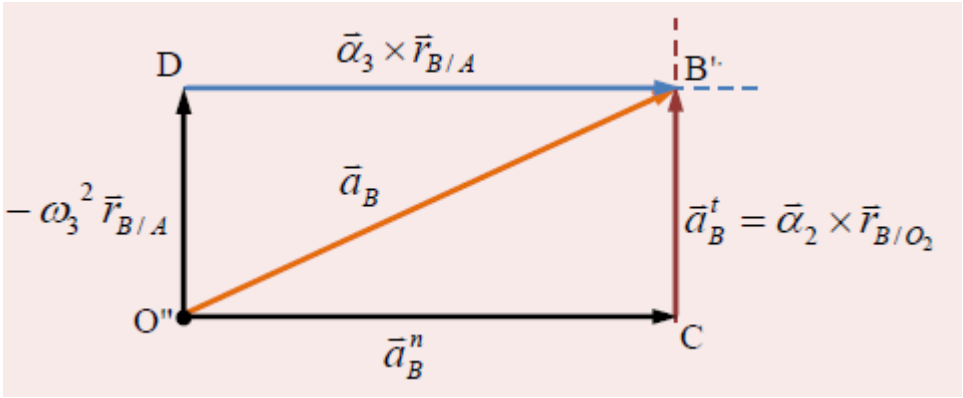
$f_e = 236$ m/sec^2

O_2B= 80 mm, AB=150 mm

θ=45°

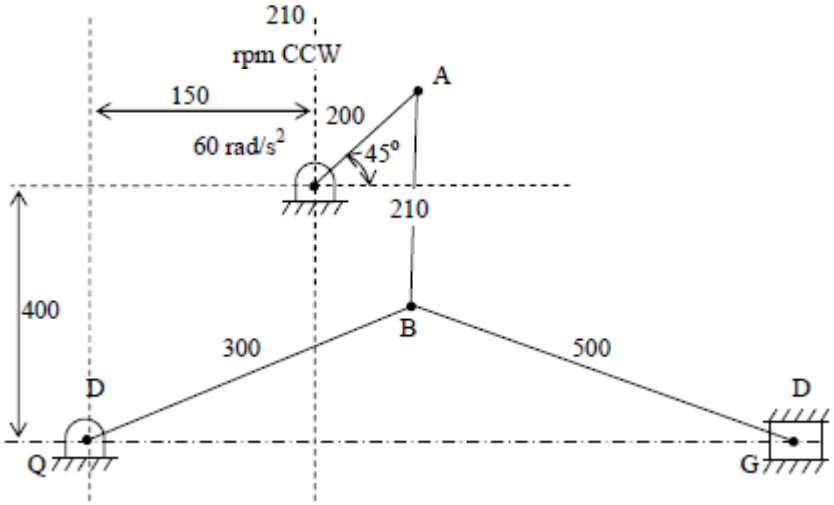


$$\vec{\omega}_3 = -0.377\vec{k}\ \text{rad/s}, \quad \vec{\omega}_2 = -0.707\vec{k}\ \text{rad/s}$$



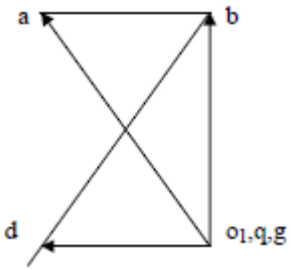
$$\alpha_2 = \frac{a_B^t}{\overline{O_2B}} = \frac{0.021}{0.08} = 0.267\ \text{rad/s}^2$$

$$\alpha_3 = \frac{\left|\vec{\alpha}_3 \times \vec{r}_{B/A}\right|}{\overline{AB}} = \frac{0.04}{0.15} = 0.267\ \text{rad/s}^2$$



$V_a = \omega_{OA} \times OA$

$V_a = \frac{2\pi\,(210)}{60} \times 0.2 = 4.4$ m/s



Sl. No.	Link	Magnitude m/s²	Direction	Sense
1.	AO	$f^c_{aO} = \omega^2 r = 96.8$ $f^t_{aO} = \alpha r = 12$	Parallel to OA $\perp^r$ to OA	→ O –
2.	AB	$f^c_{ab} = \omega^2 r = 5.93$ $f^t_{ab} = \alpha r =$	Parallel to AB $\perp^r$ to AB	→ A –
3.	BQ	$f^c_{bq} = \omega^2 r = 38.3$ $f^t_{bq} = \alpha r =$	Parallel to BQ $\perp^r$ to BQ	→ Q –
4.	BD	$f^c_{bd} = \omega^2 r = 20$	$\perp^r$ to BD	→ B
5.	Slider D	$f^t_{bd} = \alpha r =$ –	$\perp^r$ to BD Parallel to slider motion	– –

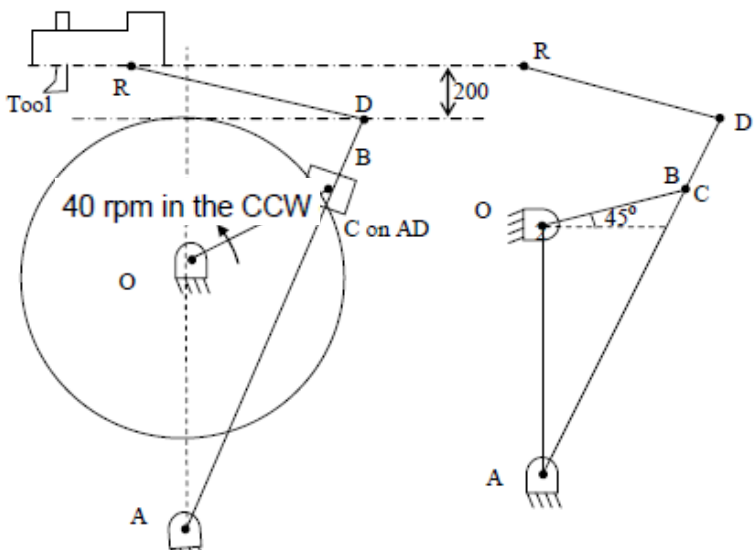
$V_d = 2.54$ m/s

$\omega_{bd} = 6.32$ rad/s

$F_d = 16.4$ m/s²

$\alpha_{bd} = 109.2$ rad/s²

O_1O_2 = 800 mm, O_1B = 300 mm, O_2D = 1300 mm and DR = 400 mm

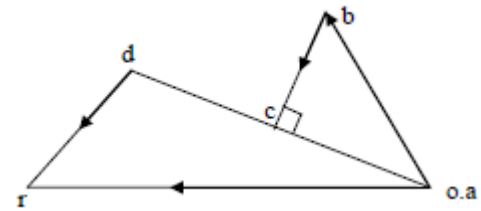


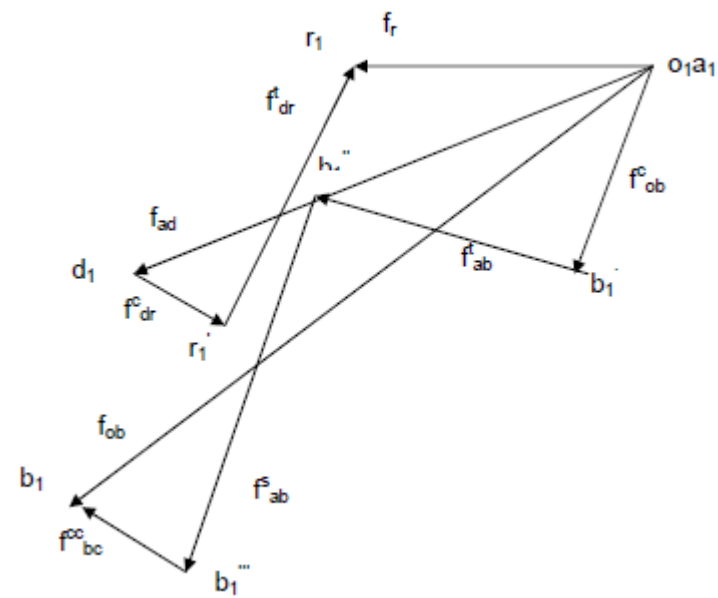
Sl. No.	Link	Magnitude m/s^2	Direction	Sense
1.	OB	$f^c_{ob} = \omega^2 r = 5.24$	Parallel to OB	→ O –
2.	AC	$f^c_{ac} = \omega^2 r$ $f^t_{ac} = \alpha r$	Parallel to AB $\perp^r$ to AB	→ A –
3.	BC	$f^s_{bc} = \alpha r$ $f^{cc}_{bc} = 2v\omega =$	Parallel to AB $\perp^r$ to AC	– –
4.	DR	$f^c_{bd} = \omega^2 r = 20$ $f^t_{bd} = \alpha\ r$	Parallel to DR $\perp^r$ to BD	→ D –
5.	Slider R	$f^t_{bd} = \alpha r$	Parallel to slider motion	–

$V_b = \omega_{OB} \times OB$

$$\omega_{OB} = \frac{2\pi N_{O1B}}{60} = \frac{2\pi \times 40}{60} = 4.18\,\text{rad/sec}$$

$V_b = 4.18 \times 0.3 = 1.254$ m/sec





Thank you for your attention!

Questions, please!