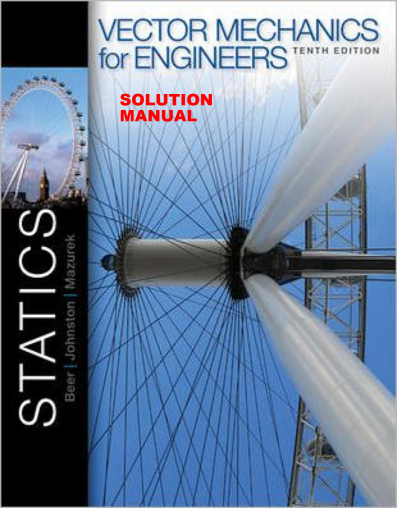
VECTOR MECHANICS for ENGINEERS

TENTH EDITION

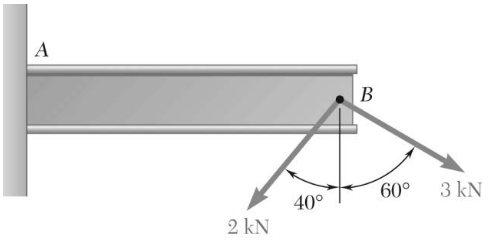
SOLUTION MANUAL



STATICS

Beer | Johnston | Mazurek

CHAPTER 2

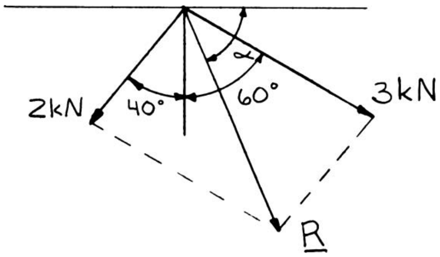


PROBLEM 2.1

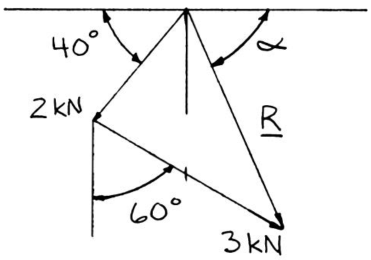
Two forces are applied at point *B* of beam *AB*. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

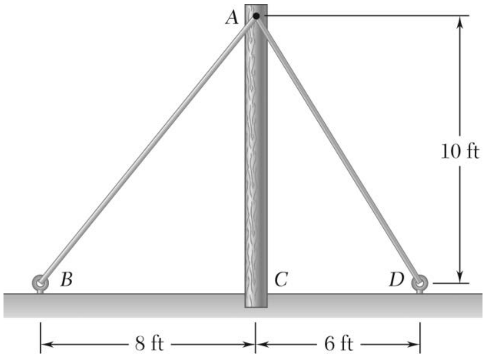
(*a*) Parallelogram law:



(*b*) Triangle rule:



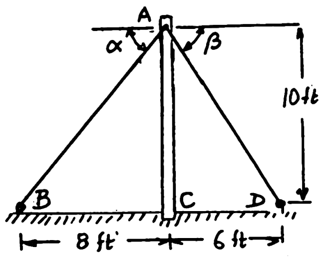
We measure: $R = 3.30$ kN, $\alpha = 66.6°$ **R** $= 3.30$ kN ↘ 66.6° ◀



PROBLEM 2.2

The cable stays *AB* and *AD* help support pole *AC*. Knowing that the tension is 120 lb in *AB* and 40 lb in *AD*, determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at *A* using (*a*) the parallelogram law, (*b*) the triangle rule.

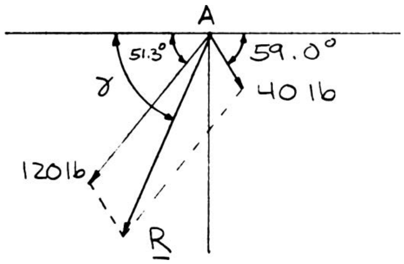
SOLUTION



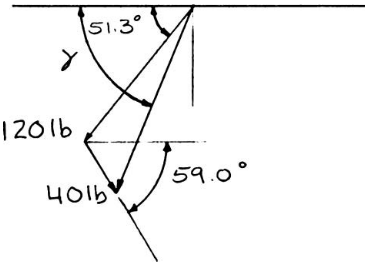
We measure: $\alpha = 51.3°$

$\beta = 59.0°$

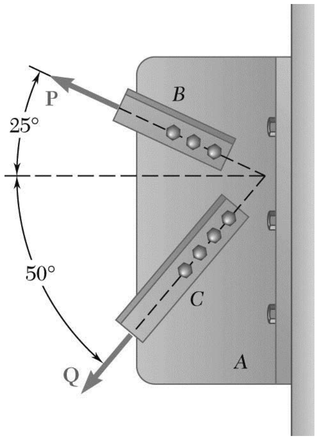
(*a*) Parallelogram law:



(*b*) Triangle rule:



We measure: $R = 139.1$ lb, $\gamma = 67.0°$ $R = 139.1$ lb ↙ 67.0° ◀

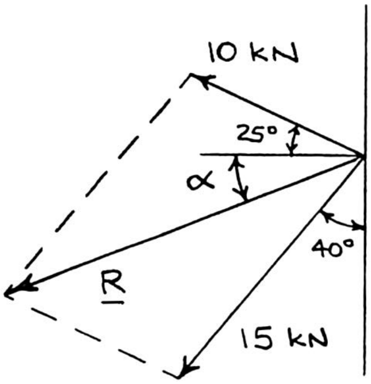


PROBLEM 2.3

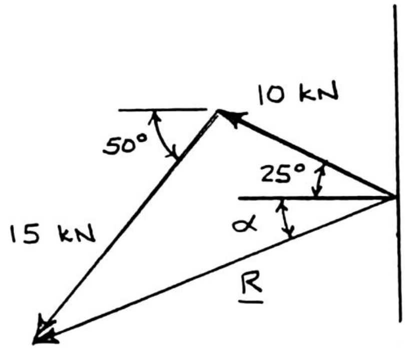
Two structural members *B* and *C* are bolted to bracket *A*. Knowing that both members are in tension and that $P = 10$ kN and $Q = 15$ kN, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

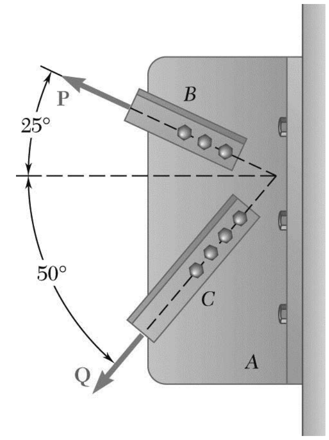
(*a*) Parallelogram law:



(*b*) Triangle rule:



We measure: $R = 20.1$ kN, $\alpha = 21.2°$ $\mathbf{R} = 20.1$ kN ∡ 21.2° ◀

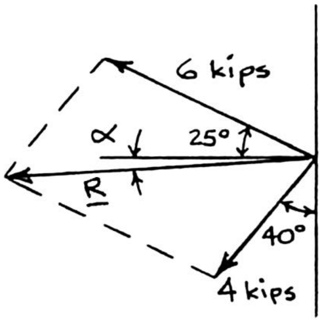


PROBLEM 2.4

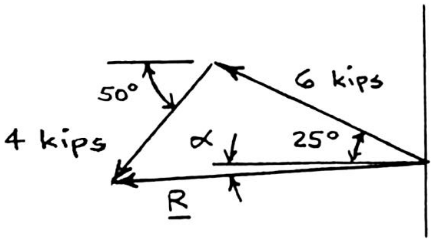
Two structural members B and C are bolted to bracket A. Knowing that both members are in tension and that $P = 6$ kips and $Q = 4$ kips, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

(a) Parallelogram law:

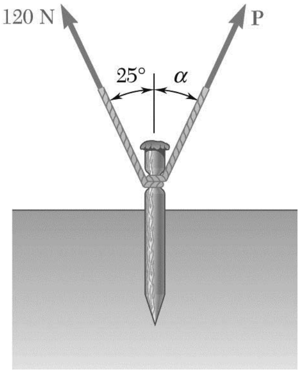


(b) Triangle rule:



We measure: $R = 8.03$ kips, $\alpha = 3.8°$

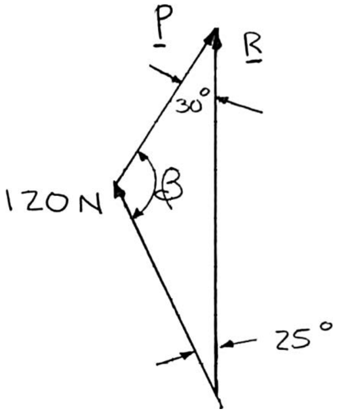
$\mathbf{R} = 8.03$ kips ↙ 3.8° ◀



PROBLEM 2.5

A stake is being pulled out of the ground by means of two ropes as shown. Knowing that $\alpha = 30°$, determine by trigonometry (*a*) the magnitude of the force **P** so that the resultant force exerted on the stake is vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION

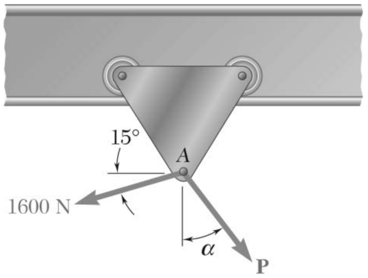


Using the triangle rule and the law of sines:

(*a*) $$\frac{120\text{ N}}{\sin 30°} = \frac{P}{\sin 25°}$$ $P = 101.4\text{ N}$ ◀

(*b*) $$30° + \beta + 25° = 180°$$
$$\beta = 180° - 25° - 30°$$
$$= 125°$$

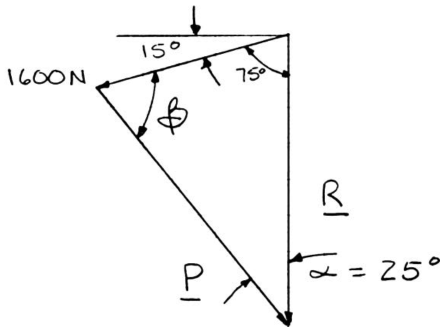
$$\frac{120\text{ N}}{\sin 30°} = \frac{R}{\sin 125°}$$ $R = 196.6\text{ N}$ ◀



PROBLEM 2.6

A trolley that moves along a horizontal beam is acted upon by two forces as shown. (*a*) Knowing that $\alpha = 25°$, determine by trigonometry the magnitude of the force **P** so that the resultant force exerted on the trolley is vertical. (*b*) What is the corresponding magnitude of the resultant?

SOLUTION



Using the triangle rule and the law of sines:

(*a*)

$$\frac{1600\text{ N}}{\sin 25°} = \frac{P}{\sin 75°}$$

$P = 3660\text{ N}$ ◀

(*b*)

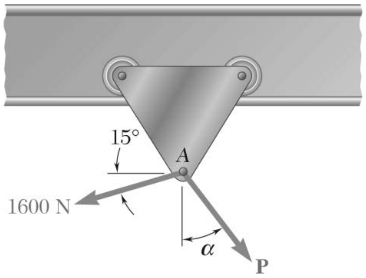
$$25° + \beta + 75° = 180°$$

$$\beta = 180° - 25° - 75°$$

$$= 80°$$

$$\frac{1600\text{ N}}{\sin 25°} = \frac{R}{\sin 80°}$$

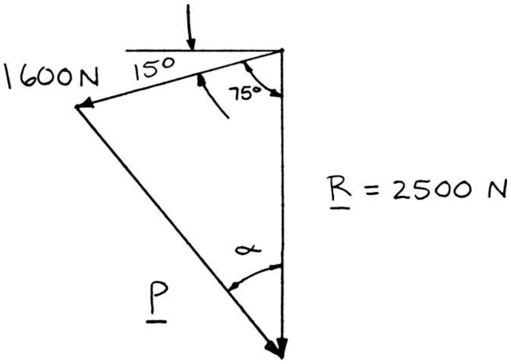
$R = 3730\text{ N}$ ◀



PROBLEM 2.7

A trolley that moves along a horizontal beam is acted upon by two forces as shown. Determine by trigonometry the magnitude and direction of the force **P** so that the resultant is a vertical force of 2500 N.

SOLUTION



Using the law of cosines: $P^2 = (1600\text{ N})^2 + (2500\text{ N})^2 - 2(1600\text{ N})(2500\text{ N})\cos 75°$

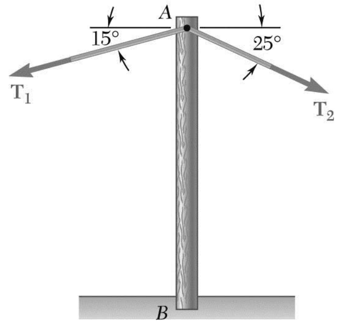
$P = 2596\text{ N}$

Using the law of sines: $\dfrac{\sin\alpha}{1600\text{ N}} = \dfrac{\sin 75°}{2596\text{ N}}$

$\alpha = 36.5°$

P is directed $90° - 36.5°$ or $53.5°$ below the horizontal. **P** = 2600 N ⦪ 53.5° ◀

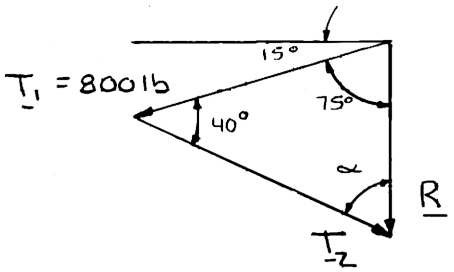
PROPRIETARY MATERIAL.



PROBLEM 2.8

A telephone cable is clamped at *A* to the pole *AB*. Knowing that the tension in the left-hand portion of the cable is $T_1 = 800$ lb, determine by trigonometry (*a*) the required tension T_2 in the right-hand portion if the resultant **R** of the forces exerted by the cable at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

(*a*)

$$75° + 40° + \alpha = 180°$$

$$\alpha = 180° - 75° - 40°$$

$$= 65°$$

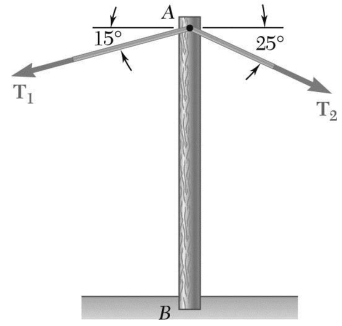
$$\frac{800\text{ lb}}{\sin 65°} = \frac{T_2}{\sin 75°}$$

$T_2 = 853$ lb ◀

(*b*)

$$\frac{800\text{ lb}}{\sin 65°} = \frac{R}{\sin 40°}$$

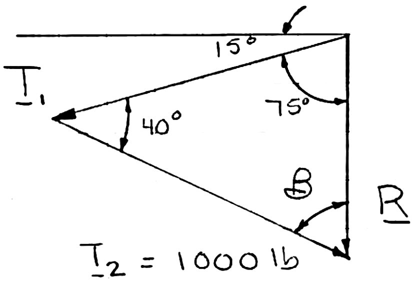
$R = 567$ lb ◀



PROBLEM 2.9

A telephone cable is clamped at *A* to the pole *AB*. Knowing that the tension in the right-hand portion of the cable is $T_2 = 1000$ lb, determine by trigonometry (*a*) the required tension T_1 in the left-hand portion if the resultant **R** of the forces exerted by the cable at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

(*a*)

$$75° + 40° + \beta = 180°$$

$$\beta = 180° - 75° - 40°$$

$$= 65°$$

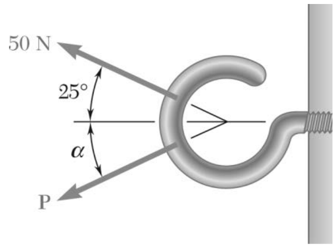
$$\frac{1000 \text{ lb}}{\sin 75°} = \frac{T_1}{\sin 65°}$$

$T_1 = 938$ lb ◀

(*b*)

$$\frac{1000 \text{ lb}}{\sin 75°} = \frac{R}{\sin 40°}$$

$R = 665$ lb ◀

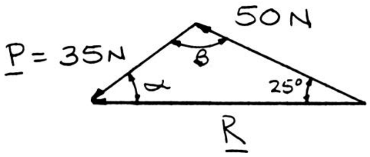


PROBLEM 2.10

Two forces are applied as shown to a hook support. Knowing that the magnitude of **P** is 35 N, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

SOLUTION

Using the triangle rule and law of sines:



(*a*)

$$\frac{\sin\alpha}{50\text{ N}} = \frac{\sin 25^\circ}{35\text{ N}}$$

$$\sin\alpha = 0.60374$$

$$\alpha = 37.138^\circ$$

$\alpha = 37.1^\circ$ ◀

(*b*)

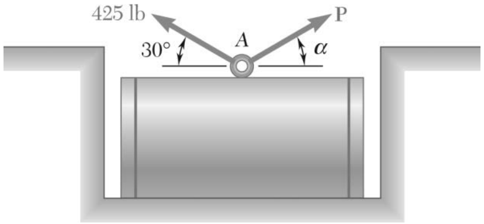
$$\alpha + \beta + 25^\circ = 180^\circ$$

$$\beta = 180^\circ - 25^\circ - 37.138^\circ$$

$$= 117.862^\circ$$

$$\frac{R}{\sin 117.862^\circ} = \frac{35\text{ N}}{\sin 25^\circ}$$

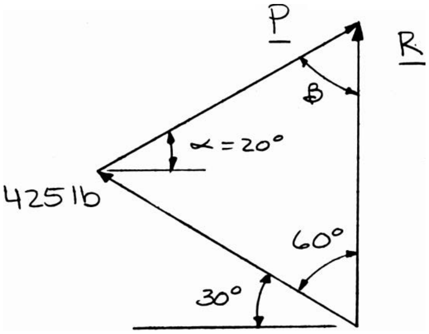
$R = 73.2\text{ N}$ ◀



PROBLEM 2.11

A steel tank is to be positioned in an excavation. Knowing that $\alpha = 20°$, determine by trigonometry (*a*) the required magnitude of the force **P** if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

(*a*)

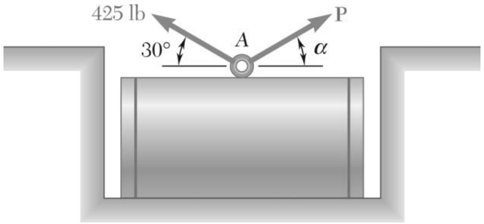
$$\beta + 50° + 60° = 180°$$

$$\beta = 180° - 50° - 60° = 70°$$

$$\frac{425\text{ lb}}{\sin 70°} = \frac{P}{\sin 60°} \qquad P = 392\text{ lb} \blacktriangleleft$$

(*b*)

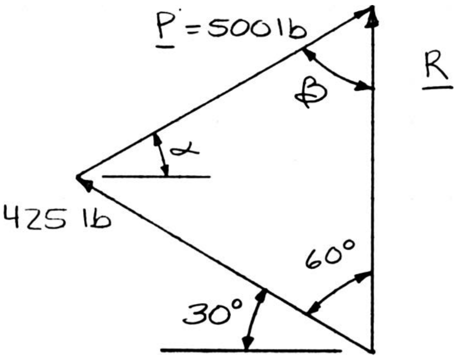
$$\frac{425\text{ lb}}{\sin 70°} = \frac{R}{\sin 50°} \qquad R = 346\text{ lb} \blacktriangleleft$$



PROBLEM 2.12

A steel tank is to be positioned in an excavation. Knowing that the magnitude of **P** is 500 lb, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

(*a*) $$(\alpha + 30°) + 60° + \beta = 180°$$

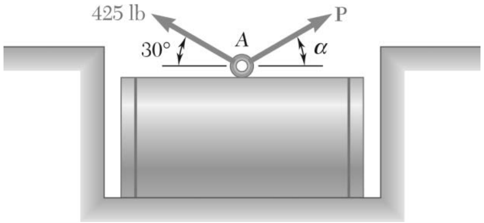
$$\beta = 180° - (\alpha + 30°) - 60°$$

$$\beta = 90° - \alpha$$

$$\frac{\sin(90° - \alpha)}{425 \text{ lb}} = \frac{\sin 60°}{500 \text{ lb}}$$

$$90° - \alpha = 47.402° \qquad \alpha = 42.6° \blacktriangleleft$$

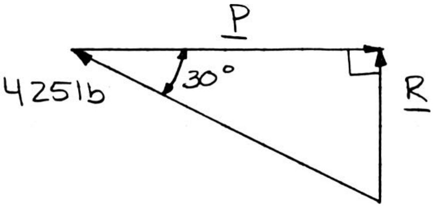
(*b*) $$\frac{R}{\sin(42.598° + 30°)} = \frac{500 \text{ lb}}{\sin 60°} \qquad R = 551 \text{ lb} \blacktriangleleft$$



PROBLEM 2.13

A steel tank is to be positioned in an excavation. Determine by trigonometry (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied at *A* is vertical, (*b*) the corresponding magnitude of **R**.

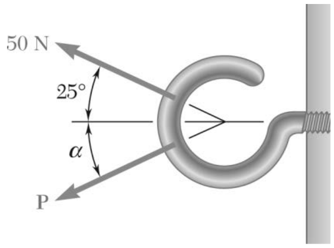
SOLUTION



The smallest force *P* will be perpendicular to *R*.

(*a*) $P = (425\text{ lb})\cos 30°$ $\qquad$ $\mathbf{P} = 368\text{ lb} \longrightarrow$ ◀

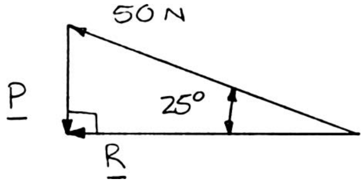
(*b*) $R = (425\text{ lb})\sin 30°$ $\qquad$ $R = 213\text{ lb}$ ◀



PROBLEM 2.14

For the hook support of Prob. 2.10, determine by trigonometry (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied to the support is horizontal, (*b*) the corresponding magnitude of **R**.

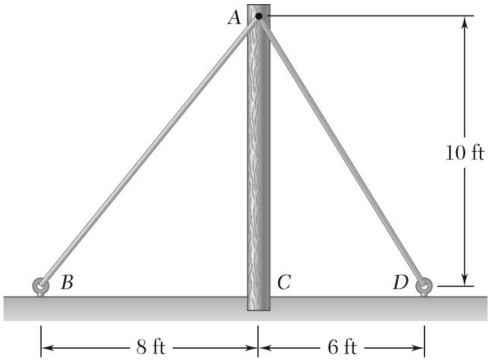
SOLUTION



The smallest force *P* will be perpendicular to *R*.

(*a*) $P = (50\text{ N})\sin 25°$ $\mathbf{P} = 21.1\text{ N}\downarrow$ ◀

(*b*) $R = (50\text{ N})\cos 25°$ $R = 45.3\text{ N}$ ◀

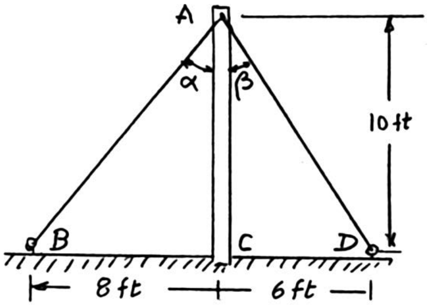


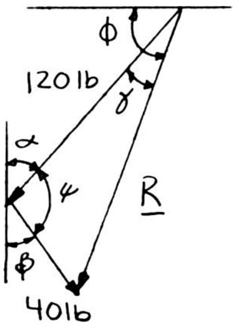
PROBLEM 2.15

Solve Problem 2.2 by trigonometry.

PROBLEM 2.2 The cable stays *AB* and *AD* help support pole *AC*. Knowing that the tension is 120 lb in *AB* and 40 lb in *AD*, determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at *A* using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION





$$\tan\alpha = \frac{8}{10}$$

$$\alpha = 38.66^\circ$$

$$\tan\beta = \frac{6}{10}$$

$$\beta = 30.96^\circ$$

Using the triangle rule:

$$\alpha + \beta + \psi = 180^\circ$$

$$38.66^\circ + 30.96^\circ + \psi = 180^\circ$$

$$\psi = 110.38^\circ$$

Using the law of cosines:

$$R^2 = (120\text{ lb})^2 + (40\text{ lb})^2 - 2(120\text{ lb})(40\text{ lb})\cos 110.38^\circ$$

$$R = 139.08\text{ lb}$$

Using the law of sines:

$$\frac{\sin\gamma}{40\text{ lb}} = \frac{\sin 110.38^\circ}{139.08\text{ lb}}$$

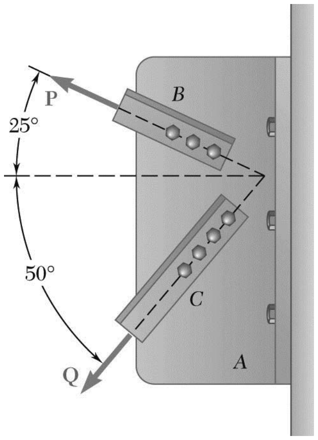
$$\gamma = 15.64^\circ$$

$$\phi = (90^\circ - \alpha) + \gamma$$

$$\phi = (90^\circ - 38.66^\circ) + 15.64^\circ$$

$$\phi = 66.98^\circ$$

R = 139.1 lb ↙ 67.0° ◀



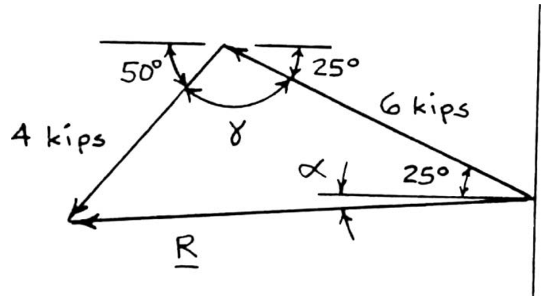
PROBLEM 2.16

Solve Problem 2.4 by trigonometry.

PROBLEM 2.4 Two structural members *B* and *C* are bolted to bracket *A*. Knowing that both members are in tension and that $P = 6$ kips and $Q = 4$ kips, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

Using the force triangle and the laws of cosines and sines:



We have:

$$\gamma = 180° - (50° + 25°)$$
$$= 105°$$

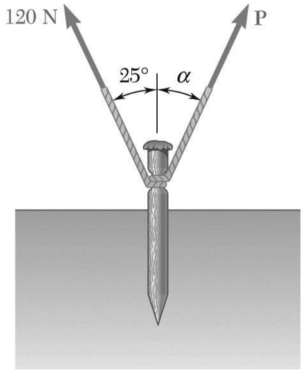
Then

$$R^2 = (4\text{ kips})^2 + (6\text{ kips})^2 - 2(4\text{ kips})(6\text{ kips})\cos 105°$$
$$= 64.423\text{ kips}^2$$
$$R = 8.0264\text{ kips}$$

And

$$\frac{4\text{ kips}}{\sin(25° + \alpha)} = \frac{8.0264\text{ kips}}{\sin 105°}$$
$$\sin(25° + \alpha) = 0.48137$$
$$25° + \alpha = 28.775°$$
$$\alpha = 3.775°$$

$\mathbf{R} = 8.03\text{ kips}$ ⦪ 3.8° ◀

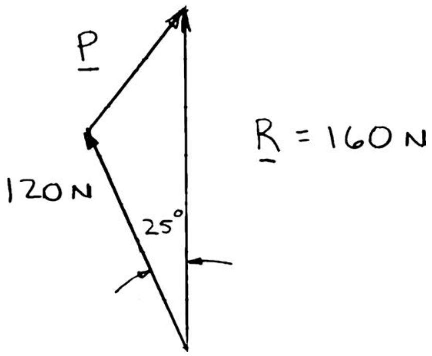


PROBLEM 2.17

For the stake of Prob. 2.5, knowing that the tension in one rope is 120 N, determine by trigonometry the magnitude and direction of the force **P** so that the resultant is a vertical force of 160 N.

PROBLEM 2.5 A stake is being pulled out of the ground by means of two ropes as shown. Knowing that $\alpha = 30°$, determine by trigonometry (*a*) the magnitude of the force **P** so that the resultant force exerted on the stake is vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION



Using the laws of cosines and sines:

$$P^2 = (120\text{ N})^2 + (160\text{ N})^2 - 2(120\text{ N})(160\text{ N})\cos 25°$$

$$P = 72.096\text{ N}$$

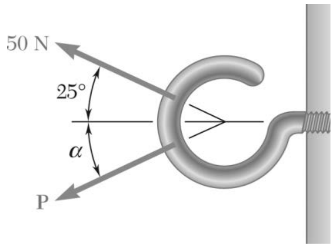
And

$$\frac{\sin\alpha}{120\text{ N}} = \frac{\sin 25°}{72.096\text{ N}}$$

$$\sin\alpha = 0.70343$$

$$\alpha = 44.703°$$

P = 72.1 N ⦨ 44.7° ◀



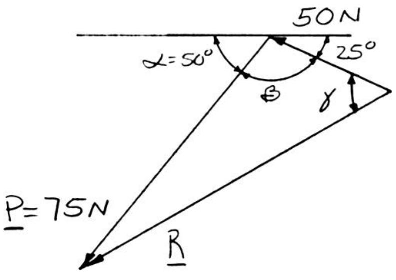
PROBLEM 2.18

For the hook support of Prob. 2.10, knowing that P = 75 N and α = 50°, determine by trigonometry the magnitude and direction of the resultant of the two forces applied to the support.

PROBLEM 2.10 Two forces are applied as shown to a hook support. Knowing that the magnitude of **P** is 35 N, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

SOLUTION

Using the force triangle and the laws of cosines and sines:



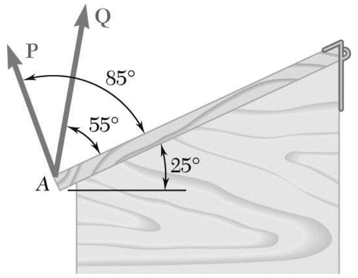
We have $\beta = 180° - (50° + 25°)$

$$= 105°$$

Then
$$R^2 = (75\text{ N})^2 + (50\text{ N})^2$$
$$-2(75\text{ N})(50\text{ N})\cos 105°$$
$$R^2 = 10{,}066.1\text{ N}^2$$
$$R = 100.330\text{ N}$$

and
$$\frac{\sin\gamma}{75\text{ N}} = \frac{\sin 105°}{100.330\text{ N}}$$
$$\sin\gamma = 0.72206$$
$$\gamma = 46.225°$$

Hence: $\gamma - 25° = 46.225° - 25° = 21.225°$ **R** = 100.3 N ↙ 21.2° ◀

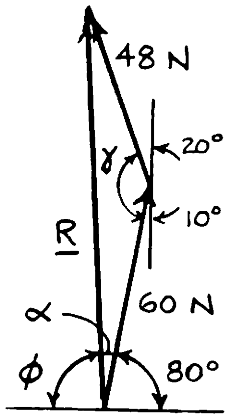


PROBLEM 2.19

Two forces **P** and **Q** are applied to the lid of a storage bin as shown. Knowing that P = 48 N and Q = 60 N, determine by trigonometry the magnitude and direction of the resultant of the two forces.

SOLUTION

Using the force triangle and the laws of cosines and sines:



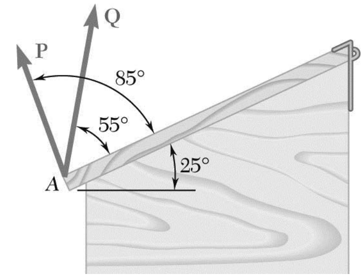
We have
$$\gamma = 180° - (20° + 10°)$$
$$= 150°$$

Then
$$R^2 = (48\text{ N})^2 + (60\text{ N})^2$$
$$-2(48\text{ N})(60\text{ N})\cos 150°$$
$$R = 104.366\text{ N}$$

and
$$\frac{48\text{ N}}{\sin\alpha} = \frac{104.366\text{ N}}{\sin 150°}$$
$$\sin\alpha = 0.22996$$
$$\alpha = 13.2947°$$

Hence:
$$\phi = 180° - \alpha - 80°$$
$$= 180° - 13.2947° - 80°$$
$$= 86.705°$$

$\mathbf{R} = 104.4\text{ N}$ ∡ 86.7° ◀

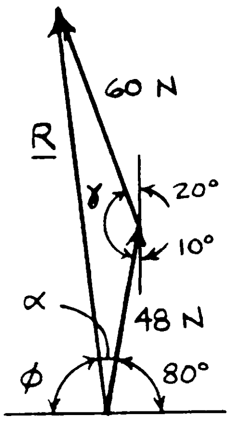


PROBLEM 2.20

Two forces **P** and **Q** are applied to the lid of a storage bin as shown. Knowing that P = 60 N and Q = 48 N, determine by trigonometry the magnitude and direction of the resultant of the two forces.

SOLUTION

Using the force triangle and the laws of cosines and sines:



We have

$$\gamma = 180° - (20° + 10°)$$
$$= 150°$$

Then

$$R^2 = (60\text{ N})^2 + (48\text{ N})^2$$
$$-2(60\text{ N})(48\text{ N})\cos 150°$$
$$R = 104.366\text{ N}$$

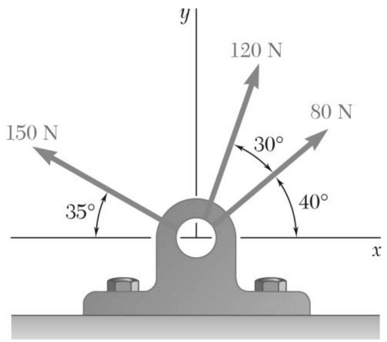
and

$$\frac{60\text{ N}}{\sin\alpha} = \frac{104.366\text{ N}}{\sin 150°}$$
$$\sin\alpha = 0.28745$$
$$\alpha = 16.7054°$$

Hence:

$$\phi = 180° - \alpha - 180°$$
$$= 180° - 16.7054° - 80°$$
$$= 83.295°$$

$\mathbf{R} = 104.4\text{ N} \measuredangle 83.3°$ ◀



PROBLEM 2.21

Determine the *x* and *y* components of each of the forces shown.

SOLUTION

80-N Force: $F_x = +(80\text{ N})\cos 40^\circ$ $F_x = 61.3\text{ N}$ ◀

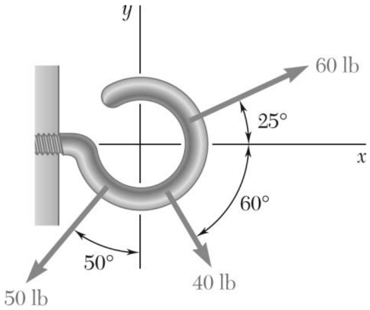
$F_y = +(80\text{ N})\sin 40^\circ$ $F_y = 51.4\text{ N}$ ◀

120-N Force: $F_x = +(120\text{ N})\cos 70^\circ$ $F_x = 41.0\text{ N}$ ◀

$F_y = +(120\text{ N})\sin 70^\circ$ $F_y = 112.8\text{ N}$ ◀

150-N Force: $F_x = -(150\text{ N})\cos 35^\circ$ $F_x = -122.9\text{ N}$ ◀

$F_y = +(150\text{ N})\sin 35^\circ$ $F_y = 86.0\text{ N}$ ◀



PROBLEM 2.22

Determine the x and y components of each of the forces shown.

SOLUTION

40-lb Force: $F_x = +(40\text{ lb})\cos 60°$ $\quad F_x = 20.0\text{ lb}$ ◀

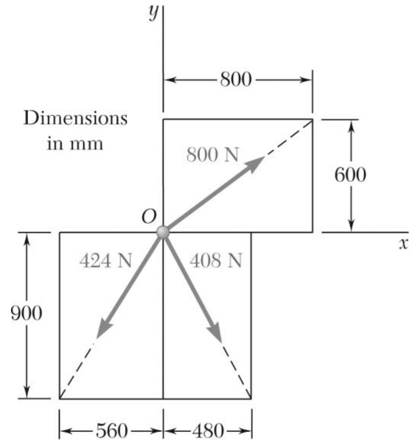
$F_y = -(40\text{ lb})\sin 60°$ $\quad F_y = -34.6\text{ lb}$ ◀

50-lb Force: $F_x = -(50\text{ lb})\sin 50°$ $\quad F_x = -38.3\text{ lb}$ ◀

$F_y = -(50\text{ lb})\cos 50°$ $\quad F_y = -32.1\text{ lb}$ ◀

60-lb Force: $F_x = +(60\text{ lb})\cos 25°$ $\quad F_x = 54.4\text{ lb}$ ◀

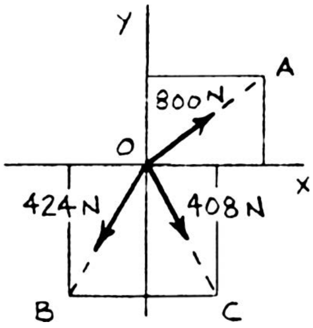
$F_y = +(60\text{ lb})\sin 25°$ $\quad F_y = 25.4\text{ lb}$ ◀



PROBLEM 2.23

Determine the *x* and *y* components of each of the forces shown.

SOLUTION



Compute the following distances:

$$OA = \sqrt{(600)^2 + (800)^2}$$
$$= 1000 \text{ mm}$$
$$OB = \sqrt{(560)^2 + (900)^2}$$
$$= 1060 \text{ mm}$$
$$OC = \sqrt{(480)^2 + (900)^2}$$
$$= 1020 \text{ mm}$$

800-N Force: $F_x = +(800 \text{ N})\dfrac{800}{1000}$ $F_x = +640$ N ◀

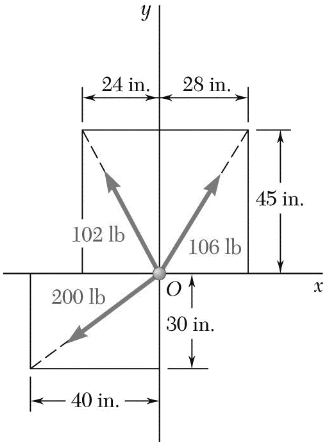
$F_y = +(800 \text{ N})\dfrac{600}{1000}$ $F_y = +480$ N ◀

424-N Force: $F_x = -(424 \text{ N})\dfrac{560}{1060}$ $F_x = -224$ N ◀

$F_y = -(424 \text{ N})\dfrac{900}{1060}$ $F_y = -360$ N ◀

408-N Force: $F_x = +(408 \text{ N})\dfrac{480}{1020}$ $F_x = +192.0$ N ◀

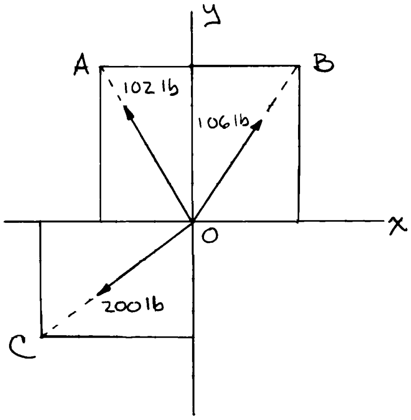
$F_y = -(408 \text{ N})\dfrac{900}{1020}$ $F_y = -360$ N ◀



PROBLEM 2.24

Determine the *x* and *y* components of each of the forces shown.

SOLUTION



Compute the following distances:

$$OA = \sqrt{(24\text{ in.})^2 + (45\text{ in.})^2}$$
$$= 51.0\text{ in.}$$
$$OB = \sqrt{(28\text{ in.})^2 + (45\text{ in.})^2}$$
$$= 53.0\text{ in.}$$
$$OC = \sqrt{(40\text{ in.})^2 + (30\text{ in.})^2}$$
$$= 50.0\text{ in.}$$

102-lb Force: $F_x = -102\text{ lb}\dfrac{24\text{ in.}}{51.0\text{ in.}}$ $F_x = -48.0\text{ lb}$ ◀

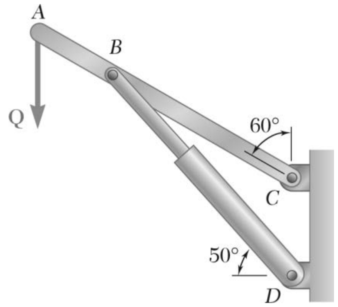
$F_y = +102\text{ lb}\dfrac{45\text{ in.}}{51.0\text{ in.}}$ $F_y = +90.0\text{ lb}$ ◀

106-lb Force: $F_x = +106\text{ lb}\dfrac{28\text{ in.}}{53.0\text{ in.}}$ $F_x = +56.0\text{ lb}$ ◀

$F_y = +106\text{ lb}\dfrac{45\text{ in.}}{53.0\text{ in.}}$ $F_y = +90.0\text{ lb}$ ◀

200-lb Force: $F_x = -200\text{ lb}\dfrac{40\text{ in.}}{50.0\text{ in.}}$ $F_x = -160.0\text{ lb}$ ◀

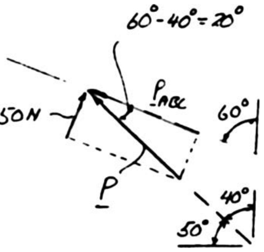
$F_y = -200\text{ lb}\dfrac{30\text{ in.}}{50.0\text{ in.}}$ $F_y = -120.0\text{ lb}$ ◀



PROBLEM 2.25

The hydraulic cylinder *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 750-N component perpendicular to member *ABC*, determine (*a*) the magnitude of the force **P**, (*b*) its component parallel to *ABC*.

SOLUTION

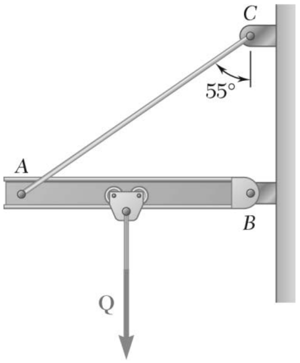


(*a*) $$750 \text{ N} = P \sin 20°$$

$$P = 2192.9 \text{ N}$$ $P = 2190$ N ◀

(*b*) $$P_{ABC} = P \cos 20°$$

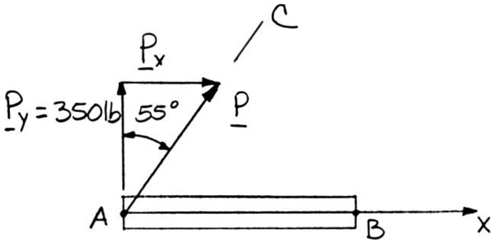
$$= (2192.9 \text{ N}) \cos 20°$$ $P_{ABC} = 2060$ N ◀



PROBLEM 2.26

Cable *AC* exerts on beam *AB* a force **P** directed along line *AC*. Knowing that **P** must have a 350-lb vertical component, determine (*a*) the magnitude of the force **P**, (*b*) its horizontal component.

SOLUTION



(*a*)

$$P = \frac{P_y}{\cos 55°}$$

$$= \frac{350 \text{ lb}}{\cos 55°}$$

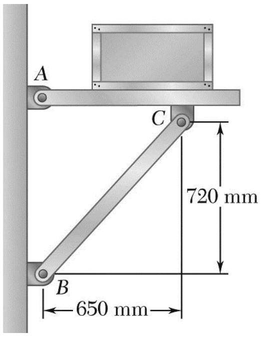
$$= 610.21 \text{ lb} \qquad P = 610 \text{ lb} \blacktriangleleft$$

(*b*)

$$P_x = P \sin 55°$$

$$= (610.21 \text{ lb}) \sin 55°$$

$$= 499.85 \text{ lb} \qquad P_x = 500 \text{ lb} \blacktriangleleft$$



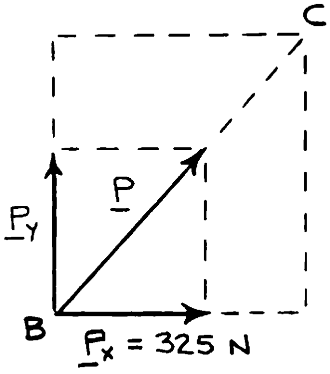
PROBLEM 2.27

Member *BC* exerts on member *AC* a force **P** directed along line *BC*. Knowing that **P** must have a 325-N horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

SOLUTION

$$BC = \sqrt{(650\text{ mm})^2 + (720\text{ mm})^2}$$

$$= 970\text{ mm}$$



(*a*)

$$P_x = P\left(\frac{650}{970}\right)$$

or

$$P = P_x\left(\frac{970}{650}\right)$$

$$= 325\text{ N}\left(\frac{970}{650}\right)$$

$$= 485\text{ N}$$

$P = 485$ N ◀

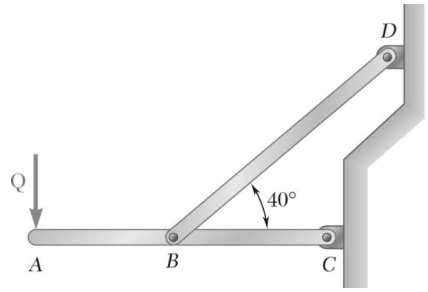
(*b*)

$$P_y = P\left(\frac{720}{970}\right)$$

$$= 485\text{ N}\left(\frac{720}{970}\right)$$

$$= 360\text{ N}$$

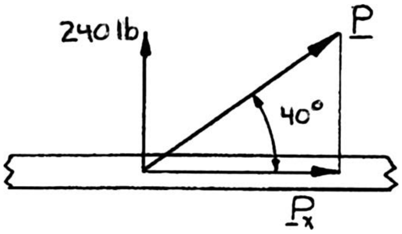
$P_y = 970$ N ◀



PROBLEM 2.28

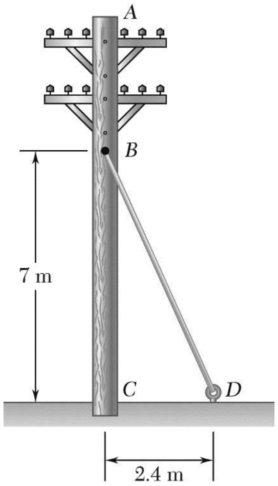
Member *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 240-lb vertical component, determine (*a*) the magnitude of the force **P**, (*b*) its horizontal component.

SOLUTION



(*a*) $$P = \frac{P_y}{\sin 40^\circ} = \frac{240\text{ lb}}{\sin 40^\circ}$$ or $P = 373$ lb ◀

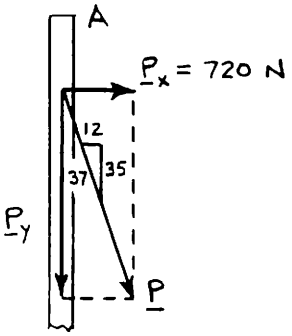
(*b*) $$P_x = \frac{P_y}{\tan 40^\circ} = \frac{240\text{ lb}}{\tan 40^\circ}$$ or $P_x = 286$ lb ◀



PROBLEM 2.29

The guy wire *BD* exerts on the telephone pole *AC* a force **P** directed along *BD*. Knowing that **P** must have a 720-N component perpendicular to the pole *AC*, determine (*a*) the magnitude of the force **P**, (*b*) its component along line *AC*.

SOLUTION



(*a*)

$$P = \frac{37}{12}P_x$$

$$= \frac{37}{12}(720\text{ N})$$

$$= 2220\text{ N}$$

$P = 2.22$ kN ◀

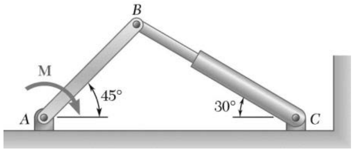
(*b*)

$$P_y = \frac{35}{12}P_x$$

$$= \frac{35}{12}(720\text{ N})$$

$$= 2100\text{ N}$$

$P_y = 2.10$ kN ◀



PROBLEM 2.30

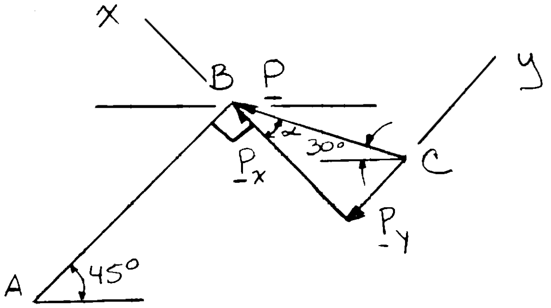
The hydraulic cylinder *BC* exerts on member *AB* a force **P** directed along line *BC*. Knowing that **P** must have a 600-N component perpendicular to member *AB*, determine (*a*) the magnitude of the force **P**, (*b*) its component along line *AB*.

SOLUTION

$$180° = 45° + \alpha + 90° + 30°$$

$$\alpha = 180° - 45° - 90° - 30°$$

$$= 15°$$



(*a*)

$$\cos\alpha = \frac{P_x}{P}$$

$$P = \frac{P_x}{\cos\alpha}$$

$$= \frac{600\text{ N}}{\cos 15°}$$

$$= 621.17\text{ N}$$

$P = 621\text{ N}$ ◀

(*b*)

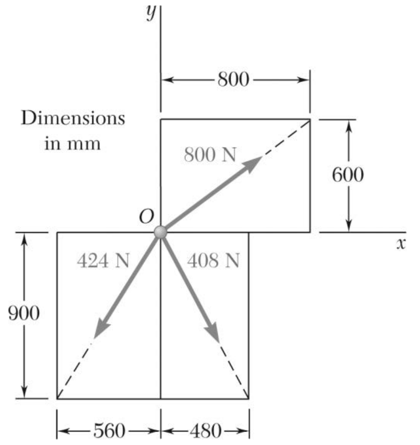
$$\tan\alpha = \frac{P_y}{P_x}$$

$$P_y = P_x \tan\alpha$$

$$= (600\text{ N})\tan 15°$$

$$= 160.770\text{ N}$$

$P_y = 160.8\text{ N}$ ◀



PROBLEM 2.31

Determine the resultant of the three forces of Problem 2.23.

PROBLEM 2.23 Determine the *x* and *y* components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.23:

Force	*x* Comp. (N)	*y* Comp. (N)
800 lb	+640	+480
424 lb	–224	–360
408 lb	+192	–360
	$R_x = +608$	$R_y = -240$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$

$$= (608\text{ lb})\mathbf{i} + (-240\text{ lb})\mathbf{j}$$

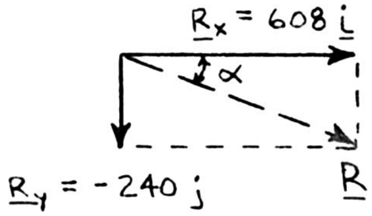
$$\tan\alpha = \frac{R_y}{R_x}$$

$$= \frac{240}{608}$$

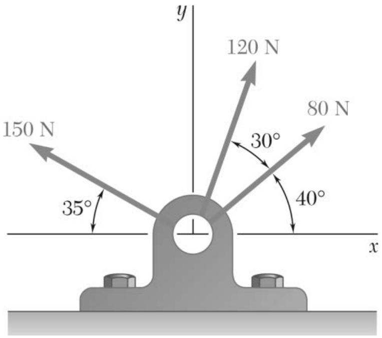
$$\alpha = 21.541°$$

$$R = \frac{240\text{ N}}{\sin(21.541°)}$$

$$= 653.65\text{ N}$$



R = 654 N ↘ 21.5° ◀



PROBLEM 2.32

Determine the resultant of the three forces of Problem 2.21.

PROBLEM 2.21 Determine the x and y components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.21:

Force	x Comp. (N)	y Comp. (N)
80 N	+61.3	+51.4
120 N	+41.0	+112.8
150 N	–122.9	+86.0
	$R_x = -20.6$	$R_y = +250.2$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$

$$= (-20.6\text{ N})\mathbf{i} + (250.2\text{ N})\mathbf{j}$$

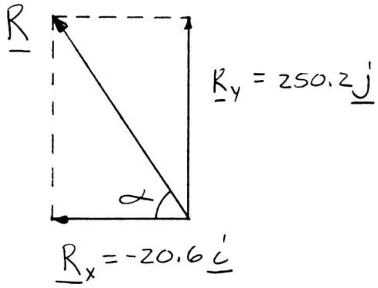
$$\tan\alpha = \frac{R_y}{R_x}$$

$$\tan\alpha = \frac{250.2\text{ N}}{20.6\text{ N}}$$

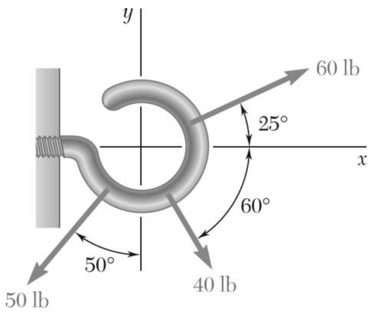
$$\tan\alpha = 12.1456$$

$$\alpha = 85.293^\circ$$

$$R = \frac{250.2\text{ N}}{\sin 85.293^\circ}$$



R = 251 N ∠ 85.3° ◀



PROBLEM 2.33

Determine the resultant of the three forces of Problem 2.22.

PROBLEM 2.22 Determine the *x* and *y* components of each of the forces shown.

SOLUTION

Force	x Comp. (lb)	y Comp. (lb)
40 lb	+20.00	–34.64
50 lb	–38.30	–32.14
60 lb	+54.38	+25.36
	$R_x = +36.08$	$R_y = -41.42$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$

$$= (+36.08 \text{ lb})\mathbf{i} + (-41.42 \text{ lb})\mathbf{j}$$

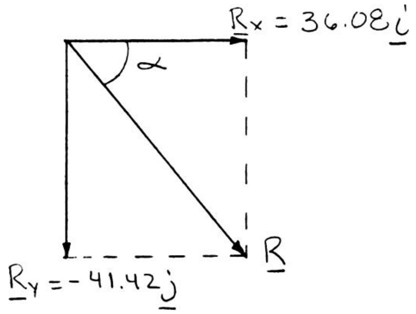
$$\tan\alpha = \frac{R_y}{R_x}$$

$$\tan\alpha = \frac{41.42 \text{ lb}}{36.08 \text{ lb}}$$

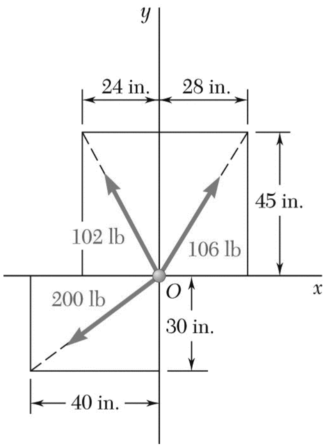
$$\tan\alpha = 1.14800$$

$$\alpha = 48.942°$$

$$R = \frac{41.42 \text{ lb}}{\sin 48.942°}$$



R = 54.9 lb ⦩ 48.9° ◀



PROBLEM 2.34

Determine the resultant of the three forces of Problem 2.24.

PROBLEM 2.24 Determine the x and y components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.24:

Force	x Comp. (lb)	y Comp. (lb)
102 lb	–48.0	+90.0
106 lb	+56.0	+90.0
200 lb	–160.0	–120.0
	$R_x = -152.0$	$R_y = 60.0$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$

$$= (-152 \text{ lb})\mathbf{i} + (60.0 \text{ lb})\mathbf{j}$$

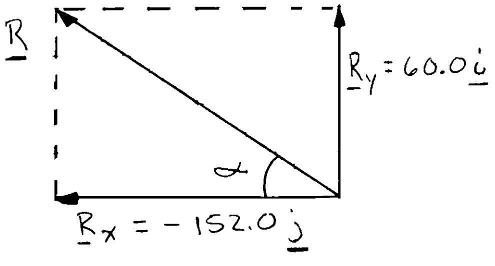
$$\tan\alpha = \frac{R_y}{R_x}$$

$$\tan\alpha = \frac{60.0 \text{ lb}}{152.0 \text{ lb}}$$

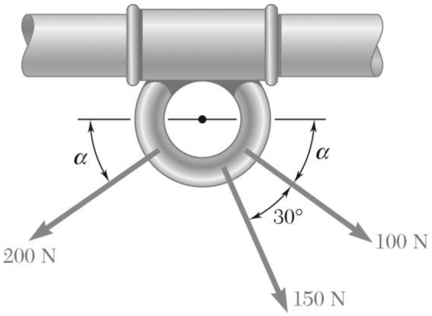
$$\tan\alpha = 0.39474$$

$$\alpha = 21.541°$$

$$R = \frac{60.0 \text{ lb}}{\sin 21.541°}$$



$\mathbf{R} = 163.4$ lb ⦪ 21.5° ◀



PROBLEM 2.35

Knowing that $\alpha = 35°$, determine the resultant of the three forces shown.

SOLUTION

100-N Force: $F_x = +(100\text{ N})\cos 35° = +81.915\text{ N}$

$F_y = -(100\text{ N})\sin 35° = -57.358\text{ N}$

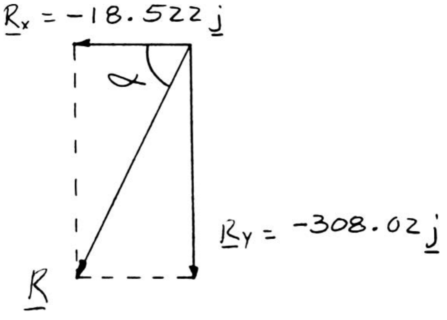
150-N Force: $F_x = +(150\text{ N})\cos 65° = +63.393\text{ N}$

$F_y = -(150\text{ N})\sin 65° = -135.946\text{ N}$

200-N Force: $F_x = -(200\text{ N})\cos 35° = -163.830\text{ N}$

$F_y = -(200\text{ N})\sin 35° = -114.715\text{ N}$

Force	x Comp. (N)	y Comp. (N)
100 N	+81.915	−57.358
150 N	+63.393	−135.946
200 N	−163.830	−114.715
	$R_x = -18.522$	$R_y = -308.02$



$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$

$$= (-18.522\text{ N})\mathbf{i} + (-308.02\text{ N})\mathbf{j}$$

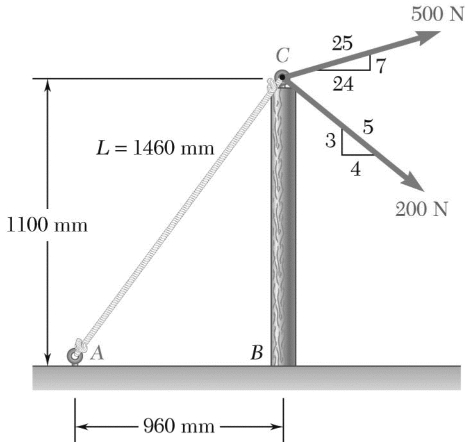
$$\tan\alpha = \frac{R_y}{R_x}$$

$$= \frac{308.02}{18.522}$$

$$\alpha = 86.559°$$

$$R = \frac{308.02\text{ N}}{\sin 86.559}$$

$\mathbf{R} = 309\text{ N}$ ↙ 86.6° ◀

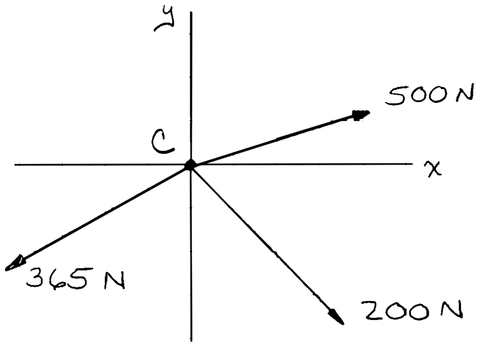


PROBLEM 2.36

Knowing that the tension in rope AC is 365 N, determine the resultant of the three forces exerted at point C of post BC.

SOLUTION

Determine force components:



Cable force AC:

$$F_x = -(365\text{ N})\frac{960}{1460} = -240\text{ N}$$

$$F_y = -(365\text{ N})\frac{1100}{1460} = -275\text{ N}$$

500-N Force:

$$F_x = (500\text{ N})\frac{24}{25} = 480\text{ N}$$

$$F_y = (500\text{ N})\frac{7}{25} = 140\text{ N}$$

200-N Force:

$$F_x = (200\text{ N})\frac{4}{5} = 160\text{ N}$$

$$F_y = -(200\text{ N})\frac{3}{5} = -120\text{ N}$$

and

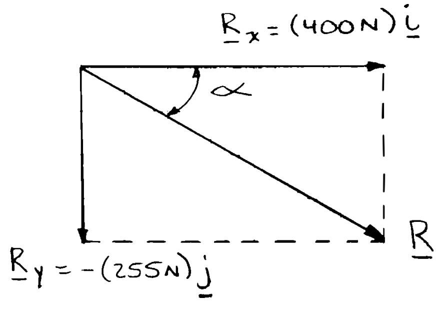
$$R_x = \Sigma F_x = -240\text{ N} + 480\text{ N} + 160\text{ N} = 400\text{ N}$$

$$R_y = \Sigma F_y = -275\text{ N} + 140\text{ N} - 120\text{ N} = -255\text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(400\text{ N})^2 + (-255\text{ N})^2}$$

$$= 474.37\text{ N}$$

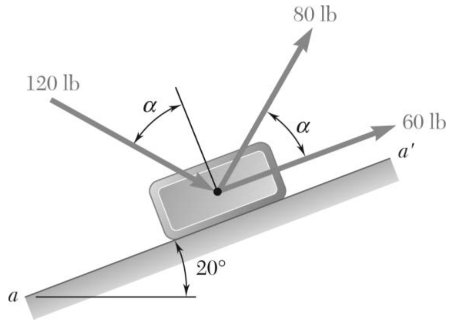


Further:

$$\tan\alpha = \frac{255}{400}$$

$$\alpha = 32.5^\circ$$

$\mathbf{R} = 474$ N ⦨ 32.5° ◀



PROBLEM 2.37

Knowing that $\alpha = 40°$, determine the resultant of the three forces shown.

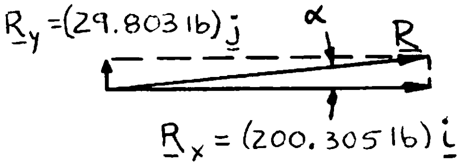
SOLUTION

60-lb Force:

$$F_x = (60\text{ lb})\cos 20° = 56.382\text{ lb}$$
$$F_y = (60\text{ lb})\sin 20° = 20.521\text{ lb}$$

80-lb Force:

$$F_x = (80\text{ lb})\cos 60° = 40.000\text{ lb}$$
$$F_y = (80\text{ lb})\sin 60° = 69.282\text{ lb}$$



120-lb Force:

$$F_x = (120\text{ lb})\cos 30° = 103.923\text{ lb}$$
$$F_y = -(120\text{ lb})\sin 30° = -60.000\text{ lb}$$

and

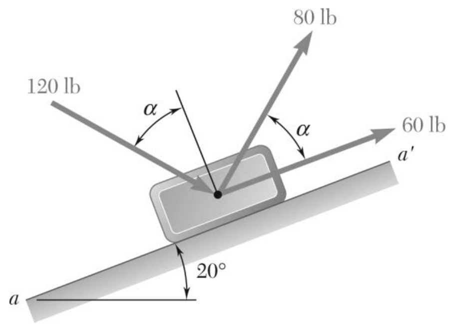
$$R_x = \Sigma F_x = 200.305\text{ lb}$$
$$R_y = \Sigma F_y = 29.803\text{ lb}$$
$$R = \sqrt{(200.305\text{ lb})^2 + (29.803\text{ lb})^2}$$
$$= 202.510\text{ lb}$$

Further:

$$\tan\alpha = \frac{29.803}{200.305}$$

$$\alpha = \tan^{-1}\frac{29.803}{200.305}$$
$$= 8.46°$$

R = 203 lb $\measuredangle$ 8.46° ◀



PROBLEM 2.38

Knowing that $\alpha = 75°$, determine the resultant of the three forces shown.

SOLUTION

60-lb Force:

$$F_x = (60\text{ lb})\cos 20° = 56.382\text{ lb}$$
$$F_y = (60\text{ lb})\sin 20° = 20.521\text{ lb}$$

80-lb Force:

$$F_x = (80\text{ lb})\cos 95° = -6.9725\text{ lb}$$
$$F_y = (80\text{ lb})\sin 95° = 79.696\text{ lb}$$

120-lb Force:

$$F_x = (120\text{ lb})\cos 5° = 119.543\text{ lb}$$
$$F_y = (120\text{ lb})\sin 5° = 10.459\text{ lb}$$

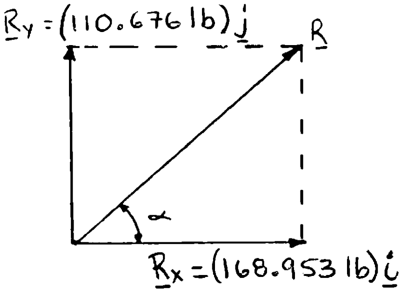
Then

$$R_x = \Sigma F_x = 168.953\text{ lb}$$
$$R_y = \Sigma F_y = 110.676\text{ lb}$$

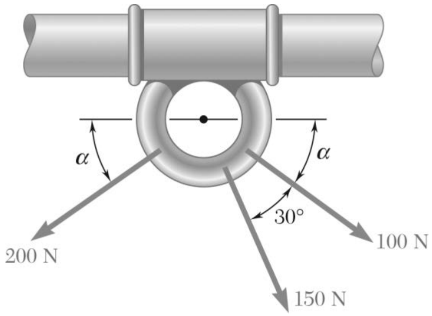
and

$$R = \sqrt{(168.953\text{ lb})^2 + (110.676\text{ lb})^2}$$
$$= 201.976\text{ lb}$$

$$\tan\alpha = \frac{110.676}{168.953}$$
$$\tan\alpha = 0.65507$$
$$\alpha = 33.228°$$



R = 202 lb ∠ 33.2° ◀



PROBLEM 2.39

For the collar of Problem 2.35, determine (*a*) the required value of α if the resultant of the three forces shown is to be vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION

$$\begin{aligned} R_x &= \Sigma F_x \\ &= (100\text{ N})\cos\alpha + (150\text{ N})\cos(\alpha+30^\circ) - (200\text{ N})\cos\alpha \\ R_x &= -(100\text{ N})\cos\alpha + (150\text{ N})\cos(\alpha+30^\circ) \qquad (1) \end{aligned}$$

$$\begin{aligned} R_y &= \Sigma F_y \\ &= -(100\text{ N})\sin\alpha - (150\text{ N})\sin(\alpha+30^\circ) - (200\text{ N})\sin\alpha \\ R_y &= -(300\text{ N})\sin\alpha - (150\text{ N})\sin(\alpha+30^\circ) \qquad (2) \end{aligned}$$

(*a*) For **R** to be vertical, we must have $R_x = 0.$ We make $R_x = 0$ in Eq. (1):

$$-100\cos\alpha + 150\cos(\alpha+30^\circ) = 0$$

$$-100\cos\alpha + 150(\cos\alpha\cos 30^\circ - \sin\alpha\sin 30^\circ) = 0$$

$$29.904\cos\alpha = 75\sin\alpha$$

$$\begin{aligned} \tan\alpha &= \frac{29.904}{75} \\ &= 0.39872 \\ \alpha &= 21.738^\circ \end{aligned}$$

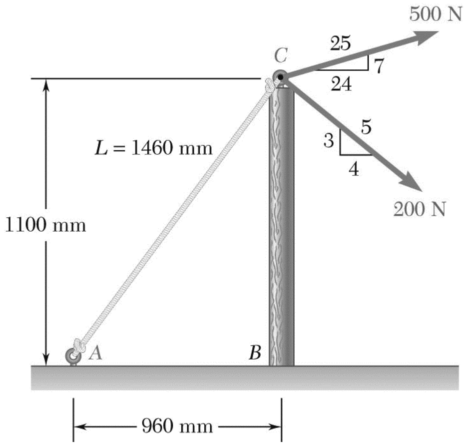
$\alpha = 21.7^\circ$ ◀

(*b*) Substituting for α in Eq. (2):

$$\begin{aligned} R_y &= -300\sin 21.738^\circ - 150\sin 51.738^\circ \\ &= -228.89\text{ N} \end{aligned}$$

$$R = |R_y| = 228.89\text{ N}$$

$R = 229$ N ◀



PROBLEM 2.40

For the post of Prob. 2.36, determine (*a*) the required tension in rope *AC* if the resultant of the three forces exerted at point *C* is to be horizontal, (*b*) the corresponding magnitude of the resultant.

SOLUTION

$$R_x = \Sigma F_x = -\frac{960}{1460}T_{AC} + \frac{24}{25}(500\text{ N}) + \frac{4}{5}(200\text{ N})$$

$$R_x = -\frac{48}{73}T_{AC} + 640\text{ N} \qquad (1)$$

$$R_y = \Sigma F_y = -\frac{1100}{1460}T_{AC} + \frac{7}{25}(500\text{ N}) - \frac{3}{5}(200\text{ N})$$

$$R_y = -\frac{55}{73}T_{AC} + 20\text{ N} \qquad (2)$$

(*a*) For **R** to be horizontal, we must have $R_y = 0$.

Set $R_y = 0$ in Eq. (2): $$-\frac{55}{73}T_{AC} + 20\text{ N} = 0$$

$$T_{AC} = 26.545\text{ N}$$

$T_{AC} = 26.5\text{ N}$ ◀

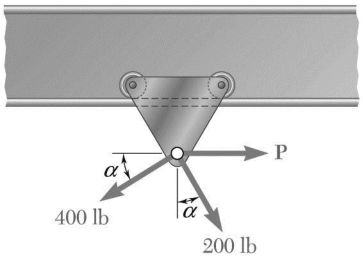
(*b*) Substituting for T_{AC} into Eq. (1) gives

$$R_x = -\frac{48}{73}(26.545\text{ N}) + 640\text{ N}$$

$$R_x = 622.55\text{ N}$$

$$R = R_x = 623\text{ N}$$

$R = 623\text{ N}$ ◀



PROBLEM 2.41

A hoist trolley is subjected to the three forces shown. Knowing that $\alpha = 40°$, determine (*a*) the required magnitude of the force **P** if the resultant of the three forces is to be vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION

$$R_x = \xrightarrow{+} \Sigma F_x = P + (200 \text{ lb}) \sin 40° - (400 \text{ lb}) \cos 40°$$

$$R_x = P - 177.860 \text{ lb} \qquad (1)$$

$$R_y = +\!\!\downarrow \Sigma F_y = (200 \text{ lb}) \cos 40° + (400 \text{ lb}) \sin 40°$$

$$R_y = 410.32 \text{ lb} \qquad (2)$$

(*a*) For **R** to be vertical, we must have $R_x = 0$.

Set $R_x = 0$ in Eq. (1)

$$0 = P - 177.860 \text{ lb}$$

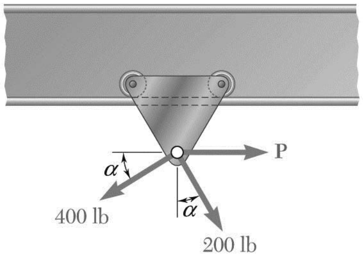
$$P = 177.860 \text{ lb}$$

$P = 177.9$ lb ◀

(*b*) Since **R** is to be vertical:

$$R = R_y = 410 \text{ lb}$$

$R = 410$ lb ◀



PROBLEM 2.42

A hoist trolley is subjected to the three forces shown. Knowing that P = 250 lb, determine (*a*) the required value of α if the resultant of the three forces is to be vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION

$$R_x = \xrightarrow{+} \Sigma F_x = 250 \text{ lb} + (200 \text{ lb})\sin\alpha - (400 \text{ lb})\cos\alpha$$

$$R_x = 250 \text{ lb} + (200 \text{ lb})\sin\alpha - (400 \text{ lb})\cos\alpha \qquad (1)$$

$$R_y = +\!\downarrow \Sigma F_y = (200 \text{ lb})\cos\alpha + (400 \text{ lb})\sin\alpha$$

(*a*) For **R** to be vertical, we must have $R_x = 0$.

Set $R_x = 0$ in Eq. (1)

$$0 = 250 \text{ lb} + (200 \text{ lb})\sin\alpha - (400 \text{ lb})\cos\alpha$$

$$(400 \text{ lb})\cos\alpha = (200 \text{ lb})\sin\alpha + 250 \text{ lb}$$

$$2\cos\alpha = \sin\alpha + 1.25$$

$$4\cos^2\alpha = \sin^2\alpha + 2.5\sin\alpha + 1.5625$$

$$4(1-\sin^2\alpha) = \sin^2\alpha + 2.5\sin\alpha + 1.5625$$

$$0 = 5\sin^2\alpha + 2.5\sin\alpha - 2.4375$$

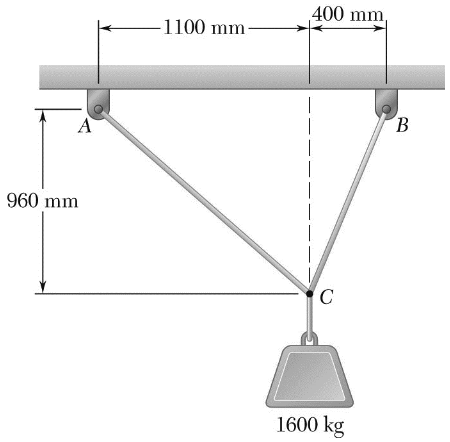
Using the quadratic formula to solve for the roots gives

$$\sin\alpha = 0.49162$$

or $\alpha = 29.447°$ $\qquad \alpha = 29.4°$ ◀

(*b*) Since **R** is to be vertical:

$R = R_y = (200 \text{ lb})\cos 29.447° + (400 \text{ lb})\sin 29.447°$ $\qquad$ **R** = 371 lb ◀

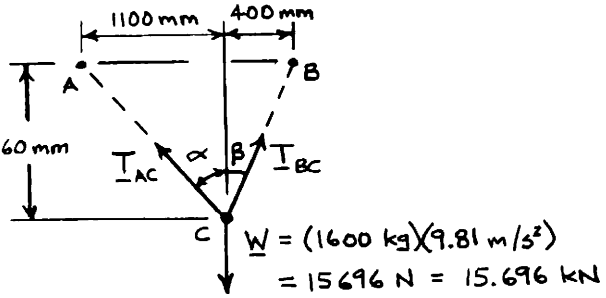


PROBLEM 2.43

Two cables are tied together at *C* and are loaded as shown. Determine the tension (*a*) in cable *AC*, (*b*) in cable *BC*.

SOLUTION

Free-Body Diagram



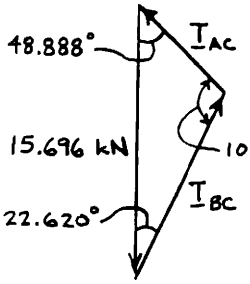
$$\tan\alpha = \frac{1100}{960}$$

$$\alpha = 48.888°$$

$$\tan\beta = \frac{400}{960}$$

$$\beta = 22.620°$$

Force Triangle



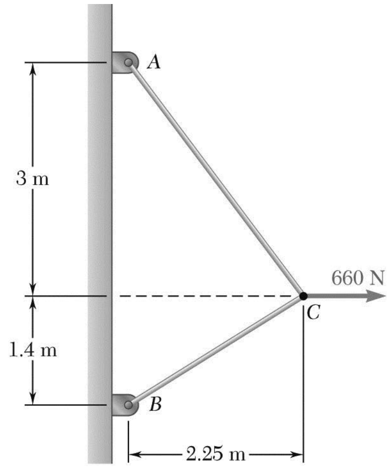
Law of sines:

$$\frac{T_{AC}}{\sin 22.620°} = \frac{T_{BC}}{\sin 48.888°} = \frac{15.696 \text{ kN}}{\sin 108.492°}$$

(*a*) $$T_{AC} = \frac{15.696 \text{ kN}}{\sin 108.492°}(\sin 22.620°)$$ $T_{AC} = 6.37$ kN ◀

(*b*) $$T_{BC} = \frac{15.696 \text{ kN}}{\sin 108.492°}(\sin 48.888°)$$ $T_{BC} = 12.47$ kN ◀

PROBLEM 2.44



Two cables are tied together at *C* and are loaded as shown. Determine the tension (*a*) in cable *AC*, (*b*) in cable *BC*.

SOLUTION

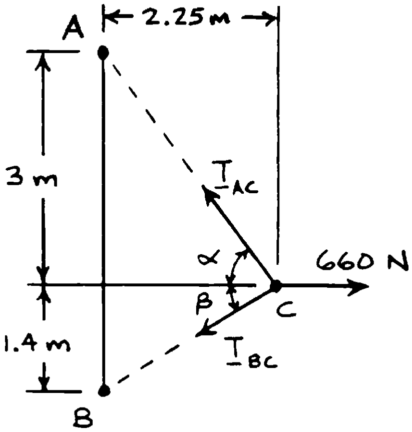
$$\tan\alpha = \frac{3}{2.25}$$

$$\alpha = 53.130°$$

$$\tan\beta = \frac{1.4}{2.25}$$

$$\beta = 31.891°$$

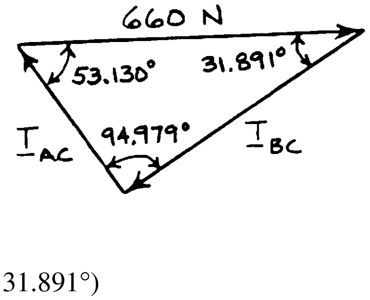
Free-Body Diagram



Law of sines:

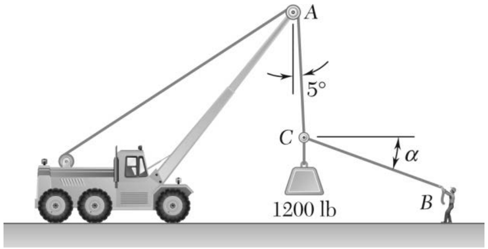
$$\frac{T_{AC}}{\sin 31.891°} = \frac{T_{BC}}{\sin 53.130°} = \frac{660\text{ N}}{\sin 94.979°}$$

Force-Triangle



(*a*) $$T_{AC} = \frac{660\text{ N}}{\sin 94.979°}(\sin 31.891°)$$ $T_{AC} = 350\text{ N}$ ◀

(*b*) $$T_{BC} = \frac{660\text{ N}}{\sin 94.979°}(\sin 53.130°)$$ $T_{BC} = 530\text{ N}$ ◀

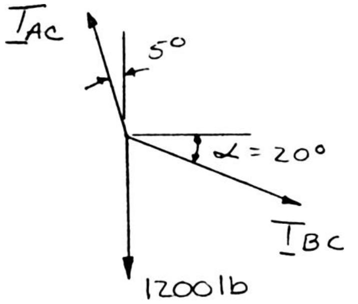


PROBLEM 2.45

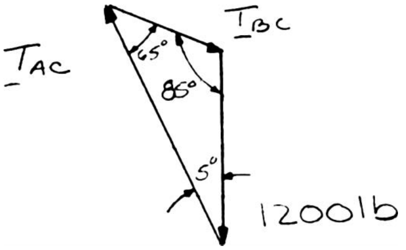
Knowing that $\alpha = 20°$, determine the tension (*a*) in cable *AC*, (*b*) in rope *BC*.

SOLUTION

Free-Body Diagram



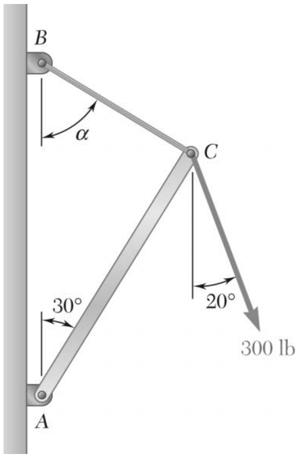
Force Triangle



Law of sines: $$\frac{T_{AC}}{\sin 110°} = \frac{T_{BC}}{\sin 5°} = \frac{1200 \text{ lb}}{\sin 65°}$$

(*a*) $$T_{AC} = \frac{1200 \text{ lb}}{\sin 65°} \sin 110°$$ $T_{AC} = 1244$ lb ◀

(*b*) $$T_{BC} = \frac{1200 \text{ lb}}{\sin 65°} \sin 5°$$ $T_{BC} = 115.4$ lb ◀

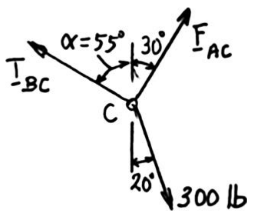


PROBLEM 2.46

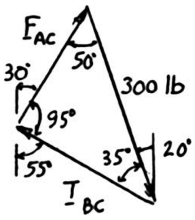
Knowing that $\alpha = 55°$ and that boom AC exerts on pin C a force directed along line AC, determine (a) the magnitude of that force, (b) the tension in cable BC.

SOLUTION

Free-Body Diagram



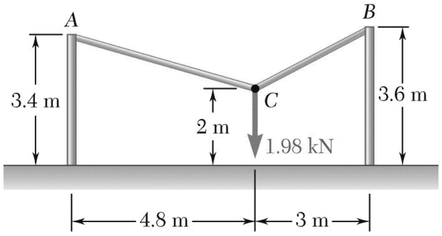
Force Triangle



Law of sines: $$\frac{F_{AC}}{\sin 35°} = \frac{T_{BC}}{\sin 50°} = \frac{300 \text{ lb}}{\sin 95°}$$

(a) $$F_{AC} = \frac{300 \text{ lb}}{\sin 95°} \sin 35°$$ $F_{AC} = 172.7$ lb ◀

(b) $$T_{BC} = \frac{300 \text{ lb}}{\sin 95°} \sin 50°$$ $T_{BC} = 231$ lb ◀



PROBLEM 2.47

Two cables are tied together at *C* and loaded as shown. Determine the tension (*a*) in cable *AC*, (*b*) in cable *BC*.

SOLUTION

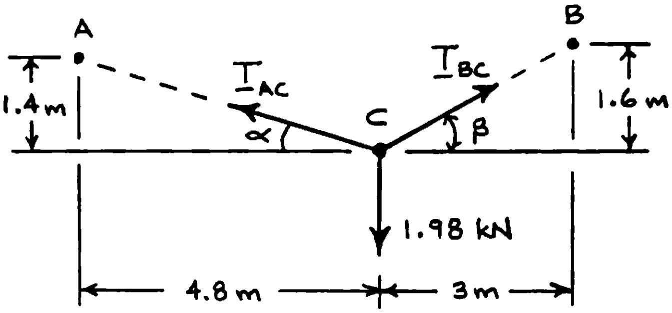
Free-Body Diagram

$$\tan\alpha = \frac{1.4}{4.8}$$

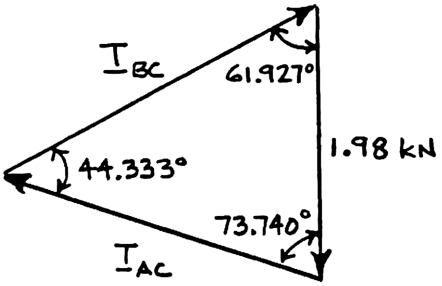
$$\alpha = 16.2602°$$

$$\tan\beta = \frac{1.6}{3}$$

$$\beta = 28.073°$$



Force Triangle

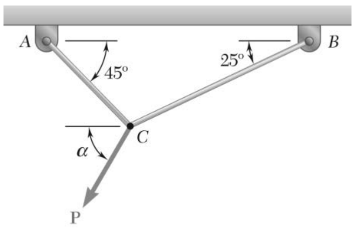


Law of sines:

$$\frac{T_{AC}}{\sin 61.927°} = \frac{T_{BC}}{\sin 73.740°} = \frac{1.98 \text{ kN}}{\sin 44.333°}$$

(*a*) $$T_{AC} = \frac{1.98 \text{ kN}}{\sin 44.333°}\sin 61.927°$$ $T_{AC} = 2.50$ kN ◀

(*b*) $$T_{BC} = \frac{1.98 \text{ kN}}{\sin 44.333°}\sin 73.740°$$ $T_{BC} = 2.72$ kN ◀

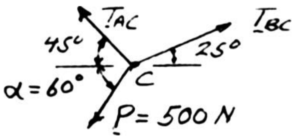


PROBLEM 2.48

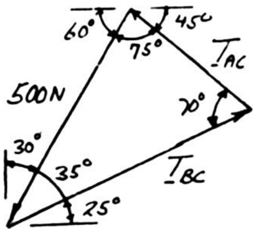
Two cables are tied together at *C* and are loaded as shown. Knowing that **P** = 500 N and $\alpha = 60°$, determine the tension in (*a*) in cable *AC*, (*b*) in cable *BC*.

SOLUTION

Free-Body Diagram



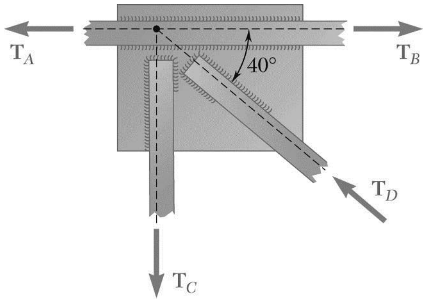
Force Triangle



Law of sines: $$\frac{T_{AC}}{\sin 35°} = \frac{T_{BC}}{\sin 75°} = \frac{500 \text{ N}}{\sin 70°}$$

(*a*) $$T_{AC} = \frac{500 \text{ N}}{\sin 70°} \sin 35°$$ $T_{AC} = 305 \text{ N}$ ◀

(*b*) $$T_{BC} = \frac{500 \text{ N}}{\sin 70°} \sin 75°$$ $T_{BC} = 514 \text{ N}$ ◀

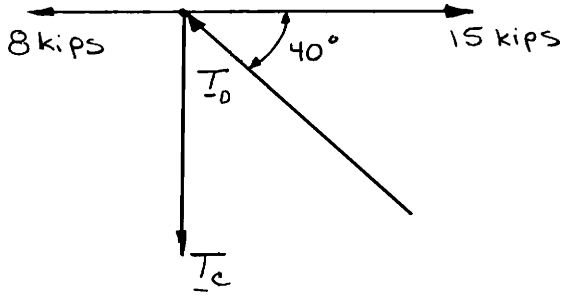


PROBLEM 2.49

Two forces of magnitude $T_A = 8$ kips and $T_B = 15$ kips are applied as shown to a welded connection. Knowing that the connection is in equilibrium, determine the magnitudes of the forces T_C and T_D.

SOLUTION

Free-Body Diagram



$\xrightarrow{+} \Sigma F_x = 0 \qquad 15 \text{ kips} - 8 \text{ kips} - T_D \cos 40° = 0$

$$T_D = 9.1379 \text{ kips}$$

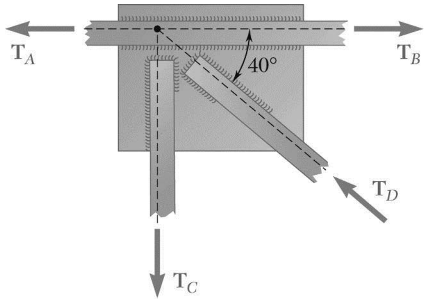
$+\uparrow \Sigma F_y = 0 \qquad T_D \sin 40° - T_C = 0$

$$(9.1379 \text{ kips}) \sin 40° - T_C = 0$$

$$T_C = 5.8737 \text{ kips}$$

$T_C = 5.87$ kips ◀

$T_D = 9.14$ kips ◀

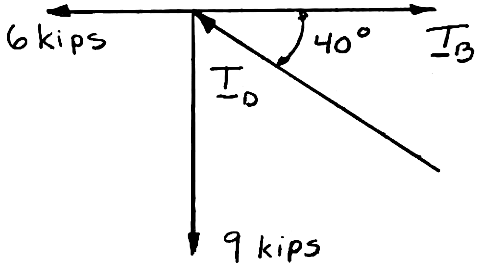


PROBLEM 2.50

Two forces of magnitude $T_A = 6$ kips and $T_C = 9$ kips are applied as shown to a welded connection. Knowing that the connection is in equilibrium, determine the magnitudes of the forces T_B and T_D.

SOLUTION

Free-Body Diagram



$\xrightarrow{+} \Sigma F_x = 0 \qquad T_B - 6\text{ kips} - T_D \cos 40° = 0 \qquad (1)$

$+\uparrow \Sigma F_y = 0 \qquad T_D \sin 40° - 9\text{ kips} = 0$

$$T_D = \frac{9\text{ kips}}{\sin 40°}$$

$$T_D = 14.0015\text{ kips}$$

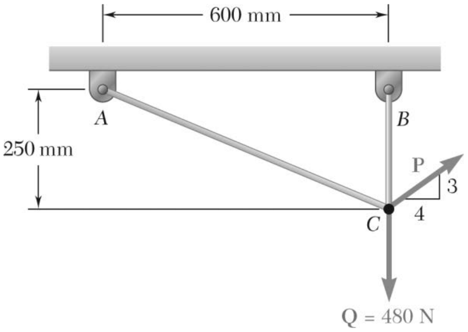
Substituting for T_D into Eq. (1) gives:

$$T_B - 6\text{ kips} - (14.0015\text{ kips})\cos 40° = 0$$

$$T_B = 16.7258\text{ kips}$$

$T_B = 16.73$ kips ◀

$T_D = 14.00$ kips ◀

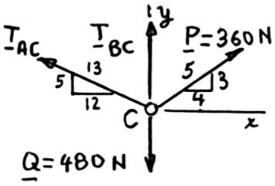


PROBLEM 2.51

Two cables are tied together at C and loaded as shown. Knowing that $P = 360$ N, determine the tension (*a*) in cable AC, (*b*) in cable BC.

SOLUTION

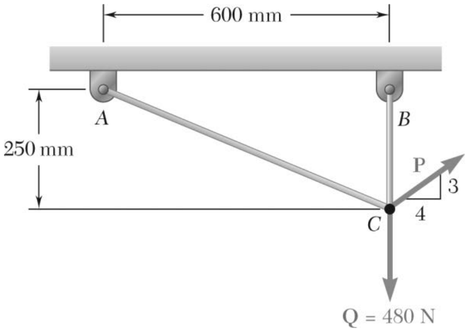
Free Body: *C*



(*a*) $\Sigma F_x = 0: \quad -\frac{12}{13}T_{AC} + \frac{4}{5}(360 \text{ N}) = 0$ $\quad T_{AC} = 312 \text{ N}$ ◀

(*b*) $\Sigma F_y = 0: \quad \frac{5}{13}(312 \text{ N}) + T_{BC} + \frac{3}{5}(360 \text{ N}) - 480 \text{ N} = 0$

$T_{BC} = 480 \text{ N} - 120 \text{ N} - 216 \text{ N}$ $\quad T_{BC} = 144 \text{ N}$ ◀

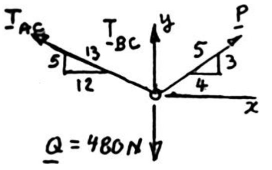


PROBLEM 2.52

Two cables are tied together at C and loaded as shown. Determine the range of values of P for which both cables remain taut.

SOLUTION

Free Body: *C*



$$\Sigma \mathbf{F}_x = 0: \quad -\frac{12}{13}T_{AC} + \frac{4}{5}\mathbf{P} = 0$$

$$T_{AC} = \frac{13}{15}P \qquad (1)$$

$$\Sigma \mathbf{F}_y = 0: \quad \frac{5}{13}T_{AC} + T_{BC} + \frac{3}{5}P - 480 \text{ N} = 0$$

Substitute for T_{AC} from (1):

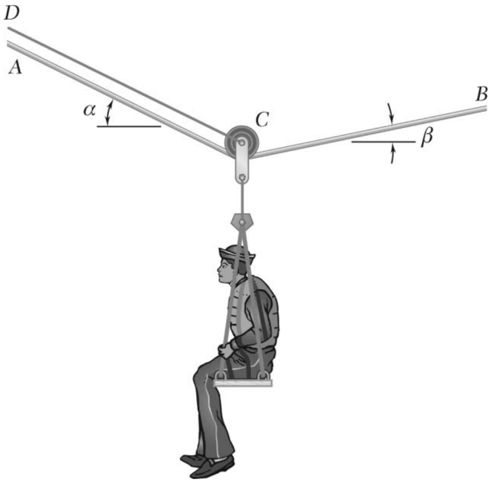
$$\left(\frac{5}{13}\right)\left(\frac{13}{15}\right)P + T_{BC} + \frac{3}{5}P - 480 \text{ N} = 0$$

$$T_{BC} = 480 \text{ N} - \frac{14}{15}P \qquad (2)$$

From (1), $T_{AC} > 0$ requires $P > 0$.

From (2), $T_{BC} > 0$ requires $\frac{14}{15}P < 480 \text{ N}, \quad P < 514.29 \text{ N}$

Allowable range: $0 < P < 514 \text{ N}$ ◀

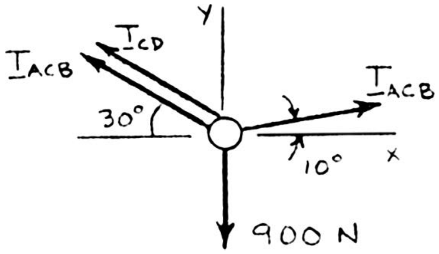


PROBLEM 2.53

A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable *ACB* and is pulled at a constant speed by cable *CD*. Knowing that $\alpha = 30°$ and $\beta = 10°$ and that the combined weight of the boatswain's chair and the sailor is 900 N, determine the tension (*a*) in the support cable *ACB*, (*b*) in the traction cable *CD*.

SOLUTION

Free-Body Diagram



$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB} \cos 10° - T_{ACB} \cos 30° - T_{CD} \cos 30° = 0$$

$$T_{CD} = 0.137158 T_{ACB} \tag{1}$$

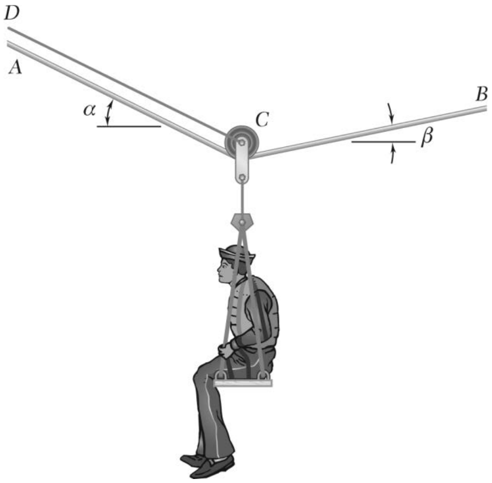
$$+\!\uparrow \Sigma F_y = 0: \quad T_{ACB} \sin 10° + T_{ACB} \sin 30° + T_{CD} \sin 30° - 900 = 0$$

$$0.67365 T_{ACB} + 0.5 T_{CD} = 900 \tag{2}$$

(*a*) Substitute (1) into (2): $0.67365 T_{ACB} + 0.5(0.137158 T_{ACB}) = 900$

$T_{ACB} = 1212.56$ N $\qquad T_{ACB} = 1213$ N ◀

(*b*) From (1): $T_{CD} = 0.137158(1212.56 \text{ N})$ $\qquad T_{CD} = 166.3$ N ◀

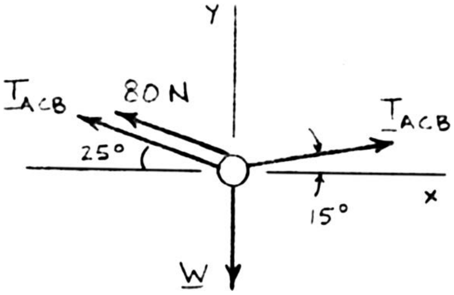


PROBLEM 2.54

A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable *ACB* and is pulled at a constant speed by cable *CD*. Knowing that $\alpha = 25°$ and $\beta = 15°$ and that the tension in cable *CD* is 80 N, determine (*a*) the combined weight of the boatswain's chair and the sailor, (*b*) in tension in the support cable *ACB*.

SOLUTION

Free-Body Diagram



$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}\cos 15° - T_{ACB}\cos 25° - (80\text{ N})\cos 25° = 0$$

$$T_{ACB} = 1216.15\text{ N}$$

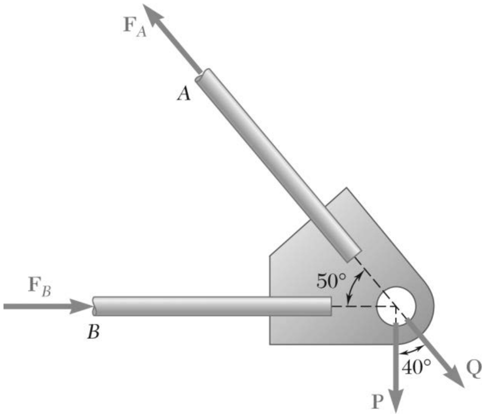
$$+\!\uparrow \Sigma F_y = 0: \quad (1216.15\text{ N})\sin 15° + (1216.15\text{ N})\sin 25°$$

$$+(80\text{ N})\sin 25° - W = 0$$

$$W = 862.54\text{ N}$$

(*a*) $W = 863\text{ N}$ ◀

(*b*) $T_{ACB} = 1216\text{ N}$ ◀

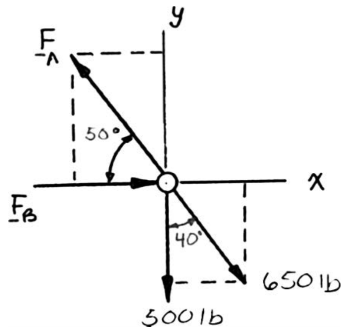


PROBLEM 2.55

Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that $P = 500$ lb and $Q = 650$ lb, determine the magnitudes of the forces exerted on the rods A and B.

SOLUTION

Free-Body Diagram



Resolving the forces into x- and y-directions:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0$$

Substituting components:

$$\begin{aligned}\mathbf{R} = &-(500\text{ lb})\mathbf{j} + [(650\text{ lb})\cos 50°]\mathbf{i}\\ &-[(650\text{ lb})\sin 50°]\mathbf{j}\\ &+F_B\mathbf{i} - (F_A\cos 50°)\mathbf{i} + (F_A\sin 50°)\mathbf{j} = 0\end{aligned}$$

In the y-direction (one unknown force):

$$-500\text{ lb} - (650\text{ lb})\sin 50° + F_A\sin 50° = 0$$

Thus,

$$F_A = \frac{500\text{ lb} + (650\text{ lb})\sin 50°}{\sin 50°}$$

$$= 1302.70\text{ lb}$$

$F_A = 1303$ lb ◀

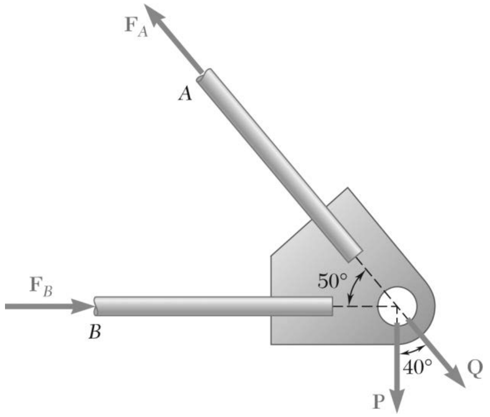
In the x-direction:

$$(650\text{ lb})\cos 50° + F_B - F_A\cos 50° = 0$$

Thus,

$$\begin{aligned}F_B &= F_A\cos 50° - (650\text{ lb})\cos 50°\\ &= (1302.70\text{ lb})\cos 50° - (650\text{ lb})\cos 50°\\ &= 419.55\text{ lb}\end{aligned}$$

$F_B = 420$ lb ◀

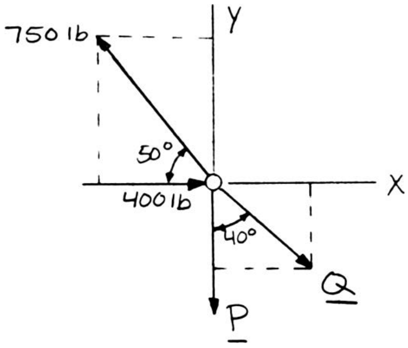


PROBLEM 2.56

Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that the magnitudes of the forces exerted on rods *A* and *B* are $F_A = 750$ lb and $F_B = 400$ lb, determine the magnitudes of **P** and **Q**.

SOLUTION

Free-Body Diagram



Resolving the forces into *x*- and *y*-directions:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0$$

Substituting components:

$$\begin{aligned}\mathbf{R} = &-P\mathbf{j} + Q\cos 50°\mathbf{i} - Q\sin 50°\mathbf{j}\\ &-[(750 \text{ lb})\cos 50°]\mathbf{i}\\ &+[(750 \text{ lb})\sin 50°]\mathbf{j} + (400 \text{ lb})\mathbf{i}\end{aligned}$$

In the *x*-direction (one unknown force):

$$Q\cos 50° - [(750 \text{ lb})\cos 50°] + 400 \text{ lb} = 0$$

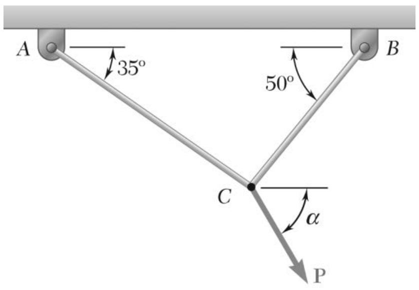
$$\begin{aligned}Q &= \frac{(750 \text{ lb})\cos 50° - 400 \text{ lb}}{\cos 50°}\\ &= 127.710 \text{ lb}\end{aligned}$$

In the *y*-direction:

$$-P - Q\sin 50° + (750 \text{ lb})\sin 50° = 0$$

$$\begin{aligned}P &= -Q\sin 50° + (750 \text{ lb})\sin 50°\\ &= -(127.710 \text{ lb})\sin 50° + (750 \text{ lb})\sin 50°\\ &= 476.70 \text{ lb}\end{aligned}$$

$P = 477$ lb; $Q = 127.7$ lb ◀

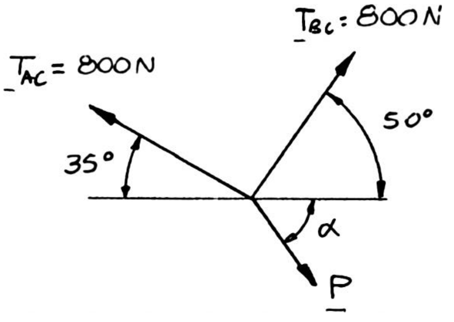


PROBLEM 2.57

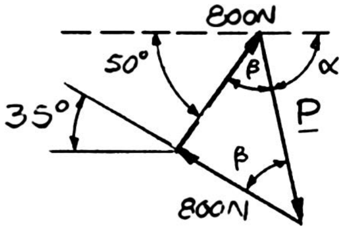
Two cables tied together at *C* are loaded as shown. Knowing that the maximum allowable tension in each cable is 800 N, determine (*a*) the magnitude of the largest force **P** that can be applied at *C*, (*b*) the corresponding value of α.

SOLUTION

Free-Body Diagram: *C*



Force Triangle



Force triangle is isosceles with

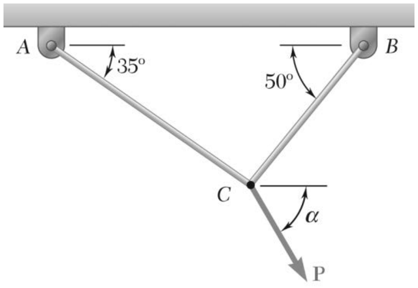
$$2\beta = 180° - 85°$$

$$\beta = 47.5°$$

(*a*) $$P = 2(800\text{ N})\cos 47.5° = 1081\text{ N}$$

Since $P > 0$, the solution is correct. $P = 1081\text{ N}$ ◀

(*b*) $$\alpha = 180° - 50° - 47.5° = 82.5°$$ $\alpha = 82.5°$ ◀

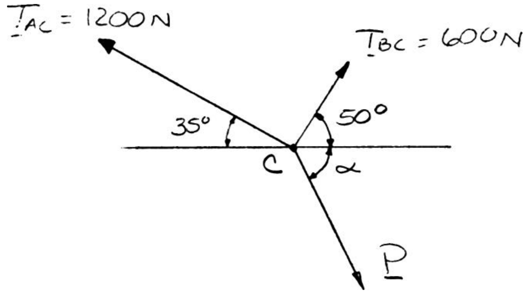


PROBLEM 2.58

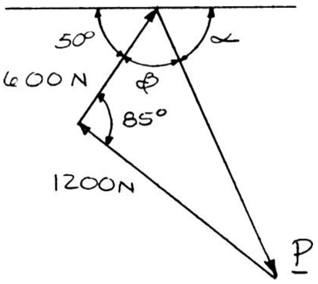
Two cables tied together at C are loaded as shown. Knowing that the maximum allowable tension is 1200 N in cable AC and 600 N in cable BC, determine (*a*) the magnitude of the largest force **P** that can be applied at C, (*b*) the corresponding value of α.

SOLUTION

Free-Body Diagram



Force Triangle



(*a*) Law of cosines:

$$P^2 = (1200\text{ N})^2 + (600\text{ N})^2 - 2(1200\text{ N})(600\text{ N})\cos 85^\circ$$

$$P = 1294.02\text{ N}$$

Since $P > 1200$ N, the solution is correct.

$P = 1294$ N ◀

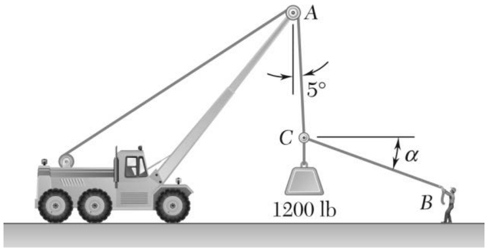
(*b*) Law of sines:

$$\frac{\sin\beta}{1200\text{ N}} = \frac{\sin 85^\circ}{1294.02\text{ N}}$$

$$\beta = 67.5^\circ$$

$$\alpha = 180^\circ - 50^\circ - 67.5^\circ$$

$\alpha = 62.5^\circ$ ◀



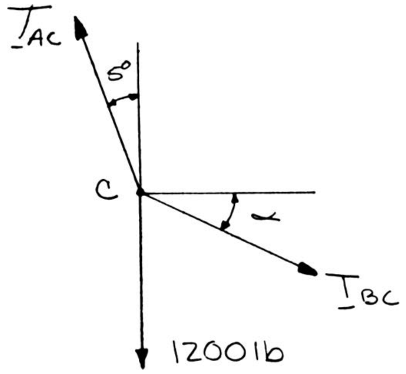
PROBLEM 2.59

For the situation described in Figure P2.45, determine (*a*) the value of α for which the tension in rope *BC* is as small as possible, (*b*) the corresponding value of the tension.

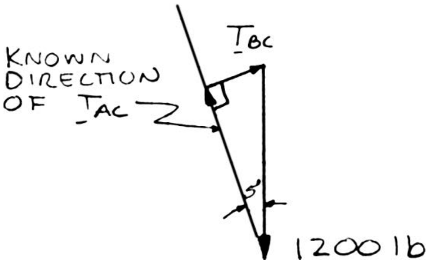
PROBLEM 2.45 Knowing that $\alpha = 20°$, determine the tension (*a*) in cable *AC*, (*b*) in rope *BC*.

SOLUTION

Free-Body Diagram



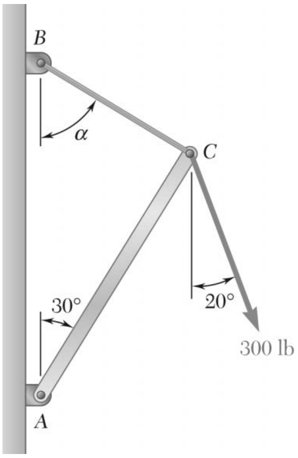
Force Triangle



To be smallest, T_{BC} must be perpendicular to the direction of T_{AC}.

(*a*) Thus, $\alpha = 5°$ $\qquad$ $\alpha = 5.00°$ ∠ ◀

(*b*) $T_{BC} = (1200 \text{ lb}) \sin 5°$ $\qquad$ $T_{BC} = 104.6 \text{ lb}$ ◀



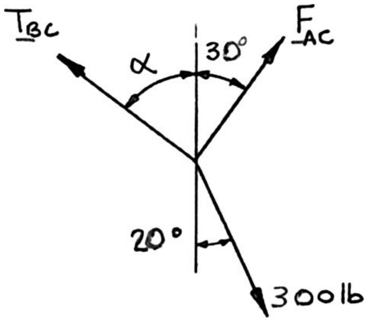
PROBLEM 2.60

For the structure and loading of Problem 2.46, determine (*a*) the value of α for which the tension in cable *BC* is as small as possible, (*b*) the corresponding value of the tension.

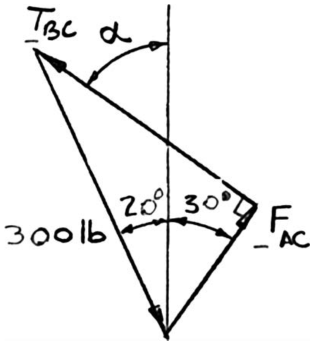
SOLUTION

T_{BC} must be perpendicular to F_{AC} to be as small as possible.

Free-Body Diagram: *C*



Force Triangle is a right triangle

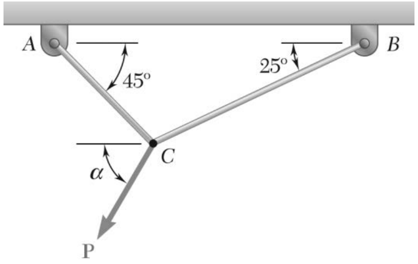


To be a minimum, T_{BC} must be perpendicular to F_{AC}.

(*a*) We observe: $\alpha = 90° - 30°$ $\qquad \alpha = 60.0°$ ◀

(*b*) $T_{BC} = (300 \text{ lb}) \sin 50°$

or $T_{BC} = 229.81 \text{ lb}$ $\qquad T_{BC} = 230 \text{ lb}$ ◀

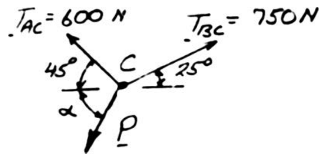


PROBLEM 2.61

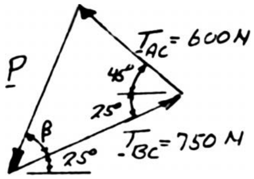
For the cables of Problem 2.48, it is known that the maximum allowable tension is 600 N in cable *AC* and 750 N in cable *BC*. Determine (*a*) the maximum force **P** that can be applied at *C*, (*b*) the corresponding value of *α*.

SOLUTION

Free-Body Diagram



Force Triangle



(*a*) Law of cosines

$$P^2 = (600)^2 + (750)^2 - 2(600)(750)\cos(25° + 45°)$$

$$P = 784.02 \text{ N}$$

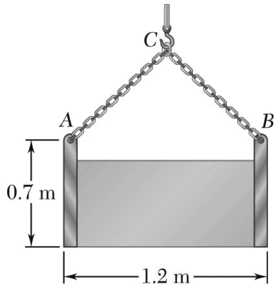
$P = 784$ N ◀

(*b*) Law of sines

$$\frac{\sin\beta}{600 \text{ N}} = \frac{\sin(25° + 45°)}{784.02 \text{ N}}$$

$$\beta = 46.0° \qquad \therefore \quad \alpha = 46.0° + 25°$$

$\alpha = 71.0°$ ◀

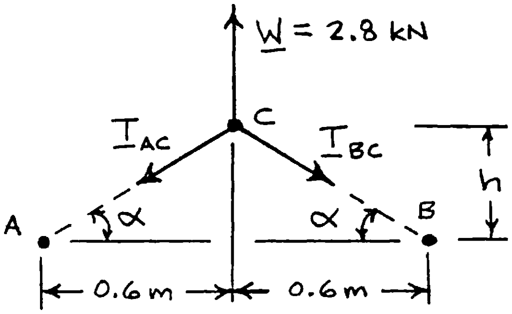


PROBLEM 2.62

A movable bin and its contents have a combined weight of 2.8 kN. Determine the shortest chain sling *ACB* that can be used to lift the loaded bin if the tension in the chain is not to exceed 5 kN.

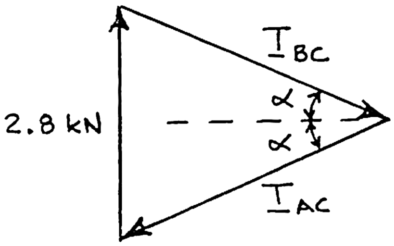
SOLUTION

Free-Body Diagram



$$\tan\alpha = \frac{h}{0.6\text{ m}} \qquad (1)$$

Isosceles Force Triangle



Law of sines: $\sin\alpha = \dfrac{\frac{1}{2}(2.8\text{ kN})}{T_{AC}}$

$$T_{AC} = 5\text{ kN}$$

$$\sin\alpha = \frac{\frac{1}{2}(2.8\text{ kN})}{5\text{ kN}}$$

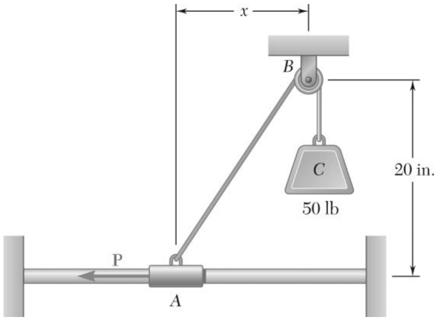
$$\alpha = 16.2602°$$

From Eq. (1): $\tan 16.2602° = \dfrac{h}{0.6\text{ m}} \qquad \therefore\ \ h = 0.175000\text{ m}$

Half length of chain $= AC = \sqrt{(0.6\text{ m})^2 + (0.175\text{ m})^2}$

$= 0.625\text{ m}$

Total length: $= 2\times 0.625\text{ m}$ 1.250 m ◀

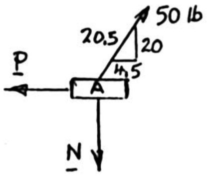


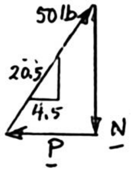
PROBLEM 2.63

Collar A is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the magnitude of the force **P** required to maintain the equilibrium of the collar when (a) $x = 4.5$ in., (b) $x = 15$ in.

SOLUTION

(a) **Free Body: Collar A**



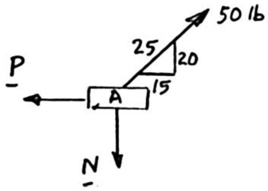


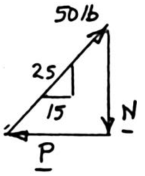
Force Triangle

$$\frac{P}{4.5} = \frac{50 \text{ lb}}{20.5}$$

$P = 10.98$ lb ◀

(b) **Free Body: Collar A**

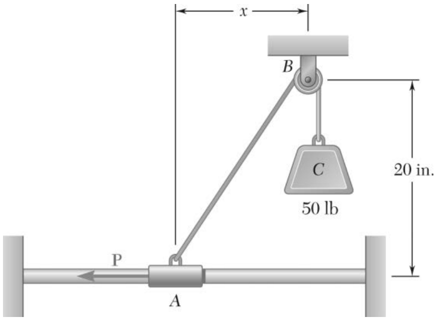




Force Triangle

$$\frac{P}{15} = \frac{50 \text{ lb}}{25}$$

$P = 30.0$ lb ◀

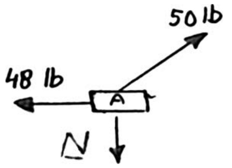


PROBLEM 2.64

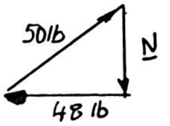
Collar A is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the distance x for which the collar is in equilibrium when $P = 48$ lb.

SOLUTION

Free Body: Collar *A*



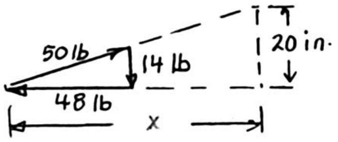
Force Triangle



$$N^2 = (50)^2 - (48)^2 = 196$$

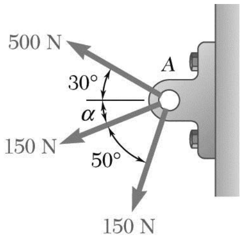
$$N = 14.00 \text{ lb}$$

Similar Triangles



$$\frac{x}{20 \text{ in.}} = \frac{48 \text{ lb}}{14 \text{ lb}}$$

$x = 68.6$ in. ◀

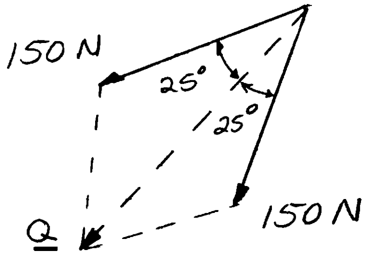


PROBLEM 2.65

Three forces are applied to a bracket as shown. The directions of the two 150-N forces may vary, but the angle between these forces is always 50°. Determine the range of values of α for which the magnitude of the resultant of the forces acting at A is less than 600 N.

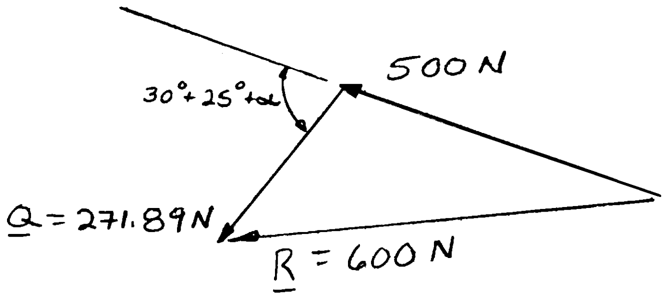
SOLUTION

Combine the two 150-N forces into a resultant force Q:



$$\begin{aligned} Q &= 2(150\text{ N})\cos 25^\circ \\ &= 271.89\text{ N} \end{aligned}$$

Equivalent loading at A:



Using the law of cosines:

$$(600\text{ N})^2 = (500\text{ N})^2 + (271.89\text{ N})^2 + 2(500\text{ N})(271.89\text{ N})\cos(55^\circ + \alpha)$$

$$\cos(55^\circ + \alpha) = 0.132685$$

Two values for α: $55^\circ + \alpha = 82.375$

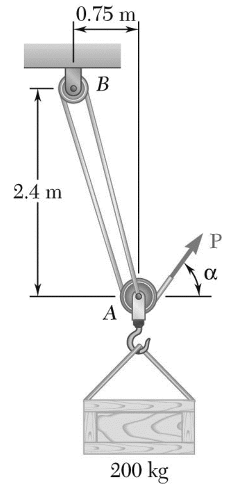
$$\alpha = 27.4^\circ$$

or $55^\circ + \alpha = -82.375^\circ$

$$55^\circ + \alpha = 360^\circ - 82.375^\circ$$

$$\alpha = 222.6^\circ$$

For $R < 600$ lb: $27.4^\circ < \alpha < 222.6$ ◀

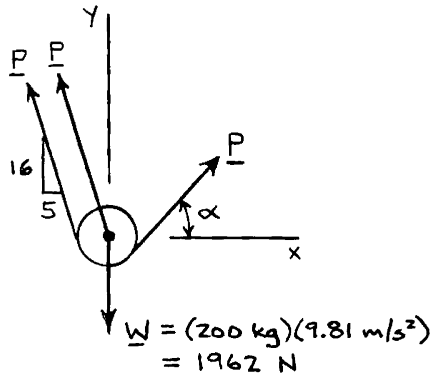


PROBLEM 2.66

A 200-kg crate is to be supported by the rope-and-pulley arrangement shown. Determine the magnitude and direction of the force **P** that must be exerted on the free end of the rope to maintain equilibrium. (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Ch. 4.)

SOLUTION

Free-Body Diagram: Pulley *A*



$$\xrightarrow{+} \Sigma F_x = 0: \quad -2P\left(\frac{5}{\sqrt{281}}\right) + P\cos\alpha = 0$$

$$\cos\alpha = 0.59655$$

$$\alpha = \pm 53.377°$$

For $\alpha = +53.377°$:

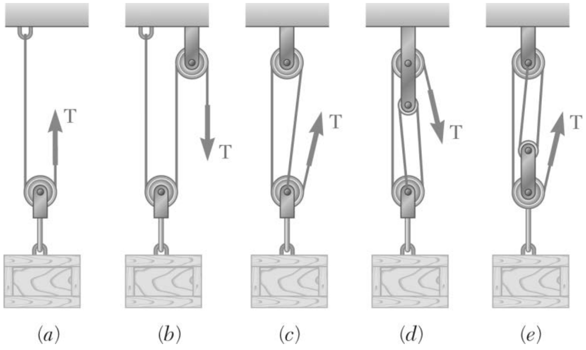
$$+\!\uparrow \Sigma F_y = 0: \quad 2P\left(\frac{16}{\sqrt{281}}\right) + P\sin 53.377° - 1962\text{ N} = 0$$

$\mathbf{P} = 724\text{ N}$ ∡ 53.4° ◀

For $\alpha = -53.377°$:

$$+\!\uparrow \Sigma F_y = 0: \quad 2P\left(\frac{16}{\sqrt{281}}\right) + P\sin(-53.377°) - 1962\text{ N} = 0$$

$\mathbf{P} = 1773$ ⦣ 53.4° ◀



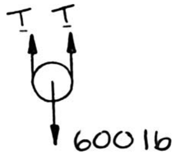
PROBLEM 2.67

A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.66.)

SOLUTION

Free-Body Diagram of Pulley

(*a*)

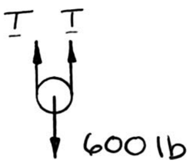


$$+\uparrow \Sigma F_y = 0: \quad 2T - (600\text{ lb}) = 0$$

$$T = \frac{1}{2}(600\text{ lb})$$

$$T = 300\text{ lb} \blacktriangleleft$$

(*b*)

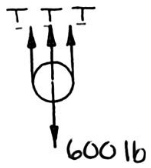


$$+\uparrow \Sigma F_y = 0: \quad 2T - (600\text{ lb}) = 0$$

$$T = \frac{1}{2}(600\text{ lb})$$

$$T = 300\text{ lb} \blacktriangleleft$$

(*c*)

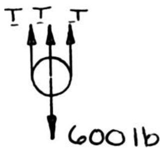


$$+\uparrow \Sigma F_y = 0: \quad 3T - (600\text{ lb}) = 0$$

$$T = \frac{1}{3}(600\text{ lb})$$

$$T = 200\text{ lb} \blacktriangleleft$$

(*d*)

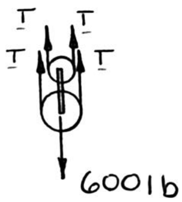


$$+\uparrow \Sigma F_y = 0: \quad 3T - (600\text{ lb}) = 0$$

$$T = \frac{1}{3}(600\text{ lb})$$

$$T = 200\text{ lb} \blacktriangleleft$$

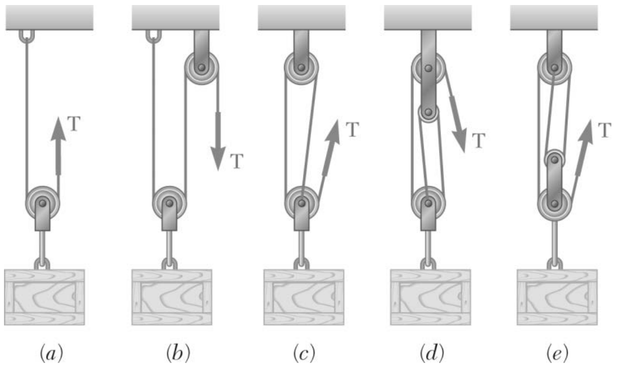
(*e*)



$$+\uparrow \Sigma F_y = 0: \quad 4T - (600\text{ lb}) = 0$$

$$T = \frac{1}{4}(600\text{ lb})$$

$$T = 150.0\text{ lb} \blacktriangleleft$$



PROBLEM 2.68

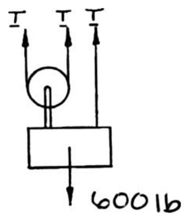
Solve Parts *b* and *d* of Problem 2.67, assuming that the free end of the rope is attached to the crate.

PROBLEM 2.67 A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.66.)

SOLUTION

Free-Body Diagram of Pulley and Crate

(*b*)

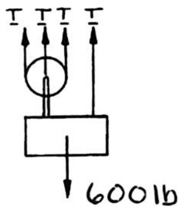


$$+\uparrow \Sigma F_y = 0: \quad 3T - (600\text{ lb}) = 0$$

$$T = \frac{1}{3}(600\text{ lb})$$

$$T = 200\text{ lb} \ \blacktriangleleft$$

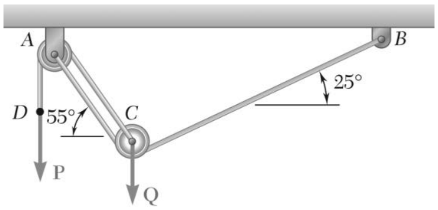
(*d*)



$$+\uparrow \Sigma F_y = 0: \quad 4T - (600\text{ lb}) = 0$$

$$T = \frac{1}{4}(600\text{ lb})$$

$$T = 150.0\text{ lb} \ \blacktriangleleft$$

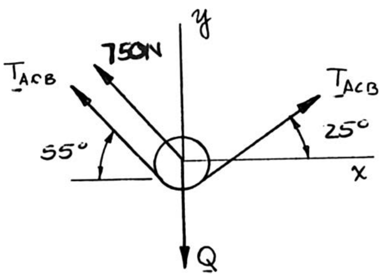


PROBLEM 2.69

A load **Q** is applied to the pulley *C*, which can roll on the cable *ACB*. The pulley is held in the position shown by a second cable *CAD*, which passes over the pulley *A* and supports a load **P**. Knowing that $P = 750\text{ N}$, determine (*a*) the tension in cable *ACB*, (*b*) the magnitude of load **Q**.

SOLUTION

Free-Body Diagram: Pulley *C*



(*a*) $\xrightarrow{+} \Sigma F_x = 0\text{:}\quad T_{ACB}(\cos 25° - \cos 55°) - (750\text{ N})\cos 55° = 0$

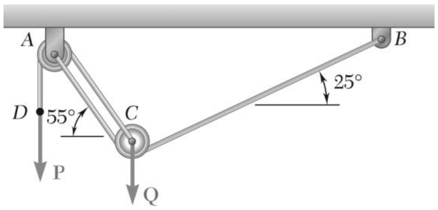
Hence: $T_{ACB} = 1292.88\text{ N}$

$T_{ACB} = 1293\text{ N}$ ◀

(*b*) $+\!\uparrow \Sigma F_y = 0\text{:}\quad T_{ACB}(\sin 25° + \sin 55°) + (750\text{ N})\sin 55° - Q = 0$

$(1292.88\text{ N})(\sin 25° + \sin 55°) + (750\text{ N})\sin 55° - Q = 0$

or $Q = 2219.8\text{ N}$ $\qquad Q = 2220\text{ N}$ ◀

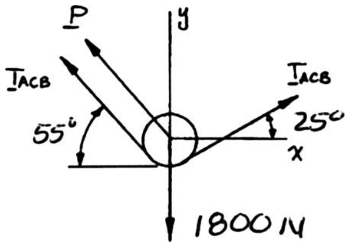


PROBLEM 2.70

An 1800-N load **Q** is applied to the pulley *C*, which can roll on the cable *ACB*. The pulley is held in the position shown by a second cable *CAD*, which passes over the pulley *A* and supports a load **P**. Determine (*a*) the tension in cable *ACB*, (*b*) the magnitude of load **P**.

SOLUTION

Free-Body Diagram: Pulley *C*



$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}(\cos 25° - \cos 55°) - P\cos 55° = 0$$

or $$P = 0.58010 T_{ACB} \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad T_{ACB}(\sin 25° + \sin 55°) + P\sin 55° - 1800 \text{ N} = 0$$

or $$1.24177 T_{ACB} + 0.81915 P = 1800 \text{ N} \qquad (2)$$

(*a*) Substitute Equation (1) into Equation (2):

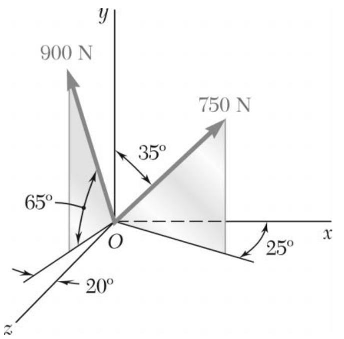
$$1.24177 T_{ACB} + 0.81915(0.58010 T_{ACB}) = 1800 \text{ N}$$

Hence: $$T_{ACB} = 1048.37 \text{ N}$$

$$T_{ACB} = 1048 \text{ N} \blacktriangleleft$$

(*b*) Using (1), $$P = 0.58010(1048.37 \text{ N}) = 608.16 \text{ N}$$

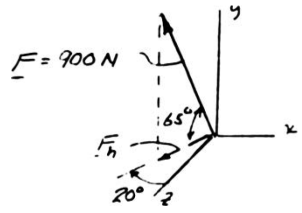
$$P = 608 \text{ N} \blacktriangleleft$$



PROBLEM 2.71

Determine (*a*) the *x*, *y*, and *z* components of the 900-N force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION



$$F_h = F\cos 65^\circ$$
$$= (900\text{ N})\cos 65^\circ$$
$$F_h = 380.36\text{ N}$$

(*a*)
$$F_x = F_h \sin 20^\circ$$
$$= (380.36\text{ N})\sin 20^\circ$$
$$F_x = -130.091\text{ N}, \qquad F_x = -130.1\text{ N} ◀$$

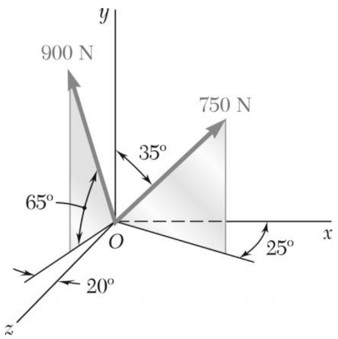
$$F_y = F\sin 65^\circ$$
$$= (900\text{ N})\sin 65^\circ$$
$$F_y = +815.68\text{ N}, \qquad F_y = +816\text{ N} ◀$$

$$F_z = F_h\cos 20^\circ$$
$$= (380.36\text{ N})\cos 20^\circ$$
$$F_z = +357.42\text{ N} \qquad F_z = +357\text{ N} ◀$$

(*b*)
$$\cos\theta_x = \frac{F_x}{F} = \frac{-130.091\text{ N}}{900\text{ N}} \qquad \theta_x = 98.3^\circ ◀$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{+815.68\text{ N}}{900\text{ N}} \qquad \theta_y = 25.0^\circ ◀$$

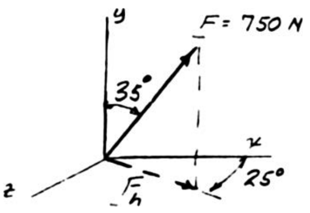
$$\cos\theta_z = \frac{F_z}{F} = \frac{+357.42\text{ N}}{900\text{ N}} \qquad \theta_z = 66.6^\circ ◀$$



PROBLEM 2.72

Determine (*a*) the x, y, and z components of the 750-N force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION



$$F_h = F\sin 35^\circ$$
$$= (750\text{ N})\sin 35^\circ$$
$$F_h = 430.18\text{ N}$$

(*a*)

$$F_x = F_h\cos 25^\circ$$
$$= (430.18\text{ N})\cos 25^\circ$$
$$F_x = +389.88\text{ N},$$

$F_x = +390\text{ N}$ ◀

$$F_y = F\cos 35^\circ$$
$$= (750\text{ N})\cos 35^\circ$$
$$F_y = +614.36\text{ N},$$

$F_y = +614\text{ N}$ ◀

$$F_z = F_h\sin 25^\circ$$
$$= (430.18\text{ N})\sin 25^\circ$$

$F_z = +181.8\text{ N}$ ◀

$$F_z = +181.802\text{ N}$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{+389.88\text{ N}}{750\text{ N}}$$

$\theta_x = 58.7^\circ$ ◀

$$\cos\theta_y = \frac{F_y}{F} = \frac{+614.36\text{ N}}{750\text{ N}}$$

$\theta_y = 35.0^\circ$ ◀

$$\cos\theta_z = \frac{F_z}{F} = \frac{+181.802\text{ N}}{750\text{ N}}$$

$\theta_z = 76.0^\circ$ ◀

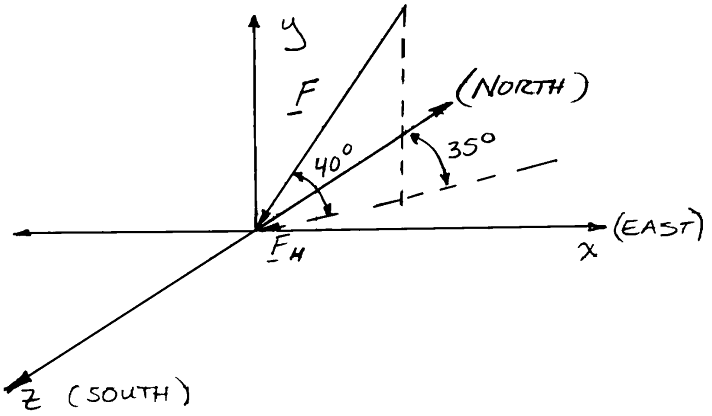
PROBLEM 2.73

A gun is aimed at a point A located 35° east of north. Knowing that the barrel of the gun forms an angle of 40° with the horizontal and that the maximum recoil force is 400 N, determine (a) the x, y, and z components of that force, (b) the values of the angles θ_x, θ_y, and θ_z defining the direction of the recoil force. (Assume that the x, y, and z axes are directed, respectively, east, up, and south.)

SOLUTION

Recoil force $F = 400\text{ N}$

$$\therefore\ F_H = (400\text{ N})\cos 40^\circ = 306.42\text{ N}$$



(a)

$$F_x = -F_H \sin 35^\circ = -(306.42\text{ N})\sin 35^\circ = -175.755\text{ N}$$

$F_x = -175.8\text{ N}$ ◀

$$F_y = -F\sin 40^\circ = -(400\text{ N})\sin 40^\circ = -257.12\text{ N}$$

$F_y = -257\text{ N}$ ◀

$$F_z = +F_H\cos 35^\circ = +(306.42\text{ N})\cos 35^\circ = +251.00\text{ N}$$

$F_z = +251\text{ N}$ ◀

(b)

$$\cos\theta_x = \frac{F_x}{F} = \frac{-175.755\text{ N}}{400\text{ N}}$$

$\theta_x = 116.1^\circ$ ◀

$$\cos\theta_y = \frac{F_y}{F} = \frac{-257.12\text{ N}}{400\text{ N}}$$

$\theta_y = 130.0^\circ$ ◀

$$\cos\theta_z = \frac{F_z}{F} = \frac{251.00\text{ N}}{400\text{ N}}$$

$\theta_z = 51.1^\circ$ ◀

PROBLEM 2.74

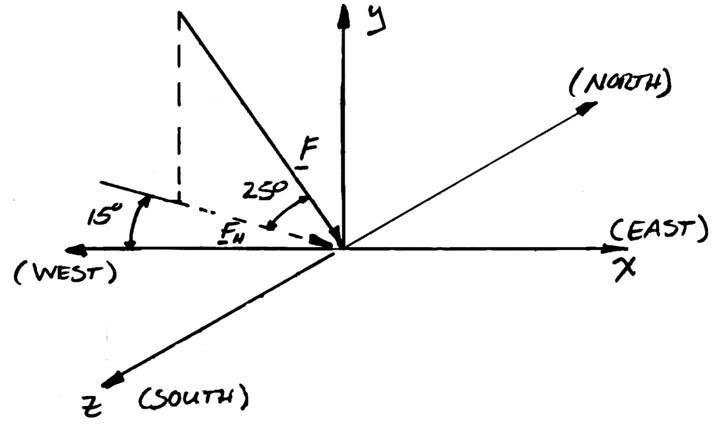
Solve Problem 2.73, assuming that point *A* is located 15° north of west and that the barrel of the gun forms an angle of 25° with the horizontal.

PROBLEM 2.73 A gun is aimed at a point *A* located 35° east of north. Knowing that the barrel of the gun forms an angle of 40° with the horizontal and that the maximum recoil force is 400 N, determine (*a*) the *x*, *y*, and *z* components of that force, (*b*) the values of the angles θ_x, θ_y, and θ_z defining the direction of the recoil force. (Assume that the *x*, *y*, and *z* axes are directed, respectively, east, up, and south.)

SOLUTION

Recoil force $F = 400\text{ N}$

$$\therefore\ F_H = (400\text{ N})\cos 25^\circ = 362.52\text{ N}$$



(*a*)

$$F_x = +F_H\cos 15^\circ = +(362.52\text{ N})\cos 15^\circ = +350.17\text{ N}$$

$F_x = +350\text{ N}$ ◀

$$F_y = -F\sin 25^\circ = -(400\text{ N})\sin 25^\circ = -169.047\text{ N}$$

$F_y = -169.0\text{ N}$ ◀

$$F_z = +F_H\sin 15^\circ = +(362.52\text{ N})\sin 15^\circ = +93.827\text{ N}$$

$F_z = +93.8\text{ N}$ ◀

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{+350.17\text{ N}}{400\text{ N}}$$

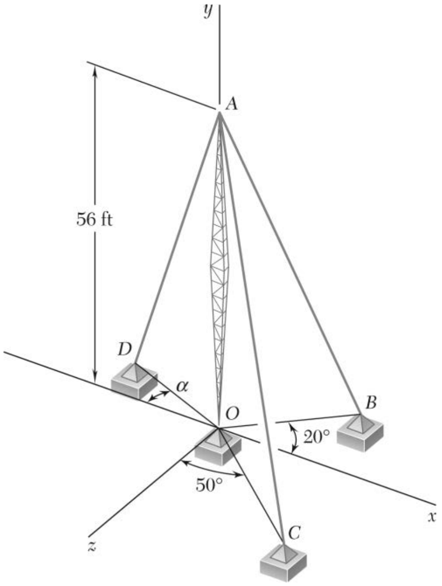
$\theta_x = 28.9^\circ$ ◀

$$\cos\theta_y = \frac{F_y}{F} = \frac{-169.047\text{ N}}{400\text{ N}}$$

$\theta_y = 115.0^\circ$ ◀

$$\cos\theta_z = \frac{F_z}{F} = \frac{+93.827\text{ N}}{400\text{ N}}$$

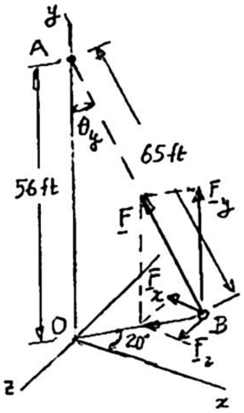
$\theta_z = 76.4^\circ$ ◀



PROBLEM 2.75

Cable *AB* is 65 ft long, and the tension in that cable is 3900 lb. Determine (*a*) the *x*, *y*, and *z* components of the force exerted by the cable on the anchor *B*, (*b*) the angles θ_x, θ_y, and θ_z defining the direction of that force.

SOLUTION



From triangle *AOB*:

$$\cos\theta_y = \frac{56\text{ ft}}{65\text{ ft}} = 0.86154$$

$$\theta_y = 30.51°$$

(*a*)

$$F_x = -F\sin\theta_y\cos 20° = -(3900\text{ lb})\sin 30.51°\cos 20°$$

$F_x = -1861$ lb ◀

$$F_y = +F\cos\theta_y = (3900\text{ lb})(0.86154)$$

$F_y = +3360$ lb ◀

$$F_z = +(3900\text{ lb})\sin 30.51°\sin 20°$$

$F_z = +677$ lb ◀

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = -\frac{1861\text{ lb}}{3900\text{ lb}} = -0.4771$$

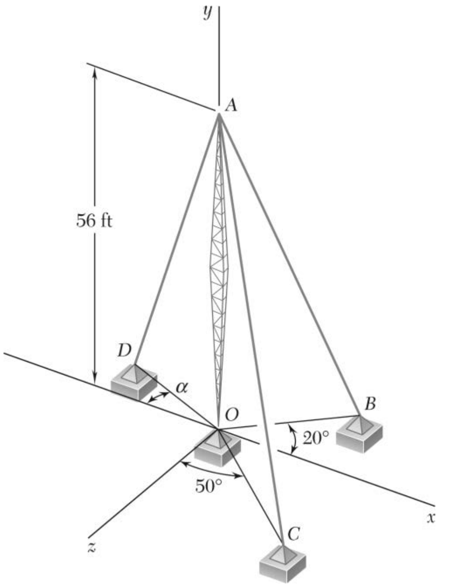
$\theta_x = 118.5°$ ◀

From above: $\theta_y = 30.51°$

$\theta_y = 30.5°$ ◀

$$\cos\theta_z = \frac{F_z}{F} = +\frac{677\text{ lb}}{3900\text{ lb}} = +0.1736$$

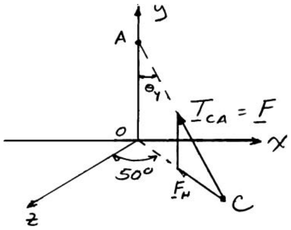
$\theta_z = 80.0°$ ◀



PROBLEM 2.76

Cable AC is 70 ft long, and the tension in that cable is 5250 lb. Determine (a) the x, y, and z components of the force exerted by the cable on the anchor C, (b) the angles θ_x, θ_y, and θ_z defining the direction of that force.

SOLUTION



In triangle AOB:

$$AC = 70 \text{ ft}$$
$$OA = 56 \text{ ft}$$
$$F = 5250 \text{ lb}$$

$$\cos\theta_y = \frac{56 \text{ ft}}{70 \text{ ft}}$$
$$\theta_y = 36.870°$$
$$F_H = F\sin\theta_y$$
$$= (5250 \text{ lb})\sin 36.870°$$
$$= 3150.0 \text{ lb}$$

(a) $F_x = -F_H \sin 50° = -(3150.0 \text{ lb})\sin 50° = -2413.04 \text{ lb}$ $\quad F_x = -2413 \text{ lb}$ ◀

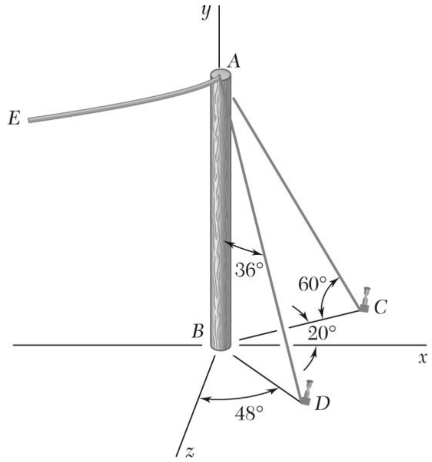
$F_y = +F\cos\theta_y = +(5250 \text{ lb})\cos 36.870° = +4200.0 \text{ lb}$ $\quad F_y = +4200 \text{ lb}$ ◀

$F_z = -F_H \cos 50° = -3150\cos 50° = -2024.8 \text{ lb}$ $\quad F_z = -2025 \text{ lb}$ ◀

(b) $\cos\theta_x = \dfrac{F_x}{F} = \dfrac{-2413.04 \text{ lb}}{5250 \text{ lb}}$ $\quad \theta_x = 117.4°$ ◀

From above: $\theta_y = 36.870°$ $\quad \theta_y = 36.9°$ ◀

$\theta_z = \dfrac{F_z}{F} = \dfrac{-2024.8 \text{ lb}}{5250 \text{ lb}}$ $\quad \theta_z = 112.7°$ ◀



PROBLEM 2.77

The end of the coaxial cable *AE* is attached to the pole *AB*, which is strengthened by the guy wires *AC* and *AD*. Knowing that the tension in wire *AC* is 120 lb, determine (*a*) the components of the force exerted by this wire on the pole, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = (120\text{ lb})\cos 60^\circ \cos 20^\circ$$

$$F_x = 56.382\text{ lb}$$

$F_x = +56.4\text{ lb}$ ◀

$$F_y = -(120\text{ lb})\sin 60^\circ$$

$$F_y = -103.923\text{ lb}$$

$F_y = -103.9\text{ lb}$ ◀

$$F_z = -(120\text{ lb})\cos 60^\circ \sin 20^\circ$$

$$F_z = -20.521\text{ lb}$$

$F_z = -20.5\text{ lb}$ ◀

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{56.382\text{ lb}}{120\text{ lb}}$$

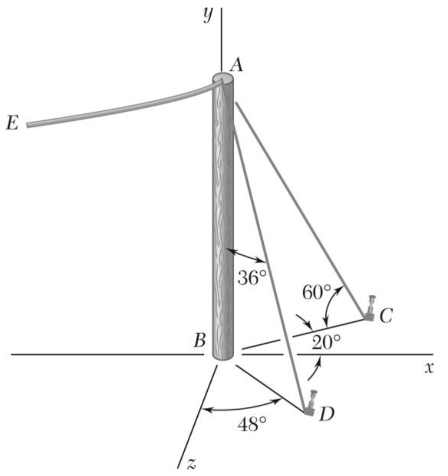
$\theta_x = 62.0^\circ$ ◀

$$\cos\theta_y = \frac{F_y}{F} = \frac{-103.923\text{ lb}}{120\text{ lb}}$$

$\theta_y = 150.0^\circ$ ◀

$$\cos\theta_z = \frac{F_z}{F} = \frac{-20.52\text{ lb}}{120\text{ lb}}$$

$\theta_z = 99.8^\circ$ ◀



PROBLEM 2.78

The end of the coaxial cable *AE* is attached to the pole *AB*, which is strengthened by the guy wires *AC* and *AD*. Knowing that the tension in wire *AD* is 85 lb, determine (*a*) the components of the force exerted by this wire on the pole, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = (85\text{ lb})\sin 36° \sin 48°$$

$$= 37.129\text{ lb} \qquad F_x = 37.1\text{ lb} \blacktriangleleft$$

$$F_y = -(85\text{ lb})\cos 36°$$

$$= -68.766\text{ lb} \qquad F_y = -68.8\text{ lb} \blacktriangleleft$$

$$F_z = (85\text{ lb})\sin 36° \cos 48°$$

$$= 33.431\text{ lb} \qquad F_z = 33.4\text{ lb} \blacktriangleleft$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{37.129\text{ lb}}{85\text{ lb}} \qquad \theta_x = 64.1° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-68.766\text{ lb}}{85\text{ lb}} \qquad \theta_y = 144.0° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{33.431\text{ lb}}{85\text{ lb}} \qquad \theta_z = 66.8° \blacktriangleleft$$

PROBLEM 2.79

Determine the magnitude and direction of the force $\mathbf{F} = (690\text{ lb})\mathbf{i} + (300\text{ lb})\mathbf{j} - (580\text{ lb})\mathbf{k}$.

SOLUTION

$$\mathbf{F} = (690\text{ N})\mathbf{i} + (300\text{ N})\mathbf{j} - (580\text{ N})\mathbf{k}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$= \sqrt{(690\text{ N})^2 + (300\text{ N})^2 + (-580\text{ N})^2}$$

$$= 950\text{ N} \qquad F = 950\text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{690\text{ N}}{950\text{ N}} \qquad \theta_x = 43.4° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{300\text{ N}}{950\text{ N}} \qquad \theta_y = 71.6° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-580\text{ N}}{950\text{ N}} \qquad \theta_z = 127.6° \blacktriangleleft$$

PROBLEM 2.80

Determine the magnitude and direction of the force **F** = (650 N)**i** – (320 N)**j** + (760 N)**k**.

SOLUTION

$$\mathbf{F} = (650\text{ N})\mathbf{i} - (320\text{ N})\mathbf{j} + (760\text{ N})\mathbf{k}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$= \sqrt{(650\text{ N})^2 + (-320\text{ N})^2 + (760\text{ N})^2} \qquad F = 1050\text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{650\text{ N}}{1050\text{ N}} \qquad \theta_x = 51.8° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-320\text{ N}}{1050\text{ N}} \qquad \theta_y = 107.7° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{760\text{ N}}{1050\text{ N}} \qquad \theta_z = 43.6° \blacktriangleleft$$

PROBLEM 2.81

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_x = 75°$ and $\theta_z = 130°$. Knowing that the y component of the force is +300 lb, determine (*a*) the angle θ_y, (*b*) the other components and the magnitude of the force.

SOLUTION

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos^2(75°) + \cos^2\theta_y + \cos^2(130°) = 1$$

$$\cos\theta_y = \pm 0.72100$$

(*a*) Since $F_y > 0$, we choose $\cos\theta_y = +0.72100$ $\qquad \therefore \quad \theta_y = 43.9°$ ◀

(*b*)

$$F_y = F\cos\theta_y$$

$$300\text{ lb} = F(0.72100)$$

$$F = 416.09\text{ lb}$$ $\qquad F = 416\text{ lb}$ ◀

$$F_x = F\cos\theta_x = 416.09\text{ lb}\cos 75°$$ $\qquad F_x = +107.7\text{ lb}$ ◀

$$F_z = F\cos\theta_z = 416.09\text{ lb}\cos 130°$$ $\qquad F_z = -267\text{ lb}$ ◀

PROBLEM 2.82

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_y = 55°$ and $\theta_z = 45°$. Knowing that the x component of the force is -500 N, determine (*a*) the angle θ_x, (*b*) the other components and the magnitude of the force.

SOLUTION

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos^2\theta_x + \cos^2 55° + \cos^2 45° = 1$$

$$\cos\theta_x = \pm 0.41353$$

(*a*) Since $F_y < 0,$ we choose $\cos\theta_x = -0.41353$ $\quad\therefore\ \theta_x = 114.4° \blacktriangleleft$

(*b*)

$$F_x = F\cos\theta_x$$

$$-500\text{ N} = F(-0.41353)$$

$$F = 1209.10\text{ N} \qquad F = 1209.1\text{ N} \blacktriangleleft$$

$$F_y = F\cos\theta_y = 1209.10\text{ N}\cos 55° \qquad F_y = +694\text{ N} \blacktriangleleft$$

$$F_z = F\cos\theta_z = 1209.10\text{ N}\cos 45° \qquad F_z = +855\text{ N} \blacktriangleleft$$

PROBLEM 2.83

A force **F** of magnitude 230 N acts at the origin of a coordinate system. Knowing that $\theta_x = 32.5°$, $F_y = -60$ N, and $F_z > 0$, determine (*a*) the components F_x and F_z, (*b*) the angles θ_y and θ_z.

SOLUTION

(*a*) We have

$$F_x = F\cos\theta_x = (230\text{ N})\cos 32.5° \qquad F_x = -194.0\text{ N} \blacktriangleleft$$

Then:

$$F_x = 193.980\text{ N}$$

$$F^2 = F_x^2 + F_y^2 + F_z^2$$

So:

$$(230\text{ N})^2 = (193.980\text{ N})^2 + (-60\text{ N})^2 + F_z^2$$

Hence:

$$F_z = +\sqrt{(230\text{ N})^2 - (193.980\text{ N})^2 - (-60\text{ N})^2} \qquad F_z = 108.0\text{ N} \blacktriangleleft$$

(*b*)

$$F_z = 108.036\text{ N}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-60\text{ N}}{230\text{ N}} = -0.26087 \qquad \theta_y = 105.1° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{108.036\text{ N}}{230\text{ N}} = 0.46972 \qquad \theta_z = 62.0° \blacktriangleleft$$

PROBLEM 2.84

A force **F** of magnitude 210 N acts at the origin of a coordinate system. Knowing that $F_x = 80$ N, $\theta_z = 151.2°$, and $F_y < 0$, determine (*a*) the components F_y and F_z, (*b*) the angles θ_x and θ_y.

SOLUTION

(*a*)

$$F_z = F\cos\theta_z = (210\text{ N})\cos 151.2°$$

$$= -184.024\text{ N} \qquad F_z = -184.0\text{ N} \blacktriangleleft$$

Then:

$$F^2 = F_x^2 + F_y^2 + F_z^2$$

So:

$$(210\text{ N})^2 = (80\text{ N})^2 + (F_y)^2 + (184.024\text{ N})^2$$

Hence:

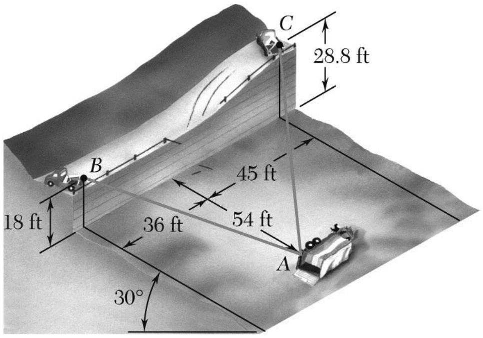
$$F_y = -\sqrt{(210\text{ N})^2 - (80\text{ N})^2 - (184.024\text{ N})^2}$$

$$= -61.929\text{ N} \qquad F_y = -62.0\text{ lb} \blacktriangleleft$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{80\text{ N}}{210\text{ N}} = 0.38095 \qquad \theta_x = 67.6° \blacktriangleleft$$

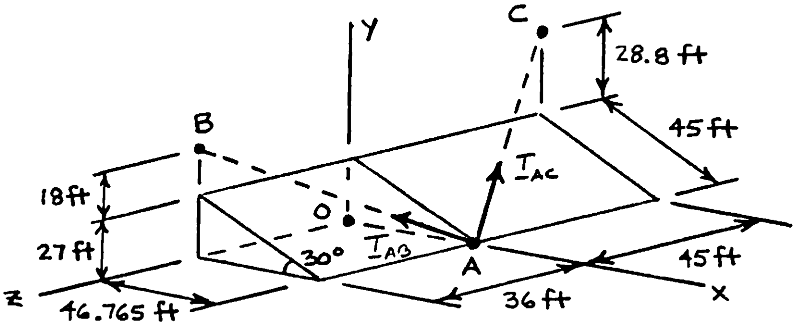
$$\cos\theta_y = \frac{F_y}{F} = \frac{61.929\text{ N}}{210\text{ N}} = -0.29490 \qquad \theta_y = 107.2° \blacktriangleleft$$



PROBLEM 2.85

In order to move a wrecked truck, two cables are attached at *A* and pulled by winches *B* and *C* as shown. Knowing that the tension in cable *AB* is 2 kips, determine the components of the force exerted at *A* by the cable.

SOLUTION



AB = 74.216 ft   AC = 85.590 ft

Cable *AB*:

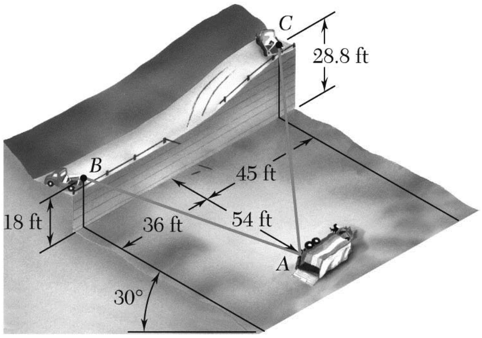
$$\lambda_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{(-46.765\text{ ft})\mathbf{i} + (45\text{ ft})\mathbf{j} + (36\text{ ft})\mathbf{k}}{74.216\text{ ft}}$$

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = \frac{-46.765\mathbf{i} + 45\mathbf{j} + 36\mathbf{k}}{74.216}$$

$(T_{AB})_x = -1.260$ kips ◀

$(T_{AB})_y = +1.213$ kips ◀

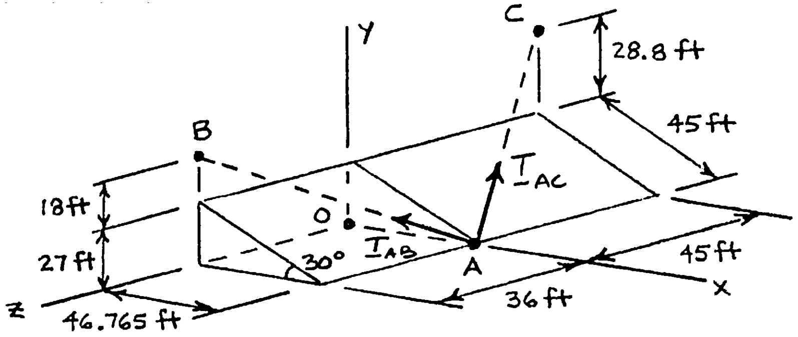
$(T_{AB})_z = +0.970$ kips ◀



PROBLEM 2.86

In order to move a wrecked truck, two cables are attached at A and pulled by winches B and C as shown. Knowing that the tension in cable AC is 1.5 kips, determine the components of the force exerted at A by the cable.

SOLUTION



$\underline{AB} = 74.216$ ft $\qquad \underline{AC} = 85.590$ ft

Cable AB:

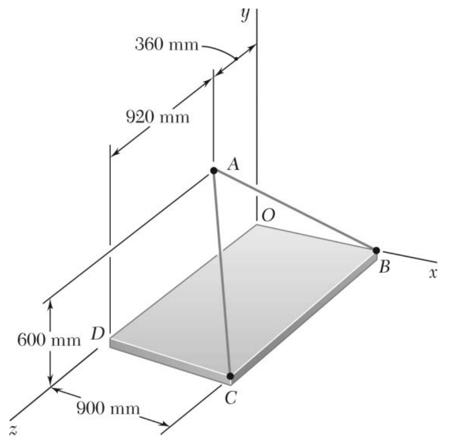
$$\lambda_{AC} = \frac{\overrightarrow{AC}}{AC} = \frac{(-46.765\text{ ft})\mathbf{i} + (55.8\text{ ft})\mathbf{j} + (-45\text{ ft})\mathbf{k}}{85.590\text{ ft}}$$

$$\mathbf{T}_{AC} = T_{AC}\,\lambda_{AC} = (1.5\text{ kips})\frac{-46.765\mathbf{i} + 55.8\mathbf{j} - 45\mathbf{k}}{85.590}$$

$(T_{AC})_x = -0.820$ kips ◀

$(T_{AC})_y = +0.978$ kips ◀

$(T_{AC})_z = -0.789$ kips ◀



PROBLEM 2.87

Knowing that the tension in cable *AB* is 1425 N, determine the components of the force exerted on the plate at *B*.

SOLUTION

$$\overrightarrow{BA} = -(900\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}$$

$$BA = \sqrt{(900\text{ mm})^2 + (600\text{ mm})^2 + (360\text{ mm})^2}$$

$$= 1140\text{ mm}$$

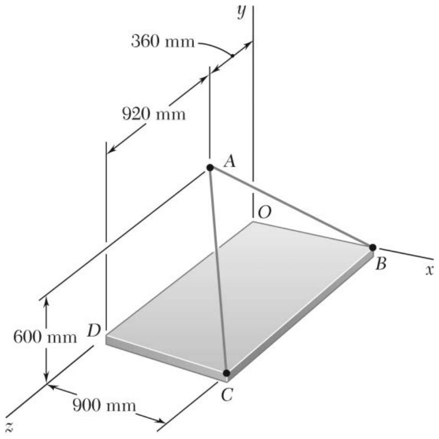
$$\mathbf{T}_{BA} = T_{BA}\boldsymbol{\lambda}_{BA}$$

$$= T_{BA}\frac{\overrightarrow{BA}}{BA}$$

$$\mathbf{T}_{BA} = \frac{1425\text{ N}}{1140\text{ mm}}[-(900\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}]$$

$$= -(1125\text{ N})\mathbf{i} + (750\text{ N})\mathbf{j} + (450\text{ N})\mathbf{k}$$

$$(T_{BA})_x = -1125\text{ N},\quad (T_{BA})_y = 750\text{ N},\quad (T_{BA})_z = 450\text{ N} \blacktriangleleft$$



PROBLEM 2.88

Knowing that the tension in cable AC is 2130 N, determine the components of the force exerted on the plate at C.

SOLUTION

$$\overrightarrow{CA} = -(900\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} - (920\text{ mm})\mathbf{k}$$

$$CA = \sqrt{(900\text{ mm})^2 + (600\text{ mm})^2 + (920\text{ mm})^2}$$

$$= 1420\text{ mm}$$

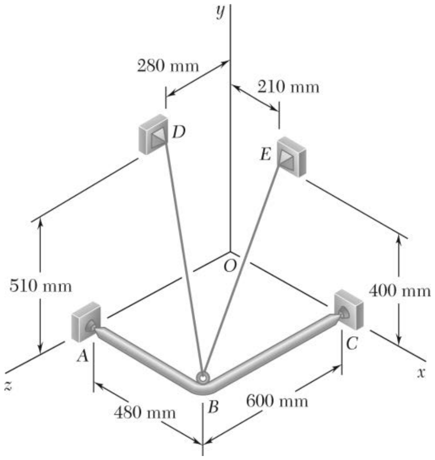
$$\mathbf{T}_{CA} = T_{CA}\boldsymbol{\lambda}_{CA}$$

$$= T_{CA}\frac{\overrightarrow{CA}}{CA}$$

$$\mathbf{T}_{CA} = \frac{2130\text{ N}}{1420\text{ mm}}[-(900\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} - (920\text{ mm})\mathbf{k}]$$

$$= -(1350\text{ N})\mathbf{i} + (900\text{ N})\mathbf{j} - (1380\text{ N})\mathbf{k}$$

$$(T_{CA})_x = -1350\text{ N},\quad (T_{CA})_y = 900\text{ N},\quad (T_{CA})_z = -1380\text{ N} \blacktriangleleft$$



PROBLEM 2.89

A frame *ABC* is supported in part by cable *DBE* that passes through a frictionless ring at *B*. Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at *D*.

SOLUTION

$$\overrightarrow{DB} = (480 \text{ mm})\mathbf{i} - (510 \text{ mm})\mathbf{j} + (320 \text{ mm})\mathbf{k}$$

$$DB = \sqrt{(480 \text{ mm})^2 + (510 \text{ mm}^2) + (320 \text{ mm})^2}$$

$$= 770 \text{ mm}$$

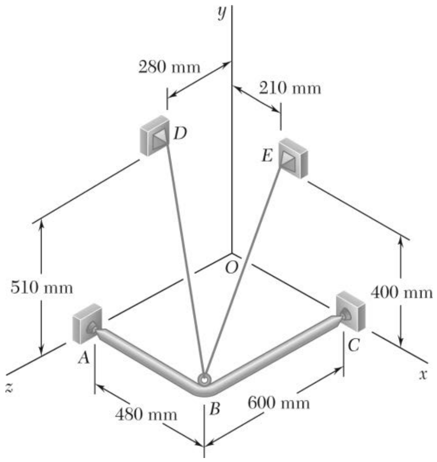
$$\mathbf{F} = F\boldsymbol{\lambda}_{DB}$$

$$= F\frac{\overrightarrow{DB}}{DB}$$

$$= \frac{385 \text{ N}}{770 \text{ mm}}[(480 \text{ mm})\mathbf{i} - (510 \text{ mm})\mathbf{j} + (320 \text{ mm})\mathbf{k}]$$

$$= (240 \text{ N})\mathbf{i} - (255 \text{ N})\mathbf{j} + (160 \text{ N})\mathbf{k}$$

$$F_x = +240 \text{ N}, \quad F_y = -255 \text{ N}, \quad F_z = +160.0 \text{ N} \blacktriangleleft$$



PROBLEM 2.90

For the frame and cable of Problem 2.89, determine the components of the force exerted by the cable on the support at *E*.

PROBLEM 2.89 A frame *ABC* is supported in part by cable *DBE* that passes through a frictionless ring at *B*. Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at *D*.

SOLUTION

$$\overrightarrow{EB} = (270\text{ mm})\mathbf{i} - (400\text{ mm})\mathbf{j} + (600\text{ mm})\mathbf{k}$$

$$EB = \sqrt{(270\text{ mm})^2 + (400\text{ mm})^2 + (600\text{ mm})^2}$$

$$= 770\text{ mm}$$

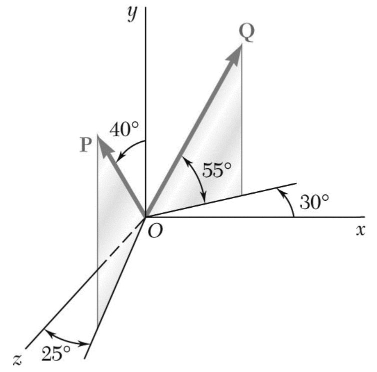
$$\mathbf{F} = F\boldsymbol{\lambda}_{EB}$$

$$= F\frac{\overrightarrow{EB}}{EB}$$

$$= \frac{385\text{ N}}{770\text{ mm}}[(270\text{ mm})\mathbf{i} - (400\text{ mm})\mathbf{j} + (600\text{ mm})\mathbf{k}]$$

$$\mathbf{F} = (135\text{ N})\mathbf{i} - (200\text{ N})\mathbf{j} + (300\text{ N})\mathbf{k}$$

$F_x = +135.0\text{ N},\quad F_y = -200\text{ N},\quad F_z = +300\text{ N}$ ◀



PROBLEM 2.91

Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 600$ N and $Q = 450$ N.

SOLUTION

$$\mathbf{P} = (600\text{ N})[\sin 40^\circ \sin 25^\circ \mathbf{i} + \cos 40^\circ \mathbf{j} + \sin 40^\circ \cos 25^\circ \mathbf{k}]$$

$$= (162.992\text{ N})\mathbf{i} + (459.63\text{ N})\mathbf{j} + (349.54\text{ N})\mathbf{k}$$

$$\mathbf{Q} = (450\text{ N})[\cos 55^\circ \cos 30^\circ \mathbf{i} + \sin 55^\circ \mathbf{j} - \cos 55^\circ \sin 30^\circ \mathbf{k}]$$

$$= (223.53\text{ N})\mathbf{i} + (368.62\text{ N})\mathbf{j} - (129.055\text{ N})\mathbf{k}$$

$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$

$$= (386.52\text{ N})\mathbf{i} + (828.25\text{ N})\mathbf{j} + (220.49\text{ N})\mathbf{k}$$

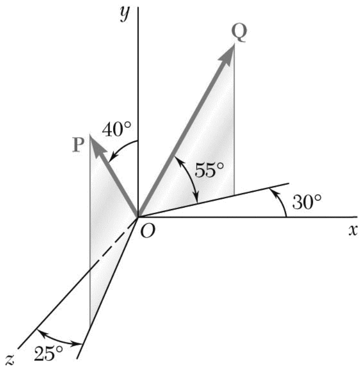
$$R = \sqrt{(386.52\text{ N})^2 + (828.25\text{ N})^2 + (220.49\text{ N})^2}$$

$$= 940.22\text{ N} \qquad R = 940\text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{R_x}{R} = \frac{386.52\text{ N}}{940.22\text{ N}} \qquad \theta_x = 65.7^\circ \blacktriangleleft$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{828.25\text{ N}}{940.22\text{ N}} \qquad \theta_y = 28.2^\circ \blacktriangleleft$$

$$\cos\theta_z = \frac{R_z}{R} = \frac{220.49\text{ N}}{940.22\text{ N}} \qquad \theta_z = 76.4^\circ \blacktriangleleft$$



PROBLEM 2.92

Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 450$ N and $Q = 600$ N.

SOLUTION

$$\mathbf{P} = (450\text{ N})[\sin 40^\circ \sin 25^\circ \mathbf{i} + \cos 40^\circ \mathbf{j} + \sin 40^\circ \cos 25^\circ \mathbf{k}]$$

$$= (122.244\text{ N})\mathbf{i} + (344.72\text{ N})\mathbf{j} + (262.154\text{ N})\mathbf{k}$$

$$\mathbf{Q} = (600\text{ N})[\cos 55^\circ \cos 30^\circ \mathbf{i} + \sin 55^\circ \mathbf{j} - \cos 55^\circ \sin 30^\circ \mathbf{k}]$$

$$= (298.04\text{ N})\mathbf{i} + (491.49\text{ N})\mathbf{j} - (172.073\text{ N})\mathbf{k}$$

$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$

$$= (420.28\text{ N})\mathbf{i} + (836.21\text{ N})\mathbf{j} + (90.081\text{ N})\mathbf{k}$$

$$R = \sqrt{(420.28\text{ N})^2 + (836.21\text{ N})^2 + (90.081\text{ N})^2}$$

$$= 940.21\text{ N}$$

$R = 940$ N ◀

$$\cos\theta_x = \frac{R_x}{R} = \frac{420.28}{940.21}$$

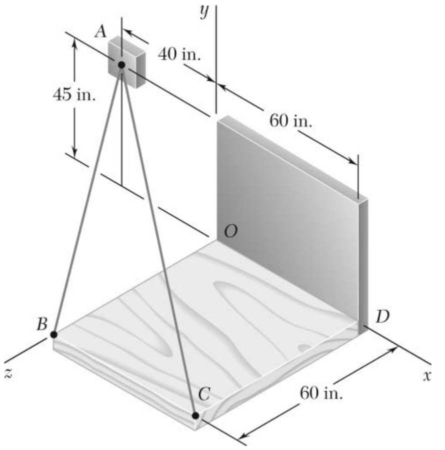
$\theta_x = 63.4^\circ$ ◀

$$\cos\theta_y = \frac{R_y}{R} = \frac{836.21}{940.21}$$

$\theta_y = 27.2^\circ$ ◀

$$\cos\theta_z = \frac{R_z}{R} = \frac{90.081}{940.21}$$

$\theta_z = 84.5^\circ$ ◀



PROBLEM 2.93

Knowing that the tension is 425 lb in cable *AB* and 510 lb in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted at *A* by the two cables.

SOLUTION

$$\overrightarrow{AB} = (40\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}$$

$$AB = \sqrt{(40\text{ in.})^2 + (45\text{ in.})^2 + (60\text{ in.})^2} = 85\text{ in.}$$

$$\overrightarrow{AC} = (100\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}$$

$$AC = \sqrt{(100\text{ in.})^2 + (45\text{ in.})^2 + (60\text{ in.})^2} = 125\text{ in.}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (425\text{ lb})\left[\frac{(40\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}}{85\text{ in.}}\right]$$

$$\mathbf{T}_{AB} = (200\text{ lb})\mathbf{i} - (225\text{ lb})\mathbf{j} + (300\text{ lb})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (510\text{ lb})\left[\frac{(100\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}}{125\text{ in.}}\right]$$

$$\mathbf{T}_{AC} = (408\text{ lb})\mathbf{i} - (183.6\text{ lb})\mathbf{j} + (244.8\text{ lb})\mathbf{k}$$

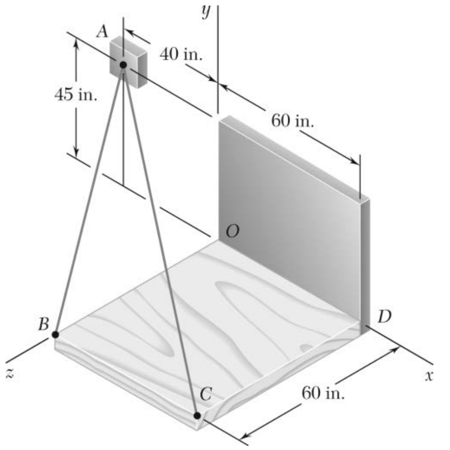
$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (608)\mathbf{i} - (408.6\text{ lb})\mathbf{j} + (544.8\text{ lb})\mathbf{k}$$

Then: $R = 912.92\text{ lb}$ $\quad R = 913\text{ lb}$ ◀

and $\cos\theta_x = \dfrac{608\text{ lb}}{912.92\text{ lb}} = 0.66599$ $\quad \theta_x = 48.2°$ ◀

$\cos\theta_y = \dfrac{408.6\text{ lb}}{912.92\text{ lb}} = -0.44757$ $\quad \theta_y = 116.6°$ ◀

$\cos\theta_z = \dfrac{544.8\text{ lb}}{912.92\text{ lb}} = 0.59677$ $\quad \theta_z = 53.4°$ ◀



PROBLEM 2.94

Knowing that the tension is 510 lb in cable *AB* and 425 lb in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted at *A* by the two cables.

SOLUTION

$$\overrightarrow{AB} = (40\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}$$

$$AB = \sqrt{(40\text{ in.})^2 + (45\text{ in.})^2 + (60\text{ in.})^2} = 85\text{ in.}$$

$$\overrightarrow{AC} = (100\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}$$

$$AC = \sqrt{(100\text{ in.})^2 + (45\text{ in.})^2 + (60\text{ in.})^2} = 125\text{ in.}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (510\text{ lb})\left[\frac{(40\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}}{85\text{ in.}}\right]$$

$$\mathbf{T}_{AB} = (240\text{ lb})\mathbf{i} - (270\text{ lb})\mathbf{j} + (360\text{ lb})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (425\text{ lb})\left[\frac{(100\text{ in.})\mathbf{i} - (45\text{ in.})\mathbf{j} + (60\text{ in.})\mathbf{k}}{125\text{ in.}}\right]$$

$$\mathbf{T}_{AC} = (340\text{ lb})\mathbf{i} - (153\text{ lb})\mathbf{j} + (204\text{ lb})\mathbf{k}$$

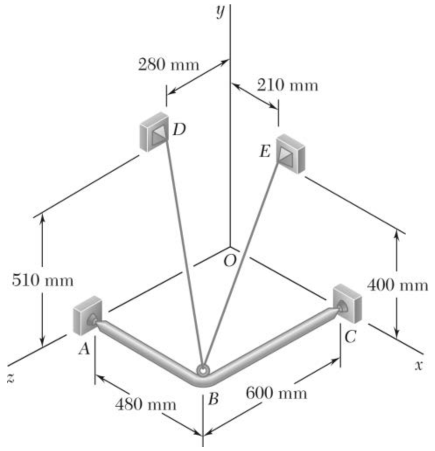
$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (580\text{ lb})\mathbf{i} - (423\text{ lb})\mathbf{j} + (564\text{ lb})\mathbf{k}$$

Then: $R = 912.92\text{ lb}$ $\qquad R = 913\text{ lb}$ ◀

and $\cos\theta_x = \dfrac{580\text{ lb}}{912.92\text{ lb}} = 0.63532$ $\qquad \theta_x = 50.6°$ ◀

$\cos\theta_y = \dfrac{-423\text{ lb}}{912.92\text{ lb}} = -0.46335$ $\qquad \theta_y = 117.6°$ ◀

$\cos\theta_z = \dfrac{564\text{ lb}}{912.92\text{ lb}} = 0.61780$ $\qquad \theta_z = 51.8°$ ◀



PROBLEM 2.95

For the frame of Problem 2.89, determine the magnitude and direction of the resultant of the forces exerted by the cable at *B* knowing that the tension in the cable is 385 N.

PROBLEM 2.89 A frame *ABC* is supported in part by cable *DBE* that passes through a frictionless ring at *B*. Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at *D*.

SOLUTION

$$\overrightarrow{BD} = -(480\text{ mm})\mathbf{i} + (510\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k}$$

$$BD = \sqrt{(480\text{ mm})^2 + (510\text{ mm})^2 + (320\text{ mm})^2} = 770\text{ mm}$$

$$\begin{aligned}\mathbf{F}_{BD} &= T_{BD}\boldsymbol{\lambda}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD}\\ &= \frac{(385\text{ N})}{(770\text{ mm})}[-(480\text{ mm})\mathbf{i} + (510\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k}]\\ &= -(240\text{ N})\mathbf{i} + (255\text{ N})\mathbf{j} - (160\text{ N})\mathbf{k}\end{aligned}$$

$$\overrightarrow{BE} = -(270\text{ mm})\mathbf{i} + (400\text{ mm})\mathbf{j} - (600\text{ mm})\mathbf{k}$$

$$BE = \sqrt{(270\text{ mm})^2 + (400\text{ mm})^2 + (600\text{ mm})^2} = 770\text{ mm}$$

$$\begin{aligned}\mathbf{F}_{BE} &= T_{BE}\boldsymbol{\lambda}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE}\\ &= \frac{(385\text{ N})}{(770\text{ mm})}[-(270\text{ mm})\mathbf{i} + (400\text{ mm})\mathbf{j} - (600\text{ mm})\mathbf{k}]\\ &= -(135\text{ N})\mathbf{i} + (200\text{ N})\mathbf{j} - (300\text{ N})\mathbf{k}\end{aligned}$$

$$\mathbf{R} = \mathbf{F}_{BD} + \mathbf{F}_{BE} = -(375\text{ N})\mathbf{i} + (455\text{ N})\mathbf{j} - (460\text{ N})\mathbf{k}$$

$$R = \sqrt{(375\text{ N})^2 + (455\text{ N})^2 + (460\text{ N})^2} = 747.83\text{ N}$$

$R = 748\text{ N}$ ◀

$$\cos\theta_x = \frac{-375\text{ N}}{747.83\text{ N}}$$

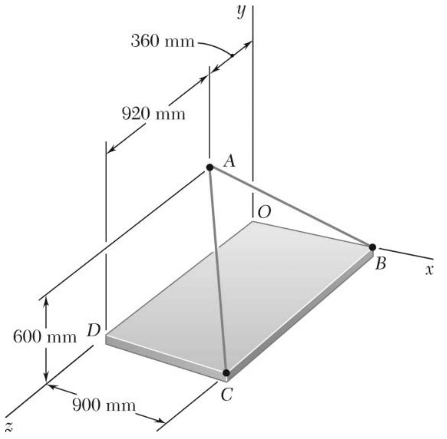
$\theta_x = 120.1°$ ◀

$$\cos\theta_y = \frac{455\text{ N}}{747.83\text{ N}}$$

$\theta_y = 52.5°$ ◀

$$\cos\theta_z = \frac{-460\text{ N}}{747.83\text{ N}}$$

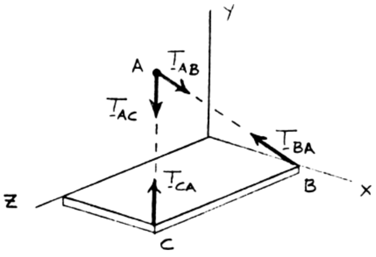
$\theta_z = 128.0°$ ◀



PROBLEM 2.96

For the cables of Problem 2.87, knowing that the tension is 1425 N in cable *AB* and 2130 N in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted at *A* by the two cables.

SOLUTION



$$T_{AB} = -T_{BA} \quad \text{(use results of Problem 2.87)}$$

$$(T_{AB})_x = +1125 \text{ N} \quad (T_{AB})_y = -750 \text{ N} \quad (T_{AB})_z = -450 \text{ N}$$

$$T_{AC} = -T_{CA} \quad \text{(use results of Problem 2.88)}$$

$$(T_{AC})_x = +1350 \text{ N} \quad (T_{AC})_y = -900 \text{ N} \quad (T_{AC})_z = +1380 \text{ N}$$

Resultant:

$$R_x = \Sigma F_x = +1125 + 1350 = +2475 \text{ N}$$

$$R_y = \Sigma F_y = -750 - 900 = -1650 \text{ N}$$

$$R_z = \Sigma F_z = -450 + 1380 = +930 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$= \sqrt{(+2475)^2 + (-1650)^2 + (+930)^2}$$

$$= 3116.6 \text{ N}$$

$R = 3120$ N ◀

$$\cos\theta_x = \frac{R_x}{R} = \frac{+2475}{3116.6}$$

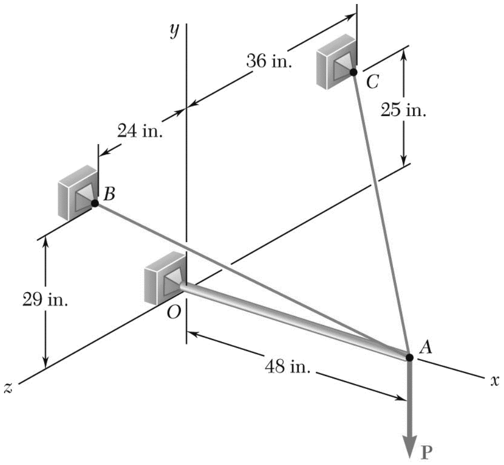
$\theta_x = 37.4°$ ◀

$$\cos\theta_y = \frac{R_y}{R} = \frac{-1650}{3116.6}$$

$\theta_y = 122.0°$ ◀

$$\cos\theta_z = \frac{R_z}{R} = \frac{+930}{3116.6}$$

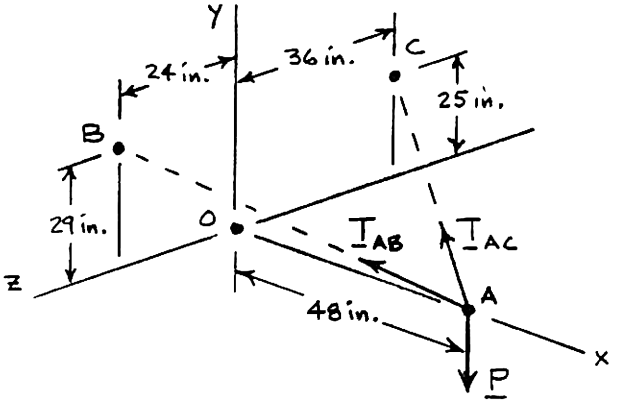
$\theta_z = 72.6°$ ◀



PROBLEM 2.97

The boom OA carries a load **P** and is supported by two cables as shown. Knowing that the tension in cable AB is 183 lb and that the resultant of the load **P** and of the forces exerted at A by the two cables must be directed along OA, determine the tension in cable AC.

SOLUTION



Cable AB: $T_{AB} = 183\text{ lb}$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (183\text{ lb})\frac{(-48\text{ in.})\mathbf{i} + (29\text{ in.})\mathbf{j} + (24\text{ in.})\mathbf{k}}{61\text{ in.}}$$

$$\mathbf{T}_{AB} = -(144\text{ lb})\mathbf{i} + (87\text{ lb})\mathbf{j} + (72\text{ lb})\mathbf{k}$$

Cable AC:
$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\frac{(-48\text{ in.})\mathbf{i} + (25\text{ in.})\mathbf{j} + (-36\text{ in.})\mathbf{k}}{65\text{ in.}}$$

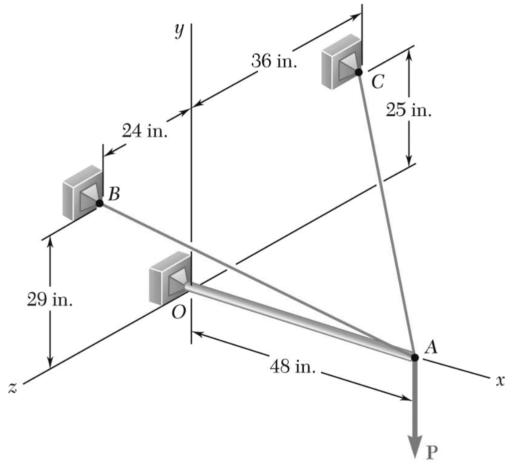
$$\mathbf{T}_{AC} = -\frac{48}{65}T_{AC}\mathbf{i} + \frac{25}{65}T_{AC}\mathbf{j} - \frac{36}{65}T_{AC}\mathbf{k}$$

Load P: $\mathbf{P} = P\mathbf{j}$

For resultant to be directed along OA, i.e., x-axis

$$R_z = 0: \quad \Sigma F_z = (72\text{ lb}) - \frac{36}{65}T'_{AC} = 0$$

$T_{AC} = 130.0\text{ lb}$ ◀



PROBLEM 2.98

For the boom and loading of Problem. 2.97, determine the magnitude of the load **P**.

PROBLEM 2.97 The boom *OA* carries a load **P** and is supported by two cables as shown. Knowing that the tension in cable *AB* is 183 lb and that the resultant of the load **P** and of the forces exerted at *A* by the two cables must be directed along *OA*, determine the tension in cable *AC*.

SOLUTION

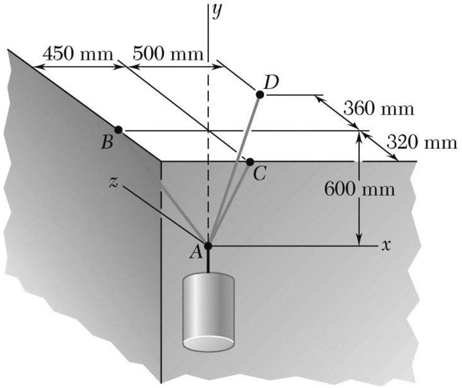
See Problem 2.97. Since resultant must be directed along *OA*, i.e., the *x*-axis, we write

$$R_y = 0: \quad \Sigma F_y = (87 \text{ lb}) + \frac{25}{65} T_{AC} - P = 0$$

$T_{AC} = 130.0 \text{ lb}$ from Problem 2.97.

Then $$(87 \text{ lb}) + \frac{25}{65}(130.0 \text{ lb}) - P = 0$$

$P = 137.0 \text{ lb}$ ◀

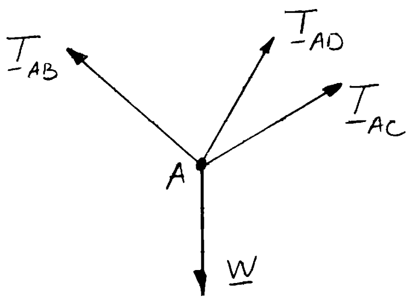


PROBLEM 2.99

A container is supported by three cables that are attached to a ceiling as shown. Determine the weight W of the container, knowing that the tension in cable AB is 6 kN.

SOLUTION

Free-Body Diagram at A:



The forces applied at A are: $\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD}$, and $\mathbf{W}$

where $\mathbf{W} = W\mathbf{j}$. To express the other forces in terms of the unit vectors **i**, **j**, **k**, we write

$$\overrightarrow{AB} = -(450\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} \qquad AB = 750\text{ mm}$$

$$\overrightarrow{AC} = +(600\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k} \qquad AC = 680\text{ mm}$$

$$\overrightarrow{AD} = +(500\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k} \qquad AD = 860\text{ mm}$$

and

$$\mathbf{T}_{AB} = \boldsymbol{\lambda}_{AB}T_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\frac{(-450\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j}}{750\text{ mm}}$$

$$= \left(-\frac{45}{75}\mathbf{i} + \frac{60}{75}\mathbf{j}\right)T_{AB}$$

$$\mathbf{T}_{AC} = \boldsymbol{\lambda}_{AC}T_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\frac{(600\text{ mm})\mathbf{i} - (320\text{ mm})\mathbf{j}}{680\text{ mm}}$$

$$= \left(\frac{60}{68}\mathbf{j} - \frac{32}{68}\mathbf{k}\right)T_{AC}$$

$$\mathbf{T}_{AD} = \boldsymbol{\lambda}_{AD}T_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = T_{AD}\frac{(500\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}}{860\text{ mm}}$$

$$= \left(\frac{50}{86}\mathbf{i} + \frac{60}{86}\mathbf{j} + \frac{36}{86}\mathbf{k}\right)T_{AD}$$

PROBLEM 2.99 (Continued)

Equilibrium condition: $\Sigma F = 0\text{:} \quad \therefore \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{W} = 0$

Substituting the expressions obtained for $\mathbf{T}_{AB}, \mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$; factoring **i**, **j**, and **k**; and equating each of the coefficients to zero gives the following equations:

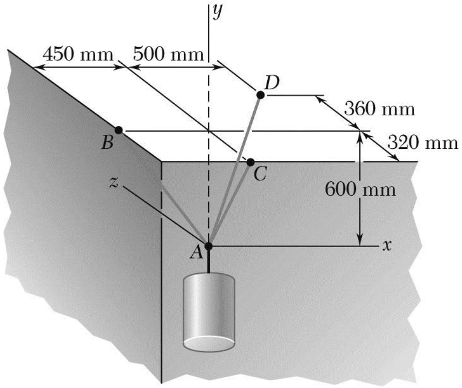
From **i**: $$-\frac{45}{75}T_{AB} + \frac{50}{86}T_{AD} = 0 \qquad (1)$$

From **j**: $$\frac{60}{75}T_{AB} + \frac{60}{68}T_{AC} + \frac{60}{86}T_{AD} - W = 0 \qquad (2)$$

From **k**: $$-\frac{32}{68}T_{AC} + \frac{36}{86}T_{AD} = 0 \qquad (3)$$

Setting $T_{AB} = 6 \text{ kN}$ in (1) and (2), and solving the resulting set of equations gives

$$T_{AC} = 6.1920 \text{ kN}$$
$$T_{AC} = 5.5080 \text{ kN} \qquad W = 13.98 \text{ kN} \blacktriangleleft$$



PROBLEM 2.100

A container is supported by three cables that are attached to a ceiling as shown. Determine the weight W of the container, knowing that the tension in cable AD is 4.3 kN.

SOLUTION

See Problem 2.99 for the figure and analysis leading to the following set of linear algebraic equations:

$$-\frac{45}{75}T_{AB}+\frac{50}{86}T_{AD}=0 \tag{1}$$

$$\frac{60}{75}T_{AB}+\frac{60}{68}T_{AC}+\frac{60}{86}T_{AD}-W=0 \tag{2}$$

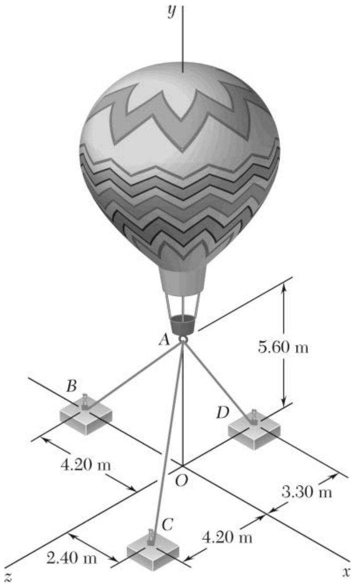
$$-\frac{32}{68}T_{AC}+\frac{36}{86}T_{AD}=0 \tag{3}$$

Setting $T_{AD}=4.3\text{ kN}$ into the above equations gives

$$T_{AB}=4.1667\text{ kN}$$

$$T_{AC}=3.8250\text{ kN}$$

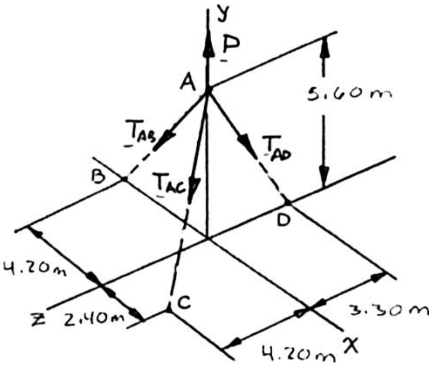
$W=9.71\text{ kN}$ ◀



PROBLEM 2.101

Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at *A* knowing that the tension in cable *AD* is 481 N.

SOLUTION



The forces applied at *A* are: $\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD}$, and **P**

where $\mathbf{P} = P\mathbf{j}$. To express the other forces in terms of the unit vectors **i**, **j**, **k**, we write

$$\overrightarrow{AB} = -(4.20\text{ m})\mathbf{i} - (5.60\text{ m})\mathbf{j} \qquad AB = 7.00\text{ m}$$

$$\overrightarrow{AC} = (2.40\text{ m})\mathbf{i} - (5.60\text{ m})\mathbf{j} + (4.20\text{ m})\mathbf{k} \qquad AC = 7.40\text{ m}$$

$$\overrightarrow{AD} = -(5.60\text{ m})\mathbf{j} - (3.30\text{ m})\mathbf{k} \qquad AD = 6.50\text{ m}$$

and

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (-0.6\mathbf{i} - 0.8\mathbf{j})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (0.32432\mathbf{i} - 0.75676\mathbf{j} + 0.56757\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = (-0.86154\mathbf{j} - 0.50769\mathbf{k})T_{AD}$$

PROBLEM 2.101 (Continued)

Equilibrium condition: $\Sigma F = 0$: $\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$

Substituting the expressions obtained for $\mathbf{T}_{AB}$, $\mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$ and factoring **i**, **j**, and **k**:

$$(-0.6T_{AB} + 0.32432T_{AC})\mathbf{i} + (-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P)\mathbf{j}$$
$$+(0.56757T_{AC} - 0.50769T_{AD})\mathbf{k} = 0$$

Equating to zero the coefficients of **i**, **j**, **k**:

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \qquad (1)$$

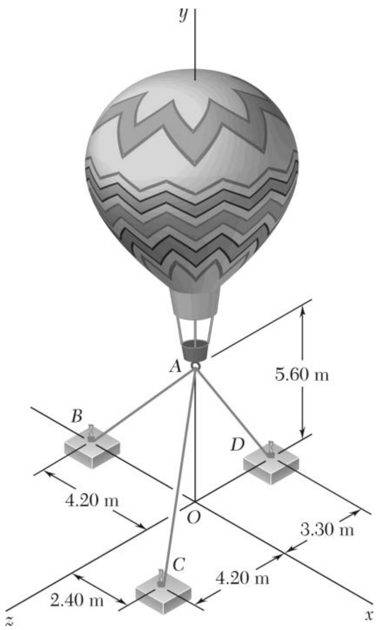
$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \qquad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \qquad (3)$$

Setting $T_{AD} = 481\text{ N}$ in (2) and (3), and solving the resulting set of equations gives

$$T_{AC} = 430.26\text{ N}$$
$$T_{AD} = 232.57\text{ N} \qquad \mathbf{P} = 926\text{ N}\uparrow ◀$$

PROPRIETARY MATERIAL.



PROBLEM 2.102

Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800-N vertical force at *A*, determine the tension in each cable.

SOLUTION

See Problem 2.101 for the figure and analysis leading to the linear algebraic Equations (1), (2), and (3).

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \qquad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \qquad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \qquad (3)$$

From Eq. (1): $T_{AB} = 0.54053T_{AC}$

From Eq. (3): $T_{AD} = 1.11795T_{AC}$

Substituting for T_{AB} and T_{AD} in terms of T_{AC} into Eq. (2) gives

$$-0.8(0.54053T_{AC}) - 0.75676T_{AC} - 0.86154(1.11795T_{AC}) + P = 0$$

$$2.1523T_{AC} = P; \quad P = 800 \text{ N}$$

$$T_{AC} = \frac{800 \text{ N}}{2.1523}$$

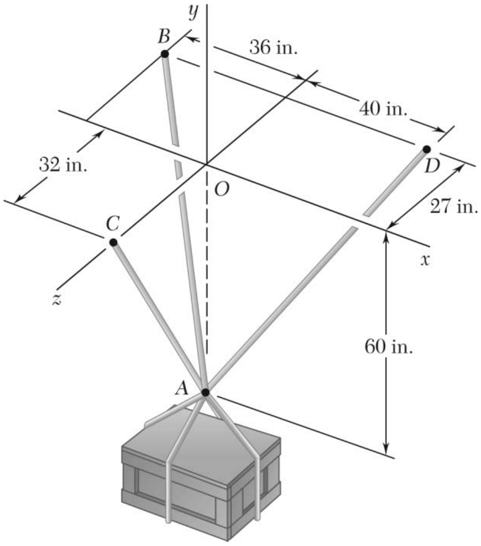
$$= 371.69 \text{ N}$$

Substituting into expressions for T_{AB} and T_{AD} gives

$$T_{AB} = 0.54053(371.69 \text{ N})$$

$$T_{AD} = 1.11795(371.69 \text{ N})$$

$$T_{AB} = 201 \text{ N}, \quad T_{AC} = 372 \text{ N}, \quad T_{AD} = 416 \text{ N} \blacktriangleleft$$



PROBLEM 2.103

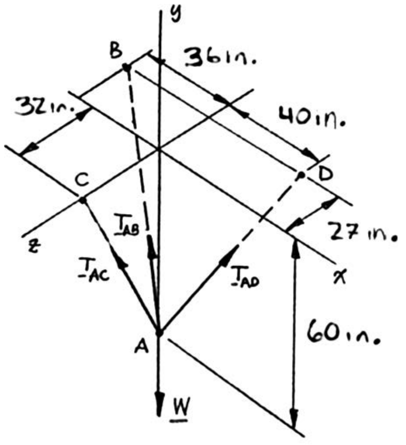
A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable *AB* is 750 lb.

SOLUTION

The forces applied at *A* are:

$$\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD} \text{ and } \mathbf{W}$$

where $\mathbf{P} = P\mathbf{j}$. To express the other forces in terms of the unit vectors **i**, **j**, **k**, we write



$$\overrightarrow{AB} = -(36 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j} - (27 \text{ in.})\mathbf{k}$$
$$AB = 75 \text{ in.}$$
$$\overrightarrow{AC} = (60 \text{ in.})\mathbf{j} + (32 \text{ in.})\mathbf{k}$$
$$AC = 68 \text{ in.}$$
$$\overrightarrow{AD} = (40 \text{ in.})\mathbf{i} + (60 \text{ in.})\mathbf{j} - (27 \text{ in.})\mathbf{k}$$
$$AD = 77 \text{ in.}$$

and

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (-0.48\mathbf{i} + 0.8\mathbf{j} - 0.36\mathbf{k})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (0.88235\mathbf{j} + 0.47059\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = (0.51948\mathbf{i} + 0.77922\mathbf{j} - 0.35065\mathbf{k})T_{AD}$$

Equilibrium Condition with $\mathbf{W} = -W\mathbf{j}$

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} - W\mathbf{j} = 0$$

PROBLEM 2.103 (Continued)

Substituting the expressions obtained for $\mathbf{T}_{AB}$, $\mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$ and factoring **i**, **j**, and **k**:

$$(-0.48T_{AB} + 0.51948T_{AD})\mathbf{i} + (0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W)\mathbf{j}$$
$$+ (-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD})\mathbf{k} = 0$$

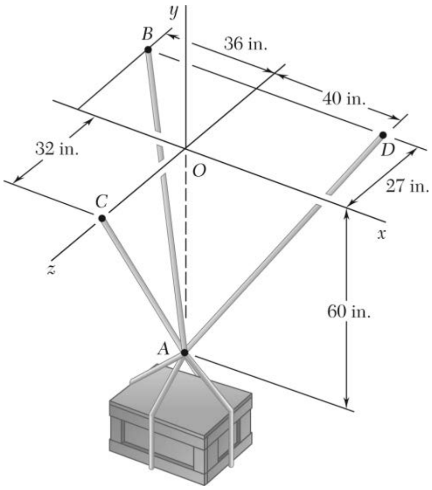
Equating to zero the coefficients of **i**, **j**, **k**:

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$
$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

Substituting $T_{AB} = 750\text{ lb}$ in Equations (1), (2), and (3) and solving the resulting set of equations, using conventional algorithms for solving linear algebraic equations, gives:

$$T_{AC} = 1090.1\text{ lb}$$
$$T_{AD} = 693\text{ lb}$$

$W = 2100\text{ lb}$ ◀



PROBLEM 2.104

A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable *AD* is 616 lb.

SOLUTION

See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0 \tag{1}$$

$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0 \tag{2}$$

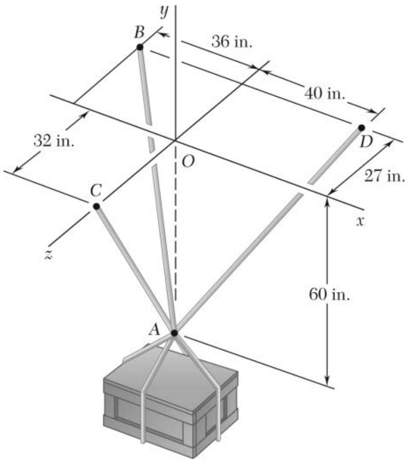
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0 \tag{3}$$

Substituting $T_{AD} = 616 \text{ lb}$ in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

$$T_{AB} = 667.67 \text{ lb}$$

$$T_{AC} = 969.00 \text{ lb}$$

$W = 1868 \text{ lb}$ ◀



PROBLEM 2.105

A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable AC is 544 lb.

SOLUTION

See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB}+0.51948T_{AD}=0 \qquad (1)$$

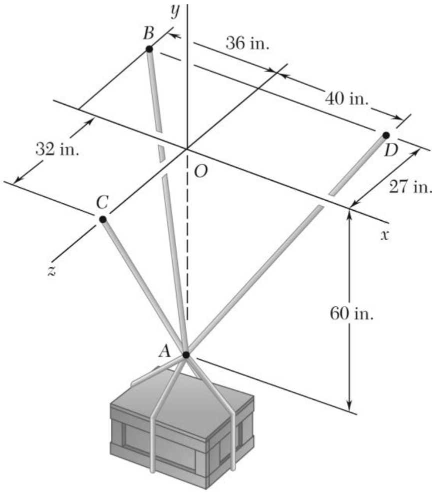
$$0.8T_{AB}+0.88235T_{AC}+0.77922T_{AD}-W=0 \qquad (2)$$

$$-0.36T_{AB}+0.47059T_{AC}-0.35065T_{AD}=0 \qquad (3)$$

Substituting $T_{AC}=544\text{ lb}$ in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

$$T_{AB}=374.27\text{ lb}$$

$$T_{AD}=345.82\text{ lb} \qquad W=1049\text{ lb} \blacktriangleleft$$



PROBLEM 2.106

A 1600-lb crate is supported by three cables as shown. Determine the tension in each cable.

SOLUTION

See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB}+0.51948T_{AD}=0 \qquad (1)$$

$$0.8T_{AB}+0.88235T_{AC}+0.77922T_{AD}-W=0 \qquad (2)$$

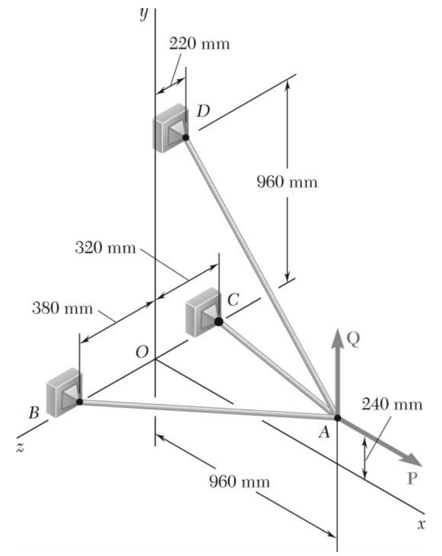
$$-0.36T_{AB}+0.47059T_{AC}-0.35065T_{AD}=0 \qquad (3)$$

Substituting $W=1600$ lb in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives

$T_{AB}=571$ lb ◀

$T_{AC}=830$ lb ◀

$T_{AD}=528$ lb ◀



PROBLEM 2.107

Three cables are connected at A, where the forces **P** and **Q** are applied as shown. Knowing that $Q = 0$, find the value of P for which the tension in cable AD is 305 N.

SOLUTION

$$\Sigma \mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} = 0 \quad \text{where} \quad \mathbf{P} = P\mathbf{i}$$

$$\overrightarrow{AB} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} + (380\text{ mm})\mathbf{k} \qquad AB = 1060\text{ mm}$$

$$\overrightarrow{AC} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k} \qquad AC = 1040\text{ mm}$$

$$\overrightarrow{AD} = -(960\text{ mm})\mathbf{i} + (720\text{ mm})\mathbf{j} - (220\text{ mm})\mathbf{k} \qquad AD = 1220\text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = \frac{305\text{ N}}{1220\text{ mm}}[(-960\text{ mm})\mathbf{i} + (720\text{ mm})\mathbf{j} - (220\text{ mm})\mathbf{k}]$$

$$= -(240\text{ N})\mathbf{i} + (180\text{ N})\mathbf{j} - (55\text{ N})\mathbf{k}$$

Substituting into $\Sigma \mathbf{F}_A = 0$, factoring $\mathbf{i}, \mathbf{j}, \mathbf{k}$, and setting each coefficient equal to ϕ gives:

$$\mathbf{i}: \quad P = \frac{48}{53}T_{AB} + \frac{12}{13}T_{AC} + 240\text{ N} \tag{1}$$

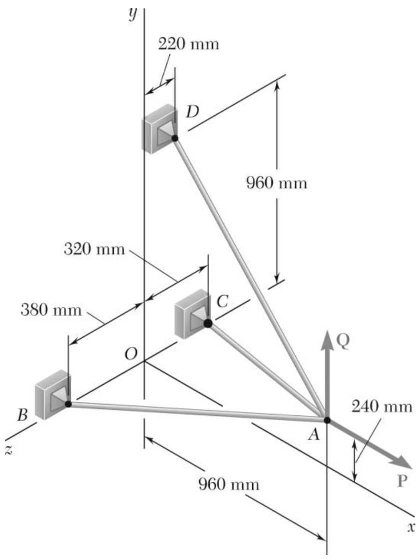
$$\mathbf{j}: \quad \frac{12}{53}T_{AB} + \frac{3}{13}T_{AC} = 180\text{ N} \tag{2}$$

$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 55\text{ N} \tag{3}$$

Solving the system of linear equations using conventional algorithms gives:

$$T_{AB} = 446.71\text{ N}$$

$$T_{AC} = 341.71\text{ N} \qquad\qquad P = 960\text{ N} \blacktriangleleft$$



PROBLEM 2.108

Three cables are connected at A, where the forces **P** and **Q** are applied as shown. Knowing that $P = 1200$ N, determine the values of Q for which cable AD is taut.

SOLUTION

We assume that $T_{AD} = 0$ and write $\Sigma\mathbf{F}_A = 0{:}\quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + Q\mathbf{j} + (1200\text{ N})\mathbf{i} = 0$

$$\overrightarrow{AB} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} + (380\text{ mm})\mathbf{k} \quad AB = 1060\text{ mm}$$

$$\overrightarrow{AC} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k} \quad AC = 1040\text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)T_{AC}$$

Substituting into $\Sigma\mathbf{F}_A = 0$, factoring $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, and setting each coefficient equal to ϕ gives:

$$\mathbf{i}{:}\quad -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} + 1200\text{ N} = 0 \tag{1}$$

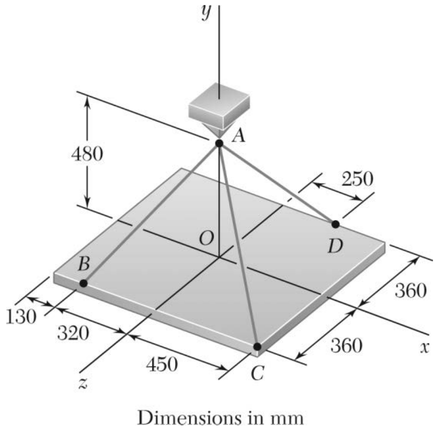
$$\mathbf{j}{:}\quad -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + Q = 0 \tag{2}$$

$$\mathbf{k}{:}\quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 0 \tag{3}$$

Solving the resulting system of linear equations using conventional algorithms gives:

$$T_{AB} = 605.71\text{ N}$$
$$T_{AC} = 705.71\text{ N}$$
$$Q = 300.00\text{ N} \qquad 0 \le Q < 300\text{ N} \blacktriangleleft$$

Note: This solution assumes that Q is directed upward as shown $(Q \ge 0)$, if negative values of Q are considered, cable AD remains taut, but AC becomes slack for $Q = -460$ N.



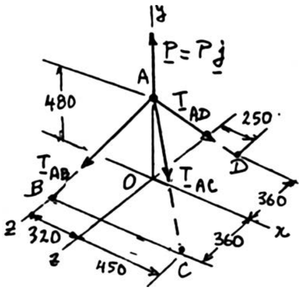
PROBLEM 2.109

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AC is 60 N, determine the weight of the plate.

SOLUTION

We note that the weight of the plate is equal in magnitude to the force **P** exerted by the support on Point A.

Free Body A:



Dimensions in mm

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

We have:

$$\overrightarrow{AB} = -(320\text{ mm})\mathbf{i} - (480\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k} \qquad AB = 680\text{ mm}$$

$$\overrightarrow{AC} = (450\text{ mm})\mathbf{i} - (480\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k} \qquad AC = 750\text{ mm}$$

$$\overrightarrow{AD} = (250\text{ mm})\mathbf{i} - (480\text{ mm})\mathbf{j} - \left(360\text{ mm}\right)\mathbf{k} \qquad AD = 650\text{ mm}$$

Thus:

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(-\frac{8}{17}\mathbf{i} - \frac{12}{17}\mathbf{j} + \frac{9}{17}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \left(0.6\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}\right)T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \left(\frac{5}{13}\mathbf{i} - \frac{9.6}{13}\mathbf{j} - \frac{7.2}{13}\mathbf{k}\right)T_{AD}$$

Substituting into the Eq. $\Sigma F = 0$ and factoring $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$:

$$\begin{aligned}&\left(-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD}\right)\mathbf{i}\\ &+\left(-\frac{12}{17}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P\right)\mathbf{j}\\ &+\left(\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD}\right)\mathbf{k} = 0\end{aligned}$$

PROBLEM 2.109 (Continued)

Setting the coefficient of **i**, **j**, **k** equal to zero:

$$\mathbf{i}: \qquad -\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \tag{1}$$

$$\mathbf{j}: \qquad -\frac{12}{7}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \tag{2}$$

$$\mathbf{k}: \qquad \frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \tag{3}$$

Making $T_{AC} = 60\text{ N}$ in (1) and (3):

$$-\frac{8}{17}T_{AB} + 36\text{ N} + \frac{5}{13}T_{AD} = 0 \tag{1$'$}$$

$$\frac{9}{17}T_{AB} + 28.8\text{ N} - \frac{7.2}{13}T_{AD} = 0 \tag{3$'$}$$

Multiply (1′) by 9, (3′) by 8, and add:

$$554.4\text{ N} - \frac{12.6}{13}T_{AD} = 0 \quad T_{AD} = 572.0\text{ N}$$

Substitute into (1′) and solve for T_{AB}:

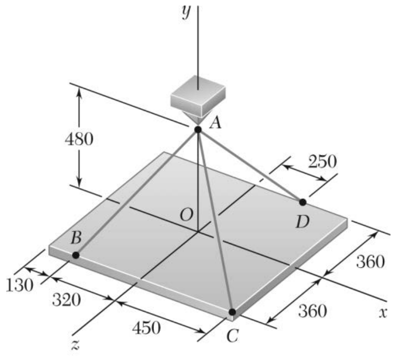
$$T_{AB} = \frac{17}{8}\left(36 + \frac{5}{13}\times 572\right) \qquad T_{AB} = 544.0\text{ N}$$

Substitute for the tensions in Eq. (2) and solve for P:

$$P = \frac{12}{17}(544\text{ N}) + 0.64(60\text{ N}) + \frac{9.6}{13}(572\text{ N})$$

$$= 844.8\text{ N}$$

Weight of plate $= P = 845\text{ N}$ ◀



Dimensions in mm

PROBLEM 2.110

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable *AD* is 520 N, determine the weight of the plate.

SOLUTION

See Problem 2.109 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \tag{1}$$

$$-\frac{12}{17}T_{AB} + 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \tag{2}$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \tag{3}$$

Making $T_{AD} = 520\text{ N}$ in Eqs. (1) and (3):

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + 200\text{ N} = 0 \tag{1$'$}$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - 288\text{ N} = 0 \tag{3$'$}$$

Multiply (1′) by 9, (3′) by 8, and add:

$$9.24T_{AC} - 504\text{ N} = 0 \quad T_{AC} = 54.5455\text{ N}$$

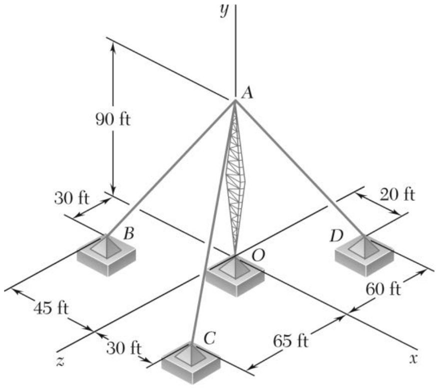
Substitute into (1′) and solve for T_{AB}:

$$T_{AB} = \frac{17}{8}(0.6 \times 54.5455 + 200) \quad T_{AB} = 494.545\text{ N}$$

Substitute for the tensions in Eq. (2) and solve for *P*:

$$P = \frac{12}{17}(494.545\text{ N}) + 0.64(54.5455\text{ N}) + \frac{9.6}{13}(520\text{ N})$$
$$= 768.00\text{ N}$$

Weight of plate $= P = 768\text{ N}$ ◀

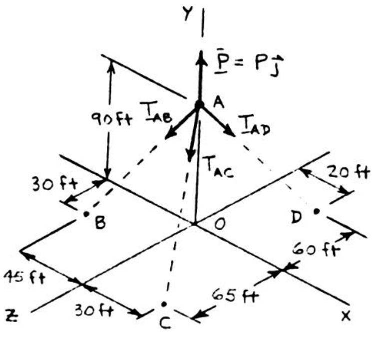


PROBLEM 2.111

A transmission tower is held by three guy wires attached to a pin at *A* and anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AB* is 630 lb, determine the vertical force **P** exerted by the tower on the pin at *A*.

SOLUTION

Free Body *A*:



$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

$$\overrightarrow{AB} = -45\mathbf{i} - 90\mathbf{j} + 30\mathbf{k} \quad AB = 105 \text{ ft}$$

$$\overrightarrow{AC} = 30\mathbf{i} - 90\mathbf{j} + 65\mathbf{k} \quad AC = 115 \text{ ft}$$

$$\overrightarrow{AD} = 20\mathbf{i} - 90\mathbf{j} - 60\mathbf{k} \quad AD = 110 \text{ ft}$$

We write

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$

$$= \left(-\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC}$$

$$= \left(\frac{6}{23}\mathbf{i} - \frac{18}{23}\mathbf{j} + \frac{13}{23}\mathbf{k}\right)T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD}$$

$$= \left(\frac{2}{11}\mathbf{i} - \frac{9}{11}\mathbf{j} - \frac{6}{11}\mathbf{k}\right)T_{AD}$$

Substituting into the Eq. $\Sigma \mathbf{F} = 0$ and factoring $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\left(-\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD}\right)\mathbf{i}$$

$$+\left(-\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P\right)\mathbf{j}$$

$$+\left(\frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD}\right)\mathbf{k} = 0$$

PROBLEM 2.111 (Continued)

Setting the coefficients of **i**, **j**, **k**, equal to zero:

$$\mathbf{i}: \quad -\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \tag{1}$$

$$\mathbf{j}: \quad -\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P = 0 \tag{2}$$

$$\mathbf{k}: \quad \frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \tag{3}$$

Set $T_{AB} = 630$ lb in Eqs. (1) – (3):

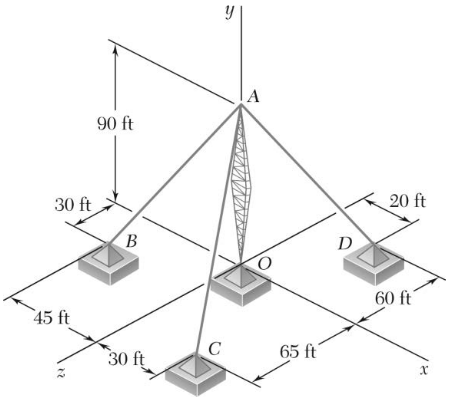
$$-270\text{ lb} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \tag{1$'$}$$

$$-540\text{ lb} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P = 0 \tag{2$'$}$$

$$180\text{ lb} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \tag{3$'$}$$

Solving, $T_{AC} = 467.42$ lb $T_{AD} = 814.35$ lb $P = 1572.10$ lb

$P = 1572$ lb ◀



PROBLEM 2.112

A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C, and D. If the tension in wire AC is 920 lb, determine the vertical force **P** exerted by the tower on the pin at A.

SOLUTION

See Problem 2.111 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \tag{1}$$

$$-\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P = 0 \tag{2}$$

$$\frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \tag{3}$$

Substituting for $T_{AC} = 920\text{ lb}$ in Equations (1), (2), and (3) above and solving the resulting set of equations using conventional algorithms gives:

$$-\frac{3}{7}T_{AB} + 240\text{ lb} + \frac{2}{11}T_{AD} = 0 \tag{1$'$}$$

$$-\frac{6}{7}T_{AB} - 720\text{ lb} - \frac{9}{11}T_{AD} + P = 0 \tag{2$'$}$$

$$\frac{2}{7}T_{AB} + 520\text{ lb} - \frac{6}{11}T_{AD} = 0 \tag{3$'$}$$

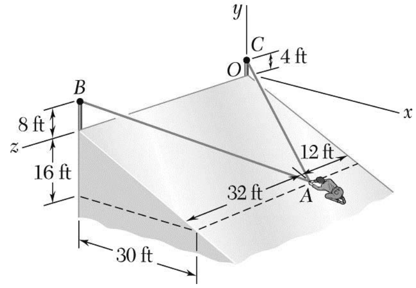
Solving,

$$T_{AB} = 1240.00\text{ lb}$$

$$T_{AD} = 1602.86\text{ lb}$$

$$P = 3094.3\text{ lb}$$

$P = 3090\text{ lb}$ ◀

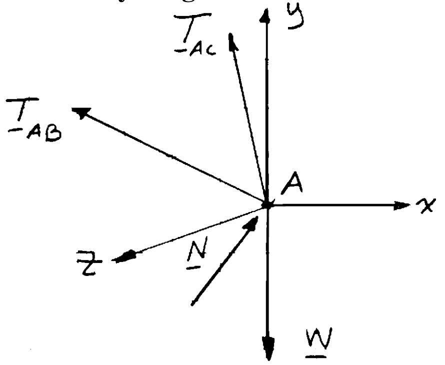


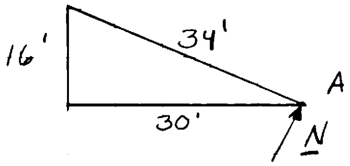
PROBLEM 2.113

In trying to move across a slippery icy surface, a 180-lb man uses two ropes AB and AC. Knowing that the force exerted on the man by the icy surface is perpendicular to that surface, determine the tension in each rope.

SOLUTION

Free-Body Diagram at *A*





$$\mathbf{N} = N\left(\frac{16}{34}\mathbf{i} + \frac{30}{34}\mathbf{j}\right)$$

and $\mathbf{W} = W\,\mathbf{j} = -(180\text{ lb})\mathbf{j}$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\frac{(-30\text{ ft})\mathbf{i} + (20\text{ ft})\mathbf{j} - (12\text{ ft})\mathbf{k}}{38\text{ ft}}$$

$$= T_{AC}\left(-\frac{15}{19}\mathbf{i} + \frac{10}{19}\mathbf{j} - \frac{6}{19}\mathbf{k}\right)$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\frac{(-30\text{ ft})\mathbf{i} + (24\text{ ft})\mathbf{j} + (32\text{ ft})\mathbf{k}}{50\text{ ft}}$$

$$= T_{AB}\left(-\frac{15}{25}\mathbf{i} + \frac{12}{25}\mathbf{j} + \frac{16}{25}\mathbf{k}\right)$$

Equilibrium condition: $\Sigma\mathbf{F} = 0$

$$\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{N} + \mathbf{W} = 0$$

PROBLEM 2.113 (Continued)

Substituting the expressions obtained for $\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{N},$ and $\mathbf{W}$; factoring **i**, **j**, and **k**; and equating each of the coefficients to zero gives the following equations:

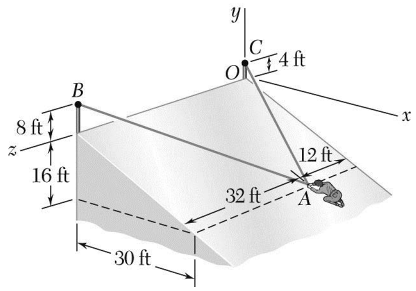
From **i**:
$$-\frac{15}{25}T_{AB} - \frac{15}{19}T_{AC} + \frac{16}{34}N = 0 \qquad (1)$$

From **j**:
$$\frac{12}{25}T_{AB} + \frac{10}{19}T_{AC} + \frac{30}{34}N - (180\text{ lb}) = 0 \qquad (2)$$

From **k**:
$$\frac{16}{25}T_{AB} - \frac{6}{19}T_{AC} = 0 \qquad (3)$$

Solving the resulting set of equations gives:

$T_{AB} = 31.7\text{ lb};\ T_{AC} = 64.3\text{ lb}$ ◀



PROBLEM 2.114

Solve Problem 2.113, assuming that a friend is helping the man at *A* by pulling on him with a force **P** = –(60 lb)**k**.

PROBLEM 2.113 In trying to move across a slippery icy surface, a 180-lb man uses two ropes *AB* and *AC*. Knowing that the force exerted on the man by the icy surface is perpendicular to that surface, determine the tension in each rope.

SOLUTION

Refer to Problem 2.113 for the figure and analysis leading to the following set of equations, Equation (3) being modified to include the additional force **P** = (–60 lb)**k**.

$$-\frac{15}{25}T_{AB} - \frac{15}{19}T_{AC} + \frac{16}{34}N = 0 \tag{1}$$

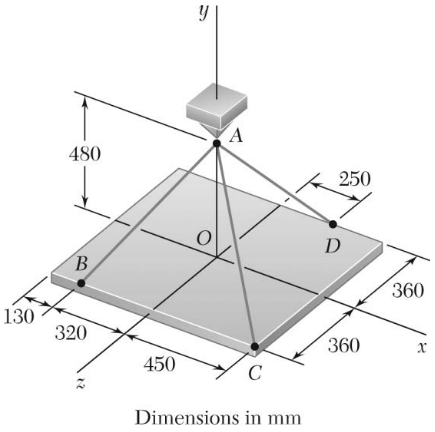
$$\frac{12}{25}T_{AB} + \frac{10}{19}T_{AC} + \frac{30}{34}N - (180\text{ lb}) = 0 \tag{2}$$

$$\frac{16}{25}T_{AB} - \frac{6}{19}T_{AC} - (60\text{ lb}) = 0 \tag{3}$$

Solving the resulting set of equations simultaneously gives:

$T_{AB} = 99.0$ lb ◀

$T_{AC} = 10.55$ lb ◀



PROBLEM 2.115

For the rectangular plate of Problems 2.109 and 2.110, determine the tension in each of the three cables knowing that the weight of the plate is 792 N.

SOLUTION

See Problem 2.109 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below. Setting $P = 792 \text{ N}$ gives:

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \tag{1}$$

$$-\frac{12}{17}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + 792 \text{ N} = 0 \tag{2}$$

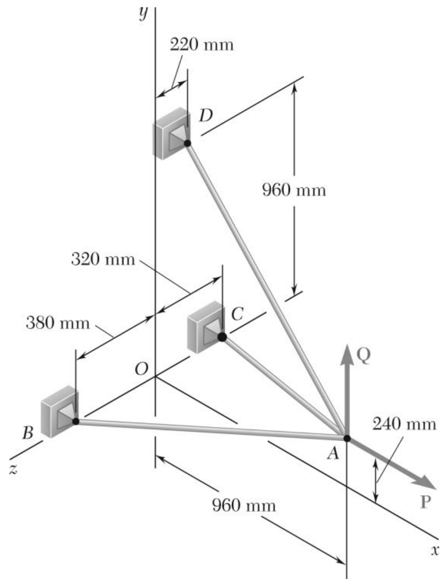
$$\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \tag{3}$$

Solving Equations (1), (2), and (3) by conventional algorithms gives

$T_{AB} = 510.00 \text{ N}$ $T_{AB} = 510 \text{ N}$ ◀

$T_{AC} = 56.250 \text{ N}$ $T_{AC} = 56.2 \text{ N}$ ◀

$T_{AD} = 536.25 \text{ N}$ $T_{AD} = 536 \text{ N}$ ◀



PROBLEM 2.116

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that $P = 2880$ N and $Q = 0$.

SOLUTION

$$\Sigma \mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} + \mathbf{Q} = 0$$

Where $\quad \mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{j}$

$$\overrightarrow{AB} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} + (380 \text{ mm})\mathbf{k} \quad AB = 1060 \text{ mm}$$

$$\overrightarrow{AC} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 1040 \text{ mm}$$

$$\overrightarrow{AD} = -(960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k} \quad AD = 1220 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = T_{AD}\left(-\frac{48}{61}\mathbf{i} + \frac{36}{61}\mathbf{j} - \frac{11}{61}\mathbf{k}\right)$$

Substituting into $\Sigma \mathbf{F}_A = 0$, setting $P = (2880 \text{ N})\mathbf{i}$ and $Q = 0$, and setting the coefficients of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ equal to 0, we obtain the following three equilibrium equations:

$$\mathbf{i}: \quad -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880 \text{ N} = 0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} = 0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \qquad (3)$$

PROBLEM 2.116 (Continued)

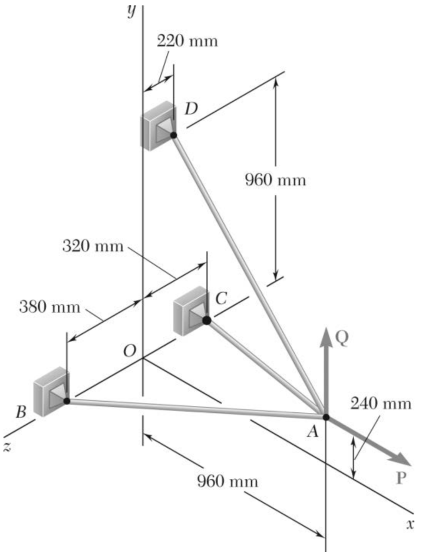
Solving the system of linear equations using conventional algorithms gives:

$$T_{AB} = 1340.14 \text{ N}$$
$$T_{AC} = 1025.12 \text{ N}$$
$$T_{AD} = 915.03 \text{ N}$$

$T_{AB} = 1340 \text{ N}$ ◀

$T_{AC} = 1025 \text{ N}$ ◀

$T_{AD} = 915 \text{ N}$ ◀



PROBLEM 2.117

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that $P = 2880$ N and $Q = 576$ N.

SOLUTION

See Problem 2.116 for the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + P = 0 \tag{1}$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + Q = 0 \tag{2}$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \tag{3}$$

Setting $P = 2880$ N and $Q = 576$ N gives:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880 \text{ N} = 0 \tag{1$'$}$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + 576 \text{ N} = 0 \tag{2$'$}$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \tag{3$'$}$$

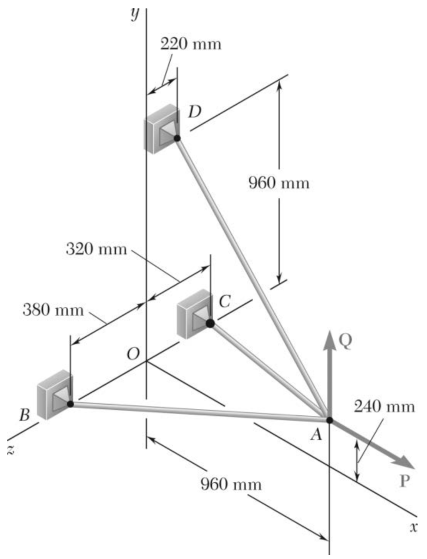
Solving the resulting set of equations using conventional algorithms gives:

$$T_{AB} = 1431.00 \text{ N}$$
$$T_{AC} = 1560.00 \text{ N}$$
$$T_{AD} = 183.010 \text{ N}$$

$T_{AB} = 1431$ N ◀

$T_{AC} = 1560$ N ◀

$T_{AD} = 183.0$ N ◀



PROBLEM 2.118

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that $P = 2880$ N and $Q = -576$ N. (**Q** is directed downward).

SOLUTION

See Problem 2.116 for the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + P = 0 \qquad (1)$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + Q = 0 \qquad (2)$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \qquad (3)$$

Setting $P = 2880$ N and $Q = -576$ N gives:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880\text{ N} = 0 \qquad (1')$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} - 576\text{ N} = 0 \qquad (2')$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \qquad (3')$$

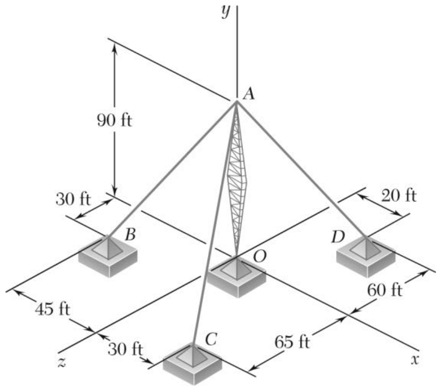
Solving the resulting set of equations using conventional algorithms gives:

$$T_{AB} = 1249.29\text{ N}$$
$$T_{AC} = 490.31\text{ N}$$
$$T_{AD} = 1646.97\text{ N}$$

$T_{AB} = 1249$ N ◀

$T_{AC} = 490$ N ◀

$T_{AD} = 1647$ N ◀



PROBLEM 2.119

For the transmission tower of Problems 2.111 and 2.112, determine the tension in each guy wire knowing that the tower exerts on the pin at A an upward vertical force of 2100 lb.

SOLUTION

See Problem 2.111 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \tag{1}$$

$$-\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + P = 0 \tag{2}$$

$$\frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \tag{3}$$

Substituting for $P = 2100\text{ lb}$ in Equations (1), (2), and (3) above and solving the resulting set of equations using conventional algorithms gives:

$$-\frac{3}{7}T_{AB} + \frac{6}{23}T_{AC} + \frac{2}{11}T_{AD} = 0 \tag{1$'$}$$

$$-\frac{6}{7}T_{AB} - \frac{18}{23}T_{AC} - \frac{9}{11}T_{AD} + 2100\text{ lb} = 0 \tag{2$'$}$$

$$\frac{2}{7}T_{AB} + \frac{13}{23}T_{AC} - \frac{6}{11}T_{AD} = 0 \tag{3$'$}$$

$$T_{AB} = 841.55\text{ lb}$$

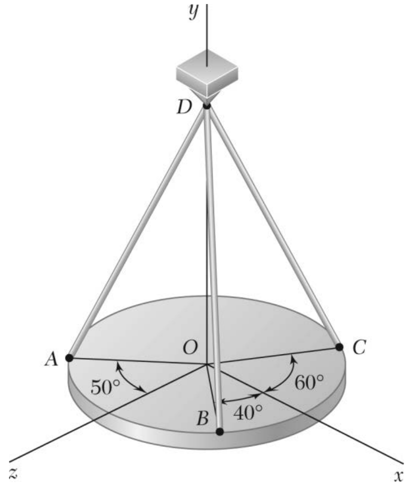
$$T_{AC} = 624.38\text{ lb}$$

$$T_{AD} = 1087.81\text{ lb}$$

$T_{AB} = 842\text{ lb}$ ◀

$T_{AC} = 624\text{ lb}$ ◀

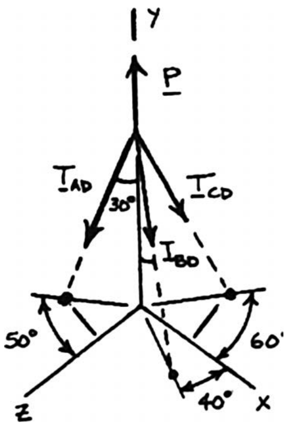
$T_{AD} = 1088\text{ lb}$ ◀



PROBLEM 2.120

A horizontal circular plate weighing 60 lb is suspended as shown from three wires that are attached to a support at *D* and form 30° angles with the vertical. Determine the tension in each wire.

SOLUTION



$\Sigma F_x = 0$:

$$-T_{AD}(\sin 30°)(\sin 50°) + T_{BD}(\sin 30°)(\cos 40°) + T_{CD}(\sin 30°)(\cos 60°) = 0$$

Dividing through by $\sin 30°$ and evaluating:

$$-0.76604T_{AD} + 0.76604T_{BD} + 0.5T_{CD} = 0 \quad (1)$$

$\Sigma F_y = 0$: $\quad -T_{AD}(\cos 30°) - T_{BD}(\cos 30°) - T_{CD}(\cos 30°) + 60 \text{ lb} = 0$

or $$T_{AD} + T_{BD} + T_{CD} = 69.282 \text{ lb} \quad (2)$$

$\Sigma F_z = 0$: $\quad T_{AD}\sin 30°\cos 50° + T_{BD}\sin 30°\sin 40° - T_{CD}\sin 30°\sin 60° = 0$

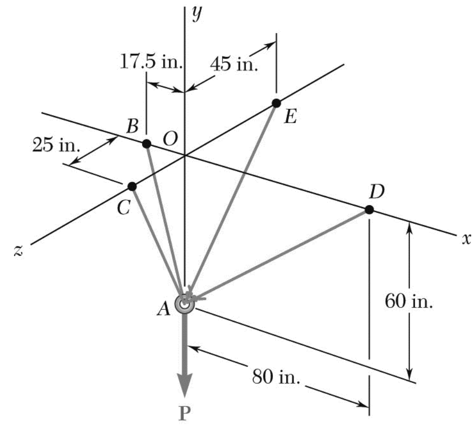
or $$0.64279T_{AD} + 0.64279T_{BD} - 0.86603T_{CD} = 0 \quad (3)$$

Solving Equations (1), (2), and (3) simultaneously:

$$T_{AD} = 29.5 \text{ lb} \blacktriangleleft$$

$$T_{BD} = 10.25 \text{ lb} \blacktriangleleft$$

$$T_{CD} = 29.5 \text{ lb} \blacktriangleleft$$

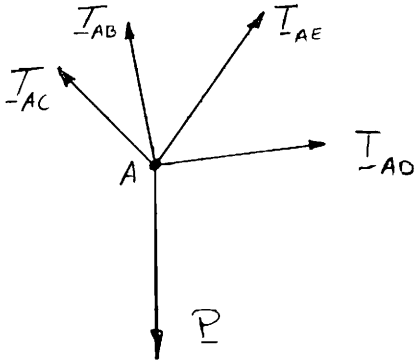


PROBLEM 2.121

Cable *BAC* passes through a frictionless ring *A* and is attached to fixed supports at *B* and *C*, while cables *AD* and *AE* are both tied to the ring and are attached, respectively, to supports at *D* and *E*. Knowing that a 200-lb vertical load **P** is applied to ring *A*, determine the tension in each of the three cables.

SOLUTION

Free Body Diagram at *A*:



Since T_{BAC} = tension in cable *BAC*, it follows that

$$T_{AB} = T_{AC} = T_{BAC}$$

$$\mathbf{T}_{AB} = T_{BAC}\boldsymbol{\lambda}_{AB} = T_{BAC}\frac{(-17.5\text{ in.})\mathbf{i} + (60\text{ in.})\mathbf{j}}{62.5\text{ in.}} = T_{BAC}\left(\frac{-17.5}{62.5}\mathbf{i} + \frac{60}{62.5}\mathbf{j}\right)$$

$$\mathbf{T}_{AC} = T_{BAC}\boldsymbol{\lambda}_{AC} = T_{BAC}\frac{(60\text{ in.})\mathbf{i} + (25\text{ in.})\mathbf{k}}{65\text{ in.}} = T_{BAC}\left(\frac{60}{65}\mathbf{j} + \frac{25}{65}\mathbf{k}\right)$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{(80\text{ in.})\mathbf{i} + (60\text{ in.})\mathbf{j}}{100\text{ in.}} = T_{AD}\left(\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}\right)$$

$$\mathbf{T}_{AE} = T_{AE}\boldsymbol{\lambda}_{AE} = T_{AE}\frac{(60\text{ in.})\mathbf{j} - (45\text{ in.})\mathbf{k}}{75\text{ in.}} = T_{AE}\left(\frac{4}{5}\mathbf{j} - \frac{3}{5}\mathbf{k}\right)$$

PROBLEM 2.121 (Continued)

Substituting into $\Sigma \mathbf{F}_A = 0$, setting $\mathbf{P} = (-200 \text{ lb})\mathbf{j}$, and setting the coefficients of **i**, **j**, **k** equal to ϕ, we obtain the following three equilibrium equations:

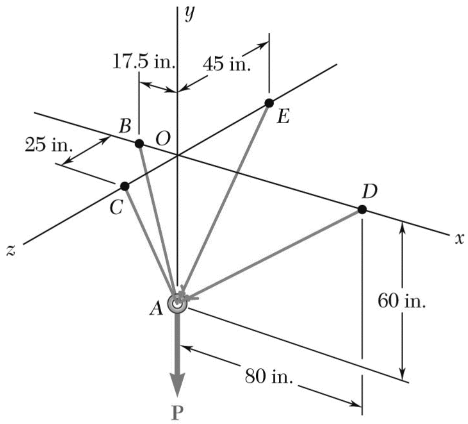
From **i**: $$-\frac{17.5}{62.5}T_{BAC} + \frac{4}{5}T_{AD} = 0 \qquad (1)$$

From **j**: $$\left(\frac{60}{62.5} + \frac{60}{65}\right)T_{BAC} + \frac{3}{5}T_{AD} + \frac{4}{5}T_{AE} - 200 \text{ lb} = 0 \qquad (2)$$

From **k**: $$\frac{25}{65}T_{BAC} - \frac{3}{5}T_{AE} = 0 \qquad (3)$$

Solving the system of linear equations using convential acgorithms gives:

$T_{BAC} = 76.7 \text{ lb}$; $T_{AD} = 26.9 \text{ lb}$; $T_{AE} = 49.2 \text{ lb}$ ◀



PROBLEM 2.122

Knowing that the tension in cable *AE* of Prob. 2.121 is 75 lb, determine (*a*) the magnitude of the load **P**, (*b*) the tension in cables *BAC* and *AD*.

PROBLEM 2.121 Cable *BAC* passes through a frictionless ring *A* and is attached to fixed supports at *B* and *C*, while cables *AD* and *AE* are both tied to the ring and are attached, respectively, to supports at *D* and *E*. Knowing that a 200-lb vertical load **P** is applied to ring *A*, determine the tension in each of the three cables.

SOLUTION

Refer to the solution to Problem 2.121 for the figure and analysis leading to the following set of equilibrium equations, Equation (2) being modified to include $P\mathbf{j}$ as an unknown quantity:

$$-\frac{17.5}{62.5}T_{BAC}+\frac{4}{5}T_{AD}=0 \qquad (1)$$

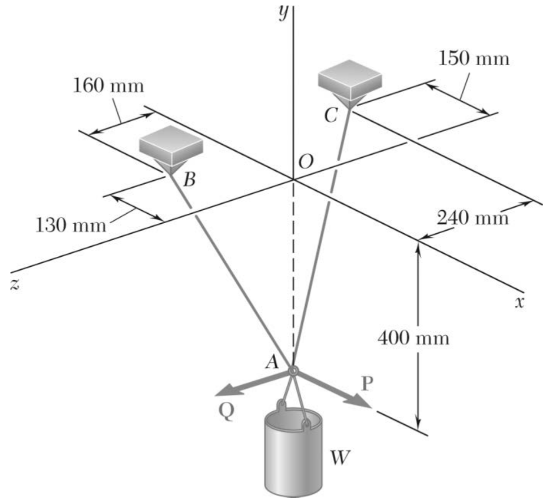
$$\left(\frac{60}{62.5}+\frac{60}{65}\right)T_{BAC}+\frac{3}{5}T_{AD}+\frac{4}{5}T_{AE}-P=0 \qquad (2)$$

$$\frac{25}{65}T_{BAC}-\frac{3}{5}T_{AE}=0 \quad (3)$$

Substituting for $T_{AE}=75\text{ lb}$ and solving simultaneously gives:

$P=305\text{ lb};\ T_{BAC}=117.0\text{ lb};\ T_{AD}=40.9\text{ lb}$ ◀

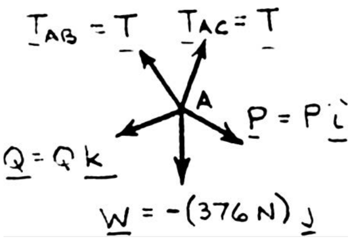
PROBLEM 2.123



A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 376$ N, determine P and Q. (*Hint:* The tension is the same in both portions of cable BAC.)

SOLUTION

Free-Body A:



$$\begin{aligned}\mathbf{T}_{AB} &= T\boldsymbol{\lambda}_{AB}\\ &= T\frac{\overrightarrow{AB}}{AB}\\ &= T\frac{(-130\text{ mm})\mathbf{i} + (400\text{ mm})\mathbf{j} + (160\text{ mm})\mathbf{k}}{450\text{ mm}}\\ &= T\left(-\frac{13}{45}\mathbf{i} + \frac{40}{45}\mathbf{j} + \frac{16}{45}\mathbf{k}\right)\end{aligned}$$

$$\begin{aligned}\mathbf{T}_{AC} &= T\boldsymbol{\lambda}_{AC}\\ &= T\frac{\overrightarrow{AC}}{AC}\\ &= T\frac{(-150\text{ mm})\mathbf{i} + (400\text{ mm})\mathbf{j} + (-240\text{ mm})\mathbf{k}}{490\text{ mm}}\\ &= T\left(-\frac{15}{49}\mathbf{i} + \frac{40}{49}\mathbf{j} - \frac{24}{49}\mathbf{k}\right)\end{aligned}$$

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{Q} + \mathbf{P} + \mathbf{W} = 0$$

Setting coefficients of **i**, **j**, **k** equal to zero:

$$\mathbf{i}: \quad -\frac{13}{45}T - \frac{15}{49}T + P = 0 \qquad 0.59501T = P \tag{1}$$

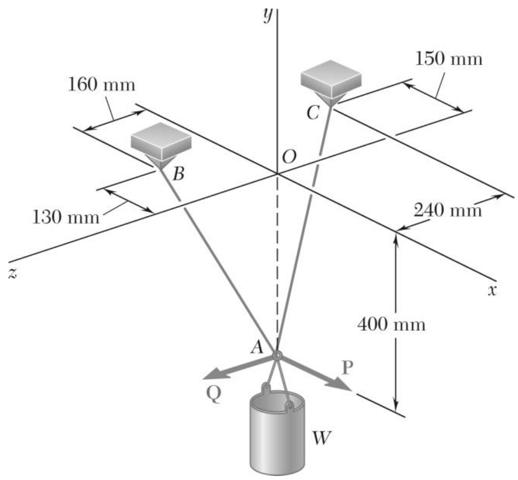
$$\mathbf{j}: \quad +\frac{40}{45}T + \frac{40}{49}T - W = 0 \qquad 1.70521T = W \tag{2}$$

$$\mathbf{k}: \quad +\frac{16}{45}T - \frac{24}{49}T + Q = 0 \qquad 0.134240T = Q \tag{3}$$

Data: $W = 376\text{ N} \quad 1.70521T = 376\text{ N} \quad T = 220.50\text{ N}$

$0.59501(220.50\text{ N}) = P$ $\qquad P = 131.2\text{ N}$ ◀

$0.134240(220.50\text{ N}) = Q$ $\qquad Q = 29.6\text{ N}$ ◀



PROBLEM 2.124

For the system of Problem 2.123, determine W and Q knowing that $P = 164$ N.

PROBLEM 2.123 A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 376$ N, determine P and Q. (*Hint:* The tension is the same in both portions of cable BAC.)

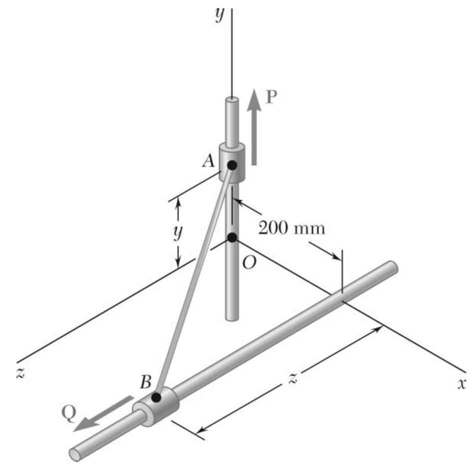
SOLUTION

Refer to Problem 2.123 for the figure and analysis resulting in Equations (1), (2), and (3) for P, W, and Q in terms of T below. Setting $P = 164$ N we have:

Eq. (1): $0.59501T = 164 \text{ N}$ $T = 275.63 \text{ N}$

Eq. (2): $1.70521(275.63 \text{ N}) = W$ $W = 470 \text{ N}$ ◀

Eq. (3): $0.134240(275.63 \text{ N}) = Q$ $Q = 37.0 \text{ N}$ ◀



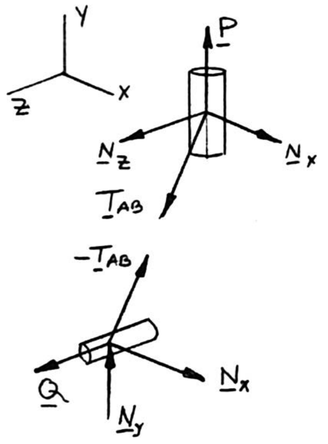
PROBLEM 2.125

Collars A and B are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force $\mathbf{P} = (341\text{ N})\mathbf{j}$ is applied to collar A, determine (*a*) the tension in the wire when $y = 155$ mm, (*b*) the magnitude of the force $\mathbf{Q}$ required to maintain the equilibrium of the system.

SOLUTION

For both Problems 2.125 and 2.126:

Free-Body Diagrams of Collars:



$$(AB)^2 = x^2 + y^2 + z^2$$

Here $$(0.525\text{ m})^2 = (0.20\text{ m})^2 + y^2 + z^2$$

or $$y^2 + z^2 = 0.23563\text{ m}^2$$

Thus, when y given, z is determined,

Now
$$\begin{aligned}\boldsymbol{\lambda}_{AB} &= \frac{\overrightarrow{AB}}{AB}\\ &= \frac{1}{0.525\text{ m}}(0.20\mathbf{i} - y\mathbf{j} + z\mathbf{k})\text{m}\\ &= 0.38095\mathbf{i} - 1.90476y\mathbf{j} + 1.90476z\mathbf{k}\end{aligned}$$

Where y and z are in units of meters, m.

From the F.B. Diagram of collar A: $\quad \Sigma\mathbf{F} = 0\text{:}\quad N_x\mathbf{i} + N_z\mathbf{k} + P\mathbf{j} + T_{AB}\lambda_{AB} = 0$

Setting the $\mathbf{j}$ coefficient to zero gives $\quad P - (1.90476y)T_{AB} = 0$

With
$$P = 341\text{ N}$$
$$T_{AB} = \frac{341\text{ N}}{1.90476y}$$

Now, from the free body diagram of collar B: $\quad \Sigma\mathbf{F} = 0\text{:}\quad N_x\mathbf{i} + N_y\mathbf{j} + Q\mathbf{k} - T_{AB}\boldsymbol{\lambda}_{AB} = 0$

Setting the $\mathbf{k}$ coefficient to zero gives $\quad Q - T_{AB}(1.90476z) = 0$

And using the above result for T_{AB}, we have $\quad Q = T_{AB}z = \dfrac{341\text{ N}}{(1.90476)y}(1.90476z) = \dfrac{(341\text{ N})(z)}{y}$

Then from the specifications of the problem, $y = 155 \text{ mm} = 0.155 \text{ m}$

$$z^2 = 0.23563 \text{ m}^2 - (0.155 \text{ m})^2$$
$$z = 0.46 \text{ m}$$

and

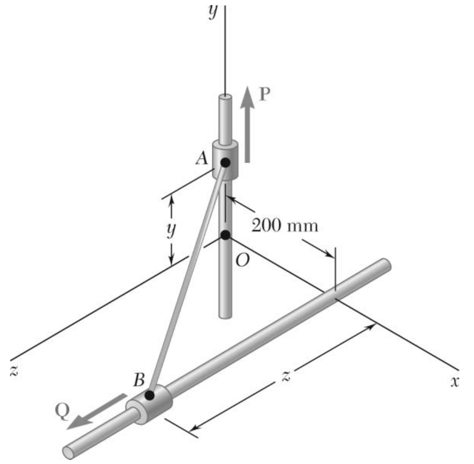
(*a*)
$$T_{AB} = \frac{341 \text{ N}}{0.155(1.90476)}$$
$$= 1155.00 \text{ N}$$

or $T_{AB} = 1155 \text{ N}$ ◀

and

(*b*)
$$Q = \frac{341 \text{ N}(0.46 \text{ m})(0.866)}{(0.155 \text{ m})}$$
$$= (1012.00 \text{ N})$$

or $Q = 1012 \text{ N}$ ◀



PROBLEM 2.126

Solve Problem 2.125 assuming that $y = 275$ mm.

PROBLEM 2.125 Collars *A* and *B* are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force $\mathbf{P} = (341\text{ N})\mathbf{j}$ is applied to collar *A*, determine (*a*) the tension in the wire when $y = 155$ mm, (*b*) the magnitude of the force **Q** required to maintain the equilibrium of the system.

SOLUTION

From the analysis of Problem 2.125, particularly the results:

$$y^2 + z^2 = 0.23563\text{ m}^2$$

$$T_{AB} = \frac{341\text{ N}}{1.90476y}$$

$$Q = \frac{341\text{ N}}{y}z$$

With $y = 275\text{ mm} = 0.275\text{ m}$, we obtain:

$$z^2 = 0.23563\text{ m}^2 - (0.275\text{ m})^2$$

$$z = 0.40\text{ m}$$

and

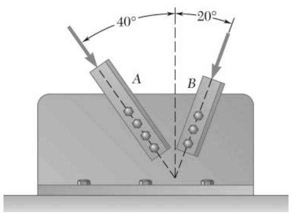
(*a*) $$T_{AB} = \frac{341\text{ N}}{(1.90476)(0.275\text{ m})} = 651.00$$

or $$T_{AB} = 651\text{ N} \blacktriangleleft$$

and

(*b*) $$Q = \frac{341\text{ N}(0.40\text{ m})}{(0.275\text{ m})}$$

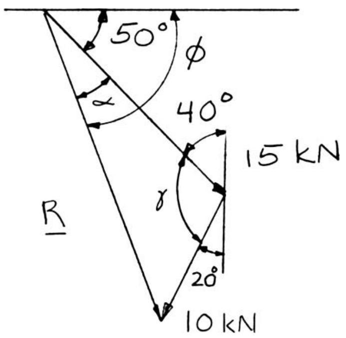
or $$Q = 496\text{ N} \blacktriangleleft$$



PROBLEM 2.127

Two structural members *A* and *B* are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 15 kN in member *A* and 10 kN in member *B*, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members *A* and *B*.

SOLUTION



Using the force triangle and the laws of cosines and sines,

we have $\gamma = 180° - (40° + 20°)$

$= 120°$

Then

$$R^2 = (15\text{ kN})^2 + (10\text{ kN})^2 - 2(15\text{ kN})(10\text{ kN})\cos 120°$$

$$= 475\text{ kN}^2$$

$$R = 21.794\text{ kN}$$

and

$$\frac{10\text{ kN}}{\sin\alpha} = \frac{21.794\text{ kN}}{\sin 120°}$$

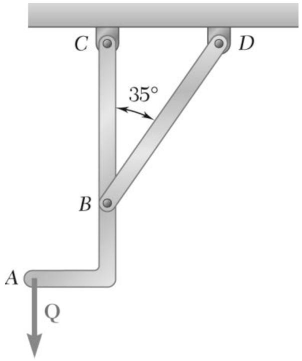
$$\sin\alpha = \left(\frac{10\text{ kN}}{21.794\text{ kN}}\right)\sin 120°$$

$$= 0.39737$$

$$\alpha = 23.414$$

Hence: $\phi = \alpha + 50° = 73.414$

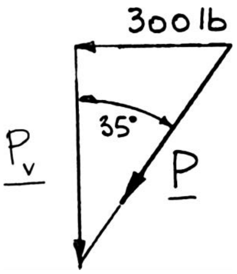
R = 21.8 kN ∠ 73.4° ◀



PROBLEM 2.128

Member *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 300-lb horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

SOLUTION



(*a*) $P \sin 35° = 300 \text{ lb}$

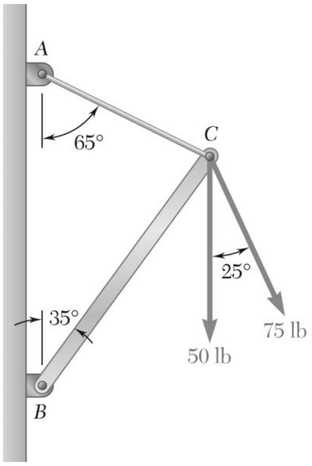
$$P = \frac{300 \text{ lb}}{\sin 35°}$$

$P = 523 \text{ lb}$ ◀

(*b*) Vertical component $P_v = P \cos 35°$

$= (523 \text{ lb}) \cos 35°$

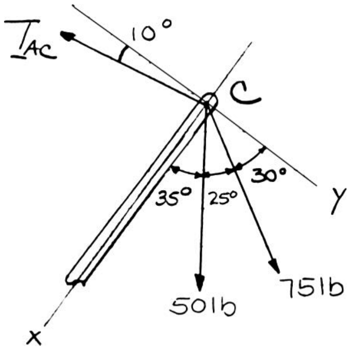
$P_v = 428 \text{ lb}$ ◀



PROBLEM 2.129

Determine (*a*) the required tension in cable *AC*, knowing that the resultant of the three forces exerted at Point *C* of boom *BC* must be directed along *BC*, (*b*) the corresponding magnitude of the resultant.

SOLUTION



Using the *x* and *y* axes shown:

$$R_x = \Sigma F_x = T_{AC}\sin 10° + (50\text{ lb})\cos 35° + (75\text{ lb})\cos 60°$$

$$= T_{AC}\sin 10° + 78.458\text{ lb} \qquad (1)$$

$$R_y = \Sigma F_y = (50\text{ lb})\sin 35° + (75\text{ lb})\sin 60° - T_{AC}\cos 10°$$

$$R_y = 93.631\text{ lb} - T_{AC}\cos 10° \qquad (2)$$

(*a*) Set $R_y = 0$ in Eq. (2):

$$93.631\text{ lb} - T_{AC}\cos 10° = 0$$

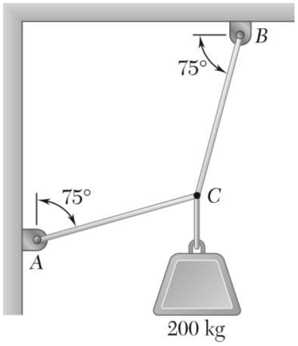
$$T_{AC} = 95.075\text{ lb} \qquad T_{AC} = 95.1\text{ lb} \blacktriangleleft$$

(*b*) Substituting for T_{AC} in Eq. (1):

$$R_x = (95.075\text{ lb})\sin 10° + 78.458\text{ lb}$$

$$= 94.968\text{ lb}$$

$$R = R_x \qquad R = 95.0\text{ lb} \blacktriangleleft$$

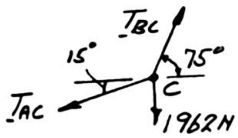


PROBLEM 2.130

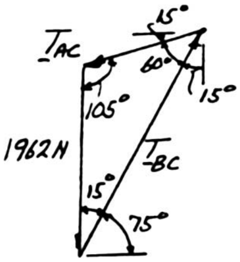
Two cables are tied together at *C* and are loaded as shown. Determine the tension (*a*) in cable *AC*, (*b*) in cable *BC*.

SOLUTION

Free-Body Diagram



Force Triangle



$$\begin{aligned} W &= \text{mg} \\ &= (200\text{ kg})(9.81\text{ m/s}^2) \\ &= 1962\text{ N} \end{aligned}$$

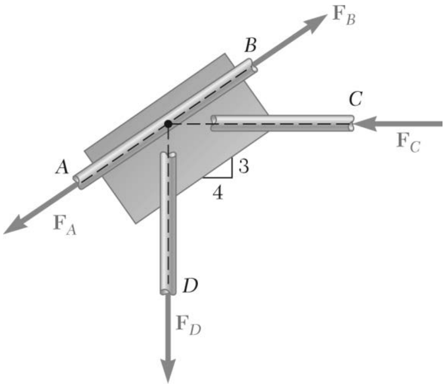
Law of sines:
$$\frac{T_{AC}}{\sin 15°} = \frac{T_{BC}}{\sin 105°} = \frac{1962\text{ N}}{\sin 60°}$$

(*a*)
$$T_{AC} = \frac{(1962\text{ N})\ \sin 15°}{\sin 60°}$$

$T_{AC} = 586\text{ N}$ ◀

(*b*)
$$T_{BC} = \frac{(1962\text{ N})\sin 105°}{\sin 60°}$$

$T_{BC} = 2190\text{ N}$ ◀

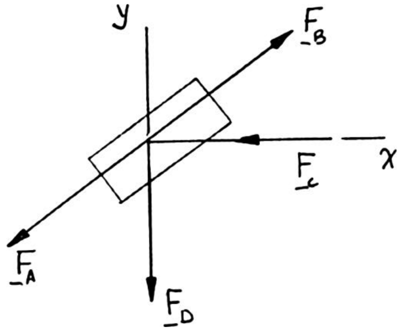


PROBLEM 2.131

A welded connection is in equilibrium under the action of the four forces shown. Knowing that $F_A = 8\text{ kN}$ and $F_B = 16\text{ kN}$, determine the magnitudes of the other two forces.

SOLUTION

Free-Body Diagram of Connection



$$\Sigma F_x = 0\text{:}\quad \frac{3}{5}F_B - F_C - \frac{3}{5}F_A = 0$$

With $F_A = 8\text{ kN}$

$F_B = 16\text{ kN}$

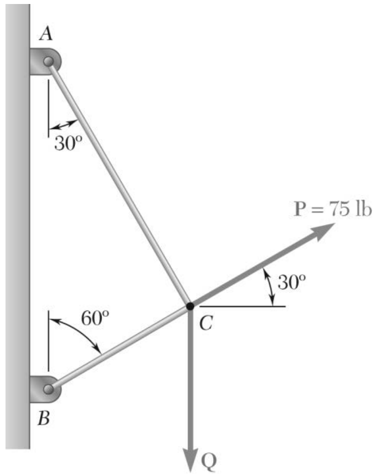
$$F_C = \frac{4}{5}(16\text{ kN}) - \frac{4}{5}(8\text{ kN})$$

$F_C = 6.40\text{ kN}$ ◀

$$\Sigma F_y = 0\text{:}\quad -F_D + \frac{3}{5}F_B - \frac{3}{5}F_A = 0$$

With F_A and F_B as above: $$F_D = \frac{3}{5}(16\text{ kN}) - \frac{3}{5}(8\text{ kN})$$

$F_D = 4.80\text{ kN}$ ◀

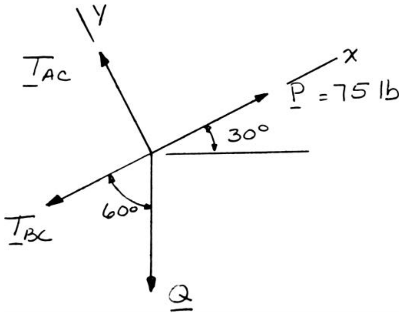


PROBLEM 2.132

Two cables tied together at C are loaded as shown. Determine the range of values of Q for which the tension will not exceed 60 lb in either cable.

SOLUTION

Free-Body Diagram



$$\Sigma F_x = 0: \quad -T_{BC} - Q\cos 60° + 75 \text{ lb} = 0$$

$$T_{BC} = 75 \text{ lb} - Q \cos 60° \qquad (1)$$

$$\Sigma F_y = 0: \quad T_{AC} - Q\sin 60° = 0$$

$$T_{AC} = Q\sin 60° \qquad (2)$$

Requirement: $T_{AC} \leq 60 \text{ lb}$:

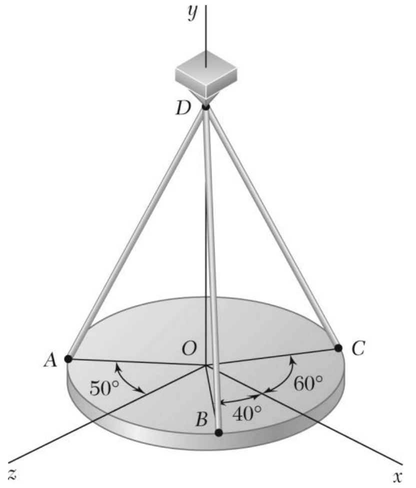
From Eq. (2): $Q\sin 60° \leq 60 \text{ lb}$

$$Q \leq 69.3 \text{ lb}$$

Requirement: $T_{BC} \leq 60 \text{ lb}$:

From Eq. (1): $75 \text{ lb} - Q\sin 60° \leq 60 \text{ lb}$

$$Q \geq 30.0 \text{ lb} \qquad 30.0 \text{ lb} \leq Q \leq 69.3 \text{ lb} \blacktriangleleft$$



PROBLEM 2.133

A horizontal circular plate is suspended as shown from three wires that are attached to a support at *D* and form 30° angles with the vertical. Knowing that the *x* component of the force exerted by wire *AD* on the plate is 110.3 N, determine (*a*) the tension in wire *AD*, (*b*) the angles θ_x, θ_y, and θ_z that the force exerted at *A* forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = F \sin 30° \sin 50° = 110.3 \text{ N} \quad \text{(Given)}$$

$$F = \frac{110.3 \text{ N}}{\sin 30° \sin 50°} = 287.97 \text{ N} \qquad F = 288 \text{ N} \blacktriangleleft$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{110.3 \text{ N}}{287.97 \text{ N}} = 0.38303 \qquad \theta_x = 67.5° \blacktriangleleft$$

$$F_y = F \cos 30° = 249.39$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{249.39 \text{ N}}{287.97 \text{ N}} = 0.86603 \qquad \theta_y = 30.0° \blacktriangleleft$$

$$\begin{aligned} F_z &= -F \sin 30° \cos 50° \\ &= -(287.97 \text{ N}) \sin 30° \cos 50° \\ &= -92.552 \text{ N} \end{aligned}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-92.552 \text{ N}}{287.97 \text{ N}} = -0.32139 \qquad \theta_z = 108.7° \blacktriangleleft$$

PROBLEM 2.134

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_y = 55°$ and $\theta_z = 45°$. Knowing that the x component of the force is −500 lb, determine (*a*) the angle θ_x, (*b*) the other components and the magnitude of the force.

SOLUTION

(*a*) We have

$$(\cos\theta_x)^2 + (\cos\theta_y)^2 + (\cos\theta_z)^2 = 1 \Rightarrow (\cos\theta_y)^2 = 1 - (\cos\theta_y)^2 - (\cos\theta_z)^2$$

Since $F_x < 0,$ we must have $\cos\theta_x < 0.$

Thus, taking the negative square root, from above, we have

$$\cos\theta_x = -\sqrt{1-(\cos 55)^2 - (\cos 45)^2} = 0.41353$$

$\theta_x = 114.4°$ ◀

(*b*) Then

$$F = \frac{F_x}{\cos\theta_x} = \frac{500 \text{ lb}}{0.41353} = 1209.10 \text{ lb}$$

$F = 1209 \text{ lb}$ ◀

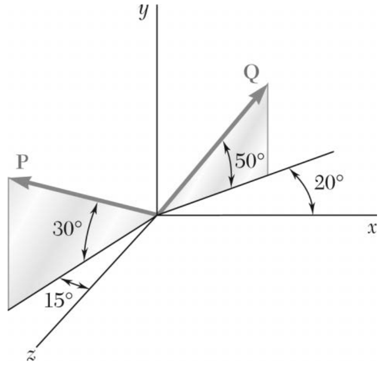
and

$$F_y = F\cos\theta_y = (1209.10 \text{ lb})\cos 55°$$

$F_y = 694 \text{ lb}$ ◀

$$F_z = F\cos\theta_z = (1209.10 \text{ lb})\cos 45°$$

$F_z = 855 \text{ lb}$ ◀



PROBLEM 2.135

Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 300$ N and $Q = 400$ N.

SOLUTION

$$\mathbf{P} = (300\text{ N})[-\cos 30°\sin 15°\mathbf{i} + \sin 30°\mathbf{j} + \cos 30°\cos 15°\mathbf{k}]$$
$$= -(67.243\text{ N})\mathbf{i} + (150\text{ N})\mathbf{j} + (250.95\text{ N})\mathbf{k}$$
$$\mathbf{Q} = (400\text{ N})[\cos 50°\cos 20°\mathbf{i} + \sin 50°\mathbf{j} - \cos 50°\sin 20°\mathbf{k}]$$
$$= (400\text{ N})[0.60402\mathbf{i} + 0.76604\mathbf{j} - 0.21985]$$
$$= (241.61\text{ N})\mathbf{i} + (306.42\text{ N})\mathbf{j} - (87.939\text{ N})\mathbf{k}$$
$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$
$$= (174.367\text{ N})\mathbf{i} + (456.42\text{ N})\mathbf{j} + (163.011\text{ N})\mathbf{k}$$
$$R = \sqrt{(174.367\text{ N})^2 + (456.42\text{ N})^2 + (163.011\text{ N})^2}$$
$$= 515.07\text{ N}$$

$R = 515$ N ◀

$$\cos\theta_x = \frac{R_x}{R} = \frac{174.367\text{ N}}{515.07\text{ N}} = 0.33853$$

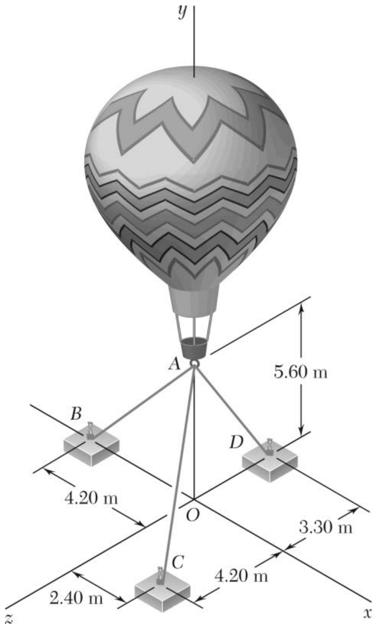
$\theta_x = 70.2°$ ◀

$$\cos\theta_y = \frac{R_y}{R} = \frac{456.42\text{ N}}{515.07\text{ N}} = 0.88613$$

$\theta_y = 27.6°$ ◀

$$\cos\theta_z = \frac{R_z}{R} = \frac{163.011\text{ N}}{515.07\text{ N}} = 0.31648$$

$\theta_z = 71.5°$ ◀



PROBLEM 2.136

Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at *A* knowing that the tension in cable *AC* is 444 N.

SOLUTION

See Problem 2.101 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \tag{1}$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \tag{2}$$

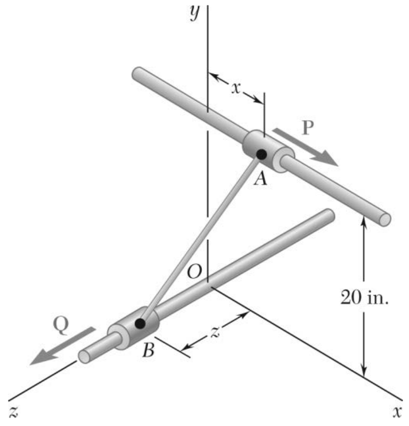
$$0.56757T_{AC} - 0.50769T_{AD} = 0 \tag{3}$$

Substituting $T_{AC} = 444\text{ N}$ in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives

$$T_{AB} = 240\text{ N}$$

$$T_{AD} = 496.36\text{ N}$$

$\mathbf{P} = 956\text{ N}\uparrow$ ◀



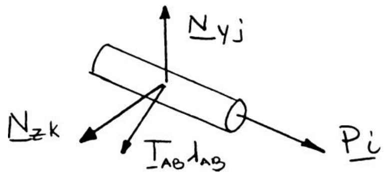
PROBLEM 2.137

Collars *A* and *B* are connected by a 25-in.-long wire and can slide freely on frictionless rods. If a 60-lb force **Q** is applied to collar *B* as shown, determine (*a*) the tension in the wire when $x = 9$ in., (*b*) the corresponding magnitude of the force **P** required to maintain the equilibrium of the system.

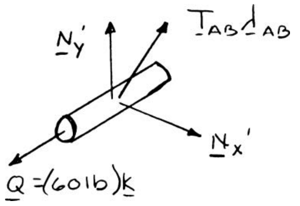
SOLUTION

Free-Body Diagrams of Collars:

***A*:**



***B*:**



$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{-x\mathbf{i} - (20 \text{ in.})\mathbf{j} + z\mathbf{k}}{25 \text{ in.}}$$

Collar *A*: $\Sigma \mathbf{F} = 0: \quad P\mathbf{i} + N_y\mathbf{j} + N_z\mathbf{k} + T_{AB}\boldsymbol{\lambda}_{AB} = 0$

Substitute for $\boldsymbol{\lambda}_{AB}$ and set coefficient of **i** equal to zero:

$$P - \frac{T_{AB}x}{25 \text{ in.}} = 0 \tag{1}$$

Collar *B*: $\Sigma \mathbf{F} = 0: \quad (60 \text{ lb})\mathbf{k} + N_x'\mathbf{i} + N_y'\mathbf{j} - T_{AB}\boldsymbol{\lambda}_{AB} = 0$

Substitute for $\boldsymbol{\lambda}_{AB}$ and set coefficient of **k** equal to zero:

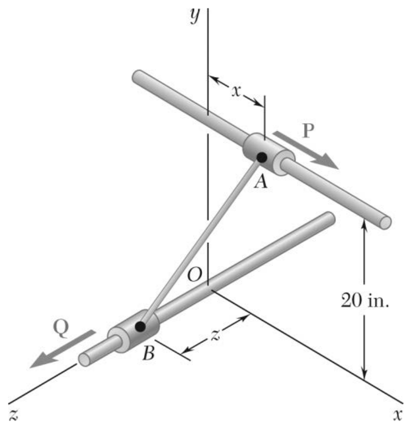
$$60 \text{ lb} - \frac{T_{AB}z}{25 \text{ in.}} = 0 \tag{2}$$

(*a*) $x = 9$ in. $\quad (9 \text{ in.})^2 + (20 \text{ in.})^2 + z^2 = (25 \text{ in.})^2$

$$z = 12 \text{ in.}$$

From Eq. (2): $\dfrac{60 \text{ lb} - T_{AB}(12 \text{ in.})}{25 \text{ in.}}$ $\qquad T_{AB} = 125.0 \text{ lb}$ ◀

(*b*) From Eq. (1): $P = \dfrac{(125.0 \text{ lb})(9 \text{ in.})}{25 \text{ in.}}$ $\qquad P = 45.0 \text{ lb}$ ◀



PROBLEM 2.138

Collars A and B are connected by a 25-in.-long wire and can slide freely on frictionless rods. Determine the distances x and z for which the equilibrium of the system is maintained when $P = 120$ lb and $Q = 60$ lb.

SOLUTION

See Problem 2.137 for the diagrams and analysis leading to Equations (1) and (2) below:

$$P = \frac{T_{AB}x}{25 \text{ in.}} = 0 \tag{1}$$

$$60 \text{ lb} - \frac{T_{AB}z}{25 \text{ in.}} = 0 \tag{2}$$

For $P = 120$ lb, Eq. (1) yields $$T_{AB}x = (25 \text{ in.})(20 \text{ lb}) \tag{1'}$$

From Eq. (2): $$T_{AB}z = (25 \text{ in.})(60 \text{ lb}) \tag{2'}$$

Dividing Eq. (1′) by (2′), $$\frac{x}{z} = 2 \tag{3}$$

Now write $$x^2 + z^2 + (20 \text{ in.})^2 = (25 \text{ in.})^2 \tag{4}$$

Solving (3) and (4) simultaneously,

$$4z^2 + z^2 + 400 = 625$$

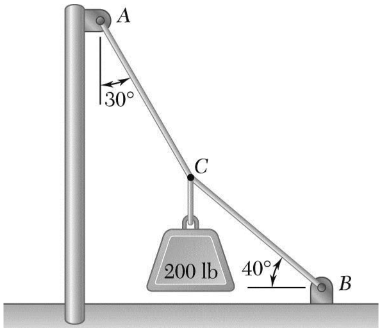
$$z^2 = 45$$

$$z = 6.7082 \text{ in.}$$

From Eq. (3): $$x = 2z = 2(6.7082 \text{ in.})$$

$$= 13.4164 \text{ in.}$$

$$x = 13.42 \text{ in.}, \quad z = 6.71 \text{ in.} \blacktriangleleft$$

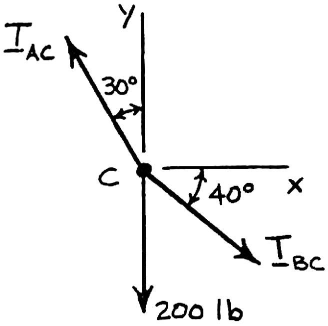


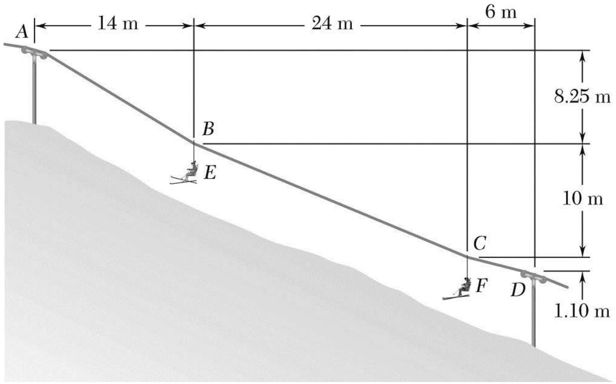
PROBLEM 2F1

Two cables are tied together at *C* and loaded as shown. Draw the free-body diagram needed to determine the tension in *AC* and *BC*.

SOLUTION

Free-Body Diagram of Point *C*:



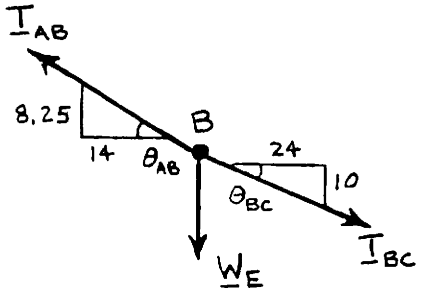


PROBLEM 2.F2

A chairlift has been stopped in the position shown. Knowing that each chair weighs 250 N and that the skier in chair *E* weighs 765 N, draw the free-body diagrams needed to determine the weight of the skier in chair *F*.

SOLUTION

Free-Body Diagram of Point *B*:



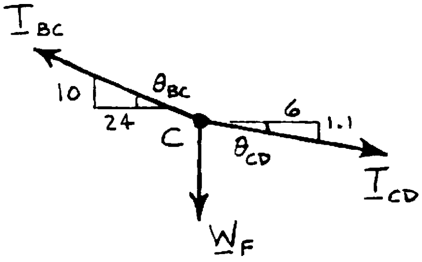
$$W_E = 250\text{ N} + 765\text{ N} = 1015\text{ N}$$

$$\theta_{AB} = \tan^{-1}\frac{8.25}{14} = 30.510°$$

$$\theta_{BC} = \tan^{-1}\frac{10}{24} = 22.620°$$

Use this free body to determine T_{AB} and T_{BC}.

Free-Body Diagram of Point *C*:

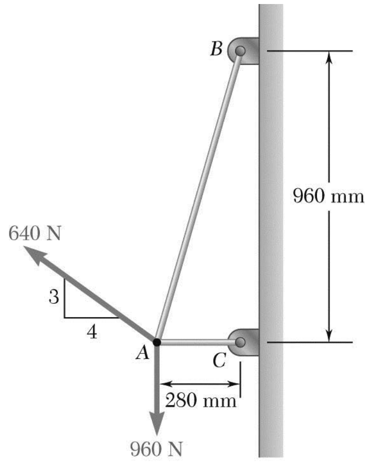


$$\theta_{CD} = \tan^{-1}\frac{1.1}{6} = 10.3889°$$

Use this free body to determine T_{CD} and W_F.

Then weight of skier W_S is found by

$$W_S = W_F - 250\text{ N} \blacktriangleleft$$

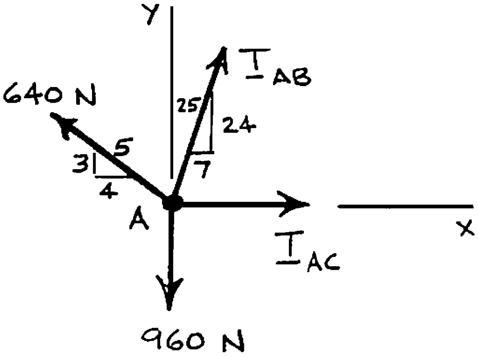


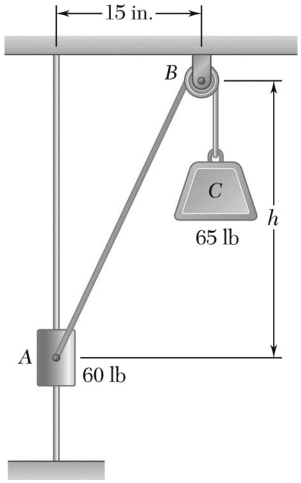
PROBLEM 2.F3

Two cables are tied together at *A* and loaded as shown. Draw the free-body diagram needed to determine the tension in each cable.

SOLUTION

Free-Body Diagram of Point *A*:



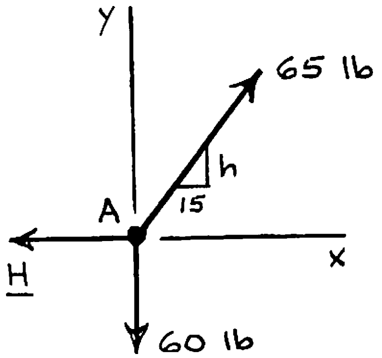


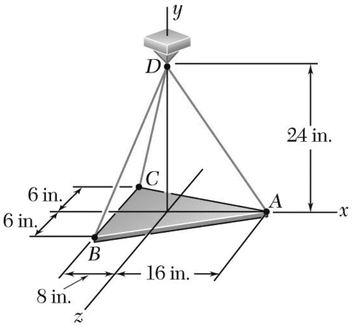
PROBLEM 2.F4

The 60-lb collar *A* can slide on a frictionless vertical rod and is connected as shown to a 65-lb counterweight *C*. Draw the free-body diagram needed to determine the value of *h* for which the system is in equilibrium.

SOLUTION

Free-Body Diagram of Point *A*:



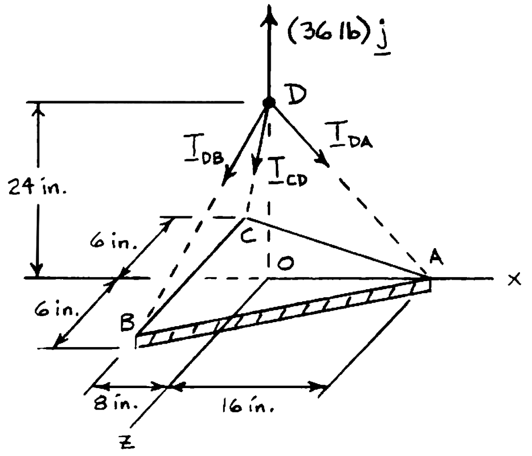


PROBLEM 2.F5

A 36-lb triangular plate is supported by three cables as shown. Draw the free-body diagram needed to determine the tension in each wire.

SOLUTION

Free-Body Diagram of Point *D*:



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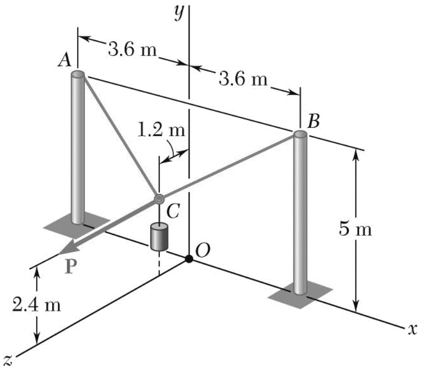
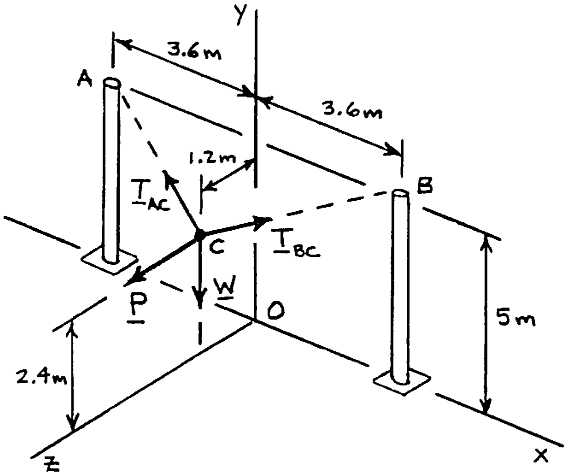


PROBLEM 2.F6

A 70-kg cylinder is supported by two cables *AC* and *BC*, which are attached to the top of vertical posts. A horizontal force **P**, perpendicular to the plane containing the posts, holds the cylinder in the position shown. Draw the free-body diagram needed to determine the magnitude of **P** and the force in each cable.

SOLUTION

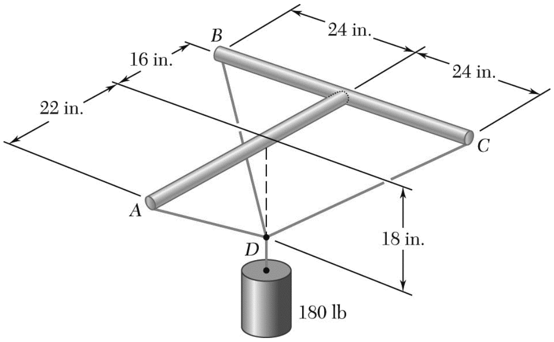
Free-Body Diagram of Point *C*:



$$W = (70\text{ kg})(9.81\text{ m/s}^2)$$

$$= 686.7\text{ N}$$

$$\mathbf{W} = -(686.7\text{ N})\mathbf{j}$$

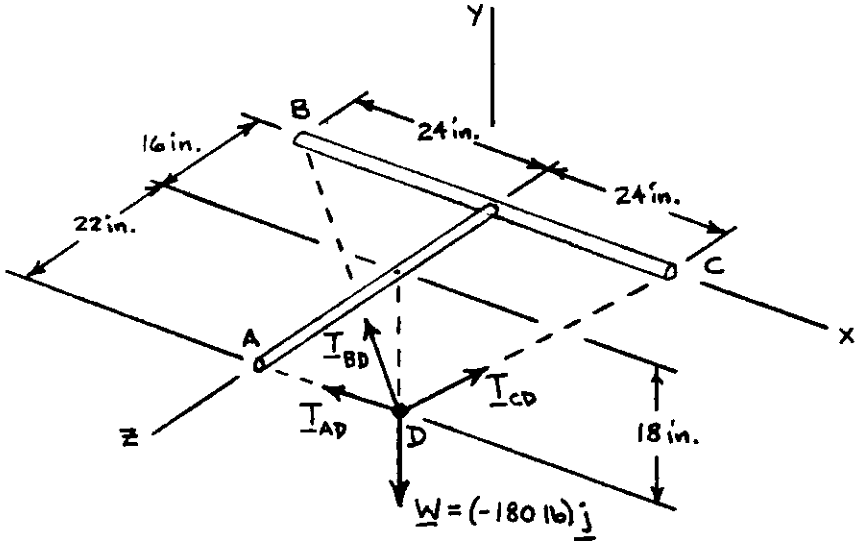


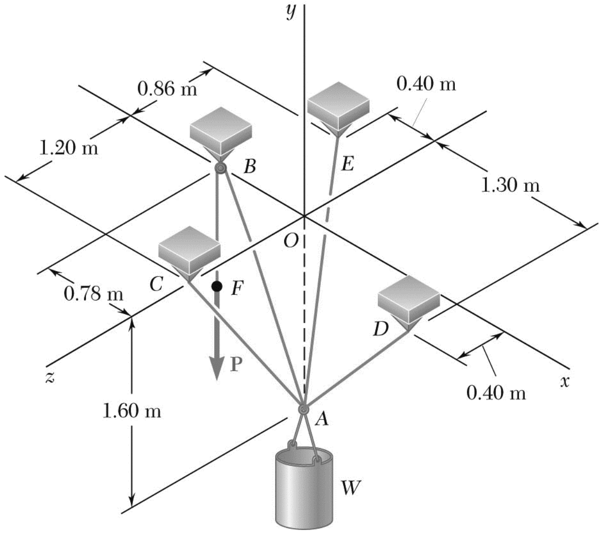
PROBLEM 2.F7

Three cables are connected at point *D*, which is located 18 in. below the T-shaped pipe support *ABC*. The cables support a 180-lb cylinder as shown. Draw the free-body diagram needed to determine the tension in each cable.

SOLUTION

Free-Body Diagram of Point *D*:



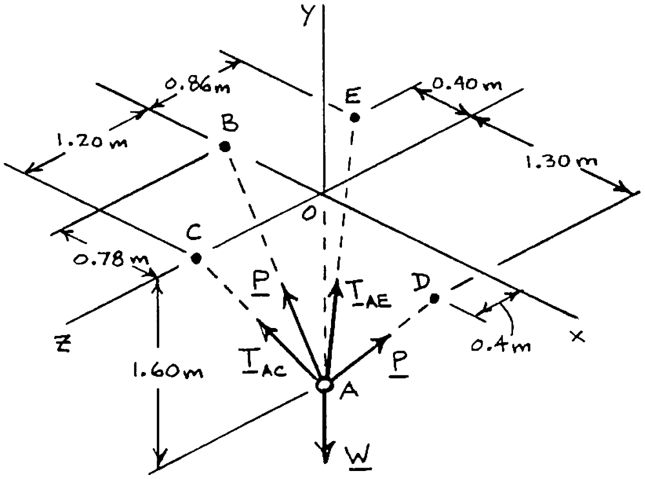


PROBLEM 2.F8

A 100-kg container is suspended from ring *A*, to which cables *AC* and *AE* are attached. A force *P* is applied to end *F* of a third cable that passes over a pulley at *B* and through ring *A* and then is attached to a support at *D*. Draw the free-body diagram needed to determine the magnitude of *P*. (Hint: The tension is the same in all portions of cable *FBAD*.)

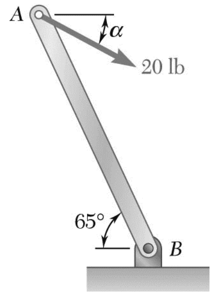
SOLUTION

Free-Body Diagram of Ring *A*:



$$W = (100\text{ kg})(9.81\text{ m/s}^2)$$
$$= 981\text{ N}$$
$$\mathbf{W} = -(681\text{ N})\mathbf{j}$$

CHAPTER 3

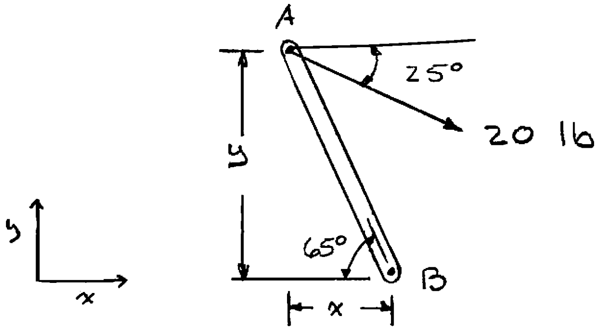


PROBLEM 3.1

A 20-lb force is applied to the control rod *AB* as shown. Knowing that the length of the rod is 9 in. and that $\alpha = 25°$, determine the moment of the force about Point *B* by resolving the force into horizontal and vertical components.

SOLUTION

Free-Body Diagram of Rod *AB*:



$$x = (9 \text{ in.})\cos 65° = 3.8036 \text{ in.}$$

$$y = (9 \text{ in.})\sin 65° = 8.1568 \text{ in.}$$

$$\begin{aligned}\mathbf{F} &= F_x\mathbf{i} + F_y\mathbf{j}\\ &= (20 \text{ lb} \cos 25°)\mathbf{i} + (-20 \text{ lb} \sin 25°)\mathbf{j}\\ &= (18.1262 \text{ lb})\mathbf{i} - (8.4524 \text{ lb})\mathbf{j}\end{aligned}$$

$$\mathbf{r}_{A/B} = \overrightarrow{BA} = (-3.8036 \text{ in.})\mathbf{i} + (8.1568 \text{ in.})\mathbf{j}$$

$$\begin{aligned}\mathbf{M}_B &= \mathbf{r}_{A/B} \times \mathbf{F}\\ &= (-3.8036\mathbf{i} + 8.1568\mathbf{j}) \times (18.1262\mathbf{i} - 8.4524\mathbf{j})\\ &= 32.150\mathbf{k} - 147.852\mathbf{k}\\ &= -115.702 \text{ lb-in.}\end{aligned}$$

$\mathbf{M}_B = 115.7$ lb-in. $\circlearrowright$ ◀

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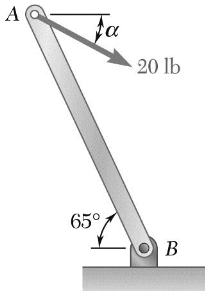
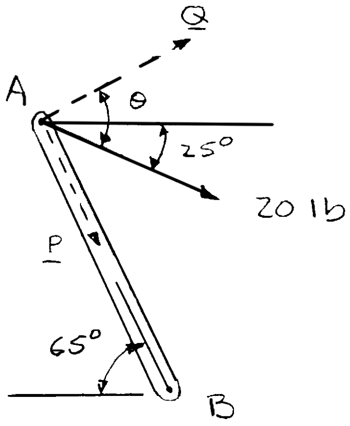


PROBLEM 3.2

A 20-lb force is applied to the control rod *AB* as shown. Knowing that the length of the rod is 9 in. and that $\alpha = 25°$, determine the moment of the force about Point *B* by resolving the force into components along *AB* and in a direction perpendicular to *AB*.

SOLUTION

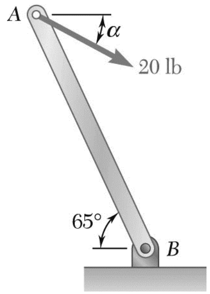
Free-Body Diagram of Rod *AB*:



$$\theta = 90° - (65° - 25°)$$
$$= 50°$$

$$Q = (20\text{ lb})\cos 50°$$
$$= 12.8558\text{ lb}$$
$$M_B = Q(9\text{ in.})$$
$$= (12.8558\text{ lb})(9\text{ in.})$$
$$= 115.702\text{ lb-in.}$$

$\mathbf{M}_B = 115.7\text{ lb-in.} \circlearrowright$ ◀

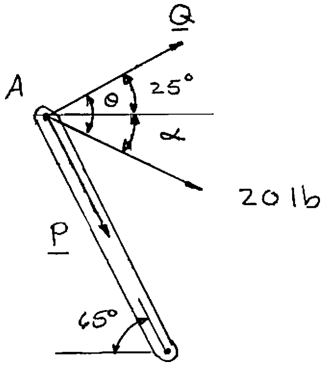


PROBLEM 3.3

A 20-lb force is applied to the control rod *AB* as shown. Knowing that the length of the rod is 9 in. and that the moment of the force about *B* is 120 lb · in. clockwise, determine the value of *α*.

SOLUTION

Free-Body Diagram of Rod *AB*:



$$\alpha = \theta - 25°$$

$$Q = (20 \text{ lb})\cos\theta$$

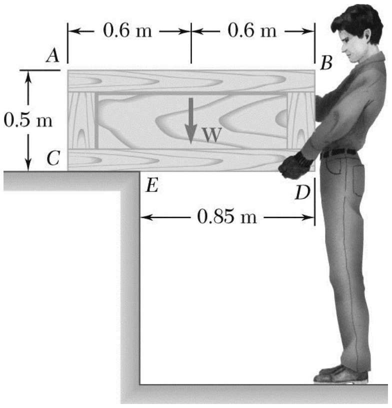
and $$M_B = (Q)(9 \text{ in.})$$

Therefore, $$120 \text{ lb-in.} = (20 \text{ lb})(\cos\theta)(9 \text{ in.})$$

$$\cos\theta = \frac{120 \text{ lb-in.}}{180 \text{ lb-in.}}$$

or $$\theta = 48.190°$$

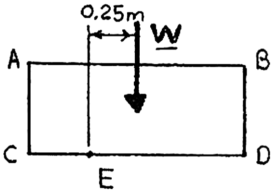
Therefore, $$\alpha = 48.190° - 25°$$ $$\alpha = 23.2° \blacktriangleleft$$



PROBLEM 3.4

A crate of mass 80 kg is held in the position shown. Determine (*a*) the moment produced by the weight **W** of the crate about *E*, (*b*) the smallest force applied at *B* that creates a moment of equal magnitude and opposite sense about *E*.

SOLUTION

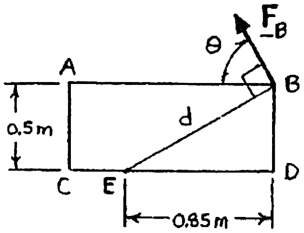


(*a*) By definition, $W = mg = 80\text{ kg}(9.81\text{ m/s}^2) = 784.8\text{ N}$

We have $\Sigma M_E\text{:}\quad M_E = (784.8\text{ N})(0.25\text{ m})$

$\mathbf{M}_E = 196.2\text{ N}\cdot\text{m} \circlearrowright$ ◀

(*b*) For the force at *B* to be the smallest, resulting in a moment ($\mathbf{M}_E$) about *E*, the line of action of force $\mathbf{F}_B$ must be perpendicular to the line connecting *E* to *B*. The sense of $\mathbf{F}_B$ must be such that the force produces a counterclockwise moment about *E*.



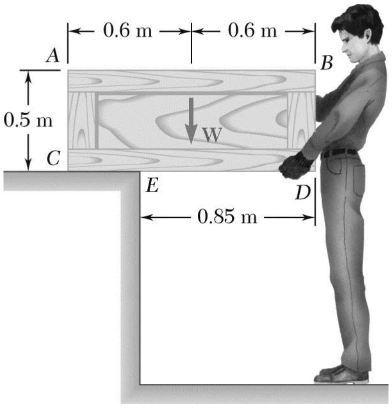
Note: $d = \sqrt{(0.85\text{ m})^2 + (0.5\text{ m})^2} = 0.98615\text{ m}$

We have $\Sigma M_E\text{:}\quad 196.2\text{ N}\cdot\text{m} = F_B(0.98615\text{ m})$

$F_B = 198.954\text{ N}$

and $\theta = \tan^{-1}\left(\dfrac{0.85\text{ m}}{0.5\text{ m}}\right) = 59.534^\circ$

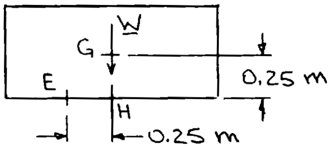
or $\mathbf{F}_B = 199.0\text{ N} \measuredangle\ 59.5^\circ$ ◀



PROBLEM 3.5

A crate of mass 80 kg is held in the position shown. Determine (*a*) the moment produced by the weight **W** of the crate about *E*, (*b*) the smallest force applied at *A* that creates a moment of equal magnitude and opposite sense about *E*, (*c*) the magnitude, sense, and point of application on the bottom of the crate of the smallest vertical force that creates a moment of equal magnitude and opposite sense about *E*.

SOLUTION

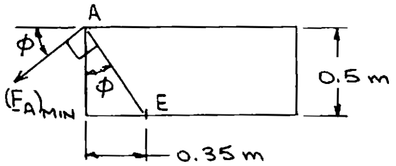


First note. . .

$$W = \text{mg} = (80\text{ kg})(9.81\text{ m/s}^2) = 784.8\text{ N}$$

(*a*) We have $M_E = r_{H/E}W = (0.25\text{ m})(784.8\text{ N}) = 196.2\text{ N}\cdot\text{m}$ or $\mathbf{M}_E = 196.2\text{ N}\cdot\text{m} \circlearrowright$ ◀

(*b*)



For $\mathbf{F}_A$ to be minimum, it must be perpendicular to the line joining Points *A* and *E*. Then with $\mathbf{F}_A$ directed as shown, we have $(-M_E) = r_{A/E}(F_A)_{\text{min}}$.

Where $r_{A/E} = \sqrt{(0.35\text{ m})^2 + (0.5\text{ m})^2} = 0.61033\text{ m}$

then $196.2\text{ N}\cdot\text{m} = (0.61033\text{ m})(F_A)_{\text{min}}$

or $(F_A)_{\text{min}} = 321\text{ N}$

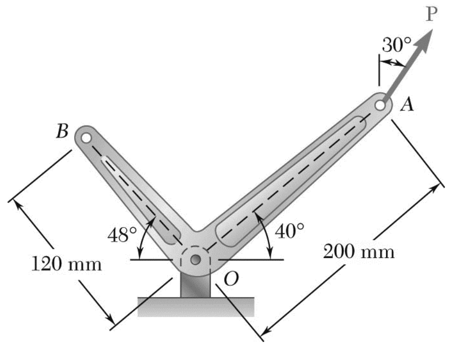
Also $\tan\phi = \dfrac{0.35\text{ m}}{0.5\text{ m}}$ or $\phi = 35.0^\circ$ $(\mathbf{F}_A)_{\text{min}} = 321\text{ N} \swarrow 35.0^\circ$ ◀

(*c*) For $\mathbf{F}_{vertical}$ to be minimum, the perpendicular distance from its line of action to Point *E* must be maximum. Thus, apply $(\mathbf{F}_{vertical})_{\text{min}}$ at Point *D*, and then

$$(-M_E) = r_{D/E}(F_{vertical})_{\text{min}}$$

$$196.2\text{ N}\cdot\text{m} = (0.85\text{ m})(F_{vertical})_{\text{min}}$$

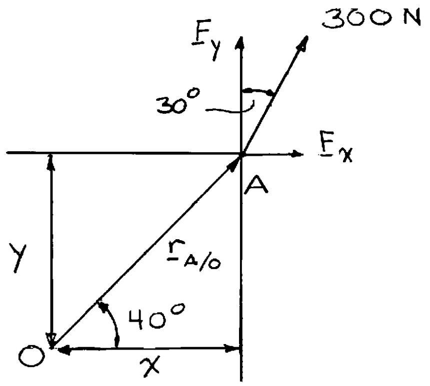
or $(\mathbf{F}_{vertical})_{\text{min}} = 231\text{ N} \uparrow$ at Point *D* ◀



PROBLEM 3.6

A 300-N force **P** is applied at Point *A* of the bell crank shown. (*a*) Compute the moment of the force **P** about *O* by resolving it into horizontal and vertical components. (*b*) Using the result of part (*a*), determine the perpendicular distance from *O* to the line of action of **P**.

SOLUTION



$$x = (0.2\text{ m})\cos 40°$$
$$= 0.153209\text{ m}$$
$$y = (0.2\text{ m})\sin 40°$$
$$= 0.128558\text{ m}$$
$$\therefore\quad \mathbf{r}_{A/O} = (0.153209\text{ m})\mathbf{i} + (0.128558\text{ m})\mathbf{j}$$

(*a*) $F_x = (300\text{ N})\sin 30°$

$= 150\text{ N}$

$F_y = (300\text{ N})\cos 30°$

$= 259.81\text{ N}$

$\mathbf{F} = (150\text{ N})\mathbf{i} + (259.81\text{ N})\mathbf{j}$

$\mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{F}$

$= (0.153209\mathbf{i} + 0.128558\mathbf{j})\text{ m} \times (150\mathbf{i} + 259.81\mathbf{j})\text{ N}$

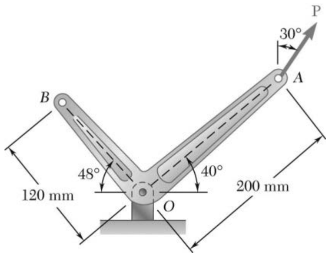
$= (39.805\mathbf{k} - 19.2837\mathbf{k})\text{ N}\cdot\text{m}$

$= (20.521\text{ N}\cdot\text{m})\mathbf{k}$ $\qquad \mathbf{M}_O = 20.5\text{ N}\cdot\text{m} \circlearrowleft$ ◀

(*b*) $M_O = Fd$

$20.521\text{ N}\cdot\text{m} = (300\text{ N})(d)$

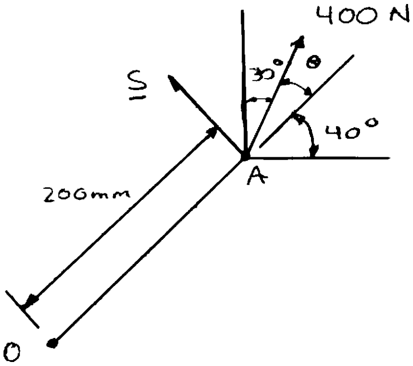
$d = 0.068403\text{ m}$ $\qquad d = 68.4\text{ mm}$ ◀



PROBLEM 3.7

A 400-N force **P** is applied at Point *A* of the bell crank shown. (*a*) Compute the moment of the force **P** about *O* by resolving it into components along line *OA* and in a direction perpendicular to that line. (*b*) Determine the magnitude and direction of the smallest force **Q** applied at *B* that has the same moment as **P** about *O*.

SOLUTION



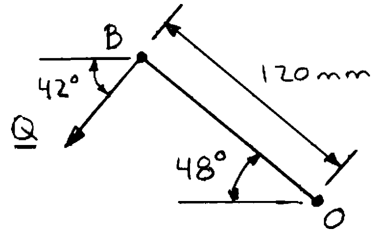
(*a*) Portion *OA* of crank:

$$\theta = 90° - 30° - 40°$$
$$\theta = 20°$$
$$S = P\sin\theta$$
$$= (400\ \text{N})\sin 20°$$
$$= 136.81\ \text{N}$$
$$\text{M}_O = r_{O/A}S$$
$$= (0.2\ \text{m})(136.81\ \text{N})$$
$$= 27.362\ \text{N}\cdot\text{m}$$

$\mathbf{M}_O = 27.4\ \text{N}\cdot\text{m}$ ↻ ◀

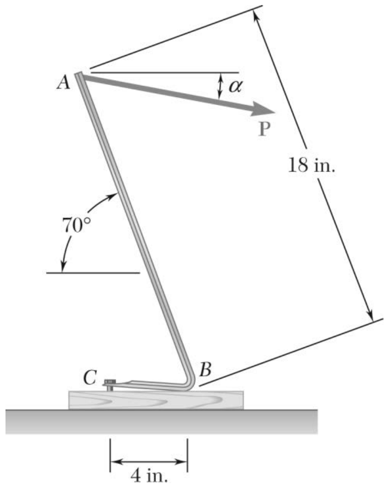
(*b*) Smallest force *Q* must be perpendicular to *OB*.

Portion *OB* of crank:



$$M_O = r_{O/B}Q$$
$$M_O = (0.120\ \text{m})Q$$
$$27.362\ \text{N}\cdot\text{m} = (0.120\ \text{m})Q$$

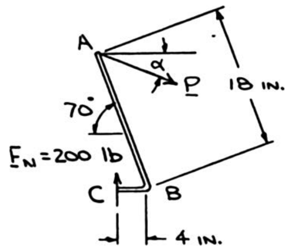
$\mathbf{Q} = 228\ \text{N} \nearrow 42.0°$ ◀



PROBLEM 3.8

It is known that a vertical force of 200 lb is required to remove the nail at *C* from the board. As the nail first starts moving, determine (*a*) the moment about *B* of the force exerted on the nail, (*b*) the magnitude of the force **P** that creates the same moment about *B* if $\alpha = 10°$, (*c*) the smallest force **P** that creates the same moment about *B*.

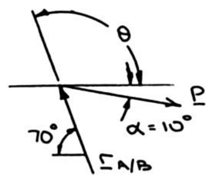
SOLUTION



(*a*) We have

$$\begin{aligned} M_B &= r_{C/B} F_N \\ &= (4 \text{ in.})(200 \text{ lb}) \\ &= 800 \text{ lb} \cdot \text{in.} \end{aligned}$$

or $M_B = 800 \text{ lb} \cdot \text{in.} \circlearrowright$ ◀

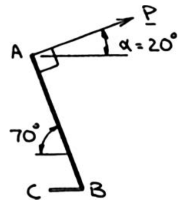


(*b*) By definition,

$$\begin{aligned} M_B &= r_{A/B} P \sin\theta \\ \theta &= 10° + (180° - 70°) \\ &= 120° \end{aligned}$$

Then $800 \text{ lb} \cdot \text{in.} = (18 \text{ in.}) \times P \sin 120°$

or $P = 51.3 \text{ lb}$ ◀



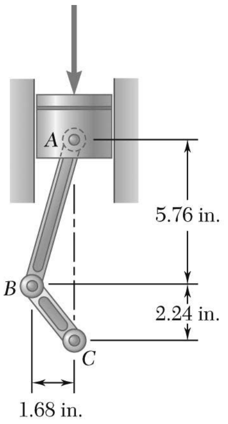
(*c*) For **P** to be minimum, it must be perpendicular to the line joining Points *A* and *B*. Thus, **P** must be directed as shown.

Thus

$$\begin{aligned} M_B &= dP_{\min} \\ d &= r_{A/B} \end{aligned}$$

or $800 \text{ lb} \cdot \text{in.} = (18 \text{ in.}) P_{\min}$

or $P_{\min} = 44.4 \text{ lb}$ $\mathbf{P}_{\min} = 44.4 \text{ lb} \measuredangle 20°$ ◀



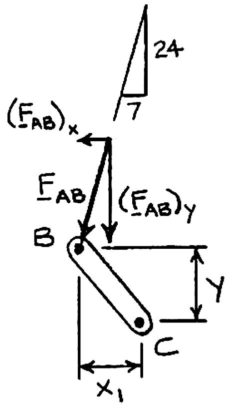
PROBLEM 3.9

It is known that the connecting rod *AB* exerts on the crank *BC* a 500-lb force directed down and to the left along the centerline of *AB*. Determine the moment of the force about *C*.

SOLUTION

Using (*a*):

$$
\begin{aligned}
M_C &= y_1(F_{AB})_x + x_1(F_{AB})_y \\
&= (2.24\text{ in.})\left(\frac{7}{25}\times 500\text{ lb}\right) + (1.68\text{ in.})\left(\frac{24}{25}\times 500\text{ lb}\right) \\
&= 1120\text{ lb}\cdot\text{in.}
\end{aligned}
$$

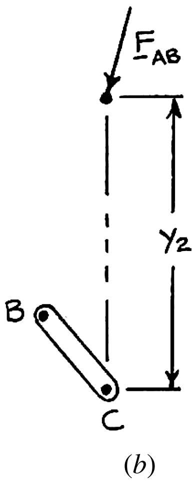


(*a*)

$$\mathbf{M}_C = 1.120\text{ kip}\cdot\text{in.}\circlearrowleft \blacktriangleleft$$

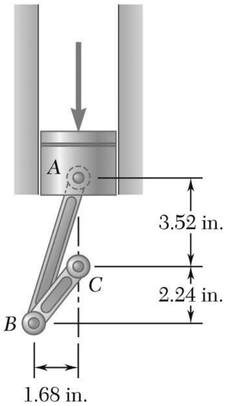
Using (*b*):

$$
\begin{aligned}
M_C &= y_2(F_{AB})_x \\
&= (8\text{ in.})\left(\frac{7}{25}\times 500\text{ lb}\right) \\
&= 1120\text{ lb}\cdot\text{in.}
\end{aligned}
$$



(*b*)

$$\mathbf{M}_C = 1.120\text{ kip}\cdot\text{in.}\circlearrowleft \blacktriangleleft$$



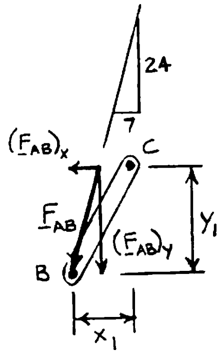
PROBLEM 3.10

It is known that the connecting rod *AB* exerts on the crank *BC* a 500-lb force directed down and to the left along the centerline of *AB*. Determine the moment of the force about *C*.

SOLUTION

Using (*a*):

$$
\begin{aligned}
M_C &= -y_1(F_{AB})_x + x_1(F_{AB})_y \\
&= -(2.24\text{ in.})\left(\frac{7}{25}\times 500\text{ lb}\right) + (1.68\text{ in.})\left(\frac{24}{25}\times 500\text{ lb}\right) \\
&= +492.8\text{ lb}\cdot\text{in.}
\end{aligned}
$$

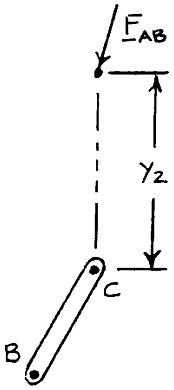


(*a*)

$\mathbf{M}_C = 493\text{ lb}\cdot\text{in.}$ ↺ ◀

Using (*b*):

$$
\begin{aligned}
M_C &= y_2(F_{AB})_x \\
&= (3.52\text{ in.})\left(\frac{7}{25}\times 500\text{ lb}\right) \\
&= +492.8\text{ lb}\cdot\text{in.}
\end{aligned}
$$

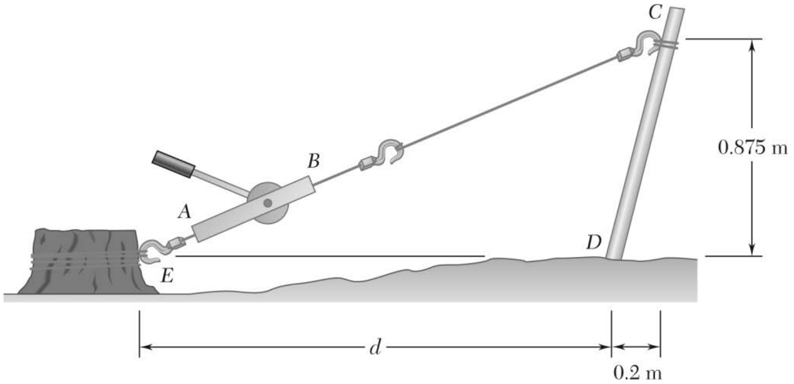


(*b*)

$\mathbf{M}_C = 493\text{ lb}\cdot\text{in.}$ ↺ ◀

PROBLEM 3.11

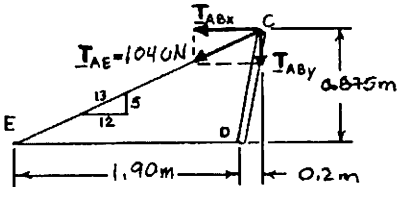
A winch puller *AB* is used to straighten a fence post. Knowing that the tension in cable *BC* is 1040 N and length *d* is 1.90 m, determine the moment about *D* of the force exerted by the cable at *C* by resolving that force into horizontal and vertical components applied (*a*) at Point *C*, (*b*) at Point *E*.



SOLUTION

(*a*) Slope of line:

$$EC = \frac{0.875\text{ m}}{1.90\text{ m} + 0.2\text{ m}} = \frac{5}{12}$$



(*a*)

Then

$$T_{ABx} = \frac{12}{13}(T_{AB})$$

$$= \frac{12}{13}(1040\text{ N})$$

$$= 960\text{ N}$$

and

$$T_{ABy} = \frac{5}{13}(1040\text{ N})$$

$$= 400\text{ N}$$

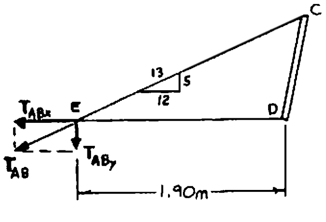
Then

$$M_D = T_{ABx}(0.875\text{ m}) - T_{ABy}(0.2\text{ m})$$

$$= (960\text{ N})(0.875\text{ m}) - (400\text{ N})(0.2\text{ m})$$

$$= 760\text{ N}\cdot\text{m}$$

or $\mathbf{M}_D = 760\text{ N}\cdot\text{m} \circlearrowright$ ◀



(*b*)

(*b*) We have

$$M_D = T_{ABx}(y) + T_{ABx}(x)$$

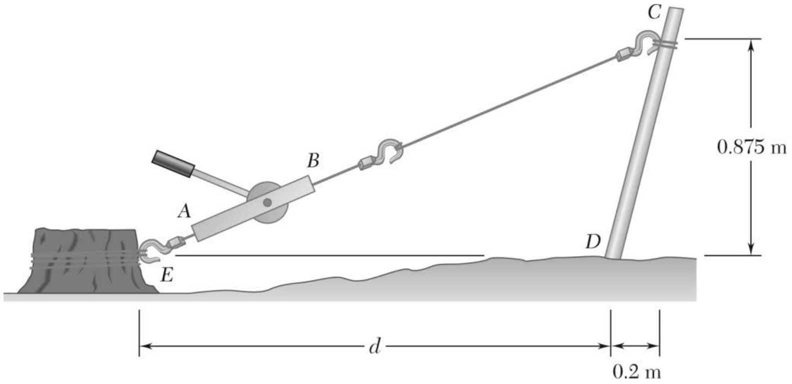
$$= (960\text{ N})(0) + (400\text{ N})(1.90\text{ m})$$

$$= 760\text{ N}\cdot\text{m}$$

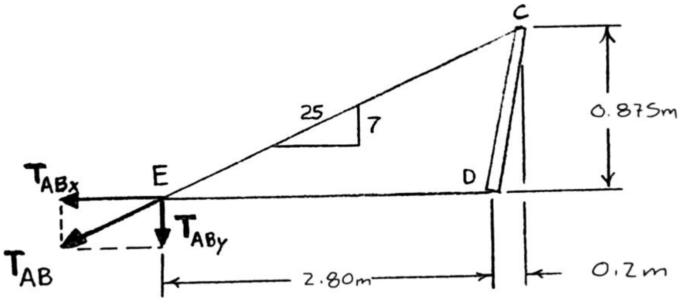
or $\mathbf{M}_D = 760\text{ N}\cdot\text{m} \circlearrowright$ ◀

PROBLEM 3.12

It is known that a force with a moment of 960 N · m about *D* is required to straighten the fence post *CD*. If *d* = 2.80 m, determine the tension that must be developed in the cable of winch puller *AB* to create the required moment about Point *D*.



SOLUTION



Slope of line: $$EC = \frac{0.875 \text{ m}}{2.80 \text{ m} + 0.2 \text{ m}} = \frac{7}{24}$$

Then $$T_{ABx} = \frac{24}{25} T_{AB}$$

and $$T_{ABy} = \frac{7}{25} T_{AB}$$

We have $$M_D = T_{ABx}(y) + T_{ABy}(x)$$

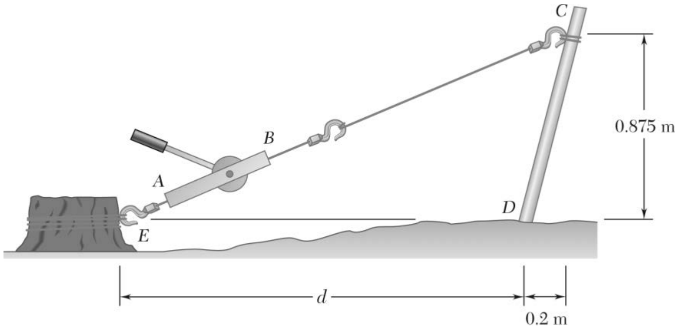
$$960 \text{ N} \cdot \text{m} = \frac{24}{25} T_{AB}(0) + \frac{7}{25} T_{AB}(2.80 \text{ m})$$

$$T_{AB} = 1224 \text{ N}$$

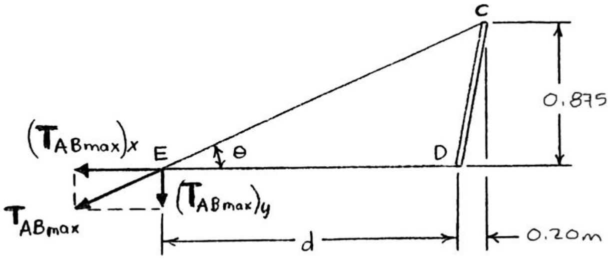
or $T_{AB} = 1224$ N ◀

PROBLEM 3.13

It is known that a force with a moment of 960 N · m about *D* is required to straighten the fence post *CD*. If the capacity of winch puller *AB* is 2400 N, determine the minimum value of distance *d* to create the specified moment about Point *D*.



SOLUTION



The minimum value of *d* can be found based on the equation relating the moment of the force $\mathbf{T}_{AB}$ about *D*:

$$M_D = (T_{AB\max})_y(d)$$

where $M_D = 960\ \text{N}\cdot\text{m}$

$$(T_{AB\max})_y = T_{AB\max}\sin\theta = (2400\ \text{N})\sin\theta$$

Now
$$\sin\theta = \frac{0.875\ \text{m}}{\sqrt{(d+0.20)^2+(0.875)^2}\ \text{m}}$$

$$960\ \text{N}\cdot\text{m} = 2400\ \text{N}\left[\frac{0.875}{\sqrt{(d+0.20)^2+(0.875)^2}}\right](d)$$

or
$$\sqrt{(d+0.20)^2+(0.875)^2} = 2.1875d$$

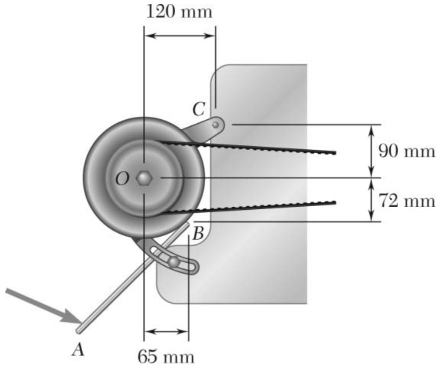
or
$$(d+0.20)^2+(0.875)^2 = 4.7852d^2$$

or
$$3.7852d^2 - 0.40d - 0.8056 = 0$$

Using the quadratic equation, the minimum values of *d* are 0.51719 m and -0.41151 m.

Since only the positive value applies here, $d = 0.51719$ m

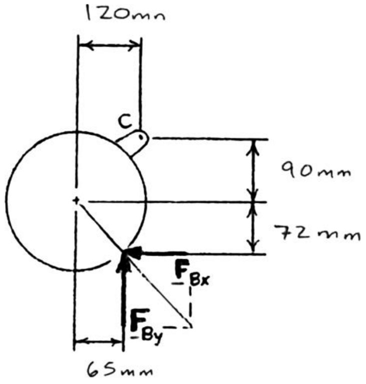
or $d = 517$ mm ◀



PROBLEM 3.14

A mechanic uses a piece of pipe *AB* as a lever when tightening an alternator belt. When he pushes down at *A*, a force of 485 N is exerted on the alternator at *B*. Determine the moment of that force about bolt *C* if its line of action passes through *O*.

SOLUTION



We have $\mathbf{M}_C = \mathbf{r}_{B/C} \times \mathbf{F}_B$

Noting the direction of the moment of each force component about *C* is clockwise,

$$M_C = xF_{By} + yF_{Bx}$$

where $x = 120 \text{ mm} - 65 \text{ mm} = 55 \text{ mm}$

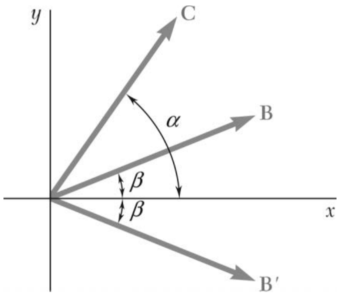
$y = 72 \text{ mm} + 90 \text{ mm} = 162 \text{ mm}$

and $$F_{Bx} = \frac{65}{\sqrt{(65)^2 + (72)^2}}(485 \text{ N}) = 325 \text{ N}$$

$$F_{By} = \frac{72}{\sqrt{(65)^2 + (72)^2}}(485 \text{ N}) = 360 \text{ N}$$

$$M_C = (55 \text{ mm})(360 \text{ N}) + (162)(325 \text{ N})$$
$$= 72450 \text{ N}\cdot\text{m}$$
$$= 72.450 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_C = 72.5 \text{ N}\cdot\text{m} \circlearrowright$ ◀



PROBLEM 3.15

Form the vector products **B** × **C** and **B′** × **C**, where $B = B'$, and use the results obtained to prove the identity

$$\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha+\beta) + \frac{1}{2}\sin(\alpha-\beta).$$

SOLUTION

Note:

$$\mathbf{B} = B(\cos\beta\,\mathbf{i} + \sin\beta\,\mathbf{j})$$
$$\mathbf{B}' = B(\cos\beta\,\mathbf{i} - \sin\beta\,\mathbf{j})$$
$$\mathbf{C} = C(\cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j})$$

By definition,

$$|\mathbf{B}\times\mathbf{C}| = BC\sin(\alpha-\beta) \tag{1}$$

$$|\mathbf{B}'\times\mathbf{C}| = BC\sin(\alpha+\beta) \tag{2}$$

Now

$$\mathbf{B}\times\mathbf{C} = B(\cos\beta\mathbf{i} + \sin\beta\mathbf{j})\times C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$

$$= BC(\cos\beta\sin\alpha - \sin\beta\cos\alpha)\mathbf{k} \tag{3}$$

and

$$\mathbf{B}'\times\mathbf{C} = B(\cos\beta\mathbf{i} - \sin\beta\mathbf{j})\times C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$

$$= BC(\cos\beta\sin\alpha + \sin\beta\cos\alpha)\mathbf{k} \tag{4}$$

Equating the magnitudes of $\mathbf{B}\times\mathbf{C}$ from Equations (1) and (3) yields:

$$BC\sin(\alpha-\beta) = BC(\cos\beta\sin\alpha - \sin\beta\cos\alpha) \tag{5}$$

Similarly, equating the magnitudes of $\mathbf{B}'\times\mathbf{C}$ from Equations (2) and (4) yields:

$$BC\sin(\alpha+\beta) = BC(\cos\beta\sin\alpha + \sin\beta\cos\alpha) \tag{6}$$

Adding Equations (5) and (6) gives:

$$\sin(\alpha-\beta) + \sin(\alpha+\beta) = 2\cos\beta\sin\alpha$$

or $\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha+\beta) + \frac{1}{2}\sin(\alpha-\beta)$ ◀

PROBLEM 3.16

The vectors **P** and **Q** are two adjacent sides of a parallelogram. Determine the area of the parallelogram when (*a*) $\mathbf{P} = -7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ and $\mathbf{Q} = 2\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, (*b*) $\mathbf{P} = 6\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$ and $\mathbf{Q} = -2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.

SOLUTION

(*a*) We have $A = |\mathbf{P} \times \mathbf{Q}|$

where $\mathbf{P} = -7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$

$\mathbf{Q} = 2\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$

Then
$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7 & 3 & -3 \\ 2 & 2 & 5 \end{vmatrix}$$

$$= [(15+6)\mathbf{i} + (-6+35)\mathbf{j} + (-14-6)\mathbf{k}]$$

$$= (21)\mathbf{i} + (29)\mathbf{j}(-20)\mathbf{k}$$

$$A = \sqrt{(20)^2 + (29)^2 + (-20)^2}$$

or $A = 41.0$ ◀

(*b*) We have $A = |\mathbf{P} \times \mathbf{Q}|$

where $\mathbf{P} = 6\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$

$\mathbf{Q} = -2\mathbf{i} + 5\mathbf{j} - 1\mathbf{k}$

Then
$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -5 & -2 \\ -2 & 5 & -1 \end{vmatrix}$$

$$= [(5+10)\mathbf{i} + (4+6)\mathbf{j} + (30-10)\mathbf{k}]$$

$$= (15)\mathbf{i} + (10)\mathbf{j} + (20)\mathbf{k}$$

$$A = \sqrt{(15)^2 + (10)^2 + (20)^2}$$

or $A = 26.9$ ◀

PROBLEM 3.17

A plane contains the vectors **A** and **B**. Determine the unit vector normal to the plane when **A** and **B** are equal to, respectively, (*a*) **i** + 2**j** – 5**k** and 4**i** – 7**j** – 5**k**, (*b*) 3**i** – 3**j** + 2**k** and –2**i** + 6**j** – 4**k**.

SOLUTION

(*a*) We have $\boldsymbol{\lambda} = \dfrac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$

where $\mathbf{A} = 1\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$

$\mathbf{B} = 4\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$

Then
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & +2 & -5 \\ 4 & -7 & -5 \end{vmatrix}$$
$$= (-10 - 35)\mathbf{i} + (20 + 5)\mathbf{j} + (-7 - 8)\mathbf{k}$$
$$= 15(3\mathbf{i} - 1\mathbf{j} - 1\mathbf{k})$$

and $|\mathbf{A} \times \mathbf{B}| = 15\sqrt{(-3)^2 + (-1)^2 + (-1)^2} = 15\sqrt{11}$

$$\boldsymbol{\lambda} = \frac{15(-3\mathbf{i} - 1\mathbf{j} - 1\mathbf{k})}{15\sqrt{11}}$$

or $\boldsymbol{\lambda} = \dfrac{1}{\sqrt{11}}(-3\mathbf{i} - \mathbf{j} - \mathbf{k})$ ◀

(*b*) We have $\boldsymbol{\lambda} = \dfrac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$

where $\mathbf{A} = 3\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

$\mathbf{B} = -2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$

Then
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -3 & 2 \\ -2 & 6 & -4 \end{vmatrix}$$
$$= (12 - 12)\mathbf{i} + (-4 + 12)\mathbf{j} + (18 - 6)\mathbf{k}$$
$$= (8\mathbf{j} + 12\mathbf{k})$$

and $|\mathbf{A} \times \mathbf{B}| = 4\sqrt{(2)^2 + (3)^2} = 4\sqrt{13}$

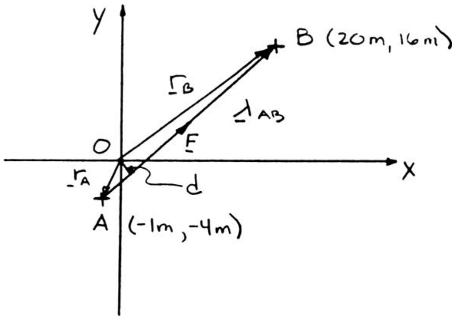
$$\boldsymbol{\lambda} = \frac{4(2\mathbf{j} + 3\mathbf{k})}{4\sqrt{13}}$$

or $\boldsymbol{\lambda} = \dfrac{1}{\sqrt{13}}(2\mathbf{j} + 3\mathbf{k})$ ◀

PROBLEM 3.18

A line passes through the Points (20 m, 16 m) and (–1 m, –4 m). Determine the perpendicular distance *d* from the line to the origin *O* of the system of coordinates.

SOLUTION



$$\begin{aligned} d_{AB} &= \sqrt{[20\text{ m} - (-1\text{ m})]^2 + [16\text{ m} - (-4\text{ m})]^2} \\ &= 29\text{ m} \end{aligned}$$

Assume that a force **F**, or magnitude *F*(N), acts at Point *A* and is directed from *A* to *B*.

Then $\quad \mathbf{F} = F\boldsymbol{\lambda}_{AB}$

where $\quad \begin{aligned} \boldsymbol{\lambda}_{AB} &= \frac{\mathbf{r}_B - \mathbf{r}_A}{d_{AB}} \\ &= \frac{1}{29}(21\mathbf{i} + 20\mathbf{j}) \end{aligned}$

By definition, $\quad \mathbf{M}_O = |\mathbf{r}_A \times \mathbf{F}| = dF$

where $\quad \mathbf{r}_A = -(1\text{ m})\mathbf{i} - (4\text{ m})\mathbf{j}$

Then $\quad \begin{aligned} \mathbf{M}_O &= [-(-1\text{ m})\mathbf{i} - (4\text{ m})\mathbf{j}] \times \frac{F}{29\text{ m}}[(21\text{ m})\mathbf{i} + (20\text{ m})\mathbf{j}] \\ &= \frac{F}{29}[-(20)\mathbf{k} + (84)\mathbf{k}] \\ &= \left(\frac{64}{29}F\right)\mathbf{k}\text{ N}\cdot\text{m} \end{aligned}$

Finally, $\quad \left(\frac{64}{29}F\right) = d(F)$

$$d = \frac{64}{29}\text{ m}$$

$d = 2.21$ m ◀

PROBLEM 3.19

Determine the moment about the origin O of the force $\mathbf{F} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ that acts at a Point A. Assume that the position vector of A is (*a*) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, (*b*) $\mathbf{r} = -8\mathbf{i} + 6\mathbf{j} - 10\mathbf{k}$, (c) $\mathbf{r} = 8\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$.

SOLUTION

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

(*a*)
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & -4 \\ 4 & -3 & 5 \end{vmatrix}$$

$$= (15-12)\mathbf{i} + (-16-10)\mathbf{j} + (-6-12)\mathbf{k} \qquad \mathbf{M}_O = 3\mathbf{i} - 26\mathbf{j} - 18\mathbf{k} \blacktriangleleft$$

(*b*)
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & 6 & -10 \\ 4 & -3 & 5 \end{vmatrix}$$

$$= (30-30)\mathbf{i} + (-40+40)\mathbf{j} + (24-24)\mathbf{k} \qquad \mathbf{M}_O = 0 \blacktriangleleft$$

(*c*)
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & -6 & 5 \\ 4 & -3 & 5 \end{vmatrix}$$

$$= (-30+15)\mathbf{i} + (20-40)\mathbf{j} + (-24+24)\mathbf{k} \qquad \mathbf{M}_O = -15\mathbf{i} - 20\mathbf{j} \blacktriangleleft$$

Note: The answer to Part (*b*) could have been anticipated since the elements of the last two rows of the determinant are proportional.

PROBLEM 3.20

Determine the moment about the origin *O* of the force **F** = 2**i** + 3**j** – 4**k** that acts at a Point *A*. Assume that the position vector of *A* is (*a*) **r** = 3**i** – 6**j** + 5**k**, (*b*) **r** = **i** – 4**j** – 2**k**, (*c*) **r** = 4**i** + 6**j** – 8**k**.

SOLUTION

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

(*a*)

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -6 & 5 \\ 2 & 3 & -4 \end{vmatrix}$$

$$= (24-15)\mathbf{i} + (10+12)\mathbf{j} + (9+12)\mathbf{k} \qquad \mathbf{M}_O = 9\mathbf{i} + 22\mathbf{j} + 21\mathbf{k} \blacktriangleleft$$

(*b*)

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & -2 \\ 2 & 3 & -4 \end{vmatrix}$$

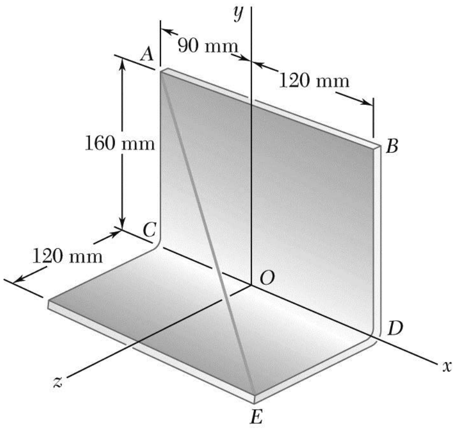
$$= (16+6)\mathbf{i} + (-4+4)\mathbf{j} + (3+8)\mathbf{k} \qquad \mathbf{M}_O = 22\mathbf{i} + 11\mathbf{k} \blacktriangleleft$$

(*c*)

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 6 & -8 \\ 2 & 3 & -4 \end{vmatrix}$$

$$= (-24+24)\mathbf{i} + (-16+16)\mathbf{j} + (12-12)\mathbf{k} \qquad \mathbf{M}_O = 0 \blacktriangleleft$$

Note: The answer to Part (*c*) could have been anticipated since the elements of the last two rows of the determinant are proportional.



PROBLEM 3.21

The wire *AE* is stretched between the corners *A* and *E* of a bent plate. Knowing that the tension in the wire is 435 N, determine the moment about *O* of the force exerted by the wire (*a*) on corner *A*, (*b*) on corner *E*.

SOLUTION

$$\overline{AE} = (0.21\text{ m})\mathbf{i} - (0.16\text{ m})\mathbf{j} + (0.12\text{ m})\mathbf{k}$$

$$AE = \sqrt{(0.21\text{ m})^2 + (-0.16\text{ m})^2 + (0.12\text{ m})^2} = 0.29\text{ m}$$

(*a*)

$$\mathbf{F}_A = F_A \boldsymbol{\lambda}_{AE} = F\frac{\overline{AE}}{AE}$$

$$= (435\text{ N})\frac{0.21\mathbf{i} - 0.16\mathbf{j} + 0.12\mathbf{k}}{0.29}$$

$$= (315\text{ N})\mathbf{i} - (240\text{ N})\mathbf{j} + (180\text{ N})\mathbf{k}$$

$$\mathbf{r}_{A/O} = -(0.09\text{ m})\mathbf{i} + (0.16\text{ m})\mathbf{j}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.09 & 0.16 & 0 \\ 315 & -240 & 180 \end{vmatrix}$$

$$= 28.8\mathbf{i} + 16.20\mathbf{j} + (21.6 - 50.4)\mathbf{k} \qquad \mathbf{M}_O = (28.8\text{ N}\cdot\text{m})\mathbf{i} + (16.20\text{ N}\cdot\text{m})\mathbf{j} - (28.8\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

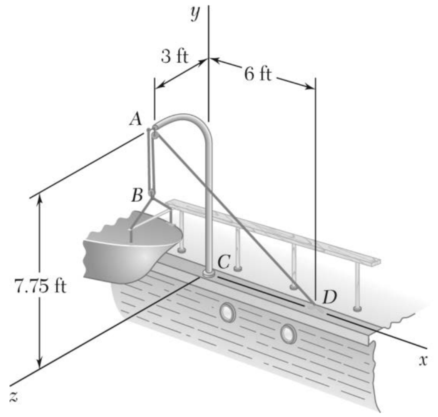
(*b*)

$$\mathbf{F}_E = -\mathbf{F}_A = -(315\text{ N})\mathbf{i} + (240\text{ N})\mathbf{j} - (180\text{ N})\mathbf{k}$$

$$\mathbf{r}_{E/O} = (0.12\text{ m})\mathbf{i} + (0.12\text{ m})\mathbf{k}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.12 & 0 & 0.12 \\ -315 & 240 & -180 \end{vmatrix}$$

$$= -28.8\mathbf{i} + (-37.8 + 21.6)\mathbf{j} + 28.8\mathbf{k} \qquad \mathbf{M}_O = -(28.8\text{ N}\cdot\text{m})\mathbf{i} - (16.20\text{ N}\cdot\text{m})\mathbf{j} + (28.8\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.22

A small boat hangs from two davits, one of which is shown in the figure. The tension in line *ABAD* is 82 lb. Determine the moment about *C* of the resultant force $\mathbf{R}_A$ exerted on the davit at *A*.

SOLUTION

We have $\mathbf{R}_A = 2\mathbf{F}_{AB} + \mathbf{F}_{AD}$

where $\mathbf{F}_{AB} = -(82 \text{ lb})\mathbf{j}$

and
$$\mathbf{F}_{AD} = \mathbf{F}_{AD}\frac{\overrightarrow{AD}}{AD} = (82 \text{ lb})\frac{6\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k}}{10.25}$$
$$\mathbf{F}_{AD} = (48 \text{ lb})\mathbf{i} - (62 \text{ lb})\mathbf{j} - (24 \text{ lb})\mathbf{k}$$

Thus $\mathbf{R}_A = 2\mathbf{F}_{AB} + \mathbf{F}_{AD} = (48 \text{ lb})\mathbf{i} - (226 \text{ lb})\mathbf{j} - (24 \text{ lb})\mathbf{k}$

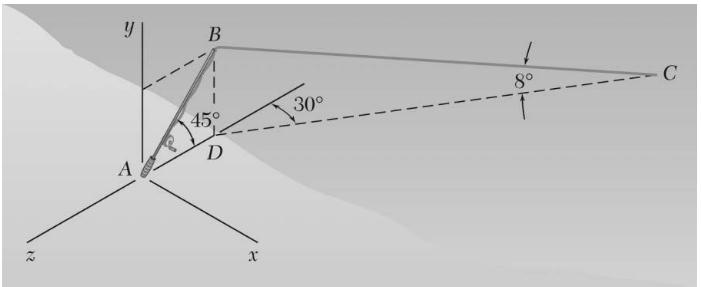
Also $\mathbf{r}_{A/C} = (7.75 \text{ ft})\mathbf{j} + (3 \text{ ft})\mathbf{k}$

Using Eq. (3.21):
$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 7.75 & 3 \\ 48 & -226 & -24 \end{vmatrix}$$
$$= (492 \text{ lb}\cdot\text{ft})\mathbf{i} + (144.0 \text{ lb}\cdot\text{ft})\mathbf{j} - (372 \text{ lb}\cdot\text{ft})\mathbf{k}$$

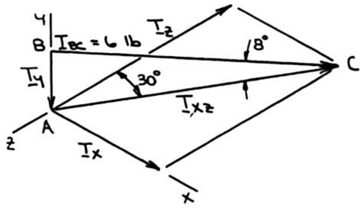
$$\mathbf{M}_C = (492 \text{ lb}\cdot\text{ft})\mathbf{i} + (144.0 \text{ lb}\cdot\text{ft})\mathbf{j} - (372 \text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.23

A 6-ft-long fishing rod *AB* is securely anchored in the sand of a beach. After a fish takes the bait, the resulting force in the line is 6 lb. Determine the moment about *A* of the force exerted by the line at *B*.



SOLUTION



We have $T_{xz} = (6\text{ lb})\cos 8° = 5.9416\text{ lb}$

Then $T_x = T_{xz}\sin 30° = 2.9708\text{ lb}$

$$T_y = T_{BC}\sin 8° = -0.83504\text{ lb}$$

$$T_z = T_{xz}\cos 30° = -5.1456\text{ lb}$$

Now $\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BC}$

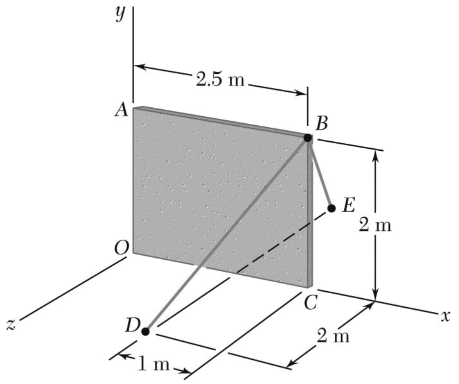
where $\mathbf{r}_{B/A} = (6\sin 45°)\mathbf{j} - (6\cos 45°)\mathbf{k}$

$$= \frac{6\text{ ft}}{\sqrt{2}}(\mathbf{j}-\mathbf{k})$$

Then $$\mathbf{M}_A = \frac{6}{\sqrt{2}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & -1 \\ 2.9708 & -0.83504 & -5.1456 \end{vmatrix}$$

$$= \frac{6}{\sqrt{2}}(-5.1456 - 0.83504)\mathbf{i} - \frac{6}{\sqrt{2}}(2.9708)\mathbf{j} - \frac{6}{\sqrt{2}}(2.9708)\mathbf{k}$$

or $\mathbf{M}_A = -(25.4\text{ lb}\cdot\text{ft})\mathbf{i} - (12.60\text{ lb}\cdot\text{ft})\mathbf{j} - (12.60\text{ lb}\cdot\text{ft})\mathbf{k}$ ◀



PROBLEM 3.24

A precast concrete wall section is temporarily held by two cables as shown. Knowing that the tension in cable BD is 900 N, determine the moment about Point O of the force exerted by the cable at B.

SOLUTION

$$\mathbf{F} = F\frac{\overrightarrow{BD}}{BD} \quad \text{where} \quad F = 900 \text{ N}$$

$$\overrightarrow{BD} = -(1 \text{ m})\mathbf{i} - (2 \text{ m})\mathbf{j} + (2 \text{ m})\mathbf{k}$$

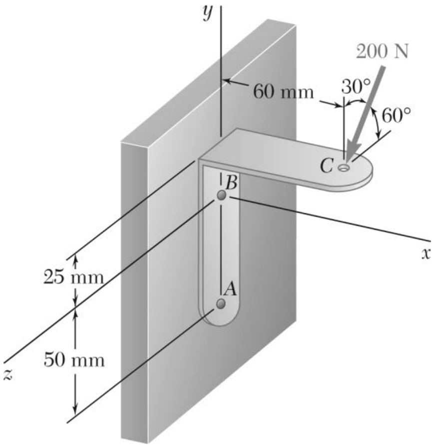
$$\begin{aligned} BD &= \sqrt{(-1 \text{ m})^2 + (-2 \text{ m})^2 + (2 \text{ m})^2} \\ &= 3 \text{ m} \end{aligned}$$

$$\begin{aligned} \mathbf{F} &= (900 \text{ N})\frac{-\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}}{3} \\ &= -(300 \text{ N})\mathbf{i} - (600 \text{ N})\mathbf{j} + (600 \text{ N})\mathbf{k} \end{aligned}$$

$$\mathbf{r}_{B/O} = (2.5 \text{ m})\mathbf{i} + (2 \text{ m})\mathbf{j}$$

$$\begin{aligned} \mathbf{M}_O &= \mathbf{r}_{B/O} \times \mathbf{F} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.5 & 2 & 0 \\ -300 & -600 & 600 \end{vmatrix} \\ &= 1200\mathbf{i} - 1500\mathbf{j} + (-1500 + 600)\mathbf{k} \end{aligned}$$

$$\mathbf{M}_O = (1200 \text{ N}\cdot\text{m})\mathbf{i} - (1500 \text{ N}\cdot\text{m})\mathbf{j} - (900 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.25

A 200-N force is applied as shown to the bracket *ABC*. Determine the moment of the force about *A*.

SOLUTION

We have $\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F}_C$

where $\mathbf{r}_{C/A} = (0.06\text{ m})\mathbf{i} + (0.075\text{ m})\mathbf{j}$

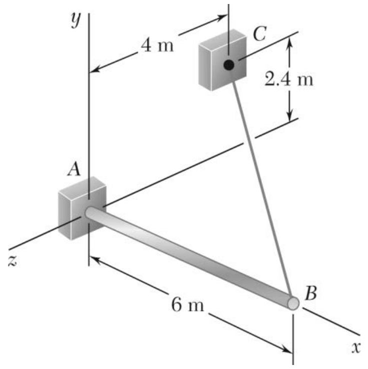
$$\mathbf{F}_C = -(200\text{ N})\cos 30°\mathbf{j} + (200\text{ N})\sin 30°\mathbf{k}$$

Then

$$\mathbf{M}_A = 200\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.06 & 0.075 & 0 \\ 0 & -\cos 30° & \sin 30° \end{vmatrix}$$

$$= 200[(0.075\sin 30°)\mathbf{i} - (0.06\sin 30°)\mathbf{j} - (0.06\cos 30°)\mathbf{k}]$$

or $\mathbf{M}_A = (7.50\text{ N}\cdot\text{m})\mathbf{i} - (6.00\text{ N}\cdot\text{m})\mathbf{j} - (10.39\text{ N}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 3.26

The 6-m boom *AB* has a fixed end *A*. A steel cable is stretched from the free end *B* of the boom to a Point *C* located on the vertical wall. If the tension in the cable is 2.5 kN, determine the moment about *A* of the force exerted by the cable at *B*.

SOLUTION

First note $$d_{BC} = \sqrt{(-6)^2 + (2.4)^2 + (-4)^2}$$
$$= 7.6 \text{ m}$$

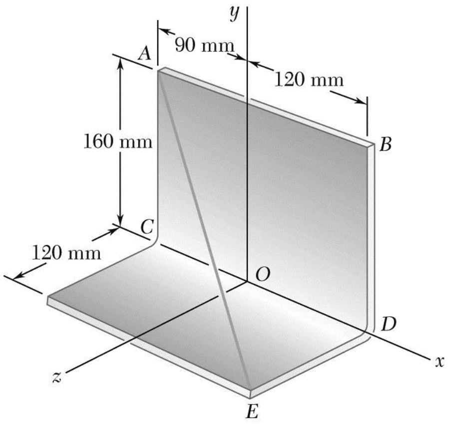
Then $$\mathbf{T}_{BC} = \frac{2.5 \text{ kN}}{7.6}(-6\mathbf{i} + 2.4\mathbf{j} - 4\mathbf{k})$$

We have $$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BC}$$

where $$\mathbf{r}_{B/A} = (6 \text{ m})\mathbf{i}$$

Then $$\mathbf{M}_A = (6 \text{ m})\mathbf{i} \times \frac{2.5 \text{ kN}}{7.6}(-6\mathbf{i} + 2.4\mathbf{j} - 4\mathbf{k})$$

or $\mathbf{M}_A = (7.89 \text{ kN}\cdot\text{m})\mathbf{j} + (4.74 \text{ kN}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 3.27

In Prob. 3.21, determine the perpendicular distance from point *O* to wire *AE*.

PROBLEM 3.21 The wire *AE* is stretched between the corners *A* and *E* of a bent plate. Knowing that the tension in the wire is 435 N, determine the moment about *O* of the force exerted by the wire (*a*) on corner *A*, (*b*) on corner *E*.

SOLUTION

From the solution to Prob. 3.21

$$\mathbf{M}_O = (28.8 \text{ N}\cdot\text{m})\mathbf{i} + (16.20 \text{ N}\cdot\text{m})\mathbf{j} - (28.8 \text{ N}\cdot\text{m})\mathbf{k}$$

$$M_O = \sqrt{(28.8)^2 + (16.20)^2 + (28.8)^2}$$

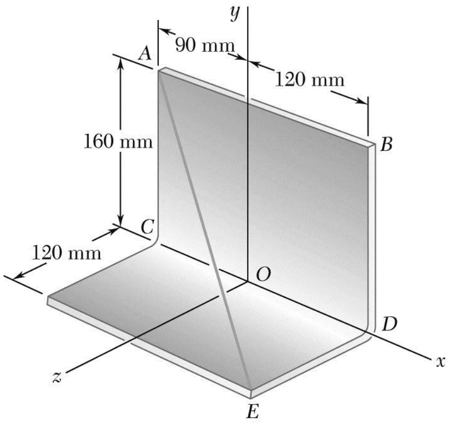
$$= 43.8329 \text{ N}\cdot\text{m}$$

But $\quad M_O = F_A d \quad$ or $\quad d = \dfrac{M_O}{F_A}$

$$d = \frac{43.8329 \text{ N}\cdot\text{m}}{435 \text{ N}}$$

$$= 0.100765 \text{ m}$$

$d = 100.8$ mm ◀



PROBLEM 3.28

In Prob. 3.21, determine the perpendicular distance from point *B* to wire *AE*.

PROBLEM 3.21 The wire *AE* is stretched between the corners *A* and *E* of a bent plate. Knowing that the tension in the wire is 435 N, determine the moment about *O* of the force exerted by the wire (*a*) on corner *A*, (*b*) on corner *E*.

SOLUTION

From the solution to Prob. 3.21

$$\mathbf{F}_A = (315\text{ N})\mathbf{i} - (240\text{ N})\mathbf{j} + (180\text{ N})\mathbf{k}$$

$$\mathbf{r}_{A/B} = -(0.210\text{ m})\mathbf{i}$$

$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F}_A = -0.21\mathbf{i} \times (315\mathbf{i} - 240\mathbf{j} + 180\mathbf{k})$$

$$= 50.4\mathbf{k} + 37.8\mathbf{j}$$

$$M_B = \sqrt{(50.4)^2 + (37.8)^2}$$

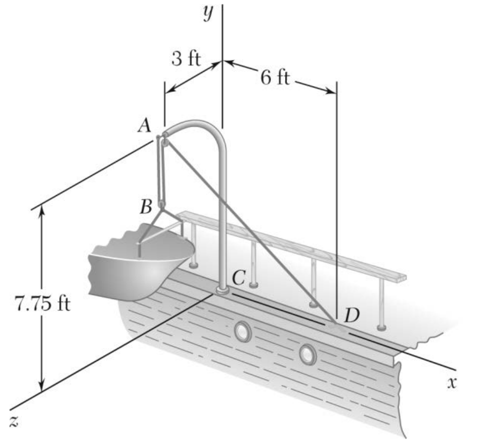
$$= 63.0\text{ N}\cdot\text{m}$$

$$M_B = F_A d \qquad \text{or} \qquad d = \frac{M_B}{F_A}$$

$$d = \frac{63.0\text{ N}\cdot\text{m}}{435\text{ N}}$$

$$= 0.144829\text{ m}$$

$d = 144.8$ mm ◀



PROBLEM 3.29

In Problem 3.22, determine the perpendicular distance from point *C* to portion *AD* of the line *ABAD*.

PROBLEM 3.22 A small boat hangs from two davits, one of which is shown in the figure. The tension in line *ABAD* is 82 lb. Determine the moment about *C* of the resultant force $\mathbf{R}_A$ exerted on the davit at *A*.

SOLUTION

First compute the moment about *C* of the force $\mathbf{F}_{DA}$ exerted by the line on *D*:

From Problem 3.22:

$$\mathbf{F}_{DA} = -\mathbf{F}_{AD}$$
$$= -(48 \text{ lb})\mathbf{i} + (62 \text{ lb})\mathbf{j} + (24 \text{ lb})\mathbf{k}$$
$$\mathbf{M}_C = \mathbf{r}_{D/C} \times \mathbf{F}_{DA}$$
$$= +(6 \text{ ft})\mathbf{i} \times [-(48 \text{ lb})\mathbf{i} + (62 \text{ lb})\mathbf{j} + (24 \text{ lb})\mathbf{k}]$$
$$= -(144 \text{ lb}\cdot\text{ft})\mathbf{j} + (372 \text{ lb}\cdot\text{ft})\mathbf{k}$$
$$\mathbf{M}_C = \sqrt{(144)^2 + (372)^2}$$
$$= 398.90 \text{ lb}\cdot\text{ft}$$

Then

$$\mathbf{M}_C = \mathbf{F}_{DA} d$$

Since

$$F_{DA} = 82 \text{ lb}$$

$$d = \frac{M_C}{F_{DA}}$$
$$= \frac{398.90 \text{ lb}\cdot\text{ft}}{82 \text{ lb}}$$

$d = 4.86$ ft ◀

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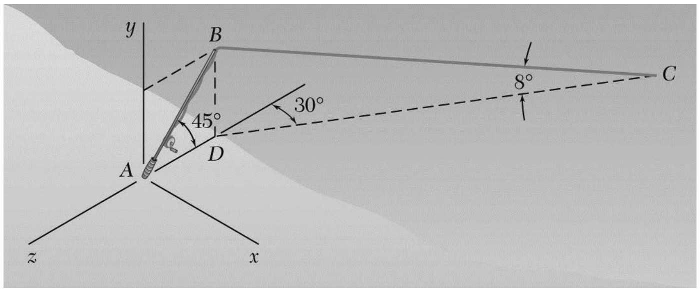
PROBLEM 3.30

In Prob. 3.23, determine the perpendicular distance from point *A* to a line drawn through points *B* and *C*.

PROBLEM 3.23 A 6-ft-long fishing rod *AB* is securely anchored in the sand of a beach. After a fish takes the bait, the resulting force in the line is 6 lb. Determine the moment about *A* of the force exerted by the line at *B*.



SOLUTION

From the solution to Prob. 3.23:

$$\mathbf{M}_A = -(25.4\ \text{lb}\cdot\text{ft})\mathbf{i} - (12.60\ \text{lb}\cdot\text{ft})\mathbf{j} - (12.60\ \text{lb}\cdot\text{ft})\mathbf{k}$$

$$M_A = \sqrt{(-25.4)^2 + (-12.60)^2 + (-12.60)^2}$$

$$= 31.027\ \text{lb}\cdot\text{ft}$$

$$M_A = T_{BC}d \qquad \text{or} \qquad d = \frac{M_A}{T_{BC}}$$

$$= \frac{31.027\ \text{lb}\cdot\text{ft}}{6\ \text{lb}}$$

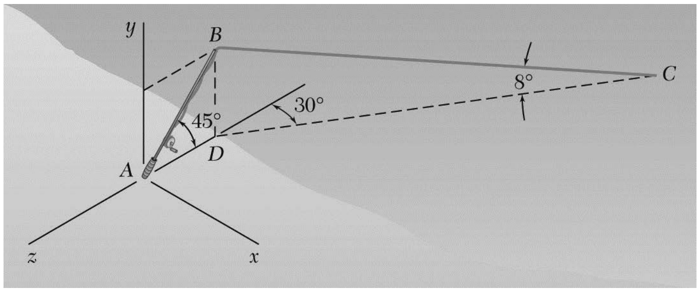
$$= 5.1712\ \text{ft}$$

$d = 5.17$ ft ◀

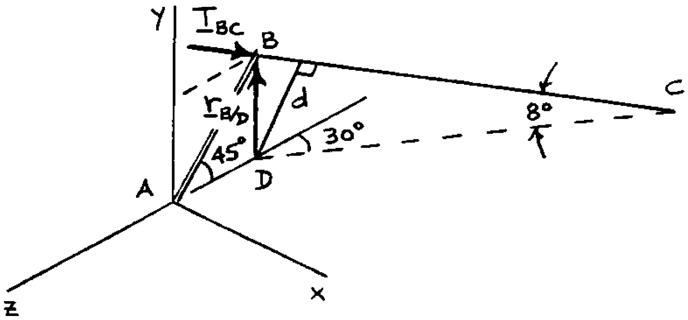
PROBLEM 3.31

In Prob. 3.23, determine the perpendicular distance from point *D* to a line drawn through points *B* and *C*.

PROBLEM 3.23 A 6-ft-long fishing rod *AB* is securely anchored in the sand of a beach. After a fish takes the bait, the resulting force in the line is 6 lb. Determine the moment about *A* of the force exerted by the line at *B*.



SOLUTION



$\overline{AB} = 6$ ft

$T_{BC} = 6$ lb

We have $|\mathbf{M}_D| = T_{BC}d$

where d = perpendicular distance from *D* to line *BC*.

$$\mathbf{M}_D = \mathbf{r}_{B/D} \times \mathbf{T}_{BC} \qquad \mathbf{r}_{B/D} = (6\sin 45° \text{ ft})\mathbf{j} = (4.2426 \text{ ft})$$

$$\mathbf{T}_{BC}: \quad (T_{BC})_x = (6 \text{ lb})\cos 8° \sin 30° = 2.9708 \text{ lb}$$

$$(T_{BC})_y = -(6 \text{ lb})\sin 8° = -0.83504 \text{ lb}$$

$$(T_{BC})_z = -(6 \text{ lb})\cos 8° \cos 30° = -5.1456 \text{ lb}$$

$$\mathbf{T}_{BC} = (2.9708 \text{ lb})\mathbf{i} - (0.83504 \text{ lb})\mathbf{j} - (5.1456 \text{ lb})\mathbf{k}$$

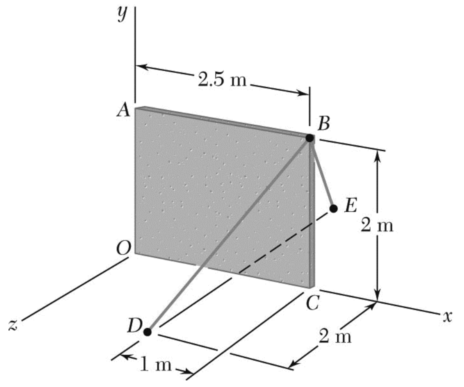
$$\mathbf{M}_D = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4.2426 & 0 \\ 2.9708 & -0.83504 & -5.1456 \end{vmatrix}$$

$$= -(21.831 \text{ lb}\cdot\text{ft})\mathbf{i} - (12.6039 \text{ lb}\cdot\text{ft})$$

$$|M_D| = \sqrt{(-21.831)^2 + (-12.6039)^2} = 25.208 \text{ lb}\cdot\text{ft}$$

$$25.208 \text{ lb}\cdot\text{ft} = (6 \text{ lb})d$$

$d = 4.20$ ft ◀



PROBLEM 3.32

In Prob. 3.24, determine the perpendicular distance from point *O* to cable *BD*.

PROBLEM 3.24 A precast concrete wall section is temporarily held by two cables as shown. Knowing that the tension in cable *BD* is 900 N, determine the moment about Point *O* of the force exerted by the cable at *B*.

SOLUTION

From the solution to Prob. 3.24 we have

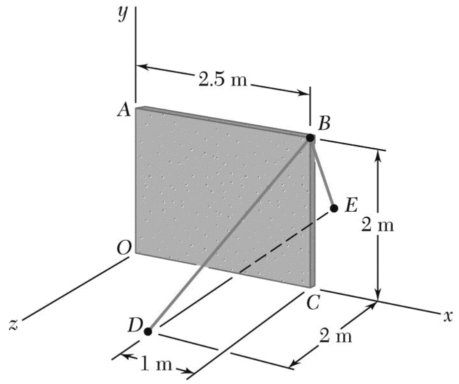
$$\mathbf{M}_O = (1200\ \text{N}\cdot\text{m})\mathbf{i} - (1500\ \text{N}\cdot\text{m})\mathbf{j} - (900\ \text{N}\cdot\text{m})\mathbf{k}$$

$$M_O = \sqrt{(1200)^2 + (-1500)^2 + (-900)^2} = 2121.3\ \text{N}\cdot\text{m}$$

$$M_O = Fd \qquad d = \frac{M_O}{F}$$

$$= \frac{2121.3\ \text{N}\cdot\text{m}}{900\ \text{N}}$$

$d = 2.36$ m ◀



PROBLEM 3.33

In Prob. 3.24, determine the perpendicular distance from point *C* to cable *BD*.

PROBLEM 3.24 A precast concrete wall section is temporarily held by two cables as shown. Knowing that the tension in cable *BD* is 900 N, determine the moment about Point *O* of the force exerted by the cable at *B*.

SOLUTION

From the solution to Prob. 3.24 we have

$$\mathbf{F} = -(300\text{ N})\mathbf{i} - (600\text{ N})\mathbf{j} + (600\text{ N})\mathbf{k}$$

$$\mathbf{r}_{B/C} = (2\text{ m})\mathbf{j}$$

$$\mathbf{M}_C = \mathbf{r}_{B/C} \times \mathbf{F} = (2\text{ m})\mathbf{j} \times (-300\text{ N}\mathbf{i} - 600\text{ N}\mathbf{j} + 600\text{ N}\mathbf{k})$$

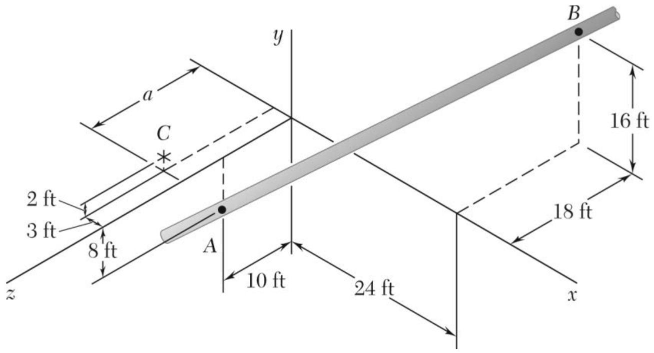
$$= (600\text{ N}\cdot\text{m})\mathbf{k} + (1200\text{ N}\cdot\text{m})\mathbf{i}$$

$$M_C = \sqrt{(600)^2 + (1200)^2} = 1341.64\text{ N}\cdot\text{m}$$

$$M_C = Fd \qquad d = \frac{M_C}{F}$$

$$= \frac{1341.64\text{ N}\cdot\text{m}}{900\text{ N}}$$

$$d = 1.491\text{ m} \blacktriangleleft$$



PROBLEM 3.34

Determine the value of *a* that minimizes the perpendicular distance from Point *C* to a section of pipeline that passes through Points *A* and *B*.

SOLUTION

Assuming a force **F** acts along *AB*,

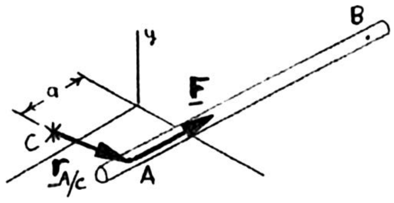
$$|\mathbf{M}_C| = |\mathbf{r}_{A/C} \times \mathbf{F}| = F(d)$$

where $d =$ perpendicular distance from C to line AB

$$\mathbf{F} = \boldsymbol{\lambda}_{AB} F$$

$$= \frac{(24\text{ ft})\,\mathbf{i} + (24\text{ ft})\,\mathbf{j} - (28\text{ ft})\,\mathbf{k}}{\sqrt{(24)^2 + (24)^2 + (28)^2}\text{ ft}} F$$

$$= \frac{F}{11}(6)\,\mathbf{i} + (6)\,\mathbf{j} - (7)\,\mathbf{k}$$



$$\mathbf{r}_{A/C} = (3\text{ ft})\mathbf{i} - (10\text{ ft})\mathbf{j} - (a - 10\text{ ft})\mathbf{k}$$

$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -10 & 10a \\ 6 & 6 & -7 \end{vmatrix} \frac{F}{11}$$

$$= [(10 + 6a)\mathbf{i} + (81 - 6a)\mathbf{j} + 78\,\mathbf{k}]\,\frac{F}{11}$$

Since $|\mathbf{M}_C| = \sqrt{|\mathbf{r}_{A/C} \times \mathbf{F}^2|}$ or $|\mathbf{r}_{A/C} \times \mathbf{F}^2| = (dF)^2$

$$\frac{1}{121}(10 + 6a)^2 + (81 - 6a)^2 + (78)^2 = d^2$$

Setting $\frac{d}{da}(d^2) = 0$ to find a to minimize d:

$$\frac{1}{121}[2(6)(10 + 6a) + 2(-6)(81 - 6a)] = 0$$

Solving $a = 5.92$ ft or $a = 5.92$ ft ◀

PROBLEM 3.35

Given the vectors $\mathbf{P} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{Q} = 4\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, and $\mathbf{S} = -2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, compute the scalar products $\mathbf{P} \cdot \mathbf{Q}$, $\mathbf{P} \cdot \mathbf{S}$, and $\mathbf{Q} \cdot \mathbf{S}$.

SOLUTION

$$\begin{aligned}\mathbf{P} \cdot \mathbf{Q} &= (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \cdot (4\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})\\ &= (3)(4) + (-1)(5) + (2)(-3)\\ &= 12 - 5 - 6\end{aligned}$$

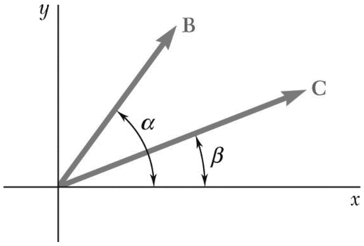
$$\mathbf{P} \cdot \mathbf{Q} = +1 \blacktriangleleft$$

$$\begin{aligned}\mathbf{P} \cdot \mathbf{S} &= (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \cdot (-2\mathbf{i} + 3\mathbf{j} - \mathbf{k})\\ &= (3)(-2) + (-1)(3) + (2)(-1)\\ &= -6 - 3 - 2\end{aligned}$$

$$\mathbf{P} \cdot \mathbf{S} = -11 \blacktriangleleft$$

$$\begin{aligned}\mathbf{Q} \cdot \mathbf{S} &= (4\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}) \cdot (-2\mathbf{i} + 3\mathbf{j} - \mathbf{k})\\ &= (4)(-2) + (5)(3) + (-3)(-1)\\ &= -8 + 15 + 3\end{aligned}$$

$$\mathbf{Q} \cdot \mathbf{S} = +10 \blacktriangleleft$$

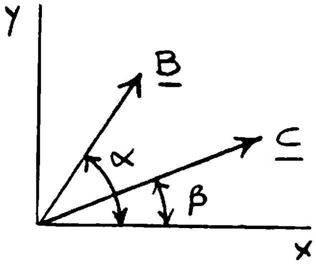


PROBLEM 3.36

Form the scalar product $\mathbf{B} \cdot \mathbf{C}$ and use the result obtained to prove the identity

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta .$$

SOLUTION



$$\mathbf{B} = B\cos\alpha\mathbf{i} + B\sin\alpha\mathbf{j} \qquad (1)$$

$$\mathbf{C} = C\cos\beta\mathbf{i} + C\sin\beta\mathbf{j} \qquad (2)$$

By definition:

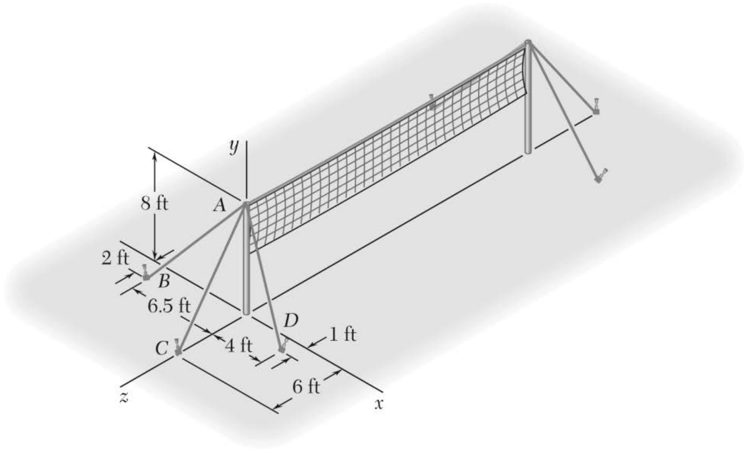
$$\mathbf{B} \cdot \mathbf{C} = BC\cos(\alpha - \beta) \qquad (3)$$

From (1) and (2):

$$\begin{aligned}\mathbf{B} \cdot \mathbf{C} &= (B\cos\alpha\mathbf{i} + B\sin\alpha\mathbf{j}) \cdot (C\cos\beta\mathbf{i} + C\sin\beta\mathbf{j}) \\ &= BC(\cos\alpha\cos\beta + \sin\alpha\sin\beta) \qquad (4)\end{aligned}$$

Equating the right-hand members of (3) and (4),

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \blacktriangleleft$$



PROBLEM 3.37

Consider the volleyball net shown. Determine the angle formed by guy wires AB and AC.

SOLUTION

First note: $AB = \sqrt{(-6.5)^2 + (-8)^2 + (2)^2} = 10.5 \text{ ft}$

$$AC = \sqrt{(0)^2 + (-8)^2 + (6)^2} = 10 \text{ ft}$$

and $\overrightarrow{AB} = -(6.5 \text{ ft})\mathbf{i} - (8 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}$

$$\overrightarrow{AC} = -(8 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k}$$

By definition, $\overrightarrow{AB} \cdot \overrightarrow{AC} = (AB)(AC)\cos\theta$

or $(-6.5\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) \cdot (-8\mathbf{j} + 6\mathbf{k}) = (10.5)(10)\cos\theta$

$$(-6.5)(0) + (-8)(-8) + (2)(6) = 105\cos\theta$$

or $\cos\theta = 0.72381$ or $\theta = 43.6°$ ◀

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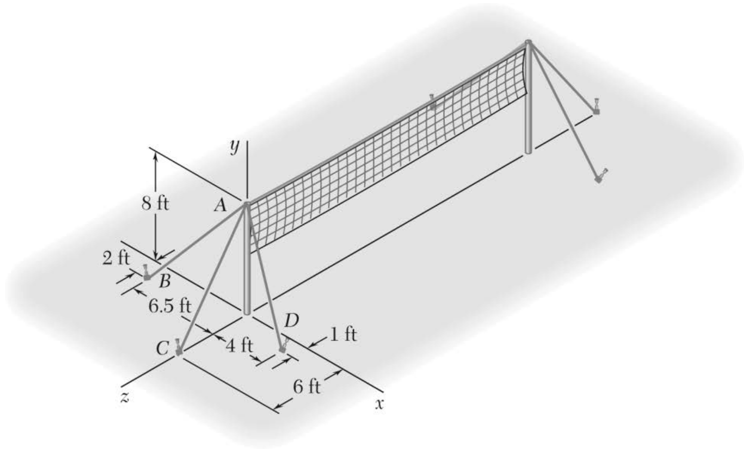


PROBLEM 3.38

Consider the volleyball net shown. Determine the angle formed by guy wires *AC* and *AD*.

SOLUTION

First note:

$$AC = \sqrt{(0)^2 + (-8)^2 + (6)^2}$$
$$= 10 \text{ ft}$$
$$AD = \sqrt{(4)^2 + (-8)^2 + (1)^2}$$
$$= 9 \text{ ft}$$

and

$$\overrightarrow{AC} = -(8 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k}$$
$$\overrightarrow{AD} = (4 \text{ ft})\mathbf{i} - (8 \text{ ft})\mathbf{j} + (1 \text{ ft})\mathbf{k}$$

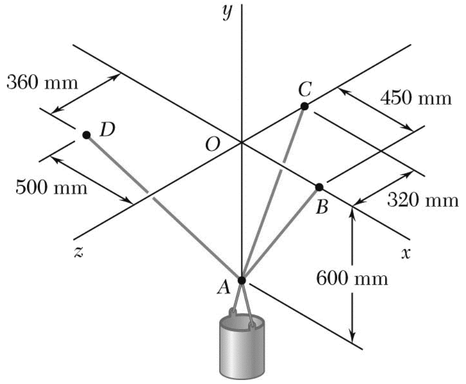
By definition,

$$\overrightarrow{AC} \cdot \overrightarrow{AD} = (AC)(AD)\cos\theta$$

or

$$(-8\mathbf{j} + 6\mathbf{k}) \cdot (4\mathbf{i} - 8\mathbf{j} + \mathbf{k}) = (10)(9)\cos\theta$$
$$(0)(4) + (-8)(-8) + (6)(1) = 90\cos\theta$$

or $\cos\theta = 0.77778$ or $\theta = 38.9°$ ◀



PROBLEM 3.39

Three cables are used to support a container as shown. Determine the angle formed by cables *AB* and *AD*.

SOLUTION

First note:

$$AB = \sqrt{(450\text{ mm})^2 + (600\text{ mm})^2}$$
$$= 750\text{ mm}$$

$$AD = \sqrt{(-500\text{ mm})^2 + (600\text{ mm})^2 + (360\text{ mm})^2}$$
$$= 860\text{ mm}$$

and

$$\overrightarrow{AB} = (450\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j}$$
$$\overrightarrow{AD} = (-500\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}$$

By definition,

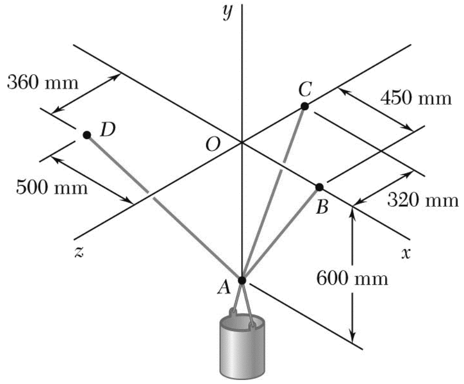
$$\overrightarrow{AB} \cdot \overrightarrow{AD} = (AB)(AD)\cos\theta$$

$$(450\mathbf{i} + 600\mathbf{j}) \cdot (-500\mathbf{i} - 600\mathbf{j} + 360\mathbf{k}) = (750)(860)\cos\theta$$

$$(450)(-500) + (600)(600) + (0)(360) = (750)(860)\cos\theta$$

or

$$\cos\theta = 0.20930 \qquad \theta = 77.9° ◀$$



PROBLEM 3.40

Three cables are used to support a container as shown. Determine the angle formed by cables AC and AD.

SOLUTION

First note:

$$AC = \sqrt{(600\text{ mm})^2 + (-320\text{ mm})^2}$$
$$= 680\text{ mm}$$
$$AD = \sqrt{(-500\text{ mm})^2 + (600\text{ mm})^2 + (360\text{ mm})^2}$$
$$= 860\text{ mm}$$

and

$$\overrightarrow{AC} = (600\text{ mm})\mathbf{j} + (-320\text{ mm})\mathbf{k}$$
$$\overrightarrow{AD} = (-500\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}$$

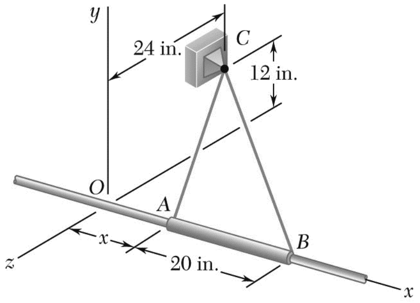
By definition,

$$\overrightarrow{AC} \cdot \overrightarrow{AD} = (AC)(AD)\cos\theta$$

$$(600\mathbf{j} - 320\mathbf{k}) \cdot (-500\mathbf{i} + 600\mathbf{j} + 360\mathbf{k}) = (680)(860)\cos\theta$$

$$0(-500) + (600)(600) + (-320)(360) = (680)(860)\cos\theta$$

$$\cos\theta = 0.41860 \qquad \theta = 65.3° \blacktriangleleft$$



PROBLEM 3.41

The 20-in. tube *AB* can slide along a horizontal rod. The ends *A* and *B* of the tube are connected by elastic cords to the fixed point *C*. For the position corresponding to $x = 11$ in., determine the angle formed by the two cords (*a*) using Eq. (3.32), (*b*) applying the law of cosines to triangle *ABC*.

SOLUTION

(*a*) Using Eq. (3.32):

$$\overline{CA} = 11\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}$$

$$CA = \sqrt{(11)^2 + (-12)^2 + (24)^2} = 29 \text{ in.}$$

$$\overline{CB} = 31\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}$$

$$CB = \sqrt{(31)^2 + (-12)^2 + (24)^2} = 41 \text{ in.}$$

$$\begin{aligned}\cos\theta &= \frac{\overline{CA}\cdot\overline{CB}}{(CA)(CB)}\\ &= \frac{(11\mathbf{i} - 12\mathbf{j} + 24\mathbf{k})\cdot(31\mathbf{i} - 12\mathbf{j} + 24\mathbf{k})}{(29)(41)}\\ &= \frac{(11)(31) + (-12)(-12) + (24)(24)}{(29)(41)}\\ &= 0.89235\end{aligned}$$

$\theta = 26.8°$ ◀

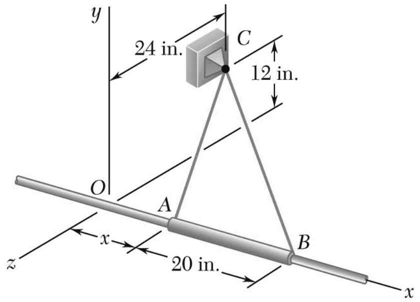
(*b*) Law of cosines:

$$(AB)^2 = (CA)^2 + (CB)^2 - 2(CA)(CB)\cos\theta$$

$$(20)^2 = (29)^2 + (41)^2 - 2(29)(41)\cos\theta$$

$$\cos\theta = 0.89235$$

$\theta = 26.8°$ ◀



PROBLEM 3.42

Solve Prob. 3.41 for the position corresponding to $x = 4$ in.

PROBLEM 3.41 The 20-in. tube *AB* can slide along a horizontal rod. The ends *A* and *B* of the tube are connected by elastic cords to the fixed point *C*. For the position corresponding to $x = 11$ in., determine the angle formed by the two cords (*a*) using Eq. (3.32), (*b*) applying the law of cosines to triangle *ABC*.

SOLUTION

(*a*) Using Eq. (3.32):

$$\overrightarrow{CA} = 4\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}$$

$$CA = \sqrt{(4)^2 + (-12)^2 + (24)^2} = 27.129 \text{ in.}$$

$$\overrightarrow{CB} = 24\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}$$

$$CB = \sqrt{(24)^2 + (-12)^2 + (24)^2} = 36 \text{ in.}$$

$$\cos\theta = \frac{\overrightarrow{CA}\cdot\overrightarrow{CB}}{(CA)(CB)}$$

$$= \frac{(4\mathbf{i} - 12\mathbf{j} + 24\mathbf{k})\cdot(24\mathbf{i} - 12\mathbf{j} + 24\mathbf{k})}{(27.129)(36)}$$

$$= 0.83551 \qquad \theta = 33.3° \blacktriangleleft$$

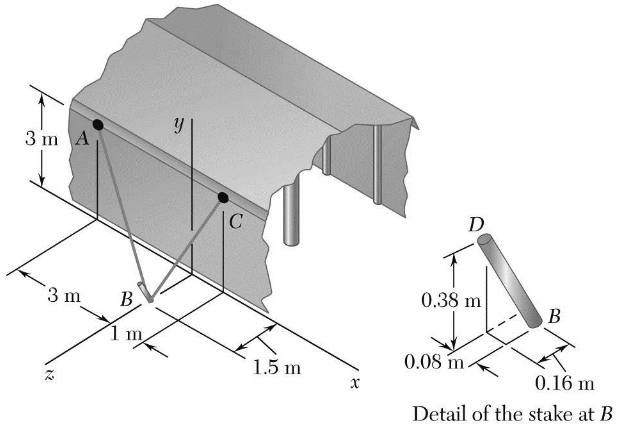
(*b*) Law of cosines:

$$(AB)^2 = (CA)^2 + (CB)^2 - 2(CA)(CB)\cos\theta$$

$$(20)^2 = (27.129)^2 + (36)^2 - 2(27.129)(36)\cos\theta$$

$$\cos\theta = 0.83551$$

$$\theta = 33.3° \blacktriangleleft$$



Detail of the stake at B

PROBLEM 3.43

Ropes AB and BC are two of the ropes used to support a tent. The two ropes are attached to a stake at B. If the tension in rope AB is 540 N, determine (*a*) the angle between rope AB and the stake, (*b*) the projection on the stake of the force exerted by rope AB at Point B.

SOLUTION

First note:

$$BA = \sqrt{(-3)^2 + (3)^2 + (-1.5)^2} = 4.5 \text{ m}$$

$$BD = \sqrt{(-0.08)^2 + (0.38)^2 + (0.16)^2} = 0.42 \text{ m}$$

Then

$$\mathbf{T}_{BA} = \frac{T_{BA}}{4.5}(-3\mathbf{i} + 3\mathbf{j} - 1.5\mathbf{k})$$

$$= \frac{T_{BA}}{3}(-2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$$

$$\boldsymbol{\lambda}_{BD} = \frac{\overrightarrow{BD}}{BD} = \frac{1}{0.42}(-0.08\mathbf{i} + 0.38\mathbf{j} + 0.16\mathbf{k})$$

$$= \frac{1}{21}(-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k})$$

(*a*) We have

$$\mathbf{T}_{BA} \cdot \boldsymbol{\lambda}_{BD} = T_{BA}\cos\theta$$

or

$$\frac{T_{BA}}{3}(-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot \frac{1}{21}(-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k}) = T_{BA}\cos\theta$$

or

$$\cos\theta = \frac{1}{63}[(-2)(-4) + (2)(19) + (-1)(8)]$$

$$= 0.60317$$

or $\theta = 52.9°$ ◀

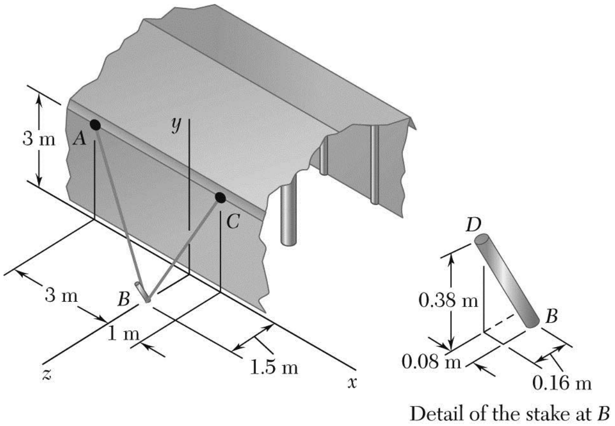
(*b*) We have

$$(T_{BA})_{BD} = \mathbf{T}_{BA} \cdot \boldsymbol{\lambda}_{BD}$$

$$= T_{BA}\cos\theta$$

$$= (540 \text{ N})(0.60317)$$

or $(T_{BA})_{BD} = 326 \text{ N}$ ◀



Detail of the stake at B

PROBLEM 3.44

Ropes *AB* and *BC* are two of the ropes used to support a tent. The two ropes are attached to a stake at *B*. If the tension in rope *BC* is 490 N, determine (*a*) the angle between rope *BC* and the stake, (*b*) the projection on the stake of the force exerted by rope *BC* at Point *B*.

SOLUTION

First note:

$$BC = \sqrt{(1)^2 + (3)^2 + (-1.5)^2} = 3.5 \text{ m}$$

$$BD = \sqrt{(-0.08)^2 + (0.38)^2 + (0.16)^2} = 0.42 \text{ m}$$

$$\mathbf{T}_{BC} = \frac{T_{BC}}{3.5}(\mathbf{i} + 3\mathbf{j} - 1.5\mathbf{k})$$

$$= \frac{T_{BC}}{7}(2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})$$

$$\lambda_{BD} = \frac{\overrightarrow{BD}}{BD} = \frac{1}{0.42}(-0.08\mathbf{i} + 0.38\mathbf{j} + 0.16\mathbf{k})$$

$$= \frac{1}{21}(-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k})$$

(*a*)

$$\mathbf{T}_{BC} \cdot \lambda_{BD} = T_{BC} \cos\theta$$

$$\frac{T_{BC}}{7}(2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) \cdot \frac{1}{21}(-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k}) = T_{BC}\cos\theta$$

$$\cos\theta = \frac{1}{147}[(2)(-4) + (6)(19) + (-3)(8)]$$

$$= 0.55782$$

$$\theta = 56.1° \blacktriangleleft$$

(*b*)

$$(T_{BC})_{BD} = \mathbf{T}_{BC} \cdot \lambda_{BD}$$

$$= T_{BC}\cos\theta$$

$$= (490 \text{ N})(0.55782)$$

$$(T_{BC})_{BD} = 273 \text{ N} \blacktriangleleft$$

PROBLEM 3.45

Given the vectors $\mathbf{P} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}, \mathbf{Q} = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$, and $\mathbf{S} = S_x\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, determine the value of S_x for which the three vectors are coplanar.

SOLUTION

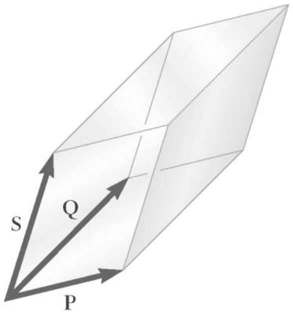
If **P**, **Q**, and **S** are coplanar, then **P** must be perpendicular to $(\mathbf{Q} \times \mathbf{S})$.

$$\mathbf{P} \cdot (\mathbf{Q} \times \mathbf{S}) = 0$$

(or, the volume of a parallelepiped defined by **P**, **Q**, and **S** is zero).

Then
$$\begin{vmatrix} 4 & -2 & 3 \\ 2 & 4 & -5 \\ S_x & -1 & 2 \end{vmatrix} = 0$$

or
$$32 + 10S_x - 6 - 20 + 8 - 12S_x = 0 \qquad S_x = 7 \blacktriangleleft$$



PROBLEM 3.46

Determine the volume of the parallelepiped of Fig. 3.25 when (*a*) $\mathbf{P} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, $\mathbf{Q} = -2\mathbf{i} - 5\mathbf{j} + \mathbf{k}$, and $\mathbf{S} = 7\mathbf{i} + \mathbf{j} - \mathbf{k}$, (*b*) $\mathbf{P} = 5\mathbf{i} - \mathbf{j} + 6\mathbf{k}$, $\mathbf{Q} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, and $\mathbf{S} = -3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$.

SOLUTION

Volume of a parallelepiped is found using the mixed triple product.

(*a*)

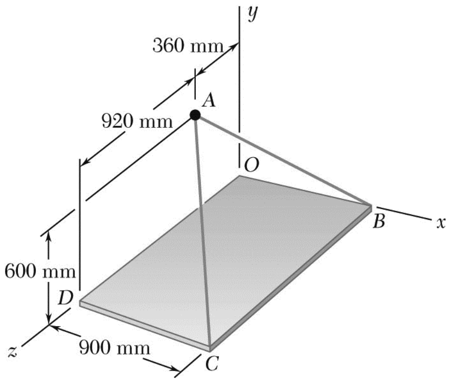
$$\begin{aligned}\text{Vol.} &= \mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S})\\ &= \begin{vmatrix} 4 & -3 & 2\\ -2 & -5 & 1\\ 7 & 1 & -1\end{vmatrix}\text{in.}^3\\ &= (20-21-4+70+6-4)\\ &= 67\end{aligned}$$

or $\text{Volume} = 67.0$ ◀

(*b*)

$$\begin{aligned}\text{Vol.} &= \mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S})\\ &= \begin{vmatrix} 5 & -1 & 6\\ 2 & 3 & 1\\ -3 & -2 & 4\end{vmatrix}\text{in.}^3\\ &= (60+3-24+54+8+10)\\ &= 111\end{aligned}$$

or $\text{Volume} = 111.0$ ◀



PROBLEM 3.47

Knowing that the tension in cable AB is 570 N, determine the moment about each of the coordinate axes of the force exerted on the plate at B.

SOLUTION

$$\overline{BA} = (-900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}$$

$$BA = \sqrt{(-900)^2 + (600)^2 + (360)^2} = 1140 \text{ mm}$$

$$\mathbf{F}_B = F_B \frac{\overline{BA}}{BA}$$

$$= (570 \text{ N})\frac{-900\mathbf{i} + 600\mathbf{j} + 360\mathbf{k}}{1140}$$

$$= -(450 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} + (180 \text{ N})\mathbf{k}$$

$$\mathbf{r}_B = (0.9 \text{ m})\mathbf{i}$$

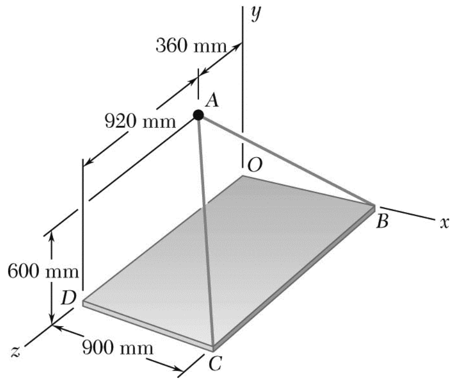
$$\mathbf{M}_O = \mathbf{r}_B \times \mathbf{F}_B = 0.9\mathbf{i} \times (-450\mathbf{i} + 300\mathbf{j} + 180\mathbf{k})$$

$$= 270\mathbf{k} - 162\mathbf{j}$$

$$\mathbf{M}_O = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

$$= -(162 \text{ N}\cdot\text{m})\mathbf{j} + (270 \text{ N}\cdot\text{m})\mathbf{k}$$

Therefore, $M_x = 0,\ M_y = -162.0 \text{ N}\cdot\text{m},\ M_z = +270 \text{ N}\cdot\text{m}$ ◀



PROBLEM 3.48

Knowing that the tension in cable *AC* is 1065 N, determine the moment about each of the coordinate axes of the force exerted on the plate at *C*.

SOLUTION

$$\overline{CA} = (-900\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (-920\text{ mm})\mathbf{k}$$

$$CA = \sqrt{(-900)^2 + (600)^2 + (-920)^2} = 1420\text{ mm}$$

$$\mathbf{F}_C = F_C\frac{\overline{CA}}{CA}$$

$$= (1065\text{ N})\frac{-900\mathbf{i} + 600\mathbf{j} - 920\mathbf{k}}{1420}$$

$$= -(675\text{ N})\mathbf{i} + (450\text{ N})\mathbf{j} - (690\text{ N})\mathbf{k}$$

$$\mathbf{r}_C = (0.9\text{ m})\mathbf{i} + (1.28\text{ m})\mathbf{k}$$

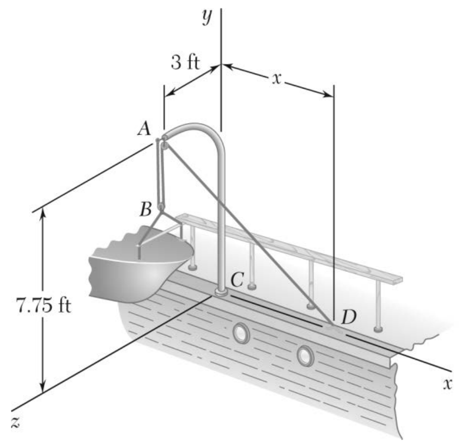
Using Eq. (3.19):

$$\mathbf{M}_O = \mathbf{r}_C \times \mathbf{F}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 0 & 1.28 \\ -675 & 450 & -690 \end{vmatrix}$$

$$\mathbf{M}_O = -(576\text{ N}\cdot\text{m})\mathbf{i} - (243\text{ N}\cdot\text{m})\mathbf{j} + (405\text{ N}\cdot\text{m})\mathbf{k}$$

But $\mathbf{M}_O = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$

Therefore, $M_x = -576\text{ N}\cdot\text{m},\ M_y = -243\text{ N}\cdot\text{m},\ M_z = +405\text{ N}\cdot\text{m}$ ◀



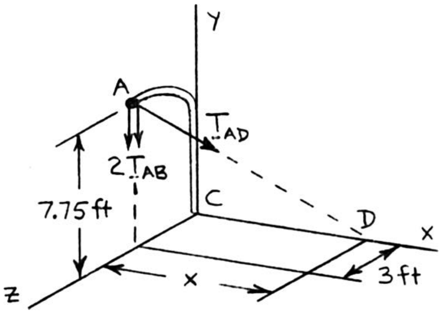
PROBLEM 3.49

A small boat hangs from two davits, one of which is shown in the figure. It is known that the moment about the *z*-axis of the resultant force $\mathbf{R}_A$ exerted on the davit at *A* must not exceed 279 lb·ft in absolute value. Determine the largest allowable tension in line *ABAD* when $x = 6$ ft.

SOLUTION

First note: $\mathbf{R}_A = 2\mathbf{T}_{AB} + \mathbf{T}_{AD}$

Also note that only $\mathbf{T}_{AD}$ will contribute to the moment about the *z*-axis.



Now

$$AD = \sqrt{(6)^2 + (-7.75)^2 + (-3)^2}$$
$$= 10.25 \text{ ft}$$

Then

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD}$$
$$= \frac{T}{10.25}(6\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k})$$

Now

$$M_z = \mathbf{k} \cdot (\mathbf{r}_{A/C} \times \mathbf{T}_{AD})$$

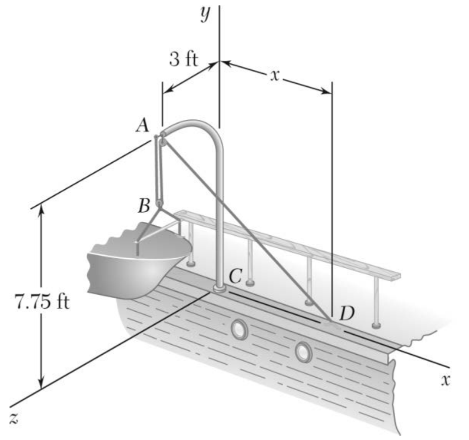
where

$$\mathbf{r}_{A/C} = (7.75 \text{ ft})\mathbf{j} + (3 \text{ ft})\mathbf{k}$$

Then for $T_{\max}$,

$$279 = \left| \frac{T_{\max}}{10.25} \begin{vmatrix} 0 & 0 & 1 \\ 0 & 7.75 & 3 \\ 6 & -7.75 & -3 \end{vmatrix} \right|$$
$$= \frac{T_{\max}}{10.25}\left| -(1)(7.75)(6) \right|$$

or $T_{\max} = 61.5$ lb ◀



PROBLEM 3.50

For the davit of Problem 3.49, determine the largest allowable distance x when the tension in line $ABAD$ is 60 lb.

SOLUTION

From the solution of Problem 3.49, $\mathbf{T}_{AD}$ is now

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD}$$

$$= \frac{60 \text{ lb}}{\sqrt{x^2 + (-7.75)^2 + (-3)^2}}(x\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k})$$

Then $M_z = \mathbf{k} \cdot (\mathbf{r}_{A/C} \times \mathbf{T}_{AD})$ becomes

$$279 = \left| \frac{60}{\sqrt{x^2 + (-7.75)^2 + (-3)^2}} \begin{vmatrix} 0 & 0 & 1 \\ 0 & 7.75 & 3 \\ x & -7.75 & -3 \end{vmatrix} \right|$$

$$279 = \frac{60}{\sqrt{x^2 + 69.0625}} | -(1)(7.75)(x) |$$

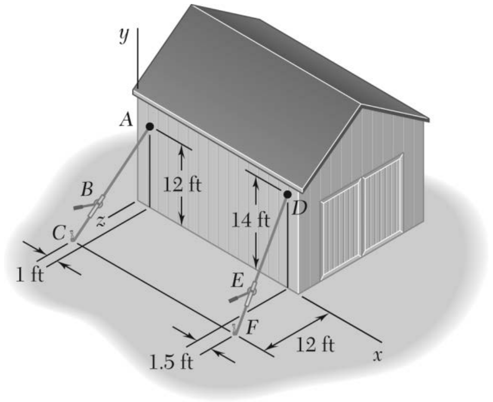
$$279\sqrt{x^2 + 69.0625} = 465x$$

$$0.6\sqrt{x^2 + 69.0625} = x$$

Squaring both sides: $0.36x^2 + 24.8625 = x^2$

$$x^2 = 38.848$$

$x = 6.23$ ft ◀

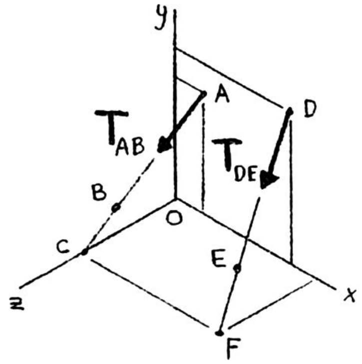


PROBLEM 3.51

A farmer uses cables and winch pullers B and E to plumb one side of a small barn. If it is known that the sum of the moments about the x-axis of the forces exerted by the cables on the barn at Points A and D is equal to 4728 lb · ft, determine the magnitude of $\mathbf{T}_{DE}$ when $T_{AB} = 255$ lb.

SOLUTION

The moment about the x-axis due to the two cable forces can be found using the z components of each force acting at their intersection with the xy plane (A and D). The x components of the forces are parallel to the x-axis, and the y components of the forces intersect the x-axis. Therefore, neither the x or y components produce a moment about the x-axis.



We have $\Sigma M_x\colon \quad (T_{AB})_z(y_A) + (T_{DE})_z(y_D) = M_x$

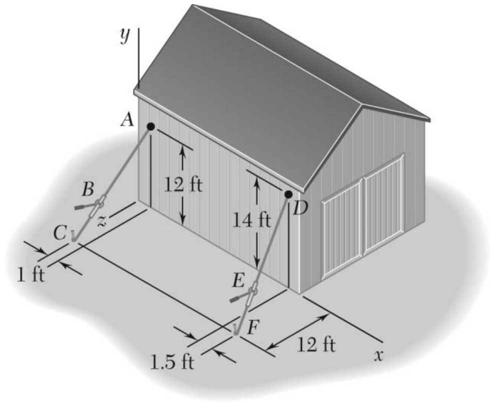
where

$$\begin{aligned}(T_{AB})_z &= \mathbf{k}\cdot\mathbf{T}_{AB}\\ &= \mathbf{k}\cdot(T_{AB}\lambda_{AB})\\ &= \mathbf{k}\cdot\left[255\text{ lb}\left(\frac{-\mathbf{i}-12\mathbf{j}+12\mathbf{k}}{17}\right)\right]\\ &= 180\text{ lb}\end{aligned}$$

$$\begin{aligned}(T_{DE})_z &= \mathbf{k}\cdot\mathbf{T}_{DE}\\ &= \mathbf{k}\cdot(T_{DE}\lambda_{DE})\\ &= \mathbf{k}\cdot\left[T_{DE}\left(\frac{1.5\mathbf{i}-14\mathbf{j}+12\mathbf{k}}{18.5}\right)\right]\\ &= 0.64865T_{DE}\\ y_A &= 12\text{ ft}\\ y_D &= 14\text{ ft}\\ M_x &= 4728\text{ lb}\cdot\text{ft}\end{aligned}$$

$$(180\text{ lb})(12\text{ ft}) + (0.64865T_{DE})(14\text{ ft}) = 4728\text{ lb}\cdot\text{ft}$$

and $T_{DE} = 282.79$ lb or $T_{DE} = 283$ lb ◀



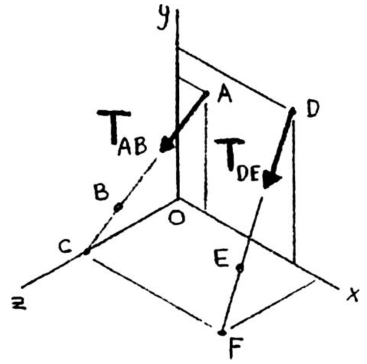
PROBLEM 3.52

Solve Problem 3.51 when the tension in cable *AB* is 306 lb.

PROBLEM 3.51 A farmer uses cables and winch pullers *B* and *E* to plumb one side of a small barn. If it is known that the sum of the moments about the *x*-axis of the forces exerted by the cables on the barn at Points *A* and *D* is equal to 4728 lb · ft, determine the magnitude of $\mathbf{T}_{DE}$ when $T_{AB} = 255$ lb.

SOLUTION

The moment about the *x*-axis due to the two cable forces can be found using the *z* components of each force acting at the intersection with the *xy* plane (*A* and *D*). The *x* components of the forces are parallel to the *x*-axis, and the *y* components of the forces intersect the *x*-axis. Therefore, neither the *x* or *y* components produce a moment about the *x*-axis.



We have ΣM_x: $(T_{AB})_z(y_A) + (T_{DE})_z(y_D) = M_x$

Where

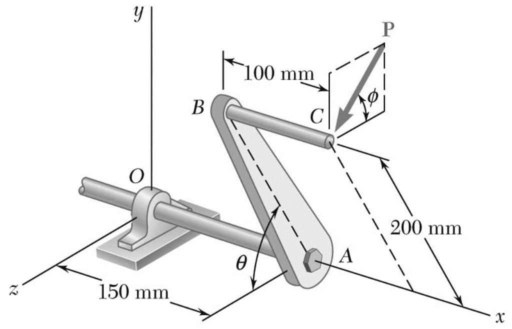
$$(T_{AB})_z = \mathbf{k}\cdot\mathbf{T}_{AB}$$
$$= \mathbf{k}\cdot(T_{AB}\boldsymbol{\lambda}_{AB})$$
$$= \mathbf{k}\cdot\left[306\text{ lb}\left(\frac{-\mathbf{i}-12\mathbf{j}+12\mathbf{k}}{17}\right)\right]$$
$$= 216\text{ lb}$$

$$(T_{DE})_z = \mathbf{k}\cdot\mathbf{T}_{DE}$$
$$= \mathbf{k}\cdot(T_{DE}\boldsymbol{\lambda}_{DE})$$
$$= \mathbf{k}\cdot\left[T_{DE}\left(\frac{1.5\mathbf{i}-14\mathbf{j}+12\mathbf{k}}{18.5}\right)\right]$$
$$= 0.64865T_{DE}$$

$$y_A = 12\text{ ft}$$
$$y_D = 14\text{ ft}$$
$$M_x = 4728\text{ lb}\cdot\text{ft}$$

$$(216\text{ lb})(12\text{ ft}) + (0.64865T_{DE})(14\text{ ft}) = 4728\text{ lb}\cdot\text{ft}$$

and $T_{DE} = 235.21$ lb or $T_{DE} = 235$ lb ◀



PROBLEM 3.53

A single force **P** acts at *C* in a direction perpendicular to the handle *BC* of the crank shown. Knowing that $M_x = +20$ N · m and $M_y = -8.75$ N · m, and $M_z = -30$ N · m, determine the magnitude of **P** and the values of ϕ and θ.

SOLUTION

$$\mathbf{r}_C = (0.25\text{ m})\mathbf{i} + (0.2\text{ m})\sin\theta\mathbf{j} + (0.2\text{ m})\cos\theta\mathbf{k}$$
$$\mathbf{P} = -P\sin\phi\mathbf{j} + P\cos\phi\mathbf{k}$$
$$\mathbf{M}_O = \mathbf{r}_C \times \mathbf{P} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.25 & 0.2\sin\theta & 0.2\cos\theta \\ 0 & -P\sin\phi & P\cos\phi \end{vmatrix}$$

Expanding the determinant, we find

$$M_x = (0.2)P(\sin\theta\cos\phi + \cos\theta\sin\phi)$$
$$M_x = (0.2)P\sin(\theta+\phi) \qquad (1)$$
$$M_y = -(0.25)P\cos\phi \qquad (2)$$
$$M_z = -(0.25)P\sin\phi \qquad (3)$$

Dividing Eq. (3) by Eq. (2) gives: $\tan\phi = \dfrac{M_z}{M_y}$ (4)

$$\tan\phi = \frac{-30\text{ N}\cdot\text{m}}{-8.75\text{ N}\cdot\text{m}}$$
$$\phi = 73.740 \qquad \phi = 73.7° ◀$$

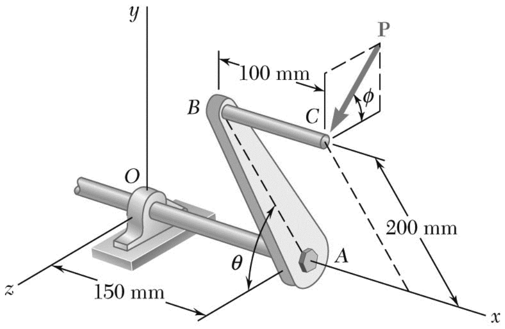
Squaring Eqs. (2) and (3) and adding gives:

$$M_y^2 + M_z^2 = (0.25)^2P^2 \quad \text{or} \quad P = 4\sqrt{M_y^2 + M_z^2} \qquad (5)$$
$$P = 4\sqrt{(8.75)^2 + (30)^2}$$
$$= 125.0\text{ N} \qquad P = 125.0\text{ N} ◀$$

Substituting data into Eq. (1):

$$(+20\text{ N}\cdot\text{m}) = 0.2\text{ m}(125.0\text{ N})\sin(\theta+\phi)$$
$$(\theta+\phi) = 53.130° \quad \text{and} \quad (\theta+\phi) = 126.87°$$
$$\theta = -20.6° \quad \text{and} \quad \theta = 53.1°$$

$$Q = 53.1° ◀$$



PROBLEM 3.54

A single force **P** acts at *C* in a direction perpendicular to the handle *BC* of the crank shown. Determine the moment M_x of **P** about the *x*-axis when $\theta = 65°$, knowing that $M_y = -15\ \text{N}\cdot\text{m}$ and $M_z = -36\ \text{N}\cdot\text{m}$.

SOLUTION

See the solution to Prob. 3.53 for the derivation of the following equations:

$$M_x = (0.2)P\sin(\theta+\phi) \qquad (1)$$

$$\tan\phi = \frac{M_z}{M_y} \qquad (4)$$

$$P = 4\sqrt{M_y^2 + M_z^2} \qquad (5)$$

Substituting for known data gives:

$$\tan\phi = \frac{-36\ \text{N}\cdot\text{m}}{-15\ \text{N}\cdot\text{m}}$$

$$\phi = 67.380°$$

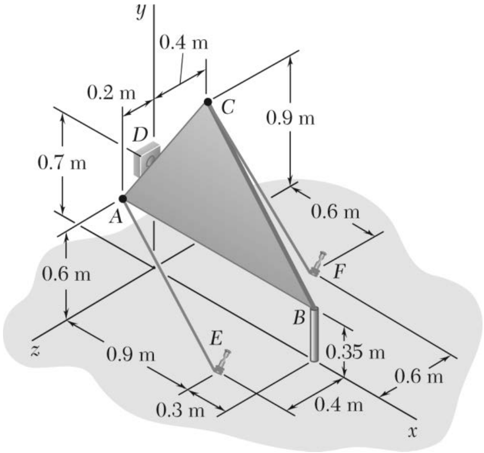
$$P = 4\sqrt{(-15)^2 + (-36)^2}$$

$$P = 156.0\ \text{N}$$

$$M_x = 0.2\ \text{m}(156.0\ \text{N})\sin(65° + 67.380°)$$

$$= 23.047\ \text{N}\cdot\text{m}$$

$$M_x = 23.0\ \text{N}\cdot\text{m} \blacktriangleleft$$



PROBLEM 3.55

The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *AE* at *A* is 55 N, determine the moment of that force about the line joining Points *D* and *B*.

SOLUTION

First note:

$$\mathbf{T}_{AE} = T_{AE}\frac{\overrightarrow{AE}}{AE}$$

$$AE = \sqrt{(0.9)^2 + (-0.6)^2 + (0.2)^2} = 1.1 \text{ m}$$

Then

$$\mathbf{T}_{AE} = \frac{55 \text{ N}}{1.1}(0.9\mathbf{i} - 0.6\mathbf{j} + 0.2\mathbf{k})$$

$$= 5[(9 \text{ N})\mathbf{i} - (6 \text{ N})\mathbf{j} + (2 \text{ N})\mathbf{k}]$$

Also,

$$DB = \sqrt{(1.2)^2 + (-0.35)^2 + (0)^2}$$

$$= 1.25 \text{ m}$$

Then

$$\boldsymbol{\lambda}_{DB} = \frac{\overrightarrow{DB}}{DB}$$

$$= \frac{1}{1.25}(1.2\mathbf{i} - 0.35\mathbf{j})$$

$$= \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

Now

$$M_{DB} = \boldsymbol{\lambda}_{DB} \cdot (\mathbf{r}_{A/D} \times \mathbf{T}_{AE})$$

where

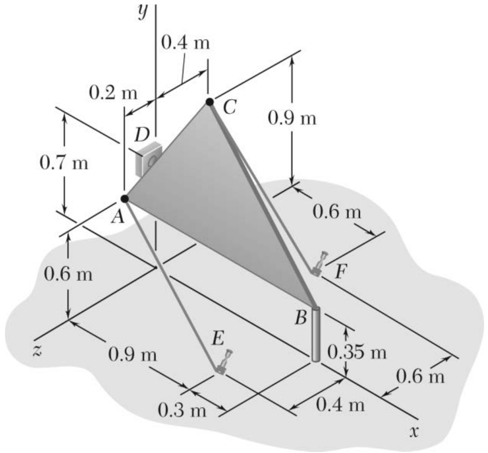
$$\mathbf{r}_{A/D} = -(0.1 \text{ m})\mathbf{j} + (0.2 \text{ m})\mathbf{k}$$

Then

$$M_{DB} = \frac{1}{25}(5)\begin{vmatrix} 24 & -7 & 0 \\ 0 & -0.1 & 0.2 \\ 9 & -6 & 2 \end{vmatrix}$$

$$= \frac{1}{5}(-4.8 - 12.6 + 28.8)$$

or $M_{DB} = 2.28 \text{ N}\cdot\text{m}$ ◀



PROBLEM 3.56

The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *CF* at *C* is 33 N, determine the moment of that force about the line joining Points *D* and *B*.

SOLUTION

First note:

$$\mathbf{T}_{CF} = T_{CF}\frac{\overrightarrow{CF}}{CF}$$

$$CF = \sqrt{(0.6)^2 + (-0.9)^2 + (-0.2)^2} = 1.1 \text{ m}$$

Then

$$\mathbf{T}_{CF} = \frac{33 \text{ N}}{1.1}(0.6\mathbf{i} - 0.9\mathbf{j} + 0.2\mathbf{k})$$

$$= 3[(6 \text{ N})\mathbf{i} - (9 \text{ N})\mathbf{j} - (2 \text{ N})\mathbf{k}]$$

Also,

$$DB = \sqrt{(1.2)^2 + (-0.35)^2 + (0)^2}$$

$$= 1.25 \text{ m}$$

Then

$$\boldsymbol{\lambda}_{DB} = \frac{\overrightarrow{DB}}{DB}$$

$$= \frac{1}{1.25}(1.2\mathbf{i} - 0.35\mathbf{j})$$

$$= \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

Now

$$M_{DB} = \boldsymbol{\lambda}_{DB} \cdot (\mathbf{r}_{C/D} \times \mathbf{T}_{CF})$$

where

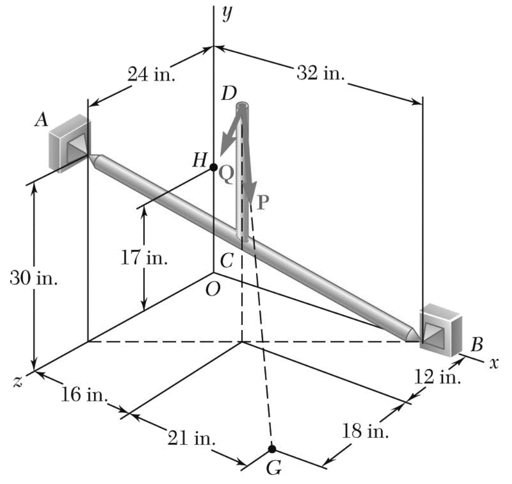
$$\mathbf{r}_{C/D} = (0.2 \text{ m})\mathbf{j} - (0.4 \text{ m})\mathbf{k}$$

Then

$$M_{DB} = \frac{1}{25}(3)\begin{vmatrix} 24 & -7 & 0 \\ 0 & 0.2 & -0.4 \\ 6 & -9 & -2 \end{vmatrix}$$

$$= \frac{3}{25}(-9.6 + 16.8 - 86.4)$$

or $M_{DB} = -9.50 \text{ N}\cdot\text{m}$ ◀



PROBLEM 3.57

The 23-in. vertical rod *CD* is welded to the midpoint *C* of the 50-in. rod *AB*. Determine the moment about *AB* of the 235-lb force **P**.

SOLUTION

$$\overline{AB} = (32\text{ in.})\mathbf{i} - (30\text{ in.})\mathbf{j} - (24\text{ in.})\mathbf{k}$$

$$AB = \sqrt{(32)^2 + (-30)^2 + (-24)^2} = 50\text{ in.}$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overline{AB}}{AB} = 0.64\mathbf{i} - 0.60 - 0.48\mathbf{k}$$

We shall apply the force *P* at Point *G*:

$$\mathbf{r}_{G/B} = (5\text{ in.})\mathbf{i} + (30\text{ in.})\mathbf{k}$$

$$\overrightarrow{DG} = (21\text{ in.})\mathbf{i} - (38\text{ in.})\mathbf{j} + (18\text{ in.})\mathbf{k}$$

$$DG = \sqrt{(21)^2 + (-38)^2 + (18)^2} = 47\text{ in.}$$

$$\mathbf{P} = P\frac{\overrightarrow{DG}}{DG} = (235\text{ lb})\frac{21\mathbf{i} - 38\mathbf{j} + 18\mathbf{k}}{47}$$

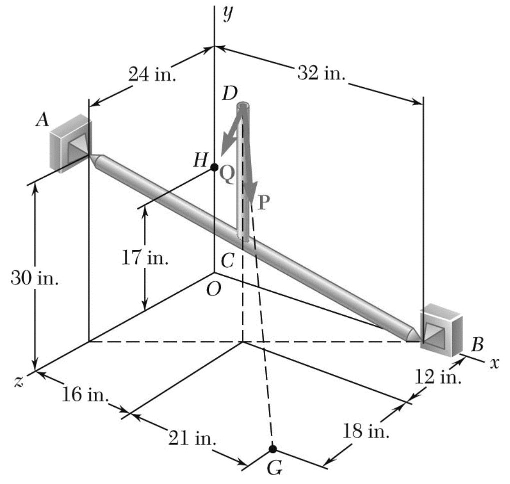
$$\mathbf{P} = (105\text{ lb})\mathbf{i} - (190\text{ lb})\mathbf{j} + (90\text{ lb})\mathbf{k}$$

The moment of **P** about *AB* is given by Eq. (3.46):

$$\mathbf{M}_{AB} = \boldsymbol{\lambda}_{AB} \cdot (\mathbf{r}_{G/B} \times P) = \begin{vmatrix} 0.64 & -0.60 & -0.48 \\ 5\text{ in.} & 0 & 30\text{ in.} \\ 105\text{ lb} & -190\text{ lb} & 90\text{ lb} \end{vmatrix}$$

$$\begin{aligned} \mathbf{M}_{AB} &= 0.64[0 - (30\text{ in.})(-190\text{ lb})] \\ &\quad - 0.60[(30\text{ in.})(105\text{ lb}) - (5\text{ in.})(90\text{ lb})] \\ &\quad - 0.48[(5\text{ in.})(-190\text{ lb}) - 0] \\ &= +2484\text{ lb}\cdot\text{in.} \end{aligned}$$

$$\mathbf{M}_{AB} = +207\text{ lb}\cdot\text{ft} \blacktriangleleft$$



PROBLEM 3.58

The 23-in. vertical rod *CD* is welded to the midpoint *C* of the 50-in. rod *AB*. Determine the moment about *AB* of the 174-lb force **Q**.

SOLUTION

$$\overline{AB} = (32\text{ in.})\mathbf{i} - (30\text{ in.})\mathbf{j} - (24\text{ in.})\mathbf{k}$$

$$AB = \sqrt{(32)^2 + (-30)^2 + (-24)^2} = 50\text{ in.}$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overline{AB}}{AB} = 0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}$$

We shall apply the force **Q** at Point *H*:

$$\mathbf{r}_{H/B} = -(32\text{ in.})\mathbf{i} + (17\text{ in.})\mathbf{j}$$

$$\overline{DH} = -(16\text{ in.})\mathbf{i} - (21\text{ in.})\mathbf{j} - (12\text{ in.})\mathbf{k}$$

$$DH = \sqrt{(16)^2 + (-21)^2 + (-12)^2} = 29\text{ in.}$$

$$\mathbf{Q} = \frac{\overline{DH}}{DH} = (174\text{ lb})\frac{-16\mathbf{i} - 21\mathbf{j} - 12\mathbf{k}}{29}$$

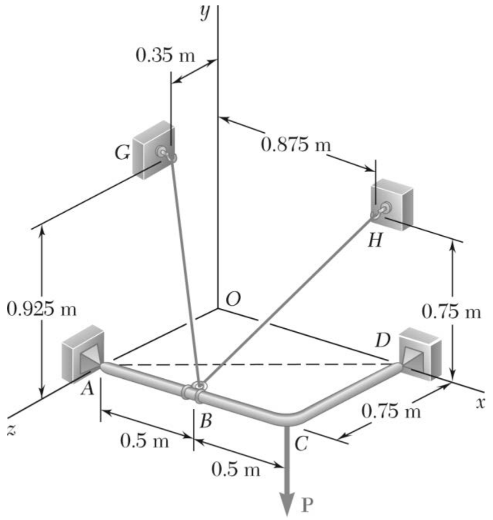
$$Q = -(96\text{ lb})\mathbf{i} - (126\text{ lb})\mathbf{j} - (72\text{ lb})\mathbf{k}$$

The moment of **Q** about *AB* is given by Eq. (3.46):

$$\mathbf{M}_{AB} = \boldsymbol{\lambda}_{AB} \cdot (\mathbf{r}_{H/B} \times \mathbf{Q}) = \begin{vmatrix} 0.64 & -0.60 & -0.48 \\ -32\text{ in.} & 17\text{ in.} & 0 \\ -96\text{ lb} & -126\text{ lb} & -72\text{ lb} \end{vmatrix}$$

$$\begin{aligned} \mathbf{M}_{AB} &= 0.64[(17\text{ in.})(-72\text{ lb}) - 0] \\ &\quad - 0.60[(0 - (-32\text{ in.})(-72\text{ lb})] \\ &\quad - 0.48[(-32\text{ in.})(-126\text{ lb}) - (17\text{ in.})(-96\text{ lb})] \\ &= -2119.7\text{ lb}\cdot\text{in.} \end{aligned}$$

$$\mathbf{M}_{AB} = 176.6\text{ lb}\cdot\text{ft} \blacktriangleleft$$



PROBLEM 3.59

The frame *ACD* is hinged at *A* and *D* and is supported by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the tension in the cable is 450 N, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BH* of the cable.

SOLUTION

$$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BH})$$

Where

$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

$$\mathbf{r}_{B/A} = (0.5\text{ m})\mathbf{i}$$

and

$$d_{BH} = \sqrt{(0.375)^2 + (0.75)^2 + (-0.75)^2}$$
$$= 1.125\text{ m}$$

Then

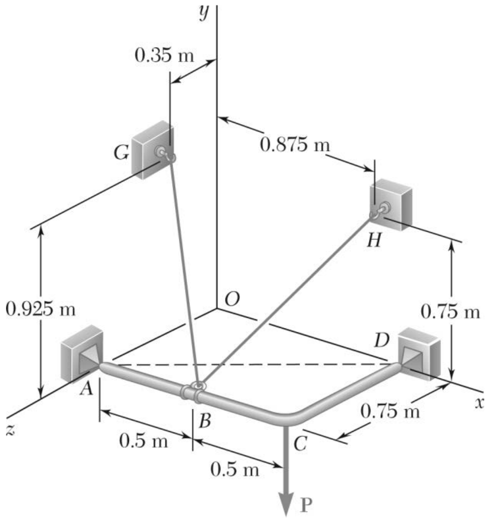
$$\mathbf{T}_{BH} = \frac{450\text{ N}}{1.125}(0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k})$$
$$= (150\text{ N})\mathbf{i} + (300\text{ N})\mathbf{j} - (300\text{ N})\mathbf{k}$$

Finally,

$$M_{AD} = \frac{1}{5}\begin{vmatrix} 4 & 0 & -3 \\ 0.5 & 0 & 0 \\ 150 & 300 & -300 \end{vmatrix}$$

$$= \frac{1}{5}[(-3)(0.5)(300)]$$

or $M_{AD} = -90.0\text{ N}\cdot\text{m}$ ◀



PROBLEM 3.60

In Problem 3.59, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BG* of the cable.

PROBLEM 3.59 The frame *ACD* is hinged at *A* and *D* and is supported by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the tension in the cable is 450 N, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BH* of the cable.

SOLUTION

$$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BG})$$

Where

$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

$$\mathbf{r}_{B/A} = (0.5 \text{ m})\mathbf{j}$$

and

$$BG = \sqrt{(-0.5)^2 + (0.925)^2 + (-0.4)^2}$$

$$= 1.125 \text{ m}$$

Then

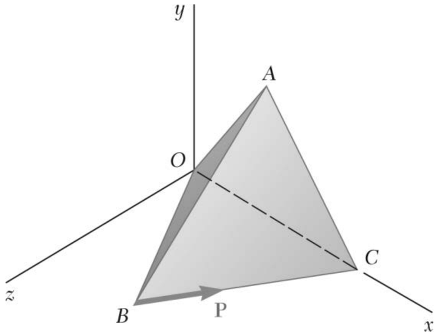
$$\mathbf{T}_{BG} = \frac{450 \text{ N}}{1.125}(-0.5\mathbf{i} + 0.925\mathbf{j} - 0.4\mathbf{k})$$

$$= -(200 \text{ N})\mathbf{i} + (370 \text{ N})\mathbf{j} - (160 \text{ N})\mathbf{k}$$

Finally,

$$M_{AD} = \frac{1}{5}\begin{vmatrix} 4 & 0 & -3 \\ 0.5 & 0 & 0 \\ -200 & 370 & -160 \end{vmatrix}$$

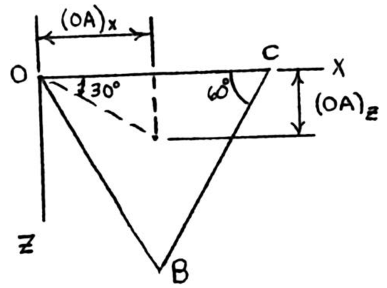
$$= \frac{1}{5}[(-3)(0.5)(370)] \qquad M_{AD} = -111.0 \text{ N}\cdot\text{m} \blacktriangleleft$$



PROBLEM 3.61

A regular tetrahedron has six edges of length a. A force **P** is directed as shown along edge BC. Determine the moment of **P** about edge OA.

SOLUTION



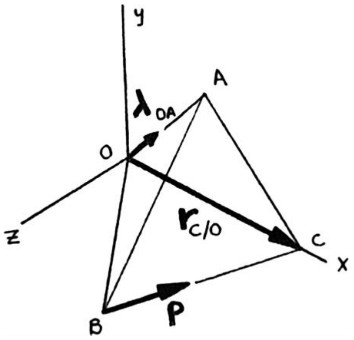
We have $$M_{OA} = \boldsymbol{\lambda}_{OA} \cdot (\mathbf{r}_{C/O} \times \mathbf{P})$$

From triangle OBC: $$(OA)_x = \frac{a}{2}$$

$$(OA)_z = (OA)_x \tan 30° = \frac{a}{2}\left(\frac{1}{\sqrt{3}}\right) = \frac{a}{2\sqrt{3}}$$

Since $$(OA)^2 = (OA)_x^2 + (OA)_y^2 + (OA_z)^2$$

or $$a^2 = \left(\frac{a}{2}\right)^2 + (OA)_y^2 + \left(\frac{a}{2\sqrt{3}}\right)^2$$



$$(OA)_y = \sqrt{a^2 - \frac{a^2}{4} - \frac{a^2}{12}} = a\sqrt{\frac{2}{3}}$$

Then $$\mathbf{r}_{A/O} = \frac{a}{2}\mathbf{i} + a\sqrt{\frac{2}{3}}\,\mathbf{j} + \frac{a}{2\sqrt{3}}\mathbf{k}$$

and $$\boldsymbol{\lambda}_{OA} = \frac{1}{2}\mathbf{i} + \sqrt{\frac{2}{3}}\mathbf{j} + \frac{1}{2\sqrt{3}}\mathbf{k}$$

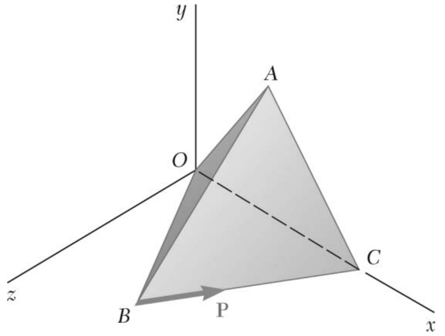
$$\mathbf{P} = \boldsymbol{\lambda}_{BC} P = \frac{(a\sin 30°)\mathbf{i} - (a\cos 30°)\mathbf{k}}{a}(P) = \frac{P}{2}(\mathbf{i} - \sqrt{3}\mathbf{k})$$

$$\mathbf{r}_{C/O} = a\mathbf{i}$$

$$M_{OA} = \begin{vmatrix} \frac{1}{2} & \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}} \\ 1 & 0 & 0 \\ 1 & 0 & -\sqrt{3} \end{vmatrix} (a)\left(\frac{P}{2}\right)$$

$$= \frac{aP}{2}\left(-\sqrt{\frac{2}{3}}\right)(1)(-\sqrt{3}) = \frac{aP}{\sqrt{2}}$$

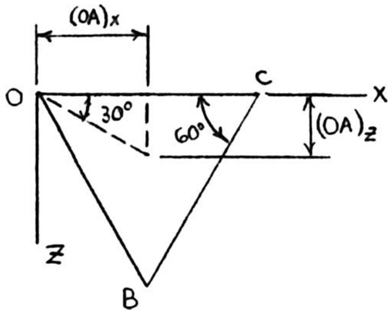
$$M_{OA} = \frac{aP}{\sqrt{2}} ◀$$

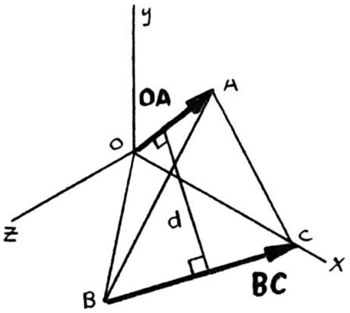


PROBLEM 3.62

A regular tetrahedron has six edges of length *a*. (*a*) Show that two opposite edges, such as *OA* and *BC*, are perpendicular to each other. (*b*) Use this property and the result obtained in Problem 3.61 to determine the perpendicular distance between edges *OA* and *BC*.

SOLUTION





(*a*) For edge *OA* to be perpendicular to edge *BC*,

$$\overrightarrow{OA} \cdot \overrightarrow{BC} = 0$$

From triangle *OBC*: $(OA)_x = \dfrac{a}{2}$

$$(OA)_z = (OA)_x \tan 30° = \frac{a}{2}\left(\frac{1}{\sqrt{3}}\right) = \frac{a}{2\sqrt{3}}$$

$$\overrightarrow{OA} = \left(\frac{a}{2}\right)\mathbf{i} + (OA)_y\,\mathbf{j} + \left(\frac{a}{2\sqrt{3}}\right)\mathbf{k}$$

and $\overrightarrow{BC} = (a\sin 30°)\,\mathbf{i} - (a\cos 30°)\,\mathbf{k}$

$$= \frac{a}{2}\mathbf{i} - \frac{a\sqrt{3}}{2}\mathbf{k} = \frac{a}{2}(\mathbf{i} - \sqrt{3}\,\mathbf{k})$$

Then $\left[\dfrac{a}{2}\mathbf{i} + (OA)_y\,\mathbf{j} + \left(\dfrac{a}{2\sqrt{3}}\right)\mathbf{k}\right] \cdot (\mathbf{i} - \sqrt{3}\mathbf{k})\dfrac{a}{2} = 0$

or $\dfrac{a^2}{4} + (OA)_y(0) - \dfrac{a^2}{4} = 0$

$$\overrightarrow{OA} \cdot \overrightarrow{BC} = 0$$

so that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{BC}$. ◀

(*b*) We have $M_{OA} = Pd$, with *P* acting along *BC* and *d* the perpendicular distance from $\overrightarrow{OA}$ to $\overrightarrow{BC}$.

From the results of Problem 3.57,

$$M_{OA} = \frac{Pa}{\sqrt{2}}$$

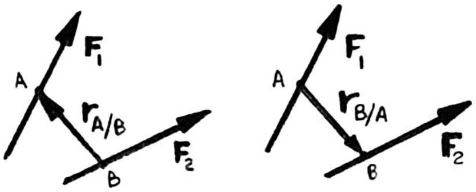
$$\frac{Pa}{\sqrt{2}} = Pd$$

or $d = \dfrac{a}{\sqrt{2}}$ ◀

PROBLEM 3.63

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in space have the same magnitude F. Prove that the moment of $\mathbf{F}_1$ about the line of action of $\mathbf{F}_2$ is equal to the moment of $\mathbf{F}_2$ about the line of action of $\mathbf{F}_1$.

SOLUTION



First note that $\mathbf{F}_1 = F_1\boldsymbol{\lambda}_1$ and $\mathbf{F}_2 = F_2\boldsymbol{\lambda}_2$

Let $M_1 =$ moment of $\mathbf{F}_2$ about the line of action of $\mathbf{F}_1$ and $M_2 =$ moment of $\mathbf{F}_1$ about the line of action of $\mathbf{F}_2$.

Now, by definition,

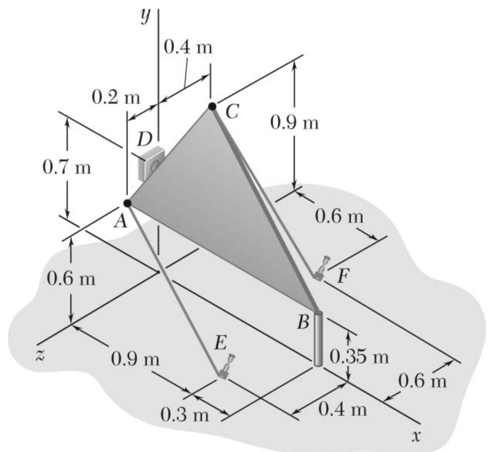
$$M_1 = \boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \mathbf{F}_2)$$
$$= \boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2)F_2$$
$$M_2 = \boldsymbol{\lambda}_2 \cdot (\mathbf{r}_{A/B} \times \mathbf{F}_1)$$
$$= \boldsymbol{\lambda}_2 \cdot (\mathbf{r}_{A/B} \times \boldsymbol{\lambda}_1)F_1$$

Since $F_1 = F_2 = F$ and $\mathbf{r}_{A/B} = -\mathbf{r}_{B/A}$

$$M_1 = \boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2)F$$
$$M_2 = \boldsymbol{\lambda}_2 \cdot (-\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_1)F$$

Using Equation (3.39): $\boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2) = \boldsymbol{\lambda}_2 \cdot (-\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_1)$

so that $M_2 = \boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2)F$ $\quad M_{12} = M_{21}$ ◀



PROBLEM 3.64

In Problem 3.55, determine the perpendicular distance between cable *AE* and the line joining Points *D* and *B*.

PROBLEM 3.55 The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *AE* at *A* is 55 N, determine the moment of that force about the line joining Points *D* and *B*.

SOLUTION

From the solution to Problem 3.55:

$$\mathbf{T}_{AE} = 55 \text{ N}$$

$$\mathbf{T}_{AE} = 5[(9\text{ N})\mathbf{i} - (6\text{ N})\mathbf{j} + (2\text{ N})\mathbf{k}]$$

$$|M_{DB}| = 2.28 \text{ N}\cdot\text{m}$$

$$\boldsymbol{\lambda}_{DB} = \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{AE}$ will contribute to the moment of $\mathbf{T}_{AE}$ about line $\overrightarrow{DB}$.

Now

$$(T_{AE})_{\text{parallel}} = \mathbf{T}_{AE} \cdot \boldsymbol{\lambda}_{DB}$$

$$= 5(9\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}) \cdot \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

$$= \frac{1}{5}[(9)(24) + (-6)(-7)]$$

$$= 51.6 \text{ N}$$

Also,

$$\mathbf{T}_{AE} = (\mathbf{T}_{AE})_{\text{parallel}} + (\mathbf{T}_{AE})_{\text{perpendicular}}$$

so that

$$(\mathbf{T}_{AE})_{\text{perpendicular}} = \sqrt{(55)^2 + (51.6)^2} = 19.0379 \text{ N}$$

Since $\boldsymbol{\lambda}_{DB}$ and $(\mathbf{T}_{AE})_{\text{perpendicular}}$ are perpendicular, it follows that

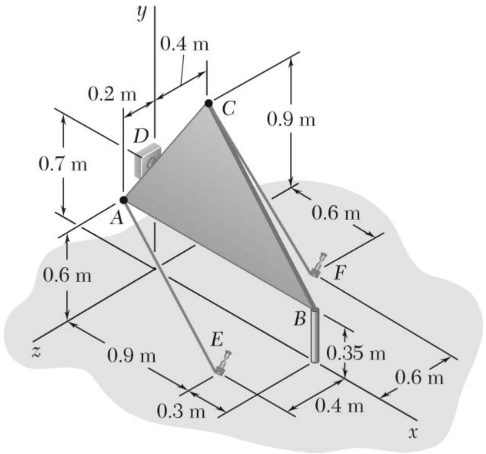
$$M_{DB} = d(T_{AE})_{\text{perpendicular}}$$

or

$$2.28 \text{ N}\cdot\text{m} = d(19.0379 \text{ N})$$

$$d = 0.119761$$

$d = 0.1198$ m ◀



PROBLEM 3.65

In Problem 3.56, determine the perpendicular distance between cable *CF* and the line joining Points *D* and *B*.

PROBLEM 3.56 The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *CF* at *C* is 33 N, determine the moment of that force about the line joining Points *D* and *B*.

SOLUTION

From the solution to Problem 3.56: $\mathbf{T}_{CF} = 33\text{ N}$

$$\mathbf{T}_{CF} = 3[(6\text{ N})\mathbf{i} - (9\text{ N})\mathbf{j} - (2\text{ N})\mathbf{k}]$$

$$|M_{DB}| = 9.50\text{ N}\cdot\text{m}$$

$$\boldsymbol{\lambda}_{DB} = \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{CF}$ will contribute to the moment of $\mathbf{T}_{CF}$ about line $\overrightarrow{DB}$.

Now $$(\mathbf{T}_{CF})_{\text{parallel}} = \mathbf{T}_{CF}\cdot\boldsymbol{\lambda}_{DB}$$
$$= 3(6\mathbf{i} - 9\mathbf{j} - 2\mathbf{k})\cdot\frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$
$$= \frac{3}{25}[(6)(24) + (-9)(-7)]$$
$$= 24.84\text{ N}$$

Also, $$\mathbf{T}_{CF} = (\mathbf{T}_{CF})_{\text{parallel}} + (\mathbf{T}_{CF})_{\text{perpendicular}}$$

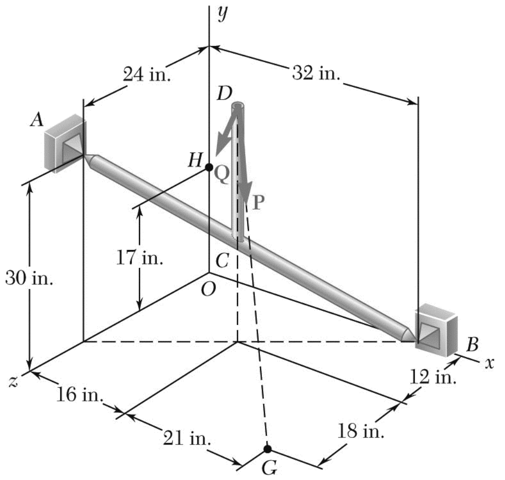
so that $$(\mathbf{T}_{CF})_{\text{perpendicular}} = \sqrt{(33)^2 - (24.84)^2}$$
$$= 21.725\text{ N}$$

Since $\boldsymbol{\lambda}_{DB}$ and $(\mathbf{T}_{CF})_{\text{perpendicular}}$ are perpendicular, it follows that

$$|M_{DB}| = d(T_{CF})_{\text{perpendicular}}$$

or $$9.50\text{ N}\cdot\text{m} = d\times 21.725\text{ N}$$

or $d = 0.437\text{ m}$ ◀



PROBLEM 3.66

In Prob. 3.57, determine the perpendicular distance between rod *AB* and the line of action of **P**.

PROBLEM 3.57 The 23-in. vertical rod *CD* is welded to the midpoint *C* of the 50-in. rod *AB*. Determine the moment about *AB* of the 235-lb force **P**.

SOLUTION

$$\overline{AB} = (32\text{ in.})\mathbf{i} - (30\text{ in.})\mathbf{j} - (24\text{ in.})\mathbf{k}$$

$$AB = \sqrt{(32)^2 + (-30)^2 + (-24)^2} = 50\text{ in.}$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overline{AB}}{AB} = 0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}$$

$$\boldsymbol{\lambda}_P = \frac{\mathbf{P}}{P} = \frac{105\mathbf{i} - 190\mathbf{j} + 90\mathbf{k}}{235}$$

Angle θ between *AB* and **P**:

$$\begin{aligned}\cos\theta &= \boldsymbol{\lambda}_{AB} \cdot \boldsymbol{\lambda}_P \\ &= (0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}) \cdot \frac{105\mathbf{i} - 190\mathbf{j} + 90\mathbf{k}}{235} \\ &= 0.58723\end{aligned}$$

$$\therefore \quad \theta = 54.039°$$

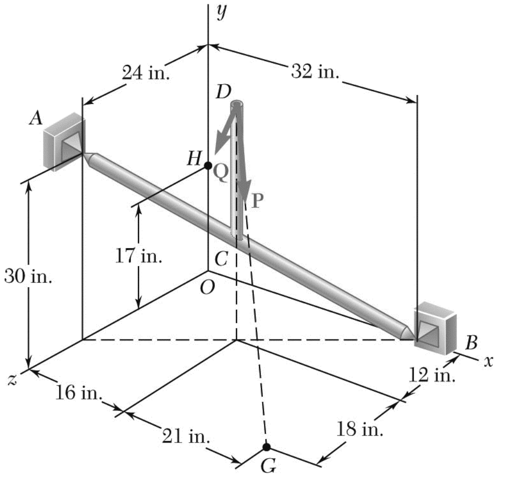
The moment of **P** about *AB* may be obtained by multiplying the projection of **P** on a plane perpendicular to *AB* by the perpendicular distance *d* from *AB* to **P**:

$$\mathbf{M}_{AB} = (P\sin\theta)d$$

From the solution to Prob. 3.57: $\mathbf{M}_{AB} = 207\text{ lb}\cdot\text{ft} = 2484\text{ lb}\cdot\text{in.}$

We have $\qquad 2484\text{ lb}\cdot\text{in.} = (235\text{ lb})(\sin 54.039)d$

$$d = 13.06\text{ in.} \blacktriangleleft$$



PROBLEM 3.67

In Prob. 3.58, determine the perpendicular distance between rod *AB* and the line of action of **Q**.

PROBLEM 3.58 The 23-in. vertical rod *CD* is welded to the midpoint *C* of the 50-in. rod *AB*. Determine the moment about *AB* of the 174-lb force **Q**.

SOLUTION

$$\overline{AB} = (32\text{ in.})\mathbf{i} - (30\text{ in.})\mathbf{j} - (24\text{ in.})\mathbf{k}$$

$$AB = \sqrt{(32)^2 + (-30)^2 + (-24)^2} = 50\text{ in.}$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overline{AB}}{AB} = 0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}$$

$$\boldsymbol{\lambda}_Q = \frac{\mathbf{Q}}{Q} = \frac{-96\mathbf{i} - 126\mathbf{j} - 72\mathbf{k}}{174}$$

Angle θ between *AB* and **Q**:

$$\begin{aligned} \cos\theta &= \boldsymbol{\lambda}_{AB} \cdot \boldsymbol{\lambda}_Q \\ &= (0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}) \cdot \frac{(-96\mathbf{i} - 126\mathbf{j} - 72\mathbf{k})}{174} \\ &= 0.28000 \\ \therefore\quad \theta &= 73.740^\circ \end{aligned}$$

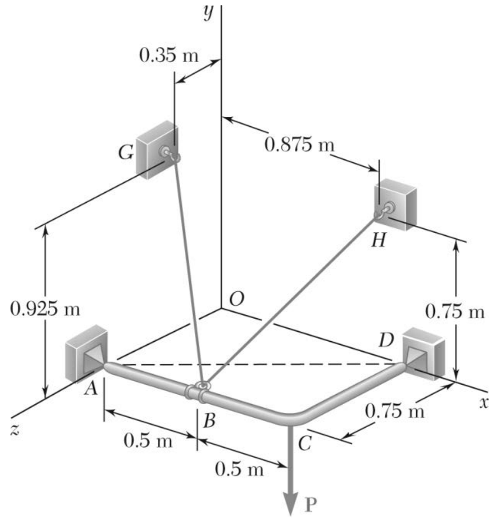
The moment of **Q** about *AB* may be obtained by multiplying the projection of **Q** on a plane perpendicular to *AB* by the perpendicular distance *d* from *AB* to **Q**:

$$\mathbf{M}_{AB} = (Q\sin\theta)d$$

From the solution to Prob. 3.58: $\mathbf{M}_{AB} = 176.6\text{ lb}\cdot\text{ft} = 2119.2\text{ lb}\cdot\text{in.}$

$$2119.2\text{ lb}\cdot\text{in.} = (174\text{ lb})(\sin 73.740^\circ)d$$

$d = 12.69$ in. ◀



PROBLEM 3.68

In Problem 3.59, determine the perpendicular distance between portion *BH* of the cable and the diagonal *AD*.

PROBLEM 3.59 The frame *ACD* is hinged at *A* and *D* and is supported by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the tension in the cable is 450 N, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BH* of the cable.

SOLUTION

From the solution to Problem 3.59: $T_{BH} = 450 \text{ N}$

$$\mathbf{T}_{BH} = (150 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} - (300 \text{ N})\mathbf{k}$$

$$|M_{AD}| = 90.0 \text{ N}\cdot\text{m}$$

$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{BH}$ will contribute to the moment of $\mathbf{T}_{BH}$ about line $\overrightarrow{AD}$.

Now $$(T_{BH})_{\text{parallel}} = \mathbf{T}_{BH} \cdot \boldsymbol{\lambda}_{AD}$$

$$= (150\mathbf{i} + 300\mathbf{j} - 300\mathbf{k}) \cdot \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

$$= \frac{1}{5}[(150)(4) + (-300)(-3)]$$

$$= 300 \text{ N}$$

Also, $$\mathbf{T}_{BH} = (\mathbf{T}_{BH})_{\text{parallel}} + (\mathbf{T}_{BH})_{\text{perpendicular}}$$

so that $$(T_{BH})_{\text{perpendicular}} = \sqrt{(450)^2 - (300)^2} = 335.41 \text{ N}$$

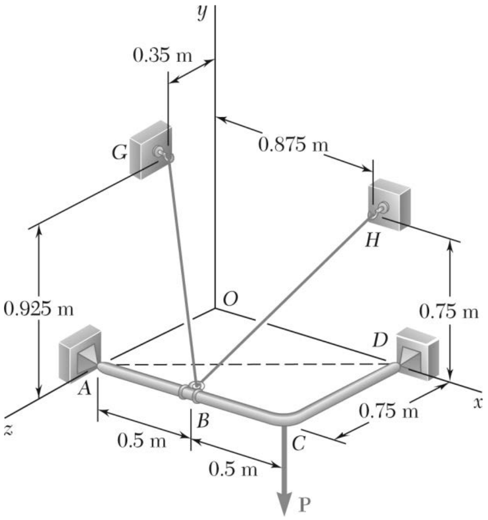
Since $\boldsymbol{\lambda}_{AD}$ and $(\mathbf{T}_{BH})_{\text{perpendicular}}$ are perpendicular, it follows that

$$M_{AD} = d(T_{BH})_{\text{perpendicular}}$$

or $$90.0 \text{ N}\cdot\text{m} = d(335.41 \text{ N})$$

$$d = 0.26833 \text{ m}$$

$d = 0.268$ m ◀



PROBLEM 3.69

In Problem 3.60, determine the perpendicular distance between portion *BG* of the cable and the diagonal *AD*.

PROBLEM 3.60 In Problem 3.59, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BG* of the cable.

SOLUTION

From the solution to Problem 3.60: $\mathbf{T}_{BG} = 450 \text{ N}$

$$\mathbf{T}_{BG} = -(200 \text{ N})\mathbf{i} + (370 \text{ N})\mathbf{j} - (160 \text{ N})\mathbf{k}$$

$$|M_{AD}| = 111 \text{ N}\cdot\text{m}$$

$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{BG}$ will contribute to the moment of $\mathbf{T}_{BG}$ about line $\overline{AD}$.

Now $$(T_{BG})_{\text{parallel}} = \mathbf{T}_{BG} \cdot \boldsymbol{\lambda}_{AD}$$

$$= (-200\mathbf{i} + 370\mathbf{j} - 160\mathbf{k}) \cdot \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

$$= \frac{1}{5}[(-200)(4) + (-160)(-3)]$$

$$= -64 \text{ N}$$

Also, $$\overline{\mathbf{T}}_{BG} = (\mathbf{T}_{BG})_{\text{parallel}} + (\mathbf{T}_{BG})_{\text{perpendicular}}$$

so that $$(\mathbf{T}_{BG})_{\text{perpendicular}} = \sqrt{(450)^2 - (-64)^2} = 445.43 \text{ N}$$

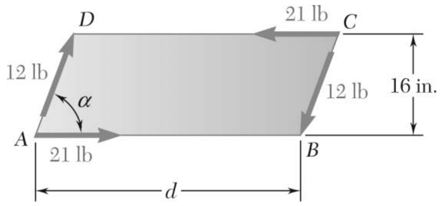
Since $\boldsymbol{\lambda}_{AD}$ and $(\mathbf{T}_{BG})_{\text{perpendicular}}$ are perpendicular, it follows that

$$M_{AD} = d(T_{BG})_{\text{perpendicular}}$$

or $$111 \text{ N}\cdot\text{m} = d(445.43 \text{ N})$$

$$d = 0.24920 \text{ m}$$

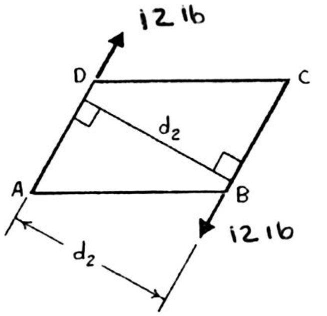
$d = 0.249$ m ◀



PROBLEM 3.70

A plate in the shape of a parallelogram is acted upon by two couples. Determine (*a*) the moment of the couple formed by the two 21-lb forces, (*b*) the perpendicular distance between the 12-lb forces if the resultant of the two couples is zero, (*c*) the value of α if the resultant couple is $72\text{ lb}\cdot\text{in.}$ clockwise and *d* is 42 in.

SOLUTION



(*a*) We have $M_1 = d_1 F_1$

where $d_1 = 16\text{ in.}$

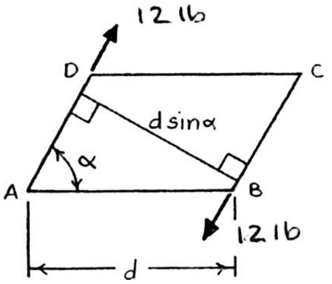
$F_1 = 21\text{ lb}$

$M_1 = (16\text{ in.})(21\text{ lb})$

$= 336\text{ lb}\cdot\text{in.}$ or $\mathbf{M}_1 = 336\text{ lb}\cdot\text{in.}\circlearrowleft$ ◀

(*b*) We have $\mathbf{M}_1 + \mathbf{M}_2 = 0$

or $336\text{ lb}\cdot\text{in.} - d_2(12\text{ lb}) = 0$ $d_2 = 28.0\text{ in.}$ ◀

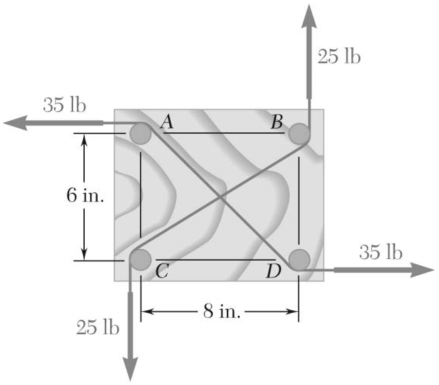


(*c*) We have $\mathbf{M}_{\text{total}} = \mathbf{M}_1 + \mathbf{M}_2$

or $-72\text{ lb}\cdot\text{in.} = 336\text{ lb}\cdot\text{in.} - (42\text{ in.})(\sin\alpha)(12\text{ lb})$

$\sin\alpha = 0.80952$

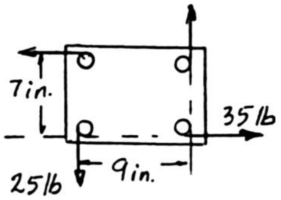
and $\alpha = 54.049°$ or $\alpha = 54.0°$ ◀



PROBLEM 3.71

Four 1-in.-diameter pegs are attached to a board as shown. Two strings are passed around the pegs and pulled with the forces indicated. (*a*) Determine the resultant couple acting on the board. (*b*) If only one string is used, around which pegs should it pass and in what directions should it be pulled to create the same couple with the minimum tension in the string? (*c*) What is the value of that minimum tension?

SOLUTION



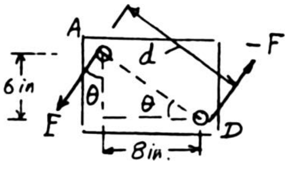
(*a*)

$$+\circlearrowleft M = (35\text{ lb})(7\text{ in.}) + (25\text{ lb})(9\text{ in.})$$
$$= 245\text{ lb}\cdot\text{in.} + 225\text{ lb}\cdot\text{in.}$$

$$M = 470\text{ lb}\cdot\text{in.}\ \circlearrowleft \blacktriangleleft$$

(*b*) With only one string, pegs *A* and *D*, or *B* and *C* should be used. We have

$$\tan\theta = \frac{6}{8} \qquad \theta = 36.9° \qquad 90° - \theta = 53.1°$$



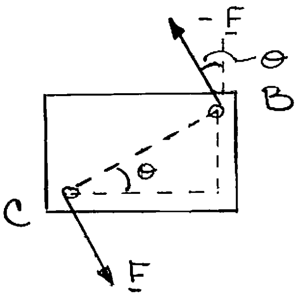
Direction of forces:

With pegs *A* and *D*: $\theta = 53.1°$ ◀

With pegs *B* and *C*: $\theta = 53.1°$ ◀

(*c*) The distance between the centers of the two pegs is

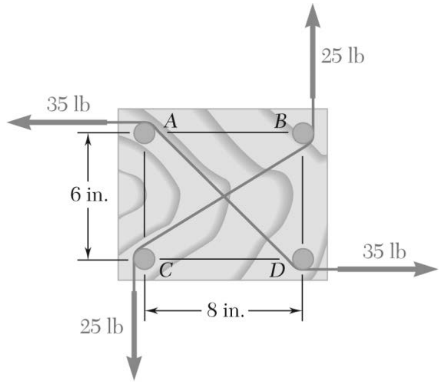
$$\sqrt{8^2 + 6^2} = 10\text{ in.}$$



Therefore, the perpendicular distance *d* between the forces is

$$d = 10\text{ in.} + 2\left(\frac{1}{2}\text{ in.}\right)$$
$$= 11\text{ in.}$$

We must have $M = Fd \quad 470\text{ lb}\cdot\text{in.} = F(11\text{ in.}) \qquad F = 42.7\text{ lb}$ ◀



PROBLEM 3.72

Four pegs of the same diameter are attached to a board as shown. Two strings are passed around the pegs and pulled with the forces indicated. Determine the diameter of the pegs knowing that the resultant couple applied to the board is 485 lb·in. counterclockwise.

SOLUTION

$$M = d_{AD}F_{AD} + d_{BC}F_{BC}$$

$$485\text{ lb}\cdot\text{in.} = [(6+d)\text{ in.}](35\text{ lb}) + [(8+d)\text{ in.}](25\text{ lb}) \qquad d = 1.250\text{ in.} \blacktriangleleft$$

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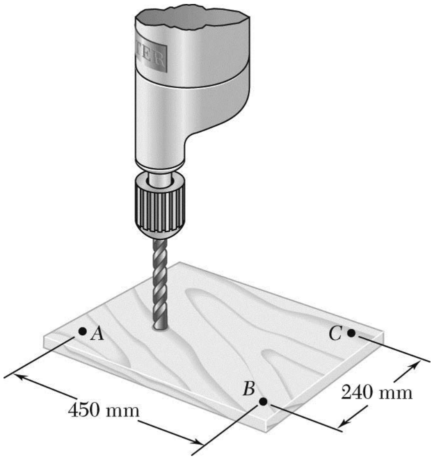
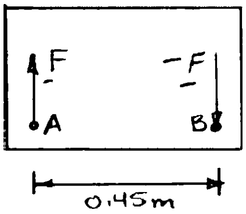


PROBLEM 3.73

A piece of plywood in which several holes are being drilled successively has been secured to a workbench by means of two nails. Knowing that the drill exerts a 12-N·m couple on the piece of plywood, determine the magnitude of the resulting forces applied to the nails if they are located (*a*) at *A* and *B*, (*b*) at *B* and *C*, (*c*) at *A* and *C*.

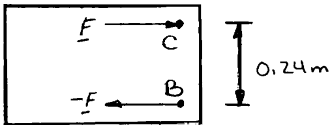
SOLUTION



(*a*) $M = Fd$

$$12\ \text{N}\cdot\text{m} = F(0.45\ \text{m})$$

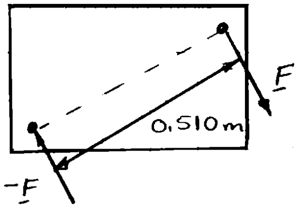
$F = 26.7\ \text{N}$ ◀



(*b*) $M = Fd$

$$12\ \text{N}\cdot\text{m} = F(0.24\ \text{m})$$

$F = 50.0\ \text{N}$ ◀

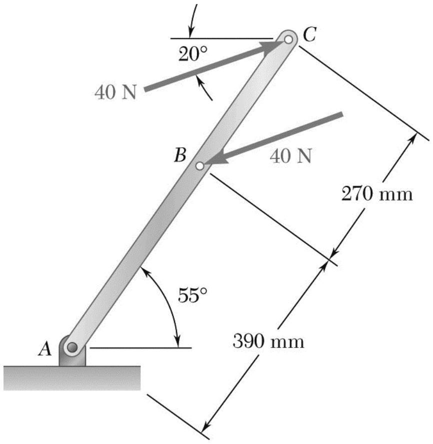


(*c*) $M = Fd \qquad d = \sqrt{(0.45\ \text{m})^2 + (0.24\ \text{m})^2}$

$$= 0.510\ \text{m}$$

$$12\ \text{N}\cdot\text{m} = F(0.510\ \text{m})$$

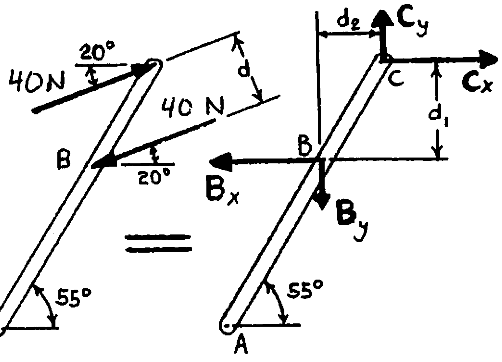
$F = 23.5\ \text{N}$ ◀



PROBLEM 3.74

Two parallel 40-N forces are applied to a lever as shown. Determine the moment of the couple formed by the two forces (*a*) by resolving each force into horizontal and vertical components and adding the moments of the two resulting couples, (*b*) by using the perpendicular distance between the two forces, (*c*) by summing the moments of the two forces about Point *A*.

SOLUTION



(*a*) We have $\Sigma \mathbf{M}_B\!: \; -d_1C_x + d_2C_y = \mathbf{M}$

where

$$d_1 = (0.270\text{ m})\sin 55^\circ = 0.22117\text{ m}$$

$$d_2 = (0.270\text{ m})\cos 55^\circ = 0.154866\text{ m}$$

$$C_x = (40\text{ N})\cos 20^\circ = 37.588\text{ N}$$

$$C_y = (40\text{ N})\sin 20^\circ = 13.6808\text{ N}$$

$$\mathbf{M} = -(0.22117\text{ m})(37.588\text{ N})\mathbf{k} + (0.154866\text{ m})(13.6808\text{ N})\mathbf{k}$$

$$= -(6.1946\text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = 6.19\text{ N}\cdot\text{m} \circlearrowright$ ◀

(*b*) We have

$$\mathbf{M} = Fd(-\mathbf{k}) = 40\text{ N}[(0.270\text{ m})\sin(55^\circ - 20^\circ)](-\mathbf{k}) = -(6.1946\text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = 6.19\text{ N}\cdot\text{m} \circlearrowright$ ◀

(*c*) We have $\Sigma \mathbf{M}_A\!: \; \Sigma(\mathbf{r}_A \times \mathbf{F}) = \mathbf{r}_{B/A} \times \mathbf{F}_B + \mathbf{r}_{C/A} \times \mathbf{F}_C = \mathbf{M}$

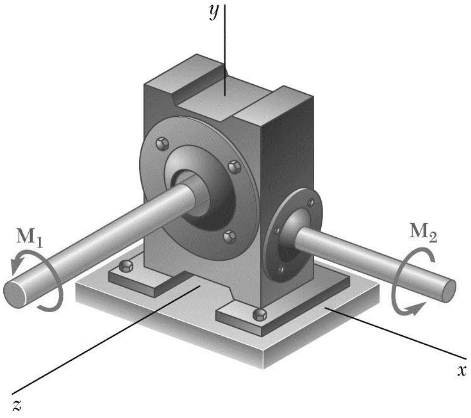
$$M = (0.390\text{ m})(40\text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55^\circ & \sin 55^\circ & 0 \\ -\cos 20^\circ & -\sin 20^\circ & 0 \end{vmatrix} + (0.660\text{ m})(40\text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55^\circ & \sin 55^\circ & 0 \\ \cos 20^\circ & \sin 20^\circ & 0 \end{vmatrix}$$

$$= (8.9478\text{ N}\cdot\text{m} - 15.1424\text{ N}\cdot\text{m})\mathbf{k}$$

$$= -(6.1946\text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = 6.19\text{ N}\cdot\text{m} \circlearrowright$ ◀

PROBLEM 3.75



The two shafts of a speed-reducer unit are subjected to couples of magnitude M_1 = 15 lb·ft and M_2 = 3 lb·ft, respectively. Replace the two couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

$$M_1 = (15 \text{ lb}\cdot\text{ft})\mathbf{k}$$

$$M_2 = (3 \text{ lb}\cdot\text{ft})\mathbf{i}$$

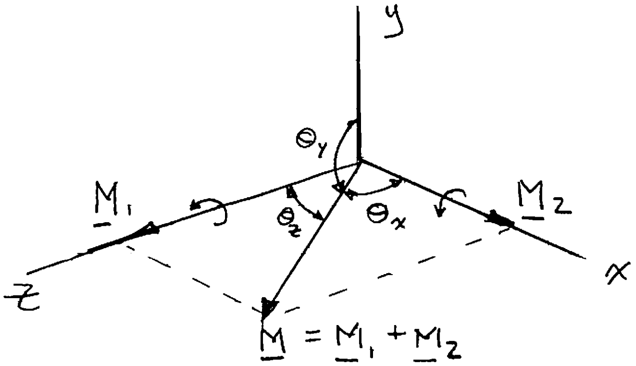
$$\begin{aligned} M &= \sqrt{M_1^2 + M_2^2} \\ &= \sqrt{(15)^2 + (3)^2} \\ &= 15.30 \text{ lb}\cdot\text{ft} \end{aligned}$$

$$\tan\theta_x = \frac{15}{3} = 5$$

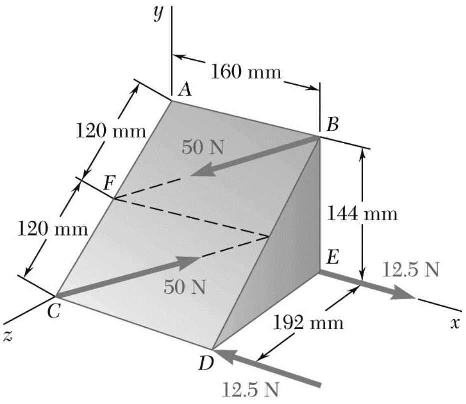
$$\theta_x = 78.7°$$

$$\theta_y = 90°$$

$$\begin{aligned} \theta_z &= 90° - 78.7° \\ &= 11.30° \end{aligned}$$



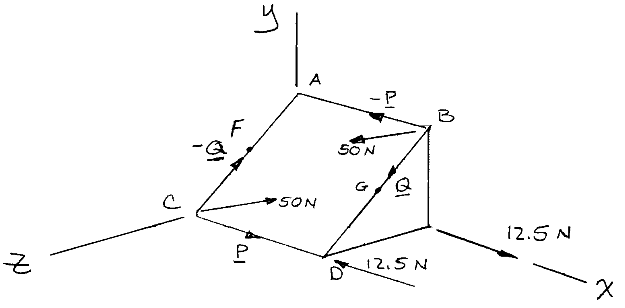
$M = 15.30$ lb·ft; $\theta_x = 78.7°$, $\theta_y = 90.0°$, $\theta_z = 11.30°$ ◀



PROBLEM 3.76

Replace the two couples shown with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



Replace the couple in the *ABCD* plane with two couples *P* and *Q* shown:

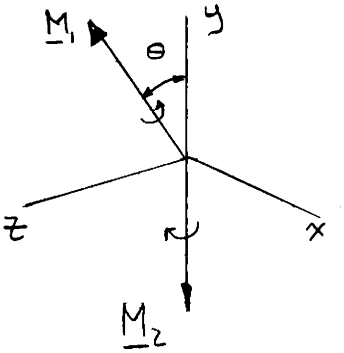
$$P = (50\text{ N})\frac{CD}{CG} = (50\text{ N})\left(\frac{160\text{ mm}}{200\text{ mm}}\right) = 40\text{ N}$$

$$Q = (50\text{ N})\frac{CF}{CG} = (50\text{ N})\left(\frac{120\text{ mm}}{200\text{ mm}}\right) = 30\text{ N}$$

Couple vector $\mathbf{M}_1$ perpendicular to plane *ABCD*:

$$+\circlearrowleft M_1 = (40\text{ N})(0.24\text{ m}) - (30\text{ N})(0.16\text{ m}) = 4.80\text{ N}\cdot\text{m}$$

Couple vector $\mathbf{M}_2$ in the *xy* plane:



$$+\circlearrowleft M_2 = -(12.5\text{ N})(0.192\text{ m}) = -2.40\text{ N}\cdot\text{m}$$

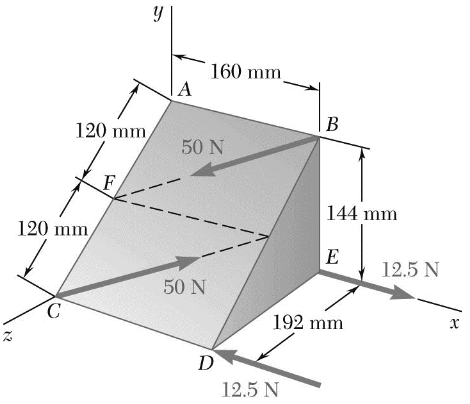
$$\tan\theta = \frac{144\text{ mm}}{192\text{ mm}} \qquad \theta = 36.870^\circ$$

$$\mathbf{M}_1 = (4.80\cos 36.870^\circ)\mathbf{j} + (4.80\sin 36.870^\circ)\mathbf{k}$$
$$= 3.84\mathbf{j} + 2.88\mathbf{k}$$

$$\mathbf{M}_2 = -2.40\mathbf{j}$$

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 = 1.44\mathbf{j} + 2.88\mathbf{k}$$

$$\mathbf{M} = 3.22\text{ N}\cdot\text{m};\ \theta_x = 90.0^\circ,\ \theta_y = 53.1^\circ,\ \theta_z = 36.9^\circ \blacktriangleleft$$

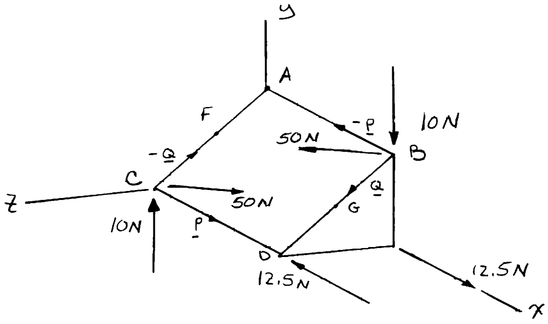


PROBLEM 3.77

Solve Prob. 3.76, assuming that two 10-N vertical forces have been added, one acting upward at *C* and the other downward at *B*.

PROBLEM 3.76 Replace the two couples shown with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



Replace the couple in the *ABCD* plane with two couples *P* and *Q* shown.

$$P = (50\text{ N})\frac{CD}{CG} = (50\text{ N})\left(\frac{160\text{ mm}}{200\text{ mm}}\right) = 40\text{ N}$$

$$Q = (50\text{ N})\frac{CF}{CG} = (50\text{ N})\left(\frac{120\text{ mm}}{200\text{ mm}}\right) = 30\text{ N}$$

Couple vector $\mathbf{M}_1$ perpendicular to plane *ABCD*.

$$+\circlearrowleft M_1 = (40\text{ N})(0.24\text{ m}) - (30\text{ N})(0.16\text{ m}) = 4.80\text{ N}\cdot\text{m}$$

$$\tan\theta = \frac{144\text{ mm}}{192\text{ mm}} \quad \theta = 36.870°$$

$$\begin{aligned}\mathbf{M}_1 &= (4.80\cos 36.870°)\mathbf{j} + (4.80\sin 36.870°)\mathbf{k}\\ &= 3.84\mathbf{j} + 2.88\mathbf{k}\end{aligned}$$

$$\begin{aligned}+\circlearrowleft M_2 &= -(12.5\text{ N})(0.192\text{ m}) = -2.40\text{ N}\cdot\text{m}\\ &= -2.40\mathbf{j}\end{aligned}$$

$$\begin{aligned}\mathbf{M}_3 &= \mathbf{r}_{B/C} \times M_3;\ \mathbf{r}_{B/C} = (0.16\text{ m})\mathbf{i} + (0.144\text{ m})\mathbf{j} - (0.192\text{ m})\mathbf{k}\\ &= (0.16\text{ m})\mathbf{i} + (0.144\text{ m})\mathbf{j} - (0.192\text{ m})\mathbf{k} \times (-10\text{ N})\mathbf{j}\\ &= -1.92\mathbf{i} - 1.6\mathbf{k}\end{aligned}$$

PROBLEM 3.77 (Continued)

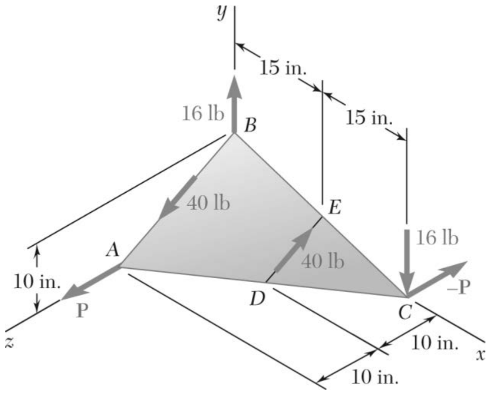
$$M = M_1 + M_2 + M_3 = (3.84\mathbf{j} + 2.88\mathbf{k}) - 2.40\mathbf{j} + (-1.92\mathbf{i} - 1.6\mathbf{k})$$
$$= -(1.92 \text{ N}\cdot\text{m})\mathbf{i} + (1.44 \text{ N}\cdot\text{m})\mathbf{j} + (1.28 \text{ N}\cdot\text{m})\mathbf{k}$$

$$M = \sqrt{(-1.92)^2 + (1.44)^2 + (1.28)^2} = 2.72 \text{ N}\cdot\text{m} \qquad M = 2.72 \text{ N}\cdot\text{m} \blacktriangleleft$$

$$\cos\theta_x = -1.92/2.72$$

$$\cos\theta_y = 1.44/2.72$$

$$\cos\theta_z = 1.28/2.72 \qquad \theta_x = 134.9° \quad \theta_y = 58.0° \quad \theta_z = 61.9° \blacktriangleleft$$



PROBLEM 3.78

If $P = 0$, replace the two remaining couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2; \quad F_1 = 16 \text{ lb}, \quad F_2 = 40 \text{ lb}$$

$$\mathbf{M_1} = \mathbf{r}_C \times \mathbf{F}_1 = (30 \text{ in.})\mathbf{i} \times [-(16 \text{ lb})\mathbf{j}] = -(480 \text{ lb} \cdot \text{in.})\mathbf{k}$$

$$\mathbf{M_2} = \mathbf{r}_{E/B} \times \mathbf{F}_2; \quad \mathbf{r}_{E/B} = (15 \text{ in.})\mathbf{i} - (5 \text{ in.})\mathbf{j}$$

$$d_{DE} = \sqrt{(0)^2 + (5)^2 + (10)^2} = 5\sqrt{5} \text{ in.}$$

$$F_2 = \frac{40 \text{ lb}}{5\sqrt{5}}(5\mathbf{j} - 10\mathbf{k})$$

$$= 8\sqrt{5}[(1 \text{ lb})\mathbf{j} - (2 \text{ lb})\mathbf{k}]$$

$$\mathbf{M}_2 = 8\sqrt{5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 15 & -5 & 0 \\ 0 & 1 & -2 \end{vmatrix}$$

$$= 8\sqrt{5}[(10 \text{ lb} \cdot \text{in.})\mathbf{i} + (30 \text{ lb} \cdot \text{in.})\mathbf{j} + (15 \text{ lb} \cdot \text{in.})\mathbf{k}]$$

$$\mathbf{M} = -(480 \text{ lb} \cdot \text{in.})\mathbf{k} + 8\sqrt{5}[(10 \text{ lb} \cdot \text{in.})\mathbf{i} + (30 \text{ lb} \cdot \text{in.})\mathbf{j} + (15 \text{ lb} \cdot \text{in.})\mathbf{k}]$$

$$= (178.885 \text{ lb} \cdot \text{in.})\mathbf{i} + (536.66 \text{ lb} \cdot \text{in.})\mathbf{j} - (211.67 \text{ lb} \cdot \text{in.})\mathbf{k}$$

$$M = \sqrt{(178.885)^2 + (536.66)^2 + (-211.67)^2}$$

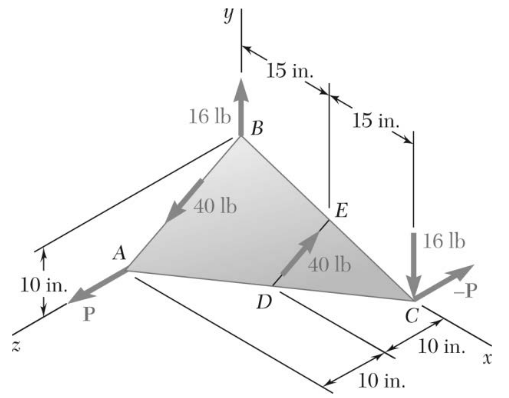
$$= 603.99 \text{ lb} \cdot \text{in} \qquad M = 604 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

$$\boldsymbol{\lambda}_{\text{axis}} = \frac{\mathbf{M}}{M} = 0.29617\mathbf{i} + 0.88852\mathbf{j} - 0.35045\mathbf{k}$$

$$\cos\theta_x = 0.29617$$

$$\cos\theta_y = 0.88852$$

$$\cos\theta_z = -0.35045 \qquad \theta_x = 72.8° \quad \theta_y = 27.3° \quad \theta_z = 110.5° \blacktriangleleft$$



PROBLEM 3.79

If $P = 20\text{ lb}$, replace the three couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

From the solution to Problem. 3.78:

16-lb force: $M_1 = -(480\text{ lb}\cdot\text{in.})\mathbf{k}$

40-lb force: $M_2 = 8\sqrt{5}[(10\text{ lb}\cdot\text{in.})\mathbf{i} + (30\text{ lb}\cdot\text{in.})\mathbf{j} + (15\text{ lb}\cdot\text{in.})\mathbf{k}]$

$P = 20\text{ lb}$

$$\begin{aligned} M_3 &= \mathbf{r}_C \times P \\ &= (30\text{ in.})\mathbf{i} \times (20\text{ lb})\mathbf{k} \\ &= (600\text{ lb}\cdot\text{in.})\mathbf{j} \end{aligned}$$

$$\begin{aligned} \mathbf{M} &= \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 \\ &= -(480)\mathbf{k} + 8\sqrt{5}\,(10\mathbf{i} + 30\mathbf{j} + 15\mathbf{k}) + 600\mathbf{j} \\ &= (178.885\text{ lb}\cdot\text{in.})\mathbf{i} + (1136.66\text{ lb}\cdot\text{in.})\mathbf{j} - (211.67\text{ lb}\cdot\text{in.})\mathbf{k} \end{aligned}$$

$$\begin{aligned} M &= \sqrt{(178.885)^2 + (113.66)^2 + (211.67)^2} \\ &= 1169.96\text{ lb}\cdot\text{in.} \end{aligned}$$

$M = 1170\text{ lb}\cdot\text{in.}$ ◀

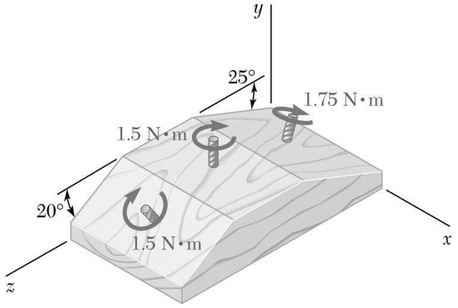
$$\lambda_{\text{axis}} = \frac{\mathbf{M}}{M} = 0.152898\mathbf{i} + 0.97154\mathbf{j} - 0.180921\mathbf{k}$$

$\cos\theta_x = 0.152898$

$\cos\theta_y = 0.97154$

$\cos\theta_z = -0.180921$

$\theta_x = 81.2°$ $\theta_y = 13.70°$ $\theta_z = 100.4°$ ◀



PROBLEM 3.80

In a manufacturing operation, three holes are drilled simultaneously in a workpiece. If the holes are perpendicular to the surfaces of the workpiece, replace the couples applied to the drills with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

$$\begin{aligned}\mathbf{M} &= \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3\\ &= (1.5\text{ N}\cdot\text{m})(-\cos 20°\mathbf{j} + \sin 20°\mathbf{k}) - (1.5\text{ N}\cdot\text{m})\mathbf{j}\\ &\quad + (1.75\text{ N}\cdot\text{m})(-\cos 25°\mathbf{j} + \sin 25°\mathbf{k})\\ &= -(4.4956\text{ N}\cdot\text{m})\mathbf{j} + (0.22655\text{ N}\cdot\text{m})\mathbf{k}\end{aligned}$$

$$\begin{aligned}M &= \sqrt{(0)^2 + (-4.4956)^2 + (0.22655)^2}\\ &= 4.5013\text{ N}\cdot\text{m}\end{aligned}$$

$M = 4.50\text{ N}\cdot\text{m}$ ◀

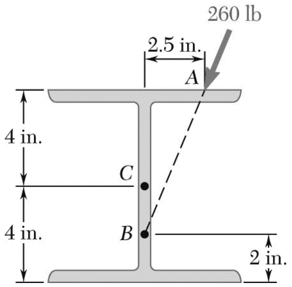
$$\boldsymbol{\lambda}_{\text{axis}} = \frac{\mathbf{M}}{M} = -(0.99873\mathbf{j} + 0.050330\mathbf{k})$$

$$\cos\theta_x = 0$$

$$\cos\theta_y = -0.99873$$

$$\cos\theta_z = 0.050330$$

$\theta_x = 90.0°, \quad \theta_y = 177.1°, \quad \theta_z = 87.1°$ ◀



PROBLEM 3.81

A 260-lb force is applied at *A* to the rolled-steel section shown. Replace that force with an equivalent force-couple system at the center *C* of the section.

SOLUTION

$$AB = \sqrt{(2.5 \text{ in.})^2 + (6.0 \text{ in.})^2} = 6.50 \text{ in.}$$

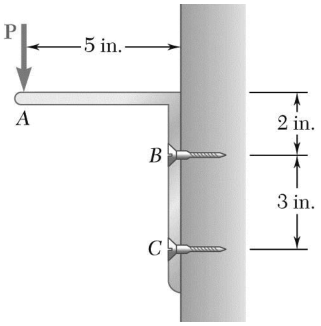
$$\sin\alpha = \frac{2.5 \text{ in.}}{6.5 \text{ in.}} = \frac{5}{13}$$

$$\cos\alpha = \frac{6.0 \text{ in.}}{6.5 \text{ in.}} = \frac{12}{13} \qquad \alpha = 22.6^\circ$$

$$\begin{aligned}\mathbf{F} &= -F\sin\alpha\mathbf{i} - F\cos\alpha\mathbf{j}\\ &= -(260 \text{ lb})\frac{5}{13}\mathbf{i} - (260 \text{ lb})\frac{12}{13}\mathbf{j}\\ &= -(100.0 \text{ lb})\mathbf{i} - (240 \text{ lb})\mathbf{j}\end{aligned}$$

$$\begin{aligned}\mathbf{M}_C &= \mathbf{r}_{A/C} \times \mathbf{F}\\ &= (2.5\mathbf{i} + 4.0\mathbf{j}) \times (-100.0\mathbf{i} - 240\mathbf{j})\\ &= 400\mathbf{k} - 600\mathbf{k}\\ &= -(200 \text{ lb}\cdot\text{in.})\mathbf{k}\end{aligned}$$

$$\mathbf{F} = 260 \text{ lb} \swarrow 67.4^\circ; \quad \mathbf{M}_C = 200 \text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$



PROBLEM 3.82

A 30-lb vertical force **P** is applied at *A* to the bracket shown, which is held by screws at *B* and *C*. (*a*) Replace **P** with an equivalent force-couple system at *B*. (*b*) Find the two horizontal forces at *B* and *C* that are equivalent to the couple obtained in part *a*.

SOLUTION

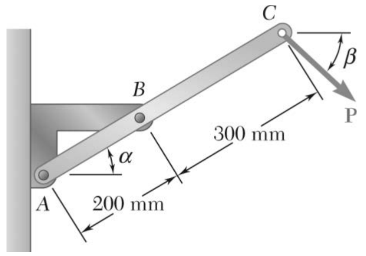
(*a*) $M_B = (30 \text{ lb})(5 \text{ in.})$

$= 150.0 \text{ lb} \cdot \text{in.}$

$\mathbf{F} = 30.0 \text{ lb} \downarrow, \quad \mathbf{M}_B = 150.0 \text{ lb} \cdot \text{in.} \circlearrowright$ ◀

(*b*) $B = C = \dfrac{150 \text{ lb} \cdot \text{in.}}{3.0 \text{ in.}} = 50.0 \text{ lb}$

$\mathbf{B} = 50.0 \text{ lb} \longleftarrow; \quad \mathbf{C} = 50.0 \text{ lb} \longrightarrow$ ◀



PROBLEM 3.83

The force **P** has a magnitude of 250 N and is applied at the end *C* of a 500-mm rod *AC* attached to a bracket at *A* and *B*. Assuming $\alpha = 30^\circ$ and $\beta = 60^\circ$, replace **P** with (*a*) an equivalent force-couple system at *B*, (*b*) an equivalent system formed by two parallel forces applied at A and *B*.

SOLUTION

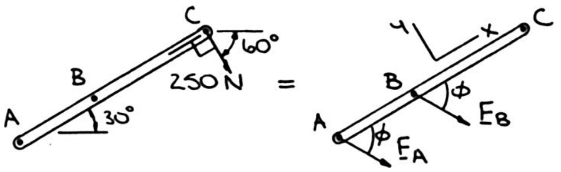
(*a*) Equivalence requires $\Sigma \mathbf{F}: \quad \mathbf{F} = \mathbf{P} \quad \text{or} \quad \mathbf{F} = 250\text{ N} \searrow 60^\circ$

$$\Sigma \mathbf{M}_B: \quad M = -(0.3\text{ m})(250\text{ N}) = -75\text{ N}\cdot\text{m}$$

The equivalent force-couple system at *B* is

$\mathbf{F}_B = 250\text{ N} \searrow 60^\circ$ $\qquad \mathbf{M}_B = 75.0\text{ N}\cdot\text{m} \circlearrowright$ ◀

(*b*) We require



Equivalence then requires

$$\Sigma F_x: \quad 0 = F_A \cos\phi + F_B \cos\phi$$

$$F_A = -F_B \quad \text{or} \quad \cos\phi = 0$$

$$\Sigma F_y: \quad -250 = -F_A \sin\phi - F_B \sin\phi$$

Now if $F_A = -F_B \Rightarrow -250 = 0$, reject.

$$\cos\phi = 0$$

or $\phi = 90^\circ$

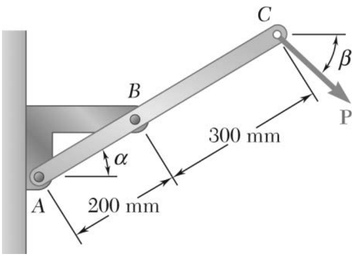
and $F_A + F_B = 250$

Also, $\Sigma M_B: \quad -(0.3\text{ m})(250\text{ N}) = (0.2\text{m})F_A$

or $F_A = -375\text{ N}$

and $F_B = 625\text{ N}$

$\mathbf{F}_A = 375\text{ N} \nwarrow 60^\circ$ $\qquad \mathbf{F}_B = 625\text{ N} \searrow 60.0^\circ$ ◀

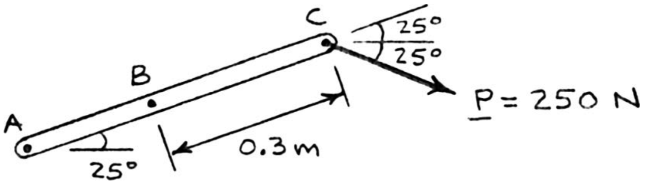


PROBLEM 3.84

Solve Problem 3.83, assuming $\alpha = \beta = 25°$.

PROBLEM 3.83 The force **P** has a magnitude of 250 N and is applied at the end *C* of a 500-mm rod *AC* attached to a bracket at *A* and *B*. Assuming $\alpha = 30°$ and $\beta = 60°$, replace **P** with (*a*) an equivalent force-couple system at *B*, (*b*) an equivalent system formed by two parallel forces applied at A and *B*.

SOLUTION



(*a*) Equivalence requires

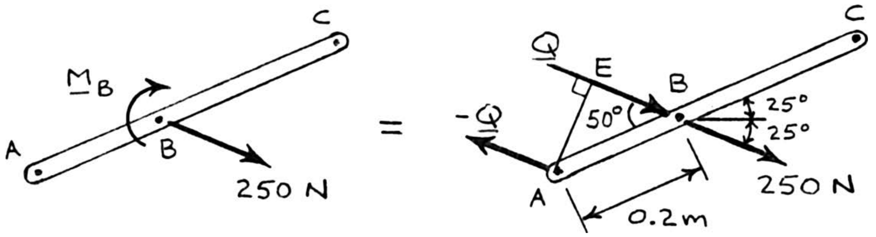
$$\Sigma \mathbf{F}: \quad \mathbf{F}_B = \mathbf{P} \quad \text{or} \quad \mathbf{F}_B = 250 \text{ N} \searrow 25.0°$$

$$\Sigma \mathbf{M}_B: \quad M_B = -(0.3 \text{ m})[(250 \text{ N})\sin 50°] = -57.453 \text{ N}\cdot\text{m}$$

The equivalent force-couple system at *B* is

$$\mathbf{F}_B = 250 \text{ N} \searrow 25.0° \qquad \mathbf{M}_B = 57.5 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

(*b*) We require

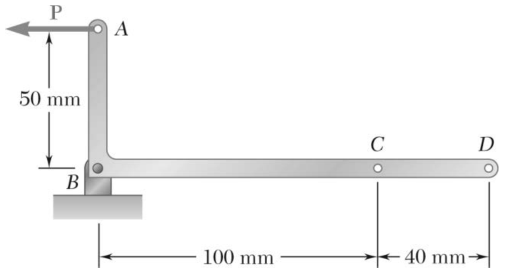


Equivalence requires

$$\begin{aligned} M_B &= d_{AE}Q \quad (0.3 \text{ m})[(250 \text{ N})\sin 50°] \\ &= [(0.2 \text{ m})\sin 50°]Q \\ Q &= 375 \text{ N} \end{aligned}$$

Adding the forces at *B*:

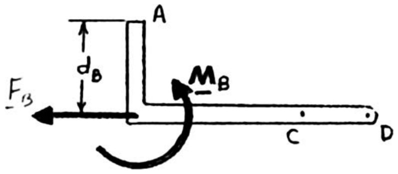
$$\mathbf{F}_A = 375 \text{ N} \nwarrow 25.0° \qquad \mathbf{F}_B = 625 \text{ N} \searrow 25.0° \blacktriangleleft$$



PROBLEM 3.85

The 80-N horizontal force **P** acts on a bell crank as shown. (*a*) Replace **P** with an equivalent force-couple system at *B*. (*b*) Find the two vertical forces at *C* and *D* that are equivalent to the couple found in part *a*.

SOLUTION



(*a*) Based on $\Sigma F: \; F_B = F = 80\ \text{N}$ or $\mathbf{F}_B = 80.0\ \text{N} \longleftarrow$ ◀

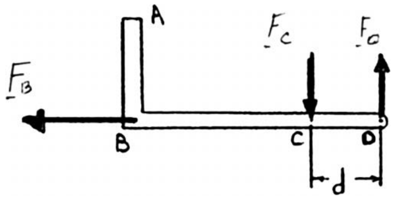
$$\Sigma M: \; M_B = Fd_B$$

$$= 80\ \text{N}\,(0.05\ \text{m})$$

$$= 4.0000\ \text{N}\cdot\text{m}$$

or $\mathbf{M}_B = 4.00\ \text{N}\cdot\text{m}$ ↺ ◀

(*b*) If the two vertical forces are to be equivalent to $\mathbf{M}_B$, they must be a couple. Further, the sense of the moment of this couple must be counterclockwise.



Then with F_C and F_D acting as shown,

$$\Sigma M: \; M_D = F_C d$$

$$4.0000\ \text{N}\cdot\text{m} = F_C\,(0.04\ \text{m})$$

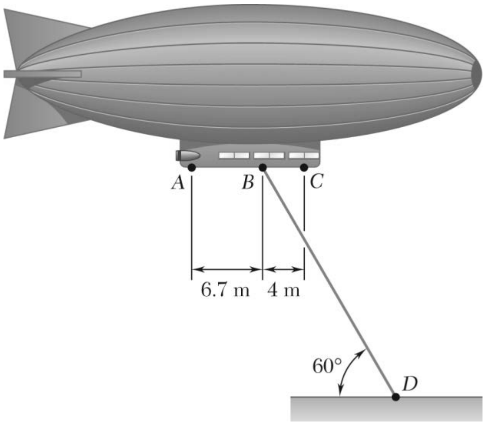
$$F_C = 100.000\ \text{N}$$

or $\mathbf{F}_C = 100.0\ \text{N} \downarrow$ ◀

$$\Sigma F_y: \; 0 = F_D - F_C$$

$$F_D = 100.000\ \text{N}$$

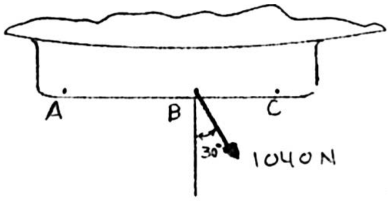
or $\mathbf{F}_D = 100.0\ \text{N} \uparrow$ ◀



PROBLEM 3.86

A dirigible is tethered by a cable attached to its cabin at B. If the tension in the cable is 1040 N, replace the force exerted by the cable at B with an equivalent system formed by two parallel forces applied at A and C.

SOLUTION

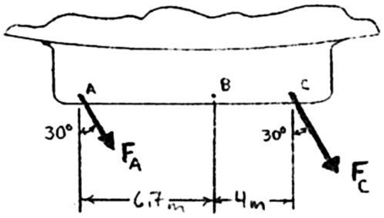


Require the equivalent forces acting at A and C be parallel and at an angle of α with the vertical.

Then for equivalence,

$$\Sigma F_x: \quad (1040\text{ N})\sin 30° = F_A \sin\alpha + F_B \sin\alpha \qquad (1)$$

$$\Sigma F_y: \quad -(1040\text{ N})\cos 30° = -F_A \cos\alpha - F_B \cos\alpha \qquad (2)$$



Dividing Equation (1) by Equation (2),

$$\frac{(1040\text{ N})\sin 30°}{-(1040\text{ N})\cos 30°} = \frac{(F_A + F_B)\sin\alpha}{-(F_A + F_B)\cos\alpha}$$

Simplifying yields $\alpha = 30°$.

Based on

$$\Sigma M_C: \quad [(1040\text{ N})\cos 30°](4\text{ m}) = (F_A \cos 30°)(10.7\text{ m})$$

$$F_A = 388.79\text{ N}$$

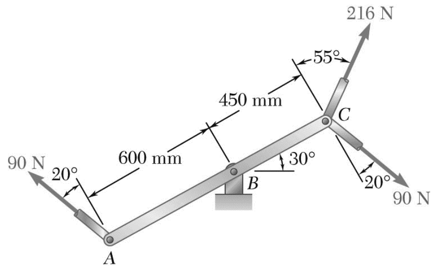
or $\mathbf{F}_A = 389\text{ N} \searrow 60.0°$ ◀

Based on

$$\Sigma M_A: \quad -[(1040\text{ N})\cos 30°](6.7\text{ m}) = (F_C \cos 30°)(10.7\text{ m})$$

$$F_C = 651.21\text{ N}$$

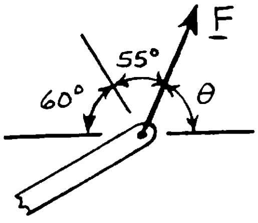
or $\mathbf{F}_C = 651\text{ N} \searrow 60.0°$ ◀



PROBLEM 3.87

Three control rods attached to a lever *ABC* exert on it the forces shown. (*a*) Replace the three forces with an equivalent force-couple system at *B*. (*b*) Determine the single force that is equivalent to the force-couple system obtained in part *a*, and specify its point of application on the lever.

SOLUTION



(*a*) First note that the two 90-N forces form a couple. Then

$$\mathbf{F} = 216 \text{ N} \measuredangle\, \theta$$

where $\theta = 180° - (60° + 55°) = 65°$

and

$$\begin{aligned} M &= \Sigma M_B \\ &= (0.450 \text{ m})(216 \text{ N})\cos 55° - (1.050 \text{ m})(90 \text{ N})\cos 20° \\ &= -33.049 \text{ N}\cdot\text{m} \end{aligned}$$

The equivalent force-couple system at *B* is

$$\mathbf{F} = 216 \text{ N} \measuredangle\, 65.0°;\ \ \mathbf{M} = 33.0 \text{ N}\cdot\text{m} \circlearrowright$$ ◀

(*b*) The single equivalent force **F′** is equal to **F**. Further, since the sense of **M** is clockwise, **F′** must be applied between *A* and *B*. For equivalence,

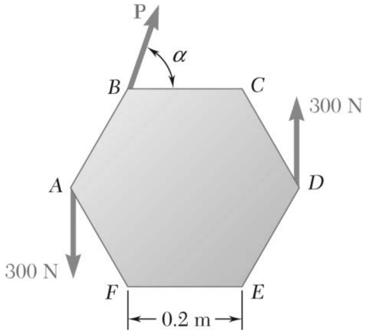
$$\Sigma M_B\!:\quad M = aF'\cos 55°$$

where *a* is the distance from *B* to the point of application of **F′**. Then

$$-33.049 \text{ N}\cdot\text{m} = -a(216 \text{ N})\cos 55°$$

$$a = 0.26676 \text{ m}$$

or $\mathbf{F}' = 216 \text{ N} \measuredangle\, 65.0°$ applied to the lever 267 mm to the left of *B* ◀

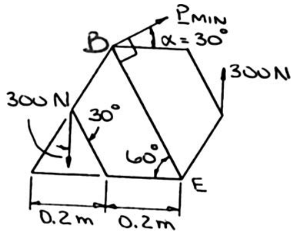


PROBLEM 3.88

A hexagonal plate is acted upon by the force **P** and the couple shown. Determine the magnitude and the direction of the smallest force **P** for which this system can be replaced with a single force at *E*.

SOLUTION

From the statement of the problem, it follows that $\Sigma M_E = 0$ for the given force-couple system. Further, for $\mathbf{P}_{\text{min}}$, we must require that **P** be perpendicular to $\mathbf{r}_{B/E}$. Then

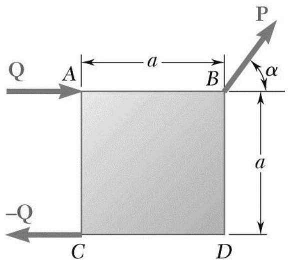


$$\begin{aligned}\Sigma M_E:\quad &(0.2 \sin 30° + 0.2)\text{m} \times 300\text{ N}\\ &+ (0.2\text{ m})\sin 30° \times 300\text{ N}\\ &- (0.4\text{ m})P_{\text{min}} = 0\end{aligned}$$

or

$$P_{\text{min}} = 300\text{ N}$$

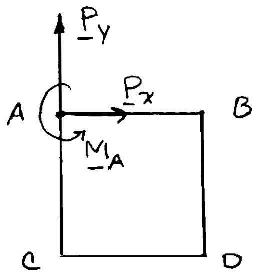
$\mathbf{P}_{\text{min}} = 300\text{ N} \measuredangle 30.0°$ ◀



PROBLEM 3.89

A force and couple act as shown on a square plate of side $a = 25$ in. Knowing that $P = 60$ lb, $Q = 40$ lb, and $\alpha = 50°$, replace the given force and couple by a single force applied at a point located (*a*) on line *AB*, (*b*) on line *AC*. In each case determine the distance from *A* to the point of application of the force.

SOLUTION



Replace the given force-couple system with an equivalent force-couple system at *A*.

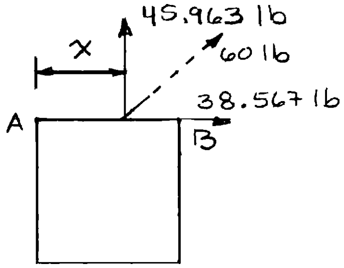
$$P_x = (60\text{ lb})(\cos 50°) = 38.567\text{ lb}$$

$$P_y = (60\text{ lb})(\sin 50°) = 45.963\text{ lb}$$

$$+\circlearrowleft M_A = P_y a - Qa$$

$$= (45.963\text{ lb})(25\text{ in.}) - (40\text{ lb})(25\text{ in.})$$

$$= 149.075\text{ lb}\cdot\text{in.} \circlearrowleft$$

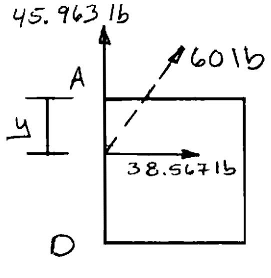


(*a*) Equating moments about *A* gives:

$$149.075\text{ lb}\cdot\text{in.} = (45.963\text{ lb})x$$

$$x = 3.24\text{ in.}$$

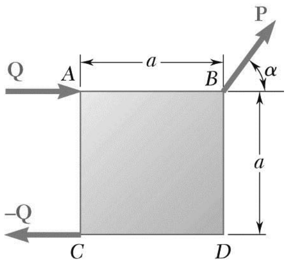
P = 60.0 lb ⦣ 50.0°; 3.24 in. from *A* ◀



(*b*) $149.075\text{ lb}\cdot\text{in.} = (38.567\text{ lb})y$

$$y = 3.87\text{ in.}$$

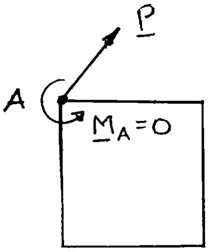
P = 60.0 lb ⦣ 50.0°; 3.87 in. below *A* ◀



PROBLEM 3.90

The force and couple shown are to be replaced by an equivalent single force. Knowing that $P = 2Q$, determine the required value of α if the line of action of the single equivalent force is to pass through (*a*) Point A, (*b*) Point C.

SOLUTION

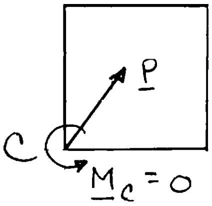


(*a*) We must have $M_A = 0$

$$(P\sin\alpha)a - Q(a) = 0$$

$$\sin\alpha = \frac{Q}{P} = \frac{Q}{2Q} = \frac{1}{2}$$

$\alpha = 30.0°$ ◀



(*b*) We must have $M_C = 0$

$$(P\sin\alpha)a - (P\cos\alpha)a - Q(a) = 0$$

$$\sin\alpha - \cos\alpha = \frac{Q}{P} = \frac{Q}{2Q} = \frac{1}{2}$$

$$\sin\alpha = \cos\alpha + \frac{1}{2} \qquad (1)$$

$$\sin^2\alpha = \cos^2\alpha + \cos\alpha + \frac{1}{4}$$

$$1 - \cos^2\alpha = \cos^2\alpha + \cos\alpha + \frac{1}{4}$$

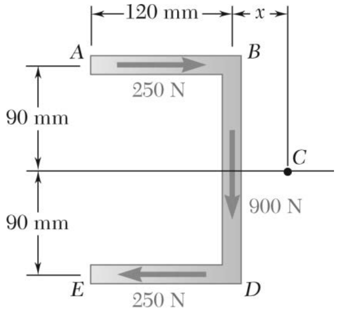
$$2\cos^2\alpha + \cos\alpha - 0.75 = 0 \qquad (2)$$

Solving the quadratic in $\cos\alpha$:

$$\cos\alpha = \frac{-1 \pm \sqrt{7}}{4} \qquad \alpha = 65.7° \text{ or } 155.7°$$

Only the first value of α satisfies Eq. (1),

therefore $\alpha = 65.7°$ ◀

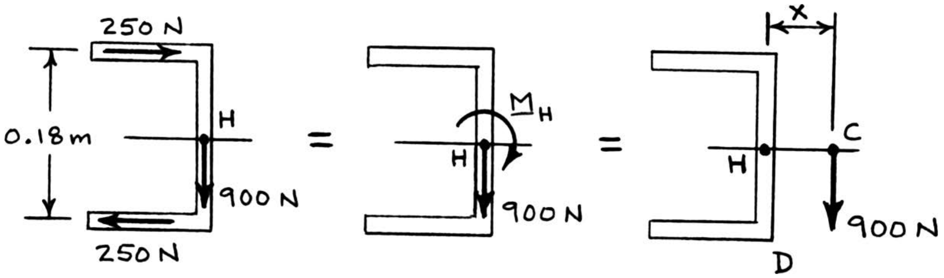


PROBLEM 3.91

The shearing forces exerted on the cross section of a steel channel can be represented by a 900-N vertical force and two 250-N horizontal forces as shown. Replace this force and couple with a single force **F** applied at Point *C*, and determine the distance *x* from *C* to line *BD*. (Point *C* is defined as the *shear center* of the section.)

SOLUTION

Replace the 250-N forces with a couple and move the 900-N force to Point *C* such that its moment about *H* is equal to the moment of the couple



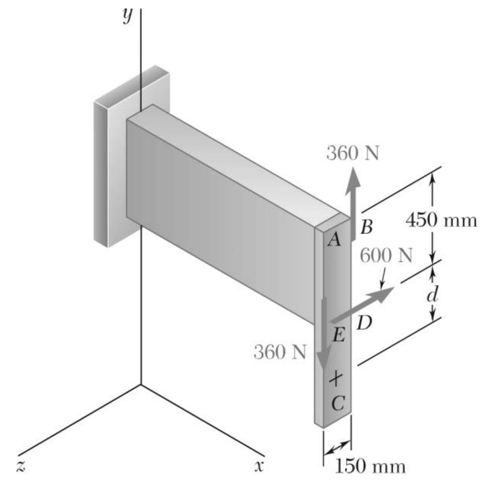
$$M_H = (0.18)(250 \text{ N}) = 45 \text{ N} \cdot \text{m}$$

Then $$M_H = x(900 \text{ N})$$

or $$45 \text{ N} \cdot \text{m} = x(900 \text{ N})$$

$$x = 0.05 \text{ m}$$

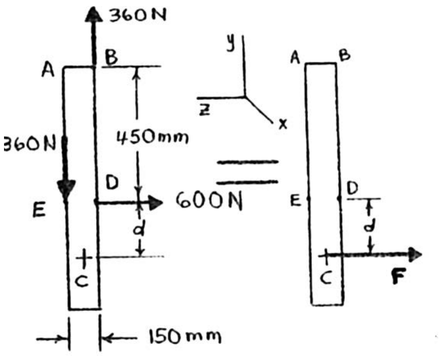
$\mathbf{F} = 900 \text{ N}\downarrow \quad x = 50.0 \text{ mm}$ ◀

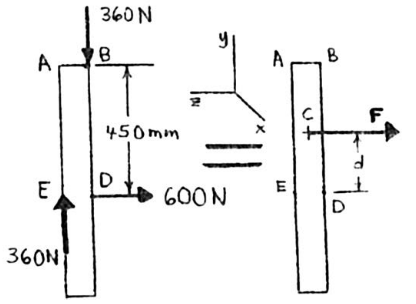


PROBLEM 3.92

A force and a couple are applied as shown to the end of a cantilever beam. (*a*) Replace this system with a single force **F** applied at Point *C*, and determine the distance *d* from *C* to a line drawn through Points *D* and *E*. (*b*) Solve part *a* if the directions of the two 360-N forces are reversed.

SOLUTION





(*a*) We have $\Sigma\mathbf{F}$: $\mathbf{F} = (360\text{ N})\mathbf{j} - (360\text{ N})\mathbf{j} - (600\text{ N})\mathbf{k}$

or $\mathbf{F} = -(600\text{ N})\mathbf{k}$ ◀

and ΣM_D: $(360\text{ N})(0.15\text{ m}) = (600\text{ N})(d)$

$d = 0.09\text{ m}$

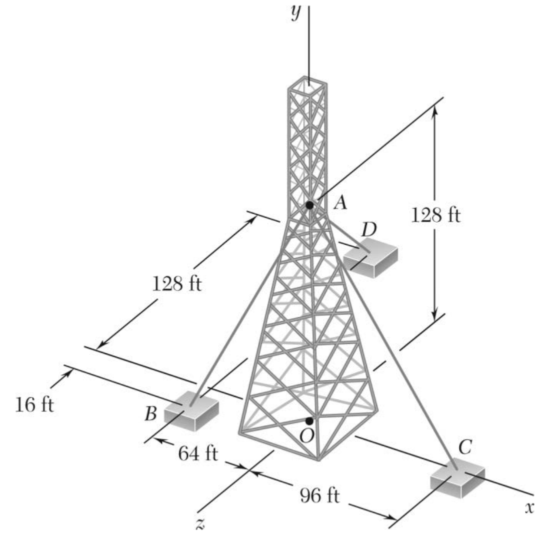
or $d = 90.0$ mm below *ED* ◀

(*b*) We have from part *a*: $\mathbf{F} = -(600\text{ N})\mathbf{k}$ ◀

and ΣM_D: $-(360\text{ N})(0.15\text{ m}) = -(600\text{ N})(d)$

$d = 0.09\text{ m}$

or $d = 90.0$ mm above *ED* ◀



PROBLEM 3.93

An antenna is guyed by three cables as shown. Knowing that the tension in cable *AB* is 288 lb, replace the force exerted at *A* by cable *AB* with an equivalent force-couple system at the center *O* of the base of the antenna.

SOLUTION

We have $$d_{AB} = \sqrt{(-64)^2 + (-128)^2 + (16)^2} = 144 \text{ ft}$$

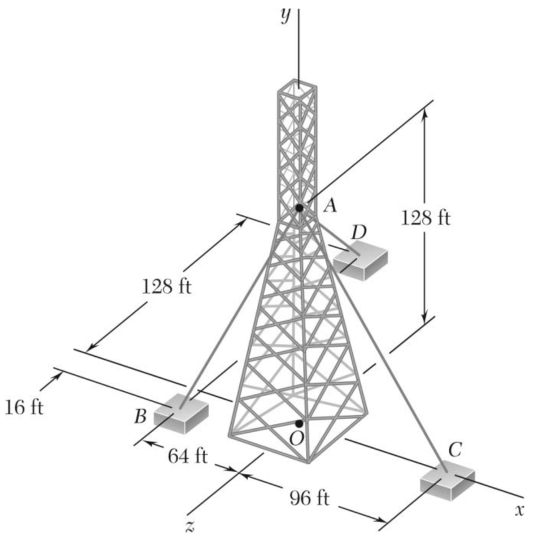
Then $$\mathbf{T}_{AB} = \frac{288 \text{ lb}}{144}(-64\mathbf{i} - 128\mathbf{j} + 16\mathbf{k})$$
$$= (32 \text{ lb})(-4\mathbf{i} - 8\mathbf{j} + \mathbf{k})$$

Now $$\mathbf{M} = \mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{AB}$$
$$= 128\mathbf{j} \times 32(-4\mathbf{i} - 8\mathbf{j} + \mathbf{k})$$
$$= (4096 \text{ lb} \cdot \text{ft})\mathbf{i} + (16,384 \text{ lb} \cdot \text{ft})\mathbf{k}$$

The equivalent force-couple system at *O* is

$$\mathbf{F} = -(128.0 \text{ lb})\mathbf{i} - (256 \text{ lb})\mathbf{j} + (32.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

$$\mathbf{M} = (4.10 \text{ kip} \cdot \text{ft})\mathbf{i} + (16.38 \text{ kip} \cdot \text{ft})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.94

An antenna is guyed by three cables as shown. Knowing that the tension in cable AD is 270 lb, replace the force exerted at A by cable AD with an equivalent force-couple system at the center O of the base of the antenna.

SOLUTION

We have

$$d_{AD} = \sqrt{(-64)^2 + (-128)^2 + (-128)^2}$$
$$= 192 \text{ ft}$$

Then

$$\mathbf{T}_{AD} = \frac{270 \text{ lb}}{192}(-64\mathbf{i} - 128\mathbf{j} + 128\mathbf{k})$$
$$= (90 \text{ lb})(-\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$

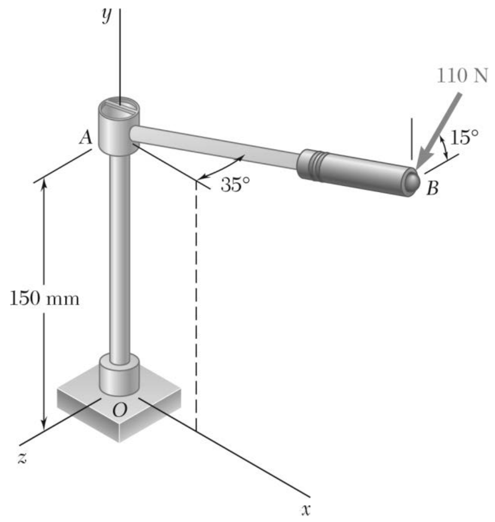
Now

$$\mathbf{M} = \mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{AD}$$
$$= 128\mathbf{j} \times 90(-\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$
$$= -(23{,}040 \text{ lb} \cdot \text{ft})\mathbf{i} + (11{,}520 \text{ lb} \cdot \text{ft})\mathbf{k}$$

The equivalent force-couple system at O is

$$\mathbf{F} = -(90.0 \text{ lb})\mathbf{i} - (180.0 \text{ lb})\mathbf{j} - (180.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

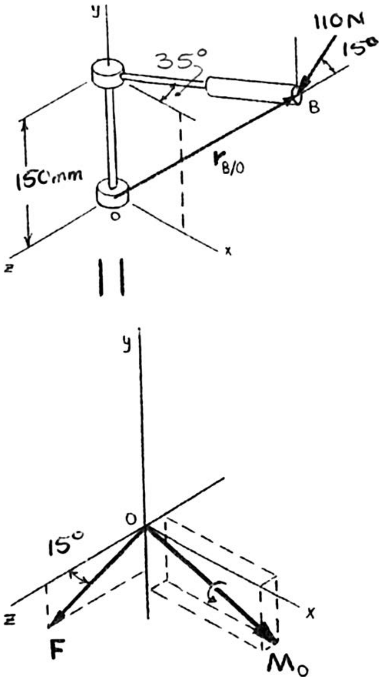
$$\mathbf{M} = -(23.0 \text{ kip} \cdot \text{ft})\mathbf{i} + (11.52 \text{ kip} \cdot \text{ft})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.95

A 110-N force acting in a vertical plane parallel to the *yz*-plane is applied to the 220-mm-long horizontal handle *AB* of a socket wrench. Replace the force with an equivalent force-couple system at the origin *O* of the coordinate system.

SOLUTION



We have $\quad \Sigma\mathbf{F}: \quad \mathbf{P}_B = \mathbf{F}$

where
$$\mathbf{P}_B = 110\text{ N}[-(\sin 15°)\mathbf{j} + (\cos 15°)\mathbf{k}]$$
$$= -(28.470\text{ N})\mathbf{j} + (106.252\text{ N})\mathbf{k}$$

or $\mathbf{F} = -(28.5\text{ N})\mathbf{j} + (106.3\text{ N})\mathbf{k}$ ◀

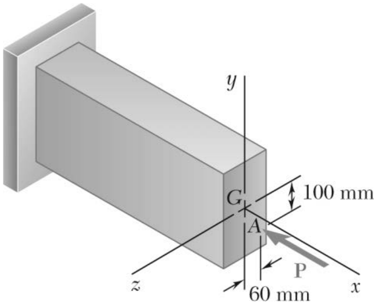
We have $\quad \Sigma M_O: \quad \mathbf{r}_{B/O} \times \mathbf{P}_B = \mathbf{M}_O$

where
$$\mathbf{r}_{B/O} = [(0.22\cos 35°)\mathbf{i} + (0.15)\mathbf{j} - (0.22\sin 35°)\mathbf{k}]\text{ m}$$
$$= (0.180213\text{ m})\mathbf{i} + (0.15\text{ m})\mathbf{j} - (0.126187\text{ m})\mathbf{k}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.180213 & 0.15 & 0.126187 \\ 0 & -28.5 & 106.3 \end{vmatrix} \text{N}\cdot\text{m} = \mathbf{M}_O$$

$$\mathbf{M}_O = [(12.3487)\mathbf{i} - (19.1566)\mathbf{j} - (5.1361)\mathbf{k}]\text{ N}\cdot\text{m}$$

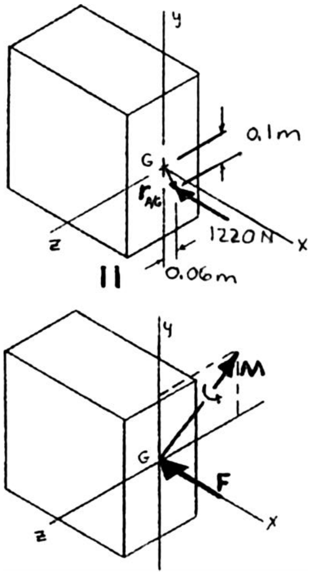
or $\mathbf{M}_O = (12.35\text{ N}\cdot\text{m})\mathbf{i} - (19.16\text{ N}\cdot\text{m})\mathbf{j} - (5.13\text{ N}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 3.96

An eccentric, compressive 1220-N force **P** is applied to the end of a cantilever beam. Replace **P** with an equivalent force-couple system at *G*.

SOLUTION



We have

$$\Sigma \mathbf{F}: \quad -(1220\text{ N})\mathbf{i} = \mathbf{F}$$

$$\mathbf{F} = -(1220\text{ N})\mathbf{i} \blacktriangleleft$$

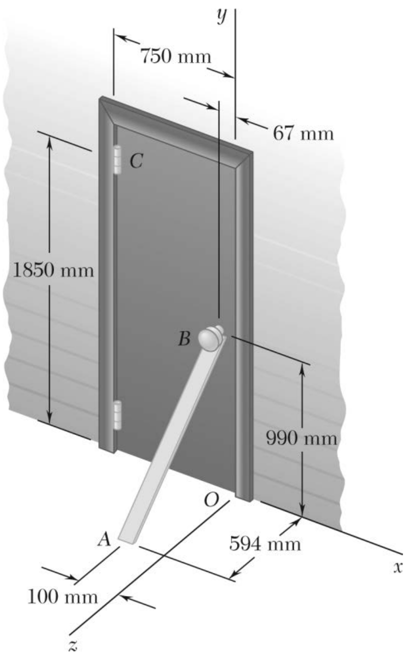
Also, we have

$$\Sigma \mathbf{M}_G: \quad \mathbf{r}_{A/G} \times \mathbf{P} = \mathbf{M}$$

$$1220\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.1 & -0.06 \\ -1 & 0 & 0 \end{vmatrix} \text{N} \cdot \text{m} = \mathbf{M}$$

$$\mathbf{M} = (1220\text{ N} \cdot \text{m})[(-0.06)(-1)\mathbf{j} - (-0.1)(-1)\mathbf{k}]$$

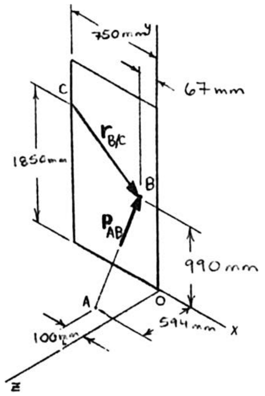
$$\text{or} \quad \mathbf{M} = (73.2\text{ N} \cdot \text{m})\mathbf{j} - (122\text{ N} \cdot \text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.97

To keep a door closed, a wooden stick is wedged between the floor and the doorknob. The stick exerts at *B* a 175-N force directed along line *AB*. Replace that force with an equivalent force-couple system at *C*.

SOLUTION



We have

$$\Sigma \mathbf{F}: \quad \mathbf{P}_{AB} = \mathbf{F}_C$$

where

$$\mathbf{P}_{AB} = \boldsymbol{\lambda}_{AB} P_{AB}$$

$$= \frac{(33 \text{ mm})\mathbf{i} + (990 \text{ mm})\mathbf{j} - (594 \text{ mm})\mathbf{k}}{1155.00 \text{ mm}}(175 \text{ N})$$

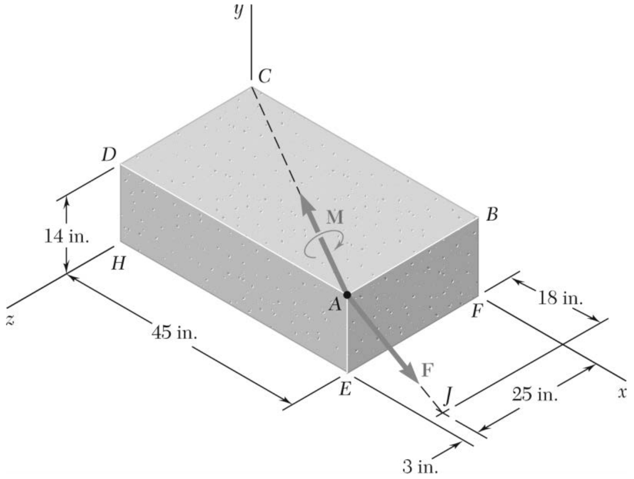
$$\text{or} \quad \mathbf{F}_C = (5.00 \text{ N})\mathbf{i} + (150.0 \text{ N})\mathbf{j} - (90.0 \text{ N})\mathbf{k} \quad ◀$$

We have $\Sigma \mathbf{M}_C: \quad \mathbf{r}_{B/C} \times \mathbf{P}_{AB} = \mathbf{M}_C$

$$\mathbf{M}_C = 5\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.683 & -0.860 & 0 \\ 1 & 30 & -18 \end{vmatrix} \text{N} \cdot \text{m}$$

$$= (5)\{(-0.860)(-18)\mathbf{i} - (0.683)(-18)\mathbf{j} + [(0.683)(30) - (0.860)(1)]\mathbf{k}\}$$

$$\text{or} \quad \mathbf{M}_C = (77.4 \text{ N} \cdot \text{m})\mathbf{i} + (61.5 \text{ N} \cdot \text{m})\mathbf{j} + (106.8 \text{ N} \cdot \text{m})\mathbf{k} \quad ◀$$



PROBLEM 3.98

A 46-lb force **F** and a 2120-lb·in. couple **M** are applied to corner *A* of the block shown. Replace the given force-couple system with an equivalent force-couple system at corner *H*.

SOLUTION

We have $$d_{AJ} = \sqrt{(18)^2 + (-14)^2 + (-3)^2} = 23 \text{ in.}$$

Then $$\mathbf{F} = \frac{46 \text{ lb}}{23}(18\mathbf{i} - 14\mathbf{j} - 3\mathbf{k})$$
$$= (36 \text{ lb})\mathbf{i} - (28 \text{ lb})\mathbf{j} - (6 \text{ lb})\mathbf{k}$$

Also $$d_{AC} = \sqrt{(-45)^2 + (0)^2 + (-28)^2} = 53 \text{ in.}$$

Then $$\mathbf{M} = \frac{2120 \text{ lb}\cdot\text{in.}}{53}(-45\mathbf{i} - 28\mathbf{k})$$
$$= -(1800 \text{ lb}\cdot\text{in.})\mathbf{i} - (1120 \text{ lb}\cdot\text{in.})\mathbf{k}$$

Now $$\mathbf{M}' = \mathbf{M} + \mathbf{r}_{A/H} \times \mathbf{F}$$

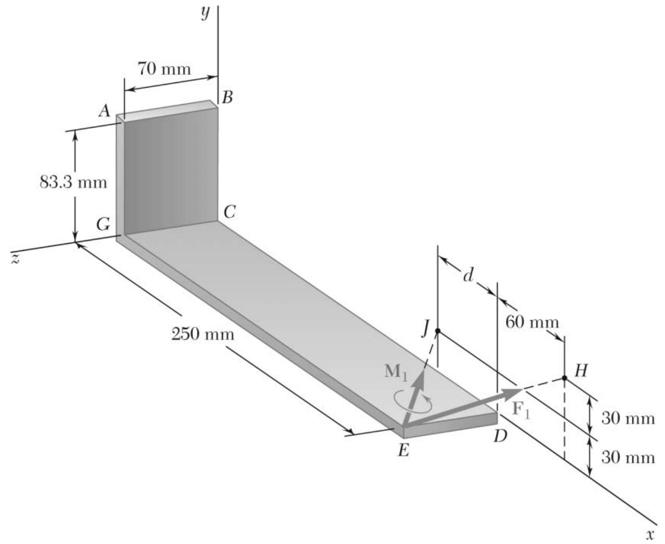
where $$\mathbf{r}_{A/H} = (45 \text{ in.})\mathbf{i} + (14 \text{ in.})\mathbf{j}$$

Then $$\mathbf{M}' = (-1800\mathbf{i} - 1120\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 45 & 14 & 0 \\ 36 & -28 & -6 \end{vmatrix}$$

$$= (-1800\mathbf{i} - 1120\mathbf{k}) + \{[(14)(-6)]\mathbf{i} + [-(45)(-6)]\mathbf{j} + [(45)(-28) - (14)(36)]\mathbf{k}\}$$
$$= (-1800 - 84)\mathbf{i} + (270)\mathbf{j} + (-1120 - 1764)\mathbf{k}$$
$$= -(1884 \text{ lb}\cdot\text{in.})\mathbf{i} + (270 \text{ lb}\cdot\text{in.})\mathbf{j} - (2884 \text{ lb}\cdot\text{in.})\mathbf{k}$$
$$= -(157 \text{ lb}\cdot\text{ft})\mathbf{i} + (22.5 \text{ lb}\cdot\text{ft})\mathbf{j} - (240 \text{ lb}\cdot\text{ft})\mathbf{k}$$

The equivalent force-couple system at *H* is $$\mathbf{F}' = (36.0 \text{ lb})\mathbf{i} - (28.0 \text{ lb})\mathbf{j} - (6.00 \text{ lb})\mathbf{k} \blacktriangleleft$$

$$\mathbf{M}' = -(157.0 \text{ lb}\cdot\text{ft})\mathbf{i} + (22.5 \text{ lb}\cdot\text{ft})\mathbf{j} - (240 \text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.99

A 77-N force $\mathbf{F}_1$ and a 31-N · m couple $\mathbf{M}_1$ are applied to corner E of the bent plate shown. If $\mathbf{F}_1$ and $\mathbf{M}_1$ are to be replaced with an equivalent force-couple system ($\mathbf{F}_2$, $\mathbf{M}_2$) at corner B and if $(M_2)_z = 0$, determine (*a*) the distance d, (*b*) $\mathbf{F}_2$ and $\mathbf{M}_2$.

SOLUTION

(*a*) We have $\Sigma M_{Bz}: \quad M_{2z} = 0$

$$\mathbf{k} \cdot (\mathbf{r}_{H/B} \times \mathbf{F}_1) + M_{1z} = 0 \qquad (1)$$

where $\mathbf{r}_{H/B} = (0.31\text{ m})\mathbf{i} - (0.0233)\mathbf{j}$

$$\mathbf{F}_1 = \boldsymbol{\lambda}_{EH} F_1$$

$$= \frac{(0.06\text{ m})\mathbf{i} + (0.06\text{ m})\mathbf{j} - (0.07\text{ m})\mathbf{k}}{0.11\text{ m}}(77\text{ N})$$

$$= (42\text{ N})\mathbf{i} + (42\text{ N})\mathbf{j} - (49\text{ N})\mathbf{k}$$

$$M_{1z} = \mathbf{k} \cdot \mathbf{M}_1$$

$$\mathbf{M}_1 = \boldsymbol{\lambda}_{EJ} M_1$$

$$= \frac{-d\mathbf{i} + (0.03\text{ m})\mathbf{j} - (0.07\text{ m})\mathbf{k}}{\sqrt{d^2 + 0.0058}\text{ m}}(31\text{ N}\cdot\text{m})$$

Then from Equation (1),

$$\begin{vmatrix} 0 & 0 & 1 \\ 0.31 & -0.0233 & 0 \\ 42 & 42 & -49 \end{vmatrix} + \frac{(-0.07\text{ m})(31\text{ N}\cdot\text{m})}{\sqrt{d^2 + 0.0058}} = 0$$

Solving for d, Equation (1) reduces to

$$(13.0200 + 0.9786) - \frac{2.17\text{ N}\cdot\text{m}}{\sqrt{d^2 + 0.0058}} = 0$$

from which $d = 0.1350\text{ m}$ or $d = 135.0\text{ mm}$ ◀

PROBLEM 3.99 (Continued)

(*b*)

$$\mathbf{F}_2 = \mathbf{F}_1 = (42\mathbf{i} + 42\mathbf{j} - 49\mathbf{k})\text{ N} \qquad \text{or} \quad \mathbf{F}_2 = (42.0\text{ N})\mathbf{i} + (42.0\text{ N})\mathbf{j} - (49.0\text{ N})\mathbf{k} \blacktriangleleft$$

$$\mathbf{M}_2 = \mathbf{r}_{H/B} \times \mathbf{F}_1 + \mathbf{M}_1$$

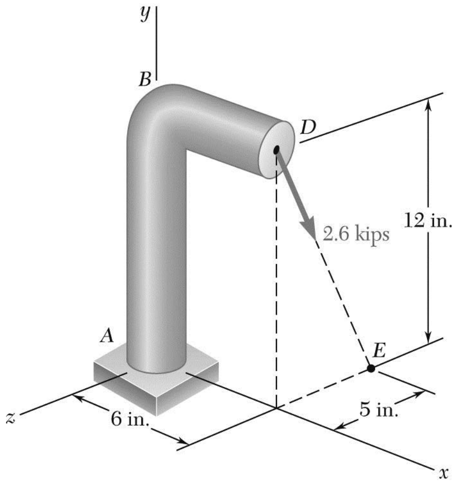
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.31 & -0.0233 & 0 \\ 42 & 42 & -49 \end{vmatrix} + \frac{(0.1350)\mathbf{i} + 0.03\mathbf{j} - 0.07\mathbf{k}}{0.155000}(31\text{ N}\cdot\text{m})$$

$$= (1.14170\mathbf{i} + 15.1900\mathbf{j} + 13.9986\mathbf{k})\text{ N}\cdot\text{m}$$

$$+ (-27.000\mathbf{i} + 6.0000\mathbf{j} - 14.0000\mathbf{k})\text{ N}\cdot\text{m}$$

$$\mathbf{M}_2 = -(25.858\text{ N}\cdot\text{m})\mathbf{i} + (21.190\text{ N}\cdot\text{m})\mathbf{j}$$

$$\text{or} \quad \mathbf{M}_2 = -(25.9\text{ N}\cdot\text{m})\mathbf{i} + (21.2\text{ N}\cdot\text{m})\mathbf{j} \blacktriangleleft$$



PROBLEM 3.100

A 2.6-kip force is applied at Point *D* of the cast iron post shown. Replace that force with an equivalent force-couple system at the center *A* of the base section.

SOLUTION

$$\overrightarrow{DE} = -(12\text{ in.})\mathbf{j} - (5\text{ in.})\mathbf{k}; \quad DE = 13.00\text{ in.}$$

$$\mathbf{F} = (2.6\text{ kips})\frac{\overrightarrow{DE}}{DE}$$

$$\mathbf{F} = (2.6\text{ kips})\frac{-12\mathbf{j} - 5\mathbf{k}}{13}$$

$$\mathbf{F} = -(2.40\text{ kips})\mathbf{j} - (1.000\text{ kip})\mathbf{k} \blacktriangleleft$$

$$\mathbf{M}_A = \mathbf{r}_{D/A} \times \mathbf{F}$$

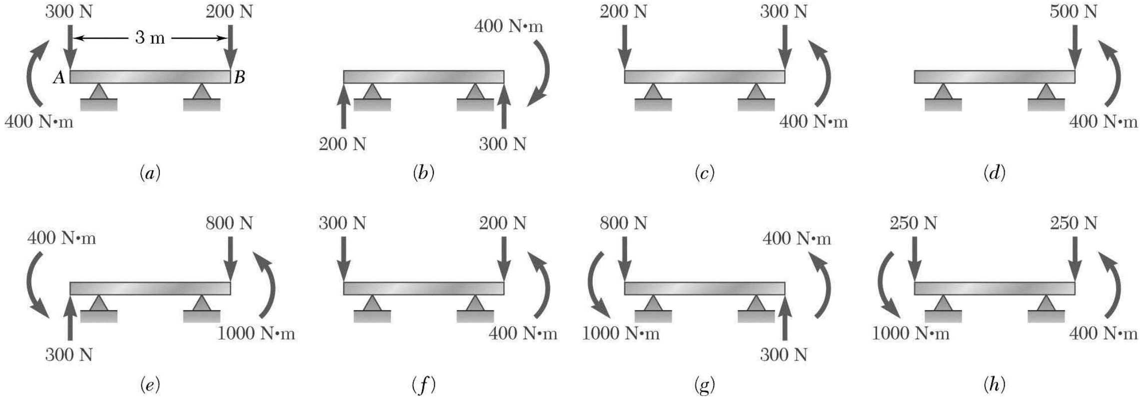
where $\mathbf{r}_{D/A} = (6\text{ in.})\mathbf{i} + (12\text{ in.})\mathbf{j}$

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6\text{ in.} & 12\text{ in.} & 0 \\ 0 & -2.4\text{ kips} & -1.0\text{ kips} \end{vmatrix}$$

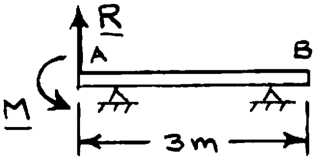
$$\mathbf{M}_A = -(12.00\text{ kip}\cdot\text{in.})\mathbf{i} + (6.00\text{ kip}\cdot\text{in.})\mathbf{j} - (14.40\text{ kip}\cdot\text{in.})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.101

A 3-m-long beam is subjected to a variety of loadings. (*a*) Replace each loading with an equivalent force-couple system at end *A* of the beam. (*b*) Which of the loadings are equivalent?



SOLUTION



(*a*) (*a*) We have $\Sigma F_Y\text{:}\quad -300\text{ N}-200\text{ N}=R_a$

or $\mathbf{R}_a = 500\text{ N}\downarrow$ ◀

and $\Sigma M_A\text{:}\quad -400\text{ N}\cdot\text{m}-(200\text{ N})(3\text{ m})=M_a$

or $\mathbf{M}_a = 1000\text{ N}\cdot\text{m}\circlearrowright$ ◀

(*b*) We have $\Sigma F_Y\text{:}\quad 200\text{ N}+300\text{ N}=R_b$

or $\mathbf{R}_b = 500\text{ N}\uparrow$ ◀

and $\Sigma M_A\text{:}\quad -400\text{ N}\cdot\text{m}+(300\text{ N})(3\text{ m})=M_b$

or $\mathbf{M}_b = 500\text{ N}\cdot\text{m}\circlearrowleft$ ◀

(*c*) We have $\Sigma F_Y\text{:}\quad -200\text{ N}-300\text{ N}=R_c$

or $\mathbf{R}_c = 500\text{ N}\downarrow$ ◀

and $\Sigma M_A\text{:}\quad 400\text{ N}\cdot\text{m}-(300\text{ N})(3\text{ m})=M_c$

or $\mathbf{M}_c = 500\text{ N}\cdot\text{m}\circlearrowright$ ◀

PROBLEM 3.101 (Continued)

(*d*) We have $\Sigma F_Y\text{:}\quad -500\text{ N} = R_d$

or $\mathbf{R}_d = 500\text{ N}\downarrow$ ◀

and $\Sigma M_A\text{:}\quad 400\text{ N}\cdot\text{m} - (500\text{ N})(3\text{ m}) = M_d$

or $\mathbf{M}_d = 1100\text{ N}\cdot\text{m}$ ↻ ◀

(*e*) We have $\Sigma F_Y\text{:}\quad 300\text{ N} - 800\text{ N} = R_e$

or $\mathbf{R}_e = 500\text{ N}\downarrow$ ◀

and $\Sigma M_A\text{:}\quad 400\text{ N}\cdot\text{m} + 1000\text{ N}\cdot\text{m} - (800\text{ N})(3\text{ m}) = M_e$

or $\mathbf{M}_e = 1000\text{ N}\cdot\text{m}$ ↻ ◀

(*f*) We have $\Sigma F_Y\text{:}\quad -300\text{ N} - 200\text{ N} = R_f$

or $\mathbf{R}_f = 500\text{ N}\downarrow$ ◀

and $\Sigma M_A\text{:}\quad 400\text{ N}\cdot\text{m} - (200\text{ N})(3\text{ m}) = M_f$

or $\mathbf{M}_f = 200\text{ N}\cdot\text{m}$ ↻ ◀

(*g*) We have $\Sigma F_Y\text{:}\quad -800\text{ N} + 300\text{ N} = R_g$

or $\mathbf{R}_g = 500\text{ N}\downarrow$ ◀

and $\Sigma M_A\text{:}\quad 1000\text{ N}\cdot\text{m} + 400\text{ N}\cdot\text{m} + (300\text{ N})(3\text{ m}) = M_g$

or $\mathbf{M}_g = 2300\text{ N}\cdot\text{m}$ ↺ ◀

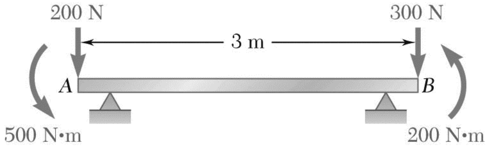
(*h*) We have $\Sigma F_Y\text{:}\quad -250\text{ N} - 250\text{ N} = R_h$

or $\mathbf{R}_h = 500\text{ N}\downarrow$ ◀

and $\Sigma M_A\text{:}\quad 1000\text{ N}\cdot\text{m} + 400\text{ N}\cdot\text{m} - (250\text{ N})(3\text{ m}) = M_h$

or $\mathbf{M}_h = 650\text{ N}\cdot\text{m}$ ↺ ◀

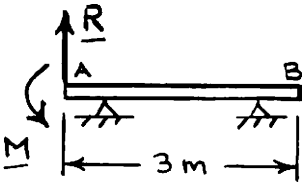
(*b*) Therefore, loadings (*a*) and (*e*) are equivalent.



PROBLEM 3.102

A 3-m-long beam is loaded as shown. Determine the loading of Prob. 3.101 that is equivalent to this loading.

SOLUTION



We have $\Sigma F_Y: \quad -200\text{ N} - 300\text{ N} = R$

or $\mathbf{R} = 500\text{ N} \downarrow$

and $\Sigma M_A: \quad 500\text{ N}\cdot\text{m} + 200\text{ N}\cdot\text{m} - (300\text{ N})(3\text{ m}) = M$

or $\mathbf{M} = 200\text{ N}\cdot\text{m} \circlearrowright$

Problem 3.101 equivalent force-couples at *A*:

Case	$\overline{\mathbf{R}}$	$\overline{\mathbf{M}}$
(*a*)	500 N ↓	1000 N·m ↻
(*b*)	500 N ↑	500 N·m ↺
(*c*)	500 N ↓	500 N·m ↻
(*d*)	500 N ↓	1100 N·m ↻
(*e*)	500 N ↓	1000 N·m ↻
(*f*)	500 N ↓	200 N·m ↻ ←
(*g*)	500 N ↓	2300 N·m ↺
(*h*)	500 N ↓	650 N·m ↺

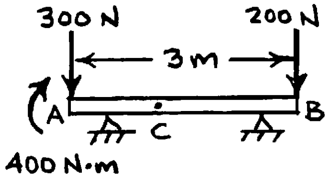
Equivalent to case (*f*) of Problem 3.101 ◀

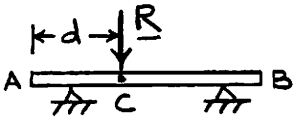
PROBLEM 3.103

Determine the single equivalent force and the distance from Point A to its line of action for the beam and loading of (*a*) Prob. 3.101*a*, (*b*) Prob. 3.101*b*, (*c*) Prob. 3.102.

SOLUTION

For equivalent single force at distance d from A:



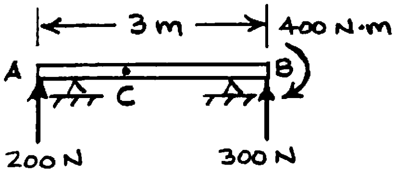


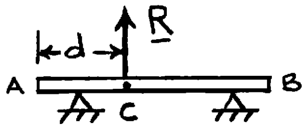
(*a*) We have $\Sigma F_Y:\ -300\text{ N} - 200\text{ N} = R$

or $\mathbf{R} = 500\text{ N}\downarrow$ ◀

and $\Sigma M_C:\ -400\text{ N}\cdot\text{m} + (300\text{ N})(d) - (200\text{ N})(3-d) = 0$

or $d = 2.00\text{ m}$ ◀



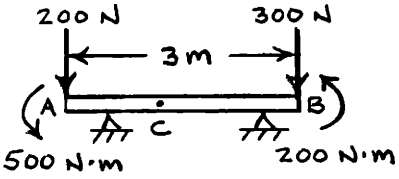


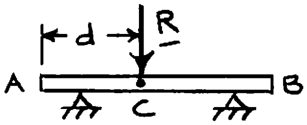
(*b*) We have $\Sigma F_Y:\ 200\text{ N} + 300\text{ N} = R$

or $\mathbf{R} = 500\text{ N}\uparrow$ ◀

and $\Sigma M_C:\ -400\text{ N}\cdot\text{m} - (200\text{ N})(d) + (300\text{ N})(3-d) = 0$

or $d = 1.000\text{ m}$ ◀





(*c*) We have $\Sigma F_Y:\ -200\text{ N} - 300\text{ N} = R$

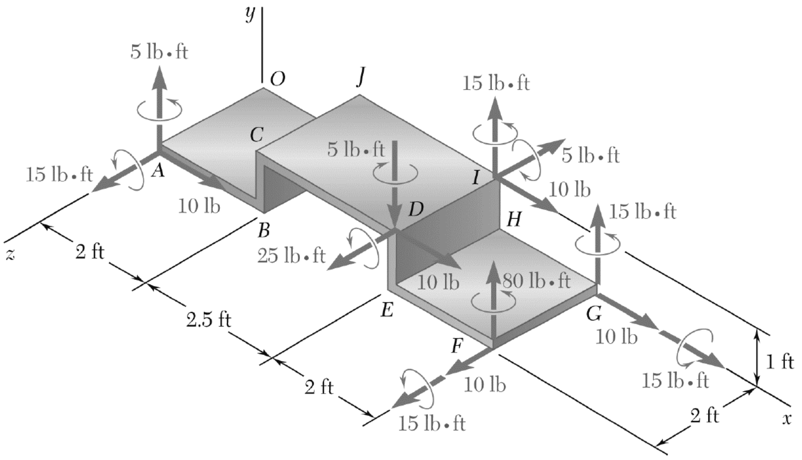
or $\mathbf{R} = 500\text{ N}\downarrow$ ◀

and $\Sigma M_C:\ 500\text{ N}\cdot\text{m} + 200\text{ N}\cdot\text{m} + (200\text{ N})(d) - (300\text{ N})(3-d) = 0$

or $d = 0.400\text{ m}$ ◀

PROBLEM 3.104

Five separate force-couple systems act at the corners of a piece of sheet metal, which has been bent into the shape shown. Determine which of these systems is equivalent to a force **F** = (10 lb)**i** and a couple of moment **M** = (15 lb · ft)**j** + (15 lb · ft)**k** located at the origin.



SOLUTION

First note that the force-couple system at *F* cannot be equivalent because of the direction of the force [The force of the other four systems is (10 lb)**i**]. Next, move each of the systems to the origin *O*; the forces remain unchanged.

$$A:\quad \mathbf{M}_A = \Sigma\mathbf{M}_O = (5\text{ lb}\cdot\text{ft})\mathbf{j} + (15\text{ lb}\cdot\text{ft})\mathbf{k} + (2\text{ ft})\mathbf{k}\times(10\text{ lb})\mathbf{i}$$

$$= (25\text{ lb}\cdot\text{ft})\mathbf{j} + (15\text{ lb}\cdot\text{ft})\mathbf{k}$$

$$D:\quad \mathbf{M}_D = \Sigma\mathbf{M}_O = -(5\text{ lb}\cdot\text{ft})\mathbf{j} + (25\text{ lb}\cdot\text{ft})\mathbf{k}$$

$$+[(4.5\text{ ft})\mathbf{i} + (1\text{ ft})\mathbf{j} + (2\text{ ft})\mathbf{k}]\times 10\text{ lb})\mathbf{i}$$

$$= (15\text{ lb}\cdot\text{ft})\mathbf{i} + (15\text{ lb}\cdot\text{ft})\mathbf{k}$$

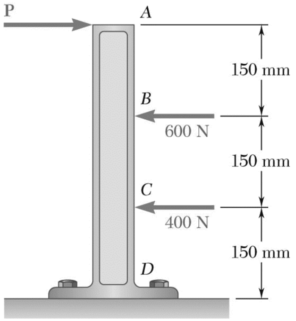
$$G:\quad \mathbf{M}_G = \Sigma\mathbf{M}_O = (15\text{ lb}\cdot\text{ft})\mathbf{i} + (15\text{ lb}\cdot\text{ft})\mathbf{j}$$

$$I:\quad \mathbf{M}_I = \Sigma\mathbf{M}_I = (15\text{ lb}\cdot\text{ft})\mathbf{j} - (5\text{ lb}\cdot\text{ft})\mathbf{k}$$

$$+[(4.5\text{ ft})\mathbf{i} + (1\text{ ft})\mathbf{j}]\times(10\text{ lb})\mathbf{j}$$

$$= (15\text{ lb}\cdot\text{ft})\mathbf{j} - (15\text{ lb}\cdot\text{ft})\mathbf{k}$$

The equivalent force-couple system is the system at corner *D*. ◀

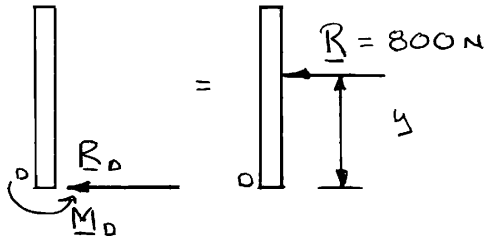


PROBLEM 3.105

Three horizontal forces are applied as shown to a vertical cast iron arm. Determine the resultant of the forces and the distance from the ground to its line of action when (*a*) $P = 200$ N, (*b*) $P = 2400$ N, (*c*) $P = 1000$ N.

SOLUTION

(*a*)



$$+\longrightarrow R_D = +200\text{ N} - 600\text{ N} - 400\text{ N} = -800\text{ N}$$

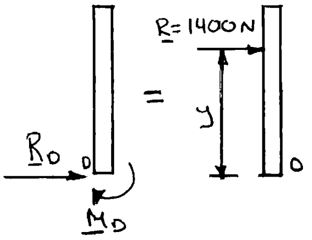
$$+\circlearrowleft M_D = -(200\text{ N})(0.450\text{ m}) + (600\text{ N})(0.300\text{ m}) + (400\text{ N})(0.1500\text{ m})$$

$$= +150.0\text{ N}\cdot\text{m}$$

$$y = \frac{M_D}{R} = \frac{150\text{ N}\cdot\text{m}}{800\text{ N}} = 0.1875\text{ m}$$

R = 800 N ←; $y = 187.5$ mm ◀

(*b*)



$$+\longrightarrow R_D = +2400\text{ N} - 600\text{ N} - 400\text{ N} = +1400\text{ N}$$

$$+\circlearrowleft M_D = -(2400\text{ N})(0.450\text{ m}) + (600\text{ N})(0.300\text{ m}) + (400\text{ N})(0.1500\text{ m})$$

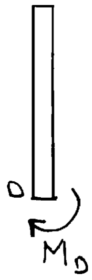
$$= -840\text{ N}\cdot\text{m}$$

$$y = \frac{M_D}{R} = \frac{840\text{ N}\cdot\text{m}}{1400\text{ N}} = 0.600\text{ m}$$

R = 1400 N →; $y = 600$ mm ◀

PROBLEM 3.105 (Continued)

(*c*)

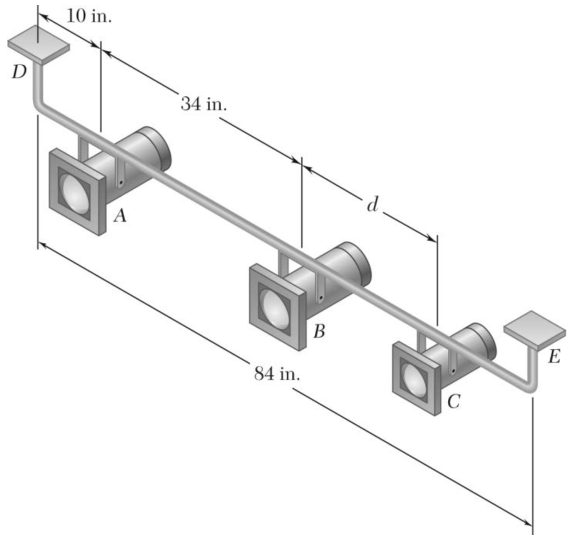


$$\xrightarrow{+} R_D = +1000 - 600 - 400 = 0$$

$$\stackrel{+}{\circlearrowleft} M_D = -(1000\text{ N})(0.450\text{ m}) + (600\text{ N})(0.300\text{ m}) + (400\text{ N})(0.1500\text{ m})$$
$$= -210\text{ N}\cdot\text{m}$$

$\therefore \quad y = \infty$ System reduces to a couple.

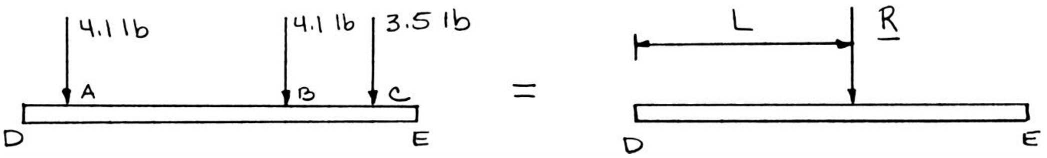
$\mathbf{M}_D = 210\text{ N}\cdot\text{m} \circlearrowright$ ◀



PROBLEM 3.106

Three stage lights are mounted on a pipe as shown. The lights at *A* and *B* each weigh 4.1 lb, while the one at *C* weighs 3.5 lb. (*a*) If d = 25 in., determine the distance from *D* to the line of action of the resultant of the weights of the three lights. (*b*) Determine the value of d so that the resultant of the weights passes through the midpoint of the pipe.

SOLUTION



For equivalence,

$$\Sigma F_y: \quad -4.1 - 4.1 - 3.5 = -R \quad \text{or} \quad \mathbf{R} = 11.7 \text{ lb}\downarrow$$

$$\Sigma F_D: \quad -(10 \text{ in.})(4.1 \text{ lb}) - (44 \text{ in.})(4.1 \text{ lb})$$
$$-[(4.4 + d) \text{ in.}](3.5 \text{ lb}) = -(L \text{ in.})(11.7 \text{ lb})$$

or $$375.4 + 3.5d = 11.7L \quad (d, L \text{ in in.})$$

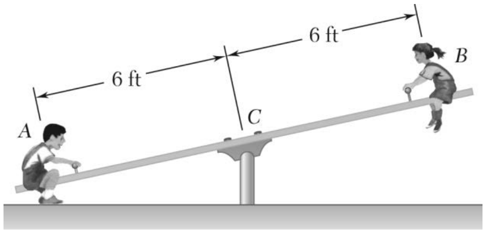
(*a*) $$d = 25 \text{ in.}$$

We have $$375.4 + 3.5(25) = 11.7L \quad \text{or} \quad L = 39.6 \text{ in.}$$

The resultant passes through a point 39.6 in. to the right of *D*. ◀

(*b*) $$L = 42 \text{ in.}$$

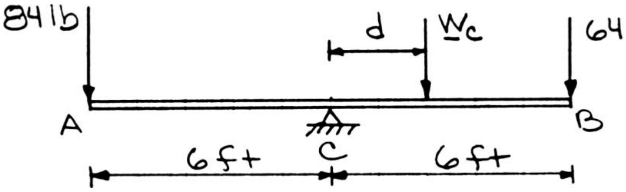
We have $$375.4 + 3.5d = 11.7(42) \qquad \text{or} \quad d = 33.1 \text{ in.}$$ ◀



PROBLEM 3.107

The weights of two children sitting at ends *A* and *B* of a seesaw are 84 lb and 64 lb, respectively. Where should a third child sit so that the resultant of the weights of the three children will pass through *C* if she weighs (*a*) 60 lb, (*b*) 52 lb.

SOLUTION



(*a*) For the resultant weight to act at *C*, $\Sigma M_C = 0 \quad W_C = 60 \text{ lb}$

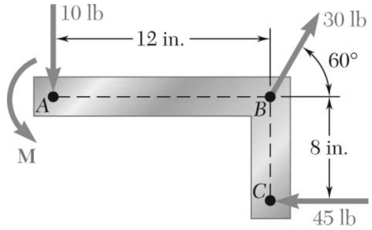
Then $(84 \text{ lb})(6 \text{ ft}) - 60 \text{ lb}(d) - 64 \text{ lb}(6 \text{ ft}) = 0$

$d = 2.00$ ft to the right of *C* ◀

(*b*) For the resultant weight to act at *C*, $\Sigma M_C = 0 \quad W_C = 52 \text{ lb}$

Then $(84 \text{ lb})(6 \text{ ft}) - 52 \text{ lb}(d) - 64 \text{ lb}(6 \text{ ft}) = 0$

$d = 2.31$ ft to the right of *C* ◀



PROBLEM 3.108

A couple of magnitude $M = 54$ lb · in. and the three forces shown are applied to an angle bracket. (*a*) Find the resultant of this system of forces. (*b*) Locate the points where the line of action of the resultant intersects line *AB* and line *BC*.

SOLUTION

(*a*) We have

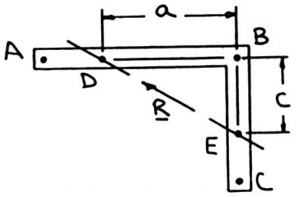
$$\Sigma\mathbf{F}: \quad \mathbf{R} = (-10\mathbf{j}) + (30\cos 60°)\mathbf{i} + 30\sin 60°\mathbf{j} + (-45\mathbf{i})$$
$$= -(30\text{ lb})\mathbf{i} + (15.9808\text{ lb})\mathbf{j}$$

or $\mathbf{R} = 34.0\text{ lb} \nwarrow 28.0°$ ◀

(*b*) First reduce the given forces and couple to an equivalent force-couple system $(\mathbf{R}, \mathbf{M}_B)$ at *B*.

We have

$$\Sigma M_B: \quad M_B = (54\text{ lb}\cdot\text{in}) + (12\text{ in.})(10\text{ lb}) - (8\text{ in.})(45\text{ lb})$$
$$= -186\text{ lb}\cdot\text{in.}$$



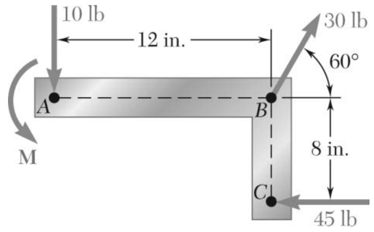
Then with **R** at *D*, $\Sigma M_B: \quad -186\text{ lb}\cdot\text{in} = a(15.9808\text{ lb})$

or $a = 11.64$ in.

and with **R** at *E*, $\Sigma M_B: \quad -186\text{ lb}\cdot\text{in} = C(30\text{ lb})$

or $C = 6.2$ in.

The line of action of **R** intersects line *AB* 11.64 in. to the left of *B* and intersects line *BC* 6.20 in. below *B*. ◀



PROBLEM 3.109

A couple **M** and the three forces shown are applied to an angle bracket. Find the moment of the couple if the line of action of the resultant of the force system is to pass through (*a*) Point *A*, (*b*) Point *B*, (*c*) Point *C*.

SOLUTION

In each case, we must have $\mathbf{M}_1^R = 0$

(*a*) $+\circlearrowleft M_A^B = \Sigma M_A = M + (12 \text{ in.})[(30 \text{ lb})\sin 60°] - (8 \text{ in.})(45 \text{ lb}) = 0$

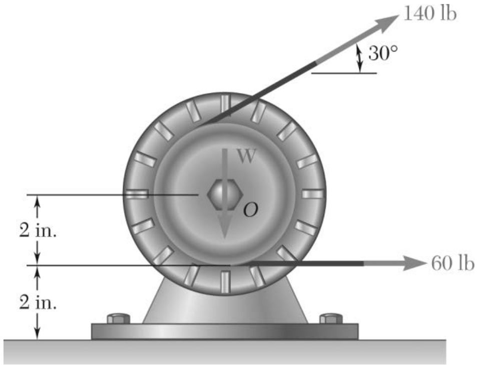
$M = +48.231 \text{ lb} \cdot \text{in.}$ $\qquad \mathbf{M} = 48.2 \text{ lb} \cdot \text{in.} \circlearrowleft$ ◀

(*b*) $+\circlearrowleft M_B^R = \Sigma M_B = M + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})(45 \text{ lb}) = 0$

$M = +240 \text{ lb} \cdot \text{in.}$ $\qquad \mathbf{M} = 240 \text{ lb} \cdot \text{in.} \circlearrowleft$ ◀

(*c*) $+\circlearrowleft M_C^R = \Sigma M_C = M + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})[(30 \text{ lb})\cos 60°] = 0$

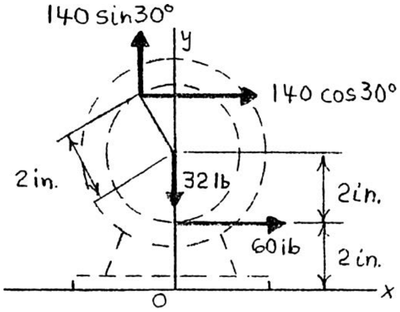
$M = 0$ $\qquad \mathbf{M} = 0$ ◀



PROBLEM 3.110

A 32-lb motor is mounted on the floor. Find the resultant of the weight and the forces exerted on the belt, and determine where the line of action of the resultant intersects the floor.

SOLUTION



We have

$$\Sigma\mathbf{F}:\quad (60\text{ lb})\mathbf{i} - (32\text{ lb})\mathbf{j} + (140\text{ lb})(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = \mathbf{R}$$

$$\mathbf{R} = (181.244\text{ lb})\mathbf{i} + (38.0\text{ lb})\mathbf{j}$$

$$\text{or}\quad \mathbf{R} = 185.2\text{ lb} \measuredangle 11.84° \blacktriangleleft$$

We have $\Sigma M_O:\quad \Sigma M_O = xR_y$

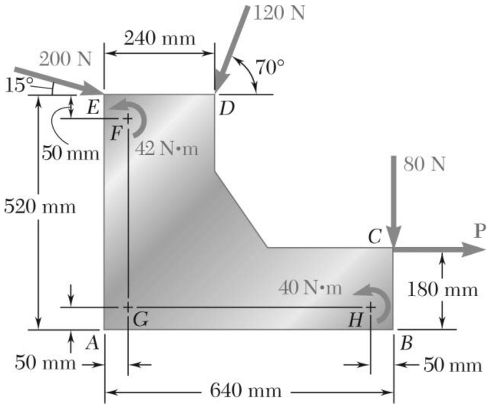
$$-[(140\text{ lb})\cos 30°][(4 + 2\cos 30°)\text{in.}] - [(140\text{ lb})\sin 30°][(2\text{ in.})\sin 30°]$$

$$-(60\text{ lb})(2\text{ in.}) = x(38.0\text{ lb})$$

$$x = \frac{1}{38.0}(-694.97 - 70.0 - 120)\text{ in.}$$

and $x = -23.289$ in.

Or resultant intersects the base (x-axis) 23.3 in. to the left of the vertical centerline (y-axis) of the motor. ◀

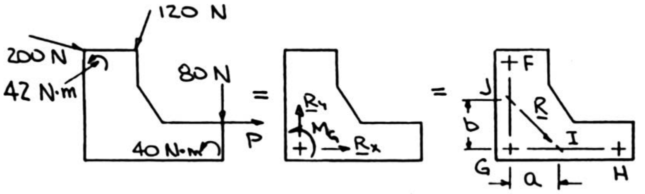


PROBLEM 3.111

A machine component is subjected to the forces and couples shown. The component is to be held in place by a single rivet that can resist a force but not a couple. For $P = 0$, determine the location of the rivet hole if it is to be located (*a*) on line *FG*, (*b*) on line *GH*.

SOLUTION

We have



First replace the applied forces and couples with an equivalent force-couple system at *G*.

Thus, ΣF_x: $200\cos 15° - 120\cos 70° + P = R_x$

or $R_x = (152.142 + P)$ N

ΣF_y: $-200\sin 15° - 120\sin 70° - 80 = R_y$

or $R_y = -244.53$ N

$$\begin{aligned}\Sigma M_G:\ & -(0.47\text{ m})(200\text{ N})\cos 15° + (0.05\text{ m})(200\text{ N})\sin 15° \\ & + (0.47\text{ m})(120\text{ N})\cos 70° - (0.19\text{ m})(120\text{ N})\sin 70° \\ & - (0.13\text{ m})(P\text{ N}) - (0.59\text{ m})(80\text{ N}) + 42\text{ N}\cdot\text{m} \\ & + 40\text{ N}\cdot\text{m} = M_G\end{aligned}$$

or $M_G = -(55.544 + 0.13P)$ N·m (1)

Setting $P = 0$ in Eq. (1):

Now with **R** at *I*, ΣM_G: $-55.544\text{ N}\cdot\text{m} = -a(244.53\text{ N})$

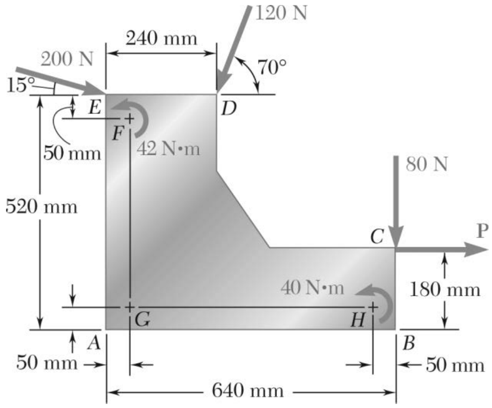
or $a = 0.227$ m

and with **R** at *J*, ΣM_G: $-55.544\text{ N}\cdot\text{m} = -b(152.142\text{ N})$

or $b = 0.365$ m

(*a*) The rivet hole is 0.365 m above *G*. ◀

(*b*) The rivet hole is 0.227 m to the right of *G*. ◀



PROBLEM 3.112

Solve Problem 3.111, assuming that $P = 60$ N.

PROBLEM 3.111 A machine component is subjected to the forces and couples shown. The component is to be held in place by a single rivet that can resist a force but not a couple. For $P = 0$, determine the location of the rivet hole if it is to be located (*a*) on line *FG*, (*b*) on line *GH*.

SOLUTION

See the solution to Problem 3.111 leading to the development of Equation (1):

$$M_G = -(55.544 + 0.13P)\ \text{N}\cdot\text{m}$$

and $$R_x = (152.142 + P)\ \text{N}$$

For $$P = 60\ \text{N}$$

we have $$R_x = (152.142 + 60)$$
$$= 212.14\ \text{N}$$
$$M_G = -[55.544 + 0.13(60)]$$
$$= -63.344\ \text{N}\cdot\text{m}$$

Then with **R** at *I*, $$\Sigma M_G:\quad -63.344\ \text{N}\cdot\text{m} = -a(244.53\ \text{N})$$

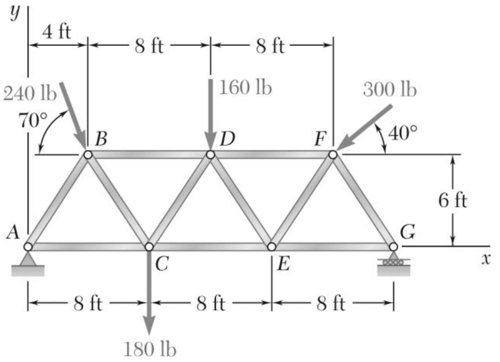
or $$a = 0.259\ \text{m}$$

and with **R** at *J*, $$\Sigma M_G:\quad -63.344\ \text{N}\cdot\text{m} = -b(212.14\ \text{N})$$

or $$b = 0.299\ \text{m}$$

(*a*) The rivet hole is 0.299 m above *G*. ◀

(*b*) The rivet hole is 0.259 m to the right of *G*. ◀



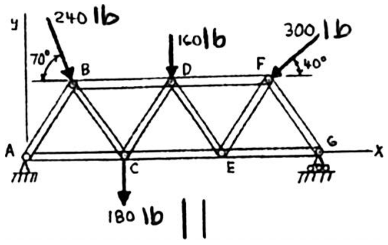
PROBLEM 3.113

A truss supports the loading shown. Determine the equivalent force acting on the truss and the point of intersection of its line of action with a line drawn through Points *A* and *G*.

SOLUTION

We have $\mathbf{R} = \Sigma\mathbf{F}$

$$\mathbf{R} = (240 \text{ lb})(\cos 70°\mathbf{i} - \sin 70°\mathbf{j}) - (160 \text{ lb})\mathbf{j}$$
$$+ (300 \text{ lb})(-\cos 40°\mathbf{i} - \sin 40°\mathbf{j}) - (180 \text{ lb})\mathbf{j}$$



$$\mathbf{R} = -(147.728 \text{ lb})\mathbf{i} - (758.36 \text{ lb})\mathbf{j}$$

$$R = \sqrt{R_x^2 + R_y^2}$$
$$= \sqrt{(147.728)^2 + (758.36)^2}$$
$$= 772.62 \text{ lb}$$

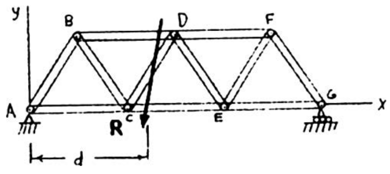
$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$
$$= \tan^{-1}\left(\frac{-758.36}{-147.728}\right)$$
$$= 78.977°$$

or $\mathbf{R} = 773$ lb ↙ 79.0° ◀

We have $\Sigma M_A = dR_y$

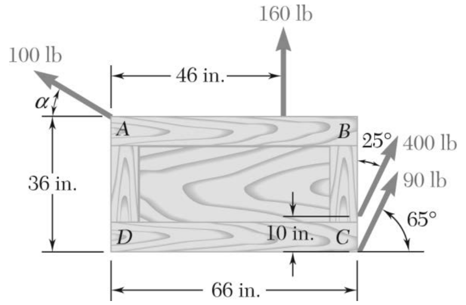
where

$$\Sigma M_A = -[240 \text{ lb} \cos 70°](6 \text{ ft}) - [240 \text{ lb} \sin 70°](4 \text{ ft})$$
$$-(160 \text{ lb})(12 \text{ ft}) + [300 \text{ lb} \cos 40°](6 \text{ ft})$$
$$-[300 \text{ lb} \sin 40°](20 \text{ ft}) - (180 \text{ lb})(8 \text{ ft})$$
$$= -7232.5 \text{ lb} \cdot \text{ft}$$



$$d = \frac{-7232.5 \text{ lb} \cdot \text{ft}}{-758.36 \text{ lb}}$$
$$= 9.5370 \text{ ft}$$

or $d = 9.54$ ft to the right of *A* ◀

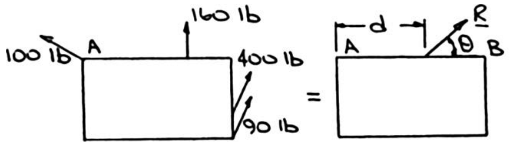


PROBLEM 3.114

Four ropes are attached to a crate and exert the forces shown. If the forces are to be replaced with a single equivalent force applied at a point on line *AB*, determine (*a*) the equivalent force and the distance from *A* to the point of application of the force when $\alpha = 30°$, (*b*) the value of α so that the single equivalent force is applied at Point *B*.

SOLUTION

We have



(*a*) For equivalence, ΣF_x: $-100\cos 30° + 400\cos 65° + 90\cos 65° = R_x$

or $R_x = 120.480 \text{ lb}$

ΣF_y: $100\sin\alpha + 160 + 400\sin 65° + 90\sin 65° = R_y$

or $R_y = (604.09 + 100\sin\alpha) \text{ lb}$ (1)

With $\alpha = 30°$, $R_y = 654.09 \text{ lb}$

Then $R = \sqrt{(120.480)^2 + (654.09)^2}$ $\tan\theta = \dfrac{654.09}{120.480}$

$= 665 \text{ lb}$ or $\theta = 79.6°$

Also

$$\begin{aligned}\Sigma M_A:\ &(46 \text{ in.})(160 \text{ lb}) + (66 \text{ in.})(400 \text{ lb})\sin 65° \\ &+ (26 \text{ in.})(400 \text{ lb})\cos 65° + (66 \text{ in.})(90 \text{ lb})\sin 65° \\ &+ (36 \text{ in.})(90 \text{ lb})\cos 65° = d(654.09 \text{ lb})\end{aligned}$$

or $\Sigma M_A = 42{,}435 \text{ lb}\cdot\text{in.}$ and $d = 64.9 \text{ in.}$ $R = 665 \text{ lb} \measuredangle 79.6°$ ◀

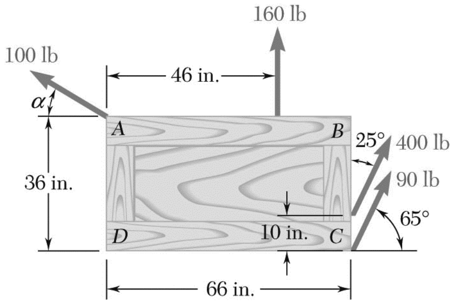
and **R** is applied 64.9 in. to the right of *A*. ◀

(*b*) We have $d = 66$ in.

Then ΣM_A: $42{,}435 \text{ lb}\cdot\text{in} = (66 \text{ in.})R_y$

or $R_y = 642.95 \text{ lb}$

Using Eq. (1): $642.95 = 604.09 + 100\sin\alpha$ or $\alpha = 22.9°$ ◀

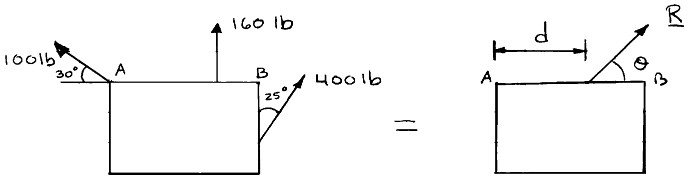


PROBLEM 3.115

Solve Prob. 3.114, assuming that the 90-lb force is removed.

PROBLEM 3.114 Four ropes are attached to a crate and exert the forces shown. If the forces are to be replaced with a single equivalent force applied at a point on line *AB*, determine (*a*) the equivalent force and the distance from *A* to the point of application of the force when $\alpha = 30°$, (*b*) the value of α so that the single equivalent force is applied at Point *B*.

SOLUTION



(*a*) For equivalence, ΣF_x: $-(100 \text{ lb})\cos 30° + (400 \text{ lb})\sin 25° = R_x$

or $R_x = 82.445$ lb

ΣF_y: $160 \text{ lb} + (100 \text{ lb})\sin 30° + (400 \text{ lb})\cos 25° = R_y$

or $R_y = 572.52$ lb

$$R = \sqrt{(82.445)^2 + (572.52)} = 578.43 \text{ lb}$$

$$\tan\theta = \frac{572.52}{82.445} \qquad \text{or} \qquad \theta = 81.806°$$

$$\Sigma M_A: \quad (46 \text{ in.})(160 \text{ lb}) + (66 \text{ in.})(400 \text{ lb})\cos 25° + (26 \text{ in.})(400 \text{ lb})\sin 25° = d(527.52 \text{ lb})$$

$$d = 62.3 \text{ in.}$$

R = 578 lb ⦢ 81.8° and is applied 62.3 in. to the right of *A*. ◀

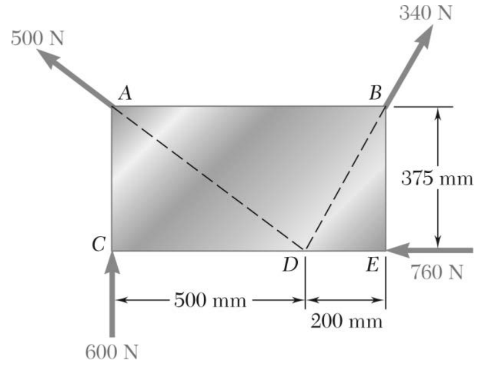
(*b*) We have $d = 66.0$ in. For *R* applied at *B*,

$$\Sigma M_A: \quad R_y(66 \text{ in.}) = (160 \text{ lb})(46 \text{ in.}) + (66 \text{ in.})(400 \text{ lb})\cos 25° + (26 \text{ in.})(400 \text{ lb})\sin 25°$$

$$R_y = 540.64 \text{ lb}$$

$$\Sigma F_Y: \quad 160 \text{ lb} + (100 \text{ lb})\sin\alpha + (400 \text{ lb})\cos 25° = 540.64 \text{ lb}$$

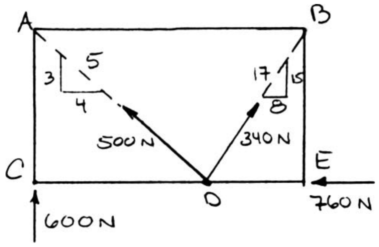
$\alpha = 10.44°$ ◀



PROBLEM 3.116

Four forces act on a 700 × 375-mm plate as shown. (*a*) Find the resultant of these forces. (*b*) Locate the two points where the line of action of the resultant intersects the edge of the plate.

SOLUTION



(*a*)

$$\mathbf{R} = \Sigma\mathbf{F}$$

$$= (-400\text{ N} + 160\text{ N} - 760\text{ N})\mathbf{i} + (600\text{ N} + 300\text{ N} + 300\text{ N})\mathbf{j}$$

$$= -(1000\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j}$$

$$R = \sqrt{(1000\text{ N})^2 + (1200\text{ N})^2}$$

$$= 1562.09\text{ N}$$

$$\tan\theta = \left(-\frac{1200\text{ N}}{1000\text{ N}}\right)$$

$$= -1.20000$$

$$\theta = -50.194°$$

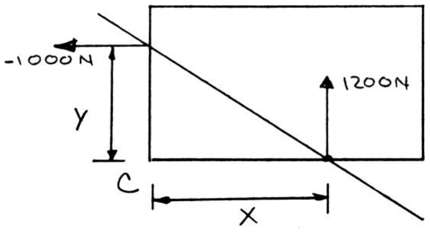
$\mathbf{R} = 1562\text{ N} \measuredangle 50.2°$ ◀

(*b*)

$$\mathbf{M}_C^R = \Sigma\mathbf{r}\times\mathbf{F}$$

$$= (0.5\text{ m})\mathbf{i}\times(300\text{ N} + 300\text{ N})\mathbf{j}$$

$$= (300\text{ N}\cdot\text{m})\mathbf{k}$$



$$(300\text{ N}\cdot\text{m})\mathbf{k} = x\mathbf{i}\times(1200\text{ N})\mathbf{j}$$

$$x = 0.25000\text{ m}$$

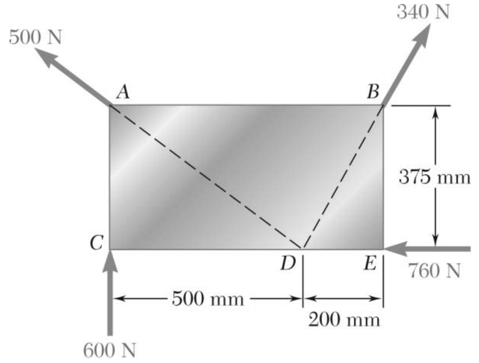
$$x = 250\text{ mm}$$

$$(300\text{ N}\cdot\text{m}) = y\mathbf{j}\times(-1000\text{ N})\mathbf{i}$$

$$y = 0.30000\text{ m}$$

$$y = 300\text{ mm}$$

Intersection 250 mm to right of *C* and 300 mm above *C* ◀

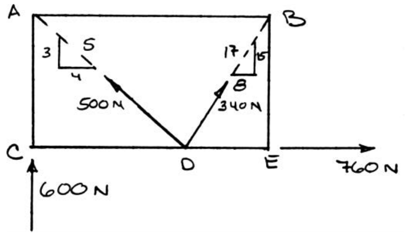


PROBLEM 3.117

Solve Problem 3.116, assuming that the 760-N force is directed to the right.

PROBLEM 3.116 Four forces act on a 700 × 375-mm plate as shown. (*a*) Find the resultant of these forces. (*b*) Locate the two points where the line of action of the resultant intersects the edge of the plate.

SOLUTION



(*a*)

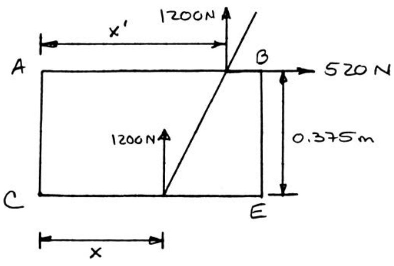
$$\begin{aligned}\mathbf{R} &= \Sigma\mathbf{F}\\ &= (-400\text{N} + 160\text{ N} + 760\text{ N})\mathbf{i}\\ &\quad + (600\text{ N} + 300\text{ N} + 300\text{ N})\mathbf{j}\\ &= (520\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j}\end{aligned}$$

$$R = \sqrt{(520\text{ N})^2 + (1200\text{ N})^2} = 1307.82\text{ N}$$

$$\tan\theta = \left(\frac{1200\text{ N}}{520\text{ N}}\right) = 2.3077$$

$$\theta = 66.5714°$$

$\mathbf{R} = 1308\text{ N} \measuredangle 66.6°$ ◀



(*b*)

$$\begin{aligned}\mathbf{M}_C^R &= \Sigma\mathbf{r}\times\mathbf{F}\\ &= (0.5\text{ m})\mathbf{i}\times(300\text{ N} + 300\text{ N})\mathbf{j}\\ &= (300\text{ N}\cdot\text{m})\mathbf{k}\end{aligned}$$

$$(300\text{ N}\cdot\text{m})\mathbf{k} = x\mathbf{i}\times(1200\text{ N})\mathbf{j}$$

$$x = 0.25000\text{ m}$$

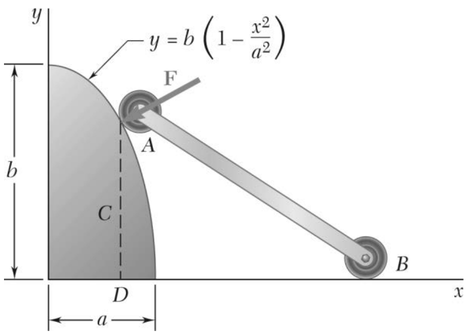
or $x = 0.250\text{ mm}$

$$\begin{aligned}(300\text{ N}\cdot\text{m})\mathbf{k} &= [x'\mathbf{i} + (0.375\text{ m})\mathbf{j}]\times[(520\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j}]\\ &= (1200x' - 195)\mathbf{k}\end{aligned}$$

$$x' = 0.41250\text{ m}$$

or $x' = 412.5\text{ mm}$

Intersection 412 mm to the right of *A* and 250 mm to the right of *C* ◀

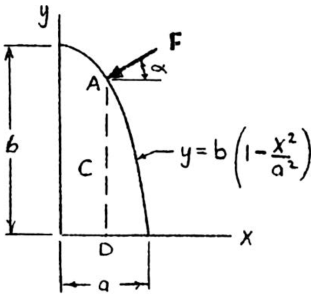


PROBLEM 3.118

As follower *AB* rolls along the surface of member *C*, it exerts a constant force **F** perpendicular to the surface. (*a*) Replace **F** with an equivalent force-couple system at Point *D* obtained by drawing the perpendicular from the point of contact to the *x*-axis. (*b*) For *a* = 1 m and *b* = 2 m, determine the value of *x* for which the moment of the equivalent force-couple system at *D* is maximum.

SOLUTION

(*a*) The slope of any tangent to the surface of member *C* is

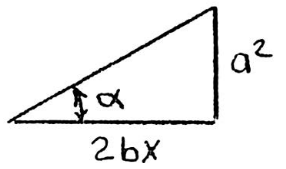


$$\frac{dy}{dx} = \frac{d}{dx}\left[b\left(1 - \frac{x^2}{a^2}\right)\right] = \frac{-2b}{a^2}x$$

Since the force **F** is perpendicular to the surface,

$$\tan\alpha = -\left(\frac{dy}{dx}\right)^{-1} = \frac{a^2}{2b}\left(\frac{1}{x}\right)$$

For equivalence,



$$\Sigma F: \quad \mathbf{F} = \mathbf{R}$$

$$\Sigma M_D: \quad (F\cos\alpha)(y_A) = M_D$$

where

$$\cos\alpha = \frac{2bx}{\sqrt{(a^2)^2 + (2bx)^2}}$$

$$y_A = b\left(1 - \frac{x^2}{a^2}\right)$$

$$M_D = \frac{2Fb^2\left(x - \frac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}}$$

Therefore, the equivalent force-couple system at *D* is

$$\mathbf{R} = F \;\measuredangle\; \tan^{-1}\left(\frac{a^2}{2bx}\right) \blacktriangleleft$$

$$\mathbf{M} = \frac{2Fb^2\left(x - \frac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}} \blacktriangleleft$$

(*b*) To maximize *M*, the value of *x* must satisfy $\dfrac{dM}{dx} = 0$

where for $a = 1\text{ m}, \quad b = 2\text{ m}$

$$M = \frac{8F(x - x^3)}{\sqrt{1 + 16x^2}}$$

$$\frac{dM}{dx} = 8F\,\frac{\sqrt{1+16x^2}\,(1-3x^2) - (x - x^3)\left[\frac{1}{2}(32x)(1+16x^2)^{-1/2}\right]}{(1+16x^2)} = 0$$

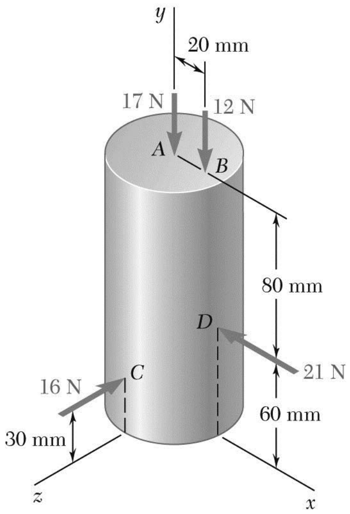
$$(1+16x^2)(1-3x^2) - 16x(x - x^3) = 0$$

or

$$32x^4 + 3x^2 - 1 = 0$$

$$x^2 = \frac{-3 \pm \sqrt{9 - 4(32)(-1)}}{2(32)} = 0.136011\text{ m}^2 \quad \text{and} \quad -0.22976\text{ m}^2$$

Using the positive value of x^2: $x = 0.36880\text{ m}$ or $x = 369\text{ mm}$ ◀



PROBLEM 3.119

As plastic bushings are inserted into a 60-mm-diameter cylindrical sheet metal enclosure, the insertion tools exert the forces shown on the enclosure. Each of the forces is parallel to one of the coordinate axes. Replace these forces with an equivalent force-couple system at *C*.

SOLUTION

For equivalence, $\Sigma\mathbf{F}$: $\mathbf{R} = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D$

$$= -(17\text{ N})\mathbf{j} - (12\text{ N})\mathbf{j} - (16\text{ N})\mathbf{k} - (21\text{ N})\mathbf{i}$$

$$= -(21\text{ N})\mathbf{i} - (29\text{ N})\mathbf{j} - (16\text{ N})\mathbf{k}$$

$$\Sigma M_C: \quad \mathbf{M} = \mathbf{r}_{A/C} \times \mathbf{F}_A + \mathbf{r}_{B/C} \times \mathbf{F}_B + \mathbf{r}_{D/C} \times \mathbf{F}_D$$

$$M = [(0.11\text{ m})\mathbf{j} - (0.03\text{ m})\mathbf{k}] \times [-(17\text{ N})]\mathbf{j}$$

$$+ [(0.02\text{ m})\mathbf{i} + (0.11\text{ m})\mathbf{j} - (0.03\text{ m})\mathbf{k}] \times [-(12\text{ N})]\mathbf{j}$$

$$+ [(0.03\text{ m})\mathbf{i} + (0.03\text{ m})\mathbf{j} - (0.03\text{ m})\mathbf{k}] \times [-(21\text{ N})]\mathbf{i}$$

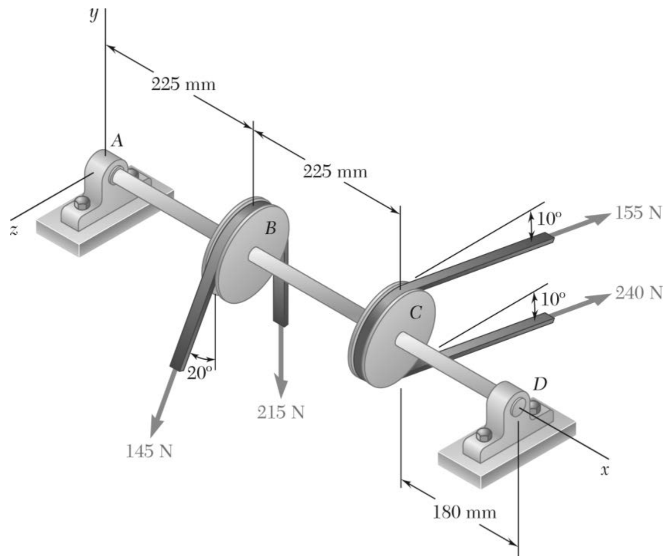
$$= -(0.51\text{ N}\cdot\text{m})\mathbf{i} + [-(0.24\text{ N}\cdot\text{m})\mathbf{k} - (0.36\text{ N}\cdot\text{m})\mathbf{i}]$$

$$+ [(0.63\text{ N}\cdot\text{m})\mathbf{k} + (0.63\text{ N}\cdot\text{m})\mathbf{j}]$$

∴ The equivalent force-couple system at *C* is

$$\mathbf{R} = -(21.0\text{ N})\mathbf{i} - (29.0\text{ N})\mathbf{j} - (16.00\text{ N})\mathbf{k} \blacktriangleleft$$

$$\mathbf{M} = -(0.870\text{ N}\cdot\text{m})\mathbf{i} + (0.630\text{ N}\cdot\text{m})\mathbf{j} + (0.390\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

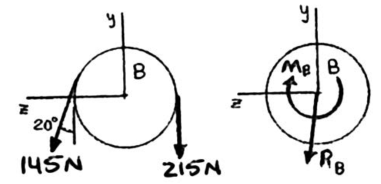


PROBLEM 3.120

Two 150-mm-diameter pulleys are mounted on line shaft *AD*. The belts at *B* and *C* lie in vertical planes parallel to the *yz*-plane. Replace the belt forces shown with an equivalent force-couple system at *A*.

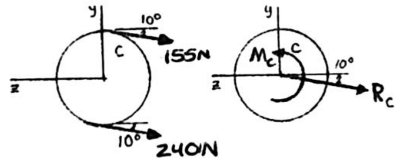
SOLUTION

Equivalent force-couple at each pulley:



Pulley *B*:

$$\mathbf{R}_B = (145\text{ N})(-\cos 20°\mathbf{j} + \sin 20°\mathbf{k}) - 215\text{ N}\mathbf{j}$$
$$= -(351.26\text{ N})\mathbf{j} + (49.593\text{ N})\mathbf{k}$$
$$\mathbf{M}_B = -(215\text{ N} - 145\text{ N})(0.075\text{ m})\mathbf{i}$$
$$= -(5.25\text{ N}\cdot\text{m})\mathbf{i}$$



Pulley *C*:

$$\mathbf{R}_C = (155\text{ N} + 240\text{ N})(-\sin 10°\mathbf{j} - \cos 10°\mathbf{k})$$
$$= -(68.591\text{ N})\mathbf{j} - (389.00\text{ N})\mathbf{k}$$
$$\mathbf{M}_C = (240\text{ N} - 155\text{ N})(0.075\text{ m})\mathbf{i}$$
$$= (6.3750\text{ N}\cdot\text{m})\mathbf{i}$$

Then

$$\mathbf{R} = \mathbf{R}_B + \mathbf{R}_C = -(419.85\text{ N})\mathbf{j} - (339.41)\mathbf{k} \qquad \text{or } \mathbf{R} = (420\text{ N})\mathbf{j} - (339\text{ N})\mathbf{k} \blacktriangleleft$$

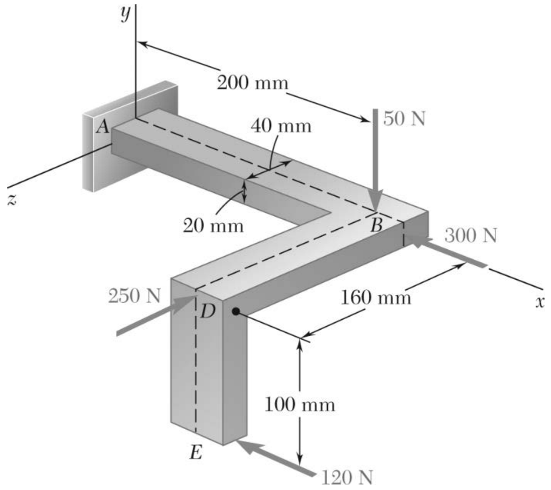
$$\mathbf{M}_A = \mathbf{M}_B + \mathbf{M}_C + \mathbf{r}_{B/A} \times \mathbf{R}_B + \mathbf{r}_{C/A} \times \mathbf{R}_C$$

$$= -(5.25\text{ N}\cdot\text{m})\mathbf{i} + (6.3750\text{ N}\cdot\text{m})\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.225 & 0 & 0 \\ 0 & -351.26 & 49.593 \end{vmatrix} \text{N}\cdot\text{m}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0 \\ 0 & -68.591 & -389.00 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (1.12500\text{ N}\cdot\text{m})\mathbf{i} + (163.892\text{ N}\cdot\text{m})\mathbf{j} - (109.899\text{ N}\cdot\text{m})\mathbf{k}$$

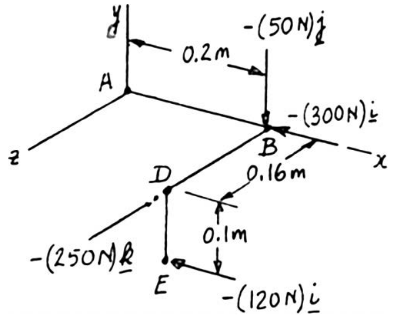
$$\text{or } \mathbf{M}_A = (1.125\text{ N}\cdot\text{m})\mathbf{i} + (163.9\text{ N}\cdot\text{m})\mathbf{j} - (109.9\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.121

Four forces are applied to the machine component *ABDE* as shown. Replace these forces with an equivalent force-couple system at *A*.

SOLUTION



$$\mathbf{R} = -(50\text{ N})\mathbf{j} - (300\text{ N})\mathbf{i} - (120\text{ N})\mathbf{i} - (250\text{ N})\mathbf{k}$$

$$\mathbf{R} = -(420\text{ N})\mathbf{i} - (50\text{ N})\mathbf{j} - (250\text{ N})\mathbf{k}$$

$$\mathbf{r}_B = (0.2\text{ m})\mathbf{i}$$

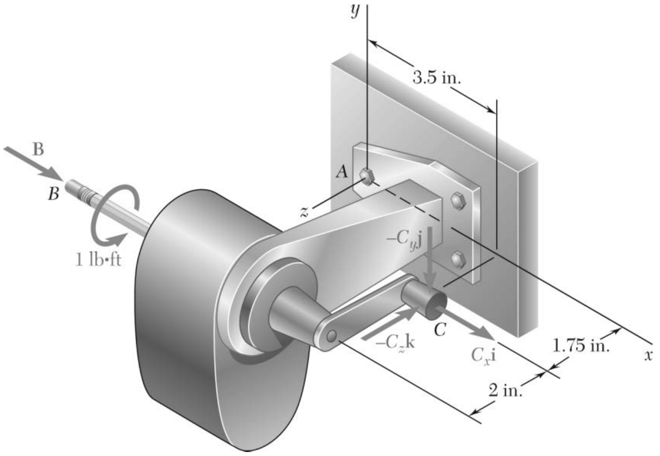
$$\mathbf{r}_D = (0.2\text{ m})\mathbf{i} + (0.16\text{ m})\mathbf{k}$$

$$\mathbf{r}_E = (0.2\text{ m})\mathbf{i} - (0.1\text{ m})\mathbf{j} + (0.16\text{ m})\mathbf{k}$$

$$\begin{aligned}\mathbf{M}_A^R &= \mathbf{r}_B \times [-(300\text{ N})\mathbf{i} - (50\text{ N})\mathbf{j}] \\ &\quad + \mathbf{r}_D \times (-250\text{ N})\mathbf{k} + \mathbf{r} \times (-120\text{ N})\mathbf{i} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2\text{ m} & 0 & 0 \\ -300\text{ N} & -50\text{ N} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2\text{ m} & 0 & 0.16\text{ m} \\ 0 & 0 & -250\text{ N} \end{vmatrix} \\ &\quad + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2\text{ m} & -0.1\text{ m} & 0.16\text{ m} \\ -120\text{ N} & 0 & 0 \end{vmatrix} \\ &= -(10\text{ N}\cdot\text{m})\mathbf{k} + (50\text{ N}\cdot\text{m})\mathbf{j} - (19.2\text{ N}\cdot\text{m})\mathbf{j} - (12\text{ N}\cdot\text{m})\mathbf{k}\end{aligned}$$

Force-couple system at *A* is

$$\mathbf{R} = -(420\text{ N})\mathbf{i} - (50\text{ N})\mathbf{j} - (250\text{ N})\mathbf{k} \quad \mathbf{M}_A^R = (30.8\text{ N}\cdot\text{m})\mathbf{j} - (220\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.122

While using a pencil sharpener, a student applies the forces and couple shown. (*a*) Determine the forces exerted at *B* and *C* knowing that these forces and the couple are equivalent to a force-couple system at *A* consisting of the force $\mathbf{R} = (2.6\text{ lb})\mathbf{i} + R_y\mathbf{j} - (0.7\text{ lb})\mathbf{k}$ and the couple $\mathbf{M}_A^R = M_x\mathbf{i} + (1.0\text{ lb}\cdot\text{ft})\mathbf{j} - (0.72\text{ lb}\cdot\text{ft})\mathbf{k}$.
(*b*) Find the corresponding values of R_y and M_x.

SOLUTION

(*a*) From the statement of the problem, equivalence requires

$$\Sigma\mathbf{F}:\quad \mathbf{B}+\mathbf{C}=\mathbf{R}$$

or $$\Sigma F_x:\quad B_x + C_x = 2.6\text{ lb} \qquad (1)$$

$$\Sigma F_y:\quad -C_y = R_y \qquad (2)$$

$$\Sigma F_z:\quad -C_z = -0.7\text{ lb} \quad \text{or} \quad C_z = 0.7\text{ lb}$$

and $$\Sigma\mathbf{M}_A:\quad (\mathbf{r}_{B/A}\times\mathbf{B}+\mathbf{M}_B)+\mathbf{r}_{C/A}\times\mathbf{C}=M_A^R$$

or $$\Sigma\mathbf{M}_x:\quad (1\text{ lb}\cdot\text{ft})+\left(\frac{1.75}{12}\text{ ft}\right)(C_y)=M_x \qquad (3)$$

$$\Sigma M_y:\quad \left(\frac{3.75}{12}\text{ ft}\right)(B_x)+\left(\frac{1.75}{12}\text{ ft}\right)(C_x)+\left(\frac{3.5}{12}\text{ ft}\right)(0.7\text{ lb})=1\text{ lb}\cdot\text{ft}$$

or $$3.75B_x + 1.75C_x = 9.55$$

Using Eq. (1): $$3.75B_x + 1.75(2.6B_x) = 9.55$$

or $$B_x = 2.5\text{ lb}$$

and $$C_x = 0.1\text{ lb}$$

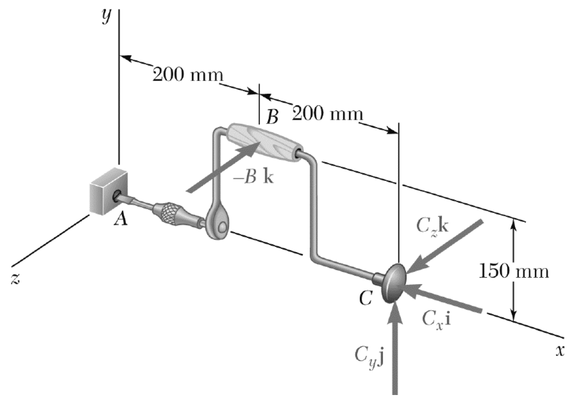
$$\Sigma M_z:\quad -\left(\frac{3.5}{12}\text{ ft}\right)(C_y) = -0.72\text{ lb}\cdot\text{ft}$$

or $$C_y = 2.4686\text{ lb}$$

$$\mathbf{B}=(2.50\text{ lb})\mathbf{i} \quad \mathbf{C}=(0.1000\text{ lb})\mathbf{i}-(2.47\text{ lb})\mathbf{j}-(0.700\text{ lb})\mathbf{k} \ \blacktriangleleft$$

(*b*) Eq. (2) ⇒ $$R_y = -2.47\text{ lb} \ \blacktriangleleft$$

Using Eq. (3): $$1+\left(\frac{1.75}{12}\right)(2.4686)=M_x \qquad \text{or} \quad M_x = 1.360\text{ lb}\cdot\text{ft} \ \blacktriangleleft$$



PROBLEM 3.123

A blade held in a brace is used to tighten a screw at A. (a) Determine the forces exerted at B and C, knowing that these forces are equivalent to a force-couple system at A consisting of $\mathbf{R} = -(30\text{ N})\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k}$ and $\mathbf{M}_A^R = -(12\text{ N}\cdot\text{m})\mathbf{i}$. ($b$) Find the corresponding values of R_y and R_z. (c) What is the orientation of the slot in the head of the screw for which the blade is least likely to slip when the brace is in the position shown?

SOLUTION

(a) Equivalence requires $\Sigma\mathbf{F}$: $\mathbf{R} = \mathbf{B} + \mathbf{C}$

or $-(30\text{ N})\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} = -B\mathbf{k} + (-C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k})$

Equating the **i** coefficients: $\mathbf{i}$: $-30\text{ N} = -C_x$ or $C_x = 30\text{ N}$

Also, $\Sigma\mathbf{M}_A$: $\mathbf{M}_A^R = \mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{r}_{C/A} \times \mathbf{C}$

or

$$-(12\text{ N}\cdot\text{m})\mathbf{i} = [(0.2\text{ m})\mathbf{i} + (0.15\text{ m})\mathbf{j}] \times (-B)\mathbf{k} + (0.4\text{ m})\mathbf{i} \times [-(30\text{ N})\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k}]$$

Equating coefficients:

$\mathbf{i}$: $-12\text{ N}\cdot\text{m} = -(0.15\text{ m})B$ or $B = 80\text{ N}$

$\mathbf{k}$: $0 = (0.4\text{ m})C_y$ or $C_y = 0$

$\mathbf{j}$: $0 = (0.2\text{ m})(80\text{ N}) - (0.4\text{ m})C_z$ or $C_z = 40\text{ N}$

$\mathbf{B} = -(80.0\text{ N})\mathbf{k}$ $\mathbf{C} = -(30.0\text{ N})\mathbf{i} + (40.0\text{ N})\mathbf{k}$ ◀

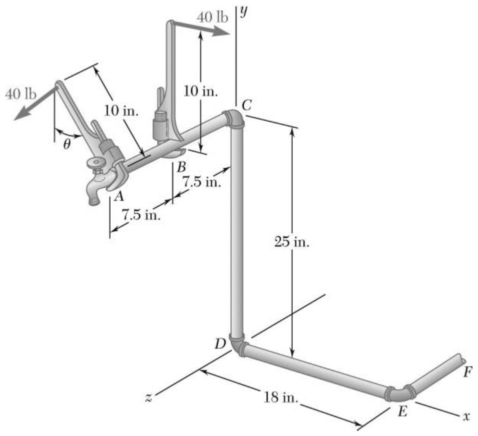
(b) Now we have for the equivalence of forces

$$-(30\text{ N})\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} = -(80\text{ N})\mathbf{k} + [(-30\text{ N})\mathbf{i} + (40\text{ N})\mathbf{k}]$$

Equating coefficients: $\mathbf{j}$: $R_y = 0$ $R_y = 0$ ◀

$\mathbf{k}$: $R_z = -80 + 40$ or $R_z = -40.0\text{ N}$ ◀

(c) First note that $\mathbf{R} = -(30\text{ N})\mathbf{i} - (40\text{ N})\mathbf{k}$. Thus, the screw is best able to resist the lateral force R_z when the slot in the head of the screw is vertical. ◀



PROBLEM 3.124

In order to unscrew the tapped faucet *A*, a plumber uses two pipe wrenches as shown. By exerting a 40-lb force on each wrench, at a distance of 10 in. from the axis of the pipe and in a direction perpendicular to the pipe and to the wrench, he prevents the pipe from rotating, and thus avoids loosening or further tightening the joint between the pipe and the tapped elbow *C*. Determine (*a*) the angle *θ* that the wrench at *A* should form with the vertical if elbow *C* is not to rotate about the vertical, (*b*) the force-couple system at *C* equivalent to the two 40-lb forces when this condition is satisfied.

SOLUTION

We first reduce the given forces to force-couple systems at *A* and *B*, noting that

$$|\mathbf{M}_A| = |\mathbf{M}_B| = (40\text{ lb})(10\text{ in.})$$
$$= 400\text{ lb}\cdot\text{in.}$$

We now determine the equivalent force-couple system at *C*.

$$\mathbf{R} = (40\text{ lb})(1-\cos\theta)\mathbf{i} - (40\text{ lb})\sin\theta\mathbf{j} \qquad (1)$$

$$\begin{aligned}\mathbf{M}_C^R &= \mathbf{M}_A + \mathbf{M}_B + (15\text{ in.})\mathbf{k}\times[-(40\text{ lb})\cos\theta\mathbf{i} - (40\text{ lb})\sin\theta\mathbf{j}]\\ &\quad + (7.5\text{ in.})\mathbf{k}\times(40\text{ lb})\mathbf{i}\\ &= +400 - 400 - 600\cos\theta\mathbf{j} + 600\sin\theta\mathbf{i} + 300\mathbf{j}\\ &= (600\text{ lb}\cdot\text{in.})\sin\theta\mathbf{i} + (300\text{ lb}\cdot\text{in.})(1-2\cos\theta)\mathbf{j} \qquad (2)\end{aligned}$$

(*a*) For no rotation about vertical, *y* component of $\mathbf{M}_C^R$ must be zero.

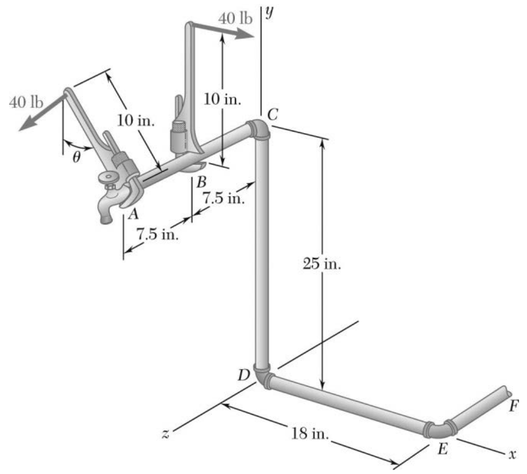
$$1 - 2\cos\theta = 0$$
$$\cos\theta = 1/2$$

$$\theta = 60.0° \blacktriangleleft$$

(*b*) For $\theta = 60.0°$ in Eqs. (1) and (2),

$$\mathbf{R} = (20.0\text{ lb})\mathbf{i} - (34.641\text{ lb})\mathbf{j};\ \mathbf{M}_C^R = (519.62\text{ lb}\cdot\text{in.})\mathbf{i}$$

$$\mathbf{R} = (20.0\text{ lb})\mathbf{i} - (34.6\text{ lb})\mathbf{j};\ \mathbf{M}_C^R = (520\text{ lb}\cdot\text{in.})\mathbf{i} \blacktriangleleft$$



PROBLEM 3.125

Assuming $\theta = 60°$ in Prob. 3.124, replace the two 40-lb forces with an equivalent force-couple system at *D* and determine whether the plumber's action tends to tighten or loosen the joint between (*a*) pipe *CD* and elbow *D*, (*b*) elbow *D* and pipe *DE*. Assume all threads to be right-handed.

PROBLEM 3.124 In order to unscrew the tapped faucet *A*, a plumber uses two pipe wrenches as shown. By exerting a 40-lb force on each wrench, at a distance of 10 in. from the axis of the pipe and in a direction perpendicular to the pipe and to the wrench, he prevents the pipe from rotating, and thus avoids loosening or further tightening the joint between the pipe and the tapped elbow *C*. Determine (*a*) the angle θ that the wrench at *A* should form with the vertical if elbow *C* is not to rotate about the vertical, (*b*) the force-couple system at *C* equivalent to the two 40-lb forces when this condition is satisfied.

SOLUTION

The equivalent force-couple system at *C* for $\theta = 60°$ was obtained in the solution to Prob. 3.124:

$$\mathbf{R} = (20.0 \text{ lb})\mathbf{i} - (34.641 \text{ lb})\mathbf{j}$$

$$\mathbf{M}_C^R = (519.62 \text{ lb} \cdot \text{in.})\mathbf{i}$$

The equivalent force-couple system at *D* is made of $\mathbf{R}$ and $\mathbf{M}_D^R$ where

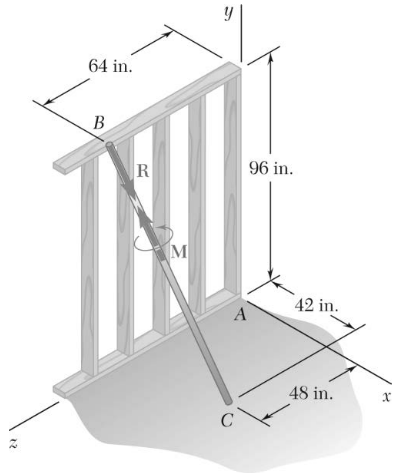
$$\begin{aligned}\mathbf{M}_D^R &= \mathbf{M}_C^R + \mathbf{r}_{C/D} \times \mathbf{R} \\ &= (519.62 \text{ lb} \cdot \text{in.})\mathbf{i} + (25.0 \text{ in.})\mathbf{j} \times [(20.0 \text{ lb})\mathbf{i} - (34.641 \text{ lb})\mathbf{j}] \\ &= (519.62 \text{ lb} \cdot \text{in.})\mathbf{i} - (500 \text{ lb} \cdot \text{in.})\mathbf{k}\end{aligned}$$

Equivalent force-couple at *D*:

$$\mathbf{R} = (20.0 \text{ lb})\mathbf{i} - (34.6 \text{ lb})\mathbf{j}; \;\; \mathbf{M}_C^R = (520 \text{ lb} \cdot \text{in.})\mathbf{i} - (500 \text{ lb} \cdot \text{in.})\mathbf{k} \;\blacktriangleleft$$

(*a*) Since $\mathbf{M}_D^R$ has no component along the *y*-axis, the plumber's action will neither loosen nor tighten the joint between pipe *CD* and elbow. ◀

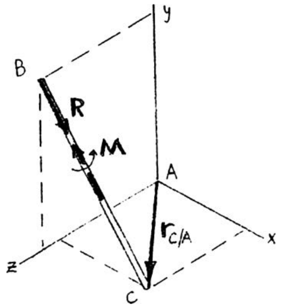
(*b*) Since the *x* component of $\mathbf{M}_D^R$ is ↷, the plumber's action will tend to tighten the joint between elbow and pipe *DE*. ◀



PROBLEM 3.126

As an adjustable brace BC is used to bring a wall into plumb, the force-couple system shown is exerted on the wall. Replace this force-couple system with an equivalent force-couple system at A if $R = 21.2$ lb and $M = 13.25$ lb · ft.

SOLUTION



We have $\quad \Sigma\mathbf{F}: \quad \mathbf{R} = \mathbf{R}_A = \mathbf{R}\lambda_{BC}$

where
$$\boldsymbol{\lambda}_{BC} = \frac{(42\text{ in.})\mathbf{i} - (96\text{ in.})\mathbf{j} - (16\text{ in.})\mathbf{k}}{106\text{ in.}}$$

$$\mathbf{R}_A = \frac{21.2\text{ lb}}{106}(42\mathbf{i} - 96\mathbf{j} - 16\mathbf{k})$$

or $\quad \mathbf{R}_A = (8.40\text{ lb})\mathbf{i} - (19.20\text{ lb})\mathbf{j} - (3.20\text{ lb})\mathbf{k}$ ◀

We have $\quad \Sigma\mathbf{M}_A: \quad \mathbf{r}_{C/A} \times \mathbf{R} + \mathbf{M} = \mathbf{M}_A$

where
$$\mathbf{r}_{C/A} = (42\text{ in.})\mathbf{i} + (48\text{ in.})\mathbf{k} = \frac{1}{12}(42\mathbf{i} + 48\mathbf{k})\text{ft}$$
$$= (3.5\text{ ft})\mathbf{i} + (4.0\text{ ft})\mathbf{k}$$

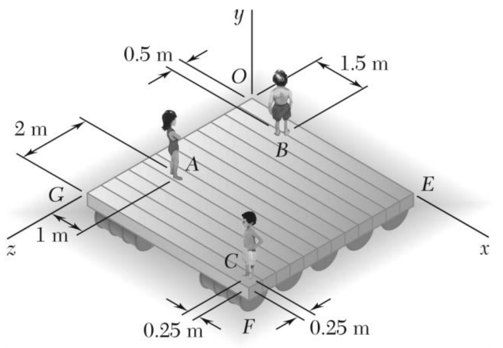
$$\mathbf{R} = (8.40\text{ lb})\mathbf{i} - (19.50\text{ lb})\mathbf{j} - (3.20\text{ lb})\mathbf{k}$$

$$\mathbf{M} = -\lambda_{BC}M$$
$$= \frac{-42\mathbf{i} + 96\mathbf{j} + 16\mathbf{k}}{106}(13.25\text{ lb}\cdot\text{ft})$$
$$= -(5.25\text{ lb}\cdot\text{ft})\mathbf{i} + (12\text{ lb}\cdot\text{ft})\mathbf{j} + (2\text{ lb}\cdot\text{ft})\mathbf{k}$$

Then
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.5 & 0 & 4.0 \\ 8.40 & -19.20 & -3.20 \end{vmatrix} \text{lb}\cdot\text{ft} + (-5.25\mathbf{i} + 12\mathbf{j} + 2\mathbf{k})\text{ lb}\cdot\text{ft} = \mathbf{M}_A$$

$$\mathbf{M}_A = (71.55\text{ lb}\cdot\text{ft})\mathbf{i} + (56.80\text{ lb}\cdot\text{ft})\mathbf{j} - (65.20\text{ lb}\cdot\text{ft})\mathbf{k}$$

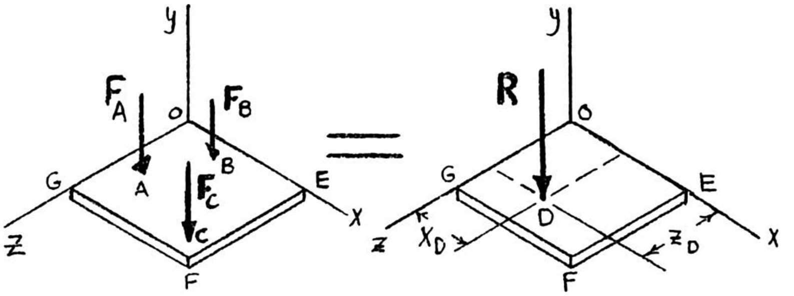
or $\quad \mathbf{M}_A = (71.6\text{ lb}\cdot\text{ft})\mathbf{i} + (56.8\text{ lb}\cdot\text{ft})\mathbf{j} - (65.2\text{ lb}\cdot\text{ft})\mathbf{k}$ ◀



PROBLEM 3.127

Three children are standing on a 5×5-m raft. If the weights of the children at Points *A*, *B*, and *C* are 375 N, 260 N, and 400 N, respectively, determine the magnitude and the point of application of the resultant of the three weights.

SOLUTION



We have $\quad \Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{R}$

$$-(375\text{ N})\mathbf{j} - (260\text{ N})\mathbf{j} - (400\text{ N})\mathbf{j} = \mathbf{R}$$

$$-(1035\text{ N})\mathbf{j} = \mathbf{R}$$

or $R = 1035$ N ◀

We have $\quad \Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) = R(z_D)$

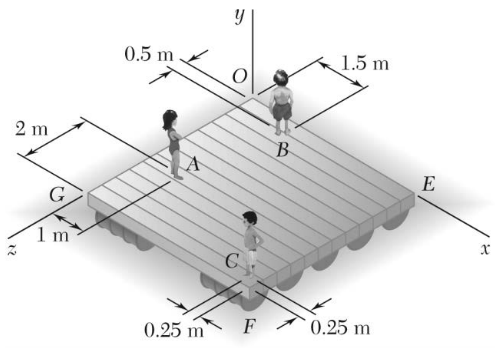
$$(375\text{ N})(3\text{ m}) + (260\text{ N})(0.5\text{ m}) + (400\text{ N})(4.75\text{ m}) = (1035\text{ N})(z_D)$$

$z_D = 3.0483$ m $\quad$ or $\quad z_D = 3.05$ m ◀

We have $\quad \Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) = R(x_D)$

$$375\text{ N}(1\text{ m}) + (260\text{ N})(1.5\text{ m}) + (400\text{ N})(4.75\text{ m}) = (1035\text{ N})(x_D)$$

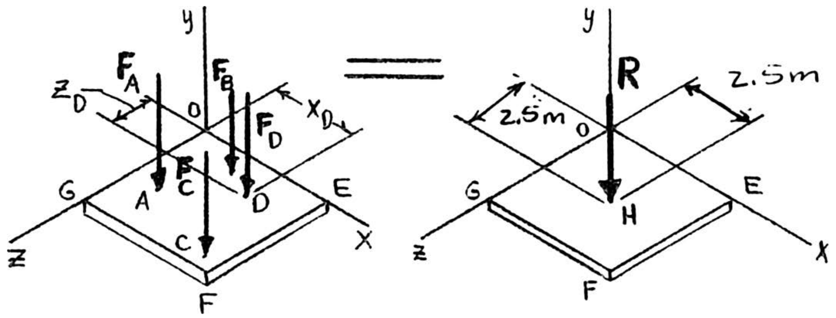
$x_D = 2.5749$ m $\quad$ or $\quad x_D = 2.57$ m ◀



PROBLEM 3.128

Three children are standing on a 5×5-m raft. The weights of the children at Points *A*, *B*, and *C* are 375 N, 260 N, and 400 N, respectively. If a fourth child of weight 425 N climbs onto the raft, determine where she should stand if the other children remain in the positions shown and the line of action of the resultant of the four weights is to pass through the center of the raft.

SOLUTION



We have $\quad \Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{R}$

$$-(375\text{ N})\mathbf{j} - (260\text{ N})\mathbf{j} - (400\text{ N})\mathbf{j} - (425\text{ N})\mathbf{j} = \mathbf{R}$$

$$\mathbf{R} = -(1460\text{ N})\mathbf{j}$$

We have $\quad \Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) + F_D(z_D) = R(z_H)$

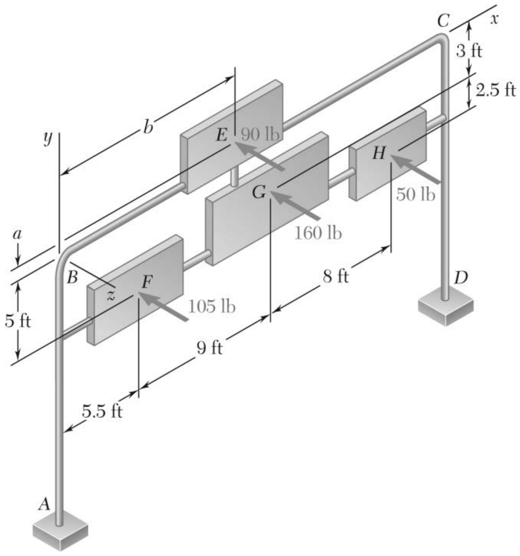
$$(375\text{ N})(3\text{ m}) + (260\text{ N})(0.5\text{ m}) + (400\text{ N})(4.75\text{ m}) + (425\text{ N})(z_D) = (1460\text{ N})(2.5\text{ m})$$

$$z_D = 1.16471\text{ m} \qquad \text{or} \quad z_D = 1.165\text{ m} \blacktriangleleft$$

We have $\quad \Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) + F_D(x_D) = R(x_H)$

$$(375\text{ N})(1\text{ m}) + (260\text{ N})(1.5\text{ m}) + (400\text{ N})(4.75\text{ m}) + (425\text{ N})(x_D) = (1460\text{ N})(2.5\text{ m})$$

$$x_D = 2.3235\text{ m} \qquad \text{or} \quad x_D = 2.32\text{ m} \blacktriangleleft$$

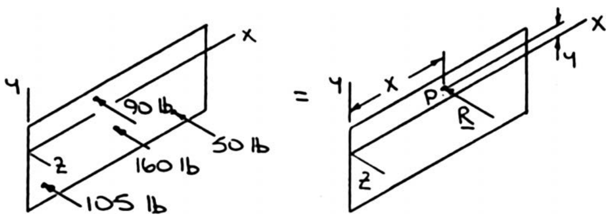


PROBLEM 3.129

Four signs are mounted on a frame spanning a highway, and the magnitudes of the horizontal wind forces acting on the signs are as shown. Determine the magnitude and the point of application of the resultant of the four wind forces when $a = 1$ ft and $b = 12$ ft.

SOLUTION

We have



Assume that the resultant **R** is applied at Point P whose coordinates are (x, y, 0).

Equivalence then requires

$$\Sigma F_z: \quad -105 - 90 - 160 - 50 = -R$$

or $R = 405$ lb ◀

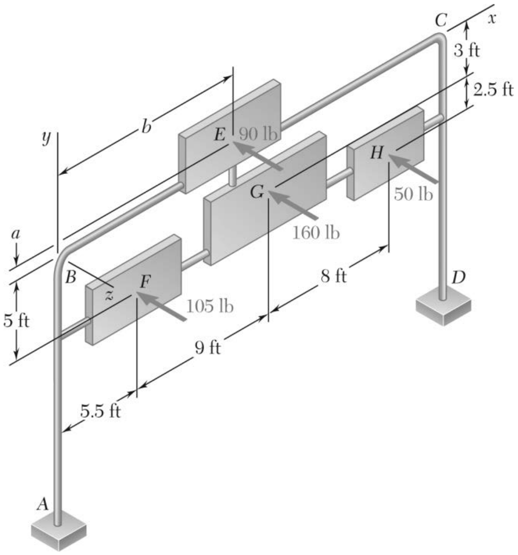
$$\Sigma M_x: \quad (5\text{ ft})(105\text{ lb}) - (1\text{ ft})(90\text{ lb}) + (3\text{ ft})(160\text{ lb}) + (5.5\text{ ft})(50\text{ lb}) = -y(405\text{ lb})$$

or $$y = -2.94\text{ ft}$$

$$\Sigma M_y: \quad (5.5\text{ ft})(105\text{ lb}) + (12\text{ ft})(90\text{ lb}) + (14.5\text{ ft})(160\text{ lb}) + (22.5\text{ ft})(50\text{ lb}) = -x(405\text{ lb})$$

or $$x = 12.60\text{ ft}$$

R acts 12.60 ft to the right of member AB and 2.94 ft below member BC. ◀



PROBLEM 3.130

Four signs are mounted on a frame spanning a highway, and the magnitudes of the horizontal wind forces acting on the signs are as shown. Determine a and b so that the point of application of the resultant of the four forces is at G.

SOLUTION

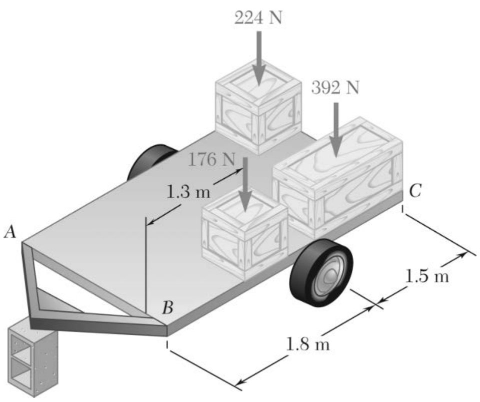
Since **R** acts at G, equivalence then requires that $\Sigma \mathbf{M}_G$ of the applied system of forces also be zero. Then at

$$G: \Sigma M_x: \quad -(a+3)\text{ ft}\times(90\text{ lb}) + (2\text{ ft})(105\text{ lb}) + (2.5\text{ ft})(50\text{ lb}) = 0$$

or $a = 0.722$ ft ◀

$$\Sigma M_y: \quad -(9\text{ ft})(105\text{ ft}) - (14.5-b)\text{ ft}\times(90\text{ lb}) + (8\text{ ft})(50\text{ lb}) = 0$$

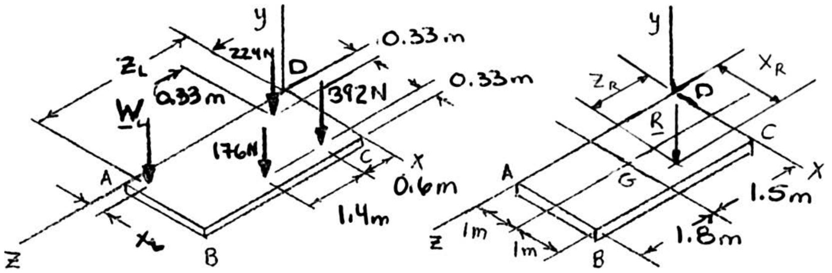
or $b = 20.6$ ft ◀



PROBLEM 3.131*

A group of students loads a 2×3.3-m flatbed trailer with two $0.66 \times 0.66 \times 0.66$-m boxes and one $0.66 \times 0.66 \times 1.2$-m box. Each of the boxes at the rear of the trailer is positioned so that it is aligned with both the back and a side of the trailer. Determine the smallest load the students should place in a second $0.66 \times 0.66 \times 1.2$-m box and where on the trailer they should secure it, without any part of the box overhanging the sides of the trailer, if each box is uniformly loaded and the line of action of the resultant of the weights of the four boxes is to pass through the point of intersection of the centerlines of the trailer and the axle. (*Hint:* Keep in mind that the box may be placed either on its side or on its end.)

SOLUTION



For the smallest weight on the trailer so that the resultant force of the four weights acts over the axle at the intersection with the center line of the trailer, the added $0.66 \times 0.66 \times 1.2$-m box should be placed adjacent to one of the edges of the trailer with the 0.66×0.66-m side on the bottom. The edges to be considered are based on the location of the resultant for the three given weights.

We have $\Sigma \mathbf{F}$: $-(224\text{ N})\mathbf{j} - (392\text{ N})\mathbf{j} - (176\text{ N})\mathbf{j} = \mathbf{R}$

$$\mathbf{R} = -(792\text{ N})\mathbf{j}$$

We have ΣM_z: $-(224\text{ N})(0.33\text{ m}) - (392\text{ N})(1.67\text{ m}) - (176\text{ N})(1.67\text{ m}) = (-792\text{ N})(x)$

$$x_R = 1.29101\text{ m}$$

We have ΣM_x: $(224\text{ N})(0.33\text{ m}) + (392\text{ N})(0.6\text{ m}) + (176\text{ N})(2.0\text{ m}) = (792\text{ N})(z)$

$$z_R = 0.83475\text{ m}$$

From the statement of the problem, it is known that the resultant of **R** from the original loading and the lightest load **W** passes through *G*, the point of intersection of the two center lines. Thus, $\Sigma \mathbf{M}_G = 0$.

Further, since the lightest load **W** is to be as small as possible, the fourth box should be placed as far from *G* as possible without the box overhanging the trailer. These two requirements imply

$$(0.33\text{ m} \le x \le 1\text{ m})\ (1.5\text{ m} \le z \le 2.97\text{ m})$$

PROBLEM 3.131* (Continued)

With $x_L = 0.33 \text{ m}$

at G: ΣM_z: $(1-0.33) \text{ m} \times W_L - (1.29101-1) \text{ m} \times (792 \text{ N}) = 0$

or $W_L = 344.00 \text{ N}$

Now we must check if this is physically possible,

at G: ΣM_x: $(z_L - 1.5) \text{ m} \times 344 \text{ N}) - (1.5 - 0.83475) \text{ m} \times (792 \text{ N}) = 0$

or $z_L = 3.032 \text{ m}$

which is **not** acceptable.

With $z_L = 2.97 \text{ m}$:

at G: ΣM_x: $(2.97 - 1.5) \text{ m} \times W_L - (1.5 - 0.83475) \text{ m} \times (792 \text{ N}) = 0$

or $W_L = 358.42 \text{ N}$

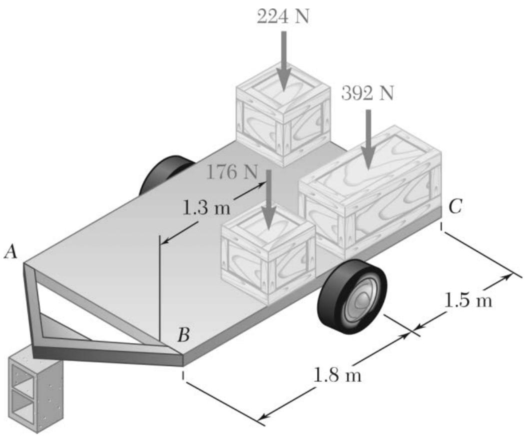
Now check if this is physically possible,

at G: ΣM_z: $(1 - x_L) \text{ m} \times (358.42 \text{ N}) - (1.29101 - 1) \text{ m} \times (792 \text{ N}) = 0$

or $x_L = 0.357 \text{ m}$ ok!

The minimum weight of the fourth box is $W_L = 358 \text{ N}$ ◀

And it is placed on end A (0.66×0.66-m side down) along side AB with the center of the box 0.357 m from side AD. ◀



PROBLEM 3.132*

Solve Problem 3.131 if the students want to place as much weight as possible in the fourth box and at least one side of the box must coincide with a side of the trailer.

PROBLEM 3.131* A group of students loads a 2×3.3-m flatbed trailer with two $0.66\times0.66\times0.66$-m boxes and one $0.66\times0.66\times1.2$-m box. Each of the boxes at the rear of the trailer is positioned so that it is aligned with both the back and a side of the trailer. Determine the smallest load the students should place in a second $0.66\times0.66\times1.2$-m box and where on the trailer they should secure it, without any part of the box overhanging the sides of the trailer, if each box is uniformly loaded and the line of action of the resultant of the weights of the four boxes is to pass through the point of intersection of the centerlines of the trailer and the axle. (*Hint:* Keep in mind that the box may be placed either on its side or on its end.)

SOLUTION

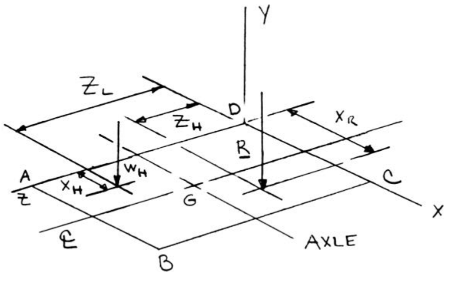
First replace the three known loads with a single equivalent force **R** applied at coordinate $(x_R,\ 0,\ z_R)$.

Equivalence requires

$$\Sigma F_y:\quad -224-392-176=-R$$

or

$$\mathbf{R}=792\text{ N}\downarrow$$



$$\Sigma M_x:\quad (0.33\text{ m})(224\text{ N})+(0.6\text{ m})(392\text{ N})$$
$$+(2\text{ m})(176\text{ N})=z_R(792\text{ N})$$

or

$$z_R=0.83475\text{ m}$$

$$\Sigma M_z:\quad -(0.33\text{ m})(224\text{ N})-(1.67\text{ m})(392\text{ N})$$
$$-(1.67\text{ m})(176\text{ N})=x_R(792\text{ N})$$

or

$$x_R=1.29101\text{ m}$$

From the statement of the problem, it is known that the resultant of **R** and the heaviest loads $\mathbf{W}_H$ passes through G, the point of intersection of the two center lines. Thus,

$$\Sigma\mathbf{M}_G=0$$

Further, since $\mathbf{W}_H$ is to be as large as possible, the fourth box should be placed as close to G as possible while keeping one of the sides of the box coincident with a side of the trailer. Thus, the two limiting cases are

$$x_H=0.6\text{ m}\quad\text{or}\quad z_H=2.7\text{ m}$$

PROBLEM 3.132* (Continued)

Now consider these two possibilities.

With $x_H = 0.6$ m

at G: ΣM_z: $(1-0.6)\text{ m}\times W_H - (1.29101-1)\text{ m}\times(792\text{ N}) = 0$

or $W_H = 576.20$ N

Checking if this is physically possible

at G: ΣM_x: $(z_H - 1.5)\text{ m}\times(576.20\text{ N}) - (1.5-0.83475)\text{ m}\times(792\text{ N}) = 0$

or $z_H = 2.414$ m

which is acceptable.

With $z_H = 2.7$ m

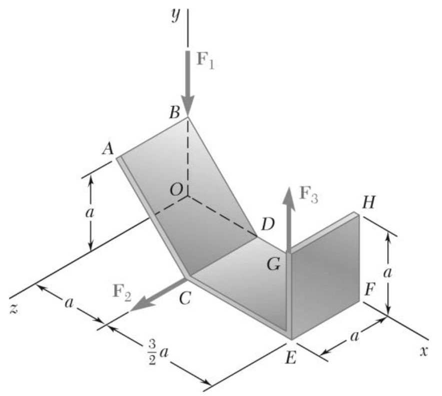
at G: ΣM_x: $(2.7-1.5)\text{ m}\times W_H - (1.5-0.83475)\text{ m}\times(792\text{ N}) = 0$

or $W_H = 439$ N

Since this is less than the first case, the maximum weight of the fourth box is

$W_H = 576$ N ◀

and it is placed with a 0.66×1.2-m side down, a 0.66-m edge along side *AD*, and the center 2.41 m from side *DC*. ◀



PROBLEM 3.133*

A piece of sheet metal is bent into the shape shown and is acted upon by three forces. If the forces have the same magnitude *P*, replace them with an equivalent wrench and determine (*a*) the magnitude and the direction of the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the axis of the wrench.

SOLUTION

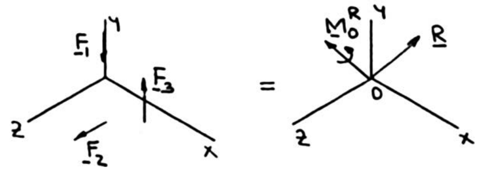
First reduce the given forces to an equivalent force-couple system $\left(\mathbf{R},\ \mathbf{M}_O^R\right)$ at the origin.

We have

$$\Sigma\mathbf{F}:\quad -P\mathbf{j}+P\mathbf{j}+P\mathbf{k}=R$$

or

$$\mathbf{R}=P\mathbf{k}$$



$$\Sigma\mathbf{M}_O:\quad -(aP)\mathbf{j}+\left[-(aP)\mathbf{i}+\left(\frac{5}{2}aP\right)\mathbf{k}\right]=M_O^R$$

or

$$\mathbf{M}_O^R=aP\left(-\mathbf{i}-\mathbf{j}+\frac{5}{2}\mathbf{k}\right)$$

(*a*) Then for the wrench,

$R=P$ ◀

and

$$\boldsymbol{\lambda}_{\text{axis}}=\frac{\mathbf{R}}{R}=\mathbf{k}$$

$$\cos\theta_x=0\quad\cos\theta_y=0\quad\cos\theta_z=1$$

or

$$\theta_x=90^\circ\quad\theta_y=90^\circ\quad\theta_z=0^\circ$$ ◀

(*b*) Now

$$\begin{aligned}M_1&=\boldsymbol{\lambda}_{\text{axis}}\cdot\mathbf{M}_O^R\\&=\mathbf{k}\cdot aP\left(-\mathbf{i}-\mathbf{j}+\frac{5}{2}\mathbf{k}\right)\\&=\frac{5}{2}aP\end{aligned}$$

Then

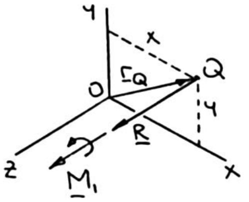
$$P=\frac{\mathbf{M}_1}{R}=\frac{\frac{5}{2}aP}{P}$$

or $P=\frac{5}{2}a$ ◀

PROBLEM 3.133* (Continued)

(*c*) The components of the wrench are $(\mathbf{R}, \mathbf{M}_1)$, where $\mathbf{M}_1 = M_1\boldsymbol{\lambda}_{\text{axis}}$, and the axis of the wrench is assumed to intersect the *xy*-plane at Point *Q*, whose coordinates are $(x, y, 0)$. Thus, we require

$$\mathbf{M}_z = \mathbf{r}_Q \times \mathbf{R}_R$$



where $$\mathbf{M}_z = \mathbf{M}_O \times \mathbf{M}_1$$

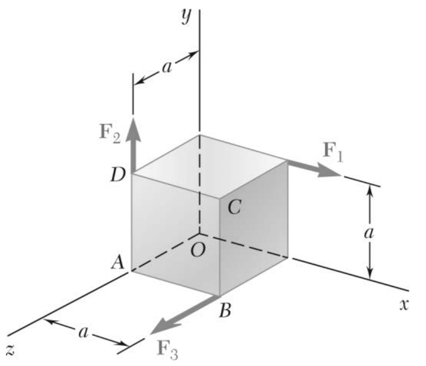
Then

$$aP\left(-\mathbf{i} - \mathbf{j} + \frac{5}{2}\mathbf{k}\right) - \frac{5}{2}aP\mathbf{k} = (x\mathbf{i} + y\mathbf{j}) + P\mathbf{k}$$

Equating coefficients:

$$\mathbf{i}: \quad -aP = yP \quad \text{or} \quad y = -a$$
$$\mathbf{j}: \quad -aP = -xP \quad \text{or} \quad x = a$$

The axis of the wrench is parallel to the *z*-axis and intersects the *xy*-plane at $x = a,\ y = -a.$ ◀

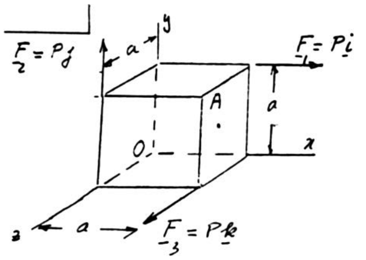


PROBLEM 3.134*

Three forces of the same magnitude P act on a cube of side a as shown. Replace the three forces by an equivalent wrench and determine (*a*) the magnitude and direction of the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the axis of the wrench.

SOLUTION

Force-couple system at O:

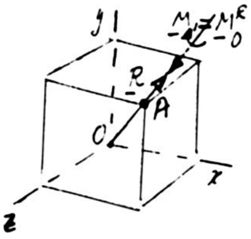


$$\mathbf{R} = P\mathbf{i} + P\mathbf{j} + P\mathbf{k} = P(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

$$\mathbf{M}_O^R = a\mathbf{j} \times P\mathbf{i} + a\mathbf{k} \times P\mathbf{j} + a\mathbf{i} \times P\mathbf{k}$$
$$= -Pa\mathbf{k} - Pa\mathbf{i} - Pa\mathbf{j}$$

$$\mathbf{M}_O^R = -Pa(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

Since **R** and $\mathbf{M}_O^R$ have the same direction, they form a wrench with $\mathbf{M}_1 = \mathbf{M}_O^R$. Thus, the axis of the wrench is the diagonal OA. We note that



$$\cos\theta_x = \cos\theta_y = \cos\theta_z = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$R = P\sqrt{3} \quad \theta_x = \theta_y = \theta_z = 54.7°$$

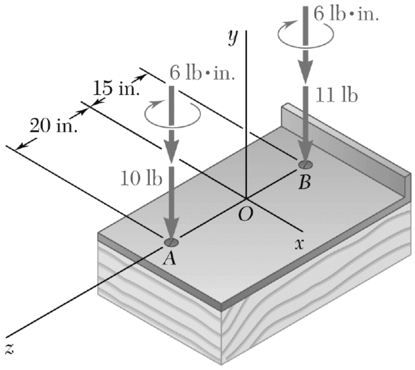
$$M_1 = M_O^R = -Pa\sqrt{3}$$

$$\text{Pitch} = p = \frac{M_1}{R} = \frac{-Pa\sqrt{3}}{P\sqrt{3}} = -a$$

(*a*) $R = P\sqrt{3} \quad \theta_x = \theta_y = \theta_z = 54.7°$ ◀

(*b*) $-a$◀

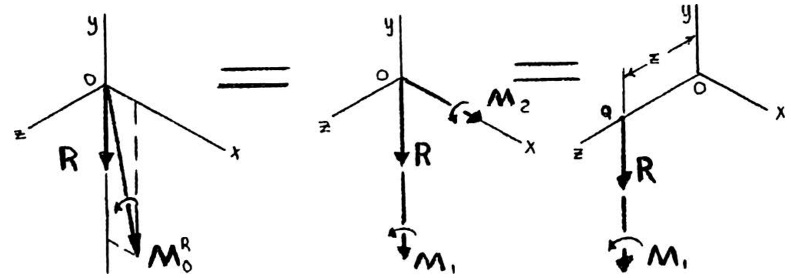
(*c*) Axis of the wrench is diagonal OA.◀



PROBLEM 3.135*

The forces and couples shown are applied to two screws as a piece of sheet metal is fastened to a block of wood. Reduce the forces and the couples to an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz*-plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

We have $\Sigma\mathbf{F}:\quad -(10\text{ lb})\mathbf{j} - (11\text{ lb})\mathbf{j} = \mathbf{R}$

$$\mathbf{R} = -(21\text{ lb})\mathbf{j}$$

We have $\Sigma\mathbf{M}_O:\quad \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma\mathbf{M}_C = \mathbf{M}_O^R$

$$\mathbf{M}_O^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 20 \\ 0 & -10 & 0 \end{vmatrix} \text{lb}\cdot\text{in.} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -15 \\ 0 & -11 & 0 \end{vmatrix} \text{lb}\cdot\text{in.} - (12\text{ lb}\cdot\text{in})\mathbf{j}$$

$$= (35\text{ lb}\cdot\text{in.})\mathbf{i} - (12\text{ lb}\cdot\text{in.})\mathbf{j}$$

(*a*) $\mathbf{R} = -(21\text{ lb})\mathbf{j}$ or $\mathbf{R} = -(21.0\text{ lb})\mathbf{j}$ ◀

(*b*) We have $M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R \quad \boldsymbol{\lambda}_R = \dfrac{\mathbf{R}}{R}$

$$= (-\mathbf{j}) \cdot [(35\text{ lb}\cdot\text{in.})\mathbf{i} - (12\text{ lb}\cdot\text{in.})\mathbf{j}]$$

$$= 12\text{ lb}\cdot\text{in.} \quad \text{and} \quad \mathbf{M}_1 = -(12\text{ lb}\cdot\text{in.})\mathbf{j}$$

and pitch $p = \dfrac{M_1}{R} = \dfrac{12\text{ lb}\cdot\text{in.}}{21\text{ lb}} = 0.57143\text{ in.}$ or $p = 0.571\text{ in.}$ ◀

PROBLEM 3.135* (Continued)

(*c*) We have $\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$

$$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = (35\text{ lb}\cdot\text{in.})\mathbf{i}$$

We require $\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$

$$(35\text{ lb}\cdot\text{in.})\mathbf{i} = (x\mathbf{i} + z\mathbf{k}) \times [-(21\text{ lb})\mathbf{j}]$$

$$35\mathbf{i} = -(21x)\mathbf{k} + (21z)\mathbf{i}$$

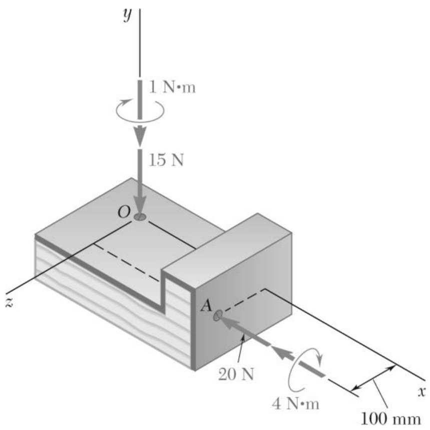
From **i**: $35 = 21z$

$$z = 1.66667\text{ in.}$$

From **k**: $0 = -21x$

$$z = 0$$

The axis of the wrench is parallel to the *y*-axis and intersects the *xz*-plane at $x = 0,\ z = 1.667$ in. ◀



PROBLEM 3.136*

The forces and couples shown are applied to two screws as a piece of sheet metal is fastened to a block of wood. Reduce the forces and the couples to an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz*-plane.

SOLUTION

First, reduce the given force system to a force-couple system.

We have $\Sigma\mathbf{F}:\ -(20\text{ N})\mathbf{i}-(15\text{ N})\mathbf{j}=\mathbf{R}\quad R=25\text{ N}$

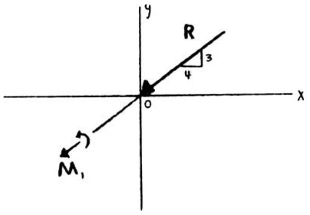
We have $\Sigma\mathbf{M}_O:\ \Sigma(\mathbf{r}_O\times\mathbf{F})+\Sigma\mathbf{M}_C=\mathbf{M}_O^R$

$$\mathbf{M}_O^R=-20\text{ N}(0.1\text{ m})\mathbf{j}-(4\text{ N}\cdot\text{m})\mathbf{i}-(1\text{ N}\cdot\text{m})\mathbf{j}$$
$$=-(4\text{ N}\cdot\text{m})\mathbf{i}-(3\text{ N}\cdot\text{m})\mathbf{j}$$

(*a*) $\mathbf{R}=-(20.0\text{ N})\mathbf{i}-(15.00\text{ N})\mathbf{j}$ ◀

(*b*) We have $M_1=\boldsymbol{\lambda}_R\cdot\mathbf{M}_O^R\quad \boldsymbol{\lambda}=\dfrac{\mathbf{R}}{R}$

$$=(-0.8\mathbf{i}-0.6\mathbf{j})\cdot[-(4\text{ N}\cdot\text{m})\mathbf{i}-(3\text{ N}\cdot\text{m})\mathbf{j}]$$
$$=5\text{ N}\cdot\text{m}$$



Pitch: $p=\dfrac{M_1}{R}=\dfrac{5\text{ N}\cdot\text{m}}{25\text{ N}}=0.200\text{ m}$

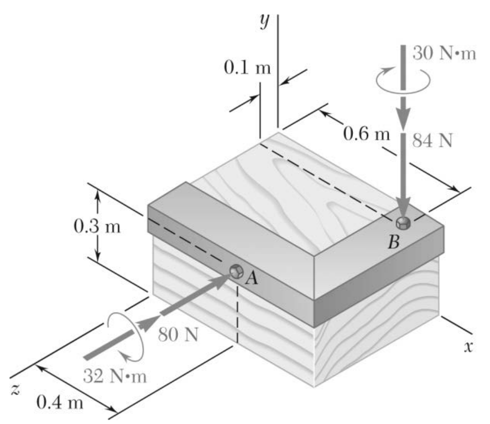
or $p=0.200\text{ m}$ ◀

(*c*) From above, note that

$$\mathbf{M}_1=\mathbf{M}_O^R$$

Therefore, the axis of the wrench goes through the origin. The line of action of the wrench lies in the *xy*-plane with a slope of

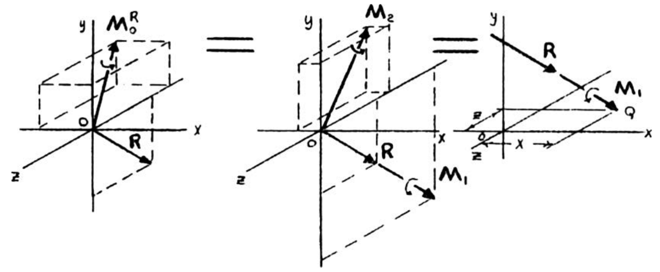
$y=\dfrac{3}{4}x$ ◀



PROBLEM 3.137*

Two bolts at *A* and *B* are tightened by applying the forces and couples shown. Replace the two wrenches with a single equivalent wrench and determine (*a*) the resultant **R**, (*b*) the pitch of the single equivalent wrench, (*c*) the point where the wrench intersects the *xz*-plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

We have $\Sigma\mathbf{F}:\quad -(84\text{ N})\mathbf{j}-(80\text{ N})\mathbf{k}=\mathbf{R}\qquad R=116\text{ N}$

and $\Sigma\mathbf{M}_O:\quad \Sigma(\mathbf{r}_O\times\mathbf{F})+\Sigma\mathbf{M}_C=\mathbf{M}_O^R$

$$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\0.6&0&0.1\\0&84&0\end{vmatrix}+\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\0.4&0.3&0\\0&0&80\end{vmatrix}+(-30\mathbf{j}-32\mathbf{k})\text{ N}\cdot\text{m}=\mathbf{M}_O^R$$

$$\mathbf{M}_O^R=-(15.6\text{ N}\cdot\text{m})\mathbf{i}+(2\text{ N}\cdot\text{m})\mathbf{j}-(82.4\text{ N}\cdot\text{m})\mathbf{k}$$

(*a*) $\mathbf{R}=-(84.0\text{ N})\mathbf{j}-(80.0\text{ N})\mathbf{k}$ ◀

(*b*) We have $M_1=\boldsymbol{\lambda}_R\cdot\mathbf{M}_O^R\qquad\boldsymbol{\lambda}_R=\dfrac{\mathbf{R}}{R}$

$$=-\frac{-84\mathbf{j}-80\mathbf{k}}{116}\cdot[-(15.6\text{ N}\cdot\text{m})\mathbf{i}+(2\text{ N}\cdot\text{m})\mathbf{j}-(82.4\text{ N}\cdot\text{m})\mathbf{k}]$$

$$=55.379\text{ N}\cdot\text{m}$$

and $\mathbf{M}_1=M_1\lambda_R=-(40.102\text{ N}\cdot\text{m})\mathbf{j}-(38.192\text{ N}\cdot\text{m})\mathbf{k}$

Then pitch $p=\dfrac{M_1}{R}=\dfrac{55.379\text{ N}\cdot\text{m}}{116\text{ N}}=0.47741\text{ m}$ or $p=0.477\text{ m}$ ◀

PROBLEM 3.137* (Continued)

(*c*) We have $$\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = [(-15.6\mathbf{i} + 2\mathbf{j} - 82.4\mathbf{k}) - (40.102\mathbf{j} - 38.192\mathbf{k})]\ \text{N}\cdot\text{m}$$
$$= -(15.6\ \text{N}\cdot\text{m})\mathbf{i} + (42.102\ \text{N}\cdot\text{m})\mathbf{j} - (44.208\ \text{N}\cdot\text{m})\mathbf{k}$$

We require $$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

$$(-15.6\mathbf{i} + 42.102\mathbf{j} - 44.208\mathbf{k}) = (x\mathbf{i} + z\mathbf{k}) \times (84\mathbf{j} - 80\mathbf{k})$$
$$= (84z)\mathbf{i} + (80x)\mathbf{j} - (84x)\mathbf{k}$$

From **i**: $$-15.6 = 84z$$
$$z = -0.185714\ \text{m}$$

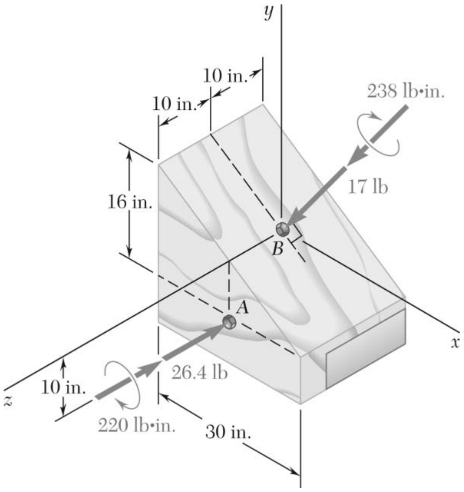
or $$z = -0.1857\ \text{m}$$

From **k**: $$-44.208 = -84x$$
$$x = 0.52629\ \text{m}$$

or $$x = 0.526\ \text{m}$$

The axis of the wrench intersects the *xz*-plane at

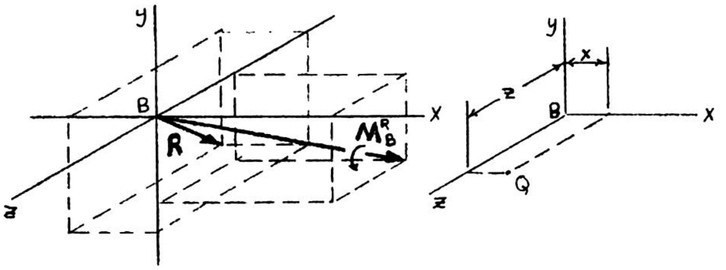
$x = 0.526$ m $y = 0$ $z = -0.1857$ m ◀



PROBLEM 3.138*

Two bolts at *A* and *B* are tightened by applying the forces and couples shown. Replace the two wrenches with a single equivalent wrench and determine (*a*) the resultant **R**, (*b*) the pitch of the single equivalent wrench, (*c*) the point where the axis of the wrench intersects the *xz*-plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin at *B*.

(*a*) We have $\Sigma\mathbf{F}:\ -(26.4\text{ lb})\mathbf{k}-(17\text{ lb})\left(\frac{8}{17}\mathbf{i}+\frac{15}{17}\mathbf{j}\right)=\mathbf{R}$

$$\mathbf{R}=-(8.00\text{ lb})\mathbf{i}-(15.00\text{ lb})\mathbf{j}-(26.4\text{ lb})\mathbf{k}\ ◀$$

and $R=31.4\text{ lb}$

We have $\Sigma\mathbf{M}_B:\ \mathbf{r}_{A/B}\times\mathbf{F}_A+\mathbf{M}_A+\mathbf{M}_B=\mathbf{M}_B^R$

$$\mathbf{M}_B^R=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\0&-10&0\\0&0&-26.4\end{vmatrix}-220\mathbf{k}-238\left(\frac{8}{17}\mathbf{i}+\frac{15}{17}\mathbf{j}\right)=264\mathbf{i}-220\mathbf{k}-14(8\mathbf{i}+15\mathbf{j})$$

$$\mathbf{M}_B^R=(152\text{ lb}\cdot\text{in.})\mathbf{i}-(210\text{ lb}\cdot\text{in.})\mathbf{j}-(220\text{ lb}\cdot\text{in.})\mathbf{k}$$

(*b*) We have $M_1=\boldsymbol{\lambda}_R\cdot\mathbf{M}_O^R\quad\boldsymbol{\lambda}_R=\frac{\mathbf{R}}{R}$

$$=\frac{-8.00\mathbf{i}-15.00\mathbf{j}-26.4\mathbf{k}}{31.4}\cdot[(152\text{ lb}\cdot\text{in.})\mathbf{i}-(210\text{ lb}\cdot\text{in.})\mathbf{j}-(220\text{ lb}\cdot\text{in.})\mathbf{k}]$$

$$=246.56\text{ lb}\cdot\text{in.}$$

and $$\mathbf{M}_1 = M_1\lambda_R = -(62.818\text{ lb}\cdot\text{in.})\mathbf{i} - (117.783\text{ lb}\cdot\text{in.})\mathbf{j} - (207.30\text{ lb}\cdot\text{in.})\mathbf{k}$$

Then pitch $$p = \frac{M_1}{R} = \frac{246.56\text{ lb}\cdot\text{in.}}{31.4\text{ lb}} = 7.8522\text{ in.}$$ or $p = 7.85$ in. ◀

(*c*) We have $$\mathbf{M}_B^R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\mathbf{M}_2 = \mathbf{M}_B^R - \mathbf{M}_1 = (152\mathbf{i} - 210\mathbf{j} - 220\mathbf{k}) - (-62.818\mathbf{i} - 117.783\mathbf{j} - 207.30\mathbf{k})$$

$$= (214.82\text{ lb}\cdot\text{in.})\mathbf{i} - (92.217\text{ lb}\cdot\text{in.})\mathbf{j} - (12.7000\text{ lb}\cdot\text{in.})\mathbf{k}$$

We require $$\mathbf{M}_2 = \mathbf{r}_{Q/B} \times \mathbf{R}$$

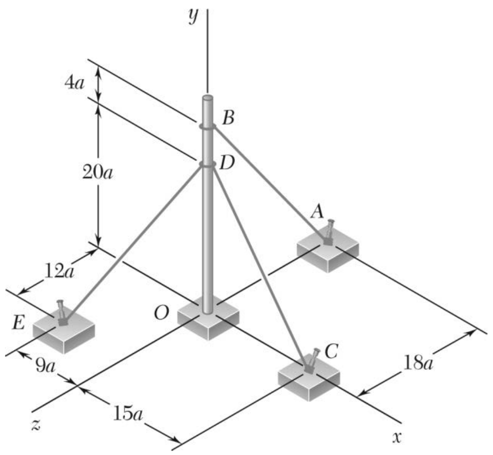
$$214.82\mathbf{i} - 92.217\mathbf{j} - 12.7000\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ -8 & -15 & -26.4 \end{vmatrix}$$

$$= (15z)\mathbf{i} - (8z)\mathbf{j} + (26.4x)\mathbf{j} - (15x)\mathbf{k}$$

From **i**: $214.82 = 15z$ $\quad z = 14.3213$ in.

From **k**: $-12.7000 = -15x$ $\quad x = 0.84667$ in.

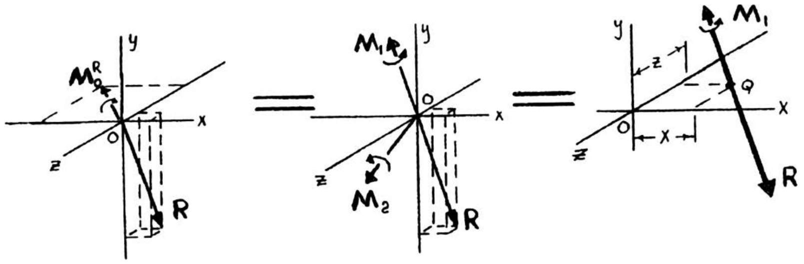
The axis of the wrench intersects the *xz*-plane at $x = 0.847$ in. $\quad y = 0 \quad z = 14.32$ in. ◀



PROBLEM 3.139*

A flagpole is guyed by three cables. If the tensions in the cables have the same magnitude P, replace the forces exerted on the pole with an equivalent wrench and determine (*a*) the resultant force $\mathbf{R}$, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz*-plane.

SOLUTION



(*a*) First reduce the given force system to a force-couple at the origin.

We have $\Sigma\mathbf{F}$: $P\boldsymbol{\lambda}_{BA} + P\boldsymbol{\lambda}_{DC} + P\boldsymbol{\lambda}_{DE} = \mathbf{R}$

$$\mathbf{R} = P\left[\left(\frac{4}{5}\mathbf{j} - \frac{3}{5}\mathbf{k}\right) + \left(\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}\right) + \left(\frac{-9}{25}\mathbf{i} - \frac{4}{5}\mathbf{j} + \frac{12}{25}\mathbf{k}\right)\right]$$

$$\mathbf{R} = \frac{3P}{25}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \blacktriangleleft$$

$$R = \frac{3P}{25}\sqrt{(2)^2 + (20)^2 + (1)^2} = \frac{27\sqrt{5}}{25}P$$

We have $\Sigma\mathbf{M}$: $\Sigma(\mathbf{r}_O \times P) = \mathbf{M}_O^R$

$$(24a)\mathbf{j} \times \left(\frac{-4P}{5}\mathbf{j} - \frac{3P}{5}\mathbf{k}\right) + (20a)\mathbf{j} \times \left(\frac{3P}{5}\mathbf{i} - \frac{4P}{5}\mathbf{j}\right) + (20a)\mathbf{j} \times \left(\frac{-9P}{25}\mathbf{i} - \frac{4P}{5}\mathbf{j} + \frac{12P}{25}\mathbf{k}\right) = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = \frac{24Pa}{5}(-\mathbf{i} - \mathbf{k})$$

PROBLEM 3.139* (Continued)

(*b*) We have $M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R$

where $\boldsymbol{\lambda}_R = \dfrac{\mathbf{R}}{R} = \dfrac{3P}{25}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})\dfrac{25}{27\sqrt{5}P} = \dfrac{1}{9\sqrt{5}}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})$

Then $M_1 = \dfrac{1}{9\sqrt{5}}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \cdot \dfrac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) = \dfrac{-8Pa}{15\sqrt{5}}$

and pitch $p = \dfrac{M_1}{R} = \dfrac{-8Pa}{15\sqrt{5}}\left(\dfrac{25}{27\sqrt{5}P}\right) = \dfrac{-8a}{81}$ or $p = -0.0988a$ ◀

(*c*) $\mathbf{M}_1 = M_1\boldsymbol{\lambda}_R = \dfrac{-8Pa}{15\sqrt{5}}\left(\dfrac{1}{9\sqrt{5}}\right)(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) = \dfrac{8Pa}{675}(-2\mathbf{i} + 20\mathbf{j} + \mathbf{k})$

Then $\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = \dfrac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) - \dfrac{8Pa}{675}(-2\mathbf{i} + 20\mathbf{j} + \mathbf{k}) = \dfrac{8Pa}{675}(-430\mathbf{i} - 20\mathbf{j} - 406\mathbf{k})$

We require $\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$

$$\left(\frac{8Pa}{675}\right)(-403\mathbf{i} - 20\mathbf{j} - 406\mathbf{k}) = (x\mathbf{i} + z\mathbf{k}) \times \left(\frac{3P}{25}\right)(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})$$

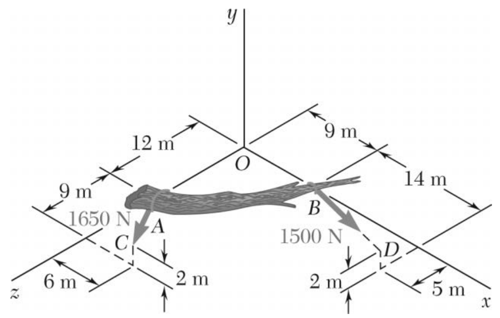
$$= \left(\frac{3P}{25}\right)[20z\mathbf{i} + (x + 2z)\mathbf{j} - 20x\mathbf{k}]$$

From **i**: $8(-403)\dfrac{Pa}{675} = 20z\left(\dfrac{3P}{25}\right) \qquad z = -1.99012a$

From **k**: $8(-406)\dfrac{Pa}{675} = -20x\left(\dfrac{3P}{25}\right) \qquad x = 2.0049a$

The axis of the wrench intersects the *xz*-plane at

$x = 2.00a,\ z = -1.990a$ ◀



PROBLEM 3.140*

Two ropes attached at *A* and *B* are used to move the trunk of a fallen tree. Replace the forces exerted by the ropes with an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *yz*-plane.

SOLUTION

(*a*) First replace the given forces with an equivalent force-couple system $\left(\mathbf{R},\ \mathbf{M}_O^R\right)$ at the origin.

We have

$$d_{AC} = \sqrt{(6)^2 + (2)^2 + (9)^2} = 11 \text{ m}$$
$$d_{BD} = \sqrt{(14)^2 + (2)^2 + (5)^2} = 15 \text{ m}$$

Then

$$\begin{aligned} T_{AC} &= \frac{1650 \text{ N}}{11} = (6\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}) \\ &= (900 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} + (1350 \text{ N})\mathbf{k} \end{aligned}$$

and

$$\begin{aligned} T_{BD} &= \frac{1500 \text{ N}}{15} = (14\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) \\ &= (1400 \text{ N})\mathbf{i} + (200 \text{ N})\mathbf{j} + (500 \text{ N})\mathbf{k} \end{aligned}$$

Equivalence then requires

$$\begin{aligned} \Sigma\mathbf{F}: \quad \mathbf{R} &= \mathbf{T}_{AC} + \mathbf{T}_{BD} \\ &= (900\mathbf{i} + 300\mathbf{j} + 1350\mathbf{k}) \\ &\quad + (1400\mathbf{i} + 200\mathbf{j} + 500\mathbf{k}) \\ &= (2300 \text{ N})\mathbf{i} + (500 \text{ N})\mathbf{j} + (1850 \text{ N})\mathbf{k} \end{aligned}$$

$$\begin{aligned} \Sigma\mathbf{M}_O: \quad \mathbf{M}_O^R &= \mathbf{r}_A \times \mathbf{T}_{AC} + \mathbf{r}_B \times \mathbf{T}_{BD} \\ &= (12 \text{ m})\mathbf{k} \times [(900 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} + (1350 \text{ N})\mathbf{k}] \\ &\quad + (9 \text{ m})\mathbf{i} \times [(1400 \text{ N})\mathbf{i} + (200 \text{ N})\mathbf{j} + (500 \text{ N})\mathbf{k}] \\ &= -(3600)\mathbf{i} + (10{,}800 - 4500)\mathbf{j} + (1800)\mathbf{k} \\ &= -(3600 \text{ N}\cdot\text{m})\mathbf{i} + (6300 \text{ N}\cdot\text{m})\mathbf{j} + (1800 \text{ N}\cdot\text{m})\mathbf{k} \end{aligned}$$

The components of the wrench are $(\mathbf{R},\ \mathbf{M}_1)$, where

$$\mathbf{R} = (2300 \text{ N})\mathbf{i} + (500 \text{ N})\mathbf{j} + (1850 \text{ N})\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 3.140* (Continued)

(*b*) We have

$$R = 100\sqrt{(23)^2 + (5)^2 + (18.5)^2} = 2993.7 \text{ N}$$

Let

$$\boldsymbol{\lambda}_{\text{axis}} = \frac{\mathbf{R}}{R} = \frac{1}{29.937}(23\mathbf{i} + 5\mathbf{j} + 18.5\mathbf{k})$$

Then

$$\begin{aligned} M_1 &= \boldsymbol{\lambda}_{\text{axis}} \cdot \mathbf{M}_O^R \\ &= \frac{1}{29.937}(23\mathbf{i} + 5\mathbf{j} + 18.5\mathbf{k}) \cdot (-3600\mathbf{i} + 6300\mathbf{j} + 1800\mathbf{k}) \\ &= \frac{1}{0.29937}[(23)(-36) + (5)(63) + (18.5)(18)] \\ &= -601.26 \text{ N} \cdot \text{m} \end{aligned}$$

Finally,

$$P = \frac{M_1}{R} = \frac{-601.26 \text{ N} \cdot \text{m}}{2993.7 \text{ N}}$$

or $P = -0.201$ m ◀

(*c*) We have

$$\begin{aligned} \mathbf{M}_1 &= M_1 \boldsymbol{\lambda}_{\text{axis}} \\ &= (-601.26 \text{ N} \cdot \text{m}) \times \frac{1}{29.937}(23\mathbf{i} + 5\mathbf{j} + 18.5\mathbf{k}) \end{aligned}$$

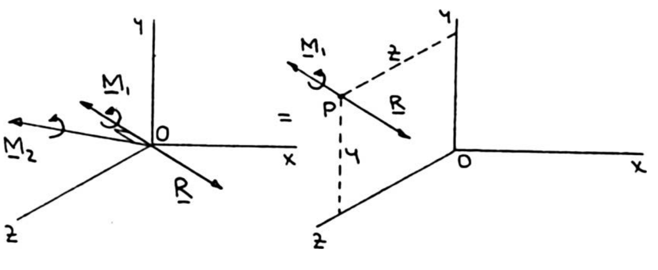
or

$$\mathbf{M}_1 = -(461.93 \text{ N} \cdot \text{m})\mathbf{i} - (100.421 \text{ N} \cdot \text{m})\mathbf{j} - (371.56 \text{ N} \cdot \text{m})\mathbf{k}$$

Now

$$\begin{aligned} \mathbf{M}_2 &= \mathbf{M}_O^R - \mathbf{M}_1 \\ &= (-3600\mathbf{i} + 6300\mathbf{j} + 1800\mathbf{k}) \\ &\quad - (-461.93\mathbf{i} - 100.421\mathbf{j} - 371.56\mathbf{k}) \\ &= -(3138.1 \text{ N} \cdot \text{m})\mathbf{i} + (6400.4 \text{ N} \cdot \text{m})\mathbf{j} + (2171.6 \text{ N} \cdot \text{m})\mathbf{k} \end{aligned}$$

For equivalence:



PROBLEM 3.140* (Continued)

Thus, we require $\mathbf{M}_2 = \mathbf{r}_P \times \mathbf{R}$ $\quad \mathbf{r} = (y\mathbf{j} + z\mathbf{k})$

Substituting:

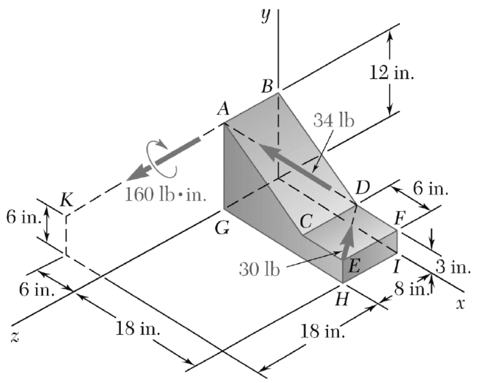
$$-3138.1\mathbf{i} + 6400.4\mathbf{j} + 2171.6\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y & z \\ 2300 & 500 & 1850 \end{vmatrix}$$

Equating coefficients:

$$\mathbf{j}: \quad 6400.4 = 2300z \qquad \text{or} \quad z = 2.78 \text{ m}$$

$$\mathbf{k}: \quad 2171.6 = -2300y \qquad \text{or} \quad y = -0.944 \text{ m}$$

The axis of the wrench intersects the yz-plane at $y = -0.944$ m $\quad z = 2.78$ m ◀



PROBLEM 3.141*

Determine whether the force-and-couple system shown can be reduced to a single equivalent force **R**. If it can, determine **R** and the point where the line of action of **R** intersects the *yz*-plane. If it cannot be so reduced, replace the given system with an equivalent wrench and determine its resultant, its pitch, and the point where its axis intersects the *yz*-plane.

SOLUTION

First determine the resultant of the forces at *D*. We have

$$d_{DA} = \sqrt{(-12)^2 + (9)^2 + (8)^2} = 17 \text{ in.}$$

$$d_{ED} = \sqrt{(-6)^2 + (0)^2 + (-8)^2} = 10 \text{ in.}$$

Then

$$\mathbf{F}_{DA} = \frac{34 \text{ lb}}{17} = (-12\mathbf{i} + 9\mathbf{j} + 8\mathbf{k})$$
$$= -(24 \text{ lb})\mathbf{i} + (18 \text{ lb})\mathbf{j} + (16 \text{ lb})\mathbf{k}$$

and

$$\mathbf{F}_{ED} = \frac{30 \text{ lb}}{10} = (-6\mathbf{i} - 8\mathbf{k})$$
$$= -(18 \text{ lb})\mathbf{i} - (24 \text{ lb})\mathbf{k}$$

Then

$$\Sigma\mathbf{F}: \quad \mathbf{R} = \mathbf{F}_{DA} + \mathbf{F}_{ED}$$
$$= (-24\mathbf{i} + 18\mathbf{j} + 16\mathbf{k} + (-18\mathbf{i} - 24\mathbf{k})$$
$$= -(42 \text{ lb})\mathbf{i} + (18 \text{ lb})\mathbf{j} - (8 \text{ lb})\mathbf{k}$$

For the applied couple

$$d_{AK} = \sqrt{(-6)^2 + (-6)^2 + (18)^2} = 6\sqrt{11} \text{ in.}$$

Then

$$\mathbf{M} = \frac{160 \text{ lb} \cdot \text{in.}}{6\sqrt{11}}(-6\mathbf{i} - 6\mathbf{j} + 18\mathbf{k})$$
$$= \frac{160}{\sqrt{11}}[-(1 \text{ lb} \cdot \text{in.})\mathbf{i} - (1 \text{ lb} \cdot \text{in.})\mathbf{j} + (3 \text{ lb} \cdot \text{in.})\mathbf{k}]$$

To be able to reduce the original forces and couple to a single equivalent force, **R** and **M** must be perpendicular. Thus

$$\mathbf{R} \cdot \mathbf{M} \overset{?}{=} 0$$

PROBLEM 3.141* (Continued)

Substituting

$$(-42\mathbf{i}+18\mathbf{j}-8\mathbf{k})\cdot\frac{160}{\sqrt{11}}(-\mathbf{i}-\mathbf{j}+3\mathbf{k})\overset{?}{=}0$$

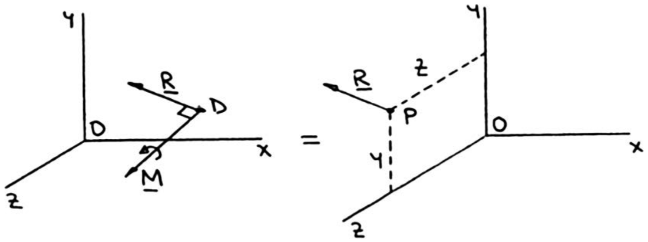
or $$\frac{160}{\sqrt{11}}[(-42)(-1)+(18)(-1)+(-8)(3)]\overset{?}{=}0$$

or $$0\overset{\checkmark}{=}0$$

R and **M** are perpendicular so that the given system can be reduced to the single equivalent force.

$$\mathbf{R}=-(42.0\text{ lb})\mathbf{i}+(18.00\text{ lb})\mathbf{j}-(8.00\text{ lb})\mathbf{k} \quad ◀$$

Then for equivalence,



Thus, we require $\mathbf{M}=\mathbf{r}_{P/D}\times\mathbf{R}$

where $\mathbf{r}_{P/D}=-(12\text{ in.})\mathbf{i}+[(y-3)\text{in.}]\mathbf{j}+(z\text{ in.})\mathbf{k}$

Substituting:

$$\frac{160}{\sqrt{11}}(-\mathbf{i}-\mathbf{j}+3\mathbf{k})=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -12 & (y-3) & z\\ -42 & 18 & -8\end{vmatrix}$$

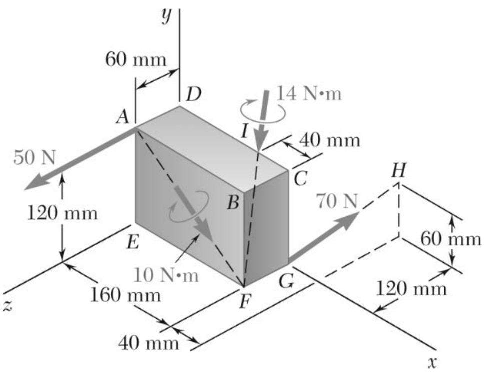
$$\begin{aligned}&=[(y-3)(-8)-(z)(18)]\mathbf{i}\\ &\quad+[(z)(-42)-(-12)(-8)]\mathbf{j}\\ &\quad+[(-12)(18)-(y-3)(-42)]\mathbf{k}\end{aligned}$$

Equating coefficients:

$$\mathbf{j}:\quad -\frac{160}{\sqrt{11}}=-42z-96 \qquad \text{or} \quad z=-1.137\text{ in.}$$

$$\mathbf{k}:\quad \frac{480}{\sqrt{11}}=-216+42(y-3) \quad \text{or} \quad y=11.59\text{ in.}$$

The line of action of **R** intersects the yz-plane at $x=0 \quad y=11.59\text{ in.} \quad z=-1.137\text{ in.}$ ◀



PROBLEM 3.142*

Determine whether the force-and-couple system shown can be reduced to a single equivalent force **R**. If it can, determine **R** and the point where the line of action of **R** intersects the *yz*-plane. If it cannot be so reduced, replace the given system with an equivalent wrench and determine its resultant, its pitch, and the point where its axis intersects the *yz*-plane.

SOLUTION

First, reduce the given force system to a force-couple at the origin.

We have $\Sigma\mathbf{F}\text{:}\quad \mathbf{F}_A + \mathbf{F}_G = \mathbf{R}$

$$\mathbf{R} = (50\text{ N})\mathbf{k} + 70\text{ N}\left[\frac{(40\text{ mm})\mathbf{i} + (60\text{ mm})\mathbf{j} - (120\text{ mm})\mathbf{k}}{140\text{ mm}}\right]$$

$$= (20\text{ N})\mathbf{i} + (30\text{ N})\mathbf{j} - (10\text{ N})\mathbf{k}$$

and $R = 37.417\text{ N}$

We have $\Sigma\mathbf{M}_O\text{:}\quad \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma\mathbf{M}_C = \mathbf{M}_O^R$

$$\mathbf{M}_O^R = [(0.12\text{ m})\mathbf{j} \times (50\text{ N})\mathbf{k}] + \{(0.16\text{ m})\mathbf{i} \times [(20\text{ N})\mathbf{i} + (30\text{ N})\mathbf{j} - (60\text{ N})\mathbf{k}]\}$$

$$+ (10\text{ N}\cdot\text{m})\left[\frac{(160\text{ mm})\mathbf{i} - (120\text{ mm})\mathbf{j}}{200\text{ mm}}\right]$$

$$+ (14\text{ N}\cdot\text{m})\left[\frac{(40\text{ mm})\mathbf{i} - (120\text{ mm})\mathbf{j} + (60\text{ mm})\mathbf{k}}{140\text{ mm}}\right]$$

$$\mathbf{M}_0^R = (18\text{ N}\cdot\text{m})\mathbf{i} - (8.4\text{ N}\cdot\text{m})\mathbf{j} + (10.8\text{ N}\cdot\text{m})\mathbf{k}$$

To be able to reduce the original forces and couples to a single equivalent force, **R** and **M** must be perpendicular. Thus, $\mathbf{R}\cdot\mathbf{M} = 0$.

Substituting

$$(20\mathbf{i} + 30\mathbf{j} - 10\mathbf{k})\cdot(18\mathbf{i} - 8.4\mathbf{j} + 10.8\mathbf{k}) \overset{?}{=} 0$$

or

$$(20)(18) + (30)(-8.4) + (-10)(10.8) \overset{?}{=} 0$$

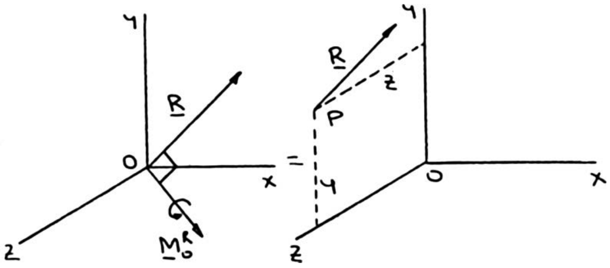
or

$$0 \overset{\checkmark}{=} 0$$

R and **M** are perpendicular so that the given system can be reduced to the single equivalent force.

$\mathbf{R} = (20.0\text{ N})\mathbf{i} + (30.0\text{ N})\mathbf{j} - (10.00\text{ N})\mathbf{k}$ ◀

Then for equivalence,



Thus, we require $\mathbf{M}_O^R = \mathbf{r}_p \times \mathbf{R} \quad \mathbf{r}_p = y\mathbf{j} + z\mathbf{k}$

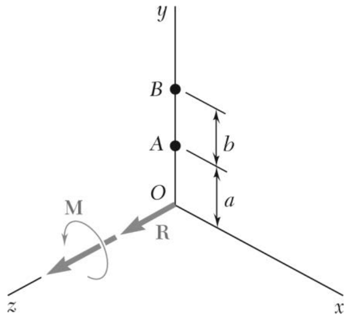
Substituting:

$$18\mathbf{i} - 8.4\mathbf{j} + 10.8\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y & z \\ 20 & 30 & -10 \end{vmatrix}$$

Equating coefficients:

$$\mathbf{j}: \quad -8.4 = 20z \qquad \text{or} \qquad z = -0.42 \text{ m}$$
$$\mathbf{k}: \quad 10.8 = -20y \qquad \text{or} \qquad y = -0.54 \text{ m}$$

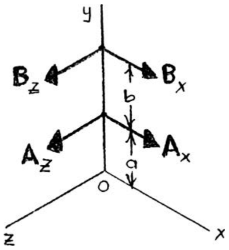
The line of action of **R** intersects the *yz*-plane at $x = 0 \quad y = -0.540 \text{ m} \quad z = -0.420 \text{ m}$ ◀



PROBLEM 3.143*

Replace the wrench shown with an equivalent system consisting of two forces perpendicular to the *y*-axis and applied respectively at *A* and *B*.

SOLUTION



Express the forces at *A* and *B* as

$$\mathbf{A} = A_x\mathbf{i} + A_z\mathbf{k}$$
$$\mathbf{B} = B_x\mathbf{i} + B_z\mathbf{k}$$

Then, for equivalence to the given force system,

$$\Sigma F_x: \quad A_x + B_x = 0 \qquad (1)$$
$$\Sigma F_z: \quad A_z + B_z = R \qquad (2)$$
$$\Sigma M_x: \quad A_z(a) + B_z(a+b) = 0 \qquad (3)$$
$$\Sigma M_z: \quad -A_x(a) - B_x(a+b) = M \qquad (4)$$

From Equation (1), $B_x = -A_x$

Substitute into Equation (4):

$$-A_x(a) + A_x(a+b) = M$$
$$A_x = \frac{M}{b} \quad \text{and} \quad B_x = -\frac{M}{b}$$

From Equation (2), $B_z = R - A_z$

and Equation (3), $A_z a + (R - A_z)(a+b) = 0$

$$A_z = R\left(1 + \frac{a}{b}\right)$$

and $B_z = R - R\left(1 + \frac{a}{b}\right)$

$$B_z = -\frac{a}{b}R$$

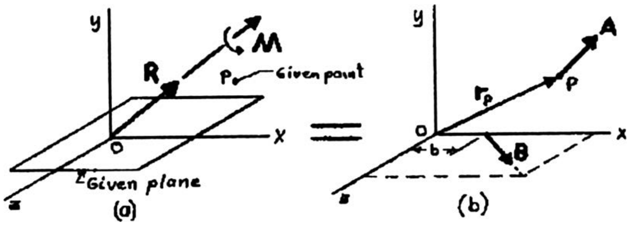
Then $$\mathbf{A} = \left(\frac{M}{b}\right)\mathbf{i} + R\left(1 + \frac{a}{b}\right)\mathbf{k} \blacktriangleleft$$

$$\mathbf{B} = -\left(\frac{M}{b}\right)\mathbf{i} - \left(\frac{a}{b}R\right)\mathbf{k} \blacktriangleleft$$

PROBLEM 3.144*

Show that, in general, a wrench can be replaced with two forces chosen in such a way that one force passes through a given point while the other force lies in a given plane.

SOLUTION



First, choose a coordinate system so that the *xy*-plane coincides with the given plane. Also, position the coordinate system so that the line of action of the wrench passes through the origin as shown in Figure *a*. Since the orientation of the plane and the components (**R**, **M**) of the wrench are known, it follows that the scalar components of **R** and **M** are known relative to the shown coordinate system.

A force system to be shown as equivalent is illustrated in Figure *b*. Let **A** be the force passing through the given Point *P* and **B** be the force that lies in the given plane. Let *b* be the *x*-axis intercept of **B**.

The known components of the wrench can be expressed as

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} \quad \text{and} \quad M = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

while the unknown forces **A** and **B** can be expressed as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_z\mathbf{k}$$

Since the position vector of Point *P* is given, it follows that the scalar components (*x*, *y*, *z*) of the position vector $\mathbf{r}_P$ are also known.

Then, for equivalence of the two systems,

$$\Sigma F_x: \quad R_x = A_x + B_x \tag{1}$$

$$\Sigma F_y: \quad R_y = A_y \tag{2}$$

$$\Sigma F_z: \quad R_z = A_z + B_z \tag{3}$$

$$\Sigma M_x: \quad M_x = yA_z - zA_y \tag{4}$$

$$\Sigma M_y: \quad M_y = zA_x - xA_z - bB_z \tag{5}$$

$$\Sigma M_z: \quad M_z = xA_y - yA_x \tag{6}$$

Based on the above six independent equations for the six unknowns $(A_x, A_y, A_z, B_x, B_z, b)$, there exists a unique solution for **A** and **B**.

From Equation (2), $A_y = R_y$ ◀

PROBLEM 3.144* (Continued)

Equation (6): $$A_x = \left(\frac{1}{y}\right)(xR_y - M_z) \blacktriangleleft$$

Equation (1): $$B_x = R_x - \left(\frac{1}{y}\right)(xR_y - M_z) \blacktriangleleft$$

Equation (4): $$A_z = \left(\frac{1}{y}\right)(M_x + zR_y) \blacktriangleleft$$

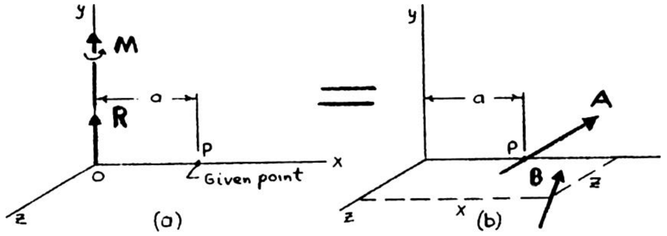
Equation (3): $$B_z = R_z - \left(\frac{1}{y}\right)(M_x + zR_y) \blacktriangleleft$$

Equation (5): $$b = \frac{(xM_x + yM_y + zM_z)}{(M_x - yR_z + zR_y)} \blacktriangleleft$$

PROBLEM 3.145*

Show that a wrench can be replaced with two perpendicular forces, one of which is applied at a given point.

SOLUTION



First, observe that it is always possible to construct a line perpendicular to a given line so that the constructed line also passes through a given point. Thus, it is possible to align one of the coordinate axes of a rectangular coordinate system with the axis of the wrench while one of the other axes passes through the given point.

See Figures *a* and *b*.

We have $\mathbf{R} = R\mathbf{j}$ and $\mathbf{M} = M\mathbf{j}$ and are known.

The unknown forces **A** and **B** can be expressed as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

The distance *a* is known. It is assumed that force **B** intersects the *xz*-plane at (*x*, 0, *z*). Then for equivalence,

$$\Sigma F_x: \quad 0 = A_x + B_x \tag{1}$$

$$\Sigma F_y: \quad R = A_y + B_y \tag{2}$$

$$\Sigma F_z: \quad 0 = A_z + B_z \tag{3}$$

$$\Sigma M_x: \quad 0 = -zB_y \tag{4}$$

$$\Sigma M_y: \quad M = -aA_z - xB_z + zB_x \tag{5}$$

$$\Sigma M_z: \quad 0 = aA_y + xB_y \tag{6}$$

Since **A** and **B** are made perpendicular,

$$\mathbf{A} \cdot \mathbf{B} = 0 \quad \text{or} \quad A_xB_x + A_yB_y + A_zB_z = 0 \tag{7}$$

There are eight unknowns: $A_x, A_y, A_z, B_x, B_y, B_z, x, z$

But only seven independent equations. Therefore, *there exists an infinite number of solutions.*

Next, consider Equation (4): $0 = -zB_y$

If $B_y = 0$, Equation (7) becomes $A_xB_x + A_zB_z = 0$

Using Equations (1) and (3), this equation becomes $A_x^2 + A_z^2 = 0$

Since the components of **A** must be real, a nontrivial solution is not possible. Thus, it is required that $B_y \neq 0$, so that from Equation (4), $z = 0$.

To obtain one possible solution, arbitrarily let $A_x = 0$.

(*Note:* Setting A_y, A_z, or B_z equal to zero results in unacceptable solutions.)

The defining equations then become

$$0 = B_x \tag{1$'$}$$

$$R = A_y + B_y \tag{2}$$

$$0 = A_z + B_z \tag{3}$$

$$M = -aA_z - xB_z \tag{5$'$}$$

$$0 = aA_y + xB_y \tag{6}$$

$$A_y B_y + A_z B_z = 0 \tag{7$'$}$$

Then Equation (2) can be written $A_y = R - B_y$

Equation (3) can be written $B_z = -A_z$

Equation (6) can be written $x = -\dfrac{aA_y}{B_y}$

Substituting into Equation (5)′,

$$M = -aA_z - \left(-a\frac{R - B_y}{B_y}\right)(-A_z)$$

or

$$A_z = -\frac{M}{aR}B_y \tag{8}$$

Substituting into Equation (7)′,

$$(R - B_y)B_y + \left(-\frac{M}{aR}B_y\right)\left(\frac{M}{aR}B_y\right) = 0$$

or

$$B_y = \frac{a^2R^3}{a^2R^2 + M^2}$$

Then from Equations (2), (8), and (3),

$$A_y = R - \frac{a^2R^2}{a^2R^2 + M^2} = \frac{RM^2}{a^2R^2 + M^2}$$

$$A_z = -\frac{M}{aR}\left(\frac{a^2R^3}{a^2R^2 + M^2}\right) = -\frac{aR^2M}{a^2R^2 + M^2}$$

$$B_z = \frac{aR^2M}{a^2R^2 + M^2}$$

PROBLEM 3.145* (Continued)

In summary,

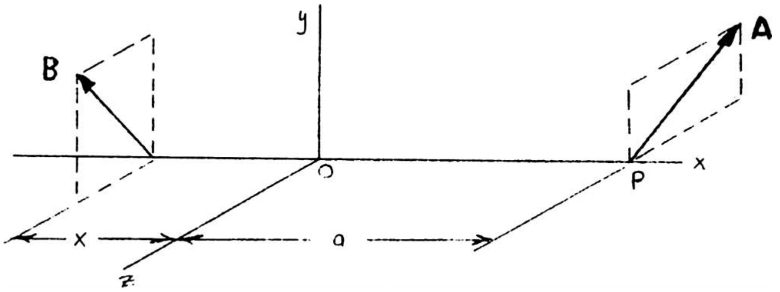
$$\mathbf{A} = \frac{RM}{a^2R^2 + M^2}(M\mathbf{j} - aR\mathbf{k}) \quad ◀$$

$$\mathbf{B} = \frac{aR^2}{a^2R^2 + M^2}(aR\mathbf{j} + M\mathbf{k}) \quad ◀$$

Which shows that it is possible to replace a wrench with two perpendicular forces, one of which is applied at a given point.

Lastly, if $R > 0$ and $M > 0$, it follows from the equations found for **A** and **B** that $A_y > 0$ and $B_y > 0$.

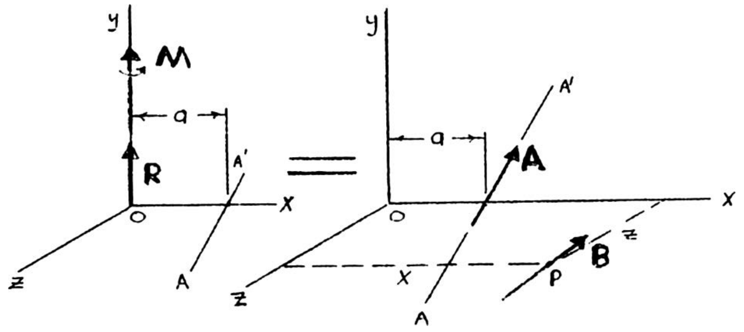
From Equation (6), $x < 0$ (assuming $a > 0$). Then, as a consequence of letting $A_x = 0$, force **A** lies in a plane parallel to the *yz*-plane and to the right of the origin, while force **B** lies in a plane parallel to the *yz*-plane but to the left to the origin, as shown in the figure below.



PROBLEM 3.146*

Show that a wrench can be replaced with two forces, one of which has a prescribed line of action.

SOLUTION



First, choose a rectangular coordinate system where one axis coincides with the axis of the wrench and another axis intersects the prescribed line of action (AA'). Note that it has been assumed that the line of action of force **B** intersects the xz-plane at Point $P(x, 0, z)$. Denoting the known direction of line AA' by

$$\lambda_A = \lambda_x \mathbf{i} + \lambda_y \mathbf{j} + \lambda_z \mathbf{k}$$

it follows that force **A** can be expressed as

$$\mathbf{A} = A\lambda_A = A(\lambda_x \mathbf{i} + \lambda_y \mathbf{j} + \lambda_z \mathbf{k})$$

Force **B** can be expressed as

$$\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$$

Next, observe that since the axis of the wrench and the prescribed line of action AA' are known, it follows that the distance a can be determined. In the following solution, it is assumed that a is known.

Then for equivalence,

$$\Sigma F_x: \quad 0 = A\lambda_x + B_x \tag{1}$$

$$\Sigma F_y: \quad R = A\lambda_y + B_y \tag{2}$$

$$\Sigma F_z: \quad 0 = A\lambda_z + B_z \tag{3}$$

$$\Sigma M_x: \quad 0 = -zB_y \tag{4}$$

$$\Sigma M_y: \quad M = -aA\lambda_z + zB_x - xB_z \tag{5}$$

$$\Sigma M_x: \quad 0 = -aA\lambda_y + xB_y \tag{6}$$

Since there are six unknowns (A, B_x, B_y, B_z, x, z) and six independent equations, it will be possible to obtain a solution.

PROBLEM 3.146* (Continued)

Case 1: Let $z = 0$ to satisfy Equation (4).

Now Equation (2): $A\lambda_y = R - B_y$

Equation (3): $B_z = -A\lambda_z$

Equation (6): $x = -\dfrac{aA\lambda_y}{B_y} = -\left(\dfrac{a}{B_y}\right)(R - B_y)$

Substitution into Equation (5):

$$M = -aA\lambda_z - \left[-\left(\frac{a}{B_y}\right)(R - B_y)(-A\lambda_z)\right]$$

$$A = -\frac{1}{\lambda_z}\left(\frac{M}{aR}\right)B_y$$

Substitution into Equation (2):

$$R = -\frac{1}{\lambda_z}\left(\frac{M}{aR}\right)B_y\lambda_y + B_y$$

$$B_y = \frac{\lambda_z aR^2}{\lambda_z aR - \lambda_y M}$$

Then $A = -\dfrac{MR}{\lambda_z aR - \lambda_y M} = \dfrac{R}{\lambda_y - \dfrac{aR}{M}\lambda_z}$

$$B_x = -A\lambda_x = \frac{\lambda_x MR}{\lambda_z aR - \lambda_y M}$$

$$B_z = -A\lambda_z = \frac{\lambda_z MR}{\lambda_z aR - \lambda_y M}$$

In summary, $\mathbf{A} = \dfrac{P}{\lambda_y - \dfrac{aR}{M}\lambda_z}\lambda_A$ ◀

$$\mathbf{B} = \frac{R}{\lambda_z aR - \lambda_y M}(\lambda_x M\,\mathbf{i} + \lambda_z aR\mathbf{j} + \lambda_z M\mathbf{k})$$ ◀

and $x = a\left(1 - \dfrac{R}{B_y}\right)$

$$= a\left[1 - R\left(\frac{\lambda_z aR - \lambda_y M}{\lambda_z aR^2}\right)\right]$$

or $x = \dfrac{\lambda_y M}{\lambda_z R}$ ◀

Note that for this case, the lines of action of both **A** and **B** intersect the *x*-axis.

Case 2: Let $B_y = 0$ to satisfy Equation (4).

Now Equation (2): $$A = \frac{R}{\lambda_y}$$

Equation (1): $$B_x = -R\left(\frac{\lambda_x}{\lambda_y}\right)$$

Equation (3): $$B_z = -R\left(\frac{\lambda_z}{\lambda_y}\right)$$

Equation (6): $aA\lambda_y = 0$ which requires $a = 0$

Substitution into Equation (5):

$$M = z\left[-R\left(\frac{\lambda_x}{\lambda_y}\right)\right] - x\left[-R\left(\frac{\lambda_z}{\lambda_y}\right)\right] \quad \text{or} \quad \lambda_z x - \lambda_x z = \left(\frac{M}{R}\right)\lambda_y$$

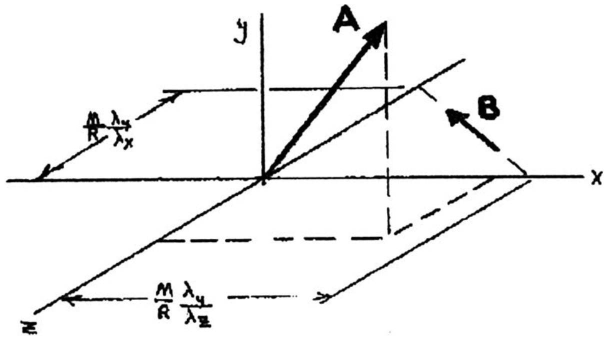
This last expression is the equation for the line of action of force **B**.

In summary,

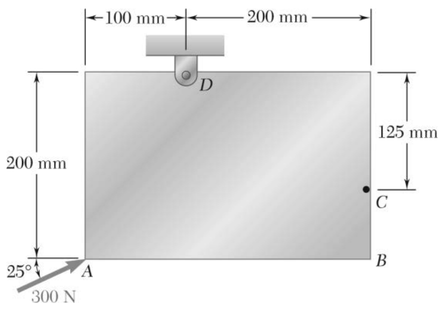
$$\mathbf{A} = \left(\frac{R}{\lambda_y}\right)\lambda_A \quad \blacktriangleleft$$

$$\mathbf{B} = \left(\frac{R}{\lambda_y}\right)(-\lambda_x\mathbf{i} - \lambda_x\mathbf{k}) \quad \blacktriangleleft$$

Assuming that $\lambda_x, \lambda_y, \lambda_z > 0$, the equivalent force system is as shown below.



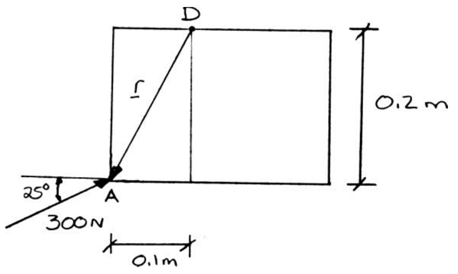
Note that the component of **A** in the *xz*-plane is parallel to **B**.



PROBLEM 3.147

A 300-N force is applied at *A* as shown. Determine (*a*) the moment of the 300-N force about *D*, (*b*) the smallest force applied at *B* that creates the same moment about *D*.

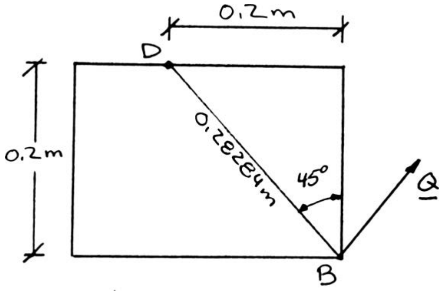
SOLUTION



(*a*)

$$F_x = (300\text{ N})\cos 25^\circ$$
$$= 271.89\text{ N}$$
$$F_y = (300\text{ N})\sin 25^\circ$$
$$= 126.785\text{ N}$$
$$\mathbf{F} = (271.89\text{ N})\mathbf{i} + (126.785\text{ N})\mathbf{j}$$
$$\mathbf{r} = \overrightarrow{DA} = -(0.1\text{ m})\mathbf{i} - (0.2\text{ m})\mathbf{j}$$
$$\mathbf{M}_D = \mathbf{r}\times\mathbf{F}$$
$$\mathbf{M}_D = [-(0.1\text{ m})\mathbf{i} - (0.2\text{ m})\mathbf{j}]\times[(271.89\text{ N})\mathbf{i} + (126.785\text{ N})\mathbf{j}]$$
$$= -(12.6785\text{ N}\cdot\text{m})\mathbf{k} + (54.378\text{ N}\cdot\text{m})\mathbf{k}$$
$$= (41.700\text{ N}\cdot\text{m})\mathbf{k}$$

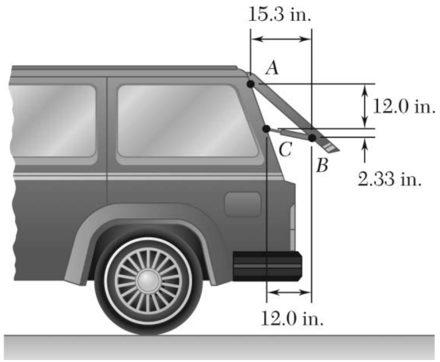
$\mathbf{M}_D = 41.7\text{ N}\cdot\text{m}\ \circlearrowleft$ ◀



(*b*) The smallest force *Q* at *B* must be perpendicular to $\overrightarrow{DB}$ at 45° $\measuredangle$

$$\mathbf{M}_D = Q(\overrightarrow{DB})$$

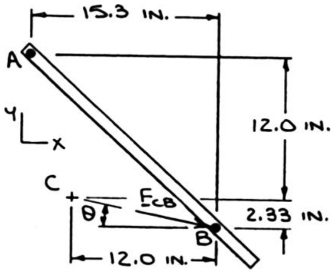
$41.700\text{ N}\cdot\text{m} = Q(0.28284\text{ m})$ $Q = 147.4\text{ N}\ \measuredangle\ 45.0^\circ$ ◀



PROBLEM 3.148

The tailgate of a car is supported by the hydraulic lift BC. If the lift exerts a 125-lb force directed along its centerline on the ball and socket at B, determine the moment of the force about A.

SOLUTION



First note
$$d_{CB} = \sqrt{(12.0\text{ in.})^2 + (2.33\text{ in.})^2}$$
$$= 12.2241\text{ in.}$$

Then
$$\cos\theta = \frac{12.0\text{ in.}}{12.2241\text{ in.}}$$
$$\sin\theta = \frac{2.33\text{ in.}}{12.2241\text{ in.}}$$

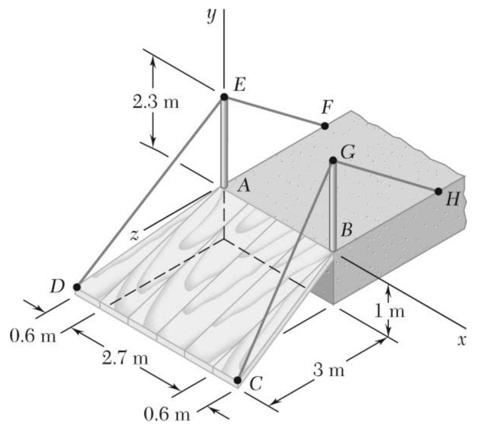
and
$$\mathbf{F}_{CB} = F_{CB}\cos\theta\,\mathbf{i} - F_{CB}\sin\theta\,\mathbf{j}$$
$$= \frac{125\text{ lb}}{12.2241\text{ in.}}[(12.0\text{ in.})\,\mathbf{i} - (2.33\text{ in.})\,\mathbf{j}]$$

Now
$$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{F}_{CB}$$

where
$$\mathbf{r}_{B/A} = (15.3\text{ in.})\,\mathbf{i} - (12.0\text{ in.} + 2.33\text{ in.})\,\mathbf{j}$$
$$= (15.3\text{ in.})\,\mathbf{i} - (14.33\text{ in.})\,\mathbf{j}$$

Then
$$\mathbf{M}_A = [(15.3\text{ in.})\mathbf{i} - (14.33\text{ in.})\mathbf{j}] \times \frac{125\text{ lb}}{12.2241\text{ in.}}(12.0\mathbf{i} - 2.33\mathbf{j})$$
$$= (1393.87\text{ lb}\cdot\text{in.})\mathbf{k}$$
$$= (116.156\text{ lb}\cdot\text{ft})\mathbf{k}$$

or $\mathbf{M}_A = 116.2\text{ lb}\cdot\text{ft}\ \circlearrowleft$ ◀



PROBLEM 3.149

The ramp *ABCD* is supported by cables at corners *C* and *D*. The tension in each of the cables is 810 N. Determine the moment about *A* of the force exerted by (*a*) the cable at *D*, (*b*) the cable at *C*.

SOLUTION

(*a*) We have $\mathbf{M}_A = \mathbf{r}_{E/A} \times \mathbf{T}_{DE}$

where

$$\mathbf{r}_{E/A} = (2.3\text{ m})\mathbf{j}$$

$$\mathbf{T}_{DE} = \boldsymbol{\lambda}_{DE} T_{DE}$$

$$= \frac{(0.6\text{ m})\mathbf{i} + (3.3\text{ m})\mathbf{j} - (3\text{ m})\mathbf{k}}{\sqrt{(0.6)^2 + (3.3)^2 + (3)^2}\text{ m}}(810\text{ N})$$

$$= (108\text{ N})\mathbf{i} + (594\text{ N})\mathbf{j} - (540\text{ N})\mathbf{k}$$

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.3 & 0 \\ 108 & 594 & -540 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= -(1242\text{ N}\cdot\text{m})\mathbf{i} - (248.4\text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M}_A = -(1242\text{ N}\cdot\text{m})\mathbf{i} - (248\text{ N}\cdot\text{m})\mathbf{k}$ ◀

(*b*) We have $\mathbf{M}_A = \mathbf{r}_{G/A} \times \mathbf{T}_{CG}$

where

$$\mathbf{r}_{G/A} = (2.7\text{ m})\mathbf{i} + (2.3\text{ m})\mathbf{j}$$

$$\mathbf{T}_{CG} = \boldsymbol{\lambda}_{CG} T_{CG}$$

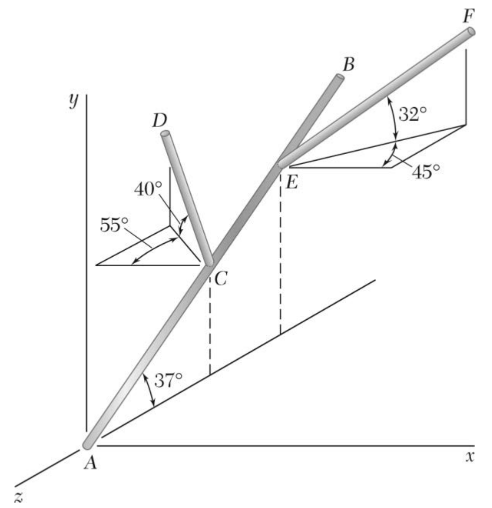
$$= \frac{-(.6\text{ m})\mathbf{i} + (3.3\text{ m})\mathbf{j} - (3\text{ m})\mathbf{k}}{\sqrt{(.6)^2 + (3.3)^2 + (3)^2}\text{ m}}(810\text{ N})$$

$$= -(108\text{ N})\mathbf{i} + (594\text{ N})\mathbf{j} - (540\text{ N})\mathbf{k}$$

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.7 & 2.3 & 0 \\ -108 & 594 & -540 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= -(1242\text{ N}\cdot\text{m})\mathbf{i} + (1458\text{ N}\cdot\text{m})\mathbf{j} + (1852\text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M}_A = -(1242\text{ N}\cdot\text{m})\mathbf{i} + (1458\text{ N}\cdot\text{m})\mathbf{j} + (1852\text{ N}\cdot\text{m})\mathbf{k}$ ◀



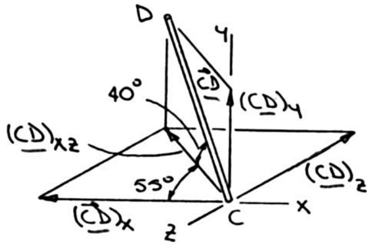
PROBLEM 3.150

Section *AB* of a pipeline lies in the *yz*-plane and forms an angle of 37° with the *z*-axis. Branch lines *CD* and *EF* join *AB* as shown. Determine the angle formed by pipes *AB* and *CD*.

SOLUTION

First note $\overrightarrow{AB} = AB(\sin 37°\mathbf{j} - \cos 37°\mathbf{k})$

$$\mathbf{CD} = CD(-\cos 40° \cos 55°\mathbf{j} + \sin 40°\mathbf{j} - \cos 40° \sin 55°\mathbf{k})$$



Now $\overrightarrow{AB} \cdot \overrightarrow{CD} = (AB)(CD)\cos\theta$

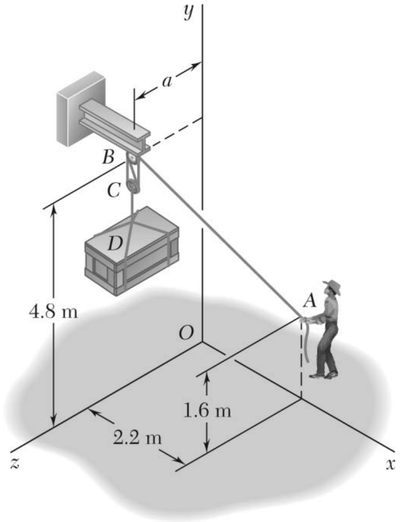
or $AB(\sin 37°\mathbf{j} - \cos 37°\mathbf{k}) \cdot CD(-\cos 40° \cos 55°\mathbf{i} + \sin 40°\mathbf{j} - \cos 40° \sin 55°\mathbf{k})$

$$= (AB)(CD)\cos\theta$$

or $\cos\theta = (\sin 37°)(\sin 40°) + (-\cos 37°)(-\cos 40° \sin 55°)$

$$= 0.88799$$

or $\theta = 27.4°$ ◀



PROBLEM 3.151

To lift a heavy crate, a man uses a block and tackle attached to the bottom of an I-beam at hook B. Knowing that the moments about the y and the z axes of the force exerted at B by portion AB of the rope are, respectively, 120 N · m and –460 N · m, determine the distance a.

SOLUTION

First note $$\overrightarrow{BA} = (2.2\text{ m})\mathbf{i} - (3.2\text{ m})\mathbf{j} - (a\text{ m})\mathbf{k}$$

Now $$\mathbf{M}_D = \mathbf{r}_{A/D} \times \mathbf{T}_{BA}$$

where $$\mathbf{r}_{A/D} = (2.2\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j}$$

$$\mathbf{T}_{BA} = \frac{T_{BA}}{d_{BA}}(2.2\mathbf{i} - 3.2\mathbf{j} - a\mathbf{k})\ (\text{N})$$

Then $$\mathbf{M}_D = \frac{T_{BA}}{d_{BA}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.2 & 1.6 & 0 \\ 2.2 & -3.2 & -a \end{vmatrix}$$

$$= \frac{T_{BA}}{d_{BA}}\{-1.6a\,\mathbf{i} + 2.2a\,\mathbf{j} + [(2.2)(-3.2) - (1.6)(2.2)]\mathbf{k}\}$$

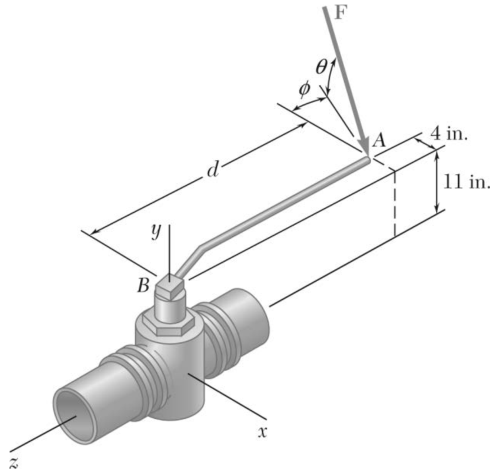
Thus $$M_y = 2.2\frac{T_{BA}}{d_{BA}}a \qquad (\text{N}\cdot\text{m})$$

$$M_z = -10.56\frac{T_{BA}}{d_{BA}} \qquad (\text{N}\cdot\text{m})$$

Then forming the ratio $$\frac{M_y}{M_z}$$

$$\frac{120\text{ N}\cdot\text{m}}{-460\text{ N}\cdot\text{m}} = \frac{2.2\frac{T_{BA}}{d_{BA}}\,(\text{N}\cdot\text{m})}{-10.56\frac{T_{BA}}{d_{BA}}\,(\text{N}\cdot\text{m})}$$

or $a = 1.252$ m ◀



PROBLEM 3.152

To loosen a frozen valve, a force **F** of magnitude 70 lb is applied to the handle of the valve. Knowing that $\theta = 25°$, $M_x = -61\ \text{lb}\cdot\text{ft}$, and $M_z = -43\ \text{lb}\cdot\text{ft}$, determine ϕ and d.

SOLUTION

We have $\Sigma \mathbf{M}_O: \quad \mathbf{r}_{A/O} \times \mathbf{F} = \mathbf{M}_O$

where $\mathbf{r}_{A/O} = -(4\text{ in.})\mathbf{i} + (11\text{ in.})\mathbf{j} - (d)\mathbf{k}$

$$\mathbf{F} = F(\cos\theta\cos\phi\mathbf{i} - \sin\theta\mathbf{j} + \cos\theta\sin\phi\mathbf{k})$$

For $F = 70\text{ lb}, \quad \theta = 25°$

$$\mathbf{F} = (70\text{ lb})[(0.90631\cos\phi)\mathbf{i} - 0.42262\mathbf{j} + (0.90631\sin\phi)\mathbf{k}]$$

$$\mathbf{M}_O = (70\text{ lb})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 11 & -d \\ -0.90631\cos\phi & -0.42262 & 0.90631\sin\phi \end{vmatrix}\text{ in.}$$

$$= (70\text{ lb})[(9.9694\sin\phi - 0.42262d)\mathbf{i} + (-0.90631d\cos\phi + 3.6252\sin\phi)\mathbf{j} + (1.69048 - 9.9694\cos\phi)\mathbf{k}]\text{ in.}$$

and

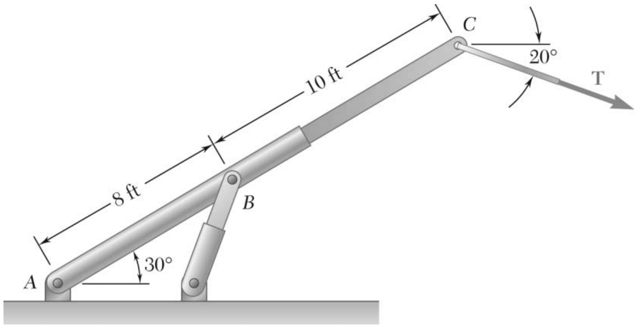
$$M_x = (70\text{ lb})(9.9694\sin\phi - 0.42262d)\text{ in.} = -(61\text{ lb}\cdot\text{ft})(12\text{ in./ft}) \qquad (1)$$

$$M_y = (70\text{ lb})(-0.90631d\cos\phi + 3.6252\sin\phi)\text{ in.} \qquad (2)$$

$$M_z = (70\text{ lb})(1.69048 - 9.9694\cos\phi)\text{ in.} = -43\text{ lb}\cdot\text{ft}(12\text{ in./ft}) \qquad (3)$$

From Equation (3): $\phi = \cos^{-1}\left(\dfrac{634.33}{697.86}\right) = 24.636°$ or $\phi = 24.6°$ ◀

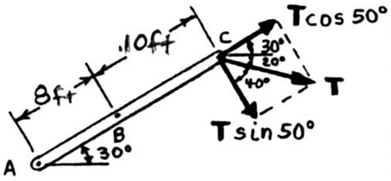
From Equation (1): $d = \left(\dfrac{1022.90}{29.583}\right) = 34.577\text{ in.}$ or $d = 34.6\text{ in.}$ ◀



PROBLEM 3.153

The tension in the cable attached to the end *C* of an adjustable boom *ABC* is 560 lb. Replace the force exerted by the cable at *C* with an equivalent force-couple system (*a*) at *A*, (*b*) at *B*.

SOLUTION



(*a*) Based on $\Sigma F: \quad F_A = T = 560 \text{ lb}$

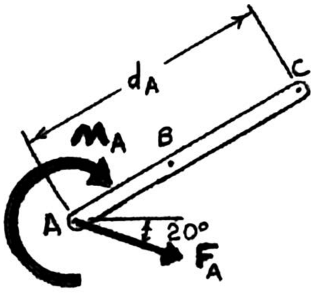
or $\mathbf{F}_A = 560$ lb ↘ 20.0° ◀

$$\Sigma M_A: \quad M_A = (T \sin 50°)(d_A)$$

$$= (560 \text{ lb}) \sin 50° (18 \text{ ft})$$

$$= 7721.7 \text{ lb} \cdot \text{ft}$$

or $\mathbf{M}_A = 7720 \text{ lb} \cdot \text{ft}$ ↻ ◀



(*b*) Based on $\Sigma F: \quad F_B = T = 560 \text{ lb}$

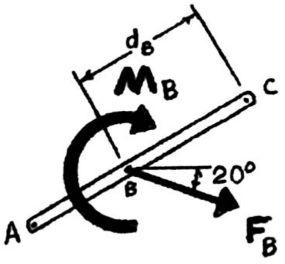
or $\mathbf{F}_B = 560$ lb ↘ 20.0° ◀

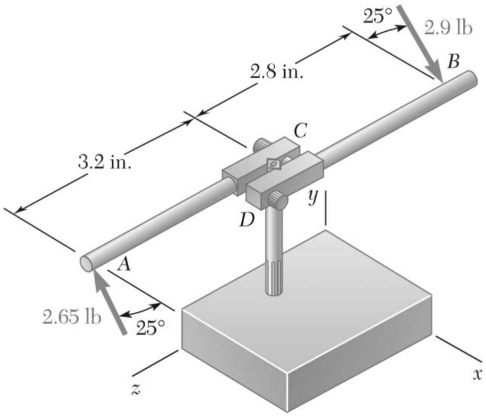
$$\Sigma M_B: \quad M_B = (T \sin 50°)(d_B)$$

$$= (560 \text{ lb}) \sin 50° (10 \text{ ft})$$

$$= 4289.8 \text{ lb} \cdot \text{ft}$$

or $\mathbf{M}_B = 4290 \text{ lb} \cdot \text{ft}$ ↻ ◀





PROBLEM 3.154

While tapping a hole, a machinist applies the horizontal forces shown to the handle of the tap wrench. Show that these forces are equivalent to a single force, and specify, if possible, the point of application of the single force on the handle.

SOLUTION

Since the forces at *A* and *B* are parallel, the force at *B* can be replaced with the sum of two forces with one of the forces equal in magnitude to the force at *A* except with an opposite sense, resulting in a force-couple.

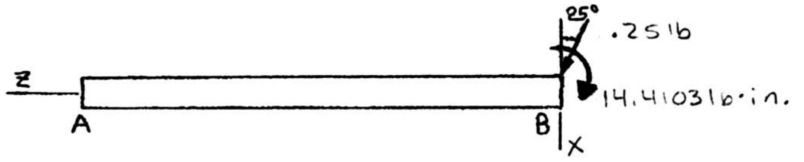
We have $F_B = 2.9\text{ lb} - 2.65\text{ lb} = 0.25\text{ lb}$, where the 2.65-lb force is part of the couple. Combining the two parallel forces,

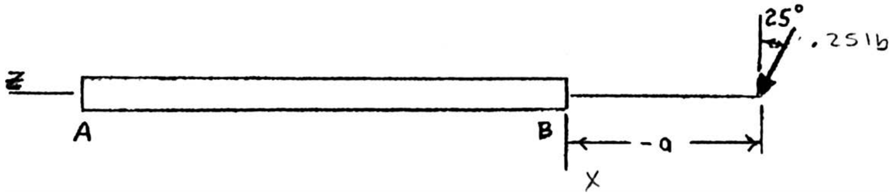
$$M_{\text{couple}} = (2.65\text{ lb})[(3.2\text{ in.} + 2.8\text{ in.})\cos 25°]$$

$$= 14.4103\text{ lb}\cdot\text{in.}$$

and

$$\mathbf{M}_{\text{couple}} = 14.4103\text{ lb}\cdot\text{in.}$$





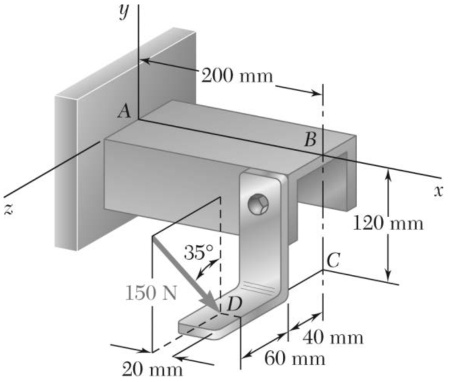
A single equivalent force will be located in the negative *z* direction.

Based on $\Sigma M_B: \quad -14.4103\text{ lb}\cdot\text{in.} = [(0.25\text{ lb})\cos 25°](a)$

$$a = 63.600\text{ in.}$$

$$\mathbf{F}' = (0.25\text{ lb})(\cos 25°\mathbf{i} + \sin 25°\mathbf{k})$$

$\mathbf{F}' = (0.227\text{ lb})\mathbf{i} + (0.1057\text{ lb})\mathbf{k}$ and is applied on an extension of handle *BD* at a distance of 63.6 in. to the right of *B*. ◀



PROBLEM 3.155

Replace the 150-N force with an equivalent force-couple system at *A*.

SOLUTION

Equivalence requires $\Sigma\mathbf{F}: \quad \mathbf{F} = (150\text{ N})(-\cos 35°\mathbf{j} - \sin 35°\mathbf{k})$

$$= -(122.873\text{ N})\mathbf{j} - (86.036\text{ N})\mathbf{k}$$

$$\Sigma\mathbf{M}_A: \quad \mathbf{M} = \mathbf{r}_{D/A} \times \mathbf{F}$$

where $\mathbf{r}_{D/A} = (0.18\text{ m})\mathbf{i} - (0.12\text{ m})\mathbf{j} + (0.1\text{ m})\mathbf{k}$

Then
$$\mathbf{M} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.18 & -0.12 & 0.1 \\ 0 & -122.873 & -86.036 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= [(-0.12)(-86.036) - (0.1)(-122.873)]\mathbf{i}$$
$$+ [-(0.18)(-86.036)]\mathbf{j}$$
$$+ [(0.18)(-122.873)]\mathbf{k}$$
$$= (22.6\text{ N}\cdot\text{m})\mathbf{i} + (15.49\text{ N}\cdot\text{m})\mathbf{j} - (22.1\text{ N}\cdot\text{m})\mathbf{k}$$

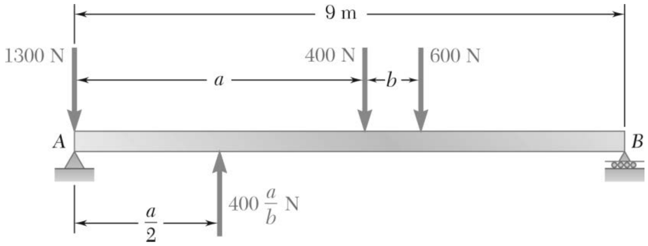
The equivalent force-couple system at *A* is

$\mathbf{F} = -(122.9\text{ N})\mathbf{j} - (86.0\text{ N})\mathbf{k}$ ◀

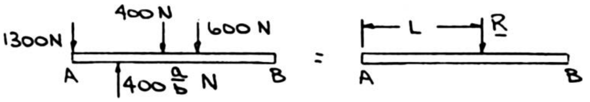
$\mathbf{M} = (22.6\text{ N}\cdot\text{m})\mathbf{i} + (15.49\text{ N}\cdot\text{m})\mathbf{j} - (22.1\text{ N}\cdot\text{m})\mathbf{k}$ ◀

PROBLEM 3.156

A beam supports three loads of given magnitude and a fourth load whose magnitude is a function of position. If $b = 1.5$ m and the loads are to be replaced with a single equivalent force, determine (*a*) the value of a so that the distance from support A to the line of action of the equivalent force is maximum, (*b*) the magnitude of the equivalent force and its point of application on the beam.



SOLUTION



For equivalence,

$$\Sigma F_y: \quad -1300 + 400\frac{a}{b} - 400 - 600 = -R$$

or
$$R = \left(2300 - 400\frac{a}{b}\right) \text{N} \tag{1}$$

$$\Sigma M_A: \quad \frac{a}{2}\left(400\frac{a}{b}\right) - a(400) - (a+b)(600) = -LR$$

or
$$L = \frac{1000a + 600b - 200\dfrac{a^2}{b}}{2300 - 400\dfrac{a}{b}}$$

Then with
$$b = 1.5 \text{ m} \qquad L = \frac{10a + 9 - \dfrac{4}{3}a^2}{23 - \dfrac{8}{3}a} \tag{2}$$

where a, L are in m.

(*a*) Find value of a to maximize L.

$$\frac{dL}{da} = \frac{\left(10 - \dfrac{8}{3}a\right)\left(23 - \dfrac{8}{3}a\right) - \left(10a + 9 - \dfrac{4}{3}a^2\right)\left(-\dfrac{8}{3}\right)}{\left(23 - \dfrac{8}{3}a\right)^2}$$

PROBLEM 3.156 (Continued)

or $$230-\frac{184}{3}a-\frac{80}{3}a+\frac{64}{9}a^2+\frac{80}{3}a+24-\frac{32}{9}a^2=0$$

or $$16a^2-276a+1143=0$$

Then $$a=\frac{276\pm\sqrt{(-276)^2-4(16)(1143)}}{2(16)}$$

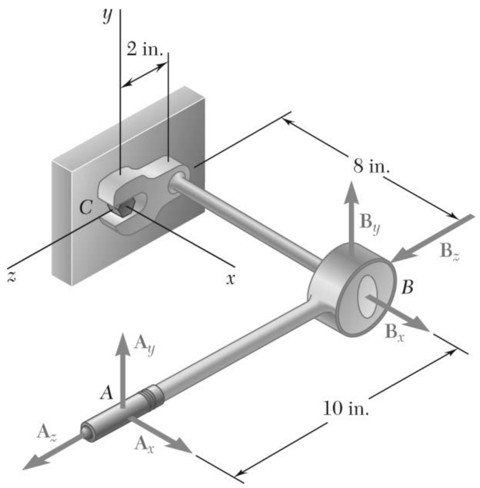
or $$a=10.3435\text{ m}\quad\text{and}\quad a=6.9065\text{ m}$$

Since $AB=9$ m, a must be less than 9 m $\qquad a=6.91$ m ◀

(*b*) Using Eq. (1), $$R=2300-400\frac{6.9065}{1.5}$$ or $R=458$ N ◀

and using Eq. (2), $$L=\frac{10(6.9065)+9-\frac{4}{3}(6.9065)^2}{23-\frac{8}{3}(6.9065)}=3.16\text{ m}$$

R is applied 3.16 m to the right of *A*. ◀



PROBLEM 3.157

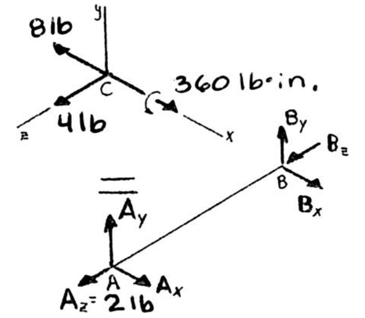
A mechanic uses a crowfoot wrench to loosen a bolt at *C*. The mechanic holds the socket wrench handle at Points *A* and *B* and applies forces at these points. Knowing that these forces are equivalent to a force-couple system at *C* consisting of the force $\mathbf{C} = (8 \text{ lb})\mathbf{i} + (4 \text{ lb})\mathbf{k}$ and the couple $\mathbf{M}_C = (360 \text{ lb}\cdot\text{in.})\mathbf{i}$, determine the forces applied at *A* and at *B* when $A_z = 2$ lb.

SOLUTION

We have $\qquad \Sigma\mathbf{F}: \quad \mathbf{A} + \mathbf{B} = \mathbf{C}$

or $\qquad F_x: \quad A_x + B_x = 8 \text{ lb}$

$$B_x = -(A_x + 8 \text{ lb}) \qquad (1)$$



$$\Sigma F_y: \quad A_y + B_y = 0$$

or $\qquad A_y = -B_y \qquad (2)$

$$\Sigma F_z: \quad 2 \text{ lb} + B_z = 4 \text{ lb}$$

or $\qquad B_z = 2 \text{ lb} \qquad (3)$

We have $\qquad \Sigma\mathbf{M}_C: \quad \mathbf{r}_{B/C} \times \mathbf{B} + \mathbf{r}_{A/C} \times \mathbf{A} = \mathbf{M}_C$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0 & 2 \\ B_x & B_y & 2 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0 & 8 \\ A_x & A_y & 2 \end{vmatrix} \text{lb}\cdot\text{in.} = (360 \text{ lb}\cdot\text{in.})\mathbf{i}$$

or
$$(2B_y - 8A_y)\mathbf{i} + (2B_x - 16 + 8A_x - 16)\mathbf{j}$$
$$+(8B_y + 8A_y)\mathbf{k} = (360 \text{ lb}\cdot\text{in.})\mathbf{i}$$

From $\quad$ **i**-coefficient: $\quad 2B_y - 8A_y = 360 \text{ lb}\cdot\text{in.} \qquad (4)$

j-coefficient: $\quad -2B_x + 8A_x = 32 \text{ lb}\cdot\text{in.} \qquad (5)$

k-coefficient: $\quad 8B_y + 8A_y = 0 \qquad (6)$

PROBLEM 3.157 (Continued)

From Equations (2) and (4): $2B_y - 8(-B_y) = 360$

$$B_y = 36 \text{ lb} \quad A_y = 36 \text{ lb}$$

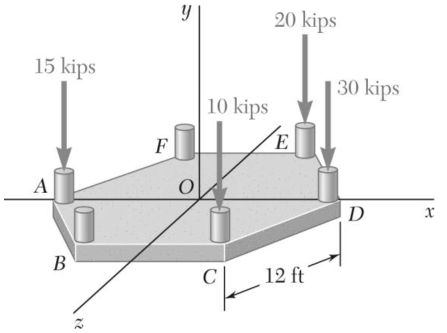
From Equations (1) and (5): $2(-A_x - 8) + 8A_x = 32$

$$A_x = 1.6 \text{ lb}$$

From Equation (1): $B_x = -(1.6 + 8) = -9.6 \text{ lb}$

$\mathbf{A} = (1.600 \text{ lb})\mathbf{i} - (36.0 \text{ lb})\mathbf{j} + (2.00 \text{ lb})\mathbf{k}$ ◀

$\mathbf{B} = -(9.60 \text{ lb})\mathbf{i} + (36.0 \text{ lb})\mathbf{j} + (2.00 \text{ lb})\mathbf{k}$ ◀



PROBLEM 3.158

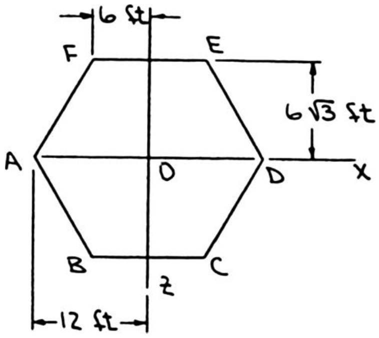
A concrete foundation mat in the shape of a regular hexagon of side 12 ft supports four column loads as shown. Determine the magnitudes of the additional loads that must be applied at *B* and *F* if the resultant of all six loads is to pass through the center of the mat.

SOLUTION

From the statement of the problem, it can be concluded that the six applied loads are equivalent to the resultant **R** at *O*. It then follows that

$$\Sigma \mathbf{M}_O = 0 \quad \text{or} \quad \Sigma M_x = 0 \quad \Sigma M_z = 0$$

For the applied loads:



Then $\Sigma \mathbf{M}_x = 0$: $(6\sqrt{3}\text{ ft})F_B + (6\sqrt{3}\text{ ft})(10\text{ kips}) - (6\sqrt{3}\text{ ft})(20\text{ kips}) - (6\sqrt{3}\text{ ft})F_F = 0$

or $F_B - F_F = 10$ (1)

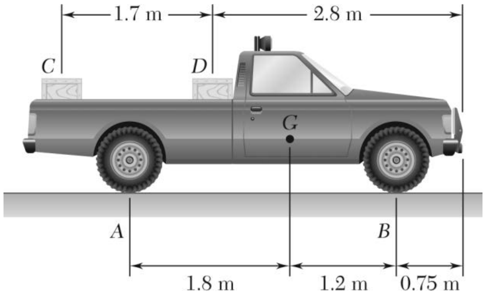
$\Sigma \mathbf{M}_z = 0$: $(12\text{ ft})(15\text{ kips}) + (6\text{ ft})F_B - (6\text{ ft})(10\text{ kips}) - (12\text{ ft})(30\text{ kips}) - (6\text{ ft})(20\text{ kips}) + (6\text{ ft})F_F = 0$

or $F_B + F_F = 60$ (2)

Then Eqs. (1) + (2) ⇒ $\mathbf{F}_B = 35.0\text{ kips}\downarrow$ ◀

and $\mathbf{F}_F = 25.0\text{ kips}\downarrow$ ◀

CHAPTER 4

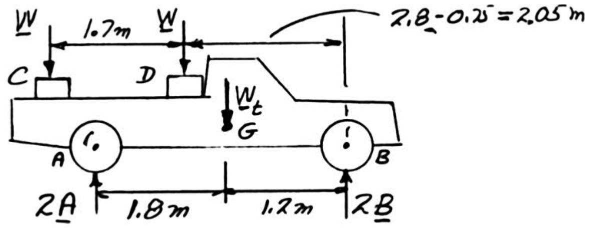


PROBLEM 4.1

Two crates, each of mass 350 kg, are placed as shown in the bed of a 1400-kg pickup truck. Determine the reactions at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

SOLUTION

Free-Body Diagram:



$$W = (350\text{ kg})(9.81\text{ m/s}^2) = 3.4335\text{ kN}$$

$$W_t = (1400\text{ kg})(9.81\text{ m/s}^2) = 13.7340\text{ kN}$$

(*a*) Rear wheels: $+\circlearrowleft \Sigma M_B = 0{:}\quad W(1.7\text{ m} + 2.05\text{ m}) + W(2.05\text{ m}) + W_t(1.2\text{ m}) - 2A(3\text{ m}) = 0$

$$(3.4335\text{ kN})(3.75\text{ m}) + (3.4335\text{ kN})(2.05\text{ m}) + (13.7340\text{ kN})(1.2\text{ m}) - 2A(3\text{ m}) = 0$$

$$A = +6.0659\text{ kN}$$

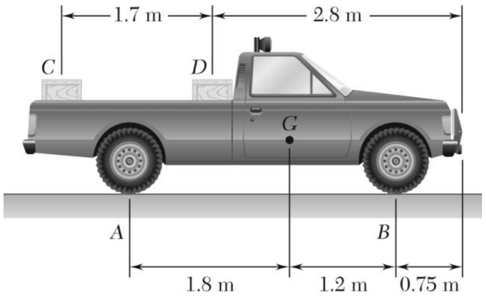
$\mathbf{A} = 6.07\text{ kN}\uparrow$ ◀

(*b*) Front wheels: $+\uparrow \Sigma F_y = 0{:}\quad -W - W - W_t + 2A + 2B = 0$

$$-3.4335\text{ kN} - 3.4335\text{ kN} - 13.7340\text{ kN} + 2(6.0659\text{ kN}) + 2B = 0$$

$$B = +4.2346\text{ kN}$$

$\mathbf{B} = 4.23\text{ kN}\uparrow$ ◀



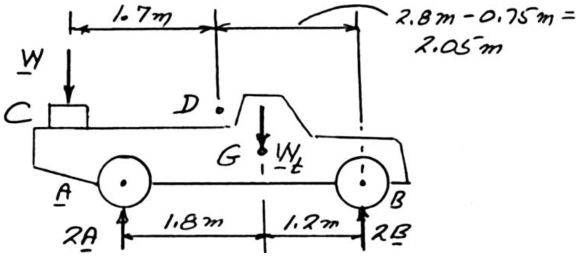
PROBLEM 4.2

Solve Problem 4.1, assuming that crate *D* is removed and that the position of crate *C* is unchanged.

PROBLEM 4.1 Two crates, each of mass 350 kg, are placed as shown in the bed of a 1400-kg pickup truck. Determine the reactions at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

SOLUTION

Free-Body Diagram:



$$W = (350\text{ kg})(9.81\text{ m/s}^2) = 3.4335\text{ kN}$$

$$W_t = (1400\text{ kg})(9.81\text{ m/s}^2) = 13.7340\text{ kN}$$

(*a*) Rear wheels: $+\circlearrowleft \Sigma M_B = 0\text{:}\quad W(1.7\text{ m} + 2.05\text{ m}) + W_t(1.2\text{ m}) - 2A(3\text{ m}) = 0$

$$(3.4335\text{ kN})(3.75\text{ m}) + (13.7340\text{ kN})(1.2\text{ m}) - 2A(3\text{ m}) = 0$$

$$A = +4.8927\text{ kN}$$

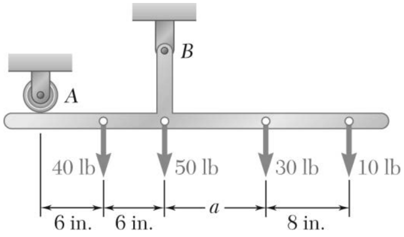
$\mathbf{A} = 4.89\text{ kN}\uparrow$ ◀

(*b*) Front wheels: $+\uparrow \Sigma M_y = 0\text{:}\quad -W - W_t + 2A + 2B = 0$

$$-3.4335\text{ kN} - 13.7340\text{ kN} + 2(4.8927\text{ kN}) + 2B = 0$$

$$B = +3.6911\text{ kN}$$

$\mathbf{B} = 3.69\text{ kN}\uparrow$ ◀

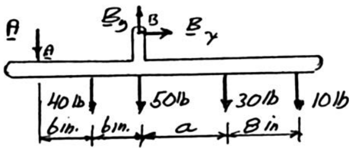


PROBLEM 4.3

A T-shaped bracket supports the four loads shown. Determine the reactions at A and B (a) if $a = 10$ in., (b) if $a = 7$ in.

SOLUTION

Free-Body Diagram:



$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (40\text{ lb})(6\text{ in.}) - (30\text{ lb})a - (10\text{ lb})(a + 8\text{ in.}) + (12\text{ in.})A = 0$$

$$A = \frac{(40a - 160)}{12} \qquad (1)$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(40\text{ lb})(6\text{ in.}) - (50\text{ lb})(12\text{ in.}) - (30\text{ lb})(a + 12\text{ in.})$$
$$-(10\text{ lb})(a + 20\text{ in.}) + (12\text{ in.})B_y = 0$$

$$B_y = \frac{(1400 + 40a)}{12}$$

Since $$B_x = 0, \quad B = \frac{(1400 + 40a)}{12} \qquad (2)$$

(a) <u>For $a = 10$ in.</u>,

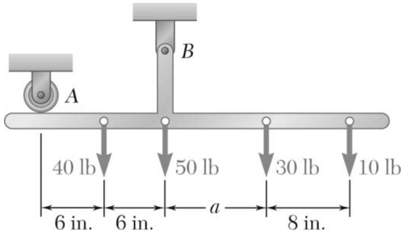
Eq. (1): $$A = \frac{(40 \times 10 - 160)}{12} = +20.0\text{ lb}$$ $\mathbf{A} = 20.0\text{ lb}\downarrow$ ◀

Eq. (2): $$B = \frac{(1400 + 40 \times 10)}{12} = +150.0\text{ lb}$$ $\mathbf{B} = 150.0\text{ lb}\uparrow$ ◀

(b) <u>For $a = 7$ in.</u>,

Eq. (1): $$A = \frac{(40 \times 7 - 160)}{12} = +10.00\text{ lb}$$ $\mathbf{A} = 10.00\text{ lb}\downarrow$ ◀

Eq. (2): $$B = \frac{(1400 + 40 \times 7)}{12} = +140.0\text{ lb}$$ $\mathbf{B} = 140.0\text{ lb}\uparrow$ ◀



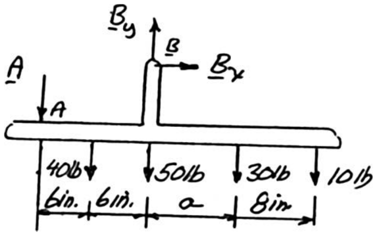
PROBLEM 4.4

For the bracket and loading of Problem 4.3, determine the smallest distance *a* if the bracket is not to move.

PROBLEM 4.3 A T-shaped bracket supports the four loads shown. Determine the reactions at *A* and *B* (*a*) if $a = 10$ in., (*b*) if $a = 7$ in.

SOLUTION

Free-Body Diagram:

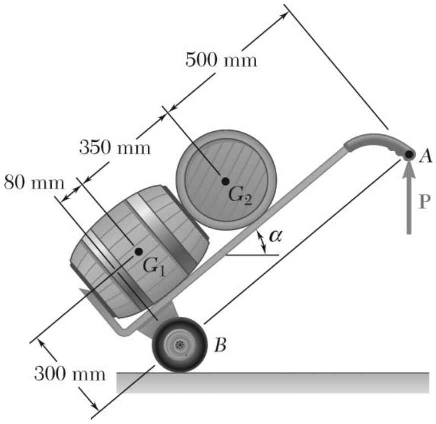


For no motion, reaction at *A* must be downward or zero; smallest distance *a* for no motion corresponds to $\mathbf{A} = 0$.

$$+\circlearrowleft \Sigma M_B = 0: \quad (40\text{ lb})(6\text{ in.}) - (30\text{ lb})a - (10\text{ lb})(a + 8\text{ in.}) + (12\text{ in.})A = 0$$

$$A = \frac{(40a - 160)}{12}$$

$$A = 0: \quad (40a - 160) = 0 \qquad a = 4.00\text{ in.} \blacktriangleleft$$

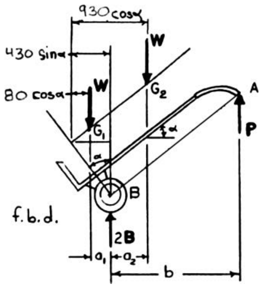


PROBLEM 4.5

A hand truck is used to move two kegs, each of mass 40 kg. Neglecting the mass of the hand truck, determine (*a*) the vertical force **P** that should be applied to the handle to maintain equilibrium when $\alpha = 35°$, (*b*) the corresponding reaction at each of the two wheels.

SOLUTION

Free-Body Diagram:



Dimensions in mm

$$W = mg = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.40 \text{ N}$$
$$a_1 = (300 \text{ mm})\sin\alpha - (80 \text{ mm})\cos\alpha$$
$$a_2 = (430 \text{ mm})\cos\alpha - (300 \text{ mm})\sin\alpha$$
$$b = (930 \text{ mm})\cos\alpha$$

From free-body diagram of hand truck,

$$+\circlearrowleft \Sigma M_B = 0: \quad P(b) - W(a_2) + W(a_1) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad P - 2W + 2B = 0 \qquad (2)$$

For $\alpha = 35°$

$$a_1 = 300\sin 35° - 80\cos 35° = 106.541 \text{ mm}$$
$$a_2 = 430\cos 35° - 300\sin 35° = 180.162 \text{ mm}$$
$$b = 930\cos 35° = 761.81 \text{ mm}$$

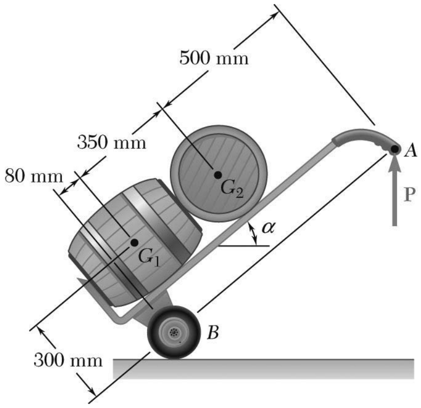
(*a*) From Equation (1):

$$P(761.81 \text{ mm}) - 392.40 \text{ N}(180.162 \text{ mm}) + 392.40 \text{ N}(106.54 \text{ mm}) = 0$$

$P = 37.921$ N or $\mathbf{P} = 37.9 \text{ N}\uparrow$ ◀

(*b*) From Equation (2):

$37.921 \text{ N} - 2(392.40 \text{ N}) + 2B = 0$ or $\mathbf{B} = 373 \text{ N}\uparrow$ ◀



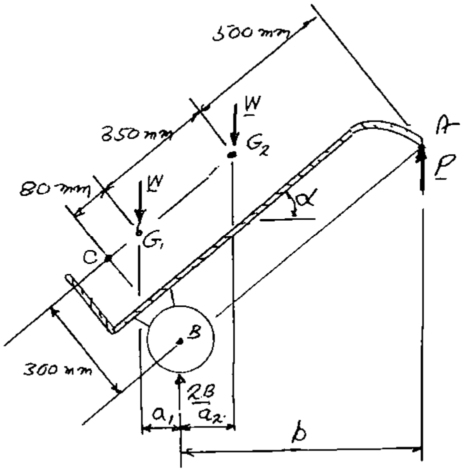
PROBLEM 4.6

Solve Problem 4.5 when $\alpha = 40°$.

PROBLEM 4.5 A hand truck is used to move two kegs, each of mass 40 kg. Neglecting the mass of the hand truck, determine (*a*) the vertical force **P** that should be applied to the handle to maintain equilibrium when $\alpha = 35°$, (*b*) the corresponding reaction at each of the two wheels.

SOLUTION

Free-Body Diagram:



$$W = mg = (40\text{ kg})(9.81\text{ m/s}^2)$$
$$W = 392.40\text{ N}$$
$$a_1 = (300\text{ mm})\sin\alpha - (80\text{ mm})\cos\alpha$$
$$a_2 = (430\text{ mm})\cos\alpha - (300\text{ mm})\sin\alpha$$
$$b = (930\text{ mm})\cos\alpha$$

From F.B.D.:

$$+\circlearrowleft \Sigma M_B = 0: \quad P(b) - W(a_2) + W(a_1) = 0$$

$$P = W(a_2 - a_1)/b \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad -W - W + P + 2B = 0$$

$$B = W - \frac{1}{2}P \tag{2}$$

For $\alpha = 40°$:

$$a_1 = 300\sin 40° - 80\cos 40° = 131.553\text{ mm}$$
$$a_2 = 430\cos 40° - 300\sin 40° = 136.563\text{ mm}$$
$$b = 930\cos 40° = 712.42\text{ mm}$$

(*a*) From Equation (1):

$$P = \frac{392.40\text{ N }(0.136563\text{ m} - 0.131553\text{ m})}{0.71242\text{ m}}$$

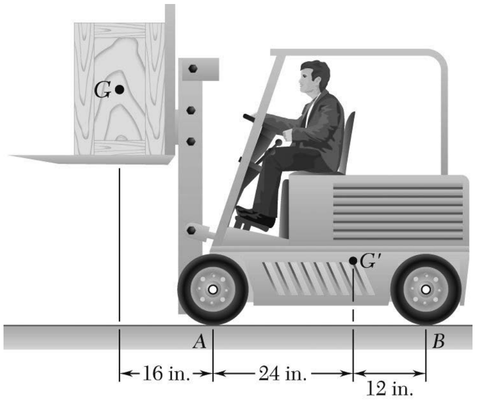
$$P = 2.7595\text{ N}$$

P = 2.76 N ↑ ◀

(*b*) From Equation (2):

$$B = 392.40\text{ N} - \frac{1}{2}(2.7595\text{ N})$$

B = 391 N ↑ ◀

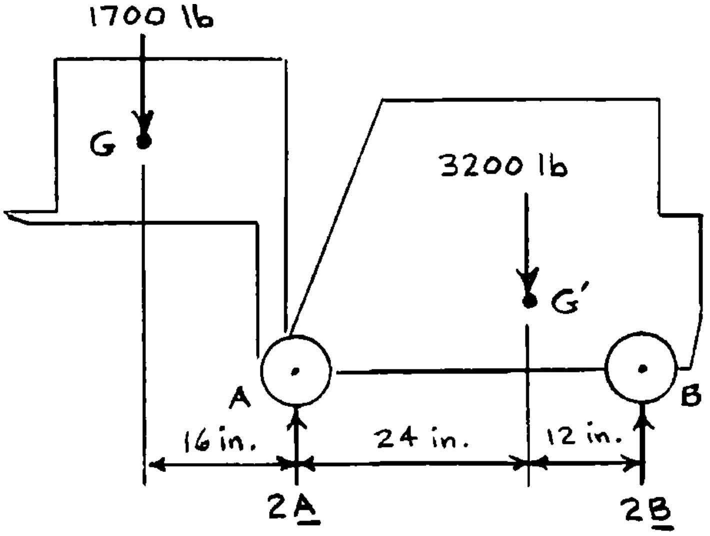


PROBLEM 4.7

A 3200-lb forklift truck is used to lift a 1700-lb crate. Determine the reaction at each of the two (*a*) front wheels *A*, (*b*) rear wheels *B*.

SOLUTION

Free-Body Diagram:

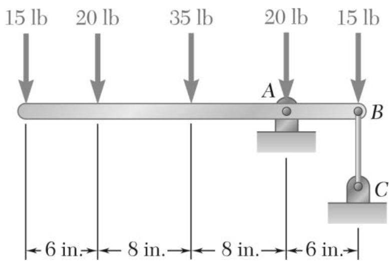


(*a*) Front wheels: $+\circlearrowleft \Sigma M_B = 0$: $(1700 \text{ lb})(52 \text{ in.}) + (3200 \text{ lb})(12 \text{ in.}) - 2A(36 \text{ in.}) = 0$

$A = +1761.11 \text{ lb}$ $\qquad \mathbf{A} = 1761 \text{ lb} \uparrow$ ◀

(*b*) Rear wheels: $+\uparrow \Sigma F_y = 0$: $-1700 \text{ lb} - 3200 \text{ lb} + 2(1761.11 \text{ lb}) + 2B = 0$

$B = +688.89 \text{ lb}$ $\qquad \mathbf{B} = 689 \text{ lb} \uparrow$ ◀

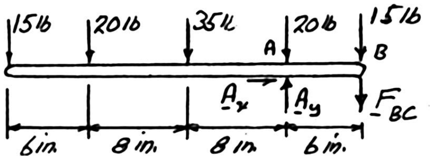


PROBLEM 4.8

For the beam and loading shown, determine (*a*) the reaction at *A*, (*b*) the tension in cable *BC*.

SOLUTION

Free-Body Diagram:



(*a*) Reaction at *A*: $\Sigma F_x = 0$: $A_x = 0$

$+\circlearrowleft \Sigma M_B = 0$: $(15\text{ lb})(28\text{ in.}) + (20\text{ lb})(22\text{ in.}) + (35\text{ lb})(14\text{ in.}) + (20\text{ lb})(6\text{ in.}) - A_y(6\text{ in.}) = 0$

$A_y = +245\text{ lb}$ **A** = 245 lb↑ ◀

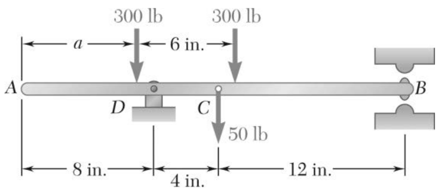
(*b*) Tension in *BC*: $+\circlearrowleft \Sigma M_A = 0$: $(15\text{ lb})(22\text{ in.}) + (20\text{ lb})(16\text{ in.}) + (35\text{ lb})(8\text{ in.}) - (15\text{ lb})(6\text{ in.}) - F_{BC}(6\text{ in.}) = 0$

$F_{BC} = +140.0\text{ lb}$ $F_{BC} = 140.0\text{ lb}$ ◀

Check: $+\uparrow \Sigma F_y = 0$: $-15\text{ lb} - 20\text{ lb} = 35\text{ lb} - 20\text{ lb} + A - F_{BC} = 0$

$-105\text{ lb} + 245\text{ lb} - 140.0 = 0$

$0 = 0$ (Checks)

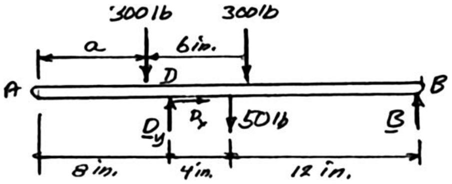


PROBLEM 4.9

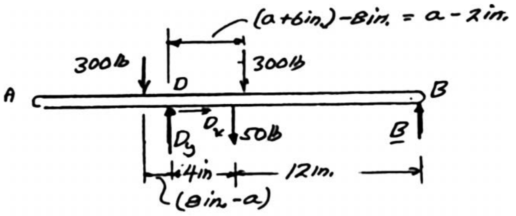
For the beam and loading shown, determine the range of the distance *a* for which the reaction at *B* does not exceed 100 lb downward or 200 lb upward.

SOLUTION

Assume *B* is positive when directed ↑.



Sketch showing distance from *D* to forces.



$$+\circlearrowleft \Sigma M_D = 0: \quad (300\text{ lb})(8\text{ in.} - a) - (300\text{ lb})(a - 2\text{ in.}) - (50\text{ lb})(4\text{ in.}) + 16B = 0$$

$$-600a + 2800 + 16B = 0$$

$$a = \frac{(2800 + 16B)}{600} \tag{1}$$

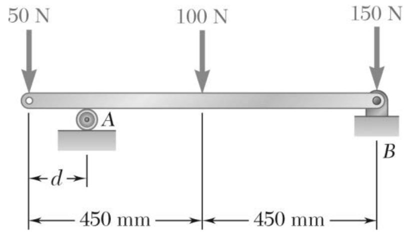
For $B = 100\text{ lb}\downarrow = -100\text{ lb}$, Eq. (1) yields:

$$a \geq \frac{[2800 + 16(-100)]}{600} = \frac{1200}{600} = 2\text{ in.} \qquad a \geq 2.00\text{ in.} \triangleleft$$

For $B = 200\uparrow = +200\text{ lb}$, Eq. (1) yields:

$$a \leq \frac{[2800 + 16(200)]}{600} = \frac{6000}{600} = 10\text{ in.} \qquad a \leq 10.00\text{ in.} \triangleleft$$

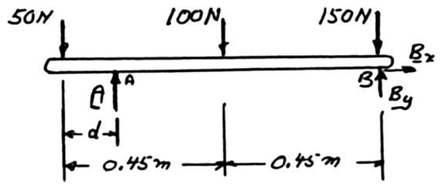
Required range: $2.00\text{ in.} \leq a \leq 10.00\text{ in.}$ ◀



PROBLEM 4.10

The maximum allowable value of each of the reactions is 180 N. Neglecting the weight of the beam, determine the range of the distance *d* for which the beam is safe.

SOLUTION



$$\Sigma F_x = 0: \quad B_x = 0$$

$$B = B_y$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (50 \text{ N})d - (100 \text{ N})(0.45 \text{ m} - d) - (150 \text{ N})(0.9 \text{ m} - d) + B(0.9 \text{ m} - d) = 0$$

$$50d - 45 + 100d - 135 + 150d + 0.9B - Bd = 0$$

$$d = \frac{180 \text{ N} \cdot \text{m} - (0.9 \text{ m})B}{300A - B} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (50 \text{ N})(0.9 \text{ m}) - A(0.9 \text{ m} - d) + (100 \text{ N})(0.45 \text{ m}) = 0$$

$$45 - 0.9A + Ad + 45 = 0$$

$$d = \frac{(0.9 \text{ m})A - 90 \text{ N} \cdot \text{m}}{A} \qquad (2)$$

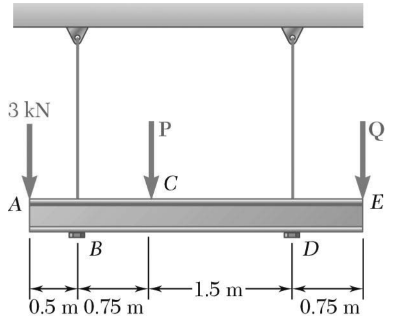
Since $B \leq 180$ N, Eq. (1) yields

$$d \geq \frac{180 - (0.9)180}{300 - 180} = \frac{18}{120} = 0.15 \text{ m} \qquad d \geq 150.0 \text{ mm} \ \triangleleft$$

Since $A \leq 180$ N, Eq. (2) yields

$$d \leq \frac{(0.9)180 - 90}{180} = \frac{72}{180} = 0.40 \text{ m} \qquad d \leq 400 \text{ mm} \ \triangleleft$$

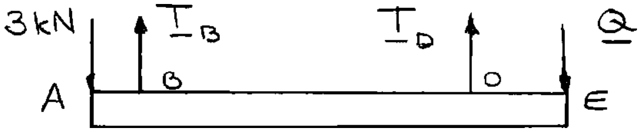
Range: $150.0 \text{ mm} \leq d \leq 400 \text{ mm}$ ◀



PROBLEM 4.11

Three loads are applied as shown to a light beam supported by cables attached at *B* and *D*. Neglecting the weight of the beam, determine the range of values of *Q* for which neither cable becomes slack when $P = 0$.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad (3.00 \text{ kN})(0.500 \text{ m}) + T_D(2.25 \text{ m}) - Q(3.00 \text{ m}) = 0$$

$$Q = 0.500 \text{ kN} + (0.750)\, T_D \tag{1}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (3.00 \text{ kN})(2.75 \text{ m}) - T_B(2.25 \text{ m}) - Q(0.750 \text{ m}) = 0$$

$$Q = 11.00 \text{ kN} - (3.00)\, T_B \tag{2}$$

For cable *B* not to be slack, $T_B \geq 0$, and from Eq. (2),

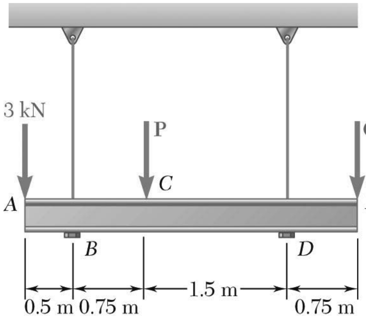
$$Q \leq 11.00 \text{ kN}$$

For cable *D* not to be slack, $T_D \geq 0$, and from Eq. (1),

$$Q \geq 0.500 \text{ kN}$$

For neither cable to be slack,

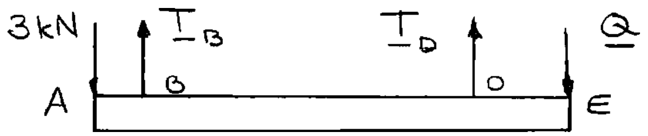
$$0.500 \text{ kN} \leq Q \leq 11.00 \text{ kN} \blacktriangleleft$$



PROBLEM 4.12

Three loads are applied as shown to a light beam supported by cables attached at *B* and *D*. Knowing that the maximum allowable tension in each cable is 4 kN and neglecting the weight of the beam, determine the range of values of *Q* for which the loading is safe when $P = 0$.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad (3.00 \text{ kN})(0.500 \text{ m}) + T_D(2.25 \text{ m}) - Q(3.00 \text{ m}) = 0$$

$$Q = 0.500 \text{ kN} + (0.750)\, T_D \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (3.00 \text{ kN})(2.75 \text{ m}) - T_B(2.25 \text{ m}) - Q(0.750 \text{ m}) = 0$$

$$Q = 11.00 \text{ kN} - (3.00)\, T_B \qquad (2)$$

For $T_B \le 4.00$ kN, Eq. (2) yields

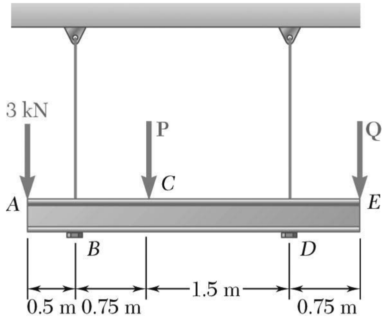
$$Q \ge 11.00 \text{ kN} - 3.00(4.00 \text{ kN}) \qquad Q \ge -1.000 \text{ kN}$$

For $T_D \le 4.00$ kN, Eq. (1) yields

$$Q \le 0.500 \text{ kN} + 0.750(4.00 \text{ kN}) \qquad Q \le 3.50 \text{ kN}$$

For loading to be safe, cables must also not be slack. Combining with the conditions obtained in Problem 4.11,

$0.500 \text{ kN} \le Q \le 3.50 \text{ kN}$ ◀

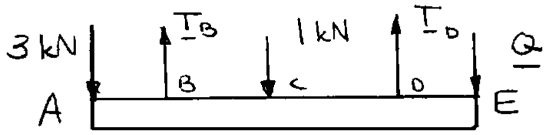


PROBLEM 4.13

For the beam of Problem 4.12, determine the range of values of Q for which the loading is safe when $P = 1$ kN.

PROBLEM 4.12 Three loads are applied as shown to a light beam supported by cables attached at B and D. Knowing that the maximum allowable tension in each cable is 4 kN and neglecting the weight of the beam, determine the range of values of Q for which the loading is safe when $P = 0$.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad (3.00 \text{ kN})(0.500 \text{ m}) - (1.000 \text{ kN})(0.750 \text{ m}) + T_D(2.25 \text{ m}) - Q(3.00 \text{ m}) = 0$$

$$Q = 0.250 \text{ kN} + 0.75\, T_D \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (3.00 \text{ kN})(2.75 \text{ m}) + (1.000 \text{ kN})(1.50 \text{ m}) - T_B(2.25 \text{ m}) - Q(0.750 \text{ m}) = 0$$

$$Q = 13.00 \text{ kN} - 3.00\, T_B \qquad (2)$$

For the loading to be safe, cables must not be slack and tension must not exceed 4.00 kN.

Making $0 \le T_B \le 4.00$ kN in Eq. (2), we have

$$13.00 \text{ kN} - 3.00(4.00 \text{ kN}) \le Q \le 13.00 \text{ kN} - 3.00(0)$$

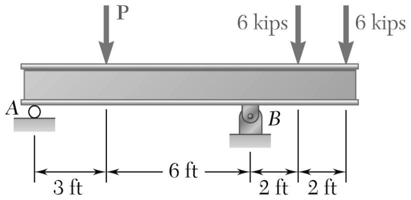
$$1.000 \text{ kN} \le Q \le 13.00 \text{ kN} \qquad (3)$$

Making $0 \le T_D \le 4.00$ kN in Eq. (1), we have

$$0.250 \text{ kN} + 0.750(0) \le Q \le 0.250 \text{ kN} + 0.750(4.00 \text{ kN})$$

$$0.250 \text{ kN} \le Q \le 3.25 \text{ kN} \qquad (4)$$

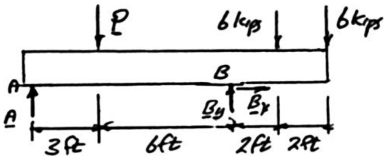
Combining Eqs. (3) and (4), $1.000 \text{ kN} \le Q \le 3.25 \text{ kN}$ ◀



PROBLEM 4.14

For the beam of Sample Problem 4.2, determine the range of values of *P* for which the beam will be safe, knowing that the maximum allowable value of each of the reactions is 30 kips and that the reaction at *A* must be directed upward.

SOLUTION



$$\Sigma F_x = 0: \quad B_x = 0$$

$$\mathbf{B} = B_y \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -P(3\text{ ft}) + B(9\text{ ft}) - (6\text{ kips})(11\text{ ft}) - (6\text{ kips})(13\text{ ft}) = 0$$

$$P = 3B - 48\text{ kips} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -A(9\text{ ft}) + P(6\text{ ft}) - (6\text{ kips})(2\text{ ft}) - (6\text{ kips})(4\text{ ft}) = 0$$

$$P = 1.5A + 6\text{ kips} \qquad (2)$$

Since $B \le 30$ kips, Eq. (1) yields

$$P \le (3)(30\text{ kips}) - 48\text{ kips} \qquad P \le 42.0\text{ kips} \quad \triangleleft$$

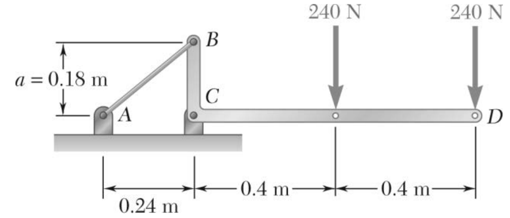
Since $0 \le A \le 30$ kips, Eq. (2) yields

$$0 + 6\text{ kips} \le P \le (1.5)(30\text{ kips})1.6\text{ kips}$$

$$6.00\text{ kips} \le P \le 51.0\text{ kips} \qquad \triangleleft$$

Range of values of *P* for which beam will be safe:

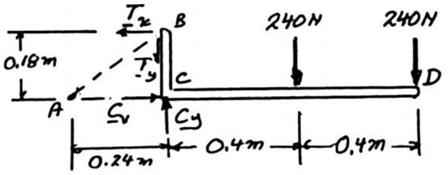
$$6.00\text{ kips} \le P \le 42.0\text{ kips} \qquad \blacktriangleleft$$



PROBLEM 4.15

The bracket *BCD* is hinged at *C* and attached to a control cable at *B*. For the loading shown, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION



At *B*:

$$\frac{T_y}{T_x} = \frac{0.18\text{ m}}{0.24\text{ m}}$$

$$T_y = \frac{3}{4}T_x \qquad (1)$$

(*a*) $+\circlearrowleft \Sigma M_C = 0{:}\quad T_x(0.18\text{ m}) - (240\text{ N})(0.4\text{ m}) - (240\text{ N})(0.8\text{ m}) = 0$

$$T_x = +1600\text{ N}$$

From Eq. (1):

$$T_y = \frac{3}{4}(1600\text{ N}) = 1200\text{ N}$$

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{1600^2 + 1200^2} = 2000\text{ N}$$

$T = 2.00\text{ kN}$ ◀

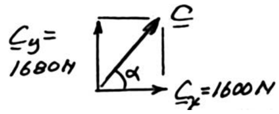
(*b*) $\xrightarrow{+} \Sigma F_x = 0{:}\quad C_x - T_x = 0$

$$C_x - 1600\text{ N} = 0 \quad C_x = +1600\text{ N}$$

$\mathbf{C}_x = 1600\text{ N} \longrightarrow$

$+\uparrow \Sigma F_y = 0{:}\quad C_y - T_y - 240\text{ N} - 240\text{ N} = 0$

$$C_y - 1200\text{ N} - 480\text{ N} = 0$$



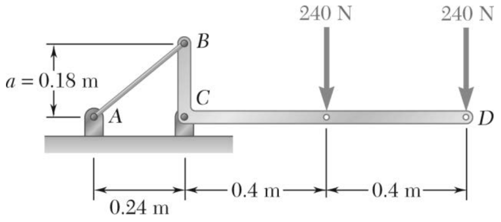
$$C_y = +1680\text{ N}$$

$\mathbf{C}_y = 1680\text{ N}\uparrow$

$$\alpha = 46.4°$$

$$C = 2320\text{ N}$$

$\mathbf{C} = 2.32\text{ kN} \measuredangle 46.4°$ ◀

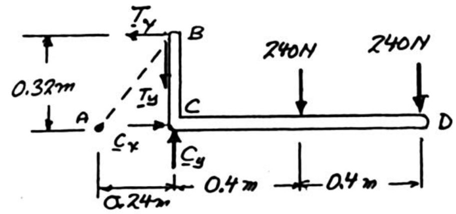


PROBLEM 4.16

Solve Problem 4.15, assuming that $a = 0.32$ m.

PROBLEM 4.15 The bracket *BCD* is hinged at *C* and attached to a control cable at *B*. For the loading shown, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION



At *B*:

$$\frac{T_y}{T_x} = \frac{0.32 \text{ m}}{0.24 \text{ m}}$$

$$T_y = \frac{4}{3} T_x$$

$$+\circlearrowleft \Sigma M_C = 0: \quad T_x(0.32 \text{ m}) - (240 \text{ N})(0.4 \text{ m}) - (240 \text{ N})(0.8 \text{ m}) = 0$$

$$T_x = 900 \text{ N}$$

From Eq. (1):

$$T_y = \frac{4}{3}(900 \text{ N}) = 1200 \text{ N}$$

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{900^2 + 1200^2} = 1500 \text{ N} \qquad T = 1.500 \text{ kN} \blacktriangleleft$$

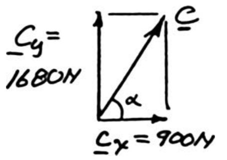
$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - T_x = 0$$

$$C_x - 900 \text{ N} = 0 \quad C_x = +900 \text{ N} \qquad \mathbf{C}_x = 900 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - T_y - 240 \text{ N} - 240 \text{ N} = 0$$

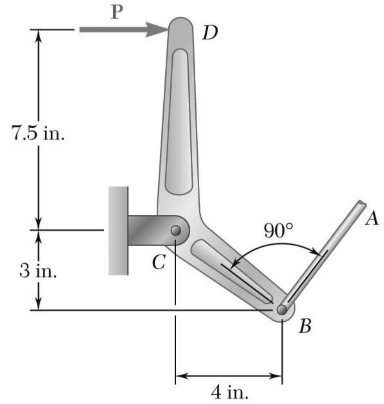
$$C_y - 1200 \text{ N} - 480 \text{ N} = 0$$

$$C_y = +1680 \text{ N} \qquad \mathbf{C}_y = 1680 \text{ N} \uparrow$$



$$\alpha = 61.8°$$

$$C = 1906 \text{ N} \qquad \mathbf{C} = 1.906 \text{ kN} \measuredangle 61.8° \blacktriangleleft$$

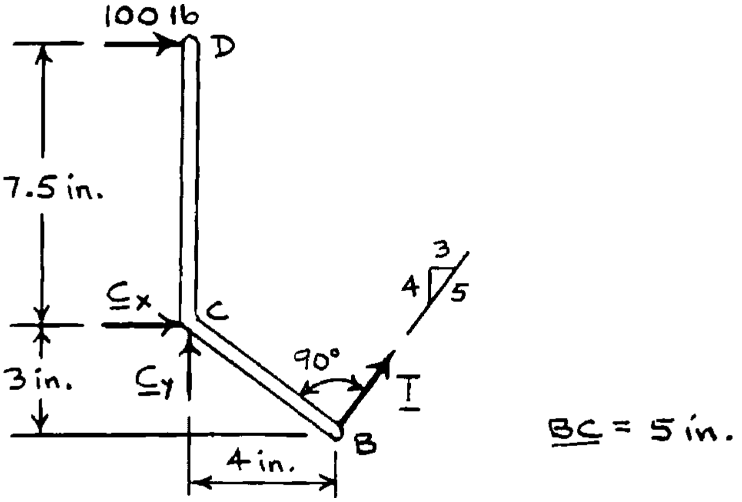


PROBLEM 4.17

The lever *BCD* is hinged at *C* and attached to a control rod at *B*. If $P = 100$ lb, determine (*a*) the tension in rod *AB*, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:

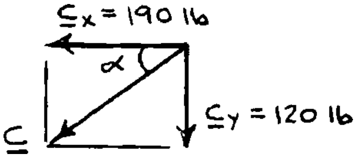


(*a*) $\overset{\curvearrowleft}{+}\Sigma M_C = 0\text{:}\quad T(5\text{ in.}) - (100\text{ lb})(7.5\text{ in.}) = 0$

$T = 150.0\text{ lb}$ ◀

(*b*) $\underrightarrow{+}\ \Sigma F_x = 0\text{:}\quad C_x + 100\text{ lb} + \frac{3}{5}(150.0\text{ lb}) = 0$

$C_x = -190\text{ lb} \qquad \mathbf{C}_x = 190\text{ lb} \longleftarrow$

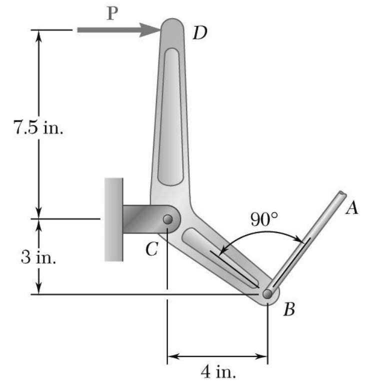


$+\!\uparrow \Sigma F_y = 0\text{:}\quad C_y + \frac{4}{5}(150.0\text{ lb}) = 0$

$C_y = -120\text{ lb} \qquad \mathbf{C}_y = 120\text{ lb}\downarrow$

$\alpha = 32.3° \qquad C = 225\text{ lb}$

$\mathbf{C} = 225\text{ lb} \measuredangle\ 32.3°$ ◀

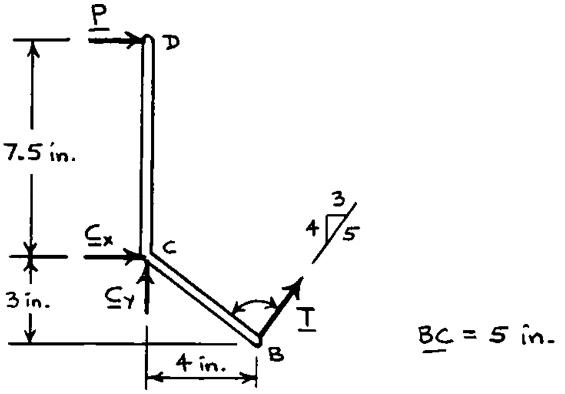


PROBLEM 4.18

The lever *BCD* is hinged at *C* and attached to a control rod at *B*. Determine the maximum force **P** that can be safely applied at *D* if the maximum allowable value of the reaction at *C* is 250 lb.

SOLUTION

Free-Body Diagram:

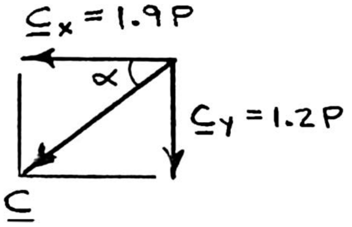


$$+\circlearrowleft \Sigma M_C = 0: \quad T(5\text{ in.}) - P(7.5\text{ in.}) = 0$$

$$T = 1.5P$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P + C_x + \frac{3}{5}(1.5P) = 0$$

$$C_x = -1.9P \qquad \mathbf{C}_x = 1.9P \longleftarrow$$



$$+\uparrow \Sigma F_y = 0: \quad C_y + \frac{4}{5}(1.5P) = 0$$

$$C_y = -1.2P \qquad \mathbf{C}_y = 1.2P \downarrow$$

$$C = \sqrt{C_x^2 + C_y^2}$$

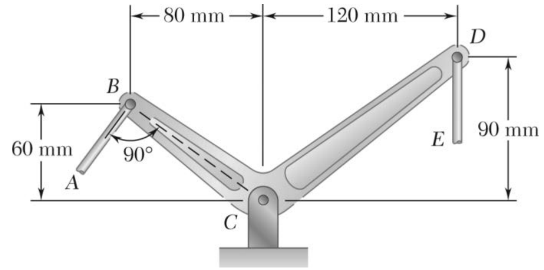
$$= \sqrt{(1.9P)^2 + (1.2P)^2}$$

$$C = 2.2472P$$

For $C = 250$ lb,

$$250\text{ lb} = 2.2472P$$

$$P = 111.2\text{ lb} \qquad \mathbf{P} = 111.2\text{ lb} \longrightarrow \blacktriangleleft$$

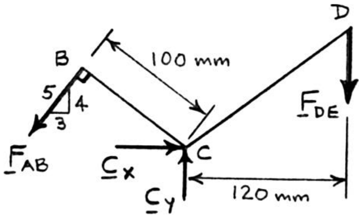


PROBLEM 4.19

Two links *AB* and *DE* are connected by a bell crank as shown. Knowing that the tension in link *AB* is 720 N, determine (*a*) the tension in link *DE*, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_C = 0: \quad F_{AB}(100 \text{ mm}) - F_{DE}(120 \text{ mm}) = 0$$

$$F_{DE} = \frac{5}{6} F_{AB} \qquad (1)$$

(*a*) For

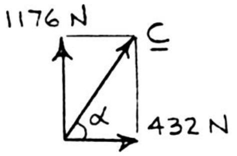
$$F_{AB} = 720 \text{ N}$$

$$F_{DE} = \frac{5}{6}(720 \text{ N}) \qquad F_{DE} = 600 \text{ N} \blacktriangleleft$$

(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}(720 \text{ N}) + C_x = 0$$

$$C_x = +432 \text{ N}$$



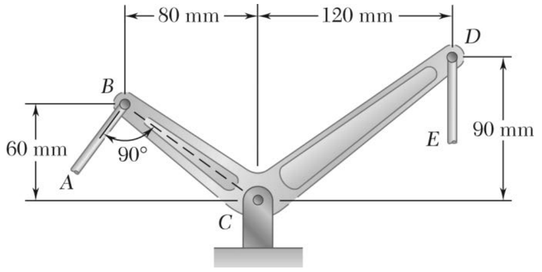
$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}(720 \text{ N}) + C_y - 600 \text{ N} = 0$$

$$C_y = +1176 \text{ N}$$

$$C = 1252.84 \text{ N}$$

$$\alpha = 69.829°$$

$$\mathbf{C} = 1253 \text{ N} \measuredangle 69.8° \blacktriangleleft$$



PROBLEM 4.20

Two links *AB* and *DE* are connected by a bell crank as shown. Determine the maximum force that may be safely exerted by link *AB* on the bell crank if the maximum allowable value for the reaction at *C* is 1600 N.

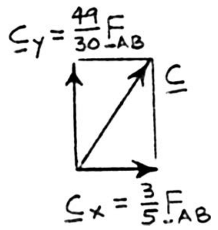
SOLUTION

See solution to Problem 4.15 for F.B.D. and derivation of Eq. (1).

$$F_{DE} = \frac{5}{6} F_{AB} \qquad (1)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5} F_{AB} + C_x = 0 \qquad C_x = \frac{3}{5} F_{AB}$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5} F_{AB} + C_y - F_{DE} = 0$$



$$-\frac{4}{5} F_{AB} + C_y - \frac{5}{6} F_{AB} = 0$$

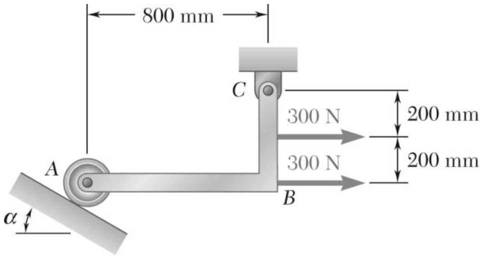
$$C_y = \frac{49}{30} F_{AB}$$

$$C = \sqrt{C_x^2 + C_y^2}$$

$$= \frac{1}{30}\sqrt{(49)^2 + (18)^2}\, F_{AB}$$

$$C = 1.74005 F_{AB}$$

For $C = 1600$ N, $1600 \text{ N} = 1.74005 F_{AB}$ $\qquad F_{AB} = 920$ N ◀



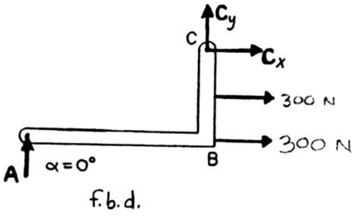
PROBLEM 4.21

Determine the reactions at A and C when (*a*) $\alpha = 0$, (*b*) $\alpha = 30°$.

SOLUTION

(*a*) $\alpha = 0$

From F.B.D. of member *ABC*:



$$+\circlearrowleft \Sigma M_C = 0: \quad (300\text{ N})(0.2\text{ m}) + (300\text{ N})(0.4\text{ m}) - A(0.8\text{ m}) = 0$$

$$A = 225\text{ N} \qquad \text{or} \qquad \mathbf{A} = 225\text{ N}\uparrow$$ ◀

$$+\uparrow \Sigma F_y = 0: \quad C_y + 225\text{ N} = 0$$

$$C_y = -225\text{ N} \quad \text{or} \quad \mathbf{C}_y = 225\text{ N}\downarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 300\text{ N} + 300\text{ N} + C_x = 0$$

$$C_x = -600\text{ N} \quad \text{or} \quad \mathbf{C}_x = 600\text{ N} \leftarrow$$

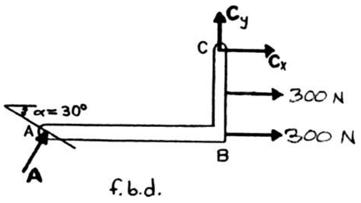
Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(600)^2 + (225)^2} = 640.80\text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-225}{-600}\right) = 20.556°$$

or $\mathbf{C} = 641\text{ N}$ ∠ 20.6° ◀

(*b*) $\alpha = 30°$

From F.B.D. of member *ABC*:



$$+\circlearrowleft \Sigma M_C = 0: \quad (300\text{ N})(0.2\text{ m}) + (300\text{ N})(0.4\text{ m}) - (A\cos 30°)(0.8\text{ m}) + (A\sin 30°)(20\text{ in.}) = 0$$

$$A = 365.24\text{ N} \qquad \text{or} \qquad \mathbf{A} = 365\text{ N} \angle\, 60.0°$$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad 300\text{ N} + 300\text{ N} + (365.24\text{ N})\sin 30° + C_x = 0$$

$$C_x = -782.62$$

PROBLEM 4.21 (Continued)

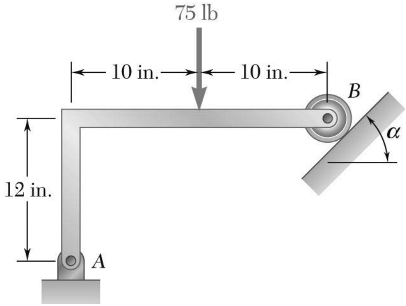
$$+\uparrow \Sigma F_y = 0: \quad C_y + (365.24\text{ N})\cos 30° = 0$$

$$C_y = -316.31\text{ N} \quad \text{or} \quad \mathbf{C}_y = 316\text{ N}\downarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(782.62)^2 + (316.31)^2} = 884.12\text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-316.31}{-782.62}\right) = 22.007°$$

or **C** = 884 N ↙ 22.0° ◀

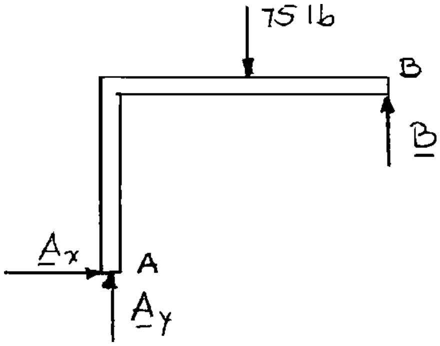


PROBLEM 4.22

Determine the reactions at *A* and *B* when (*a*) $\alpha = 0$, (*b*) $\alpha = 90°$, (*c*) $\alpha = 30°$.

SOLUTION

(*a*) $\alpha = 0$



$$+\circlearrowleft \Sigma M_A = 0\text{:}\quad B(20\text{ in.}) - 75\text{ lb}(10\text{ in.}) = 0$$
$$B = 37.5\text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0\text{:}\quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0\text{:}\quad A_y - 75\text{ lb} + 37.5\text{ lb} = 0$$
$$A_y = 37.5\text{ lb}$$

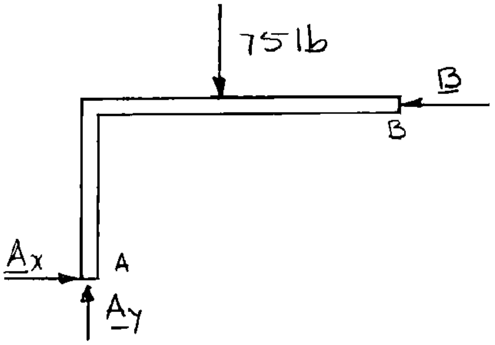
$$\mathbf{A} = \mathbf{B} = 37.5\text{ lb}\uparrow \blacktriangleleft$$

(*b*) $\alpha = 90°$

$$+\circlearrowleft \Sigma M_A = 0\text{:}\quad B(12\text{ in.}) - 75\text{ lb}(10\text{ in.}) = 0$$
$$B = 62.5\text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0\text{:}\quad A_x - B = 0$$
$$A_x = 62.5\text{ lb}$$

$$+\uparrow \Sigma F_y = 0\text{:}\quad A_y - 75\text{ lb} = 0$$
$$A_y = 75\text{ lb}$$



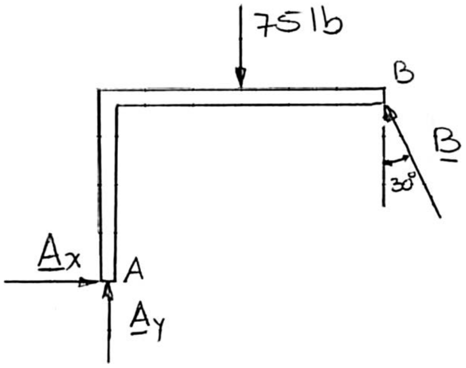
$$A = \sqrt{A_x^2 + A_y^2}$$
$$= \sqrt{(62.5\text{ lb})^2 + (75\text{ lb})^2}$$
$$= 97.6\text{ lb}$$

$$\tan\theta = \frac{75}{62.5}$$
$$\theta = 50.2°$$

$$\mathbf{A} = 97.6\text{ lb} \measuredangle 50.2°;\ \mathbf{B} = 62.51\text{ lb} \longleftarrow \blacktriangleleft$$

PROBLEM 4.22 (Continued)

(*c*) $\alpha = 30°$



$$\overset{\curvearrowleft}{+}\Sigma M_A = 0\colon\ (B\cos 30°)(20\text{ in.}) + (B\sin 30°)(12\text{ in.}) - (75\text{ lb})(10\text{ in.}) = 0$$

$$B = 32.161\text{ lb}$$

$$\xrightarrow{+}\ \Sigma F_x = 0\colon\ A_x - (32.161)\sin 30° = 0$$

$$A_x = 16.0805\text{ lb}$$

$$+\!\uparrow \Sigma F_y = 0\colon\ A_y + (32.161)\cos 30° - 75 = 0$$

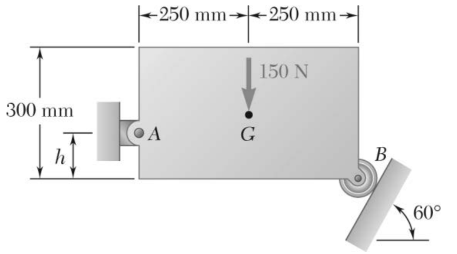
$$A_y = 47.148\text{ lb}$$

$$\begin{aligned} A &= \sqrt{A_x^2 + A_y^2} \\ &= \sqrt{(16.0805)^2 + (47.148)^2} \\ &= 49.8\text{ lb} \end{aligned}$$

$$\tan\theta = \frac{47.148}{16.0805}$$

$$\theta = 71.2°$$

A = 49.8 lb ⦣ 71.2°; **B** = 32.2 lb ⦤ 60.0° ◀

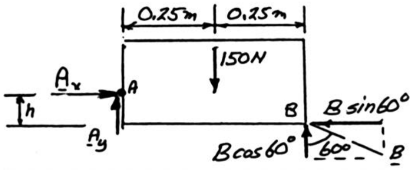


PROBLEM 4.23

Determine the reactions at A and B when (a) $h = 0$, (b) $h = 200$ mm.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad (B\cos 60°)(0.5\text{ m}) - (B\sin 60°)h - (150\text{ N})(0.25\text{ m}) = 0$$

$$B = \frac{37.5}{0.25 - 0.866h} \qquad (1)$$

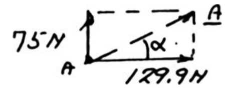
(a) When $h = 0$,

From Eq. (1): $B = \dfrac{37.5}{0.25} = 150$ N **B** $= 150.0$ N $\nwarrow$ 30.0° ◀

$\xrightarrow{+} \Sigma F_y = 0: \quad A_x - B\sin 60° = 0$

$A_x = (150)\sin 60° = 129.9$ N $\mathbf{A}_x = 129.9$ N →

$+\uparrow \Sigma F_y = 0: \quad A_y - 150 + B\cos 60° = 0$



$A_y = 150 - (150)\cos 60° = 75$ N $\mathbf{A}_y = 75$ N ↑

$\alpha = 30°$

$A = 150.0$ N **A** $= 150.0$ N $\nearrow$ 30.0° ◀

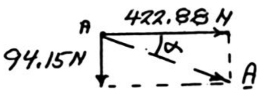
(b) When $h = 200$ mm $= 0.2$ m,

From Eq. (1): $B = \dfrac{37.5}{0.25 - 0.866(0.2)} = 488.3$ N **B** $= 488$ N $\nwarrow$ 30.0° ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B\sin 60° = 0$

$A_x = (488.3)\sin 60° = 422.88$ N $\mathbf{A}_x = 422.88$ N →

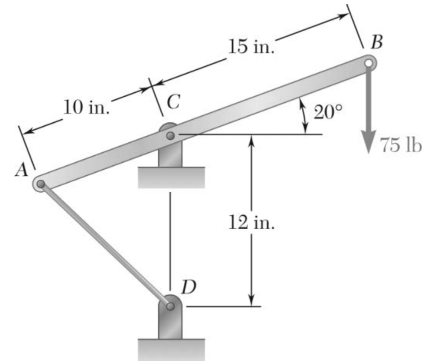
$+\uparrow \Sigma F_y = 0: \quad A_y - 150 + B\cos 60° = 0$



$A_y = 150 - (488.3)\cos 60° = -94.15$ N $\mathbf{A}_y = 94.15$ N ↓

$\alpha = 12.55°$

$A = 433.2$ N **A** $= 433$ N $\searrow$ 12.55° ◀

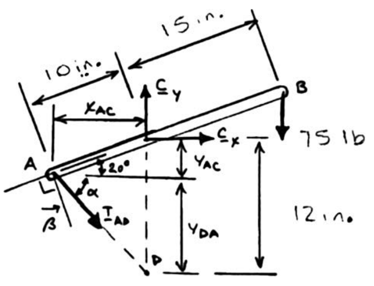


PROBLEM 4.24

A lever *AB* is hinged at *C* and attached to a control cable at *A*. If the lever is subjected to a 75-lb vertical force at *B*, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:



Geometry:

$$x_{AC} = (10\text{ in.})\cos 20° = 9.3969\text{ in.}$$
$$y_{AC} = (10\text{ in.})\sin 20° = 3.4202\text{ in.}$$

$$\Rightarrow y_{DA} = 12\text{ in.} - 3.4202\text{ in.} = 8.5798\text{ in.}$$

$$\alpha = \tan^{-1}\left(\frac{y_{DA}}{x_{AC}}\right) = \tan^{-1}\left(\frac{8.5798}{9.3969}\right) = 42.397°$$

$$\beta = 90° - 20° - 42.397° = 27.603°$$

Equilibrium for lever:

(*a*) $+\circlearrowleft \Sigma M_C = 0$: $T_{AD}\cos 27.603°(10\text{ in.}) - (75\text{ lb})[(15\text{ in.})\cos 20°] = 0$

$T_{AD} = 119.293\text{ lb}$ $\qquad T_{AD} = 119.3\text{ lb}$ ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0$: $C_x + (119.293\text{ lb})\cos 42.397° = 0$

$C_x = -88.097\text{ lb}$

$+\uparrow \Sigma F_y = 0$: $C_y - 75\text{ lb} - (119.293\text{ lb})\sin 42.397° = 0$

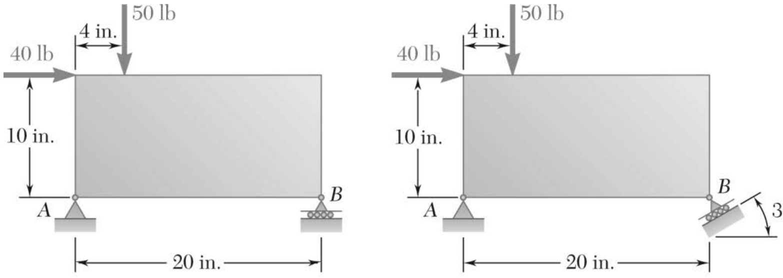
$C_y = 155.435$

Thus, $\quad C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-88.097)^2 + (155.435)^2} = 178.665\text{ lb}$

and $\quad \theta = \tan^{-1}\dfrac{C_y}{C_x} = \tan^{-1}\dfrac{155.435}{88.097} = 60.456°$ $\qquad \mathbf{C} = 178.7\text{ lb}$ ⦪ $60.5°$ ◀

PROBLEM 4.25

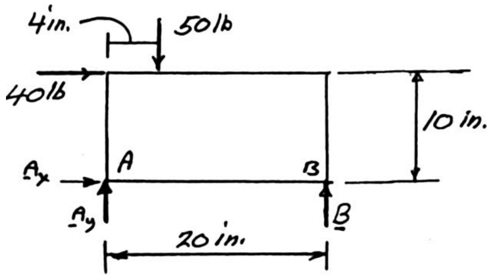
For each of the plates and loadings shown, determine the reactions at *A* and *B*.



(*a*) (*b*)

SOLUTION

(*a*) **Free-Body Diagram:**



$$+\circlearrowleft \Sigma M_A = 0: \quad B(20\text{ in.}) - (50\text{ lb})(4\text{ in.}) - (40\text{ lb})(10\text{ in.}) = 0$$

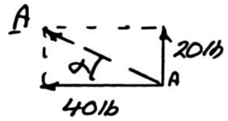
$$B = +30\text{ lb} \qquad \mathbf{B} = 30.0\text{ lb}\uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 40\text{ lb} = 0$$

$$A_x = -40\text{ lb} \qquad \mathbf{A}_x = 40.0\text{ lb} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + B - 50\text{ lb} = 0$$

$$A_y + 30\text{ lb} - 50\text{ lb} = 0$$



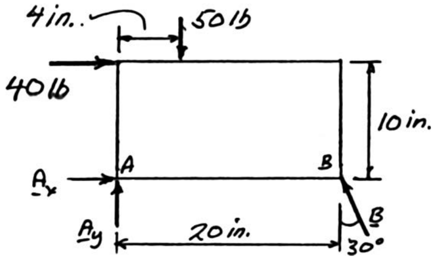
$$A_y = +20\text{ lb} \qquad \mathbf{A}_y = 20.0\text{ lb}\uparrow$$

$$\alpha = 26.56°$$

$$A = 44.72\text{ lb} \qquad \mathbf{A} = 44.7\text{ lb} \nwarrow 26.6° \blacktriangleleft$$

PROBLEM 4.25 (Continued)

(*b*) **Free-Body Diagram:**



$$+\circlearrowleft \Sigma M_A = 0: \quad (B\cos 30°)(20 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) - (50 \text{ lb})(4 \text{ in.}) = 0$$

$B = 34.64$ lb $\qquad \mathbf{B} = 34.6$ lb ∠ 60.0° ◀

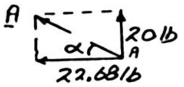
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B\sin 30° + 40 \text{ lb}$$

$$A_x - (34.64 \text{ lb})\sin 30° + 40 \text{ lb} = 0$$

$A_x = -22.68$ lb $\qquad \mathbf{A}_x = 22.68$ lb ←

$$+\uparrow \Sigma F_y = 0: \quad A_y + B\cos 30° - 50 \text{ lb} = 0$$

$$A_y + (34.64 \text{ lb})\cos 30° - 50 \text{ lb} = 0$$



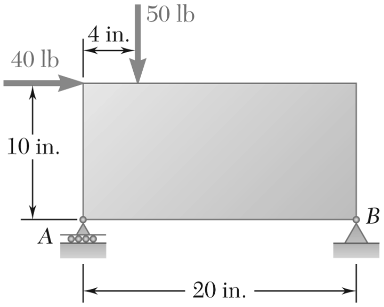
$A_y = +20$ lb $\qquad \mathbf{A}_y = 20.0$ lb ↑

$\alpha = 41.4°$

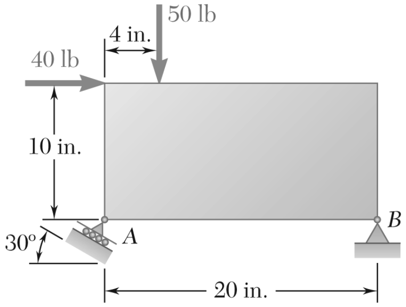
$A = 30.24$ lb $\qquad \mathbf{A} = 30.2$ lb ∠ 41.4° ◀

PROBLEM 4.26

For each of the plates and loadings shown, determine the reactions at *A* and *B*.



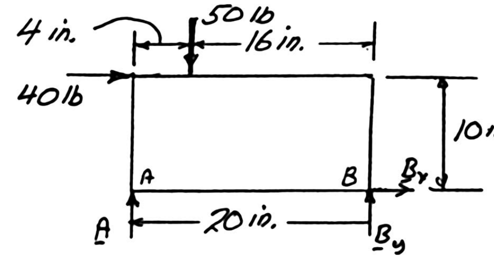
(*a*)



(*b*)

SOLUTION

(*a*) **Free-Body Diagram:**



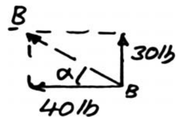
$$+\circlearrowleft \Sigma M_B = 0: \quad A(20 \text{ in.}) + (50 \text{ lb})(16 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) = 0$$

$A = +20$ lb $\qquad$ $\mathbf{A} = 20.0 \text{ lb}\uparrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad 40 \text{ lb} + B_x = 0$

$B_x = -40$ lb $\qquad$ $\mathbf{B}_x = 40 \text{ lb} \longleftarrow$

$+\uparrow \Sigma F_y = 0: \quad A + B_y - 50 \text{ lb} = 0$



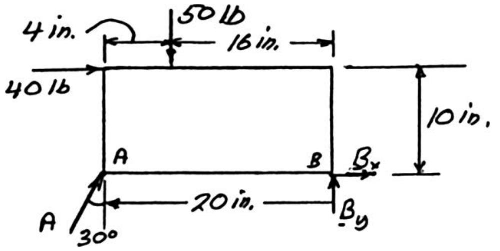
$20 \text{ lb} + B_y - 50 \text{ lb} = 0$

$B_y = +30$ lb $\qquad$ $\mathbf{B}_y = 30 \text{ lb}\uparrow$

$\alpha = 36.87°$ $\qquad$ $B = 50$ lb $\qquad$ $\mathbf{B} = 50.0 \text{ lb} \measuredangle 36.9°$ ◀

PROBLEM 4.26 (Continued)

(*b*)



$$+\circlearrowleft \Sigma M_A = 0: \quad -(A\cos 30°)(20 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) + (50 \text{ lb})(16 \text{ in.}) = 0$$

$A = 23.09$ lb $\qquad$ $\mathbf{A} = 23.1$ lb ∠ 60.0° ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A \sin 30° + 40 \text{ lb} + B_x = 0$$

$$(23.09 \text{ lb}) \sin 30° + 40 \text{ lb} + 8_x = 0$$

$B_x = -51.55$ lb $\qquad$ $\mathbf{B}_x = 51.55$ lb ←

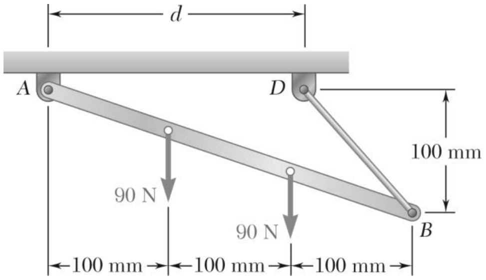
$$+\uparrow \Sigma F_y = 0: \quad A \cos 30° + B_y - 50 \text{ lb} = 0$$

$$(23.09 \text{ lb}) \cos 30° + B_y - 50 \text{ lb} = 0$$

$B_y = +30$ lb $\qquad$ $\mathbf{B}_y = 30$ lb ↑

$\alpha = 30.2°$

$B = 59.64$ lb $\qquad$ $\mathbf{B} = 59.6$ lb ⦪ 30.2° ◀

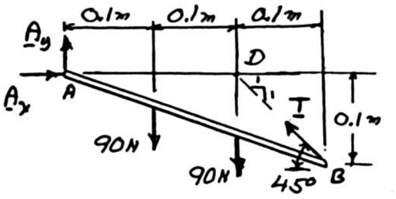


PROBLEM 4.27

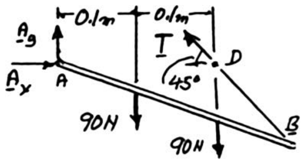
A rod *AB* hinged at *A* and attached at *B* to cable *BD* supports the loads shown. Knowing that $d = 200$ mm, determine (*a*) the tension in cable *BD*, (*b*) the reaction at *A*.

SOLUTION

Free-Body Diagram:



(*a*) Move **T** along *BD* until it acts at Point *D*.



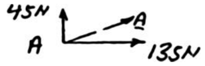
$$+\circlearrowleft \Sigma M_A = 0: \quad (T\sin 45°)(0.2 \text{ m}) + (90 \text{ N})(0.1 \text{ m}) + (90 \text{ N})(0.2 \text{ m}) = 0$$

$$T = 190.919 \text{ N} \qquad T = 190.9 \text{ N} \blacktriangleleft$$

(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (190.919 \text{ N})\cos 45° = 0$$

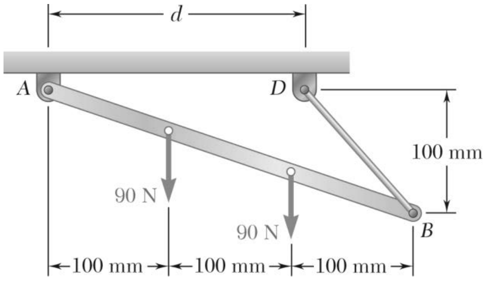
$$A_x = +135.0 \text{ N} \qquad \mathbf{A}_x = 135.0 \text{ N} \longrightarrow$$



$$+\uparrow \Sigma F_y = 0: \quad A_y - 90 \text{ N} - 90 \text{ N} + (190.919 \text{ N})\sin 45° = 0$$

$$A_y = +45.0 \text{ N} \qquad \mathbf{A}_y = 45.0 \text{ N}\uparrow$$

$$\mathbf{A} = 142.3 \text{ N} \measuredangle 18.43° \blacktriangleleft$$

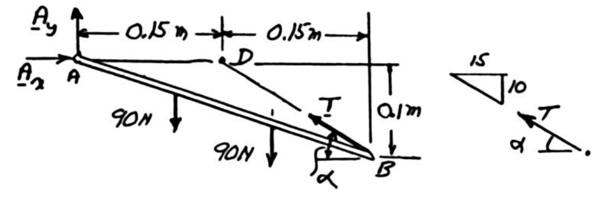


PROBLEM 4.28

A rod *AB*, hinged at *A* and attached at *B* to cable *BD*, supports the loads shown. Knowing that $d = 150$ mm, determine (*a*) the tension in cable *BD*, (*b*) the reaction at *A*.

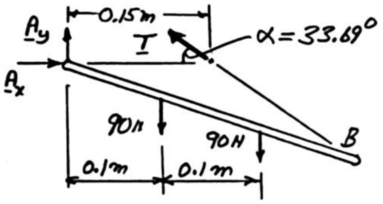
SOLUTION

Free-Body Diagram:



$$\tan\alpha = \frac{10}{15}; \quad \alpha = 33.690°$$

(*a*) Move **T** along *BD* until it acts at Point *D*.



$$+\circlearrowleft \Sigma M_A = 0: \quad (T\sin 33.690°)(0.15 \text{ m}) - (90 \text{ N})(0.1 \text{ m}) - (90 \text{ N})(0.2 \text{ m}) = 0$$

$$T = 324.50 \text{ N} \qquad T = 324 \text{ N} ◀$$

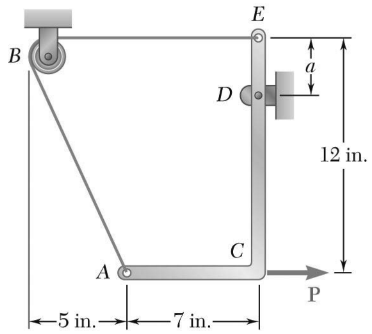
(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (324.50 \text{ N})\cos 33.690° = 0$$

$$A_x = +270 \text{ N} \qquad \mathbf{A}_x = 270 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 90 \text{ N} - 90 \text{ N} + (324.50 \text{ N})\sin 33.690° = 0$$

$$A_y = 0 \qquad \mathbf{A} = 270 \text{ N} \longrightarrow ◀$$

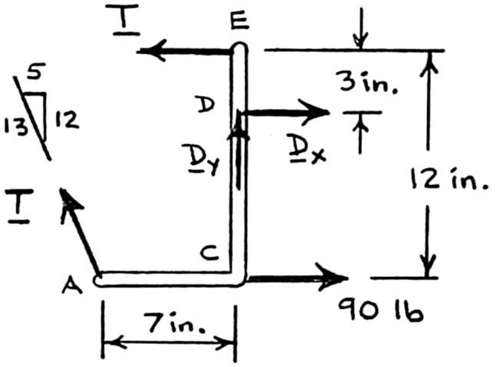


PROBLEM 4.29

A force **P** of magnitude 90 lb is applied to member *ACDE*, which is supported by a frictionless pin at *D* and by the cable *ABE*. Since the cable passes over a small pulley at *B*, the tension may be assumed to be the same in portions *AB* and *BE* of the cable. For the case when $a = 3$ in., determine (*a*) the tension in the cable, (*b*) the reaction at *D*.

SOLUTION

Free-Body Diagram:



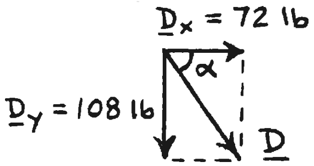
(*a*) $+\circlearrowleft \Sigma M_D = 0$: $(90\text{ lb})(9\text{ in.}) - \frac{5}{13}T(9\text{ in.}) - \frac{12}{13}T(7\text{ in.}) + T(3\text{ in.}) = 0$

$T = 117$ lb

$T = 117.0$ lb ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0$: $D_x - 117\text{ lb} - \frac{5}{13}(117\text{ lb}) + 90 = 0$

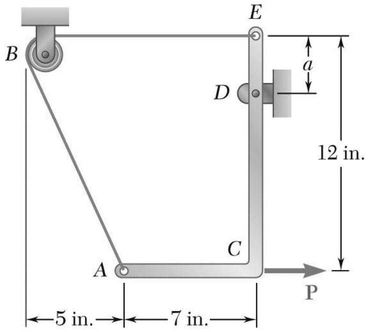
$D_x = +72$ lb



$+\uparrow \Sigma F_y = 0$: $D_y + \frac{12}{13}(117\text{ lb}) = 0$

$D_y = -108$ lb

$\mathbf{D} = 129.8$ lb ⦩ 56.3° ◀



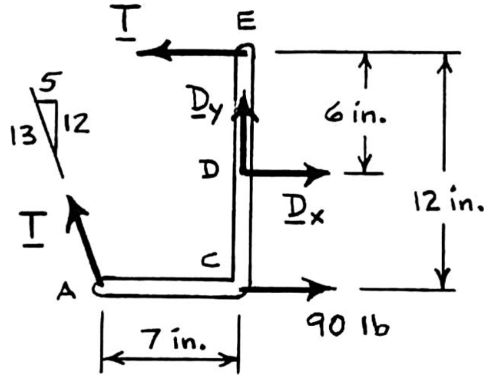
PROBLEM 4.30

Solve Problem 4.29 for $a = 6$ in.

PROBLEM 4.29 A force **P** of magnitude 90 lb is applied to member *ACDE*, which is supported by a frictionless pin at *D* and by the cable *ABE*. Since the cable passes over a small pulley at *B*, the tension may be assumed to be the same in portions *AB* and *BE* of the cable. For the case when $a = 3$ in., determine (*a*) the tension in the cable, (*b*) the reaction at *D*.

SOLUTION

Free-Body Diagram:



(*a*)

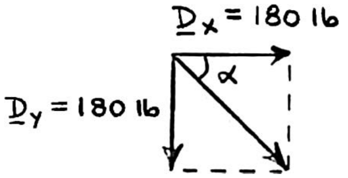
$$+\circlearrowleft \Sigma M_D = 0: \quad (90 \text{ lb})(6 \text{ in.}) - \frac{5}{13}T(6 \text{ in.}) - \frac{12}{13}T(7 \text{ in.}) + T(6 \text{ in.}) = 0$$

$T = 195$ lb　　$T = 195.0$ lb ◀

(*b*)

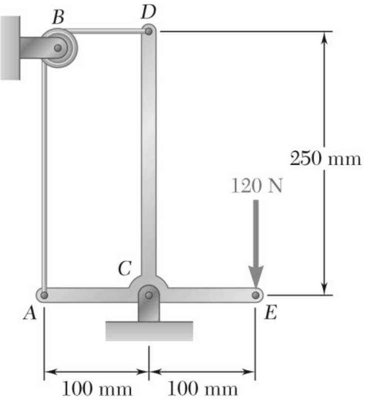
$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - 195 \text{ lb} - \frac{5}{13}(195 \text{ lb}) + 90 = 0$$

$D_x = +180$ lb



$$+\uparrow \Sigma F_y = 0: \quad D_y + \frac{12}{13}(195 \text{ lb}) = 0$$

$D_y = -180$ lb　　$\mathbf{D} = 255$ lb $\searrow$ 45.0° ◀

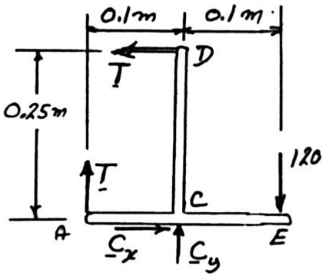


PROBLEM 4.31

Neglecting friction, determine the tension in cable *ABD* and the reaction at support *C*.

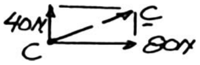
SOLUTION

Free-Body Diagram:



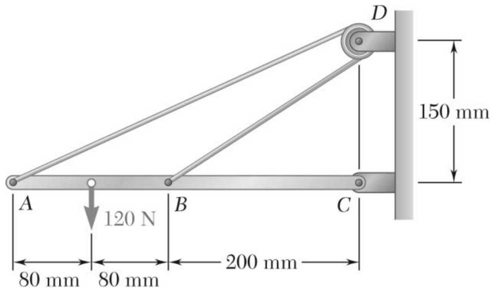
$+\circlearrowleft \Sigma M_C = 0$: $T(0.25\text{ m}) - T(0.1\text{ m}) - (120\text{ N})(0.1\text{ m}) = 0$ $T = 80.0\text{ N}$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $C_x - 80\text{ N} = 0$ $C_x = +80\text{ N}$ $\mathbf{C}_x = 80.0\text{ N} \longrightarrow$



$+\uparrow \Sigma F_y = 0$: $C_y - 120\text{ N} + 80\text{ N} = 0$ $C_y = +40\text{ N}$ $\mathbf{C}_y = 40.0\text{ N} \uparrow$

$\mathbf{C} = 89.4\text{ N} \measuredangle 26.6^\circ$ ◀



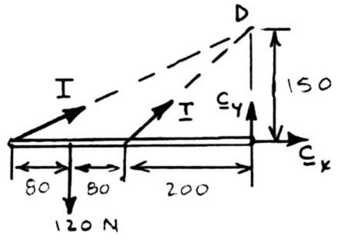
PROBLEM 4.32

Neglecting friction and the radius of the pulley, determine (*a*) the tension in cable *ADB*, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:

Dimensions in mm



Geometry:

Distance: $AD = \sqrt{(0.36)^2 + (0.150)^2} = 0.39 \text{ m}$

Distance: $BD = \sqrt{(0.2)^2 + (0.15)^2} = 0.25 \text{ m}$

Equilibrium for beam:

(*a*) $+\circlearrowleft \Sigma M_C = 0$: $(120 \text{ N})(0.28 \text{ m}) - \left(\dfrac{0.15}{0.39}T\right)(0.36 \text{ m}) - \left(\dfrac{0.15}{0.25}T\right)(0.2 \text{ m}) = 0$

$T = 130.000 \text{ N}$

or $T = 130.0 \text{ N}$ ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0$: $C_x + \left(\dfrac{0.36}{0.39}\right)(130.000 \text{ N}) + \left(\dfrac{0.2}{0.25}\right)(130.000 \text{ N}) = 0$

$C_x = -224.00 \text{ N}$

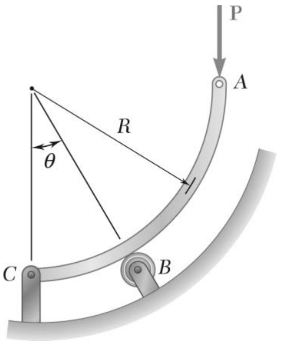
$+\uparrow \Sigma F_y = 0$: $C_y + \left(\dfrac{0.15}{0.39}\right)(130.00 \text{ N}) + \left(\dfrac{0.15}{0.25}\right)(130.00 \text{ N}) - 120 \text{ N} = 0$

$C_y = -8.0000 \text{ N}$

Thus, $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-224)^2 + (-8)^2} = 224.14 \text{ N}$

and $\theta = \tan^{-1}\dfrac{C_y}{C_x} = \tan^{-1}\dfrac{8}{224} = 2.0454°$

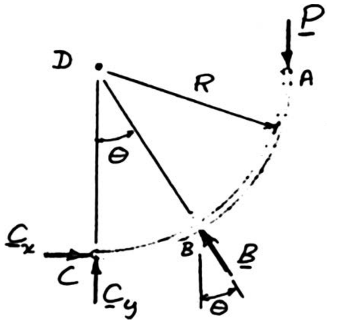
$\mathbf{C} = 224 \text{ N} \swarrow 2.05°$ ◀



PROBLEM 4.33

Rod *ABC* is bent in the shape of an arc of circle of radius *R*. Knowing the $\theta = 30°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION



Free-Body Diagram: $+\circlearrowleft \Sigma M_D = 0: \quad C_x(R) - P(R) = 0$

$$C_x = +P$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - B\sin\theta = 0$$

$$P - B\sin\theta = 0$$

$$B = P/\sin\theta$$

$$\mathbf{B} = \frac{P}{\sin\theta} \measuredangle\, \theta$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + B\cos\theta - P = 0$$

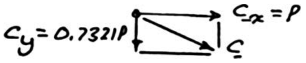
$$C_y + (P/\sin\theta)\cos\theta - P = 0$$

$$C_y = P\left(1 - \frac{1}{\tan\theta}\right)$$

For $\theta = 30°$,

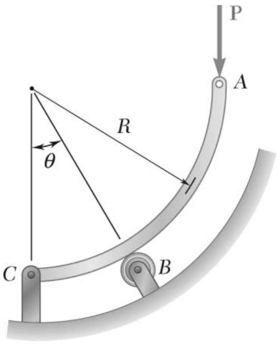
(*a*) $B = P/\sin 30° = 2P$ $\quad\mathbf{B} = 2P \measuredangle\, 60.0°$ ◀

(*b*) $C_x = +P \qquad C_x = P \longrightarrow$



$C_y = P(1 - 1/\tan 30°) = -0.732/P$ $\quad\mathbf{C}_y = 0.7321P\downarrow$

$\mathbf{C} = 1.239P \measuredangle\, 36.2°$ ◀



PROBLEM 4.34

Rod *ABC* is bent in the shape of an arc of circle of radius *R*. Knowing that $\theta = 60°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION

See the solution to Problem 4.33 for the free-body diagram and analysis leading to the following expressions:

$$C_x = +P$$

$$C_y = P\left(1 - \frac{1}{\tan\theta}\right)$$

$$B = \frac{P}{\sin\theta}$$

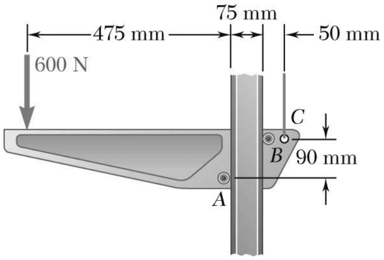
For $\theta = 60°$,

(*a*) $B = P/\sin 60° = 1.1547P$ $\mathbf{B} = 1.155P$ ↘ 30.0° ◀

(*b*) $C_x = +P$ $C_x = P \longrightarrow$

$C_y = P(1 - 1/\tan 60°) = +0.4226P$ $\mathbf{C}_y = 0.4226P \downarrow$

$\mathbf{C} = 1.086P$ ↗ 22.9° ◀

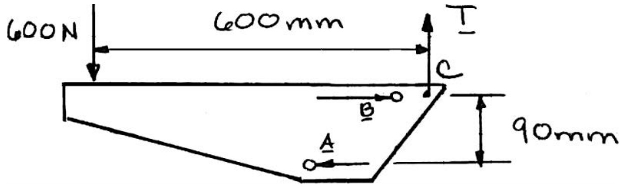


PROBLEM 4.35

A movable bracket is held at rest by a cable attached at C and by frictionless rollers at A and B. For the loading shown, determine (*a*) the tension in the cable, (*b*) the reactions at A and B.

SOLUTION

Free-Body Diagram:



(*a*) $+\uparrow \Sigma F_y = 0: \quad T - 600\text{ N} = 0$

$T = 600\text{ N}$ ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad B - A = 0 \qquad \therefore \quad B = A$

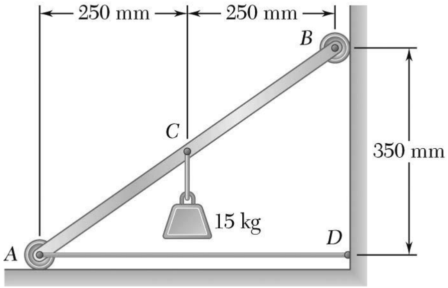
Note that the forces shown form two couples.

$$+\circlearrowleft \Sigma M = 0: \quad (600\text{ N})(600\text{ mm}) - A(90\text{ mm}) = 0$$

$$A = 4000\text{ N}$$

$$\therefore \quad B = 4000\text{ N}$$

$\mathbf{A} = 4.00\text{ kN} \longleftarrow; \ \mathbf{B} = 4.00\text{ kN} \longrightarrow$ ◀

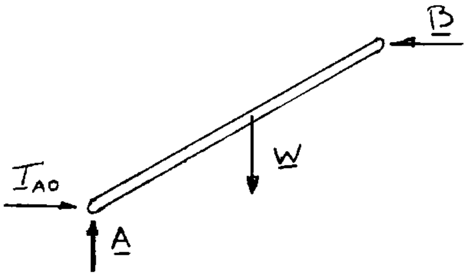


PROBLEM 4.36

A light bar *AB* supports a 15-kg block at its midpoint *C*. Rollers at *A* and *B* rest against frictionless surfaces, and a horizontal cable *AD* is attached at *A*. Determine (*a*) the tension in cable *AD*, (*b*) the reactions at *A* and *B*.

SOLUTION

Free-Body Diagram:



$$W = (15\text{ kg})(9.81\text{ m/s}^2)$$
$$= 147.150\text{ N}$$

(*a*) $\xrightarrow{+} \Sigma F_x = 0$: $T_{AD} - 105.107\text{ N} = 0$

$T_{AD} = 105.1\text{ N}$ ◀

(*b*) $+\uparrow \Sigma F_y = 0$: $A - W = 0$

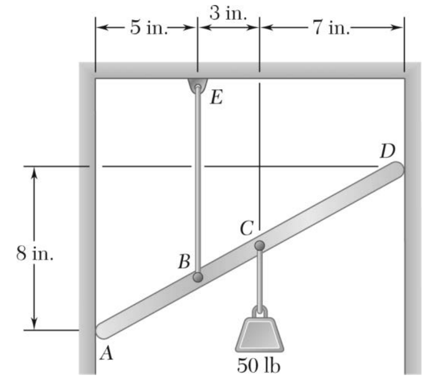
$A - 147.150\text{ N} = 0$

$\mathbf{A} = 147.2\text{ N}\uparrow$ ◀

$+\circlearrowright \Sigma M_A = 0$: $B(350\text{ mm}) - (147.150\text{ N})(250\text{ mm}) = 0$

$B = 105.107\text{ N}$

$\mathbf{B} = 105.1\text{ N} \longleftarrow$ ◀

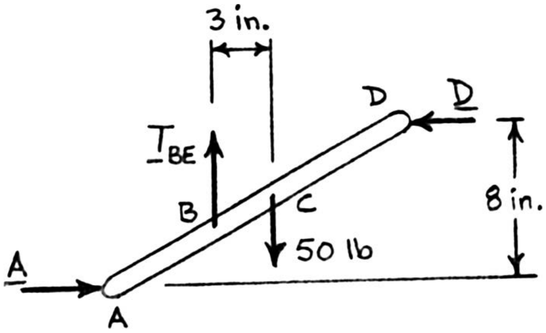


PROBLEM 4.37

A light bar *AD* is suspended from a cable *BE* and supports a 50-lb block at *C*. The ends *A* and *D* of the bar are in contact with frictionless vertical walls. Determine the tension in cable *BE* and the reactions at *A* and *D*.

SOLUTION

Free-Body Diagram:



$\Sigma F_x = 0$: $A = D$

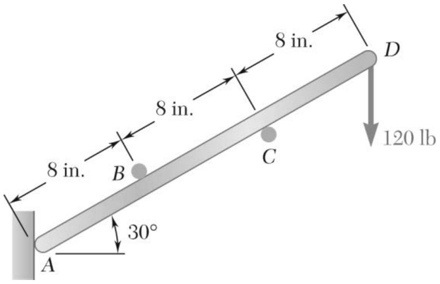
$\Sigma F_y = 0$: $T_{BE} = 50.0\text{ lb}$ ◀

We note that the forces shown form two couples.

$$+\circlearrowleft \Sigma M = 0: \quad A(8\text{ in.}) - (50\text{ lb})(3\text{ in.}) = 0$$

$$A = 18.75\text{ lb}$$

$\mathbf{A} = 18.75\text{ lb} \longrightarrow$ $\mathbf{D} = 18.75\text{ lb} \longleftarrow$ ◀

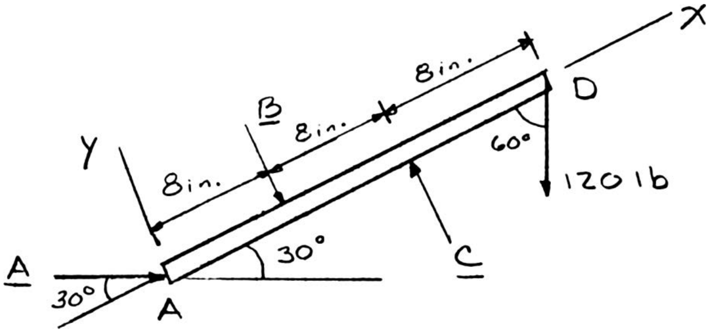


PROBLEM 4.38

A light rod *AD* is supported by frictionless pegs at *B* and *C* and rests against a frictionless wall at *A*. A vertical 120-lb force is applied at *D*. Determine the reactions at *A*, *B*, and *C*.

SOLUTION

Free-Body Diagram:



$\nearrow \Sigma F_x = 0: \quad A\cos 30° - (120\text{ lb})\cos 60° = 0$

$A = 69.28\text{ lb}$ $\qquad \mathbf{A} = 69.3\text{ lb} \longrightarrow$ ◀

$+\circlearrowleft \Sigma M_B = 0: \quad C(8\text{ in.}) - (120\text{ lb})(16\text{ in.})\cos 30°$
$+ (69.28\text{ lb})(8\text{ in.})\sin 30° = 0$

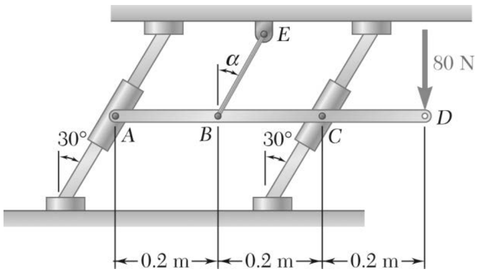
$C = 173.2\text{ lb}$ $\qquad \mathbf{C} = 173.2\text{ lb} \measuredangle 60.0°$ ◀

$+\circlearrowleft \Sigma M_C = 0: \quad B(8\text{ in.}) - (120\text{ lb})(8\text{ in.})\cos 30°$
$+ (69.28\text{ lb})(16\text{ in.})\sin 30° = 0$

$B = 34.6\text{ lb}$ $\qquad \mathbf{B} = 34.6\text{ lb} \measuredangle 60.0°$ ◀

Check: $\nwarrow \Sigma F_y = 0: \quad 173.2 - 34.6 - (69.28)\sin 30° - (120)\sin 60° = 0$

$0 = 0$ (check)

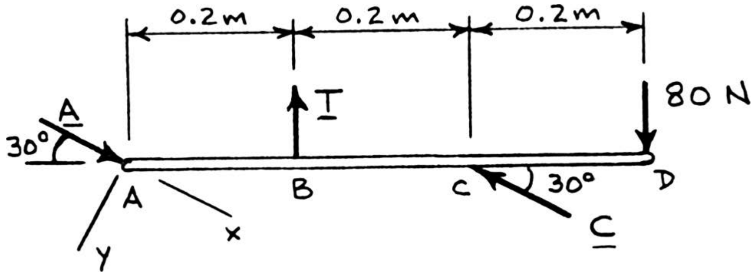


PROBLEM 4.39

Bar *AD* is attached at *A* and *C* to collars that can move freely on the rods shown. If the cord *BE* is vertical ($\alpha = 0$), determine the tension in the cord and the reactions at *A* and *C*.

SOLUTION

Free-Body Diagram:



$+\swarrow \Sigma F_y = 0: \quad -T\cos 30° + (80\text{ N})\cos 30° = 0$

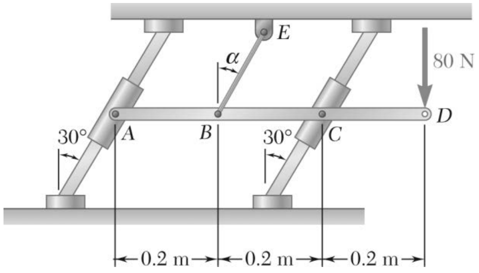
$T = 80\text{ N}$ $\qquad T = 80.0\text{ N}$ ◀

$+\circlearrowleft \Sigma M_C = 0: \quad (A\sin 30°)(0.4\text{ m}) - (80\text{ N})(0.2\text{ m}) - (80\text{ N})(0.2\text{ m}) = 0$

$A = +160\text{ N}$ $\qquad \mathbf{A} = 160.0\text{ N} \measuredangle 30.0°$ ◀

$+\circlearrowleft \Sigma M_A = 0: \quad (80\text{ N})(0.2\text{ m}) - (80\text{ N})(0.6\text{ m}) + (C\sin 30°)(0.4\text{ m}) = 0$

$C = +160\text{ N}$ $\qquad \mathbf{C} = 160.0\text{ N} \measuredangle 30.0°$ ◀



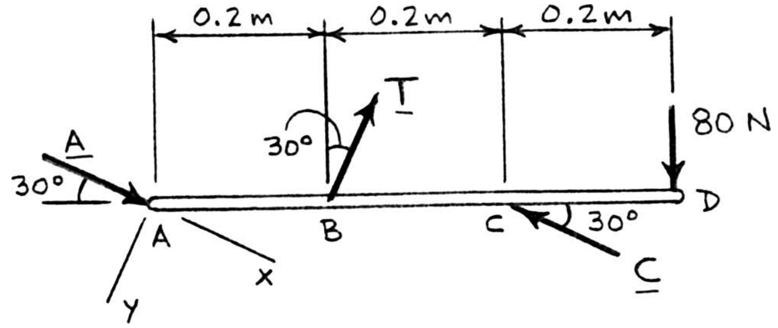
PROBLEM 4.40

Solve Problem 4.39 if the cord BE is parallel to the rods ($\alpha = 30°$).

PROBLEM 4.39 Bar *AD* is attached at *A* and *C* to collars that can move freely on the rods shown. If the cord *BE* is vertical ($\alpha = 0$), determine the tension in the cord and the reactions at *A* and *C*.

SOLUTION

Free-Body Diagram:



$$+\swarrow \Sigma F_y = 0: \quad -T + (80\text{ N})\cos 30° = 0$$

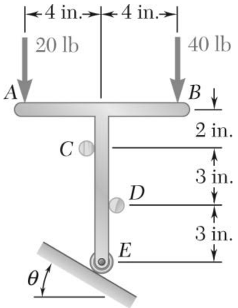
$$T = 69.282\text{ N} \qquad\qquad T = 69.3\text{ N} ◀$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -(69.282\text{ N})\cos 30°(0.2\text{ m})$$
$$-(80\text{ N})(0.2\text{ m}) + (A\sin 30°)(0.4\text{ m}) = 0$$

$$A = +140.000\text{ N} \qquad \mathbf{A} = 140.0\text{ N} \searrow 30.0° ◀$$

$$+\circlearrowleft \Sigma M_A = 0: \quad +(69.282\text{ N})\cos 30°(0.2\text{ m})$$
$$-(80\text{ N})(0.6\text{ m}) + (C\sin 30°)(0.4\text{ m}) = 0$$

$$C = +180.000\text{ N} \qquad \mathbf{C} = 180.0\text{ N} \nwarrow 30.0° ◀$$

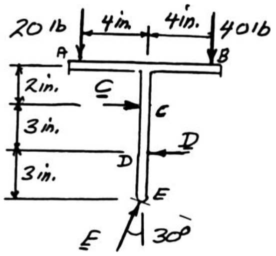


PROBLEM 4.41

The T-shaped bracket shown is supported by a small wheel at *E* and pegs at *C* and *D*. Neglecting the effect of friction, determine the reactions at *C*, *D*, and *E* when $\theta = 30°$.

SOLUTION

Free-Body Diagram:



$$+\uparrow \Sigma F_y = 0: \quad E\cos 30° - 20 - 40 = 0$$

$$E = \frac{60 \text{ lb}}{\cos 30°} = 69.282 \text{ lb}$$

$\mathbf{E} = 69.3$ lb ⦣ 60.0° ◀

$$+\circlearrowleft \Sigma M_D = 0: \quad (20 \text{ lb})(4 \text{ in.}) - (40 \text{ lb})(4 \text{ in.})$$
$$- C(3 \text{ in.}) + E\sin 30°(3 \text{ in.}) = 0$$
$$-80 - 3C + 69.282(0.5)(3) = 0$$

$$C = 7.9743 \text{ lb}$$

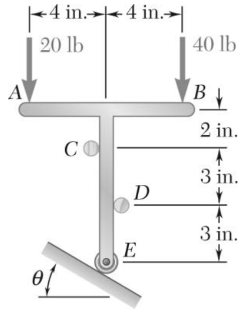
$\mathbf{C} = 7.97$ lb → ◀

$$\Sigma F_x = 0: \quad E\sin 30° + C - D = 0$$

$$(69.282 \text{ lb})(0.5) + 7.9743 \text{ lb} - D = 0$$

$$D = 42.615 \text{ lb}$$

$\mathbf{D} = 42.6$ lb ← ◀

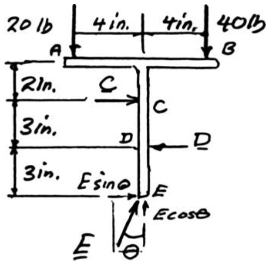


PROBLEM 4.42

The T-shaped bracket shown is supported by a small wheel at *E* and pegs at *C* and *D*. Neglecting the effect of friction, determine (*a*) the smallest value of θ for which the equilibrium of the bracket is maintained, (*b*) the corresponding reactions at *C*, *D*, and *E*.

SOLUTION

Free-Body Diagram:



$$+\uparrow \Sigma F_y = 0: \quad E\cos\theta - 20 - 40 = 0$$

$$E = \frac{60}{\cos\theta} \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (20\text{ lb})(4\text{ in.}) - (40\text{ lb})(4\text{ in.}) - C(3\text{ in.}) + \left(\frac{60}{\cos\theta}\sin\theta\right)3\text{ in.} = 0$$

$$C = \frac{1}{3}(180\tan\theta - 80)$$

(*a*) For $C = 0$, $\quad 180\tan\theta = 80$

$$\tan\theta = \frac{4}{9} \quad \theta = 23.962^\circ$$

$\theta = 24.0^\circ$ ◀

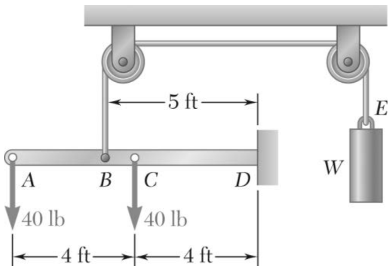
From Eq. (1): $\quad E = \dfrac{60}{\cos 23.962^\circ} = 65.659$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -D + C + E\sin\theta = 0$$

$$D = (65.659)\sin 23.962 = 26.666\text{ lb}$$

(*b*) $\mathbf{C} = 0 \quad \mathbf{D} = 26.7\text{ lb} \longleftarrow$

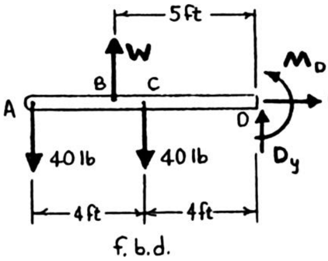
$\mathbf{E} = 65.7\text{ lb} \measuredangle\ 66.0^\circ$ ◀



PROBLEM 4.43

Beam AD carries the two 40-lb loads shown. The beam is held by a fixed support at D and by the cable BE that is attached to the counterweight W. Determine the reaction at D when (*a*) $W = 100$ lb, (*b*) $W = 90$ lb.

SOLUTION



(*a*) $W = 100 \text{ lb}$

From F.B.D. of beam AD:

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x = 0$$

$$+\!\uparrow \Sigma F_y = 0: \quad D_y - 40 \text{ lb} - 40 \text{ lb} + 100 \text{ lb} = 0$$

$$D_y = -20.0 \text{ lb} \qquad \text{or} \qquad \mathbf{D} = 20.0 \text{ lb} \downarrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M_D - (100 \text{ lb})(5 \text{ ft}) + (40 \text{ lb})(8 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) = 0$$

$$M_D = 20.0 \text{ lb}\cdot\text{ft} \qquad \text{or} \qquad \mathbf{M}_D = 20.0 \text{ lb}\cdot\text{ft} \circlearrowleft \blacktriangleleft$$

(*b*) $W = 90 \text{ lb}$

From F.B.D. of beam AD:

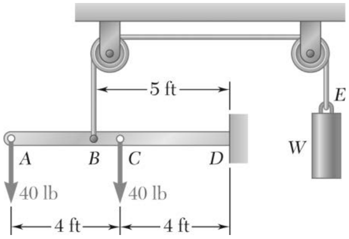
$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x = 0$$

$$+\!\uparrow \Sigma F_y = 0: \quad D_y + 90 \text{ lb} - 40 \text{ lb} - 40 \text{ lb} = 0$$

$$D_y = -10.00 \text{ lb} \qquad \text{or} \qquad \mathbf{D} = 10.00 \text{ lb} \downarrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M_D - (90 \text{ lb})(5 \text{ ft}) + (40 \text{ lb})(8 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) = 0$$

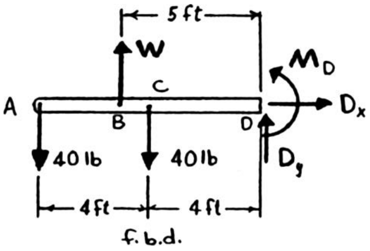
$$M_D = -30.0 \text{ lb}\cdot\text{ft} \qquad \text{or} \qquad \mathbf{M}_D = 30.0 \text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$



PROBLEM 4.44

For the beam and loading shown, determine the range of values of W for which the magnitude of the couple at D does not exceed 40 lb · ft.

SOLUTION



For $W_{\min}$, $\quad M_D = -40 \text{ lb}\cdot\text{ft}$

From F.B.D. of beam AD: $\quad +\circlearrowleft \Sigma M_D = 0\text{:} \quad (40 \text{ lb})(8 \text{ ft}) - W_{\min}(5 \text{ ft})$

$$+ (40 \text{ lb})(4 \text{ ft}) - 40 \text{ lb}\cdot\text{ft} = 0$$

$$W_{\min} = 88.0 \text{ lb}$$

For $W_{\max}$, $\quad M_D = 40 \text{ lb}\cdot\text{ft}$

From F.B.D. of beam AD: $\quad +\circlearrowleft \Sigma M_D = 0\text{:} \quad (40 \text{ lb})(8 \text{ ft}) - W_{\max}(5 \text{ ft})$

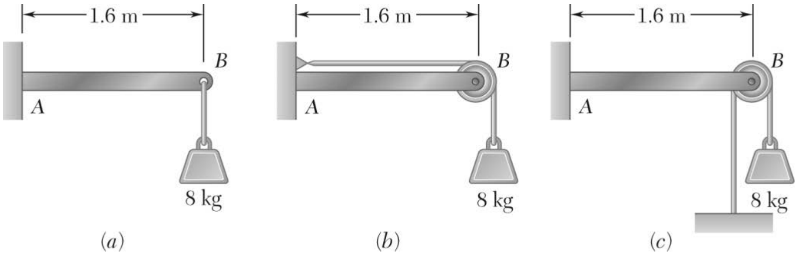
$$+ (40 \text{ lb})(4 \text{ ft}) + 40 \text{ lb}\cdot\text{ft} = 0$$

$$W_{\max} = 104.0 \text{ lb}$$

or $88.0 \text{ lb} \le W \le 104.0 \text{ lb}$ ◀

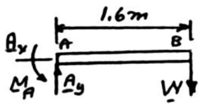
PROBLEM 4.45

An 8-kg mass can be supported in the three different ways shown. Knowing that the pulleys have a 100-mm radius, determine the reaction at *A* in each case.



SOLUTION

$$W = mg = (8\text{ kg})(9.81\text{ m/s}^2) = 78.480\text{ N}$$



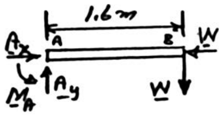
(*a*) $\Sigma F_x = 0{:}\quad A_x = 0$

$+\uparrow \Sigma F_y = 0{:}\quad A_y - W = 0$ $\qquad \mathbf{A}_y = 78.480\text{ N}\uparrow$

$+\circlearrowleft \Sigma M_A = 0{:}\quad M_A - W(1.6\text{ m}) = 0$

$M_A = +(78.480\text{ N})(1.6\text{ m})$ $\qquad \mathbf{M}_A = 125.568\text{ N}\cdot\text{m}\circlearrowleft$

$\mathbf{A} = 78.5\text{ N}\uparrow$ $\qquad \mathbf{M}_A = 125.6\text{ N}\cdot\text{m}\circlearrowleft$ ◀



(*b*) $\overset{+}{\rightarrow} \Sigma F_x = 0{:}\quad A_x - W = 0$ $\qquad \mathbf{A}_x = 78.480\uparrow$

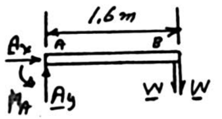
$+\uparrow \Sigma F_y = 0{:}\quad A_y - W = 0$ $\qquad \mathbf{A}_y = 78.480 \longleftarrow$

$$\mathbf{A} = (78.480\text{ N})\sqrt{2} = 110.987\text{ N} \measuredangle 45°$$

$+\circlearrowleft \Sigma M_A = 0{:}\quad M_A - W(1.6\text{ m}) = 0$

$M_A = +(78.480\text{ N})(1.6\text{ m})$ $\qquad \mathbf{M}_A = 125.568\text{ N}\cdot\text{m}\circlearrowleft$

$\mathbf{A} = 111.0\text{ N} \measuredangle 45°$ $\qquad \mathbf{M}_A = 125.6\text{ N}\cdot\text{m}\circlearrowleft$ ◀



(*c*) $\Sigma F_x = 0{:}\quad A_x = 0$

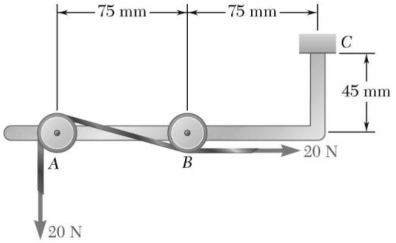
$+\uparrow \Sigma F_y = 0{:}\quad A_y - 2W = 0$

$$A_y = 2W = 2(78.480\text{ N}) = 156.960\text{ N}\uparrow$$

$+\circlearrowleft \Sigma M_A = 0{:}\quad M_A - 2W(1.6\text{ m}) = 0$

$M_A = +2(78.480\text{ N})(1.6\text{ m})$ $\qquad \mathbf{M}_A = 251.14\text{ N}\cdot\text{m}\circlearrowleft$

$\mathbf{A} = 157.0\text{ N}\uparrow$ $\qquad \mathbf{M}_A = 251\text{ N}\cdot\text{m}\circlearrowleft$ ◀

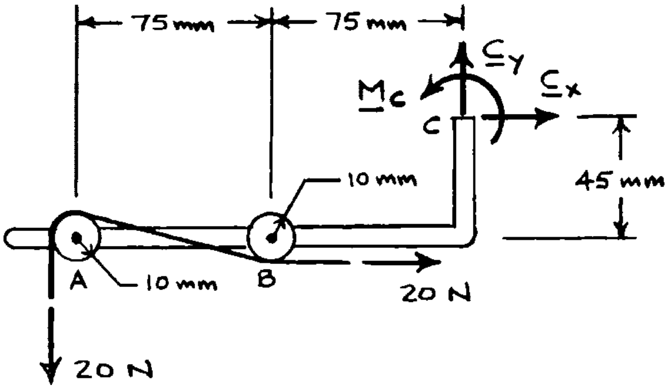


PROBLEM 4.46

A tension of 20 N is maintained in a tape as it passes through the support system shown. Knowing that the radius of each pulley is 10 mm, determine the reaction at *C*.

SOLUTION

Free-Body Diagram:



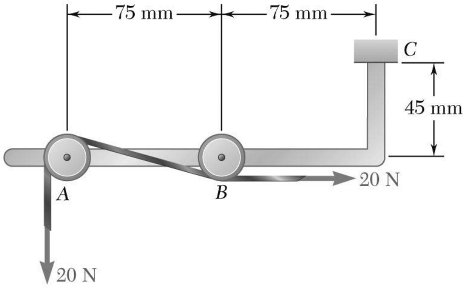
$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (20\text{ N}) = 0 \qquad C_x = -20\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - (20\text{ N}) = 0 \qquad C_y = +20\text{ N}$$

$$\mathbf{C} = 28.3\text{ N} \searrow 45.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (20\text{ N})(0.160\text{ m}) + (20\text{ N})(0.055\text{ m}) = 0$$

$$M_C = -4.30\text{ N}\cdot\text{m} \qquad \mathbf{M}_C = 4.30\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



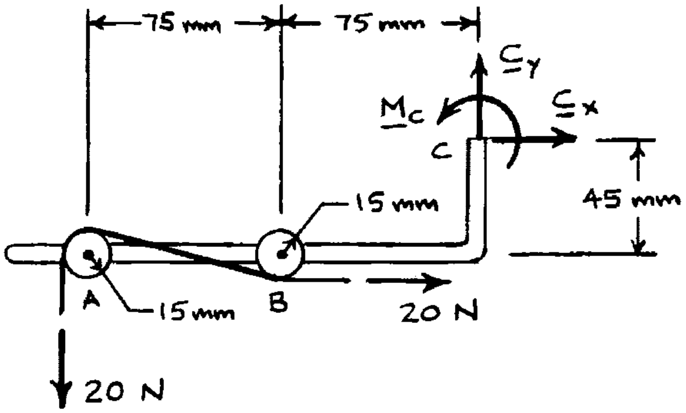
PROBLEM 4.47

Solve Problem 4.46, assuming that 15-mm-radius pulleys are used.

PROBLEM 4.46 A tension of 20 N is maintained in a tape as it passes through the support system shown. Knowing that the radius of each pulley is 10 mm, determine the reaction at *C*.

SOLUTION

Free-Body Diagram:



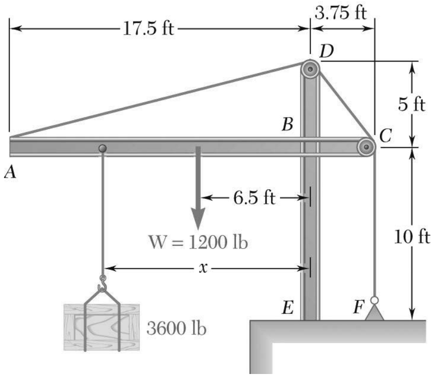
$\xrightarrow{+} \Sigma F_x = 0\text{:} \quad C_x + (20\text{ N}) = 0 \qquad C_x = -20\text{ N}$

$+\!\uparrow \Sigma F_y = 0\text{:} \quad C_y - (20\text{ N}) = 0 \qquad C_y = +20\text{ N}$

C = 28.3 N ⦣ 45.0° ◀

$+\circlearrowleft \Sigma M_C = 0\text{:} \quad M_C + (20\text{ N})(0.165\text{ m}) + (20\text{ N})(0.060\text{ m}) = 0$

$M_C = -4.50\text{ N}\cdot\text{m}$ $\qquad \mathbf{M}_C = 4.50\text{ N}\cdot\text{m} \circlearrowright$ ◀

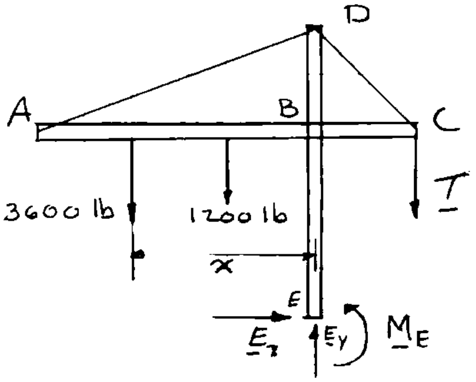


PROBLEM 4.48

The rig shown consists of a 1200-lb horizontal member *ABC* and a vertical member *DBE* welded together at *B*. The rig is being used to raise a 3600-lb crate at a distance $x = 12$ ft from the vertical member *DBE*. If the tension in the cable is 4 kips, determine the reaction at *E*, assuming that the cable is (*a*) anchored at *F* as shown in the figure, (*b*) attached to the vertical member at a point located 1 ft above *E*.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft M_E = 0: \quad M_E + (3600\text{ lb})x + (1200\text{ lb})(6.5\text{ ft}) - T(3.75\text{ ft}) = 0$$

$$M_E = 3.75T - 3600x - 7800 \qquad (1)$$

(*a*) For $x = 12$ ft and $T = 4000$ lbs,

$$M_E = 3.75(4000) - 3600(12) - 7800$$
$$= 36{,}000\text{ lb}\cdot\text{ft}$$

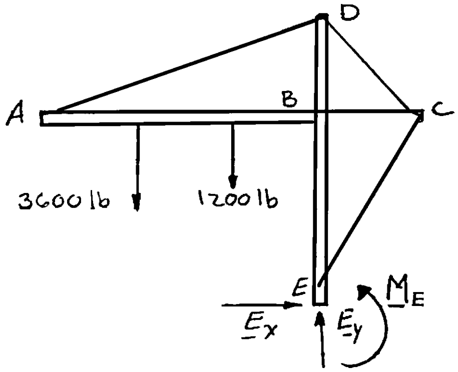
$$\xrightarrow{+} \Sigma F_x = 0 \quad \therefore \quad E_x = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad E_y - 3600\text{ lb} - 1200\text{ lb} - 4000 = 0$$

$$E_y = 8800\text{ lb}$$

$$\mathbf{E} = 8.80\text{ kips}\uparrow;\ \mathbf{M}_E = 36.0\text{ kip}\cdot\text{ft}\circlearrowright \blacktriangleleft$$

PROBLEM 4.48 (Continued)



(*b*) $$+\circlearrowleft \Sigma M_E = 0: \quad M_E + (3600 \text{ lb})(12 \text{ ft}) + (1200 \text{ lb})(6.5 \text{ ft}) = 0$$

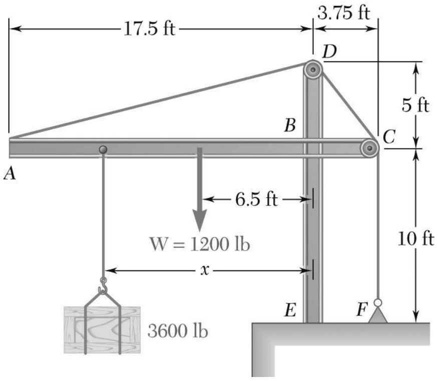
$$M_E = -51{,}000 \text{ lb} \cdot \text{ft}$$

$$\xrightarrow{+} \Sigma F_x = 0 \quad \therefore \quad E_x = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad E_y - 3600 \text{ lb} - 1200 \text{ lb} = 0$$

$$E_y = 4800 \text{ lb}$$

$\mathbf{E} = 4.80 \text{ kips} \uparrow; \ \mathbf{M}_E = 51.0 \text{ kip} \cdot \text{ft} \circlearrowright$ ◀

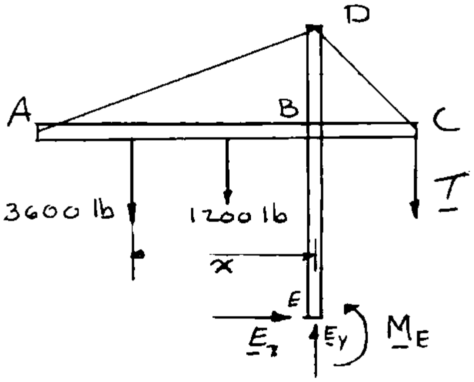


PROBLEM 4.49

For the rig and crate of Prob. 4.48, and assuming that cable is anchored at *F* as shown, determine (*a*) the required tension in cable *ADCF* if the maximum value of the couple at *E* as *x* varies from 1.5 to 17.5 ft is to be as small as possible, (*b*) the corresponding maximum value of the couple.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft M_E = 0: \quad M_E + (3600 \text{ lb})x + (1200 \text{ lb})(6.5 \text{ ft}) - T(3.75 \text{ ft}) = 0$$

$$M_E = 3.75T - 3600x - 7800 \tag{1}$$

For $x = 1.5$ ft, Eq. (1) becomes

$$(M_E)_1 = 3.75T - 3600(1.5) - 7800 \tag{2}$$

For $x = 17.5$ ft, Eq. (1) becomes

$$(M_E)_2 = 3.75T - 3600(17.5) - 7800$$

(*a*) For smallest max value of $|M_E|$, we set

$$(M_E)_1 =^{-} (M_E)_2$$

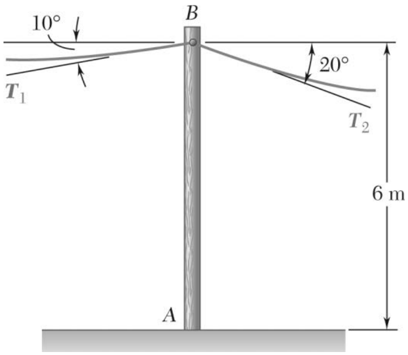
$$3.75T - 13{,}200 = -3.75T + 70{,}800$$

T $= 11.20$ kips ◀

(*b*) From Equation (2), then

$$M_E = 3.75(11.20) - 13.20$$

$|M_E| = 28.8 \text{ kip} \cdot \text{ft}$ ◀

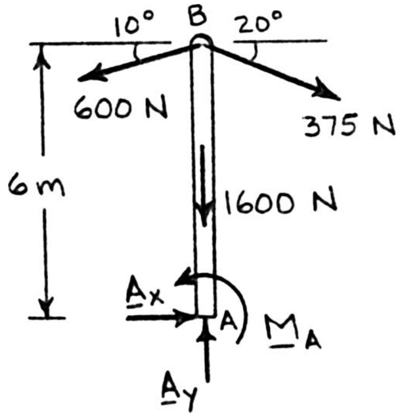


PROBLEM 4.50

A 6-m telephone pole weighing 1600 N is used to support the ends of two wires. The wires form the angles shown with the horizontal axis and the tensions in the wires are, respectively, $T_1 = 600$ N and $T_2 = 375$ N. Determine the reaction at the fixed end A.

SOLUTION

Free-Body Diagram:



$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + (375\text{ N})\cos 20° - (600\text{ N})\cos 10° = 0$$

$$A_x = +238.50\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1600\text{ N} - (600\text{ N})\sin 10° - (375\text{ N})\sin 20° = 0$$

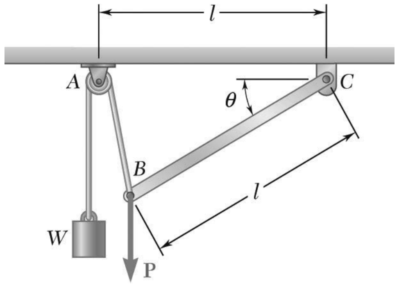
$$A_y = +1832.45\text{ N}$$

$$A = \sqrt{238.50^2 + 1832.45^2}$$

$$\theta = \tan^{-1}\frac{1832.45}{238.50} \qquad \mathbf{A} = 1848\text{ N} \measuredangle 82.6° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A + (600\text{ N})\cos 10°(6\text{ m}) - (375\text{ N})\cos 20°(6\text{ m}) = 0$$

$$M_A = -1431.00\text{ N}\cdot\text{m} \qquad \mathbf{M}_A = 1431\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

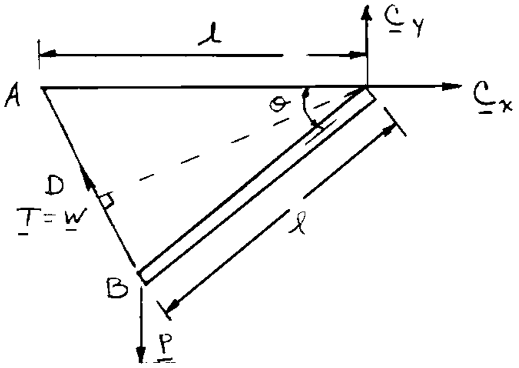


PROBLEM 4.51

A vertical load **P** is applied at end *B* of rod *BC*. (*a*) Neglecting the weight of the rod, express the angle θ corresponding to the equilibrium position in terms of *P*, *l*, and the counterweight *W*. (*b*) Determine the value of θ corresponding to equilibrium if $P = 2W$.

SOLUTION

Free-Body Diagram:



(*a*) Triangle *ABC* is isosceles. We have

$$CD = (BC)\cos\left(\frac{\theta}{2}\right) = l\cos\left(\frac{\theta}{2}\right)$$

$$+\circlearrowleft \Sigma M_C = 0: \quad P(l\cos\theta) - W\left(l\cos\frac{\theta}{2}\right) = 0$$

Setting $\cos\theta = 2\cos^2\frac{\theta}{2} - 1$:

$$Pl\left(2\cos^2\frac{\theta}{2} - 1\right) - Wl\cos\frac{\theta}{2} = 0$$

$$\cos^2\frac{\theta}{2} - \left(\frac{W}{2P}\right)\cos\frac{\theta}{2} - \frac{1}{2} = 0$$

$$\cos\frac{\theta}{2} = \frac{1}{4}\left(\frac{W}{P} \pm \sqrt{\frac{W^2}{P^2} + 8}\right)$$

$$\theta = 2\cos^{-1}\left[\frac{1}{4}\left(\frac{W}{P} \pm \sqrt{\frac{W^2}{P^2} + 8}\right)\right] \blacktriangleleft$$

PROBLEM 4.51 (Continued)

(*b*) For $P = 2W$,

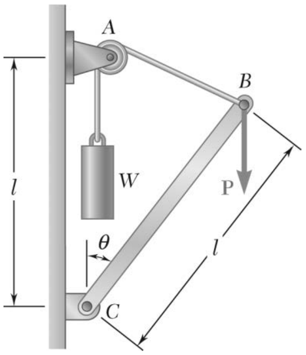
$$\cos\frac{\theta}{2} = \frac{1}{4}\left(\frac{1}{2} \pm \sqrt{\frac{1}{4} + 8}\right) = \frac{1}{8}\left(1 \pm \sqrt{33}\right)$$

$$\cos\frac{\theta}{2} = 0.84307 \quad \text{and} \quad \cos\frac{\theta}{2} = -0.59307$$

$$\frac{\theta}{2} = 32.534^\circ \qquad \frac{\theta}{2} = 126.375^\circ$$

$$\theta = 65.1^\circ \qquad \theta = 252.75^\circ \quad \text{(discard)}$$

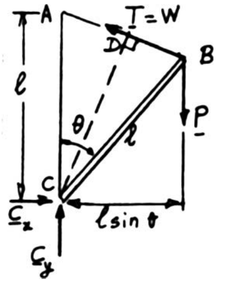
$\theta = 65.1^\circ$ ◀



PROBLEM 4.52

A vertical load **P** is applied at end B of rod BC. (a) Neglecting the weight of the rod, express the angle θ corresponding to the equilibrium position in terms of P, l, and the counterweight W. (b) Determine the value of θ corresponding to equilibrium if $P = 2W$.

SOLUTION



(a) Triangle ABC is isosceles. We have

$$CD = (BC)\cos\frac{\theta}{2} = l\cos\frac{\theta}{2}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad W\left(l\cos\frac{\theta}{2}\right) - P(l\sin\theta) = 0$$

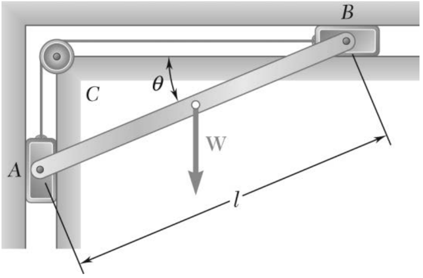
Setting $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$: $\quad Wl\cos\frac{\theta}{2} - 2Pl\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 0$

$$W - 2P\sin\frac{\theta}{2} = 0 \qquad \theta = 2\sin^{-1}\left(\frac{W}{2P}\right) ◀$$

(b) For $P = 2W$, $\quad \sin\frac{\theta}{2} = \frac{W}{2P} = \frac{W}{4W} = 0.25$

$$\frac{\theta}{2} = 14.5° \qquad \theta = 29.0° ◀$$

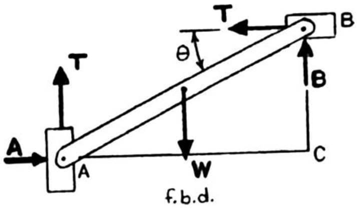
or $\quad \frac{\theta}{2} = 165.5° \quad \theta = 331° \quad$ (discard)



PROBLEM 4.53

A slender rod *AB*, of weight *W*, is attached to blocks *A* and *B*, which move freely in the guides shown. The blocks are connected by an elastic cord that passes over a pulley at *C*. (*a*) Express the tension in the cord in terms of *W* and *θ*. (*b*) Determine the value of *θ* for which the tension in the cord is equal to 3*W*.

SOLUTION



(*a*) From F.B.D. of rod *AB*:

$$+\circlearrowleft \Sigma M_C = 0: \quad T(l\sin\theta) + W\left[\left(\frac{1}{2}\right)\cos\theta\right] - T(l\cos\theta) = 0$$

$$T = \frac{W\cos\theta}{2(\cos\theta - \sin\theta)}$$

Dividing both numerator and denominator by cos *θ*,

$$T = \frac{W}{2}\left(\frac{1}{1-\tan\theta}\right) \qquad \text{or } T = \frac{\left(\frac{W}{2}\right)}{(1-\tan\theta)} \blacktriangleleft$$

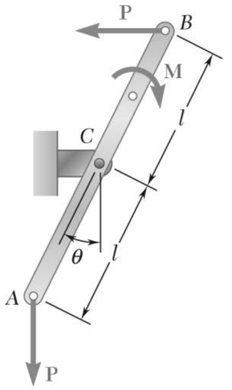
(*b*) For $T = 3W$,

$$3W = \frac{\left(\frac{W}{2}\right)}{(1-\tan\theta)}$$

$$1 - \tan\theta = \frac{1}{6}$$

or

$$\theta = \tan^{-1}\left(\frac{5}{6}\right) = 39.806° \qquad \text{or} \quad \theta = 39.8° \blacktriangleleft$$

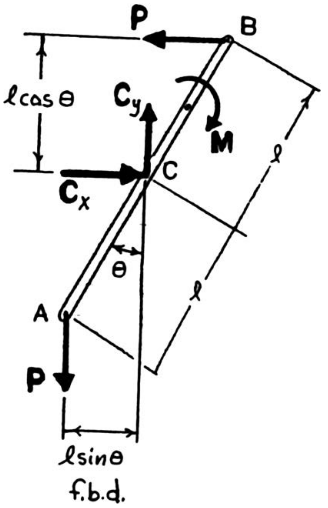


PROBLEM 4.54

Rod AB is acted upon by a couple **M** and two forces, each of magnitude P. (a) Derive an equation in θ, P, M, and l that must be satisfied when the rod is in equilibrium. (b) Determine the value of θ corresponding to equilibrium when $M = 150\text{ N}\cdot\text{m},\ P = 200\text{ N},$ and $l = 600\text{ mm}.$

SOLUTION

Free-Body Diagram:



(a) From free-body diagram of rod AB:

$$+\circlearrowleft \Sigma M_C = 0: \quad P(l\cos\theta) + P(l\sin\theta) - M = 0$$

$$\text{or } \sin\theta + \cos\theta = \frac{M}{Pl} \blacktriangleleft$$

(b) For $M = 150\text{ lb}\cdot\text{in.}$, $P = 20\text{ lb}$, and $l = 6\text{ in.}$,

$$\sin\theta + \cos\theta = \frac{150\text{ lb}\cdot\text{in.}}{(20\text{ lb})(6\text{ in.})} = \frac{5}{4} = 1.25$$

Using identity $\quad \sin^2\theta + \cos^2\theta = 1$

$$\sin\theta + (1 - \sin^2\theta)^{1/2} = 1.25$$

$$(1 - \sin^2\theta)^{1/2} = 1.25 - \sin\theta$$

$$1 - \sin^2\theta = 1.5625 - 2.5\sin\theta + \sin^2\theta$$

$$2\sin^2\theta - 2.5\sin\theta + 0.5625 = 0$$

Using quadratic formula

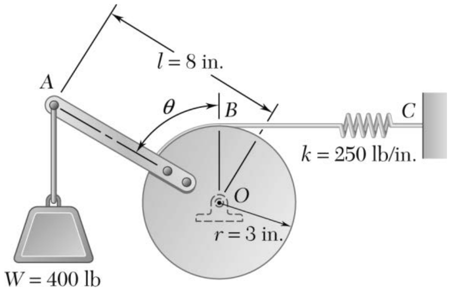
$$\sin\theta = \frac{-(-2.5) \pm \sqrt{(625) - 4(2)(0.5625)}}{2(2)}$$

$$= \frac{2.5 \pm \sqrt{1.75}}{4}$$

or $\quad \sin\theta = 0.95572 \quad \text{and} \quad \sin\theta = 0.29428$

$$\theta = 72.886° \quad \text{and} \quad \theta = 17.1144°$$

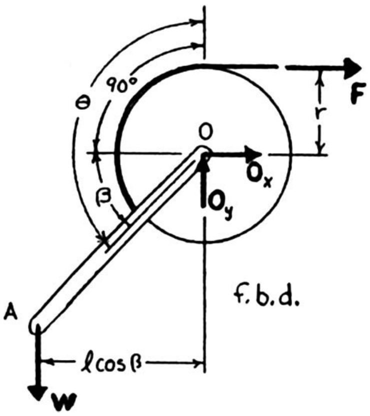
$$\text{or } \theta = 17.11° \quad \text{and} \quad \theta = 72.9° \blacktriangleleft$$



PROBLEM 4.55

Solve Sample Problem 4.5, assuming that the spring is unstretched when $\theta = 90°$.

SOLUTION



First note: $T =$ tension in spring $= ks$

where $s =$ deformation of spring

$= r\beta$

$F = kr\beta$

From F.B.D. of assembly: $+\circlearrowleft \Sigma M_0 = 0$: $W(l\cos\beta) - F(r) = 0$

or $Wl\cos\beta - kr^2\beta = 0$

$$\cos\beta = \frac{kr^2}{Wl}\beta$$

For $k = 250$ lb/in.

$r = 3$ in.

$l = 8$ in.

$W = 400$ lb

$$\cos\beta = \frac{(250 \text{ lb/in.})(3 \text{ in.})^2}{(400 \text{ lb})(8 \text{ in.})}\beta$$

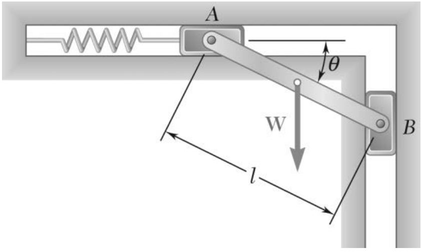
or $\cos\beta = 0.703125\beta$

Solving numerically, $\beta = 0.89245$ rad

or $\beta = 51.134°$

Then $\theta = 90° + 51.134° = 141.134°$ or $\theta = 141.1°$ ◀

PROPRIETARY MATERIAL.

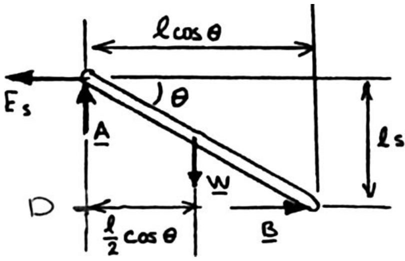


PROBLEM 4.56

A slender rod *AB*, of weight *W*, is attached to blocks *A* and *B* that move freely in the guides shown. The constant of the spring is *k*, and the spring is unstretched when $\theta = 0$. (*a*) Neglecting the weight of the blocks, derive an equation in *W*, *k*, *l*, and θ that must be satisfied when the rod is in equilibrium. (*b*) Determine the value of θ when $W = 75$ lb, $l = 30$ in., and $k = 3$ lb/in.

SOLUTION

Free-Body Diagram:



Spring force: $$F_s = ks = k(l - l\cos\theta) = kl(1-\cos\theta)$$

(*a*) $$+\circlearrowleft \Sigma M_D = 0: \quad F_s(l\sin\theta) - W\left(\frac{l}{2}\cos\theta\right) = 0$$

$$kl(1-\cos\theta)l\sin\theta - \frac{W}{2}l\cos\theta = 0$$

$$kl(1-\cos\theta)\tan\theta - \frac{W}{2} = 0 \qquad \text{or} \quad (1-\cos\theta)\tan\theta = \frac{W}{2kl} \blacktriangleleft$$

(*b*) For given values of

$W = 75$ lb

$l = 30$ in.

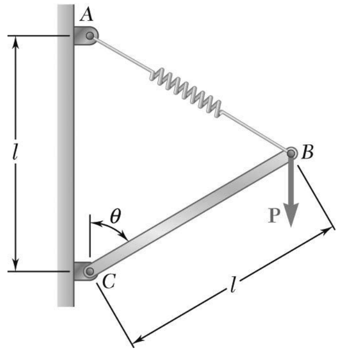
$k = 3$ lb/in.

$$(1-\cos\theta)\tan\theta = \tan\theta - \sin\theta$$

$$= \frac{75 \text{ lb}}{2(3 \text{ lb/in.})(30 \text{ in.})}$$

$$= 0.41667$$

Solving numerically, $\theta = 49.710°$ or $\theta = 49.7° \blacktriangleleft$

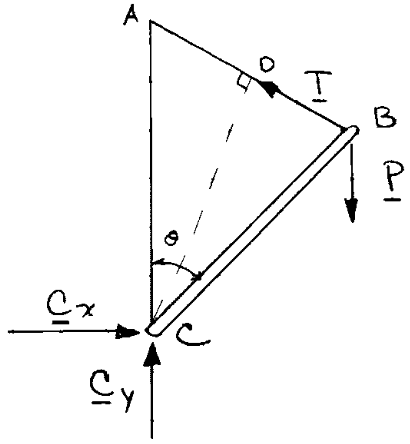


PROBLEM 4.57

A vertical load **P** is applied at end *B* of rod *BC*. The constant of the spring is *k*, and the spring is unstretched when $\theta = 60°$. (*a*) Neglecting the weight of the rod, express the angle θ corresponding to the equilibrium position terms of *P*, *k*, and *l*. (*b*) Determine the value of θ corresponding to equilibrium if $P = \frac{1}{4}kl$.

SOLUTION

Free-Body Diagram:



(*a*) Triangle *ABC* is isosceles. We have

$$AB = 2(AD) = 2l\sin\left(\frac{\theta}{2}\right);\ CD = l\cos\left(\frac{\theta}{2}\right)$$

Elongation of spring: $x = (AB)_\theta - (AB)_\theta = 60°$

$$= 2l\sin\left(\frac{\theta}{2}\right) - 2l\sin 30°$$

$$T = kx = 2kl\left(\sin\frac{\theta}{2} - \frac{1}{2}\right)$$

$$\circlearrowleft \Sigma M_C = 0{:}\quad T\left(l\cos\ \frac{\theta}{2}\right) - P(l\sin\theta) = 0$$

PROBLEM 4.57 (Continued)

$$2kl\left(\sin\frac{\theta}{2}-\frac{1}{2}\right)l\cos\frac{\theta}{2}-Pl\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)=0$$

$$\cos\frac{\theta}{2}=0 \quad \text{or} \quad 2(kl-P)\sin\frac{\theta}{2}-kl=0$$

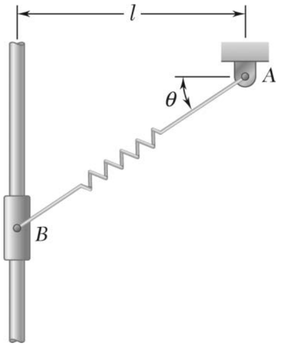
$$\theta=180^\circ \text{ (trivial)} \qquad \sin\frac{\theta}{2}=\frac{\frac{1}{2}kl}{kl-P}$$

$$\theta=2\sin^{-1}\left[\frac{1}{2}kl/(kl-P)\right] \blacktriangleleft$$

(*b*) For $P=\frac{1}{4}kl$,

$$\sin\frac{\theta}{2}=\frac{\frac{1}{2}kl}{\frac{3}{4}kl}=\frac{2}{3}$$

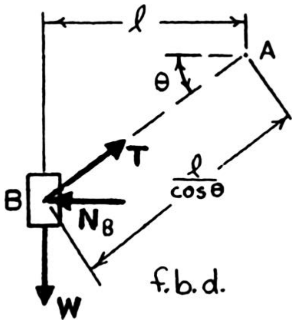
$$\frac{\theta}{2}=41.8^\circ \qquad \theta=83.6^\circ \blacktriangleleft$$



PROBLEM 4.58

A collar B of weight W can move freely along the vertical rod shown. The constant of the spring is k, and the spring is unstretched when $\theta = 0$. (*a*) Derive an equation in θ, W, k, and l that must be satisfied when the collar is in equilibrium. (*b*) Knowing that $W = 300\text{ N}$, $l = 500\text{ mm}$, and $k = 800\text{ N/m}$, determine the value of θ corresponding to equilibrium.

SOLUTION



First note: $T = ks$

where $k =$ spring constant

$s =$ elongation of spring

$$= \frac{l}{\cos\theta} - l$$

$$= \frac{l}{\cos\theta}(1 - \cos\theta)$$

$$T = \frac{kl}{\cos\theta}(1 - \cos\theta)$$

(*a*) From F.B.D. of collar *B*: $+\!\uparrow \Sigma F_y = 0\!: \quad T\sin\theta - W = 0$

or $$\frac{kl}{\cos\theta}(1 - \cos\theta)\sin\theta - W = 0$$

or $\tan\theta - \sin\theta = \dfrac{W}{kl}$ ◀

(*b*) For $W = 3\text{ lb}$

$l = 6\text{ in.}$

$k = 8\text{ lb/ft}$

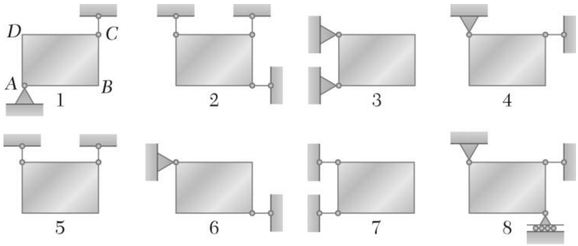
$$l = \frac{6\text{ in.}}{12\text{ in./ft}} = 0.5\text{ ft}$$

$$\tan\theta - \sin\theta = \frac{3\text{ lb}}{(8\text{ lb/ft})(0.5\text{ ft})} = 0.75$$

Solving numerically, $\theta = 57.957°$ or $\theta = 58.0°$ ◀

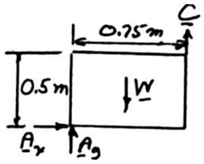
PROBLEM 4.59

Eight identical 500×750-mm rectangular plates, each of mass $m = 40$ kg, are held in a vertical plane as shown. All connections consist of frictionless pins, rollers, or short links. In each case, determine whether (*a*) the plate is completely, partially, or improperly constrained, (*b*) the reactions are statically determinate or indeterminate, (*c*) the equilibrium of the plate is maintained in the position shown. Also, wherever possible, compute the reactions.



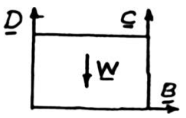
SOLUTION

1. Three non-concurrent, non-parallel reactions:
   (*a*) Plate: completely constrained
   (*b*) Reactions: determinate
   (*c*) Equilibrium maintained



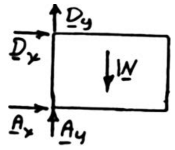
$$\mathbf{A} = \mathbf{C} = 196.2 \text{ N} \uparrow$$

2. Three non-concurrent, non-parallel reactions:
   (*a*) Plate: completely constrained
   (*b*) Reactions: determinate
   (*c*) Equilibrium maintained



$$\mathbf{B} = 0, \quad \mathbf{C} = \mathbf{D} = 196.2 \text{ N} \uparrow$$

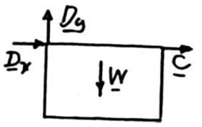
3. Four non-concurrent, non-parallel reactions:
   (*a*) Plate: completely constrained
   (*b*) Reactions: indeterminate
   (*c*) Equilibrium maintained



$$\mathbf{A}_x = 294 \text{ N} \longrightarrow, \quad \mathbf{D}_x = 294 \text{ N} \longleftarrow$$

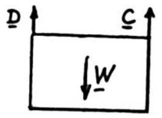
$$(\mathbf{A}_y + \mathbf{D}_y = 392 \text{ N} \uparrow)$$

4. Three concurrent reactions (through *D*):
   (*a*) Plate: improperly constrained
   (*b*) Reactions: indeterminate
   (*c*) No equilibrium $(\Sigma M_D \neq 0)$



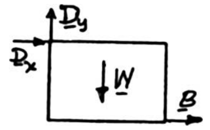
PROBLEM 4.59 (Continued)

5. Two reactions:
    - (*a*) Plate: partial constraint
    - (*b*) Reactions: determinate
    - (*c*) Equilibrium maintained



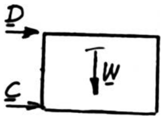
$$\mathbf{C} = \mathbf{D} = 196.2\text{ N}\uparrow$$

6. Three non-concurrent, non-parallel reactions:
    - (*a*) Plate: completely constrained
    - (*b*) Reactions: determinate
    - (*c*) Equilibrium maintained

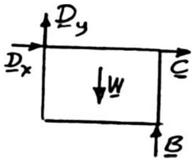


$$\mathbf{B} = 294\text{ N}\longrightarrow, \quad \mathbf{D} = 491\text{ N} \searrow 53.1^\circ$$

7. Two reactions:
    - (*a*) Plate: improperly constrained
    - (*b*) Reactions determined by dynamics
    - (*c*) No equilibrium ($\Sigma F_y \neq 0$)



8. Four non-concurrent, non-parallel reactions:
    - (*a*) Plate: completely constrained
    - (*b*) Reactions: indeterminate
    - (*c*) Equilibrium maintained

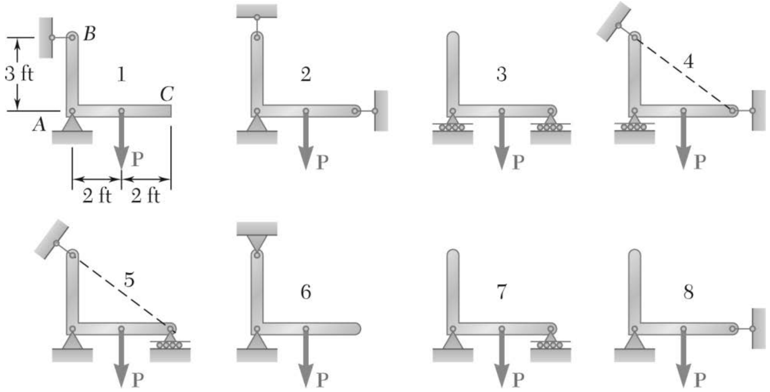


$$\mathbf{B} = \mathbf{D}_y = 196.2\text{ N}\uparrow$$

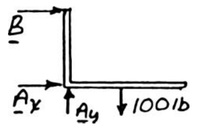
$$(\mathbf{C} + \mathbf{D}_x = 0)$$

PROBLEM 4.60

The bracket *ABC* can be supported in the eight different ways shown. All connections consist of smooth pins, rollers, or short links. For each case, answer the questions listed in Problem 4.59, and, wherever possible, compute the reactions, assuming that the magnitude of the force **P** is 100 lb.



SOLUTION



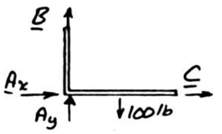
1. Three non-concurrent, non-parallel reactions:

(*a*) Bracket: complete constraint

(*b*) Reactions: determinate

(*c*) Equilibrium maintained

$$\mathbf{A} = 120.2 \text{ lb} \measuredangle 56.3^\circ, \quad \mathbf{B} = 66.7 \text{ lb} \longleftarrow$$

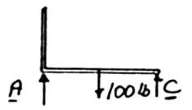


2. Four concurrent, reactions (through *A*):

(*a*) Bracket: improper constraint

(*b*) Reactions: indeterminate

(*c*) No equilibrium $(\Sigma M_A \neq 0)$



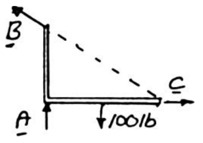
3. Two reactions:

(*a*) Bracket: partial constraint

(*b*) Reactions: indeterminate

(*c*) Equilibrium maintained

$$\mathbf{A} = 50 \text{ lb} \uparrow, \quad \mathbf{C} = 50 \text{ lb} \uparrow$$



4. Three non-concurrent, non-parallel reactions:

(*a*) Bracket: complete constraint

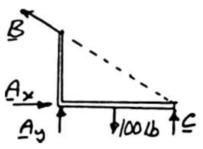
(*b*) Reactions: determinate

(*c*) Equilibrium maintained

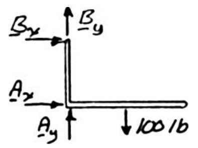
$$\mathbf{A} = 50 \text{ lb} \uparrow, \quad \mathbf{B} = 83.3 \text{ lb} \measuredangle 36.9^\circ, \quad \mathbf{C} = 66.7 \text{ lb} \longrightarrow$$

PROBLEM 4.60 (Continued)

5. Four non-concurrent, non-parallel reactions:
   - (*a*) Bracket: complete constraint
   - (*b*) Reactions: indeterminate
   - (*c*) Equilibrium maintained $(\Sigma M_C = 0)\ \mathbf{A}_y = 50\text{ lb}\uparrow$



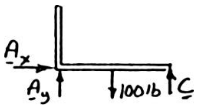
6. Four non-concurrent, non-parallel reactions:
   - (*a*) Bracket: complete constraint
   - (*b*) Reactions: indeterminate
   - (*c*) Equilibrium maintained



$$\mathbf{A}_x = 66.7\text{ lb} \rightarrow \quad \mathbf{B}_x = 66.7\text{ lb} \leftarrow$$

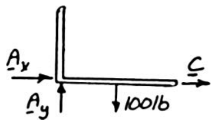
$$(\mathbf{A}_y + \mathbf{B}_y = 100\text{ lb}\uparrow)$$

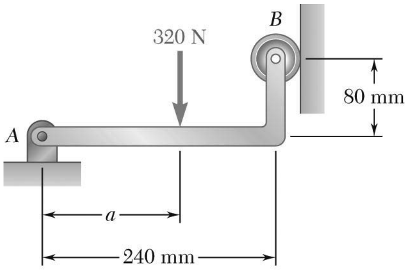
7. Three non-concurrent, non-parallel reactions:
   - (*a*) Bracket: complete constraint
   - (*b*) Reactions: determinate
   - (*c*) Equilibrium maintained



$$\mathbf{A} = \mathbf{C} = 50\text{ lb}\uparrow$$

8. Three concurrent, reactions (through *A*)
   - (*a*) Bracket: improper constraint
   - (*b*) Reactions: indeterminate
   - (*c*) No equilibrium $(\Sigma M_A \neq 0)$



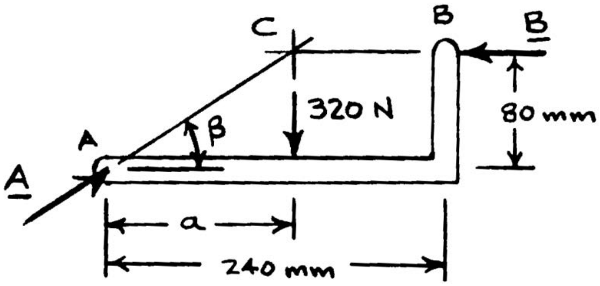


PROBLEM 4.61

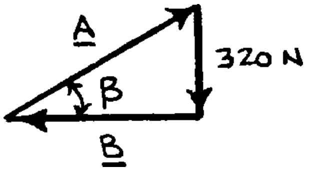
Determine the reactions at *A* and *B* when *a* = 150 mm.

SOLUTION

Free-Body Diagram:



Force triangle



$$\tan \beta = \frac{80 \text{ mm}}{a} = \frac{80 \text{ mm}}{150 \text{ mm}}$$

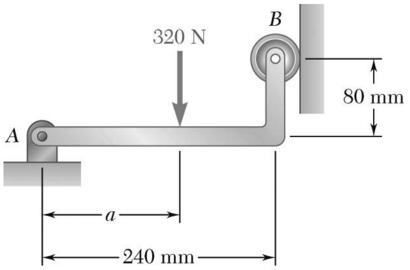
$$\beta = 28.072^\circ$$

$$A = \frac{320 \text{ N}}{\sin 28.072^\circ}$$

$\mathbf{A} = 680 \text{ N} \measuredangle 28.1^\circ$ ◀

$$B = \frac{320 \text{ N}}{\tan 28.072^\circ}$$

$\mathbf{B} = 600 \text{ N} \longleftarrow$ ◀

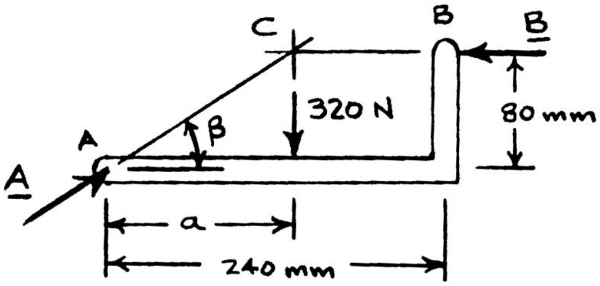


PROBLEM 4.62

Determine the value of a for which the magnitude of the reaction at B is equal to 800 N.

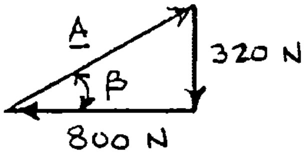
SOLUTION

Free-Body Diagram:



Force triangle

$$\tan\beta = \frac{80 \text{ mm}}{a} \qquad a = \frac{80 \text{ mm}}{\tan\beta} \qquad (1)$$



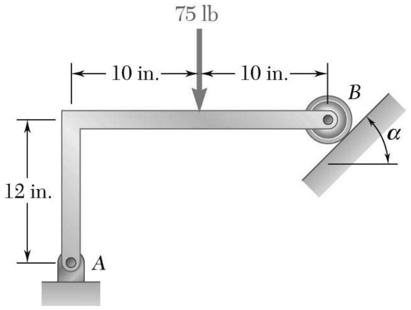
From force triangle:

$$\tan\beta = \frac{320 \text{ N}}{800 \text{ N}} = 0.4$$

From Eq. (1):

$$a = \frac{80 \text{ mm}}{0.4}$$

$a = 200$ mm ◀



PROBLEM 4.63

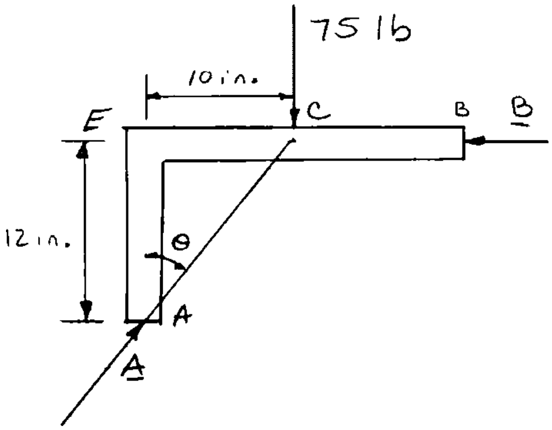
Using the method of Sec. 4.7, solve Problem 4.22*b*.

PROBLEM 4.22 Determine the reactions at *A* and *B* when (*a*) $\alpha = 0$, (*b*) $\alpha = 90°$, (*c*) $\alpha = 30°$.

SOLUTION

Free-Body Diagram:

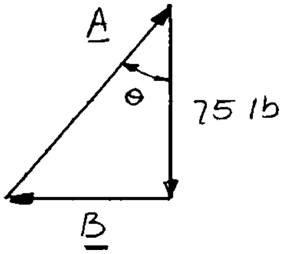
(Three-force body)



The line of action at *A* must pass through *C*, where **B** and the 75-lb load intersect.

In triangle *ACE*: $\tan\theta = \dfrac{10 \text{ in.}}{12 \text{ in.}}$ $\theta = 39.806°$

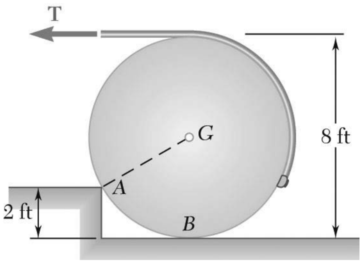
Force triangle



$$B = (75 \text{ lb}) \tan 39.806° = 62.5 \text{ lb}$$

$$A = \frac{75 \text{ lb}}{\cos 39.806°} = 97.6°$$

$\mathbf{A} = 97.6$ lb $\measuredangle$ 50.2°; $\mathbf{B} = 62.5$ lb ⟵ ◀

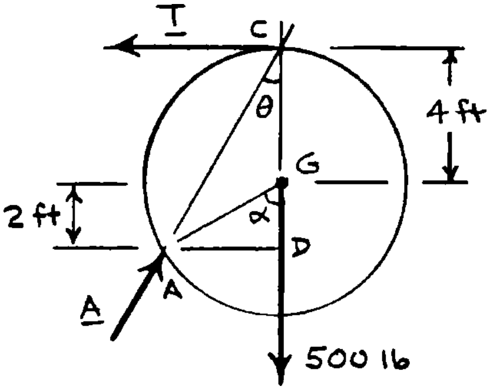


PROBLEM 4.64

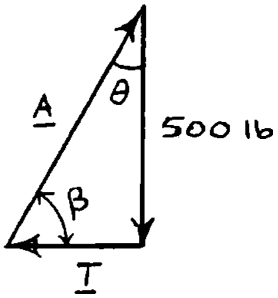
A 500-lb cylindrical tank, 8 ft in diameter, is to be raised over a 2-ft obstruction. A cable is wrapped around the tank and pulled horizontally as shown. Knowing that the corner of the obstruction at *A* is rough, find the required tension in the cable and the reaction at *A*.

SOLUTION

Free-Body Diagram:



Force triangle

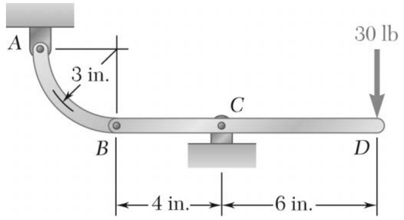


$$\cos\alpha = \frac{GD}{AG} = \frac{2\text{ ft}}{4\text{ ft}} = 0.5 \qquad \alpha = 60°$$

$$\theta = \frac{1}{2}\alpha = 30° \qquad (\beta = 60°)$$

$$T = (500\text{ lb})\tan 30° \qquad T = 289\text{ lb}$$

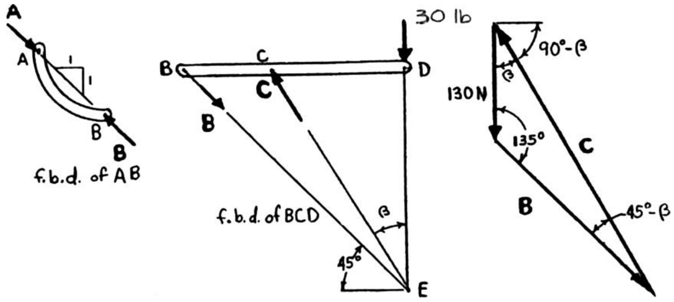
$$A = \frac{500\text{ lb}}{\cos 30°} \qquad \mathbf{A} = 577\text{ lb} \measuredangle 60.0°$$



PROBLEM 4.65

For the frame and loading shown, determine the reactions at *A* and *C*.

SOLUTION



Since member *AB* is acted upon by two forces, **A** and **B**, they must be colinear, have the same magnitude, and be opposite in direction for *AB* to be in equilibrium. The force **B** acting at *B* of member *BCD* will be equal in magnitude but opposite in direction to force **B** acting on member *AB*. Member *BCD* is a three-force body with member forces intersecting at *E*. The F.B.D.'s of members *AB* and *BCD* illustrate the above conditions. The force triangle for member *BCD* is also shown. The angle β is found from the member dimensions:

$$\beta = \tan^{-1}\left(\frac{6\text{ in.}}{10\text{ in.}}\right) = 30.964^\circ$$

Applying the law of sines to the force triangle for member *BCD*,

$$\frac{30\text{ lb}}{\sin(45^\circ - \beta)} = \frac{B}{\sin\beta} = \frac{C}{\sin 135^\circ}$$

or

$$\frac{30\text{ lb}}{\sin 14.036^\circ} = \frac{B}{\sin 30.964^\circ} = \frac{C}{\sin 135^\circ}$$

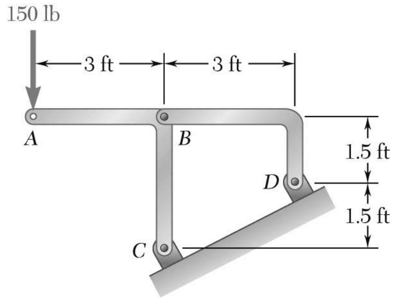
$$A = B = \frac{(30\text{ lb})\sin 30.964^\circ}{\sin 14.036^\circ} = 63.641\text{ lb}$$

or **A** = 63.6 lb ↘ 45.0° ◀

and

$$C = \frac{(30\text{ lb})\sin 135^\circ}{\sin 14.036^\circ} = 87.466\text{ lb}$$

or **C** = 87.5 lb ↖ 59.0° ◀



PROBLEM 4.66

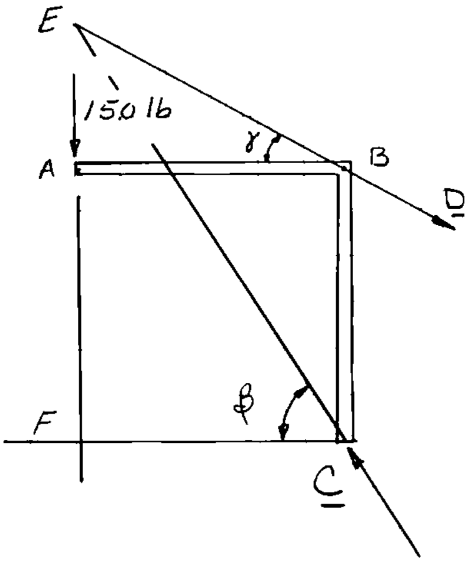
For the frame and loading shown, determine the reactions at *C* and *D*.

SOLUTION

Since *BD* is a two-force member, the reaction at *D* must pass through Points *B* and *D*.

Free-Body Diagram:

(Three-force body)



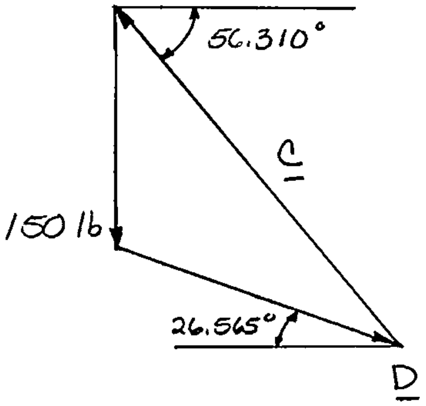
Reaction at *C* must pass through *E*, where the reaction at *D* and the 150-lb load intersect.

Triangle *CEF*: $\tan\beta = \dfrac{4.5\text{ ft}}{3\text{ ft}}$ $\beta = 56.310°$

Triangle *ABE*: $\tan\gamma = \dfrac{1}{2}$ $\gamma = 26.565°$

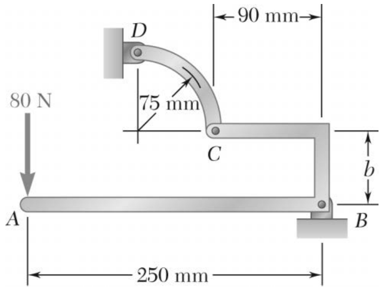
PROBLEM 4.66 (Continued)

Force Triangle



Law of sines:
$$\frac{150\text{ lb}}{\sin 29.745^\circ}=\frac{C}{\sin 116.565^\circ}=\frac{D}{\sin 33.690^\circ}$$
$$C = 270.42\text{ lb},$$
$$D = 167.704\text{ lb}$$

C = 270 lb ↘ 56.3°; **D** = 167.7 lb ↘ 26.6° ◀



PROBLEM 4.67

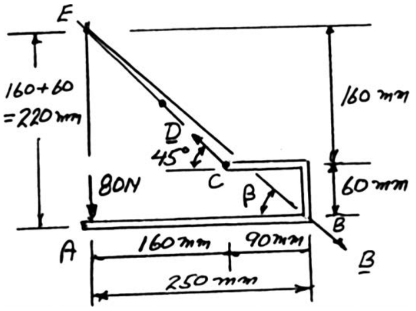
Determine the reactions at *B* and *D* when $b = 60$ mm.

SOLUTION

Since *CD* is a two-force member, the line of action of reaction at *D* must pass through Points *C* and *D*. 

Free-Body Diagram:

(Three-force body)

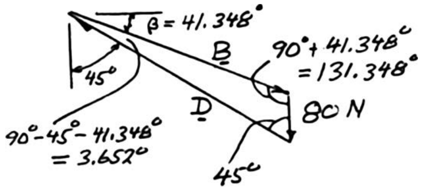


Reaction at *B* must pass through *E*, where the reaction at *D* and the 80-N force intersect.

$$\tan\beta = \frac{220 \text{ mm}}{250 \text{ mm}}$$

$$\beta = 41.348°$$

<u>Force triangle</u>



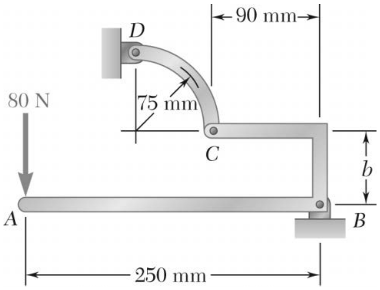
Law of sines:

$$\frac{80 \text{ N}}{\sin 3.652°} = \frac{B}{\sin 45°} = \frac{D}{\sin 131.348°}$$

$$B = 888.0 \text{ N}$$

$$D = 942.8 \text{ N}$$

B = 888 N ↖ 41.3° **D** = 943 N ↘ 45.0° ◀



PROBLEM 4.68

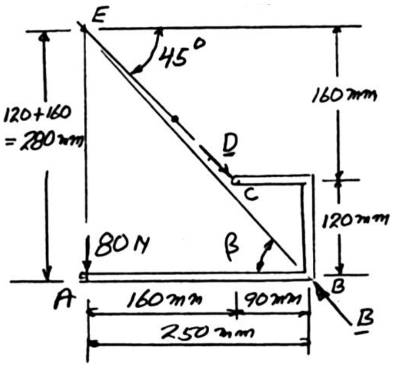
Determine the reactions at B and D when $b = 120$ mm.

SOLUTION

Since CD is a two-force member, line of action of reaction at D must pass through C and D ↘ 45°.

Free-Body Diagram:

(Three-force body)

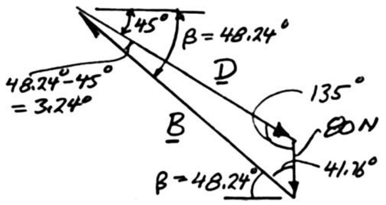


Reaction at B must pass through E, where the reaction at D and the 80-N force intersect.

$$\tan\beta = \frac{280 \text{ mm}}{250 \text{ mm}}$$

$$\beta = 48.24°$$

Force triangle



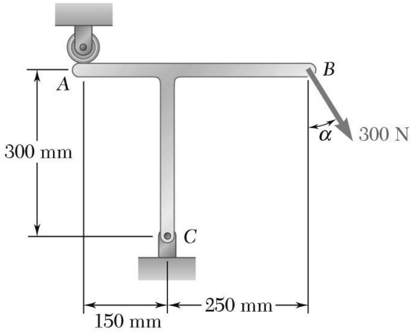
Law of sines:

$$\frac{80 \text{ N}}{\sin 3.24°} = \frac{B}{\sin 135°} = \frac{D}{\sin 41.76°}$$

$$B = 1000.9 \text{ N}$$

$$D = 942.8 \text{ N}$$

$\mathbf{B} = 1001$ N ⦪ 48.2° $\mathbf{D} = 943$ N ↘ 45.0° ◀



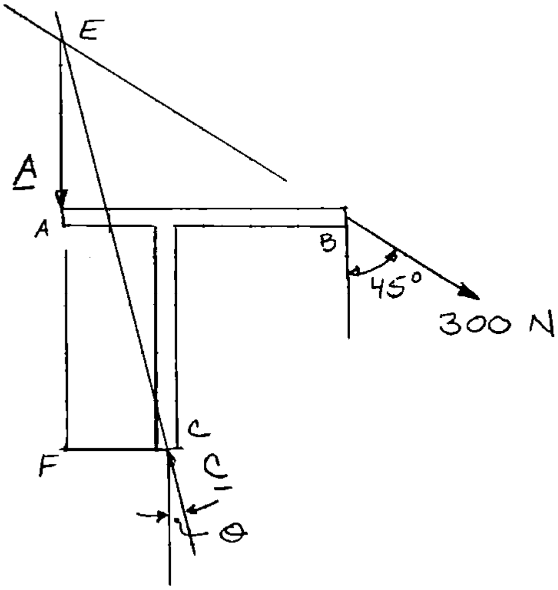
PROBLEM 4.69

A T-shaped bracket supports a 300-N load as shown. Determine the reactions at *A* and *C* when $\alpha = 45°$.

SOLUTION

Free-Body Diagram:

(Three-force body)



The line of action of **C** must pass through *E*, where **A** and the 300-N force intersect.

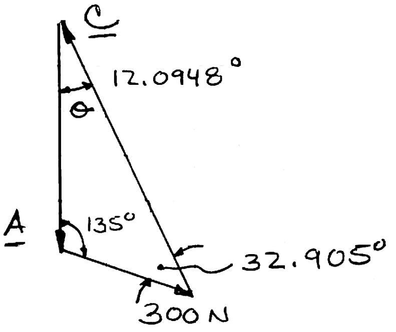
Triangle *ABE* is isosceles: $EA = AB = 400\text{ mm}$

In triangle *CEF*:

$$\tan\theta = \frac{CF}{EF} = \frac{CF}{EA + AF} = \frac{150\text{ mm}}{700\text{ mm}} \qquad \theta = 12.0948°$$

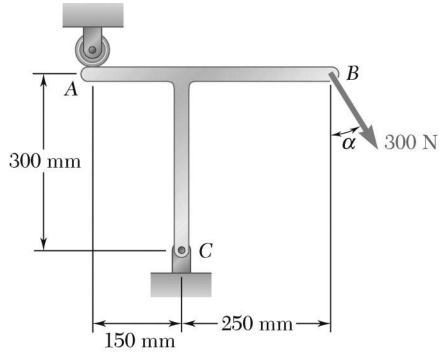
PROBLEM 4.69 (Continued)

Force Triangle



Law of sines: $$\frac{A}{\sin 32.905^\circ} = \frac{C}{\sin 135^\circ} = \frac{300\text{ N}}{\sin 12.0948^\circ}$$

$\mathbf{A} = 778\text{ N}\downarrow$; $\mathbf{C} = 1012\text{ N} \searrow 77.9^\circ$ ◀

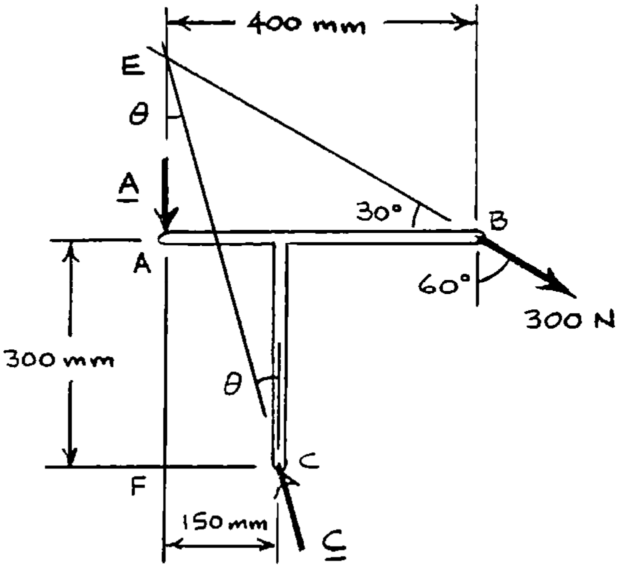


PROBLEM 4.70

A T-shaped bracket supports a 300-N load as shown. Determine the reactions at *A* and *C* when $\alpha = 60°$.

SOLUTION

Free-Body Diagram:



$$EA = (400 \text{ mm}) \tan 30°$$
$$= 230.94 \text{ mm}$$

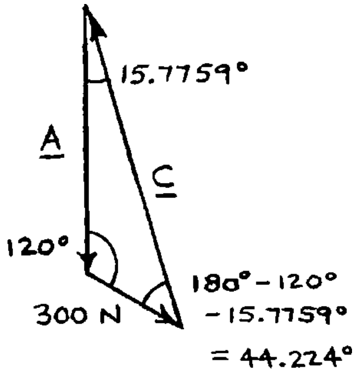
In triangle *CEF*:

$$\tan\theta = \frac{CF}{EF} = \frac{CF}{EA + AF}$$

$$\tan\theta = \frac{150}{230.94 + 300}$$
$$\theta = 15.7759°$$

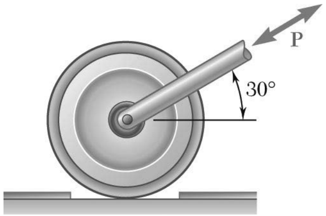
PROBLEM 4.70 (Continued)

Force Triangle



Law of sines:
$$\frac{A}{\sin 44.224^\circ} = \frac{C}{\sin 120^\circ} = \frac{300\text{ N}}{\sin 15.7759^\circ}$$
$$A = 770\text{ N}$$
$$C = 956\text{ N}$$

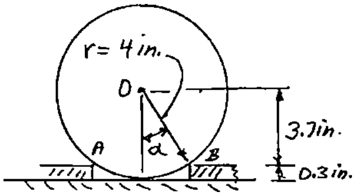
A = 770 N ↓; **C** = 956 N ⦪ 74.2° ◀



PROBLEM 4.71

A 40-lb roller, of diameter 8 in., which is to be used on a tile floor, is resting directly on the subflooring as shown. Knowing that the thickness of each tile is 0.3 in., determine the force **P** required to move the roller onto the tiles if the roller is (*a*) pushed to the left, (*b*) pulled to the right.

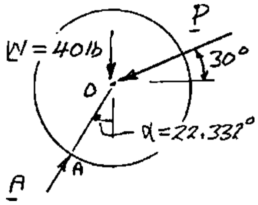
SOLUTION



Geometry: For each case as roller comes into contact with tile,

$$\alpha = \cos^{-1}\frac{3.7\text{ in.}}{4\text{ in.}}$$

$$\alpha = 22.332°$$

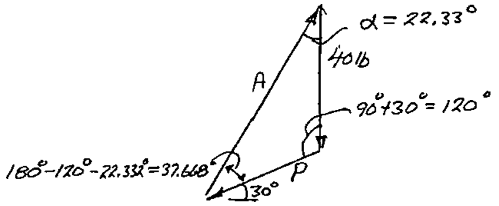


(*a*) Roller pushed to left (three-force body):
Forces must pass through *O*.

Law of sines: $\dfrac{40\text{ lb}}{\sin 37.668°} = \dfrac{P}{\sin 22.332°}$; $P = 24.87$ lb

$\mathbf{P} = 24.9$ lb ↙30.0° ◀

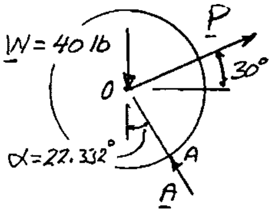
Force Triangle



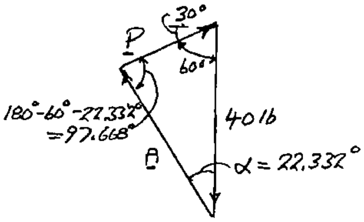
(*b*) Roller pulled to right (three-force body):
Forces must pass through *O*.

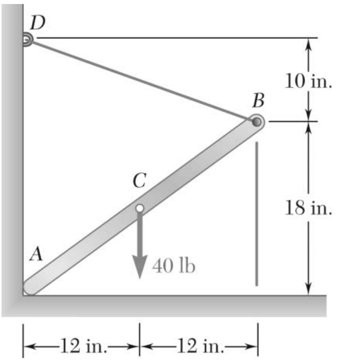
Law of sines: $\dfrac{40\text{ lb}}{\sin 97.668°} = \dfrac{P}{\sin 22.332°}$; $P = 15.3361$ lb

$\mathbf{P} = 15.34$ lb ↗30.0° ◀



Force Triangle



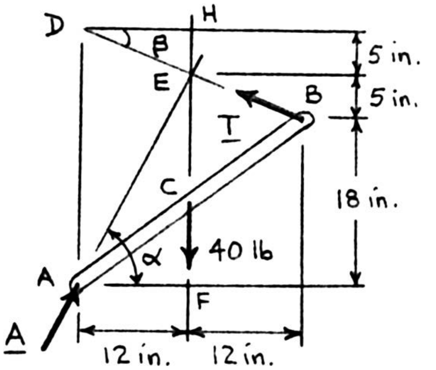


PROBLEM 4.72

One end of rod *AB* rests in the corner *A* and the other end is attached to cord *BD*. If the rod supports a 40-lb load at its midpoint *C*, find the reaction at *A* and the tension in the cord.

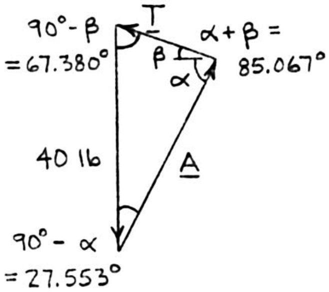
SOLUTION

Free-Body Diagram: (Three-force body)



The line of action of reaction at *A* must pass through *E*, where **T** and the 40-lb load intersect.

Force triangle



$$\tan\alpha = \frac{EF}{AF} = \frac{23}{12}$$

$$\alpha = 62.447°$$

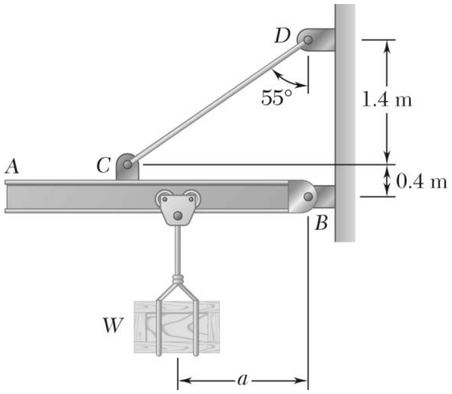
$$\tan\beta = \frac{EH}{DH} = \frac{5}{12}$$

$$\beta = 22.620°$$

$$\frac{A}{\sin 67.380°} = \frac{T}{\sin 27.553°} = \frac{40 \text{ lb}}{\sin 85.067°}$$

$\mathbf{A} = 37.1 \text{ lb} \measuredangle\ 62.4°$ ◀

$T = 18.57 \text{ lb}$ ◀



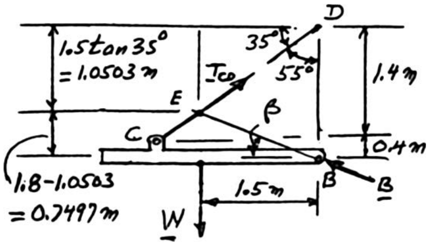
PROBLEM 4.73

A 50-kg crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ m, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *B*.

SOLUTION

Three-force body: **W** and $\mathbf{T}_{CD}$ intersect at *E*.

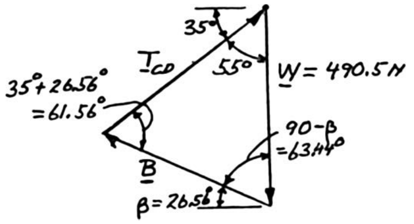
$$\tan\beta = \frac{0.7497\text{ m}}{1.5\text{ m}}$$
$$\beta = 26.556^\circ$$



Three forces intersect at *E*.

$$W = (50\text{ kg})\ 9.81\text{ m/s}^2$$
$$= 490.50\text{ N}$$

Force triangle

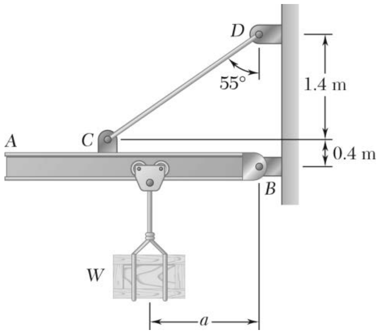


Law of sines:

$$\frac{490.50\text{ N}}{\sin 61.556^\circ} = \frac{T_{CD}}{\sin 63.444^\circ} = \frac{B}{\sin 55^\circ}$$
$$T_{CD} = 498.99\text{ N}$$
$$B = 456.96\text{ N}$$

(*a*) $T_{CD} = 499\text{ N}$ ◀

(*b*) $\mathbf{B} = 457\text{ N} \measuredangle 26.6^\circ$ ◀



PROBLEM 4.74

Solve Problem 4.73, assuming that $a = 3$ m.

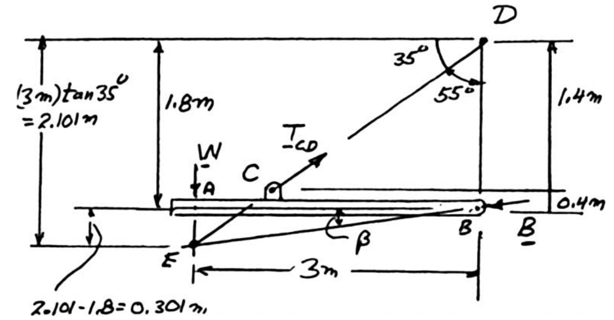
PROBLEM 4.73 A 50-kg crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ m, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *B*.

SOLUTION

W and $\mathbf{T}_{CD}$ intersect at *E*.

Free-Body Diagram:

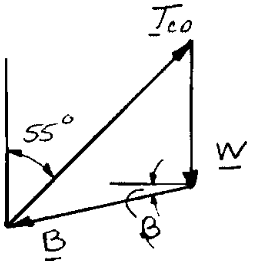
Three-Force Body



$$\tan\beta = \frac{AE}{AB} = \frac{0.301\text{ m}}{3\text{ m}}$$
$$\beta = 5.7295°$$

Three forces intersect at *E*.

Force Triangle



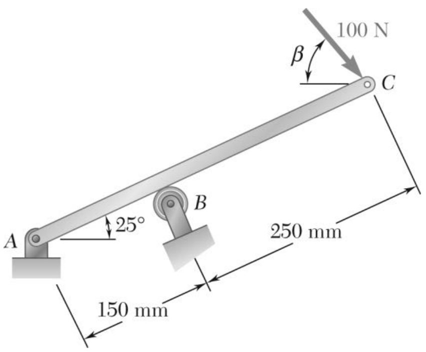
$$W = (50\text{ kg})\,9.81\text{ m/s}^2$$
$$= 490.50\text{ N}$$

Law of sines:

$$\frac{490.50\text{ N}}{\sin 29.271°} = \frac{T_{CD}}{\sin 95.730°} = \frac{B}{\sin 55°}$$
$$T_{CD} = 998.18\text{ N}$$
$$B = 821.76\text{ N}$$

(*a*) $T_{CD} = 998\text{ N}$ ◀

(*b*) $\mathbf{B} = 822\text{ N} \nearrow 5.73°$ ◀

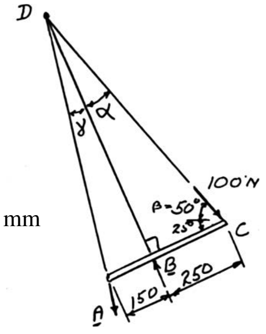


PROBLEM 4.75

Determine the reactions at A and B when $\beta = 50°$.

SOLUTION

Free-Body Diagram: (Three-force body)



Dimensions in mm

Reaction **A** must pass through Point D where the 100-N force and **B** intersect.

In right Δ BCD:

$$\alpha = 90° - 75° = 15°$$

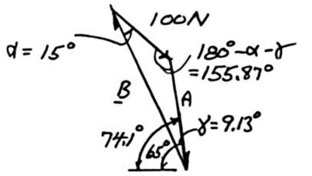
$$BD = 250 \tan 75° = 933.01 \text{ mm}$$

In right Δ ABD:

$$\tan\gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933.01 \text{ mm}}$$

$$\gamma = 9.1333°$$

Force Triangle

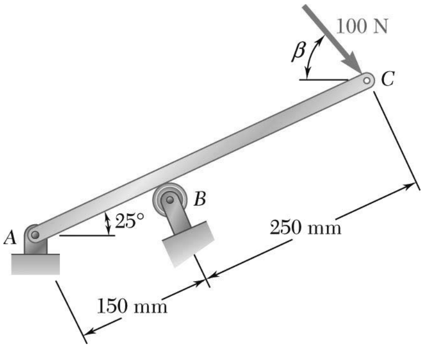


Law of sines:

$$\frac{100 \text{ N}}{\sin 9.1333°} = \frac{A}{\sin 15°} = \frac{B}{\sin 155.867°}$$

$$A = 163.1 \text{ N}; \quad B = 257.6 \text{ N}$$

A $= 163.1$ N $\nwarrow$ 74.1° **B** $= 258$ N $\searrow$ 65.0° ◀



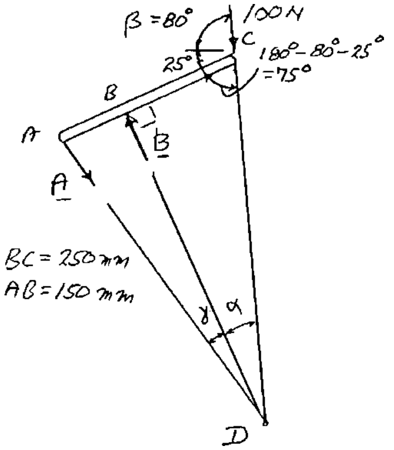
PROBLEM 4.76

Determine the reactions at A and B when $\beta = 80°$.

SOLUTION

Free-Body Diagram:

(Three-force body)



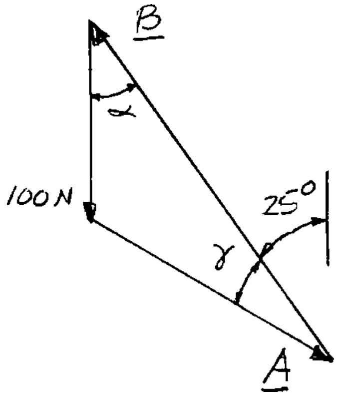
Reaction **A** must pass through D where the 100-N force and **B** intersect.

In right triangle BCD:
$$\alpha = 90° - 75° = 15°$$
$$BD = BC \tan 75° = 250 \tan 75°$$
$$BD = 933.01 \text{ mm}$$

In right triangle ABD:
$$\tan\gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933.01 \text{ mm}} \qquad \gamma = 9.1333°$$

PROBLEM 4.76 (Continued)

Force Triangle

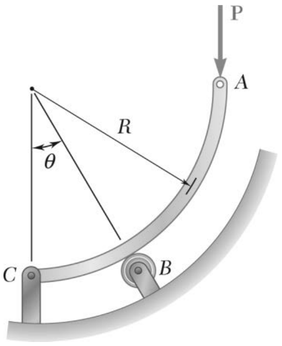


Law of sines:

$$\frac{100\text{ N}}{\sin 9.1333^\circ} = \frac{A}{\sin 15^\circ} = \frac{B}{\sin 155.867^\circ}$$

A = 163.1 N ↘ 55.9° ◀

B = 258 N ↖ 65.0° ◀

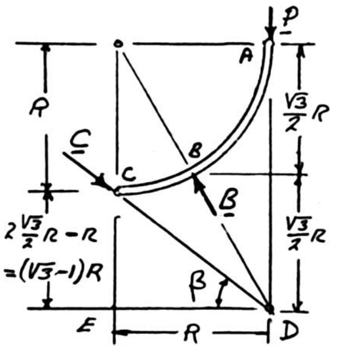


PROBLEM 4.77

Knowing that $\theta = 30°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION

Free-Body Diagram:
(Three-force body)



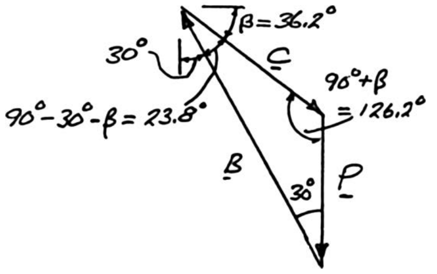
Reaction at *C* must pass through *D* where force **P** and reaction at *B* intersect.

In Δ *CDE*:

$$\tan\beta = \frac{(\sqrt{3}-1)R}{R} = \sqrt{3}-1$$

$$\beta = 36.2°$$

Force Triangle



Law of sines:

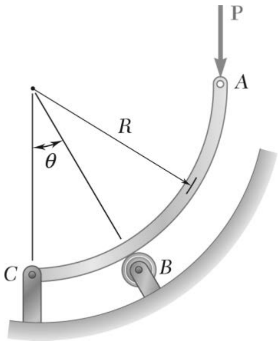
$$\frac{P}{\sin 23.8°} = \frac{B}{\sin 126.2°} = \frac{C}{\sin 30°}$$

$$B = 2.00P$$

$$C = 1.239P$$

(*a*) $\mathbf{B} = 2P$ ∠ 60.0° ◀

(*b*) $\mathbf{C} = 1.239P$ ∠ 36.2° ◀



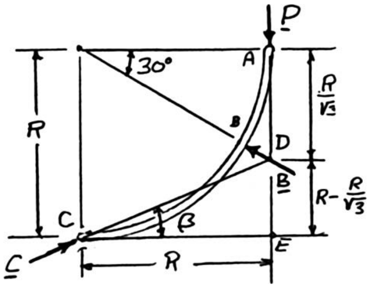
PROBLEM 4.78

Knowing that $\theta = 60°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION

Reaction at *C* must pass through *D* where force **P** and reaction at *B* intersect.

Free-Body Diagram:
(Three-force body)



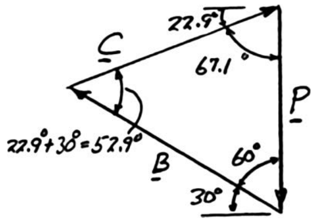
In ΔCDE:

$$\tan\beta = \frac{R - \frac{R}{\sqrt{3}}}{R}$$

$$= 1 - \frac{1}{\sqrt{3}}$$

$$\beta = 22.9°$$

Force Triangle



Law of sines:

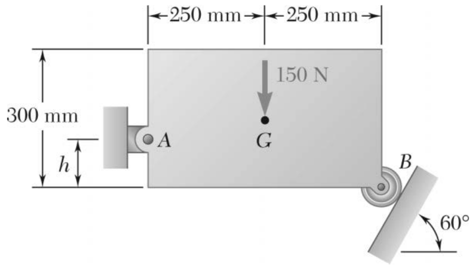
$$\frac{P}{\sin 52.9°} = \frac{B}{\sin 67.1°} = \frac{C}{\sin 60°}$$

$$B = 1.155P$$

$$C = 1.086P$$

(*a*) $\mathbf{B} = 1.155P$ ⦫ 30.0° ◀

(*b*) $\mathbf{C} = 1.086P$ ⦪ 22.9° ◀



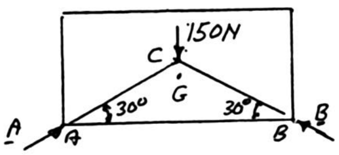
PROBLEM 4.79

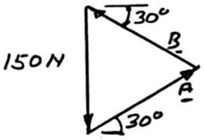
Using the method of Section 4.7, solve Problem 4.23.

PROBLEM 4.23 Determine the reactions at *A* and *B* when (*a*) $h = 0$, (*b*) $h = 200$ mm.

SOLUTION

Free-Body Diagram:





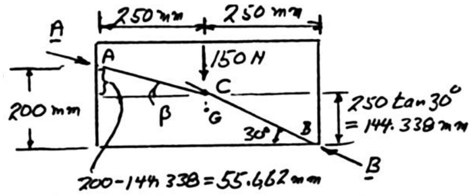
(*a*) $\underline{h = 0}$

Reaction **A** must pass through *C* where the 150-N weight and **B** interect.

Force triangle is equilateral.

A = 150.0 N ↗ 30.0° ◀

B = 150.0 N ↖ 30.0° ◀



(*b*) $\underline{h = 200\text{ mm}}$

$$\tan\beta = \frac{55.662}{250}$$

$$\beta = 12.5521°$$

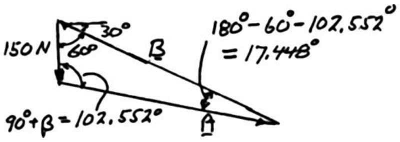
Law of sines:

$$\frac{150\text{ N}}{\sin 17.4480°} = \frac{A}{\sin 60°} = \frac{B}{\sin 102.552°}$$

$$A = 433.24\text{ N}$$

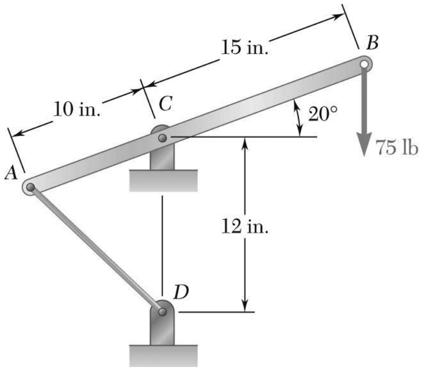
$$B = 488.31\text{ N}$$

Force Triangle



A = 433 N ↘ 12.55° ◀

B = 488 N ↖ 30.0° ◀

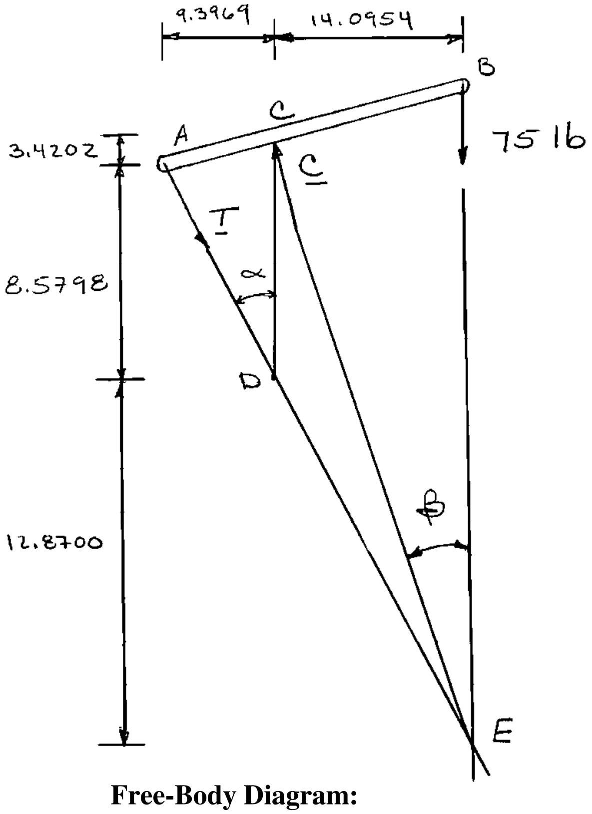


PROBLEM 4.80

Using the method of Section 4.7, solve Problem 4.24.

PROBLEM 4.24 A lever *AB* is hinged at *C* and attached to a control cable at *A*. If the lever is subjected to a 75-lb vertical force at *B*, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION



Free-Body Diagram:

Dimensions in in.

Reaction at *C* must pass through *E*, where the 75-lb force and **T** intersect.

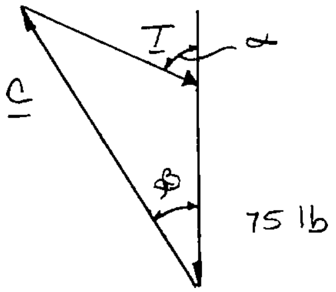
$$\tan\alpha = \frac{9.3969\text{ in.}}{8.5798\text{ in.}}$$

$$\alpha = 47.602°$$

$$\tan\beta = \frac{14.0954\text{ in.}}{24.870\text{ in.}}$$

$$\beta = 29.543°$$

Force Triangle

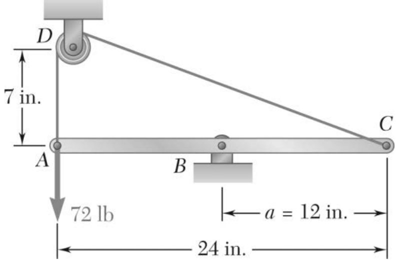


Law of sines:

$$\frac{75\text{ lb}}{\sin 18.0590°} = \frac{T}{\sin 29.543°} = \frac{C}{\sin 132.398°}$$

(*a*) $T = 119.3$ lb ◀

(*b*) $\mathbf{C} = 178.7$ lb ⦪ 60.5° ◀

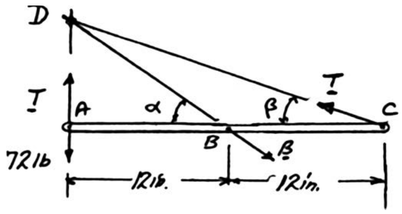


PROBLEM 4.81

Member *ABC* is supported by a pin and bracket at *B* and by an inextensible cord attached at *A* and *C* and passing over a frictionless pulley at *D*. The tension may be assumed to be the same in portions *AD* and *CD* of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at *B*.

SOLUTION

Free-Body Diagram:



Reaction at *B* must pass through *D*.

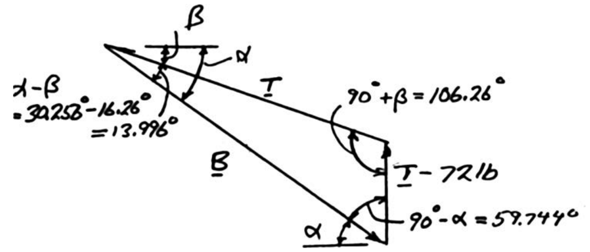
$$\tan\alpha = \frac{7\text{ in.}}{12\text{ in.}}$$

$$\alpha = 30.256°$$

$$\tan\beta = \frac{7\text{ in.}}{24\text{ in.}}$$

$$\beta = 16.26°$$

Force Triangle



Law of sines:

$$\frac{T}{\sin 59.744°} = \frac{T - 72\text{ lb}}{\sin 13.996°} = \frac{B}{\sin 106.26}$$

$$T(\sin 13.996°) = (T - 72\text{ lb})(\sin 59.744°)$$

$$T(0.24185) = (T - 72)(0.86378)$$

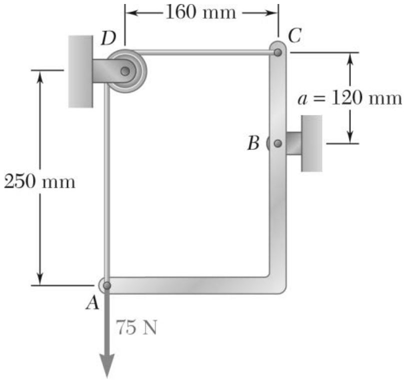
$$T = 100.00\text{ lb}$$

$T = 100.0$ lb ◀

$$B = (100\text{ lb})\frac{\sin 106.26°}{\sin 59.744°}$$

$$= 111.14\text{ lb}$$

$\mathbf{B} = 111.1\text{ lb} \searrow 30.3°$ ◀

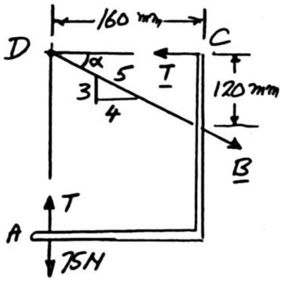


PROBLEM 4.82

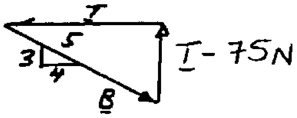
Member *ABC* is supported by a pin and bracket at *B* and by an inextensible cord attached at *A* and *C* and passing over a frictionless pulley at *D*. The tension may be assumed to be the same in portions *AD* and *CD* of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at *B*.

SOLUTION

Free-Body Diagram:



Force Triangle

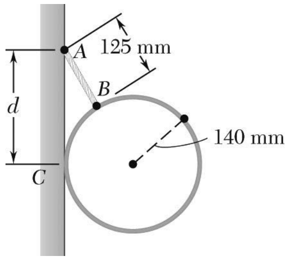


Reaction at *B* must pass through *D*.

$$\tan\alpha = \frac{120}{160}; \quad \alpha = 36.9° \qquad \frac{T}{4} = \frac{T - 75 \text{ N}}{3} = \frac{B}{5}$$

$$3T = 4T - 300; \quad T = 300 \text{ N}$$

$$B = \frac{5}{4}T = \frac{5}{4}(300 \text{ N}) = 375 \text{ N} \qquad \mathbf{B} = 375 \text{ N} \searrow 36.9°$$



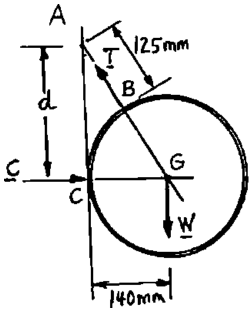
PROBLEM 4.83

A thin ring of mass 2 kg and radius $r = 140$ mm is held against a frictionless wall by a 125-mm string AB. Determine (*a*) the distance d, (*b*) the tension in the string, (*c*) the reaction at C.

SOLUTION

Free-Body Diagram:

(Three-force body)



The force T exerted at B must pass through the center G of the ring, since **C** and **W** intersect at that point.

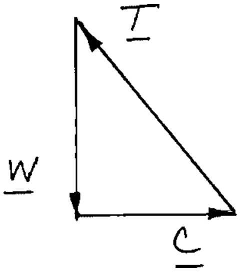
Thus, points A, B, and G are in a straight line.

(*a*) From triangle ACG:

$$d = \sqrt{(AG)^2 - (CG)^2}$$
$$= \sqrt{(265\text{ mm})^2 - (140\text{ mm})^2}$$
$$= 225.00\text{ mm}$$

$d = 225$ mm ◀

Force Triangle



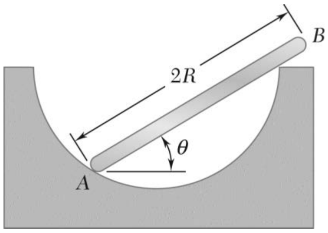
$$W = (2\text{ kg})(9.81\text{ m/s}^2) = 19.6200\text{ N}$$

Law of sines:

$$\frac{T}{265\text{ mm}} = \frac{C}{140\text{ mm}} = \frac{19.6200\text{ N}}{225.00\text{ mm}}$$

(*b*) $T = 23.1$ N ◀

(*c*) $\mathbf{C} = 12.21\text{ N} \longrightarrow$ ◀



PROBLEM 4.84

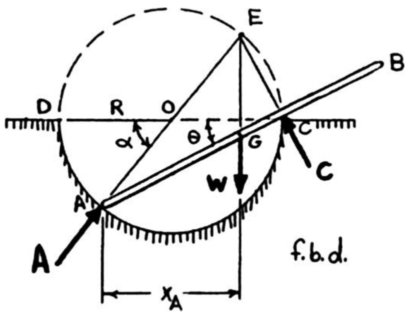
A uniform rod *AB* of length 2*R* rests inside a hemispherical bowl of radius *R* as shown. Neglecting friction, determine the angle θ corresponding to equilibrium.

SOLUTION

Based on the F.B.D., the uniform rod *AB* is a three-force body. Point *E* is the point of intersection of the three forces. Since force *A* passes through *O*, the center of the circle, and since force **C** is perpendicular to the rod, triangle *ACE* is a right triangle inscribed in the circle. Thus, *E* is a point on the circle.

Note that the angle α of triangle *DOA* is the central angle corresponding to the inscribed angle θ of triangle *DCA*.

$$\alpha = 2\theta$$



The horizontal projections of $AE, (x_{AE})$, and $AG, (x_{AG})$, are equal.

$$x_{AE} = x_{AG} = x_A$$

or $$(AE)\cos 2\theta = (AG)\cos\theta$$

and $$(2R)\cos 2\theta = R\cos\theta$$

Now $$\cos 2\theta = 2\cos^2\theta - 1$$

then $$4\cos^2\theta - 2 = \cos\theta$$

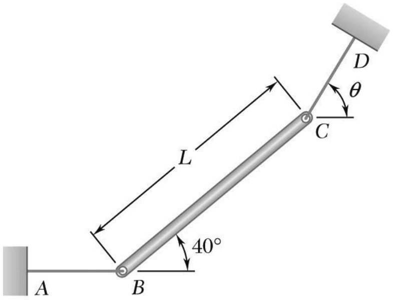
or $$4\cos^2\theta - \cos\theta - 2 = 0$$

Applying the quadratic equation,

$$\cos\theta = 0.84307 \quad \text{and} \quad \cos\theta = -0.59307$$

$$\theta = 32.534° \quad \text{and} \quad \theta = 126.375° \quad \text{(Discard)}$$

or $\theta = 32.5°$ ◀



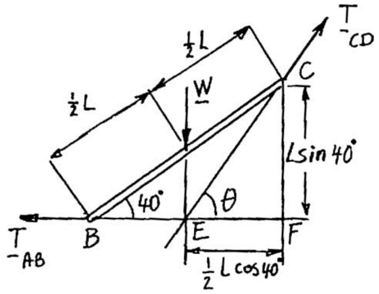
PROBLEM 4.85

A slender rod *BC* of length *L* and weight *W* is held by two cables as shown. Knowing that cable *AB* is horizontal and that the rod forms an angle of 40° with the horizontal, determine (*a*) the angle θ that cable *CD* forms with the horizontal, (*b*) the tension in each cable.

SOLUTION

Free-Body Diagram:

(Three-force body)



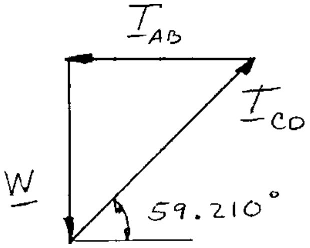
(*a*) The line of action of $\mathbf{T}_{CD}$ must pass through *E*, where $\mathbf{T}_{AB}$ and $\mathbf{W}$ intersect.

$$\tan\theta = \frac{CF}{EF}$$
$$= \frac{L\sin 40°}{\frac{1}{2}L\cos 40°}$$
$$= 2\tan 40°$$
$$= 59.210°$$

$$\theta = 59.2° \blacktriangleleft$$

(b) Force Triangle

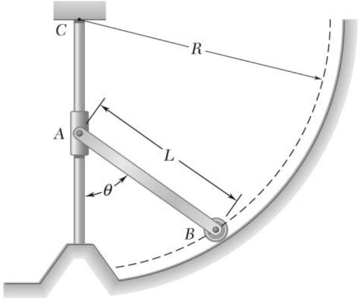
$$T_{AB} = W\tan 30.790°$$
$$= 0.59588W$$



$$T_{AB} = 0.596W \blacktriangleleft$$

$$T_{CD} = \frac{W}{\cos 30.790°}$$
$$= 1.16408W$$

$$T_{CD} = 1.164W \blacktriangleleft$$

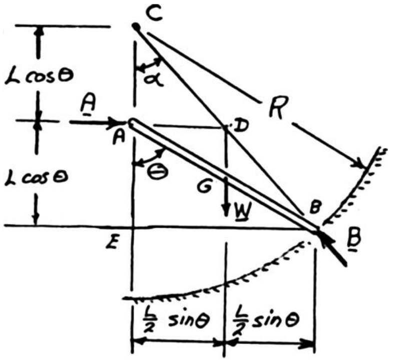


PROBLEM 4.86

A slender rod of length L and weight W is attached to a collar at A and is fitted with a small wheel at B. Knowing that the wheel rolls freely along a cylindrical surface of radius R, and neglecting friction, derive an equation in θ, L, and R that must be satisfied when the rod is in equilibrium.

SOLUTION

Free-Body Diagram (Three-force body)



Reaction **B** must pass through D where **B** and **W** intersect.

Note that ΔABC and ΔBGD are similar.

$$AC = AE = L\cos\theta$$

In ΔABC:

$$(CE)^2 + (BE)^2 = (BC)^2$$

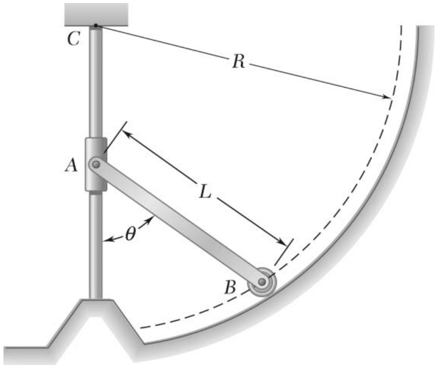
$$(2L\cos\theta)^2 + (L\sin\theta)^2 = R^2$$

$$\left(\frac{R}{L}\right)^2 = 4\cos^2\theta + \sin^2\theta$$

$$\left(\frac{R}{L}\right)^2 = 4\cos^2\theta + 1 - \cos^2\theta$$

$$\left(\frac{R}{L}\right)^2 = 3\cos^2\theta + 1$$

$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{R}{L}\right)^2 - 1\right] \blacktriangleleft$$



PROBLEM 4.87

Knowing that for the rod of Problem 4.86, L = 15 in., R = 20 in., and W = 10 lb, determine (*a*) the angle θ corresponding to equilibrium, (*b*) the reactions at A and B.

SOLUTION

See the solution to Problem 4.86 for the free-body diagram and analysis leading to the following equation:

$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{R}{L}\right)^2 - 1\right]$$

For $L = 15$ in., $R = 20$ in., and $W = 10$ lb,

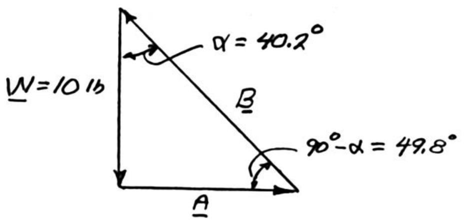
(*a*) $$\cos^2\theta = \frac{1}{3}\left[\left(\frac{20\text{ in.}}{15\text{ in.}}\right)^2 - 1\right]; \quad \theta = 59.39°$$ $\theta = 59.4°$ ◀

In ΔABC: $$\tan\alpha = \frac{BE}{CE} = \frac{L\sin\theta}{2L\cos\theta} = \frac{1}{2}\tan\theta$$

$$\tan\alpha = \frac{1}{2}\tan 59.39° = 0.8452$$

$$\alpha = 40.2°$$

Force Triangle

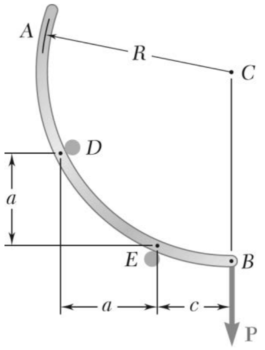


$$A = W\tan\alpha = (10\text{ lb})\tan 40.2° = 8.45\text{ lb}$$

$$B = \frac{W}{\cos\alpha} = \frac{(10\text{ lb})}{\cos 40.2°} = 13.09\text{ lb}$$

(*b*) **A** = 8.45 lb → ◀

B = 13.09 lb ↘ 49.8° ◀

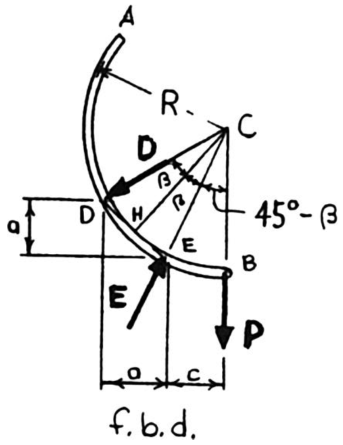


PROBLEM 4.88

Rod *AB* is bent into the shape of an arc of circle and is lodged between two pegs *D* and *E*. It supports a load **P** at end *B*. Neglecting friction and the weight of the rod, determine the distance *c* corresponding to equilibrium when $a = 20$ mm and $R = 100$ mm.

SOLUTION

Free-Body Diagram:



Since $y_{ED} = x_{ED} = a$,

slope of *ED* is ⦣ 45°;

slope of *HC* is ∠ 45°.

Also $$DE = \sqrt{2}a$$

and $$DH = HE = \left(\frac{1}{2}\right)DE = \frac{a}{\sqrt{2}}$$

For triangles *DHC* and *EHC*, $$\sin\beta = \frac{\frac{a}{\sqrt{2}}}{R} = \frac{a}{\sqrt{2}R}$$

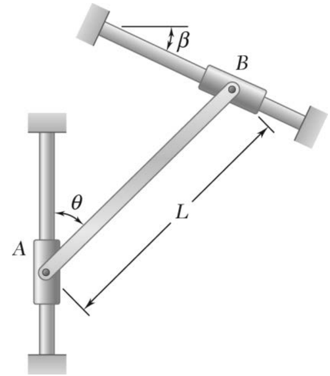
Now $$c = R\sin(45° - \beta)$$

For $$a = 20\text{ mm} \quad\text{and}\quad R = 100\text{ mm}$$

$$\sin\beta = \frac{20\text{ mm}}{\sqrt{2}(100\text{ mm})}$$
$$= 0.141421$$
$$\beta = 8.1301°$$

and $$c = (100\text{ mm})\sin(45° - 8.1301°)$$
$$= 60.00\text{ mm}$$

or $c = 60.0$ mm ◀



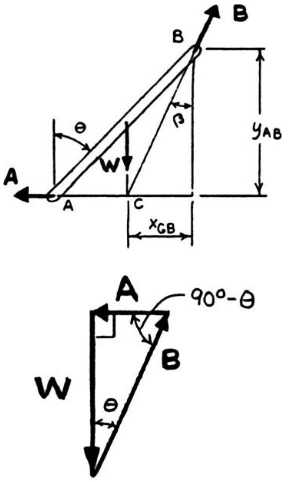
PROBLEM 4.89

A slender rod of length L is attached to collars that can slide freely along the guides shown. Knowing that the rod is in equilibrium, derive an expression for the angle θ in terms of the angle β.

SOLUTION

As shown in the free-body diagram of the slender rod AB, the three forces intersect at C. From the force geometry:

Free-Body Diagram:



$$\tan\beta = \frac{x_{GB}}{y_{AB}}$$

where

$$y_{AB} = L\cos\theta$$

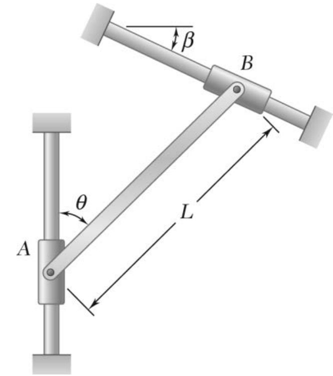
and

$$x_{GB} = \frac{1}{2}L\sin\theta$$

$$\tan\beta = \frac{\frac{1}{2}L\sin\theta}{L\cos\theta}$$

$$= \frac{1}{2}\tan\theta$$

or $\tan\theta = 2\tan\beta$ ◀



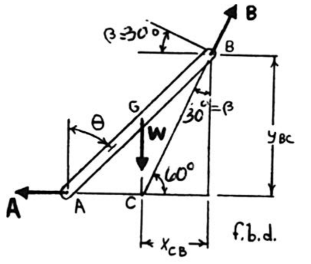
PROBLEM 4.90

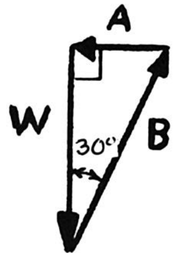
An 8-kg slender rod of length L is attached to collars that can slide freely along the guides shown. Knowing that the rod is in equilibrium and that $\beta = 30°$, determine (*a*) the angle θ that the rod forms with the vertical, (*b*) the reactions at A and B.

SOLUTION

(*a*) As shown in the free-body diagram of the slender rod AB, the three forces intersect at C. From the geometry of the forces:

Free-Body Diagram:





$$\tan\beta = \frac{x_{CB}}{y_{BC}}$$

where

$$x_{CB} = \frac{1}{2}L\sin\theta$$

and

$$y_{BC} = L\cos\theta$$

$$\tan\beta = \frac{1}{2}\tan\theta$$

or $\tan\theta = 2\tan\beta$

For $\beta = 30°$

$$\begin{aligned}\tan\theta &= 2\tan 30°\\ &= 1.15470\\ \theta &= 49.107°\end{aligned}$$

or $\theta = 49.1°$ ◀

(*b*) $W = mg = (8\text{ kg})(9.81\text{ m/s}^2) = 78.480\text{ N}$

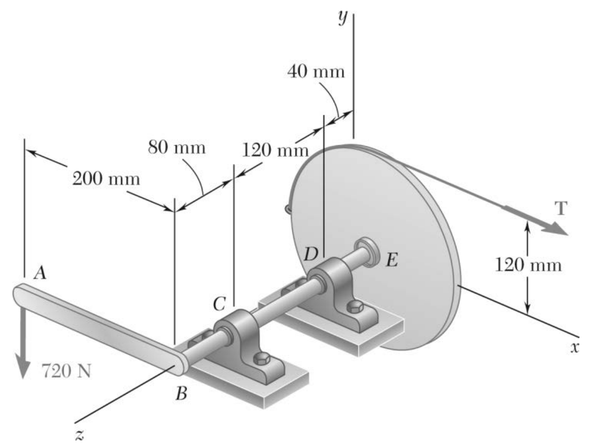
From force triangle:

$$\begin{aligned}A &= W\tan\beta\\ &= (78.480\text{ N})\tan 30°\\ &= 45.310\text{ N}\end{aligned}$$

or $\mathbf{A} = 45.3\text{ N} \longleftarrow$ ◀

and $B = \dfrac{W}{\cos\beta} = \dfrac{78.480\text{ N}}{\cos 30°} = 90.621\text{ N}$

or $\mathbf{B} = 90.6\text{ N} \measuredangle 60.0°$ ◀

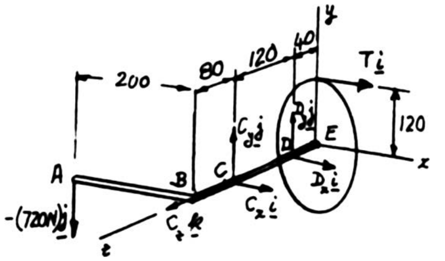


PROBLEM 4.91

A 200-mm lever and a 240-mm-diameter pulley are welded to the axle *BE* that is supported by bearings at *C* and *D*. If a 720-N vertical load is applied at *A* when the lever is horizontal, determine (*a*) the tension in the cord, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Dimensions in mm



We have six unknowns and six equations of equilibrium. —OK

$$\Sigma\mathbf{M}_C = 0: \quad (-120\mathbf{k})\times(D_x\mathbf{i}+D_y\mathbf{j})+(120\mathbf{j}-160\mathbf{k})\times T\mathbf{i}+(80\mathbf{k}-200\mathbf{i})\times(-720\mathbf{j})=0$$

$$-120D_x\mathbf{j}+120D_y\mathbf{i}-120T\mathbf{k}-160T\mathbf{j}+57.6\times10^3\mathbf{i}+144\times10^3\mathbf{k}=0$$

Equating to zero the coefficients of the unit vectors:

$\mathbf{k}$: $\quad -120T+144\times10^3=0$ $\qquad$ (*a*) $T = 1200$ N ◀

$\mathbf{i}$: $\quad 120D_y+57.6\times10^3=0 \qquad D_y=-480$ N

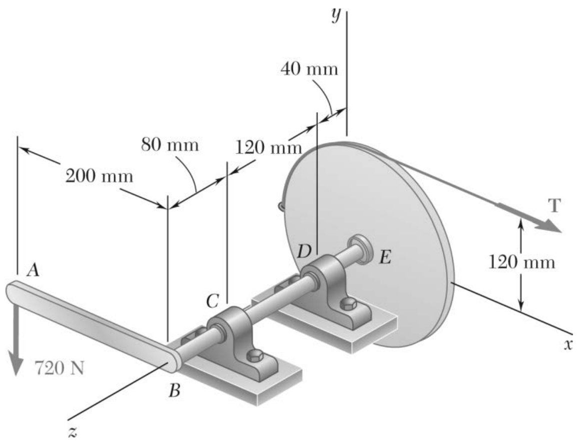
$\mathbf{j}$: $\quad -120D_x-160(1200\text{ N})=0 \qquad D_x=-1600$ N

$\Sigma F_x=0$: $\quad C_x+D_x+T=0 \qquad C_x=1600-1200=400$ N

$\Sigma F_y=0$: $\quad C_y+D_y-720=0 \qquad C_y=480+720=1200$ N

$\Sigma F_z=0$: $\quad C_z=0$

(*b*) $\qquad \mathbf{C}=(400\text{ N})\mathbf{i}+(1200\text{ N})\mathbf{j}; \quad \mathbf{D}=-(1600\text{ N})\mathbf{i}-(480\text{ N})\mathbf{j}$ ◀



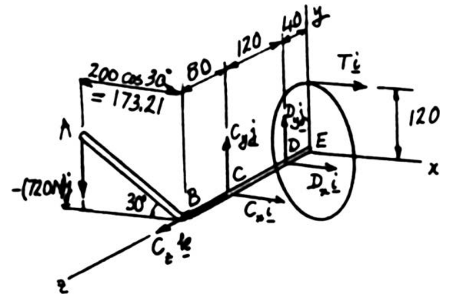
PROBLEM 4.92

Solve Problem 4.91, assuming that the axle has been rotated clockwise in its bearings by 30° and that the 720-N load remains vertical.

PROBLEM 4.91 A 200-mm lever and a 240-mm-diameter pulley are welded to the axle *BE* that is supported by bearings at *C* and *D*. If a 720-N vertical load is applied at *A* when the lever is horizontal, determine (*a*) the tension in the cord, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Dimensions in mm



We have six unknowns and six equations of equilibrium.

$$\Sigma \mathbf{M}_C = 0: \quad (-120\mathbf{k})\times(D_x\mathbf{i}+D_y\mathbf{j}) + (120\mathbf{j}-160\mathbf{k})\times T\mathbf{i} + (80\mathbf{k}-173.21\mathbf{i})\times(-720\mathbf{j}) = 0$$

$$-120D_x\mathbf{j} + 120D_y\mathbf{i} - 120T\,\mathbf{k} - 160T\mathbf{j} + 57.6\times10^3\,\mathbf{i} + 124.71\times10^3\,\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors,

$$\mathbf{k}: \quad -120T + 124.71\times10^3 = 0 \qquad T = 1039.2 \text{ N}$$

$T = 1039$ N ◀

$$\mathbf{i}: \quad 120D_y + 57.6\times10^3 = 0 \qquad D_y = -480 \text{ N}$$

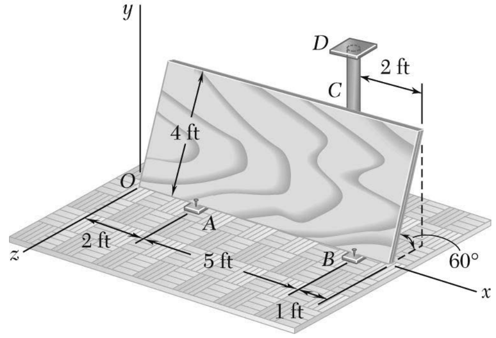
$$\mathbf{j}: \quad -120D_x - 160(1039.2) \qquad D_x = -1385.6 \text{ N}$$

$$\Sigma F_x = 0: \quad C_x + D_x + T = 0 \qquad C_x = 1385.6 - 1039.2 = 346.4$$

$$\Sigma F_y = 0: \quad C_y + D_y - 720 = 0 \qquad C_y = 480 + 720 = 1200 \text{ N}$$

$$\Sigma F_z = 0: \quad C_z = 0$$

(*b*) **C** = (346 N)**i** + (1200 N)**j**   **D** = −(1386 N)**i** − (480 N)**j** ◀



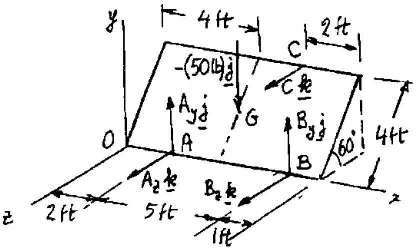
PROBLEM 4.93

A 4 × 8-ft sheet of plywood weighing 40 lb has been temporarily propped against column *CD*. It rests at *A* and *B* on small wooden blocks and against protruding nails. Neglecting friction at all surfaces of contact, determine the reactions at *A*, *B*, and *C*.

SOLUTION

Free-Body Diagram:

We have five unknowns and six equations of equilibrium. Plywood sheet is free to move in *x* direction, but equilibrium is maintained $(\Sigma F_x = 0)$.



$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times (B_y\,\mathbf{j} + B_z\mathbf{k}) + \mathbf{r}_{C/A} \times C\,\mathbf{k} + \mathbf{r}_{G/A} \times (-40\text{ lb})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 4\sin 60° & -4\cos 60° \\ 0 & 0 & C \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2\sin 60° & -2\cos 60° \\ 0 & -40 & 0 \end{vmatrix} = 0$$

$$(4C\sin 60° - 80\cos 60°)\mathbf{i} + (-5B_z - 4C)\mathbf{j} + (5B_y - 80)\mathbf{k} = 0$$

Equating the coefficients of the unit vectors to zero,

$$\mathbf{i}: \quad 4C\sin 60° - 80\cos 60° = 0 \quad C = 11.5470\text{ lb}$$

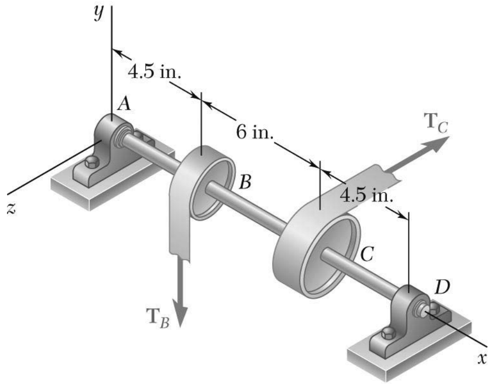
$$\mathbf{j}: \quad -5B_z - 4C = 0 \quad B_z = 9.2376\text{ lb}$$

$$\mathbf{k}: \quad 5B_y - 80 = 0 \quad B_y = 16.0000\text{ lb}$$

$$\Sigma F_y = 0: \quad A_y + B_y - 40 = 0 \quad A_y = 40 - 16.0000 = 24.000\text{ lb}$$

$$\Sigma F_z = 0: \quad A_z + B_z + C = 0 \quad A_z = 9.2376 - 11.5470 = -2.3094\text{ lb}$$

$$\mathbf{A} = (24.0\text{ lb})\mathbf{j} - (2.31\text{ lb})\mathbf{k}; \quad \mathbf{B} = (16.00\text{ lb})\mathbf{j} - (9.24\text{ lb})\mathbf{k}; \quad \mathbf{C} = (11.55\text{ lb})\mathbf{k} \blacktriangleleft$$

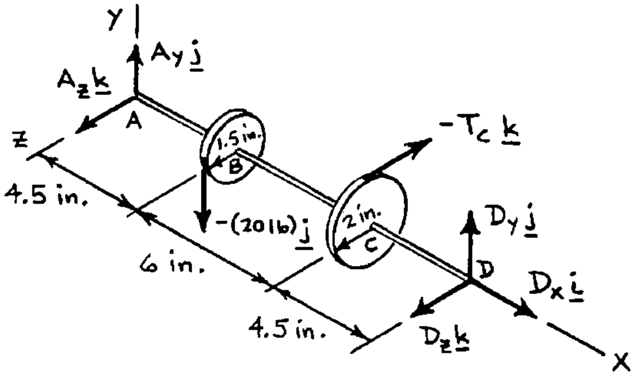


PROBLEM 4.94

Two tape spools are attached to an axle supported by bearings at *A* and *D*. The radius of spool *B* is 1.5 in. and the radius of spool *C* is 2 in. Knowing that $T_B = 20$ lb and that the system rotates at a constant rate, determine the reactions at *A* and *D*. Assume that the bearing at *A* does not exert any axial thrust and neglect the weights of the spools and axle.

SOLUTION

Free-Body Diagram:



We have six unknowns and six equations of equilibrium.

$$\Sigma M_A = 0: \quad (4.5\mathbf{i} + 1.5\mathbf{k}) \times (-20\mathbf{j}) + (10.5\mathbf{i} + 2\mathbf{j}) \times (-T_C\mathbf{k}) + (15\mathbf{i}) \times (D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-90\mathbf{k} + 30\mathbf{i} + 10.5T_C\mathbf{j} - 2T_C\mathbf{i} + 15D_y\mathbf{k} - 15D_z\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$$\vec{\mathbf{i}}: \quad 30 - 2T_C = 0 \qquad\qquad T_C = 15\text{ lb}$$

$$\vec{\mathbf{j}}: \quad 10.5T_C - 15D_z = 0 \quad 10.5(15) - 15D_z = 0 \qquad\qquad D_z = 10.5\text{ lb}$$

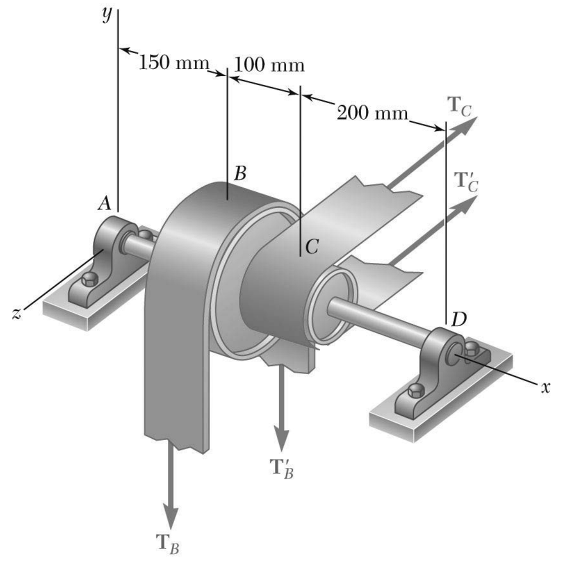
$$\vec{\mathbf{k}}: \quad -90 + 15D_y = 0 \qquad\qquad D_y = 6\text{ lb}$$

$$\Sigma F_x = 0: \quad D_x = 0$$

$$\Sigma F_y = 0: \quad A_y + D_y - 20\text{ lb} = 0 \qquad A_y = 20 - 6 = 14\text{ lb}$$

$$\Sigma F_z = 0: \quad A_z + D_z - 15\text{ lb} = 0 \qquad A_z = 15 - 10.5 = 4.5\text{ lb}$$

$$\mathbf{A} = (14.00\text{ lb})\mathbf{j} + (4.50\text{ lb})\mathbf{k}; \quad \mathbf{D} = (6.00\text{ lb})\mathbf{j} + (10.50\text{ lb})\mathbf{k} \blacktriangleleft$$

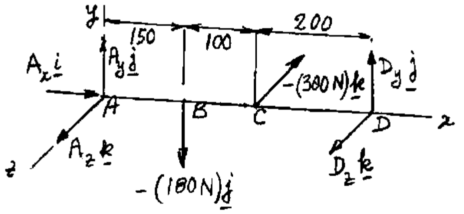


PROBLEM 4.95

Two transmission belts pass over a double-sheaved pulley that is attached to an axle supported by bearings at *A* and *D*. The radius of the inner sheave is 125 mm and the radius of the outer sheave is 250 mm. Knowing that when the system is at rest, the tension is 90 N in both portions of belt *B* and 150 N in both portions of belt *C*, determine the reactions at *A* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

We replace $\mathbf{T}_B$ and $\mathbf{T}'_B$ by their resultant $(-180\text{ N})\mathbf{j}$ and $\mathbf{T}_C$ and $\mathbf{T}'_C$ by their resultant $(-300\text{ N})\mathbf{k}$.



Dimensions in mm

We have five unknowns and six equations of equilibrium. Axle *AD* is free to rotate about the *x*-axis, but equilibrium is maintained $(\Sigma M_x = 0)$.

$$\Sigma \mathbf{M}_A = 0: \quad (150\mathbf{i})\times(-180\mathbf{j}) + (250\mathbf{i})\times(-300\mathbf{k}) + (450\mathbf{i})\times(D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-27\times10^3\mathbf{k} + 75\times10^3\mathbf{j} + 450D_y\mathbf{k} - 450D_z\mathbf{j} = 0$$

Equating coefficients of **j** and **k** to zero,

$$\mathbf{j}: \quad 75\times10^3 - 450D_z = 0 \qquad D_z = 166.7\text{ N}$$

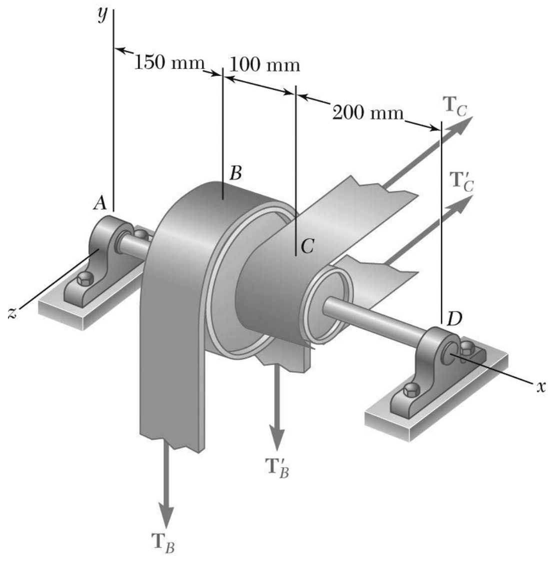
$$\mathbf{k}: \quad -27\times10^3 + 450D_y = 0 \qquad D_y = 60.0\text{ N}$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma F_y = 0: \quad A_y + D_y - 180\text{ N} = 0 \qquad A_y = 180 - 60 = 120.0\text{ N}$$

$$\Sigma F_z = 0: \quad A_z + D_z - 300\text{ N} = 0 \qquad A_z = 300 - 166.7 = 133.3\text{ N}$$

$$\mathbf{A} = (120.0\text{ N})\mathbf{j} + (133.3\text{ N})\mathbf{k}; \quad \mathbf{D} = (60.0\text{ N})\mathbf{j} + (166.7\text{ N})\mathbf{k} \blacktriangleleft$$

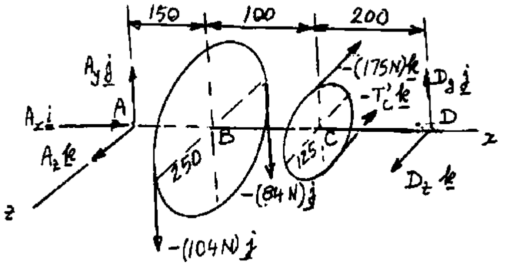


PROBLEM 4.96

Solve Problem 4.95, assuming that the pulley rotates at a constant rate and that $T_B = 104$ N, $T'_B = 84$ N, $T_C = 175$ N.

PROBLEM 4.95 Two transmission belts pass over a double-sheaved pulley that is attached to an axle supported by bearings at *A* and *D*. The radius of the inner sheave is 125 mm and the radius of the outer sheave is 250 mm. Knowing that when the system is at rest, the tension is 90 N in both portions of belt *B* and 150 N in both portions of belt *C*, determine the reactions at *A* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION



Dimensions in mm

We have six unknowns and six equations of equilibrium. —OK

$$\Sigma \mathbf{M}_A = 0: \quad (150\mathbf{i} + 250\mathbf{k}) \times (-104\mathbf{j}) + (150\mathbf{i} - 250\mathbf{k}) \times (-84\mathbf{j})$$

$$+ (250\mathbf{i} + 125\mathbf{j}) \times (-175\mathbf{k}) + (250\mathbf{i} - 125\mathbf{j}) \times (-T_C)$$

$$+ 450\mathbf{i} \times (D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-150(104 + 84)\mathbf{k} + 250(104 - 84)\mathbf{i} + 250(175 + T'_C)\,\mathbf{j} \; -125(175 - T'_C)$$

$$+ 450D_y\mathbf{k} - 450D_z\mathbf{j} = 0$$

Equating the coefficients of the unit vectors to zero,

$$\mathbf{i}: \quad 250(104 - 84) - 125(175 - T'_C) = 0 \qquad 175 = T'_C = 40 \qquad T'_C = 135;$$

$$\mathbf{j}: \quad 250(175 + 135) - 450D_z = 0 \qquad D_z = 172.2 \text{ N}$$

$$\mathbf{k}: \quad -150(104 + 84) + 450\,D_y = 0 \qquad D_y = 62.7 \text{ N}$$

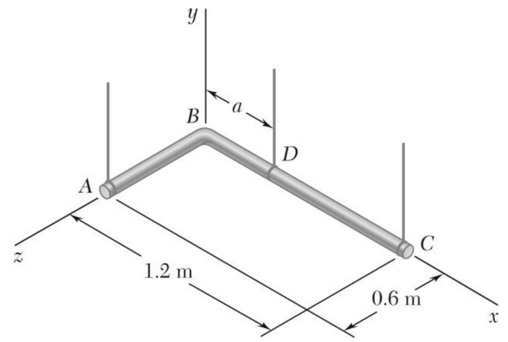
PROBLEM 4.96 (Continued)

$\Sigma F_x = 0$: $A_x = 0$

$\Sigma F_y = 0$: $A_y - 104 - 84 + 62.7 = 0$ $A_y = 125.3 \text{ N}$

$\Sigma F_z = 0$: $A_z - 175 - 135 + 172.2 = 0$ $A_z = 137.8 \text{ N}$

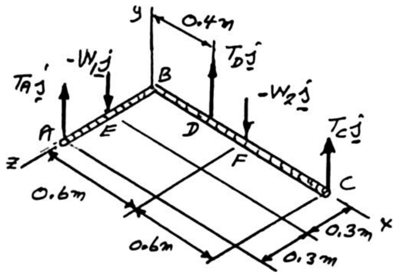
$\mathbf{A} = (125.3 \text{ N})\mathbf{j} + (137.8 \text{ N})\mathbf{k}$; $\mathbf{D} = (62.7 \text{ N})\mathbf{j} + (172.2 \text{ N})\mathbf{k}$ ◀



PROBLEM 4.97

Two steel pipes *AB* and *BC*, each having a mass per unit length of 8 kg/m, are welded together at *B* and supported by three wires. Knowing that $a = 0.4$ m, determine the tension in each wire.

SOLUTION



$$W_1 = 0.6m'g$$
$$W_2 = 1.2m'g$$

$$\Sigma M_D = 0: \quad \mathbf{r}_{A/D} \times T_A\mathbf{j} + \mathbf{r}_{E/D} \times (-W_1\mathbf{j}) + \mathbf{r}_{F/D} \times (-W_2\mathbf{j}) + \mathbf{r}_{C/D} \times T_C\mathbf{j} = 0$$

$$(-0.4\mathbf{i} + 0.6\mathbf{k}) \times T_A\mathbf{j} + (-0.4\mathbf{i} + 0.3\mathbf{k}) \times (-W_1\mathbf{j}) + 0.2\mathbf{i} \times (-W_2\mathbf{j}) + 0.8\mathbf{i} \times T_C\mathbf{j} = 0$$

$$-0.4T_A\mathbf{k} - 0.6T_A\mathbf{i} + 0.4W_1\mathbf{k} + 0.3W_1\mathbf{i} - 0.2W_2\mathbf{k} + 0.8T_C\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.6T_A + 0.3W_1 = 0; \quad T_A = \frac{1}{2}W_1 = \frac{1}{2}0.6m'g = 0.3m'g$$

$$\mathbf{k}: \quad -0.4T_A + 0.4W_1 - 0.2W_2 + 0.8T_C = 0$$

$$-0.4(0.3m'g) + 0.4(0.6m'g) - 0.2(1.2m'g) + 0.8T_C = 0$$

$$T_C = \frac{(0.12 - 0.24 - 0.24)m'g}{0.8} = 0.15m'g$$

$$\Sigma F_y = 0: \quad T_A + T_C + T_D - W_1 - W_2 = 0$$

$$0.3m'g + 0.15m'g + T_D - 0.6m'g - 1.2m'g = 0$$

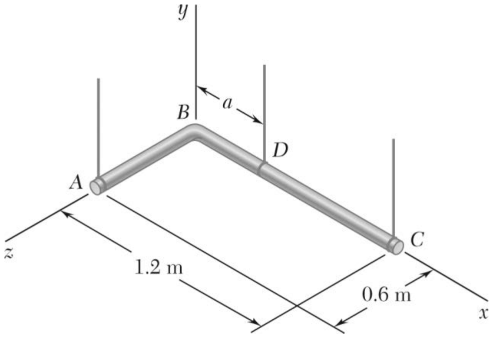
$$T_D = 1.35m'g$$

$$m'g = (8\text{ kg/m})(9.81\text{m/s}^2) = 78.48\text{ N/m}$$

$T_A = 0.3m'g = 0.3 \times 78.45$ $\quad T_A = 23.5$ N ◀

$T_B = 0.15m'g = 0.15 \times 78.45$ $\quad T_B = 11.77$ N ◀

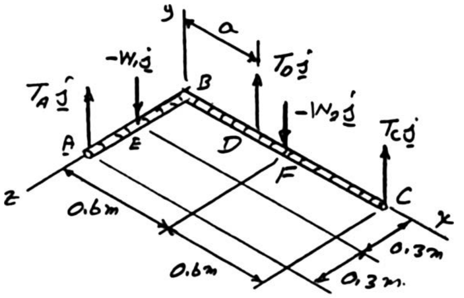
$T_C = 1.35m'g = 1.35 \times 78.45$ $\quad T_C = 105.9$ N ◀



PROBLEM 4.98

For the pipe assembly of Problem 4.97, determine (*a*) the largest permissible value of *a* if the assembly is not to tip, (*b*) the corresponding tension in each wire.

SOLUTION



$$W_1 = 0.6m'g$$
$$W_2 = 1.2m'g$$

$$\Sigma M_D = 0: \quad \mathbf{r}_{A/D} \times T_A\mathbf{j} + \mathbf{r}_{E/D} \times (-W_1\mathbf{j}) + \mathbf{r}_{F/D} \times (-W_2\mathbf{j}) + \mathbf{r}_{C/D} \times T_C\mathbf{j} = 0$$

$$(-a\mathbf{i} + 0.6\mathbf{k}) \times T_A\mathbf{j} + (-a\mathbf{i} + 0.3\mathbf{k}) \times (-W_1\mathbf{j}) + (0.6 - a)\mathbf{i} \times (-W_2\mathbf{j}) + (1.2 - a)\mathbf{i} \times T_C\mathbf{j} = 0$$

$$-T_A a\mathbf{k} - 0.6T_A\mathbf{i} + W_1 a\mathbf{k} + 0.3W_1\mathbf{i} - W_2(0.6 - a)\mathbf{k} + T_C(1.2 - a)\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.6T_A + 0.3W_1 = 0; \quad T_A = \frac{1}{2}W_1 = \frac{1}{2}0.6m'g = 0.3m'g$$

$$\mathbf{k}: \quad -T_A a + W_1 a - W_2(0.6 - a) + T_C(1.2 - a) = 0$$

$$-0.3m'ga + 0.6m'ga - 1.2m'g(0.6 - a) + T_C(1.2 - a) = 0$$

$$T_C = \frac{0.3a - 0.6a + 1.2(0.6 - a)}{1.2 - a}$$ For maximum *a* and no tipping, $T_C = 0$.

(*a*)

$$-0.3a + 1.2(0.6 - a) = 0$$
$$-0.3a + 0.72 - 1.2a = 0$$
$$1.5a = 0.72$$

$a = 0.480 \text{ m}$ ◀

PROBLEM 4.98 (Continued)

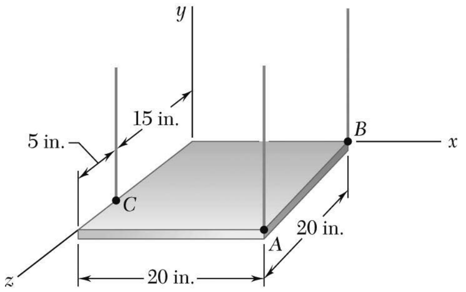
(*b*) Reactions: $m'g = (8\text{ kg/m})\,9.81\text{ m/s}^2 = 78.48\text{ N/m}$

$$T_A = 0.3m'g = 0.3 \times 78.48 = 23.544\text{ N} \qquad T_A = 23.5\text{ N} \blacktriangleleft$$

$$\Sigma F_y = 0{:}\quad T_A + T_C + T_D - W_1 - W_2 = 0$$

$$T_A + 0 + T_D - 0.6m'g - 1.2m'g = 0$$

$$T_D = 1.8m'g - T_A = 1.8 \times 78.48 - 23.544 = 117.72 \qquad T_D = 117.7\text{ N} \blacktriangleleft$$

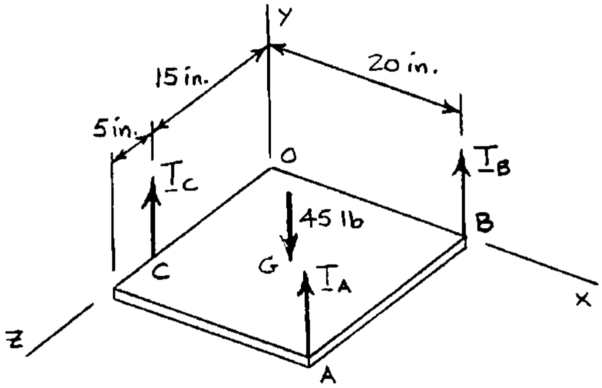


PROBLEM 4.99

The 45-lb square plate shown is supported by three vertical wires. Determine the tension in each wire.

SOLUTION

Free-Body Diagram:



$$\Sigma M_B = 0: \quad \mathbf{r}_{C/B} \times T_C\mathbf{j} + \mathbf{r}_{A/B} \times T_A\mathbf{j} + \mathbf{r}_{G/B} \times (-45\text{ lb})\mathbf{j} = 0$$

$$[-(20\text{ in.})\mathbf{i} + (15\text{ in.})\vec{\mathbf{k}}] \times T_C\mathbf{j} + (20\text{ in.})\mathbf{k} \times T_A\mathbf{j}$$

$$+ [-(10\text{ in.})\mathbf{i} + (10\text{ in.})\mathbf{k}] \times [-(45\text{ lb})\mathbf{j}] = 0$$

$$-20T_C\mathbf{k} - 15T_C\mathbf{i} - 20T_A\mathbf{i} + 450\mathbf{k} + 450\mathbf{i} = 0$$

Equating to zero the coefficients of the unit vectors,

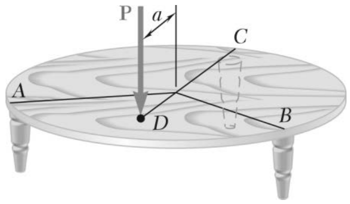
$\mathbf{k}$: $\quad -20T_C + 450 = 0 \qquad T_C = 22.5\text{ lb}$ ◀

$\mathbf{i}$: $\quad -15(22.5) - 20T_A + 450 = 0 \qquad T_A = 5.625\text{ lb}$ ◀

$\Sigma F_y = 0$: $\quad T_A + T_B + T_C - 45\text{ lb} = 0$

$5.625\text{ lb} + T_B + 22.5\text{ lb} - 45\text{ lb} = 0 \qquad T_B = 16.875\text{ lb}$ ◀

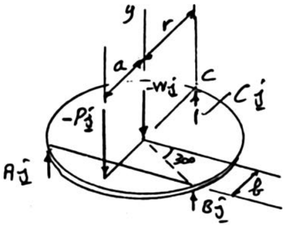
$T_A = 5.63\text{ lb};\ T_B = 16.88\text{ lb};\ T_C = 22.5\text{ lb}$ ◀



PROBLEM 4.100

The table shown weighs 30 lb and has a diameter of 4 ft. It is supported by three legs equally spaced around the edge. A vertical load **P** of magnitude 100 lb is applied to the top of the table at *D*. Determine the maximum value of *a* if the table is not to tip over. Show, on a sketch, the area of the table over which **P** can act without tipping the table.

SOLUTION



$$r = 2\text{ ft} \quad b = r\sin 30° = 1\text{ ft}$$

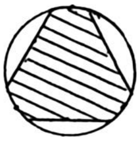
We shall sum moments about *AB*.

$$(b+r)C + (a-b)P - bW = 0$$

$$(1+2)C + (a-1)100 - (1)30 = 0$$

$$C = \frac{1}{3}[30 - (a-1)100]$$

If table is not to tip, $C \geq 0$.

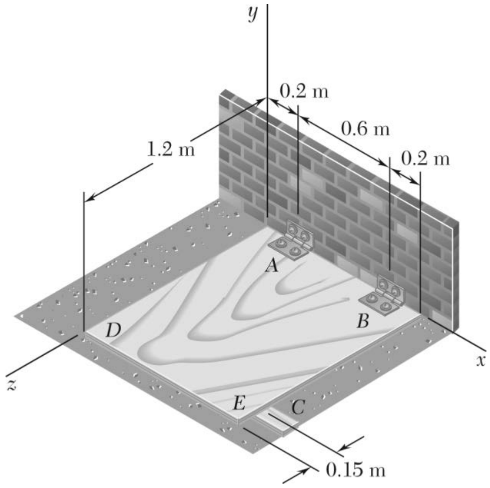


$$[30 - (a-1)100] \geq 0$$

$$30 \geq (a-1)100$$

$$a - 1 \leq 0.3 \quad a \leq 1.3\text{ ft} \quad a = 1.300\text{ ft}$$

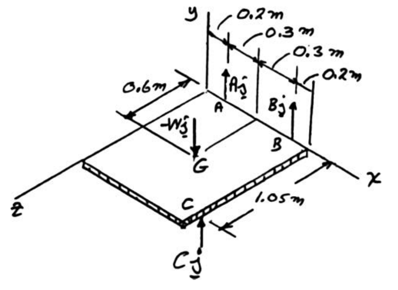
Only ⊥ distance from *P* to *AB* matters. Same condition must be satisfied for each leg. *P* must be located in shaded area for no tipping.



PROBLEM 4.101

An opening in a floor is covered by a 1×1.2-m sheet of plywood of mass 18 kg. The sheet is hinged at *A* and *B* and is maintained in a position slightly above the floor by a small block *C*. Determine the vertical component of the reaction (*a*) at *A*, (*b*) at *B*, (*c*) at *C*.

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$
$$\mathbf{r}_{C/A} = 0.8\mathbf{i} + 1.05\mathbf{k}$$
$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$
$$W = mg = (18\text{ kg})9.81$$
$$W = 176.58\text{ N}$$

$$\Sigma M_A = 0\!: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$
$$(0.6\mathbf{i}) \times B\mathbf{j} + (0.8\mathbf{i} + 1.05\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$$
$$0.6B\mathbf{k} + 0.8C\mathbf{k} - 1.05C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

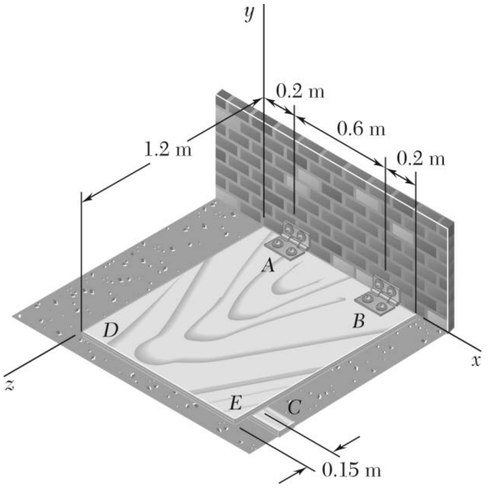
Equate coefficients of unit vectors to zero:

$$\mathbf{i}\!: \quad 1.05C + 0.6W = 0 \quad C = \left(\frac{0.6}{1.05}\right)176.58\text{ N} = 100.90\text{ N}$$

$$\mathbf{k}\!: \quad 0.6B + 0.8C - 0.3W = 0$$
$$0.6B + 0.8(100.90\text{ N}) - 0.3(176.58\text{ N}) = 0 \quad B = -46.24\text{ N}$$

$$\Sigma F_y = 0\!: \quad A + B + C - W = 0$$
$$A - 46.24\text{ N} + 100.90\text{ N} + 176.58\text{ N} = 0 \quad A = 121.92\text{ N}$$

(*a*) $A = 121.9\text{ N}$ (*b*) $B = -46.2\text{ N}$ (*c*) $C = 100.9\text{ N}$ ◀

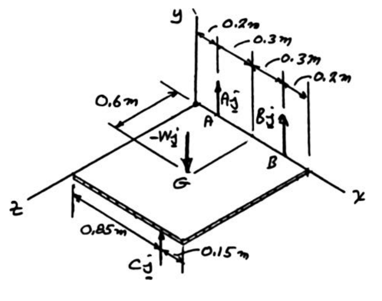


PROBLEM 4.102

Solve Problem 4.101, assuming that the small block C is moved and placed under edge DE at a point 0.15 m from corner E.

PROBLEM 4.101 An opening in a floor is covered by a 1×1.2-m sheet of plywood of mass 18 kg. The sheet is hinged at A and B and is maintained in a position slightly above the floor by a small block C. Determine the vertical component of the reaction (a) at A, (b) at B, (c) at C.

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$
$$\mathbf{r}_{C/A} = 0.65\mathbf{i} + 1.2\mathbf{k}$$
$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$

$$W = mg = (18\text{ kg})\,9.81\text{ m/s}^2$$
$$W = 176.58\text{ N}$$

$$\Sigma M_A = 0:\quad \mathbf{r}_{B/A}\times B\mathbf{j} + \mathbf{r}_{C/A}\times C\mathbf{j} + \mathbf{r}_{G/A}\times(-W\mathbf{j}) = 0$$
$$0.6\mathbf{i}\times B\mathbf{j} + (0.65\mathbf{i} + 1.2\mathbf{k})\times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k})\times(-W\mathbf{j}) = 0$$
$$0.6B\mathbf{k} + 0.65C\mathbf{k} - 1.2C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

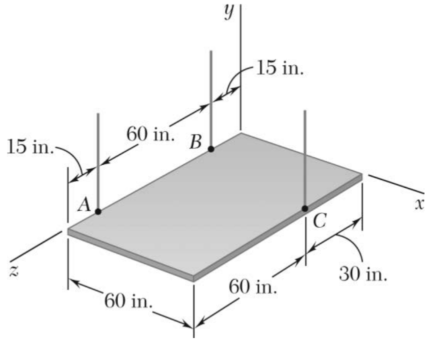
Equate coefficients of unit vectors to zero:

$$\mathbf{i}:\quad -1.2C + 0.6W = 0 \qquad C = \left(\frac{0.6}{1.2}\right)176.58\text{ N} = 88.29\text{ N}$$

$$\mathbf{k}:\quad 0.6B + 0.65C - 0.3W = 0$$
$$0.6B + 0.65(88.29\text{ N}) - 0.3(176.58\text{ N}) = 0 \qquad B = -7.36\text{ N}$$

$$\Sigma F_y = 0:\quad A + B + C - W = 0$$
$$A - 7.36\text{ N} + 88.29\text{ N} - 176.58\text{ N} = 0 \qquad A = 95.648\text{ N}$$

$$(a)\quad A = 95.6\text{ N} \quad (b)\quad B = -7.36\text{ N} \quad (c)\quad C = 88.3\text{ N} \blacktriangleleft$$

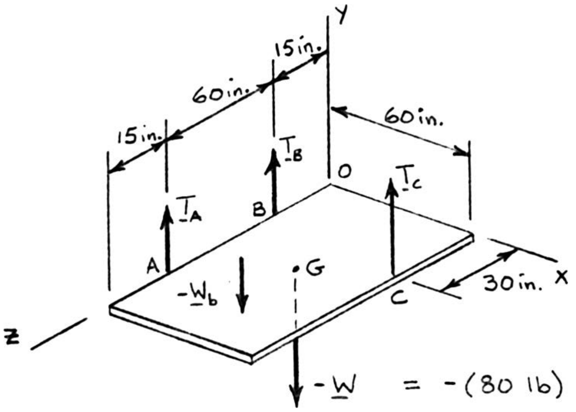


PROBLEM 4.103

The rectangular plate shown weighs 80 lb and is supported by three vertical wires. Determine the tension in each wire.

SOLUTION

Free-Body Diagram:



$$\Sigma \mathbf{M}_B = 0: \quad \mathbf{r}_{A/B} \times T_A \mathbf{j} + \mathbf{r}_{C/B} \times T_C \mathbf{j} + \mathbf{r}_{G/B} \times (-80 \text{ lb})\mathbf{j} = 0$$

$$(60 \text{ in.})\mathbf{k} \times T_A \mathbf{j} + [(60 \text{ in.})\mathbf{i} + (15 \text{ in.})\mathbf{k}] \times T_C \mathbf{j} + [(30 \text{ in.})\mathbf{i} + (30 \text{ in.})\mathbf{k}] \times (-80 \text{ lb})\mathbf{j} = 0$$

$$-60T_A\mathbf{i} + 60T_C\mathbf{k} - 15T_C\mathbf{i} - 2400\mathbf{k} + 2400\mathbf{i} = 0$$

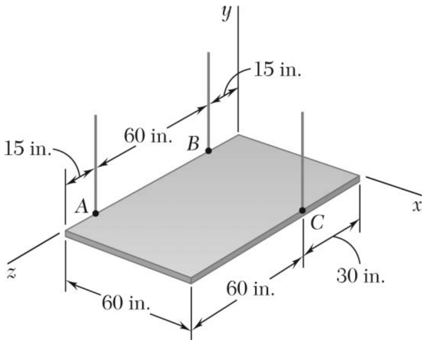
Equating to zero the coefficients of the unit vectors,

$$\mathbf{i}: \quad 60T_A - 15(40) + 2400 = 0 \qquad T_A = 30.0 \text{ lb} \blacktriangleleft$$

$$\mathbf{k}: \quad 60T_C - 2400 = 0 \qquad T_C = 40.0 \text{ lb} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - 80 \text{ lb} = 0$$

$$30 \text{ lb} + T_B + 40 \text{ lb} - 80 \text{ lb} = 0 \qquad T_B = 10.00 \text{ lb} \blacktriangleleft$$

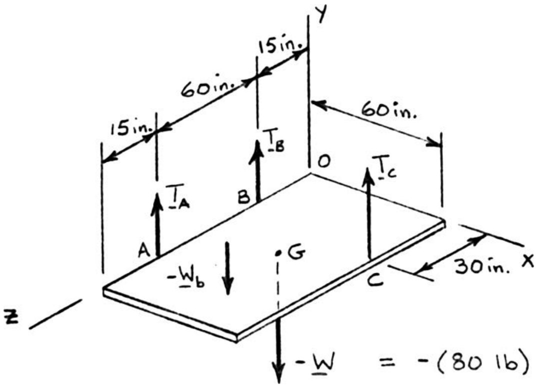


PROBLEM 4.104

The rectangular plate shown weighs 80 lb and is supported by three vertical wires. Determine the weight and location of the lightest block that should be placed on the plate if the tensions in the three wires are to be equal.

SOLUTION

Free-Body Diagram:



Let $-W_b\mathbf{j}$ be the weight of the block and x and z the block's coordinates.

Since tensions in wires are equal, let

$$T_A = T_B = T_C = T$$

$$\Sigma M_0 = 0: \quad (\mathbf{r}_A \times T\mathbf{j}) + (\mathbf{r}_B \times T\mathbf{j}) + (\mathbf{r}_C \times T\mathbf{j}) + \mathbf{r}_G \times (-W\mathbf{j}) + (x\mathbf{i} + z\mathbf{k}) \times (-W_b\mathbf{j}) = 0$$

or $$(75\,\mathbf{k}) \times T\mathbf{j} + (15\,\mathbf{k}) \times T\mathbf{j} + (60\mathbf{i} + 30\mathbf{k}) \times T\mathbf{j} + (30\mathbf{i} + 45\mathbf{k}) \times (-W\mathbf{j}) + (x\mathbf{i} + z\mathbf{k}) \times (-W_b\mathbf{j}) = 0$$

or $$-75T\mathbf{i} - 15T\mathbf{i} + 60T\mathbf{k} - 30T\mathbf{i} - 30W\mathbf{k} + 45W\mathbf{i} - W_b \times \mathbf{k} + W_b z\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -120T + 45W + W_b z = 0 \tag{1}$$

$$\mathbf{k}: \quad 60T - 30W - W_b x = 0 \tag{2}$$

Also, $$\Sigma F_y = 0: \quad 3T - W - W_b = 0 \tag{3}$$

Eq. (1) + 40 Eq. (3): $$5W + (z - 40)W_b = 0 \tag{4}$$

Eq. (2) – 20 Eq. (3): $$-10W - (x - 20)W_b = 0 \tag{5}$$

Solving Eqs. (4) and (5) for W_b/W and recalling that $0 \le x \le 60$ in., $0 \le z \le 90$ in.,

Eq. (4):
$$\frac{W_b}{W} = \frac{5}{40 - z} \ge \frac{5}{40 - 0} = 0.125$$

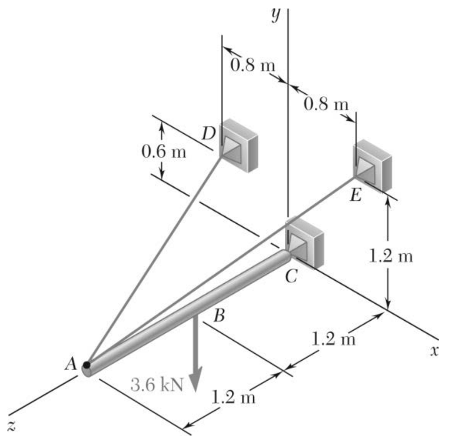
Eq. (5):
$$\frac{W_b}{W} = \frac{10}{20 - x} \ge \frac{10}{20 - 0} = 0.5$$

Thus, $(W_b)_{\min} = 0.5W = 0.5(80) = 40 \text{ lb}$ $\qquad (W_b)_{\min} = 40.0 \text{ lb}$ ◀

Making $W_b = 0.5W$ in Eqs. (4) and (5):

$$5W + (z - 40)(0.5W) = 0 \qquad z = 30.0 \text{ in.}$$ ◀

$$-10W - (x - 20)(0.5W) = 0 \qquad x = 0 \text{ in.}$$ ◀

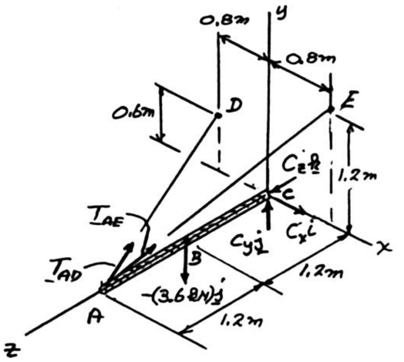


PROBLEM 4.105

A 2.4-m boom is held by a ball-and-socket joint at *C* and by two cables *AD* and *AE*. Determine the tension in each cable and the reaction at *C*.

SOLUTION

Free-Body Diagram: Five unknowns and six equations of equilibrium, but equilibrium is maintained ($\Sigma M_{AC} = 0$).



$$\mathbf{r}_B = 1.2\mathbf{k}$$
$$\mathbf{r}_A = 2.4\mathbf{k}$$

$$\overrightarrow{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \qquad AD = 2.6 \text{ m}$$
$$\overrightarrow{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \qquad AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$
$$T_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times (-3 \text{ kN})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ -0.8 & 0.6 & -2.4 \end{vmatrix} \frac{T_{AD}}{2.6} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ 0.8 & 1.2 & -2.4 \end{vmatrix} \frac{T_{AE}}{2.8} + 1.2\mathbf{k} \times (-3.6 \text{ kN})\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.55385T_{AD} - 1.02857T_{AE} + 4.32 = 0 \tag{1}$$
$$\mathbf{j}: \quad -0.73846T_{AD} + 0.68671T_{AE} = 0$$

$$T_{AD} = 0.92857T_{AE} \tag{2}$$

From Eq. (1):

$$-0.55385(0.92857)T_{AE} - 1.02857T_{AE} + 4.32 = 0$$

$$1.54286T_{AE} = 4.32$$
$$T_{AE} = 2.800 \text{ kN} \qquad T_{AE} = 2.80 \text{ kN} ◀$$

PROBLEM 4.105 (Continued)

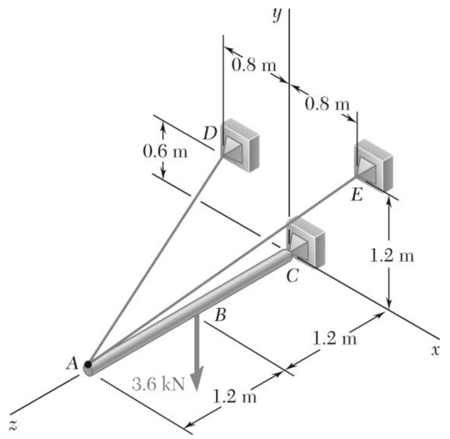
From Eq. (2): $T_{AD} = 0.92857(2.80) = 2.600\text{ kN}$ $\qquad T_{AD} = 2.60\text{ kN}$ ◀

$$\Sigma F_x = 0: \quad C_x - \frac{0.8}{2.6}(2.6\text{ kN}) + \frac{0.8}{2.8}(2.8\text{ kN}) = 0 \qquad C_x = 0$$

$$\Sigma F_y = 0: \quad C_y + \frac{0.6}{2.6}(2.6\text{ kN}) + \frac{1.2}{2.8}(2.8\text{ kN}) - (3.6\text{ kN}) = 0 \qquad C_y = 1.800\text{ kN}$$

$$\Sigma F_z = 0: \quad C_z - \frac{2.4}{2.6}(2.6\text{ kN}) - \frac{2.4}{2.8}(2.8\text{ kN}) = 0 \qquad C_z = 4.80\text{ kN}$$

$\mathbf{C} = (1.800\text{ kN})\mathbf{j} + (4.80\text{ kN})\mathbf{k}$ ◀



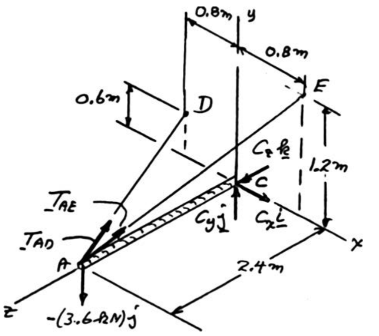
PROBLEM 4.106

Solve Problem 4.105, assuming that the 3.6-kN load is applied at Point *A*.

PROBLEM 4.105 A 2.4-m boom is held by a ball-and-socket joint at *C* and by two cables *AD* and *AE*. Determine the tension in each cable and the reaction at *C*.

SOLUTION

Free-Body Diagram: Five unknowns and six equations of equilibrium, but equilibrium is maintained $(\Sigma M_{AC} = 0)$.



$$\overrightarrow{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \qquad AD = 2.6\text{ m}$$

$$\overrightarrow{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \qquad AE = 2.8\text{ m}$$

$$T_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$

$$T_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_A \times (-3.6\text{ kN})\mathbf{j}$$

Factor r_A: $\quad \mathbf{r}_A \times (\mathbf{T}_{AD} + \mathbf{T}_{AE} - (3.6\text{ kN})\mathbf{j})$

or $\quad \mathbf{T}_{AD} + \mathbf{T}_{AE} - (3\text{ kN})\mathbf{j} = 0 \quad$ (Forces concurrent at *A*)

Coefficient of **i**:

$$-\frac{T_{AD}}{2.6}(0.8) + \frac{T_{AE}}{2.8}(0.8) = 0$$

$$T_{AD} = \frac{2.6}{2.8}T_{AE} \tag{1}$$

Coefficient of **j**:

$$\frac{T_{AD}}{2.6}(0.6) + \frac{T_{AE}}{2.8}(1.2) - 3.6\text{ kN} = 0$$

$$\frac{2.6}{2.8}T_{AE}\left(\frac{0.6}{2.6}\right) + \frac{1.2}{2.8}T_{AE} - 3.6\text{ kN} = 0$$

$$T_{AE}\left(\frac{0.6 + 1.2}{2.8}\right) = 3.6\text{ kN}$$

$$T_{AE} = 5.600\text{ kN} \qquad T_{AE} = 5.60\text{ kN} \blacktriangleleft$$

PROBLEM 4.106 (Continued)

From Eq. (1): $T_{AD} = \dfrac{2.6}{2.8}(5.6) = 5.200\text{ kN}$ $\qquad T_{AD} = 5.20\text{ kN}$ ◀

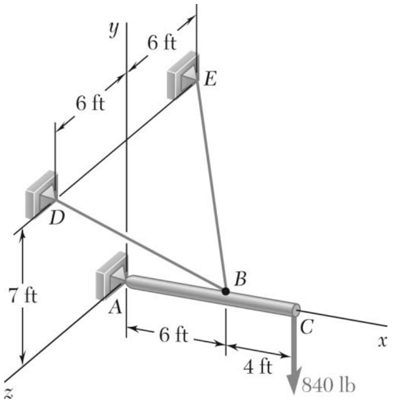
$$\Sigma F_x = 0\text{:}\quad C_x - \frac{0.8}{2.6}(5.2\text{ kN}) + \frac{0.8}{2.8}(5.6\text{ kN}) = 0 \qquad C_x = 0$$

$$\Sigma F_y = 0\text{:}\quad C_y + \frac{0.6}{2.6}(5.2\text{ kN}) + \frac{1.2}{2.8}(5.6\text{ kN}) - 3.6\text{ kN} = 0 \qquad C_y = 0$$

$$\Sigma F_z = 0\text{:}\quad C_z - \frac{2.4}{2.6}(5.2\text{ kN}) - \frac{2.4}{2.8}(5.6\text{ kN}) = 0 \qquad C_z = 9.60\text{ kN}$$

$\mathbf{C} = (9.60\text{ kN})\mathbf{k}$ ◀

Note: Since the forces and reaction are concurrent at *A*, we could have used the methods of Chapter 2.



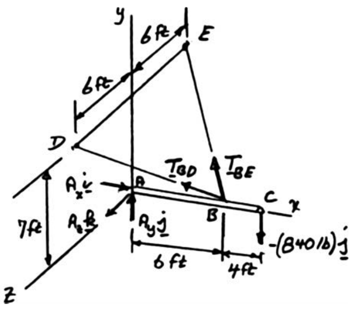
PROBLEM 4.107

A 10-ft boom is acted upon by the 840-lb force shown. Determine the tension in each cable and the reaction at the ball-and-socket joint at *A*.

SOLUTION

We have five unknowns and six equations of equilibrium, but equilibrium is maintained $(\Sigma M_x = 0)$.

Free-Body Diagram:



$$\overrightarrow{BD} = (-6\text{ ft})\mathbf{i} + (7\text{ ft})\mathbf{j} + (6\text{ ft})\mathbf{k} \quad BD = 11\text{ ft}$$

$$\overrightarrow{BE} = (-6\text{ ft})\mathbf{i} + (7\text{ ft})\mathbf{j} - (6\text{ ft})\mathbf{k} \quad BE = 11\text{ ft}$$

$$T_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{11}(-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$$

$$T_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{T_{BE}}{11}(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_B \times T_{BD} + \mathbf{r}_B \times T_{BE} + \mathbf{r}_C \times (-840\mathbf{j}) = 0$$

$$6\mathbf{i} \times \frac{T_{BD}}{11}(-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}) + 6\mathbf{i} \times \frac{T_{BE}}{11}(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}) + 10\mathbf{i} \times (-840\mathbf{j}) = 0$$

$$\frac{42}{11}T_{BD}\mathbf{k} - \frac{36}{11}T_{BD}\mathbf{j} + \frac{42}{11}T_{BE}\mathbf{k} + \frac{36}{11}T_{BE}\mathbf{j} - 8400\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -\frac{36}{11}T_{BD} + \frac{36}{11}T_{BE} = 0 \quad T_{BE} = T_{BD}$$

$$\mathbf{k}: \quad \frac{42}{11}T_{BD} + \frac{42}{11}T_{BE} - 8400 = 0$$

$$2\left(\frac{42}{11}T_{BD}\right) = 8400 \qquad T_{BD} = 1100\text{ lb} \blacktriangleleft$$

$$T_{BE} = 1100\text{ lb} \blacktriangleleft$$

PROBLEM 4.107 (Continued)

$$\Sigma F_x = 0: \quad A_x - \frac{6}{11}(1100 \text{ lb}) - \frac{6}{11}(1100 \text{ lb}) = 0$$

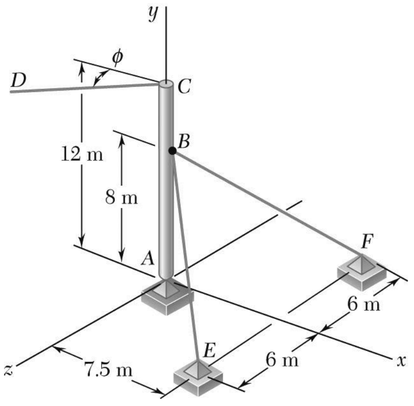
$$A_x = 1200 \text{ lb}$$

$$\Sigma F_y = 0: \quad A_y + \frac{7}{11}(1100 \text{ lb}) + \frac{7}{11}(1100 \text{ lb}) - 840 \text{ lb} = 0$$

$$A_y = -560 \text{ lb}$$

$$\Sigma F_z = 0: \quad A_z + \frac{6}{11}(1100 \text{ lb}) - \frac{6}{11}(1100 \text{ lb}) = 0$$

$$A_z = 0 \qquad \mathbf{A} = (1200 \text{ lb})\mathbf{i} - (560 \text{ lb})\mathbf{j} \blacktriangleleft$$

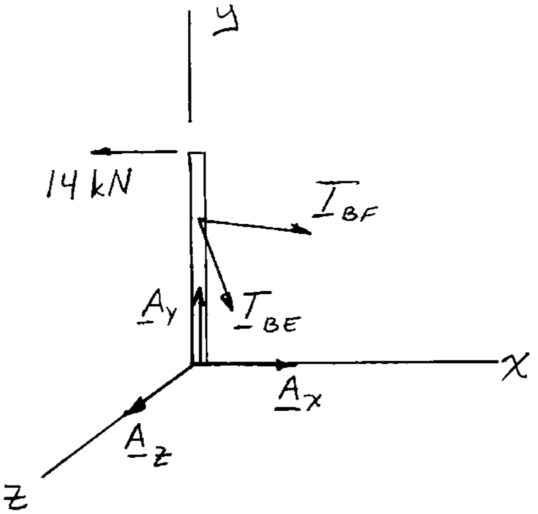


PROBLEM 4.108

A 12-m pole supports a horizontal cable *CD* and is held by a ball and socket at *A* and two cables *BE* and *BF*. Knowing that the tension in cable *CD* is 14 kN and assuming that *CD* is parallel to the *x*-axis ($\phi = 0$), determine the tension in cables *BE* and *BF* and the reaction at *A*.

SOLUTION

Free-Body Diagram:



There are five unknowns and six equations of equilibrium. The pole is free to rotate about the *y*-axis, but equilibrium is maintained under the given loading ($\Sigma M_y = 0$).

Resolve $\overrightarrow{BE}$ and $\overrightarrow{BF}$ into components:

$$\overrightarrow{BE} = (7.5 \text{ m})\mathbf{i} - (8 \text{ m})\mathbf{j} + (6 \text{ m})\mathbf{k} \qquad BE = 12.5 \text{ m}$$

$$\overrightarrow{BF} = (7.5 \text{ m})\mathbf{i} - (8 \text{ m})\mathbf{j} - (6 \text{ m})\mathbf{k} \qquad BF = 12.5 \text{ m}$$

Express $\mathbf{T}_{BE}$ and $\mathbf{T}_{BF}$ in terms of components:

$$\mathbf{T}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = T_{BE}(0.60\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}) \tag{1}$$

$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = T_{BF}(0.60\mathbf{i} - 0.64\mathbf{j} - 0.48\mathbf{k}) \tag{2}$$

PROBLEM 4.108 (Continued)

$$\Sigma M_A = 0:\quad \mathbf{r}_{B/A} \times T_{BE} + \mathbf{r}_{B/A} \times T_{BF} + \mathbf{r}_{C/A} \times (-14\text{ kN})\mathbf{i} = 0$$

$$8\mathbf{j} \times T_{BE}(0.60\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}) + 8\mathbf{j} \times T_{BF}(0.60\mathbf{i} - 0.64\mathbf{j} - 0.48\mathbf{k}) + 12\mathbf{j} \times (-14\mathbf{i}) = 0$$

$$-4.8\,T_{BE}\mathbf{k} + 3.84\,T_{BE}\mathbf{i} - 4.8T_{BF}\mathbf{k} - 3.84T_{BF}\mathbf{i} + 168\mathbf{k} = 0$$

Equating the coefficients of the unit vectors to zero,

$$\mathbf{i}:\qquad 3.84T_{BE} - 3.84T_{BF} = 0 \qquad T_{BE} = T_{BF}$$

$$\mathbf{k}:\qquad -4.8T_{BE} - 4.8T_{BF} + 168 = 0 \qquad T_{BE} = T_{BF} = 17.50\text{ kN}$$ ◀

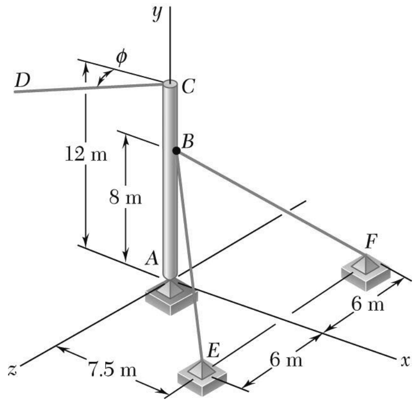
$$\Sigma F_x = 0:\qquad A_x + 2(0.60)(17.50\text{ kN}) - 14\text{ kN} = 0 \qquad A_x = 7.00\text{ kN}$$

$$\Sigma F_y = 0:\qquad A_y - z(0.64)(17.50\text{ kN}) = 0 \qquad A_y = 22.4\text{ kN}$$

$$\Sigma F_z = 0:\qquad A_z + 0 = 0 \qquad A_z = 0$$

$$\mathbf{A} = -(7.00\text{ kN})\mathbf{i} + (22.4\text{ kN})\mathbf{j}$$ ◀

Because of the symmetry, we could have noted at the outset that $T_{BF} = T_{BE}$ and eliminated one unknown.



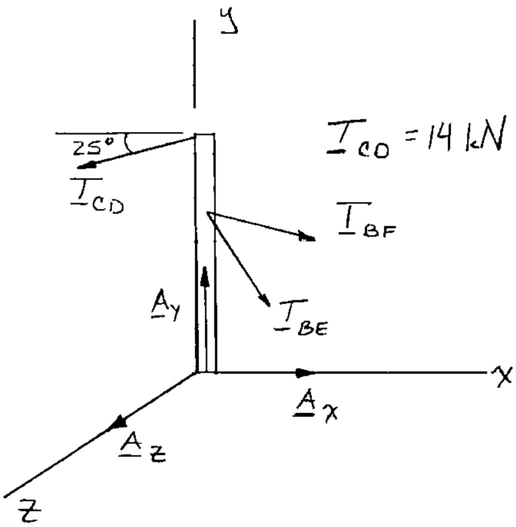
PROBLEM 4.109

Solve Problem 4.108, assuming that cable *CD* forms an angle $\phi = 25°$ with the vertical *xy* plane.

PROBLEM 4.108 A 12-m pole supports a horizontal cable *CD* and is held by a ball and socket at *A* and two cables *BE* and *BF*. Knowing that the tension in cable *CD* is 14 kN and assuming that *CD* is parallel to the *x*-axis ($\phi = 0$), determine the tension in cables *BE* and *BF* and the reaction at *A*.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{BE} = (7.5\text{ m})\mathbf{i} - (8\text{ m})\mathbf{j} + (6\text{ m})\mathbf{k}$$

$$BE = 12.5\text{ m}$$

$$\overrightarrow{BF} = (7.5\text{ m})\mathbf{i} - (8\text{ m})\mathbf{j} - (6\text{ m})\mathbf{k}$$

$$BF = 12.5\text{ m}$$

$$\mathbf{T}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = T_{BE}(0.60\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k})$$

$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = T_{BF}(0.60\mathbf{i} - 0.64\mathbf{j} - 0.48\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times \mathbf{T}_{BE} + \mathbf{r}_{B/A} \times \mathbf{T}_{BF} + \mathbf{r}_{C/A} \times \mathbf{T}_{CD} = 0$$

$$8\mathbf{j} \times T_{BE}(0.60\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}) + 8\mathbf{j} \times T_{BF}(0.60\mathbf{i} - 0.64\mathbf{j} - 0.48\mathbf{k})$$
$$+ 12\mathbf{j} \times (19\text{ kN})(-\cos 25°\,\mathbf{i} + \sin 25°\mathbf{k}) = 0$$

$$-4.8T_{BE}\,\mathbf{k} + 3.84T_{BE}\,\mathbf{i} - 4.8T_{BF}\,\mathbf{k} - 3.84T_{BF}\,\mathbf{i} + 152.6\,\mathbf{k} - 71.00\,\mathbf{i} = 0$$

Equating the coefficients of the unit vectors to zero,

$$\mathbf{i}: \quad 3.84T_{BE} - 3.84T_{BF} + 71.00 = 0; \qquad T_{BF} - T_{BE} = 18.4896$$

$$\mathbf{k}: \quad -4.8T_{BE} - 4.8\,T_{BF} + 152.26 = 0; \quad T_{BF} + T_{BE} = 31.721$$

Solving simultaneously, $T_{BE} = 6.6157$ kN; $T_{BF} = 25.105$ kN

$T_{BE} = 6.62$ kN; $T_{BF} = 25.1$ kN ◀

$$\Sigma F_x = 0: \quad A_x + (0.60)(T_{BF} + T_{BE}) - 14\cos 25° = 0$$

$$A_x = 12.6883 - 0.60(31.7207)$$

$$A_x = -6.34\text{ kN}$$

PROBLEM 4.109 (Continued)

$$\Sigma F_y = 0: \quad A_y - (0.64)(T_{BF} + T_{BE}) = 0$$

$$A_y = 0.64(31.721)$$

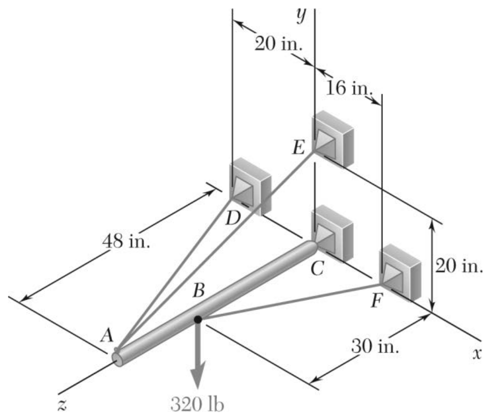
$$A_y = 20.3 \text{ kN}$$

$$\Sigma F_z = 0: \quad A_z - 0.48(T_{BF} - T_{BE}) + 14\sin 25° = 0$$

$$A_z = 0.48(18.4893) - 5.9167$$

$$A_z = 2.96 \text{ kN}$$

$$\mathbf{A} = -(6.34 \text{ kN})\mathbf{i} + (20.3 \text{ kN})\mathbf{j} + (2.96 \text{ kN})\mathbf{k} \blacktriangleleft$$

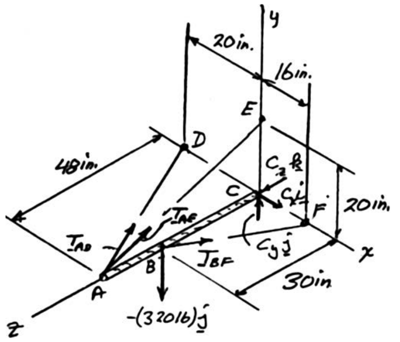


PROBLEM 4.110

A 48-in. boom is held by a ball-and-socket joint at *C* and by two cables *BF* and *DAE*; cable *DAE* passes around a frictionless pulley at *A*. For the loading shown, determine the tension in each cable and the reaction at *C*.

SOLUTION

Free-Body Diagram:



Five unknowns and six equations of equilibrium, but equilibrium is maintained $(\Sigma M_{AC} = 0)$.

$T =$ Tension in both parts of cable *DAE*.

$$\mathbf{r}_B = 30\mathbf{k}$$
$$\mathbf{r}_A = 48\mathbf{k}$$

$$\overrightarrow{AD} = -20\mathbf{i} - 48\mathbf{k} \qquad AD = 52 \text{ in.}$$
$$\overrightarrow{AE} = 20\mathbf{j} - 48\mathbf{k} \qquad AE = 52 \text{ in.}$$
$$\overrightarrow{BF} = 16\mathbf{i} - 30\mathbf{k} \qquad BF = 34 \text{ in.}$$

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD} = \frac{T}{52}(-20\mathbf{i} - 48\mathbf{k}) = \frac{T}{13}(-5\mathbf{i} - 12\mathbf{k})$$
$$\mathbf{T}_{AE} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{52}(20\mathbf{j} - 48\mathbf{k}) = \frac{T}{13}(5\mathbf{j} - 12\mathbf{k})$$
$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = \frac{T_{BF}}{34}(16\mathbf{i} - 30\mathbf{k}) = \frac{T_{BF}}{17}(8\mathbf{i} - 15\mathbf{k})$$

$$\Sigma\mathbf{M}_C = 0\text{:} \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times \mathbf{T}_{BF} + \mathbf{r}_B \times (-320 \text{ lb})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 8 & 0 & -15 \end{vmatrix}\frac{T_{BF}}{17} + (30\mathbf{k}) \times (-320\mathbf{j}) = 0$$

Coefficient of **i**: $$-\frac{240}{13}T + 9600 = 0 \qquad T = 520 \text{ lb}$$

PROBLEM 4.110 (Continued)

Coefficient of **j**: $-\frac{240}{13}T+\frac{240}{17}T_{BD}=0$

$$T_{BD}=\frac{17}{13}T=\frac{17}{13}(520) \quad T_{BD}=680\text{ lb}$$

$\Sigma\mathbf{F}=0$: $\mathbf{T}_{AD}+\mathbf{T}_{AE}+\mathbf{T}_{BF}-320\mathbf{j}+C=0$

Coefficient of **i**: $-\frac{20}{52}(520)+\frac{8}{17}(680)+C_x=0$

$$-200+320+C_x=0 \qquad C_x=-120\text{ lb}$$

Coefficient of **j**: $\frac{20}{52}(520)-320+C_y=0$

$$200-320+C_y=0 \qquad C_y=120\text{ lb}$$

Coefficient of **k**: $-\frac{48}{52}(520)-\frac{48}{52}(520)-\frac{30}{34}(680)+C_z=0$

$$-480-480-600+C_z=0$$

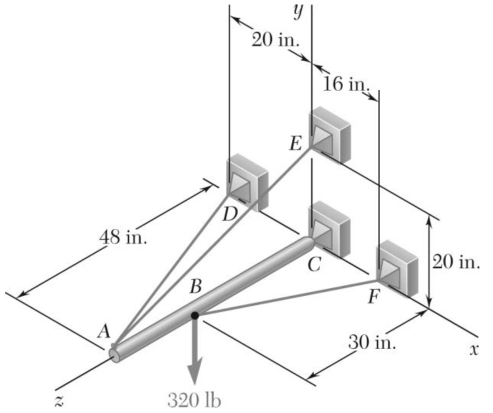
$$C_z=1560\text{ lb}$$

Answers: $T_{DAE}=T$

$T_{DAE}=520\text{ lb}$ ◀

$T_{BD}=680\text{ lb}$ ◀

$\mathbf{C}=-(120.0\text{ lb})\mathbf{i}+(120.0\text{ lb})\mathbf{j}+(1560\text{ lb})\mathbf{k}$ ◀



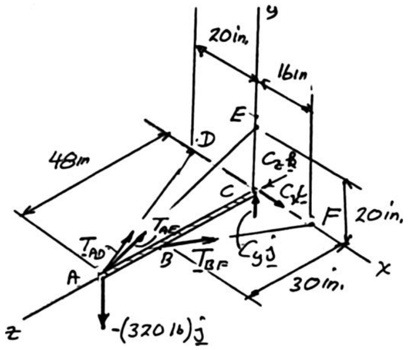
PROBLEM 4.111

Solve Problem 4.110, assuming that the 320-lb load is applied at *A*.

PROBLEM 4.110 A 48-in. boom is held by a ball-and-socket joint at *C* and by two cables *BF* and *DAE*; cable *DAE* passes around a frictionless pulley at *A*. For the loading shown, determine the tension in each cable and the reaction at *C*.

SOLUTION

Free-Body Diagram:



Five unknowns and six equations of equilibrium, but equilibrium is maintained $(\Sigma M_{AC} = 0)$.

T = tension in both parts of cable *DAE*.

$$\mathbf{r}_B = 30\mathbf{k}$$
$$\mathbf{r}_A = 48\mathbf{k}$$

$$\overrightarrow{AD} = -20\mathbf{i} - 48\mathbf{k} \qquad AD = 52\text{ in.}$$
$$\overrightarrow{AE} = 20\mathbf{j} - 48\mathbf{k} \qquad AE = 52\text{ in.}$$
$$\overrightarrow{BF} = 16\mathbf{i} - 30\mathbf{k} \qquad BF = 34\text{ in.}$$

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD} = \frac{T}{52}(-20\mathbf{i} - 48\mathbf{k}) = \frac{T}{13}(-5\mathbf{i} - 12\mathbf{k})$$
$$\mathbf{T}_{AE} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{52}(20\mathbf{j} - 48\mathbf{k}) = \frac{T}{13}(5\mathbf{j} - 12\mathbf{k})$$
$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = \frac{T_{BF}}{34}(16\mathbf{i} - 30\mathbf{k}) = \frac{T_{BF}}{17}(8\mathbf{i} - 15\mathbf{k})$$

$$\Sigma M_C = 0\text{:} \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times \mathbf{T}_{BF} + \mathbf{r}_A \times (-320\text{ lb})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 8 & 0 & -15 \end{vmatrix}\frac{T_{BF}}{17} + 48\mathbf{k} \times (-320\mathbf{j}) = 0$$

Coefficient of **i**: $$-\frac{240}{13}T + 15{,}360 = 0 \qquad T = 832\text{ lb}$$

PROBLEM 4.111 (Continued)

Coefficient of **j**: $-\frac{240}{13}T + \frac{240}{17}T_{BD} = 0$

$$T_{BD} = \frac{17}{13}T = \frac{17}{13}(832) \qquad T_{BD} = 1088 \text{ lb}$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AD} + \mathbf{T}_{AE} + \mathbf{T}_{BF} - 320\mathbf{j} + \mathbf{C} = 0$$

Coefficient of **i**: $-\frac{20}{52}(832) + \frac{8}{17}(1088) + C_x = 0$

$$-320 + 512 + C_x = 0 \qquad C_x = -192 \text{ lb}$$

Coefficient of **j**: $\frac{20}{52}(832) - 320 + C_y = 0$

$$320 - 320 + C_y = 0 \qquad C_y = 0$$

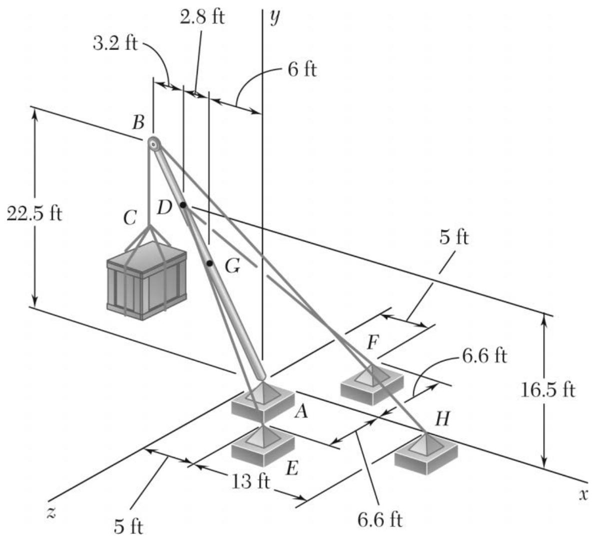
Coefficient of **k**: $-\frac{48}{52}(832) - \frac{48}{52}(852) - \frac{30}{34}(1088) + C_z = 0$

$$-768 - 768 - 960 + C_z = 0 \qquad C_z = 2496 \text{ lb}$$

Answers: $T_{DAE} = T$ $\qquad T_{DAE} = 832 \text{ lb}$ ◀

$T_{BD} = 1088 \text{ lb}$ ◀

$\mathbf{C} = -(192.0 \text{ lb})\mathbf{i} + (2496 \text{ lb})\mathbf{k}$ ◀

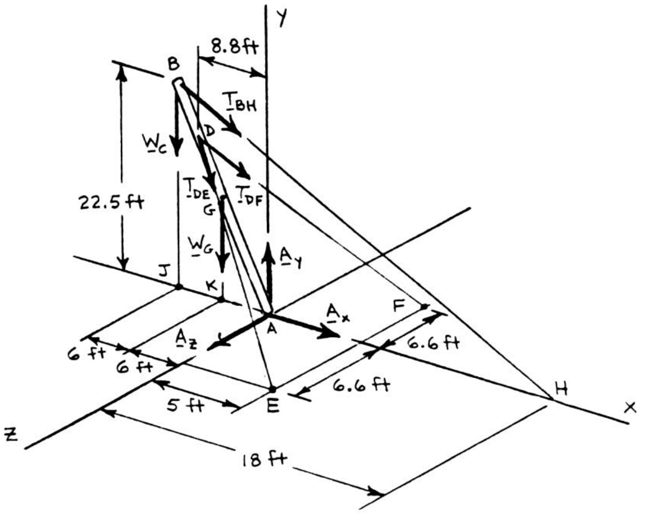


PROBLEM 4.112

A 600-lb crate hangs from a cable that passes over a pulley *B* and is attached to a support at *H*. The 200-lb boom *AB* is supported by a ball-and-socket joint at *A* and by two cables *DE* and *DF*. The center of gravity of the boom is located at *G*. Determine (*a*) the tension in cables *DE* and *DF*, (*b*) the reaction at *A*.

SOLUTION

Free-Body Diagram:



$W_C = 600\text{ lb}$

$W_G = 200\text{ lb}$

We have five unknowns $(T_{DE}, T_{DF}, A_x, A_y, A_z)$ and five equilibrium equations. The boom is free to spin about the AB axis, but equilibrium is maintained, since $\Sigma M_{AB} = 0$.

We have $\overrightarrow{BH} = (30\text{ ft})\mathbf{i} - (22.5\text{ ft})\mathbf{j} \qquad BH = 37.5\text{ ft}$

$$\overrightarrow{DE} = (13.8\text{ ft})\mathbf{i} - \frac{8.8}{12}(22.5\text{ ft})\mathbf{j} + (6.6\text{ ft})\mathbf{k}$$

$$= (13.8\text{ ft})\mathbf{i} - (16.5\text{ ft})\mathbf{j} + (6.6\text{ ft})\mathbf{k} \qquad DE = 22.5\text{ ft}$$

$$\overrightarrow{DF} = (13.8\text{ ft})\mathbf{i} - (16.5\text{ ft})\mathbf{j} - (6.6\text{ ft})\mathbf{k} \qquad DF = 22.5\text{ ft}$$

Thus:

$$\mathbf{T}_{BH} = \mathbf{T}_{BH}\frac{\overline{BH}}{BH} = (600\text{ lb})\frac{30\mathbf{i} - 22.5\mathbf{j}}{37.5} = (480\text{ lb})\mathbf{i} - (360\text{ lb})\mathbf{j}$$

$$\mathbf{T}_{DE} = \mathbf{T}_{DE}\frac{\overline{DE}}{DE} = \frac{T_{DE}}{22.5}(13.8\mathbf{i} - 16.5\mathbf{j} + 6.6\mathbf{k})$$

$$\mathbf{T}_{DF} = \mathbf{T}_{DF}\frac{\overline{DF}}{DF} = \frac{T_{DE}}{22.5}(13.8\mathbf{i} - 16.5\mathbf{j} - 6.6\mathbf{k})$$

(*a*) $\Sigma\mathbf{M}_A = 0$: $(\mathbf{r}_J \times \mathbf{W}_C) + (\mathbf{r}_K \times \mathbf{W}_G) + (\mathbf{r}_H \times \mathbf{T}_{BH}) + (\mathbf{r}_E \times \mathbf{T}_{DE}) + (\mathbf{r}_F \times \mathbf{T}_{DF}) = 0$

$$-(12\mathbf{i}) \times (-600\mathbf{j}) - (6\mathbf{i}) \times (-200\mathbf{j}) + (18\mathbf{i}) \times (480\mathbf{i} - 360\mathbf{j})$$

$$+\frac{T_{DE}}{22.5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 6.6 \\ 13.8 & -16.5 & 6.6 \end{vmatrix} + \frac{T_{DF}}{22.5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & -6.6 \\ 13.8 & -16.5 & -6.6 \end{vmatrix} = 0$$

or

$$7200\mathbf{k} + 1200\mathbf{k} - 6480\mathbf{k} + 4.84(T_{DE} - T_{DF})\mathbf{i}$$

$$+\frac{58.08}{22.5}(T_{DE} - T_{DF})\mathbf{j} - \frac{82.5}{22.5}(T_{DE} + T_{DF})\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors,

i or **j**: $T_{DE} - T_{DF} = 0 \qquad T_{DE} = T_{DF}{}^*$

k: $7200 + 1200 - 6480 - \dfrac{82.5}{22.5}(2T_{DE}) = 0 \qquad T_{DE} = 261.82\text{ lb}$

$$T_{DE} = T_{DF} = 262\text{ lb} \blacktriangleleft$$

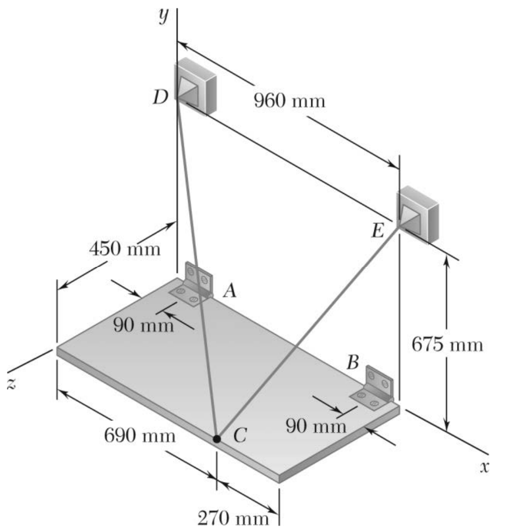
(*b*) $\Sigma F_x = 0$: $A_x + 480 + 2\left(\dfrac{13.8}{22.5}\right)(261.82) = 0 \qquad A_x = -801.17\text{ lb}$

$\Sigma F_y = 0$: $A_y - 600 - 200 - 360 - 2\left(\dfrac{16.5}{22.5}\right)(261.82) = 0 \qquad A_y = 1544.00\text{ lb}$

$\Sigma F_z = 0$: $A_z = 0$

$$\mathbf{A} = -(801\text{ lb})\mathbf{i} + (1544\text{ lb})\mathbf{j} \blacktriangleleft$$

*Remark: The fact is that $T_{DE} = T_{DF}$ could have been noted at the outset from the symmetry of structure with respect to *xy* plane.



PROBLEM 4.113

A 100-kg uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE* that passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION

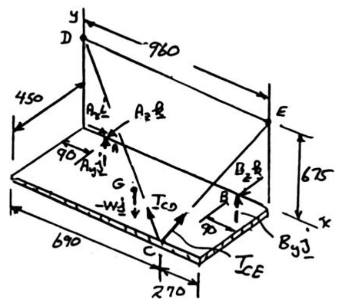
$$\mathbf{r}_{B/A}(960-180)\mathbf{i} = 780\mathbf{i}$$

$$\mathbf{r}_{G/A} = \left(\frac{960}{2} - 90\right)\mathbf{i} + \frac{450}{2}\mathbf{k}$$

$$= 390\mathbf{i} + 225\mathbf{k}$$

$$\mathbf{r}_{C/A} = 600\mathbf{i} + 450\mathbf{k}$$

Dimensions in mm



T = Tension in cable *DCE*

$$\overrightarrow{CD} = -690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \qquad CD = 1065 \text{ mm}$$

$$\overrightarrow{CE} = 270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \qquad CE = 855 \text{ mm}$$

$$\mathbf{T}_{CD} = \frac{T}{1065}(-690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{T}_{CE} = \frac{T}{855}(270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{W} = -mg\mathbf{i} = -(100 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(981 \text{ N})\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{C/A} \times \mathbf{T}_{CD} + \mathbf{r}_{C/A} \times \mathbf{T}_{CE} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ -690 & 675 & -450 \end{vmatrix} \frac{T}{1065} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix} \frac{T}{855}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

PROBLEM 4.113 (Continued)

Coefficient of **i**: $$-(450)(675)\frac{T}{1065}-(450)(675)\frac{T}{855}+220.73\times 10^3=0$$

$T = 344.64$ N $\qquad T = 345$ N ◀

Coefficient of **j**: $$(-690\times 450+600\times 450)\frac{344.64}{1065}+(270\times 450+600\times 450)\frac{344.64}{855}-780B_z=0$$

$B_z = 185.516$ N

Coefficient of **k**: $$(600)(675)\frac{344.64}{1065}+(600)(675)\frac{344.64}{855}-382.59\times 10^3+780B_y=0 \quad B_y=113.178\text{ N}$$

$\mathbf{B} = (113.2\text{ N})\mathbf{j} + (185.5\text{ N})\mathbf{k}$ ◀

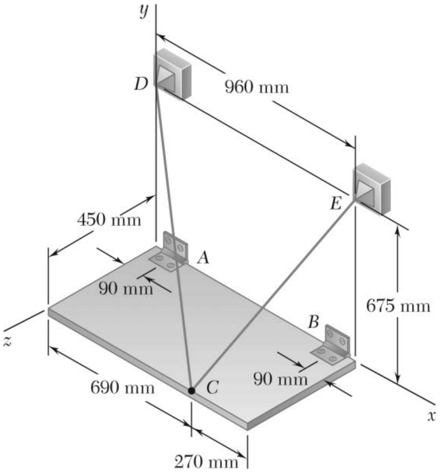
$\Sigma\mathbf{F} = 0$: $\mathbf{A} + \mathbf{B} + \mathbf{T}_{CD} + \mathbf{T}_{CE} + \mathbf{W} = 0$

Coefficient of **i**: $$A_x-\frac{690}{1065}(344.64)+\frac{270}{855}(344.64)=0 \qquad A_x=114.5\text{ N}$$

Coefficient of **j**: $$A_y+113.178+\frac{675}{1065}(344.64)+\frac{675}{855}(344.64)-981=0 \quad A_y=377\text{ N}$$

Coefficient of **k**: $$A_z+185.516-\frac{450}{1065}(344.64)-\frac{450}{855}(344.64)=0 \qquad A_z=141.5\text{ N}$$

$\mathbf{A} = (114.5\text{ N})\mathbf{i} + (377\text{ N})\mathbf{j} + (144.5\text{ N})\mathbf{k}$ ◀



PROBLEM 4.114

Solve Problem 4.113, assuming that cable *DCE* is replaced by a cable attached to Point *E* and hook *C*.

PROBLEM 4.113 A 100-kg uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE* that passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION

See solution to Problem 4.113 for free-body diagram and analysis leading to the following:

$$CD = 1065 \text{ mm}$$

$$CE = 855 \text{ mm}$$

Now,

$$\mathbf{T}_{CD} = \frac{T}{1065}(-690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{T}_{CE} = \frac{T}{855}(270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{W} = -mg\mathbf{i} = -(100 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(981 \text{ N})\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{C/A} \times T_{CE} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) + \mathbf{r}_{B/A} \times B = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix} \frac{T}{855} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Coefficient of **i**: $$-(450)(675)\frac{T}{855} + 220.73 \times 10^3 = 0$$

$$T = 621.31 \text{ N} \qquad\qquad T = 621 \text{ N} \blacktriangleleft$$

Coefficient of **j**: $$(270 \times 450 + 600 \times 450)\frac{621.31}{855} - 780B_z = 0 \quad B_z = 364.74 \text{ N}$$

Coefficient of **k**: $$(600)(675)\frac{621.31}{855} - 382.59 \times 10^3 + 780B_y = 0 \quad B_y = 113.186 \text{ N}$$

$$\mathbf{B} = (113.2 \text{ N})\mathbf{j} + (365 \text{ N})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.114 (Continued)

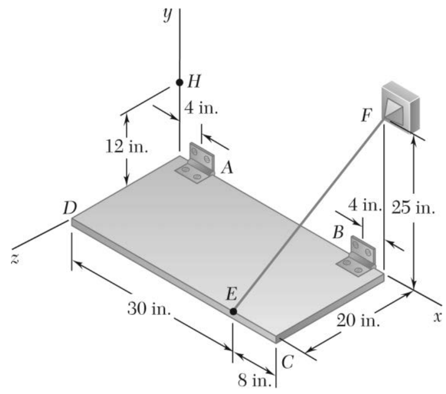
$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T}_{CE} + \mathbf{W} = 0$$

Coefficient of **i**: $A_x + \dfrac{270}{855}(621.31) = 0$ $\qquad A_x = -196.2 \text{ N}$

Coefficient of **j**: $A_y + 113.186 + \dfrac{675}{855}(621.31) - 981 = 0$ $\qquad A_y = 377.3 \text{ N}$

Coefficient of **k**: $A_z + 364.74 - \dfrac{450}{855}(621.31) = 0$ $\qquad A_z = -37.7 \text{ N}$

$$\mathbf{A} = -(196.2 \text{ N})\mathbf{i} + (377 \text{ N})\mathbf{j} - (37.7 \text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.115

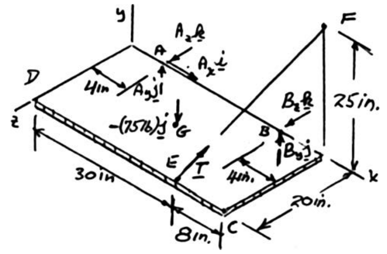
The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at *A* and *B* and by cable *EF*. Assuming that the hinge at *B* does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*.

SOLUTION

$$\mathbf{r}_{B/A} = (38-8)\mathbf{i} = 30\mathbf{i}$$

$$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k}$$

$$= 26\mathbf{i} + 20\mathbf{k}$$



$$\mathbf{r}_{G/A} = \frac{38}{2}\mathbf{i} + 10\mathbf{k}$$

$$= 19\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{EF} = 8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k}$$

$$EF = 33 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{33}(8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0\text{:}\quad \mathbf{r}_{E/A} \times T + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + \mathbf{r}_{B/A} \times B = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ 8 & 25 & -20 \end{vmatrix}\frac{T}{33} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Coefficient of **i**: $-(25)(20)\dfrac{T}{33} + 750 = 0\text{:}$ $T = 49.5 \text{ lb}$ ◀

Coefficient of **j**: $(160+520)\dfrac{49.5}{33} - 30B_z = 0\text{:}\quad B_z = 34 \text{ lb}$

Coefficient of **k**: $(26)(25)\dfrac{49.5}{33} - 1425 + 30B_y = 0\text{:}\quad B_y = 15 \text{ lb}$ $\mathbf{B} = (15.00 \text{ lb})\mathbf{j} + (34.0 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.115 (Continued)

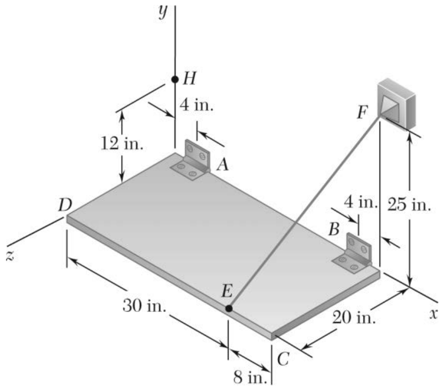
$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T} - (75 \text{ lb})\mathbf{j} = 0$$

Coefficient of **i**: $A_x + \dfrac{8}{33}(49.5) = 0 \quad A_x = -12.00 \text{ lb}$

Coefficient of **j**: $A_y + 15 + \dfrac{25}{33}(49.5) - 75 = 0 \quad A_y = 22.5 \text{ lb}$

Coefficient of **k**: $A_z + 34 - \dfrac{20}{33}(49.5) = 0 \quad A_z = -4.00 \text{ lb}$

$\mathbf{A} = -(12.00 \text{ lb})\mathbf{i} + (22.5 \text{ lb})\mathbf{j} - (4.00 \text{ lb})\mathbf{k}$ ◀

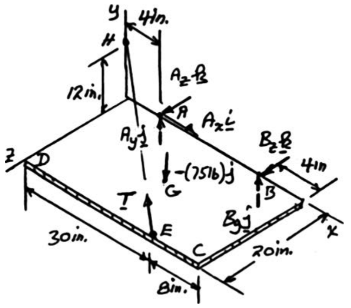


PROBLEM 4.116

Solve Problem 4.115, assuming that cable *EF* is replaced by a cable attached at points *E* and *H*.

PROBLEM 4.115 The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at *A* and *B* and by cable *EF*. Assuming that the hinge at *B* does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*.

SOLUTION



$$\mathbf{r}_{B/A} = (38-8)\mathbf{i} = 30\mathbf{i}$$

$$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k}$$
$$= 26\mathbf{i} + 20\mathbf{k}$$

$$\mathbf{r}_{G/A} = \frac{38}{2}\mathbf{i} + 10\mathbf{k}$$
$$= 19\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{EH} = -30\mathbf{i} + 12\mathbf{j} - 20\mathbf{k}$$
$$EH = 38 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{EH}}{EH} = \frac{T}{38}(-30\mathbf{i} + 12\mathbf{j} - 20\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{E/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ -30 & 12 & -20 \end{vmatrix} \frac{T}{38} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Coefficient of **i**: $-(12)(20)\dfrac{T}{38} + 750 = 0 \quad T = 118.75$ $\qquad T = 118.8 \text{ lb}$ ◀

Coefficient of **j**: $(-600 + 520)\dfrac{118.75}{38} - 30B_z = 0 \quad B_z = -8.33 \text{ lb}$

Coefficient of **k**: $(26)(12)\dfrac{118.75}{38} - 1425 + 30B_y = 0 \quad B_y = 15.00 \text{ lb}$ $\qquad \mathbf{B} = (15.00 \text{ lb})\mathbf{j} - (8.33 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.116 (Continued)

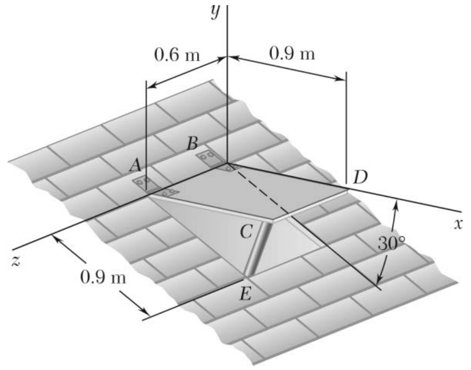
$$\Sigma \mathbf{F} = 0: \qquad \mathbf{A} + \mathbf{B} + \mathbf{T} - (75 \text{ lb})\mathbf{j} = 0$$

Coefficient of **i**: $A_x - \dfrac{30}{38}(118.75) = 0 \quad A_x = 93.75 \text{ lb}$

Coefficient of **j**: $A_y + 15 + \dfrac{12}{38}(118.75) - 75 = 0 \quad A_y = 22.5 \text{ lb}$

Coefficient of **k**: $A_z - 8.33 - \dfrac{20}{38}(118.75) = 0 \quad A_z = 70.83 \text{ lb}$

$$\mathbf{A} = (93.8 \text{ lb})\mathbf{i} + (22.5 \text{ lb})\mathbf{j} + (70.8 \text{ lb})\mathbf{k} \quad ◀$$

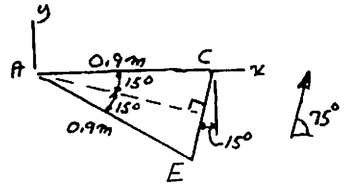


PROBLEM 4.117

A 20-kg cover for a roof opening is hinged at corners *A* and *B*. The roof forms an angle of 30° with the horizontal, and the cover is maintained in a horizontal position by the brace *CE*. Determine (*a*) the magnitude of the force exerted by the brace, (*b*) the reactions at the hinges. Assume that the hinge at *A* does not exert any axial thrust.

SOLUTION

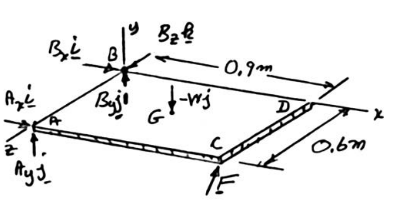
Force exerted by *CE*:



$$\mathbf{F} = F(\cos 75°)\mathbf{i} + F(\sin 75°)\mathbf{j}$$

$$\mathbf{F} = F(0.25882\mathbf{i} + 0.96593\mathbf{j})$$

$$W = mg = 20\text{ kg}(9.81\text{ m/s}^2) = 196.2\text{ N}$$



$$\mathbf{r}_{A/B} = 0.6\mathbf{k}$$

$$\mathbf{r}_{C/B} = 0.9\mathbf{i} + 0.6\mathbf{k}$$

$$\mathbf{r}_{G/B} = 0.45\mathbf{i} + 0.3\mathbf{k}$$

$$\mathbf{F} = F(0.25882\mathbf{i} + 0.96593\mathbf{j})$$

$$\Sigma\mathbf{M}_B = 0:\quad \mathbf{r}_{G/B} \times (-196.2\mathbf{j}) + \mathbf{r}_{C/B} \times \mathbf{F} + \mathbf{r}_{A/B} \times \mathbf{A} = 0$$

(*a*)

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.3 \\ 0 & -196.2 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 0 & 0.6 \\ 0.25882 & +0.96593 & 0 \end{vmatrix} F + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.6 \\ A_x & A_y & 0 \end{vmatrix} = 0$$

Coefficient of **i** : $+58.86 - 0.57956F - 0.6A_y = 0$ (1)

Coefficient of **j**: $+0.155292F + 0.6A_x = 0$ (2)

Coefficient of **k**: $-88.29 + 0.86934F = 0: \quad F = 101.56\text{ N}$

From Eq. (2): $+58.86 - 0.57956(101.56) - 0.6A_y = 0 \quad A_y = 0$

From Eq. (3): $+0.155292(101.56) + 0.6A_x = 0 \quad A_x = -26.286\text{ N}$

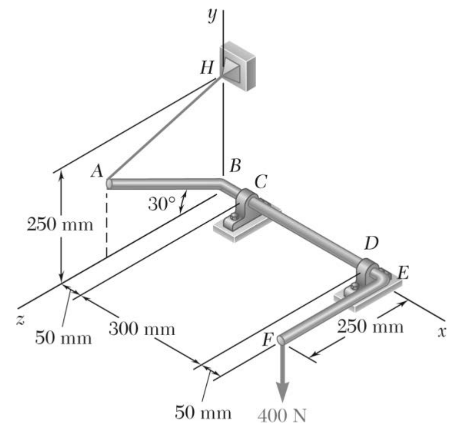
$F = (101.6\text{ N})$ ◀

(*b*) $\Sigma\mathbf{F}:\quad \mathbf{A} + \mathbf{B} + \mathbf{F} - W_{\mathbf{j}} = 0$

Coefficient of **i**: $26.286 + B_x + 0.25882(101.56) = 0 \quad B_x = 0$

Coefficient of **j**: $B_y + 0.96593(101.56) - 196.2 = 0 \quad B_y = 98.1\text{ N}$

Coefficient of **k**: $B_z = 0$ $\quad \mathbf{A} = -(26.3\text{ N})\mathbf{i};\ \mathbf{B} = (98.1\text{ N})\mathbf{j}$ ◀



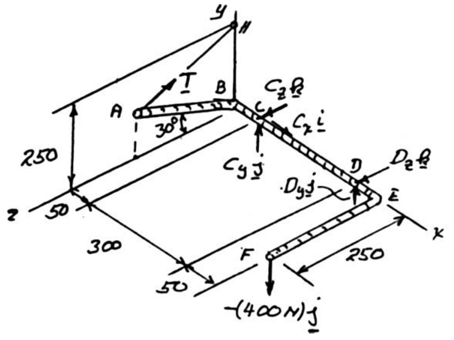
PROBLEM 4.118

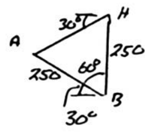
The bent rod *ABEF* is supported by bearings at *C* and *D* and by wire *AH*. Knowing that portion *AB* of the rod is 250 mm long, determine (*a*) the tension in wire *AH*, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Free-Body Diagram: ΔABH is equilateral.

Dimensions in mm





$$\mathbf{r}_{H/C} = -50\mathbf{i} + 250\mathbf{j}$$

$$\mathbf{r}_{D/C} = 300\mathbf{i}$$

$$\mathbf{r}_{F/C} = 350\mathbf{i} + 250\mathbf{k}$$

$$\mathbf{T} = T(\sin 30°)\mathbf{j} - T(\cos 30°)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k})$$

$$\Sigma\mathbf{M}_C = 0: \quad \mathbf{r}_{H/C} \times \mathbf{T} + \mathbf{r}_D \times \mathbf{D} + \mathbf{r}_{F/C} \times (-400\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 300 & 0 & 0 \\ 0 & D_y & D_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} = 0$$

Coefficient **i**: $-216.5T + 100 \times 10^3 = 0$

$$T = 461.9 \text{ N} \qquad\qquad T = 462 \text{ N} ◀$$

Coefficient of **j**: $-43.3T - 300D_z = 0$

$$-43.3(461.9) - 300D_z = 0 \qquad D_z = -66.67 \text{ N}$$

Coefficient of **k**: $-25T + 300D_y - 140\times 10^3 = 0$

$-25(461.9) + 300D_y - 140\times 10^3 = 0 \qquad D_y = 505.1\text{ N}$

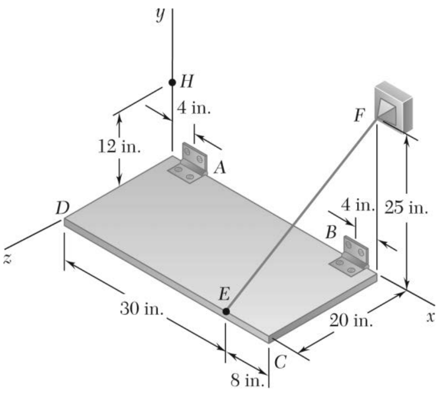
$\mathbf{D} = (505\text{ N})\mathbf{j} - (66.7\text{ N})\mathbf{k}$ ◀

$\Sigma\mathbf{F} = 0: \quad \mathbf{C} + \mathbf{D} + \mathbf{T} - 400\mathbf{j} = 0$

Coefficient **i**: $C_x = 0 \qquad C_x = 0$

Coefficient **j**: $C_y + (461.9)0.5 + 505.1 - 400 = 0 \quad C_y = -336\text{ N}$

Coefficient **k**: $C_z - (461.9)0.866 - 66.67 = 0 \qquad C_z = 467\text{ N}$ $\qquad \mathbf{C} = -(336\text{ N})\mathbf{j} + (467\text{ N})\mathbf{k}$ ◀

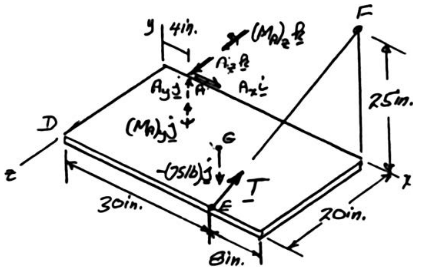


PROBLEM 4.119

Solve Problem 4.115, assuming that the hinge at *B* is removed and that the hinge at *A* can exert couples about axes parallel to the *y* and *z* axes.

PROBLEM 4.115 The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at *A* and *B* and by cable *EF*. Assuming that the hinge at *B* does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*.

SOLUTION



$$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k} = 26\mathbf{i} + 20\mathbf{k}$$

$$\mathbf{r}_{G/A} = (0.5\times 38)\mathbf{i} + 10\mathbf{k} = 19\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{AE} = 8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k}$$

$$AE = 33 \text{ in.}$$

$$T = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{33}(8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{E/A}\times\mathbf{T} + \mathbf{r}_{G/A}\times(-75\mathbf{j}) + (M_A)_y\mathbf{j} + (M_A)_z\mathbf{k} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ 8 & 25 & -20 \end{vmatrix}\frac{T}{33} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + (M_A)_y\mathbf{j} + (M_A)_z\mathbf{k} = 0$$

Coefficient of **i**: $\quad -(20)(25)\dfrac{T}{33} + 750 = 0 \qquad T = 49.5 \text{ lb}$ ◀

Coefficient of **j**: $\quad (160+520)\dfrac{49.5}{33} + (M_A)_y = 0 \quad (M_A)_y = -1020 \text{ lb}\cdot\text{in.}$

Coefficient of **k**: $\quad (26)(25)\dfrac{49.5}{33} - 1425 + (M_A)_z = 0 \quad (M_A)_z = 450 \text{ lb}\cdot\text{in.}$

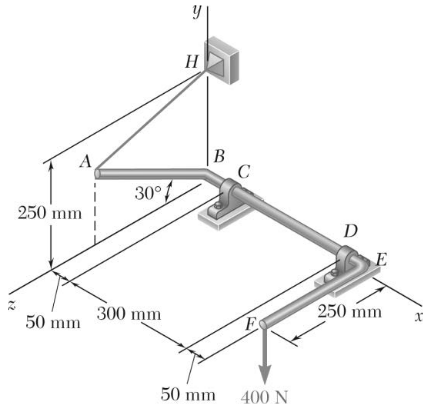
$$\Sigma F = 0: \quad A + T - 75\mathbf{j} = 0 \qquad \mathbf{M}_A = -(1020 \text{ lb}\cdot\text{in.})\mathbf{j} + (450 \text{ lb}\cdot\text{in.})\mathbf{k}$$ ◀

Coefficient of **i**: $\quad A_x + \dfrac{8}{33}(49.5) = 0 \quad A_x = 12.00 \text{ lb}$

Coefficient of **j**: $\quad A_y + \dfrac{25}{33}(49.5) - 75 = 0 \quad A_y = 37.5 \text{ lb}$

Coefficient of **k**: $\quad A_z - \dfrac{20}{33}(49.5) = 0 \quad A_z = 30.0 \text{ lb}$

$$\mathbf{A} = -(12.00 \text{ lb})\mathbf{i} + (37.5 \text{ lb})\mathbf{j} + (30.0 \text{ lb})\mathbf{k}$$ ◀



PROBLEM 4.120

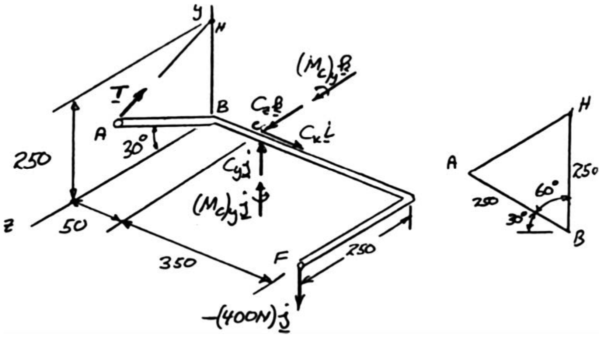
Solve Problem 4.118, assuming that the bearing at *D* is removed and that the bearing at *C* can exert couples about axes parallel to the *y* and *z* axes.

PROBLEM 4.118 The bent rod *ABEF* is supported by bearings at *C* and *D* and by wire *AH*. Knowing that portion *AB* of the rod is 250 mm long, determine (*a*) the tension in wire *AH*, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Free-Body Diagram: ΔABH is equilateral.

Dimensions in mm



$$\mathbf{r}_{H/C} = -50\mathbf{i} + 250\mathbf{j}$$
$$\mathbf{r}_{F/C} = 350\mathbf{i} + 250\mathbf{k}$$
$$\mathbf{T} = T(\sin 30°)\mathbf{j} - T(\cos 30°)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k})$$

$$\Sigma\mathbf{M}_C = 0\text{:}\quad \mathbf{r}_{F/C} \times (-400\mathbf{j}) + \mathbf{r}_{H/C} \times T + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$$

Coefficient of **i**: $+100\times 10^3 - 216.5T = 0 \quad T = 461.9\text{ N}$ $\quad T = 462\text{ N}$ ◀

Coefficient of **j**: $-43.3(461.9) + (M_C)_y = 0$

$$(M_C)_y = 20\times 10^3\text{ N}\cdot\text{mm}$$
$$(M_C)_y = 20.0\text{ N}\cdot\text{m}$$

PROBLEM 4.120 (Continued)

Coefficient of **k**: $-140\times10^3 - 25(461.9) + (M_C)_z = 0$

$$(M_C)_z = 151.54\times10^3 \text{ N}\cdot\text{mm}$$

$$(M_C)_z = 151.5 \text{ N}\cdot\text{m}$$

$$\Sigma F = 0: \quad C + T - 400\mathbf{j} = 0$$

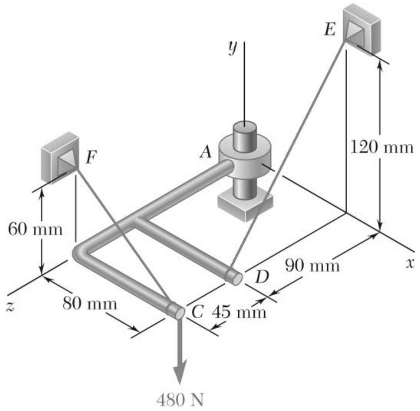
$$\mathbf{M}_C = (20.0 \text{ N}\cdot\text{m})\mathbf{j} + (151.5 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

Coefficient of **i**: $C_x = 0$

Coefficient of **j**: $C_y + 0.5(461.9) - 400 = 0 \quad C_y = 169.1 \text{ N}$

Coefficient of **k**: $C_z - 0.866(461.9) = 0 \quad C_z = 400 \text{ N}$

$$\mathbf{C} = (169.1 \text{ N})\mathbf{j} + (400 \text{ N})\mathbf{k} \blacktriangleleft$$

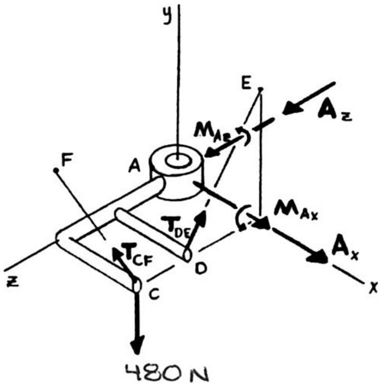


PROBLEM 4.121

The assembly shown is welded to collar *A* that fits on the vertical pin shown. The pin can exert couples about the *x* and *z* axes but does not prevent motion about or along the *y*-axis. For the loading shown, determine the tension in each cable and the reaction at *A*.

SOLUTION

Free-Body Diagram:



First note: $$\mathbf{T}_{CF} = \boldsymbol{\lambda}_{CF} T_{CF} = \frac{-(0.08\text{ m})\mathbf{i} + (0.06\text{ m})\mathbf{j}}{\sqrt{(0.08)^2 + (0.06)^2}\text{ m}} T_{CF}$$

$$= T_{CF}(-0.8\mathbf{i} + 0.6\mathbf{j})$$

$$\mathbf{T}_{DE} = \boldsymbol{\lambda}_{DE} T_{DE} = \frac{(0.12\text{ m})\mathbf{j} - (0.09\text{ m})\mathbf{k}}{\sqrt{(0.12)^2 + (0.09)^2}\text{ m}} T_{DE}$$

$$= T_{DE}(0.8\mathbf{j} - 0.6\mathbf{k})$$

(*a*) From F.B.D. of assembly:

$$\Sigma F_y = 0: \quad 0.6T_{CF} + 0.8T_{DE} - 480\text{ N} = 0$$

or $$0.6T_{CF} + 0.8T_{DE} = 480\text{ N} \qquad (1)$$

$$\Sigma M_y = 0: \quad -(0.8T_{CF})(0.135\text{ m}) + (0.6T_{DE})(0.08\text{ m}) = 0$$

or $$T_{DE} = 2.25T_{CF} \qquad (2)$$

Substituting Equation (2) into Equation (1),

$$0.6T_{CF} + 0.8[(2.25)T_{CF}] = 480\text{ N}$$

$$T_{CF} = 200.00\text{ N}$$

or $$T_{CF} = 200\text{ N} \blacktriangleleft$$

and from Equation (2): $$T_{DE} = 2.25(200.00\text{ N}) = 450.00$$

or $$T_{DE} = 450\text{ N} \blacktriangleleft$$

PROBLEM 4.121 (Continued)

(*b*) From F.B.D. of assembly:

$$\Sigma F_z = 0: \quad A_z - (0.6)(450.00\text{ N}) = 0 \quad A_z = 270.00\text{ N}$$

$$\Sigma F_x = 0: \quad A_x - (0.8)(200.00\text{ N}) = 0 \quad A_x = 160.000\text{ N}$$

or $\mathbf{A} = (160.0\text{ N})\mathbf{i} + (270\text{ N})\mathbf{k}$ ◀

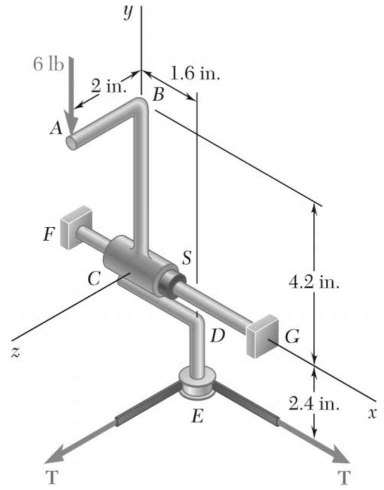
$$\Sigma M_x = 0: \quad M_{A_x} + (480\text{ N})(0.135\text{ m}) - [(200.00\text{ N})(0.6)](0.135\text{ m}) - [(450\text{ N})(0.8)](0.09\text{ m}) = 0$$

$$M_{A_x} = -16.2000\text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0: \quad M_{A_z} - (480\text{ N})(0.08\text{ m}) + [(200.00\text{ N})(0.6)](0.08\text{ m}) + [(450\text{ N})(0.8)](0.08\text{ m}) = 0$$

$$M_{A_z} = 0$$

or $\mathbf{M}_A = -(16.20\text{ N}\cdot\text{m})\mathbf{i}$ ◀

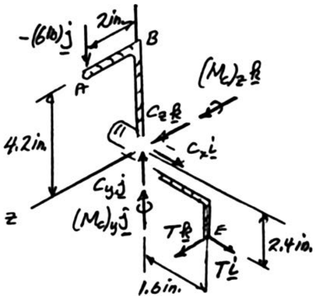


PROBLEM 4.122

The assembly shown is used to control the tension T in a tape that passes around a frictionless spool at E. Collar C is welded to rods ABC and CDE. It can rotate about shaft FG but its motion along the shaft is prevented by a washer S. For the loading shown, determine (*a*) the tension T in the tape, (*b*) the reaction at C.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{A/C} = 4.2\mathbf{j} + 2\mathbf{k}$$
$$\mathbf{r}_{E/C} = 1.6\mathbf{i} - 2.4\mathbf{j}$$

$$\Sigma M_C = 0: \quad \mathbf{r}_{A/C} \times (-6\mathbf{j}) + \mathbf{r}_{E/C} \times T(\mathbf{i} + \mathbf{k}) + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$$

$$(4.2\,\mathbf{j} + 2\mathbf{k}) \times (-6\mathbf{j}) + (1.6\mathbf{i} - 2.4\mathbf{j}) \times T(\mathbf{i} + \mathbf{k}) + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$$

Coefficient of **i**: $12 - 2.4T = 0$ $\quad T = 5.00 \text{ lb}$ ◀

Coefficient of **j**: $-1.6(5 \text{ lb}) + (M_C)_y = 0 \quad (M_C)_y = 8 \text{ lb}\cdot\text{in.}$

Coefficient of **k**: $2.4(5 \text{ lb}) + (M_C)_z = 0 \quad (M_C)_z = -12 \text{ lb}\cdot\text{in.}$

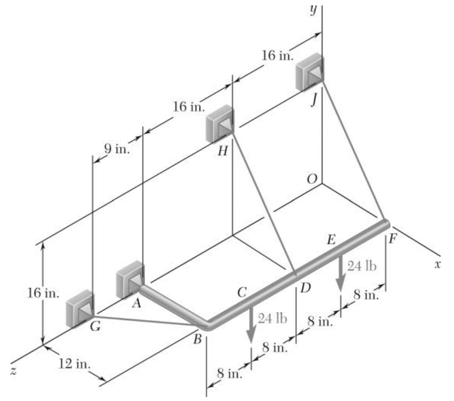
$\mathbf{M}_C = (8.00 \text{ lb}\cdot\text{in.})\mathbf{j} - (12.00 \text{ lb}\cdot\text{in.})\mathbf{k}$ ◀

$$\Sigma F = 0: \quad C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k} - (6 \text{ lb})\mathbf{j} + (5 \text{ lb})\mathbf{i} + (5 \text{ lb})\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero.

$C_x = -5 \text{ lb} \quad C_y = 6 \text{ lb} \quad C_z = -5 \text{ lb}$ ◀

$\mathbf{C} = -(5.00 \text{ lb})\mathbf{i} + (6.00 \text{ lb})\mathbf{j} - (5.00 \text{ lb})\mathbf{k}$ ◀

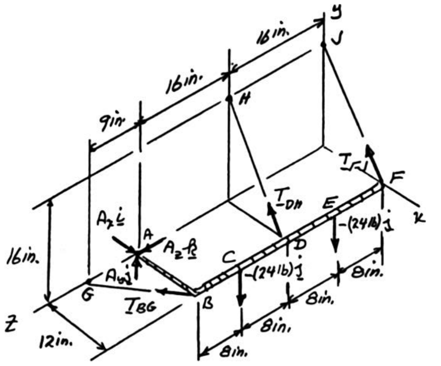


PROBLEM 4.123

The rigid L-shaped member *ABF* is supported by a ball-and-socket joint at *A* and by three cables. For the loading shown, determine the tension in each cable and the reaction at *A*.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{B/A} = 12\mathbf{i}$$

$$\mathbf{r}_{F/A} = 12\mathbf{j} - 8\mathbf{k}$$

$$\mathbf{r}_{D/A} = 12\mathbf{i} - 16\mathbf{k}$$

$$\mathbf{r}_{E/A} = 12\mathbf{i} - 24\mathbf{k}$$

$$\mathbf{r}_{F/A} = 12\mathbf{i} - 32\mathbf{k}$$

$$\overrightarrow{BG} = -12\mathbf{i} + 9\mathbf{k}$$

$$BG = 15 \text{ in.}$$

$$\boldsymbol{\lambda}_{BG} = -0.8\mathbf{i} + 0.6\mathbf{k}$$

$$\overrightarrow{DH} = -12\mathbf{i} + 16\mathbf{j}; \quad DH = 20 \text{ in.}; \quad \lambda_{DH} = -0.6\mathbf{i} + 0.8\mathbf{j}$$

$$\overrightarrow{FJ} = -12\mathbf{i} + 16\mathbf{j}; \quad FJ = 20 \text{ in.}; \quad \lambda_{FJ} = -0.6\mathbf{i} + 0.8\mathbf{j}$$

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{B/A} \times \mathbf{T}_{BG}\lambda_{BG} + \mathbf{r}_{DH} \times \mathbf{T}_{DH}\lambda_{DH} + \mathbf{r}_{F/A} \times \mathbf{T}_{FJ}\lambda_{FJ}$$
$$+\mathbf{r}_{F/A} \times (-24\mathbf{j}) + \mathbf{r}_{E/A} \times (-24\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ -0.8 & 0 & 0.6 \end{vmatrix} T_{BG} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -16 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{DH} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -32 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{FJ}$$

$$+\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -8 \\ 0 & -24 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -24 \\ 0 & -24 & 0 \end{vmatrix} = 0$$

Coefficient of **i**: $+12.8T_{DH} + 25.6T_{FJ} - 192 - 576 = 0$ (1)

Coefficient of **k**: $+9.6T_{DH} + 9.6T_{FJ} - 288 - 288 = 0$ (2)

$\frac{3}{4}$ Eq. (1) – Eq. (2): $9.6T_{FJ} = 0$ $T_{FJ} = 0$ ◀

PROBLEM 4.123 (Continued)

From Eq. (1): $12.8T_{DH} - 268 = 0$ $\qquad T_{DH} = 60\text{ lb}$ ◀

Coefficient of **j:** $-7.2T_{BG} + (16 \times 0.6)(60.0\text{ lb}) = 0$ $\qquad T_{BG} = 80.0\text{ lb}$ ◀

$$\Sigma\mathbf{F} = 0: \quad \mathbf{A} + T_{BG}\boldsymbol{\lambda}_{BG} + T_{DH}\boldsymbol{\lambda}_{DH} + T_{FJ} - 24\mathbf{j} - 24\mathbf{j} = 0$$

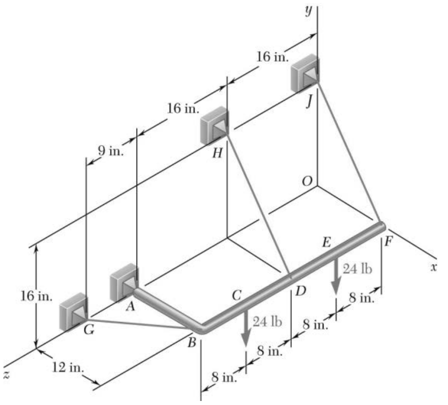
Coefficient of **i**: $A_x + (80)(-0.8) + (60.0)(-0.6) = 0 \quad A_x = 100.0\text{ lb}$

Coefficient of **j**: $A_y + (60.0)(0.8) - 24 - 24 = 0 \quad A_y = 0$

Coefficient of **k:** $A_z + (80.0)(+0.6) = 0 \quad A_z = -48.0\text{ lb}$

$\mathbf{A} = (100.0\text{ lb})\mathbf{i} - (48.0\text{ lb})\mathbf{j}$ ◀

Note: The value $A_y = 0$ can be confirmed by considering $\Sigma M_{BF} = 0$.



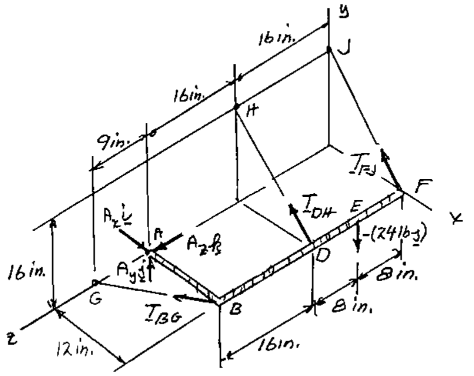
PROBLEM 4.124

Solve Problem 4.123, assuming that the load at *C* has been removed.

PROBLEM 4.123 The rigid L-shaped member *ABF* is supported by a ball-and-socket joint at *A* and by three cables. For the loading shown, determine the tension in each cable and the reaction at *A*.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{B/A} = 12\mathbf{i}$$

$$\mathbf{r}_{B/A} = 12\mathbf{i} - 16\mathbf{k}$$

$$\mathbf{r}_{E/A} = 12\mathbf{i} - 24\mathbf{k}$$

$$\mathbf{r}_{F/A} = 12\mathbf{i} - 32\mathbf{k}$$

$$\overrightarrow{BG} = -12\mathbf{i} + 9\mathbf{k}; \quad BG = 15 \text{ in.}; \quad \boldsymbol{\lambda}_{BG} = -0.8\mathbf{i} + 0.6\mathbf{k}$$

$$\overrightarrow{DH} = -12\mathbf{i} + 16\mathbf{j}; \quad DH = 20 \text{ in.}; \quad \boldsymbol{\lambda}_{DH} = -0.6\mathbf{i} + 0.8\mathbf{j}$$

$$\overrightarrow{FJ} = -12\mathbf{i} + 16\mathbf{j}; \quad FJ = 20 \text{ in.}; \quad \boldsymbol{\lambda}_{FJ} = -0.6\mathbf{i} + 0.8\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{B/A} \times T_{BG}\,\boldsymbol{\lambda}_{BG} + \mathbf{r}_{D/A} \times T_{DH}\,\boldsymbol{\lambda}_{DH} + \mathbf{r}_{F/A} \times T_{FJ}\boldsymbol{\lambda}_{FJ} + \mathbf{r}_{E/A} \times (-24\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ -0.8 & 0 & 0.6 \end{vmatrix} T_{BG} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -16 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{DH} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -32 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{FJ} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -24 \\ 0 & -24 & 0 \end{vmatrix} = 0$$

$$\mathbf{i}: \quad +12.8T_{DH} + 25.6T_{FJ} - 576 = 0 \qquad (1)$$

$$\mathbf{k}: \quad +9.6T_{DH} + 9.6T_{FJ} - 288 = 0 \qquad (2)$$

PROBLEM 4.124 (Continued)

Multiply Eq. (1) by $\frac{3}{4}$ and subtract Eq. (2):

$$9.6T_{FJ} - 144 = 0 \qquad T_{FJ} = 15.00 \text{ lb} \blacktriangleleft$$

From Eq. (1):

$$12.8T_{DH} + 25.6(15.00) - 576 = 0 \qquad T_{DH} = 15.00 \text{ lb} \blacktriangleleft$$

$$\mathbf{j}: \quad -7.2T_{BG} + (16)(0.6)(15) + (32)(0.6)(15) = 0$$

$$-7.2T_{BG} + 432 = 0 \qquad T_{BG} = 60.0 \text{ lb} \blacktriangleleft$$

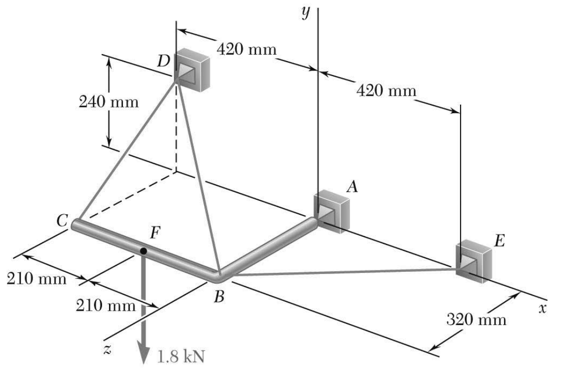
$$\Sigma F = 0: \quad \mathbf{A} + T_{BG}\boldsymbol{\lambda}_{BG} + T_{DA}\boldsymbol{\lambda}_{DH} + T_{FJ}\boldsymbol{\lambda}_{FJ} - 24\mathbf{j} = 0$$

$$\mathbf{i}: \quad A_x + (60)(-0.8) + (15)(-0.6) + (15)(-0.6) = 0 \qquad A_x = 66.0 \text{ lb}$$

$$\mathbf{j}: \quad A_y + (15)(0.8) + (15)(0.8) - 24 = 0 \qquad A_y = 0$$

$$\mathbf{k}: \quad A_z + (60)(0.6) = 0 \qquad A_z = -36.0 \text{ lb}$$

$$\mathbf{A} = (66.0 \text{ lb})\mathbf{i} - (36.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

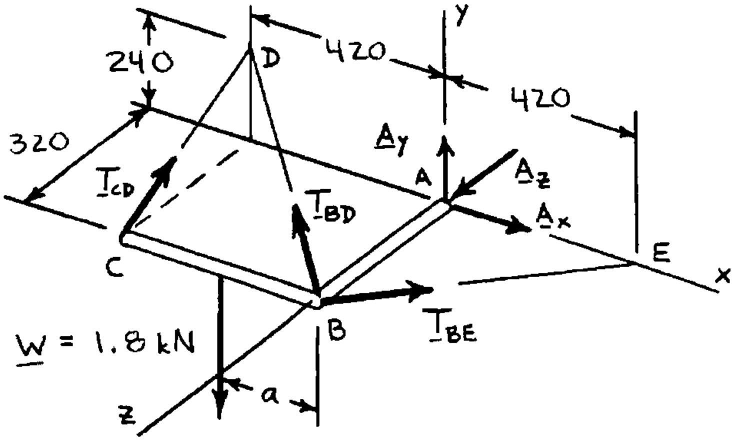


PROBLEM 4.125

The rigid L-shaped member *ABC* is supported by a ball-and-socket joint at *A* and by three cables. If a 1.8-kN load is applied at *F*, determine the tension in each cable.

SOLUTION

Free-Body Diagram: Dimensions in mm



In this problem: $a = 210$ mm

We have $\overrightarrow{CD} = (240\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k} \quad CD = 400\text{ mm}$

$\overrightarrow{BD} = -(420\text{ mm})\mathbf{i} + (240\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k} \quad BD = 580\text{ mm}$

$\overrightarrow{BE} = (420\text{ mm})\mathbf{i} - (320\text{ mm})\mathbf{k} \quad BE = 528.02\text{ mm}$

Thus,

$$T_{CD} = T_{CD}\frac{\overrightarrow{CD}}{CD} = T_{CD}(0.6\mathbf{j} - 0.8\mathbf{k})$$

$$T_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = T_{BD}(-0.72414\mathbf{i} + 0.41379\mathbf{j} - 0.55172\mathbf{k})$$

$$T_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = T_{BE}(0.79542\mathbf{i} - 0.60604\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0: \quad (\mathbf{r}_C \times \mathbf{T}_{CD}) + (\mathbf{r}_B \times \mathbf{T}_{BD}) + (\mathbf{r}_B \times \mathbf{T}_{BE}) + (\mathbf{r}_W \times \mathbf{W}) = 0$$

Noting that
$$\mathbf{r}_C = -(420\text{ mm})\mathbf{i} + (320\text{ mm})\mathbf{k}$$
$$\mathbf{r}_B = (320\text{ mm})\mathbf{k}$$
$$\mathbf{r}_W = -a\mathbf{i} + (320\text{ mm})\mathbf{k}$$

and using determinants, we write

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -420 & 0 & 320 \\ 0 & 0.6 & -0.8 \end{vmatrix} T_{CD} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 320 \\ -0.72414 & 0.41379 & -0.55172 \end{vmatrix} T_{BD}$$
$$+\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 320 \\ 0.79542 & 0 & -0.60604 \end{vmatrix} T_{BE} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -a & 0 & 320 \\ 0 & -1.8 & 0 \end{vmatrix} = 0$$

Equating to zero the coefficients of the unit vectors,

$$\mathbf{i}: \quad -192T_{CD} - 132.413T_{BD} + 576 = 0 \qquad (1)$$

$$\mathbf{j}: \quad -336T_{CD} - 231.72T_{BD} + 254.53T_{BE} = 0 \qquad (2)$$

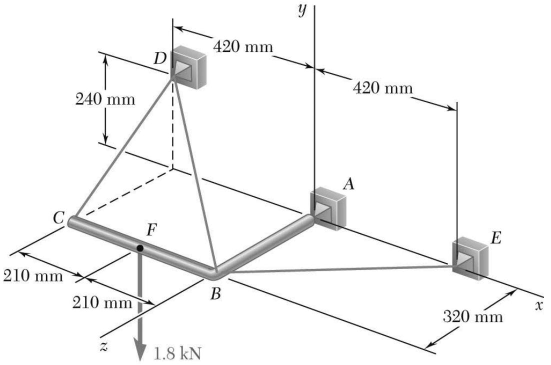
$$\mathbf{k}: \quad -252T_{CD} + 1.8a = 0 \qquad (3)$$

Recalling that $a = 210$ mm, Eq. (3) yields

$$T_{CD} = \frac{1.8(210)}{252} = 1.500\text{ kN} \qquad T_{CD} = 1.500\text{ kN} \blacktriangleleft$$

From Eq. (1):
$$-192(1.5) - 132.413T_{BD} + 576 = 0$$
$$T_{BD} = 2.1751\text{ kN} \qquad T_{BD} = 2.18\text{ kN} \blacktriangleleft$$

From Eq. (2):
$$-336(1.5) - 231.72(2.1751) + 254.53T_{BE} = 0$$
$$T_{BE} = 3.9603\text{ kN} \qquad T_{BE} = 3.96\text{ kN} \blacktriangleleft$$



PROBLEM 4.126

Solve Problem 4.125, assuming that the 1.8-kN load is applied at *C*.

PROBLEM 4.125 The rigid L-shaped member *ABC* is supported by a ball-and-socket joint at *A* and by three cables. If a 1.8-kN load is applied at *F*, determine the tension in each cable.

SOLUTION

See solution of Problem 4.125 for free-body diagram and derivation of Eqs. (1), (2), and (3):

$$-192T_{CD} - 132.413T_{BD} + 576 = 0 \tag{1}$$

$$-336T_{CD} - 231.72T_{BD} + 254.53T_{BE} = 0 \tag{2}$$

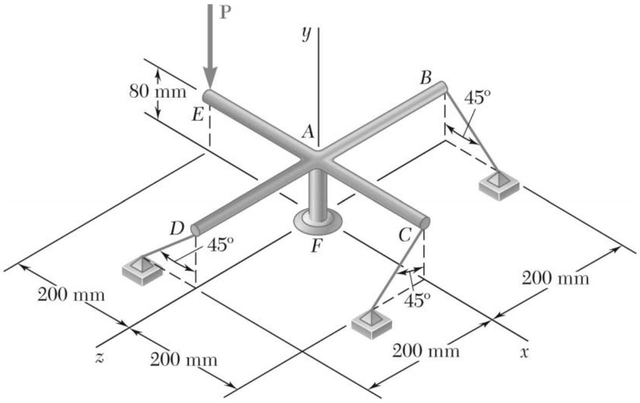
$$-252T_{CD} + 1.8a = 0 \tag{3}$$

In this problem, the 1.8-kN load is applied at *C* and we have $a = 420$ mm. Carrying into Eq. (3) and solving for T_{CD},

$$T_{CD} = 3.00 \qquad T_{CD} = 3.00 \text{ kN} \blacktriangleleft$$

From Eq. (1): $-(192)(3) - 132.413T_{BD} + 576 = 0$ $\qquad T_{BD} = 0 \blacktriangleleft$

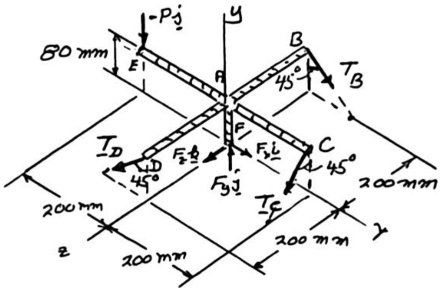
From Eq. (2): $-336(3) - 0 + 254.53T_{BE} = 0$ $\qquad T_{BE} = 3.96 \text{ kN} \blacktriangleleft$



PROBLEM 4.127

The assembly shown consists of an 80-mm rod *AF* that is welded to a cross consisting of four 200-mm arms. The assembly is supported by a ball-and-socket joint at *F* and by three short links, each of which forms an angle of 45° with the vertical. For the loading shown, determine (*a*) the tension in each link, (*b*) the reaction at *F*.

SOLUTION



$$\mathbf{r}_{E/F} = -200\,\mathbf{i} + 80\mathbf{j}$$

$$\mathbf{T}_B = T_B(\mathbf{i}-\mathbf{j})/\sqrt{2} \qquad \mathbf{r}_{B/F} = 80\mathbf{j} - 200\mathbf{k}$$

$$\mathbf{T}_C = T_C(-\mathbf{j}+\mathbf{k})/\sqrt{2} \qquad \mathbf{r}_{C/F} = 200\mathbf{i} + 80\mathbf{j}$$

$$\mathbf{T}_D = T_D(-\mathbf{i}+\mathbf{j})/\sqrt{2} \qquad \mathbf{r}_{D/E} = 80\mathbf{j} + 200\mathbf{k}$$

$$\Sigma M_F = 0: \quad \mathbf{r}_{B/F} \times \mathbf{T}_B + \mathbf{r}_{C/F} \times \mathbf{T}_C + \mathbf{r}_{D/F} \times T_D + \mathbf{r}_{E/F} \times (-P\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & -200 \\ 1 & -1 & 0 \end{vmatrix} \frac{T_B}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 200 & 80 & 0 \\ 0 & -1 & 1 \end{vmatrix} \frac{T_C}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & 200 \\ -1 & -1 & 0 \end{vmatrix} \frac{T_D}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -200 & 80 & 0 \\ 0 & -P & 0 \end{vmatrix} = 0$$

Equate coefficients of unit vectors to zero and multiply each equation by $\sqrt{2}$.

$$\mathbf{i}: \qquad -200T_B + 80T_C + 200\,T_D = 0 \tag{1}$$

$$\mathbf{j}: \qquad -200T_B - 200T_C - 200T_D = 0 \tag{2}$$

$$\mathbf{k}: \qquad -80T_B - 200T_C + 80T_D + 200\sqrt{2}P = 0 \tag{3}$$

$$\frac{80}{200}(2): \qquad -80T_B - 80T_C - 80T_D = 0 \tag{4}$$

Eqs. (3) + (4): $\quad -160T_B - 280T_C + 200\sqrt{2}P = 0 \qquad (5)$

Eqs. (1) + (2): $\quad -400T_B - 120T_C = 0$

$$T_B = -\frac{120}{400}T_C - 0.3T_C \tag{6}$$

PROBLEM 4.127 (Continued)

Eqs. (6) ⟶ (5): $-160(-0.3T_C) - 280T_C + 200\sqrt{2}P = 0$

$-232T_C + 200\sqrt{2}P = 0$

$T_C = 1.2191P$ $\qquad T_C = 1.219P$ ◀

From Eq. (6): $T_B = -0.3(1.2191P) = -0.36574 = P$ $\qquad T_B = -0.366P$ ◀

From Eq. (2): $-200(-0.3657P) - 200(1.2191P) - 200T_{\theta D} = 0$

$T_D = -0.8534P$ $\qquad T_D = -0.853P$ ◀

$\Sigma\mathbf{F} = 0$: $\quad \mathbf{F} + \mathbf{T}_B + \mathbf{T}_C + \mathbf{T}_D - P\mathbf{j} = 0$

$$\mathbf{i}: \quad F_x + \frac{(-0.36574P)}{\sqrt{2}} - \frac{(-0.8534P)}{\sqrt{2}} = 0$$

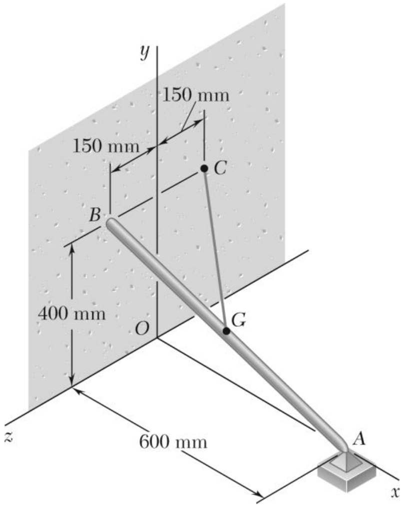
$F_x = -0.3448P \qquad F_x = -0.345P$

$$\mathbf{j}: \quad F_y - \frac{(-0.36574P)}{\sqrt{2}} - \frac{(1.2191P)}{\sqrt{2}} - \frac{(-0.8534P)}{\sqrt{2}} - 200 = 0$$

$F_y = P \qquad F_y = P$

$$\mathbf{k}: \quad F_z + \frac{(1.2191P)}{\sqrt{2}} = 0$$

$F_z = -0.8620P \quad F_z = -0.862P$ $\qquad \mathbf{F} = -0.345P\mathbf{i} + P\mathbf{j} - 0.862P\mathbf{k}$ ◀

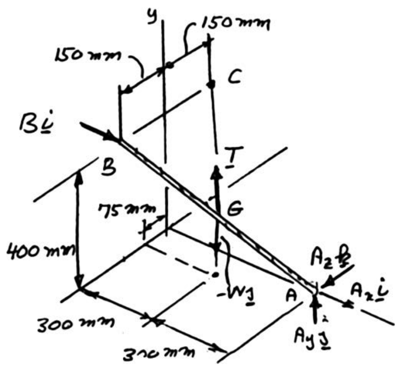


PROBLEM 4.128

The uniform 10-kg rod *AB* is supported by a ball-and-socket joint at *A* and by the cord *CG* that is attached to the midpoint *G* of the rod. Knowing that the rod leans against a frictionless vertical wall at *B*, determine (*a*) the tension in the cord, (*b*) the reactions at *A* and *B*.

SOLUTION

Free-Body Diagram:



Five unknowns and six equations of equilibrium, but equilibrium is maintained $(\Sigma M_{AB} = 0)$.

$$W = mg$$

$$= (10\text{ kg})\,9.81\text{ m/s}^2$$

$$W = 98.1\text{ N}$$

$$\overrightarrow{GC} = -300\mathbf{i} + 200\mathbf{j} - 225\mathbf{k} \quad GC = 425\text{ mm}$$

$$\mathbf{T} = T\frac{\overrightarrow{GC}}{GC} = \frac{T}{425}(-300\mathbf{i} + 200\mathbf{j} - 225\mathbf{k})$$

$$\mathbf{r}_{B/A} = -600\mathbf{i} + 400\mathbf{j} + 150\text{ mm}$$

$$\mathbf{r}_{G/A} = -300\mathbf{i} + 200\mathbf{j} + 75\text{ mm}$$

$$\Sigma M_A = 0\text{:} \quad \mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{r}_{G/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -600 & 400 & 150 \\ B & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -300 & 200 & 75 \\ -300 & 200 & -225 \end{vmatrix}\frac{T}{425} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -300 & 200 & 75 \\ 0 & -98.1 & 0 \end{vmatrix}$$

Coefficient of **i**: $(-105.88 - 35.29)T + 7357.5 = 0$

$$T = 52.12\text{ N}$$

$T = 52.1\text{ N}$ ◀

PROBLEM 4.128 (Continued)

Coefficient of $\mathbf{j}$: $150B - (300\times 75 + 300\times 225)\dfrac{52.12}{425} = 0$

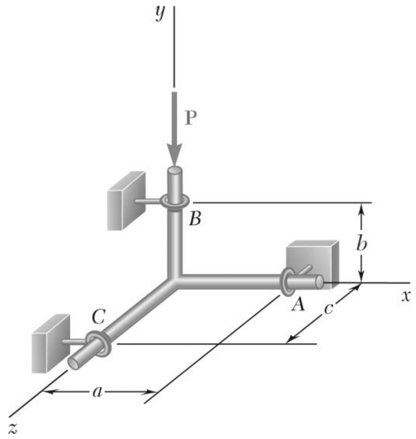
$B = 73.58\text{ N}$ $\qquad\qquad$ $\mathbf{B} = (73.6\text{ N})\mathbf{i}$ ◀

$\Sigma\mathbf{F} = 0$: $\mathbf{A} + \mathbf{B} + \mathbf{T} - W\mathbf{j} = 0$

Coefficient of $\mathbf{i}$: $A_x + 73.58 - 52.15\dfrac{300}{425} = 0$ $\qquad\qquad$ $A_x = -36.8\text{ N}$ ◀

Coefficient of $\mathbf{j}$: $A_y + 52.15\dfrac{200}{425} - 98.1 = 0$ $\qquad\qquad$ $A_y = 73.6\text{ N}$ ◀

Coefficient of $\mathbf{k}$: $A_z - 52.15\dfrac{225}{425} = 0$ $\qquad\qquad$ $A_z = 27.6\text{ N}$ ◀

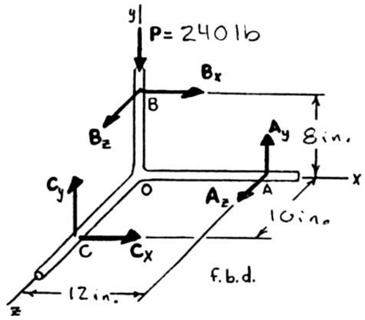


PROBLEM 4.129

Three rods are welded together to form a "corner" that is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240\text{ lb}$, $a = 12\text{ in.}$, $b = 8\text{ in.}$, and $c = 10\text{ in.}$

SOLUTION

From F.B.D. of weldment:



$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ C_x & C_y & 0 \end{vmatrix} = 0$$

$$(-12A_z\mathbf{j} + 12A_y\mathbf{k}) + (8B_z\mathbf{i} - 8B_x\mathbf{k}) + (-10C_y\mathbf{i} + 10C_x\mathbf{j}) = 0$$

From **i**-coefficient: $8B_z - 10C_y = 0$

or $B_z = 1.25C_y$ (1)

j-coefficient: $-12A_z + 10C_x = 0$

or $C_x = 1.2A_z$ (2)

k-coefficient: $12A_y - 8B_x = 0$

or $B_x = 1.5A_y$ (3)

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} - \mathbf{P} = 0$$

or $(B_x + C_x)\mathbf{i} + (A_y + C_y - 240\text{ lb})\mathbf{j} + (A_z + B_z)\mathbf{k} = 0$

From **i**-coefficient: $B_x + C_x = 0$

or $C_x = -B_x$ (4)

j-coefficient: $A_y + C_y - 240\text{ lb} = 0$

or $A_y + C_y = 240\text{ lb}$ (5)

PROBLEM 4.129 (Continued)

k-coefficient: $A_z + B_z = 0$

or $$A_z = -B_z \tag{6}$$

Substituting C_x from Equation (4) into Equation (2),

$$-B_z = 1.2A_z \tag{7}$$

Using Equations (1), (6), and (7),

$$C_y = \frac{B_z}{1.25} = \frac{-A_z}{1.25} = \frac{1}{1.25}\left(\frac{B_x}{1.2}\right) = \frac{B_x}{1.5} \tag{8}$$

From Equations (3) and (8):

$$C_y = \frac{1.5A_y}{1.5} \quad \text{or} \quad C_y = A_y$$

and substituting into Equation (5),

$$2A_y = 240 \text{ lb}$$

$$A_y = C_y = 120 \text{ lb} \tag{9}$$

Using Equation (1) and Equation (9),

$$B_z = 1.25(120 \text{ lb}) = 150.0 \text{ lb}$$

Using Equation (3) and Equation (9),

$$B_x = 1.5(120 \text{ lb}) = 180.0 \text{ lb}$$

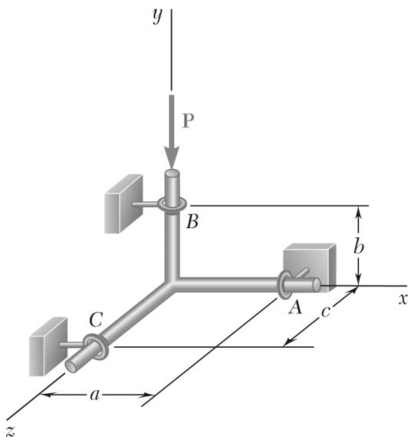
From Equation (4): $C_x = -180.0 \text{ lb}$

From Equation (6): $A_z = -150.0 \text{ lb}$

Therefore, $\mathbf{A} = (120.0 \text{ lb})\mathbf{j} - (150.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{B} = (180.0 \text{ lb})\mathbf{i} + (150.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{C} = -(180.0 \text{ lb})\mathbf{i} + (120.0 \text{ lb})\mathbf{j}$ ◀

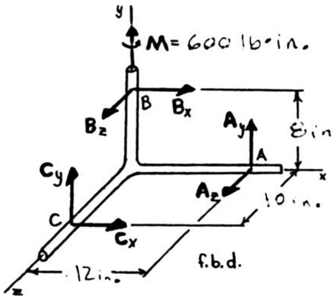


PROBLEM 4.130

Solve Problem 4.129, assuming that the force **P** is removed and is replaced by a couple $\mathbf{M} = +(600 \text{ lb}\cdot\text{in.})\mathbf{j}$ acting at B.

PROBLEM 4.129 Three rods are welded together to form a “corner” that is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240 \text{ lb}$, $a = 12 \text{ in.}$, $b = 8 \text{ in.}$, and $c = 10 \text{ in.}$

SOLUTION



From F.B.D. of weldment:

$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} + \mathbf{M} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ C_x & C_y & 0 \end{vmatrix} + (600 \text{ lb}\cdot\text{in.})\mathbf{j} = 0$$

$$(-12A_z\mathbf{j} + 12A_y\mathbf{k}) + (8B_z\mathbf{j} - 8B_x\mathbf{k}) + (-10C_y\mathbf{i} + 10C_x\mathbf{j}) + (600 \text{ lb}\cdot\text{in.})\mathbf{j} = 0$$

From **i**-coefficient: $8B_z - 10C_y = 0$

or $C_y = 0.8B_z$ (1)

j-coefficient: $-12A_z + 10C_x + 600 = 0$

or $C_x = 1.2A_z - 60$ (2)

k-coefficient: $12A_y - 8B_x = 0$

or $B_x = 1.5A_y$ (3)

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} = 0$$

$$(B_x + C_x)\mathbf{i} + (A_y + C_y)\mathbf{j} + (A_z + B_z)\mathbf{k} = 0$$

From **i**-coefficient: $C_x = -B_x$ (4)

j-coefficient: $C_y = -A_y$ (5)

k-coefficient: $A_z = -B_z$ (6)

PROBLEM 4.130 (Continued)

Substituting C_x from Equation (4) into Equation (2),

$$A_z = 50 - \left(\frac{B_x}{1.2}\right) \tag{7}$$

Using Equations (1), (6), and (7),

$$C_y = 0.8B_z = -0.8A_z = \left(\frac{2}{3}\right)B_x - 40 \tag{8}$$

From Equations (3) and (8):

$$C_y = A_y - 40$$

Substituting into Equation (5), $2A_y = 40$

$$A_y = 20.0 \text{ lb}$$

From Equation (5): $C_y = -20.0 \text{ lb}$

Equation (1): $B_z = -25.0 \text{ lb}$

Equation (3): $B_x = 30.0 \text{ lb}$

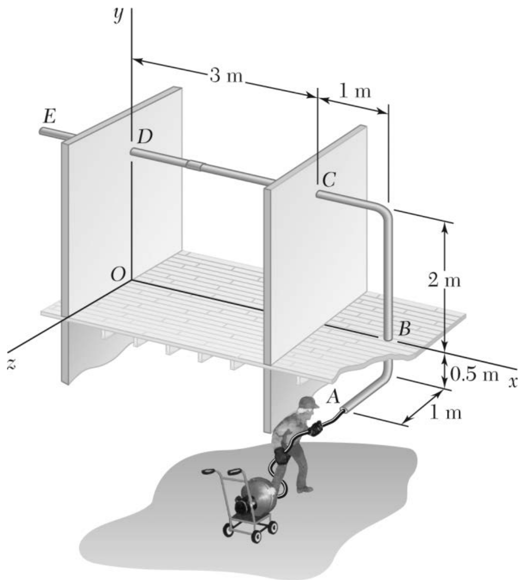
Equation (4): $C_x = -30.0 \text{ lb}$

Equation (6): $A_z = 25.0 \text{ lb}$

Therefore, $\mathbf{A} = (20.0 \text{ lb})\mathbf{j} + (25.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{B} = (30.0 \text{ lb})\mathbf{i} - (25.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{C} = -(30.0 \text{ lb})\mathbf{i} - (20.0 \text{ lb})\mathbf{j}$ ◀

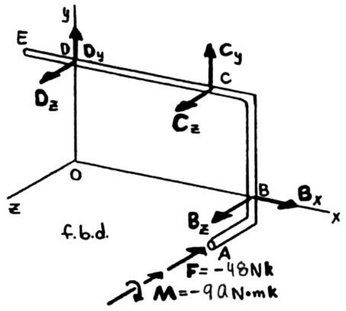


PROBLEM 4.131

In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor that rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(48\text{ N})\mathbf{k}, \mathbf{M} = -(90\text{ N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION

From F.B.D. of pipe assembly *ABCD*:



$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (48\text{ N})(2.5\text{ m}) - B_z(2\text{ m}) = 0$$

$$B_z = 60.0\text{ N}$$

$$\text{and } \mathbf{B} = (60.0\text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\text{ m}) - 90\text{ N}\cdot\text{m} = 0$$

$$C_y = 30.0\text{ N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad -C_z(3\text{ m}) - (60.0\text{ N})(4\text{ m}) + (48\text{ N})(4\text{ m}) = 0$$

$$C_z = -16.00\text{ N}$$

$$\text{and } \mathbf{C} = (30.0\text{ N})\mathbf{j} - (16.00\text{ N})\mathbf{k} \blacktriangleleft$$

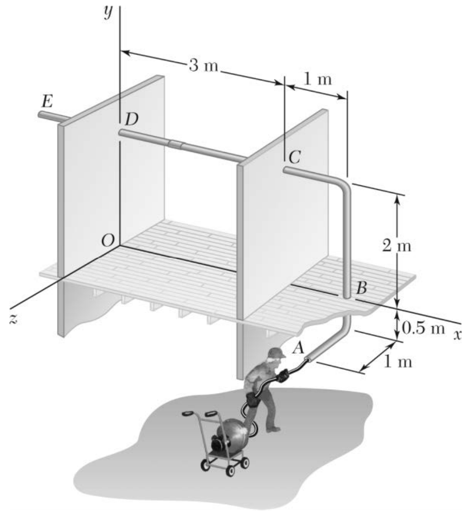
$$\Sigma F_y = 0: \quad D_y + 30.0 = 0$$

$$D_y = -30.0\text{ N}$$

$$\Sigma F_z = 0: \quad D_z - 16.00\text{ N} + 60.0\text{ N} - 48\text{ N} = 0$$

$$D_z = 4.00\text{ N}$$

$$\text{and } \mathbf{D} = -(30.0\text{ N})\mathbf{j} + (4.00\text{ N})\mathbf{k} \blacktriangleleft$$



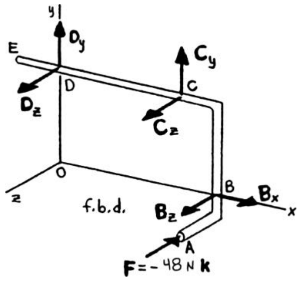
PROBLEM 4.132

Solve Problem 4.131, assuming that the plumber exerts a force $\mathbf{F} = -(48\text{ N})\mathbf{k}$ and that the motor is turned off $(\mathbf{M} = 0)$.

PROBLEM 4.131 In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor that rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(48\text{ N})\mathbf{k}, \mathbf{M} = -(90\text{ N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION

From F.B.D. of pipe assembly *ABCD*:



$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (48\text{ N})(2.5\text{ m}) - B_z(2\text{ m}) = 0$$

$$B_z = 60.0\text{ N} \qquad \text{and } \mathbf{B} = (60.0\text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\text{ m}) - B_x(2\text{ m}) = 0$$

$$C_y = 0$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(3\text{ m}) - (60.0\text{ N})(4\text{ m}) + (48\text{ N})(4\text{ m}) = 0$$

$$C_z = -16.00\text{ N} \qquad \text{and } \mathbf{C} = -(16.00\text{ N})\mathbf{k} \blacktriangleleft$$

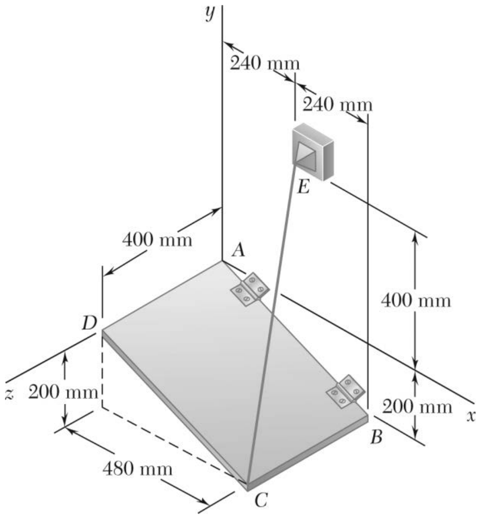
$$\Sigma F_y = 0: \quad D_y + C_y = 0$$

$$D_y = 0$$

$$\Sigma F_z = 0: \quad D_z + B_z + C_z - F = 0$$

$$D_z + 60.0\text{ N} - 16.00\text{ N} - 48\text{ N} = 0$$

$$D_z = 4.00\text{ N} \qquad \text{and } \mathbf{D} = (4.00\text{ N})\mathbf{k} \blacktriangleleft$$

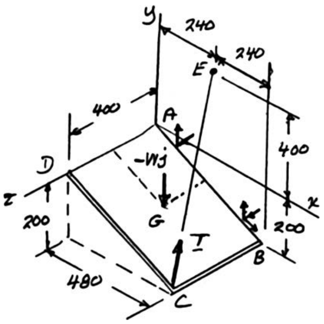


PROBLEM 4.133

The 50-kg plate *ABCD* is supported by hinges along edge *AB* and by wire *CE*. Knowing that the plate is uniform, determine the tension in the wire.

SOLUTION

Free-Body Diagram:



$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2)$$

$$W = 490.50\text{ N}$$

$$\overrightarrow{CE} = -240\mathbf{i} + 600\mathbf{j} - 400\mathbf{k}$$

$$CE = 760\text{ mm}$$

$$\mathbf{T} = T\frac{\overrightarrow{CE}}{CE} = \frac{T}{760}(-240\mathbf{i} + 600\mathbf{j} - 400\mathbf{k})$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{480\mathbf{i} - 200\mathbf{j}}{520} = \frac{1}{13}(12\mathbf{i} - 5\mathbf{j})$$

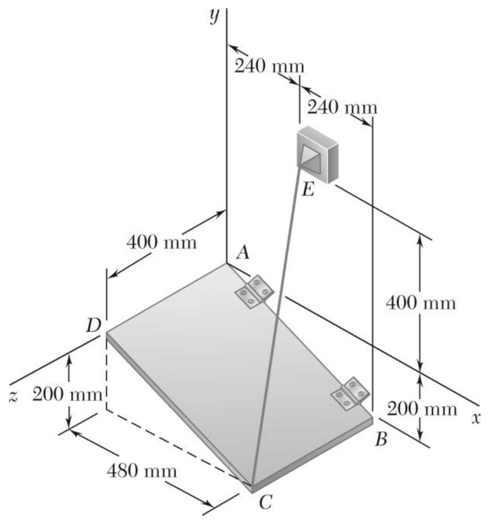
$$\Sigma\mathbf{M}_{AB} = 0: \quad \boldsymbol{\lambda}_{AB}\cdot(\mathbf{r}_{E/A}\times T) + \boldsymbol{\lambda}_{AB}\cdot(\mathbf{r}_{G/A}\times -W\mathbf{j}) = 0$$

$$\mathbf{r}_{E/A} = 240\mathbf{i} + 400\mathbf{j}; \quad \mathbf{r}_{G/A} = 240\mathbf{i} - 100\mathbf{j} + 200\mathbf{k}$$

$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & 400 & 0 \\ -240 & 600 & -400 \end{vmatrix}\frac{T}{13\times 20} + \begin{vmatrix} 12 & -5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix}\frac{1}{13} = 0$$

$$(-12\times 400\times 400 - 5\times 240\times 400)\frac{T}{760} + 12\times 200W = 0$$

$$T = 0.76W = 0.76(490.50\text{ N}) \qquad T = 373\text{ N} ◀$$



PROBLEM 4.134

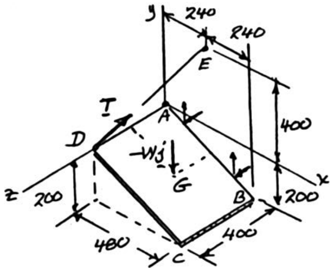
Solve Problem 4.133, assuming that wire *CE* is replaced by a wire connecting *E* and *D*.

PROBLEM 4.133 The 50-kg plate *ABCD* is supported by hinges along edge *AB* and by wire *CE*. Knowing that the plate is uniform, determine the tension in the wire.

SOLUTION

Free-Body Diagram:

Dimensions in mm



$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2)$$

$$W = 490.50\text{ N}$$

$$\overrightarrow{DE} = -240\mathbf{i} + 400\mathbf{j} - 400\mathbf{k}$$

$$DE = 614.5\text{ mm}$$

$$\mathbf{T} = T\frac{\overrightarrow{DE}}{DE} = \frac{T}{614.5}(240\mathbf{i} + 400\mathbf{j} - 400\mathbf{k})$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{480\mathbf{i} - 200\mathbf{j}}{520} = \frac{1}{13}(12\mathbf{i} - 5\mathbf{j})$$

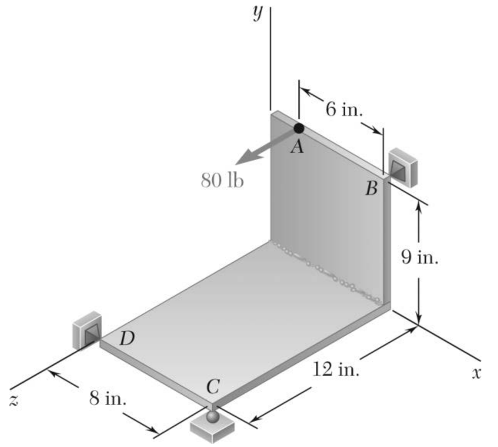
$$\mathbf{r}_{E/A} = 240\mathbf{i} + 400\mathbf{j}; \quad \mathbf{r}_{G/A} = 240\mathbf{i} - 100\mathbf{j} + 200\mathbf{k}$$

$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & 400 & 0 \\ 240 & 400 & -400 \end{vmatrix} \frac{T}{13 \times 614.5} + \begin{vmatrix} 12 & 5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix} \frac{1}{13} = 0$$

$$(-12 \times 400 \times 400 - 5 \times 240 \times 400)\frac{T}{614.5} + 12 \times 200 \times W = 0$$

$$T = 0.6145W = 0.6145(490.50\text{ N})$$

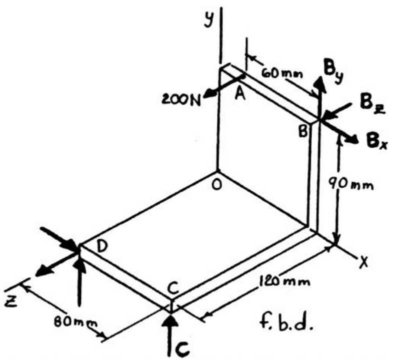
$$T = 301\text{ N} \blacktriangleleft$$



PROBLEM 4.135

Two rectangular plates are welded together to form the assembly shown. The assembly is supported by ball-and-socket joints at *B* and *D* and by a ball on a horizontal surface at *C*. For the loading shown, determine the reaction at *C*.

SOLUTION



First note:

$$\boldsymbol{\lambda}_{BD} = \frac{-(6\text{ in.})\mathbf{i} - (9\text{ in.})\mathbf{j} + (12\text{ in.})\mathbf{k}}{\sqrt{(6)^2 + (9)^2 + (12)^2}\text{ in.}}$$

$$= \frac{1}{16.1555}(-6\mathbf{i} - 9\mathbf{j} + 12\mathbf{k})$$

$$\mathbf{r}_{A/B} = -(6\text{ in.})\mathbf{i}$$

$$\mathbf{P} = (80\text{ lb})\mathbf{k}$$

$$\mathbf{r}_{C/D} = (8\text{ in.})\mathbf{i}$$

$$\mathbf{C} = (C)\mathbf{j}$$

From the F.B.D. of the plates:

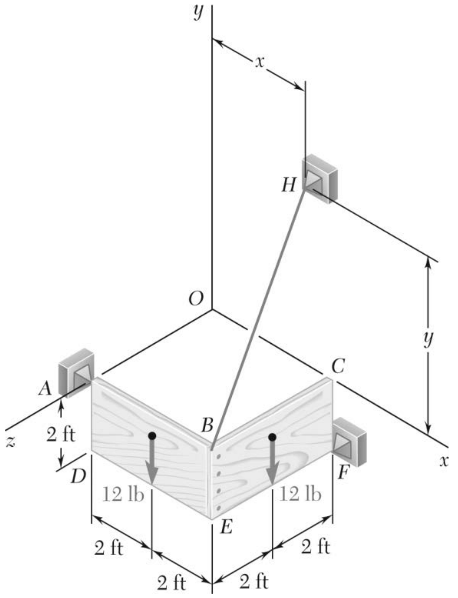
$$\Sigma M_{BD} = 0: \quad \boldsymbol{\lambda}_{BD} \cdot (\mathbf{r}_{A/B} \times \text{P}) + \boldsymbol{\lambda}_{BD} \cdot (\mathbf{r}_{C/D} \times \text{C}) = 0$$

$$\begin{vmatrix} -6 & -9 & 12 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \left[\frac{6(80)}{16.1555}\right] + \begin{vmatrix} -6 & -9 & 12 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \left[\frac{C(8)}{16.1555}\right] = 0$$

$$(-9)(6)(80) + (12)(8)C = 0$$

$C = 45.0$ lb or $\mathbf{C} = (45.0\text{ lb})\mathbf{j}$ ◀

PROPRIETARY MATERIAL.

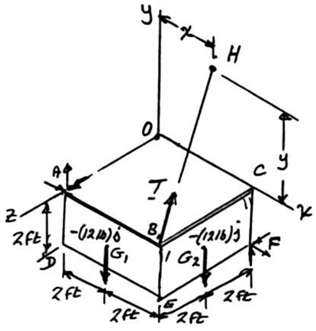


PROBLEM 4.136

Two 2×4-ft plywood panels, each of weight 12 lb, are nailed together as shown. The panels are supported by ball-and-socket joints at *A* and *F* and by the wire *BH*. Determine (*a*) the location of *H* in the *xy* plane if the tension in the wire is to be minimum, (*b*) the corresponding minimum tension.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{AF} = 4\mathbf{i} - 2\mathbf{j} - 4\mathbf{k} \quad AF = 6\,\text{ft}$$

$$\boldsymbol{\lambda}_{AF} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G_1/A} = 2\mathbf{i} - \mathbf{j}$$

$$\mathbf{r}_{G_2/A} = 4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{B/A} = 4\mathbf{i}$$

$$\Sigma M_{AF} = 0: \quad \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{G_1/A} \times (-12\mathbf{j}) + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{G_2/A} \times (-12\mathbf{j})) + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times T) = 0$$

$$\begin{vmatrix} 2 & -1 & -2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \begin{vmatrix} 2 & -1 & -2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$(2 \times 2 \times 12)\frac{1}{3} + (-2 \times 2 \times 12 + 2 \times 4 \times 12)\frac{1}{3} + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$\boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = -32 \quad \text{or} \quad \mathbf{T} \cdot (\boldsymbol{\lambda}_{A/F} \times \mathbf{r}_{B/A}) = -32 \qquad (1)$$

PROBLEM 4.136 (Continued)

Projection of **T** on $(\boldsymbol{\lambda}_{AF} \times \mathbf{r}_{B/A})$ is constant. Thus, $T_{\min}$ is parallel to

$$\boldsymbol{\lambda}_{AF} \times \mathbf{r}_{B/A} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \times 4\mathbf{i} = \frac{1}{3}(-8\mathbf{j} + 4\mathbf{k})$$

Corresponding unit vector is $\frac{1}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k})$.

$$T_{\min} = T(-2\mathbf{j} + \mathbf{k})\frac{1}{\sqrt{5}} \qquad (2)$$

From Eq. (1):

$$\frac{T}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k}) \cdot \left[\frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \times 4\mathbf{i}\right] = -32$$

$$\frac{T}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k}) \cdot \frac{1}{3}(-8\mathbf{j} + 4\mathbf{k}) = -32$$

$$\frac{T}{3\sqrt{5}}(16 + 4) = -32 \qquad T = -\frac{3\sqrt{5}(32)}{20} = 4.8\sqrt{5}$$

$$T = 10.7331 \text{ lb}$$

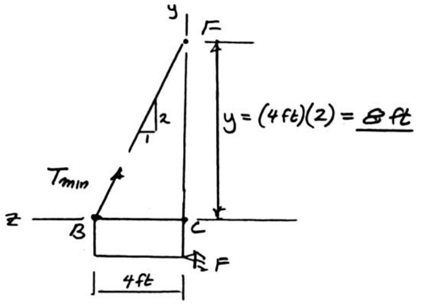
From Eq. (2):

$$T_{\min} = T(-2\mathbf{j} + \mathbf{k})\frac{1}{\sqrt{5}}$$

$$= 4.8\sqrt{5}(-2\mathbf{j} + \mathbf{k})\frac{1}{\sqrt{5}}$$

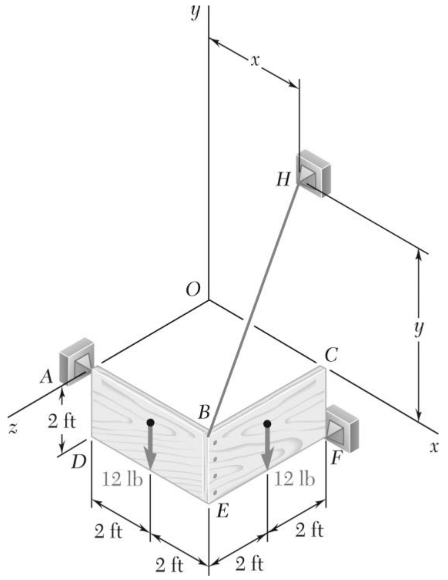
$$\mathbf{T}_{\min} = -(9.6 \text{ lb})\mathbf{j} + (4.8 \text{ lb } \mathbf{k})$$

Since $T_{\min}$ has no **i** component, wire BH is parallel to the yz plane, and $x = 4$ ft.



(*a*) $x = 4.00$ ft; $y = 8.00$ ft ◀

(*b*) $T_{\min} = 10.73$ lb ◀



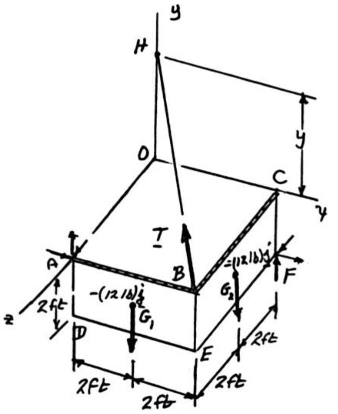
PROBLEM 4.137

Solve Problem 4.136, subject to the restriction that *H* must lie on the *y*-axis.

PROBLEM 4.136 Two 2×4-ft plywood panels, each of weight 12 lb, are nailed together as shown. The panels are supported by ball-and-socket joints at *A* and *F* and by the wire *BH*. Determine (*a*) the location of *H* in the *xy* plane if the tension in the wire is to be minimum, (*b*) the corresponding minimum tension.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{AF} = 4\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$$

$$\boldsymbol{\lambda}_{AF} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G_1/A} = 2\mathbf{i} - \mathbf{j}$$

$$\mathbf{r}_{G_2/A} = 4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{B/A} = 4\mathbf{i}$$

$$\Sigma M_{AF} = 0: \quad \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{G/A} \times (-12\mathbf{j}) + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{G_2/A} \times (-12\mathbf{j})) + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times T) = 0$$

$$\begin{vmatrix} 2 & -1 & 2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \begin{vmatrix} 2 & -1 & -2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$(2 \times 2 \times 12)\frac{1}{3} + (-2 \times 2 \times 12 + 2 \times 4 \times 12)\frac{1}{3} + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$\boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = -32 \qquad (1)$$

$$\overrightarrow{BH} = -4\mathbf{i} + y\mathbf{j} - 4\mathbf{k} \qquad BH = (32 + y^2)^{1/2}$$

$$\mathbf{T} = T\frac{\overrightarrow{BH}}{BH} = T\frac{-4\mathbf{i} + y\mathbf{j} - 4\mathbf{k}}{(32 + y^2)^{1/2}}$$

PROBLEM 4.137 (Continued)

From Eq. (1):

$$\boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times T)=\begin{vmatrix}2 & -1 & -2\\ 4 & 0 & 0\\ -4 & y & -4\end{vmatrix}\frac{T}{3(32+y^2)^{1/2}}=-32$$

$$(-16-8y)T=-3\times 32(32+y^2)^{1/2} \qquad T=96\frac{(32+y^2)^{1/2}}{8y+16} \tag{2}$$

$$\frac{dT}{dy}=0: \quad 96\frac{(8y+16)\frac{1}{2}(32+y^2)^{-1/2}(2y)+(32+y^2)^{1/2}(8)}{(8y+16)^2}$$

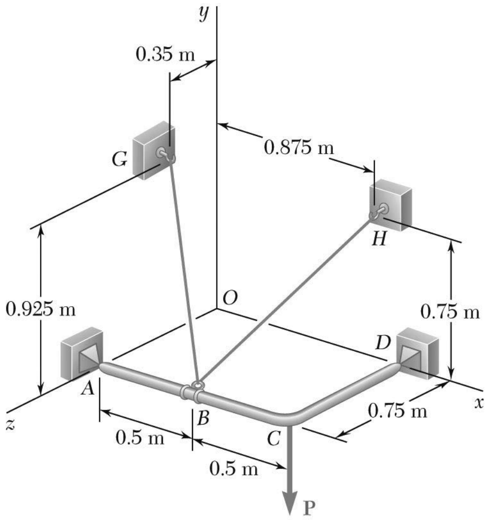
Numerator = 0: $(8y+16)y=(32+y^2)8$

$8y^2+16y=32\times 8+8y^2$

$x=0$ ft; $y=16.00$ ft ◀

From Eq. (2): $T=96\dfrac{(32+16^2)^{1/2}}{8\times 16+16}=11.3137$ lb

$T_{\min}=11.31$ lb ◀

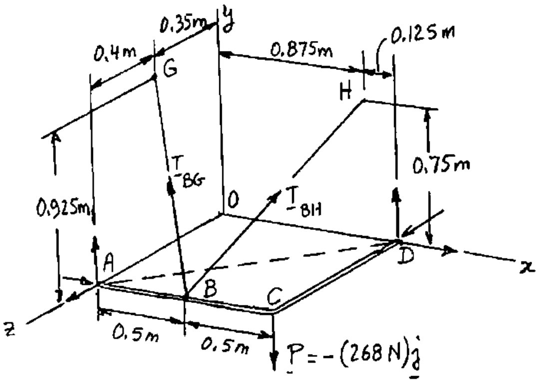


PROBLEM 4.138

The frame *ACD* is supported by ball-and-socket joints at *A* and *D* and by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the frame supports at Point *C* a load of magnitude $P = 268$ N, determine the tension in the cable.

SOLUTION

Free-Body Diagram:



$$\boldsymbol{\lambda}_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{(1\text{ m})\mathbf{i} - (0.75\text{ m})\mathbf{k}}{1.25\text{ m}}$$

$$\boldsymbol{\lambda}_{AD} = 0.8\mathbf{i} - 0.6\mathbf{k}$$

$$\mathbf{T}_{BG} = T_{BG}\frac{\overrightarrow{BG}}{BG}$$

$$= T_{BG}\frac{-0.5\mathbf{i} + 0.925\mathbf{j} - 0.4\mathbf{k}}{1.125}$$

$$T_{BH} = T_{BH}\frac{\overrightarrow{BH}}{BH}$$

$$= T_{BH}\frac{0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k}}{1.125}$$

$$\mathbf{r}_{B/A} = (0.5\text{ m})\mathbf{i}; \quad \mathbf{r}_{C/A} = (1\text{ m})\mathbf{i}; \quad \mathbf{P} = -(268\text{ N})\mathbf{j}$$

To eliminate the reactions at A and D, we shall write

$$\Sigma\mathbf{M}_{AD} = 0: \quad \boldsymbol{\lambda}_{AD}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}_{BG}) + \boldsymbol{\lambda}_{AD}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}_{BH}) + \boldsymbol{\lambda}_{AD}\cdot(\mathbf{r}_{C/A}\times\mathbf{P}) = 0 \qquad (1)$$

Substituting for terms in Eq. (1) and using determinants,

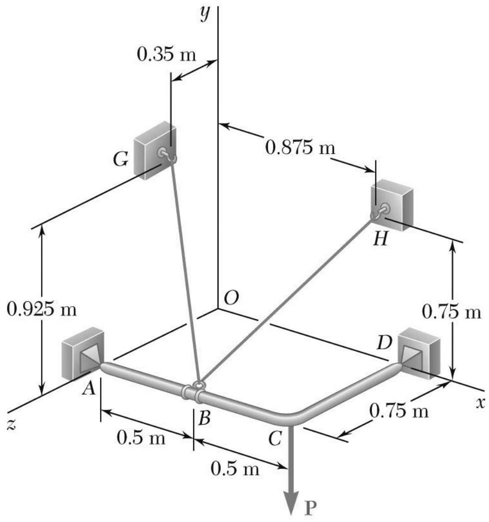
$$\begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ -0.5 & 0.925 & -0.4 \end{vmatrix}\frac{T_{BG}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ 0.375 & 0.75 & -0.75 \end{vmatrix}\frac{T_{BH}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 1 & 0 & 0 \\ 0 & -268 & 0 \end{vmatrix} = 0$$

Multiplying all terms by (–1.125),

$$0.27750T_{BG} + 0.22500T_{BH} = 180.900 \qquad (2)$$

For this problem, $T_{BG} = T_{BH} = T$

$$(0.27750 + 0.22500)T = 180.900 \qquad T = 360\text{ N} \blacktriangleleft$$



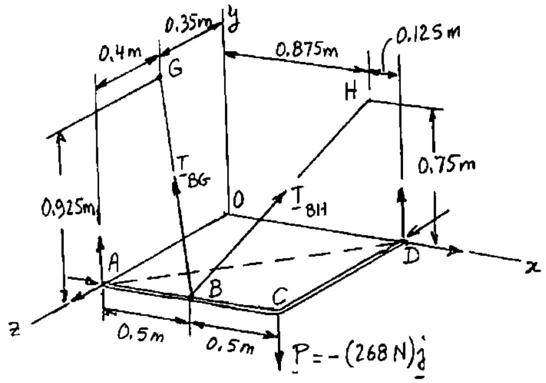
PROBLEM 4.139

Solve Prob. 4.138, assuming that cable *GBH* is replaced by a cable *GB* attached at *G* and *B*.

PROBLEM 4.138 The frame *ACD* is supported by ball-and-socket joints at *A* and *D* and by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the frame supports at Point *C* a load of magnitude *P* = 268 N, determine the tension in the cable.

SOLUTION

Free-Body Diagram:



$$\boldsymbol{\lambda}_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{(1\text{ m})\mathbf{i} - (0.75\text{ m})\mathbf{k}}{1.25\text{ m}}$$

$$\boldsymbol{\lambda}_{AD} = 0.8\mathbf{i} - 0.6\mathbf{k}$$

$$\mathbf{T}_{BG} = T_{BG}\frac{\overrightarrow{BG}}{BG}$$

$$= T_{BG}\frac{-0.5\mathbf{i} + 0.925\mathbf{j} - 0.4\mathbf{k}}{1.125}$$

$$\mathbf{T}_{BH} = T_{BH}\frac{\overrightarrow{BH}}{BH}$$

$$= T_{BH}\frac{0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k}}{1.125}$$

PROBLEM 4.139 (Continued)

$$\mathbf{r}_{B/A} = (0.5\text{ m})\mathbf{i}; \quad \mathbf{r}_{C/A} = (1\text{ m})\mathbf{i}; \quad \mathbf{P} = -(268\text{ N})\mathbf{j}$$

To eliminate the reactions at *A* and *D*, we shall write

$$\Sigma \mathbf{M}_{AD} = 0: \quad \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BG}) + \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BH}) + \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{C/A} \times \mathbf{P}) = 0 \tag{1}$$

Substituting for terms in Eq. (1) and using determinants,

$$\begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ -0.5 & 0.925 & -0.4 \end{vmatrix} \frac{T_{BG}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ 0.375 & 0.75 & -0.75 \end{vmatrix} \frac{T_{BH}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 1 & 0 & 0 \\ 0 & -268 & 0 \end{vmatrix} = 0$$

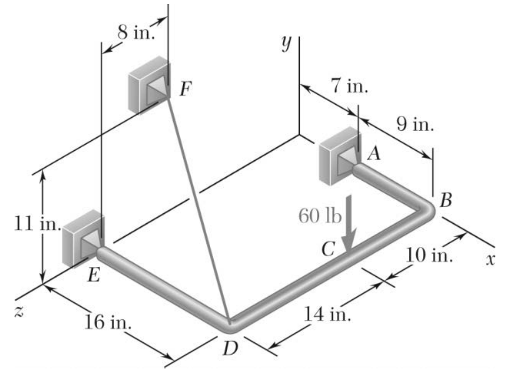
Multiplying all terms by (–1.125),

$$0.27750T_{BG} + 0.22500T_{BH} = 180.900 \tag{2}$$

For this problem, $T_{BH} = 0.$

Thus, Eq. (2) reduces to

$$0.27750T_{BG} = 180.900 \qquad T_{BG} = 652\text{ N} \blacktriangleleft$$

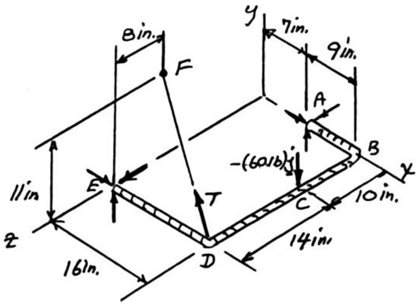


PROBLEM 4.140

The bent rod *ABDE* is supported by ball-and-socket joints at *A* and *E* and by the cable *DF*. If a 60-lb load is applied at *C* as shown, determine the tension in the cable.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{DF} = -16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k} \qquad DF = 21\text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{DE}}{DF} = \frac{T}{21}(-16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k})$$

$$\mathbf{r}_{D/E} = 16\mathbf{i}$$

$$\mathbf{r}_{C/E} = 16\mathbf{i} - 14\mathbf{k}$$

$$\boldsymbol{\lambda}_{EA} = \frac{\overrightarrow{EA}}{EA} = \frac{7\mathbf{i} - 24\mathbf{k}}{25}$$

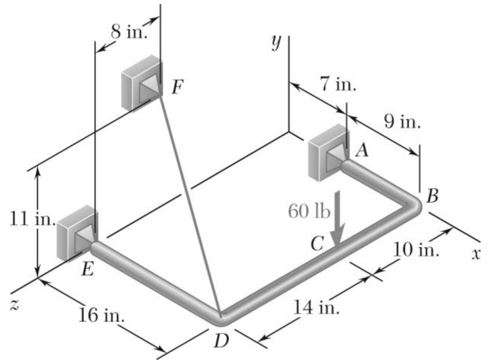
$$\Sigma M_{EA} = 0: \quad \boldsymbol{\lambda}_{EA} \cdot (\mathbf{r}_{B/E} \times \mathbf{T}) + \boldsymbol{\lambda}_{EA} \cdot (\mathbf{r}_{C/E} \cdot (-60\mathbf{j})) = 0$$

$$\begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T}{21 \times 25} + \begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & -14 \\ 0 & -60 & 0 \end{vmatrix} \frac{1}{25} = 0$$

$$-\frac{24 \times 16 \times 11}{21 \times 25} T + \frac{-7 \times 14 \times 60 + 24 \times 16 \times 60}{25} = 0$$

$$201.14T + 17{,}160 = 0$$

$$T = 85.314\text{ lb} \qquad\qquad T = 85.3\text{ lb} \blacktriangleleft$$

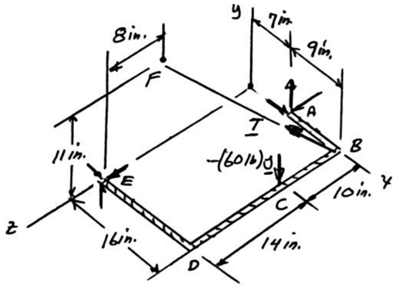


PROBLEM 4.141

Solve Problem 4.140, assuming that cable *DF* is replaced by a cable connecting *B* and *F*.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{B/A} = 9\mathbf{i}$$

$$\mathbf{r}_{C/A} = 9\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{BF} = -16\mathbf{i} + 11\mathbf{j} + 16\mathbf{k} \qquad BF = 25.16 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{BF}}{BF} = \frac{T}{25.16}(-16\mathbf{i} + 11\mathbf{j} + 16\mathbf{k})$$

$$\boldsymbol{\lambda}_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{7\mathbf{i} - 24\mathbf{k}}{\mathbf{25}}$$

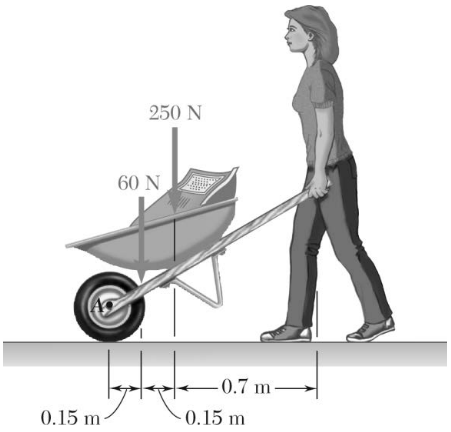
$$\Sigma M_{AE} = 0: \quad \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) + \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{C/A} \cdot (-60\mathbf{j})) = 0$$

$$\begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 0 \\ -16 & 11 & 16 \end{vmatrix} \frac{T}{25 \times 25.16} + \begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 10 \\ 0 & -60 & 0 \end{vmatrix} \frac{1}{25} = 0$$

$$-\frac{24 \times 9 \times 11}{25 \times 25.16}T + \frac{24 \times 9 \times 60 + 7 \times 10 \times 60}{25} = 0$$

$$94.436T - 17{,}160 = 0$$

$T = 181.7$ lb ◀

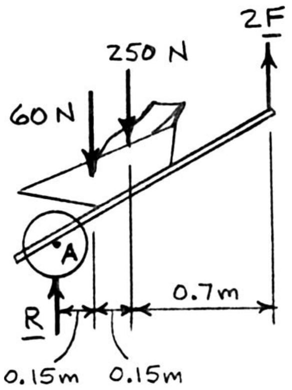


PROBLEM 4.142

A gardener uses a 60-N wheelbarrow to transport a 250-N bag of fertilizer. What force must she exert on each handle?

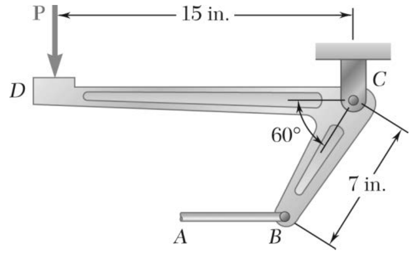
SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad (2F)(1\text{ m}) - (60\text{ N})(0.15\text{ m}) - (250\text{ N})(0.3\text{ m}) = 0$$

$$\mathbf{F} = 42.0\text{ N}\uparrow \blacktriangleleft$$

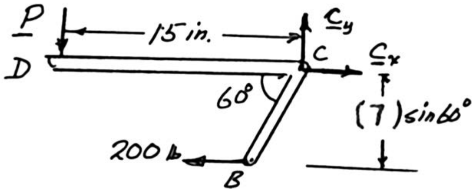


PROBLEM 4.143

The required tension in cable *AB* is 200 lb. Determine (*a*) the vertical force **P** that must be applied to the pedal, (*b*) the corresponding reaction at *C*.

SOLUTION

Free-Body Diagram:



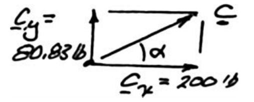
$BC = 7$ in.

(*a*) $+\circlearrowleft \Sigma M_C = 0: \quad P(15 \text{ in.}) - (200 \text{ lb})(6.062 \text{ in.}) = 0$

$P = 80.83$ lb $\quad$ $\mathbf{P} = 80.8 \text{ lb} \downarrow$ ◀

(*b*) $\xrightarrow{+} \Sigma F_y = 0: \quad C_x - 200 \text{ lb} = 0$ $\quad$ $\mathbf{C}_x = 200 \text{ lb} \longrightarrow$

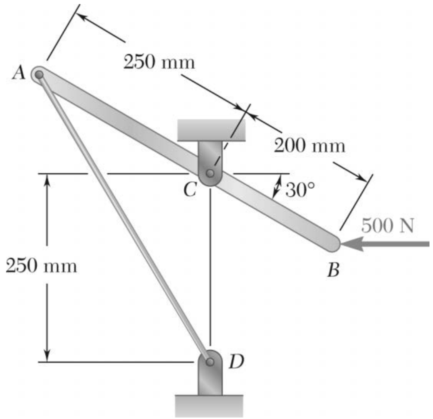
$+\uparrow \Sigma F_y = 0: \quad C_y - P = 0 \quad C_y - 80.83 \text{ lb} = 0$ $\quad$ $\mathbf{C}_y = 80.83 \text{ lb} \uparrow$



$\alpha = 22.0°$

$C = 215.7$ lb

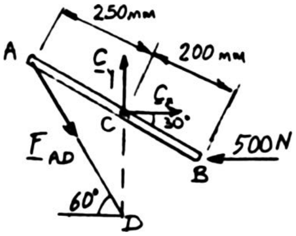
$\mathbf{C} = 216 \text{ lb} \measuredangle\, 22.0°$ ◀



PROBLEM 4.144

A lever *AB* is hinged at *C* and attached to a control cable at *A*. If the lever is subjected to a 500-N horizontal force at *B*, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

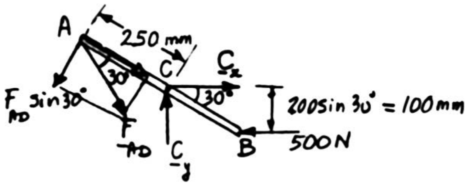
SOLUTION



Triangle *ACD* is isosceles with $\sphericalangle C = 90° + 30° = 120°$ $\sphericalangle A = \sphericalangle D = \frac{1}{2}(180° - 120°) = 30°.$

Thus, *DA* forms angle of 60° with the horizontal axis.

(*a*) We resolve $\mathbf{F}_{AD}$ into components along *AB* and perpendicular to *AB*.



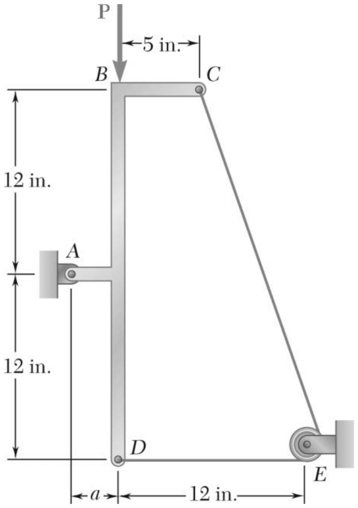
$$+\circlearrowleft \Sigma M_C = 0: \quad (F_{AD}\sin 30°)(250\text{ mm}) - (500\text{ N})(100\text{ mm}) = 0 \qquad F_{AD} = 400\text{ N} ◀$$

(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad -(400\text{ N})\cos 60° + C_x - 500\text{ N} = 0 \qquad C_x = +300\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad -(400\text{ N})\sin 60° + C_y = 0 \qquad C_y = +346.4\text{ N}$$

$$\mathbf{C} = 458\text{ N} \measuredangle 49.1° ◀$$

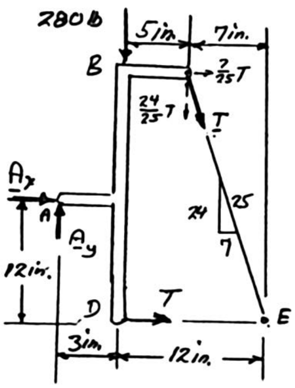


PROBLEM 4.145

A force **P** of magnitude 280 lb is applied to member *ABCD*, which is supported by a frictionless pin at *A* and by the cable *CED*. Since the cable passes over a small pulley at *E*, the tension may be assumed to be the same in portions *CE* and *ED* of the cable. For the case when $a = 3$ in., determine (*a*) the tension in the cable, (*b*) the reaction at *A*.

SOLUTION

Free-Body Diagram:



(*a*) $+\circlearrowleft \Sigma M_A = 0: \quad -(280\text{ lb})(8\text{ in.})$

$$T(12\text{ in.}) - \frac{7}{25}T(12\text{ in.})$$

$$-\frac{24}{25}T(8\text{ in.}) = 0$$

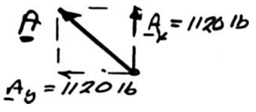
$$(12 - 11.04)T = 840 \qquad T = 875\text{ lb} ◀$$

(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad \frac{7}{25}(875\text{ lb}) + 875\text{ lb} + A_x = 0$

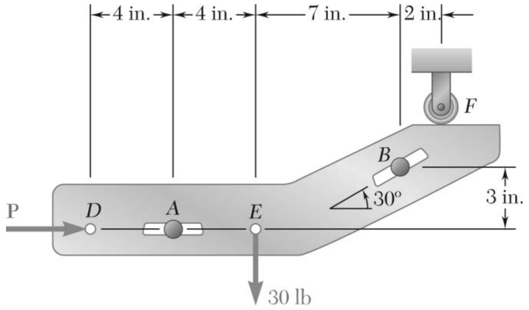
$$A_x = -1120 \qquad \mathbf{A}_x = 1120\text{ lb} \longleftarrow$$

$+\uparrow \Sigma F_y = 0: \quad A_y - 280\text{ lb} - \frac{24}{25}(875\text{ lb}) = 0$

$$A_y = +1120 \qquad \mathbf{A}_y = 1120\text{ lb}\uparrow$$



$$\mathbf{A} = 1584\text{ lb} \nwarrow 45.0^\circ ◀$$

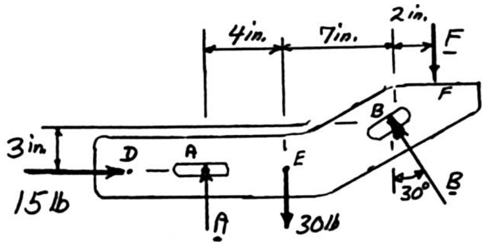


PROBLEM 4.146

Two slots have been cut in plate *DEF*, and the plate has been placed so that the slots fit two fixed, frictionless pins *A* and *B*. Knowing that $P = 15$ lb, determine (*a*) the force each pin exerts on the plate, (*b*) the reaction at *F*.

SOLUTION

Free-Body Diagram:



(*a*) $\xrightarrow{+} \Sigma F_x = 0$: $15\text{ lb} - B\sin 30° = 0$ $\qquad \mathbf{B} = 30.0\text{ lb} \searrow 60.0°$ ◀

(*b*) $+\circlearrowleft \Sigma M_A = 0$: $-(30\text{ lb})(4\text{ in.}) + B\sin 30°(3\text{ in.}) + B\cos 30°(11\text{ in.}) - F(13\text{ in.}) = 0$

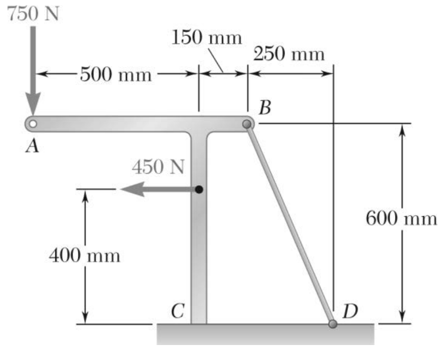
$$-120\text{ lb}\cdot\text{in.} + (30\text{ lb})\sin 30°(3\text{ in.}) + (30\text{ lb})\cos 30°(11\text{ in.}) - F(13\text{ in.}) = 0$$

$$F = +16.2145\text{ lb}$$ $\qquad \mathbf{F} = 16.21\text{ lb}\downarrow$ ◀

(*a*) $+\uparrow \Sigma F_y = 0$: $A - 30\text{ lb} + B\cos 30° - F = 0$

$$A - 30\text{ lb} + (30\text{ lb})\cos 30° - 16.2145\text{ lb} = 0$$

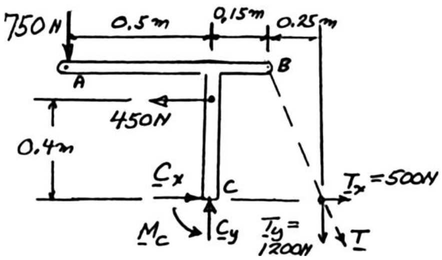
$$A = +20.23\text{ lb}$$ $\qquad \mathbf{A} = 20.2\text{ lb}\uparrow$ ◀



PROBLEM 4.147

Knowing that the tension in wire *BD* is 1300 N, determine the reaction at the fixed support *C* of the frame shown.

SOLUTION



$$T = 1300\ \text{N}$$

$$T_x = \frac{5}{13}T$$

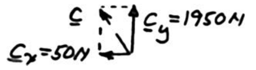
$$= 500\ \text{N}$$

$$T_y = \frac{12}{13}T$$

$$= 1200\ \text{N}$$

$\xrightarrow{+} \Sigma M_x = 0{:}\quad C_x - 450\ \text{N} + 500\ \text{N} = 0 \qquad C_x = -50\ \text{N}$ $\qquad \mathbf{C}_x = 50\ \text{N} \longleftarrow$

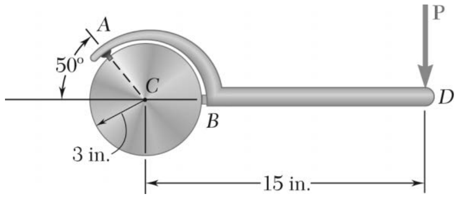
$+\uparrow \Sigma F_y = 0{:}\quad C_y - 750\ \text{N} - 1200\ \text{N} = 0 \quad C_y = +1950\ \text{N}$ $\qquad \mathbf{C}_y = 1950\ \text{N} \uparrow$



$\mathbf{C} = 1951\ \text{N} \measuredangle 88.5^\circ$ ◀

$+\circlearrowleft \Sigma M_C = 0{:}\quad M_C + (750\ \text{N})(0.5\ \text{m}) + (4.50\ \text{N})(0.4\ \text{m})$
$-(1200\ \text{N})(0.4\ \text{m}) = 0$

$M_C = -75.0\ \text{N}\cdot\text{m}$ $\qquad \mathbf{M}_C = 75.0\ \text{N}\cdot\text{m} \circlearrowright$ ◀



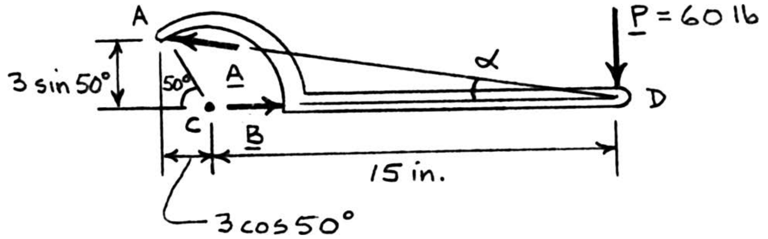
PROBLEM 4.148

The spanner shown is used to rotate a shaft. A pin fits in a hole at *A*, while a flat, frictionless surface rests against the shaft at *B*. If a 60-lb force **P** is exerted on the spanner at *D*, find the reactions at *A* and *B*.

SOLUTION

Free-Body Diagram:

(Three-force body)



The line of action of **A** must pass through *D*, where **B** and **P** intersect.

$$\tan\alpha = \frac{3\sin 50°}{3\cos 50° + 15}$$

$$= 0.135756$$

$$\alpha = 7.7310°$$

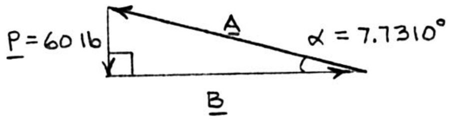
$$A = \frac{60\text{ lb}}{\sin 7.7310°}$$

$$= 446.02\text{ lb}$$

$$B = \frac{60\text{ lb}}{\tan 7.7310°}$$

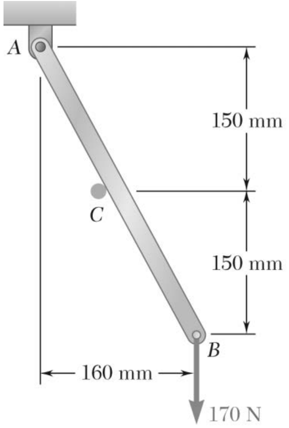
$$= 441.97\text{ lb}$$

Force triangle



A = 446 lb ↘ 7.73° ◀

B = 442 lb → ◀

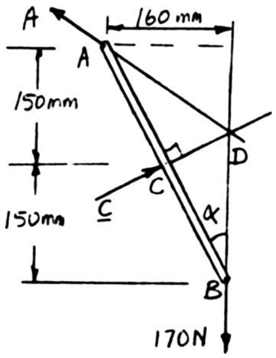


PROBLEM 4.149

Rod *AB* is supported by a pin and bracket at *A* and rests against a frictionless peg at *C*. Determine the reactions at *A* and *C* when a 170-N vertical force is applied at *B*.

SOLUTION

Free-Body Diagram:
(Three-force body)



The reaction at *A* must pass through *D* where **C** and the 170-N force intersect.

$$\tan\alpha = \frac{160\text{ mm}}{300\text{ mm}}$$

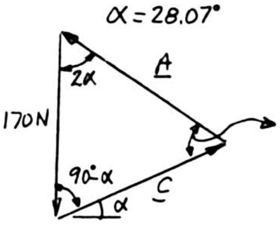
$$\alpha = 28.07°$$

We note that triangle *ABD* is isosceles (since $AC = BC$) and, therefore,

$$\sphericalangle CAD = \alpha = 28.07°$$

Also, since $CD \perp CB$, reaction **C** forms angle $\alpha = 28.07°$ with the horizontal axis.

Force triangle



We note that **A** forms angle 2α with the vertical axis. Thus, **A** and **C** form angle

$$180° - (90° - \alpha) - 2\alpha = 90° - \alpha$$

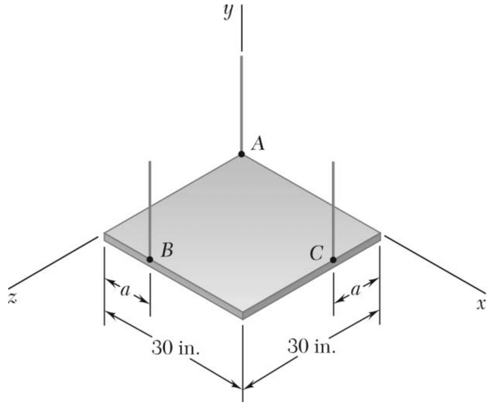
Force triangle is isosceles, and we have

$$A = 170\text{ N}$$

$$C = 2(170\text{ N})\sin\alpha$$

$$= 160.0\text{ N}$$

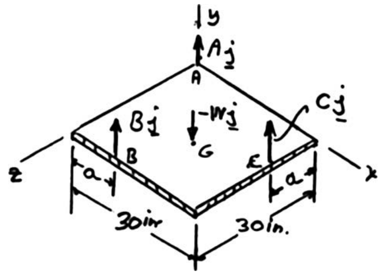
$\mathbf{A} = 170.0\text{ N} \searrow 33.9°; \quad \mathbf{C} = 160.0\text{ N} \nearrow 28.1°$ ◀



PROBLEM 4.150

The 24-lb square plate shown is supported by three vertical wires. Determine (*a*) the tension in each wire when $a = 10$ in., (*b*) the value of *a* for which the tension in each wire is 8 lb.

SOLUTION



$$\mathbf{r}_{B/A} = a\mathbf{i} + 30\mathbf{k}$$
$$\mathbf{r}_{C/A} = 30\mathbf{i} + a\mathbf{k}$$
$$\mathbf{r}_{G/A} = 15\mathbf{i} + 15\mathbf{k}$$

By symmetry, $B = C$.

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_C \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$(a\mathbf{i} + 30\mathbf{k}) \times B\mathbf{j} + (30\mathbf{i} + a\mathbf{k}) \times B\mathbf{j} + (15\mathbf{i} + 15\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$Ba\mathbf{k} - 30B\mathbf{i} + 30B\mathbf{k} - Ba\mathbf{i} - 15W\mathbf{k} + 15W\mathbf{i} = 0$$

Equate coefficient of unit vector **i** to zero:

$$\mathbf{i}: \quad -30B - Ba + 15W = 0$$

$$B = \frac{15W}{30 + a} \qquad C = B = \frac{15W}{30 + a} \tag{1}$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A + 2\left[\frac{15W}{30 + a}\right] - W = 0; \quad A = \frac{aW}{30 + a} \tag{2}$$

(*a*) For $a = 10$ in.

From Eq. (1): $C = B = \dfrac{15(24 \text{ lb})}{30 + 10} = 9.00 \text{ lb}$

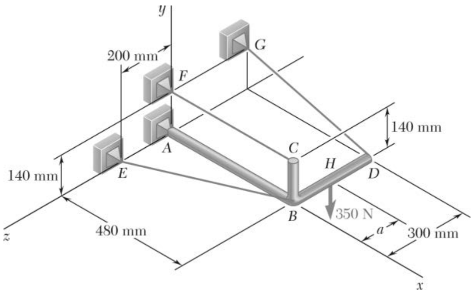
From Eq. (2): $A = \dfrac{10(24 \text{ lb})}{30 + 10} = 6.00 \text{ lb}$ $\qquad A = 6.00 \text{ lb}; \quad B = C = 9.00 \text{ lb}$ ◀

PROBLEM 4.150 (Continued)

(*b*) For tension in each wire = 8 lb,

From Eq. (1): $$8\text{ lb} = \frac{15(24\text{ lb})}{30 + a}$$

$$30\text{ in.} + a = 45 \qquad a = 15.00\text{ in.} \blacktriangleleft$$



PROBLEM 4.151

Frame *ABCD* is supported by a ball-and-socket joint at *A* and by three cables. For $a = 150$ mm, determine the tension in each cable and the reaction at *A*.

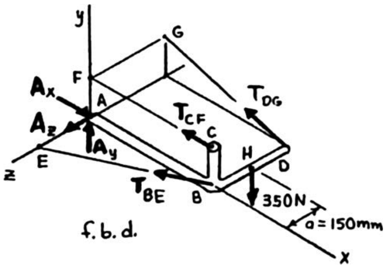
SOLUTION

First note:

$$\mathbf{T}_{DG} = \boldsymbol{\lambda}_{DG} T_{DG} = \frac{-(0.48\text{ m})\mathbf{i} + (0.14\text{ m})\mathbf{j}}{\sqrt{(0.48)^2 + (0.14)^2}\text{ m}} T_{DG}$$

$$= \frac{-0.48\mathbf{i} + 0.14\mathbf{j}}{0.50} T_{DG}$$

$$= \frac{T_{DG}}{25}(24\mathbf{i} + 7\mathbf{j})$$



$$\mathbf{T}_{BE} = \boldsymbol{\lambda}_{BE} T_{BE} = \frac{-(0.48\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.2)^2}\text{ m}} T_{BE}$$

$$= \frac{-0.48\mathbf{i} + 0.2\mathbf{k}}{0.52} T_{BE}$$

$$= \frac{T_{BE}}{13}(-12\mathbf{j} + 5\mathbf{k})$$

From F.B.D. of frame *ABCD*:

$$\Sigma M_x = 0\text{:} \quad \left(\frac{7}{25}T_{DG}\right)(0.3\text{ m}) - (350\text{ N})(0.15\text{ m}) = 0$$

or $T_{DG} = 625\text{ N}$ ◀

$$\Sigma M_y = 0\text{:} \quad \left(\frac{24}{25}\times 625\text{ N}\right)(0.3\text{ m}) - \left(\frac{5}{13}T_{BE}\right)(0.48\text{ m}) = 0$$

or $T_{BE} = 975\text{ N}$ ◀

$$\Sigma M_z = 0\text{:} \quad T_{CF}(0.14\text{ m}) + \left(\frac{7}{25}\times 625\text{ N}\right)(0.48\text{ m}) - (350\text{ N})(0.48\text{ m}) = 0$$

or $T_{CF} = 600\text{ N}$ ◀

PROBLEM 4.151 (Continued)

$$\Sigma F_x = 0: \quad A_x + T_{CF} + (T_{BE})_x + (T_{DG})_x = 0$$

$$A_x - 600\text{ N} - \left(\frac{12}{13}\times 975\text{ N}\right) - \left(\frac{24}{25}\times 625\text{ N}\right) = 0$$

$$A_x = 2100\text{ N}$$

$$\Sigma F_y = 0: \quad A_y + (T_{DG})_y - 350\text{ N} = 0$$

$$A_y + \left(\frac{7}{25}\times 625\text{ N}\right) - 350\text{ N} = 0$$

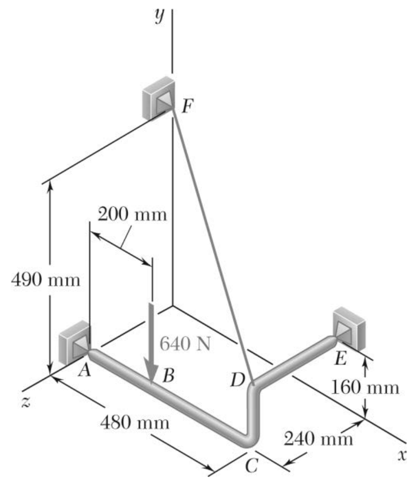
$$A_y = 175.0\text{ N}$$

$$\Sigma F_z = 0: \quad A_z + (T_{BE})_z = 0$$

$$A_z + \left(\frac{5}{13}\times 975\text{ N}\right) = 0$$

$$A_z = -375\text{ N}$$

Therefore, $\mathbf{A} = (2100\text{ N})\mathbf{i} + (175.0\text{ N})\mathbf{j} - (375\text{ N})\mathbf{k}$ ◀



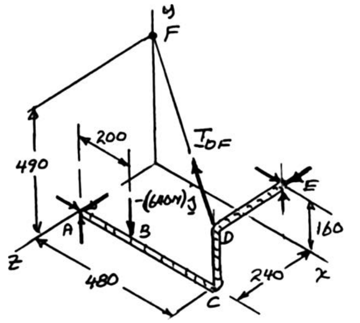
PROBLEM 4.152

The pipe *ACDE* is supported by ball-and-socket joints at *A* and *E* and by the wire *DF*. Determine the tension in the wire when a 640-N load is applied at *B* as shown.

SOLUTION

Free-Body Diagram:

Dimensions in mm



$$\overrightarrow{AE} = 480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}$$

$$AE = 560 \text{ mm}$$

$$\boldsymbol{\lambda}_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}}{560}$$

$$\boldsymbol{\lambda}_{AE} = \frac{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}}{7}$$

$$\mathbf{r}_{B/A} = 200\mathbf{i}$$

$$\mathbf{r}_{D/A} = 480\mathbf{i} + 160\mathbf{j}$$

$$\overrightarrow{DF} = -480\mathbf{i} + 330\mathbf{j} - 240\mathbf{k}; \qquad DF = 630 \text{ mm}$$

$$\mathbf{T}_{DF} = T_{DF}\frac{\overrightarrow{DF}}{DF} = T_{DF}\frac{-480\mathbf{i} + 330\mathbf{j} - 240\mathbf{k}}{630} = T_{DF}\frac{-16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}}{21}$$

$$\Sigma M_{AE} = \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{D/A} \times \mathbf{T}_{DF}) + \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{B/A} \times (-600\mathbf{j})) = 0$$

$$\begin{vmatrix} 6 & 2 & -3 \\ 480 & 160 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T_{DF}}{21 \times 7} + \begin{vmatrix} 6 & 2 & -3 \\ 200 & 0 & 0 \\ 0 & -640 & 0 \end{vmatrix} \frac{1}{7} = 0$$

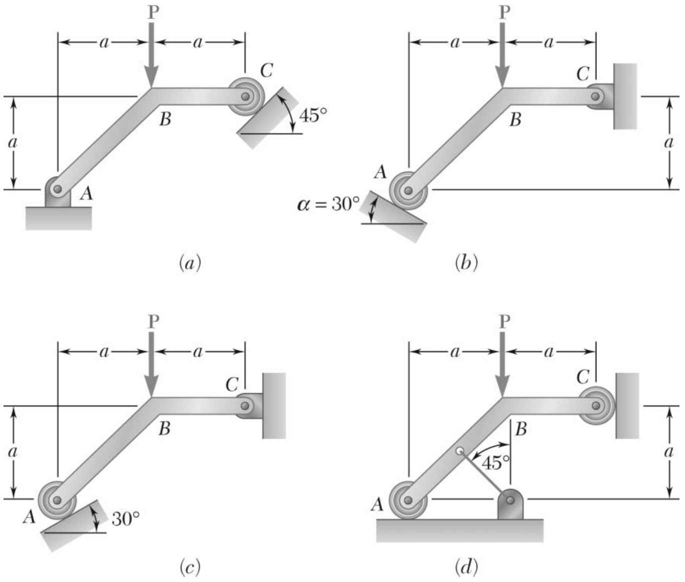
$$\frac{-6 \times 160 \times 8 + 2 \times 480 \times 8 - 3 \times 480 \times 11 - 3 \times 160 \times 16}{21 \times 7} T_{DF} + \frac{3 \times 200 \times 640}{7} = 0$$

$$-1120 T_{DF} + 384 \times 10^3 = 0$$

$$T_{DF} = 342.86 \text{ N} \qquad\qquad T_{DF} = 343 \text{ N} \blacktriangleleft$$

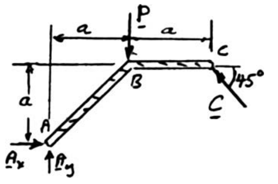
PROBLEM 4.153

A force **P** is applied to a bent rod *ABC*, which may be supported in four different ways as shown. In each case, if possible, determine the reactions at the supports.



SOLUTION

(*a*)



$$+\circlearrowleft \Sigma M_A = 0: \quad -P_a + (C\sin 45°)2a + (\cos 45°)a = 0$$

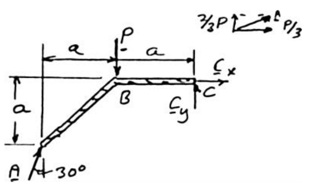
$$3\frac{C}{\sqrt{2}} = P \qquad C = \frac{\sqrt{2}}{3}P \qquad \mathbf{C} = 0.471P \searrow 45° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - \left(\frac{\sqrt{2}}{3}P\right)\frac{1}{\sqrt{2}} \qquad A_x = \frac{P}{3} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - P + \left(\frac{\sqrt{2}}{3}P\right)\frac{1}{\sqrt{2}} \qquad A_y = \frac{2P}{3}\uparrow$$

$$\mathbf{A} = 0.745P \nearrow 63.4° \blacktriangleleft$$

(*b*)



$$+\circlearrowleft \Sigma M_C = 0: \quad +Pa - (A\cos 30°)2a + (A\sin 30°)a = 0$$

$$A(1.732 - 0.5) = P \quad A = 0.812P$$

$$\mathbf{A} = 0.812P \nearrow 60.0° \blacktriangleleft$$

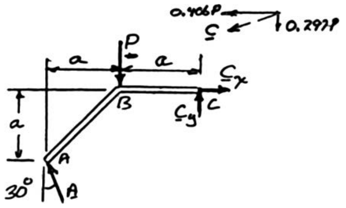
$$\xrightarrow{+} \Sigma F_x = 0: \quad (0.812P)\sin 30° + C_x = 0 \qquad C_x = -0.406P$$

$$+\uparrow \Sigma F_y = 0: \quad (0.812P)\cos 30° - P + C_y = 0 \quad C_y = -0.297P$$

$$\mathbf{C} = 0.503P \swarrow 36.2° \blacktriangleleft$$

PROBLEM 4.153 (Continued)

(*c*)



$$+\circlearrowleft \Sigma M_C = 0: \quad +Pa - (A\cos 30°)2a + (A\sin 30°)a = 0$$

$$A(1.732 + 0.5) = P \quad A = 0.448P$$

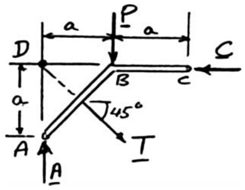
$$A = 0.448P \searrow 60.0° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -(0.448P)\sin 30° + C_x = 0 \qquad C_x = 0.224P \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad (0.448P)\cos 30° - P + C_y = 0 \quad C_y = 0.612P \uparrow$$

$$\mathbf{C} = 0.652P \nearrow 69.9° \blacktriangleleft$$

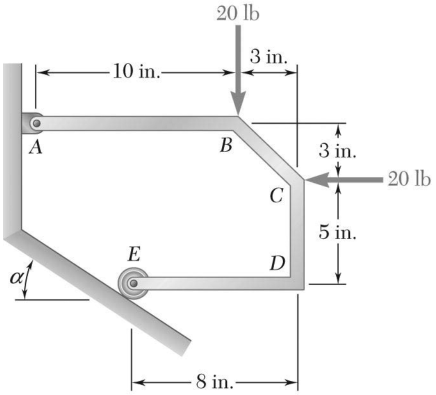
(*d*) Force **T** exerted by wire and reactions **A** and **C** all intersect at Point *D*.



$$+\circlearrowleft \Sigma M_D = 0: \quad P_a = 0$$

Equilibrium is not maintained.

Rod is improperly constrained. ◀

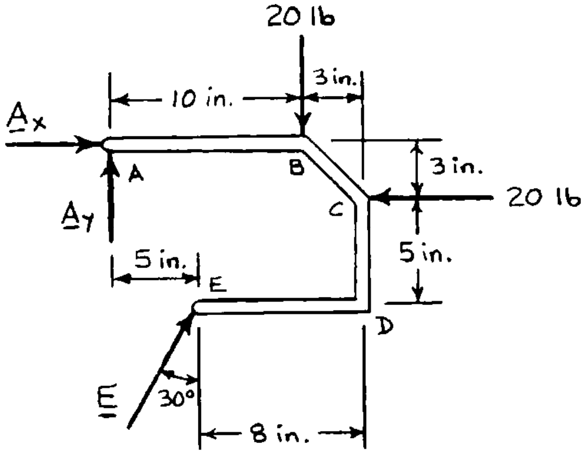


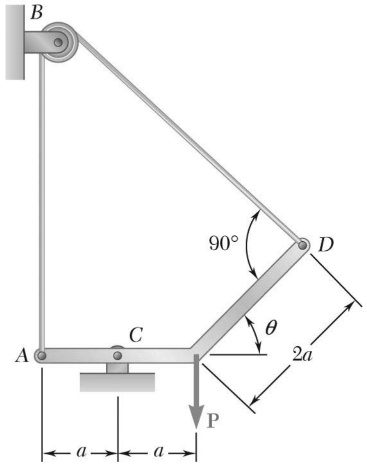
PROBLEM 4.F1

For the frame and loading shown, draw the free-body diagram needed to determine the reactions at *A* and *E* when $\alpha = 30°$.

SOLUTION

Free-Body Diagram of Frame:



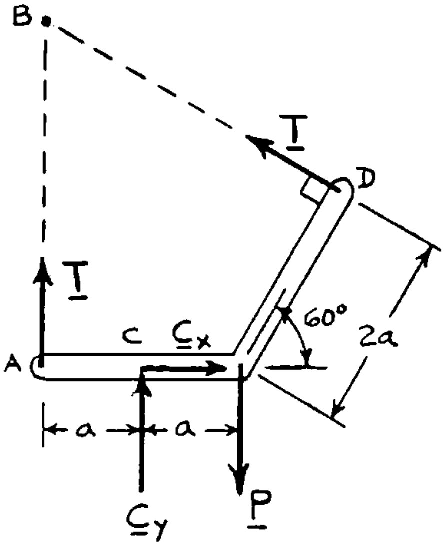


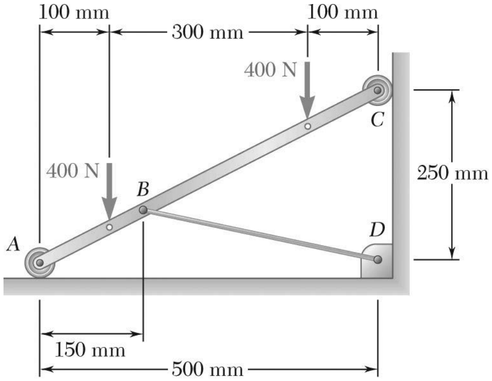
PROBLEM 4.F2

Neglecting friction, draw the free-body diagram needed to determine the tension in cable *ABD* and the reaction at *C* when $\theta = 60°$.

SOLUTION

Free-Body Diagram of Member *ACD*:



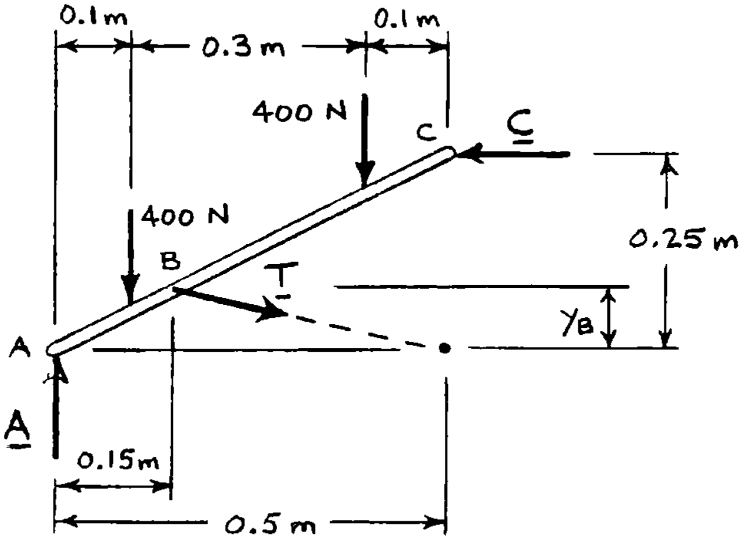


PROBLEM 4.F3

Bar *AC* supports two 400-N loads as shown. Rollers at *A* and *C* rest against frictionless surfaces and a cable *BD* is attached at *B*. Draw the free-body diagram needed to determine the tension in cable *BD* and the reactions at *A* and *C*.

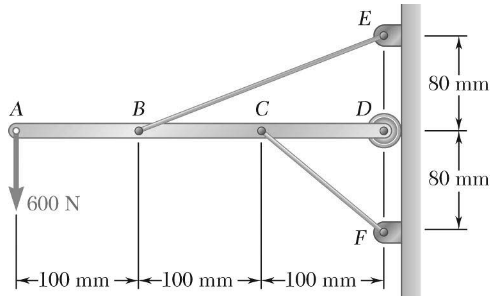
SOLUTION

Free-Body Diagram of Bar *AC*:



Note: By similar triangles

$$\frac{y_B}{0.25\text{ m}} = \frac{0.15\text{ m}}{0.5\text{ m}} \qquad y_B = 0.075\text{ m}$$

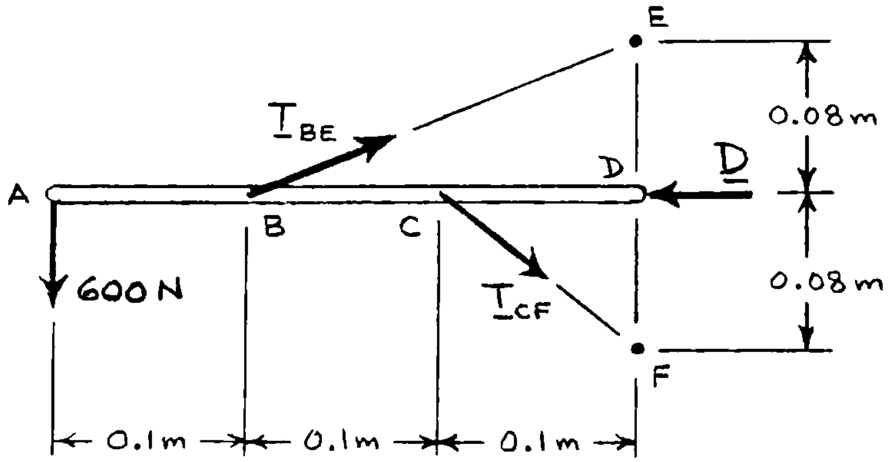


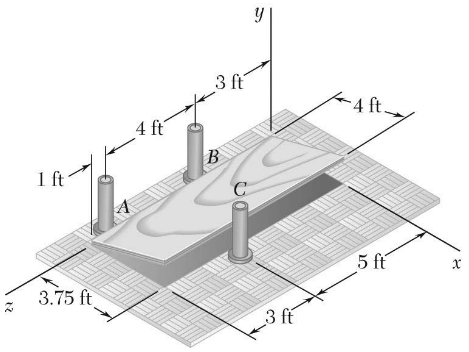
PROBLEM 4.F4

Draw the free-body diagram needed to determine the tension in each cable and the reaction at *D*.

SOLUTION

Free-Body Diagram of Member *ABCD*:



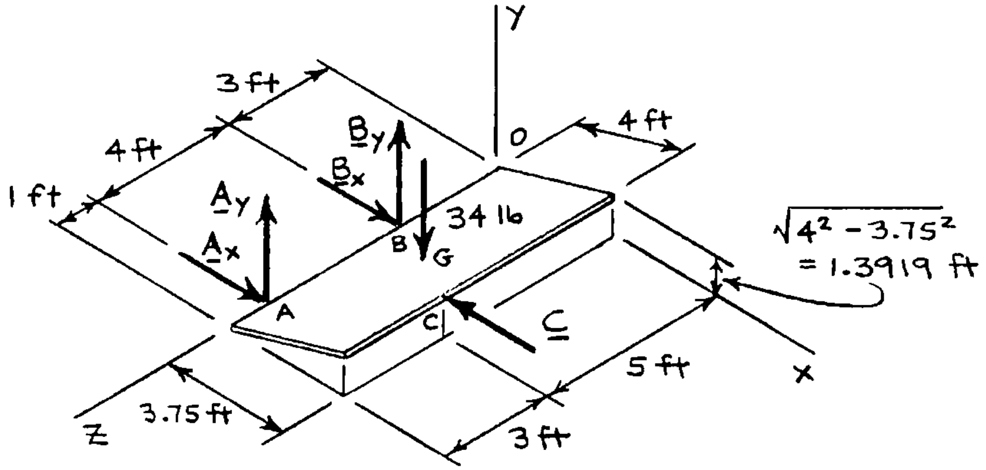


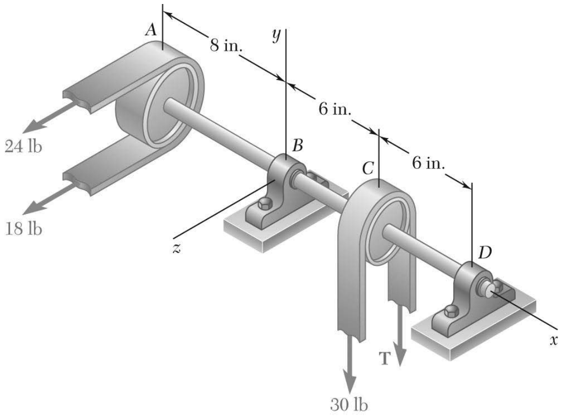
PROBLEM 4.F5

A 4 × 8-ft sheet of plywood weighing 34 lb has been temporarily placed among three pipe supports. The lower edge of the sheet rests on small collars at *A* and *B* and its upper edge leans against pipe *C*. Neglecting friction on all surfaces, draw the free-body diagram needed to determine the reactions at *A*, *B*, and *C*.

SOLUTION

Free-Body Diagram of Plywood sheet:



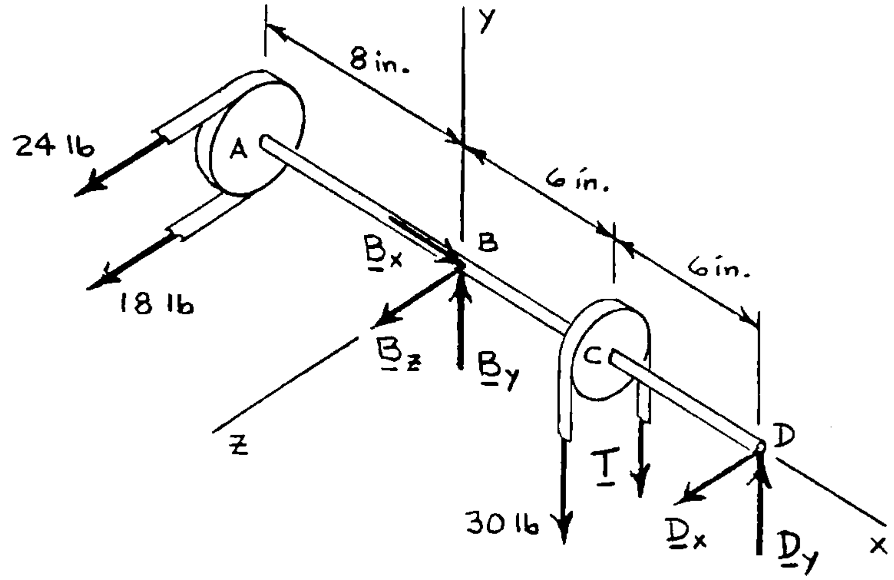


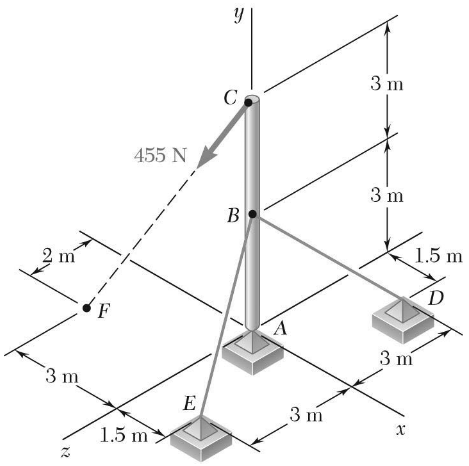
PROBLEM 4.F6

Two transmission belts pass over sheaves welded to an axle supported by bearings at B and D. The sheave at A has a radius of 2.5 in. and the sheave at C has a radius of 2 in. Knowing that the system rotates at a constant rate, draw the free-body diagram needed to determine the tension T and the reactions at B and D. Assume that the bearing at D does not exert any axial thrust and neglect the weights of the sheaves and axle.

SOLUTION

Free-Body Diagram of axle-sheave system:



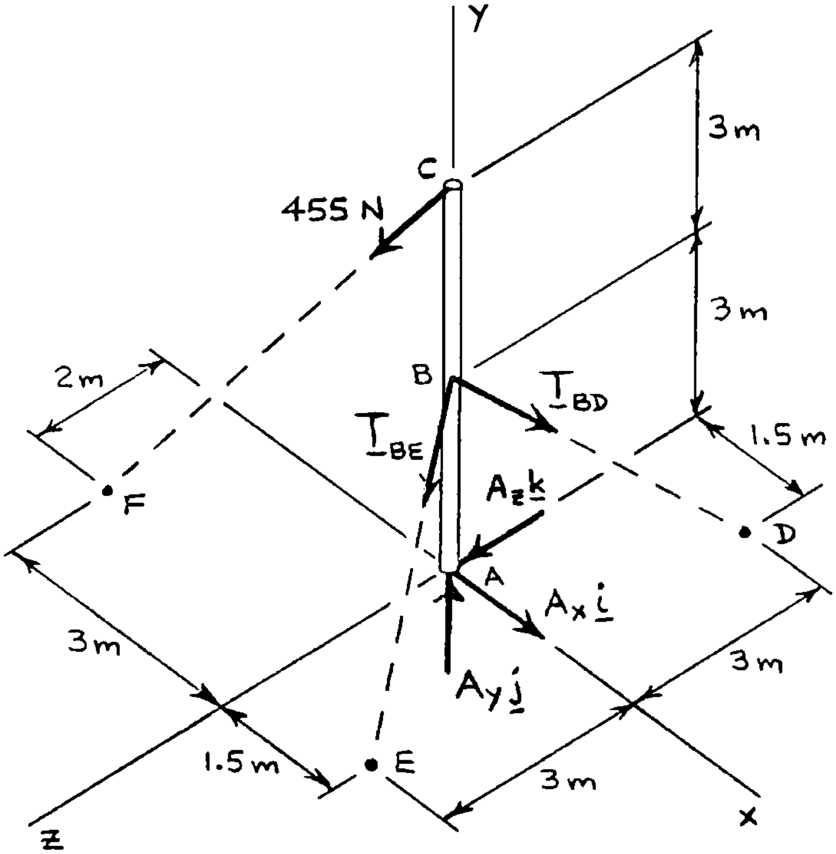


PROBLEM 4.F7

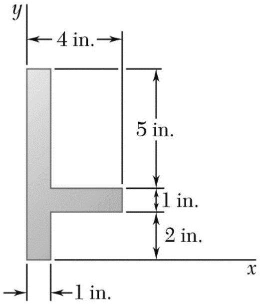
The 6-m pole *ABC* is acted upon by a 455-N force as shown. The pole is held by a ball-and-socket joint at *A* and by two cables *BD* and *BE*. Draw the free-body diagram needed to determine the tension in each cable and the reaction at *A*.

SOLUTION

Free-Body Diagram of Pole:



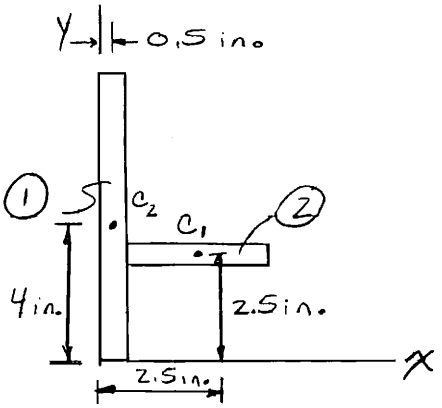
CHAPTER 5



PROBLEM 5.1

Locate the centroid of the plane area shown.

SOLUTION



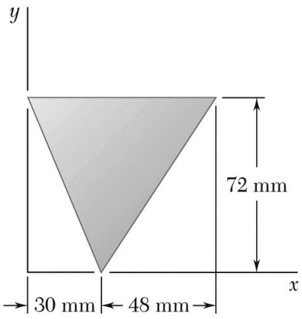
	A, in^2	$\bar{x}$, in	$\bar{y}$, in	$\bar{x}A, \text{in}^3$	$\bar{y}A, \text{in}^3$
1	8	0.5	4	4	32
2	3	2.5	2.5	7.5	7.5
Σ	11			11.5	39.5

$$\bar{X}\Sigma A = \bar{x}A$$

$$\bar{X}(11\,\text{in}^2) = 11.5\,\text{in}^3 \qquad \bar{X} = 1.045\text{ in.} \blacktriangleleft$$

$$\bar{Y}\Sigma A = \Sigma\bar{y}A$$

$$\bar{Y}(11) = 39.5 \qquad \bar{Y} = 3.59\text{ in.} \blacktriangleleft$$



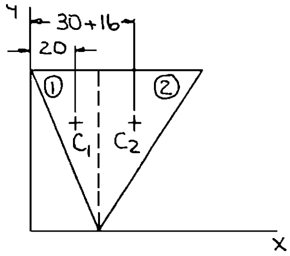
PROBLEM 5.2

Locate the centroid of the plane area shown.

SOLUTION

For the area as a whole, it can be concluded by observation that

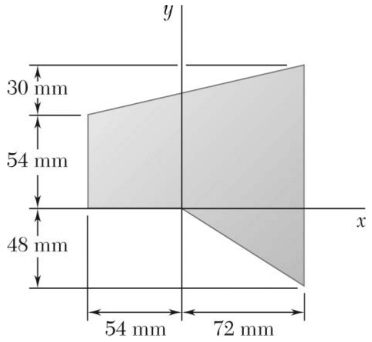
$$\overline{Y} = \frac{2}{3}(72\text{ mm}) \qquad \text{or } \overline{Y} = 48.0\text{ mm} ◀$$



Dimensions in mm

	A, mm^2	$\overline{x}$, mm	$\overline{x}A$, mm^3
1	$\frac{1}{2}\times 30\times 72 = 1080$	20	21,600
2	$\frac{1}{2}\times 48\times 72 = 1728$	46	79,488
Σ	2808		101,088

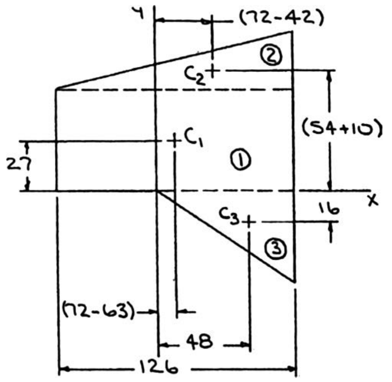
Then $\overline{X}A = \Sigma\overline{x}A \qquad \overline{X}(2808) = 101,088 \qquad$ or $\overline{X} = 36.0$ mm ◀



PROBLEM 5.3

Locate the centroid of the plane area shown.

SOLUTION



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$126 \times 54 = 6804$	9	27	61,236	183,708
2	$\frac{1}{2} \times 126 \times 30 = 1890$	30	64	56,700	120,960
3	$\frac{1}{2} \times 72 \times 48 = 1728$	48	−16	82,944	−27,648
Σ	10,422			200,880	277,020

Then

$$\bar{X}\Sigma A = \Sigma \bar{x} A$$

$$\bar{X}(10,422\ \text{m}^2) = 200,880\ \text{mm}^2$$

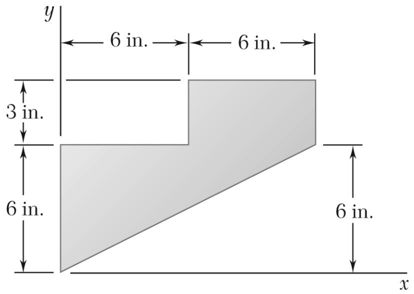
or $\bar{X} = 19.27$ mm ◀

and

$$\bar{Y}\Sigma A = \Sigma \bar{y} A$$

$$\bar{Y}(10,422\ \text{m}^2) = 270,020\ \text{mm}^3$$

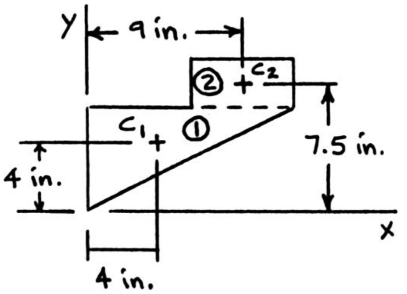
or $\bar{Y} = 26.6$ mm ◀



PROBLEM 5.4

Locate the centroid of the plane area shown.

SOLUTION



	A, in^2	$\bar{x}$, in	$\bar{y}$, in	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$\frac{1}{2}(12)(6)=36$	4	4	144	144
2	$(6)(3)=18$	9	7.5	162	135
Σ	54			306	279

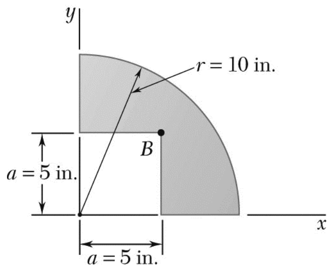
Then

$$\bar{X}A = \Sigma\bar{x}A$$

$$\bar{X}(54) = 306 \qquad \bar{X} = 5.67 \text{ in.} \blacktriangleleft$$

$$\bar{Y}A = \Sigma\bar{y}A$$

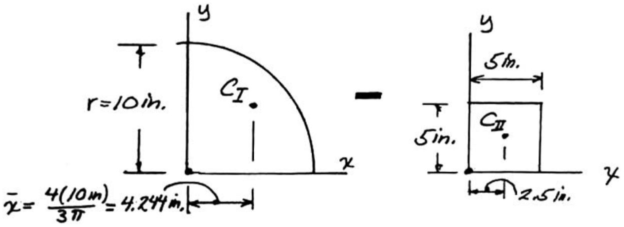
$$\bar{Y}(54) = 279 \qquad \bar{Y} = 5.17 \text{ in.} \blacktriangleleft$$



PROBLEM 5.5

Locate the centroid of the plane area shown.

SOLUTION



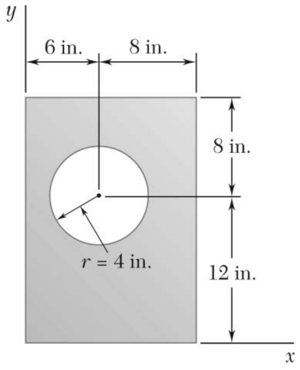
By symmetry, $\overline{X}=\overline{Y}$

	Component	A, in^2	$\overline{x}$, in.	$\overline{x}A$, in^3
I	Quarter circle	$\frac{\pi}{4}(10)^2=78.54$	4.2441	333.33
II	Square	$-(5)^2=-25$	2.5	-62.5
Σ		53.54		270.83

$$\overline{X}\Sigma A=\Sigma\overline{x}A: \quad \overline{X}(53.54\text{ in}^2)=270.83\text{ in}^3$$

$$\overline{X}=5.0585\text{ in.}$$

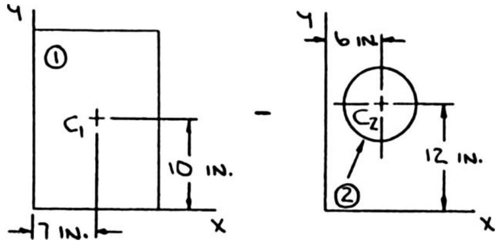
$\overline{X}=\overline{Y}=5.06$ in. ◀



PROBLEM 5.6

Locate the centroid of the plane area shown.

SOLUTION

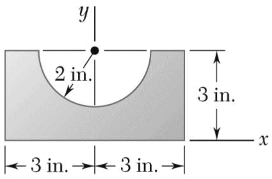


	A, in^2	$\overline{x}$, in.	$\overline{y}$, in.	$\overline{x}A$, in^3	$\overline{y}A$, in^3
1	$14 \times 20 = 280$	7	10	1960	2800
2	$-\pi(4)^2 = -16\pi$	6	12	–301.59	–603.19
Σ	229.73			1658.41	2196.8

Then

$$\overline{X} = \frac{\Sigma \overline{x}A}{\Sigma A} = \frac{1658.41}{229.73} \qquad \overline{X} = 7.22 \text{ in.} \blacktriangleleft$$

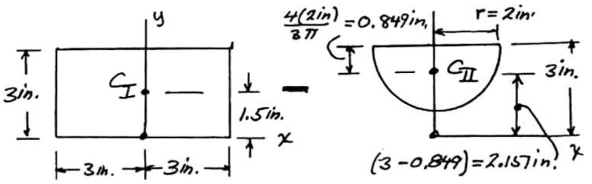
$$\overline{Y} = \frac{\Sigma \overline{y}A}{\Sigma A} = \frac{2196.8}{229.73} \qquad \overline{Y} = 9.56 \text{ in.} \blacktriangleleft$$



PROBLEM 5.7

Locate the centroid of the plane area shown.

SOLUTION



By symmetry, $\bar{X} = 0$

	Component	A, in^2	$\bar{y}$, in.	$\bar{y}A$, in^3
I	Rectangle	$(3)(6) = 18$	1.5	27.0
II	Semicircle	$-\dfrac{\pi}{2}(2)^2 = -6.28$	2.151	−13.51
Σ		11.72		13.49

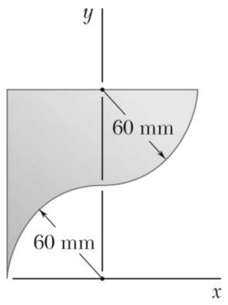
$$\bar{Y}\,\Sigma A = \Sigma\,\bar{y}A$$

$$\bar{Y}(11.72\text{ in.}^2) = 13.49\text{ in}^3$$

$$\bar{Y} = 1.151\text{ in.}$$

$\bar{X} = 0$

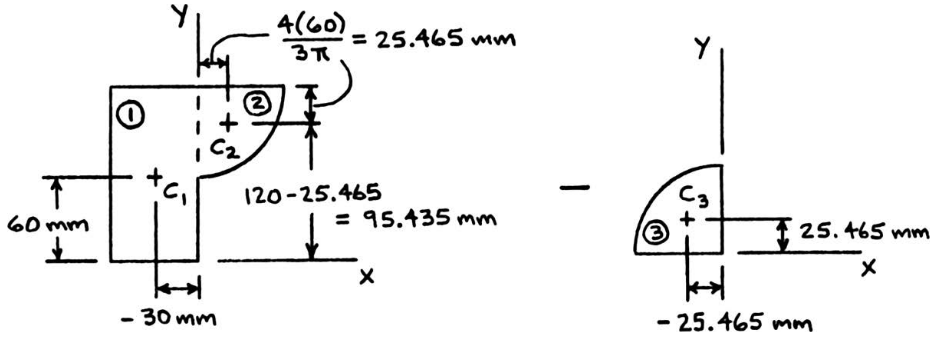
$\bar{Y} = 1.151$ in. ◀



PROBLEM 5.8

Locate the centroid of the plane area shown.

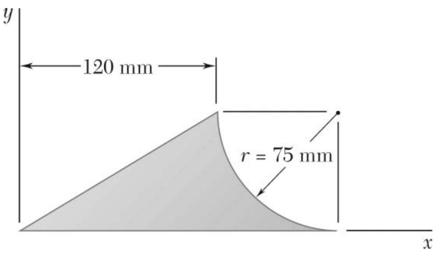
SOLUTION



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$(60)(120) = 7200$	–30	60	-216×10^3	432×10^3
2	$\frac{\pi}{4}(60)^2 = 2827.4$	25.465	95.435	72.000×10^3	269.83×10^3
3	$-\frac{\pi}{4}(60)^2 = -2827.4$	–25.465	25.465	72.000×10^3	-72.000×10^3
Σ	7200			-72.000×10^3	629.83×10^3

Then $\bar{X}A = \Sigma\bar{x}A \qquad \bar{X}(7200) = -72.000\times10^3 \qquad \bar{X} = -10.00\text{ mm}$ ◀

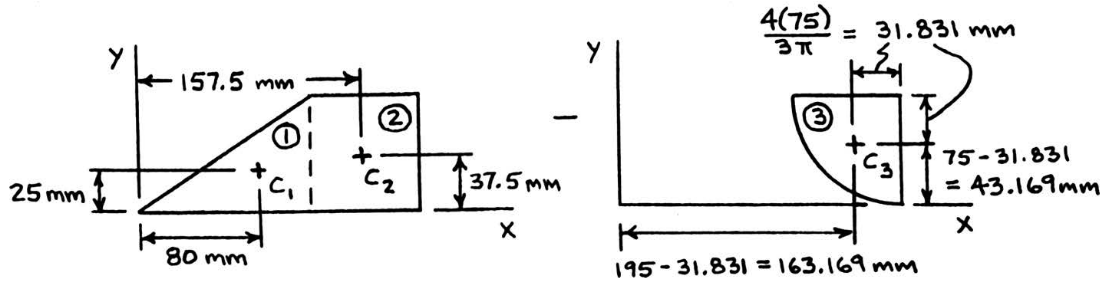
$\bar{Y}A = \Sigma\bar{y}A \qquad \bar{Y}(7200) = 629.83\times10^3 \qquad \bar{Y} = 87.5\text{ mm}$ ◀



PROBLEM 5.9

Locate the centroid of the plane area shown.

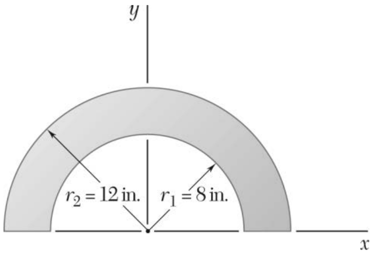
SOLUTION



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{1}{2}(120)(75) = 4500$	80	25	360×10^3	112.5×10^3
2	$(75)(75) = 5625$	157.5	37.5	885.94×10^3	210.94×10^3
3	$-\frac{\pi}{4}(75)^2 = -4417.9$	163.169	43.169	-720.86×10^3	-190.716×10^3
Σ	5707.1			525.08×10^3	132.724×10^3

Then $\bar{X}A = \Sigma \bar{x} A \qquad \bar{X}(5707.1) = 525.08 \times 10^3 \qquad \bar{X} = 92.0 \text{ mm}$ ◀

$\bar{Y}A = \Sigma \bar{y} A \qquad \bar{Y}(5707.1) = 132.724 \times 10^3 \qquad \bar{Y} = 23.3 \text{ mm}$ ◀

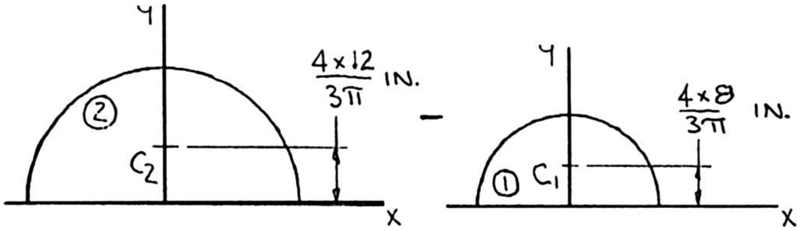


PROBLEM 5.10

Locate the centroid of the plane area shown.

SOLUTION

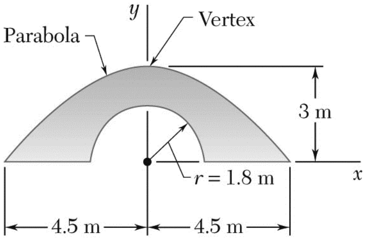
First note that symmetry implies $\bar{X} = 0$ ◀



	A, in^2	$\bar{y}$, in.	$\bar{y}A$, in^3
1	$-\dfrac{\pi(8)^2}{2} = -100.531$	3.3953	−341.33
2	$\dfrac{\pi(12)^2}{2} = 226.19$	5.0930	1151.99
Σ	125.659		810.66

Then $$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{810.66 \text{ in}^3}{125.66 \text{ in}^2}$$ or $\bar{Y} = 6.45$ in. ◀

PROPRIETARY MATERIAL.

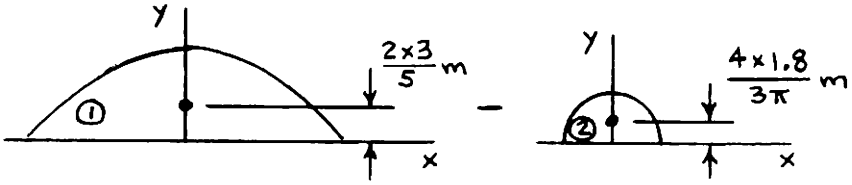


PROBLEM 5.11

Locate the centroid of the plane area shown.

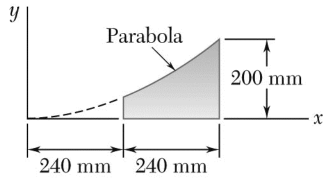
SOLUTION

First note that symmetry implies $\bar{X} = 0$ ◀



	A, m^2	$\bar{y}$, m	$\bar{y}A$, m^3
1	$\frac{4}{3}\times4.5\times3=18$	1.2	21.6
2	$-\frac{\pi}{2}(1.8)^2=-5.0894$	0.76394	−3.8880
Σ	12.9106		17.7120

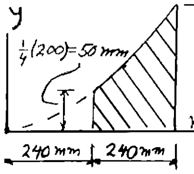
Then $\bar{Y} = \dfrac{\Sigma\bar{y}A}{\Sigma A} = \dfrac{17.7120\text{ m}^3}{12.9106\text{ m}^2}$ or $\bar{Y} = 1.372$ m ◀

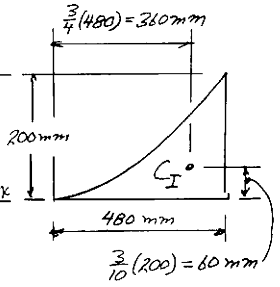


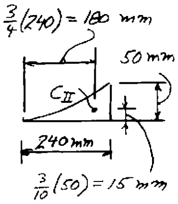
PROBLEM 5.12

Locate the centroid of the plane area shown.

SOLUTION







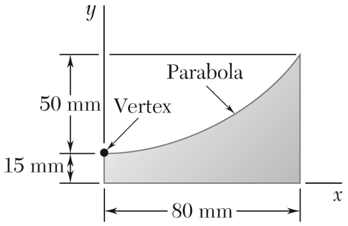
	Area mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{1}{3}(200)(480)=32\times10^3$	360	60	11.52×10^6	1.92×10^6
2	$-\frac{1}{3}(50)(240)=4\times10^3$	180	15	-0.72×10^6	-0.06×10^6
Σ	28×10^3			10.80×10^6	1.86×10^6

$\bar{X}\Sigma A=\Sigma\bar{x}A$: $\quad \bar{X}(28\times10^3\,\text{mm}^2)=10.80\times10^6\,\text{mm}^3$

$\bar{X}=385.7$ mm $\qquad \bar{X}=386$ mm ◀

$\bar{Y}\Sigma A=\Sigma\bar{y}A$: $\quad \bar{Y}(28\times10^3\,\text{mm}^2)=1.86\times10^6\,\text{mm}^3$

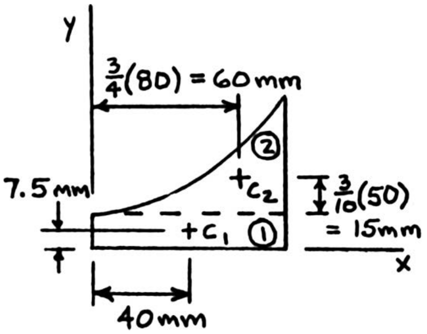
$\bar{Y}=66.43$ mm $\qquad \bar{Y}=66.4$ mm ◀



PROBLEM 5.13

Locate the centroid of the plane area shown.

SOLUTION



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$(15)(80) = 1200$	40	7.5	48×10^3	9×10^3
2	$\frac{1}{3}(50)(80) = 1333.33$	60	30	80×10^3	40×10^3
Σ	2533.3			128×10^3	49×10^3

Then
$$\bar{X}A = \Sigma \bar{x}A$$

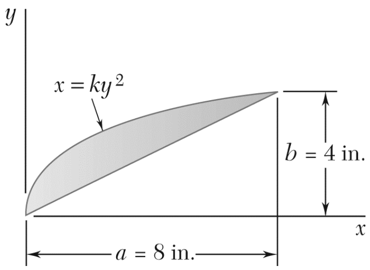
$$\bar{X}(2533.3) = 128 \times 10^3$$

$\bar{X} = 50.5$ mm ◀

$$\bar{Y}A = \Sigma \bar{y}A$$

$$\bar{Y}(2533.3) = 49 \times 10^3$$

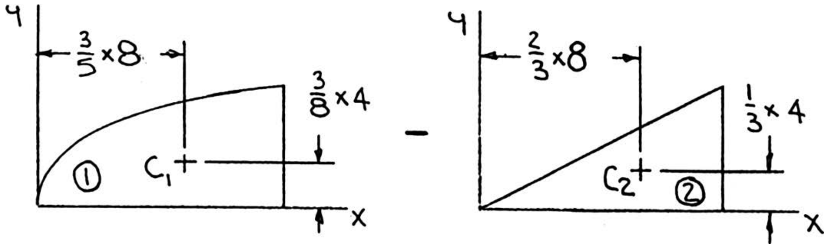
$\bar{Y} = 19.34$ mm ◀



PROBLEM 5.14

Locate the centroid of the plane area shown.

SOLUTION

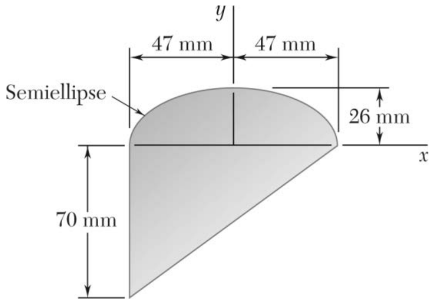


Dimensions in in.

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$\frac{2}{3}(4)(8) = 21.333$	4.8	1.5	102.398	32.000
2	$-\frac{1}{2}(4)(8) = -16.0000$	5.3333	1.33333	85.333	−21.333
Σ	5.3333			17.0650	10.6670

Then
$$\bar{X}\Sigma A = \Sigma \bar{x}A$$
$$\bar{X}(5.3333\text{ in}^2) = 17.0650\text{ in}^3 \qquad \text{or} \quad \bar{X} = 3.20\text{ in.} \blacktriangleleft$$

and
$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$
$$\bar{Y}(5.3333\text{ in}^2) = 10.6670\text{ in}^3 \qquad \text{or} \quad \bar{Y} = 2.00\text{ in.} \blacktriangleleft$$

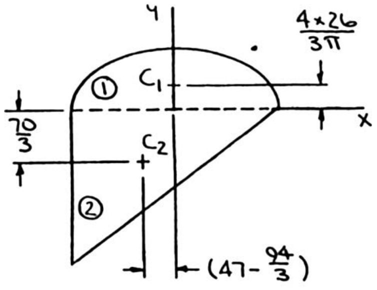


PROBLEM 5.15

Locate the centroid of the plane area shown.

SOLUTION

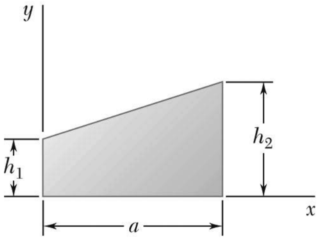
Dimensions in mm



	A, mm^2	$\overline{x}$, mm	$\overline{y}$, mm	$\overline{x}A$, mm^3	$\overline{y}A$, mm^3
1	$\frac{\pi}{2}\times47\times26=1919.51$	0	11.0347	0	21,181
2	$\frac{1}{2}\times94\times70=3290$	–15.6667	–23.333	–51,543	–76,766
Σ	5209.5			–51,543	–55,584

Then $\overline{X}=\frac{\Sigma\overline{x}A}{\Sigma A}=\frac{-51{,}543}{5209.5}$ $\overline{X}=-9.89$ mm ◀

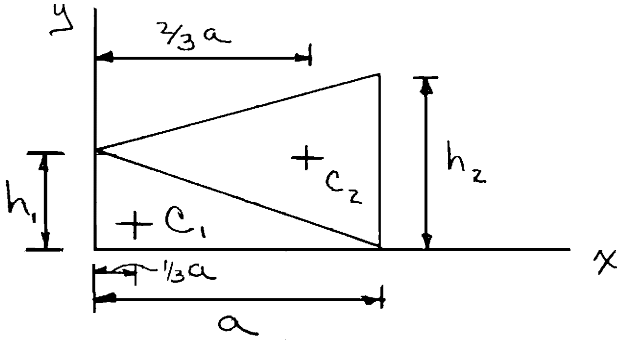
$\overline{Y}=\frac{\Sigma\overline{y}A}{\Sigma A}=\frac{-55{,}584}{5209.5}$ $\overline{Y}=-10.67$ mm ◀



PROBLEM 5.16

Determine the x coordinate of the centroid of the trapezoid shown in terms of h_1, h_2, and a.

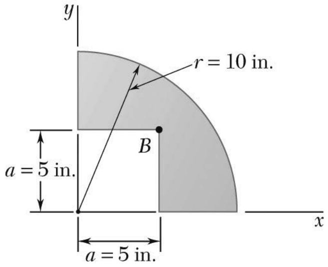
SOLUTION



	A	$\overline{x}$	$\overline{x}A$
1	$\frac{1}{2}h_1a$	$\frac{1}{3}a$	$\frac{1}{6}h_1a^2$
2	$\frac{1}{2}h_2a$	$\frac{2}{3}a$	$\frac{2}{6}h_2a^2$
Σ	$\frac{1}{2}a(h_1+h_2)$		$\frac{1}{6}a^2(h_1+2h_2)$

$$\overline{X} = \frac{\Sigma\overline{x}A}{\Sigma A} = \frac{\frac{1}{6}a^2(h_1+2h_2)}{\frac{1}{2}a(h_1+h_2)}$$

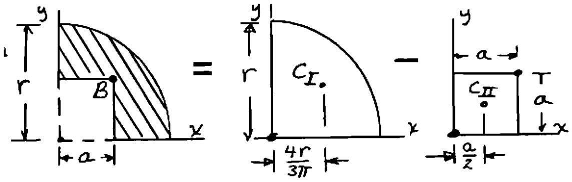
$$\overline{X} = \frac{1}{3}a\frac{h_1+2h_2}{h_1+h_2} \blacktriangleleft$$



PROBLEM 5.17

For the plane area of Problem 5.5, determine the ratio *a*/*r* so that the centroid of the area is located at point *B*.

SOLUTION



By symmetry, $\bar{X} = \bar{Y}$. For centroid to be at *B*, $\bar{X} = a$.

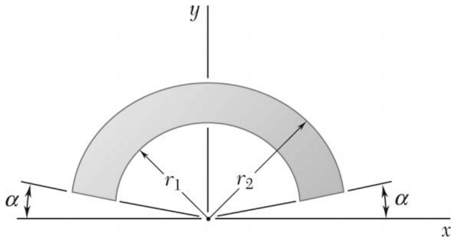
		Area	$\bar{x}$	$\bar{x}A$
I	Quarter circle	$\frac{1}{4}\pi r^2$	$\frac{4r}{3\pi}$	$\frac{1}{3}r^3$
II	Square	$-a^2$	$\frac{1}{2}a$	$-\frac{1}{2}a^3$
Σ		$\frac{\pi}{4}r^2 - a^2$		$\frac{1}{3}r^3 - \frac{1}{2}a^3$

$$\bar{X}\Sigma A = \Sigma \bar{x} A: \qquad \bar{X}\left(\frac{\pi}{4}r^2 - a^2\right) = \frac{1}{3}r^3 - \frac{1}{2}a^3$$

$$\text{Set } \bar{X} = a: \qquad a\left(\frac{\pi}{4}r^2 - a^2\right) = \frac{1}{3}r^3 - \frac{1}{2}a^3$$

$$\frac{1}{2}a^3 - \frac{\pi}{4}r^2 a + \frac{1}{3}r^3 = 0$$

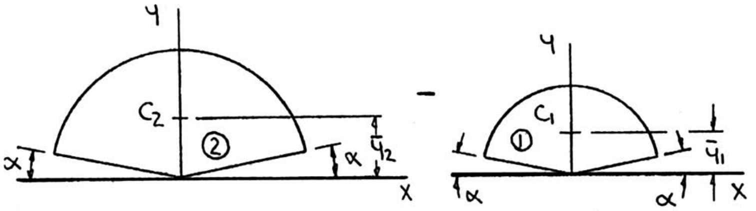
Divide by $\frac{1}{2}r^3$:
$$\left(\frac{a}{r}\right)^3 - \frac{\pi}{2}\frac{a}{r} + \frac{2}{3} = 0 \qquad \frac{a}{r} = 0.508 \blacktriangleleft$$



PROBLEM 5.18

Determine the y coordinate of the centroid of the shaded area in terms of r_1, r_2, and α.

SOLUTION



First, determine the location of the centroid.

From Figure 5.8A:

$$\bar{y}_2 = \frac{2}{3} r_2 \frac{\sin\left(\frac{\pi}{2}-\alpha\right)}{\left(\frac{\pi}{2}-\alpha\right)} \qquad A_2 = \left(\frac{\pi}{2}-\alpha\right) r_2^2$$

$$= \frac{2}{3} r_2 \frac{\cos\alpha}{\left(\frac{\pi}{2}-\alpha\right)}$$

Similarly,

$$\bar{y}_1 = \frac{2}{3} r_1 \frac{\cos\alpha}{\left(\frac{\pi}{2}-\alpha\right)} \qquad A_1 = \left(\frac{\pi}{2}-\alpha\right) r_1^2$$

Then

$$\Sigma\bar{y}A = \frac{2}{3} r_2 \frac{\cos\alpha}{\left(\frac{\pi}{2}-\alpha\right)}\left[\left(\frac{\pi}{2}-\alpha\right) r_2^2\right] - \frac{2}{3} r_1 \frac{\cos\alpha}{\left(\frac{\pi}{2}-\alpha\right)}\left[\left(\frac{\pi}{2}-\alpha\right) r_1^2\right]$$

$$= \frac{2}{3}\left(r_2^3 - r_1^3\right)\cos\alpha$$

and

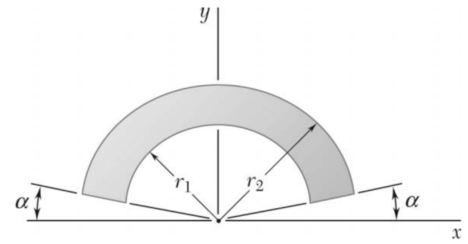
$$\Sigma A = \left(\frac{\pi}{2}-\alpha\right) r_2^2 - \left(\frac{\pi}{2}-\alpha\right) r_1^2$$

$$= \left(\frac{\pi}{2}-\alpha\right)\left(r_2^2 - r_1^2\right)$$

Now

$$\bar{Y}\Sigma A = \Sigma\,\bar{y}A$$

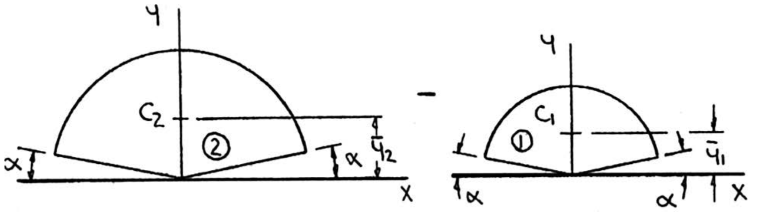
$$\bar{Y}\left[\left(\frac{\pi}{2}-\alpha\right)\left(r_2^2 - r_1^2\right)\right] = \frac{2}{3}\left(r_2^3 - r_1^3\right)\cos\alpha \qquad \bar{Y} = \frac{2}{3}\left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right)\left(\frac{2\cos\alpha}{\pi - 2\alpha}\right) \blacktriangleleft$$



PROBLEM 5.19

Show that as r_1 approaches r_2, the location of the centroid approaches that for an arc of circle of radius $(r_1 + r_2)/2$.

SOLUTION



First, determine the location of the centroid.

From Figure 5.8A:

$$\bar{y}_2 = \frac{2}{3}r_2\frac{\sin\left(\frac{\pi}{2}-\alpha\right)}{\left(\frac{\pi}{2}-\alpha\right)} \qquad A_2 = \left(\frac{\pi}{2}-\alpha\right)r_2^2$$

$$= \frac{2}{3}r_2\frac{\cos\alpha}{\left(\frac{\pi}{2}-\alpha\right)}$$

Similarly,

$$\bar{y}_1 = \frac{2}{3}r_1\frac{\cos\alpha}{\left(\frac{\pi}{2}-\alpha\right)} \qquad A_1 = \left(\frac{\pi}{2}-\alpha\right)r_1^2$$

Then

$$\Sigma\bar{y}A = \frac{2}{3}r_2\frac{\cos\alpha}{\left(\frac{\pi}{2}-\alpha\right)}\left[\left(\frac{\pi}{2}-\alpha\right)r_2^2\right] - \frac{2}{3}r_1\frac{\cos\alpha}{\left(\frac{\pi}{2}-\alpha\right)}\left[\left(\frac{\pi}{2}-\alpha\right)r_1^2\right]$$

$$= \frac{2}{3}\left(r_2^3 - r_1^3\right)\cos\alpha$$

and

$$\Sigma A = \left(\frac{\pi}{2}-\alpha\right)r_2^2 - \left(\frac{\pi}{2}-\alpha\right)r_1^2$$

$$= \left(\frac{\pi}{2}-\alpha\right)\left(r_2^2 - r_1^2\right)$$

Now

$$\bar{Y}\Sigma A = \Sigma\,\bar{y}A$$

$$\bar{Y}\left[\left(\frac{\pi}{2}-\alpha\right)\left(r_2^2 - r_1^2\right)\right] = \frac{2}{3}\left(r_2^3 - r_1^3\right)\cos\alpha$$

$$\bar{Y} = \frac{2}{3}\left(\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right)\left(\frac{2\cos\alpha}{\pi - 2\alpha}\right)$$

PROBLEM 5.19 (Continued)

Using Figure 5.8B, $\bar{Y}$ of an arc of radius $\frac{1}{2}(r_1 + r_2)$ is

$$\bar{Y} = \frac{1}{2}(r_1 + r_2)\frac{\sin(\frac{\pi}{2} - \alpha)}{(\frac{\pi}{2} - \alpha)}$$

$$= \frac{1}{2}(r_1 + r_2)\frac{\cos\alpha}{(\frac{\pi}{2} - \alpha)} \qquad (1)$$

Now

$$\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} = \frac{(r_2 - r_1)\left(r_2^2 + r_1 r_2 + r_1^2\right)}{(r_2 - r_1)(r_2 + r_1)}$$

$$= \frac{r_2^2 + r_1 r_2 + r_1^2}{r_2 + r_1}$$

Let

$$r_2 = r + \Delta$$

$$r_1 = r - \Delta$$

Then

$$r = \frac{1}{2}(r_1 + r_2)$$

and

$$\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} = \frac{(r + \Delta)^2 + (r + \Delta)(r - \Delta)(r - \Delta)^2}{(r + \Delta) + (r - \Delta)}$$

$$= \frac{3r^2 + \Delta^2}{2r}$$

In the limit as $\Delta \longrightarrow 0$ (i.e., $r_1 = r_2$), then

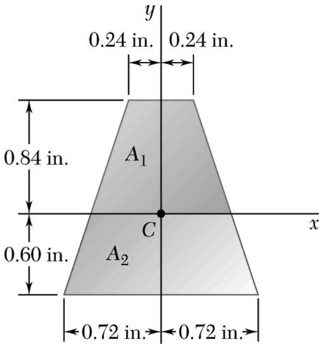
$$\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} = \frac{3}{2}r$$

$$= \frac{3}{2} \times \frac{1}{2}(r_1 + r_2)$$

So that

$$\bar{Y} = \frac{2}{3} \times \frac{3}{4}(r_1 + r_2)\frac{\cos\alpha}{\frac{\pi}{2} - \alpha} \qquad \text{or} \quad \bar{Y} = (r_1 + r_2)\frac{\cos\alpha}{\pi - 2\alpha} \blacktriangleleft$$

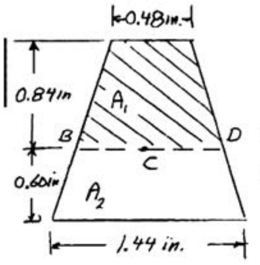
which agrees with Equation (1).

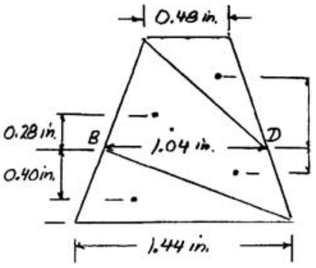


PROBLEM 5.20

The horizontal *x*-axis is drawn through the centroid *C* of the area shown, and it divides the area into two component areas A_1 and A_2. Determine the first moment of each component area with respect to the *x*-axis, and explain the results obtained.

SOLUTION





Length of *BD*:

$$BD = 0.48\text{ in.} + (1.44\text{ in.} - 0.48\text{ in.})\frac{0.84\text{in.}}{0.84\text{ in.}\times 0.60\text{ in.}} = 0.48 + 0.56 = 1.04\text{ in.}$$

Area above *x*-axis (consider two triangular areas):

$$Q_1 = \Sigma\,\overline{y}_A = (0.28\text{ in.})\left[\frac{1}{2}(0.84\text{ in.})(1.04\text{ in.})\right] + (0.56\text{ in.})\left[\frac{1}{2}(0.84\text{ in.})(0.48\text{ in.})\right]$$

$$= 0.122304\text{ in}^3 + 0.112896\text{ in}^3$$

$$Q_1 = 0.2352\text{ in}^3 \blacktriangleleft$$

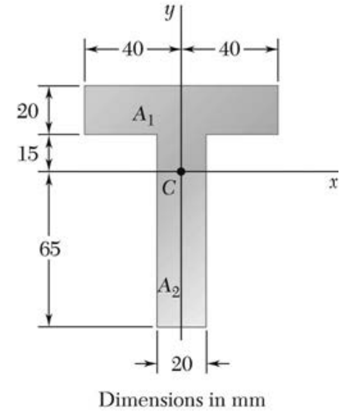
Area below *x*-axis:

$$Q_2 = \Sigma\,\overline{y}A = -(0.40\text{ in.})\left[\frac{1}{2}(0.60\text{ in.})(1.44\text{ in.})\right] - (0.20\text{ in.})\left[\frac{1}{2}(0.60\text{ in.})\right]$$

$$= -0.1728\text{ in}^3 - 0.0624\text{ in}^3$$

$$Q_2 = -0.2352\text{ in}^3 \blacktriangleleft$$

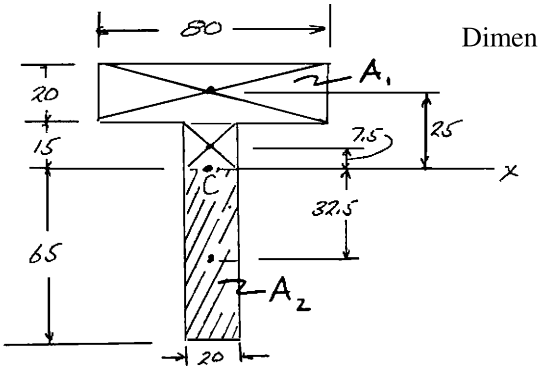
$|Q| = |Q_2|$, since *C* is centroid and thus, $Q = \Sigma\,\overline{y}A = 0$



PROBLEM 5.21

The horizontal x-axis is drawn through the centroid C of the area shown, and it divides the area into two component areas A_1 and A_2. Determine the first moment of each component area with respect to the x-axis, and explain the results obtained.

SOLUTION



Dimensions in mm

Area above x-axis (Area A_1):

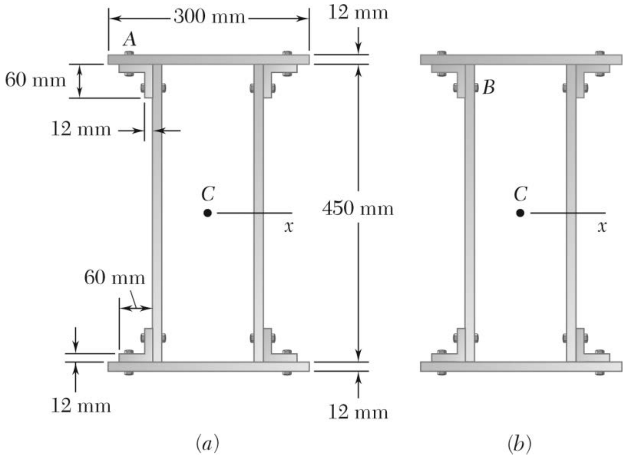
$$Q_1 = \Sigma \bar{y} A = (25)(20 \times 80) + (7.5)(15 \times 20)$$
$$= 40 \times 10^3 + 2.25 \times 10^3$$

$$Q_1 = 42.3 \times 10^3 \text{ mm}^3 \blacktriangleleft$$

Area below x-axis (Area A_2):

$$Q_2 = \Sigma \bar{y} A = (-32.5)(65 \times 20) \qquad Q_2 = -42.3 \times 10^3 \text{ mm}^3 \blacktriangleleft$$

$|Q_1| = |Q_2|$, since C is centroid and thus, $Q = \Sigma \bar{y} A = 0$

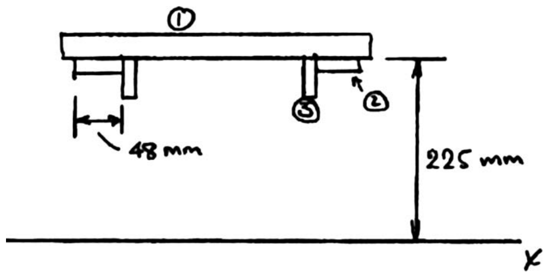


(a) (b)

PROBLEM 5.22

A composite beam is constructed by bolting four plates to four 60 × 60 × 12-mm angles as shown. The bolts are equally spaced along the beam, and the beam supports a vertical load. As proved in mechanics of materials, the shearing forces exerted on the bolts at *A* and *B* are proportional to the first moments with respect to the centroidal *x*-axis of the red-shaded areas shown, respectively, in parts *a* and *b* of the figure. Knowing that the force exerted on the bolt at *A* is 280 N, determine the force exerted on the bolt at *B*.

SOLUTION



From the problem statement, *F* is proportional to Q_x.

Therefore, $$\frac{F_A}{(Q_x)_A} = \frac{F_B}{(Q_x)_B}, \quad \text{or} \quad F_B = \frac{(Q_x)_B}{(Q_x)_A} F_A$$

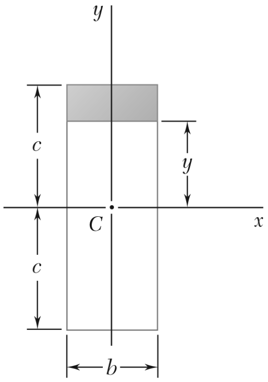
For the first moments, $$(Q_x)_A = \left(225 + \frac{12}{2}\right)(300 \times 12)$$

$$= 831{,}600 \text{ mm}^3$$

$$(Q_x)_B = (Q_x)A + 2\left(225 - \frac{12}{2}\right)(48 \times 12) + 2(225 - 30)(12 \times 60)$$

$$= 1{,}364{,}688 \text{ mm}^3$$

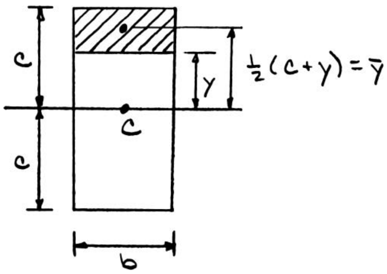
Then $$F_B = \frac{1{,}364{,}688}{831{,}600}(280 \text{ N})$$ or $F_B = 459$ N ◀



PROBLEM 5.23

The first moment of the shaded area with respect to the x-axis is denoted by Q_x. (a) Express Q_x in terms of b, c, and the distance y from the base of the shaded area to the x-axis. (b) For what value of y is Q_x maximum, and what is that maximum value?

SOLUTION



Shaded area:

$$A = b(c-y)$$

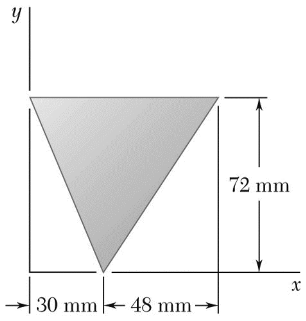
$$Q_x = \bar{y}A$$

$$= \frac{1}{2}(c+y)[b(c-y)]$$

(a) $$Q_x = \frac{1}{2}b(c^2 - y^2)$$ ◀

(b) For $Q_{\max}$, $$\frac{dQ}{dy} = 0 \quad \text{or} \quad \frac{1}{2}b(-2y) = 0$$ $y = 0$ ◀

For $y = 0$, $$(Q_x) = \frac{1}{2}bc^2$$ $(Q_x) = \frac{1}{2}bc^2$ ◀

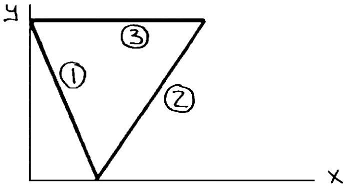


PROBLEM 5.24

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION

First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.



	L, mm	$\overline{x}$, mm	$\overline{y}$, mm	$\overline{y}L$, mm^2	$\overline{y}L$, mm^2
1	$\sqrt{30^2+72^2}=78$	15	36	1170.0	2808.0
2	$\sqrt{48^2+72^2}=86.533$	54	36	4672.8	3115.2
3	78	39	72	3042.0	5616.0
Σ	242.53			8884.8	11,539.2

Then

$$\overline{X}\,\Sigma L=\Sigma\overline{x}\,L$$

$$\overline{X}(242.53)=8884.8$$

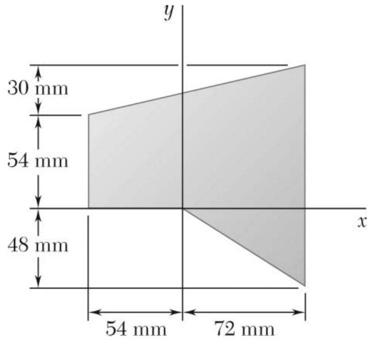
or $\overline{X}=36.6$ mm ◀

and

$$\overline{Y}\,\Sigma L=\Sigma\overline{y}\,L$$

$$\overline{Y}(242.53)=11{,}539.2$$

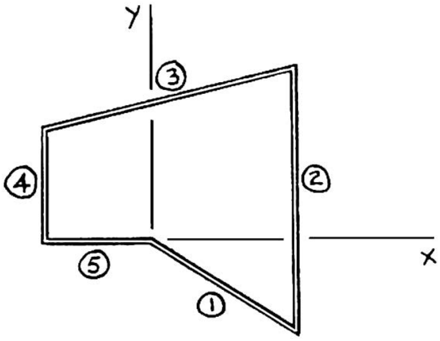
or $\overline{Y}=47.6$ mm ◀



PROBLEM 5.25

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION



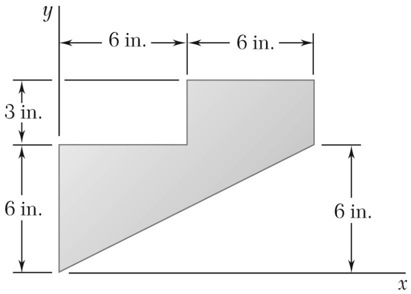
	L, mm	$\overline{x}$, mm	$\overline{y}$, mm	$\overline{x}L$, mm^2	$\overline{y}L$, mm^2
1	$\sqrt{72^2+48^2}=86.533$	36	–24	3115.2	–2076.8
2	132	72	18	9504.0	2376.0
3	$\sqrt{126^2+30^2}=129.522$	9	69	1165.70	8937.0
4	54	–54	27	–2916.0	1458.0
5	54	–27	0	–1458.0	0
Σ	456.06			9410.9	10,694.2

Then $$\overline{X}\,\Sigma L=\Sigma\,\overline{x}\,L$$

$$\overline{X}(456.06)=9410.9$$ or $\overline{X}=20.6$ mm ◀

$$\overline{Y}\,\Sigma L=\Sigma\,\overline{y}\,L$$

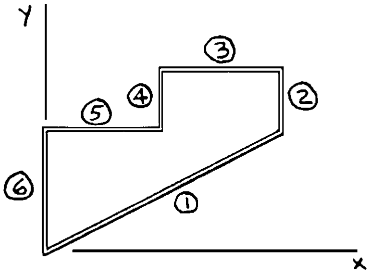
$$\overline{Y}(456.06)=10{,}694.2$$ or $\overline{Y}=23.4$ mm ◀



PROBLEM 5.26

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION

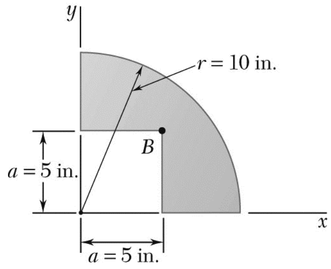


	L, in.	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}L$, in^2	$\bar{y}L$, in^2
1	$\sqrt{12^2+6^2}=13.4164$	6	3	80.498	40.249
2	3	12	7.5	36	22.5
3	6	9	9	54	54.0
4	3	6	7.5	18	22.5
5	6	3	6	18	36.0
6	6	0	3	0	18.0
Σ	37.416			206.50	193.249

Then $\bar{X}\Sigma L=\Sigma\bar{x}L \qquad \bar{X}(37.416)=206.50 \qquad \bar{X}=5.52\text{ in.}$ ◀

$\bar{Y}\Sigma L=\Sigma\bar{y}L \qquad \bar{Y}(37.416)=193.249 \qquad \bar{Y}=5.16\text{ in.}$ ◀

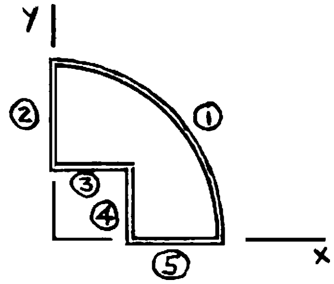
PROBLEM 5.27



A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION

By symmetry, $\overline{X} = \overline{Y}$.



	L, in.	$\overline{x}$, in.	$\overline{y}L$, in^2
1	$\frac{1}{2}\pi(10) = 15.7080$	$\frac{2(10)}{\pi} = 6.3662$	100
2	5	0	0
3	5	2.5	12.5
4	5	5	25
5	5	7.5	37.5
Σ	35.708		175

Then $\quad \overline{X}\,\Sigma L = \Sigma \overline{x}\,L \qquad \overline{X}(35.708) = 175 \qquad \overline{X} = \overline{Y} = 4.90$ in. ◀

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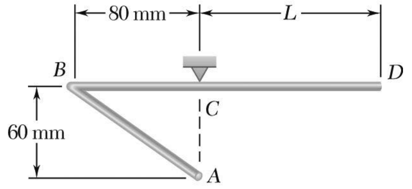


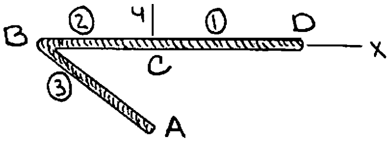
PROBLEM 5.28

The homogeneous wire *ABCD* is bent as shown and is attached to a hinge at *C*. Determine the length *L* for which portion *BCD* of the wire is horizontal.

SOLUTION

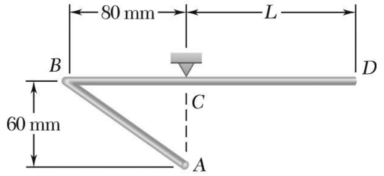
First note that for equilibrium, the center of gravity of the wire must lie on a vertical line through *C*. Further, because the wire is homogeneous, the center of gravity of the wire will coincide with the centroid of the corresponding line. Thus,

$$\overline{X} = 0 \text{ so that } \Sigma \overline{X} L = 0$$



Then
$$\frac{L}{2} + (-40 \text{ mm})(80 \text{ mm}) + (-40 \text{ mm})(100 \text{ mm}) = 0$$

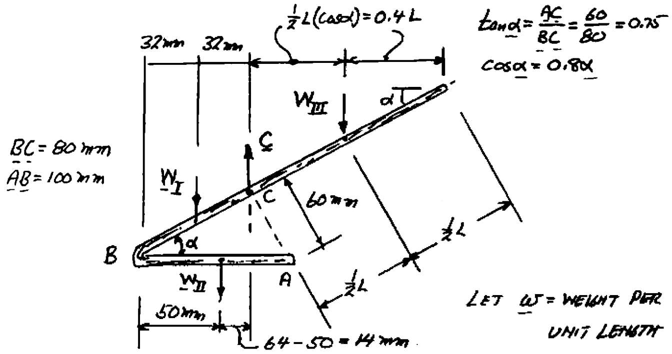
$$L^2 = 14,400 \text{ mm}^2 \qquad L = 120.0 \text{ mm} \blacktriangleleft$$



PROBLEM 5.29

The homogeneous wire *ABCD* is bent as shown and is attached to a hinge at *C*. Determine the length *L* for which portion *AB* of the wire is horizontal.

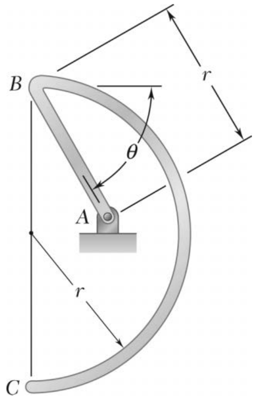
SOLUTION



$$W_{\mathrm{I}} = 80w \qquad W_{\mathrm{II}} = 100w \qquad W_{\mathrm{III}} = Lw$$

$$+\circlearrowleft \Sigma M_C = 0: \quad (80w)(32) + (100w)(14) - (Lw)(0.4L) = 0$$

$$L^2 = 9900 \qquad\qquad L = 99.5 \text{ mm} \blacktriangleleft$$

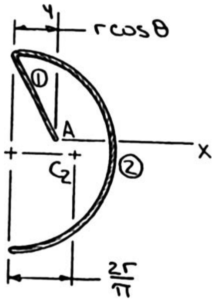


PROBLEM 5.30

The homogeneous wire *ABC* is bent into a semicircular arc and a straight section as shown and is attached to a hinge at *A*. Determine the value of θ for which the wire is in equilibrium for the indicated position.

SOLUTION

First note that for equilibrium, the center of gravity of the wire must lie on a vertical line through *A*. Further, because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line. Thus,



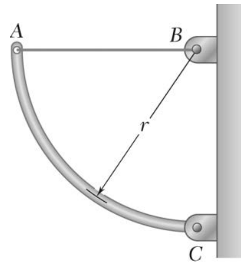
$$\bar{X} = 0$$

so that $$\Sigma \bar{x} L = 0$$

Then $$\left(-\frac{1}{2} r\cos\theta\right)(r) + \left(\frac{2r}{\pi} - r\cos\theta\right)(\pi r) = 0$$

or $$\cos\theta = \frac{4}{1+2\pi}$$
$$= 0.54921$$

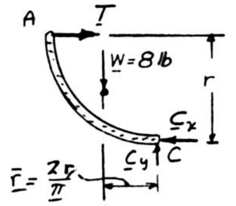
or $\theta = 56.7°$ ◀



PROBLEM 5.31

A uniform circular rod of weight 8 lb and radius 10 in. is attached to a pin at *C* and to the cable *AB*. Determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION



For quarter circle, $\bar{r} = \dfrac{2r}{\pi}$

(*a*) $+\circlearrowleft \Sigma M_C = 0\text{:} \quad W\left(\dfrac{2r}{\pi}\right) - Tr = 0$

$$T = W\left(\frac{2}{\pi}\right) = (8 \text{ lb})\left(\frac{2}{\pi}\right)$$

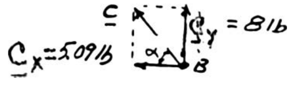
$T = 5.09$ lb ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0\text{:} \quad T - C_x = 0 \quad 5.09 \text{ lb} - C_x = 0$

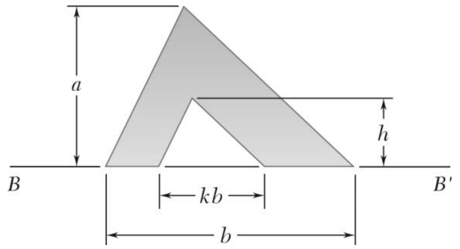
$\mathbf{C}_x = 5.09 \text{ lb} \longleftarrow$

$+\uparrow \Sigma F_y = 0\text{:} \quad C_y - W = 0 \qquad C_y - 8 \text{ lb} = 0$

$\mathbf{C}_y = 8 \text{ lb} \uparrow$



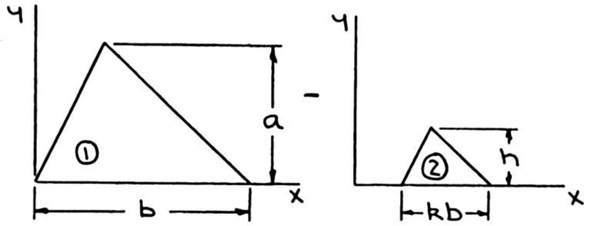
$\mathbf{C} = 9.48$ lb ⦪ 57.5° ◀



PROBLEM 5.32

Determine the distance h for which the centroid of the shaded area is as far above line BB' as possible when (*a*) $k = 0.10$, (*b*) $k = 0.80$.

SOLUTION



	A	$\bar{y}$	$\bar{y}A$
1	$\frac{1}{2}ba$	$\frac{1}{3}a$	$\frac{1}{6}a^2b$
2	$-\frac{1}{2}(kb)h$	$\frac{1}{3}h$	$-\frac{1}{6}kbh^2$
Σ	$\frac{b}{2}(a-kh)$		$\frac{b}{6}(a^2-kh^2)$

Then
$$\bar{Y}\Sigma A = \Sigma\bar{y}A$$
$$\bar{Y}\left[\frac{b}{2}(a-kh)\right] = \frac{b}{6}(a^2-kh^2)$$

or
$$\bar{Y} = \frac{a^2-kh^2}{3(a-kh)} \qquad (1)$$

and
$$\frac{d\bar{Y}}{dh} = \frac{1}{3}\frac{-2kh(a-kh)-(a^2-kh^2)(-k)}{(a-kh)^2} = 0$$

or
$$2h(a-kh) - a^2 + kh^2 = 0 \qquad (2)$$

Simplifying Eq. (2) yields

$$kh^2 - 2ah + a^2 = 0$$

PROBLEM 5.32 (Continued)

Then
$$h = \frac{2a \pm \sqrt{(-2a)^2 - 4(k)(a^2)}}{2k}$$
$$= \frac{a}{k}\left[1 \pm \sqrt{1-k}\right]$$

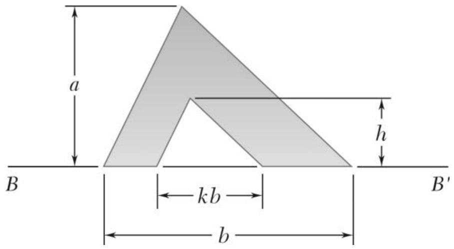
Note that only the negative root is acceptable since $h < a$. Then

(*a*) $k = 0.10$

$$h = \frac{a}{0.10}\left[1 - \sqrt{1-0.10}\right] \qquad \text{or } h = 0.513a \blacktriangleleft$$

(*b*) $k = 0.80$

$$h = \frac{a}{0.80}\left[1 - \sqrt{1-0.80}\right] \qquad \text{or } h = 0.691a \blacktriangleleft$$



PROBLEM 5.33

Knowing that the distance h has been selected to maximize the distance $\overline{y}$ from line BB' to the centroid of the shaded area, show that $\overline{y} = 2h/3$.

SOLUTION

See solution to Problem 5.32 for analysis leading to the following equations:

$$\overline{Y} = \frac{a^2 - kh^2}{3(a - kh)} \tag{1}$$

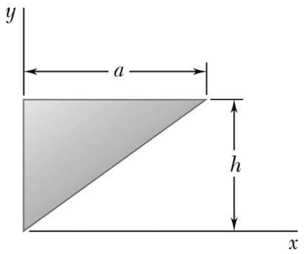
$$2h(a - kh) - a^2 + kh^2 = 0 \tag{2}$$

Rearranging Eq. (2) (which defines the value of h which maximizes $\overline{Y}$) yields

$$a^2 - kh^2 = 2h(a - kh)$$

Then substituting into Eq. (1) (which defines $\overline{Y}$),

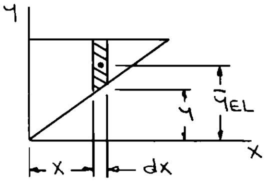
$$\overline{Y} = \frac{1}{3(a - kh)} \times 2h(a - kh) \qquad \text{or } \overline{Y} = \frac{2}{3}h \blacktriangleleft$$



PROBLEM 5.34

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *h*.

SOLUTION



We have $$y = \frac{h}{a}x$$

and $$dA = (h - y)dx$$

$$= h\left(1 - \frac{x}{a}\right)dx$$

$$\bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}(h + y)$$

$$= \frac{h}{2}\left(1 + \frac{x}{a}\right)$$

Then $$A = \int dA = \int_0^a h\left(1 - \frac{x}{a}\right)dx = h\left[x - \frac{x^2}{2a}\right]_0^a = \frac{1}{2}ah$$

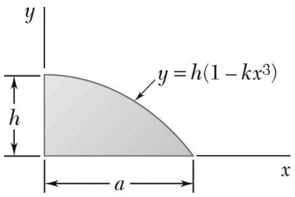
and $$\int \bar{x}_{EL} dA = \int_0^a x\left[h\left(1 - \frac{x}{a}\right)dx\right] = h\left[\frac{x^2}{2} - \frac{x^3}{3a}\right]_0^a = \frac{1}{6}a^2h$$

$$\int \bar{y}_{EL} dA = \int_0^a \frac{h}{2}\left(1 + \frac{x}{a}\right)\left[h\left(1 - \frac{x}{a}\right)dx\right]$$

$$= \frac{h^2}{2}\int_0^a \left(1 - \frac{x^2}{a^2}\right)dx = \frac{h^2}{2}\left[x - \frac{x^3}{3a^2}\right]_0^a = \frac{1}{3}ah^2$$

$$\bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x}\left(\frac{1}{2}ah\right) = \frac{1}{6}a^2h \qquad \bar{x} = \frac{2}{3}a \blacktriangleleft$$

$$\bar{y}A = \int y_{EL} dA: \quad \bar{y}\left(\frac{1}{2}ah\right) = \frac{1}{3}ah^2 \qquad \bar{y} = \frac{2}{3}h \blacktriangleleft$$



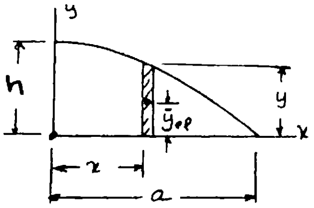
PROBLEM 5.35

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *h*.

SOLUTION

$$y = h(1 - kx^3)$$

For $x = a,\ y = 0.$



$$0 = h(1 - k\,a^3)$$

$$\therefore\quad k = \frac{1}{a^3}$$

$$y = h\left(1 - \frac{x^3}{a^3}\right)$$

$$\bar{x}_{EL} = x,\quad \bar{y}_{EL} = \frac{1}{2}y \quad dA = ydx$$

$$A = \int dA = \int_0^a ydx = \int_0^a h\left(1 - \frac{x^3}{a^3}\right)dx = h\left[x - \frac{x^4}{4a^3}\right]_0^a = \frac{3}{4}ah$$

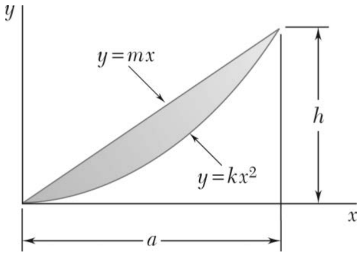
$$\int \bar{x}_{EL}dA = \int_0^a xydx = \int_0^a h\left(x - \frac{x^4}{a^3}\right)dx = h\left[\frac{x^2}{2} - \frac{x^5}{5a^3}\right]_0^a = \frac{3}{10}a^2h$$

$$\int \bar{y}_{EL}dA = \int_0^a \left(\frac{1}{2}y\right)ydx = \frac{1}{2}\int_0^a h^2\left(1 - \frac{x^3}{a^3}\right)dx = \frac{h^2}{2}\int_0^a \left(1 - \frac{2x^3}{a^3} + \frac{x^6}{a^6}\right)dx$$

$$= \frac{h^2}{2}\left[x - \frac{x^4}{2a^3} + \frac{x^7}{7a^6}\right]_0^a = \frac{9}{28}ah^2$$

$$\bar{x}A = \int \bar{x}_{EL}dA:\quad \bar{x}\left(\frac{3}{4}ah\right) = \frac{3}{10}a^2h \qquad \bar{x} = \frac{2}{5}a \ ◀$$

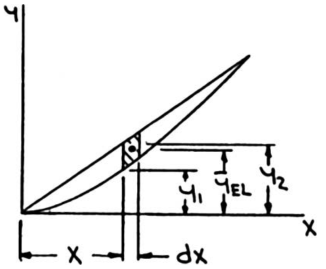
$$\bar{y}A = \int \bar{y}_{EL}dA:\quad \bar{y}\left(\frac{3}{4}ah\right) = \frac{9}{28}ah^2 \qquad \bar{y} = \frac{3}{7}h \ ◀$$



PROBLEM 5.36

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *h*.

SOLUTION



At (a, h), $y_1: \quad h = ka^2$

or $k = \dfrac{h}{a^2}$

$y_2: \quad h = ma$

or $m = \dfrac{h}{a}$

Now $\bar{x}_{EL} = x$

$$\bar{y}_{EL} = \frac{1}{2}(y_1 + y_2)$$

and $$dA = (y_2 - y_1)dx = \left[\frac{h}{a}x - \frac{h}{a^2}x^2\right]dx$$

$$= \frac{h}{a^2}(ax - x^2)dx$$

Then $$A = \int dA = \int_0^a \frac{h}{a^2}(ax - x^2)dx = \frac{h}{a^2}\left[\frac{a}{2}x^2 - \frac{1}{3}x^3\right]_0^a = \frac{1}{6}ah$$

and $$\int \bar{x}_{EL}dA = \int_0^a x\left[\frac{h}{a^2}(ax - x^2)dx = \frac{h}{a^2}\left[\frac{a}{3}x^3 - \frac{1}{4}x^4\right]_0^a = \frac{1}{12}a^2h\right.$$

$$\int \bar{y}_{EL}dA = \int \frac{1}{2}(y_1 + y_2)[(y_2 - y_1)dx] = \int \frac{1}{2}\left(y_2^2 - y_1^2\right)dx$$

$$= \frac{1}{2}\int_0^a \left(\frac{h^2}{a^2}x^2 - \frac{h^2}{a^4}x^4\right)dx$$

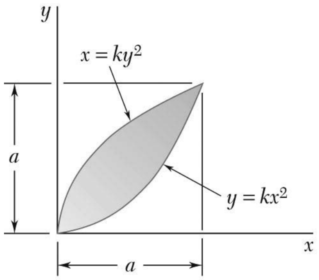
$$= \frac{1}{2}\frac{h^2}{a^4}\left[\frac{a^2}{3}x^3 - \frac{1}{5}x^5\right]_0^a$$

$$= \frac{1}{15}ah^2$$

PROBLEM 5.36 (Continued)

$$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x}\left(\frac{1}{6}ah\right) = \frac{1}{12}a^2h \qquad \bar{x} = \frac{1}{2}a \blacktriangleleft$$

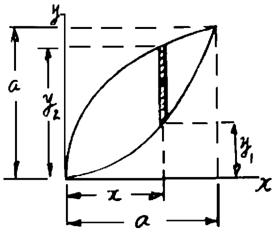
$$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y}\left(\frac{1}{6}ah\right) = \frac{1}{15}ah^2 \qquad \bar{y} = \frac{2}{5}h \blacktriangleleft$$



PROBLEM 5.37

Determine by direct integration the centroid of the area shown.

SOLUTION



$$y_2 = \sqrt{\frac{x}{k}}, \quad y_1 = kx^2$$

But $$a = ka^2, \text{ thus, } k = \frac{1}{a^2}$$

$$y_2 = \sqrt{ax}, \quad y_1 = \frac{x^2}{a}$$

$$\overline{x}_{EL} = x$$

$$dA = (y_2 - y_1)dx = \left(\sqrt{ax} - \frac{x^2}{a}\right)dx$$

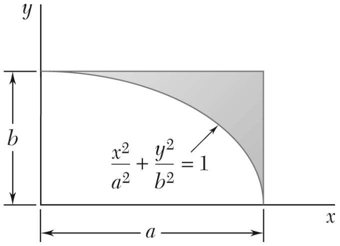
$$A = \int dA = \int_0^a \left(\sqrt{ax} - \frac{x^2}{a}\right)dx$$

$$= \left[\frac{2}{3}\sqrt{a}x^{3/2} - \frac{x^3}{3a}\right]_0^a = \frac{1}{3}a^2$$

$$\int \overline{x}_{EL}dA = \int_0^a x\left(\sqrt{ax} - \frac{x^2}{a}\right)dx = \int_0^a \left(\sqrt{a}x^{3/2} - \frac{x^3}{a}\right)dx = \left[\frac{2}{5}\sqrt{a}x^{5/2} - \frac{x^4}{4a}\right]_0^a = \frac{3}{20}a^3$$

$$\overline{x}A = \int x_{EL}dA: \quad \overline{x}\left(\frac{1}{3}a^2\right) = \frac{3}{20}a^3 \qquad \overline{x} = \frac{9a}{20}$$

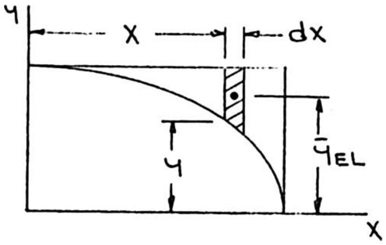
By symmetry, $$\overline{y} = \overline{x} = \frac{9a}{20} \blacktriangleleft$$



PROBLEM 5.38

Determine by direct integration the centroid of the area shown.

SOLUTION



For the element (EL) shown, $y=\frac{b}{a}\sqrt{a^2-x^2}$

and
$$dA=(b-y)dx$$
$$=\frac{b}{a}\left(a-\sqrt{a^2-x^2}\right)dx$$
$$\bar{x}_{EL}=x$$
$$\bar{y}_{EL}=\frac{1}{2}(y+b)$$
$$=\frac{b}{2a}\left(a+\sqrt{a^2-x^2}\right)$$

Then
$$A=\int dA=\int_0^a\frac{b}{a}\left(a-\sqrt{a^2-x^2}\right)dx$$

To integrate, let $x=a\sin\theta$: $\sqrt{a^2-x^2}=a\cos\theta$, $dx=a\cos\theta\, d\theta$

Then
$$A=\int_0^{\pi/2}\frac{b}{a}(a-a\cos\theta)(a\cos\theta d\theta)$$
$$=\frac{b}{a}\left[a^2\sin\theta-a^2\left(\frac{\theta}{2}+\sin\frac{2\theta}{4}\right)\right]_0^{\pi/2}$$
$$=ab\left(1-\frac{\pi}{4}\right)$$

and
$$\int\bar{x}_{EL}dA=\int_0^a x\left[\frac{b}{a}\left(a-\sqrt{a^2-x^2}\right)dx\right]$$
$$=\frac{b}{a}\left[\left(\frac{a}{2}x^2+\frac{1}{3}(a^2-x^2)^{3/2}\right)\right]_0^{\pi/2}$$
$$=\frac{1}{6}a^3b$$

PROBLEM 5.38 (Continued)

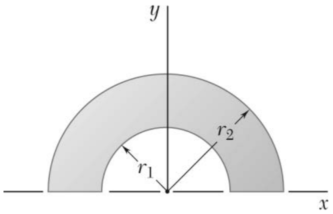
$$\int \overline{y}_{EL}\,dA = \int_0^a \frac{b}{2a}\left(a+\sqrt{a^2-x^2}\right)\left[\frac{b}{a}\left(a-\sqrt{a^2-x^2}\right)dx\right]$$

$$= \frac{b^2}{2a^2}\int_0^a (x^2)\ dx = \frac{b^2}{2a^2}\left(\frac{x^3}{3}\right)\Bigg|_0^a$$

$$= \frac{1}{6}ab^2$$

$$\overline{x}A = \int \overline{x}_{EL}\,dA: \quad \overline{x}\left[ab\left(1-\frac{\pi}{4}\right)\right] = \frac{1}{6}a^2b \qquad \text{or } \overline{x} = \frac{2a}{3(4-\pi)} \blacktriangleleft$$

$$\overline{y}A = \int \overline{y}_{EL}\,dA: \quad \overline{y}\left[ab\left(1-\frac{\pi}{4}\right)\right] = \frac{1}{6}ab^2 \qquad \text{or } \overline{y} = \frac{2b}{3(4-\pi)} \blacktriangleleft$$



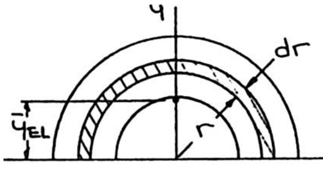
PROBLEM 5.39

Determine by direct integration the centroid of the area shown.

SOLUTION

First note that symmetry implies $\overline{x} = 0$ ◀

For the element (EL) shown,



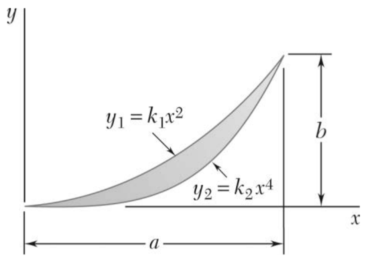
$$\overline{y}_{EL} = \frac{2r}{\pi} \quad \text{(Figure 5.8B)}$$

$$dA = \pi r dr$$

Then $A = \int dA = \int_{r_1}^{r_2} \pi r dr = \pi\left(\frac{r^2}{2}\right)\bigg|_{r_1}^{r_2} = \frac{\pi}{2}\left(r_2^2 - r_1^2\right)$

and $\int \overline{y}_{EL} dA = \int_{r_1}^{r_2} \frac{2r}{\pi}(\pi r dr) = 2\left(\frac{1}{3}r^3\right)\bigg|_{r_1}^{r_2} = \frac{2}{3}\left(r_2^3 - r_1^3\right)$

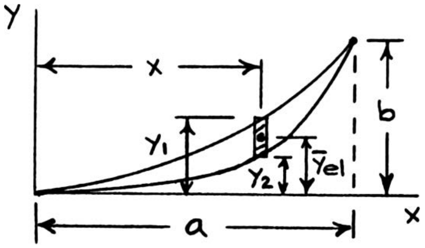
So $\overline{y}A = \int \overline{y}_{EL} dA$: $\overline{y}\left[\frac{\pi}{2}\left(r_2^2 - r_1^2\right)\right] = \frac{2}{3}\left(r_2^3 - r_1^3\right)$ or $\overline{y} = \frac{4}{3\pi}\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$ ◀



PROBLEM 5.40

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *b*.

SOLUTION



$$y_1 = k_1x^2 \quad \text{but} \quad b = k_1a^2 \qquad y_1 = \frac{b}{a^2}x^2$$

$$y_2 = k_2x^4 \quad \text{but} \quad b = k_2a^4 \qquad y_2 = \frac{b}{a^4}x^4$$

$$dA = (y_2 - y_1)dx = \frac{b}{a^2}\left(x^2 - \frac{x^4}{a^2}\right)dx$$

$$\overline{x}_{EL} = x$$

$$\overline{y}_{EL} = \frac{1}{2}(y_1 + y_2)$$

$$= \frac{b}{2a^2}\left(x^2 + \frac{x^4}{a^2}\right)$$

$$A = \int dA = \frac{b}{a^2}\int_0^a \left(x^2 - \frac{x^4}{a^2}\right)dx$$

$$= \frac{b}{a^2}\left[\frac{x^3}{3} - \frac{x^5}{5a^2}\right]_0^a$$

$$= \frac{2}{15}ba$$

$$\int \overline{x}_{EL}dA = \int_0^a x\frac{b}{a^2}\left(x^2 - \frac{x^4}{a^2}\right)dx$$

$$= \frac{b}{a^2}\int_0^a \left(x^3 - \frac{x^5}{a^2}\right)dx$$

$$= \frac{b}{a^2}\left[\frac{x^4}{4} - \frac{x^6}{6a^2}\right]_0^a$$

$$= \frac{1}{12}a^2b$$

PROBLEM 5.40 (Continued)

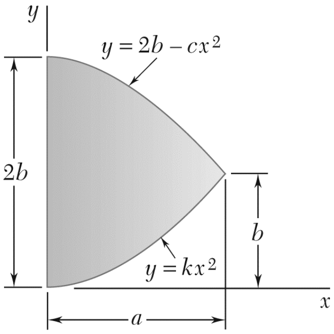
$$\int \overline{y}_{EL}\,dA = \int_0^a \frac{b}{2a^2}\left(x^2+\frac{x^4}{a^2}\right)\frac{b}{a^2}\left(x^2-\frac{x^4}{a^2}\right)dx$$

$$= \frac{b^2}{2a^4}\int_0^a \left(x^4-\frac{x^8}{a^4}\right)dx$$

$$= \frac{b^2}{2a^4}\left[\frac{x^5}{5}-\frac{x^9}{9a^4}\right]_0^a = \frac{2}{45}ab^2$$

$$\overline{x}A = \int \overline{x}_{EL}\,dA:\quad \overline{x}\left(\frac{2}{15}ba\right) = \frac{1}{12}a^2b \qquad \overline{x} = \frac{5}{8}a \blacktriangleleft$$

$$\overline{y}A = \int \overline{y}_{EL}\,dA:\quad \overline{y}\left(\frac{2}{15}ba\right) = \frac{2}{45}ab^2 \qquad \overline{y} = \frac{1}{3}b \blacktriangleleft$$



PROBLEM 5.41

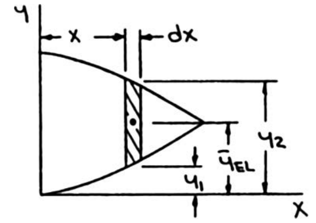
Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *b*.

SOLUTION

First note that symmetry implies $\bar{y} = b$ ◀

At $x = a, \quad y = b$

$$y_1: \quad b = ka^2 \quad \text{or} \quad k = \frac{b}{a^2}$$



Then $y_1 = \frac{b}{a^2}x^2$

$$y_2: \quad b = 2b - ca^2$$

or $c = \frac{b}{a^2}$

Then $y_2 = b\left(2 - \frac{x^2}{a^2}\right)$

Now

$$dA = (y_2 - y_1)dx_2 = \left[b\left(2 - \frac{x^2}{a^2}\right) - \frac{b}{a^2}x^2\right]dx$$

$$= 2b\left(1 - \frac{x^2}{a^2}\right)dx$$

and $\bar{x}_{EL} = x$

Then

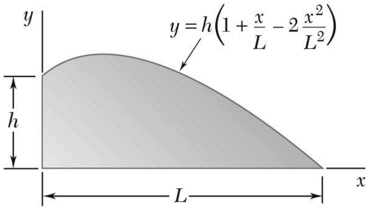
$$A = \int dA \int_0^a 2b\left(1 - \frac{x^2}{a^2}\right)dx = 2b\left[x - \frac{x^3}{3a^2}\right]_0^a = \frac{4}{3}ab$$

and

$$\int \bar{x}_{EL}dA = \int_0^a x\left[2b\left(1 - \frac{x^2}{a^2}\right)dx\right] = 2b\left[\frac{x^2}{2} - \frac{x^4}{4a^2}\right]_0^a = \frac{1}{2}a^2b$$

$$\bar{x}A = \int \bar{x}_{EL}dA: \quad \bar{x}\left(\frac{4}{3}ab\right) = \frac{1}{2}a^2b$$

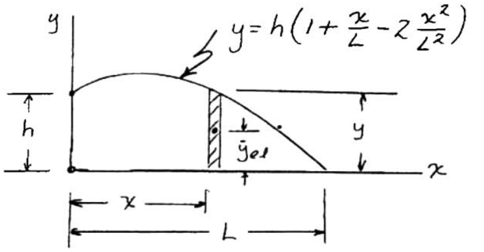
$\bar{x} = \frac{3}{8}a$ ◀



PROBLEM 5.42

Determine by direct integration the centroid of the area shown.

SOLUTION



$$\bar{x}_{EL} = x \quad \bar{y}_{EL} = \frac{1}{2}y \quad dA = y\,dx$$

$$A = \int dA = \int_0^L h\left(1 + \frac{x}{L} - 2\frac{x^2}{L^2}\right)dx = h\left[x + \frac{x^2}{2L} - \frac{2}{3}\frac{x^3}{L^2}\right]_0^L = \frac{5}{6}hL$$

$$\int x_{EL}dA = \int_0^L xh\left(1 + \frac{x}{L} - 2\frac{x^2}{L^2}\right)dx = h\int_0^L\left(x + \frac{x^2}{L} - 2\frac{x^3}{L^2}\right)dx$$

$$= h\left[\frac{x^2}{2} + \frac{1}{3}\frac{x^3}{L} - \frac{2}{4}\frac{x^4}{L^2}\right]_0^L = \frac{1}{3}hL^2$$

$$\bar{x}A = \int x_{EL}dA: \quad \bar{x}\left(\frac{5}{6}hL\right) = \frac{1}{3}hL^2 \qquad \bar{x} = \frac{2}{5}L \blacktriangleleft$$

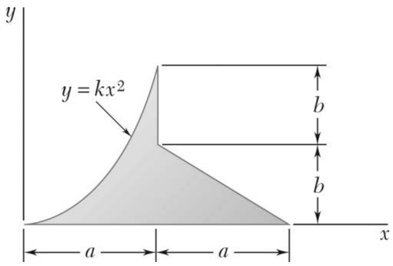
$$A = \frac{5}{6}hL \quad \bar{y}_{EL} = \frac{1}{2}y \quad y = h\left(1 + \frac{x}{L} - 2\frac{x^2}{L^2}\right)$$

$$\int \bar{y}_{EL}dA = \frac{1}{2}\int y^2dx = \frac{h^2}{2}\int_0^L\left(1 + \frac{x}{L} - 2\frac{x^2}{L^2}\right)^2 dx$$

$$= \frac{h^2}{2}\int_0^L\left(1 + \frac{x^2}{L^2} + 4\frac{x^4}{L^4} + 2\frac{x}{L} - 4\frac{x^2}{L^2} - 4\frac{x^3}{L^3}\right)dx$$

$$= \frac{h^2}{2}\left[x + \frac{x^3}{3L^2} + \frac{4x^5}{5L^4} + \frac{x^2}{L} - \frac{4x^3}{3L^2} - \frac{x^4}{L^3}\right]_0^L = \frac{4}{10}h^2L$$

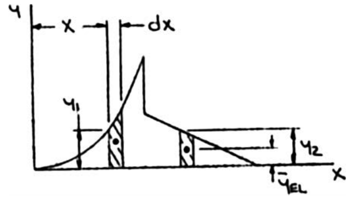
$$\bar{y}A = \int y_{EL}dA: \quad \bar{y}\left(\frac{5}{6}hL\right) = \frac{4}{10}h^2L \qquad \bar{y} = \frac{12}{25}h \blacktriangleleft$$



PROBLEM 5.43

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *b*.

SOLUTION



For y_1 at $x = a$, $\quad y = 2b, \quad 2b = ka^2, \quad \text{or} \quad k = \dfrac{2b}{a^2}$

Then $\quad y_1 = \dfrac{2b}{a^2}x^2$

By observation, $\quad y_2 = -\dfrac{b}{a}(x + 2b) = b\left(2 - \dfrac{x}{a}\right)$

Now $\quad \bar{x}_{EL} = x$

and for $0 \le x \le a$, $\quad \bar{y}_{EL} = \dfrac{1}{2}y_1 = \dfrac{b}{a^2}x^2 \quad \text{and} \quad dA = y_1\,dx = \dfrac{2b}{a^2}x^2dx$

For $a \le x \le 2a$, $\quad \bar{y}_{EL} = \dfrac{1}{2}y_2 = \dfrac{b}{2}\left(2 - \dfrac{x}{a}\right) \quad \text{and} \quad dA = y_2\,dx = b\left(2 - \dfrac{x}{a}\right)dx$

Then

$$A = \int dA = \int_0^a \frac{2b}{a^2}x^2dx + \int_a^{2a} b\left(2 - \frac{x}{a}\right)dx$$

$$= \frac{2b}{a^2}\left[\frac{x^3}{3}\right]_0^a + b\left[-\frac{a}{2}\left(2 - \frac{x}{a}\right)^2\right]_0^{2a} = \frac{7}{6}ab$$

and

$$\int \bar{x}_{EL}dA = \int_0^a x\left(\frac{2b}{a^2}x^2dx\right) + \int_a^{2a} x\left[b\left(2 - \frac{x}{a}\right)dx\right]$$

$$= \frac{2b}{a^2}\left[\frac{x^4}{4}\right]_0^a + b\left[x^2 - \frac{x^3}{3a}\right]_0^{2a}$$

$$= \frac{1}{2}a^2b + b\left\{\left[(2a)^2 - (a)^2\right] + \frac{1}{3a}\left[(2a^2) - (a)^3\right]\right\}$$

$$= \frac{7}{6}a^2b$$

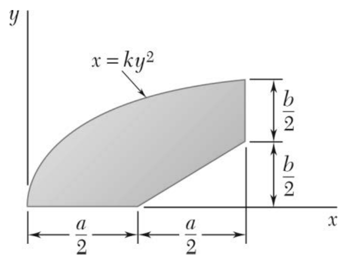
$$\int \overline{y}_{EL}dA = \int_0^a \frac{b}{a^2}x^2\left[\frac{2b}{a^2}x^2dx\right] + \int_0^{2a}\frac{b}{2}\left(2-\frac{x}{a}\right)\left[b\left(2-\frac{x}{a}\right)dx\right]$$

$$= \frac{2b^2}{a^4}\left[\frac{x^5}{5}\right]_0^a + \frac{b^2}{2}\left[-\frac{a}{3}\left(2-\frac{x}{a}\right)^3\right]_a^{2a}$$

$$= \frac{17}{30}ab^2$$

Hence, $\overline{x}A = \int \overline{x}_{EL}dA: \quad \overline{x}\left(\frac{7}{6}ab\right) = \frac{7}{6}a^2b$ $\qquad \overline{x} = a$ ◀

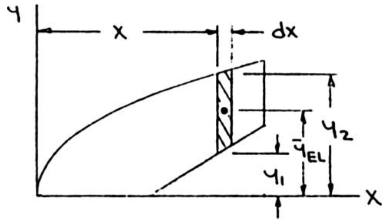
$\overline{y}A = \int \overline{y}_{EL}dA: \quad \overline{y}\left(\frac{7}{6}ab\right) = \frac{17}{30}ab^2$ $\qquad \overline{y} = \frac{17}{35}b$ ◀



PROBLEM 5.44

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *b*.

SOLUTION



For y_2 at $x = a$, $\quad y = b, \quad a = kb^2, \quad \text{or} \quad k = \frac{a}{b^2}$

Then $\quad y_2 = \frac{b}{\sqrt{a}} x^{1/2}$

Now $\quad \bar{x}_{EL} = x$

and for $0 \le x \le \frac{a}{2}$, $\quad \bar{y}_{EL} = \frac{y_2}{2} = \frac{b}{2}\frac{x^{1/2}}{\sqrt{a}}$

$$dA = y_2\,dx = b\frac{x^{1/2}}{\sqrt{a}}\,dx$$

For $\frac{a}{2} \le x \le a$, $\quad \bar{y}_{EL} = \frac{1}{2}(y_1 + y_2) = \frac{b}{2}\left(\frac{x}{a} - \frac{1}{2} + \frac{x^{1/2}}{\sqrt{a}}\right)$

$$dA = (y_2 - y_1)dx = b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx$$

Then

$$A = \int dA = \int_0^{a/2} b\frac{x^{1/2}}{\sqrt{a}}dx + \int_{a/2}^{a} b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx$$

$$= \frac{b}{\sqrt{a}}\left[\frac{2}{3}x^{3/2}\right]_0^{a/2} + b\left[\frac{2}{3}\frac{x^{3/2}}{\sqrt{a}} - \frac{x^2}{2a} + \frac{1}{2}x\right]_{a/2}^{a}$$

$$= \frac{2}{3}\frac{b}{\sqrt{a}}\left[\left(\frac{a}{2}\right)^{3/2} + (a)^{3/2} - \left(\frac{a}{2}\right)^{3/2}\right]$$

$$+ b\left\{-\frac{1}{2a}\left[(a^2) - \left(\frac{a}{2}\right)^2\right] + \frac{1}{2}\left[(a) - \left(\frac{a}{2}\right)\right]\right\}$$

$$= \frac{13}{24}ab$$

PROBLEM 5.44 (Continued)

and
$$\int \overline{x}_{EL}\,dA = \int_0^{a/2} x\left(b\frac{x^{1/2}}{\sqrt{a}}dx\right) + \int_{a/2}^{a} x\left[b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)\right]dx$$

$$= \frac{b}{\sqrt{a}}\left[\frac{2}{5}x^{5/2}\right]_0^{a/2} + b\left[\frac{2}{5}\frac{x^{5/2}}{\sqrt{a}} - \frac{x^3}{3a} + \frac{x^4}{4}\right]_{a/2}^{a}$$

$$= \frac{2}{5}\frac{b}{\sqrt{a}}\left[\left(\frac{a}{2}\right)^{5/2} + (a)^{5/2} - \left(\frac{a}{2}\right)^{5/2}\right]$$

$$+b\left\{-\frac{1}{3a}\left[(a)^3 - \left(\frac{a}{2}\right)^3\right] + \frac{1}{4}\left[(a)^2 - \left(\frac{a}{2}\right)^2\right]\right\}$$

$$= \frac{71}{240}a^2b$$

$$\int \overline{y}_{EL}\,dA = \int_0^{a/2} \frac{b}{2}\frac{x^{1/2}}{\sqrt{a}}\left[b\frac{x^{1/2}}{\sqrt{a}}dx\right]$$

$$+\int_{a/2}^{a} \frac{b}{2}\left(\frac{x}{a} - \frac{1}{2} + \frac{x^{1/2}}{\sqrt{a}}\right)\left[b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx\right]$$

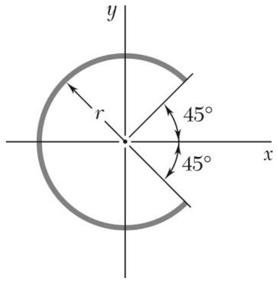
$$= \frac{b^2}{2a}\left[\frac{1}{2}x^2\right]_0^{a/2} + \frac{b^2}{2}\left[\left(\frac{x^2}{2a} - \frac{1}{3a}\left(\frac{x}{a} - \frac{1}{2}\right)^3\right)\right]_{a/2}^{a}$$

$$= \frac{b}{4a}\left[\left(\frac{a}{2}\right)^2 + (a)^2 - \left(\frac{a}{2}\right)^2\right] - \frac{b^2}{6a}\left(\frac{a}{2} - \frac{1}{2}\right)^3$$

$$= \frac{11}{48}ab^2$$

Hence,
$$\overline{x}A = \int \overline{x}_{EL}\,dA: \quad \overline{x}\left(\frac{13}{24}ab\right) = \frac{71}{240}a^2b \qquad \overline{x} = \frac{17}{130}a = 0.546a \blacktriangleleft$$

$$\overline{y}A = \int \overline{y}_{EL}\,dA: \quad \overline{y}\left(\frac{13}{24}ab\right) = \frac{11}{48}ab^2 \qquad \overline{y} = \frac{11}{26}b = 0.423b \blacktriangleleft$$

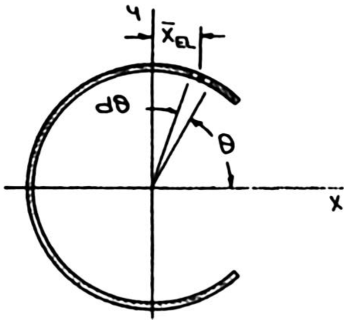


PROBLEM 5.45

A homogeneous wire is bent into the shape shown. Determine by direct integration the *x* coordinate of its centroid.

SOLUTION

First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line.

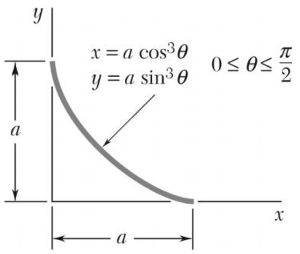


Now $\bar{x}_{EL} = r\cos\theta \quad \text{and} \quad dL = r\,d\theta$

Then $L = \int dL = \int_{\pi/4}^{7\pi/4} r\,d\theta = r[\theta]_{\pi/4}^{7\pi/4} = \frac{3}{2}\pi r$

and
$$\int \bar{x}_{EL}\,dL = \int_{\pi/4}^{7\pi/4} r\cos\theta(r\,d\theta)$$
$$= r^2[\sin\theta]_{\pi/4}^{7\pi/4}$$
$$= r^2\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$
$$= -r^2\sqrt{2}$$

Thus $\bar{x}L = \int \bar{x}\,dL: \quad \bar{x}\left(\frac{3}{2}\pi r\right) = -r^2\sqrt{2}$ $\qquad \bar{x} = -\frac{2\sqrt{2}}{3\pi} r$ ◀

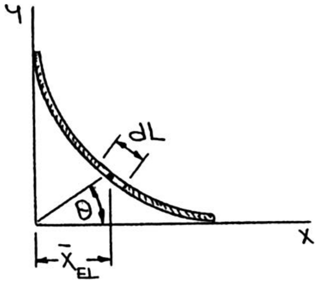


PROBLEM 5.46

A homogeneous wire is bent into the shape shown. Determine by direct integration the *x* coordinate of its centroid.

SOLUTION

First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line.



Now $\bar{x}_{EL} = a\cos^3\theta$ and $dL = \sqrt{dx^2 + dy^2}$

where

$$x = a\cos^3\theta: \quad dx = -3a\cos^2\theta\sin\theta d\theta$$

$$y = a\sin^3\theta: \quad dy = 3a\sin^2\theta\cos\theta d\theta$$

Then

$$\begin{aligned} dL &= [(-3a\cos^2\theta\sin\theta d\theta)^2 + (3a\sin^2\theta\cos\theta d\theta)^2]^{1/2} \\ &= 3a\cos\theta\sin\theta(\cos^2\theta + \sin^2\theta)^{1/2} d\theta \\ &= 3a\cos\theta\sin\theta d\theta \end{aligned}$$

$$\begin{aligned} L = \int dL &= \int_0^{\pi/2} 3a\cos\theta\sin\theta d\theta = 3a\left[\frac{1}{2}\sin^2\theta\right]_0^{\pi/2} \\ &= \frac{3}{2}a \end{aligned}$$

and

$$\begin{aligned} \int \bar{x}_{EL} dL &= \int_0^{\pi/2} a\cos^3\theta(3a\cos\theta\sin\theta d\theta) \\ &= 3a^2\left[-\frac{1}{5}\cos^5\theta\right]_0^{\pi/2} = \frac{3}{5}a^2 \end{aligned}$$

Hence, $\bar{x}L = \int \bar{x}_{EL} dL: \quad \bar{x}\left(\frac{3}{2}a\right) = \frac{3}{5}a^2$ $\quad\bar{x} = \frac{2}{5}a$ ◀

Alternative Solution:

$$x = a\cos^3\theta \Rightarrow \cos^2\theta = \left(\frac{x}{a}\right)^{2/3}$$

$$y = a\sin^3\theta \Rightarrow \sin^2\theta = \left(\frac{y}{a}\right)^{2/3}$$

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1 \quad \text{or} \quad y = (a^{2/3} - x^{2/3})^{3/2}$$

PROBLEM 5.46 (Continued)

Then $$\frac{dy}{dx} = (a^{2/3} - x^{2/3})^{1/2}(-x^{-1/3})$$

Now $$\overline{x}_{EL} = x$$

and $$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

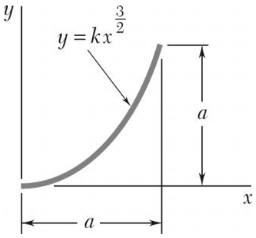
$$dx = \left\{1 + \left[(a^{2/3} - x^{2/3})^{1/2}(-x^{-1/3})\right]^2\right\}^{1/2} dx$$

Then $$L = \int dL = \int_0^a \frac{a^{1/3}}{x^{1/3}} dx = a^{1/3}\left[\frac{3}{2}x^{2/3}\right]_0^a = \frac{3}{2}a$$

and $$\int \overline{x}_{EL} dL = \int_0^a x\left(\frac{a^{1/3}}{x^{1/3}} dx\right) = a^{1/3}\left[\frac{3}{5}x^{5/3}\right]_0^a = \frac{3}{5}a^2$$

Hence $$\overline{x}L = \int \overline{x}_{EL} dL: \quad \overline{x}\left(\frac{3}{2}a\right) = \frac{3}{5}a^2$$

$$\overline{x} = \frac{2}{5}a \blacktriangleleft$$

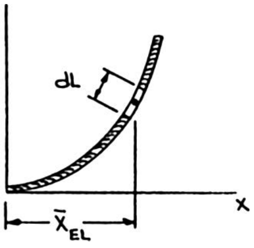


PROBLEM 5.47*

A homogeneous wire is bent into the shape shown. Determine by direct integration the *x* coordinate of its centroid. Express your answer in terms of *a*.

SOLUTION

First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.



We have at $x = a$, $$y = a, \quad a = ka^{3/2}, \quad \text{or} \quad k = \frac{1}{\sqrt{a}}$$

Then $$y = \frac{1}{\sqrt{a}} x^{3/2}$$

and $$\frac{dy}{dx} = \frac{3}{2\sqrt{a}} x^{1/2}$$

Now $$\overline{x}_{EL} = x$$

and $$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$
$$= \left[1 + \left(\frac{3}{2\sqrt{a}} x^{1/2}\right)^2\right]^{1/2} dx$$
$$= \frac{1}{2\sqrt{a}} \sqrt{4a + 9x}\, dx$$

Then $$L = \int dL = \int_0^a \frac{1}{2\sqrt{a}} \sqrt{4a + 9x}\, dx$$
$$= \frac{1}{2\sqrt{a}} \left[\frac{2}{3} \times \frac{1}{9} (4a + 9x)^{3/2}\right]_0^a$$
$$= \frac{a}{27} [(13)^{3/2} - 8]$$
$$= 1.43971a$$

and $$\int \overline{x}_{EL} dL = \int_0^a x \left[\frac{1}{2\sqrt{a}} \sqrt{4a + 9x}\, dx\right]$$

PROBLEM 5.47* (Continued)

Use integration by parts with

$$u = x \qquad dv = \sqrt{4a+9x}\,dx$$

$$du = dx \qquad v = \frac{2}{27}(4a+9x)^{3/2}$$

Then

$$\int \overline{x}_{EL}\,dL = \frac{1}{2\sqrt{a}}\left\{\left[x\times\frac{2}{27}(4a+9x)^{3/2}\right]_0^a - \int_0^a \frac{2}{27}(4a+9x)^{3/2}\,dx\right\}$$

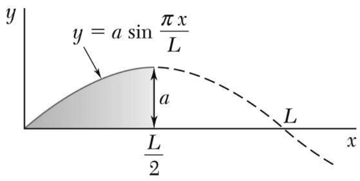
$$= \frac{(13)^{3/2}}{27}a^2 - \frac{1}{27\sqrt{a}}\left[\frac{2}{45}(4a+9x)^{5/2}\right]_0^a$$

$$= \frac{a^2}{27}\left\{(13)^{3/2} - \frac{2}{45}[(13)^{5/2} - 32]\right\}$$

$$= 0.78566a^2$$

$$\overline{x}L = \int x_{EL}\,dL: \quad \overline{x}(1.43971a) = 0.78566a^2$$

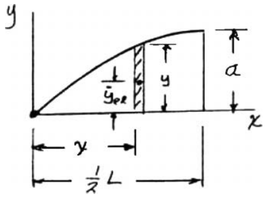
or $\overline{x} = 0.546a$ ◀



PROBLEM 5.48

Determine by direct integration the centroid of the area shown.

SOLUTION



$$y = a\sin\frac{\pi x}{L}$$

$$\overline{x}_{EL} = x, \quad \overline{y}_{EL} = \frac{1}{2}y, \quad dA = y\,dx$$

$$A = \int_0^{L/2} y\,dx = a\int_0^{L/2} \sin\frac{\pi x}{L}dx$$

$$A = a\left[\frac{L}{\pi}\left(-\cos\frac{\pi x}{L}\right)\right]_0^{L/2} = \frac{aL}{\pi}$$

$$\int \overline{x}_{EL}dA = \int_0^{L/2} xydx = \int_0^{L/2} xa\sin\frac{\pi x}{L}dx$$

Setting $u = \frac{\pi x}{L}$, we have $x = \frac{L}{\pi}u, \quad dx = \frac{L}{\pi}du,$

$$\int \overline{x}_{EL}dA = \int_0^{\pi/2}\left(\frac{L}{\pi}u\right)a\sin u\left(\frac{L}{\pi}du\right) = a\left(\frac{L}{\pi}\right)^2\int_0^{\pi/2} u\sin x\,du$$

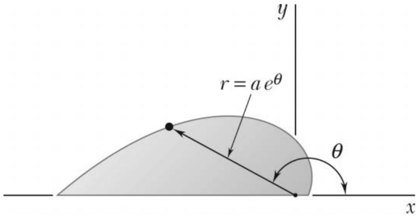
Integrating by parts, $$\int \overline{x}_{EL}dA = a\left(\frac{L}{\pi}\right)^2\left\{[-u\cos u]_0^{\pi/2} + \int_0^{\pi/2}\cos u\,du\right\} = \frac{aL^2}{\pi^2}$$

$$\int \overline{y}_{EL}dA = \int_0^{L/2}\frac{1}{2}y^2dx = \frac{1}{2}a^2\int_0^{L/2}\sin^2\frac{\pi x}{L}dx = \frac{a^2L}{2\pi}\int_0^{\pi/2}\sin^2 u\,du$$

$$= \frac{a^2L}{2\pi^2}\int_0^{\pi/2}\frac{1}{2}(1-\cos 2u)du = \frac{a^2L}{4\pi}\left[u - \frac{1}{2}\sin 2u\right]_0^{\pi/2} = \frac{1}{8}a^2L$$

$$\overline{x}A = \int \overline{x}_{EL}dA: \quad \overline{x}\left(\frac{aL}{\pi}\right) = \frac{aL^2}{\pi^2} \qquad \overline{x} = \frac{L}{\pi} \blacktriangleleft$$

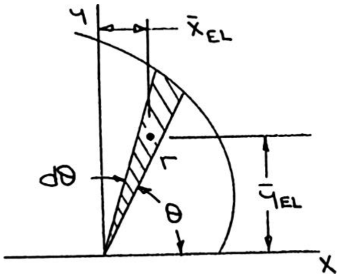
$$\overline{y}A = \int \overline{y}_{EL}dA: \quad \overline{y}\left(\frac{aL}{\pi}\right) = \frac{1}{8}a^2L \qquad \overline{y} = \frac{\pi}{8}a \blacktriangleleft$$



PROBLEM 5.49*

Determine by direct integration the centroid of the area shown.

SOLUTION



We have
$$\bar{x}_{EL} = \frac{2}{3}r\cos\theta = \frac{2}{3}ae^{\theta}\cos\theta$$
$$\bar{y}_{EL} = \frac{2}{3}r\sin\theta = \frac{2}{3}ae^{\theta}\sin\theta$$

and
$$dA = \frac{1}{2}(r)(rd\theta) = \frac{1}{2}a^2e^{2\theta}d\theta$$

Then
$$A = \int dA = \int_0^{\pi}\frac{1}{2}a^2e^{2\theta}d\theta = \frac{1}{2}a^2\left[\frac{1}{2}e^{2\theta}\right]_0^{\pi}$$
$$= \frac{1}{4}a^2(e^{2\pi}-1)$$
$$= 133.623a^2$$

and
$$\int \bar{x}_{EL}dA = \int_0^{\pi}\frac{2}{3}ae^{\theta}\cos\theta\left(\frac{1}{2}a^2e^{2\theta}d\theta\right)$$
$$= \frac{1}{3}a^3\int_0^{\pi}e^{3\theta}\cos\theta d\theta$$

To proceed, use integration by parts, with
$$u = e^{3\theta} \quad \text{and} \quad du = 3e^{3\theta}d\theta$$
$$dv = \cos\theta d\theta \quad \text{and} \quad v = \sin\theta$$

Then
$$\int e^{3\theta}\cos\theta d\theta = e^{3\theta}\sin\theta - \int\sin\theta(3e^{3\theta}d\theta)$$

Now let
$$u = e^{3\theta} \quad \text{then} \quad du = 3e^{3\theta}d\theta$$
$$dv = \sin\theta d\theta, \quad \text{then} \quad v = -\cos\theta$$

Then
$$\int e^{3\theta}\cos\theta d\theta = e^{3\theta}\sin\theta - 3\left[-e^{3\theta}\cos\theta - \int(-\cos\theta)(3e^{3\theta}d\theta)\right]$$

PROBLEM 5.49* (Continued)

so that
$$\int e^{3\theta}\cos\theta d\theta = \frac{e^{3\theta}}{10}(\sin\theta + 3\cos\theta)$$

$$\int x_{EL}dA = \frac{1}{3}a^3\left[\frac{e^{3\theta}}{10}(\sin\theta + 3\cos\theta)\right]_0^{\pi}$$

$$= \frac{a^3}{30}(-3e^{3\pi} - 3) = -1239.26a^3$$

Also,
$$\int \overline{y}_{EL}dA = \int_0^{\pi}\frac{2}{3}ae^{\theta}\sin\theta\left(\frac{1}{2}a^2e^{2\theta}d\theta\right)$$

$$= \frac{1}{3}a^3\int_0^{\pi}e^{3\theta}\sin\theta d\theta$$

Use integration by parts, as above, with

$$u = e^{3\theta} \quad \text{and} \quad du = 3e^{3\theta}d\theta$$

$$dv = \int\sin\theta d\theta \quad \text{and} \quad v = -\cos\theta$$

Then
$$\int e^{3\theta}\sin\theta d\theta = -e^{3\theta}\cos\theta - \int(-\cos\theta)(3e^{3\theta}d\theta)$$

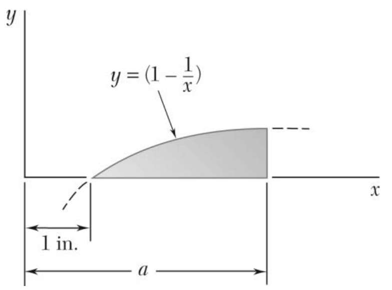
so that
$$\int e^{3\theta}\sin\theta d\theta = \frac{e^{3\theta}}{10}(-\cos\theta + 3\sin\theta)$$

$$\int \overline{y}_{EL}dA = \frac{1}{3}a^3\left[\frac{e^{3\theta}}{10}(-\cos\theta + 3\sin\theta)\right]_0^{\pi}$$

$$= \frac{a^3}{30}(e^{3\pi} + 1) = 413.09a^3$$

Hence,
$$\overline{x}A = \int x_{EL}dA: \quad \overline{x}(133.623a^2) = -1239.26a^3$$
or $\overline{x} = -9.27a$ ◀

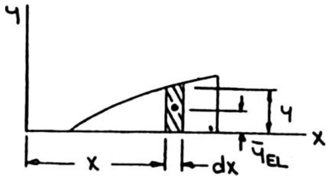
$$\overline{y}A = \int \overline{y}_{EL}dA: \quad \overline{y}(133.623a^2) = 413.09a^3$$
or $\overline{y} = 3.09a$ ◀



PROBLEM 5.50

Determine the centroid of the area shown when $a = 2$ in.

SOLUTION



We have $$\bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{2}\left(1 - \frac{1}{x}\right)$$

and $$dA = y\,dx = \left(1 - \frac{1}{x}\right)dx$$

Then $$A = \int dA = \int_1^a \left(1 - \frac{1}{x}\right)\frac{dx}{2} = [x - \ln x]_1^a = (a - \ln a - 1)\text{ in}^2$$

and $$\int \bar{x}_{EL}\,dA = \int_1^a x\left[\left(1 - \frac{1}{x}\right)dx\right] = \left[\frac{x^2}{2} - x\right]_1^a = \left(\frac{a^2}{2} - a + \frac{1}{2}\right)\text{in}^3$$

$$\int \bar{y}_{EL}\,dA = \int_1^a \frac{1}{2}\left(1 - \frac{1}{x}\right)\left[\left(1 - \frac{1}{x}\right)dx\right] = \frac{1}{2}\int_1^a \left(1 - \frac{2}{x} + \frac{1}{x^2}\right)dx$$

$$= \frac{1}{2}\left[x - 2\ln x - \frac{1}{x}\right]_1^a = \frac{1}{2}\left(a - 2\ln a - \frac{1}{a}\right)\text{in}^3$$

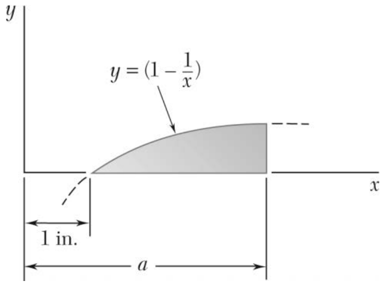
$$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x} = \frac{\frac{a^2}{2} - a + \frac{1}{2}}{a - \ln a - 1}\text{ in.}$$

$$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y} = \frac{a - 2\ln a - \frac{1}{a}}{2(a - \ln a - 1)}\text{ in.}$$

Find $\bar{x}$ and $\bar{y}$ when $a = 2$ in.

We have $$\bar{x} = \frac{\frac{1}{2}(2)^2 - 2 + \frac{1}{2}}{2 - \ln 2 - 1}$$ or $\bar{x} = 1.629$ in. ◀

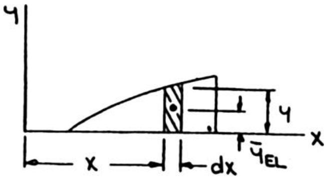
and $$\bar{y} = \frac{2 - 2\ln 2 - \frac{1}{2}}{2(2 - \ln 2 - 1)}$$ or $\bar{y} = 0.1853$ in. ◀



PROBLEM 5.51

Determine the value of a for which the ratio $\overline{x}/\overline{y}$ is 9.

SOLUTION



We have $$\overline{x}_{EL} = x$$

$$\overline{y}_{EL} = \frac{1}{2}y = \frac{1}{2}\left(1 - \frac{1}{x}\right)$$

and $$dA = y\,dx = \left(1 - \frac{1}{x}\right)dx$$

Then $$A = \int dA = \int_1^a \left(1 - \frac{1}{x}\right)\frac{dx}{2} = [x - \ln x]_1^a$$

$$= (a - \ln a - 1)\ \text{in}^2$$

and $$\int \overline{x}_{EL}dA = \int_1^a x\left[\left(1 - \frac{1}{x}\right)dx\right] = \left[\frac{x^2}{2} - x\right]_1^a$$

$$= \left(\frac{a^2}{2} - a + \frac{1}{2}\right)\text{in}^3$$

$$\int \overline{y}_{EL}dA = \int_1^a \frac{1}{2}\left(1 - \frac{1}{x}\right)\left[\left(1 - \frac{1}{x}\right)dx\right] = \frac{1}{2}\int_1^a \left(1 - \frac{2}{x} + \frac{1}{x^2}\right)dx$$

$$= \frac{1}{2}\left[x - 2\ln x - \frac{1}{x}\right]_1^a$$

$$= \frac{1}{2}\left(a - 2\ln a - \frac{1}{a}\right)\text{in}^3$$

$$\overline{x}A = \int \overline{x}_{EL}dA: \quad \overline{x} = \frac{\frac{a^2}{2} - a + \frac{1}{2}}{a - \ln a - 1}\ \text{in.}$$

$$\overline{y}A = \int \overline{y}_{EL}dA: \quad \overline{y} = \frac{a - 2\ln a - \frac{1}{a}}{2(a - \ln a - 1)}\ \text{in.}$$

Find a so that $\dfrac{\bar{x}}{\bar{y}} = 9$.

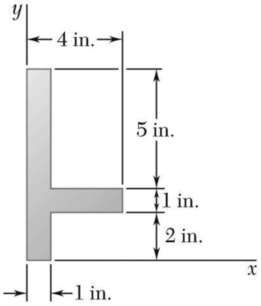
We have
$$\frac{\bar{x}}{\bar{y}} = \frac{\bar{x}A}{\bar{y}A} = \frac{\int \bar{x}_{EL}\,dA}{\int \bar{y}_{EL}\,dA}$$

Then
$$\frac{\frac{1}{2}a^2 - a + \frac{1}{2}}{\frac{1}{2}\left(a - 2\ln a - \frac{1}{a}\right)} = 9$$

or
$$a^3 - 11a^2 + a + 18a\ln a + 9 = 0$$

Using trial and error or numerical methods, and ignoring the trivial solution $a = 1$ in., we find

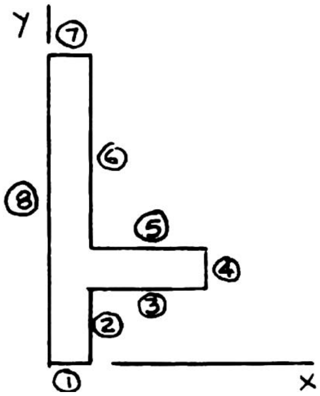
$a = 1.901\text{ in.}$ and $a = 3.74\text{ in.}$ ◀



PROBLEM 5.52

Determine the volume and the surface area of the solid obtained by rotating the area of Problem 5.1 about (*a*) the *x*-axis, (*b*) the *y*-axis.

SOLUTION



From the solution of Problem 5.1, we have

$$A = 11\text{ in}^2$$

$$\Sigma \bar{x}A = 11.5\text{ in}^3$$

$$\Sigma \bar{y}A = 39.5\text{ in}^3$$

Applying the theorems of Pappus-Guldinus, we have

(*a*) Rotation about the *x*-axis:

$$\text{Volume} = 2\pi \bar{y}_{\text{area}} A = 2\pi \Sigma \bar{y}A$$

$$= 2\pi(39.5\text{ in}^3)$$

or $\text{Volume} = 248\text{ in}^3$ ◀

$$\text{Area} = 2\pi\, \bar{y}_{\text{line}} L = 2\pi \Sigma(\bar{y}_{\text{line}})L$$

$$= 2\pi(\bar{y}_2 L_2 + \bar{y}_3 L_3 + \bar{y}_4 L_4 + \bar{y}_5 L_5 + \bar{y}_6 L_6 + \bar{y}_7 L_7 + \bar{y}_8 L_8)$$

$$= 2\pi[(1)(2) + (2)(3) + (2.5)(1) + (3)(3) + (5.5)(5) + (8)(1) + (4)(8)]$$

or $\text{Area} = 547\text{ in}^2$ ◀

(*b*) Rotation about the *y*-axis:

$$\text{Volume} = 2\pi \bar{x}_{\text{area}} A = 2\pi \Sigma \bar{x}A$$

$$= 2\pi(11.5\text{ in}^3)$$

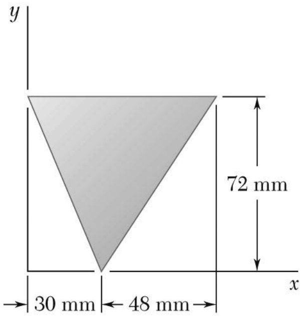
or $\text{Volume} = 72.3\text{ in}^3$ ◀

$$\text{Area} = 2\pi\, \bar{x}_{\text{line}} L = 2\pi\, \Sigma(\bar{x}_{\text{line}})L$$

$$= 2\pi(\bar{x}_1 L_1 + \bar{x}_2 L_2 + \bar{x}_3 L_3 + \bar{x}_4 L_4 + \bar{x}_5 L_5 + \bar{x}_6 L_6 + \bar{x}_7 L_7)$$

$$= 2\pi[(0.5)(1) + (1)(2) + (2.5)(3) + (4)(1) + (2.5)(3) + (1)(5) + (0.5)(1)]$$

or $\text{Area} = 169.6\text{ in}^2$ ◀

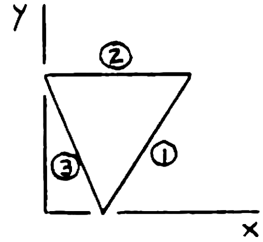


PROBLEM 5.53

Determine the volume and the surface area of the solid obtained by rotating the area of Problem 5.2 about (*a*) the line $y = 72$ mm, (*b*) the *x*-axis.

SOLUTION

From the solution of Problem 5.2, we have



$$A = 2808\ \text{mm}^2$$
$$\bar{x} = 36\ \text{mm}$$
$$\bar{y} = 48\ \text{mm}$$

Applying the theorems of Pappus-Guldinus, we have

(*a*) Rotation about the line $y = 72$ mm:

$$\text{Volume} = 2\pi(72 - \bar{y})A$$
$$= 2\pi(72 - 48)(2808)$$

$\text{Volume} = 423 \times 10^3\ \text{mm}^3$ ◀

$$\text{Area} = 2\pi\bar{y}_{\text{line}}L$$
$$= 2\pi\Sigma(\bar{y}_{\text{line}})L$$
$$= 2\pi(\bar{y}_1 L_1 + \bar{y}_3 L_3)$$

where $\bar{y}_1$ and $\bar{y}_3$ are measured with respect to line $y = 72$ mm.

$$\text{Area} = 2\pi\left[(36)\left(\sqrt{48^2 + 72^2}\right) + (36)\left(\sqrt{30^2 + 72^2}\right)\right]$$

$\text{Area} = 37.2 \times 10^3\ \text{mm}^2$ ◀

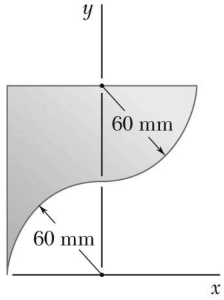
(*b*) Rotation about the *x*-axis:

$$\text{Volume} = 2\pi\bar{y}_{\text{area}}A$$
$$= 2\pi(48)(2808)$$

$\text{Volume} = 847 \times 10^3\ \text{mm}^3$ ◀

$$\text{Area} = 2\pi\bar{y}_{\text{line}}L = 2\pi\Sigma(\bar{y}_{\text{line}})L$$
$$= 2\pi(\bar{y}_1 L_1 + \bar{y}_2 L_2 + \bar{y}_3 L_3)$$
$$= 2\pi\left[(36)\left(\sqrt{48^2 + 72^2}\right) + (72)(78) + (36)\left(\sqrt{30^2 + 72^2}\right)\right]$$

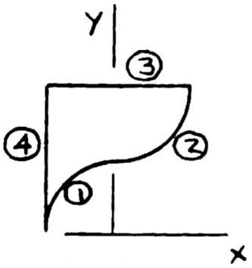
$\text{Area} = 72.5 \times 10^3\ \text{mm}^2$ ◀



PROBLEM 5.54

Determine the volume and the surface area of the solid obtained by rotating the area of Problem 5.8 about (*a*) the line $x = -60$ mm, (*b*) the line $y = 120$ mm.

SOLUTION



From the solution of Problem 5.8, we have

$$A = 7200\ \text{mm}^2$$
$$\Sigma \overline{x} A = -72\times 10^3\ \text{mm}^3$$
$$\Sigma \overline{y} A = 629.83\times 10^3\ \text{mm}^3$$

Applying the theorems of Pappus-Guldinus, we have

(*a*) Rotation about line $x = -60$ mm:

$$\text{Volume} = 2\pi(\overline{x}+60)A = 2\pi(\Sigma\overline{x}A + 60A)$$
$$= 2\pi[-72\times 10^3 + 60(7200)] \qquad \text{Volume} = 2.26\times 10^6\ \text{mm}^3 \blacktriangleleft$$

$$\text{Area} = 2\pi\overline{x}_{\text{line}}L = 2\pi\Sigma(\overline{x}_{\text{line}})L$$
$$= 2\pi(\overline{x}_1L_1 + \overline{x}_2L_2 + \overline{x}_3L_3)$$
$$= 2\pi\left[\left(60 - \frac{2(60)}{\pi}\right)\left(\frac{\pi(60)}{2}\right) + \left(60 + \frac{2(60)}{\pi}\right)\left(\frac{\pi(60)}{2}\right) + (60)(120)\right]$$

where $\overline{x}_1, \overline{x}_2, \overline{x}_3$ are measured with respect to line $x = -60$ mm. $\text{Area} = 116.3\times 10^3\ \text{mm}^2 \blacktriangleleft$

(*b*) Rotation about line $y = 120$ mm:

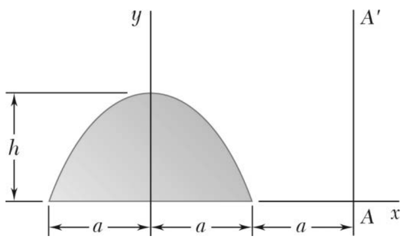
$$\text{Volume} = 2\pi(120 - \overline{y})A = 2\pi(120A - \Sigma\overline{y}A)$$
$$= 2\pi[120(7200) - 629.83\times 10^3] \qquad \text{Volume} = 1.471\times 10^6\ \text{mm}^3 \blacktriangleleft$$

$$\text{Area} = 2\pi\overline{y}_{\text{line}}L = 2\pi\Sigma(\overline{y}_{\text{line}})L$$
$$= 2\pi(\overline{y}_1L_1 + \overline{y}_2L_2 + \overline{y}_4L_4)$$

where $\overline{y}_1, \overline{y}_2, \overline{y}_4$ are measured with respect to line $y = 120$ mm.

$$\text{Area} = 2\pi\left[\left(120 - \frac{2(60)}{\pi}\right)\left(\frac{\pi(60)}{2}\right) + \left(\frac{2(60)}{\pi}\right)\left(\frac{\pi(60)}{2}\right) + (60)(120)\right]$$

$\text{Area} = 116.3\times 10^3\ \text{mm}^2 \blacktriangleleft$



PROBLEM 5.55

Determine the volume of the solid generated by rotating the parabolic area shown about (*a*) the *x*-axis, (*b*) the axis *AA*′.

SOLUTION

First, from Figure 5.8a, we have $A = \frac{4}{3}ah$

$$\bar{y} = \frac{2}{5}h$$

Applying the second theorem of Pappus-Guldinus, we have

(*a*) Rotation about the *x*-axis:

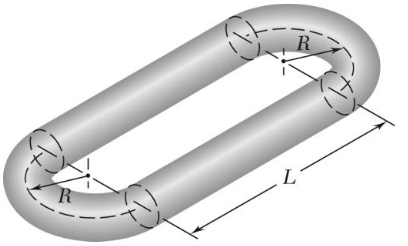
$$\text{Volume} = 2\pi \bar{y} A$$

$$= 2\pi\left(\frac{2}{5}h\right)\left(\frac{4}{3}ah\right) \qquad \text{or } \text{Volume} = \frac{16}{15}\pi a h^2 \blacktriangleleft$$

(*b*) Rotation about the line *AA*′:

$$\text{Volume} = 2\pi(2a)A$$

$$= 2\pi(2a)\left(\frac{4}{3}ah\right) \qquad \text{or } \text{Volume} = \frac{16}{3}\pi a^2 h \blacktriangleleft$$



PROBLEM 5.56

Determine the volume and the surface area of the chain link shown, which is made from a 6-mm-diameter bar, if $R = 10$ mm and $L = 30$ mm.

SOLUTION

The area A and circumference C of the cross section of the bar are

$$A = \frac{\pi}{4}d^2 \quad \text{and} \quad C = \pi d.$$

Also, the semicircular ends of the link can be obtained by rotating the cross section through a horizontal semicircular arc of radius R. Now, applying the theorems of Pappus-Guldinus, we have for the volume V,

$$\begin{aligned} V &= 2(V_{\text{side}}) + 2(V_{\text{end}}) \\ &= 2(AL) + 2(\pi RA) \\ &= 2(L + \pi R)A \end{aligned}$$

or

$$\begin{aligned} V &= 2[30\text{ mm} + \pi(10\text{ mm})]\left[\frac{\pi}{4}(6\text{ mm})^2\right] \\ &= 3470\text{ mm}^3 \end{aligned}$$

or $V = 3470\text{ mm}^3$ ◀

For the area A,

$$\begin{aligned} A &= 2(A_{\text{side}}) + 2(A_{\text{end}}) \\ &= 2(CL) + 2(\pi RC) \\ &= 2(L + \pi R)C \end{aligned}$$

or

$$\begin{aligned} A &= 2[30\text{ mm} + \pi(10\text{ mm})][\pi(6\text{ mm})] \\ &= 2320\text{ mm}^2 \end{aligned}$$

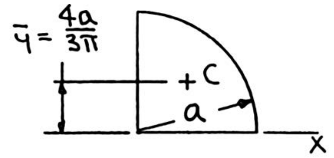
or $A = 2320\text{ mm}^2$ ◀

PROBLEM 5.57

Verify that the expressions for the volumes of the first four shapes in Figure 5.21 on Page 264 are correct.

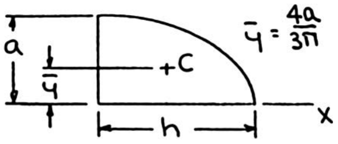
SOLUTION

Following the second theorem of Pappus-Guldinus, in each case, a specific generating area A will be rotated about the x-axis to produce the given shape. Values of $\overline{y}$ are from Figure 5.8a.



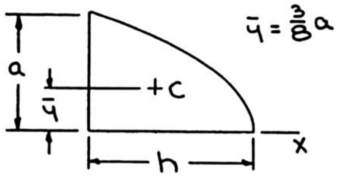
(1) Hemisphere: the generating area is a quarter circle.

We have $V = 2\pi\overline{y}A = 2\pi\left(\dfrac{4a}{3\pi}\right)\left(\dfrac{\pi}{4}a^2\right)$ or $V = \dfrac{2}{3}\pi a^3$ ◀



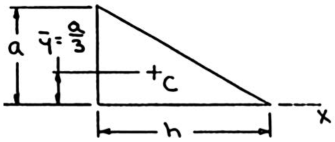
(2) Semiellipsoid of revolution: the generating area is a quarter ellipse.

We have $V = 2\pi\overline{y}A = 2\pi\left(\dfrac{4a}{3\pi}\right)\left(\dfrac{\pi}{4}ha\right)$ or $V = \dfrac{2}{3}\pi a^2 h$ ◀



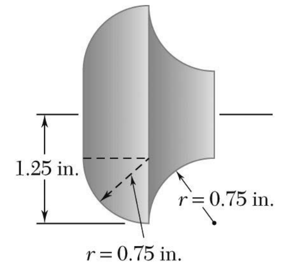
(3) Paraboloid of revolution: the generating area is a quarter parabola.

We have $V = 2\pi\overline{y}A = 2\pi\left(\dfrac{3}{8}a\right)\left(\dfrac{2}{3}ah\right)$ or $V = \dfrac{1}{2}\pi a^2 h$ ◀



(4) Cone: the generating area is a triangle.

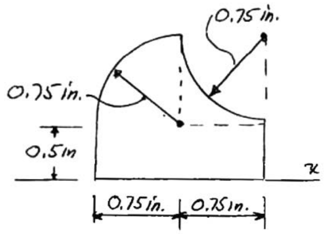
We have $V = 2\pi\overline{y}A = 2\pi\left(\dfrac{a}{3}\right)\left(\dfrac{1}{2}ha\right)$ or $V = \dfrac{1}{3}\pi a^2 h$ ◀



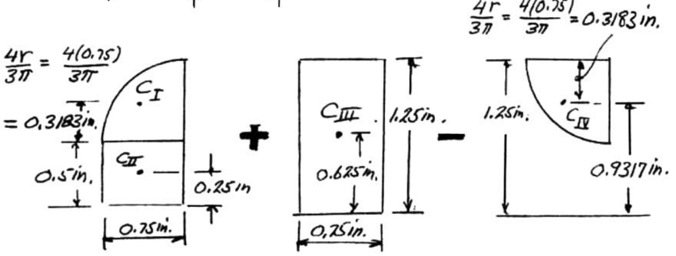
PROBLEM 5.58

Determine the volume and weight of the solid brass knob shown, knowing that the specific weight of brass is 0.306 lb/in^3.

SOLUTION



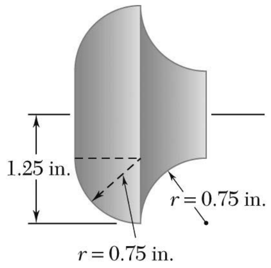
Volume of knob is obtained by rotating area at left about the x-axis. Consider area as made of components shown below.



	Area, in^2	$\overline{y}$, in.	$\overline{y}A$, in^3
1	$\frac{\pi}{4}(0.75)^2 = 0.4418$	0.8183	0.3615
2	$(0.5)(0.75) = 0.375$	0.25	0.0938
3	$(1.25)(0.75) = 0.9375$	0.625	0.5859
4	$\frac{-\pi}{4}(0.75)^2 = -0.4418$	0.9317	−0.4116
Σ			0.6296

$$V = 2\pi\Sigma\,\overline{y}A = 2\pi(0.6296\text{ in}^3) = 3.9559\text{ in}^3 \qquad V = 3.96\text{ in}^3 \blacktriangleleft$$

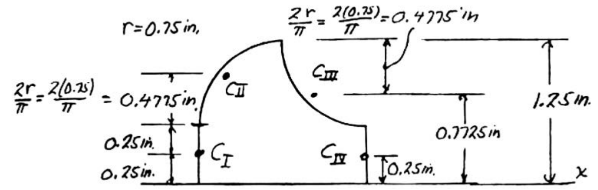
$$W = \gamma V = (0.306\text{ lb/in}^3)(3.9559\text{ in}^3) \qquad W = 1.211\text{ lb} \blacktriangleleft$$



PROBLEM 5.59

Determine the total surface area of the solid brass knob shown.

SOLUTION



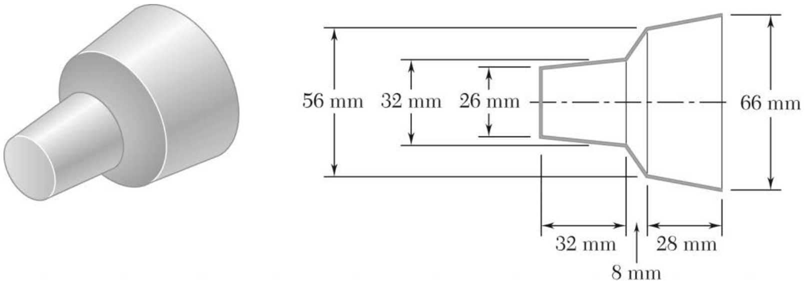
Area is obtained by rotating lines shown about the *x*-axis.

	L, in.	$\overline{y}$, in.	$\overline{y}L$, in^2
1	0.5	0.25	0.1250
2	$\frac{\pi}{2}(0.75) = 1.1781$	0.9775	1.1516
3	$\frac{\pi}{2}(0.75) = 1.1781$	0.7725	0.9101
4	0.5	0.25	0.1250
Σ			2.3117

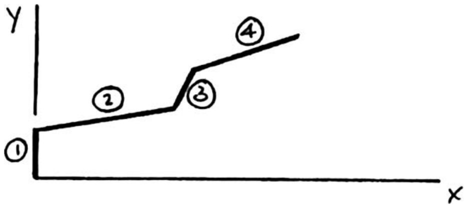
$$A = 2\pi\Sigma\,\overline{y}\,L = 2\pi(2.3117\text{ in}^2) \qquad A = 14.52\text{ in}^2 \blacktriangleleft$$

PROBLEM 5.60

The aluminum shade for the small high-intensity lamp shown has a uniform thickness of 1 mm. Knowing that the density of aluminum is 2800 kg/m^3, determine the mass of the shade.



SOLUTION



The mass of the lamp shade is given by

$$m = \rho V = \rho A t$$

where A is the surface area and t is the thickness of the shade. The area can be generated by rotating the line shown about the x-axis. Applying the first theorem of Pappus Guldinus, we have

$$A = 2\pi \overline{y} L = 2\pi \Sigma \overline{y} L$$
$$= 2\pi(\overline{y}_1 L_1 + \overline{y}_2 L_2 + \overline{y}_3 L_3 + \overline{y}_4 L_4)$$

or

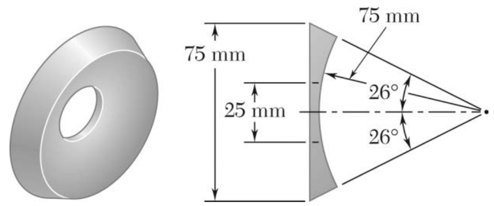
$$A = 2\pi\left[\frac{13\text{ mm}}{2}(13\text{ mm}) + \left(\frac{13+16}{2}\right)\text{mm} \times \sqrt{(32\text{ mm})^2 + (3\text{ mm})^2} + \left(\frac{16+28}{2}\right)\text{mm} \times \sqrt{(8\text{ mm})^2 + (12\text{ mm})^2} + \left(\frac{28+33}{2}\right)\text{mm} \times \sqrt{(28\text{ mm})^2 + (5\text{ mm})^2}\right]$$
$$= 2\pi(84.5 + 466.03 + 317.29 + 867.51)$$
$$= 10{,}903.4\text{ mm}^2$$

Then

$$m = \rho A t$$
$$= (2800\text{ kg/m}^3)(10.9034 \times 10^{-3}\text{ m}^2)(0.001\text{ m})$$

or

$$m = 0.0305\text{ kg} \blacktriangleleft$$

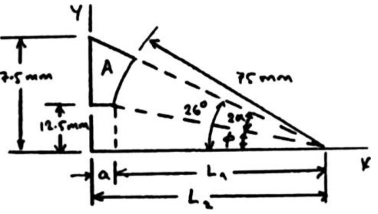


PROBLEM 5.61

The escutcheon (a decorative plate placed on a pipe where the pipe exits from a wall) shown is cast from brass. Knowing that the density of brass is 8470 kg/m^3, determine the mass of the escutcheon.

SOLUTION

The mass of the escutcheon is given by $m = (\text{density})V$, where V is the volume. V can be generated by rotating the area A about the x-axis.



From the figure:

$$L_1 = \sqrt{75^2 - 12.5^2} = 73.9510 \text{ m}$$

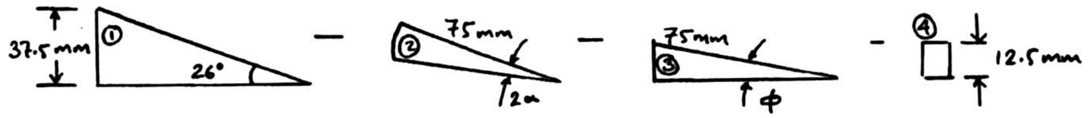
$$L_2 = \frac{37.5}{\tan 26°} = 76.8864 \text{ mm}$$

$$a = L_2 - L_1 = 2.9324 \text{ mm}$$

$$\phi = \sin^{-1}\frac{12.5}{75} = 9.5941°$$

$$\alpha = \frac{26° - 9.5941°}{2} = 8.2030° = 0.143168 \text{ rad}$$

Area A can be obtained by combining the following four areas:



Applying the second theorem of Pappus-Guldinus and using Figure 5.8a, we have

$$V = 2\pi\bar{y}A = 2\pi\Sigma\,\bar{y}A$$

Seg.	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$\frac{1}{2}(76.886)(37.5) = 1441.61$	$\frac{1}{3}(37.5) = 12.5$	18,020.1
2	$-\alpha(75)^2 = -805.32$	$\frac{2(75)\sin\alpha}{3\alpha}\sin(\alpha+\phi) = 15.2303$	−12,265.3
3	$-\frac{1}{2}(73.951)(12.5) = -462.19$	$\frac{1}{3}(12.5) = 4.1667$	−1925.81
4	$-(2.9354)(12.5) = -36.693$	$\frac{1}{2}(12.5) = 6.25$	−229.33
Σ			3599.7

PROBLEM 5.61 (Continued)

Then $V = 2\pi\Sigma\overline{y}A$

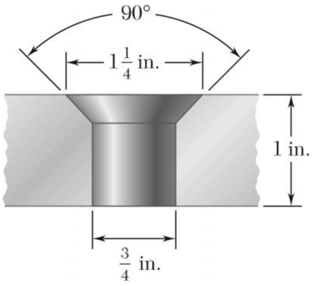
$= 2\pi(3599.7\text{ mm}^3)$

$= 22{,}618\text{ mm}^3$

$m = (\text{density})V$

$= (8470\text{ kg/m}^3)(22.618\times10^{-6}\text{ m}^3)$

$= 0.191574\text{ kg}$ or $m = 0.1916\text{ kg}$ ◀

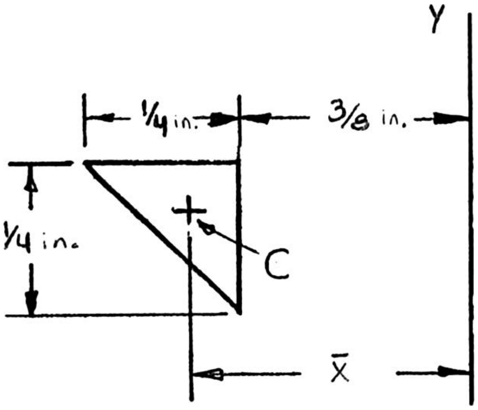


PROBLEM 5.62

A $\frac{3}{4}$- in.-diameter hole is drilled in a piece of 1-in.-thick steel; the hole is then countersunk as shown. Determine the volume of steel removed during the countersinking process.

SOLUTION

The required volume can be generated by rotating the area shown about the *y*-axis. Applying the second theorem of Pappus-Guldinus, we have



$$V = 2\pi \bar{x} A$$

$$= 2\pi\left[\frac{3}{8}+\frac{1}{3}\left(\frac{1}{4}\right)\text{in.}\right]\times\left[\frac{1}{2}\times\frac{1}{4}\text{ in.}\times\frac{1}{4}\text{ in.}\right]$$

$V = 0.0900\,\text{in}^3$ ◀

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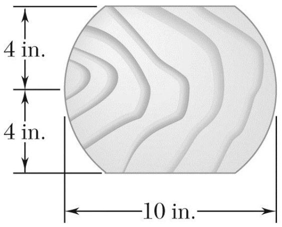
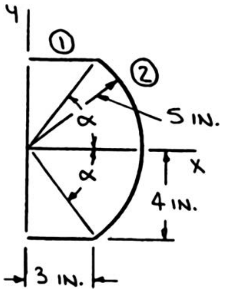


PROBLEM 5.63

Knowing that two equal caps have been removed from a 10-in.-diameter wooden sphere, determine the total surface area of the remaining portion.

SOLUTION

The surface area can be generated by rotating the line shown about the *y*-axis. Applying the first theorem of Pappus-Guldinus, we have



$$A = 2\pi \bar{X} L = 2\pi \Sigma \bar{x} L$$
$$= 2\pi(2\bar{x}_1 L_1 + \bar{x}_2 L_2)$$

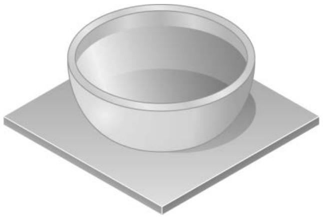
Now
$$\tan\alpha = \frac{4}{3}$$

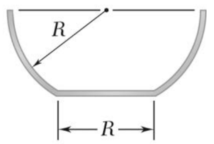
or
$$\alpha = 53.130^\circ$$

Then
$$\bar{x}_2 = \frac{5\text{ in.} \times \sin 53.130^\circ}{53.130^\circ \times \frac{\pi}{180^\circ}}$$
$$= 4.3136\text{ in.}$$

and
$$L_2 = 2\left(53.130^\circ \times \frac{\pi}{180^\circ}\right)(5\text{ in.})$$
$$= 9.2729\text{ in.}$$
$$A = 2\pi\left[2\left(\frac{3}{2}\text{ in.}\right)(3\text{ in.}) + (4.3136\text{ in.})(9.2729\text{ in.})\right]$$

or
$$A = 308\text{ in}^2 \blacktriangleleft$$



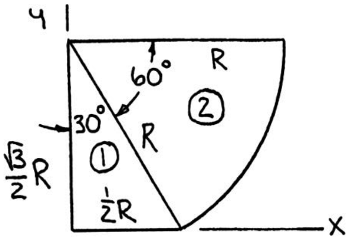


PROBLEM 5.64

Determine the capacity, in liters, of the punch bowl shown if $R = 250$ mm.

SOLUTION

The volume can be generated by rotating the triangle and circular sector shown about the *y*-axis. Applying the second theorem of Pappus-Guldinus and using Figure 5.8a, we have

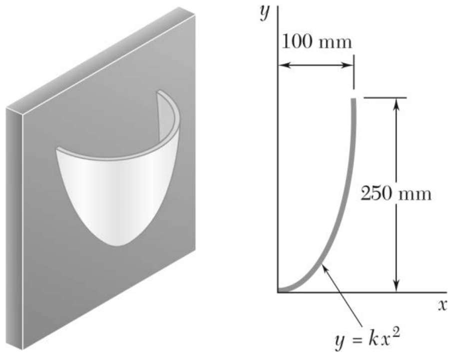


$$
\begin{aligned}
V &= 2\pi\bar{x}A = 2\pi\Sigma\bar{x}A \\
&= 2\pi(\bar{x}_1A_1 + \bar{x}_2A_2) \\
&= 2\pi\left[\left(\frac{1}{3}\times\frac{1}{2}R\right)\left(\frac{1}{2}\times\frac{1}{2}R\times\frac{\sqrt{3}}{2}R\right)+\left(\frac{2R\sin 30^\circ}{3\times\frac{\pi}{6}}\right)\left(\frac{\pi}{6}R^2\right)\right] \\
&= 2\pi\left(\frac{R^3}{16\sqrt{3}}+\frac{R^3}{2\sqrt{3}}\right) \\
&= \frac{3\sqrt{3}}{8}\pi R^3 \\
&= \frac{3\sqrt{3}}{8}\pi(0.25\text{ m})^3 \\
&= 0.031883\text{ m}^3
\end{aligned}
$$

Since $10^3\,\text{l} = 1\text{ m}^3$

$$V = 0.031883\text{ m}^3\times\frac{10^3\,\text{l}}{1\text{ m}^3}$$

$V = 31.9\,\text{l}$ ◀



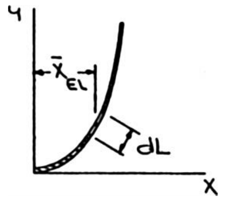
PROBLEM 5.65*

The shade for a wall-mounted light is formed from a thin sheet of translucent plastic. Determine the surface area of the outside of the shade, knowing that it has the parabolic cross section shown.

SOLUTION

First note that the required surface area A can be generated by rotating the parabolic cross section through π radians about the y-axis. Applying the first theorem of Pappus-Guldinus, we have

$$A = \pi \bar{x} L$$



Now at $x = 100 \text{ mm}, \qquad y = 250 \text{ mm}$

$$250 = k(100)^2 \quad \text{or} \quad k = 0.025 \text{ mm}^{-1}$$

and $x_{EL} = x$

$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

where $\dfrac{dy}{dx} = 2kx$

Then $dL = \sqrt{1 + 4k^2x^2}\, dx$

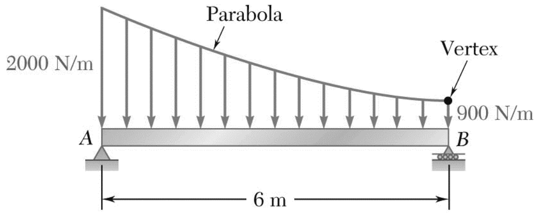
We have $xL = \int x_{EL}\, dL = \int_0^{100} x\left(\sqrt{1+4k^2x^2 dx}\right)$

$$xL = \left[\frac{1}{3}\frac{1}{4k^2}(1+4k^2x^2)^{3/2}\right]_0^{100}$$

$$= \frac{1}{12}\frac{1}{(0.025)^2}\left\{[1+4(0.025)^2(100)^2]^{3/2} - (1)^{3/2}\right\}$$

$$= 17{,}543.3 \text{ mm}^2$$

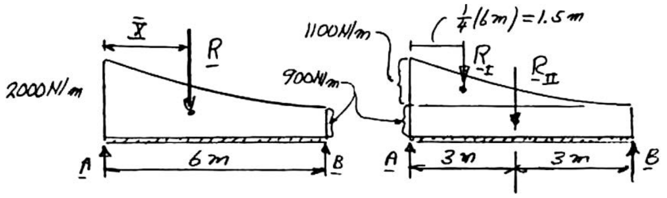
Finally, $A = \pi(17{,}543.3 \text{ mm}^2)$ or $A = 55.1 \times 10^3 \text{ mm}^2$ ◀



PROBLEM 5.66

For the beam and loading shown, determine (*a*) the magnitude and location of the resultant of the distributed load, (*b*) the reactions at the beam supports.

SOLUTION



(*a*) $$R_{\text{I}} = \frac{1}{3}(1100\text{ N/m})(6\text{ m}) = 2200\text{ N}$$

$$R_{\text{II}} = (900\text{ N/m})(6\text{ m}) = 5400\text{ N}$$

$$R = R_{\text{I}} + R_{\text{II}} = 2200 + 5400 = 7600\text{ N}$$

$$XR = \Sigma xR: \quad X(7600) = (2200)(1.5) + (5400)(3)$$

$$X = 2.5658\text{ m}$$

$\mathbf{R} = 7.60\text{ kN}\downarrow, \quad X = 2.57\text{ m}$ ◀

(*b*) $$+\circlearrowleft \Sigma M_A = 0: \quad B(6\text{ m}) - (7600\text{ N})(2.5658\text{ m}) = 0$$

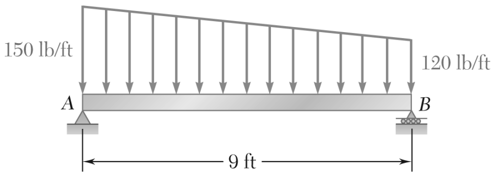
$$B = 3250.0\text{ N}$$

$\mathbf{B} = 3.25\text{ kN}\uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A + 3250.0\text{ N} - 7600\text{ N} = 0$$

$$A = 4350.0\text{ N}$$

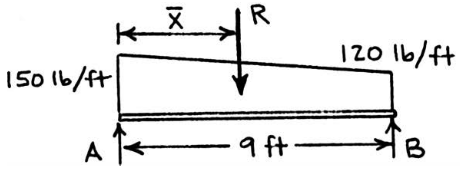
$\mathbf{A} = 4.35\text{ kN}\uparrow$ ◀

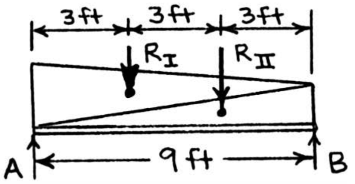


PROBLEM 5.67

For the beam and loading shown, determine (*a*) the magnitude and location of the resultant of the distributed load, (*b*) the reactions at the beam supports.

SOLUTION





$$R_{\text{I}} = \frac{1}{2}(150\,\text{lb/ft})(9\text{ ft}) = 675\text{ lb}$$

$$R_{\text{II}} = \frac{1}{2}(120\text{ lb/ft})(9\,\text{ft}) = 540\,\text{lb}$$

$$R = R_{\text{I}} + R_{\text{II}} = 675 + 540 = 1215\text{ lb}$$

$$\bar{X}R = \Sigma\bar{x}\,R: \quad \bar{X}(1215) = (3)(675) + (6)(540) \quad \bar{X} = 4.3333\text{ ft}$$

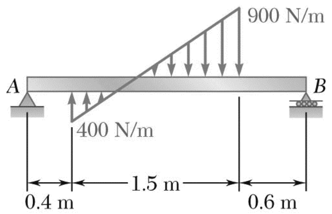
(*a*) $\mathbf{R} = 1215\text{ lb}\downarrow \quad \bar{X} = 4.33\text{ ft}$ ◀

(*b*) Reactions: $+\circlearrowleft \Sigma M_A = 0: \quad B(9\text{ ft}) - (1215\text{ lb})(4.3333\text{ ft}) = 0$

$B = 585.00\text{ lb}$ $\mathbf{B} = 585\text{ lb}\uparrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad A + 585.00\text{ lb} - 1215\text{ lb} = 0$

$A = 630.00\text{ lb}$ $\mathbf{A} = 630\text{ lb}\uparrow$ ◀

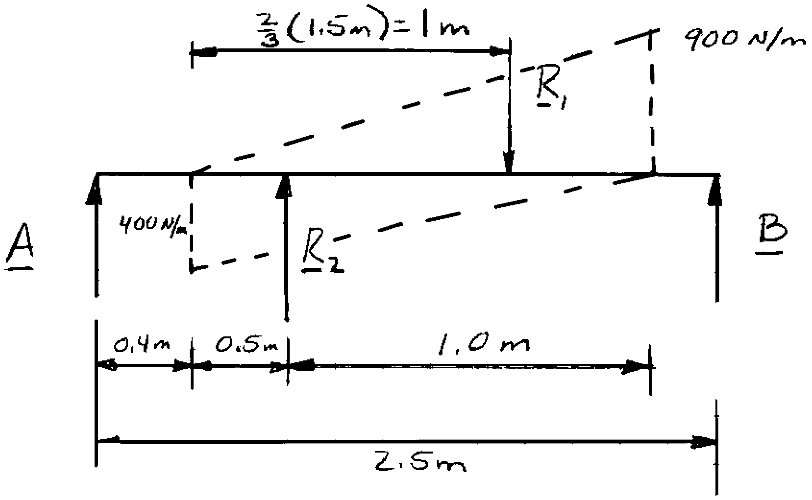


PROBLEM 5.68

Determine the reactions at the beam supports for the given loading.

SOLUTION

First replace the given loading by the loadings shown below. Both loading are equivalent since they are both defined by a linear relation between load and distance and have the same values at the end points.



$$R_1 = \frac{1}{2}(900\text{ N/m})(1.5\text{ m}) = 675\text{ N}$$

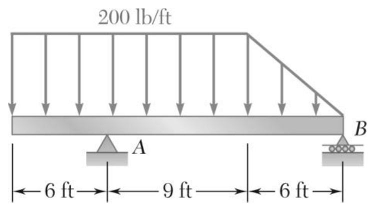
$$R_2 = \frac{1}{2}(400\text{ N/m})(1.5\text{ m}) = 300\text{ N}$$

$+\circlearrowleft \Sigma M_A = 0$: $\quad -(675\text{ N})(1.4\text{ m}) + (300\text{ N})(0.9\text{ m}) + B(2.5\text{ m}) = C$

$B = 270\text{ N}$ $\qquad$ $\mathbf{B} = 270\text{ N}\uparrow$ ◀

$+\uparrow \Sigma F_y = 0$: $\quad A - 675\text{ N} + 300\text{ N} + 270\text{ N} = 0$

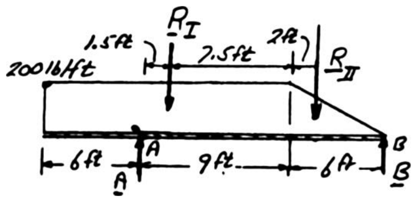
$A = 105.0\text{ N}$ $\qquad$ $\mathbf{A} = 105.0\text{ N}\uparrow$ ◀



PROBLEM 5.69

Determine the reactions at the beam supports for the given loading.

SOLUTION



$$R_{\text{I}} = (200\text{ lb/ft})(15\text{ ft})$$

$$R_{\text{I}} = 3000\text{ lb}$$

$$R_{\text{II}} = \frac{1}{2}(200\text{ lb/ft})(6\text{ ft})$$

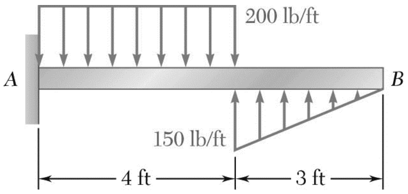
$$R_{\text{II}} = 600\text{ lb}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(3000\text{ lb})(1.5\text{ ft}) - (600\text{ lb})(9\text{ ft} + 2\text{ ft}) + B(15\text{ ft}) = 0$$

$$B = 740\text{ lb} \qquad \mathbf{B} = 740\text{ lb}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A + 740\text{ lb} - 3000\text{ lb} - 600\text{ lb} = 0$$

$$A = 2860\text{ lb} \qquad \mathbf{A} = 2860\text{ lb}\uparrow \blacktriangleleft$$



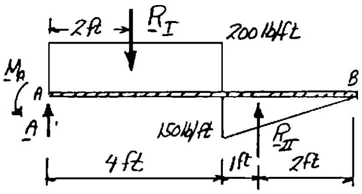
PROBLEM 5.70

Determine the reactions at the beam supports for the given loading.

SOLUTION

$$R_{\mathrm{I}} = (200\ \text{lb/ft})(4\ \text{ft}) = 800\ \text{lb}$$

$$R_{\mathrm{II}} = \frac{1}{2}(150\ \text{lb/ft})(3\ \text{ft}) = 225\ \text{lb}$$

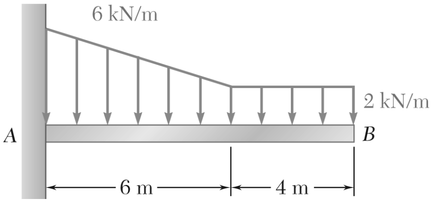


$$+\uparrow \Sigma F_y = 0: \quad A - 800\ \text{lb} + 225\ \text{lb} = 0$$

$$\mathbf{A} = 575\ \text{lb}\uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (800\ \text{lb})(2\ \text{ft}) + (225\ \text{lb})(5\ \text{ft}) = 0$$

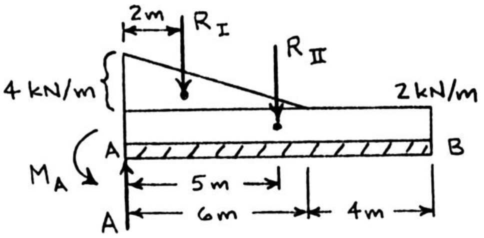
$$\mathbf{M}_A = 475\ \text{lb}\cdot\text{ft} \circlearrowleft \blacktriangleleft$$



PROBLEM 5.71

Determine the reactions at the beam supports for the given loading.

SOLUTION



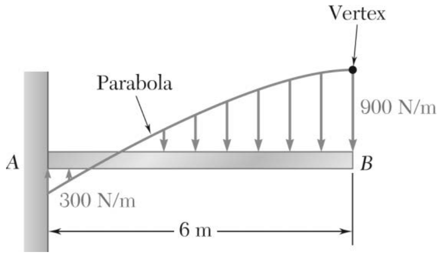
$$R_{\text{I}} = \frac{1}{2}(4\text{ kN/m})(6\text{ m})$$
$$= 12\text{ kN}$$
$$R_{\text{II}} = (2\text{ kN/m})(10\text{ m})$$
$$= 20\text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad A - 12\text{ kN} - 20\text{ kN} = 0$$

$$\mathbf{A} = 32.0\text{ kN}\uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (12\text{ kN})(2\text{ m}) - (20\text{ kN})(5\text{ m}) = 0$$

$$\mathbf{M}_A = 124.0\text{ kN}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

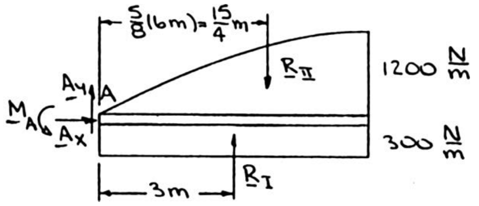


PROBLEM 5.72

Determine the reactions at the beam supports for the given loading.

SOLUTION

First replace the given loading with the loading shown below. The two loadings are equivalent because both are defined by a parabolic relation between load and distance and the values at the end points are the same.



We have $R_{\mathrm{I}} = (6\text{ m})(300\text{ N/m}) = 1800\text{ N}$

$$R_{\mathrm{II}} = \frac{2}{3}(6\text{ m})(1200\text{ N/m}) = 4800\text{ N}$$

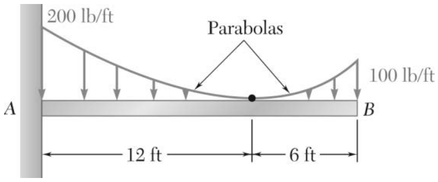
Then $\xrightarrow{+} \Sigma F_x = 0{:}\quad A_x = 0$

$$+\uparrow \Sigma F_y = 0{:}\quad A_y + 1800\text{ N} - 4800\text{ N} = 0$$

or $A_y = 3000\text{ N}$ $\qquad \mathbf{A} = 3.00\text{ kN}\uparrow$ ◀

$$+\circlearrowleft \Sigma M_A = 0{:}\quad M_A + (3\text{ m})(1800\text{ N}) - \left(\frac{15}{4}\text{ m}\right)(4800\text{ N}) = 0$$

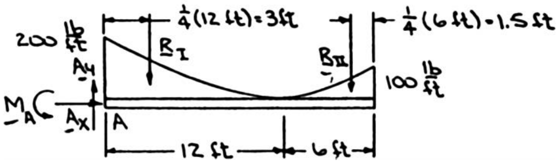
or $M_A = 12.6\text{ kN}\cdot\text{m}$ $\qquad \mathbf{M}_A = 12.60\text{ kN}\cdot\text{m}\circlearrowleft$ ◀



PROBLEM 5.73

Determine the reactions at the beam supports for the given loading.

SOLUTION



We have $R_{\mathrm{I}} = \dfrac{1}{3}(12\text{ ft})(200\text{ lb/ft}) = 800\text{ lb}$

$$R_{\mathrm{II}} = \frac{1}{3}(6\text{ ft})(100\text{ lb/ft}) = 200\text{ lb}$$

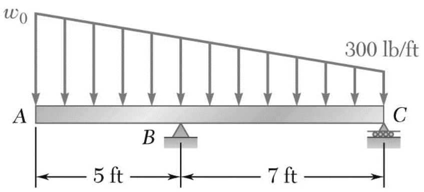
Then $\xrightarrow{+} \Sigma F_x = 0\!: \quad A_x = 0$

$$+\!\uparrow \Sigma F_y = 0\!: \quad A_y - 800\text{ lb} - 200\text{ lb} = 0$$

or $A_y = 1000\text{ lb}$ $\qquad$ $\mathbf{A} = 1000\text{ lb}\uparrow$ ◀

$$+\circlearrowleft \Sigma M_A = 0\!: \quad M_A - (3\text{ ft})(800\text{ lb}) - (16.5\text{ ft})(200\text{ lb}) = 0$$

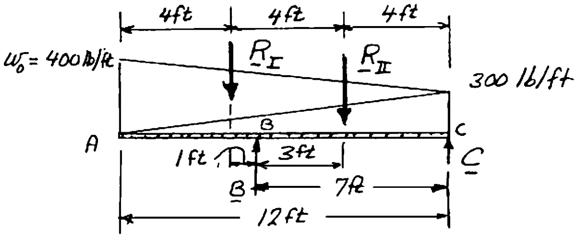
or $M_A = 5700\text{ lb}\cdot\text{ft}$ $\qquad$ $\mathbf{M}_A = 5700\text{ lb}\cdot\text{ft}\circlearrowleft$ ◀



PROBLEM 5.74

Determine the reactions at the beam supports for the given loading when $w_O = 400$ lb/ft.

SOLUTION



$$R_{\mathrm{I}} = \frac{1}{2} w_O (12\text{ ft}) = \frac{1}{2}(400\text{ lb/ft})(12\text{ ft}) = 2400\text{ lb}$$

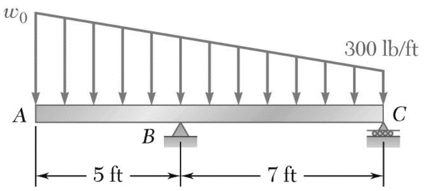
$$R_{\mathrm{II}} = \frac{1}{2}(300\text{ lb/ft})(12\text{ ft}) = 1800\text{ lb}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (2400\text{ lb})(1\text{ ft}) - (1800\text{ lb})(3\text{ ft}) + C(7\text{ ft}) = 0$$

$C = 428.57$ lb **C** $= 429$ lb ↑ ◀

$$+\uparrow \Sigma F_y = 0: \quad B + 428.57\text{ lb} - 2400\text{ lb} - 1800\text{ lb} = 0$$

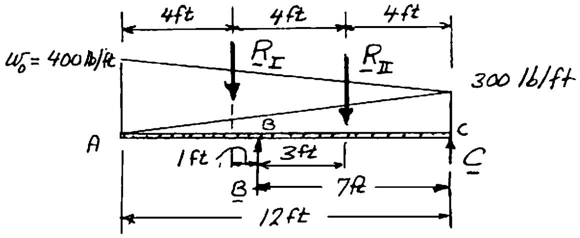
$B = 3771$ lb **B** $= 3770$ lb ↑ ◀



PROBLEM 5.75

Determine (*a*) the distributed load w_O at the end *A* of the beam *ABC* for which the reaction at *C* is zero, (*b*) the corresponding reaction at *B*.

SOLUTION



For w_O, $$R_\text{I} = \frac{1}{2}w_O(12 \text{ ft}) = 6\,w_O$$

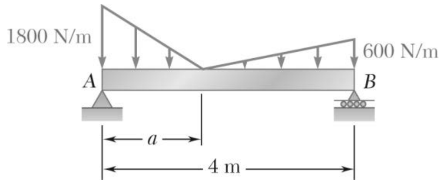
$$R_\text{II} = \frac{1}{2}(300 \text{ lb/ft})(12 \text{ ft}) = 1800 \text{ lb}$$

(*a*) For $C = 0$, $+\circlearrowleft \Sigma M_B = 0$: $(6\,w_O)(1 \text{ ft}) - (1800 \text{ lb})(3 \text{ ft}) = 0$ $\quad w_O = 900 \text{ lb/ft}$ ◀

(*b*) Corresponding value of R_I:

$$R_\text{I} = 6(900) = 5400 \text{ lb}$$

$+\uparrow \Sigma F_y = 0$: $B - 5400 \text{ lb} - 1800 \text{ lb} = 0$ $\quad \mathbf{B} = 7200 \text{ lb} \uparrow$ ◀

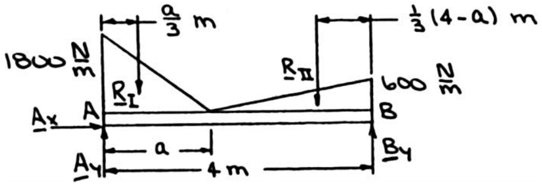


PROBLEM 5.76

Determine (*a*) the distance *a* so that the vertical reactions at supports *A* and *B* are equal, (*b*) the corresponding reactions at the supports.

SOLUTION

(*a*)



We have $$R_{\text{I}} = \frac{1}{2}(a\text{ m})(1800\text{ N/m}) = 900a\text{ N}$$

$$R_{\text{II}} = \frac{1}{2}[(4-a)\text{ m}](600\text{ N/m}) = 300(4-a)\text{ N}$$

Then $$+\uparrow \Sigma F_y = 0\text{:}\quad A_y - 900a - 300(4-a) + B_y = 0$$

or $$A_y + B_y = 1200 + 600a$$

Now $$A_y = B_y \Rightarrow A_y = B_y = 600 + 300a\text{ (N)} \qquad (1)$$

Also, $$+\circlearrowleft \Sigma M_B = 0\text{:}\quad -(4\text{ m})A_y + \left[\left(4 - \frac{a}{3}\right)\text{m}\right][(900a)\text{ N}]$$

$$+\left[\frac{1}{3}(4-a)\text{ m}\right][300(4-a)\text{ N}] = 0$$

or $$A_y = 400 + 700a - 50a^2 \qquad (2)$$

Equating Eqs. (1) and (2), $$600 + 300a = 400 + 700a - 50a^2$$

or $$a^2 - 8a + 4 = 0$$

Then $$a = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(4)}}{2}$$

or $$a = 0.53590\text{ m} \qquad a = 7.4641\text{ m}$$

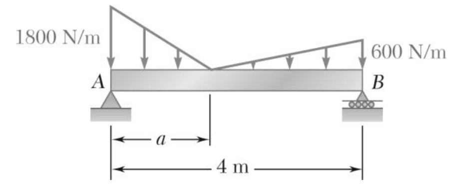
Now $$a \le 4\text{ m} \Rightarrow \qquad a = 0.536\text{ m} \blacktriangleleft$$

(*b*) We have $\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

From Eq. (1):

$$\begin{aligned} A_y &= B_y \\ &= 600 + 300(0.53590) \\ &= 761\text{ N} \end{aligned}$$

$\mathbf{A} = \mathbf{B} = 761\text{ N}\uparrow$ ◀

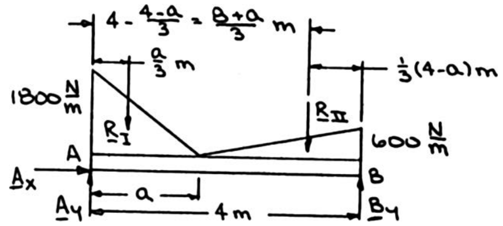


PROBLEM 5.77

Determine (*a*) the distance *a* so that the reaction at support *B* is minimum, (*b*) the corresponding reactions at the supports.

SOLUTION

(*a*)



We have $$R_\text{I}=\frac{1}{2}(a\text{ m})(1800\text{ N/m})=900a\text{ N}$$

$$R_\text{II}=\frac{1}{2}[(4-a)\text{m}](600\text{ N/m})=300(4-a)\text{ N}$$

Then $$+\circlearrowleft\Sigma M_A=0:\quad -\left(\frac{a}{3}\text{m}\right)(900a\text{ N})-\left(\frac{8+a}{3}\text{m}\right)[300(4-a)\text{N}]+(4\text{ m})B_y=0$$

or $$B_y=50a^2-100a+800 \tag{1}$$

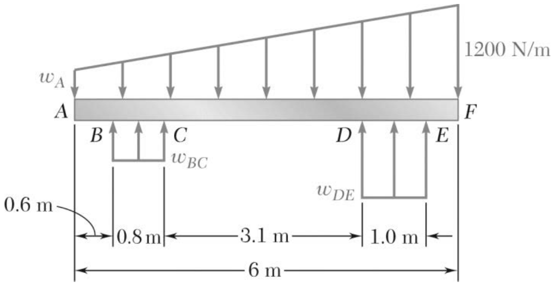
Then $$\frac{dB_y}{da}=100a-100=0$$ or $a=1.000$ m ◀

(*b*) From Eq. (1): $$B_y=50(1)^2-100(1)+800=750\text{ N}$$ $\mathbf{B}=750\text{ N}\uparrow$ ◀

and $$\xrightarrow{+}\Sigma F_x=0:\quad A_x=0$$

$$+\uparrow\Sigma F_y=0:\quad A_y-900(1)\text{ N}-300(4-1)\text{ N}+750\text{ N}=0$$

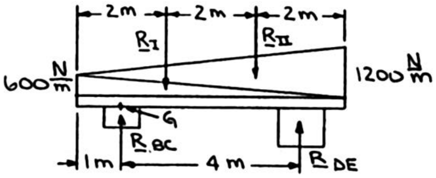
or $$A_y=1050\text{ N}$$ $\mathbf{A}=1050\text{ N}\uparrow$ ◀



PROBLEM 5.78

A beam is subjected to a linearly distributed downward load and rests on two wide supports *BC* and *DE*, which exert uniformly distributed upward loads as shown. Determine the values of w_{BC} and w_{DE} corresponding to equilibrium when $w_A = 600$ N/m.

SOLUTION



We have $$R_{\mathrm{I}} = \frac{1}{2}(6\text{ m})(600\text{ N/m}) = 1800\text{ N}$$

$$R_{\mathrm{II}} = \frac{1}{2}(6\text{ m})(1200\text{ N/m}) = 3600\text{ N}$$

$$R_{BC} = (0.8\text{ m})(w_{BC}\text{ N/m}) = (0.8w_{BC})\text{ N}$$

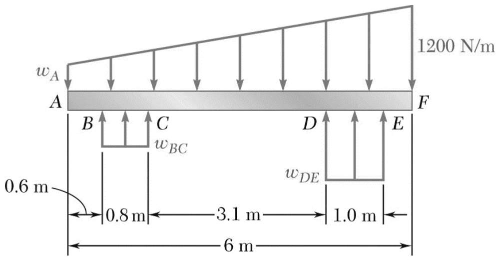
$$R_{DE} = (1.0\text{ m})(w_{DE}\text{ N/m}) = (w_{DE})\text{ N}$$

Then $$+\circlearrowleft \Sigma M_G = 0: \quad -(1\text{ m})(1800\text{ N}) - (3\text{ m})(3600\text{ N}) + (4\text{ m})(w_{DE}\text{ N}) = 0$$

or $$w_{DE} = 3150\text{ N/m} \blacktriangleleft$$

and $$+\uparrow \Sigma F_y = 0: \quad (0.8w_{BC})\text{ N} - 1800\text{ N} - 3600\text{ N} + 3150\text{ N} = 0$$

or $$w_{BC} = 2812.5\text{ N/m} \qquad w_{BC} = 2810\text{ N/m} \blacktriangleleft$$

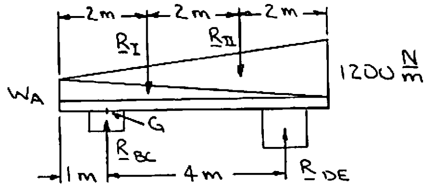


PROBLEM 5.79

A beam is subjected to a linearly distributed downward load and rests on two wide supports *BC* and *DE*, which exert uniformly distributed upward loads as shown. Determine (*a*) the value of w_A so that $w_{BC} = w_{DE}$, (*b*) the corresponding values of w_{BC} and w_{DE}.

SOLUTION

(*a*)



We have $$R_{\text{I}} = \frac{1}{2}(6\text{ m})(w_A\text{ N/m})\cdot(3\,w_A)\text{ N}$$

$$R_{\text{II}} = \frac{1}{2}(6\text{ m})(1200\text{ N/m}) = 3600\text{ N}$$

$$R_{BC} = (0.8\text{ m})(w_{BC}\text{ N/m}) = (0.8w_{BC})\text{ N}$$

$$R_{DE} = (1\text{ m})(w_{DE}\text{ N/m}) = (w_{DE})\text{ N}$$

Then $$+\!\uparrow \Sigma F_y = 0\!: \quad (0.8w_{BC})\text{ N} - (3w_A)\text{ N} - 3600\text{ N} + (w_{DE})\text{ N} = 0$$

or $$0.8w_{BC} + w_{DE} = 3600 + 3w_A$$

Now $$w_{BC} = w_{DE} \Rightarrow w_{BC} = w_{DE} = 2000 + \frac{5}{3}w_A \tag{1}$$

Also, $$+\circlearrowleft \Sigma M_G = 0\!: \quad -(1\text{ m})(3w_A\text{ N}) - (3\text{ m})(3600\text{ N}) + (4\text{ m})(w_{DE}\text{ N}) = 0$$

or $$w_{DE} = 2700 + \frac{3}{4}w_A \tag{2}$$

Equating Eqs. (1) and (2),

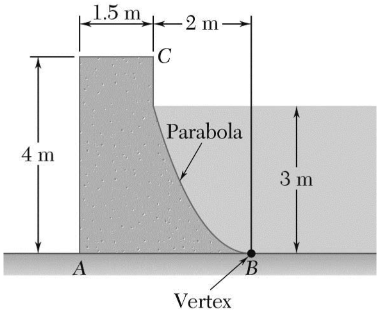
$$2000 + \frac{5}{3}w_A = 2700 + \frac{3}{4}w_A$$

or $$w_A = \frac{8400}{11}\text{ N/m}$$

$w_A = 764\text{ N/m}$ ◀

(*b*) Eq. (1)$\Rightarrow$ $$w_{BC} = w_{DE} = 2000 + \frac{5}{3}\left(\frac{8400}{11}\right)$$

or $w_{BC} = w_{DE} = 3270\text{ N/m}$ ◀

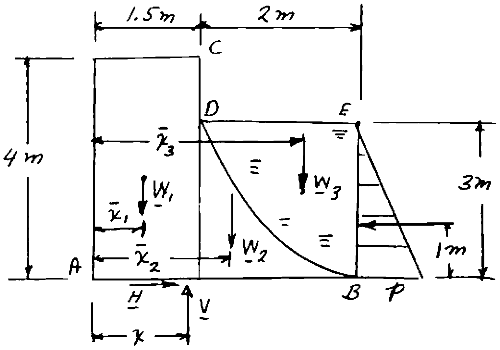


PROBLEM 5.80

The cross section of a concrete dam is as shown. For a 1-m-wide dam section, determine (*a*) the resultant of the reaction forces exerted by the ground on the base *AB* of the dam, (*b*) the point of application of the resultant of part *a*, (*c*) the resultant of the pressure forces exerted by the water on the face *BC* of the dam.

SOLUTION

(*a*) Consider free body made of dam and section *BDE* of water. (Thickness = 1 m)



$$p = (3\text{ m})(10\text{ kg/m}^3)(9.81\text{ m/s}^2)$$

$$W_1 = (1.5\text{ m})(4\text{ m})(1\text{ m})(2.4\times 10^3\text{ kg/m}^3)(9.81\text{ m/s}^2) = 144.26\text{ kN}$$

$$W_2 = \frac{1}{3}(2\text{ m})(3\text{ m})(1\text{ m})(2.4\times 10^3\text{ kg/m}^3)(9.81\text{ m/s}^2) = 47.09\text{ kN}$$

$$W_3 = \frac{2}{3}(2\text{ m})(3\text{ m})(1\text{ m})(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2) = 39.24\text{ kN}$$

$$P = \frac{1}{2}Ap = \frac{1}{2}(3\text{ m})(1\text{ m})(3\text{ m})(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2) = 44.145\text{ kN}$$

$\xrightarrow{+} \Sigma F_x = 0\text{:} \quad H - 44.145\text{ kN} = 0$

$H = 44.145\text{ kN}$ $\qquad$ $\mathbf{H} = 44.1\text{ kN} \longrightarrow$ ◀

$+\!\uparrow \Sigma F_y = 0\text{:} \quad V - 141.26 - 47.09 - 39.24 = 0$

$V = 227.6\text{ kN}$ $\qquad$ $\mathbf{V} = 228\text{ kN} \uparrow$ ◀

PROBLEM 5.80 (Continued)

$$\bar{x}_1 = \frac{1}{2}(1.5\text{ m}) = 0.75\text{ m}$$

$$\bar{x}_2 = 1.5\text{ m} + \frac{1}{4}(2\text{ m}) = 2\text{ m}$$

$$\bar{x}_3 = 1.5\text{ m} + \frac{5}{8}(2\text{ m}) = 2.75\text{ m}$$

$$+\circlearrowleft \Sigma M_A = 0\text{:} \quad xV - \Sigma \bar{x}W + P(1\text{ m}) = 0$$

$$x(227.6\text{ kN}) - (141.26\text{ kN})(0.75\text{ m}) - (47.09\text{ kN})(2\text{ m}) - (39.24\text{ kN})(2.75\text{ m}) + (44.145\text{ kN})(1\text{ m}) = 0$$

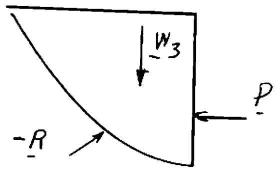
$$x(227.6\text{ kN}) - 105.9 - 94.2 - 107.9 + 44.145 = 0$$

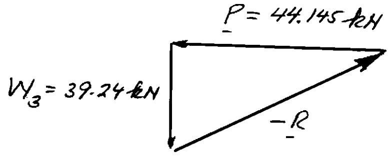
$$x(227.6) - 263.9 = 0$$

$x = 1.159$ m (to right of A) ◀

(*b*) Resultant of face *BC*:

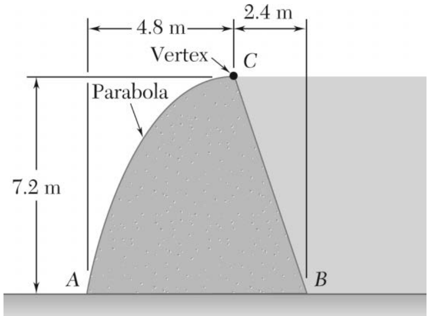
Consider free body of section *BDE* of water.





$-\mathbf{R} = 59.1\text{ kN} \measuredangle 41.6^\circ$

$\mathbf{R} = 59.1\text{ kN} \measuredangle 41.6^\circ$ ◀

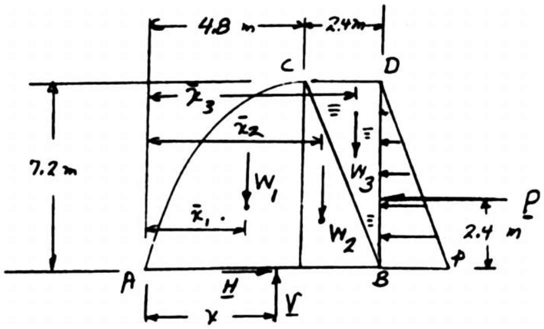


PROBLEM 5.81

The cross section of a concrete dam is as shown. For a 1-m-wide dam section, determine (*a*) the resultant of the reaction forces exerted by the ground on the base *AB* of the dam, (*b*) the point of application of the resultant of part *a*, (*c*) the resultant of the pressure forces exerted by the water on the face *BC* of the dam.

SOLUTION

(*a*) Consider free body made of dam and triangular section of water shown. (Thickness = 1 m.)



$$p = (7.2\,\text{m})(10^3\,\text{kg/m}^3)(9.81\,\text{m/s}^2)$$

$$W_1 = \frac{2}{3}(4.8\text{ m})(7.2\text{ m})(1\text{ m})(2.4\times10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)$$
$$= 542.5\text{ kN}$$

$$W_2 = \frac{1}{2}(2.4\text{ m})(7.2\text{ m})(1\text{ m})(2.4\times10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)$$
$$= 203.4\text{ kN}$$

$$W_3 = \frac{1}{2}(2.4\text{ m})(7.2\text{ m})(1\text{ m})(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)$$
$$= 84.8\text{ kN}$$

$$P = \frac{1}{2}Ap = \frac{1}{2}(7.2\text{ m})(1\text{ m})(7.2\text{ m})(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)$$
$$= 254.3\text{ kN}$$

$\xrightarrow{+} \Sigma F_x = 0$: $H - 254.3\text{ kN} = 0$ $\qquad$ $\mathbf{H} = 254\text{ kN}\longrightarrow$ ◀

$+\!\uparrow \Sigma F_y = 0$: $V - 542.5 - 203.4 - 84.8 = 0$

$V = 830.7\text{ kN}$ $\qquad$ $\mathbf{V} = 831\text{ kN}\uparrow$ ◀

PROBLEM 5.81 (Continued)

(*b*)

$$\bar{x}_1 = \frac{5}{8}(4.8\text{ m}) = 3\text{ m}$$

$$\bar{x}_2 = 4.8 + \frac{1}{3}(2.4) = 5.6\text{ m}$$

$$\bar{x}_3 = 4.8 + \frac{2}{3}(2.4) = 6.4\text{ m}$$

$$+\circlearrowleft \Sigma M_A = 0\text{:}\quad xV - \Sigma\bar{x}W + P(2.4\text{ m}) = 0$$

$$x(830.7\text{ kN}) - (3\text{ m})(542.5\text{ kN}) - (5.6\text{ m})(203.4\text{ kN})$$
$$- (6.4\text{ m})(84.8\text{ kN}) + (2.4\text{ m})(254.3\text{ kN}) = 0$$

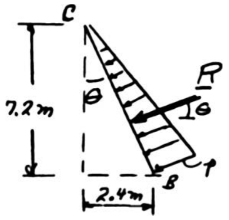
$$x(830.7) - 1627.5 - 1139.0 - 542.7 + 610.3 = 0$$

$$x(830.7) - 2698.9 = 0$$

$x = 3.25$ m (to right of A) ◀

(*c*) Resultant on face *BC*:

Direct computation:



$$P = \rho g h = (10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(7.2\text{ m})$$

$$P = 70.63\text{ kN/m}^2$$

$$BC = \sqrt{(2.4)^2 + (7.2)^2}$$

$$= 7.589\text{ m}$$

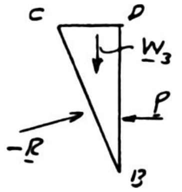
$$\theta = 18.43°$$

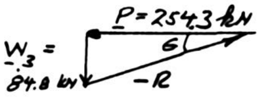
$$R = \frac{1}{2}PA$$

$$= \frac{1}{2}(70.63\text{ kN/m}^2)(7.589\text{ m})(1\text{ m})$$

$\mathbf{R} = 268$ kN ↙ 18.43° ◀

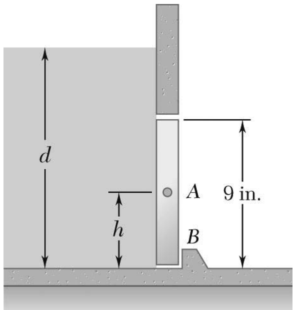
Alternate computation: Use free body of water section *BCD*.





$-\mathbf{R} = 268$ kN ↗ 18.43°

$\mathbf{R} = 268$ kN ↙ 18.43° ◀



PROBLEM 5.82

An automatic valve consists of a 9 × 9-in. square plate that is pivoted about a horizontal axis through A located at a distance $h = 3.6$ in. above the lower edge. Determine the depth of water d for which the valve will open.

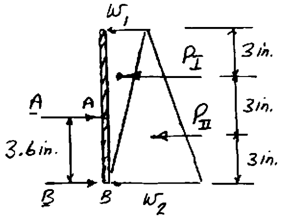
SOLUTION

Since valve is 9 in. wide, $w = 9p = 9\gamma h$, where all dimensions are in inches.

$$w_1 = 9\gamma(d-9), \quad w_2 = 9\gamma d$$

$$P_{\text{I}} = \frac{1}{2}(9\text{ in.})w_1 = \frac{1}{2}(9)(9\gamma)(d-9)$$

$$P_{\text{II}} = \frac{1}{2}(9\text{ in.})w_2 = \frac{1}{2}(9)(9\gamma d)$$



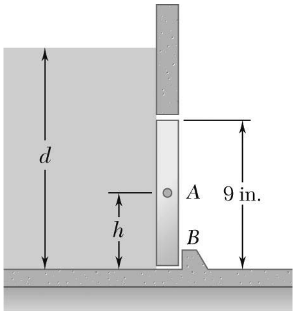
Valve opens when $B = 0$.

$$+\circlearrowleft \Sigma M_A = 0: \quad P_{\text{I}}(6\text{ in.} - 3.6\text{ in.}) - P_{\text{II}}(3.6\text{ in.} - 3\text{ in.}) = 0$$

$$\left[\frac{1}{2}(9)(9\gamma)(d-9)\right](2.4) - \left[\frac{1}{2}(9)(9\gamma d)\right](0.6) = 0$$

$$(d-9)(2.4) - d(0.6) = 0$$

$$1.8d - 21.6 = 0 \qquad d = 12.00\text{ in.} \blacktriangleleft$$



PROBLEM 5.83

An automatic valve consists of a 9 × 9-in. square plate that is pivoted about a horizontal axis through A. If the valve is to open when the depth of water is d = 18 in., determine the distance h from the bottom of the valve to the pivot A.

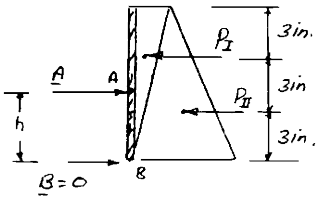
SOLUTION

Since valve is 9 in. wide, $w = 9p = 9\gamma h$, where all dimensions are in inches.

$$w_1 = 9\gamma(d-9)$$
$$w_2 = 9\gamma d$$

For $d = 18$ in.,

$$w_1 = 9\gamma(18-9) = 81\gamma$$
$$w_2 = 9\gamma(18) = 162\gamma$$
$$P_{\text{I}} = \frac{1}{2}(9)(9\gamma)(18-9) = \frac{1}{2}(729\gamma)$$
$$P_{\text{II}} = \frac{1}{2}(9)(9\gamma)(18) = 729\gamma$$



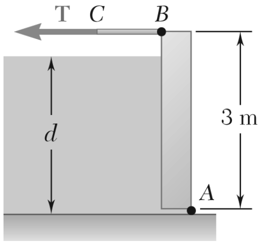
Valve opens when $B = 0$.

$$+\circlearrowleft \Sigma M_A = 0\text{:}\quad P_1(6-h) - P_{\text{II}}(h-3) = 0$$

$$\frac{1}{2}729\gamma(6-h) - 729(h-3) = 0$$

$$3 - \frac{1}{2}h - h + 3 = 0$$

$$6 - 1.5h = 0 \qquad h = 4.00 \text{ in.} \blacktriangleleft$$

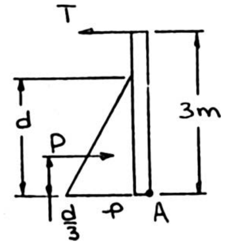


PROBLEM 5.84

The 3 × 4-m side *AB* of a tank is hinged at its bottom *A* and is held in place by a thin rod *BC*. The maximum tensile force the rod can withstand without breaking is 200 kN, and the design specifications require the force in the rod not to exceed 20 percent of this value. If the tank is slowly filled with water, determine the maximum allowable depth of water *d* in the tank.

SOLUTION

Consider the free-body diagram of the side.



We have $P = \frac{1}{2}Ap = \frac{1}{2}A(\rho g d)$

Now $+\circlearrowleft \Sigma M_A = 0: \quad hT - \frac{d}{3}P = 0$

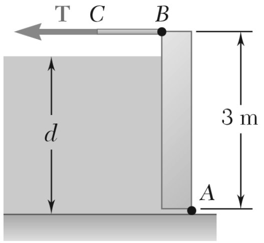
where $h = 3\text{ m}$

Then for d_{max},

$$(3\text{ m})(0.2\ \times 200\times 10^3\text{N}) - \frac{d_{max}}{3}\left[\frac{1}{2}(4\text{ m}\times d_{max})\times(10^3\text{ kg/m}^3\times 9.81\text{ m/s}^2\times d_{max})\right] = 0$$

or $$120\text{ N}\cdot\text{m} - 6.54 d_{max}^3 \text{N/m}^2 = 0$$

or $$d_{max} = 2.64\text{ m} \blacktriangleleft$$

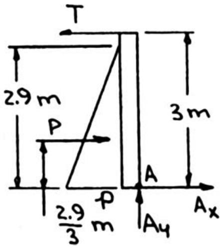


PROBLEM 5.85

The 3 × 4-m side of an open tank is hinged at its bottom *A* and is held in place by a thin rod *BC*. The tank is to be filled with glycerine, whose density is 1263 kg/m^3. Determine the force **T** in the rod and the reactions at the hinge after the tank is filled to a depth of 2.9 m.

SOLUTION

Consider the free-body diagram of the side.



We have $$P = \frac{1}{2}Ap = \frac{1}{2}A(\rho g d)$$

$$= \frac{1}{2}[(2.9\text{ m})(4\text{ m})]\,[(1263\text{ kg/m}^3)(9.81\text{ m/s}^2)(2.9\text{ m})]$$

$$= 208.40\text{ kN}$$

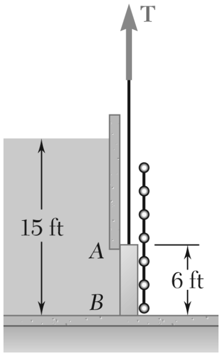
Then $$+\!\uparrow \Sigma F_y = 0:\quad A_y = 0$$

$$+\circlearrowleft \Sigma M_A = 0:\quad (3\text{ m})T - \left(\frac{2.9}{3}\text{m}\right)(208.4\text{ kN}) = 0$$

or $$T = 67.151\text{ kN} \qquad \mathbf{T} = 67.2\text{ kN} \longleftarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0:\quad A_x + 208.40\text{ kN} - 67.151\text{ kN} = 0$$

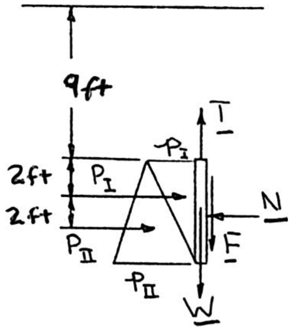
or $$A_x = -141.249\text{ kN} \qquad \mathbf{A} = 141.2\text{ kN} \longleftarrow \blacktriangleleft$$



PROBLEM 5.86

The friction force between a 6 × 6-ft square sluice gate *AB* and its guides is equal to 10 percent of the resultant of the pressure forces exerted by the water on the face of the gate. Determine the initial force needed to lift the gate if it weighs 1000 lb.

SOLUTION



Consider the free-body diagram of the gate.

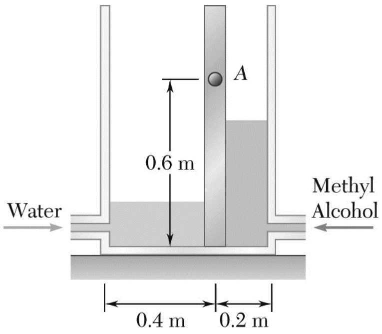
Now
$$P_{\text{I}} = \frac{1}{2}Ap_{\text{I}} = \frac{1}{2}[(6\times 6)\ \text{ft}^2][(62.4\ \text{lb/ft}^3)(9\ \text{ft})]$$
$$= 10{,}108.8\ \text{lb}$$

$$P_{\text{II}} = \frac{1}{2}Ap_{\text{II}} = \frac{1}{2}[(6\times 6)\ \text{ft}^2][(62.4\ \text{lb/ft}^3)(15\ \text{ft})]$$
$$= 16{,}848\ \text{lb}$$

Then
$$F = 0.1P = 0.1(P_{\text{I}} + P_{\text{II}})$$
$$= 0.1(10{,}108.8 + 16{,}848)\ \text{lb}$$
$$= 2695.7\ \text{lb}$$

Finally $+\uparrow \Sigma F_y = 0: \quad T - 2695.7\ \text{lb} - 1000\ \text{lb} = 0$

or $\mathbf{T} = 3.70\ \text{kips} \uparrow$ ◀



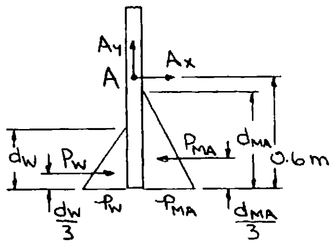
PROBLEM 5.87

A tank is divided into two sections by a 1 × 1-m square gate that is hinged at A. A couple of magnitude 490 N · m is required for the gate to rotate. If one side of the tank is filled with water at the rate of 0.1 m^3/min and the other side is filled simultaneously with methyl alcohol (density $\rho_{ma} = 789\ \text{kg/m}^3$) at the rate of 0.2 m^3/min, determine at what time and in which direction the gate will rotate.

SOLUTION

Consider the free-body diagram of the gate.

First note $V = A_{\text{base}}d$ and $V = rt$.



Then
$$d_W = \frac{0.1\ \text{m}^3/\text{min} \times t(\text{min})}{(0.4\ \text{m})(1\ \text{m})} = 0.25t(\text{m})$$

$$d_{MA} = \frac{0.2\ \text{m}^3/\text{min} \times t(\text{min})}{(0.2\ \text{m})(1\ \text{m})} = t(\text{m})$$

Now
$$P = \frac{1}{2}Ap = \frac{1}{2}A(\rho g h) \text{ so that}$$

$$P_W = \frac{1}{2}[(0.25t)\ \text{m} \times (1\ \text{m})][(10^3\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.25t)\ \text{m}]$$
$$= 306.56t^2\ \text{N}$$

$$P_{MA} = \frac{1}{2}[(t)\ \text{m} \times (1\ \text{m})][(789\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(t)\ \text{m}]$$
$$= 3870t^2\ \text{N}$$

Now assume that the gate will rotate clockwise and when $d_{MA} \le 0.6$ m. When rotation of the gate is impending, we require

$$\Sigma M_A:\quad M_R = \left(0.6\ \text{m} - \frac{1}{3}d_{MA}\right)P_{MA} - \left(0.6\ \text{m} - \frac{1}{3}d_W\right)P_W$$

Substituting

$$490\ \text{N}\cdot\text{m} = \left(0.6 - \frac{1}{3}t\right)\text{m} \times (3870t^2)\ \text{N} - \left(0.6 - \frac{1}{3}\times 0.25t\right)\text{m} \times (306.56t^2)\ \text{N}$$

Simplifying $1264.45t^3 - 2138.1t^2 + 490 = 0$

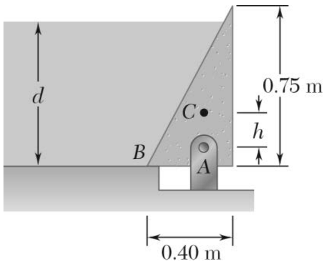
Solving (positive roots only)

$$t = 0.59451 \text{ min} \;\text{ and }\; t = 1.52411 \text{ min}$$

Now check assumption using the smaller root. We have

$$d_{MA} = (t)\text{ m} = 0.59451\text{ m} < 0.6\text{ m} \qquad \therefore\; t = 0.59451\text{min} = 35.7\text{ s} \blacktriangleleft$$

and the gate rotates clockwise. ◀



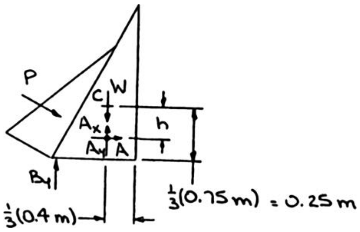
PROBLEM 5.88

A prismatically shaped gate placed at the end of a freshwater channel is supported by a pin and bracket at *A* and rests on a frictionless support at *B*. The pin is located at a distance $h = 0.10$ m below the center of gravity *C* of the gate. Determine the depth of water *d* for which the gate will open.

SOLUTION

First note that when the gate is about to open (clockwise rotation is impending), $B_y \longrightarrow 0$ and the line of action of the resultant **P** of the pressure forces passes through the pin at *A*. In addition, if it is assumed that the gate is homogeneous, then its center of gravity *C* coincides with the centroid of the triangular area. Then

$$a = \frac{d}{3} - (0.25 - h)$$

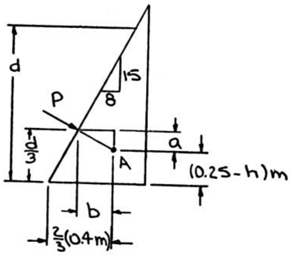


and

$$b = \frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)$$

Now

$$\frac{a}{b} = \frac{8}{15}$$



so that

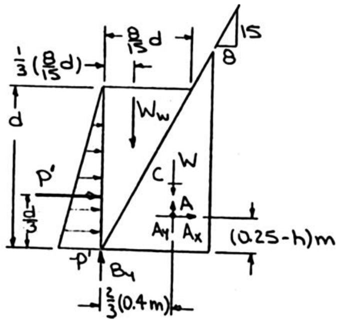
$$\frac{\frac{d}{3} - (0.25 - h)}{\frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)} = \frac{8}{15}$$

Simplifying yields

$$\frac{289}{45}d + 15h = \frac{70.6}{12} \qquad (1)$$

Alternative solution:

Consider a free body consisting of a 1-m thick section of the gate and the triangular section *BDE* of water above the gate.



Now

$$P' = \frac{1}{2}Ap' = \frac{1}{2}(d \times 1\text{ m})(\rho g d)$$

$$= \frac{1}{2}\rho g d^2 \text{ (N)}$$

$$W' = \rho g V = \rho g\left(\frac{1}{2} \times \frac{8}{15}d \times d \times 1\text{ m}\right)$$

$$= \frac{4}{15}\rho g d^2 \text{ (N)}$$

PROBLEM 5.88 (Continued)

Then with $B_y = 0$ (as explained above), we have

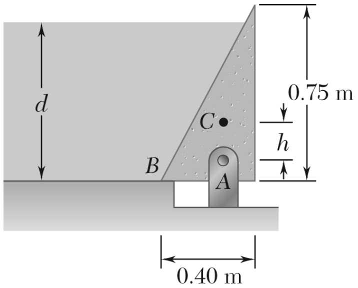
$$+\circlearrowleft \Sigma M_A = 0: \quad \left[\frac{2}{3}(0.4) - \frac{1}{3}\left(\frac{8}{15}d\right)\right]\left(\frac{4}{15}\rho g d^2\right) - \left[\frac{d}{3} - (0.25 - h)\right]\left(\frac{1}{2}\rho g d^2\right) = 0$$

Simplifying yields $$\frac{289}{45}d + 15h = \frac{70.6}{12}$$

as above.

Find d: $$h = 0.10 \text{ m}$$

Substituting into Eq. (1), $$\frac{289}{45}d + 15(0.10) = \frac{70.6}{12}$$ or $d = 0.683 \text{ m}$ ◀



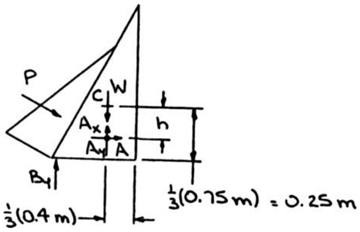
PROBLEM 5.89

A prismatically shaped gate placed at the end of a freshwater channel is supported by a pin and bracket at *A* and rests on a frictionless support at *B*. Determine the distance *h* if the gate is to open when $d = 0.75$ m.

SOLUTION

First note that when the gate is about to open (clockwise rotation is impending), $B_y \longrightarrow 0$ and the line of action of the resultant **P** of the pressure forces passes through the pin at *A*. In addition, if it is assumed that the gate is homogeneous, then its center of gravity *C* coincides with the centroid of the triangular area. Then

$$a = \frac{d}{3} - (0.25 - h)$$

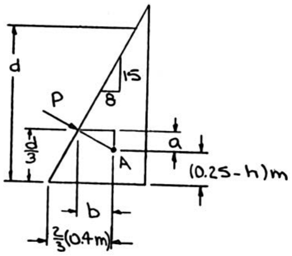


and

$$b = \frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)$$

Now

$$\frac{a}{b} = \frac{8}{15}$$



so that

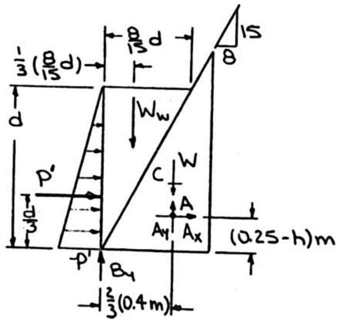
$$\frac{\frac{d}{3} - (0.25 - h)}{\frac{2}{3}(0.4) - \frac{8}{15}\left(\frac{d}{3}\right)} = \frac{8}{15}$$

Simplifying yields

$$\frac{289}{45}d + 15h = \frac{70.6}{12} \tag{1}$$

Alternative solution:

Consider a free body consisting of a 1-m thick section of the gate and the triangular section *BDE* of water above the gate.



Now

$$P' = \frac{1}{2}Ap' = \frac{1}{2}(d \times 1\text{ m})(\rho g d)$$

$$= \frac{1}{2}\rho g d^2 \text{ (N)}$$

$$W' = \rho g V = \rho g\left(\frac{1}{2} \times \frac{8}{15}d \times d \times 1\text{ m}\right)$$

$$= \frac{4}{15}\rho g d^2 \text{ (N)}$$

PROBLEM 5.89 (Continued)

Then with $B_y = 0$ (as explained above), we have

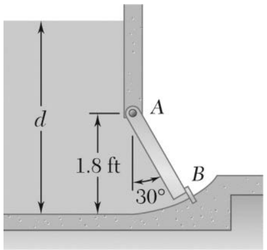
$$+\circlearrowleft \Sigma M_A = 0: \quad \left[\frac{2}{3}(0.4) - \frac{1}{3}\left(\frac{8}{15}d\right)\right]\left(\frac{4}{15}\rho g d^2\right) - \left[\frac{d}{3} - (0.25 - h)\right]\left(\frac{1}{2}\rho g d^2\right) = 0$$

Simplifying yields $$\frac{289}{45}d + 15h = \frac{70.6}{12}$$

as above.

Find *h*: $$d = 0.75 \text{ m}$$

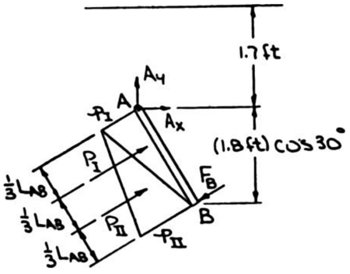
Substituting into Eq. (1), $$\frac{289}{45}(0.75) + 15h = \frac{70.6}{12}$$ or $h = 0.0711 \text{ m}$ ◀



PROBLEM 5.90

The square gate *AB* is held in the position shown by hinges along its top edge *A* and by a shear pin at *B*. For a depth of water $d = 3.5\,\text{ft}$, determine the force exerted on the gate by the shear pin.

SOLUTION



First consider the force of the water on the gate. We have

$$P = \frac{1}{2}Ap$$

$$= \frac{1}{2}A(\gamma h)$$

Then

$$P_{\text{I}} = \frac{1}{2}(1.8\ \text{ft})^2(62.4\ \text{lb/ft}^3)(1.7\ \text{ft})$$

$$= 171.850\ \text{lb}$$

$$P_{\text{II}} = \frac{1}{2}(1.8\ \text{ft})^2(62.4\ \text{lb/ft}^3)\times(1.7 + 1.8\cos 30°)\ \text{ft}$$

$$= 329.43\ \text{lb}$$

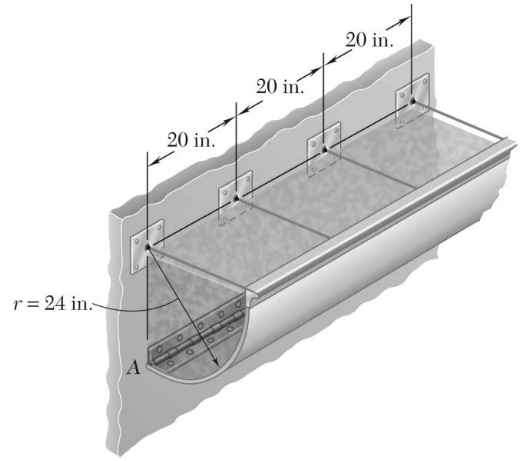
Now

$$\Sigma M_A = 0{:}\quad \left(\frac{1}{3}L_{AB}\right)P_{\text{I}} + \left(\frac{2}{3}L_{AB}\right)P_{\text{II}} - L_{AB}F_B = 0$$

or

$$\frac{1}{3}(171.850\ \text{lb}) + \frac{2}{3}(329.43\ \text{lb}) - F_B = 0$$

or $F_B = 276.90\ \text{lb}$ $\qquad \mathbf{F}_B = 277\ \text{lb}$ ∠ 30.0° ◀

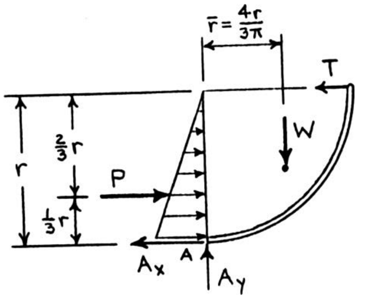


PROBLEM 5.91

A long trough is supported by a continuous hinge along its lower edge and by a series of horizontal cables attached to its upper edge. Determine the tension in each of the cables, at a time when the trough is completely full of water.

SOLUTION

Consider free body consisting of 20-in. length of the trough and water.



$$l = 20\text{-in. length of free body}$$

$$W = \gamma v = \gamma\left[\frac{\pi}{4}r^2 l\right]$$

$$P_A = \gamma r$$

$$P = \frac{1}{2}P_A r l = \frac{1}{2}(\gamma r) r l = \frac{1}{2}\gamma r^2 l$$

$$+\circlearrowleft \Sigma M_A = 0: \quad Tr - Wr - P\left(\frac{1}{3}r\right) = 0$$

$$Tr - \left(\gamma\frac{\pi}{4}r^2 l\right)\left(\frac{4r}{3\pi}\right) - \left(\frac{1}{2}\gamma r^2 l\right)\left(\frac{1}{3}r\right) = 0$$

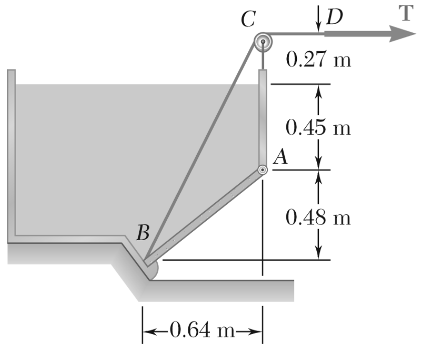
$$T = \frac{1}{3}\gamma r^2 l + \frac{1}{6}\gamma r^2 l = \frac{1}{2}\gamma r^2 l$$

Data: $\gamma = 62.4 \text{ lb/ft}^3 \quad r = \frac{24}{12}\text{ ft} = 2\text{ ft} \quad l = \frac{20}{12}\text{ ft}$

Then $T = \frac{1}{2}(62.4 \text{ lb/ft}^3)(2\text{ ft})^2\left(\frac{20}{12}\text{ ft}\right)$

$= 208.00 \text{ lb}$

$T = 208 \text{ lb}$ ◀

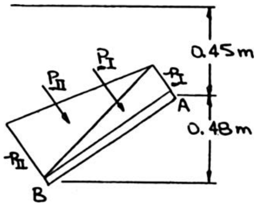


PROBLEM 5.92

A 0.5 × 0.8-m gate *AB* is located at the bottom of a tank filled with water. The gate is hinged along its top edge *A* and rests on a frictionless stop at *B*. Determine the reactions at *A* and *B* when cable *BCD* is slack.

SOLUTION

First consider the force of the water on the gate.



We have $P = \dfrac{1}{2}Ap = \dfrac{1}{2}A(\rho gh)$

so that $P_{\text{I}} = \dfrac{1}{2}[(0.5\text{ m})(0.8\text{ m})]\times[(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.45\text{ m})]$

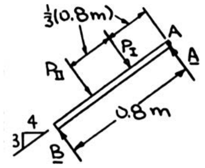
$= 882.9\text{ N}$

$P_{\text{II}} = \dfrac{1}{2}[(0.5\text{ m})(0.8\text{ m})]\times[(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.93\text{ m})]$

$= 1824.66\text{ N}$

Reactions at *A* and *B* when *T* = 0:

We have



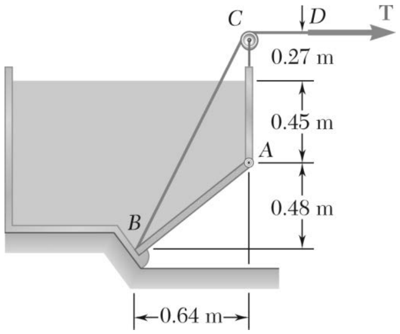
$$+\circlearrowleft \Sigma M_A = 0:\quad \frac{1}{3}(0.8\text{ m})(882.9\text{ N}) + \frac{2}{3}(0.8\text{ m})(1824.66\text{ N}) - (0.8\text{ m})B = 0$$

or $B = 1510.74\text{ N}$

or $\mathbf{B} = 1511\text{ N} \measuredangle 53.1^\circ$ ◀

$$+\nwarrow \Sigma F = 0:\quad A + 1510.74\text{ N} - 882.9\text{ N} - 1824.66\text{ N} = 0$$

or $\mathbf{A} = 1197\text{ N} \measuredangle 53.1^\circ$ ◀

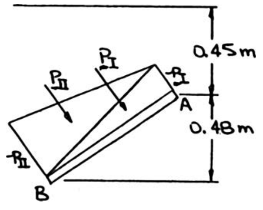


PROBLEM 5.93

A 0.5 × 0.8-m gate *AB* is located at the bottom of a tank filled with water. The gate is hinged along its top edge *A* and rests on a frictionless stop at *B*. Determine the minimum tension required in cable *BCD* to open the gate.

SOLUTION

First consider the force of the water on the gate.



We have $P = \frac{1}{2}Ap = \frac{1}{2}A(\rho g h)$

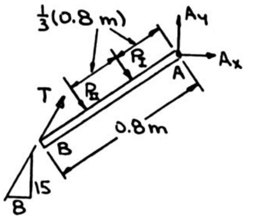
so that $P_{\text{I}} = \frac{1}{2}[(0.5\text{ m})(0.8\text{ m})]\times[(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.45\text{ m})]$

$= 882.9\text{ N}$

$P_{\text{II}} = \frac{1}{2}[(0.5\text{ m})(0.8\text{ m})]\times[(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.93\text{ m})]$

$= 1824.66\text{ N}$

<u>*T* to open gate</u>:



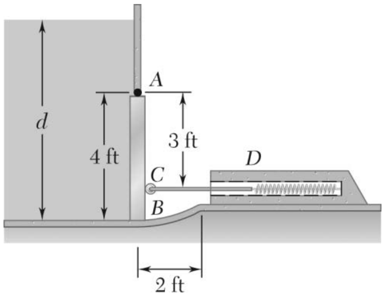
First note that when the gate begins to open, the reaction at $B \longrightarrow 0$.

Then $+\circlearrowleft \Sigma M_A = 0\text{:}\quad \frac{1}{3}(0.8\text{ m})(882.9\text{ N}) + \frac{2}{3}(0.8\text{ m})(1824.66\text{ N})$

$-(0.45+0.27)\text{m}\times\left(\frac{8}{17}T\right) = 0$

or $235.44 + 973.152 - 0.33882T = 0$

or $\mathbf{T} = 3570\text{ N}$ ◀

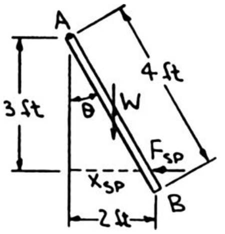


PROBLEM 5.94

A 4×2-ft gate is hinged at *A* and is held in position by rod *CD*. End *D* rests against a spring whose constant is 828 lb/ft. The spring is undeformed when the gate is vertical. Assuming that the force exerted by rod *CD* on the gate remains horizontal, determine the minimum depth of water *d* for which the bottom *B* of the gate will move to the end of the cylindrical portion of the floor.

SOLUTION

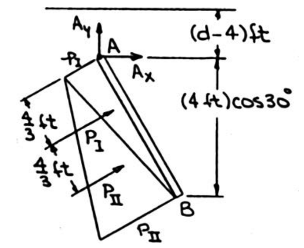
First determine the forces exerted on the gate by the spring and the water when *B* is at the end of the cylindrical portion of the floor.



We have $\sin\theta = \dfrac{2}{4} \quad \theta = 30°$

Then $x_{SP} = (3\text{ ft})\tan 30°$

and
$$F_{SP} = kx_{SP}$$
$$= 828\text{ lb/ft} \times 3\text{ ft} \times \tan 30°$$
$$= 1434.14\text{ lb}$$



Assume $d \geq 4\text{ ft}$

We have $P = \dfrac{1}{2}Ap = \dfrac{1}{2}A(\gamma h)$

Then
$$P_{\text{I}} = \frac{1}{2}[(4\text{ ft})(2\text{ ft})] \times [(62.4\text{ lb/ft}^3)(d-4)\text{ ft}]$$
$$= 249.6(d-4)\text{ lb}$$

$$P_{\text{II}} = \frac{1}{2}[(4\text{ ft})(2\text{ ft})] \times [(62.4\text{ lb/ft}^3)(d - 4 + 4\cos 30°)]$$
$$= 249.6(d - 0.53590°)\text{ lb}$$

For $d_{\min}$ so that the gate opens, $W = 0$

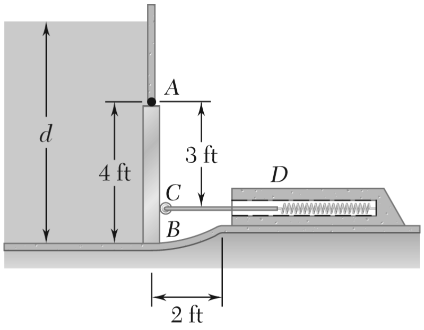
Using the above free-body diagrams of the gate, we have

$$+\circlearrowleft \Sigma M_A = 0\!: \quad \left(\frac{4}{3}\text{ ft}\right)[249.6(d-4)\text{ lb}] + \left(\frac{8}{3}\text{ ft}\right)[249.6(d-0.53590)\text{ lb}]$$
$$-(3\text{ ft})(1434.14\text{ lb}) = 0$$

or
$$(332.8d - 1331.2) + (665.6d - 356.70) - 4302.4 = 0$$

or
$$d = 6.00\text{ ft}$$

$d \geq 4\text{ ft} \Rightarrow$ assumption correct $\qquad d = 6.00\text{ ft}$ ◀



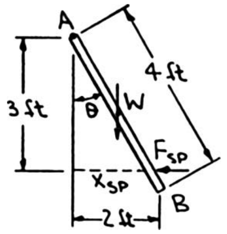
PROBLEM 5.95

Solve Problem 5.94 if the gate weighs 1000 lb.

PROBLEM 5.94 A 4×2-ft gate is hinged at *A* and is held in position by rod *CD*. End *D* rests against a spring whose constant is 828 lb/ft. The spring is undeformed when the gate is vertical. Assuming that the force exerted by rod *CD* on the gate remains horizontal, determine the minimum depth of water *d* for which the bottom *B* of the gate will move to the end of the cylindrical portion of the floor.

SOLUTION

First determine the forces exerted on the gate by the spring and the water when *B* is at the end of the cylindrical portion of the floor.

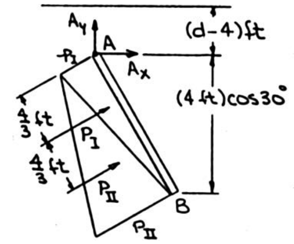


We have $\sin\theta = \dfrac{2}{4} \quad \theta = 30°$

Then $x_{SP} = (3\text{ ft})\tan 30°$

and
$$F_{SP} = kx_{SP} = 828\text{ lb/ft} \times 3\text{ ft} \times \tan 30°$$
$$= 1434.14\text{ lb}$$

Assume $d \geq 4\text{ ft}$



We have $P = \dfrac{1}{2}Ap = \dfrac{1}{2}A(\gamma h)$

Then
$$P_{\text{I}} = \frac{1}{2}[(4\text{ ft})(2\text{ ft})] \times [(62.4\text{ lb/ft}^3)(d-4)\text{ ft}]$$
$$= 249.6(d-4)\text{ lb}$$
$$P_{\text{II}} = \frac{1}{2}[(4\text{ ft})(2\text{ ft})] \times [(62.4\text{ lb/ft}^3)(d-4+4\cos 30°)]$$
$$= 249.6(d-0.53590°)\text{ lb}$$

For $d_{\min}$ so that the gate opens, $W = 1000$ lb

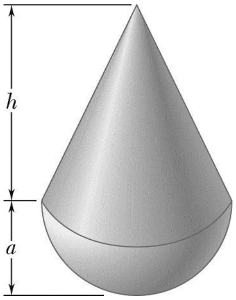
Using the above free-body diagrams of the gate, we have

$$+\circlearrowleft \Sigma M_A = 0\text{:} \quad \left(\frac{4}{3}\text{ ft}\right)[249.6(d-4)\text{ lb}] + \left(\frac{8}{3}\text{ ft}\right)[249.6(d-0.53590)\text{ lb}]$$
$$-(3\text{ ft})(1434.14\text{ lb}) - (1\text{ ft})(1000\text{ lb}) = 0$$

or
$$(332.8d - 1331.2) + (665.6d - 356.70) - 4302.4 - 1000 = 0$$

or
$$d = 7.00\text{ ft}$$

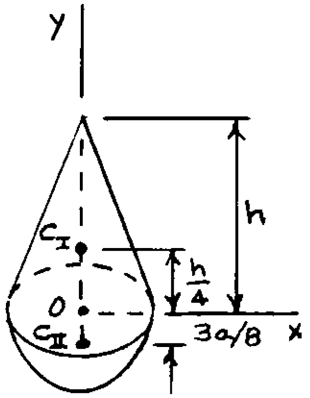
$d \geq 4\text{ ft} \Rightarrow$ assumption correct $\qquad d = 7.00\text{ ft}$ ◀



PROBLEM 5.96

A hemisphere and a cone are attached as shown. Determine the location of the centroid of the composite body when (*a*) $h = 1.5a$, (*b*) $h = 2a$.

SOLUTION



	V	$\bar{y}$	$\bar{y}V$
Cone I	$\frac{1}{3}\pi a^2 h$	$\frac{h}{4}$	$\frac{1}{12}\pi a^2 h^2$
Hemisphere II	$\frac{2}{3}\pi a^3$	$-\frac{3a}{8}$	$-\frac{1}{4}\pi a^4$

$$V = \frac{1}{3}\pi a^2 (h + 2a)$$

$$\Sigma \bar{y}V = \frac{1}{12}\pi a^2 (h^2 - 3a^2)$$

(*a*) For $h = 1.5a$,

$$V = \frac{1}{3}\pi a^2 (1.5a + 2a) = \frac{7}{6}\pi a^2$$

$$\Sigma \bar{y}V = \frac{1}{12}\pi a^2 [(1.5a)^2 - 3a^2] = -\frac{1}{16}\pi a^4$$

$$\bar{Y}V = \Sigma \bar{y}V: \quad \bar{Y}\left(\frac{7}{6}\pi a^3\right) = -\frac{1}{16}\pi a^4 \quad \bar{Y} = -\frac{3}{56}a$$

Centroid is 0.0536*a* below base of cone. ◀

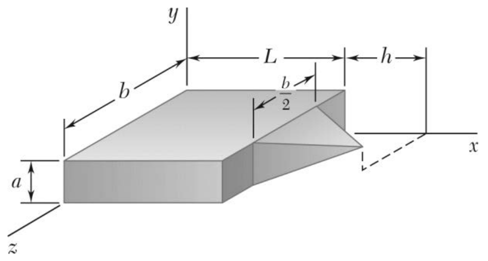
PROBLEM 5.96 (Continued)

(*b*) For $h = 2a$, $$V = \frac{1}{3}\pi a^2(2a + 2a) = \frac{4}{3}\pi a^3$$

$$\Sigma\overline{y}V = \frac{1}{12}\pi a^2[(2a)^2 - 3a^2] = \frac{1}{12}\pi a^4$$

$$\overline{Y}V = \Sigma\overline{y}V: \quad \overline{Y}\left(\frac{4}{3}\pi a^3\right) = \frac{1}{12}\pi a^4 \quad \overline{Y} = \frac{1}{16}a$$

Centroid is $0.0625a$ above base of cone. ◀



PROBLEM 5.97

Consider the composite body shown. Determine (*a*) the value of $\overline{x}$ when $h = L/2$, (*b*) the ratio h/L for which $\overline{x} = L$.

SOLUTION

	V	$\overline{x}$	$\overline{x}V$
Rectangular prism	Lab	$\frac{1}{2}L$	$\frac{1}{2}L^2ab$
Pyramid	$\frac{1}{3}a\left(\frac{b}{2}\right)h$	$L+\frac{1}{4}h$	$\frac{1}{6}abh\left(L+\frac{1}{4}h\right)$

Then
$$\Sigma V = ab\left(L+\frac{1}{6}h\right)$$

$$\Sigma\overline{x}V = \frac{1}{6}ab\left[3L^2 + h\left(L+\frac{1}{4}h\right)\right]$$

Now
$$\overline{X}\Sigma V = \Sigma\overline{x}V$$

so that
$$\overline{X}\left[ab\left(L+\frac{1}{6}h\right)\right] = \frac{1}{6}ab\left(3L^2 + hL + \frac{1}{4}h^2\right)$$

or
$$\overline{X}\left(1+\frac{1}{6}\frac{h}{L}\right) = \frac{1}{6}L\left(3+\frac{h}{L}+\frac{1}{4}\frac{h^2}{L^2}\right) \qquad (1)$$

(*a*) $\overline{X} = ?$ when $h = \frac{1}{2}L.$

Substituting $\frac{h}{L} = \frac{1}{2}$ into Eq. (1),

$$\overline{X}\left[1+\frac{1}{6}\left(\frac{1}{2}\right)\right] = \frac{1}{6}L\left[3+\left(\frac{1}{2}\right)+\frac{1}{4}\left(\frac{1}{2}\right)^2\right]$$

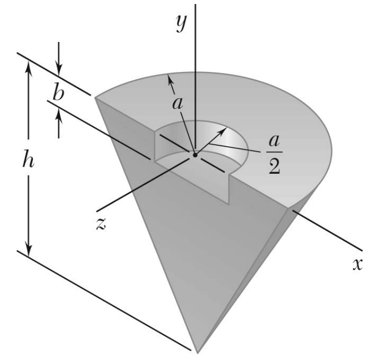
or
$$\overline{X} = \frac{57}{104}L \qquad \overline{X} = 0.548L \blacktriangleleft$$

(*b*) $\dfrac{h}{L} = ?$ when $\bar{X} = L.$

Substituting into Eq. (1),
$$L\left(1+\frac{1}{6}\frac{h}{L}\right) = \frac{1}{6}L\left(3+\frac{h}{L}+\frac{1}{4}\frac{h^2}{L^2}\right)$$

or
$$1+\frac{1}{6}\frac{h}{L} = \frac{1}{2}+\frac{1}{6}\frac{h}{L}+\frac{1}{24}\frac{h^2}{L^2}$$

or
$$\frac{h^2}{L^2} = 12 \qquad \frac{h}{L} = 2\sqrt{3} \blacktriangleleft$$

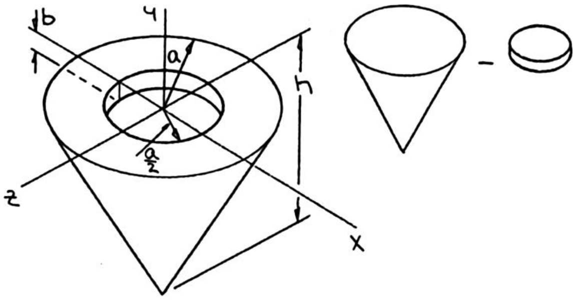


PROBLEM 5.98

Determine the *y* coordinate of the centroid of the body shown.

SOLUTION

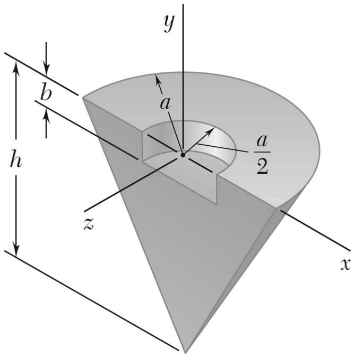
First note that the values of $\overline{Y}$ will be the same for the given body and the body shown below. Then



	V	$\overline{y}$	$\overline{y}V$
Cone	$\frac{1}{3}\pi a^2 h$	$-\frac{1}{4}h$	$-\frac{1}{12}\pi a^2 h^2$
Cylinder	$-\pi\left(\frac{a}{2}\right)^2 b = -\frac{1}{4}\pi a^2 b$	$-\frac{1}{2}b$	$\frac{1}{8}\pi a^2 b^2$
Σ	$\frac{\pi}{12}a^2(4h-3b)$		$-\frac{\pi}{24}a^2(2h^2-3b^2)$

We have $$\overline{Y}\Sigma V = \Sigma\overline{y}V$$

Then $$\overline{Y}\left[\frac{\pi}{12}a^2(4h-3b)\right] = -\frac{\pi}{24}a^2(2h^2-3b^2)$$ or $$\overline{Y} = -\frac{2h^2-3b^2}{2(4h-3b)} \blacktriangleleft$$

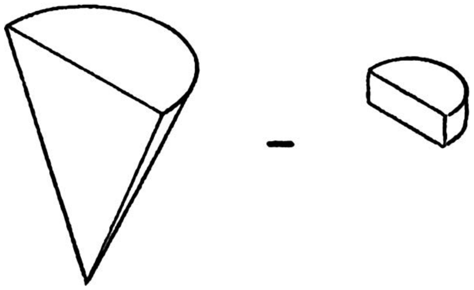


PROBLEM 5.99

Determine the z coordinate of the centroid of the body shown. (*Hint:* Use the result of Sample Problem 5.13.)

SOLUTION

First note that the body can be formed by removing a half cylinder from a half cone, as shown.



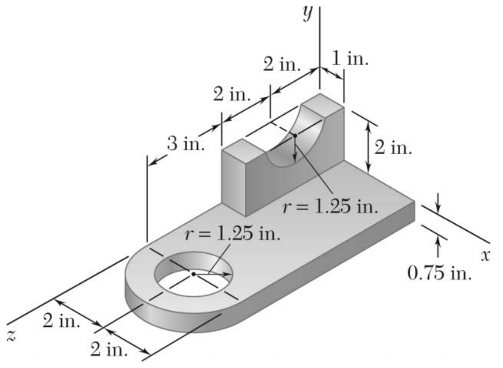
	V	$\bar{z}$	$\bar{z}V$
Half cone	$\frac{1}{6}\pi a^2 h$	$-\frac{a}{\pi}$	$-\frac{1}{6}a^3 h$
Half cylinder	$-\frac{\pi}{2}\left(\frac{a}{2}\right)^2 b = -\frac{\pi}{8}a^2 b$	$-\frac{4}{3\pi}\left(\frac{a}{2}\right) = -\frac{2a}{3\pi}$	$\frac{1}{12}a^3 b$
Σ	$\frac{\pi}{24}a^2(4h-3b)$		$-\frac{1}{12}a^3(2h-b)$

From Sample Problem 5.13:

We have $$\bar{Z}\Sigma V = \Sigma \bar{z} V$$

Then $$\bar{Z}\left[\frac{\pi}{24}a^2(4h-3b)\right] = -\frac{1}{12}a^3(2h-b)$$

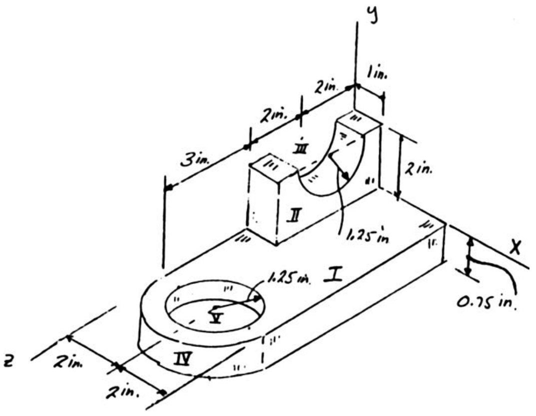
or $\bar{Z} = -\frac{a}{\pi}\left(\frac{4h-2b}{4h-3b}\right)$ ◀



PROBLEM 5.100

For the machine element shown, locate the y coordinate of the center of gravity.

SOLUTION



For half-cylindrical hole,

$$r = 1.25 \text{ in.}$$

$$\bar{y}_{\text{III}} = 2 - \frac{4(1.25)}{3\pi}$$

$$= 1.470 \text{ in.}$$

For half-cylindrical plate, $r = 2$ in.

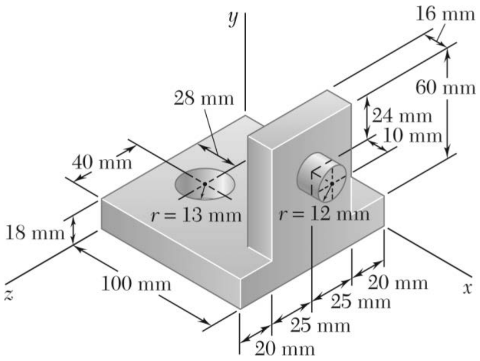
$$\bar{z}_{\text{IV}} = 7 + \frac{4(2)}{3\pi} = 7.85 \text{ in.}$$

		V, in^3	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{y}V$, in^4	$\bar{z}V$, in^4
I	Rectangular plate	$(7)(4)(0.75) = 21.0$	–0.375	3.5	–7.875	73.50
II	Rectangular plate	$(4)(2)(1) = 8.0$	1.0	2	8.000	16.00
III	–(Half cylinder)	$-\frac{\pi}{2}(1.25)^2(1) = 2.454$	1.470	2	–3.607	–4.908
IV	Half cylinder	$\frac{\pi}{2}(2)^2(0.75) = 4.712$	–0.375	–7.85	–1.767	36.99
V	–(Cylinder)	$-\pi(1.25)^2(0.75) = -3.682$	–0.375	7	1.381	–25.77
	Σ	27.58			–3.868	95.81

$$\bar{Y}\Sigma V = \Sigma \bar{y} V$$

$$\bar{Y}(27.58 \text{ in}^3) = -3.868 \text{ in}^4$$

$\bar{Y} = -0.1403$ in. ◀

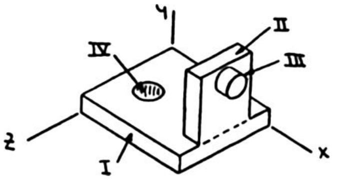


PROBLEM 5.101

For the machine element shown, locate the *y* coordinate of the center of gravity.

SOLUTION

First assume that the machine element is homogeneous so that its center of gravity will coincide with the centroid of the corresponding volume.

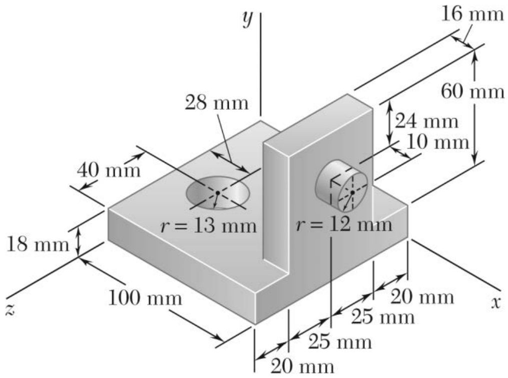


	V, mm^3	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}V$, mm^4	$\bar{y}V$, mm^4
I	$(100)(18)(90) = 162{,}000$	50	9	8,100,000	1,458,000
II	$(16)(60)(50) = 48{,}000$	92	48	4,416,000	2,304,000
III	$\pi(12)^2(10) = 4523.9$	105	54	475,010	244,290
IV	$-\pi(13)^2(18) = -9556.7$	28	9	–267,590	–86,010
Σ	204,967.2			12,723,420	3,920,280

We have $$\bar{Y}\Sigma V = \Sigma\bar{y}V$$

$$\bar{Y}(204{,}967.2 \text{ mm}^3) = 3{,}920{,}280 \text{ mm}^4$$

or $\bar{Y} = 19.13$ mm ◀

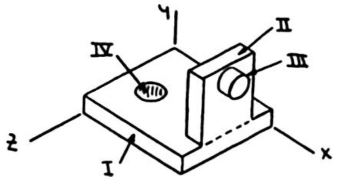


PROBLEM 5.102

For the machine element shown, locate the *x* coordinate of the center of gravity.

SOLUTION

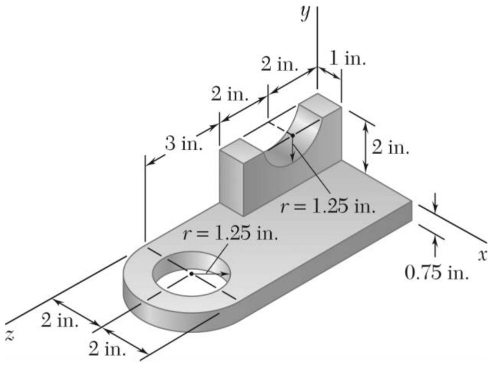
First assume that the machine element is homogeneous so that its center of gravity will coincide with the centroid of the corresponding volume.



	V, mm^3	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}V$, mm^4	$\bar{y}V$, mm^4
I	$(100)(18)(90) = 162,000$	50	9	8,100,000	1,458,000
II	$(16)(60)(50) = 48,000$	92	48	4,416,000	2,304,000
III	$\pi(12)^2(10) = 4523.9$	105	54	475,010	244,290
IV	$-\pi(13)^2(18) = -9556.7$	28	9	–267,590	–86,010
Σ	204,967.2			12,723,420	3,920,280

We have $$\bar{X}\Sigma V = \Sigma\bar{x}V$$

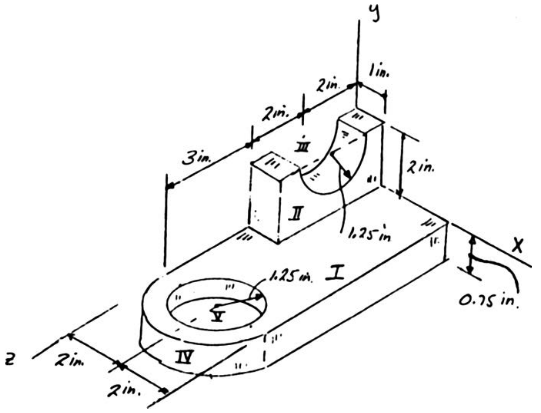
$$\bar{X}(204,967.2 \text{ mm}^3) = 12,723,420 \text{ mm}^4$$ $\bar{X} = 62.1$ mm ◀



PROBLEM 5.103

For the machine element shown, locate the z coordinate of the center of gravity.

SOLUTION



For half-cylindrical hole,

$$r = 1.25\text{ in.}$$

$$\bar{y}_{\text{III}} = 2 - \frac{4(1.25)}{3\pi}$$

$$= 1.470\text{ in.}$$

For half-cylindrical plate, $r = 2$ in.

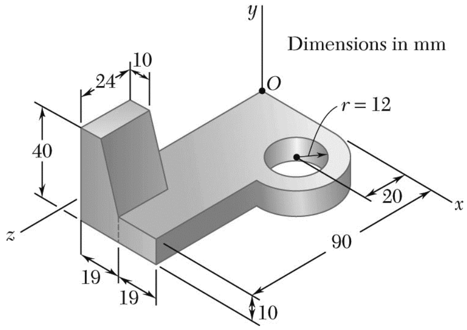
$$\bar{z}_{\text{IV}} = 7 + \frac{4(2)}{3\pi} = 7.85\text{ in.}$$

		V, in^3	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{y}V$, in^4	$\bar{z}V$, in^4
I	Rectangular plate	$(7)(4)(0.75) = 21.0$	–0.375	3.5	–7.875	73.50
II	Rectangular plate	$(4)(2)(1) = 8.0$	1.0	2	8.000	16.00
III	–(Half cylinder)	$-\frac{\pi}{2}(1.25)^2(1) = 2.454$	1.470	2	–3.607	–4.908
IV	Half cylinder	$\frac{\pi}{2}(2)^2(0.75) = 4.712$	–0.375	–7.85	–1.767	36.99
V	–(Cylinder)	$-\pi(1.25)^2(0.75) = -3.682$	–0.375	7	1.381	–25.77
	Σ	27.58			–3.868	95.81

Now $$\bar{Z}\Sigma V = \bar{z}V$$

$$\bar{Z}(27.58\text{ in}^3) = 95.81\text{ in}^4$$

$\bar{Z} = 3.47$ in. ◀

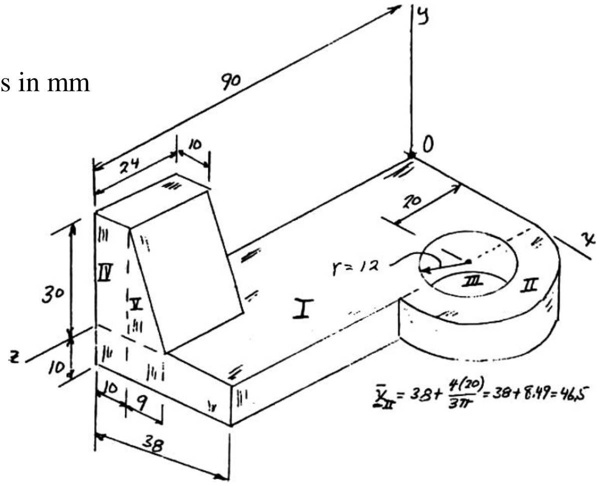


PROBLEM 5.104

For the machine element shown, locate the *x* coordinate of the center of gravity.

SOLUTION

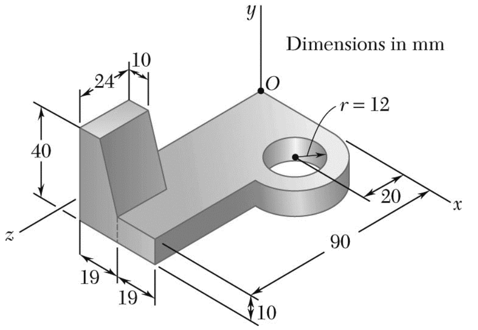
Dimensions in mm



		V, mm^3	$\bar{x}$, mm	$\bar{z}$, mm	$\bar{x}V$, mm^4	$\bar{z}V$, mm^4
I	Rectangular plate	$(10)(90)(38) = 34.2\times10^3$	19	45	649.8×10^3	1539×10^3
II	Half cylinder	$\frac{\pi}{2}(20)^2(10) = 6.2832\times10^3$	46.5	20	292.17×10^3	125.664×10^3
III	–(Cylinder)	$-\pi(12)^2(10) = -4.5239\times10^3$	38	20	-171.908×10^3	-90.478×10^3
IV	Rectangular prism	$(30)(10)(24) = 7.2\times10^3$	5	78	36×10^3	561.6×10^3
V	Triangular prism	$\frac{1}{2}(30)(9)(24) = 3.24\times10^3$	13	78	42.12×10^3	252.72×10^3
	Σ	46.399×10^3			848.18×10^3	2388.5×10^3

$$\bar{X}\Sigma V = \Sigma\bar{x}V$$

$$\bar{X} = \frac{\Sigma\bar{x}V}{\Sigma V} = \frac{848.18\times10^3\,\text{mm}^4}{46.399\times10^3\,\text{mm}^3} \qquad \bar{X} = 18.28\text{ mm} \blacktriangleleft$$

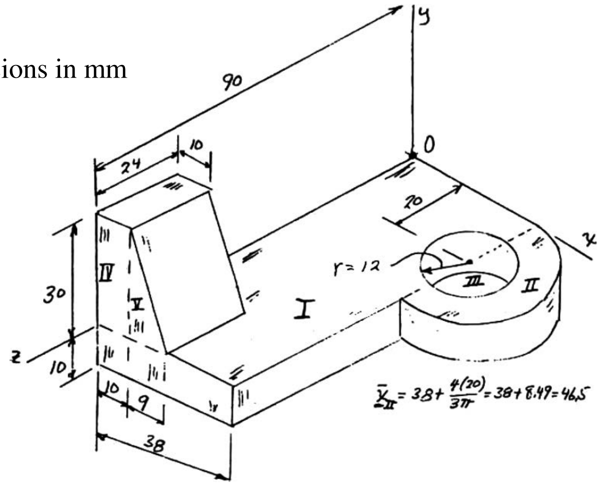


PROBLEM 5.105

For the machine element shown, locate the *z* coordinate of the center of gravity.

SOLUTION

Dimensions in mm



		V, mm^3	$\bar{x}$, mm	$\bar{z}$, mm	$\bar{x}V$, mm^4	$\bar{z}V$, mm^4
I	Rectangular plate	$(10)(90)(38) = 34.2\times10^3$	19	45	649.8×10^3	1539×10^3
II	Half cylinder	$\frac{\pi}{2}(20)^2(10) = 6.2832\times10^3$	46.5	20	292.17×10^3	125.664×10^3
III	–(Cylinder)	$-\pi(12)^2(10) = -4.5239\times10^3$	38	20	-171.908×10^3	-90.478×10^3
IV	Rectangular prism	$(30)(10)(24) = 7.2\times10^3$	5	78	36×10^3	561.6×10^3
V	Triangular prism	$\frac{1}{2}(30)(9)(24) = 3.24\times10^3$	13	78	42.12×10^3	252.72×10^3
	Σ	46.399×10^3			848.18×10^3	2388.5×10^3

$$\bar{Z}\Sigma V = \Sigma\bar{z}V$$

$$\bar{Z} = \frac{\Sigma\bar{z}V}{\Sigma V} = \frac{2388.5\times10^3\,\text{mm}^4}{46.399\times10^3\,\text{mm}^3}$$

$\bar{Z} = 51.5$ mm ◀

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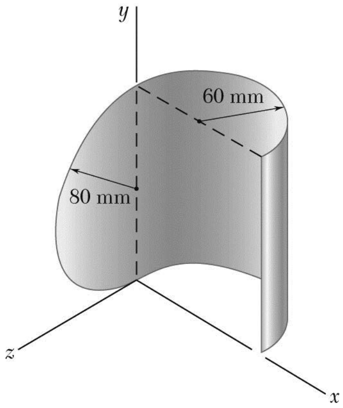
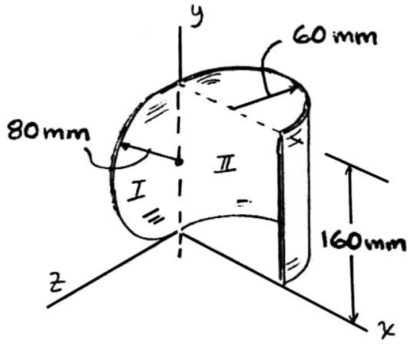


PROBLEM 5.106

Locate the center of gravity of the sheet-metal form shown.

SOLUTION

By symmetry, $\bar{Y} = 80.0 \text{ mm}$ ◀



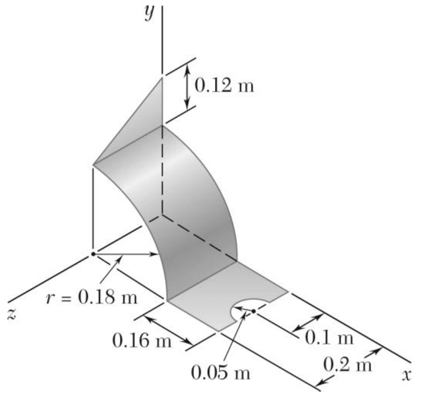
$$\bar{z}_{\text{I}} = \frac{4(80)}{3\pi} = 33.953 \text{ mm}$$

$$\bar{z}_{\text{II}} = -\frac{2(60)}{\pi} = -38.197 \text{ mm}$$

	A, mm^2	$\bar{x}$, mm	$\bar{z}$, mm	$\bar{x}A$, mm^3	$\bar{z}A$, mm^3
I	$\frac{\pi}{2}(80)^2 = 10{,}053$	0	33.953	0	341.33×10^3
II	$\pi(60)(160) = 30{,}159$	60	−38.197	1809.54×10^3	-1151.98×10^3
Σ	40,212			1809.54×10^3	-810.65×10^3

$\bar{X}\Sigma A = \Sigma\bar{x}A$: $\bar{X}(40{,}212) = 1809.54\times10^3$ $\bar{X} = 45.0 \text{ mm}$ ◀

$\bar{Z}\Sigma A = \Sigma\bar{z}A$: $\bar{Z}(40{,}212) = -810.65\times10^3$ $\bar{Z} = -20.2 \text{ mm}$ ◀



PROBLEM 5.107

Locate the center of gravity of the sheet-metal form shown.

SOLUTION

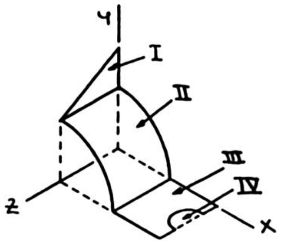
First assume that the sheet metal is homogeneous so that the center of gravity of the form will coincide with the centroid of the corresponding area.

$$\overline{y}_{\mathrm{I}} = 0.18 + \frac{1}{3}(0.12) = 0.22 \text{ m}$$

$$\overline{z}_{\mathrm{I}} = \frac{1}{3}(0.2 \text{ m})$$

$$\overline{x}_{\mathrm{II}} = \overline{y}_{\mathrm{II}} = \frac{2\times 0.18}{\pi} = \frac{0.36}{\pi} \text{ m}$$

$$\overline{x}_{\mathrm{IV}} = 0.34 - \frac{4\times 0.05}{3\pi} = 0.31878 \text{ m}$$



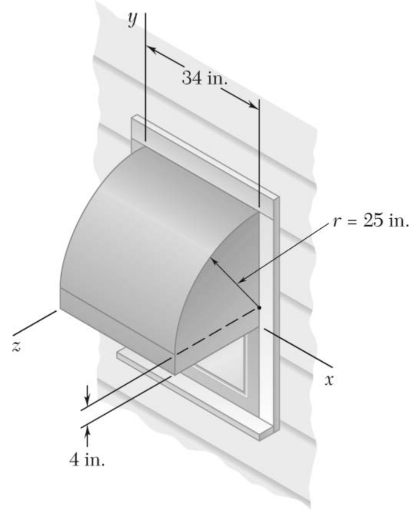
	A, m^2	$\overline{x}$, m	$\overline{y}$, m	$\overline{z}$, m	$\overline{x}A$, m^3	$\overline{y}A$, m^3	$\overline{z}A$, m^3
I	$\frac{1}{2}(0.2)(0.12) = 0.012$	0	0.22	$\frac{0.2}{3}$	0	0.00264	0.0008
II	$\frac{\pi}{2}(0.18)(0.2) = 0.018\pi$	$\frac{0.36}{\pi}$	$\frac{0.36}{\pi}$	0.1	0.00648	0.00648	0.005655
III	$(0.16)(0.2) = 0.032$	0.26	0	0.1	0.00832	0	0.0032
IV	$-\frac{\pi}{2}(0.05)^2 = -0.00125\pi$	0.31878	0	0.1	–0.001258	0	–0.000393
Σ	0.096622				0.013542	0.00912	0.009262

PROBLEM 5.107 (Continued)

We have $\bar{X}\Sigma V = \Sigma\bar{x}V$: $\bar{X}(0.096622\text{ m}^2) = 0.013542\text{ m}^3$ or $\bar{X} = 0.1402\text{ m}$ ◀

$\bar{Y}\Sigma V = \Sigma\bar{y}V$: $\bar{Y}(0.096622\text{ m}^2) = 0.00912\text{ m}^3$ or $\bar{Y} = 0.0944\text{ m}$ ◀

$\bar{Z}\Sigma V = \Sigma\bar{z}V$: $\bar{Z}(0.096622\text{ m}^2) = 0.009262\text{ m}^3$ or $\bar{Z} = 0.0959\text{ m}$ ◀

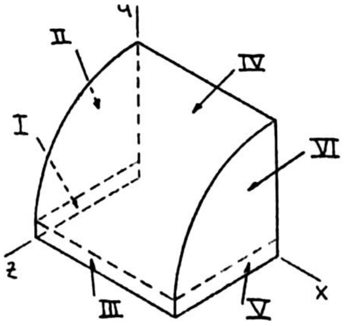


PROBLEM 5.108

A window awning is fabricated from sheet metal of uniform thickness. Locate the center of gravity of the awning.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the awning coincides with the centroid of the corresponding area.



$$\bar{y}_{\text{II}} = \bar{y}_{\text{VI}} = 4 + \frac{(4)(25)}{3\pi} = 14.6103 \text{ in.}$$

$$\bar{z}_{\text{II}} = \bar{z}_{\text{VI}} = \frac{(4)(25)}{3\pi} = \frac{100}{3\pi} \text{ in.}$$

$$\bar{y}_{\text{IV}} = 4 + \frac{(2)(25)}{\pi} = 19.9155 \text{ in.}$$

$$\bar{z}_{\text{IV}} = \frac{(2)(25)}{\pi} = \frac{50}{\pi} \text{ in.}$$

$$A_{\text{II}} = A_{\text{VI}} = \frac{\pi}{4}(25)^2 = 490.87 \text{ in}^2$$

$$A_{\text{IV}} = \frac{\pi}{2}(25)(34) = 1335.18 \text{ in}^2$$

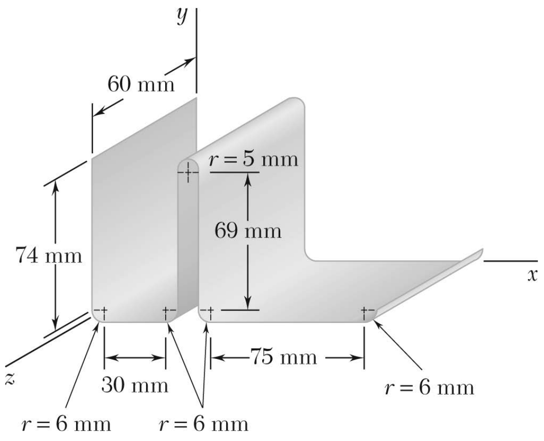
	A, in^2	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{y}A$, in^3	$\bar{z}A$, in^3
I	$(4)(25) = 100$	2	12.5	200	1250
II	490.87	14.6103	$\frac{100}{3\pi}$	7171.8	5208.3
III	$(4)(34) = 136$	2	25	272	3400
IV	1335.18	19.9155	$\frac{50}{\pi}$	26,591	21,250
V	$(4)(25) = 100$	2	12.5	200	1250
VI	490.87	14.6103	$\frac{100}{3\pi}$	7171.8	5208.3
Σ	2652.9			41,607	37,567

PROBLEM 5.108 (Continued)

Now, symmetry implies $\bar{X} = 17.00 \text{ in.}$ ◀

and $\bar{Y}\Sigma A = \Sigma\bar{y}A$: $\bar{Y}(2652.9 \text{ in}^2) = 41,607 \text{ in}^3$ or $\bar{Y} = 15.68 \text{ in.}$ ◀

$\bar{Z}\Sigma A = \Sigma\bar{z}A$: $\bar{Z}(2652.9 \text{ in}^2) = 37,567 \text{ in}^3$ or $\bar{Z} = 14.16 \text{ in.}$ ◀



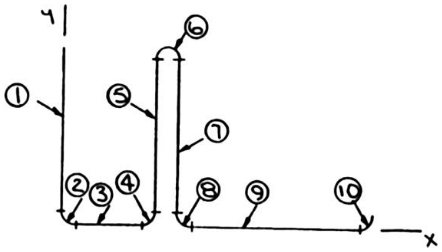
PROBLEM 5.109

A thin sheet of plastic of uniform thickness is bent to form a desk organizer. Locate the center of gravity of the organizer.

SOLUTION

First assume that the plastic is homogeneous so that the center of gravity of the organizer will coincide with the centroid of the corresponding area. Now note that symmetry implies

$$\bar{Z} = 30.0 \text{ mm} \blacktriangleleft$$



$$\bar{x}_2 = 6 - \frac{2\times 6}{\pi} = 2.1803 \text{ mm}$$

$$\bar{x}_4 = 36 + \frac{2\times 6}{\pi} = 39.820 \text{ mm}$$

$$\bar{x}_8 = 58 - \frac{2\times 6}{\pi} = 54.180 \text{ mm}$$

$$\bar{x}_{10} = 133 + \frac{2\times 6}{\pi} = 136.820 \text{ mm}$$

$$\bar{y}_2 = \bar{y}_4 = \bar{y}_8 = \bar{y}_{10} = 6 - \frac{2\times 6}{\pi} = 2.1803 \text{ mm}$$

$$\bar{y}_6 = 75 + \frac{2\times 5}{\pi} = 78.183 \text{ mm}$$

$$A_2 = A_4 = A_8 = A_{10} = \frac{\pi}{2}\times 6\times 60 = 565.49 \text{ mm}^2$$

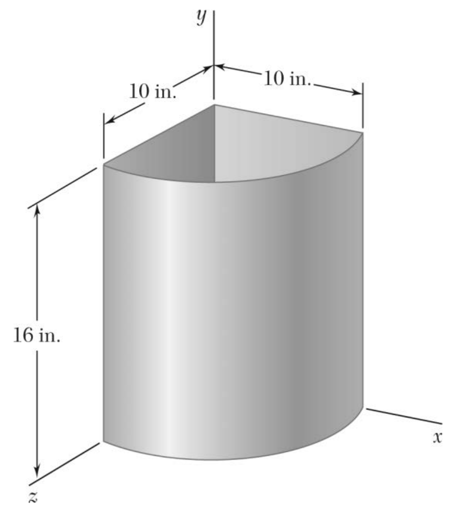
$$A_6 = \pi\times 5\times 60 = 942.48 \text{ mm}^2$$

PROBLEM 5.109 (Continued)

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$(74)(60) = 4440$	0	43	0	190,920
2	565.49	2.1803	2.1803	1233	1233
3	$(30)(60) = 1800$	21	0	37,800	0
4	565.49	39.820	2.1803	22,518	1233
5	$(69)(60) = 4140$	42	40.5	173,880	167,670
6	942.48	47	78.183	44,297	73,686
7	$(69)(60) = 4140$	52	40.5	215,280	167,670
8	565.49	54.180	2.1803	30,638	1233
9	$(75)(60) = 4500$	95.5	0	429,750	0
10	565.49	136.820	2.1803	77,370	1233
Σ	22,224.44			1,032,766	604,878

We have $\bar{X}\Sigma A = \Sigma\bar{x}A$: $\bar{X}(22,224.44 \text{ mm}^2) = 1,032,766 \text{ mm}^3$ or $\bar{X} = 46.5$ mm ◀

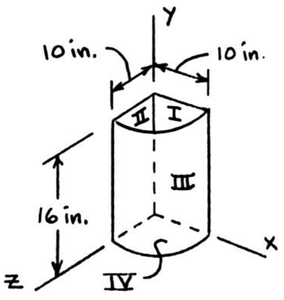
$\bar{Y}\Sigma A = \Sigma\bar{y}A$: $\bar{Y}(22,224.44 \text{ mm}^2) = 604,878 \text{ mm}^3$ or $\bar{Y} = 27.2$ mm ◀



PROBLEM 5.110

A wastebasket, designed to fit in the corner of a room, is 16 in. high and has a base in the shape of a quarter circle of radius 10 in. Locate the center of gravity of the wastebasket, knowing that it is made of sheet metal of uniform thickness.

SOLUTION



By symmetry, $\overline{X} = \overline{Z}$

For III (Cylindrical surface), $\overline{x} = \dfrac{2r}{\pi} = \dfrac{2(10)}{\pi} = 6.3662 \text{ in.}$

$$A = \frac{\pi}{2} rh = \frac{\pi}{2}(10)(16) = 251.33 \text{ in}^2$$

For IV (Quarter-circle bottom), $\overline{x} = \dfrac{4r}{3\pi} = \dfrac{4(10)}{3\pi} = 4.2441 \text{ in.}$

$$A = \frac{\pi}{4} r^2 = \frac{\pi}{4}(10)^2 = 78.540 \text{ in}^2$$

	A, in^2	$\overline{x}$, in.	$\overline{x}$, in.	$\overline{x}A$, in^3	$\overline{y}A$, in^3
I	$(10)(16) = 160$	5	8	800	1280
II	$(10)(16) = 160$	0	8	0	1280
III	251.33	6.3662	8	1600.0	2010.6
IV	78.540	4.2441	0	333.33	0
Σ	649.87			2733.3	4570.6

$\overline{X}\Sigma A = \Sigma \overline{x}A$: $\overline{X}(649.87 \text{ in}^2) = 2733.3 \text{ in}^3$

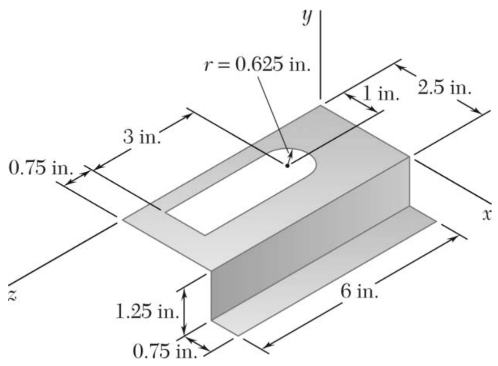
$$\overline{X} = 4.2059 \text{ in.}$$

$\overline{X} = \overline{Z} = 4.21 \text{ in.}$ ◀

$\overline{Y}\Sigma A = \Sigma \overline{y}A$: $\overline{Y}(649.87 \text{ in}^2) = 4570.6 \text{ in}^3$

$$\overline{Y} = 7.0331 \text{ in.}$$

$\overline{Y} = 7.03 \text{ in.}$ ◀

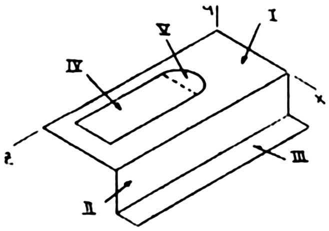


PROBLEM 5.111

A mounting bracket for electronic components is formed from sheet metal of uniform thickness. Locate the center of gravity of the bracket.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the bracket coincides with the centroid of the corresponding area. Then (see diagram)



$$\bar{z}_V = 2.25 - \frac{4(0.625)}{3\pi} = 1.98474 \text{ in.}$$

$$A_V = -\frac{\pi}{2}(0.625)^2 = -0.61359 \text{ in}^2$$

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3	$\bar{z}A$, in^3
I	$(2.5)(6) = 15$	1.25	0	3	18.75	0	45
II	$(1.25)(6) = 7.5$	2.5	–0.625	3	18.75	–4.6875	22.5
III	$(0.75)(6) = 4.5$	2.875	–1.25	3	12.9375	–5.625	13.5
IV	$-\left(\frac{5}{4}\right)(3) = -3.75$	1.0	0	3.75	3.75	0	–14.0625
V	-0.61359	1.0	0	1.9847	0.61359	0	–1.21782
Σ	22.6364				46.0739	10.3125	65.7197

We have $\bar{X}\Sigma A = \Sigma \bar{x}A$

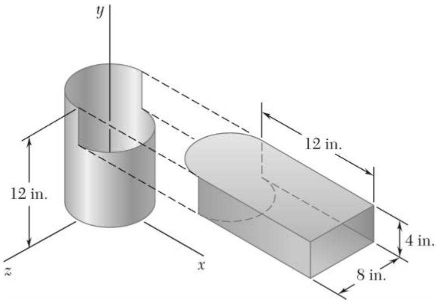
$\bar{X}(22.6364 \text{ in}^2) = 46.0739 \text{ in}^3$ or $\bar{X} = 2.04$ in. ◀

$\bar{Y}\Sigma A = \Sigma \bar{y}A$

$\bar{Y}(22.6364 \text{ in}^2) = -10.3125 \text{ in}^3$ or $\bar{Y} = -0.456$ in. ◀

$\bar{Z}\Sigma A = \Sigma \bar{z}A$

$\bar{Z}(22.6364 \text{ in}^2) = 65.7197 \text{ in}^3$ or $\bar{Z} = 2.90$ in. ◀



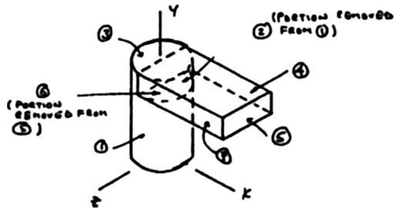
PROBLEM 5.112

An 8-in.-diameter cylindrical duct and a 4 × 8-in. rectangular duct are to be joined as indicated. Knowing that the ducts were fabricated from the same sheet metal, which is of uniform thickness, locate the center of gravity of the assembly.

SOLUTION

Assume that the body is homogeneous so that its center of gravity coincides with the centroid of the area.

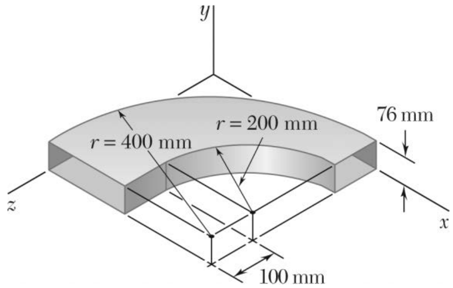
By symmetry, $\bar{z} = 0$.



	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$\pi(8)(12) = 96\pi$	0	6	0	576π
2	$-\frac{\pi}{2}(8)(4) = -16\pi$	$\frac{2(4)}{\pi} = \frac{8}{\pi}$	10	-128	-160π
3	$\frac{\pi}{2}(4)^2 = 8\pi$	$-\frac{4(4)}{3\pi} = -\frac{16}{3\pi}$	12	-42.667	96π
4	$(8)(12) = 96$	6	12	576	1152
5	$(8)(12) = 96$	6	8	576	768
6	$-\frac{\pi}{2}(4)^2 = -8\pi$	$\frac{4(4)}{3\pi} = \frac{16}{3\pi}$	8	-42.667	-64π
7	$(4)(12) = 48$	6	10	288	480
8	$(4)(12) = 48$	6	10	288	480
Σ	539.33			1514.6	4287.4

Then $$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{1514.67}{539.33} \text{ in.} \qquad \text{or } \bar{X} = 2.81 \text{ in.} \blacktriangleleft$$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{4287.4}{539.33} \text{ in.} \qquad \text{or } \bar{Y} = 7.95 \text{ in.} \blacktriangleleft$$



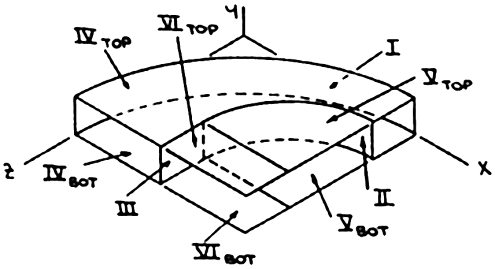
PROBLEM 5.113

An elbow for the duct of a ventilating system is made of sheet metal of uniform thickness. Locate the center of gravity of the elbow.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the duct coincides with the centroid of the corresponding area. Also, note that the shape of the duct implies

$$\overline{Y} = 38.0 \text{ mm} \blacktriangleleft$$



Note that

$$\overline{x}_{\text{I}} = \overline{z}_{\text{I}} = 400 - \frac{2}{\pi}(400) = 145.352 \text{ mm}$$

$$\overline{x}_{\text{II}} = 400 - \frac{2}{\pi}(200) = 272.68 \text{ mm}$$

$$\overline{z}_{\text{II}} = 300 - \frac{2}{\pi}(200) = 172.676 \text{ mm}$$

$$\overline{x}_{\text{IV}} = \overline{z}_{\text{IV}} = 400 - \frac{4}{3\pi}(400) = 230.23 \text{ mm}$$

$$\overline{x}_{\text{V}} = 400 - \frac{4}{3\pi}(200) = 315.12 \text{ mm}$$

$$\overline{z}_{\text{V}} = 300 - \frac{4}{3\pi}(200) = 215.12 \text{ mm}$$

Also note that the corresponding top and bottom areas will contribute equally when determining $\overline{x}$ and $\overline{z}$.

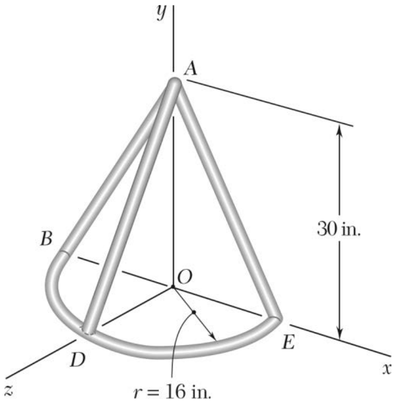
Thus,

	A, mm^2	$\overline{x}$, mm	$\overline{z}$, mm	$\overline{x}A$, mm^3	$\overline{z}A$, mm^3
I	$\frac{\pi}{2}(400)(76) = 47,752$	145.352	145.352	6,940,850	6,940,850
II	$\frac{\pi}{2}(200)(76) = 23,876$	272.68	172.676	6,510,510	4,122,810
III	$100(76) = 7600$	200	350	1,520,000	2,660,000
IV	$2\left(\frac{\pi}{4}\right)(400)^2 = 251,327$	230.23	230.23	57,863,020	57,863,020
V	$-2\left(\frac{\pi}{4}\right)(200)^2 = -62,832$	315.12	215.12	–19,799,620	–13,516,420
VI	$-2(100)(200) = -40,000$	300	350	–12,000,000	–14,000,000
Σ	227,723			41,034,760	44,070,260

PROBLEM 5.113 (Continued)

We have $\overline{X}\Sigma A = \Sigma\overline{x}A$: $\overline{X}(227,723\text{ mm}^2) = 41,034,760\text{ mm}^3$ or $\overline{X} = 180.2\text{ mm}$ ◀

$\overline{Z}\Sigma A = \Sigma\overline{z}A$: $\overline{Z}(227,723\text{ mm}^2) = 44,070,260\text{ mm}^3$ or $\overline{Z} = 193.5\text{ mm}$ ◀

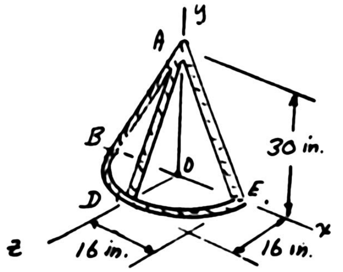


PROBLEM 5.114

Locate the center of gravity of the figure shown, knowing that it is made of thin brass rods of uniform diameter.

SOLUTION

By symmetry, $\bar{X} = 0$ ◀



	L, in.	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{y}L$, in^2	$\bar{z}L$, in^2
AB	$\sqrt{30^2+16^2}=34$	15	0	510	0
AD	$\sqrt{30^2+16^2}=34$	15	8	510	272
AE	$\sqrt{30^2+16^2}=34$	15	0	510	0
BDE	$\pi(16)=50.265$	0	$\dfrac{2(16)}{\pi}=10.186$	0	512
Σ	152.265			1530	784

$$\bar{Y}\Sigma L = \Sigma\bar{y}L: \quad \bar{Y}(152.265 \text{ in.}) = 1530 \text{ in}^2$$

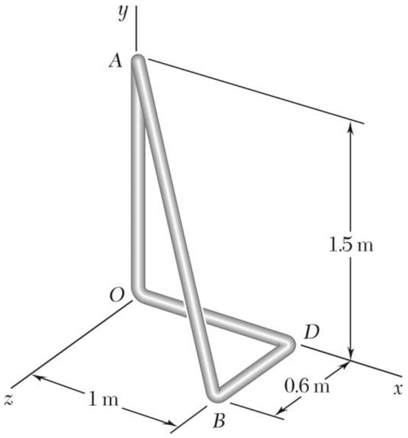
$$\bar{Y} = 10.048 \text{ in.}$$

$\bar{Y} = 10.05$ in. ◀

$$\bar{Z}\Sigma L = \Sigma\bar{z}L: \quad \bar{Z}(152.265 \text{ in.}) = 784 \text{ in}^2$$

$$\bar{Z} = 5.149 \text{ in.}$$

$\bar{Z} = 5.15$ in. ◀

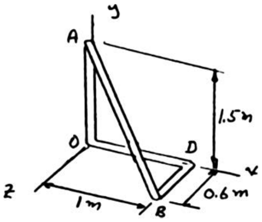


PROBLEM 5.115

Locate the center of gravity of the figure shown, knowing that it is made of thin brass rods of uniform diameter.

SOLUTION

Uniform rod:



$$AB^2 = (1\text{ m})^2 + (0.6\text{ m})^2 + (1.5\text{ m})^2$$

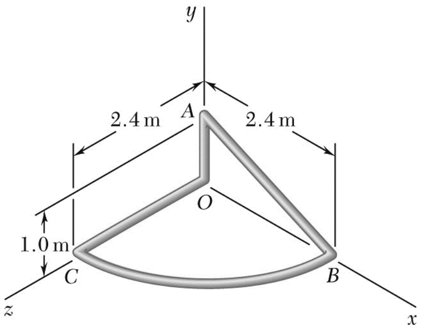
$$AB = 1.9\text{ m}$$

	L, m	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}L$, m^2	$\bar{y}L$, m^2	ΣL, m
AB	1.9	0.5	0.75	0.3	0.95	1.425	0.57
BD	0.6	1.0	0	0.3	0.60	0	0.18
DO	1.0	0.5	0	0	0.50	0	0
OA	1.5	0	0.75	0	0	1.125	0
Σ	5.0				2.05	2.550	0.75

$\bar{X}\Sigma L = \Sigma \bar{x} L$: $\bar{X}(5.0\text{ m}) = 2.05\text{ m}^2$ $\quad \bar{X} = 0.410\text{ m}$ ◀

$\bar{Y}\Sigma L = \Sigma \bar{y} L$: $\bar{Y}(5.0\text{ m}) = 2.55\text{ m}^2$ $\quad \bar{Y} = 0.510\text{ m}$ ◀

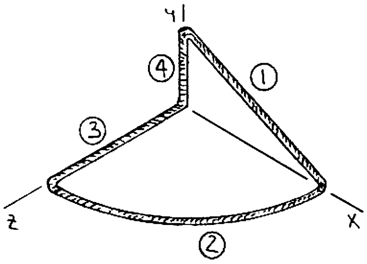
$\bar{Z}\Sigma L = \Sigma \bar{z} L$: $\bar{Z}(5.0\text{ m}) = 0.75\text{ m}^2$ $\quad \bar{Z} = 0.1500\text{ m}$ ◀



PROBLEM 5.116

A thin steel wire of uniform cross section is bent into the shape shown. Locate its center of gravity.

SOLUTION



First assume that the wire is homogeneous so that its center of gravity will coincide with the centroid of the corresponding line.

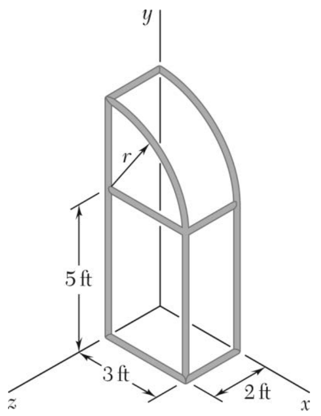
$$\bar{x}_2 = \bar{z}_2 = \frac{2 \times 2.4}{\pi} = \frac{4.8}{\pi} \text{ m}$$

	L, m	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}L$, m^2	$\bar{y}L$, m^2	$\bar{z}L$, m^2
1	2.6	1.2	0.5	0	3.12	1.3	0
2	$\frac{\pi}{2} \times 2.4 = 1.2\pi$	$\frac{4.8}{\pi}$	0	$\frac{4.8}{\pi}$	5.76	0	5.76
3	2.4	0	0	1.2	0	0	2.88
4	1.0	0	0.5	0	0	0.5	0
Σ	9.7699				8.88	1.8	8.64

We have $\bar{X}\Sigma L = \Sigma \bar{x} L$: $\bar{X}(9.7699 \text{ m}) = 8.88 \text{ m}^2$ or $\bar{X} = 0.909 \text{ m}$ ◀

$\bar{Y}\Sigma L = \Sigma \bar{y} L$: $\bar{Y}(9.7699 \text{ m}) = 1.8 \text{ m}^2$ or $\bar{Y} = 0.1842 \text{ m}$ ◀

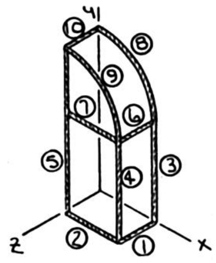
$\bar{Z}\Sigma L = \Sigma \bar{z} L$: $\bar{Z}(9.7699 \text{ m}) = 8.64 \text{ m}^2$ or $\bar{Z} = 0.884 \text{ m}$ ◀



PROBLEM 5.117

The frame of a greenhouse is constructed from uniform aluminum channels. Locate the center of gravity of the portion of the frame shown.

SOLUTION



First assume that the channels are homogeneous so that the center of gravity of the frame will coincide with the centroid of the corresponding line.

$$\bar{x}_8 = \bar{x}_9 = \frac{2\times 3}{\pi} = \frac{6}{\pi}\text{ ft}$$

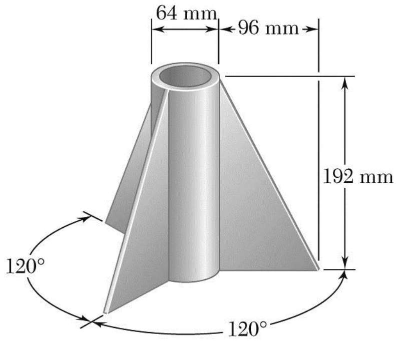
$$\bar{y}_8 = \bar{y}_9 = 5 + \frac{2\times 3}{\pi} = 6.9099\text{ ft}$$

	L, ft	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$\bar{x}L$, ft^2	$\bar{y}L$, ft^2	$\bar{z}L$, ft^2
1	2	3	0	1	6	0	2
2	3	1.5	0	2	4.5	0	6
3	5	3	2.5	0	15	12.5	0
4	5	3	2.5	2	15	12.5	10
5	8	0	4	2	0	32	16
6	2	3	5	1	6	10	2
7	3	1.5	5	2	4.5	15	6
8	$\frac{\pi}{2}\times 3 = 4.7124$	$\frac{6}{\pi}$	6.9099	0	9	32.562	0
9	$\frac{\pi}{2}\times 3 = 4.7124$	$\frac{6}{\pi}$	6.9099	2	9	32.562	9.4248
10	2	0	8	1	0	16	2
Σ	39.4248				69	163.124	53.4248

We have $\bar{X}\Sigma L = \Sigma\bar{x}L$: $\bar{X}(39.4248\text{ ft}) = 69\text{ ft}^2$ or $\bar{X} = 1.750\text{ ft}$ ◀

$\bar{Y}\Sigma L = \Sigma\bar{y}L$: $\bar{Y}(39.4248\text{ ft}) = 163.124\text{ ft}^2$ or $\bar{Y} = 4.14\text{ ft}$ ◀

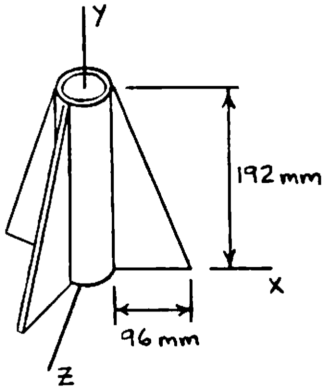
$\bar{Z}\Sigma L = \Sigma\bar{z}L$: $\bar{Z}(39.4248\text{ ft}) = 53.4248\text{ ft}^2$ or $\bar{Z} = 1.355\text{ ft}$ ◀



PROBLEM 5.118

Three brass plates are brazed to a steel pipe to form the flagpole base shown. Knowing that the pipe has a wall thickness of 8 mm and that each plate is 6 mm thick, determine the location of the center of gravity of the base. (Densities: brass = 8470 kg/m^3; steel = 7860 kg/m^3.)

SOLUTION



Since brass plates are equally spaced, we note that the center of gravity lies on the y-axis.

Thus, $\bar{x} = \bar{z} = 0$ ◀

Steel pipe: $V = \dfrac{\pi}{4}[(0.064\text{ m})^2 - (0.048\text{ m})^2](0.192\text{ m})$

$= 270.22\times 10^{-6}\text{ m}^3$

$m = \rho V = (7860\text{ kg/m}^3)(270.22\times 10^{-6}\text{ m}^3)$

$= 2.1239\text{ kg}$

Each brass plate: $V = \dfrac{1}{2}(0.096\text{ m})(0.192\text{ m})(0.006\text{ m}) = 55.296\times 10^{-6}\text{ m}^3$

$m = \rho V = (8470\text{ kg/m}^3)(55.296\times 10^{-6}\text{ m}^3) = 0.46836\text{ kg}$

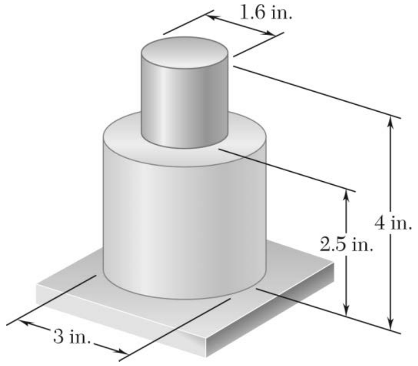
Flagpole base:

$$\Sigma m = 2.1239\text{ kg} + 3(0.46836\text{ kg}) = 3.5290\text{ kg}$$

$$\Sigma \bar{y} m = (0.096\text{ m})(2.1239\text{ kg}) + 3[(0.064\text{ m})(0.46836\text{ kg})] = 0.29382\text{ kg}\cdot\text{m}$$

$$\bar{Y}\Sigma m = \Sigma \bar{y} m: \quad \bar{Y}(3.5290\text{ kg}) = 0.29382\text{ kg}\cdot\text{m}$$

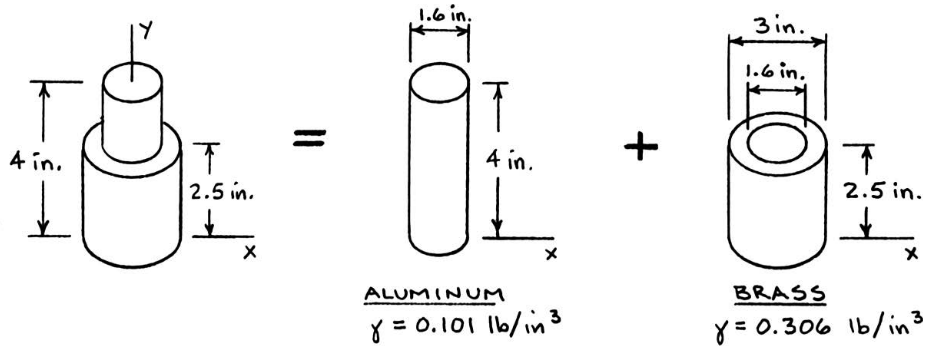
$\bar{Y} = 0.083259\text{ m}$ $\bar{Y} = 83.3\text{ mm}$ above the base ◀



PROBLEM 5.119

A brass collar, of length 2.5 in., is mounted on an aluminum rod of length 4 in. Locate the center of gravity of the composite body. (Specific weights: brass = 0.306 lb/in^3, aluminum = 0.101 lb/in^3)

SOLUTION



Aluminum rod:

$$W = \gamma V$$

$$= (0.101\ \text{lb/in}^3)\left[\frac{\pi}{4}(1.6\ \text{in.})^2(4\ \text{in.})\right]$$

$$= 0.81229\ \text{lb}$$

Brass collar:

$$W = \gamma V$$

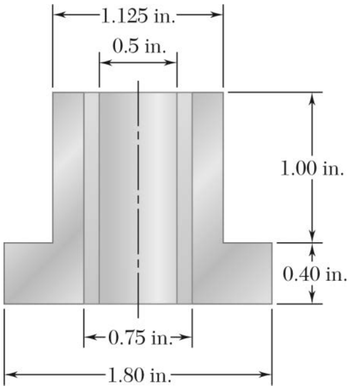
$$= (0.306\ \text{lb/in.}^3)\frac{\pi}{4}[(3\ \text{in.})^2 - (1.6\ \text{in.})^2](2.5\ \text{in.})$$

$$= 3.8693\ \text{lb}$$

Component	W(lb)	$\overline{y}$ (in.)	$\overline{y}W$ (lb · in.)
Rod	0.81229	2	1.62458
Collar	3.8693	1.25	4.8366
Σ	4.6816		6.4612

$$\overline{Y}\Sigma W = \Sigma\overline{y}W: \quad \overline{Y}(4.6816\ \text{lb}) = 6.4612\ \text{lb}\cdot\text{in.}$$

$$\overline{Y} = 1.38013\ \text{in.} \qquad \overline{Y} = 1.380\ \text{in.}\ \blacktriangleleft$$

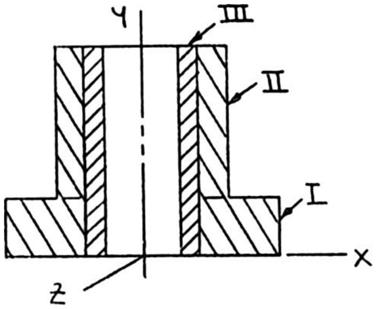


PROBLEM 5.120

A bronze bushing is mounted inside a steel sleeve. Knowing that the specific weight of bronze is 0.318 lb/in^3 and of steel is 0.284 lb/in^3, determine the location of the center of gravity of the assembly.

SOLUTION

First, note that symmetry implies $\bar{X} = \bar{Z} = 0$ ◀



Now $W = (\rho g)V$

$$\bar{y}_{\text{I}} = 0.20 \text{ in.} \quad W_{\text{I}} = (0.284 \text{ lb/in}^3)\left\{\left(\frac{\pi}{4}\right)[(1.8^2 - 0.75^2) \text{ in}^2](0.4 \text{ in.})\right\} = 0.23889 \text{ lb}$$

$$\bar{y}_{\text{II}} = 0.90 \text{ in.} \quad W_{\text{II}} = (0.284 \text{ lb/in}^3)\left\{\left(\frac{\pi}{4}\right)[(1.125^2 - 0.75^2) \text{ in}^2](1 \text{ in.})\right\} = 0.156834 \text{ lb}$$

$$\bar{y}_{\text{III}} = 0.70 \text{ in.} \quad W_{\text{III}} = (0.318 \text{ lb/in}^3)\left\{\left(\frac{\pi}{4}\right)[(0.75^2 - 0.5^2) \text{ in}^2](1.4 \text{ in.})\right\} = 0.109269 \text{ lb}$$

We have $\bar{Y}\Sigma W = \Sigma \bar{y} W$

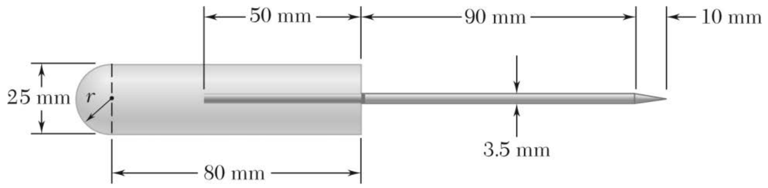
$$\bar{Y} = \frac{(0.20 \text{ in.})(0.23889 \text{ lb}) + (0.90 \text{ in.})(0.156834 \text{ lb}) + (0.70 \text{ in.})(0.109269 \text{ lb})}{0.23889 \text{ lb} + 0.156834 \text{ lb} + 0.109269 \text{ lb}}$$

or $\bar{Y} = 0.526 \text{ in.}$ ◀

(above base)

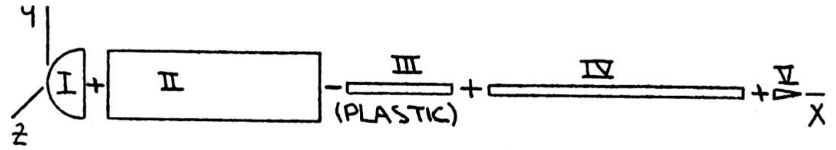
PROBLEM 5.121

A scratch awl has a plastic handle and a steel blade and shank. Knowing that the density of plastic is 1030 kg/m^3 and of steel is $7860\ \text{kg/m}^3$, locate the center of gravity of the awl.



SOLUTION

First, note that symmetry implies $\bar{Y} = \bar{Z} = 0$ ◀



$$\bar{x}_{\text{I}} = \frac{5}{8}(12.5\ \text{mm}) = 7.8125\ \text{mm}$$

$$W_{\text{I}} = (1030\ \text{kg/m}^3)\left(\frac{2\pi}{3}\right)(0.0125\ \text{m})^3$$

$$= 4.2133\times10^{-3}\ \text{kg}$$

$$\bar{x}_{\text{II}} = 52.5\ \text{mm}$$

$$W_{\text{II}} = (1030\ \text{kg/m}^3)\left(\frac{\pi}{4}\right)(0.025\ \text{m})^2(0.08\ \text{m})$$

$$= 40.448\times10^{-3}\ \text{kg}$$

$$\bar{x}_{\text{III}} = 92.5\ \text{mm} - 25\ \text{mm} = 67.5\ \text{mm}$$

$$W_{\text{III}} = -(1030\ \text{kg/m}^3)\left(\frac{\pi}{4}\right)(0.0035\ \text{m})^2(0.05\ \text{m})$$

$$= -0.49549\times10^{-3}\ \text{kg}$$

$$\bar{x}_{\text{IV}} = 182.5\ \text{mm} - 70\ \text{mm} = 112.5\ \text{mm}$$

$$W_{\text{IV}} = (7860\ \text{kg/m}^3)\left(\frac{\pi}{4}\right)(0.0035\ \text{m})^2(0.14\ \text{m})^2 = 10.5871\times10^{-3}\ \text{kg}$$

$$\bar{x}_{\text{V}} = 182.5\ \text{mm} + \frac{1}{4}(10\ \text{mm}) = 185\ \text{mm}$$

$$W_{\text{V}} = (7860\ \text{kg/m}^3)\left(\frac{\pi}{3}\right)(0.00175\ \text{m})^2(0.01\ \text{m}) = 0.25207\times10^{-3}\ \text{kg}$$

PROBLEM 5.121 (Continued)

	W, kg	$\bar{x}$, mm	$\bar{x}W$, kg $\cdot$ mm
I	4.123×10^{-3}	7.8125	32.916×10^{-3}
II	40.948×10^{-3}	52.5	2123.5×10^{-3}
III	-0.49549×10^{-3}	67.5	-33.447×10^{-3}
IV	10.5871×10^{-3}	112.5	1191.05×10^{-3}
V	0.25207×10^{-3}	185	46.633×10^{-3}
Σ	55.005×10^{-3}		3360.7×10^{-3}

We have $\bar{X}\Sigma W = \Sigma\bar{x}W: \quad \bar{X}(55.005\times10^{-3}\text{ kg}) = 3360.7\times10^{-3}\text{ kg}\cdot\text{mm}$

or $\bar{X} = 61.1\text{ mm}$ ◀

(from the end of the handle)

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PROBLEM 5.122

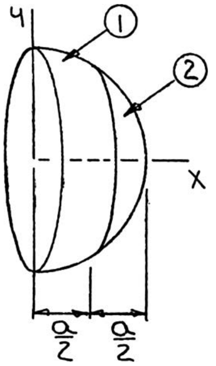
Determine by direct integration the values of $\bar{x}$ for the two volumes obtained by passing a vertical cutting plane through the given shape of Figure 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A hemisphere

SOLUTION

Choose as the element of volume a disk of radius *r* and thickness *dx*. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$



The equation of the generating curve is $x^2 + y^2 = a^2$ so that $r^2 = a^2 - x^2$ and then

$$dV = \pi(a^2 - x^2)dx$$

Component 1:

$$V_1 = \int_0^{a/2} \pi(a^2 - x^2)dx = \pi\left[a^2 x - \frac{x^3}{3}\right]_0^{a/2}$$

$$= \frac{11}{24}\pi a^3$$

and

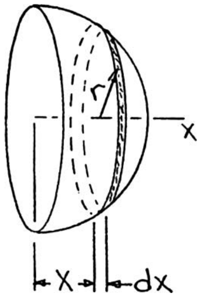
$$\int_1 \bar{x}_{EL} dV = \int_0^{a/2} x\left[\pi(a^2 - x^2)dx\right]$$

$$= \pi\left[a^2\frac{x^2}{2} - \frac{x^4}{4}\right]_0^{a/2}$$

$$= \frac{7}{64}\pi a^4$$

Now $\bar{x}_1 V_1 = \int_1 \bar{x}_{EL} dV: \quad \bar{x}_1\left(\frac{11}{24}\pi a^3\right) = \frac{7}{64}\pi a^4$ or $\bar{x}_1 = \frac{21}{88}a$ ◀

Component 2:



$$V_2 = \int_{a/2}^{a} \pi(a^2 - x^2)dx = \pi\left[a^2 x - \frac{x^3}{3}\right]_{a/2}^{a}$$

$$= \pi\left\{\left[a^2(a) - \frac{a^3}{3}\right] - \left[a^2\left(\frac{a}{2}\right) - \frac{\left(\frac{a}{2}\right)^3}{3}\right]\right\}$$

$$= \frac{5}{24}\pi a^3$$

PROBLEM 5.122 (Continued)

and
$$\int_2 \overline{x}_{EL}\,dV = \int_{a/2}^{a} x\left[\pi(a^2 - x^2)dx\right] = \pi\left[a^2\frac{x^2}{2} - \frac{x^4}{4}\right]_{a/2}^{a}$$

$$= \pi\left\{\left[a^2\frac{(a)^2}{2} - \frac{(a)^4}{4}\right] - \left[a^2\frac{\left(\frac{a}{2}\right)^2}{2} - \frac{\left(\frac{a}{2}\right)^4}{4}\right]\right\}$$

$$= \frac{9}{64}\pi a^4$$

Now
$$\overline{x}_2V_2 = \int_2 \overline{x}_{EL}\,dV: \quad \overline{x}_2\left(\frac{5}{24}\pi a^3\right) = \frac{9}{64}\pi a^4$$

or $\overline{x}_2 = \dfrac{27}{40}a$ ◀

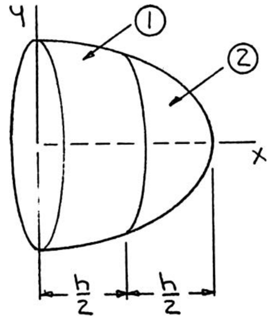
PROBLEM 5.123

Determine by direct integration the values of $\bar{x}$ for the two volumes obtained by passing a vertical cutting plane through the given shape of Figure 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A semiellipsoid of revolution

SOLUTION

Choose as the element of volume a disk of radius *r* and thickness *dx*. Then



$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

The equation of the generating curve is $\dfrac{x^2}{h^2} + \dfrac{y^2}{a^2} = 1$ so that

$$r^2 = \frac{a^2}{h^2}(h^2 - x^2)$$

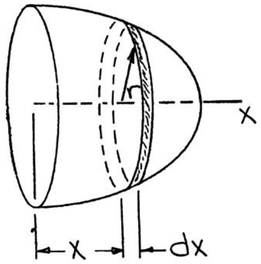
and then
$$dV = \pi \frac{a^2}{h^2}(h^2 - x^2)dx$$

Component 1:
$$V_1 = \int_0^{h/2} \pi \frac{a^2}{h^2}(h^2 - x^2)dx = \pi \frac{a^2}{h^2}\left[h^2 x - \frac{x^3}{3}\right]_0^{h/2}$$
$$= \frac{11}{24}\pi a^2 h$$

and
$$\int_1 \bar{x}_{EL} dV = \int_0^{h/2} x\left[\pi \frac{a^2}{h^2}(h^2 - x^2)dx\right]$$
$$= \pi \frac{a^2}{h^2}\left[h^2 \frac{x^2}{2} - \frac{x^4}{4}\right]_0^{h/2}$$
$$= \frac{7}{64}\pi a^2 h^2$$

Now
$$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL} dV: \quad \bar{x}_1\left(\frac{11}{24}\pi a^2 h\right) = \frac{7}{64}\pi a^2 h^2$$
or $\bar{x}_1 = \dfrac{21}{88}h$ ◀

Component 2:
$$V_2 = \int_{h/2}^{h} \pi \frac{a^2}{h^2}(h^2 - x^2)dx = \pi \frac{a^2}{h^2}\left[h^2 x - \frac{x^3}{3}\right]_{h/2}^{h}$$
$$= \pi \frac{a^2}{h^2}\left\{\left[h^2(h) - \frac{(h)^3}{3}\right] - \left[h^2\left(\frac{h}{2}\right) - \frac{\left(\frac{h}{2}\right)^3}{3}\right]\right\}$$
$$= \frac{5}{24}\pi a^2 h$$



PROBLEM 5.123 (Continued)

and
$$\int_2 \overline{x}_{EL}\,dV = \int_{h/2}^{h} x\left[\pi\frac{a^2}{h^2}(h^2-x^2)dx\right]$$
$$=\pi\frac{a^2}{h^2}\left[h^2\frac{x^2}{2}-\frac{x^4}{4}\right]_{h/2}^{h}$$
$$=\pi\frac{a^2}{h^2}\left\{\left[h^2\frac{(h)^2}{2}-\frac{(h)^4}{4}\right]-\left[h^2\frac{\left(\frac{h}{2}\right)^2}{2}-\frac{\left(\frac{h}{2}\right)^4}{4}\right]\right\}$$
$$=\frac{9}{64}\pi a^2h^2$$

Now
$$\overline{x}_2V_2=\int_2 \overline{x}_{EL}\,dV:\quad \overline{x}_2\left(\frac{5}{24}\pi a^2h\right)=\frac{9}{64}\pi a^2h^2$$

or $\overline{x}_2=\dfrac{27}{40}h$ ◀

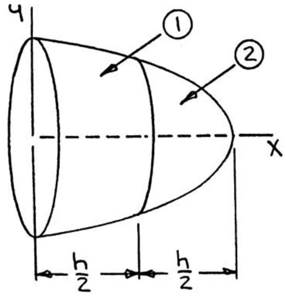
PROBLEM 5.124

Determine by direct integration the values of $\bar{x}$ for the two volumes obtained by passing a vertical cutting plane through the given shape of Figure 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A paraboloid of revolution

SOLUTION

Choose as the element of volume a disk of radius *r* and thickness *dx*. Then



$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

The equation of the generating curve is $x = h - \frac{h}{a^2}y^2$ so that $r^2 = \frac{a^2}{h}(h-x)$.

and then $$dV = \pi\frac{a^2}{h}(h-x)dx$$

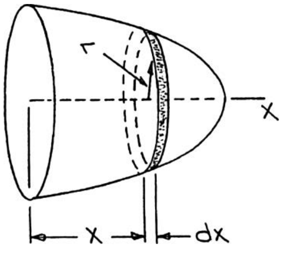
Component 1: $$V_1 = \int_0^{h/2} \pi\frac{a^2}{h}(h-x)dx$$
$$= \pi\frac{a^2}{h}\left[hx - \frac{x^2}{2}\right]_0^{h/2}$$
$$= \frac{3}{8}\pi a^2 h$$

and $$\int_1 \bar{x}_{EL} dV = \int_0^{h/2} x\left[\pi\frac{a^2}{h}(h-x)dx\right]$$
$$= \pi\frac{a^2}{h}\left[h\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{h/2} = \frac{1}{12}\pi a^2 h^2$$

Now $$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL} dV: \quad \bar{x}_1\left(\frac{3}{8}\pi a^2 h\right) = \frac{1}{12}\pi a^2 h^2$$ or $\bar{x}_1 = \frac{2}{9}h$ ◀

Component 2:

$$V_2 = \int_{h/2}^{h} \pi\frac{a^2}{h}(h-x)dx = \pi\frac{a^2}{h}\left[hx - \frac{x^2}{2}\right]_{h/2}^{h}$$
$$= \pi\frac{a^2}{h}\left\{\left[h(h) - \frac{(h)^2}{2}\right] - \left[h\left(\frac{h}{2}\right) - \frac{\left(\frac{h}{2}\right)^2}{2}\right]\right\}$$
$$= \frac{1}{8}\pi a^2 h$$



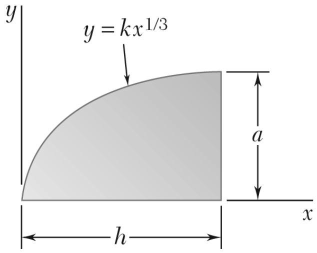
PROBLEM 5.124 (Continued)

and
$$\int_2 \overline{x}_{EL}\,dV = \int_{h/2}^{h} x\left[\pi\frac{a^2}{h}(h-x)\,dx\right] = \pi\frac{a^2}{h}\left[h\frac{x^2}{2}-\frac{x^3}{3}\right]_{h/2}^{h}$$

$$= \pi\frac{a^2}{h}\left\{\left[h\frac{(h)^2}{2}-\frac{(h)^3}{3}\right]-\left[h\frac{\left(\frac{h}{2}\right)^2}{2}-\frac{\left(\frac{h}{2}\right)^3}{3}\right]\right\}$$

$$= \frac{1}{12}\pi a^2 h^2$$

Now
$$\overline{x}_2 V_2 = \int_2 \overline{x}_{EL}\,dV: \quad \overline{x}_2\left(\frac{1}{8}\pi a^2 h\right) = \frac{1}{12}\pi a^2 h^2$$
or $\overline{x}_2 = \frac{2}{3}h$ ◀



PROBLEM 5.125

Locate the centroid of the volume obtained by rotating the shaded area about the *x*-axis.

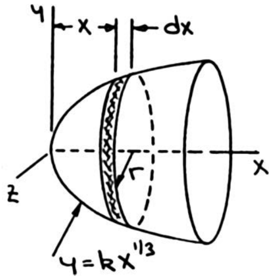
SOLUTION

First note that symmetry implies $\overline{y} = 0$ ◀

$\overline{z} = 0$ ◀

Choose as the element of volume a disk of radius *r* and thickness *dx*. Then

$$dV = \pi r^2 dx, \quad x_{EL} = x$$



Now $r = kx^{1/3}$

so that $dV = \pi k^2 x^{2/3} dx$

at $x = h,\ y = a,$ $a = kh^{1/3}$

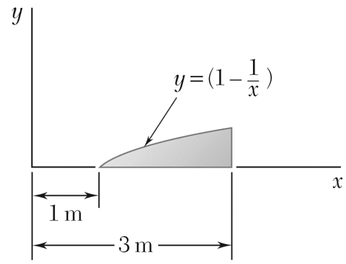
or $k = \dfrac{a}{h^{1/3}}$

Then $$dV = \pi \frac{a^2}{h^{2/3}} x^{2/3} dx$$

and $$V = \int_0^h \pi \frac{a^2}{h^{2/3}} x^{2/3} dx = \pi \frac{a^2}{h^{2/3}} \left[\frac{3}{5} x^{5/3} \right]_0^h = \frac{3}{5} \pi a^2 h$$

Also $$\int \overline{x}_{EL} dV = \int_0^h x \left(\pi \frac{a^2}{h^{2/3}} x^{2/3} dx \right) = \pi \frac{a^2}{h^{2/3}} \left[\frac{3}{8} x^{8/3} \right]_0^h = \frac{3}{8} \pi a^2 h^2$$

Now $\overline{x} V = \int \overline{x} dV$: $\quad \overline{x} \left(\dfrac{3}{5} \pi a^2 h \right) = \dfrac{3}{8} \pi a^2 h^2$ or $\overline{x} = \dfrac{5}{8} h$ ◀



PROBLEM 5.126

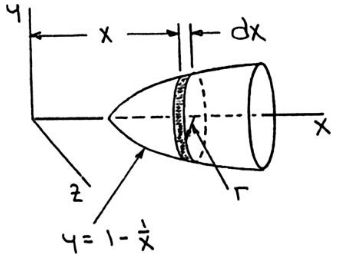
Locate the centroid of the volume obtained by rotating the shaded area about the *x*-axis.

SOLUTION

First, note that symmetry implies $\bar{y}=0$ ◀

$\bar{z}=0$ ◀

Choose as the element of volume a disk of radius *r* and thickness *dx*.



Then $dV=\pi r^2 dx, \quad \bar{x}_{EL}=x$

Now $r=1-\frac{1}{x}$ so that

$$dV=\pi\left(1-\frac{1}{x}\right)^2 dx$$

$$=\pi\left(1-\frac{2}{x}+\frac{1}{x^2}\right)dx$$

Then

$$V=\int_1^3 \pi\left(1-\frac{2}{x}+\frac{1}{x^2}\right)dx=\pi\left[x-2\ln x-\frac{1}{x}\right]_1^3$$

$$=\pi\left[\left(3-2\ln 3-\frac{1}{3}\right)-\left(1-2\ln 1-\frac{1}{1}\right)\right]$$

$$=(0.46944\pi)\ \text{m}^3$$

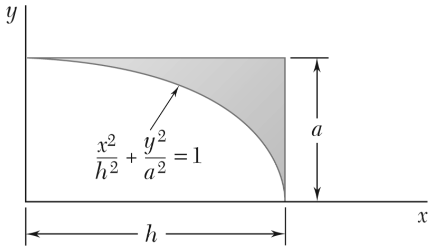
and

$$\int \bar{x}_{EL}dV=\int_1^3 x\left[\pi\left(1-\frac{2}{x}+\frac{1}{x^2}\right)dx\right]=\pi\left[\frac{x^2}{2}-2x+\ln x\right]_1^3$$

$$=\pi\left\{\left[\frac{3^2}{2}-2(3)+\ln 3\right]-\left[\frac{1^3}{2}-2(1)+\ln 1\right]\right\}$$

$$=(1.09861\pi)\ \text{m}$$

Now $\bar{x}V=\int \bar{x}_{EL}dV: \quad \bar{x}(0.46944\pi\ \text{m}^3)=1.09861\pi\ \text{m}^4$ or $\bar{x}=2.34$ m ◀



PROBLEM 5.127

Locate the centroid of the volume obtained by rotating the shaded area about the line $x = h$.

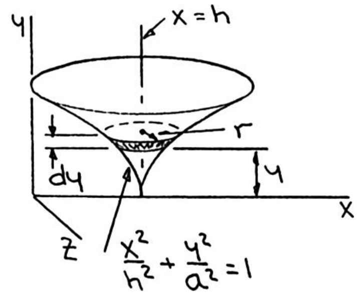
SOLUTION

First, note that symmetry implies $\bar{x} = h$ ◀

$\bar{z} = 0$ ◀

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dy, \quad \bar{y}_{EL} = y$$



Now $x^2 = \frac{h^2}{a^2}(a^2 - y^2)$ so that $r = h - \frac{h}{a}\sqrt{a^2 - y^2}$.

Then
$$dV = \pi \frac{h^2}{a^2}\left(a - \sqrt{a^2 - y^2}\right)^2 dy$$

and
$$V = \int_0^a \pi \frac{h^2}{a^2}\left(a - \sqrt{a^2 - y^2}\right)^2 dy$$

Let
$$y = a\sin\theta \Rightarrow dy = a\cos\theta\, d\theta$$

Then
$$V = \pi\frac{h^2}{a^2}\int_0^{\pi/2}\left(a - \sqrt{a^2 - a^2\sin^2\theta}\right)^2 a\cos\theta\, d\theta$$

$$= \pi\frac{h^2}{a^2}\int_0^{\pi/2}\left[a^2 - 2a(a\cos\theta) + (a^2 - a^2\sin^2\theta)\right]a\cos\theta\, d\theta$$

$$= \pi a h^2\int_0^{\pi/2}(2\cos\theta - 2\cos^2\theta - \sin^2\theta\cos\theta)\,d\theta$$

$$= \pi a h^2\left[2\sin\theta - 2\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) - \frac{1}{3}\sin^3\theta\right]_0^{\pi/2}$$

$$= \pi a h^2\left[2 - 2\left(\frac{\frac{\pi}{2}}{2}\right) - \frac{1}{3}\right]$$

$$= 0.095870\pi a h^2$$

PROBLEM 5.127 (Continued)

and

$$\int \bar{y}_{EL}\,dV = \int_0^a y\left[\pi\frac{h^2}{a^2}\left(a-\sqrt{a^2-y^2}\right)^2 dy\right]$$

$$= \pi\frac{h^2}{a^2}\int_0^a\left(2a^2y - 2ay\sqrt{a^2-y^2} - y^3\right)dy$$

$$= \pi\frac{h^2}{a^2}\left[a^2y^2 + \frac{2}{3}a(a^2-y^2)^{3/2} - \frac{1}{4}y^4\right]_0^a$$

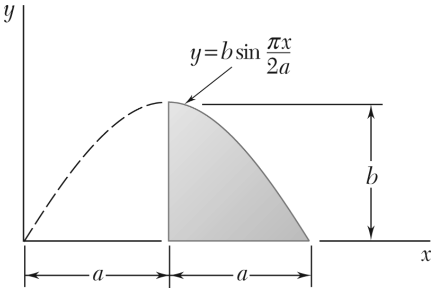
$$= \pi\frac{h^2}{a^2}\left\{\left[a^2(a)^2 - \frac{1}{4}a^4\right] - \left[\frac{2}{3}a(a^2)^{3/2}\right]\right\}$$

$$= \frac{1}{12}\pi a^2h^2$$

Now

$$\bar{y}V = \int \bar{y}_{EL}\,dV: \qquad \bar{y}(0.095870\pi a h^2) = \frac{1}{12}\pi a^2 h^2$$

or $\bar{y} = 0.869a$ ◀



PROBLEM 5.128*

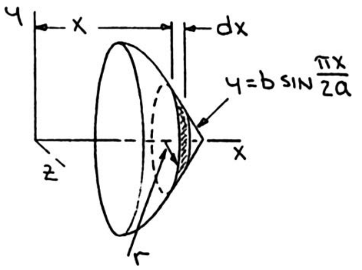
Locate the centroid of the volume generated by revolving the portion of the sine curve shown about the *x*-axis.

SOLUTION

First, note that symmetry implies $\bar{y} = 0$ ◀

$\bar{z} = 0$ ◀

Choose as the element of volume a disk of radius *r* and thickness *dx*.



Then $dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$

Now $r = b\sin\frac{\pi x}{2a}$

so that $dV = \pi b^2 \sin^2\frac{\pi x}{2a}dx$

Then

$$V = \int_a^{2a} \pi b^2 \sin^2\frac{\pi x}{2a}dx$$

$$= \pi b^2\left[\frac{x}{2} - \frac{\sin\frac{\pi x}{2a}}{2\frac{\pi}{a}}\right]_a^{2a}$$

$$= \pi b^2\left[\left(\tfrac{2a}{2}\right) - \left(\tfrac{a}{2}\right)\right]$$

$$= \frac{1}{2}\pi ab^2$$

and

$$\int \bar{x}_{EL}dV = \int_a^{2a} x\left(\pi b^2 \sin^2\frac{\pi x}{2a}dx\right)$$

Use integration by parts with

$$u = x \qquad dV = \sin^2\frac{\pi x}{2a}$$

$$du = dx \qquad V = \frac{x}{2} - \frac{\sin\frac{\pi x}{a}}{\frac{2\pi}{a}}$$

Then
$$\int \overline{x}_{EL} dV = \pi b^2 \left\{ \left[x\left(\frac{x}{2} - \frac{\sin\frac{\pi x}{a}}{\frac{2\pi}{a}} \right) \right]_a^{2a} - \int_a^{2a} \left(\frac{x}{2} - \frac{\sin\frac{\pi x}{a}}{\frac{2\pi}{a}} \right) dx \right\}$$

$$= \pi b^2 \left\{ \left[2a\left(\frac{2a}{2} \right) - a\left(\frac{a}{2} \right) \right] - \left[\frac{1}{4}x^2 + \frac{a^2}{2\pi^2}\cos\frac{\pi x}{a} \right]_a^{2a} \right\}$$

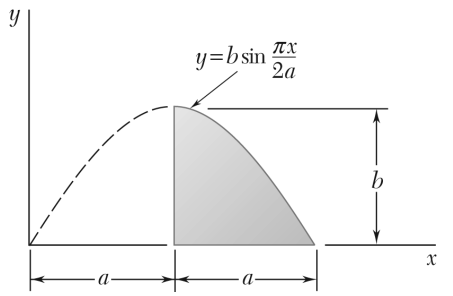
$$= \pi b^2 \left\{ \left(\frac{3}{2}a^2 \right) - \left[\frac{1}{4}(2a)^2 + \frac{a^2}{2\pi^2} - \frac{1}{4}(a)^2 + \frac{a^2}{2\pi^2} \right] \right\}$$

$$= \pi a^2 b^2 \left(\frac{3}{4} - \frac{1}{\pi^2} \right)$$

$$= 0.64868\pi a^2 b^2$$

Now
$$\overline{x}V = \int \overline{x}_{EL} dV: \quad \overline{x}\left(\frac{1}{2}\pi ab^2 \right) = 0.64868\pi a^2 b^2$$

or $\overline{x} = 1.297a$ ◀



PROBLEM 5.129*

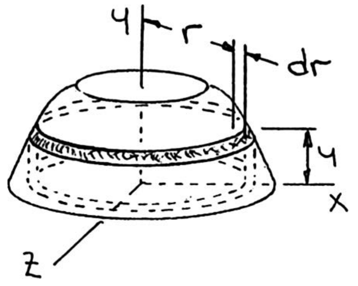
Locate the centroid of the volume generated by revolving the portion of the sine curve shown about the *y*-axis. (*Hint:* Use a thin cylindrical shell of radius *r* and thickness *dr* as the element of volume.)

SOLUTION

First note that symmetry implies $\bar{x} = 0$ ◀

$\bar{z} = 0$ ◀

Choose as the element of volume a cylindrical shell of radius *r* and thickness *dr*.



Then $$dV = (2\pi r)(y)(dr), \quad \bar{y}_{EL} = \frac{1}{2}y$$

Now $$y = b\sin\frac{\pi r}{2a}$$

so that $$dV = 2\pi br\sin\frac{\pi r}{2a}dr$$

Then $$V = \int_a^{2a} 2\pi br\sin\frac{\pi r}{2a}dr$$

Use integration by parts with

$$u = rd \qquad dv = \sin\frac{\pi r}{2a}dr$$

$$du = dr \qquad v = -\frac{2a}{\pi}\cos\frac{\pi r}{2a}$$

Then $$V = 2\pi b\left\{\left[(r)\left(-\frac{2a}{\pi}\cos\frac{\pi r}{2a}\right)\right]_a^{2a} - \int_a^{2a}\left(\frac{2a}{\pi}\cos\frac{\pi r}{2a}\right)dr\right\}$$

$$= 2\pi b\left\{-\frac{2a}{\pi}[(2a)(-1)] + \left[\frac{4a^2}{\pi^2}\sin\frac{\pi r}{2a}\right]_a^{2a}\right\}$$

$$V = 2\pi b\left(\frac{4a^2}{\pi} - \frac{4a^2}{\pi^2}\right)$$

$$= 8a^2b\left(1 - \frac{1}{\pi}\right)$$

$$= 5.4535a^2b$$

PROBLEM 5.129* (Continued)

Also
$$\int \overline{y}_{EL}\,dV = \int_a^{2a}\left(\tfrac{1}{2}b\sin\frac{\pi r}{2a}\right)\left(2\pi br\sin\frac{\pi r}{2a}dr\right)$$
$$= \pi b^2\int_a^{2a} r\sin^2\frac{\pi r}{2a}dr$$

Use integration by parts with

$$u = r \qquad dv = \sin^2\frac{\pi r}{2a}dr$$
$$du = dr \qquad v = \frac{r}{2} - \frac{\sin\frac{\pi r}{a}}{\frac{2\pi}{a}}$$

Then
$$\int \overline{y}_{EL}\,dV = \pi b^2\left\{\left[(r)\left(\frac{r}{2} - \frac{\sin\frac{\pi r}{a}}{\frac{2\pi}{a}}\right)\right]_a^{2a} - \int_a^{2a}\left(\frac{r}{2} - \frac{\sin\frac{\pi r}{a}}{\frac{2\pi}{a}}\right)dr\right\}$$
$$= \pi b^2\left\{\left[(2a)\left(\frac{2a}{2}\right) - (a)\left(\frac{a}{2}\right)\right] - \left[\frac{r^2}{4} + \frac{a^2}{2\pi^2}\cos\frac{\pi r}{a}\right]_a^{2a}\right\}$$
$$= \pi b^2\left\{\frac{3}{2}a^2 - \left[\frac{(2a)^2}{4} + \frac{a^2}{2\pi^2} - \frac{(a)^2}{4} + \frac{a^2}{2\pi^2}\right]\right\}$$
$$= \pi a^2b^2\left(\frac{3}{4} - \frac{1}{\pi^2}\right)$$
$$= 2.0379a^2b^2$$

Now $\overline{y}V = \int \overline{y}_{EL}\,dV$: $\quad \overline{y}(5.4535a^2b) = 2.0379a^2b^2$ $\qquad$ or $\overline{y} = 0.374b$ ◀

PROBLEM 5.130*

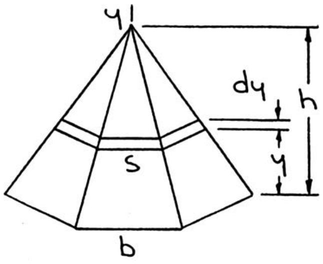
Show that for a regular pyramid of height h and n sides $(n = 3, 4, \ldots)$ the centroid of the volume of the pyramid is located at a distance $h/4$ above the base.

SOLUTION

Choose as the element of a horizontal slice of thickness dy. For any number N of sides, the area of the base of the pyramid is given by

$$A_{\text{base}} = kb^2$$

where $k = k(N)$; see note below. Using similar triangles, we have



$$\frac{s}{b} = \frac{h-y}{h}$$

or

$$s = \frac{b}{h}(h-y)$$

Then

$$dV = A_{\text{slice}}dy = ks^2dy = k\frac{b^2}{h^2}(h-y)^2\,dy$$

and

$$V = \int_0^h k\frac{b^2}{h^2}(h-y)^2\,dy = k\frac{b^2}{h^2}\left[-\frac{1}{3}(h-y)^3\right]_0^h$$

$$= \frac{1}{3}kb^2h$$

Also,

$$\bar{y}_{EL} = y$$

so that

$$\int \bar{y}_{EL}dV = \int_0^h y\left[k\frac{b^2}{h^2}(h-y)^2\,dy\right] = k\frac{b^2}{h^2}\int_0^h (h^2y - 2hy^2 + y^3)\,dy$$

$$= k\frac{b^2}{h^2}\left[\frac{1}{2}h^2y^2 - \frac{2}{3}hy^3 + \frac{1}{4}y^4\right]_0^h = \frac{1}{12}kb^2h^2$$

Now

$$\bar{y}V = \int \bar{y}_{EL}dV\colon \quad \bar{y}\left(\frac{1}{3}kb^2h\right) = \frac{1}{12}kb^2h^2$$

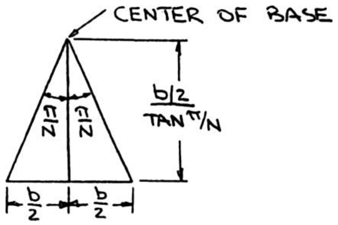
or $y = \frac{1}{4}h$ Q.E.D. ◀

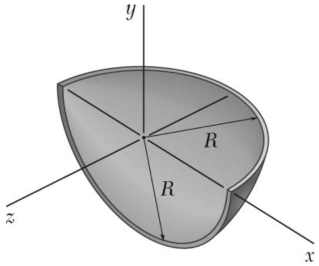
Note:

$$A_{\text{base}} = N\left(\frac{1}{2}\times b\times\frac{\frac{b}{2}}{\tan\frac{\pi}{N}}\right)$$

$$= \frac{N}{4\tan\frac{\pi}{N}}b^2$$

$$= k(N)b^2$$





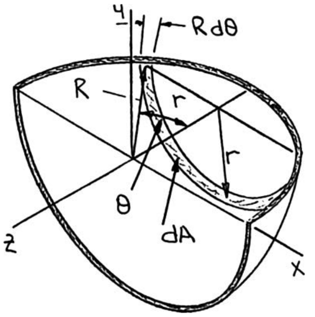
PROBLEM 5.131

Determine by direct integration the location of the centroid of one-half of a thin, uniform hemispherical shell of radius *R*.

SOLUTION

First note that symmetry implies $\bar{x} = 0$ ◀

The element of area *dA* of the shell shown is obtained by cutting the shell with two planes parallel to the *xy* plane. Now



$$dA = (\pi r)(R d\theta)$$

$$\bar{y}_{EL} = -\frac{2r}{\pi}$$

where $r = R\sin\theta$

so that $dA = \pi R^2 \sin\theta d\theta$

$$\bar{y}_{EL} = -\frac{2R}{\pi}\sin\theta$$

Then $A = \int_0^{\pi/2} \pi R^2 \sin\theta\, d\theta = \pi R^2[-\cos\theta]_0^{\pi/2}$

$$= \pi R^2$$

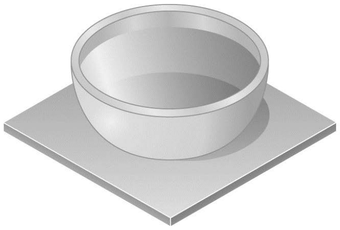
and $\int \bar{y}_{EL} dA = \int_0^{\pi/2}\left(-\frac{2R}{\pi}\sin\theta\right)(\pi R^2 \sin\theta d\theta)$

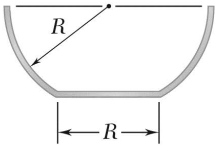
$$= -2R^3\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/2}$$

$$= -\frac{\pi}{2}R^3$$

Now $\bar{y}A = \int \bar{y}_{EL} dA$: $\bar{y}(\pi R^2) = -\frac{\pi}{2}R^3$ or $\bar{y} = -\frac{1}{2}R$ ◀

Symmetry implies $\bar{z} = \bar{y}$ $\bar{z} = -\frac{1}{2}R$ ◀





PROBLEM 5.132

The sides and the base of a punch bowl are of uniform thickness t. If $t << R$ and $R = 250$ mm, determine the location of the center of gravity of (*a*) the bowl, (*b*) the punch.

SOLUTION

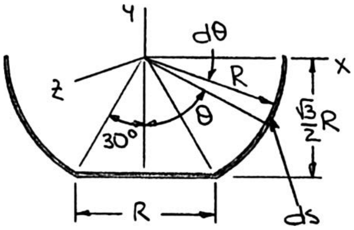
(*a*) Bowl:

First note that symmetry implies $\bar{x} = 0$ ◀

$\bar{z} = 0$ ◀

for the coordinate axes shown below. Now assume that the bowl may be treated as a shell; the center of gravity of the bowl will coincide with the centroid of the shell. For the walls of the bowl, an element of area is obtained by rotating the arc ds about the y-axis. Then

$$dA_{\text{wall}} = (2\pi R\sin\theta)(R\,d\theta)$$



and $$(\bar{y}_{EL})_{\text{wall}} = -R\cos\theta$$

Then
$$\begin{aligned}A_{\text{wall}} &= \int_{\pi/6}^{\pi/2} 2\pi R^2\sin\theta\,d\theta\\ &= 2\pi R^2[-\cos\theta]_{\pi/6}^{\pi/2}\\ &= \pi\sqrt{3}R^2\end{aligned}$$

and
$$\begin{aligned}\bar{y}_{\text{wall}}A_{\text{wall}} &= \int(\bar{y}_{EL})_{\text{wall}}\,dA\\ &= \int_{\pi/6}^{\pi/2}(-R\cos\theta)(2\pi R^2\sin\theta d\theta)\\ &= \pi R^3[\cos^2\theta]_{\pi/6}^{\pi/2}\\ &= -\frac{3}{4}\pi R^3\end{aligned}$$

By observation, $$A_{\text{base}} = \frac{\pi}{4}R^2, \quad \bar{y}_{\text{base}} = -\frac{\sqrt{3}}{2}R$$

Now $$\bar{y}\Sigma A = \Sigma\bar{y}A$$

or $$\bar{y}\left(\pi\sqrt{3}R^2 + \frac{\pi}{4}R^2\right) = -\frac{3}{4}\pi R^3 + \frac{\pi}{4}R^2\left(-\frac{\sqrt{3}}{2}R\right)$$

or $$\bar{y} = -0.48763R \quad R = 250\text{ mm}$$

$\bar{y} = -121.9$ mm ◀

PROBLEM 5.132 (Continued)

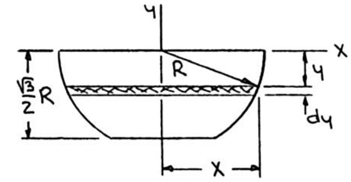
(*b*) <u>Punch</u>:

First note that symmetry implies $\bar{x} = 0$ ◀

$\bar{z} = 0$ ◀

and that because the punch is homogeneous, its center of gravity will coincide with the centroid of the corresponding volume. Choose as the element of volume a disk of radius *x* and thickness *dy*. Then

$$dV = \pi x^2 dy, \quad \bar{y}_{EL} = y$$



Now $\quad x^2 + y^2 = R^2$

so that $\quad dV = \pi(R^2 - y^2)dy$

Then

$$V = \int_{-\sqrt{3}/2R}^{0} \pi(R^2 - y^2)\,dy$$

$$= \pi\left[R^2 y - \frac{1}{3}y^3\right]_{-\sqrt{3}/2R}^{0}$$

$$= -\pi\left[R^2\left(-\frac{\sqrt{3}}{2}R\right) - \frac{1}{3}\left(-\frac{\sqrt{3}}{2}R\right)^3\right] = \frac{3}{8}\pi\sqrt{3}R^3$$

and

$$\int \bar{y}_{EL}dV = \int_{-\sqrt{3}/2R}^{0} (y)\left[\pi\left(R^2 - y^2\right)dy\right]$$

$$= \pi\left[\frac{1}{2}R^2y^2 - \frac{1}{4}y^4\right]_{-\sqrt{3}/2R}^{0}$$

$$= -\pi\left[\frac{1}{2}R^2\left(-\frac{\sqrt{3}}{2}R\right)^2 - \frac{1}{4}\left(-\frac{\sqrt{3}}{2}R\right)^4\right] = -\frac{15}{64}\pi R^4$$

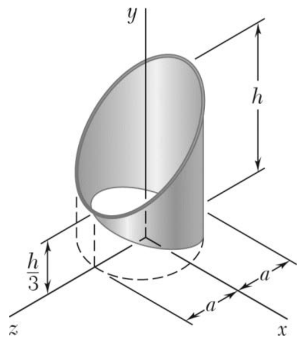
Now

$$\bar{y}V = \int \bar{y}_{EL}dV: \quad \bar{y}\left(\frac{3}{8}\pi\sqrt{3}R^3\right) = -\frac{15}{64}\pi R^4$$

or

$$\bar{y} = -\frac{5}{8\sqrt{3}}R \quad R = 250\text{ mm}$$

$\bar{y} = -90.2$ mm ◀



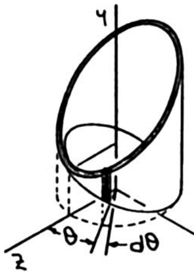
PROBLEM 5.133

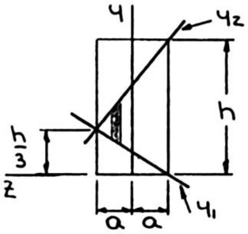
Locate the centroid of the section shown, which was cut from a thin circular pipe by two oblique planes.

SOLUTION

First note that symmetry implies $\overline{x} = 0$ ◀

Assume that the pipe has a uniform wall thickness t and choose as the element of volume a vertical strip of width $ad\theta$ and height $(y_2 - y_1)$. Then





$$dV = (y_2 - y_1)ta\,d\theta, \quad \overline{y}_{EL} = \frac{1}{2}(y_1 + \overline{y}_2)\,\overline{z}_{EL} = z$$

Now

$$y_1 = \frac{\frac{h}{3}}{2a}z + \frac{h}{6} \qquad y_2 = -\frac{\frac{2h}{3}}{2a}z + \frac{2}{3}h$$

$$= \frac{h}{6a}(z + a) \qquad = \frac{h}{3a}(-z + 2a)$$

and

$$z = a\cos\theta$$

Then

$$(y_2 - y_1) = \frac{h}{3a}(-a\cos\theta + 2a) - \frac{h}{6a}(a\cos\theta + a)$$

$$= \frac{h}{2}(1 - \cos\theta)$$

and

$$(y_1 + y_2) = \frac{h}{6a}(a\cos\theta + a) + \frac{h}{3a}(-a\cos\theta + 2a)$$

$$= \frac{h}{6}(5 - \cos\theta)$$

$$dV = \frac{aht}{2}(1 - \cos\theta)d\theta \quad \overline{y}_{EL} = \frac{h}{12}(5 - \cos\theta), \quad \overline{z}_{EL} = a\cos\theta$$

PROBLEM 5.133 (Continued)

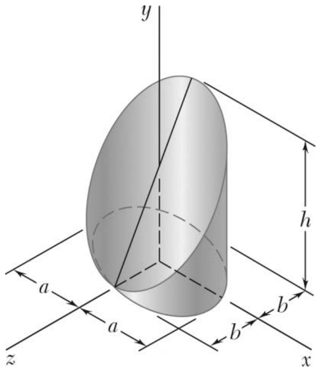
Then
$$V = 2\int_0^{\pi} \frac{aht}{2}(1-\cos\theta)d\theta = aht[\theta - \sin\theta]_0^{\pi}$$
$$= \pi aht$$

and
$$\int \overline{y}_{EL} dV = 2\int_0^{\pi} \frac{h}{12}(5-\cos\theta)\left[\frac{aht}{2}(1-\cos\theta)d\theta\right]$$
$$= \frac{ah^2t}{12}\int_0^{\pi}(5 - 6\cos\theta + \cos^2\theta)d\theta$$
$$= \frac{ah^2t}{12}\left[5\theta - 6\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\pi}$$
$$= \frac{11}{24}\pi ah^2t$$

$$\int \overline{z}_{EL} dV = 2\int_0^{\pi} a\cos\theta\left[\frac{aht}{2}(1-\cos\theta)d\theta\right]$$
$$= a^2ht\left[\sin\theta - \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}$$
$$= -\frac{1}{2}\pi a^2ht$$

Now
$$\overline{y}V = \int \overline{y}_{EL} dV: \quad \overline{y}(\pi aht) = \frac{11}{24}\pi ah^2t \qquad \text{or } \overline{y} = \frac{11}{24}h \blacktriangleleft$$

and
$$\overline{z}V = \int \overline{z}_{EL} dV: \quad \overline{z}(\pi aht) = -\frac{1}{2}\pi a^2ht \qquad \text{or } \overline{z} = -\frac{1}{2}a \blacktriangleleft$$

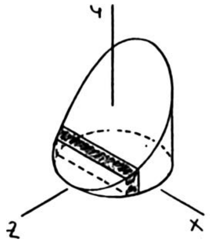


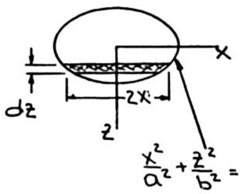
PROBLEM 5.134*

Locate the centroid of the section shown, which was cut from an elliptical cylinder by an oblique plane.

SOLUTION

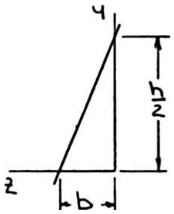
First note that symmetry implies $x = 0$ ◀





Choose as the element of volume a vertical slice of width *zx*, thickness *dz*, and height *y*. Then

$$dV = 2xy\,dz, \quad \bar{y}_{EL} = \frac{1}{24}, \quad \bar{z}_{EL} = z$$



Now $\quad x = \frac{a}{b}\sqrt{b^2 - z^2}$

and $\quad y = -\frac{h/2}{b}z + \frac{h}{2} = \frac{h}{2b}(b - z)$

Then $\quad V = \int_{-b}^{b}\left(2\frac{a}{b}\sqrt{b^2 - z^2}\right)\left[\frac{h}{2b}(b - z)\right]dz$

Let $\quad z = b\sin\theta \quad dz = b\cos\theta\, d\theta$

Then $\quad V = \frac{ah}{b^2}\int_{\pi/2}^{\pi/2}(b\cos\theta)[b(1 - \sin\theta)]b\cos\theta\, d\theta$

$$= abh\int_{-\pi/2}^{\pi/2}(\cos^2\theta - \sin\theta\cos^2\theta)\,d\theta$$

$$= abh\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} + \frac{1}{3}\cos^3\theta\right]_{-\pi/2}^{\pi/2}$$

$$V = \frac{1}{2}\pi abh$$

PROBLEM 5.134* (Continued)

and
$$\int \overline{y}_{EL} dV = \int_{-b}^{b} \left[\frac{1}{2} \times \frac{h}{2b}(b-z)\right]\left\{\left(2\frac{a}{b}\sqrt{b^2-z^2}\right)\left[\frac{h}{2b}(b-z)\right]dz\right\}$$
$$= \frac{1}{4}\frac{ah^2}{b^3}\int_{-b}^{b}(b-z)^2\sqrt{b^2-z^2}\,dz$$

Let
$$z = b\sin\theta \quad dz = b\cos\theta\, d\theta$$

Then
$$\int \overline{y}_{EL} dV = \frac{1}{4}\frac{ah^2}{b^3}\int_{-\pi/2}^{\pi/2}[b(1-\sin\theta)]^2(b\cos\theta)\times(b\cos\theta\, d\theta)$$
$$= \frac{1}{4}abh^2\int_{-\pi/2}^{\pi/2}(\cos^2\theta - 2\sin\theta\cos^2\theta + \sin^2\theta\cos^2\theta)\,d\theta$$

Now
$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \quad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

so that
$$\sin^2\theta\cos^2\theta = \frac{1}{4}(1-\cos^2 2\theta)$$

Then
$$\int \overline{y}_{EL} dV = \frac{1}{4}abh^2\int_{-\pi/2}^{\pi/2}\left[\cos^2\theta - 2\sin\theta\cos^2\theta + \frac{1}{4}(1-\cos^2 2\theta)\right]d\theta$$
$$= \frac{1}{4}abh^2\left[\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right)+\frac{1}{3}\cos^3\theta + \frac{1}{4}\theta - \frac{1}{4}\left(\frac{\theta}{2}+\frac{\sin 4\theta}{8}\right)\right]_{-\pi/2}^{\pi/2}$$
$$= \frac{5}{32}\pi abh^2$$

Also,
$$\int \overline{z}_{EL} dV = \int_{-b}^{b} z\left\{2\frac{a}{b}\sqrt{a^2-z^2}\left[\frac{h}{2b}(b-z)\right]dz\right\}$$
$$= \frac{ah}{b^2}\int_{-b}^{b} z(b-z)\sqrt{b^2-z^2}\,dz$$

Let
$$z = b\sin\theta \quad dz = b\cos\theta\, d\theta$$

Then
$$\int \overline{z}_{EL} dV = \frac{ah}{b^2}\int_{-\pi/2}^{\pi/2}(b\sin\theta)[b(1-\sin\theta)](b\cos\theta)\times(b\cos\theta\, d\theta)$$
$$= ab^2h\int_{-\pi/2}^{\pi/2}(\sin\theta\cos^2\theta - \sin^2\theta\cos^2\theta)d\theta$$

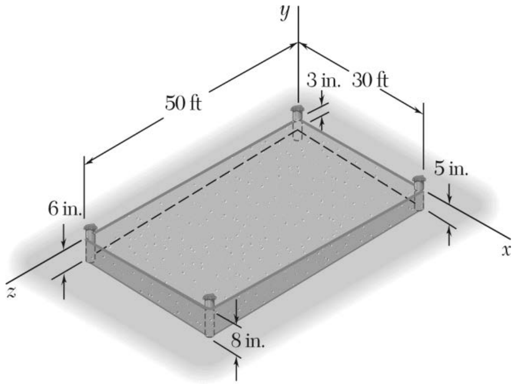
Using
$$\sin^2\theta\cos^2\theta = \frac{1}{4}(1-\cos^2 2\theta) \text{ from above,}$$

$$\int z_{EL} dV = ab^2h\int_{-\pi/2}^{\pi/2}\left[\sin\theta\cos^2\theta - \frac{1}{4}(1-\cos^2 2\theta)\right]d\theta$$
$$= ab^2h\left[-\frac{1}{3}\cos^3\theta - \frac{1}{4}\theta + \frac{1}{4}\left(\frac{\theta}{2}+\frac{\sin 4\theta}{8}\right)\right]_{-\pi/2}^{\pi/2} = -\frac{1}{8}\pi ab^2h$$

PROBLEM 5.134* (Continued)

Now $\quad \bar{y}V = \int \bar{y}_{EL}dV: \quad \bar{y}\left(\frac{1}{2}\pi abh\right) = \frac{5}{32}\pi abh^2 \quad$ or $\quad \bar{y} = \frac{5}{16}h$ ◀

and $\quad \bar{z}V = \int \bar{z}_{EL}dV: \quad \bar{z}\left(\frac{1}{2}\pi abh\right) = -\frac{1}{8}\pi ab^2h \quad$ or $\quad \bar{z} = -\frac{1}{4}b$ ◀

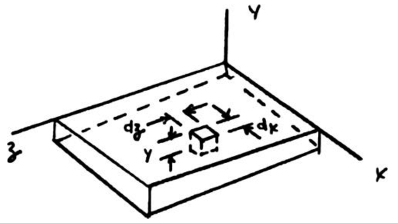


PROBLEM 5.135

After grading a lot, a builder places four stakes to designate the corners of the slab for a house. To provide a firm, level base for the slab, the builder places a minimum of 3 in. of gravel beneath the slab. Determine the volume of gravel needed and the *x* coordinate of the centroid of the volume of the gravel. (*Hint:* The bottom surface of the gravel is an oblique plane, which can be represented by the equation $y = a + bx + cz$.)

SOLUTION

The centroid can be found by integration. The equation for the bottom of the gravel is $y = a + bx + cz$, where the constants a, b, and c can be determined as follows:



For $x = 0$ and $z = 0$, $y = -3$ in., and therefore,

$$-\frac{3}{12}\text{ ft} = a, \quad \text{or} \quad a = -\frac{1}{4}\text{ ft}$$

For $x = 30$ ft and $z = 0$, $y = -5$ in., and therefore,

$$-\frac{5}{12}\text{ ft} = -\frac{1}{4}\text{ ft} + b(30\text{ ft}), \quad \text{or} \quad b = -\frac{1}{180}$$

For $x = 0$ and $z = 50$ ft, $y = -6$ in., and therefore,

$$-\frac{6}{12}\text{ ft} = -\frac{1}{4}\text{ ft} + c(50\text{ ft}), \quad \text{or} \quad c = -\frac{1}{200}$$

Therefore,

$$y = -\frac{1}{4}\text{ ft} - \frac{1}{180}x - \frac{1}{200}z$$

Now

$$\bar{x} = \frac{\int x_{EL}\,dV}{V}$$

A volume element can be chosen as

$$dV = |y|\,dx\,dz$$

or

$$dV = \frac{1}{4}\left(1 + \frac{1}{45}x + \frac{1}{50}z\right)dx\,dz$$

and

$$\bar{x}_{EL} = x$$

PROBLEM 5.135 (Continued)

Then
$$\int x_{EL}dV = \int_0^{50}\int_0^{30}\frac{x}{4}\left(1+\frac{1}{45}x+\frac{1}{50}z\right)dx\,dz$$
$$=\frac{1}{4}\int_0^{50}\left[\frac{x^2}{2}+\frac{1}{135}x^3+\frac{z}{100}x^2\right]_0^{30}dz$$
$$=\frac{1}{4}\int_0^{50}(650+9z)\,dz$$
$$=\frac{1}{4}\left[650\,z+\frac{9}{2}z^2\right]_0^{50}$$
$$=10937.5\text{ ft}^4$$

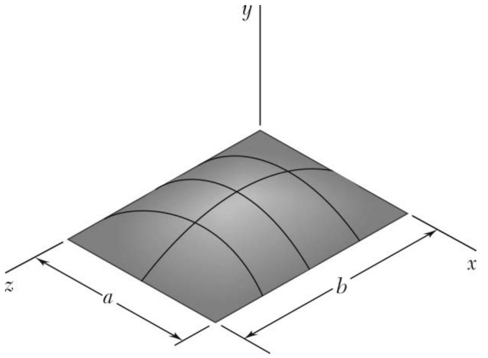
The volume is
$$V\int dV = \int_0^{50}\int_0^{30}\frac{1}{4}\left(1+\frac{1}{45}x+\frac{1}{50}z\right)dx\,dz$$
$$=\frac{1}{4}\int_0^{50}\left[x+\frac{1}{90}x^2+\frac{z}{50}x\right]_0^{30}dz$$
$$=\frac{1}{4}\int_0^{50}\left(40+\frac{3}{5}z\right)dz$$
$$=\frac{1}{4}\left[40z+\frac{3}{10}z^2\right]_0^{50}$$
$$=687.50\text{ ft}^3$$

Then
$$\bar{x}=\frac{\int \bar{x}_{EL}dV}{V}=\frac{10937.5\text{ ft}^4}{687.5\text{ ft}^3}=15.9091\text{ ft}$$

Therefore,

$V = 688\text{ ft}^3$ ◀

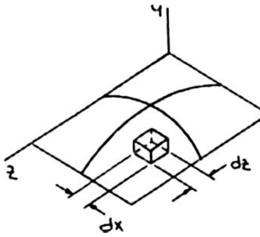
$\bar{x} = 15.91\text{ ft}$ ◀



PROBLEM 5.136

Determine by direct integration the location of the centroid of the volume between the *xz* plane and the portion shown of the surface $y = 16h(ax - x^2)(bz - z^2)/a^2b^2$.

SOLUTION



First note that symmetry implies $\bar{x} = \dfrac{a}{2}$ ◀

$\bar{z} = \dfrac{b}{2}$ ◀

Choose as the element of volume a filament of base $dx \times dz$ and height *y*. Then

$$dV = y\,dx\,dz, \quad \bar{y}_{EL} = \frac{1}{2}y$$

or $$dV = \frac{16h}{a^2b^2}(ax - x^2)(bz - z^2)dx\,dz$$

Then $$V = \int_0^b \int_0^a \frac{16h}{a^2b^2}(ax - x^2)(bz - z^2)dx\,dz$$

$$V = \frac{16h}{a^2b^2}\int_0^b (bz - z^2)\left[\frac{a}{z}x^2 - \frac{1}{3}x^3\right]_0^a dz$$

$$= \frac{16h}{a^2b^2}\left[\frac{a}{2}(a^2) - \frac{1}{3}(a)^3\right]\left[\frac{b}{2}z^2 - \frac{1}{3}z^3\right]_0^b$$

$$= \frac{8ah}{3b^2}\left[\frac{b}{2}(b)^2 - \frac{1}{3}(b)^3\right]$$

$$= \frac{4}{9}abh$$

PROBLEM 5.136 (Continued)

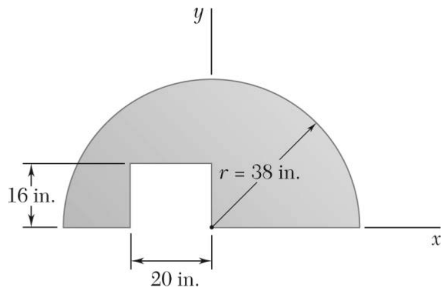
and

$$\begin{aligned}
\int \bar{y}_{EL}dV &= \int_0^b\int_0^a \frac{1}{2}\left[\frac{16h}{a^2b^2}(ax-x^2)(bz-z^2)\right]\left[\frac{16h}{a^2b^2}(ax-x^2)(bz-z^2)dx\,dz\right] \\
&= \frac{128h^2}{a^4b^4}\int_0^b\int_0^a (a^2x^2-2ax^3+x^4)(b^2z^2-2bz^3+z^4)dx\,dz \\
&= \frac{128h^2}{a^2b^4}\int_0^b (b^2z^2-2bz^3+z^4)\left[\frac{a^2}{3}x^3-\frac{a}{2}x^4+\frac{1}{5}x^5\right]_0^a dz \\
&= \frac{128h^2}{a^4b^4}\left[\frac{a^2}{3}(a)^3-\frac{a}{2}(a)^4+\frac{1}{5}(a)^5\right]\left[\frac{b^2}{3}z^3-\frac{b}{z}z^4+\frac{1}{5}z^5\right]_0^b \\
&= \frac{64ah^2}{15b^4}\left[\frac{b^3}{3}(b)^3-\frac{b}{2}(b)^4+\frac{1}{5}(b)^5\right] = \frac{32}{225}abh^2
\end{aligned}$$

Now

$$\bar{y}V = \int \bar{y}_{EL}dV: \quad \bar{y}\left(\frac{4}{9}abh\right) = \frac{32}{225}abh^2$$

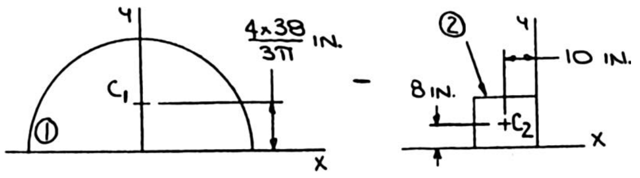
or $\bar{y} = \dfrac{8}{25}h$ ◀



PROBLEM 5.137

Locate the centroid of the plane area shown.

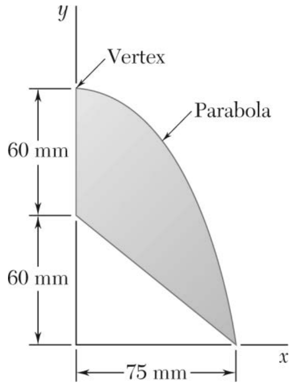
SOLUTION



	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$\frac{\pi}{2}(38)^2 = 2268.2$	0	16.1277	0	36,581
2	$-20\times 16 = 320$	−10	8	3200	−2560
Σ	1948.23			3200	34,021

Then
$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{3200}{1948.23} \qquad \bar{X} = 1.643 \text{ in.} ◀$$

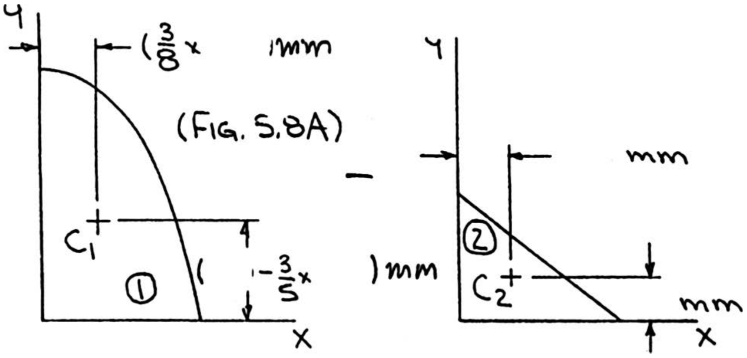
$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{34,021}{1948.23} \qquad \bar{Y} = 17.46 \text{ in.} ◀$$



PROBLEM 5.138

Locate the centroid of the plane area shown.

SOLUTION



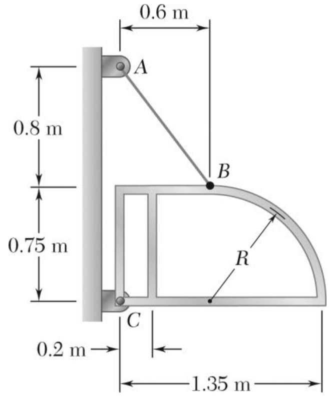
	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{2}{3}(75)(120) = 6000$	28.125	48	168,750	288,000
2	$-\frac{1}{2}(75)(60) = -2250$	25	20	–56,250	–45,000
Σ	3750			112,500	243,000

Then $\bar{X}\Sigma A = \Sigma \bar{x}A$

$\bar{X}(3750 \text{ mm}^2) = 112{,}500 \text{ mm}^3$ or $\bar{X} = 30.0$ mm ◀

and $\bar{Y}\Sigma A = \Sigma \bar{y}A$

$\bar{Y}(3750 \text{ mm}^2) = 243{,}000 \text{ mm}^3$ or $\bar{Y} = 64.8$ mm ◀

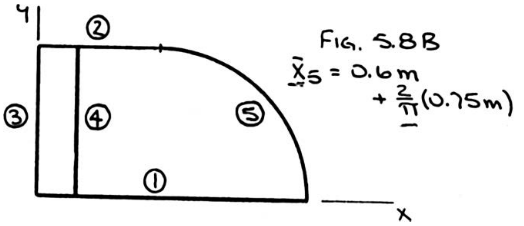


PROBLEM 5.139

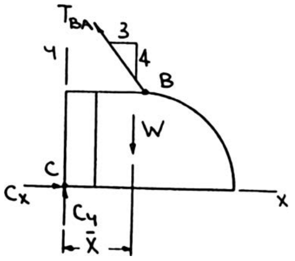
The frame for a sign is fabricated from thin, flat steel bar stock of mass per unit length 4.73 kg/m. The frame is supported by a pin at *C* and by a cable *AB*. Determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION

First note that because the frame is fabricated from uniform bar stock, its center of gravity will coincide with the centroid of the corresponding line.



	L, m	$\bar{x}$, m	$\bar{x}L$, m^2
1	1.35	0.675	0.91125
2	0.6	0.3	0.18
3	0.75	0	0
4	0.75	0.2	0.15
5	$\frac{\pi}{2}(0.75) = 1.17810$	1.07746	1.26936
Σ	4.62810		2.5106



Then $$\bar{X}\Sigma L = \Sigma\bar{x}L$$

$$\bar{X}(4.62810) = 2.5106$$

or $$\bar{X} = 0.54247 \text{ m}$$

The free-body diagram of the frame is then

where $$W = (m'\Sigma L)g$$

$$= 4.73 \text{ kg/m} \times 4.62810 \text{ m} \times 9.81 \text{ m/s}^2$$

$$= 214.75 \text{ N}$$

PROBLEM 5.139 (Continued)

Equilibrium then requires

(*a*) $$\Sigma M_C = 0: \quad (1.55\text{ m})\left(\frac{3}{5}T_{BA}\right) - (0.54247\text{ m})(214.75\text{ N}) = 0$$

or $T_{BA} = 125.264\text{ N}$ or $T_{BA} = 125.3\text{ N}$ ◀

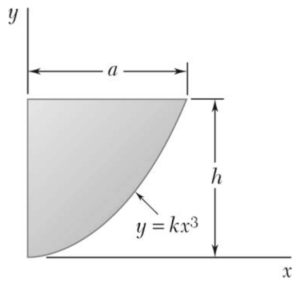
(*b*) $$\Sigma F_x = 0: \quad C_x - \frac{3}{5}(125.264\text{ N}) = 0$$

or $\mathbf{C}_x = 75.158\text{ N} \longrightarrow$

$$\Sigma F_y = 0: \quad C_y + \frac{4}{5}(125.264\text{ N}) - (214.75\text{ N}) = 0$$

or $\mathbf{C}_y = 114.539\text{ N} \uparrow$

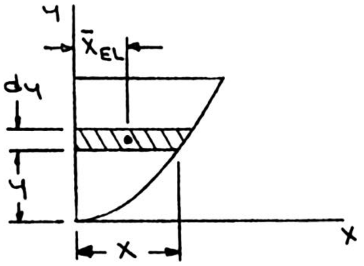
Then $\mathbf{C} = 137.0\text{ N} \measuredangle 56.7^\circ$ ◀



PROBLEM 5.140

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *h*.

SOLUTION



For the element (EL) shown,

at $x = a,\ y = h,\quad h = ka^3\quad$ or $\quad k = \dfrac{h}{a^3}$

Then $$x = \frac{a}{h^{1/3}}y^{1/3}$$

Now $$dA = x\,dy = \frac{a}{h^{1/3}}y^{1/3}dy$$

$$\bar{x}_{EL} = \frac{1}{2}x = \frac{1}{2}\frac{a}{h^{1/3}}y^{1/3}$$

$$\bar{y}_{EL} = y$$

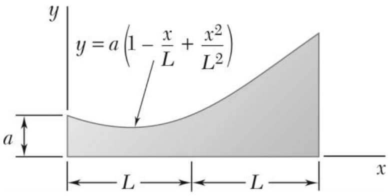
Then $$A = \int dA = \int_0^h \frac{a}{h^{1/3}}y^{1/3}dy = \frac{3}{4}\frac{a}{h^{1/3}}\left(y^{4/3}\right)\Big|_0^h = \frac{3}{4}ah$$

and $$\int \bar{x}_{EL}dA = \int_0^h \frac{1}{2}\frac{a}{h^{1/3}}y^{1/3}\left(\frac{a}{h^{1/3}}y^{1/3}dy\right) = \frac{1}{2}\frac{a}{h^{2/3}}\left(\frac{3}{5}y^{5/3}\right)\Big|_0^h = \frac{3}{10}a^2h$$

$$\int \bar{y}_{EL}dA = \int_0^h y\left(\frac{a}{h^{1/3}}y^{1/3}dy\right) = \frac{a}{h^{1/3}}\left(\frac{3}{7}y^{7/3}\right)\Big|_0^h = \frac{3}{7}ah^2$$

Hence $$\bar{x}A = \int \bar{x}_{EL}dA:\quad \bar{x}\left(\frac{3}{4}ah\right) = \frac{3}{10}a^2h \qquad \bar{x} = \frac{2}{5}a \blacktriangleleft$$

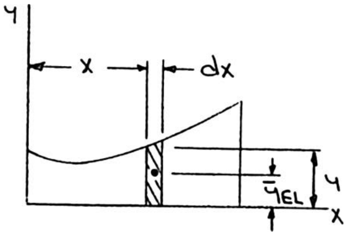
$$\bar{y}A = \int \bar{y}_{EL}dA:\quad \bar{y}\left(\frac{3}{4}ah\right) = \frac{3}{7}ah^2 \qquad \bar{y} = \frac{4}{7}h \blacktriangleleft$$



PROBLEM 5.141

Determine by direct integration the centroid of the area shown.

SOLUTION



We have

$$\bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}y = \frac{a}{2}\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)$$

$$dA = y\,dx = a\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)dx$$

Then

$$A = \int dA = \int_0^{2L} a\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)dx = a\left[x - \frac{x^2}{2L} + \frac{x^3}{3L^2}\right]_0^{2L}$$

$$= \frac{8}{3}aL$$

and

$$\int \bar{x}_{EL}\,dA = \int_0^{2L} x\left[a\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)dx\right] = a\left[\frac{x^2}{2} - \frac{x^3}{3L} + \frac{x^4}{4L^2}\right]_0^{2L}$$

$$= \frac{10}{3}aL^2$$

$$\int \bar{y}_{EL}\,dA = \int_0^{2L} \frac{a}{2}\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)\left[a\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)dx\right]$$

$$= \frac{a^2}{2}\int_0^{EL}\left(1 - 2\frac{x}{L} + 3\frac{x^2}{L^2} - 2\frac{x^3}{L^3} + \frac{x^4}{L^4}\right)dx$$

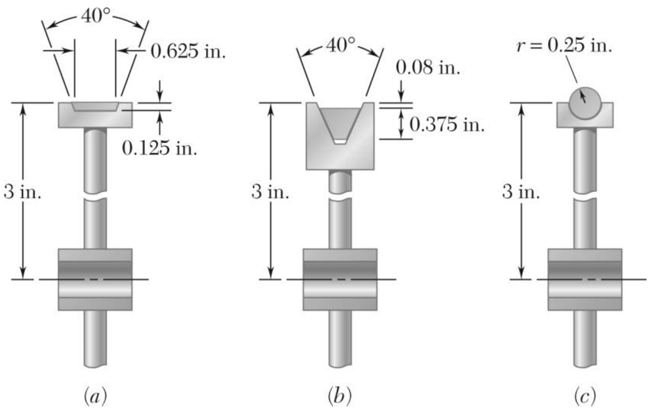
$$= \frac{a^2}{2}\left[x - \frac{x^2}{L} + \frac{x^3}{L^2} - \frac{x^4}{2L^3} + \frac{x^5}{5L^4}\right]_0^{2L}$$

$$= \frac{11}{5}a^2L$$

Hence,

$$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x}\left(\frac{8}{3}aL\right) = \frac{10}{3}aL^2 \qquad \bar{x} = \frac{5}{4}L \blacktriangleleft$$

$$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y}\left(\frac{1}{8}a\right) = \frac{11}{5}a^2 \qquad \bar{y} = \frac{33}{40}a \blacktriangleleft$$



(*a*) (*b*) (*c*)

PROBLEM 5.142

Three different drive belt profiles are to be studied. If at any given time each belt makes contact with one-half of the circumference of its pulley, determine the *contact area* between the belt and the pulley for each design.

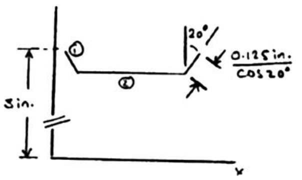
SOLUTION

SOLUTION

Applying the first theorem of Pappus-Guldinus, the contact area A_C of a belt is given by

$$A_C = \pi \overline{y} L = \pi \Sigma \overline{y} L$$

where the individual lengths are the lengths of the belt cross section that are in contact with the pulley.

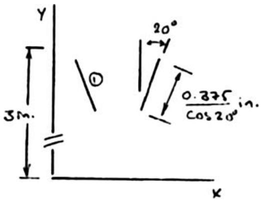


(*a*)

$$A_C = \pi[2(\overline{y}_1 L_1) + \overline{y}_2 L_2]$$

$$= \pi\left\{2\left[\left(3 - \frac{0.125}{2}\right)\text{in.}\right]\left[\frac{0.125\text{ in.}}{\cos 20°}\right] + [(3 - 0.125)\text{ in.}](0.625\text{ in.})\right\}$$

or $A_C = 8.10\text{ in}^2$ ◀

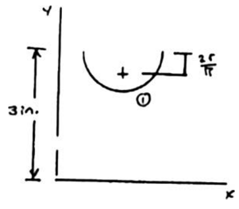


(*b*)

$$A_C = \pi[2(\overline{y}_1 L_1)]$$

$$= 2\pi\left[\left(3 - 0.08 - \frac{0.375}{2}\right)\text{in.}\right]\left(\frac{0.375\text{ in.}}{\cos 20°}\right)$$

or $A_C = 6.85\text{ in}^2$ ◀

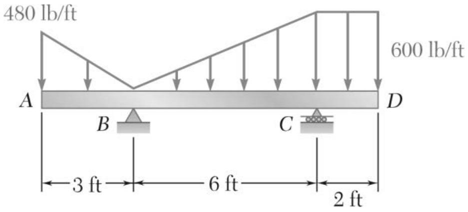


(*c*)

$$A_C = \pi[2(\overline{y}_1 L_1)]$$

$$= \pi\left[\left(3 - \frac{2(0.25)}{\pi}\right)\text{in.}\right][\pi(0.25\text{ in.})]$$

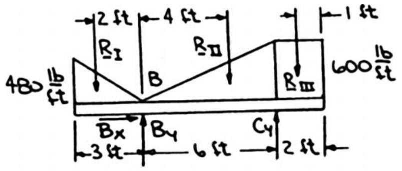
or $A_C = 7.01\text{ in}^2$ ◀



PROBLEM 5.143

Determine the reactions at the beam supports for the given loading.

SOLUTION



We have $$R_{\mathrm{I}} = \frac{1}{2}(3\,\text{ft})(480\text{ lb/ft}) = 720\text{ lb}$$

$$R_{\mathrm{II}} = \frac{1}{2}(6\text{ ft})(600\text{ lb/ft}) = 1800\text{ lb}$$

$$R_{\mathrm{III}} = (2\,\text{ft})(600\text{ lb/ft}) = 1200\text{ lb}$$

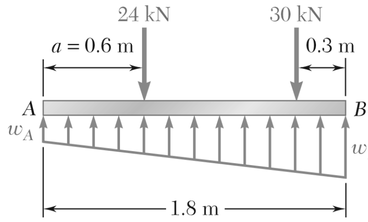
Then $$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (2\text{ ft})(720\text{ lb}) - (4\text{ ft})(1800\text{ lb}) + (6\text{ ft})C_y - (7\text{ ft})(1200\text{ lb}) = 0$$

or $$C_y = 2360\text{ lb} \qquad \mathbf{C} = 2360\text{ lb}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -720\text{ lb} + B_y - 1800\text{ lb} + 2360\text{ lb} - 1200\text{ lb} = 0$$

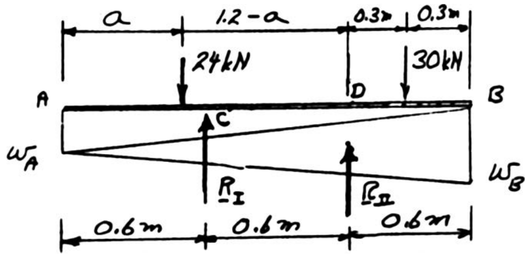
or $$B_y = 1360\text{ lb} \qquad \mathbf{B} = 1360\text{ lb}\uparrow \blacktriangleleft$$



PROBLEM 5.144

The beam *AB* supports two concentrated loads and rests on soil that exerts a linearly distributed upward load as shown. Determine the values of ω_A and ω_B corresponding to equilibrium.

SOLUTION



$$R_{\text{I}} = \frac{1}{2}\omega_A(1.8 \text{ m}) = 0.9\omega_A$$

$$R_{\text{II}} = \frac{1}{2}\omega_B(1.8 \text{ m}) = 0.9\omega_B$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (24 \text{ kN})(1.2 - a) - (30 \text{ kN})(0.3 \text{ m}) - (0.9\omega_A)(0.6 \text{ m}) = 0 \qquad (1)$$

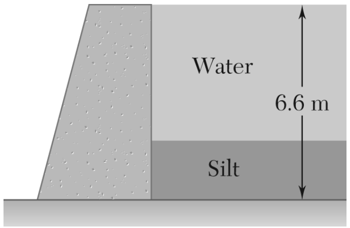
For $a = 0.6$ m,

$$24(1.2 - 0.6) - (30)(0.3) - 0.54\omega_a = 0$$

$$14.4 - 9 - 0.54\omega_A = 0 \qquad \omega_A = 10.00 \text{ kN/m} \blacktriangleleft$$

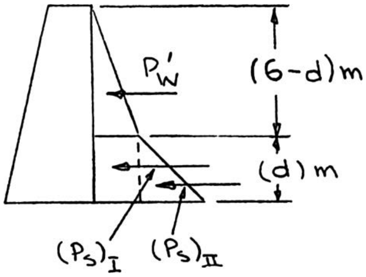
$$+\uparrow \Sigma F_y = 0: \quad -24 \text{ kN} - 30 \text{ kN} + 0.9(10 \text{ kN/m}) + 0.9\omega_B = 0 \qquad \omega_B = 50.0 \text{ kN/m} \blacktriangleleft$$

PROBLEM 5.145



The base of a dam for a lake is designed to resist up to 120 percent of the horizontal force of the water. After construction, it is found that silt (that is equivalent to a liquid of density $\rho_s = 1.76\times10^3$ kg/m^3) is settling on the lake bottom at the rate of 12 mm/year. Considering a 1-m-wide section of dam, determine the number of years until the dam becomes unsafe.

SOLUTION



First determine force on dam without the silt,

$$P_w = \frac{1}{2}A_{p_w} = \frac{1}{2}A(\rho gh)$$

$$= \frac{1}{2}[(6.6\text{ m})(1\text{ m})][(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(6.6\text{ m})]$$

$$= 213.66\text{ kN}$$

$$P_{\text{allow}} = 1.2P_w = (1.5)(213.66\text{ kN}) = 256.39\text{ kN}$$

Next determine the force P' on the dam face after a depth d of silt has settled.

We have $$P'_w = \frac{1}{2}[(6.6-d)\text{ m}\times(1\text{ m})][(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(6.6-d)\text{ m}]$$

$$= 4.905(6.6-d)^2\text{ kN}$$

$$(P_s)_{\text{I}} = [d(1\text{ m})][(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(6.6-d)\text{ m}]$$

$$= 9.81(6.6d-d^2)\text{ kN}$$

$$(P_s)_{\text{II}} = \frac{1}{2}[d(1\text{ m})][(1.76\times10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(d)\text{ m}]$$

$$= 8.6328d^2\text{ kN}$$

$$P' = P'_w + (P_s)_{\text{I}} + (P_s)_{\text{II}} = [4.905(43.560 - 13.2000d + d^2) + 9.81(6.6d - d^2) + 8.6328d^2]\text{ kN}$$

$$= [3.7278d^2 + 213.66]\text{ kN}$$

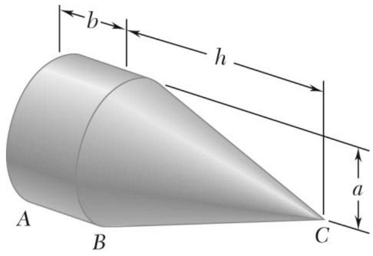
Now it's required that $P' = P_{\text{allow}}$ to determine the maximum value of d.

$$(3.7278d^2 + 213.66)\text{ kN} = 256.39\text{ kN}$$

or $$d = 3.3856\text{ m}$$

Finally, $$3.3856\text{ m} = 12\times10^{-3}\,\frac{\text{m}}{\text{year}}\times\text{N}$$ or $N = 282$ years ◀

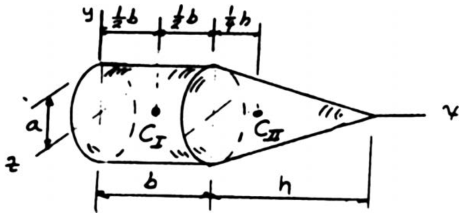
PROPRIETARY MATERIAL.



PROBLEM 5.146

Determine the location of the centroid of the composite body shown when (*a*) $h = 2b$, (*b*) $h = 2.5b$.

SOLUTION



	V	$\bar{x}$	$\bar{x}V$
Cylinder I	$\pi a^2 b$	$\frac{1}{2}b$	$\frac{1}{2}\pi a^2 b^2$
Cone II	$\frac{1}{3}\pi a^2 h$	$b + \frac{1}{4}h$	$\frac{1}{3}\pi a^2 h\left(b + \frac{1}{4}h\right)$

$$V = \pi a^2\left(b + \frac{1}{3}h\right)$$

$$\Sigma\bar{x}V = \pi a^2\left(\frac{1}{2}b^2 + \frac{1}{3}hb + \frac{1}{12}h^2\right)$$

(*a*) For $h = 2b$,

$$V = \pi a^2\left[b + \frac{1}{3}(2b)\right] = \frac{5}{3}\pi a^2 b$$

$$\Sigma\bar{x}V = \pi a^2\left[\frac{1}{2}b^2 + \frac{1}{3}(2b)b + \frac{1}{12}(2b)^2\right]$$

$$= \pi a^2 b^2\left[\frac{1}{2} + \frac{2}{3} + \frac{1}{3}\right] = \frac{3}{2}\pi a^2 b^2$$

$$\bar{X}V = \Sigma\bar{x}V: \quad \bar{X}\left(\frac{5}{3}\pi a^2 b\right) = \frac{3}{2}\pi a^2 b^2 \quad \bar{X} = \frac{9}{10}b$$

Centroid is $\frac{1}{10}b$ to left of base of cone. ◀

PROBLEM 5.146 (Continued)

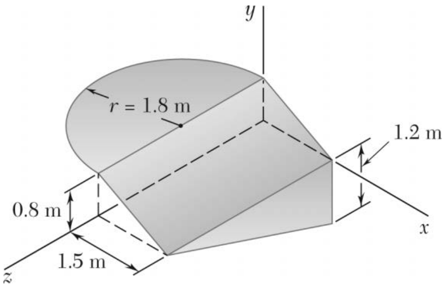
(*b*) For $h = 2.5b$, $$V = \pi a^2\left[b + \frac{1}{3}(2.5b)\right] = 1.8333\pi a^2 b$$

$$\Sigma\bar{x}V = \pi a^2\left[\frac{1}{2}b^2 + \frac{1}{3}(2.5b)b + \frac{1}{12}(2.5b)^2\right]$$
$$= \pi a^2 b^2[0.5 + 0.8333 + 0.52083]$$
$$= 1.85416\pi a^2 b^2$$

$\bar{X}V = \Sigma\bar{x}V$: $\bar{X}(1.8333\pi a^2 b) = 1.85416\pi a^2 b^2$ $\bar{X} = 1.01136b$

Centroid is 0.01136*b* to right of base of cone. ◀

Note: Centroid is at base of cone for $h = \sqrt{6}b = 2.449b$.

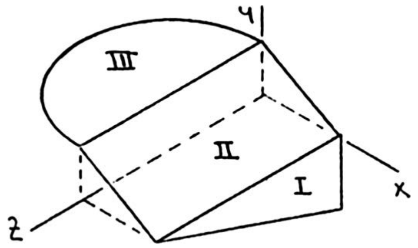


PROBLEM 5.147

Locate the center of gravity of the sheet-metal form shown.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the form will coincide with the centroid of the corresponding area.



$$\bar{y}_{\mathrm{I}} = -\frac{1}{3}(1.2) = -0.4 \text{ m}$$

$$\bar{z}_{\mathrm{I}} = \frac{1}{3}(3.6) = 1.2 \text{ m}$$

$$\bar{x}_{\mathrm{III}} = -\frac{4(1.8)}{3\pi} = -\frac{2.4}{\pi} \text{ m}$$

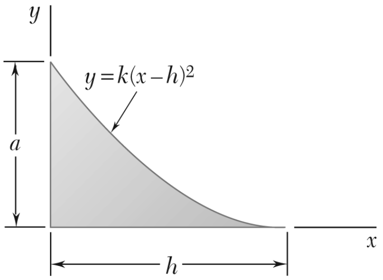
	A, m^2	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}A$, m^3	$\bar{y}A$, m^3	$\bar{z}A$, m^3
I	$\frac{1}{2}(3.6)(1.2) = 2.16$	1.5	–0.4	1.2	3.24	–0.864	2.592
II	$(3.6)(1.7) = 6.12$	0.75	0.4	1.8	4.59	2.448	11.016
III	$\frac{\pi}{2}(1.8)^2 = 5.0894$	$-\frac{2.4}{\pi}$	0.8	1.8	–3.888	4.0715	9.1609
Σ	13.3694				3.942	5.6555	22.769

We have

$\bar{X}\Sigma V = \Sigma \bar{x} V$: $\bar{X}(13.3694 \text{ m}^2) = 3.942 \text{ m}^3$ or $\bar{X} = 0.295$ m ◀

$\bar{Y}\Sigma V = \Sigma \bar{y} V$: $\bar{Y}(13.3694 \text{ m}^2) = 5.6555 \text{ m}^3$ or $\bar{Y} = 0.423$ m ◀

$\bar{Z}\Sigma V = \Sigma \bar{z} V$: $\bar{Z}(13.3694 \text{ m}^2) = 22.769 \text{ m}^3$ or $\bar{Z} = 1.703$ m ◀



PROBLEM 5.148

Locate the centroid of the volume obtained by rotating the shaded area about the *x*-axis.

SOLUTION

First note that symmetry implies $\overline{y}=0$ ◀

and $\overline{z}=0$ ◀

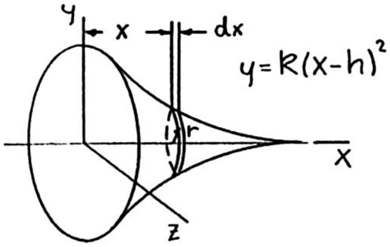
We have $y=k(X-h)^2$

At $x=0,\ y=a,$ $a=k(-h)^2$

or $k=\dfrac{a}{h^2}$

Choose as the element of volume a disk of radius *r* and thickness *dx*. Then

$$dV=\pi r^2dx,\ \ \overline{X}_{EL}=x$$



Now $r=\dfrac{a}{h^2}(x-h)^2$

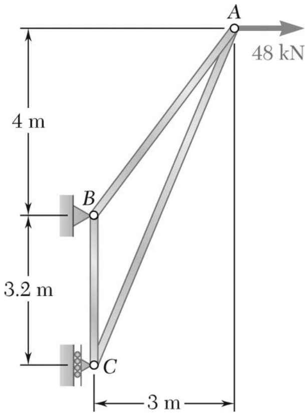
so that $dV=\pi\dfrac{a^2}{h^4}(x-h)^4dx$

Then
$$V=\int_0^h\pi\frac{a^2}{h^4}(x-h)^4dx=\frac{\pi}{5}\frac{a^2}{h^4}[(x-h)^5]_0^h$$
$$=\frac{1}{5}\pi a^2h$$

and
$$\int\overline{x}_{EL}dV=\int_0^h x\left[\pi\frac{a^2}{h^4}(x-h)^4dx\right]$$
$$=\pi\frac{a^2}{h^4}\int_0^h(x^5-4hx^4+6h^2x^3-4h^3x^2+h^4x)dx$$
$$=\pi\frac{a^2}{h^4}\left[\frac{1}{6}x^6-\frac{4}{5}hx^5+\frac{3}{2}h^2x^4-\frac{4}{3}h^3x^3+\frac{1}{2}h^4x^2\right]_0^h$$
$$=\frac{1}{30}\pi a^2h^2$$

Now $\overline{x}V=\int\overline{x}_{EL}dV:\quad \overline{x}\left(\dfrac{\pi}{5}a^2h\right)=\dfrac{\pi}{30}a^2h^2$ or $\overline{x}=\dfrac{1}{6}h$ ◀

CHAPTER 6



PROBLEM 6.1

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Entire truss:

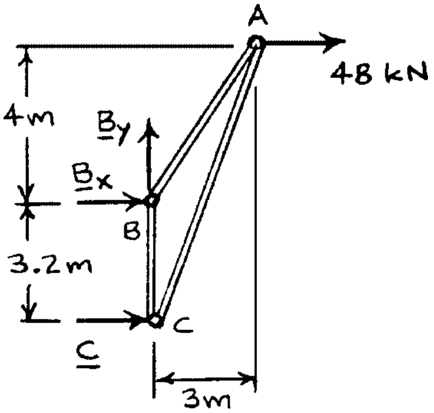
$$+\uparrow \Sigma F_y = 0: \quad B_y = 0 \qquad \mathbf{B}_y = 0$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -B_x(3.2\text{ m}) - (48\text{ kN})(7.2\text{ m}) = 0$$

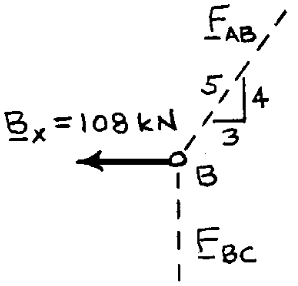
$$B_x = -108\text{ kN} \quad \mathbf{B}_x = 108\text{ kN} \longleftarrow$$

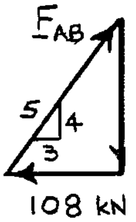
$$\xrightarrow{+} \Sigma F_x = 0: \quad C - 108\text{ kN} + 48\text{ kN} = 0$$

$$C = 60\text{ kN} \qquad \mathbf{C} = 60\text{ kN} \longrightarrow$$



Free body: Joint B:



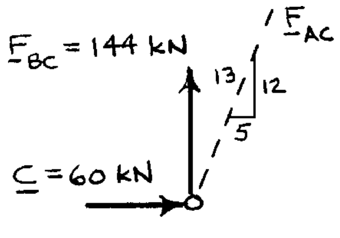


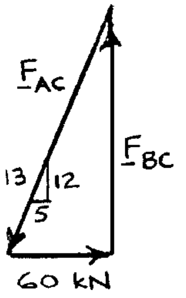
$$\frac{F_{AB}}{5} = \frac{F_{BC}}{4} = \frac{108\text{ kN}}{3}$$

$F_{AB} = 180.0\text{ kN} \quad T$ ◀

$F_{BC} = 144.0\text{ kN} \quad T$ ◀

Free body: Joint C:

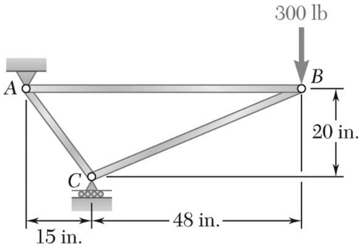




$$\frac{F_{AC}}{13} = \frac{F_{BC}}{12} = \frac{60\text{ kN}}{5}$$

$F_{AC} = 156.0\text{ kN} \quad C$ ◀

$F_{BC} = 144\text{ kN}$ (checks)



PROBLEM 6.2

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

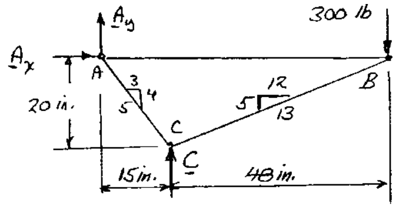
SOLUTION

Reactions:

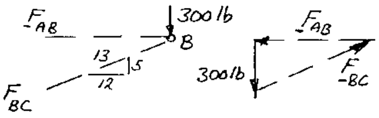
$\Sigma M_A = 0: \quad \mathbf{C} = 1260 \text{ lb}\uparrow$

$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$

$\Sigma F_y = 0: \quad \mathbf{A}_y = 960 \text{ lb}\downarrow$



Joint B:

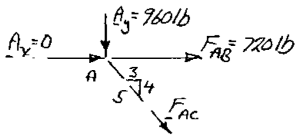


$$\frac{F_{AB}}{12} = \frac{F_{BC}}{13} = \frac{300 \text{ lb}}{5}$$

$F_{AB} = 720 \text{ lb} \quad T$ ◀

$F_{BC} = 780 \text{ lb} \quad C$ ◀

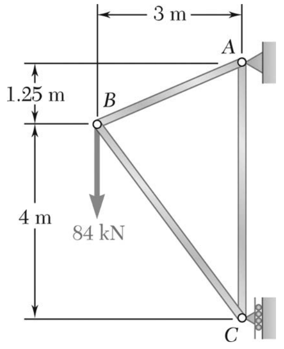
Joint A:



$+\uparrow \Sigma F_y = 0: \quad -960 \text{ lb} - \frac{4}{5}F_{AC} = 0$

$F_{AC} = 1200 \text{ lb} \qquad F_{AC} = 1200 \text{ lb} \quad C$ ◀

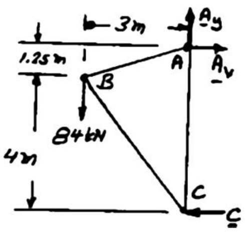
$\xrightarrow{+} \Sigma F_x = 0: \quad 720 \text{ lb} - (1200 \text{ lb})\frac{3}{5} = 0 \quad \text{(checks)}$



PROBLEM 6.3

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



$$AB = \sqrt{3^2 + 1.25^2} = 3.25 \text{ m}$$

$$BC = \sqrt{3^2 + 4^2} = 5 \text{ m}$$

Reactions:

$$+\circlearrowleft \Sigma M_A = 0: \quad (84 \text{ kN})(3 \text{ m}) - C(5.25 \text{ m}) = 0$$

$$\mathbf{C} = 48 \text{ kN} \longleftarrow$$

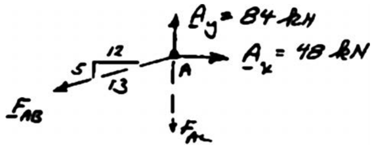
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - C = 0$$

$$\mathbf{A}_x = 48 \text{ kN} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y = 84 \text{ kN} = 0$$

$$\mathbf{A}_y = 84 \text{ kN} \uparrow$$

Joint A:



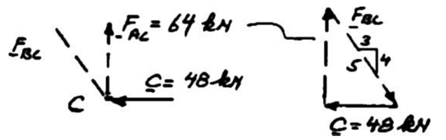
$$\xrightarrow{+} \Sigma F_x = 0: \quad 48 \text{ kN} - \frac{12}{13} F_{AB} = 0$$

$$F_{AB} = +52 \text{ kN} \qquad F_{AB} = 52.0 \text{ kN} \quad T \blacktriangleleft$$

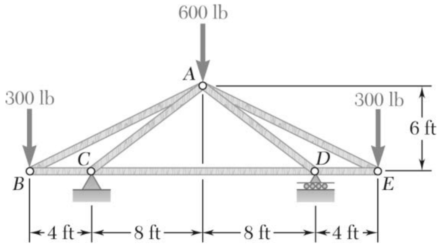
$$+\uparrow \Sigma F_y = 0: \quad 84 \text{ kN} - \frac{5}{13}(52 \text{ kN}) - F_{AC} = 0$$

$$F_{AC} = +64.0 \text{ kN} \qquad F_{AC} = 64.0 \text{ kN} \quad T \blacktriangleleft$$

Joint C:



$$\frac{F_{BC}}{5} = \frac{48 \text{ kN}}{3} \qquad F_{BC} = 80.0 \text{ kN} \quad C \blacktriangleleft$$



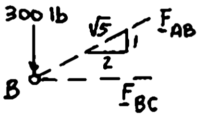
PROBLEM 6.4

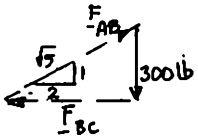
Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

<u>Free body: Truss</u>:

From the symmetry of the truss and loading, we find **C** = **D** = 600 lb $\uparrow$



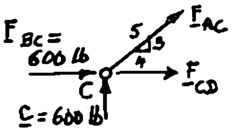


<u>Free body: Joint *B*</u>:

$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{BC}}{2} = \frac{300\text{ lb}}{1}$$

$F_{AB} = 671\text{ lb}\quad T \qquad F_{BC} = 600\text{ lb}\quad C$ ◀

<u>Free body: Joint *C*</u>:



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{AC} + 600\text{ lb} = 0$$

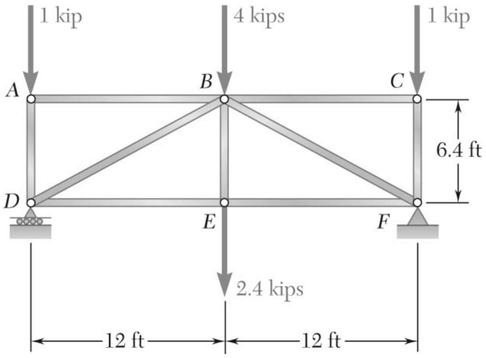
$F_{AC} = -1000\text{ lb} \qquad F_{AC} = 1000\text{ lb}\quad C$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}(-1000\text{ lb}) + 600\text{ lb} + F_{CD} = 0$$

$F_{CD} = 200\text{ lb}\quad T$ ◀

From symmetry:

$F_{AD} = F_{AC} = 1000\text{ lb}\quad C, \quad F_{AE} = F_{AB} = 671\text{ lb}\quad T, \quad F_{DE} = F_{BC} = 600\text{ lb}\quad C$ ◀

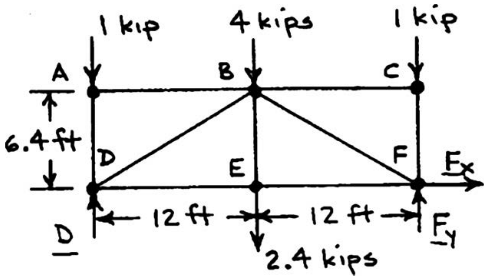


PROBLEM 6.5

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Reactions:



$$+\circlearrowleft \Sigma M_D = 0: \quad F_y(24) - (4+2.4)(12) - (1)(24) = 0$$

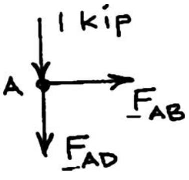
$$\mathbf{F}_y = 4.2 \text{ kips} \uparrow$$

$$\Sigma F_x = 0: \quad \mathbf{F}_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad D - (1+4+1+2.4) + 4.2 = 0$$

$$\mathbf{D} = 4.2 \text{ kips} \uparrow$$

Joint A:

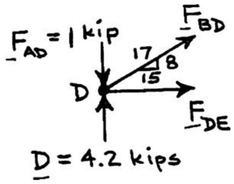


$$\Sigma F_x = 0: \quad F_{AB} = 0 \qquad F_{AB} = 0 \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -1 - F_{AD} = 0$$

$$F_{AD} = -1 \text{ kip} \qquad F_{AD} = 1.000 \text{ kip} \quad C \blacktriangleleft$$

Joint D:



$$+\uparrow \Sigma F_y = 0: \quad -1 + 4.2 + \frac{8}{17} F_{BD} = 0$$

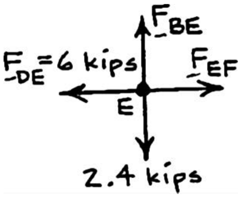
$$F_{BD} = -6.8 \text{ kips} \qquad F_{BD} = 6.80 \text{ kips} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{15}{17}(-6.8) + F_{DE} = 0$$

$$F_{DE} = +6 \text{ kips} \qquad F_{DE} = 6.00 \text{ kips} \quad T \blacktriangleleft$$

PROBLEM 6.5 (Continued)

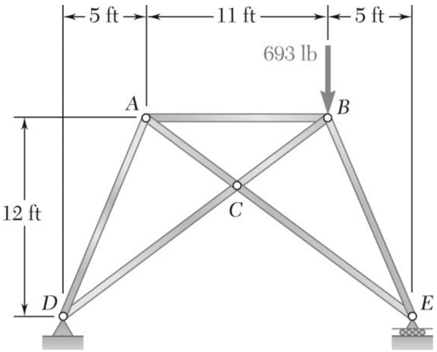
Joint E:



$$+\uparrow \Sigma F_y = 0: \quad F_{BE} - 2.4 = 0$$

$$F_{BE} = +2.4 \text{ kips} \qquad\qquad F_{BE} = 2.40 \text{ kips} \quad T \blacktriangleleft$$

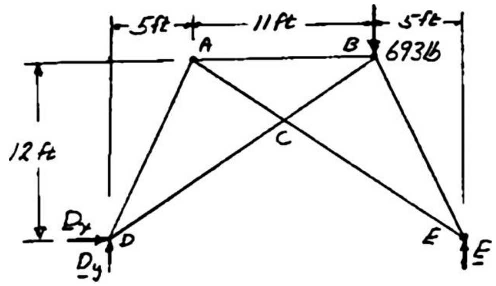
Truss and loading symmetrical about ℄.



PROBLEM 6.6

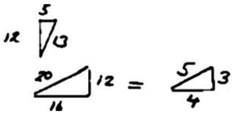
Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



$$AD = \sqrt{5^2 + 12^2} = 13\text{ ft}$$

$$BCD = \sqrt{12^2 + 16^2} = 20\text{ ft}$$

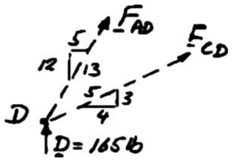


Reactions: $\Sigma F_x = 0{:}\quad D_x = 0$

$$+\circlearrowleft \Sigma M_E = 0{:}\quad D_y(21\text{ ft}) - (693\text{ lb})(5\text{ ft}) = 0 \qquad \mathbf{D}_y = 165\text{ lb}\uparrow$$

$$+\uparrow \Sigma F_y = 0{:}\quad 165\text{ lb} - 693\text{ lb} + E = 0 \qquad \mathbf{E} = 528\text{ lb}\uparrow$$

Joint D:



$$\xrightarrow{+} \Sigma F_x = 0{:}\quad \frac{5}{13}F_{AD} + \frac{4}{5}F_{DC} = 0 \tag{1}$$

$$+\uparrow \Sigma F_y = 0{:}\quad \frac{12}{13}F_{AD} + \frac{3}{5}F_{DC} + 165\text{ lb} = 0 \tag{2}$$

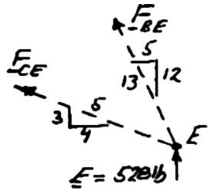
Solving Eqs. (1) and (2) simultaneously,

$$F_{AD} = -260\text{ lb} \qquad F_{AD} = 260\text{ lb} \quad C \blacktriangleleft$$

$$F_{DC} = +125\text{ lb} \qquad F_{DC} = 125\text{ lb} \quad T \blacktriangleleft$$

PROBLEM 6.6 (Continued)

Joint E:



$$\xleftarrow{+}\Sigma F_x = 0:\quad \frac{5}{13}F_{BE} + \frac{4}{5}F_{CE} = 0 \qquad (3)$$

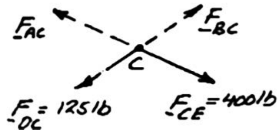
$$+\big\uparrow \Sigma F_y = 0:\quad \frac{12}{13}F_{BE} + \frac{3}{5}F_{CE} + 528\text{ lb} = 0 \qquad (4)$$

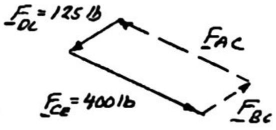
Solving Eqs. (3) and (4) simultaneously,

$$F_{BE} = -832\text{ lb} \qquad F_{BE} = 832\text{ lb}\quad C \blacktriangleleft$$

$$F_{CE} = +400\text{ lb} \qquad F_{CE} = 400\text{ lb}\quad T \blacktriangleleft$$

Joint C:





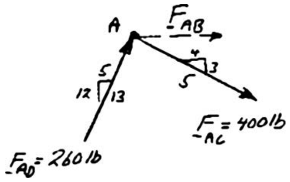
Force polygon is a parallelogram (see Fig. 6.11, p. 209).

$$F_{AC} = 400\text{ lb}\quad T \blacktriangleleft$$

$$F_{BC} = 125.0\text{ lb}\quad T \blacktriangleleft$$

Joint A:

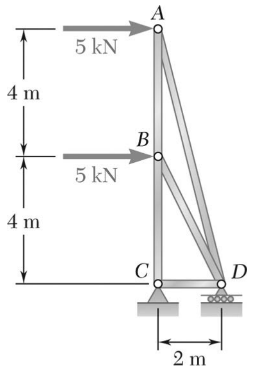
$$\xrightarrow{+}\Sigma F_x = 0:\quad \frac{5}{13}(260\text{ lb}) + \frac{4}{5}(400\text{ lb}) + F_{AB} = 0$$



$$F_{AB} = -420\text{ lb} \qquad F_{AB} = 420\text{ lb}\quad C \blacktriangleleft$$

$$+\big\uparrow \Sigma F_y = 0:\quad \frac{12}{13}(260\text{ lb}) - \frac{3}{5}(400\text{ lb}) = 0$$

$$0 = 0 \quad \text{(Checks)}$$

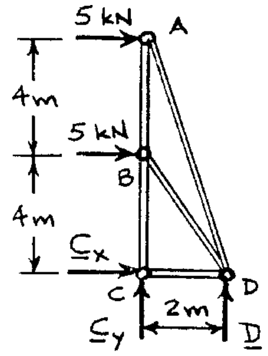


PROBLEM 6.7

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Entire truss:



$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 2(5\text{ kN}) = 0$$

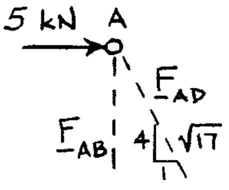
$$C_x = -10\text{ kN} \qquad \mathbf{C}_x = 10\text{ kN} \longleftarrow$$

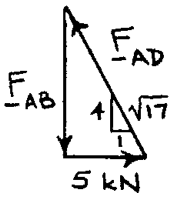
$$+\circlearrowleft \Sigma M_C = 0: \quad D(2\text{ m}) - (5\text{ kN})(8\text{ m}) - (5\text{ kN})(4\text{ m}) = 0$$

$$D = +30\text{ kN} \qquad \mathbf{D} = 30\text{ kN}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 30\text{ kN} = 0 \quad C_y = -30\text{ kN} \quad \mathbf{C}_y = 30\text{ kN}\downarrow$$

Free body: Joint A:



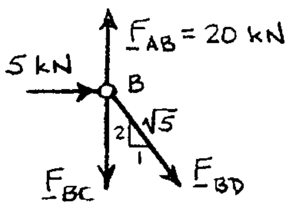


$$\frac{F_{AB}}{4} = \frac{F_{AD}}{\sqrt{17}} = \frac{5\text{ kN}}{1}$$

$$F_{AB} = 20.0\text{ kN} \quad T \blacktriangleleft$$

$$F_{AD} = 20.6\text{ kN} \quad C \blacktriangleleft$$

Free body: Joint B:



$$\xrightarrow{+} \Sigma F_x = 0: \quad 5\text{ kN} + \frac{1}{\sqrt{5}} F_{BD} = 0$$

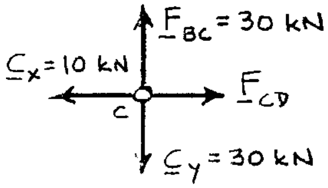
$$F_{BD} = -5\sqrt{5}\text{ kN} \qquad F_{BD} = 11.18\text{ kN} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 20\text{ kN} - F_{BC} - \frac{2}{\sqrt{5}}(-5\sqrt{5}\text{ kN}) = 0$$

$$F_{BC} = +30\text{ kN} \qquad F_{BC} = 30.0\text{ kN} \quad T \blacktriangleleft$$

PROBLEM 6.7 (Continued)

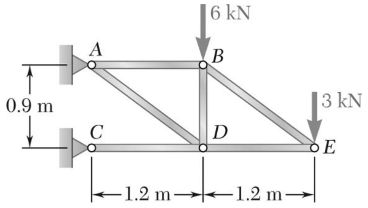
Free body: Joint *C*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CD} - 10\text{ kN} = 0$$

$$F_{CD} = +10\text{ kN} \qquad\qquad F_{CD} = 10.00\text{ kN} \quad T \blacktriangleleft$$

$$+\big\uparrow \Sigma F_y = 0: \quad 30\text{ kN} - 30\text{ kN} = 0 \quad \text{(checks)}$$



PROBLEM 6.8

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

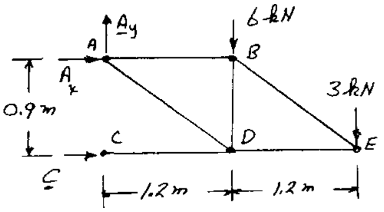
SOLUTION

Reactions:

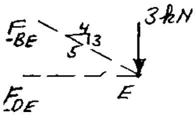
$$\Sigma M_C = 0: \quad \mathbf{A}_x = 16\text{ kN} \longleftarrow$$

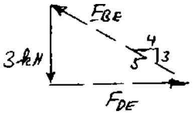
$$\Sigma F_y = 0: \quad \mathbf{A}_y = 9\text{ kN} \uparrow$$

$$\Sigma F_x = 0: \quad \mathbf{C} = 16\text{ kN} \longrightarrow$$



Joint E:



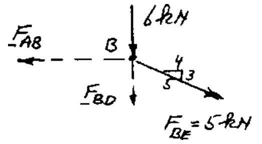


$$\frac{F_{BE}}{5} = \frac{F_{DE}}{4} = \frac{3\text{ kN}}{3}$$

$F_{BE} = 5.00\text{ kN}$ T ◀

$F_{DE} = 4.00\text{ kN}$ C ◀

Joint B:



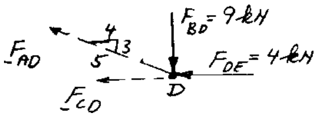
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}(5\text{ kN}) - F_{AB} = 0$$

$F_{AB} = +4\text{ kN}$ $\qquad F_{AB} = 4.00\text{ kN}$ T ◀

$$+\!\uparrow \Sigma F_y = 0: \quad -6\text{ kN} - \frac{3}{5}(5\text{ kN}) - F_{BD} = 0$$

$F_{BD} = -9\text{ kN}$ $\qquad F_{BD} = 9.00\text{ kN}$ C ◀

Joint D:

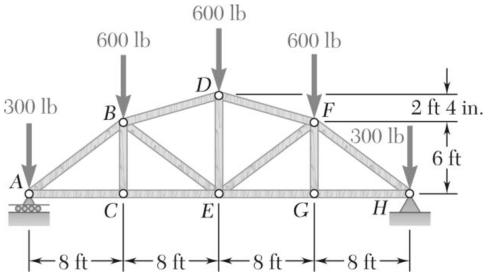


$$+\!\uparrow \Sigma F_y = 0: \quad -9\text{ kN} + \frac{3}{5}F_{AD} = 0$$

$F_{AD} = +15\text{ kN}$ $\qquad F_{AD} = 15.00\text{ kN}$ T ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad -4\text{ kN} - \frac{4}{5}(15\text{ kN}) - F_{CD} = 0$$

$F_{CD} = -16\text{ kN}$ $\qquad F_{CD} = 16.00\text{ kN}$ C ◀

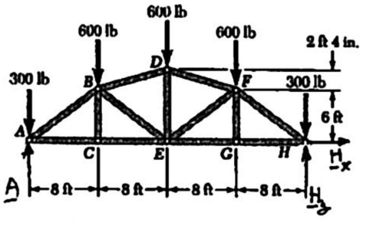


PROBLEM 6.9

Determine the force in each member of the Gambrel roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:



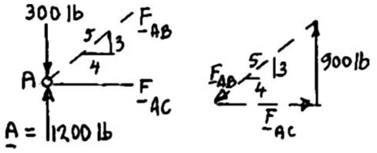
$$\Sigma F_x = 0: \quad \mathbf{H}_x = 0$$

Because of the symmetry of the truss and loading,

$$\mathbf{A} = \mathbf{H}_y = \frac{1}{2}\text{total load}$$

$$\mathbf{A} = \mathbf{H}_y = 1200 \text{ lb}\uparrow$$

Free body: Joint A:

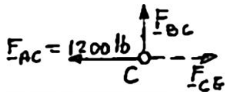


$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{900 \text{ lb}}{3}$$

$\mathbf{F}_{AB} = 1500 \text{ lb} \quad C$ ◀

$\mathbf{F}_{AC} = 1200 \text{ lb} \quad T$ ◀

Free body: Joint C:

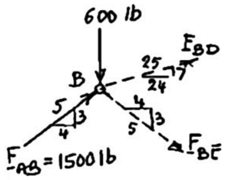


BC is a zero-force member.

$$\mathbf{F}_{BC} = 0$$

$F_{CE} = 1200 \text{ lb} \quad T$ ◀

Free body: Joint B:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{24}{25}F_{BD} + \frac{4}{5}F_{BE} + \frac{4}{5}(1500 \text{ lb}) = 0$$

or $$24F_{BD} + 20F_{BE} = -30{,}000 \text{ lb} \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{7}{25}F_{BD} - \frac{3}{5}F_{BE} + \frac{3}{5}(1500) - 600 = 0$$

or $$7F_{BD} - 15F_{BE} = -7{,}500 \text{ lb} \qquad (2)$$

PROBLEM 6.9 (Continued)

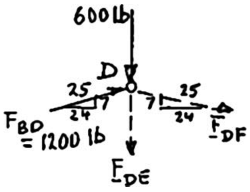
Multiply Eq. (1) by 3, Eq. (2) by 4, and add:

$$100F_{BD} = -120{,}000 \text{ lb} \qquad F_{BD} = 1200 \text{ lb} \quad C \blacktriangleleft$$

Multiply Eq. (1) by 7, Eq. (2) by –24, and add:

$$500F_{BE} = -30{,}000 \text{ lb} \qquad F_{BE} = 60.0 \text{ lb} \quad C \blacktriangleleft$$

Free body: Joint *D*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{24}{25}(1200 \text{ lb}) + \frac{24}{25}F_{DF} = 0$$

$$F_{DF} = -1200 \text{ lb} \qquad F_{DF} = 1200 \text{ lb} \quad C \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{7}{25}(1200 \text{ lb}) - \frac{7}{25}(-1200 \text{ lb}) - 600 \text{ lb} - F_{DE} = 0$$

$$F_{DE} = 72.0 \text{ lb} \qquad F_{DE} = 72.0 \text{ lb} \quad T \blacktriangleleft$$

Because of the symmetry of the truss and loading, we deduce that

$$F_{EF} = F_{BE} \qquad F_{EF} = 60.0 \text{ lb} \quad C \blacktriangleleft$$

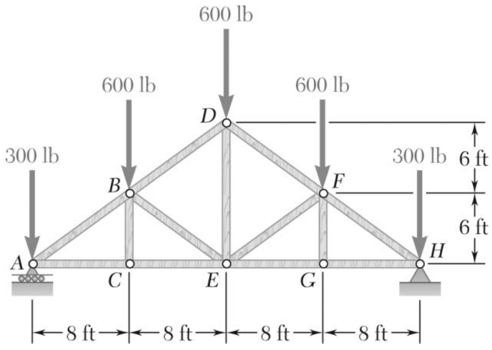
$$F_{EG} = F_{CE} \qquad F_{EG} = 1200 \text{ lb} \quad T \blacktriangleleft$$

$$F_{FG} = F_{BC} \qquad F_{FG} = 0 \blacktriangleleft$$

$$F_{FH} = F_{AB} \qquad F_{FH} = 1500 \text{ lb} \quad C \blacktriangleleft$$

$$F_{GH} = F_{AC} \qquad F_{GH} = 1200 \text{ lb} \quad T \blacktriangleleft$$

Note: Compare results with those of Problem 6.11.

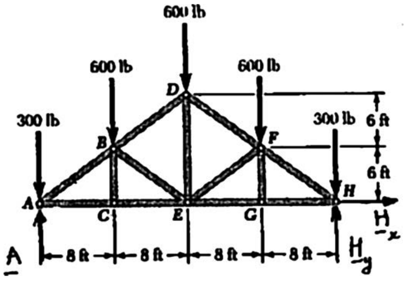


PROBLEM 6.10

Determine the force in each member of the Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:



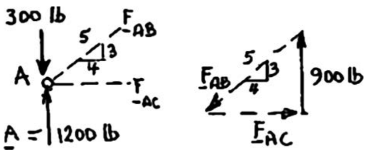
$$\Sigma F_x = 0: \quad \mathbf{H}_x = 0$$

Because of the symmetry of the truss and loading,

$$A = H_y = \frac{1}{2}\text{total load}$$

$$\mathbf{A} = \mathbf{H}_y = 1200 \text{ lb}\uparrow$$

Free body: Joint *A*:



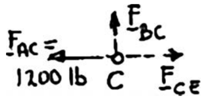
$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{900 \text{ lb}}{3}$$

$\mathbf{F}_{AB} = 1500 \text{ lb}$ *C* ◀

$\mathbf{F}_{AC} = 1200 \text{ lb}$ *T* ◀

Free body: Joint *C*:

BC is a zero-force member.

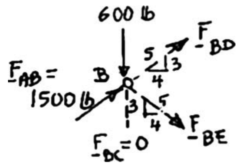


$\mathbf{F}_{BC} = 0$

$\mathbf{F}_{CE} = 1200 \text{ lb}$ *T* ◀

PROBLEM 6.10 (Continued)

Free body: Joint *B*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{BD} + \frac{4}{5}F_{BC} + \frac{4}{5}(1500\text{ lb}) = 0$$

or $$F_{BD} + F_{BE} = -1500\text{ lb} \qquad (1)$$

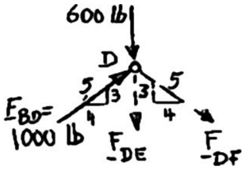
$$+\!\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} - \frac{3}{5}F_{BE} + \frac{3}{5}(1500\text{ lb}) - 600\text{ lb} = 0$$

or $$F_{BD} - F_{BE} = -500\text{ lb} \qquad (2)$$

Add Eqs. (1) and (2): $$2F_{BD} = -2000\text{ lb} \qquad F_{BD} = 1000\text{ lb} \quad C \ \blacktriangleleft$$

Subtract Eq. (2) from Eq. (1): $$2F_{BE} = -1000\text{ lb} \qquad F_{BE} = 500\text{ lb} \quad C \ \blacktriangleleft$$

Free Body: Joint *D*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}(1000\text{ lb}) + \frac{4}{5}F_{DF} = 0$$

$$F_{DF} = -1000\text{ lb} \qquad F_{DF} = 1000\text{ lb} \quad C \ \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{3}{5}(1000\text{ lb}) - \frac{3}{5}(-1000\text{ lb}) - 600\text{ lb} - F_{DE} = 0$$

$$F_{DE} = +600\text{ lb} \qquad F_{DE} = 600\text{ lb} \quad T \ \blacktriangleleft$$

Because of the symmetry of the truss and loading, we deduce that

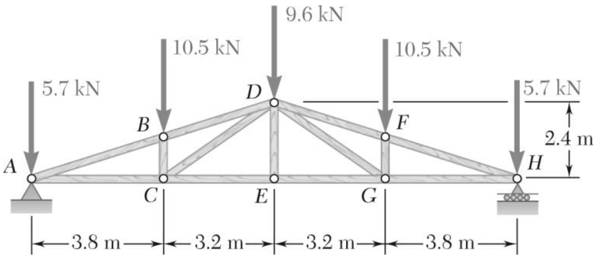
$$F_{EF} = F_{BE} \qquad F_{EF} = 500\text{ lb} \quad C \ \blacktriangleleft$$

$$F_{EG} = F_{CE} \qquad F_{EG} = 1200\text{ lb} \quad T \ \blacktriangleleft$$

$$F_{FG} = F_{BC} \qquad F_{FG} = 0 \ \blacktriangleleft$$

$$F_{FH} = F_{AB} \qquad F_{FH} = 1500\text{ lb} \quad C \ \blacktriangleleft$$

$$F_{GH} = F_{AC} \qquad F_{GH} = 1200\text{ lb} \quad T \ \blacktriangleleft$$

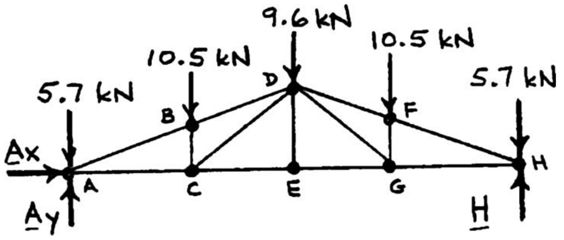


PROBLEM 6.11

Determine the force in each member of the Pratt roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:

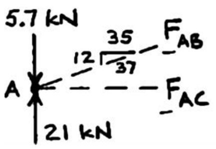


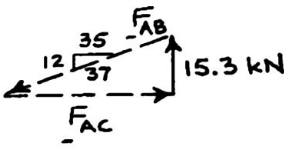
$$\Sigma F_x = 0: \quad A_x = 0$$

Due to symmetry of truss and load,

$$A_y = H = \frac{1}{2} \text{ total load} = 21 \text{ kN} \uparrow$$

Free body: Joint A:





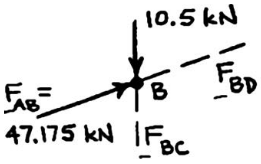
$$\frac{F_{AB}}{37} = \frac{F_{AC}}{35} = \frac{15.3 \text{ kN}}{12}$$

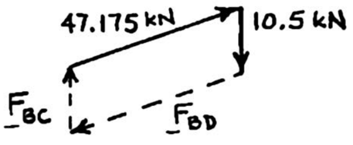
$$F_{AB} = 47.175 \text{ kN} \quad F_{AC} = 44.625 \text{ kN}$$

$F_{AB} = 47.2$ kN C ◀

$F_{AC} = 44.6$ kN T ◀

Free body: Joint B:





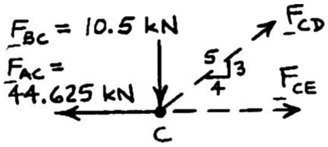
From force polygon: $F_{BD} = 47.175 \text{ kN}, \quad F_{BC} = 10.5 \text{ kN}$

$F_{BC} = 10.50$ kN C ◀

$F_{BD} = 47.2$ kN C ◀

PROBLEM 6.11 (Continued)

Free body: Joint *C*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{CD} - 10.5 = 0 \qquad F_{CD} = 17.50 \text{ kN} \quad T \blacktriangleleft$$

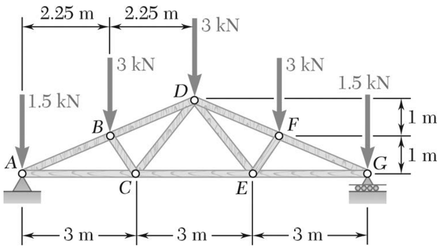
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} + \frac{4}{5}(17.50) - 44.625 = 0$$

$$F_{CE} = 30.625 \text{ kN} \qquad F_{CE} = 30.6 \text{ kN} \quad T \blacktriangleleft$$

Free body: Joint *E*:

DE is a zero-force member. $F_{DE} = 0 \blacktriangleleft$

Truss and loading symmetrical about ℄.

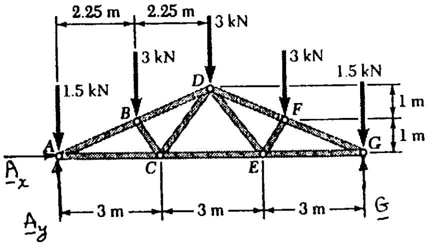


PROBLEM 6.12

Determine the force in each member of the Fink roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:



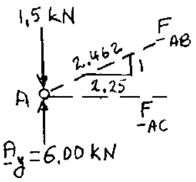
$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

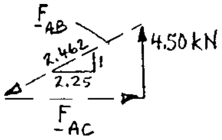
Because of the symmetry of the truss and loading,

$$\mathbf{A}_y = \mathbf{G} = \frac{1}{2} \text{ total load}$$

$$\mathbf{A}_y = \mathbf{G} = 6.00 \text{ kN} \uparrow$$

Free body: Joint *A*:



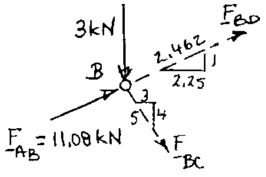


$$\frac{F_{AB}}{2.462} = \frac{F_{AC}}{2.25} = \frac{4.50 \text{ kN}}{1}$$

$F_{AB} = 11.08 \text{ kN} \quad C$ ◀

$F_{AC} = 10.125 \text{ kN}$ $\qquad$ $F_{AC} = 10.13 \text{ kN} \quad T$ ◀

Free body: Joint *B*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{3}{5}F_{BC} + \frac{2.25}{2.462}F_{BD} + \frac{2.25}{2.462}(11.08 \text{ kN}) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}F_{BC} + \frac{F_{BD}}{2.462} + \frac{11.08 \text{ kN}}{2.462} - 3 \text{ kN} = 0 \qquad (2)$$

Multiply Eq. (2) by –2.25 and add to Eq. (1):

$$\frac{12}{5}F_{BC} + 6.75 \text{ kN} = 0 \qquad F_{BC} = -2.8125$$

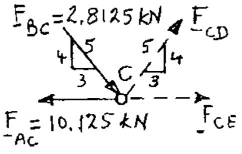
$F_{BC} = 2.81 \text{ kN} \quad C$ ◀

Multiply Eq. (1) by 4, Eq. (2) by 3, and add:

$$\frac{12}{2.462}F_{BD} + \frac{12}{2.462}(11.08 \text{ kN}) - 9 \text{ kN} = 0$$

$F_{BD} = -9.2335 \text{ kN}$ $\qquad$ $F_{BD} = 9.23 \text{ kN} \quad C$ ◀

Free body: Joint *C*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{4}{5}F_{CD} - \frac{4}{5}(2.8125 \text{ kN}) = 0$$

$$F_{CD} = 2.8125 \text{ kN}, \qquad F_{CD} = 2.81 \text{ kN} \quad T$$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} - 10.125 \text{ kN} + \frac{3}{5}(2.8125 \text{ kN}) + \frac{3}{5}(2.8125 \text{ kN}) = 0$$

$$F_{CE} = +6.7500 \text{ kN} \qquad F_{CE} = 6.75 \text{ kN} \quad T$$ ◀

Because of the symmetry of the truss and loading, we deduce that

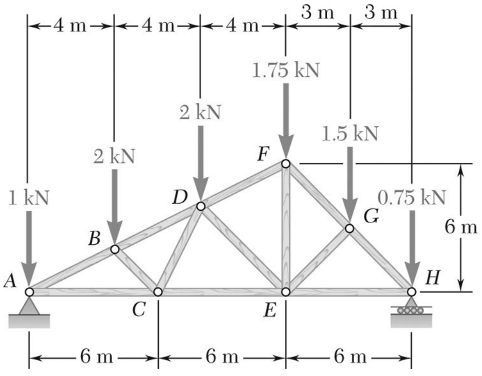
$$F_{DE} = F_{CD} \qquad F_{CD} = 2.81 \text{ kN} \quad T$$ ◀

$$F_{DF} = F_{BD} \qquad F_{DF} = 9.23 \text{ kN} \quad C$$ ◀

$$F_{EF} = F_{BC} \qquad F_{EF} = 2.81 \text{ kN} \quad C$$ ◀

$$F_{EG} = F_{AC} \qquad F_{EG} = 10.13 \text{ kN} \quad T$$ ◀

$$F_{FG} = F_{AB} \qquad F_{FG} = 11.08 \text{ kN} \quad C$$ ◀

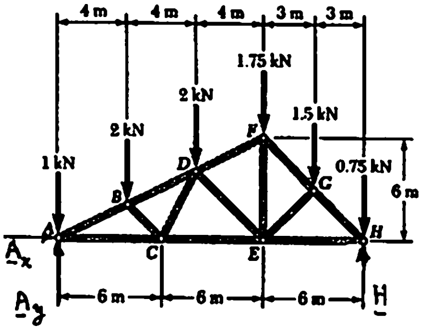


PROBLEM 6.13

Using the method of joints, determine the force in each member of the double-pitch roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:



$$+\circlearrowleft \Sigma M_A = 0: \quad H(18\text{ m}) - (2\text{ kN})(4\text{ m}) - (2\text{ kN})(8\text{ m}) - (1.75\text{ kN})(12\text{ m}) - (1.5\text{ kN})(15\text{ m}) - (0.75\text{ kN})(18\text{ m}) = 0$$

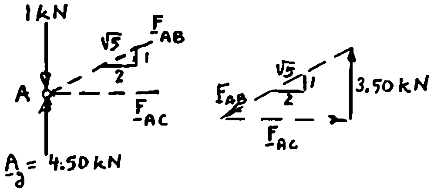
$$\mathbf{H} = 4.50\text{ kN}\uparrow$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma F_y = 0: \quad A_y + H - 9 = 0$$

$$A_y = 9 - 4.50 \qquad \mathbf{A}_y = 4.50\text{ kN}\uparrow$$

Free body: Joint *A*:

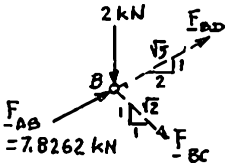


$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{3.50\text{ kN}}{1}$$

$$F_{AB} = 7.8262\text{ kN} \quad C \qquad F_{AB} = 7.83\text{ kN} \quad C \blacktriangleleft$$

$$F_{AC} = 7.00\text{ kN} \quad T \blacktriangleleft$$

PROBLEM 6.13 (Continued)



Free body: Joint *B*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}}(7.8262 \text{ kN}) + \frac{1}{\sqrt{2}} F_{BC} = 0$$

or $$F_{BD} + 0.79057 F_{BC} = -7.8262 \text{ kN} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{BD} + \frac{1}{\sqrt{5}}(7.8262 \text{ kN}) - \frac{1}{\sqrt{2}} F_{BC} - 2 \text{ kN} = 0$$

or $$F_{BD} - 1.58114 F_{BC} = -3.3541 \qquad (2)$$

Multiply Eq. (1) by 2 and add Eq. (2):

$$3F_{BD} = -19.0065$$

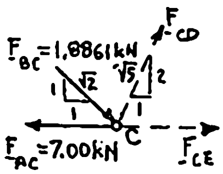
$$F_{BD} = -6.3355 \text{ kN} \qquad F_{BD} = 6.34 \text{ kN} \quad C \blacktriangleleft$$

Subtract Eq. (2) from Eq. (1):

$$2.37111 F_{BC} = -4.4721$$

$$F_{BC} = -1.8861 \text{ kN} \qquad F_{BC} = 1.886 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *C*:

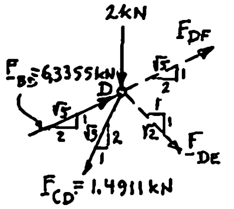


$$+\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{5}} F_{CD} - \frac{1}{\sqrt{2}}(1.8861 \text{ kN}) = 0$$

$$F_{CD} = +1.4911 \text{ kN} \qquad F_{CD} = 1.491 \text{ kN} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} - 7.00 \text{ kN} + \frac{1}{\sqrt{2}}(1.8861 \text{ kN}) + \frac{1}{\sqrt{5}}(1.4911 \text{ kN}) = 0$$

$$F_{CE} = +5.000 \text{ kN} \qquad F_{CE} = 5.00 \text{ kN} \quad T \blacktriangleleft$$



Free body: Joint *D*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{DF} + \frac{1}{\sqrt{2}} F_{DE} + \frac{2}{\sqrt{5}}(6.3355 \text{ kN}) - \frac{1}{\sqrt{5}}(1.4911 \text{ kN}) = 0$$

or $$F_{DF} + 0.79057 F_{DE} = -5.5900 \text{ kN} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{2}} F_{DE} + \frac{1}{\sqrt{5}}(6.3355 \text{ kN}) - \frac{2}{\sqrt{5}}(1.4911 \text{ kN}) - 2 \text{ kN} = 0$$

or $$F_{DF} - 0.79057 F_{DE} = -1.1188 \text{ kN} \qquad (2)$$

Add Eqs. (1) and (2): $$2F_{DF} = -6.7088 \text{ kN}$$

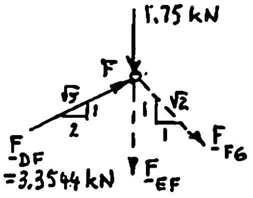
$$F_{DF} = -3.3544 \text{ kN} \qquad F_{DF} = 3.35 \text{ kN} \quad C \blacktriangleleft$$

Subtract Eq. (2) from Eq. (1): $$1.58114 F_{DE} = -4.4712 \text{ kN}$$

$$F_{DE} = -2.8278 \text{ kN} \qquad F_{DE} = 2.83 \text{ kN} \quad C \blacktriangleleft$$

PROBLEM 6.13 (Continued)

Free body: Joint *F*:



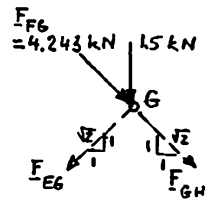
$$\xrightarrow{+}\Sigma F_x = 0: \quad \frac{1}{\sqrt{2}} F_{FG} + \frac{2}{\sqrt{5}}(3.3544 \text{ kN}) = 0$$

$$F_{FG} = -4.243 \text{ kN} \qquad F_{FG} = 4.24 \text{ kN} \quad C \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_{EF} - 1.75 \text{ kN} + \frac{1}{\sqrt{5}}(3.3544 \text{ kN}) - \frac{1}{\sqrt{2}}(-4.243 \text{ kN}) = 0$$

$$F_{EF} = 2.750 \text{ kN} \qquad F_{EF} = 2.75 \text{ kN} \quad T \blacktriangleleft$$

Free body: Joint *G*:



$$\xrightarrow{+}\Sigma F_x = 0: \quad \frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{EG} + \frac{1}{\sqrt{2}}(4.243 \text{ kN}) = 0$$

or

$$F_{GH} - F_{EG} = -4.243 \text{ kN} \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad -\frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{EG} - \frac{1}{\sqrt{2}}(4.243 \text{ kN}) - 1.5 \text{ kN} = 0$$

or

$$F_{GH} + F_{EG} = -6.364 \text{ kN} \qquad (2)$$

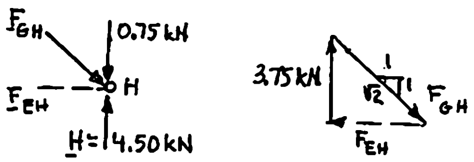
Add Eqs. (1) and (2): $2F_{GH} = -10.607$

$$F_{GH} = -5.303 \qquad F_{GH} = 5.30 \text{ kN} \quad C \blacktriangleleft$$

Subtract Eq. (1) from Eq. (2): $2F_{EG} = -2.121 \text{ kN}$

$$F_{EG} = -1.0605 \text{ kN} \qquad F_{EG} = 1.061 \text{ kN} \quad C \blacktriangleleft$$

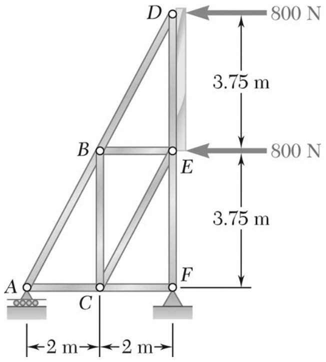
Free body: Joint *H*:



$$\frac{F_{EH}}{1} = \frac{3.75 \text{ kN}}{1} \qquad F_{EH} = 3.75 \text{ kN} \quad T \blacktriangleleft$$

We can also write

$$\frac{F_{GH}}{\sqrt{2}} = \frac{3.75 \text{ kN}}{1} \qquad F_{GH} = 5.30 \text{ kN} \quad C \quad \text{(Checks)}$$

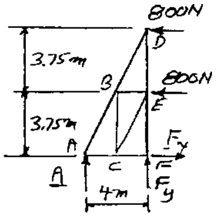


PROBLEM 6.14

The truss shown is one of several supporting an advertising panel. Determine the force in each member of the truss for a wind load equivalent to the two forces shown. State whether each member is in tension or compression.

SOLUTION

Free body: Entire truss:



$$+\circlearrowleft \Sigma M_F = 0: \quad (800\text{ N})(7.5\text{ m}) + (800\text{ N})(3.75\text{ m}) - A(2\text{ m}) = 0$$

$$A = +2250\text{ N} \qquad \mathbf{A} = 2250\text{ N}\uparrow$$

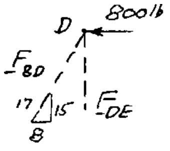
$$+\uparrow \Sigma F_y = 0: \quad 2250\text{ N} + F_y = 0$$

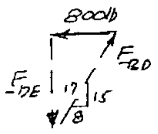
$$F_y = -2250\text{ N} \quad \mathbf{F}_y = 2250\text{ N}\downarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -800\text{ N} - 800\text{ N} + F_x = 0$$

$$F_x = +1600\text{ N} \quad \mathbf{F}_x = 1600\text{ N}\longrightarrow$$

Joint D:



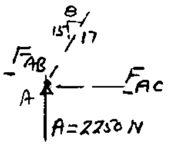


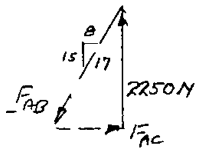
$$\frac{800\text{ N}}{8} = \frac{F_{DE}}{15} = \frac{F_{BD}}{17}$$

$F_{BD} = 1700\text{ N}$ *C* ◀

$F_{DE} = 1500\text{ N}$ *T* ◀

Joint A:



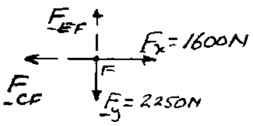


$$\frac{2250\text{ N}}{15} = \frac{F_{AB}}{17} = \frac{F_{AC}}{8}$$

$F_{AB} = 2250\text{ N}$ *C* ◀

$F_{AC} = 1200\text{ N}$ *T* ◀

Joint F:



$$\xrightarrow{+} \Sigma F_x = 0: \quad 1600\text{ N} - F_{CF} = 0$$

$$F_{CF} = +1600\text{ N}$$

$F_{CF} = 1600\text{ N}$ *T* ◀

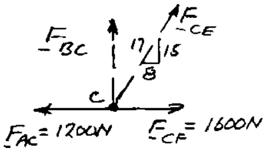
$$+\uparrow \Sigma F_y = 0: \quad F_{EF} - 2250\text{ N} = 0$$

$$F_{EF} = +2250\text{ N}$$

$F_{EF} = 2250\text{ N}$ *T* ◀

PROBLEM 6.14 (Continued)

Joint *C*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{8}{17}F_{CE} - 1200\text{ N} + 1600\text{ N} = 0$$

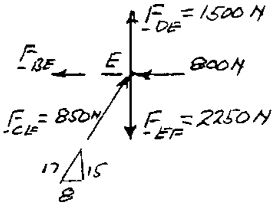
$$F_{CE} = -850\text{ N} \qquad F_{CE} = 850\text{ N} \quad C \ ◀$$

$$+\uparrow \Sigma F_y = 0: \quad F_{BC} + \frac{15}{17}F_{CE} = 0$$

$$F_{BC} = -\frac{15}{17}F_{CE} = -\frac{15}{17}(-850\text{ N})$$

$$F_{BC} = +750\text{ N} \qquad F_{BC} = 750\text{ N} \quad T \ ◀$$

Joint *E*:

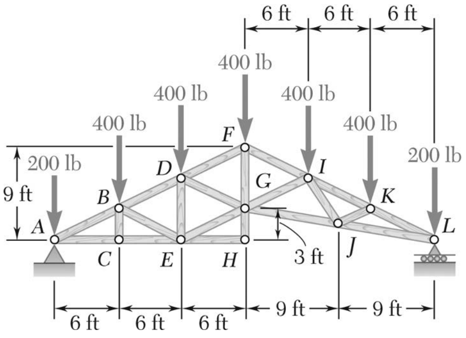


$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{BE} - 800\text{ N} + \frac{8}{17}(850\text{ N}) = 0$$

$$F_{BE} = -400\text{ N} \qquad F_{BE} = 400\text{ N} \quad C \ ◀$$

$$+\uparrow \Sigma F_y = 0: \quad 1500\text{ N} - 2250\text{ N} + \frac{15}{17}(850\text{ N}) = 0$$

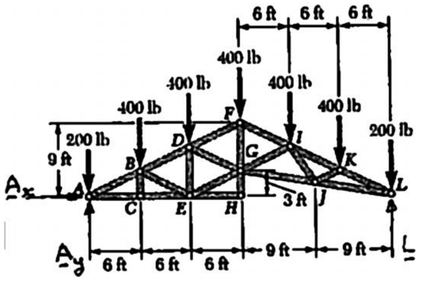
$$0 = 0 \quad \text{(checks)}$$



PROBLEM 6.15

Determine the force in each of the members located to the left of line *FGH* for the studio roof truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss: $\Sigma F_x = 0$: $\mathbf{A}_x = 0$

Because of symmetry of loading,

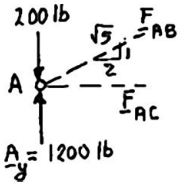
$$A_y = L = \frac{1}{2}\text{ total load}$$

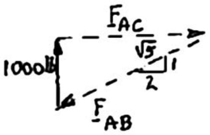
$$\mathbf{A}_y = \mathbf{L} = 1200\text{ lb}\uparrow$$

Zero-Force Members: Examining joints *C* and *H*, we conclude that *BC*, *EH*, and *GH* are zero-force members. Thus,

$$\mathbf{F}_{BC} = \mathbf{F}_{EH} = 0 \quad ◀$$

Also, $F_{CE} = F_{AC}$ (1)





Free body: Joint *A*: $\dfrac{F_{AB}}{\sqrt{5}} = \dfrac{F_{AC}}{2} = \dfrac{1000\text{ lb}}{1}$

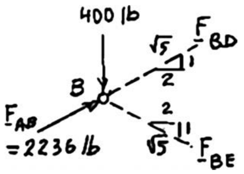
$$F_{AB} = 2236\text{ lb} \quad C$$

$$F_{AB} = 2240\text{ lb} \quad C \quad ◀$$

$$F_{AC} = 2000\text{ lb} \quad T \quad ◀$$

From Eq. (1): $F_{CE} = 2000\text{ lb} \quad T$ ◀

Free body: Joint *B*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}}F_{BD} + \frac{2}{\sqrt{5}}F_{BE} + \frac{2}{\sqrt{5}}(2236\text{ lb}) = 0$$

or $F_{BD} + F_{BE} = -2236\text{ lb}$ (2)

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{BD} - \frac{1}{\sqrt{5}}F_{BE} + \frac{1}{\sqrt{5}}(2236\text{ lb}) - 400\text{ lb} = 0$$

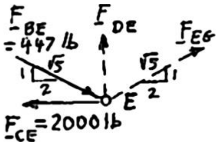
or $F_{BD} - F_{BE} = -1342\text{ lb}$ (3)

PROBLEM 6.15 (Continued)

Add Eqs. (2) and (3): $2F_{BD} = -3578 \text{ lb} \quad F_{BD} = 1789 \text{ lb} \quad C$ ◀

Subtract Eq. (3) from Eq. (1): $2F_{BE} = -894 \text{ lb} \quad F_{BE} = 447 \text{ lb} \quad C$ ◀

<u>Free body: Joint *E*:</u>



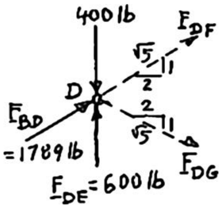
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{EG} + \frac{2}{\sqrt{5}}(447 \text{ lb}) - 2000 \text{ lb} = 0$$

$$F_{EG} = 1789 \text{ lb} \quad T \quad ◀$$

$$+\uparrow \Sigma F_y = 0: \quad F_{DE} + \frac{1}{\sqrt{5}}(1789 \text{ lb}) - \frac{1}{\sqrt{5}}(447 \text{ lb}) = 0$$

$$F_{DE} = -600 \text{ lb} \qquad F_{DE} = 600 \text{ lb} \quad C \quad ◀$$

<u>Free body: Joint *D*:</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{DF} + \frac{2}{\sqrt{5}} F_{DG} + \frac{2}{\sqrt{5}}(1789 \text{ lb}) = 0$$

or $$F_{DF} + F_{DG} = -1789 \text{ lb} \qquad (4)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{5}} F_{DG} + \frac{1}{\sqrt{5}}(1789 \text{ lb}) + 600 \text{ lb} - 400 \text{ lb} = 0$$

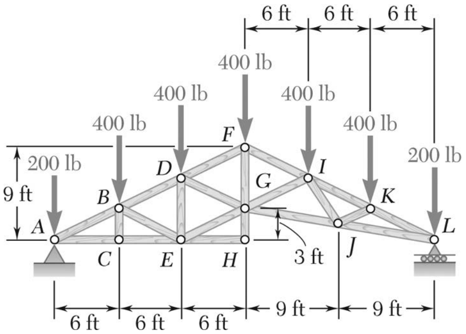
or $$F_{DF} - F_{DG} = -2236 \text{ lb} \qquad (5)$$

Add Eqs. (4) and (5): $2F_{DF} = -4025 \text{ lb} \quad F_{DF} = 2010 \text{ lb} \quad C$ ◀

Subtract Eq. (5) from Eq. (4): $2F_{DG} = 447 \text{ lb} \quad F_{DG} = 224 \text{ lb} \quad T$ ◀

PROBLEM 6.16

Determine the force in member *FG* and in each of the members located to the right of *FG* for the studio roof truss shown. State whether each member is in tension or compression.



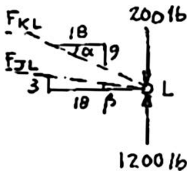
SOLUTION

Reaction at *L*: Because of the symmetry of the loading,

$$L = \frac{1}{2}\text{ total load}, \quad \mathbf{L} = 1200\text{ lb}\uparrow$$

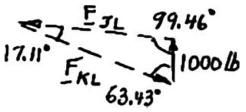
(See *F.B.* diagram to the left for more details.)

Free body: Joint *L*:



$$\alpha = \tan^{-1}\frac{9}{18} = 26.57°$$

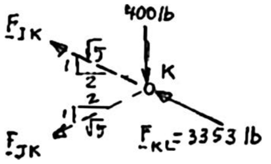
$$\beta = \tan^{-1}\frac{3}{18} = 9.46°$$



$$\frac{F_{JL}}{\sin 63.43°} = \frac{F_{KL}}{\sin 99.46°} = \frac{1000\text{ lb}}{\sin 17.11°} \qquad F_{JL} = 3040\text{ lb} \quad T ◄$$

$$F_{KL} = 3352.7\text{ lb} \quad C \qquad F_{KL} = 3350\text{ lb} \quad C ◄$$

Free body: Joint *K*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{2}{\sqrt{5}}F_{IK} - \frac{2}{\sqrt{5}}F_{JK} - \frac{2}{\sqrt{5}}(3352.7\text{ lb}) = 0$$

or $$F_{IK} + F_{JK} = -3352.7\text{ lb} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{IK} - \frac{1}{\sqrt{5}}F_{JK} + \frac{1}{\sqrt{5}}(3352.7) - 400 = 0$$

or $$F_{IK} - F_{JK} = -2458.3\text{ lb} \qquad (2)$$

Add Eqs. (1) and (2): $$2F_{IK} = -5811.0$$

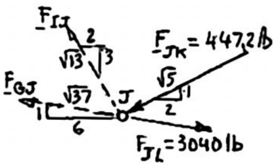
$$F_{IK} = -2905.5\text{ lb} \qquad F_{IK} = 2910\text{ lb} \quad C ◄$$

Subtract Eq. (2) from Eq. (1): $$2F_{JK} = -894.4$$

$$F_{JK} = -447.2\text{ lb} \qquad F_{JK} = 447\text{ lb} \quad C ◄$$

PROBLEM 6.16 (Continued)

Free body: Joint *J*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{2}{\sqrt{13}}F_{IJ} - \frac{6}{\sqrt{37}}F_{GJ} + \frac{6}{\sqrt{37}}(3040\text{ lb}) - \frac{2}{\sqrt{5}}(447.2) = 0 \qquad (3)$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{3}{\sqrt{13}}F_{IJ} + \frac{1}{\sqrt{37}}F_{GJ} - \frac{1}{\sqrt{37}}(3040\text{ lb}) - \frac{1}{\sqrt{5}}(447.2) = 0 \qquad (4)$$

Multiply Eq. (4) by 6 and add to Eq. (3):

$$\frac{16}{\sqrt{13}}F_{IJ} - \frac{8}{\sqrt{5}}(447.2) = 0$$

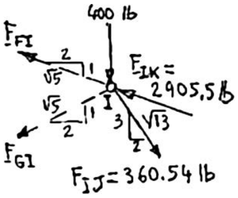
$$F_{IJ} = 360.54\text{ lb} \qquad F_{IJ} = 361\text{ lb} \quad T \blacktriangleleft$$

Multiply Eq. (3) by 3, Eq. (4) by 2, and add:

$$-\frac{16}{\sqrt{37}}(F_{GJ} - 3040) - \frac{8}{\sqrt{5}}(447.2) = 0$$

$$F_{GJ} = 2431.7\text{ lb} \qquad F_{GJ} = 2430\text{ lb} \quad T \blacktriangleleft$$

Free body: Joint *I*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{2}{\sqrt{5}}F_{FI} - \frac{2}{\sqrt{5}}F_{GI} - \frac{2}{\sqrt{5}}(2905.5) + \frac{2}{\sqrt{13}}(360.54) = 0$$

or $$F_{FI} + F_{GI} = -2681.9\text{ lb} \qquad (5)$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{FI} - \frac{1}{\sqrt{5}}F_{GI} + \frac{1}{\sqrt{5}}(2905.5) - \frac{3}{\sqrt{13}}(360.54) - 400 = 0$$

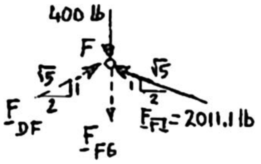
or $$F_{FI} - F_{GI} = -1340.3\text{ lb} \qquad (6)$$

Add Eqs. (5) and (6): $2F_{FI} = -4022.2$

$$F_{FI} = -2011.1\text{ lb} \qquad F_{FI} = 2010\text{ lb} \quad C \blacktriangleleft$$

Subtract Eq. (6) from Eq. (5): $2F_{GI} = -1341.6$ lb $\qquad F_{GI} = 671\text{ lb} \quad C \blacktriangleleft$

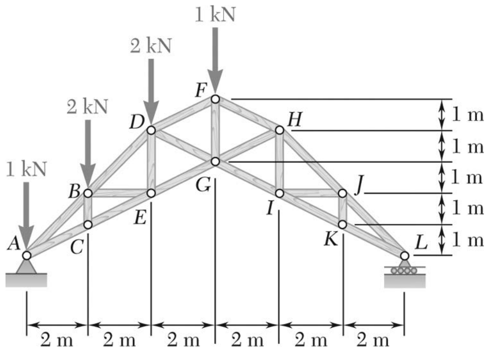
Free body: Joint *F*:



From $\quad \Sigma F_x = 0: \quad F_{DF} = F_{FI} = 2011.1\text{ lb} \quad C$

$$+\!\uparrow \Sigma F_y = 0: \quad F_{FG} - 400\text{ lb} + 2\left(\frac{1}{\sqrt{5}}2011.1\text{ lb}\right) = 0$$

$$F_{FG} = +1400\text{ lb} \qquad F_{FG} = 1400\text{ lb} \quad T \blacktriangleleft$$

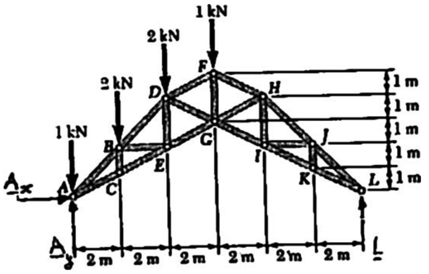


PROBLEM 6.17

Determine the force in each of the members located to the left of *FG* for the scissors roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free Body: Truss:



$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

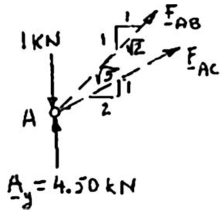
$$+\circlearrowleft \Sigma M_L = 0: \quad (1\text{ kN})(12\text{ m}) + (2\text{ kN})(10\text{ m}) + (2\text{ kN})(8\text{ m}) + (1\text{ kN})(6\text{ m}) - A_y(12\text{ m}) = 0$$

$$\mathbf{A}_y = 4.50\text{ kN}\uparrow$$

We note that *BC* is a zero-force member: $F_{BC} = 0$ ◀

Also, $F_{CE} = F_{AC}$ (1)

Free body: Joint *A*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{2}}F_{AB} + \frac{2}{\sqrt{5}}F_{AC} = 0 \qquad (2)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}}F_{AB} + \frac{1}{\sqrt{5}}F_{AC} + 3.50\text{ kN} = 0 \qquad (3)$$

Multiply Eq. (3) by –2 and add Eq. (2):

$$-\frac{1}{\sqrt{2}}F_{AB} - 7\text{ kN} = 0 \qquad F_{AB} = 9.90\text{ kN} \quad C \blacktriangleleft$$

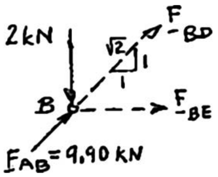
PROBLEM 6.17 (Continued)

Subtract Eq. (3) from Eq. (2):

$$\frac{1}{\sqrt{5}}F_{AC} - 3.50\text{ kN} = 0 \quad F_{AC} = 7.826\text{ kN} \qquad F_{AC} = 7.83\text{ kN} \quad T \blacktriangleleft$$

From Eq. (1): $F_{CE} = F_{AC} = 7.826\text{ kN}$ $\qquad F_{CE} = 7.83\text{ kN} \quad T \blacktriangleleft$

Free body: Joint *B*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}}F_{BD} + \frac{1}{\sqrt{2}}(9.90\text{ kN}) - 2\text{ kN} = 0$$

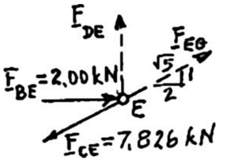
$$F_{BD} = -7.071\text{ kN} \qquad F_{BD} = 7.07\text{ kN} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BE} + \frac{1}{\sqrt{2}}(9.90 - 7.071)\text{ kN} = 0$$

$$F_{BE} = -2.000\text{ kN} \qquad F_{BE} = 2.00\text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *E*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}}(F_{EG} - 7.826\text{ kN}) + 2.00\text{ kN} = 0$$

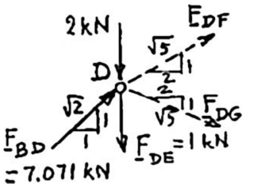


$$F_{EG} = 5.590\text{ kN} \qquad F_{EG} = 5.59\text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F_{DE} - \frac{1}{\sqrt{5}}(7.826 - 5.590)\text{ kN} = 0$$

$$F_{DE} = 1.000\text{ kN} \qquad F_{DE} = 1.000\text{ kN} \quad T \blacktriangleleft$$

Free body: Joint *D*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}}(F_{DF} + F_{DG}) + \frac{1}{\sqrt{2}}(7.071\text{ kN})$$

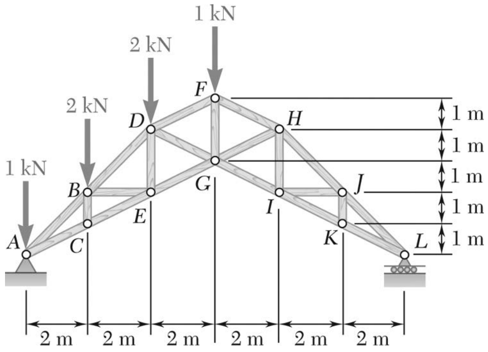
or $\qquad F_{DF} + F_{DG} = -5.590\text{ kN} \qquad (4)$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}(F_{DF} - F_{DG}) + \frac{1}{\sqrt{2}}(7.071\text{ kN}) = 2\text{ kN} - 1\text{ kN} = 0$$

or $\qquad F_{DE} - F_{DG} = -4.472 \qquad (5)$

Add Eqs. (4) and (5): $\qquad 2F_{DF} = -10.062\text{ kN} \qquad F_{DF} = 5.03\text{ kN} \quad C \blacktriangleleft$

Subtract Eq. (5) from Eq. (4): $\qquad 2F_{DG} = -1.1180\text{ kN} \qquad F_{DG} = 0.559\text{ kN} \quad C \blacktriangleleft$

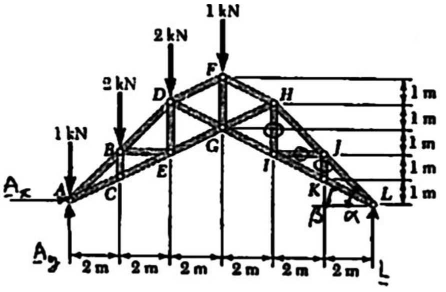


PROBLEM 6.18

Determine the force in member *FG* and in each of the members located to the right of *FG* for the scissors roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:



$$+\circlearrowleft \Sigma M_A = 0: \quad L(12\text{ m}) - (2\text{ kN})(2\text{ m}) - (2\text{ kN})(4\text{ m}) - (1\text{ kN})(6\text{ m}) = 0$$

$$\mathbf{L} = 1.500\text{ kN}\uparrow$$

Angles:

$$\tan\alpha = 1 \qquad \alpha = 45°$$

$$\tan\beta = \frac{1}{2} \qquad \beta = 26.57°$$

Zero-force members:

Examining successively joints *K*, *J*, and *I*, we note that the following members to the right of *FG* are zero-force members: *JK*, *IJ*, and *HI*.

Thus, $F_{HI} = F_{IJ} = F_{JK} = 0$ ◀

We also note that

$$F_{GI} = F_{IK} = F_{KL} \qquad (1)$$

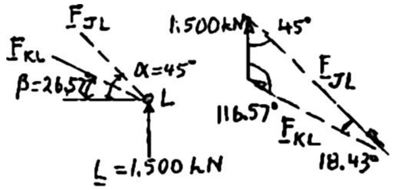
and

$$F_{HJ} = F_{JL} \qquad (2)$$

PROPRIETARY MATERIAL.

PROBLEM 6.18 (Continued)

<u>Free body: Joint *L*</u>:



$$\frac{F_{JL}}{\sin 116.57^\circ} = \frac{F_{KL}}{\sin 45^\circ} = \frac{1.500\text{ kN}}{\sin 18.43^\circ}$$

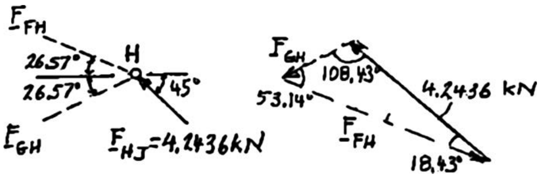
$$F_{JL} = 4.2436\text{ kN} \qquad F_{JL} = 4.24\text{ kN} \quad C \blacktriangleleft$$

$$F_{KL} = 3.35\text{ kN} \quad T \blacktriangleleft$$

From Eq. (1): $F_{GI} = F_{IK} = F_{KL}$ $\qquad F_{GI} = F_{IK} = 3.35\text{ kN} \quad T \blacktriangleleft$

From Eq. (2): $F_{HJ} = F_{JL} = 4.2436\text{ kN}$ $\qquad F_{HJ} = 4.24\text{ kN} \quad C \blacktriangleleft$

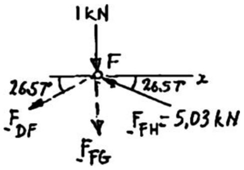
<u>Free body: Joint *H*</u>:



$$\frac{F_{FH}}{\sin 108.43^\circ} = \frac{F_{GH}}{\sin 18.43^\circ} = \frac{4.2436}{\sin 53.14^\circ} \qquad F_{FH} = 5.03\text{ kN} \quad C \blacktriangleleft$$

$$F_{GH} = 1.677\text{ kN} \quad T \blacktriangleleft$$

<u>Free body: Joint *F*</u>:

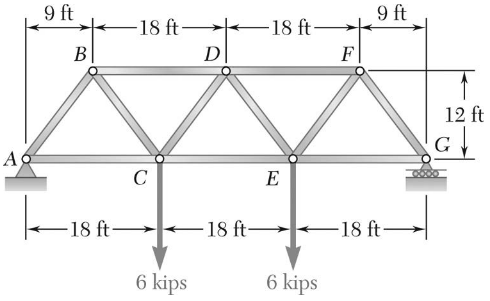


$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{DF}\cos 26.57^\circ - (5.03\text{ kN})\cos 26.57^\circ = 0$$

$$F_{DF} = -5.03\text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{FG} - 1\text{ kN} + (5.03\text{ kN})\sin 26.57^\circ - (-5.03\text{ kN})\sin 26.57^\circ = 0$$

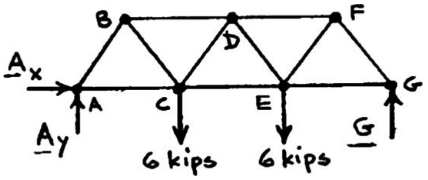
$$F_{FG} = 3.500\text{ kN} \qquad F_{FG} = 3.50\text{ kN} \quad T \blacktriangleleft$$



PROBLEM 6.19

Determine the force in each member of the Warren bridge truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss: $\Sigma F_x = 0\text{:} \quad A_x = 0$

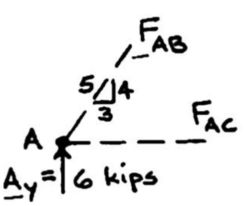
Due to symmetry of truss and loading,

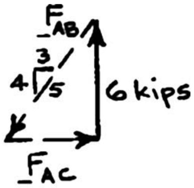
$$A_y = G = \frac{1}{2}\text{ total load} = 6 \text{ kips}\uparrow$$

Free body: Joint *A*: $\dfrac{F_{AB}}{5} = \dfrac{F_{AC}}{3} = \dfrac{6\text{ kips}}{4}$

$F_{AB} = 7.50$ kips C ◀

$F_{AC} = 4.50$ kips T ◀

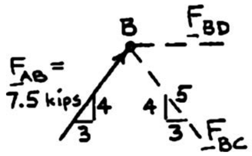


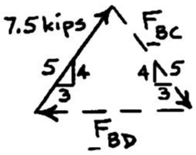


Free body: Joint *B*: $\dfrac{F_{BC}}{5} = \dfrac{F_{BD}}{6} = \dfrac{7.5\text{ kips}}{5}$

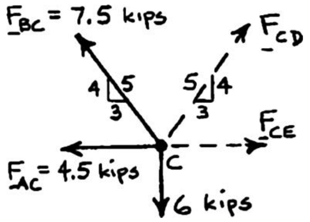
$F_{BC} = 7.50$ kips T ◀

$F_{BD} = 9.00$ kips C ◀





Free body: Joint *C*:



$$+\uparrow \Sigma F_y = 0\text{:} \quad \frac{4}{5}(7.5) + \frac{4}{5}F_{CD} - 6 = 0$$

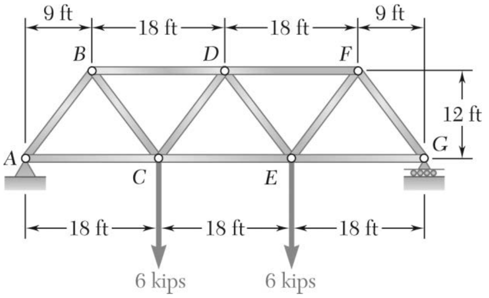
$F_{CD} = 0$ ◀

$$\xrightarrow{+} \Sigma F_x = 0\text{:} \quad F_{CE} - 4.5 - \frac{3}{5}(7.5) = 0$$

$+\uparrow F_{CE} = +9$ kips

$F_{CE} = 9.00$ kips T ◀

Truss and loading is symmetrical about ℄.



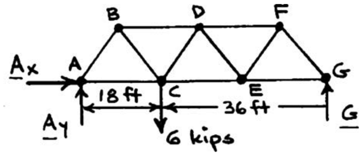
PROBLEM 6.20

Solve Problem 6.19 assuming that the load applied at *E* has been removed.

PROBLEM 6.19 Determine the force in each member of the Warren bridge truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:

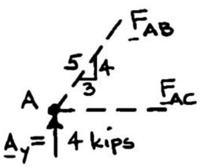


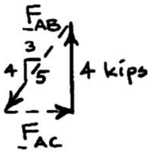
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_G = 0: \quad 6(36) - A_y(54) = 0 \quad \mathbf{A}_y = 4 \text{ kips}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 4 - 6 + G = 0 \quad \mathbf{G} = 2 \text{ kips}\uparrow$$

Free body: Joint *A*:



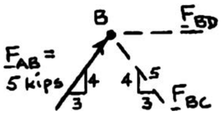


$$\frac{F_{AB}}{5} = \frac{F_{AC}}{3} = \frac{4 \text{ kips}}{4}$$

$F_{AB} = 5.00 \text{ kips} \quad C$ ◀

$F_{AC} = 3.00 \text{ kips} \quad T$ ◀

Free body Joint *B*:

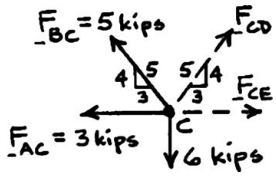


$$\frac{F_{BC}}{5} = \frac{F_{BD}}{6} = \frac{5 \text{ kips}}{5}$$

$F_{BC} = 5.00 \text{ kips} \quad T$ ◀

$F_{BD} = 6.00 \text{ kips} \quad C$ ◀

Free body Joint *C*:



$$+\uparrow \Sigma M_y = 0: \quad \frac{4}{5}(5) + \frac{4}{5}F_{CD} - 6 = 0$$

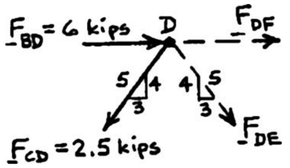
$F_{CD} = 2.50 \text{ kips} \quad T$ ◀

$$+\uparrow \Sigma F_x = 0: \quad F_{CE} + \frac{3}{5}(2.5) - \frac{3}{5}(5) - 3 = 0$$

$F_{CE} = 4.50 \text{ kips} \quad T$ ◀

PROBLEM 6.20 (Continued)

Free body: Joint *D*:



$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}(2.5) - \frac{4}{5}F_{DE} = 0$$

$$F_{DE} = -2.5 \text{ kips} \qquad F_{DE} = 2.50 \text{ kips} \quad C \blacktriangleleft$$

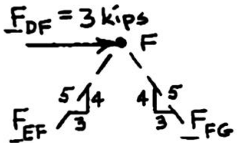
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{DF} + 6 - \frac{3}{5}(2.5) - \frac{3}{5}(2.5) = 0$$

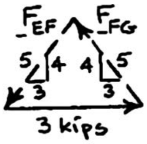
$$F_{DF} = -3 \text{ kips} \qquad F_{DF} = 3.00 \text{ kips} \quad C \blacktriangleleft$$

Free body: Joint *F*:

$$\frac{F_{EF}}{5} = \frac{F_{FG}}{5} = \frac{3 \text{ kips}}{6} \qquad F_{EF} = 2.50 \text{ kips} \quad T \blacktriangleleft$$

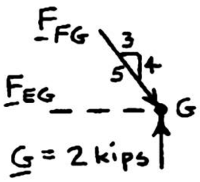
$$F_{FG} = 2.50 \text{ kips} \quad C \blacktriangleleft$$

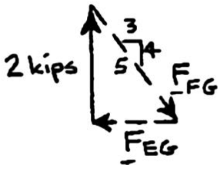




Free body: Joint *G*:

$$\frac{F_{EG}}{3} = \frac{2 \text{ kips}}{4} \qquad F_{EG} = 1.500 \text{ kips} \quad T \blacktriangleleft$$

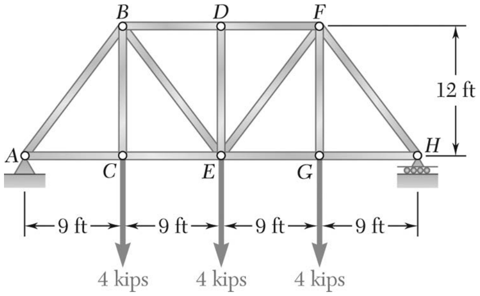




Also,

$$\frac{F_{FG}}{5} = \frac{2 \text{ kips}}{4}$$

$$F_{FG} = 2.50 \text{ kips} \quad C \quad \text{(Checks)}$$

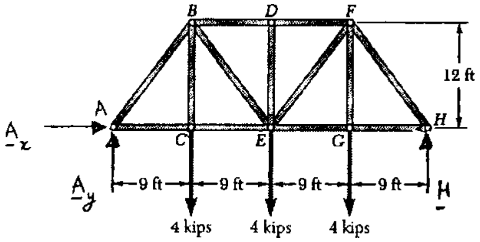


PROBLEM 6.21

Determine the force in each member of the Pratt bridge truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:



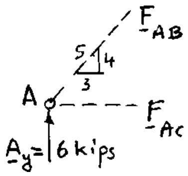
$$\xrightarrow{+} \Sigma F_z = 0: \quad \mathbf{A}_x = 0$$

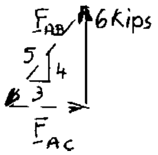
$$+\circlearrowleft \Sigma M_A = 0: \quad H(36 \text{ ft}) - (4 \text{ kips})(9 \text{ ft}) - (4 \text{ kips})(18 \text{ ft}) - (4 \text{ kips})(27 \text{ ft}) = 0$$

$$\mathbf{H} = 6 \text{ kips} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 6 \text{ kips} - 12 \text{ kips} = 0 \qquad \mathbf{A}_y = 6 \text{ kips} \uparrow$$

Free body: Joint A:



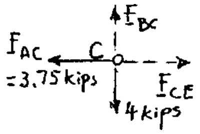


$$\frac{F_{AB}}{5} = \frac{F_{AC}}{3} = \frac{6 \text{ kips}}{4}$$

$F_{AB} = 7.50 \text{ kips} \quad C$ ◀

$F_{AC} = 4.50 \text{ kips} \quad T$ ◀

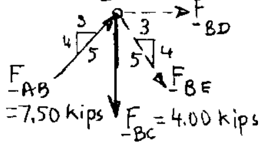
Free body: Joint C:



$\Sigma F_x = 0: \quad F_{CE} = 4.50 \text{ kips} \quad T$ ◀

$\Sigma F_y = 0: \quad F_{BC} = 4.00 \text{ kips} \quad T$ ◀

Free body: Joint B:



$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}F_{BE} + \frac{4}{5}(7.50 \text{ kips}) - 4.00 \text{ kips} = 0$$

$F_{BE} = 2.50 \text{ kips} \quad T$ ◀

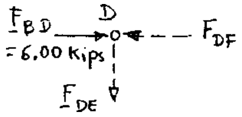
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{8}{5}(7.50 \text{ kips}) + \frac{3}{5}(2.50 \text{ kips}) + F_{BD} = 0$$

$$F_{BD} = -6.00 \text{ kips}$$

$F_{BD} = 6.00 \text{ kips} \quad C$ ◀

PROBLEM 6.21 (Continued)

Free body: Joint *D*:



We note that *DE* is a zero-force member: $F_{DE} = 0$ ◀

Also, $F_{DF} = 6.00$ kips C ◀

From symmetry:

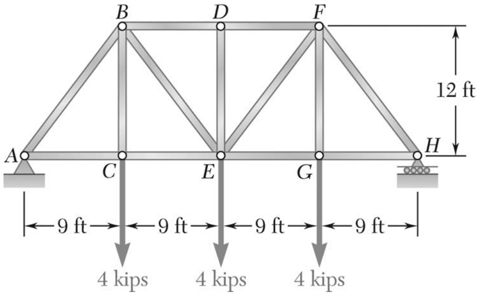
$F_{FE} = F_{BE}$ $\qquad F_{EF} = 2.50$ kips T ◀

$F_{EG} = F_{CE}$ $\qquad F_{EG} = 4.50$ kips T ◀

$F_{FG} = F_{BC}$ $\qquad F_{FG} = 4.00$ kips T ◀

$F_{FH} = F_{AB}$ $\qquad F_{FH} = 7.50$ kips C ◀

$F_{GH} = F_{AC}$ $\qquad F_{GH} = 4.50$ kips T ◀



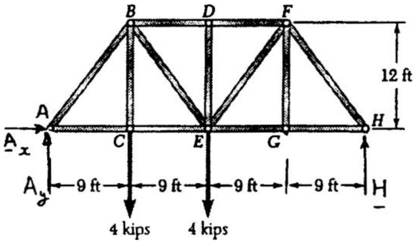
PROBLEM 6.22

Solve Problem 6.21 assuming that the load applied at G has been removed.

PROBLEM 6.21 Determine the force in each member of the Pratt bridge truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:



$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

$$+\circlearrowleft \Sigma \mathbf{M}_A = 0: \quad H(36 \text{ ft}) - (4 \text{ kips})(9 \text{ ft}) - (4 \text{ kips})(18 \text{ ft}) = 0$$

$$\mathbf{H} = 3.00 \text{ kips}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad \mathbf{A}_y + 5.00 \text{ kips}\uparrow$$

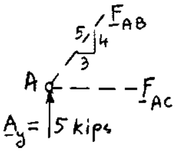
We note that DE and FG are zero-force members.

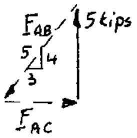
Therefore, $F_{DE} = 0, \quad F_{FG} = 0$ ◀

Also, $F_{BD} = F_{DF}$ (1)

and $F_{EG} = F_{GH}$ (2)

Free body: Joint A:



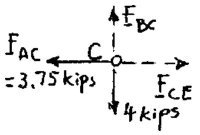


$$\frac{F_{AB}}{5} = \frac{F_{AC}}{3} = \frac{5 \text{ kips}}{4}$$

$F_{AB} = 6.25$ kips C ◀

$F_{AC} = 3.75$ kips T ◀

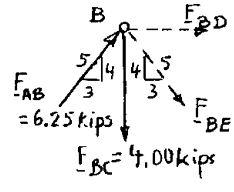
Free body: Joint C:



$\Sigma F_x = 0: \quad F_{CE} = 3.75$ kips T ◀

$\Sigma F_y = 0: \quad F_{BC} = 4.00$ kips T ◀

Free body: Joint B:



$$+\uparrow \Sigma F_x = 0: \quad \frac{4}{5}(6.25 \text{ kips}) - 4.00 \text{ kips} - \frac{4}{5} F_{BE} = 0$$

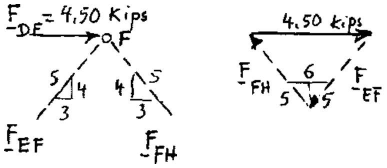
$F_{BE} = 1.250$ kips T ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BD} + \frac{3}{5}(6.25 \text{ kips}) + \frac{3}{5}(1.250 \text{ kips}) = 0$$

$F_{BD} = -4.50$ kips $\qquad F_{BD} = 4.50$ kips C ◀

PROBLEM 6.22 (Continued)

Free body: Joint F:



We recall that $F_{FG} = 0$, and from Eq. (1) that

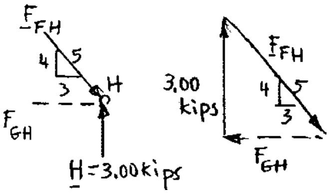
$F_{DF} = F_{BD}$ $\qquad F_{DF} = 4.50\text{ kips} \quad C$ ◀

$$\frac{F_{EF}}{5} = \frac{F_{FH}}{5} = \frac{4.50\text{ kips}}{6}$$

$F_{EF} = 3.75\text{ kips} \quad T$ ◀

$F_{FH} = 3.75\text{ kips} \quad C$ ◀

Free body: Joint H:



$$\frac{F_{GH}}{3} = \frac{3.00\text{ kips}}{4}$$

$F_{GH} = 2.25\text{ kips} \quad T$ ◀

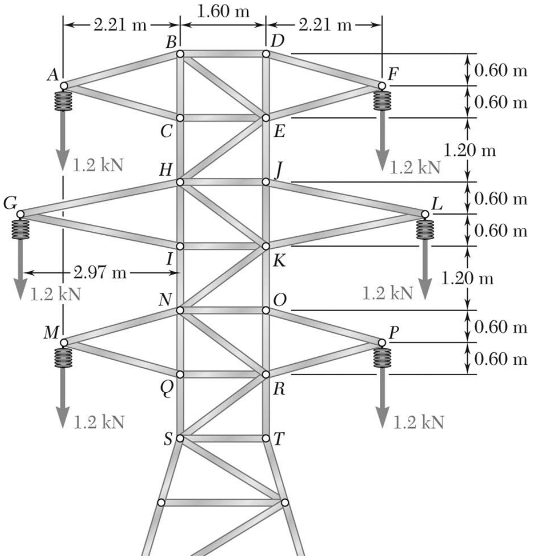
Also,

$$\frac{F_{FH}}{5} = \frac{3.00\text{ kips}}{4}$$

$F_{FH} = 3.75\text{ kips} \quad C$ (checks)

From Eq. (2):

$F_{EG} = F_{GH}$ $\qquad F_{EG} = 2.25\text{ kips} \quad T$ ◀



PROBLEM 6.23

The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

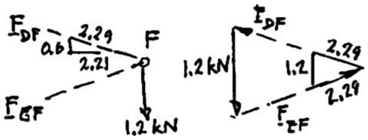
SOLUTION

Free body: Joint *A*:

$$\frac{F_{AB}}{2.29}=\frac{F_{AC}}{2.29}=\frac{1.2\text{ kN}}{1.2}$$

$F_{AB}=2.29\text{ kN}\quad T$ ◀

$F_{AC}=2.29\text{ kN}\quad C$ ◀

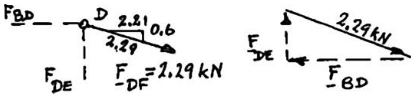


Free body: Joint *F*:

$$\frac{F_{DF}}{2.29}=\frac{F_{EF}}{2.29}=\frac{1.2\text{ kN}}{2.1}$$

$F_{DF}=2.29\text{ kN}\quad T$ ◀

$F_{EF}=2.29\text{ kN}\quad C$ ◀



Free body: Joint *D*:

$$\frac{F_{BD}}{2.21}=\frac{F_{DE}}{0.6}=\frac{2.29\text{ kN}}{2.29}$$

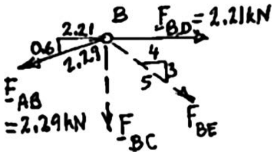
$F_{BD}=2.21\text{ kN}\quad T$ ◀

$F_{DE}=0.600\text{ kN}\quad C$ ◀

Free body: Joint *B*:

$$\xrightarrow{+}\Sigma F_x=0:\quad \frac{4}{5}F_{BE}+2.21\text{ kN}-\frac{2.21}{2.29}(2.29\text{ kN})=0$$

$F_{BE}=0$ ◀

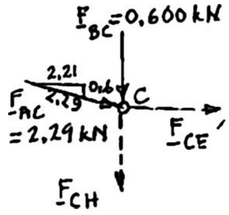


$$+\uparrow\Sigma F_y=0:\quad -F_{BC}-\frac{3}{5}(0)-\frac{0.6}{2.29}(2.29\text{ kN})=0$$

$F_{BC}=-0.600\text{ kN}\qquad F_{BC}=0.600\text{ kN}\quad C$ ◀

PROBLEM 6.23 (Continued)

Free body: Joint *C*:



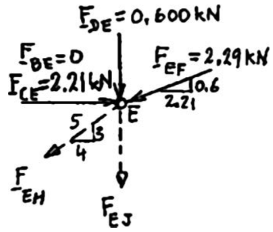
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} + \frac{2.21}{2.29}(2.29\text{ kN}) = 0$$

$$F_{CE} = -2.21\text{ kN} \qquad F_{CE} = 2.21\text{ kN} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{CH} - 0.600\text{ kN} - \frac{0.6}{2.29}(2.29\text{ kN}) = 0$$

$$F_{CH} = -1.200\text{ kN} \qquad F_{CH} = 1.200\text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *E*:

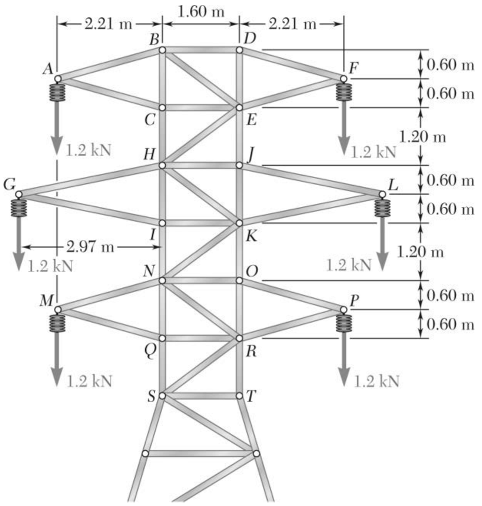


$$\xrightarrow{+} \Sigma F_x = 0: \quad 2.21\text{ kN} - \frac{2.21}{2.29}(2.29\text{ kN}) - \frac{4}{5}F_{EH} = 0$$

$$F_{EH} = 0 \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{EJ} - 0.600\text{ kN} - \frac{0.6}{2.29}(2.29\text{ kN}) - 0 = 0$$

$$F_{EJ} = -1.200\text{ kN} \qquad F_{EJ} = 1.200\text{ kN} \quad C \blacktriangleleft$$

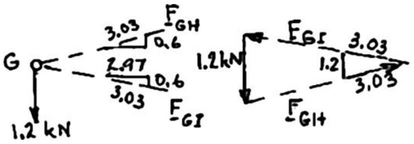


PROBLEM 6.24

For the tower and loading of Problem 6.23 and knowing that $F_{CH} = F_{EJ} = 1.2$ kN C and $F_{EH} = 0$, determine the force in member *HJ* and in each of the members located between *HJ* and *NO*. State whether each member is in tension or compression.

PROBLEM 6.23 The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

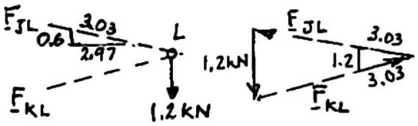
SOLUTION



Free body: Joint *G*:

$$\frac{F_{GH}}{3.03} = \frac{F_{GI}}{3.03} = \frac{1.2\text{ kN}}{1.2} \qquad F_{GH} = 3.03\text{ kN} \quad T ◀$$

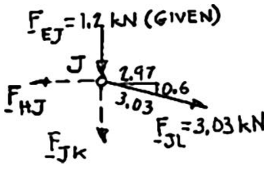
$$F_{GI} = 3.03\text{ kN} \quad C ◀$$



Free body: Joint *L*:

$$\frac{F_{JL}}{3.03} = \frac{F_{KL}}{3.03} = \frac{1.2\text{ kN}}{1.2} \qquad F_{JL} = 3.03\text{ kN} \quad T ◀$$

$$F_{KL} = 3.03\text{ kN} \quad C ◀$$



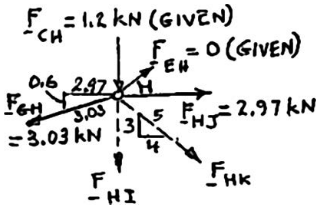
Free body: Joint *J*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{HJ} + \frac{2.97}{3.03}(3.03\text{ kN}) = 0$$

$$F_{HJ} = 2.97\text{ kN} \quad T ◀$$

$$+\uparrow F_y = 0: \quad -F_{JK} - 1.2\text{ kN} - \frac{0.6}{3.03}(3.03\text{ kN}) = 0$$

$$F_{JK} = -1.800\text{ kN} \qquad F_{JK} = 1.800\text{ kN} \quad C ◀$$



Free body: Joint *H*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{HK} + 2.97\text{ kN} - \frac{2.97}{3.03}(3.03\text{ kN}) = 0$$

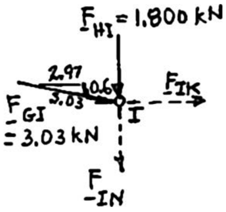
$$F_{HK} = 0 ◀$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{HI} - 1.2\text{ kN} - \frac{0.6}{3.03}(3.03)\text{ kN} - \frac{3}{5}(0) = 0$$

$$F_{HI} = -1.800\text{ kN} \qquad F_{HI} = 1.800\text{ kN} \quad C ◀$$

PROBLEM 6.24 (Continued)

<u>Free body: Joint *I*:</u>



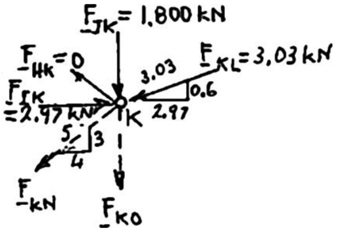
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{IK} + \frac{2.97}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{IK} = -2.97 \text{ kN} \qquad F_{IK} = 2.97 \text{ kN} \quad C \blacktriangleleft$$

$$+\big\uparrow \Sigma F_y = 0: \quad -F_{IN} - 1.800 \text{ kN} - \frac{0.6}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{IN} = -2.40 \text{ kN} \qquad F_{IN} = 2.40 \text{ kN} \quad C \blacktriangleleft$$

<u>Free body: Joint *K*:</u>

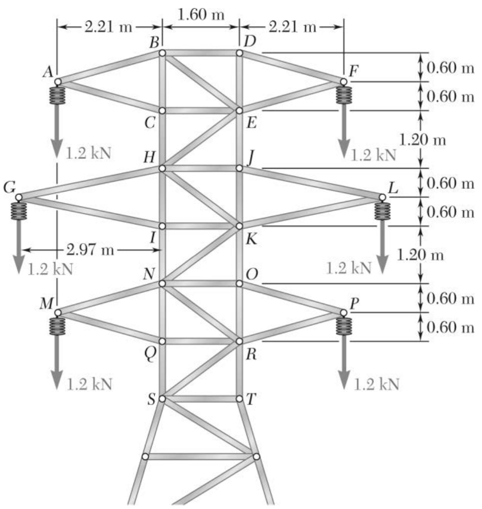


$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{4}{5}F_{KN} + 2.97 \text{ kN} - \frac{2.97}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{KN} = 0 \blacktriangleleft$$

$$+\big\uparrow \Sigma F_y = 0: \quad -F_{KO} - \frac{0.6}{3.03}(3.03 \text{ kN}) - 1.800 \text{ kN} - \frac{3}{5}(0) = 0$$

$$F_{KO} = -2.40 \text{ kN} \qquad F_{KO} = 2.40 \text{ kN} \quad C \blacktriangleleft$$

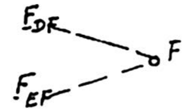


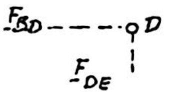
PROBLEM 6.25

Solve Problem 6.23 assuming that the cables hanging from the right side of the tower have fallen to the ground.

PROBLEM 6.23 The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

SOLUTION





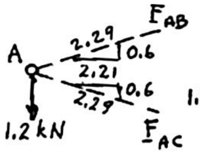
Zero-Force Members:

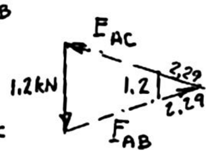
Considering joint *F*, we note that *DF* and *EF* are zero-force members:

$$F_{DF} = F_{EF} = 0 \blacktriangleleft$$

Considering next joint *D*, we note that *BD* and *DE* are zero-force members:

$$F_{BD} = F_{DE} = 0 \blacktriangleleft$$

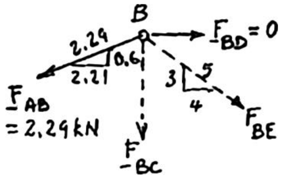




Free body: Joint *A*:

$$\frac{F_{AB}}{2.29} = \frac{F_{AC}}{2.29} = \frac{1.2\text{ kN}}{1.2} \qquad F_{AB} = 2.29\text{ kN}\quad T \blacktriangleleft$$

$$F_{AC} = 2.29\text{ kN}\quad C \blacktriangleleft$$



Free body: Joint *B*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{BE} - \frac{2.21}{2.29}(2.29\text{ kN}) = 0$$

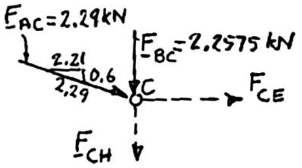
$$F_{BE} = 2.7625\text{ kN} \qquad F_{BE} = 2.76\text{ kN}\quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{BC} - \frac{0.6}{2.29}(2.29\text{ kN}) - \frac{3}{5}(2.7625\text{ kN}) = 0$$

$$F_{BC} = -2.2575\text{ kN} \qquad F_{BC} = 2.26\text{ kN}\quad C \blacktriangleleft$$

PROBLEM 6.25 (Continued)

Free body: Joint C:



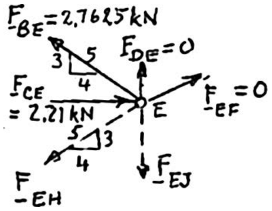
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} + \frac{2.21}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CE} = 2.21 \text{ kN} \quad C \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_{CH} - 2.2575 \text{ kN} - \frac{0.6}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CH} = -2.8575 \text{ kN} \qquad F_{CH} = 2.86 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint E:

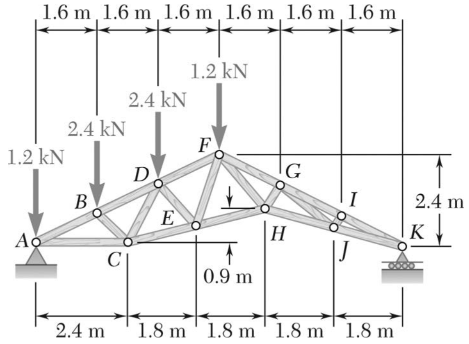


$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{4}{5}F_{EH} - \frac{4}{5}(2.7625 \text{ kN}) + 2.21 \text{ kN} = 0$$

$$F_{EH} = 0 \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_{EJ} + \frac{3}{5}(2.7625 \text{ kN}) - \frac{3}{5}(0) = 0$$

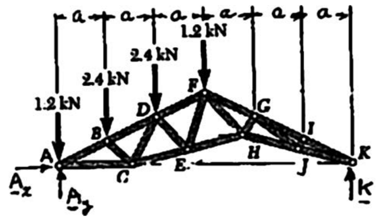
$$F_{EJ} = +1.6575 \text{ kN} \qquad F_{EJ} = 1.658 \text{ kN} \quad T \blacktriangleleft$$



PROBLEM 6.26

Determine the force in each of the members connecting joints A through F of the vaulted roof truss shown. State whether each member is in tension or compression.

SOLUTION

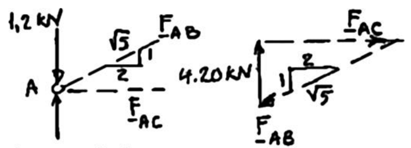


Free body: Truss:

$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

$$+\circlearrowleft \Sigma M_K = 0: \quad (1.2\text{ kN})6a + (2.4\text{ kN})5a + (2.4\text{ kN})4a + (1.2\text{ kN})3a$$

$$-A_y(6a) = 0 \quad \mathbf{A}_y = 5.40\text{ kN}\uparrow$$



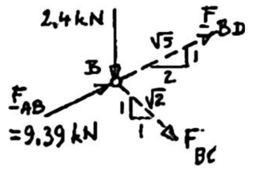
Free body: Joint A:

$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{4.20\text{ kN}}{1}$$

$F_{AB} = 9.3915\text{ kN}$ $\quad$ $F_{AB} = 9.39\text{ kN} \quad C$ ◀

$F_{AC} = 8.40\text{ kN} \quad T$ ◀

Free body: Joint B:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}}F_{BD} + \frac{1}{\sqrt{2}}F_{BC} + \frac{2}{\sqrt{5}}(9.3915) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{BD} - \frac{1}{\sqrt{2}}F_{BC} + \frac{1}{\sqrt{5}}(9.3915) - 2.4 = 0 \qquad (2)$$

Add Eqs. (1) and (2):

$$\frac{3}{\sqrt{5}}F_{BD} + \frac{3}{\sqrt{5}}(9.3915\text{ kN}) - 2.4\text{ kN} = 0$$

$F_{BD} = -7.6026\text{ kN}$ $\quad$ $F_{BD} = 7.60\text{ kN} \quad C$ ◀

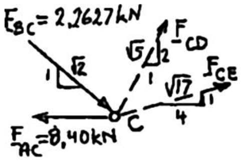
Multiply Eq. (2) by –2 and add Eq. (1):

$$\frac{3}{\sqrt{2}}F_B + 4.8\text{ kN} = 0$$

$F_{BC} = -2.2627\text{ kN}$ $\quad$ $F_{BC} = 2.26\text{ kN} \quad C$ ◀

PROBLEM 6.26 (Continued)

<u>Free body: Joint C:</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{5}}F_{CD} + \frac{4}{\sqrt{17}}F_{CE} + \frac{1}{\sqrt{2}}(2.2627) - 8.40 = 0 \qquad (3)$$

$$+\big\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{5}}F_{CD} + \frac{1}{\sqrt{17}}F_{CE} - \frac{1}{\sqrt{2}}(2.2627) = 0 \qquad (4)$$

Multiply Eq. (4) by −4 and add Eq. (1):

$$-\frac{7}{\sqrt{5}}F_{CD} + \frac{5}{\sqrt{2}}(2.2627) - 8.40 = 0$$

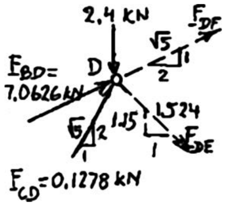
$F_{CD} = -0.1278$ kN $\qquad F_{CD} = 0.128$ kN C ◀

Multiply Eq. (1) by 2 and subtract Eq. (2):

$$\frac{7}{\sqrt{17}}F_{CE} + \frac{3}{\sqrt{2}}(2.2627) - 2(8.40) = 0$$

$F_{CE} = 7.068$ kN $\qquad F_{CE} = 7.07$ kN T ◀

<u>Free body: Joint D:</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}}F_{DF} + \frac{1}{1.524}F_{DE} + \frac{2}{\sqrt{5}}(7.6026)$$

$$+\frac{1}{\sqrt{5}}(0.1278) = 0 \qquad (5)$$

$$+\big\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{DF} - \frac{1.15}{1.524}F_{DE} + \frac{1}{\sqrt{5}}(7.6026)$$

$$+\frac{2}{\sqrt{5}}(0.1278) - 2.4 = 0 \qquad (6)$$

Multiply Eq. (5) by 1.15 and add Eq. (6):

$$\frac{3.30}{\sqrt{5}}F_{DF} + \frac{3.30}{\sqrt{5}}(7.6026) + \frac{3.15}{\sqrt{5}}(0.1278) - 2.4 = 0$$

$F_{DF} = -6.098$ kN $\qquad F_{DF} = 6.10$ kN C ◀

Multiply Eq. (6) by –2 and add Eq. (5):

$$\frac{3.30}{1.524}F_{DE} - \frac{3}{\sqrt{5}}(0.1278) + 4.8 = 0$$

$F_{DE} = -2.138$ kN $\qquad F_{DE} = 2.14$ kN C ◀

PROBLEM 6.26 (Continued)

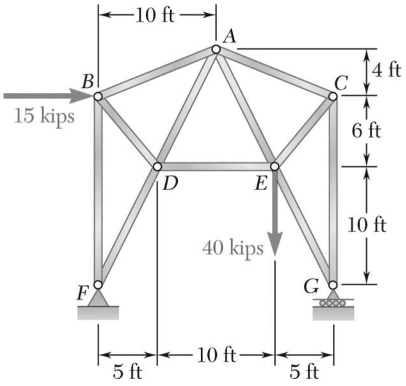
Free body: Joint E:

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{0.6}{2.04}F_{EF} + \frac{4}{\sqrt{17}}(F_{EH} - F_{CE}) + \frac{1}{1.524}(2.138) = 0 \qquad (7)$$

$$+\big\uparrow \Sigma F_y = 0: \quad \frac{1.95}{2.04}F_{EF} + \frac{1}{\sqrt{17}}(F_{EH} - F_{CE}) - \frac{1.15}{1.524}(2.138) = 0 \qquad (8)$$

Multiply Eq. (8) by 4 and subtract Eq. (7):

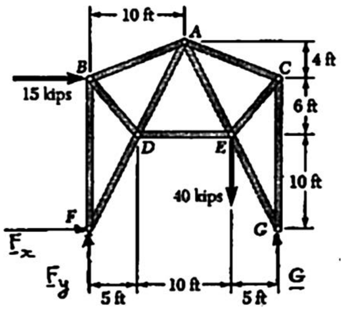
$$\frac{7.2}{2.04}F_{EF} - 7.856 \text{ kN} = 0 \qquad\qquad F_{EF} = 2.23 \text{ kN} \quad T \blacktriangleleft$$



PROBLEM 6.27

Determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss:

$$+\circlearrowleft \Sigma M_F = 0: \quad G(20\text{ ft}) - (15\text{ kips})(16\text{ ft}) - (40\text{ kips})(15\text{ ft}) = 0$$

$$\mathbf{G} = 42\text{ kips}\uparrow$$

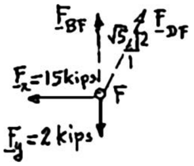
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_x + 15\text{ kips} = 0$$

$$\mathbf{F}_x = 15\text{ kips} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad F_y - 40\text{ kips} + 42\text{ kips} = 0$$

$$\mathbf{F}_y = 2\text{ kips}\downarrow$$

Free body: Joint *F*:



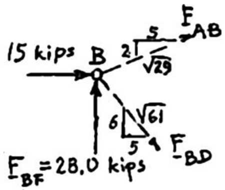
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{5}} F_{DF} - 15\text{ kips} = 0$$

$$F_{DF} = 33.54\text{ kips} \qquad F_{DF} = 33.5\text{ kips} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F_{BF} - 2\text{ kips} + \frac{2}{\sqrt{5}}(33.54\text{ kips}) = 0$$

$$F_{BF} = -28.00\text{ kips} \qquad F_{BF} = 28.0\text{ kips} \quad C \blacktriangleleft$$

Free body: Joint *B*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{5}{\sqrt{29}} F_{AB} + \frac{5}{\sqrt{61}} F_{BD} + 15\text{ kips} = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{29}} F_{AB} - \frac{6}{\sqrt{61}} F_{BD} + 28\text{ kips} = 0 \qquad (2)$$

PROBLEM 6.27 (Continued)

Multiply Eq. (1) by 6, Eq. (2) by 5, and add:

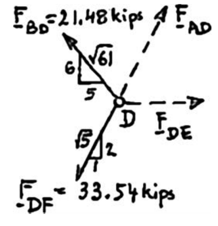
$$\frac{40}{\sqrt{29}}F_{AB} + 230\text{ kips} = 0$$

$F_{AB} = -30.96\text{ kips}$ $\qquad$ $F_{AB} = 31.0\text{ kips}$ C ◀

Multiply Eq. (1) by 2, Eq. (2) by –5, and add:

$$\frac{40}{\sqrt{61}}F_{BD} - 110\text{ kips} = 0$$

$F_{BD} = 21.48\text{ kips}$ $\qquad$ $F_{BD} = 21.5\text{ kips}$ T ◀



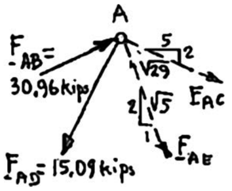
Free body: Joint D:

$$+\uparrow\Sigma F_y = 0:\quad \frac{2}{\sqrt{5}}F_{AD} - \frac{2}{\sqrt{5}}(33.54) + \frac{6}{\sqrt{61}}(21.48) = 0$$

$F_{AD} = 15.09\text{ kips}$ T ◀

$$\xrightarrow{+}\Sigma F_x = 0:\quad F_{DE} + \frac{1}{\sqrt{5}}(15.09 - 33.54) - \frac{5}{\sqrt{61}}(21.48) = 0$$

$F_{DE} = 22.0\text{ kips}$ T ◀



Free body: Joint A:

$$\xrightarrow{+}\Sigma F_x = 0:\quad \frac{5}{\sqrt{29}}F_{AC} + \frac{1}{\sqrt{5}}F_{AE} + \frac{5}{\sqrt{29}}(30.36) - \frac{1}{\sqrt{5}}(15.09) = 0 \qquad (3)$$

$$+\uparrow\Sigma F_y = 0:\quad -\frac{2}{\sqrt{29}}F_{AC} - \frac{2}{\sqrt{5}}F_{AE} + \frac{2}{\sqrt{29}}(30.96) - \frac{2}{\sqrt{5}}(15.09) = 0 \qquad (4)$$

Multiply Eq. (3) by 2 and add Eq. (4):

$$\frac{8}{\sqrt{29}}F_{AC} + \frac{12}{\sqrt{29}}(30.96) - \frac{4}{\sqrt{5}}(15.09) = 0$$

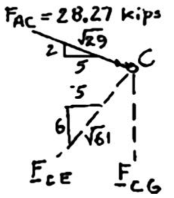
$F_{AC} = -28.27\text{ kips},$ $\qquad$ $F_{AC} = 28.3\text{ kips}$ C ◀

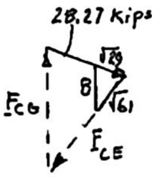
Multiply Eq. (3) by 2, Eq. (4) by 5, and add:

$$-\frac{8}{\sqrt{5}}F_{AE} + \frac{20}{\sqrt{29}}(30.96) - \frac{12}{\sqrt{5}}(15.09) = 0$$

$F_{AE} = 9.50\text{ kips}$ T ◀

PROBLEM 6.27 (Continued)



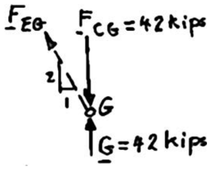


Free body: Joint *C*:

From force triangle:

$$\frac{F_{CE}}{\sqrt{61}} = \frac{F_{CG}}{8} = \frac{28.27 \text{ kips}}{\sqrt{29}} \qquad F_{CE} = 41.0 \text{ kips} \quad T \blacktriangleleft$$

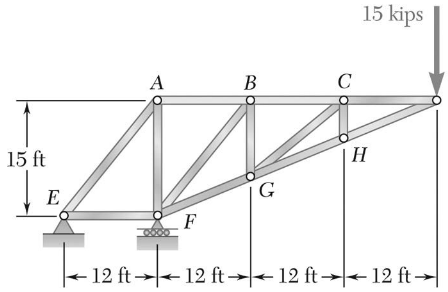
$$F_{CG} = 42.0 \text{ kips} \quad C \blacktriangleleft$$



Free body: Joint *G*:

$$\xrightarrow{+} \Sigma F_x = 0: \qquad F_{EG} = 0 \blacktriangleleft$$

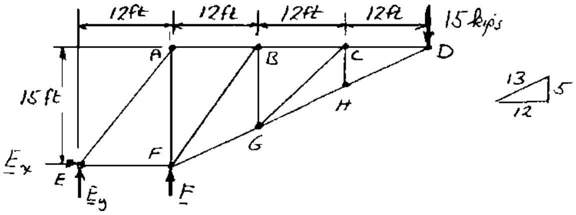
$$+\!\uparrow \Sigma F_y = 0: \quad 42 \text{ kips} - 42 \text{ kips} = 0 \qquad \text{(Checks)}$$



PROBLEM 6.28

Determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

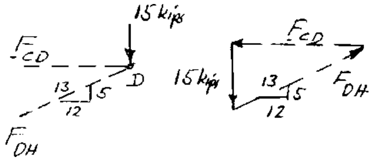


Reactions: $\Sigma F_x = 0$: $\mathbf{E}_x = 0$

$\Sigma M_F = 0$: $\mathbf{E}_y = 45 \text{ kips}\downarrow$

$\Sigma F_y = 0$: $\mathbf{F} = 60 \text{ kips}\uparrow$

Joint D:

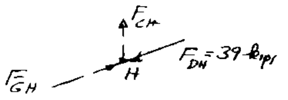


$$\frac{F_{CD}}{12} = \frac{F_{DH}}{13} = \frac{15 \text{ kips}}{5}$$

$F_{CD} = 36.0 \text{ kips}$ T ◀

$F_{DH} = 39.0 \text{ kips}$ C ◀

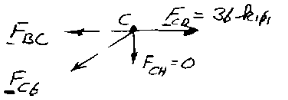
Joint H:



$\nwarrow \Sigma F = 0$: $F_{CH} = 0$ ◀

$\nearrow \Sigma F = 0$: $F_{GH} = 39.0 \text{ kips}$ C ◀

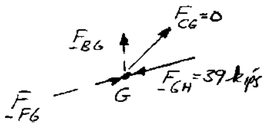
Joint C:



$\uparrow \Sigma F = 0$: $F_{CG} = 0$ ◀

$\rightarrow \Sigma F = 0$: $F_{BC} = 36.0 \text{ kips}$ T ◀

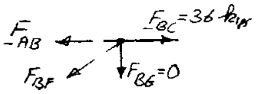
Joint G:



$\nwarrow \Sigma F = 0$: $F_{BG} = 0$ ◀

$\nearrow \Sigma F = 0$: $F_{FG} = 39.0 \text{ kips}$ C ◀

Joint B:

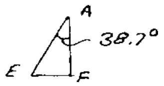


$\uparrow \Sigma F = 0$: $F_{BF} = 0$ ◀

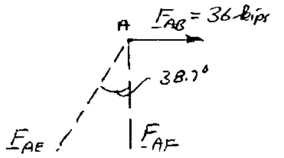
$\rightarrow \Sigma F = 0$: $F_{AB} = 36.0 \text{ kips}$ T ◀

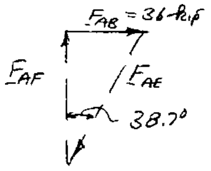
PROBLEM 6.28 (Continued)

Joint *A*:



$$AE = \sqrt{12^2 + 15^2} = 19.21 \text{ ft}$$

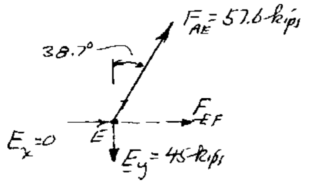




$$\tan 38.7° = \frac{36 \text{ kips}}{F_{AF}} \qquad F_{AF} = 45.0 \text{ kips} \quad C ◀$$

$$\sin 38.7° = \frac{36 \text{ kips}}{F_{AE}} \qquad F_{AE} = 57.6 \text{ kips} \quad T ◀$$

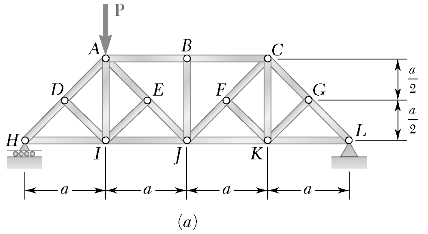
Joint *E*:



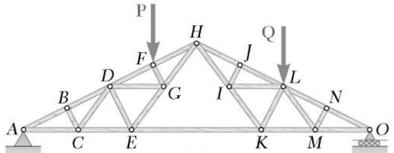
$$\xrightarrow{+} \Sigma F_x = 0: \quad +(57.6 \text{ kips})\sin 38.7° + F_{EF} = 0$$

$$F_{EF} = -36.0 \text{ kips} \qquad F_{EF} = 36.0 \text{ kips} \quad C ◀$$

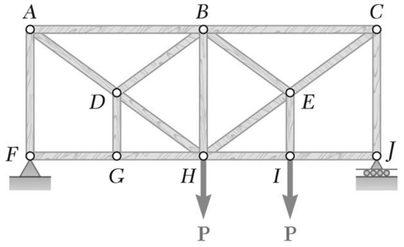
$$+\!\uparrow \Sigma F_y = 0: \quad (57.6 \text{ kips})\cos 38.7° - 45 \text{ kips} = 0 \quad \text{(Checks)}$$



(a)



(a)

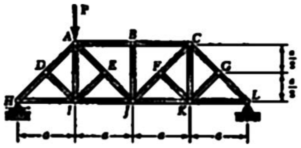


(a)

PROBLEM 6.29

Determine whether the trusses of Problems 6.31*a*, 6.32*a*, and 6.33*a* are simple trusses.

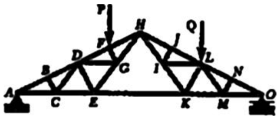
SOLUTION



Truss of Problem 6.31*a*:

Starting with triangle *HDI* and adding two members at a time, we obtain successively joints *A*, *E*, *J*, and *B*, but cannot go further. Thus, this truss

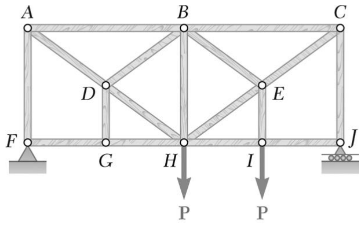
is not a simple truss. ◀



Truss of Problem 6.32*a*:

Starting with triangle *ABC* and adding two members at a time, we obtain joints *D*, *E*, *G*, *F*, and *H*, but cannot go further. Thus, this truss

is not a simple truss. ◀



(a)

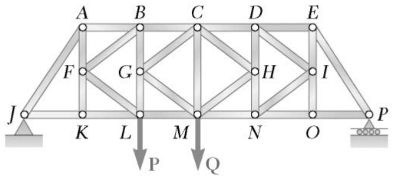
Truss of Problem 6.33*a*:

Starting with triangle *ABD* and adding two members at a time, we obtain successively joints *H*, *G*, *F*, *E*, *I*, *C*, and *J*, thus completing the truss.

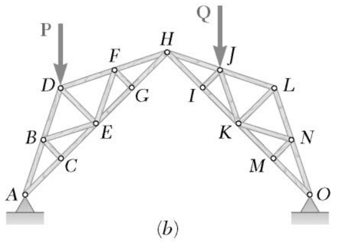
Therefore, this is a simple truss. ◀

PROBLEM 6.30

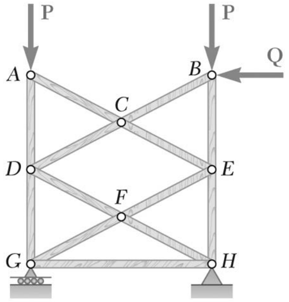
Determine whether the trusses of Problems 6.31*b*, 6.32*b*, and 6.33*b* are simple trusses.



(*b*)

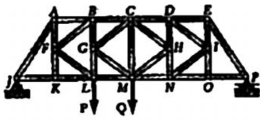


(*b*)



(*b*)

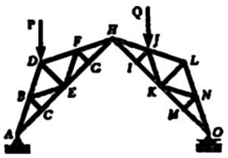
SOLUTION



Truss of Problem 6.31*b*:

Starting with triangle *CGM* and adding two members at a time, we obtain successively joints *B*, *L*, *F*, *A*, *K*, *J*, then *H*, *D*, *N*, *I*, *E*, *O*, and *P*, thus completing the truss.

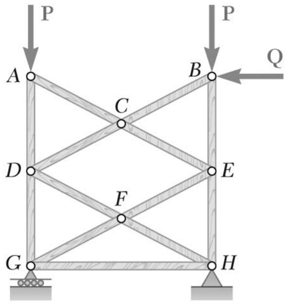
Therefore, this truss is a simple truss. ◀



Truss of Problem 6.32*b*:

Starting with triangle *ABC* and adding two members at a time, we obtain successively joints *E*, *D*, *F*, *G*, and *H*, but cannot go further. Thus, this truss

is not a simple truss. ◀

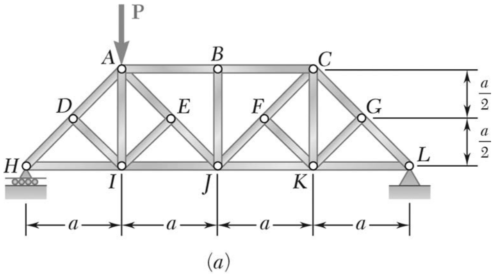


(*b*)

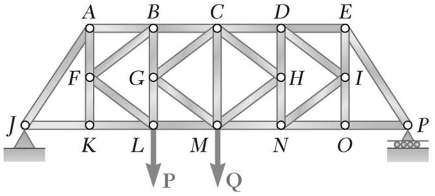
Truss of Problem 6.33*b*:

Starting with triangle *GFH* and adding two members at a time, we obtain successively joints *D*, *E*, *C*, *A*, and *B*, thus completing the truss.

Therefore, this is a simple truss. ◀



(a)

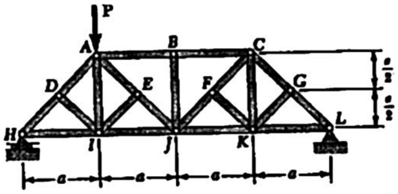


(b)

PROBLEM 6.31

For the given loading, determine the zero-force members in each of the two trusses shown.

SOLUTION



(a)

<u>Truss (*a*)</u>:

FB: Joint *B*: $F_{BJ} = 0$

FB: Joint *D*: $F_{DI} = 0$

FB: Joint *E*: $F_{EI} = 0$

FB: Joint *I*: $F_{AI} = 0$

FB: Joint *F*: $F_{FK} = 0$

FB: Joint *G*: $F_{GK} = 0$

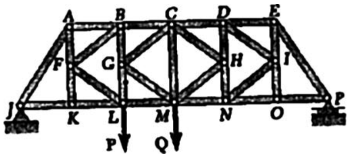
FB: Joint *K*: $F_{CK} = 0$

The zero-force members, therefore, are $AI, BJ, CK, DI, EI, FK, GK$ ◀

<u>Truss (*b*)</u>:

FB: Joint *K*: $F_{FK} = 0$

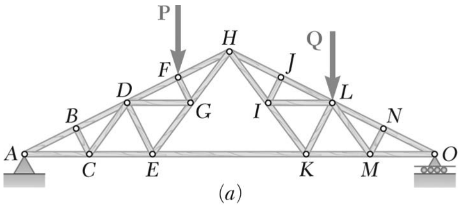
FB: Joint *O*: $F_{IO} = 0$



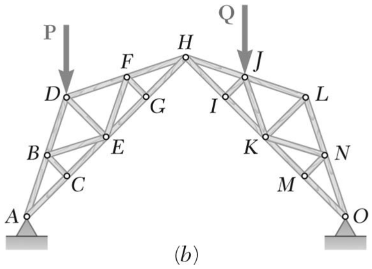
(b)

The zero-force members, therefore, are *FK* and *IO* ◀

All other members are either in tension or compression.



(*a*)



(*b*)

PROBLEM 6.32

For the given loading, determine the zero-force members in each of the two trusses shown.

SOLUTION

Truss (*a*):

FB: Joint B: $F_{BC} = 0$

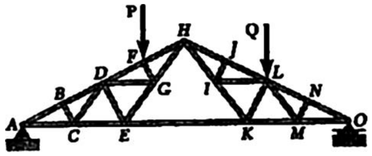
FB: Joint C: $F_{CD} = 0$

FB: Joint J: $F_{IJ} = 0$

FB: Joint I: $F_{IL} = 0$

FB: Joint N: $F_{MN} = 0$

FB: Joint M: $F_{LM} = 0$



(*a*)

The zero-force members, therefore, are BC, CD, IJ, IL, LM, MN ◀

Truss (*b*):

FB: Joint C: $F_{BC} = 0$

FB: Joint B: $F_{BE} = 0$

FB: Joint G: $F_{FG} = 0$

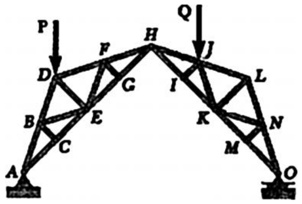
FB: Joint F: $F_{EF} = 0$

FB: Joint E: $F_{DE} = 0$

FB: Joint I: $F_{IJ} = 0$

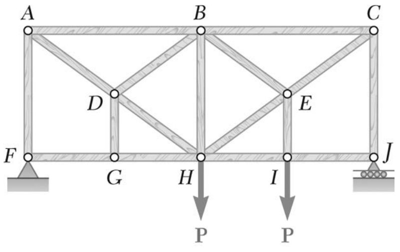
FB: Joint M: $F_{MN} = 0$

FB: Joint N: $F_{KN} = 0$

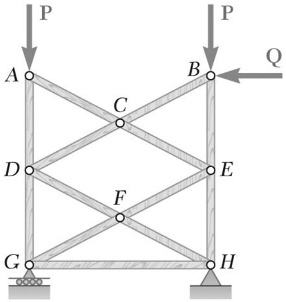


(*b*)

The zero-force members, therefore, are $BC, BE, DE, EF, FG, IJ, KN, MN$ ◀



(a)



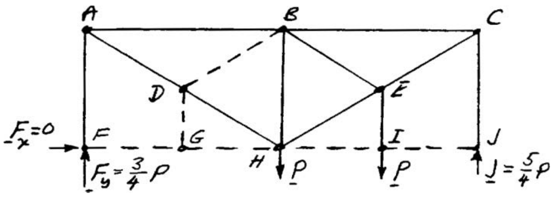
(b)

PROBLEM 6.33

For the given loading, determine the zero-force members in each of the two trusses shown.

SOLUTION

Truss (*a*):



Note: Reason at F is vertical $(F_x = 0)$.

Joint G: $\downarrow \Sigma F = 0,$ $F_{DG} = 0$ ◀

Joint D: $\nearrow \Sigma F = 0,$ $F_{DB} = 0$ ◀

Joint F: $\xrightarrow{} \Sigma F = 0,$ $F_{FG} = 0$ ◀

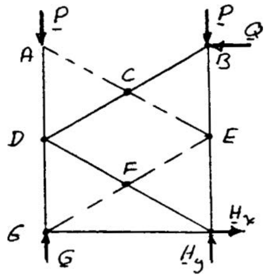
Joint G: $\xrightarrow{} \Sigma F = 0,$ $F_{GH} = 0$ ◀

Joint J: $\xrightarrow{} \Sigma F = 0,$ $F_{IJ} = 0$ ◀

Joint I: $\xrightarrow{} \Sigma F = 0,$ $F_{HI} = 0$ ◀

PROBLEM 6.33 (Continued)

Truss (*b*):



Joint A: $\xrightarrow{} \Sigma F = 0,$ $F_{AC} = 0$ ◀

Joint C: $\searrow \Sigma F = 0,$ $F_{CE} = 0$ ◀

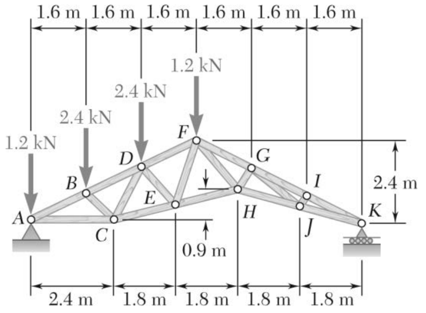
Joint E: $\xrightarrow{} \Sigma F = 0,$ $F_{EF} = 0$ ◀

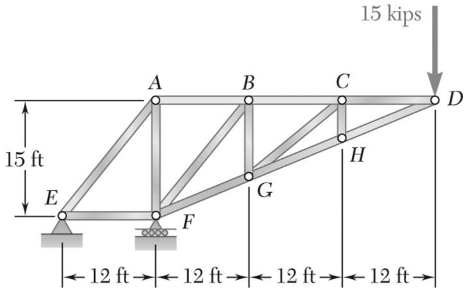
Joint F: $\swarrow \Sigma F = 0,$ $F_{FG} = 0$ ◀

Joint G: $\xrightarrow{} \Sigma F = 0,$ $F_{GH} = 0$ ◀

PROBLEM 6.34

Determine the zero-force members in the truss of (*a*) Problem 6.26, (*b*) Problem 6.28.





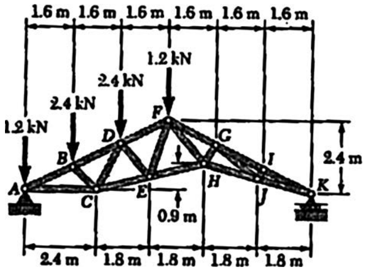
SOLUTION

(*a*) Truss of Problem 6.26:

FB: Joint I: $F_{IJ} = 0$

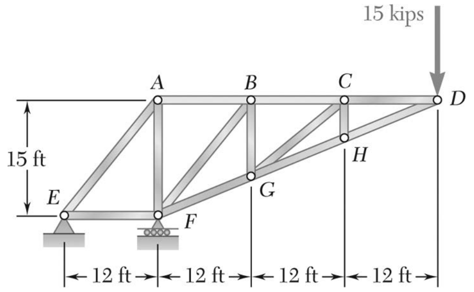
FB: Joint J: $F_{GJ} = 0$

FB: Joint G: $F_{GH} = 0$



The zero-force members, therefore, are GH, GJ, IJ ◀

(*b*) Truss of Problem 6.28:



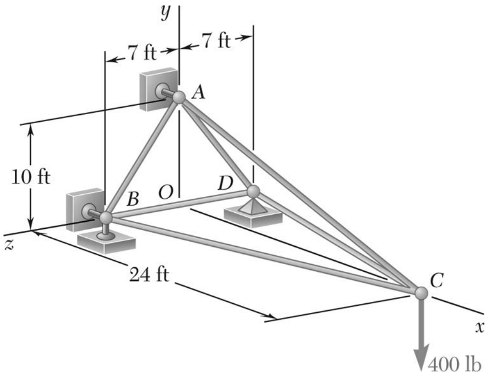
FB: Joint B: $F_{BF} = 0$

FB: Joint B: $F_{BG} = 0$

FB: Joint C: $F_{CG} = 0$

FB: Joint C: $F_{CH} = 0$

The zero-force members, therefore, are BF, BG, CG, CH ◀

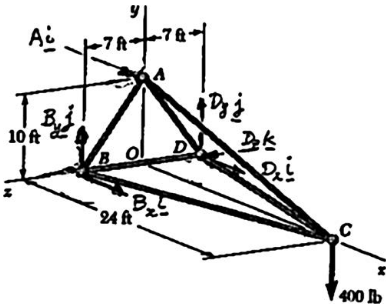


PROBLEM 6.35*

The truss shown consists of six members and is supported by a short link at *A*, two short links at *B*, and a ball and socket at *D*. Determine the force in each of the members for the given loading.

SOLUTION

<u>Free body: Truss</u>:



From symmetry:

$$D_x = B_x \quad \text{and} \quad D_y = B_y$$

$$\Sigma M_z = 0: \quad -A(10\text{ ft}) - (400\text{ lb})(24\text{ ft}) = 0$$

$$A = -960\text{ lb}$$

$$\Sigma F_x = 0: \quad B_x + D_x + A = 0$$

$$2B_x - 960\text{ lb} = 0, \quad B_x = 480\text{ lb}$$

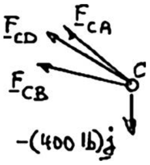
$$\Sigma F_y = 0: \quad B_y + D_y - 400\text{ lb} = 0$$

$$2B_y = 400\text{ lb}$$

$$B_y = +200\text{ lb}$$

Thus, $\mathbf{B} = (480\text{ lb})\mathbf{i} + (200\text{ lb})\mathbf{j} \ \triangleleft$

<u>Free body: *C*</u>:



$$F_{CA} = F_{AC}\frac{\overrightarrow{CA}}{CA} = \frac{F_{AC}}{26}(-24\mathbf{i} + 10\mathbf{j})$$

$$F_{CB} = F_{BC}\frac{\overrightarrow{CB}}{CB} = \frac{F_{BC}}{25}(-24\mathbf{i} + 7\mathbf{k})$$

$$F_{CD} = F_{CD}\frac{\overrightarrow{CD}}{CD} = \frac{F_{CD}}{25}(-24\mathbf{i} - 7\mathbf{k})$$

$$\mathbf{\Sigma F} = 0: \quad \mathbf{F}_{CA} + \mathbf{F}_{CB} + \mathbf{F}_{CD} - (400\text{ lb})\mathbf{j} = 0$$

PROBLEM 6.35* (Continued)

Substituting for $\mathbf{F}_{CA}, \mathbf{F}_{CB}, \mathbf{F}_{CD}$, and equating to zero the coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\mathbf{i}: \quad -\frac{24}{26}F_{AC} - \frac{24}{25}(F_{BC} + F_{CD}) = 0 \qquad (1)$$

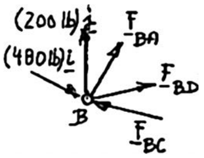
$$\mathbf{j}: \quad \frac{10}{26}F_{AC} - 400 \text{ lb} = 0 \qquad F_{AC} = 1040 \text{ lb} \quad T \blacktriangleleft$$

$$\mathbf{k}: \quad \frac{7}{25}(F_{BC} - F_{CD}) = 0 \quad F_{CD} = F_{BC}$$

Substitute for F_{AC} and F_{CD} in Eq. (1):

$$-\frac{24}{26}(10.40 \text{ lb}) - \frac{24}{25}(2F_{BC}) = 0 \quad F_{BC} = -500 \text{ lb} \qquad F_{BC} = F_{CD} = 500 \text{ lb} \quad C \blacktriangleleft$$

Free body: *B*:



$$F_{BC} = (500 \text{ lb})\frac{\overrightarrow{CB}}{CB} = -(480 \text{ lb})\mathbf{i} + (140 \text{ lb})\mathbf{k}$$

$$F_{BA} = F_{AB}\frac{\overrightarrow{BA}}{BA} = \frac{F_{AB}}{12.21}(10\mathbf{j} - 7\mathbf{k})$$

$$F_{BD} = -F_{BD}\mathbf{k}$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{BD} + \mathbf{F}_{BC} + (480 \text{ lb})\mathbf{i} + (200 \text{ lb})\mathbf{j} = 0$$

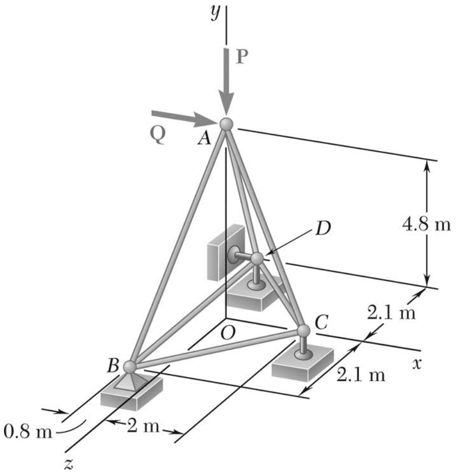
Substituting for $\mathbf{F}_{BA}, \mathbf{F}_{BD}, \mathbf{F}_{BC}$ and equating to zero the coefficients of $\mathbf{j}$ and $\mathbf{k}$:

$$\mathbf{j}: \quad \frac{10}{12.21}F_{AB} + 200 \text{ lb} = 0 \quad F_{AB} = -244.2 \text{ lb} \qquad F_{AB} = 244 \text{ lb} \quad C \blacktriangleleft$$

$$\mathbf{k}: \quad -\frac{7}{12.21}F_{AB} - F_{BD} + 140 \text{ lb} = 0$$

$$F_{BD} = -\frac{7}{12.21}(-244.2 \text{ lb}) + 140 \text{ lb} = +280 \text{ lb} \qquad F_{BD} = 280 \text{ lb} \quad T \blacktriangleleft$$

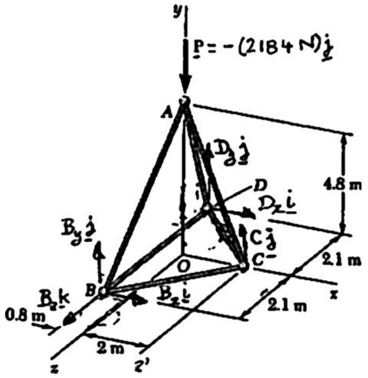
From symmetry: $F_{AD} = F_{AB}$ $\qquad F_{AD} = 244 \text{ lb} \quad C \blacktriangleleft$



PROBLEM 6.36*

The truss shown consists of six members and is supported by a ball and socket at *B*, a short link at *C*, and two short links at *D*. Determine the force in each of the members for **P** = (−2184 N)**j** and **Q** = 0.

SOLUTION



Free body: Truss:

From symmetry:

$$D_x = B_x \text{ and } D_y = B_y$$

$$\Sigma F_x = 0: \quad 2B_x = 0$$

$$B_x = D_x = 0$$

$$\Sigma F_z = 0: \quad B_z = 0$$

$$\Sigma M_{cz} = 0: \quad -2B_y(2.8\text{ m}) + (2184\text{ N})(2\text{ m}) = 0$$

$$B_y = 780\text{ N}$$

Thus, $\mathbf{B} = (780\text{ N})\mathbf{j}$ ◁

Free body: *A*:

$$F_{AB} = F_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{F_{AB}}{5.30}(-0.8\mathbf{i} - 4.8\mathbf{j} + 2.1\mathbf{k})$$

$$F_{AC} = F_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{F_{AC}}{5.20}(2\mathbf{i} - 4.8\mathbf{j})$$

$$F_{AD} = F_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{F_{AD}}{5.30}(+0.8\mathbf{i} - 4.8\mathbf{j} - 2.1\mathbf{k})$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} - (2184\text{ N})\mathbf{j} = 0$$

PROBLEM 6.36* (Continued)

Substituting for $\mathbf{F}_{AB}, \mathbf{F}_{AC}, \mathbf{F}_{AD},$ and equating to zero the coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\mathbf{i}: \quad -\frac{0.8}{5.30}(F_{AB}+F_{AD})+\frac{2}{5.20}F_{AC}=0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{4.8}{5.30}(F_{AB}+F_{AD})-\frac{4.8}{5.20}F_{AC}-2184\text{ N}=0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{2.1}{5.30}(F_{AB}-F_{AD})=0 \qquad F_{AD}=F_{AB}$$

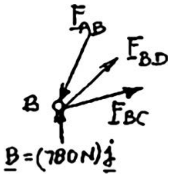
Multiply Eq. (1) by –6 and add Eq. (2):

$$-\left(\frac{16.8}{5.20}\right)F_{AC}-2184\text{ N}=0, \quad F_{AC}=-676\text{ N} \qquad F_{AC}=676\text{ N} \quad C ◀$$

Substitute for F_{AC} and F_{AD} in Eq. (1):

$$-\left(\frac{0.8}{5.30}\right)2F_{AB}+\left(\frac{2}{5.20}\right)(-676\text{ N})=0, \quad F_{AB}=-861.25\text{ N} \qquad F_{AB}=F_{AD}=861\text{ N} \quad C ◀$$

Free body: *B*:



$$\mathbf{F}_{AB}=(861.25\text{ N})\frac{\overrightarrow{AB}}{AB}=-(130\text{ N})\mathbf{i}-(780\text{ N})\mathbf{j}+(341.25\text{ N})\mathbf{k}$$

$$\mathbf{F}_{BC}=F_{BC}\left(\frac{2.8\mathbf{i}-2.1\mathbf{k}}{3.5}\right)=F_{BC}(0.8\mathbf{i}-0.6\mathbf{k})$$

$$\mathbf{F}_{BD}=-F_{BD}\mathbf{k}$$

$$\Sigma\mathbf{F}=0: \quad \mathbf{F}_{AB}+\mathbf{F}_{BC}+\mathbf{F}_{BD}+(780\text{ N})\mathbf{j}=0$$

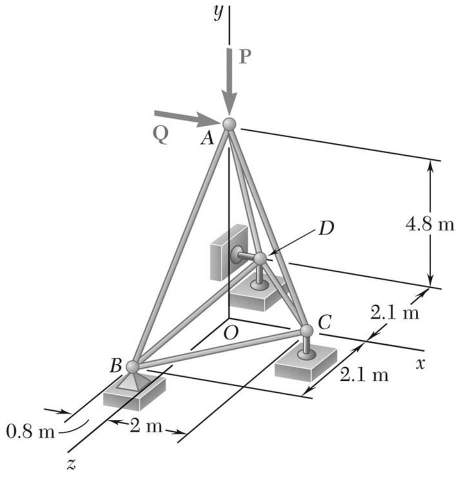
Substituting for $\mathbf{F}_{AB}, \mathbf{F}_{BC}, \mathbf{F}_{BD}$ and equating to zero the coefficients of $\mathbf{i}$ and $\mathbf{k}$,

$$\mathbf{i}: \quad -130\text{ N}+0.8F_{BC}=0 \quad F_{BC}=+162.5\text{ N} \qquad F_{BC}=162.5\text{ N} \quad T ◀$$

$$\mathbf{k}: \quad 341.25\text{ N}-0.6F_{BC}-F_{BD}=0$$

$$F_{BD}=341.25-0.6(162.5)=+243.75\text{ N} \qquad F_{BD}=244\text{ N} \quad T ◀$$

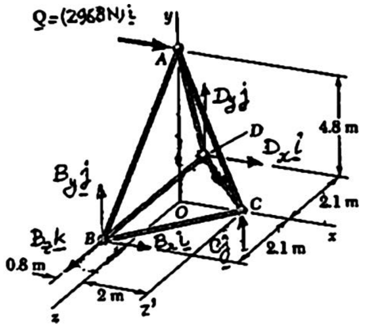
From symmetry: $F_{CD}=F_{BC}$ $\qquad F_{CD}=162.5\text{ N} \quad T$ ◀



PROBLEM 6.37*

The truss shown consists of six members and is supported by a ball and socket at B, a short link at C, and two short links at D. Determine the force in each of the members for $\mathbf{P} = 0$ and $\mathbf{Q} = (2968\text{ N})\mathbf{i}$.

SOLUTION



Free body: Truss:

From symmetry:

$$D_x = B_x \text{ and } D_y = B_y$$

$$\Sigma F_x = 0:\quad 2B_x + 2968\text{ N} = 0$$

$$B_x = D_x = -1484\text{ N}$$

$$\Sigma M_{cz'} = 0:\quad -2B_y(2.8\text{ m}) - (2968\text{ N})(4.8\text{ m}) = 0$$

$$B_y = -2544\text{ N}$$

Thus, $\mathbf{B} = -(1484\text{ N})\mathbf{i} - (2544\text{ N})\mathbf{j}$ ◁

Free body: A:

$$F_{AB} = F_{AB}\frac{\overrightarrow{AB}}{AB}$$

$$= \frac{F_{AB}}{5.30}(-0.8\mathbf{i} - 4.8\mathbf{j} + 2.1\mathbf{k})$$

$$F_{AC} = F_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{F_{AC}}{5.20}(2\mathbf{i} - 4.8\mathbf{j})$$

$$F_{AD} = F_{AD}\frac{\overrightarrow{AD}}{AD}$$

$$= \frac{F_{AD}}{5.30}(-0.8\mathbf{i} - 4.8\mathbf{j} - 2.1\mathbf{k})$$

$$\Sigma\mathbf{F} = 0:\quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + (2968\text{ N})\mathbf{i} = 0$$

PROBLEM 6.37* (Continued)

Substituting for $\mathbf{F}_{AB}, \mathbf{F}_{AC}, \mathbf{F}_{AD}$, and equating to zero the coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$,

$$\mathbf{i}: \quad -\frac{0.8}{5.30}(F_{AB}+F_{AD})+\frac{2}{5.20}F_{AC}+2968\text{ N}=0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{4.8}{5.30}(F_{AB}+F_{AD})-\frac{4.8}{5.20}F_{AC}=0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{2.1}{5.30}(F_{AB}-F_{AD})=0 \qquad F_{AD}=F_{AB}$$

Multiply Eq. (1) by –6 and add Eq. (2):

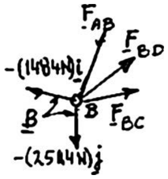
$$-\left(\frac{16.8}{5.20}\right)F_{AC}-6(2968\text{ N})=0, \quad F_{AC}=-5512\text{ N} \qquad F_{AC}=5510\text{ N} \quad C \blacktriangleleft$$

Substitute for F_{AC} and F_{AD} in Eq. (2):

$$-\left(\frac{4.8}{5.30}\right)2F_{AB}-\left(\frac{4.8}{5.20}\right)(-5512\text{ N})=0, \quad F_{AB}=+2809\text{ N}$$

$$F_{AB}=F_{AD}=2810\text{ N} \quad T \blacktriangleleft$$

<u>Free body: *B*</u>:



$$\mathbf{F}_{AB}=(2809\text{ N})\frac{\overrightarrow{BA}}{BA}=(424\text{ N})\mathbf{i}+(2544\text{ N})\mathbf{j}-(1113\text{ N})\mathbf{k}$$

$$\mathbf{F}_{BC}=F_{BC}\left(\frac{2.8\mathbf{i}-2.1\mathbf{k}}{3.5}\right)=F_{BC}(0.8\mathbf{i}-0.6\mathbf{k})$$

$$\mathbf{F}_{BD}=-F_{BD}\mathbf{k}$$

$$\Sigma\mathbf{F}=0: \quad \mathbf{F}_{AB}+\mathbf{F}_{BC}+\mathbf{F}_{BD}-(1484\text{ N})\mathbf{i}-(2544\text{ N})\mathbf{j}=0$$

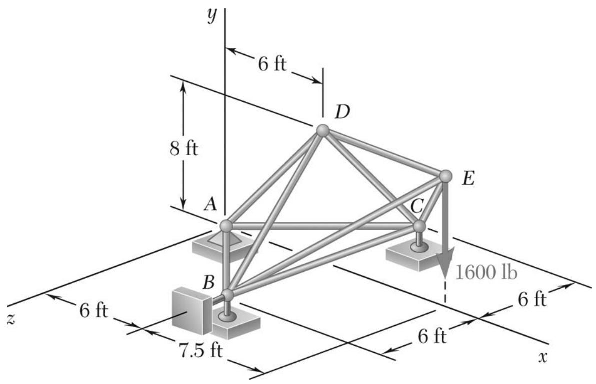
Substituting for $\mathbf{F}_{AB}, \mathbf{F}_{BC}, \mathbf{F}_{BD}$ and equating to zero the coefficients of $\mathbf{i}$ and $\mathbf{k}$,

$$\mathbf{i}: \quad +24\text{ N}+0.8F_{BC}-1484\text{ N}=0, \quad F_{BC}=+1325\text{ N} \qquad F_{BC}=1325\text{ N} \quad T \blacktriangleleft$$

$$\mathbf{k}: \quad -1113\text{ N}-0.6F_{BC}-F_{BD}=0$$

$$F_{BD}=-1113\text{ N}-0.6(1325\text{ N})=-1908\text{ N}, \qquad F_{BD}=1908\text{ N} \quad C \blacktriangleleft$$

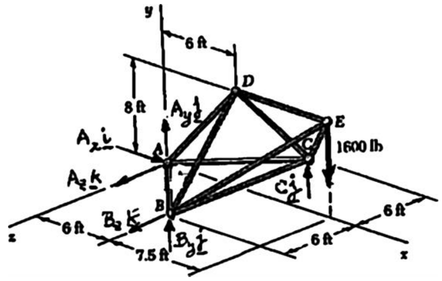
From symmetry: $\quad F_{CD}=F_{BC} \qquad F_{CD}=1325\text{ N} \quad T \blacktriangleleft$



PROBLEM 6.38*

The truss shown consists of nine members and is supported by a ball and socket at *A*, two short links at *B*, and a short link at *C*. Determine the force in each of the members for the given loading.

SOLUTION



Free body: Truss:

From symmetry:

$$A_z = B_z = 0$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma M_{BC} = 0: \quad A_y(6\text{ ft}) + (1600\text{ lb})(7.5\text{ ft}) = 0$$

$$A_y = -2000\text{ lb} \qquad \mathbf{A} = -(2000\text{ lb})\mathbf{j} \ \triangleleft$$

From symmetry: $B_y = C$

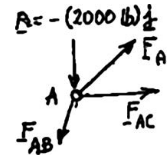
$$\Sigma F_y = 0: \quad 2B_y - 2000\text{ lb} - 1600\text{ lb} = 0$$

$$B_y = 1800\text{ lb} \qquad \mathbf{B} = (1800\text{ lb})\mathbf{j} \ \triangleleft$$

Free body: *A*:

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} - (2000\text{ lb})\mathbf{j} = 0$$

$$F_{AB}\frac{\mathbf{i}+\mathbf{k}}{\sqrt{2}} + F_{AC}\frac{\mathbf{i}-\mathbf{k}}{\sqrt{2}} + F_{AD}(0.6\mathbf{i} + 0.8\mathbf{j}) - (2000\text{ lb})\mathbf{j} = 0$$



Factoring **i**, **j**, **k** and equating their coefficient to zero,

$$\frac{1}{\sqrt{2}}F_{AB} + \frac{1}{\sqrt{2}}F_{AC} + 0.6F_{AD} = 0 \qquad (1)$$

$$0.8F_{AD} - 2000\text{ lb} = 0 \qquad F_{AD} = 2500\text{ lb} \quad T \ \blacktriangleleft$$

$$\frac{1}{\sqrt{2}}F_{AB} - \frac{1}{\sqrt{2}}F_{AC} = 0 \qquad F_{AC} = F_{AB}$$

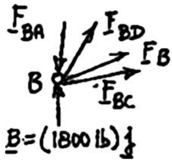
PROBLEM 6.38* (Continued)

Substitute for F_{AD} and F_{AC} into Eq. (1):

$$\frac{2}{\sqrt{2}}F_{AB} + 0.6(2500\text{ lb}) = 0, \quad \mathbf{F}_{AB} = -1060.7\text{ lb}, \qquad F_{AB} = F_{AC} = 1061\text{ lb} \quad C \blacktriangleleft$$

Free body: *B*:

$$\mathbf{F}_{BA} = F_{AB}\frac{\overrightarrow{BA}}{BA} = +(1060.7\text{ lb})\frac{\mathbf{i}+\mathbf{k}}{\sqrt{2}} = (750\text{ lb})(\mathbf{i}+\mathbf{k})$$



$$\mathbf{F}_{BC} = -F_{BC}\mathbf{k}$$

$$\mathbf{F}_{BD} = F_{BD}(0.8\mathbf{j} - 0.6\mathbf{k})$$

$$\mathbf{F}_{BE} = F_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{F_{BE}}{12.5}(7.5\mathbf{i} + 8\mathbf{j} - 6\mathbf{k})$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{BC} + \mathbf{F}_{BD} + \mathbf{F}_{BE} + (1800\text{ lb})\mathbf{j} = 0$$

Substituting for $\mathbf{F}_{BA}, \mathbf{F}_{BC}, \mathbf{F}_{BD}$, and $\mathbf{F}_{BE}$ and equating to zero the coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$,

$$\mathbf{i}: \quad 750\text{ lb} + \left(\frac{7.5}{12.5}\right)F_{BE} = 0, \quad F_{BE} = -1250\text{ lb} \qquad F_{BE} = 1250\text{ lb} \quad C \blacktriangleleft$$

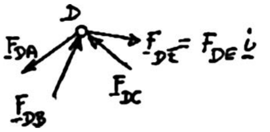
$$\mathbf{j}: \quad 0.8\,F_{BD} + \left(\frac{8}{12.5}\right)(-1250\text{ lb}) + 1800\text{ lb} = 0 \qquad F_{BD} = 1250\text{ lb} \quad C \blacktriangleleft$$

$$\mathbf{k}: \quad 750\text{ lb} - F_{BC} - 0.6(-1250\text{ lb}) - \frac{6}{12.5}(-1250\text{ lb}) = 0$$

$$F_{BC} = 2100\text{ lb} \quad T \blacktriangleleft$$

From symmetry: $F_{BD} = F_{CD} = 1250\text{ lb} \quad C \blacktriangleleft$

Free body: *D*:

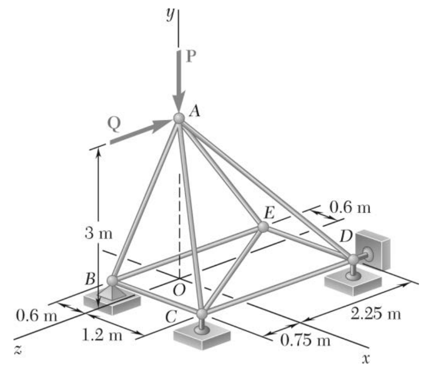


$$\Sigma\mathbf{F} = 0: \ \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC} + \mathbf{F}_{DE}\mathbf{i} = 0$$

We now substitute for $\mathbf{F}_{DA}, \mathbf{F}_{DB}, \mathbf{F}_{DC}$ and equate to zero the coefficient of **i**. Only $\mathbf{F}_{DA}$ contains **i** and its coefficient is

$$-0.6\,F_{AD} = -0.6(2500\text{ lb}) = -1500\text{ lb}$$

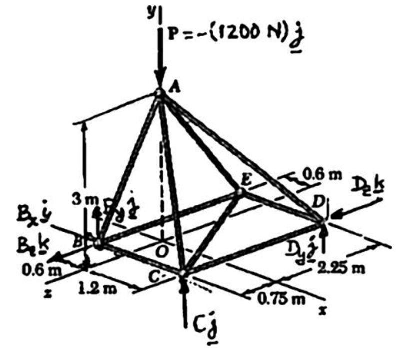
$$\mathbf{i}: \quad -1500\text{ lb} + F_{DE} = 0 \qquad F_{DE} = 1500\text{ lb} \quad T \blacktriangleleft$$



PROBLEM 6.39*

The truss shown consists of nine members and is supported by a ball and socket at *B*, a short link at *C*, and two short links at *D*. (*a*) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (*b*) Determine the force in each member for **P** = (−1200 N)**j** and **Q** = 0.

SOLUTION



Free body: Truss:

$$\Sigma \mathbf{M}_B = 0: \quad 1.8\mathbf{i}\times C\mathbf{j} + (1.8\mathbf{i} - 3\mathbf{k})\times(D_y\mathbf{j} + D\mathbf{k})$$

$$+(0.6\mathbf{i} - 0.75\mathbf{k})\times(-1200\mathbf{j}) = 0$$

$$-1.8\,C\mathbf{k} + 1.8\,D_y\mathbf{k} - 1.8\,D_z\mathbf{j}$$

$$+3D_y\mathbf{i} - 720\mathbf{k} - 900\mathbf{i} = 0$$

Equate to zero the coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

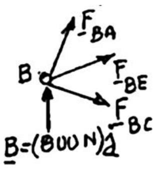
$$\mathbf{i}: \quad 3D_y - 900 = 0, \quad D_y = 300\text{ N}$$

$$\mathbf{j}: \quad D_z = 0, \qquad \mathbf{D} = (300\text{ N})\mathbf{j} \ \triangleleft$$

$$\mathbf{k}: \quad 1.8C + 1.8(300) - 720 = 0 \qquad \mathbf{C} = (100\text{ N})\mathbf{j} \ \triangleleft$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{B} + 300\mathbf{j} + 100\mathbf{j} - 1200\mathbf{j} = 0 \qquad \mathbf{B} = (800\text{ N})\mathbf{j} \ \triangleleft$$

Free body: *B*:



$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{BC} + \mathbf{F}_{BE} + (800\text{ N})\mathbf{j} = 0, \text{ with}$$

$$\mathbf{F}_{BA} = \mathbf{F}_{AB}\frac{\overrightarrow{BA}}{BA} = \frac{F_{AB}}{3.15}(0.6\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k})$$

$$\mathbf{F}_{BC} = F_{BC}\mathbf{i} \qquad \mathbf{F}_{BE} = -F_{BE}\mathbf{k}$$

Substitute and equate to zero the coefficient of $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

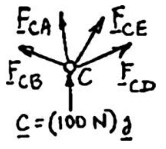
$$\mathbf{j}: \quad \left(\frac{3}{3.315}\right)F_{AB} + 800\text{ N} = 0, \quad F_{AB} = -840\text{ N}, \qquad F_{AB} = 840\text{ N} \quad C \ \blacktriangleleft$$

$$\mathbf{i}: \quad \left(\frac{0.6}{3.15}\right)(-840\text{ N}) + F_{BC} = 0 \qquad F_{BC} = 160.0\text{ N} \quad T \ \blacktriangleleft$$

$$\mathbf{k}: \quad \left(-\frac{0.75}{3.15}\right)(-840\text{ N}) - F_{BE} = 0 \qquad F_{BE} = 200\text{ N} \quad T \ \blacktriangleleft$$

PROBLEM 6.39* (Continued)

Free body: *C*:



$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{CA} + \mathbf{F}_{CB} + \mathbf{F}_{CD} + \mathbf{F}_{CE} + (100\text{ N})\mathbf{j} = 0, \text{ with}$$

$$\mathbf{F}_{CA} = F_{AC}\frac{\overrightarrow{CA}}{CA} = \frac{F_{AC}}{3.317}(-1.2\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k})$$

$$F_{CB} = -(160\text{ N})\,\mathbf{i}$$

$$\mathbf{F}_{CD} = -F_{CD}\mathbf{k} \quad \mathbf{F}_{CE} = \mathbf{F}_{CE}\frac{\overrightarrow{CE}}{CE} = \frac{F_{CE}}{3{,}499}(-1.8\mathbf{i} - 3\mathbf{k})$$

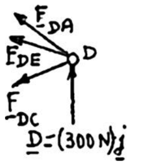
Substitute and equate to zero the coefficient of $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

$\mathbf{j}$: $\left(\frac{3}{3.317}\right)F_{AC} + 100\text{ N} = 0, \quad F_{AC} = -110.57\text{ N}$ $\qquad F_{AC} = 110.6\text{ N} \quad C$ ◀

$\mathbf{i}$: $-\frac{1.2}{3.317}(-110.57) - 160 - \frac{1.8}{3.499}F_{CE} = 0, \quad F_{CE} = -233.3$ $\qquad F_{CE} = 233\text{ N} \quad C$ ◀

$\mathbf{k}$: $-\frac{0.75}{3.317}(-110.57) - F_{CD} - \frac{3}{3.499}(-233.3) = 0$ $\qquad F_{CD} = 225\text{ N} \quad T$ ◀

Free body: *D*:



$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{DA} + \mathbf{F}_{DC} + \mathbf{F}_{DE} + (300\text{ N})\mathbf{j} = 0, \quad \text{with}$$

$$\mathbf{F}_{DA} = F_{AD}\frac{\overrightarrow{DA}}{DA} = \frac{F_{AD}}{3.937}(-1.2\mathbf{i} + 3\mathbf{j} + 2.25\mathbf{k})$$

$$\mathbf{F}_{DC} = F_{CD}\mathbf{k} = (225\text{ N})\mathbf{k} \qquad F_{DE} = -F_{DE}\mathbf{i}$$

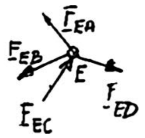
Substitute and equate to zero the coefficient of $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

$\mathbf{j}$: $\left(\frac{3}{3.937}\right)F_{AD} + 300\text{ N} = 0, \qquad F_{AD} = -393.7\text{ N},$ $\qquad F_{AD} = 394\text{ N} \quad C$ ◀

$\mathbf{i}$: $\left(-\frac{1.2}{3.937}\right)(-393.7\text{ N}) - F_{DE} = 0$ $\qquad F_{DE} = 120.0\text{ N} \quad T$ ◀

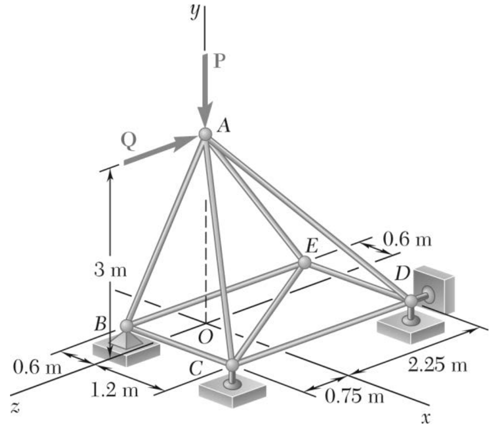
$\mathbf{k}$: $\left(\frac{2.25}{3.937}\right)(-393.7\text{ N}) + 225\text{ N} = 0$ (Checks)

Free body: *E*:



Member *AE* is the only member at *E* which does not lie in the *xz* plane.
Therefore, it is a zero-force member.

$F_{AE} = 0$ ◀



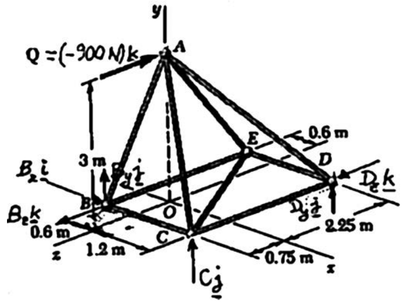
PROBLEM 6.40*

Solve Problem 6.39 for $\mathbf{P} = 0$ and $\mathbf{Q} = (-900\text{ N})\mathbf{k}$.

PROBLEM 6.39* The truss shown consists of nine members and is supported by a ball and socket at *B*, a short link at *C*, and two short links at *D*. (*a*) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (*b*) Determine the force in each member for $\mathbf{P} = (-1200\text{ N})\mathbf{j}$ and $\mathbf{Q} = 0$.

SOLUTION

Free body: Truss:



$$\Sigma \mathbf{M}_B = 0: \quad 1.8\mathbf{i} \times C\mathbf{j} + (1.8\mathbf{i} - 3\mathbf{k}) \times (D_y\mathbf{j} + D_z\mathbf{k})$$

$$+ (0.6\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k}) \times (-900\text{N})\mathbf{k} = 0$$

$$1.8C\mathbf{k} + 1.8D_y\mathbf{k} - 1.8D_z\mathbf{j}$$

$$+ 3D_y\mathbf{i} + 540\mathbf{j} - 2700\mathbf{i} = 0$$

Equate to zero the coefficient of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$3D_y - 2700 = 0 \qquad D_y = 900\text{ N}$$

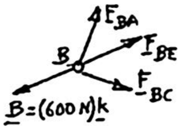
$$-1.8D_z + 540 = 0 \qquad D_z = 300\text{ N}$$

$$1.8C + 1.8D_y = 0 \qquad C = -D_y = -900\text{ N}$$

Thus, $\mathbf{C} = -(900\text{ N})\mathbf{j} \quad \mathbf{D} = (900\text{ N})\mathbf{j} + (300\text{ N})\mathbf{k}$ ◁

$\Sigma \mathbf{F} = 0: \quad \mathbf{B} - 900\mathbf{j} + 900\mathbf{j} + 300\mathbf{k} - 900\mathbf{k} = 0$ $\qquad \mathbf{B} = (600\text{ N})\mathbf{k}$ ◁

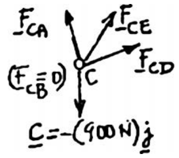
Free body: *B*:



Since **B** is aligned with member *BE*,

$F_{AB} = F_{BC} = 0,$ $\qquad F_{BE} = 600\text{ N} \quad T$ ◀

Free body: *C*:



$\Sigma \mathbf{F} = 0: \ \mathbf{F}_{CA} + \mathbf{F}_{CD} + \mathbf{F}_{CE} - (900\text{ N})\mathbf{j} = 0,$ with

PROBLEM 6.40* (Continued)

$$\mathbf{F}_{CA} = F_{AC}\frac{\overrightarrow{CA}}{CA} = \frac{F_{AC}}{3.317}(-1.2\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k})$$

$$\mathbf{F}_{CD} = -F_{CD}\mathbf{k} \quad \mathbf{F}_{CE} = F_{CE}\frac{\overrightarrow{CE}}{CE} = \frac{F_{CE}}{3.499}(-1.8\mathbf{i} - 3\mathbf{k})$$

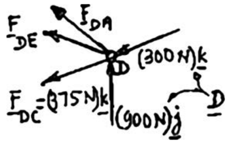
Substitute and equate to zero the coefficient of $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

$$\mathbf{j}: \quad \left(\frac{3}{3.317}\right)F_{AC} - 900\text{ N} = 0, \qquad F_{AC} = 995.1\text{ N} \qquad F_{AC} = 995\text{ N} \quad T \blacktriangleleft$$

$$\mathbf{i}: \quad -\frac{1.2}{3.317}(995.1) - \frac{1.8}{3.499}F_{CE} = 0, \quad F_{CE} = -699.8\text{ N} \qquad F_{CE} = 700\text{ N} \quad C \blacktriangleleft$$

$$\mathbf{k}: \quad -\frac{0.75}{3.317}(995.1) - F_{CD} - \frac{3}{3.499}(-699.8) = 0 \qquad F_{CD} = 375\text{ N} \quad T \blacktriangleleft$$

Free body: D:



$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{DA} + \mathbf{F}_{DE} + (375\text{ N})\mathbf{k} + (900\text{ N})\mathbf{j} + (300\text{ N})\mathbf{k} = 0$$

with $$\mathbf{F}_{DA} = \mathbf{F}_{AD}\frac{\overrightarrow{DA}}{DA} = \frac{F_{AD}}{3.937}(-1.2\mathbf{i} + 3\mathbf{j} + 2.25\mathbf{k})$$

and $$F_{DE} = -F_{DE}\,\mathbf{i}$$

Substitute and equate to zero the coefficient $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

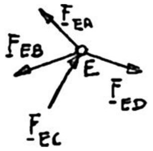
$$\mathbf{j}: \quad \left(\frac{3}{3.937}\right)F_{AD} + 900\text{ N} = 0, \quad F_{AD} = -1181.1\text{ N} \qquad F_{AD} = 1181\text{ N} \quad C \blacktriangleleft$$

$$\mathbf{i}: \quad -\left(\frac{1.2}{3.937}\right)(-1181.1\text{ N}) - F_{DE} = 0 \qquad F_{DE} = 360\text{ N} \quad T \blacktriangleleft$$

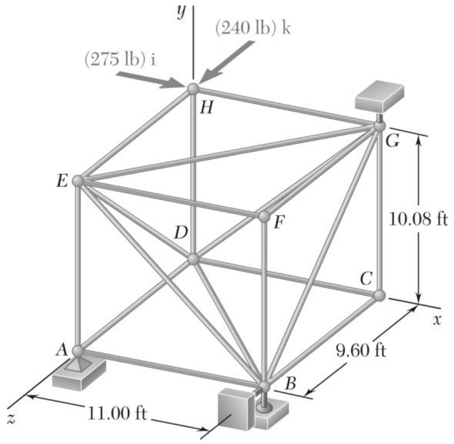
$$\mathbf{k}: \quad \left(\frac{2.25}{3.937}\right)(-1181.1\text{ N} + 375\text{ N} + 300\text{ N} = 0) \qquad \text{(Checks)}$$

Free body: E:

Member AE is the only member at E which does not lie in the xz plane. Therefore, it is a zero-force member.



$$F_{AE} = 0 \blacktriangleleft$$



PROBLEM 6.41*

The truss shown consists of 18 members and is supported by a ball and socket at *A*, two short links at *B*, and one short link at *G*. (*a*) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (*b*) For the given loading, determine the force in each of the six members joined at *E*.

SOLUTION

(*a*) Check simple truss.

(1) Start with tetrahedron *BEFG*.

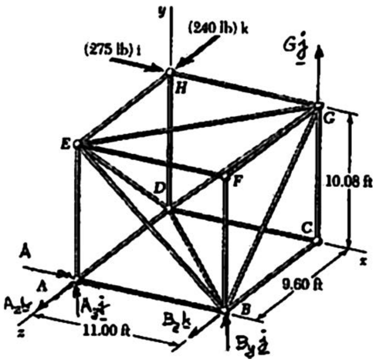
(2) Add members *BD*, *ED*, *GD* joining at *D*.

(3) Add members *BA*, *DA*, *EA* joining at *A*.

(4) Add members *DH*, *EH*, *GH* joining at *H*.

(5) Add members *BC*, *DC*, *GC* joining at *C*.

Truss has been completed: It is a simple truss.



Free body: Truss:

Check constraints and reactions.

Six unknown reactions—ok; however, supports at *A* and *B* constrain truss to rotate about *AB* and support at *G* prevents such a rotation. Thus,

Truss is completely constrained and reactions are statically determinate.

Determination of reactions:

$$\Sigma \mathbf{M}_A = 0: \quad 11\mathbf{i} \times (B_y\mathbf{j} + B_z\mathbf{k}) + (11\mathbf{i} - 9.6\mathbf{k}) \times G\mathbf{j}$$

$$+ (10.08\mathbf{j} - 9.6\mathbf{k}) \times (275\mathbf{i} + 240\mathbf{k}) = 0$$

$$11B_y\mathbf{k} - 11B_y\mathbf{j} + 11G\mathbf{k} + 9.6G\mathbf{i} - (10.08)(275)\mathbf{k}$$

$$+ (10.08)(240)\mathbf{i} - (9.6)(275)\mathbf{j} = 0$$

Equate to zero the coefficient of **i**, **j**, **k**:

$$\mathbf{i}: \quad 9.6G + (10.08)(240) = 0 \quad G = -252 \text{ lb} \qquad \mathbf{G} = (-252 \text{ lb})\mathbf{j} \ \triangleleft$$

$$\mathbf{j}: \quad -11B_z - (9.6)(275) = 0 \quad B_z = -240 \text{ lb}$$

$$\mathbf{k}: \quad 11B_y + 11(-252) - (10.08)(275) = 0, \quad B_y = 504 \text{ lb} \qquad \mathbf{B} = (504 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k} \ \triangleleft$$

PROBLEM 6.41* (Continued)

$$\Sigma\mathbf{F} = 0: \quad \mathbf{A} + (504\text{ lb})\mathbf{j} - (240\text{ lb})\mathbf{k} - (252\text{ lb})\mathbf{j} + (275\text{ lb})\mathbf{i} + (240\text{ lb})\mathbf{k} = 0$$

$\mathbf{A} = -(275\text{ lb})\mathbf{i} - (252\text{ lb})\mathbf{j}$ ◁

Zero-force members.

The determination of these members will facilitate our solution.

FB: C: Writing $\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma F_z = 0$ yields $F_{BC} = F_{CD} = F_{CG} = 0$ ◁

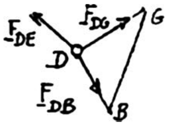
FB: F: Writing $\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma F_z = 0$ yields $F_{BF} = F_{EF} = F_{FG} = 0$ ◁

FB: A: Since $A_z = 0$, writing $\Sigma F_z = 0$ yields $F_{AD} = 0$ ◁

FB: H: Writing $\Sigma F_y = 0$ yields $F_{DH} = 0$ ◁

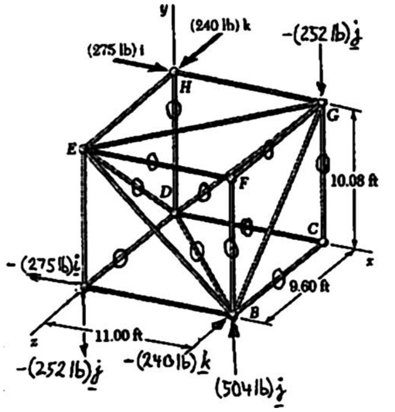
FB: D: Since $F_{AD} = F_{CD} = F_{DH} = 0$, we need consider only members *DB*, *DE*, and *DG*.

Since F_{DE} is the only force not contained in plane *BDG*, it must be zero. Simple reasonings show that the other two forces are also zero.



$F_{BD} = F_{DE} = F_{DG} = 0$ ◁

The results obtained for the reactions at the supports and for the zero-force members are shown on the figure below. Zero-force members are indicated by a zero ("0").



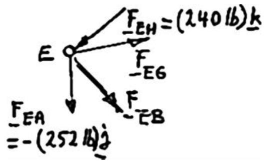
(*b*) Force in each of the members joined at *E*.

We already found that $F_{DE} = F_{EF} = 0$ ◀

Free body: *A*: $\Sigma F_y = 0$ yields $F_{AE} = 252\text{ lb}\quad T$ ◀

Free body: *H*: $\Sigma F_z = 0$ yields $F_{EH} = 240\text{ lb}\quad C$ ◀

PROBLEM 6.41* (Continued)



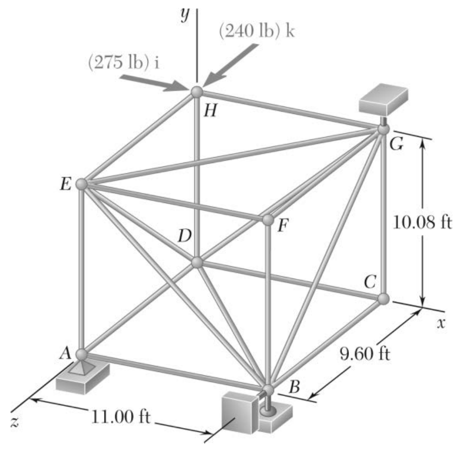
Free body: E: $\Sigma \mathbf{F} = 0$: $\mathbf{F}_{EB} + \mathbf{F}_{EG} + (240\text{ lb})\mathbf{k} - (252\text{ lb})\mathbf{j} = 0$

$$\frac{F_{BE}}{14.92}(11\mathbf{i} - 10.08\mathbf{j}) + \frac{F_{EG}}{14.6}(11\mathbf{i} - 9.6\mathbf{k}) + 240\mathbf{k} - 252\mathbf{j} = 0$$

Equate to zero the coefficient of **y** and **k**:

$$\mathbf{j}: \quad -\left(\frac{10.08}{14.92}\right)F_{BE} - 252 = 0 \qquad F_{BE} = 373\text{ lb} \quad C \blacktriangleleft$$

$$\mathbf{k}: \quad -\left(\frac{9.6}{14.6}\right)F_{EG} + 240 = 0 \qquad F_{EG} = 365\text{ lb} \quad T \blacktriangleleft$$



PROBLEM 6.42*

The truss shown consists of 18 members and is supported by a ball and socket at *A*, two short links at *B*, and one short link at *G*. (*a*) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (*b*) For the given loading, determine the force in each of the six members joined at *G*.

SOLUTION

See solution to Problem 6.41 for part (a) and for reactions and zero-force members.

(*b*) Force in each of the members joined at *G*.

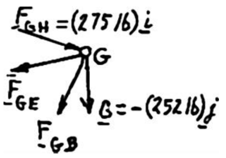
We already know that

$$F_{CG} = F_{DG} = F_{FG} = 0 \blacktriangleleft$$

Free body: *H*: $\Sigma F_x = 0$ yields $F_{GH} = 275 \text{ lb} \quad C \blacktriangleleft$

Free body: *G*: $\Sigma \mathbf{F} = 0\!: \quad \mathbf{F}_{GB} + \mathbf{F}_{GE} + (275 \text{ lb})\mathbf{i} - (252 \text{ lb})\mathbf{j} = 0$

$$\frac{F_{BG}}{13.92}(-10.08\mathbf{j} + 9.6\mathbf{k}) + \frac{F_{EG}}{14.6}(-11\mathbf{i} + 9.6\mathbf{k}) + 275\mathbf{i} - 252\mathbf{j} = 0$$

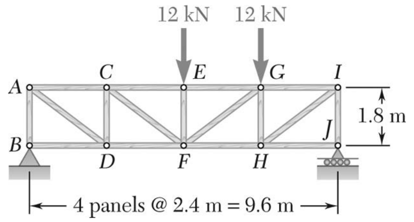


Equate to zero the coefficient of **i**, **j**, **k**:

$$\mathbf{i}\!: \quad -\left(\frac{11}{14.6}\right)F_{EG} + 275 = 0 \qquad F_{EG} = 365 \text{ lb} \quad T \blacktriangleleft$$

$$\mathbf{j}\!: \quad -\left(\frac{10.08}{13.92}\right)F_{BG} - 252 = 0 \qquad F_{BG} = 348 \text{ lb} \quad C \blacktriangleleft$$

$$\mathbf{k}\!: \quad \left(\frac{9.6}{13.92}\right)(-348) + \left(\frac{9.6}{14.6}\right)(365) = 0 \qquad \text{(Checks)}$$

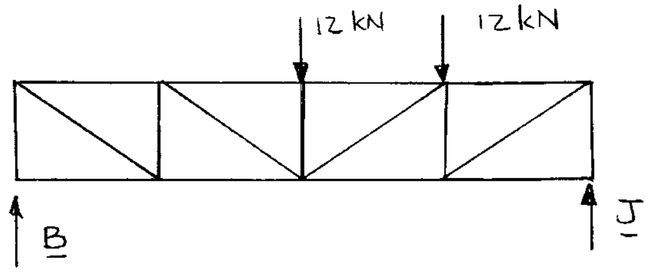


PROBLEM 6.43

Determine the force in members *CD* and *DF* of the truss shown.

SOLUTION

Reactions:



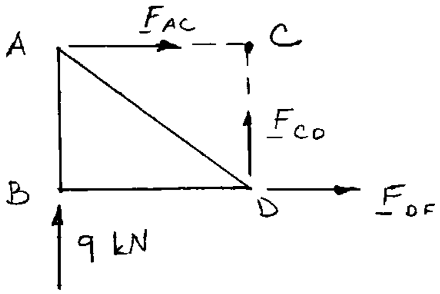
$$+\circlearrowleft \Sigma M_J = 0: \quad (12\text{ kN})(4.8\text{ m}) + (12\text{ kN})(2.4\text{ m}) - B(9.6\text{ m}) = 0$$

$$B = 9.00\text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad 9.00\text{ kN} - 12.00\text{ kN} - 12.00\text{ kN} + J = 0$$

$$J = 15.00\text{ kN}$$

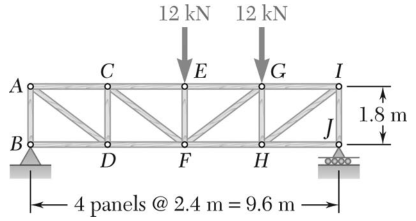
Member *CD*:



$$+\uparrow \Sigma F_y = 0: \quad 9.00\text{ kN} + F_{CD} = 0 \qquad F_{CD} = 9.00\text{ kN} \quad C \blacktriangleleft$$

Member *DF*:

$$+\circlearrowleft \Sigma M_C = 0: \quad F_{DF}(1.8\text{ m}) - (9.00\text{ kN})(2.4\text{ m}) = 0 \qquad F_{DF} = 12.00\text{ kN} \quad T \blacktriangleleft$$

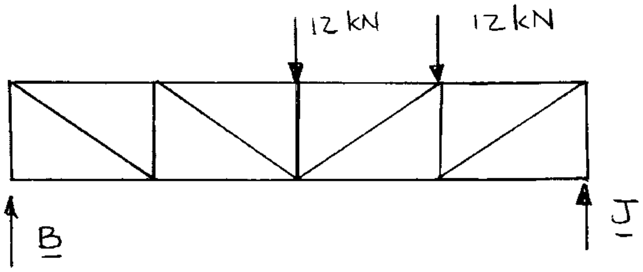


PROBLEM 6.44

Determine the force in members *FG* and *FH* of the truss shown.

SOLUTION

<u>Reactions</u>:



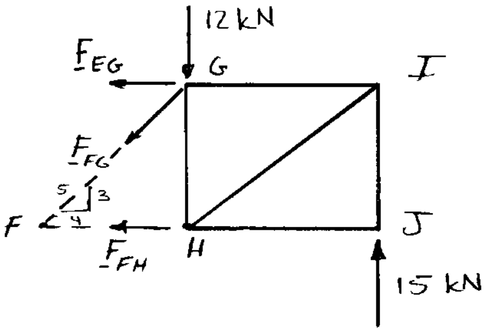
$$+\circlearrowleft \Sigma M_J = 0: \quad (12 \text{ kN})(4.8 \text{ m}) + (12 \text{ kN})(2.4 \text{ m}) - B(9.6 \text{ m}) = 0$$

$$B = 9.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad 9.00 \text{ kN} - 12.00 \text{ kN} - 12.00 \text{ kN} + J = 0$$

$$J = 15.00 \text{ kN}$$

<u>Member *FG*</u>:



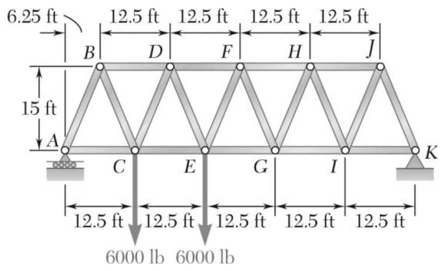
$$+\uparrow \Sigma F_y = 0: \quad -\frac{3}{5}F_{FG} - 12.00 \text{ kN} + 15.00 \text{ kN} = 0$$

$$F_{FG} = 5.00 \text{ kN} \quad T \blacktriangleleft$$

<u>Member *FH*</u>:

$$+\circlearrowleft \Sigma M_G = 0: \quad (15.00 \text{ kN})(2.4 \text{ m}) - F_{FH}(1.8 \text{ m}) = 0$$

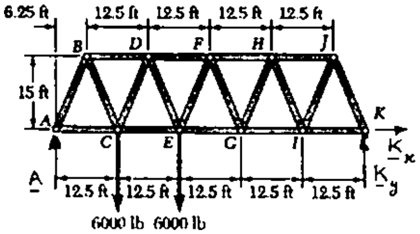
$$F_{FH} = 20.0 \text{ kN} \quad T \blacktriangleleft$$



PROBLEM 6.45

A Warren bridge truss is loaded as shown. Determine the force in members *CE*, *DE*, and *DF*.

SOLUTION



Free body: Truss:

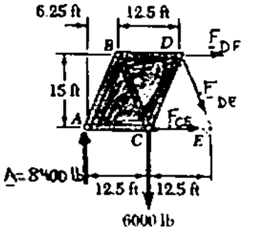
$$\xrightarrow{+} \Sigma F_x = 0: \quad \mathbf{k}_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad k_y(62.5\text{ ft}) - (6000\text{ lb})(12.5\text{ ft}) - (6000\text{ lb})(25\text{ ft}) = 0$$

$$\mathbf{k} = \mathbf{k}_y = 3600\text{ lb}\uparrow \ \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A + 3600\text{ lb} - 6000\text{ lb} - 6000\text{ lb} = 0$$

$$\mathbf{A} = 8400\text{ lb}\uparrow \ \triangleleft$$



We pass a section through members *CE*, *DE*, and *DF* and use the free body shown.

$$+\circlearrowleft \Sigma M_D = 0: \quad F_{CE}(15\text{ ft}) - (8400\text{ lb})(18.75\text{ ft}) + (6000\text{ lb})(6.25\text{ ft}) = 0$$

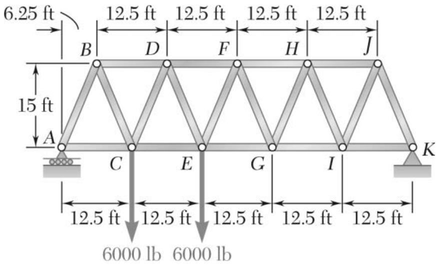
$$F_{CE} = +8000\text{ lb} \qquad F_{CE} = 8000\text{ lb} \quad T \ \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 8400\text{ lb} - 6000\text{ lb} - \frac{15}{16.25}F_{DE} = 0$$

$$F_{DE} = +2600\text{ lb} \qquad F_{DE} = 2600\text{ lb} \quad T \ \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_E = 0: \quad 6000\text{ lb}(12.5\text{ ft}) - (8400\text{ lb})(25\text{ ft}) - F_{DF}(15\text{ ft}) = 0$$

$$F_{DF} = -9000\text{ lb} \qquad F_{DF} = 9000\text{ lb} \quad C \ \blacktriangleleft$$



PROBLEM 6.46

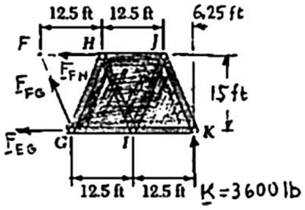
A Warren bridge truss is loaded as shown. Determine the force in members *EG*, *FG*, and *FH*.

SOLUTION

See solution of Problem 6.45 for free-body diagram of truss and determination of reactions:

$$\mathbf{A} = 8400\ \text{lb} \qquad \mathbf{k} = 3600\ \text{lb} \ \triangleleft$$

We pass a section through members *EG*, *FG*, and *FH*, and use the free body shown.



$$+\circlearrowleft \Sigma M_F = 0: \quad (3600\ \text{lb})(31.25\ \text{ft}) - F_{EG}(15\ \text{ft}) = 0$$

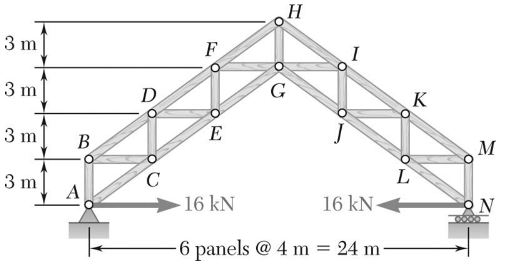
$$F_{EG} = +7500\ \text{lb} \qquad F_{EG} = 7500\ \text{lb} \quad T \ \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{15}{16.25} F_{FG} + 3600\ \text{lb} = 0$$

$$F_{FG} = -3900\ \text{lb} \qquad F_{FG} = 3900\ \text{lb} \quad C \ \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_G = 0: \quad F_{FH}(15\ \text{ft}) + (3600\ \text{lb})(25\ \text{ft}) = 0$$

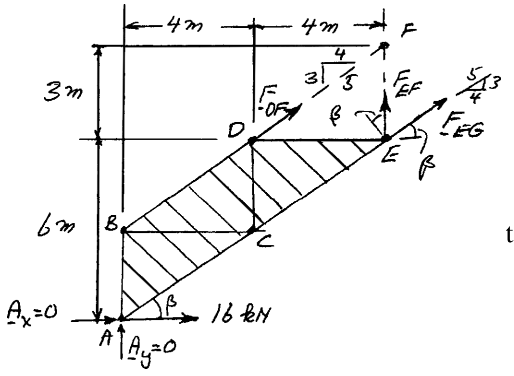
$$F_{FH} = -6000\ \text{lb} \qquad F_{FH} = 6000\ \text{lb} \quad C \ \blacktriangleleft$$



PROBLEM 6.47

Determine the force in members *DF*, *EF*, and *EG* of the truss shown.

SOLUTION



$$\tan\beta = \frac{3}{4}$$

Reactions:

$$\mathbf{A} = \mathbf{N} = 0$$

Member DF: $+\circlearrowleft \Sigma M_E = 0: \quad +(16\text{ kN})(6\text{ m}) - \frac{3}{5}F_{DF}(4\text{ m}) = 0$

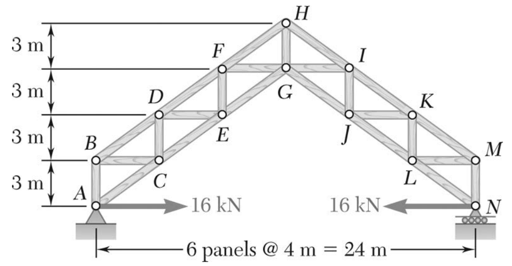
$F_{DF} = +40\text{ kN}$ $\qquad F_{DF} = 40.0\text{ kN}\quad T$ ◀

Member EF: $+\searrow_\beta \Sigma F = 0: \quad (16\text{ kN})\sin\beta - F_{EF}\cos\beta = 0$

$F_{EF} = 16\tan\beta = 16(0.75) = 12\text{ kN}$ $\qquad F_{EF} = 12.00\text{ kN}\quad T$ ◀

Member EG: $+\circlearrowleft \Sigma M_F = 0: \quad (16\text{ kN})(9\text{ m}) + \frac{4}{5}F_{EG}(3\text{ m}) = 0$

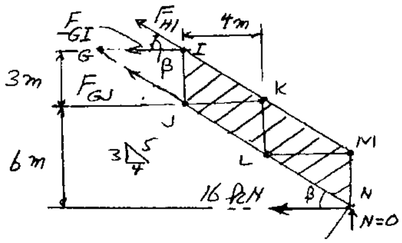
$F_{EG} = -60\text{ kN}$ $\qquad F_{EG} = 60.0\text{ kN}\quad C$ ◀



PROBLEM 6.48

Determine the force in members *GI*, *GJ*, and *HI* of the truss shown.

SOLUTION



Reactions:

$$\mathbf{A} = \mathbf{N} = 0$$

Member *GI*: $+\nearrow_{\beta}\Sigma F = 0:\quad (16\text{ kN})\sin\beta + F_{GI}\sin\beta = 0$

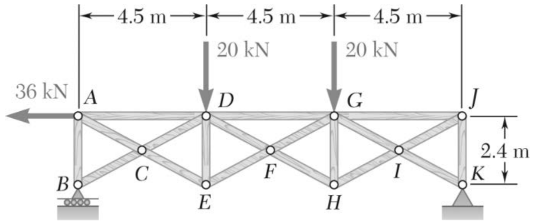
$F_{GI} = -16\text{ kN}$ $\qquad F_{GI} = 16.00\text{ kN}\quad C$ ◀

Member *GJ*: $+\circlearrowleft\Sigma M_I = 0:\quad -(16\text{ kN})(9\text{ m}) - \frac{4}{5}F_{GJ}(3\text{ m}) = 0$

$F_{GJ} = -60\text{ kN}$ $\qquad F_{GJ} = 60.0\text{ kN}\quad C$ ◀

Member *HI*: $+\circlearrowleft\Sigma M_G = 0:\quad -(16\text{ kN})(9\text{ m}) + \frac{3}{5}F_{HI}(4\text{ m}) = 0$

$F_{HI} = +60\text{ kN}$ $\qquad F_{HI} = 60.0\text{ kN}\quad T$ ◀

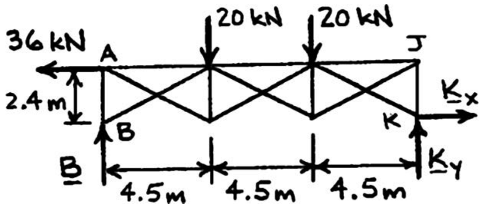


PROBLEM 6.49

Determine the force in members *AD*, *CD*, and *CE* of the truss shown.

SOLUTION

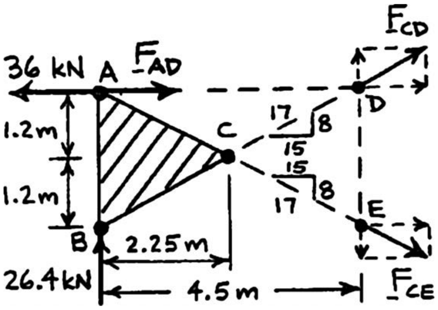
Reactions:



$$+\circlearrowleft \Sigma M_k = 0: \quad 36(2.4) - B(13.5) + 20(9) + 20(4.5) = 0 \qquad \mathbf{B} = 26.4 \text{ kN} \uparrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -36 + K_x = 0 \qquad \mathbf{K}_x = 36 \text{ kN} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad 26.4 - 20 - 20 + K_y = 0 \quad \mathbf{K}_y = 13.6 \text{ kN} \uparrow$$



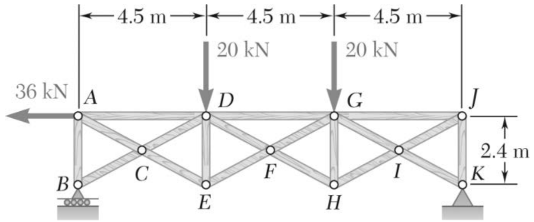
$$+\circlearrowleft \Sigma M_C = 0: \quad 36(1.2) - 26.4(2.25) - F_{AD}(1.2) = 0$$

$$F_{AD} = -13.5 \text{ kN} \qquad F_{AD} = 13.5 \text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad \left(\frac{8}{17}F_{CD}\right)(4.5) = 0 \qquad F_{CD} = 0 \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad \left(\frac{15}{17}F_{CE}\right)(2.4) - 26.4(4.5) = 0$$

$$F_{CE} = +56.1 \text{ kN} \qquad F_{CE} = 56.1 \text{ kN} \quad T \blacktriangleleft$$



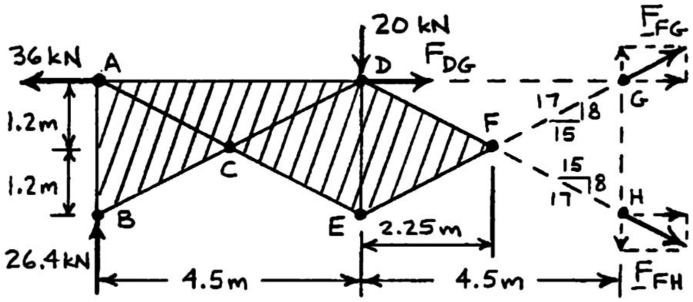
PROBLEM 6.50

Determine the force in members *DG*, *FG*, and *FH* of the truss shown.

SOLUTION

See the solution to Problem 6.49 for free-body diagram and analysis to determine the reactions at the supports *B* and *K*.

$$\mathbf{B} = 26.4 \text{ kN}\uparrow; \quad \mathbf{K}_x = 36.0 \text{ kN}\longrightarrow; \quad \mathbf{K}_y = 13.60 \text{ kN}\uparrow$$



$$+\circlearrowleft \Sigma M_F = 0: \quad 36(1.2) - 26.4(6.75) + 20(2.25) - F_{DG}(1.2) = 0$$

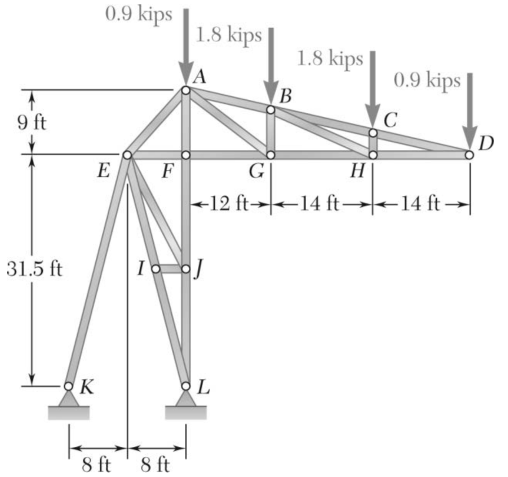
$$F_{DG} = -75 \text{ kN} \qquad F_{DG} = 75.0 \text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -26.4(4.5) + \left(\frac{8}{17}F_{FG}\right)(4.5) = 0$$

$$F_{FG} = +56.1 \text{ kN} \qquad F_{FG} = 56.1 \text{ kN} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_G = 0: \quad 20(4.5) - 26.4(9) + \left(\frac{15}{17}F_{FH}\right)(2.4) = 0$$

$$F_{FH} = +69.7 \text{ kN} \qquad F_{FH} = 69.7 \text{ kN} \quad T \blacktriangleleft$$

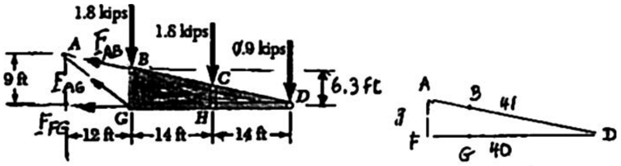


PROBLEM 6.51

A stadium roof truss is loaded as shown. Determine the force in members *AB*, *AG*, and *FG*.

SOLUTION

We pass a section through members *AB*, *AG*, and *FG*, and use the free body shown.



$$+\circlearrowleft \Sigma M_G = 0: \quad \left(\frac{40}{41}F_{AB}\right)(6.3\text{ ft}) - (1.8\text{ kips})(14\text{ ft})$$

$$-(0.9\text{ kips})(28\text{ ft}) = 0$$

$$F_{AB} = +8.20\text{ kips} \qquad\qquad F_{AB} = 8.20\text{ kips}\quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -\left(\frac{3}{5}F_{AG}\right)(28\text{ ft}) + (1.8\text{ kips})(28\text{ ft})$$

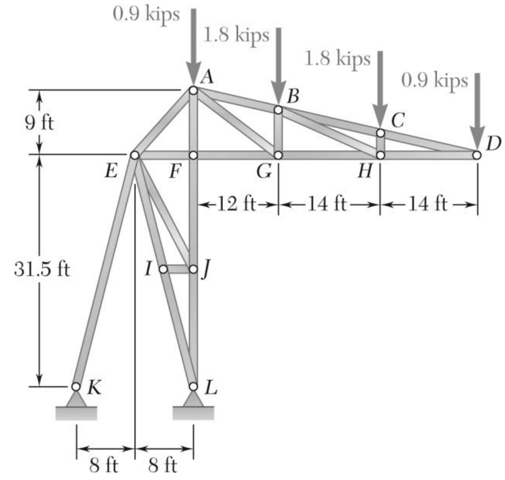
$$+(1.8\text{ kips})(14\text{ ft}) = 0$$

$$F_{AG} = +4.50\text{ kips} \qquad\qquad F_{AG} = 4.50\text{ kips}\quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -F_{FG}(9\text{ ft}) - (1.8\text{ kips})(12\text{ ft}) - (1.8\text{ kips})(26\text{ ft})$$

$$-(0.9\text{ kips})(40\text{ ft}) = 0$$

$$F_{FG} = -11.60\text{ kips} \qquad\qquad F_{FG} = 11.60\text{ kips}\quad C \blacktriangleleft$$

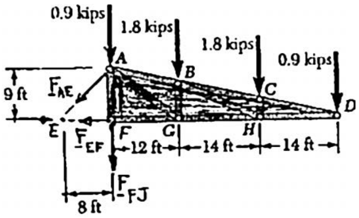


PROBLEM 6.52

A stadium roof truss is loaded as shown. Determine the force in members *AE*, *EF*, and *FJ*.

SOLUTION

We pass a section through members *AE*, *EF*, and *FJ*, and use the free body shown.



$$+\circlearrowleft \Sigma M_F = 0: \quad \left(\frac{8}{\sqrt{8^2+9^2}}F_{AE}\right)(9\text{ ft}) - (1.8\text{ kips})(12\text{ ft}) - (1.8\text{ kips})(26\text{ ft}) - (0.9\text{ kips})(40\text{ ft}) = 0$$

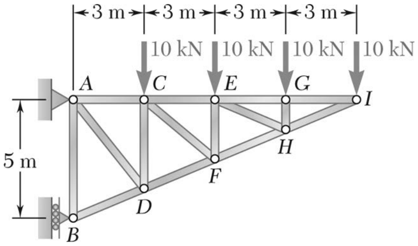
$$F_{AE} = +17.46\text{ kips} \qquad F_{AE} = 17.46\text{ kips} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -F_{EF}(9\text{ ft}) - (1.8\text{ kips})(12\text{ ft}) - (1.8\text{ kips})(26\text{ ft}) - (0.9\text{ kips})(40\text{ ft}) = 0$$

$$F_{EF} = -11.60\text{ kips} \qquad F_{EF} = 11.60\text{ kips} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_E = 0: \quad -F_{FJ}(8\text{ ft}) - (0.9\text{ kips})(8\text{ ft}) - (1.8\text{ kips})(20\text{ ft}) - (1.8\text{ kips})(34\text{ ft}) - (0.9\text{ kips})(48\text{ ft}) = 0$$

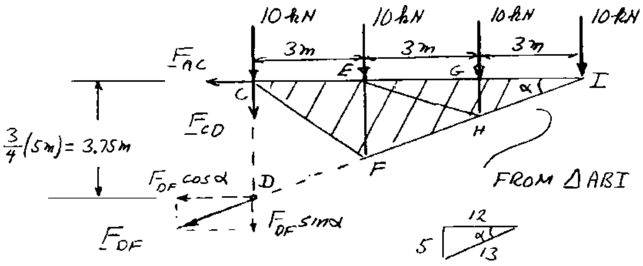
$$F_{FJ} = -18.45\text{ kips} \qquad F_{FJ} = 18.45\text{ kips} \quad C \blacktriangleleft$$



PROBLEM 6.53

Determine the force in members *CD* and *DF* of the truss shown.

SOLUTION



$$\tan\alpha = \frac{5}{12} \qquad \alpha = 22.62°$$

$$\sin\alpha = \frac{5}{13} \quad \cos\alpha = \frac{12}{13}$$

Member *CD*:

$$+\circlearrowleft \Sigma M_I = 0\text{:} \quad F_{CD}(9\text{ m}) + (10\text{ kN})(9\text{ m}) + (10\text{ kN})(6\text{ m}) + (10\text{ kN})(3\text{ m}) = 0$$

$$F_{CD} = -20\text{ kN} \qquad\qquad F_{CD} = 20.0\text{ kN} \quad C \blacktriangleleft$$

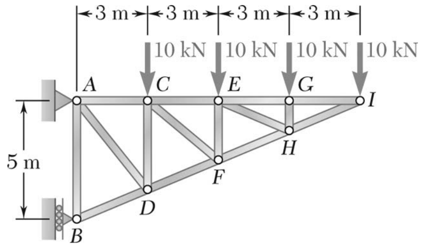
Member *DF*:

$$+\circlearrowright \Sigma M_C = 0\text{:} \quad (F_{DF}\cos\alpha)(3.75\text{ m}) + (10\text{ kN})(3\text{ m}) + (10\text{ kN})(6\text{ m}) + (10\text{ kN})(9\text{ m}) = 0$$

$$F_{DF}\cos\alpha = -48\text{ kN}$$

$$F_{DF}\left(\frac{12}{13}\right) = -48\text{ kN} \quad F_{DF} = -52.0\text{ kN} \qquad\qquad F_{DF} = 52.0\text{ kN} \quad C \blacktriangleleft$$

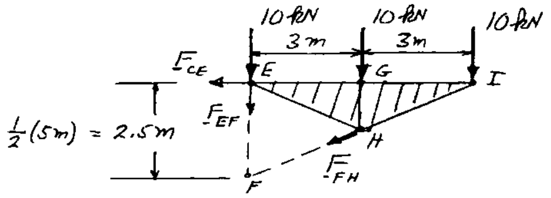
PROPRIETARY MATERIAL.



PROBLEM 6.54

Determine the force in members *CE* and *EF* of the truss shown.

SOLUTION



Member *CE*:

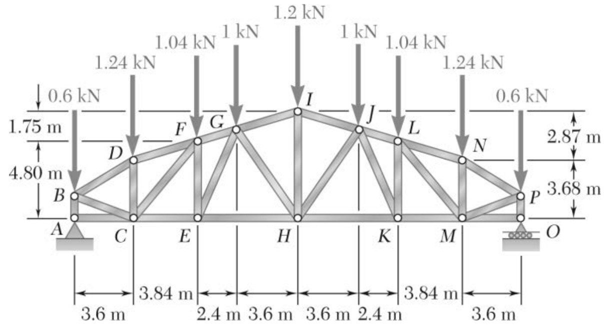
$$+\circlearrowleft \Sigma M_F = 0: \quad F_{CE}(2.5 \text{ m}) - (10 \text{ kN})(3 \text{ m}) - (10 \text{ kN})(6 \text{ m}) = 0$$

$$F_{CE} = +36 \text{ kN} \qquad\qquad F_{CE} = 36.0 \text{ kN} \quad T \blacktriangleleft$$

Member *EF*:

$$+\circlearrowleft \Sigma M_I = 0: \quad F_{EF}(6 \text{ m}) + (10 \text{ kN})(6 \text{ m}) + (10 \text{ kN})(3 \text{ m}) = 0$$

$$F_{EF} = -15 \text{ kN} \qquad\qquad F_{EF} = 15.00 \text{ kN} \quad C \blacktriangleleft$$



PROBLEM 6.55

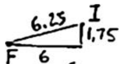
The truss shown was designed to support the roof of a food market. For the given loading, determine the force in members *FG*, *EG*, and *EH*.

SOLUTION

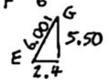
Reactions at supports. Because of the symmetry of the loading,

$$A_x = 0, \quad A_y = O = \frac{1}{2} \text{ total load} \qquad \mathbf{A} = \mathbf{O} = 4.48 \text{ kN}\uparrow \blacktriangleleft$$

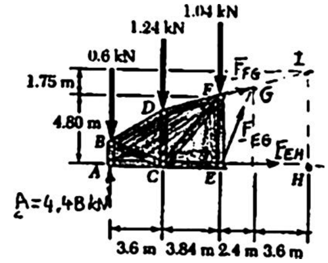
We pass a section through members *FG*, *EG*, and *EH*, and use the free body shown.



$$\text{Slope } FG = \text{Slope } FI = \frac{1.75 \text{ m}}{6 \text{ m}}$$



$$\text{Slope } EG = \frac{5.50 \text{ m}}{2.4 \text{ m}}$$



$$+\circlearrowleft \Sigma M_E = 0: \quad (0.6 \text{ kN})(7.44 \text{ m}) + (1.24 \text{ kN})(3.84 \text{ m})$$

$$-(4.48 \text{ kN})(7.44 \text{ m})$$

$$-\left(\frac{6}{6.25} F_{FG}\right)(4.80 \text{ m}) = 0$$

$$F_{FG} = -5.231 \text{ kN} \qquad F_{FG} = 5.23 \text{ kN} \quad C \blacktriangleleft$$

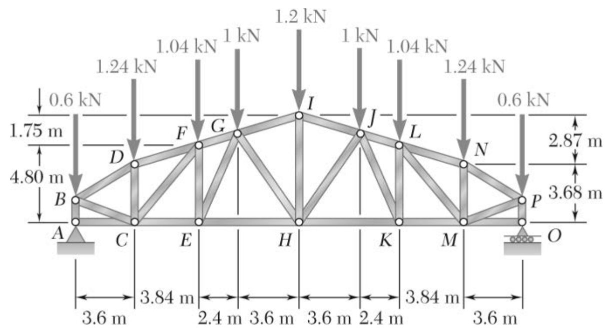
$$+\circlearrowleft \Sigma M_G = 0: \quad F_{EH}(5.50 \text{ m}) + (0.6 \text{ kN})(9.84 \text{ m})$$

$$+(1.24 \text{ kN})(6.24 \text{ m}) + (1.04 \text{ kN})(2.4 \text{ m})$$

$$-(4.48 \text{ kN})(9.84 \text{ m}) = 0 \qquad F_{EH} = 5.08 \text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{5.50}{6.001} F_{EG} + \frac{1.75}{6.25}(-5.231 \text{ kN}) + 4.48 \text{ kN} - 0.6 \text{ kN} - 1.24 \text{ kN} - 1.04 \text{ kN} = 0$$

$$F_{EG} = -0.1476 \text{ kN} \qquad F_{EG} = 0.1476 \text{ kN} \quad C \blacktriangleleft$$



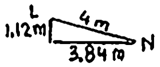
PROBLEM 6.56

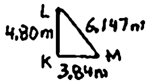
The truss shown was designed to support the roof of a food market. For the given loading, determine the force in members *KM*, *LM*, and *LN*.

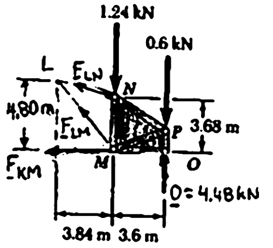
SOLUTION

Because of symmetry of loading, $\mathbf{O} = \frac{1}{2}\text{load}$ $\qquad \mathbf{O} = 4.48\text{ kN}\uparrow$ ◁

We pass a section through *KM*, *LM*, *LN*, and use free body shown.







$$+\circlearrowleft \Sigma M_M = 0:\quad \left(\frac{3.84}{4}F_{LN}\right)(3.68\text{ m}) + (4.48\text{ kN} - 0.6\text{ kN})(3.6\text{ m}) = 0$$

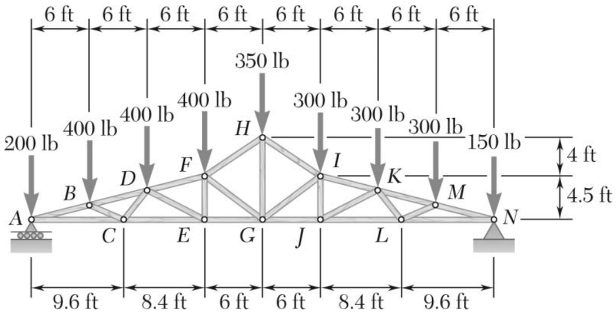
$$F_{LN} = -3.954\text{ kN} \qquad F_{LN} = 3.95\text{ kN}\quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_L = 0:\quad -F_{KM}(4.80\text{ m}) - (1.24\text{ kN})(3.84\text{ m}) + (4.48\text{ kN} - 0.6\text{ kN})(7.44\text{ m}) = 0$$

$$F_{KM} = +5.022\text{ kN} \qquad F_{KM} = 5.02\text{ kN}\quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0:\quad \frac{4.80}{6.147}F_{LM} + \frac{1.12}{4}(-3.954\text{ kN}) - 1.24\text{ kN} - 0.6\text{ kN} + 4.48\text{ kN} = 0$$

$$F_{LM} = -1.963\text{ kN} \qquad F_{LM} = 1.963\text{ kN}\quad C \blacktriangleleft$$

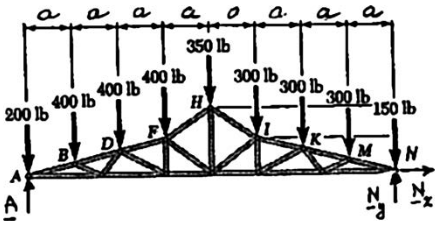


PROBLEM 6.57

A Polynesian, or duopitch, roof truss is loaded as shown. Determine the force in members *DF*, *EF*, and *EG*.

SOLUTION

Free body: Truss:



$\Sigma F_x = 0: \quad N_x = 0$

$+\circlearrowleft \Sigma M_N = 0: \quad (200\text{ lb})(8a) + (400\text{ lb})(7a + 6a + 5a) + (350\text{ lb})(4a) + (300\text{ lb})(3a + 2a + a) - A(8a) = 0$

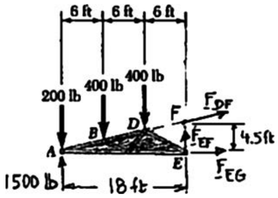
$\mathbf{A} = 1500\text{ lb}\uparrow$ ◁

$+\uparrow \Sigma F_y = 0: \quad 1500\text{ lb} - 200\text{ lb} - 3(400\text{ lb}) - 350\text{ lb} - 3(300\text{ lb}) - 150\text{ lb} + N_y = 0$

$N_y = 1300\text{ lb} \quad \mathbf{N} = 1300\text{ lb}\uparrow$ ◁

We pass a section through *DF*, *EF*, and *EG,* and use the free body shown.

(We apply $\mathbf{F}_{DF}$ at *F*.)



$$+\circlearrowleft \Sigma M_E = 0: \quad (200\text{ lb})(18\text{ ft}) + (400\text{ lb})(12\text{ ft}) + (400\text{ lb})(6\text{ ft}) - (1500\text{ lb})(18\text{ ft}) - \left(\frac{18}{\sqrt{18^2 + 4.5^2}} F_{DF}\right)(4.5\text{ ft}) = 0$$

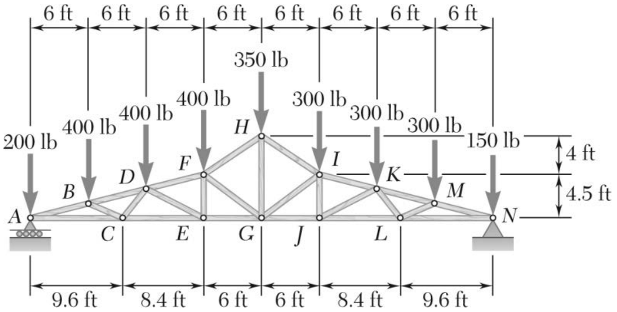
$F_{DF} = -3711\text{ lb} \quad F_{DF} = 3710\text{ lb} \quad C$ ◀

$+\circlearrowleft \Sigma M_A = 0: \quad F_{EF}(18\text{ ft}) - (400\text{ lb})(6\text{ ft}) - (400\text{ lb})(12\text{ ft}) = 0$

$F_{EF} = +400\text{ lb} \qquad F_{EF} = 400\text{ lb} \quad T$ ◀

$+\circlearrowleft \Sigma M_F = 0: \quad F_{EG}(4.5\text{ ft}) - (1500\text{ lb})(18\text{ ft}) + (200\text{ lb})(18\text{ ft}) + (400\text{ lb})(12\text{ ft}) + (400\text{ lb})(6\text{ ft}) = 0$

$F_{EG} = +3600\text{ lb} \quad F_{EG} = 3600\text{ lb} \quad T$ ◀



PROBLEM 6.58

A Polynesian, or duopitch, roof truss is loaded as shown. Determine the force in members *HI*, *GI*, and *GJ*.

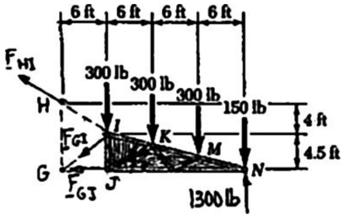
SOLUTION

See solution of Problem 6.57 for reactions:

$$\mathbf{A} = 1500 \text{ lb}\uparrow, \quad \mathbf{N} = 1300 \text{ lb}\uparrow \triangleleft$$

We pass a section through *HI*, *GI*, and *GJ*, and use the free body shown.

(We apply F_{HI} at *H*.)



$$+\circlearrowleft \Sigma M_G = 0: \quad \left(\frac{6}{\sqrt{6^2+4^2}}F_{HI}\right)(8.5 \text{ ft}) + (1300 \text{ lb})(24 \text{ ft}) - (300 \text{ lb})(6 \text{ ft})$$
$$- (300 \text{ lb})(12 \text{ ft}) - (300 \text{ lb})(18 \text{ ft}) - (150 \text{ lb})(24 \text{ ft}) = 0$$

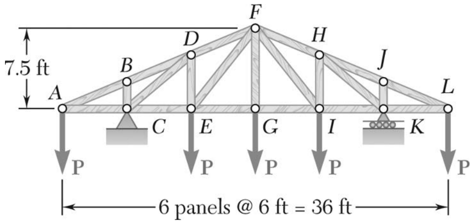
$$F_{HI} = -2375.4 \text{ lb} \quad F_{HI} = 2375 \text{ lb} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_I = 0: \quad (1300 \text{ lb})(18 \text{ ft}) - (300 \text{ lb})(6 \text{ ft}) - (300 \text{ lb})(12 \text{ ft})$$
$$- (150 \text{ lb})(18 \text{ ft}) - F_{GJ}(4.5 \text{ ft}) = 0$$

$$F_{GJ} = +3400 \text{ lb} \quad F_{GJ} = 3400 \text{ lb} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{4}{5}F_{GI} - \frac{6}{\sqrt{6^2+4^2}}(-2375.4 \text{ lb}) - 3400 \text{ lb} = 0$$

$$F_{GI} = -1779.4 \text{ lb} \qquad F_{GI} = 1779 \text{ lb} \quad C \blacktriangleleft$$



PROBLEM 6.59

Determine the force in members *DE* and *DF* of the truss shown when $P = 20$ kips.

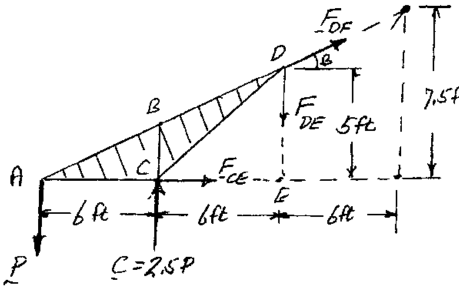
SOLUTION

Reactions:

$$\mathbf{C} = \mathbf{K} = 2.5P \uparrow$$

$$\tan\beta = \frac{7.5}{18}$$

$$\beta = 22.62°$$



Member *DE*:

$$+\circlearrowleft \Sigma M_A = 0: \quad (2.5P)(6\text{ ft}) - F_{DE}(12\text{ ft}) = 0$$

$$F_{DE} = +1.25P$$

For $P = 20$ kips, $\quad F_{DE} = +1.25(20) = +25$ kips $\quad F_{DE} = 25.0$ kips T ◀

Member *DF*:

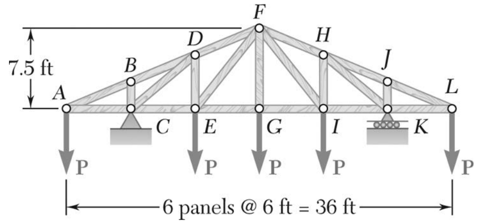
$$+\circlearrowleft \Sigma M_E = 0: \quad P(12\text{ ft}) - (2.5P)(6\text{ ft}) - F_{DF}\cos\beta(5\text{ ft}) = 0$$

$$12P - 15P - F_{DF}\cos 22.62°(5\text{ ft}) = 0$$

$$F_{DF} = -0.65P$$

For $P = 20$ kips, $\quad F_{DF} = -0.65(20) = -13$ kips

$F_{DF} = 13.00$ kips C ◀



PROBLEM 6.60

Determine the force in members EG and EF of the truss shown when $P = 20$ kips.

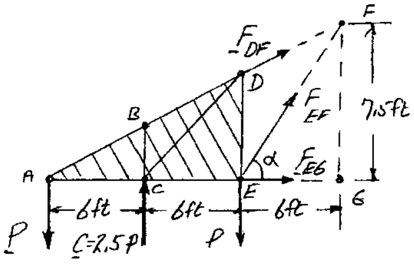
SOLUTION

Reactions:

$$\mathbf{C} = \mathbf{K} = 2.5P$$

$$\tan\alpha = \frac{7.5}{6}$$

$$\alpha = 51.34°$$



Member EG:

$$+\circlearrowleft \Sigma M_F = 0: \quad P(18\text{ ft}) - 2.5P(12\text{ ft}) - P(6\text{ ft}) + F_{EG}(7.5\text{ ft}) = 0$$

$$F_{EG} = +0.8P;$$

For $P = 20$ kips, $\qquad F_{EG} = 0.8(20) = +16$ kips $\qquad F_{EG} = 16.00$ kips T ◀

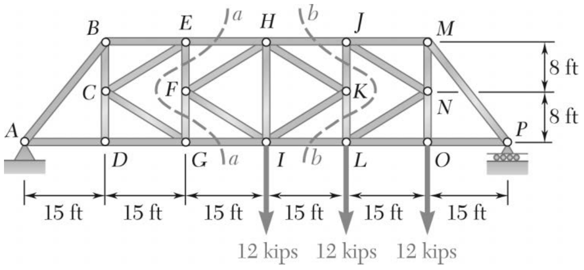
Member EF:

$$+\circlearrowleft \Sigma M_A = 0: \quad 2.5P(6\text{ ft}) - P(12\text{ ft}) + F_{EF}\sin 51.34°(12\text{ ft}) = 0$$

$$F_{EF} = -0.320P;$$

For $P = 20$ kips, $\qquad F_{EF} = -0.320(20) = -6.4$ kips

$F_{EF} = 6.40$ kips C ◀

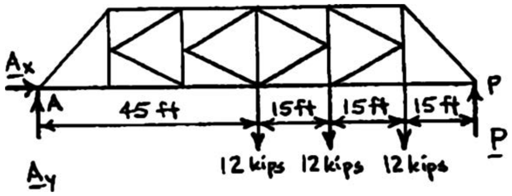


PROBLEM 6.61

Determine the force in members *EH* and *GI* of the truss shown. (*Hint:* Use section *aa*.)

SOLUTION

<u>Reactions</u>:

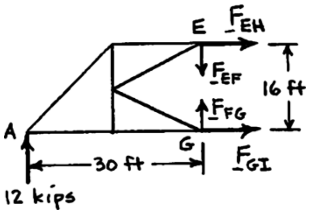


$\Sigma F_x = 0: \quad A_x = 0$

$+\circlearrowleft \Sigma M_P = 0: \quad 12(45) + 12(30) + 12(15) - A_y(90) = 0$

$\mathbf{A}_y = 12 \text{ kips} \uparrow$

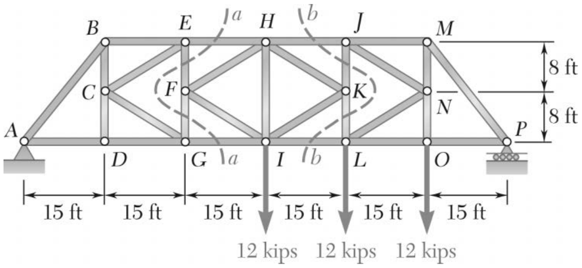
$+\uparrow \Sigma F_y = 0: \quad 12 - 12 - 12 - 12 + P = 0 \qquad \mathbf{P} = 24 \text{ kips} \uparrow$



$+\circlearrowleft \Sigma M_G = 0: \quad -(12 \text{ kips})(30 \text{ ft}) - F_{EH}(16 \text{ ft}) = 0$

$F_{EH} = -22.5 \text{ kips} \qquad F_{EH} = 22.5 \text{ kips} \quad C$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad F_{GI} - 22.5 \text{ kips} = 0 \qquad F_{GI} = 22.5 \text{ kips} \quad T$ ◀

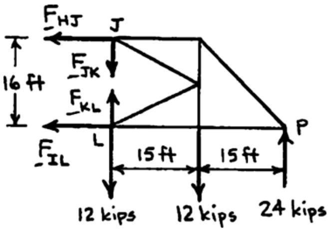


PROBLEM 6.62

Determine the force in members *HJ* and *IL* of the truss shown. (*Hint:* Use section *bb*.)

SOLUTION

See the solution to Problem 6.61 for free body diagram and analysis to determine the reactions at supports *A* and *P*.



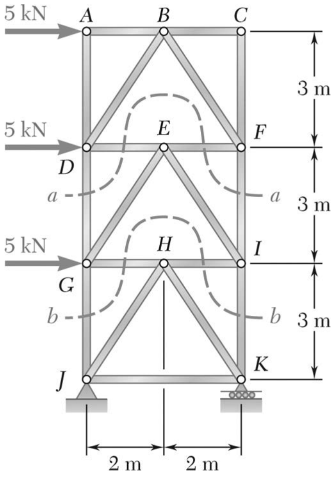
$$\mathbf{A}_x = 0; \quad \mathbf{A}_y = 12.00 \text{ kips} \uparrow; \quad \mathbf{P} = 24.0 \text{ kips} \uparrow$$

$$+\circlearrowleft \Sigma M_L = 0: \quad F_{HJ}(16 \text{ ft}) - (12 \text{ kips})(15 \text{ ft}) + (24 \text{ kips})(30 \text{ ft}) = 0$$

$$F_{HJ} = -33.75 \text{ kips} \qquad F_{HJ} = 33.8 \text{ kips} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 33.75 \text{ kips} - F_{IL} = 0$$

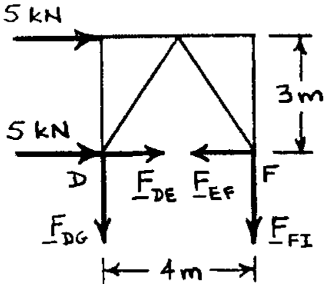
$$F_{IL} = +33.75 \text{ kips} \qquad F_{IL} = 33.8 \text{ kips} \quad T \blacktriangleleft$$



PROBLEM 6.63

Determine the force in members *DG* and *FI* of the truss shown. (*Hint:* Use section *aa*.)

SOLUTION

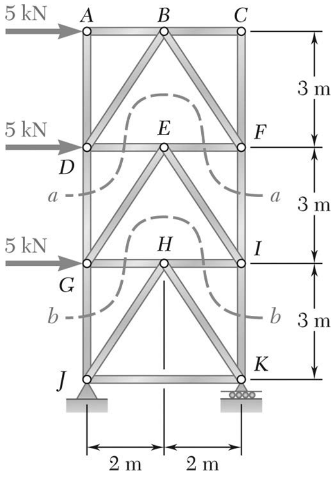


$$+\circlearrowleft \Sigma M_F = 0: \quad F_{DG}(4 \text{ m}) - (5 \text{ kN})(3 \text{ m}) = 0$$

$$F_{DG} = +3.75 \text{ kN} \qquad\qquad F_{DG} = 3.75 \text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -3.75 \text{ kN} - F_{FI} = 0$$

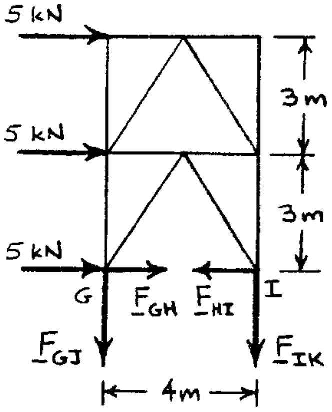
$$F_{FI} = -3.75 \text{ kN} \qquad\qquad F_{FI} = 3.75 \text{ kN} \quad C \blacktriangleleft$$



PROBLEM 6.64

Determine the force in members *GJ* and *IK* of the truss shown. (*Hint:* Use section *bb*.)

SOLUTION

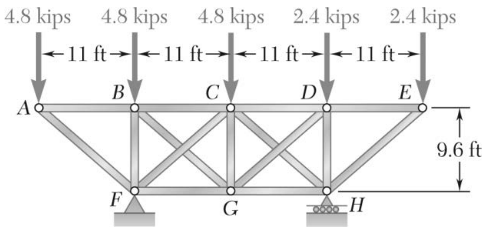


$$+\circlearrowleft \Sigma M_I = 0: \quad F_{GJ}(4\text{ m}) - (5\text{ kN})(6\text{ m}) - (5\text{ kN})(3\text{ m}) = 0$$

$$F_{GJ} = +11.25\text{ kN} \qquad F_{GJ} = 11.25\text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -11.25\text{ kN} - F_{IK} = 0$$

$$F_{IK} = -11.25\text{ kN} \qquad F_{IK} = 11.25\text{ kN} \quad C \blacktriangleleft$$

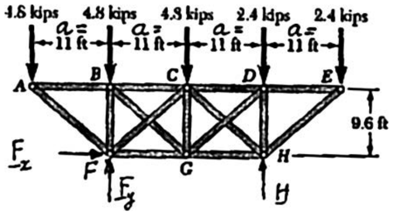


PROBLEM 6.65

The diagonal members in the center panels of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the forces in the counters that are acting under the given loading.

SOLUTION

Free body: Truss:



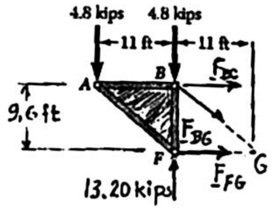
$$\Sigma F_x = 0: \quad F_x = 0$$

$$+\circlearrowleft \Sigma M_H = 0: \quad 4.8(3a) + 4.8(2a) + 4.8a - 2.4a - F_y(2a) = 0$$

$$F_y = +13.20 \text{ kips} \qquad \mathbf{F} = 13.20 \text{ kips}\uparrow \ \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad H + 13.20 \text{ kips} - 3(4.8 \text{ kips}) - 2(2.4 \text{ kips}) = 0$$

$$H = +6.00 \text{ kips} \qquad \mathbf{H} = 6.00 \text{ kips}\uparrow \ \triangleleft$$



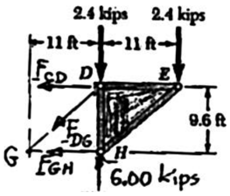
Free body: *ABF*:

We assume that counter *BG* is acting.

$$+\uparrow \Sigma F_y = 0: \quad -\frac{9.6}{14.6} F_{BG} + 13.20 - 2(4.8) = 0$$

$$F_{BG} = +5.475 \qquad F_{BG} = 5.48 \text{ kips} \quad T \ \blacktriangleleft$$

Since *BG* is in tension, our assumption was correct.



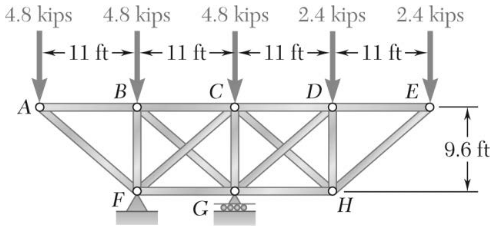
Free body: *DEH*:

We assume that counter *DG* is acting.

$$+\uparrow \Sigma F_y = 0: \quad -\frac{9.6}{14.6} F_{DG} + 6.00 - 2(2.4) = 0$$

$$F_{DG} = +1.825 \qquad F_{DG} = 1.825 \text{ kips} \quad T \ \blacktriangleleft$$

Since *DG* is in tension, O.K.

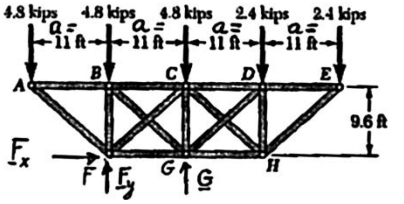


PROBLEM 6.66

The diagonal members in the center panels of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the forces in the counters that are acting under the given loading.

SOLUTION

Free body: Truss:

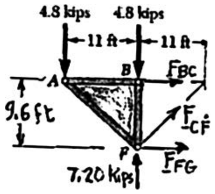


$$\Sigma F_x = 0: \quad F_x = 0$$

$$+\circlearrowleft \Sigma M_G = 0: \quad -F_y a + 4.8(2a) + 4.8a - 2.4a - 2.4(2a) = 0$$

$$F_y = 7.20 \qquad F = 7.20 \text{ kips} \uparrow \triangleleft$$

Free body: *ABF*:



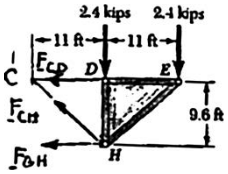
We assume that counter *CF* is acting.

$$+\uparrow \Sigma F_y = 0: \quad \frac{9.6}{14.6} F_{CF} + 7.20 - 2(4.8) = 0$$

$$F_{CF} = +3.65 \qquad F_{CF} = 3.65 \text{ kips} \quad T \blacktriangleleft$$

Since *CF* is in tension, O.K.

Free body: *DEH*:

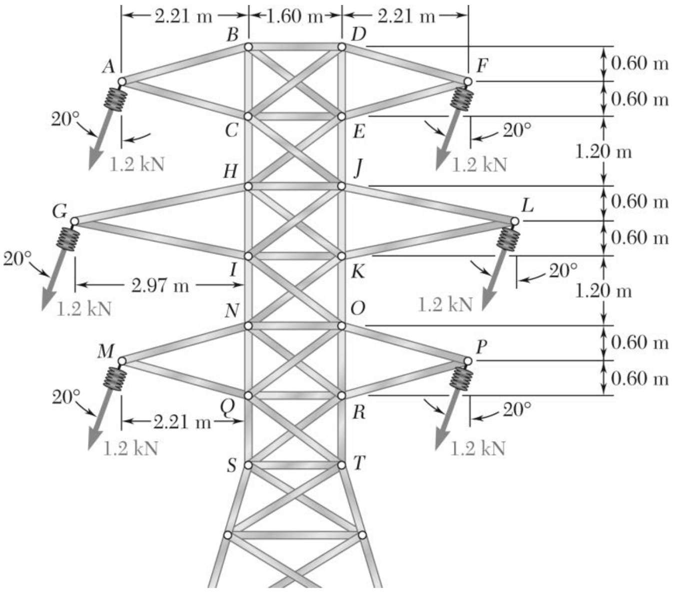


We assume that counter *CH* is acting.

$$+\uparrow \Sigma F_y = 0: \quad \frac{9.6}{14.6} F_{CH} - 2(2.4 \text{ kips}) = 0$$

$$F_{CH} = +7.30 \qquad F_{CH} = 7.30 \text{ kips} \quad T \blacktriangleleft$$

Since *CH* is in tension, O.K.



PROBLEM 6.67

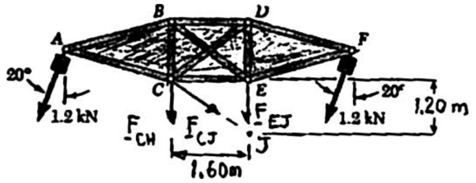
The diagonal members in the center panels of the power transmission line tower shown are very slender and can act only in tension; such members are known as *counters*. For the given loading, determine (*a*) which of the two counters listed below is acting, (*b*) the force in that counter.

Counters *CJ* and *HE*

SOLUTION

<u>Free body: Portion *ABDFEC* of tower</u>.

We assume that counter *CJ* is acting and show the forces exerted by that counter and by members *CH* and *EJ*.

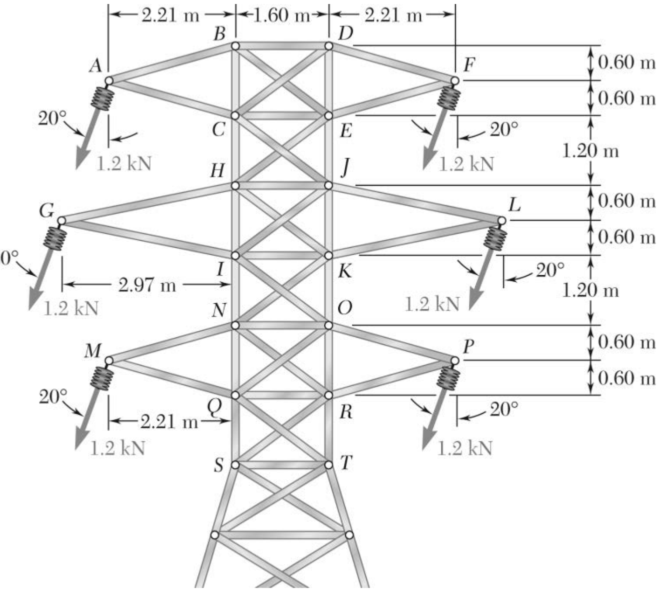


$$\xrightarrow{+}\Sigma F_x = 0: \quad \frac{4}{5}F_{CJ} - 2(1.2\text{ kN})\sin 20^\circ = 0 \qquad F_{CJ} = +1.026\text{ kN}$$

Since *CJ* is found to be in tension, our assumption was correct. Thus, the answers are

(*a*) *CJ* ◀

(*b*) 1.026 kN *T* ◀



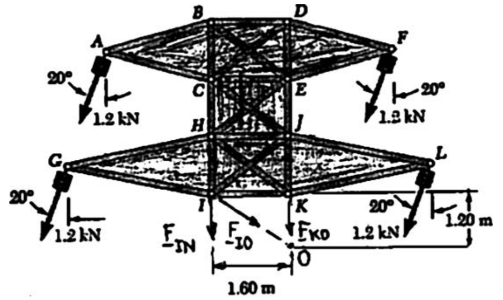
PROBLEM 6.68

The diagonal members in the center panels of the power transmission line tower shown are very slender and can act only in tension; such members are known as *counters*. For the given loading, determine (*a*) which of the two counters listed below is acting, (*b*) the force in that counter.

Counters *IO* and *KN*

SOLUTION

Free body: Portion of tower shown.



We assume that counter *IO* is acting and show the forces exerted by that counter and by members *IN* and *KO*.

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{IO} - 4(1.2 \text{ kN})\sin 20° = 0 \quad F_{IO} = +2.05 \text{ kN}$$

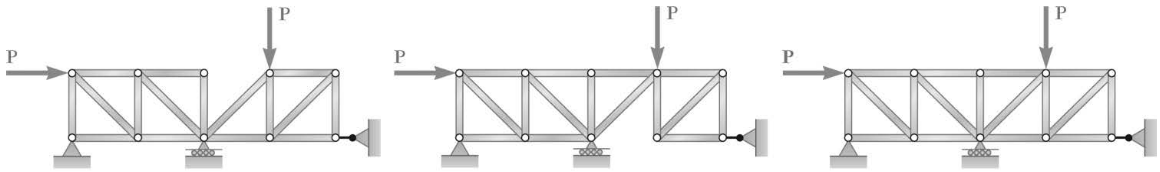
Since *IO* is found to be in tension, our assumption was correct. Thus, the answers are

(*a*) *IO* ◀

(*b*) 2.05 kN *T* ◀

PROBLEM 6.69

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



(*a*) (*b*) (*c*)

SOLUTION

Structure (a)

Number of members:

$$m = 16$$

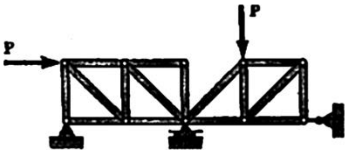
Number of joints:

$$n = 10$$

Reaction components:

$$r = 4$$

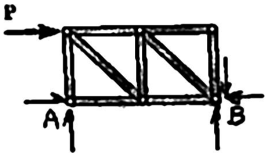
$$m + r = 20, \quad 2n = 20$$



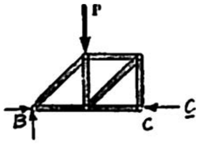
(a)

Thus, $m + r = 2n$ ◁

To determine whether the structure is actually completely constrained and determinate, we must try to find the reactions at the supports. We divide the structure into two simple trusses and draw the free-body diagram of each truss.



This is a properly supported simple truss – O.K.

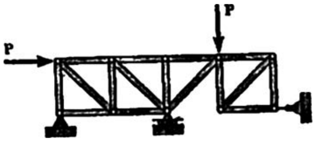


This is an improperly supported simple truss. (Reaction at *C* passes through *B*. Thus, Eq. $\Sigma M_B = 0$ cannot be satisfied.)

Structure is improperly constrained. ◀

Structure (b)

$$m = 16$$
$$n = 10$$
$$r = 4$$
$$m + r = 20, \quad 2n = 20$$

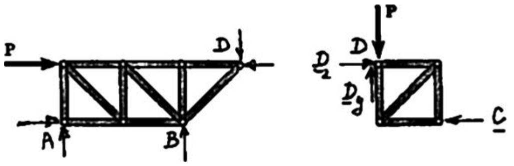


(b)

Thus, $m + r = 2n$ ◁

PROBLEM 6.69 (Continued)

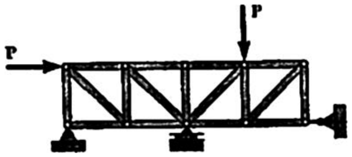
We must again try to find the reactions at the supports dividing the structure as shown.



Both portions are simply supported simple trusses.

Structure is completely constrained and determinate. ◀

Structure (c)



(c)

$$m = 17$$

$$n = 10$$

$$r = 4$$

$$m + r = 21, \quad 2n = 20$$

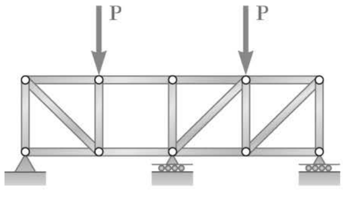
Thus, $m + r > 2n$ ◁

This is a simple truss with an extra support which causes reactions (and forces in members) to be indeterminate.

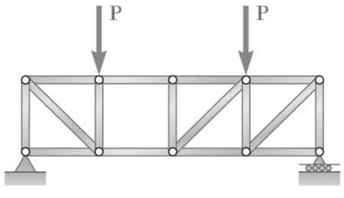
Structure is completely constrained and indeterminate. ◀

PROBLEM 6.70

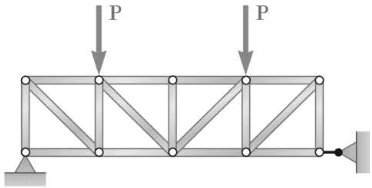
Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



(*a*)



(*b*)



(*c*)

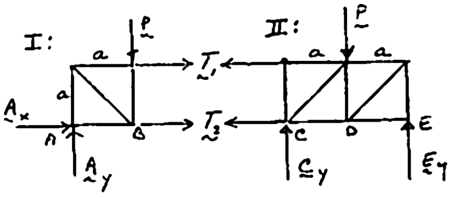
SOLUTION

Structure (a):

Nonsimple truss with $r = 4,\ \ m = 16,\ \ n = 10,$

so $m + r = 20 = 2n,$ but we must examine further.

FBD Sections:



FBD I: $\Sigma M_A = 0 \Rightarrow T_1$

II: $\Sigma F_x = 0 \Rightarrow T_2$

I: $\Sigma F_x = 0 \Rightarrow A_x$

I: $\Sigma F_y = 0 \Rightarrow A_y$

II: $\Sigma M_E = 0 \Rightarrow C_y$

II: $\Sigma F_y = 0 \Rightarrow E_y$

Since each section is a simple truss with reactions determined,

structure is completely constrained and determinate. ◀

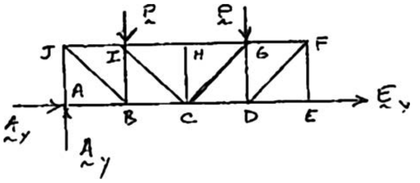
Structure (b):

Nonsimple truss with $r = 3,\ \ m = 16,\ \ n = 10,$

so $m + r = 19 < 2n = 20$ Structure is partially constrained. ◀

PROBLEM 6.70 (Continued)

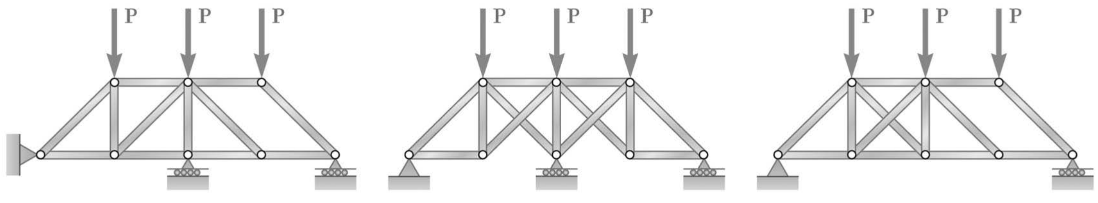
Structure (c):



Simple truss with $r = 3$, $m = 17$, $n = 10$, $m + r = 20 = 2n$, but the horizontal reaction forces A_x and E_x are collinear and no equilibrium equation will resolve them, so the structure is improperly constrained and indeterminate. ◀

PROBLEM 6.71

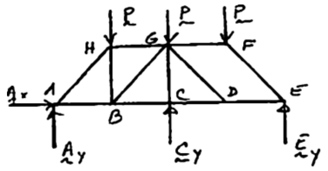
Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)

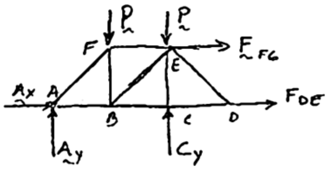


(*a*) (*b*) (*c*)

SOLUTION

Structure (a):





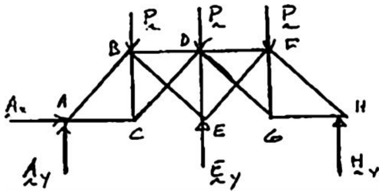
Nonsimple truss with $r = 4,\ m = 12,\ n = 8$ so $r + m = 16 = 2n$.
Check for determinacy:

One can solve joint *F* for forces in *EF*, *FG* and then solve joint *E* for $\mathbf{E}_y$ and force in *DE*.

This leaves a simple truss *ABCDGH* with

$r = 3, m = 9, n = 6$ so $r + m = 12 = 2n$ Structure is completely constrained and determinate. ◀

Structure (b):

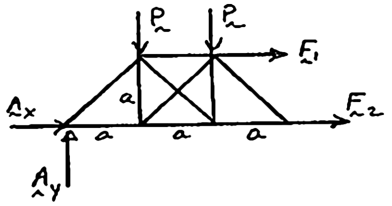


Simple truss (start with *ABC* and add joints alphabetically to complete truss) with $r = 4,\ m = 13,\ n = 8$

so $r + m = 17 > 2n = 16$ Constrained but indeterminate ◀

PROBLEM 6.71 (Continued)

Structure (c):



Nonsimple truss with $r = 3,\ m = 13,\ n = 8$ so $r + m = 16 = 2n.$ To further examine, follow procedure in part (*a*) above to get truss at left.

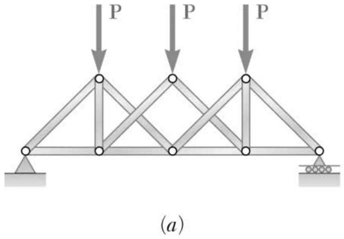
Since $\mathbf{F}_1 \neq 0$ (from solution of joint *F*),

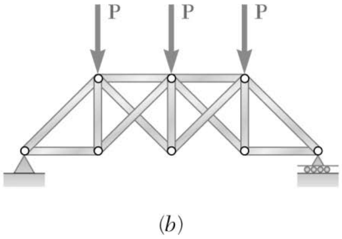
$\Sigma M_A = aF_1 \circlearrowright \neq 0$ and there is no equilibrium.

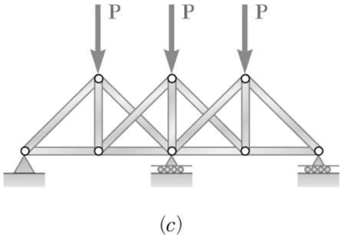
Structure is improperly constrained. ◀

PROBLEM 6.72

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)

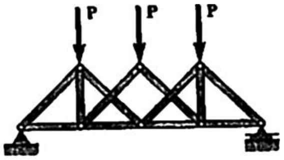






SOLUTION

Structure (a)



Number of members:

$$m = 12$$

Number of joints:

$$n = 8$$

Reaction components:

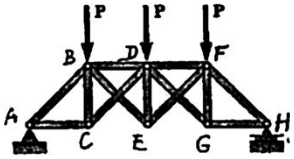
$$r = 3$$

$$m + r = 15, \quad 2n = 16$$

Thus, $m + r < 2n$ ◁

Structure is partially constrained. ◀

Structure (b)



$$m = 13, \; n = 8$$

$$r = 3$$

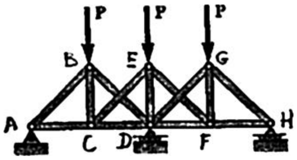
$$m + r = 16, \quad 2n = 16$$

Thus, $m + r = 2n$ ◁

To verify that the structure is actually completely constrained and determinate, we observe that it is a simple truss (follow lettering to check this) and that it is simply supported by a pin-and-bracket and a roller. Thus,

structure is completely constrained and determinate. ◀

Structure (c)



$$m = 13, \quad n = 8$$

$$r = 4$$

$$m + r = 17, \ \ 2n = 16$$

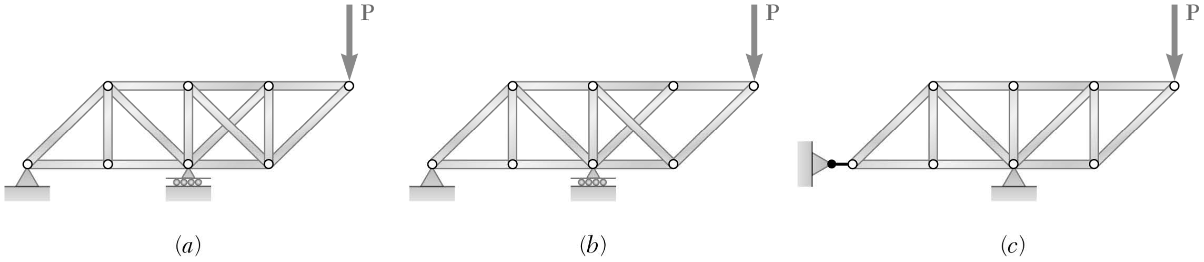
Thus, $m + r > 2n$ ◁

Structure is completely constrained and indeterminate. ◀

This result can be verified by observing that the structure is a simple truss (follow lettering to check this), therefore it is rigid, and that its supports involve four unknowns.

PROBLEM 6.73

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



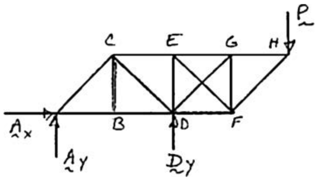
SOLUTION

Structure (a): Rigid truss with $r = 3,\ m = 14,\ n = 8,$

so $r + m = 17 > 2n = 16$

so completely constrained but indeterminate ◀

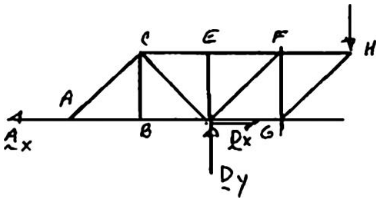
Structure (b): Simple truss (start with *ABC* and add joints alphabetically), with



$r = 3, m = 13, n = 8,$ so $r + m = 16 = 2n$

so completely constrained and determinate ◀

Structure (c):

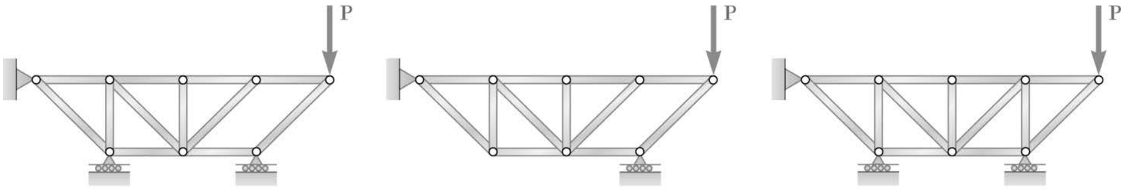


Simple truss with $r = 3,\ \ m = 13,\ \ n = 8,$ so $r + m = 16 = 2n,$ but horizontal reactions $(A_x$ and $D_x)$ are collinear, so cannot be resolved by any equilibrium equation.

Structure is improperly constrained. ◀

PROBLEM 6.74

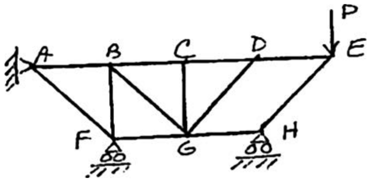
Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



(*a*) (*b*) (*c*)

SOLUTION

Structure (a):

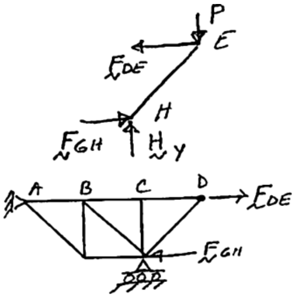


No. of members $m = 12$

No. of joints $n = 8 \quad m + r = 16 = 2n$

No. of reaction components $r = 4 \quad \text{unknows} = \text{equations}$

FBD of EH:

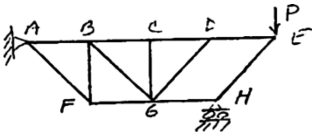


$$\Sigma M_H = 0 \longrightarrow F_{DE};\ \Sigma F_x = 0 \longrightarrow F_{GH};\ \Sigma F_y = 0 \longrightarrow H_y$$

Then *ABCDGF* is a simple truss and all forces can be determined.

This example is completely constrained and determinate. ◀

Structure (b):



No. of members $m = 12$

No. of joints $n = 8 \quad m + r = 15 < 2n = 16$

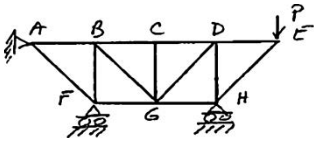
No. of reaction components $r = 3 \quad \text{unknows} < \text{equations}$

partially constrained ◀

Note: Quadrilateral *DEHG* can collapse with joint *D* moving downward; in (a), the roller at *F* prevents this action.

PROBLEM 6.74 (Continued)

Structure (c):

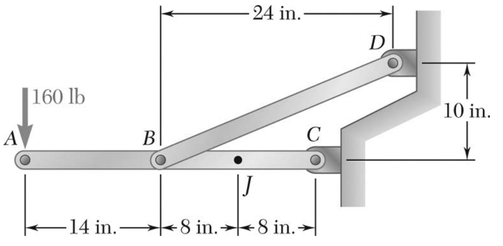


No. of members $m = 13$

No. of joints $n = 8$ $m + r = 17 > 2n = 16$

No. of reaction components $r = 4$ unknows > equations

completely constrained but indeterminate ◀



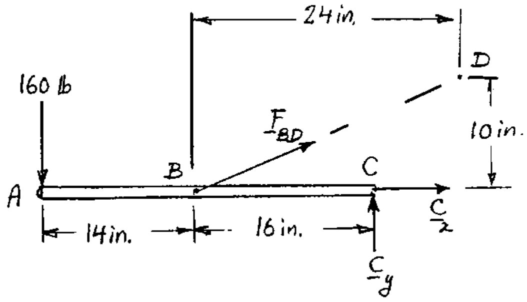
PROBLEM 6.75

Determine the force in member *BD* and the components of the reaction at *C*.

SOLUTION

We note that *BD* is a two-force member. The force it exerts on *ABC*, therefore, is directed along line *BD*.

Free body: *ABC*:



$$BD = \sqrt{(24)^2 + (10)^2} = 26 \text{ in.}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad (160 \text{ lb})(30 \text{ in.}) - \left(\frac{10}{26} F_{BD}\right)(16 \text{ in.}) = 0$$

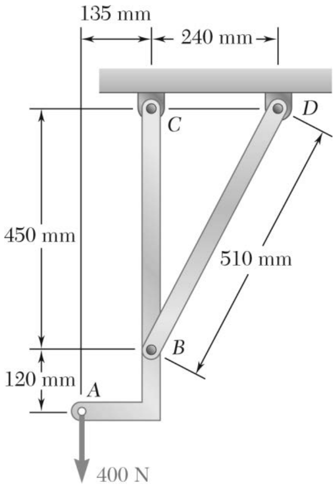
$F_{BD} = +780$ lb $\qquad$ $F_{BD} = 780$ lb T ◀

$$\xrightarrow{+} \Sigma M_x = 0: \quad C_x + \frac{24}{26}(780 \text{ lb}) = 0$$

$C_x = -720$ lb $\qquad$ $\mathbf{C}_x = 720$ lb ⟵ ◀

$$+\uparrow \Sigma F_y = 0: \quad C_y - 160 \text{ lb} + \frac{10}{26}(780 \text{ lb}) = 0$$

$C_y = -140.0$ lb $\qquad$ $\mathbf{C}_y = 140.0$ lb ↓ ◀



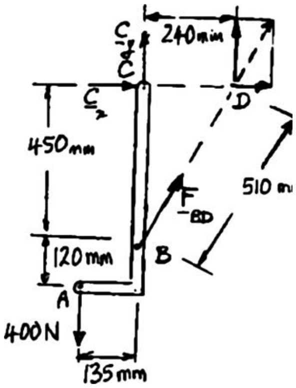
PROBLEM 6.76

Determine the force in member *BD* and the components of the reaction at *C*.

SOLUTION

We note that *BD* is a two-force member. The force it exerts on *ABC*, therefore, is directed along line *BD*.

Free body: *ABC*:



Attaching $\mathbf{F}_{BD}$ at *D* and resolving it into components, we write

$$+\circlearrowleft \Sigma M_C = 0: \quad (400\text{ N})(135\text{ mm}) + \left(\frac{450}{510}F_{BD}\right)(240\text{ mm}) = 0$$

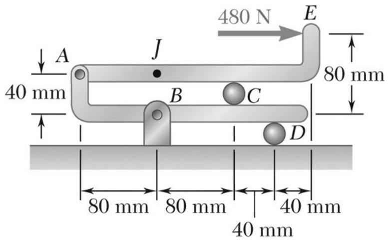
$$F_{BD} = -255\text{ N} \qquad F_{BD} = 255\text{ N} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + \frac{240}{510}(-255\text{ N}) = 0$$

$$C_x = +120.0\text{ N} \qquad \mathbf{C}_x = 120.0\text{ N} \longrightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 400\text{ N} + \frac{450}{510}(-255\text{ N}) = 0$$

$$C_y = +625\text{ N} \qquad \mathbf{C}_y = 625\text{ N}\uparrow \blacktriangleleft$$

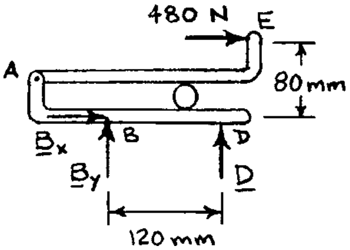


PROBLEM 6.77

Determine the components of all forces acting on member *ABCD* of the assembly shown.

SOLUTION

Free body: Entire assembly:



$$+\circlearrowleft \Sigma M_B = 0: \quad D(120\text{ mm}) - (480\text{ N})(80\text{ mm}) = 0$$

$$\mathbf{D} = 320\text{ N}\uparrow \blacktriangleleft$$

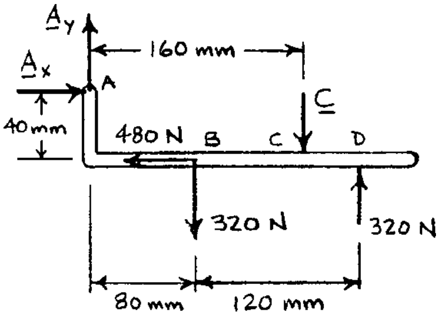
$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 480\text{ N} = 0$$

$$\mathbf{B}_x = 480\text{ N} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 320\text{ N} = 0$$

$$\mathbf{B}_y = 320\text{ N}\downarrow \blacktriangleleft$$

Free body: Member *ABCD*:



$$+\circlearrowleft \Sigma M_A = 0: \quad (320\text{ N})(200\text{ mm}) - C(160\text{ mm}) - (320\text{ N})(80\text{ mm})$$
$$- (480\text{ N})(40\text{ mm}) = 0$$

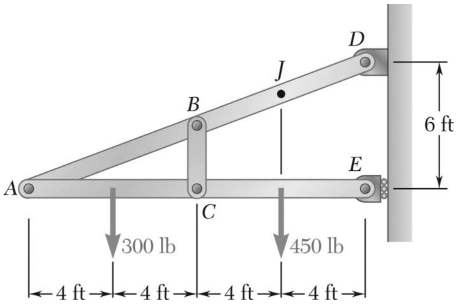
$$\mathbf{C} = 120.0\text{ N}\downarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 480\text{ N} = 0$$

$$\mathbf{A}_x = 480\text{ N} \longrightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 320\text{ N} - 120\text{ N} + 320\text{ N} = 0$$

$$\mathbf{A}_y = 120.0\text{ N}\uparrow \blacktriangleleft$$

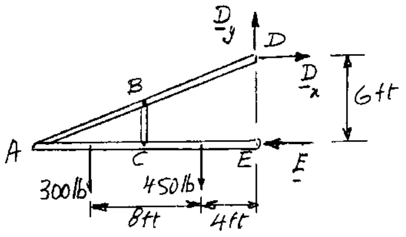


PROBLEM 6.78

Determine the components of all forces acting on member *ABD* of the frame shown.

SOLUTION

<u>Free body: Entire frame</u>:



$$+\circlearrowleft \Sigma M_D = 0: \quad (300 \text{ lb}) - (12 \text{ ft}) + (450 \text{ lb})(4 \text{ ft}) - E(6 \text{ ft}) = 0$$

$\mathbf{E} = +900$ lb $\quad \mathbf{E} = 900$ lb ← ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - 900 \text{ lb} = 0$$

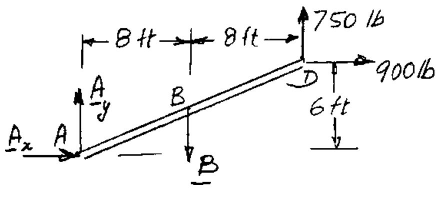
$\mathbf{D}_x = 900$ lb → ◀

$$+\uparrow \Sigma F_y = 0: \quad D_y - 300 \text{ lb} - 450 \text{ lb} = 0$$

$\mathbf{D}_y = 750$ lb ↑ ◀

<u>Free body: Member *ABD*</u>:

We note that *BC* is a two-force member and that **B** is directed along *BC*.



$$+\circlearrowleft \Sigma M_A = 0: \quad (750 \text{ lb})(16 \text{ ft}) - (900 \text{ lb})(6 \text{ ft}) - B(8 \text{ ft}) = 0$$

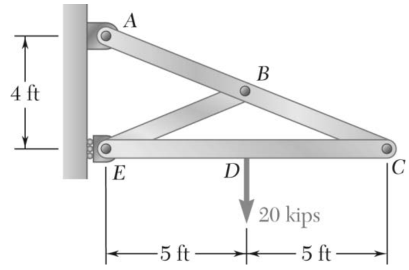
$B = +825$ lb $\quad \mathbf{B} = 825$ lb ↓ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 900 \text{ lb} = 0$$

$A_x = -900$ lb $\quad \mathbf{A}_x = 900$ lb ← ◀

$$+\uparrow \Sigma F_y = 0: \quad A_y + 750 \text{ lb} - 825 \text{ lb} = 0$$

$A_y = +75$ lb $\quad \mathbf{A}_y = 75.0$ lb ↑ ◀

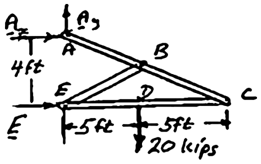


PROBLEM 6.79

For the frame and loading shown, determine the components of all forces acting on member *ABC*.

SOLUTION

Free body: Entire frame:



$+\circlearrowleft \Sigma M_E = 0: \quad -A_x(4) - (20 \text{ kips})(5) = 0$

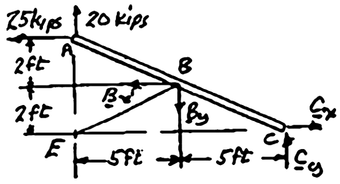
$A_x = -25 \text{ kips}, \qquad \mathbf{A}_x = 25.0 \text{ kips} \longleftarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad A_y - 20 \text{ kips} = 0$

$A_y = 20 \text{ kips} \qquad \mathbf{A}_y = 20.0 \text{ kips} \uparrow$ ◀

Free body: Member *ABC*:

Note: BE is a two-force member, thus **B** is directed along line *BE* and $B_y = \frac{2}{5} B_x$.



$+\circlearrowleft \Sigma M_C = 0: \quad (25 \text{ kips})(4 \text{ ft}) - (20 \text{ kips})(10 \text{ ft}) + B_x(2 \text{ ft}) + B_y(5 \text{ ft}) = 0$

$-100 \text{ kip} \cdot \text{ft} + B_x(2 \text{ ft}) + \frac{2}{5} B_x(5 \text{ ft}) = 0$

$B_x = 25 \text{ kips} \qquad \mathbf{B}_x = 25.0 \text{ kips} \longleftarrow$ ◀

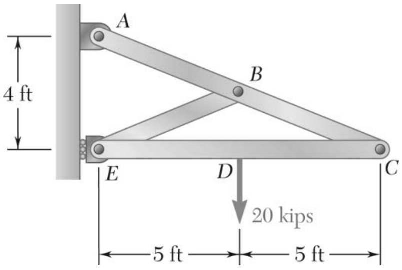
$B_y = \frac{2}{5}(B_x) = \frac{2}{5}(25) = 10 \text{ kips} \qquad \mathbf{B}_y = 10.00 \text{ kips} \downarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 25 \text{ kips} - 25 \text{ kips} = 0$

$C_x = 50 \text{ kips} \qquad \mathbf{C}_x = 50.0 \text{ kips} \longrightarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad C_y + 20 \text{ kips} - 10 \text{ kips} = 0$

$C_y = -10 \text{ kips} \qquad \mathbf{C}_y = 10.00 \text{ kips} \downarrow$ ◀



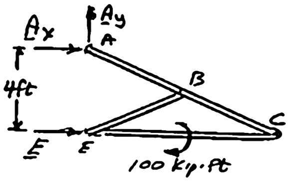
PROBLEM 6.80

Solve Problem 6.79 assuming that the 20-kip load is replaced by a clockwise couple of magnitude 100 kip · ft applied to member *EDC* at Point *D*.

PROBLEM 6.79 For the frame and loading shown, determine the components of all forces acting on member *ABC*.

SOLUTION

Free body: Entire frame:



$$+\uparrow \Sigma F_y = 0: \quad A_y = 0$$

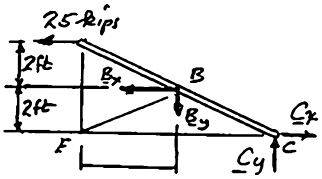
$$+\circlearrowright \Sigma M_E = 0: \quad -A_x(4\text{ ft}) - 100\text{ kip}\cdot\text{ft} = 0$$

$A_x = -25$ kips $\qquad$ $\mathbf{A}_x = 25.0$ kips ←

$\mathbf{A} = 25.0$ kips ← ◀

Free body: Member *ABC*:

Note: BE is a two-force member, thus **B** is directed along line *BE* and $B_y = \dfrac{2}{5}B_x$.



$$+\circlearrowright \Sigma M_C = 0: \quad (25\text{ kips})(4\text{ ft}) + B_x(2\text{ ft}) + B_y(5\text{ ft}) = 0$$

$$100\text{ kip}\cdot\text{ft} + B_x(2\text{ ft}) + \frac{2}{5}B_x(5\text{ ft}) = 0$$

$B_x = -25$ kips $\qquad$ $\mathbf{B}_x = 25.0$ kips → ◀

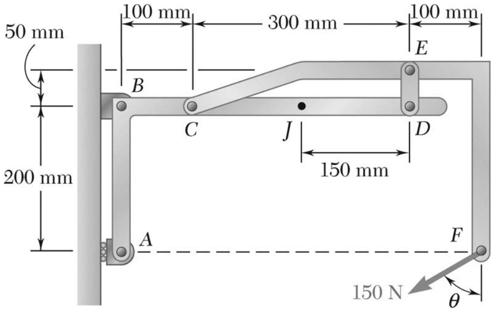
$B_y = \dfrac{2}{5}B_x = \dfrac{2}{5}(-25) = -10$ kips; $\qquad$ $\mathbf{B}_y = 10.00$ kips ↑ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad -25\text{ kips} + 25\text{ kips} + C_x = 0 \qquad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad +10\text{ kips} + C_y = 0$$

$C_y = -10$ kips $\qquad$ $C_y = 10$ kips ↓

$\mathbf{C} = 10.00$ kips ↓ ◀

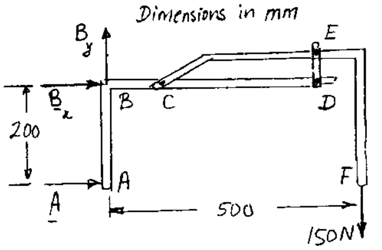


PROBLEM 6.81

Determine the components of all forces acting on member *ABCD* when $\theta = 0$.

SOLUTION

Free body: Entire assembly:



$$+\circlearrowleft \Sigma M_B = 0: \quad A(200) - (150\text{ N})(500) = 0$$

$$A = +375\text{ N} \qquad \mathbf{A} = 375\text{ N} \longrightarrow ◀$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 375\text{ N} = 0$$

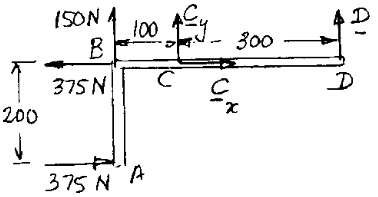
$$B_x = -375\text{ N} \qquad \mathbf{B}_x = 375\text{ N} \longleftarrow ◀$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 150\text{ N} = 0$$

$$B_y = +150\text{ N} \qquad \mathbf{B}_y = 150\text{ N}\uparrow ◀$$

Free body: Member *ABCD*:

We note that **D** is directed along *DE*, since *DE* is a two-force member.



$$+\circlearrowleft \Sigma M_C = 0: \quad D(300) - (150\text{ N})(100) + (375\text{ N})(200) = 0$$

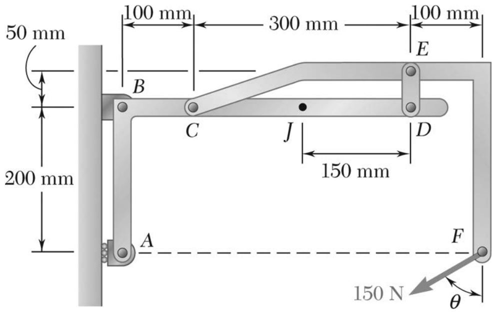
$$D = -200\text{ N} \qquad \mathbf{D} = 200\text{ N}\downarrow ◀$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 375 - 375 = 0$$

$$C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 150 - 200 = 0$$

$$C_y = +50.0\text{ N} \qquad \mathbf{C} = 50.0\text{ N}\uparrow ◀$$

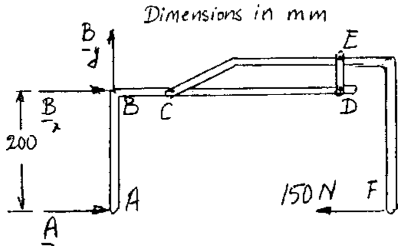


PROBLEM 6.82

Determine the components of all forces acting on member *ABCD* when $\theta = 90°$.

SOLUTION

Free body: Entire assembly:



$$+\circlearrowleft \Sigma M_B = 0: \quad A(200) - (150 \text{ N})(200) = 0$$

$$A = +150.0 \text{ N} \qquad \mathbf{A} = 150.0 \text{ N} \longrightarrow \blacktriangleleft$$

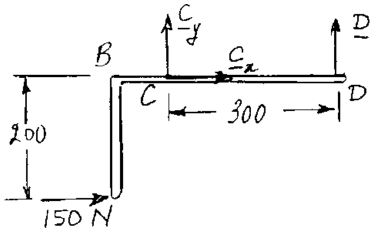
$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 150 - 150 = 0$$

$$B_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad B_y = 0 \qquad \mathbf{B} = 0 \blacktriangleleft$$

Free body: Member *ABCD*:

We note that **D** is directed along *DE*, since *DE* is a two-force member.



$$+\circlearrowleft \Sigma M_C = 0: \quad D(300) + (150 \text{ N})(200) = 0$$

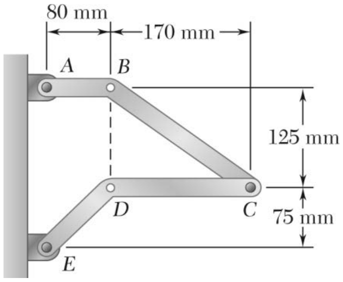
$$D = -100.0 \text{ N} \qquad \mathbf{D} = 100.0 \text{ N} \downarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 150 \text{ N} = 0$$

$$C_x = -150 \text{ N} \qquad \mathbf{C}_x = 150.0 \text{ N} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 100 \text{ N} = 0$$

$$C_y = +100.0 \text{ N} \qquad \mathbf{C}_y = 100.0 \text{ N} \uparrow \blacktriangleleft$$



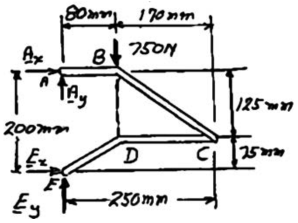
PROBLEM 6.83

Determine the components of the reactions at *A* and *E* if a 750-N force directed vertically downward is applied (*a*) at *B*, (*b*) at *D*.

SOLUTION

Free body: Entire frame:

The following analysis is valid for both parts (*a*) and (*b*) since position of load on its line of action is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: \quad -(750\text{ N})(80\text{ mm}) - A_x(200\text{ mm}) = 0$$

$$A_x = -300\text{ N} \quad \mathbf{A}_x = 300\text{ N} \longleftarrow$$

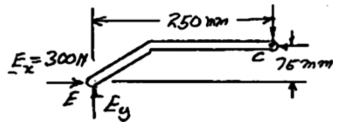
$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x - 300\text{ N} = 0 \quad E_x = 300\text{ N} \quad \mathbf{E}_x = 300\text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y - 750\text{ N} = 0 \tag{1}$$

(*a*) Load applied at *B*.

Free body: Member *CE*:

CE is a two-force member. Thus, the reaction at *E* must be directed along *CE*.



$$\frac{E_y}{300\text{ N}} = \frac{75\text{ mm}}{250\text{ mm}} \quad E_y = 90\text{ N}\uparrow$$

From Eq. (1): $A_y + 90\text{ N} - 750\text{ N} = 0$ $\qquad A_y = 660\text{ N}\uparrow$

Thus, reactions are

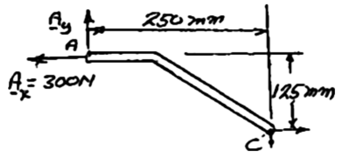
$$\mathbf{A}_x = 300\text{ N} \longleftarrow, \qquad \mathbf{A}_y = 660\text{ N}\uparrow \blacktriangleleft$$

$$\mathbf{E}_x = 300\text{ N} \longrightarrow, \qquad \mathbf{E}_y = 90.0\text{ N}\uparrow \blacktriangleleft$$

(*b*) Load applied at *D*.

Free body: Member *AC*:

AC is a two-force member. Thus, the reaction at *A* must be directed along *AC*.



$$\frac{A_y}{300\text{ N}} = \frac{125\text{ mm}}{250\text{ mm}} \quad A_y = 150\text{ N}\uparrow$$

PROBLEM 6.83 (Continued)

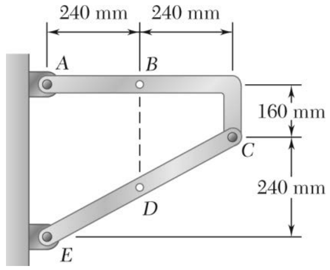
From Eq. (1): $A_y + E_y - 750\text{ N} = 0$

$$150\text{ N} + E_y - 750\text{ N} = 0$$

$$E_y = 600\text{ N} \quad \mathbf{E}_y = 600\text{ N}\uparrow$$

Thus, reactions are $\mathbf{A}_x = 300\text{ N} \leftarrow, \quad \mathbf{A}_y = 150.0\text{ N}\uparrow$ ◀

$\mathbf{E}_x = 300\text{ N} \rightarrow, \quad \mathbf{E}_y = 600\text{ N}\uparrow$ ◀



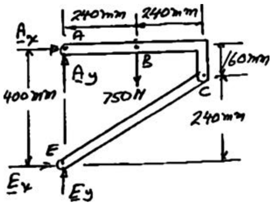
PROBLEM 6.84

Determine the components of the reactions at *A* and *E* if a 750-N force directed vertically downward is applied (*a*) at *B*, (*b*) at *D*.

SOLUTION

Free-body: Entire frame:

The following analysis is valid for both parts (*a*) and (*b*) since position of load on its line of action is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: \quad -(750\text{ N})(240\text{ mm}) - A_x(400\text{ mm}) = 0$$

$$A_x = -450\text{ N} \quad \mathbf{A}_x = 450\text{ N} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x - 450\text{ N} = 0 \quad E_x = 450\text{ N} \quad \mathbf{E}_x = 450\text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y - 750\text{ N} = 0 \tag{1}$$

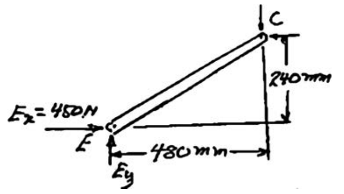
(*a*) Load applied at *B*.

Free body: Member *CE*:

CE is a two-force member. Thus, the reaction at *E* must be directed along *CE*.

$$\frac{E_y}{450\text{ N}} = \frac{240\text{ mm}}{480\text{ mm}}; \quad \mathbf{E}_y = 225\text{ N}\uparrow$$

From Eq. (1): $A_y + 225 - 750 = 0; \quad \mathbf{A}_y = 525\text{ N}\uparrow$



Thus, reactions are

$\mathbf{A}_x = 450\text{ N} \longleftarrow, \qquad \mathbf{A}_y = 525\text{ N}\uparrow$ ◀

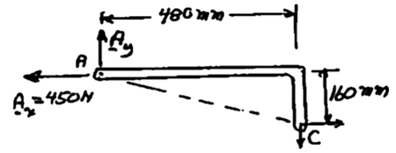
$\mathbf{E}_x = 450\text{N} \longrightarrow, \qquad \mathbf{E}_y = 225\text{ N}\uparrow$ ◀

(*b*) Load applied at *D*.

Free body: Member *AC*:

AC is a two-force member. Thus, the reaction at *A* must be directed along *AC*.

$$\frac{A_y}{450\text{ N}} = \frac{160\text{ mm}}{480\text{ mm}} \quad \mathbf{A}_y = 150.0\text{ N}\uparrow$$



From Eq. (1):

$$A_y + E_y - 750\text{ N} = 0$$

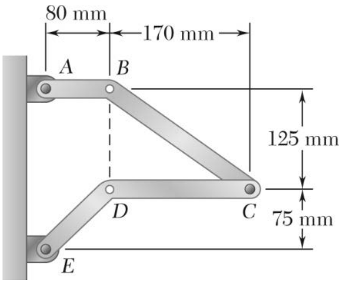
$$150\text{ N} + E_y - 750\text{ N} = 0$$

$$E_y = 600\text{ N} \quad \mathbf{E}_y = 600\text{ N}\uparrow$$

Thus, reactions are

$\mathbf{A}_x = 450\text{ N} \longleftarrow, \qquad \mathbf{A}_y = 150.0\text{ N}\uparrow$ ◀

$\mathbf{E}_x = 450\text{ N} \longrightarrow, \qquad \mathbf{E}_y = 600\text{ N}\uparrow$ ◀



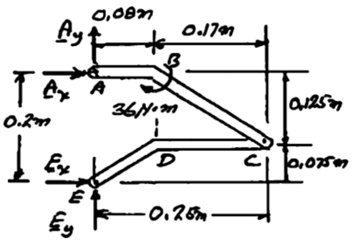
PROBLEM 6.85

Determine the components of the reactions at A and E if the frame is loaded by a clockwise couple of magnitude 36 N · m applied (a) at B, (b) at D.

SOLUTION

Free body: Entire frame:

The following analysis is valid for both parts (a) and (b) since the point of application of the couple is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: \quad -36\text{ N}\cdot\text{m} - A_x(0.2\text{ m}) = 0$$

$$A_x = -180\text{ N} \quad \mathbf{A}_x = 180\text{ N} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -180\text{ N} + E_x = 0$$

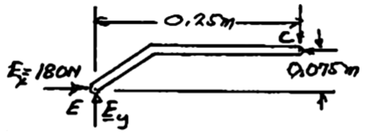
$$E_x = 180\text{ N} \quad \mathbf{E}_x = 180\text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y = 0 \tag{1}$$

(a) Couple applied at B.

Free body: Member CE:

AC is a two-force member. Thus, the reaction at E must be directed along EC.



$$\frac{E_y}{180\text{ N}} = \frac{0.075\text{ m}}{0.25\text{ m}} \quad \mathbf{E}_y = 54\text{ N}\uparrow$$

From Eq. (1): $\quad A_y + 54\text{ N} = 0$

$$A_y = -54\text{ N} \quad \mathbf{A}_y = 54.0\text{ N}\downarrow$$

Thus, reactions are

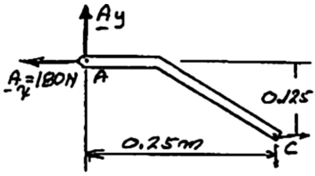
$$\mathbf{A}_x = 180.0\text{ N} \longleftarrow, \quad \mathbf{A}_y = 54.0\text{ N}\downarrow \blacktriangleleft$$

$$\mathbf{E}_x = 180.0\text{ N} \longrightarrow, \quad \mathbf{E}_y = 54.0\text{ N}\uparrow \blacktriangleleft$$

(b) Couple applied at D.

Free body: Member AC:

AC is a two-force member. Thus, the reaction at A must be directed along EC.



$$\frac{A_y}{180\text{ N}} = \frac{0.125\text{ m}}{0.25\text{ m}} \quad A_y = 90\text{ N}\uparrow$$

PROBLEM 6.85 (Continued)

From Eq. (1):

$$A_y + E_y = 0$$

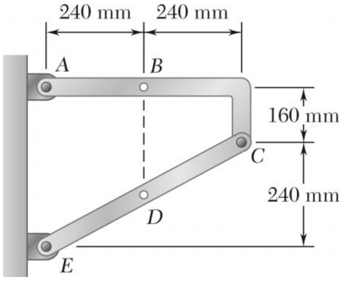
$$90\text{ N} + E_y = 0$$

$$E_y = -90\text{ N} \quad \mathbf{E}_y = 90\text{ N}\downarrow$$

Thus, reactions are

$$\mathbf{A}_x = 180.0\text{ N}\longleftarrow, \qquad \mathbf{A}_y = 90.0\text{ N}\uparrow \blacktriangleleft$$

$$\mathbf{E}_x = -180.0\text{ N}\longrightarrow, \qquad \mathbf{E}_y = 90.0\text{ N}\downarrow \blacktriangleleft$$



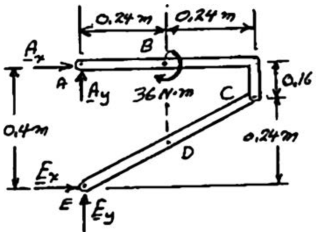
PROBLEM 6.86

Determine the components of the reactions at *A* and *E* if the frame is loaded by a clockwise couple of magnitude 36 N · m applied (*a*) at *B*, (*b*) at *D*.

SOLUTION

Free body: Entire frame:

The following analysis is valid for both parts (*a*) and (*b*) since the point of application of the couple is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: \quad -36\text{ N}\cdot\text{m} - A_x(0.4\text{ m}) = 0$$

$$A_x = -90\text{ N} \quad \mathbf{A}_x = 90.0\text{ N} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -90 + E_x = 0$$

$$E_x = 90\text{ N} \quad \mathbf{E}_x = 90.0\text{ N} \longrightarrow$$

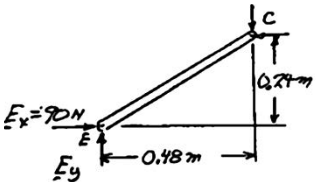
$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y = 0 \qquad (1)$$

(*a*) Couple applied at *B*.

Free body: Member *CE*:

AC is a two-force member. Thus, the reaction at *E* must be directed along *EC*.

$$\frac{E_y}{90\text{ N}} = \frac{0.24\text{ m}}{0.48\text{ m}}; \quad \mathbf{E}_y = 45.0\text{ N}\uparrow$$



From Eq. (1): $A_y + 45\text{ N} = 0$

$$A_y = -45\text{ N} \quad \mathbf{A}_y = 45.0\text{ N}\downarrow$$

Thus, reactions are

$$\mathbf{A}_x = 90.0\text{ N} \longleftarrow, \qquad \mathbf{A}_y = 45.0\text{ N}\downarrow \blacktriangleleft$$

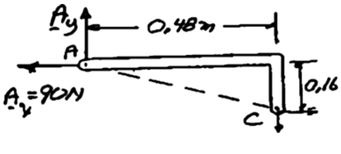
$$\mathbf{E}_x = 90.0\text{ N} \longrightarrow, \qquad \mathbf{E}_y = 45.0\text{ N}\uparrow \blacktriangleleft$$

(*b*) Couple applied at *D*.

Free body: Member *AC*:

AC is a two-force member. Thus, the reaction at *A* must be directed along *AC*.

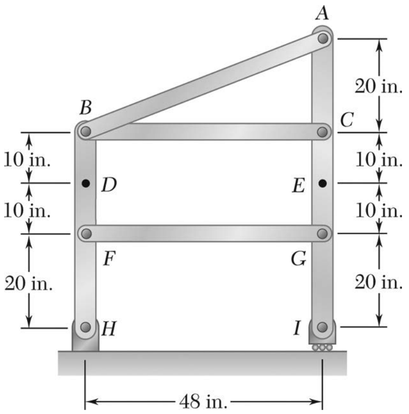
$$\frac{A_y}{90\text{ N}} = \frac{0.16\text{ m}}{0.48\text{ m}}; \quad \mathbf{A}_y = 30\text{ N}\uparrow$$



PROBLEM 6.86 (Continued)

From Eq. (1):
$$A_y + E_y = 0$$
$$30\text{ N} + E_y = 0$$
$$E_y = -30\text{ N} \quad \mathbf{E}_y = 30\text{ N}\downarrow$$

Thus, reactions are
$$\mathbf{A}_x = 90.0\text{ N} \longleftarrow, \qquad \mathbf{A}_y = 30.0\text{ N}\uparrow \blacktriangleleft$$
$$\mathbf{E}_x = -90.0\text{ N} \longrightarrow, \qquad \mathbf{E}_y = 30.0\text{ N}\downarrow \blacktriangleleft$$



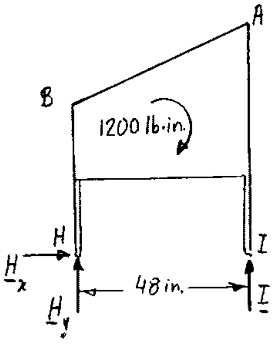
PROBLEM 6.87

Determine all the forces exerted on member *AI* if the frame is loaded by a clockwise couple of magnitude 1200 lb · in. applied (*a*) at Point *D*, (*b*) at Point *E*.

SOLUTION

Free body: Entire frame:

Location of couple is immaterial.



$$+\circlearrowleft \Sigma M_H = 0: \quad I(48\text{ in.}) - 1200\text{ lb}\cdot\text{in.} = 0$$

$$I = +25.0\text{ lb}$$

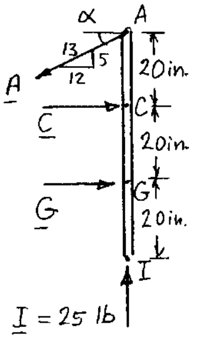
(*a*) and (*b*) $\mathbf{I} = 25.0\text{ lb}\uparrow$ ◀

We note that *AB*, *BC*, and *FG* are two-force members.

Free body: Member *AI*:

$$\tan\alpha = \frac{20}{48} = \frac{5}{12} \qquad \alpha = 22.6^\circ$$

(*a*) Couple applied at *D*.



$$+\uparrow \Sigma F_y = 0: \quad -\frac{5}{13}A + 25\text{ lb} = 0$$

$$A = +65.0\text{ lb} \qquad \mathbf{A} = 65.0\text{ lb} \swarrow 22.6^\circ$$ ◀

$$+\circlearrowleft \Sigma M_G = 0: \quad \frac{12}{13}(65\text{ lb})(40\text{ in.}) - C(20\text{ in.}) = 0$$

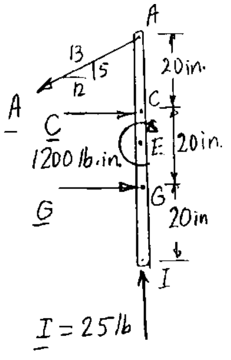
$$C = +120\text{ lb} \qquad \mathbf{C} = 120\text{ lb} \longrightarrow$$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{12}{13}(65\text{ lb}) + 120\text{ lb} + G = 0$$

$$G = -60.0\text{ lb} \qquad \mathbf{G} = 60\text{ lb} \longleftarrow$$ ◀

PROBLEM 6.87 (Continued)

(*b*) Couple applied at *E*.



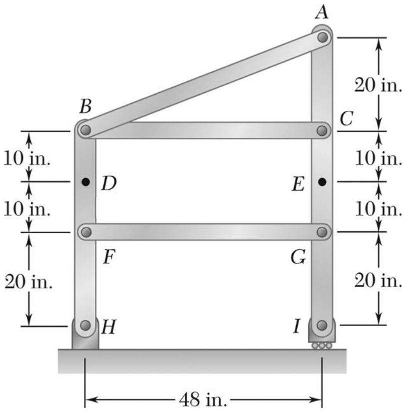
$$+\uparrow \Sigma F_y = 0: \quad -\frac{5}{13}A + 25 \text{ lb} = 0$$

$$A = +65.0 \text{ lb} \qquad \mathbf{A} = 65.0 \text{ lb} \swarrow 22.6° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_G = 0: \quad \frac{12}{13}(65 \text{ lb}) + (40 \text{ in.}) - C(20 \text{ in.}) - 1200 \text{ lb}\cdot\text{in.} = 0$$

$$C = +60.0 \text{ lb} \qquad \mathbf{C} = 60.0 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\underrightarrow{+} \Sigma F_x = 0: \quad -\frac{12}{13}(65 \text{ lb}) + 60 \text{ lb} + G = 0 \qquad \mathbf{G} = 0 \blacktriangleleft$$



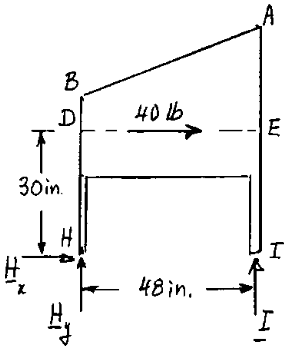
PROBLEM 6.88

Determine all the forces exerted on member *AI* if the frame is loaded by a 40-lb force directed horizontally to the right and applied (*a*) at Point *D*, (*b*) at Point *E*.

SOLUTION

Free body: Entire frame:

Location of 40-lb force on its line of action *DE* is immaterial.



$$+\circlearrowleft \Sigma M_H = 0: \quad I(48 \text{ in.}) - (40 \text{ lb})(30 \text{ in.}) = 0$$

$$I = +25.0 \text{ lb}$$

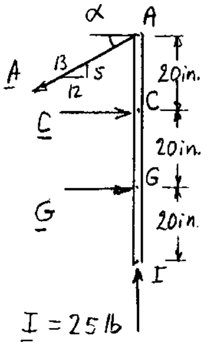
(*a*) and (*b*) $\mathbf{I} = 25.0 \text{ lb} \uparrow$ ◀

We note that *AB*, *BC*, and *FG* are two-force members.

Free body: Member *AI*:

$$\tan\alpha = \frac{20}{48} = \frac{5}{12} \qquad \alpha = 22.6°$$

(*a*) Force applied at *D*.



$$+\uparrow \Sigma F_y = 0: \quad -\frac{5}{13}A + 25 \text{ lb} = 0$$

$$A = +65.0 \text{ lb} \qquad \mathbf{A} = 65.0 \text{ lb} \swarrow 22.6° ◀$$

$$+\circlearrowleft \Sigma M_G = 0: \quad \frac{12}{13}(65 \text{ lb})(40 \text{ in.}) - C(20 \text{ in.}) = 0$$

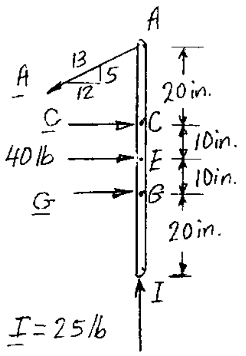
$$C = +120.0 \text{ lb} \qquad \mathbf{C} = 120.0 \text{ lb} \longrightarrow ◀$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{12}{13}(65 \text{ lb}) + 120 \text{ lb} + G = 0$$

$$G = -60.0 \text{ lb} \qquad \mathbf{G} = 60.0 \text{ lb} \longleftarrow ◀$$

PROBLEM 6.88 (Continued)

(*b*) Force applied at *E*.



$$+\uparrow \Sigma F_y = 0: \quad -\frac{5}{13}A + 25\text{ lb} = 0$$

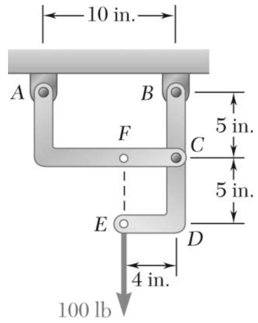
$A = +65.0\text{ lb}$ $\qquad$ $\mathbf{A} = 65.0\text{ lb} \measuredangle 22.6^\circ$ ◀

$$+\circlearrowleft \Sigma M_G = 0: \quad \frac{12}{13}(65\text{ lb})(40\text{ in.}) - C(20\text{ in.}) - (40\text{ lb})(10\text{ in.}) = 0$$

$C = +100.0\text{ lb}$ $\qquad$ $\mathbf{C} = 100.0\text{ lb} \longrightarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{12}{13}(65\text{ lb}) + 100\text{ lb} + 40\text{ lb} + G = 0$$

$G = -80.0\text{ lb}$ $\qquad$ $\mathbf{G} = 80.0\text{ lb} \longleftarrow$ ◀

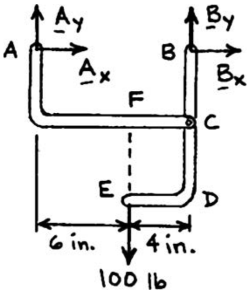


PROBLEM 6.89

Determine the components of the reactions at *A* and *B*, (*a*) if the 500-N load is applied as shown, (*b*) if the 500-N load is moved along its line of action and is applied at Point *F*.

SOLUTION

Free body: Entire frame:



Analysis is valid for either parts (*a*) or (*b*), since position of 100-lb load on its line of action is immaterial.

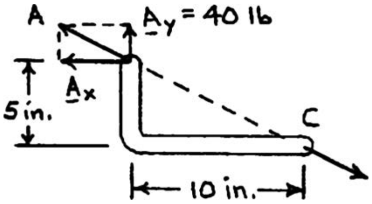
$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(10) - (100\text{ lb})(6) = 0 \quad B_y = +60\text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 60 - 100 = 0 \qquad A_y = +40\text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + B_x = 0 \tag{1}$$

(*a*) Load applied at *E*.

Free body: Member *AC*:



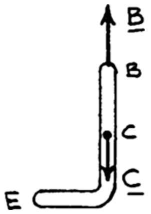
Since *AC* is a two-force member, the reaction at *A* must be directed along *CA*. We have

$$\frac{A_x}{10\text{ in.}} = \frac{40\text{ lb}}{5\text{ in.}} \qquad \mathbf{A}_x = 80.0\text{ lb} \longleftarrow, \qquad \mathbf{A}_y = 40.0\text{ lb}\uparrow \blacktriangleleft$$

From Eq. (1): $-80 + B_x = 0 \quad B_x = +80\text{ lb}$

Thus, $\mathbf{B}_x = 80.0\text{ lb} \longrightarrow, \qquad \mathbf{B}_y = 60.0\text{ lb}\uparrow \blacktriangleleft$

PROBLEM 6.89 (Continued)



(*b*) Load applied at *F*.

Free body: Member *BCD*:

Since *BCD* is a two-force member (with forces applied at *B* and *C* only), the reaction at *B* must be directed along *CB*. We have, therefore,

$$B_x = 0$$

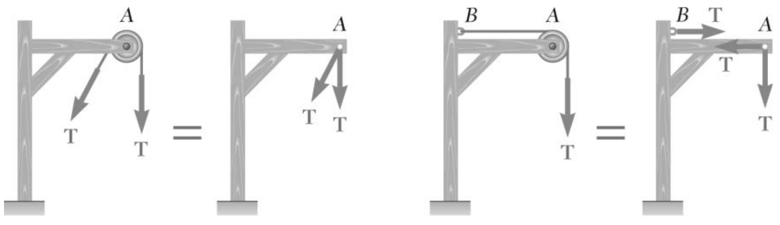
The reaction at *B* is $\mathbf{B}_x = 0$ $\quad$ $\mathbf{B}_y = 60.0 \text{ lb} \uparrow$ ◀

From Eq. (1): $A_x + 0 = 0 \quad A_x = 0$

The reaction at *A* is $\mathbf{A}_x = 0$ $\quad$ $\mathbf{A}_y = 40.0 \text{ lb} \uparrow$ ◀

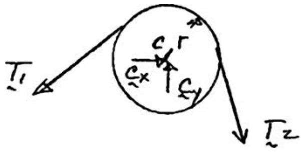
PROBLEM 6.90

(*a*) Show that when a frame supports a pulley at *A*, an equivalent loading of the frame and of each of its component parts can be obtained by removing the pulley and applying at *A* two forces equal and parallel to the forces that the cable exerted on the pulley. (*b*) Show that if one end of the cable is attached to the frame at a Point *B*, a force of magnitude equal to the tension in the cable should also be applied at *B*.



(*a*) (*b*)

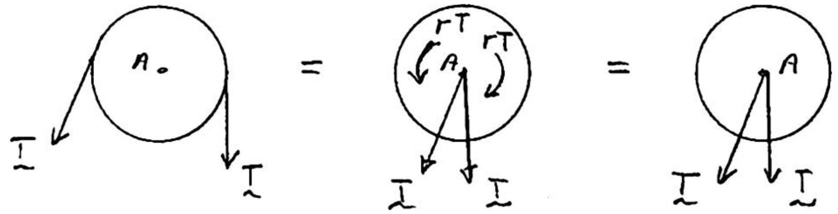
SOLUTION



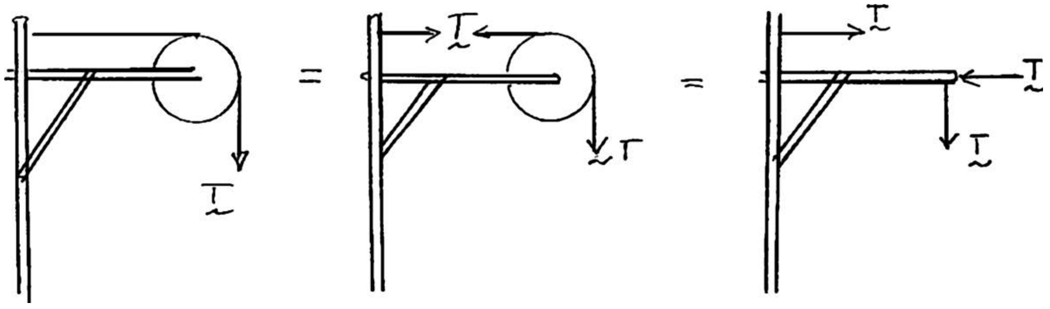
First note that, when a cable or cord passes over a *frictionless, motionless* pulley, the tension is unchanged.

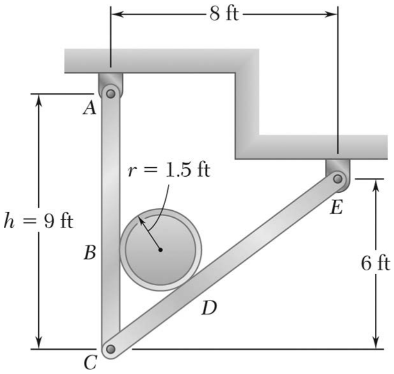
$$\circlearrowleft + \Sigma M_C = 0: \quad rT_1 - rT_2 = 0 \quad T_1 = T_2$$

(*a*) Replace each force with an equivalent force-couple.



(*b*) Cut the cable and replace the forces on pulley with equivalent pair of forces at *A* as above.





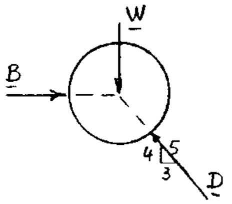
PROBLEM 6.91

A 3-ft-diameter pipe is supported every 16 ft by a small frame like that shown. Knowing that the combined weight of the pipe and its contents is 500 lb/ft and assuming frictionless surfaces, determine the components (*a*) of the reaction at *E*, (*b*) of the force exerted at *C* on member *CDE*.

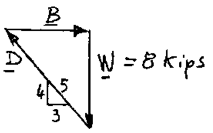
SOLUTION

Free body: 16-ft length of pipe:

$$W = (500\text{ lb/ft})(16\text{ ft}) = 8\text{ kips}$$



Force Triangle

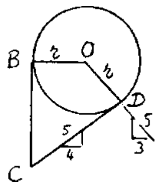


$$\frac{B}{3} = \frac{D}{5} = \frac{8\text{ kips}}{4}$$

$$B = 6\text{ kips} \qquad D = 10\text{ kips}$$

Determination of *CB* = *CD*.

We note that horizontal projection of $\overrightarrow{\mathbf{BO}} + \overrightarrow{\mathbf{OD}}$ = horizontal projection of $\overrightarrow{\mathbf{CD}}$

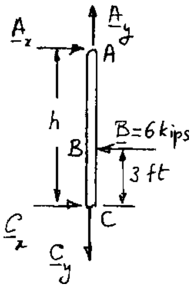


$$r + \frac{3}{5}r = \frac{4}{5}(CD)$$

Thus,

$$CB = CD = \frac{8}{4}r = 2(1.5\text{ ft}) = 3\text{ ft}$$

Free body: Member *ABC*:



$$+\circlearrowright \Sigma M_A = 0: \quad C_x h - (6\text{ kips})(h-3)$$

$$C_x = \frac{h-3}{h}(6\text{ kips}) \qquad (1)$$

For $h = 9$ ft,

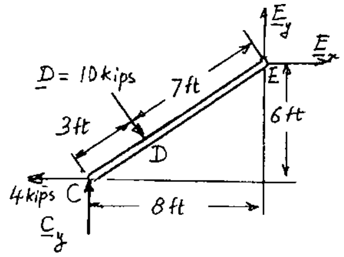
$$C_x = \frac{9-3}{9}(6\text{ kips}) = 4\text{ kips}$$

PROBLEM 6.91 (Continued)

Free body: Member *CDE*:

From above, we have

$$\mathbf{C}_x = 4.00 \text{ kips} \longleftarrow \blacktriangleleft$$



$$+\circlearrowleft \Sigma M_E = 0\text{:} \quad (10 \text{ kips})(7 \text{ ft}) - (4 \text{ kips})(6 \text{ ft}) - C_y(8 \text{ ft}) = 0$$

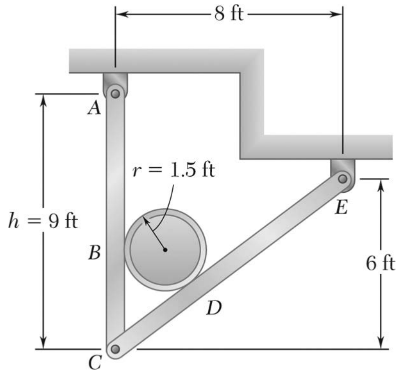
$$C_y = +5.75 \text{ kips,} \qquad \mathbf{C}_y = 5.75 \text{ kips} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0\text{:} \quad -4 \text{ kips} + \frac{3}{5}(10 \text{ kips}) + E_x = 0$$

$$E_x = -2 \text{ kips,} \qquad \mathbf{E}_x = 2.00 \text{ kips} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0\text{:} \quad 5.75 \text{ kips} - \frac{4}{5}(10 \text{ kips}) + E_y = 0,$$

$$\mathbf{E}_y = 2.25 \text{ kips} \uparrow \blacktriangleleft$$



PROBLEM 6.92

Solve Problem 6.91 for a frame where $h = 6$ ft.

PROBLEM 6.91 A 3-ft-diameter pipe is supported every 16 ft by a small frame like that shown. Knowing that the combined weight of the pipe and its contents is 500 lb/ft and assuming frictionless surfaces, determine the components (*a*) of the reaction at *E*, (*b*) of the force exerted at *C* on member *CDE*.

SOLUTION

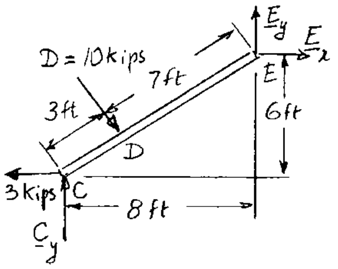
See solution of Problem 6.91 for derivation of Eq. (1).

For $h = 6$ ft, $\quad C_x = \dfrac{h-3}{h}(6 \text{ kips}) = \dfrac{6-3}{6} = 3 \text{ kips}$

Free body: Member *CDE*:

From above, we have

$$\mathbf{C}_x = 3.00 \text{ kips} \longleftarrow \blacktriangleleft$$



$$+\circlearrowright \Sigma M_E = 0: \quad (10 \text{ kips})(7 \text{ ft}) - (3 \text{ kips})(6 \text{ ft}) - C_y(8 \text{ ft}) = 0$$

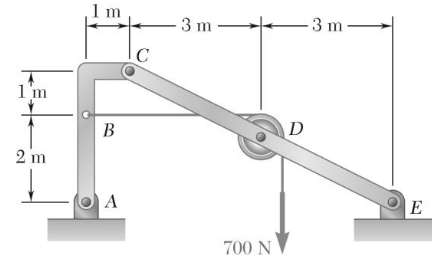
$$C_y = +6.50 \text{ kips}, \qquad \mathbf{C}_y = 6.50 \text{ kips} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -3 \text{ kips} + \frac{3}{5}(10 \text{ kips}) + E_x = 0$$

$$E_x = -3.00 \text{ kips}, \qquad \mathbf{E}_x = 3.00 \text{ kips} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 6.5 \text{ kips} - \frac{4}{5}(10 \text{ kips}) + E_y = 0$$

$$E_y = 1.500 \text{ kips} \qquad \mathbf{E}_y = 1.500 \text{ kips} \uparrow \blacktriangleleft$$

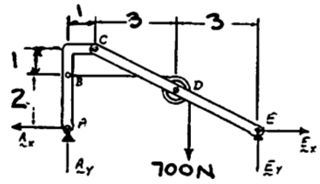


PROBLEM 6.93

Knowing that the pulley has a radius of 0.5 m, determine the components of the reactions at *A* and *E*.

SOLUTION

FBD Frame:



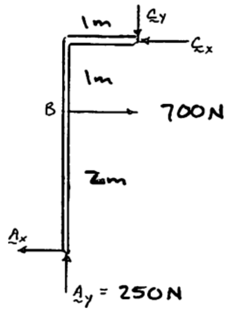
Dimensions in *m*

$\circlearrowleft \Sigma M_A = 0: \quad (7\text{ m})E_y - (4.5\text{ m})(700\text{ N}) = 0 \qquad \mathbf{E}_y = 450\text{ N}\uparrow$ ◀

$\uparrow \Sigma F_y = 0: \quad A_y - 700\text{ N} + 450\text{ N} = 0 \qquad \mathbf{A}_y = 250\text{ N}\uparrow$ ◀

$\rightarrow \Sigma F_x = 0: \quad A_x - E_x = 0 \qquad A_x = E_x$

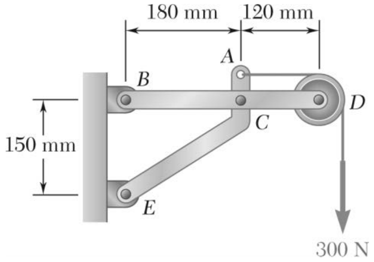
FBD Member ABC:



$\circlearrowleft \Sigma M_C = 0: \quad (1\text{ m})(700\text{ N}) - (1\text{ m})(250\text{ N}) - (3\text{ m})\,A_x = 0$

$\mathbf{A}_x = 150.0\text{ N} \longleftarrow$ ◀

so $\mathbf{E}_x = 150.0\text{ N} \longrightarrow$ ◀

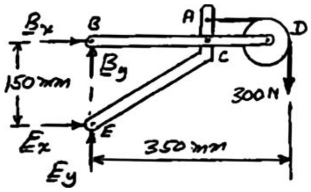


PROBLEM 6.94

Knowing that the pulley has a radius of 50 mm, determine the components of the reactions at *B* and *E*.

SOLUTION

Free body: Entire assembly:



$$+\circlearrowleft \Sigma M_E = 0: \quad -(300\text{ N})(350\text{ mm}) - B_x(150\text{ mm}) = 0$$

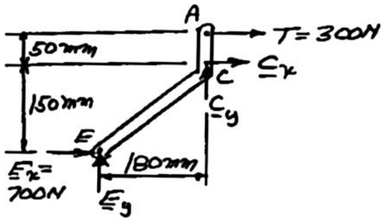
$$B_x = -700\text{ N} \qquad \mathbf{B}_x = 700\text{ N} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -700N + E_x = 0$$

$$E_x = 700\text{ N} \qquad \mathbf{E}_x = 700\text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + E_y - 300\text{ N} = 0 \qquad (1)$$

Free body: Member *ACE*:



$$+\circlearrowleft \Sigma M_C = 0: \quad (700\text{ N})(150\text{ mm}) - (300\text{ N})(50\text{ mm}) - E_y(180\text{ mm}) = 0$$

$$E_y = 500\text{ N} \qquad \mathbf{E}_y = 500\text{ N}\uparrow$$

From Eq. (1): $B_y + 500\text{ N} - 300\text{ N} = 0$

$$B_y = -200\text{ N} \qquad \mathbf{B}_y = 200\text{ N}\downarrow$$

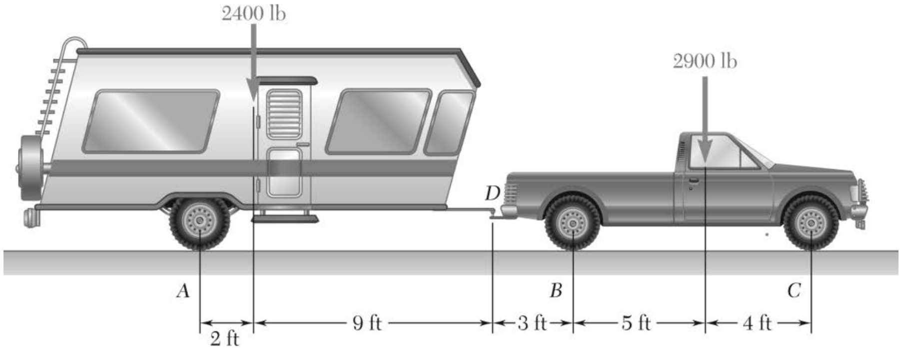
Thus, reactions are

$\mathbf{B}_x = 700\text{ N} \longleftarrow, \qquad \mathbf{B}_y = 200\text{ N}\downarrow$ ◀

$\mathbf{E}_x = 700\text{ N} \longrightarrow, \qquad \mathbf{E}_y = 500\text{ N}\uparrow$ ◀

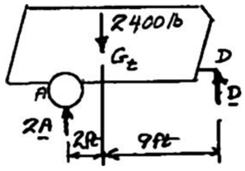
PROBLEM 6.95

A trailer weighing 2400 lb is attached to a 2900-lb pickup truck by a ball-and-socket truck hitch at *D*. Determine (*a*) the reactions at each of the six wheels when the truck and trailer are at rest, (*b*) the additional load on each of the truck wheels due to the trailer.



SOLUTION

(*a*) Free body: Trailer:



(We shall denote by *A*, *B*, *C* the reaction at one wheel.)

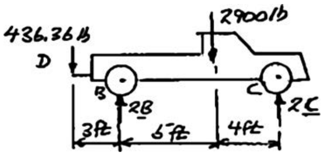
$$+\circlearrowleft \Sigma M_A = 0: \quad -(2400 \text{ lb})(2 \text{ ft}) + D(11 \text{ ft}) = 0$$

$$D = 436.36 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad 2A - 2400 \text{ lb} + 436.36 \text{ lb} = 0$$

$$A = 981.82 \text{ lb} \qquad \mathbf{A} = 982 \text{ lb}\uparrow \blacktriangleleft$$

Free body: Truck.



$$+\circlearrowleft \Sigma M_B = 0: \quad (436.36 \text{ lb})(3 \text{ ft}) - (2900 \text{ lb})(5 \text{ ft}) + 2C(9 \text{ ft}) = 0$$

$$C = 732.83 \text{ lb} \qquad \mathbf{C} = 733 \text{ lb}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2B - 436.36 \text{ lb} - 2900 \text{ lb} + 2(732.83 \text{ lb}) = 0$$

$$B = 935.35 \text{ lb} \qquad \mathbf{B} = 935 \text{ lb}\uparrow \blacktriangleleft$$

(*b*) Additional load on truck wheels.

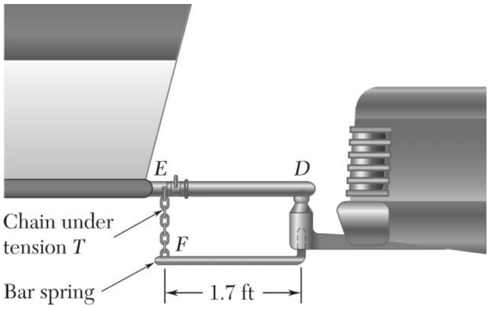
Use free body diagram of truck without 2900 lb.

$$+\circlearrowleft \Sigma M_B = 0: \quad (436.36 \text{ lb})(3 \text{ ft}) + 2C(9 \text{ ft}) = 0$$

$$C = -72.73 \text{ lb} \qquad \Delta C = -72.7 \text{ lb} \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2B - 436.36 \text{ lb} - 2(72.73 \text{ lb}) = 0$$

$$B = 290.9 \text{ lb} \qquad \Delta B = +291 \text{ lb} \blacktriangleleft$$



PROBLEM 6.96

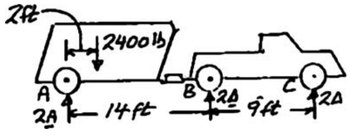
In order to obtain a better weight distribution over the four wheels of the pickup truck of Problem 6.95, a compensating hitch of the type shown is used to attach the trailer to the truck. The hitch consists of two bar springs (only one is shown in the figure) that fit into bearings inside a support rigidly attached to the truck. The springs are also connected by chains to the trailer frame, and specially designed hooks make it possible to place both chains in tension. (*a*) Determine the tension T required in each of the two chains if the additional load due to the trailer is to be evenly distributed over the four wheels of the truck. (*b*) What are the resulting reactions at each of the six wheels of the trailer-truck combination?

PROBLEM 6.95 A trailer weighing 2400 lb is attached to a 2900-lb pickup truck by a ball-and-socket truck hitch at D. Determine (*a*) the reactions at each of the six wheels when the truck and trailer are at rest, (*b*) the additional load on each of the truck wheels due to the trailer.

SOLUTION

(*a*) We small first find the additional reaction Δ at each wheel due to the trailer.

<u>Free body diagram</u>: (Same Δ at each truck wheel)



$$+\circlearrowleft \Sigma M_A = 0: \quad -(2400 \text{ lb})(2 \text{ ft}) + 2\Delta(14 \text{ ft}) + 2\Delta(23 \text{ ft}) = 0$$

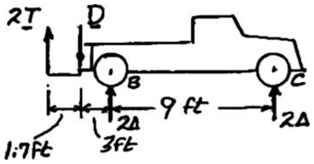
$$\Delta = 64.86 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad 2A - 2400 \text{ lb} + 4(64.86 \text{ lb}) = 0;$$

$$A = 1070 \text{ lb}; \qquad \mathbf{A} = 1070 \text{ lb} \uparrow$$

<u>Free body: Truck</u>:

(Trailer loading only)



$$+\circlearrowleft \Sigma M_D = 0: \quad 2\Delta(12 \text{ ft}) + 2\Delta(3 \text{ ft}) - 2T(1.7 \text{ ft}) = 0$$

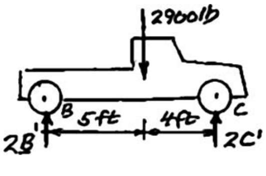
$$T = 8.824\Delta$$

$$= 8.824(64.86 \text{ lb})$$

$$T = 572.3 \text{ lb} \qquad T = 572 \text{ lb} \blacktriangleleft$$

<u>Free body: Truck</u>:

(Truck weight only)



$$+\circlearrowleft \Sigma M_B = 0: \quad -(2900 \text{ lb})(5 \text{ ft}) + 2C'(9 \text{ ft}) = 0$$

$$C' = 805.6 \text{ lb} \qquad \mathbf{C}' = 805.6 \text{ lb} \uparrow$$

PROBLEM 6.96 (Continued)

$$+\uparrow \Sigma F_y = 0: \quad 2B' - 2900\text{ lb} + 2(805.6\text{ lb}) = 0$$

$B' = 644.4\text{ lb}$ $\qquad$ $\mathbf{B}' = 644.4\text{ lb}\uparrow$

Actual reactions:

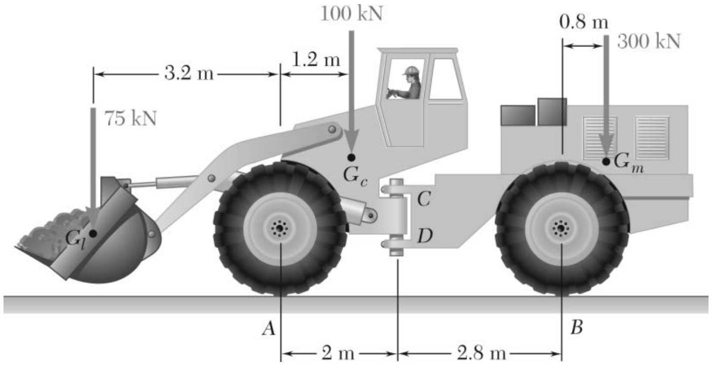
$B = B' + \Delta = 644.4\text{ lb} + 64.86 = 709.2\text{ lb}$ $\qquad$ $\mathbf{B} = 709\text{ lb}\uparrow$ ◀

$C = C' + \Delta = 805.6\text{ lb} + 64.86 = 870.46\text{ lb}$ $\qquad$ $\mathbf{C} = 870\text{ lb}\uparrow$ ◀

From part *a*: $\qquad$ $\mathbf{A} = 1070\text{ lb}\uparrow$ ◀

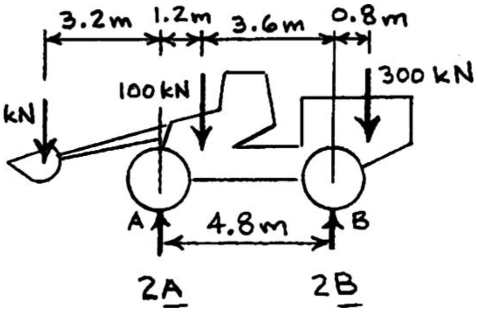
PROBLEM 6.97

The cab and motor units of the front-end loader shown are connected by a vertical pin located 2 m behind the cab wheels. The distance from C to D is 1 m. The center of gravity of the 300-kN motor unit is located at G_m, while the centers of gravity of the 100-kN cab and 75-kN load are located, respectively, at G_c and G_l. Knowing that the machine is at rest with its brakes released, determine (*a*) the reactions at each of the four wheels, (*b*) the forces exerted on the motor unit at C and D.



SOLUTION

(*a*) Free body: Entire machine:



A = Reaction at each front wheel

B = Reaction at each rear wheel

$$+\circlearrowleft \Sigma M_A = 0: \quad 75(3.2 \text{ m}) - 100(1.2 \text{ m}) + 2B(4.8 \text{ m}) - 300(5.6 \text{ m}) = 0$$

$$2B = 325 \text{ kN} \qquad \mathbf{B} = 162.5 \text{ kN}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2A + 325 - 75 - 100 - 300 = 0$$

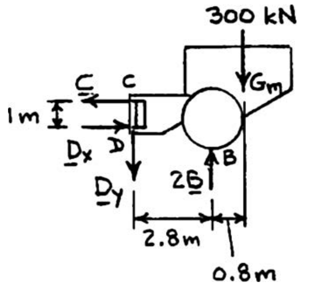
$$2A = 150 \text{ kN} \qquad \mathbf{A} = 75.0 \text{ kN}\uparrow \blacktriangleleft$$

PROBLEM 6.97 (Continued)

(*b*) Free body: Motor unit:

$$+\circlearrowleft \Sigma M_D = 0: \quad C(1 \text{ m}) + 2B(2.8 \text{ m}) - 300(3.6 \text{ m}) = 0$$

$$C = 1080 - 5.6B \qquad (1)$$



Recalling $B = 162.5 \text{ kN}, \quad C = 1080 - 5.6(162.5) = 170 \text{ kN}$

$\mathbf{C} = 170.0 \text{ kN} \leftarrow$ ◀

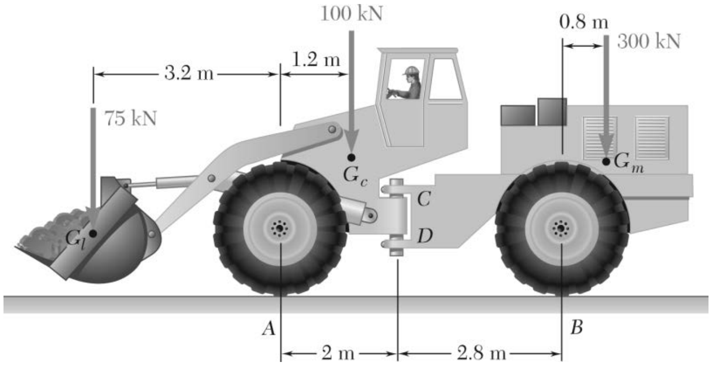
$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - 170 = 0$ $\qquad \mathbf{D}_x = 170.0 \text{ kN} \rightarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad 2(162.5) - D_y - 300 = 0$ $\qquad \mathbf{D}_y = 25.0 \text{ kN} \downarrow$ ◀

PROBLEM 6.98

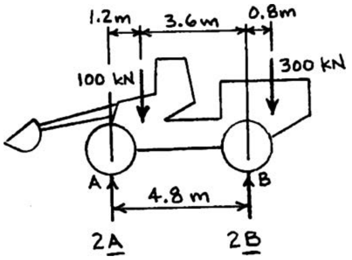
Solve Problem 6.97 assuming that the 75-kN load has been removed.

PROBLEM 6.97 The cab and motor units of the front-end loader shown are connected by a vertical pin located 2 m behind the cab wheels. The distance from C to D is 1 m. The center of gravity of the 300-kN motor unit is located at G_m, while the centers of gravity of the 100-kN cab and 75-kN load are located, respectively, at G_c and G_l. Knowing that the machine is at rest with its brakes released, determine (*a*) the reactions at each of the four wheels, (*b*) the forces exerted on the motor unit at C and D.



SOLUTION

(*a*) Free body: Entire machine:



$\mathbf{A}$ = Reaction at each front wheel

$\mathbf{B}$ = Reaction at each rear wheel

$$+\circlearrowleft \Sigma M_A = 0: \quad 2B(4.8\text{ m}) - 100(1.2\text{ m}) - 300(5.6\text{ m}) = 0$$

$$2B = 375\text{ kN} \qquad \mathbf{B} = 187.5\text{ kN}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2A + 375 - 100 - 300 = 0$$

$$2A = 25\text{ kN} \qquad \mathbf{A} = 12.50\text{ kN}\uparrow \blacktriangleleft$$

(*b*) Free body: Motor unit:

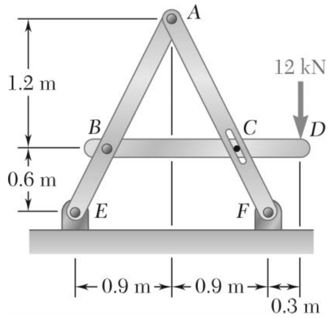
See solution of Problem 6.97 for free body diagram and derivation of Eq. (1). With $B = 187.5$ kN, we have

$$C = 1080 - 5.6(187.5) = 30\text{ kN}$$

$$\mathbf{C} = 30.0\text{ kN} \longleftarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - 30 = 0 \qquad \mathbf{D}_x = 30.0\text{ kN} \longrightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2(187.5) - D_y - 300 = 0 \qquad \mathbf{D}_y = 75.0\text{ kN}\downarrow \blacktriangleleft$$

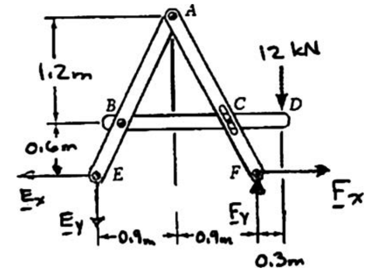


PROBLEM 6.99

For the frame and loading shown, determine the components of all forces acting on member *ABE*.

SOLUTION

FBD Frame:



$$\circlearrowleft + \Sigma M_E = 0: \quad (1.8\text{ m})F_y - (2.1\text{ m})(12\text{ kN}) = 0$$

$$\mathbf{F}_y = 14.00\text{ kN}\uparrow$$

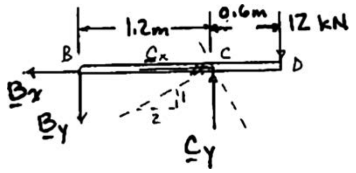
$$+\uparrow \Sigma F_y = 0: \quad -E_y + 14.00\text{ kN} - 12\text{ kN} = 0$$

$$E_y = 2\text{ kN}\downarrow$$

$$\mathbf{E}_y = 2.00\text{ kN}\downarrow$$ ◀

FBD member *BCD*:

$$\circlearrowleft + \Sigma M_B = 0: \quad (1.2\text{ m})C_y - (12\text{ kN})(1.8\text{ m}) = 0 \quad C_y = 18.00\text{ kN}\uparrow$$



But **C** is $\perp ACF$, so $C_x = 2C_y$; $C_x = 36.0\text{ kN}\longrightarrow$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -B_x + C_x = 0 \quad B_x = C_x = 36.0\text{ kN}$$

$$B_x = 36.0\text{ kN}\longleftarrow \text{ on } BCD$$

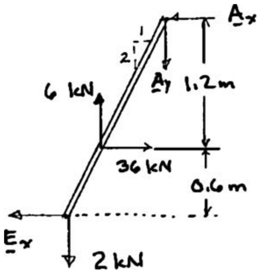
$$+\uparrow \Sigma F_y = 0: \quad -B_y + 18.00\text{ kN} - 12\text{ kN} = 0 \quad B_y = 6.00\text{ kN}\downarrow \text{ on } BCD$$

On *ABE*:

$$\mathbf{B}_x = 36.0\text{ kN}\longrightarrow$$ ◀

$$\mathbf{B}_y = 6.00\text{ kN}\uparrow$$ ◀

FBD member *ABE*:



$$\circlearrowleft + \Sigma M_A = 0: \quad (1.2\text{ m})(36.0\text{ kN}) - (0.6\text{ m})(6.00\text{ kN}) + (0.9\text{ m})(2.00\text{ kN}) - (1.8\text{ m})(E_x) = 0$$

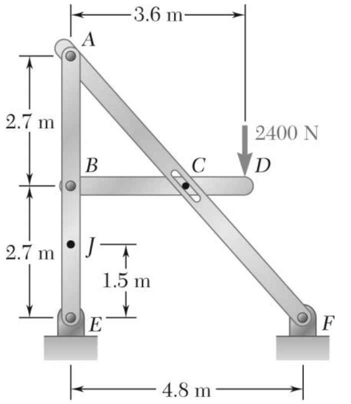
$$\mathbf{E}_x = 23.0\text{ kN}\longleftarrow$$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad -23.0\text{ kN} + 36.0\text{ kN} - A_x = 0$$

$$\mathbf{A}_x = 13.00\text{ kN}\longleftarrow$$ ◀

$$+\uparrow \Sigma F_y = 0: \quad -2.00\text{ kN} + 6.00\text{ kN} - A_y = 0$$

$$\mathbf{A}_y = 4.00\text{ kN}\downarrow$$ ◀



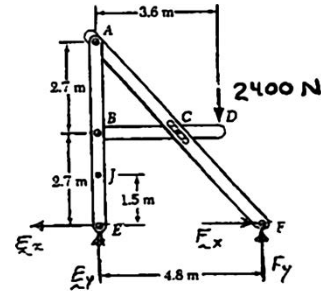
PROBLEM 6.100

For the frame and loading shown, determine the components of all forces acting on member *ABE*.

PROBLEM 6.99 For the frame and loading shown, determine the components of all forces acting on member *ABE*.

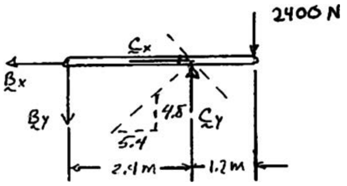
SOLUTION

FBD Frame:



$$\circlearrowleft + \Sigma M_F = 0\colon \quad (1.2\text{ m})(2400\text{ N}) - (4.8\text{ m})E_y = 0 \qquad \mathbf{E}_y = 600\text{ N}\uparrow \blacktriangleleft$$

FBD member *BC*:



$$C_y = \frac{4.8}{5.4}C_x = \frac{8}{9}C_x$$

$$\circlearrowleft + \Sigma M_C = 0\colon \quad (2.4\text{ m})B_y - (1.2\text{ m})(2400\text{ N}) = 0 \quad B_y = 1200\text{ N}\downarrow$$

On *ABE*: $\mathbf{B}_y = 1200\text{ N}\uparrow \blacktriangleleft$

$$+\uparrow \Sigma F_y = 0\colon \quad -1200\text{ N} + C_y - 2400\text{ N} = 0 \quad C_y = 3600\text{ N}\uparrow$$

so

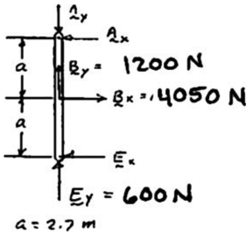
$$C_x = \frac{9}{8}C_y \quad C_x = 4050\text{ N}\longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = 0\colon \quad -B_x + C_x = 0 \quad B_x = 4050\text{ N}\longleftarrow \text{ on } BC$$

On *ABE*: $\mathbf{B}_x = 4050\text{ N}\longrightarrow \blacktriangleleft$

PROBLEM 6.100 (Continued)

FBD member *AB0E*:

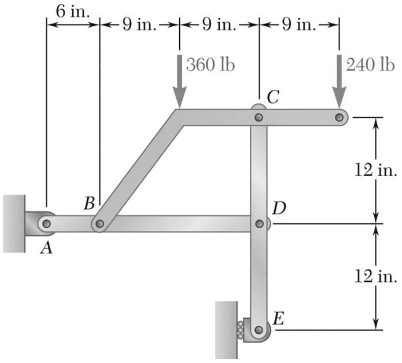


$$\circlearrowleft + \Sigma M_A = 0: \quad a(4050\text{ N}) - 2aE_x = 0$$

$$E_x = 2025\text{ N} \qquad \mathbf{E}_x = 2025\text{ N} \longleftarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + (4050 - 2025)\text{ N} = 0 \qquad \mathbf{A}_x = 2025\text{ N} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 600\text{ N} + 1200\text{ N} - A_y = 0 \qquad \mathbf{A}_y = 1800\text{ N}\downarrow \blacktriangleleft$$

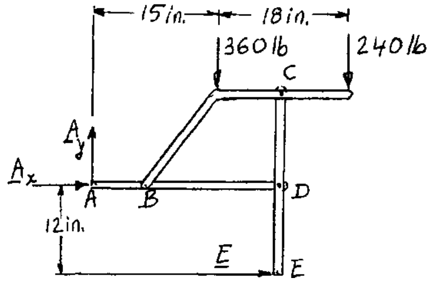


PROBLEM 6.101

For the frame and loading shown, determine the components of all forces acting on member *ABD*.

SOLUTION

Free body: Entire frame:



$+\circlearrowleft \Sigma M_A = 0\text{:}\quad E(12\text{ in.}) - (360\text{ lb})(15\text{ in.}) - (240\text{ lb})(33\text{ in.}) = 0$

$E = +1110\text{ lb}$ $\qquad\mathbf{E} = +1110\text{ lb} \longrightarrow$

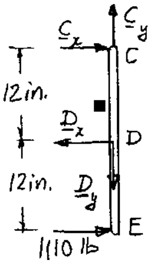
$\xrightarrow{+} \Sigma F_x = 0\text{:}\quad A_x + 1110\text{ lb} = 0$

$A_x = -1110\text{ lb}$ $\qquad\mathbf{A}_x = 1110\text{ lb} \longleftarrow$ ◀

$+\uparrow \Sigma F_y = 0\text{:}\quad A_y - 360\text{ lb} - 240\text{ lb} = 0$

$A_y = +600\text{ lb}$ $\qquad\mathbf{A}_y = 600\text{ lb} \uparrow$ ◀

Free body: Member *CDE*:



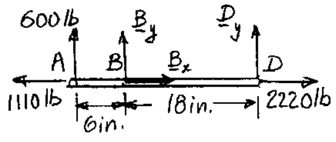
$+\circlearrowleft \Sigma M_C = 0\text{:}\quad (1110\text{ lb})(24\text{ in.}) - D_x(12\text{ in.}) = 0$

$D_x = +2220\text{ lb}$

PROBLEM 6.101 (Continued)

<u>Free body: Member *ABD*</u>:

From above: $\mathbf{D}_x = 2220\text{ lb} \longrightarrow$ ◀



$$+\circlearrowleft \Sigma M_B = 0: \quad D_y(18\text{ in.}) - (600\text{ lb})(6\text{ in.}) = 0$$

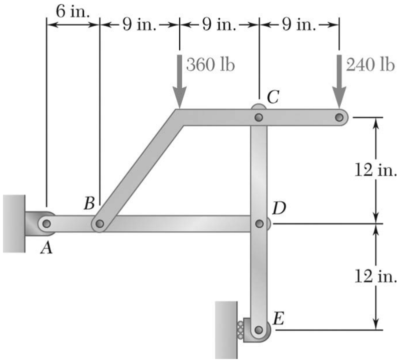
$$D_y = +200\text{ lb} \qquad \mathbf{D}_y = 200\text{ lb}\uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 2220\text{ lb} - 1110\text{ lb} = 0$$

$$B_x = -1110\text{ lb} \qquad \mathbf{B}_x = 1110\text{ lb} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 200\text{ lb} + 600\text{ lb} = 0$$

$$B_y = -800\text{ lb} \qquad \mathbf{B}_y = 800\text{ lb}\downarrow \blacktriangleleft$$



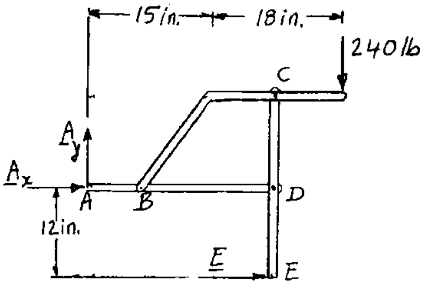
PROBLEM 6.102

Solve Problem 6.101 assuming that the 360-lb load has been removed.

PROBLEM 6.101 For the frame and loading shown, determine the components of all forces acting on member *ABD*.

SOLUTION

Free body diagram of entire frame.



$$+\circlearrowleft \Sigma M_A = 0: \quad E(12\text{ in.}) - (240\text{ lb})(33\text{ in.}) = 0$$

$$E = +660\text{ lb} \qquad \mathbf{E} = 660\text{ lb} \longrightarrow$$

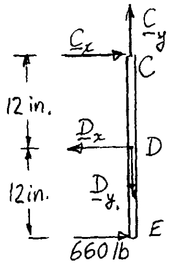
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 660\text{ lb} = 0$$

$$A_x = -660\text{ lb} \qquad \mathbf{A}_x = 660\text{ lb} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 240\text{ lb} = 0$$

$$A_y = +240\text{ lb} \qquad \mathbf{A}_y = 240\text{ lb}\uparrow \blacktriangleleft$$

Free body: Member *CDE*:



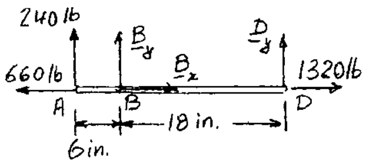
$$+\circlearrowleft \Sigma M_C = 0: \quad (660\text{ lb})(24\text{ in.}) - D_x(12\text{ in.}) = 0$$

$$D_x = +1320\text{ lb}$$

PROBLEM 6.102 (Continued)

Free body: Member *ABD*:

From above: $\mathbf{D}_x = 1320\text{ lb} \longrightarrow$ ◀



$$+\circlearrowleft \Sigma M_B = 0: \quad D_y(18\text{ in.}) - (240\text{ lb})(6\text{ in.}) = 0$$

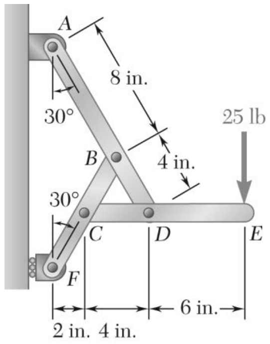
$$D_y = +80\text{ lb} \qquad \mathbf{D}_y = 80.0\text{ lb}\uparrow$$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 1320\text{ lb} - 660\text{ lb} = 0$$

$$B_x = -660\text{ lb} \qquad \mathbf{B}_x = 660\text{ lb} \longleftarrow$$ ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y + 80\text{ lb} + 240\text{ lb} = 0$$

$$B_y = -320\text{ lb} \qquad \mathbf{B}_y = 320\text{ lb}\downarrow$$ ◀

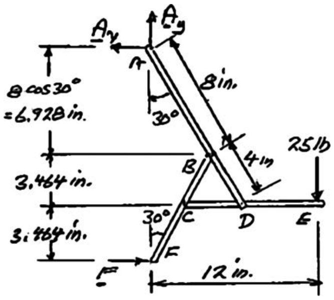


PROBLEM 6.103

For the frame and loading shown, determine the components of the forces acting on member *CDE* at *C* and *D*.

SOLUTION

<u>Free body: Entire frame</u>:



$+\uparrow \Sigma M_y = 0$: $A_y - 25\text{ lb} = 0$

$A_y = 25\text{ lb}$ $\qquad$ $\mathbf{A}_y = 25\text{ lb}\uparrow$

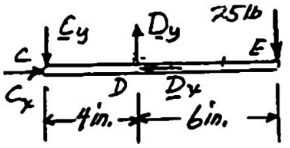
$+\circlearrowleft \Sigma M_F = 0$: $A_x(6.928 + 2\times 3.464) - (25\text{ lb})(12\text{ in.}) = 0$

$A_x = 21.651\text{ lb}$ $\qquad$ $\mathbf{A}_y = 21.65\text{ lb} \longleftarrow$

$\xrightarrow{+} \Sigma F_x = 0$: $F - 21.651\text{ lb} = 0$

$F = 21.651\text{ lb}$ $\qquad$ $\mathbf{F} = 21.65\text{ lb} \longrightarrow$

<u>Free body: Member *CDE*</u>:



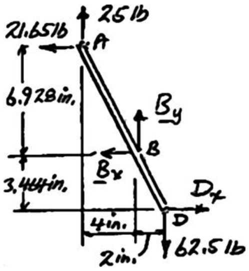
$+\circlearrowleft \Sigma M_C = 0$: $D_y(4\text{ in.}) - (25\text{ lb})(10\text{ in.}) = 0$

$D_y = +62.5\text{ lb}$ $\qquad$ $\mathbf{D}_y = 62.5\text{ lb}\uparrow$ ◀

$+\uparrow \Sigma F_y = 0$: $-C_y + 62.5\text{ lb} - 25\text{ lb} = 0$

$C_y = +37.5\text{ lb}$ $\qquad$ $\mathbf{C}_y = 37.5\text{ lb}\downarrow$ ◀

<u>Free body: Member *ABD*</u>:

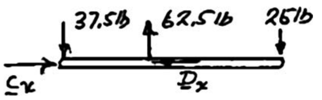


$$+\circlearrowleft \Sigma M_B = 0: \quad D_x(3.464\text{ in.}) + (21.65\text{ lb})(6.928\text{ in.}) - (25\text{ lb})(4\text{ in.}) - (62.5\text{ lb})(2\text{ in.}) = 0$$

$D_x = +21.65\text{ lb}$

<u>Return to free body: Member *CDE*</u>:

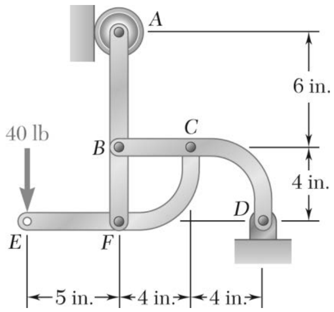
From above:



$D_x = +21.65\text{ lb}$ $\qquad$ $\mathbf{D}_x = 21.7\text{ lb} \longleftarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $C_x - 21.65\text{ lb}$

$C_x = +21.65\text{ lb}$ $\qquad$ $\mathbf{C}_x = 21.7\text{ lb} \longrightarrow$ ◀

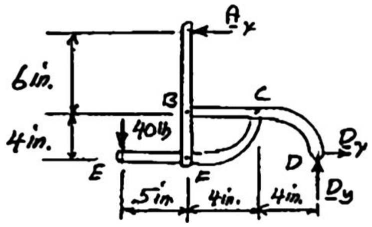


PROBLEM 6.104

For the frame and loading shown, determine the components of the forces acting on member *CFE* at *C* and *F*.

SOLUTION

Free body: Entire frame:



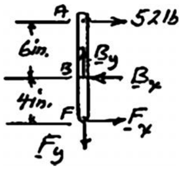
$$+\circlearrowleft \Sigma M_D = 0: \quad (40\text{ lb})(13\text{ in.}) + A_x(10\text{ in.}) = 0$$

$$A_x = -52\text{ lb}, \qquad \mathbf{A}_x = 52\text{ lb} \longrightarrow$$

Free body: Member *ABF*:

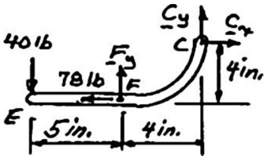
$$+\circlearrowleft \Sigma M_B = 0: \quad -(52\text{ lb})(6\text{ in.}) + F_x(4\text{ in.}) = 0$$

$$F_x = +78\text{ lb}$$



Free body: Member *CFE*:

From above: $$\mathbf{F}_x = 78.0\text{ lb} \longleftarrow \blacktriangleleft$$



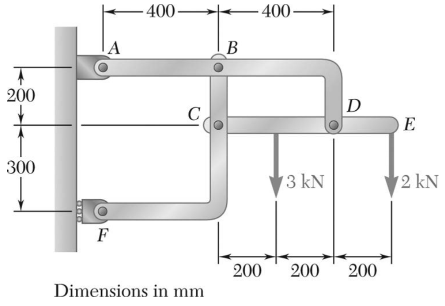
$$+\circlearrowleft \Sigma M_C = 0: \quad (40\text{ lb})(9\text{ in.}) - (78\text{ lb})(4\text{ in.}) - F_y(4\text{ in.}) = 0$$

$$F_y = +12\text{ lb} \qquad \mathbf{F}_y = 12.00\text{ lb}\uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 78\text{ lb} = 0$$

$$C_x = +78\text{ lb} \qquad \mathbf{C}_x = 78.0\text{ lb} \longrightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -40\text{ lb} + 12\text{ lb} + C_y = 0; \quad C_y = +28\text{ lb} \qquad \mathbf{C}_y = 28.0\text{ lb}\uparrow \blacktriangleleft$$



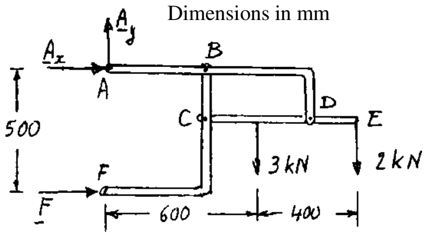
PROBLEM 6.105

For the frame and loading shown, determine the components of all forces acting on member *ABD*.

SOLUTION

Free body: Entire frame:

Dimensions in mm



$+\circlearrowleft \Sigma M_A = 0$: $F(500) - (3\text{ kN})(600) - (2\text{ kN})(1000) = 0$

$F = +7.60\text{ kN}$ $\quad$ $\mathbf{F} = 7.60\text{ kN} \longrightarrow$

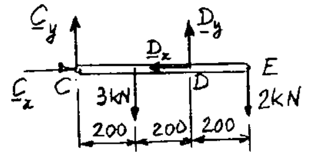
$\xrightarrow{+} \Sigma F_x = 0$: $A_x + 7.60\text{ N} = 0,$

$A_x = -7.60\text{ N}$ $\quad$ $\mathbf{A}_x = 7.60\text{ kN} \longleftarrow$ ◀

$+\!\uparrow \Sigma F_y = 0$: $A_y - 3\text{ kN} - 2\text{ kN} = 0$

$A_y = +5\text{ kN}$ $\quad$ $\mathbf{A}_y = 5.00\text{ kN}\uparrow$ ◀

Free body: Member *CDE*:

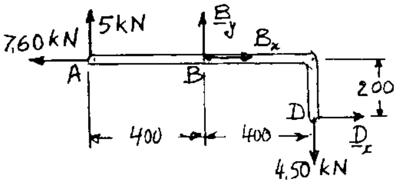


$+\circlearrowleft \Sigma M_C = 0$: $D_y(400) - (3\text{ kN})(200) - (2\text{ kN})(600) = 0$

$D_y = +4.50\text{ kN}$

Free body: Member *ABD*:

From above: $\quad$ $\mathbf{D}_y = 4.50\text{ kN}\downarrow$ ◀



$+\circlearrowleft \Sigma M_B = 0$: $D_x(200) - (4.50\text{ kN})(400) - (5\text{ kN})(400) = 0$

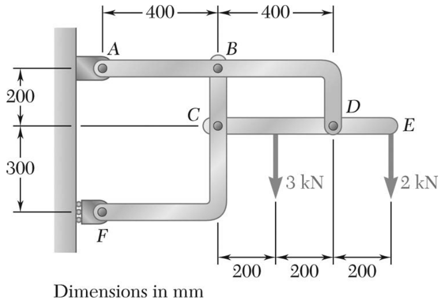
$D_x = +19\text{ kN}$ $\quad$ $\mathbf{D}_x = 19.00\text{ kN} \longrightarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $B_x + 19\text{ kN} - 7.60\text{ kN} = 0$

$B_x = -11.40\text{ kN}$ $\quad$ $\mathbf{B}_x = 11.40\text{ kN} \longleftarrow$ ◀

$+\!\uparrow \Sigma F_y = 0$: $B_y + 5\text{ kN} - 4.50\text{ kN} = 0$

$B_y = -0.50\text{ kN}$ $\quad$ $\mathbf{B}_y = 0.500\text{ kN}\downarrow$ ◀

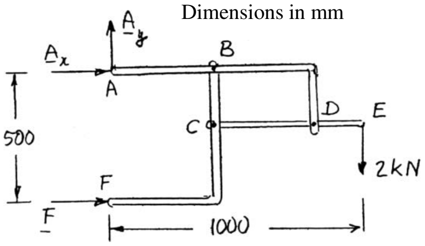


PROBLEM 6.106

Solve Problem 6.105 assuming that the 3-kN load has been removed.

PROBLEM 6.105 For the frame and loading shown, determine the components of all forces acting on member *ABD*.

SOLUTION



Free body: Entire frame:

$$+\circlearrowleft \Sigma M_A = 0: \quad F(500) - (2\text{ kN})(1000) = 0$$

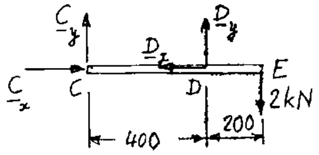
$$F = +4\text{ kN} \qquad \mathbf{F} = 4.00\text{ kN} \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 4\text{ kN} = 0,$$

$$A_x = -4\text{ kN} \qquad \mathbf{A}_x = 4.00\text{ kN} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 2\text{ kN} = 0,$$

$$A_y = +2\text{ kN} \qquad \mathbf{A}_y = 2.00\text{ kN} \uparrow \blacktriangleleft$$



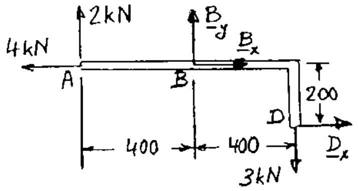
Free body: Member *CDE*:

$$+\circlearrowleft \Sigma M_C = 0: \quad D_y(400) - (2\text{ kN})(600) = 0$$

$$D_y = +3.00\text{ kN}$$

Free body: Member *ABD*:

From above: $\mathbf{D}_y = 3.00\text{ kN} \downarrow \blacktriangleleft$



$$+\circlearrowleft \Sigma M_B = 0: \quad D_x(200) - (3\text{ kN})(400) - (2\text{ kN})(400) = 0$$

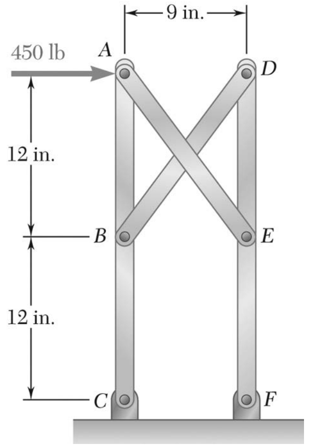
$$D_x = +10.00\text{ kN} \qquad \mathbf{D}_x = 10.00\text{ kN} \longrightarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 10\text{ kN} - 4\text{ kN} = 0$$

$$B_x = -6\text{ kN} \qquad \mathbf{B}_x = 6.00\text{ kN} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 2\text{ kN} - 3\text{ kN} = 0$$

$$B_y = +1\text{ kN} \qquad \mathbf{B}_y = 1.000\text{ kN} \uparrow \blacktriangleleft$$

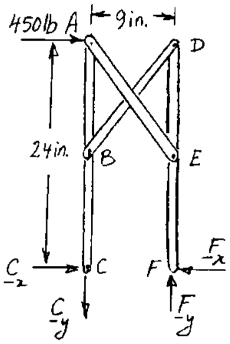


PROBLEM 6.107

Determine the reaction at F and the force in members AE and BD.

SOLUTION

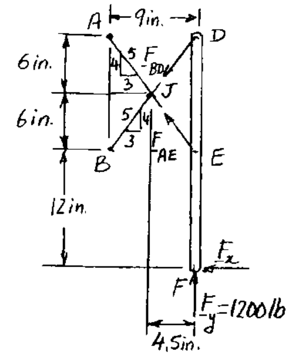
Free body: Entire frame:



$$+\circlearrowleft \Sigma M_C = 0\text{:}\quad F_y(9\text{ in.}) - (450\text{ lb})(24\text{ in.}) = 0$$

$$F_y = 1200\text{ lb} \qquad \mathbf{F}_y = 1200\text{ lb}\uparrow$$

Free body: Member *DEF*:



$$+\circlearrowleft \Sigma M_J = 0\text{:}\quad (1200\text{ lb})(4.5\text{ in.}) - F_x(18\text{ in.}) = 0$$

$$F_x = 300\text{ lb} \qquad \mathbf{F}_x = 300\text{ lb} \longleftarrow \blacktriangleleft$$

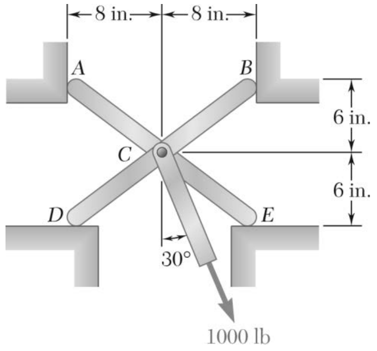
$$+\circlearrowleft \Sigma M_D = 0\text{:}\quad -F_x(24\text{ in.}) - \left(\frac{3}{5}F_{AE}\right)(12\text{ in.}) = 0$$

$$F_{AE} = -\frac{10}{3}F_x = -\frac{10}{3}(300\text{ lb})$$

$$F_{AE} = -1000\text{ lb} \qquad F_{AE} = 1000\text{ lb} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0\text{:}\quad 1200\text{ lb} + \frac{4}{5}(-1000\text{ lb}) - \frac{4}{5}F_{BD} = 0$$

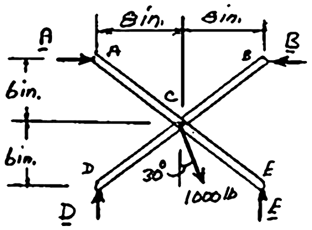
$$F_{BD} = \frac{5}{4}(1200\text{ lb}) - 1000\text{ lb} = +500\text{ lb} \qquad F_{BD} = 500\text{ lb} \quad T \blacktriangleleft$$



PROBLEM 6.108

For the frame and loading shown, determine the reactions at *A*, *B*, *D*, and *E*. Assume that the surface at each support is frictionless.

SOLUTION



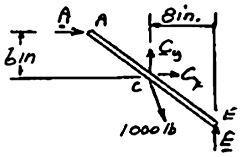
Free body: Entire frame:

$$\xrightarrow{+} \Sigma F_x = 0: \quad A - B + (1000\text{ lb})\sin 30° = 0$$

$$A - B + 500 = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad D + E - (1000\text{ lb})\cos 30° = 0$$

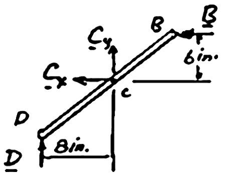
$$D + E - 866.03 = 0 \qquad (2)$$



Free body: Member *ACE*:

$$+\circlearrowleft \Sigma M_C = 0: \quad -A(6\text{ in.}) + E(8\text{ in.}) = 0$$

$$E = \frac{3}{4}A \qquad (3)$$



Free body: Member *BCD*:

$$+\circlearrowleft \Sigma M_C = 0: \quad -D(8\text{ in.}) + B(6\text{ in.}) = 0$$

$$D = \frac{3}{4}B \qquad (4)$$

Substitute *E* and *D* from Eqs. (3) and (4) into Eq. (2):

$$-\frac{3}{4}A + \frac{3}{4}B - 866.06 = 0$$

$$A + B - 1154.71 = 0 \qquad (5)$$

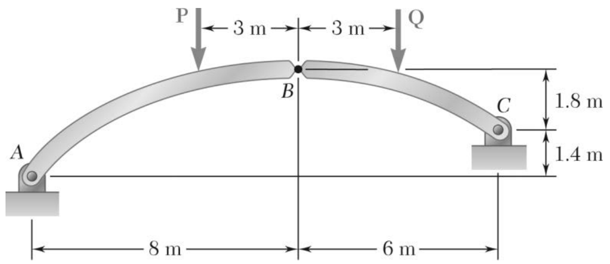
From Eq. (1): $A - B + 500 = 0$ (6)

Eqs. (5) + (6): $2A - 654.71 = 0$ $\quad A = 327.4$ lb $\quad$ **A** = 327 lb ⟶ ◀

Eqs. (5) − (6): $2B - 1654.71 = 0$ $\quad B = 827.4$ lb $\quad$ **B** = 827 lb ⟵ ◀

From Eq. (4): $D = \frac{3}{4}(827.4)$ $\quad D = 620.5$ lb $\quad$ **D** = 621 lb ↑ ◀

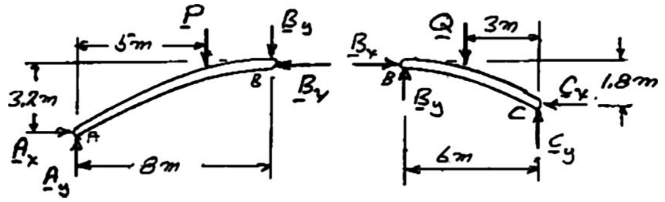
From Eq. (3): $E = \frac{3}{4}(327.4)$ $\quad E = 245.5$ lb $\quad$ **E** = 246 lb ↑ ◀



PROBLEM 6.109

The axis of the three-hinge arch *ABC* is a parabola with the vertex at *B*. Knowing that $P = 112$ kN and $Q = 140$ kN, determine (*a*) the components of the reaction at *A*, (*b*) the components of the force exerted at *B* on segment *AB*.

SOLUTION



<u>Free body: Segment *AB*:</u>

$$+\circlearrowleft \Sigma M_A = 0: \quad B_x(3.2 \text{ m}) - B_y(8 \text{ m}) - P(5 \text{ m}) = 0 \tag{1}$$

0.75 (Eq. 1):
$$B_x(2.4 \text{ m}) - B_y(6 \text{ m}) - P(3.75 \text{ m}) = 0 \tag{2}$$

<u>Free body: Segment *BC*:</u>

$$+\circlearrowright \Sigma M_C = 0: \quad B_x(1.8 \text{ m}) + B_y(6 \text{ m}) - Q(3 \text{ m}) = 0 \tag{3}$$

Add Eqs. (2) and (3):
$$4.2B_x - 3.75P - 3Q = 0$$

$$B_x = (3.75P + 3Q)/4.2 \tag{4}$$

From Eq. (1):
$$(3.75P + 3Q)\frac{3.2}{4.2} - 8B_y - 5P = 0$$

$$B_y = (-9P + 9.6Q)/33.6 \tag{5}$$

given that $P = 112$ kN and $Q = 140$ kN.

(*a*) <u>Reaction at *A*.</u>

Considering again *AB* as a free body,

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B_x = 0; \quad A_x = B_x = 200 \text{ kN}$$
$\mathbf{A}_x = 200 \text{ kN} \longrightarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A_y - P - B_y = 0$$

$$A_y - 112 \text{ kN} - 10 \text{ kN} = 0$$

$$A_y = +122 \text{ kN}$$
$\mathbf{A}_y = 122.0 \text{ kN} \uparrow$ ◀

PROBLEM 6.109 (Continued)

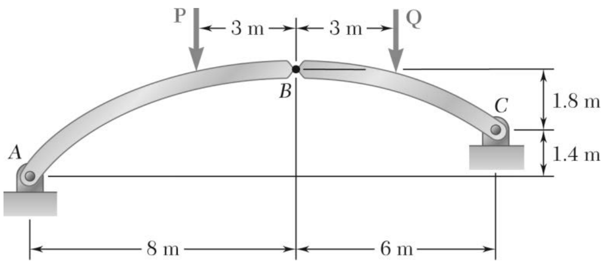
(*b*) Force exerted at B on AB.

From Eq. (4): $B_x = (3.75 \times 112 + 3 \times 140)/4.2 = 200 \text{ kN}$

$$\mathbf{B}_x = 200 \text{ kN} \longleftarrow \blacktriangleleft$$

From Eq. (5): $B_y = (-9 \times 112 + 9.6 \times 140)/33.6 = +10 \text{ kN}$

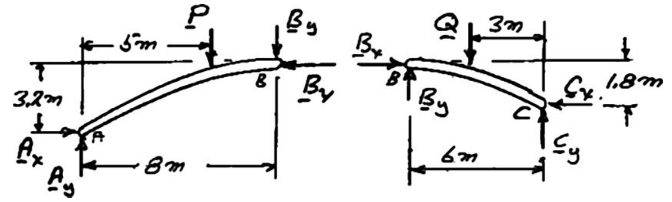
$$\mathbf{B}_y = 10.00 \text{ kN} \downarrow \blacktriangleleft$$



PROBLEM 6.110

The axis of the three-hinge arch *ABC* is a parabola with the vertex at *B*. Knowing that *P* = 140 kN and *Q* = 112 kN, determine (*a*) the components of the reaction at *A*, (*b*) the components of the force exerted at *B* on segment *AB*.

SOLUTION



Free body: Segment *AB*:

$$+\circlearrowleft \Sigma M_A = 0: \quad B_x(3.2\text{ m}) - B_y(8\text{ m}) - P(5\text{ m}) = 0 \qquad (1)$$

0.75 (Eq. 1): $$B_x(2.4\text{ m}) - B_y(6\text{ m}) - P(3.75\text{ m}) = 0 \qquad (2)$$

Free body: Segment *BC*:

$$+\circlearrowleft \Sigma M_C = 0: \quad B_x(1.8\text{ m}) + B_y(6\text{ m}) - Q(3\text{ m}) = 0 \qquad (3)$$

Add Eqs. (2) and (3): $$4.2B_x - 3.75P - 3Q = 0$$

$$B_x = (3.75P + 3Q)/4.2 \qquad (4)$$

From Eq. (1): $$(3.75P + 3Q)\frac{3.2}{4.2} - 8B_y - 5P = 0$$

$$B_y = (-9P + 9.6Q)/33.6 \qquad (5)$$

given that $P = 140\text{ kN}$ and $Q = 112\text{ kN}$.

(*a*) Reaction at *A*.

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B_x = 0; \quad A_x = B_x = 205\text{ kN} \qquad \mathbf{A}_x = 205\text{ kN} \longrightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - P - B_y = 0$$

$$A_y - 140\text{ kN} - (-5.5\text{ kN}) = 0$$

$$A_y = 134.5\text{ kN} \qquad \mathbf{A}_y = 134.5\text{ kN}\uparrow \blacktriangleleft$$

PROBLEM 6.110 (Continued)

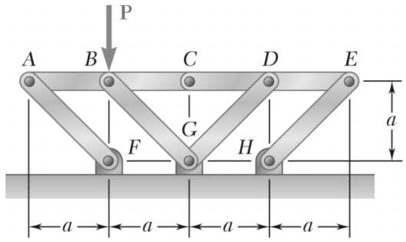
(*b*) Force exerted at *B* on *AB*.

From Eq. (4): $B_x = (3.75 \times 140 + 3 \times 112)/4.2 = 205 \text{ kN}$

$$\mathbf{B}_x = 205 \text{ kN} \longleftarrow \blacktriangleleft$$

From Eq. (5): $B_y = (-9 \times 140 + 9.6 \times 112)/33.6 = -5.5 \text{ kN}$

$$\mathbf{B}_y = 5.50 \text{ kN} \uparrow \blacktriangleleft$$

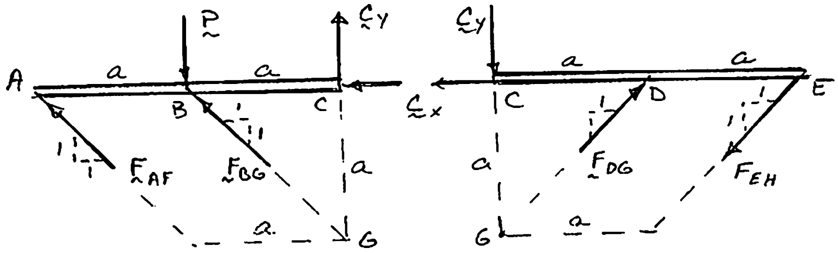


PROBLEM 6.111

Members *ABC* and *CDE* are pin-connected at *C* and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

Member FBDs:



I II

FBD I: $\circlearrowleft \Sigma M_B = 0: \quad aC_y - a\frac{1}{\sqrt{2}}F_{AF} = 0 \quad F_{AF} = \sqrt{2}C_y$

FBD II: $\circlearrowleft M_D = 0: \quad aC_y - a\frac{1}{\sqrt{2}}F_{EH} = 0 \quad F_{EH} = \sqrt{2}C_y$

FBDs combined: $\circlearrowleft \Sigma M_G = 0: \quad aP - a\frac{1}{\sqrt{2}}F_{AF} - a\frac{1}{\sqrt{2}}F_{EH} = 0 \quad P = \frac{1}{\sqrt{2}}\sqrt{2}C_y + \frac{1}{\sqrt{2}}\sqrt{2}C_y$

$$C_y = \frac{P}{2}$$

so $F_{AF} = \frac{\sqrt{2}}{2}P \quad C$ ◀

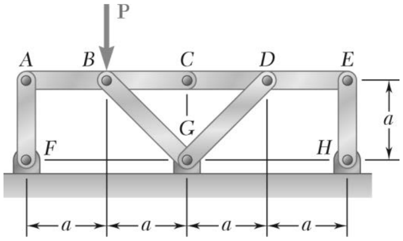
$F_{EH} = \frac{\sqrt{2}}{2}P \quad T$ ◀

FBD I: $\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}}F_{AF} + \frac{1}{\sqrt{2}}F_{BG} - P + C_y = 0 \quad \frac{P}{2} + \frac{1}{\sqrt{2}}F_{BG} - P + \frac{P}{2} = 0$

$F_{BG} = 0$ ◀

FBD II: $\uparrow \Sigma F_y = 0: \quad -C_y + \frac{1}{\sqrt{2}}F_{DG} - \frac{1}{\sqrt{2}}F_{EH} = 0 \quad -\frac{P}{2} + \frac{1}{\sqrt{2}}F_{DG} - \frac{P}{2} = 0$

$F_{DG} = \sqrt{2}P \quad C$ ◀

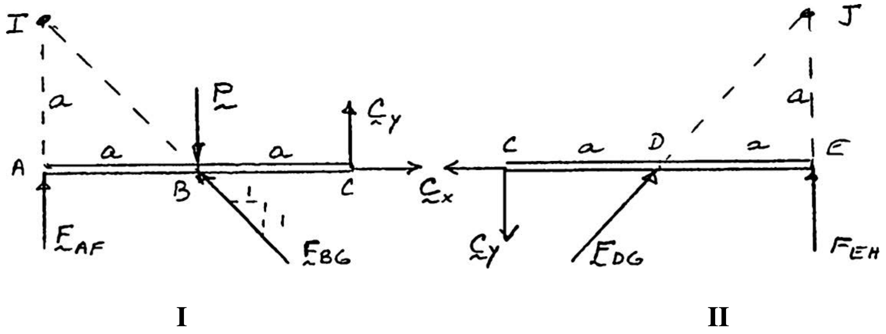


PROBLEM 6.112

Members *ABC* and *CDE* are pin-connected at *C* and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

Member FBDs:



FBD I: $\circlearrowleft \Sigma M_I = 0\colon\quad 2aC_y + aC_x - aP = 0 \quad 2C_y + C_x = P$

FBD II: $\circlearrowleft \Sigma M_J = 0\colon\quad 2aC_y - aC_x = 0 \quad 2C_y - C_x = 0$

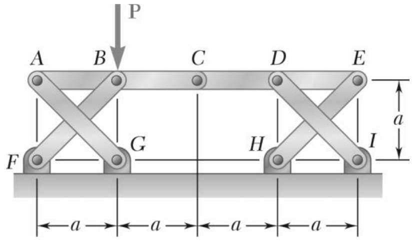
Solving, $C_x = \dfrac{P}{2};\quad C_y = \dfrac{P}{4}$ as shown.

FBD I: $\longrightarrow \Sigma F_x = 0\colon\quad -\dfrac{1}{\sqrt{2}}F_{BG} + C_x = 0 \quad F_{BG} = C_x\sqrt{2}$ $\qquad F_{BG} = \dfrac{P}{\sqrt{2}}$ C ◀

$\uparrow \Sigma F_y = 0\colon\quad F_{AF} - P + \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{2}P\right) + \dfrac{P}{4} = 0$ $\qquad F_{AF} = \dfrac{P}{4}$ C ◀

FBD II: $\longrightarrow \Sigma F_x = 0\colon\quad -C_x + \dfrac{1}{\sqrt{2}}F_{DG} = 0 \quad F_{DG} = C_x\sqrt{2}$ $\qquad F_{DG} = \dfrac{P}{\sqrt{2}}$ C ◀

$\uparrow \Sigma F_y = 0\colon\quad -C_y + \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{2}P\right) + F_{EH} = 0 \quad F_{EH} = \dfrac{P}{4} - \dfrac{P}{2} = -\dfrac{P}{4}$ $\qquad F_{EH} = \dfrac{P}{4}$ T ◀

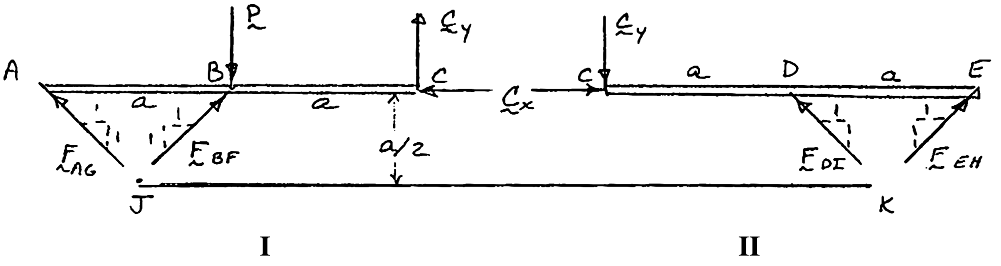


PROBLEM 6.113

Members *ABC* and *CDE* are pin-connected at *C* and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

Member FBDs:



From FBD I: $\circlearrowleft \Sigma M_J = 0$: $\frac{a}{2}C_x + \frac{3a}{2}C_y - \frac{a}{2}P = 0 \quad C_x + 3C_y = P$

FBD II: $\circlearrowleft \Sigma M_K = 0$: $\frac{a}{2}C_x - \frac{3a}{2}C_y = 0 \quad C_x - 3C_y = 0$

Solving, $C_x = \frac{P}{2}$; $C_y = \frac{P}{6}$ as drawn.

FBD I: $\circlearrowleft \Sigma M_B = 0$: $aC_y - a\frac{1}{\sqrt{2}}F_{AG} = 0 \quad F_{AG} = \sqrt{2}C_y = \frac{\sqrt{2}}{6}P$

$$F_{AG} = \frac{\sqrt{2}}{6}P \quad C \blacktriangleleft$$

$\longrightarrow \Sigma F_x = 0$: $-\frac{1}{\sqrt{2}}F_{AG} + \frac{1}{\sqrt{2}}F_{BF} - C_x = 0 \quad F_{BF} = F_{AG} + C_x\sqrt{2} = \frac{\sqrt{2}}{6}P + \frac{\sqrt{2}}{2}P$

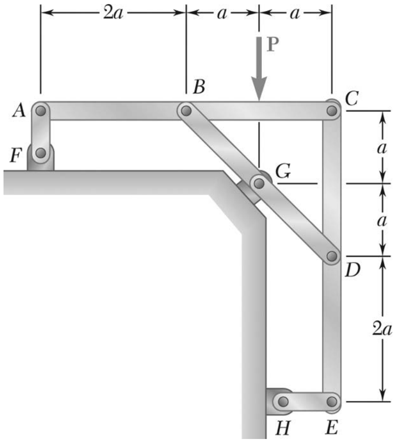
$$F_{BF} = \frac{2\sqrt{2}}{3}P \quad C \blacktriangleleft$$

FBD II: $\circlearrowleft \Sigma M_D = 0$: $a\frac{1}{\sqrt{2}}F_{EH} + aC_y = 0 \quad F_{EH} = -\sqrt{2}C_y = -\frac{\sqrt{2}}{6}P$

$$F_{EH} = \frac{\sqrt{2}}{6}P \quad T \blacktriangleleft$$

$\longrightarrow \Sigma F_x = 0$: $C_x - \frac{1}{\sqrt{2}}F_{DI} + \frac{1}{\sqrt{2}}F_{EH} = 0 \quad F_{DI} = F_{EH} + C_x\sqrt{2} = -\frac{\sqrt{2}}{6}P + \frac{\sqrt{2}}{2}P$

$$F_{DI} = \frac{\sqrt{2}}{3}P \quad C \blacktriangleleft$$

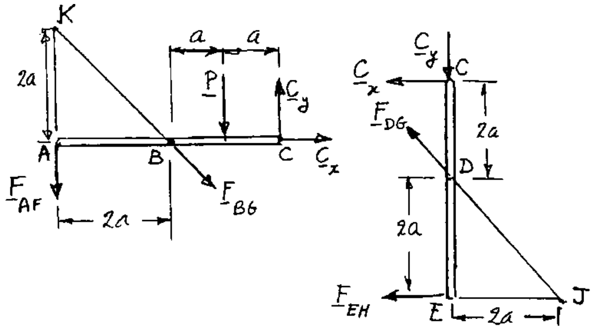


PROBLEM 6.114

Members *ABC* and *CDE* are pin-connected at *C* and supported by the four links *AF*, *BG*, *DG*, and *EH*. For the loading shown, determine the force in each link.

SOLUTION

We consider members *ABC* and *CDE*:



<u>Free body: *CDE*:</u>

$$+\circlearrowleft \Sigma M_J = 0: \quad C_x(4a) + C_y(2a) = 0 \qquad C_y = -2C_x \tag{1}$$

<u>Free body: *ABC*:</u>

$$+\circlearrowleft \Sigma M_K = 0: \quad C_x(2a) + C_y(4a) - P(3a) = 0$$

Substituting for C_y from Eq. (1): $\quad C_x(2a) - 2C_x(4a) - P(3a) = 0$

$$C_x = -\frac{1}{2}P \qquad C_y = -2\left(-\frac{1}{2}P\right) = +P$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{2}}F_{BG} + C_x = 0$$

$$F_{BG} = -\sqrt{2}C_x = -\sqrt{2}\left(-\frac{1}{2}P\right) = +\frac{P}{\sqrt{2}}, \qquad F_{BG} = \frac{P}{\sqrt{2}} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{AF} - \frac{1}{\sqrt{2}}F_{BG} - P + C_y = 0$$

$$F_{AF} = -\frac{1}{\sqrt{2}}\frac{P}{\sqrt{2}} - P + P = -\frac{P}{2} \qquad F_{AF} = \frac{P}{2} \quad C \blacktriangleleft$$

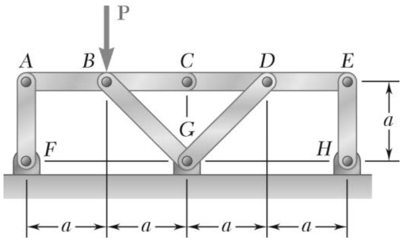
Free body: *CDE*:

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}}F_{DG} - C_y = 0$$

$$F_{DG} = \sqrt{2}C_y = +\sqrt{2}P \qquad F_{DG} = \sqrt{2}P \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{EH} - C_x - \frac{1}{\sqrt{2}}F_{DG} = 0$$

$$F_{EH} = -\left(-\frac{1}{2}P\right) - \frac{1}{\sqrt{2}}\sqrt{2}P = -\frac{P}{2} \qquad F_{EH} = \frac{P}{2} \quad C \blacktriangleleft$$



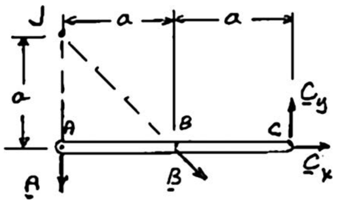
PROBLEM 6.115

Solve Problem 6.112 assuming that the force **P** is replaced by a clockwise couple of moment $\mathbf{M}_0$ applied to member *CDE* at *D*.

PROBLEM 6.112 Members *ABC* and *CDE* are pin-connected at *C* and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

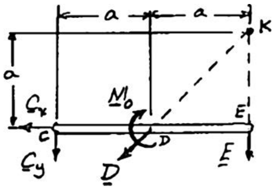
Free body: Member *ABC*:



$$+\circlearrowright \Sigma M_J = 0: \quad C_y(2a) + C_x(a) = 0$$

$$C_x = -2C_y$$

Free body: Member *CDE*:



$$+\circlearrowright \Sigma M_K = 0: \quad C_y(2a) - C_x(a) - M_0 = 0$$

$$C_y(2a) - (-2C_y)(a) - M_0 = 0 \qquad C_y = \frac{M_0}{4a} \; \triangleleft$$

$$C_x = -2C_y: \qquad C_x = -\frac{M_0}{2a} \; \triangleleft$$

$$\xleftarrow{+} \Sigma F_x = 0: \quad \frac{D}{\sqrt{2}} + C_x = 0; \quad \frac{D}{\sqrt{2}} - \frac{M_0}{2a} = 0$$

$$D = \frac{M_0}{\sqrt{2}a} \qquad F_{DG} = \frac{M_0}{\sqrt{2}a} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad E(a) - C_y(a) + M_0 = 0$$

$$E(a) - \left(\frac{M_0}{4a}\right)(a) + M_0 = 0$$

$$E = -\frac{3}{4}\frac{M_0}{a} \qquad F_{EH} = \frac{3}{4}\frac{M_0}{a} \quad C \blacktriangleleft$$

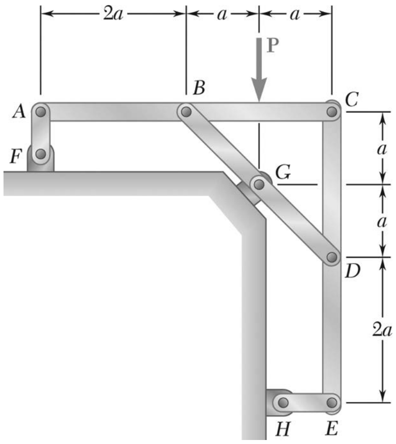
Return to free body of *ABC*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{B}{\sqrt{2}} + C_x = 0; \quad \frac{B}{\sqrt{2}} - \frac{M_0}{2a} = 0$$

$$B = \frac{M_0}{\sqrt{2}a} \qquad F_{BG} = \frac{M_0}{\sqrt{2}a} \quad T \blacktriangleleft$$

$$+\circlearrowright \Sigma M_B = 0: \quad A(a) + C_y(a); \quad A(a) + \frac{M_0}{4a}(a) = 0$$

$$A = -\frac{M_0}{4a} \qquad F_{AF} = \frac{M_0}{4a} \quad C \blacktriangleleft$$



PROBLEM 6.116

Solve Problem 6.114 assuming that the force **P** is replaced by a clockwise couple of moment $\mathbf{M}_0$ applied at the same point.

PROBLEM 6.114 Members *ABC* and *CDE* are pin-connected at *C* and supported by the four links *AF*, *BG*, *DG*, and *EH*. For the loading shown, determine the force in each link.

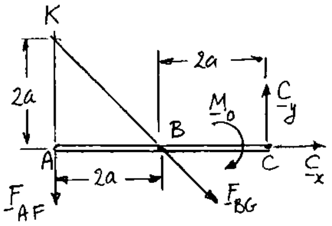
SOLUTION

Free body: *CDE*: (same as for Problem 6.114)

$$+\circlearrowleft \Sigma M_J = 0: \quad C_x(4a) + C_y(2a) = 0 \qquad C_y = -2C_x \qquad (1)$$

Free body: *ABC*:

$$+\circlearrowleft \Sigma M_K = 0: \quad C_x(2a) + C_y(4a) - M_0 = 0$$



Substituting for C_y from Eq. (1): $C_x(2a) - 2C_x(4a) - M_0 = 0$

$$C_x = -\frac{M_0}{6a}$$

$$C_y = -2\left(-\frac{M_0}{6a}\right) = +\frac{M_0}{3a}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{2}} F_{BG} + C_x = 0$$

$$F_{BG} = -\sqrt{2}C_x = +\frac{\sqrt{2}M_0}{6a} \qquad F_{BG} = \frac{\sqrt{2}M_0}{6a} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{AF} - \frac{1}{\sqrt{2}} F_{BG} + C_y = 0$$

$$F_{AF} = -\frac{1}{\sqrt{2}} \frac{\sqrt{2}M_0}{6a} + \frac{M_0}{3a} = +\frac{M_0}{6a} \qquad F_{AF} = \frac{M_0}{6a} \quad T \blacktriangleleft$$

PROBLEM 6.116 (Continued)

Free body: *CDE*: (Use F.B. diagram of Problem 6.114.)

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}} F_{DG} - C_y = 0$$

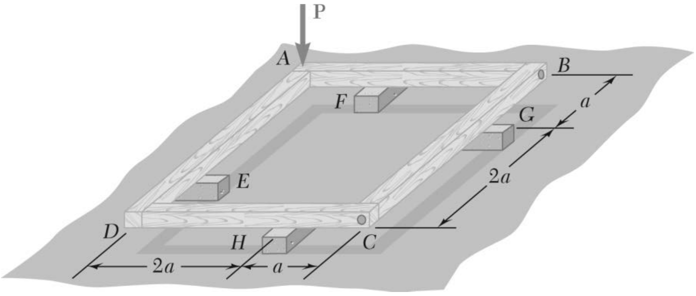
$$F_{DG} = \sqrt{2} C_y = +\frac{\sqrt{2} M_0}{3a} \qquad F_{DG} = \frac{\sqrt{2} M_0}{3a} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{EH} - C_x - \frac{1}{\sqrt{2}} F_{DG} = 0$$

$$F_{EH} = -\left(-\frac{M_0}{6a}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{2} M_0}{3a}\right) = -\frac{M_0}{6a}, \quad F_{EH} = \frac{M_0}{6a} \quad C \blacktriangleleft$$

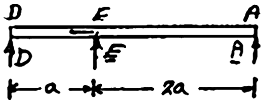
PROBLEM 6.117

Four beams, each of length $3a$, are held together by single nails at A, B, C, and D. Each beam is attached to a support located at a distance a from an end of the beam as shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at E, F, G, and H.



SOLUTION

We shall draw the free body of each member. Force **P** will be applied to member AFB. Starting with member AED, we shall express all forces in terms of reaction E.



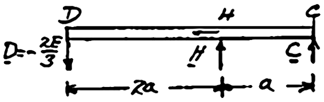
Member AFB:

$$+\circlearrowleft \Sigma M_D = 0: \quad A(3a) + E(a) = 0$$

$$A = -\frac{E}{3}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -D(3a) - E(2a) = 0$$

$$D = -\frac{2E}{3}$$



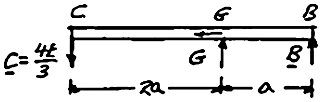
Member DHC:

$$+\circlearrowleft \Sigma M_C = 0: \quad \left(-\frac{2E}{3}\right)(3a) - H(a) = 0$$

$$H = -2E \qquad (1)$$

$$+\circlearrowleft \Sigma M_H = 0: \quad \left(-\frac{2E}{3}\right)(2a) + C(a) = 0$$

$$C = +\frac{4E}{3}$$



Member CGB:

$$+\circlearrowleft \Sigma M_B = 0: \quad +\left(\frac{4E}{3}\right)(3a) - G(a) = 0$$

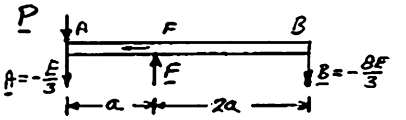
$$G = +4E \qquad (2)$$

$$+\circlearrowleft \Sigma M_G = 0: \quad +\left(\frac{4E}{3}\right)(2a) + B(a) = 0$$

$$B = -\frac{8E}{3}$$

PROBLEM 6.117 (Continued)

Member *AFB*:



$$+\uparrow \Sigma F_y = 0: \quad F - A - B - P = 0$$

$$F - \left(-\frac{E}{3}\right) - \left(-\frac{8E}{3}\right) - P = 0$$

$$F = P - 3E \qquad (3)$$

$$+\circlearrowleft \Sigma M_A = 0: \quad F(a) - B(3a) = 0$$

$$(P - 3E)(a) - \left(-\frac{8E}{3}\right)(3a) = 0$$

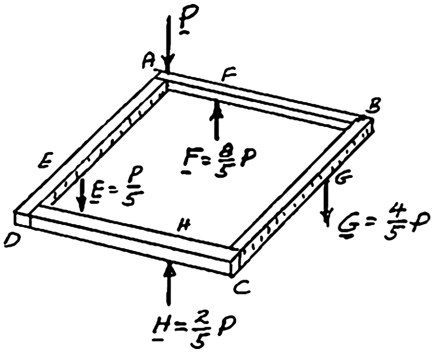
$$P - 3E + 8E = 0; \quad E = -\frac{P}{5} \qquad \mathbf{E} = \frac{P}{5}\downarrow \blacktriangleleft$$

Substitute $E = -\frac{P}{5}$ into Eqs. (1), (2), and (3).

$$H = -2E = -2\left(-\frac{P}{5}\right) \qquad H = +\frac{2P}{5} \qquad \mathbf{H} = \frac{2P}{5}\uparrow \blacktriangleleft$$

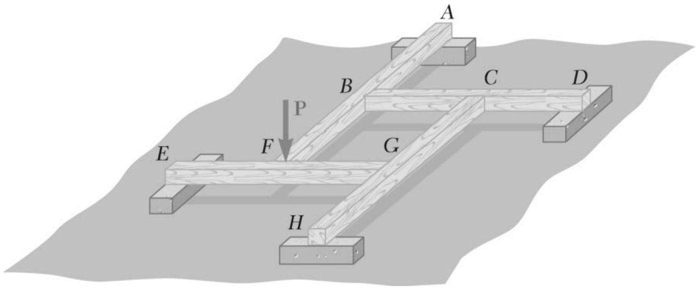
$$G = +4E = 4\left(-\frac{P}{5}\right) \qquad G = -\frac{4P}{5} \qquad \mathbf{G} = \frac{4P}{5}\downarrow \blacktriangleleft$$

$$F = P - 3E = P - 3\left(-\frac{P}{5}\right) \qquad F = +\frac{8P}{5} \qquad \mathbf{F} = \frac{8P}{5}\uparrow \blacktriangleleft$$



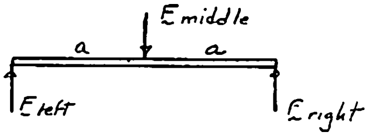
PROBLEM 6.118

Four beams, each of length $2a$, are nailed together at their midpoints to form the support system shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at A, D, E, and H.



SOLUTION

Note that, if we assume P is applied to EG, each individual member FBD looks like



so $$2F_{\text{left}} = 2F_{\text{right}} = F_{\text{middle}}$$

Labeling each interaction force with the letter corresponding to the joint of its application, we see that

$$B = 2A = 2F$$

$$C = 2B = 2D$$

$$G = 2C = 2H$$

$$P + F = 2G(= 4C = 8B = 16F) = 2E$$

From $P + F = 16F$, $$F = \frac{P}{15}$$

so $\mathbf{A} = \dfrac{P}{15}\uparrow$ ◀

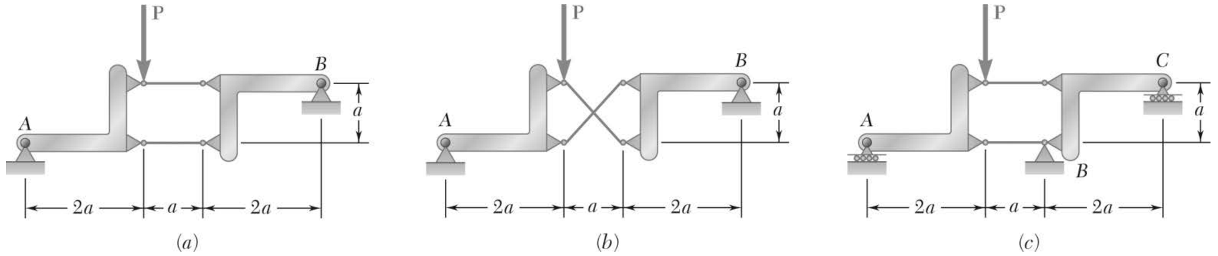
$\mathbf{D} = \dfrac{2P}{15}\uparrow$ ◀

$\mathbf{H} = \dfrac{4P}{15}\uparrow$ ◀

$\mathbf{E} = \dfrac{8P}{15}\uparrow$ ◀

PROBLEM 6.119

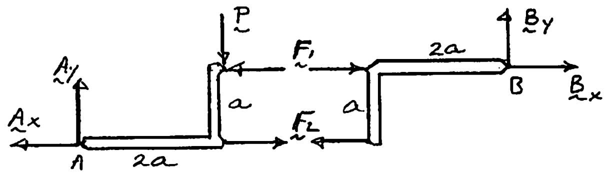
Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.



(*a*) (*b*) (*c*)

SOLUTION

(*a*) **Member FBDs:**



FBD I: $\circlearrowleft \Sigma M_A = 0$: $aF_1 - 2aP = 0 \quad F_1 = 2P$; $\uparrow \Sigma F_y = 0$: $A_y - P = 0 \quad \mathbf{A}_y = P \uparrow$

FBD II: $\circlearrowleft \Sigma M_B = 0$: $-aF_2 = 0 \quad F_2 = 0$

$\longrightarrow \Sigma F_x = 0$: $B_x + F_1 = 0, \quad B_x = -F_1 = -2P$ $\qquad \mathbf{B}_x = 2P \longrightarrow$

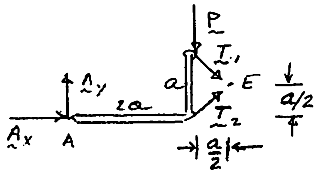
$\uparrow \Sigma F_y = 0$: $B_y = 0$ $\qquad$ so $\mathbf{B} = 2P \longrightarrow$ ◀

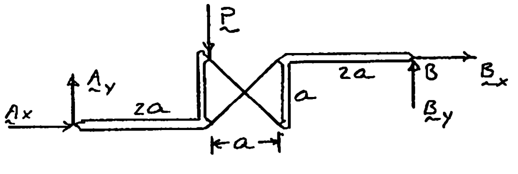
FBD I: $\longrightarrow \Sigma F_x = 0$: $-A_x - F_1 + F_2 = 0 \quad A_x = F_2 - F_1 = 0 - 2P$ $\qquad \mathbf{A}_x = 2P \longrightarrow$

so $\mathbf{A} = 2.24P \measuredangle 26.6°$ ◀

Frame is rigid. ◀

(*b*) **FBD left:** **FBD whole:**





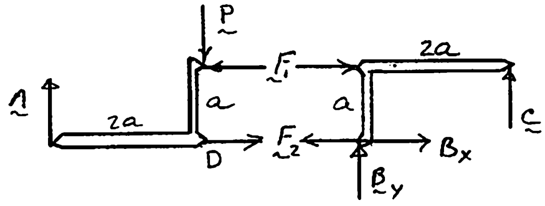
FBD I: $\circlearrowleft \Sigma M_E = 0$: $\dfrac{a}{2}P + \dfrac{a}{2}A_x - \dfrac{5a}{2}A_y = 0 \quad A_x - 5A_y = -P$

FBD II: $\circlearrowleft \Sigma M_B = 0$: $3aP + aA_x - 5aA_y = 0 \quad A_x - 5A_y = -3P$

This is impossible unless $P = 0$. $\qquad$ not rigid ◀

PROBLEM 6.119 (Continued)

(*c*) **Member FBDs:**



I II

FBD I: $\Sigma F_y = 0\text{:}\quad A - P = 0$ $\mathbf{A} = P\uparrow$ ◀

$\circlearrowleft \Sigma M_D = 0\text{:}\quad aF_1 - 2aA = 0 \quad F_1 = 2P$

$\xrightarrow{} \Sigma F_x = 0\text{:}\quad F_2 - F_1 = 0 \quad F_2 = 2P$

FBD II: $\circlearrowleft \Sigma M_B = 0\text{:}\quad 2aC - aF_1 = 0 \quad C = \frac{F_1}{2} = P$ $\mathbf{C} = P\uparrow$ ◀

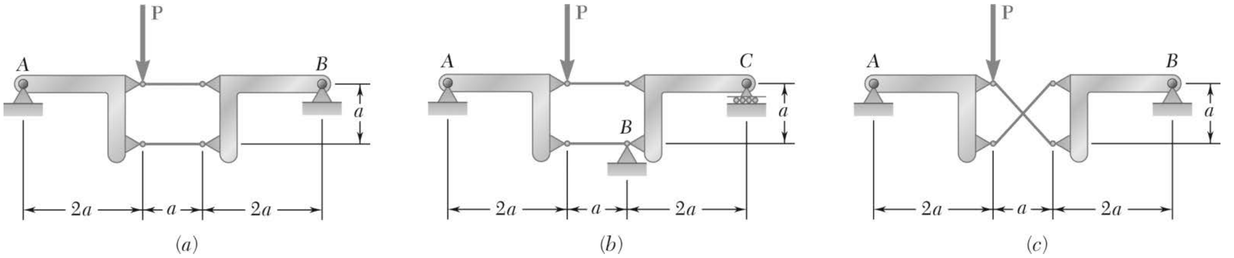
$\xrightarrow{} \Sigma F_x = 0\text{:}\quad F_1 - F_2 + B_x = 0 \quad B_x = P - P = 0$

$\uparrow \Sigma F_x = 0\text{:}\quad B_y + C = 0 \quad B_y = -C = -P$ $\mathbf{B} = P$ ◀

Frame is rigid. ◀

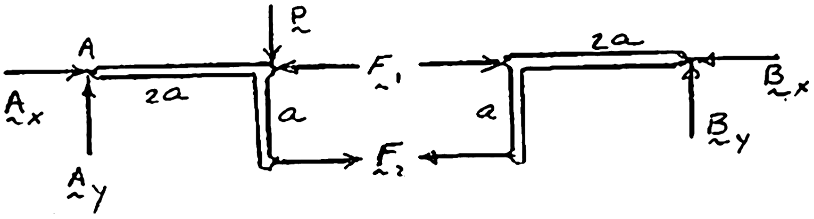
PROBLEM 6.120

Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.



SOLUTION

(*a*) **Member FBDs:**



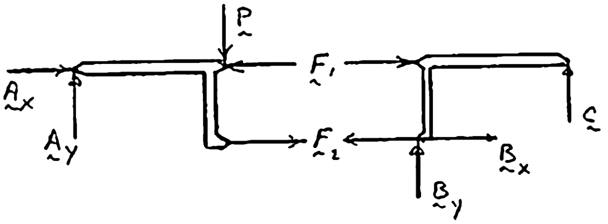
I II

FBD II: $\uparrow \Sigma F_y = 0$: $B_y = 0$ $\circlearrowleft \Sigma M_B = 0$: $aF_2 = 0$ $F_2 = 0$

FBD I: $\circlearrowleft \Sigma M_A = 0$: $aF_2 - 2aP = 0$, but $F_2 = 0$

so $P = 0$ not rigid for $P \neq 0$ ◀

(*b*) **Member FBDs:**



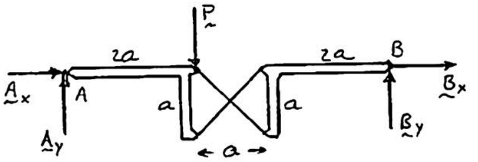
Note: Seven unknowns $(A_x, A_y, B_x, B_y, F_1, F_2, C)$, but only six independent equations

System is statically indeterminate. ◀

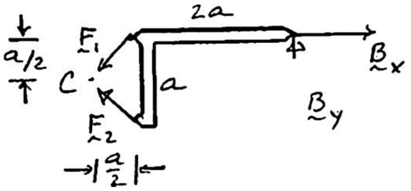
System is, however, rigid. ◀

PROBLEM 6.120 (Continued)

(*c*) **FBD whole:** **FBD right:**



I



II

FBD I: $\circlearrowright \Sigma M_A = 0$: $5aB_y - 2aP = 0$ $\quad \mathbf{B}_y = \frac{2}{5}P \uparrow$

$\uparrow \Sigma F_y = 0$: $A_y - P + \frac{2}{5}P = 0$ $\quad \mathbf{A}_y = \frac{3}{5}P \uparrow$

FBD II: $\circlearrowleft \Sigma M_C = 0$: $\frac{a}{2}B_x - \frac{5a}{2}B_y = 0 \quad B_x = 5B_y$ $\quad \mathbf{B}_x = 2P \longrightarrow$

FBD I: $\longrightarrow \Sigma F_x = 0$: $A_x + B_x = 0 \quad A_x = -B_x$ $\quad \mathbf{A}_x = 2P \longleftarrow$

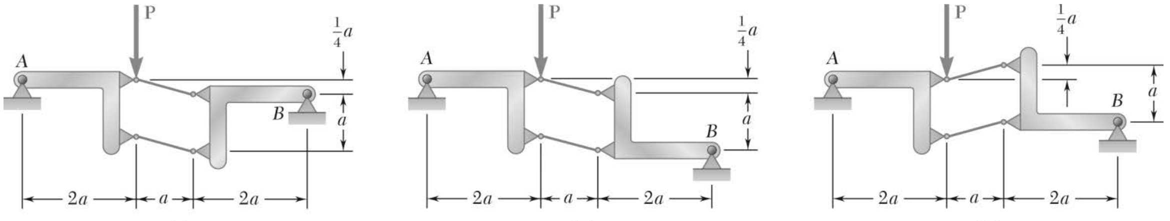
$\mathbf{A} = 2.09P \searrow 16.70°$ ◀

$\mathbf{B} = 2.04P \nearrow 11.31°$ ◀

System is rigid. ◀

PROBLEM 6.121

Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.

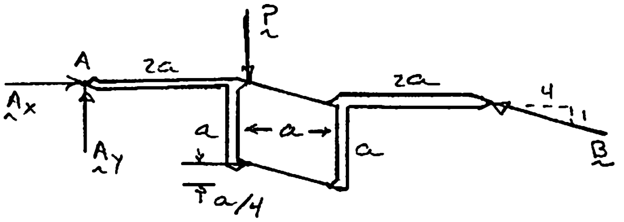


(a) (b) (c)

SOLUTION

Note: In all three cases, the right member has only three forces acting, two of which are parallel. Thus, the third force, at *B*, must be parallel to the link forces.

(*a*) **FBD whole:**



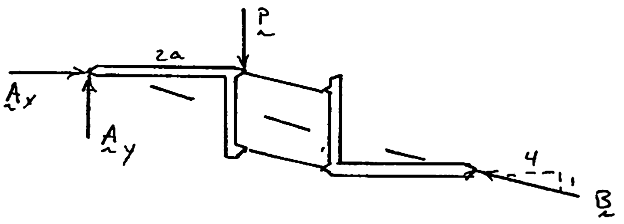
$\curvearrowleft \Sigma M_A = 0{:}\quad -2aP - \frac{a}{4}\frac{4}{\sqrt{17}}B + 5a\frac{1}{\sqrt{17}}B = 0 \quad B = 2.06P$ $\qquad$ $\mathbf{B} = 2.06P$ ⦨ 14.04° ◀

$\longrightarrow \Sigma F_x = 0{:}\quad A_x - \frac{4}{\sqrt{17}}B = 0 \qquad \mathbf{A}_x = 2P \longleftarrow$

$\uparrow \Sigma F_y = 0{:}\quad A_y - P + \frac{1}{\sqrt{17}}B = 0 \quad \mathbf{A}_y = \frac{P}{2}\uparrow$ $\qquad$ $\mathbf{A} = 2.06P$ ∠ 14.04° ◀

rigid ◀

(*b*) **FBD whole:**

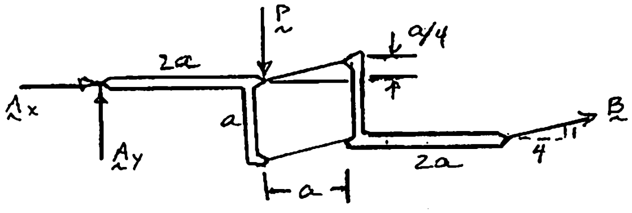


Since **B** passes through *A*, $\curvearrowleft \Sigma M_A = 2aP = 0$ only if $P = 0$.

no equilibrium if $P \neq 0$ $\qquad$ not rigid ◀

PROBLEM 6.121 (Continued)

(*c*) **FBD whole:**

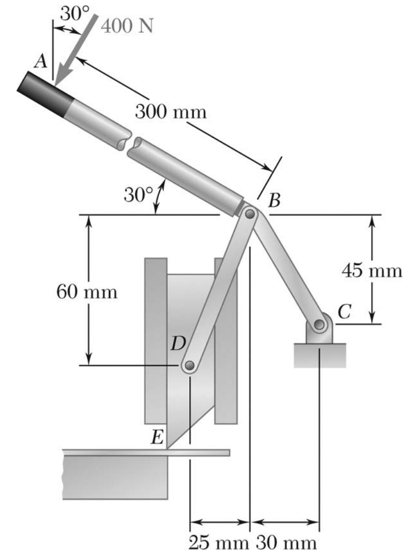


$$\circlearrowleft \Sigma M_A = 0: \quad 5a\frac{1}{\sqrt{17}}B + \frac{3a}{4}\frac{4}{\sqrt{17}}B - 2aP = 0 \quad B = \frac{\sqrt{17}}{4}P \qquad \mathbf{B} = 1.031P \measuredangle 14.04^\circ \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: \quad A_x + \frac{4}{\sqrt{17}}B = 0 \quad A_x = -P$$

$$\uparrow \Sigma F_y = 0: \quad A_y - P + \frac{1}{\sqrt{17}}B = 0 \quad A_y = P - \frac{P}{4} = \frac{3P}{4} \qquad \mathbf{A} = 1.250P \searrow 36.9^\circ \blacktriangleleft$$

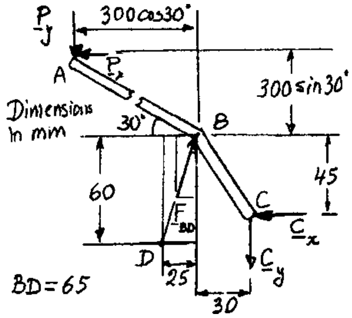
System is rigid. ◀



PROBLEM 6.122

The shear shown is used to cut and trim electronic circuit board laminates. For the position shown, determine (*a*) the vertical component of the force exerted on the shearing blade at *D*, (*b*) the reaction at *C*.

SOLUTION



We note that *BD* is a two-force member.

Free body: Member *ABC*: We have the components:

$$\mathbf{P}_x = (400\text{ N})\sin 30° = 200\text{ N} \longleftarrow$$

$$\mathbf{P}_y = (400\text{ N})\cos 30° = 346.41\text{ N}\downarrow$$

$$(\mathbf{F}_{BD})_x = \frac{25}{65}F_{BD} \longrightarrow$$

$$(\mathbf{F}_{BD})_y = \frac{60}{65}F_{BD}\uparrow$$

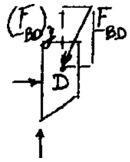
$$+\circlearrowleft \Sigma M_C = 0: \quad (F_{BD})_x(45) + (F_{BD})_y(30) - P_x(45 + 300\sin 30°) - P_y(30 + 300\cos 30°) = 0$$

$$\left(\frac{25}{65}F_{BD}\right)(45) + \left(\frac{60}{65}F_{BD}\right)(30) = (200)(195) + (346.41)(289.81)$$

$$45F_{BD} = 139.39\times 10^3$$

$$F_{BD} = 3097.6\text{ N}$$

(*a*) Vertical component of force exerted on shearing blade at *D*.



$$(F_{BD})_y = \frac{60}{65}F_{BD} = \frac{60}{65}(3097.6\text{ N}) = 2859.3\text{ N}$$

$$(\mathbf{F}_{BD})_y = 2860\text{ N}\downarrow \blacktriangleleft$$

PROBLEM 6.122 (Continued)

(*b*) Returning to *FB* diagram of member *ABC*,

$$\xrightarrow{+} \Sigma F_x = 0: \quad (F_{BD})_x - P_x - C_x = 0$$

$$C_x = (F_{BD})_x - P_x = \frac{25}{65} F_{BD} - P_x$$

$$= \frac{25}{65}(3097.6) - 200$$

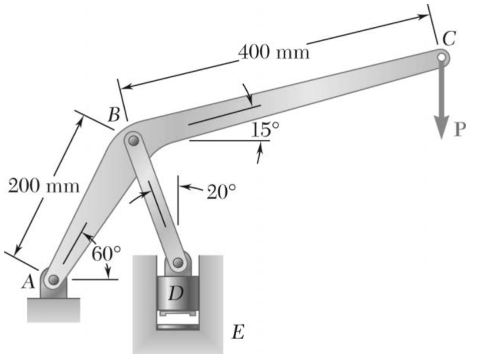
$$C_x = +991.39 \qquad \mathbf{C}_x = 991.39\text{ N} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad (F_{BD})_y - P_y - C_y = 0$$

$$C_y = (F_{BD})_y - P_y = \frac{60}{65} F_{BD} - P_y = \frac{60}{65}(3097.6) - 346.41$$

$$C_y = +2512.9\text{ N} \qquad \mathbf{C}_y = 2512.9 \downarrow$$

$$C = 2295\text{ N} \qquad \mathbf{C} = 2700\text{ N} \swarrow 68.5° \blacktriangleleft$$



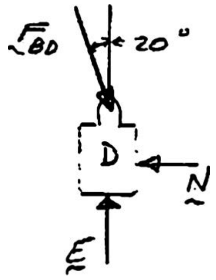
PROBLEM 6.123

The press shown is used to emboss a small seal at *E*. Knowing that $P = 250$ N, determine (*a*) the vertical component of the force exerted on the seal, (*b*) the reaction at *A*.

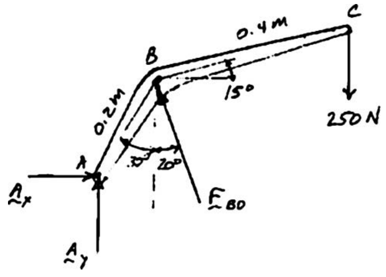
SOLUTION

FBD Stamp *D*:

$$\uparrow \Sigma F_y = 0: \quad E - F_{BD}\cos 20° = 0, \quad E = F_{BD}\cos 20°$$



FBD *ABC*:



$$\circlearrowleft \Sigma M_A = 0: \quad (0.2\text{ m})(\sin 30°)(F_{BD}\cos 20°) + (0.2\text{ m})(\cos 30°)(F_{BD}\sin 20°)$$

$$- [(0.2\text{ m})\sin 30° + (0.4\text{ m})\cos 15°](250\text{ N}) = 0$$

$$F_{BD} = 793.64\text{ N} \quad C$$

and, from above,

$$E = (793.64\text{ N})\cos 20°$$

(*a*) $\mathbf{E} = 746\text{ N}\downarrow$ ◀

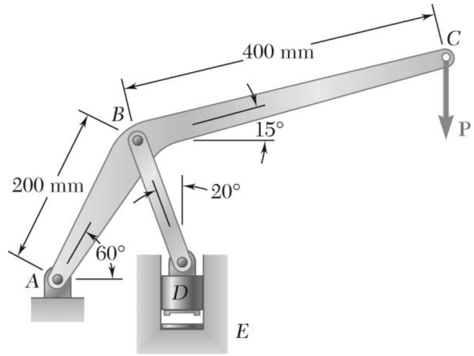
$$\rightarrow \Sigma F_x = 0: \quad A_x - (793.64\text{ N})\sin 20° = 0$$

$$\mathbf{A}_x = 271.44\text{ N} \rightarrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y + (793.64\text{ N})\cos 20° - 250\text{ N} = 0$$

$$\mathbf{A}_y = 495.78\text{ N}\downarrow$$

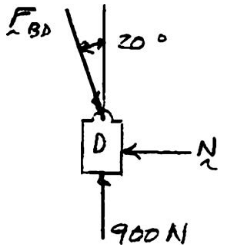
so (*b*) $\mathbf{A} = 565\text{ N} \searrow 61.3°$ ◀



PROBLEM 6.124

The press shown is used to emboss a small seal at *E*. Knowing that the vertical component of the force exerted on the seal must be 900 N, determine (*a*) the required vertical force **P**, (*b*) the corresponding reaction at *A*.

SOLUTION

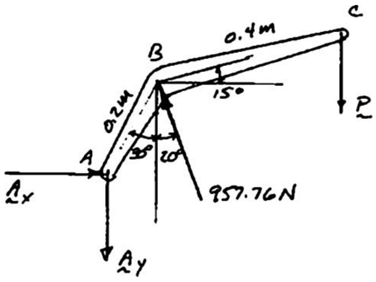


FBD Stamp *D*:

$$+\uparrow \Sigma F_y = 0: \quad 900\text{ N} - F_{BD}\cos 20° = 0, \quad F_{BD} = 957.76\text{ N} \quad C$$

(*a*)

FBD *ABC*:



$$+\circlearrowleft \Sigma M_A = 0: \quad [(0.2\text{ m})(\sin 30°)](957.76\text{ N})\cos 20° + [(0.2\text{ m})(\cos 30°)](957.76\text{ N})\sin 20°$$

$$-[(0.2\text{ m})\sin 30° + (0.4\text{ m})\cos 15°]P = 0$$

$$P = 301.70\text{ N}, \qquad \mathbf{P} = 302\text{ N}\downarrow \blacktriangleleft$$

(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (957.76\text{ N})\sin 20° = 0$$

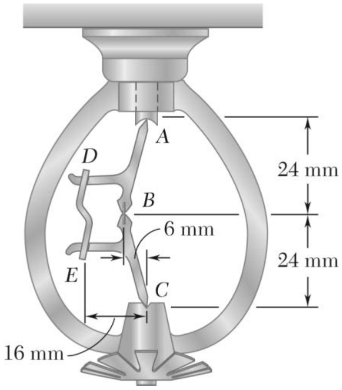
$$\mathbf{A}_x = 327.57\text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad -A_y + (957.76\text{ N})\cos 20° - 301.70\text{ N} = 0$$

$$\mathbf{A}_y = 598.30\text{ N}\downarrow$$

so

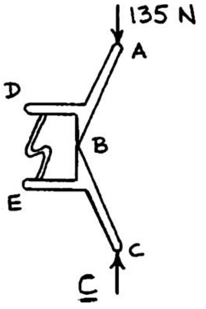
$$\mathbf{A} = 682\text{ N} \searrow 61.3° \blacktriangleleft$$



PROBLEM 6.125

Water pressure in the supply system exerts a downward force of 135 N on the vertical plug at *A*. Determine the tension in the fusible link *DE* and the force exerted on member *BCE* at *B*.

SOLUTION

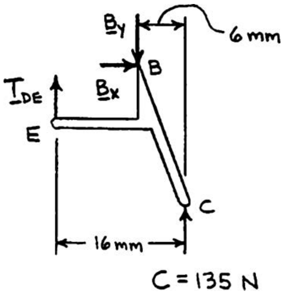


Free body: Entire linkage:

$$+\Sigma F_y = 0: \quad C - 135 = 0$$

$$C = +135 \text{ N}$$

Free body: Member *BCE*:



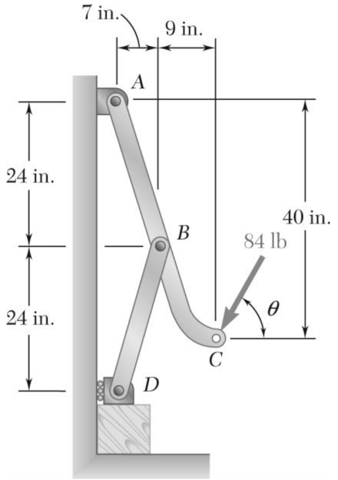
$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (135 \text{ N})(6 \text{ mm}) - T_{DE}(10 \text{ mm}) = 0$$

$$T_{DE} = 81.0 \text{ N} \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 135 + 81 - B_y = 0$$

$$B_y = +216 \text{ N} \qquad \mathbf{B} = 216 \text{ N}\downarrow \blacktriangleleft$$



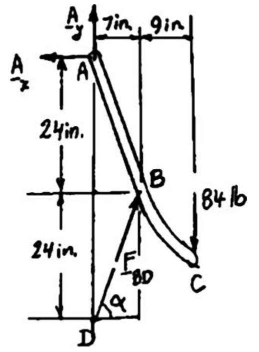
PROBLEM 6.126

An 84-lb force is applied to the toggle vise at *C*. Knowing that $\theta = 90°$, determine (*a*) the vertical force exerted on the block at *D*, (*b*) the force exerted on member *ABC* at *B*.

SOLUTION

We note that *BD* is a two-force member.

Free body: Member *ABC*:



We have $$BD = \sqrt{(7)^2 + (24)^2} = 25 \text{ in.}$$

$$(F_{BD})_x = \frac{7}{25}F_{BD}, \quad (F_{BD})_y = \frac{24}{25}F_{BD}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (F_{BD})_x(24) + (F_{BD})_y(7) - 84(16) = 0$$

$$\left(\frac{7}{25}F_{BD}\right)(24) + \left(\frac{24}{25}F_{BD}\right)(7) = 84(16)$$

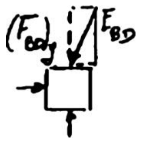
$$\frac{336}{25}F_{BD} = 1344$$

$$F_{BD} = 100 \text{ lb}$$

$$\tan\alpha = \frac{24}{7} \quad \alpha = 73.7°$$

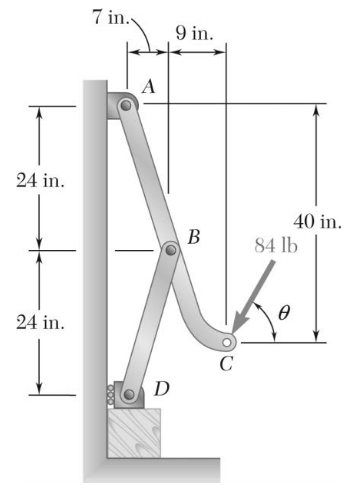
(*b*) Force exerted at *B*. $\mathbf{F}_{BD} = 100.0 \text{ lb} \measuredangle 73.7°$ ◀

(*a*) Vertical force exerted on block.



$$(F_{BD})_y = \frac{24}{25}F_{BD} = \frac{24}{25}(100 \text{ lb}) = 96 \text{ lb}$$

$(\mathbf{F}_{BD})_y = 96.0 \text{ lb} \downarrow$ ◀



PROBLEM 6.127

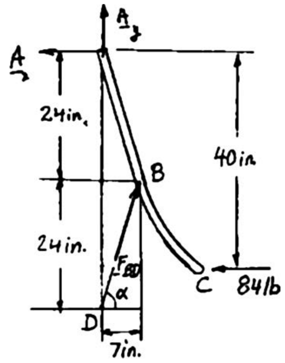
Solve Problem 6.126 when $\theta = 0$.

PROBLEM 6.126 An 84-lb force is applied to the toggle vise at *C*. Knowing that $\theta = 90°$, determine (*a*) the vertical force exerted on the block at *D*, (*b*) the force exerted on member *ABC* at *B*.

SOLUTION

We note that *BD* is a two-force member.

Free body: Member *ABC*:



We have $$BD = \sqrt{(7)^2 + (24)^2} = 25 \text{ in.}$$

$$(F_{BD})_x = \frac{7}{25} F_{BD}, \quad (F_{BD})_y = \frac{24}{25} F_{BD}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (F_{BD})_x(24) + (F_{BD})_y(7) - 84(40) = 0$$

$$\left(\frac{7}{25} F_{BD}\right)(24) + \left(\frac{24}{25} F_{BD}\right)(7) = 84(40)$$

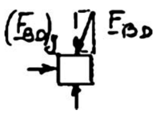
$$\frac{336}{25} F_{BD} = 3360$$

$$F_{BD} = 250 \text{ lb}$$

$$\tan\alpha = \frac{24}{7} \quad \alpha = 73.7°$$

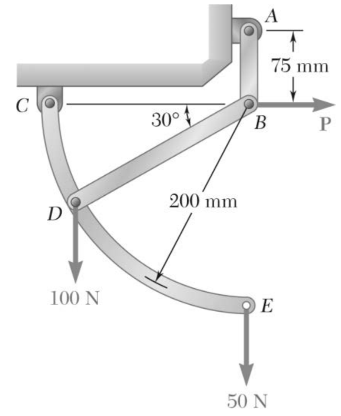
(*b*) Force exerted at *B*. $\mathbf{F}_{BD} = 250.0 \text{ lb} \measuredangle 73.7°$ ◀

(*a*) Vertical force exerted on block.



$$(F_{BD})_y = \frac{24}{25} F_{BD} = \frac{24}{25}(250 \text{ lb}) = 240 \text{ lb}$$

$(\mathbf{F}_{BD})_y = 240 \text{ lb} \downarrow$ ◀



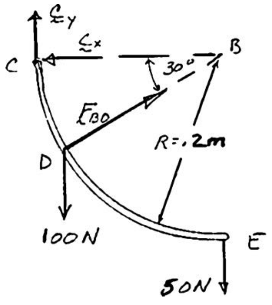
PROBLEM 6.128

For the system and loading shown, determine (*a*) the force **P** required for equilibrium, (*b*) the corresponding force in member *BD*, (*c*) the corresponding reaction at *C*.

SOLUTION

Member FBDs:

I:



FBD I:

$$\circlearrowleft \Sigma M_C = 0: \quad R(F_{BD}\sin 30°) - [R(1-\cos 30°)](100\text{ N}) - R(50\text{ N}) = 0$$

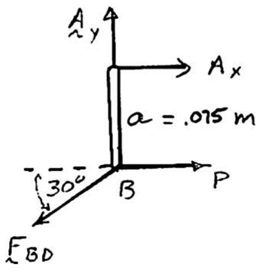
$$F_{BD} = 126.795\text{ N} \qquad (b)\quad F_{BD} = 126.8\text{ N} \quad T \;\blacktriangleleft$$

$$\xrightarrow{} \Sigma F_x = 0: \quad -C_x + (126.795\text{ N})\cos 30° = 0 \qquad \mathbf{C}_x = 109.808\text{ N} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: \quad C_y + (126.795\text{ N})\sin 30° - 100\text{ N} - 50\text{ N} = 0$$

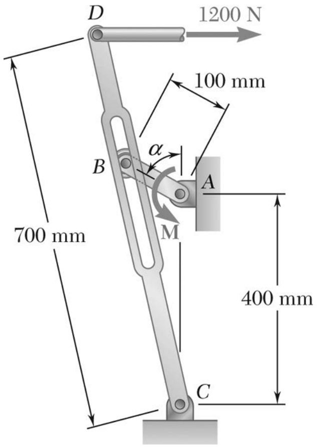
$$\mathbf{C}_y = 86.603\text{ N}\uparrow \qquad \text{so} \quad (c)\quad \mathbf{C} = 139.8\text{ N} \measuredangle 38.3° \;\blacktriangleleft$$

II:



FBD II:

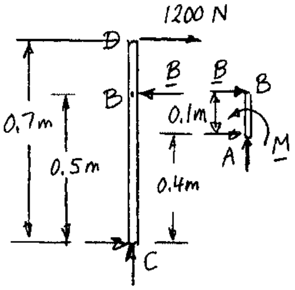
$$\circlearrowleft \Sigma M_A = 0: \quad aP - a[(126.795\text{ N})\cos 30°] = 0 \qquad (a)\quad \mathbf{P} = 109.8\text{ N} \longrightarrow \;\blacktriangleleft$$



PROBLEM 6.129

The Whitworth mechanism shown is used to produce a quick-return motion of Point *D*. The block at *B* is pinned to the crank *AB* and is free to slide in a slot cut in member *CD*. Determine the couple **M** that must be applied to the crank *AB* to hold the mechanism in equilibrium when (*a*) $\alpha = 0$, (*b*) $\alpha = 30°$.

SOLUTION



(*a*) Free body: Member *CD*:

$$+\circlearrowleft \Sigma M_C = 0: \quad B(0.5 \text{ m}) - (1200 \text{ N})(0.7 \text{ m}) = 0$$

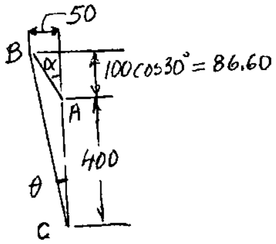
$$B = 1680 \text{ N}$$

Free body: Crank *AB*:

$$+\circlearrowleft \Sigma M_A = 0: \quad M - (1680 \text{ N})(0.1 \text{ m}) = 0$$

$$M = 168.0 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 168.0 \text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

(*b*) Geometry:



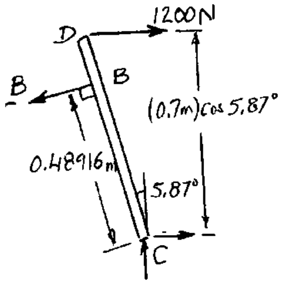
$$AB = 100 \text{ mm}, \quad \alpha = 30°$$

$$\tan\theta = \frac{50}{486.6} \qquad \theta = 5.87°$$

$$BC = \sqrt{(50)^2 + (486.6)^2}$$

$$= 489.16 \text{ mm}$$

Free body: Member *CD*:

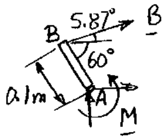


$$+\circlearrowleft \Sigma M_C = 0: \quad B(0.48916) - (1200 \text{ N})(0.7)\cos 5.87° = 0$$

$$B = 1708.2 \text{ N}$$

PROBLEM 6.129 (Continued)

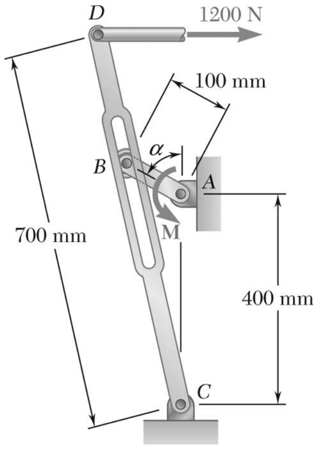
Free body: Crank *AB*:



$$+\circlearrowleft \Sigma M_A = 0: \quad M - (B \sin 65.87°)(0.1\text{ m}) = 0$$

$$M = (1708.2\text{ N})(0.1\text{ m})\sin 65.87°$$

$$M = 155.90\text{ N}\cdot\text{m} \qquad \mathbf{M} = 155.9\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$



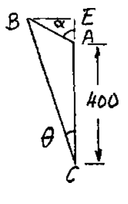
PROBLEM 6.130

Solve Problem 6.129 when (*a*) $\alpha = 60°$, (*b*) $\alpha = 90°$.

PROBLEM 6.129 The Whitworth mechanism shown is used to produce a quick-return motion of Point *D*. The block at *B* is pinned to the crank *AB* and is free to slide in a slot cut in member *CD*. Determine the couple **M** that must be applied to the crank *AB* to hold the mechanism in equilibrium when (*a*) $\alpha = 0$, (*b*) $\alpha = 30°$.

SOLUTION

(*a*) Geometry:



$$AB = 100 \text{ mm}, \quad \alpha = 60°$$

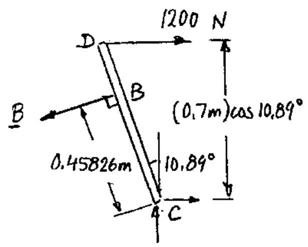
$$BE = 100\cos 60° = 86.60$$

$$CE = 400 + 100\sin 60° = 450$$

$$\tan\theta = \frac{86.60}{450} \quad \theta = 10.89°$$

$$BC = \sqrt{(86.60)^2 + (450)^2} = 458.26 \text{ mm}$$

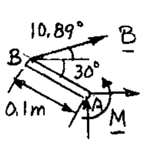
Free body: Member *CD*:



$$+\circlearrowleft \Sigma M_C = 0: \quad B(0.45826 \text{ m}) - (1200 \text{ N})(0.7 \text{ m})\cos 10.89° = 0$$

$$B = 1800.0 \text{ N}$$

Free body: Crank *AB*:



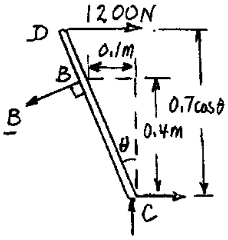
$$+\circlearrowleft \Sigma M_A = 0: \quad M - (B\sin 40.89°)(0.1 \text{ m}) = 0$$

$$M = (1800.0 \text{ N})(0.1 \text{ m})\sin 40.89°$$

$$M = 117.83 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 117.8 \text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

PROBLEM 6.130 (Continued)

(*b*) Free body: Member *CD*:



$$\tan\theta = \frac{0.1\text{ m}}{0.4\text{ m}}$$

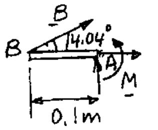
$$\theta = 14.04°$$

$$BC = \sqrt{(0.1)^2 + (0.4)^2} = 0.41231\text{ m}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad B(0.41231) - (1200\text{ N})(0.7\cos 14.04°) = 0$$

$$B = 1976.4\text{ N}$$

Free body: Crank *AB*:



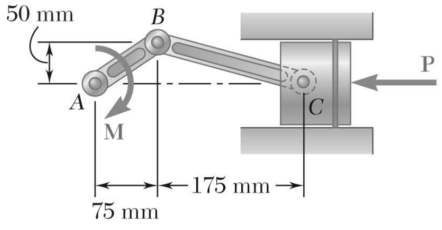
$$+\circlearrowleft \Sigma M_A = 0: \quad M - (B\sin 14.04°)(0.1\text{ m}) = 0$$

$$M = (1976.4\text{ N})(0.1\text{ m})\sin 14.04°$$

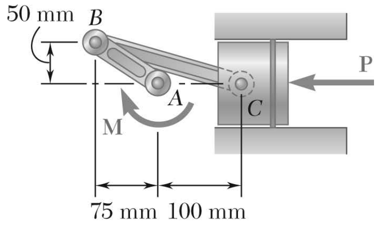
$$M = 47.948\text{ N}\cdot\text{m} \qquad \mathbf{M} = 47.9\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

PROBLEM 6.131

A couple **M** of magnitude 1.5 kN · m is applied to the crank of the engine system shown. For each of the two positions shown, determine the force **P** required to hold the system in equilibrium.



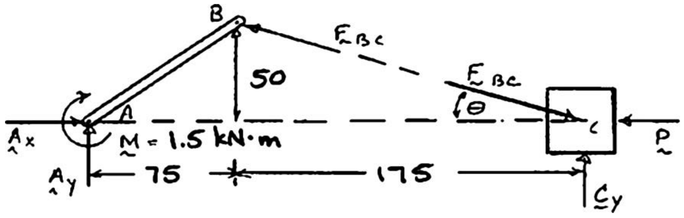
(*a*)



(*b*)

SOLUTION

(*a*) **FBDs:**



Dimensions in mm

Note: $\tan\theta = \dfrac{50 \text{ mm}}{175 \text{ mm}}$

$$= \frac{2}{7}$$

FBD whole: $\circlearrowleft \Sigma M_A = 0$: $(0.250 \text{ m})C_y - 1.5 \text{ kN} \cdot \text{m} = 0 \quad C_y = 6.00 \text{ kN}$

FBD piston: $\uparrow \Sigma F_y = 0$: $C_y - F_{BC}\sin\theta = 0 \quad F_{BC} = \dfrac{C_y}{\sin\theta} = \dfrac{6.00 \text{ kN}}{\sin\theta}$

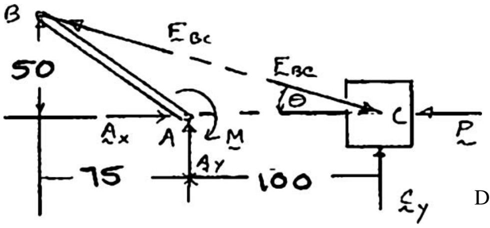
$\longrightarrow \Sigma F_x = 0$: $F_{BC}\cos\theta - P = 0$

$$P = F_{BC}\cos\theta = \frac{6.00 \text{ kN}}{\tan\theta} = 7 \text{ kips}$$

P = 21.0 kN ⟵ ◀

PROBLEM 6.131 (Continued)

(*b*) **FBDs:**



Dimensions in mm

Note: $\tan\theta = \frac{2}{7}$ as above

FBD whole: $\circlearrowleft \Sigma M_A = 0$: $(0.100\text{ m})C_y - 1.5\text{ kN}\cdot\text{m} = 0 \quad C_y = 15\text{ kN}$

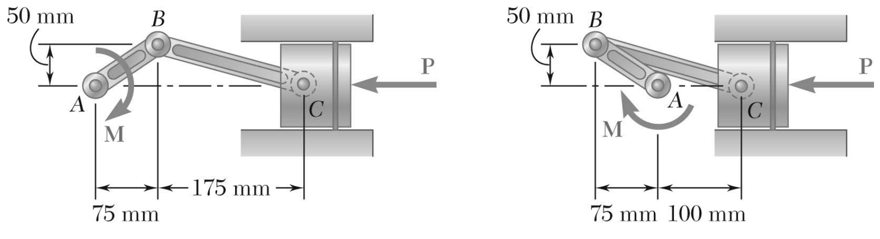
$$\uparrow \Sigma F_y = 0: \quad C_y - F_{BC}\sin\theta = 0 \quad F_{BC} = \frac{C_y}{\sin\theta}$$

$$\xrightarrow{} \Sigma F_x = 0: \quad F_{BC}\cos\theta - P = 0$$

$$P = F_{BC}\cos\theta = \frac{C_y}{\tan\theta} = \frac{15\text{ kN}}{2/7} \qquad \mathbf{P} = 52.5\text{ kN} \longleftarrow \blacktriangleleft$$

PROBLEM 6.132

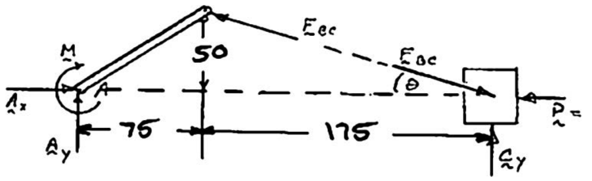
A force **P** of magnitude 16 kN is applied to the piston of the engine system shown. For each of the two positions shown, determine the couple **M** required to hold the system in equilibrium.



(*a*) (*b*)

SOLUTION

(*a*) **FBDs:**



Dimensions in mm

Note: $$\tan\theta = \frac{50\text{ mm}}{175\text{ mm}} = \frac{2}{7}$$

FBD piston: $$\xrightarrow{} \Sigma F_x = 0: \quad F_{BC}\cos\theta - P = 0 \quad F_{BC} = \frac{P}{\cos\theta}$$

$$\uparrow \Sigma F_y = 0: \quad C_y - F_{BC}\sin\theta = 0 \quad C_y = F_{BC}\sin\theta = P\tan\theta = \frac{2}{7}P$$

FBD whole: $$\circlearrowleft \Sigma M_A = 0: \quad (0.250\text{ m})C_y - M = 0$$

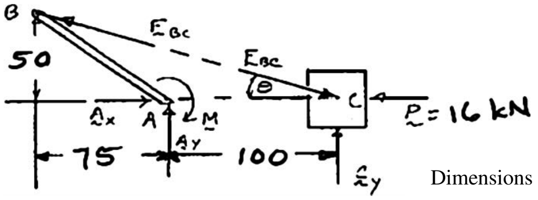
$$M = (0.250\text{ m})\left(\frac{2}{7}\right)(16\text{ kN})$$

$$= 1.14286\text{ kN}\cdot\text{m}$$

$\mathbf{M} = 1143\text{ N}\cdot\text{m} \circlearrowright$ ◀

PROBLEM 6.132 (Continued)

(*b*) **FBDs:**



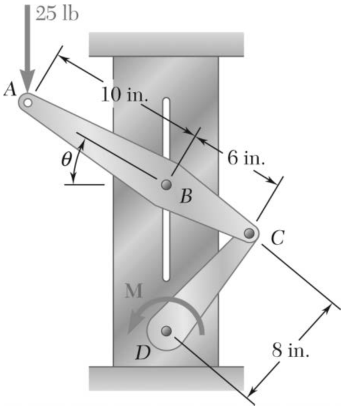
Dimensions in mm

Note: $\tan\theta = \dfrac{2}{7}$ as above

FBD piston, as above: $C_y = P\tan\theta = \dfrac{2}{7}P$

FBD whole: $\circlearrowleft \Sigma M_A = 0$: $(0.100\text{ m})C_y - M = 0 \quad M = (0.100\text{ m})\dfrac{2}{7}(16\text{ kN})$

$M = 0.45714\text{ kN}\cdot\text{m}$ $\mathbf{M} = 457\text{ N}\cdot\text{m} \circlearrowright$ ◀

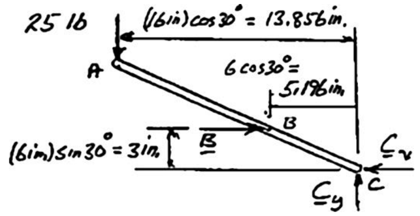


PROBLEM 6.133

The pin at B is attached to member ABC and can slide freely along the slot cut in the fixed plate. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 30°$.

SOLUTION

<u>Free body: Member *ABC*:</u>



$$+\circlearrowleft \Sigma M_C = 0: \quad (25\text{ lb})(13.856\text{ in.}) - B(3\text{ in.}) = 0$$

$$B = +115.47\text{ lb}$$

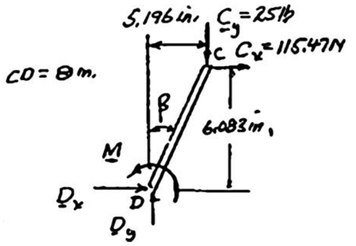
$$+\uparrow \Sigma F_y = 0: \quad -25\text{ lb} + C_y = 0$$

$$C_y = +25\text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 115.47\text{ lb} - C_x = 0$$

$$C_x = +115.47\text{ lb}$$

<u>Free body: Member *CD*:</u>



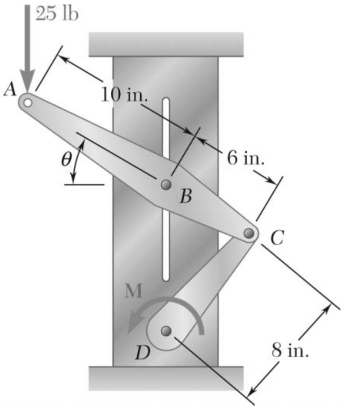
$$\beta = \sin^{-1}\frac{5.196}{8}; \quad \beta = 40.505°$$

$$CD\cos\beta = (8\text{ in.})\cos 40.505° = 6.083\text{ in.}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M - (25\text{ lb})(5.196\text{ in.}) - (115.47\text{ lb})(6.083\text{ in.}) = 0$$

$$M = +832.3\text{ lb}\cdot\text{in.}$$

$$\mathbf{M} = 832\text{ lb}\cdot\text{in.} \blacktriangleleft$$

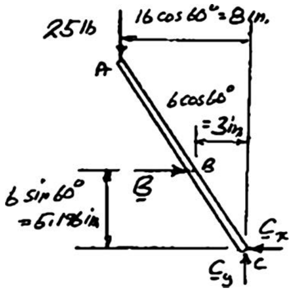


PROBLEM 6.134

The pin at B is attached to member ABC and can slide freely along the slot cut in the fixed plate. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 60°$.

SOLUTION

<u>Free body: Member ABC:</u>



$$+\circlearrowleft \Sigma M_C = 0: \quad (25\text{ lb})(8\text{ in.}) - B(5.196\text{ in.}) = 0$$

$$B = +38.49\text{ lb}$$

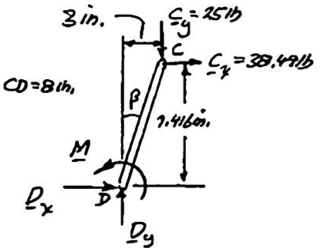
$$\xrightarrow{+} \Sigma F_x = 0: \quad 38.49\text{ lb} - C_x = 0$$

$$C_x = +38.49\text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad -25\text{ lb} + C_y = 0$$

$$C_y = +25\text{ lb}$$

<u>Free body: Member CD:</u>

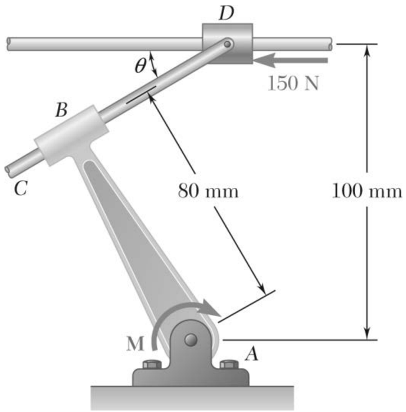


$$\beta = \sin^{-1}\frac{3}{8}; \quad \beta = 22.024°$$

$$CD\cos\beta = (8\text{ in.})\cos 22.024° = 7.416\text{ in.}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M - (25\text{ lb})(3\text{ in.}) - (38.49\text{ lb})(7.416\text{ in.}) = 0$$

$$M = +360.4\text{ lb}\cdot\text{in.} \qquad \mathbf{M} = 360\text{ lb}\cdot\text{in.}\circlearrowleft \blacktriangleleft$$

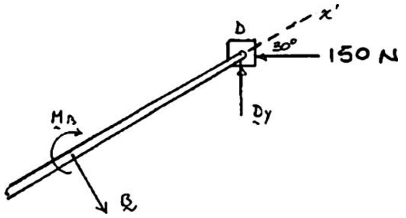


PROBLEM 6.135

Rod *CD* is attached to the collar *D* and passes through a collar welded to end *B* of lever *AB*. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 30°$.

SOLUTION

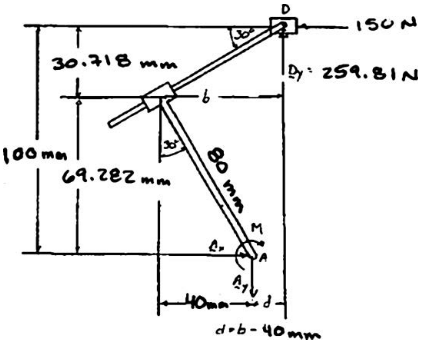
FBD *DC*:



$$\nearrow \Sigma F_{x'} = 0: \quad D_y \sin 30° - (150\text{ N})\cos 30° = 0$$

$$D_y = (150\text{ N})\text{ ctn } 30° = 259.81\text{ N}$$

FBD machine:



$$\circlearrowleft \Sigma M_A = 0: \quad (0.100\text{ m})(150\text{ N}) + d(259.81\text{ N}) - M = 0$$

$$d = b - 0.040\text{ m}$$

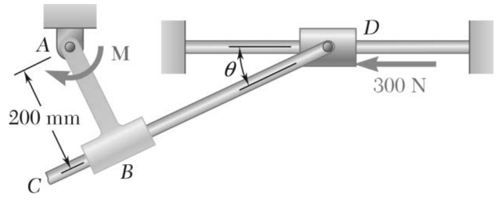
$$b = \frac{0.030718\text{ m}}{\tan 30}$$

so $$b = 0.053210\text{ m}$$

$$d = 0.0132100\text{ m}$$

$$M = 18.4321\text{ N}\cdot\text{m}$$

$\mathbf{M} = 18.43\text{ N}\cdot\text{m.} \circlearrowright$ ◀

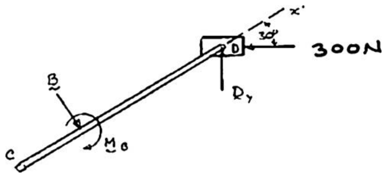


PROBLEM 6.136

Rod *CD* is attached to the collar *D* and passes through a collar welded to end *B* of lever *AB*. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 30°$.

SOLUTION

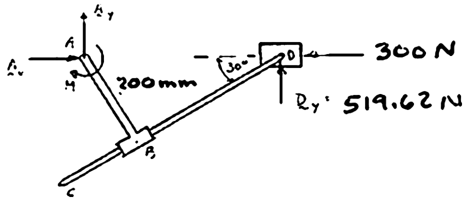
Note: $\mathbf{B} \perp CD$



FBD *DC*: $\nearrow \Sigma F_{x'} = 0\colon \quad D_y \sin 30° - (300 \text{ N})\cos 30° = 0$

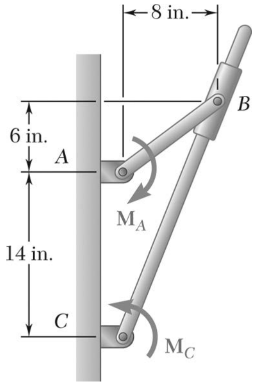
$$D_y = \frac{300 \text{ N}}{\tan 30°} = 519.62 \text{ N}$$

FBD machine:



$$\circlearrowleft \Sigma M_A = 0\colon \quad \frac{0.200 \text{ m}}{\sin 30°} 519.62 \text{ N} - M = 0$$

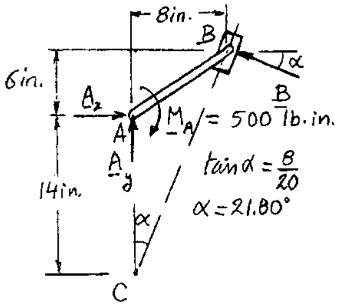
$M = 207.85 \text{ N}\cdot\text{m}$ $\quad\quad$ $\mathbf{M} = 208 \text{ N}\cdot\text{m} \circlearrowright$ ◀

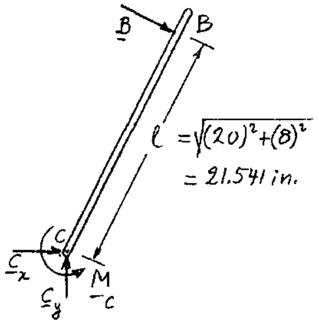


PROBLEM 6.137

Two rods are connected by a frictionless collar *B*. Knowing that the magnitude of the couple $\mathbf{M}_A$ is 500 lb · in., determine (*a*) the couple $\mathbf{M}_C$ required for equilibrium, (*b*) the corresponding components of the reaction at *C*.

SOLUTION





(*a*) Free body: Rod *AB* & collar:

$$+\circlearrowleft \Sigma M_A = 0: \quad (B\cos\alpha)(6\text{ in.}) + (B\sin\alpha)(8\text{ in.}) - M_A = 0$$

$$B = (6\cos 21.8° + 8\sin 21.8°) - 500 = 0$$

$$B = 58.535\text{ lb}$$

Free body: Rod *BC*:

$$+\Sigma M_C = 0: \quad M_C - B\ell = 0$$

$$M_C = B\ell = (58.535\text{ lb})(21.541\text{ in.}) = 1260.9\text{ lb}\cdot\text{in.} \qquad \mathbf{M}_C = 1261\text{ lb}\cdot\text{in.} \circlearrowleft \blacktriangleleft$$

(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + B\cos\alpha = 0$$

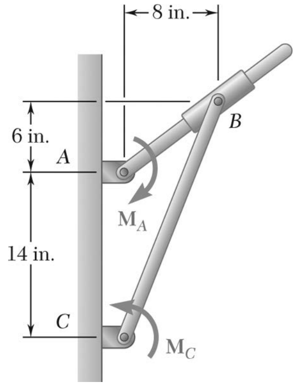
$$C_x = -B\cos\alpha = -(58.535\text{ lb})\cos 21.8° = -54.3\text{ lb}$$

$$\mathbf{C}_x = 54.3\text{ lb} \longleftarrow \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad C_y - B\sin\alpha = 0$$

$$C_y = B\sin\alpha = (58.535\text{ lb})\sin 21.8° = +21.7\text{ lb}$$

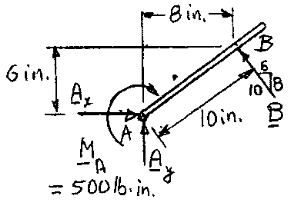
$$\mathbf{C}_y = 21.7\text{ lb}\uparrow \blacktriangleleft$$

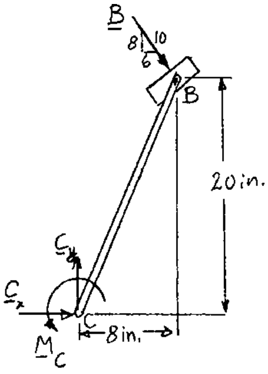


PROBLEM 6.138

Two rods are connected by a frictionless collar *B*. Knowing that the magnitude of the couple $\mathbf{M}_A$ is 500 lb · in., determine (*a*) the couple $\mathbf{M}_C$ required for equilibrium, (*b*) the corresponding components of the reaction at *C*.

SOLUTION





(*a*) Free body: Rod *AB*:

$$+\circlearrowleft \Sigma M_A = 0: \quad B(10 \text{ in.}) - 500 \text{ lb} \cdot \text{in.} = 0$$

$$B = 50.0$$

Free body: Rod *BC* & collar:

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C - (0.6B)(20 \text{ in.}) - (0.8B)(8 \text{ in.}) = 0$$

$$M_C - (30 \text{ lb})(20 \text{ in.}) - (40 \text{ lb})(8 \text{ in.}) = 0$$

$$M_C = 920 \text{ lb} \cdot \text{in.}$$

$\mathbf{M}_C = 920 \text{ lb} \cdot \text{in.} \circlearrowleft$ ◀

(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 0.6B = 0$$

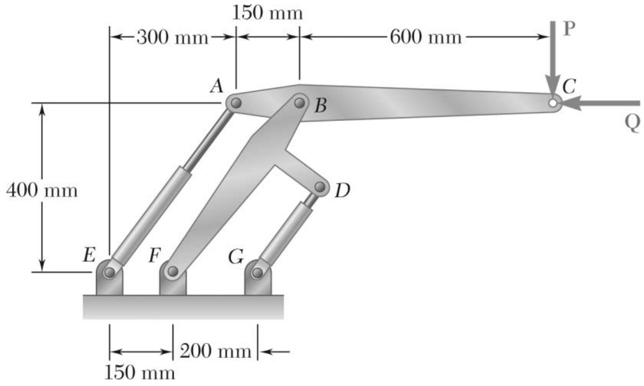
$$C_x = -0.6B = -0.6(50.0 \text{ lb}) = -30.0 \text{ lb}$$

$\mathbf{C}_x = 30.0 \text{ lb} \longleftarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad C_y - 0.8B = 0$$

$$C_y = 0.8B = 0.8(50.0 \text{ lb}) = +40.0 \text{ lb}$$

$\mathbf{C}_y = 40.0 \text{ lb} \uparrow$ ◀

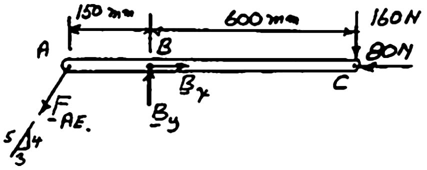


PROBLEM 6.139

Two hydraulic cylinders control the position of the robotic arm *ABC*. Knowing that in the position shown the cylinders are parallel, determine the force exerted by each cylinder when $P = 160\text{ N}$ and $Q = 80\text{ N}$.

SOLUTION

<u>Free body: Member *ABC*:</u>



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{4}{5}F_{AE}(150\text{ mm}) - (160\text{ N})(600\text{ mm}) = 0$$

$$F_{AE} = +800\text{ N} \qquad\qquad F_{AE} = 800\text{ N} \quad T \blacktriangleleft$$

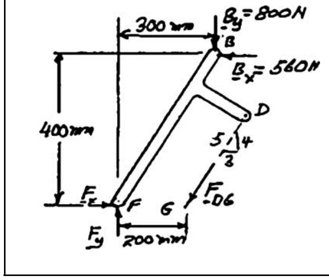
$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}(800\text{ N}) + B_x - 80\text{ N} = 0$$

$$B_x = +560\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}(800\text{ N}) + B_y - 160\text{ N} = 0$$

$$B_y = +800\text{ N}$$

<u>Free body: Member *BDF*:</u>

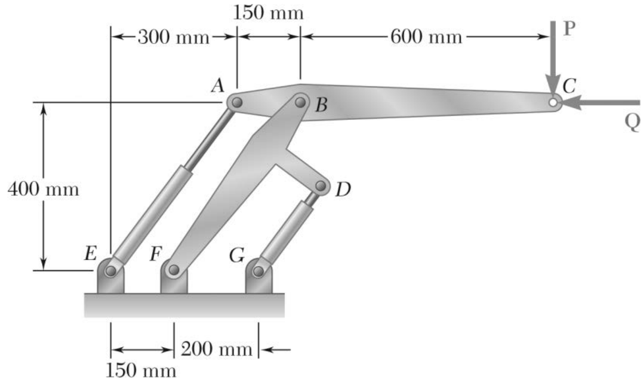


$$+\circlearrowleft \Sigma M_F = 0: \quad (560\text{ N})(400\text{ mm}) - (800\text{ N})(300\text{ mm}) - \frac{4}{5}F_{DG}(200\text{ mm}) = 0$$

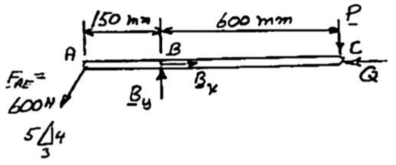
$$F_{DG} = -100\text{ N} \qquad\qquad F_{DG} = 100.0\text{ N} \quad C \blacktriangleleft$$

PROBLEM 6.140

Two hydraulic cylinders control the position of the robotic arm *ABC*. In the position shown, the cylinders are parallel and both are in tension. Knowing the $F_{AE} = 600\text{ N}$ and $F_{DG} = 50\text{ N}$, determine the forces **P** and **Q** applied at *C* to arm *ABC*.



SOLUTION



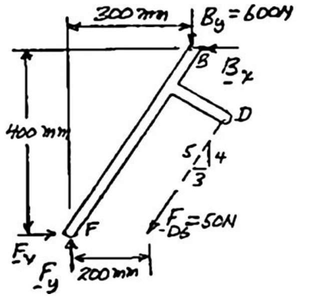
Free body: Member *ABC*:

$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{4}{5}(600\text{ N})(150\text{ mm}) - P(600\text{ mm}) = 0$$

$$P = +120\text{ N} \qquad \mathbf{P} = 120.0\text{ N}\downarrow$$ ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{4}{5}(600)(750\text{ mm}) - B_y(600\text{ mm}) = 0$$

$$B_y = +600\text{ N}$$



Free body: Member *BDF*:

$$+\circlearrowleft \Sigma M_F = 0: \quad B_x(400\text{ mm}) - (600\text{ N})(300\text{ mm})$$

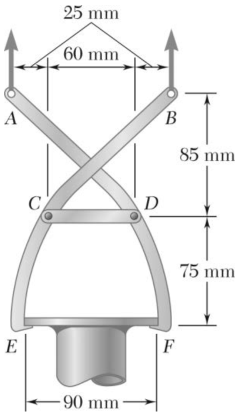
$$-\frac{4}{5}(50\text{ N})(200\text{ mm}) = 0$$

$$B_x = +470\text{ N}$$

Return to free body: Member *ABC*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}(600\text{ N}) + 470\text{ N} - Q = 0$$

$$Q = +110\text{ N} \qquad \mathbf{Q} = 110.0\text{ N} \longleftarrow$$ ◀

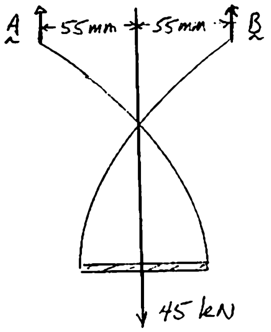


PROBLEM 6.141

The tongs shown are used to apply a total upward force of 45 kN on a pipe cap. Determine the forces exerted at *D* and *F* on tong *ADF*.

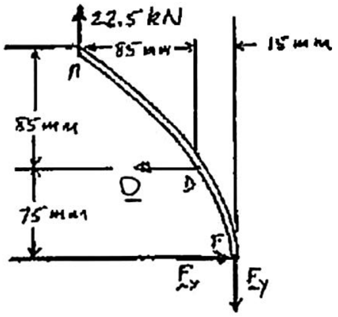
SOLUTION

FBD whole:



By symmetry, $\mathbf{A} = \mathbf{B} = 22.5\text{ kN}\uparrow$

FBD *ADF*:



$$\circlearrowleft \Sigma M_F = 0\text{:}\quad (75\text{ mm})D - (100\text{ mm})(22.5\text{ kN}) = 0$$

$$\mathbf{D} = 30.0\text{ kN} \longleftarrow \blacktriangleleft$$

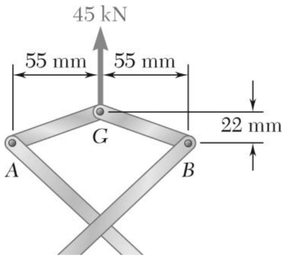
$$\xrightarrow{} \Sigma F_x = 0\text{:}\quad F_x - D = 0$$

$$F_x = D = 30\text{ kN}$$

$$\uparrow \Sigma F_y = 0\text{:}\quad 22.5\text{ kN} - F_y = 0$$

$$F_y = 22.5\text{ kN}$$

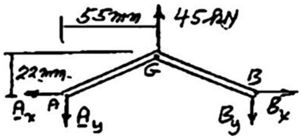
so $$\mathbf{F} = 37.5\text{ kN} \searrow 36.9° \blacktriangleleft$$



PROBLEM 6.142

If the toggle shown is added to the tongs of Problem 6.141 and a single vertical force is applied at *G*, determine the forces exerted at *D* and *F* on tong *ADF*.

SOLUTION



Free body: Toggle:

By symmetry,

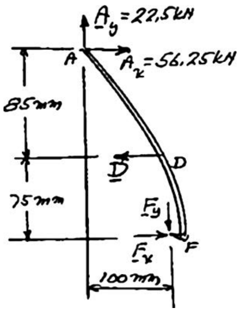
$$A_y = \frac{1}{2}(45\text{ kN}) = 22.5\text{ kN}$$

AG is a two-force member.

$$\frac{22.5\text{ kN}}{22\text{ mm}} = \frac{A_x}{55\text{ mm}}$$

$$A_x = 56.25\text{ kN}$$

Free body: Tong *ADF*:



$$+\uparrow \Sigma F_y = 0: \quad 22.5\text{ kN} - F_y = 0$$

$$F_y = +22.5\text{ kN}$$

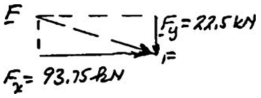
$$+\circlearrowleft \Sigma M_F = 0: \quad D(75\text{ mm}) - (22.5\text{ kN})(100\text{ mm}) - (56.25\text{ kN})(160\text{ mm}) = 0$$

$$D = +150\text{ kN}$$

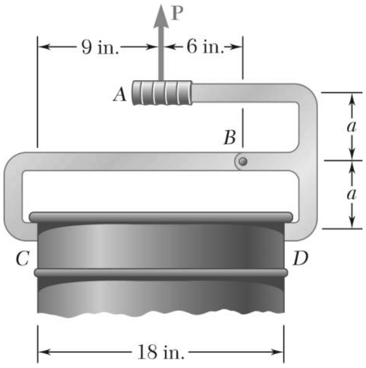
$\mathbf{D} = 150.0\text{ kN} \longleftarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad 56.25\text{ kN} - 150\text{ kN} + F_x = 0$$

$$F_x = 93.75\text{ kN}$$



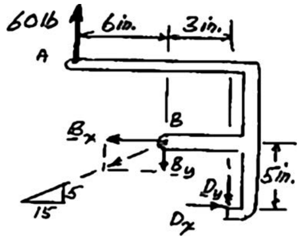
$\mathbf{F} = 96.4\text{ kN} \searrow 13.50^\circ$ ◀



PROBLEM 6.143

A small barrel weighing 60 lb is lifted by a pair of tongs as shown. Knowing that $a = 5$ in., determine the forces exerted at *B* and *D* on tong *ABD*.

SOLUTION



We note that *BC* is a two-force member.

Free body: Tong *ABD*:

$$\frac{B_x}{15} = \frac{B_y}{5} \quad B_x = 3B_y$$

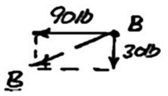
$$+\circlearrowleft \Sigma M_D = 0: \quad B_y(3\text{ in.}) + 3B_y(5\text{ in.}) - (60\text{ lb})(9\text{ in.}) = 0$$

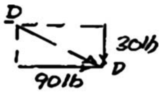
$$B_y = 30\text{ lb}\downarrow$$

$$B_x = 3B_y: \quad B_x = 90\text{ lb} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -90\text{ lb} + D_x = 0 \qquad \mathbf{D}_x = 90\text{ lb} \longrightarrow$$

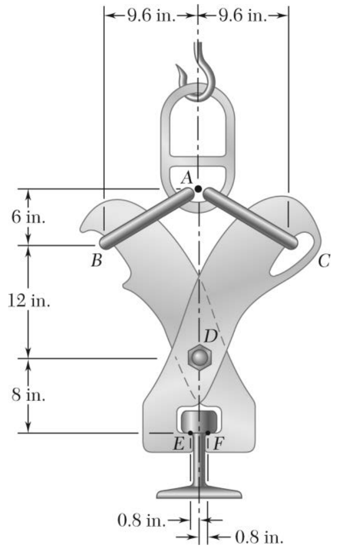
$$+\uparrow \Sigma F_y = 0: \quad 60\text{ lb} - 30\text{ lb} - D_y = 0 \qquad \mathbf{D}_y = 30\text{ lb}\downarrow$$





$\mathbf{B} = 94.9\text{ lb} \swarrow 18.43°$ ◀

$\mathbf{D} = 94.9\text{ lb} \searrow 18.43°$ ◀

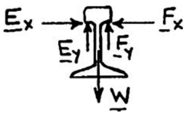


PROBLEM 6.144

A 39-ft length of railroad rail of weight 44 lb/ft is lifted by the tongs shown. Determine the forces exerted at *D* and *F* on tong *BDF*.

SOLUTION

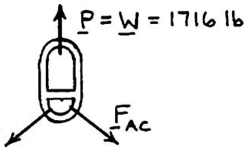
Free body: Rail:



$$W = (39\text{ ft})(44\text{ lb/ft}) = 1716\text{ lb}$$

By symmetry, $$E_y = F_y = \frac{1}{2}W = 858\text{ lb}$$

Free body: Upper link:

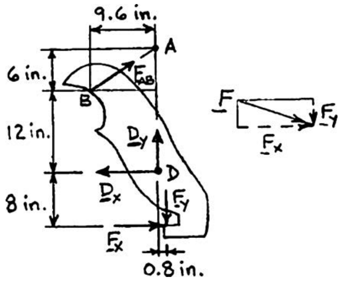


By symmetry, $$(F_{AB})_y = (F_{AC})_y = \frac{1}{2}W = 858\text{ lb}$$

Since *AB* is a two-force member,

$$\frac{(F_{AB})_x}{9.6} = \frac{(F_{AB})_y}{6} \qquad (F_{AB})_x = \frac{9.6}{6}(858) = 1372.8\text{ lb}$$

Free Body: Tong *BDF*:



$$+\circlearrowleft \Sigma M_D = 0: \quad (\text{Attach } F_{AB} \text{ at } A.)$$

$$F_x(8) - (F_{AB})_x(18) - F_y(0.8) = 0$$

$$F_x(8) - (1372.8\text{ lb})(18) - (858\text{ lb})(0.8) = 0$$

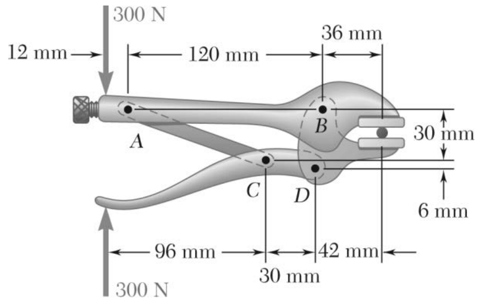
$$F_x = +3174.6\text{ lb} \qquad \mathbf{F} = 3290\text{ lb} \searrow 15.12° ◀$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -D_x + (F_{AB})_x + F_x = 0$$

$$D_x = (F_{AB})_x + F_x = 1372.8 + 3174.6 = 4547.4\text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad D_y + (F_{AB})_y - F_y = 0$$

$$D_y = 0 \qquad \mathbf{D} = 4550\text{ lb} \longleftarrow ◀$$



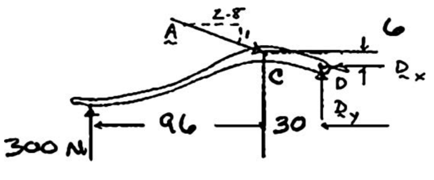
PROBLEM 6.145

Determine the magnitude of the gripping forces produced when two 300-N forces are applied as shown.

SOLUTION

We note that AC is a two-force member.

FBD handle *CD*:



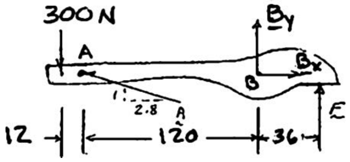
Dimensions in mm

$$\curvearrowleft \Sigma M_D = 0: \quad -(126\text{ mm})(300\text{ N}) - (6\text{ mm})\frac{2.8}{\sqrt{8.84}}A$$

$$+ (30\text{ mm})\left(\frac{1}{\sqrt{8.84}}A\right) = 0$$

$$A = 2863.6\sqrt{8.84}\text{ N}$$

FBD handle *AB*:

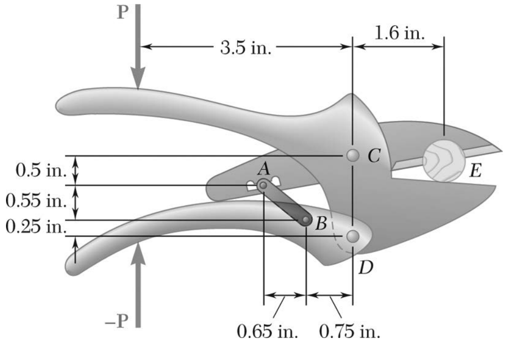


Dimensions in mm

$$\curvearrowleft \Sigma M_B = 0: \quad (132\text{ mm})(300\text{ N}) - (120\text{ mm})\frac{1}{\sqrt{8.84}}(2863.6\sqrt{8.84}\text{ N})$$

$$+(36\text{ mm})F = 0$$

$$F = 8.45\text{ kN} \blacktriangleleft$$

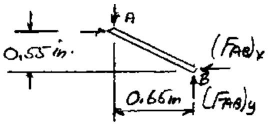


PROBLEM 6.146

The compound-lever pruning shears shown can be adjusted by placing pin *A* at various ratchet positions on blade *ACE*. Knowing that 300-lb vertical forces are required to complete the pruning of a small branch, determine the magnitude *P* of the forces that must be applied to the handles when the shears are adjusted as shown.

SOLUTION

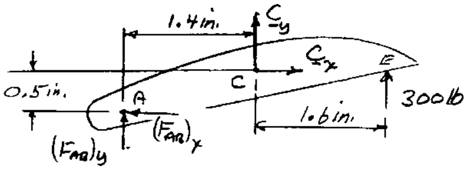
We note that *AB* is a two-force member.



$$\frac{(F_{AB})_x}{0.65\text{ in.}} = \frac{(F_{AB})_y}{0.55\text{ in.}}$$

$$(F_{AB})_y = \frac{11}{13}(F_{AB})_x \qquad (1)$$

Free body: Blade *ACE*:



$$+\circlearrowleft \Sigma M_C = 0: \quad (300\text{ lb})(1.6\text{ in.}) - (F_{AB})_x(0.5\text{ in.}) - (F_{AB})_y(1.4\text{ in.}) = 0$$

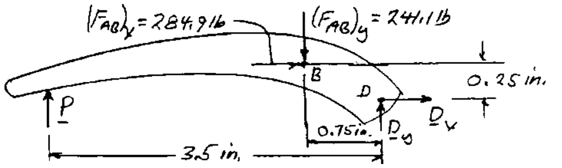
Use Eq. (1):

$$(F_{AB})_x(0.5\text{ in.}) + \frac{11}{13}(F_{AB})_x(1.4\text{ in.}) = 480\text{ lb}\cdot\text{in.}$$

$$1.6846(F_{AB})_x = 480 \qquad (F_{AB})_x = 284.9\text{ lb}$$

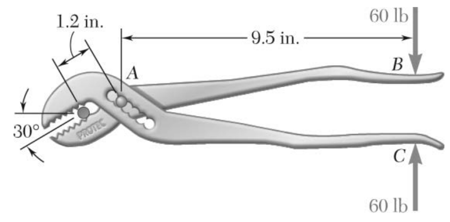
$$(F_{AB})_y = \frac{11}{13}(284.9\text{ lb}) \qquad (F_{AB})_y = 241.1\text{ lb}$$

Free body: Lower handle:



$$+\circlearrowleft \Sigma M_D = 0: \quad (241.1\text{ lb})(0.75\text{ in.}) - (284.9\text{ lb})(0.25\text{ in.}) - P(3.5\text{ in.}) = 0$$

$$P = 31.3\text{ lb} \blacktriangleleft$$



PROBLEM 6.147

The pliers shown are used to grip a 0.3-in.-diameter rod. Knowing that two 60-lb forces are applied to the handles, determine (*a*) the magnitude of the forces exerted on the rod, (*b*) the force exerted by the pin at *A* on portion *AB* of the pliers.

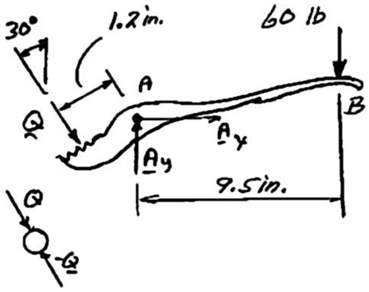
SOLUTION

<u>Free body: Portion *AB*:</u>

(*a*)

$$+\circlearrowleft \Sigma M_A = 0: \quad Q(1.2\text{ in.}) - (60\text{ lb})(9.5\text{ in.}) = 0$$

$$Q = 475\text{ lb} \quad ◀$$



(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad Q(\sin 30°) + A_x = 0$$

$$(475\text{ lb})(\sin 30°) + A_x = 0$$

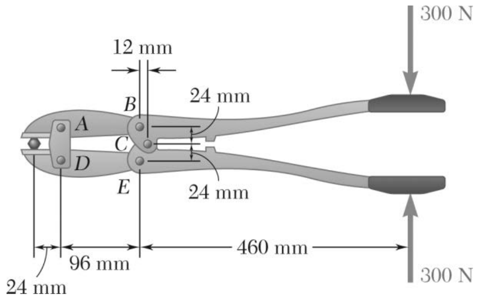
$$A_x = -237.5\text{ lb} \qquad \mathbf{A}_x = 237.5\text{ lb} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad -Q(\cos 30°) + A_y - 60\text{ lb} = 0$$

$$-(475\text{ lb})(\cos 30°) + A_y - 60\text{ lb} = 0$$

$$A_y = +471.4\text{ lb} \qquad \mathbf{A}_y = 471.4\text{ lb} \uparrow$$

$$\mathbf{A} = 528\text{ lb} \measuredangle\, 63.3° \quad ◀$$

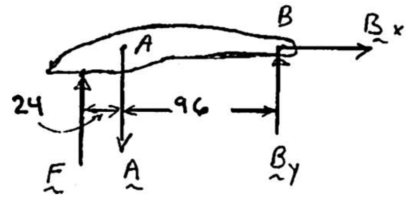


PROBLEM 6.148

In using the bolt cutter shown, a worker applies two 300-N forces to the handles. Determine the magnitude of the forces exerted by the cutter on the bolt.

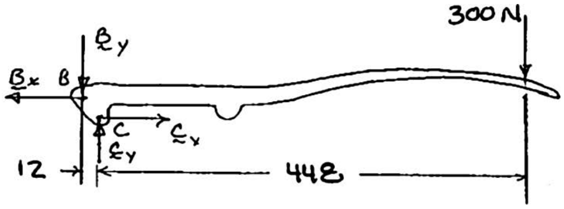
SOLUTION

FBD cutter *AB*:



I

FBD handle *BC*:



II

Dimensions in mm

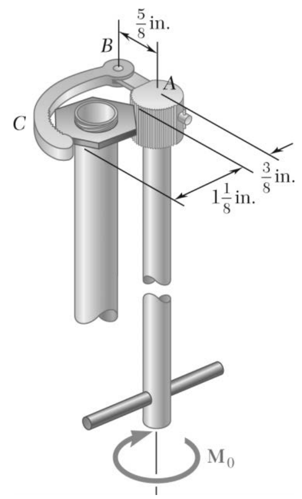
FBD I: $\xrightarrow{} \Sigma F_x = 0{:} \quad B_x = 0$

FBD II: $\circlearrowleft \Sigma M_C = 0{:} \quad (12\text{ mm})B_y - (448\text{ mm})300\text{ N} = 0$

$$B_y = 11{,}200.0\text{ N}$$

Then FBD I: $\circlearrowleft \Sigma M_A = 0{:} \quad (96\text{ mm})B_y - (24\text{ mm})F = 0 \qquad F = 4B_y$

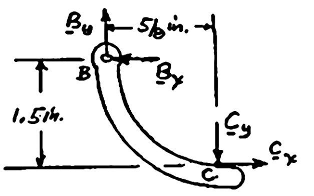
$$F = 44{,}800\text{ N} = 44.8\text{ kN} \blacktriangleleft$$



PROBLEM 6.149

The specialized plumbing wrench shown is used in confined areas (e.g., under a basin or sink). It consists essentially of a jaw *BC* pinned at *B* to a long rod. Knowing that the forces exerted on the nut are equivalent to a clockwise (when viewed from above) couple of magnitude 135 lb · in., determine (*a*) the magnitude of the force exerted by pin *B* on jaw *BC*, (*b*) the couple $\mathbf{M}_0$ that is applied to the wrench.

SOLUTION



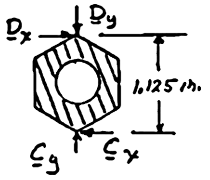
Free body: Jaw *BC*:

This is a two-force member.

$$\frac{C_y}{1.5\text{ in.}} = \frac{C_x}{\frac{5}{8}\text{ in.}} \quad C_y = 2.4C_x$$

$$\Sigma F_x = 0: \quad B_x = C_x \qquad (1)$$

$$\Sigma F_y = 0: \quad B_y = C_y = 2.4\,C_x \qquad (2)$$



Free body: Nut: $\Sigma F_x = 0: \quad C_x = D_x$

$$\circlearrowright \Sigma M = 135\text{ lb}\cdot\text{in.}$$

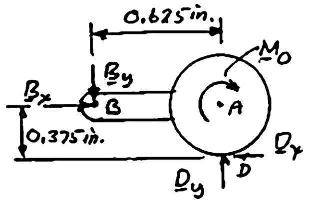
$$C_x(1.125\text{ in.}) = 135\text{ lb}\cdot\text{in.}$$

$$C_x = 120\text{ lb}$$

(*a*) Eq. (1): $B_x = C_x = 120\text{ lb}$

Eq. (2): $B_y = C_y = 2.4(120\text{ lb}) = 288\text{ lb}$

$$B = \left(B_x^2 + B_y^2\right)^{1/2} = (120^2 + 288^2)^{1/2} \qquad B = 312\text{ lb} \blacktriangleleft$$

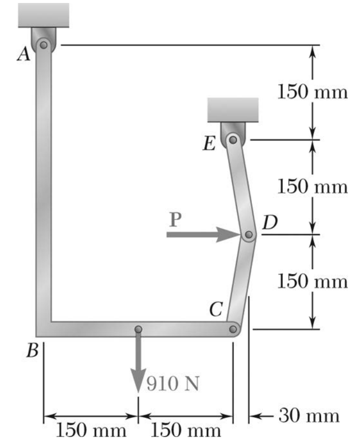


(*b*) Free body: Rod:

$$+\circlearrowright \Sigma M_D = 0: \quad -M_0 + B_y(0.625\text{ in.}) - B_x(0.375\text{ in.}) = 0$$

$$-M_0 + (288)(0.625) - (120)(0.375) = 0$$

$$\mathbf{M}_0 = 135.0\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$



PROBLEM 6.150

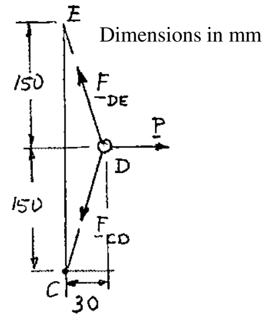
Determine the force **P** that must be applied to the toggle *CDE* to maintain bracket *ABC* in the position shown.

SOLUTION

We note that *CD* and *DE* are two-force members.

Free body: Joint *D*:

$$\frac{(F_{CD})_x}{30} = \frac{(F_{CD})_y}{150} \qquad (F_{CD})_y = 5(F_{CD})_x$$



Similarly, $(F_{DE})_y = 5(F_{DE})_x$

$$+\uparrow \Sigma F_y = 0: \quad (F_{DE})_y = (F_{CD})_y$$

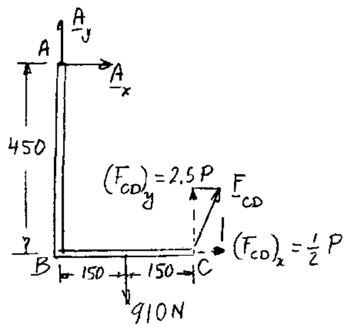
It follows that $(F_{DE})_x = (F_{CD})_x$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - (F_{DE})_x - (F_{CD})_x = 0$$

$$(F_{DE})_x = (F_{CD})_x = \frac{1}{2}P$$

Also, $(F_{DE})_y = (F_{CD})_y = 5\left(\frac{1}{2}P\right) = 2.5P$

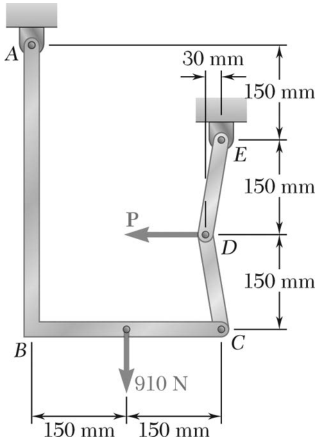
Free body: Member *ABC*:



$$+\circlearrowleft \Sigma M_A = 0: \quad (2.5P)(300) + \left(\frac{1}{2}P\right)(450) - (910\text{ N})(150) = 0$$

$$(750 + 225)P = (910\text{ N})(150)$$

$$P = 140.0\text{ N} \blacktriangleleft$$



PROBLEM 6.151

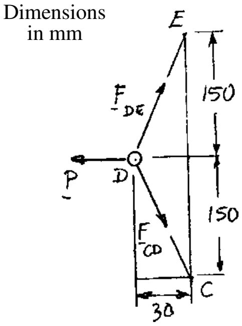
Determine the force **P** that must be applied to the toggle *CDE* to maintain bracket *ABC* in the position shown.

SOLUTION

We note that *CD* and *DE* are two-force members.

Free body: Joint *D*:

$$\frac{(F_{CD})_x}{30} = \frac{(F_{CD})_y}{150} \qquad (F_{CD})_y = 5(F_{CD})_x$$



Similarly, $(F_{DE})_y = 5(F_{DE})_x$

$$+\uparrow \Sigma F_y = 0: \quad (F_{DE})_y = (F_{CD})_y$$

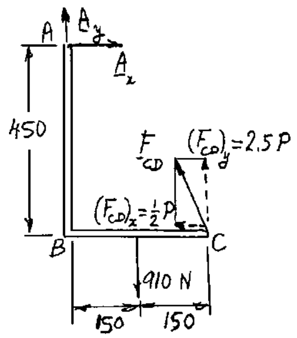
It follows that $(F_{DE})_x = (F_{CD})_x$

$$\xrightarrow{+} \Sigma F_x = 0: \quad (F_{DE})_x + (F_{CD})_x - P = 0$$

$$(F_{DE})_x = (F_{CD})_x = \frac{1}{2}P$$

Also, $(F_{DE})_y = (F_{CD})_y = 5\left(\frac{1}{2}P\right) = 2.5P$

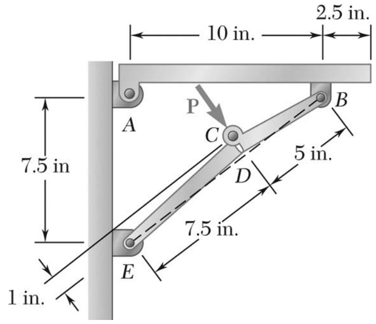
Free body: Member *ABC*:



$$+\circlearrowleft \Sigma M_A = 0: \quad (2.5P)(300) - \left(\frac{1}{2}P\right)(450) - (910 \text{ N})(150) = 0$$

$$(750 - 225)P = (910 \text{ N})(150)$$

$P = 260$ N ◀

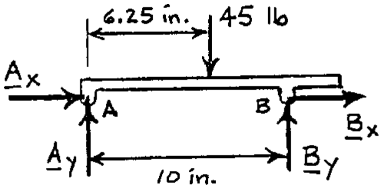


PROBLEM 6.152

A 45-lb shelf is held horizontally by a self-locking brace that consists of two parts *EDC* and *CDB* hinged at *C* and bearing against each other at *D*. Determine the force **P** required to release the brace.

SOLUTION

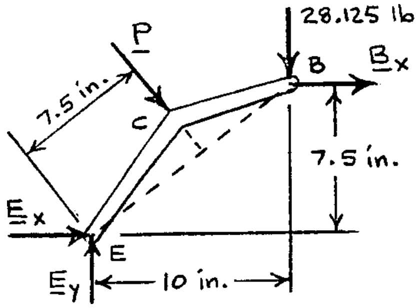
Free body: Shelf:



$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(10\text{ in.}) - (45\text{ lb})(6.25\text{ in.}) = 0$$

$$B_y = 28.125\text{ lb}$$

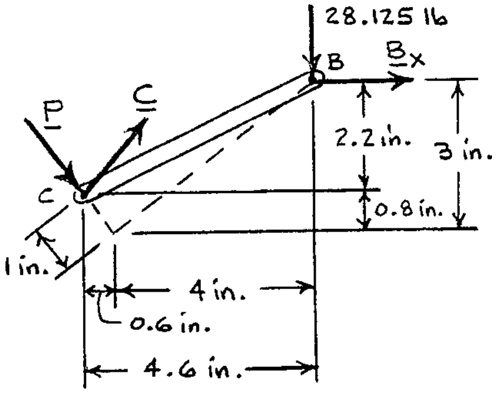
Free body: Portion *ECB*:



$$+\circlearrowleft \Sigma M_E = 0: \quad -B_x(7.5\text{ in.}) - P(7.5\text{ in.}) - (28.125\text{ lb})(10\text{ in.}) = 0$$

$$B_x = -37.5 - P$$

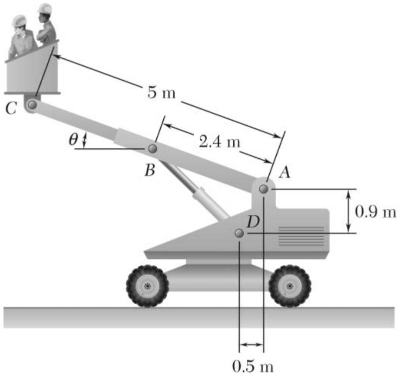
Free body: Portion *CDB*:



$$+\circlearrowleft \Sigma M_C = 0: \quad -(28.125\text{ lb})(4.6\text{ in.}) - B_x(2.2\text{ in.}) = 0$$

$$-(28.125\text{ lb})(4.6\text{ in.}) - (-37.5 - P)(2.2\text{ in.}) = 0$$

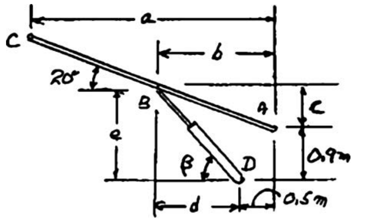
$P = 21.3$ lb **P** = 21.3 lb ↘ ◀



PROBLEM 6.153

The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together have a mass of 200 kg and have a combined center of gravity located directly above *C*. For the position when $\theta = 20°$, determine (*a*) the force exerted at *B* by the single hydraulic cylinder *BD*, (*b*) the force exerted on the supporting carriage at *A*.

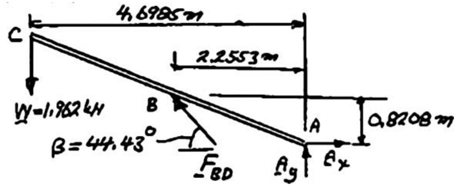
SOLUTION



Geometry:

$$a = (5\text{ m})\cos 20° = 4.6985\text{ m}$$
$$b = (2.4\text{ m})\cos 20° = 2.2553\text{ m}$$
$$c = (2.4\text{ m})\sin 20° = 0.8208\text{ m}$$
$$d = b - 0.5 = 1.7553\text{ m}$$
$$e = c + 0.9 = 1.7208\text{ m}$$

$$\tan\beta = \frac{e}{d} = \frac{1.7208}{1.7553}; \quad \beta = 44.43°$$



Free body: Arm *ABC*:

We note that *BD* is a two-force member.

$$W = (200\text{ kg})(9.81\text{ m/s}^2) = 1.962\text{ kN}$$

(*a*) $+\circlearrowleft \Sigma M_A = 0$: $(1.962\text{ kN})(4.6985\text{ m}) - F_{BD}\sin 44.43°(2.2553\text{ m}) + F_{BD}\cos 44.43(0.8208\text{ m}) = 0$

$$9.2185 - F_{BD}(0.9927) = 0: \quad F_{BD} = 9.2867\text{ kN}$$

(*b*) $\mathbf{F}_{BD} = 9.29\text{ kN} \searrow 44.4°$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $A_x - F_{BD}\cos\beta = 0$

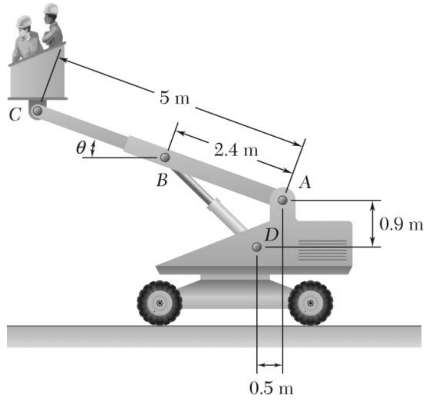
$A_x = (9.2867\text{ kN})\cos 44.43° = 6.632\text{ kN}$ $\quad \mathbf{A}_x = 6.632\text{ kN} \longrightarrow$

$+\uparrow \Sigma F_y = 0$: $A_y - 1.962\text{ kN} + F_{BD}\sin\beta = 0$

$A_y = 1.962\text{ kN} - (9.2867\text{ kN})\sin 44.43° = -4.539\text{ kN}$

$\mathbf{A}_y = 4.539\text{ kN}\downarrow$

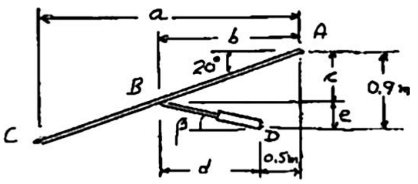
$\mathbf{A} = 8.04\text{ kN} \nwarrow 34.4°$ ◀



PROBLEM 6.154

The telescoping arm *ABC* can be lowered until end *C* is close to the ground, so that workers can easily board the platform. For the position when $\theta = -20°$, determine (*a*) the force exerted at *B* by the single hydraulic cylinder *BD*, (*b*) the force exerted on the supporting carriage at *A*.

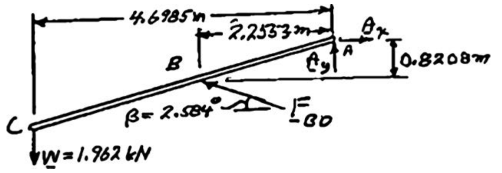
SOLUTION



Geometry:

$$a = (5\text{ m})\cos 20° = 4.6985\text{ m}$$
$$b = (2.4\text{ m})\cos 20° = 2.2552\text{ m}$$
$$c = (2.4\text{ m})\sin 20° = 0.8208\text{ m}$$
$$d = b - 0.5 = 1.7553\text{ m}$$
$$e = 0.9 - c = 0.0792\text{ m}$$

$$\tan\beta = \frac{e}{d} = \frac{0.0792}{1.7552}; \quad \beta = 2.584°$$



Free body: Arm *ABC*:

We note that *BD* is a two-force member.

$$W = (200\text{ kg})(9.81\text{ m/s}^2)$$
$$W = 1962\text{ N} = 1.962\text{ kN}$$

(*a*) $+\circlearrowleft \Sigma M_A = 0$: $(1.962\text{ kN})(4.6985\text{ m}) - F_{BD}\sin 2.584°(2.2553\text{ m}) - F_{BD}\cos 2.584°(0.8208\text{ m}) = 0$

$$9.2185 - F_{BD}(0.9216) = 0 \quad F_{BD} = 10.003\text{ kN}$$

(*b*)

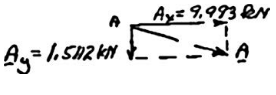
$\mathbf{F}_{BD} = 10.00\text{ kN} \searrow 2.58°$ ◀

$\xrightarrow{+} \Sigma F_x = 0$: $A_x - F_{BD}\cos\beta = 0$

$A_x = (10.003\text{ kN})\cos 2.583° = 9.993\text{ kN}$ $\qquad \mathbf{A}_x = 9.993\text{ kN} \longrightarrow$

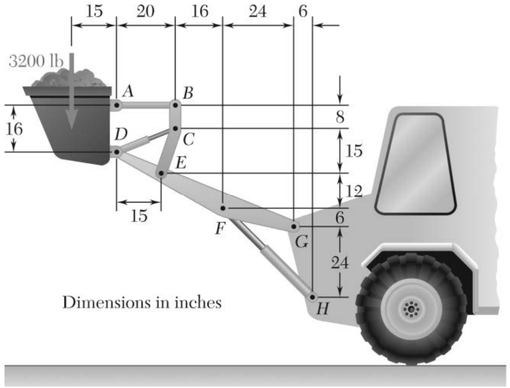
$+\!\uparrow \Sigma F_y = 0$: $A_y - 1.962\text{ kN} + F_{BD}\sin\beta = 0$

$$A_y = 1.962\text{ kN} - (10.003\text{ kN})\sin 2.583° = -1.5112\text{ kN}$$



$\mathbf{A}_y = 1.5112\text{ kN}\downarrow$

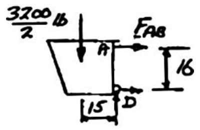
$\mathbf{A} = 10.11\text{ kN} \searrow 8.60°$ ◀



PROBLEM 6.155

The bucket of the front-end loader shown carries a 3200-lb load. The motion of the bucket is controlled by two identical mechanisms, only one of which is shown. Knowing that the mechanism shown supports one-half of the 3200-lb load, determine the force exerted (*a*) by cylinder *CD*, (*b*) by cylinder *FH*.

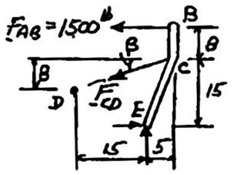
SOLUTION



Free body: Bucket: (one mechanism)

$$+\circlearrowleft \Sigma M_D = 0: \quad (1600\text{ lb})(15\text{ in.}) - F_{AB}(16\text{ in.}) = 0$$

$$F_{AB} = 1500\text{ lb}$$



Note: There are two identical support mechanisms.

Free body: One arm *BCE*:

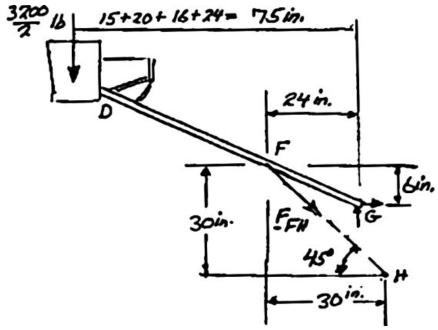
$$\tan\beta = \frac{8}{20}$$

$$\beta = 21.8°$$

$$+\circlearrowleft \Sigma M_E = 0: \quad (1500\text{ lb})(23\text{ in.}) + F_{CD}\cos 21.8°(15\text{ in.}) - F_{CD}\sin 21.8°(5\text{ in.}) = 0$$

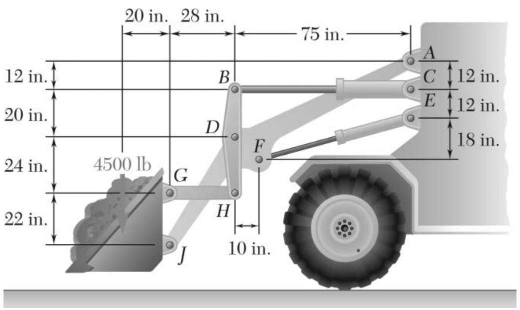
$$F_{CD} = -2858\text{ lb} \qquad F_{CD} = 2.86\text{ kips} \quad C \blacktriangleleft$$

Free body: Arm *DFG*:



$$+\circlearrowleft \Sigma M_G = 0: \quad (1600\text{ lb})(75\text{ in.}) + F_{FH}\sin 45°(24\text{ in.}) - F_{FH}\cos 45°(6\text{ in.}) = 0$$

$$F_{FH} = -9.428\text{ kips} \qquad F_{FH} = 9.43\text{ kips} \quad C \blacktriangleleft$$

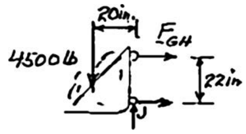


PROBLEM 6.156

The motion of the bucket of the front-end loader shown is controlled by two arms and a linkage that are pin-connected at *D*. The arms are located symmetrically with respect to the central, vertical, and longitudinal plane of the loader; one arm *AFJ* and its control cylinder *EF* are shown. The single linkage *GHDB* and its control cylinder *BC* are located in the plane of symmetry. For the position and loading shown, determine the force exerted (*a*) by cylinder *BC*, (*b*) by cylinder *EF*.

SOLUTION

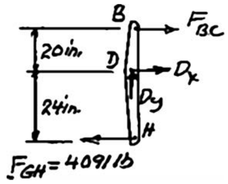
Free body: Bucket



$$+\circlearrowleft \Sigma M_J = 0: \quad (4500 \text{ lb})(20 \text{ in.}) - F_{GH}(22 \text{ in.}) = 0$$

$$F_{GH} = 4091 \text{ lb}$$

Free body: Arm *BDH*



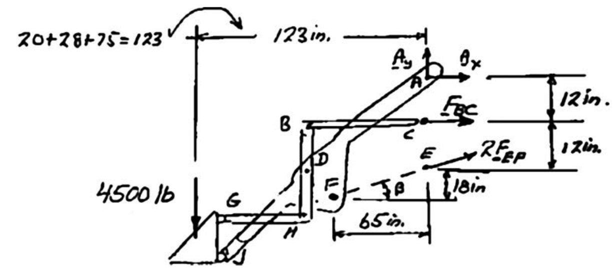
$$+\circlearrowleft \Sigma M_D = 0: \quad -(4091 \text{ lb})(24 \text{ in.}) - F_{BC}(20 \text{ in.}) = 0$$

$$F_{BC} = -4909 \text{ lb}$$

$$F_{BC} = 4.91 \text{ kips} \quad C \blacktriangleleft$$

Free body: Entire mechanism

(Two arms and cylinders *AFJE*)



Note: Two arms thus $2F_{EF}$

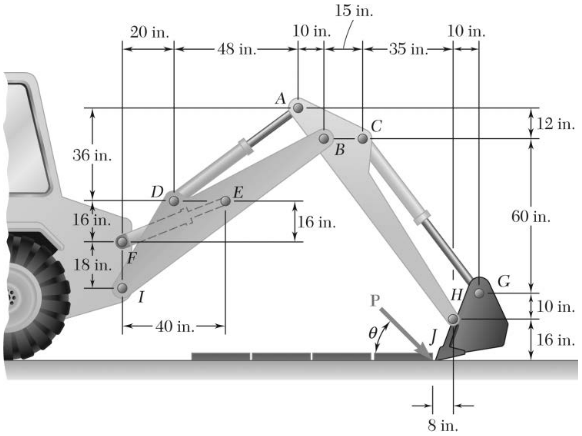
$$\tan\beta = \frac{18 \text{ in.}}{65 \text{ in.}}$$

$$\beta = 15.48^\circ$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (4500 \text{ lb})(123 \text{ in.}) + F_{BC}(12 \text{ in.}) + 2F_{EF}\cos\beta(24 \text{ in.}) = 0$$

$$(4500 \text{ lb})(123 \text{ in.}) - (4909 \text{ lb})(12 \text{ in.}) + 2F_{EF}\cos 15.48^\circ(24 \text{ in.}) = 0$$

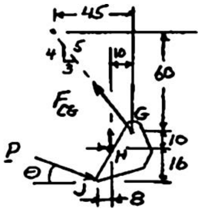
$$F_{EF} = -10.690 \text{ lb} \qquad F_{EF} = 10.69 \text{ kips} \quad C \blacktriangleleft$$



PROBLEM 6.157

The motion of the backhoe bucket shown is controlled by the hydraulic cylinders *AD*, *CG*, and *EF*. As a result of an attempt to dislodge a portion of a slab, a 2-kip force **P** is exerted on the bucket teeth at *J*. Knowing that $\theta = 45°$, determine the force exerted by each cylinder.

SOLUTION



Free body: Bucket:

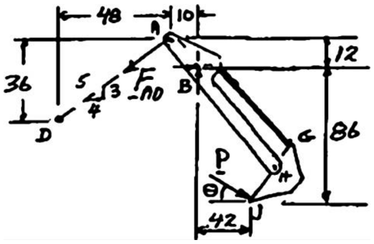
$$+\circlearrowleft \Sigma M_H = 0: \quad \text{(Dimensions in inches)}$$

$$\frac{4}{5}F_{CG}(10) + \frac{3}{5}F_{CG}(10) + P\cos\theta(16) + P\sin\theta(8) = 0$$

$$F_{CG} = -\frac{P}{14}(16\cos\theta + 8\sin\theta) \qquad (1)$$

Free body: Arm *ABH* and bucket:

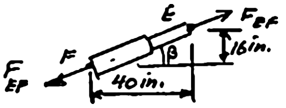
(Dimensions in inches)



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{4}{5}F_{AD}(12) + \frac{3}{5}F_{AD}(10) + P\cos\theta(86) - P\sin\theta(42) = 0$$

$$F_{AD} = -\frac{P}{15.6}(86\cos\theta - 42\sin\theta) \qquad (2)$$

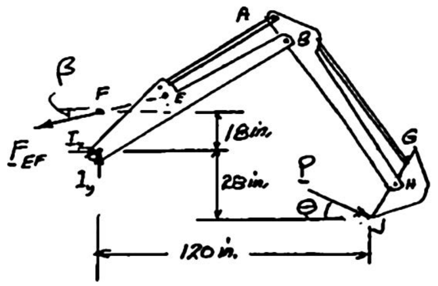
Free body: Bucket and arms *IEB* + *ABH*:



Geometry of cylinder *EF*: $\tan\beta = \dfrac{16 \text{ in.}}{40 \text{ in.}}$

$$\beta = 21.801°$$

PROBLEM 6.157 (Continued)



$$+\circlearrowleft \Sigma M_I = 0: \quad F_{EF}\cos\beta(18\text{ in.}) + P\cos\theta(28\text{ in.}) - P\sin\theta(120\text{ in.}) = 0$$

$$F_{EF} = \frac{P(120\sin\theta - 28\cos\theta)}{\cos 21.8°(18)}$$

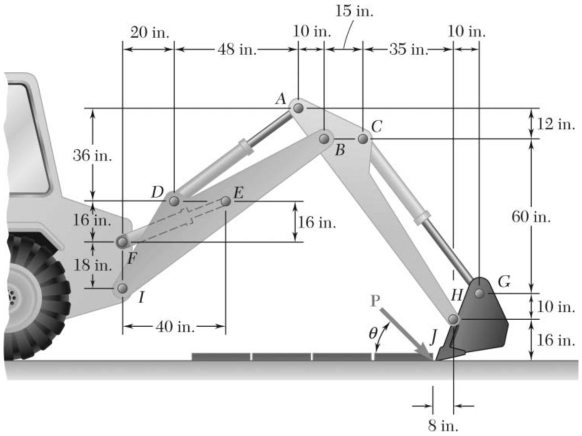
$$= \frac{P}{16.7126}(120\sin\theta - 28\cos\theta) \qquad (3)$$

For $P = 2$ kips, $\theta = 45°$

From Eq. (1): $F_{CG} = -\frac{2}{14}(16\cos 45° + 8\sin 45°) = -2.42$ kips $\qquad F_{CG} = 2.42$ kips C ◀

From Eq. (2): $F_{AD} = -\frac{2}{15.6}(86\cos 45° - 42\sin 45°) = -3.99$ kips $\qquad F_{AD} = 3.99$ kips C ◀

From Eq. (3): $F_{EF} = \frac{2}{16.7126}(120\sin 45° - 28\cos 45°) = +7.79$ kips $\qquad F_{EF} = 7.79$ kips T ◀

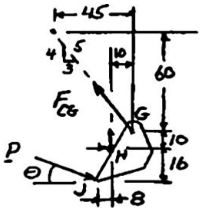


PROBLEM 6.158

Solve Problem 6.157 assuming that the 2-kip force **P** acts horizontally to the right ($\theta = 0$).

PROBLEM 6.157 The motion of the backhoe bucket shown is controlled by the hydraulic cylinders *AD*, *CG*, and *EF*. As a result of an attempt to dislodge a portion of a slab, a 2-kip force **P** is exerted on the bucket teeth at *J*. Knowing that $\theta = 45°$, determine the force exerted by each cylinder.

SOLUTION



Free body: Bucket:

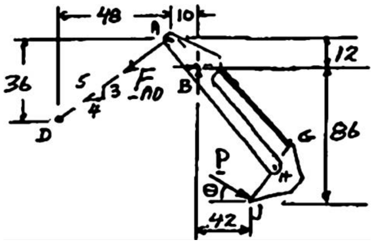
$$+\circlearrowleft \Sigma M_H = 0: \quad \text{(Dimensions in inches)}$$

$$\frac{4}{5}F_{CG}(10) + \frac{3}{5}F_{CG}(10) + P\cos\theta(16) + P\sin\theta(8) = 0$$

$$F_{CG} = -\frac{P}{14}(16\cos\theta + 8\sin\theta) \qquad (1)$$

Free body: Arm *ABH* and bucket:

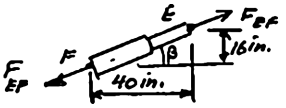
(Dimensions in inches)



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{4}{5}F_{AD}(12) + \frac{3}{5}F_{AD}(10) + P\cos\theta(86) - P\sin\theta(42) = 0$$

$$F_{AD} = -\frac{P}{15.6}(86\cos\theta - 42\sin\theta) \qquad (2)$$

Free body: Bucket and arms *IEB* + *ABH*:

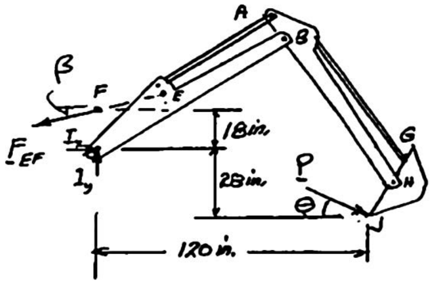


Geometry of cylinder *EF*:

$$\tan\beta = \frac{16\text{ in.}}{40\text{ in.}}$$

$$\beta = 21.801°$$

PROBLEM 6.158 (Continued)



$$+\circlearrowleft \Sigma M_I = 0: \quad F_{EF}\cos\beta(18 \text{ in.}) + P\cos\theta(28 \text{ in.}) - P\sin\theta(120 \text{ in.}) = 0$$

$$F_{EF} = \frac{P(120\sin\theta - 28\cos\theta)}{\cos 21.8°(18)}$$

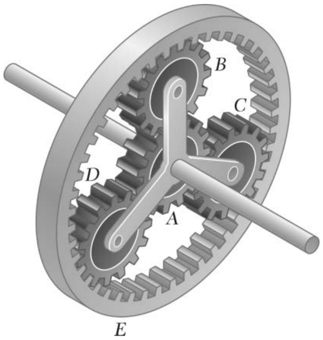
$$= \frac{P}{16.7126}(120\sin\theta - 28\cos\theta) \qquad (3)$$

For $P = 2$ kips, $\quad \theta = 0$

From Eq. (1): $\quad F_{CG} = -\dfrac{2}{14}(16\cos 0 + 8\sin 0) = -2.29$ kips $\quad F_{CG} = 2.29$ kips C ◀

From Eq. (2): $\quad F_{AD} = -\dfrac{2}{15.6}(86\cos 0 - 42\sin 0) = -11.03$ kips $\quad F_{AD} = 11.03$ kips C ◀

From Eq. (3): $\quad F_{EF} = \dfrac{2}{16.7126}(120\sin 0 - 28\cos 0) = -3.35$ kips $\quad F_{EF} = 3.35$ kips C ◀

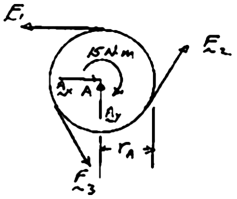


PROBLEM 6.159

In the planetary gear system shown, the radius of the central gear A is $a = 18$ mm, the radius of each planetary gear is b, and the radius of the outer gear E is $(a + 2b)$. A clockwise couple of magnitude $M_A = 10$ N · m is applied to the central gear A and a counterclockwise couple of magnitude $M_S = 50$ N · m is applied to the spider BCD. If the system is to be in equilibrium, determine (*a*) the required radius b of the planetary gears, (*b*) the magnitude M_E of the couple that must be applied to the outer gear E.

SOLUTION

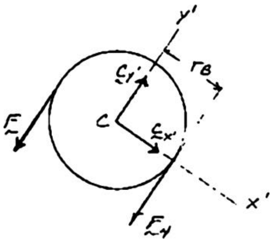
FBD Central Gear:



By symmetry, $F_1 = F_2 = F_3 = F$

$$\circlearrowleft \Sigma M_A = 0: \quad 3(r_A F) - 10 \text{ N}\cdot\text{m} = 0, \qquad F = \frac{10}{3r_A} \text{N}\cdot\text{m}$$

FBD Gear *C*:



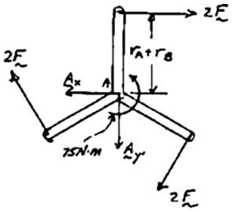
$$\circlearrowleft \Sigma M_C = 0: \quad r_B(F - F_4) = 0, \quad F_4 = F$$

$$\searrow \Sigma F_{x'} = 0: \quad C_{x'} = 0$$

$$\nearrow \Sigma F_{y'} = 0: \quad C_{y'} - 2F = 0, \qquad C_{y'} = 2F$$

Gears B and D are analogous, each having a central force of $2F$.

FBD Spider:



$$\circlearrowleft \Sigma M_A = 0: \quad 50 \text{ N}\cdot\text{m} - 3(r_A + r_B)2F = 0$$

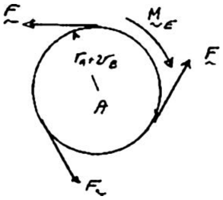
$$50 \text{ N}\cdot\text{m} - 3(r_A + r_B)\frac{20}{r_A} \text{N}\cdot\text{m} = 0$$

$$\frac{r_A + r_B}{r_A} = 2.5 = 1 + \frac{r_B}{r_A}, \qquad r_B = 1.5 r_A$$

Since $r_A = 18$ mm,

(*a*) $r_B = 27.0$ mm ◀

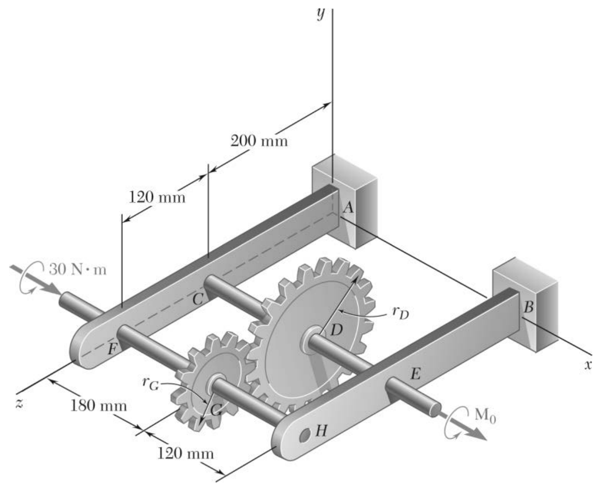
FBD Outer Gear:



$$\circlearrowleft \Sigma M_A = 0: \quad 3(r_A + 2r_B)F - M_E = 0$$

$$3(18 \text{ mm} + 54 \text{ mm})\frac{10 \text{ N}\cdot\text{m}}{54 \text{ mm}} - M_E = 0$$

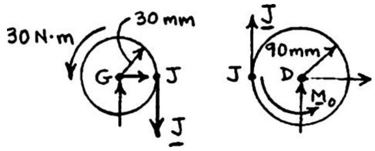
(*b*) $\mathbf{M}_E = 40.0 \text{ N}\cdot\text{m} \circlearrowright$ ◀



PROBLEM 6.160

The gears *D* and *G* are rigidly attached to shafts that are held by frictionless bearings. If $r_D = 90$ mm and $r_G = 30$ mm, determine (*a*) the couple $\mathbf{M}_0$ that must be applied for equilibrium, (*b*) the reactions at *A* and *B*.

SOLUTION



(*a*) Projections on *yz* plane.

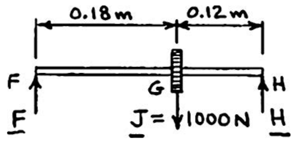
Free body: Gear *G*:

$$+\circlearrowleft \Sigma M_G = 0: \quad 30 \text{ N}\cdot\text{m} - J(0.03 \text{ m}) = 0; \quad J = 1000 \text{ N}$$

Free body: Gear *D*:

$$+\circlearrowleft \Sigma M_D = 0: \quad M_0 - (1000 \text{ N})(0.09 \text{ m}) = 0$$

$$M_0 = 90 \text{ N}\cdot\text{m} \qquad \mathbf{M}_0 = (90.0 \text{ N}\cdot\text{m})\mathbf{i} \blacktriangleleft$$



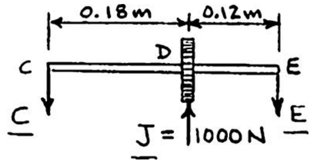
(*b*) Gear *G* and axle *FH*:

$$+\circlearrowleft \Sigma M_F = 0: \quad H(0.3 \text{ m}) - (1000 \text{ N})(0.18 \text{ m}) = 0$$

$$H = 600 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad F + 600 - 1000 = 0$$

$$F = 400 \text{ N}$$



Gear *D* and axle *CE*:

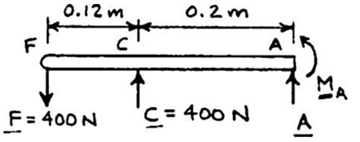
$$+\circlearrowleft \Sigma M_C = 0: \quad (1000 \text{ N})(0.18 \text{ m}) - E(0.3 \text{ m}) = 0$$

$$E = 600 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad 1000 - C - 600 = 0$$

$$C = 400 \text{ N}$$

PROBLEM 6.160 (Continued)

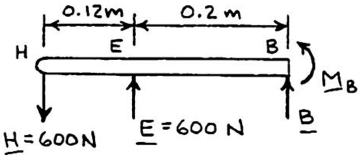


Free body: Bracket *AE*:

$$+\uparrow \Sigma F_y = 0: \quad A - 400 + 400 = 0 \qquad \mathbf{A} = 0 \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A + (400\text{ N})(0.32\text{ m}) - (400\text{ N})(0.2\text{ m}) = 0$$

$$M_A = -48\text{ N}\cdot\text{m} \qquad \mathbf{M}_A = -(48.0\text{ N}\cdot\text{m})\mathbf{i} \blacktriangleleft$$



Free body: Bracket *BH*:

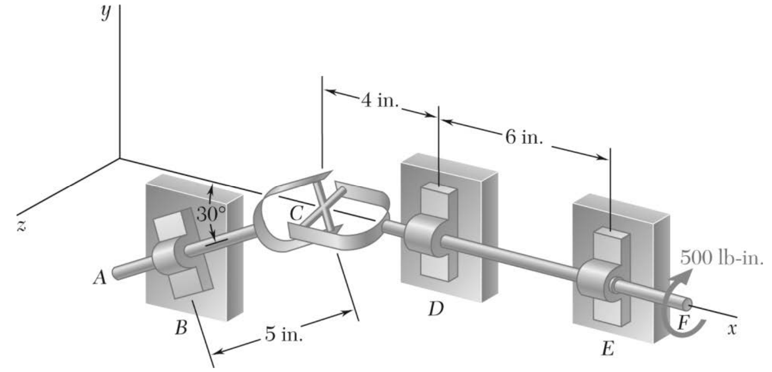
$$+\uparrow \Sigma F_y = 0: \quad B - 600 + 600 = 0 \qquad \mathbf{B} = 0 \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_B = 0: \quad M_B + (600\text{ N})(0.32\text{ m}) - (600\text{ N})(0.2\text{ m}) = 0$$

$$M_B = -72\text{ N}\cdot\text{m} \qquad \mathbf{M}_B = -(72.0\text{ N}\cdot\text{m})\mathbf{i} \blacktriangleleft$$

PROBLEM 6.161*

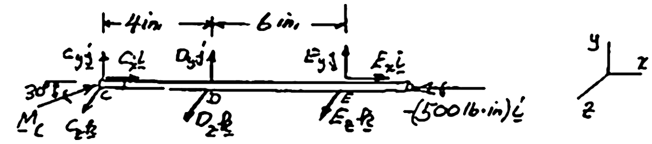
Two shafts *AC* and *CF*, which lie in the vertical *xy* plane, are connected by a universal joint at *C*. The bearings at *B* and *D* do not exert any axial force. A couple of magnitude 500 lb · in. (clockwise when viewed from the positive *x*-axis) is applied to shaft *CF* at *F*. At a time when the arm of the crosspiece attached to shaft *CF* is horizontal, determine (*a*) the magnitude of the couple that must be applied to shaft *AC* at *A* to maintain equilibrium, (*b*) the reactions at *B*, *D*, and *E*. (*Hint:* The sum of the couples exerted on the crosspiece must be zero.)



SOLUTION

We recall from Figure 4.10 that a universal joint exerts on members it connects a force of unknown direction and a couple about an axis perpendicular to the crosspiece.

Free body: Shaft *DF*:

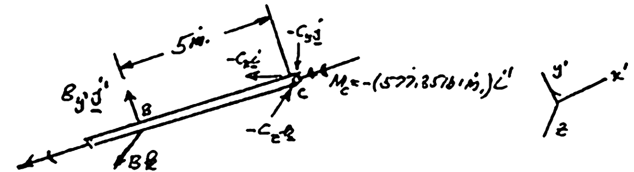


$$\Sigma M_x = 0: \quad M_C \cos 30° - 500\ \text{lb}\cdot\text{in.} = 0$$

$$M_C = 577.35\ \text{lb}\cdot\text{in.}$$

Free body: Shaft *BC*:

We use here x', y', z with x' along BC.



$$\Sigma M_C = 0: \quad -M_R i' - (577.35\ \text{lb}\cdot\text{in.})i' + (-5\ \text{in.})i' \times (B_y j' + B_z k) = 0$$

PROBLEM 6.161* (Continued)

Equate coefficients of unit vectors to zero:

i: $M_A - 577.35 \text{ lb}\cdot\text{in.} = 0$ $\qquad M_A = 577.35 \text{ lb}\cdot\text{in.}$

$M_A = 577 \text{ lb}\cdot\text{in.}$ ◀

$$\left.\begin{array}{ll}\mathbf{j}: & B_z = 0 \\ \mathbf{k}: & B_y = 0\end{array}\right\} \longrightarrow B = 0$$

$\mathbf{B} = 0$

$\Sigma\mathbf{F} = 0\text{:} \quad B + C = 0, \quad \text{since } B = 0,$ $\qquad \mathbf{C} = 0$

<u>Return to free body of shaft *DF*</u>.

$\Sigma\mathbf{M}_D = 0$ (Note that $C = 0$ and $M_C = 577.35 \text{ lb}\cdot\text{in.}$)

$$(577.35 \text{ lb}\cdot\text{in.})(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) - (500 \text{ lb}\cdot\text{in.})\mathbf{i}$$

$$+(6 \text{ in.})\mathbf{i} \times (E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k}) = 0$$

$$(500 \text{ lb}\cdot\text{in.})\mathbf{i} + (288.68 \text{ lb}\cdot\text{in.})\mathbf{j} - (500 \text{ lb}\cdot\text{in.})\mathbf{i}$$

$$+ (6 \text{ in.})E_y\mathbf{k} - (6 \text{ in.})E_z\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

j: $288.68 \text{ lb}\cdot\text{in.} - (6 \text{ in.})E_z = 0 \quad E_z = 48.1 \text{ lb}$

k: $E_y = 0$

$\Sigma\mathbf{F} = 0\text{:} \quad \mathbf{C} + \mathbf{D} + \mathbf{E} = 0$

$0 + D_y\mathbf{j} + D_z\mathbf{k} + E_x\mathbf{i} + (48.1 \text{ lb})\mathbf{k} = 0$

i: $E_x = 0$

j: $D_y = 0$

k: $D_z + 48.1 \text{ lb} = 0 \quad D_z = -48.1 \text{ lb}$

<u>Reactions are</u>:

$\mathbf{B} = 0$ ◀

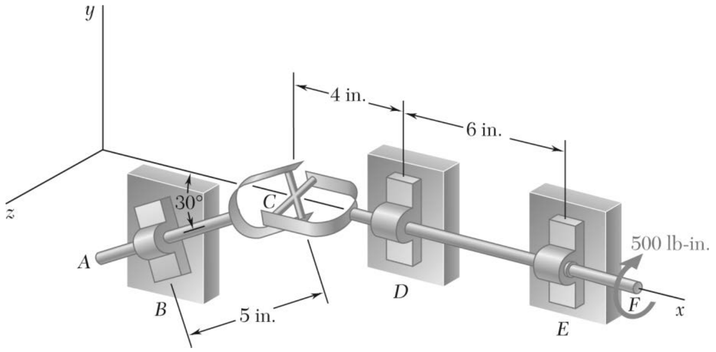
$\mathbf{D} = -(48.1 \text{ lb})\mathbf{k}$ ◀

$\mathbf{E} = (48.1 \text{ lb})\mathbf{k}$ ◀

PROBLEM 6.162*

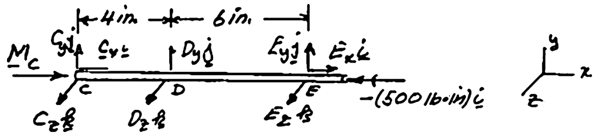
Solve Problem 6.161 assuming that the arm of the crosspiece attached to shaft *CF* is vertical.

PROBLEM 6.161 Two shafts *AC* and *CF*, which lie in the vertical *xy* plane, are connected by a universal joint at *C*. The bearings at *B* and *D* do not exert any axial force. A couple of magnitude 500 lb · in. (clockwise when viewed from the positive *x*-axis) is applied to shaft *CF* at *F*. At a time when the arm of the crosspiece attached to shaft *CF* is horizontal, determine (*a*) the magnitude of the couple that must be applied to shaft *AC* at *A* to maintain equilibrium, (*b*) the reactions at *B*, *D*, and *E*. (*Hint:* The sum of the couples exerted on the crosspiece must be zero.)



SOLUTION

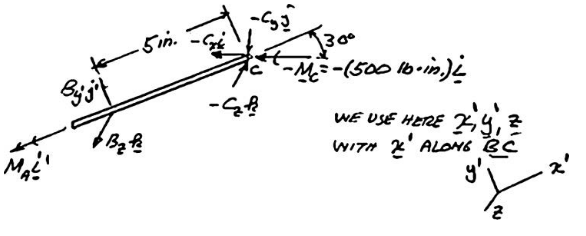
Free body: Shaft *DF*.



$$\Sigma M_x = 0: \quad M_C - 500\text{ lb}\cdot\text{in.} = 0$$

$$M_C = 500\text{ lb}\cdot\text{in.}$$

Free body: Shaft *BC*:



We resolve $-(520\text{ lb}\cdot\text{in.})\mathbf{i}$ into components along x' and y' axes:

$$-\mathbf{M}_C = -(500\text{ lb}\cdot\text{in.})(\cos 30°\mathbf{i}' + \sin 30°\mathbf{j}')$$

$$\Sigma\mathbf{M}_C = 0: \quad M_A\mathbf{i}' - (500\text{ lb}\cdot\text{in.})(\cos 30°\mathbf{i}' + \sin 30°\mathbf{j}') + (5\text{ in.})\mathbf{i}' \times (B_{y'}\mathbf{j}' + B_z\mathbf{k}) = 0$$

$$M_A\mathbf{i}' - (433\text{ lb}\cdot\text{in.})\mathbf{i}' - (250\text{ lb}\cdot\text{in.})\mathbf{j} + (5\text{ in.})B_{y'}\mathbf{k} - (5\text{ in.})B_z\mathbf{j}' = 0$$

PROBLEM 6.162* (Continued)

Equate to zero coefficients of unit vectors:

$$\mathbf{i}': \quad M_A - 433\text{ lb}\cdot\text{in.} = 0 \qquad\qquad M_A = 433\text{ lb}\cdot\text{in.} \blacktriangleleft$$

$$\mathbf{j}': \quad -250\text{ lb}\cdot\text{in.} - (5\text{ in.})B_z = 0 \qquad\qquad B_z = -50\text{ lb}$$

$$\mathbf{k}: \quad \mathbf{B}_{y'} = 0$$

Reactions at *B*. $\mathbf{B} = -(50\text{ lb})\mathbf{k}$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{B} - \mathbf{C} = 0$$

$$-(50\text{ lb})\mathbf{k} - C = 0 \qquad\qquad \mathbf{C} = -(50\text{ lb})\mathbf{k}$$

Return to free body of shaft *DF*.

$$\Sigma\mathbf{M}_D = 0: \quad (6\text{ in.})\mathbf{i}\times(E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k}) - (4\text{ in.})\mathbf{i}\times(-50\text{ lb})\mathbf{k}$$
$$-(500\text{ lb}\cdot\text{in.})\mathbf{i} + (500\text{ lb}\cdot\text{in.})\mathbf{i} = 0$$
$$(6\text{ in.})E_y\mathbf{k} - (6\text{ in.})E_z\mathbf{j} - (200\text{ lb}\cdot\text{in.})\mathbf{j} = 0$$

$$\mathbf{k}: \quad E_y = 0$$

$$\mathbf{j}: \quad -(6\text{ in.})E_z - 200\text{ lb}\cdot\text{in.} = 0 \qquad\qquad E_z = -33.3\text{ lb}$$

$$\Sigma F = 0: \quad \mathbf{C} + \mathbf{D} + \mathbf{E} = 0$$

$$-(50\text{ lb})\mathbf{k} + D_y\mathbf{j} + D_z\mathbf{k} + E_x\mathbf{i} - (33.3\text{ lb})\mathbf{k} = 0$$

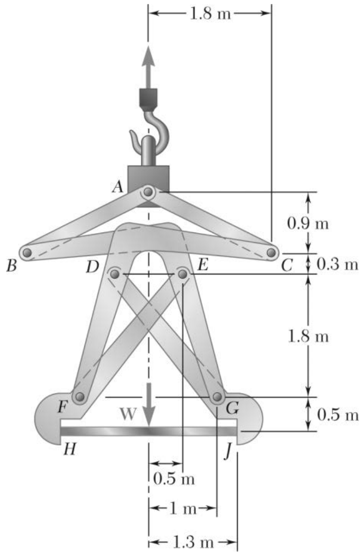
$$\mathbf{i}: \quad E_x = 0$$
$$\mathbf{k}: \quad -50\text{ lb} - 33.3\text{ lb} + D_z = 0$$

$$D_z = 83.3\text{ lb}$$

Reactions are $\mathbf{B} = -(50\text{ lb})\mathbf{k} \blacktriangleleft$

$\mathbf{D} = (83.3\text{ lb})\mathbf{k} \blacktriangleleft$

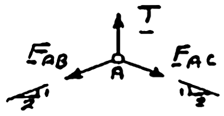
$\mathbf{E} = -(33.3\text{ lb})\mathbf{k} \blacktriangleleft$



PROBLEM 6.163*

The large mechanical tongs shown are used to grab and lift a thick 7500-kg steel slab *HJ*. Knowing that slipping does not occur between the tong grips and the slab at *H* and *J*, determine the components of all forces acting on member *EFH*. (*Hint:* Consider the symmetry of the tongs to establish relationships between the components of the force acting at *E* on *EFH* and the components of the force acting at *D* on *DGJ*.)

SOLUTION

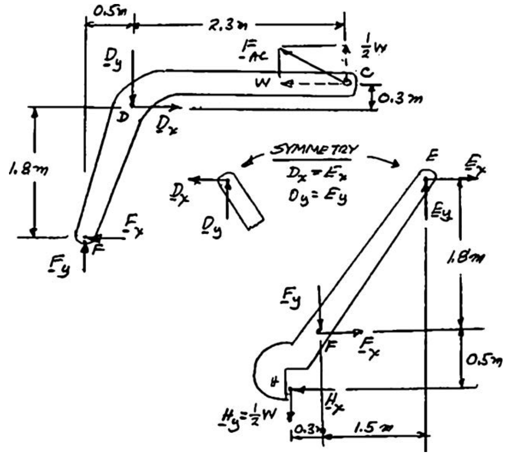


Free body: Pin *A*: $T = W = mg = (7500 \text{ kg})(9.81 \text{ m/s}^2) = 73.575 \text{ kN}$

$$\Sigma F_x = 0: \quad (F_{AB})_x = (F_{AC})_x$$

$$\Sigma F_y = 0: \quad (F_{AB})_y = (F_{AC})_y = \frac{1}{2}W$$

Also, $(F_{AC})_x = 2(F_{AC})_y = W$



Free body: Member *CDF*:

$$+\circlearrowleft \Sigma M_D = 0: \quad W(0.3) + \frac{1}{2}W(2.3) - F_x(1.8) - F_y(0.5 \text{ m}) = 0$$

or $$1.8F_x + 0.5F_y = 1.45W \qquad (1)$$

PROBLEM 6.163* (Continued)

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - F_x - W = 0$$

or $$E_x - F_x = W \qquad (2)$$

$$+\uparrow \Sigma F_y = 0: \quad F_y - D_y + \frac{1}{2}W = 0$$

or $$E_y - F_y = \frac{1}{2}W \qquad (3)$$

<u>Free body: Member *EFH*</u>:

$$+\circlearrowleft \Sigma M_E = 0: \quad F_x(1.8) + F_y(1.5) - H_x(2.3) + \frac{1}{2}W(1.8 \text{ m}) = 0$$

or $$1.8F_x + 1.5F_y = 2.3H_x - 0.9W \qquad (4)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x + F_x - H_x = 0$$

or $$E_x + F_x = H_x \qquad (5)$$

Subtract Eq. (2) from Eq. (5): $$2F_x = H_x - W \qquad (6)$$

Subtract Eq. (4) from 3 × (1): $$3.6F_x = 5.25W - 2.3H_x \qquad (7)$$

Add Eq. (7) to 2.3 × Eq. (6): $$8.2F_x = 2.95W$$

$$F_x = 0.35976W \qquad (8)$$

Substitute from Eq. (8) into Eq. (1):

$$(1.8)(0.35976W) + 0.5F_y = 1.45W$$

$$0.5F_y = 1.45W - 0.64756W = 0.80244W$$

$$F_y = 1.6049W \qquad (9)$$

Substitute from Eq. (8) into Eq. (2): $E_x - 0.35976W = W; \quad E_x = 1.35976W$

Substitute from Eq. (9) into Eq. (3): $E_y - 1.6049W = \frac{1}{2}W \quad E_y = 2.1049W$

From Eq. (5): $H_x = E_x + F_x = 1.35976W + 0.35976W = 1.71952W$

Recall that $H_y = \frac{1}{2}W$

PROBLEM 6.163* (Continued)

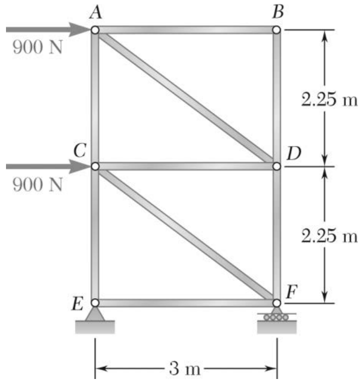
Since all expressions obtained are positive, all forces are directed as shown on the free-body diagrams.

Substitute $W = 73.575 \text{ kN}$

$\mathbf{E}_x = 100.0 \text{ kN} \longrightarrow$ $\mathbf{E}_y = 154.9 \text{ kN} \uparrow$ ◀

$\mathbf{F}_x = 26.5 \text{ kN} \longrightarrow$ $\mathbf{F}_y = 118.1 \text{ kN} \downarrow$ ◀

$\mathbf{H}_x = 126.5 \text{ kN} \longleftarrow$ $\mathbf{H}_y = 36.8 \text{ kN} \downarrow$ ◀

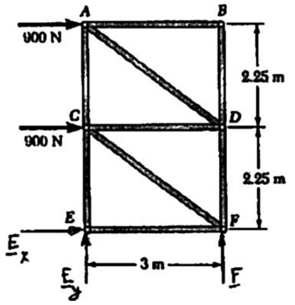


PROBLEM 6.164

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free Body: Truss:



$$+\circlearrowleft \Sigma M_E = 0: \quad F(3\text{ m}) - (900\text{ N})(2.25\text{ m}) - (900\text{ N})(4.5\text{ m}) = 0$$

$$\mathbf{F} = 2025\text{ N}\uparrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x + 900\text{ N} + 900\text{ N} = 0$$

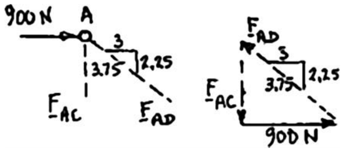
$$E_x = -1800\text{ N} \quad \mathbf{E}_x = 1800\text{ N} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad E_y + 2025\text{ N} = 0$$

$$E_y = -2025\text{ N} \quad \mathbf{E}_y = 2025\text{ N}\downarrow$$

We note that *AB* and *BD* are zero-force members. $F_{AB} = F_{BD} = 0$ ◀

Free body: Joint *A*:

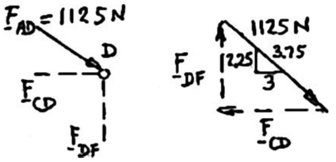


$$\frac{F_{AC}}{2.25} = \frac{F_{AD}}{3.75} = \frac{900\text{ N}}{3}$$

$F_{AC} = 675\text{ N}$ *T* ◀

$F_{AD} = 1125\text{ N}$ *C* ◀

Free body: Joint *D*:

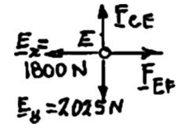


$$\frac{F_{CD}}{3} = \frac{F_{DE}}{2.23} = \frac{1125\text{ N}}{3.75}$$

$F_{CD} = 900\text{ N}$ *T* ◀

$F_{DF} = 675\text{ N}$ *C* ◀

Free body: Joint *E*:



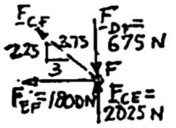
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{EF} - 1800\text{ N} = 0$$

$F_{EF} = 1800\text{ N}$ *T* ◀

$$+\uparrow \Sigma F_y = 0: \quad F_{CE} - 2025\text{ N} = 0$$

$F_{CE} = 2025\text{ N}$ *T* ◀

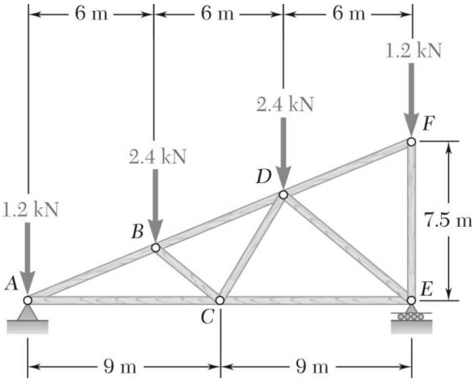
Free body: Joint *F*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{2.25}{3.75} F_{CF} + 2025\text{ N} - 675\text{ N} = 0$$

$$F_{CF} = -2250\text{ N} \qquad F_{CF} = 2250\text{ N} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = -\frac{3}{3.75}(-2250\text{ N}) - 1800\text{ N} = 0 \quad \text{(Checks)}$$

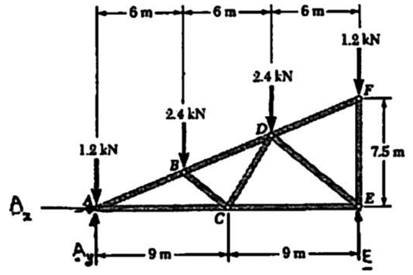


PROBLEM 6.165

Using the method of joints, determine the force in each member of the roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:



$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

From symmetry of loading:

$$\mathbf{A}_y = \mathbf{E} = \frac{1}{2} \quad \text{total load}$$

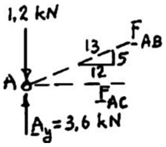
$$\mathbf{A}_y = \mathbf{E} = 3.6 \text{ kN} \uparrow$$

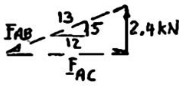
We note that *DF* is a zero-force member and that *EF* is aligned with the load. Thus,

$$F_{DF} = 0 \blacktriangleleft$$

$$F_{EF} = 1.2 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *A*:



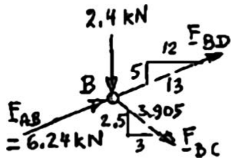


$$\frac{F_{AB}}{13} = \frac{F_{AC}}{12} = \frac{2.4 \text{ kN}}{5}$$

$$F_{AB} = 6.24 \text{ kN} \quad C \blacktriangleleft$$

$$F_{AC} = 2.76 \text{ kN} \quad T \blacktriangleleft$$

Free body: Joint *B*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{3}{3.905}F_{BC} + \frac{12}{13}F_{BD} + \frac{12}{13}(6.24 \text{ kN}) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{2.5}{3.905}F_{BC} + \frac{5}{13}F_{BD} + \frac{5}{13}(6.24 \text{ kN}) - 2.4 \text{ kN} = 0 \qquad (2)$$

PROBLEM 6.165 (Continued)

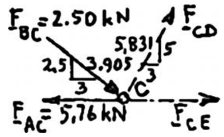
Multiply Eq. (1) by 2.5, Eq. (2) by 3, and add:

$$\frac{45}{13}F_{BD}+\frac{45}{13}(6.24\text{ kN})-7.2\text{ kN}=0,\quad F_{BD}=-4.16\text{ kN},\qquad F_{BD}=4.16\text{ kN}\quad C \blacktriangleleft$$

Multiply Eq. (1) by 5, Eq. (2) by –12, and add:

$$\frac{45}{3.905}F_{BC}+28.8\text{ kN}=0,\quad F_{BC}=-2.50\text{ kN},\qquad F_{BC}=2.50\text{ kN}\quad C \blacktriangleleft$$

Free body: Joint *C*:



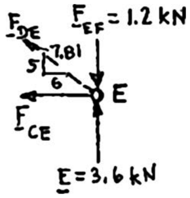
$$+\uparrow\Sigma F_y=0:\quad \frac{5}{5.831}F_{CD}-\frac{2.5}{3.905}(2.50\text{ kN})=0$$

$$F_{CD}=1.867\text{ kN}\quad T \blacktriangleleft$$

$$\xrightarrow{+}\Sigma F_x=0:\quad F_{CE}-5.76\text{ kN}+\frac{3}{3.905}(2.50\text{ kN})+\frac{3}{5.831}(1.867\text{ kN})=0$$

$$F_{CE}=2.88\text{ kN}\quad T$$

Free body: Joint *E*:

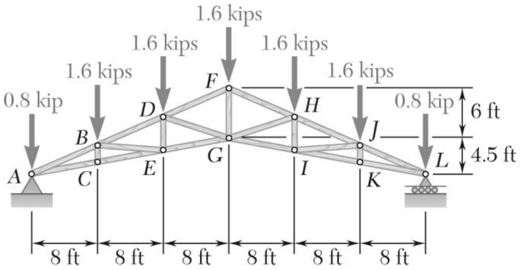


$$+\uparrow\Sigma F_y=0:\quad \frac{5}{7.81}F_{DE}+3.6\text{ kN}-1.2\text{ kN}=0$$

$$F_{DE}=-3.75\text{ kN}\qquad F_{DE}=3.75\text{ kN}\quad C \blacktriangleleft$$

$$\xrightarrow{+}\Sigma F_x=0:\quad -F_{CE}-\frac{6}{7.81}(-3.75\text{ kN})=0$$

$$F_{CE}=+2.88\text{ kN}\quad F_{CE}=2.88\text{ kN}\quad T\qquad \text{(Checks)}$$



PROBLEM 6.166

A Howe scissors roof truss is loaded as shown. Determine the force in members *DF*, *DG*, and *EG*.

SOLUTION

Reactions at supports.

Because of symmetry of loading,

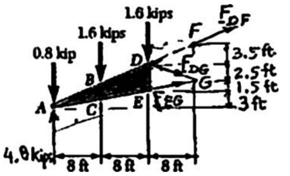
$$A_x = 0, \quad A_y = L = \frac{1}{2}(\text{total load}) = \frac{1}{2}(9.60 \text{ kips}) = 4.80 \text{ kips}$$

$$\mathbf{A} = \mathbf{L} = 4.80 \text{ kips}\uparrow \blacktriangleleft$$

We pass a section through members *DF*, *DG*, and *EG*, and use the free body shown.

We slide $\mathbf{F}_{DF}$ to apply it at *F*:

$$+\circlearrowleft \Sigma M_G = 0: \quad (0.8 \text{ kip})(24 \text{ ft}) + (1.6 \text{ kips})(16 \text{ ft}) + (1.6 \text{ kips})(8 \text{ ft}) - (4.8 \text{ kips})(24 \text{ ft}) - \frac{8F_{DF}}{\sqrt{8^2+3.5^2}}(6 \text{ ft}) = 0$$



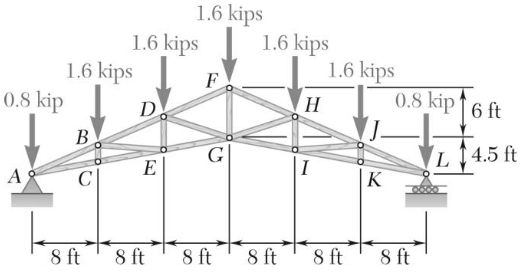
$$F_{DF} = -10.48 \text{ kips}, \quad F_{DF} = 10.48 \text{ kips} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(1.6 \text{ kips})(8 \text{ ft}) - (1.6 \text{ kips})(16 \text{ ft}) - \frac{2.5F_{DG}}{\sqrt{8^2+2.5^2}}(16 \text{ ft}) - \frac{8F_{DG}}{\sqrt{8^2+2.5^2}}(7 \text{ ft}) = 0$$

$$F_{DG} = -3.35 \text{ kips}, \quad F_{DG} = 3.35 \text{ kips} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (0.8 \text{ kips})(16 \text{ ft}) + (1.6 \text{ kips})(8 \text{ ft}) - (4.8 \text{ kips})(16 \text{ ft}) - \frac{8F_{EG}}{\sqrt{8^2+1.5^2}}(4 \text{ ft}) = 0$$

$$F_{EG} = +13.02 \text{ kips}, \quad F_{EG} = 13.02 \text{ kips} \quad T \blacktriangleleft$$



PROBLEM 6.167

A Howe scissors roof truss is loaded as shown. Determine the force in members *GI*, *HI*, and *HJ*.

SOLUTION

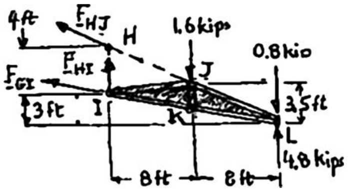
Reactions at supports. Because of symmetry of loading,

$$A_x = 0, \qquad A_y = L = \frac{1}{2}(\text{Total load})$$

$$= \frac{1}{2}(9.60 \text{ kips})$$

$$= 4.80 \text{ kips} \qquad \mathbf{A} = \mathbf{L} = 4.80 \text{ kips}\uparrow \blacktriangleleft$$

We pass a section through members *GI*, *HI*, and *HJ*, and use the free body shown.



$$+\circlearrowleft \Sigma M_H = 0: \quad -\frac{16F_{GI}}{\sqrt{16^2+3^2}}(4 \text{ ft}) + (4.8 \text{ kips})(16 \text{ ft}) - (0.8 \text{ kip})(16 \text{ ft}) - (1.6 \text{ kips})(8 \text{ ft}) = 0$$

$$F_{GI} = +13.02 \text{ kips} \qquad F_{GI} = 13.02 \text{ kips} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_L = 0: \quad (1.6 \text{ kips})(8 \text{ ft}) - F_{HI}(16 \text{ ft}) = 0$$

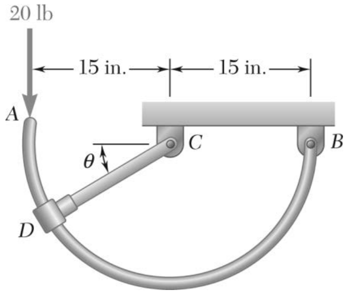
$$F_{HI} = +0.800 \text{ kips}$$

$$F_{HI} = 0.800 \text{ kips} \quad T \blacktriangleleft$$

We slide $\mathbf{F}_{HG}$ to apply it at *H*.

$$+\circlearrowleft \Sigma M_I = 0: \quad \frac{8F_{HJ}}{\sqrt{8^2+3.5^2}}(4 \text{ ft}) + (4.8 \text{ kips})(16 \text{ ft}) - (1.6 \text{ kips})(8 \text{ ft}) - (0.8 \text{ kip})(16 \text{ ft}) = 0$$

$$F_{HJ} = -13.97 \text{ kips} \qquad F_{HJ} = 13.97 \text{ kips} \quad C \blacktriangleleft$$

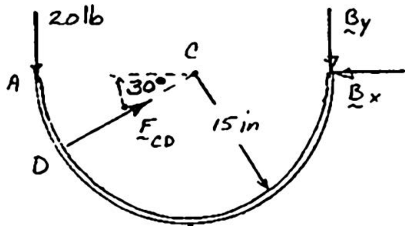


PROBLEM 6.168

Rod *CD* is fitted with a collar at *D* that can be moved along rod *AB*, which is bent in the shape of an arc of circle. For the position when $\theta = 30°$, determine (*a*) the force in rod *CD*, (*b*) the reaction at *B*.

SOLUTION

FBD:



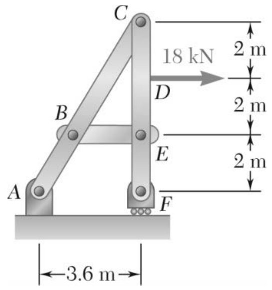
(*a*) $\circlearrowleft \Sigma M_C = 0$: $(15\text{ in.})(20\text{ lb} - B_y) = 0$ $\quad \mathbf{B}_y = 20\text{ lb}\downarrow$

$\uparrow \Sigma F_y = 0$: $-20\text{ lb} + F_{CD}\sin 30° - 20\text{ lb} = 0$ $\quad F_{CD} = 80.0\text{ lb} \quad T$ ◀

(*b*) $\longrightarrow \Sigma F_x = 0$: $(80\text{ lb})\cos 30° - B_x = 0$

$\mathbf{B}_x = 69.282\text{ lb} \longleftarrow$

so $\mathbf{B} = 72.1\text{ lb} \measuredangle\ 16.10°$ ◀

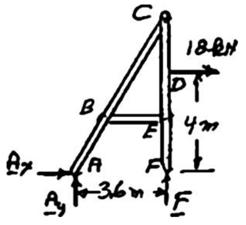


PROBLEM 6.169

For the frame and loading shown, determine the components of all forces acting on member *ABC*.

SOLUTION

Free body: Entire frame:



$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 18 \text{ kN} = 0$

$A_x = -18 \text{ kN}$ $\quad \mathbf{A}_x = 18.00 \text{ kN} \longleftarrow$ ◀

$+\circlearrowleft \Sigma M_E = 0: \quad -(18 \text{ kN})(4 \text{ m}) - A_y(3.6 \text{ m}) = 0$

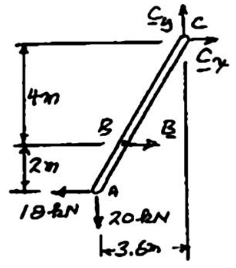
$A_y = -20 \text{ kN}$ $\quad \mathbf{A}_y = 20.0 \text{ kN} \downarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad -20 \text{ kN} + F = 0$

$F = +20 \text{ kN} \quad \mathbf{F} = 20 \text{ kN} \uparrow$

Free body: Member *ABC*

Note: BE is a two-force member, thus **B** is directed along line *BE*.



$+\circlearrowleft \Sigma M_C = 0: \quad B(4 \text{ m}) - (18 \text{ kN})(6 \text{ m}) + (20 \text{ kN})(3.6 \text{ m}) = 0$

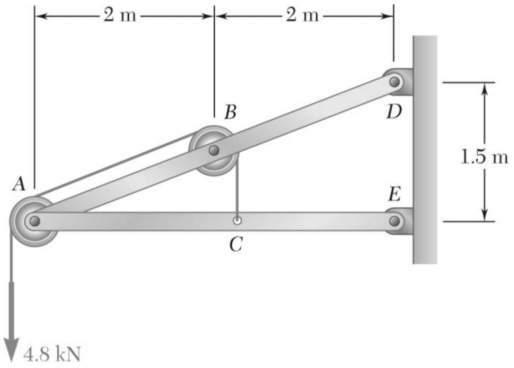
$B = 9 \text{ kN}$ $\quad \mathbf{B} = 9.00 \text{ kN} \longrightarrow$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 18 \text{ kN} + 9 \text{ kN} = 0$

$C_x = 9 \text{ kN}$ $\quad \mathbf{C}_x = 9.00 \text{ kN} \longrightarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad C_y - 20 \text{ kN} = 0$

$C_y = 20 \text{ kN}$ $\quad \mathbf{C}_y = 20.0 \text{ kN} \uparrow$ ◀

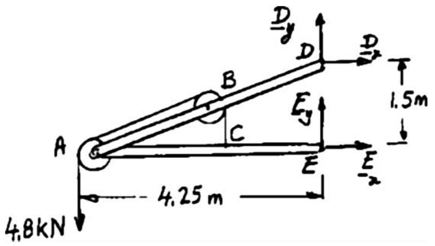


PROBLEM 6.170

Knowing that each pulley has a radius of 250 mm, determine the components of the reactions at *D* and *E*.

SOLUTION

<u>Free body: Entire assembly</u>:



$$+\circlearrowleft \Sigma M_E = 0: \quad (4.8\text{ kN})(4.25\text{ m}) - D_x(1.5\text{ m}) = 0$$

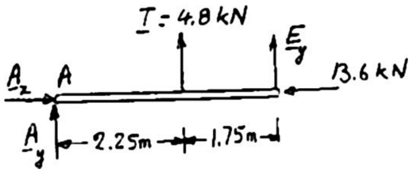
$D_x = +13.60\text{ kN}$ $\quad\quad \mathbf{D}_x = 13.60\text{ kN} \longrightarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x + 13.60\text{ kN} = 0$$

$E_x = -13.60\text{ kN}$ $\quad\quad \mathbf{E}_x = 13.60\text{ kN} \longleftarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad D_y + E_y - 4.8\text{ kN} = 0 \qquad (1)$$

<u>Free body: Member *ACE*</u>:

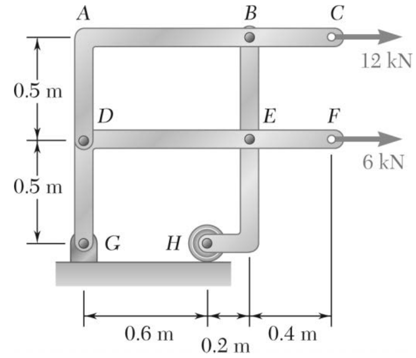


$$+\circlearrowleft \Sigma M_A = 0: \quad (4.8\text{ kN})(2.25\text{ m}) + E_y(4\text{ m}) = 0$$

$E_y = -2.70\text{ kN}$ $\quad\quad \mathbf{E}_y = 2.70\text{ kN}\downarrow$ ◀

From Eq. (1): $\quad D_y - 2.70\text{ kN} - 4.80\text{ kN} = 0$

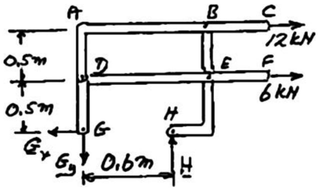
$D_y = +7.50\text{ kN}$ $\quad\quad \mathbf{D}_y = 7.50\text{ kN}\uparrow$ ◀



PROBLEM 6.171

For the frame and loading shown, determine the components of the forces acting on member *DABC* at *B* and *D*.

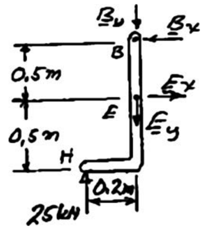
SOLUTION



Free body: Entire frame:

$$+\circlearrowleft \Sigma M_G = 0: \quad H(0.6\text{ m}) - (12\text{ kN})(1\text{ m}) - (6\text{ kN})(0.5\text{ m}) = 0$$

$$H = 25\text{ kN} \quad \mathbf{H} = 25\text{ kN}\uparrow$$



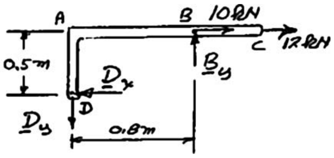
Free body: Member *BEH*:

$$+\circlearrowleft \Sigma M_F = 0: \quad B_x(0.5\text{ m}) - (25\text{ kN})(0.2\text{ m}) = 0$$

$$B_x = +10\text{ kN}$$

Free body: Member *DABC*:

From above: $\mathbf{B}_x = 10.00\text{ kN} \longrightarrow$ ◀



$$+\circlearrowleft \Sigma M_D = 0: \quad -B_y(0.8\text{ m}) + (10\text{ kN} + 12\text{ kN})(0.5\text{ m}) = 0$$

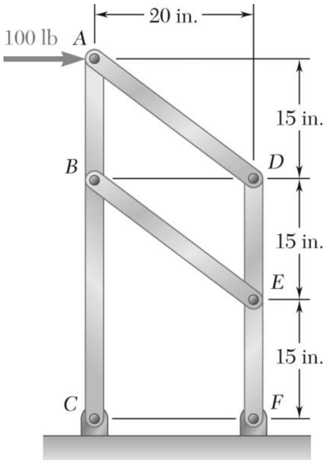
$$B_y = +13.75\text{ kN} \qquad \mathbf{B}_y = 13.75\text{ kN}\uparrow$$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad -D_x + 10\text{ kN} + 12\text{ kN} = 0$$

$$D_x = +22\text{ kN} \qquad \mathbf{D}_x = 22.0\text{ kN} \longleftarrow$$ ◀

$$+\uparrow \Sigma F_y = 0: \quad -D_y + 13.75\text{ kN} = 0$$

$$D_y = +13.75\text{ kN} \qquad \mathbf{D}_y = 13.75\text{ kN}\downarrow$$ ◀

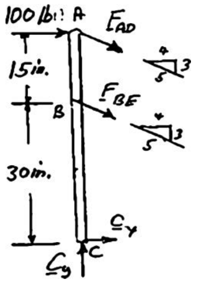


PROBLEM 6.172

For the frame and loading shown, determine (*a*) the reaction at C, (*b*) the force in member AD.

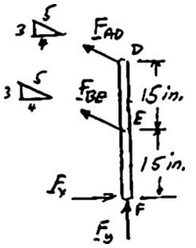
SOLUTION

Free body: Member ABC:



$$\overset{+}{\curvearrowleft}\Sigma M_C = 0: \quad +(100\text{ lb})(45\text{ in.}) + \frac{4}{5}F_{AD}(45\text{ in.}) + \frac{4}{5}F_{BE}(30\text{ in.}) = 0$$

$$3F_{AD} + 2F_{BF} = -375\text{ lb} \qquad (1)$$



Free Body: Member DEF:

$$\overset{+}{\curvearrowleft}\Sigma M_F = 0: \quad \frac{4}{5}F_{AD}(30\text{ in.}) + \frac{4}{5}F_{BF}(15\text{ in.}) = 0$$

$$F_{BE} = -2F_{AD} \qquad (2)$$

(*b*) Substitute from Eq. (2) into Eq. (1):

$$3F_{AD} + 2(-2F_{AD}) = -375\text{ lb}$$

$$F_{AD} = +375\text{ lb} \qquad F_{AD} = 375\text{ lb} \quad T \blacktriangleleft$$

From Eq. (2):

$$F_{BE} = -2F_{AD} = -2(375\text{ lb})$$

$$F_{BE} = -750\text{ lb} \qquad F_{BE} = 750\text{ lb} \quad C.$$

(*a*) Return to free body of member ABC.

$$\xrightarrow{+}\Sigma F_x = 0: \quad C_x + 100\text{ lb} + \frac{4}{5}F_{AD} + \frac{4}{5}F_{BE} = 0$$

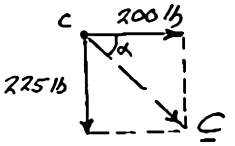
$$C_x + 100 + \frac{4}{5}(375) + \frac{4}{5}(-750) = 0$$

$$C_x = +200\text{ lb} \quad \mathbf{C}_x = 200\text{ lb} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - \frac{3}{5}F_{AD} - \frac{3}{5}F_{BF} = 0$$

$$C_y - \frac{3}{5}(375) - \frac{3}{5}(-750) = 0$$

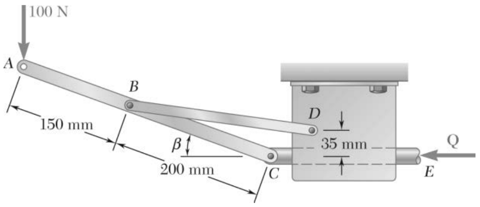
$$C_y = -225 \text{ lb} \quad \mathbf{C}_y = 225 \text{ lb}\downarrow$$



$$\alpha = 48.37°$$

$$C = 301.0 \text{ lb}$$

$\mathbf{C} = 301 \text{ lb} \searrow 48.4°$ ◀



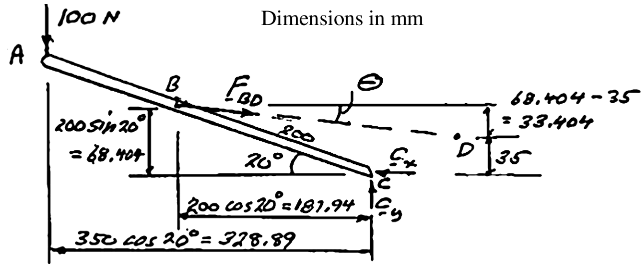
PROBLEM 6.173

The control rod *CE* passes through a horizontal hole in the body of the toggle system shown. Knowing that link *BD* is 250 mm long, determine the force **Q** required to hold the system in equilibrium when $\beta = 20°$.

SOLUTION

We note that *BD* is a two-force member.

Free body: Member *ABC*:



Dimensions in mm

$$\text{Since } BD = 250, \qquad \theta = \sin^{-1}\frac{33.404}{250}; \quad \theta = 7.679°$$

$$+\circlearrowleft \Sigma M_C = 0: \quad (F_{BD}\sin\theta)187.94 - (F_{BD}\cos\theta)68.404 + (100\text{ N})328.89 = 0$$

$$F_{BD}[187.94\sin 7.679° - 68.404\cos 7.679°] = 32{,}889$$

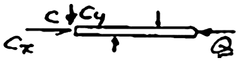
$$F_{BD} = 770.6\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad (770.6\text{ N})\cos 7.679° = C_x = 0$$

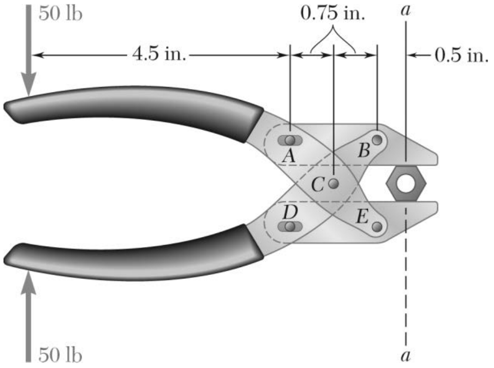
$$C_x = +763.7\text{ N}$$

Member *CQ*:

$$\Sigma F_x = 0: \quad Q = C_x = 763.7\text{ N}$$



Q = 764 N ⟵ ◀

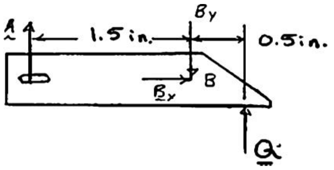


PROBLEM 6.174

Determine the magnitude of the gripping forces exerted along line *aa* on the nut when two 50-lb forces are applied to the handles as shown. Assume that pins *A* and *D* slide freely in slots cut in the jaws.

SOLUTION

FBD jaw *AB*:



$$\xrightarrow{} \Sigma F_x = 0: \quad B_x = 0$$

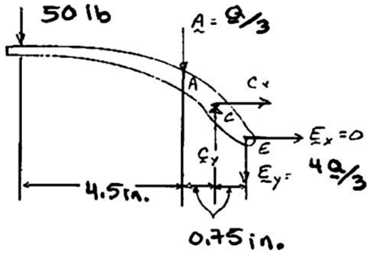
$$\circlearrowleft \Sigma M_B = 0: \quad (0.5\text{ in.})Q - (1.5\text{ in.})A = 0$$

$$A = \frac{Q}{3}$$

$$\uparrow \Sigma F_y = 0: \quad A + Q - B_y = 0$$

$$B_y = A + Q = \frac{4Q}{3}$$

FBD handle *ACE*:



By symmetry and FBD jaw *DE*,

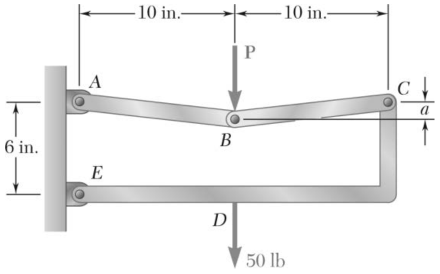
$$D = A = \frac{Q}{3}$$

$$E_x = B_x = 0$$

$$E_y = B_y = \frac{4Q}{3}$$

$$\circlearrowleft \Sigma M_C = 0: \quad (5.25\text{ in.})(50\text{ lb}) + (0.75\text{ in.})\frac{Q}{3} - (0.75\text{ in.})\frac{4Q}{3} = 0$$

$$Q = 350\text{ lb} \blacktriangleleft$$

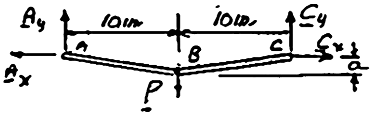


PROBLEM 6.175

Knowing that the frame shown has a sag at B of $a = 1$ in., determine the force **P** required to maintain equilibrium in the position shown.

SOLUTION

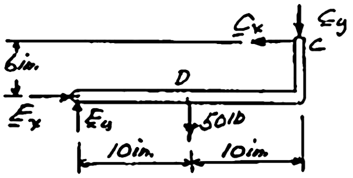
We note that AB and BC are two-force members.



Free body: Toggle:

By symmetry,
$$C_y = \frac{P}{2}$$
$$\frac{C_x}{10\text{ in.}} = \frac{C_y}{a}$$
$$C_x = \frac{10}{a}C_y = \frac{10}{a}\cdot\frac{P}{2} = \frac{5P}{a}$$

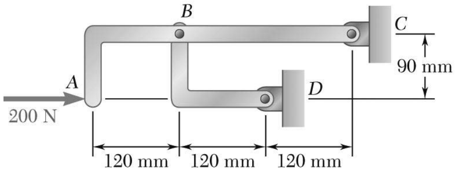
Free body: Member CDE:



$$+\circlearrowleft\Sigma M_E = 0:\quad C_x(6\text{ in.}) - C_y(20\text{ in.}) - (50\text{ lb})(10\text{ in.}) = 0$$
$$\frac{5P}{a}(b) - \frac{P}{2}(20) = 500$$
$$P\left(\frac{30}{a} - 10\right) = 500 \qquad (1)$$

For
$$a = 1.0\text{ in.}$$
$$P\left(\frac{30}{1} - 10\right) = 500$$
$$20P = 500$$

P = 25.0 lb↓ ◀



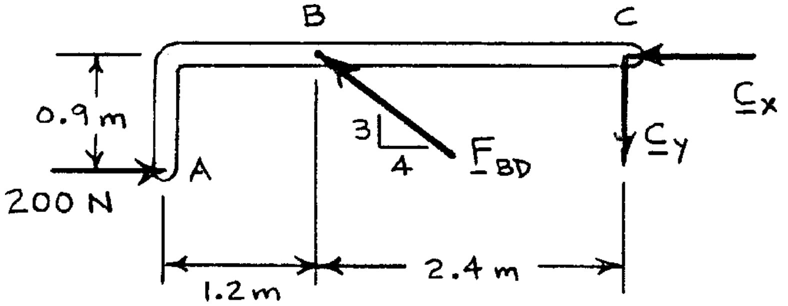
PROBLEM 6.F1

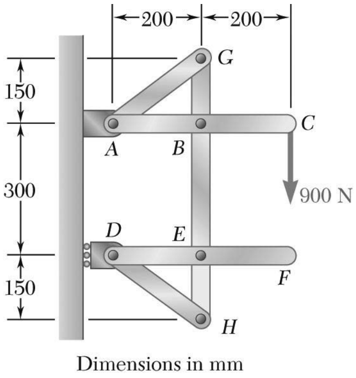
For the frame and loading shown, draw the free-body diagram(s) needed to determine the forces acting on member *ABC* at *B* and *C*.

SOLUTION

We note that *BD* is a two-force member.

Free body: *ABC*





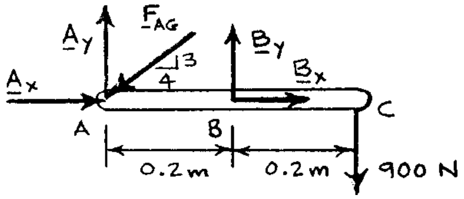
PROBLEM 6.F2

For the frame and loading shown, draw the free-body diagram(s) needed to determine all forces acting on member *GBEH*.

SOLUTION

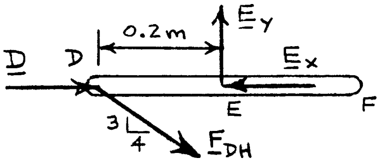
We note that *AG*, *DEF*, and *DH* are two-force members.

Free body: *ABC*



Note: Sum moments about *A* to determine $\mathbf{B}_y$.

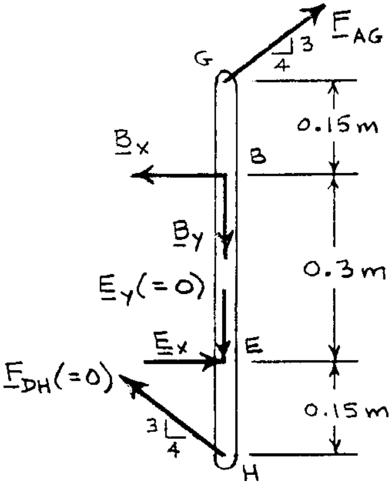
Free body: *DEF*



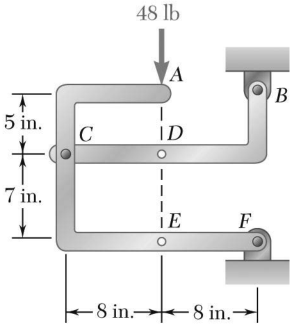
Note: Sum moments about *E* to determine $\mathbf{F}_{DH}$ (which becomes zero).

Sum forces in *y* to determine $\mathbf{E}_y$ (which also becomes zero).

Free body: *GBEH*



Note: With $\mathbf{B}_y$, $\mathbf{E}_y$, and $\mathbf{F}_{DH}$ previously determined, apply three equilibrium equations to determine three remaining forces on *GBEH*.



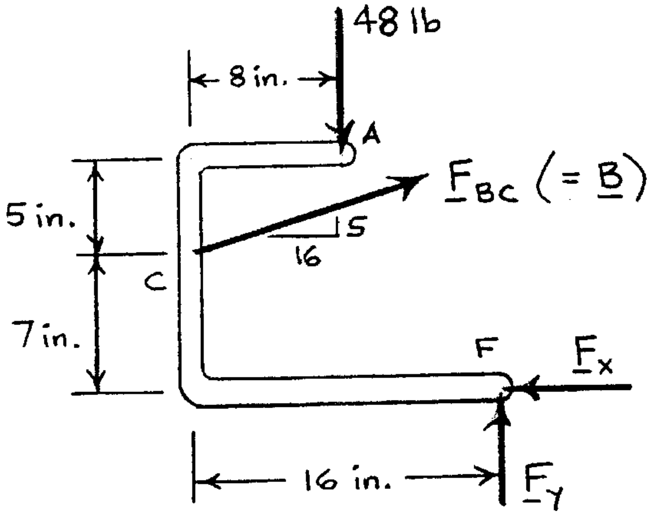
PROBLEM 6.F3

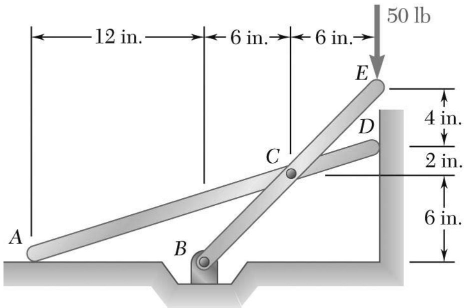
For the frame and loading shown, draw the free-body diagram(s) needed to determine the reactions at *B* and *F*.

SOLUTION

We note that *BC* is two-force member.

Free body: *ACF*



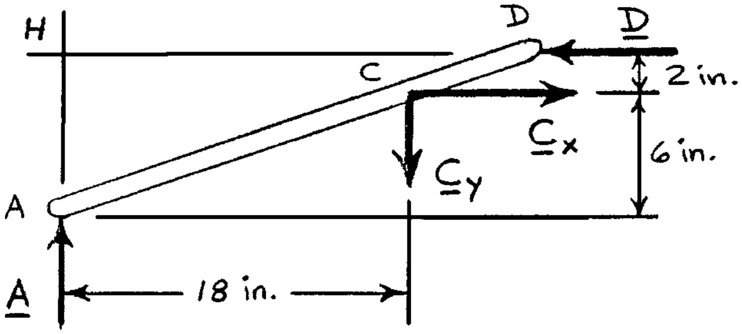


PROBLEM 6.F4

Knowing that the surfaces at *A* and *D* are frictionless, draw the free-body diagram(s) needed to determine the forces exerted at *B* and *C* on member *BCE*.

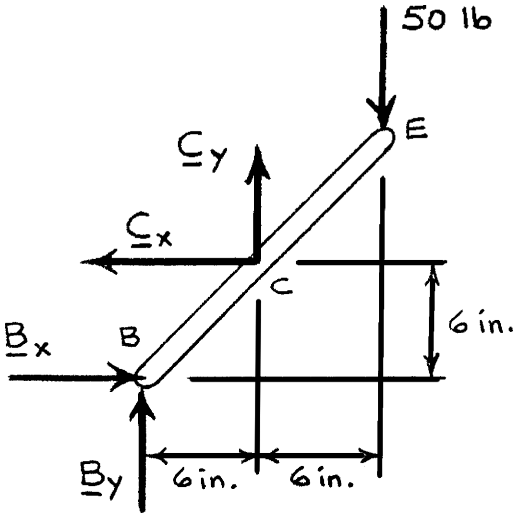
SOLUTION

Free body: *ACD*

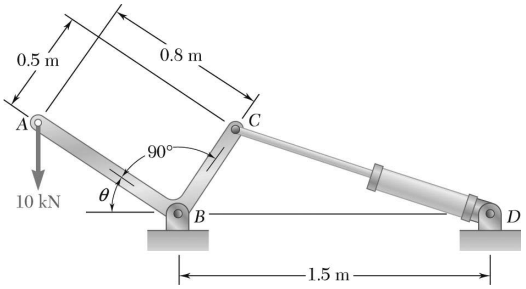


Note: Sum moments about *H* to relate $\mathbf{C}_x$ and $\mathbf{C}_y$.

Free body: *BCE*



Note: Sum moments about *B* to relate $\mathbf{C}_x$ and $\mathbf{C}_y$; combine with relation from free body *ACD* to determine $\mathbf{C}_x$ and $\mathbf{C}_y$; finally, sum forces in *x* and *y* to determine $\mathbf{B}_x$ and $\mathbf{B}_y$.



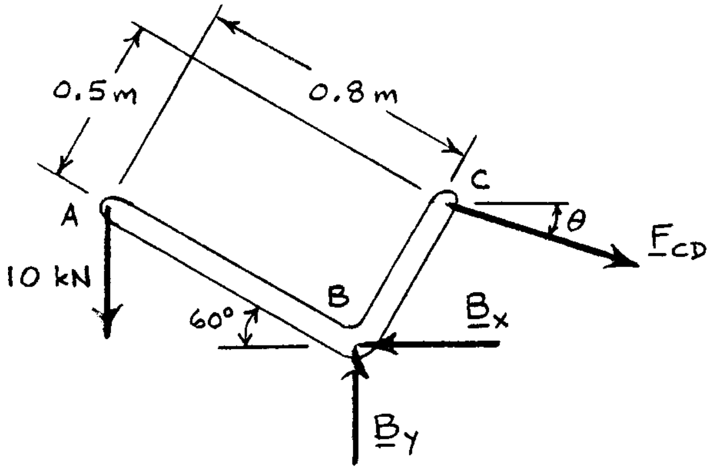
PROBLEM 6.F5

The position of member *ABC* is controlled by the hydraulic cylinder *CD*. Knowing that $\theta = 30°$, draw the free-body diagram(s) needed to determine the force exerted by the hydraulic cylinder on pin *C*, and the reaction at *B*.

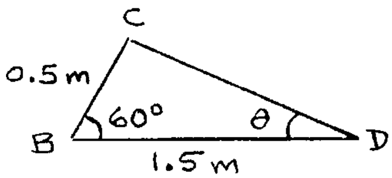
SOLUTION

We note that *CD* is a two-force member.

Free body: *ABC*



Note: To find θ, consider geometry of triangle *BCD*:

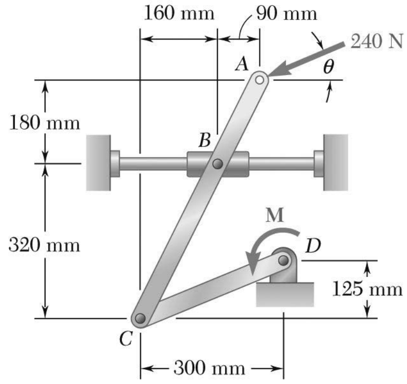


Law of cosines: $(CD)^2 = (0.5)^2 + (1.5)^2 - 2(0.5)(1.5)\cos 60°$

$$CD = 1.32288 \text{ m}$$

Law of sines: $\dfrac{\sin\theta}{0.5 \text{ m}} = \dfrac{\sin 60°}{1.32288 \text{ m}}$

$$\theta = 19.1066°$$

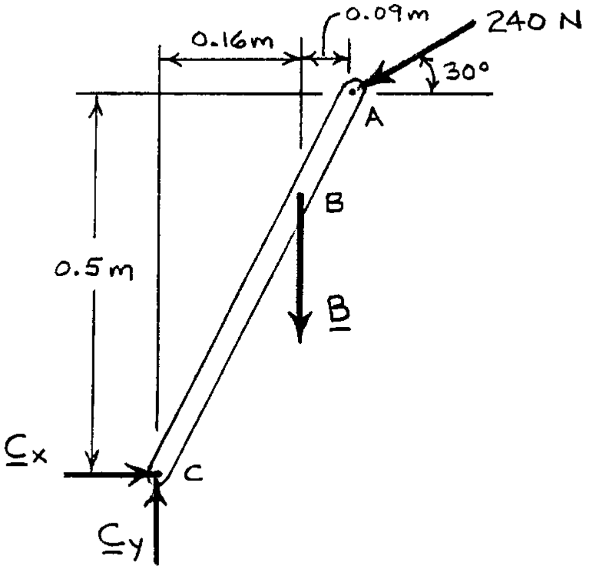


PROBLEM 6.F6

Arm *ABC* is connected by pins to a collar at *B* and to crank *CD* at *C*. Neglecting the effect of friction, draw the free-body diagram(s) needed to determine the couple **M** to hold the system in equilibrium when $\theta = 30°$.

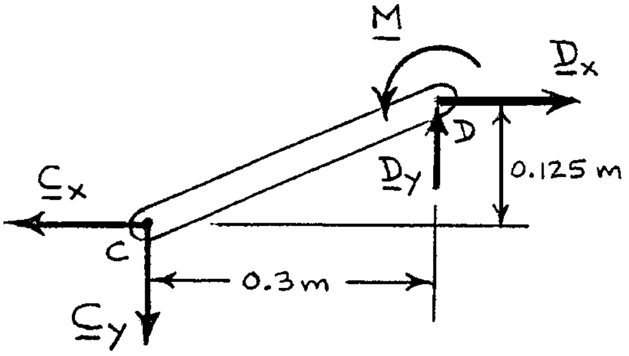
SOLUTION

Free body: *ABC*

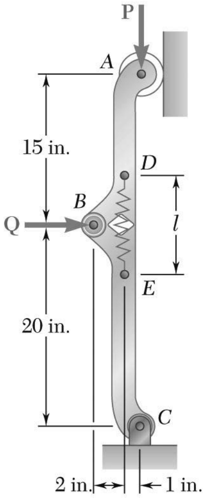


Note: Sum forces in *x* to determine $\mathbf{C}_x$; sum moments about *B* to determine $\mathbf{C}_y$.

Free body: *CD*



Note: Sum moments about *D* to determine **M**.

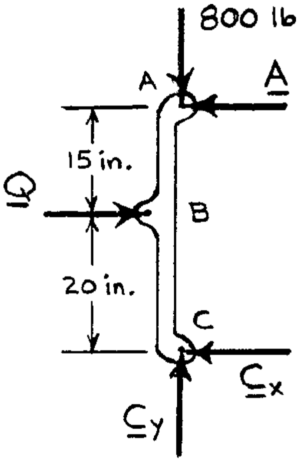


PROBLEM 6.F7

Since the brace shown must remain in position even when the magnitude of **P** is very small, a single safety spring is attached at *D* and *E*. The spring *DE* has a constant of 50 lb/in. and an unstretched length of 7 in. Knowing that l = 10 in. and that the magnitude of **P** is 800 lb, draw the free-body diagram(s) needed to determine the force **Q** required to release the brace.

SOLUTION

Free body: *ABC*



Note: Sum moments about *C* to relate **Q** to **A**.

Spring force.

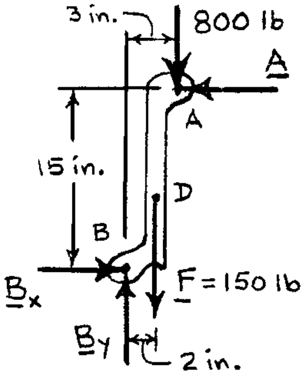
Unstretched length: $\ell_0 = 7$ in.

Stretched length: $\ell = 10$ in.

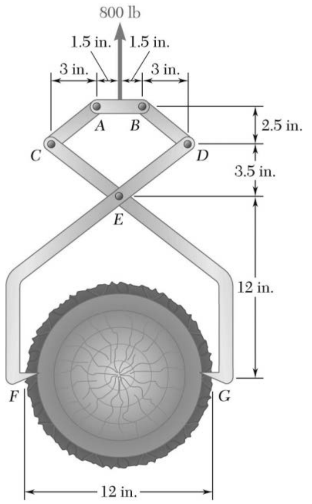
$k = 50$ lb/in.

$$F = kx = k(\ell - \ell_0)$$
$$= 50(10 - 7) = 150 \text{ lb}$$

Free body: *ADB*



Note: Sum moments about *B* to determine **A**; use relation from free body *ABC* to determine **Q**.



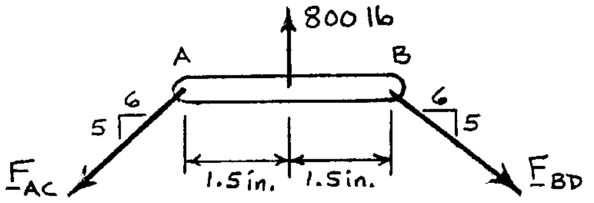
PROBLEM 6.F8

A log weighing 800 lb is lifted by a pair of tongs as shown. Draw the free-body diagram(s) needed to determine the forces exerted at *E* and *F* on tong *DEF*.

SOLUTION

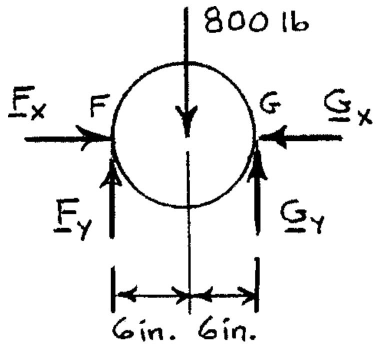
We note that *AC* and *BD* are two-force members.

Free body: *AB*



Note: Sum moments about *A* to determine $\mathbf{F}_{BD}$.

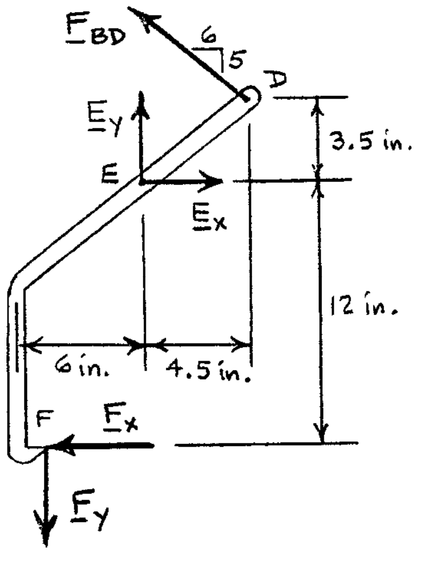
Free body: *LOG*



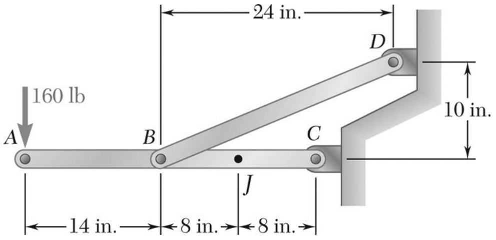
Note: Sum moments about *G* to determine $\mathbf{F}_y$.

Free body: *DEF*

Note: Sum moments about *E* to find $\mathbf{F}_x$; sum forces in *x* and *y* to find $\mathbf{E}_x$ and $\mathbf{E}_y$.



CHAPTER 7



PROBLEM 7.1

Determine the internal forces (axial force, shearing force, and bending moment) at Point *J* of the structure indicated.

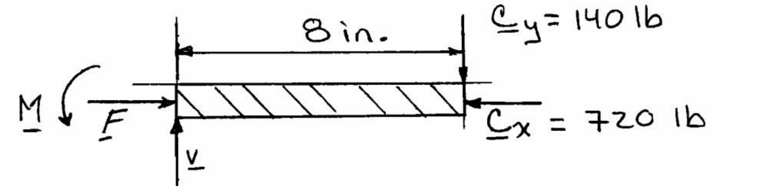
Frame and loading of Problem 6.75.

SOLUTION

From Problem 6.75: $\mathbf{C}_x = 720 \text{ lb} \longleftarrow$

$\mathbf{C}_y = 140 \text{ lb} \downarrow$

FBD of *JC*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad F - 720 \text{ lb} = 0$$

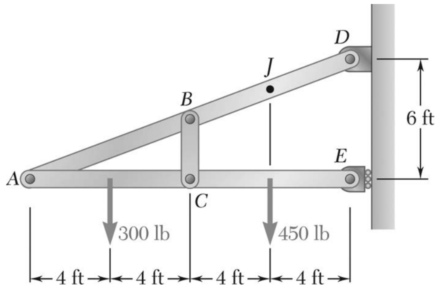
$$F = +720 \text{ lb} \qquad \mathbf{F} = 720 \text{ lb} \longrightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad V - 140 \text{ lb} = 0$$

$$V = 140 \text{ lb} \qquad \mathbf{V} = 140.0 \text{ lb} \uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (140 \text{ lb})(8 \text{ in.}) = 0$$

$$M = +1120 \text{ lb} \cdot \text{in.} \qquad \mathbf{M} = 1120 \text{ lb} \cdot \text{in.} \circlearrowleft \blacktriangleleft$$



PROBLEM 7.2

Determine the internal forces (axial force, shearing force, and bending moment) at Point *J* of the structure indicated.

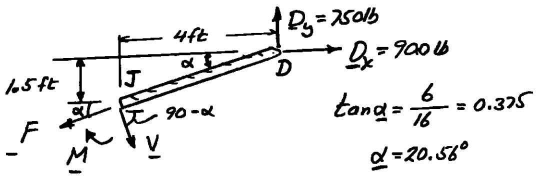
Frame and loading of Problem 6.78.

SOLUTION

From Problem 6.78: $\mathbf{D}_x = 900\text{ lb} \longrightarrow$

$\mathbf{D}_y = 750\text{ lb} \uparrow$

FBD of *JD*:



$$+\circlearrowleft \Sigma M_J = 0: \quad -M + (750\text{ lb})(4\text{ ft}) - (900\text{ lb})(1.5\text{ ft}) = 0$$

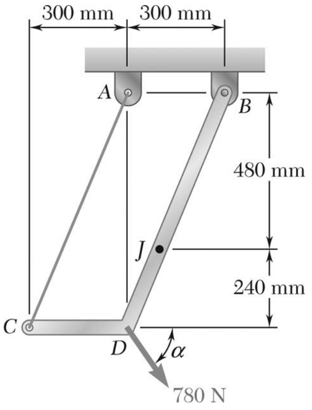
$$M = +1650\text{ lb}\cdot\text{ft} \qquad \mathbf{M} = 1650\text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

$$+\nwarrow \Sigma F = 0: \quad -V + (750\text{ lb})\cos 20.56^\circ - (900\text{ lb})\sin 20.56^\circ = 0$$

$$V = +386.2\text{ lb} \qquad \mathbf{V} = 386\text{ lb} \searrow 69.4^\circ \blacktriangleleft$$

$$+\swarrow \Sigma F = 0: \quad F - (750\text{ lb})\sin 20.56^\circ + (900\text{ lb})\cos 20.56^\circ = 0$$

$$F = +1106.1\text{ lb} \qquad \mathbf{F} = 1106\text{ lb} \swarrow 20.6^\circ \blacktriangleleft$$

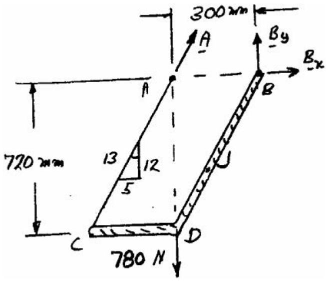


PROBLEM 7.3

Determine the internal forces at Point J when $\alpha = 90°$.

SOLUTION

Reactions $(\alpha = 90°)$



$$\Sigma M_A = 0: \quad \mathbf{B}_y = 0$$

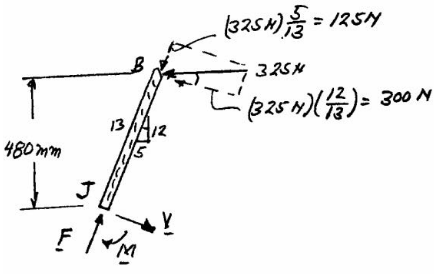
$$+\uparrow \Sigma F_y = 0: \quad A\left(\frac{12}{13}\right) - 780\text{ N} = 0$$

$$A = 845\text{ N} \qquad \mathbf{A} = 845\text{ N} \nearrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad (845\text{ N})\frac{5}{13} + B_x = 0$$

$$B_x = -325\text{ N} \qquad \mathbf{B}_x = 325\text{ N} \longleftarrow$$

FBD BJ:

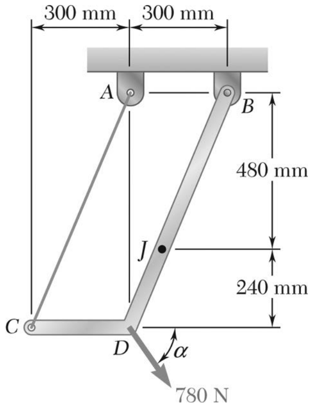


$$+\nearrow \Sigma F = 0: \quad 125\text{ N} - F = 0 \qquad \mathbf{F} = 125.0\text{ N} \angle 67.4° ◀$$

$$+\searrow \Sigma F = 0: \quad V - 300\text{ N} = 0 \qquad \mathbf{V} = 300\text{ N} \searrow 22.6° ◀$$

$$+\circlearrowleft \Sigma M = 0: \quad (325\text{ N})(0.480\text{ m}) - M = 0$$

$$M = +156\text{ N}\cdot\text{m} \qquad \mathbf{M} = 156.0\text{ N}\cdot\text{m} \circlearrowleft ◀$$

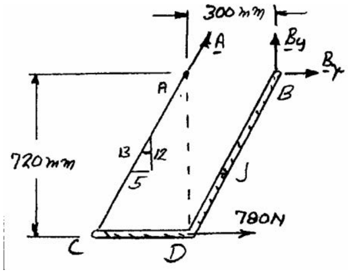


PROBLEM 7.4

Determine the internal forces at Point J when $\alpha = 0$.

SOLUTION

Reactions ($\alpha = 0$)



$$+\circlearrowleft \Sigma M_A = 0: \quad (780\text{ N})(0.720\text{ m}) + B_y(0.3\text{ m}) = 0$$

$$B_y = -1872\text{ N} \qquad \mathbf{B}_y = 1872\text{ N}\downarrow$$

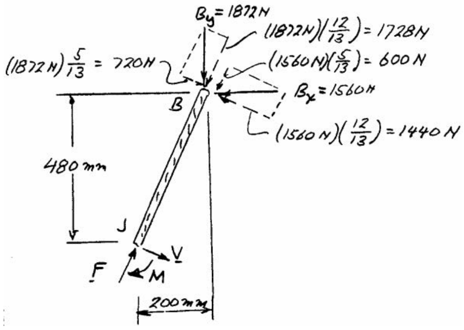
$$+\uparrow \Sigma F_y = 0: \quad A\left(\frac{12}{13}\right) - 1872\text{ N} = 0$$

$$\mathbf{A} = 2028\text{ N} \nearrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad (2028\text{ N})\left(\frac{5}{13}\right) + 780\text{ N} + B_x = 0$$

$$B_x = -1560\text{ N} \qquad \mathbf{B}_x = 1560\text{ N} \longleftarrow$$

FBD BJ:



$$\swarrow \Sigma F = 0: \quad 1728\text{ N} + 600\text{ N} - F = 0$$

$$F = +2328\text{ N} \qquad \mathbf{F} = 2330\text{ N} \measuredangle 67.4° \blacktriangleleft$$

$$\overset{+}{\searrow} \Sigma F = 0: \quad 720\text{ N} - 1440\text{ N} + V = 0$$

$$V = +720\text{ N} \qquad \mathbf{V} = 720\text{ N} \searrow 22.6° \blacktriangleleft$$

$$BJ = \sqrt{480^2 + 200^2} = 520\text{ mm}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad (1440\text{ N})(0.520\text{ m}) - (720\text{ N})(0.520\text{ m}) - M = 0$$

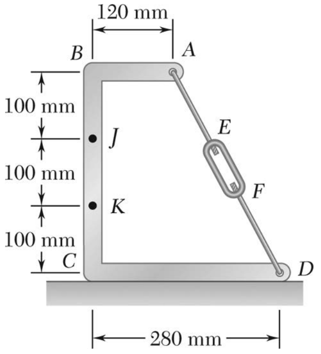
$$M = +374.4\text{ N}\cdot\text{m} \qquad \mathbf{M} = 374\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

Alternate

Computation of M using $\mathbf{B}_x + \mathbf{B}_y$:

$$+\circlearrowleft \Sigma M_J = 0: \quad (1560\text{ N})(0.48\text{ m}) - (1872\text{ N})(0.2\text{ m}) - M = 0$$

$$M = +374.4\text{ N}\cdot\text{m} \qquad \mathbf{M} = 374\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

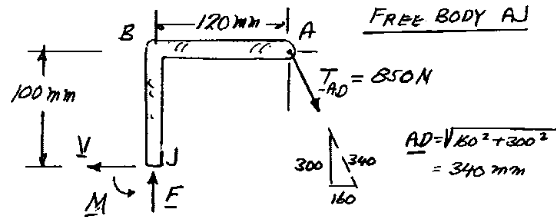


PROBLEM 7.5

Knowing that the turnbuckle has been tightened until the tension in wire *AD* is 850 N, determine the internal forces at point indicated:

Point *J*.

SOLUTION



$$\xrightarrow{+} \Sigma F_x = 0\colon \quad -V + \left(\frac{160}{340}\right)(850\ \text{N}) = 0$$

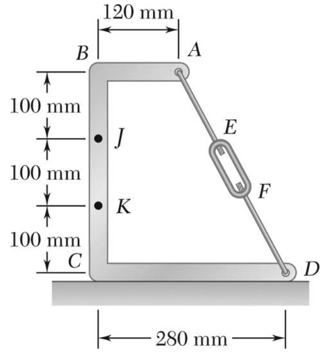
$$V = +400\ \text{N} \qquad \mathbf{V} = 400\ \text{N} \longleftarrow \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0\colon \quad F - \left(\frac{300}{340}\right)(850\ \text{N}) = 0$$

$$F = +750\ \text{N} \qquad \mathbf{F} = 750\ \text{N}\uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0\colon \quad M - \left(\frac{300}{340}\right)(850\ \text{N})(120\ \text{mm}) - \left(\frac{160}{340}\right)(850\ \text{N})(100\ \text{mm}) = 0$$

$$M = +130\ \text{N}\cdot\text{m} \qquad \mathbf{M} = 130\ \text{N}\cdot\text{m}\circlearrowleft \blacktriangleleft$$



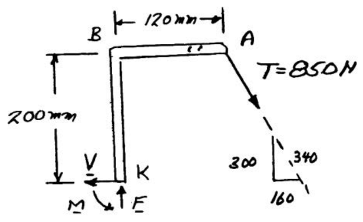
PROBLEM 7.6

Knowing that the turnbuckle has been tightened until the tension in wire *AD* is 850 N, determine the internal forces at point indicated:

Point *K*.

SOLUTION

<u>Free body *AK*:</u>



$$AD = \sqrt{160^2 + 300^2}$$
$$= 340 \text{ mm}$$

<u>On portion *KBA*:</u>

$$\xrightarrow{+} \Sigma F_x = 0: \quad -V + \left(\frac{160}{340}\right)(850 \text{ N}) = 0$$

$$V = +400 \text{ N} \qquad \mathbf{V} = 400 \text{ N} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F - \left(\frac{300}{340}\right)(850 \text{ N}) = 0$$

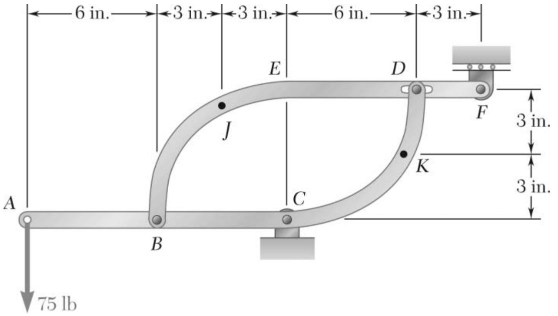
$$F = +750 \text{ N} \qquad \mathbf{F} = 750 \text{ N} \uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - \left(\frac{300}{340}\right)(850 \text{ N})(120 \text{ mm}) - \left(\frac{160}{340}\right)(850 \text{ N})(200 \text{ mm}) = 0$$

$$M = +170 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 170.0 \text{ N}\cdot\text{m} \circlearrowleft$$

<u>Internal forces acting on *KCD*</u> are equal and opposite

$$\mathbf{F} = 750 \text{ N} \downarrow, \quad \mathbf{V} = 400 \text{ N} \longrightarrow, \quad \mathbf{M} = 170.0 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

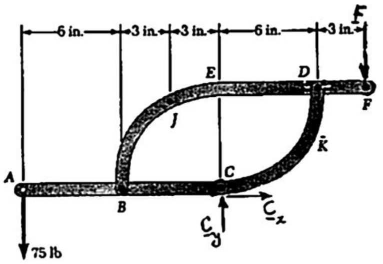


PROBLEM 7.7

Two members, each consisting of a straight and a quarter-circular portion of rod, are connected as shown and support a 75-lb load at *A*. Determine the internal forces at Point *J*.

SOLUTION

Free body: Entire frame



$$+\circlearrowleft \Sigma M_C = 0: \quad (75\text{ lb})(12\text{ in.}) - F(9\text{ in.}) = 0$$

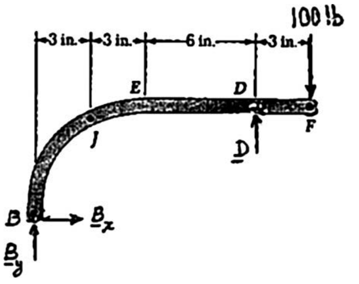
$\mathbf{F} = 100\text{ lb}\downarrow$ ◁

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 75\text{ lb} - 100\text{ lb} = 0$$

$C_y = +175\text{ lb}$ $\qquad$ $\mathbf{C} = 175\text{ lb}\uparrow$ ◁

Free body: Member *BEDF*



$$+\circlearrowleft \Sigma M_B = 0: \quad D(12\text{ in.}) - (100\text{ lb})(15\text{ in.}) = 0$$

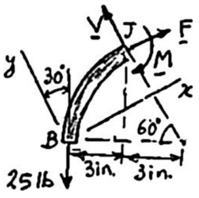
$\mathbf{D} = 125\text{ lb}\uparrow$ ◁

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 125\text{ lb} - 100\text{ lb} = 0$$

$B_y = -25\text{ lb}$ $\qquad$ $\mathbf{B} = 25\text{ lb}\downarrow$ ◁

Free body: *BJ*



$$\nearrow\!\!+\ \Sigma F_x = 0: \quad F - (25\text{ lb})\sin 30° = 0$$

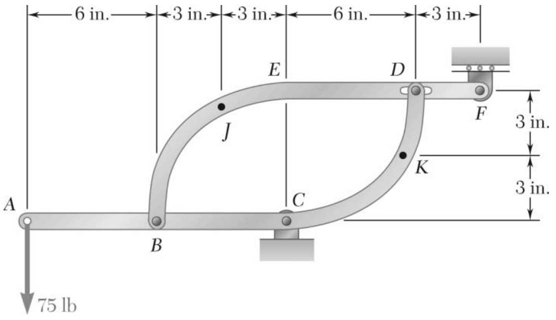
$\mathbf{F} = 12.50\text{ lb} \measuredangle\ 30.0°$ ◀

$$+\nwarrow \Sigma F_y = 0: \quad V - (25\text{ lb})\cos 30° = 0$$

$\mathbf{V} = 21.7\text{ lb} \searrow 60.0°$ ◀

$$+\circlearrowleft \Sigma M_J = 0: \quad -M + (25\text{ lb})(3\text{ in.}) = 0$$

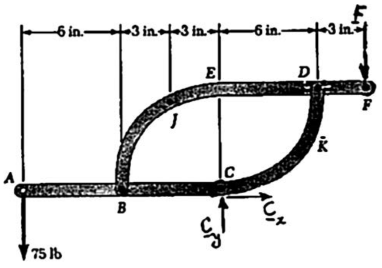
$\mathbf{M} = 75.0\text{ lb}\cdot\text{in.} \circlearrowright$ ◀



PROBLEM 7.8

Two members, each consisting of a straight and a quarter-circular portion of rod, are connected as shown and support a 75-lb load at A. Determine the internal forces at Point K.

SOLUTION



Free body: Entire frame

$$+\circlearrowleft \Sigma M_C = 0: \quad (75\text{ lb})(12\text{ in.}) - F(9\text{ in.}) = 0$$

$\mathbf{F} = 100\text{ lb}\downarrow$ ◁

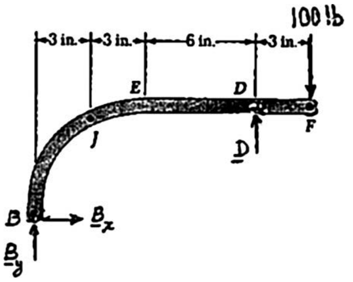
$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 75\text{ lb} - 100\text{ lb} = 0$$

$$C_y = +175\text{ lb}$$

$\mathbf{C} = 175\text{ lb}\uparrow$ ◁

Free body: Member *BEDF*



$$+\circlearrowleft \Sigma M_B = 0: \quad D(12\text{ in.}) - (100\text{ lb})(15\text{ in.}) = 0$$

$\mathbf{D} = 125\text{ lb}\uparrow$ ◁

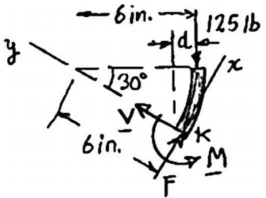
$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 125\text{ lb} - 100\text{ lb} = 0$$

$$B_y = -25\text{ lb}$$

$\mathbf{B} = 25\text{ lb}\downarrow$ ◁

Free body: *DK*



$d = 6\text{in.}(1-\cos 30°)$
$d = 0.8038\text{ in.}$

We found in Problem 7.11 that

$$\mathbf{D} = 125\text{ lb}\uparrow \text{ on } BEDF.$$

Thus $\mathbf{D} = 125\text{ lb}\downarrow$ on DK. ◁

$$+\nearrow \Sigma F_x = 0: \quad F - (125\text{ lb})\cos 30° = 0$$

$\mathbf{F} = 108.3\text{ lb} \measuredangle 60.0°$ ◀

PROBLEM 7.8 (Continued)

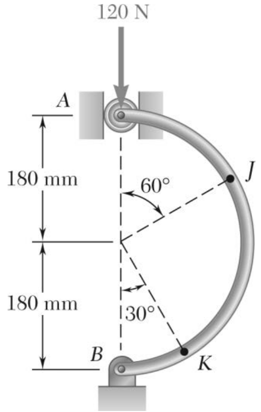
$$\nwarrow^{+} \Sigma F_y = 0: \quad V - (125\text{ lb})\sin 30^\circ = 0$$

$$\mathbf{V} = 62.5\text{ lb} \searrow 30.0^\circ \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_K = 0: \quad M - (125\text{ lb})d = 0$$

$$M = (125\text{ lb})d = (125\text{ lb})(0.8038\text{ in.})$$
$$= 100.5\text{ lb}\cdot\text{in.}$$

$$\mathbf{M} = 100.5\text{ lb}\cdot\text{in.} \circlearrowleft \blacktriangleleft$$

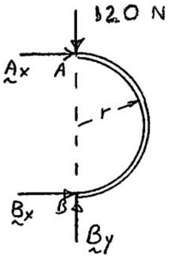


PROBLEM 7.9

A semicircular rod is loaded as shown. Determine the internal forces at Point *J*.

SOLUTION

FBD Rod:



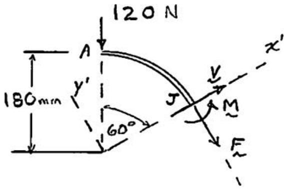
$$\circlearrowleft \Sigma M_B = 0: \quad A_x(2r) = 0$$

$$\mathbf{A}_x = 0$$

$$\nearrow \Sigma F_{x'} = 0: \quad V - (120\text{ N})\cos 60° = 0$$

$$\mathbf{V} = 60.0\text{ N} \nearrow \blacktriangleleft$$

FBD AJ:



$$\searrow \Sigma F_{y'} = 0: \quad F + (120\text{ N})\sin 60° = 0$$

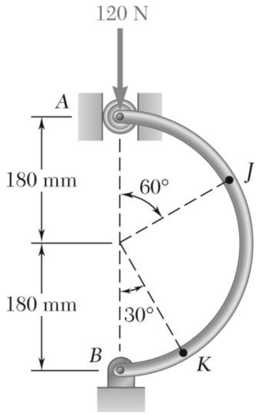
$$F = -103.923\text{ N}$$

$$\mathbf{F} = 103.9\text{ N} \nwarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - [(0.180\text{ m})\sin 60°](120\text{ N}) = 0$$

$$M = 18.7061$$

$$\mathbf{M} = 18.71 \circlearrowright \blacktriangleleft$$

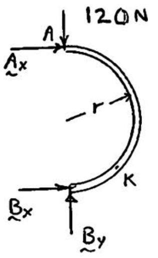


PROBLEM 7.10

A semicircular rod is loaded as shown. Determine the internal forces at Point *K*.

SOLUTION

FBD Rod:



$$\uparrow \Sigma F_y = 0: \quad B_y - 120\text{ N} = 0 \quad \mathbf{B}_y = 120\text{ N}\uparrow$$

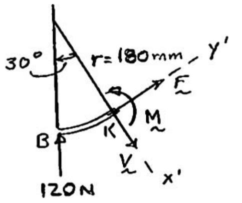
$$\circlearrowleft \Sigma M_A = 0: \quad 2rB_x = 0 \quad \mathbf{B}_x = 0$$

$$\searrow \Sigma F_{x'} = 0: \quad V - (120\text{ N})\cos 30° = 0$$

$$V = 103.923\text{ N}$$

$$\mathbf{V} = 103.9\text{ N} \searrow \blacktriangleleft$$

FBD BK:



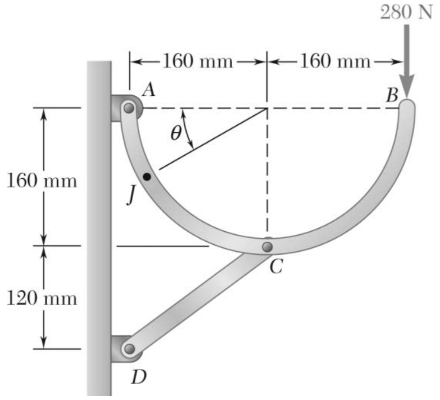
$$\nearrow \Sigma F_{y'} = 0: \quad F + (120\text{ N})\sin 30° = 0$$

$$F = -60\text{ N}$$

$$\mathbf{F} = 60.0\text{ N} \swarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: \quad M - [(0.180\text{ m})\sin 30°](120\text{ N}) = 0$$

$$\mathbf{M} = 10.80\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

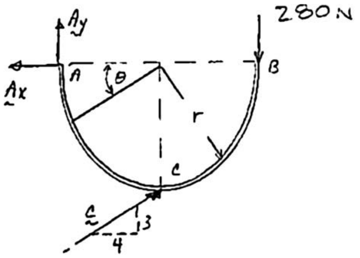


PROBLEM 7.11

A semicircular rod is loaded as shown. Determine the internal forces at Point J knowing that $\theta = 30°$.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: \quad r\left(\frac{4}{5}C\right) + r\left(\frac{3}{5}C\right) - 2r(280\text{ N}) = 0$$

$$\mathbf{C} = 400\text{ N} \nearrow$$

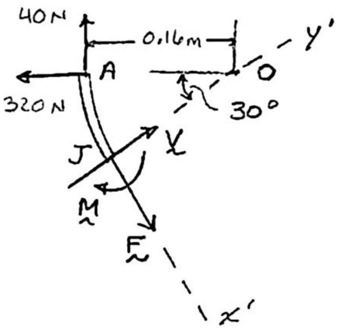
$$\rightarrow \Sigma F_x = 0: \quad -A_x + \frac{4}{5}(400\text{ N}) = 0$$

$$\mathbf{A}_x = 320\text{ N} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y + \frac{3}{5}(400\text{ N}) - 280\text{ N} = 0$$

$$\mathbf{A}_y = 40.0\text{ N} \uparrow$$

FBD AJ:



$$\searrow \Sigma F_{x'} = 0: \quad F - (320\text{ N})\sin 30° - (40.0\text{ N})\cos 30° = 0$$

$$F = 194.641\text{ N}$$

$$\mathbf{F} = 194.6\text{ N} \nwarrow 60.0° \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: \quad V - (320\text{ N})\cos 30° + (40\text{ N})\sin 30° = 0$$

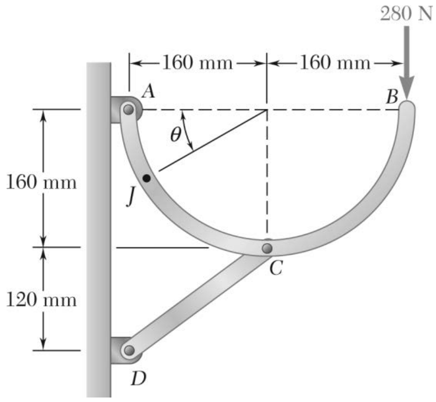
$$V = 257.13\text{ N}$$

$$\mathbf{V} = 257\text{ N} \measuredangle 30.0° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_0 = 0: \quad (0.160\text{ m})(194.641\text{ N}) - (0.160\text{ m})(40.0\text{ N}) - M = 0$$

$$M = 24.743$$

$$\mathbf{M} = 24.7\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

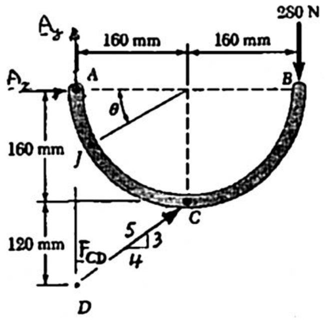


PROBLEM 7.12

A semicircular rod is loaded as shown. Determine the magnitude and location of the maximum bending moment in the rod.

SOLUTION

Free body: Rod *ACB*



$$+\circlearrowleft \Sigma M_A = 0: \quad \left(\frac{4}{5}F_{CD}\right)(0.16\text{ m}) + \left(\frac{3}{5}F_{CD}\right)(0.16\text{ m}) - (280\text{ N})(0.32\text{ m}) = 0$$

$$\mathbf{F}_{CD} = 400\text{ N} \nearrow \quad ◁$$

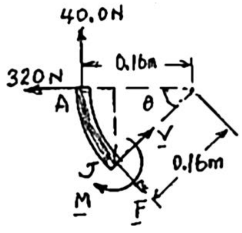
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + \frac{4}{5}(400\text{ N}) = 0$$

$$A_x = -320\text{ N} \qquad \mathbf{A}_x = 320\text{ N} \leftarrow \quad ◁$$

$$+\Sigma F_y = 0: \quad A_y + \frac{3}{5}(400\text{ N}) - 280\text{ N} = 0$$

$$A_y = +40.0\text{ N} \qquad \mathbf{A}_y = 40.0\text{ N} \uparrow \quad ◁$$

Free body: *AJ* (For $\theta < 90°$)



$$+\circlearrowleft \Sigma M_J = 0: \quad (320\text{ N})(0.16\text{ m})\sin\theta - (40.0\text{ N})(0.16\text{ m})(1-\cos\theta) - M = 0$$

$$M = 51.2\sin\theta + 6.4\cos\theta - 6.4 \qquad (1)$$

For maximum value between *A* and *C*:

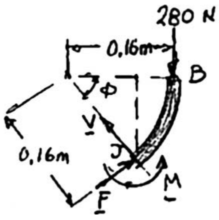
$$\frac{dM}{d\theta} = 0: \quad 51.2\cos\theta - 6.4\sin\theta = 0$$

$$\tan\theta = \frac{51.2}{6.4} = 8 \qquad \theta = 82.87° \quad ◀$$

Carrying into (1):

$$M = 51.2\sin 82.87° + 6.4\cos 82.87° - 6.4 = +45.20\text{ N}\cdot\text{m} \quad ◀$$

PROBLEM 7.12 (Continued)



Free body: BJ (For $\theta > 90°$)

$$\circlearrowleft\!+\ \Sigma M_J = 0:\quad M - (280\text{ N})(0.16\text{ m})(1-\cos\phi) = 0$$

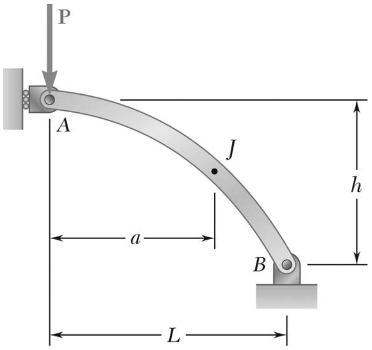
$$M = (44.8\text{ N}\cdot\text{m})(1-\cos\phi)$$

Largest value occurs for $\phi = 90°$, that is, at C, and is

$$M_C = 44.8\text{ N}\cdot\text{m} \ \triangleleft$$

We conclude that

$$M_{\max} = 45.2\text{ N}\cdot\text{m} \quad \text{for} \quad \theta = 82.9° \ \blacktriangleleft$$

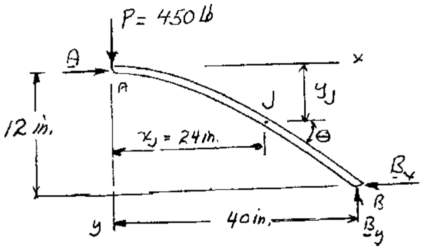


PROBLEM 7.13

The axis of the curved member *AB* is a parabola with vertex at *A*. If a vertical load **P** of magnitude 450 lb is applied at *A*, determine the internal forces at *J* when $h = 12$ in., $L = 40$ in., and $a = 24$ in.

SOLUTION

Free body *AB*



$$+\uparrow \Sigma F_y = 0: \quad -450\text{ lb} + B_y = 0$$

$$\mathbf{B}_y = 450\text{ lb}\uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad B_x(12\text{ in.}) - (450\text{ lb})(40\text{ in.}) = 0$$

$$\mathbf{B}_x = 1500\text{ lb} \longleftarrow$$

$$\Sigma F_x = 0: \quad \mathbf{A} = 1500\text{ lb} \longrightarrow$$

Parabola: $y = kx^2$

At *B*: $12\text{ in.} = k(40\text{ in.})^2 \qquad k = 0.0075$

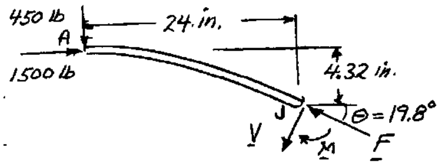
Equation of parabola: $y = 0.0075x^2$

$$\text{slope} = \frac{dy}{dx} = 0.015x$$

At *J*: $x_J = 24\text{ in.} \qquad y_J = 0.0075(24)^2 = 4.32\text{ in.}$

$$\text{slope} = 0.015(24) = 0.36, \quad \tan\theta = 0.36, \quad \theta = 19.8°$$

Free body *AJ*



$$+\circlearrowleft \Sigma M_J = 0: \quad (450\text{ lb})(24\text{ in.}) - (1500\text{ lb})(4.32\text{ in.}) - M = 0$$

$$M = 4320\text{ lb}\cdot\text{in.} \qquad \mathbf{M} = 4320\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

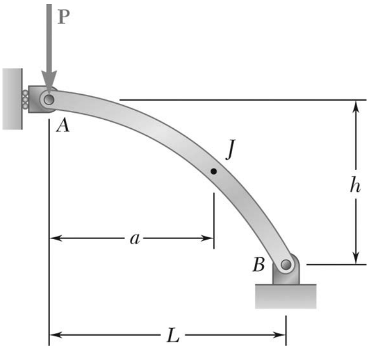
PROBLEM 7.13 (Continued)

$$+\nwarrow \Sigma F = 0: \quad F - (450 \text{ lb})\sin 19.8^\circ - (1500 \text{ lb})\cos 19.8^\circ = 0$$

$$F = +1563.8 \text{ lb} \qquad \mathbf{F} = 1564 \text{ lb} \nwarrow 19.8^\circ \blacktriangleleft$$

$$+\nearrow \Sigma F = 0: \quad -V - (450 \text{ lb})\cos 19.8^\circ + (1500 \text{ lb})\sin 19.8^\circ = 0$$

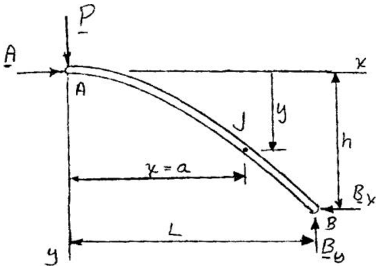
$$V = +84.71 \text{ lb} \qquad \mathbf{V} = 84.7 \text{ lb} \swarrow 70.2^\circ \blacktriangleleft$$



PROBLEM 7.14

Knowing that the axis of the curved member AB is a parabola with vertex at A, determine the magnitude and location of the maximum bending moment.

SOLUTION



Parabola $y = kx^2$

At B: $h = kL^2$

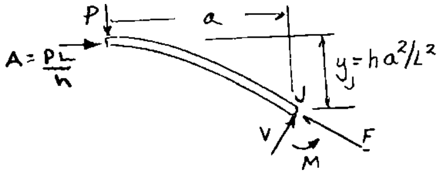
$k = h/L^2$

Equation of parabola $y = hx^2/L^2$

$$+\circlearrowleft \Sigma M_B = 0: \quad P(L) - A(h) = 0$$

$\mathbf{A} = PL/h \longrightarrow$

Free body AJ At J: $x_J = a \qquad y_J = ha^2/L^2$



$$+\circlearrowleft \Sigma M_J = 0: \quad P_a - (PL/h)(ha^2/L^2) + M = 0$$

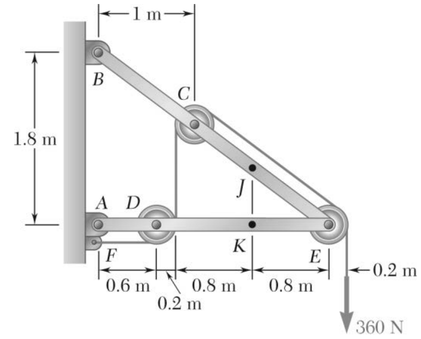
$$M = P\left(\frac{a^2}{L} - a\right)$$

For maximum: $$\frac{dM}{da} = P\left(\frac{2a}{L} - 1\right) = 0$$

$M_{\max}$ occurs at: $a = \frac{1}{2}L$ ◀

$$M_{\max} = P\left[\frac{(L/2)^2}{L} - \frac{L}{2}\right] = -\frac{PL}{4}$$

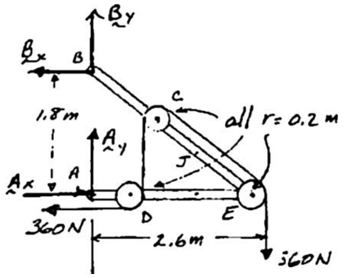
$|M|_{\max} = \frac{1}{4}PL$ ◀



PROBLEM 7.15

Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at Point J of the frame shown.

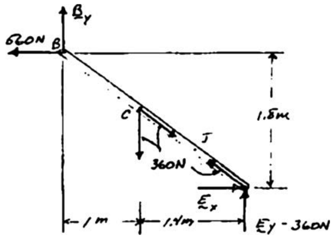
SOLUTION



FBD Frame with pulley and cord:

$$\circlearrowleft \Sigma M_A = 0: \quad (1.8\text{ m})B_x - (2.6\text{ m})(360\text{ N}) - (0.2\text{ m})(360\text{ N}) = 0$$

$$\mathbf{B}_x = 560\text{ N} \longleftarrow$$

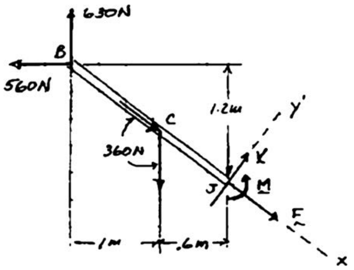


FBD BE:

Note: Cord forces have been moved to pulley hub as per Problem 6.91.

$$\circlearrowleft \Sigma M_E = 0: \quad (1.4\text{ m})(360\text{ N}) + (1.8\text{ m})(560\text{ N}) - (2.4\text{ m})B_y = 0$$

$$\mathbf{B}_y = 630\text{ N} \uparrow$$



FBD BJ:

$$\searrow \Sigma F_{x'} = 0: \quad F + 360\text{ N} - \frac{3}{5}(630\text{ N} - 360\text{ N}) - \frac{4}{5}(560\text{ N}) = 0$$

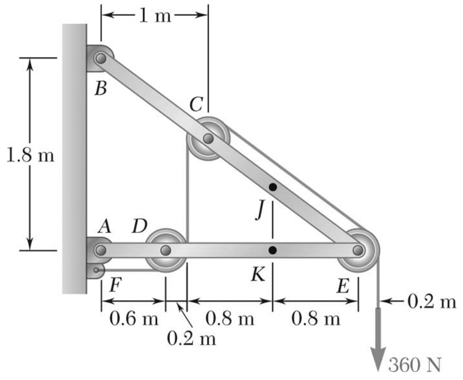
$$\mathbf{F} = 250\text{ N} \nwarrow 36.9° \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: \quad V + \frac{4}{5}(630\text{ N} - 360\text{ N}) - \frac{3}{5}(560\text{ N}) = 0$$

$$\mathbf{V} = 120.0\text{ N} \nearrow 53.1° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + (0.6\text{ m})(360\text{ N}) + (1.2\text{ m})(560\text{ N}) - (1.6\text{ m})(630\text{ N}) = 0$$

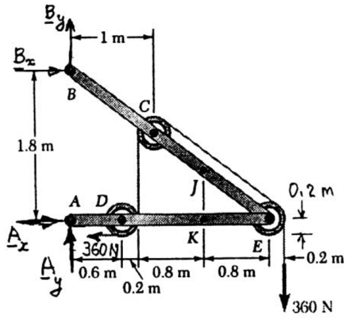
$$\mathbf{M} = 120.0\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



PROBLEM 7.16

Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at Point K of the frame shown.

SOLUTION



Free body: frame and pulleys

$$+\circlearrowleft \Sigma M_A = 0: \quad -B_x(1.8 \text{ m}) - (360 \text{ N})(0.2 \text{ m}) - (360 \text{ N})(2.6 \text{ m}) = 0$$

$$B_x = -560 \text{ N} \qquad \mathbf{B}_x = 560 \text{ N} \longleftarrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 560 \text{ N} - 360 \text{ N} = 0$$

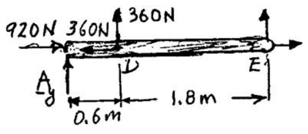
$$A_x = +920 \text{ N} \qquad \mathbf{A}_x = +920 \text{ N} \longrightarrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + B_y - 360 \text{ N} = 0$$

$$A_y + B_y = 360 \text{ N} \qquad (1)$$

Free body: member AE

We recall that the forces applied to a pulley may be applied directly to the axle of the pulley.



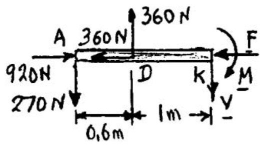
$$+\circlearrowleft \Sigma M_E = 0: \quad -A_y(2.4 \text{ m}) - (360 \text{ N})(1.8 \text{ m}) = 0$$

$$A_y = -270 \text{ N} \qquad \mathbf{A}_y = 270 \text{ N}\downarrow \triangleleft$$

From (1): $B_y = 360 \text{ N} + 270 \text{ N}$

$$B_y = 630 \text{ N} \qquad \mathbf{B}_y = 630 \text{ N}\uparrow \triangleleft$$

PROBLEM 7.16 (Continued)



Free body: *AK*

$\xrightarrow{+} \Sigma F_x = 0$: $920\text{ N} - 360\text{ N} - F = 0$

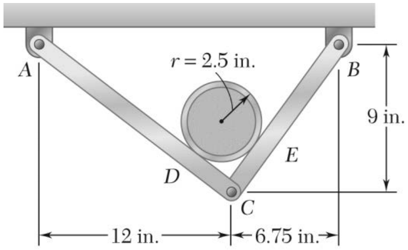
$F = +560\text{ N}$ $\qquad$ $\mathbf{F} = 560\text{ N} \longleftarrow$ ◀

$+\!\uparrow \Sigma F_y = 0$: $360\text{ N} - 270\text{ N} - V = 0$

$V = +90.0\text{ N}$ $\qquad$ $\mathbf{V} = 90.0\text{ N}\downarrow$ ◀

$+\circlearrowright \Sigma M_K = 0$: $(270\text{ N})(1.6\text{ m}) - (360\text{ N})(1\text{ m}) - M = 0$

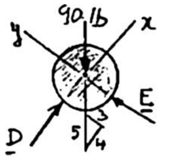
$M = +72.0\text{ N}\cdot\text{m}$ $\qquad$ $\mathbf{M} = 72.0\text{ N}\cdot\text{m} \circlearrowright$ ◀



PROBLEM 7.17

A 5-in.-diameter pipe is supported every 9 ft by a small frame consisting of two members as shown. Knowing that the combined weight of the pipe and its contents is 10 lb/ft and neglecting the effect of friction, determine the magnitude and location of the maximum bending moment in member *AC*.

SOLUTION



Free body: 10-ft section of pipe

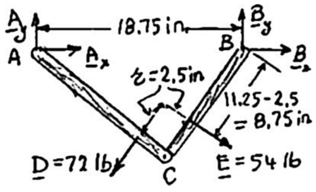
$$+\nearrow \Sigma F_x = 0: \quad D - \frac{4}{5}(90\text{ lb}) = 0 \qquad \mathbf{D} = 72\text{ lb} \nearrow \triangleleft$$

$$\nwarrow + \Sigma F_y = 0: \quad E - \frac{3}{5}(90\text{ lb}) = 0 \qquad \mathbf{E} = 54\text{ lb} \nwarrow \triangleleft$$

Free body: Frame

$$+\circlearrowleft \Sigma M_B = 0: \quad -A_y(18.75\text{ in.}) + (72\text{ lb})(2.5\text{ in.}) + (54\text{ lb})(8.75\text{ in.}) = 0$$

$$A_y = +34.8\text{ lb} \qquad \mathbf{A}_y = 34.8\text{ lb}\uparrow \triangleleft$$



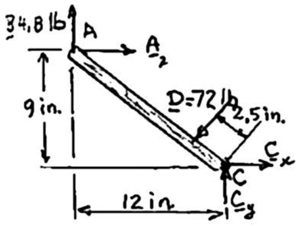
$$+\uparrow \Sigma F_y = 0: \quad B_y + 34.8\text{ lb} - \frac{4}{5}(72\text{ lb}) - \frac{3}{5}(54\text{ lb}) = 0$$

$$B_y = +55.2\text{ lb} \qquad \mathbf{B}_y = 55.2\text{ lb}\uparrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + B_x - \frac{3}{5}(72\text{ lb}) + \frac{4}{5}(54\text{ lb}) = 0$$

$$A_x + B_x = 0 \qquad (1)$$

Free body: Member *AC*



$$+\circlearrowleft \Sigma M_C = 0: \quad (72\text{ lb})(2.5\text{ in.}) - (34.8\text{ lb})(12\text{ in.}) - A_x(9\text{ in.}) = 0$$

$$A_x = -26.4\text{ lb} \qquad \mathbf{A}_x = 26.4\text{ lb} \leftarrow \triangleleft$$

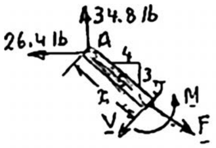
From (1): $B_x = -A_x = +26.4\text{ lb}$

$$\mathbf{B}_x = 26.4\text{ lb} \rightarrow \triangleleft$$

PROBLEM 7.17 (Continued)

Free body: Portion AJ

For $x \le 12.5$ in. $(AJ \le AD)$:

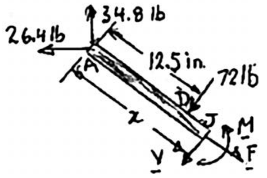


$$+\circlearrowleft \Sigma M_J = 0: \quad (26.4\text{ lb})\frac{3}{5}x - (34.8\text{ lb})\frac{4}{5}x + M = 0$$

$$M = 12x$$

$$M_{\max} = 150\text{ lb}\cdot\text{in. for } x = 12.5\text{ in.}$$

$$M_{\max} = 150.0\text{ lb}\cdot\text{in. at } D \; \triangleleft$$



For $x > 12.5$ in.$(AJ > AD)$:

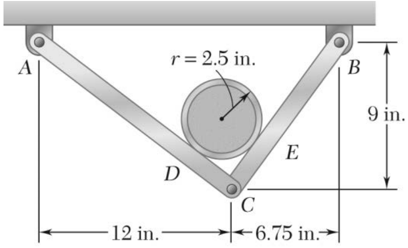
$$+\circlearrowleft \Sigma M_J = 0: \quad (26.4\text{ lb})\frac{3}{5}x - (34.8\text{ lb})\frac{4}{5}x + (72\text{ lb})(x - 12.5) + M = 0$$

$$M = 900 - 60x$$

$$M_{\max} = 150\text{ lb}\cdot\text{in. for } x = 12.5\text{ in.}$$

Thus: $M_{\max} = 150.0\text{ lb}\cdot\text{in. at } D$ ◀

PROPRIETARY MATERIAL.



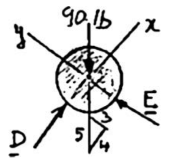
PROBLEM 7.18

For the frame of Problem 7.17, determine the magnitude and location of the maximum bending moment in member BC.

PROBLEM 7.17 A 5-in.-diameter pipe is supported every 9 ft by a small frame consisting of two members as shown. Knowing that the combined weight of the pipe and its contents is 10 lb/ft and neglecting the effect of friction, determine the magnitude and location of the maximum bending moment in member AC.

SOLUTION

Free body: 10-ft section of pipe



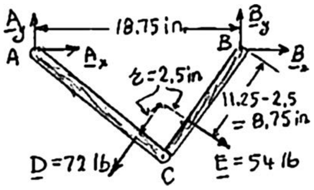
$$+\nearrow \Sigma F_x = 0: \quad D - \frac{4}{5}(90\text{ lb}) = 0 \qquad \mathbf{D} = 72\text{ lb} \nearrow \quad \triangleleft$$

$$\nwarrow + \Sigma F_y = 0: \quad E - \frac{3}{5}(90\text{ lb}) = 0 \qquad \mathbf{E} = 54\text{ lb} \nwarrow \quad \triangleleft$$

Free body: Frame

$$+\circlearrowleft \Sigma M_B = 0: \quad -A_y(18.75\text{ in.}) + (72\text{ lb})(2.5\text{ in.}) + (54\text{ lb})(8.75\text{ in.}) = 0$$

$$A_y = +34.8\text{ lb} \qquad \mathbf{A}_y = 34.8\text{ lb}\uparrow \quad \triangleleft$$



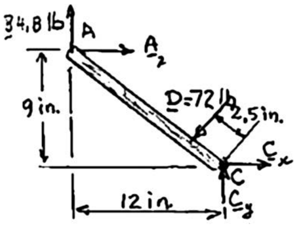
$$+\uparrow \Sigma F_y = 0: \quad B_y + 34.8\text{ lb} - \frac{4}{5}(72\text{ lb}) - \frac{3}{5}(54\text{ lb}) = 0$$

$$B_y = +55.2\text{ lb} \qquad \mathbf{B}_y = 55.2\text{ lb}\uparrow \quad \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + B_x - \frac{3}{5}(72\text{ lb}) + \frac{4}{5}(54\text{ lb}) = 0$$

$$A_x + B_x = 0 \qquad (1)$$

Free body: Member AC



$$+\circlearrowleft \Sigma M_C = 0: \quad (72\text{ lb})(2.5\text{ in.}) - (34.8\text{ lb})(12\text{ in.}) - A_x(9\text{ in.}) = 0$$

$$A_x = -26.4\text{ lb} \qquad \mathbf{A}_x = 26.4\text{ lb} \longleftarrow \quad \triangleleft$$

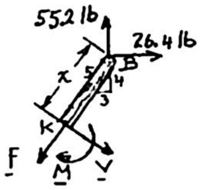
From (1): $\quad B_x = -A_x = +26.4\text{ lb}$

$$\mathbf{B}_x = 26.4\text{ lb} \longrightarrow \quad \triangleleft$$

PROBLEM 7.18 (Continued)

Free body: Portion BK

For $x \le 8.75$ in. ($BK \le BE$):

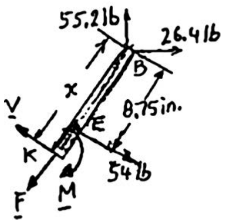


$$+\circlearrowleft \Sigma M_K = 0: \quad (55.2 \text{ lb})\frac{3}{5}x - (26.4 \text{ lb})\frac{4}{5}x - M = 0$$

$$M = 12x$$

$$M_{\max} = 105.0 \text{ lb} \cdot \text{in.} \quad \text{for} \quad x = 8.75 \text{ in.}$$

$$M_{\max} = 105.0 \text{ lb} \cdot \text{in. at } E \ \triangleleft$$



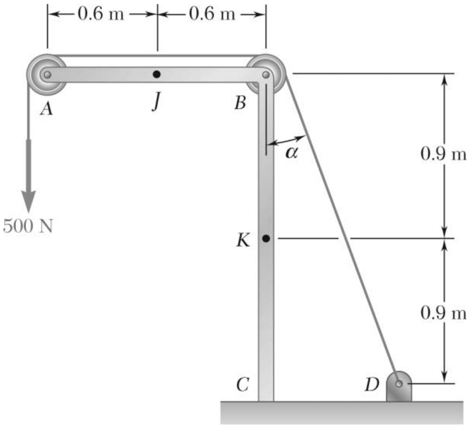
For $x > 8.75$ in. ($BK > BE$):

$$+\circlearrowleft \Sigma M_K = 0: \quad (55.2 \text{ lb})\frac{3}{5}x - (26.4 \text{ lb})\frac{4}{5}x - (54 \text{ lb})(x - 8.75 \text{ in.}) - M = 0$$

$$M = 472.5 - 42x$$

$$M_{\max} = 105.0 \text{ lb} \cdot \text{in.} \quad \text{for} \quad x = 8.75 \text{ in.}$$

Thus $$M_{\max} = 105.0 \text{ lb} \cdot \text{in. at } E \ \blacktriangleleft$$



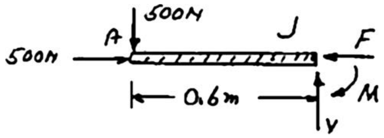
PROBLEM 7.19

Knowing that the radius of each pulley is 150 mm, that $\alpha = 20°$, and neglecting friction, determine the internal forces at (*a*) Point *J*, (*b*) Point *K*.

SOLUTION

Tension in cable = 500 N. Replace cable tension by forces at pins *A* and *B*. Radius does not enter computations: (cf. Problem 6.90)

(*a*) Free body: *AJ*



$$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - F = 0$$

$$F = 500\text{ N} \qquad \mathbf{F} = 500\text{ N} \longleftarrow \blacktriangleleft$$

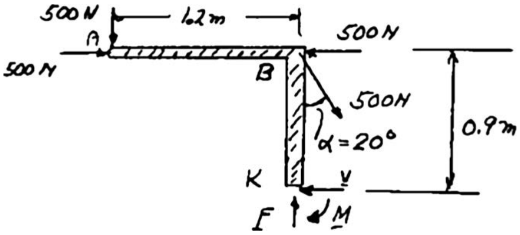
$$+\!\uparrow \Sigma F_y = 0: \quad V - 500\text{ N} = 0$$

$$V = 500\text{ N} \qquad \mathbf{V} = 500\text{ N} \uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad (500\text{ N})(0.6\text{ m}) = 0$$

$$M = 300\text{ N}\cdot\text{m} \qquad \mathbf{M} = 300\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

(*b*) Free body: *ABK*



$$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - 500\text{ N} + (500\text{ N})\sin 20° - V = 0$$

$$V = 171.01\text{ N} \qquad \mathbf{V} = 171.0\text{ N} \longleftarrow \blacktriangleleft$$

PROBLEM 7.19 (Continued)

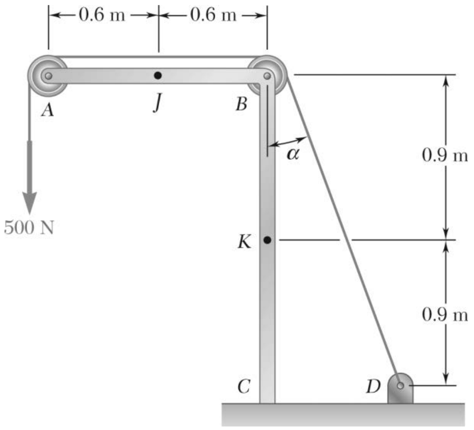
$$+\uparrow \Sigma F_y = 0: \quad -500\,\text{N} - (500\,\text{N})\cos 20° + F = 0$$

$$F = 969.8\,\text{N} \qquad\qquad \mathbf{F} = 970\,\text{N}\uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_K = 0: \quad (500\,\text{N})(1.2\,\text{m}) - (500\,\text{N})\sin 20°(0.9\,\text{m}) - M = 0$$

$$M = 446.1\,\text{N}\cdot\text{m}$$

$$\mathbf{M} = 446\,\text{N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



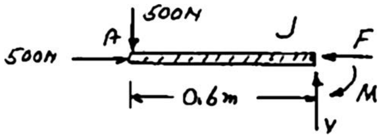
PROBLEM 7.20

Knowing that the radius of each pulley is 150 mm, that $\alpha = 30°$, and neglecting friction, determine the internal forces at (*a*) Point *J*, (*b*) Point *K*.

SOLUTION

Tension in cable = 500 N. Replace cable tension by forces at pins *A* and *B*. Radius does not enter computations: (cf. Problem 6.90)

(*a*) <u>Free body: *AJ*</u>:



$$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - F = 0$$

$$F = 500\text{ N} \qquad \mathbf{F} = 500\text{ N} \longleftarrow ◀$$

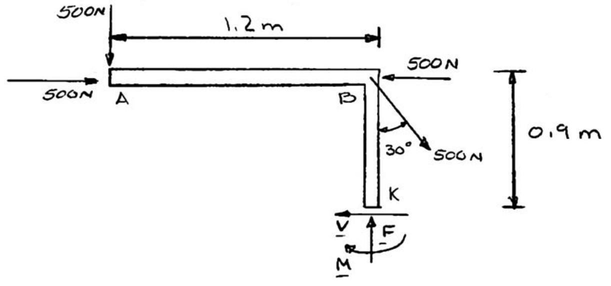
$$+\!\uparrow \Sigma F_y = 0: \quad V - 500\text{ N} = 0$$

$$V = 500\text{ N} \qquad \mathbf{V} = 500\text{ N}\uparrow ◀$$

$$+\!\circlearrowleft \Sigma M_J = 0: \quad (500\text{ N})(0.6\text{ m}) = 0$$

$$M = 300\text{ N}\cdot\text{m} \qquad \mathbf{M} = 300\text{ N}\cdot\text{m} \circlearrowright ◀$$

(*b*) <u>FBD: Portion *ABK*</u>:



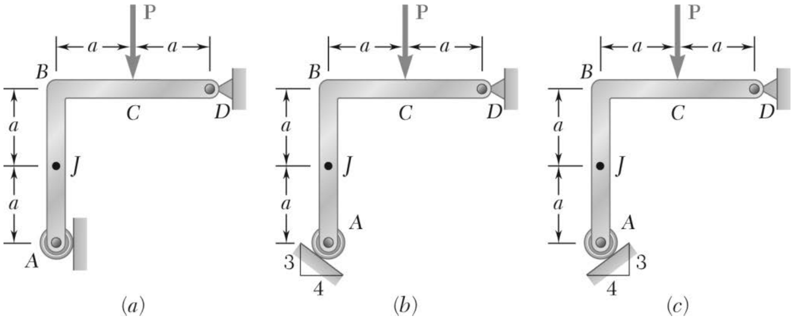
$$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - 500\text{ N} + (500\text{ N})\sin 30° - V \qquad \mathbf{V} = 250\text{ N} \longleftarrow ◀$$

$$+\!\uparrow \Sigma F_y = 0: \quad -500\text{ N} - (500\text{ N})\cos 30° + F = 0 \qquad \mathbf{F} = 933\text{ N}\uparrow ◀$$

$$+\!\circlearrowleft \Sigma M_K = 0: \quad (500\text{ N})(1.2\text{ m}) - (500\text{ N})\sin 30°(0.9\text{ m}) - M = 0 \qquad \mathbf{M} = 375\text{ N}\cdot\text{m} \circlearrowright ◀$$

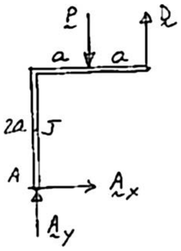
PROBLEM 7.21

A force **P** is applied to a bent rod that is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at Point *J*.



SOLUTION

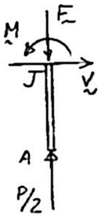
(*a*) **FBD Rod:**



$$\xrightarrow{}\Sigma F_x = 0: \quad A_x = 0$$

$$\circlearrowleft \Sigma M_D = 0: \quad aP - 2aA_y = 0 \qquad A_y = \frac{P}{2}$$

FBD AJ:

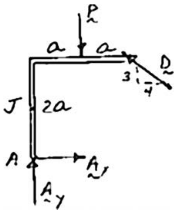


$$\xrightarrow{}\Sigma F_x = 0: \quad \mathbf{V} = 0 \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{P}{2} - F = 0 \qquad \mathbf{F} = \frac{P}{2}\downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad \mathbf{M} = 0 \blacktriangleleft$$

(*b*) **FBD Rod:**



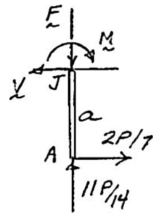
$$\circlearrowleft \Sigma M_A = 0: \quad 2a\left(\frac{4}{5}D\right) + 2a\left(\frac{3}{5}D\right) - aP = 0 \qquad D = \frac{5P}{14}$$

$$\xrightarrow{}\Sigma F_x = 0: \quad A_x - \frac{4}{5}\frac{5}{14}P = 0 \qquad A_x = \frac{2P}{7}$$

$$\uparrow \Sigma F_y = 0: \quad A_y - P + \frac{3}{5}\frac{5}{14}P = 0 \qquad A_y = \frac{11P}{14}$$

PROBLEM 7.21 (Continued)

FBD AJ:

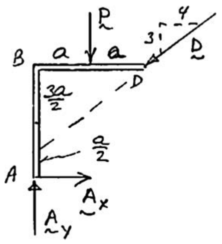


$\longrightarrow \Sigma F_x = 0: \quad \frac{2}{7}P - V = 0$ $\qquad \mathbf{V} = \frac{2P}{7} \longleftarrow$ ◀

$\uparrow \Sigma F_y = 0: \quad \frac{11P}{14} - F = 0$ $\qquad \mathbf{F} = \frac{11P}{14} \downarrow$ ◀

$\circlearrowleft \Sigma M_J = 0: \quad a\frac{2P}{7} - M = 0$ $\qquad \mathbf{M} = \frac{2}{7}aP \circlearrowright$ ◀

(*c*) **FBD Rod:**

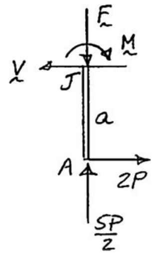


$\circlearrowleft \Sigma M_A = 0: \quad \frac{a}{2}\left(\frac{4D}{5}\right) - aP = 0$ $\qquad D = \frac{5P}{2}$

$\longrightarrow \Sigma F_x = 0: \quad A_x - \frac{4}{5}\frac{5P}{2} = 0$ $\qquad A_x = 2P$

$\uparrow \Sigma F_y = 0: \quad A_y - P - \frac{3}{5}\frac{5P}{2} = 0$ $\qquad A_y = \frac{5P}{2}$

FBD AJ:



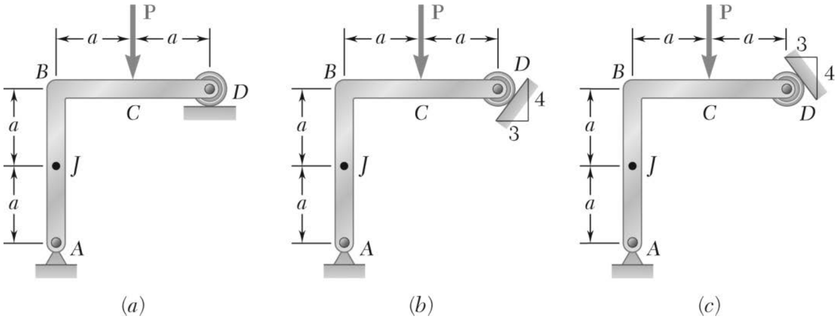
$\longrightarrow \Sigma F_x = 0: \quad 2P - V = 0$ $\qquad \mathbf{V} = 2P \longleftarrow$ ◀

$\uparrow \Sigma F_y = 0: \quad \frac{5P}{2} - F = 0$ $\qquad \mathbf{F} = \frac{5P}{2} \downarrow$ ◀

$\circlearrowleft \Sigma M_J = 0: \quad a(2P) - M = 0$ $\qquad \mathbf{M} = 2aP \circlearrowright$ ◀

PROBLEM 7.22

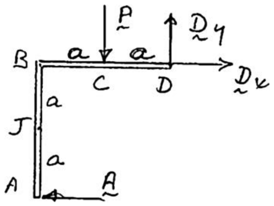
A force **P** is applied to a bent rod that is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at Point *J*.



(*a*) (*b*) (*c*)

SOLUTION

(*a*) **FBD Rod:**



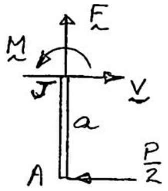
$$\circlearrowleft \Sigma M_D = 0: \quad aP - 2aA = 0$$

$$\mathbf{A} = \frac{P}{2} \longleftarrow$$

$$\longrightarrow \Sigma F_x = 0: \quad V - \frac{P}{2} = 0$$

$$\mathbf{V} = \frac{P}{2} \longrightarrow \blacktriangleleft$$

FBD AJ:



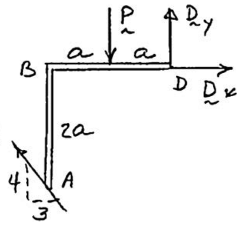
$$\uparrow \Sigma F_y = 0:$$

$$\mathbf{F} = 0 \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - a\frac{P}{2} = 0$$

$$\mathbf{M} = \frac{aP}{2} \circlearrowleft \blacktriangleleft$$

(*b*) **FBD Rod:**

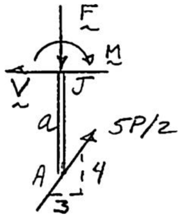


$$\circlearrowleft \Sigma M_D = 0: \quad aP - \frac{a}{2}\left(\frac{4}{5}A\right) = 0$$

$$\mathbf{A} = \frac{5P}{2} \nearrow$$

PROBLEM 7.22 (Continued)

FBD AJ:

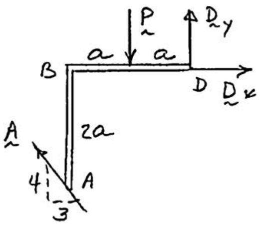


$$\xrightarrow{} \Sigma F_x = 0: \quad \frac{3}{5}\frac{5P}{2} - V = 0 \qquad \mathbf{V} = \frac{3P}{2} \longleftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{4}{5}\frac{5P}{2} - F = 0 \qquad \mathbf{F} = 2P \downarrow \blacktriangleleft$$

$$\mathbf{M} = \frac{3}{2}aP \circlearrowright \blacktriangleleft$$

(*c*) **FBD Rod:**



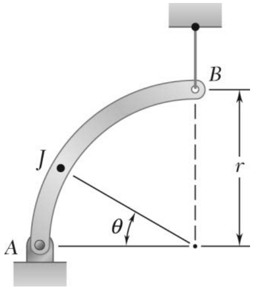
$$\circlearrowleft \Sigma M_D = 0: \quad aP - 2a\left(\frac{3}{5}A\right) - 2a\left(\frac{4}{5}A\right) = 0$$

$$A = \frac{5P}{14}$$

$$\xrightarrow{} \Sigma F_x = 0: \quad V - \left(\frac{3}{5}\frac{5P}{14}\right) = 0 \qquad \mathbf{V} = \frac{3P}{14} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{4}{5}\frac{5P}{14} - F = 0 \qquad \mathbf{F} = \frac{2P}{7} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - a\left(\frac{3}{5}\frac{5P}{14}\right) = 0 \qquad \mathbf{M} = \frac{3}{14}aP \circlearrowleft \blacktriangleleft$$

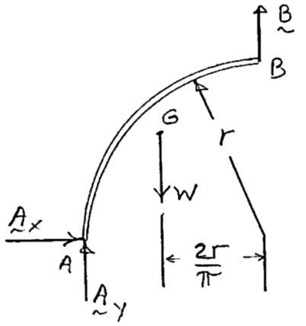


PROBLEM 7.23

A quarter-circular rod of weight *W* and uniform cross section is supported as shown. Determine the bending moment at Point *J* when $\theta = 30°$.

SOLUTION

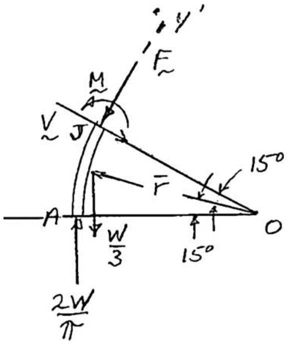
FBD Rod:



$$\longrightarrow \Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad \frac{2r}{\pi}W - rA_y = 0 \qquad \mathbf{A}_y = \frac{2W}{\pi}\uparrow$$

FBD AJ:



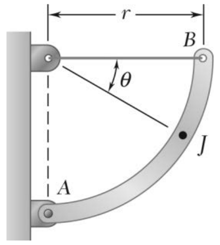
$$\alpha = 15°, \text{ weight of segment} = W\frac{30°}{90°} = \frac{W}{3}$$

$$\bar{r} = \frac{r}{\alpha}\sin\alpha = \frac{r}{\frac{\pi}{12}}\sin 15° = 0.9886r$$

$$\nearrow \Sigma F_{y'} = 0: \quad \frac{2W}{\pi}\cos 30° - \frac{W}{3}\cos 30° - F = 0$$

$$\mathbf{F} = \frac{W\sqrt{3}}{2}\left(\frac{2}{\pi} - \frac{1}{3}\right)\swarrow$$

$$\circlearrowleft \Sigma M_0 = M + r\left(F - \frac{2W}{\pi}\right) + \bar{r}\cos 15°\frac{W}{3} = 0 \qquad \mathbf{M} = 0.0557Wr \circlearrowleft \blacktriangleleft$$

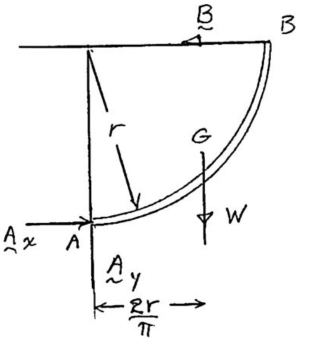


PROBLEM 7.24

A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

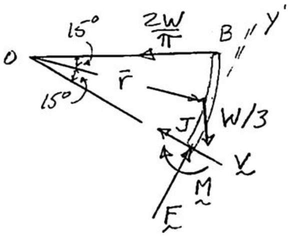
FBD Rod:



$$\circlearrowleft \Sigma M_A = 0: \quad rB - \frac{2r}{\pi}W = 0$$

$$\mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

FBD BJ:



$$\alpha = 15° = \frac{\pi}{12}$$

$$\overline{r} = \frac{r}{\frac{\pi}{12}}\sin 15° = 0.98862r$$

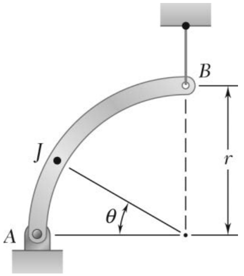
$$\text{Weight of segment } = W\frac{30°}{90°} = \frac{W}{3}$$

$$\nearrow \Sigma F_{y'} = 0: \quad F - \frac{W}{3}\cos 30° - \frac{2W}{\pi}\sin 30° = 0$$

$$\mathbf{F} = \left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right)W \nearrow$$

$$\circlearrowleft \Sigma M_0 = 0: \quad rF - (\overline{r}\cos 15°)\frac{W}{3} - M = 0$$

$$M = rW\left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right) - \left(0.98862\frac{\cos 15°}{3}\right)Wr \qquad \mathbf{M} = 0.289Wr \circlearrowright \blacktriangleleft$$



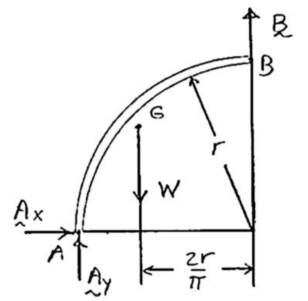
PROBLEM 7.25

For the rod of Problem 7.23, determine the magnitude and location of the maximum bending moment.

PROBLEM 7.23 A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

FBD Rod:



$$\xrightarrow{} \Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma M_B = 0: \quad \frac{2r}{\pi}W - rA_y = 0 \qquad A_y = \frac{2W}{\pi}$$

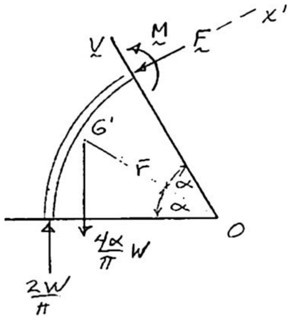
$$\alpha = \frac{\theta}{2}, \qquad \bar{r} = \frac{r}{\alpha}\sin\alpha$$

$$\text{Weight of segment } = W\frac{2\alpha}{\frac{\pi}{2}} = \frac{4\alpha}{\pi}W$$

$$\Sigma F_{x'} = 0: \quad -F - \frac{4\alpha}{\pi}W\cos 2\alpha + \frac{2W}{\pi}\cos 2\alpha = 0$$

$$F = \frac{2W}{\pi}(1-2\alpha)\cos 2\alpha = \frac{2W}{\pi}(1-\theta)\cos\theta$$

FBD AJ: $\Sigma M_0 = 0: \quad M + \left(F - \frac{2W}{\pi}\right)r + (\bar{r}\cos\alpha)\frac{4\alpha}{\pi}W = 0$



$$M = \frac{2W}{\pi}(1 + \theta\cos\theta - \cos\theta)r - \frac{4\alpha W}{\pi}\frac{r}{\alpha}\sin\alpha\cos\alpha$$

But, $\sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha = \frac{1}{2}\sin\theta$

so $M = \frac{2r}{\pi}W(1 - \cos\theta + \theta\cos\theta - \sin\theta)$

$$\frac{dM}{d\theta} = \frac{2rW}{\pi}(\sin\theta - \theta\sin\theta + \cos\theta - \cos\theta) = 0$$

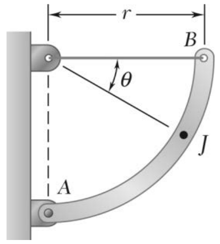
for $(1-\theta)\sin\theta = 0$

$$\frac{dM}{d\theta} = 0 \quad \text{for} \quad \theta = 0, 1, n\pi\,(n = 1, 2, \cdots)$$

Only 0 and 1 in valid range

At $\theta = 0 \quad M = 0,$ at $\theta = 1\text{ rad}$

at $\theta = 57.3°$ $\qquad M = M_{\max} = 0.1009Wr$ ◀



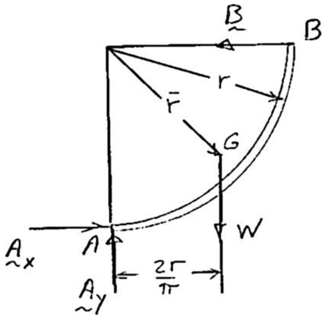
PROBLEM 7.26

For the rod of Problem 7.24, determine the magnitude and location of the maximum bending moment.

PROBLEM 7.24 A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

FBD Bar:



$$\circlearrowleft \Sigma M_A = 0: \quad rB - \frac{2r}{\pi}W = 0 \qquad \mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

$$\alpha = \frac{\theta}{2} \qquad \text{so} \qquad 0 \le \alpha \le \frac{\pi}{4}$$

$$\bar{r} = \frac{r}{\alpha}\sin\alpha$$

$$\text{Weight of segment } = W\frac{2\alpha}{\frac{\pi}{2}}$$

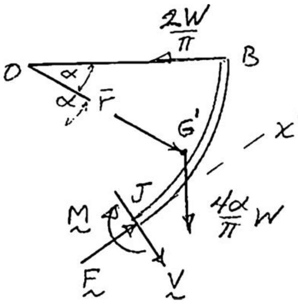
$$= \frac{4\alpha}{\pi}W$$

$$\nearrow \Sigma F_{x'} = 0: \quad F - \frac{4\alpha}{\pi}W\cos 2\alpha - \frac{2W}{\pi}\sin 2\alpha = 0$$

$$F = \frac{2W}{\pi}(\sin 2\alpha + 2\alpha\cos 2\alpha)$$

$$= \frac{2W}{\pi}(\sin\theta + \theta\cos\theta)$$

FBD BJ:



$$\circlearrowleft \Sigma M_0 = 0: \quad rF - (\bar{r}\cos\alpha)\frac{4\alpha}{\pi}W - M = 0$$

$$M = \frac{2}{\pi}Wr(\sin\theta + \theta\cos\theta) - \left(\frac{r}{\alpha}\sin\alpha\cos\alpha\right)\frac{4\alpha}{\pi}W$$

But, $\qquad \sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha = \frac{1}{2}\sin\theta$

PROBLEM 7.26 (Continued)

so $$M = \frac{2Wr}{\pi}(\sin\theta + \theta\cos\theta - \sin\theta)$$

or $$M = \frac{2}{\pi}Wr\theta\cos\theta$$

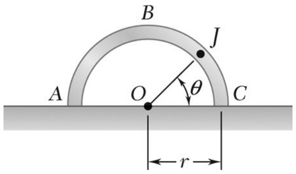
$$\frac{dM}{d\theta} = \frac{2}{\pi}Wr(\cos\theta - \theta\sin\theta) = 0 \quad \text{at } \theta\tan\theta = 1$$

Solving numerically $\theta = 0.8603$ rad

and $\mathbf{M} = 0.357Wr \circlearrowright$ ◀

at $\theta = 49.3°$ ◀

(Since $M = 0$ at both limits, this is the maximum)



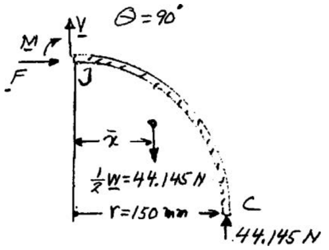
PROBLEM 7.27

A half section of pipe rests on a frictionless horizontal surface as shown. If the half section of pipe has a mass of 9 kg and a diameter of 300 mm, determine the bending moment at Point J when $\theta = 90°$.

SOLUTION

For half section $m = 9\text{ kg}$

$$W = mg = (9)(9.81) = 88.29\text{ N}$$



Portion JC:

$$\text{Weight} = \frac{1}{2}W = 44.145\text{ N}$$

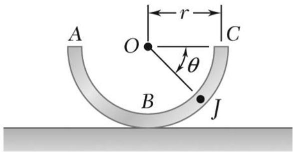
From Fig. 5.8B:

$$x = \frac{2r}{\pi} = \frac{2(150)}{\pi}$$

$$x = 95.49\text{ mm}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad (44.145\text{ N})(0.15\text{ m}) - (44.145\text{ N})(0.09549\text{ m}) - M = 0$$

$$M = +2.406\text{ N}\cdot\text{m} \qquad \mathbf{M} = 2.41\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



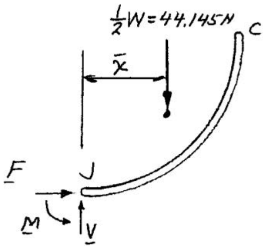
PROBLEM 7.28

A half section of pipe rests on a frictionless horizontal surface as shown. If the half section of pipe has a mass of 9 kg and a diameter of 300 mm, determine the bending moment at point J when $\theta = 90°$.

SOLUTION

For half section $m = 9\text{ kg}$

$$W = mg = (9\text{ kg})(9.81\text{ m/s}^2) = 88.29\text{ N}$$



Free body JC

Weight of portion JC, $= \dfrac{1}{2}W = 44.145\text{ N}$

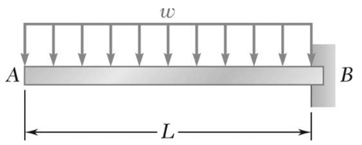
$$r = 150\text{ mm}$$

From Fig. 5.8B:

$$x = \frac{2r}{\pi} = \frac{2(150)}{\pi} = 95.49\text{ mm}$$

$$+\circlearrowleft \Sigma M_J = 0\text{:}\quad M - (44.145\text{ N})(0.09549\text{ m}) = 0$$

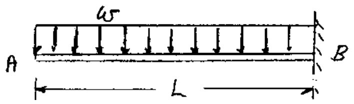
$$M = +4.2154\text{ N}\cdot\text{m} \qquad \mathbf{M} = 4.22\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

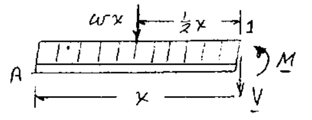


PROBLEM 7.29

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



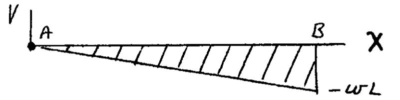


$$+\!\uparrow \Sigma F_y = 0: \quad -wx - V = 0$$

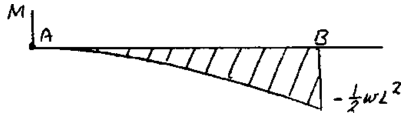
$$V = -wx$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad wx\left(\frac{x}{2}\right) + M = 0$$

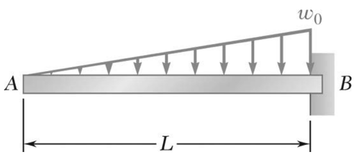
$$M = -\frac{1}{2}wx^2$$



$$|V|_{\max} = wL \blacktriangleleft$$



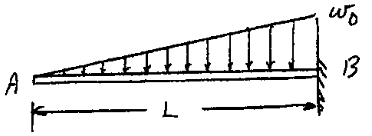
$$|M|_{\max} = \frac{1}{2}wL^2 \blacktriangleleft$$



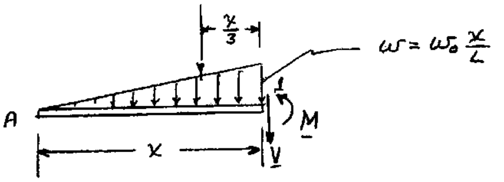
PROBLEM 7.30

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

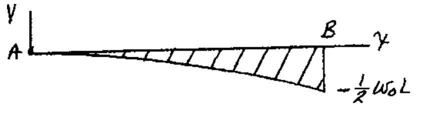


$$\frac{1}{2}wx = \frac{1}{2}\left(w_0\frac{x}{L}\right)x = \frac{1}{2}w_0\frac{x^2}{L}$$ By similar Δ's

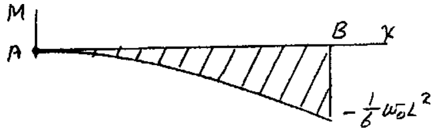


$$+\uparrow \Sigma F_y = 0\colon\quad -\frac{1}{2}w_0\frac{x^2}{L} - V = 0 \qquad V = -\frac{1}{2}w_0\frac{x^2}{L}$$

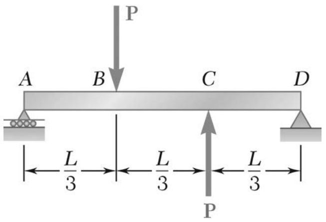
$$+\circlearrowleft \Sigma F_y = 0\colon\quad \left(\frac{1}{2}w_0\frac{x^2}{L}\right)\frac{x}{3} + M = 0 \qquad M = -\frac{1}{6}w_0\frac{x^3}{L}$$



$$|V|_{\max} = \frac{1}{2}w_0L \blacktriangleleft$$



$$|M|_{\max} = \frac{1}{6}w_0L^2 \blacktriangleleft$$

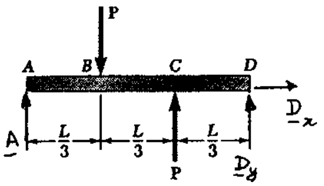


PROBLEM 7.31

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: entire beam



$$+\circlearrowleft \Sigma M_D = 0: \quad P\left(\frac{2L}{3}\right) - P\left(\frac{L}{3}\right) - AL = 0$$

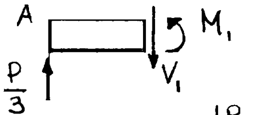
$$\mathbf{A} = P/3 \uparrow$$

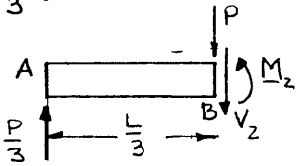
$$\Sigma F_x = 0: \quad D_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{P}{3} - P + P + D_y = 0$$

$$D_y = -P/3 \qquad \mathbf{D} = P/3 \downarrow$$

(*a*) Shear and bending moment. Since the loading consists of concentrated loads, the shear diagram is made of horizontal straight-line segments and the B. M. diagram is made of oblique straight-line segments.





Just to the right of *A*:

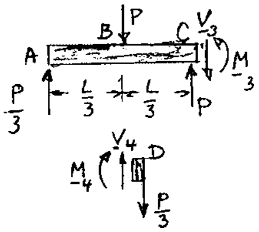
$$+\uparrow \Sigma F_y = 0: \quad -V_1 + \frac{P}{3} = 0 \qquad V_1 = +P/3 \ \triangleleft$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad M_1 - \frac{P}{3}(0) = 0 \qquad M_1 = 0 \ \triangleleft$$

Just to the right of *B*:

$$+\uparrow \Sigma F_y = 0: \quad -V_2 + \frac{P}{3} - P = 0, \qquad V_2 = -2P/3 \ \triangleleft$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - \frac{P}{3}\left(\frac{L}{3}\right) + P(0) = 0 \qquad M_2 = +PL/9 \ \triangleleft$$



Just to the right of *C*:

$$+\uparrow \Sigma F_y = 0: \quad \frac{P}{3} - P + P - V_3 = 0 \qquad V_3 = +P/3 \ \triangleleft$$

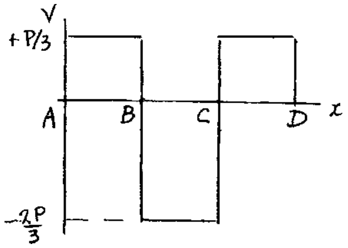
$$+\circlearrowleft \Sigma M_3 = 0: \quad M_3 - \frac{P}{3}\left(\frac{2L}{3}\right) + P\frac{L}{3} - P(0) = 0 \qquad M_3 = -PL/9 \ \triangleleft$$

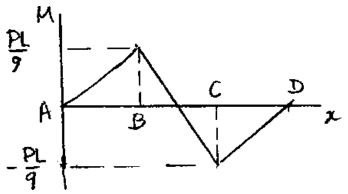
PROBLEM 7.31 (Continued)

Just to the left of D:

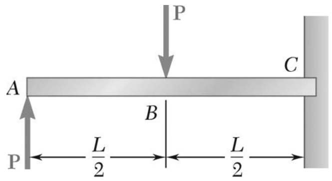
$$+\uparrow \Sigma F_y = 0: \quad V_4 - \frac{P}{3} = 0 \qquad V_4 = +\frac{P}{3} \quad \triangleleft$$

$$+\circlearrowleft \Sigma M_4 = 0: \quad -M_4 - \frac{P}{3}(0) = 0 \qquad M_4 = 0 \quad \triangleleft$$





(*b*) $|V|_{\max} = 2P/3; \; |M|_{\max} = PL/9$ ◀



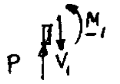
PROBLEM 7.32

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

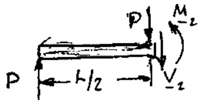
(*a*) Shear and bending moment.

Just to the right of *A*:



$V_1 = +P;$ $\qquad M_1 = 0$ ◁

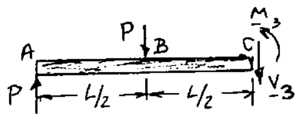
Just to the right of *B*:



$+\uparrow \Sigma F_y = 0: \quad P - P - V_2 = 0;$ $\qquad V_2 = 0$ ◁

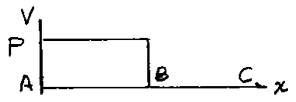
$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - P\left(\dfrac{L}{2}\right) = 0;$ $\qquad M_2 = +PL/2$ ◁

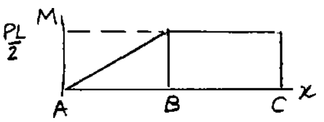
Just to the left of *C*:



$+\uparrow \Sigma F_y = 0: \quad P - P - V_3 = 0,$ $\qquad V_3 = 0$ ◁

$+\circlearrowleft \Sigma M_3 = 0: \quad M_3 + P\left(\dfrac{L}{2}\right) - PL = 0,$ $\qquad M_3 = +PL/2$ ◁





(*b*) $|V|_{\max} = P$ ◀

$|M|_{\max} = PL/2$ ◀

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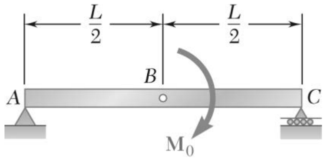


PROBLEM 7.33

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

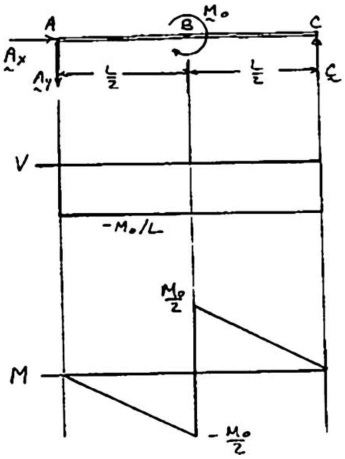
(*a*) **FBD Beam:**

$$\circlearrowleft \Sigma M_C = 0: \quad LA_y - M_0 = 0$$

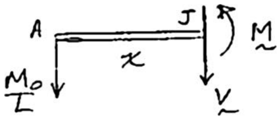
$$\mathbf{A}_y = \frac{M_0}{L} \downarrow$$

$$\uparrow \Sigma F_y = 0: \quad -A_y + C = 0$$

$$\mathbf{C} = \frac{M_0}{L} \uparrow$$



Along AB:



$$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0$$

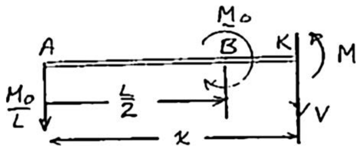
$$V = -\frac{M_0}{L}$$

$$\circlearrowleft \Sigma M_J = 0: \quad x\frac{M_0}{L} + M = 0$$

$$M = -\frac{M_0}{L}x$$

Straight with $M = -\dfrac{M_0}{2}$ at B

Along BC:



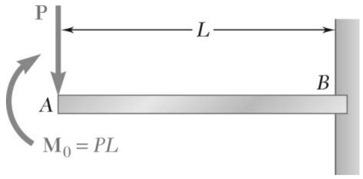
$$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0 \qquad V = -\frac{M_0}{L}$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + x\frac{M_0}{L} - M_0 = 0 \qquad M = M_0\left(1 - \frac{x}{L}\right)$$

Straight with $M = \dfrac{M_0}{2}$ at B $\qquad M = 0$ at C

(*b*) From diagrams: $|V|_{\max} = M_0/L$ ◀

$|M|_{\max} = \dfrac{M_0}{2}$ at B ◀

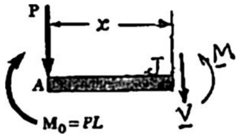


PROBLEM 7.34

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

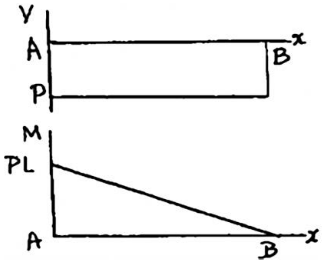
Free body: Portion *AJ*



$+\uparrow \Sigma F_y = 0: \quad -P - V = 0$ $\qquad V = -P$ ◁

$+\circlearrowleft \Sigma M_J = 0: \quad M + P_x - PL = 0$ $\qquad M = P(L - x)$ ◁

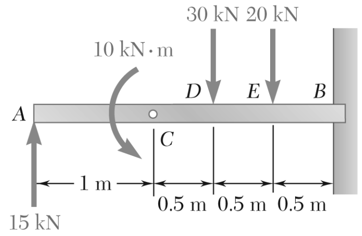
(*a*) The *V* and *M* diagrams are obtained by plotting the functions *V* and *M*.



(*b*)

$|V|_{\max} = P$ ◀

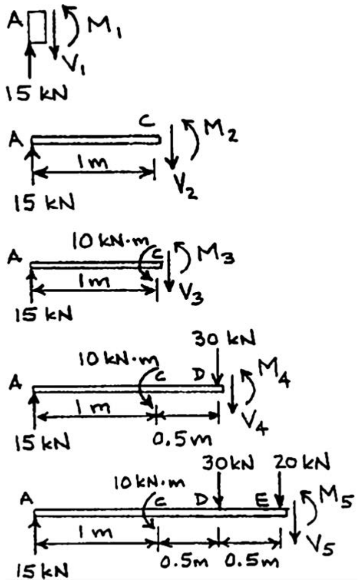
$|M|_{\max} = PL$ ◀



PROBLEM 7.35

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) Just to the right of *A*:

$$+\uparrow \Sigma F_y = 0 \quad V_1 = +15 \text{ kN} \quad M_1 = 0$$

Just to the left of *C*:

$$V_2 = +15 \text{ kN} \quad M_2 = +15 \text{ kN} \cdot \text{m}$$

Just to the right of *C*:

$$V_3 = +15 \text{ kN} \quad M_3 = +5 \text{ kN} \cdot \text{m}$$

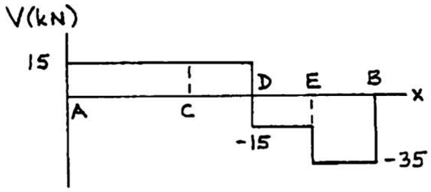
Just to the right of *D*:

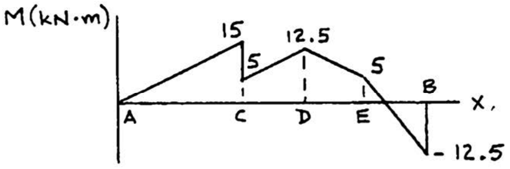
$$V_4 = -15 \text{ kN} \quad M_4 = +12.5 \text{ kN} \cdot \text{m}$$

Just to the right of *E*:

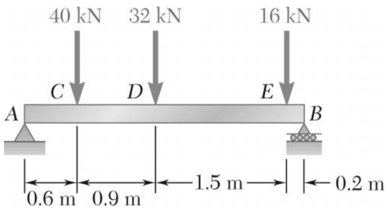
$$V_5 = -35 \text{ kN} \quad M_5 = +5 \text{ kN} \cdot \text{m}$$

At *B*: $M_B = -12.5 \text{ kN} \cdot \text{m}$





(*b*) $|V|_{\max} = 35.0 \text{ kN}$ $|M|_{\max} = 12.50 \text{ kN} \cdot \text{m}$ ◀

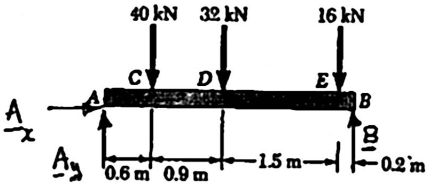


PROBLEM 7.36

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad B(3.2 \text{ m}) - (40 \text{ kN})(0.6 \text{ m}) - (32 \text{ kN})(1.5 \text{ m}) - (16 \text{ kN})(3 \text{ m}) = 0$$

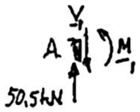
$B = +37.5 \text{ kN}$ $\qquad$ $\mathbf{B} = 37.5 \text{ kN}\uparrow$ ◁

$\Sigma F_x = 0: \quad A_x = 0$

$+\uparrow \Sigma F_y = 0: \quad A_y + 37.5 \text{ kN} - 40 \text{ kN} - 32 \text{ kN} - 16 \text{ kN} = 0$

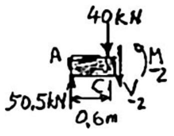
$A_y = +50.5 \text{ kN}$ $\qquad$ $\mathbf{A} = 50.5 \text{ kN}\uparrow$ ◁

(*a*) Shear and bending moment



Just to the right of *A*: $\qquad V_1 = 50.5 \text{ kN}$ $\qquad M_1 = 0$ ◁

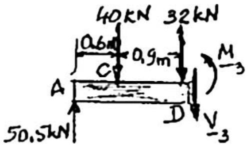
Just to the right of *C*:



$+\uparrow \Sigma F_y = 0: \quad 50.5 \text{ kN} - 40 \text{ kN} - V_2 = 0$ $\qquad V_2 = +10.5 \text{ kN}$ ◁

$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - (50.5 \text{ kN})(0.6 \text{ m}) = 0$ $\qquad M_2 = +30.3 \text{ kN} \cdot \text{m}$ ◁

Just to the right of *D*:

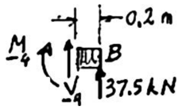


$+\uparrow \Sigma F_y = 0: \quad 50.5 - 40 - 32 - V_3 = 0$ $\qquad V_3 = -21.5 \text{ kN}$ ◁

$+\circlearrowleft \Sigma M_3 = 0: \quad M_3 - (50.5)(1.5) + (40)(0.9) = 0$ $\qquad M_3 = +39.8 \text{ kN} \cdot \text{m}$ ◁

PROBLEM 7.36 (Continued)

Just to the right of *E*:

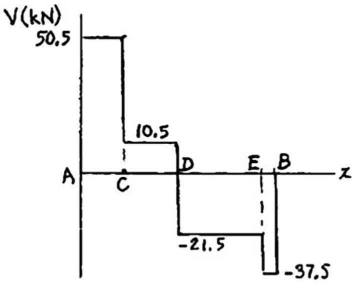


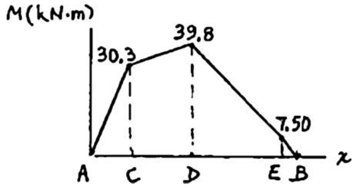
$+\!\uparrow \Sigma F_y = 0: \quad V_4 + 37.5 = 0$ $\qquad V_4 = -37.5 \text{ kN}$ ◁

$+\circlearrowleft \Sigma M_4 = 0: \quad -M_4 + (37.5)(0.2) = 0$ $\qquad M_4 = +7.50 \text{ kN}\cdot\text{m}$ ◁

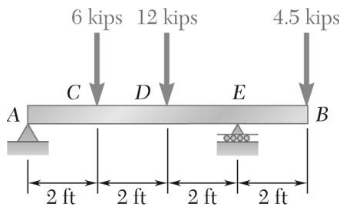
At *B*: $\qquad V_B = M_B = 0$ ◁

(*b*) $\qquad |V|_{\max} = 50.5 \text{ kN}$ ◀





$|M|_{\max} = 39.8 \text{ kN}\cdot\text{m}$ ◀

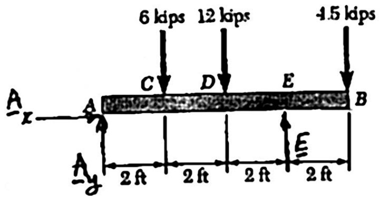


PROBLEM 7.37

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad E(6\text{ ft}) - (6\text{ kips})(2\text{ ft}) - (12\text{ kips})(4\text{ ft}) - (4.5\text{ kips})(8\text{ ft}) = 0$$

$$E = +16\text{ kips} \qquad \mathbf{E} = 16\text{ kips}\uparrow \triangleleft$$

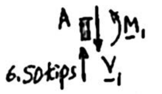
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 16\text{ kips} - 6\text{ kips} - 12\text{ kips} - 4.5\text{ kips} = 0$$

$$A_y = +6.50\text{ kips} \qquad \mathbf{A} = 6.50\text{ kips}\uparrow \triangleleft$$

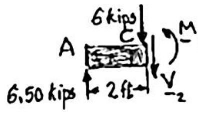
(*a*) Shear and bending moment

Just to the right of *A*:



$$V_1 = +6.50\text{ kips} \quad M_1 = 0 \qquad \triangleleft$$

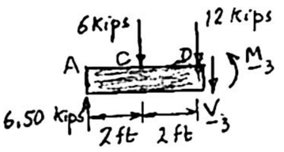
Just to the right of *C*:



$$+\uparrow \Sigma F_y = 0: \quad 6.50\text{ kips} - 6\text{ kips} - V_2 = 0 \qquad V_2 = +0.50\text{ kips} \triangleleft$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - (6.50\text{ kips})(2\text{ ft}) = 0 \qquad M_2 = +13\text{ kip}\cdot\text{ft} \triangleleft$$

Just to the right of *D*:

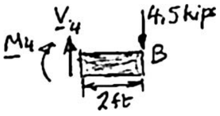


$$+\uparrow \Sigma F_y = 0: \quad 6.50 - 6 - 12 - V_3 = 0 \qquad V_3 = +11.5\text{ kips} \triangleleft$$

$$+\circlearrowleft \Sigma M_3 = 0: \quad M_3 - (6.50)(4) - (6)(2) = 0 \qquad M_3 = +14\text{ kip}\cdot\text{ft} \triangleleft$$

PROBLEM 7.37 (Continued)

Just to the right of *E*:

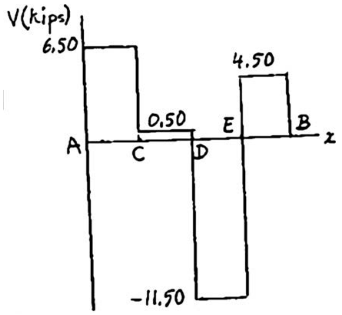


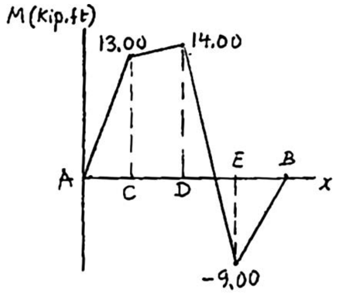
$+\uparrow \Sigma F_y = 0\text{:}\quad V_4 - 4.5 = 0 \qquad V_4 = +4.5 \text{ kips}$ ◁

$+\circlearrowleft \Sigma M_4 = 0\text{:}\quad -M_4 - (4.5)2 = 0 \qquad M_4 = -9 \text{ kip} \cdot \text{ft}$ ◁

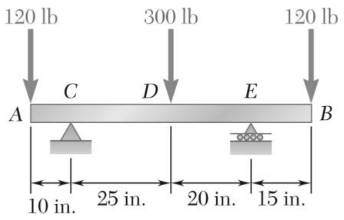
At *B*: $V_B = M_B = 0$ ◁

(*b*) $|V|_{\max} = 11.50 \text{ kips}$ ◀





$|M|_{\max} = 14.00 \text{ kip} \cdot \text{ft}$ ◀

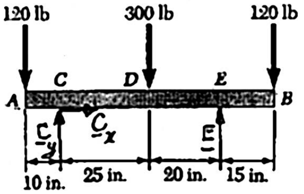


PROBLEM 7.38

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$+\circlearrowleft \Sigma M_C = 0$: $(120 \text{ lb})(10 \text{ in.}) - (300 \text{ lb})(25 \text{ in.}) + E(45 \text{ in.}) - (120 \text{ lb})(60 \text{ in.}) = 0$

$E = +300 \text{ lb}$ $\qquad$ $\mathbf{E} = 300 \text{ lb} \uparrow$ ◁

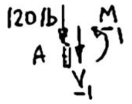
$\Sigma F_x = 0$: $C_x = 0$

$+\uparrow \Sigma F_y = 0$: $C_y + 300 \text{ lb} - 120 \text{ lb} - 300 \text{ lb} - 120 \text{ lb} = 0$

$C_y = +240 \text{ lb}$ $\qquad$ $\mathbf{C} = 240 \text{ lb} \uparrow$ ◁

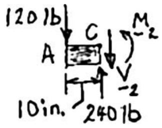
(*a*) Shear and bending moment

Just to the right of *A*:



$+\uparrow \Sigma F_y = 0$: $-120 \text{ lb} - V_1 = 0$ $\qquad$ $V_1 = -120 \text{ lb}, \quad M_1 = 0$ ◁

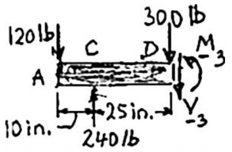
Just to the right of *C*:



$+\uparrow \Sigma F_y = 0$: $240 \text{ lb} - 120 \text{ lb} - V_2 = 0$ $\qquad$ $V_2 = +120 \text{ lb}$ ◁

$+\circlearrowleft \Sigma M_C = 0$: $M_2 + (120 \text{ lb})(10 \text{ in.}) = 0$ $\qquad$ $M_2 = -1200 \text{ lb} \cdot \text{in.}$ ◁

Just to the right of *D*:

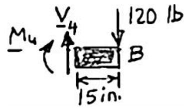


$+\uparrow \Sigma F_y = 0$: $240 - 120 - 300 - V_3 = 0$ $\qquad$ $V_3 = -180 \text{ lb}$ ◁

$+\circlearrowleft \Sigma M_3 = 0$: $M_3 + (120)(35) - (240)(25) = 0,$ $\qquad$ $M_3 = +1800 \text{ lb} \cdot \text{in.}$ ◁

PROBLEM 7.38 (Continued)

Just to the right of *E*:

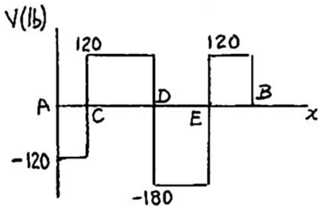


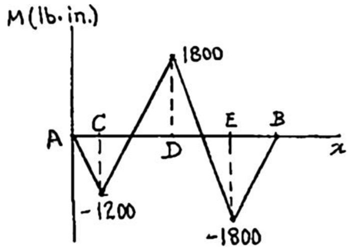
$+\Sigma F_y = 0\text{:}\quad V_4 - 120\text{ lb} = 0$ $\qquad V_4 = +120\text{ lb}$ ◁

$+\circlearrowleft \Sigma M_4 = 0\text{:}\quad -M_4 - (120\text{ lb})(15\text{ in.}) = 0$ $\qquad M_4 = -1800\text{ lb}\cdot\text{in.}$ ◁

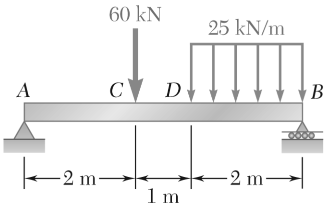
At *B*: $\qquad V_B = M_B = 0$ ◁

(*b*) $\qquad |V|_{\max} = 180.0\text{ lb}$ ◀





$|M|_{\max} = 1800\text{ lb}\cdot\text{in.}$ ◀

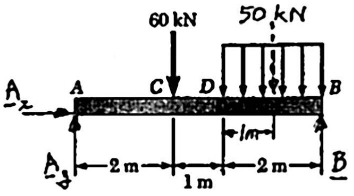


PROBLEM 7.39

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$+\circlearrowleft \Sigma M_A = 0: \quad B(5\text{ m}) - (60\text{ kN})(2\text{ m}) - (50\text{ kN})(4\text{ m}) = 0$

$B = +64.0\text{ kN}$ $\qquad \mathbf{B} = 64.0\text{ kN}\uparrow$ ◁

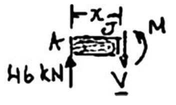
$\Sigma F_x = 0: \quad A_x = 0$

$+\uparrow \Sigma F_y = 0: \quad A_y + 64.0\text{ kN} - 6.0\text{ kN} - 50\text{ kN} = 0$

$A_y = +46.0\text{ kN}$ $\qquad \mathbf{A} = 46.0\text{ kN}\uparrow$ ◁

(*a*) Shear and bending-moment diagrams.

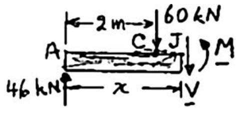
From *A* to *C*:



$+\uparrow \Sigma F_y = 0: \quad 46 - V = 0$ $\qquad V = +46\text{ kN}$ ◁

$+\circlearrowleft \Sigma M_y = 0: \quad M - 46x = 0$ $\qquad M = (46x)\text{kN}\cdot\text{m}$ ◁

From *C* to *D*:



$+\uparrow \Sigma F_y = 0: \quad 46 - 60 - V = 0$ $\qquad V = -14\text{ kN}$ ◁

$+\circlearrowleft \Sigma M_j = 0: \quad M - 46x + 60(x-2) = 0$

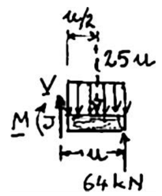
$M = (120 - 14x)\text{kN}\cdot\text{m}$

For $\qquad x = 2\text{ m}: \quad M_C = +92.0\text{ kN}\cdot\text{m}$ ◁

For $\qquad x = 3\text{ m}: \quad M_D = +78.0\text{ kN}\cdot\text{m}$ ◁

PROBLEM 7.39 (Continued)

From D to B:



$$+\uparrow \Sigma F_y = 0: \quad V + 64 - 25\mu = 0$$

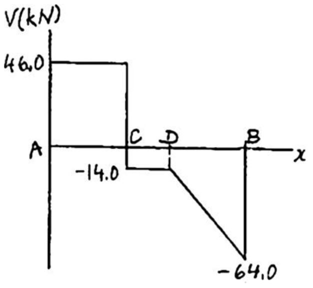
$$V = (25\mu - 64)\text{kN}$$

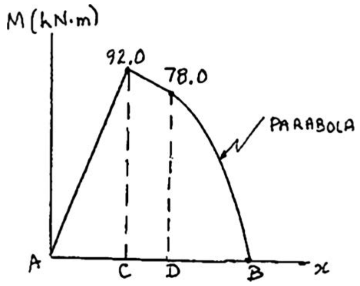
$$+\circlearrowleft \Sigma M_j = 0: \quad 64\mu - (25\mu)\left(\frac{\mu}{2}\right) - M = 0$$

$$M = (64\mu - 12.5\mu^2)\text{kN}\cdot\text{m}$$

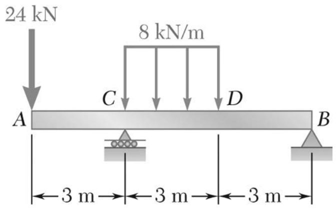
For $\quad \mu = 0: \quad V_B = -64 \text{ kN} \qquad M_B = 0 \ \triangleleft$

(*b*) $\qquad |V|_{\max} = 64.0 \text{ kN} \ \blacktriangleleft$





$$|M|_{\max} = 92.0 \text{ kN}\cdot\text{m} \ \blacktriangleleft$$

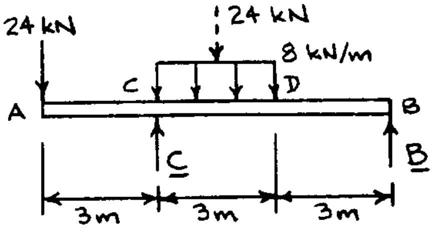


PROBLEM 7.40

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam

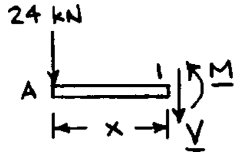


$+\circlearrowleft \Sigma M_B = 0$: $(24\text{ kN})(9\text{ m}) - C(6\text{ m}) + (24\text{ kN})(4.5\text{ m}) = 0$ $\qquad \mathbf{C} = 54\text{ kN}\uparrow$

$+\uparrow \Sigma F_y = 0$: $54 - 24 - 24 + B = 0$

$B = -6\text{ kN}$ $\qquad \mathbf{B} = 6\text{ kN}\downarrow$

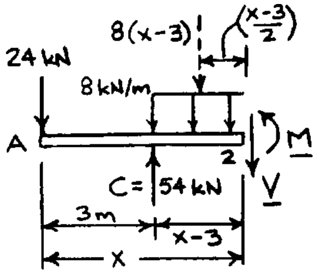
From *A* to *C*:



$+\uparrow \Sigma F_y = 0$: $-24 - V = 0$ $\qquad V = -24\text{ kN}$

$+\circlearrowleft \Sigma M_1 = 0$: $(24)(x) + M = 0$ $\qquad M = (-24x)\text{kN}\cdot\text{m}$

From *C* to *D*:



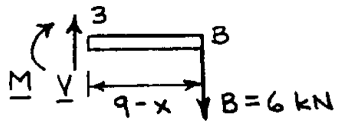
$+\uparrow \Sigma F_y = 0$: $-24 - 8(x-3) - V + 54 = 0$

$$V = (-8x + 54)\text{kN}$$

$+\circlearrowleft \Sigma M_2 = 0$: $(24)(x) + 8(x-3)\left(\dfrac{x-3}{2}\right) - (54)(x-3) + M = 0$

$$M = (-4x^2 + 54x - 198)\text{kN}\cdot\text{m}$$

From *D* to *B*:



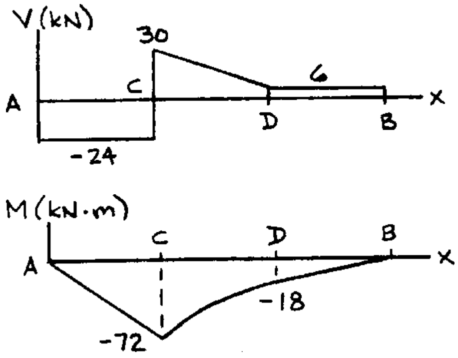
$+\uparrow \Sigma F_y = 0$: $V - 6 = 0$ $\qquad V = +6\text{ kN}$

$+\circlearrowleft \Sigma M_3 = 0$: $-M - (6)(9-x) = 0$ $\qquad M = (6x - 54)\text{kN}\cdot\text{m}$

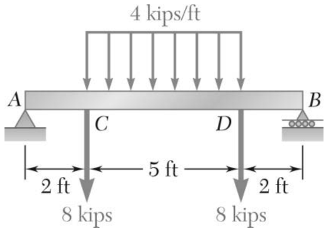
PROBLEM 7.40 (Continued)

(*b*)

$|V|_{\max} = 30.0 \text{ kN}$ ◀



$|M|_{\max} = 72.0 \text{ kN} \cdot \text{m}$ ◀



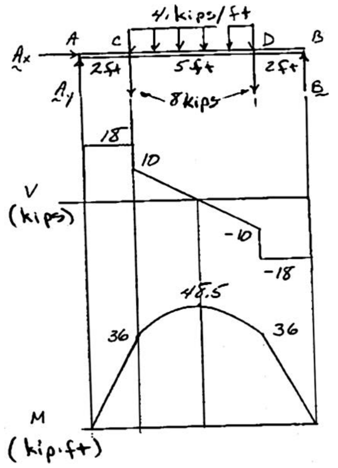
PROBLEM 7.41

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

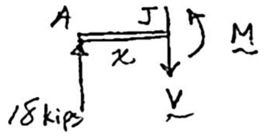
SOLUTION

(*a*) By symmetry:

$$A_y = B = 8\text{ kips} + \frac{1}{2}(4\text{ kips})(5\text{ ft}) \quad \mathbf{A}_y = \mathbf{B} = 18\text{ kips}\uparrow$$



Along AC:



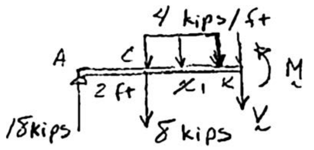
$$+\!\uparrow \Sigma F_y = 0: \quad 18\text{ kips} - V = 0 \quad V = 18\text{ kips}$$

$$\circlearrowleft + \Sigma M_J = 0: \quad M - x(18\text{ kips}) \quad M = (18\text{ kips})x$$

$$M = 36\text{ kip}\cdot\text{ft at } C(x = 2\text{ ft})$$

Along CD:

$$+\!\uparrow \Sigma F_y = 0: \quad 18\text{ kips} - 8\text{ kips} - (4\text{ kips/ft})x_1 - V = 0$$



$$V = 10\text{ kips} - (4\text{ kips/ft})x_1$$

$$V = 0 \text{ at } x_1 = 2.5\text{ ft (at center)}$$

$$\circlearrowleft + \Sigma M_K = 0: \quad M + \frac{x_1}{2}(4\text{ kips/ft})x_1 + (8\text{ kips})x_1 - (2\text{ ft} + x_1)(18\text{ kips}) = 0$$

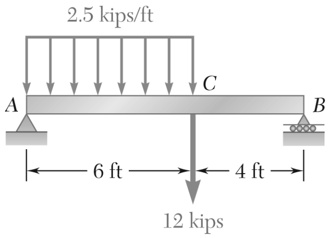
$$M = 36\text{ kip}\cdot\text{ft} + (10\text{ kips/ft})x_1 - (2\text{ kips/ft})x_1^2$$

$$M = 48.5\text{ kip}\cdot\text{ft} \text{ at } x_1 = 2.5\text{ ft}$$

Complete diagram by symmetry

(*b*) From diagrams: $|V|_{\max} = 18.00\text{ kips}$ ◀

$|M|_{\max} = 48.5\text{ kip}\cdot\text{ft}$ ◀

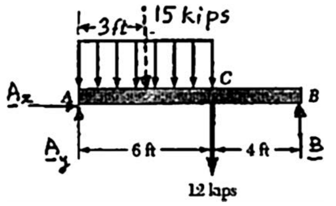


PROBLEM 7.42

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad B(10\text{ ft}) - (15\text{ kips})(3\text{ ft}) - (12\text{ kips})(6\text{ ft}) = 0$$

$$B = +11.70\text{ kips} \qquad \mathbf{B} = 11.70\text{ kips}\uparrow \triangleleft$$

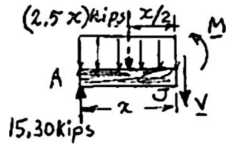
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 15 - 12 + 11.70 = 0$$

$$A_y = +15.30\text{ kips} \qquad \mathbf{A} = 15.30\text{ kips}\uparrow \triangleleft$$

(*a*) Shear and bending-moment diagrams

From *A* to *C*:



$$+\uparrow \Sigma F_y = 0: \quad 15.30 - 2.5x - V = 0$$

$$V = (15.30 - 2.5x)\text{ kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + (2.5x)\left(\frac{x}{2}\right) - 15.30x = 0$$

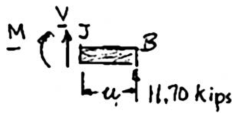
$$M = 15.30x - 1.25x^2$$

For $x = 0$: $V_A = +15.30\text{ kips}$ $\quad M_A = 0 \triangleleft$

For $x = 6\text{ ft}$: $V_C = +0.300\text{ kip}$ $\quad M_C = +46.8\text{ kip}\cdot\text{ft} \triangleleft$

PROBLEM 7.42 (Continued)

<u>From C to B:</u>



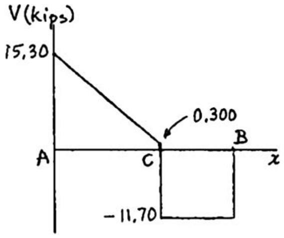
$+\uparrow \Sigma F_y = 0: \quad V + 11.70 = 0 \qquad V = -11.70 \text{ kips}$ ◁

$+\circlearrowleft \Sigma M_J = 0: \quad 11.70\mu - M = 0$

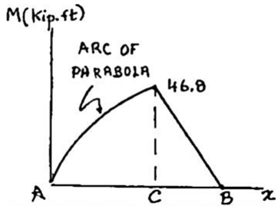
$$M = (11.70\mu) \text{ kip} \cdot \text{ft}$$

For $\mu = 4$ ft: $\qquad M_C = +46.8 \text{ kip} \cdot \text{ft}$ ◁

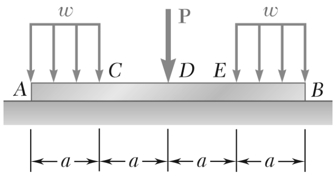
For $\mu = 0$: $\qquad M_B = 0$ ◁



(*b*) $\qquad |V|_{\max} = 15.30 \text{ kips}$ ◀



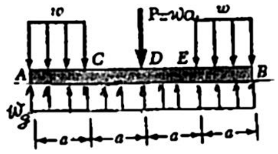
$|M|_{\max} = 46.8 \text{ kip} \cdot \text{ft}$ ◀



PROBLEM 7.43

Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $P = wa$, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



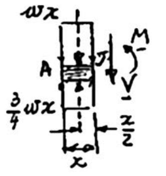
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - wa = 0$$

$$w_g = \frac{3}{4}w \quad \triangleleft$$

(*a*) Shear and bending-moment diagrams

From *A* to *C*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{4}wx - wx - V = 0$$

$$V = -\frac{1}{4}wx$$

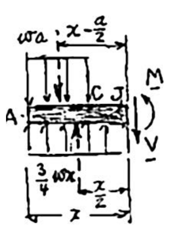
$$+\circlearrowleft \Sigma M_J = 0: \quad M + (wx)\frac{x}{2} - \left(\frac{3}{4}wx\right)\frac{x}{2} = 0$$

$$M = -\frac{1}{8}wx^2$$

For $x = 0$: $\qquad V_A = M_A = 0 \quad \triangleleft$

For $x = a$: $\qquad V_C = -\frac{1}{4}wa \qquad M_C = -\frac{1}{8}wa^2 \quad \triangleleft$

From *C* to *D*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{4}wx - wa - V = 0$$

$$V = \left(\frac{3}{4}x - a\right)w$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \frac{3}{4}wx\left(\frac{x}{2}\right) = 0$$

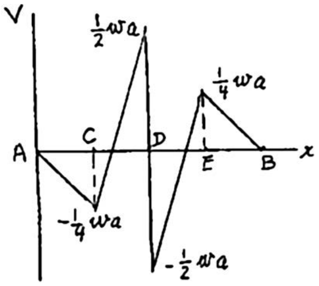
$$M = \frac{3}{8}wx^2 - wa\left(x - \frac{a}{2}\right) \qquad (1)$$

PROBLEM 7.43 (Continued)

For $x = a$: $\qquad V_C = -\frac{1}{4}wa \qquad\qquad M_C = -\frac{1}{8}wa^2$ ◁

For $x = 2a$: $\qquad V_D = +\frac{1}{2}wa \qquad\qquad M_D = 0$ ◁

Because of the symmetry of the loading, we can deduce the values of V and M for the right-hand half of the beam from the values obtained for its left-hand half.

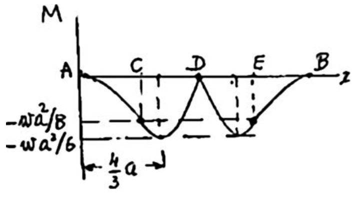


(*b*) $\qquad |V|_{\max} = \frac{1}{2}wa$ ◀

To find $|M|_{\max}$, we differentiate Eq. (1) and set $\frac{dM}{dx} = 0$:

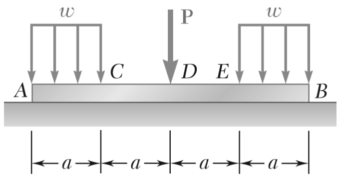
$$\frac{dM}{dx} = \frac{3}{4}wx - wa = 0, \quad x = \frac{4}{3}a$$

$$M = \frac{3}{8}w\left(\frac{4}{3}a\right)^2 - wa^2\left(\frac{4}{3} - \frac{1}{2}\right) = -\frac{wa^2}{6}$$



$|M|_{\max} = \frac{1}{6}wa^2$ ◀

Bending-moment diagram consists of four distinct arcs of parabola.

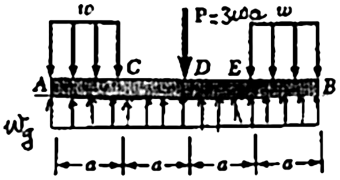


PROBLEM 7.44

Solve Problem 7.43 knowing that $P = 3wa$.

PROBLEM 7.43 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $P = wa$, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



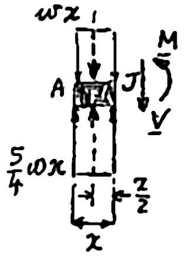
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - 3wa = 0$$

$$w_g = \frac{5}{4}w \quad \triangleleft$$

(*a*) Shear and bending-moment diagrams

From *A* to *C*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{4}wx - wx - V = 0$$

$$V = +\frac{1}{4}wx$$

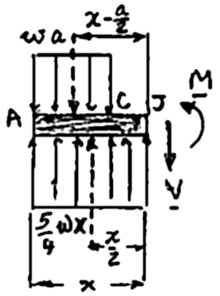
$$+\circlearrowleft \Sigma M_J = 0: \quad M + (wx)\frac{x}{2} - \left(\frac{5}{4}wx\right)\frac{x}{2} = 0$$

$$M = +\frac{1}{8}wx^2$$

For $x = 0$: $\quad V_A = M_A = 0 \quad \triangleleft$

For $x = a$: $\quad V_C = +\frac{1}{4}wa \qquad M_C = +\frac{1}{8}wa^2 \quad \triangleleft$

From *C* to *D*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{4}wx - wa - V = 0$$

$$V = \left(\frac{5}{4}x - a\right)w$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \frac{5}{4}wx\left(\frac{x}{2}\right) = 0$$

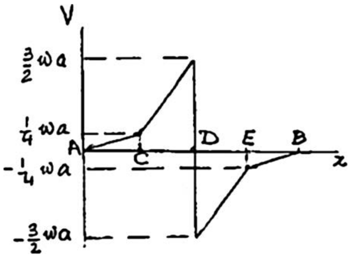
$$M = \frac{5}{8}wx^2 - wa\left(x - \frac{a}{2}\right) \qquad (1)$$

PROBLEM 7.44 (Continued)

For $x = a$: $$V_C = +\frac{1}{4}wa, \;\; M_C = +\frac{1}{8}wa^2 \;\triangleleft$$

For $x = 2a$: $$V_D = +\frac{3}{2}wa, \;\; M_D = +wa^2 \;\triangleleft$$

Because of the symmetry of the loading, we can deduce the values of V and M for the right-hand half of the beam from the values obtained for its left-hand half.

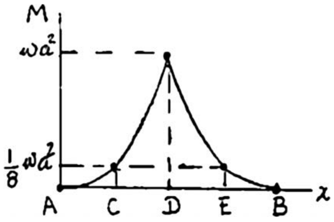


(*b*) $$|V|_{\max} = \frac{3}{2}wa \;\blacktriangleleft$$

To find $|M|_{\max}$, we differentiate Eq. (1) and set $\frac{dM}{dx} = 0$:

$$\frac{dM}{dx} = \frac{5}{4}wx - wa = 0$$

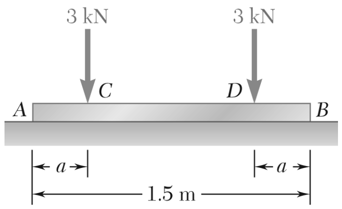
$$x = \frac{4}{5}a < a \quad \text{(outside portion } CD\text{)}$$



The maximum value of $|M|$ occurs at D:

$$|M|_{\max} = wa^2 \;\blacktriangleleft$$

Bending-moment diagram consists of four distinct arcs of parabola.



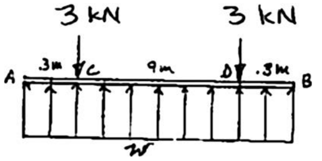
PROBLEM 7.45

Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that a = 0.3 m, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

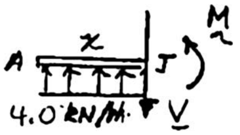
SOLUTION

(*a*) **FBD Beam:** $\uparrow \Sigma F_y = 0\text{:}\quad w(1.5\text{ m}) - 2(3.0\text{ kN}) = 0$

$$w = 4.0\text{ kN/m}$$



Along AC:

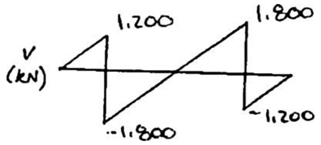


$\uparrow \Sigma F_y = 0\text{:}\quad (4.0\text{ kN/m})x - V = 0$

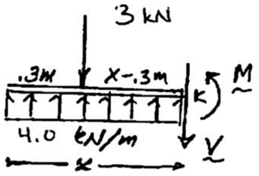
$$V = (4.0\text{ kN/m})x$$

$\circlearrowleft \Sigma M_J = 0\text{:}\quad M - \dfrac{x}{2}(4.0\text{ kN/m})x = 0$

$$M = (2.0\text{ kN/m})x^2$$



Along CD:

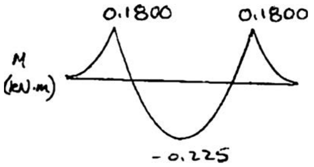


$\uparrow \Sigma F_y = 0\text{:}\quad (4.0\text{ kN/m})x - 3.0\text{ kN} - V = 0$

$$V = (4.0\text{ kN/m})x - 3.0\text{ kN}$$

$\circlearrowleft \Sigma M_K = 0\text{:}\quad M + (x - 0.3\text{ m})(3.0\text{ kN}) - \dfrac{x}{2}(4.0\text{ kN/m})x = 0$

$$M = 0.9\text{ kN}\cdot\text{m} - (3.0\text{ kN})x + (2.0\text{ kN/m})x^2$$

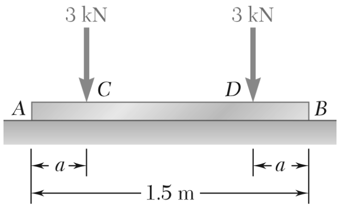


Note: $V = 0$ at $x = 0.75$ m, where $M = -0.225\text{ kN}\cdot\text{m}$

Complete diagrams using symmetry.

(*b*) $|V|_{\max} = 1.800\text{ kN}$ ◀

$|M|_{\max} = 0.225\text{ kN}\cdot\text{m}$ ◀



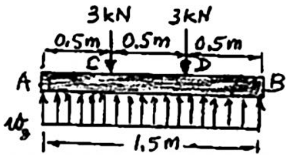
PROBLEM 7.46

Solve Problem 7.45 knowing that $a = 0.5$ m.

PROBLEM 7.45 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $a = 0.3$ m, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

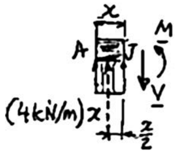
Free body: Entire beam



$$+\uparrow \Sigma F_y = 0: \quad w_g(1.5\text{ m}) - 3\text{ kN} - 3\text{ kN} = 0 \qquad w_g = 4\text{ kN/m} \ \triangleleft$$

(*a*) Shear and bending moment

From *A* to *C*:



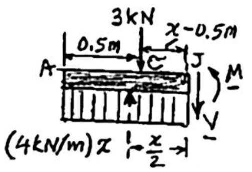
$$+\uparrow \Sigma F_y = 0: \quad 4x - V = 0 \quad V = (4x)\text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (4x)\frac{x}{2} = 0, \quad M = (2x^2)\text{ kN}\cdot\text{m}$$

For $x = 0$: $V_A = M_A = 0 \ \triangleleft$

For $x = 0.5$ m: $V_C = 2$ kN, $M_C = 0.500\text{ kN}\cdot\text{m} \ \triangleleft$

From *C* to *D*:



$$+\uparrow \Sigma F_y = 0: \quad 4x - 3\text{ kN} - V = 0$$

$$V = (4x - 3)\text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + (3\text{ kN})(x - 0.5) - (4x)\frac{x}{2} = 0$$

$$M = (2x^2 - 3x + 1.5)\text{ kN}\cdot\text{m}$$

For $x = 0.5$ m: $V_C = -1.00$ kN, $M_C = 0.500\text{ kN}\cdot\text{m} \ \triangleleft$

For $x = 0.75$ m: $V_C = 0$, $M_C = 0.375\text{ kN}\cdot\text{m} \ \triangleleft$

For $x = 1.0$ m: $V_C = 1.00$ kN, $M_C = 0.500\text{ kN}\cdot\text{m} \ \triangleleft$

PROBLEM 7.46 (Continued)

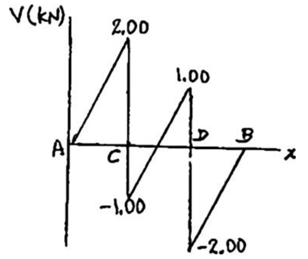
From D to B:

$$+\uparrow \Sigma F_y = 0: \quad V + 4\mu = 0 \quad V = -(4\mu)\text{ kN}$$

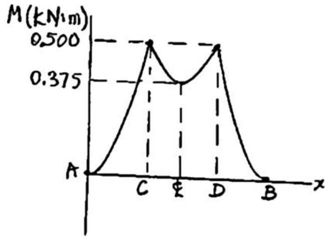
$$+\circlearrowleft \Sigma M_J = 0: \quad (4\mu)\frac{\mu}{2} - M = 0, \quad M = 2\mu^2$$

For $\mu = 0$: $V_B = M_B = 0$ ◁

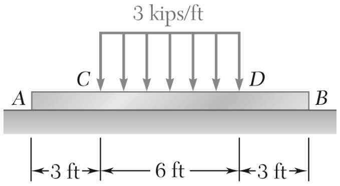
For $\mu = 0.5$ m: $V_D = -2$ kN, $M_D = 0.500\text{ kN}\cdot\text{m}$ ◁



(*b*) $|V|_{\max} = 2.00$ kN ◀



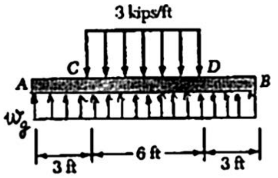
$|M|_{\max} = 0.500\text{ kN}\cdot\text{m}$ ◀



PROBLEM 7.47

Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

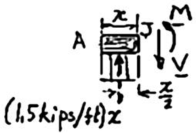


Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(12\text{ ft}) - (3\text{ kips/ft})(6\text{ ft}) = 0$$

$$w_g = 1.5\text{ kips/ft} \quad ◁$$

(*a*) Shear and bending-moment diagrams from *A* to *C*:



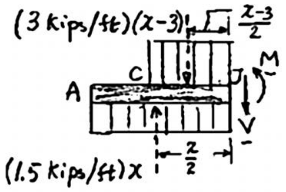
$$+\uparrow \Sigma F_y = 0: \quad 1.5x - V = 0 \qquad V = (1.5x)\text{kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (1.5x)\frac{x}{2} \quad M = (0.75x^2)\text{kip}\cdot\text{ft}$$

For $x = 0$: $\qquad V_A = M_A = 0$ ◁

For $x = 3$ ft: $V_C = 4.5$ kips, $\qquad M_C = 6.75$ kip · ft ◁

From *C* to *D*:



$$+\uparrow \Sigma F_y = 0: \quad 1.5x - 3(x-3) - V = 0$$

$$V = (9 - 1.5x)\text{kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + 3(x-3)\frac{x-3}{2} - (1.5x)\frac{x}{2} = 0$$

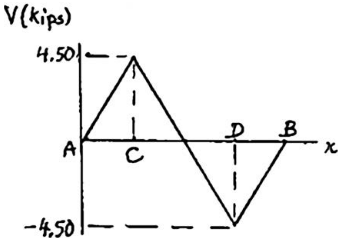
$$M = [0.75x^2 - 1.5(x-3)^2]\text{kip}\cdot\text{ft}$$

For $x = 3$ ft: $V_C = 4.5$ kips, $\qquad M_C = 6.75$ kip · ft ◁

For $x = 6$ ft: $V_{℄} = 0$, $\qquad M_{℄} = 13.50$ kip · ft ◁

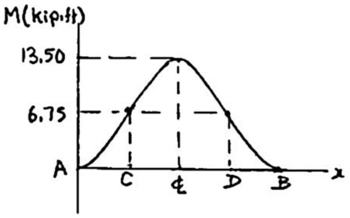
For $x = 9$ ft: $V_D = -4.5$ kips, $\qquad M_D = 6.75$ kip · ft ◁

At *B*: $\qquad V_B = M_B = 0$ ◁



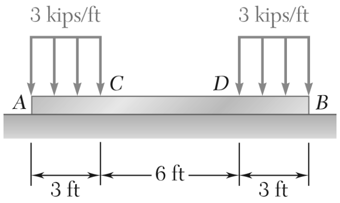
(*b*) $\qquad |V|_{\max} = 4.50$ kips ◀

$|M|_{\max} = 13.50$ kip · ft ◀



Bending-moment diagram consists or three distinct arcs of parabola, all located above the *x* axis.

Thus: $M \geq 0$ everywhere ◀

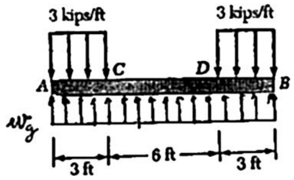


PROBLEM 7.48

Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

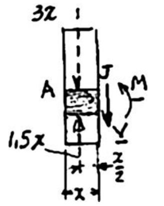
<u>Free body: Entire beam</u>



$$+\!\uparrow\Sigma F_y = 0: \quad w_g\,(12\text{ ft}) - (3\text{ kips/ft})(6\text{ ft}) = 0 \qquad\qquad w_g = 1.5\text{ kips/ft} \ \triangleleft$$

(*a*) <u>Shear and bending-moment diagrams</u>.

<u>From *A* to *C*:</u>



$$+\!\uparrow\Sigma F_y = 0: \quad 1.5x - 3x - V = 0$$

$$V = (-1.5x)\text{ kips}$$

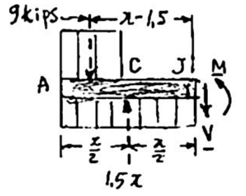
$$+\circlearrowleft\Sigma M_J = 0: \quad M + (3x)\frac{x}{2} - (1.5x)\frac{x}{2} = 0$$

$$M = (-0.75x^2)\text{ kip}\cdot\text{ft}$$

For $x = 0$: $\qquad V_A = M_A = 0 \ \triangleleft$

For $x = 3\text{ ft}$: $\qquad V_C = -4.5\text{ kips} \quad M_C = -6.75\text{ kip}\cdot\text{ft} \ \triangleleft$

<u>From *C* to *D*:</u>



$$+\!\uparrow\Sigma F_y = 0: \quad 1.5x = 9 - V = 0, \quad V = (1.5x - 9)\text{ kips}$$

$$+\circlearrowleft\Sigma M_J = 0: \quad M + 9(x - 1.5) - (1.5x)\frac{x}{2} = 0$$

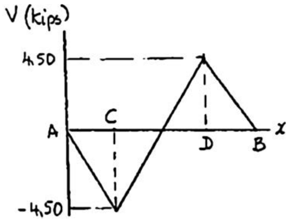
$$M = 0.75x^2 - 9x + 13.5$$

For $x = 3\text{ ft}$: $\qquad V_C = -4.5\text{ kips}, \qquad M_C = -6.75\text{ kip}\cdot\text{ft} \ \triangleleft$

For $x = 6\text{ ft}$: $\qquad V_C = 0, \qquad M_C = -13.50\text{ kip}\cdot\text{ft} \ \triangleleft$

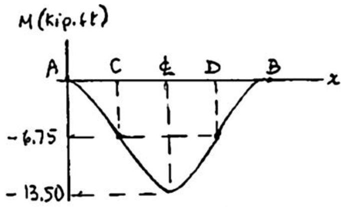
For $x = 9\text{ ft}$: $V_D = 4.5\text{ kips}, \quad M_D = -6.75\text{ kip}\cdot\text{ft}$ ◁

At B: $V_B = M_B = 0$ ◁



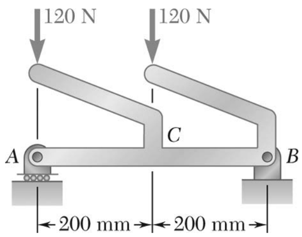
(b) $|V|_{\max} = 4.50\text{ kips}$ ◀

Bending-moment diagram consists of three distinct arcs of parabola.



$|M|_{\max} = 13.50\text{ kip}\cdot\text{ft}$ ◀

Since entire diagram is below the x axis: $M \leq 0$ everywhere ◀

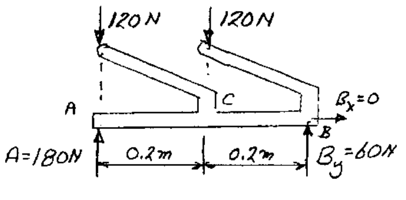


PROBLEM 7.49

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

Reactions:



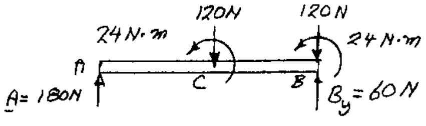
$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(0.4) - (120)(0.2) = 0$$

$$\mathbf{B}_y = 60\text{ N}\uparrow$$

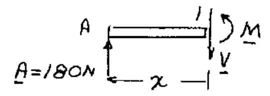
$$\Sigma F_x = 0: \qquad \mathbf{B}_x = 0$$

$$\Sigma F_y = 0: \qquad \mathbf{A} = 180\text{ N}\uparrow$$

Equivalent loading on straight part of beam *AB*.



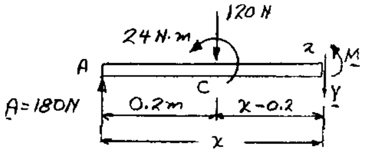
From *A* to *C*:



$$+\uparrow \Sigma F_y = 0: \quad V = +180\text{ N}$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad M = +180x$$

From *C* to *B*:

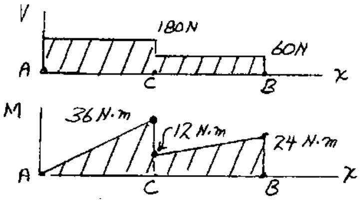


$$+\uparrow \Sigma F_y = 0: \quad 180 - 120 - V = 0$$

$$V = 60\text{ N}$$

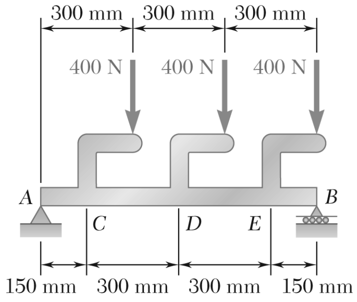
$$+\circlearrowleft \Sigma M_x = 0: \quad -(180\text{ N})(x) + 24\text{ N}\cdot\text{m} + (120\text{ N})(x - 0.2) + M = 0$$

$$M = +60x$$



$$|V|_{\max} = 180.0\text{ N} \blacktriangleleft$$

$$|M|_{\max} = 36.0\text{ N}\cdot\text{m} \blacktriangleleft$$

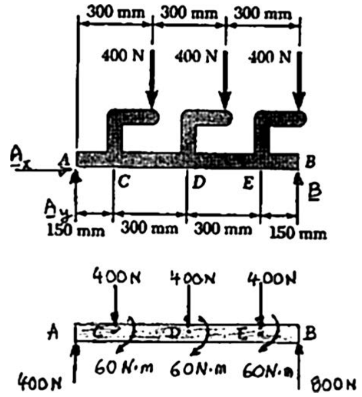


PROBLEM 7.50

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad B(0.9 \text{ m}) - (400 \text{ N})(0.3 \text{ m}) - (400 \text{ N})(0.6 \text{ m}) - (400 \text{ N})(0.9 \text{ m}) = 0$$

$$B = +800 \text{ N} \qquad \mathbf{B} = 800 \text{ N} \uparrow \triangleleft$$

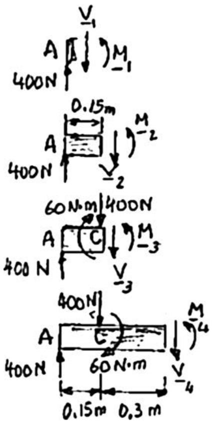
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 800 \text{ N} - 3(400 \text{ N}) = 0$$

$$A_y = +400 \text{ N} \qquad \mathbf{A} = 400 \text{ N} \uparrow \triangleleft$$

We replace the loads by equivalent force-couple systems at *C*, *D*, and *E*.

We consider successively the following *F-B* diagrams.



$V_1 = +400 \text{ N}$
$M_1 = 0$

$V_2 = +400 \text{ N}$
$M_2 = +60 \text{ N}\cdot\text{m}$

$V_3 = 0$
$M_3 = +120 \text{ N}\cdot\text{m}$

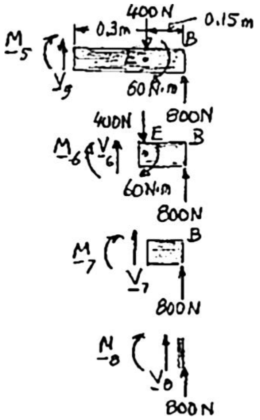
$V_4 = 0$
$M_4 = +120 \text{ N}\cdot\text{m}$

$V_5 = -400 \text{ N}$
$M_5 = +180 \text{ N}\cdot\text{m}$

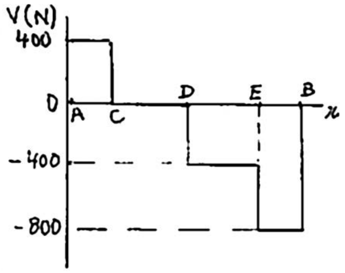
$V_6 = -400 \text{ N}$
$M_6 = +60 \text{ N}\cdot\text{m}$

$V_7 = -800 \text{ N}$
$M_7 = +120 \text{ N}\cdot\text{m}$

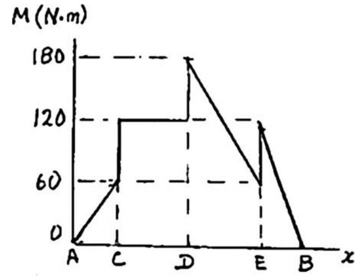
$V_8 = -800 \text{ N}$
$M_8 = 0$



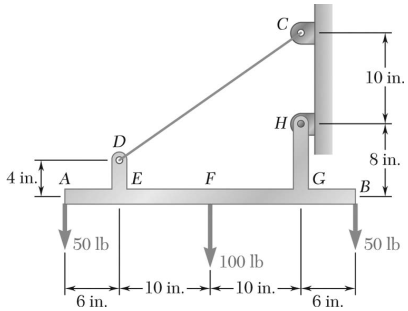
PROBLEM 7.50 (Continued)



(*b*) $|V|_{\max} = 800 \text{ N}$ ◀



$|M|_{\max} = 180.0 \text{ N} \cdot \text{m}$ ◀

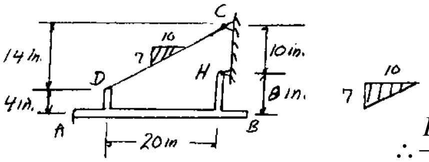


PROBLEM 7.51

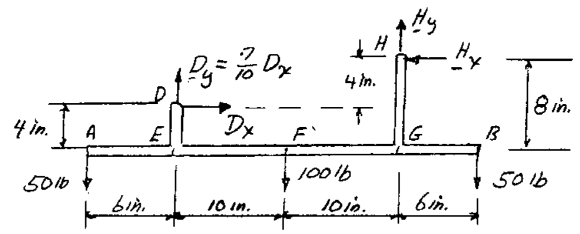
Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

Slope of cable *CD* is



$$\therefore \frac{D_y}{7} = \frac{D_x}{10}$$



$$+\circlearrowleft \Sigma M_H = 0: \quad (50 \text{ lb})(26 \text{ in.}) + D_x(4 \text{ in.}) - \left(\frac{7}{10}D_x\right)(20 \text{ in.}) + (100 \text{ lb})(10 \text{ in.}) - (50 \text{ lb})(6 \text{ in.}) = 0$$

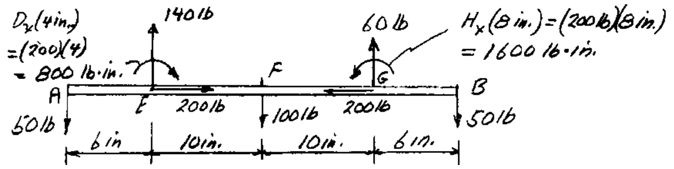
$$2000 + (4 - 14)D_x = 0 \qquad \mathbf{D}_x = 200 \text{ lb} \longrightarrow$$

$$D_y = \frac{7}{10}D_x = \frac{7}{10}(200) \qquad \mathbf{D}_y = 140 \text{ lb} \uparrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 200 \text{ lb} - H_x = 0 \qquad H_x = 200 \text{ lb} \longleftarrow$$

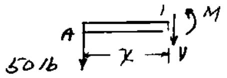
$$+\uparrow \Sigma F_y = 0: \quad 140 - 50 - 100 - 50 + H_y = 0 \qquad H_y = 60 \text{ lb} \uparrow$$

Equivalent loading on straight part of beam *AB*.



PROBLEM 7.51 (Continued)

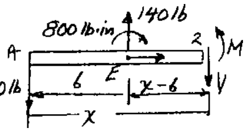
From A to E:



$$\Sigma F_y = 0: \quad V = -50\text{ lb}$$

$$\Sigma M_1 = 0: \quad M = -50x$$

From E to F:



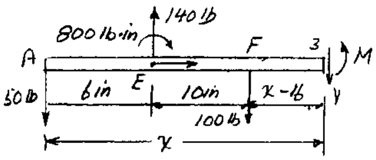
$$+\uparrow \Sigma F_y = 0: \quad -50+140-V=0$$

$$V = +90\text{ lb}$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad (50\text{ lb})x-(140\text{ lb})(x-6)-800\text{ lb}\cdot\text{in.}+M=0$$

$$M = -40+90x$$

From F to G:



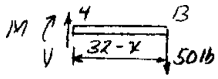
$$+\uparrow \Sigma F_y = 0: \quad -50+140-100-V=0$$

$$V = -10\text{ lb}$$

$$+\circlearrowleft \Sigma M_3 = 0: \quad 50x-140(x-6)+100(x-16)-800+M=0$$

$$M = 1560-10x$$

From G to B:

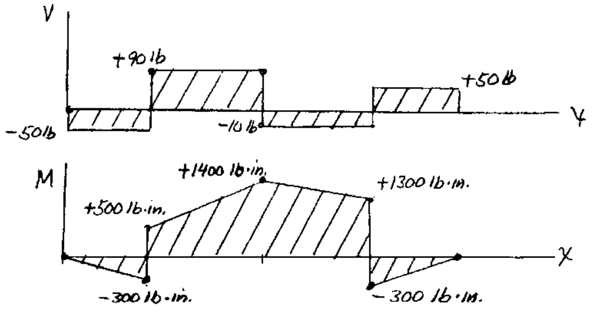


$$+\uparrow \Sigma F_y = 0: \quad V-50=0$$

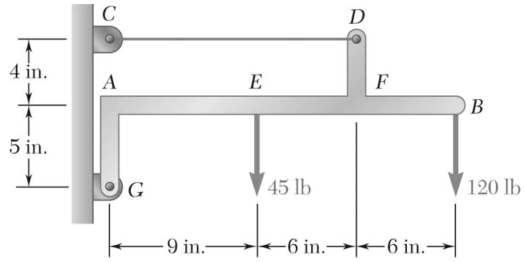
$$V = 50\text{ lb}$$

$$+\circlearrowleft \Sigma M_4 = 0: \quad -M-(50)(32-x)=0$$

$$M = -1600+50x$$



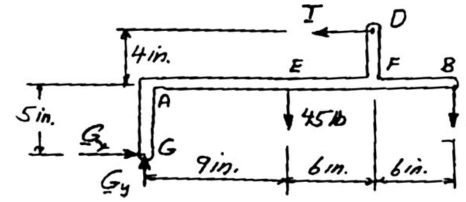
$$|V|_{\max} = 90.0\text{ lb}; \quad |M|_{\max} = 1400\text{ lb}\cdot\text{in.} \blacktriangleleft$$



PROBLEM 7.52

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

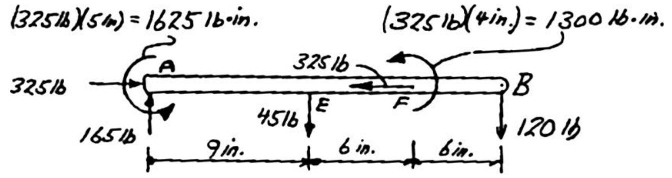


$$+\circlearrowleft \Sigma F_G = 0: \quad T(9\text{ in.}) - (45\text{ lb})(9\text{ in.}) - (120\text{ lb})(21\text{ in.}) = 0 \qquad T = 325\text{ lb}$$

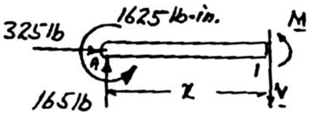
$$\xrightarrow{+} \Sigma F_x = 0: \quad -325\text{ lb} + G_x = 0 \qquad \mathbf{G}_x = 325\text{ lb} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad G_y - 45\text{ lb} - 120\text{ lb} = 0 \qquad \mathbf{G}_y = 165\text{ lb}\uparrow$$

Equivalent loading on straight part of beam *AB*



From *A* to *E*:

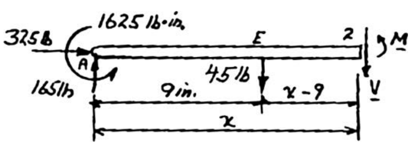


$$\Sigma F_y = 0: \quad V = +165\text{ lb}$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad +1625\text{ lb}\cdot\text{in.} - (165\text{ lb})x + M = 0$$

$$M = -1625 + 165x$$

From *E* to *F*:



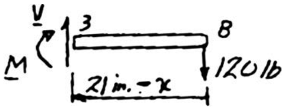
$$+\uparrow \Sigma F_y = 0: \quad 165 - 45 - V = 0$$

$$V = +120\text{ lb}$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad +1625\text{ lb}\cdot\text{in.} - (165\text{ lb})x + (45\text{ lb})(x-9) + M = 0$$

$$M = -1220 + 120x$$

PROBLEM 7.52 (Continued)

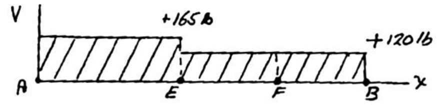


From F to B:

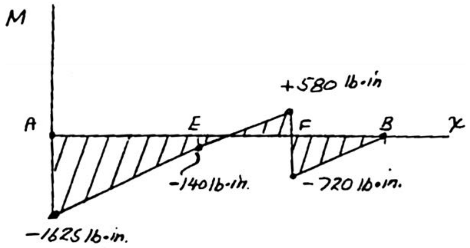
$$\Sigma F_y = 0 \quad V = +120\text{ lb}$$

$$+\circlearrowleft \Sigma M_3 = 0: \quad -(120)(21-x) - M = 0$$

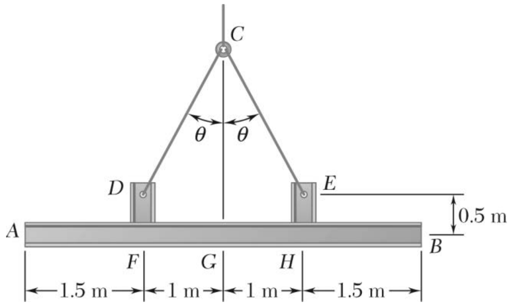
$$M = -2520 + 120x$$



$$|V|_{\max} = 165.0\text{ lb} \blacktriangleleft$$



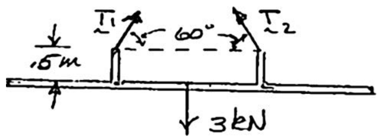
$$|M|_{\max} = 1625\text{ lb}\cdot\text{in.} \blacktriangleleft$$



PROBLEM 7.53

Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION



FBD Beam + channels:

(*a*) By symmetry: $T_1 = T_2 = T$

$$\uparrow \Sigma F_y = 0: \quad 2T \sin 60° - 3 \text{ kN} = 0$$

$$T = \frac{3}{\sqrt{3}} \text{ kN}$$

$$T_{1x} = \frac{3}{2\sqrt{3}}$$

$$T_{1y} = \frac{3}{2} \text{ kN}$$

FBD Beam:

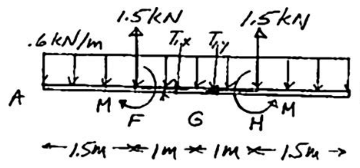
$$M = (0.5 \text{ m})\frac{3}{2\sqrt{3}} \text{ kN}$$

$$= 0.433 \text{ kN} \cdot \text{m}$$

With cable force replaced by equivalent force-couple system at *F* and *G*

Shear Diagram: *V* is piecewise linear

$$\left(\frac{dV}{dx} = -0.6 \text{ kN/m}\right) \text{with } 1.5 \text{ kN}$$



discontinuities at *F* and *H*.

$$V_{F^-} = -(0.6 \text{ kN/m})(1.5 \text{ m}) = 0.9 \text{ kN}$$

V increases by 1.5 kN to $+0.6$ kN at F^+

$$V_G = 0.6 \text{ kN} - (0.6 \text{ kN/m})(1 \text{ m}) = 0$$

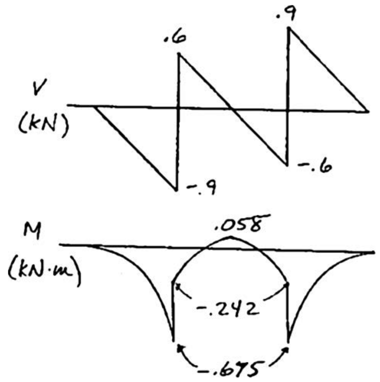
Finish by invoking symmetry

PROBLEM 7.53 (Continued)

Moment diagram: M is piecewise parabolic

$$\left(\frac{dM}{dx}\text{ decreasing with }V\right)$$

with discontinuities of .433 kN at F and H.



$$M_{F^-} = -\frac{1}{2}(0.9\text{ kN})(1.5\text{ m})$$
$$= -0.675\text{ kN}\cdot\text{m}$$

M increases by 0.433 kN m to –0.242 kN · m at F^+

$$M_G = -0.242\text{ kN}\cdot\text{m} + \frac{1}{2}(0.6\text{ kN})(1\text{ m})$$
$$= 0.058\text{ kN}\cdot\text{m}$$

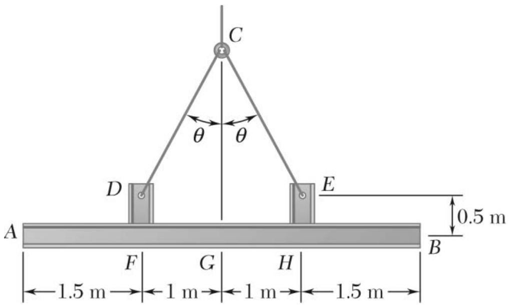
Finish by invoking symmetry

(*b*) $|V|_{\max} = 900\text{ N}$ ◀

at F^- and G^+

$|M|_{\max} = 675\text{ N}\cdot\text{m}$ ◀

at F and G

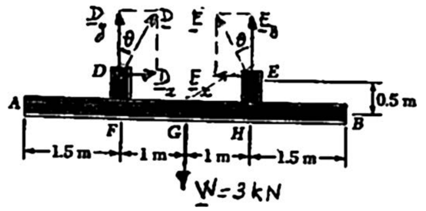


PROBLEM 7.54

Solve Problem 7.53 when $\theta = 60°$.

PROBLEM 7.53 Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION



Free body: Beam and channels

From symmetry:

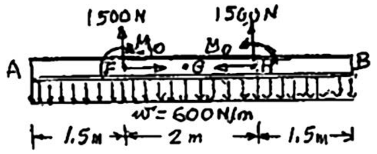
$$E_y = D_y$$

Thus: $$E_x = D_x = D_y \tan\theta \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad D_y + E_y - 3\text{ kN} = 0 \qquad \mathbf{D}_y = \mathbf{E}_y = 1.5\text{ kN}\uparrow \ \triangleleft$$

From (1): $$\mathbf{D}_x = (1.5\text{ kN})\tan\theta \longrightarrow \qquad \mathbf{E} = (1.5\text{ kN})\tan\theta \longleftarrow \ \triangleleft$$

We replace the forces at *D* and *E* by equivalent force-couple systems at *F* and *H*, where



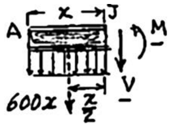
$$M_0 = (1.5\text{ kN}\tan\theta)(0.5\text{ m}) = (750\text{ N}\cdot\text{m})\tan\theta \qquad (2)$$

We note that the weight of the beam per unit length is

$$w = \frac{W}{L} = \frac{3\text{ kN}}{5\text{ m}} = 0.6\text{ kN/m} = 600\text{ N/m}$$

(*a*) Shear and bending moment diagrams

From *A* to *F*:



$$+\!\uparrow \Sigma F_y = 0: \quad -V - 600x = 0 \quad V = (-600x)\text{ N}$$

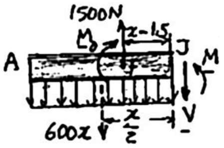
$$+\!\circlearrowleft \Sigma M_J = 0: \quad M + (600x)\frac{x}{2} = 0, \quad M = (-300x^2)\text{ N}\cdot\text{m}$$

For $x = 0$: $$V_A = M_A = 0 \ \triangleleft$$

For $x = 1.5$ m: $$V_F = -900\text{ N}, \quad M_F = -675\text{ N}\cdot\text{m} \ \triangleleft$$

PROBLEM 7.54 (Continued)

From F to H:



$$+\uparrow \Sigma F_y = 0: \quad 1500 - 600x - V = 0$$

$$V = (1500 - 600x)\ \text{N}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + (600x)\frac{x}{2} - 1500(x - 1.5) - M_0 = 0$$

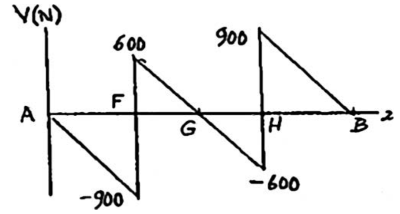
$$M = M_0 - 300x^2 + 1500(x - 1.5)\ \text{N}\cdot\text{m}$$

For $x = 1.5$ m: $V_F = +600$ N, $M_F = (M_0 - 675)$ N·m ◁

For $x = 2.5$ m: $V_G = 0$, $M_G = (M_0 - 375)$ N·m ◁

From G to B, The V and M diagrams will be obtained by symmetry,

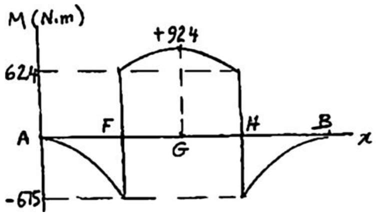
(b) $|V|_{\max} = 900$ N ◀



Making $\theta = 60°$ in Eq. (2): $M_0 = 750 \tan 60° = 1299$ N·m

Thus, just to the right of F: $M = 1299 - 675 = 624$ N·m ◁

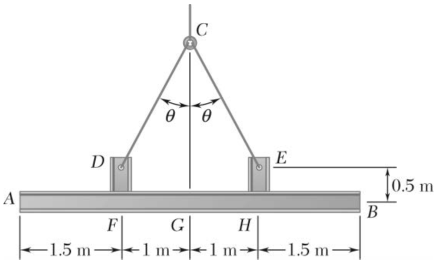
and $M_G = 1299 - 375 = 924$ N·m ◁



(b) $|V|_{\max} = 900$ N ◀

$|M|_{\max} = 924$ N·m ◀

PROBLEM 7.55

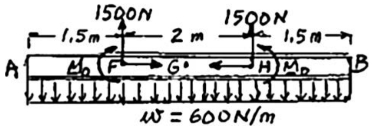


For the structural member of Problem 7.53, determine (*a*) the angle θ for which the maximum absolute value of the bending moment in beam *AB* is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (*Hint:* Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)

PROBLEM 7.53 Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

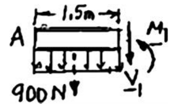
See solution of Problem 7.50 for reduction of loading or beam *AB* to the following:



where $M_0 = (750\ \text{N}\cdot\text{m})\tan\theta$ ◁

[Equation (2)]

The largest negative bending moment occurs <u>Just to the left of *F*</u>:

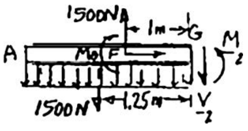


$$+\circlearrowleft \Sigma M_1 = 0: \quad M_1 + (900\ \text{N})\left(\frac{1.5\ \text{m}}{2}\right) = 0$$

$M_1 = -675\ \text{N}\cdot\text{m}$ ◁

The largest positive bending moment occurs

<u>At *G*</u>:



$$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - M_0 + (1500\ \text{N})(1.25\ \text{m} - 1\ \text{m}) = 0$$

$M_2 = M_0 - 375\ \text{N}\cdot\text{m}$ ◁

Equating M_2 and $-M_1$:

$$M_0 - 375 = +675$$

$$M_0 = 1050\ \text{N}\cdot\text{m}$$

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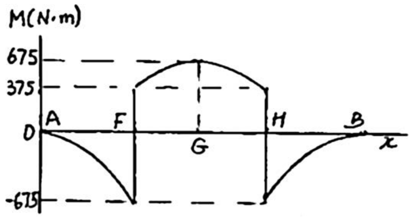
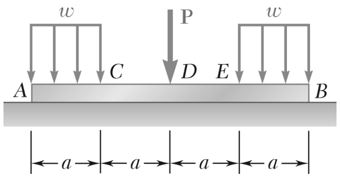
PROBLEM 7.55 (Continued)

(*a*) From Equation (2): $\tan\theta = \dfrac{1050}{750} = 1.400$ $\theta = 54.5° \blacktriangleleft$



(*b*) $|M|_{\max} = 675\ \text{N}\cdot\text{m} \blacktriangleleft$

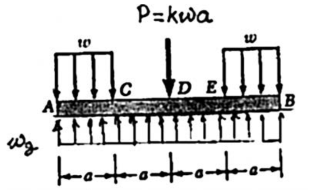


PROBLEM 7.56

For the beam of Problem 7.43, determine (*a*) the ratio $k = P/wa$ for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Problem 7.55.)

PROBLEM 7.43 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $P = wa$, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Entire beam

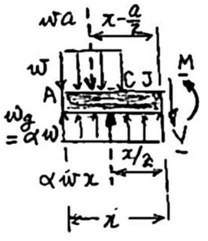
$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - kwa = 0$$

$$w_g = \frac{w}{4}(2+k)$$

Setting $$\frac{w_g}{w} = \alpha \tag{1}$$

We have $$k = 4\alpha - 2 \tag{2}$$

Minimum value of B.M. For *M* to have negative values, we must have $w_g < w$. We verify that *M* will then be negative and keep decreasing in the portion *AC* of the beam. Therefore, $M_{\min}$ will occur between *C* and *D*.



From *C* to *D*:

$$+\circlearrowleft \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \alpha w x\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{2}w(\alpha x^2 - 2ax + a^2) \tag{3}$$

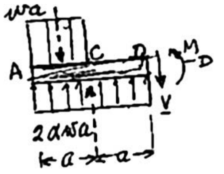
We differentiate and set $\frac{dM}{dx} = 0$: $$\alpha x - a = 0 \qquad x_{\min} = \frac{a}{\alpha} \tag{4}$$

Substituting in (3):

$$M_{\min} = \frac{1}{2}wa^2\left(\frac{1}{\alpha} - \frac{2}{\alpha} + 1\right)$$

$$M_{\min} = -wa^2\frac{1-\alpha}{2\alpha} \tag{5}$$

Maximum value of bending moment occurs at D



$$+\circlearrowleft \Sigma M_D = 0: \quad M_D + wa\left(\frac{3a}{2}\right) - (2\alpha wa)a = 0$$

$$M_{\max} = M_D = wa^2\left(2\alpha - \frac{3}{2}\right) \qquad (6)$$

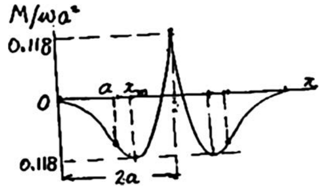
Equating $-M_{\min}$ and $M_{\max}$:

$$wa^2\frac{1-\alpha}{2\alpha} = wa^2\left(2\alpha - \frac{3}{2}\right)$$

$$4\alpha^2 - 2\alpha - 1 = 0$$

$$\alpha = \frac{2+\sqrt{20}}{8} \qquad \alpha = \frac{1+\sqrt{5}}{4} = 0.809$$

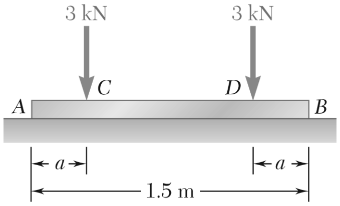
(*a*) Substitute in (2): $k = 4(0.809) - 2$ $\qquad k = 1.236$ ◀



(*b*) Substitute for α in (5):

$$|M|_{\max} = -M_{\min} = -wa^2\frac{1-0.809}{2(0.809)} \qquad |M|_{\max} = 0.1180wa^2 ◀$$

Substitute for α in (4): $\qquad x_{\min} = \dfrac{a}{0.809}1.236a$ ◁



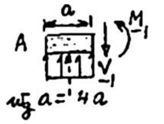
PROBLEM 7.57

For the beam of Problem 7.45, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Problem 7.55.)

PROBLEM 7.45 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $a = 0.3$ m, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Force per unit length exerted by ground:

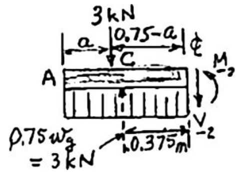


$$w_g = \frac{6 \text{ kN}}{1.5 \text{ m}} = 4 \text{ kN/m}$$

The largest positive bending moment occurs Just to the left of *C*:

$$+\circlearrowleft \Sigma M_1 = 0: \quad M_1 = (4a)\frac{a}{2} \qquad M_1 = 2a^2 \lhd$$

The largest negative bending moment occurs



At the center line:

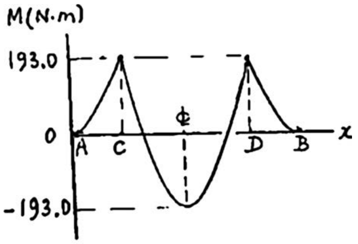
$$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 + 3(0.75 - a) - 3(0.375) = 0 \qquad M_2 = -(1.125 - 3a) \lhd$$

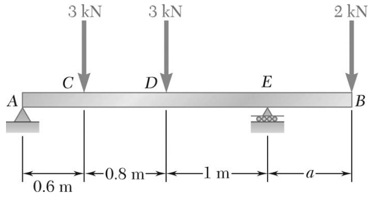
Equating M_1 and $-M_2$: $$2a^2 = 1.125 - 3a$$

$$a^2 + 1.5a - 0.5625 = 0$$

(*a*) Solving the quadratic equation: $a = 0.31066$, $\qquad a = 0.311 \text{ m} \blacktriangleleft$

(*b*) Substituting: $|M|_{\max} = M_1 = 2(0.31066)^2 \qquad |M|_{\max} = 193.0 \text{ N} \cdot \text{m} \blacktriangleleft$



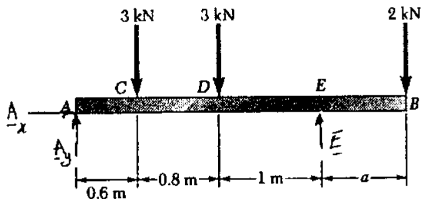


PROBLEM 7.58

For the beam and loading shown, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Problem 7.55.)

SOLUTION

Free body: Entire beam

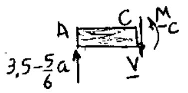


$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_E = 0: \quad -A_y(2.4) + (3)(1.8) + 3(1) - (2)a = 0$$

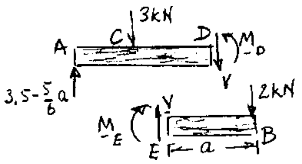
$$A_y = 3.5\text{ kN} - \frac{5}{6}a \qquad \mathbf{A} = 3.5\text{ kN} - \frac{5}{6}a \uparrow$$ ◁

Free body: *AC*



$$+\circlearrowleft \Sigma M_C = 0: \quad M_C - \left(3.5 - \frac{5}{6}a\right)(0.6\text{ m}) = 0, \qquad M_C = +2.1 - \frac{a}{2}$$ ◁

Free body: *AD*



$$+\circlearrowleft \Sigma M_D = 0: \quad M_D - \left(3.5 - \frac{5}{6}a\right)(1.4\text{ m}) + (3\text{ kN})(0.8\text{ m}) = 0$$

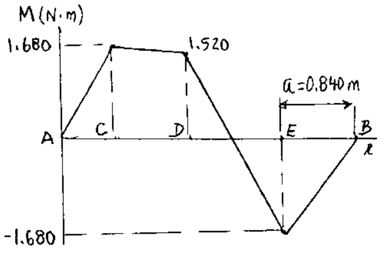
$$M_D = +2.5 - \frac{7}{6}a$$ ◁

Free body: *EB* $$+\circlearrowleft \Sigma M_E = 0: \quad -M_E - (2\text{ kN})a = 0 \qquad M_E = -2a$$ ◁

We shall assume that $M_C > M_D$ and, thus, that $M_{\max} = M_C$.

We set $M_{\max} = |M_{\min}|$ or $M_C = |M_E| = 2.1 - \frac{a}{2} = 2a$ $\qquad a = 0.840\text{ m}$ ◁

$$|M|_{\max} = M_C = |M_E| = 2a = 2(0.840) \qquad |M|_{\max} = 1.680\text{ N}\cdot\text{m}$$ ◁



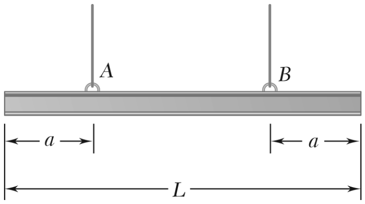
We must check our assumption.

$$M_D = 2.5 - \frac{7}{6}(0.840) = 1.520\text{ N}\cdot\text{m}$$

Thus, $M_C > M_D$, O.K.

The answers are (*a*) $a = 0.840\text{ m}$ ◀

(*b*) $|M|_{\max} = 1.680\text{ N}\cdot\text{m}$ ◀

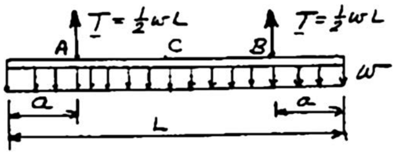


PROBLEM 7.59

A uniform beam is to be picked up by crane cables attached at *A* and *B*. Determine the distance *a* from the ends of the beam to the points where the cables should be attached if the maximum absolute value of the bending moment in the beam is to be as small as possible. (*Hint:* Draw the bending-moment diagram in terms of *a*, *L*, and the weight *w* per unit length, and then equate the absolute values of the largest positive and negative bending moments obtained.)

SOLUTION

w = weight per unit length

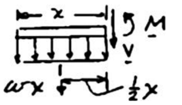


To the left of *A*:

$$\overset{+}{\circlearrowleft}\Sigma M_1 = 0: \quad M + wx\left(\frac{x}{2}\right) = 0$$

$$M = -\frac{1}{2}wx^2$$

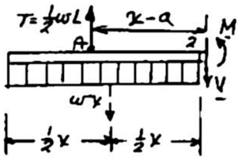
$$M_A = -\frac{1}{2}wa^2$$



Between *A* and *B*:

$$\overset{+}{\circlearrowleft}\Sigma M_2 = 0: \quad M - \left(\frac{1}{2}wL\right)(x-a) + (wx)\left(\frac{1}{2}x\right) = 0$$

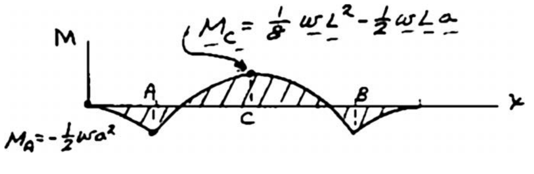
$$M = -\frac{1}{2}wx^2 + \frac{1}{2}wLx - \frac{1}{2}wLa$$



At center *C*:

$$x = \frac{L}{2}$$

$$M_C = -\frac{1}{2}w\left(\frac{L}{2}\right)^2 + \frac{1}{2}wL\left(\frac{L}{2}\right) - \frac{1}{2}wLa$$



PROBLEM 7.59 (Continued)

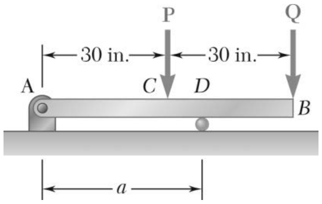
We set $|M_A| = |M_C|$: $\left|-\frac{1}{2}wa^2\right| = \left|\frac{1}{8}wL^2 - \frac{1}{2}wLa\right| + \frac{1}{2}wa^2 = \frac{1}{8}wL^2 - \frac{1}{2}wLa$

$$a^2 + La - 0.25L^2 = 0$$

$$a = \frac{1}{2}(L \pm \sqrt{L^2 + L^2}) = \frac{1}{2}(\sqrt{2} - 1)L$$

$$M_{\max} = \frac{1}{2}w(0.207L)^2 = 0.0214wL^2$$

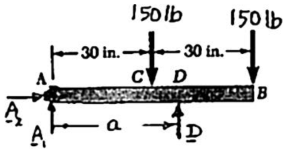
$a = 0.207L$ ◀



PROBLEM 7.60

Knowing that $P = Q = 150$ lb, determine (*a*) the distance a for which the maximum absolute value of the bending moment in beam AB is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Problem 7.55.)

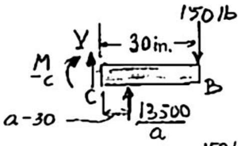
SOLUTION



Free body: Entire beam

$$+\circlearrowleft \Sigma M_A = 0: \quad Da - (150)(30) - (150)(60) = 0$$

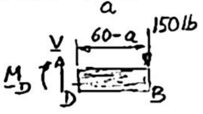
$$D = \frac{13{,}500}{a} \qquad \triangleleft$$



Free body: CB

$$+\circlearrowleft \Sigma M_C = 0: \quad -M_C - (150)(30) + \frac{13{,}500}{a}(a-30) = 0$$

$$M_C = 9000\left(1 - \frac{45}{a}\right) \qquad \triangleleft$$



Free body: DB

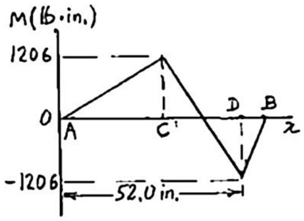
$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - (150)(60-a) = 0$$

$$M_D = -150(60-a) \qquad \triangleleft$$

(*a*) We set

$$M_{\max} = |M_{\min}| \quad \text{or} \quad M_C = -M_D: \quad 9000\left(1 - \frac{45}{a}\right) = 150(60-a)$$

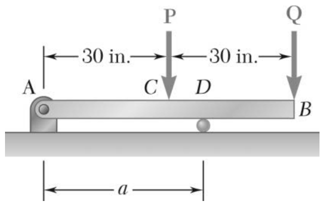
$$60 - \frac{2700}{a} = 60 - a$$



$$a^2 = 2700 \quad a = 51.96 \text{ in.} \qquad a = 52.0 \text{ in.} \blacktriangleleft$$

(*b*) $|M|_{\max} = -M_D = 150(60 - 51.96)$

$$|M|_{\max} = 1206 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

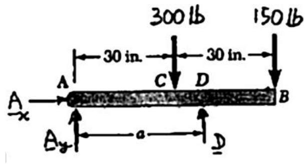


PROBLEM 7.61

Solve Problem 7.60 assuming that $P = 300$ lb and $Q = 150$ lb.

PROBLEM 7.60 Knowing that $P = Q = 150$ lb, determine (*a*) the distance a for which the maximum absolute value of the bending moment in beam AB is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Problem 7.55.)

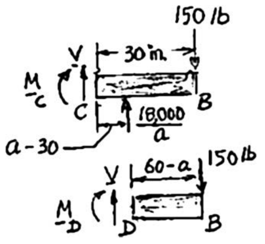
SOLUTION



Free body: Entire beam

$$+\circlearrowleft \Sigma M_A = 0: \quad Da - (300)(30) - (150)(60) = 0$$

$$D = \frac{18{,}000}{a} \quad \triangleleft$$



Free body: *CB*

$$+\circlearrowleft \Sigma M_C = 0: \quad -M_C - (150)(30) + \frac{18{,}000}{a}(a - 30) = 0$$

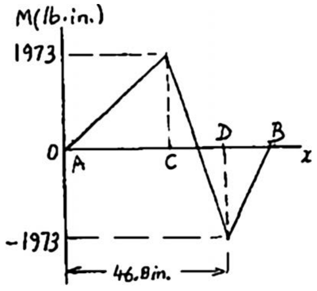
$$M_C = 13{,}500\left(1 - \frac{40}{a}\right) \quad \triangleleft$$

Free body: *DB*

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - (150)(60 - a) = 0$$

$$M_D = -150(60 - a) \quad \triangleleft$$

(*a*) We set



$$M_{\max} = |M_{\min}| \quad \text{or} \quad M_C = -M_D: \quad 13{,}500\left(1 - \frac{40}{a}\right) = 150(60 - a)$$

$$90 - \frac{3600}{a} = 60 - a$$

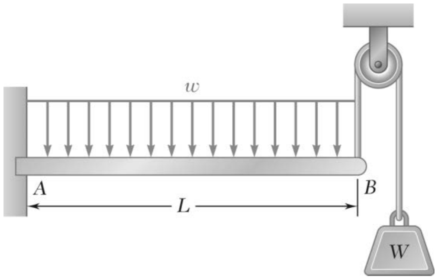
$$a^2 + 30a - 3600 = 0$$

$$a = \frac{-30 + \sqrt{15.300}}{2} = 46.847$$

$$a = 46.8 \text{ in.} \quad \blacktriangleleft$$

(*b*) $|M|_{\max} = -M_D = 150(60 - 46.847)$

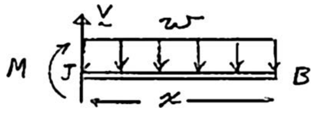
$$|M|_{\max} = 1973 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$



PROBLEM 7.62*

In order to reduce the bending moment in the cantilever beam *AB*, a cable and counterweight are permanently attached at end *B*. Determine the magnitude of the counterweight for which the maximum absolute value of the bending moment in the beam is as small as possible and the corresponding value of $|M|_{\max}$. Consider (*a*) the case when the distributed load is permanently applied to the beam, (*b*) the more general case when the distributed load may either be applied or removed.

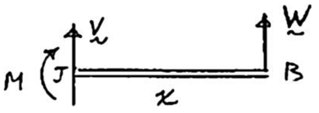
SOLUTION



M due to distributed load:

$$\circlearrowleft \Sigma M_J = 0: \quad -M - \frac{x}{2}wx = 0$$

$$M = -\frac{1}{2}wx^2$$

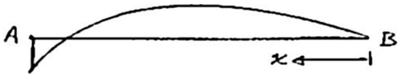


M due to counter weight:

$$\circlearrowleft \Sigma M_J = 0: \quad -M + xw = 0$$

$$M = wx$$

(*a*) **Both applied:**



$$M = W_x - \frac{w}{2}x^2$$

$$\frac{dM}{dx} = W - wx = 0 \text{ at } x = \frac{W}{w}$$

And here $M = \dfrac{W^2}{2w} > 0$ so $M_{\max}$; $M_{\min}$ must be at $x = L$

So $M_{\min} = WL - \frac{1}{2}wL^2$. For minimum $|M|_{\max}$ set $M_{\max} = -M_{\min}$,

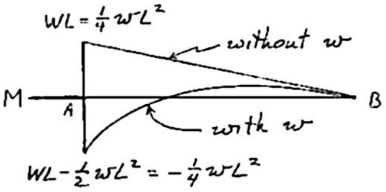
so $$\frac{W^2}{2w} = -WL + \frac{1}{2}wL^2 \quad \text{or} \quad W^2 + 2wLW - w^2L^2 = 0$$

$$W = -wL \pm \sqrt{2w^2L^2}\ (\text{need}+) \qquad W = (\sqrt{2} - 1)wL = 0.414\, wL \blacktriangleleft$$

PROBLEM 7.62* (Continued)

(*b*) **w may be removed**

$$M_{\max} = \frac{W^2}{2w} = \frac{(\sqrt{2}-1)^2}{2}wL^2 \qquad M_{\max} = 0.0858\,wL^2 \blacktriangleleft$$



Without w, $\quad M = Wx$

$$M_{\max} = WL \text{ at } A$$

With w (see Part a) $\quad M = Wx - \frac{w}{2}x^2$

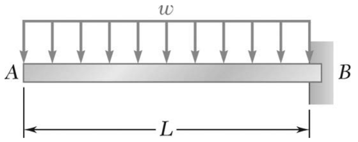
$$M_{\max} = \frac{W^2}{2w} \text{ at } x = \frac{W}{w}$$

$$M_{\min} = WL - \frac{1}{2}wL^2 \text{ at } x = L$$

For minimum $M_{\max}$, set $M_{\max}\,(\text{no } w) = -M_{\min}\,(\text{with } w)$

$$WL = -WL + \frac{1}{2}wL^2 \rightarrow W = \frac{1}{4}wL \rightarrow \qquad M_{\max} = \frac{1}{4}wL^2 \blacktriangleleft$$

With $\qquad W = \frac{1}{4}wL \blacktriangleleft$

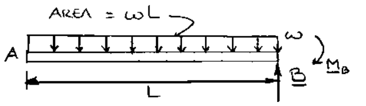


PROBLEM 7.63

Using the method of Section 7.6, solve Problem 7.29.

PROBLEM 7.29 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

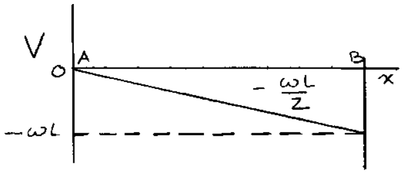
SOLUTION



Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad wL - B = 0$$

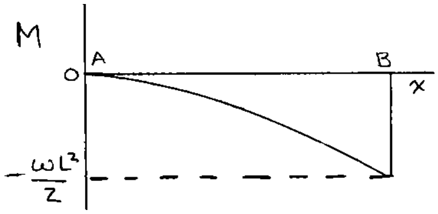
$$\mathbf{B} = wL \uparrow$$



Shear diagram

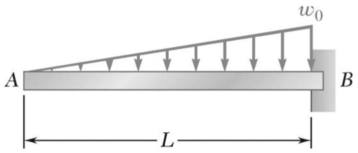
$$+\circlearrowleft \Sigma M_B = 0: \quad (wL)\left(\frac{L}{2}\right) - M_B = 0$$

$$\mathbf{M}_B = \frac{wL^2}{2} \circlearrowleft$$



Moment diagram

(*b*) $|V|_{\max} = wL$; $\qquad |M|_{\max} = \dfrac{wL^2}{2}$ ◀

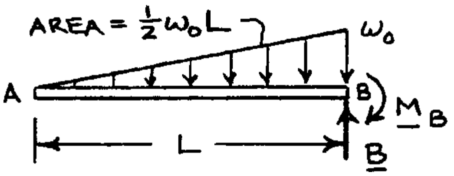


PROBLEM 7.64

Using the method of Section 7.6, solve Problem 7.30.

PROBLEM 7.30 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

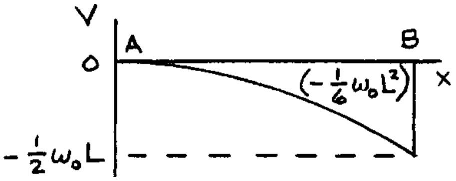
SOLUTION



Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad B - \frac{1}{2}(w_0)(L) = 0$$

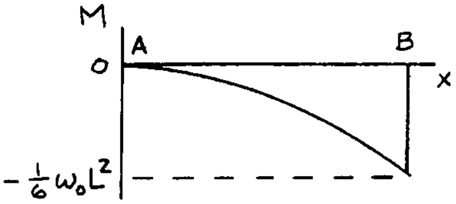
$$\mathbf{B} = \frac{1}{2}w_0L \uparrow$$



Shear diagram

$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{1}{2}(w_0)(L)\left(\frac{L}{3}\right) - M_B = 0$$

$$\mathbf{M}_B = \frac{1}{6}w_0L^2 \circlearrowright$$

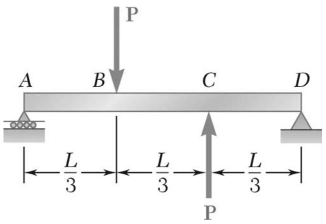


Moment diagram

(*b*)

$$|V|_{\max} = w_0L/2 \blacktriangleleft$$

$$|M|_{\max} = w_0L^2/6 \blacktriangleleft$$

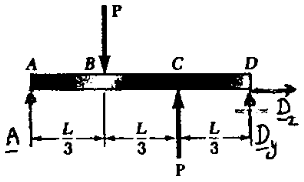


PROBLEM 7.65

Using the method of Section 7.6, solve Problem 7.31.

PROBLEM 7.31 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

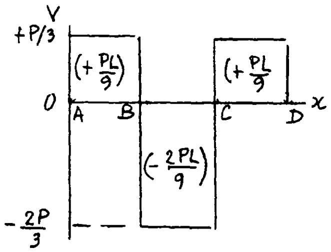
SOLUTION



Free body: Entire beam

$$+\circlearrowleft \Sigma M_D = 0: \quad P\left(\frac{2L}{3}\right) - P\left(\frac{L}{3}\right) - AL = 0$$

$$\mathbf{A} = P/3 \uparrow$$

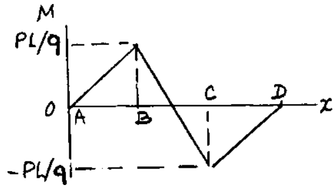


Shear diagram

We note that

$$V_A = A = +P/3$$

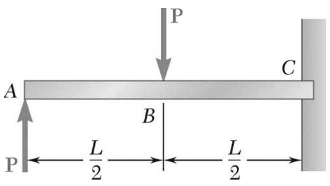
$$|V|_{\max} = 2P/3 \blacktriangleleft$$



Bending diagram

We note that $M_A = 0$

$$|M|_{\max} = PL/9 \blacktriangleleft$$

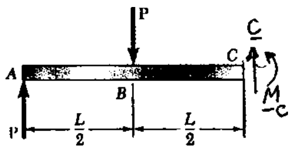


PROBLEM 7.66

Using the method of Section 7.6, solve Problem 7.32.

PROBLEM 7.32 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

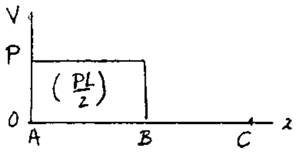
SOLUTION



Free body: Entire beam

$$\Sigma F_y = 0: \quad \mathbf{C} = 0$$

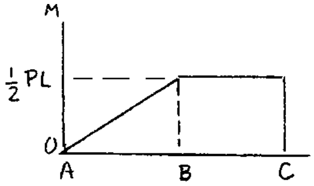
$$\Sigma M_C = 0: \quad \mathbf{M}_C = \frac{1}{2}PL \circlearrowleft$$



Shear diagram

At A: $\quad V_A = +P$

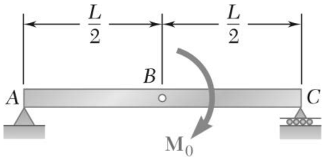
$$|V|_{\max} = P \blacktriangleleft$$



Moment diagram

At A: $\quad M_A = 0$

$$|M|_{\max} = \frac{1}{2}PL \blacktriangleleft$$

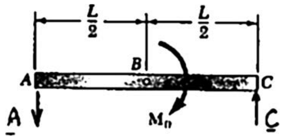


PROBLEM 7.67

Using the method of Section 7.6, solve Problem 7.33.

PROBLEM 7.33 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

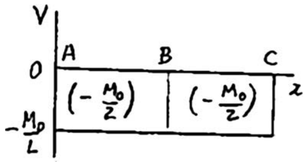


Free body: Entire beam

$$\Sigma F_y = 0: \quad A = C$$

$$+\circlearrowleft \Sigma M_C = 0: \quad Al - M_0 = 0$$

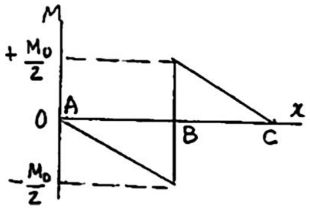
$$A = C = \frac{M_0}{L}$$



Shear diagram

At A: $$V_A = -\frac{M_0}{L}$$

$$|V|_{\max} = \frac{M_0}{L} \blacktriangleleft$$

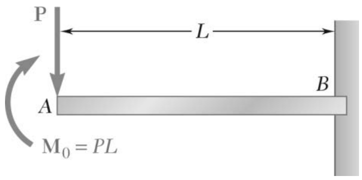


Bending-moment diagram

At A: $$M_A = 0$$

At B, M increases by M_0 on account of applied couple.

$$|M|_{\max} = M_0/2 \blacktriangleleft$$

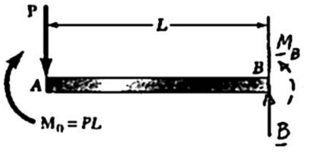


PROBLEM 7.68

Using the method of Section 7.6, solve Problem 7.34.

PROBLEM 7.34 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



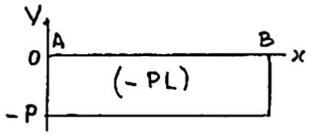
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad B - P = 0$$

$$\mathbf{B} = P \uparrow$$

$$+\circlearrowleft \Sigma M_B = 0: \quad M_B - M_0 + PL = 0$$

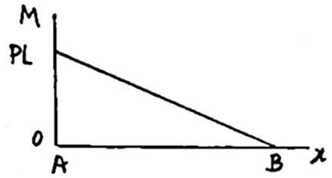
$$\mathbf{M}_B = 0$$



Shear diagram

At A: $V_A = -P$

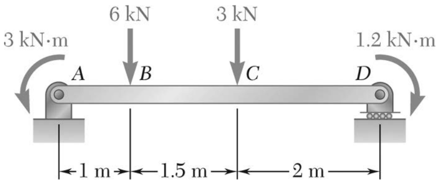
$$|V|_{\max} = P \blacktriangleleft$$



Bending-moment diagram

At A: $M_A = M_0 = PL$

$$|M|_{\max} = PL \blacktriangleleft$$



PROBLEM 7.69

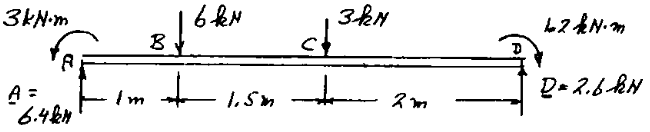
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

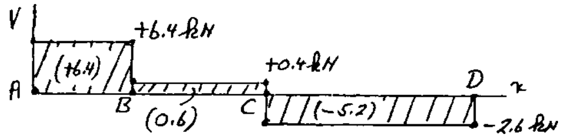
SOLUTION

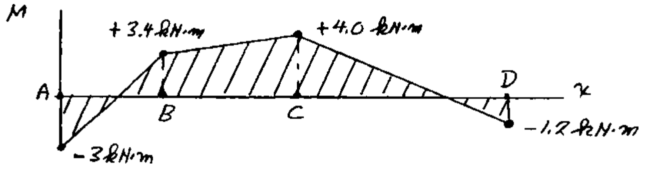
Reactions $\Sigma F_x = 0 \quad A_x = 0$

$$+\circlearrowleft \Sigma M_D = 0{:} \quad +3\text{ kN}\cdot\text{m} + (6\text{ kN})(3.5\text{ m}) + (3\text{ kN})(2\text{ m}) - 1.2\text{ kN}\cdot\text{m} - A_y(4.5\text{ m}) = 0$$

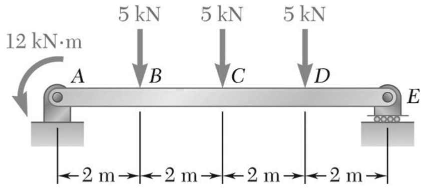
$$A_y = +6.4\text{ kN} \qquad A_y = +6.4\text{ kN}\uparrow$$







(*b*) $|V|_{\max} = 6.40\text{ kN};$ $|M|_{\max} = 4.00\text{ kN}\cdot\text{m}$ ◀



PROBLEM 7.70

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

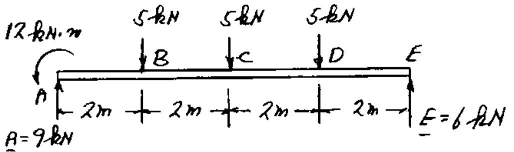
Reactions

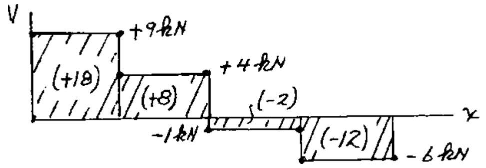
$$+\circlearrowleft \Sigma M_A = 0: \quad 12\text{ kN}\cdot\text{m} - 3(5\text{ kN})(4\text{ m}) + E(8\text{ m}) = 0$$

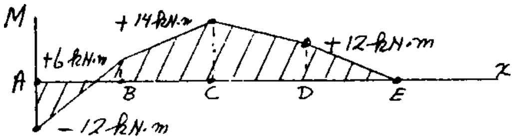
$$E = 6\text{ kN}\uparrow$$

$$\Sigma F_y = 0$$

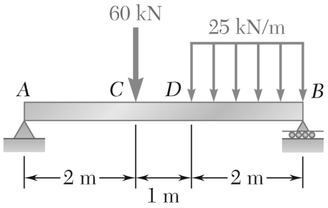
$$A = 9\text{ kN}\uparrow$$







(*b*) $|V|_{\max} = 9.00\text{ kN};$ $|M|_{\max} = 14.00\text{ kN}\cdot\text{m}$ ◀

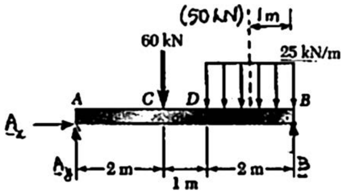


PROBLEM 7.71

Using the method of Section 7.6, solve Problem 7.39.

PROBLEM 7.39 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Beam

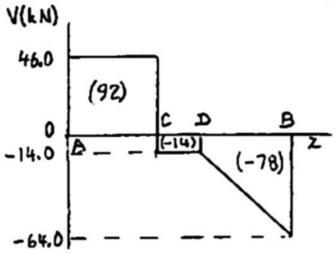
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (60\text{ kN})(3\text{ m}) + (50\text{ kN})(1\text{ m}) - A_y(5\text{ m}) = 0$$

$$A_y = +46.0\text{ kN} \ \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B + 46.0\text{ kN} - 60\text{ kN} - 50\text{ kN} = 0$$

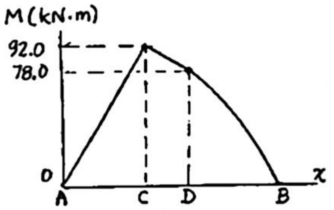
$$B = +64.0\text{ kN} \ \triangleleft$$



Shear diagram

At A: $\quad V_A = A_y = +46.0\text{ kN}$

$$|V|_{\max} = 64.0\text{ kN} \ \blacktriangleleft$$

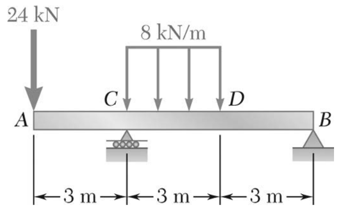


Bending-moment diagram

At A: $\quad M_A = 0$

$$|M|_{\max} = 92.0\text{ kN}\cdot\text{m} \ \blacktriangleleft$$

Parabola from D to B. Its slope at D is same as that of straight-line segment CD since V has no discontinuity at D.

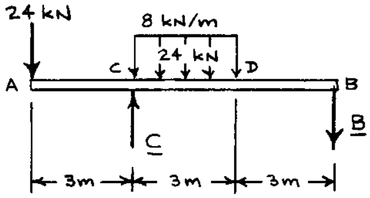


PROBLEM 7.72

Using the method of Section 7.6, solve Problem 7.40.

PROBLEM 7.40 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

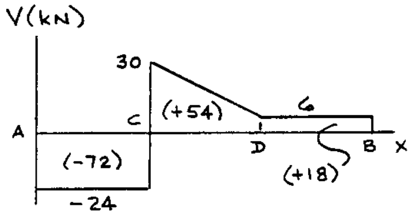
SOLUTION



Free body: Entire beam

$$+\circlearrowleft \Sigma M_B = 0: \quad (24\text{ kN})(9\text{ m}) - C(6\text{ m}) + (24\text{ kN})(4.5\text{ m}) = 0$$

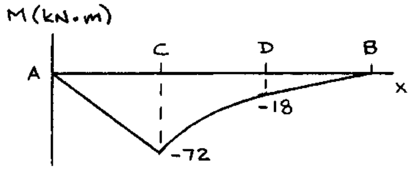
$$\mathbf{C} = 54\text{ kN}\uparrow$$



Shear diagram

$$+\uparrow \Sigma F_y = 0: \quad 54 - 24 - 24 - B = 0$$

$$\mathbf{B} = 6\text{ kN}\downarrow$$



(*b*) $|V|_{\max} = 30.0\text{ kN};$ $|M|_{\max} = 72.0\text{ kN}\cdot\text{m}$ ◀

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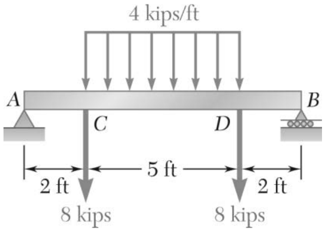
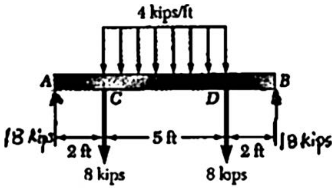


PROBLEM 7.73

Using the method of Section 7.6, solve Problem 7.41.

PROBLEM 7.41 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

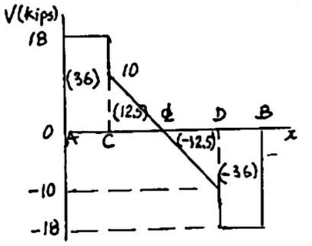


<u>Reactions at supports</u>.

Because of the symmetry:

$$A = B = \frac{1}{2}(8 + 8 + 4 \times 5)\,\text{kips}$$

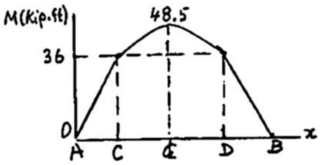
$$\mathbf{A} = \mathbf{B} = 18\ \text{kips}\uparrow \ \triangleleft$$



<u>Shear diagram</u>

At A: $V_A = +18$ kips

$$|V|_{\max} = 18.00\ \text{kips}\ \blacktriangleleft$$

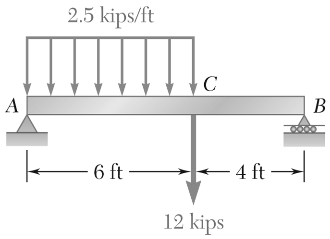


<u>Bending-moment diagram</u>

At A: $M_A = 0$

$$|M|_{\max} = 48.5\ \text{kip}\cdot\text{ft}\ \blacktriangleleft$$

Discontinuities in slope at C and D, due to the discontinuities of V.

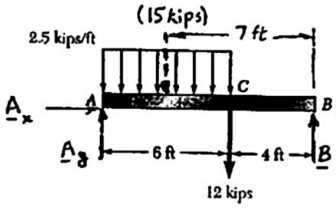


PROBLEM 7.74

Using the method of Section 7.6, solve Problem 7.42.

PROBLEM 7.42 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Beam

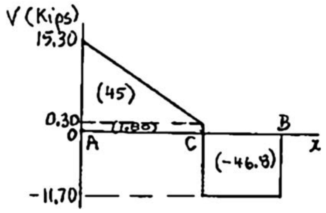
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (12\text{ kips})(4\text{ ft}) + (15\text{ kips})(7\text{ ft}) - A_y(10\text{ ft}) = 0$$

$$A_y = +15.3\text{ kips} \quad \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B + 15.3 - 15 - 12 = 0$$

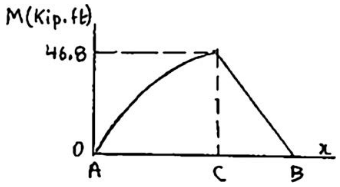
$$B = +11.7\text{ kips} \quad \triangleleft$$



Shear diagram

At A: $V_A = A_y = 15.3\text{ kips}$

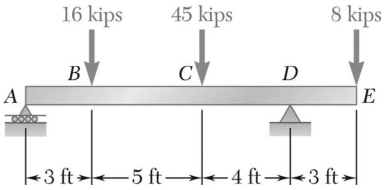
$$|V|_{\max} = 15.30\text{ kips} \quad \blacktriangleleft$$



Bending-moment diagram

At A: $M_A = 0$

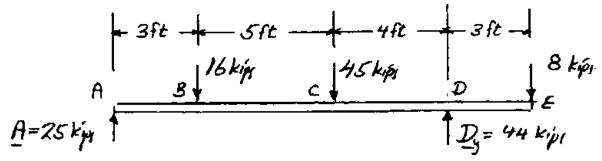
$$|M|_{\max} = 46.8\text{ kip}\cdot\text{ft} \quad \blacktriangleleft$$



PROBLEM 7.75

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Reactions

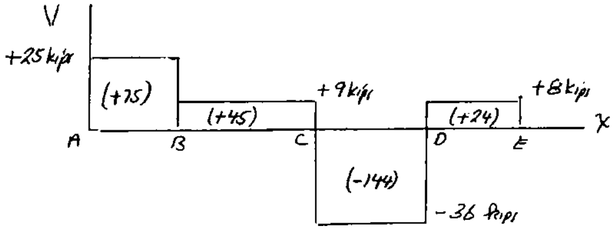
$$+\circlearrowleft \Sigma M_D = 0: \quad (16)(9) + (45)(4) - (8)(3) - A(12) = 0$$

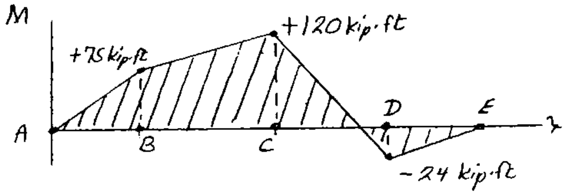
$A = +25$ kips $\qquad$ $\mathbf{A} = 25 \text{ kips} \uparrow$

$$+\uparrow \Sigma F_y = 0: \quad 25 - 16 - 45 - 8 + D_y = 0$$

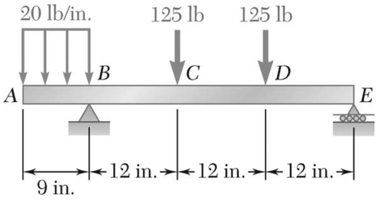
$D_y = +44,$ $\qquad$ $D_y = 44 \text{ kips} \uparrow$

$$\Sigma F_x = 0: \quad D_x = 0$$





(*b*) $\quad |V|_{\max} = 36.0$ kips; $\qquad$ $|M|_{\max} = 120.0 \text{ kip} \cdot \text{ft}$ ◀



PROBLEM 7.76

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Reactions

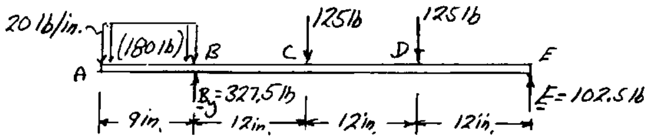
$$\Sigma F_x = 0: \quad B_x = 0$$

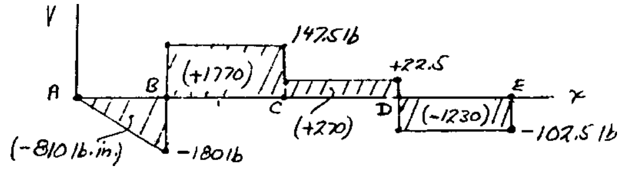
$$+\circlearrowleft \Sigma M_E = 0: \quad (20 \text{ lb/in.})(9 \text{ in.})(40.5 \text{ in.}) + (125 \text{ in.})(24 \text{ in.}) + (125 \text{ in.})(12 \text{ in.}) - B(36 \text{ in.}) = 0$$

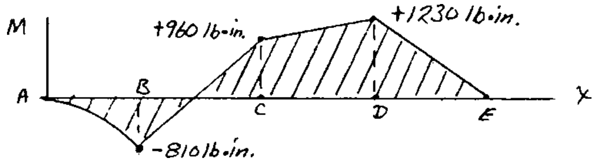
$$B_y = +327.5 \text{ lb} \qquad \mathbf{B}_y = 327.5 \text{ lb}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad -(20 \text{ lb/in.})(9 \text{ in.}) - 125 \text{ lb} - 125 \text{ lb} + 327.5 \text{ lb} + E = 0$$

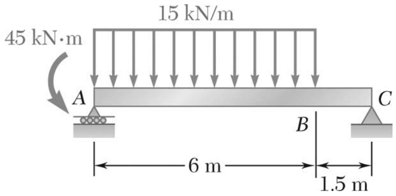
$$E = +102.5 \text{ lb} \qquad \mathbf{E} = 102.5 \text{ lb}\uparrow$$







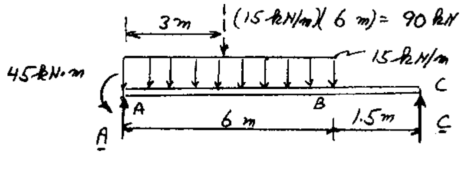
(*b*) $|V|_{\max} = 180.0$ lb; $|M|_{\max} = 1230$ lb · in. ◀



PROBLEM 7.77

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



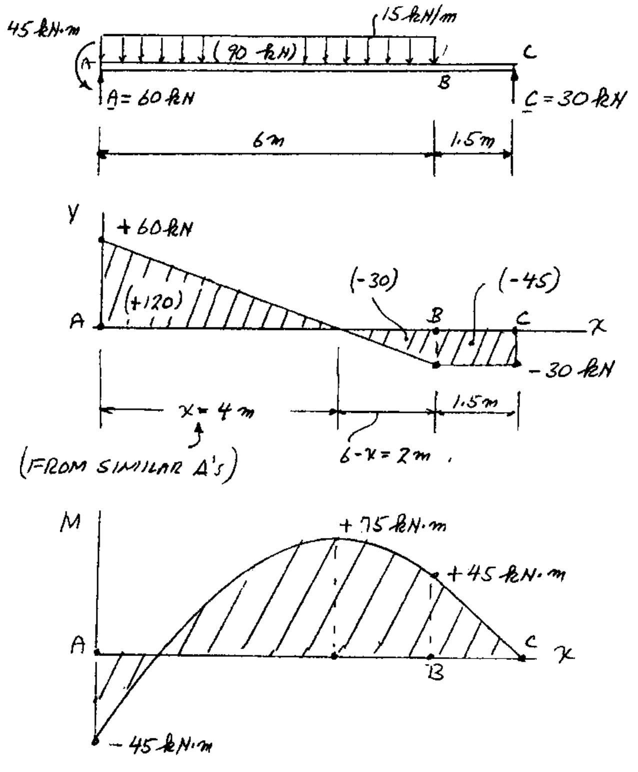
Reactions

$$+\circlearrowleft \Sigma M_A = 0: \quad +45\ \text{kN}\cdot\text{m} - (90\ \text{kN})(3\ \text{m}) + C(7.5\ \text{m}) = 0$$

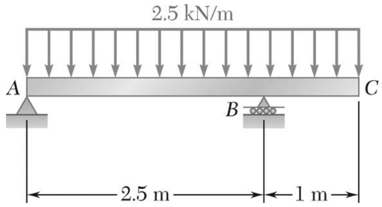
$C = +30\ \text{kN}$ $\qquad$ $\mathbf{C} = 30\ \text{kN}\uparrow$

$$+\uparrow \Sigma F_y = 0: \quad A - 90\ \text{kN} + 30\ \text{kN} = 0$$

$\mathbf{A} = 60\ \text{kN}\uparrow$



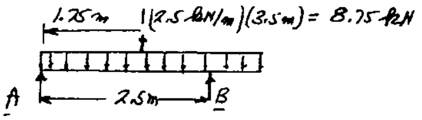
(*b*) $\qquad$ $75.0\ \text{kN}\cdot\text{m}$, 4.00 m from *A* ◀



PROBLEM 7.78

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

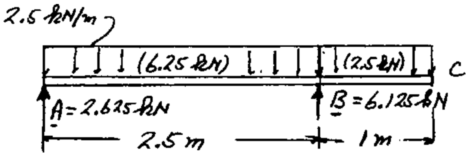


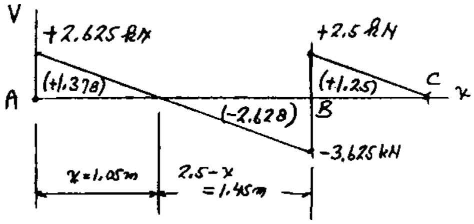
$$+\circlearrowleft \Sigma M_A = 0: \quad (8.75)(1.75) - B(2.5) = 0$$

$$\mathbf{B} = 6.125\text{ kN}\uparrow$$

$$\Sigma F_y = 0:$$

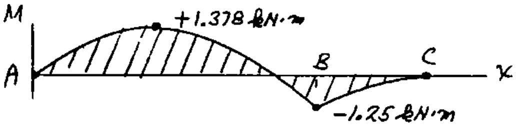
$$\mathbf{A} = 2.625\text{ kN}\uparrow$$



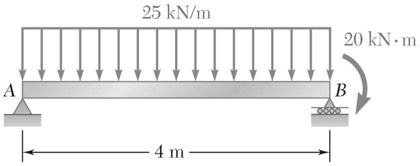


Similar Δ's

$$\underbrace{\frac{x}{2.625} = \frac{2.5-x}{3.625}}_{\substack{\text{Add numerators}\\ \text{and denominators}\\ x = 1.05\text{ m}}} = \frac{2.5}{6.25}$$



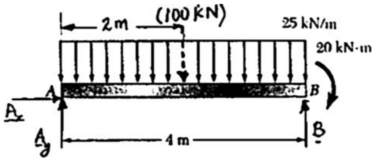
(*b*) 1.378 kN · m, 1.050 m from *A* ◀



PROBLEM 7.79

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



Free body: Beam

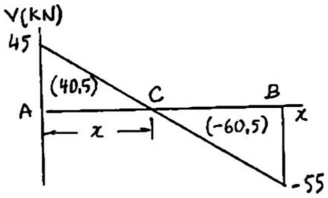
$$+\circlearrowleft \Sigma M_A = 0: \quad B(4\text{ m}) - (100\text{ kN})(2\text{ m}) - 20\text{ kN}\cdot\text{m} = 0$$

$$B = +55\text{ kN} \quad \triangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 55 - 100 = 0$$

$$A_y = +45\text{ kN} \quad \triangleleft$$



Shear diagram

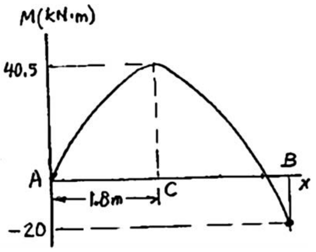
At *A*: $V_A = A_y = +45\text{ kN}$

To determine Point *C* where $V = 0$:

$$V_C - V_A = -wx$$
$$0 - 45\text{ kN} = -(25\text{ kN}\cdot\text{m})x$$

$$x = 1.8\text{ m} \quad \triangleleft$$

We compute all areas bending-moment



Bending-moment diagram

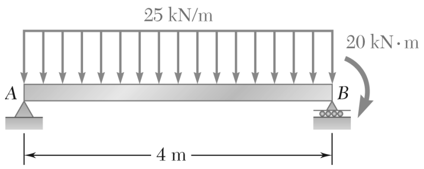
At *A*: $M_A = 0$

At *B*: $M_B = -20\text{ kN}\cdot\text{m}$

$$|M|_{\max} = 40.5\text{ kN}\cdot\text{m} \quad ◀$$

$$1.800\text{ m from } A \quad ◀$$

Single arc of parabola

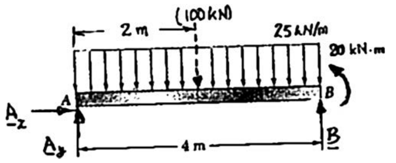


PROBLEM 7.80

Solve Problem 7.79 assuming that the 20-kN · m couple applied at B is counterclockwise.

PROBLEM 7.79 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



Free body: Beam

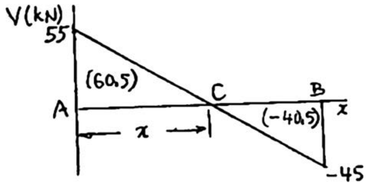
$$+\circlearrowleft \Sigma M_A = 0: \quad B(4\text{ m}) - (100\text{ kN})(2\text{ m}) - 20\text{ kN}\cdot\text{m} = 0$$

$$B = +45\text{ kN} \quad \triangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 45 - 100 = 0$$

$$A_y = +55\text{ kN} \quad \triangleleft$$



Shear diagram

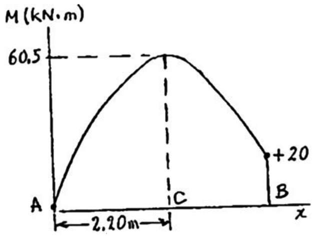
At A: $\quad V_A = A_y = +55\text{ kN}$

To determine Point C where $V = 0$:

$$V_C - V_A = -wx$$
$$0 - 55\text{ kN} = -(25\text{ kN/m})x$$

$$x = 2.20\text{ m} \quad \triangleleft$$

We compute all areas bending-moment



Bending-moment diagram

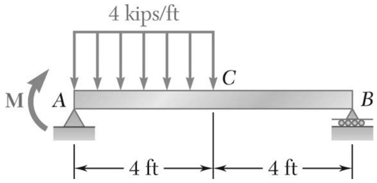
At A: $\quad M_A = 0$

At B: $\quad M_B = +20\text{ kN}\cdot\text{m}$

$$|M|_{\max} = 60.5\text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$2.20\text{ m from } A \quad \blacktriangleleft$$

Single arc of parabola

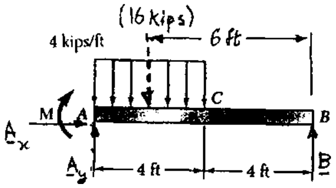


PROBLEM 7.81

For the beam shown, draw the shear and bending-moment diagrams, and determine the magnitude and location of the maximum absolute value of the bending moment, knowing that (*a*) $M = 0$, (*b*) $M = 24$ kip · ft.

SOLUTION

Free body: Beam



$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (16 \text{ kips})(6 \text{ ft}) - A_y(8 \text{ ft}) - M = 0$$

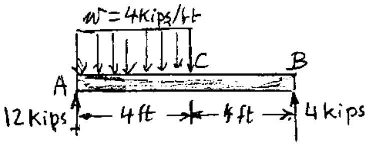
$$A_y = 12 \text{ kips} - \frac{1}{8}M \qquad (1) \ \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B + 12 - \frac{1}{8}M - 16 = 0$$

$$B = 4 \text{ kips} + \frac{1}{8}M \qquad (2) \ \triangleleft$$

(*a*) $\underline{M = 0}$:

Load diagram

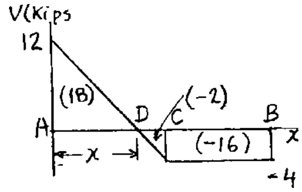


Making $M = 0$ in. (1) and (2).

$$A_y = +12 \text{ kips}$$

$$B = +4 \text{ kips}$$

Shear diagram $\qquad V_A = A_y = +12$ kips

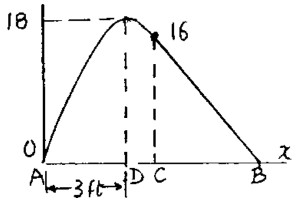


To determine Point D where $V = 0$:

$$V_D - V_A = -wx$$

$$0 - 12 \text{ kips} = -(4 \text{ kips/ft})x$$

$x = 3$ ft ◁



We compute all areas

B. M. Diagram

At A: $\qquad M_A = 0$

$|M|_{\max} = 18.00$ kip·ft, ◀

3.00 ft from A ◀

Parabola from A to C

PROBLEM 7.81 (Continued)

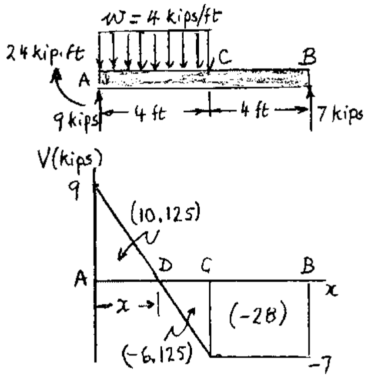
(*b*) $\underline{M = 24\text{ kip}\cdot\text{ft}}$

Load diagram

Making $M = 24\text{ kip}\cdot\text{ft}$ in (1) and (2)

$$A = 12 - \frac{1}{8}(24) = +9\text{ kips}$$

$$B = 4 + \frac{1}{8}(24) = +7\text{ kips}$$

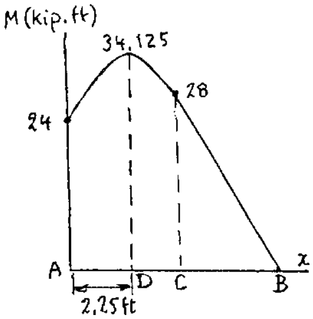


Shear diagram $\qquad V_A = A_y = +9\text{ kips}$

To determine Point *D* where $V = 0$:

$$V_D = V_A = -wx$$

$$0 - 9\text{ kips} = -(4\text{ kips/ft})\,x \qquad x = 2.25\text{ ft} \;\triangleleft$$



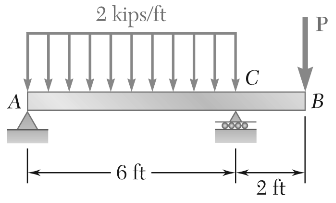
B. M. Diagram

At *A*: $\qquad M_A = +24\text{ kip}\cdot\text{ft}$

$|M|_{\max} = 34.1\text{ kip}\cdot\text{ft},$ ◀

2.25 ft from *A* ◀

Parabola from *A* to *C*.

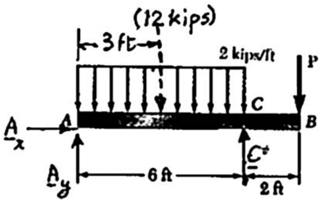


PROBLEM 7.82

For the beam shown, draw the shear and bending-moment diagrams, and determine the magnitude and location of the maximum absolute value of the bending moment, knowing that (*a*) $P = 6$ kips, (*b*) $P = 3$ kips.

SOLUTION

Free body: Beam



$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad C(6 \text{ ft}) - (12 \text{ kips})(3 \text{ ft}) - P(8 \text{ ft}) = 0$$

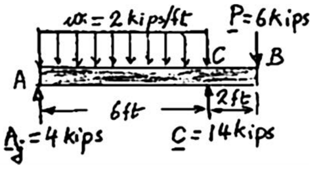
$$C = 6 \text{ kips} + \frac{4}{3}P \qquad (1) \ \triangleleft$$

$$\Sigma F_y = 0: \quad A_y + \left(6 + \frac{4}{3}P\right) - 12 - P = 0$$

$$A_y = 6 \text{ kips} - \frac{1}{3}P \qquad (2) \ \triangleleft$$

(*a*) $P = 6$ kips.

Load diagram

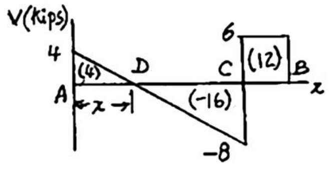


Substituting for P in Eqs. (2) and (1):

$$A_y = 6 - \frac{1}{3}(6) = 4 \text{ kips}$$

$$C = 6 + \frac{4}{3}(6) = 14 \text{ kips}$$

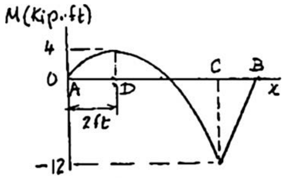
Shear diagram $\quad V_A = A_y = +4$ kips



To determine Point D where $V = 0$:

$$V_D - V_A = -wx$$

$$0 - 4 \text{ kips} = (2 \text{ kips/ft})x \qquad x = 2 \text{ ft} \ \triangleleft$$



We compute all areas

Bending-moment diagram

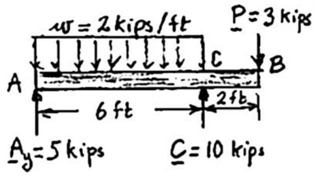
At A: $\quad M_A = 0$

$$|M|_{\max} = 12.00 \text{ kip} \cdot \text{ft, at } C \ \blacktriangleleft$$

Parabola from A to C

PROBLEM 7.82 (Continued)

(*b*) $\underline{P = 3 \text{ kips}}$

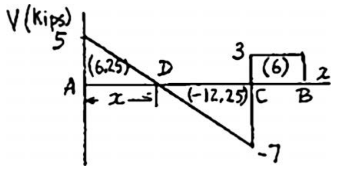


Load diagram

Substituting for P in Eqs. (2) and (1):

$$A = 6 - \frac{1}{3}(3) = 5 \text{ kips}$$

$$C = 6 + \frac{4}{3}(3) = 10 \text{ kips}$$



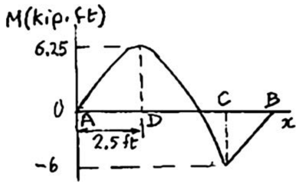
Shear diagram $\qquad V_A = A_y = +5 \text{ kips}$

To determine D where $V = 0$:

$$V_D - V_A = -wx$$

$$0 - (5 \text{ kips}) = -(2 \text{ kips/ft})\, x$$

$x = 2.5 \text{ ft}$ ◁



We compute all areas

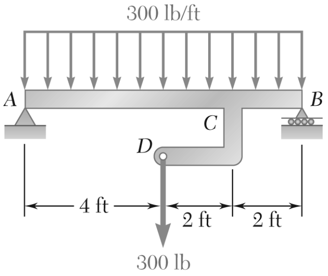
Bending-moment diagram

At A: $\qquad M_A = 0$

$|M|_{\max} = 6.25 \text{ kip} \cdot \text{ft}$ ◀

2.50 ft from A ◀

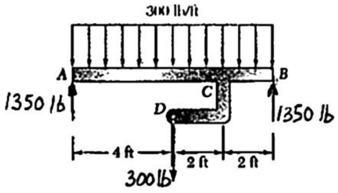
Parabola from A to C.



PROBLEM 7.83

(*a*) Draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

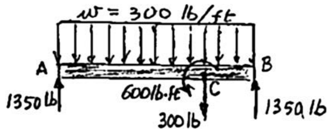


Reactions at supports

Because of symmetry of load

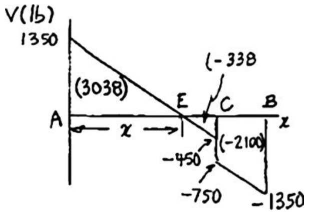
$$A = B = \frac{1}{2}(300 \times 8 + 300)$$

$\mathbf{A} = \mathbf{B} = 1350\text{ lb}\uparrow$ ◁



Load diagram for *AB*

The 300-lb force at *D* is replaced by an equivalent force-couple system at *C*.



Shear diagram

At *A*: $V_A = A = 1350\text{ lb}$

To determine Point *E* where $V = 0$:

$$V_E - V_A = -wx$$
$$0 - 1350\text{ lb} = -(300\text{ lb/ft})x$$

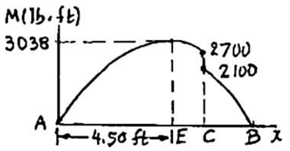
$x = 4.50\text{ ft}$ ◁

We compute all areas

Bending-moment diagram

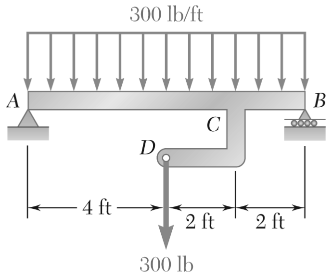
At *A*: $M_A = 0$

Note $600-\text{lb}\cdot\text{ft}$ drop at *C* due to couple



$|M|_{\max} = 3040\text{ lb}\cdot\text{ft}$ ◀

4.50 ft from *A* ◀

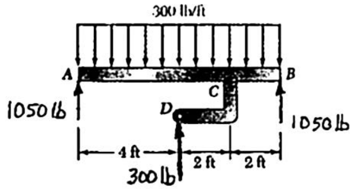


PROBLEM 7.84

Solve Problem 7.83 assuming that the 300-lb force applied at *D* is directed upward.

PROBLEM 7.83 (*a*) Draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

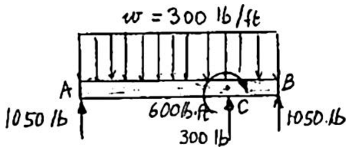


Reactions at supports

Because of symmetry of load:

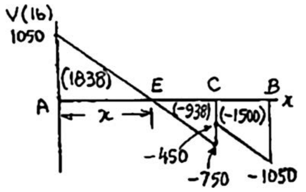
$$A = B = \frac{1}{2}(300 \times 8 - 300)$$

$\mathbf{A} = \mathbf{B} = 1050\text{ lb}\uparrow$ ◁



Load diagram

The 300-lb force at *D* is replaced by an equivalent force-couple system at *C*.



Shear diagram

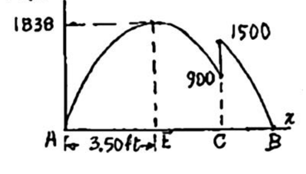
At *A*: $V_A = A = 1050\text{ lb}$

To determine Point *E* where $V = 0$:

$$V_E - V_A = -wx$$
$$0 - 1050\text{ lb} = -(300\text{ lb/ft})x$$

$x = 3.50\text{ ft}$ ◁

We compute all areas



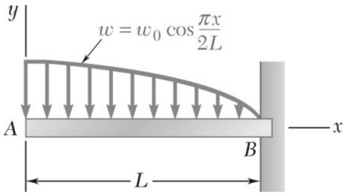
Bending-moment diagram

At *A*: $M_A = 0$

Note $600 - \text{lb}\cdot\text{ft}$ increase at *C* due to couple

$|M|_{\max} = 1838\text{ lb}\cdot\text{ft}$ ◀

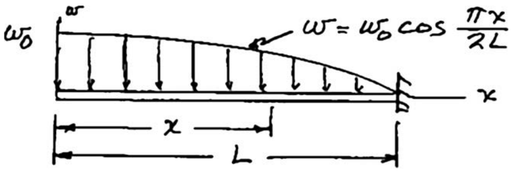
3.50 ft from *A* ◀



PROBLEM 7.85

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION



$$\frac{dv}{dx} = -w = w_0 \cos \frac{\pi}{2} \frac{x}{L}$$

$$V = -\int w dx = -w_0 \left(\frac{2L}{\pi}\right) \sin \frac{\pi x}{2L} + C_1 \qquad (1)$$

$$\frac{dM}{dx} = V = -w_0 \left(\frac{2L}{\pi}\right) \sin \frac{\pi x}{2L} + C_1$$

$$M = \int V dx = +w_0 \left(\frac{2L}{\pi}\right)^2 \cos \frac{\pi x}{2L} + C_1 x + C_2 \qquad (2)$$

Boundary conditions

At $x = 0$: $V = C_1 = 0 \quad C_1 = 0$

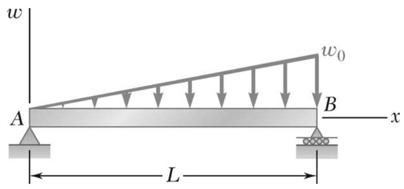
At $x = 0$: $M = +w_0 \left(\frac{2L}{\pi}\right)^2 \cos(0) + C_2 = 0$

$$C_2 = -w_0 \left(\frac{2L}{\pi}\right)^2$$

Eq. (1) $V = -w_0 \left(\frac{2L}{\pi}\right) \sin \frac{\pi x}{2L}$ ◀

$M = w_0 \left(\frac{2L}{\pi}\right)^2 \left(-1 + \cos \frac{\pi x}{2L}\right)$ ◀

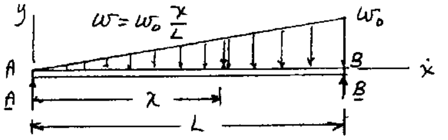
$M_{\max}$ at $x = L$: $|M_{\max}| = w_0 \left(\frac{2L}{\pi}\right)^2 |-1+0| = \frac{4}{\pi^2} w_0 L^2$ ◀



PROBLEM 7.86

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION



Eq. (7.1): $$\frac{dV}{dx} = -w = -w_0\frac{x}{L}$$

$$V = \int -w_0\frac{x}{L}dx = -\frac{1}{2}w_0\frac{x^2}{L} + C_1 \qquad (1)$$

Eq. (7.3): $$\frac{dM}{dx} = V = -\frac{1}{2}w_0\frac{x^2}{L} + C_1$$

$$M = \int\left(-\frac{1}{2}w_0\frac{x^2}{L} + C_1\right)dx = -\frac{1}{6}w_0 = \frac{x^3}{L} + C_1x + C_2 \qquad (2)$$

(*a*) Boundary conditions

$$\text{At}\quad x = 0:\quad M = 0 = 0 + 0 + C_2 \qquad C_2 = 0$$

$$x = L:\quad M = 0 = -\frac{1}{6}w_0L^2 + C_1L \qquad C_1 = \frac{1}{6}w_0L$$

Substituting C_1 and C_2 into (1) and (2):

$$V = -\frac{1}{2}w_0\frac{x^2}{L} + \frac{1}{6}w_0L \qquad V = \frac{1}{2}w_0L\left(\frac{1}{3} - \frac{x^2}{L^2}\right) \blacktriangleleft$$

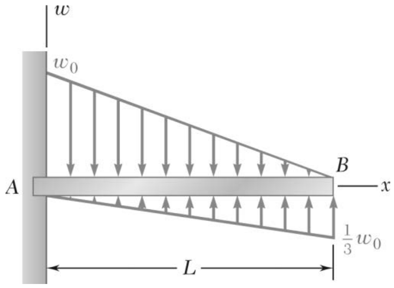
$$M = -\frac{1}{6}w_0\frac{x^3}{L} + \frac{1}{6}w_0Lx \qquad M = \frac{1}{6}w_0L^2\left(\frac{x}{L} - \frac{x^3}{L^3}\right) \blacktriangleleft$$

(*b*) Max moment occurs when $V = 0$:

$$1 - 3\frac{x^2}{L^2} = 0 \qquad \frac{x}{L} = \frac{1}{\sqrt{3}} \qquad x = 0.577L \blacktriangleleft$$

$$M_{\max} = \frac{1}{6}w_0L^2\left[\frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)^3\right] \qquad M_{\max} = 0.0642w_0L^2 \blacktriangleleft$$

Note: At $x = 0$, $$A = V_A = \frac{1}{2}w_0L\left(\frac{1}{3}\right) \qquad \mathbf{A} = \frac{1}{6}w_0L\uparrow$$



PROBLEM 7.87

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION

(*a*) We note that at $B(x = L)$: $V_B = 0, \quad M_B = 0$ (1)

Load: $$w(x) = w_0\left(1 - \frac{x}{L}\right) - \frac{1}{3}w_0\left(\frac{x}{L}\right) = w_0\left(1 - \frac{4x}{3L}\right)$$

Shear: We use Eq. (7.2) between $C(x = x)$ and $B(x = L)$:

$$V_B - V_C = -\int_x^L w(x)dx \quad 0 - V(x) = -\int_x^L w(x)dx$$

$$V(x) = w_0\int_x^L\left(1 - \frac{4x}{3L}\right)dx$$

$$= w_0\left[x - \frac{2x^2}{3L}\right]_x^L = w_0\left(L - \frac{2L}{3} - x + \frac{2x^2}{3L}\right)$$

$$V(x) = \frac{w_0}{3L}(2x^2 - 3Lx + L^2) \quad (2) \blacktriangleleft$$

Bending moment: We use to Eq. (7.4) between $C(x = x)$ and $B(x = L)$:

$$M_B - M_C = \int_x^L V(x)dx \quad 0 - M(x)$$

$$= \frac{w_0}{3L}\int_x^L (2x^2 - 3Lx + L^2)dx$$

$$M(x) = -\frac{w_0}{3L}\left[\frac{2}{3}x^3 - \frac{3}{2}Lx^2 + L^2x\right]^L$$

$$= -\frac{w_0}{18L}[4x^3 - 9Lx^2 + 6L^2x]_x^L$$

$$= -\frac{w_0}{18L}[(4L^3 - 9L^3 + 6L^3) - (4x^3 - 9Lx^2 + 6L^2x)]$$

$$M(x) = \frac{w_0}{18L}(4x^3 - 9Lx^2 + 6L^2x - L^3) \quad (3) \blacktriangleleft$$

PROBLEM 7.87 (Continued)

(*b*) Maximum bending moment

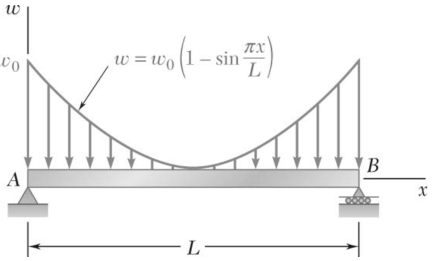
$$\frac{dM}{dx} = V = 0$$

Eq. (2): $2x^2 - 3Lx + L^2 = 0$

$$x = \frac{3 - \sqrt{9-8}}{4} L = \frac{L}{2}$$

Carrying into (3): $M_{\max} = \dfrac{w_0 L^2}{72}$, At $x = \dfrac{L}{2}$ ◀

Note: $|M|_{\max} = \dfrac{w_0 L^2}{18}$ At $x = 0$



PROBLEM 7.88

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the maximum bending moment.

SOLUTION

(*a*) Reactions at supports: $A = B = \frac{1}{2}W$, where $\frac{W}{L} = \text{Total load}$

$$W = \int_0^L w\,dx = w_0\int_0^L\left(1 - \sin\frac{\pi x}{L}\right)dx$$

$$= w_0\left[x + \frac{L}{x}\cos\frac{\pi x}{L}\right]_0^L$$

$$= w_0L\left(1 - \frac{2}{\pi}\right)$$

Thus $$V_A = A = \frac{1}{2}W = \frac{1}{2}w_0L\left(1 - \frac{2}{\pi}\right)$$

$$M_A = 0 \qquad (1)$$

Load: $$w(x) = w_0\left(1 - \sin\frac{\pi x}{L}\right)$$

Shear: From Eq. (7.2):

$$V(x) - V_A = -\int_0^x w(x)dx$$

$$= -w_0\int_0^x\left(1 - \sin\frac{\pi x}{L}\right)dx$$

Integrating and recalling first of Eqs. (1),

$$V(x) - \frac{1}{2}w_0L\left(1 - \frac{2}{\pi}\right) = -w_0\left[x + \frac{L}{\pi}\cos\frac{\pi x}{L}\right]_0^x$$

$$V(x) = \frac{1}{2}w_0L\left(1 - \frac{2}{\pi}\right) - w_0\left(2 + \frac{L}{\pi}\cos\frac{\pi x}{L}\right) + w_0\frac{L}{\pi}$$

$$V(x) = w_0\left(\frac{L}{2} - x - \frac{L}{\pi}\cos\frac{\pi x}{L}\right) \qquad (2) ◀$$

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PROBLEM 7.88 (Continued)

Bending moment: From Eq. (7.4) and recalling that $M_A = 0$.

$$M(x) - M_A = \int_0^x V(x)dx$$

$$= w_0 \left[\frac{L}{2}x - \frac{1}{2}x^2 - \left(\frac{L}{\pi}\right)^2 \sin\frac{\pi x}{L} \right]_0^x$$

$$M(x) = \frac{1}{2}w_0 \left(Lx - x^2 - \frac{2L^2}{\pi^2}\sin\frac{\pi x}{L} \right) \qquad (3) ◀$$

(*b*) Maximum bending moment

$$\frac{dM}{dx} = V = 0.$$

This occurs at $x = \frac{L}{2}$ as we may check from (2):

$$V\left(\frac{L}{2}\right) = w_0\left(\frac{L}{2} - \frac{L}{2} - \frac{L}{\pi}\cos\frac{\pi}{2}\right) = 0$$

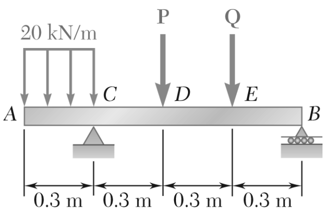
From (3):

$$M\left(\frac{L}{2}\right) = \frac{1}{2}w_0\left(\frac{L^2}{2} - \frac{L^2}{4} - \frac{2L^2}{\pi^2}\sin\frac{\pi}{2}\right)$$

$$= \frac{1}{8}w_0L^2\left(1 - \frac{8}{\pi^2}\right)$$

$$= 0.0237w_0L^2$$

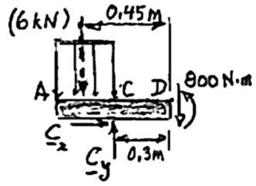
$$M_{\max} = 0.0237w_0L^2, \quad \text{at} \quad x = \frac{L}{2} \qquad ◀$$



PROBLEM 7.89

The beam AB is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+800\text{ N}\cdot\text{m}$ at D and $+1300\text{ N}\cdot\text{m}$ at E, (a) determine **P** and **Q**, (b) draw the shear and bending-moment diagrams for the beam.

SOLUTION

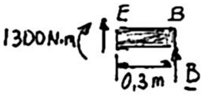


(a) Free body: Portion AD

$$\Sigma F_x = 0: \quad C_x = 0$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -C_y(0.3\text{ m}) + 0.800\text{ kN}\cdot\text{m} + (6\text{ kN})(0.45\text{ m}) = 0$$

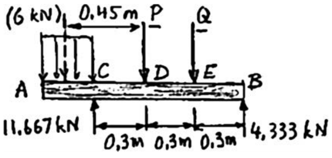
$$C_y = +11.667\text{ kN} \qquad \mathbf{C} = 11.667\text{ kN}\uparrow \triangleleft$$



Free body: Portion EB

$$+\circlearrowleft \Sigma M_E = 0: \quad B(0.3\text{ m}) - 1.300\text{ kN}\cdot\text{m} = 0$$

$$\mathbf{B} = 4.333\text{ kN}\uparrow \triangleleft$$



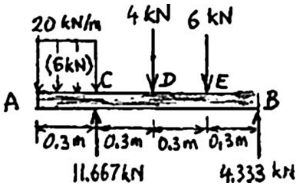
Free body: Entire beam

$$+\circlearrowleft \Sigma M_D = 0: \quad (6\text{ kN})(0.45\text{ m}) - (11.667\text{ kN})(0.3\text{ m})$$
$$- Q(0.3\text{ m}) + (4.333\text{ kN})(0.6\text{ m}) = 0$$

$$\mathbf{Q} = 6.00\text{ kN}\downarrow \blacktriangleleft$$

$$+\uparrow \Sigma M_y = 0: \quad 11.667\text{ kN} + 4.333\text{ kN}$$
$$-6\text{ kN} - P - 6\text{ kN} = 0$$

$$\mathbf{P} = 4.00\text{ kN}\downarrow \blacktriangleleft$$

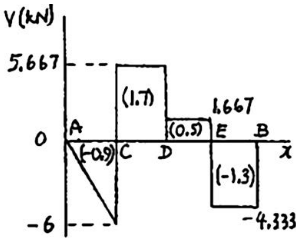


Load diagram

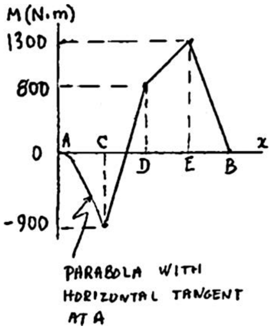
(b) Shear diagram

At A: $V_A = 0$

$$|V|_{\max} = 6\text{ kN} \triangleleft$$



PROBLEM 7.89 (Continued)



Bending-moment diagram

At A: $M_A = 0$

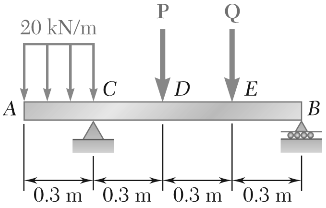
$|M|_{\max} = 1300 \text{ N}\cdot\text{m}$ ◁

We check that

$$M_D = +800 \text{ N}\cdot\text{m} \quad \text{and} \quad M_E = +1300 \text{ N}\cdot\text{m}$$

As given:

At C: $M_C = -900 \text{ N}\cdot\text{m}$ ◀



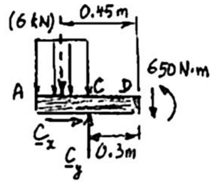
PROBLEM 7.90

Solve Problem 7.89 assuming that the bending moment was found to be $+650\ \text{N}\cdot\text{m}$ at *D* and $+1450\ \text{N}\cdot\text{m}$ at *E*.

PROBLEM 7.89 The beam *AB* is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+800\ \text{N}\cdot\text{m}$ at *D* and $+1300\ \text{N}\cdot\text{m}$ at *E*, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

SOLUTION

(*a*) Free body: Portion *AD*

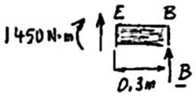


$$\Sigma F_x = 0: \quad C_x = 0$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -C(0.3\text{ m}) + 0.650\text{ kN}\cdot\text{m} + (6\text{ kN})(0.45\text{ m}) = 0$$

$$C_y = +11.167\text{ kN} \qquad \mathbf{C} = 11.167\text{ kN}\uparrow \ \triangleleft$$

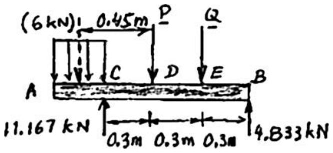
Free body: Portion *EB*



$$+\circlearrowleft \Sigma M_E = 0: \quad B(0.3\text{ m}) - 1.450\text{ kN}\cdot\text{m} = 0$$

$$\mathbf{B} = 4.833\text{ kN}\uparrow \ \triangleleft$$

Free body: Entire beam



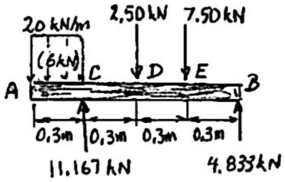
$$+\circlearrowleft \Sigma M_D = 0: \quad (6\text{ kN})(0.45\text{ m}) - (11.167\text{ kN})(0.3\text{ m}) - Q(0.3\text{ m}) + (4.833\text{ kN})(0.6\text{ m}) = 0$$

$$\mathbf{Q} = 7.50\text{ kN}\downarrow \ \blacktriangleleft$$

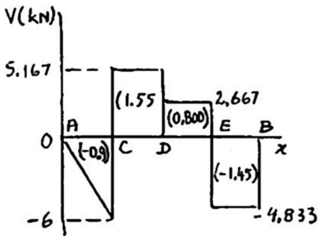
$$+\uparrow \Sigma M_y = 0: \quad 11.167\text{ kN} + 4.833\text{ kN} - 6\text{ kN} - P - 7.50\text{ kN} = 0$$

$$\mathbf{P} = 2.50\text{ kN}\downarrow \ \blacktriangleleft$$

Load diagram



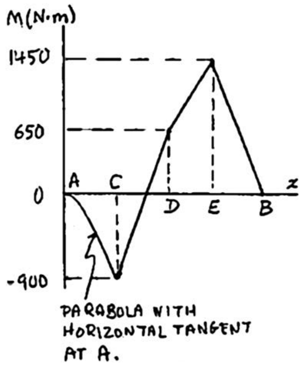
(*b*) Shear diagram



At *A*: $V_A = 0$

$$|V|_{\max} = 6\text{ kN} \ \triangleleft$$

PROBLEM 7.90 (Continued)



Bending-moment diagram

At A: $M_A = 0$

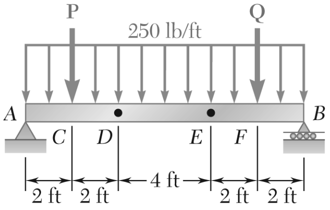
$|M|_{\max} = 1450 \text{ N} \cdot \text{m}$ ◁

We check that

$$M_D = +650 \text{ N} \cdot \text{m} \quad \text{and} \quad M_E = +1450 \text{ N} \cdot \text{m}$$

As given:

At C: $M_C = -900 \text{ N} \cdot \text{m}$ ◀

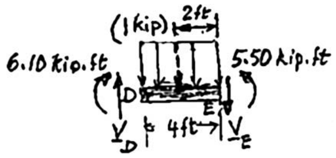


PROBLEM 7.91*

The beam AB is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+6.10\text{ kip}\cdot\text{ft}$ at D and $+5.50\text{ kip}\cdot\text{ft}$ at E, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

SOLUTION

(*a*) Free body: Portion *DE*



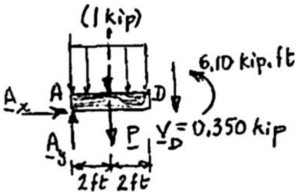
$$+\circlearrowleft \Sigma M_E = 0:\quad 5.50\text{ kip}\cdot\text{ft} - 6.10\text{ kip}\cdot\text{ft} + (1\text{ kip})(2\text{ ft}) - V_D(4\text{ ft}) = 0$$

$$V_D = +0.350\text{ kip}$$

$$+\uparrow \Sigma F_y = 0:\quad 0.350\text{ kip} - 1\text{kip} - V_E = 0$$

$$V_E = -0.650\text{ kip}$$

Free body: Portion *AD*



$$+\circlearrowleft \Sigma M_A = 0:\quad 6.10\text{ kip}\cdot\text{ft} - P(2\text{ft}) - (1\text{ kip})(2\text{ ft}) - (0.350\text{ kip})(4\text{ ft}) = 0$$

P $= 1.350$ kips ↓ ◀

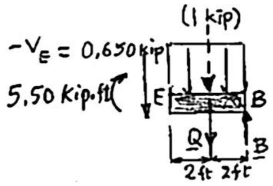
$$\Sigma F_x = 0:\quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0:\quad A_y - 1\text{ kip} - 1.350\text{ kip} - 0.350\text{ kip} = 0$$

$$A_y = +2.70\text{ kips}$$

A $= 2.70$ kips ↑ ◀

Free body: Portion *EB*



$$+\circlearrowleft \Sigma M_B = 0:\quad (0.650\text{ kip})(4\text{ ft}) + (1\text{ kip})(2\text{ ft}) + Q(2\text{ ft}) - 5.50\text{ kip}\cdot\text{ft} = 0$$

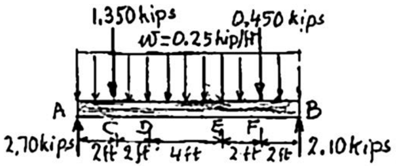
Q $= 0.450$ kip ↓ ◀

$$+\uparrow \Sigma F_y = 0:\quad B - 0.450 - 1 - 0.650 = 0$$

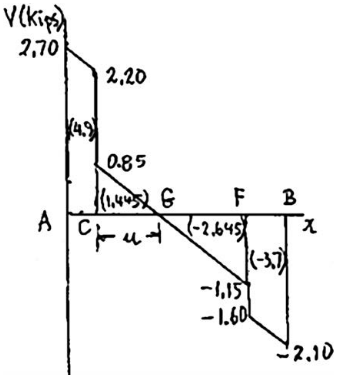
B $= 2.10$ kips ↑ ◀

PROBLEM 7.91* (Continued)

(*b*) Load diagram



Shear diagram



At A: $$V_A = A = +2.70 \text{ kips}$$

To determine Point G where $V = 0$, we write

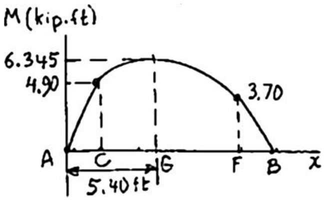
$$V_G - V_C = -w\mu$$
$$0 - 0.85 \text{ kips} = -(0.25 \text{ kip/ft})\mu$$

$\mu = 3.40$ ft ◀

We next compute all areas

$|V|_{\max} = 2.70$ kips at A ◀

Bending-moment diagram



At A: $M_A = 0$

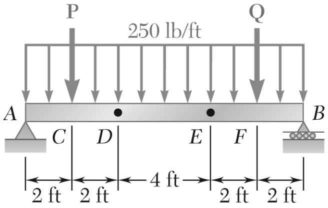
Largest value occurs at G with

$$AG = 2 + 3.40 = 5.40 \text{ ft}$$

$|M|_{\max} = 6.345$ kip·ft ◀

5.40 ft from A ◀

Bending-moment diagram consists of 3 distinct arcs of parabolas.



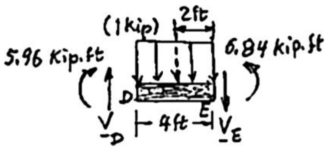
PROBLEM 7.92*

Solve Problem 7.91 assuming that the bending moment was found to be +5.96 kip·ft at *D* and +6.84 kip·ft at *E*.

PROBLEM 7.91* The beam *AB* is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is +6.10 kip·ft at *D* and +5.50 kip·ft at *E*, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

SOLUTION

(*a*) Free body: Portion *DE*



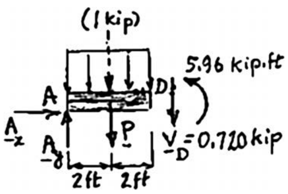
$$+\circlearrowleft \Sigma M_E = 0: \quad 6.84 \text{ kip}\cdot\text{ft} - 5.96 \text{ kip}\cdot\text{ft} + (1 \text{ kip})(2 \text{ ft}) - V_D(4 \text{ ft}) = 0$$

$$V_D = +0.720 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0: \quad 0.720 \text{ kip} - 1 \text{kip} - V_E = 0$$

$$V_E = -0.280 \text{ kip}$$

Free body: Portion *AD*



$$+\circlearrowleft \Sigma M_A = 0: \quad 5.96 \text{ kip}\cdot\text{ft} - P(2 \text{ ft}) - (1 \text{ kip})(2 \text{ ft}) - (0.720 \text{ kip})(4 \text{ ft}) = 0$$

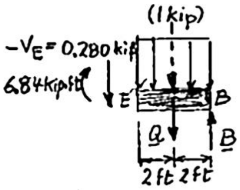
$$\mathbf{P} = 0.540 \text{ kip}\downarrow \blacktriangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1 \text{ kip} - 0.540 \text{ kip} - 0.720 \text{ kip} = 0$$

$$A_y = +2.26 \text{ kips} \qquad \mathbf{A} = 2.26 \text{ kips}\uparrow \triangleleft$$

Free body: Portion *EB*



$$+\circlearrowleft \Sigma M_B = 0: \quad (0.280 \text{ kip})(4 \text{ ft}) + (1 \text{ kip})(2 \text{ ft}) + Q(2 \text{ ft}) - 6.84 \text{ kip}\cdot\text{ft} = 0$$

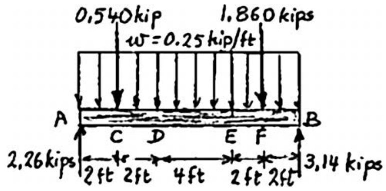
$$\mathbf{Q} = 1.860 \text{ kips}\downarrow \blacktriangleleft$$

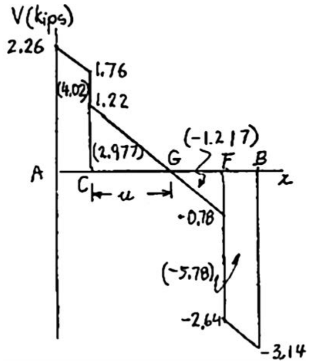
$$+\uparrow \Sigma F_y = 0: \quad B - 1.860 - 1 - 0.280 = 0$$

$$\mathbf{B} = 3.14 \text{ kips}\uparrow \blacktriangleleft$$

PROBLEM 7.92* (Continued)

(*b*) Load diagram





Shear diagram

At *A*: $V_A = A = +2.26$ kips

To determine Point *G* where $V = 0$, we write

$$V_G - V_C = -w\mu$$

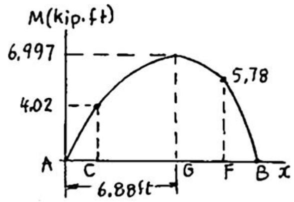
$$0 - (1.22 \text{ kips}) = -(0.25 \text{ kip/ft})\mu$$

$\mu = 4.88$ ft ◁

We next compute all areas

$|V|_{\max} = 3.14$ kips at *B* ◀

Bending-moment diagram



At *A*: $M_A = 0$

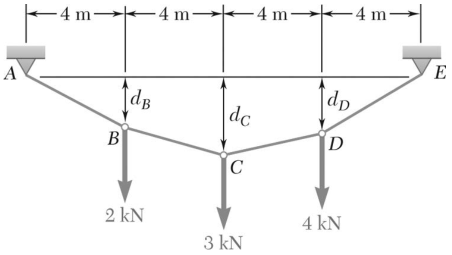
Largest value occurs at *G* with

$$AG = 2 + 4.88 = 6.88 \text{ ft}$$

$|M|_{\max} = 6.997$ kip·ft ◀

6.88 ft from *A* ◀

Bending-moment diagram consists of 3 distinct arcs of parabolas.

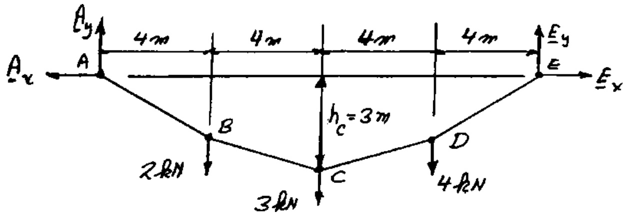


PROBLEM 7.93

Three loads are suspended as shown from the cable *ABCDE*. Knowing that $d_C = 3$ m, determine (*a*) the components of the reaction at *E*, (*b*) the maximum tension in the cable.

SOLUTION

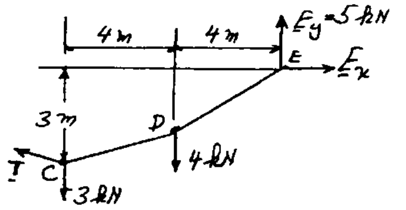
(*a*)



$$+\circlearrowleft \Sigma M_A = 0: \quad E_y(16\text{ m}) - (2\text{ kN})(4\text{ m}) - (3\text{ kN})(8\text{ m}) - (4\text{ kN})(12\text{ m}) = 0$$

$$E_y = +5\text{ kN} \qquad \mathbf{E}_y = 5.00\text{ kN}\uparrow \blacktriangleleft$$

Portion <u>*CDE*</u>:

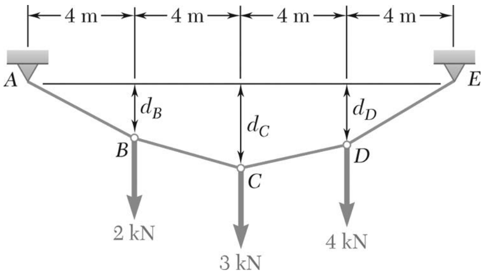


$$+\circlearrowleft \Sigma M_C = 0: \quad (5\text{ kN})(8\text{ m}) - (4\text{ kN})(4\text{ m}) - E_x(3\text{ m}) = 0$$

$$E_x = 8\text{ kN} \qquad \mathbf{E}_x = 8.00\text{ kN} \longrightarrow \blacktriangleleft$$

(*b*) Maximum tension occurs in *DE*:

$$T_m = \sqrt{E_x^2 + E_y^2} = \sqrt{8^2 + 5^2} \qquad T_m = 9.43\text{ kN} \blacktriangleleft$$

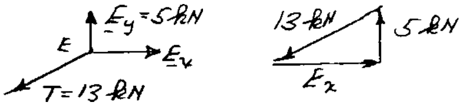


PROBLEM 7.94

Knowing that the maximum tension in cable *ABCDE* is 13 kN, determine the distance d_C.

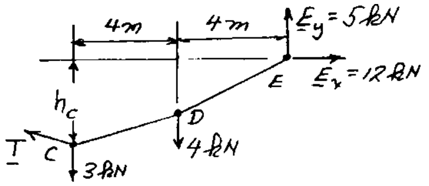
SOLUTION

Maximum tension of 13 kN occurs in *DE*. See solution of Problem 7.93 for the determination of $\mathbf{E}_y = 5.00 \text{ kN}\uparrow$



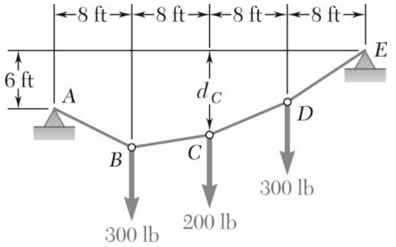
From force triangle $E_x^2 + 5^2 = 13^2$

Portion CDE: $E_x = 12 \text{ kN}$



$$+\circlearrowleft \Sigma M_C = 0: \quad (5 \text{ kN})(8 \text{ m}) - (12 \text{ kN})h_C - (4 \text{ kN})(4 \text{ m}) = 0$$

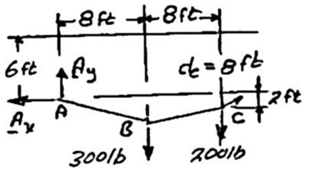
$h_C = 2.00 \text{ m}$ ◀



PROBLEM 7.95

If $d_C = 8$ ft, determine (*a*) the reaction at *A*, (*b*) the reaction at *E*.

SOLUTION



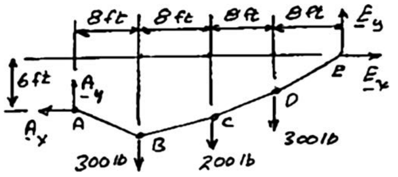
Free body: Portion *ABC*

$$+\circlearrowleft \Sigma M_C = 0$$

$$2A_x - 16A_y + 300(8) = 0$$

$$A_x = 8A_y - 1200 \qquad (1)$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_E = 0: \quad +6A_x + 32A_y - (300\text{ lb} + 200\text{ lb} + 300\text{ lb})16\text{ ft} = 0$$

$$3A_x + 16A_y - 6400 = 0$$

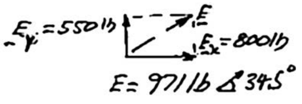
Substitute from Eq. (1):

$$3(8A_y - 1200) + 16A_y - 6400 = 0 \qquad \mathbf{A}_y = 250\text{ lb}\uparrow$$

Eq. (1) $$A_x = 8(250) - 1200 \qquad \mathbf{A}_x = 800\text{ lb}\leftarrow$$

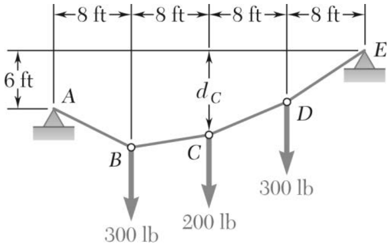
$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + E_x = 0 \quad -800\text{ lb} + E_x = 0 \qquad \mathbf{E}_x = 800\text{ lb}\rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad 250 + E_y - 300 - 200 - 300 = 0 \qquad \mathbf{E}_y = 550\text{ lb}\uparrow$$



(*a*) $\mathbf{A} = 838\text{ lb} \searrow 17.35^\circ$ ◀

(*b*) $\mathbf{E} = 971\text{ lb} \nearrow 34.5^\circ$ ◀

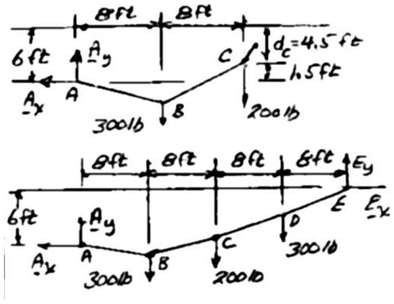


PROBLEM 7.96

If $d_C = 4.5$ ft, determine (*a*) the reaction at *A*, (*b*) the reaction at *E*.

SOLUTION

Free body: Portion *ABC*



$$\stackrel{+}{\circlearrowleft}\Sigma M_C = 0: \quad -1.5A_x - 16A_y + 300 \times 8 = 0$$

$$A_x = \frac{(2400 - 16A_y)}{1.5} \qquad (1)$$

Free body: Entire cable $\qquad \stackrel{+}{\circlearrowleft}\Sigma M_E = 0: \quad +6A_x + 32A_y - (300\text{ lb} + 200\text{ lb} + 300\text{ lb})16\text{ ft} = 0$

$$3A_x + 16A_y - 6400 = 0$$

Substitute from Eq. (1):

$$\frac{3(2400 - 16A_y)}{1.5} + 16A_y - 6400 = 0$$

$$A_y = -100\text{ lb}$$

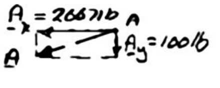
Thus A_y acts downward $\qquad \mathbf{A}_y = 100\text{ lb}\downarrow$

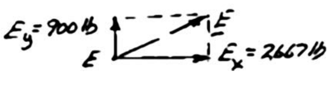
Eq. (1) $\qquad A_x = \frac{(2400 - 16(-100))}{1.5} = 2667\text{ lb} \qquad \mathbf{A}_x = 2667\text{ lb} \longleftarrow$

$$\xrightarrow{+}\Sigma F_x = 0: \quad -A_x + E_x = 0 \quad -2667 + E_x = 0 \qquad \mathbf{E}_x = 2667\text{ lb} \longrightarrow$$

$$+\!\uparrow \Sigma F_y = 0: \quad A_y + E_y - 300 - 200 - 300 = 0$$

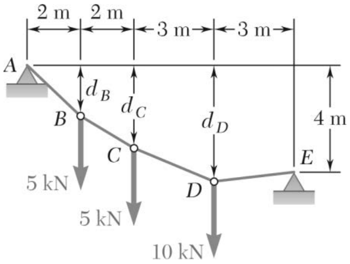
$$-100\text{ lb} + E_y - 800\text{ lb} = 0 \qquad \mathbf{E}_y = 900\text{ lb}\uparrow$$





(*a*) $\qquad \mathbf{A} = 2670\text{ lb} \angle 2.10°$ ◀

(*b*) $\qquad \mathbf{E} = 2810\text{ lb} \angle 18.65°$ ◀

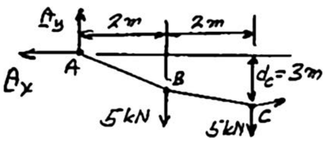


PROBLEM 7.97

Knowing that $d_C = 3$ m, determine (*a*) the distances d_B and d_D (*b*) the reaction at *E*.

SOLUTION

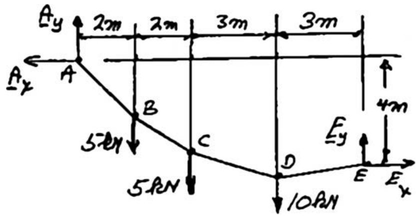
Free body: Portion *ABC*



$$+\circlearrowleft \Sigma M_C = 0: \quad 3A_x - 4A_y + (5\text{ kN})(2\text{ m}) = 0$$

$$A_x = \frac{4}{3}A_y - \frac{10}{3} \qquad (1)$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_E = 0: \quad 4A_x - 10A_y + (5\text{ kN})(8\text{ m}) + (5\text{ kN})(6\text{ m}) + (10\text{ kN})(3\text{ m}) = 0$$

$$4A_x - 10A_y + 100 = 0$$

Substitute from Eq. (1):

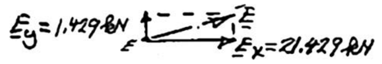
$$4\left(\frac{4}{3}A_y - \frac{10}{3}\right) - 10A_y + 100 = 0$$

$$A_y = +18.571\text{ kN} \qquad \mathbf{A}_y = 18.571\text{ kN}\uparrow$$

Eq. (1) $$A_x = \frac{4}{3}(18.511) - \frac{10}{3} = +21.429\text{ kN} \qquad \mathbf{A}_x = 21.429\text{ kN}\leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + E_x = 0 \quad -21.429 + E_x = 0 \qquad \mathbf{E}_x = 21.429\text{ kN}\rightarrow$$

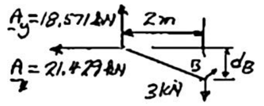
$$+\uparrow \Sigma F_y = 0: \quad 18.571\text{ kN} + E_y + 5\text{ kN} + 5\text{ kN} + 10\text{ kN} = 0 \qquad \mathbf{E}_y = 1.429\text{ kN}\uparrow$$



(*b*) $\mathbf{E} = 21.5\text{ kN} \measuredangle 3.81^\circ$ ◀

PROBLEM 7.97 (Continued)

(*a*) Portion *AB*

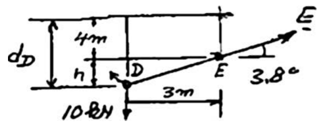


$$+\circlearrowleft \Sigma M_B = 0: \quad (18.571 \text{ kN})(2 \text{ m}) - (21.429 \text{ kN})d_B = 0$$

$d_B = 1.733$ m ◀

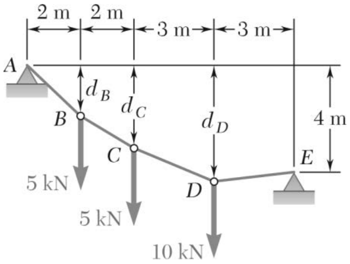
Portion *DE*

Geometry



$$h = (3 \text{ m}) \tan 3.8°$$
$$= 0.199 \text{ m}$$
$$d_D = 4 \text{ m} + 0.199 \text{ m}$$

$d_D = 4.20$ m ◀

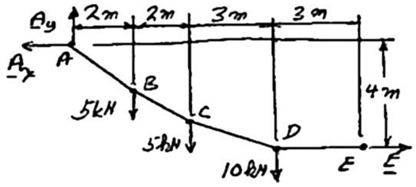


PROBLEM 7.98

Determine (*a*) distance d_C for which portion *DE* of the cable is horizontal, (*b*) the corresponding reactions at *A* and *E*.

SOLUTION

Free body: Entire cable



(*b*)

$$+\uparrow \Sigma F_y = 0: \quad A_y - 5\text{ kN} - 5\text{ kN} - 10\text{ kN} = 0 \qquad \mathbf{A}_y = 20\text{ kN}\uparrow$$

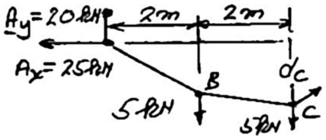
$$+\circlearrowleft \Sigma M_A = 0: \quad E(4\text{ m}) - (5\text{ kN})(2\text{ m}) - (5\text{ kN})(4\text{ m}) - (10\text{ kN})(7\text{ m}) = 0$$

$$E = +25\text{ kN} \qquad \mathbf{E} = 25.0\text{ kN} \longrightarrow \blacktriangleleft$$

$$\Sigma F_x = 0: \quad -A_x + 25\text{ kN} = 0 \qquad \mathbf{A}_x = 25\text{ kN} \longleftarrow$$

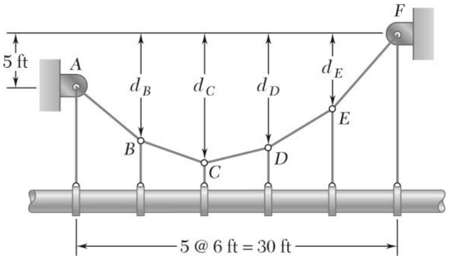
$$\mathbf{A} = 32.0\text{ kN} \measuredangle 38.7^\circ \blacktriangleleft$$

(*a*) Free body: Portion *ABC*



$$+\circlearrowleft \Sigma M_C = 0: \quad (25\text{ kN})d_C - (20\text{ kN})(4\text{ m}) + (5\text{ kN})(2\text{ m}) = 0$$

$$25d_C - 70 = 0 \qquad d_C = 2.80\text{ m} \blacktriangleleft$$

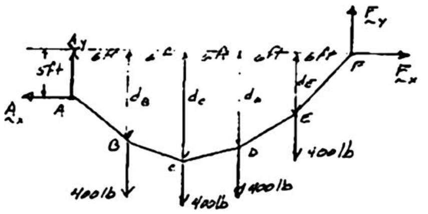


PROBLEM 7.99

An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 12$ ft, determine (*a*) the maximum tension in the cable, (*b*) the distance d_D.

SOLUTION

FBD Cable: Hanger forces at *A* and *F* act on the supports, so $\mathbf{A}_y$ and $\mathbf{F}_y$ act on the cable.

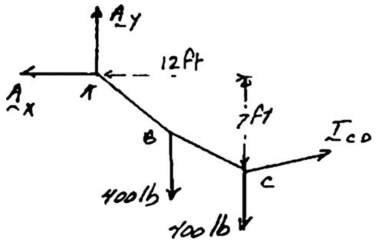


$$\circlearrowleft \Sigma M_F = 0: \quad (6\text{ ft} + 12\text{ ft} + 18\text{ ft} + 24\text{ ft})(400\text{ lb})$$

$$-(30\text{ ft})A_y - (5\text{ ft})A_x = 0$$

$$A_x + 6A_y = 4800\text{ lb} \qquad (1)$$

FBD ABC: $\circlearrowleft \Sigma M_C = 0: \quad (7\text{ ft})A_x - (12\text{ ft})A_y + (6\text{ ft})(400\text{ lb}) = 0 \qquad (2)$



Solving (1) and (2)

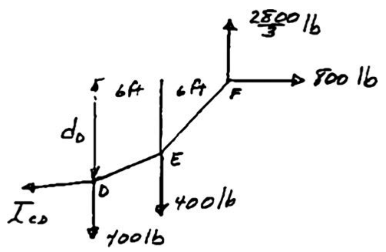
$$\mathbf{A}_x = 800\text{ lb} \longleftarrow$$

$$\mathbf{A}_y = \frac{2000}{3}\text{ lb}\uparrow$$

From FBD Cable: $\longrightarrow \Sigma F_x = 0: \quad -800\text{ lb} + F_x = 0$

$$\mathbf{F}_x = 800\text{ lb} \longrightarrow$$

FBD DEF:



$$\uparrow \Sigma F_y = 0: \quad \frac{200}{3}\text{ lb} - 4(400\text{ lb}) + F_y = 0$$

$$\mathbf{F}_y = \frac{2800}{3}\text{ lb}\uparrow$$

Since $A_x = F_x$ and $F_y > A_y$, $\quad T_{\max} = T_{EF} = \sqrt{(800\text{ lb})^2 + \left(\frac{2800}{3}\text{ lb}\right)^2}$

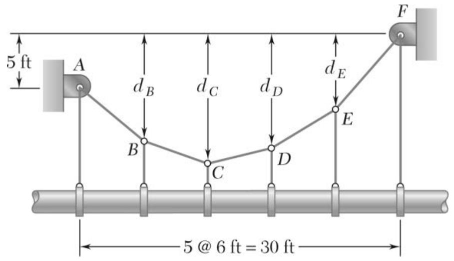
PROPRIETARY MATERIAL.

PROBLEM 7.99 (Continued)

(*a*) $T_{\max} = 1229.27 \text{ lb},$ $T_{\max} = 1229 \text{ lb}$ ◀

$$\circlearrowleft \Sigma M_D = 0: \quad (12 \text{ ft})\left(\frac{2800}{3} \text{lb}\right) - d_D(800 \text{ lb}) - (6 \text{ ft})(400 \text{ lb}) = 0$$

(*b*) $d_D = 11.00 \text{ ft}$ ◀



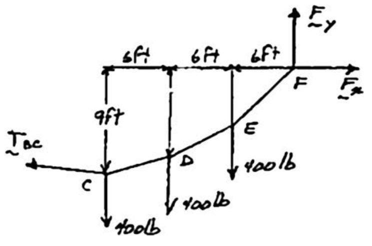
PROBLEM 7.100

Solve Problem 7.99 assuming that $d_C = 9$ ft.

PROBLEM 7.99 An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 12$ ft, determine (*a*) the maximum tension in the cable, (*b*) the distance d_D.

SOLUTION

FBD CDEF:



$$\circlearrowleft \Sigma M_C = 0: \quad (18\text{ ft})F_y - (9\text{ ft})F_y - (6\text{ ft} + 12\text{ ft})(400\text{ lb}) = 0$$

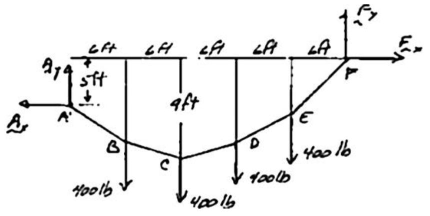
$$F_x - 2F_y = -800\text{ lb} \qquad (1)$$

FBD Cable:

$$\circlearrowleft EM_A = 0: \quad (30\text{ ft})F_y - (5\text{ ft})F_x$$
$$-(6\text{ ft})(1+2+3+4)(400\text{ lb}) = 0$$

$$F_x - 6F_y = -4800\text{ lb} \qquad (2)$$

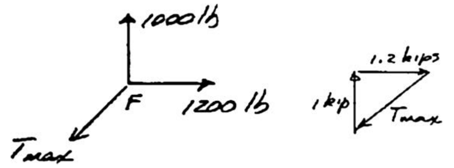
Solving (1) and (2),



$$\mathbf{F}_x = 1200\text{ lb} \longrightarrow, \quad \mathbf{F}_y = 1000\text{ lb}\uparrow$$

$$\longrightarrow \Sigma F_x = 0: \quad -A_x + 1200\text{ lb} = 0, \quad \mathbf{A}_x = 1200\text{ lb} \longrightarrow$$

Point F:



$$\uparrow \Sigma F_y = 0: \quad A_y + 1000\text{ lb} - 4(400\text{ lb}) = 0, \qquad \mathbf{A}_y = 600\text{ lb}\uparrow$$

PROBLEM 7.100 (Continued)

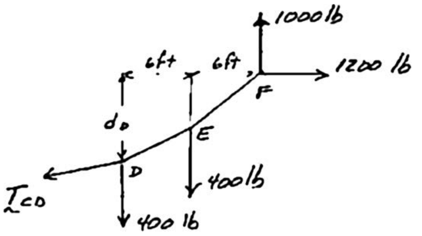
Since

$$A_x = A_y \quad \text{and} \quad F_y > A_y,\ T_{\max} = T_{EF}$$

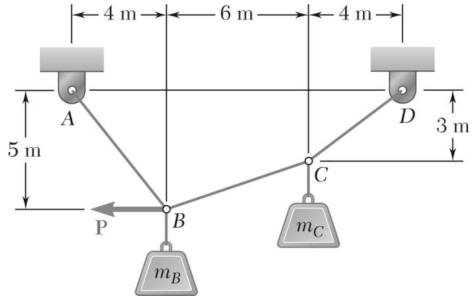
$$T_{\max} = \sqrt{(1 \text{ kip})^2 + (1.2 \text{ kips})^2}$$

(*a*) $T_{\max} = 1.562 \text{ kips}$ ◀

FBD DEF: $\circlearrowleft \Sigma M_D = 0\text{:}\quad (12 \text{ ft})(1000 \text{ lb}) - d_D(1200 \text{ lb}) - (6 \text{ ft})(400 \text{ lb}) = 0$



(*b*) $d_D = 8.00 \text{ ft}$ ◀



PROBLEM 7.101

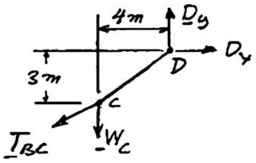
Knowing that $m_B = 70$ kg and $m_C = 25$ kg, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

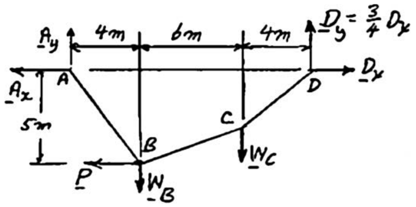
Free body: Portion *CD*

$$+\circlearrowleft \Sigma M_C = 0: \quad D_y(4\text{ m}) - D_x(3\text{ m}) = 0$$

$$D_y = \frac{3}{4}D_x$$

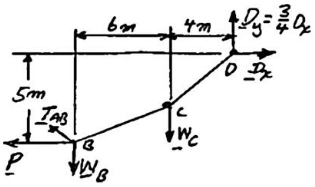


Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{3}{4}D_x(14\text{ m}) - W_B(4\text{ m}) - W_C(10\text{ m}) - P(5\text{ m}) = 0 \qquad (1)$$

Free body: Portion *BCD*



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{3}{4}D_x(10\text{ m}) - D_x(5\text{ m}) - W_C(6\text{ m}) = 0$$

$$D_x = 2.4W_C \qquad (2)$$

For $\quad m_B = 70\text{ kg} \quad m_C = 25\text{ kg}$

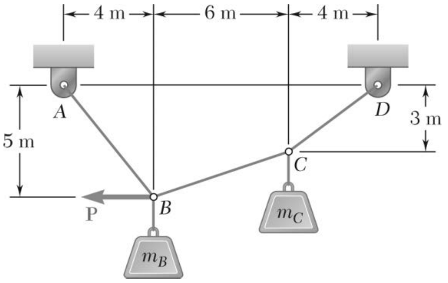
$g = 9.81\text{ m/s}^2$: $\quad W_B = 70g \quad W_C = 25g$

Eq. (2): $\quad D_x = 2.4W_C = 2.4(25g) = 60g$

Eq. (1): $\quad \frac{3}{4}60g(14) - 70g(4) - 25g(10) - 5P = 0$

$$100g - 5P = 0: \quad P = 20g$$

$$P = 20(9.81) = 196.2\text{ N} \qquad P = 196.2\text{ N} \blacktriangleleft$$

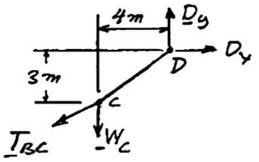


PROBLEM 7.102

Knowing that $m_B = 18$ kg and $m_C = 10$ kg, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

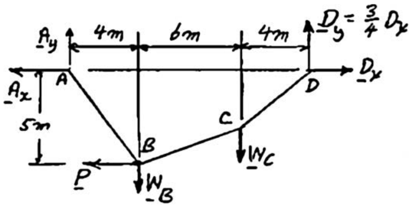
Free body: Portion *CD*



$$+\circlearrowleft \Sigma M_C = 0: \quad D_y(4\text{ m}) - D_x(3\text{ m}) = 0$$

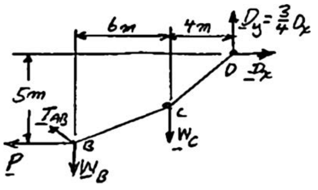
$$D_y = \frac{3}{4}D_x$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{3}{4}D_x(14\text{ m}) - W_B(4\text{ m}) - W_C(10\text{ m}) - P(5\text{ m}) = 0 \qquad (1)$$

Free body: Portion *BCD*



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{3}{4}D_x(10\text{ m}) - D_x(5\text{ m}) - W_C(6\text{ m}) = 0$$

$$D_x = 2.4W_C \qquad (2)$$

For $\quad m_B = 18\text{ kg} \quad m_C = 10\text{ kg}$

$g = 9.81\text{ m/s}^2$: $\quad W_B = 18g \quad W_C = 10g$

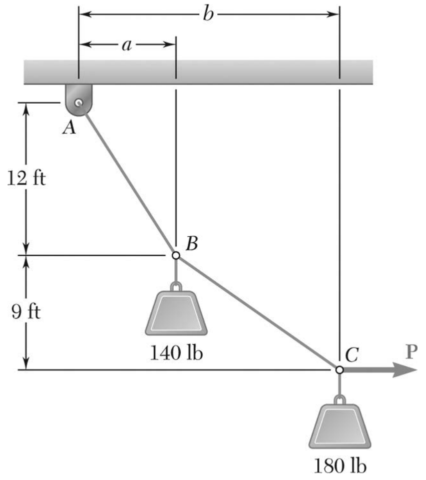
Eq. (2): $\quad D_x = 2.4W_C = 2.4(10g) = 24g$

Eq. (1): $\quad \frac{3}{4}24g(14) - (18g)(4) - (10g)(10) - 5P = 0$

$$80g - 5P: \quad P = 16g$$

$$P = 16(9.81) = 156.96\text{ N}$$

$P = 157.0$ N ◀

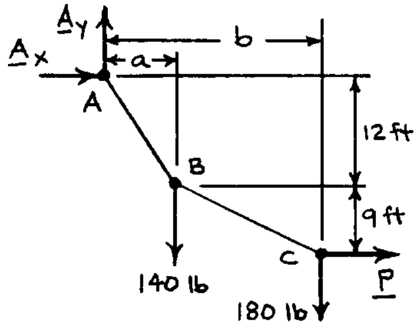


PROBLEM 7.103

Cable *ABC* supports two loads as shown. Knowing that $b = 21$ ft, determine (*a*) the required magnitude of the horizontal force **P**, (*b*) the corresponding distance *a*.

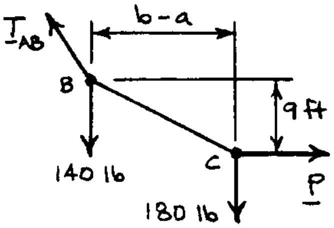
SOLUTION

Free body: *ABC*



$$+\circlearrowleft \Sigma M_A = 0: \quad P(12\text{ ft}) - (140\text{ lb})a - (180\text{ lb})b = 0 \qquad (1)$$

Free body: *BC*



$$+\circlearrowleft \Sigma M_B = 0: \quad P(9\text{ ft}) - (180\text{ lb})(b-a) = 0 \qquad (2)$$

Data $\quad b = 21$ ft

Eq. (1): $\quad 21P - 140a - 180(21) = 0 \qquad P = \frac{20}{3}a + 180 \qquad (3)$

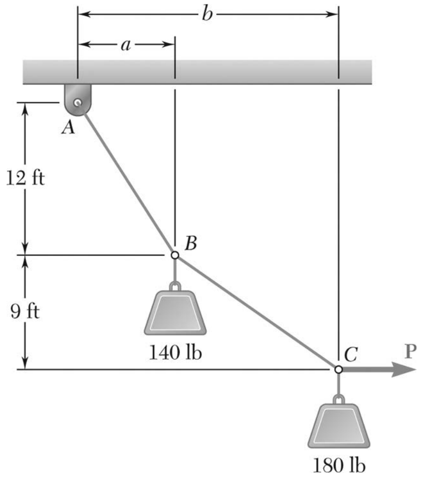
Eq. (2): $\quad 9P - 180(21-a) = 0 \qquad P = -20a + 420 \qquad (4)$

Equate (3) and (4) through P: $\quad \frac{20}{3}a + 180 = -20a + 420$

(*b*) $\quad a = 9.00\text{ ft}$ ◀

Eq. (3): $\quad P = \frac{20}{3}(9.00) + 180$

(*a*) $\quad P = 240\text{ lb}$ ◀

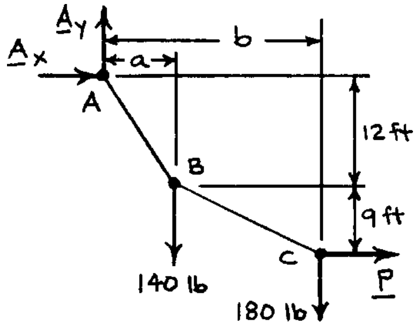


PROBLEM 7.104

Cable ABC supports two loads as shown. Determine the distances a and b when a horizontal force $\mathbf{P}$ of magnitude 200 lb is applied at A.

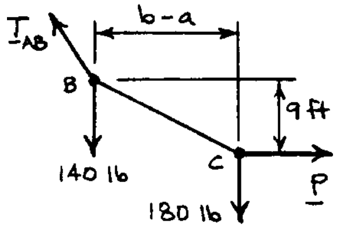
SOLUTION

Free body: ABC



$$+\circlearrowleft \Sigma M_A = 0: \quad P(12\text{ ft}) - (140\text{ lb})a - (180\text{ lb})b = 0 \tag{1}$$

Free body: BC



$$+\circlearrowleft \Sigma M_B = 0: \quad P(9\text{ ft}) - (180\text{ lb})(b-a) = 0 \tag{2}$$

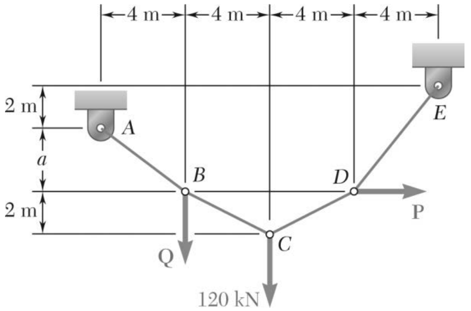
Data $\quad P = 200\text{ lb}$

Eq. (1): $\quad 200(21) - 140a - 180b = 0 \quad (3)$

Eq. (2): $\quad 200(9) + 180a - 180b = 0 \quad (4)$

$(3)-(4)$: $\quad 2400 - 320a = 0 \qquad a = 7.50\text{ ft}$ ◀

Eq. (2): $\quad (200\text{ lb})(9\text{ ft}) - (180\text{ lb})(b - 7.50\text{ ft}) = 0 \qquad b = 17.50\text{ ft}$ ◀

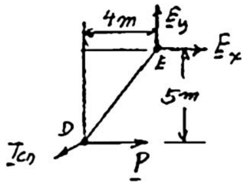


PROBLEM 7.105

If $a = 3$ m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

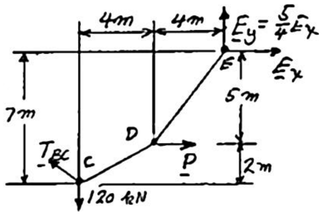
Free body: Portion *DE*



$$+\circlearrowleft \Sigma M_D = 0: \quad E_y(4\text{ m}) - E_x(5\text{ m}) = 0$$

$$E_y = \frac{5}{4}E_x$$

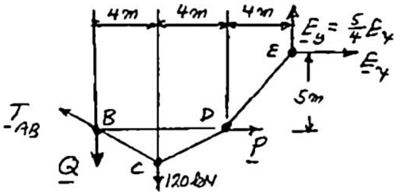
Free body: Portion *CDE*



$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{5}{4}E_x(8\text{ m}) - E_x(7\text{ m}) - P(2\text{ m}) = 0$$

$$E_x = \frac{2}{3}P \qquad (1)$$

Free body: Portion *BCDE*



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{5}{4}E_x(12\text{ m}) - E_x(5\text{ m}) - (120\text{ kN})(4\text{ m}) = 0$$

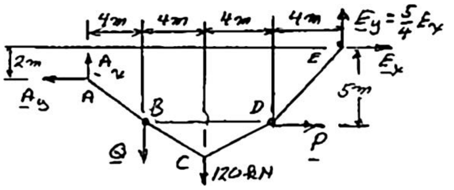
$$10E_x - 480 = 0; \quad E_x = 48\text{ kN}$$

Eq. (1):

$$48\text{ kN} = \frac{2}{3}P \qquad P = 72.0\text{ kN} \blacktriangleleft$$

PROBLEM 7.105 (Continued)

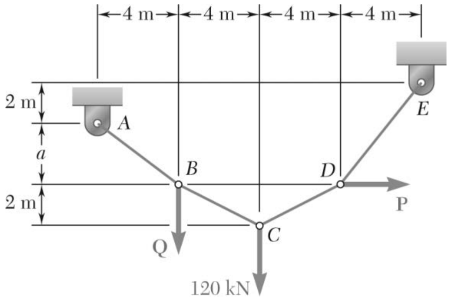
Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{5}{4}E_x(16\text{ m}) - E_x(2\text{ m}) + P(3\text{ m}) - Q(4\text{ m}) - (120\text{ kN})(8\text{ m}) = 0$$

$$(48\text{ kN})(20\text{ m} - 2\text{ m}) + (72\text{ kN})(3\text{ m}) - Q(4\text{ m}) - 960\text{ kN}\cdot\text{m} = 0$$

$$4Q = 120 \qquad Q = 30.0\text{ kN} \blacktriangleleft$$

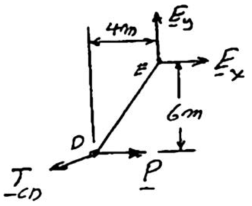


PROBLEM 7.106

If $a = 4$ m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

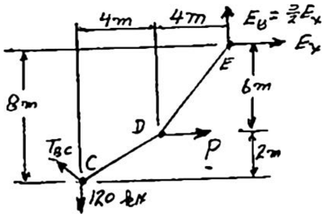
Free body: Portion *DE*



$$+\circlearrowleft \Sigma M_D = 0: \quad E_y(4\text{ m}) - E_x(6\text{ m}) = 0$$

$$E_y = \frac{3}{2}E_x$$

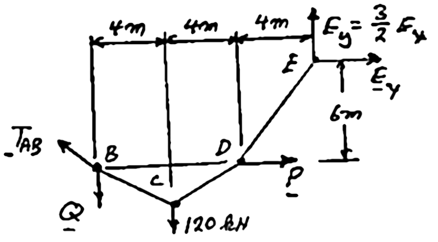
Free body: Portion *CDE*



$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{3}{2}E_x(8\text{ m}) - E_x(8\text{ m}) - P(2\text{ m}) = 0$$

$$E_x = \frac{1}{2}P \qquad (1)$$

Free body: Portion *BCDE*



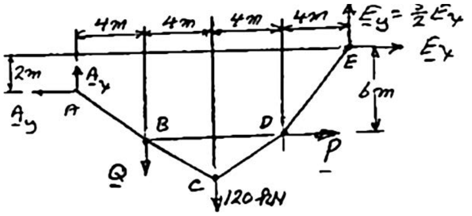
$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{3}{2}E_x(12\text{ m}) - E_x(6\text{ m}) + (120\text{ kN})(4\text{ m}) = 0$$

$$12E_x = 480 \qquad E_x = 40\text{ kN}$$

Eq (1): $\quad E_x = \frac{1}{2}P; \qquad 40\text{ kN} = \frac{1}{2}P \qquad\qquad P = 80.0\text{ kN}$ ◀

PROBLEM 7.106 (Continued)

<u>Free body: Entire cable</u>



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{3}{2}E_x(16\text{ m}) - E_x(2\text{ m}) + P(4\text{ m}) - Q(4\text{ m}) - (120\text{ kN})(8\text{ m}) = 0$$

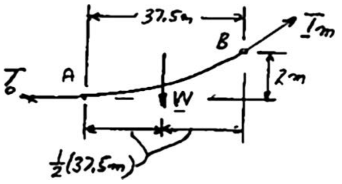
$$(40\text{ kN})(24\text{ m} - 2\text{ m}) + (80\text{ kN})(4\text{ m}) - Q(4\text{ m}) - 960\text{ kN}\cdot\text{m} = 0$$

$$4Q = 240 \qquad Q = 60.0\text{ kN} \blacktriangleleft$$

PROBLEM 7.107

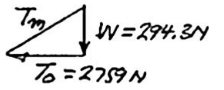
A transmission cable having a mass per unit length of 0.8 kg/m is strung between two insulators at the same elevation that are 75 m apart. Knowing that the sag of the cable is 2 m, determine (*a*) the maximum tension in the cable, (*b*) the length of the cable.

SOLUTION



$$w = (0.8\text{ kg/m})(9.81\text{ m/s}^2)$$
$$= 7.848\text{ N/m}$$
$$W = (7.848\text{ N/m})(37.5\text{ m})$$
$$W = 294.3\text{ N}$$

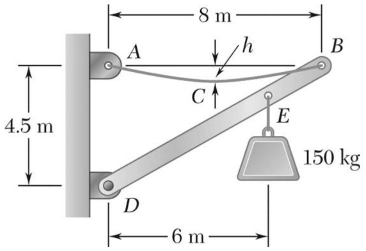
(*a*)
$$+\circlearrowleft \Sigma M_B = 0\text{:}\quad T_0(2\text{ m}) - W\left(\frac{1}{2}37.5\text{ m}\right) = 0$$



$$T_0(2\text{ m}) - (294.3\text{ N})\frac{1}{2}(37.5\text{ m}) = 0$$
$$T_0 = 2759\text{ N}$$
$$T_m^2 = (294.3\text{ N})^2 + (2759\text{ N})^2 \qquad T_m = 2770\text{ N} \blacktriangleleft$$

(*b*)
$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 + \cdots\right]$$
$$= 37.5\text{ m}\left[1 + \frac{2}{3}\left(\frac{2\text{ m}}{37.5\text{ m}}\right)^2 + \cdots\right]$$
$$= 37.57\text{ m}$$

$$\text{Length} = 2s_B = 2(37.57\text{ m}) \qquad \text{Length} = 75.14\text{ m} \blacktriangleleft$$

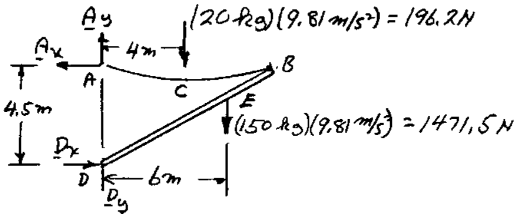


PROBLEM 7.108

The total mass of cable *ACB* is 20 kg. Assuming that the mass of the cable is distributed uniformly along the horizontal, determine (*a*) the sag *h*, (*b*) the slope of the cable at *A*.

SOLUTION

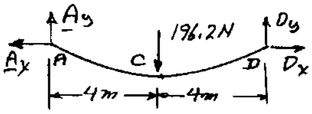
Free body: Entire frame



$$+\circlearrowleft \Sigma M_D = 0: \quad A_x(4.5\text{ m}) - (196.2\text{ N})(4\text{ m}) - (1471.5\text{ N})(6\text{ m}) = 0$$

$$A_x = 2136.4\text{ N}$$

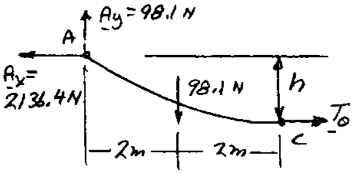
Free body: Entire cable



$$+\circlearrowleft \Sigma M_D = 0: \quad A_y(8\text{ m}) - (196.2\text{ N})(4\text{ m}) = 0$$

$$A_y = 98.1\text{ N}$$

(*a*) Free body: Portion *AC*



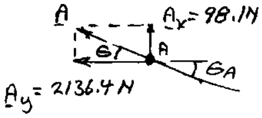
$$\Sigma F_x = 0: \quad T_0 = A_x = 2136.4\text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad T_0 h - (98.1\text{ N})(2\text{ m}) = 0$$

$$(2136.4\text{ N})h - 196.2\text{ N}\cdot\text{m} = 0$$

$$h = 0.09183\text{ m} \qquad h = 91.8\text{ mm} \blacktriangleleft$$

(*b*)



$$\tan\theta_A = \frac{A_x}{A_y} = \frac{98.1\text{ N}}{2136.4\text{ N}}$$

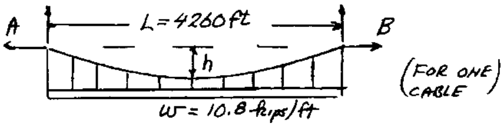
$$\tan\theta_A = 0.045918$$

$$\theta_A = 2.629^\circ \qquad \theta_A = 2.63^\circ \blacktriangleleft$$

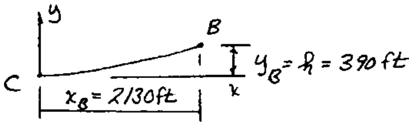
PROBLEM 7.109

The center span of the Verrazano-Narrows Bridge consists of two uniform roadways suspended from four cables. The uniform load supported by each cable is $w = 10.8$ kips/ft along the horizontal. Knowing that the span L is 4260 ft and that the sag h is 390 ft, determine (*a*) the maximum tension in each cable, (*b*) the length of each cable.

SOLUTION



(*a*)



At B:

$$y_B = \frac{wx_B^2}{2T_0}$$

$$390\text{ ft} = \frac{(10.8\text{ kips/ft})(2130\text{ ft})^2}{2T_0} \qquad T_0 = 62{,}819\text{ kips}$$

$$T_m = \sqrt{T_0^2 + w^2 x_B^2}$$

$$= \sqrt{(62{,}819\text{ kips})^2 + (10.8\text{ kips/ft})^2(2130\text{ ft})^2}$$

$$= \sqrt{62{,}819^2 + (23{,}004)^2} = 66{,}898\text{ kips} \qquad T_m = 66{,}900\text{ kips} \blacktriangleleft$$

(*b*) Length of cable

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4\right]$$

$$= (2130\text{ ft})\left[1 + \frac{2}{3}\left(\frac{390\text{ ft}}{2130\text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{390\text{ ft}}{2130\text{ ft}}\right)^4\right] = 2176.65\text{ ft}$$

Total length: $2S_B = 2(2176.65\text{ ft}) = 4353.3\text{ ft}$ $\qquad L = 4353\text{ ft} \blacktriangleleft$

PROBLEM 7.110

The center span of the Verrazano-Narrows Bridge consists of two uniform roadways suspended from four cables. The design of the bridge allows for the effect of extreme temperature changes that cause the sag of the center span to vary from $h_w = 386$ ft in winter to $h_s = 394$ ft in summer. Knowing that the span is $L = 4260$ ft, determine the change in length of the cables due to extreme temperature changes.

SOLUTION

Eq. 7.10:

$$s_B = x_B\left[1+\frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \cdots\right]$$

Winter:

$$y_B = h = 386\text{ ft},\quad x_B = \frac{1}{2}L = 2130\text{ ft}$$

$$s_B = (2130)\left[1+\frac{2}{3}\left(\frac{386}{2130}\right)^2 - \frac{2}{5}\left(\frac{386}{2130}\right)^4 + \cdots\right] = 2175.715\text{ ft}$$

Summer:

$$y_B = h = 394\text{ ft},\quad x_B = \frac{1}{2}L = 2130\text{ ft}$$

$$s_B = (2130)\left[1+\frac{2}{3}\left(\frac{394}{2130}\right)^2 - \frac{2}{5}\left(\frac{394}{2130}\right)^4 + \cdots\right] = 2177.59\text{ ft}$$

$$\Delta = 2(\Delta\, s_B) = 2(2177.59\text{ ft} - 2175.715\text{ ft}) = 2(1.875\text{ ft})$$

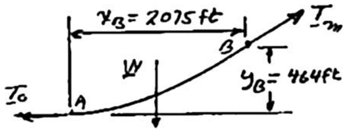
Change in length $= 3.75$ ft ◀

PROBLEM 7.111

Each cable of the Golden Gate Bridge supports a load $w = 11.1$ kips/ft along the horizontal. Knowing that the span L is 4150 ft and that the sag h is 464 ft, determine (*a*) the maximum tension in each cable, (*b*) the length of each cable.

SOLUTION

Eq. (7.8) Page 386:

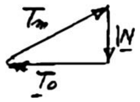


At B:

$$y_B = \frac{wx_B^2}{2T_0}$$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(11.1\text{ kip/ft})(2075\text{ ft})^2}{2(464\text{ ft})}$$

(*a*)



$$T_0 = 51.500\text{ kips}$$

$$W = wx_B = (11.1\text{ kips/ft})(2075\text{ ft}) = 23.033\text{ kips}$$

$$T_m = \sqrt{T_0^2 + W^2} = \sqrt{(51.500\text{ kips})^2 + (23.033\text{ kips})^2}$$

$$T_m = 56{,}400\text{ kips} \blacktriangleleft$$

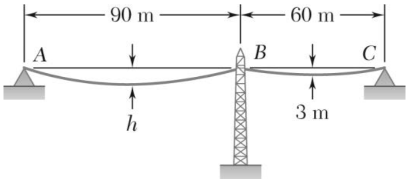
(*b*)

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{y_B}\right)^4 + \cdots\right] \qquad \frac{y_B}{x_B} = \frac{464\text{ ft}}{2075\text{ ft}} = 0.22361$$

$$s_B = (2075\text{ ft})\left[1 + \frac{2}{3}(0.22361)^2 - \frac{2}{5}(0.22361)^4 + \cdots\right] = 2142.1\text{ ft}$$

$\text{Length} = 2s_B = 2(2142.1\text{ ft})$

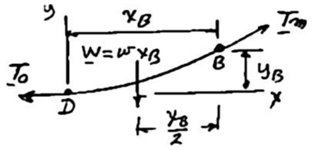
Length $= 4284$ ft ◀



PROBLEM 7.112

Two cables of the same gauge are attached to a transmission tower at *B*. Since the tower is slender, the horizontal component of the resultant of the forces exerted by the cables at *B* is to be zero. Knowing that the mass per unit length of the cables is 0.4 kg/m, determine (*a*) the required sag *h*, (*b*) the maximum tension in each cable.

SOLUTION



$$W = wx_B$$

$$+\circlearrowleft \Sigma M_B = 0: \quad T_0 \ y_B - (wx_B)\frac{y_B}{2} = 0$$

(*a*) $\quad$ Horiz. comp. $= T_0 = \dfrac{wx_B^2}{2y_B}$

<u>Cable *AB*</u>: $\quad x_B = 45 \text{ m}$

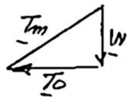
$$T_0 = \frac{w(45 \text{ m})^2}{2h}$$

<u>Cable *BC*</u>: $\quad x_B = 30 \text{ m}, \quad y_B = 3 \text{ m}$

$$T_0 = \frac{w(30 \text{ m})^2}{2(3 \text{ m})}$$

Equate $T_0 = T_0$ $\quad \dfrac{w(45 \text{ m})^2}{2h} = \dfrac{w(30 \text{ m})^2}{2(3 \text{ m})}$ $\qquad h = 6.75 \text{ m}$ ◀

(*b*) $\quad T_m^2 = T_0^2 + W^2$



<u>Cable *AB*</u>: $\quad w = (0.4 \text{ kg/m})(9.81 \text{ m/s}) = 3.924 \text{ N/m}$

$$x_B = 45 \text{ m}, \quad y_B = h = 6.75 \text{ m}$$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(3.924 \text{ N/m})(45 \text{ m})^2}{2(6.75 \text{ m})} = 588.6 \text{ N}$$

$$W = wx_B = (3.924 \text{ N/m})(45 \text{ m}) = 176.58 \text{ N}$$

$$T_m^2 = (588.6 \text{ N})^2 + (176.58 \text{ N})^2$$

<u>For *AB*</u>: $\qquad T_m = 615 \text{ N}$ ◀

PROBLEM 7.112 (Continued)

Cable *BC*: $x_B = 30 \text{ m}, \quad y_B = 3 \text{ m}$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(3.924 \text{ N/m})(30 \text{ m})^2}{2(3 \text{ m})} = 588.6 \text{ N} \quad \text{(Checks)}$$

$$W = wx_B = (3.924 \text{ N/m})(30 \text{ m}) = 117.72 \text{ N}$$

$$T_m^2 = (588.6 \text{ N})^2 + (117.72 \text{ N})^2$$

For *BC*: $T_m = 600 \text{ N}$ ◀

PROBLEM 7.113

A 50.5-m length of wire having a mass per unit length of 0.75 kg/m is used to span a horizontal distance of 50 m. Determine (*a*) the approximate sag of the wire, (*b*) the maximum tension in the wire. [*Hint:* Use only the first two terms of Eq. (7.10).]

SOLUTION

First two terms of Eq. 7.10

(*a*)

$$s_B = \frac{1}{2}(50.5\text{ m}) = 25.25\text{ m},$$

$$x_B = \frac{1}{2}(50\text{ m}) = 25\text{ m}$$

$$y_B = h$$

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

$$25.25\text{ m} = 25\text{ m}\left[1 - \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

$$\left(\frac{y_B}{x_B}\right)^2 = 0.01\left(\frac{3}{2}\right)^2 = \sqrt{0.015}$$

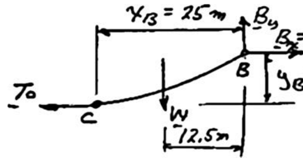
$$\frac{y_B}{x_B} = 0.12247$$

$$\frac{h}{25\text{ m}} = 0.12247$$

$$h = 3.0619\text{ m}$$

$h = 3.06$ m ◀

(*b*) Free body: Portion *CB*



$$w = (0.75\text{ kg/m})(9.81\text{m}) = 7.3575\text{ N/m}$$

$$W = s_B\ w = (25.25\text{ m})(7.3575\text{ N/m})$$

$$W = 185.78\text{ N}$$

$$+\circlearrowleft \Sigma M_0 = 0: \quad T_0\,(3.0619\text{ m}) - (185.78\text{ N})(12.5\text{ m}) = 0$$

$$T_0 = 758.4\text{ N}$$

$$B_x = T_0 = 758.4\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 185.78\text{ N} = 0 \quad B_y = 185.78\text{ N}$$

$$T_m = \sqrt{B_x^2 + B_y^2} = \sqrt{(758.4\text{ N})^2 + (185.78\text{ N})^2}$$

$T_m = 781$ N ◀

PROBLEM 7.114

A cable of length $L + \Delta$ is suspended between two points that are at the same elevation and a distance L apart. (*a*) Assuming that Δ is small compared to L and that the cable is parabolic, determine the approximate sag in terms of L and Δ. (*b*) If $L = 100$ ft and $\Delta = 4$ ft, determine the approximate sag. [*Hint:* Use only the first two terms of Eq. (7.10).

SOLUTION

Eq. 7.10 (First two terms)

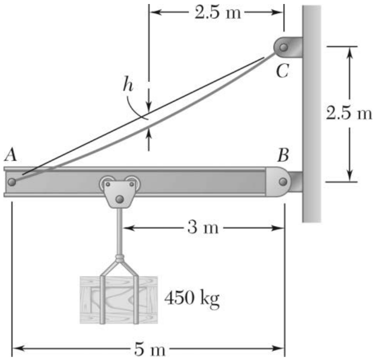
(*a*)
$$s_B = x_B\left[1+\frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

$$x_B = L/2$$
$$s_B = \frac{1}{2}(L+\Delta)$$
$$y_B = h$$

$$\frac{1}{2}(L+\Delta) = \frac{L}{2}\left[1+\frac{2}{3}\left(\frac{h}{\frac{L}{2}}\right)^2\right]$$

$$\frac{\Delta}{2} = \frac{4}{3}\frac{h^2}{L}; \quad h^2 = \frac{3}{8}L\Delta; \qquad h = \sqrt{\frac{3}{8}L\Delta} \blacktriangleleft$$

(*b*)
$$L = 100 \text{ ft}, \quad h = 4 \text{ ft}. \qquad h = \sqrt{\frac{3}{8}(100)(4)}; \qquad h = 12.25 \text{ ft} \blacktriangleleft$$



PROBLEM 7.115

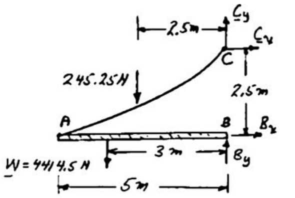
The total mass of cable *AC* is 25 kg. Assuming that the mass of the cable is distributed uniformly along the horizontal, determine the sag *h* and the slope of the cable at *A* and *C*.

SOLUTION

Cable: $m = 25\text{ kg}$

$W = 25\,(9.81)$

$= 245.25\text{ N}$



Block: $m = 450\text{ kg}$

$W = 4414.5\text{ N}$

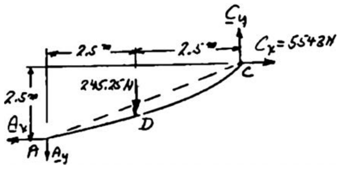
$$+\circlearrowleft \Sigma M_B = 0{:}\quad (245.25)(2.5) + (4414.5)(3) - C_x(2.5) = 0$$

$$C_x = 5543\text{ N}$$

$$\Sigma F_x = 0{:}\quad A_x = C_x = 5543\text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0{:}\quad C_y(5) - (5543)(2.5) - (245.25)(2.5) = 0$$

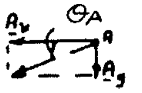
$$\mathbf{C}_y = 2894\text{ N}\uparrow$$



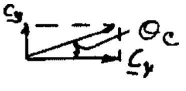
$$+\uparrow \Sigma F_y = 0{:}\quad C_y - A_y - 245.25\text{ N} = 0$$

$$2894\,N - A_y - 245.25\text{ N} = 0 \qquad \mathbf{A}_y = 2649\text{ N}\downarrow$$

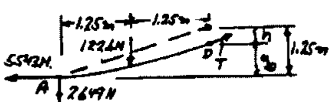
Point *A*: $\tan\theta_A = \dfrac{A_y}{A_x} = \dfrac{2649}{5543} = 0.4779;$ $\theta_A = 25.5°$ ◀



Point *C*: $\tan\theta_C = \dfrac{C_y}{C_x} = \dfrac{2894}{5543} = 0.5221;$ $\theta_A = 27.6°$ ◀



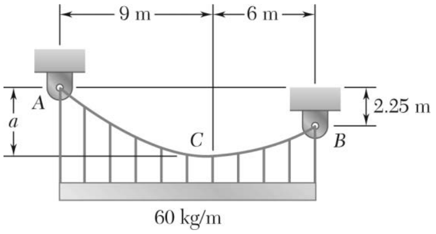
Free body: Half cable $W = (12.5\text{ kg})\,g = 122.6$



$$+\circlearrowleft \Sigma M_0 = 0{:}\quad (122.6\text{ N})(1.25\text{ M}) + (2649\text{ N})(2.5\text{ m}) - (5543\text{ N})\,y_d = 0$$

$$y_d = 1.2224\text{ m};\ \text{sag} = h = 1.25\text{ m} - 1.2224\text{ m}$$

$$h = 0.0276\text{ m} = 27.6\text{ mm} \quad ◀$$

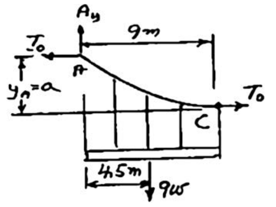


PROBLEM 7.116

Cable *ACB* supports a load uniformly distributed along the horizontal as shown. The lowest Point *C* is located 9 m to the right of *A*. Determine (*a*) the vertical distance *a*, (*b*) the length of the cable, (*c*) the components of the reaction at *A*.

SOLUTION

Free body: Portion *AC*

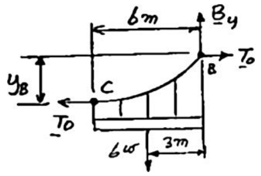


$$+\uparrow \Sigma F_y = 0: \quad A_y - 9w = 0$$

$$\mathbf{A}_y = 9w \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad T_0 a - (9w)(4.5\text{ m}) = 0 \qquad (1)$$

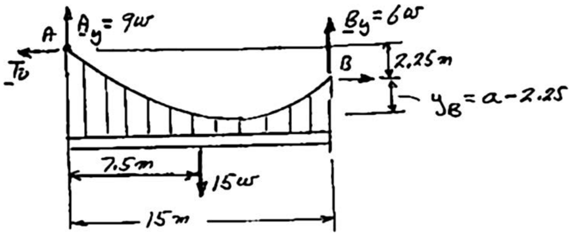
Free body: Portion *CB*



$$+\uparrow \Sigma F_y = 0: \quad B_y - 6w = 0$$

$$\mathbf{B}_y = 6w \uparrow$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad 15w(7.5\text{ m}) - 6w(15\text{ m}) - T_0(2.25\text{ m}) = 0$$

(*a*) $$T_0 = 10w$$

Eq. (1): $$T_0 a - (9w)(4.5\text{ m}) = 0$$

$$10wa = (9w)(4.5) = 0 \qquad a = 4.05\text{ m} ◀$$

PROBLEM 7.116 (Continued)

(*b*) Length $= AC + CB$

Portion *AC*: $x_A = 9\text{ m},\quad y_A = a = 4.05\text{ m};\quad \dfrac{y_A}{x_A} = \dfrac{4.05}{9} = 0.45$

$$s_{AC} = x_B\left[1 + \frac{2}{3}\left(\frac{y_A}{x_A}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_A}\right)^4 + \cdots\right]$$

$$s_{AC} = 9\text{ m}\left(1 + \frac{2}{3}0.45^2 - \frac{2}{5}0.45^4 + \cdots\right) = 10.067\text{ m}$$

Portion *CB*: $x_B = 6\text{ m},\quad y_B = 4.05 - 2.25 = 1.8\text{ m};\quad \dfrac{y_B}{x_B} = 0.3$

$$s_{CB} = 6\text{ m}\left(1 + \frac{2}{3}0.3^2 - \frac{2}{5}0.3^4 + \cdots\right) = 6.341\text{ m}$$

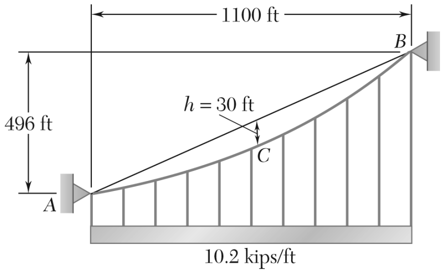
Total length $= 10.067\text{ m} + 6.341\text{ m}$ Total length $= 16.41\text{ m}$ ◀

(*c*) Components of reaction at *A*.

$$A_y = 9w = 9(60\text{ kg/m})(9.81\text{ m/s}^2)$$
$$= 5297.4\text{ N}$$
$$A_x = T_0 = 10w = 10(60\text{ kg/m})(9.81\text{ m/s}^2)$$
$$= 5886\text{ N}$$

$\mathbf{A}_x = 5890\text{ N} \longleftarrow$ ◀

$\mathbf{A}_y = 5300\text{ N}\uparrow$ ◀

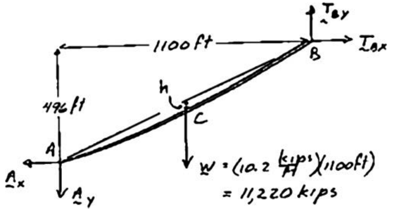


PROBLEM 7.117

Each cable of the side spans of the Golden Gate Bridge supports a load $w = 10.2$ kips/ft along the horizontal. Knowing that for the side spans the maximum vertical distance h from each cable to the chord AB is 30 ft and occurs at midspan, determine (*a*) the maximum tension in each cable, (*b*) the slope at B.

SOLUTION

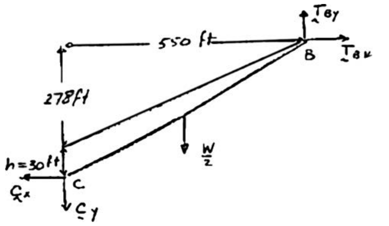
FBD AB:



$$\circlearrowleft \Sigma M_A = 0\text{:}\quad (1100\text{ ft})T_{By} - (496\text{ ft})T_{Bx} - (550\text{ ft})W = 0$$

$$11T_{By} - 4.96T_{Bx} = 5.5W \qquad (1)$$

FBD CB:



$$\circlearrowleft \Sigma M_C = 0\text{:}\quad (550\text{ ft})T_{By} - (278\text{ ft})T_{Bx} - (275\text{ ft})\frac{W}{2} = 0$$

$$11T_{By} - 5.56T_{Bx} = 2.75W \qquad (2)$$

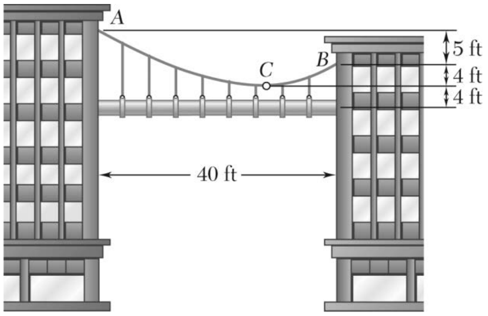
Solving (1) and (2) $\qquad T_{By} = 28{,}798$ kips

$$T_{Bx} = 51{,}425 \text{ kips}$$

$$T_{\max} = T_B = \sqrt{T_{B_x}^2 + T_{B_y}^2} \qquad \tan\theta_B = \frac{T_{B_y}}{T_{B_x}}$$

So that $\qquad$ (*a*) $\quad T_{\max} = 58{,}900$ kips ◀

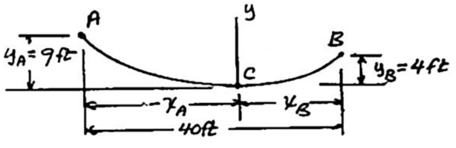
(*b*) $\quad \theta_B = 29.2°$ ◀



PROBLEM 7.118

A steam pipe weighting 45 lb/ft that passes between two buildings 40 ft apart is supported by a system of cables as shown. Assuming that the weight of the cable system is equivalent to a uniformly distributed loading of 5 lb/ft, determine (*a*) the location of the lowest Point *C* of the cable, (*b*) the maximum tension in the cable.

SOLUTION



Note: $x_B - x_A = 40\text{ ft}$

or $x_A = x_B - 40\text{ ft}$

(*a*) Use Eq. 7.8

Point *A*: $$y_A = \frac{wx_A^2}{2T_0};\quad 9 = \frac{w(x_B-40)^2}{2T_0} \qquad (1)$$

Point *B*: $$y_B = \frac{wx_B^2}{2T_0};\quad 4 = \frac{wx_B^2}{2T_0} \qquad (2)$$

Dividing (1) by (2): $$\frac{9}{4} = \frac{(x_B-40)^2}{x_B^2};\quad x_B = 16\text{ ft}$$

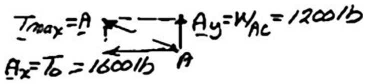
Point *C* is 16.00 ft to left of *B* ◀

(*b*) Maximum slope and thus $T_{\max}$ is at *A*

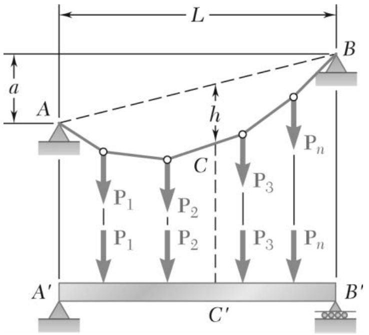
$$x_A = x_B - 40 = 16 - 40 = -24\text{ ft}$$

$$y_A = \frac{wx_A^2}{2T_0};\quad 9\text{ ft} = \frac{(50\text{ lb/ft})(-24\text{ ft})^2}{2T_0};\quad T_0 = 1600\text{ lb}$$

$$W_{AC} = (50\text{ lb/ft})(24\text{ ft}) = 1200\text{ lb}$$



$T_{\max} = 2000\text{ lb}$ ◀



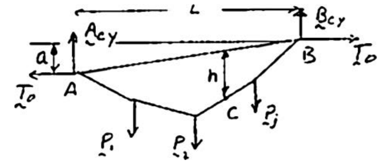
PROBLEM 7.119*

A cable AB of span L and a simple beam $A'B'$ of the same span are subjected to identical vertical loadings as shown. Show that the magnitude of the bending moment at a point C' in the beam is equal to the product T_0h, where T_0 is the magnitude of the horizontal component of the tension force in the cable and h is the vertical distance between Point C and the chord joining the points of support A and B.

SOLUTION

$$\curvearrowleft \Sigma M_B = 0: \quad LA_{Cy} + aT_0 - \Sigma \overset{\curvearrowright}{M}_{B\text{ loads}} = 0 \qquad (1)$$

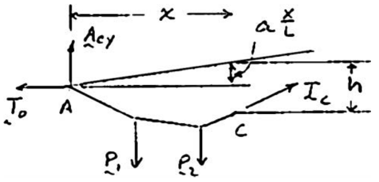
FBD Cable:



(Where $\Sigma M_{B\text{ loads}}$ includes all applied loads)

$$\curvearrowleft \Sigma M_C = 0: \quad xA_{Cy} - \left(h - a\frac{x}{L}\right)T_0 - \Sigma \overset{\curvearrowright}{M}_{C\text{ left}} = 0 \qquad (2)$$

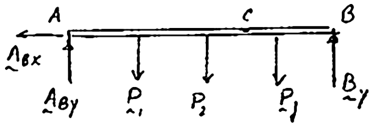
FBD AC:



(Where $\Sigma M_{C\text{ left}}$ includes all loads left of C)

$$\frac{x}{L}(1) - (2): \quad hT_0 - \frac{x}{L}\Sigma \overset{\curvearrowright}{M}_{B\text{ loads}} + \Sigma \overset{\curvearrowright}{M}_{C\text{ left}} = 0 \qquad (3)$$

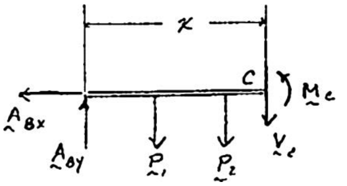
FBD Beam:



$$\curvearrowleft \Sigma M_B = 0: \quad LA_{By} - \Sigma \overset{\curvearrowright}{M}_{B\text{ loads}} = 0 \qquad (4)$$

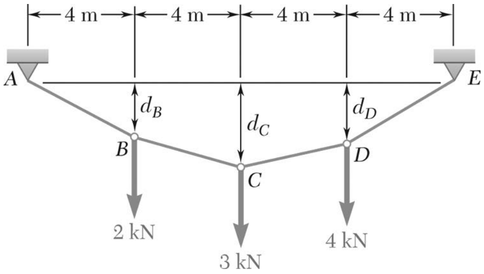
$$\curvearrowleft \Sigma M_C = 0: \quad xA_{By} - \Sigma \overset{\curvearrowright}{M}_{C\text{ left}} - M_C = 0 \qquad (5)$$

FBD AC:



$$\frac{x}{L}(4) - (5): \quad -\frac{x}{L}\Sigma \overset{\curvearrowright}{M}_{B\text{ loads}} + \Sigma \overset{\curvearrowright}{M}_{C\text{ left}} + M_C = 0 \qquad (6)$$

Comparing (3) and (6) $\qquad M_C = hT_0 \qquad$ Q.E.D.



PROBLEM 7.120

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

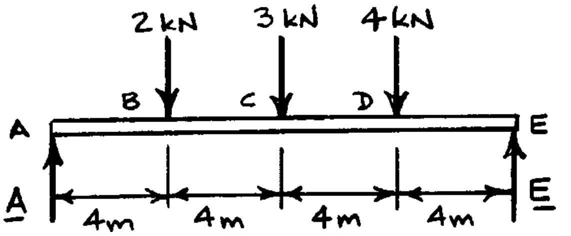
PROBLEM 7.94 Knowing that the maximum tension in cable *ABCDE* is 13 kN, determine the distance d_C.

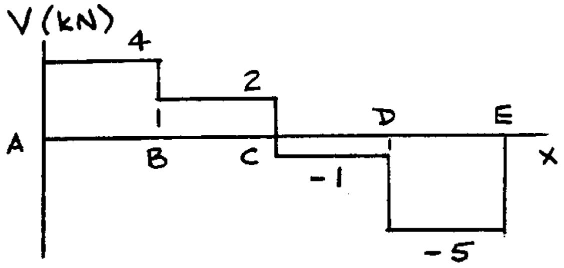
SOLUTION

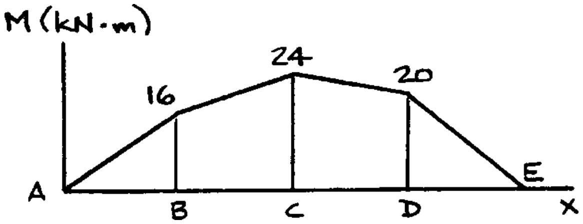
Free body: beam *AE*

$$+\circlearrowleft \Sigma M_E = 0: \quad -A(16) + 2(12) + 3(8) + 4(4) = 0 \qquad \mathbf{A} = 4 \text{ kN}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 4 - 2 - 3 - 4 + E = 0 \qquad \mathbf{E} = 5 \text{ kN}\uparrow$$







At *E*: $T_m^2 = T_0^2 + E^2 \qquad 13^2 = T_0^2 + 5^2 \qquad T_0 = 12 \text{ kN}$

At *C*: $M_C = T_0 h_C; \qquad 24 \text{ kN}\cdot\text{m} = (12 \text{ kN})h_C$

$$h_C = d_C = 2.00 \text{ m} \qquad 2.00 \text{ m} ◀$$

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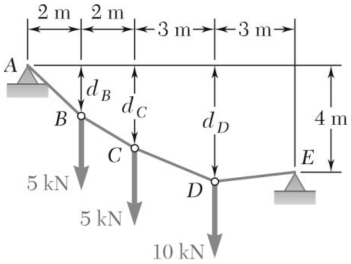
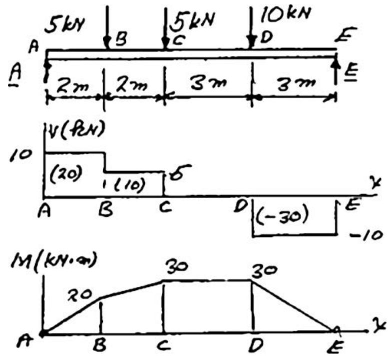


PROBLEM 7.121

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.97 (*a*) Knowing that $d_C = 3$ m, determine the distances d_B and d_D.

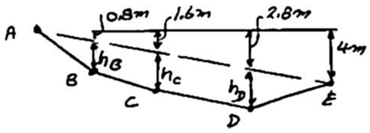
SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad A(10\text{ m}) - (5\text{ kN})(8\text{ m}) - (5\text{ kN})(6\text{ m}) - (10\text{ kN})(3\text{ m}) = 0$$

$$A = 10\text{ kN}$$

Geometry:



$$d_C = 1.6\text{ m} + h_C$$
$$3\text{ m} = 1.6\text{ m} + h_C$$
$$h_C = 1.4\text{ m}$$

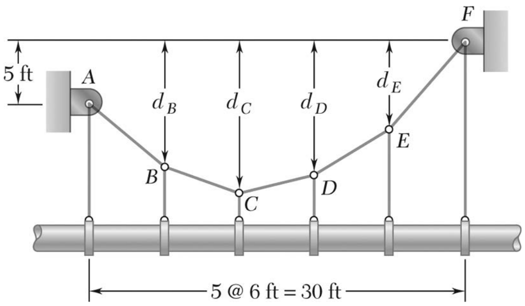
Since $M = T_0 h$, h is proportional to M, thus

$$\frac{h_B}{M_B} = \frac{h_C}{M_C} = \frac{h_D}{M_D}; \quad \frac{h_B}{20\text{ kN}\cdot\text{m}} = \frac{1.4\text{ m}}{30\text{ kN}\cdot\text{m}} = \frac{h_D}{30\text{ kN}\cdot\text{m}}$$

$$h_B = 1.4\left(\frac{20}{30}\right) = 0.9333\text{ m} \qquad \Big\| \qquad h_D = 1.4\left(\frac{30}{30}\right) = 1.4\text{ m}$$

$$d_B = 0.8\text{ m} + 0.9333\text{ m} \qquad \Big\| \qquad d_D = 2.8\text{ m} + 1.4\text{ m}$$

$$d_B = 1.733\text{ m} \blacktriangleleft \qquad d_D = 4.20\text{ m} \blacktriangleleft$$

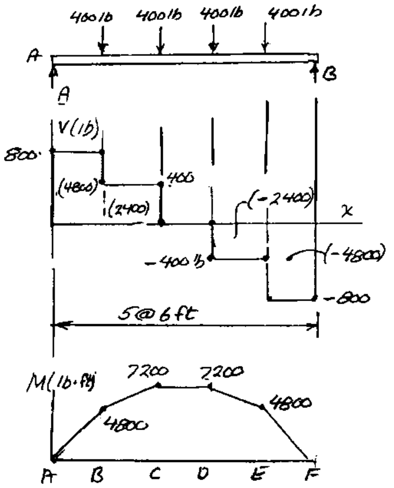


PROBLEM 7.122

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

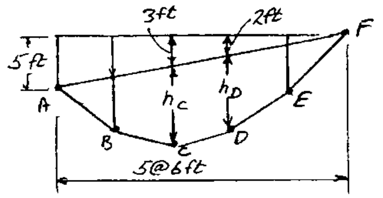
PROBLEM 7.99 An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 12$ ft, determine (a) the maximum tension in the cable.

SOLUTION



$$A = B = \frac{1}{2}(4 \times 400) = 800 \text{ lb}$$

Geometry



$$d_C = h_C + 3 \text{ ft}$$

$$12 \text{ ft} = h_C + 3 \text{ ft}$$

$$h_C = 9 \text{ ft}$$

$$d_D = h_D + 2 \text{ ft} \qquad (1)$$

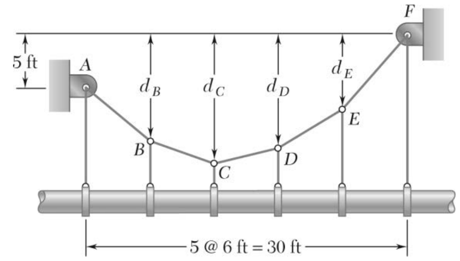
At C: $\quad M_C = T_0 h_C$

$7200 \text{ lb}\cdot\text{ft} = T_0(9 \text{ ft}) \qquad T_0 = 800 \text{ lb}$

At D: $\quad M_D = T_0 h_D$

$7200 \text{ lb}\cdot\text{ft} = (800 \text{ lb})h_D \qquad h_0 = 9 \text{ ft}$

Eq. (1): $\quad d_D = 9 \text{ ft} + 2 \text{ ft}$ $\qquad d_D = 11.00 \text{ ft}$ ◀

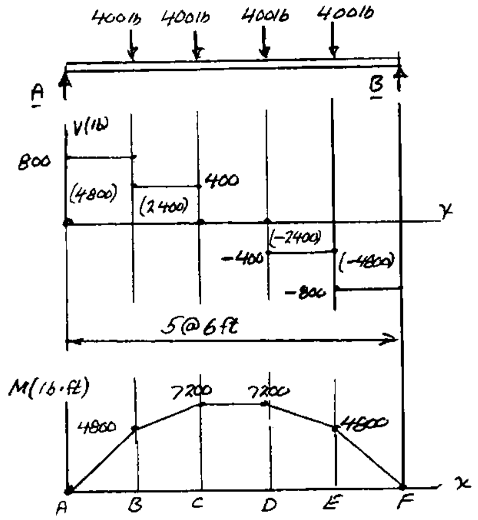


PROBLEM 7.123

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.100 An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 9$ ft, determine (*b*) the distance d_D.

SOLUTION



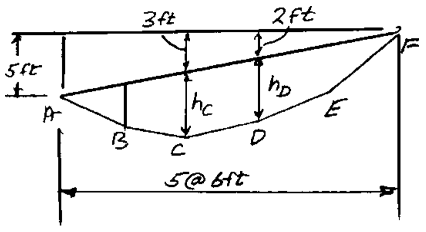
$$A = B = \frac{1}{2}(4 \times 400)$$

$$A = B = 800 \text{ lb}$$

At any point: $M = T_0 h$

We note that since $M_C = M_D$, we have $h_C = h_D$

Geometry



$$d_C = h_C + 3 \text{ ft}$$

$$9 \text{ ft} = h_C + 3 \text{ ft}$$

$$h_C = 6 \text{ ft}$$

and $h_D = 6 \text{ ft}$

$$d_D = h_D + 2 \text{ ft} = 6 \text{ ft} + 2 \text{ ft}$$

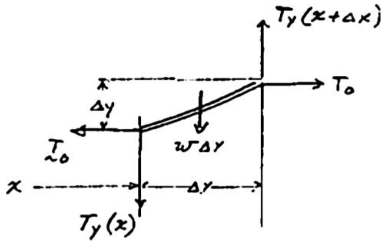
$$d_D = 8.00 \text{ ft} \blacktriangleleft$$

PROBLEM 7.124*

Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION

FBD Elemental segment:



$$\uparrow \Sigma F_y = 0: \quad T_y(x+\Delta x) - T_y(x) - w(x)\Delta x = 0$$

So
$$\frac{T_y(x+\Delta x)}{T_0} - \frac{T_y(x)}{T_0} = \frac{w(x)}{T_0}\Delta x$$

But
$$\frac{T_y}{T_0} = \frac{dy}{dx}$$

So
$$\frac{\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}}{\Delta x} = \frac{w(x)}{T_0}$$

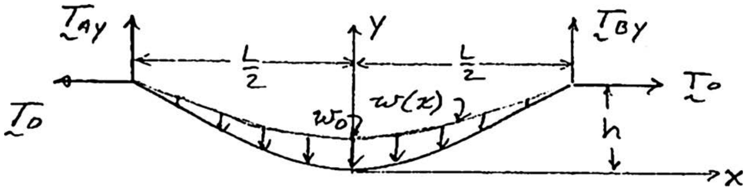
In $\lim\limits_{\Delta x \to 0}$:
$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} \qquad \text{Q.E.D.}$$

PROBLEM 7.125*

Using the property indicated in Problem 7.124, determine the curve assumed by a cable of span L and sag h carrying a distributed load $w = w_0 \cos(\pi x/L)$, where x is measured from mid-span. Also determine the maximum and minimum values of the tension in the cable.

PROBLEM 7.124 Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION



$$w(x) = w_0 \cos\frac{\pi x}{L}$$

From Problem 7.124

$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} = \frac{w_0}{T_0}\cos\frac{\pi x}{L}$$

So $$\frac{dy}{dx} = \frac{W_0 L}{T_0 \pi}\sin\frac{\pi x}{L} \quad \left(\text{using } \left.\frac{dy}{dx}\right|_0 = 0\right)$$

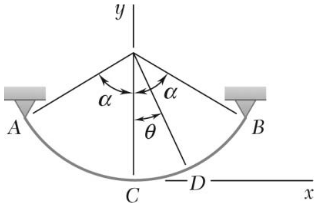
$$y = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi x}{L}\right) \quad [\text{using } y(0) = 0] \blacktriangleleft$$

But $$y\left(\frac{L}{2}\right) = h = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi}{2}\right) \quad \text{so} \quad T_0 = \frac{w_0 L^2}{\pi^2 h}$$

And $$T_0 = T_{\min} \quad \text{so} \qquad T_{\min} = \frac{w_0 L^2}{\pi^2 h} \blacktriangleleft$$

$$T_{\max} = T_A = T_B: \quad \frac{T_{By}}{T_0} = \left.\frac{dy}{dx}\right|_{x=L/2} = \frac{w_0 L}{T_0 \pi}$$

$$T_{By} = \frac{w_0 L}{\pi} \qquad T_B = \sqrt{T_{By}^2 + T_0^2} = \frac{w_0 L}{\pi}\sqrt{1 + \left(\frac{L}{\pi h}\right)^2} \blacktriangleleft$$



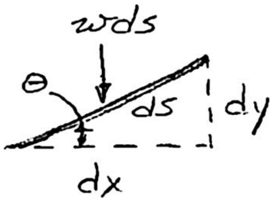
PROBLEM 7.126*

If the weight per unit length of the cable AB is $w_0/\cos^2 \theta$, prove that the curve formed by the cable is a circular arc. (*Hint:* Use the property indicated in Problem 7.124.)

PROBLEM 7.124 Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION

Elemental Segment:



Load on segment* $$w(x)dx = \frac{w_0}{\cos^2\theta} ds$$

But $$dx = \cos\theta ds, \quad \text{so} \quad w(x) = \frac{w_0}{\cos^3\theta}$$

From Problem 7.119 $$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} = \frac{w_0}{T_0\cos^3\theta}$$

In general $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan\theta) = \sec^2\theta\frac{d\theta}{dx}$$

So $$\frac{d\theta}{dx} = \frac{w_0}{T_0\cos^3\theta\sec^2\theta} = \frac{w_0}{T_0\cos\theta}$$

or $$\frac{T_0}{w_0}\cos\theta d\theta = dx = rd\theta\cos\theta$$

Giving $r = \dfrac{T_0}{w_0} = \text{constant}$. So curve is circular arc Q.E.D.

*For large sag, it is not appropriate to approximate ds by dx.

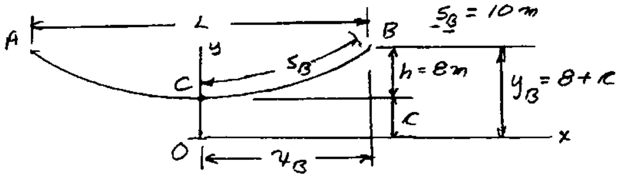
PROBLEM 7.127

A 20-m chain of mass 12 kg is suspended between two points at the same elevation. Knowing that the sag is 8 m, determine (*a*) the distance between the supports, (*b*) the maximum tension in the chain.

SOLUTION

$$\text{mass/meter} = (12\text{ kg})/(20\text{ m}) = 0.6\text{ kg/m}$$

$$w = (0.6\text{ kg/m})(9.81\text{ m/s}^2) = 5.886\text{ N/m}$$



Eq. 7.17: $y_B^2 - s_B^2 = c^2$; $(8+c)^2 - 10^2 = c^2$

$$64 + 16c + c^2 - 100 = c^2$$

$$16c = 36 \quad c = 2.25\text{ m}$$

Eq. 7.18: $T_m = wy_B = (5.886\text{ N/m})(8\text{ m} + 2.25\text{ m})$

$$T_m = 60.33\text{ N}$$

$T_m = 60.3$ N ◀

Eq. 7.15: $s_B = c\sinh\dfrac{x_B}{c}$; $10 = (2.25\text{ m})\sinh\dfrac{x_B}{c}$

$$\sinh\frac{x_B}{c} = 4.444; \quad \frac{x_B}{c} = 2.197$$

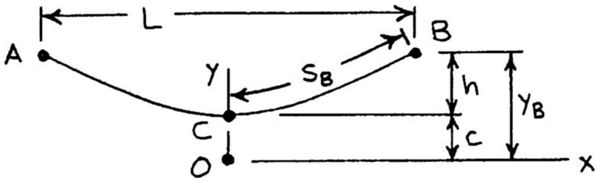
$$x_B = 2.197(2.25\text{ m}) = 4.944\text{ m}; \quad L = 2x_B = 2(4.944\text{ m}) = 9.888\text{ m}$$

$L = 9.89$ m ◀

PROBLEM 7.128

A 600-ft-long aerial tramway cable having a weight per unit length of 3.0 lb/ft is suspended between two points at the same elevation. Knowing that the sag is 150 ft, find (*a*) the horizontal distance between the supports, (*b*) the maximum tension in the cable.

SOLUTION



Given:

$$\text{Length} = 600 \text{ ft}$$

$$\text{Unit mass} = 3.0 \text{ lb/ft}$$

$$h = 150 \text{ ft}$$

Then,

$$s_B = 300 \text{ ft}$$

$$y_B = h + c = 150 \text{ ft} + c$$

$$y_B^2 - s_B^2 = c^2; \quad (150 + c)^2 - (300)^2 = c^2$$

$$150^2 + 300c + c^2 - 300^2 = c^2$$

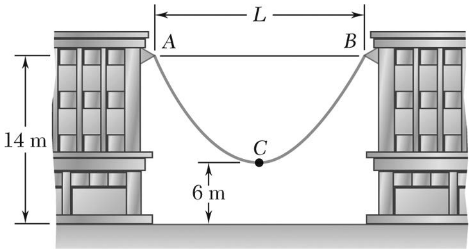
$$c = 225 \text{ ft}$$

$$s_B = c \sinh\frac{x_B}{c}; \quad 300 = 225\sinh\frac{x_B}{225}$$

$$x_B = 247.28 \text{ ft}$$

$$\text{span} = L = 2x_B = 2(247.28 \text{ ft}) \qquad L = 495 \text{ ft} ◀$$

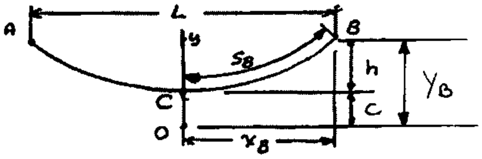
$$T_m = wy_B = (3 \text{ lb/ft})(150 \text{ ft} + 225 \text{ ft}) \qquad T_m = 1125 \text{ lb} ◀$$



PROBLEM 7.129

A 40-m cable is strung as shown between two buildings. The maximum tension is found to be 350 N, and the lowest point of the cable is observed to be 6 m above the ground. Determine (*a*) the horizontal distance between the buildings, (*b*) the total mass of the cable.

SOLUTION



$$s_B = 20 \text{ m}$$
$$T_m = 350 \text{ N}$$
$$h = 14 \text{ m} - 6 \text{ m} = 8 \text{ m}$$
$$y_B = h + c = 8 \text{ m} + c$$

Eq. 7.17: $y_B^2 - s_B^2 = c^2; \quad (8+c)^2 - 20^2 = c^2$

$$64 + 16c + c^2 - 400 = c^2$$
$$c = 21.0 \text{ m}$$

Eq. 7.15: $s_B = c \sinh \dfrac{x_B}{c}; \quad 20 = (21.0)\sinh \dfrac{x_B}{21.0}$

(*a*) $x_B = 17.7933 \text{ m}; \quad L = 2x_B$ $\qquad L = 35.6 \text{ m}$ ◀

(*b*) Eq. 7.18: $T_m = wy_B; \quad 350 \text{ N} = w(8 + 21.0)$

$$w = 12.0690 \text{ N/m}$$

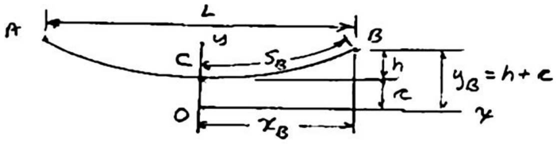
$$W = 2s_B w = (40 \text{ m})(12.0690 \text{ N/m})$$
$$= 482.76 \text{ N}$$

$m = \dfrac{W}{g} = \dfrac{482.76 \text{ N}}{9.81 \text{ m/s}^2}$ $\qquad$ Total mass $= 49.2$ kg ◀

PROBLEM 7.130

A 200-ft steel surveying tape weighs 4 lb. If the tape is stretched between two points at the same elevation and pulled until the tension at each end is 16 lb, determine the horizontal distance between the ends of the tape. Neglect the elongation of the tape due to the tension.

SOLUTION



$$s_B = 100 \text{ ft}$$

$$w = \left(\frac{4 \text{ lb}}{200 \text{ ft}}\right) = 0.02 \text{ lb/ft} \quad T_m = 16 \text{ m}$$

Eq. 7.18: $T_m = wy_B; \quad 16 \text{ lb} = (0.02 \text{ lb/ft})y_B; \quad y_B = 800 \text{ ft}$

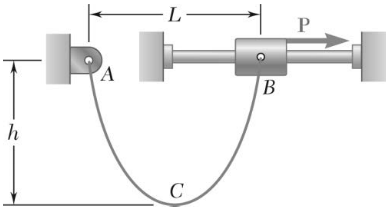
Eq. 7.17: $y_B^2 - s_B^2 = c^2; \quad (800)^2 - (100)^2 = c^2; \quad c = 793.73 \text{ ft}$

Eq. 7.15: $s_B = c \sinh \dfrac{x_B}{c}; \quad 100 = 793.73 \sinh \dfrac{x_B}{c}$

$$\frac{x_B}{c} = 0.12566; \quad x_B = 99.737 \text{ ft}$$

$$L = 2x_B = 2(99.737 \text{ ft})$$

$L = 199.5$ ft ◀

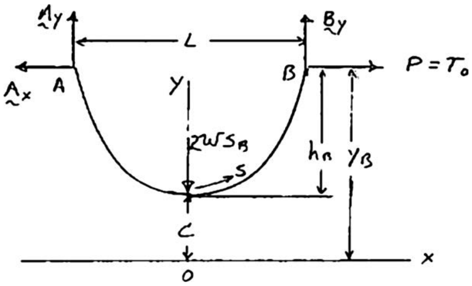


PROBLEM 7.131

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at *A* and to a collar at *B*. Neglecting the effect of friction, determine (*a*) the force **P** for which $h = 8$ m, (*b*) the corresponding span *L*.

SOLUTION

FBD Cable:



$$s_T = 20 \text{ m} \quad \left(\text{so } s_B = \frac{20 \text{ m}}{2} = 10 \text{ m}\right)$$

$$w = (0.2 \text{ kg/m})(9.81 \text{ m/s}^2)$$

$$= 1.96200 \text{ N/m}$$

$$h_B = 8 \text{ m}$$

$$y_B^2 = (c + h_B)^2 = c^2 + s_B^2$$

So
$$c = \frac{s_B^2 - h_B^2}{2h_B}$$

$$c = \frac{(10 \text{ m})^2 - (8 \text{ m})^2}{2(8 \text{ m})}$$

$$= 2.250 \text{ m}$$

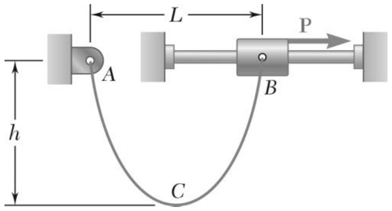
Now
$$s_B = c \sinh \frac{x_B}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c}$$

$$= (2.250 \text{ m}) \sinh^{-1}\left(\frac{10 \text{ m}}{2.250 \text{ m}}\right)$$

$$x_B = 4.9438 \text{ m}$$

$P = T_0 = wc = (1.96200 \text{ N/m})(2.250 \text{ m})$ (*a*) $\mathbf{P} = 4.41 \text{ N} \longrightarrow$ ◀

$L = 2x_B = 2(4.9438 \text{ m})$ (*b*) $L = 9.89 \text{ m}$ ◀

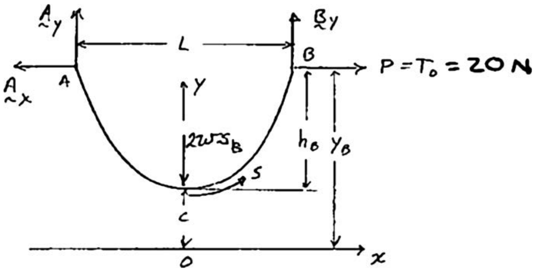


PROBLEM 7.132

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at *A* and to a collar at *B*. Knowing that the magnitude of the horizontal force applied to the collar is *P* = 20 N, determine (*a*) the sag *h*, (*b*) the span *L*.

SOLUTION

FBD Cable:



$$s_T = 20 \text{ m}, \quad w = (0.2 \text{ kg/m})(9.81 \text{ m/s}^2) = 1.96200 \text{ N/m}$$

$$P = T_0 = wc \quad c = \frac{P}{w}$$

$$c = \frac{20 \text{ N}}{1.9620 \text{ N/m}} = 10.1937 \text{ m}$$

$$y_B^2 = (h_B + c)^2 = c^2 + s_B^2$$

$$h^2 + 2ch - s_B^2 = 0 \quad s_B = \frac{20 \text{ m}}{2} = 10 \text{ m}$$

$$h^2 + 2(10.1937 \text{ m})h - 100 \text{ m}^2 = 0$$

$$h = 4.0861 \text{ m}$$

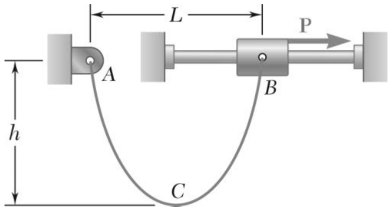
(*a*) $h = 4.09$ m ◀

$$s_B = c \sinh \frac{x_A}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c} = (10.1937 \text{ m}) \sinh^{-1}\left(\frac{10 \text{ m}}{10.1937 \text{ m}}\right)$$

$$= 8.8468 \text{ m}$$

$$L = 2x_B = 2(8.8468 \text{ m})$$

(*b*) $L = 17.69$ m ◀

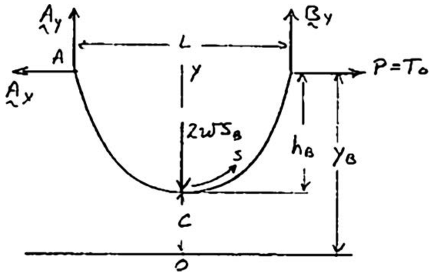


PROBLEM 7.133

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at A and to a collar at B. Neglecting the effect of friction, determine (*a*) the sag h for which L = 15 m, (*b*) the corresponding force **P**.

SOLUTION

FBD Cable:



$$s_T = 20 \text{ m} \rightarrow s_B = \frac{20 \text{ m}}{2} = 10 \text{ m}$$

$$w = (0.2 \text{ kg/m})(9.81 \text{ m/s}^2) = 1.96200 \text{ N/m}$$

$$L = 15 \text{ m}$$

$$s_B = c \sinh \frac{x_B}{c} = c \sinh \frac{\frac{L}{2}}{c}$$

$$10 \text{ m} = c \sinh \frac{7.5 \text{ m}}{c}$$

Solving numerically: $c = 5.5504 \text{ m}$

$$y_B = c \cosh\left(\frac{x_B}{c}\right) = (5.5504)\cosh\left(\frac{7.5}{5.5504}\right)$$

$$y_B = 11.4371 \text{ m}$$

$$h_B = y_B - c = 11.4371 \text{ m} - 5.5504 \text{ m}$$

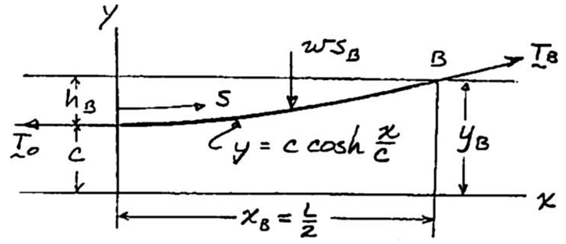
(*a*) $h_B = 5.89 \text{ m}$ ◀

$P = wc = (1.96200 \text{ N/m})(5.5504 \text{ m})$ (*b*) $\mathbf{P} = 10.89 \text{ N} \longrightarrow$ ◀

PROBLEM 7.134

Determine the sag of a 30-ft chain that is attached to two points at the same elevation that are 20 ft apart.

SOLUTION



$$s_B = \frac{30\text{ ft}}{2} = 15\text{ ft} \quad L = 20\text{ ft}$$

$$x_B = \frac{L}{2} = 10\text{ ft}$$

$$s_B = c\sinh\frac{x_B}{c}$$

$$15\text{ ft} = c\sinh\frac{10\text{ ft}}{c}$$

Solving numerically: $c = 6.1647\text{ ft}$

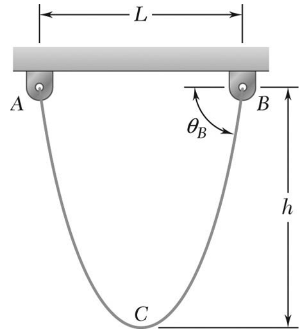
$$y_B = c\cosh\frac{x_B}{c}$$

$$= (6.1647\text{ ft})\cosh\frac{10\text{ ft}}{6.1647\text{ ft}}$$

$$= 16.2174\text{ ft}$$

$$h_B = y_B - c = 16.2174\text{ ft} - 6.1647\text{ ft}$$

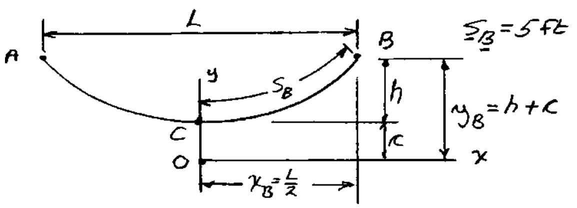
$h_B = 10.05\text{ ft}$ ◀



PROBLEM 7.135

A 10-ft rope is attached to two supports *A* and *B* as shown. Determine (*a*) the span of the rope for which the span is equal to the sag, (*b*) the corresponding angle θ_B.

SOLUTION



Note: Since $L = h$,

$$x_B = \frac{L}{2} = \frac{h}{2}$$

Eq. 7.16:

$$y_B = \cosh\frac{x_B}{c}$$

$$h + c = c\cosh\frac{h/2}{c}$$

$$\frac{h}{c} + 1 = \cosh\left(\frac{1}{2}\frac{h}{c}\right)$$

Solve for *h*/*c*:

$$\frac{h}{c} = 4.933$$

Eq. 7.16:

$$y = c\cosh\frac{x}{c}$$

$$\frac{dy}{dx} = \sinh\frac{x}{c}$$

At *B*:

$$\tan\theta_B = \left.\frac{dy}{dx}\right|_B = \sinh\frac{x_B}{c}$$

Substitute

$$x_B = \frac{h}{2}: \quad \tan\theta_B = \sinh\left(\frac{1}{2}\frac{h}{c}\right) = \sinh\left(\frac{1}{2}\times 4.933\right)$$

$$\tan\theta_B = 5.848 \qquad \theta_B = 80.3° \blacktriangleleft$$

PROBLEM 7.135 (Continued)

<u>Eq. 7.17</u>:

$$s_B = c\sinh\frac{x_B}{c} = c\sinh\left(\frac{1}{2}\frac{h}{c}\right)$$

$$5\text{ ft} = c\sinh\left(\frac{1}{2}\times 4.933\right)$$

$$5\text{ ft} = c(5.848)$$

$$c = 0.855$$

Recall that

$$\frac{h}{c} = 4.933$$

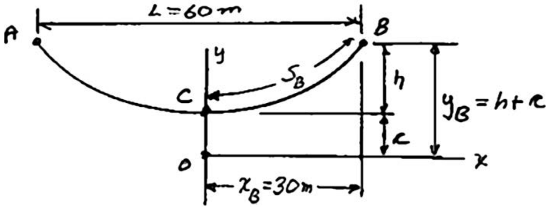
$$h = 4.933(0.855) = 4.218$$

$$h = 4.22\text{ ft} \blacktriangleleft$$

PROBLEM 7.136

A 90-m wire is suspended between two points at the same elevation that are 60 m apart. Knowing that the maximum tension is 300 N, determine (*a*) the sag of the wire, (*b*) the total mass of the wire.

SOLUTION



$$s_B = 45 \text{ m}$$

Eq. 7.17:

$$s_B = c\sinh\frac{x_B}{c}$$

$$45 = c\sinh\frac{30}{c}; \quad c = 18.494 \text{ m}$$

Eq. 7.16:

$$y_B = c\cosh\frac{x_B}{c}$$

$$y_B = (18.494)\cosh\frac{30}{18.494}$$

$$y_B = 48.652 \text{ m}$$

$$y_B = h + c$$

$$48.652 = h + 18.494$$

$$h = 30.158 \text{ m}$$

$h = 30.2$ m ◀

Eq. 7.18:

$$T_m = wy_B$$

$$300 \text{ N} = w(48.652 \text{ m})$$

$$w = 6.166 \text{ N/m}$$

Total weight of cable

$$W = w(\text{Length})$$

$$= (6.166 \text{ N/m})(90 \text{ m})$$

$$= 554.96 \text{ N}$$

Total mass of cable

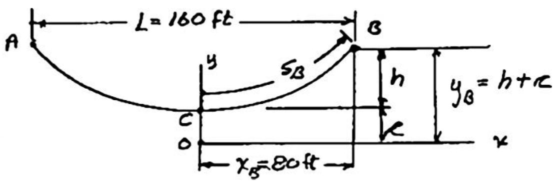
$$m = \frac{W}{g} = \frac{554.96 \text{ N}}{9.81 \text{ m/s}} = 56.57 \text{ kg}$$

$m = 56.6$ kg ◀

PROBLEM 7.137

A cable weighing 2 lb/ft is suspended between two points at the same elevation that are 160 ft apart. Determine the smallest allowable sag of the cable if the maximum tension is not to exceed 400 lb.

SOLUTION



Eq. 7.18: $T_m = wy_B; \quad 400\text{ lb} = (2\text{ lb/ft})y_B; \quad y_B = 200\text{ ft}$

Eq. 7.16: $$y_B = c\cosh\frac{x_B}{c}$$

$$200\text{ ft} = c\cosh\frac{80\text{ ft}}{c}$$

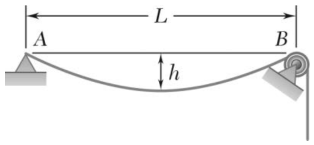
Solve for c: $c = 182.148$ ft and $c = 31.592$ ft

$$y_B = h + c; \quad h = y_B - c$$

For $c = 182.148$ ft; $h = 200 - 182.147 = 17.852$ ft ◁

For $c = 31.592$ ft; $h = 200 - 31.592 = 168.408$ ft ◁

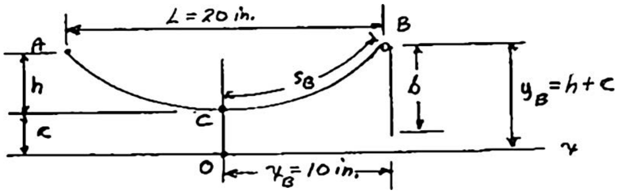
For $T_m \leq 400$ lb: smallest $h = 17.85$ ft ◀



PROBLEM 7.138

A uniform cord 50 in. long passes over a pulley at *B* and is attached to a pin support at *A*. Knowing that $L = 20$ in. and neglecting the effect of friction, determine the smaller of the two values of *h* for which the cord is in equilibrium.

SOLUTION



Length of overhang: $b = 50\text{ in.} - 2s_B$

Weight of overhang equals max. tension

$$T_m = T_B = wb = w(50\text{ in.} - 2s_B)$$

Eq. 7.15: $s_B = c\sinh\dfrac{x_B}{c}$

Eq. 7.16: $y_B = c\cosh\dfrac{x_B}{c}$

Eq. 7.18: $T_m = wy_B$

$$w(50\text{ in.} - 2s_B) = wy_B$$

$$w\left(50\text{ in.} - 2c\sinh\frac{x_B}{c}\right) = wc\cosh\frac{x_B}{c}$$

$$x_B = 10:\quad 50 - 2c\sinh\frac{10}{c} = c\cosh\frac{10}{c}$$

Solve by trial and error: $c = 5.549$ in. and $c = 27.742$ in.

For $c = 5.549$ in.

$$y_B = (5.549\text{ in.})\cosh\frac{10\text{ in.}}{5.549\text{ in.}} = 17.277\text{ in.}$$

$$y_B = h + c;\quad 17.277\text{ in.} = h + 5.549\text{ in.}$$

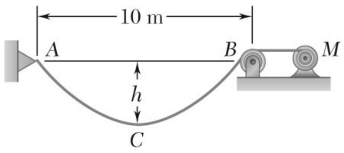
$h = 11.728$ in. $h = 11.73$ in. ◀

For $c = 27.742$ in.

$$y_B = (27.742\text{ in.})\cosh\frac{10\text{ in.}}{27.742\text{ in.}} = 29.564\text{ in.}$$

$$y_B = h + c;\quad 29.564\text{ in.} = h + 27.742\text{ in.}$$

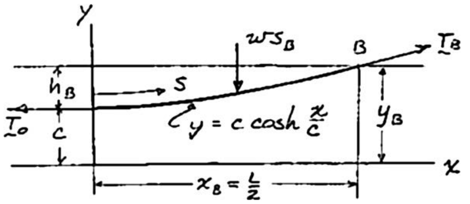
$h = 1.8219$ in. $h = 1.822$ in. ◀



PROBLEM 7.139

A motor *M* is used to slowly reel in the cable shown. Knowing that the mass per unit length of the cable is 0.4 kg/m, determine the maximum tension in the cable when $h = 5$ m.

SOLUTION



$$w = 0.4 \text{ kg/m} \quad L = 10 \text{ m} \quad h_B = 5 \text{ m}$$

$$y_B = c \cosh \frac{x_B}{c}$$

$$h_B + c = c \cosh \frac{L}{2c}$$

$$5 \text{ m} = c\left(\cosh \frac{5 \text{ m}}{c} - 1\right)$$

Solving numerically: $c = 3.0938$ m

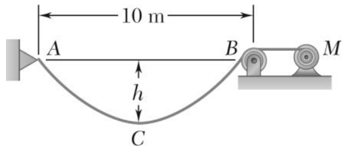
$$y_B = h_B + c = 5 \text{ m} + 3.0938 \text{ m}$$

$$= 8.0938 \text{ m}$$

$$T_{\max} = T_B = w y_B$$

$$= (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(8.0938 \text{ m})$$

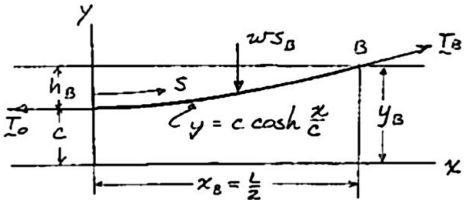
$T_{\max} = 31.8$ N ◀



PROBLEM 7.140

A motor M is used to slowly reel in the cable shown. Knowing that the mass per unit length of the cable is 0.4 kg/m, determine the maximum tension in the cable when $h = 3$ m.

SOLUTION



$$w = 0.4 \text{ kg/m}, \quad L = 10 \text{ m}, \quad h_B = 3 \text{ m}$$

$$y_B = h_B + c = c\cosh\frac{x_B}{c} = c\cosh\frac{L}{2c}$$

$$3 \text{ m} = c\left(c\cosh\frac{5 \text{ m}}{c} - 1\right)$$

Solving numerically: $c = 4.5945$ m

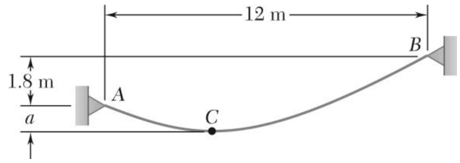
$$y_B = h_B + c = 3 \text{ m} + 4.5945 \text{ m}$$

$$= 7.5945 \text{ m}$$

$$T_{\max} = T_B = wy_B$$

$$= (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(7.5945 \text{ m})$$

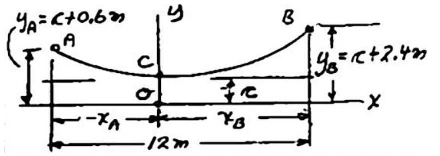
$T_{\max} = 29.8$ N ◀



PROBLEM 7.141

The cable *ACB* has a mass per unit length of 0.45 kg/m. Knowing that the lowest point of the cable is located at a distance $a = 0.6$ m below the support *A*, determine (*a*) the location of the lowest Point *C*, (*b*) the maximum tension in the cable.

SOLUTION



Note: $x_B - x_A = 12 \text{ m}$

or, $-x_A = 12 \text{ m} - x_B$

Point *A*: $y_A = c\cosh\dfrac{-x_A}{c}; \quad c + 0.6 = c\cosh\dfrac{12 - x_B}{c}$ (1)

Point *B*: $y_B = c\cosh\dfrac{x_B}{c}; \quad c + 2.4 = c\cosh\dfrac{x_B}{c}$ (2)

From (1): $\dfrac{12}{c} - \dfrac{x_B}{c} = \cosh^{-1}\left(\dfrac{c + 0.6}{c}\right)$ (3)

From (2): $\dfrac{x_B}{c} = \cosh^{-1}\left(\dfrac{c + 2.4}{c}\right)$ (4)

Add (3) + (4): $\dfrac{12}{c} = \cosh^{-1}\left(\dfrac{c + 0.6}{c}\right) + \cosh^{-1}\left(\dfrac{c + 2.4}{c}\right)$

Solve by trial and error: $c = 13.6214 \text{ m}$

Eq. (2): $13.6214 + 2.4 = 13.6214\cosh\dfrac{x_B}{c}$

$$\cosh\frac{x_B}{c} = 1.1762; \quad \frac{x_B}{c} = 0.58523$$

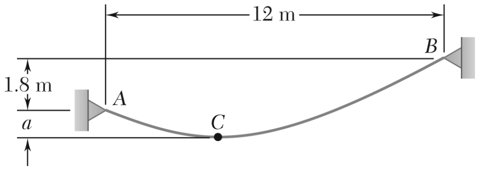
$$x_B = 0.58523(13.6214 \text{ m}) = 7.9717 \text{ m}$$

Point *C* is 7.97 m to left of *B* ◀

$$y_B = c + 2.4 = 13.6214 + 2.4 = 16.0214 \text{ m}$$

Eq. 7.18: $T_m = wy_B = (0.45 \text{ kg/m})(9.81 \text{ m/s}^2)(16.0214 \text{ m})$

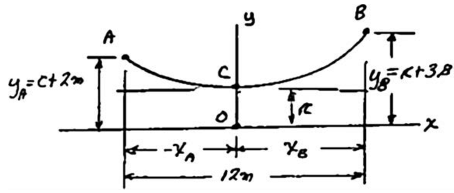
$T_m = 70.726 \text{ N}$ $\qquad T_m = 70.7 \text{ N}$ ◀



PROBLEM 7.142

The cable *ACB* has a mass per unit length of 0.45 kg/m. Knowing that the lowest point of the cable is located at a distance $a = 2$ m below the support *A*, determine (*a*) the location of the lowest Point *C*, (*b*) the maximum tension in the cable.

SOLUTION



Note:

$$x_B - x_A = 12\text{ m}$$

or

$$-x_A = 12\text{ m} - x_B$$

Point *A*: $$y_A = c\cosh\frac{-x_A}{c};\quad c+2 = c\cosh\frac{12-x_B}{c} \qquad (1)$$

Point *B*: $$y_B = c\cosh\frac{x_B}{c};\quad c+3.8 = c\cosh\frac{x_B}{c} \qquad (2)$$

From (1): $$\frac{12}{c} - \frac{x_B}{c} = \cosh^{-1}\left(\frac{c+2}{c}\right) \qquad (3)$$

From (2): $$\frac{x_B}{c} = \cosh^{-1}\left(\frac{c+3.8}{c}\right) \qquad (4)$$

Add (3) + (4): $$\frac{12}{c} = \cosh^{-1}\left(\frac{c+2}{c}\right) + \cosh^{-1}\left(\frac{c+3.8}{c}\right)$$

Solve by trial and error: $c = 6.8154\text{ m}$

Eq. (2): $$6.8154\text{ m} + 3.8\text{ m} = (6.8154\text{ m})\cosh\frac{x_B}{c}$$

$$\cosh\frac{x_B}{c} = 1.5576 \quad \frac{x_B}{c} = 1.0122$$

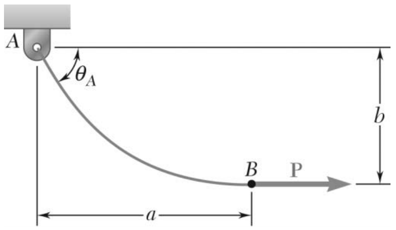
$$x_B = 1.0122(6.8154\text{ m}) = 6.899\text{ m}$$

Point *C* is 6.90 m to left of *B* ◀

$$y_B = c + 3.8 = 6.8154 + 3.8 = 10.6154\text{ m}$$

Eq. (7.18): $$T_m = wy_B = (0.45\text{ kg/m})(9.81\text{ m/s}^2)(10.6154\text{ m})$$

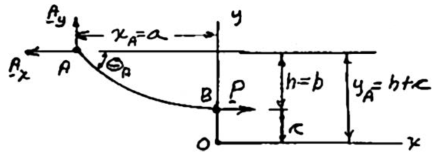
$$T_m = 46.86\text{ N}$$ $T_m = 46.9$ N ◀



PROBLEM 7.143

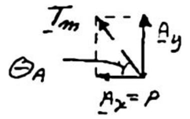
A uniform cable weighing 3 lb/ft is held in the position shown by a horizontal force **P** applied at *B*. Knowing that *P* = 180 lb and $\theta_A = 60°$, determine (*a*) the location of Point *B*, (*b*) the length of the cable.

SOLUTION



Eq. 7.18: $$T_0 = P = cw$$

$$c = \frac{P}{w} = \frac{180\text{ lb}}{3\text{ lb/ft}} \quad c = 60\text{ ft}$$



At *A*: $$T_m = \frac{P}{\cos 60°}$$

$$= \frac{cw}{0.5} = 2cw$$

(*a*) Eq. 7.18: $$T_m = w(h+c)$$

$$2cw = w(h+c)$$

$$2c = h + c \quad h = b = c$$

$b = 60.0$ ft ◀

Eq. 7.16: $$y_A = c\cosh\frac{x_A}{c}$$

$$h + c = c\cosh\frac{x_A}{c}$$

$$(60\text{ ft} + 60\text{ ft}) = (60\text{ ft}) \quad \cosh\frac{x_A}{60}$$

$$\cosh\frac{x_A}{60\text{ m}} = 2 \quad \frac{x_A}{60\text{ m}} = 1.3170$$

$$x_A = 79.02\text{ ft}$$

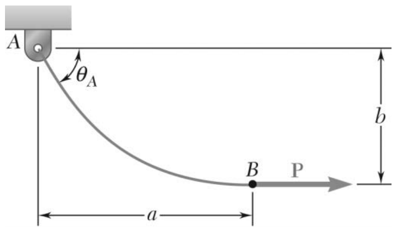
$a = 79.0$ ft ◀

(*b*) Eq. 7.15: $$s_A = c\sinh\frac{x_B}{c} = (60\text{ ft})\sinh\frac{79.02\text{ ft}}{60\text{ ft}}$$

$$s_A = 103.92\text{ ft}$$

$$\text{length} = s_A$$

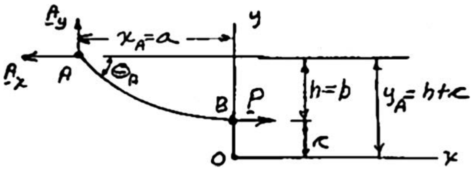
$s_A = 103.9$ ft ◀



PROBLEM 7.144

A uniform cable weighing 3 lb/ft is held in the position shown by a horizontal force **P** applied at *B*. Knowing that *P* = 150 lb and $\theta_A = 60°$, determine (*a*) the location of Point *B*, (*b*) the length of the cable.

SOLUTION



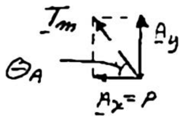
<u>Eq. 7.18</u>: $T_0 = P = cw$

$$c = \frac{P}{w} = \frac{150 \text{ lb}}{3 \text{ lb/ft}} = 50 \text{ ft}$$

<u>At *A*</u>:

$$T_m = \frac{P}{\cos 60°}$$

$$= \frac{cw}{0.5} = 2\,cw$$



(*a*) <u>Eq. 7.18</u>:

$$T_m = w(h + c)$$

$$2cw = w(h + c)$$

$$2c = h + c \quad h = c = b$$

$b = 50.0 \text{ ft}$ ◀

<u>Eq. 7.16</u>:

$$y_A = c \cosh \frac{x_A}{c}$$

$$h + c = c \cosh \frac{x_A}{c}$$

$$(50 \text{ ft} + 50 \text{ ft}) = (50 \text{ ft}) \cosh \frac{x_A}{c}$$

$$\cosh \frac{x_A}{c} = 2 \quad \frac{x_A}{c} = 1.3170$$

$$x_A = 1.3170(50 \text{ ft}) = 65.85 \text{ ft}$$

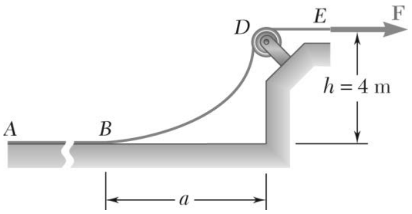
$a = 65.8 \text{ ft}$ ◀

(*b*) <u>Eq. 7.15</u>:

$$s_A = c \sinh \frac{x_A}{c} = (50 \text{ ft}) \sinh \frac{65.85 \text{ ft}}{50 \text{ ft}}$$

$$s_A = 86.6 \text{ ft}$$

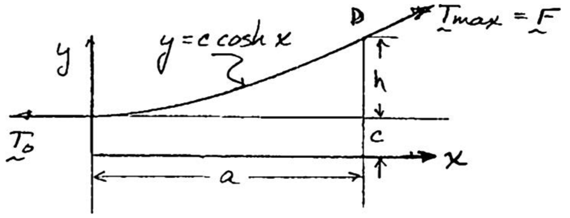
length $= s_A = 86.6 \text{ ft}$ ◀



PROBLEM 7.145

To the left of Point *B* the long cable *ABDE* rests on the rough horizontal surface shown. Knowing that the mass per unit length of the cable is 2 kg/m, determine the force **F** when $a = 3.6$ m.

SOLUTION



$$x_D = a = 3.6 \text{ m} \quad h = 4 \text{ m}$$

$$y_D = c\cosh\frac{x_D}{c}$$

$$h + c = c\cosh\frac{a}{c}$$

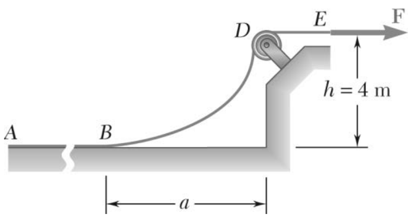
$$4 \text{ m} = c\left(\cosh\frac{3.6 \text{ m}}{c} - 1\right)$$

Solving numerically: $c = 2.0712$ m

Then $y_B = h + c = 4 \text{ m} + 2.0712 \text{ m} = 6.0712 \text{ m}$

$$F = T_{\max} = wy_B = (2 \text{ kg/m})(9.81 \text{ m/s}^2)(6.0712 \text{ m})$$

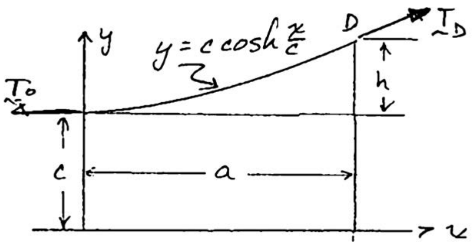
$\mathbf{F} = 119.1 \text{ N} \longrightarrow$ ◀



PROBLEM 7.146

To the left of Point *B* the long cable *ABDE* rests on the rough horizontal surface shown. Knowing that the mass per unit length of the cable is 2 kg/m, determine the force **F** when $a = 6$ m.

SOLUTION



$$x_D = a = 6\text{ m} \quad h = 4\text{ m}$$

$$y_D = c\cosh\frac{x_D}{c}$$

$$h + c = c\cosh\frac{a}{c}$$

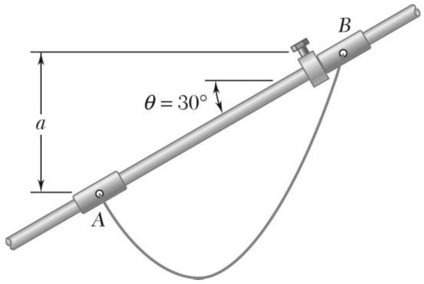
$$4\text{ m} = c\left(\cosh\frac{6\text{ m}}{c} - 1\right)$$

Solving numerically: $c = 5.054$ m

$$y_B = h + c = 4\text{ m} + 5.054\text{ m} = 9.054\text{ m}$$

$$F = T_D = wy_D = (2\text{ kg/m})(9.81\text{ m/s}^2)(9.054\text{ m})$$

$\mathbf{F} = 177.6\text{ N} \longrightarrow$ ◀

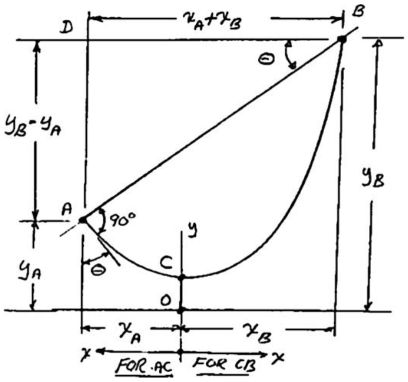


PROBLEM 7.147*

The 10-ft cable *AB* is attached to two collars as shown. The collar at *A* can slide freely along the rod; a stop attached to the rod prevents the collar at *B* from moving on the rod. Neglecting the effect of friction and the weight of the collars, determine the distance *a*.

SOLUTION

Collar at *A*: Since $\mu = 0$, cable $\perp$ rod



Point *A*: $$y = c\cosh\frac{x}{c};\quad \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$\tan\theta = \left|\frac{dy}{dx}\right|_A = \sinh\frac{x_A}{c}$$

$$\frac{x_A}{c} = \sinh(\tan(90° - \theta))$$

$$x_A = c\sinh(\tan(90° - \theta)) \tag{1}$$

Length of cable = 10 ft $$10\text{ ft} = AC + CB$$

$$10 = c\sinh\frac{x_A}{c} + c\sinh\frac{x_B}{c}$$

$$\sinh\frac{x_B}{c} = \frac{10}{c} - \sinh\frac{x_A}{c}$$

$$x_B = c\sinh^{-1}\left[\frac{10}{c} - \sinh\frac{x_A}{c}\right] \tag{2}$$

$$y_A = c\cosh\frac{x_A}{c} \quad y_B = c\cosh\frac{x_B}{c} \tag{3}$$

In Δ *ABD*: $$\tan\theta = \frac{y_B - y_A}{x_B + x_A} \tag{4}$$

PROBLEM 7.147* (Continued)

Method of solution:

For given value of θ, choose trial value of c and calculate:

From Eq. (1): x_A

Using value of x_A and c, calculate:

From Eq. (2): x_B

From Eq. (3): y_A and y_B

Substitute values obtained for x_A, x_B, y_A, y_B into Eq. (4) and calculate θ

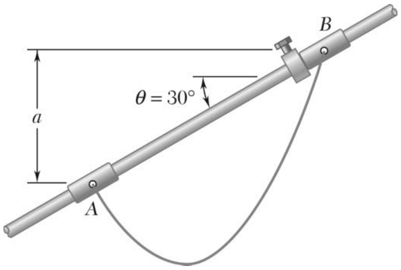
Choose new trial value of θ and repeat above procedure until calculated value of θ is equal to given value of θ.

For $\theta = 30°$

Result of trial and error procedure:

$$\begin{aligned}
c &= 1.803 \text{ ft} \\
x_A &= 2.3745 \text{ ft} \\
x_B &= 3.6937 \text{ ft} \\
y_A &= 3.606 \text{ ft} \\
y_B &= 7.109 \text{ ft} \\
a &= y_B - y_A \\
&= 7.109 \text{ ft} - 3.606 \text{ ft} \\
&= 3.503 \text{ ft}
\end{aligned}$$

$a = 3.50 \text{ ft}$ ◀



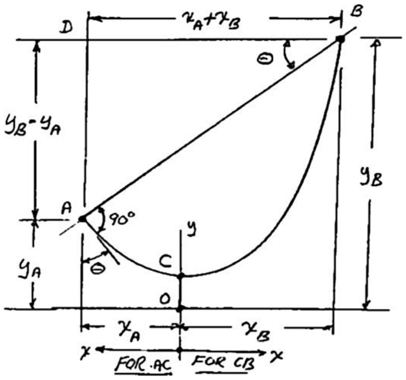
PROBLEM 7.148*

Solve Problem 7.147 assuming that the angle θ formed by the rod and the horizontal is 45°.

PROBLEM 7.147 The 10-ft cable *AB* is attached to two collars as shown. The collar at *A* can slide freely along the rod; a stop attached to the rod prevents the collar at *B* from moving on the rod. Neglecting the effect of friction and the weight of the collars, determine the distance *a*.

SOLUTION

Collar at *A*: Since $\mu = 0$, cable $\perp$ rod



Point *A*: $$y = c\cosh\frac{x}{c}; \quad \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$\tan\theta = \left|\frac{dy}{dx}\right|_A = \sinh\frac{x_A}{c}$$

$$\frac{x_A}{c} = \sinh(\tan(90° - \theta))$$

$$x_A = c\sinh(\tan(90° - \theta)) \qquad (1)$$

Length of cable = 10 ft $$10\text{ ft} = AC + CB$$

$$10 = c\sinh\frac{x_A}{c} + c\sinh\frac{x_B}{c}$$

$$\sinh\frac{x_B}{c} = \frac{10}{c} - \sinh\frac{x_A}{c}$$

$$x_B = c\sinh^{-1}\left[\frac{10}{c} - \sinh\frac{x_A}{c}\right] \qquad (2)$$

$$y_A = c\cosh\frac{x_A}{c} \quad y_B = c\cosh\frac{x_B}{c} \qquad (3)$$

PROBLEM 7.148* (Continued)

In $\Delta\, ABD$: $$\tan\theta = \frac{y_B - y_A}{x_B + x_A} \qquad (4)$$

Method of solution:

For given value of θ, choose trial value of c and calculate:

From Eq. (1): x_A

Using value of x_A and c, calculate:

From Eq. (2): x_B

From Eq. (3): y_A and y_B

Substitute values obtained for x_A, x_B, y_A, y_B into Eq. (4) and calculate θ

Choose new trial value of θ and repeat above procedure until calculated value of θ is equal to given value of θ.

For $\theta = 45°$

Result of trial and error procedure:

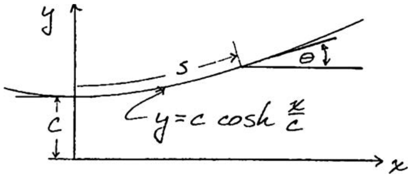
$$c = 1.8652 \text{ ft}$$
$$x_A = 1.644 \text{ ft}$$
$$x_B = 4.064 \text{ ft}$$
$$y_A = 2.638 \text{ ft}$$
$$y_B = 8.346 \text{ ft}$$
$$\begin{aligned} a &= y_B - y_A \\ &= 8.346 \text{ ft} - 2.638 \text{ ft} \\ &= 5.708 \text{ ft} \end{aligned}$$

$a = 5.71 \text{ ft}$ ◀

PROBLEM 7.149

Denoting by θ the angle formed by a uniform cable and the horizontal, show that at any point (*a*) $s = c \tan \theta$, (*b*) $y = c \sec \theta$.

SOLUTION



(*a*) $$\tan\theta = \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$s = c\sinh\frac{x}{c} = c\tan\theta \qquad \text{Q.E.D.}$$

(*b*) Also $$y^2 = s^2 + c^2(\cosh^2 x = \sinh^2 x + 1)$$

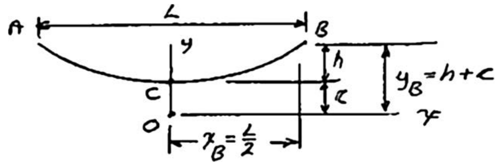
so $$y^2 = c^2(\tan^2\theta + 1)c^2\sec^2\theta$$

and $$y = c\sec\theta \qquad \text{Q.E.D.}$$

PROBLEM 7.150*

(*a*) Determine the maximum allowable horizontal span for a uniform cable of weight per unit length w if the tension in the cable is not to exceed a given value T_m. (*b*) Using the result of part *a*, determine the maximum span of a steel wire for which $w = 0.25$ lb/ft and $T_m = 8000$ lb.

SOLUTION



(*a*)

$$T_m = wy_B$$

$$= wc\cosh\frac{x_B}{c}$$

$$= wx_B\left(\frac{1}{\frac{x_B}{c}}\right)\cosh\frac{x_B}{c}$$

We shall find ratio $\left(\frac{x_B}{c}\right)$ for when T_m is minimum

$$\frac{dT_m}{d\left(\frac{x_B}{c}\right)} = wx_B\left[\frac{1}{\frac{x_B}{c}}\sinh\frac{x_B}{c} - \left(\frac{1}{\frac{x_B}{c}}\right)^2\cosh\frac{x_B}{c}\right] = 0$$

$$\frac{\sinh\frac{x_B}{c}}{\cosh\frac{x_B}{c}} = \frac{1}{\frac{x_B}{c}}$$

$$\tanh\frac{x_B}{c} = \frac{c}{x_B}$$

Solve by trial and error for: $\frac{x_B}{c} = 1.200$ (1)

$s_B = c\sinh\frac{x_B}{c} = c\sinh(1.200)$: $\frac{s_B}{c} = 1.509$

<u>Eq. 7.17</u>: $y_B^2 - s_B^2 = c^2$

$$y_B^2 = c^2\left[1 + \left(\frac{s_B}{c}\right)^2\right] = c^2(1 + 1.509^2)$$

$$y_B = 1.810c$$

PROBLEM 7.150* (Continued)

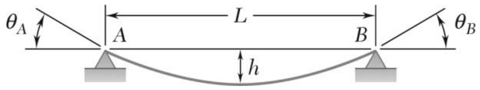
Eq. 7.18:
$$T_m = wy_B$$
$$= 1.810\,wc$$
$$c = \frac{T_m}{1.810\,w}$$

Eq. (1):
$$x_B = 1.200c = 1.200\frac{T_m}{1.810w} = 0.6630\frac{T_m}{w}$$

Span:
$$L = 2x_B = 2(0.6630)\frac{T_m}{w}$$
$$L = 1.326\frac{T_m}{w} \blacktriangleleft$$

(*b*) For $w = 0.25\,\text{lb/ft}$ and $T_m = 8000\,\text{lb}$

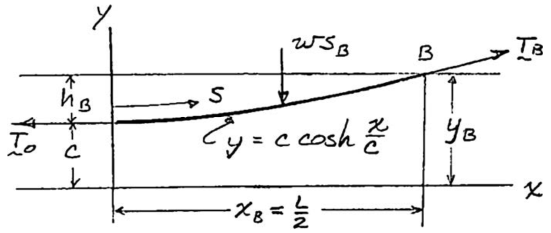
$$L = 1.326\frac{8000\text{ lb}}{0.25\,\text{lb/ft}}$$
$$= 42,432\,\text{ft}$$
$$L = 8.04\ \text{miles} \blacktriangleleft$$



PROBLEM 7.151*

A cable has a mass per unit length of 3 kg/m and is supported as shown. Knowing that the span L is 6 m, determine the *two* values of the sag h for which the maximum tension is 350 N.

SOLUTION



$$y_{\max} = c\cosh\frac{L}{2c} = h + c$$

$$w = (3\text{ kg/m})(9.81\text{ m/s}^2) = 29.43\text{ N/m}$$

$$T_{\max} = wy_{\max}$$

$$y_{\max} = \frac{T_{\max}}{w}$$

$$y_{\max} = \frac{350\text{ N}}{29.43\text{ N/m}} = 11.893\text{ m}$$

$$c\cosh\frac{3\text{ m}}{c} = 11.893\text{ m}$$

Solving numerically:

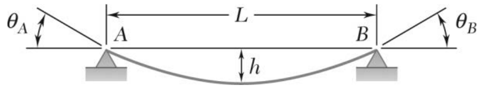
$$c_1 = 0.9241\text{ m}$$

$$c_2 = 11.499\text{ m}$$

$$h = y_{\max} - c$$

$$h_1 = 11.893\text{ m} - 0.9241\text{ m} \qquad h_1 = 10.97\text{ m} \blacktriangleleft$$

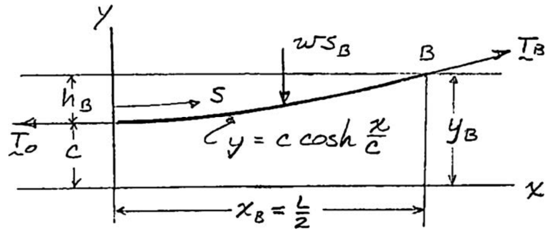
$$h_2 = 11.893\text{ m} - 11.499\text{ m} \qquad h_2 = 0.394\text{ m} \blacktriangleleft$$



PROBLEM 7.152*

Determine the sag-to-span ratio for which the maximum tension in the cable is equal to the total weight of the entire cable *AB*.

SOLUTION



$$T_{\max} = wy_B = 2ws_B$$

$$y_B = 2s_B$$

$$c\cosh\frac{L}{2c} = 2c\sinh\frac{L}{2c}$$

$$\tanh\frac{L}{2c} = \frac{1}{2}$$

$$\frac{L}{2c} = \tanh^{-1}\frac{1}{2} = 0.549306$$

$$\frac{h_B}{c} = \frac{y_B - c}{c} = \cosh\frac{L}{2c} - 1 = 0.154701$$

$$\frac{h_B}{L} = \frac{\frac{h_B}{c}}{2\left(\frac{L}{2c}\right)} = \frac{0.5(0.154701)}{0.549306} = 0.14081$$

$$\frac{h_B}{L} = 0.1408 \blacktriangleleft$$

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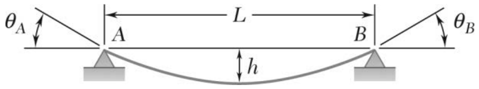
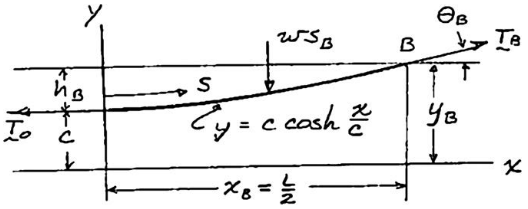
PROBLEM 7.153*



A cable of weight per unit length w is suspended between two points at the same elevation that are a distance L apart. Determine (a) the sag-to-span ratio for which the maximum tension is as small as possible, (b) the corresponding values of θ_B and T_m.

SOLUTION



(a)
$$T_{\max} = wy_B = wc\cosh\frac{L}{2c}$$

$$\frac{dT_{\max}}{dc} = w\left(\cosh\frac{L}{2c} - \frac{L}{2c}\sinh\frac{L}{2c}\right)$$

For $\quad \min T_{\max}, \quad \dfrac{dT_{\max}}{dc} = 0$

$$\tanh\frac{L}{2c} = \frac{2c}{L} \rightarrow \frac{L}{2c} = 1.1997$$

$$\frac{y_B}{c} = \cosh\frac{L}{2c} = 1.8102$$

$$\frac{h}{c} = \frac{y_B}{c} - 1 = 0.8102$$

$$\frac{h}{L} = \left[\frac{1}{2}\frac{h}{c}\left(\frac{2c}{L}\right)\right] = \frac{0.8102}{2(1.1997)} = 0.3375$$

$\dfrac{h}{L} = 0.338$ ◀

(b)
$$T_0 = wc \quad T_{\max} = wc\cosh\frac{L}{2c} \quad \frac{T_{\max}}{T_0} = \cosh\frac{L}{2c} = \frac{y_B}{c}$$

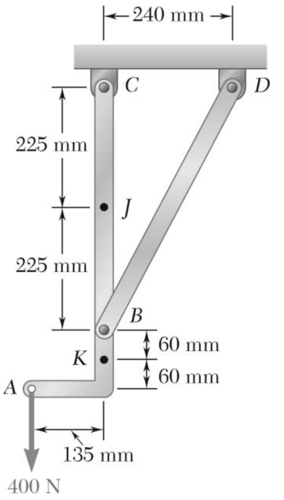
But $\quad T_0 = T_{\max}\cos\theta_B \quad \dfrac{T_{\max}}{T_0} = \sec\theta_B$

So $\quad \theta_B = \sec^{-1}\left(\dfrac{y_B}{c}\right) = \sec^{-1}(1.8102) = 56.46°$

$\theta_B = 56.5°$ ◀

$$T_{\max} = wy_B = w\frac{y_B}{c}\left(\frac{2c}{L}\right)\left(\frac{L}{2}\right) = w(1.8102)\frac{L}{2(1.1997)}$$

$T_{\max} = 0.755wL$ ◀

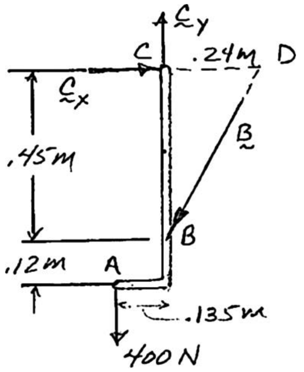


PROBLEM 7.154

Determine the internal forces at Point J of the structure shown.

SOLUTION

FBD ABC:



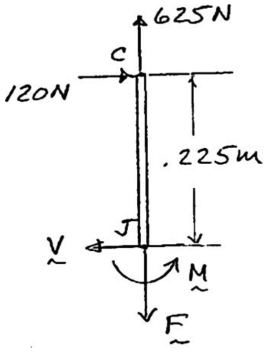
$$\circlearrowleft \Sigma M_D = 0\text{:} \quad (0.375\text{ m})(400\text{ N}) - (0.24\text{ m})C_y = 0$$

$$\mathbf{C}_y = 625\text{ N}\uparrow$$

$$\circlearrowleft \Sigma M_B = 0\text{:} \quad -(0.45\text{ m})C_x + (0.135\text{ m})(400\text{ N}) = 0$$

$$\mathbf{C}_x = 120\text{ N}\longrightarrow$$

FBD CJ:

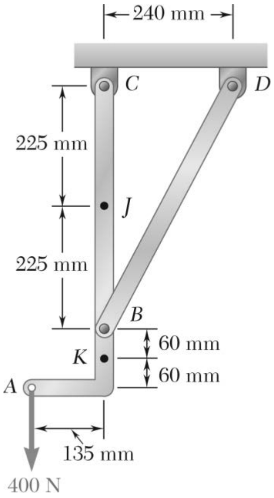


$$\uparrow \Sigma F_y = 0\text{:} \quad 625\text{ N} - F = 0 \qquad \mathbf{F} = 625\text{ N}\downarrow ◀$$

$$\longrightarrow \Sigma F_x = 0\text{:} \quad 120\text{ N} - V = 0 \qquad \mathbf{V} = 120.0\text{ N}\longleftarrow ◀$$

$$\circlearrowleft \Sigma M_J = 0\text{:} \quad M - (0.225\text{ m})(120\text{ N}) = 0$$

$$\mathbf{M} = 27.0\text{ N}\cdot\text{m}\circlearrowleft ◀$$

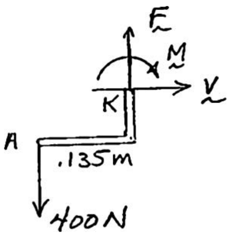


PROBLEM 7.155

Determine the internal forces at Point *K* of the structure shown.

SOLUTION

FBD AK:



$$\xrightarrow{} \Sigma F_x = 0: \quad V = 0$$

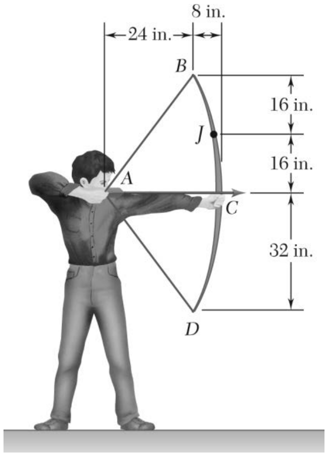
$$\mathbf{V} = 0 \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad F - 400 \text{ N} = 0$$

$$\mathbf{F} = 400 \text{ N} \uparrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: \quad (0.135 \text{ m})(400 \text{ N}) - M = 0$$

$$\mathbf{M} = 54.0 \text{ N} \cdot \text{m} \circlearrowright \blacktriangleleft$$

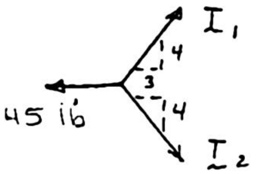


PROBLEM 7.156

An archer aiming at a target is pulling with a 45-lb force on the bowstring. Assuming that the shape of the bow can be approximated by a parabola, determine the internal forces at Point *J*.

SOLUTION

FBD Point A:

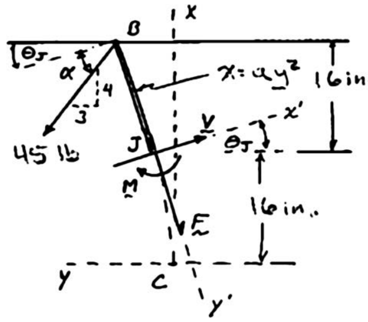


By symmetry $T_1 = T_2$

$$\xrightarrow{} \Sigma F_x = 0: \quad 2\left(\frac{3}{5}T_1\right) - 45 \text{ lb} = 0 \quad T_1 = T_2 = 37.5 \text{ lb}$$

Curve *CJB* is parabolic: $x = ay^2$

FBD BJ:



At B: $x = 8$ in. $y = 32$ in.

$$a = \frac{8 \text{ in.}}{(32 \text{ in.})^2} = \frac{1}{128 \text{ in.}}$$

$$x = \frac{y^2}{128}$$

$$\text{Slope of parabola} = \tan\theta = \frac{dx}{dy} = \frac{2y}{128} = \frac{y}{64}$$

At J: $$\theta_J = \tan^{-1}\left[\frac{16}{64}\right] = 14.036°$$

So $$\alpha = \tan^{-1}\frac{4}{3} - 14.036° = 39.094°$$

$$\nearrow \Sigma F_{x'} = 0: \quad V - (37.5 \text{ lb})\cos(39.094°) = 0$$

$\mathbf{V} = 29.1 \text{ lb} \measuredangle 14.04°$ ◀

PROBLEM 7.156 (Continued)

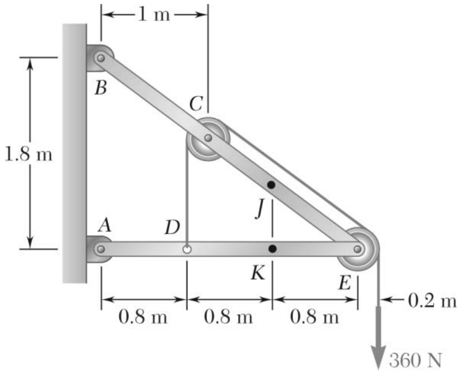
$$\searrow \Sigma F_{y'} = 0: \quad F + (37.5\text{ lb})\sin(39.094°) = 0$$

$$F = -23.647$$

$$\mathbf{F} = 23.6\text{ lb} \searrow 76.0° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + (16\text{ in.})\left[\frac{3}{5}(37.5\text{ lb})\right] + [(8-2)\text{ in.}]\left[\frac{4}{5}(37.5\text{ lb})\right] = 0$$

$$\mathbf{M} = 540\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

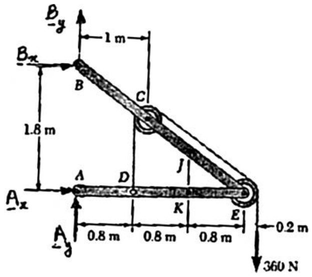


PROBLEM 7.157

Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at Point J of the frame shown.

SOLUTION

Free body: Frame and pulleys



$$\circlearrowleft + \Sigma M_A = 0: \quad -B_x(1.8\text{ m}) - (360\text{ N})(2.6\text{ m}) = 0$$

$$B_x = -520\text{ N} \qquad \mathbf{B}_x = 520\text{ N} \longleftarrow \triangleleft$$

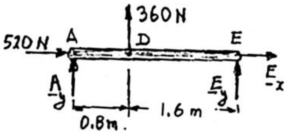
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 520\text{ N} = 0$$

$$A_x = +520\text{ N} \qquad \mathbf{A}_x = 520\text{ N} \longrightarrow \triangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad A_y + B_y - 360\text{ N} = 0$$

$$A_y + B_y = 360\text{ N} \qquad (1)$$

Free body: Member AE



$$\circlearrowleft + \Sigma M_E = 0: \quad -A_y(2.4\text{ m}) - (360\text{ N})(1.6\text{ m}) = 0$$

$$A_y = -240\text{ N} \qquad \mathbf{A}_y = 240\text{ N}\downarrow \triangleleft$$

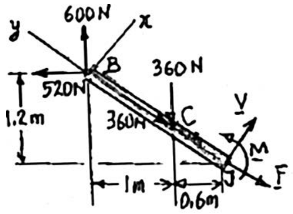
From (1):

$$B_y = 360\text{ N} + 240\text{ N}$$

$$B_y = +600\text{ N} \qquad \mathbf{B}_y = 600\text{ N}\uparrow \triangleleft$$

Free body: BJ

We recall that the forces applied to a pulley may be applied directly to its axle.



$$\nwarrow\!+ \Sigma F_y = 0: \quad \frac{3}{5}(600\text{ N}) + \frac{4}{5}(520\text{ N})$$

$$-360\text{ N} - \frac{3}{5}(360\text{ N}) - F = 0$$

$$F = +200\text{ N} \qquad \mathbf{F} = 200\text{ N} \searrow 36.9° \blacktriangleleft$$

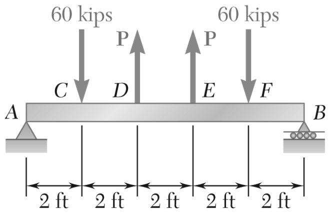
PROBLEM 7.157 (Continued)

$$+\nearrow \Sigma F_x = 0: \quad \frac{4}{5}(600\text{ N}) - \frac{3}{5}(520\text{ N}) - \frac{4}{5}(360\text{ N}) + V = 0$$

$$V = +120.0\text{ N} \qquad \mathbf{V} = 120.0\text{ N} \measuredangle\ 53.1^\circ \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad (520\text{ N})(1.2\text{ m}) - (600\text{ N})(1.6\text{ m}) + (360\text{ N})(0.6\text{ m}) + M = 0$$

$$M = +120.0\text{ N}\cdot\text{m} \qquad \mathbf{M} = 120.0\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

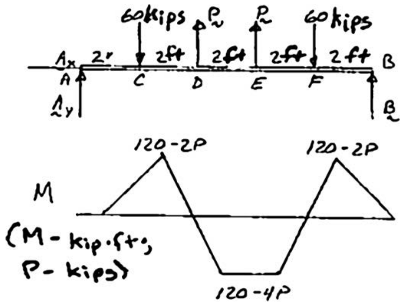


PROBLEM 7.158

For the beam shown, determine (*a*) the magnitude P of the two upward forces for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{\max}$.

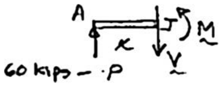
SOLUTION

By symmetry: $A_y = B = 60\text{ kips} - P$



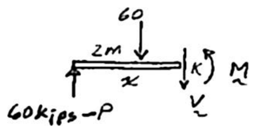
Along AC:

$$\circlearrowleft \Sigma M_J = 0: \quad M - x(60\text{ kips} - P) = 0$$
$$M = (60\text{ kips} - P)x$$
$$M = 120\text{ kips}\cdot\text{ft} - (2\text{ ft})P \quad \text{at} \quad x = 2\text{ ft}$$



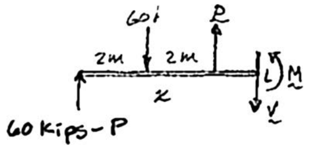
Along CD:

$$\circlearrowleft \Sigma M_K = 0: \quad M + (x - 2\text{ ft})(60\text{ kips}) - x(60\text{ kips} - P) = 0$$
$$M = 120\text{ kip}\cdot\text{ft} - Px$$
$$M = 120\text{ kip}\cdot\text{ft} - (4\text{ ft})P \quad \text{at} \quad x = 4\text{ ft}$$



Along DE:

$$\circlearrowleft \Sigma M_L = 0: \quad M - (x - 4\text{ ft})P + (x - 2\text{ ft})(60\text{ kips})$$
$$-x(60\text{ kips} - P) = 0$$
$$M = 120\text{ kip}\cdot\text{ft} - (4\text{ ft})P \quad \text{(const)}$$



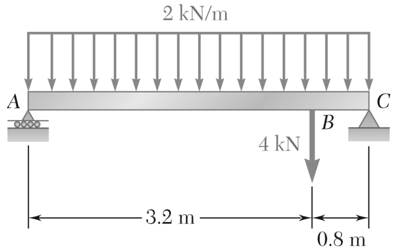
Complete diagram by symmetry

For minimum $|M|_{\max}$, set $M_{\max} = -M_{\min}$

$$120\text{ kip}\cdot\text{ft} - (2\text{ ft})P = -[120\text{ kip}\cdot\text{ft} - (4\text{ ft})P]$$

(*a*) $P = 40.0\text{ kips}$ ◀

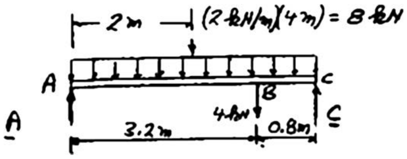
$M_{\min} = 120\text{ kip}\cdot\text{ft} - (4\text{ ft})P$ (*b*) $|M|_{\max} = 40.0\text{ kip}\cdot\text{ft}$ ◀



PROBLEM 7.159

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

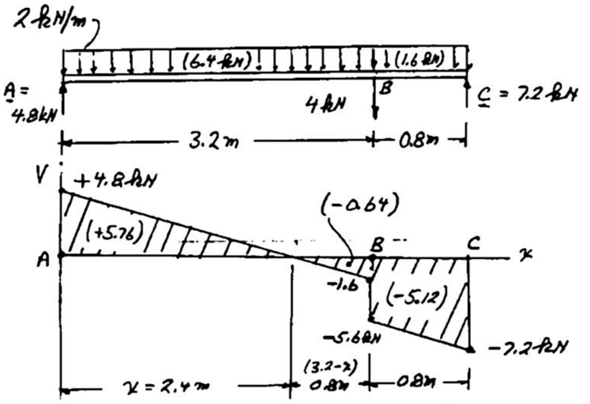


$$+\circlearrowleft \Sigma M_A = 0: \quad (8)(2) + (4)(3.2) - 4C = 0$$

$$\mathbf{C} = 7.2\text{ kN}\uparrow$$

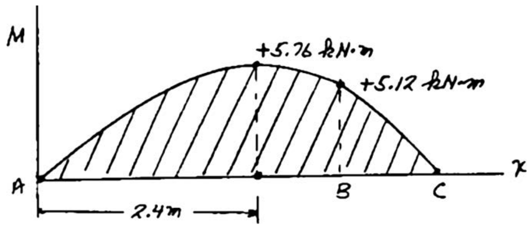
$$\Sigma F_y = 0: \quad \mathbf{A} = 4.8\text{ kN}\uparrow$$

(*a*) Shear diagram



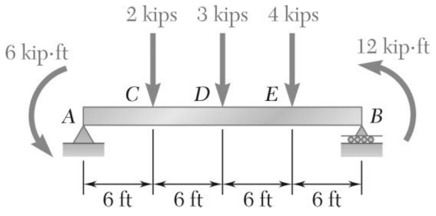
Similar Triangles: $\dfrac{x}{4.8} = \underset{\substack{\uparrow \\ \text{Add num. \& den.}}}{\dfrac{3.2-x}{1.6}} = \dfrac{3.2}{6.4}; \quad x = 2.4\text{ m}$

<u>Bending-moment diagram</u>



(*b*) $|V|_{\max} = 7.20\text{ kN}$ ◀

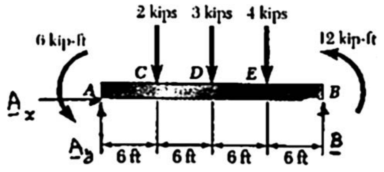
$|M|_{\max} = 5.76\text{ kN}\cdot\text{m}$ ◀



PROBLEM 7.160

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

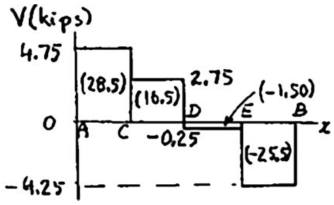


Free body: Beam

$$+\circlearrowleft \Sigma M_B = 0: \quad 6\text{ kip}\cdot\text{ft} + 12\text{ kip}\cdot\text{ft} + (2\text{ kips})(18\text{ ft})$$
$$+(3\text{ kips})(12\text{ ft}) + (4\text{ kips})(6\text{ ft}) - A_y(24\text{ ft}) = 0$$

$$A_y = +4.75\text{ kips} \quad \triangleleft$$

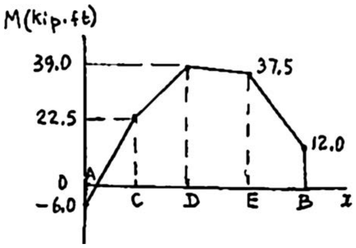
$$\Sigma F_x = 0: \quad A_x = 0$$



Shear diagram

At A: $V_A = A_y = +4.75\text{ kips}$

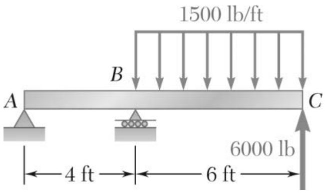
$$|V|_{\max} = 4.75\text{ kips} \quad \blacktriangleleft$$



Bending-moment diagram

At A: $M_A = -6\text{ kip}\cdot\text{ft}$

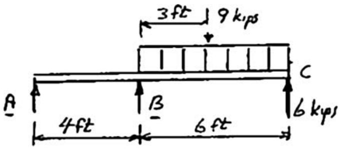
$$|M|_{\max} = 39.0\text{ kip}\cdot\text{ft} \quad \blacktriangleleft$$



PROBLEM 7.161

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



Free body: Entire beam

$$+\circlearrowleft \Sigma M_A = 0: \quad (6\text{ kips})(10\text{ ft}) - (9\text{ kips})(7\text{ ft}) + 8(4\text{ ft}) = 0$$

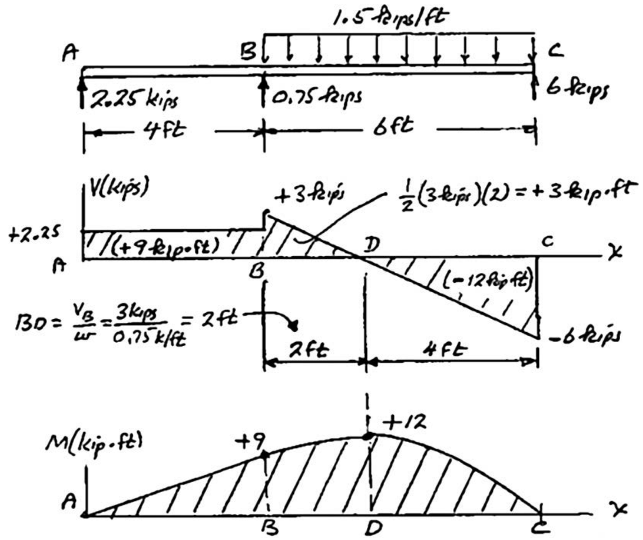
$$B = +0.75\text{ kips}$$

$$\mathbf{B} = 0.75\text{ kips}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad A + 0.75\text{ kips} - 9\text{ kips} + 6\text{ kips} = 0$$

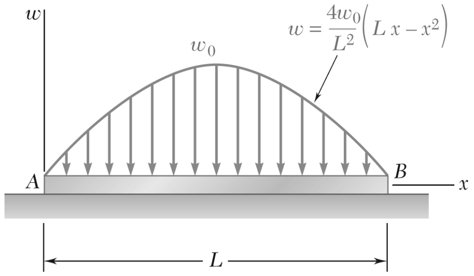
$$A = +2.25\text{ kips}$$

$$\mathbf{A} = 2.25\text{ kips}\uparrow$$



$$M_{\max} = 12.00\text{ kip}\cdot\text{ft} \blacktriangleleft$$

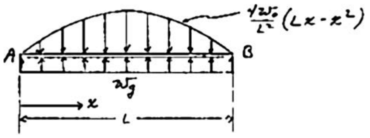
6.00 ft from *A*



PROBLEM 7.162

The beam AB, which lies on the ground, supports the parabolic load shown. Assuming the upward reaction of the ground to be uniformly distributed, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the maximum bending moment.

SOLUTION



(*a*)

$$\uparrow\Sigma F_y = 0:\quad w_g L - \int_0^L \frac{4w_0}{L^2}(Lx - x^2)dx = 0$$

$$w_g L = \frac{4w_0}{L^2}\left(\frac{1}{2}LL^2 - \frac{1}{3}L^3\right) = \frac{2}{3}w_0 L$$

$$w_g = \frac{2w_0}{3}$$

Define $\xi = \dfrac{x}{L}$ so $d\xi = \dfrac{dx}{L} \longrightarrow$ net load $w = 4w_0\left[\dfrac{x}{L} - \left(\dfrac{x}{L}\right)^2\right] - \dfrac{2}{3}w_0$

or $w = 4w_0\left(-\dfrac{1}{6} + \xi - \xi^2\right)$

$$V = V(0) - \int_0^\xi 4w_0 L\left(-\frac{1}{6} + \xi - \xi^2\right)d\xi$$

$$= 0 + 4w_0 L\left(\frac{1}{6}\xi + \frac{1}{2}\xi^2 - \frac{1}{3}\xi^3\right)$$

$$V = \frac{2}{3}w_0 L\,(\xi - 3\xi^2 + 2\xi^3) \blacktriangleleft$$

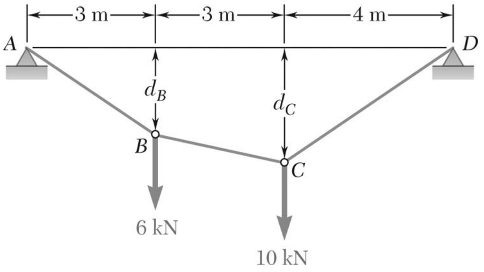
$$M = M_0 + \int_0^x V dx = 0 + \frac{2}{3}w_0 L^2 \int_0^\xi (\xi - 3\xi^2 + 2\xi^3)d\xi$$

$$= \frac{2}{3}w_0 L^2\left(\frac{1}{2}\xi^2 - \xi^3 + \frac{1}{2}\xi^4\right) = \frac{1}{3}w_0 L^2(\xi^2 - 2\xi^3 + \xi^4) \blacktriangleleft$$

(*b*) Max M occurs where $V = 0 \longrightarrow 1 - 3\xi + 2\xi^2 = 0 \longrightarrow \xi = \dfrac{1}{2}$

$$M\left(\xi = \frac{1}{2}\right) = \frac{1}{3}w_0 L^2\left(\frac{1}{4} - \frac{2}{8} + \frac{1}{16}\right) = \frac{w_0 L^2}{48}$$

$M_{\text{max}} = \dfrac{w_0 L^2}{48}$ at center of beam ◀

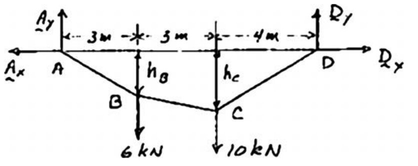


PROBLEM 7.163

Two loads are suspended as shown from the cable *ABCD*. Knowing that $d_B = 1.8$ m, determine (*a*) the distance d_C, (*b*) the components of the reaction at *D*, (*c*) the maximum tension in the cable.

SOLUTION

FBD Cable:



$$\xrightarrow{} \Sigma F_x = 0: \quad -A_x + D_x = 0 \quad A_x = D_x$$

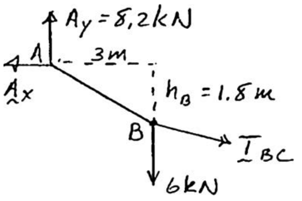
$$\circlearrowleft \Sigma M_A = 0: \quad (10 \text{ m})D_y - (6 \text{ m})(10 \text{ kN}) - (3 \text{ m})(6 \text{ kN}) = 0$$

$$\mathbf{D}_y = 7.8 \text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - 6 \text{ kN} - 10 \text{ kN} + 7.8 \text{ kN} = 0$$

$$\mathbf{A}_y = 8.2 \text{ kN} \uparrow$$

FDB AB:

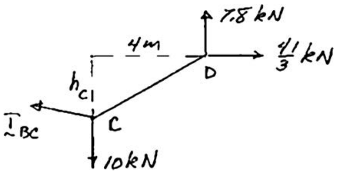


$$\circlearrowleft \Sigma M_B = 0: \quad (1.8 \text{ m})A_x - (3 \text{ m})(8.2 \text{ kN}) = 0$$

$$\mathbf{A}_x = \frac{41}{3} \text{ kN} \longleftarrow$$

From above $\quad D_x = A_x = \dfrac{41}{3}$ kN

FBD CD:



$$\circlearrowleft \Sigma M_C = 0: \quad (4 \text{ m})(7.8 \text{ kN}) - d_C\left(\frac{41}{3} \text{ kN}\right) = 0$$

$$d_C = 2.283 \text{ m}$$

(*a*) $\quad d_C = 2.28 \text{ m}$ ◀

(*b*) $\quad \mathbf{D}_x = 13.67 \text{ kN} \longrightarrow$ ◀

$\mathbf{D}_y = 7.80 \text{ kN} \uparrow$ ◀

Since $A_x = B_x$ and $A_y > B_y$, max T is T_{AB}

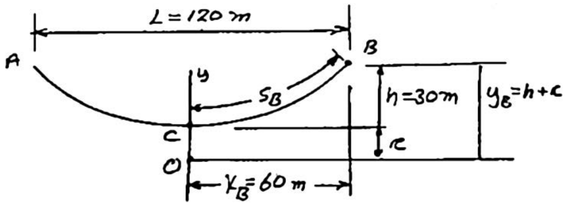
$$T_{AB} = \sqrt{A_x^2 + A_y^2} = \sqrt{\left(\frac{41}{3} \text{ kN}\right)^2 + (8.2 \text{ kN})^2}$$

(*c*) $\quad T_{\max} = 15.94 \text{ kN}$ ◀

PROBLEM 7.164

A wire having a mass per unit length of 0.65 kg/m is suspended from two supports at the same elevation that are 120 m apart. If the sag is 30 m, determine (*a*) the total length of the wire, (*b*) the maximum tension in the wire.

SOLUTION



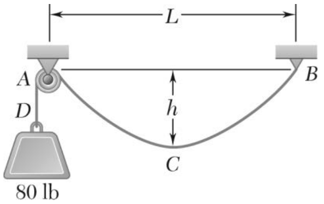
Eq. 7.16:
$$y_B = c\cosh\frac{x_B}{c}$$
$$30m + c = c\cosh\frac{60}{c}$$

Solve by trial and error: $c = 64.459\text{ m}$

Eq. 7.15:
$$s_B = c\sin h\frac{x_B}{c}$$
$$s_B = (64.456\text{ m})\sinh\frac{60\text{ m}}{64.459\text{ m}}$$
$$s_B = 69.0478\text{ m}$$

Length $= 2s_B = 2(69.0478\text{ m}) = 138.0956\text{ m}$ $L = 138.1\text{ m}$ ◀

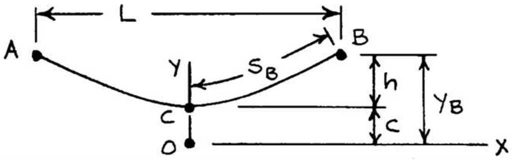
Eq. 7.18:
$$T_m = wy_B = w(h+c)$$
$$= (0.65\text{ kg/m})(9.81\text{ m/s}^2)(30\text{ m} + 64.459\text{ m})$$
$$T_m = 602.32\text{ N}$$
$T_m = 602\text{ N}$ ◀



PROBLEM 7.165

A counterweight D is attached to a cable that passes over a small pulley at A and is attached to a support at B. Knowing that L = 45 ft and h = 15 ft, determine (*a*) the length of the cable from A to B, (*b*) the weight per unit length of the cable. Neglect the weight of the cable from A to D.

SOLUTION



Given:
$$L = 45 \text{ ft}$$
$$h = 15 \text{ ft}$$
$$T_A = 80 \text{ lb}$$
$$x_B = 22.5 \text{ ft}$$

By symmetry: $T_B = T_A = T_m = 80 \text{ lb}$

We have $y_B = c\cosh\frac{x_B}{c} = c\cosh\frac{22.5}{c}$

and $y_B = h + c = 15 + c$

Then $c\cosh\frac{22.5}{c} = 15 + c$

or $\cosh\frac{22.5}{c} = \frac{15}{c} + 1$

Solve by trial for c: $c = 18.9525 \text{ ft}$

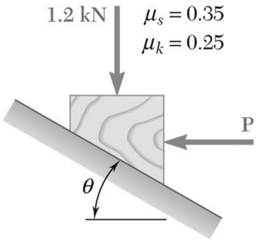
(*a*)
$$s_B = c\sinh\frac{x_B}{c}$$
$$= (18.9525 \text{ ft})\sinh\frac{22.5}{18.9525}$$
$$= 28.170 \text{ ft}$$

Length $= 2s_B = 2(28.170 \text{ ft}) = 56.3 \text{ ft}$ ◀

(*b*)
$$T_m = wy_B = w(h + c)$$
$$80 \text{ lb} = w(15 \text{ ft} + 18.9525 \text{ ft})$$

$w = 2.36 \text{ lb/ft}$ ◀

CHAPTER 8



PROBLEM 8.1

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 25°$ and $P = 750$ N.

SOLUTION

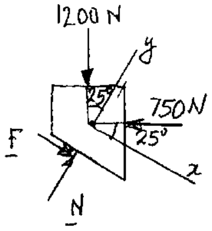
Assume equilibrium:

$$\overset{+}{\searrow} \Sigma F_x = 0: \quad F + (1200\text{ N})\sin 25° - (750\text{ N})\cos 25° = 0$$

$$F = +172.6\text{ N} \qquad \mathbf{F} = 172.6\text{ N} \searrow$$

$$+\nearrow \Sigma F_y = 0: \quad N - (1200\text{ N})\cos 25° - (750\text{ N})\sin 25° = 0$$

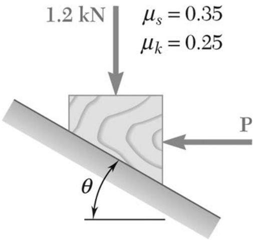
$$N = 1404.5\text{ N}$$



Maximum friction force: $F_m = \mu_s N = 0.35(1404.5\text{ N}) = 491.6\text{ N}$

Since $F < F_m$, block is in equilibrium ◀

Friction force: $\mathbf{F} = 172.6\text{ N} \searrow 25.0°$ ◀

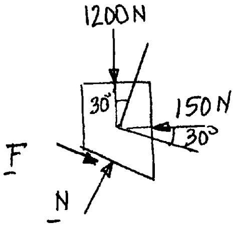


PROBLEM 8.2

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 30°$ and $P = 150$ N.

SOLUTION

Assume equilibrium:



$$\overset{+}{\searrow} \Sigma F_x = 0: \quad F + (1200\text{ N})\sin 30° - (150\text{ N})\cos 30° = 0$$

$$F = -470.1\text{ N} \qquad \mathbf{F} = 470.1\text{ N} \nwarrow$$

$$+\nearrow \Sigma F_y = 0: \quad N - (1200\text{ N})\cos 30° - (150\text{ N})\sin 30° = 0$$

$$N = 1114.2\text{ N}$$

(a) Maximum friction force:

$$F_m = \mu_s N$$
$$= 0.35(1114.2\text{ N})$$
$$= 390.0\text{ N}$$

Since **F** is ↖ and $F > F_m$, <u>block moves down</u> ◀

(*b*) Actual friction force: $F = F_k = \mu_k N = 0.25(1114.2\text{ N}) = 279\text{ N}$ $\qquad$ $\mathbf{F} = 279\text{ N} \nwarrow 30.0°$ ◀

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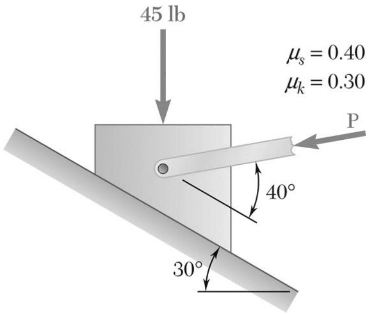
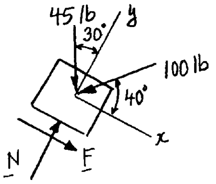


PROBLEM 8.3

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $P = 100$ lb.

SOLUTION

Assume equilibrium:



$$\overset{+}{\searrow}\Sigma F_x = 0: \quad F + (45\text{ lb})\sin 30° - (100\text{ lb})\cos 40° = 0$$

$$F = +54.0\text{ lb}$$

$$+\nearrow \Sigma F_y = 0: \quad N - (45\text{ lb})\cos 30° - (100\text{ lb})\sin 40° = 0$$

$$N = 103.2\text{ lb}$$

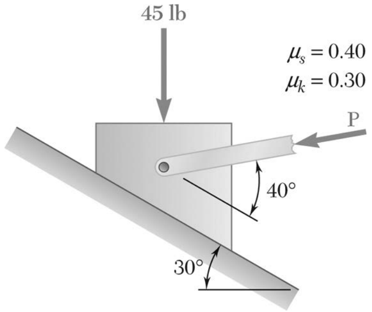
(*a*) Maximum friction force:

$$\begin{aligned} F_m &= \mu_s N \\ &= 0.40(103.2\text{ lb}) \\ &= 41.30\text{ lb} \end{aligned}$$

We note that $F > F_m$. Thus, <u>block moves up</u> ◀

(*b*) Actual friction force:

$F = F_k = \mu_k N = 0.30(103.2\text{ lb}) = 30.97\text{ lb},$ $\mathbf{F} = 31.0\text{ lb} \nwarrow 30.0°$ ◀

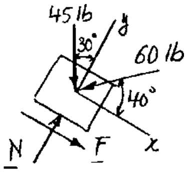


PROBLEM 8.4

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $P = 60$ lb.

SOLUTION

Assume equilibrium:



$$\overset{+}{\searrow} \Sigma F_x = 0: \quad F + (45\text{ lb})\sin 30^\circ - (60\text{ lb})\cos 40^\circ = 0$$

$$F = +23.46\text{ lb}$$

$$+\nearrow \Sigma F_y = 0: \quad N - (45\text{ lb})\cos 30^\circ - (60\text{ lb})\sin 40^\circ = 0$$

$$N = 77.54\text{ lb}$$

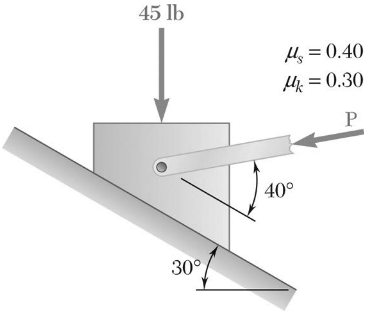
(*a*) Maximum friction force:

$$\begin{aligned} F_m &= \mu_s N \\ &= 0.40(77.54\text{ lb}) \\ &= 31.02\text{ lb} \end{aligned}$$

We check that $F < F_m$. Thus, block is in equilibrium ◀

(*b*) Thus $F = +23.46$ lb $\quad \mathbf{F} = 23.5\text{ lb} \searrow 30.0^\circ$ ◀

Note: We have $F_k = \mu_k N = 0.30(77.54) = 23.26$ lb. Thus $F > F_k$. If block originally in motion, it will keep moving with $F_k = 23.26$ lb.



PROBLEM 8.5

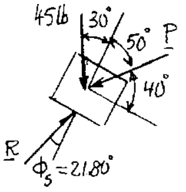
Determine the smallest value of P required to (*a*) start the block up the incline, (*b*) keep it moving up, (*c*) prevent it from moving down.

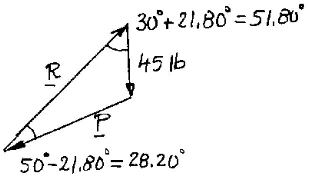
SOLUTION

(*a*) To start block up the incline:

$$\mu_s = 0.40$$

$$\phi_s = \tan^{-1} 0.40 = 21.80°$$





From force triangle:

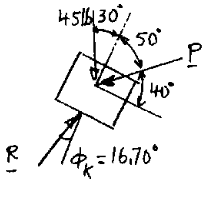
$$\frac{P}{\sin 51.80°} = \frac{45 \text{ lb}}{\sin 28.20°}$$

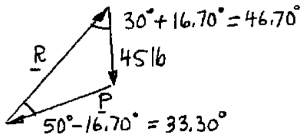
$P = 74.8$ lb ◀

(*b*) To keep block moving up:

$$\mu_k = 0.30$$

$$\phi_k = \tan^{-1} 0.30 = 16.70°$$





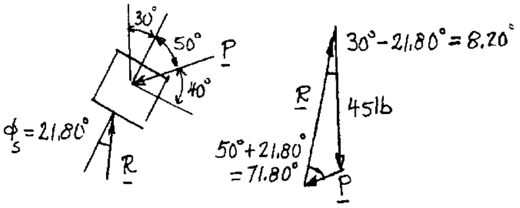
From force triangle:

$$\frac{P}{\sin 46.70°} = \frac{45 \text{ lb}}{\sin 33.30°}$$

$P = 59.7$ lb ◀

PROBLEM 8.5 (Continued)

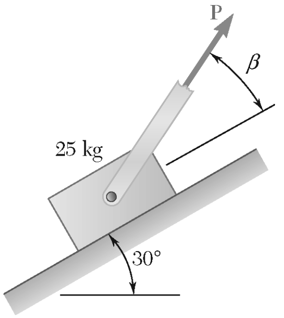
(*c*) <u>To prevent block from moving down</u>:



From force triangle:

$$\frac{P}{\sin 8.20°} = \frac{45 \text{ lb}}{\sin 71.80°}$$

$P = 6.76$ lb ◀

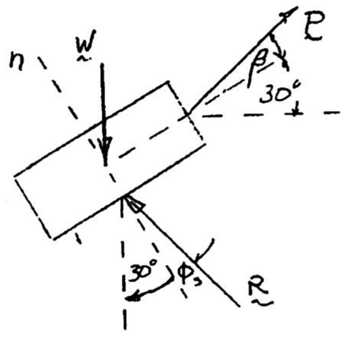


PROBLEM 8.6

Knowing that the coefficient of friction between the 25-kg block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of *P* required to start the block moving up the incline, (*b*) the corresponding value of β.

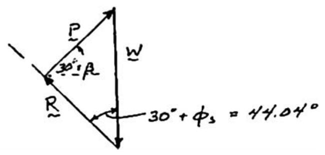
SOLUTION

FBD block (Impending motion up)



$$W = mg$$
$$= (25\text{ kg})(9.81\text{ m/s}^2)$$
$$= 245.25\text{ N}$$

$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1}(0.25)$$
$$= 14.04°$$



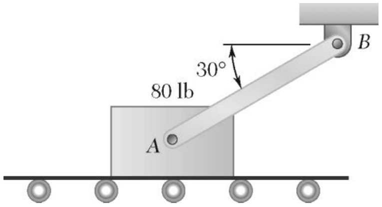
(*a*) (*Note:* For minimum *P*, $\mathbf{P} \perp \mathbf{R}$ so $\beta = \phi_s$.)

Then

$$P = W\sin(30° + \phi_s)$$
$$= (245.25\text{ N})\sin 44.04°$$

$P_{\min} = 170.5\text{ N}$ ◀

(*b*) We have $\beta = \phi_s$ $\beta = 14.04°$ ◀



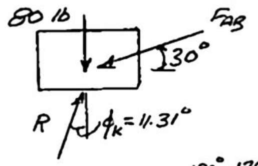
PROBLEM 8.7

The 80-lb block is attached to link *AB* and rests on a moving belt. Knowing that $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the magnitude of the horizontal force **P** that should be applied to the belt to maintain its motion (*a*) to the right, (*b*) to the left.

SOLUTION

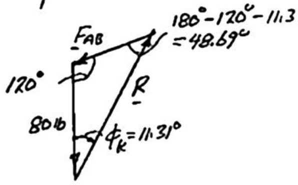
We note that link *AB* is a two-force member, since there is motion between belt and block $\mu_k = 0.20$ and $\phi_k = \tan^{-1} 0.20 = 11.31°$

(*a*) <u>Belt moves to right</u>



<u>Free body: Block</u>

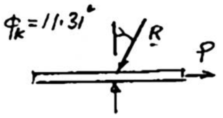
Force triangle:



$$\frac{R}{\sin 120°} = \frac{80\text{ lb}}{\sin 48.69°}$$

$$R = 92.23\text{ lb}$$

<u>Free body: Belt</u>

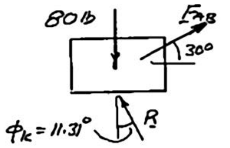


$$\xrightarrow{+} \Sigma F_x = 0: \quad P - (92.23\text{ lb})\sin 11.31°$$

$$P = 18.089\text{ lb}$$

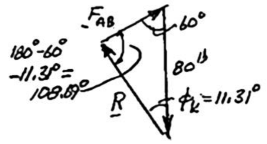
$\mathbf{P} = 18.09\text{ lb} \longrightarrow$ ◀

(*b*) <u>Belt moves to left</u>



<u>Free body: Block</u>

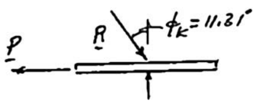
Force triangle:



$$\frac{R}{\sin 60°} = \frac{80\text{ lb}}{\sin 108.69°}$$

$$R = 73.139\text{ lb}$$

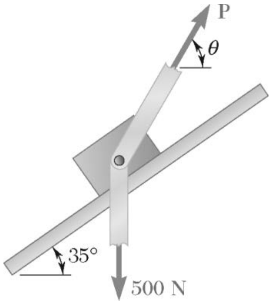
<u>Free body: Belt</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad (73.139\text{ lb})\sin 11.31° - P = 0$$

$$P = 14.344\text{ lb}$$

$\mathbf{P} = 14.34\text{ lb} \longleftarrow$ ◀



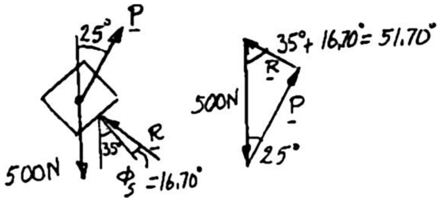
PROBLEM 8.8

The coefficients of friction between the block and the rail are $\mu_s = 0.30$ and $\mu_k = 0.25$. Knowing that $\theta = 65°$, determine the smallest value of P required (*a*) to start the block moving up the rail, (*b*) to keep it from moving down.

SOLUTION

(*a*) To start block up the rail:

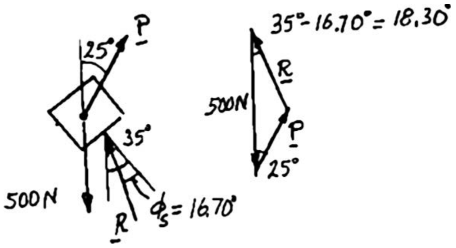
$$\mu_s = 0.30$$
$$\phi_s = \tan^{-1} 0.30 = 16.70°$$



Force triangle:

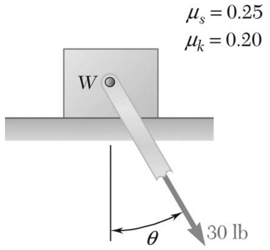
$$\frac{P}{\sin 51.70°} = \frac{500 \text{ N}}{\sin (180° - 25° - 51.70°)} \qquad P = 403 \text{ N} \blacktriangleleft$$

(*b*) To prevent block from moving down:



Force triangle:

$$\frac{P}{\sin 18.30°} = \frac{500 \text{ N}}{\sin (180° - 25° - 18.30°)} \qquad P = 229 \text{ N} \blacktriangleleft$$

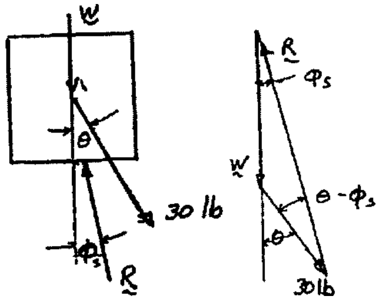


PROBLEM 8.9

Considering only values of θ less than 90°, determine the smallest value of θ required to start the block moving to the right when (a) $W = 75$ lb, (b) $W = 100$ lb.

SOLUTION

FBD block (Motion impending):



$$\phi_s = \tan^{-1}\mu_s = 14.036^\circ$$

$$\frac{30\text{ lb}}{\sin\phi_s} = \frac{W}{\sin(\theta - \phi_s)}$$

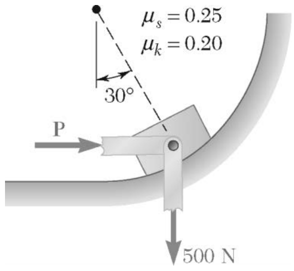
$$\sin(\theta - \phi_s) = \frac{W\sin 14.036^\circ}{30\text{ lb}}$$

or

$$\sin(\theta - 14.036^\circ) = \frac{W}{123.695\text{ lb}}$$

(a) $W = 75$ lb: $\theta = 14.036^\circ + \sin^{-1}\dfrac{75\text{ lb}}{123.695\text{ lb}}$ $\theta = 51.4^\circ$ ◀

(b) $W = 100$ lb: $\theta = 14.036^\circ + \sin^{-1}\dfrac{100\text{ lb}}{123.695\text{ lb}}$ $\theta = 68.0^\circ$ ◀



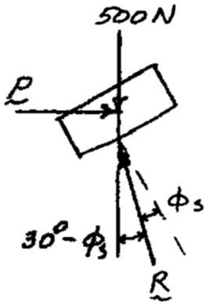
PROBLEM 8.10

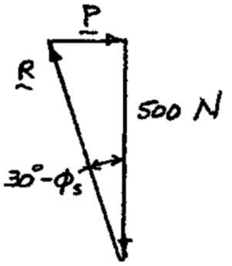
Determine the range of values of P for which equilibrium of the block shown is maintained.

SOLUTION

FBD block:

(Impending motion down):

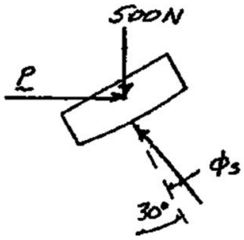


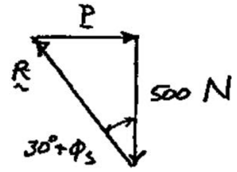


$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25$$

$$P = (500\text{ N})\tan(30° - \tan^{-1}0.25)$$
$$= 143.03\text{ N}$$

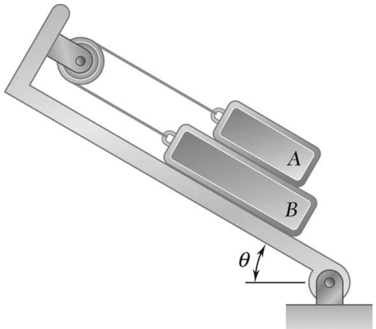
(Impending motion up):





$$P = (500\text{ N})\tan(30° + \tan^{-1}0.25)$$
$$= 483.46\text{ N}$$

Equilibrium is maintained for $143.0\text{ N} \le P \le 483\text{ N}$ ◀



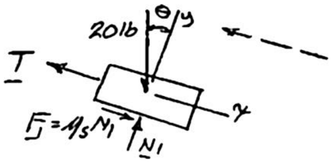
PROBLEM 8.11

The 20-lb block *A* and the 30-lb block *B* are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between the two blocks and zero between block *B* and the incline, determine the value of θ for which motion is impending.

SOLUTION

Since motion impends, $F = \mu_s N$ between $A + B$

Free body: Block *A*



Impending motion:

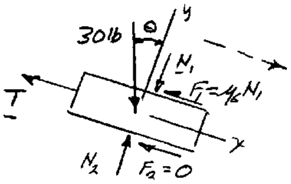
$$\Sigma F_y = 0: \quad N_1 = 20\cos\theta$$

$$\overset{+}{\nwarrow} \Sigma F_x = 0: \quad T - 20\sin\theta - \mu_s N_1 = 0$$

$$T = 20\sin\theta + 0.15(20\cos\theta)$$

$$T = 20\sin\theta + 3\cos\theta \qquad (1)$$

Free body: Block *B*



Impending motion:

$$\overset{+}{\nwarrow} \Sigma F_x = 0: \quad T - 30\sin\theta + \mu_s N_1 = 0$$

$$T = 30\sin\theta - \mu_s N_1$$

$$= 30\sin\theta - 0.15(20\cos\theta)$$

$$T = 30\sin\theta - 3\cos\theta \qquad (2)$$

Eq. (1)-Eq. (2): $\quad 20\sin\theta + 3\cos\theta - 30\sin\theta + 3\cos\theta = 0$

$$6\cos\theta = 10\sin\theta: \quad \tan\theta = \frac{6}{10}; \quad \theta = 30.96^\circ \qquad \theta = 31.0^\circ \blacktriangleleft$$

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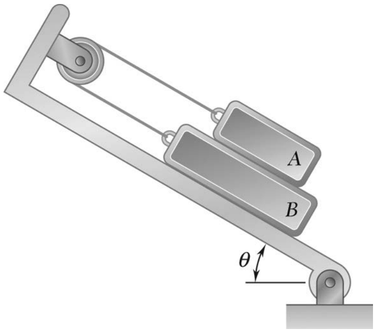


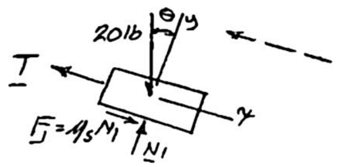
PROBLEM 8.12

The 20-lb block *A* and the 30-lb block *B* are supported by an incline that is held in the position shown. Knowing that the coefficient of static friction is 0.15 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

Since motion impends, $F = \mu_s N$ at all surfaces.

Free body: Block *A*



Impending motion:

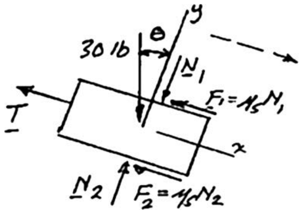
$$\Sigma F_y = 0: \quad N_1 = 20\cos\theta$$

$$\overset{+}{\nwarrow} \Sigma F_x = 0: \quad T - 20\sin\theta - \mu_s N_1 = 0$$

$$T = 20\sin\theta + 0.15(20\cos\theta)$$

$$T = 20\sin\theta + 3\cos\theta \qquad (1)$$

Free body: Block *B*



Impending motion:

$$+\nearrow \Sigma F_y = 0: \quad N_2 - 30\cos\theta - N_1 = 0$$

$$N_2 = 30\cos\theta + 20\cos\theta = 50\cos\theta$$

$$F_2 = \mu_s N_2 = 0.15(50\cos\theta) = 6\cos\theta$$

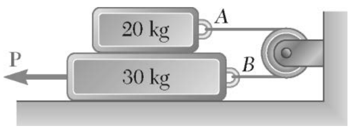
$$\overset{+}{\nwarrow} \Sigma F_x = 0: \quad T - 30\sin\theta + \mu_s N_1 + \mu_s N_2 = 0$$

$$T = 30\sin\theta - 0.15(20\cos\theta) - 0.15(50\cos\theta)$$

$$T = 30\sin\theta - 3\cos\theta - 7.5\cos\theta \qquad (2)$$

Eq. (1) subtracted by Eq. (2): $\quad 20\sin\theta + 3\cos\theta - 30\sin\theta + 3\cos\theta + 7.5\cos\theta = 0$

$$13.5\cos\theta = 10\sin\theta, \quad \tan\theta = \frac{13.5°}{10} \qquad \theta = 53.5° \blacktriangleleft$$

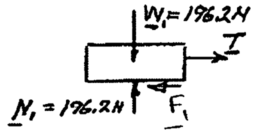


PROBLEM 8.13

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force **P** required to start the 30-kg block moving if cable *AB* (*a*) is attached as shown, (*b*) is removed.

SOLUTION

(*a*) Free body: 20-kg block

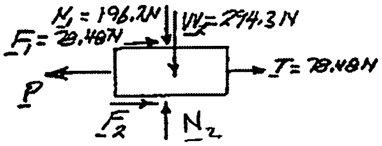


$$W_1 = (20\text{ kg})(9.81\text{ m/s}^2) = 196.2\text{ N}$$

$$F_1 = \mu_s N_1 = 0.4(196.2\text{ N}) = 78.48\text{ N}$$

$$\xrightarrow{+} \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48\text{ N}$$

Free body: 30-kg block



$$W_2 = (30\text{ kg})(9.81\text{ m/s}^2) = 294.3\text{ N}$$

$$N_2 = 196.2\text{ N} + 294.3\text{ N} = 490.5\text{ N}$$

$$F_2 = \mu_s N_2 = 0.4(490.5\text{ N}) = 196.2\text{ N}$$

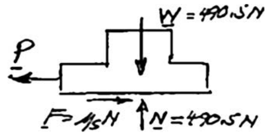
$$\xleftarrow{+} \Sigma F = 0: \quad P - F_1 - F_2 - T = 0$$

$$P = 78.48\text{ N} + 196.2\text{ N} + 78.48\text{ N} = 353.2\text{ N}$$

$\mathbf{P} = 353\text{ N} \longleftarrow$ ◀

(*b*) Free body: Both blocks

Blocks move together

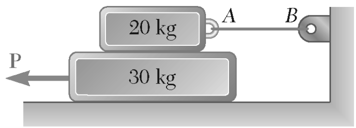


$$W = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N}$$

$$\xleftarrow{+} \Sigma F = 0: \quad P - F = 0$$

$$P = \mu_s N = 0.4(490.5\text{ N}) = 196.2\text{ N}$$

$\mathbf{P} = 196.2\text{ N} \longleftarrow$ ◀

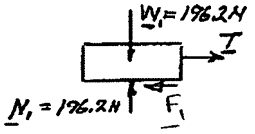


PROBLEM 8.14

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the smallest force **P** required to start the 30-kg block moving if cable *AB* (*a*) is attached as shown, (*b*) is removed.

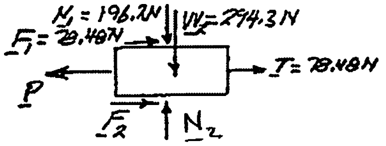
SOLUTION

(*a*) Free body: 20-kg block



$$W_1 = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$
$$F_1 = \mu_s N_1 = 0.4(196.2 \text{ N}) = 78.48 \text{ N}$$
$$\xrightarrow{+} \Sigma F = 0: \quad T - F_1 = 0 \quad T = F_1 = 78.48 \text{ N}$$

Free body: 30-kg block

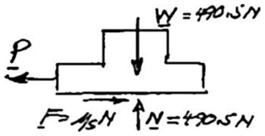


$$W_2 = (30 \text{ kg})(9.81 \text{ m/s}^2) = 294.3 \text{ N}$$
$$N_2 = 196.2 \text{ N} + 294.3 \text{ N} = 490.5 \text{ N}$$
$$F_2 = \mu_s N_2 = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$
$$\xleftarrow{+} \Sigma F = 0: \quad P - F_1 - F_2 = 0$$
$$P = 78.48 \text{ N} + 196.2 \text{ N} = 274.7 \text{ N}$$

P = 275 N ⟵ ◀

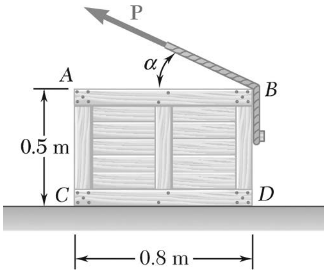
(*b*) Free body: Both blocks

Blocks move together



$$W = (50 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 490.5 \text{ N}$$
$$\xleftarrow{+} \Sigma F = 0: \quad P - F = 0$$
$$P = \mu_s N = 0.4(490.5 \text{ N}) = 196.2 \text{ N}$$

P = 196.2 N ⟵ ◀



PROBLEM 8.15

A 40-kg packing crate must be moved to the left along the floor without tipping. Knowing that the coefficient of static friction between the crate and the floor is 0.35, determine (*a*) the largest allowable value of α, (*b*) the corresponding magnitude of the force **P**.

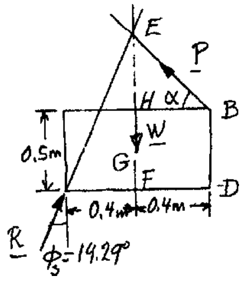
SOLUTION

(*a*) Free-body diagram

If the crate is about to tip about *C*, contact between crate and ground is only at *C* and the reaction **R** is applied at *C*. As the crate is about to slide, **R** must form with the vertical an angle

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}$$

$$0.35 = 19.29^\circ$$



Since the crate is a 3-force body. **P** must pass through *E* where **R** and **W** intersect.

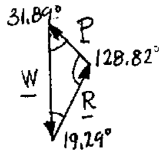
$$EF = \frac{CF}{\tan\theta_s} = \frac{0.4\text{ m}}{0.35} = 1.1429\text{ m}$$

$$EH = EF - HF = 1.1429 - 0.5 = 0.6429\text{ m}$$

$$\tan\alpha = \frac{EH}{HB} = \frac{0.6429\text{ m}}{0.4\text{ m}}$$

$$\alpha = 58.11^\circ \qquad \alpha = 58.1^\circ \blacktriangleleft$$

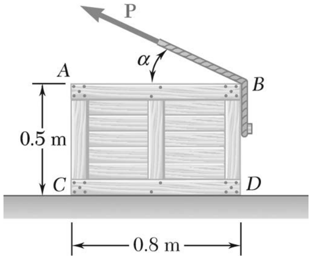
(*b*) Force Triangle



$$\frac{P}{\sin 19.29^\circ} = \frac{W}{\sin 128.82^\circ} \qquad P = 0.424\,W$$

$$P = 0.424(40\text{ kg})(9.81\text{ m/s}^2), \qquad P = 166.4\text{ N} \blacktriangleleft$$

Note: After the crate starts moving, μ_s should be replaced by the lower value μ_k. This will yield a larger value of α.



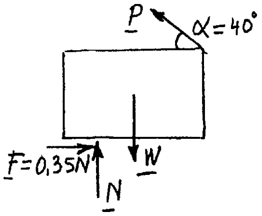
PROBLEM 8.16

A 40-kg packing crate is pulled by a rope as shown. The coefficient of static friction between the crate and the floor is 0.35. If $\alpha = 40°$, determine (*a*) the magnitude of the force **P** required to move the crate, (*b*) whether the crate will slide or tip.

SOLUTION

Force P for which sliding is impending

(We assume that crate does not tip)



$$W = (40\text{ kg})(9.81\text{ m/s}^2) = 392.4\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad N - W + P\sin 40° = 0$$

$$N = W - P\sin 40° \qquad (1)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 0.35\,N - P\cos 40° = 0$$

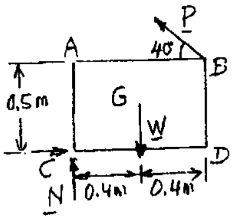
Substitute for N from Eq. (1):

$$0.35(W - P\sin 40°) - P\cos 40° = 0$$

$$P = \frac{0.35W}{0.35\sin 40° + \cos 40°} \qquad P = 0.3532W \ \triangleleft$$

Force P for which crate rotates about C

(We assume that crate does not slide)



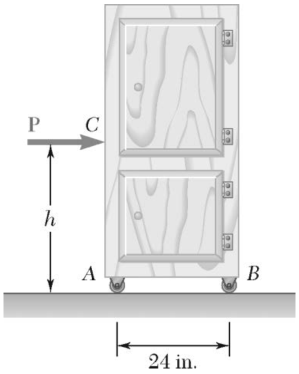
$$+\circlearrowleft \Sigma M_C = 0: \quad (P\sin 40°)(0.8\text{ m}) + (P\cos 40°)(0.5\text{ m})$$

$$-W(0.4\text{ m}) = 0$$

$$P = \frac{0.4W}{0.8\sin 40° + 0.5\cos 40°} = 0.4458W \ \blacktriangleleft$$

Crate will first slide ◁

$$P = 0.3532(392.4\text{ N}) \qquad P = 138.6\text{ N} \ \blacktriangleleft$$



PROBLEM 8.17

A 120-lb cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. If $h = 32$ in., determine the magnitude of the force **P** required to move the cabinet to the right (*a*) if all casters are locked, (*b*) if the casters at *B* are locked and the casters at *A* are free to rotate, (*c*) if the casters at *A* are locked and the casters at *B* are free to rotate.

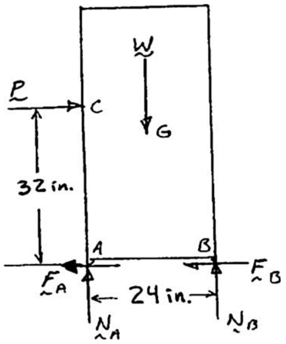
SOLUTION

FBD cabinet: *Note:* for tipping,

$$N_A = F_A = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad (12 \text{ in.})W - (32 \text{ in.})P_{\text{tip}} = 0$$

$$P_{\text{tip}} = 2.66667$$



(*a*) All casters locked. Impending slip:

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0$$

$$N_A + N_B = W \qquad W = 120 \text{ lb}$$

So $F_A + F_B = \mu_s W \qquad \mu_s = 0.3$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0$$

$$P = F_A + F_B = \mu_s W$$

$$P = 0.3(120 \text{ lb}) \qquad \text{or} \quad \mathbf{P} = 36.0 \text{ lb} \longrightarrow \blacktriangleleft$$

$$(P = 0.3W < P_{\text{tip}} \quad \text{OK})$$

(*b*) Casters at *A* free, so $F_A = 0$

Impending slip: $F_B = \mu_s N_B$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_B = 0$$

$$P = F_B = \mu_s N_B \quad N_B = \frac{P}{\mu_s}$$

PROBLEM 8.17 (Continued)

$$\circlearrowleft \Sigma M_A = 0: \quad (32 \text{ in.})P + (12 \text{ in.})W - (24 \text{ in.})N_B = 0$$

$$8P + 3W - 6\frac{P}{0.3} = 0 \quad P = 0.25W$$

$$(P = 0.25W < P_{\text{tip}} \quad \text{OK})$$

$$P = 0.25(120 \text{ lb}) \qquad \text{or} \quad \mathbf{P} = 30.0 \text{ lb} \longrightarrow \blacktriangleleft$$

(*c*) Casters at *B* free, so $F_B = 0$

Impending slip: $F_A = \mu_s N_A$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_A = 0 \quad P = F_A = \mu_s N_A$$

$$N_A = \frac{P}{\mu_s} = \frac{P}{0.3}$$

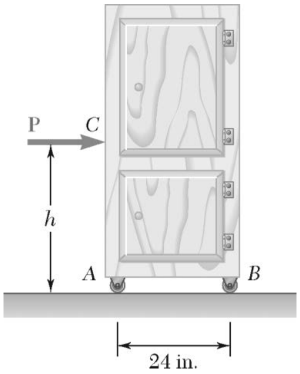
$$\circlearrowright \Sigma M_B = 0: \quad (12 \text{ in.})W - (32 \text{ in.})P - (24 \text{ in.})N_A = 0$$

$$3W - 8P - 6\frac{P}{0.3} = 0$$

$$P = 0.107143W = 12.8572$$

$$(P < P_{\text{tip}} \quad \text{OK})$$

$$\mathbf{P} = 12.86 \text{ lb} \longrightarrow \blacktriangleleft$$

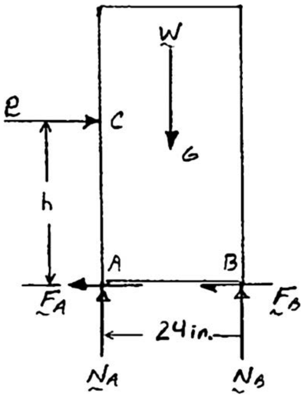


PROBLEM 8.18

A 120-lb cabinet is mounted on casters that can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Assuming that the casters at both *A* and *B* are locked, determine (*a*) the force **P** required to move the cabinet to the right, (*b*) the largest allowable value of *h* if the cabinet is not to tip over.

SOLUTION

FBD cabinet:



(*a*)

$$\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0$$

$$N_A + N_B = W$$

Impending slip:

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

So

$$F_A + F_B = \mu_s W$$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0$$

$$P = F_A + F_B = \mu_s W$$

$W = 120\text{ lb}$

$\mu_s = 0.3$

$$P = 0.3(120\text{ lb}) = 141.26\text{ N}$$

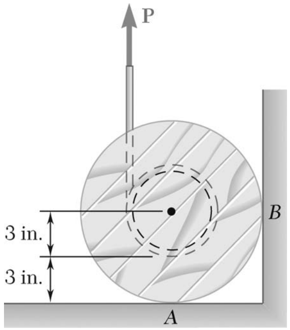
$\mathbf{P} = 36.0\text{ lb} \longrightarrow$ ◀

(*b*) For tipping, $N_A = F_A = 0$

$$\circlearrowleft \Sigma M_B = 0: \quad hP - (12\text{ in.})W = 0$$

$$h_{\max} = (12\text{ in.})\frac{W}{P} = (12\text{ in.})\frac{1}{\mu_s} = \frac{12\text{ in.}}{0.3}$$

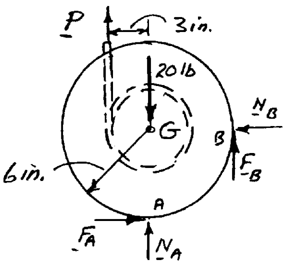
$h_{\max} = 40.0\text{ in.}$ ◀



PROBLEM 8.19

Wire is being drawn at a constant rate from a spool by applying a vertical force **P** to the wire as shown. The spool and the wire wrapped on the spool have a combined weight of 20 lb. Knowing that the coefficients of friction at both *A* and *B* are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine the required magnitude of the force **P**.

SOLUTION



Since spool is rotating

$$F_A = \mu_k N_A \qquad F_B = \mu_k N_B$$

$$+\circlearrowleft \Sigma M_G = 0{:}\quad P(3\text{ in.}) - F_A(6\text{ in.}) - F_B(6\text{ in.}) = 0$$

$$3P - 6\mu_k(N_A + N_B) = 0 \tag{1}$$

$$\xrightarrow{+} \Sigma F_x = 0{:}\quad F_A - N_B = 0$$

$$N_B = \mu_k N_A \tag{2}$$

$$+\uparrow \Sigma F_y = 0{:}\quad P + N_A + F_B - 20\text{ lb} = 0$$

$$P + N_A + \mu_k N_B - 20 = 0$$

Substitute for N_B from (2): $$P + N_A + \mu_k N_A - 20 = 0$$

$$N_A = \frac{20 - P}{1 + \mu_{k^2}} \tag{3}$$

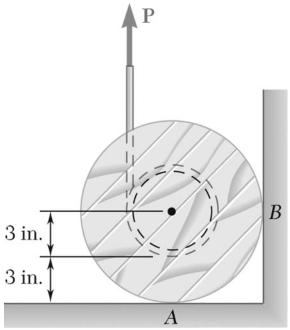
Substitute from (2) into (1): $$3P - 6\mu_k(N_A + \mu_k N_A) = 0$$

$$N_A = \frac{1}{2}\frac{P}{\mu_k(1+\mu_k)} \tag{4}$$

(3)=(4): $$\frac{20-P}{1+\mu_k^2} = \frac{P}{2(\mu_k + \mu_k^2)}$$

Substitute $\mu_k = 0.30$: $$\frac{20-P}{1+(0.3)^2} \quad \frac{P}{2(0.3)(1.03)}$$

$$20 - P = 1.3974P; \quad 2.3974P = 20; \qquad P = 8.34\text{ lb} \blacktriangleleft$$

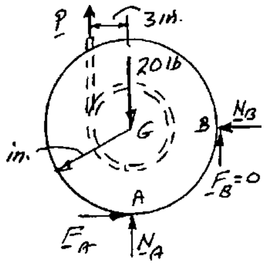


PROBLEM 8.20

Solve Problem 8.19 assuming that the coefficients of friction at *B* are zero.

PROBLEM 8.19 Wire is being drawn at a constant rate from a spool by applying a vertical force **P** to the wire as shown. The spool and the wire wrapped on the spool have a combined weight of 20 lb. Knowing that the coefficients of friction at both *A* and *B* are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine the required magnitude of the force **P**.

SOLUTION



Since spool is rotating

$$F_A = \mu_k N_A$$

$$+\circlearrowleft \Sigma M_G = 0: \quad P(3\text{ in.}) - F_A(6\text{ in.}) = 0$$

$$P = 2F_A = 2\mu_k N_A \qquad (1)$$

$$+\uparrow \Sigma F_y = 20: \quad P - 20\text{ lb} + N_A = 0$$

$$N_A = 20 - P \qquad (2)$$

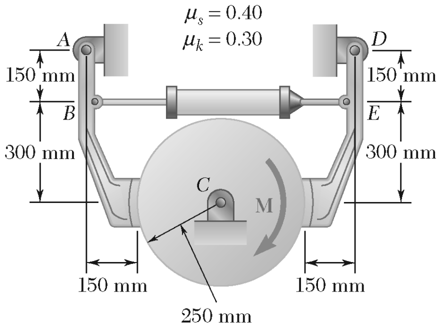
Substitute for N_A from (2) into (1) $\quad P = 2\mu_k(20 - P)$

Substitute $\mu_k = 0.30$: $\quad P = 2(0.3)(20 - P)$

$$1.667P = 20 - P$$

$$2.667P = 20$$

$$P = 7.50\text{ lb} \qquad\qquad P = 7.50\text{ lb} \blacktriangleleft$$

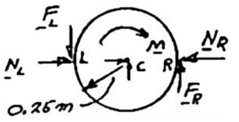


PROBLEM 8.21

The hydraulic cylinder shown exerts a force of 3 kN directed to the right on Point *B* and to the left on Point *E*. Determine the magnitude of the couple **M** required to rotate the drum clockwise at a constant speed.

SOLUTION

Free body: Drum



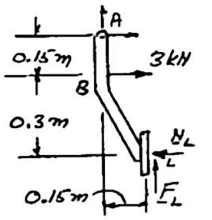
$$+\circlearrowright \Sigma M_C = 0: \quad M - (0.25 \text{ m})(F_L + F_R) = 0$$

$$M = (0.25 \text{ m})(F_L + F_R) \qquad (1)$$

Since drum is rotating

$$F_L = \mu_k N_L = 0.3 N_L$$
$$F_R = \mu_k N_R = 0.3 N_R$$

Free body: Left arm *ABL*



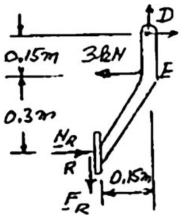
$$+\circlearrowright \Sigma M_A = 0: \quad (3 \text{ kN})(0.15 \text{ m}) + F_L(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.45 \text{ kN} \cdot \text{m} + (0.3 N_L)(0.15 \text{ m}) - N_L(0.45 \text{ m}) = 0$$

$$0.405 N_L = 0.45$$

$$N_L = 1.111 \text{ kN}$$
$$F_L = 0.3 N_L = 0.3(1.111 \text{ kN})$$
$$= 0.3333 \text{ kN} \qquad (2)$$

Free body: Right arm *DER*



$$+\circlearrowright \Sigma M_D = 0: \quad (3 \text{ kN})(0.15 \text{ m}) - F_R(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.45 \text{ kN} \cdot \text{m} - (0.3 N_R)(0.15 \text{ m}) - N_R(0.45 \text{ m}) = 0$$

$$0.495 N_R = 0.45$$

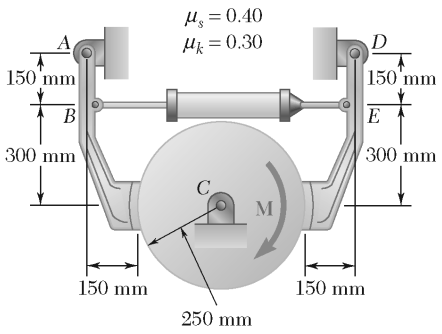
$$N_R = 0.9091 \text{ kN}$$
$$F_R = \mu_k N_R = 0.3(0.9091 \text{ kN})$$
$$= 0.2727 \text{ kN} \qquad (3)$$

Substitute for F_L and F_R into (1):

$$M = (0.25 \text{ m})(0.333 \text{ kN} + 0.2727 \text{ kN})$$

$$M = 0.1515 \text{ kN} \cdot \text{m}$$

$$M = 151.5 \text{ N} \cdot \text{m} \blacktriangleleft$$

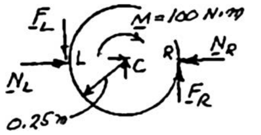


PROBLEM 8.22

A couple **M** of magnitude 100 N · m is applied to the drum as shown. Determine the smallest force that must be exerted by the hydraulic cylinder on joints B and E if the drum is not to rotate.

SOLUTION

Free body: Drum



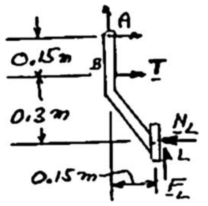
$$+\circlearrowleft \Sigma M_C = 0: \quad 100\ \text{N}\cdot\text{m} - (0.25\ \text{m})(F_L + F_R) = 0$$

$$F_L + F_R = 400\ \text{N} \qquad (1)$$

Since motion impends

$$F_L = \mu_s N_L = 0.4 N_L$$
$$F_R = \mu_s N_R = 0.4 N_R$$

Free body: Left arm *ABL*



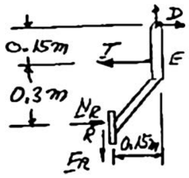
$$+\circlearrowright \Sigma M_A = 0: \quad T(0.15\ \text{m}) + F_L(0.15\ \text{m}) - N_L(0.45\ \text{m}) = 0$$

$$0.15T + (0.4N_L)(0.15\ \text{m}) - N_L(0.45\ \text{m}) = 0$$

$$0.39N_L = 0.15T; \quad N_L = 0.38462T$$

$$F_L = 0.4N_L = 0.4(0.38462T)$$
$$F_L = 0.15385T \qquad (2)$$

Free body: Right arm *DER*



$$+\circlearrowleft \Sigma M_D = 0: \quad T(0.15\ \text{m}) - F_R(0.15\ \text{m}) - N_R(0.45\ \text{m}) = 0$$

$$0.15T - (0.4N_R)(0.15\ \text{m}) - N_R(0.45\ \text{m}) = 0$$

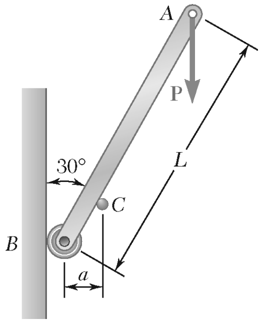
$$0.51N_R = 0.15T; \quad N_R = 0.29412T$$

$$F_R = 0.4N_R = 0.4(0.29412T)$$
$$F_R = 0.11765T \qquad (3)$$

Substitute for F_L and F_R into Eq. (1):

$$0.15385T + 0.11765T = 400$$
$$T = 1473.3\ \text{N}$$

$$T = 1.473\ \text{kN} \blacktriangleleft$$



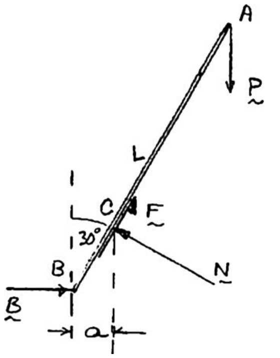
PROBLEM 8.23

A slender rod of length L is lodged between peg C and the vertical wall and supports a load **P** at end A. Knowing that the coefficient of static friction between the peg and the rod is 0.15 and neglecting friction at the roller, determine the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

FBD rod:

<u>Free-body diagram</u>: <u>For motion of *B* impending upward</u>:



$$+\circlearrowleft \Sigma M_B = 0: \quad PL\sin\theta - N_C\left(\frac{a}{\sin\theta}\right) = 0$$

$$N_C = \frac{PL}{a}\sin^2\theta \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_C\sin\theta - \mu_s N_C\cos\theta - P = 0$$

$$N_C(\sin\theta - \mu\cos\theta) = P$$

Substitute for N_C from Eq. (1), and solve for a/L.

$$\frac{a}{L} = \sin^2\theta(\sin\theta - \mu_s\cos\theta) \qquad (2)$$

For $\theta = 30°$ and $\mu_s = 0.15$:

$$\frac{a}{L} = \sin^2 30°(\sin 30° - 0.15\cos 30°)$$

$$\frac{a}{L} = 0.092524 \qquad \frac{L}{a} = 10.808$$

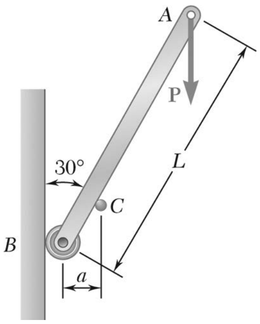
<u>For motion of *B* impending downward</u>, reverse sense of friction force F_C. To do this we make $\mu_s = -0.15$ in. Eq. (2).

Eq. (2):

$$\frac{a}{L} = \sin^2 30°(\sin 30° - (-0.15)\cos 30°)$$

$$\frac{a}{L} = 0.15748 \qquad \frac{L}{a} = 6.350$$

Range of values of L/a for equilibrium: $6.35 \le \frac{L}{a} \le 10.81$ ◀



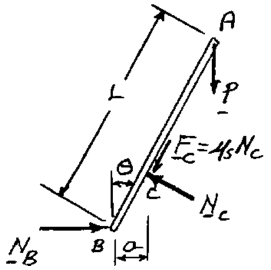
PROBLEM 8.24

Solve Problem 8.23 assuming that the coefficient of static friction between the peg and the rod is 0.60.

PROBLEM 8.23 A slender rod of length L is lodged between peg C and the vertical wall and supports a load **P** at end A. Knowing that the coefficient of static friction between the peg and the rod is 0.15 and neglecting friction at the roller, determine the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

Free-body diagram: For motion of B impending upward



$$+\circlearrowleft \Sigma M_B = 0: \quad PL\sin\theta - N_C\left(\frac{a}{\sin\theta}\right) = 0$$

$$N_C = \frac{PL}{a}\sin^2\theta \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_C\sin\theta - \mu_s N_C\cos\theta - P = 0$$

$$N_C(\sin\theta - \mu_s\cos\theta) = P$$

Substitute for N_C from (1), and solve for $\frac{a}{L}$

$$\frac{a}{L} = \sin^2\theta(\sin\theta - \mu_s\cos\theta)$$

For $\theta = 30°$ and $\mu_s = 0.60$:

$$\frac{a}{L} = \sin^2 30°(\sin 30° - 0.60\cos 30°) \qquad (2)$$

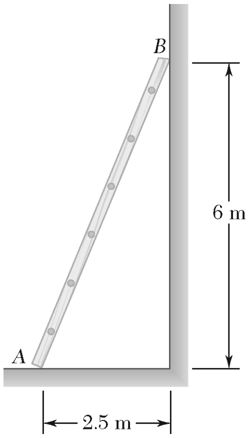
$$\frac{a}{L} = -0.0049 < 0$$

Thus, slipping of B upward does not occur for motion of B impending downward, reverse sense of friction force F_C. To do this we make $\mu_C = -0.60$ in Eq. (2).

$$\frac{a}{L} = \sin^2 30°(\sin 30° - (-0.60)\cos 30°)$$

$$\frac{a}{L} = 0.2459 \qquad \frac{L}{a} = 3.923$$

Range of L/a for equilibrium: $L/a \geq 3.92$ ◀



PROBLEM 8.25

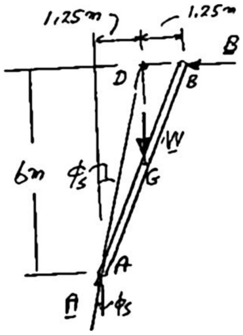
A 6.5-m ladder *AB* leans against a wall as shown. Assuming that the coefficient of static friction μ_s is zero at *B*, determine the smallest value of μ_s at *A* for which equilibrium is maintained.

SOLUTION

Free body: Ladder

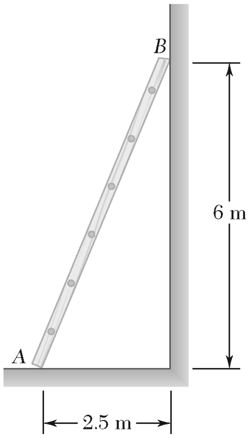
Three-force body.

Line of action of **A** must pass through *D*, where **W** and **B** intersect.



At *A*: $\mu_s = \tan\phi_s = \dfrac{1.25 \text{ m}}{6 \text{ m}} = 0.2083$

$\mu_s = 0.208$ ◀



PROBLEM 8.26

A 6.5-m ladder *AB* leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at *A* and *B*, determine the smallest value of μ_s for which equilibrium is maintained.

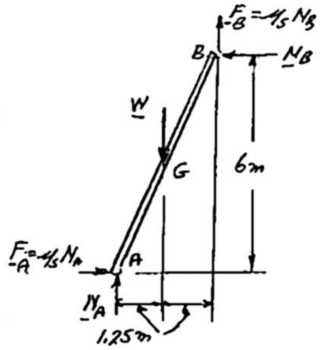
SOLUTION

Free body: Ladder

Motion impending:

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$



$$+\circlearrowleft \Sigma M_A = 0: \quad W(1.25 \text{ m}) - N_B(6 \text{ m}) - \mu_s N_B(2.5 \text{ m}) = 0$$

$$N_B = \frac{1.25W}{6 + 2.5\mu_s} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_A + \mu_s N_B - W = 0$$

$$N_A = W - \mu_s N_B$$

$$N_A = W - \frac{1.25\mu_s W}{6 + 2.5\mu_s} \qquad (2)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad \mu_s N_A - N_B = 0$$

Substitute for N_A and N_B from Eqs. (1) and (2):

$$\mu_s W - \frac{1.25\mu_s^2 W}{6 + 2.5\mu_s} = \frac{1.25W}{6 + 2.5\mu_s}$$

$$6\mu_s + 2.5\mu_s^2 - 1.25\mu_s^2 = 1.25$$

$$1.25\mu_s^2 + 6\mu_s - 1.25 = 0$$

$$\mu_s = 0.2$$

and $\mu_s = -5$ (Discard)

$\mu_s = 0.200$ ◀

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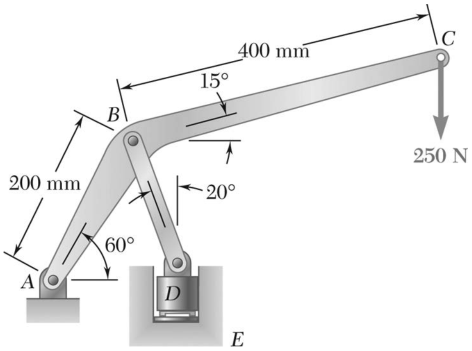
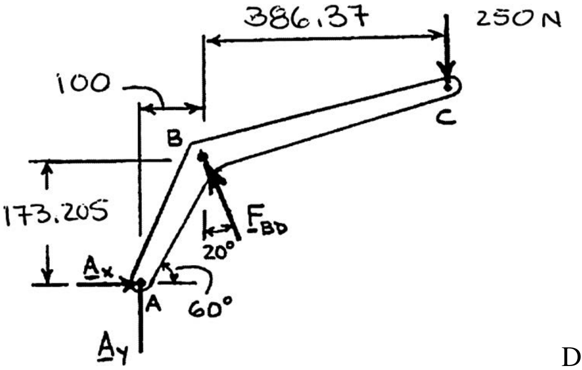


PROBLEM 8.27

The press shown is used to emboss a small seal at *E*. Knowing that the coefficient of static friction between the vertical guide and the embossing die *D* is 0.30, determine the force exerted by the die on the seal.

SOLUTION

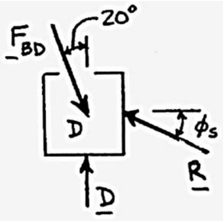
Free body: Member *ABC*



Dimensions in mm

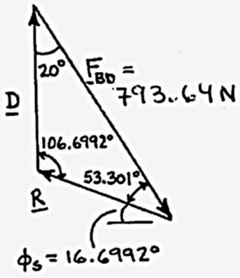
$$+\circlearrowleft \Sigma M_A = 0: \quad F_{BD}\cos 20°(100) + F_{BD}\sin 20°(173.205) - (250\text{ N})(100 + 386.37) = 0$$

$$F_{BD} = 793.64\text{ N}$$



Free body: Die *D*

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.3 = 16.6992°$$

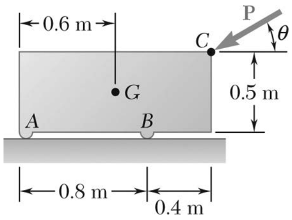


Force triangle:

$$\frac{D}{\sin 53.301°} = \frac{793.64\text{ N}}{\sin 106.6992°}$$

$$D = 664.35\text{ N}$$

On seal: $\mathbf{D} = 664\text{ N}\downarrow$ ◀



PROBLEM 8.28

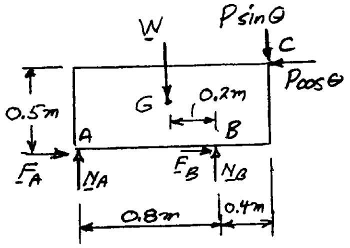
The machine base shown has a mass of 75 kg and is fitted with skids at *A* and *B*. The coefficient of static friction between the skids and the floor is 0.30. If a force **P** of magnitude 500 N is applied at corner *C*, determine the range of values of *θ* for which the base will not move.

SOLUTION

Free-body: Machine base

$$m = (75 \text{ kg})(9.81 \text{ m/s}^2) = 735.75 \text{ N}$$

Assume sliding impends



$$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$$

$$+\uparrow \Sigma F_y = 0: \quad N_A + N_B - W - P\sin\theta = 0$$

$$(N_A + N_B) = W + P\sin\theta \tag{1}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_A + F_B - P\cos\theta = 0$$

$$\mu_s(N_A + N_B) = P\cos\theta = 0 \tag{2}$$

$\dfrac{\text{Eq. (2)}}{\text{Eq. (1)}}$:
$$\mu_s = \frac{P\cos\theta}{W + P\sin\theta}$$

$$\mu_s W + \mu_s P\sin\theta = P\cos\theta$$

$$0.30(735.75 \text{ N}) + 0.30(500 \text{ N})\sin\theta = 500\cos\theta$$

$$500\cos\theta - 150\sin\theta = 220.73$$

Solve for θ: $\theta = 48.28°$

Assume tipping about *B* impends: $\therefore N_A = 0$

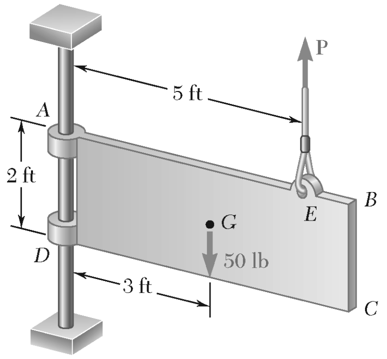
$$+\circlearrowleft \Sigma M_B = 0: \quad P\sin\theta(0.4 \text{ m}) - P\cos\theta(0.5 \text{ m}) - W(0.2 \text{ m}) = 0$$

$$500\sin\theta(0.4) - 500\cos\theta(0.5) - 735.75(0.2 \text{ m}) = 0$$

$$200\sin\theta - 250\cos\theta = 147.15$$

Solve for θ: $\theta = 78.70°$

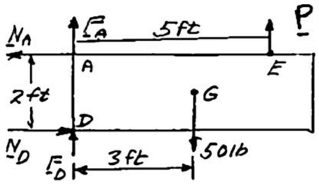
Range for no motion: $48.3° \le \theta \le 78.7°$ ◀



PROBLEM 8.29

The 50-lb plate *ABCD* is attached at *A* and *D* to collars that can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at *E* is (*a*) $P = 0$, (*b*) $P = 20$ lb.

SOLUTION



(*a*) $\underline{P = 0}$

$$\circlearrowleft\!+\ \Sigma M_D = 0: \quad N_A(2\text{ ft}) - (50\text{ lb})(3\text{ ft}) = 0$$

$$N_A = 75\text{ lb}$$

$$\Sigma F_x = 0: \quad N_D = N_A = 75\text{ lb}$$

$$+\!\uparrow \Sigma F_y = 0: \quad F_A + F_D - 50\text{ lb} = 0$$

$$F_A + F_D = 50\text{ lb}$$

But:

$$(F_A)_m = \mu_s N_A = 0.40(75\text{ lb}) = 30\text{ lb}$$
$$(F_D)_m = \mu_s N_D = 0.40(75\text{ lb}) = 30\text{ lb}$$

Thus: $(F_A)_m + (F_D)_m = 60\text{ lb}$

and $(F_A)_m + (F_D)_m > F_A + F_D$ Plate is in equilibrium ◀

(*b*) $\underline{P = 20\text{ lb}}$

$$\circlearrowleft\!+\ \Sigma M_D = 0: \quad N_A(2\text{ ft}) - (50\text{ lb})(3\text{ ft}) + (20\text{ lb})(5\text{ ft}) = 0$$

$$N_A = 25\text{ lb}$$

$$\Sigma F_x = 0: \quad N_D = N_A = 25\text{ lb}$$

$$+\!\uparrow \Sigma F_y = 0: \quad F_A + F_D - 50\text{ lb} + 20\text{ lb} = 0$$

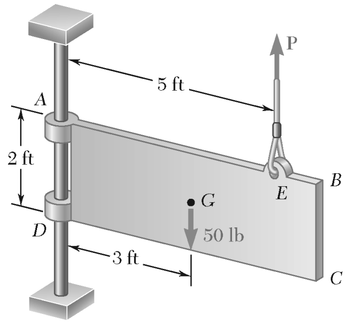
$$F_A + F_D = 30\text{ lb}$$

But:

$$(F_A)_m = \mu_s N_A = 0.4(25\text{ lb}) = 10\text{ lb}$$
$$(F_D)_m = \mu_s N_D = 0.4(25\text{ lb}) = 10\text{ lb}$$

Thus: $(F_A)_m + (F_D)_m = 20\text{ lb}$

and $F_A + F_D > (F_A)_m + (F_D)_m$ Plate moves downward ◀

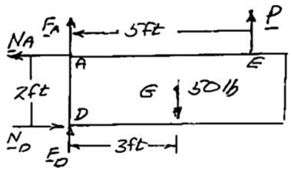


PROBLEM 8.30

In Problem 8.29, determine the range of values of the magnitude P of the vertical force applied at E for which the plate will move downward.

PROBLEM 8.29 The 50-lb plate $ABCD$ is attached at A and D to collars that can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at E is (*a*) $P = 0$, (*b*) $P = 20$ lb.

SOLUTION



We shall consider the following two cases:

(1) $0 < P < 30$ lb

$$+\circlearrowleft \Sigma M_D = 0: \quad N_A(2\text{ ft}) - (50\text{ lb})(3\text{ ft}) + P(5\text{ ft}) = 0$$

$$N_A = 75\text{ lb} - 2.5P$$

(*Note:* $N_A \geq 0$ and directed ⟵ for $P \leq 30$ lb as assumed here)

$$\Sigma F_x = 0: \quad N_A = N_D$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_D + P - 50 = 0$$

$$F_A + F_D = 50 - P$$

But:

$$\begin{aligned}(F_A)_m = (F_0)_m &= \mu_s N_A \\ &= 0.40(75 - 2.5P) \\ &= 30 - P\end{aligned}$$

<u>Plate moves ↓ if</u>: $\quad F_A + F_D > (F_A)_m + (F_D)_m$

or $\quad 50 - P > (30 - P) + (30 - P)$ $\quad P > 10\text{ lb}$ ◁

(2) $30\text{ lb} < P < 50$ lb

$$+\circlearrowleft \Sigma M_D = 0: \quad -N_A(2\text{ ft}) - (50\text{ lb})(3\text{ ft}) + P(5\text{ ft}) = 0$$

$$N_A = 2.5P - 75$$

(*Note:* $N_A >$ and directed ⟶ for $P > 30$ lb as assumed)

$$\Sigma F_x = 0: \quad N_A = N_D$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_D + P - 50 = 0$$

$$F_A + F_D = 50 - P$$

PROBLEM 8.30 (Continued)

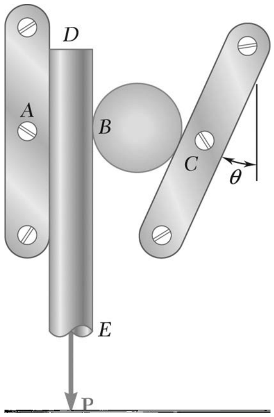
But:
$$\begin{aligned}(F_A)_m &= (F_D)_m = \mu_s N_A\\ &= 0.40(2.5P - 75)\\ &= P - 30 \text{ lb}\end{aligned}$$

<u>Plate moves ↓ if</u>: $F_A + F_D > (F_A)_m + (F_D)_m$

$$50 - P > (P - 30) + (P - 30) \qquad P < \frac{110}{3} = 36.7 \text{ lb} \ \triangleleft$$

Thus, plate moves downward for: $10.00 \text{ lb} < P < 36.7 \text{ lb}$ ◀

(*Note:* For $P > 50$ lb, plate is in equilibrium)



PROBLEM 8.31

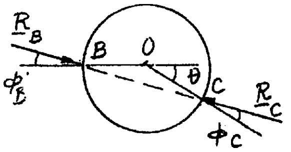
A rod *DE* and a small cylinder are placed between two guides as shown. The rod is not to slip downward, however large the force **P** may be; i.e., the arrangement is said to be self-locking. Neglecting the weight of the cylinder, determine the minimum allowable coefficients of static friction at *A*, *B*, and *C*.

SOLUTION

<u>Free body: Cylinder</u>

Since cylinder is a two-force body, $\mathbf{R}_B$ and $\mathbf{R}_C$ have the same line of action. Thus $\phi_B = \phi_C$:

From triangle *OBC*:



$$\phi_B + \phi_C = \theta$$

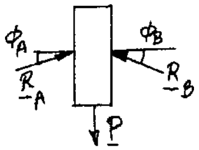
Thus: $$\phi_B = \phi_C = \frac{\theta}{2}$$

For no sliding, we must have $\tan\phi_B \le (\mu_B)_s$, $\tan\phi_C \le (\mu_C)_s$

Therefore: $$(\mu_B)_s \ge \tan\frac{\theta}{2}, \qquad (\mu_c)_s \ge \tan\frac{\theta}{2} \blacktriangleleft$$

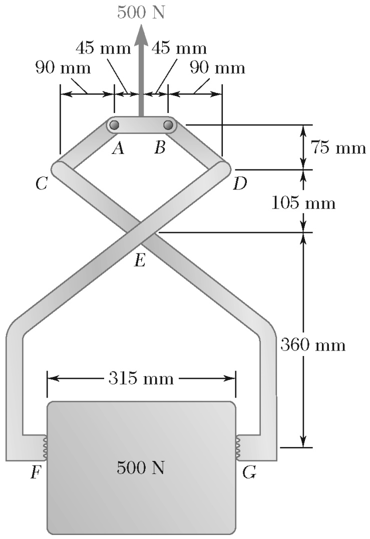
We also note that R_B and R_C are indeterminate.

<u>Free body: Rod</u>



Since R_B is indeterminate, it may be as large as necessary to satisfy equation $\Sigma F_y = 0$, no matter how large **P** is or how small ϕ_A is. Therefore

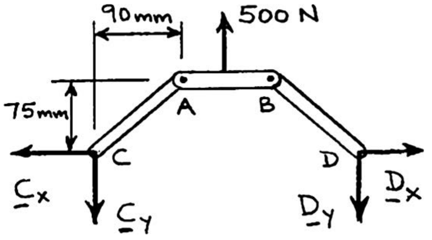
$(\mu_A)_s$ may have any value ◀



PROBLEM 8.32

A 500-N concrete block is to be lifted by the pair of tongs shown. Determine the smallest allowable value of the coefficient of static friction between the block and the tongs at *F* and *G*.

SOLUTION



Free body: Members *CA*, *AB*, *BD*

By symmetry:
$$C_y = D_y = \frac{1}{2}(500) = 250 \text{ N}$$

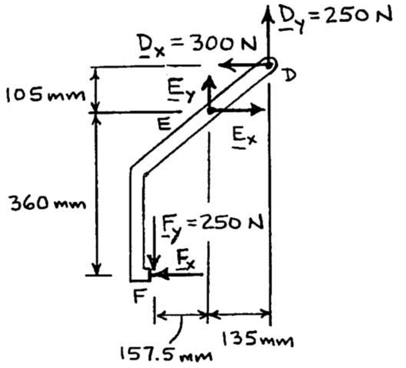
Since *CA* is a two-force member,

$$\frac{C_x}{90 \text{ mm}} = \frac{C_y}{75 \text{ mm}} = \frac{250 \text{ N}}{75 \text{ mm}}$$

$$C_x = 300 \text{ N}$$

$$\Sigma F_x = 0: \quad D_x = C_x$$

$$D_x = 300 \text{ N}$$



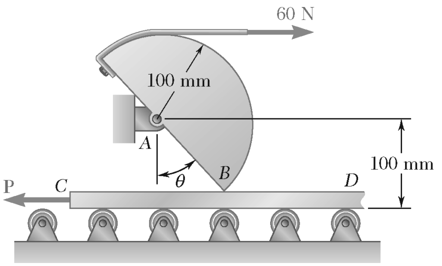
Free body: Tong *DEF*

$$+\circlearrowleft \Sigma M_E = 0: \quad (300 \text{ N})(105 \text{ mm}) + (250 \text{ N})(135 \text{ mm}) + (250 \text{ N})(157.5 \text{ mm}) - F_x(360 \text{ mm}) = 0$$

$$F_x = +290.625 \text{ N}$$

Minimum value of μ_s:
$$\mu_s = \frac{F_y}{F_x} = \frac{250 \text{ N}}{290.625 \text{ N}}$$

$\mu_s = 0.860$ ◀

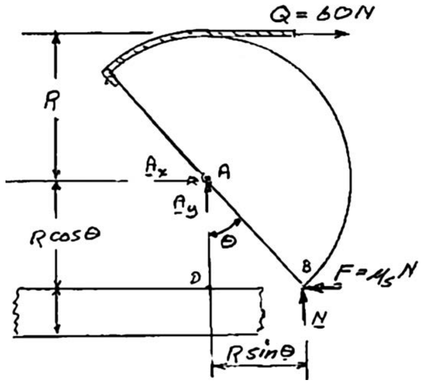


PROBLEM 8.33

The 100-mm-radius cam shown is used to control the motion of the plate *CD*. Knowing that the coefficient of static friction between the cam and the plate is 0.45 and neglecting friction at the roller supports, determine (*a*) the force **P** required to maintain the motion of the plate, knowing that the plate is 20 mm thick, (*b*) the largest thickness of the plate for which the mechanism is self-locking (i.e., for which the plate cannot be moved however large the force **P** may be).

SOLUTION

Free body: Cam



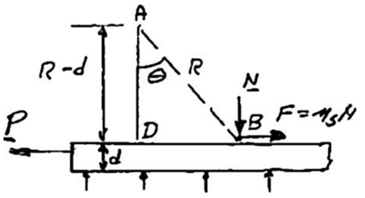
Impending motion: $F = \mu_s N$

$$+\circlearrowright \Sigma M_A = 0: \quad QR - NR\sin\theta + (\mu_s N)R\cos\theta = 0$$

$$N = \frac{Q}{\sin\theta - \mu_s \cos\theta} \qquad (1)$$

Free body: Plate $\quad \Sigma F_x = 0 \quad P = \mu_s N \qquad (2)$

Geometry in ΔABD with $R = 100$ mm and $d = 20$ mm



$$\cos\theta = \frac{R-d}{R} = \frac{80 \text{ mm}}{100 \text{ mm}} = 0.8$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = 0.6$$

PROBLEM 8.33 (Continued)

(*a*) Eq. (1) using $Q = 60\text{ N}\quad\text{and}\quad \mu_s = 0.45$

$$N = \frac{60\text{ N}}{0.6 - (0.45)(0.8)}$$

$$= \frac{60}{0.24} = 250\text{ N}$$

Eq. (2) $P = \mu_s N = (0.45)(250\text{ N})$

$P = 112.5\text{ N}$ ◀

(*b*) For $P = \infty$, $N = \infty$. Denominator is zero in Eq. (1).

$$\sin\theta - \mu_s \cos\theta = 0$$

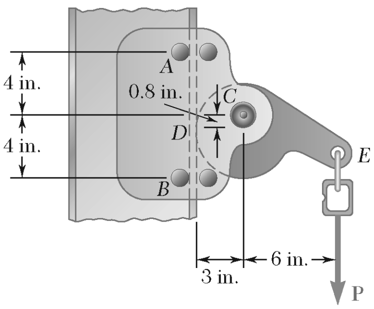
$$\tan\theta = \mu_s = 0.45$$

$$\theta = 24.23°$$

$$\cos\theta = \frac{R-d}{R}$$

$$\cos 24.23 = \frac{100-d}{100}$$

$d = 8.81\text{ mm}$ ◀

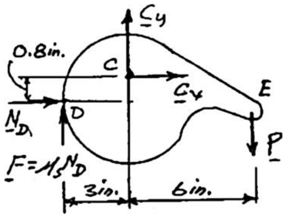


PROBLEM 8.34

A safety device used by workers climbing ladders fixed to high structures consists of a rail attached to the ladder and a sleeve that can slide on the flange of the rail. A chain connects the worker's belt to the end of an eccentric cam that can be rotated about an axle attached to the sleeve at *C*. Determine the smallest allowable common value of the coefficient of static friction between the flange of the rail, the pins at *A* and *B*, and the eccentric cam if the sleeve is not to slide down when the chain is pulled vertically downward.

SOLUTION

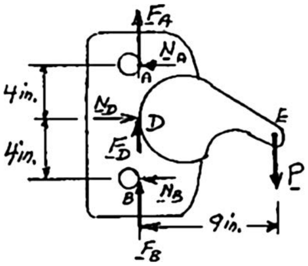
Free body: Cam



$$+\circlearrowleft \Sigma M_C = 0: \quad N_D(0.8 \text{ in.}) - \mu_s N_D(3\text{in.}) - P(6 \text{ in.}) = 0$$

$$N_D = \frac{6P}{0.8 - 3\mu_s} \qquad (1)$$

Free body: Sleeve and cam



$$\xrightarrow{+} \Sigma F_x = 0: \quad N_D - N_A - N_B = 0$$

$$N_A + N_B = N_D \qquad (2)$$

$$+\uparrow \Sigma F_y = 0: \quad F_A + F_B + F_D - P = 0$$

or

$$\mu_s(N_A + N_B + N_D) = P \qquad (3)$$

Substitute from Eq. (2) into Eq. (3):

$$\mu_s(2N_D) = P \qquad N_D = \frac{P}{2\mu_s} \qquad (4)$$

Equate expressions for N_D from Eq. (1) and Eq. (4):

$$\frac{P}{2\mu_s} = \frac{6P}{0.8 - 3\mu_s}; \quad 0.8 - 3\mu_s = 12\mu_s$$

$$\mu_s = \frac{0.8}{15} \qquad \mu_s = 0.0533 \blacktriangleleft$$

(*Note:* To verify that contact at pins *A* and *B* takes places as assumed, we shall check that $N_A > 0$ and $N_B = 0$.)

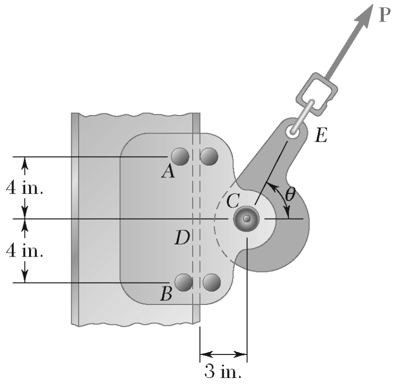
PROBLEM 8.34 (Continued)

From Eq. (4):
$$N_D = \frac{P}{2\mu_s} = \frac{P}{2(0.0533)} = 9.375P$$

From free body of cam and sleeve:

$$+\circlearrowleft \Sigma M_B = 0: \quad N_A(8 \text{ in.}) - N_D(4 \text{ in.}) - P(9 \text{ in.}) = 0$$
$$8N_A = (9.375P)(4) + 9P$$
$$N_A = 5.8125P > 0 \quad \text{OK}$$

From Eq. (2):
$$N_A + N_B = N_D$$
$$5.8125P + N_B = 9.375P$$
$$N_B = 3.5625P > 0 \quad \text{OK}$$



PROBLEM 8.35

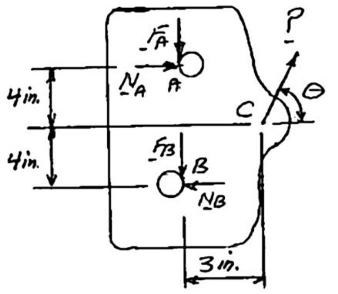
To be of practical use, the safety sleeve described in Problem 8.34 must be free to slide along the rail when pulled upward. Determine the largest allowable value of the coefficient of static friction between the flange of the rail and the pins at A and B if the sleeve is to be free to slide when pulled as shown in the figure, assuming (*a*) $\theta = 60°$, (*b*) $\theta = 50°$, (*c*) $\theta = 40°$.

SOLUTION

Note the cam is a two-force member.

Free body: Sleeve

We assume contact between rail and pins as shown.



$$+\circlearrowleft \Sigma M_C = 0: \quad F_A(3\text{ in.}) + F_B(3\text{ in.}) - N_A(4\text{ in.}) - N_B(4\text{ in.}) = 0$$

But

$$F_A = \mu_s N_A$$
$$F_B = \mu_s N_B$$

We find

$$3\mu_s(N_A + N_B) - 4(N_A + N_B) = 0$$

$$\mu_s = \frac{4}{3} = 1.33333$$

We now verify that our assumption was correct.

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B + P\cos\theta = 0$$

$$N_B - N_A = P\cos\theta \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad -F_A - F_B + P\sin\theta = 0$$

$$\mu_s N_A + \mu_s N_B = P\sin\theta$$

$$N_A + N_B = \frac{P\sin\theta}{\mu_s} \qquad (2)$$

Add Eqs. (1) and (2):

$$2N_B = P\left(\cos\theta + \frac{\sin\theta}{\mu_s}\right) > 0 \quad \text{OK}$$

PROBLEM 8.35 (Continued)

Subtract Eq. (1) from Eq. (2): $$2N_A = P\left(\frac{\sin\theta}{\mu_s} - \cos\theta\right)$$

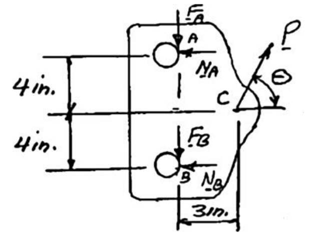
$N_A > 0$ only if $$\frac{\sin\theta}{\mu_s} - \cos\theta > 0$$

$$\tan\theta > \mu_s = 1.33333$$
$$\theta = 53.130°$$

(*a*) For case (*a*): Condition is satisfied, contact takes place as shown. Answer is correct.

$\mu_s = 1.333$ ◀

But for (*b*) and (*c*): $\theta < 53.130°$ and our assumption is wrong, N_A is directed to left.



$$\xrightarrow{+} \Sigma F_x = 0: \quad -N_A - N_B + P\cos\theta = 0$$

$$N_A + N_B = P\cos\theta \qquad (3)$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_A - F_A + P\sin\theta = 0$$

$$\mu_s(N_A + N_B) = P\sin\theta \qquad (4)$$

Divide Eq. (4) by Eq. (3):

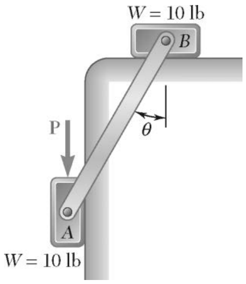
$$\mu_s = \tan\theta \qquad (5)$$

(*b*) We make $\theta = 50°$ in Eq. (5):

$$\mu_s = \tan 50°$$ $\mu_s = 1.192$ ◀

(*c*) We make $\theta = 40°$ in Eq. (5):

$$\mu_s = \tan 40°$$ $\mu_s = 0.839$ ◀



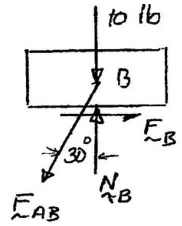
PROBLEM 8.36

Two 10-lb blocks *A* and *B* are connected by a slender rod of negligible weight. The coefficient of static friction is 0.30 between all surfaces of contact, and the rod forms an angle $\theta = 30°$. with the vertical. (*a*) Show that the system is in equilibrium when $P = 0$. (*b*) Determine the largest value of *P* for which equilibrium is maintained.

SOLUTION

FBD block ***B***:

(*a*) Since $P = 2.69$ lb to initiate motion, equilibrium exists with $P = 0$ ◀



(*b*) For $P_{\max}$, motion impends at both surfaces:

Block *B*: $\uparrow \Sigma F_y = 0$: $N_B - 10\,\text{lb} - F_{AB}\cos 30° = 0$

$$N_B = 10\ \text{lb} + \frac{\sqrt{3}}{2}F_{AB} \qquad (1)$$

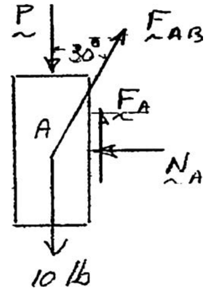
Impending motion: $F_B = \mu_s N_B = 0.3N_B$

$\longrightarrow \Sigma F_x = 0$: $F_B - F_{AB}\sin 30° = 0$

$$F_{AB} = 2F_B = 0.6N_B \qquad (2)$$

Solving Eqs. (1) and (2): $N_B = 10\ \text{lb} + \frac{\sqrt{3}}{2}(0.6N_B) = 20.8166\ \text{lb}$

FBD block ***A***:



Then $F_{AB} = 0.6N_B = 12.4900\ \text{lb}$

Block *A*: $\longrightarrow \Sigma F_x = 0$: $F_{AB}\sin 30° - N_A = 0$

$$N_A = \frac{1}{2}F_{AB} = \frac{1}{2}(12.4900\ \text{lb}) = 6.2450\ \text{lb}$$

Impending motion: $F_A = \mu_s N_A = 0.3(6.2450\ \text{lb}) = 1.8735\ \text{lb}$

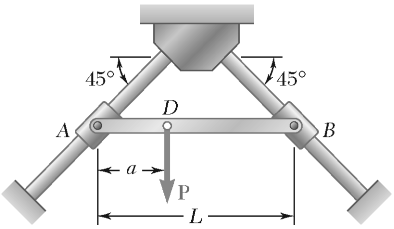
$\uparrow \Sigma F_y = 0$: $F_A + F_{AB}\cos 30° - P - 10\ \text{lb} = 0$

$$P = F_A + \frac{\sqrt{3}}{2}F_{AB} - 10\ \text{lb}$$

$$= 1.8735\ \text{lb} + \frac{\sqrt{3}}{2}(12.4900\ \text{lb}) - 10\ \text{lb}$$

$$= 2.69\ \text{lb}$$

$P = 2.69$ lb ◀

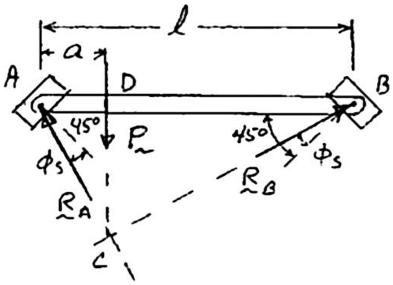


PROBLEM 8.37

Bar *AB* is attached to collars that can slide on the inclined rods shown. A force **P** is applied at Point *D* located at a distance *a* from end *A*. Knowing that the coefficient of static friction μ_s between each collar and the rod upon which it slides is 0.30 and neglecting the weights of the bar and of the collars, determine the smallest value of the ratio *a*/*L* for which equilibrium is maintained.

SOLUTION

FBD bar and collars:



Impending motion:

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.6992°$$

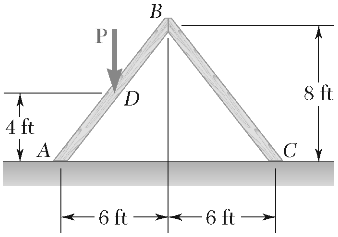
Neglect weights: 3-force *FBD* and $\measuredangle\, ACB = 90°$

so

$$AC = \frac{a}{\cos(45° + \phi_s)} = l\sin(45° - \phi_s)$$

$$\frac{a}{l} = \sin(45° - 16.6992°)\cos(45° + 16.6992°)$$

$$\frac{a}{l} = 0.225 \blacktriangleleft$$

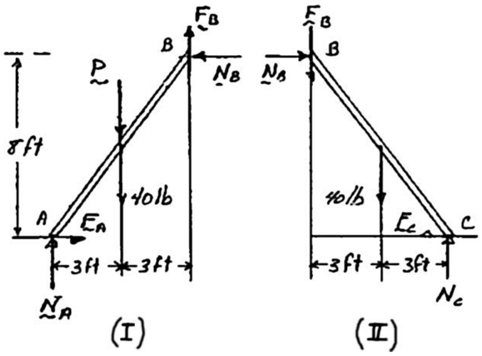


PROBLEM 8.38

Two identical uniform boards, each of weight 40 lb, are temporarily leaned against each other as shown. Knowing that the coefficient of static friction between all surfaces is 0.40, determine (*a*) the largest magnitude of the force **P** for which equilibrium will be maintained, (*b*) the surface at which motion will impend.

SOLUTION

Board FBDs:



Assume impending motion at *C*, so

$$F_C = \mu_s N_C = 0.4 N_C$$

FBD II: $\circlearrowleft \Sigma M_B = 0$: $(6\text{ ft})N_C - (8\text{ ft})F_C - (3\text{ ft})(40\text{ lb}) = 0$

$$[6\text{ ft} - 0.4(8\text{ ft})]N_C = (3\text{ ft})(40\text{ lb})$$

or $$N_C = 42.857\text{ lb}$$

and $$F_C = 0.4N_C = 17.143\text{ lb}$$

$\rightarrow \Sigma F_x = 0$: $N_B - F_C = 0$

$$N_B = F_C = 17.143\text{ lb}$$

$\uparrow \Sigma M_y = 0$: $-F_B - 40\text{ lb} + N_C = 0$

$$F_B = N_C - 40\text{ lb} = 2.857\text{ lb}$$

Check for motion at *B*: $\dfrac{F_B}{N_B} = \dfrac{2.857\text{ lb}}{17.143\text{ lb}} = 0.167 < \mu_s$, OK, no motion.

FBD I: $\circlearrowleft \Sigma M_A = 0$: $(8\text{ ft})N_B + (6\text{ ft})F_B - (3\text{ ft})(P + 40\text{ lb}) = 0$

$$P = \frac{(8\text{ ft})(17.143\text{ lb}) + (6\text{ ft})(2.857\text{ lb})}{3\text{ ft}} - 40\text{ lb}$$

$$= 11.429\text{ lb}$$

Check for slip at A (unlikely because of P):

$$\xrightarrow{} \Sigma F_x = 0: \quad F_A - N_B = 0 \quad \text{or} \quad F_A = N_B = 17.143 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad N_A - P - 40 \text{ lb} + F_B = 0$$

or

$$N_A = 11.429 \text{ lb} + 40 \text{ lb} - 2.857 \text{ lb}$$
$$= 48.572 \text{ lb}$$

Then

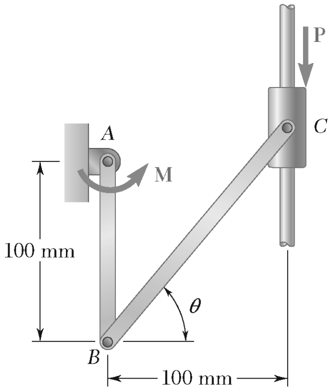
$$\frac{F_A}{N_A} = \frac{17.143 \text{ lb}}{48.572 \text{ lb}} = 0.353 < \mu_s$$

OK, no slip $\Rightarrow$ assumption is correct.

Therefore

(*a*) $P_{\text{max}} = 11.43 \text{ lb}$ ◀

(*b*) Motion impends at C ◀

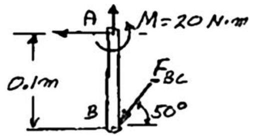


PROBLEM 8.39

Knowing that the coefficient of static friction between the collar and the rod is 0.35, determine the range of values of P for which equilibrium is maintained when $\theta = 50°$ and $M = 20\ \text{N}\cdot\text{m}$.

SOLUTION

Free body member *AB*:

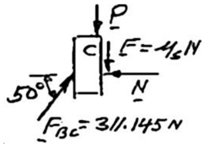


BC is a two-force member.

$$+\circlearrowleft \Sigma M_A = 0: \quad 20\ \text{N}\cdot\text{m} - F_{BC}\cos 50°(0.1\ \text{m}) = 0$$

$$F_{BC} = 311.145\ \text{N}$$

Motion of C impending upward:



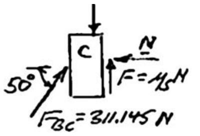
$$\xrightarrow{+} \Sigma F_x = 0: \quad (311.145\ \text{N})\cos 50° - N = 0$$

$$N = 200\ \text{N}$$

$$+\uparrow \Sigma F_y = 0: \quad (311.145\ \text{N})\sin 50° - P - (0.35)(200\ \text{N}) = 0$$

$$P = 168.351\ \text{N} \quad \triangleleft$$

Motion of C impending downward:



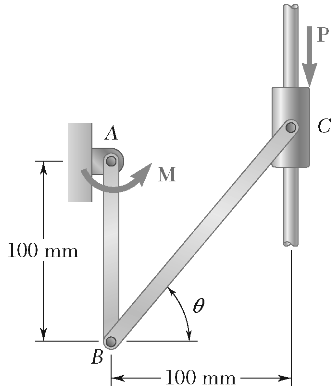
$$\xrightarrow{+} \Sigma F_x = 0: \quad (311.145\ \text{N})\cos 50° - N = 0$$

$$N = 200\ \text{N}$$

$$+\uparrow \Sigma F_y = 0: \quad (311.145\ \text{N})\sin 50° - P + (0.35)(200\ \text{N}) = 0$$

$$P = 308.35\ \text{N} \quad \triangleleft$$

Range of P: $168.4\ \text{N} \le P \le 308\ \text{N}$ ◀

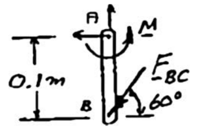


PROBLEM 8.40

Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of M for which equilibrium is maintained when $\theta = 60°$ and $P = 200$ N.

SOLUTION

Free body member *AB*:

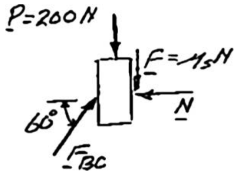


BC is a two-force member.

$$+\circlearrowleft \Sigma M_A = 0: \quad M - F_{BC}\cos 60°(0.1\text{ m}) = 0$$

$$M = 0.05F_{BC} \qquad (1)$$

<u>Motion of *C* impending upward</u>:



$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC}\cos 60° - N = 0$$

$$N = 0.5F_{BC}$$

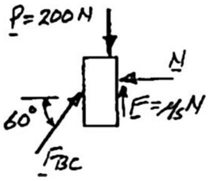
$$+\!\uparrow \Sigma F_y = 0: \quad F_{BC}\sin 60° - 200\text{ N} - (0.40)(0.5F_{BC}) = 0$$

$$F_{BC} = 300.29\text{ N}$$

Eq. (1): $\quad M = 0.05(300.29)$

$$M = 15.014\text{ N}\cdot\text{m} \quad ◁$$

<u>Motion of *C* impending downward</u>:



$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC}\cos 60° - N = 0$$

$$N = 0.5F_{BC}$$

$$+\!\uparrow \Sigma F_y = 0: \quad F_{BC}\sin 60° - 200\text{ N} + (0.40)(0.5F_{BC}) = 0$$

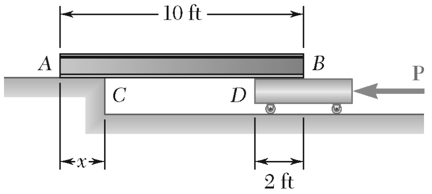
$$F_{BC} = 187.613\text{ N}$$

Eq. (1): $\quad M = 0.05(187.613)$

$$M = 9.381\text{ N}\cdot\text{m} \quad ◁$$

Range of M: $\quad 9.38\text{ N}\cdot\text{m} \le M \le 15.01\text{ N}\cdot\text{m}$ ◀

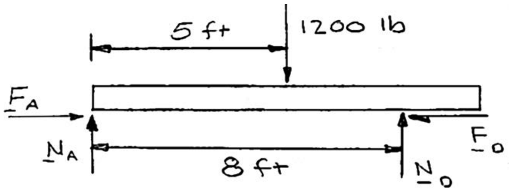
PROBLEM 8.41



A 10-ft beam, weighing 1200 lb, is to be moved to the left onto the platform. A horizontal force **P** is applied to the dolly, which is mounted on frictionless wheels. The coefficients of friction between all surfaces are $\mu_s = 0.30$ and $\mu_k = 0.25$, and initially $x = 2$ ft. Knowing that the top surface of the dolly is slightly higher than the platform, determine the force **P** required to start moving the beam. (*Hint:* The beam is supported at *A* and *D*.)

SOLUTION

FBD beam:



$$+\circlearrowleft \Sigma M_A = 0: \quad N_D(8\,\text{ft}) - (1200\text{ lb})(5\,\text{ft}) = 0$$

$$\mathbf{N}_D = 750\text{ lb}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad N_A - 1200 + 750 = 0$$

$$\mathbf{N}_A = 450\text{ lb}\uparrow$$

$$(F_A)_m = \mu_s N_A = 0.3(450) = 135.0\text{ lb}$$

$$(F_D)_m = \mu_s N_D = 0.3(750) = 225\text{ lb}$$

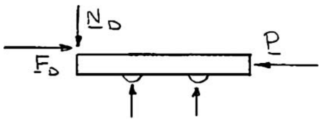
Since $(F_A)_m < (F_D)_m$, sliding first impends at *A* with

$$F_A = (F_A)_m = 135\text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_A - F_D = 0$$

$$F_D = F_A = 135.0\text{ lb}$$

FBD dolly:

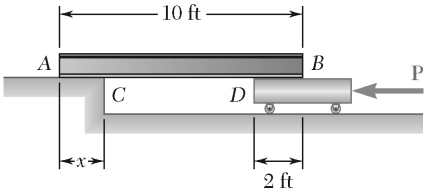


From *FBD* of dolly:

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_D - P = 0$$

$$P = F_D = 135.0\text{ lb}$$

$P = 135.0$ lb ◀



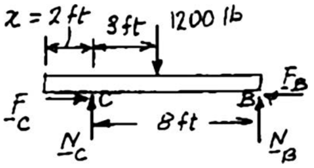
PROBLEM 8.42

(*a*) Show that the beam of Problem 8.41 *cannot* be moved if the top surface of the dolly is slightly *lower* than the platform. (*b*) Show that the beam *can* be moved if two 175-lb workers stand on the beam at *B* and determine how far to the left the beam can be moved.

PROBLEM 8.41 A 10-ft beam, weighing 1200 lb, is to be moved to the left onto the platform. A horizontal force **P** is applied to the dolly, which is mounted on frictionless wheels. The coefficients of friction between all surfaces are $\mu_s = 0.30$ and $\mu_k = 0.25$, and initially $x = 2$ ft. Knowing that the top surface of the dolly is slightly higher than the platform, determine the force **P** required to start moving the beam. (*Hint:* The beam is supported at *A* and *D*.)

SOLUTION

(*a*) Beam alone



$$+\circlearrowleft \Sigma M_C = 0: \quad N_B(8 \text{ ft}) - (1200 \text{ lb})(3 \text{ ft}) = 0$$

$$\mathbf{N}_B = 450 \text{ lb} \uparrow$$

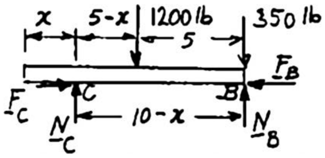
$$+\uparrow F_y = 0: \quad N_C + 450 - 1200 = 0$$

$$\mathbf{N}_C = 750 \text{ lb} \uparrow$$

$$(F_C)_m = \mu_s N_C = 0.3(750) = 225 \text{ lb}$$
$$(F_B)_m = \mu_s N_B = 0.3(450) = 135 \text{ lb}$$

Since $(F_B)_m < (F_C)_m$, sliding first impends at *B*, and Beam cannot be moved ◀

(*b*) Beam with workers standing at *B*



$$+\circlearrowleft \Sigma M_C = 0: \quad N_B(10 - x) - (1200)(5 - x) - 350(10 - x) = 0$$

$$N_B = \frac{9500 - 1550x}{10 - x}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (1200)(5) - N_C(10 - x) = 0$$

$$N_C = \frac{6000}{10 - x}$$

PROBLEM 8.42 (Continued)

Check that beam starts moving for $x = 2$ ft:

For $x = 2$ ft: $$N_B = \frac{9500 - 1550(2)}{10-2} = 800 \text{ lb}$$

$$N_C = \frac{6000}{10-2} = 750 \text{ lb}$$

$$(F_C)_m = \mu_s N_C = 0.3(750) = 225 \text{ lb}$$

$$(F_B)_m = \mu_s N_B = 0.3(800) = 240 \text{ lb}$$

Since $(F_C)_m < (F_B)_m$, sliding first impends at C, Beam moves ◀

How far does beam move?

Beam will stop moving when

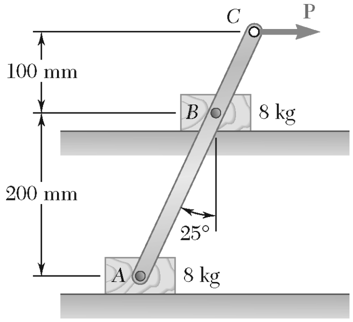
$$F_C = (F_B)_m$$

But $$F_C = \mu_k N_C = 0.25 \frac{6000}{10-x} = \frac{1500}{10-x}$$

and $$(F_B)_m = \mu_s N_B = 0.30 \frac{9500 - 1550x}{10-x} = \frac{2850 - 465x}{10-x}$$

Setting $F_C = (F_B)_m$: $1500 = 2850 - 465x$ $x = 2.90 \text{ ft}$ ◀

(*Note:* We have assumed that, once started, motion is continuous and uniform (no acceleration).)

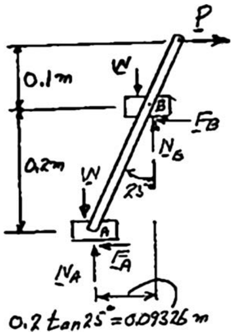


PROBLEM 8.43

Two 8-kg blocks A and B resting on shelves are connected by a rod of negligible mass. Knowing that the magnitude of a horizontal force **P** applied at C is slowly increased from zero, determine the value of P for which motion occurs, and what that motion is, when the coefficient of static friction between all surfaces is (a) $\mu_s = 0.40$, (b) $\mu_s = 0.50$.

SOLUTION

(a) $\underline{\mu_s = 0.40}$: Assume blocks slide to right.



$$W = mg = (8\text{ kg})(9.81\text{ m/s}^2) = 78.48\text{ N}$$

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$+\uparrow \Sigma F_y = 0: \quad N_A + N_B - 2W = 0$$

$$N_A + N_B = 2W$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - F_A - F_B = 0$$

$$P = F_A + F_B = \mu_s(N_A + N_B) = \mu_s(2W) \qquad (1)$$

$$P = 0.40(2)(78.48\text{ N}) = 62.78\text{ N}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad P(0.1\text{ m}) - (N_A - W)(0.09326\text{ m}) + F_A(0.2\text{ m}) = 0$$

$$(62.78)(0.1) - (N_A - 78.48)(0.09326) + (0.4)(N_B)(0.2) = 0$$

$$0.17326 N_A = 1.041$$

$$N_A = 6.01\text{ N} > 0 \quad \text{OK}$$

<u>System slides</u>: $P = 62.8$ N ◀

(b) $\underline{\mu_s = 0.50}$: See part a.

Eq. (1): $P = 0.5(2)(78.48\text{ N}) = 78.48\text{ N}$

$$+\circlearrowleft \Sigma M_B = 0: \quad P(0.1\text{ m}) + (N_A - W)(0.09326\text{ m}) + F_A(0.2\text{ m}) = 0$$

$$(78.48)(0.1) + (N_A - 78.48)(0.09326) + (0.5)N_A(0.2) = 0$$

$$0.19326 N_A = -0.529$$

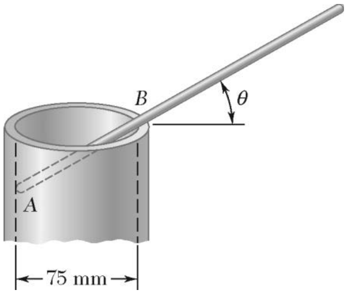
$$N_A = -2.73\text{ N} < 0 \text{ uplift, rotation about } B$$

PROBLEM 8.43 (Continued)

For $N_A = 0$: $\circlearrowright \Sigma M_B = 0$: $P(0.1\text{ m}) - W(0.09326\text{ m}) = 0$

$$P = (78.48\text{ N})(0.09326\text{ m})/(0.1) = 73.19$$

System rotates about *B*: $P = 73.2\text{ N}$ ◀

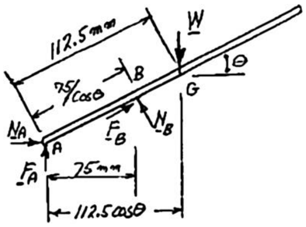


PROBLEM 8.44

A slender steel rod of length 225 mm is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

<u>Motion of rod impends down at *A* and to left at *B*.</u>



$$F_A = \mu_s N_A \quad F_B = \mu_s N_B$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B \sin\theta + F_B \cos\theta = 0$$

$$N_A - N_B \sin\theta + \mu_s N_B \cos\theta = 0$$

$$N_A = N_B(\sin\theta - \mu_s \cos\theta) \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad F_A + N_B \cos\theta + F_B \sin\theta - W = 0$$

$$\mu_s N_A + N_B \cos\theta + \mu_s N_B \sin\theta - W = 0 \qquad (2)$$

Substitute for N_A from Eq. (1) into Eq. (2):

$$\mu_s N_B(\sin\theta - \mu_s \cos\theta) + N_B \cos\theta + \mu_s N_B \sin\theta - W = 0$$

$$N_B = \frac{W}{(1-\mu_s^2)\cos\theta + 2\mu_s \sin\theta} = \frac{W}{(1-0.2^2)\cos\theta + 2(0.2)\sin\theta} \qquad (3)$$

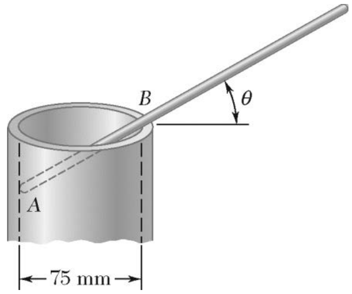
$$+\circlearrowleft \Sigma M_A = 0: \quad N_B\left(\frac{75}{\cos\theta}\right) - W(112.5\cos\theta) = 0$$

Substitute for N_B from Eq. (3), cancel *W*, and simplify to find

$$9.6\cos^3\theta + 4\sin\theta\cos^2\theta - 6.6667 = 0$$

$$\cos^3\theta(2.4 + \tan\theta) = 1.6667$$

Solve by trial & error: $\theta = 35.8°$ ◀



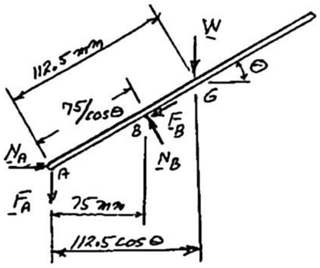
PROBLEM 8.45

In Problem 8.44, determine the smallest value of θ for which the rod will not fall out the pipe.

PROBLEM 8.44 A slender steel rod of length 225 mm is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

Motion of rod impends up at A and right at B.



$$F_A = \mu_s N_A \quad F_B = \mu_s N_B$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B \sin\theta - F_B \cos\theta = 0$$

$$N_A - N_B \sin\theta - \mu_s N_B \cos\theta = 0$$

$$N_A = N_B(\sin\theta + \mu_s \cos\theta) \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad -F_A + N_B \cos\theta - F_B \sin\theta - W = 0$$

$$-\mu_s N_A + N_B \cos\theta - \mu_s N_B \sin\theta - W = 0 \qquad (2)$$

Substitute for N_A from Eq. (1) into Eq. (2):

$$-\mu_s N_B(\sin\theta + \mu_s \cos\theta) + N_B \cos\theta - \mu_s N_B \sin\theta - W = 0$$

$$N_B = \frac{W}{(1-\mu_s^2)\cos\theta - 2\mu_s \sin\theta} = \frac{W}{(1-0.2^2)\cos\theta - 2(0.2)\sin\theta} \qquad (3)$$

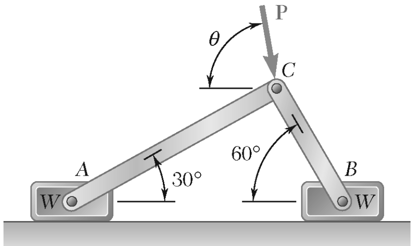
$$+\circlearrowleft \Sigma M_A = 0: \quad N_B\left(\frac{75}{\cos\theta}\right) - W(112.5\cos\theta) = 0$$

Substitute for N_B from Eq. (3), cancel W, and simplify to find

$$9.6\cos^3\theta - 4\sin\theta\cos^2\theta - 6.6667 = 0$$

$$\cos^3\theta(2.4 - \tan\theta) = 1.6667$$

Solve by trial + error: $\theta = 20.5°$ ◀

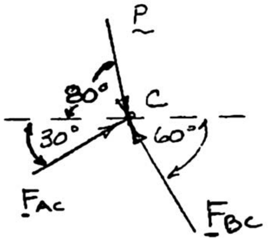


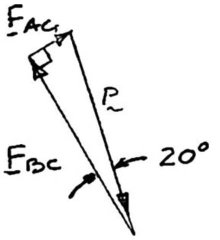
PROBLEM 8.46

Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B, each of weight W. Knowing that $\theta = 80°$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD pin C:



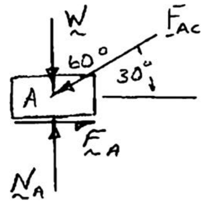


$$F_{AC} = P\sin 20° = 0.34202P$$
$$F_{BC} = P\cos 20° = 0.93969P$$

$\uparrow \Sigma F_y = 0$: $N_A - W - F_{AC}\sin 30° = 0$

or $N_A = W + 0.34202P\sin 30° = W + 0.171010P$

FBD block A: $\rightarrow \Sigma F_x = 0$: $F_A - F_{AC}\cos 30° = 0$



or $F_A = 0.34202P\cos 30° = 0.29620P$

For impending motion at A: $F_A = \mu_s N_A$

Then $N_A = \dfrac{F_A}{\mu_s}$: $W + 0.171010P = \dfrac{0.29620}{0.3}P$

or $P = 1.22500W$

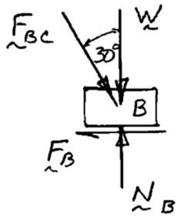
$\uparrow \Sigma F_y = 0$: $N_B - W - F_{BC}\cos 30° = 0$

$$N_B = W + 0.93969P\cos 30° = W + 0.81380P$$

$\rightarrow \Sigma F_x = 0$: $F_{BC}\sin 30° - F_B = 0$

$$F_B = 0.93969P\sin 30° = 0.46985P$$

PROBLEM 8.46 (Continued)



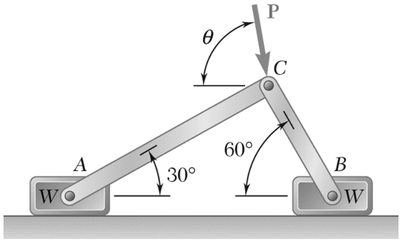
FBD block *B*:

For impending motion at *B*: $F_B = \mu_s N_B$

Then $N_B = \dfrac{F_B}{\mu_s}$: $W + 0.81380P = \dfrac{0.46985P}{0.3}$

or $P = 1.32914W$

Thus, maximum *P* for equilibrium $P_{\text{max}} = 1.225W$ ◀



PROBLEM 8.47

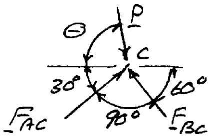
Two slender rods of negligible weight are pin-connected at *C* and attached to blocks *A* and *B*, each of weight *W*. Knowing that $P = 1.260W$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the range of values of θ, between 0 and 180°, for which equilibrium is maintained.

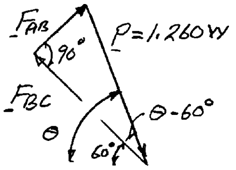
SOLUTION

AC and *BC* are two-force members

Free body: Joint *C*

Force triangle:





From Force triangle:

$$F_{AB} = P\sin(\theta - 60°) = 1.26W\sin(\theta - 60°) \qquad (1)$$

$$F_{BC} = P\cos(\theta - 60°) = 1.26W\cos(\theta - 60°) \qquad (2)$$

We shall, in turn, seek θ corresponding to impending motion of each block

For motion of *A* impending to left

from solution of Prob. 8.46: $F_{AC} = 0.419W$

EQ (1):

$$F_{AC} = 0.419W = 1.26W\sin(\theta - 60°)$$

$$\sin(\theta - 60°) = 0.33254$$

$$\theta - 60° = 19.423°$$

$$\theta = 79.42° \quad \triangleleft$$

For motion of *B* impending to right.

from solution of Prob. 8.46: $F_{BC} = 1.249W$

Eq. (2):

$$F_{BC} = 1.249W = 1.26W\cos(\theta - 60°)$$

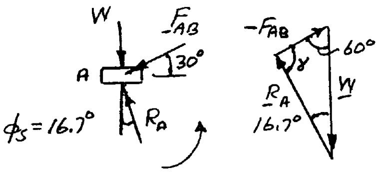
$$\cos(\theta - 60°) = 0.99127$$

$$\theta - 60° = \pm 7.58°$$

$$\theta - 60° = +7.58° \qquad \theta = 67.6° \quad \triangleleft$$

$$\theta - 60° = -7.58° \qquad \theta = 52.4° \quad \triangleleft$$

PROBLEM 8.47 (Continued)



For motion of *A* impending to right

$$\gamma = 180° - 60° - 16.7° = 103.3°$$

Law of sines:

$$\frac{-F_{AB}}{\sin 16.7°} = \frac{W}{\sin 103.3°}$$

$$F_{AB} = -0.29528W$$

Note: Direction of $+F_{AB}$ is kept same as in free body of Joint *C*.

Eq. (1):

$$F_{AB} = -0.29528W = 1.26W \sin(\theta - 60°)$$

$$\sin(\theta - 60°) = -0.23435$$

$$(\theta - 60°) = -13.553°$$

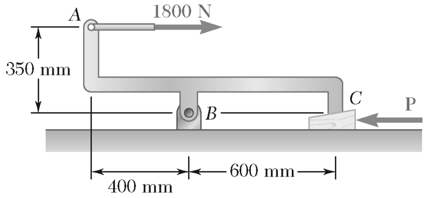
$\theta = 46.4°$ ◁

Summary:

A moves to right	No motion	B moves to right	No motion	A moves to left

46.4° 52.4° 67.6° 79.4° θ

No motion for: $46.4° \le \theta \le 52.4°$ and $67.6° \le \theta \le 79.4°$ ◀



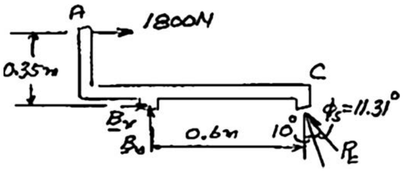
PROBLEM 8.48

The machine part *ABC* is supported by a frictionless hinge at *B* and a 10° wedge at *C*. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine (*a*) the force **P** required to move the wedge, (*b*) the components of the corresponding reaction at *B*.

SOLUTION

$$\phi_s = \tan^{-1} 0.20 = 11.31°$$

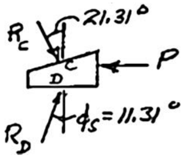
Free body: Part *ABC*



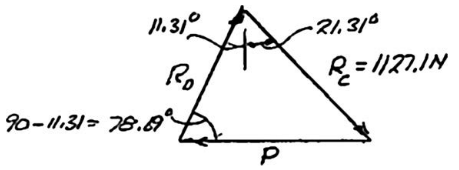
$$+\circlearrowleft \Sigma M_B = 0 \quad (1800 \text{ N})(0.35 \text{ m}) - R\cos 21.31°(0.6 \text{ m}) = 0$$

$$R_C = 1127.1 \text{ N}$$

Free body: Wedge



Force triangle:



(*a*) Law of sines:

$$\frac{P}{\sin(11.31° + 21.31°)} = \frac{1127.1 \text{ N}}{\sin 78.69°}$$

$$P = 619.6 \text{ N}$$

$\mathbf{P} = 620 \text{ N} \leftarrow$ ◀

(*b*) Return to part *ABC*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 1800 \text{ N} - R_C \sin 21.31° = 0$$

$$B_x + 1800 \text{ N} - (1127.1 \text{ N})\sin 21.31°$$

$$B_x = -1390.4 \text{ N}$$

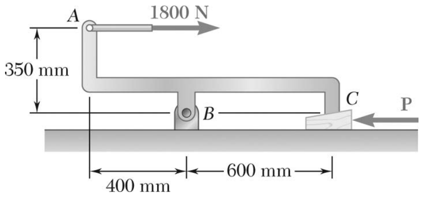
$\mathbf{B}_x = 1390 \text{ N} \leftarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y + R_C \cos 21.31° = 0$$

$$B_y + (1127.1 \text{ N})\cos 21.31° = 0$$

$$B_y = -1050 \text{ N}$$

$\mathbf{B}_y = 1050 \text{ N} \downarrow$ ◀



PROBLEM 8.49

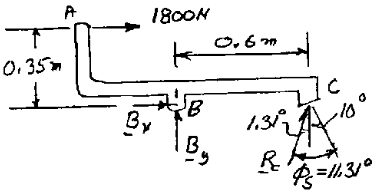
Solve Problem 8.48 assuming that the force **P** is directed to the right.

PROBLEM 8.48 The machine part *ABC* is supported by a frictionless hinge at *B* and a 10° wedge at *C*. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine (*a*) the force **P** required to move the wedge, (*b*) the components of the corresponding reaction at *B*.

SOLUTION

$$\phi_s = \tan^{-1} 0.20 = 11.31°$$

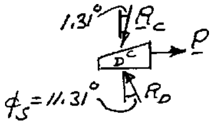
Free body: Part *ABC*



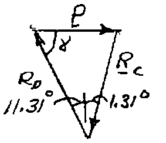
$$+\circlearrowright \Sigma M_B = 0: \quad (1800 \text{ N})(0.35 \text{ m}) - R_C \cos 1.31°(0.6 \text{ m}) = 0$$

$$R_C = 1050.3 \text{ N}$$

Free body: Wedge



Force triangle:



$$\gamma = 90° - 11.31° = 78.69°$$

(*a*) Law of sines:

$$\frac{P}{\sin(11.31° + 1.31°)} = \frac{1050.3 \text{ N}}{\sin 78.69°}$$

$$P = 234 \text{ N}$$

P = 234 N ⟶ ◀

(*b*) Return to Part *ABC*:

$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad B_x + 1800 \text{ N} + R_C \sin 1.31° = 0$$

$$B_x + 1800 \text{ N} + (1050.3 \text{ N}) \sin 1.31° = 0$$

$$B_x = -1824 \text{ N}$$

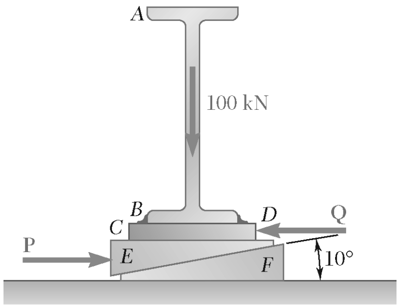
$\mathbf{B}_x = 1824$ N ⟵ ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y + R_C \cos 1.31° = 0$$

$$B_y + (1050.3 \text{ N}) \cos 1.31° = 0$$

$$B_y = -1050 \text{ N}$$

$\mathbf{B}_y = 1050$ N ↓ ◀

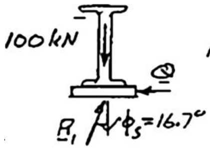


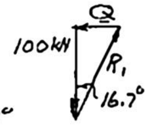
PROBLEM 8.50

The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 100 kN. The coefficient of static friction is 0.30 between two steel surfaces and 0.60 between steel and concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

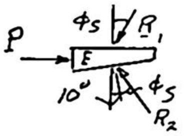
Free body: Beam and plate *CD*

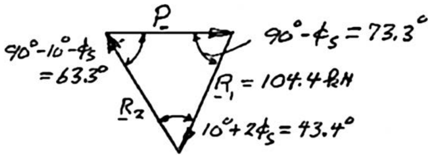




$$R_1 = \frac{(100\text{ kN})}{\cos 16.7°}$$

$$R_1 = 104.4\text{ kN}$$





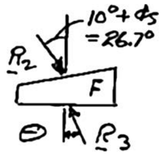
(*a*) $$\frac{P}{\sin 43.4°} = \frac{104.4\text{ kN}}{\sin 63.3°}$$ **P** = 80.3 kN ⟶ ◀

(*b*) $$\phi_s = \tan^{-1} 0.3 = 16.7°$$

$$Q = (100\text{ kN})\tan 16.7°$$ **Q** = 30 kN ⟵ ◀

Free body: Wedge *F*

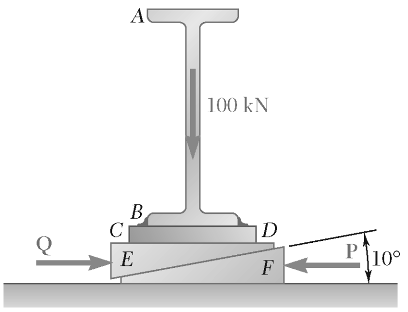
(To check that it does not move.)



Since wedge *F* is a two-force body, R_2 and R_3 are colinear

Thus $\theta = 26.7°$

But $\phi_{\text{concrete}} = \tan^{-1} 0.6 = 31.0° > \theta$ OK

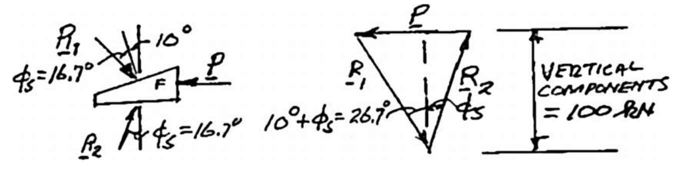


PROBLEM 8.51

The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 100 kN. The coefficient of static friction is 0.30 between two steel surfaces and 0.60 between steel and concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

Free body: Wedge *F* $\quad \phi_s = \tan^{-1} 0.30 = 16.7°$



(*a*)

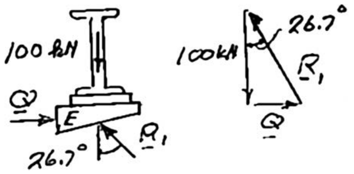
$$P = (100 \text{ kN}) \tan 26.7° + (100 \text{ kN}) \tan \phi_s$$

$$P = 50.29 \text{ kN} + 30 \text{ kN}$$

$$P = 80.29 \text{ kN} \qquad \mathbf{P} = 80.3 \text{ kN} \longleftarrow \blacktriangleleft$$

$$R_1 = \frac{(100 \text{ kN})}{\cos 26.7°} = 111.94 \text{ kN}$$

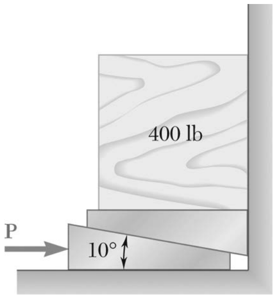
Free body: Beam, plate, and wedge *E*



(*b*)

$$Q = W \tan 26.7° = (100 \text{ kN}) \tan 26.7°$$

$$Q = 50.29 \text{ kN} \qquad \mathbf{Q} = 50.3 \text{ kN} \longrightarrow \blacktriangleleft$$



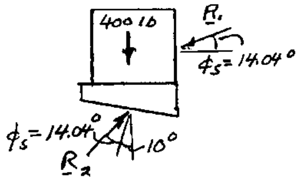
PROBLEM 8.52

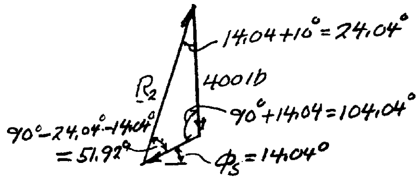
Two 10° wedges of negligible weight are used to move and position the 400-lb block. Knowing that the coefficient of static friction is 0.25 at all surfaces of contact, determine the smallest force **P** that should be applied as shown to one of the wedges.

SOLUTION

Free body: Block and top wedge $\phi_s = \tan^{-1} 0.25 = 14.04°$

Force triangle



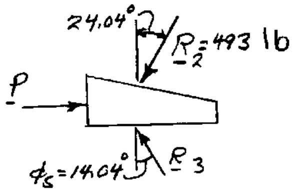


Law of sines:

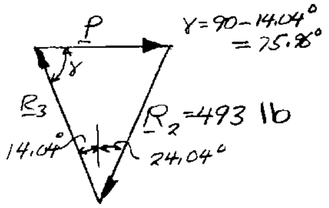
$$\frac{R_2}{\sin 104.04°} = \frac{400 \text{ lb}}{\sin 51.92°}$$

$$R_2 = 493 \text{ lb}$$

Free body: Lower wedge



Force triangle

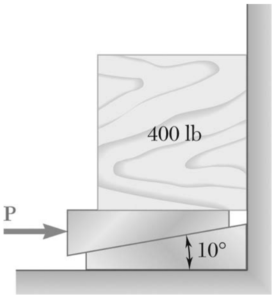


Law of sine:

$$\frac{P}{\sin(14.04° + 24.04°)} = \frac{493 \text{ lb}}{\sin 75.96°}$$

$$P = 313.4 \text{ N}$$

P = 313 lb ⟶ ◀



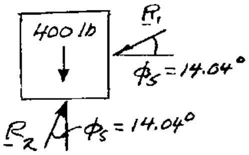
PROBLEM 8.53

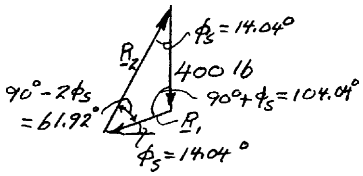
Two 10° wedges of negligible weight are used to move and position the 400-lb block. Knowing that the coefficient of static friction is 0.25 at all surfaces of contact, determine the smallest force **P** that should be applied as shown to one of the wedges.

SOLUTION

Free body: Block $\qquad \phi_s = \tan 0.25 = 14.04°$

Force triangle



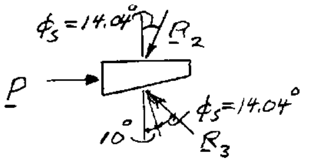


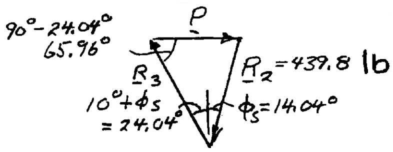
Law of sines:

$$\frac{R_2}{\sin 104.04°} = \frac{400 \text{ lb}}{\sin 61.92°}$$

$$R_2 = 439.8 \text{ lb}$$

Free body: Wedge $\qquad$ Force triangle





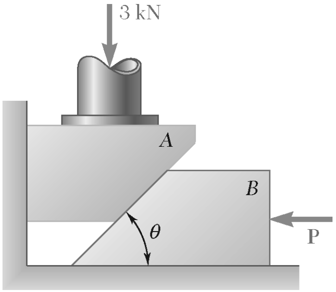
Law of sines:

$$\frac{P}{\sin(24.04° + 14.04°)} = \frac{439.8 \text{ lb}}{\sin 65.96°}$$

$$P = 297.0 \text{ lb}$$

$P = 297$ lb ⟶ ◀

PROPRIETARY MATERIAL.



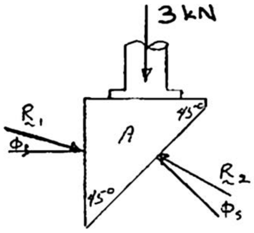
PROBLEM 8.54

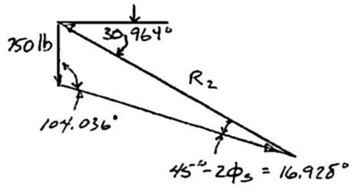
Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** required to raise block *A*.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.25 = 14.036°$$

FBD block *A*:

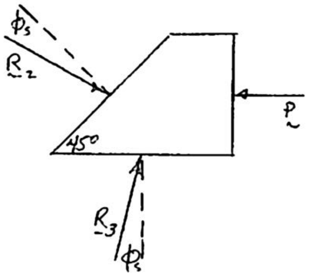


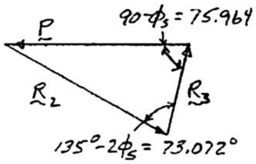


$$\frac{R_2}{\sin 104.036°} = \frac{3\text{ kN}}{\sin 16.928°}$$

$$R_2 = 10.0000\text{ kN}$$

FBD wedge *B*:

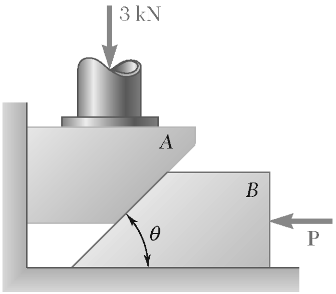




$$\frac{P}{\sin 73.072°} = \frac{10.0000\text{ kN}}{\sin 75.964°}$$

$$P = 9.8611\text{ kN}$$

P = 9.86 kN ← ◀



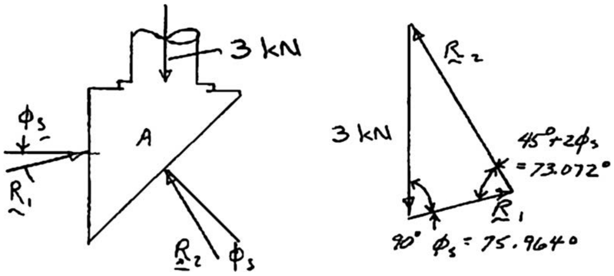
PROBLEM 8.55

Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** for which equilibrium is maintained.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036°$$

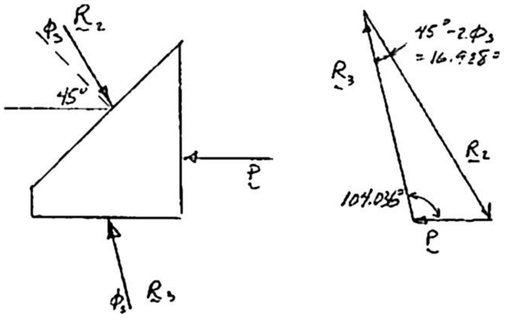
FBD block *A*:



$$\frac{R_2}{\sin(75.964°)} = \frac{3\text{ kN}}{\sin(73.072°)}$$

$$R_2 = 3.0420\text{ kN}$$

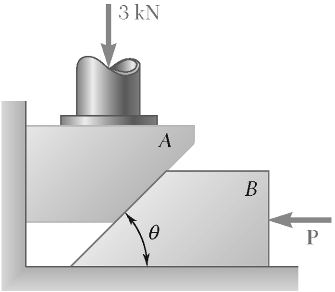
FBD wedge *B*:



$$\frac{P}{\sin 16.928°} = \frac{3.0420\text{ kN}}{\sin 104.036°}$$

$$P = 0.91300\text{ kN}$$

P = 913 N ⟵ ◀



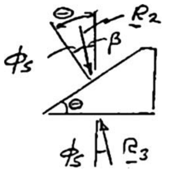
PROBLEM 8.56

Block *A* supports a pipe column and rests as shown on wedge *B*. The coefficient of static friction at all surfaces of contact is 0.25. If $\mathbf{P} = 0$, determine (*a*) the angle θ for which sliding is impending, (*b*) the corresponding force exerted on the block by the vertical wall.

SOLUTION

Free body: Wedge *B* $\qquad \phi_s = \tan^{-1} 0.25 = 14.04°$

(*a*) Since wedge is a two-force body, $\mathbf{R}_2$ and $\mathbf{R}_3$ must be equal and opposite. Therefore, they form equal angles with vertical



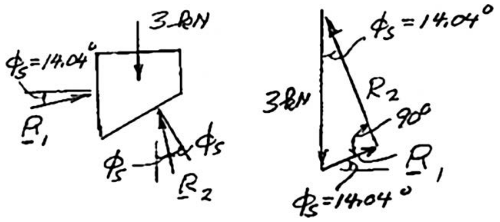
$$\beta = \phi_s$$

and

$$\theta - \phi_s = \phi_s$$
$$\theta = 2\phi_s = 2(14.04°)$$

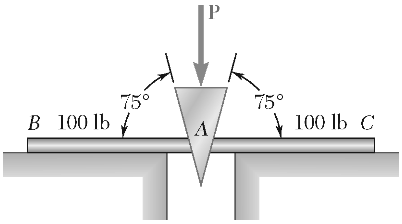
$\theta = 28.1°$ ◀

Free body: Block *A*



$$R_1 = (3\text{ kN})\sin 14.04° = 0.7278\text{ kN}$$

(*b*) Force exerted by wall: $\mathbf{R}_1 = 728$ N ∠ 14.04° ◀



PROBLEM 8.57

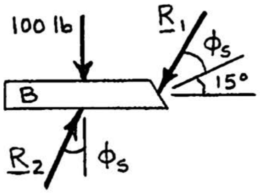
A wedge *A* of negligible weight is to be driven between two 100-lb plates *B* and *C*. The coefficient of static friction between all surfaces of contact is 0.35. Determine the magnitude of the force **P** required to start moving the wedge (*a*) if the plates are equally free to move, (*b*) if plate *C* is securely bolted to the surface.

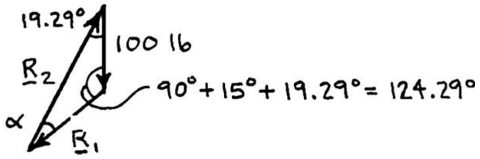
SOLUTION

(*a*) With plates equally free to move

Free body: Plate *B* $\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.35 = 19.2900°$

Force triangle:





$$\alpha = 180° - 124.29° - 19.29° = 36.42°$$

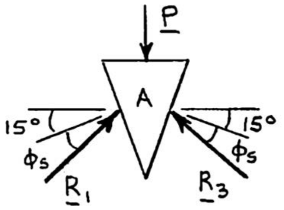
Law of sines:

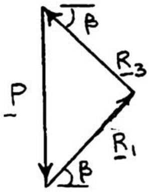
$$\frac{R_1}{\sin 19.29°} = \frac{100\text{ lb}}{\sin 36.42°}$$

$$R_1 = 55.643\text{ lb}$$

Free body: Wedge *A*

Force triangle:





By symmetry, $R_3 = R_1 = 55.643\text{ lb}$

$$\beta = 19.29° + 15° = 34.29°$$

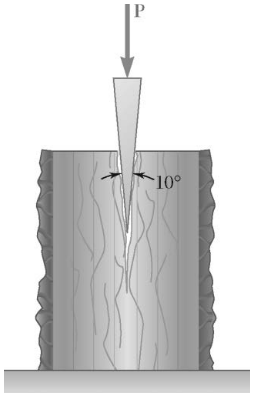
Then $P = 2R_1 \sin\beta$

or $P = 2(55.643)\sin 34.29°$ $P = 62.7\text{ lb}$ ◀

(*b*) With plate *C* bolted

The free body diagrams of plate *B* and wedge *A* (the only members to move) are same as above. Answer is thus the same.

$P = 62.7\text{ lb}$ ◀

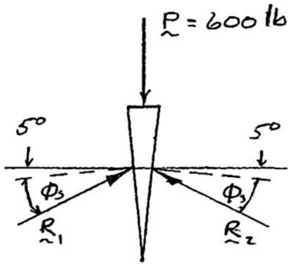


PROBLEM 8.58

A 10° wedge is used to split a section of a log. The coefficient of static friction between the wedge and the log is 0.35. Knowing that a force **P** of magnitude 600 lb was required to insert the wedge, determine the magnitude of the forces exerted on the wood by the wedge after insertion.

SOLUTION

FBD wedge (impending motion ↓):



$$\phi_s = \tan^{-1}\mu_s$$
$$= \tan^{-1} 0.35$$
$$= 19.29°$$

By symmetry: $R_1 = R_2$

$$+\uparrow \Sigma F_y = 0: \quad 2R_1 \sin(5° + \phi_s) - 600 \text{ lb} = 0$$

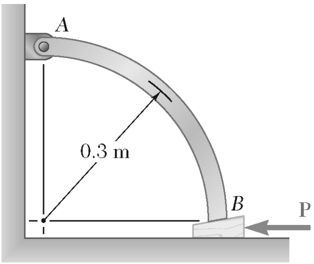
or $$R_1 = R_2 = \frac{300 \text{ lb}}{\sin(5° + 19.29°)} = 729.30 \text{ lb}$$

When P is removed, the vertical components of R_1 and R_2 vanish, leaving the horizontal components

$$R_{1x} = R_{2x}$$
$$= R_1 \cos(5° + \phi_s)$$
$$= (729.30 \text{ lb})\cos(5° + 19.29°)$$

$R_{1x} = R_{2x} = 665$ lb ◀

(Note that $\phi_s > 5°$, so wedge is self-locking.)

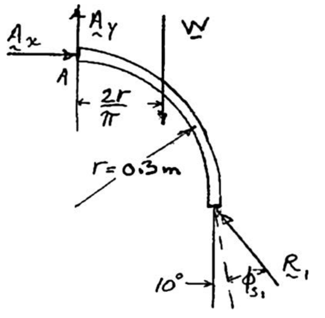


PROBLEM 8.59

A 10° wedge is to be forced under end *B* of the 5-kg rod *AB*. Knowing that the coefficient of static friction is 0.40 between the wedge and the rod and 0.20 between the wedge and the floor, determine the smallest force **P** required to raise end *B* of the rod.

SOLUTION

FBD *AB*:



$$W = mg$$

$$W = (5\text{ kg})(9.81\text{ m/s}^2)$$

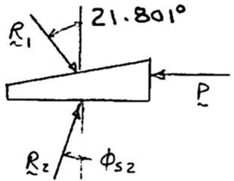
$$W = 49.050\text{ N}$$

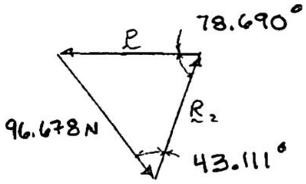
$$\phi_{s1} = \tan^{-1}(\mu_s)_1 = \tan^{-1} 0.40 = 21.801°$$

$$\circlearrowleft \Sigma M_A = 0: \quad rR_1\cos(10° + 21.801°) - rR_1\sin(10° + 21.801°) - \frac{2r}{\pi}(49.050\text{ N}) = 0$$

$$R_1 = 96.678\text{ N}$$

FBD wedge:

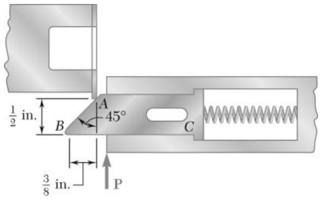




$$\phi_{s2} = \tan^{-1}(\mu_s)_2 = \tan^{-1} 0.20 = 11.3099$$

$$\frac{P}{\sin(43.111°)} = \frac{96.678\text{ N}}{\sin 78.690°}$$

P = 67.4 N ⟵ ◀

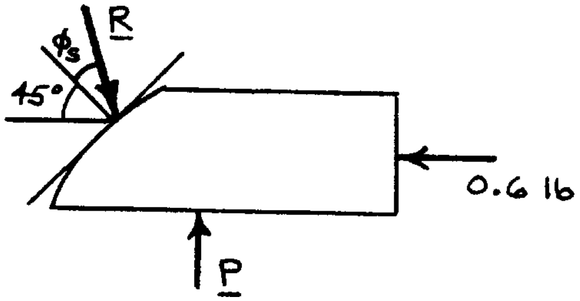


PROBLEM 8.60

The spring of the door latch has a constant of 1.8 lb/in. and in the position shown exerts a 0.6-lb force on the bolt. The coefficient of static friction between the bolt and the strike plate is 0.40; all other surfaces are well lubricated and may be assumed frictionless. Determine the magnitude of the force **P** required to start closing the door.

SOLUTION

Free body: Bolt

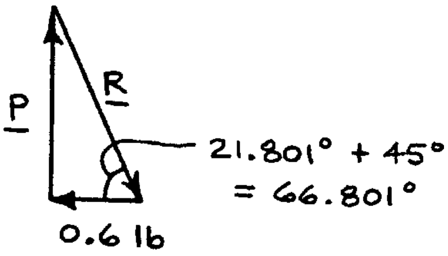


$$\mu_s = 0.40$$

$$\phi_s = \tan^{-1} 0.40$$

$$= 21.801°$$

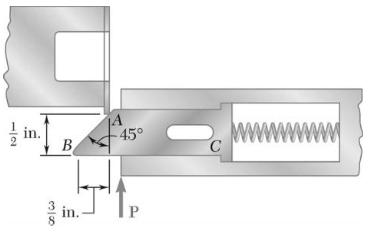
Force triangle:



From force triangle,

$$P = (0.6 \text{ lb}) \tan 66.801°$$

$$P = 1.39997 \text{ lb} \qquad P = 1.400 \text{ lb} \blacktriangleleft$$



PROBLEM 8.61

In Problem 8.60, determine the angle that the face of the bolt should form with the line *BC* if the force **P** required to close the door is to be the same for both the position shown and the position when *B* is almost at the strike plate.

PROBLEM 8.60 The spring of the door latch has a constant of 1.8 lb/in. and in the position shown exerts a 0.6-lb force on the bolt. The coefficient of static friction between the bolt and the strike plate is 0.40; all other surfaces are well lubricated and may be assumed frictionless. Determine the magnitude of the force **P** required to start closing the door.

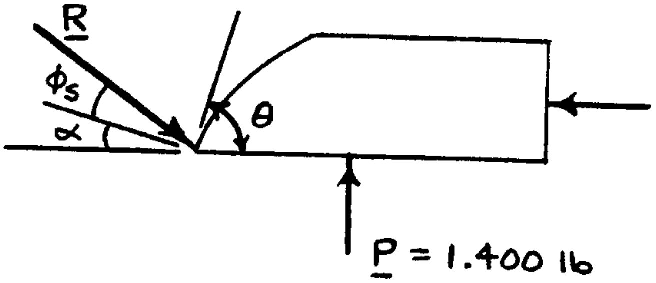
SOLUTION

For position shown in figure:

From Prob. 8.60: $P = 1.400$ lb

For position when *B* reaches strike plate:

Free body: Bolt



$$\mu_s = 0.40$$

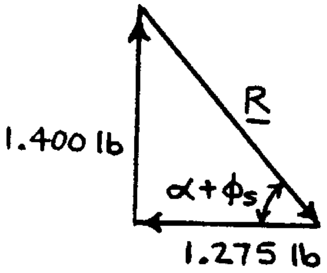
$$\phi_s = \tan^{-1} 0.40 = 21.801°$$

$$F = 0.6 \text{ lb} + kx$$

$$= 0.6 \text{ lb} + (1.8 \text{ lb/in.})\left(\frac{3}{8} \text{ in.}\right)$$

$$= 1.275 \text{ lb}$$

Force triangle:



From force triangle,

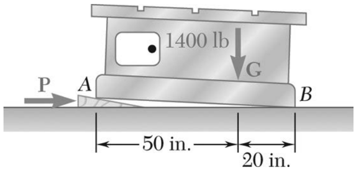
$$\tan(\alpha - \phi_s) = \frac{1.400 \text{ lb}}{1.275 \text{ lb}} = 1.09804$$

$$\alpha + \phi_s = 47.675°$$

$$\alpha = 47.675° - 21.801° = 25.874°$$

$$\theta = 180° - 90° - \alpha = 90° - \alpha = 90° - 25.874°$$

$\theta = 64.1°$ ◀

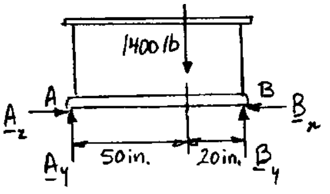


PROBLEM 8.62

A 5° wedge is to be forced under a 1400-lb machine base at *A*. Knowing that the coefficient of static friction at all surfaces is 0.20, (*a*) determine the force **P** required to move the wedge, (*b*) indicate whether the machine base will move.

SOLUTION

Free body: Machine base



$$+\circlearrowleft \Sigma M_B = 0: \quad (1400\text{ lb})(20\text{ in}) - A_y(70\text{ in}) = 0$$

$$A_y = 400\text{ lb}$$

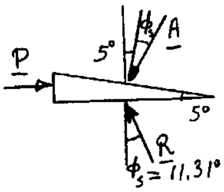
$$\xrightarrow{+} \Sigma F_y = 0: \quad A_y + B_y - 1400\text{ lb} = 0$$

$$B_y = 1400 - 400 = 1000\text{ lb}$$

Free body: Wedge

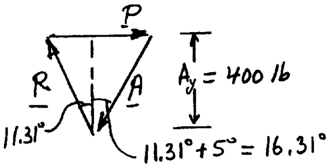
(Assume machine will not move)

$$\mu_s = 0.20, \quad \phi_s = \tan 0.20 = 11.31^\circ$$



We know that $A_y = 400\text{ lb}$

Force triangle:

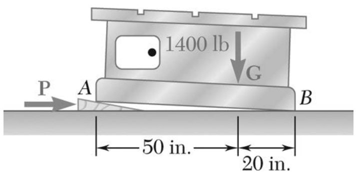


(*a*) $P = (400\text{ lb})(\tan 11.31^\circ + \tan 16.31^\circ) = 197.0\text{ lb}$ $\quad$ $\mathbf{P} = 197.0\text{ lb} \longrightarrow$ ◀

(*b*) Total maximum friction force at *A* and *B*:

$$F_m = \mu_c W = 0.20(1400\text{ lb}) = 280\text{ lb}$$

Since $P < F_m$: Machine will not move ◀



PROBLEM 8.63

Solve Problem 8.62 assuming that the wedge is to be forced under the machine base at *B* instead of *A*.

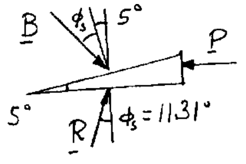
PROBLEM 8.62 A 5° wedge is to be forced under a 1400-lb machine base at *A*. Knowing that the coefficient of static friction at all surfaces is 0.20, (*a*) determine the force **P** required to move the wedge, (*b*) indicate whether the machine base will move.

SOLUTION

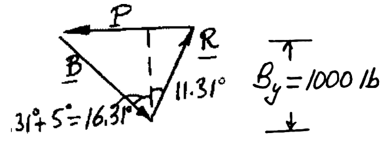
See solution to Prob. 8.62 for F.B.D. of machine base and determination of $B_y = 1000$ lb.

Free body: Wedge (Assume machine will not move)

$$\mu_s = 0.20 \qquad \phi_s = \tan^{-1} 0.20 = 11.31°$$



Force triangle:



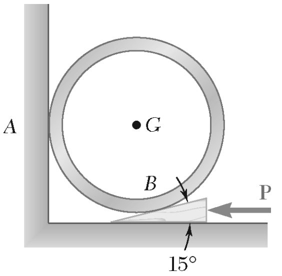
$$P = (1000 \text{ lb})(\tan 11.31° + \tan 16.31°) = 493 \text{ lb}$$

Total maximum friction force at *A* and *B*:

$$F_m = \mu_s W = 0.20(1400 \text{ lb}) = 280 \text{ lb} < 493 \text{ lb}$$

(*b*) Since $P > F_m$, machine will move with wedge ◀

(*a*) We then have $P = F_m$, **P** = 280 lb ⟵ ◀



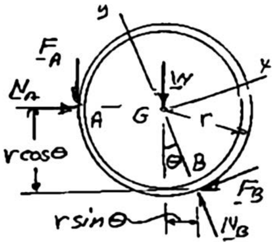
PROBLEM 8.64

A 15° wedge is forced under a 50-kg pipe as shown. The coefficient of static friction at all surfaces is 0.20. (*a*) Show that slipping will occur between the pipe and the vertical wall. (*b*) Determine the force **P** required to move the wedge.

SOLUTION

Free body: Pipe

$$+\circlearrowleft \Sigma M_B = 0: \quad Wr\sin\theta + F_A r(1+\sin\theta) - N_A r\cos\theta = 0$$



Assume slipping at A:

$$F_A = \mu_s N_A$$

$$N_A\cos\theta - \mu_s N_A(1+\sin\theta) = W\sin\theta$$

$$N_A = \frac{W\sin\theta}{\cos\theta - \mu_s(1+\sin\theta)}$$

$$N_A = \frac{W\sin 15°}{\cos 15° - (0.20)(1+\sin 15°)}$$

$$= 0.36241W$$

$$+\nearrow \Sigma F_x = 0: \quad -F_B - W\sin\theta - F_A\sin\theta + N_A\cos\theta = 0$$

$$F_B = N_A\cos\theta - \mu_s N_A\sin\theta - W\sin\theta$$

$$F_B = 0.36241W\cos 15° - 0.20(0.36241W)\sin 15° - W\sin 15°$$

$$F_B = 0.072482W$$

$$+\nwarrow \Sigma F_y = 0: \quad N_B - W\cos\theta - F_A\cos\theta - N_A\sin\theta = 0$$

$$N_B = N_A\sin\theta + \mu_s N_A\cos\theta + W\cos\theta$$

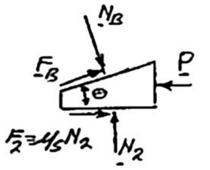
$$N_B = (0.36241W)\sin 15° + 0.20(0.36241W)\cos 15° + W\cos 15°$$

$$N_B = 1.12974W$$

Maximum available: $F_B = \mu_s N_B = 0.22595W$

(*a*) We note that $F_B < F_{\max}$ No slip at B ◀

(*b*) Free body: Wedge $+\uparrow \Sigma F_y = 0: \quad N_2 - N_B\cos\theta + F_B\sin\theta = 0$



$$N_2 = N_B\cos\theta - F_B\sin\theta$$

$$N_2 = (1.12974W)\cos 15° - (0.07248W)\sin 15°$$

$$N_2 = 1.07249W$$

PROBLEM 8.64 (Continued)

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_B \cos\theta + N_B \sin\theta + \mu_s N_2 - P = 0$$

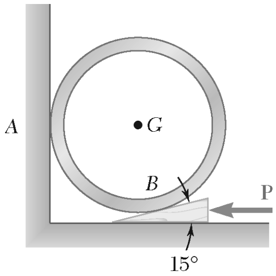
$$P = F_B \cos\theta + N_B \sin\theta + \mu_s N_2$$

$$P = (0.07248W)\cos 15° + (1.12974W)\sin 15° + 0.2(1.07249W)$$

$$P = 0.5769W$$

$$W = mg: \quad P = 0.5769(50\text{ kg})(9.81\text{ m/s}^2)$$

$\mathbf{P} = 283\text{ N} \longleftarrow$ ◀

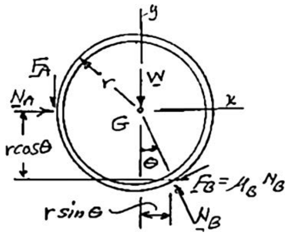


PROBLEM 8.65

A 15° wedge is forced under a 50-kg pipe as shown. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine the largest coefficient of static friction between the pipe and the vertical wall for which slipping will occur at *A*.

SOLUTION

<u>Free body: Pipe</u>



$$+\circlearrowleft \Sigma M_A = 0: \quad N_B r\cos\theta - \mu_B N_B r - (\mu_B N_B \sin\theta)r - Wr = 0$$

$$N_B = \frac{W}{\cos\theta - \mu_B(1+\sin\theta)}$$

$$N_B = \frac{W}{\cos 15° - 0.2(1+\sin 15°)}$$

$$N_B = 1.4002W$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad N_A - N_B \sin\theta - \mu_B N_B \cos\theta = 0$$

$$N_A = N_B(\sin\theta + \mu_B\cos\theta)$$

$$= (1.4002W)(\sin 15° + 0.2\times\cos 15°)$$

$$N_A = 0.63293W$$

$$+\uparrow \Sigma F_y = 0: \quad -F_A - W + N_B\cos\theta - \mu_B N_B \sin\theta = 0$$

$$F_A = N_B(\cos\theta - \mu_B\sin\theta) - W$$

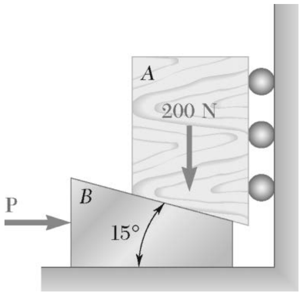
$$F_A = (1.4002W)(\cos 15° - 0.2\times\sin\theta) - W$$

$$F_A = 0.28001W$$

For slipping at *A*: $F_A = \mu_A N_A$

$$\mu_A = \frac{F_A}{N_A} = \frac{0.28001W}{0.63293W}$$

$\mu_A = 0.442$ ◀



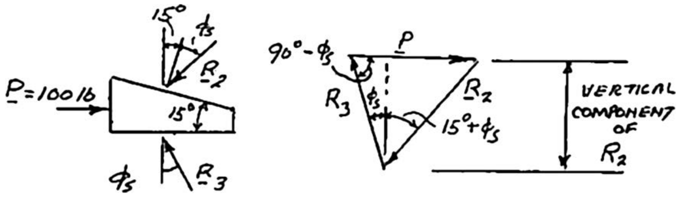
PROBLEM 8.66*

A 200-N block rests as shown on a wedge of negligible weight. The coefficient of static friction μ_s is the same at both surfaces of the wedge, and friction between the block and the vertical wall may be neglected. For $P = 100$ N, determine the value of μ_s for which motion is impending. (*Hint:* Solve the equation obtained by trial and error.)

SOLUTION

Free body: Wedge

Force triangle:

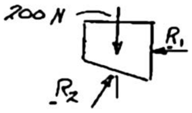


Law of sines:

$$\frac{R_2}{\sin(90° - \phi_s)} = \frac{P}{\sin(15° + 2\phi_s)}$$

$$R_2 = P\frac{\sin(90° - \phi_s)}{\sin(15° + 2\phi_s)} \tag{1}$$

Free body: Block



$$\Sigma F_y = 0$$

Vertical component of R_2 is 200 N

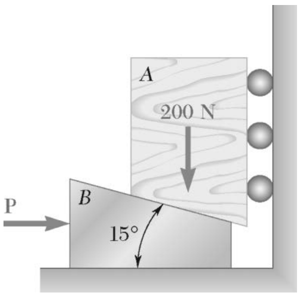
Return to force triangle of wedge. Note $P = 100$ N

$$100\,\text{N} = (200\,\text{N})\tan\phi + (200\,\text{N})\tan(15° + \phi_s)$$

$$0.5 = \tan\phi + \tan(15° + \phi_s)$$

Solve by trial and error $\quad \phi_s = 6.292$

$$\mu_s = \tan\phi_s = \tan 6.292° \qquad \mu_s = 0.1103 \blacktriangleleft$$



PROBLEM 8.67*

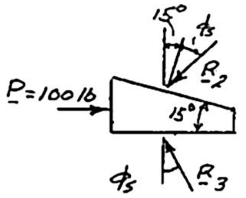
Solve Problem 8.66 assuming that the rollers are removed and that μ_s is the coefficient of friction at all surfaces of contact.

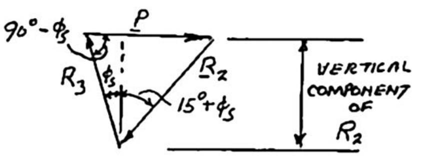
PROBLEM 8.66* A 200-N block rests as shown on a wedge of negligible weight. The coefficient of static friction μ_s is the same at both surfaces of the wedge, and friction between the block and the vertical wall may be neglected. For P = 100 N, determine the value of μ_s for which motion is impending. (*Hint:* Solve the equation obtained by trial and error.)

SOLUTION

Free body: Wedge

Force triangle:





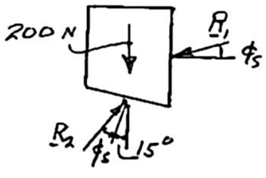
Law of sines:

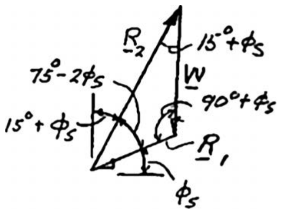
$$\frac{R_2}{\sin(90^\circ - \phi_s)} = \frac{P}{\sin(15^\circ + 2\phi_s)}$$

$$R_2 = P\frac{\sin(90^\circ - \phi_s)}{\sin(15^\circ + 2\phi_s)} \qquad (1)$$

Free body: Block (Rollers removed)

Force triangle:





Law of sines:

$$\frac{R_2}{\sin(90^\circ + \phi_s)} = \frac{W}{\sin(75^\circ - 2\phi_s)}$$

$$R_2 = W\frac{\sin(90^\circ + \phi_s)}{\sin(75^\circ - 2\theta_s)} \qquad (2)$$

PROBLEM 8.67* (Continued)

Equate R_2 from Eq. (1) and Eq. (2):

$$P\frac{\sin(90^\circ-\phi_s)}{\sin(15^\circ+2\phi_s)}=W\frac{\sin(90^\circ+\phi_s)}{\sin(75^\circ-2\phi_s)}$$

$$P=100\text{ lb}$$

$$W=200\text{ N}$$

$$0.5=\frac{\sin(90^\circ+\phi_s)\sin(15^\circ+2\phi_s)}{\sin(75^\circ-2\phi_s)\sin(90^\circ-\phi_s)}$$

Solve by trial and error: $\phi_s=5.784^\circ$

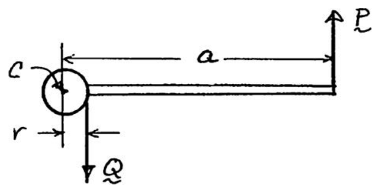
$\mu_s=\tan\phi_s=\tan 5.784^\circ$ $\mu_s=0.1013$ ◀

PROBLEM 8.68

Derive the following formulas relating the load **W** and the force **P** exerted on the handle of the jack discussed in Section 8.6. (*a*) $P = (Wr/a)\tan(\theta + \phi_s)$, to raise the load; (*b*) $P = (Wr/a)\tan(\phi_s - \theta)$, to lower the load if the screw is self-locking; (*c*) $P = (Wr/a)\tan(\theta - \phi_s)$, to hold the load if the screw is not self-locking.

SOLUTION

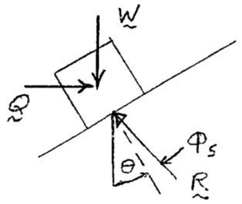
FBD jack handle:

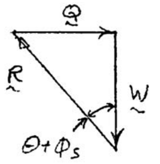


See Section 8.6. $\circlearrowleft \Sigma M_C = 0$: $aP - rQ = 0$ or $P = \dfrac{r}{a}Q$

FBD block on incline:

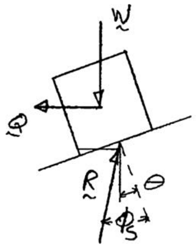
(*a*) Raising load

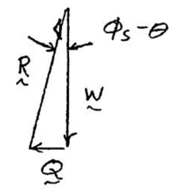




$$Q = W\tan(\theta + \phi_s) \qquad\qquad P = \frac{r}{a}W\tan(\theta + \phi_s) \ ◀$$

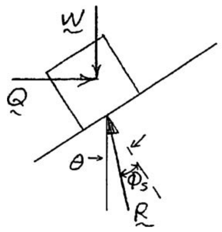
(*b*) Lowering load if screw is self-locking (i.e., if $\phi_s > \theta$)

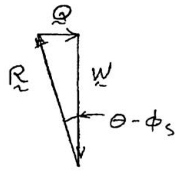




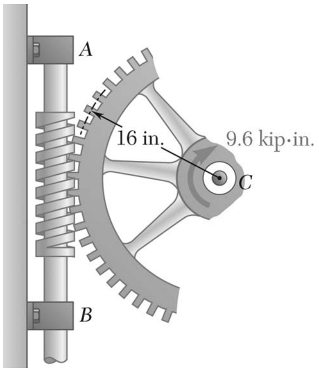
$$Q = W\tan(\phi_s - \theta) \qquad\qquad P = \frac{r}{a}W\tan(\phi_s - \theta) \ ◀$$

(*c*) Holding load is screw is not self-locking (i.e., if $\phi_s < \theta$)





$$Q = W\tan(\theta - \phi_s) \qquad\qquad P = \frac{r}{a}W\tan(\theta - \phi_s) \ ◀$$



PROBLEM 8.69

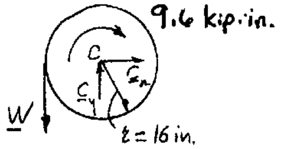
The square-threaded worm gear shown has a mean radius of 2 in. and a lead of 0.5 in. The large gear is subjected to a constant clockwise couple of 9.6 kip · in. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft *AB* in order to rotate the large gear counterclockwise. Neglect friction in the bearings at *A*, *B*, and *C*.

SOLUTION

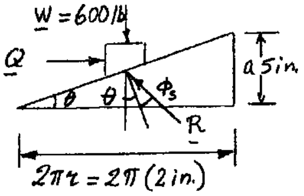
Free body: Large gear

$$+\circlearrowleft \Sigma M_C = 0: \quad W(16\text{ in.}) - 9.6\text{ kip}\cdot\text{in.} = 0$$

$$W = 0.6\text{ kip} = 600\text{ lb}$$



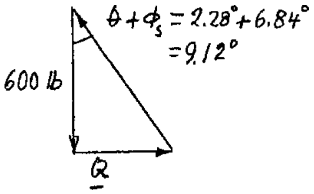
Block-and-Incline analysis of worn gear



$$\tan\theta = \frac{0.5\text{ in.}}{2\pi(2\text{ in.})} = 0.039789$$

$$\theta = 2.28°$$

$$\mu_s = 0.12, \; \phi_s = \tan^{-1} 0.12 = 6.84°$$



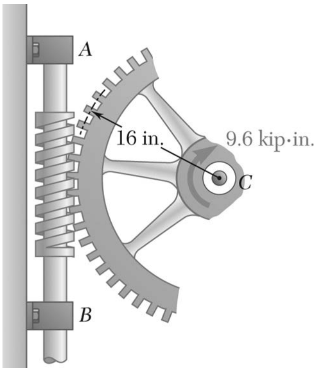
$$Q = (600\text{ lb})\tan 9.12°$$

$$= 96.32\text{ lb}$$

$$\text{Torque} = Q_r = (96.32\text{ lb})(2\text{ in.})$$

$$= 192.6\text{ lb}\cdot\text{in.}$$

$$\text{Torque} = 16.05\text{ lb}\cdot\text{ft} \blacktriangleleft$$



PROBLEM 8.70

In Problem 8.69, determine the couple that must be applied to shaft *AB* in order to rotate the large gear clockwise.

PROBLEM 8.69 The square-threaded worm gear shown has a mean radius of 2 in. and a lead of 0.5 in. The large gear is subjected to a constant clockwise couple of 9.6 kip · in. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft *AB* in order to rotate the large gear counterclockwise. Neglect friction in the bearings at *A*, *B*, and *C*.

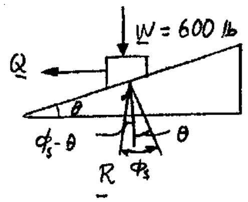
SOLUTION

Free body: Large gear

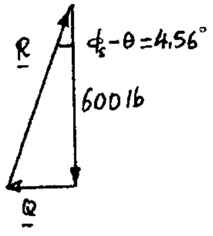
See solution to Prob. 8.69. We find $W = 600$ lb

Block-and-incline analysis of worm gear

From Prob. 8.69 we have $\theta = 2.28°$ and $\phi_s = 6.84°$. Since $\theta < \phi_s$, gear is self-looking and a torque must be applied to it to rotate large gear clockwise.



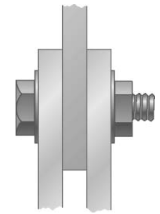
$$\phi_s - \theta = 6.84° - 2.28° = 4.56°$$



$$Q = (600 \text{ lb}) \tan 4.56°$$
$$= 47.85 \text{ lb}$$

$$\text{Torque} = Q_r = (47.85 \text{ lb})(2 \text{ in.})$$
$$= 95.70 \text{ lb} \cdot \text{in.}$$

$$\text{Torque} = 7.98 \text{ lb} \cdot \text{ft}$$ ◀

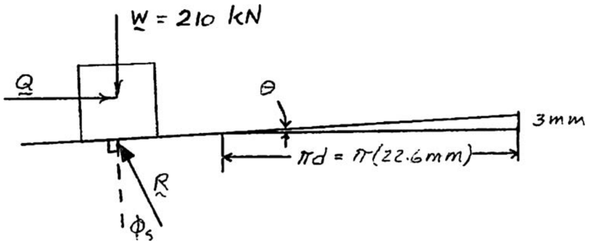


PROBLEM 8.71

High-strength bolts are used in the construction of many steel structures. For a 24-mm-nominal-diameter bolt the required minimum bolt tension is 210 kN. Assuming the coefficient of friction to be 0.40, determine the required couple that should be applied to the bolt and nut. The mean diameter of the thread is 22.6 mm, and the lead is 3 mm. Neglect friction between the nut and washer, and assume the bolt to be square-threaded.

SOLUTION

FBD block on incline:



$$\theta = \tan^{-1}\frac{3\text{ mm}}{(22.6\text{ mm})\pi}$$

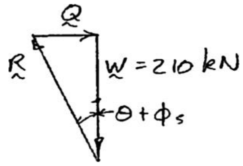
$$= 2.4195°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.40$$

$$\phi_s = 21.801°$$

$$Q = (210\text{ kN})\tan(21.801° + 2.4195°)$$

$$Q = 94.468\text{ kN}$$

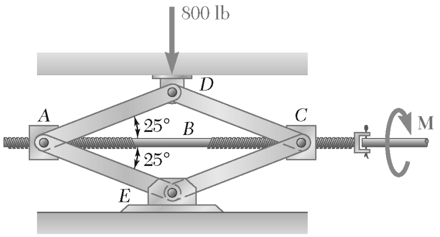


$$\text{Torque} = \frac{d}{2}Q$$

$$= \frac{22.6\text{ mm}}{2}(94.468\text{ kN})$$

$$= 1067.49\text{ N}\cdot\text{m}$$

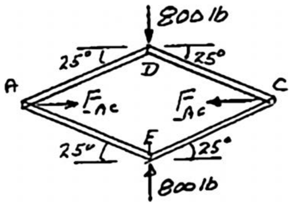
Torque = 1068 N · m ◀



PROBLEM 8.72

The position of the automobile jack shown is controlled by a screw *ABC* that is single-threaded at each end (right-handed thread at *A*, left-handed thread at *C*). Each thread has a pitch of 0.1 in. and a mean diameter of 0.375 in. If the coefficient of static friction is 0.15, determine the magnitude of the couple **M** that must be applied to raise the automobile.

SOLUTION

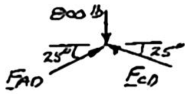


Free body: Parts *A*, *D*, *C*, *E*

Two-force members

Joint *D*:

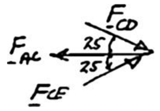
Symmetry: $F_{AD} = F_{CD}$



$$+\uparrow \Sigma F_y = 0: \quad 2F_{CD}\sin 25° - 800 \text{ lb} = 0$$

$$F_{CD} = 946.5 \text{ lb}$$

Joint *C*:



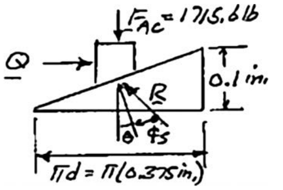
Symmetry: $F_{CE} = F_{CD}$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 2F_{CD}\cos 25° - F_{AC} = 0$$

$$F_{AC} = 2(946.5 \text{ lb})\cos 25°$$

$$F_{AC} = 1715.6 \text{ lb}$$

Block-and-incline analysis of *one* screw:

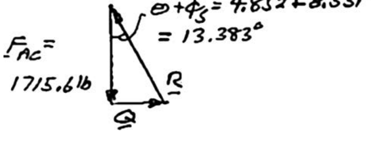


$$\tan\theta = \frac{0.1 \text{ in.}}{\pi(0.375 \text{ in.})}$$

$$\theta = 4.852°$$

$$\phi_s = \tan^{-1} 0.15$$

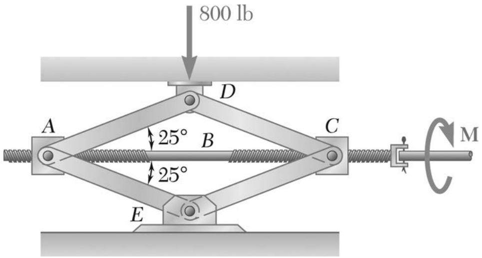
$$= 8.531°$$



$$Q = (1715.6 \text{ lb})\tan 13.383°$$

$$Q = 408.2 \text{ lb}$$

But, we have *two* screws: $\text{Torque} = 2Qr = 2(408.2 \text{ lb})\left(\frac{0.375 \text{ in.}}{2}\right)$ $\qquad \text{Torque} = 153.1 \text{ lb}\cdot\text{in.}$ ◀



PROBLEM 8.73

For the jack of Problem 8.72, determine the magnitude of the couple **M** that must be applied to lower the automobile.

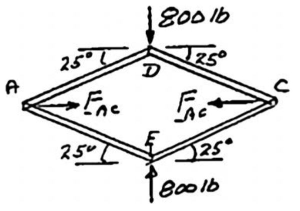
PROBLEM 8.72 The position of the automobile jack shown is controlled by a screw *ABC* that is single-threaded at each end (right-handed thread at *A*, left-handed thread at *C*). Each thread has a pitch of 0.1 in. and a mean diameter of 0.375 in. If the coefficient of static friction is 0.15, determine the magnitude of the couple **M** that must be applied to raise the automobile.

SOLUTION

Free body: Parts *A*, *D*, *C*, *E*

Two-force members

Joint *D*:

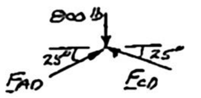


Symmetry: $F_{AD} = F_{CD}$

$$+\uparrow \Sigma F_y = 0: \quad 2F_{CD}\sin 25° - 800\text{ lb} = 0$$

$$F_{CD} = 946.5\text{ lb}$$

Joint *C*:

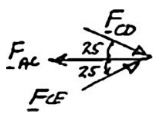


Symmetry: $F_{CE} = F_{CD}$

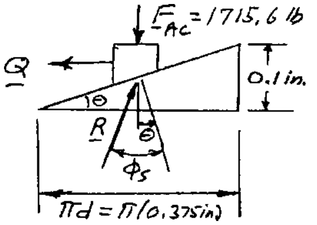
$$\xrightarrow{+} \Sigma F_x = 0: \quad 2F_{CD}\cos 25° - F_{AC} = 0$$

$$F_{AC} = 2(946.5\text{ lb})\cos 25°;$$

$$F_{AC} = 1715.6\text{ lb}$$



Block-and-incline analysis of *one* screw:



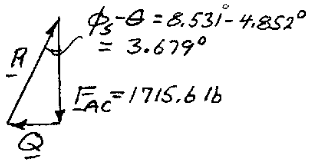
$$\tan\theta = \frac{0.1\text{ in.}}{\pi(0.375\text{ in.})}$$

$$\theta = 4.852°$$

$$\phi_s = \tan^{-1} 0.15$$

$$= 8.531°$$

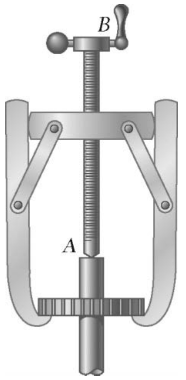
Since $\phi_s > \theta$, the screw is self-locking



$$Q = (1715.6\text{ lb})\tan 3.679°$$

$$Q = 110.3\text{ lb}$$

For two screws: $\text{Torque} = 2(110.3\text{ lb})\dfrac{1}{2}(0.375\text{ in.})$ $\quad$ $\text{Torque} = 41.4\text{ lb}\cdot\text{in.}$ ◀

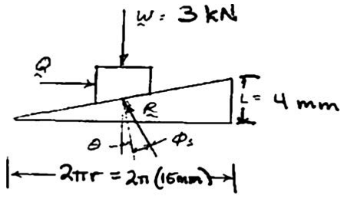


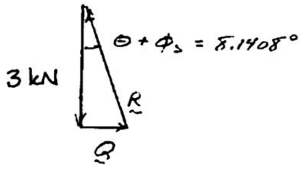
PROBLEM 8.74

In the gear-pulling assembly shown the square-threaded screw *AB* has a mean radius of 15 mm and a lead of 4 mm. Knowing that the coefficient of static friction is 0.10, determine the couple that must be applied to the screw in order to produce a force of 3 kN on the gear. Neglect friction at end *A* of the screw.

SOLUTION

Block/Incline:





$$\theta = \tan^{-1}\frac{4\text{ mm}}{30\pi\text{ mm}}$$

$$= 2.4302°$$

$$\phi_s = \tan^{-1}\mu_s$$

$$= \tan^{-1}(0.10)$$

$$= 5.7106°$$

$$Q = (3000\text{ N})\tan(8.1408°)$$

$$= 429.14\text{ N}$$

$$\text{Couple} = rQ$$

$$= (0.015\text{ m})(429.14\text{ N})$$

$$= 6.4371\text{ N}\cdot\text{m}$$

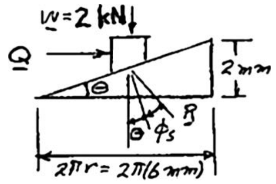
$M = 6.44\text{ N}\cdot\text{m}$ ◀

PROPRIETARY MATERIAL.

PROBLEM 8.75

The ends of two fixed rods *A* and *B* are each made in the form of a single-threaded screw of mean radius 6 mm and pitch 2 mm. Rod *A* has a right-handed thread and rod *B* has a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.

SOLUTION



To draw rods together:

Screw at *A*

$$\tan\theta = \frac{2\text{ mm}}{2\pi(6\text{ mm})}$$

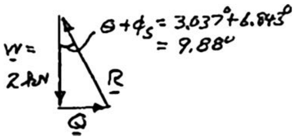
$$\theta = 3.037°$$

$$\phi_s = \tan^{-1} 0.12$$

$$= 6.843°$$

$$Q = (2\text{ kN})\tan 9.88°$$

$$= 348.3\text{ N}$$



$$\text{Torque at } A = Qr$$

$$= (348.3\text{ N})(0.006\text{ m})$$

$$= 2.0898\text{ N}\cdot\text{m}$$

Same torque required at *B* Total torque $= 4.18\text{ N}\cdot\text{m}$ ◀

PROBLEM 8.76

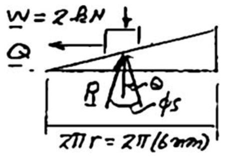
Assuming that in Problem 8.75 a right-handed thread is used on *both* rods A and B, determine the magnitude of the couple that must be applied to the sleeve in order to rotate it.

PROBLEM 8.75 The ends of two fixed rods A and B are each made in the form of a single-threaded screw of mean radius 6 mm and pitch 2 mm. Rod A has a right-handed thread and rod B has a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.

SOLUTION

From the solution to Problem 8.70,

$$\text{Torque at } A = 2.09\ \text{N}\cdot\text{m}$$



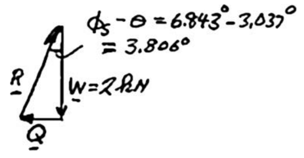
Screw at B: Loosening

$$\theta = 3.037°$$

$$\phi_s = 6.843°$$

$$Q = (2\ \text{kN})\tan 3.806°$$

$$= 133.1\ \text{N}$$



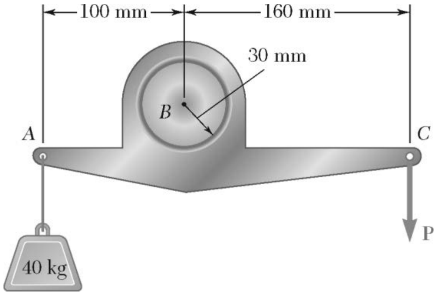
$$\text{Torque at } B = Qr$$

$$= (133.1\ \text{N})(0.006\ \text{m})$$

$$= 0.79860\ \text{N}\cdot\text{m}$$

$$\text{Total torque } = 2.0848\ \text{N}\cdot\text{m} + 0.79860\ \text{N}\cdot\text{m}$$

Total torque $= 2.89\ \text{N}\cdot\text{m}$ ◀

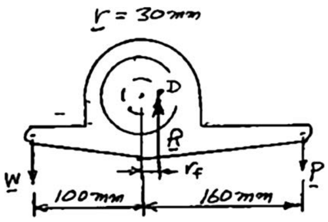


PROBLEM 8.77

A lever of negligible weight is loosely fitted onto a 30-mm-radius fixed shaft as shown. Knowing that a force **P** of magnitude 275 N will just start the lever rotating clockwise, determine (*a*) the coefficient of static friction between the shaft and the lever, (*b*) the smallest force **P** for which the lever does not start rotating counterclockwise.

SOLUTION

(*a*) Impending motion $\circlearrowright$



$$W = (40\text{ kg})(9.81\text{ m/s}^2) = 392.4\text{ N}$$

$$+\circlearrowright \Sigma M_D = 0: \quad P(160 - r_f) - W(100 + r_f) = 0$$

$$r_f = \frac{160P - 100W}{P + W}$$

$$r_f = \frac{(160\text{ mm})(275\text{ N}) - (100\text{ mm})(392.4\text{ N})}{275\text{ N} + 392.4\text{ N}}$$

$$r_f = 7.132\text{ mm}$$

$$r_f = r\sin\phi_s = r\mu_s$$

$$\mu_s = \frac{r_f}{r} = \frac{7.132\text{ mm}}{30\text{ mm}} = 0.2377$$

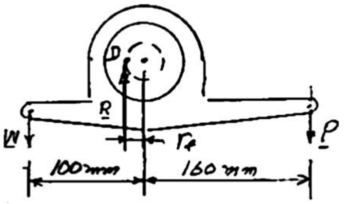
$\mu_s = 0.238$ ◀

(*b*) Impending motion $\circlearrowleft$

$$r_f = r\sin\phi_s = r\mu_s$$

$$= (30\text{ mm})(0.2377)$$

$$r_f = 7.132\text{ mm}$$



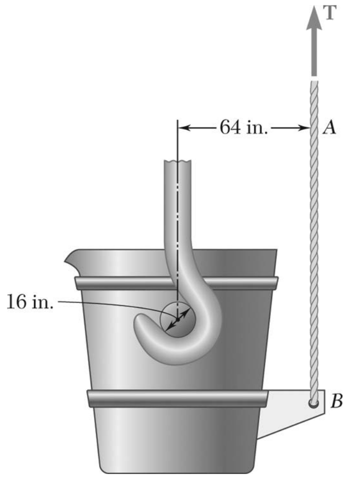
$$+\circlearrowright \Sigma M_D = 0: \quad P(160 + r_f) - W(100 - r_f) = 0$$

$$P = W\frac{100 - r_f}{160 + r_f}$$

$$P = (392.4\text{ N})\frac{100\text{ mm} - 7.132\text{ mm}}{160\text{ mm} + 7.132\text{ mm}}$$

$$P = 218.04\text{ N}$$

P = 218 N ↓ ◀

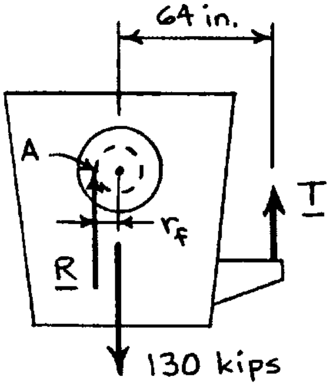


PROBLEM 8.78

A hot-metal ladle and its contents weigh 130 kips. Knowing that the coefficient of static friction between the hooks and the pinion is 0.30, determine the tension in cable *AB* required to start tipping the ladle.

SOLUTION

Free body: Ladle



$$\sin\phi_s \approx \tan\phi_s = \mu_s = 0.30$$

$$r_{\text{bearing}} = 8 \text{ in.}$$

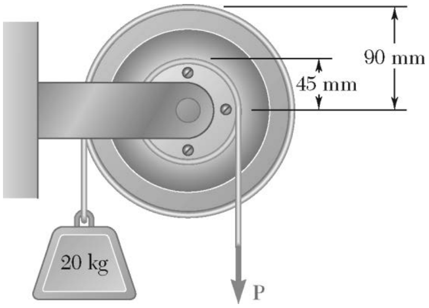
$$r_f = r_{\text{bearing}} \sin\phi_s = (8 \text{ in.})\,(0.30) = 2.4 \text{ in.}$$

R is tangent to friction circle at *A*.

$$+\circlearrowleft \Sigma M_A = 0: \quad T(64 \text{ in.} + r_f) - (130 \text{ kips})r_f = 0$$

$$T(64 + 2.4) - (130)(2.4) = 0$$

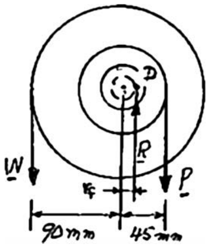
$$T = 4.70 \text{ kips} \quad \blacktriangleleft$$



PROBLEM 8.79

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION



$$+\circlearrowleft \Sigma M_D = 0: \quad P(45 - r_f) - W(90 + r_f) = 0$$

$$P = W\frac{90 + r_f}{45 - r_f}$$

$$= (196.2\text{ N})\frac{90\text{ mm} + 4\text{ mm}}{45\text{ mm} - 4\text{ mm}}$$

$$P = 449.82\text{ N}$$

$P = 450$ N ◀

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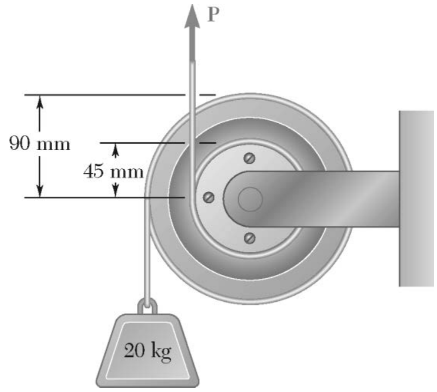
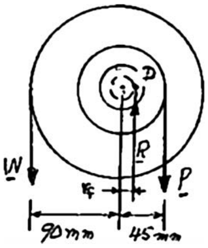


PROBLEM 8.80

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION

<u>Find **P** required to start raising load</u>



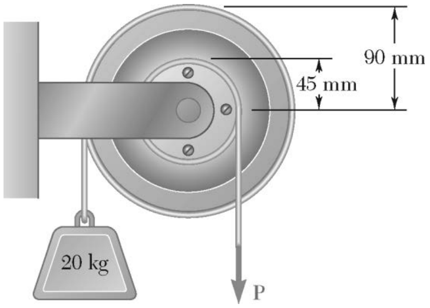
$$+\circlearrowleft \Sigma M_D = 0: \quad P(45 - r_f) - W(90 - r_f) = 0$$

$$P = W\frac{90 - r_f}{45 - r_f}$$

$$= (196.2\text{ N})\frac{90\text{ mm} - 4\text{ mm}}{45\text{ mm} - 4\text{ mm}}$$

$$P = 411.54\text{ N}$$

$P = 412$ N ◀

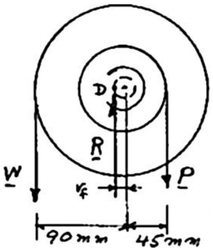


PROBLEM 8.81

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the smallest force **P** required to maintain equilibrium.

SOLUTION

Find smallest **P** to maintain equilibrium



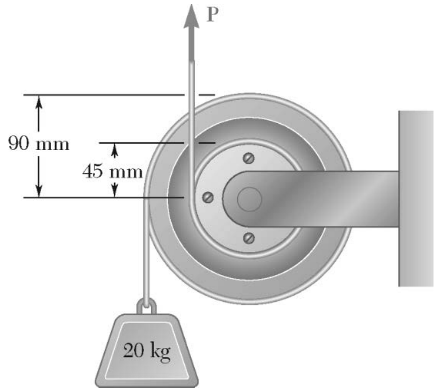
$$+\circlearrowright \Sigma M_D = 0: \quad P(45 + r_f) - W(90 - r_f) = 0$$

$$P = W\frac{90 - r_f}{45 + r_f}$$

$$= (196.2\text{ N})\frac{90\text{ mm} - 4\text{ mm}}{45\text{ mm} + 4\text{ mm}}$$

$$P = 344.35\text{ N}$$

$P = 344$ N ◀

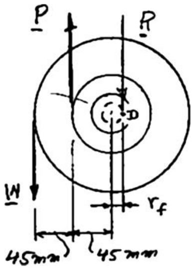


PROBLEM 8.82

The double pulley shown is attached to a 10-mm-radius shaft that fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the smallest force **P** required to maintain equilibrium.

SOLUTION

Find smallest **P** to maintain equilibrium



$$+\circlearrowleft \Sigma M_D = 0: \quad P(45 + r_f) - W(90 + r_f) = 0$$

$$P = W\frac{90 + r_f}{45 + r_f}$$

$$= (196.2\text{ N})\frac{90\text{ mm} + 4\text{ mm}}{45\text{ mm} + 4\text{ mm}}$$

$$P = 376.38\text{ N} \qquad\qquad P = 376\text{ N} \blacktriangleleft$$

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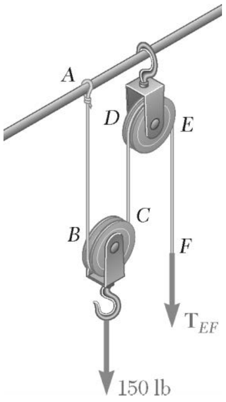


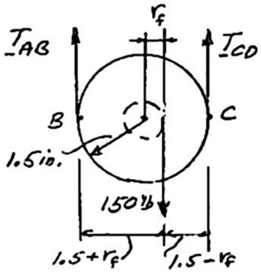
PROBLEM 8.83

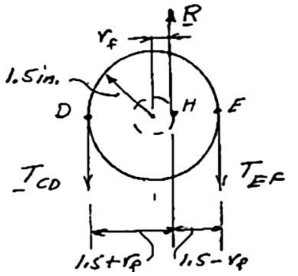
The block and tackle shown are used to raise a 150-lb load. Each of the 3-in.-diameter pulleys rotates on a 0.5-in.-diameter axle. Knowing that the coefficient of static friction is 0.20, determine the tension in each portion of the rope as the load is slowly raised.

SOLUTION

For each pulley: Axle diameter $= 0.5$ in.

$$r_f = r\sin\phi_s \approx \mu_s r = 0.20\left(\frac{0.5\text{ in.}}{2}\right) = 0.05\text{ in.}$$





Pulley *BC*:

$$+\circlearrowleft \Sigma M_B = 0:\quad T_{CD}(3\text{ in.}) - (150\text{ lb})(1.5\text{ in.} + r_f) = 0$$

$$T_{CD} = \frac{1}{3}(150\text{ lb})(1.5\text{ in.} + 0.05\text{ in.}) \qquad T_{CD} = 77.5\text{ lb} \blacktriangleleft$$

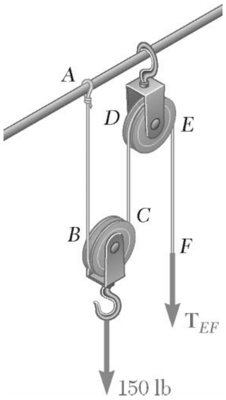
$$+\uparrow \Sigma F_y = 0:\quad T_{AB} + 77.5\text{ lb} - 150\text{ lb} = 0 \qquad T_{AB} = 72.5\text{ lb} \blacktriangleleft$$

Pulley *DE*:

$$+\circlearrowleft \Sigma M_B = 0:\quad T_{CD}(1.5 + r_f) - T_{EF}(1.5 - r_f) = 0$$

$$T_{EF} = T_{CD}\frac{1.5 + r_f}{1.5 - r_f}$$

$$= (77.5\text{ lb})\frac{1.5\text{ in.} + 0.05\text{ in.}}{1.5\text{ in.} - 0.05\text{ in.}} \qquad T_{EF} = 82.8\text{ lb} \blacktriangleleft$$



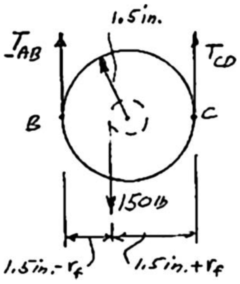
PROBLEM 8.84

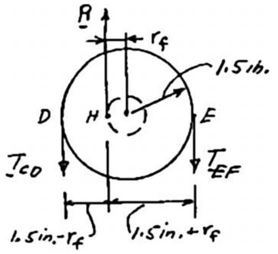
The block and tackle shown are used to lower a 150-lb load. Each of the 3-in.-diameter pulleys rotates on a 0.5-in.-diameter axle. Knowing that the coefficient of static friction is 0.20, determine the tension in each portion of the rope as the load is slowly lowered.

SOLUTION

For each pulley:

$$r_f = r\mu_s = \left(\frac{0.5\text{ in.}}{2}\right)0.2 = 0.05\text{ in.}$$





Pulley BC:

$$+\circlearrowleft \Sigma M_B = 0: \quad T_{CD}(3\text{ in.}) - (150\text{ lb})(1.5\text{ in.} - r_f) = 0$$

$$T_{CD} = \frac{(150\text{ lb})(1.5\text{ in.} - 0.05\text{ in.})}{3\text{ in.}}$$

$T_{CD} = 72.5\text{ lb}$ ◀

$$+\uparrow \Sigma F_y = 0: \quad T_{AB} + 72.5\text{ lb} - 150\text{ lb} = 0$$

$T_{AB} = 77.5\text{ lb}$ ◀

Pulley DE:

$$T_{CD}(1.5\text{ in.} - r_f) - T_{EF}(1.5\text{ in.} + r_f) = 0$$

$$T_{EF} = T_{CD}\frac{1.5\text{ in.} - r_f}{1.5\text{ in.} + r_f}$$

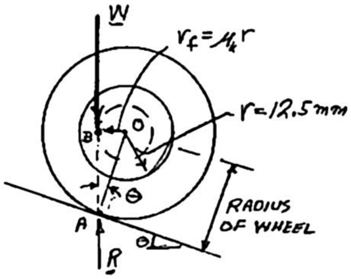
$$= (72.5\text{ lb})\frac{1.5\text{ in.} - 0.05\text{ in.}}{1.5\text{ in.} + 0.05\text{ in.}}$$

$T_{EF} = 67.8\text{ lb}$ ◀

PROBLEM 8.85

A scooter is to be designed to roll down a 2 percent slope at a constant speed. Assuming that the coefficient of kinetic friction between the 25-mm-diameter axles and the bearings is 0.10, determine the required diameter of the wheels. Neglect the rolling resistance between the wheels and the ground.

SOLUTION



$$\tan\theta = \frac{2}{100} = 0.02$$

Since a scooter rolls at constant speed, each wheel is in equilibrium. Thus, **W** and **R** must have a common line of action tangent to the friction circle.

$$r_f = \mu_k r = (0.10)(12.5 \text{ mm})$$

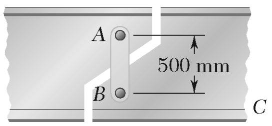
$$= 1.25 \text{ mm}$$

$$OA = \frac{OB}{\tan\theta} = \frac{r_f}{\tan\theta} = \frac{1.25 \text{ mm}}{0.02}$$

$$= 62.5 \text{ mm}$$

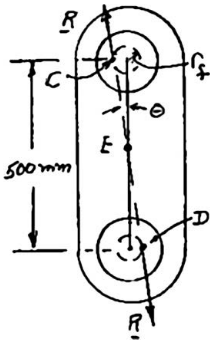
Diameter of wheel $= 2(OA) = 125.0$ mm ◀

PROBLEM 8.86



The link arrangement shown is frequently used in highway bridge construction to allow for expansion due to changes in temperature. At each of the 60-mm-diameter pins A and B the coefficient of static friction is 0.20. Knowing that the vertical component of the force exerted by BC on the link is 200 kN, determine (a) the horizontal force that should be exerted on beam BC to just move the link, (b) the angle that the resulting force exerted by beam BC on the link will form with the vertical.

SOLUTION



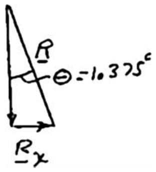
Bearing:

$$r = 30 \text{ mm}$$

$$\begin{aligned} r_f &= \mu_s r \\ &= 0.20(30 \text{ mm}) \\ &= 6 \text{ mm} \end{aligned}$$

Resultant forces **R** must be tangent to friction circles at Points C and D.

(a)

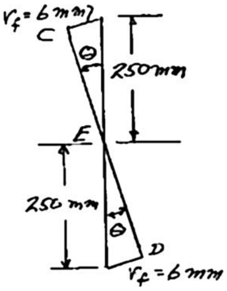


$$R_y = \text{Vertical component} = 200 \text{ kN}$$

$$\begin{aligned} R_x &= R_y \tan\theta \\ &= (200 \text{ kN}) \tan 1.375° \\ &= 4.80 \text{ kN} \end{aligned}$$

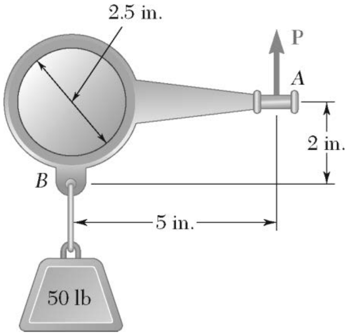
Horizontal force = 4.80 kN ◀

(b)



$$\sin\theta = \frac{6 \text{ mm}}{250 \text{ mm}}$$

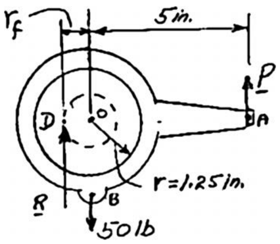
$$\sin\theta = 0.024 \qquad \theta = 1.375° \blacktriangleleft$$



PROBLEM 8.87

A lever AB of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating counterclockwise.

SOLUTION

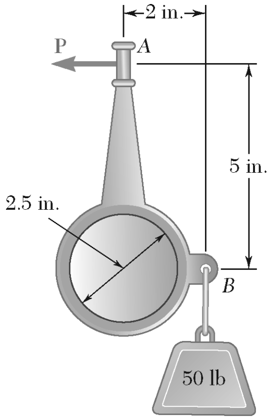


$$\begin{aligned} r_f &= \mu_s r \\ &= 0.15(1.25 \text{ in.}) \\ &= 0.1875 \text{ in.} \end{aligned}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad P(5 \text{ in.} + r_f) - (50 \text{ lb})r_f = 0$$

$$P = \frac{50(0.1875)}{5.1875}$$
$$= 1.807 \text{ lb}$$

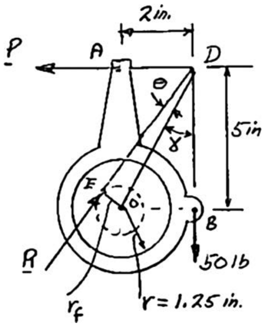
$\mathbf{P} = 1.807 \text{ lb}\uparrow$ ◀



PROBLEM 8.88

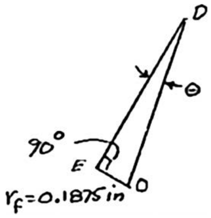
A lever *AB* of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating counterclockwise.

SOLUTION



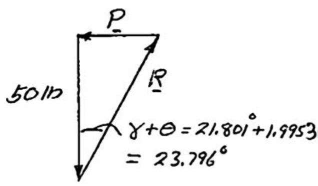
$$r_f = \mu_s r$$
$$= 0.15(1.25 \text{ in.})$$
$$r_f = 0.1875 \text{ in.}$$
$$\tan\gamma = \frac{2 \text{ in.}}{5 \text{ in.}}$$
$$\gamma = 21.801°$$

In ΔEOD:



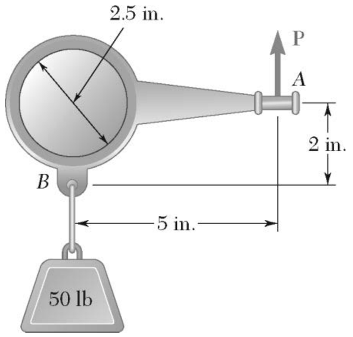
$$OD = \sqrt{(2 \text{ in.})^2 + (5 \text{ in.})^2}$$
$$= 5.3852 \text{ in.}$$
$$\sin\theta = \frac{OE}{OD} = \frac{r_f}{OD}$$
$$= \frac{0.1875 \text{ in.}}{5.3852 \text{ in.}}$$
$$\theta = 1.99531°$$

Force triangle:



$$P = (50 \text{ lb})\tan(\gamma + \theta)$$
$$= (50 \text{ lb})\tan 23.796°$$
$$= 22.049 \text{ lb}$$

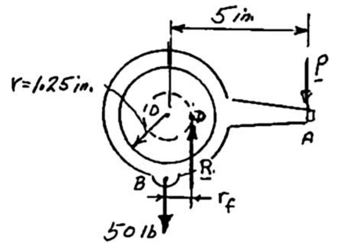
P = 22.0 lb ⟵ ◀



PROBLEM 8.89

A lever *AB* of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating clockwise.

SOLUTION



$$r_f = \mu_s r$$

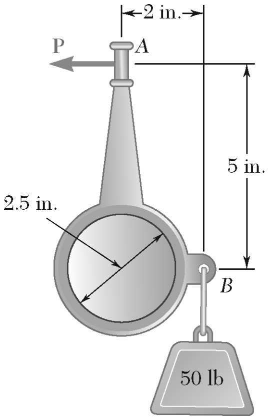
$$= 0.15(1.25\text{ in.})$$

$$r_f = 0.1875\text{ in.}$$

$$+\circlearrowright \Sigma M_D = 0: \quad P(5\text{ in.} - r_f) - (50\text{ lb})r_f = 0$$

$$P = \frac{50(0.1875)}{5 - 0.1875}$$

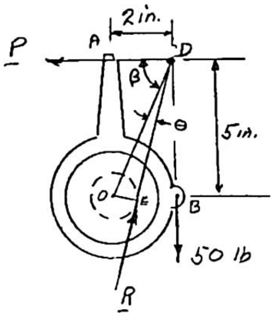
$$= 1.948\text{ lb} \qquad \mathbf{P} = 1.948\text{ lb}\downarrow \blacktriangleleft$$



PROBLEM 8.90

A lever *AB* of negligible weight is loosely fitted onto a 2.5-in.-diameter fixed shaft. Knowing that the coefficient of static friction between the fixed shaft and the lever is 0.15, determine the force **P** required to start the lever rotating clockwise.

SOLUTION

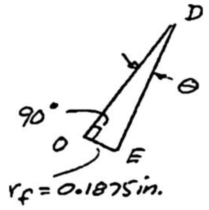


$$\begin{aligned} r_f &= \mu_s r \\ &= 0.15(1.25\text{ in.}) \\ &= 0.1875\text{ in.} \end{aligned}$$

$$\tan\beta = \frac{5\text{ in.}}{2\text{ in.}}$$

$$\beta = 68.198°$$

In ΔEOD:



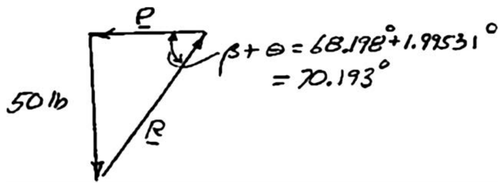
$$OD = \sqrt{(2\text{ in.})^2 + (5\text{ in.})^2}$$

$$OD = 5.3852\text{ in.}$$

$$\sin\theta = \frac{OE}{OD} = \frac{0.1875\text{ in.}}{5.3852\text{ in.}}$$

$$\theta = 1.99531°$$

Force triangle:



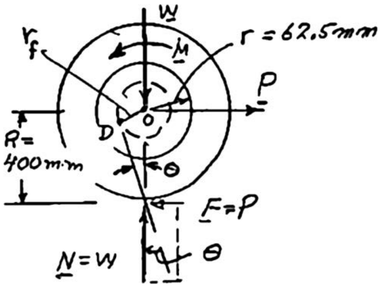
$$P = \frac{50}{\tan(\beta+\theta)} = \frac{50\text{ lb}}{\tan 70.193°}$$

P = 18.01 lb ← ◀

PROBLEM 8.91

A loaded railroad car has a mass of 30 Mg and is supported by eight 800-mm-diameter wheels with 125-mm-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (*a*) to start the car moving, (*b*) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the track.

SOLUTION



$$r_f = \mu r; \quad R = 400 \text{ mm}$$

$$\sin\theta = \tan\theta = \frac{r_f}{R} = \frac{\mu r}{R}$$

$$P = W\tan\theta = W\frac{\mu r}{R}$$

$$P = W\mu\frac{62.5 \text{ mm}}{400 \text{ mm}}$$

$$= 0.15625 W\mu$$

<u>For one wheel</u>: $$W = \frac{1}{8}(30 \text{ mg})(9.81 \text{ m/s}^2)$$

$$= \frac{1}{8}(294.3 \text{ kN})$$

<u>For eight wheels of railroad car</u>: $$\Sigma P = 8(0.15625)\frac{1}{8}(294.3 \text{ kN})\mu$$

$$= (45.984\mu) \text{ kN}$$

(*a*) To start motion: $\mu_s = 0.020$

$$\Sigma P = (45.984)(0.020)$$

$$= 0.9197 \text{ kN}$$

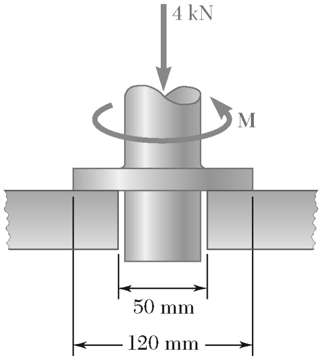
$\Sigma P = 920 \text{ N}$ ◀

(*b*) To maintain motion: $\mu_k = 0.015$

$$\Sigma P = (45.984)(0.015)$$

$$= 0.6897 \text{ kN}$$

$\Sigma P = 690 \text{ N}$ ◀



PROBLEM 8.92

Knowing that a couple of magnitude 30 N · m is required to start the vertical shaft rotating, determine the coefficient of static friction between the annular surfaces of contact.

SOLUTION

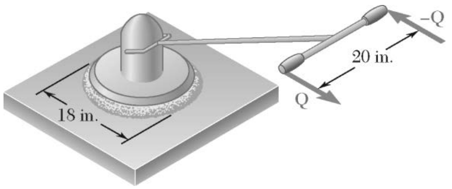
For annular contact regions, use Equation 8.8 with impending slipping:

$$M = \frac{2}{3}\mu_s N \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

So,

$$30\text{ N}\cdot\text{m} = \frac{2}{3}\mu_s(4000\text{ N})\frac{(0.06\text{ m})^3 - (0.025\text{ m})^3}{(0.06\text{ m})^2 - (0.025\text{ m})^2}$$

$\mu_s = 0.1670$ ◀



PROBLEM 8.93

A 50-lb electric floor polisher is operated on a surface for which the coefficient of kinetic friction is 0.25. Assuming that the normal force per unit area between the disk and the floor is uniformly distributed, determine the magnitude Q of the horizontal forces required to prevent motion of the machine.

SOLUTION

See Figure 8.12 and Eq. (8.9).

Using:

$$R = 9\text{ in.}$$

$$P = 50\text{ lb}$$

and

$$\mu_k = 0.25$$

$$M = \frac{2}{3}\mu_k PR = \frac{2}{3}(0.25)(50\text{ lb})(9\text{ in.})$$

$$= 75\text{ lb}\cdot\text{in.}$$

$\Sigma M_y = 0$ yields:

$$M = Q(20\text{ in.})$$

$$75\text{ lb}\cdot\text{in.} = Q(20\text{ in.})$$

$Q = 3.75$ lb ◀

PROBLEM 8.94*

The frictional resistance of a thrust bearing decreases as the shaft and bearing surfaces wear out. It is generally assumed that the wear is directly proportional to the distance traveled by any given point of the shaft and thus to the distance *r* from the point to the axis of the shaft. Assuming, then, that the normal force per unit area is inversely proportional to *r*, show that the magnitude *M* of the couple required to overcome the frictional resistance of a worn-out end bearing (with contact over the full circular area) is equal to 75 percent of the value given by Eq. (8.9) for a new bearing.

SOLUTION

Using Figure 8.12, we assume

$$\Delta N = \frac{k}{r}\Delta A: \quad \Delta A = r\Delta\theta\,\Delta r$$

$$\Delta N = \frac{k}{r} r\Delta\theta\Delta r = k\Delta\theta\Delta r$$

We write $P = \Sigma\Delta N \quad \text{or} \quad P = \int dN$

$$P = \int_0^{2\pi}\int_0^R k\Delta\theta\Delta r = 2\pi Rk; \quad k = \frac{P}{2\pi R}$$

$$\Delta N = \frac{P\Delta\theta\Delta r}{2\pi R}$$

$$\Delta M = r\Delta F = r\mu_k\Delta N = r\mu_k\frac{P\Delta\theta\Delta r}{2\pi R}$$

$$M = \int_0^{2\pi}\int_0^R \frac{\mu_k P}{2\pi R} r dr d\theta = \frac{2\pi\mu_k P}{2\pi R}\cdot\frac{R^2}{2} = \frac{1}{2}\mu_k PR$$

From Eq. (8.9) for a new bearing,

$$M_{\text{new}} = \frac{2}{3}\mu_k PR$$

Thus, $$\frac{M}{M_{\text{new}}} = \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{3}{4}$$

$M = 0.75M_{\text{new}}$ ◀

PROBLEM 8.95*

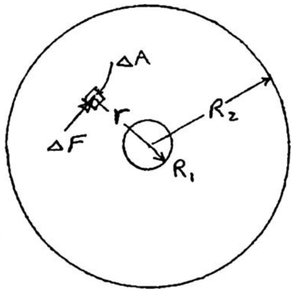
Assuming that bearings wear out as indicated in Problem 8.94, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out collar bearing is

$$M = \tfrac{1}{2}\mu_k P(R_1 + R_2)$$

where P = magnitude of the total axial force

R_1, R_2 = inner and outer radii of collar

SOLUTION



Let normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = \dfrac{k}{r}.$

As in the text, $\Delta F = \mu \Delta N \quad \Delta M = r\Delta F$

The total normal force P is

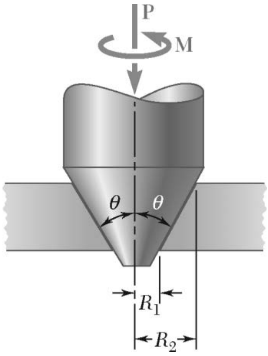
$$P = \lim_{\Delta A \to 0} \Sigma \Delta N = \int_0^{2\pi} \left(\int_{R_1}^{R_2} \frac{k}{r} r dr \right) d\theta$$

$$P = 2\pi \int_{R_1}^{R_2} k dr = 2\pi k(R_2 - R_1) \quad \text{or} \quad k = \frac{P}{2\pi(R_2 - R_1)}$$

Total couple:
$$M_{\text{worn}} = \lim_{\Delta A \to 0} \Sigma \Delta M = \int_0^{2\pi} \left(\int_{R_1}^{R_2} r\mu \frac{k}{r} r dr \right) d\theta$$

$$M_{\text{worn}} = 2\pi\mu k \int_{R_1}^{R_2} (r dr) = \pi\mu k \left(R_2^2 - R_1^2 \right) = \frac{\pi\mu P \left(R_2^2 - R_1^2 \right)}{2\pi(R_2 - R_1)}$$

$$M_{\text{worn}} = \frac{1}{2}\mu P(R_2 + R_1) \blacktriangleleft$$



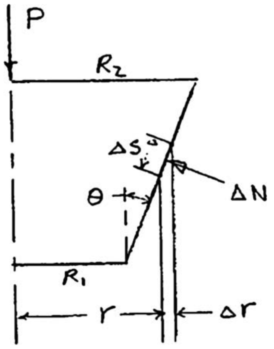
PROBLEM 8.96*

Assuming that the pressure between the surfaces of contact is uniform, show that the magnitude M of the couple required to overcome frictional resistance for the conical bearing shown is

$$M = \frac{2}{3}\frac{\mu_k P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

SOLUTION

Let normal force on ΔA be ΔN and $\dfrac{\Delta N}{\Delta A} = k.$



So $\qquad \Delta N = k\Delta A \qquad \Delta A = r\Delta s\Delta\phi \qquad \Delta s = \dfrac{\Delta r}{\sin\theta}$

where ϕ is the azimuthal angle around the symmetry axis of rotation.

$$\Delta F_y = \Delta N \sin\theta = kr\Delta r\Delta\phi$$

Total vertical force: $\qquad P = \lim_{\Delta A \to 0} \Sigma\Delta F_y$

$$P = \int_0^{2\pi}\left(\int_{R_1}^{R_2} krdr\right)d\phi = 2\pi k\int_{R_1}^{R_2} rdr$$

$$P = \pi k\left(R_2^2 - R_1^2\right) \quad \text{or} \quad k = \frac{P}{\pi\left(R_2^2 - R_1^2\right)}$$

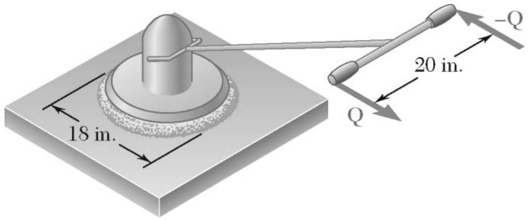
Friction force: $\qquad \Delta F = \mu\Delta N = \mu k\Delta A$

Moment: $\qquad \Delta M = r\Delta F = r\mu kr\dfrac{\Delta r}{\sin\theta}\Delta\phi$

Total couple: $\qquad M = \lim_{\Delta A \to 0}\Sigma\Delta M = \displaystyle\int_0^{2\pi}\left(\int_{R_1}^{R_2}\frac{\mu k}{\sin\theta}r^2dr\right)d\phi$

$$M = 2\pi\frac{\mu k}{\sin\theta}\int_{R_1}^{R_2} r^2dr = \frac{2}{3}\frac{\pi\mu}{\sin\theta}\frac{P}{\pi\left(R_2^2 - R_3^2\right)}\left(R_2^3 - R_3^3\right)$$

$$M = \frac{2}{3}\frac{\mu P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \blacktriangleleft$$



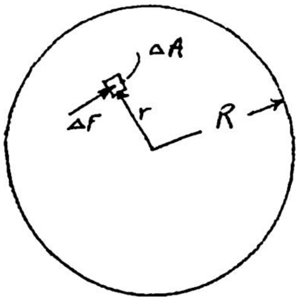
PROBLEM 8.97

Solve Problem 8.93 assuming that the normal force per unit area between the disk and the floor varies linearly from a maximum at the center to zero at the circumference of the disk.

PROBLEM 8.93 A 50-lb electric floor polisher is operated on a surface for which the coefficient of kinetic friction is 0.25. Assuming that the normal force per unit area between the disk and the floor is uniformly distributed, determine the magnitude Q of the horizontal forces required to prevent motion of the machine.

SOLUTION

Let normal force on ΔA be ΔN and $\dfrac{\Delta N}{\Delta A} = k\left(1-\dfrac{r}{R}\right)$.



$$\Delta F = \mu \Delta N = \mu k\left(1-\frac{r}{R}\right)\Delta A = \mu k\left(1-\frac{r}{R}\right) r\Delta r\Delta\theta$$

$$P = \lim_{\Delta A\to 0} \Sigma\Delta N = \int_0^{2\pi}\left[\int_0^R k\left(1-\frac{r}{R}\right) rdr\right] d\theta$$

$$P = 2\pi k\int_0^R \left(1-\frac{r}{R}\right) rdr = 2\pi k\left(\frac{R^2}{2}-\frac{R^3}{3R}\right)$$

$$P = \frac{1}{3}\pi k R^2 \quad \text{or} \quad k = \frac{3P}{\pi R^2}$$

$$M = \lim_{\Delta A\to 0} \Sigma r\Delta F = \int_0^{2\pi}\left[\int_0^R r\mu k\left(1-\frac{r}{R}\right) rdr\right] d\theta$$

$$= 2\pi\mu k\int_0^R \left(r^2-\frac{r^3}{R}\right) dr$$

$$= 2\pi\mu k\left(\frac{R^3}{3}-\frac{R^4}{4R}\right) = \frac{1}{6}\pi\mu k R^3$$

$$= \frac{\pi\mu}{6}\frac{3P}{\pi R^2}R^3 = \frac{1}{2}\mu PR$$

where $\mu = \mu_k = 0.25 \quad R = 9\text{ in.}$

PROBLEM 8.97 (Continued)

$$P = W = 50 \text{ lb}$$

Then
$$M = \frac{1}{2}(0.25)(50 \text{ lb})(9 \text{ in.})$$
$$= 56.250 \text{ lb} \cdot \text{in.}$$

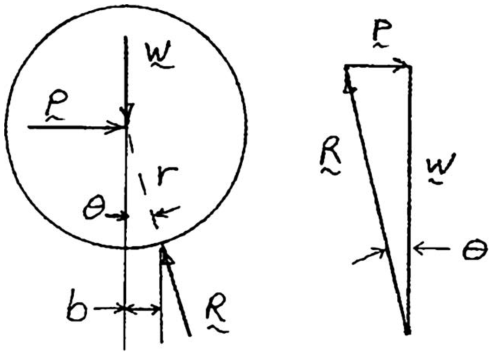
Finally
$$Q = \frac{M}{d} = \frac{56.250 \text{ lb} \cdot \text{in.}}{20 \text{ in.}}$$
$Q = 2.81$ lb ◀

PROBLEM 8.98

Determine the horizontal force required to move a 2500-lb automobile with 23-in.-diameter tires along a horizontal road at a constant speed. Neglect all forms of friction except rolling resistance, and assume the coefficient of rolling resistance to be 0.05 in.

SOLUTION

FBD wheel:



$$r = 11.5 \text{ in.}$$

$$b = 0.05 \text{ in.}$$

$$\theta = \sin^{-1}\frac{b}{r}$$

$P = W\tan\theta = W\tan\left(\sin^{-1}\frac{b}{r}\right)$ for each wheel, so for total

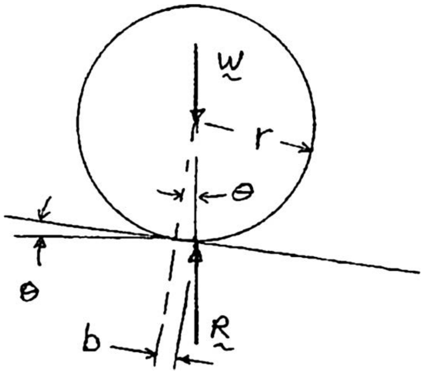
$$P = 2500 \text{ lb}\tan\left(\sin^{-1}\frac{0.05}{11.5}\right) \qquad P = 10.87 \text{ lb} \blacktriangleleft$$

PROBLEM 8.99

Knowing that a 6-in.-diameter disk rolls at a constant velocity down a 2 percent incline, determine the coefficient of rolling resistance between the disk and the incline.

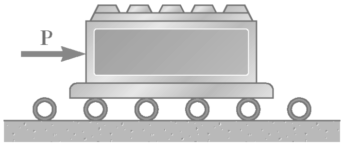
SOLUTION

FBD disk:



$$\tan\theta = \text{slope} = 0.02$$

$$b = r\tan\theta = (3 \text{ in.})(0.02) \qquad b = 0.0600 \text{ in.} \blacktriangleleft$$

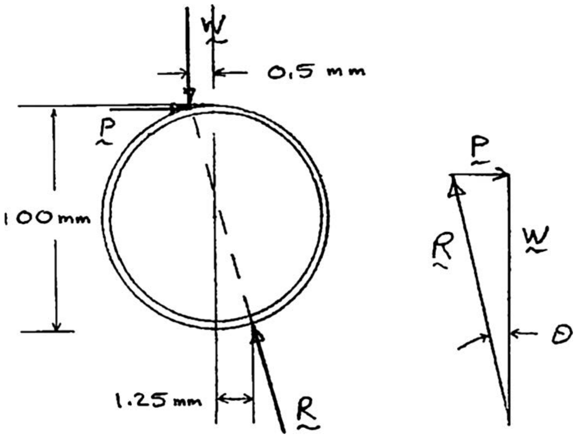


PROBLEM 8.100

A 900-kg machine base is rolled along a concrete floor using a series of steel pipes with outside diameters of 100 mm. Knowing that the coefficient of rolling resistance is 0.5 mm between the pipes and the base and 1.25 mm between the pipes and the concrete floor, determine the magnitude of the force **P** required to slowly move the base along the floor.

SOLUTION

FBD pipe:



$$W = mg$$

$$= (900 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 8829.0 \text{ N}$$

$$\theta = \sin^{-1}\frac{.5 \text{ mm} + 1.25 \text{ mm}}{100 \text{ mm}}$$

$$= 1.00273°$$

$P = W\tan\theta$ for each pipe, so also for total

$$P = (8829.0 \text{ N})\tan(1.00273°)$$

$P = 154.4$ N ◀

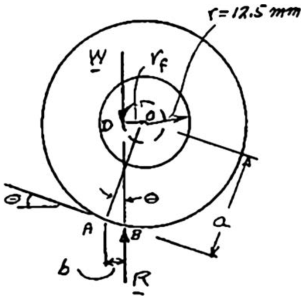
PROBLEM 8.101

Solve Problem 8.85 including the effect of a coefficient of rolling resistance of 1.75 mm.

PROBLEM 8.85 A scooter is to be designed to roll down a 2 percent slope at a constant speed. Assuming that the coefficient of kinetic friction between the 25-mm-diameter axles and the bearings is 0.10, determine the required diameter of the wheels. Neglect the rolling resistance between the wheels and the ground.

SOLUTION

Since the scooter rolls at a constant speed, each wheel is in equilibrium. Thus, **W** and **R** must have a common line of action tangent to the friction circle.



$$a = \text{Radius of wheel}$$

$$\tan\theta = \frac{2}{100} = 0.02$$

Since b and r_f are small compared to a,

$$\tan\theta \approx \frac{r_f + b}{a} = \frac{\mu_k r + b}{a} = 0.02$$

Data: $\mu_k = 0.10, \quad b = 1.75 \text{ mm}, \quad r = 12.5 \text{ mm}$

$$\frac{(0.10)(12.5 \text{ mm}) + 1.75 \text{ mm}}{a} = 0.02$$

$a = 150$ mm Diameter $= 2a = 300$ mm ◀

PROPRIETARY MATERIAL.

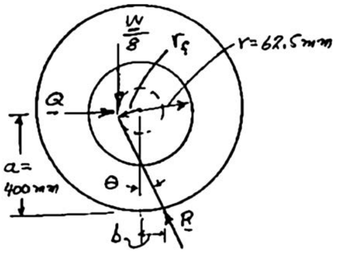
PROBLEM 8.102

Solve Problem 8.91 including the effect of a coefficient of rolling resistance of 0.5 mm.

PROBLEM 8.91 A loaded railroad car has a mass of 30 Mg and is supported by eight 800-mm-diameter wheels with 125-mm-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (*a*) to start the car moving, (*b*) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the track.

SOLUTION

For one wheel:



$$r_f = \mu r$$

$$\tan\theta \approx \sin\theta \approx \frac{r_f + b}{a}$$

$$\tan\theta = \frac{\mu r + b}{a}$$

$$Q = \frac{W}{8}\tan\theta = \frac{W}{8}\frac{\mu r + b}{a}$$

For eight wheels of car:

$$P = W\frac{\mu r + b}{a}$$

$$W = mg = (30 \text{ Mg})(9.81 \text{ m/s}^2) = 294.3 \text{ kN}$$

$$a = 400 \text{ mm}, \quad r = 62.5 \text{ mm}, \quad b = 0.5 \text{ mm}$$

(*a*) To start motion:

$$\mu = \mu_s = 0.02$$

$$P = (294.3 \text{ kN})\frac{(0.020)(62.5 \text{ mm}) + 0.5 \text{ mm}}{400 \text{ mm}}$$

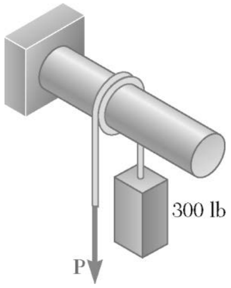
$$P = 1.288 \text{ kN} \blacktriangleleft$$

(*b*) To maintain constant speed

$$\mu = \mu_k = 0.015$$

$$P = (294.3 \text{ kN})\frac{(0.015)(62.5 \text{ mm}) + 0.5 \text{ mm}}{400 \text{ mm}}$$

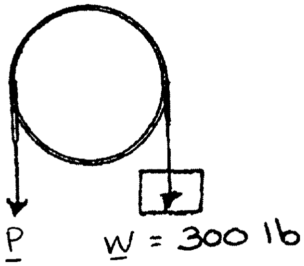
$$P = 1.058 \text{ kN} \blacktriangleleft$$



PROBLEM 8.103

A 300-lb block is supported by a rope that is wrapped $1\frac{1}{2}$ times around a horizontal rod. Knowing that the coefficient of static friction between the rope and the rod is 0.15, determine the range of values of P for which equilibrium is maintained.

SOLUTION



$$\beta = 1.5 \text{ turns} = 3\pi \text{ rad}$$

For impending motion of W up,

$$P = We^{\mu_s\beta} = (300 \text{ lb})e^{(0.15)3\pi}$$
$$= 1233.36 \text{ lb}$$

For impending motion of W down,

$$P = We^{-\mu_s\beta} = (300 \text{ lb})e^{-(0.15)3\pi}$$
$$= 72.971 \text{ lb}$$

For equilibrium, $73.0 \text{ lb} \le P \le 1233 \text{ lb}$ ◀

PROBLEM 8.104

A hawser is wrapped two full turns around a bollard. By exerting an 80-lb force on the free end of the hawser, a dockworker can resist a force of 5000 lb on the other end of the hawser. Determine (*a*) the coefficient of static friction between the hawser and the bollard, (*b*) the number of times the hawser should be wrapped around the bollard if a 20,000-lb force is to be resisted by the same 80-lb force.

SOLUTION

(*a*)

$$\beta = 2 \text{ turns} = 2(2\pi) = 4\pi$$

$$T_1 = 80 \text{ lb}, \qquad T_2 = 5000 \text{ lb}$$

$$\ln\frac{T_2}{T_1} = \mu_s\beta$$

$$\mu_s = \frac{1}{\beta}\ln\frac{T_2}{T_1} = \frac{1}{4\pi}\ln\frac{5000 \text{ lb}}{80 \text{ lb}}$$

$$\mu_s = \frac{1}{4\pi}\ln 62.5 = \frac{4.1351}{4\pi}$$

$\mu_s = 0.329$ ◀

(*b*)

$$T_1 = 80 \text{ lb}, \quad T_2 = 20{,}000 \text{ lb}, \quad \mu_s = 0.329$$

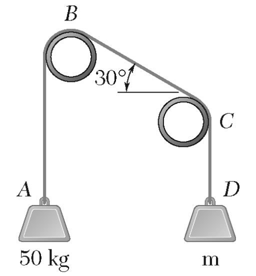
$$\ln\frac{T_2}{T_1} = \mu_s\beta$$

$$\beta = \frac{1}{\mu}\ln\frac{T_2}{T_1} = \frac{1}{0.329}\ln\frac{20{,}000 \text{ lb}}{80 \text{ lb}}$$

$$\beta = \frac{1}{0.329}\ln(250) = \frac{5.5215}{0.329} = 16.783$$

$$\text{Number of turns} = \frac{16.783}{2\pi}$$

Number of turns = 2.67 ◀



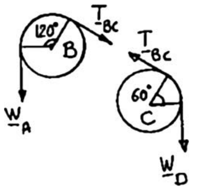
PROBLEM 8.105

A rope *ABCD* is looped over two pipes as shown. Knowing that the coefficient of static friction is 0.25, determine (*a*) the smallest value of the mass *m* for which equilibrium is possible, (*b*) the corresponding tension in portion *BC* of the rope.

SOLUTION

We apply Eq. (8.14) to pipe *B* and pipe *C*.

$$\frac{T_2}{T_1} = e^{\mu_s \beta} \qquad (8.14)$$



Pipe *B*:

$$T_2 = W_A, \quad T_1 = T_{BC}$$

$$\mu_s = 0.25, \quad \beta = \frac{2\pi}{3}$$

$$\frac{W_A}{T_{BC}} = e^{0.25(2\pi/3)} = e^{\pi/6} \qquad (1)$$

Pipe *C*:

$$T_2 = T_{BC}, \quad T_1 = W_D, \quad \mu_s = 0.25, \quad \beta = \frac{\pi}{3}$$

$$\frac{T_{BC}}{W_D} = e^{0.025(\pi/3)} = e^{\pi/12} \qquad (2)$$

(*a*) Multiplying Eq. (1) by Eq. (2):

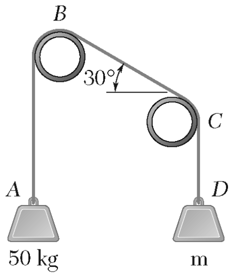
$$\frac{W_A}{W_D} = e^{\pi/6} \cdot e^{\pi/12} = e^{\pi/6+\pi/12} = e^{\pi/4} = 2.193$$

$$W_D = \frac{W_A}{2.193} \quad m = \frac{W_D}{g} = \frac{\frac{W_A}{g}}{2.193} = \frac{m_A}{2.193} = \frac{50 \text{ kg}}{2.193}$$

$m = 22.8$ kg ◀

(*b*) From Eq. (1):

$$T_{BC} = \frac{W_A}{e^{\pi/6}} = \frac{(50 \text{ kg})(9.81 \text{ m/s}^2)}{1.688} = 291 \text{ N} \blacktriangleleft$$



PROBLEM 8.106

A rope *ABCD* is looped over two pipes as shown. Knowing that the coefficient of static friction is 0.25, determine (*a*) the largest value of the mass *m* for which equilibrium is possible, (*b*) the corresponding tension in portion *BC* of the rope.

SOLUTION

See *FB* diagrams of Problem 8.105. We apply Eq. (8.14) to pipes *B* and *C*.

Pipe *B*: $T_1 = W_A, \quad T_2 = T_{BC}, \quad \mu_s = 0.25, \quad \beta = \dfrac{2\pi}{3}$

$$\frac{T_2}{T_1} = e^{\mu_s \beta}: \quad \frac{T_{BC}}{W_A} = e^{0.25(2\pi/3)} = e^{\pi/6} \tag{1}$$

Pipe *C*: $T_1 = T_{BC}, \quad T_2 = W_D, \quad \mu_s = 0.25, \quad \beta = \dfrac{\pi}{3}$

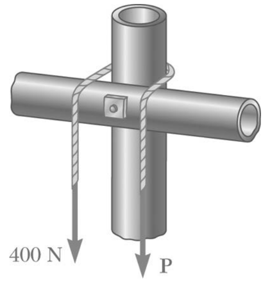
$$\frac{T_2}{T_1} = e^{\mu_s \beta}: \quad \frac{W_D}{T_{BC}} = e^{0.25(\pi/3)} = e^{\pi/12} \tag{2}$$

(*a*) Multiply Eq. (1) by Eq. (2):

$$\frac{W_D}{W_A} = e^{\pi/6} \cdot e^{\pi/12} = e^{\pi/6 + \pi/12} = e^{\pi/4} = 2.193$$

$$W_0 = 2.193 W_A \quad m = 2.193 m_A = 2.193(50 \text{ kg}) \qquad m = 109.7 \text{ kg} \blacktriangleleft$$

(*b*) From Eq. (1): $T_{BC} = W_A e^{\pi/6} = (50 \text{ kg})(9.81 \text{ m/s}^2)(1.688) = 828 \text{ N}$ ◀



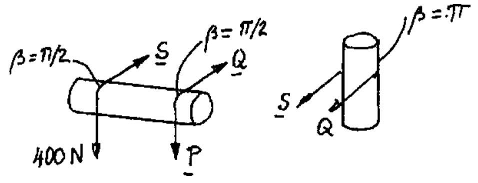
PROBLEM 8.107

Knowing that the coefficient of static friction is 0.25 between the rope and the horizontal pipe and 0.20 between the rope and the vertical pipe, determine the range of values of P for which equilibrium is maintained.

SOLUTION

Horizontal pipe $\mu_s = \mu_h$

Vertical pipe $\mu_s = \mu_v$



For motion of P to be impending downward:

$$\frac{P}{Q} = e^{\mu_h(\pi/2)}, \quad \frac{Q}{S} = e^{\mu_v \pi}, \quad \frac{S}{400\text{ N}} = e^{\mu_h(\pi/2)}$$

Multiply equation member by member:

$$\frac{P}{Q}\frac{Q}{S}\frac{S}{400} = e^{(\mu_h + 2\mu_v + \mu_h)(\pi/2)}$$

or:

$$\frac{P}{400\text{ N}} = e^{(\mu_h + \mu_v)\pi} \qquad (1)$$

For motion of **P** to be impending upward, we find in a similar way

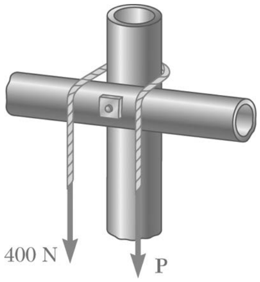
$$\frac{P}{400\text{ N}} = e^{-(\mu_h + \mu_v)\pi} \qquad (2)$$

Given data: $\mu_h = 0.25, \quad \mu_v = 0.20$

From (1): $P = (400\text{ N})e^{0.45\pi} = 1644\text{ N}$

From (2): $P = (400\text{ N})e^{-0.45\pi} = 97.3\text{ N}$

Range for equilibrium: $97.3\text{ N} \le P \le 1644\text{ N}$ ◀



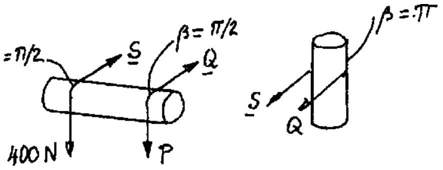
PROBLEM 8.108

Knowing that the coefficient of static friction is 0.30 between the rope and the horizontal pipe and that the smallest value of *P* for which equilibrium is maintained is 80 N, determine (*a*) the largest value of *P* for which equilibrium is maintained, (*b*) the coefficient of static friction between the rope and the vertical pipe.

SOLUTION

Horizontal pipe: $\mu_s = \mu_h$

Vertical pipe: $\mu_s = \mu_v$



For motion of *P* to be impending downward:

$$\frac{P}{Q} = e^{\mu_h(\pi/2)}, \quad \frac{Q}{S} = e^{\mu_v \pi}, \quad \frac{S}{400\text{ N}} = e^{\mu_h(\pi/2)}$$

Multiply equation member by member:

$$\frac{P}{Q}\frac{Q}{S}\frac{S}{400} = e^{(\mu_h + 2\mu_v + \mu_h)(\pi/2)}$$

or:

$$\frac{P}{400\text{ N}} = e^{(\mu_h + \mu_v)\pi} \qquad (1)$$

For motion of **P** to be impending upward, we find in a similar way

$$\frac{P}{400\text{ N}} = e^{-(\mu_h + \mu_v)\pi} \qquad (2)$$

Setting $P = P_{\max}$ in Eq. (1), $P = 80\text{ N}$ in Eq. (2) and $\mu_h = 0.30$ in both, we get

$$\frac{P_{\max}}{400\text{ N}} = e^{(0.30+\mu_v)\pi} \ (1'); \quad \frac{80\text{ N}}{400\text{ N}} = e^{-(0.30+\mu_v)\pi} \ (2')$$

(a) Multiplying (1′) by (2′):

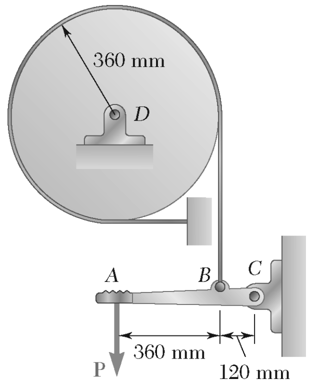
$$\frac{P_{\max}}{400} \cdot \frac{80}{400} = e^0 = 1$$

$P_{\max} = 2000\text{ N}$ ◀

(b) From $(2'): (0.30 + \mu_v)\pi = \ln\frac{400}{80} = \ln 5 = 1.60944$

$$0.30 + \mu_v = 0.512$$

$\mu_v = 0.212$ ◀

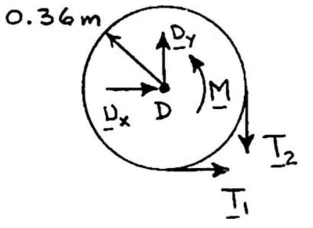


PROBLEM 8.109

A band brake is used to control the speed of a flywheel as shown. The coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.25$. Determine the magnitude of the couple being applied to the flywheel, knowing that $P = 45$ N and that the flywheel is rotating counterclockwise at a constant speed.

SOLUTION

<u>Free body: Cylinder</u>



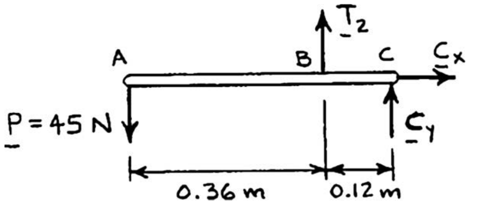
Since slipping of band relative to cylinder is clockwise, $\mathbf{T}_1$ and $\mathbf{T}_2$ are located as shown.

From free body: Lever *ABC*

$$+\circlearrowleft \Sigma M_C = 0: \quad (45\text{ N})(0.48\text{ m}) - T_2(0.12\text{ m}) = 0$$

$$T_2 = 180\text{ N}$$

<u>Free body: Lever *ABC*</u>



From free body: Cylinder

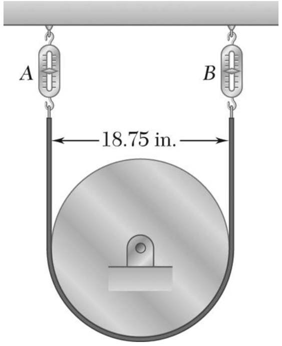
Using Eq. (8.14) with $\mu_k = 0.25$ and $\beta = 270° = \dfrac{3\pi}{2}$ rad:

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{(0.25)(3\pi/2)} = e^{3\pi/8}$$

$$T_1 = \frac{T_2}{e^{3\pi/8}} = \frac{180\text{ N}}{3.2482} = 55.415\text{ N}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (55.415\text{ N})(0.36\text{ m}) - (180\text{ N})(0.36\text{ m}) + M = 0$$

$$\mathbf{M} = 44.9\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$



PROBLEM 8.110

The setup shown is used to measure the output of a small turbine. When the flywheel is at rest, the reading of each spring scale is 14 lb. If a 105-lb · in. couple must be applied to the flywheel to keep it rotating clockwise at a constant speed, determine (*a*) the reading of each scale at that time, (*b*) the coefficient of kinetic friction. Assume that the length of the belt does not change.

SOLUTION

(*a*) Since the length of the belt is constant, the spring in scale *B* will increase in length by δ and the spring in scale *A* will decrease by the same amount. Thus, the sum of the readings in scales *A* and *B* remains constant:

$$T_A + T_B = 14 \text{ lb} + 14 \text{ lb} \qquad T_A + T_B = 28 \text{ lb} \quad (1)$$

On the other hand, the sum of the moments of T_A and T_B about axle must be equal to moment of couple:

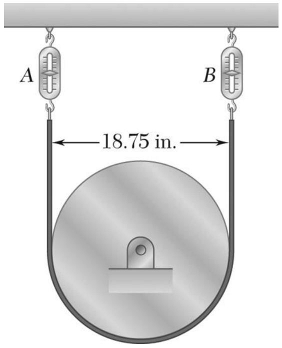
$$(T_B - T_A)(9.375 \text{ in.}) = 105 \text{ lb}\cdot\text{in.}, \qquad T_B - T_A = 11.2 \text{ lb} \quad (2)$$

Solving (1) and (2) simultaneously

$$T_A = 8.40 \text{ lb}; \qquad T_B = 19.60 \text{ lb} \blacktriangleleft$$

(*b*) Apply Eq. (8.13) with $T_2 = T_B$, $T_1 = T_A$, $\beta = 180° = \pi$ rad.

$$\ln\frac{T_2}{T_1} = \mu_s\beta: \qquad \mu_s\pi = \ln\frac{19.60}{8.40} = 0.84730 \qquad \mu_s = 0.270 \blacktriangleleft$$



PROBLEM 8.111

The setup shown is used to measure the output of a small turbine. The coefficient of kinetic friction is 0.20 and the reading of each spring scale is 16 lb when the flywheel is at rest. Determine (*a*) the reading of each scale when the flywheel is rotating clockwise at a constant speed, (*b*) the couple that must be applied to the flywheel. Assume that the length of the belt does not change.

SOLUTION

(*a*) Since the length of the belt is constant, the spring in scale *B* will increase in length by δ and the spring in scale *A* will decrease by the same amount. Thus, the sum of the readings in scales *A* and *B* remains constant:

$$T_A + T_B = 16 \text{ lb} + 16 \text{ lb} \qquad T_A + T_B = 32 \text{ lb} \quad (1)$$

We now apply Eq. (8.14) with

$$T_1 = T_A, \quad T_2 = T_B, \quad \mu_k = 0.20, \quad \beta = 180° = \pi \text{ rad.}$$

$$\frac{T_2}{T_1} = e^{\mu_k B}: \quad \frac{T_B}{T_A} = e^{0.20\pi} = 1.87446$$

$$T_B = 1.87446\ T_A \quad (2)$$

Substituting (2) into (1):

$$T_A + 1.87446 T_A = 32 \text{ lb}$$

$$T_A = 11.1325 \text{ lb} \qquad T_A = 11.13 \text{ lb} ◀$$

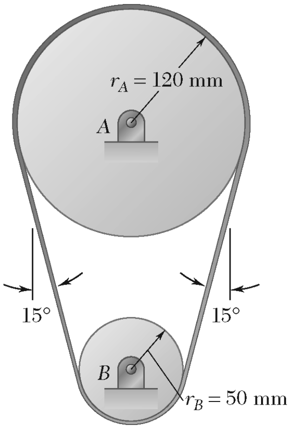
From (1): $T_B = 32 \text{ lb} - 11.1325 \text{ lb}$

$$T_B = 20.868 \text{ lb} \qquad T_B = 20.9 \text{ lb} ◀$$

(*b*) Couple applied to flywheel:

$$M = (T_B - T_A)r$$
$$= (20.868 \text{ lb} - 11.1325 \text{ lb})(9.375 \text{ in.})$$

$$M = 91.3 \text{ lb} \cdot \text{in.} \circlearrowright ◀$$

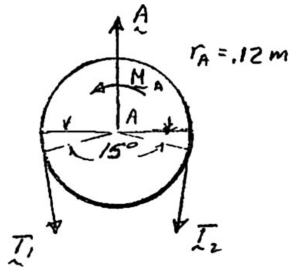


PROBLEM 8.112

A flat belt is used to transmit a couple from drum *B* to drum *A*. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest couple that can be exerted on drum *A*.

SOLUTION

FBD's drums:



$$\beta_A = 180° + 30° = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

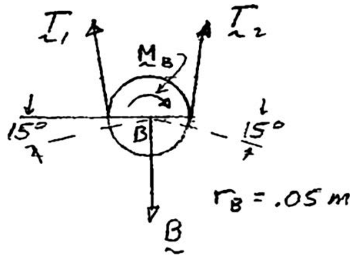
$$\beta_B = 180° - 30° = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Since $\beta_B < \beta_A$, slipping will impend first on *B* (friction coefficients being equal)

So

$$T_2 = T_{\max} = T_1 e^{\mu_s \beta_B}$$

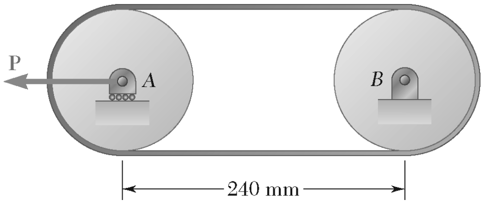
$$450\text{ N} = T_1 e^{(0.4)5\pi/6} \quad \text{or} \quad T_1 = 157.914\text{ N}$$



$$\circlearrowleft \Sigma M_A = 0: \quad M_A + (0.12\text{ m})(T_1 - T_2) = 0$$

$$M_A = (0.12\text{ m})(450\text{ N} - 157.914\text{ N}) = 35.05\text{ N}\cdot\text{m}$$

$$M_A = 35.1\text{ N}\cdot\text{m} \blacktriangleleft$$

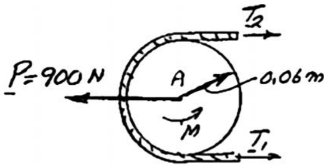


PROBLEM 8.113

A flat belt is used to transmit a couple from pulley *A* to pulley *B*. The radius of each pulley is 60 mm, and a force of magnitude $P = 900$ N is applied as shown to the axle of pulley *A*. Knowing that the coefficient of static friction is 0.35, determine (*a*) the largest couple that can be transmitted, (*b*) the corresponding maximum value of the tension in the belt.

SOLUTION

<u>Drum *A*:</u>



$$\frac{T_2}{T_1} = e^{\mu_s \pi} = e^{(0.35)\pi}$$

$$T_2 = 3.0028 T_1$$

$$\beta = 180° = \pi \text{ radians}$$

(*a*) <u>Torque:</u> $\overset{+}{\curvearrowleft}\Sigma M_A = 0: \quad M - (675.15\text{ N})(0.06\text{ m}) + (224.84\text{ N})(0.06\text{ m})$

$$M = 27.0\text{ N}\cdot\text{m} \blacktriangleleft$$

(*b*) $\xrightarrow{+}\Sigma F_x = 0: \quad T_1 + T_2 - 900\text{ N} = 0$

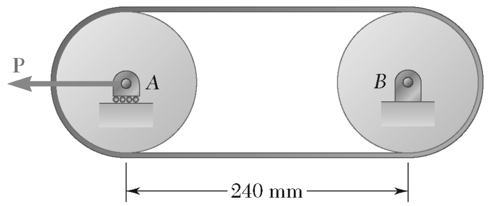
$$T_1 + 3.0028T_1 - 900\text{ N} = 0$$

$$4.00282T_1 = 900$$

$$T_1 = 224.841\text{ N}$$

$$T_2 = 3.0028(224.841\text{ N}) = 675.15\text{ N}$$

$$T_{\max} = 675\text{ N} \blacktriangleleft$$



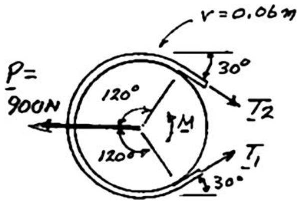
PROBLEM 8.114

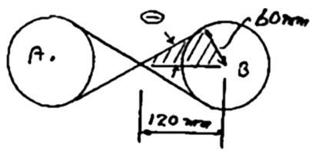
Solve Problem 8.113 assuming that the belt is looped around the pulleys in a figure eight.

PROBLEM 8.113 A flat belt is used to transmit a couple from pulley *A* to pulley *B*. The radius of each pulley is 60 mm, and a force of magnitude *P* = 900 N is applied as shown to the axle of pulley *A*. Knowing that the coefficient of static friction is 0.35, determine (*a*) the largest couple that can be transmitted, (*b*) the corresponding maximum value of the tension in the belt.

SOLUTION

Drum *A*:





$$\beta = 240° = 240°\frac{\pi}{180°} = \frac{4}{3}\pi$$

$$\sin\theta = \frac{60}{120} = \frac{1}{2}$$

$$\theta = 30°$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{0.35(4/3\pi)}$$

$$T_2 = 4.3322T_1$$

(*a*) Torque: $+\circlearrowleft \Sigma M_B = 0$: $M - (844.3\text{ N})(0.06\text{ m}) + (194.9\text{ N})(0.06\text{ m}) = 0$

$M = 39.0\text{ N}\cdot\text{m}$ ◀

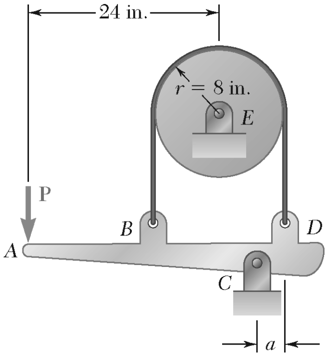
(*b*) $\xrightarrow{+} \Sigma F_x = 0$: $(T_1 + T_2)\cos 30° - 900\text{ N}$

$$(T_1 + 4.3322T_1)\cos 30° = 900$$

$$T_1 = 194.90\text{ N}$$

$$T_2 = 4.3322(194.90\text{ N}) = 844.3\text{ N}$$

$T_{\max} = 844\text{ N}$ ◀



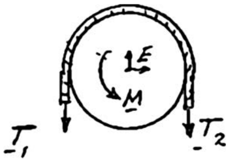
PROBLEM 8.115

The speed of the brake drum shown is controlled by a belt attached to the control bar *AD*. A force **P** of magnitude 25 lb is applied to the control bar at *A*. Determine the magnitude of the couple being applied to the drum, knowing that the coefficient of kinetic friction between the belt and the drum is 0.25, that $a = 4$ in., and that the drum is rotating at a constant speed (*a*) counterclockwise, (*b*) clockwise.

SOLUTION

(*a*) Counterclockwise rotation

Free body: Drum



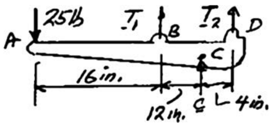
$$r = 8 \text{ in.} \quad \beta = 180^\circ = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933 T_1$$

Free body: Control bar

$$+\circlearrowleft \Sigma M_C = 0: \quad T_1(12 \text{ in.}) - T_2(4 \text{ in.}) - (25 \text{ lb})(28 \text{ in.}) = 0$$



$$T_1(12) - 2.1933T_1(4) - 700 = 0$$

$$T_1 = 216.93 \text{ lb}$$

$$T_2 = 2.1933(216.93 \text{ lb}) = 475.80 \text{ lb}$$

Return to free body of drum

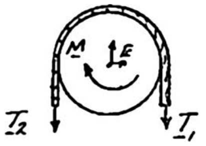
$$+\circlearrowleft \Sigma M_E = 0: \quad M + T_1(8 \text{ in.}) - T_2(8 \text{ in.}) = 0$$

$$M + (216.96 \text{ lb})(8 \text{ in.}) - (475.80 \text{ lb})(8 \text{ in.}) = 0$$

$$M = 2070.9 \text{ lb} \cdot \text{in.}$$

$M = 2070$ lb · in. ◀

(*b*) Clockwise rotation



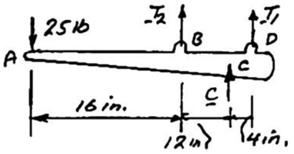
$$r = 8 \text{ in.} \quad \beta = \pi \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

$$T_2 = 2.1933 T_1$$

PROBLEM 8.115 (Continued)

Free body: Control rod



$$+\circlearrowright \Sigma M_C = 0: \quad T_2(12 \text{ in.}) - T_1(4 \text{ in.}) - (25 \text{ lb})(28 \text{ in.}) = 0$$

$$2.1933T_1(12) - T_1(4) - 700 = 0$$

$$T_1 = 31.363 \text{ lb}$$

$$T_2 = 2.1933(31.363 \text{ lb})$$

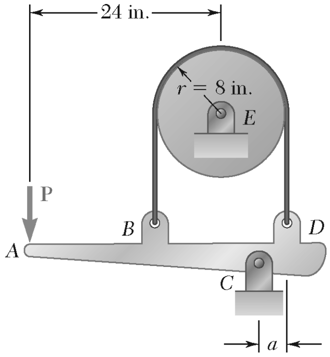
$$T_2 = 68.788 \text{ lb}$$

Return to free body of drum

$$+\circlearrowright \Sigma M_E = 0: \quad M + T_1(8 \text{ in.}) - T_2(8 \text{ in.}) = 0$$

$$M + (31.363 \text{ lb})(8 \text{ in.}) - (68.788 \text{ lb})(8 \text{ in.}) = 0$$

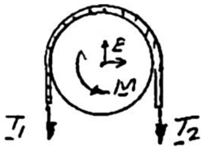
$M = 299.4 \text{ lb} \cdot \text{in.}$ $\qquad$ $M = 299 \text{ lb} \cdot \text{in.}$ ◀



PROBLEM 8.116

The speed of the brake drum shown is controlled by a belt attached to the control bar *AD*. Knowing that $a = 4$ in., determine the maximum value of the coefficient of static friction for which the brake is not self-locking when the drum rotates counterclockwise.

SOLUTION

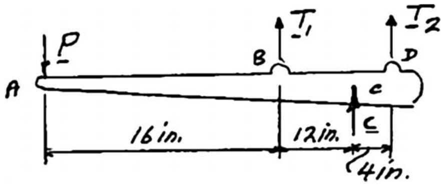


$$r = 8 \text{ in.}, \quad \beta = 180° = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta} = e^{\mu_s \pi}$$

$$T_2 = e^{\mu_s \pi} T_1$$

Free body: Control rod



$$+\circlearrowleft \Sigma M_C = 0: \quad P(28 \text{ in.}) - T_1(12 \text{ in.}) + T_2(4 \text{ in.}) = 0$$

$$28P - 12T_1 + e^{\mu\pi} T_1(4) = 0$$

For self-locking brake: $P = 0$

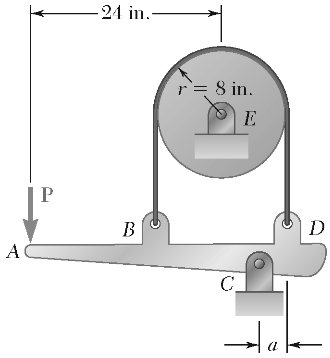
$$12T_1 = 4T_1 e^{\mu_s \pi}$$

$$e^{\mu_s \pi} = 3$$

$$\mu_s \pi = \ln 3 = 1.0986$$

$$\mu_s = \frac{1.0986}{\pi} = 0.3497$$

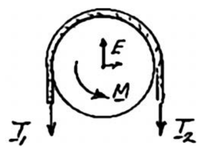
$\mu_s = 0.350$ ◀



PROBLEM 8.117

The speed of the brake drum shown is controlled by a belt attached to the control bar *AD*. Knowing that the coefficient of static friction is 0.30 and that the brake drum is rotating counterclockwise, determine the minimum value of *a* for which the brake is not self-locking.

SOLUTION

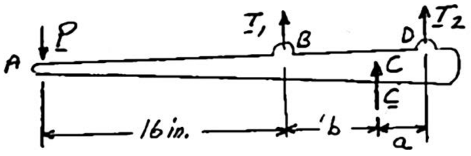


$$r = 8\text{ in.},\quad \beta = \pi \text{ radians}$$

$$\frac{T_2}{T_1} = e^{\mu_s\beta} = e^{0.30\pi} = 2.5663$$

$$T_2 = 2.5663T_1$$

Free body: Control rod



$$b = 16\text{ in.} - a$$

$$+\circlearrowleft \Sigma M_C = 0:\quad P(16\text{ in.} + b) - T_1 b + T_2 a = 0$$

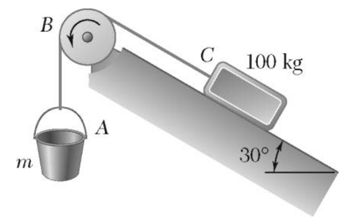
For brake to be self-locking, $P = 0$

$$T_2 a = T_1 b;\quad 2.5663T_1 a = T_1(16 - a)$$

$$2.5663a = 16 - a$$

$$3.5663a = 16$$

$a = 4.49$ in. ◀

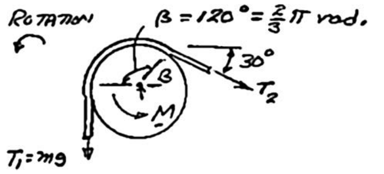


PROBLEM 8.118

Bucket *A* and block *C* are connected by a cable that passes over drum *B*. Knowing that drum *B* rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined mass *m* of the bucket and its contents for which block *C* will (*a*) remain at rest, (*b*) start moving up the incline, (*c*) continue moving up the incline at a constant speed.

SOLUTION

Free body: Drum



$$\frac{T_2}{mg} = e^{\mu \frac{2}{3}\pi}$$

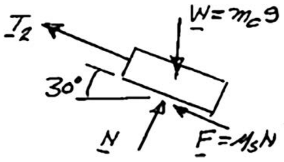
$$T_2 = mge^{2\mu\pi/3} \qquad (1)$$

(*a*) Smallest *m* for block *C* to remain at rest

Cable slips on drum.

Eq. (1) with $\mu_k = 0.25$; $T_2 = mge^{2(0.25)\pi/3} = 1.6881mg$

Block *C*: At rest, motion impending ↘



$$+\nearrow \Sigma F = 0: \quad N - m_C g \cos 30°$$

$$N = m_C g \cos 30°$$

$$F = \mu_s N = 0.35 m_C g \cos 30°$$

$$m_C = 100 \text{ kg}$$

$$+\nwarrow \Sigma F = 0: \quad T_2 + F - m_C g \sin 30° = C$$

$$1.6881mg + 0.35 m_C g \cos 30° - m_C g \sin 30° = 0$$

$$1.6881m = 0.19689 m_C$$

$$m = 0.11663 m_C = 0.11663(100 \text{ kg}); \qquad m = 11.66 \text{ kg} ◀$$

(*b*) Smallest *m* to start block moving up

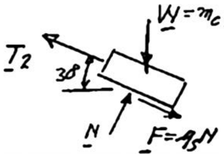
No slipping at both drum and block: $\mu_s = 0.35$

Eq. (1): $T_2 = mge^{2(0.35)\pi/3} = 2.0814mg$

PROBLEM 8.118 (Continued)

<u>Block *C*:</u>

Motion impending $\nwarrow$



$$m_C = 100\,\text{kg}$$

$$+\nearrow \Sigma F = 0: \quad N - mg\cos 30°$$

$$N = m_C g \cos 30°$$
$$F = \mu_s N = 0.35 m_C g \cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_2 - F - m_C g \sin 30° = 0$$

$$2.0814mg - 0.35m_C g\cos 30° - m_C g \sin 30° = 0$$
$$2.0814m = 0.80311m_C$$
$$m = 0.38585m_C = 0.38585(100\text{ kg})$$

$m = 38.6$ kg ◀

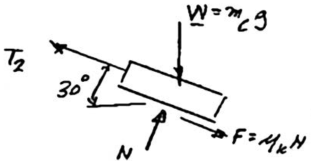
(*c*) <u>Smallest *m* to keep block moving up drum</u>: No slipping: $\mu_s = 0.35$

Eq. (1) with $\mu_s = 0.35$

$$T_2 = mg^{2\mu_s\pi/3} = mge^{2(0.35)\pi/3}$$
$$T_2 = 2.0814mg$$

<u>Block *C*</u>: Moving up plane, thus $\mu_k = 0.25$

Motion up $\nwarrow$



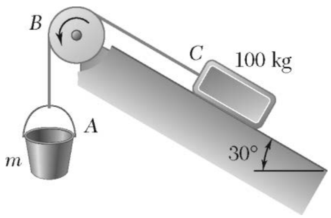
$$+\nearrow \Sigma F = 0: \quad N - m_C g\cos 30° = 0$$

$$N = m_C g \cos 30°$$
$$F = \mu_k N = 0.25m_C g\cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_2 - F - m_C g \sin 30° = 0$$

$$2.0814mg - 0.25m_C g\cos 30° - m_C g\sin 30° = 0$$
$$2.0814m = 0.71651m_C$$
$$m = 0.34424m_C = 0.34424(100\text{ kg})$$

$m = 34.4$ kg ◀



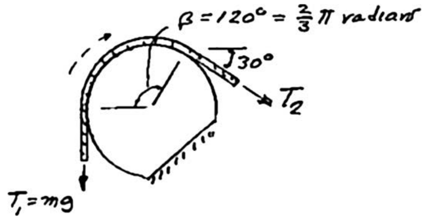
PROBLEM 8.119

Solve Problem 8.118 assuming that drum *B* is frozen and cannot rotate.

PROBLEM 8.118 Bucket *A* and block *C* are connected by a cable that passes over drum *B*. Knowing that drum *B* rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined mass *m* of the bucket and its contents for which block *C* will (*a*) remain at rest, (*b*) start moving up the incline, (*c*) continue moving up the incline at a constant speed.

SOLUTION

(*a*) Block *C* remains at rest: Motion impends ↘

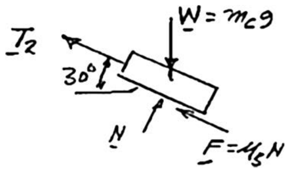


Drum:

$$\frac{T_2}{mg} = e^{\mu_k \beta} = e^{0.35(2\pi/3)}$$

$$T_2 = 2.0814mg$$

Block *C*: Motion impends ↘



$$\nearrow \Sigma F = 0: \quad N - m_C g \cos 30° = 0$$

$$N = m_C g \cos 30°$$

$$F = \mu_s N = 0.35 m_C g \cos 30°$$

$$+\nwarrow \Sigma F = 0: \quad T_2 + F - m_C g \sin 30° = 0$$

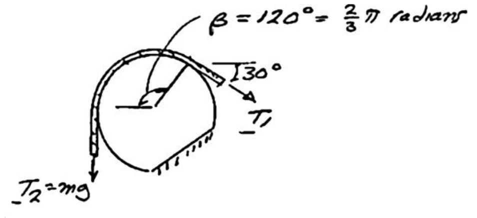
$$2.0814mg + 0.35 m_C g \cos 30° - m_C g \sin 30° = 0$$

$$2.0814mg = 0.19689 m_C$$

$$m = 0.09459 m_C = 0.09459(100 \text{ kg}) \qquad m = 9.46 \text{ kg} \blacktriangleleft$$

(*b*) Block *C*: Starts moving up $\qquad \mu_s = 0.35$

Drum: Impending motion of cable ↻



$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

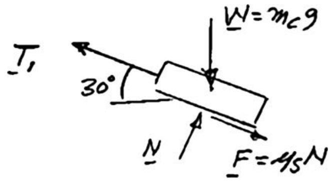
$$\frac{mg}{T_1} = e^{0.35(2/3\pi)}$$

$$T_1 = \frac{mg}{2.0814}$$

$$= 0.48045mg$$

PROBLEM 8.119 (Continued)

Block *C*: Motion impends ↖



$+\nearrow \Sigma F = 0$: $N - m_C g \cos 30°$

$$N = m_C g \cos 30°$$
$$F = \mu_s N = 0.35 m_C g \cos 30°$$

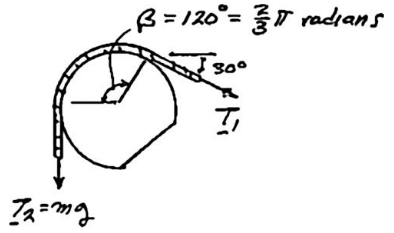
$+\nwarrow \Sigma F = 0$: $T_1 - F - m_C g \sin 30° = 0$

$$0.48045mg - 0.35 m_C g \cos 30° - 0.5 m_C g = 0$$
$$0.48045m = 0.80311 m_C$$
$$m = 1.67158 m_C = 1.67158(100 \text{ kg})$$

$m = 167.2$ kg ◀

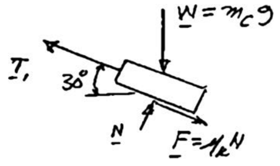
(*c*) Smallest *m* to keep block moving ↖

Drum: Motion of cable ↺



$$\mu_k = 0.25$$
$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25(2/3\pi)}$$
$$\frac{mg}{T_1} = 1.6881$$
$$T_1 = \frac{mg}{1.6881} = 0.59238mg$$

Block *C*: Block moves ↖



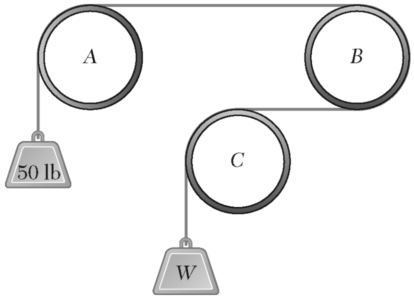
$+\nearrow \Sigma F = 0$: $N - m_C g \cos 30° = 0$

$$N = m_C g \cos 30°$$
$$F = \mu_k N = 0.25 m_C g \cos 30°$$

$+\nwarrow \Sigma F = 0$: $T_1 - F - m_C g \sin 30° = 0$

$$0.59238mg - 0.25 m_C g \cos 30° - 0.5 m_C g = 0$$
$$0.59238m = 0.71651 m_C$$
$$m = 1.20954 m_C = 1.20954(100 \text{ kg})$$

$m = 121.0$ kg ◀

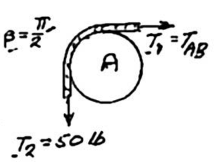


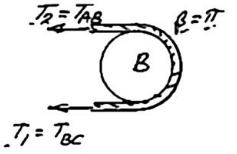
PROBLEM 8.120

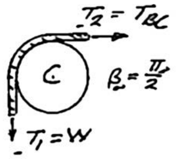
A cable is placed around three parallel pipes. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (*a*) the smallest weight *W* for which equilibrium is maintained, (*b*) the largest weight *W* that can be raised if pipe *B* is slowly rotated counterclockwise while pipes *A* and *C* remain fixed.

SOLUTION

(*a*) $\mu = \mu_s = 0.25$ at all pipes.







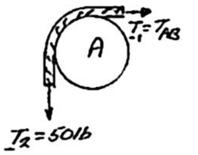
$$\frac{50\text{ lb}}{T_{AB}} = e^{0.25\pi/2} \qquad \frac{T_{AC}}{T_{BC}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.25\pi/2}$$

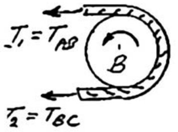
$$\frac{50\text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/8} \cdot e^{\pi/4} \cdot e^{\pi/8} = e^{\pi/8+\pi/4+\pi/8} = e^{\pi/2} = 4.8105$$

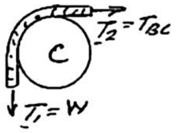
$$\frac{50\text{ lb}}{W} = 4.8015; \quad W = 10.394\text{ lb}$$

$W = 10.39$ lb ◀

(*b*) Pipe *B* rotated ↺ $\qquad \beta = \frac{\pi}{2};\ \mu = \mu_k \qquad \beta = \pi;\ \mu = \mu_s \qquad \beta = \frac{\pi}{2};\ \mu = \mu_k$







$$\frac{50\text{ lb}}{T_{AB}} = e^{0.2\pi/2} \qquad \frac{T_{BC}}{T_{AB}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi/2}$$

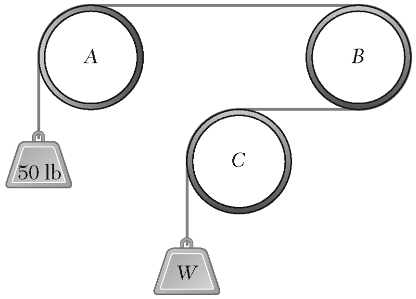
$$\frac{50\text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/10} \cdot e^{-\pi/4} \cdot e^{\pi/10}$$

$$= e^{\pi/10-\pi/4+\pi/10} = e^{-\pi/20} = 0.85464$$

$$\frac{50\text{ lb}}{W} = 0.85464$$

$$W = \frac{50\text{ lb}}{0.85464} = 58.504\text{ lb}$$

$W = 58.5$ lb ◀

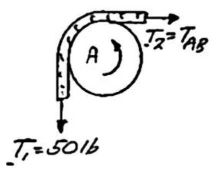


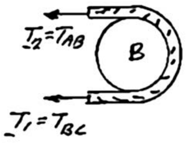
PROBLEM 8.121

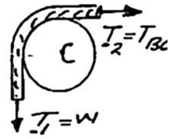
A cable is placed around three parallel pipes. Two of the pipes are fixed and do not rotate; the third pipe is slowly rotated. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the largest weight W that can be raised (*a*) if only pipe *A* is rotated counterclockwise, (*b*) if only pipe *C* is rotated clockwise.

SOLUTION

(*a*) Pipe *A* rotates ↺ $\beta = \frac{\pi}{2};\ \mu = \mu_s$ $\beta = \pi,\ \mu = \mu_k$ $\beta = \frac{\pi}{2};\ \mu = \mu_k$







$$\frac{T_{AB}}{50\text{ lb}} = e^{0.25\pi/2} \qquad \frac{T_{AB}}{T_{BC}} = e^{0.2\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi/2}$$

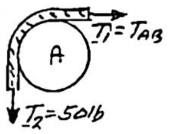
$$\frac{T_{AB}}{50\text{ lb}} \cdot \frac{T_{BC}}{T_{AB}} \cdot \frac{W}{T_{BC}} = e^{\pi/8} \cdot e^{-\pi/5} \cdot e^{-\pi/10}$$

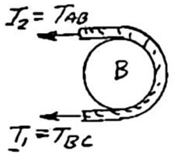
$$= e^{\pi(1/8-1/5-1/10)} = e^{-7\pi/40} = 0.57708$$

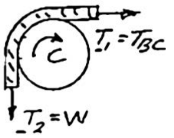
$$\frac{W}{50\text{ lb}} = 0.57708; \quad W = 28.854\text{ lb}$$

$W = 28.9\text{ lb}$ ◀

(*b*) Pipe *C* rotates ↻ $\beta = \frac{\pi}{2};\ \mu = \mu_k$ $\beta = \pi;\ \mu = \mu_k$ $\beta = \frac{\pi}{2},\ \mu = \mu_s$







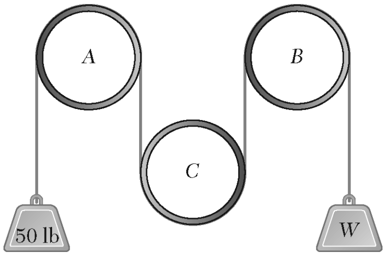
$$\frac{50\text{ lb}}{T_{AB}} = e^{0.2\pi/2} \qquad \frac{T_{AB}}{T_{BC}} = e^{0.2\pi} \qquad \frac{W}{T_{BC}} = e^{0.25\pi/2}$$

$$\frac{50\text{ lb}}{T_{AB}} \cdot \frac{T_{AB}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/10} \cdot e^{\pi/5} \cdot e^{-\pi/8} = e^{7\pi/40} = 0.57708$$

$$\frac{50\text{ lb}}{W} = 0.57708$$

$$W = 28.854\text{ lb}$$

$W = 28.9\text{ lb}$ ◀

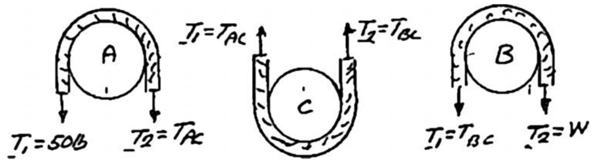


PROBLEM 8.122

A cable is placed around three parallel pipes. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (*a*) the smallest weight *W* for which equilibrium is maintained, (*b*) the largest weight *W* that can be raised if pipe *B* is slowly rotated counterclockwise while pipes *A* and *C* remain fixed.

SOLUTION

(*a*) Smallest *W* for equilibrium $B = \pi, \quad \mu = \mu_s$



$$\frac{T_{AC}}{50\text{ lb}} = e^{0.25\pi} \qquad \frac{T_{BC}}{T_{AC}} = e^{0.25\pi} \qquad \frac{W}{T_{BC}} = e^{0.25\pi}$$

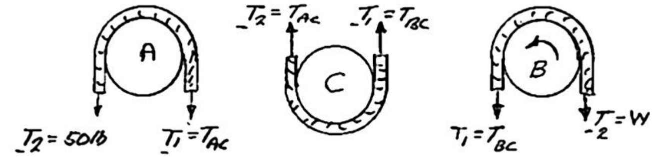
$$\frac{T_{AC}}{50\text{ lb}} \cdot \frac{T_{BC}}{T_{AC}} \cdot \frac{W}{T_{BC}} = e^{\pi/4} \cdot e^{\pi/4} \cdot e^{\pi/4} = e^{3\pi/4} = 10.551$$

$$\frac{W}{50\text{ lb}} = 10.551; \qquad W = 4.739\text{ lb}$$

$W = 4.74$ lb ◀

(*b*) Largest *W* which can be raised by pipe *B* rotated ↺

$\beta = \pi, \mu = \mu_k \qquad \beta = \pi, \mu = \mu_k \qquad \beta = \pi, \mu = \mu_s$



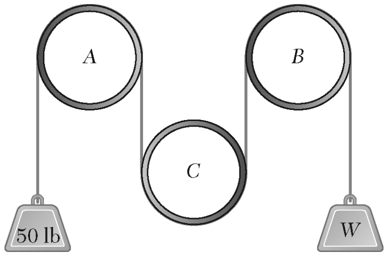
$$\frac{50\text{ lb}}{T_{AC}} = e^{0.2\pi} \qquad \frac{T_{AC}}{T_{BC}} = e^{0.2\pi} \qquad \frac{W}{T_{BC}} = e^{0.25\pi}$$

$$\frac{50\text{ lb}}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/5} \cdot e^{\pi/5} \cdot e^{-\pi/4} = e^{\pi(1/5+1/5-1/4)}$$

$$= e^{3\pi/20} = 1.602$$

$$\frac{50\text{ lb}}{W} = 1.602; \qquad W = \frac{50\text{ lb}}{1.602} = 31.21\text{ lb}$$

$W = 31.2$ lb ◀



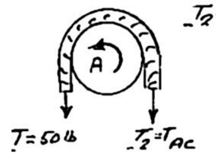
PROBLEM 8.123

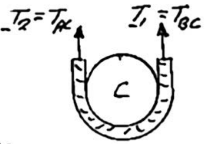
A cable is placed around three parallel pipes. Two of the pipes are fixed and do not rotate; the third pipe is slowly rotated. Knowing that the coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine the largest weight W that can be raised (*a*) if only pipe *A* is rotated counterclockwise, (*b*) if only pipe *C* is rotated clockwise.

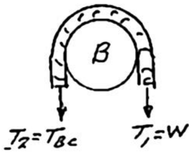
SOLUTION

(*a*) Pipe *A* rotates $\circlearrowleft$

$\beta = \pi, \mu = \mu_s$ $\qquad$ $\beta = \pi, \mu = \mu_k$ $\qquad$ $\beta = \pi, \mu = \mu_k$







$$\frac{T_{AC}}{50\text{ lb}} = e^{0.25\pi} \qquad \frac{T_{AC}}{T_{BC}} = e^{0.2\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi}$$

$$\frac{T_{AC}}{50\text{ lb}} \cdot \frac{T_{BC}}{T_{AC}} \cdot \frac{W}{T_{BC}} = e^{\pi/4} \cdot e^{-\pi/5} \cdot e^{-\pi/5}$$

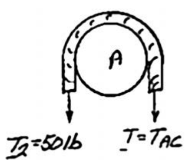
$$= e^{\pi(1/4-1/5-1/5)} = e^{-3\pi/20} = 0.62423$$

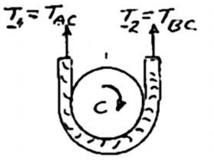
$$\frac{W}{50\text{ lb}} = 0.62423; \quad W = 31.21\text{ lb}$$

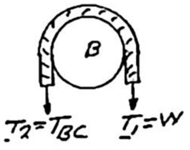
$W = 31.2$ lb ◀

(*b*) Pipe *C* rotates $\circlearrowright$

$\beta = \pi, \mu = \mu_k$ $\qquad$ $\beta = \pi, \mu = \mu_s$ $\qquad$ $\beta = \pi, \mu = \mu_k$







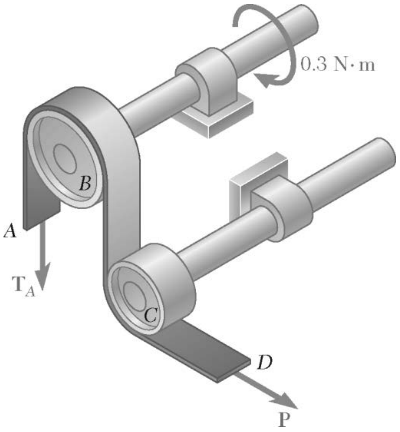
$$\frac{50\text{ lb}}{T_{AC}} = e^{0.2\pi} \qquad \frac{T_{BC}}{T_{AC}} = e^{0.25\pi} \qquad \frac{T_{BC}}{W} = e^{0.2\pi}$$

$$\frac{50\text{ lb}}{T_{AC}} \cdot \frac{T_{AC}}{T_{BC}} \cdot \frac{T_{BC}}{W} = e^{\pi/5} \cdot e^{-\pi/4} \cdot e^{\pi/5} = e^{\pi(1/5-1/4+1/5)} = e^{3\pi/20}$$

$$\frac{50\text{ lb}}{W} = e^{3\pi/20} = 1.602$$

$$W = \frac{50\text{ lb}}{1.602} = 31.21\text{ lb}$$

$W = 31.2$ lb ◀

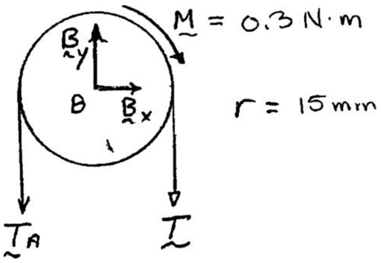


PROBLEM 8.124

A recording tape passes over the 20-mm-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

$$T_A - T = \frac{M}{r} = \frac{300 \text{ N}\cdot\text{mm}}{20 \text{ mm}} = 15.0000 \text{ N}$$

Impending slipping: $T_A = Te^{\mu_s \beta} = Te^{0.4\pi}$

So $T(e^{0.4\pi} - 1) = 15.0000$ N

or $T = 5.9676$ N

If C is free to rotate, $P = T$ $\qquad P = 5.97$ N ◀

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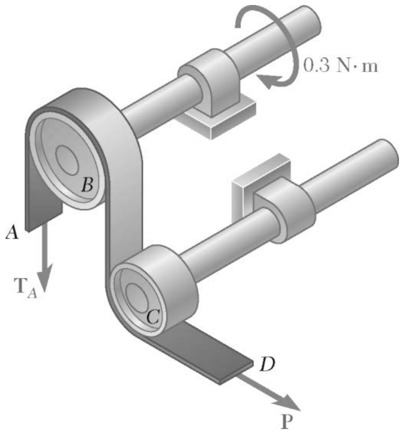


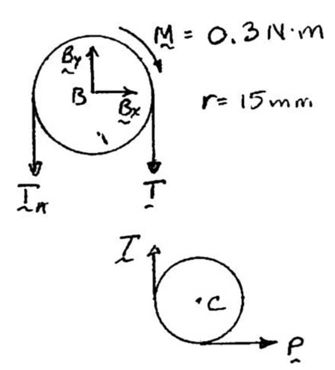
PROBLEM 8.125

Solve Problem 8.124 assuming that the idler drum C is frozen and cannot rotate.

PROBLEM 8.124 A recording tape passes over the 20-mm-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

$$T_A - T = \frac{M}{r} = 300 \text{ N}\cdot\text{mm} = 15.0000 \text{ N}$$

Impending slipping: $\quad T_A = Te^{\mu_s \beta} = Te^{0.4\pi}$

So $\quad (e^{0.4\pi} - 1)T = 15.000 \text{ N}$

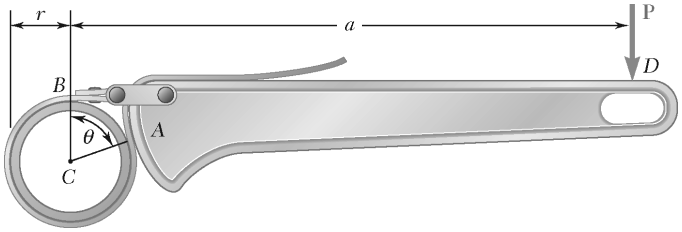
or $\quad T = 5.9676 \text{ N}$

If C is fixed, the tape must slip $\circlearrowleft$

So $\quad P = Te^{\mu_k \beta_C} = (5.9676 \text{ N})e^{0.3\pi/2} = 9.5600 \text{ N} \qquad P = 9.56 \text{ N}$ ◀

PROBLEM 8.126

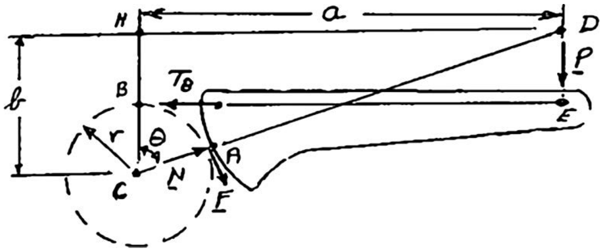
The strap wrench shown is used to grip the pipe firmly without marring the external surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 200$ mm, $r = 30$ mm, and $\theta = 65°$.



SOLUTION

For wrench to be self-locking $(P = 0)$, the value of μ_s must prevent slipping of strap which is in contact with the pipe from Point A to Point B and must be large enough so that at Point A the strap tension can increase from zero to the minimum tension required to develop "belt friction" between strap and pipe.

Free body: Wrench handle



Geometry In ΔCDH:

$$CH = \frac{a}{\tan\theta}$$

$$CD = \frac{a}{\sin\theta}$$

$$DE = BH = CH - BC$$

$$DE = \frac{a}{\tan\theta} - r$$

$$AD = CD - CA = \frac{a}{\sin\theta} - r$$

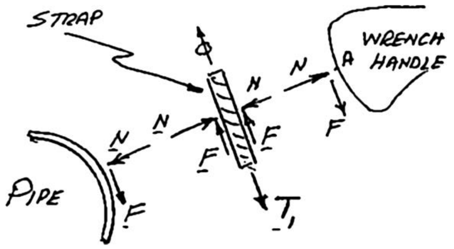
On wrench handle

$$+\circlearrowleft \Sigma M_D = 0: \quad T_B(DE) - F(AD) = 0$$

$$\frac{T_B}{F} = \frac{AD}{DE} = \frac{\dfrac{a}{\sin\theta} - r}{\dfrac{a}{\tan\theta} - r} \qquad (1)$$

PROBLEM 8.126 (Continued)

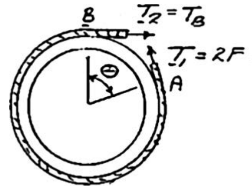
Free body: Strap at Point *A*



$$+\searrow \Sigma F = 0: \quad T_1 - 2F = 0$$

$$T_1 = 2F \qquad (2)$$

Pipe and strap



$$\beta = (2\pi - \theta) \text{ radians}$$

Eq. (8.13):

$$\mu_s \beta = \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_B}{2F} \qquad (3)$$

Return to free body of wrench handle

$$\xrightarrow{+} \Sigma F_x = 0: \quad N \sin\theta + F\cos\theta - T_B = 0$$

$$\frac{N}{F}\sin\theta = \frac{T_B}{F} - \cos\theta$$

Since $F = \mu_s N$, we have

$$\frac{1}{\mu_s}\sin\theta = \frac{T_B}{F} - \cos\theta$$

or

$$\mu_s = \frac{\sin\theta}{\frac{T_B}{F} - \cos\theta} \qquad (4)$$

(*Note:* For a given set of data, we seek the larger of the values of μ_s from Eqs. (3) and (4).)

For $a = 200 \text{ mm}, \quad r = 30 \text{ mm}, \quad \theta = 65°$

Eq. (1):

$$\frac{T_B}{F} = \frac{\frac{200 \text{ mm}}{\sin 65°} - 30 \text{ mm}}{\frac{200 \text{ mm}}{\tan 65°} - 30 \text{ mm}}$$

$$= \frac{190.676 \text{ mm}}{63.262 \text{ mm}} = 3.0141$$

$$\beta = 2\pi - \theta = 2\pi - 65°\frac{\pi}{180°} = 5.1487 \text{ radians}$$

PROBLEM 8.126 (Continued)

Eq. (3):
$$\mu_s = \frac{1}{5.1487\text{ rad}}\ln\frac{3.0141}{2}$$
$$= \frac{0.41015}{5.1487}$$
$$= 0.0797 \qquad \triangleleft$$

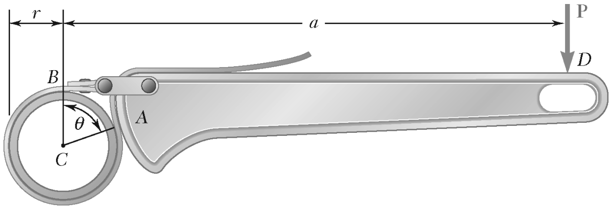
Eq. (4):
$$\mu_s = \frac{\sin 65^\circ}{3.0141 - \cos 65^\circ}$$
$$= \frac{0.90631}{2.1595}$$
$$= 0.3497 \qquad \triangleleft$$

We choose the larger value: $\mu_s = 0.350$ ◀

PROBLEM 8.127

Solve Problem 8.126 assuming that $\theta = 75°$.

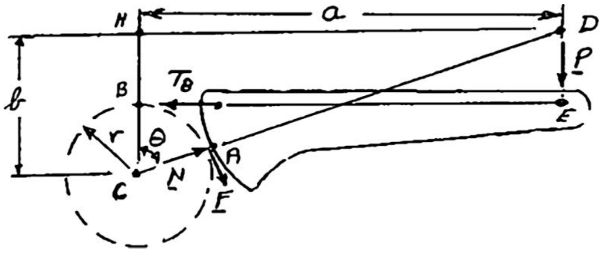
PROBLEM 8.126 The strap wrench shown is used to grip the pipe firmly without marring the external surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 200$ mm, $r = 30$ mm, and $\theta = 65°$.



SOLUTION

For wrench to be self-locking $(P = 0)$, the value of μ_s must prevent slipping of strap which is in contact with the pipe from Point A to Point B <u>and</u> must be large enough so that at Point A the strap tension can increase from zero to the minimum tension required to develop “belt friction” between strap and pipe.

<u>Free body: Wrench handle</u>



<u>Geometry</u> In ΔCDH:

$$CH = \frac{a}{\tan\theta}$$

$$CD = \frac{a}{\sin\theta}$$

$$DE = BH = CH - BC$$

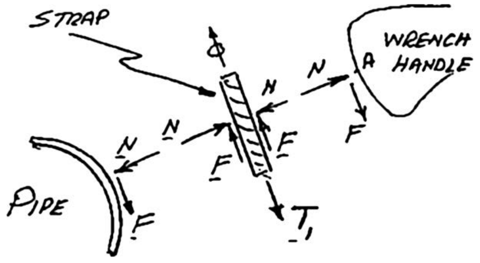
$$DE = \frac{a}{\tan\theta} - r$$

$$AD = CD - CA = \frac{a}{\sin\theta} - r$$

<u>On wrench handle</u> $+\circlearrowright \Sigma M_D = 0\text{:} \quad T_B(DE) - F(AD) = 0$

$$\frac{T_B}{F} = \frac{AD}{DE} = \frac{\dfrac{a}{\sin\theta} - r}{\dfrac{a}{\tan\theta} - r} \qquad (1)$$

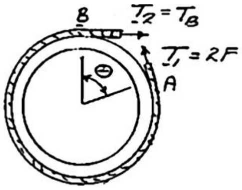
Free body: Strap at Point *A*



$$+\searrow \Sigma F = 0: \quad T_1 - 2F = 0$$

$$T_1 = 2F \tag{2}$$

Pipe and strap



$$\beta = (2\pi - \theta) \text{ radians}$$

Eq. (8.13):

$$\mu_s \beta = \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{\beta} \ln \frac{T_B}{2F} \tag{3}$$

Return to free body of wrench handle

$$\xrightarrow{+} \Sigma F_x = 0: \quad N\sin\theta + F\cos\theta - T_B = 0$$

$$\frac{N}{F}\sin\theta = \frac{T_B}{F} - \cos\theta$$

Since $F = \mu_s N$, we have

$$\frac{1}{\mu_s}\sin\theta = \frac{T_B}{F} - \cos\theta$$

or

$$\mu_s = \frac{\sin\theta}{\dfrac{T_B}{F} - \cos\theta} \tag{4}$$

(*Note:* For a given set of data, we seek the larger of the values of μ_s from Eqs. (3) and (4).)

For $a = 200 \text{ mm}, \quad r = 30 \text{ mm}, \quad \theta = 75°$

Eq. (1):

$$\frac{T_B}{F} = \frac{\dfrac{200 \text{ mm}}{\sin 75°} - 30 \text{ mm}}{\dfrac{200 \text{ mm}}{\tan 75°} - 30 \text{ mm}}$$

$$= \frac{177.055 \text{ mm}}{23.590 \text{ mm}} = 7.5056$$

$$\beta = 2\pi - \theta = 2\pi - 75° \frac{\pi}{180°} = 4.9742$$

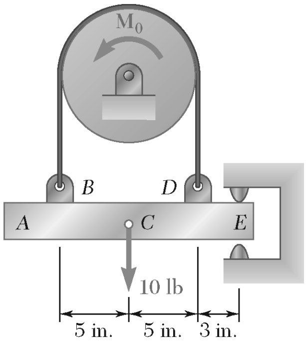
PROBLEM 8.127 (Continued)

Eq. (3):
$$\mu_s = \frac{1}{4.9742 \text{ rad}} \ln \frac{7.5056}{2}$$
$$= \frac{1.3225}{4.9742}$$
$$= 0.2659 \quad \triangleleft$$

Eq. (4):
$$\mu_s = \frac{\sin 75°}{7.5056 - \cos 75°}$$
$$= \frac{0.96953}{7.2468}$$
$$= 0.1333 \quad \triangleleft$$

We choose the larger value: $\mu_s = 0.266$ ◀

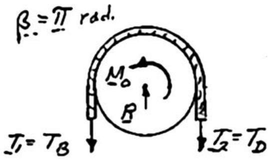
PROPRIETARY MATERIAL.



PROBLEM 8.128

The 10-lb bar *AE* is suspended by a cable that passes over a 5-in.-radius drum. Vertical motion of end *E* of the bar is prevented by the two stops shown. Knowing that $\mu_s = 0.30$ between the cable and the drum, determine (*a*) the largest counterclockwise couple $\mathbf{M}_0$ that can be applied to the drum if slipping is not to occur, (*b*) the corresponding force exerted on end *E* of the bar.

SOLUTION

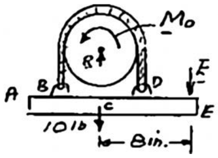


Drum: Slipping impends $\qquad \mu_s = 0.30$

$$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{T_D}{T_B} = e^{0.30\pi} = 2.5663$$

$$T_D = 2.5663T_B$$

(*a*) Free-body: Drum and bar

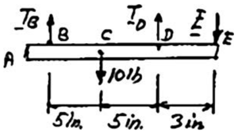


$$+\circlearrowleft \Sigma M_C = 0: \quad M_0 - E(8 \text{ in.}) = 0$$

$$M_0 = (3.78649 \text{ lb})(8 \text{ in.})$$

$$= 30.27 \text{ lb} \cdot \text{in.} \qquad \mathbf{M}_0 = 30.3 \text{ lb} \cdot \text{in.} \circlearrowleft \blacktriangleleft$$

(*b*) Bar *AE*:



$$+\uparrow \Sigma F_y = 0: \quad T_B + T_D - E - 10 \text{ lb} = 0$$

$$T_B + 2.5663T_B - E - 10 \text{ lb} = 0$$

$$3.5663T_B - E - 10 \text{ lb} = 0$$

$$E = 3.5663T_B - 10 \text{ lb} \qquad (1)$$

$$+\circlearrowright \Sigma M_D = 0: \quad E(3 \text{ in.}) - (10 \text{ lb})(5 \text{ in.}) + T_B(10 \text{ in.}) = 0$$

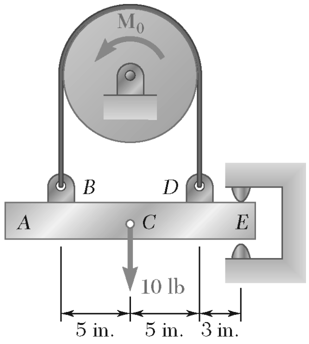
$$(3.5663T_B - 10 \text{ lb})(3 \text{ in.}) - 50 \text{ lb} \cdot \text{in.} + T_B(10 \text{ in.}) = 0$$

$$20.699T_B = 80 \quad T_B = 3.8649 \text{ lb}$$

Eq. (1):

$$E = 3.5663(3.8649 \text{ lb}) - 10 \text{ lb}$$

$$E = +3.78347 \text{ lb} \qquad \mathbf{E} = 3.78 \text{ lb} \downarrow \blacktriangleleft$$

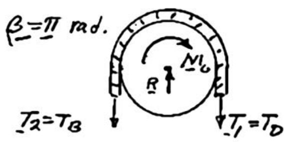


PROBLEM 8.129

Solve Problem 8.128 assuming that a clockwise couple $\mathbf{M}_0$ is applied to the drum.

PROBLEM 8.128 The 10-lb bar *AE* is suspended by a cable that passes over a 5-in.-radius drum. Vertical motion of end *E* of the bar is prevented by the two stops shown. Knowing that $\mu_s = 0.30$ between the cable and the drum, determine (*a*) the largest counterclockwise couple $\mathbf{M}_0$ that can be applied to the drum if slipping is not to occur, (*b*) the corresponding force exerted on end *E* of the bar.

SOLUTION



Drum: Slipping impends

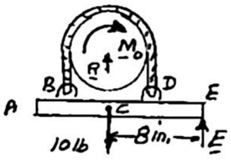
$$\mu_s = 0.30$$

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$\frac{T_B}{T_D} = e^{0.30\pi} = 2.5663$$

$$T_B = 2.5663 T_D$$

(*a*) Free body: Drum and bar

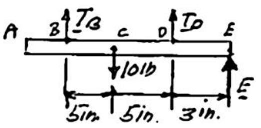


$$+\circlearrowright \Sigma M_C = 0: \quad M_0 - E(8\text{ in.}) = 0$$

$$M_0 = (2.1538\text{ lb})(8\text{ in.})$$

$$\mathbf{M}_0 = 17.23\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

(*b*) Bar *AE*:



$$+\uparrow \Sigma F_y = 0: \quad T_B + T_D + E - 10\text{ lb} = 0$$

$$= 2.5663 T_D + T_D + E - 10\text{ lb}$$

$$E = -3.5663 T_D + 10\text{ lb} \qquad (1)$$

$$+\circlearrowright \Sigma M_B = 0: \quad T_D(10\text{ in.}) - (10\text{ lb})(5\text{ in.}) + E(13\text{ in.}) = 0$$

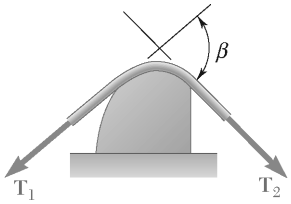
$$T_D(10\text{ in.}) - 50\text{ lb}\cdot\text{in.} + (-3.5663 T_D + 10\text{ lb})(13\text{ in.}) = 0$$

$$-36.362 T_D + 80\text{ lb}\cdot\text{in.} = 0; \quad T_D = 2.200\text{ lb}$$

Eq. (1):

$$E = -3.5633(2.200\text{ lb}) + 10\text{ lb}$$

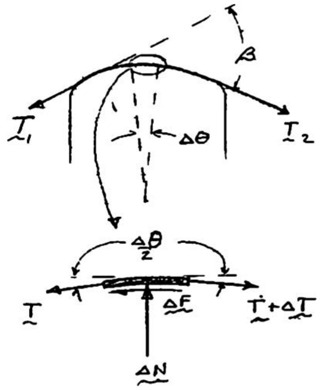
$$E = +2.1538\text{ lb} \qquad \mathbf{E} = 2.15\text{ lb}\uparrow \blacktriangleleft$$



PROBLEM 8.130

Prove that Eqs. (8.13) and (8.14) are valid for any shape of surface provided that the coefficient of friction is the same at all points of contact.

SOLUTION



$$\uparrow \Sigma F_n = 0: \quad \Delta N - [T + (T + \Delta T)]\sin\frac{\Delta\theta}{2} = 0$$

or
$$\Delta N = (2T + \Delta T)\sin\frac{\Delta\theta}{2}$$

$$\longrightarrow \Sigma F_t = 0: \quad [(T + \Delta T) - T]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

or
$$\Delta F = \Delta T\cos\frac{\Delta\theta}{2}$$

Impending slipping: $\Delta F = \mu_s \Delta N$

So
$$\Delta T\cos\frac{\Delta\theta}{2} = \mu_s 2T\sin\frac{\Delta\theta}{2} + \mu_s \Delta T\frac{\sin\Delta\theta}{2}$$

In limit as $\Delta\theta \longrightarrow 0$: $dT = \mu_s T d\theta$ or $\frac{dT}{T} = \mu_s d\theta$

So
$$\int_{T_1}^{T_2}\frac{dT}{T} = \int_0^{\beta}\mu_s d\theta$$

and
$$\ln\frac{T_2}{T_1} = \mu_s\beta \qquad \text{or} \quad T_2 = T_1 e^{\mu_s\beta} \blacktriangleleft$$

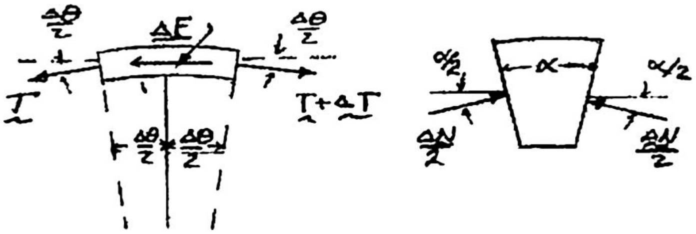
(*Note:* Nothing above depends on the shape of the surface, except it is assumed to be a smooth curve.)

PROBLEM 8.131

Complete the derivation of Eq. (8.15), which relates the tension in both parts of a V belt.

SOLUTION

Small belt section:



Side view: **End view:**

$$\uparrow \Sigma F_y = 0: \quad 2\frac{\Delta N}{2}\sin\frac{\alpha}{2} - [T + (T + \Delta T)]\sin\frac{\Delta\theta}{2} = 0$$

$$\longrightarrow \Sigma F_x = 0: \quad [(T + \Delta T) - T]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

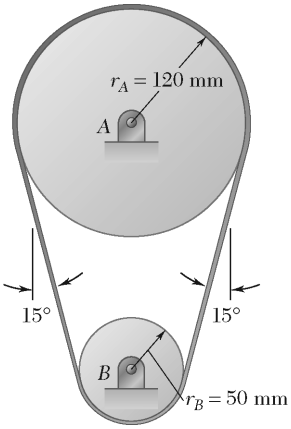
Impending slipping: $$\Delta F = \mu_s \Delta N \Rightarrow \Delta T\cos\frac{\Delta\theta}{2} = \mu_s\frac{2T + \Delta T}{\sin\frac{\alpha}{2}}\sin\frac{\Delta\theta}{2}$$

In limit as $$\Delta\theta \longrightarrow 0: \quad dT = \frac{\mu_s T d\theta}{\sin\frac{\alpha}{2}} \quad \text{or} \quad \frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}}d\theta$$

so $$\int_{T_1}^{T_2}\frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}}\int_0^{\beta} d\theta$$

or $$\ln\frac{T_2}{T_1} = \frac{\mu_s\beta}{\sin\frac{\alpha}{2}}$$

or $$T_2 = T_1 e^{\mu_s\beta/\sin\frac{\alpha}{2}} \blacktriangleleft$$



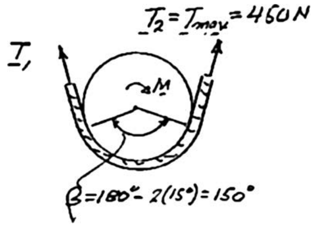
PROBLEM 8.132

Solve Problem 8.112 assuming that the flat belt and drums are replaced by a V belt and V pulleys with $\alpha = 36°$. (The angle α is as shown in Figure 8.15*a*.)

PROBLEM 8.112 A flat belt is used to transmit a couple from drum *B* to drum *A*. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest couple that can be exerted on drum *A*.

SOLUTION

Since β is smaller for pulley *B*. The belt will slip first at *B*.



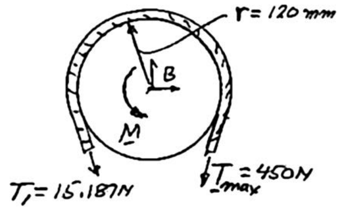
$$\beta = 150°\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{5}{6}\pi \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu_s \beta / \sin\frac{\alpha}{2}}$$

$$\frac{450 \text{ N}}{T_1} = e^{(0.4)\frac{5}{6}\pi/\sin 18°} = e^{3.389}$$

$$\frac{450 \text{ N}}{T_1} = 29.63; \quad T_1 = 15.187 \text{ N}$$

<u>Torque on pulley *A*</u>:

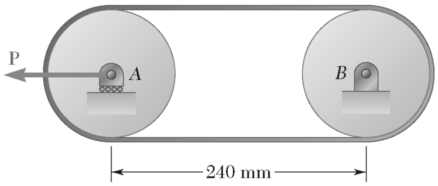


$$+\!\circlearrowleft \Sigma M_B = 0: \quad M - (T_{\max} - T_1)(0.12 \text{ m}) = 0$$

$$M - (450 \text{ N} - 15.187 \text{ N})(0.12 \text{ m}) = 0$$

$$M = 52.18 \text{ N}\cdot\text{m}$$

$M = 52.2$ N · m ◀



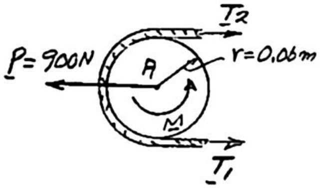
PROBLEM 8.133

Solve Problem 8.113 assuming that the flat belt and pulleys are replaced by a V belt and V pulleys with $\alpha = 36°$. (The angle α is as shown in Figure 8.15*a*.)

PROBLEM 8.113 A flat belt is used to transmit a couple from pulley *A* to pulley *B*. The radius of each pulley is 60 mm, and a force of magnitude $P = 900\text{ N}$ is applied as shown to the axle of pulley *A*. Knowing that the coefficient of static friction is 0.35, determine (*a*) the largest couple that can be transmitted, (*b*) the corresponding maximum value of the tension in the belt.

SOLUTION

Pulley *A*: $\beta = \pi \text{ rad}$



$$\frac{T_2}{T_1} = e^{\mu_s \beta / \sin\frac{\alpha}{2}}$$

$$\frac{T_2}{T_1} = e^{0.35\pi/\sin 18°}$$

$$\frac{T_2}{T_1} = e^{3.558} = 35.1$$

$$T_2 = 35.1T_1$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad T_1 + T_2 - 900\text{ N} = 0$$

$$T_1 + 35.1T_1 - 900\text{ N} = 0$$

$$T_1 = 24.93\text{ N}$$

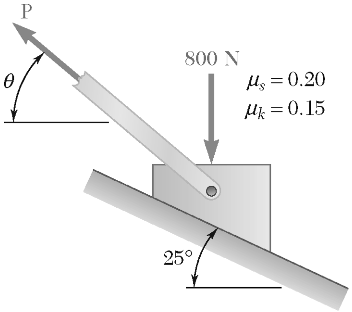
$$T_2 = 35.1(24.93\text{ N}) = 875.03\text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M - T_2(0.06\text{ m}) + T_1(0.06\text{ m}) = 0$$

$$M - (875.03\text{ N})(0.06\text{ m}) + (24.93\text{ N})(0.06\text{ m}) = 0$$

$$M = 51.0\text{ N}\cdot\text{m} \blacktriangleleft$$

$$T_{\max} = T_2 \qquad T_{\max} = 875\text{ N} \blacktriangleleft$$

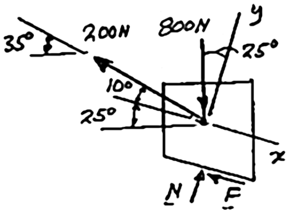


PROBLEM 8.134

Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 35°$ and $P = 200$ N.

SOLUTION

Assume equilibrium:



+↗ $\Sigma F_y = 0$: $N - (800\text{ N})\cos 25° + (200\text{ N})\sin 10° = 0$

$N = 690.3$ N **N** = 690.3 N ↑

+↘ $\Sigma F_x = 0$: $-F + (800\text{ N})\sin 25° - (200\text{ N})\cos 10° = 0$

$F = 141.13$ N **F** = 141.13 N ↖

Maximum friction force:

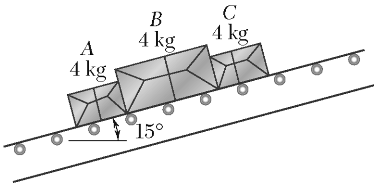
$$F_m = \mu_s N$$
$$= (0.20)(690.3\text{ N})$$
$$= 138.06\text{ N}$$

Since $F > F_m$, <u>Block moves down</u> ↘ ◀

Friction force:

$$F = \mu_k N$$
$$= (0.15)(690.3\text{ N})$$
$$= 103.547\text{ N}$$

F = 103.5 N ↖ ◀

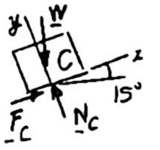


PROBLEM 8.135

Three 4-kg packages *A*, *B*, and *C* are placed on a conveyor belt that is at rest. Between the belt and both packages *A* and *C* the coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.20$; between package *B* and the belt the coefficients are $\mu_s = 0.10$ and $\mu_k = 0.08$. The packages are placed on the belt so that they are in contact with each other and at rest. Determine which, if any, of the packages will move and the friction force acting on each package.

SOLUTION

<u>Consider *C* by itself</u>: Assume equilibrium



$$+\nwarrow \Sigma F_y = 0: \quad N_C - W\cos 15° = 0$$

$$N_C = W\cos 15° = 0.966W$$

$$+\nearrow \Sigma F_x = 0: \quad F_C - W\sin 15° = 0$$

$$F_C = W\sin 15° = 0.259W$$

But $F_m = \mu_s N_C$

$$= 0.30(0.966W)$$
$$= 0.290W$$

Thus, $F_C < F_m$ <u>Package *C* does not move</u> ◀

$$F_C = 0.259W$$
$$= 0.259(4\text{ kg})(9.81\text{ m/s}^2)$$
$$= 10.16\text{ N}$$

$\mathbf{F}_C = 10.16\text{ N} \nearrow$ ◀

<u>Consider *B* by itself</u>: Assume equilibrium. We find,

$$F_B = 0.259W$$
$$N_B = 0.966W$$

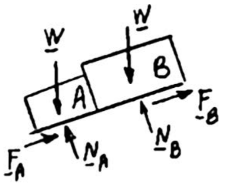
But $F_m = \mu_s N_B$

$$= 0.10(0.966W)$$
$$= 0.0966W$$

Thus, $F_B > F_m$. <u>Package *B* would move if alone</u> ◀

PROBLEM 8.135 (Continued)

<u>Consider *A* and *B* together</u>: Assume equilibrium



$$F_A = F_B = 0.259W$$

$$N_A = N_B = 0.966W$$

$$F_A + F_B = 2(0.259W) = 0.518W$$

$$(F_A)_m + (F_B)_m = 0.3N_A + 0.1N_B = 0.386W$$

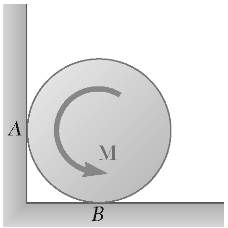
Thus, $F_A + F_B > (F_A)_m + (F_B)_m$ <u>*A* and *B* move</u> ◀

$$F_A = \mu_k N_A = 0.2(0.966)(4)(9.81)$$

$\mathbf{F}_A = 7.58\text{ N}$ ↗ ◀

$$F_B = \mu_k N_B = 0.08(0.966)(4)(9.81)$$

$\mathbf{F}_B = 3.03\text{ N}$ ↗ ◀

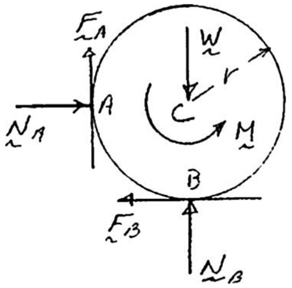


PROBLEM 8.136

The cylinder shown is of weight W and radius r. Express in terms W and r the magnitude of the largest couple **M** that can be applied to the cylinder if it is not to rotate, assuming the coefficient of static friction to be (*a*) zero at A and 0.30 at B, (*b*) 0.25 at A and 0.30 at B.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_A N_A$$
$$F_B = \mu_B N_B$$

$$\xrightarrow{} \Sigma F_x = 0: \quad N_A - F_B = 0$$
$$N_A = F_B = \mu_B N_B$$
$$F_A = \mu_A N_A = \mu_A \mu_B N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0$$
$$N_B(1 + \mu_A \mu_B) = W$$

or
$$N_B = \frac{1}{1 + \mu_A \mu_B} W$$

and
$$F_B = \mu_B N_B = \frac{\mu_B}{1 + \mu_A \mu_B} W$$
$$F_A = \mu_A \mu_B N_B = \frac{\mu_A \mu_B}{1 + \mu_A \mu_B} W$$

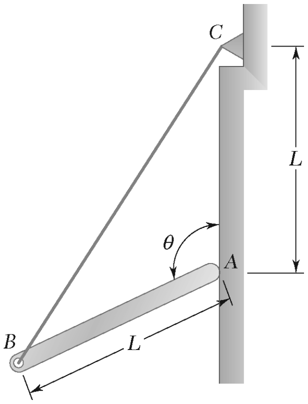
$$\circlearrowleft \Sigma M_C = 0: \quad M - r(F_A + F_B) = 0$$
$$M = Wr\mu_B \frac{1 + \mu_A}{1 + \mu_A \mu_B}$$

(*a*) For $\mu_A = 0$ and $\mu_B = 0.30$:

$M = 0.300Wr$ ◀

(*b*) For $\mu_A = 0.25$ and $\mu_B = 0.30$:

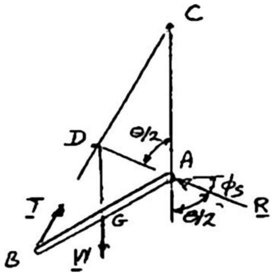
$M = 0.349Wr$ ◀



PROBLEM 8.137

End A of a slender, uniform rod of length L and weight W bears on a surface as shown, while end B is supported by a cord BC. Knowing that the coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine (*a*) the largest value of θ for which motion is impending, (*b*) the corresponding value of the tension in the cord.

SOLUTION



Free-body diagram

Three-force body. Line of action of **R** must pass through D, where **T** and **R** intersect.

Motion impends:

$$\tan\phi_s = 0.4$$

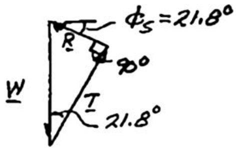
$$\phi_s = 21.80^\circ$$

(*a*) Since $BG = GA$, it follows that $BD = DC$ and AD bisects $\angle BAC$

$$\frac{\theta}{2} + \phi_s = 90^\circ$$

$$\frac{\theta}{2} + 21.8^\circ = 90^\circ$$

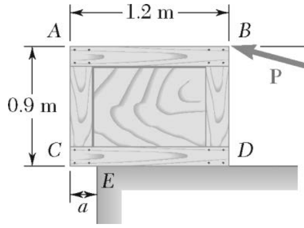
$\theta = 136.4^\circ$ ◀



(*b*) Force triangle (right triangle):

$$T = W\cos 21.8^\circ$$

$T = 0.928W$ ◀



PROBLEM 8.138

A worker slowly moves a 50-kg crate to the left along a loading dock by applying a force **P** at corner B as shown. Knowing that the crate starts to tip about the edge E of the loading dock when $a = 200$ mm, determine (*a*) the coefficient of kinetic friction between the crate and the loading dock, (*b*) the corresponding magnitude P of the force.

SOLUTION

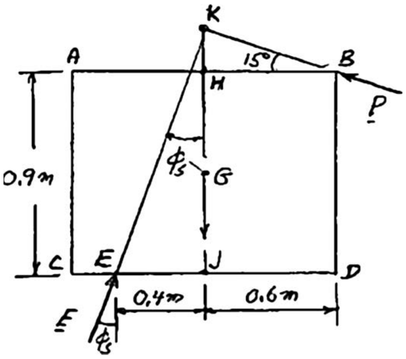
Free body: Crate Three-force body.

Reaction E must pass through K where **P** and **W** intersect.

Geometry:

(*a*)

$$HK = (0.6\text{ m})\tan 15° = 0.16077\text{ m}$$

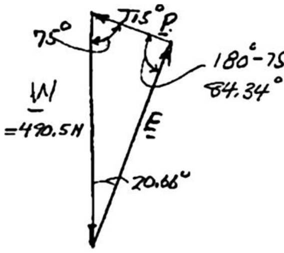


$$JK = 0.9\text{ m} + HK = 1.06077\text{ m}$$

$$\tan\phi_s = \frac{0.4\text{ m}}{1.06077\text{ m}} = 0.37708 \qquad \mu_s = \tan\phi_s = 0.377 \blacktriangleleft$$

$$\phi_s = 20.66°$$

Force triangle:



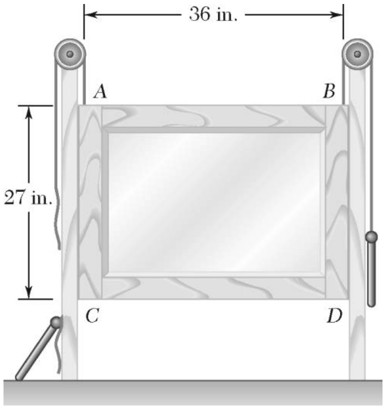
(*b*)

$$W = (50\text{ kg})(9.81\text{ m/s})$$
$$= 490.5\text{ N}$$

Law of sines:

$$\frac{P}{\sin 20.66°} = \frac{490.5\text{ N}}{\sin 84.34°}$$

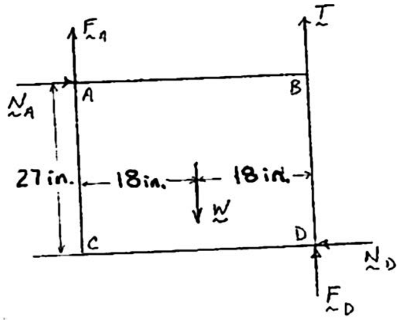
$$P = 173.91\text{ N} \qquad P = 173.9\text{ N} \blacktriangleleft$$



PROBLEM 8.139

A window sash weighing 10 lb is normally supported by two 5-lb sash weights. Knowing that the window remains open after one sash cord has broken, determine the smallest possible value of the coefficient of static friction. (Assume that the sash is slightly smaller than the frame and will bind only at Points *A* and *D*.)

SOLUTION



FBD window: $T = 5\text{ lb}$

$$\xrightarrow{} \Sigma F_x = 0: \quad N_A - N_D = 0$$

$$N_A = N_D$$

Impending motion:

$$F_A = \mu_s N_A$$

$$F_D = \mu_s N_D$$

$$\circlearrowleft \Sigma M_D = 0: \quad (18\text{ in.})W - (27\text{ in.})N_A - (36\text{ in.})F_A = 0$$

$W = 10\text{ lb}$

$$W = \frac{3}{2}N_A + 2\mu_s N_A$$

$$N_A = \frac{2W}{3+4\mu_s}$$

$$\uparrow \Sigma F_y = 0: \quad F_A - W + T + F_D = 0$$

$$F_A + F_D = W - T = \frac{W}{2}$$

Now

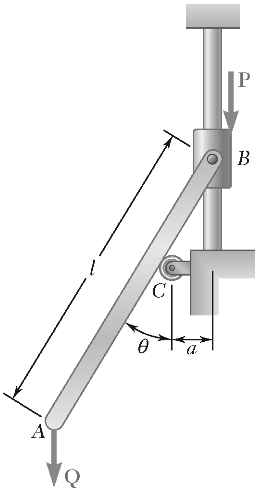
$$F_A + F_D = \mu_s(N_A + N_D)$$

$$= 2\mu_s N_A$$

Then

$$\frac{W}{2} = 2\mu_s \frac{2W}{3+4\mu_s}$$

or $\mu_s = 0.750$ ◀



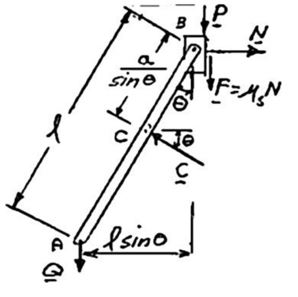
PROBLEM 8.140

The slender rod AB of length $l = 600$ mm is attached to a collar at B and rests on a small wheel located at a horizontal distance $a = 80$ mm from the vertical rod on which the collar slides. Knowing that the coefficient of static friction between the collar and the vertical rod is 0.25 and neglecting the radius of the wheel, determine the range of values of P for which equilibrium is maintained when $Q = 100$ N and $\theta = 30°$.

SOLUTION

For motion of collar at B impending upward:

$$\mathbf{F} = \mu_s N \downarrow$$



$$+\circlearrowleft \Sigma M_B = 0: \quad Ql\sin\theta - \frac{Ca}{\sin\theta} = 0$$

$$C = Q\left(\frac{l}{a}\right)\sin^2\theta$$

$$\Sigma F_x = 0: \quad N = C\cos\theta = Q\left(\frac{l}{a}\right)\sin^2\theta\cos\theta$$

$$+\downarrow \Sigma F_y = 0: \quad P + Q - C\sin\theta - \mu_s N = 0$$

$$P + Q - Q\left(\frac{l}{a}\right)\sin^3\theta - \mu_s Q\left(\frac{l}{a}\right)\sin^2\theta\cos\theta = 0$$

$$P = Q\left[\frac{l}{a}\sin^2\theta(\sin\theta - \mu_s\cos\theta) - 1\right] \tag{1}$$

Substitute data:

$$P = (100\text{ N})\left[\frac{600\text{ mm}}{80\text{ mm}}\sin^2 30°(\sin 30° - 0.25\cos 30°) - 1\right]$$

$$P = -46.84\text{ N } (P \text{ is directed } \uparrow)$$

$$P = -46.8\text{ N} \triangleleft$$

PROBLEM 8.140 (Continued)

For motion of collar, impending downward:

$$\mathbf{F} = \mu_s N \uparrow$$

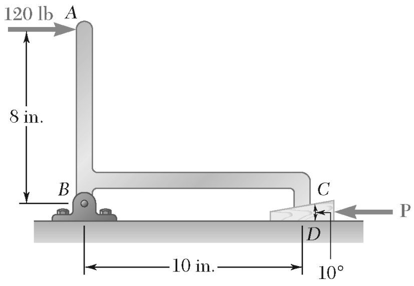
In Eq. (1) we substitute $-\mu_s$ for μ_s.

$$P = Q\left[\frac{l}{a}\sin^2\theta(\sin\theta + \mu_s\cos\theta) - 1\right]$$

$$P = (100\text{ N})\left[\frac{600\text{ mm}}{80\text{ mm}}\sin^2 30°(\sin 30° + 0.25\cos\theta) - 1\right]$$

$$P = +34.34\text{ N} \quad \triangleleft$$

For equilibrium: $-46.8\text{ N} \le P \le 34.3\text{ N}$ ◀



PROBLEM 8.141

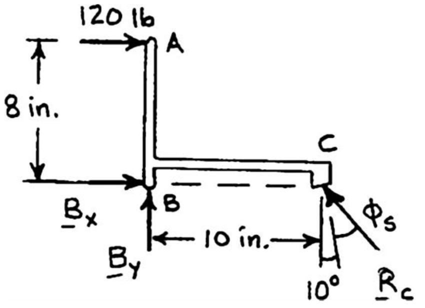
The machine part *ABC* is supported by a frictionless hinge at *B* and a 10° wedge at *C*. Knowing that the coefficient of static friction is 0.20 at both surfaces of the wedge, determine (*a*) the force **P** required to move the wedge to the left, (*b*) the components of the corresponding reaction at *B*.

SOLUTION

$$\mu_s = 0.20$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.20 = 11.3099^\circ$$

Free body: *ABC*

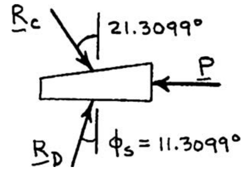


$$10^\circ + 11.3099^\circ = 21.3099^\circ$$

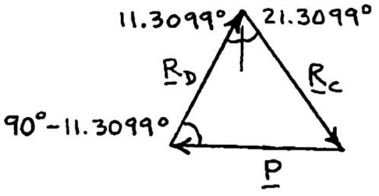
$$+\circlearrowleft \Sigma M_B = 0: \quad (R_C \cos 21.3099^\circ)(10) - (120\text{ lb})(8) = 0$$

$$R_C = 103.045\text{ lb}$$

Free body: Wedge



Force triangle:



PROBLEM 8.141 (Continued)

Law of sines:

$$\frac{P}{\sin 32.6198^\circ} = \frac{(R_C = 103.045 \text{ lb})}{\sin 78.690^\circ}$$

(*a*) $\mathbf{P} = 56.6 \text{ lb} \leftarrow$ ◀

(*b*) Returning to free body of *ABC*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 120 - (103.045)\sin 21.3099^\circ = 0$$

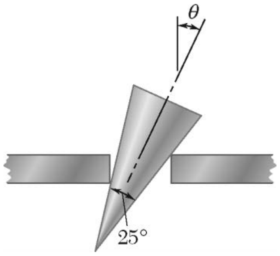
$$B_x = -82.552 \text{ lb}$$

$\mathbf{B}_x = 82.6 \text{ lb} \leftarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y + (103.045)\cos 21.3099^\circ = 0$$

$$B_y = -96.000 \text{ lb}$$

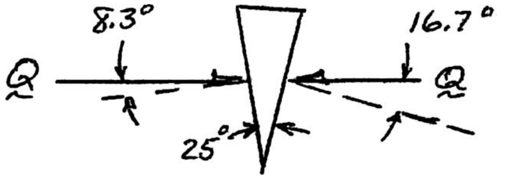
$\mathbf{B}_y = 96.0 \text{ lb} \downarrow$ ◀



PROBLEM 8.142

A conical wedge is placed between two horizontal plates that are then slowly moved toward each other. Indicate what will happen to the wedge (*a*) if $\mu_s = 0.20$, (*b*) if $\mu_s = 0.30$.

SOLUTION



As the plates are moved, the angle θ will decrease.

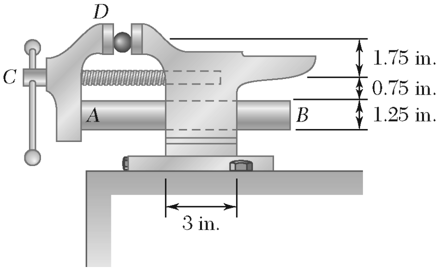
(*a*) $\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.2 = 11.31°$.

As θ decrease, the minimum angle at the contact approaches

$12.5° > \phi_s = 11.31°$, so the wedge will slide up and out from the slot. ◀

(*b*) $\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.3 = 16.70°$.

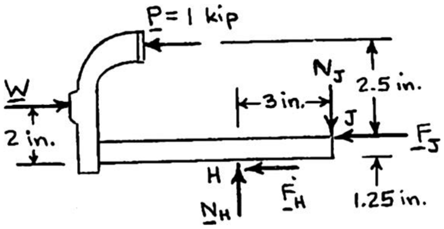
As θ decreases, the angle at one contact reaches 16.7°. (At this time the angle at the other contact is $25° - 16.7° = 8.3° < \phi_s$). The wedge binds in the slot. ◀



PROBLEM 8.143

In the machinist's vise shown, the movable jaw *D* is rigidly attached to the tongue *AB* that fits loosely into the fixed body of the vise. The screw is single-threaded into the fixed base and has a mean diameter of 0.75 in. and a pitch of 0.25 in. The coefficient of static friction is 0.25 between the threads and also between the tongue and the body. Neglecting bearing friction between the screw and the movable head, determine the couple that must be applied to the handle in order to produce a clamping force of 1 kip.

SOLUTION



Free body: Jaw *D* and tongue *AB*

P is due to elastic forces in clamped object.

W is force exerted by screw.

$$+\uparrow \Sigma F_y = 0: \quad N_H - N_J = 0 \quad N_J = N_H = N$$

For final tightening,

$$F_H = F_J = \mu_s N = 0.25\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad W - P - 2(0.25\text{ N}) = 0$$

$$N = 2(W - P) \qquad (1)$$

$$+\circlearrowleft \Sigma M_H = 0: \quad P(3.75) - W(2) - N(3) + (0.25\text{ N})(1.25) = 0$$

$$3.75P - 2W - 2.6875\text{ N} = 0 \qquad (2)$$

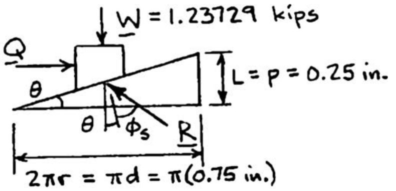
Substitute Eq. (1) into Eq. (2):

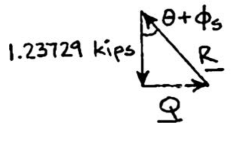
$$3.75P - 2W - 2.6875[2(W - P)] = 0$$

$$7.375W = 9.125P = 9.125(1\text{ kip})$$

$$W = 1.23729\text{ kips}$$

Block-and-incline analysis of screw:





$$\tan\phi_s = \mu_s = 0.25$$

$$\phi_s = 14.0362°$$

$$\tan\theta = \frac{0.25\text{ in.}}{\pi(0.75\text{ in.})}$$

$$\theta = 6.0566°$$

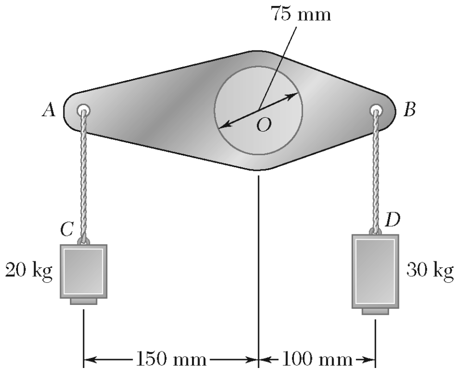
$$\theta + \phi_s = 20.093°$$

$$Q = (1.23729\text{ kips})\tan 20.093°$$

$$= 0.45261\text{ kip}$$

$$T = Qr = (452.61\text{ lb})\left(\frac{0.75\text{ in.}}{2}\right)$$

$$T = 169.7\text{ lb}\cdot\text{in.} \blacktriangleleft$$

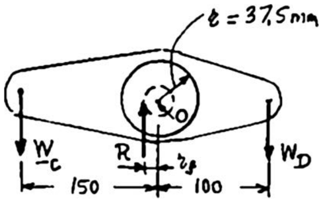


PROBLEM 8.144

A lever of negligible weight is loosely fitted onto a 75-mm-diameter fixed shaft. It is observed that the lever will just start rotating if a 3-kg mass is added at *C*. Determine the coefficient of static friction between the shaft and the lever.

SOLUTION

$$+\circlearrowleft \Sigma M_O = 0: \quad W_C(150) - W_D(100) - Rr_f = 0$$



But $$W_C = (23\text{ kg})(9.81\text{ m/s}^2)$$

$$W_D = (30\text{ kg})(9.81\text{ m/s}^2)$$

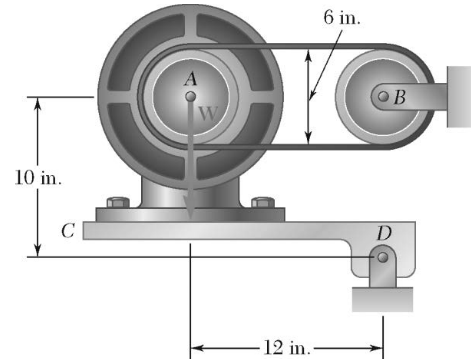
$$R = W_C + W_D = (53\text{ kg})(9.81)$$

Thus, after dividing by 9.81,

$$23(150) - 30(100) - 53\,r_f = 0$$

$$r_f = 8.49\text{ mm}$$

But $$\mu_s \approx \frac{r_f}{r} = \frac{8.49\text{ mm}}{37.5\text{ mm}} \qquad \mu_s \approx 0.226 \blacktriangleleft$$

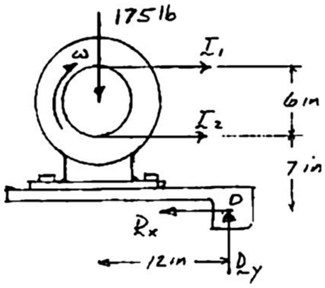


PROBLEM 8.145

In the pivoted motor mount shown the weight **W** of the 175-lb motor is used to maintain tension in the drive belt. Knowing that the coefficient of static friction between the flat belt and drums *A* and *B* is 0.40, and neglecting the weight of platform *CD*, determine the largest couple that can be transmitted to drum *B* when the drive drum *A* is rotating clockwise.

SOLUTION

FBD motor and mount:



Impending belt slip: cw rotation

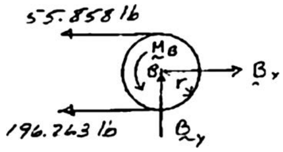
$$T_2 = T_1 e^{\mu_s \beta} = T_1 e^{0.40\pi} = 3.5136 T_1$$

$$\circlearrowleft \Sigma M_D = 0: \quad (12\text{ in.})(175\text{ lb}) - (7\text{ in.})T_2 - (13\text{ in.})T_1 = 0$$

$$2100\text{ lb} = [(7\text{ in.})(3.5136) + 13\text{ in.}]T_1$$

$$T_1 = 55.858\text{ lb}, \quad T_2 = 3.5136 T_1 = 196.263\text{ lb}$$

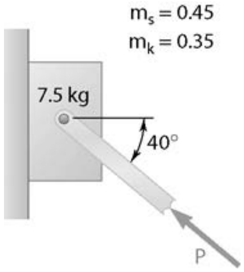
FBD drum at *B*:



$$\circlearrowleft \Sigma M_B = 0: \quad M_B - (3\text{ in.})(196.263\text{ lb} - 55.858\text{ lb}) = 0$$

$r = 3\text{ in.}$

$M_B = 421\text{ lb}\cdot\text{in.}$ ◀



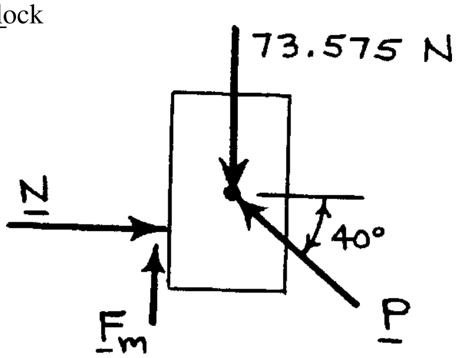
PROBLEM 8.F1

Draw the free-body diagram needed to determine the smallest force **P** for which equilibrium of the 7.5-kg block is maintained.

SOLUTION

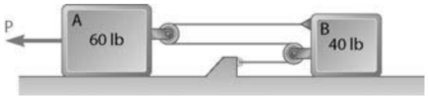
$$W = (7.5 \text{ kg})(9.81 \text{ m/s}^2) = 73.575 \text{ N}$$

Free body: Block



Since we seek smallest P, motion impends downward, with

$$F_m = \mu_s N = 0.45 \text{ N}$$

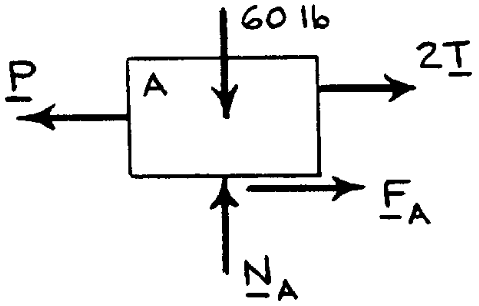


PROBLEM 8.F2

Two blocks *A* and *B* are connected by a cable as shown. Knowing that the coefficient of static friction at all surfaces of contact is 0.30 and neglecting the friction of the pulleys, draw the free-body diagrams needed to determine the smallest force **P** required to move the blocks.

SOLUTION

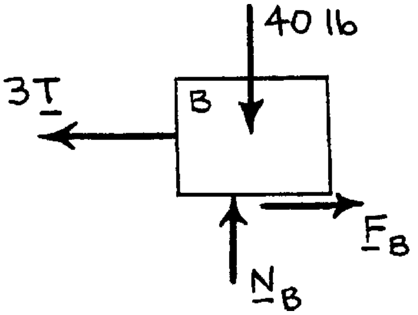
Free body: Block *A*



Motion impends to left, with

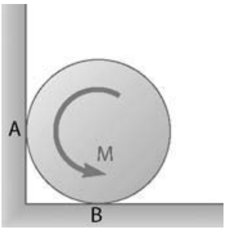
$F_A = \mu_S N_A = 0.30 N_A$

Free body: Block *B*



Motion impends to left, with

$F_B = \mu_s N_B = 0.30\ N_B$

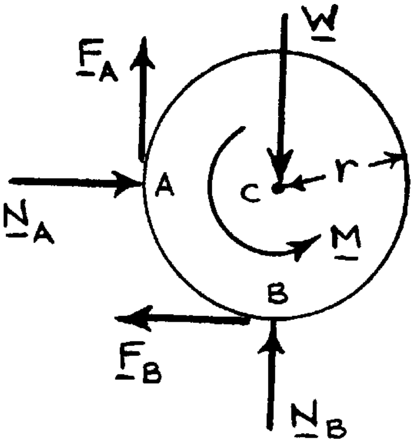


PROBLEM 8.F3

The cylinder shown is of weight W and radius r, and the coefficient of static friction μ_s is the same at A and B. Draw the free-body diagram needed to determine the largest couple **M** that can be applied to the cylinder if it is not to rotate.

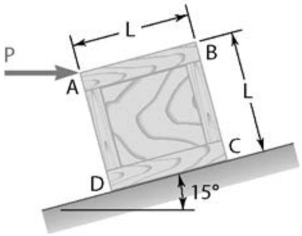
SOLUTION

Free body: Cylinder



For maximum **M**, motion impends at both A and B, with

$$F_A = \mu_s N_A$$
$$F_B = \mu_S N_B$$



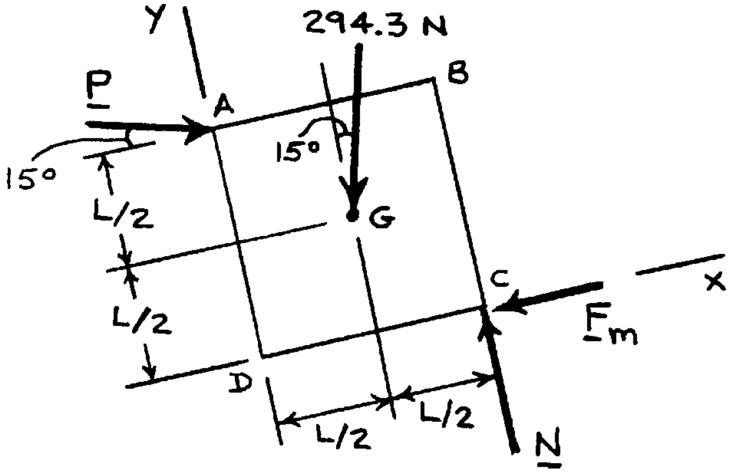
PROBLEM 8.F4

A uniform crate of mass 30 kg must be moved up along the 15° incline without tipping. Knowing that the force **P** is horizontal, draw the free-body diagram needed to determine the largest allowable coefficient of static friction between the crate and the incline, and the corresponding force **P**.

SOLUTION

$$W = (30\text{ kg})\,(9.81\text{ m/s}^2) = 294.3\text{ N}$$

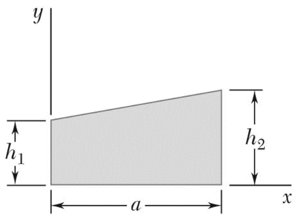
Free body: Crate



For impending tip, **N** must act at C.

For impending slip, $F_m = \mu_{\max} N$

CHAPTER 9

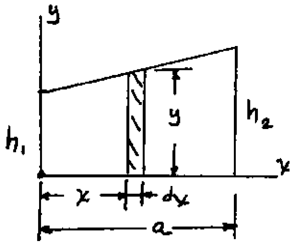


PROBLEM 9.1

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION

$$y = h_1 + (h_2 - h_1)\frac{x}{a};\ dA = y\,dx$$



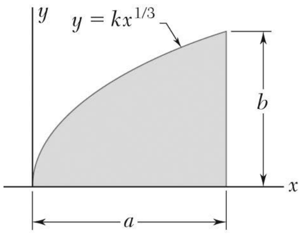
$$dI_y = x^2 dA = x^2\left[h_1 + (h_2 - h_1)\frac{x}{a}\right]dx$$

$$I_y = \int_0^a \left[h_1 x^2 + \frac{h_2 - h_1}{a}x^3\right]dx$$

$$= h_1\frac{a^3}{3} + \frac{h_2 - h_1}{a}\frac{a^4}{4}$$

$$= \frac{b_1 a^3}{12} + \frac{h_2 a^3}{4}$$

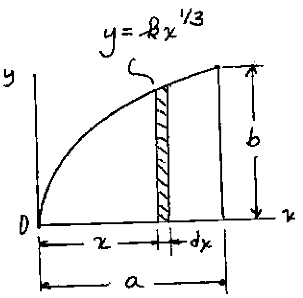
$$I_y = \frac{a^3}{12}(h_1 + 3h_2) \blacktriangleleft$$



PROBLEM 9.2

Determine by direct integration the moment of inertia of the shaded area with respect to the *y* axis.

SOLUTION



$$y = kx^{1/3}$$

For $x = a$: $$b = ka^{1/3}$$

$$k = b/a^{1/3}$$

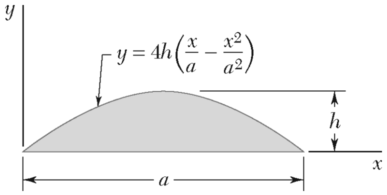
Thus: $$y = \frac{b}{a^{1/3}}x^{1/3}$$

$$dI_y = x^2 dA = x^2 y dx$$

$$dI_y = x^2 \frac{b}{a^{1/3}}x^{1/3}dx = \frac{b}{a^{1/3}}x^{7/3}dx$$

$$I_y = \int dI_y = \frac{b}{a^{1/3}}\int_0^a x^{7/3}dx = \frac{b}{a^{1/3}}\left(\frac{3}{10}a^{10/3}\right)$$

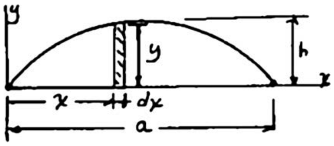
$$I_y = \frac{3}{10}a^3 b \blacktriangleleft$$



PROBLEM 9.3

Determine by direct integration the moment of inertia of the shaded area with respect to the *y* axis.

SOLUTION



$$y = 4h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)$$

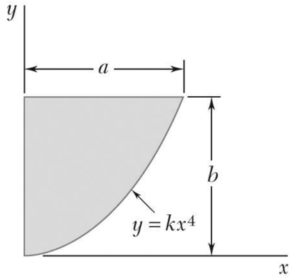
$$dA = ydx$$

$$dI_y = x^2 dA = 4hx^2\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx$$

$$I_y = 4h\int_0^a\left(\frac{x^3}{a} - \frac{x^4}{a^2}\right)dx$$

$$I_y = 4h\left[\frac{x^4}{4a} - \frac{x^5}{5a^2}\right]_0^a = 4h\left(\frac{a^3}{4} - \frac{a^3}{5}\right)$$

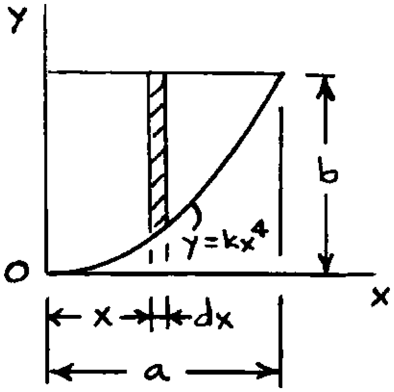
$$I_y = \frac{1}{5}ha^3 \blacktriangleleft$$



PROBLEM 9.4

Determine by direct integration the moment of inertia of the shaded area with respect to the *y* axis.

SOLUTION



$$y = kx^4$$

For $x = a$:

$$b = ka^4$$

$$k = \frac{b}{a^4}$$

Thus:

$$y = \frac{b}{a^4}x^4$$

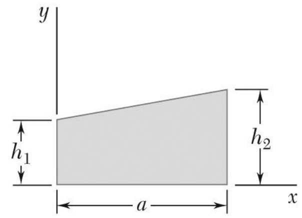
$$dA = (b - y)dx$$

$$dI_y = x^2 da = x^2(b - y)dx$$

$$= x^2\left(b - \frac{b}{a^4}x^4\right)dx$$

$$I_y = \int dI_y = \int_0^a \left(bx^2 - \frac{b}{a^4}x^6\right)dx = \frac{1}{3}a^3b - \frac{1}{7}a^3b$$

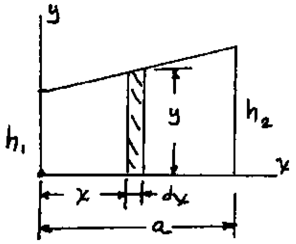
$I_y = 4a^3b/21$ ◀



PROBLEM 9.5

Determine by direct integration the moment of inertia of the shaded area with respect to the *x* axis.

SOLUTION



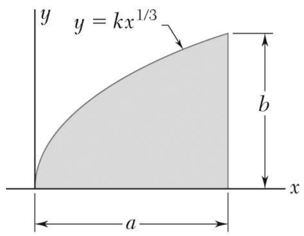
$$y = h_1 + (h_2 - h_1)\frac{x}{a} \qquad dI_x = \frac{1}{3}y^3 dx$$

$$I_x = \int dI_y = \frac{1}{3}\int_0^a \left[h_1 + (h_2 - h_1)\frac{x}{a}\right]^3 dx$$

$$= \frac{1}{12}\left|\left[h_1 + (h_2 - h_1)\frac{x}{a}\right]^4 \left(\frac{a}{h_2 - h_1}\right)\right|_0^a$$

$$= \frac{a}{12(h_2 - h_1)}\left(h_2^4 - h_1^4\right) = \frac{a}{12}\cdot\frac{(h_2^2 + h_1^2)(h_2 + h_1)(h_2 - h_1)}{h_2 - h_1}$$

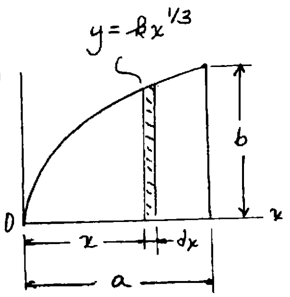
$$I_x = \frac{a}{12}(h_1^2 + h_2^2)(h_1 + h_2) \blacktriangleleft$$



PROBLEM 9.6

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



$$y = \frac{b}{a^{1/3}} x^{1/3}$$

$$dI_x = \frac{1}{3} y^3 dx = \frac{1}{3}\left(\frac{b}{a^{1/3}} x^{1/3}\right)^3 dx = \frac{1}{3}\frac{b^3}{a} x dx$$

$$I_x = \int dI_x = \int_0^a \frac{1}{3}\frac{b^3}{a} x dx = \frac{1}{3}\frac{b^3}{a}\frac{a^2}{2}$$

$$I_x = \frac{1}{6} ab^3 \blacktriangleleft$$

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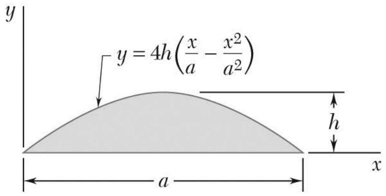
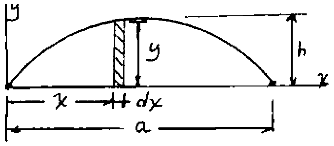


PROBLEM 9.7

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



$$y = 4h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)$$

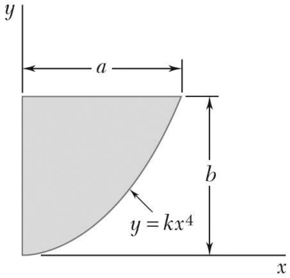
$$dI_x = \frac{1}{3}y^3dx = \frac{1}{3}\left[4h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)\right]^3 dx$$

$$I_x = \int dI_x = \frac{64h^3}{3}\int_0^a\left(\frac{x^3}{a^3} - 3\frac{x^4}{a^4} + 3\frac{x^5}{a^5} - \frac{x^6}{a^6}\right)dx$$

$$= \frac{64h^3}{3}\left|\frac{1}{4}\frac{x^4}{a^3} - \frac{3}{5}\frac{x^5}{a^4} + \frac{1}{2}\frac{x^6}{a^5} - \frac{1}{7}\frac{x^7}{a^6}\right|_0^a$$

$$= \frac{64h^3}{3}a\left(\frac{1}{4} - \frac{3}{5} + \frac{1}{2} - \frac{1}{7}\right) = \frac{64h^3}{3}a\left(\frac{3}{420}\right)$$

$$I_x = \frac{16}{105}ah^3 \blacktriangleleft$$

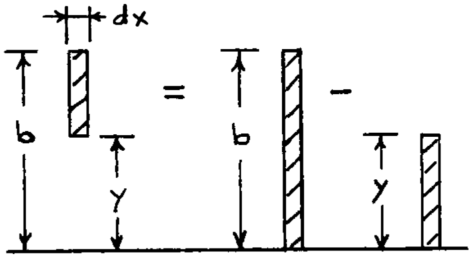


PROBLEM 9.8

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION

See figure of solution of Problem 9.4.



$$y = \frac{b}{a^4}x^4$$

$$dI_x = \frac{1}{3}b^3dx - \frac{1}{3}y^3dx = \frac{1}{3}b^3dx - \frac{1}{3}\frac{b^3}{a^{12}}x^{12}dx$$

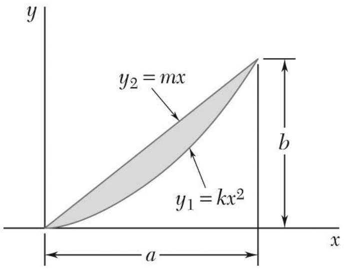
$$I_x = \int dI_x = \int_0^a\left(\frac{1}{3}b^3 - \frac{1}{3}\frac{b^3}{a^{12}}x^{12}\right)dx$$

$$= \frac{1}{3}b^3a - \frac{1}{3}\frac{b^3}{a^{12}}\frac{a^{13}}{13}$$

$$= \left(\frac{1}{3} - \frac{1}{39}\right)ab^3$$

$$= \left(\frac{13}{39} - \frac{1}{39}\right)ab^3 = \frac{12}{39}ab^3$$

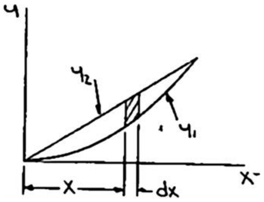
$$I_x = 4ab^3/13 \blacktriangleleft$$



PROBLEM 9.9

Determine by direct integration the moment of inertia of the shaded area with respect to the *x* axis.

SOLUTION



At $$x_1 = a, \quad y_1 = y_2 = b$$

$$y_1: \quad b = ka^2 \quad \text{or} \quad k = \frac{b}{a^2}$$

$$y_2: \quad b = ma \quad \text{or} \quad m = \frac{b}{a}$$

Then $$y_1 = \frac{b}{a^2}x^2$$

$$y_2 = \frac{b}{a}x$$

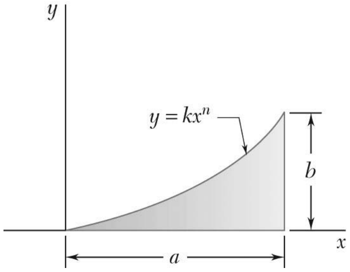
Now $$dI_x = \frac{1}{3}\left(y_2^3 - y_1^3\right)dx$$

$$= \frac{1}{3}\left(\frac{b^3}{a^3}x^3 - \frac{b^3}{a^6}x^6\right)dx$$

Then $$I_x = \int dI_x = \int_0^a \frac{b^3}{3}\left(\frac{1}{a^3}x^3 - \frac{1}{a^6}x^6\right)dx$$

$$= \frac{b^3}{3}\left[\frac{1}{4a^3}x^4 - \frac{1}{7a^6}x^7\right]_0^a$$

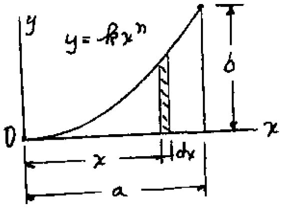
or $I_x = \frac{1}{28}ab^3$ ◀



PROBLEM 9.10

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



$$y = kx^n$$

For $x = a$: $b = ka^n$

$$k = b/a^n$$

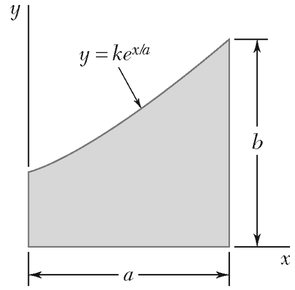
Thus: $y = \frac{b}{a^n}x^n$

$$y = \frac{b}{a^n}x^n$$

$$dI_x = \frac{1}{3}y^3 dx = \frac{1}{3}\frac{b^3}{a^{3n}}x^{3n}dx$$

$$I_x = \int dI_x = \frac{b^3}{3a^{3n}}\int_0^a x^{3n}dx = \frac{b^3}{3a^{3n}}\frac{a^{3n+1}}{(3n+1)}$$

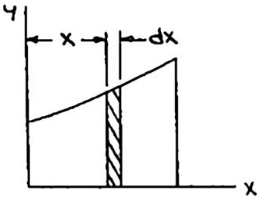
$$I_x = \frac{ab^3}{3(3n+1)} \blacktriangleleft$$



PROBLEM 9.11

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



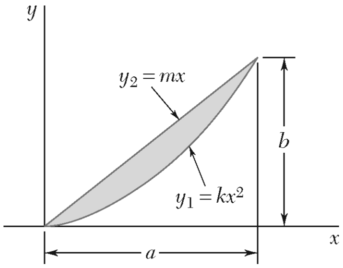
At $x = a,\ y = b$: $$b = ke^{a/a} \quad \text{or} \quad k = \frac{b}{e}$$

Then $$y = \frac{b}{e}e^{x/a} = be^{x/a-1}$$

Now $$dI_x = \frac{1}{3}y^3dx = \frac{1}{3}(be^{x/a-1})^3dx$$
$$= \frac{1}{3}b^3e^{3(x/a-1)}dx$$

Then $$I_x = \int dI_x = \int_0^a \frac{1}{3}b^3e^{3(x/a-1)}dx = \frac{b^3}{3}\left[\frac{a}{3}e^{3(x/a-1)}\right]_0^a$$
$$= \frac{1}{9}ab^3(1 - e^{-3})$$

or $I_x = 0.1056ab^3$ ◀



PROBLEM 9.12

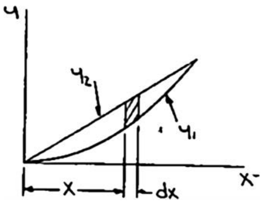
Determine by direct integration the moment of inertia of the shaded area with respect to the *y* axis.

SOLUTION

At $$x_1 = a, \quad y_1 = y_2 = b$$

$$y_1: \qquad b = ka^2 \quad \text{or} \quad k = \frac{b}{a^2}$$

$$y_2: \qquad b = ma \quad \text{or} \quad m = \frac{b}{a}$$



Then $$y_1 = \frac{b}{a^2}x^2$$

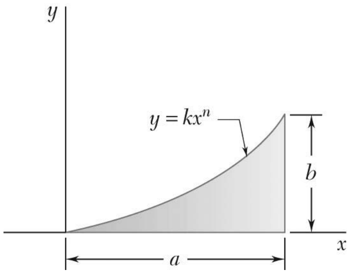
$$y_2 = \frac{b}{a}x$$

Now $$dI_y = x^2 dA = x^2[(y_2 - y_1)dx = x^2\left[\left(\frac{b}{a}x - \frac{b}{a^2}x^2\right)dx\right]$$

Then $$I_y = \int dI_y = \int_0^a b\left(\frac{1}{a}x^3 - \frac{1}{a^2}x^4\right)dx$$

$$= b\left[\frac{1}{4a}x^4 - \frac{1}{5a^2}x^5\right]_0^a$$

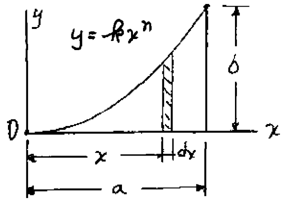
or $$I_y = \frac{1}{20}a^3b \blacktriangleleft$$



PROBLEM 9.13

Determine by direct integration the moment of inertia of the shaded area with respect to the *x* axis.

SOLUTION



$$y = kx^n$$

For $x = a$: $$b = ka^n$$

$$k = b/a^n$$

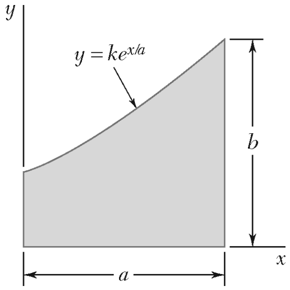
Thus: $$y = \frac{b}{a^n} x^n$$

$$dI_y = x^2 dA = x^2 y dx$$

$$dI_y = x^2 \frac{b}{a^n} x^n dx = \frac{b}{a^n} x^{n+2} dx$$

$$I_y = \int dI_y = \frac{b}{a^n} \int_0^a x^{n+2} dx = \frac{b}{a^n} \frac{a^{n+3}}{n+3}$$

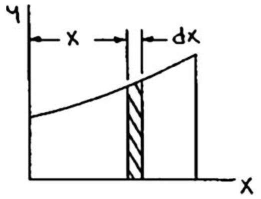
$$I_y = \frac{a^3 b}{n+3} \blacktriangleleft$$



PROBLEM 9.14

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION



At $x = a, \quad y = b$: $\quad b = ke^{a/a}$

or $\quad k = \dfrac{b}{e}$

Then $\quad y = \dfrac{b}{e}e^{x/a} = be^{x/a-1}$

Now
$$dI_y = x^2 dA = x^2(ydx)$$
$$= x^2(be^{x/a-1}dx)$$

Then
$$I_y = \int dI_y = \int_0^a bx^2 e^{x/a-1}dx$$

Now use integration by parts with

$$u = x^2 \qquad dv = e^{x/a-1}dx$$
$$du = 2xdx \qquad v = ae^{x/a-1}$$

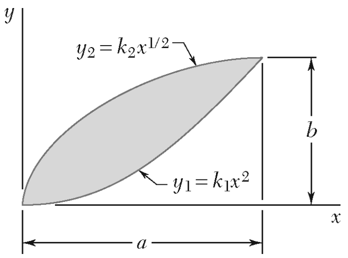
Then
$$\int_0^a x^2 e^{x/a-1}dx = \left[x^2 ae^{x/a-1}\right]_0^a - \int_0^a (ae^{x/a-1})2xdx$$
$$= a^3 - 2a\int_0^a xe^{x/a-1}dx$$

Using integration by parts with

$$u = x \qquad dv = e^{x/a-1}dx$$
$$du = dx \qquad v = ae^{x/a-1}$$

Then
$$I_y = b\left\{a^3 - 2a\left[(xae^{x/a-1})\big|_0^a - \int_0^a (ae^{x/a-1})dx\right]\right\}$$
$$= b\left\{a^3 - 2a\left[a^2 - (a^2e^{x/a-1})\big|_0^a\right]\right\}$$
$$= b\left\{a^3 - 2a\left[a^2 - (a^2 - a^2e^{-1})\right]\right\}$$

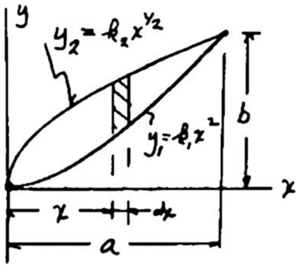
or $\quad I_y = 0.264a^3b$ ◀



PROBLEM 9.15

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *x* axis.

SOLUTION



$$y_1 = k_1 x^2 \qquad y_2 = k_2 x^{1/2}$$

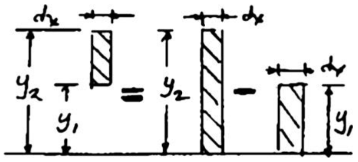
For $x = 0$ and $y_1 = y_2 = b$

$$b = k_1 a^2 \qquad b = k_2 a^{1/2}$$

$$k_1 = \frac{b}{a^2} \qquad k_2 = \frac{b}{a^{1/2}}$$

Thus,

$$y_1 = \frac{b}{a^2} x^2 \qquad y_2 = \frac{b}{a^{1/2}} x^{1/2}$$



$$dA = (y_2 - y_1)dx$$

$$A = \int_0^a \left[\frac{b}{a^{1/2}} x^{1/2} - \frac{b}{a^2} x^2\right] dx$$

$$A = \frac{2}{3}\frac{ba^{3/2}}{a^{1/2}} - \frac{ba^3}{3a^2}$$

$$A = \frac{1}{3} ab$$

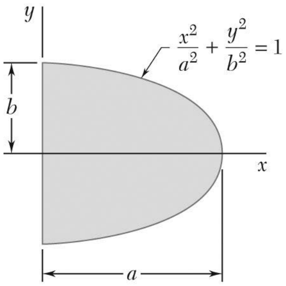
$$dI_x = \frac{1}{3} y_2^3 dx - \frac{1}{3} y_1^3 dx$$

$$= \frac{1}{3}\frac{b^3}{a^{3/2}} x^{3/2} dx - \frac{1}{3}\frac{b^3}{a^6} x^6 dx$$

$$I_x = \int dI_x = \frac{b^3}{3a^{3/2}} \int_0^a x^{3/2} dx - \frac{b^3}{3a^6} \int_0^a x^6 dx$$

$$= \frac{b^3}{3a^{3/2}} \frac{a^{5/2}}{\left(\frac{5}{2}\right)} - \frac{b^3}{3a^6}\frac{a^7}{7} = \left(\frac{2}{15} - \frac{1}{21}\right) ab^3 \qquad I_x = \frac{3}{35} ab^3 \blacktriangleleft$$

$$k_x^2 = \frac{I_x}{A} = \frac{\left(\frac{3}{35} ab^3\right)}{\frac{ab}{b}} \qquad k_x = b\sqrt{\frac{9}{35}} \blacktriangleleft$$



PROBLEM 9.16

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *x* axis.

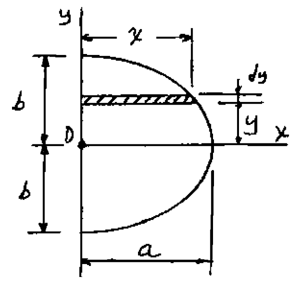
SOLUTION

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$x=a\sqrt{1-\frac{y^2}{b^2}}$$

$$dA=xdy$$

$$dI_x=y^2dA=y^2xdy$$



$$I_x=\int dI_x=\int_{-b}^{b}xy^2dy=a\int_{-b}^{b}y^2\sqrt{1-\frac{y^2}{b^2}}\,dy$$

Set: $y=b\sin\theta \qquad dy=b\cos\theta\, d\theta$

$$I_x=a\int_{-\pi/2}^{\pi/2}b^2\sin^2\theta\sqrt{1-\sin^2\theta}\quad b\cos\theta\, d\theta$$

$$=ab^3\int_{-\pi/2}^{\pi/2}\sin^2\theta\cos^2\theta\, d\theta=ab^3\int_{-\pi/2}^{\pi/2}\frac{1}{4}\sin^2 2\theta\, d\theta$$

$$=\frac{1}{4}ab^3\int_{-\pi/2}^{\pi/2}\frac{1}{2}(1-\cos 4\theta)d\theta=\frac{1}{8}ab^3\left[\theta-\frac{1}{4}\sin 4\theta\right]_{-\pi/2}^{\pi/2}$$

$$=\frac{1}{8}ab^3\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right]=\frac{\pi}{8}ab^2$$

$I_x=\frac{1}{8}\pi ab^3$ ◀

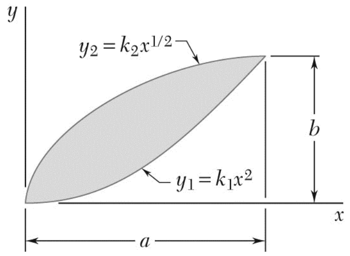
$$A=\int dA=\int_{-b}^{+b}xdy=a\int_{-b}^{b}\sqrt{1-\frac{y^2}{b^2}}\,dy=a\int_{-\pi/2}^{\pi/2}\sqrt{1-\sin^2\theta}\;b\cos\theta\, d\theta$$

$$=ab\int_{-\pi/2}^{\pi/2}\cos^2\theta\, d\theta=ab\int_{-\pi/2}^{\pi/2}\frac{1}{2}(1+\cos 2\theta)\, d\theta$$

$$=ab\left|\theta+\frac{1}{2}\sin 2\theta\right|_{-\pi/2}^{\pi/2}=\frac{ab}{2}\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right]=\frac{1}{2}\pi ab$$

$$I_x=k_x^2A \quad k_x^2=\frac{I_x}{A}=\frac{\frac{1}{8}\pi ab^3}{\frac{1}{2}\pi ab}=\frac{1}{4}b^2$$

$k_x=\frac{1}{2}b$ ◀



PROBLEM 9.17

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *y* axis.

SOLUTION

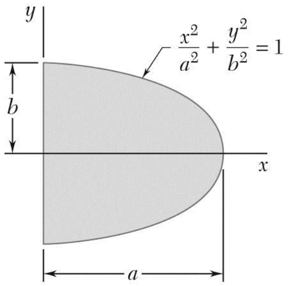
See figure of solution on Problem 9.15.

$$A = \frac{1}{3}ab \qquad dI_y = x^2 dA = x^2(y_2 - y_1)dx$$

$$I_y = \int_0^a x^2\left(\frac{b}{a^{1/2}}x^{1/2} - \frac{b}{a^2}x^2\right)dx = \frac{b}{a^{1/2}}\int_0^a x^{5/2}dx - \frac{b}{a^2}\int_0^a x^4 dx$$

$$I_y = \frac{b}{a^{1/2}} \cdot \frac{b^{7/2}}{\left(\frac{7}{2}\right)} - \frac{b}{a^2} \cdot \frac{a^5}{5} = \left(\frac{2}{7} - \frac{1}{5}\right)a^3 b \qquad I_y = \frac{3}{35}a^3 b \blacktriangleleft$$

$$k_y^2 = \frac{I_y}{A} = \frac{\left(\frac{3}{35}a^3 b\right)}{\frac{ab}{3}} \qquad k_y = a\sqrt{\frac{9}{35}} \blacktriangleleft$$



PROBLEM 9.18

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *y* axis.

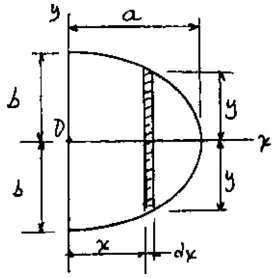
SOLUTION

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$y=b\sqrt{1-\frac{x^2}{a^2}}$$

$$dA=2y\,dx$$

$$dI_y=x^2dA=2x^2y\,dx$$



$$I_y=\int dI_y=\int_0^a 2x^2y\,dx=2b\int_0^a x^2\sqrt{1-\frac{x^2}{a^2}}\,dx$$

Set: $x=a\sin\theta \qquad dx=a\cos\theta\,d\theta$

$$I_y=2b\int_0^{\pi/2}a^2\sin^2\theta\sqrt{1-\sin^2\theta}\quad a\cos\theta\,d\theta$$

$$=2a^3b\int_0^{\pi/2}\sin^2\theta\cos^2\theta\,d\theta=2a^3b\int_0^{\pi/2}\frac{1}{4}\sin^2 2\theta\,d\theta$$

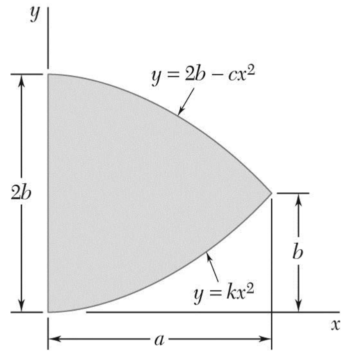
$$=\frac{1}{2}a^3b\int_0^{\pi/2}\frac{1}{2}(1-\cos 4\theta)d\theta=\frac{1}{4}a^3b\left|\theta-\frac{1}{4}\sin 4\theta\right|_0^{\pi/2}$$

$$=\frac{1}{4}a^3b\left[\frac{\pi}{2}-0\right]=\frac{\pi}{8}a^3b \qquad I_y=\frac{1}{8}\pi a^3b \blacktriangleleft$$

From solution of Problem 9.16: $A=\frac{1}{2}\pi ab$

Thus:

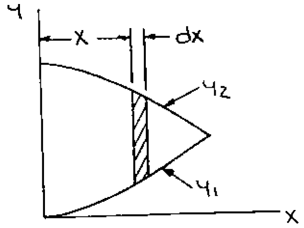
$$I_y=k_y^2A \quad k_y^2=\frac{I_y}{A}=\frac{\frac{1}{8}\pi a^3b}{\frac{1}{2}\pi ab}=\frac{1}{4}a^2 \qquad k_y=\frac{1}{2}a \blacktriangleleft$$



PROBLEM 9.19

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the x axis.

SOLUTION



At $x = a_1,\ y_1 = y_2 = b$: $\quad y_1:\ b = ka^2 \quad \text{or} \quad k = \dfrac{b}{a^2}$

$$y_2:\ b = 2b - ca^2 \quad \text{or} \quad c = \frac{b}{a^2}$$

Then $$y_1 = \frac{b}{a^2}x^2 \qquad y_2 = b\left(2 - \frac{x^2}{a^2}\right)$$

Now $$dA = (y_2 - y_1)dx = \left[b\left(2 - \frac{x^2}{a^2}\right) - \frac{b}{a^2}x^2\right]dx = \frac{2b}{a^2}(a^2 - x^2)dx$$

Then $$A = \int dA = \int_0^a \frac{2b}{a^2}(a^2 - x^2)dx = \frac{2b}{a^2}\left[a^2x - \frac{1}{3}x^3\right]_0^a = \frac{4}{3}ab$$

Now $$dI_x = \left(\frac{1}{3}y_2^3 - \frac{1}{3}y_1^3\right)dx = \frac{1}{3}\left\{\left[b\left(2 - \frac{x^2}{a^2}\right)\right]^3 - \left[\frac{b}{a^2}x^2\right]^3\right\}dx$$

$$= \frac{1}{3}\frac{b^3}{a^6}(8a^6 - 12a^4x^2 + 6a^2x^4 - x^6 - x^6)dx$$

$$= \frac{2}{3}\frac{b^3}{a^6}(4a^6 - 6a^4x^2 + 3a^2x^4 - x^6)dx$$

Then $$I_x = \int dI_x = \int_0^a \frac{2}{3}\frac{b^3}{a^6}(4a^6 - 6a^4x^2 + 3a^2x^4 - x^6)dx$$

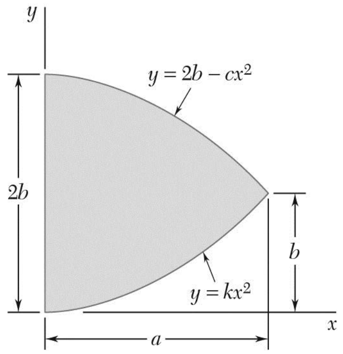
$$= \frac{2}{3}\frac{b^3}{a^6}\left[4a^6x - 2a^4x^3 + \frac{3}{5}a^6x^5 - \frac{1}{7}x^7\right]_0^a$$

$$= \frac{172}{105}ab^3$$

or $I_x = 1.638ab^3$ ◀

and $$k_x^2 = \frac{I_x}{A} = \frac{\frac{172}{105}ab^3}{\frac{4}{3}ab} = \frac{43}{35}b^2$$

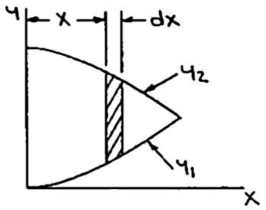
or $k_x = 1.108b$ ◀



PROBLEM 9.20

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *y* axis.

SOLUTION



At $x = a, \quad y_1 = y_2 = b$: $\quad y_1: \quad b = ka^2 \quad \text{or} \quad k = \dfrac{b}{a^2}$

$$y_2: \quad b = 2b - ca^2 \quad \text{or} \quad c = \frac{b}{a^2}$$

Then $$y_1 = \frac{b}{a^2}x^2$$

$$y_2 = b\left(2 - \frac{x^2}{a^2}\right)$$

Now $$dA = (y_2 - y_1)dx$$

$$= \left[b\left(2 - \frac{x^2}{a^2}\right) - \frac{b}{a^2}x^2\right]dx$$

$$= \frac{2b}{a^2}(a^2 - x^2)\,dx$$

Then $$A = \int dA = \int_0^a \frac{2b}{a^2}(a^2 - x^2)dx$$

$$= \frac{2b}{a^2}\left[a^2x - \frac{1}{3}x^3\right]_0^a$$

$$= \frac{4}{3}ab$$

Now $$dI_y = x^2 dA = x^2\left[\frac{2b}{a^2}(a^2 - x^2)dx\right]$$

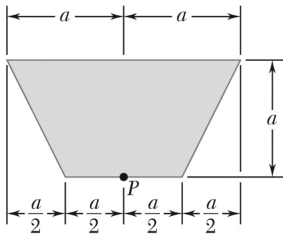
PROBLEM 9.20 (Continued)

Then
$$I_y = \int dI_y = \int_0^a \frac{2b}{a^2} x^2 (a^2 - x^2)\,dx$$
$$= \frac{2b}{a^2}\left[\frac{1}{3}a^2 x^3 - \frac{1}{5}x^5\right]_0^a$$

or $I_y = \frac{4}{15}a^3 b$ ◀

and
$$k_y^2 = \frac{I_y}{A} = \frac{\frac{4}{15}a^3 b}{\frac{4}{3}ab} = \frac{1}{5}a^2$$

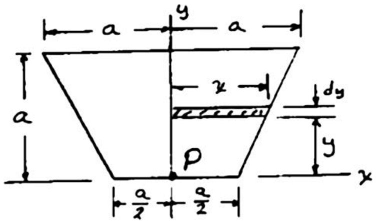
or $k_y = \frac{a}{\sqrt{5}}$ ◀



PROBLEM 9.21

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point *P*.

SOLUTION



$$x = \frac{a}{2} + \frac{a}{2}\frac{y}{a} = \frac{1}{2}(a+y)$$

$$dA = x\,dy = \frac{1}{2}(a+y)dy$$

$$\frac{1}{2}A = \int_0^a dA = \int_0^a \frac{1}{2}(a+y)dy = \frac{1}{2}\left| ay + \frac{y^2}{2} \right|_0^a = \frac{1}{2}\left(a^2 + \frac{a^2}{2}\right) = \frac{3}{4}a^2 \qquad A = \frac{3}{2}a^2 \ \triangleleft$$

$$\frac{1}{2}I_x = \int y^2 dA = \int_0^a y^2 \frac{1}{2}(a+y)dy = \frac{1}{2}\int_0^a (ay^2 + y^3)dy$$

$$= \frac{1}{2}\left| a\frac{y^3}{3} + \frac{y^4}{4} \right|_0^a = \frac{1}{2}\left(\frac{1}{3} + \frac{1}{4}\right)a^4 = \frac{1}{2}\frac{7}{12}a^4 \qquad I_x = \frac{7}{12}a^4 \ \triangleleft$$

$$\frac{1}{2}I_y = \int_0^a \frac{1}{3}x^3 dy = \frac{1}{3}\int_0^a \left(\frac{1}{2}(a+y)\right)^3 dy$$

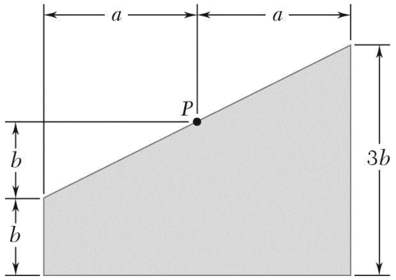
$$\frac{1}{2}I_y = \frac{1}{24}\int_0^a (a+y)^3 dy = \frac{1}{24}\left| \frac{1}{4}(a+y)^4 \right|_0^a = \frac{1}{96}\left[(2a)^4 - a^4\right] = \frac{15}{96}a^4$$

$$\frac{1}{2}I_y = \frac{5}{32}a^4 \qquad I_y = \frac{5}{16}a^4 \ \triangleleft$$

From Eq. (9.4):

$$J_O = I_x + I_y = \frac{7}{12}a^4 + \frac{5}{16}a^4 = \left(\frac{28+15}{48}\right)a^4 \qquad J_O = \frac{43}{48}a^4 \ \blacktriangleleft$$

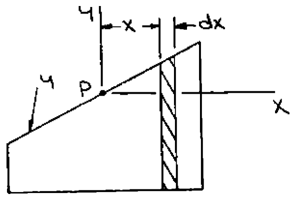
$$J_O = k_O^2 A \qquad k_O^2 = \frac{J_O}{A} = \frac{\frac{43}{48}a^4}{\frac{3}{2}a^2} = \frac{43}{72}a^2 \qquad k_O = a\sqrt{\frac{43}{72}} \qquad k_O = 0.773a \ \blacktriangleleft$$



PROBLEM 9.22

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point *P*.

SOLUTION



By observation $$y = \frac{b}{a}x$$

First note
$$dA = (y + 2b)dx$$
$$= \frac{b}{a}(x + 2a)dx$$

Now
$$dI_x = \left(\frac{1}{3}y_{\text{top}}^3 - \frac{1}{3}y_{\text{bottom}}^3\right)dx$$
$$= \frac{1}{3}\left[\left(\frac{b}{a}x\right)^3 - (-2b)^3\right]dx$$
$$= \frac{1}{3}\frac{b^3}{a^3}(x^3 + 8a^3)dx$$

Then
$$I_x = \int dI_x = \int_{-a}^{a} \frac{1}{3}\frac{b^3}{a^3}(x^2 + 8a^3)dx$$
$$= \frac{1}{3}\frac{b^3}{a^3}\left[\frac{1}{4}x^4 + 8a^3x\right]_{-a}^{a}$$
$$= \frac{1}{3}\frac{b^3}{a^3}\left\{\left[\frac{1}{4}(a)^4 + 8a^3(a)\right] - \left[\frac{1}{4}(-a)^4 + 8a^3(-a)\right]\right\} = \frac{16}{3}ab^3$$

Also
$$dI_y = x^2 dA = x^2\left[\frac{b}{a}(x + 2a)dx\right]$$

Then
$$I_y = \int dI_y = \int_{-a}^{a} \frac{b}{a}x^2(x + 2a)dx$$
$$= \frac{b}{a}\left[\frac{1}{4}x^4 + \frac{2}{3}ax^3\right]_{-a}^{a}$$
$$= \frac{b}{a}\left\{\left[\frac{1}{4}(a)^4 + \frac{2}{3}a(a)^3\right] - \left[\frac{1}{4}(-a)^4 + \frac{2}{3}a(-a)^3\right]\right\} = \frac{4}{3}a^3b$$

PROBLEM 9.22 (Continued)

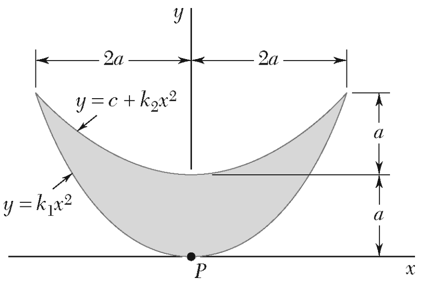
Now $$J_P = I_x + I_y = \frac{16}{3}ab^3 + \frac{4}{3}a^3b$$

$$J_P = \frac{4}{3}ab(a^2 + 4b^2) \blacktriangleleft$$

and $$k_P^2 = \frac{J_P}{A} = \frac{\frac{4}{3}ab(a^2 + 4b^2)}{(2a)(3b) - \frac{1}{2}(2a)(2b)}$$

$$= \frac{1}{3}(a^2 + 4b^2)$$

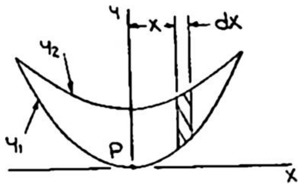
or $$k_P = \sqrt{\frac{a^2 + 4b^2}{3}} \blacktriangleleft$$



PROBLEM 9.23

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point *P*.

SOLUTION



y_1: At $x = 2a$, $y = 2a$:

$$2a = k_1(2a)^2 \quad \text{or} \quad k_1 = \frac{1}{2a}$$

y_2: At $x = 0$, $y = a$: $\quad a = c$

At $x = 2a$, $y = 2a$:

$$2a = a + k_2(2a)^2 \quad \text{or} \quad k_2 = \frac{1}{4a}$$

Then

$$y_1 = \frac{1}{2a}x^2 \qquad y_2 = a + \frac{1}{4a}x^2$$

$$= \frac{1}{4a}(4a^2 + x^2)$$

Now

$$dA = (y_2 - y_1)dx = \left[\frac{1}{4a}(4a^2 + x^2) - \frac{1}{2a}x^2\right]dx$$

$$= \frac{1}{4a}(4a^2 - x^2)dx$$

Then

$$A = \int dA = 2\int_0^{2a} \frac{1}{4a}(4a^2 - x^2)dx = \frac{1}{2a}\left[4a^2x - \frac{1}{3}x^3\right]_0^{2a} = \frac{8}{3}a^2$$

Now

$$dI_x = \left(\frac{1}{3}y_2^3 - \frac{1}{3}y_1^3\right)dx = \frac{1}{3}\left\{\left[\frac{1}{4a}(4a^2 + x^2)\right]^3 - \left[\frac{1}{2a}x^2\right]^3\right\}dx$$

$$= \frac{1}{3}\left[\frac{1}{64a^3}(64a^6 + 48a^4x^2 + 12a^2x^4 + x^6) - \frac{1}{8a^3}x^6\right]dx$$

$$= \frac{1}{192a^3}(64a^6 + 48a^4x^2 + 12a^2x^4 - 7x^6)dx$$

PROBLEM 9.23 (Continued)

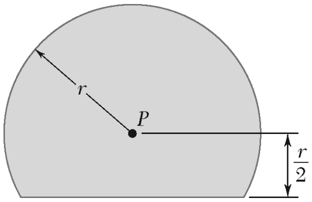
Then
$$I_x = \int dI_x = 2\int_0^{2a} \frac{1}{192a^3}(64a^6 + 48a^4x^2 + 12a^2x^4 - 7x^6)dx$$
$$= \frac{1}{96a^3}\left[64a^6x + 16a^4x^3 + \frac{12}{5}a^2x^5 - x^7\right]_0^{2a}$$
$$= \frac{1}{96a^3}\left[64a^6(2a) + 16a^4(2a)^3 + \frac{12}{5}a^2(2a)^5 - (2a)^7\right]$$
$$= \frac{1}{96}a^4\left(128 + 128 + \frac{12}{5}\times 32 - 128\right) = \frac{32}{15}a^4$$

Also
$$dI_y = x^2 dA = x^2\left[\frac{1}{4a}(4a^2 - x^2)dx\right]$$

Then
$$I_y = \int dI_y = 2\int_0^{2a} \frac{1}{4a}x^2(4a^2 - x^2)dx = \frac{1}{2a}\left[\frac{4}{3}a^2x^3 - \frac{1}{5}x^5\right]_0^{2a}$$
$$= \frac{1}{2a}\left[\frac{4}{3}a^2(2a)^3 - \frac{1}{5}(2a)^5\right] = \frac{32}{2}a^4\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{32}{15}a^4$$

Now
$$J_P = I_x + I_y = \frac{32}{15}a^4 + \frac{32}{15}a^4$$
or $J_P = \frac{64}{15}a^4$ ◀

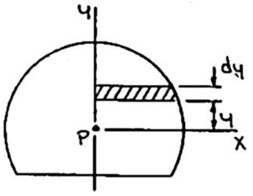
and
$$k_P^2 = \frac{J_P}{A} = \frac{\frac{64}{15}a^4}{\frac{8}{3}a^2} = \frac{8}{5}a^2$$
or $k_P = 1.265a$ ◀



PROBLEM 9.24

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point *P*.

SOLUTION



The equation of the circle is

$$x^2 + y^2 = r^2$$

So that $$x = \sqrt{r^2 - y^2}$$

Now $$dA = xdy = \sqrt{r^2 - y^2}\,dy$$

Then $$A = \int dA = 2\int_{-r/2}^{r} \sqrt{r^2 - y^2}\,dy$$

Let $$y = r\sin\theta; \quad dy = r\cos\theta\, d\theta$$

Then $$A = 2\int_{-\pi/6}^{\pi/2} \sqrt{r^2 - (r\sin\theta)^2}\, r\cos\, d\theta$$

$$= 2\int_{-\pi/6}^{\pi/2} r^2\cos^2\theta\, d\theta = 2r^2\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_{-\pi/6}^{\pi/2}$$

$$= 2r^2\left[\frac{\frac{\pi}{2}}{2} - \left(\frac{-\frac{\pi}{6}}{2} + \frac{\sin -\frac{\pi}{3}}{4}\right)\right] = 2r^2\left(\frac{\pi}{3} + \frac{\sqrt{3}}{8}\right)$$

$$= 2.5274r^2$$

Now $$dI_x = y^2 dA = y^2\left(\sqrt{r^2 - y^2}\,dy\right)$$

Then $$I_x = \int dI_x = 2\int_{-r/2}^{r} y^2\sqrt{r^2 - y^2}\,dy$$

Let $$y = r\sin\theta; \quad dy = r\cos\theta\, d\theta$$

Then $$I_x = 2\int_{-\pi/6}^{\pi/2} (r\sin\theta)^2\sqrt{r^2 - (r\sin\theta)^2}\, r\cos\theta\, d\theta$$

$$= 2\int_{-\pi/6}^{\pi/2} r^2\sin^2\theta(r\cos\theta) r\cos\theta\, d\theta$$

Now $$\sin 2\theta = 2\sin\theta\cos\theta \Rightarrow \sin^2\theta\cos^2\theta = \frac{1}{4}\sin 2\theta$$

PROBLEM 9.24 (Continued)

Then
$$I_x = 2\int_{-\pi/6}^{\pi/2} r^4\left(\frac{1}{4}\sin^2 2\theta\right)d\theta = \frac{r^4}{2}\left[\frac{\theta}{2}-\frac{\sin 4\theta}{8}\right]_{-\pi/6}^{\pi/2}$$
$$= \frac{r^4}{2}\left[\frac{\frac{\pi}{2}}{2}-\left(\frac{\frac{\pi}{6}}{2}-\frac{\sin -\frac{2\pi}{3}}{8}\right)\right]$$
$$= \frac{r^4}{2}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{16}\right)$$

Also
$$dI_y = \frac{1}{3}x^3 dy = \frac{1}{3}\left(\sqrt{r^2-y^2}\right)^3 dy$$

Then
$$I_y = \int dI_y = 2\int_{-r/2}^{r}\frac{1}{3}(r^2-y^2)^{3/2}dy$$

Let
$$y = r\sin\theta; \quad dy = r\cos\theta\, d\theta$$

Then
$$I_y = \frac{2}{3}\int_{-\pi/6}^{\pi/2}[r^2-(r\sin\theta)^2]^{3/2} r\cos\theta\, d\theta$$
$$I_y = \frac{2}{3}\int_{-\pi/6}^{\pi/2}(r^3\cos^3\theta) r\cos\theta\, d\theta$$

Now
$$\cos^4\theta = \cos^2\theta(1-\sin^2\theta) = \cos^2\theta - \frac{1}{4}\sin^2 2\theta$$

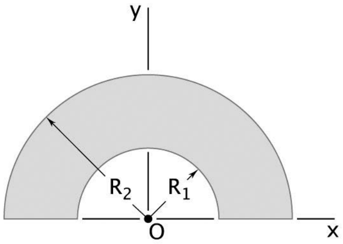
Then
$$I_y = \frac{2}{3}\int_{-\pi/6}^{\pi/2} r^4\left(\cos^2\theta-\frac{1}{4}\sin^2 2\theta\right)d\theta$$
$$= \frac{2}{3}r^4\left[\left(\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right)-\frac{1}{4}\left(\frac{\theta}{2}-\frac{\sin 4\theta}{8}\right)\right]_{-\pi/6}^{\pi/2}$$
$$= \frac{2}{3}r^4\left\{\left[\frac{\frac{\pi}{2}}{2}-\frac{1}{4}\left(\frac{\frac{\pi}{2}}{2}\right)\right]-\left[\frac{-\frac{\pi}{6}}{2}+\frac{\sin -\frac{\pi}{3}}{4}-\frac{1}{4}\left(\frac{-\frac{\pi}{6}}{2}-\frac{\sin -\frac{2\pi}{3}}{8}\right)\right]\right\}$$
$$= \frac{2}{3}r^4\left[\frac{\pi}{4}-\frac{\pi}{16}+\frac{\pi}{12}+\frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)-\frac{\pi}{48}+\frac{1}{32}\left(\frac{\sqrt{3}}{2}\right)\right]$$
$$= \frac{2}{3}r^4\left(\frac{\pi}{4}+\frac{9\sqrt{3}}{64}\right)$$

PROBLEM 9.24 (Continued)

Now $$J_P = I_x + I_y = \frac{r^4}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{16}\right) + \frac{2}{3}r^4\left(\frac{\pi}{4} + \frac{9\sqrt{3}}{64}\right)$$

$$= r^4\left(\frac{\pi}{3} + \frac{\sqrt{3}}{16}\right) = 1.15545r^4 \qquad \text{or} \quad J_P = 1.155r^4 \blacktriangleleft$$

and $$k_P^2 = \frac{J_P}{A} = \frac{1.15545r^4}{2.5274r^2} \qquad \text{or} \quad k_P = 0.676r \blacktriangleleft$$



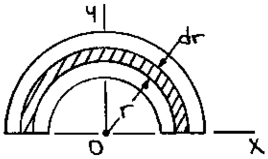
PROBLEM 9.25

(*a*) Determine by direct integration the polar moment of inertia of the semiannular area shown with respect to Point *O*. (*b*) Using the result of part *a*, determine the moments of inertia of the given area with respect to the *x* and *y* axes.

SOLUTION

(*a*) By definition

$$dJ_O = r^2 dA = r^2(\pi r dr) = \pi r^3 dr$$



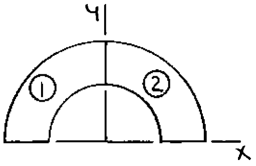
Then

$$J_O = \int dJ_O = \int_{R_1}^{R_2} \pi r^3 dr = \left[\frac{1}{4}r^4\right]_{R_1}^{R_2}$$

$$\text{or} \quad J_O = \frac{\pi}{4}\left(R_2^4 - R_1^4\right) \blacktriangleleft$$

(*b*) First note that symmetry implies

$$(I_x)_1 = (I_y)_1 \qquad (I_x)_2 = (I_y)_2$$



Also have

$$I_x = (I_x)_1 + (I_x)_2$$

and

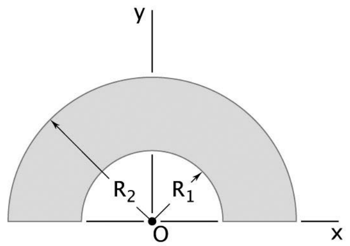
$$I_y = (I_y)_1 + (I_y)_2 = (I_x)_1 + (I_x)_2 = I_x$$

Now

$$J_O = I_x + I_y \qquad I_x = I_y$$

$$= 2I_x$$

$$\therefore \quad I_x = I_y = \frac{1}{2}J_O \qquad \text{or} \quad I_x = I_y = \frac{\pi}{8}\left(R_2^4 - R_1^4\right) \blacktriangleleft$$



PROBLEM 9.26

(*a*) Show that the polar radius of gyration k_O of the semiannular area shown is approximately equal to the mean radius $R_m = (R_1 + R_2)/2$ for small values of the thickness $t = R_2 - R_1$. (*b*) Determine the percentage error introduced by using R_m in place of k_O for the following values of t/R_m: $1, \frac{1}{2}$, and $\frac{1}{10}$.

SOLUTION

(*a*) From the solution to Problem 9.25 have

$$J_O = \frac{\pi}{4}\left(R_2^4 - R_1^4\right)$$

Now

$$k_O^2 = \frac{J_O}{A} = \frac{\frac{\pi}{4}\left(R_2^4 - R_1^4\right)}{\frac{\pi}{2}\left(R_2^2 - R_1^2\right)} = \frac{1}{2}\frac{\left(R_2^2 - R_1^2\right)\left(R_2^2 + R_1^2\right)}{R_2^2 - R_1^1}$$

$$= \frac{1}{2}\left(R_2^2 + R_1^2\right) \qquad (1)$$

Now

$$R_m = \frac{1}{2}(R_1 + R_2) \qquad t = R_2 - R_1$$

Then

$$R_1 = R_m - \frac{1}{2}t \qquad R_2 = R_m + \frac{1}{2}t$$

Substituting into Eq. (1),

$$k_O^2 = \frac{1}{2}\left[\left(R_m + \frac{1}{2}t\right)^2 + \left(R_m - \frac{1}{2}t\right)^2\right]$$

$$= \frac{1}{2}\left(R_m^2 + R_m t + \frac{1}{4}t^2 + R_m^2 - R_m t + \frac{1}{4}t^2\right)$$

$$= R_m^2 + \frac{1}{4}t^2 \qquad \left(k_O = \sqrt{R_m^2 + \frac{1}{4}t^2}\right)$$

If $t << R_1, R_2$

Then $t << R_m$

So that

$$k_O^2 \approx R_m^2 \qquad \text{or} \quad k_O \approx R_m \ \triangleleft$$

PROBLEM 9.26 (Continued)

(*b*) The percentage error, % error, is given by

$$\%\ \text{error} = \frac{R_m - k_O}{k_O} \times 100\%$$

$$= \frac{R_m - \sqrt{R_m^2 + \frac{1}{4}t^2}}{\sqrt{R_m^2 + \frac{1}{4}t^2}} \times 100\%$$

$$= \left(\frac{1}{\sqrt{1 + \left(\frac{1}{2}\frac{t}{R_m}\right)^2}} - 1 \right) \times 100\%$$

Then, for

$$\frac{t}{R_m} = 1: \quad \%\ \text{error} = \left(\frac{1}{\sqrt{1 + \left(\frac{1}{2} + 1\right)^2}} - 1 \right) \times 100\%$$

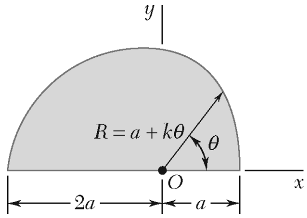
or $\%\ \text{error} = -10.56\%$ ◀

$$\frac{t}{R_m} = \frac{1}{2}: \quad \%\ \text{error} = \left(\frac{1}{\sqrt{1 + \left(\frac{1}{2} \times \frac{1}{2}\right)^2}} - 1 \right) \times 100\%$$

or $\%\ \text{error} = -2.99\%$ ◀

$$\frac{t}{R_m} = \frac{1}{10}: \quad \%\ \text{error} = \left(\frac{1}{\sqrt{1 + \left(\frac{1}{2} \times \frac{1}{10}\right)^2}} - 1 \right) \times 100\%$$

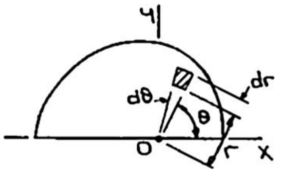
or $\%\ \text{error} = -0.1248\%$ ◀



PROBLEM 9.27

Determine the polar moment of inertia and the polar radius of gyration of the shaded area shown with respect to Point O.

SOLUTION



At $\theta = \pi, \quad R = 2a$: $\quad 2a = a + k(\pi)$

or $\quad k = \dfrac{a}{\pi}$

Then $\quad R = a + \dfrac{a}{\pi}\theta = a\left(1 + \dfrac{\theta}{\pi}\right)$

Now
$$dA = (dr)(r\,d\theta) = r\,dr\,d\theta$$

Then
$$A = \int dA = \int_0^{\pi}\int_0^{a(1+\theta/\pi)} r\,dr\,d\theta = \int_0^{\pi}\left[\frac{1}{2}r^2\right]_0^{a(1+\theta/\pi)} d\theta$$

$$A = \int_0^{\pi}\frac{1}{2}a^2\left(1+\frac{\theta}{\pi}\right)^2 d\theta = \frac{1}{2}a^2\left[\frac{\pi}{3}\left(1+\frac{\theta}{\pi}\right)^3\right]_0^{\pi}$$

$$= \frac{1}{6}\pi a^2\left[\left(1+\frac{\pi}{\pi}\right)^3 - (1)^3\right] = \frac{7}{6}\pi a^2$$

Now
$$dJ_O = r^2 dA = r^2(r\,dr\,d\theta)$$

Then
$$J_O = \int dJ_O = \int_0^{\pi}\int_0^{a(1+\theta/\pi)} r^3\,dr\,d\theta$$

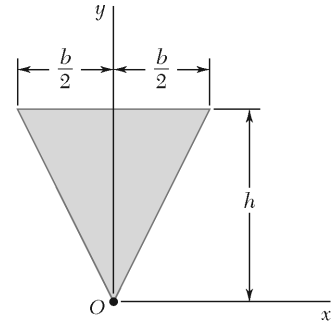
$$= \int_0^{\pi}\left[\frac{1}{4}r^4\right]_0^{a(1+\theta/\pi)} d\theta = \int_0^{\pi}\frac{1}{4}a^4\left(1+\frac{\theta}{\pi}\right)^4 d\theta$$

$$= \frac{1}{4}a^4\left[\frac{\pi}{5}\left(1+\frac{\theta}{\pi}\right)^5\right]_0^{\pi} = \frac{1}{20}\pi a^4\left[\left(1+\frac{\pi}{\pi}\right)^5 - (1)^5\right]$$

or $\quad J_O = \dfrac{31}{20}\pi a^4$ ◀

and
$$k_O^2 = \frac{J_O}{A} = \frac{\frac{31}{20}\pi a^4}{\frac{7}{6}\pi a^2} = \frac{93}{70}a^2$$

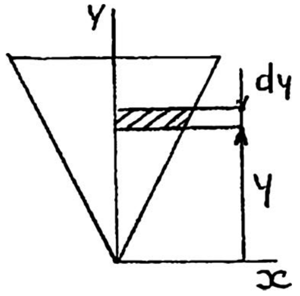
or $\quad k_O = 1.153a$ ◀



PROBLEM 9.28

Determine the polar moment of inertia and the polar radius of gyration of the isosceles triangle shown with respect to Point *O*.

SOLUTION



By observation: $y = \frac{h}{\frac{b}{2}}x$

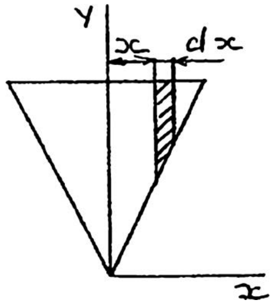
or $x = \frac{b}{2h}y$

Now $dA = xdy = \left(\frac{b}{2h}y\right)dy$

and $dI_x = y^2 dA = \frac{b}{2h}y^3 dy$

Then $I_x = \int dI_x = 2\int_0^h \frac{b}{2h}y^3 dy$

$$= \frac{b}{h}\frac{y^4}{4}\bigg|_0^h = \frac{1}{4}bh^3$$



From above: $y = \frac{2h}{b}x$

Now $dA = (h-y)dx = \left(h - \frac{2h}{b}x\right)dx$

$$= \frac{h}{b}(b-2x)dx$$

and $dI_y = x^2 dA = x^2\frac{h}{b}(b-2x)dx$

PROBLEM 9.28 (Continued)

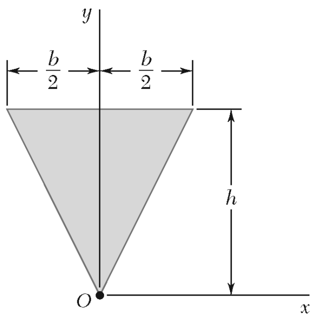
Then $$I_y = \int dI_y = 2\int_0^{b/2} \frac{h}{b} x^2 (b-2x)dx$$

$$= 2\frac{h}{b}\left[\frac{1}{3}bx^3 - \frac{1}{2}x^4\right]_0^{b/2}$$

$$= 2\frac{h}{b}\left[\frac{b}{3}\left(\frac{b}{2}\right)^3 - \frac{1}{2}\left(\frac{b}{2}\right)^4\right] = \frac{1}{48}b^3h$$

Now $$J_O = I_x + I_y = \frac{1}{4}bh^3 + \frac{1}{48}b^3h$$ or $$J_O = \frac{bh}{48}(12h^2 + b^2)$$ ◀

and $$k_O^2 = \frac{J_O}{A} = \frac{\frac{bh}{48}(12h^2+b^2)}{\frac{1}{2}bh} = \frac{1}{24}(12h^2+b^2)$$ or $$k_O = \frac{\sqrt{12h^2+b^2}}{24}$$ ◀



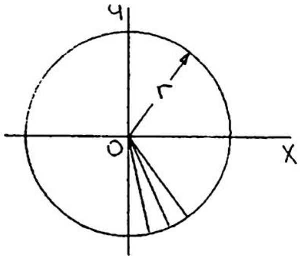
PROBLEM 9.29*

Using the polar moment of inertia of the isosceles triangle of Problem 9.28, show that the centroidal polar moment of inertia of a circular area of radius r is $\pi r^4/2$. (*Hint:* As a circular area is divided into an increasing number of equal circular sectors, what is the approximate shape of each circular sector?)

PROBLEM 9.28 Determine the polar moment of inertia and the polar radius of gyration of the isosceles triangle shown with respect to Point *O*.

SOLUTION

First the circular area is divided into an increasing number of identical circular sectors. The sectors can be approximated by isosceles triangles. For a large number of sectors the approximate dimensions of one of the isosceles triangles are as shown.



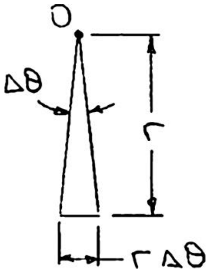
For an isosceles triangle (see Problem 9.28):

$$J_O = \frac{bh}{48}(12h^2 + b^2)$$

Then with $\qquad b = r\Delta\theta \quad \text{and} \quad h = r$

$$(\Delta J_O)_{\text{sector}} \simeq \frac{1}{48}(r\Delta\theta)(r)[12r^2 + (r\Delta\theta)^2]$$

$$= \frac{1}{48}r^4\Delta\theta\left[(12 + \Delta\theta^2)\right]$$



Now $$\frac{dJ_{O\,\text{sector}}}{d\theta} = \lim_{\Delta\theta\to 0}\left(\frac{\Delta J_{O\,\text{sector}}}{\Delta\theta}\right) = \lim_{\Delta\theta\to 0}\left\{\frac{1}{48}r^4[12 + (\Delta\theta)^2]\right\}$$

$$= \frac{1}{4}r^4$$

Then $$(J_O)_{\text{circle}} = \int dJ_{O\,\text{sector}} = \int_0^{2\pi}\frac{1}{4}r^4 d\theta = \frac{1}{4}r^4\,[\theta]_0^{2\pi}$$

or $\qquad (J_O)_{\text{circle}} = \frac{\pi}{2}r^4$ ◀

PROBLEM 9.30*

Prove that the centroidal polar moment of inertia of a given area A cannot be smaller than $A^2/2\pi$. (*Hint:* Compare the moment of inertia of the given area with the moment of inertia of a circle that has the same area and the same centroid.)

SOLUTION

From the solution to sample Problem 9.2, the centroidal polar moment of inertia of a circular area is

$$(J_C)_{\text{cir}} = \frac{\pi}{2} r^4$$

The area of the circle is

$$A_{\text{cir}} = \pi r^2$$

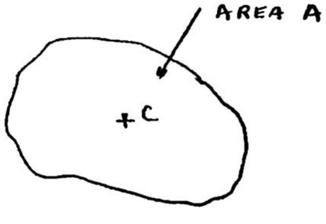
So that
$$[J_C(A)]_{\text{cir}} = \frac{A^2}{2\pi}$$

Two methods of solution will be presented. However, both methods depend upon the observation that as a given element of area dA is moved closer to some Point C. The value of J_C will be decreased ($J_C = \int r^2 dA$; as r decreases, so must J_C).

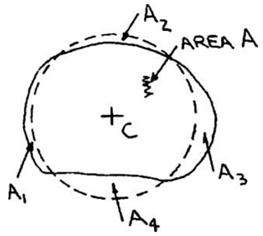
Solution 1

Imagine taking the area A and drawing it into a thin strip of negligible width and of sufficient length so that its area is equal to A. To minimize the value of $(J_C)_A$, the area would have to be distributed as closely as possible about C. This is accomplished by winding the strip into a tightly wound roll with C as its center; any voids in the roll would place the corresponding area farther from C than is necessary, thus increasing the value of $(J_C)_A$. (The process is analogous to rewinding a length of tape back into a roll.) Since the shape of the roll is circular, with the centroid of its area at C, it follows that

$$(J_C)_A \geq \frac{A^2}{2\pi} \quad \text{Q.E.D.} \blacktriangleleft$$



where the equality applies when the original area is circular.



PROBLEM 9.30* (Continued)

Solution 2

Consider an area A with its centroid at Point C and a circular area of area A with its center (and centroid) at Point C. Without loss of generality, assume that

$$A_1 = A_2 \qquad A_3 = A_4$$

It then follows that

$$(J_C)_A = (J_C)_{\text{cir}} + [J_C(A_1) - J_C(A_2) + J_C(A_3) - J_C(A_4)]$$

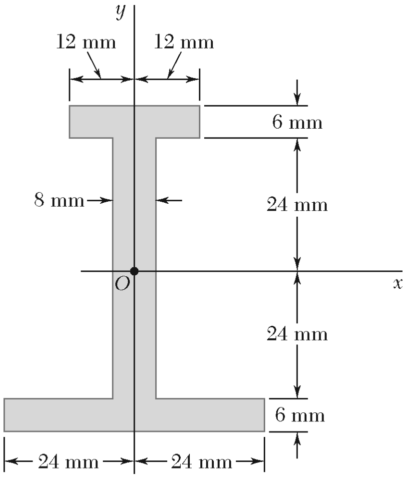
Now observe that

$$J_C(A_1) - J_C(A_2) \geq 0$$

$$J_C(A_3) - J_C(A_4) \geq 0$$

since as a given area is moved farther away from C its polar moment of inertia with respect to C must increase.

$$(J_C)_A \geq (J_C)_{\text{cir}} \qquad \text{or} \quad (J_C)_A \geq \frac{A^2}{2\pi} \quad \text{Q.E.D.} ◀$$



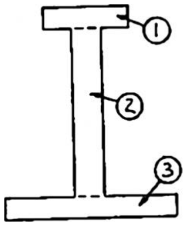
PROBLEM 9.31

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the x axis.

SOLUTION

First note that

$$
\begin{aligned}
A &= A_1 + A_2 + A_3 \\
&= [(24)(6) + (8)(48) + (48)(6)]\text{ mm}^2 \\
&= (144 + 384 + 288)\text{ mm}^2 \\
&= 816\text{ mm}^2
\end{aligned}
$$



Now

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

where

$$
\begin{aligned}
(I_x)_1 &= \frac{1}{12}(24\text{ mm})(6\text{ mm})^3 + (144\text{ mm}^2)(27\text{ mm})^2 \\
&= (432 + 104{,}976)\text{ mm}^4 \\
&= 105{,}408\text{ mm}^4
\end{aligned}
$$

$$(I_x)_2 = \frac{1}{12}(8\text{ mm})(48\text{ mm})^3 = 73{,}728\text{ mm}^4$$

$$
\begin{aligned}
(I_x)_3 &= \frac{1}{12}(48\text{ mm})(6\text{ mm})^3 + (288\text{ mm}^2)(27\text{ mm})^2 \\
&= (864 + 209{,}952)\text{ mm}^4 = 210{,}816\text{ mm}^4
\end{aligned}
$$

Then

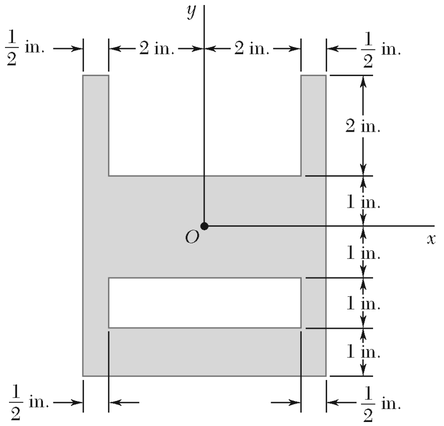
$$
\begin{aligned}
I_x &= (105{,}408 + 73{,}728 + 210{,}816)\text{ mm}^4 \\
&= 389{,}952\text{ mm}^4
\end{aligned}
$$

or $I_x = 390 \times 10^3\text{ mm}^4$ ◀

and

$$k_x^2 = \frac{I_x}{A} = \frac{389{,}952\text{ mm}^4}{816\text{ mm}^2}$$

or $k_x = 21.9\text{ mm}$ ◀



PROBLEM 9.32

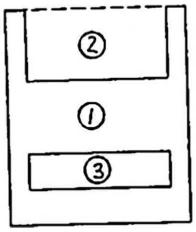
Determine the moment of inertia and the radius of gyration of the shaded area with respect to the x axis.

SOLUTION

First note that

$$A = A_1 - A_2 - A_3$$
$$= [(5)(6) - (4)(2) - (4)(1)]\text{ in}^2$$
$$= (30 - 8 - 4)\text{ in}^2$$
$$= 18\text{ in}^2$$

Now $\qquad I_x = (I_x)_1 - (I_x)_2 - (I_x)_3$



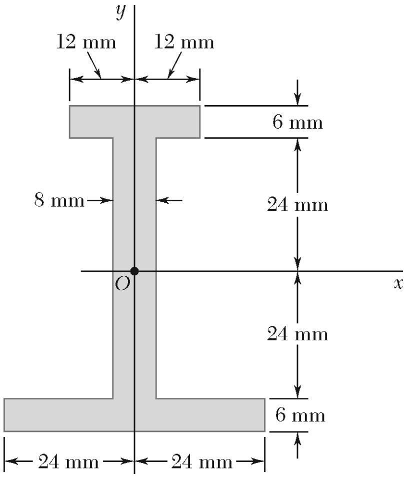
where $\qquad (I_x)_1 = \dfrac{1}{12}(5\text{ in.})(6\text{ in.})^3 = 90\text{ in}^4$

$$(I_x)_2 = \frac{1}{12}(4\text{ in.})(2\text{ in.})^3 + (8\text{ in}^2)(2\text{ in.})^2$$
$$= 34\frac{2}{3}\text{ in}^4$$

$$(I_x)_3 = \frac{1}{12}(4\text{ in.})(1\text{ in.})^3 + (4\text{ in}^2)\left(\frac{3}{2}\text{ in.}\right)^2$$
$$= 9\frac{1}{3}\text{ in}^4$$

Then $\qquad I_x = \left(90 - 34\dfrac{2}{3} - 9\dfrac{1}{3}\right)\text{in}^4 \qquad$ or $\quad I_x = 46.0\text{ in}^4$ ◀

and $\qquad k_x^2 = \dfrac{I_x}{A} = \dfrac{46.0\text{ in}^4}{18\text{ in}^4} \qquad$ or $\quad k_x = 1.599\text{ in.}$ ◀



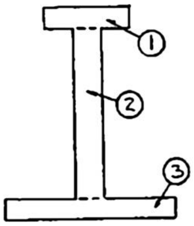
PROBLEM 9.33

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the y axis.

SOLUTION

First note that

$$\begin{aligned}A &= A_1 + A_2 + A_3\\ &= [(24\times 6) + (8)(48) + (48)(6)]\ \text{mm}^2\\ &= (144 + 384 + 288)\ \text{mm}^2\\ &= 816\ \text{mm}^2\end{aligned}$$



Now $I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$

where

$$(I_y)_1 = \frac{1}{12}(6\ \text{mm})(24\ \text{mm})^3 = 6912\ \text{mm}^4$$

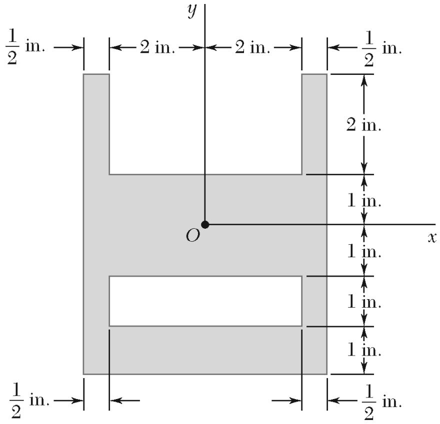
$$(I_y)_2 = \frac{1}{12}(48\ \text{mm})(8\ \text{mm})^3 = 2048\ \text{mm}^4$$

$$(I_y)_3 = \frac{1}{12}(6\ \text{mm})(48\ \text{mm})^3 = 55,296\ \text{mm}^4$$

Then $I_y = (6912 + 2048 + 55,296)\ \text{mm}^4 = 64,256\ \text{mm}^4$

or $I_y = 64.3\times 10^3\ \text{mm}^4$ ◀

and $k_y^2 = \dfrac{I_y}{A} = \dfrac{64,256\ \text{mm}^4}{816\ \text{mm}^2}$ or $k_y = 8.87\ \text{mm}$ ◀



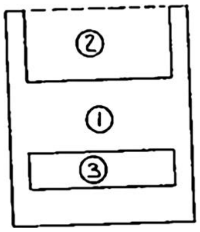
PROBLEM 9.34

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the *y* axis.

SOLUTION

First note that

$$A = A_1 - A_2 - A_3$$
$$= [(5)(6) - (4)(2) - (4)(1)]\text{ in}^2$$
$$= (30 - 8 - 4)\text{ in}^2$$
$$= 18\text{ in}^2$$



Now $$I_y = (I_y)_1 - (I_y)_2 - (I_y)_3$$

where

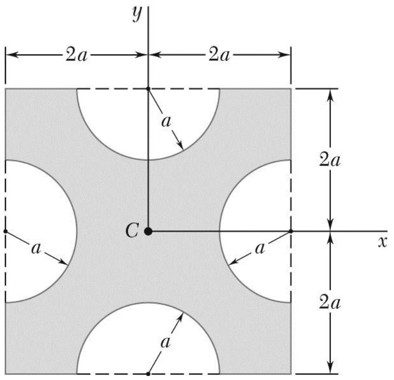
$$(I_y)_1 = \frac{1}{12}(6\text{ in.})(5\text{ in.})^3 = 62.5\text{ in}^4$$

$$(I_y)_2 = \frac{1}{12}(2\text{ in.})(4\text{ in.})^3 = 10\frac{2}{3}\text{ in}^4$$

$$(I_y)_3 = \frac{1}{12}(1\text{ in.})(4\text{ in.})^3 = 5\frac{1}{3}\text{in}^4$$

Then $$I_y = \left(62.5 - 10\frac{2}{3} - 5\frac{1}{3}\right)\text{in}^4$$ or $I_y = 46.5\text{ in}^4$ ◀

and $$k_y^2 = \frac{I_y}{A} = \frac{46.5\text{ in}^4}{18\text{ in}^2}$$ or $k_y = 1.607$ in. ◀



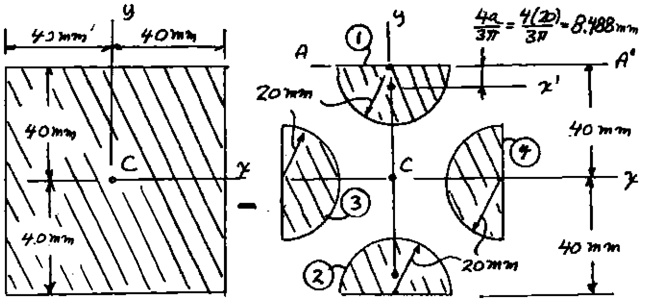
PROBLEM 9.35

Determine the moments of inertia of the shaded area shown with respect to the *x* and *y* axes when $a = 20$ mm.

SOLUTION

By symmetry: $I_x = I_y$

Given area = square − 4(semicircles)



Square $$I_x = \frac{1}{12}(80 \text{ mm})^4 = 3.413 \times 10^6 \text{ mm}^4$$

Semicircle 1: $$I_{AA'} = \frac{\pi}{8}(20 \text{ mm})^4 = 62.83 \times 10^3 \text{ mm}^4$$

$$I_{AA'} = \bar{I}_{x'} + Ad^2$$

$$62.83 \times 10^3 \text{ mm}^4 = \bar{I}_{x'} + \frac{\pi}{2}(20 \text{ mm})^2(8.488 \text{ mm})^2$$

$$\bar{I}_{x'} = 17.56 \times 10^3 \text{ mm}^4$$

$$I_x = \bar{I}_{x'} + A(40 \text{ mm} - 8.488 \text{ mm})^2$$

$$= 17.56 \times 10^3 \text{ mm}^4 + \frac{\pi}{2}(20 \text{ mm})^2(31.512 \text{ mm})^2$$

$$I_x = 641.5 \times 10^3 \text{ mm}^4$$

PROBLEM 9.35 (Continued)

Semicircle 2: Same as 1: $I_x = 641.5\times 10^3 \text{ mm}^4$

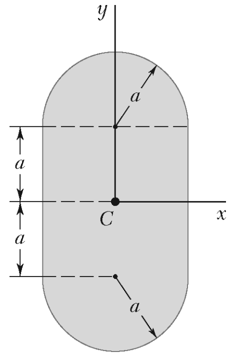
Semicircles 3 and 4 are equivalent to one 20-mm circle when computing I_x

$$I_x = \frac{\pi}{4}(20 \text{ mm})^4 = 125.66\times 10^3 \text{ mm}^4$$

Entire area = square − 4 (semicircles)

$$\begin{aligned}\overline{I}_x &= 3.413\times 10^6 \text{ mm}^4 - 2(641.5\times 10^3 \text{ mm}^4) - 125.66\times 10^{-3} \text{ mm}^4 \\ &= 2.0047\times 10^6 \text{ mm}^4\end{aligned}$$

$$\overline{I}_x = \overline{I}_y = 2.00\times 10^6 \text{ mm}^4 \blacktriangleleft$$



PROBLEM 9.36

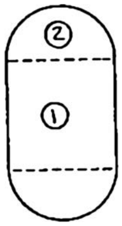
Determine the moments of inertia of the shaded area shown with respect to the x and y axes when $a = 20$ mm.

SOLUTION

We have $I_x = (I_x)_1 + 2(I_x)_2$

where $(I_x)_1 = \frac{1}{12}(40 \text{ mm})(40 \text{ mm})^3$

$= 213.33 \times 10^3 \text{ mm}^4$



$$(I_x)_2 = \left[\frac{\pi}{8}(20 \text{ mm})^4 - \frac{\pi}{2}(20 \text{ mm})^2\left(\frac{4\times 20}{3\pi} \text{ mm}\right)^2\right]$$

$$+\frac{\pi}{2}(20 \text{ mm})^2\left[\left(\frac{4\times 20}{3\pi} + 20\right)\text{mm}\right]^2$$

$= 527.49 \times 10^3 \text{ mm}^4$

Then $I_x = [213.33 + 2(527.49)] \times 10^3 \text{ mm}^4$

or $I_x = 1.268 \times 10^6 \text{ mm}^4$ ◀

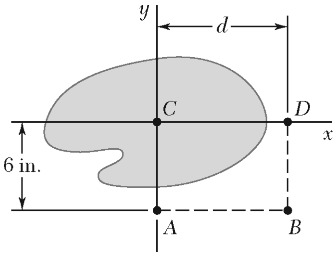
Also $I_y = (I_y)_1 + 2(I_y)_2$

where $(I_y)_1 = \frac{1}{12}(40 \text{ mm})(40 \text{ mm})^3 = 213.33 \times 10^3 \text{ mm}^4$

$(I_y)_2 = \frac{\pi}{8}(20 \text{ mm})^4 = 62.83 \times 10^3 \text{ mm}^4$

Then $I_y = [213.33 + 2(62.83)] \times 10^3 \text{ mm}^4$

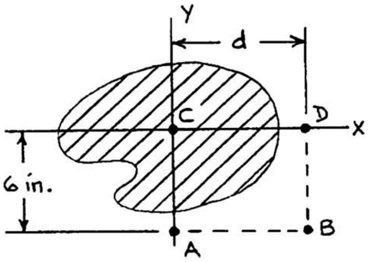
or $I_y = 339 \times 10^3 \text{ mm}^4$ ◀



PROBLEM 9.37

The shaded area is equal to 50 in^2. Determine its centroidal moments of inertia $\bar{I}_x$ and $\bar{I}_y$, knowing that $\bar{I}_y = 2\bar{I}_x$ and that the polar moment of inertia of the area about Point A is $J_A = 2250\text{ in}^4$.

SOLUTION



Given: $A = 50\text{ in}^2 \quad \bar{I}_y = 2\bar{I}_x, \quad J_A = 2250\text{ in}^4$

$$J_A = \bar{J}_C + A(6\text{ in.})^2$$

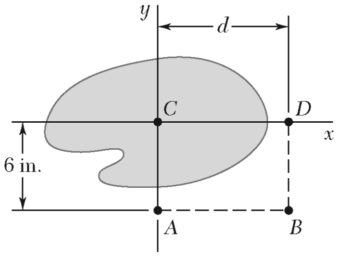
$$2250\text{ in}^4 = \bar{J}_C + (50\text{ in}^2)(6\text{ in.})^2$$

$$\bar{J}_C = 450\text{ in}^4$$

$$\bar{J}_C = \bar{I}_x + \bar{I}_y \quad \text{with} \quad \bar{I}_y = 2\bar{I}_x$$

$$450\text{ in}^4 = \bar{I}_x + 2\bar{I}_x \qquad \bar{I}_x = 150.0\text{ in}^4 \blacktriangleleft$$

$$\bar{I}_y = 2\bar{I}_x = 300\text{ in}^4 \blacktriangleleft$$



PROBLEM 9.38

The polar moments of inertia of the shaded area with respect to Points A, B, and D are, respectively, $J_A = 2880\ \text{in}^4$, $J_B = 6720\ \text{in}^4$, and $J_D = 4560\ \text{in}^4$. Determine the shaded area, its centroidal moment of inertia $\bar{J}_C$, and the distance d from C to D.

SOLUTION

See figure at solution of Problem 9.39.

Given: $J_A = 2880\ \text{in}^4, \quad J_B = 6720\ \text{in}^4, \quad J_D = 4560\ \text{in}^4$

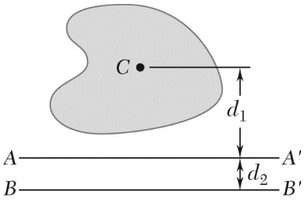
$$J_B = \bar{J}_C + A(CB)^2; \quad 6720\ \text{in}^4 = \bar{J}_C + A(6^2 + d^2) \qquad (1)$$

$$J_D = \bar{J}_C + A(CD)^2; \quad 4560\ \text{in}^4 = \bar{J}_C + Ad^2 \qquad (2)$$

Eq. (1) subtracted by Eq. (2): $J_B - J_D = 2160\ \text{in}^4 = A(6)^2$ $\qquad A = 60.0\ \text{in}^2$ ◀

$J_A = \bar{J}_C + A(AC)^2; \quad 2880\ \text{in}^4 = \bar{J}_C + (60\ \text{in}^2)(6\ \text{in.})^2$ $\qquad \bar{J}_C = 720\ \text{in}^4$ ◀

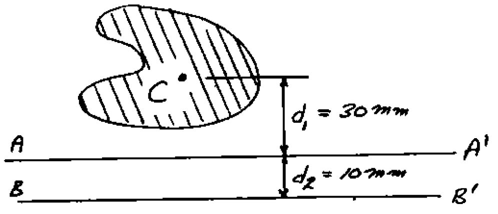
Eq. (2): $4560\ \text{in}^4 = 720\ \text{in}^4 + (60\ \text{in}^2)d^2$ $\qquad d = 8.00\ \text{in.}$ ◀



PROBLEM 9.39

Determine the shaded area and its moment of inertia with respect to the centroidal axis parallel to AA', knowing that $d_1 = 30$ mm and $d_2 = 10$ mm, and that the moments of inertia with respect to AA' and BB' are 4.1×10^6 mm^4 and 6.9×10^6 mm^4, respectively.

SOLUTION



$$I_{AA'} = 4.1\times10^6 \text{ mm}^4 = \overline{I} + A(30 \text{ mm})^2 \quad (1)$$

$$I_{BB'} = 6.9\times10^6 \text{ mm}^4 = \overline{I} + A(40 \text{ mm})^2$$

$$I_{BB'} - I_{AA'} = (6.9-4.1)\times10^6 = A(40^2 - 30^2)$$

$$2.8\times10^6 = A(700) \qquad A = 4000 \text{ mm}^2 \blacktriangleleft$$

Eq. (1): $$4.1\times10^6 = \overline{I} + (4000)(30)^2 \qquad \overline{I} = 500\times10^3 \text{ mm}^4 \blacktriangleleft$$

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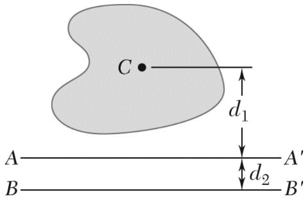
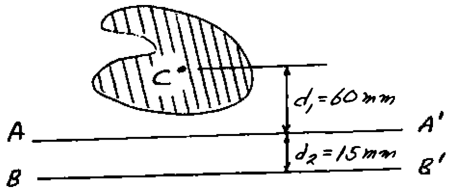


PROBLEM 9.40

Knowing that the shaded area is equal to 7500 mm^2 and that its moment of inertia with respect to AA' is 31×10^6 mm^4, determine its moment of inertia with respect to BB', for $d_1 = 60$ mm and $d_2 = 15$ mm.

SOLUTION

Given: $A = 7500 \text{ mm}^2$

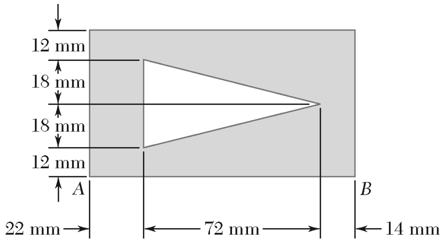


$$I_{AA'} = \overline{I} + Ad_1^2; \quad 31\times10^6 \text{ mm}^2 = \overline{I} + (7500 \text{ mm}^2)(60 \text{ mm})^2$$

$$\overline{I} = 4\times10^6 \text{ mm}^4$$

$$I_{BB'} = \overline{I} + Ad^2 = 4\times10^6 \text{ mm}^4 + (7500 \text{ mm}^2)(60 \text{ mm} + 15 \text{ mm})^2$$
$$= 4\times10^6 + 7500(75)^2 = 46.188\times10^6 \text{ mm}^4$$

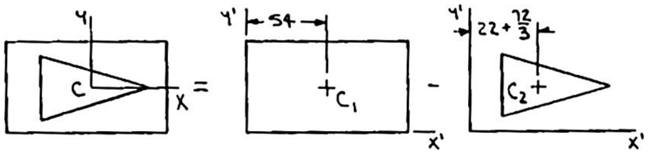
$$I_{BB'} = 46.2\times10^6 \text{ mm}^4 \blacktriangleleft$$



PROBLEM 9.41

Determine the moments of inertia $\overline{I}_x$ and $\overline{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

SOLUTION



Dimensions in mm

First locate centroid *C* of the area.

Symmetry implies $\overline{Y} = 30$ mm.

	A, mm^2	$\overline{x}$, mm	$\overline{x}A$, mm^3
1	$108 \times 60 = 6480$	54	349,920
2	$-\frac{1}{2} \times 72 \times 36 = -1296$	46	–59,616
Σ	5184		290,304

Then $\quad \overline{X}\,\Sigma A = \Sigma \overline{x}A: \quad \overline{X}(5184\text{ mm}^2) = 290{,}304\text{ mm}^3$

or $\quad \overline{X} = 56.0$ mm

Now $\quad \overline{I}_x = (I_x)_1 - (I_x)_2$

where

$$(I_x)_1 = \frac{1}{12}(108\text{ mm})(60\text{ mm})^3 = 1.944 \times 10^6\,\text{mm}^4$$

$$(I_x)_2 = 2\left[\frac{1}{36}(72\text{ mm})(18\text{ mm})^3 + \left(\frac{1}{2} \times 72\text{ mm} \times 18\text{ mm}\right)(6\text{ mm})^2\right]$$

$$= 2(11{,}664 + 23{,}328)\text{ mm}^4 = 69.984 \times 10^3\,\text{mm}^4$$

[$(I_x)_2$ is obtained by dividing A_2 into ]

Then $\quad \overline{I}_x = (1.944 - 0.069984) \times 10^6\,\text{mm}^4$

or $\quad \overline{I}_x = 1.874 \times 10^6\,\text{mm}^4$ ◀

PROBLEM 9.41 (Continued)

Also $\bar{I}_y = (I_y)_1 - (I_y)_2$

where $(I_y)_1 = \frac{1}{12}(60\text{ mm})(108\text{ mm})^3 + (6480\text{ mm}^2)[(56.54)\text{ mm}]^2$

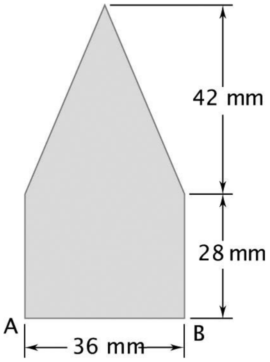
$= (6,298,560 + 25,920)\text{ mm}^4 = 6.324\times 10^6\text{ mm}^4$

$(I_y)_2 = \frac{1}{36}(36\text{ mm})(72\text{ mm})^3 + (1296\text{ mm}^2)[(56-46)\text{ mm}]^2$

$= (373,248 + 129,600)\text{ mm}^4 = 0.502\times 10^6\text{ mm}^4$

Then $\bar{I}_y = (6.324 - 0.502)10^6\text{ mm}^4$

or $\bar{I}_y = 5.82\times 10^6\text{ mm}^4$ ◀



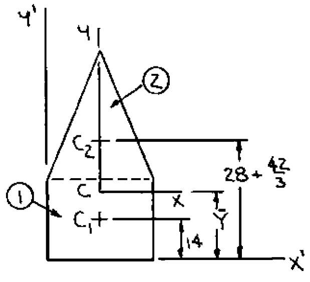
PROBLEM 9.42

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

SOLUTION

First locate *C* of the area.

Symmetry implies $\bar{X} = 18$ mm.



Dimensions in mm

	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$36\times 28 = 1008$	14	14,112
2	$\frac{1}{2}\times 36\times 42 = 756$	42	31,752
Σ	1764		45,864

Then $\quad \bar{Y}\Sigma A = \Sigma\bar{y}A: \quad \bar{Y}(1764 \text{ mm}^2) = 45{,}864 \text{ mm}^3$

or $\quad \bar{Y} = 26.0$ mm

Now $\quad \bar{I}_x = (I_x)_1 + (I_x)_2$

where

$$(I_x)_1 = \frac{1}{12}(36 \text{ mm})(28 \text{ mm})^3 + (1008 \text{ mm}^2)[(26-14) \text{ mm}]^2$$

$$= (65{,}856 + 145{,}152) \text{ mm}^4 = 211.008\times 10^3 \text{ mm}^4$$

$$(I_x)_2 = \frac{1}{36}(36 \text{ mm})(42 \text{ mm})^3 + (756 \text{ mm}^2)[(42-26) \text{ mm}]^2$$

$$= (74{,}088 + 193{,}536) \text{ mm}^4 = 267.624\times 10^3 \text{ mm}^4$$

Then $\quad \bar{I}_x = (211.008 + 267.624)\times 10^3 \text{ mm}^4$

or $\quad \bar{I}_x = 479\times 10^3 \text{ mm}^4$ ◀

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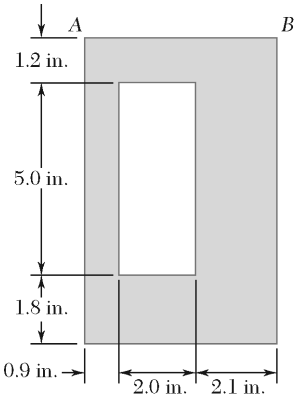
PROBLEM 9.42 (Continued)

Also $\bar{I}_y = (I_y)_1 + (I_y)_2$

where
$$(I_y)_1 = \frac{1}{12}(28\text{ mm})(36\text{ mm})^3 = 108.864 \times 10^3\text{ mm}^4$$
$$(I_y)_2 = 2\left[\frac{1}{36}(42\text{ mm})(18\text{ mm})^3 + \left(\frac{1}{2} \times 18\text{ mm} \times 42\text{ mm}\right)(6\text{ mm})^2\right]$$
$$= 2(6804 + 13,608)\text{ mm}^4 = 40.824 \times 10^3\text{ mm}^4$$

[$(I_y)_2$ is obtained by dividing A_2 into ]

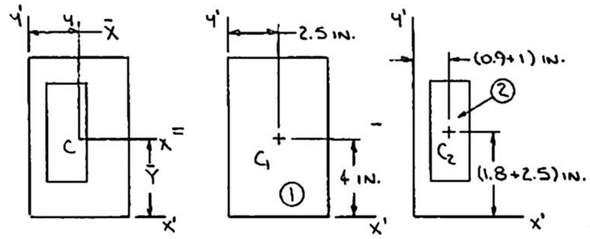
Then $\bar{I}_y = (108.864 + 40.824 \times 10^3\text{ mm}^4$ or $\bar{I}_y = 149.7 \times 10^3\text{ mm}^4$ ◀



PROBLEM 9.43

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side AB.

SOLUTION



First locate centroid C of the area.

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$5 \times 8 = 40$	2.5	4	100	160
2	$-2 \times 5 = -10$	1.9	4.3	–19	–43
Σ	30			81	117

Then $\quad \bar{X}\Sigma A = \Sigma \bar{x}A: \quad \bar{X}(30\text{ in}^2) = 81\text{ in}^3$

or $\quad \bar{X} = 2.70\text{ in.}$

and $\quad \bar{Y}\Sigma A = \Sigma \bar{y}A: \quad \bar{Y}(30\text{ in}^2) = 117\text{ in}^3$

or $\quad \bar{Y} = 3.90\text{ in.}$

Now $\quad \bar{I}_x = (I_x)_1 - (I_x)_2$

where

$$(I_x)_1 = \frac{1}{12}(5\text{ in.})(8\text{ in.})^3 + (40\text{ in}^2)[(4-3.9)\text{ in.}]^2$$
$$= (213.33 + 0.4)\text{ in}^4 = 213.73\text{ in}^4$$

$$(I_x)_2 = \frac{1}{12}(2\text{ in.})(5\text{ in.})^3 + (10\text{ in}^2)[(4.3-3.9)\text{ in.}]^2$$
$$= (20.83 + 1.60) = 22.43\text{ in}^4$$

PROBLEM 9.43 (Continued)

Then $\bar{I}_x = (213.73 - 22.43)\text{ in}^4$ or $\bar{I}_x = 191.3\text{ in}^4$ ◀

Also $\bar{I}_y = (I_y)_1 - (I_y)_2$

where

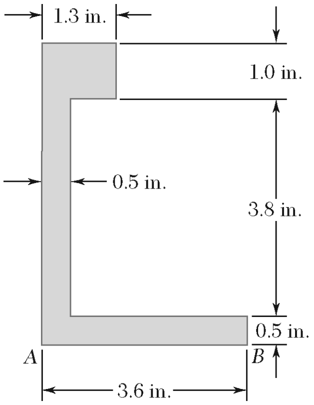
$$(I_y)_1 = \frac{1}{12}(8\text{ in.})(5\text{ in.})^3 + (40\text{ in}^2)[(2.7 - 2.5)\text{ in.}]^2$$

$$= (83.333 + 1.6)\text{ in}^4 = 84.933\text{ in}^4$$

$$(I_y)_2 = \frac{1}{12}(5\text{ in.})(2\text{ in.})^3 + (10\text{ in}^2)[(2.7 - 1.9)\text{ in.}]^2$$

$$= (3.333 + 6.4)\text{ in}^4 = 9.733\text{ in}^4$$

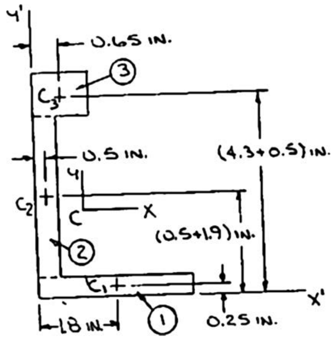
Then $\bar{I}_y = (84.933 - 9.733)\text{ in}^4$ or $\bar{I}_y = 75.2\text{ in}^4$ ◀



PROBLEM 9.44

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

SOLUTION



First locate centroid *C* of the area.

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$3.6\times 0.5 = 1.8$	1.8	0.25	3.24	0.45
2	$0.5\times 3.8 = 1.9$	0.25	2.4	0.475	4.56
3	$1.3\times 1 = 1.3$	0.65	4.8	0.845	6.24
Σ	5.0			4.560	11.25

Then $\qquad \bar{X}\Sigma A = \Sigma\bar{x}A\text{:} \quad \bar{X}(5\text{ in}^2) = 4.560\text{ in}^3$

or $\qquad \bar{X} = 0.912\text{ in.}$

and $\qquad \bar{Y}\Sigma A = \Sigma\bar{y}A\text{:} \quad \bar{Y}(5\text{ in}^2) = 11.25\text{ in}^3$

or $\qquad \bar{Y} = 2.25\text{ in.}$

PROBLEM 9.44 (Continued)

Now $$\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

where $$(I_x)_1 = \frac{1}{12}(3.6\text{ in.})(0.5\text{ in.})^3 + (1.8\text{ in}^2)[(2.25 - 0.25)\text{ in.}]^2$$
$$= (0.0375 + 7.20)\text{ in}^4 = 7.2375\text{ in}^4$$
$$(I_x)_2 = \frac{1}{12}(0.5\text{ in.})(3.8\text{ in.})^3 + (1.9\text{ in}^2)[(2.4 - 2.25)\text{ in.}]^2$$
$$= (2.2863 + 0.0428)\text{in}^4 = 2.3291\text{ in}^4$$
$$(I_x)_3 = \frac{1}{12}(1.3\text{ in.})(1\text{ in.})^3 + (1.3\text{ in}^2)[(4.8 - 2.25\text{ in.})]^2$$
$$= (0.1083 + 8.4533)\text{ in}^4 = 8.5616\text{ in}^4$$

Then $$\bar{I}_x = (7.2375 + 2.3291 + 8.5616)\text{ in}^4 = 18.1282\text{ in}^4$$

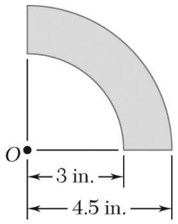
or $\bar{I}_x = 18.13\text{ in}^4$ ◀

Also $$\bar{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

where $$(I_y)_1 = \frac{1}{12}(0.5\text{ in.})(3.6\text{ in.})^3 + (1.8\text{ in}^2)[(1.8 - 0.912)\text{ in.}]^2$$
$$= (1.9440 + 1.4194)\text{ in}^4 = 3.3634\text{ in}^4$$
$$(I_y)_2 = \frac{1}{12}(3.8\text{ in.})(0.5\text{ in.})^3 + (1.9\text{ in}^2)[(0.912 - 0.25)\text{ in.}]^2$$
$$= (0.0396 + 0.8327)\text{ in}^4 = 0.8723\text{ in}^4$$
$$(I_y)_3 = \frac{1}{12}(1\text{ in.})(1.3\text{ in.})^3 + (1.3\text{ in}^2)[(0.912 - 0.65)\text{ in.}]^2$$
$$= (0.1831 + 0.0892)\text{ in}^4 = 0.2723\text{ in}^4$$

Then $$\bar{I}_y = (3.3634 + 0.8723 + 0.2723)\text{in}^4 = 4.5080\text{ in}^4$$

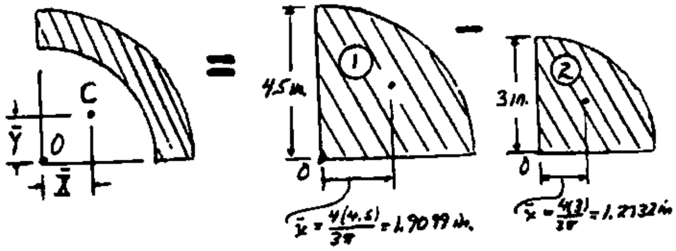
or $\bar{I}_y = 4.51\text{ in}^4$ ◀



PROBLEM 9.45

Determine the polar moment of inertia of the area shown with respect to (*a*) Point *O*, (*b*) the centroid of the area.

SOLUTION



Section	Area, in^2	$\bar{x}$, in.	$\bar{x}A$, in^3
1	$\frac{\pi}{4}(4.5)^2 = 15.904$	1.9099	30.375
2	$-\frac{\pi}{4}(3)^2 = -7.069$	1.2732	–9.00
Σ	8.835		21.375

Then $\bar{X}A = \Sigma\bar{x}A$: $\bar{X}(8.835\text{ in}^2) = 21.375\text{ in}^3$

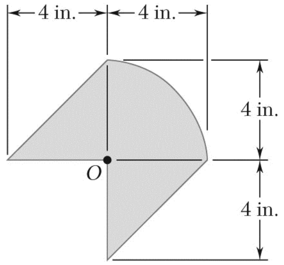
or $\bar{X} = 2.419$ in.

Then $J_O = \frac{\pi}{8}(4.5\text{ in.})^4 - \frac{\pi}{8}(3\text{ in.})^4 = 129.22\text{ in}^4$ $\qquad J_O = 129.2\text{ in}^4$ ◀

$$\overline{OC} = \sqrt{2}\bar{X} = \sqrt{2}(2.419\text{ in.}) = 3.421\text{ in.}$$

$$J_O = \bar{J}_C + A(\overline{OC})^2:$$

$129.22\text{ in}^4 = \bar{J}_C + (8.835\text{ in.})(3.421\text{ in.})^2$ $\qquad \bar{J}_C = 25.8\text{ in}^4$ ◀



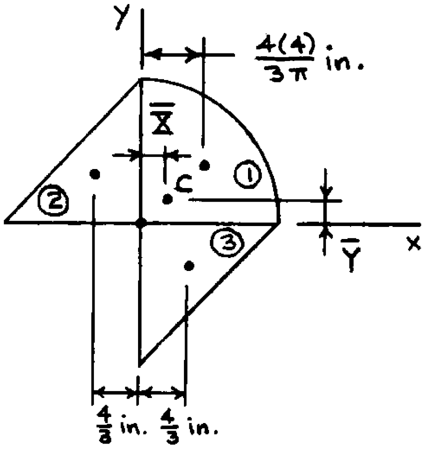
PROBLEM 9.46

Determine the polar moment of inertia of the area shown with respect to (*a*) Point *O*, (*b*) the centroid of the area.

SOLUTION

Determination of centroid *C* of entire section:

Section	Area, in^2	$\bar{x}$, in.	$\bar{x}A$, in^3
1	$\frac{\pi}{4}(4)^2 = 4\pi$	$\frac{16}{3\pi}$	21.333
2	$\frac{1}{2}(4)(4) = 8$	$-\frac{4}{3}$	−10.6667
3	$\frac{1}{2}(4)(4) = 8$	$\frac{4}{3}$	10.6667
Σ	28.566		21.333



$$\bar{X}\Sigma A = \Sigma \bar{x}A: \quad \bar{X}(28.566\ \text{in}^2) = 21.333\ \text{in}^3$$

$$\bar{X} = 0.74680\ \text{in.}; \quad \text{by symmetry,} \quad \bar{Y} = \bar{X} = 0.74680\ \text{in.}$$

(*a*) For section ①: $I_x = \frac{1}{4}\left(\frac{1}{4}\pi r^4\right) = \frac{1}{16}\pi(4)^4 = 50.265\ \text{in}^4$

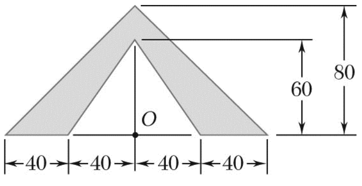
For sections ②,③: $I_x = \frac{1}{12}bh^3 = \frac{1}{12}(4)(4)^3 = 21.333\ \text{in}^4$

For total area, $I_x = 50.265 + 2(21.333) = 92.931\ \text{in}^4$

By symmetry, $I_y = I_x; \quad J_O = I_x + I_y = 185.862\ \text{in}^4$ $\qquad J_O = 185.9\ \text{in}^4$ ◀

(*b*) Parallel axis theorem: $J_C = J_O - A(OC)^2 = J_O - A(\bar{X}^2 + \bar{Y}^2)$

$J_C = 185.862 - (28.566)(0.74680^2 + 0.74680^2)$ $\qquad J_C = 154.0\ \text{in}^4$ ◀



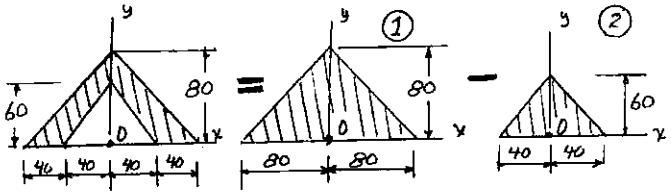
Dimensions in mm

PROBLEM 9.47

Determine the polar moment of inertia of the area shown with respect to (*a*) Point *O*, (*b*) the centroid of the area.

SOLUTION

Dimensions in mm Symmetry: $\bar{X} = 0$



Determination of centroid *C* of entire section

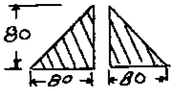
$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$
$$\bar{Y}(4000\text{ mm}^2) = 122.67\times 10^3\text{ mm}^3$$
$$\bar{Y} = 30.667\text{ mm}$$

	Area mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$\frac{1}{2}(160)(80) = 6400$	$\frac{80}{3}$	170.67×10^3
2	$-\frac{1}{2}(80)(60) = -2400$	20	-48×10^3
Σ	4000		122.67×10^3

(*a*) Polar moment of inertia J_O:

Section ①: $I_x = \frac{1}{12}(160)(80)^3 = 6.8267\times 10^6\text{ mm}^4$

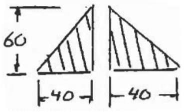
For I_y consider the following two triangles



$$I_y = 2\left[\frac{1}{12}(80)(80)^3\right] = 6.8267\times 10^6\text{ mm}^4$$
$$J_O = I_x + I_y = (6.8267 + 6.8267)10^6 = 13.653\times 10^6\text{ mm}^4$$

(*b*) Section ②: $I_x = \frac{1}{12}(80)(60)^3 = 1.44\times 10^6 \text{ mm}^4$

For I_y consider the following two triangles



$$I_y = 2\left[\frac{1}{12}(60)(40)^3\right] = 0.640\times 10^6 \text{ mm}^4$$

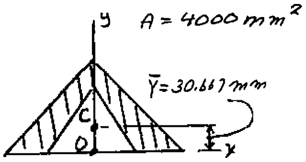
$$J_O = I_x + I_y = (1.44 + 0.640)10^6 = 2.08\times 10^6 \text{ mm}^4$$

Entire section $J_O = (J_O)_1 - (J_O)_2 = 13.653\times 10^6 - 2.08\times 10^6$

$$J_O = 11.573\times 10^6 \text{ mm}^4$$

$J_O = 11.57\times 10^6 \text{ mm}^4$ ◀

(*c*) Polar moment of inertia J_O of intire area

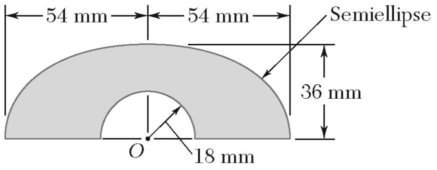


$$J_O = J_C + A\bar{Y}^2$$

$$11.573\times 10^6 = J_C + (4000)(30.667)^2$$

$$J_C = 7.811\times 10^6 \text{ mm}^4$$

$J_C = 7.81\times 10^6 \text{ mm}^4$ ◀

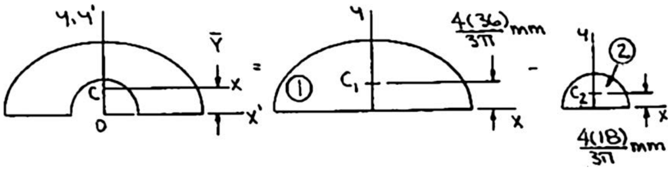


PROBLEM 9.48

Determine the polar moment of inertia of the area shown with respect to (*a*) Point *O*, (*b*) the centroid of the area.

SOLUTION

First locate centroid *C* of the area.



	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$\frac{\pi}{2}(54)(36) = 3053.6$	$\frac{48}{\pi} = 15.2789$	$46,656$
2	$-\frac{\pi}{2}(18)^2 = -508.9$	$\frac{24}{\pi} = 7.6394$	-3888
Σ	2544.7		$42,768$

Then $\bar{Y}\Sigma A = \Sigma \bar{y} A$: $\bar{Y}(2544.7 \text{ mm}^2) = 42,768 \text{ mm}^3$

or $\bar{Y} = 16.8067 \text{ mm}$ ◁

(*a*)

$$J_O = (J_O)_1 - (J_O)_2$$
$$= \frac{\pi}{8}(54 \text{ mm})(36 \text{ mm})[(54 \text{ mm})^2 + (36 \text{ mm})^2] - \frac{\pi}{4}(18 \text{ mm})^4$$
$$= (3.2155\times10^6 - 0.0824\times10^6) \text{ mm}^4$$
$$= 3.1331\times10^6 \text{ mm}^4$$

or $J_O = 3.13\times10^6 \text{ mm}^4$ ◀

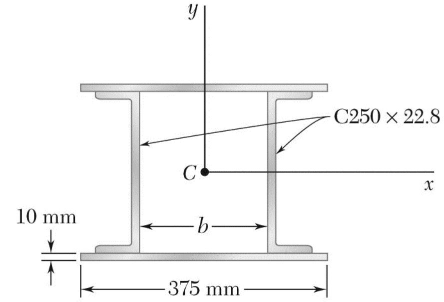
(*b*)

$$J_O = \bar{J}_C + A(\bar{Y})^2$$

or

$$\bar{J}_C = 3.1331\times10^6 \text{ mm}^4 - (2544.7 \text{ mm}^2)(16.8067 \text{ mm})^2$$

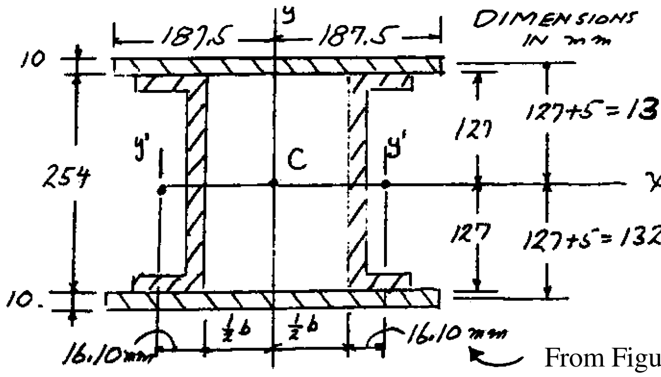
or $\bar{J}_C = 2.41\times10^6 \text{ mm}^4$ ◀



PROBLEM 9.49

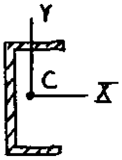
Two channels and two plates are used to form the column section shown. For $b = 200$ mm, determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal x and y axes.

SOLUTION



From Figure 9.13B

For C250 × 22.8



$$A = 2890 \text{ mm}^2$$
$$I_x = 28.0\times10^6 \text{ mm}^4$$
$$I_y = 0.945\times10^6 \text{ mm}^4$$

Total area $\quad A = 2[2890 \text{ mm}^4 + (10 \text{ mm})(375 \text{ mm})] = 13.28\times10^3 \text{ mm}^2$

Given $b = 200$ mm: $\quad \bar{I}_x = 2[28.0\times10^6 \text{ mm}^4] + 2\left[\frac{1}{12}(375 \text{ mm})(10 \text{ mm})^3 + (375 \text{ mm})(10 \text{ mm})(132 \text{ mm})^2\right]$

$$= 186.743\times10^6 \qquad \bar{I}_x = 186.7\times10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{k}_x^2 = \frac{\bar{I}_x}{A} = \frac{186.743\times10^6}{13.28\times10^3} = 14.0620\times10^3 \text{ mm}^2 \qquad \bar{k}_x = 118.6 \text{ mm} \blacktriangleleft$$

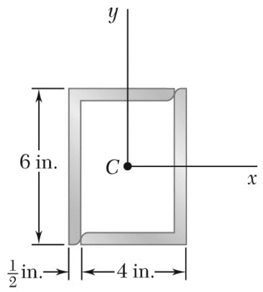
$$\bar{I}_y = \Sigma(\bar{I}_y + Ad^2) = 2[\underset{\text{Channel}}{\bar{I}_y + Ad^2}] + 2\underset{\text{Plate}}{\left[\frac{1}{12}(10\text{ mm})(375\text{ mm})^3\right]}$$

$$= 2\left[0.945\times10^6\text{ mm}^4 + (2890\text{ mm}^2)\left(\frac{200\text{ mm}}{2} + 16.10\text{ mm}\right)^2\right] + 87.891\times10^6\text{ mm}^4$$

$$= 2[0.945\times10^6 + 38.955\times10^6] + 87.891\times10^6$$

$$= 167.691\times10^6\text{ mm}^4 \qquad \bar{I}_y = 167.7\times10^6\text{ mm}^4 \blacktriangleleft$$

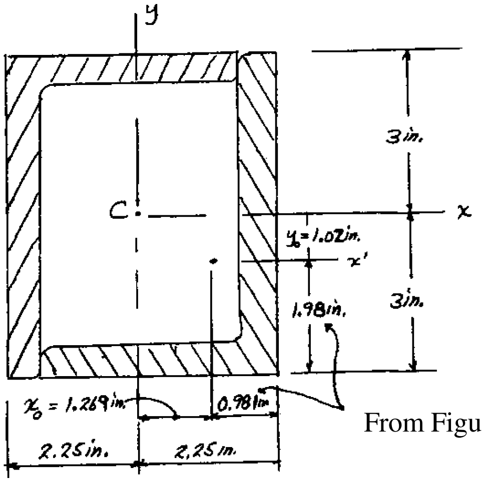
$$k_y^2 = \frac{I_y}{A} = \frac{167.691\times10^6}{13.28\times10^3} = 12.6273\times10^3 \qquad \bar{k}_y = 112.4\text{ mm} \blacktriangleleft$$



PROBLEM 9.50

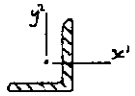
Two L6 × 4 × $\frac{1}{2}$-in. angles are welded together to form the section shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal x and y axes.

SOLUTION



From Figure 9.13A

From Figure 9.13A: Area $= A = 4.75 \text{ in}^2$



$$I_{x'} = 17.3 \text{ in}^4$$

$$I_{y'} = 6.22 \text{ in}^4$$

$$\bar{I}_x = 2[\bar{I}_{x'} + Ay_O^2] = 2[17.3 \text{ in}^4 + (4.75 \text{ in}^2)(1.02 \text{ in.})^2] = 44.484 \text{ in}^4$$

$$\bar{I}_x = 44.5 \text{ in}^4 \blacktriangleleft$$

Total area = 2(4.75) = 9.50 in^2

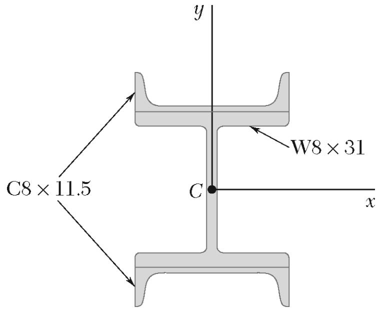
$$\bar{k}_x^2 = \frac{\bar{I}_x}{\text{area}} = \frac{44.484 \text{ in}^4}{9.50 \text{ in}^2} = 4.6825 \text{ in}^2 \qquad \bar{k}_x = 2.16 \text{ in.} \blacktriangleleft$$

$$\bar{I}_y = 2[\bar{I}_{y'} + Ax_O^2] = 2[6.22 \text{ in}^4 + (4.75 \text{ in}^2)(1.269 \text{ in.})^2]$$

$$= 27.738 \text{ in}^4 \qquad \bar{I}_y = 27.7 \text{ in}^4 \blacktriangleleft$$

Total area = 2(4.75) = 9.50 in^2

$$\bar{k}_y^2 = \frac{\bar{I}_y}{\text{area}} = \frac{27.738 \text{ in}^4}{9.50 \text{ in}^2} = 2.9198 \qquad \bar{k}_y = 1.709 \text{ in} \blacktriangleleft$$

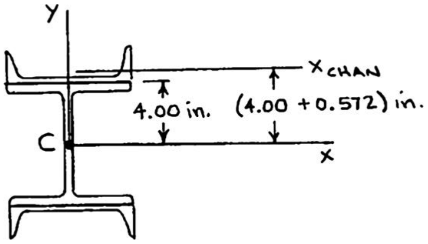


PROBLEM 9.51

Two channels are welded to a rolled W section as shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal x and y axes.

SOLUTION

W section:
$$A = 9.12 \text{ in}^2$$
$$\bar{I}_x = 110 \text{ in}^4$$
$$\bar{I}_y = 37.1 \text{ in}^4$$



Channel:
$$A = 3.37 \text{ in}^2$$
$$\bar{I}_x = 1.31 \text{ in}^4$$
$$\bar{I}_y = 32.5 \text{ in}^4$$

$$A_{\text{total}} = A_{\text{W}} + 2A_{\text{chan}}$$
$$= 9.12 + 2(3.37) = 15.86 \text{ in}^2$$

Now
$$\bar{I}_x = (\bar{I}_x)_{\text{W}} + 2(I_x)_{\text{chan}}$$

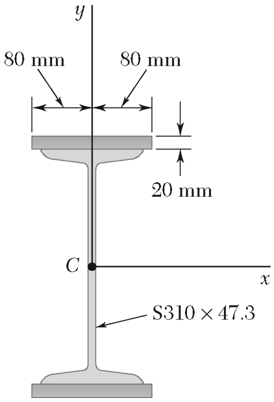
where
$$(I_x)_{\text{chan}} = \bar{I}_{x_{\text{chan}}} + Ad^2$$
$$= 1.31 \text{ in}^4 + (3.37 \text{ in}^2)(4.572 \text{ in.})^2 = 71.754 \text{ in}^4$$

Then
$$\bar{I}_x = (110 + 2 \times 71.754) \text{ in}^4 = 253.51 \text{ in}^4$$
$\bar{I}_x = 254 \text{ in}^4$ ◀

and
$$\bar{k}_x^2 = \frac{\bar{I}_x}{A_{\text{total}}} = \frac{253.51 \text{ in}^4}{15.86 \text{ in}^2}$$
$\bar{k}_x = 4.00$ in. ◀

Also
$$\bar{I}_y = (\bar{I}_y)_{\text{W}} + 2(\bar{I}_y)_{\text{chan}}$$
$$= (37.1 + 2 \times 32.5) \text{ in}^4 = 102.1 \text{ in}^4$$
$\bar{I}_y = 102.1 \text{ in}^4$ ◀

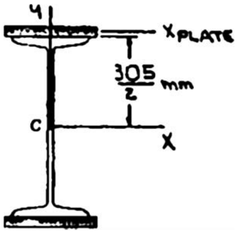
and
$$\bar{k}_y^2 = \frac{\bar{I}_y}{A_{\text{total}}} = \frac{102.1 \text{ in}^4}{15.86 \text{ in}^2}$$
$\bar{k}_y = 2.54$ in. ◀



PROBLEM 9.52

Two 20-mm steel plates are welded to a rolled S section as shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal x and y axes.

SOLUTION



S section: $A = 6010\text{ mm}^2$

$\bar{I}_x = 90.3\times 10^6\text{ mm}^4$

$\bar{I}_y = 3.88\times 10^6\text{ mm}^4$

Note: $A_{\text{total}} = A_{\text{S}} + 2A_{\text{plate}}$

$= 6010\text{ mm}^2 + 2(160\text{ mm})(20\text{ mm})$

$= 12,410\text{ mm}^2$

Now $\bar{I}_x = (\bar{I}_x)_{\text{S}} + 2(I_x)_{\text{plate}}$

where $(I_x)_{\text{plate}} = \bar{I}_{x_{\text{plate}}} + Ad^2$

$$= \frac{1}{12}(160\text{ mm})(20\text{ mm})^3 + (3200\text{ mm}^2)[(152.5+10)\text{ mm}]^2$$

$= 84.6067\times 10^6\text{ mm}^4$

Then $\bar{I}_x = (90.3 + 2\times 84.6067)\times 10^6\text{ mm}^4$

$= 259.5134\times 10^6\text{ mm}^4$ or $\bar{I}_x = 260\times 10^6\text{ mm}^4$ ◀

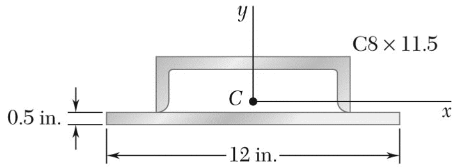
and $\bar{k}_x^2 = \dfrac{\bar{I}_x}{A_{\text{total}}} = \dfrac{259.5134\times 10^6\text{ mm}^4}{12410\text{ mm}^2}$ or $\bar{k}_x = 144.6\text{ mm}$ ◀

Also $\bar{I}_y = (\bar{I}_y)_{\text{S}} + 2(\bar{I}_y)_{\text{plate}}$

$$= 3.88\times 10^6\text{ mm}^4 + 2\left[\frac{1}{12}(20\text{ mm})(160\text{ mm})^3\right]$$

$= 17.5333\times 10^6\text{ mm}^4$ or $\bar{I}_y = 17.53\times 10^6\text{ mm}^4$ ◀

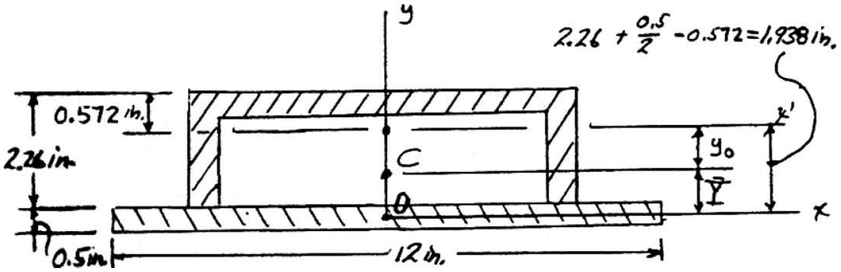
and $\bar{k}_y^2 = \dfrac{\bar{I}_y}{A_{\text{total}}} = \dfrac{17.5333\times 10^6\text{ mm}^4}{12,410\text{ mm}^2}$ or $\bar{k}_y = 37.6\text{ mm}$ ◀



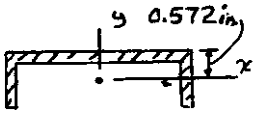
PROBLEM 9.53

A channel and a plate are welded together as shown to form a section that is symmetrical with respect to the y axis. Determine the moments of inertia of the combined section with respect to its centroidal x and y axes.

SOLUTION



From Figure 9.13B



$B = 3.37\text{ in}^2$

For C8 × 11.5: $\bar{I}_y = 1.31\text{ in}^4$

(Note change of axes) $\bar{I}_y = 32.5\text{ in}^4$

Location of centroid

$$\bar{Y}\Sigma A = \Sigma\bar{y}A: \qquad \bar{Y}[3.7\text{ in}^2 + (12\text{ in.})(0.5\text{ in.})] = (3.37\text{ in}^2)(1.938\text{ in.})$$

$$\bar{Y} = 0.69702\text{ in.}$$

Moment of inertia with respect to x axis.

Plate:

$$I_x = \bar{I}_x + A\bar{Y}^2 = \frac{1}{12}(12\text{ in.})(0.5\text{ in.})^3 + (12\text{ in.})(0.5\text{ in.})(0.69702\text{ in.})^2$$

$$= 0.125 + 2.9150 = 3.0400\text{ in}^4$$

Channel:

$$I_x = \bar{I}_{x'} + A\bar{Y}_0^2 = 1.31\text{ in}^4 + (3.37\text{ in.})(1.938\text{ in.} - 0.69702\text{ in.})^2$$

$$= 1.31 + 5.1899 = 6.4999\text{ in}^4$$

Entire section: $I_x = 3.0400\text{ in}^4 + 6.4999\text{ in}^4 = 9.5399\text{ in}^4$ $\qquad \bar{I}_x = 9.54\text{ in}^4$ ◀

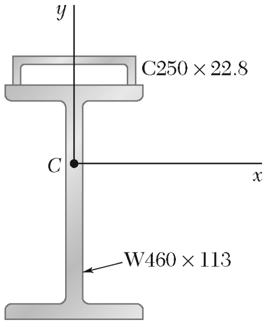
Moment of inertia with respect to y axis.

Plate:

$$\bar{I}_y = \frac{1}{12}(0.5\text{ in.})(12\text{ in.})^3 = 72\text{ in}^4$$

Channel: $\bar{I}_y = 32.5\text{ in}^4$

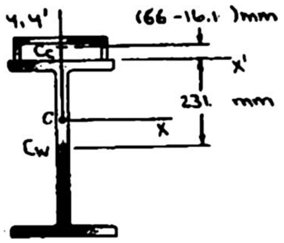
Entire section: $\bar{I}_y = 72\text{ in}^4 + 32.5\text{ in}^4 = 104.5\text{ in}^4$ $\qquad \bar{I}_y = 104.5\text{ in}^4$ ◀



PROBLEM 9.54

The strength of the rolled W section shown is increased by welding a channel to its upper flange. Determine the moments of inertia of the combined section with respect to its centroidal x and y axes.

SOLUTION



W section: $A = 14,400\text{ mm}^2$

$\bar{I}_x = 554\times10^6\text{ mm}^4$

$\bar{I}_y = 63.3\times10^6\text{ mm}^4$

Channel: $A = 2890\text{ mm}^2$

$\bar{I}_x = 0.945\times10^6\text{ mm}^4$

$\bar{I}_y = 28.0\times10^6\text{ mm}^4$

First locate centroid C of the section.

	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
W Section	14,400	−231	−33,26,400
Channel	2,890	49.9	1,44,211
Σ	17,290		−31,82,189

Then $\bar{Y}\Sigma A = \Sigma\bar{y}A$: $\bar{Y}(17,290\text{ mm}^2) = -3,182,189\text{ mm}^3$

or $\bar{Y} = -184.047\text{ mm}$

Now $\bar{I}_x = (I_x)_W + (I_x)_C$

where

$$
\begin{aligned}
(I_x)_W &= \bar{I}_x + Ad^2\\
&= 554\times10^6\text{ mm}^4 + (14,400\text{ mm}^2)(231-184.047)^2\text{ mm}^2\\
&= 585.75\times10^6\text{ mm}^4\\
(I_x)_C &= \bar{I}_x - Ad^2\\
&= 0.945\times10^6\text{ mm}^4 + (2,890\text{ mm}^2)(49.9+184.047)^2\text{ mm}^2\\
&= 159.12\times10^6\text{ mm}^4
\end{aligned}
$$

PROBLEM 9.54 (Continued)

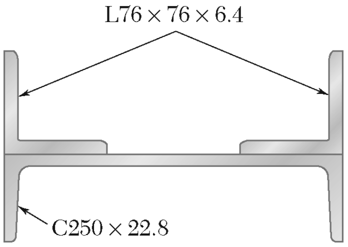
Then $\bar{I}_x = (585.75 + 159.12) \times 10^6 \text{ mm}^4$

or $\bar{I}_x = 745 \times 10^6 \text{ mm}^4$ ◀

also $\bar{I}_y = (I_y)_\text{W} + (I_y)_C$

$= (63.3 + 28.0) \times 10^6 \text{ mm}^4$

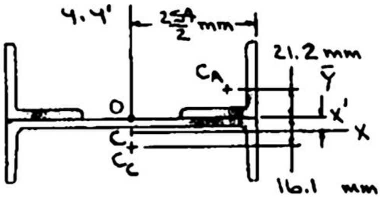
or $\bar{I}_y = 91.3 \times 10^6 \text{ mm}^4$ ◀



PROBLEM 9.55

Two L76 × 76 × 6.4-mm angles are welded to a C250 × 22.8 channel. Determine the moments of inertia of the combined section with respect to centroidal axes respectively parallel and perpendicular to the web of the channel.

SOLUTION



Angle: $A = 929\text{ mm}^2$

$\bar{I}_x = \bar{I}_y = 0.512 \times 10^6\text{ mm}^4$

Channel: $A = 2890\text{ mm}^2$

$\bar{I}_x = 0.945 \times 10^6\text{ mm}^4$

$\bar{I}_y = 28.0 \times 10^6\text{ mm}^4$

First locate centroid *C* of the section

	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
Angle	2(929) = 1858	21.2	39,389.6
Channel	2890	−16.1	−46,529
Σ	4748		−7139.4

Then $\bar{Y}\Sigma A = \Sigma \bar{y} A$: $\quad \bar{Y}(4748\text{ mm}^2) = -7139.4\text{ mm}^3$

or $\bar{Y} = -1.50366\text{ mm}$

Now $\bar{I}_x = 2(I_x)_L + (I_x)_C$

where $(I_x)_L = \bar{I}_x + Ad^2 = 0.512 \times 10^6\text{ mm}^4 + (929\text{ mm}^2)[(21.2 + 1.50366)\text{ mm}]^2$

$= 0.990859 \times 10^6\text{ mm}^4$

$(I_x)_C = \bar{I}_x + Ad^2 = 0.949 \times 10^6\text{ mm}^4 + (2890\text{ mm}^2)[(16.1 - 1.50366)\text{ mm}]^2$

$= 1.56472 \times 10^6\text{ mm}^4$

Then $\bar{I}_x = [2(0.990859) + 1.56472 \times 10^6]\text{ mm}^4$

or $\bar{I}_x = 3.55 \times 10^6\text{ mm}^4$ ◀

PROBLEM 9.55 (Continued)

Also $\bar{I}_y = 2(I_y)_L + (I_y)_C$

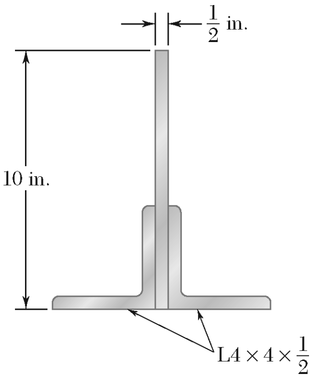
where $(I_y)_L = \bar{I}_y + Ad^2 = 0.512\times 10^6\,\text{mm}^4 + (929\text{ mm}^2)[(127-21.2)\text{ mm}]^2$

$= 10.9109\times 10^6\,\text{mm}^4$

$(I_y)_C = \bar{I}_y$

Then $\bar{I}_y = [2(10.9109) + 28.0]\times 10^6\,\text{mm}^4$

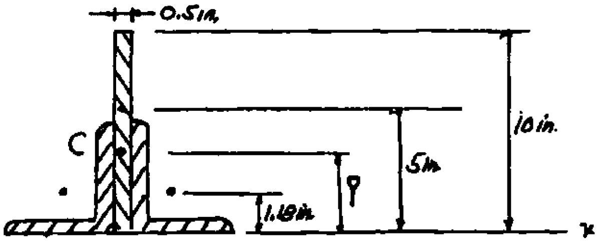
or $\bar{I}_y = 49.8\times 10^6\,\text{mm}^4$ ◀



PROBLEM 9.56

Two L4 × 4 × $\frac{1}{2}$ -in. angles are welded to a steel plate as shown. Determine the moments of inertia of the combined section with respect to centroidal axes respectively parallel and perpendicular to the plate.

SOLUTION



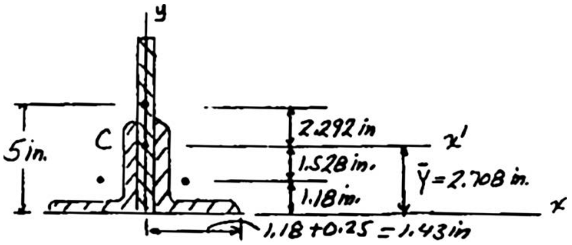
For $4\times4\times\dfrac{1}{2}$-in. angle: $A = 3.75\text{ in}^2,\quad \bar{I}_x = \bar{I}_y = 5.52\text{ in}^4$

$$\bar{Y}A = \Sigma\bar{y}\,A$$

$$\bar{Y}(12.5\text{ in}^2) = 33.85\text{ in}^3$$

$$\bar{Y} = 2.708\text{ in.}$$

Section	Area, in^2	$\bar{y}$ in.	$\bar{y}A$, in^3
Plate	(0.5)(10) = 5	5	25
Two angles	2(3.75) = 7.5	1.18	8.85
Σ	12.5		33.85



PROBLEM 9.56 (Continued)

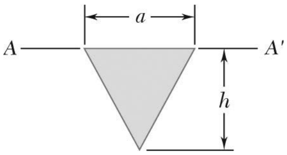
Entire section:

$$\bar{I}_x = \Sigma(\bar{I}_{x'} + Ad^2) = \left[\frac{1}{12}(0.5)(10)^3 + (0.5)(10)(2.292)^2\right] + 2[5.52 + (3.75)(1.528)^2]$$

$$= 41.667 + 26.266 + 1604 + 17.511 = 96.48\text{ in}^4 \qquad \bar{I}_x = 96.5\text{ in}^4 \blacktriangleleft$$

$$\bar{I}_y = \frac{1}{12}(10)(0.5)^3 + 2[5.52 + (3.75)(1.43)^2]$$

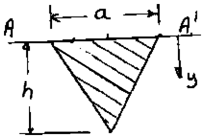
$$= 0.104 + 11.04 + 15.367 = 26.51\text{ in}^4 \qquad \bar{I}_y = 26.5\text{ in}^4 \blacktriangleleft$$



PROBLEM 9.57

The panel shown forms the end of a trough that is filled with water to the line AA'. Referring to section 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION



From section 9.2: $R = \gamma\int y\,dA, \quad M_{AA'} = \gamma\int y^2 dA$

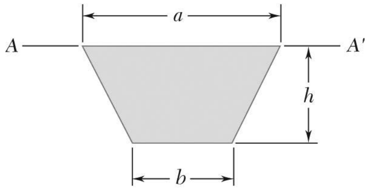
Let y_P = distance of center of pressure from AA'. We must have:

$$Ry_P = M_{AA'}: \quad y_P = \frac{M_{AA'}}{R} = \frac{\gamma\int y^2 dA}{\gamma\int y\,dA} = \frac{I_{AA'}}{\bar{y}A} \qquad (1)$$

For triangular panel:

$$I_{AA'} = \frac{1}{12}ah^3 \qquad \bar{y} = \frac{1}{3}h \qquad A = \frac{1}{2}ah$$

$$y_P = \frac{I_{AA'}}{\bar{y}A} = \frac{\frac{1}{12}ah^3}{\left(\frac{1}{3}h\right)\left(\frac{1}{2}ah\right)} \qquad y_P = \frac{1}{2}h \blacktriangleleft$$

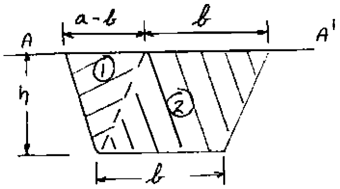


PROBLEM 9.58

The panel shown forms the end of a trough that is filled with water to the line AA'. Referring to section 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION

See solution of Problem 9.57 for derivation of Eq. (1):



$$y_P = \frac{I_{AA'}}{\bar{y}A} \tag{1}$$

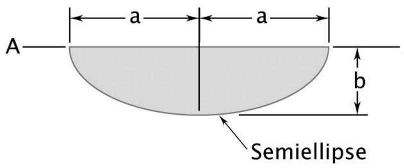
Divide trapezoid as shown:

$$I_{AA'} = \frac{1}{12}(a-b)h^3 + \frac{1}{3}bh^3$$

$$= \frac{1}{2}ah^3 + \frac{1}{4}bh^3$$

$$\bar{y}A = y_1A_1 + y_2A_2 = \frac{1}{3}h\left[\frac{1}{2}(a-b)h\right] + \frac{1}{2}h(bh) = \frac{1}{6}ah^2 + \frac{1}{3}bh^2$$

$$y_P = \frac{I_{AA'}}{\bar{y}A} = \frac{\frac{1}{12}ah^3 + \frac{1}{4}bh^3}{\frac{1}{6}ah^2 + \frac{1}{3}bh^2} \qquad y_P = \frac{a+3b}{2a+4b}h \blacktriangleleft$$



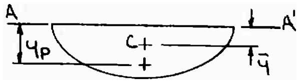
PROBLEM 9.59

The panel shown forms the end of a trough that is filled with water to the line AA'. Referring to section 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION

Using the equation developed on page 475 of the text have:

$$y_P = \frac{I_{AA'}}{\overline{y}A}$$

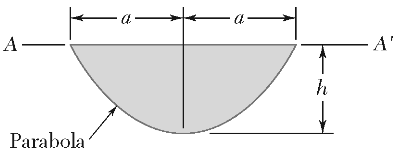


For a semi ellipse: $I_{AA'} = \dfrac{\pi}{8}ab^3$

$$\overline{y} = \frac{4b}{3\pi} \qquad A = \frac{\pi}{2}ab$$

Then $y_P = \dfrac{\frac{\pi}{8}ab^3}{\frac{4b}{3\pi} \times \frac{\pi}{2}ab}$ or

or $y_P = \dfrac{3\pi}{16}b$ ◀

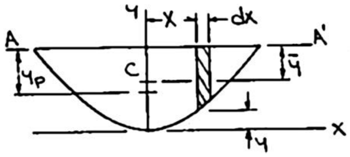


PROBLEM 9.60*

The panel shown forms the end of a trough that is filled with water to the line *AA′*. Referring to section 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION

Using the equation developed on page 491 of the text:



$$y_P = \frac{I_{AA'}}{\bar{y}A}$$

For a parabola: $\bar{y} = \frac{2}{5}h \qquad A = \frac{4}{3}ah$

Now $dI_{AA'} = \frac{1}{3}(h-y)^3\,dx$

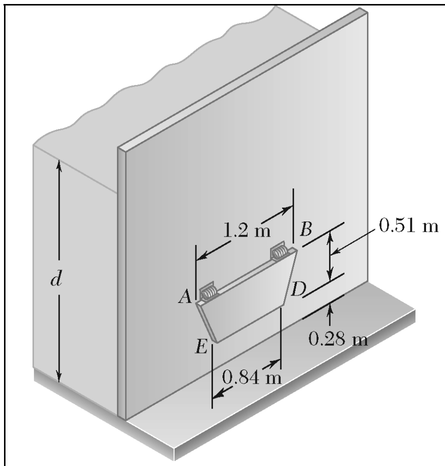
By observation $y = \frac{h}{a^2}x^2$

So that
$$dI_{AA'} = \frac{1}{3}\left(h - \frac{h}{a^2}x^2\right)^3 dx = \frac{1}{3}\frac{h^3}{a^6}(a^2 - x^2)^3\,dx$$
$$= \frac{1}{3}\frac{h}{a^6}(a^6 - 3a^4x^2 + 3a^2x^4 - x^6)dx$$

Then
$$I_{AA'} = 2\int_0^a \frac{1}{3}\frac{h^3}{a^6}(a^6 - 3a^4x^2 + 3a^2x^4 - x^6)dx$$
$$= \frac{2}{3}\frac{h^3}{a^6}\left[a^6x - a^4x^3 + \frac{3}{5}a^2x^5 - \frac{1}{7}x^7\right]_0^a$$
$$= \frac{32}{105}ah^3$$

Finally,
$$y_p = \frac{\frac{32}{105}ah^3}{\frac{2}{5}h \times \frac{4}{3}ah}$$

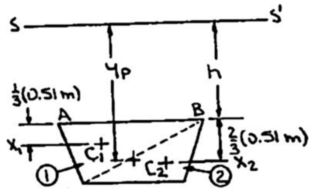
or $y_P = \frac{4}{7}h$ ◀



PROBLEM 9.61

A vertical trapezoidal gate that is used as an automatic valve is held shut by two springs attached to hinges located along edge *AB*. Knowing that each spring exerts a couple of magnitude 1470 N · m, determine the depth *d* of water for which the gate will open.

SOLUTION



From section 9.2:

$$R = \gamma \bar{y} A \qquad y_P = \frac{I_{SS'}}{\bar{y}A}$$

where *R* is the resultant of the hydrostatic forces acting on the gate and y_P is the depth to the point of application of *R*. Now

$$\bar{y}A = \Sigma \bar{y}A = [(h+0.17)\text{ m}]\left(\frac{1}{2}\times 1.2\text{ m}\times 0.51\text{ m}\right) + [(h+0.34)\text{ m}]\left(\frac{1}{2}\times 0.84\times 0.51\text{ m}\right)$$

$$= (0.5202h + 0.124848)\text{ m}^3$$

Recalling that $\gamma = py$, we have

$$R = (10^3\text{ kg/m}^3 \times 9.81\text{ m/s}^2)(0.5202h + 0.124848)\text{ m}^3$$

$$= 5103.162(h+0.24)\text{ N}$$

Also $$I_{SS'} = (I_{SS'})_1 + (I_{SS'})_2$$

where $$(I_{SS'})_1 = I_x + Ad^2$$

$$= \frac{1}{36}(1.2\text{ m})(0.51\text{ m})^3 + \left(\frac{1}{2}\times 1.2\text{ m}\times 0.51\text{ m}\right)[(h+0.17)\text{ m}]^2$$

$$= [0.0044217 + 0.306(h+0.17)^2]\text{ m}^4$$

$$= (0.306h^2 + 0.10404h + 0.0132651)\text{ m}^4$$

$$(I_{SS'})_2 = \bar{I}_{X_2} + Ad^2$$

$$= \frac{1}{36}(0.84\text{ m})(0.51\text{ m})^3 + \left(\frac{1}{2}\times 0.84\text{ m}\times 0.51\text{ m}\right)[(h+0.34)\text{ m}]^2$$

$$= [0.0030952 + 0.2142(h+0.34)^2]\text{ m}^4$$

$$= (0.2142h^2 + 0.145656h + 0.0278567)\text{ m}^4$$

PROBLEM 9.61 (Continued)

Then $$I_{SS'} = (I_{SS'})_1 + (I_{SS'})_2$$
$$= (0.5202h^2 + 0.249696h + 0.0411218)\ \text{m}^4$$

and $$y_P = \frac{(0.5202h^2 + 0.244696h + 0.0411218)\ \text{m}^4}{(0.5202h + 0.124848)\ \text{m}^3}$$
$$= \frac{h^2 + 0.48h + 0.07905}{h + 0.24}\ \text{m}$$

For the gate to open, require that

$$\Sigma M_{AB}: \quad M_{\text{open}} = (y_P - h)R$$

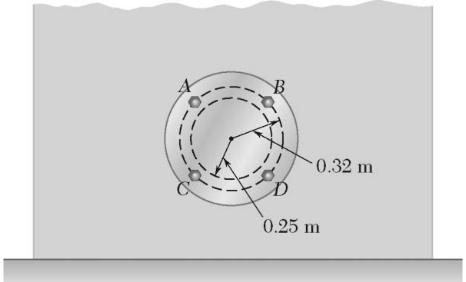
Substituting $$2940\ \text{N}\cdot\text{m} = \left(\frac{h^2 + 0.48h + 0.07905}{h + 0.24} - h\right)\text{m} \times 5103.162(h + 0.24)\ \text{N}$$

or $$5103.162(0.24h + 0.07905) = 2940$$

or $$h = 2.0711\ \text{m}$$

Then $$d = (2.0711 + 0.79)\ \text{m}$$

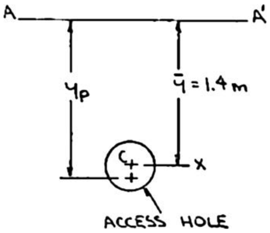
or $d = 2.86$ m ◀



PROBLEM 9.62

The cover for a 0.5-m-diameter access hole in a water storage tank is attached to the tank with four equally spaced bolts as shown. Determine the additional force on each bolt due to the water pressure when the center of the cover is located 1.4 m below the water surface.

SOLUTION



From section 9.2:

$$R = \gamma \bar{y} A \qquad y_P = \frac{I_{AA'}}{\bar{y}A}$$

where R is the resultant of the hydrostatic forces acting on the cover and y_P is the depth to the point of application of R.

Recalling that $\gamma = p \cdot y$, we have

$$R = (10^3 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2)(1.4 \text{ m})[\pi(0.25 \text{ m})^2]$$
$$= 2696.67 \text{ N}$$

Also

$$I_{AA'} = \bar{I}_x + A\bar{y}^2 = \frac{\pi}{4}(0.25 \text{ m})^4 + [\pi(0.25 \text{ m})^2](1.4 \text{ m})^2$$
$$= 0.387913 \text{ m}^4$$

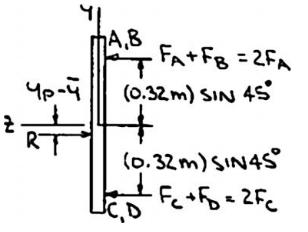
Then

$$y_P = \frac{0.387913 \text{ m}^4}{(1.4 \text{ m})[\pi(0.25 \text{ m})^2]} = 1.41116 \text{ m}$$

Now note that symmetry implies

$$F_A = F_B \qquad F_C = F_D$$

Next consider the free-body of the cover.



We have $+\circlearrowleft \Sigma M_{CD} = 0$: $[2(0.32 \text{ m}) \sin 45°](2F_A)$

$$-[0.32 \sin 45° - (1.41116 - 1.4)] \text{ m}$$
$$\times (2696.67 \text{ N}) = 0$$

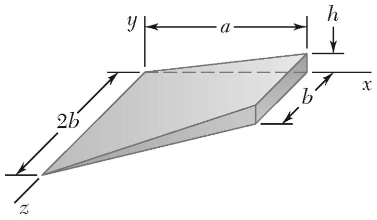
or $F_A = 640.92$ N

Then $\xleftarrow{+} \Sigma F_z = 0$: $2(640.92 \text{ N}) + 2F_C - 2696.67 \text{ N} = 0$

or $F_C = 707.42$ N

$F_A = F_B = 641$ N ◀

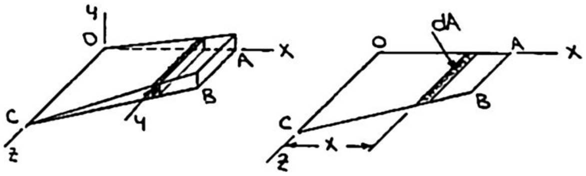
and $F_C = F_D = 707$ N ◀



PROBLEM 9.63*

Determine the x coordinate of the centroid of the volume shown. (*Hint:* The height y of the volume is proportional to the x coordinate; consider an analogy between this height and the water pressure on a submerged surface.)

SOLUTION



First note that $$y = \frac{h}{a}x$$

Now $$\bar{x}\int dV = \int \bar{x}_{EL}dV$$

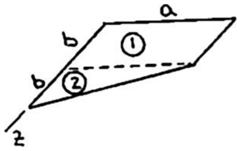
where $$\bar{x}_{EL} = x \quad dV = ydA = \left(\frac{h}{a}x\right)dA$$

Then $$\bar{x} = \frac{\int x\left(\frac{h}{a}xdA\right)}{\int \frac{h}{a}xdA} = \frac{\int x^2 dA}{\int xdA} = \frac{(I_z)_A}{(\bar{x}A)_A}$$

where $(I_z)_A$ and $(\bar{x}_A)_A$ pertain to area.

OABC: $(I_z)_A$ is the moment of inertia of the area with respect to the z axis, $\bar{x}_A$ is the x coordinate of the centroid of the area, and A is the area of *OABC*. Then

$$\begin{aligned}(I_z)_A &= (I_z)_{A_1} + (I_z)_{A_2}\\ &= \frac{1}{3}(b)(a)^3 + \frac{1}{12}(b)(a)^3\\ &= \frac{5}{12}a^3b\end{aligned}$$



and $$\begin{aligned}(\bar{x}A)_A &= \Sigma\bar{x}A\\ &= \left[\left(\frac{a}{2}\right)(a\times b)\right] + \left[\left(\frac{a}{3}\right)\left(\frac{1}{2}\times a\times b\right)\right]\\ &= \frac{2}{3}a^2b\end{aligned}$$

PROBLEM 9.63* (Continued)

Finally, $$\bar{x} = \frac{\frac{5}{12}a^3b}{\frac{2}{3}a^2b}$$

or $\bar{x} = \frac{5}{8}a$ ◀

Analogy with hydrostatic pressure on a submerged plate:

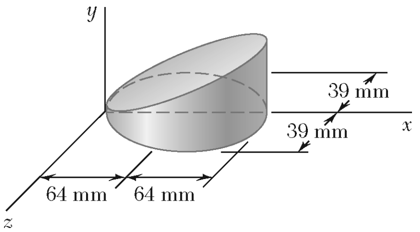
Recalling that $P = \gamma y$, it follows that the following analogies can be established.

Height $y \sim P$

$$dV = ydA \sim pdA = dF$$
$$xdV = x(ydA) \sim ydF = dM$$

Recalling that $$y_P = \frac{M_x}{R}\left(= \frac{\int dM}{\int dF}\right)$$

It can then be concluded that $x \sim y_P$



PROBLEM 9.64*

Determine the x coordinate of the centroid of the volume shown; this volume was obtained by intersecting an elliptic cylinder with an oblique plane. (*Hint:* The height y of the volume is proportional to the x coordinate; consider an analogy between this height and the water pressure on a submerged surface.)

SOLUTION

Following the "Hint," it can be shown that (see solution to Problem 9.63)

$$\bar{x} = \frac{(I_z)_A}{(\bar{x}A)_A} x$$

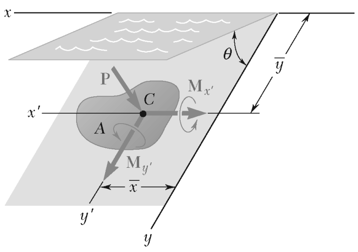
where $(I_z)_A$ and $(\bar{x}A)_A$ are the moment of inertia and the first moment of the area, respectively, of the elliptical area of the base of the volume with respect to the z axis. Then

$$\begin{aligned}(I_z)_A &= \bar{I}_z + Ad^2 \\ &= \frac{\pi}{4}(39\text{ mm})(64\text{ mm})^3 + [\pi(64\text{ mm})(39\text{ mm})](64\text{ mm})^2 \\ &= 12.779520\pi \times 10^6\text{ mm}^4\end{aligned}$$

$$\begin{aligned}(\bar{x}A)_A &= (64\text{ mm})[\pi(64\text{ mm})(39\text{ mm})] \\ &= 0.159744\pi \times 10^6\text{ mm}^3\end{aligned}$$

Finally $$\bar{x} = \frac{12.779520\pi \times 10^6\text{ mm}^4}{0.159744\pi \times 10^6\text{ mm}^4}$$

or $\bar{x} = 80.0\text{ mm}$ ◀



PROBLEM 9.65*

Show that the system of hydrostatic forces acting on a submerged plane area A can be reduced to a force **P** at the centroid C of the area and two couples. The force **P** is perpendicular to the area and is of magnitude $P = \gamma A\bar{y}\sin\theta$, where γ is the specific weight of the liquid, and the couples are $\mathbf{M}_{x'} = (\gamma\bar{I}_{x'}\sin\theta)\mathbf{i}$ and $\mathbf{M}_{y'} = (\gamma\bar{I}_{x'y'}\sin\theta)\mathbf{j}$, where $\bar{I}_{x'y'} = \int x'y'\,dA$ (see section 9.8). Note that the couples are independent of the depth at which the area is submerged.

SOLUTION

The pressure p at an arbitrary depth $(y\sin\theta)$ is

$$p = \gamma(y\sin\theta)$$

so that the hydrostatic force dF exerted on an infinitesimal area dA is

$$dF = (\gamma y\sin\theta)dA$$

Equivalence of the force **P** and the system of infinitesimal forces dF requires

$$\Sigma F:\quad P = \int dF = \int \gamma y\sin\theta\,dA = \gamma\sin\theta\int y\,dA$$

or $\quad P = \gamma A\bar{y}\sin\theta$ ◀

Equivalence of the force and couple $(\mathbf{P},\ \mathbf{M}_{x'} + \mathbf{M}_{y'})$ and the system of infinitesimal hydrostatic forces requires

$$\Sigma M_x:\quad -\bar{y}P - M_{x'} = \int(-y\,dF)$$

Now
$$-\int y\,dF = -\int y(\gamma y\sin\theta)dA = -\gamma\sin\theta\int y^2\,dA$$
$$= -(\gamma\sin\theta)I_x$$

Then
$$-\bar{y}P - M_{x'} = -(\gamma\sin\theta)I_x$$

or
$$M_{x'} = (\gamma\sin\theta)I_x - \bar{y}(\gamma A\bar{y}\sin\theta)$$
$$= \gamma\sin\theta(I_x - A\bar{y}^2)$$

or $\quad M_{x'} = \gamma\bar{I}_{x'}\sin\theta$ ◀

$$\Sigma M_y:\quad \bar{x}P + M_{y'} = \int x\,dF$$

Now
$$\int x\,dF = \int x(\gamma y\sin\theta)dA = \gamma\sin\theta\int xy\,dA$$
$$= (\gamma\sin\theta)I_{xy} \qquad \text{(Equation 9.12)}$$

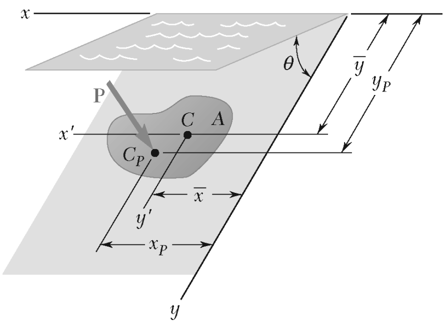
PROBLEM 9.65* (Continued)

Then $$\bar{x}P + M_{y'} = (\gamma \sin\theta) I_{xy}$$

or $$M_{y'} = (\gamma \sin\theta) I_{xy} - \bar{x}(\gamma A \bar{y} \sin\theta)$$

$$= \gamma \sin\theta (I_{xy} - A\bar{x}\,\bar{y})$$

or $M_{y'} = \gamma \bar{I}_{x'y'} \sin\theta$ ◀



PROBLEM 9.66*

Show that the resultant of the hydrostatic forces acting on a submerged plane area A is a force **P** perpendicular to the area and of magnitude $P = \gamma A\overline{y}\sin\theta = \overline{p}A$, where γ is the specific weight of the liquid and $\overline{p}$ is the pressure at the centroid C of the area. Show that **P** is applied at a Point C_P, called the center of pressure, whose coordinates are $x_P = I_{xy}/A\overline{y}$ and $y_P = I_x/A\overline{y}$, where $\overline{I}_{xy} = \int xydA$ (see section 9.8). Show also that the difference of ordinates $y_P - \overline{y}$ is equal to $\overline{k}_{x'}^2/\overline{y}$ and thus depends upon the depth at which the area is submerged.

SOLUTION

The pressure P at an arbitrary depth $(y\sin\theta)$ is

$$P = \gamma(y\sin\theta)$$

so that the hydrostatic force dP exerted on an infinitesimal area dA is

$$dP = (\gamma y\sin\theta)dA$$

The magnitude **P** of the resultant force acting on the plane area is then

$$P = \int dP = \int \gamma y\sin\theta\, dA = \gamma\sin\theta\int ydA$$
$$= \gamma\sin\theta(\overline{y}A)$$

Now $\qquad \overline{p} = \gamma\overline{y}\sin\theta \qquad\qquad P = \overline{p}A$ ◀

Next observe that the resultant **P** is equivalent to the system of infinitesimal forces d**P**. Equivalence then requires

$$\Sigma M_x: \quad -y_P P = -\int ydP$$

Now $\qquad \int ydP = \int y(\gamma y\sin\theta)dA = \gamma\sin\theta\int y^2 dA$

$$= (\gamma\sin\theta)I_x$$

Then $\qquad y_P P = (\gamma\sin\theta)I_x$

or $\qquad y_P = \dfrac{(\gamma\sin\theta)I_x}{\gamma\sin\theta(\overline{y}A)}$

or $\qquad y_P = \dfrac{I_x}{A\overline{y}}$ ◀

$$\Sigma M_y: \quad x_P P = \int x dP$$

Now $\int x dP = \int x(\gamma y \sin\theta) dA = \gamma \sin\theta \int xy dA$

$= (\gamma \sin\theta) I_{xy}$ (Equation 9.12)

Then $x_P P = (\gamma \sin\theta) I_{xy}$

or $x_P = \dfrac{(\gamma \sin\theta) I_{xy}}{\gamma \sin\theta (\bar{y} A)}$

or $x_P = \dfrac{I_{xy}}{A\bar{y}}$ ◀

Now $I_x = \bar{I}_{x'} + A\bar{y}^2$

From above $I_x = (A\bar{y}) y_P$

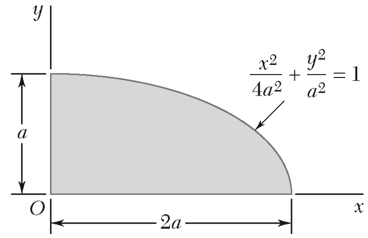
By definition $\bar{I}_{x'} = \bar{k}_{x'}^2 A$

Substituting $(A\bar{y}) y_P = \bar{k}_{x'}^2 A + A\bar{y}^2$

Rearranging yields $y_P - \bar{y} = \dfrac{\bar{k}_{x'}^2}{\bar{y}}$ ◀

Although $\bar{k}_{x'}$ is not a function of the depth of the area (it depends only on the shape of A), $\bar{y}$ is dependent on the depth.

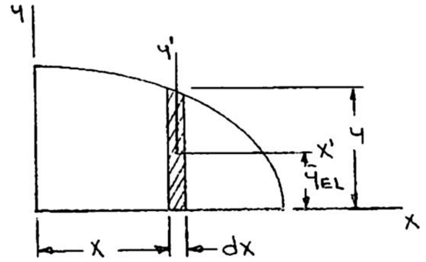
$$(y_P - \bar{y}) = f(\text{depth})$$



PROBLEM 9.67

Determine by direct integration the product of inertia of the given area with respect to the *x* and *y* axes.

SOLUTION



First note $$y = a\sqrt{1-\frac{x^2}{4a^2}}$$

$$= \frac{1}{2}\sqrt{4a^2 - x^2}$$

We have $$dI_{xy} = d\overline{I}_{x'y'} + \overline{x}_{EL}\overline{y}_{EL}dA$$

where $$d\overline{I}_{x'y'} = 0 \quad \text{(symmetry)} \quad \overline{x}_{EL} = x$$

$$\overline{y}_{EL} = \frac{1}{2}y = \frac{1}{4}\sqrt{4a^2 - x^2}$$

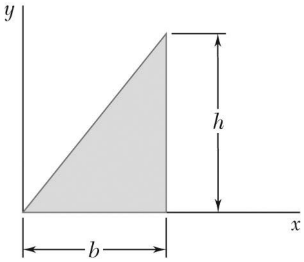
$$dA = ydx = \frac{1}{2}\sqrt{4a^2 - x^2dx}$$

Then $$I_{xy} = \int dI_{xy} = \int_0^{2a} x\left(\frac{1}{4}\sqrt{4a^2 - x^2}\right)\left(\frac{1}{2}\sqrt{4a^2 - x^2}\right)dx$$

$$= \frac{1}{8}\int_0^{2a}(4a^2x - x^3)dx = \frac{1}{8}\left[2a^2x^2 - \frac{1}{4}x^4\right]_0^{2a}$$

$$= \frac{a^4}{8}\left[2(2)^2 - \frac{1}{4}(2)^4\right]$$

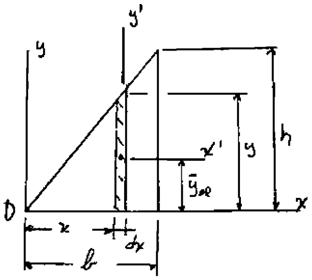
or $$I_{xy} = \frac{1}{2}a^4 \blacktriangleleft$$



PROBLEM 9.68

Determine by direct integration the product of inertia of the given area with respect to the *x* and *y* axes.

SOLUTION



$$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$$

But $$d\bar{I}_{x'y'} = 0 \quad \text{(by symmetry)} \quad \bar{x}_{EL} = x \quad \bar{y}_{EL} = \frac{1}{2}y$$

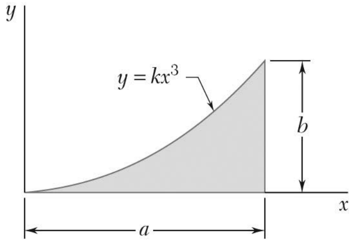
$$dA = yd_y$$

$$\frac{y}{x} = \frac{h}{b} \quad y = \frac{h}{b}x$$

$$I_{xy} = \int dI_{xy} = \int_0^b x\left(\frac{1}{2}y\right)ydx = \frac{1}{2}\int_0^b x\left(\frac{h}{b}x\right)^2 dx$$

$$= \frac{1}{2}\frac{h^2}{b^2}\int_0^b x^3dx = \frac{1}{2}\frac{h^2}{b^2}\frac{b^4}{4}$$

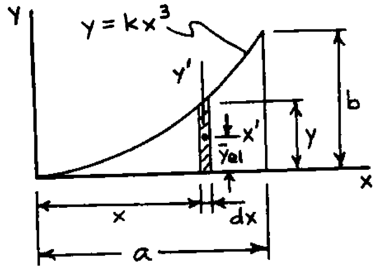
$$I_{xy} = \frac{1}{8}b^2h^2 \blacktriangleleft$$



PROBLEM 9.69

Determine by direct integration the product of inertia of the given area with respect to the x and y axes.

SOLUTION



For $\quad x = a, \quad b = ka^3 \quad \text{or} \quad k = \dfrac{b}{a^3}$

Thus, $\quad y = \dfrac{b}{a^3}x^3$

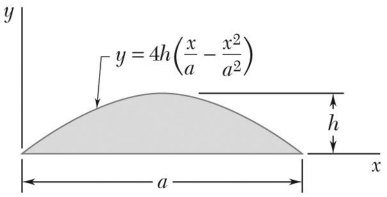
$$d\bar{I}_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA, \quad \text{But} \quad dI_{x'y'} = 0 \quad \text{(by symmetry)}$$

with $\quad \bar{x}_{EL} = x \quad \bar{y}_{EL} = \dfrac{y}{2} \quad dA = ydx$

$$I_{xy} = \int dI_{xy} = \int_0^a x\left(\frac{y}{2}\right)ydx = \frac{1}{2}\int_0^a x\left(\frac{b}{a^3}x^3\right)^2 dx$$

$$= \frac{1}{2}\frac{b^2}{a^6}\left[\frac{x^8}{8}\right]_0^a$$

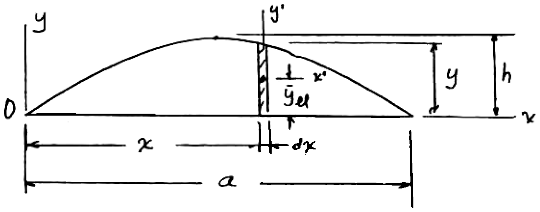
$$I_{xy} = a^2b^2/16 \blacktriangleleft$$



PROBLEM 9.70

Determine by direct integration the product of inertia of the given area with respect to the x and y axes.

SOLUTION



$$y = 4h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)$$

$$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$$

But $$d\bar{I}_{x'y'} = 0 \quad \text{(by symmetry)}$$

$$\bar{x}_{EL} = x \qquad \bar{y}_{EL} = \frac{1}{2}y \qquad dA = ydx$$

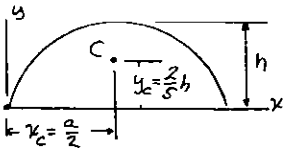
$$I_{xy} = \int dI_{xy} = \int_0^a x\left(\frac{1}{2}y\right)ydx = \frac{1}{2}\int_0^a 16h^2 x\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx$$

$$= 8h^2\int_0^a \left[\frac{x^3}{a^2} - 2\frac{x^4}{a^3} + \frac{x^5}{a^4}\right]dx = 8h^2\left|\frac{x^4}{4a^2} - \frac{2x^5}{5a^3} + \frac{x^6}{6a^4}\right|_0^a$$

$$= 8h^2a^2\left(\frac{1}{4} - \frac{2}{5} + \frac{1}{6}\right) = 8h^2a^2\left(\frac{15-24+10}{60}\right)$$

$$I_{xy} = \frac{2}{15}a^2h^2 \blacktriangleleft$$

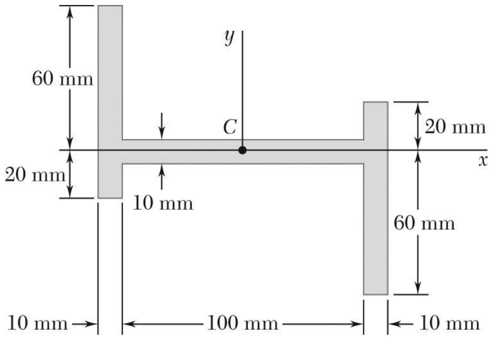
Check: Apply parallel axis theorem to area see Figure 5.8:



$$\text{Area} = \frac{2}{3}a\,h$$

$$\text{by symmetry } I_{x'y'} = 0$$

$$I_{xy} = I_{x'y'} + x_c y_c A = 0 + \left(\frac{a}{2}\right)\left(\frac{2}{5}h\right)\left(\frac{2}{3}ah\right) = \frac{2}{15}a^2h^2$$

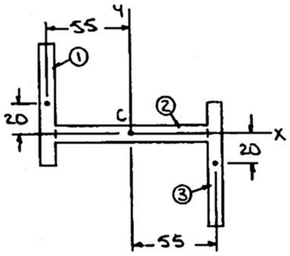


PROBLEM 9.71

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION

Dimensions in mm



We have $\bar{I}_{xy} = (I_{xy})_1 + (\bar{I}_{xy})_2 + (I_{xy})_3$

Now symmetry implies

$$(\bar{I}_{xy})_2 = 0$$

and for the other rectangles

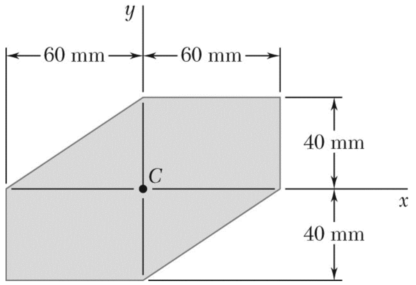
$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$$

where $I_{x'y'} = 0$ (symmetry)

Thus $\bar{I}_{xy} = (\bar{x}\,\bar{y}A)_1 + (\bar{x}\,\bar{y}A)_3$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}\,\bar{y}A$, mm^4
1	$10\times 80 = 800$	-55	20	$-880{,}000$
3	$10\times 80 = 800$	55	-20	$-880{,}000$
Σ				$-1{,}760{,}000$

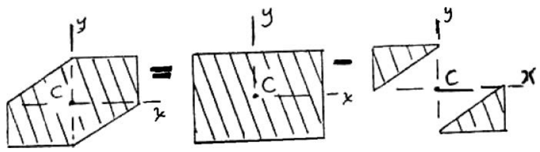
$\bar{I}_{xy} = -1.760\times 10^6\,\text{mm}^4$ ◀



PROBLEM 9.72

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal *x* and *y* axes.

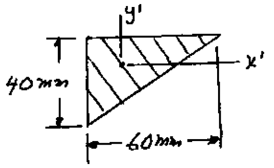
SOLUTION

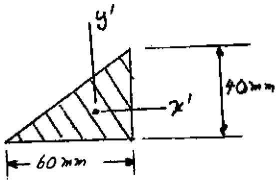


Given area = Rectangle − (Two triangles)

For rectangle $I_{xy} = 0,$ by symmetry

For each triangle:

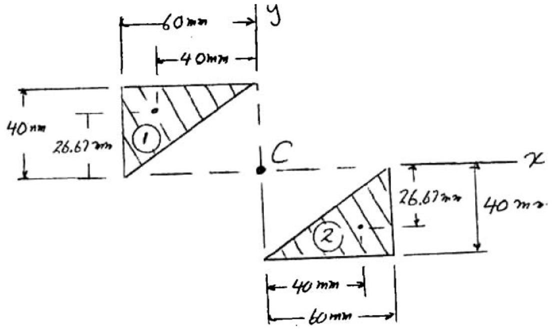




Compare these triangles with the triangle of sample Problem 9.6, where $\bar{I}_{x'y'} = -\frac{1}{72}b^2h^2.$

For orientation of axes of this problem,

$$\bar{I}_{x'y'} = +\frac{1}{72}b^2h^2 = \frac{1}{72}(60\text{ mm})^2(40\text{ mm})^2 = +80\times10^3\,\text{mm}^4$$



PROBLEM 9.72 (Continued)

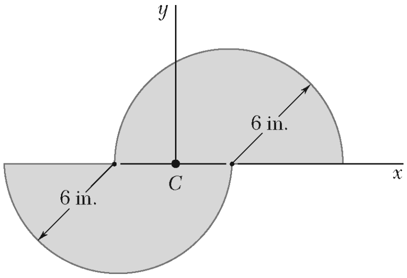
	Area, mm^2	$\overline{x}$, mm	$\overline{y}$, mm	$\overline{x}\,\overline{y}A$, mm^4
1	$\frac{1}{2}(60)(40) = 1200$	-40	$+26.67$	-1.280×10^6
2	$\frac{1}{2}(60)(40) = 1200$	$+40$	-26.67	-1.280×10^6

$$\Sigma\,\overline{x}\,\overline{y}\,A = -2.56 \times 10^6\,\text{mm}^4$$

For two triangles:

$$I_{xy} = \Sigma(\overline{I}_{x'y'} + \overline{x}\,\overline{y}\,A) = +2(80 \times 10^3) - 2.56 \times 10^6$$
$$= -2.40 \times 10^6\,\text{mm}^4$$

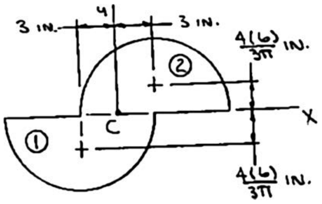
Since we must subtract triangles, $I_{xy} = +2.40 \times 10^6\,\text{mm}^4$ ◀



PROBLEM 9.73

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION



We have $\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$

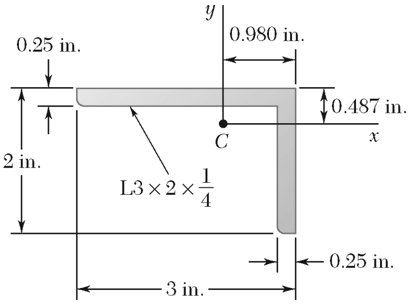
For each semicircle $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$

and $I_{x'y'} = 0$ (symmetry)

Thus $\bar{I}_{xy} = \Sigma\bar{x}\,\bar{y}A$

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\overline{xy}A$
1	$\frac{\pi}{2}(6)^2 = 18\pi$	-3	$-\frac{8}{\pi}$	432
2	$\frac{\pi}{2}(6)^2 = 18\pi$	3	$\frac{8}{\pi}$	432
Σ				864

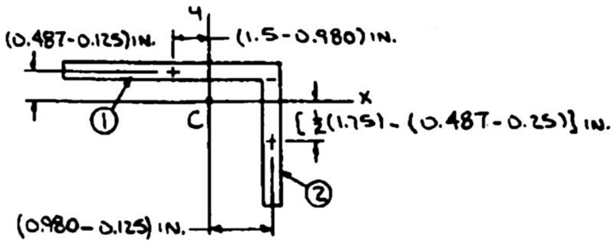
$\bar{I}_{xy} = 864\text{ in}^4$ ◀



PROBLEM 9.74

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION



We have $\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$

For each rectangle

$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$$

and $\bar{I}_{x'y'} = 0 \quad \text{(symmetry)}$

Thus $\bar{I}_{xy} = \Sigma \bar{x}\,\bar{y}A$

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}\,\bar{y}A$, in^4
1	$3\times0.25 = 0.75$	–0.520	0.362	–0.141180
2	$0.25\times1.75 = 0.4375$	0.855	–0.638	–0.238652
Σ				–0.379832

$\bar{I}_{xy} = -0.380 \text{ in}^4$ ◀

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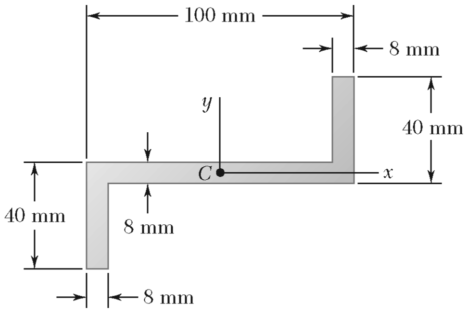
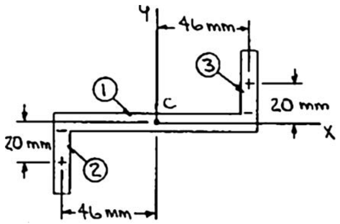


PROBLEM 9.75

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION



We have $\quad I_{xy} = (\bar{I}_{xy})_1 + (I_{xy})_2 + (I_{xy})_3$

Now symmetry implies $\quad (\bar{I}_{xy})_1 = 0$

and for the other rectangles

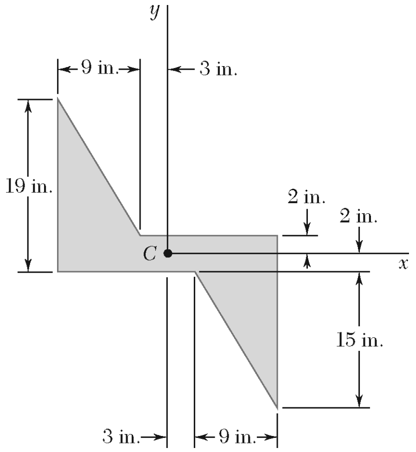
$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$$

where $\quad \bar{I}_{x'y'} = 0$ (symmetry)

Thus $\quad \bar{I}_{xy} = (\bar{x}\,\bar{y}A)_2 + (\bar{x}\,\bar{y}A)_3$

	A, mm^2	$\bar{X}$, mm	$\bar{y}$, mm	$\bar{x}\,\bar{y}A$, mm^4
2	$8\times 32 = 256$	–46	–20	235,520
3	$8\times 32 = 256$	46	20	235,520
Σ				471,040

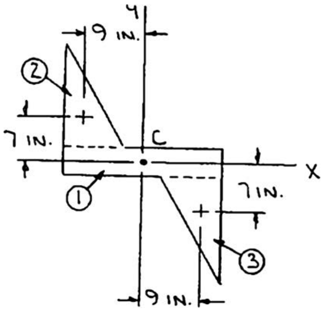
$\bar{I}_{xy} = 471\times 10^3$ mm^4 ◀



PROBLEM 9.76

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION



We have $\bar{I}_{xy} = (\bar{I}_{xy})_1 + (I_{xy})_2 + (I_{xy})_3$

Now, symmetry implies

$$(\bar{I}_{xy})_1 = 0$$

and for each triangle $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$

where, using the results of Sample Problem 9.6, $\bar{I}_{x'y'} = -\frac{1}{72}b^2h^2$ for both triangles. Note that the sign of $\bar{I}_{x'y'}$ is unchanged because the angles of rotation are 0° and 180° for triangles 2 and 3, respectively.

Now

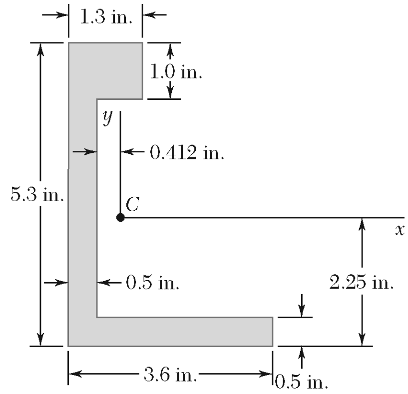
	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}\,\bar{y}A$, in^4
2	$\frac{1}{2}(9)(15) = 67.5$	–9	7	–4252.5
3	$\frac{1}{2}(9)(15) = 67.5$	9	–7	–4252.5
Σ				–8505

Then

$$\bar{I}_{xy} = 2\left[-\frac{1}{72}(9\text{ in.})^2\,(15\text{ in.})^2\right] - 8505\text{ in}^4$$

$$= -9011.25\text{ in}^4$$

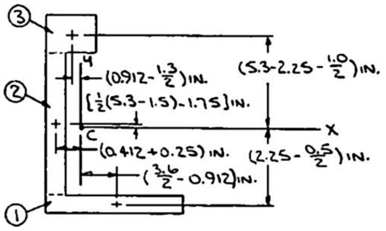
or $\bar{I}_{xy} = -9010\text{ in}^4$ ◀



PROBLEM 9.77

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal *x* and *y* axes.

SOLUTION



We have $\bar{I}_{xy} = (\bar{I}_{xy})_1 + (I_{xy})_2 + (I_{xy})_3$

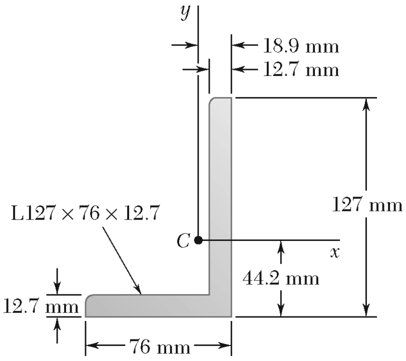
For each rectangle $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$

and $\bar{I}_{x'y'} = 0$ (symmetry)

Thus $I_{xy} = \Sigma\bar{x}\,\bar{y}A$

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}\,\bar{y}A$, in^4
1	$3.6\times0.5=1.8$	0.888	–2.00	–3.196
2	$0.5\times3.8=1.9$	–0.662	0.15	–0.18867
3	$1.3\times1.0=1.3$	–0.262	2.55	–0.86853
Σ				–4.25320

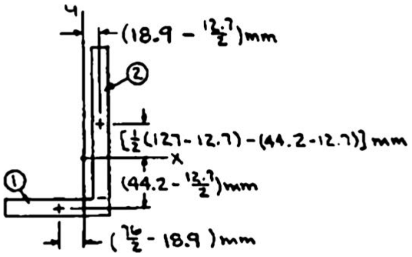
$\bar{I}_{xy} = -4.25\text{ in}^4$ ◀



PROBLEM 9.78

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal *x* and *y* axes.

SOLUTION



We have $\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$

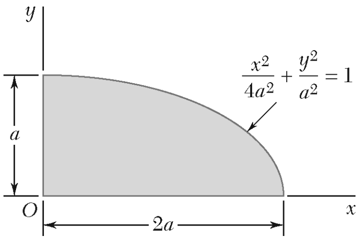
For each rectangle $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$

and $\bar{I}_{x'y'} = 0$ (symmetry)

Thus $I_{xy} = \Sigma\bar{x}\,\bar{y}A$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}\,\bar{y}A$, mm^4
1	$76\times 12.7 = 965.2$	–19.1	–37.85	697,777
2	$12.7\times(127-12.7) = 1451.61$	12.55	25.65	467,284
Σ				1,165,061

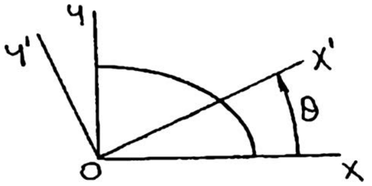
$$\bar{I}_{xy} = 1.165\times 10^6\,\text{mm}^4 \blacktriangleleft$$



PROBLEM 9.79

Determine for the quarter ellipse of Problem 9.67 the moments of inertia and the product of inertia with respect to new axes obtained by rotating the *x* and *y* axes about *O* (*a*) through 45° counterclockwise, (*b*) through 30° clockwise.

SOLUTION



From Figure 9.12:

$$I_x = \frac{\pi}{16}(2a)(a)^3 = \frac{\pi}{8}a^4$$

$$I_y = \frac{\pi}{16}(2a)^3(a) = \frac{\pi}{2}a^4$$

From Problem 9.67:

$$I_{xy} = \frac{1}{2}a^4$$

First note

$$\frac{1}{2}(I_x + I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = \frac{5}{16}\pi a^4$$

$$\frac{1}{2}(I_x - I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right) = -\frac{3}{16}\pi a^4$$

Now use Equations (9.18), (9.19), and (9.20).

Equation (9.18):

$$I_{x'} = \frac{1}{2}(I_x + I_y) + \frac{1}{2}(I_x - I_y)\cos 2\theta - I_{xy}\sin 2\theta$$

$$= \frac{5}{16}\pi a^4 - \frac{3}{16}\pi a^4 \cos 2\theta - \frac{1}{2}a^4 \sin 2\theta$$

Equation (9.19):

$$I_{y'} = \frac{1}{2}(I_x + I_y) - \frac{1}{2}(I_x - I_y)\cos 2\theta + I_{xy}\sin 2\theta$$

$$= \frac{5}{16}\pi a^4 + \frac{3}{16}\pi a^4 \cos 2\theta + \frac{1}{2}a^4 \sin 2\theta$$

Equation (9.20):

$$I_{x'y'} = \frac{1}{2}(I_x - I_y)\sin 2\theta + I_{xy}\cos 2\theta$$

$$= -\frac{3}{16}\pi a^4 \sin 2\theta + \frac{1}{2}a^4 \cos 2\theta$$

PROBLEM 9.79 (Continued)

(*a*) $\theta = +45°$:

$$I_{x'} = \frac{5}{16}\pi a^4 - \frac{3}{16}\pi a^4 \cos 90° - \frac{1}{2}a^4 \sin 90°$$

or $I_{x'} = 0.482a^4$ ◀

$$I_{y'} = \frac{5}{16}\pi + \frac{3}{16}\pi a^4 \cos 90° + \frac{1}{2}a^4$$

or $I_{y'} = 1.482a^4$ ◀

$$I_{x'y'} = -\frac{3}{16}\pi a^4 \sin 90° + \frac{1}{2}a^4 \cos 90°$$

or $I_{x'y'} = -0.589a^4$ ◀

(*b*) $\theta = -30°$:

$$I_{x'} = \frac{5}{16}\pi a^4 - \frac{3}{16}\pi a^4 \cos(-60°) - \frac{1}{2}a^4 \sin(-60°)$$

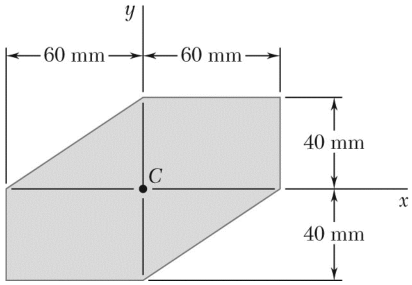
or $I_{x'} = 1.120a^4$ ◀

$$I_{y'} = \frac{5}{16}\pi a^4 + \frac{3}{16}\pi a^4 \cos(-60°) + \frac{1}{2}a^4 \sin(-60°)$$

or $I_{y'} = 0.843a^4$ ◀

$$I_{x'y'} = -\frac{3}{16}\pi a^4 \sin(-60°) + \frac{1}{2}a^4 \cos(-60°)$$

or $I_{x'y'} = 0.760a^4$ ◀

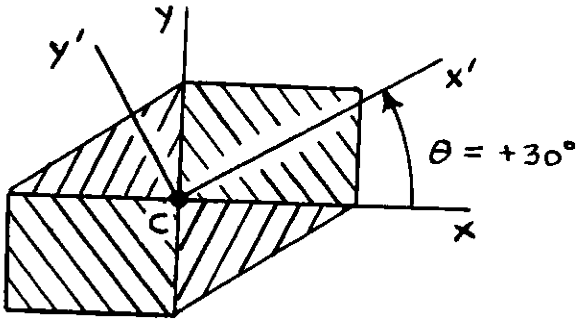


PROBLEM 9.80

Determine the moments of inertia and the product of inertia of the area of Problem 9.72 with respect to new centroidal axes obtained by rotating the *x* and *y* axes 30° counterclockwise.

SOLUTION

From Problem 9.72: $\bar{I}_{xy} = 2.40 \times 10^6 \text{ mm}^4$



Now, with two rectangles and two triangles:

$$\bar{I}_x = 2\left[\frac{1}{3}(60 \text{ mm})(40 \text{ mm})^3\right] + 2\left[\frac{1}{12}(60 \text{ mm})(40 \text{ mm})^3\right]$$

$$\bar{I}_x = 3.20 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 2\left[\frac{1}{3}(40 \text{ mm})(60 \text{ mm})^3\right] + 2\left[\frac{1}{12}(40 \text{ mm})(60 \text{ mm})^3\right] = 7.20 \times 10^6 \text{ mm}^4$$

Now $\frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(3.20 + 7.20) \times 10^6 = 5.20 \times 10^6 \text{ mm}^4$

$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(3.20 - 7.20) \times 10^6 = -2.00 \times 10^6 \text{ mm}^4$$

Using Eqs. (9.18), (9.19), (9.20):

Eq. (9.18): $\bar{I}_{x'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) + \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta - \bar{I}_{xy}\sin 2\theta$

$$= [5.20 + (-2.00)\cos 60° - (2.40)\sin 60°] \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 2.12 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.80 (Continued)

Eq. (9.19):

$$\overline{I}_{y'} = \frac{1}{2}(\overline{I}_x + \overline{I}_y) - \frac{1}{2}(\overline{I}_x - \overline{I}_y)\cos 2\theta + \overline{I}_{xy}\sin 2\theta$$

$$= [5.20 - (-2.00)\cos 60° + (2.40)\sin 60°] \times 10^6 \text{ mm}^4$$

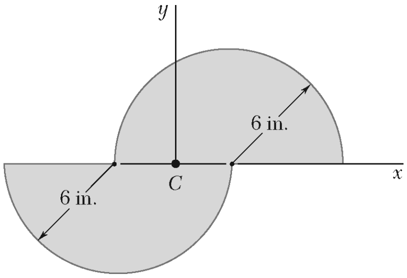
or $\overline{I}_{y'} = 8.28 \times 10^6 \text{ mm}^4$ ◀

Eq. (9.20):

$$\overline{I}_{x'y'} = \frac{1}{2}(\overline{I}_x - \overline{I}_y)\sin 2\theta + \overline{I}_{xy}\sin 2\theta$$

$$= [(-2.00)\sin 60° + (2.40)\cos 60°] \times 10^6 \text{ mm}^4$$

or $\overline{I}_{x'y'} = -0.532 \times 10^6 \text{ mm}^4$ ◀



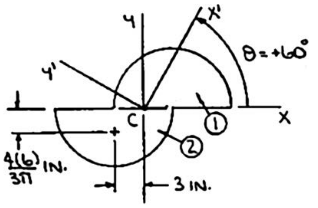
PROBLEM 9.81

Determine the moments of inertia and the product of inertia of the area of Problem 9.73 with respect to new centroidal axes obtained by rotating the x and y axes 60° counterclockwise.

SOLUTION

From Problem 9.73:

$$\bar{I}_{xy} = 864 \text{ in}^4$$



Now $\bar{I}_x = (I_x)_1 + (I_x)_2$

where
$$(I_x)_1 = (I_x)_2 = \frac{\pi}{8}(6 \text{ in.})^4$$
$$= 162\pi \text{ in}^4$$

Then $\bar{I}_x = 2(162\pi \text{ in}^4) = 324\pi \text{ in}^4$

Also $\bar{I}_y = (I_y)_1 + (I_y)_2$

where
$$(I_y)_1 = (I_y)_2 = \frac{\pi}{8}(6 \text{ in.})^4 + \left[\frac{\pi}{2}(6 \text{ in.})^2\right](3 \text{ in.})^2 = 324\pi \text{ in}^4$$

Then $\bar{I}_y = 2(324\pi \text{ in}^4) = 648\pi \text{ in}^4$

Now
$$\frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(324\pi + 648\pi) = 486\pi \text{ in}^4$$
$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(324\pi - 648\pi) = -162\pi \text{ in}^4$$

Using Eqs. (9.18), (9.19), and (9.20):

Eq. (9.18):
$$\bar{I}_{x'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) + \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$
$$= [486\pi + (-162\pi)\cos 120° - 864\sin 120°] \text{ in}^4$$

or $\bar{I}_{x'} = 1033 \text{ in}^4$ ◀

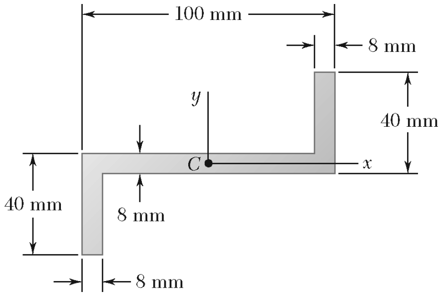
PROBLEM 9.81 (Continued)

Eq. (9.19):
$$\bar{I}_{y'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) - \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$
$$= [486\pi - (-162\pi)\cos 120° + 864\sin 120°]\ \text{in}^4$$

or $\bar{I}_{y'} = 2020\ \text{in}^4$ ◀

Eq. (9.20):
$$\bar{I}_{x'y'} = \frac{1}{2}(\bar{I}_x - \bar{I}_y)\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$
$$= [(-162\pi)\sin 120° + 864\cos 120°]\ \text{in}^4$$

or $\bar{I}_{x'y'} = -873\ \text{in}^4$ ◀

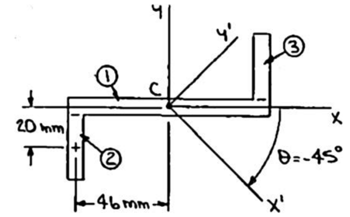


PROBLEM 9.82

Determine the moments of inertia and the product of inertia of the area of Problem 9.75 with respect to new centroidal axes obtained by rotating the *x* and *y* axes 45° clockwise.

SOLUTION

From Problem 9.75:



$$\bar{I}_{xy} = 471{,}040 \text{ mm}^4$$

Now $\bar{I}_x = (\bar{I}_x)_1 + (I_x)_2 + (I_x)_3$

where $(\bar{I}_x)_1 = \dfrac{1}{12}(100 \text{ mm})(8 \text{ mm})^3$

$$= 4266.67 \text{ mm}^4$$

$$(I_x)_2 = (I_x)_3 = \frac{1}{12}(8 \text{ mm})(32 \text{ mm})^3 + [(8 \text{ mm})(32 \text{ mm})](20 \text{ mm})^2$$

$$= 124{,}245.33$$

Then $\bar{I}_x = [4266.67 + 2(124{,}245.33)] \text{ mm}^4 = 252{,}757 \text{ mm}^4$

Also $\bar{I}_y = (\bar{I}_y)_1 + (I_y)_2 + (I_y)_3$

where $(\bar{I}_y)_1 = \dfrac{1}{12}(8 \text{ mm})(100 \text{ mm})^3 = 666{,}666.7 \text{ mm}^4$

$$(\bar{I}_y)_2 = (I_y)_3 = \frac{1}{12}(32 \text{ mm})(8 \text{ mm})^3 + [(8 \text{ mm})(32 \text{ mm})](46 \text{ mm})^2$$

$$= 543{,}061.3 \text{ mm}^4$$

Then $\bar{I}_y = [666{,}666.7 + 2(543{,}061.3)] \text{ mm}^4 = 1{,}752{,}789 \text{ mm}^4$

Now $\dfrac{1}{2}(\bar{I}_x + \bar{I}_y) = \dfrac{1}{2}(252{,}757 + 1{,}752{,}789) \text{ mm}^4 = 1{,}002{,}773 \text{ mm}^4$

$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(252{,}757 - 1{,}752{,}789) \text{ mm}^4 = -750{,}016 \text{ mm}^4$$

Using Eqs. (9.18), (9.19), and (9.20):

Eq. (9.18): $\bar{I}_{x'} = \dfrac{1}{2}(\bar{I}_x + \bar{I}_y) + \dfrac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta - \bar{I}_{xy}\sin 2\theta$

$$= [1{,}002{,}773 + (-750{,}016)\cos(-90°) - 471{,}040\sin(-90°)]$$

or $\bar{I}_{x'} = 1.474 \times 10^6 \text{ mm}^4$ ◀

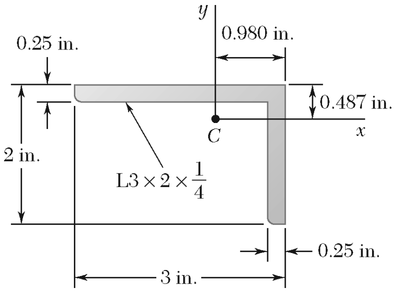
PROBLEM 9.82 (Continued)

Eq. (9.19):
$$\overline{I}_{y'} = \frac{1}{2}(\overline{I}_x + \overline{I}_y) - \frac{1}{2}(\overline{I}_x - \overline{I}_y)\cos 2\theta + \overline{I}_{xy}\sin 2\theta$$
$$= [1{,}002{,}773 - (-750{,}016)\cos(-90°) + 471{,}040\sin(-90°)]$$

or $\overline{I}_{y'} = 0.532 \times 10^6\ \text{mm}^4$ ◀

Eq. (9.20):
$$\overline{I}_{x'y'} = \frac{1}{2}(\overline{I}_x - \overline{I}_y)\sin 2\theta + \overline{I}_{xy}\cos 2\theta$$
$$= [(-750{,}016)\sin(-90°) + 471{,}040\cos(-90°)]$$

or $\overline{I}_{x'y'} = 0.750 \times 10^6\ \text{mm}^4$ ◀

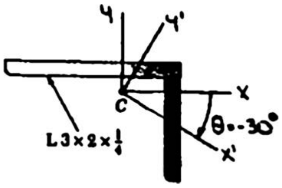


PROBLEM 9.83

Determine the moments of inertia and the product of inertia of the $L3\times 2\times \frac{1}{4}$-in. angle cross section of Problem 9.74 with respect to new centroidal axes obtained by rotating the x and y axes 30° clockwise.

SOLUTION

From Figure 9.13:



$$\bar{I}_x = 0.390\text{ in}^4$$
$$\bar{I}_y = 1.09\text{ in}^4$$

From Problem 9.74:

$$\bar{I}_{xy} = -0.37983\text{ in}^4$$

Now
$$\frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(0.390 + 1.09)\text{ in}^4 = 0.740\text{ in}^4$$
$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(0.390 - 1.09)\text{ in}^4 = -0.350\text{ in}^4$$

Using Eqs. (9.18), (9.19), and (9.20):

Eq. (9.18):
$$\bar{I}_{x'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) + \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$
$$= [0.740 + (-0.350)\cos(-60°) - (-0.37983)\sin(-60°)]$$

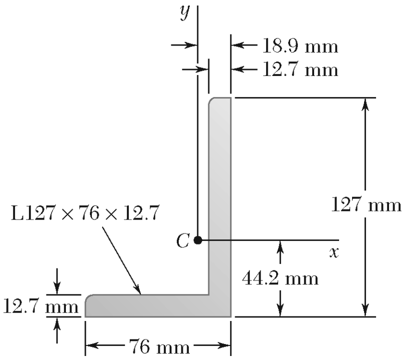
or $\bar{I}_{x'} = 0.236\text{ in}^4$ ◀

Eq. (9.19):
$$\bar{I}_{y'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) - \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$
$$= [0.740 - (-0.350)\cos(-60°) + (-0.37983)\sin(-60°)]$$

or $\bar{I}_{y'} = 1.244\text{ in}^4$ ◀

Eq. (9.20):
$$\bar{I}_{x'y'} = \frac{1}{2}(\bar{I}_x - \bar{I}_y)\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$
$$= [(-0.350)\sin(-60°) + (-0.37983)\cos(-60°)]$$

or $I_{x'y'} = 0.1132\text{ in}^4$ ◀



PROBLEM 9.84

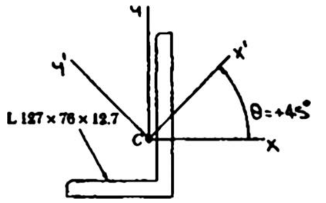
Determine the moments of inertia and the product of inertia of the L127×76×12.7-mm angle cross section of Problem 9.78 with respect to new centroidal axes obtained by rotating the x and y axes 45° counterclockwise.

SOLUTION

From Figure 9.13:

$$\bar{I}_x = 3.93\times 10^6\,\text{mm}^4$$

$$\bar{I}_y = 1.06\times 10^6\,\text{mm}^4$$



From Problem 9.78:

$$\bar{I}_{xy} = 1.165061\times 10^6\,\text{mm}^4$$

Now

$$\frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(3.93+1.06)\times 10^6\,\text{mm}^4$$

$$= 2.495\times 10^6\,\text{mm}^4$$

$$\frac{1}{2}(\bar{I}_x - \bar{I}_y) = \frac{1}{2}(3.93-1.06)\times 10^6\,\text{mm}^4 = 1.435\times 10^6\,\text{mm}^4$$

Using Eqs. (9.18), (9.19), and (9.20):

Eq. (9.18):

$$\bar{I}_{x'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) + \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$

$$= [2.495 + 1.435\cos 90° - 1.165061\sin 90°]\times 10^6\,\text{mm}^4$$

or $\bar{I}_{x'} = 1.330\times 10^6\,\text{mm}^4$ ◀

Eq. (9.19):

$$\bar{I}_{y'} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) - \frac{1}{2}(\bar{I}_x - \bar{I}_y)\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= [2.495 - 1.435\cos 90° + 1.165061\sin 90°]\times 10^6\,\text{mm}^4$$

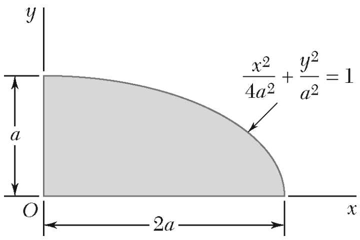
or $\bar{I}_{y'} = 3.66\times 10^6\,\text{mm}^4$ ◀

Eq. (9.20):

$$\bar{I}_{x'y'} = \frac{1}{2}(\bar{I}_x - \bar{I}_y)\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$

$$= [(1.435\sin 90° + 1.165061\cos 90°]\times 10^6\,\text{mm}^4$$

or $\bar{I}_{x'y'} = 1.435\times 10^6\,\text{mm}^4$ ◀



PROBLEM 9.85

For the quarter ellipse of Problem 9.67, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

SOLUTION

From Problem 9.79: $$I_x=\frac{\pi}{8}a^4 \qquad I_y=\frac{\pi}{2}a^4$$

Problem 9.67: $$I_{xy}=\frac{1}{2}a^4$$

Now Eq. (9.25): $$\tan 2\theta_m=-\frac{2I_{xy}}{I_x-I_y}=-\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4-\frac{\pi}{2}a^4}$$

$$=\frac{8}{3\pi}=0.84883$$

Then $$2\theta_m=40.326° \quad \text{and} \quad 220.326°$$

or $\theta_m=20.2°$ and $110.2°$ ◀

Also Eq. (9.27): $$I_{\max,\min}=\frac{I_x+I_y}{2}\pm\sqrt{\left(\frac{I_x-I_y}{2}\right)^2+I_{xy}^2}$$

$$=\frac{1}{2}\left(\frac{\pi}{8}a^4+\frac{\pi}{2}a^4\right)$$

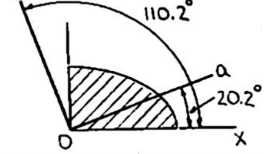
$$\pm\sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4-\frac{\pi}{2}a^4\right)\right]^2+\left(\frac{1}{2}a^4\right)^2}$$

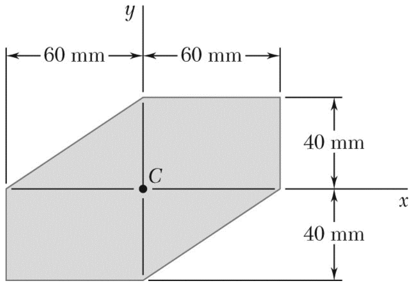
$$=(0.981{,}748\pm 0.772{,}644)a^4$$

or $I_{\max}=1.754a^4$ ◀

and $I_{\min}=0.209a^4$ ◀

By inspection, the a axis corresponds to $I_{\min}$ and the b axis corresponds to $I_{\max}$.





PROBLEM 9.86

For the area indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

Area of Problem 9.72.

SOLUTION

From Problem 9.80:

$$\bar{I}_x = 3.20 \times 10^6 \text{ mm}^4$$
$$\bar{I}_y = 7.20 \times 10^6 \text{ mm}^4$$

From Problem 9.72:

$$\bar{I}_{xy} = 2.40 \times 10^6 \text{ mm}^4$$

Now Eq. (9.25):

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(2.40 \times 10^6)}{(3.20 - 7.20) \times 10^6} = 1.200$$

Then

$$2\theta_m = 50.194° \quad \text{and} \quad 230.194°$$

or $\theta_m = 25.1°$ and $115.1°$ ◀

Also Eq. (9.27):

$$\bar{I}_{\max,\min} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + I_{xy}^2}$$

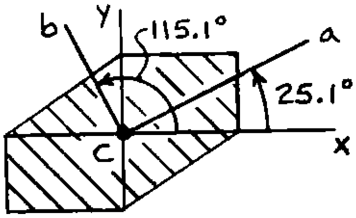
Then

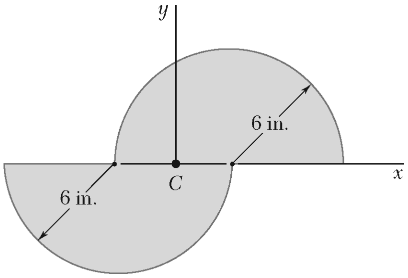
$$\bar{I}_{\max,\min} = \left[\frac{3.20 + 7.20}{2} \pm \sqrt{\left(\frac{3.20 - 7.20}{2}\right)^2 + (2.40)^2}\right] \times 10^6 \text{ mm}^4$$
$$= (5.20 \pm 3.1241) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\max} = 8.32 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\min} = 2.08 \times 10^6 \text{ mm}^4$ ◀

By inspection, the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.





PROBLEM 9.87

For the area indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

Area of Problem 9.73.

SOLUTION

From Problem 9.81: $\bar{I}_x = 324\pi \text{ in}^4 \qquad \bar{I}_y = 648\pi \text{ in}^4$

Problem 9.73: $\bar{I}_{xy} = 864 \text{ in}^4$

Now Eq. (9.25):

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(864)}{324\pi - 648\pi}$$
$$= 1.69765$$

Then $2\theta_m = 59.500° \quad \text{and} \quad 239.500°$

or $\theta_m = 29.7° \quad \text{and} \quad 119.7°$ ◀

Also Eq. (9.27):

$$\bar{I}_{\text{max, min}} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

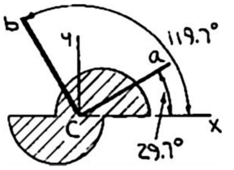
Then

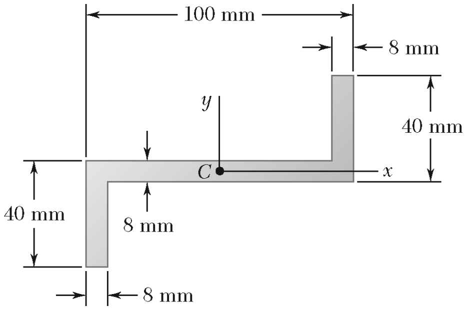
$$\bar{I}_{\text{max,min}} = \frac{324\pi + 648\pi}{2} \pm \sqrt{\left(\frac{324\pi - 648\pi}{2}\right)^2 + 864^2}$$
$$= (1526.81 \pm 1002.75) \text{ in}^4$$

or $\bar{I}_{\text{max}} = 2530 \text{ in}^4$ ◀

and $\bar{I}_{\text{min}} = 524 \text{ in}^4$ ◀

By inspection, the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.





PROBLEM 9.88

For the area indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

Area of Problem 9.75.

SOLUTION

From Problem 9.82: $\bar{I}_x = 252{,}757 \text{ mm}^4$

$\bar{I}_y = 1{,}752{,}789 \text{ mm}^4$

Problem 9.75: $\bar{I}_{xy} = 471{,}040 \text{ mm}^4$

Now Eq. (9.25):

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(471{,}040)}{252{,}757 - 1{,}752{,}789}$$

$$= 0.62804$$

Then $2\theta_m = 32.130°$ and $212.130°$

or $\theta_m = 16.07°$ and $106.1°$ ◀

Also Eq. (9.27):

$$\bar{I}_{\text{max, min}} = \frac{\bar{I}_x - \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

Then

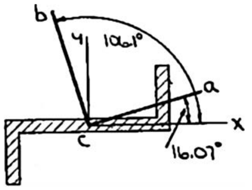
$$\bar{I}_{\text{max, min}} = \frac{252{,}757 + 1{,}752{,}789}{2} \pm \sqrt{\left(\frac{252{,}757 - 1{,}752{,}789}{2}\right)^2 + 471040^2}$$

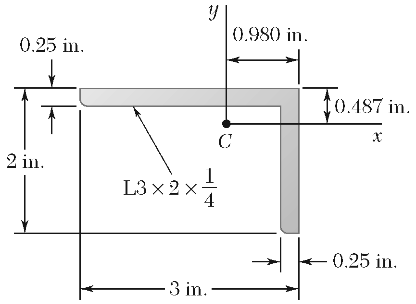
$$= (1{,}002{,}773 \pm 885{,}665) \text{ mm}^4$$

or $\bar{I}_{\text{max}} = 1.888 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\text{min}} = 0.1171 \times 10^6 \text{ mm}^4$ ◀

By inspection, the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.





PROBLEM 9.89

For the angle cross section indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

The L3×2×$\frac{1}{4}$-in. angle cross section of Problem 9.74.

SOLUTION

From Problem 9.83: $\bar{I}_x = 0.390 \text{ in}^4 \qquad \bar{I}_y = 1.09 \text{ in}^4$

Problem 9.74: $\bar{I}_{xy} = -0.37983 \text{ in}^4$

Now Eq. (9.25):

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(-0.37983)}{0.390 - 1.09}$$

$$= -1.08523$$

Then $2\theta_m = -47.341° \text{ and } 132.659°$

or $\theta_m = -23.7° \text{ and } 66.3°$ ◀

Also Eq. (9.27):

$$\bar{I}_{\text{max, min}} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

Then

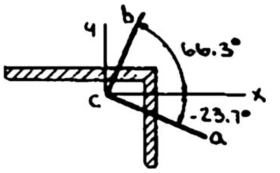
$$\bar{I}_{\text{max, min}} = \frac{0.390 + 1.09}{2} \pm \sqrt{\left(\frac{0.390 - 1.09}{2}\right)^2 + (-0.37983)^2}$$

$$= (0.740 \pm 0.51650)^2 \text{ in}^4$$

or $\bar{I}_{\text{max}} = 1.257 \text{ in}^4$ ◀

and $\bar{I}_{\text{min}} = 0.224 \text{ in}^4$ ◀

By inspection, the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.



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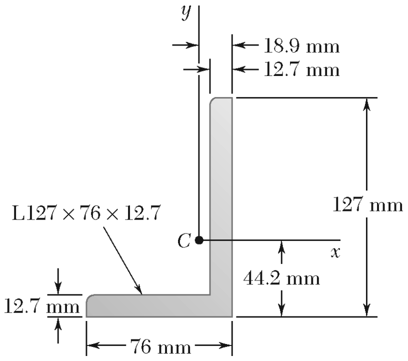


PROBLEM 9.90

For the angle cross section indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

The L127 × 76 × 12.7-mm angle cross section of Problem 9.78.

SOLUTION

From Problem 9.84: $\bar{I}_x = 3.93 \times 10^6 \text{ mm}^4$

$\bar{I}_y = 1.06 \times 10^6 \text{ mm}^4$

Problem 9.78: $\bar{I}_{xy} = 1.165061 \times 10^6 \text{ mm}^4$

Now Eq. (9.25):

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(1.165061 \times 10^6)}{(3.93 - 1.06) \times 10^6}$$

$$= -0.81189$$

Then $2\theta_m = -39.073°$ and $140.927°$

or $\theta_m = -19.54°$ and $70.5°$ ◀

Also Eq. (9.27):

$$\bar{I}_{\text{max, min}} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

Then

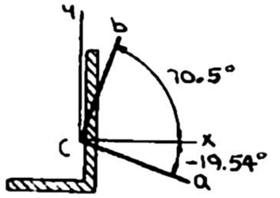
$$\bar{I}_{\text{max, min}} = \left[\frac{3.93 + 1.06}{2} \pm \sqrt{\left(\frac{3.93 - 1.06}{2}\right)^2 + 1.165061^2}\right] \times 10^6 \text{ mm}^4$$

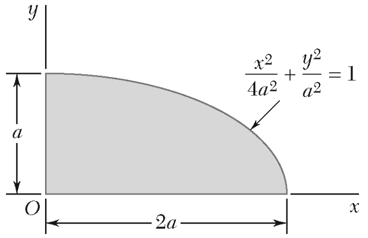
$$= (2.495 \pm 1.84840) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\text{max}} = 4.34 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\text{min}} = 0.647 \times 10^6 \text{ mm}^4$ ◀

By inspection, the *a* axis corresponds to $\bar{I}_{\text{max}}$ and the *b* axis corresponds to $\bar{I}_{\text{min}}$.

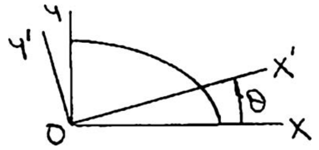




PROBLEM 9.91

Using Mohr's circle, determine for the quarter ellipse of Problem 9.67 the moments of inertia and the product of inertia with respect to new axes obtained by rotating the x and y axes about O (a) through 45° counterclockwise, (b) through 30° clockwise.

SOLUTION



From Problem 9.79: $$I_x = \frac{\pi}{8}a^4$$

$$I_y = \frac{\pi}{2}a^4$$

Problem 9.67: $$I_{xy} = \frac{1}{2}a^4$$

The Mohr's circle is defined by the diameter XY, where

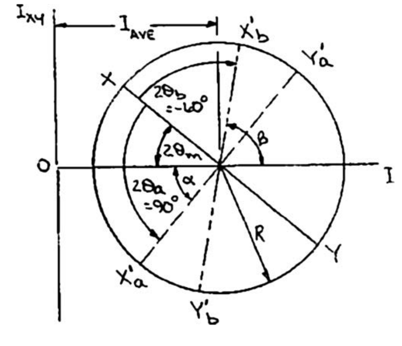
$$X\left(\frac{\pi}{8}a^4, \frac{1}{2}a^4\right) \quad \text{and} \quad Y\left(\frac{\pi}{2}a^4, -\frac{1}{2}a^4\right)$$

Now $$I_{\text{ave}} = \frac{1}{2}(I_x + I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = \frac{5}{16}\pi a^4 = 0.98175a^4$$

and $$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2}$$

$$= 0.77264a^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$

$$= -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4}$$

$$= 0.84883$$

or $$2\theta_m = 40.326°$$

PROBLEM 9.91 (Continued)

Then

$$\alpha = 90^\circ - 40.326^\circ$$
$$= 49.674^\circ$$
$$\beta = 180^\circ - (40.326^\circ + 60^\circ)$$
$$= 79.674^\circ$$

(*a*) $\theta = +45^\circ$:

$$I_{x'} = I_{\text{ave}} - R\cos\alpha = 0.98175a^4 - 0.77264a^4 \cos 49.674^\circ$$

or $I_{x'} = 0.482a^4$ ◀

$$I_{y'} = I_{\text{ave}} + R\cos\alpha = 0.98175a^4 + 0.77264a^4 \cos 49.674^\circ$$

or $I_{y'} = 1.482a^4$ ◀

$$I_{x'y'} = -R\sin\alpha = -0.77264a^4 \sin 49.674^\circ$$

or $I_{x'y'} = -0.589a^4$ ◀

(*b*) $\theta = -30^\circ$:

$$I_{x'} = I_{\text{ave}} + R\cos\beta = 0.98175a^4 + 0.77264a^4 \cos 79.674^\circ$$

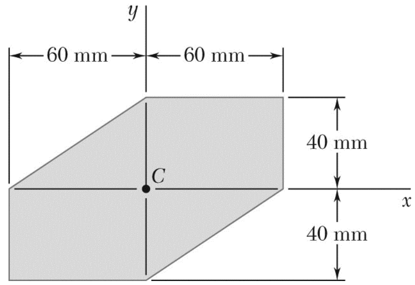
or $I_{x'} = 1.120a^4$ ◀

$$I_{y'} = I_{\text{ave}} - R\cos\beta = 0.98175a^4 - 0.77264a^4 \cos 79.674^\circ$$

or $I_{y'} = 0.843a^4$ ◀

$$I_{x'y'} = R\sin\beta = 0.77264a^4 \sin 79.674^\circ$$

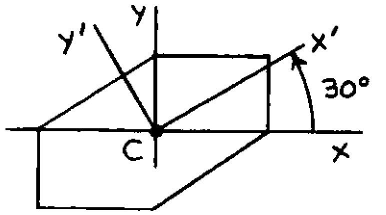
or $I_{x'y'} = 0.760a^4$ ◀



PROBLEM 9.92

Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.72 with respect to new centroidal axes obtained by rotating the *x* and *y* axes 30° counterclockwise.

SOLUTION



From Problem 9.80: $\bar{I}_x = 3.20\times10^6\,\text{mm}^4$

$\bar{I}_y = 7.20\times10^6\,\text{mm}^4$

From Problem 9.72: $\bar{I}_{xy} = 2.40\times10^6\,\text{mm}^4$

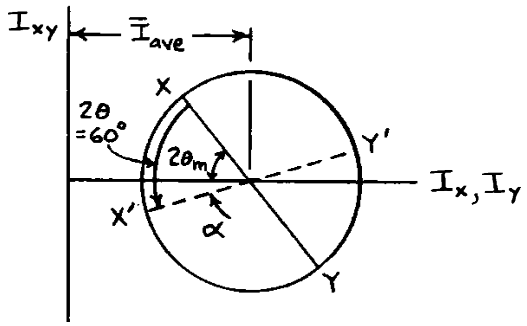
The Mohr's circle is defined by the diameter *XY*, where $X(3.20\times10^6,\ 2.40\times10^6)$ and $Y(7.20\times10^6,\ -2.40\times10^6)$.

Now $$I_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(3.20+7.20)\times10^6 = 5.20\times10^6\,\text{mm}^4$$

and $$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + I_{xy}^2} = \left\{\sqrt{\left[\frac{1}{2}(3.20-7.20)\right]^2 + (2.40)^2}\right\}\times10^6\,\text{mm}^4$$

$$= 3.1241\times10^6\,\text{mm}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(2.40\times10^6)}{(3.20-7.20)\times10^6}$$

$$= 1.200$$

or $$2\theta_m = 50.1944°$$

Then $$\alpha = 60° - 50.1944° = 9.8056°$$

PROBLEM 9.92 (Continued)

Then $\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\cos\alpha = (5.20 - 3.1241\cos 9.8056°)\times 10^6$

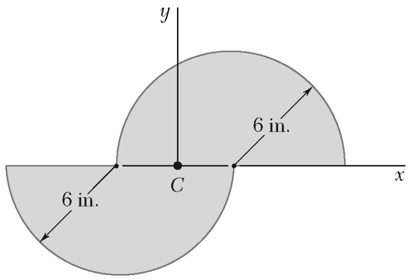
or $\bar{I}_{x'} = 2.12\times 10^6\ \text{mm}^4$ ◀

$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R\cos\alpha = (5.20 + 3.1241\cos 9.8056°)\times 10^6$

or $\bar{I}_{y'} = 8.28\times 10^6\ \text{mm}^4$ ◀

$\bar{I}_{x'y'} = -R\sin\alpha = -(3.1241\times 10^6)\sin 9.8056°$

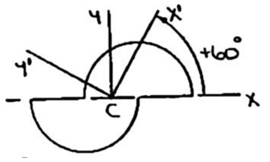
or $\bar{I}_{x'y'} = -0.532\times 10^6\ \text{mm}^4$ ◀



PROBLEM 9.93

Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.73 with respect to new centroidal axes obtained by rotating the x and y axes 60° counterclockwise.

SOLUTION



From Problem 9.81: $\bar{I}_x = 324\pi \text{ in}^4$

$\bar{I}_y = 648\pi \text{ in}^4$

Problem 9.73: $\bar{I}_{xy} = 864 \text{ in}^4$

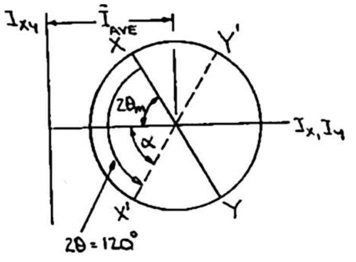
The Mohr's circle is defined by the diameter XY, where $X(324\pi,\ 864)$ and $Y(648\pi,\ -864)$.

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(324\pi + 648\pi) = 1526.81 \text{ in}^4$$

and $$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{1}{2}(324\pi + 648\pi)\right]^2 + 864^2}$$

$$= 1002.75 \text{ in}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(864)}{324\pi - 648\pi}$$

$$= 1.69765$$

or $$2\theta_m = 59.500°$$

Then $$\alpha = 120° - 59.500°$$

$$= 60.500°$$

PROBLEM 9.93 (Continued)

Then $$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\cos\alpha = 1526.81 - 1002.75\cos 60.500°$$

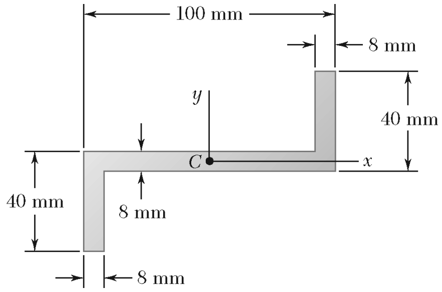
or $\bar{I}_{x'} = 1033 \text{ in}^4$ ◀

$$\bar{I}_{y} = \bar{I}_{\text{ave}} + R\cos\alpha = 1526.81 + 1002.75\cos 60.500°$$

or $\bar{I}_{y'} = 2020 \text{ in}^4$ ◀

$$\bar{I}_{x'y'} = -R\sin\alpha = -1002.75\sin 60.500°$$

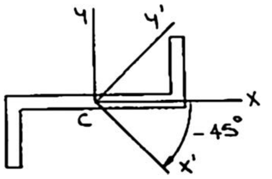
or $\bar{I}_{x'y'} = -873 \text{ in}^4$ ◀



PROBLEM 9.94

Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.75 with respect to new centroidal axes obtained by rotating the x and y axes 45° clockwise.

SOLUTION



From Problem 9.82: $\bar{I}_x = 252{,}757 \text{ mm}^4$

$\bar{I}_y = 1{,}752{,}789 \text{ mm}^4$

Problem 9.75: $\bar{I}_{xy} = 471{,}040 \text{ mm}^4$

The Mohr's circle is defined by the diameter XY, where X (252,757; 471,040) and Y (1,752,789; −471,040).

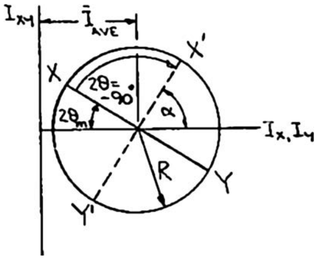
Now

$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(252{,}757 + 1{,}752{,}789)$$
$$= 1{,}002{,}773 \text{ mm}^4$$

and

$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$
$$= \sqrt{\left[\frac{1}{2}(252{,}757 - 1{,}752{,}789)^2\right] + 471{,}040^2}$$
$$= 885{,}665 \text{ mm}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$
$$= -\frac{2(471{,}040)}{252{,}757 - 1{,}752{,}789}$$
$$= 0.62804$$

or $2\theta_m = 32.130°$

Then

$$\alpha = 180° - (32.130 + 90°)$$
$$= 57.870°$$

PROBLEM 9.94 (Continued)

Then $$\bar{I}_{x'} = \bar{I}_{\text{ave}} + R\cos\alpha = 1{,}002{,}773 + 885{,}665\cos 57.870^\circ$$

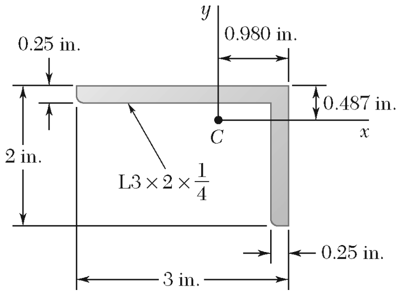
or $\bar{I}_{x'} = 1.474\times 10^6\,\text{mm}^4$ ◀

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} - R\cos\alpha = 1{,}002{,}773 - 885{,}665\cos 57.870^\circ$$

or $\bar{I}_{y'} = 0.532\times 10^6\,\text{mm}^4$ ◀

$$\bar{I}_{x'y'} = R\sin\alpha = 885{,}665\sin 57.870^\circ$$

or $\bar{I}_{x'y'} = 0.750\times 10^6\,\text{mm}^4$ ◀



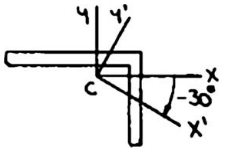
PROBLEM 9.95

Using Mohr's circle, determine the moments of inertia and the product of inertia of the $L3\times 2\times\frac{1}{4}$-in. angle cross section of Problem 9.74 with respect to new centroidal axes obtained by rotating the x and y axes 30° clockwise.

SOLUTION

From Problem 9.83: $\bar{I}_x = 0.390\text{ in}^4$

$\bar{I}_y = 1.09\text{ in}^4$



Problem 9.74:

$\bar{I}_{xy} = -0.37983\text{ in}^4$

The Mohr's circle is defined by the diameter XY, where $X(0.390, -0.37983)$ and $Y(1.09,\ 0.37983)$.

Now

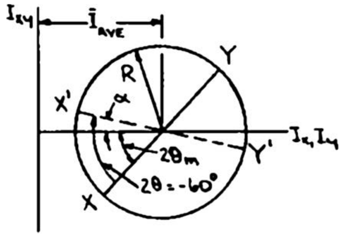
$$\begin{aligned}\bar{I}_{\text{ave}} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y)\\ &= \frac{1}{2}(0.390 + 1.09)\\ &= 0.740\text{ in}^4\end{aligned}$$

and

$$\begin{aligned}R &= \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}\\ &= \sqrt{\left[\frac{1}{2}(0.390 - 1.09)\right]^2 + (-0.37983)^2}\\ &= 0.51650\text{ in}^4\end{aligned}$$

The Mohr's circle is then drawn as shown.

$$\begin{aligned}\tan 2\theta_m &= -\frac{2I_{xy}}{I_x - I_y}\\ &= -\frac{2(-0.37983)}{0.390 - 1.09}\\ &= -1.08523\end{aligned}$$



or $2\theta_m = -47.341°$

Then $\alpha = 60° - 47.341° = 12.659°$

PROBLEM 9.95 (Continued)

Then $\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\cos\alpha = 0.740 - 0.51650\cos 12.659^\circ$

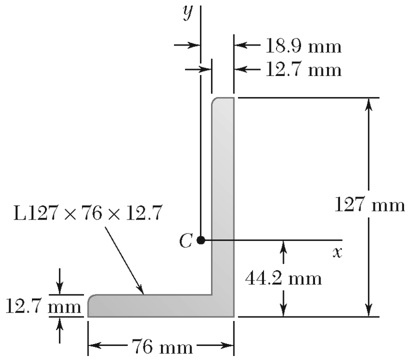
or $\bar{I}_{x'} = 0.236 \text{ in}^4$ ◀

$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R\cos\alpha = 0.740 + 0.51650\cos 12.659^\circ$

or $\bar{I}_{y} = 1.244 \text{ in}^4$ ◀

$\bar{I}_{x'y'} = R\sin\alpha = 0.51650\sin 12.659^\circ$

or $\bar{I}_{x'y'} = 0.1132 \text{ in}^4$ ◀



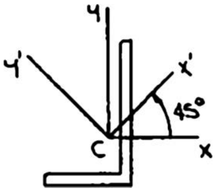
PROBLEM 9.96

Using Mohr's circle, determine the moments of inertia and the product of inertia of the $L127 \times 76 \times 12.7$-mm angle cross section of Problem 9.78 with respect to new centroidal axes obtained by rotating the *x* and *y* axes 45° counterclockwise.

SOLUTION

From Problem 9.84: $\bar{I}_x = 3.93 \times 10^6 \text{ mm}^4$

$\bar{I}_y = 1.06 \times 10^6 \text{ mm}^4$



Problem 9.78: $\bar{I}_{xy} = 1.165061 \times 10^6 \text{ mm}^4$

The Mohr's circle is defined by the diameter *XY*, where $X(3.93 \times 10^6,\ 1.165061 \times 10^6)$, $Y(1.06 \times 10^6,\ -1.165061 \times 10^6)$.

Now $\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(3.93 + 1.06) \times 10^6 = 2.495 \times 10^6 \text{ mm}^4$

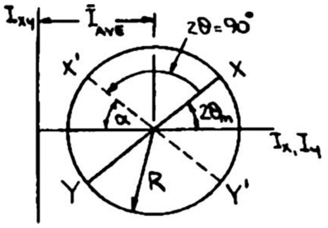
and

$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$

$$= \left\{\sqrt{\left[\frac{1}{2}(3.93 - 1.06)\right]^2 + 1.165061^2}\right\} \times 10^6 \text{ mm}^4$$

$$= 1.84840 \times 10^6 \text{ mm}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(1.165061 \times 10^6)}{(3.93 - 1.06) \times 10^6}$$

$$= -0.81189$$

or $2\theta_m = -39.073°$

Then $$\alpha = 180° - (39.073° + 90°)$$
$$= 50.927°$$

Then $$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\cos\alpha = (2.495 - 1.84840\cos 50.927°)\times 10^6$$

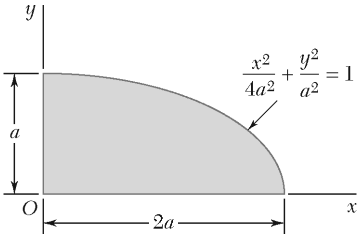
or $\bar{I}_{x'} = 1.330 \times 10^6 \text{ mm}^4$ ◀

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R\cos\alpha = (2.495 + 1.84840\cos 50.927°)\times 10^6$$

or $\bar{I}_{y'} = 3.66 \times 10^6 \text{ mm}^4$ ◀

$$\bar{I}_{x'y'} = R\sin\alpha = (1.84840\times 10^6)\sin 50.927°$$

or $\bar{I}_{x'y'} = 1.435 \times 10^6 \text{ mm}^4$ ◀



PROBLEM 9.97

For the quarter ellipse of Problem 9.67, use Mohr's circle to determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

SOLUTION

From Problem 9.79: $I_x = \frac{\pi}{8}a^4 \qquad I_y = \frac{\pi}{2}a^4$

Problem 9.67: $I_{xy} = \frac{1}{2}a^4$

The Mohr's circle is defined by the diameter *XY*, where

$$X\left(\frac{\pi}{8}a^4,\ \frac{1}{2}a^4\right) \quad \text{and} \quad Y\left(\frac{\pi}{2}a^4,\ -\frac{1}{2}a^4\right)$$

Now $$I_{\text{ave}} = \frac{1}{2}(I_x + I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = 0.98175a^4$$

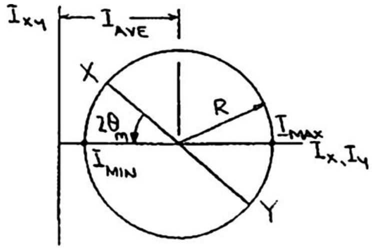
and

$$R = \sqrt{\left[\frac{1}{2}(I_x - I_y)\right]^2 + I_{xy}^2}$$

$$= \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2}$$

$$= 0.77264a^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$

$$= -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4}$$

$$= 0.84883$$

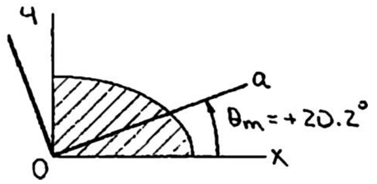
or $$2\theta_m = 40.326°$$

and $$\theta_m = 20.2°$$

The principal axes are obtained by rotating the xy axes through

20.2° counterclockwise ◀

about O.

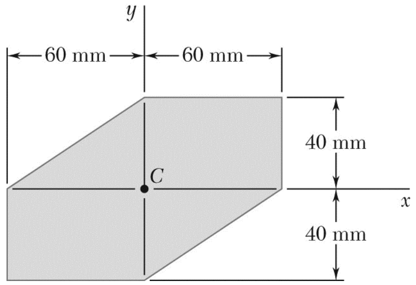


Now $$I_{\text{max, min}} = I_{\text{ave}} \pm R = 0.98175a^4 \pm 0.77264a^4$$

or $I_{\text{max}} = 1.754\,a^4$ ◀

and $I_{\text{min}} = 0.209\,a^4$ ◀

From the Mohr's circle it is seen that the a axis corresponds to I_{min} and the b axis corresponds to I_{max}.



PROBLEM 9.98

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.72.

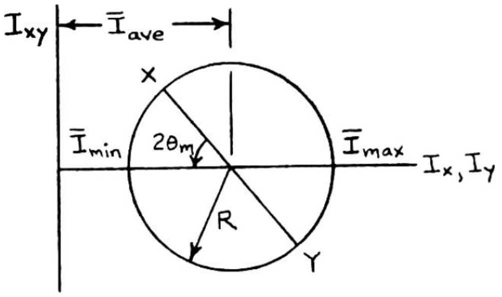
SOLUTION

From Problem 9.80: $\bar{I}_x = 3.20\times10^6\,\text{mm}^4$

$\bar{I}_y = 7.20\times10^6\,\text{mm}^4$

From Problem 9.72: $\bar{I}_{xy} = 2.40\times10^6\,\text{mm}^4$

The Mohr's circle is defined by the diameter XY, where $X(3.20\times10^6, 2.40\times10^6)$ and $Y(7.20\times10^6, -2.40\times10^6)$.



Now $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y)$$
$$= \frac{1}{2}(3.20 + 7.20)\times10^6\,\text{mm}^4$$
$$= 5.20\times10^6\,\text{mm}^4$$

and $$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$
$$= \left\{\sqrt{\left[\frac{1}{2}(3.20 - 7.20)\right]^2 + (2.40)^2}\right\}\times10^6\,\text{mm}^4$$
$$= 3.1241\times10^6\,\text{mm}^4$$

The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$
$$= -\frac{2(2.40\times10^6)}{(3.20 - 7.20)\times10^6} = 1.200$$

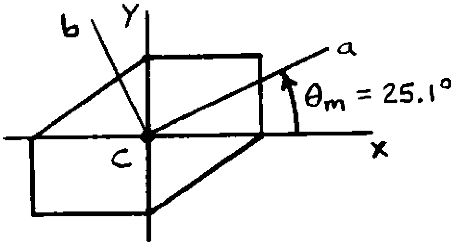
or $$2\theta_m = 50.194°$$

and $$\theta_m = 25.097°$$

The principal axes are obtained by rotating the *xy* axes through

25.1° counterclockwise ◀

about *C*.

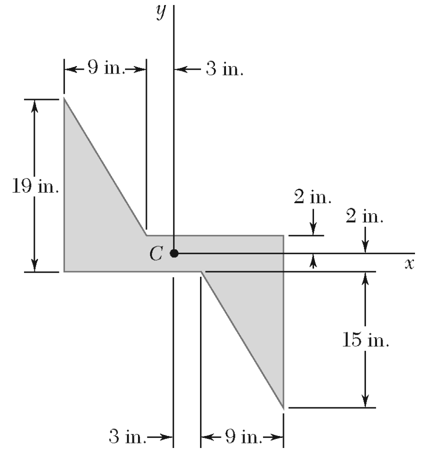


Now $\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (5.20 + 3.1241) \times 10^6$

or $\bar{I}_{\text{max}} = 8.32 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\text{min}} = 2.08 \times 10^6 \text{ mm}^4$ ◀

From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.



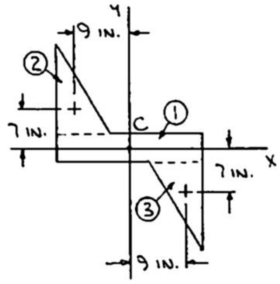
PROBLEM 9.99

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.76.

SOLUTION

From Problem 9.76:



$$\bar{I}_{xy} = -9011.25 \text{ in}^4$$

Now $\bar{I}_x = (\bar{I}_x)_1 + (I_x)_2 + (I_x)_3$

where $(\bar{I}_x)_1 = \frac{1}{12}(24 \text{ in.})(4 \text{ in.})^3 = 128 \text{ in}^4$

$$(I_x)_2 = (I_x)_3 = \frac{1}{36}(9 \text{ in.})(15 \text{ in.})^3 + \left[\frac{1}{2}(9 \text{ in.})(15 \text{ in.})\right](7 \text{ in.})^2$$

$$= 4151.25 \text{ in}^4$$

Then $\bar{I}_x = [128 + 2(4151.25)] \text{ in}^4$

$$= 8430.5 \text{ in}^4$$

Also $\bar{I}_y = (\bar{I}_y)_1 + (I_y)_2 + (I_y)_3$

where $(\bar{I}_y)_1 = \frac{1}{12}(4 \text{ in.})(24 \text{ in.})^3 = 4608 \text{ in}^4$

$$(I_y)_2 = (I_y)_3 = \frac{1}{36}(15 \text{ in.})(9 \text{ in.})^3 + \left[\frac{1}{2}(9 \text{ in.})(15 \text{ in.})\right](9 \text{ in.})^2$$

$$= 5771.25 \text{ in}^4$$

Then $\bar{I}_y = [4608 + 2(5771.25)] \text{ in}^4 = 16150.5 \text{ in}^4$

The Mohr's circle is defined by the diameter XY, where $X(8430.5,\ -9011.25)$ and $Y(16150.5,\ 9011.25)$.

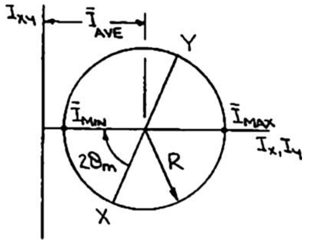
Now $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(8430.5 + 16150.5) = 12290.5 \text{ in}^4$$

and $$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$

$$= \sqrt{\left[\frac{1}{2}(8430.5 - 16150.5)\right]^2 + (-9011.25)^2}$$

$$= 9803.17 \text{ in}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(-9011.25)}{8430.5 - 16150.5}$$

$$= -2.33452$$

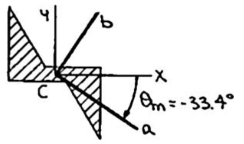
or $$2\theta_m = -66.812°$$

and $$\theta_m = -33.4°$$

The principal axes are obtained by rotating the xy axes through

33.4° clockwise ◀

about C.

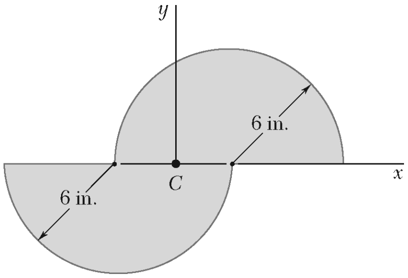


Now $$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = 12290.5 \pm 9803.17$$

or $\bar{I}_{\text{max}} = 22.1 \times 10^3 \text{ in}^4$ ◀

and $\bar{I}_{\text{min}} = 2490 \text{ in}^4$ ◀

From the Mohr's circle it is seen that the a axis corresponds to $\bar{I}_{\text{min}}$ and the b axis corresponds to $\bar{I}_{\text{max}}$.



PROBLEM 9.100

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.73

SOLUTION

From Problem 9.81: $\bar{I}_x = 324\pi \text{ in}^4 \qquad \bar{I}_y = 648\pi \text{ in}^4$

Problem 9.73: $\bar{I}_{xy} = 864 \text{ in}^4$

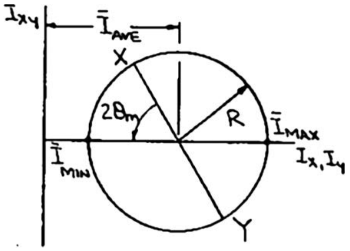
The Mohr's circle is defined by the diameter XY, where $X(324\pi,\ 864)$ and $Y(648\pi,\ -864)$.

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(324\pi + 648\pi) = 1526.81 \text{ in}^4$$

and $$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)^2\right] + I_{xy}^2}$$
$$= \sqrt{\left[\frac{1}{2}(324\pi - 648\pi)\right]^2 + 864^2}$$
$$= 1002.75 \text{ in}^4$$

The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$
$$= -\frac{2(864)}{324\pi - 648\pi}$$
$$= 1.69765$$



or $2\theta_m = 59.4998°$

and $\theta_m = 29.7°$

The principal axes are obtained by rotating the xy axes through

29.7° counterclockwise ◀

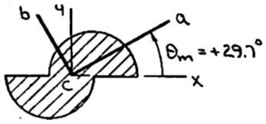
about C.

PROBLEM 9.100 (Continued)

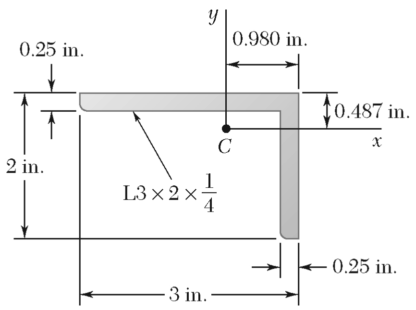
Now $$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = 1526.81 \pm 1002.75$$

or $I_{\text{max}} = 2530 \text{ in}^4$ ◀

and $\bar{I}_{\text{min}} = 524 \text{ in}^4$ ◀



From the Mohr's circle it is seen that the *a* axis corresponds to I_{min} and the *b* axis corresponds to $\bar{I}_{\text{max}}$.



PROBLEM 9.101

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.74.

SOLUTION

From Problem 9.83: $\bar{I}_x = 0.390 \text{ in}^4$

$\bar{I}_y = 1.09 \text{ in}^4$

Problem 9.74: $\bar{I}_{xy} = -0.37983 \text{ in}^4$

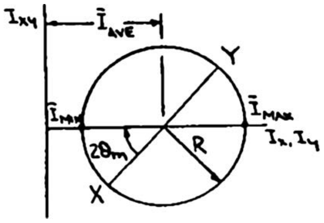
The Mohr's circle is defined by the diameter XY, where $X(0.390, -0.37983)$ and $Y(1.09, 0.37983)$.

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(0.390 + 1.09) = 0.740 \text{ in}^4$$

and
$$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$
$$= \sqrt{\left[\frac{1}{2}(0.390 - 1.09)\right]^2 + (-0.37983)^2}$$
$$= 0.51650 \text{ in}^4$$

The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(-0.37983)}{0.390 - 1.09} = -1.08523$$



Then $2\theta_m = -47.341°$

and $\theta_m = -23.7°$

The principal axes are obtained by rotating the xy axes through

23.7° clockwise ◀

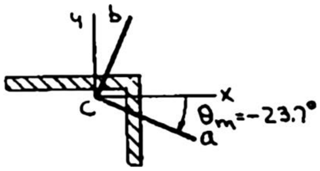
about C.

PROBLEM 9.101 (Continued)

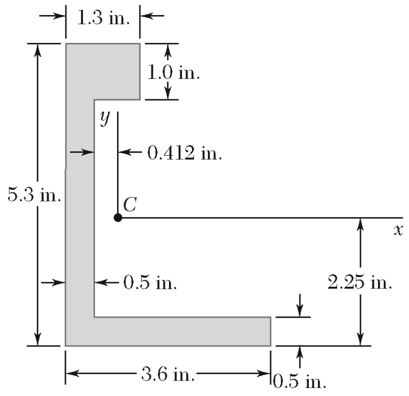
Now $$\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = 0.740 \pm 0.51650$$

or $\bar{I}_{\max} = 1.257 \text{ in}^4$ ◀

and $\bar{I}_{\min} = 0.224 \text{ in}^4$ ◀



From the Mohr's circle it is seen that the a axis corresponds to $I_{\min}$ and the b axis corresponds to $I_{\max}$.



PROBLEM 9.102

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.77

(The moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area of Problem 9.102 were determined in Problem 9.44).

SOLUTION

From Problem 9.44: $\bar{I}_x = 18.1282 \text{ in}^4$

$\bar{I}_y = 4.5080 \text{ in}^4$

Problem 9.77: $\bar{I}_{xy} = -4.25320 \text{ in}^4$

The Mohr's circle is defined by the diameter XY, where $X(18.1282, -4.25320)$ and $Y(4.5080, 4.25320)$.

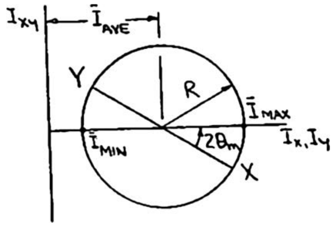
Now $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(18.1282 + 4.5080) = 11.3181 \text{ in}^4$$

and $$R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$$

$$= \sqrt{\left[\frac{1}{2}(18.1282 - 4.5080)\right]^2 + (-4.25320)^2}$$

$$= 8.02915 \text{ in}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(-4.25320)}{18.1282 - 4.5080}$$

$$= 0.62454$$

or $$2\theta_m = 31.986°$$

and $$\theta_m = 15.99°$$

The principal axes are obtained by rotating the xy axes through

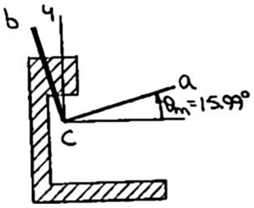
15.99° counterclockwise ◀

about C.

Now $\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = 11.3181 \pm 8.02915$

or $\bar{I}_{\text{max}} = 19.35 \text{ in}^4$ ◀

and $\bar{I}_{\text{min}} = 3.29 \text{ in}^4$ ◀



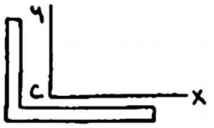
From the Mohr's circle it is seen that the *a* axis corresponds to I_{max} and the *b* axis corresponds to I_{min}.

PROBLEM 9.103

The moments and product of inertia of an L4 × 3 × $\frac{1}{4}$-in. angle cross section with respect to two rectangular axes x and y through C are, respectively, $\bar{I}_x = 1.33$ in^4, $\bar{I}_y = 2.75$ in^4, and $\bar{I}_{xy} \le 0$, with the minimum value of the moment of inertia of the area with respect to any axis through C being $\bar{I}_{\min} = 0.692$ in^4. Using Mohr's circle, determine (*a*) the product of inertia $\bar{I}_{xy}$ of the area, (*b*) the orientation of the principal axes, (*c*) the value of $\bar{I}_{\max}$.

SOLUTION

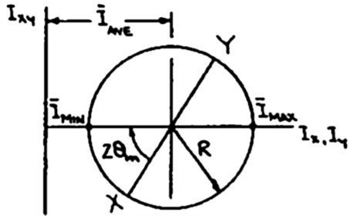
(*Note:* A review of a table of rolled-steel shapes reveals that the given values of $\bar{I}_x$ and $\bar{I}_y$ are obtained when the 4-in. leg of the angle is parallel to the x axis. Further, for $\bar{I}_{xy} < 0$, the angle must be oriented as shown.)



Now $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(1.33 + 2.75) = 2.040 \text{ in}^4$$

and $$\bar{I}_{\min} = \bar{I}_{\text{ave}} - R \quad \text{or} \quad R = 2.040 - 0.692$$
$$= 1.348 \text{ in}^4$$

Using $\bar{I}_{\text{ave}}$ and R, the Mohr's circle is then drawn as shown; note that for the diameter XY, $X(1.33,\ \bar{I}_{xy})$ and $Y(2.75,\ |\bar{I}_{xy}|)$.



(*a*) We have $$R^2 = \left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2$$

or $$\bar{I}_{xy}^2 = 1.348^2 - \left[\frac{1}{2}(1.33 - 2.75)\right]^2$$

Solving for $\bar{I}_{xy}$ and taking the negative root (since $\bar{I}_{xy} < 0$) yields $\bar{I}_{xy} = -1.14586$ in^4.

$\bar{I}_{xy} = -1.146$ in^4 ◀

(*b*) We have $$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(-1.14586)}{1.33 - 2.75}$$
$$= -1.61389$$

or $$2\theta_m = -58.217° \qquad \theta_m = -29.1°$$

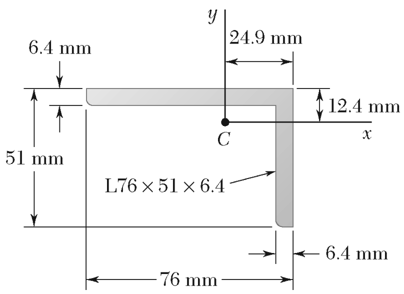
The principal axes are obtained by rotating the xy axes through

29.1° clockwise ◀

about C.

(*c*) We have $$\bar{I}_{\max} = \bar{I}_{\text{ave}} + R = 2.040 + 1.348$$

or $\bar{I}_{\max} = 3.39$ in^4 ◀



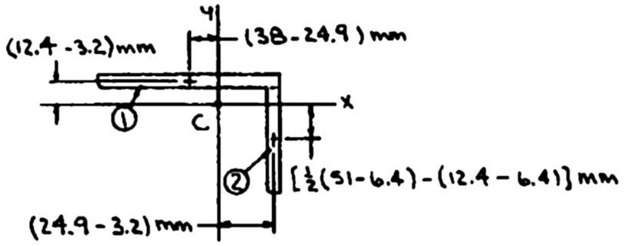
PROBLEM 9.104

Using Mohr's circle, determine for the cross section of the rolled-steel angle shown the orientation of the principal centroidal axes and the corresponding values of the moments of inertia. (Properties of the cross sections are given in Figure 9.13.)

SOLUTION

From Figure 9.13B: $\bar{I}_x = 0.162 \times 10^6\,\text{mm}^4$

$$\bar{I}_y = 0.454 \times 10^6\,\text{mm}^4$$



We have $\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2$

For each rectangle $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$

and $\bar{I}_{x'y'} = 0$ (symmetry) $\quad I_{xy} = \Sigma\bar{x}\,\bar{y}A$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}\,\bar{y}A$, mm^4
1	$76 \times 6.4 = 486.4$	−13.1	9.2	−58620.93
2	$6.4 \times (51 - 6.4) = 285.44$	21.7	−16.3	−100962.98
Σ				−159583.91

$$\bar{I}_{xy} = -159584\ \text{mm}^4$$

The Mohr's circle is defined by the diameter XY where $X(0.162 \times 10^6, -0.159584 \times 10^6)$ and $Y(0.454 \times 10^6, 0.159584 \times 10^6)$

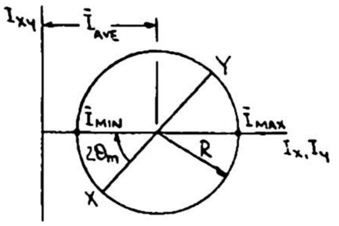
Now
$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(0.162 + 0.454) \times 10^6$$
$$= 0.3080 \times 10^6\,\text{mm}^4$$

PROBLEM 9.104 (Continued)

and $$R=\sqrt{\left[\frac{1}{2}(\bar{I}_x-\bar{I}_y)\right]^2+I_{xy}^2}=\left\{\sqrt{\left[\frac{1}{2}(0.162-0.454)\right]^2+(-0.159584)^2}\right\}\times 10^6$$

$$=0.21629\times 10^6\ \text{mm}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m=-\frac{2\bar{I}_{xy}}{\bar{I}_x-\bar{I}_y}$$

$$=-\frac{2(-0.159584\times 10^6)}{(0.162-0.454)\times 10^6}$$

$$=-1.09304$$

or $$2\theta_m=-47.545$$

and $$\theta_m=-23.8°$$

The principal axes are obtained by rotating the *xy* axes through

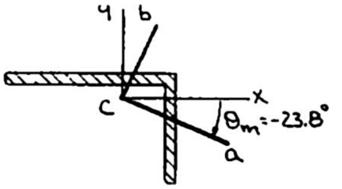
23.8° clockwise ◀

About *C*.

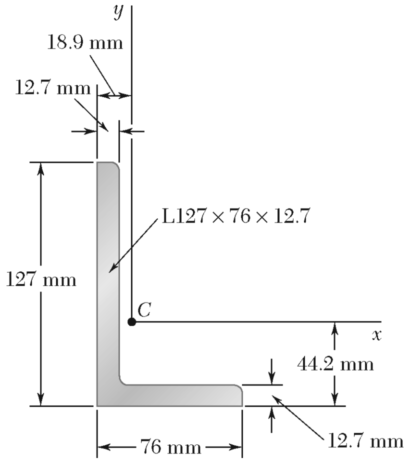
Now $$\bar{I}_{\text{max, min}}=\bar{I}_{\text{ave}}\pm R=(0.3080\pm 0.21629)\times 10^6$$

or $$\bar{I}_{\text{max}}=0.524\times 10^6\,\text{mm}^4$$ ◀

and $$\bar{I}_{\text{min}}=0.0917\times 10^6\,\text{mm}^4$$ ◀



From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.



PROBLEM 9.105

Using Mohr's circle, determine for the cross section of the rolled-steel angle shown the orientation of the principal centroidal axes and the corresponding values of the moments of inertia. (Properties of the cross sections are given in Figure 9.13.)

SOLUTION

From Figure 9.13B:

$$\bar{I}_x = 3.93\times10^6\,\text{mm}^4$$
$$\bar{I}_y = 1.06\times10^6\,\text{mm}^4$$

Problem 9.7B:

$$\bar{I}_{xy} = -1.165061\times10^6\,\text{mm}^4$$

(Note that the figure of Problem 9.105 is obtained by replacing *x* with –*x* in the figure of Problem 9.78; thus the change in sign of $\bar{I}_{xy}$.)

The Mohr's circle is defined by the diameter *XY*, where $X(3.93\times10^6, -1.165061\times10^6)$ and $Y(1.06\times10^6, 1.165061\times10^6)$.

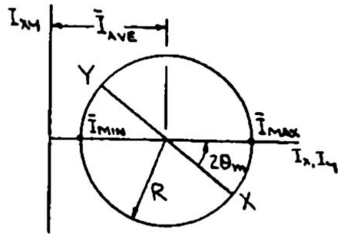
Now

$$\begin{aligned} I_{\text{ave}} &= \frac{1}{2}(\bar{I}_x + \bar{I}_y) \\ &= \frac{1}{2}(3.93 + 1.06)\times10^6 \\ &= 2.495\times10^6\,\text{mm}^4 \end{aligned}$$

and

$$\begin{aligned} R &= \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} \\ &= \left\{\sqrt{\left[\frac{1}{2}(3.93-1.06)\right]^2 + (-1.165061)^2}\right\}\times10^6 \\ &= 1.84840\times10^6\,\text{mm}^4 \end{aligned}$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(-1.165061\times10^6)}{(3.93-1.06)\times10^6}$$

$$= 0.81189$$

or $$2\theta_m = 39.073°$$

and $$\theta_m = 19.54°$$

The principal axes are obtained by rotating the *xy* axes through

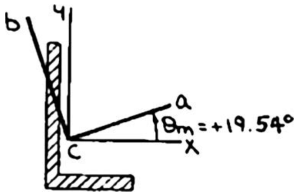
19.54° counterclockwise ◀

about *C*.

Now $$\bar{I}_{\text{max,min}} = \bar{I}_{\text{ave}} \pm R = (2.495 \pm 1.84840)\times10^6$$

or $\bar{I}_{\max} = 4.34\times10^6\text{ mm}^4$ ◀

and $\bar{I}_{\min} = 0.647\times10^6\text{ mm}^4$ ◀



From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\max}$ and the *b* axis corresponds to $\bar{I}_{\min}$.

PROBLEM 9.106*

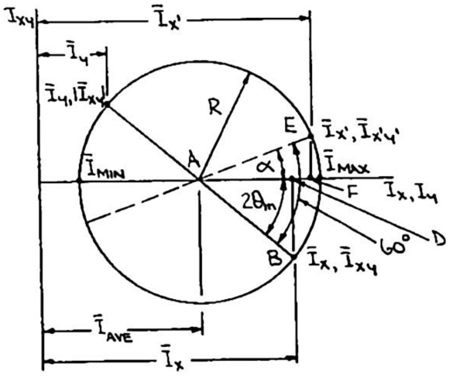
For a given area the moments of inertia with respect to two rectangular centroidal x and y axes are $\bar{I}_x = 1200$ in^4 and $\bar{I}_y = 300$ in^4, respectively. Knowing that after rotating the x and y axes about the centroid 30° counterclockwise, the moment of inertia relative to the rotated x axis is 1450 in^4, use Mohr's circle to determine (*a*) the orientation of the principal axes, (*b*) the principal centroidal moments of inertia.

SOLUTION

We have $$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(1200 + 300) = 750 \text{ in.}^4$$

Now observe that $\bar{I}_x > \bar{I}_{\text{ave}}$, $\bar{I}_{x'} > \bar{I}_x$, and $2\theta = +60°$. This is possible only if $\bar{I}_{xy} < 0$. Therefore, assume $\bar{I}_{xy} < 0$ and (for convenience) $\bar{I}_{x'y'} > 0$. Mohr's circle is then drawn as shown.

We have $$2\theta_m + \alpha = 60°$$



Now using ΔABD: $$R = \frac{\bar{I}_x - \bar{I}_{\text{ave}}}{\cos 2\theta_m} = \frac{1200 - 750}{\cos 2\theta_m} = \frac{450}{\cos 2\theta_m} \quad (\text{in}^4)$$

Using ΔAEF: $$R = \frac{\bar{I}_{x'} - \bar{I}_{\text{ave}}}{\cos \alpha} = \frac{1450 - 750}{\cos \alpha} = \frac{700}{\cos \alpha} \quad (\text{in}^4)$$

Then $$\frac{450}{\cos 2\theta_m} = \frac{700}{\cos \alpha} \qquad \alpha = 60° - 2\theta_m$$

or $$9\cos(60° - 2\theta_m) = 14\cos 2\theta_m$$

Expanding: $$9(\cos 60° \cos 2\theta_m + \sin 60° \sin 2\theta_m) = 14\cos 2\theta_m$$

or $$\tan 2\theta_m = \frac{14 - 9\cos 60°}{9\sin 60°} = 1.21885$$

or $$2\theta_m = 50.633° \quad \text{and} \quad \theta_m = 25.3°$$

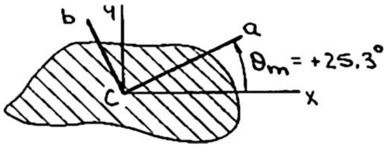
(*Note:* $2\theta_m < 60°$ implies assumption $\bar{I}_{x'y'} > 0$ is correct.)

Finally, $$R = \frac{450}{\cos 50.633°} = 709.46 \text{ in}^4$$

(*a*) From the Mohr's circle it is seen that the principal axes are obtained by rotating the given centroidal x and y axes through θ_m about the centroid C or

25.3° counterclockwise ◀

PROBLEM 9.106* (Continued)



(*b*) We have $\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = 750 \pm 709.46$

or $\bar{I}_{\text{max}} = 1459 \text{ in}^4$ ◀

and $\bar{I}_{\text{min}} = 40.5 \text{ in}^4$ ◀

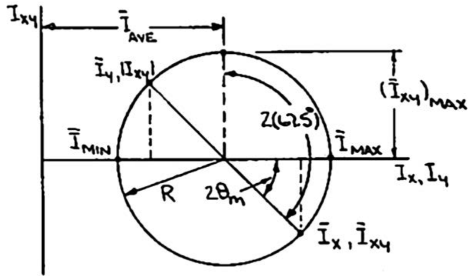
From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{max}}$ and the *b* axis corresponds to $\bar{I}_{\text{min}}$.

PROBLEM 9.107

It is known that for a given area $\bar{I}_y = 48 \times 10^6\ \text{mm}^4$ and $\bar{I}_{xy} = -20 \times 10^6\ \text{mm}^4$, where the *x* and *y* axes are rectangular centroidal axes. If the axis corresponding to the maximum product of inertia is obtained by rotating the *x* axis 67.5° counterclockwise about *C*, use Mohr's circle to determine (*a*) the moment of inertia $\bar{I}_x$ of the area, (*b*) the principal centroidal moments of inertia.

SOLUTION

First assume $\bar{I}_x > \bar{I}_y$ and then draw the Mohr's circle as shown. (*Note:* Assuming $\bar{I}_x < \bar{I}_y$ is not consistent with the requirement that the axis corresponding to $(\bar{I}_{xy})_{\max}$ is obtained after rotating the *x* axis through 67.5° CCW.)



From the Mohr's circle we have

$$2\theta_m = 2(67.5°) - 90° = 45°$$

(*a*) From the Mohr's circle we have

$$\bar{I}_x = \bar{I}_y + 2\frac{|I_{xy}|}{\tan 2\theta_m} = 48\times10^6 + 2\frac{20\times10^6}{\tan 45°}$$

or $\bar{I}_x = 88.0\times10^6\,\text{mm}^4$ ◀

(*b*) We have

$$\bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(88.0+48)\times10^6$$
$$= 68.0\times10^6\,\text{mm}^4$$

and

$$R = \frac{|I_{xy}|}{\sin 2\theta_m} = \frac{20\times10^6}{\sin 45°} = 28.284\times10^6\,\text{mm}^4$$

Now

$$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (68.0 \pm 28.284)\times10^6$$

or $\bar{I}_{\max} = 96.3\times10^6\,\text{mm}^4$ ◀

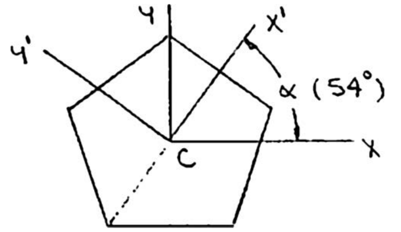
and $\bar{I}_{\min} = 39.7\times10^6\,\text{mm}^4$ ◀

PROBLEM 9.108

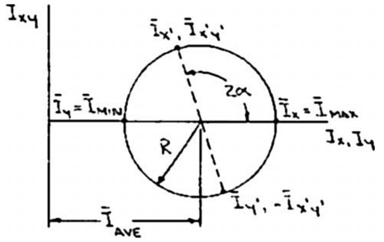
Using Mohr's circle, show that for any regular polygon (such as a pentagon) (*a*) the moment of inertia with respect to every axis through the centroid is the same, (*b*) the product of inertia with respect to every pair of rectangular axes through the centroid is zero.

SOLUTION

Consider the regular pentagon shown, with centroidal axes *x* and *y*.



Because the *y* axis is an axis of symmetry, it follows that $\bar{I}_{xy} = 0$. Since $\bar{I}_{xy} = 0$, the *x* and *y* axes must be principal axes. Assuming $\bar{I}_x = \bar{I}_{\max}$ and $\bar{I}_y = \bar{I}_{\min}$, the Mohr's circle is then drawn as shown.



Now rotate the coordinate axes through an angle α as shown; the resulting moments of inertia, $\bar{I}_{x'}$ and $\bar{I}_{y'}$, and product of inertia, $\bar{I}_{x'y'}$, are indicated on the Mohr's circle. However, the x' axis is an axis of symmetry, which implies $\bar{I}_{x'y'} = 0$. For this to be possible on the Mohr's circle, the radius *R* must be equal to zero (thus, the circle degenerates into a point). With $R = 0$, it immediately follows that

(*a*) $\bar{I}_x = \bar{I}_y = \bar{I}_{x'} = \bar{I}_{y'} = \bar{I}_{\text{ave}}$ (for all moments of inertia with respect to an axis through *C*) ◀

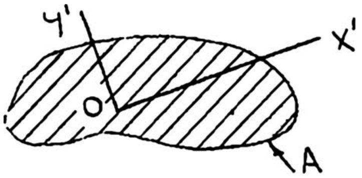
(*b*) $\bar{I}_{xy} = \bar{I}_{x'y'} = 0$ (for all products of inertia with respect to all pairs of rectangular axes with origin at *C*) ◀

PROBLEM 9.109

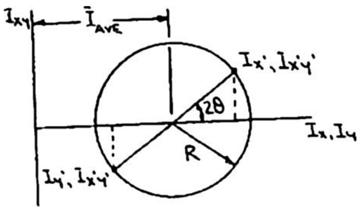
Using Mohr's circle, prove that the expression $I_{x'}I_{y'} - I_{x'y'}^2$ is independent of the orientation of the x' and y' axes, where $I_{x'}$, $I_{y'}$, and $I_{x'y'}$ represent the moments and product of inertia, respectively, of a given area with respect to a pair of rectangular axes x' and y' through a given Point O. Also show that the given expression is equal to the square of the length of the tangent drawn from the origin of the coordinate system to Mohr's circle.

SOLUTION

First observe that for a given area A and origin O of a rectangular coordinate system, the values of I_{ave} and R are the same for all orientations of the coordinate axes. Shown below is a Mohr's circle, with the moments of inertia, $I_{x'}$ and $I_{y'}$, and the product of inertia, $I_{x'y'}$, having been computed for an arbitrary orientation of the $x'y'$ axes.



From the Mohr's circle



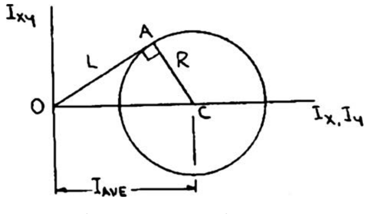
$$I_{x'} = I_{\text{ave}} + R\cos 2\theta$$
$$I_{y'} = I_{\text{ave}} - R\cos 2\theta$$
$$I_{x'y'} = R\sin 2\theta$$

Then, forming the expression $I_{x'}I_{y'} - I_{x'y'}^2$

$$I_{x'}I_{y'} - I_{x'y'}^2 = (I_{\text{ave}} + R\cos 2\theta)(I_{\text{ave}} - R\cos 2\theta) - (R\sin 2\theta)^2$$
$$= \left(I_{\text{ave}}^2 - R^2\cos^2 2\theta\right) - (R^2\sin^2 2\theta)$$
$$= I_{\text{ave}}^2 - R^2 \quad \text{which is a constant}$$

$I_{x'}I_{y'} - I_{x'y'}^2$ is independent of the orientation of the coordinate axes Q.E.D. ◀

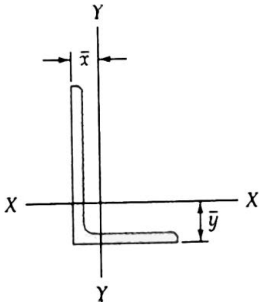
Shown is a Mohr's circle, with line $\overline{OA}$, of length L, the required tangent.



Noting that $\measuredangle OAC$ is a right angle, it follows that

$$L^2 = I_{\text{ave}}^2 - R^2$$

or $L^2 = I_{x'}I_{y'} - I_{x'y'}^2$ Q.E.D. ◀



PROBLEM 9.110

Using the invariance property established in the preceding problem, express the product of inertia I_{xy} of an area A with respect to a pair of rectangular axes through O in terms of the moments of inertia I_x and I_y of A and the principal moments of inertia $I_{\min}$ and $I_{\max}$ of A about O. Use the formula obtained to calculate the product of inertia I_{xy} of the L3 × 2 × $\frac{1}{4}$ -in. angle cross section shown in Figure 9.13A, knowing that its maximum moment of inertia is 1.257 in^4.

SOLUTION

Consider the following two sets of moments and products of inertia, which correspond to two different orientations of the coordinate axes whole origin is at Point O.

Case 1: $I_{x'} = I_x, \quad I_{y'} = I_y, \quad I_{x'y'} = I_{xy}$

Case 2: $I_{x'} = I_{\max}, \quad I_{y'} = I_{\min}, \quad I_{x'y'} = 0$

The invariance property then requires

$$I_x I_y - I_{xy}^2 = I_{\max} I_{\min} \qquad \text{or} \qquad I_{xy} = \pm\sqrt{I_x I_y - I_{\max} I_{\min}} \blacktriangleleft$$

From Figure 9.13A: $\bar{I}_x = 1.09 \text{ in}^4$

$\bar{I}_y = 0.390 \text{ in}^4$

Using Eq. (9.21): $\bar{I}_x + \bar{I}_y = \bar{I}_{\max} + \bar{I}_{\min}$

Substituting $1.09 + 0.390 = 1.257 + \bar{I}_{\min}$

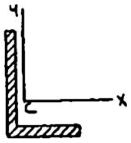
or $\bar{I}_{\min} = 0.223 \text{ in}^4$

Then

$$\bar{I}_{xy} = \sqrt{(1.09)(0.390) - (1.257)(0.223)}$$
$$= \pm 0.381 \text{ in}^4$$

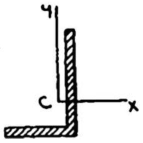
The two roots correspond to the following two orientations of the cross section.

For

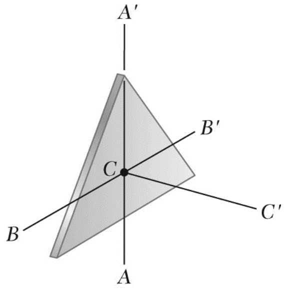


$\bar{I}_{xy} = -0.381 \text{ in}^4 \blacktriangleleft$

and for



$\bar{I}_{xy} = 0.381 \text{ in}^4 \blacktriangleleft$

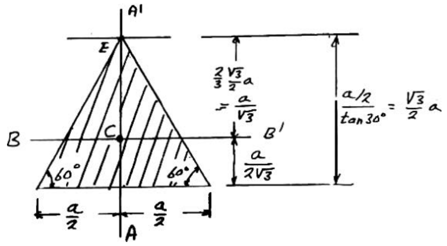


PROBLEM 9.111

A thin plate of mass m is cut in the shape of an equilateral triangle of side a. Determine the mass moment of inertia of the plate with respect to (a) the centroidal axes AA' and BB', (b) the centroidal axis CC' that is perpendicular to the plate.

SOLUTION

$$\text{Area} = \frac{1}{2}a\left(\frac{\sqrt{3}}{2}a\right) = \frac{\sqrt{3}}{4}a^2$$



$$\text{Mass} = m = \rho V = \rho t A$$

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{A} I_{\text{area}}$$

(a) Axis AA':

$$I_{AA',\text{area}} = 2\left[\frac{1}{12}\left(\frac{\sqrt{3}}{2}a\right)\left(\frac{a}{2}\right)^3\right] = \frac{\sqrt{3}}{96}a^4$$

$$I_{AA',\text{ mass}} = \frac{m}{A} I_{AA',\text{ area}} = \frac{\frac{m}{\sqrt{3}}}{4}a^2\left(\frac{\sqrt{3}}{96}a^4\right) = \frac{1}{24}ma^2 \quad \blacktriangleleft$$

Axis BB'':

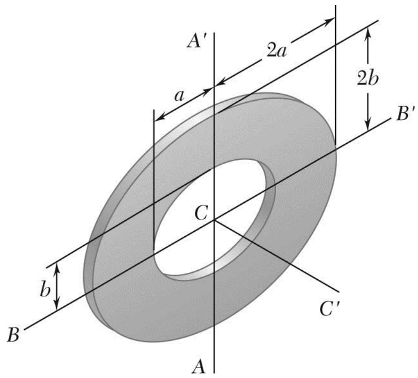
$$I_{BB',\text{ area}} = \frac{1}{36}a\left(\frac{\sqrt{3}}{2}a\right)^3 = \frac{\sqrt{3}}{96}a^4 \qquad \text{(we check that } I_{AA} = I_{BB}\text{)}$$

$$I_{BB',\text{mass}} = \frac{m}{A} I_{BB',\text{ area}} = \frac{\frac{m}{\sqrt{3}}}{4}a^2\left(\frac{\sqrt{3}}{96}a^4\right) = \frac{1}{24}ma^2 \quad \blacktriangleleft$$

(b) Eq. (9.38):

$$I_{CC'} = I_{AA'} + I_{BB'} = 2\left(\frac{1}{24}ma^2\right)$$

$$I_{CC'} = \frac{1}{12}ma^2 \quad \blacktriangleleft$$



PROBLEM 9.112

The elliptical ring shown was cut from a thin, uniform plate. Denoting the mass of the ring by *m*, determine its mass moment of inertia with respect to (*a*) the centroidal axis *BB*′, (*b*) the centroidal axis *CC*′ that is perpendicular to the plane of the ring.

SOLUTION

First note

$$\text{Mass} = m = \rho V = \rho t A$$
$$= \rho t \times \eta[(2a)(2b) - ab]$$
$$= 3\pi\, \rho t a b$$

Also

$$I_{\text{mass}} = \rho t I_{\text{area}}$$
$$= \frac{m}{3\pi ab} I_{\text{area}}$$

(*a*) Using Figure 9.12,

$$I_{BB',\text{area}} = \frac{\pi}{4}[(2a)(2b)^3 - ab^3]$$
$$= \frac{15}{4}\pi ab^3$$

Then

$$I_{BB',\text{ mass}} = \frac{m}{3\pi ab} \times \frac{15}{4}\pi ab^3$$

or $I_{BB'} = \frac{5}{4} mb^2$ ◀

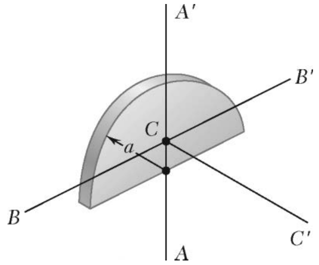
(*b*) Using Figure 9.12 and symmetry, we can conclude that

$$I_{AA',\text{ mass}} = \frac{5}{4} ma^2$$

Now

$$I_{CC',\text{mass}} = I_{AA',\text{ mass}} + I_{BB',\text{ mass}}$$
$$= \frac{5}{4} ma^2 + \frac{5}{4} mb^2$$

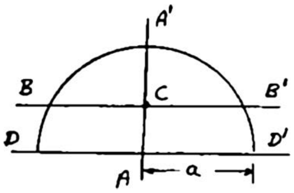
or $I_{CC'} = \frac{5}{4} m(a^2 + b^2)$ ◀



PROBLEM 9.113

A thin semicircular plate has a radius a and a mass m. Determine the mass moment of inertia of the plate with respect to (a) the centroidal axis BB', (b) the centroidal axis CC' that is perpendicular to the plate.

SOLUTION



$$\text{mass} = m = \rho t A$$

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{A} I_{\text{area}}$$

Area: $$A = \frac{1}{2}\pi a^2$$

$$I_{AA',\text{area}} = I_{DD',\text{area}} = \frac{1}{2}\left(\frac{\pi}{4}a^4\right) = \frac{1}{8}\pi a^4$$

$$I_{AA',\text{mass}} = I_{DD',\text{mass}} = \frac{m}{A} I_{AA',\text{area}} = \frac{m}{\frac{1}{2}\pi a^2}\left(\frac{1}{8}\pi a^4\right) = \frac{1}{4}ma^2$$

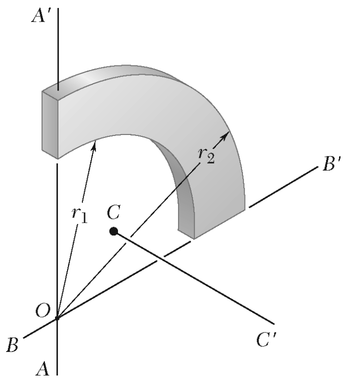
(a) $$I_{BB'} = I_{DD'} - m(AC)^2 = \frac{1}{4}ma^2 - m\left(\frac{4a}{3\pi}\right)^2$$

$$= (0.25 - 0.1801)ma^2$$

$$I_{BB'} = 0.0699ma^2 \blacktriangleleft$$

(b) Eq. (9.38): $$I_{CC'} = I_{AA'} + I_{BB'} = \frac{1}{4}ma^2 + 0.0699ma^2$$

$$I_{CC'} = 0.320ma^2 \blacktriangleleft$$



PROBLEM 9.114

The quarter ring shown has a mass m and was cut from a thin, uniform plate. Knowing that $r_1 = \frac{3}{4} r_2$, determine the mass moment of inertia of the quarter ring with respect to (*a*) the axis AA', (*b*) the centroidal axis CC' that is perpendicular to the plane of the quarter ring.

SOLUTION

First note

$$\text{mass} = m = \rho V = \rho t A = \rho t \frac{\pi}{4}\left(r_2^2 - r_1^2\right)$$

Also

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{\frac{\pi}{4}\left(r_2^2 - r_1^2\right)} I_{\text{area}}$$

(*a*) Using Figure 9.12,

$$I_{AA',\text{area}} = \frac{\pi}{16}\left(r_2^4 - r_1^4\right)$$

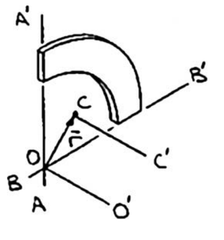
Then

$$I_{AA',\text{mass}} = \frac{m}{\frac{\pi}{4}\left(r_2^2 - r_1^2\right)} \times \frac{\pi}{16}\left(r_2^4 - r_1^4\right) = \frac{m}{4}\left(r_2^2 + r_1^2\right) = \frac{m}{4}\left[r_2^2 + \left(\frac{3}{4}r_2\right)^2\right]$$

or $I_{AA'} = \frac{25}{64} m r_2^2$ ◀

(*b*) Symmetry implies

$$I_{BB',\text{mass}} = I_{AA',\text{mass}}$$



Then

$$I_{DD'} = I_{AA'} + I_{BB'} = 2\left(\frac{25}{64} m r_2^2\right) = \frac{25}{32} m r_2^2$$

Now locate centroid C.

$$\bar{X}\,\Sigma A = \Sigma \bar{x} A$$

or
$$\bar{X}\left(\frac{\pi}{4}r_2^2 - \frac{\pi}{4}r_1^2\right) = \frac{4r_2}{3\pi}\left(\frac{\pi}{4}r_2^2\right) - \frac{4r_1}{3\pi}\left(\frac{\pi}{4}r_1^2\right)$$

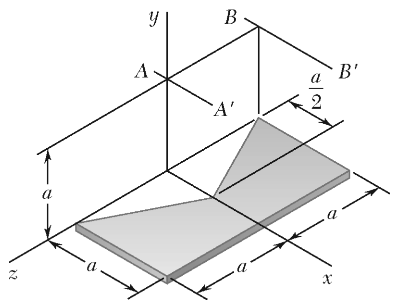
or
$$\bar{X} = \frac{4}{3\pi}\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$$

Now
$$\bar{r} = \bar{X}\sqrt{2}$$
$$= \frac{4\sqrt{2}}{3\pi}\frac{r_2^3 - \left(\frac{3}{4}r_2\right)^3}{r_2^2 - \left(\frac{3}{4}r_2\right)^2}$$
$$= \frac{37\sqrt{2}}{21\pi}r_2$$

Finally,
$$I_{DD'} = I_{CC'} + m\bar{r}^2$$

or
$$\frac{25}{32}mr_2^2 = I_{CC'} + m\left(\frac{37\sqrt{2}}{21\pi}r_2\right)^2$$

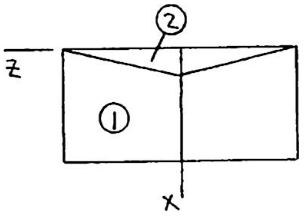
or $I_{CC'} = 0.1522mr_2^2$ ◀



PROBLEM 9.115

A piece of thin, uniform sheet metal is cut to form the machine component shown. Denoting the mass of the component by m, determine its mass moment of inertia with respect to (a) the x axis, (b) the y axis.

SOLUTION



First note

$$\text{mass} = m = \rho V = \rho t A$$
$$= \rho t\left[(2a)(a) - \frac{1}{2}(2a)\left(\frac{a}{2}\right)\right]$$
$$= \frac{3}{2}\rho t a^2$$

Also

$$I_{\text{mass}} = \rho t I_{\text{area}}$$
$$= \frac{2m}{3a^2} I_{\text{area}}$$

(a) Now

$$\bar{I}_{x,\text{area}} = (I_x)_{1,\text{area}} - 2(I_x)_{2,\text{area}}$$
$$= \frac{1}{12}(a)(2a)^3 - 2\left[\frac{1}{12}\left(\frac{a}{2}\right)(a)^3\right]$$
$$= \frac{7}{12}a^4$$

Then

$$\bar{I}_{x,\text{mass}} = \frac{2m}{3a^2} \times \frac{7}{12}a^4$$

or $\bar{I}_x = \dfrac{7}{18}ma^2$ ◀

(b) We have

$$\bar{I}_{z,\text{area}} = (I_z)_{1,\text{area}} - 2(I_z)_{2,\text{area}}$$
$$= \frac{1}{3}(2a)(a)^3 - 2\left[\frac{1}{12}(a)\left(\frac{a}{2}\right)^3\right]$$
$$= \frac{31}{48}a^4$$

Then

$$I_{z,\text{mass}} = \frac{2m}{3a^2} \times \frac{31}{48}a^4$$
$$= \frac{31}{72}ma^2$$

PROBLEM 9.115 (Continued)

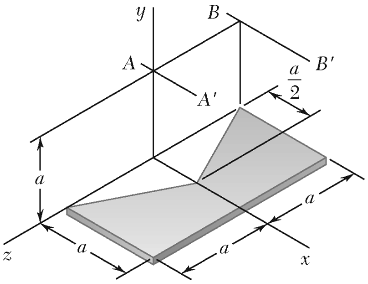
Finally,

$$I_{y,\text{mass}} = \bar{I}_{x,\text{mass}} + I_{z,\text{mass}}$$

$$= \frac{7}{18}ma^2 + \frac{31}{72}ma^2$$

$$= \frac{59}{72}ma^2$$

or $I_y = 0.819ma^2$ ◀



PROBLEM 9.116

A piece of thin, uniform sheet metal is cut to form the machine component shown. Denoting the mass of the component by m, determine its mass moment of inertia with respect to (*a*) the axis AA', (*b*) the axis BB', where the AA' and BB' axes are parallel to the x axis and lie in a plane parallel to and at a distance a above the xz plane.

SOLUTION

First note that the x axis is a centroidal axis so that

$$I = \bar{I}_{x,\text{mass}} + md^2$$

and that from the solution to Problem 9.115,

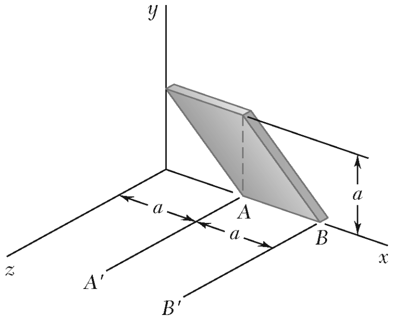
$$\bar{I}_{x,\text{mass}} = \frac{7}{18}ma^2$$

(*a*) We have $$I_{AA',\text{mass}} = \frac{7}{18}ma^2 + m(a)^2$$

or $I_{AA'} = 1.389ma^2$ ◀

(*b*) We have $$I_{BB',\text{mass}} = \frac{7}{18}ma^2 + m(a\sqrt{2})^2$$

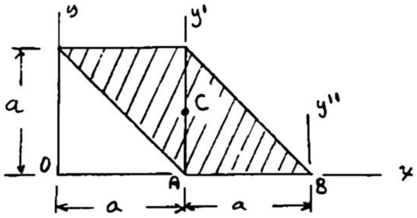
or $I_{BB'} = 2.39ma^2$ ◀



PROBLEM 9.117

A thin plate of mass m was cut in the shape of a parallelogram as shown. Determine the mass moment of inertia of the plate with respect to (*a*) the x axis, (*b*) the axis BB', which is perpendicular to the plate.

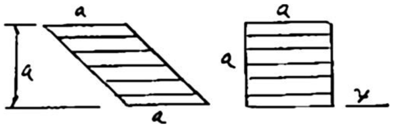
SOLUTION



$$\text{mass} = m = \rho t A$$

$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{A} I_{\text{area}}$$

(*a*) Consider parallelogram as made of horizontal strips and slide strips to form a square since distance from each strip to x axis is unchanged.



$$I_{x,\text{area}} = \frac{1}{3}a^4$$

$$I_{x,\text{mass}} = \frac{m}{A} I_{x,\text{area}} = \frac{m}{a^2}\left(\frac{1}{3}a^4\right) \qquad I_x = \frac{1}{3}ma^2 \blacktriangleleft$$

(*b*) For centroidal axis y':

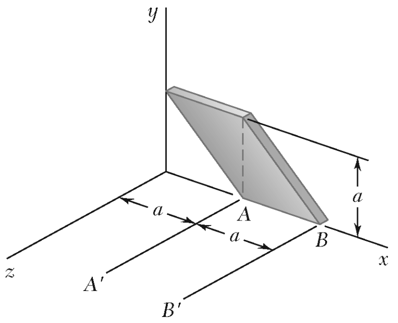
$$\bar{I}_{y',\text{area}} = 2\left[\frac{1}{12}a^4\right] = \frac{1}{6}a^4$$

$$\bar{I}_{y',\text{mass}} = \frac{m}{A} I_{y',\text{area}} = \frac{m}{a^2}\left(\frac{1}{6}a^4\right) = \frac{1}{6}ma^2$$

$$I_{y''} = \bar{I}_{y'} + ma^2 = \frac{1}{6}ma^2 + ma^2 = \frac{7}{6}ma^2$$

For axis $BB' \perp$ to plate, Eq. (9.38):

$$I_{BB'} = I_x + I_{y''} = \frac{1}{3}ma^2 + \frac{7}{6}ma^2 \qquad I_{BB'} = \frac{3}{2}ma^2 \blacktriangleleft$$



PROBLEM 9.118

A thin plate of mass m was cut in the shape of a parallelogram as shown. Determine the mass moment of inertia of the plate with respect to (*a*) the y axis, (*b*) the axis AA', which is perpendicular to the plate.

SOLUTION

See sketch of solution of Problem 9.117.

(*a*) From part *b* of solution of Problem 9.117:

$$\bar{I}_{y'} = \frac{1}{6}ma^2$$

$$I_y = \bar{I}_{y'} + ma^2 = \frac{1}{6}ma^2 + ma^2 \qquad I_y = \frac{7}{6}ma^2 \blacktriangleleft$$

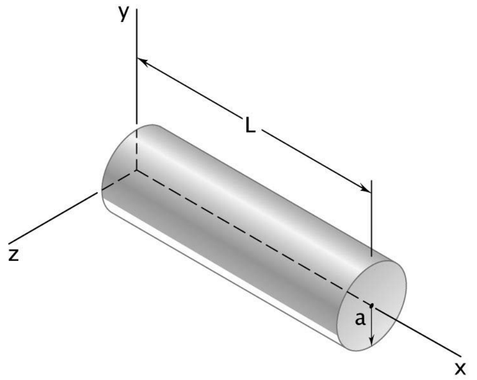
(*b*) From solution of Problem 9.115:

$$\bar{I}_{y'} = \frac{1}{6}ma^2 \quad \text{and} \quad I_x = \frac{1}{3}ma^2$$

Eq. (9.38):

$$I_{AA'} = \bar{I}_{y'} + I_x$$

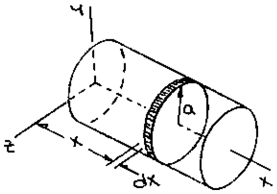
$$= \frac{1}{6}ma^2 + \frac{1}{3}ma^2 \qquad I_{AA'} = \frac{1}{2}ma^2 \blacktriangleleft$$



PROBLEM 9.119

Determine by direct integration the mass moment of inertia with respect to the *z* axis of the right circular cylinder shown, assuming that it has a uniform density and a mass *m*.

SOLUTION



For the cylinder: $m = \rho V = \rho \pi a^2 L$

For the element shown: $dm = \rho \pi a^2 dx$

$$= \frac{m}{L} dx$$

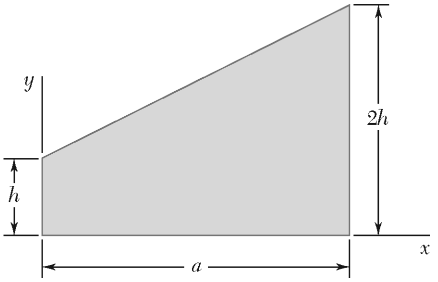
and $dI_z = dI_z + x^2 dm$

$$= \frac{1}{4} a^2 dm + x^2 dm$$

Then $I_z = \int dI_z = \int_0^L \left(\frac{1}{4} a^2 + x^2 \right) \left(\frac{m}{L} dx \right)$

$$= \frac{m}{L} \left[\frac{1}{4} a^2 x + \frac{1}{3} x^3 \right]_0^L$$

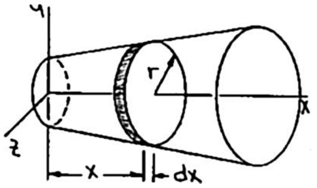
$$I_z = \frac{1}{12} m(3a^2 + 4L^2) \blacktriangleleft$$



PROBLEM 9.120

The area shown is revolved about the *x* axis to form a homogeneous solid of revolution of mass *m*. Using direct integration, express the mass moment of inertia of the solid with respect to the *x* axis in terms of *m* and *h*.

SOLUTION



We have $y = \dfrac{2h-h}{a}x + h$

so that $r = \dfrac{h}{a}(x+a)$

For the element shown:

$$dm = \rho\pi r^2 dx \quad dI_x = \frac{1}{2}r^2 dm$$

$$= \rho\pi\left[\frac{h}{a}(x+a)\right]^2 dx$$

Then $$m = \int dm = \int_0^a \rho\pi\frac{h^2}{a^2}(x+a)^2 dx = \frac{1}{3}\rho\pi\frac{h^2}{a^2}[(x+a)^3]_0^a$$

$$= \frac{1}{3}\rho\pi\frac{h^2}{a^2}(8a^3 - a^3) = \frac{7}{3}\rho\pi a h^2$$

Now $$I_x = \int dI_x = \int \frac{1}{2}r^2(\rho\pi r^2 dx) = \frac{1}{2}\rho\pi\int_0^a\left[\frac{h}{a}(x+a)\right]^4 dx$$

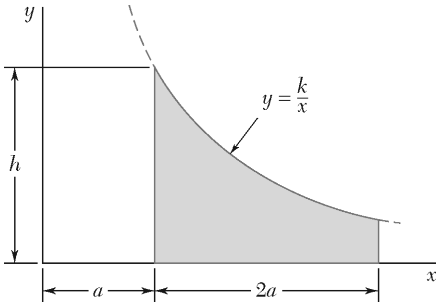
$$= \frac{1}{2}\rho\pi \times \frac{1}{5}\frac{h^4}{a^4}[(x+a)^5]_0^a = \frac{1}{10}\rho\pi\frac{h^4}{a^4}(32a^5 - a^5)$$

$$= \frac{31}{10}\rho\pi a h^4$$

From above: $\rho\pi a h^2 = \dfrac{3}{7}m$

Then $$I_x = \frac{31}{10}\left(\frac{3}{7}m\right)h^2 = \frac{93}{70}mh^2$$

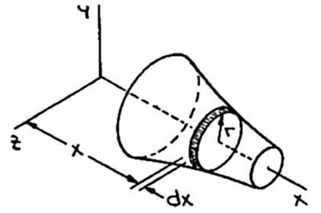
or $I_x = 1.329\, mh^2$ ◀



PROBLEM 9.121

The area shown is revolved about the x axis to form a homogeneous solid of revolution of mass m. Determine by direct integration the mass moment of inertia of the solid with respect to (a) the x axis, (b) the y axis. Express your answers in terms of m and the dimensions of the solid.

SOLUTION



We have at (a, h): $h=\frac{k}{a}$

or $k = ah$

For the element shown:

$$r = 4$$

$$dm = \rho\pi r^2 dx$$

$$= \rho\pi\left(\frac{ah}{x}\right)^2 dx$$

Then

$$m = \int dm = \int_a^{3a} \rho\pi\left(\frac{ah}{x}\right)^2 dx$$

$$= \rho\pi a^2h^2\left[-\frac{1}{x}\right]_a^{3a}$$

$$= \rho\pi a^2h^2\left[-\frac{1}{3a}-\left(-\frac{1}{a}\right)\right] = \frac{2}{3}\rho\pi a h^2$$

(a) For the element: $dI_x = \frac{1}{2}r^2 dm = \frac{1}{2}\rho\pi r^4 dx$

Then

$$I_x = \int dI_x = \int_a^{3a} \frac{1}{2}\rho\pi\left(\frac{ah}{x}\right)^4 dx = \frac{1}{2}\rho\pi a^4h^4\left[-\frac{1}{3}\frac{1}{x^3}\right]_a^{3a}$$

$$= -\frac{1}{6}\rho\pi a^4h^4\left[\left(\frac{1}{3a}\right)^3-\left(\frac{1}{a}\right)^3\right] = \frac{1}{6}\times\frac{26}{27}\rho\pi a h^4$$

$$= \frac{1}{6}\times\frac{2}{3}\rho\pi a h^2\times\frac{13}{9}h^2 = \frac{13}{54}mh^2$$

or $I_x = 0.241\,mh^2$ ◀

PROBLEM 9.121 (Continued)

(*b*) For the element: $dI_y = dI_y + x^2 dm$

$$= \frac{1}{4} r^2 dm + x^2 dm$$

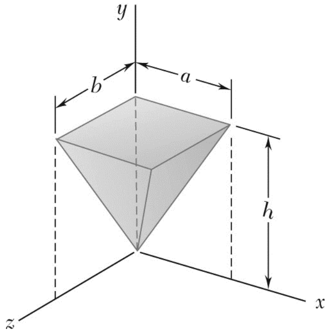
Then $I_y = \int dI_y = \int_a^{3a} \left[\frac{1}{4} \left(\frac{ah}{x} \right)^2 + x^2 \right] \rho\pi \left(\frac{ah}{x} \right)^2 dx$

$$= \rho\pi a^2 h^2 \int_a^{3a} \left(\frac{1}{4} \frac{a^2 h^2}{x^4} + 1 \right) dx = \rho\pi a^2 h^2 \left[-\frac{1}{12} \frac{a^2 h^2}{x^3} + x \right]_a^{3a}$$

$$= \left(\frac{3}{2} m \right) a \left\{ \left[-\frac{1}{12} \frac{a^2 h^2}{(3a)^3} + 3a \right] - \left[-\frac{1}{12} \frac{a^2 h^2}{(a)^3} + a \right] \right\}$$

$$= \frac{3}{2} ma \left(\frac{1}{12} \times \frac{26}{27} \frac{h^2}{a} + 2a \right) = m \left(\frac{13}{108} h^2 + 3a^2 \right)$$

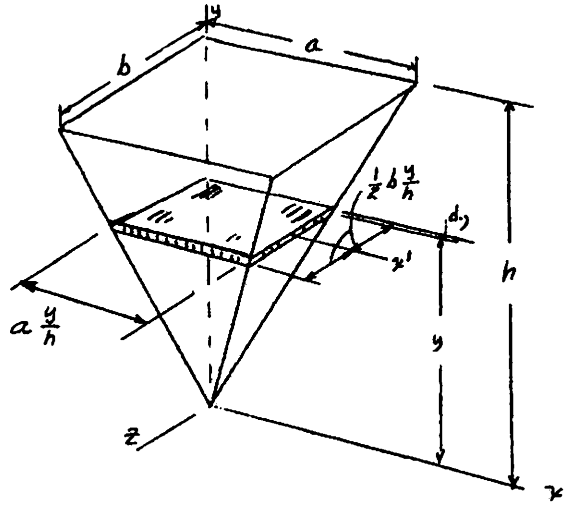
or $I_y = m(3a^2 + 0.1204h^2)$ ◀



PROBLEM 9.122

Determine by direct integration the mass moment of inertia with respect to the *x* axis of the pyramid shown, assuming that it has a uniform density and a mass *m*.

SOLUTION



For element shown:

$$dm = \rho dV = \rho\left(a\frac{y}{h}\right)\left(b\frac{y}{h}\right)dy = \rho\frac{ab}{h^2}y^2dy$$

$$dI_{x'} = \frac{1}{12}\left(b\frac{y}{h}\right)^2 dm = \frac{1}{12}\frac{b^2}{h^2}y^2\left(\rho\frac{ab}{h^2}\right)y^2dy = \frac{1}{12}\rho\frac{ab^3}{h^4}y^4dy$$

Parallel-axis theorem

$$dI_x = dI_{x'} + d^2dm \qquad \text{where} \quad d^2 = y^2 + \left(\frac{1}{2}b\frac{y}{h}\right)^2$$

$$dI_x = \frac{1}{12}\rho\frac{ab^3}{h^4}y^4dy + \left[y^2 + \frac{1}{4}\frac{b^2}{h^2}y^2\right]\rho\frac{ab}{h^2}y^2dy$$

$$= \left(\frac{\rho}{3}\frac{ab^3}{h^4} + \rho\frac{ab}{h^2}\right)y^4dy$$

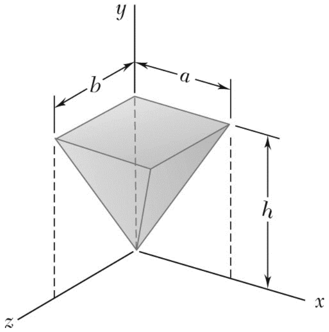
$$I_x = \int dI_x = \left(\frac{\rho}{3}\frac{ab^3}{h^4} + \rho\frac{ab}{h^2}\right)\int_0^h y^4dy = \frac{\rho ab^3h}{15} + \frac{\rho abh^3}{5}$$

PROBLEM 9.122 (Continued)

For pyramid, $m = \rho v = \frac{1}{3}\rho abh$

Thus: $I_x = \left(\frac{1}{3}\rho abh\right)\left(\frac{b^2}{5} + \frac{3h^2}{5}\right)$

$$I_x = \frac{1}{5}m(b^2 + 3h^2) \blacktriangleleft$$

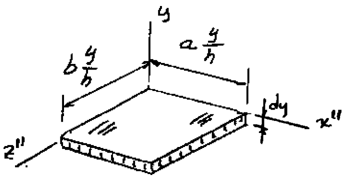


PROBLEM 9.123

Determine by direct integration the mass moment of inertia with respect to the *x* axis of the pyramid shown, assuming that it has a uniform density and a mass *m*.

SOLUTION

See figure of solution of Problem 9.122 for element shown



$$dm = \rho\left(b\frac{y}{h}\right)\left(a\frac{y}{h}\right)dy$$

$$dm = \rho\frac{ab}{h^2}y^2dy$$

For thin plate: $dI_y = dI_{x''} + dI_{z''}$

$$dI_y = \frac{1}{3}\left(b\frac{y}{h}\right)^2 dm + \frac{1}{3}\left(a\frac{y}{h}\right)^2 dm = \frac{1}{3h^2}(b^2+a^2)y^2dm$$

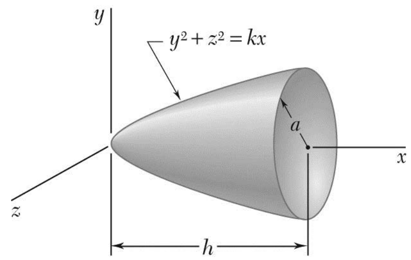
$$= \frac{1}{3h^2}(a^2+b^2)y^2\left(\rho\frac{ab}{h^2}y^2dy\right) = \frac{\rho ab}{3h^4}(a^2+b^2)y^4dy$$

$$I_y = \int dI_y = \frac{\rho ab}{3h^4}(a^2+b^2)\int_0^h y^4dy = \frac{\rho ab}{15}(a^2+b^2)h$$

For pyramid, $m = \rho v = \frac{1}{3}\rho abh$

$$I_y = \left(\frac{1}{3}\rho abh\right)\frac{1}{5}(a^2+b^2)$$

$$I_y = \frac{1}{5}m(a^2+b^2) \blacktriangleleft$$



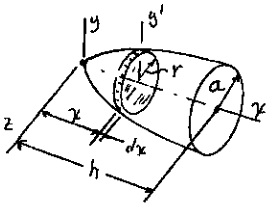
PROBLEM 9.124

Determine by direct integration the mass moment of inertia with respect to the y axis of the paraboloid shown, assuming that it has a uniform density and a mass m.

SOLUTION

$r^2 = y^2 + z^2 = kx$: at $x = h$, $r = a$; $a^2 = kh$; $k = \frac{a^2}{h}$

Thus: $r^2 = \frac{a^2}{h}x$



$$dm = \rho\pi r^2 dx = \rho\pi\frac{a^2}{h}xdy$$

$$m = \int_0^h dm = \rho\pi\frac{a^2}{h}\int_0^h xdx; \quad m = \frac{1}{2}\rho\pi a^2 h$$

$$dI_y = dI_{y'} + mx^2 = \frac{1}{4}r^2dm + x^2dm = \left(\frac{1}{4}\frac{a^2}{h}x + x^2\right)dm$$

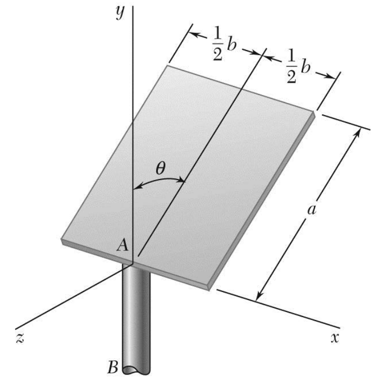
$$I_y = \int_0^a I_y = \int_0^a \left(\frac{1}{4}\frac{a^2}{h}x + x^2\right)\rho\pi\frac{a^2}{h}xdy = \rho\pi\frac{a^2}{h}\int_0^h\left(\frac{1}{4}\frac{a^2}{h}x^2 + x^3\right)dx$$

$$= \rho\pi\frac{a^2}{h}\left(\frac{1}{4}\frac{a^2}{h}\frac{h^3}{3} + \frac{h^4}{4}\right)$$

$$I_y = \frac{1}{12}\rho\pi a^2 h(a^2 + 3h^2) \ \triangleleft$$

Recall: $m = \frac{1}{2}\rho\pi a^2 h$; $I_y = \left(\frac{1}{2}\rho\pi a^2 h\right)\left(\frac{1}{6}\right)(a^2 + 3h^2)$

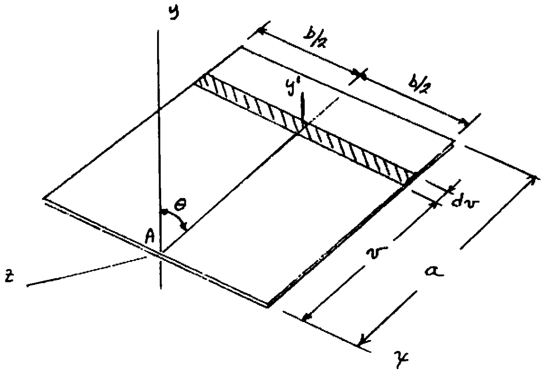
$$I_y = \frac{1}{6}m(a^2 + 3h^2) \ \blacktriangleleft$$



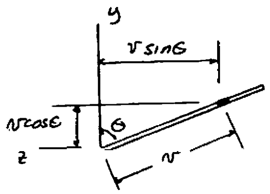
PROBLEM 9.125

A thin rectangular plate of mass *m* is welded to a vertical shaft *AB* as shown. Knowing that the plate forms an angle θ with the *y* axis, determine by direct integration the mass moment of inertia of the plate with respect to (*a*) the *y* axis, (*b*) the *z* axis.

SOLUTION



Projection on *yz* plane



Mass of plate: $m = \rho tab$

(*a*) For element shown: $dm = \rho btdv$

$$d\bar{I}_y = d\bar{I}_{y'} + (v\sin\theta)^2 dm = \frac{1}{12}b^2 dm + v^2\sin^2\theta dm$$

$$= \left(\frac{1}{12}b^2 + v^2\sin^2\theta\right)\rho btdv$$

$$I_y = \int dI_y = \rho bt\int_0^a \left(\frac{1}{12}b^2 + v^2\sin^2\theta\right)dv$$

$$= \rho bt\left|\frac{1}{12}b^2 v + \frac{v^3}{3}\sin\theta\right|_0^a = \rho bt\left(\frac{1}{12}ab^2 + \frac{a^3}{3}\sin^2\theta\right)$$

$$= (\rho tab)\frac{1}{12}(b^2 + 4a^2\sin^2\theta)$$

$$I_y = \frac{1}{12}m(b^2 + 4a^2\sin^2\theta) \blacktriangleleft$$

PROBLEM 9.125 (Continued)

(b)

$$dI_z = d\bar{I}_{z'} + (v\cos\theta)^2\,dm = \frac{1}{12}b^2dm + v^2\cos^2\theta dm$$

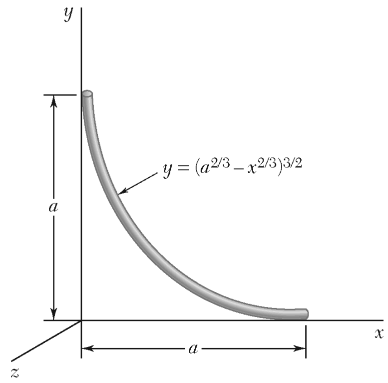
$$= \left(\frac{1}{12}b^2 + v^2\cos^2\theta\right)\rho btdv$$

$$I_z = \int dI_z = \rho bt\int_0^a\left(\frac{1}{12}b^2 + v^2\cos^2\theta\right)dv$$

$$= \rho bt\left|\frac{1}{12}b^2v + \frac{v^3}{3}\cos^2\theta\right|_0^a = \rho bt\left(\frac{1}{12}ab^2 + \frac{a^3}{3}\cos^2\theta\right)$$

$$= (\rho tab)\frac{1}{12}(b^2 + 4a^2\cos^2\theta)$$

$$I_z = \frac{1}{12}m(b^2 + 4a^2\cos^2\theta) \blacktriangleleft$$



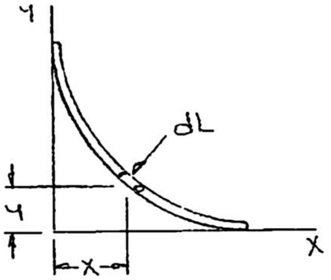
PROBLEM 9.126*

A thin steel wire is bent into the shape shown. Denoting the mass per unit length of the wire by m', determine by direct integration the mass moment of inertia of the wire with respect to each of the coordinate axes.

SOLUTION

First note
$$\frac{dy}{dx} = -x^{-1/3}(a^{2/3} - x^{2/3})^{1/2}$$

Then
$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + x^{-2/3}(a^{2/3} - x^{2/3})$$
$$= \left(\frac{a}{x}\right)^{2/3}$$



For the element shown:
$$dm = m'dL = m'\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$
$$= m'\left(\frac{a}{x}\right)^{1/3} dx$$

Then
$$m = \int dm = \int_0^a m'\frac{a^{1/3}}{x^{1/3}}dx = \frac{3}{2}m'a^{1/3}\left[x^{2/3}\right]_0^a = \frac{3}{2}m'a$$

Now
$$I_x = \int y^2 dm = \int_0^a (a^{2/3} - x^{2/3})^3\left(m'\frac{a^{1/3}}{x^{1/3}}dx\right)$$
$$= m'a^{1/3}\int_0^a\left(\frac{a^2}{x^{1/3}} - 3a^{4/3}x^{1/3} + 3a^{2/3}x - x^{5/3}\right)dx$$
$$= m'a^{1/3}\left[\frac{3}{2}a^2x^{2/3} - \frac{9}{4}a^{4/3}x^{4/3} + \frac{3}{2}a^{2/3}x^2 - \frac{3}{8}x^{8/3}\right]_0^a$$
$$= m'a^3\left(\frac{3}{2} - \frac{9}{4} + \frac{3}{2} - \frac{3}{8}\right) = \frac{3}{8}m'a^3$$

or $I_x = \frac{1}{4}ma^2$ ◀

Symmetry implies $I_y = \frac{1}{4}ma^2$ ◀

PROBLEM 9.126* (Continued)

Alternative solution:

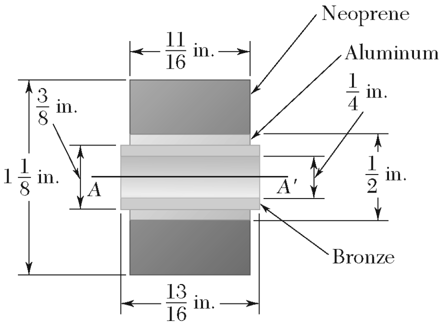
$$I_y = \int x^2 dm = \int_0^a x^2 \left(m' \frac{a^{1/3}}{x^{1/3}} dx \right) = m' a^{1/3} \int_0^a x^{5/3} dx$$

$$= m' a^{1/3} \times \frac{3}{8} \left[x^{8/3} \right]_0^a = \frac{3}{8} m' a^3$$

$$= \frac{1}{4} ma^2$$

Also $$I_z = \int (x^2 + y^2)\, dm = I_y + I_x$$

or $I_z = \dfrac{1}{2} ma^2$ ◀



PROBLEM 9.127

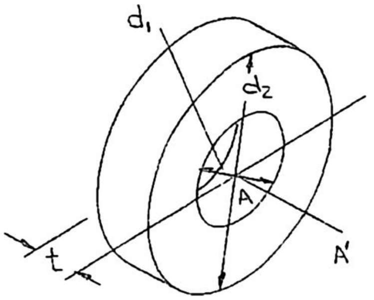
Shown is the cross section of an idler roller. Determine its mass moment of inertia and its radius of gyration with respect to the axis *AA′*. (The specific weight of bronze is 0.310 lb/in^3; of aluminum, 0.100 lb/in^3; and of neoprene, 0.0452 lb/in^3.)

SOLUTION

First note for the cylindrical ring shown that

$$m = \rho V = \rho t \times \frac{\pi}{4}\left(d_2^2 - d_1^2\right) = \frac{\pi}{4}\rho t\left(d_2^2 - d_1^2\right)$$

and, using Figure 9.28, that



$$\begin{aligned}
I_{AA'} &= \frac{1}{2}m_2\left(\frac{d_2}{2}\right)^2 - \frac{1}{2}m_1\left(\frac{d_1}{2}\right)^2 \\
&= \frac{1}{8}\left[\left(\rho t \times \frac{\pi}{4}d_2^2\right)d_2^2 - \left(\rho t \times \frac{\pi}{4}d_1^2\right)d_1^2\right] \\
&= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^4 - d_1^4\right) \\
&= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^2 - d_1^2\right)\left(d_2^2 + d_1^2\right) \\
&= \frac{1}{8}m\left(d_1^2 + d_2^2\right)
\end{aligned}$$

Now treat the roller as three concentric rings and, working from the bronze outward, we have

$$\begin{aligned}
m = \frac{\pi}{4} \times \frac{1}{32.2}\,\text{ft/s}^2 \Bigg\{ &(0.310\ \text{lb/in}^3)\left(\frac{13}{16}\text{in.}\right)\left[\left(\frac{3}{8}\right)^2 - \left(\frac{1}{4}\right)^2\right]\text{in}^2 \\
&+ (0.100\ \text{lb/in}^3)\left(\frac{11}{16}\ \text{in.}\right)\left[\left(\frac{1}{2}\right)^2 - \left(\frac{3}{8}\right)^2\right]\text{in}^2 \\
&+ (0.0452\ \text{lb/in}^3)\left(\frac{11}{16}\text{in.}\right)\left[\left(1\frac{1}{8}\right)^2 - \left(\frac{1}{2}\right)^2\right]\text{in}^2 \Bigg\} \\
= {} &(479.96 + 183.41 + 769.80) \times 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft} \\
= {} &1.4332 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}
\end{aligned}$$

PROBLEM 9.127 (Continued)

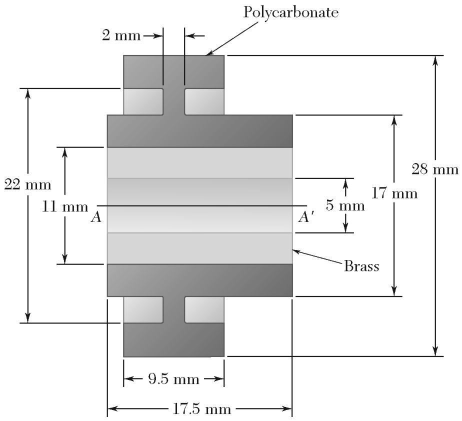
and
$$I_{AA'} = \frac{1}{8}\left\{(479.96)\left[\left(\frac{1}{4}\right)^2 + \left(\frac{3}{8}\right)^2\right] + (183.41)\left[\left(\frac{3}{8}\right)^2 + \left(\frac{1}{2}\right)^2\right]\right.$$
$$\left. + (769.80)\left[\left(1\frac{1}{8}\right)^2 + \left(\frac{1}{2}\right)^2\right]\right\} \times 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft} \times \text{in}^2 \times \frac{1\ \text{ft}^2}{144\ \text{in}^2}$$
$$= (84.628 + 62.191 + 1012.78) \times 10^{-9}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$
$$= 1.15960 \times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{AA'} = 1.160 \times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

Now
$$k_{AA'}^2 = \frac{I_{AA'}}{m} = \frac{1.15960 \times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2}{1.4332 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}} = 809.09 \times 10^{-6}\ \text{ft}^2$$

Then
$$k_{AA'} = 28.445 \times 10^{-3}\ \text{ft}$$

or $k_{AA'} = 0.341\ \text{in.}$ ◀



PROBLEM 9.128

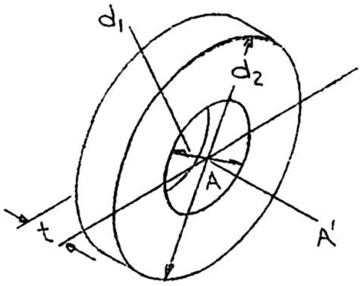
Shown is the cross section of a molded flat-belt pulley. Determine its mass moment of inertia and its radius of gyration with respect to the axis AA'. (The density of brass is 8650 kg/m^3 and the density of the fiber-reinforced polycarbonate used is 1250 kg/m^3.)

SOLUTION

First note for the cylindrical ring shown that

$$m = \rho V = \rho t \times \frac{\pi}{4}\left(d_2^2 - d_1^2\right)$$

and, using Figure 9.28, that



$$\begin{aligned}
I_{AA'} &= \frac{1}{2}m_2\left(\frac{d_2}{2}\right)^2 - \frac{1}{2}m_1\left(\frac{d_1}{2}\right)^2 \\
&= \frac{1}{8}\left[\left(\rho t \times \frac{\pi}{4}d_2^2\right)d_2^2 - \left(\rho t \times \frac{\pi}{4}d_1^2\right)d_1^2\right] \\
&= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^4 - d_1^4\right) \\
&= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^2 - d_1^2\right)\left(d_2^2 + d_1^2\right) \\
&= \frac{1}{8}m\left(d_1^2 + d_2^2\right)
\end{aligned}$$

Now treat the pulley as four concentric rings and, working from the brass outward, we have

$$\begin{aligned}
m = \frac{\pi}{4}\Big\{&8650\text{ kg/m}^3 \times (0.0175\text{ m}) \times (0.011^2 - 0.005^2)\text{ m}^2 \\
&+ 1250\text{ kg/m}^3[(0.0175\text{ m}) \times (0.017^2 - 0.011^2)\text{ m}^2 \\
&+ (0.002\text{ m}) \times (0.022^2 - 0.017^2)\text{ m}^2 \\
&+ (0.0095\text{ m}) \times (0.028^2 - 0.022^2)\text{ m}^2]\Big\} \\
= {} & (11.4134 + 2.8863 + 0.38288 + 2.7980) \times 10^{-3}\text{ kg} \\
= {} & 17.4806 \times 10^{-3}\text{ kg}
\end{aligned}$$

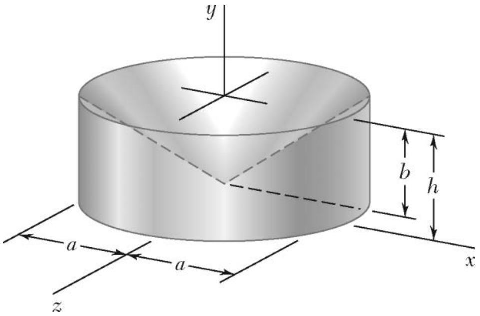
PROBLEM 9.128 (Continued)

and
$$I_{AA'} = \frac{1}{8}[(11.4134)(0.005^2 + 0.011^2) + (2.8863)(0.011^2 + 0.017^2)$$
$$+ (0.38288)(0.017^2 + 0.022^2)$$
$$+ (2.7980)(0.022^2 + 0.028^2)] \times 10^{-3}\ \text{kg} \times \text{m}^2$$
$$= (208.29 + 147.92 + 37.00 + 443.48) \times 10^{-9}\ \text{kg} \cdot \text{m}^2$$
$$= 836.69 \times 10^{-9}\ \text{kg} \cdot \text{m}^2$$

or $I_{AA'} = 837 \times 10^{-9}\ \text{kg} \cdot \text{m}^2$ ◀

Now
$$k_{AA'}^2 = \frac{I_{AA'}}{m} = \frac{836.69 \times 10^{-9}\ \text{kg} \cdot \text{m}^2}{17.4806 \times 10^{-3}\ \text{kg}} = 47.864 \times 10^{-6}\ \text{m}^2$$

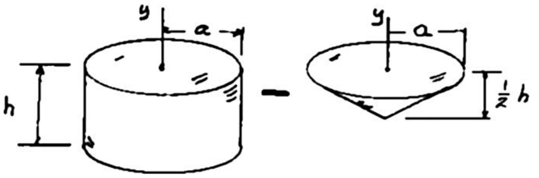
or $k_{AA'} = 6.92\ \text{mm}$ ◀



PROBLEM 9.129

The machine part shown is formed by machining a conical surface into a circular cylinder. For $b = \frac{1}{2}h$, determine the mass moment of inertia and the radius of gyration of the machine part with respect to the y axis.

SOLUTION



Mass: $$m_{\text{cyl}} = \rho\pi a^2 h \qquad m_{\text{cone}} = \frac{1}{3}\rho\pi a^2 \frac{h}{2} = \frac{1}{6}\rho\pi a^2 h$$

$$I_y:\quad I_{\text{cyl}} = \frac{1}{2}m_{\text{cyl}}a^2 \qquad I_{\text{cone}} = \frac{3}{10}m_{\text{cone}}a^2$$

$$= \frac{1}{2}\rho\pi a^4 h \qquad = \frac{1}{20}\rho\pi a^4 h$$

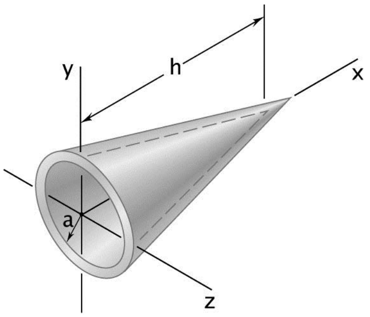
For entire machine part:

$$m = m_{\text{cyl}} - m_{\text{cone}} = \rho\pi a^2 h - \frac{1}{6}\rho\pi a^2 h = \frac{5}{6}\rho\pi a^2 h$$

$$I_y = I_{\text{cyl}} - I_{\text{cone}} = \frac{1}{2}\rho\pi a^4 h - \frac{1}{20}\rho\pi a^4 h = \frac{9}{20}\rho\pi a^4 h$$

or $$I_y = \left(\frac{5}{6}\rho\pi a^2 h\right)\left(\frac{6}{5}\right)\left(\frac{9}{20}\right)a^2 \qquad I_y = \frac{27}{50}ma^2 \blacktriangleleft$$

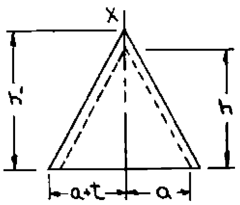
Then $$k_y^2 = \frac{I}{m} = \frac{27}{50}a^2 \qquad k_y = 0.735a \blacktriangleleft$$



PROBLEM 9.130

Given the dimensions and the mass m of the thin conical shell shown, determine the mass moment of inertia and the radius of gyration of the shell with respect to the x axis. (*Hint:* Assume that the shell was formed by removing a cone with a circular base of radius a from a cone with a circular base of radius $a + t$, where t is the thickness of the wall. In the resulting expressions, neglect terms containing t^2, t^3, etc. Do not forget to account for the difference in the heights of the two cones.)

SOLUTION



First note $$\frac{h}{a+t}=\frac{h}{a}$$

or $$h'=\frac{h}{a}(a+t)$$

For a cone of height H whose base has a radius r, have

$$I_x=\frac{3}{10}mr^2$$

$$m=\rho V=\rho\times\frac{\pi}{3}r^2H$$

Then $$I_x=\frac{3}{10}\left(\frac{\pi}{3}\rho r^2H\right)r^2$$

$$=\frac{\pi}{10}\rho r^4H$$

Now following the hint have

$$m_{\text{shell}}=m_{\text{outer}}-m_{\text{inner}}=\frac{\pi}{3}\rho[(a+t)^2h'-a^2h]$$

$$=\frac{\pi}{3}\rho\left[(a+t)^2=\frac{b}{a}(a+t)-a^2h\right]$$

$$=\frac{\pi}{3}\rho a^2h\left[\left(1+\frac{t}{a}\right)^3-1\right]=\frac{\pi}{3}\rho a^2h\left(1+3\frac{t}{a}+\cdots-1\right)$$

Neglecting the t^2 and t^3 terms obtain $m_{\text{shell}}=\pi\rho aht$

PROBLEM 9.130 (Continued)

Also
$$(I_x)_{\text{shell}} = (I_x)_{\text{outer}} - (I_x)_{\text{inner}}$$
$$= \frac{\pi}{10}\rho[(a+t)^4 h' - a^4 h]$$
$$= \frac{\pi}{10}\rho\left[(a+t)^4 \times \frac{h}{a}(a+t) - a^4 h\right]$$
$$= \frac{\pi}{10}\rho a^4 h\left[\left(1+\frac{t}{a}\right)^5 - 1\right] = \frac{\pi}{10}\rho a^4 h\left(1 + 5\frac{t}{a} + \cdots - 1\right)$$

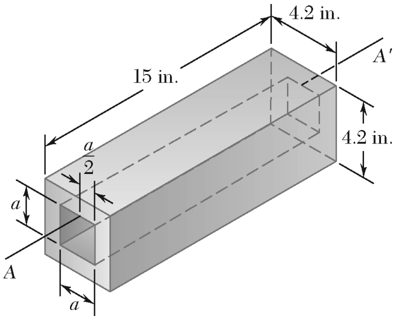
Neglecting t^2 and higher order terms, obtain

$$(I_x)_{\text{shell}} = \frac{\pi}{2}\rho a^3 ht$$

Now
$$k_x^2 = \frac{I_x}{m} = \frac{\frac{1}{2}ma^2}{m}$$

or $I_x = \dfrac{1}{2}ma^2$ ◀

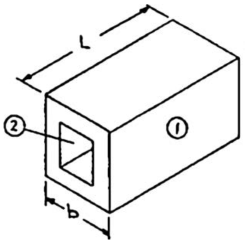
or $k_x = \dfrac{a}{\sqrt{2}}$ ◀



PROBLEM 9.131

A square hole is centered in and extends through the aluminum machine component shown. Determine (*a*) the value of *a* for which the mass moment of inertia of the component with respect to the axis *AA*′, which bisects the top surface of the hole, is maximum, (*b*) the corresponding values of the mass moment of inertia and the radius of gyration with respect to the axis *AA*′. (The specific weight of aluminum is 0.100 lb/in^3.)

SOLUTION



First note $$m_1 = \rho V_1 = \rho b^2 L$$

and $$m_2 = \rho V_2 = \rho a^2 L$$

(*a*) Using Figure 9.28 and the parallel-axis theorem, we have

$$\begin{aligned}
I_{AA'} &= (I_{AA'})_1 - (I_{AA'})_2 \\
&= \left[\frac{1}{12}m_1(b^2+b^2) + m_1\left(\frac{a}{2}\right)^2\right] \\
&\quad - \left[\frac{1}{12}m_2(a^2+a^2) + m_2\left(\frac{a}{2}\right)^2\right] \\
&= (\rho b^2 L)\left(\frac{1}{6}b^2 + \frac{1}{4}a^2\right) - (\rho a^2 L)\left(\frac{5}{12}a^2\right) \\
&= \frac{\rho L}{12}(2b^4 + 3b^2a^2 - 5a^4)
\end{aligned}$$

Then $$\frac{dI_{AA'}}{da} = \frac{\rho L}{12}(6b^2 a - 20a^3) = 0$$

or $$a = 0 \qquad \text{and} \qquad a = b\sqrt{\frac{3}{10}}$$

Also $$\frac{d^2 I_{AA'}}{da^2} = \frac{\rho L}{12}(6b^2 - 60a^2) = \frac{1}{2}\rho L(b^2 - 10a^2)$$

Now for $a = 0$, $$\frac{d^2 I_{AA'}}{da^2} > 0$$

and for $a = b\sqrt{\frac{3}{10}}$, $$\frac{d^2 I_{AA'}}{da^2} < 0$$

PROBLEM 9.131 (Continued)

$(I_{AA'})_{\max}$ occurs when $$a = b\sqrt{\frac{3}{10}}$$

Then $$a = (4.2 \text{ in.})\sqrt{\frac{3}{10}}$$

or $a = 2.30$ in. ◀

(*b*) From part *a*:

$$(I_{AA'})_{\max} = \frac{\rho L}{12}\left[2b^4 + 3b^2\left(b\sqrt{\frac{3}{10}}\right)^2 - 5\left(b\sqrt{\frac{3}{10}}\right)^4\right] = \frac{49}{240}\rho L b^4$$

$$= \frac{49}{240}\frac{\gamma_{AL}}{g}Lb^4 = \frac{49}{240} \times \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times (15 \text{ in.})(4.2 \text{ in.})^4\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

or $(I_{AA'})_{\max} = 20.6 \times 10^{-3}$ lb · ft · s^2 ◀

Now $$k_{AA'}^2 = \frac{(I_{AA'})_{\max}}{m}$$

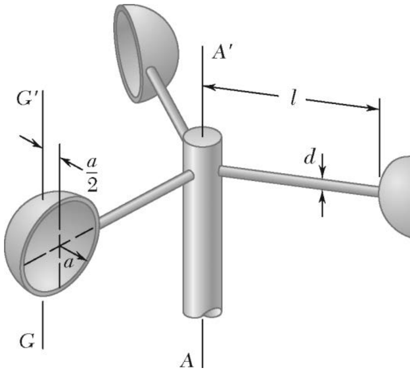
where $$m = m_1 - m_2 = \rho L(b^2 - a^2)$$

$$= \rho L\left[b^2 - \left(b\sqrt{\frac{3}{10}}\right)^2\right]$$

$$= \frac{7}{10}\rho L b^2$$

Then $$k_{AA'}^2 = \frac{\frac{49}{240}\rho L b^4}{\frac{7}{10}\rho L b^2} = \frac{7}{24}b^2 = \frac{7}{24}(4.2 \text{ in.})^2$$

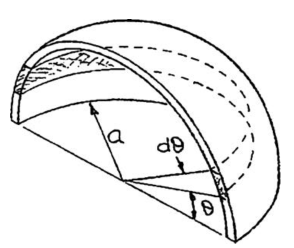
or $k_{AA'} = 2.27$ in. ◀



PROBLEM 9.132

The cups and the arms of an anemometer are fabricated from a material of density ρ. Knowing that the mass moment of inertia of a thin, hemispherical shell of mass m and thickness t with respect to its centroidal axis GG' is $5ma^2/12$, determine (*a*) the mass moment of inertia of the anemometer with respect to the axis AA', (*b*) the ratio of a to l for which the centroidal moment of inertia of the cups is equal to 1 percent of the moment of inertia of the cups with respect to the axis AA'.

SOLUTION



(*a*) First note $$m_{\text{arm}} = \rho V_{\text{arm}} = \rho \times \frac{\pi}{4} d^2 l$$

and $$dm_{\text{cup}} = \rho dV_{\text{cup}}$$
$$= \rho[(2\pi a\cos\theta)(t)(a d\theta)]$$

Then $$m_{\text{cup}} = \int dm_{\text{cup}} = \int_0^{\pi/2} 2\pi\rho a^2 t\cos\theta d\theta$$
$$= 2\pi\rho a^2 t[\sin\theta]_0^{\pi/2}$$
$$= 2\pi\rho a^2 t$$

Now $$(I_{AA'})_{\text{anem}} = (I_{AA'})_{\text{cups}} + (I_{AA'})_{\text{arms}}$$

Using the parallel-axis theorem and assuming the arms are slender rods, we have

$$(I_{AA'})_{\text{anem}} = 3\left[(I_{GG'})_{\text{cup}} + m_{\text{cup}} d_{AG}^2\right] + 3\left[\bar{I}_{\text{arm}} + m_{\text{arm}} d_{AG_{\text{arm}}}\right]$$
$$= 3\left\{\frac{5}{12} m_{\text{cup}} a^2 + m_{\text{cup}}\left[(l+a)^2 + \left(\frac{a}{2}\right)^2\right]\right\} + 3\left[\frac{1}{2} m_{\text{arm}} l^2 + m_{\text{arm}}\left(\frac{l}{2}\right)^2\right]$$
$$= 3m_{\text{cup}}\left(\frac{5}{3}a^2 + 2la + l^2\right) + m_{\text{arm}} l^2$$
$$= 3(2\pi\rho a^2 t)\left(\frac{5}{3}a^2 + 2la + l^2\right) + \left(\frac{\pi}{4}\rho d^2 l\right)(l^2)$$

or $$(I_{AA'})_{\text{anem}} = \pi\rho l^2\left[6a^2 t\left(\frac{5}{3}\frac{a^2}{l^2} + 2\frac{a}{l} + 1\right) + \frac{d^2 l}{4}\right] \blacktriangleleft$$

(*b*) We have
$$\frac{(I_{GG'})_{\text{cup}}}{(I_{AA'})_{\text{cup}}} = 0.01$$

or
$$\frac{5}{12}m_{\text{cup}}a^2 = 0.01m_{\text{cup}}\left(\frac{5}{3}a^2 + 2la + l^2\right) \quad \text{(from part } a\text{)}$$

Now let $\zeta = \frac{a}{l}$.

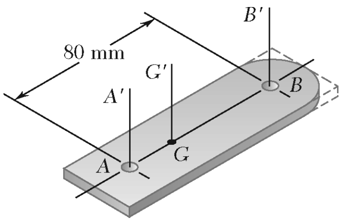
Then
$$5\zeta^2 = 0.12\left(\frac{5}{3}\zeta^2 + 2\zeta + 1\right)$$

or
$$40\zeta^2 - 2\zeta - 1 = 0$$

Then
$$\zeta = \frac{2 \pm \sqrt{(-2)^2 - 4(40)(-1)}}{2(40)}$$

or
$$\zeta = 0.1851 \quad \text{and} \quad \zeta = -0.1351$$

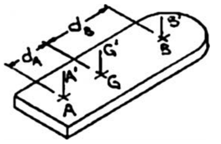
$$\frac{a}{l} = 0.1851 \blacktriangleleft$$



PROBLEM 9.133

After a period of use, one of the blades of a shredder has been worn to the shape shown and is of mass 0.18 kg. Knowing that the mass moments of inertia of the blade with respect to the *AA′* and *BB′* axes are $0.320\,\text{g}\cdot\text{m}^2$ and $0.680\,\text{g}\cdot\text{m}^2$, respectively, determine (*a*) the location of the centroidal axis *GG′*, (*b*) the radius of gyration with respect to axis *GG′*.

SOLUTION



(*a*) We have $d_B = (0.08 - d_A)\text{ m}$

and, using the parallel axis

$$I_{AA'} = \bar{I}_{GG'} + md_A^2$$
$$I_{BB'} = \bar{I}_{GG'} + md_B^2$$

Then

$$I_{BB'} - I_{AA'} = m\left(d_B^2 - d_A^2\right)$$
$$= m\left[(0.08 - d_A)^2 - d_A^2\right]$$
$$= m(0.0064 - 0.16d_A)$$

Substituting:

$$(0.68 - 0.32)\times 10^{-3}\text{ kg}\cdot\text{m}^2 = 0.18\text{ kg}(0.0064 - 0.16d_A)\text{ m}^2$$

or $d_A = 27.5\text{ mm}$ ◀

to the right of *A*

(*b*) We have $I_{AA'} = I_{GG'} + md_A^2$

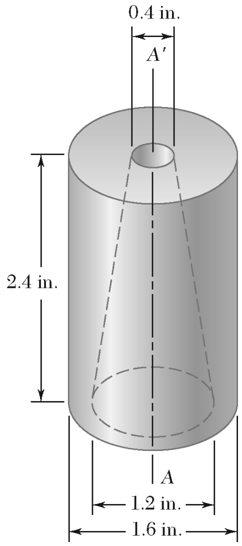
or

$$I_{GG'} = 0.32\times 10^{-3}\text{ kg}\cdot\text{m}^2 - 0.18\text{ kg}\cdot(0.0275\text{ m})^2$$
$$= 0.183875\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

Then

$$k_{GG'}^2 = \frac{I_{GG'}}{m} = \frac{0.183875\times 10^{-3}\text{ kg}\cdot\text{m}^2}{0.18\text{ kg}} = 1.02153\times 10^{-3}\text{ m}^2$$

or $k_{GG'} = 32.0\text{ mm}$ ◀

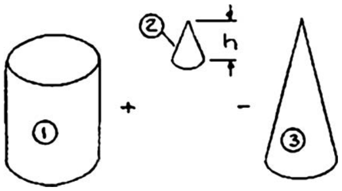


PROBLEM 9.134

Determine the mass moment of inertia of the 0.9-lb machine component shown with respect to the axis *AA′*.

SOLUTION

First note that the given shape can be formed adding a small cone to a cylinder and then removing a larger cone as indicated.



Now $$\frac{h}{0.4}=\frac{h+2.4}{1.2} \quad \text{or} \quad h=1.2 \text{ in.}$$

The weight of the body is given by

$$W=mg=g(m_1+m_2-m_3)=\rho g(V_1+V_2-V_3)$$

or $$0.9 \text{ lb}=\rho\times 32.2 \text{ ft/s}^2$$

$$\left[\pi(0.8)^2(2.4)+\frac{\pi}{3}(0.2)^2(1.2)-\frac{\pi}{3}(0.6)^2(3.6)\right] \text{in}^3\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$

$$=\rho\times 32.2 \text{ ft/s}^2(2.79253+0.02909-0.78540)\times 10^{-3}\text{ ft}^3$$

or $$\rho=13.7266 \text{ lb}\cdot\text{s}^2/\text{ft}^4$$

Then $$m_1=(13.7266)(2.79253\times 10^{-3})=0.038332 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_2=(13.7266)(0.02909\times 10^{-3})=0.000399 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_3=(13.7266)(0.78540\times 10^{-3})=0.010781 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Finally, using Figure 9.28, we have

$$
\begin{aligned}
I_{AA'} &= (I_{AA'})_1 + (I_{AA'})_2 - (I_{AA'})_3 \\
&= \frac{1}{2}m_1a_1^2 + \frac{3}{10}m_2a_2^2 - \frac{3}{10}m_3a_3^2 \\
&= \left[\frac{1}{2}(0.038332)(0.8)^2 + \frac{3}{10}(0.000399)(0.2)^2 - \frac{3}{10}(0.010781)(0.6)^2\right](\text{lb}\cdot\text{s}^2/\text{ft})\times\text{in}^2\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2 \\
&= (85.1822 + 0.0333 - 8.0858)\times 10^{-6}\,\text{lb}\cdot\text{ft}\cdot\text{s}^2
\end{aligned}
$$

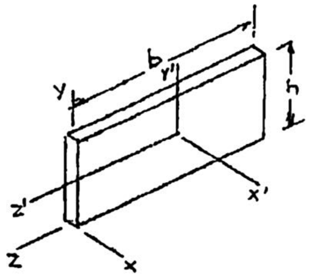
or $I_{AA'} = 77.1\times 10^{-6}\,\text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

To the instructor:

The following formulas for the mass moment of inertia of thin plates and a half cylindrical shell are derived at this time for use in the solutions of Problems 9.135 through 9.140.

Thin rectangular plate

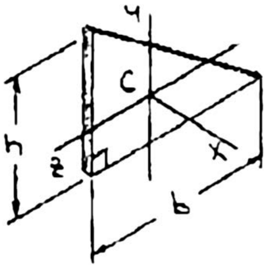
$$\begin{aligned}(I_x)_m &= (\bar{I}_{x'})_m + md^2\\ &= \frac{1}{12}m(b^2+h^2) + m\left[\left(\frac{b}{2}\right)^2 + \left(\frac{h}{2}\right)^2\right]\\ &= \frac{1}{3}m(b^2+h^2)\end{aligned}$$



$$\begin{aligned}(I_y)_m &= (\bar{I}_{y'})_m + md^2\\ &= \frac{1}{12}mb^2 + m\left(\frac{b}{2}\right)^2 = \frac{1}{3}mb^2\end{aligned}$$

$$\begin{aligned}I_z &= (\bar{I}_{z'})_m + md^2\\ &= \frac{1}{12}mh^2 + m\left(\frac{h}{2}\right)^2 = \frac{1}{3}mh^2\end{aligned}$$

Thin triangular plate



We have $$m = \rho V = \rho\left(\frac{1}{2}bht\right)$$

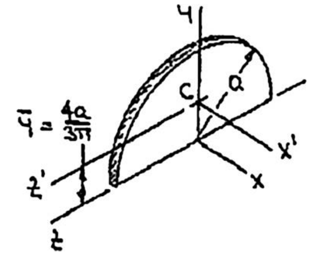
and $$\bar{I}_{z,\text{area}} = \frac{1}{36}bh^3$$

Then $$\begin{aligned}\bar{I}_{z,\text{mass}} &= \rho t I_{z,\text{area}}\\ &= \rho t \times \frac{1}{36}bh^3\\ &= \frac{1}{18}mh^2\end{aligned}$$

Similarly, $$\bar{I}_{y,\text{mass}} = \frac{1}{18}mb^2$$

Now $$\bar{I}_{x,\text{mass}} = \bar{I}_{y,\text{mass}} + \bar{I}_{z,\text{mass}} = \frac{1}{18}m(b^2+h^2)$$

Thin semicircular plate



We have $$m = \rho V = \rho\left(\frac{\pi}{2}a^2 t\right)$$

and $$\bar{I}_{y,\text{area}} = I_{z,\text{area}} = \frac{\pi}{8}a^4$$

(Continued)

Then $$\bar{I}_{y,\text{mass}} = I_{z,\text{mass}} = \rho t \bar{I}_{y,\text{area}}$$

$$= \rho t \times \frac{\pi}{8} a^4$$

$$= \frac{1}{4} m a^2$$

Now $$I_{x,\text{mass}} = \bar{I}_{y,\text{mass}} + I_{z,\text{mass}} = \frac{1}{2} m a^2$$

Also $$I_{x,\text{mass}} = \bar{I}_{x',\text{mass}} + m\bar{y}^2 \quad \text{or} \quad \bar{I}_{x',\text{mass}} = m\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)a^2$$

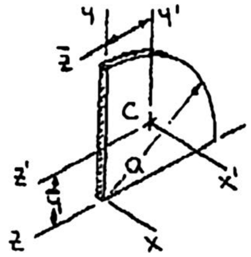
and $$I_{z,\text{mass}} = \bar{I}_{z',\text{mass}} + m\bar{y}^2 \quad \text{or} \quad \bar{I}_{z',\text{mass}} = m\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2$$

$$\bar{y} = \bar{z} = \frac{4a}{3\pi}$$

Thin Quarter-Circular Plate

We have $$m = \rho V = \rho\left(\frac{\pi}{4} a^2 t\right)$$

and $$I_{y,\text{area}} = I_{z,\text{area}} = \frac{\pi}{16} a^4$$



Then $$I_{y,\text{mass}} = I_{z,\text{mass}} = \rho t I_{y,\text{area}}$$

$$= \rho t \times \frac{\pi}{16} a^4$$

$$= \frac{1}{4} m a^2$$

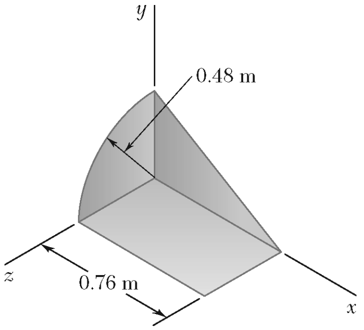
Now $$I_{x,\text{mass}} = I_{y,\text{mass}} + I_{z,\text{mass}} = \frac{1}{2} m a^2$$

Also $$I_{x,\text{mass}} = I_{x',\text{mass}} + m(\bar{y}^2 + \bar{z}^2)$$

or $$\bar{I}_{x',\text{mass}} = m\left(\frac{1}{2} - \frac{32}{9\pi^2}\right)a^2$$

and $$I_{y,\text{mass}} = I_{y',\text{mass}} + m\bar{z}^2$$

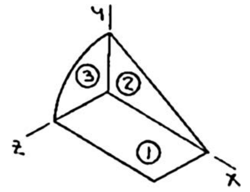
or $$\bar{I}_{y',\text{mass}} = m\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2$$



PROBLEM 9.135

A 2-mm thick piece of sheet steel is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} t A$$

Then

$$m_1 = (7850\ \text{kg/m}^3)(0.002\ \text{m})(0.76 \times 0.48)\ \text{m}^2$$
$$= 5.72736\ \text{kg}$$

$$m_2 = (7850\ \text{kg/m}^3)(0.002\ \text{m})\left(\frac{1}{2} \times 0.76 \times 0.48\right)\text{m}^2$$
$$= 2.86368\ \text{kg}$$

$$m_3 = (7850\ \text{kg/m}^3)(0.002\ \text{m})\left(\frac{\pi}{4} \times 0.48^2\right)\text{m}^2$$
$$= 2.84101\ \text{kg}$$

Using Figure 9.28 for component 1 and the equations derived above for components 2 and 3, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$
$$= \left[\frac{1}{12}(5.72736\ \text{kg})(0.48\ \text{m})^2 + (5.72736\ \text{kg})\left(\frac{0.48}{2}\ \text{m}\right)^2\right]$$
$$+ \left[\frac{1}{18}(2.86368\ \text{kg})(0.48\ \text{m})^2 + (2.86368\ \text{kg})\left(\frac{0.48}{3}\ \text{m}\right)^2\right] + \left[\frac{1}{2}(2.84101\ \text{kg})(0.48\ \text{m})^2\right]$$
$$= [(0.109965 + 0.329896) + (0.036655 + 0.073310) + (0.327284)]\ \text{kg} \cdot \text{m}^2$$
$$= (0.439861 + 0.109965 + 0.327284)\ \text{kg} \cdot \text{m}^2$$

or $I_x = 0.877\ \text{kg} \cdot \text{m}^2$ ◀

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

$$= \left\{\frac{1}{12}(5.72736\text{ kg})(0.76^2 + 0.48^2)\text{ m}^2 + (5.72736\text{ kg})\left[\left(\frac{0.76}{2}\right)^2 + \left(\frac{0.48}{2}\right)^2\right]\text{m}^2\right\}$$

$$+\left[\frac{1}{18}(2.86368\text{ kg})(0.76\text{ m})^2 + (2.86368\text{ kg})\left(\frac{0.76}{3}\text{ m}\right)^2\right] + \left[\frac{1}{4}(2.84101\text{ kg})(0.48\text{ m})^2\right]$$

$$= [(0.385642 + 1.156927) + (0.091892 + 0.183785) + (0.163642)]\text{ kg}\cdot\text{m}^2$$

$$= (1.542590 + 0.275677 + 0.163642)\text{ kg}\cdot\text{m}^2$$

or $I_y = 1.982\text{ kg}\cdot\text{m}^2$ ◀

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3$$

$$= \left[\frac{1}{12}(5.72736\text{ kg})(0.76\text{ m})^2 + (5.72736\text{ kg})\left(\frac{0.76}{2}\text{ m}\right)^2\right]$$

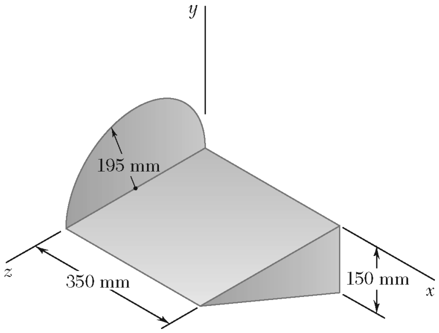
$$+\left\{\frac{1}{18}(2.86368\text{ kg})(0.76^2 + 0.48^2)\text{ m}^2 + (2.86368\text{ kg})\left[\left(\frac{0.76}{3}\right)^2 + \left(\frac{0.48}{3}\right)^2\right]\text{m}^2\right\}$$

$$+\left[\frac{1}{4}(2.84101\text{ kg})(0.48\text{ m})^2\right]$$

$$I_z = [(0.275677 + 0.827031) + (0.128548 + 0.257095) + (0.163642)]\text{ kg}\cdot\text{m}^2$$

$$= (1.102708 + 0.385643 + 0.163642)\text{ kg}\cdot\text{m}^2$$

or $I_z = 1.652\text{ kg}\cdot\text{m}^2$ ◀



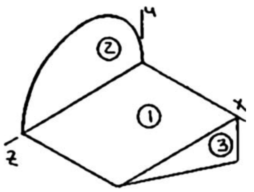
PROBLEM 9.136

A 2-mm thick piece of sheet steel is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION

First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} t A$$



Then

$$m_1 = (7850 \text{ kg/m}^3)(0.002 \text{ m})(0.35 \times 0.39) \text{ m}^2$$
$$= 2.14305 \text{ kg}$$

$$m_2 = (7850 \text{ kg/m}^3)(0.002 \text{ m})\left(\frac{\pi}{2} \times 0.195^2\right)\text{m}^2 = 0.93775 \text{ kg}$$

$$m_3 = (7850 \text{ kg/m}^3)(0.002 \text{ m})\left(\frac{1}{2} \times 0.39 \times 0.15\right)\text{m}^2 = 0.45923 \text{ kg}$$

Using Figure 9.28 for component 1 and the equations derived above for components 2 and 3, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$
$$= \left[\frac{1}{12}(2.14305 \text{ kg})(0.39 \text{ m})^2 + (2.14305 \text{ kg})\left(\frac{0.39}{2} \text{ m}\right)^2\right]$$
$$+ \left\{\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)(0.93775 \text{ kg})(0.195 \text{ m})^2 + (0.93775 \text{ kg})\left[\left(\frac{4 \times 0.195}{3\pi}\right)^2 + (0.195)^2\right]\text{m}^2\right\}$$
$$+ \left\{\frac{1}{18}(0.45923 \text{ kg})[(0.39)^2 + (0.15)^2] \text{ m}^2 + (0.45923 \text{ kg})\left[\left(\frac{0.39}{3}\right)^2 + \left(\frac{0.15}{3}\right)^2\right]\text{m}^2\right\}$$
$$= [(0.027163 + 0.081489) + (0.011406 + 0.042081) + (0.004455 + 0.008909)] \text{ kg} \cdot \text{m}^2$$
$$= (0.108652 + 0.053487 + 0.013364) \text{ kg} \cdot \text{m}^2$$
$$= 0.175503 \text{ kg} \cdot \text{m}^2$$

or $I_x = 175.5 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀

PROBLEM 9.136 (Continued)

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

$$= \left\{\frac{1}{12}(2.14305\text{ kg})[(0.35)^2 + (0.39)^2]\text{ m}^2 + (2.14305\text{ kg})\left[\left(\frac{0.35}{2}\right)^2 + \left(\frac{0.39}{2}\right)^2\right]\text{m}^2\right\}$$

$$+\left[\frac{1}{4}(0.93775\text{ kg})(0.195\text{ m})^2 + (0.93775\text{ kg})(0.195\text{ m})^2\right]$$

$$+\left\{\frac{1}{18}(0.45923\text{ kg})(0.39\text{ m})^2 + (0.45923\text{ kg})\left[(0.35)^2 + \left(\frac{0.39}{3}\right)^2\right]\text{m}^2\right\}$$

$$= [(0.049040 + 0.147120) + (0.008914 + 0.035658)$$

$$+ (0.003880 + 0.064017)]\text{ kg}\cdot\text{m}^2$$

$$= (0.196160 + 0.044572 + 0.067897)\text{ kg}\cdot\text{m}^2$$

$$= 0.308629\text{ kg}\cdot\text{m}^2$$

or $I_y = 309\times10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3$$

$$= \left[\frac{1}{12}(2.14305\text{ kg})(0.35\text{ m})^2 + (2.14305\text{ kg})\left(\frac{0.35}{2}\text{ m}\right)^2\right]$$

$$+\left[\frac{1}{4}(0.93775\text{ kg})(0.195\text{ m})^2\right]$$

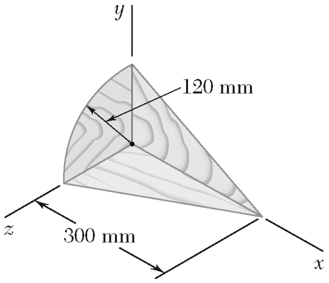
$$+\left\{\frac{1}{18}(0.45923\text{ kg})(0.15\text{ m})^2 + (0.45923\text{ kg})\left[(0.35)^2 + \left(\frac{0.15}{3}\right)^2\right]\text{m}^2\right\}$$

$$= [(0.021877 + 0.065631) + 0.008914) + (0.000574 + 0.057404)]\text{ kg}\cdot\text{m}^2$$

$$= (0.087508 + 0.008914 + 0.057978)\text{kg}\cdot\text{m}^2$$

$$= 0.154400\text{ kg}\cdot\text{m}^2$$

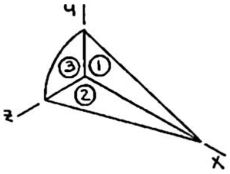
or $I_z = 154.4\times10^{-3}\text{ kg}\cdot\text{m}^2$ ◀



PROBLEM 9.137

A subassembly for a model airplane is fabricated from three pieces of 1.5-mm plywood. Neglecting the mass of the adhesive used to assemble the three pieces, determine the mass moment of inertia of the subassembly with respect to each of the coordinate axes. (The density of the plywood is $780\ \text{kg/m}^3$.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho V = \rho t A$$

Then

$$m_1 = m_2 = (780\ \text{kg/m}^3)(0.0015\ \text{m})\left(\frac{1}{2}\times 0.3\times 0.12\right)\text{m}^2$$

$$= 21.0600\times 10^{-3}\ \text{kg}$$

$$m_3 = (780\ \text{kg/m}^3)(0.0015\ \text{m})\left(\frac{\pi}{4}\times 0.12^2\right)\text{m}^2 = 13.2324\times 10^{-3}\ \text{kg}$$

Using the equations derived above and the parallel-axis theorem, we have

$$(I_x)_1 = (I_x)_2$$

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= 2\left[\frac{1}{18}(21.0600\times 10^{-3}\ \text{kg})(0.12\ \text{m})^2 + (21.0600\times 10^{-3}\ \text{kg})\left(\frac{0.12}{3}\ \text{m}\right)^2\right]$$

$$+\left[\frac{1}{2}(13.2324\times 10^{-3}\ \text{kg})(0.12\ \text{m})^2\right]$$

$$= [2(16.8480 + 33.6960) + (95.2733)]\times 10^{-6}\ \text{kg}\cdot\text{m}^2$$

$$= [2(50.5440) + (95.2733)]\times 10^{-6}\ \text{kg}\cdot\text{m}^2$$

or $\quad I_x = 196.4\times 10^{-6}\ \text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.137 (Continued)

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

$$= \left[\frac{1}{18}(21.0600\times10^{-3}\text{ kg})(0.3\text{ m})^2 + (21.0600\text{ kg})\left(\frac{0.3}{3}\text{m}\right)^2\right]$$

$$+ \left\{\frac{1}{18}(21.0600\times10^{-3}\text{ kg})[(0.3)^2 + (0.12)^2]\text{ m}^2\right.$$

$$\left.+ (21.0600\times10^{-3}\text{ kg})\left[\left(\frac{0.3}{3}\right)^2 + \left(\frac{0.12}{3}\right)^2\right]\text{m}^2\right\}$$

$$+ \left[\frac{1}{4}(13.2324\times10^{-3}\text{ kg})(0.12\text{ m})^2\right]$$

$$= [(105.300 + 210.600) + (122.148 + 244.296)$$

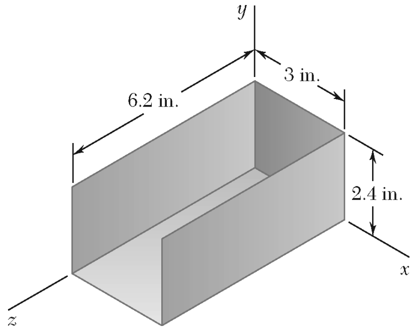
$$+ (47.637)]\times10^{-6}\text{ kg}\cdot\text{m}^2$$

$$= (315.900 + 366.444 + 47.637)\times10^{-6}\text{ kg}\cdot\text{m}^2$$

or $I_y = 730\times10^{-6}\text{ kg}\cdot\text{m}^2$ ◀

Symmetry implies $I_y = I_z$

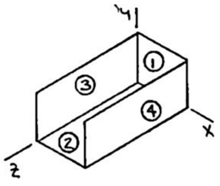
$I_z = 730\times10^{-6}\text{ kg}\cdot\text{m}^2$ ◀



PROBLEM 9.138

The cover for an electronic device is formed from sheet aluminum that is 0.05 in. thick. Determine the mass moment of inertia of the cover with respect to each of the coordinate axes. (The specific weight of aluminum is $0.100\ \text{lb/in}^3$.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho V = \frac{\gamma}{g} tA$$

Then

$$m_1 = \frac{0.100\ \text{lb/in}^3}{32.2\ \text{ft/s}^2} \times 0.05\ \text{in.} \times (3 \times\ 2.4)\ \text{in}^2 = 1.11801 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \frac{0.100\ \text{lb/in}^3}{32.2\ \text{ft/s}^2} \times 0.05\ \text{in.} \times (3 \times 6.2)\ \text{in}^2 = 2.88820 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = m_4 = \frac{0.100\ \text{lb/in}^3}{32.2\ \text{ft/s}^2} \times 0.05\ \text{in.} \times (2.4 \times 6.2)\ \text{in}^2 = 2.31056 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

Using Figure 9.28 and the parallel-axis theorem, we have

$$(I_x)_3 = (I_x)_4$$

$$\begin{aligned}
I_x &= (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 \\
&= \left[\frac{1}{12}(1.11801 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(2.4\ \text{in.})^2\right. \\
&\quad \left. + (1.11801 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{2.4}{2}\ \text{in.}\right)^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 \\
&\quad + \left[\frac{1}{12}(2.88820 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(6.2\ \text{in.})^2\right. \\
&\quad \left. + (2.88820 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{6.2}{2}\ \text{in.}\right)^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 \\
&\quad + 2\left\{\frac{1}{12}(2.31056 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[(2.4)^2 + (6.2)^2\right]\text{in}^2\right. \\
&\quad \left. + (2.31056 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2.4}{2}\right)^2 + \left(\frac{6.2}{2}\right)^2\right]\text{in}^2\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2
\end{aligned}$$

PROBLEM 9.138 (Continued)

$$\begin{aligned}
&= [(3.7267+11.1801)+(64.2491+192.7472) \\
&\quad +2(59.1011+177.3034)]\times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \\
&= [14.9068+256.9963+2(236.4045)]\times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2
\end{aligned}$$

$$\text{or}\qquad I_x = 745\times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \ \blacktriangleleft$$

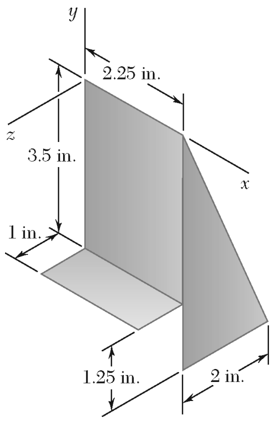
$$\begin{aligned}
I_y &= (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 \\
&= \left[\frac{1}{12}(1.11801\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(3\ \text{in.})^2 \right.\\
&\quad \left. +(1.11801\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{3}{2}\ \text{in.}\right)^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 \\
&\quad +\left\{\frac{1}{12}(2.88820\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[(3)^2+(6.2)^2]\ \text{in}^2 \right.\\
&\quad \left. +(2.88820\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{3}{2}\right)^2+\left(\frac{6.2}{2}\right)^2\right]\ \text{in}^2\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 \\
&\quad +\left[\frac{1}{12}(2.31056\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(6.2\ \text{in.})^2 \right.\\
&\quad \left. +(2.31056\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{6.2}{2}\ \text{in.}\right)^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 \\
&\quad +\left\{\frac{1}{12}(2.31056\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(6.2\ \text{in.})^2 \right.\\
&\quad \left. +(2.31056\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[(3)^2+\left(\frac{6.2}{2}\right)^2\right]\ \text{in}^2\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 \\
&= [(5.8230+17.4689)+(79.2918+237.8754)+(51.3993+154.1978) \\
&\quad +(51.3993+298.6078)]\times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \\
&= (23.2919+317.1672+205.5971+350.0071)\times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2
\end{aligned}$$

$$\text{or}\qquad I_y = 896\times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \ \blacktriangleleft$$

PROBLEM 9.138 (Continued)

$$
\begin{aligned}
I_z &= (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4\\
&= \left\{\frac{1}{12}(1.11801\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[(3)^2+(2.4)^2]\ \text{in}^2\right.\\
&\qquad \left. + (1.11801\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{3}{2}\right)^2+\left(\frac{2.4}{2}\right)^2\right]\text{in}^2\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2\\
&\quad +\left[\frac{1}{12}(2.88820\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(3\ \text{in.})^2\right.\\
&\qquad \left. + (2.88820\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{3}{2}\ \text{in.}\right)^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2\\
&\quad +\left[\frac{1}{12}(2.31056\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(2.4\ \text{in.})^2\right.\\
&\qquad \left. + (2.31056\times10^{-3}\ \text{lb}\cdot\text{s}^2/\ \text{ft})\left(\frac{2.4}{2}\text{in.}\right)^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2\\
&\quad +\left\{\frac{1}{12}(2.31056\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(2.4\ \text{in.})^2\right.\\
&\qquad \left. + (2.31056\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[(3)^2+\left(\frac{2.4}{2}\right)^2\right]\text{in}^2\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2\\
&= [(9.5497+28.6490)+(15.0427+45.1281)\\
&\qquad +(7.7019+23.1056)+(7.7019+167.5156)]\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2\\
&= (38.1987+60.1708+30.8075+175.2175)\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2
\end{aligned}
$$

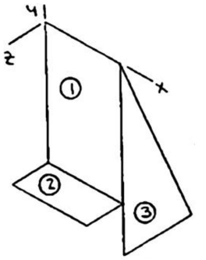
or $I_z = 304\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.139

A framing anchor is formed of 0.05-in.-thick galvanized steel. Determine the mass moment of inertia of the anchor with respect to each of the coordinate axes. (The specific weight of galvanized steel is $470\ \text{lb/ft}^3$.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho V = \frac{\gamma_{\text{G.S.}}}{g} tA$$

Then

$$m_1 = \frac{470\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2} \times 0.05\ \text{in.} \times (2.25 \times 3.5)\ \text{in}^2 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^3$$
$$= 3325.97 \times 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \frac{470\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2} \times 0.05\ \text{in.} \times\ (2.25 \times 1)\ \text{in}^2 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^3$$
$$= 950.28 \times 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \frac{470\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2} \times 0.05\ \text{in.} \times\ \left(\frac{1}{2} \times 2 \times 4.75\right)\ \text{in}^2 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^3$$
$$= 2006.14 \times 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

Using Figure 9.28 for components 1 and 2 and the equations derived above for component 3, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$
$$= \left[\frac{1}{12}(3325.97 \times 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft})(3.5\ \text{in.})^2\right.$$
$$\left. + (3325.97 \times 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{3.5}{2}\ \text{in}^2\right)\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$
$$+ \left\{\frac{1}{12}(950.28 \times 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft})(1\ \text{in.})^2\right.$$
$$\left. + (950.28 \times 10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[(3.5)^2 + \left(\frac{1}{2}\right)^2\right]\ \text{in}^2\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

PROBLEM 9.139 (Continued)

$$+\left\{\frac{1}{18}(2006.14\times10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft})[(4.75)^2+(2)^2]\ \text{in}^2\right.$$

$$\left.+(2006.14\times10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2}{3}\times4.75\right)^2+\left(\frac{1}{3}\times2\right)^2\right]\ \text{in}^2\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$=[(23.578+70.735)+(0.550+82.490)$$

$$+(20.559+145.894)]\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$=(94.313+83.040+166.453)\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 344\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_y=(I_y)_1+(I_y)_2+(I_y)_3$$

$$=\left[\frac{1}{12}(3325.97\times10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft})(2.25\ \text{in.})^2\right.$$

$$\left.+(3325.97\times10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{2.25}{2}\ \text{in.}\right)^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+\left\{\frac{1}{12}(950.28\times10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft})[(2.25)^2+(1)^2]\ \text{in}^2\right.$$

$$\left.+(950.28\times10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2.25}{2}\right)^2+\left(\frac{1}{2}\right)^2\right]\ \text{in}^2\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+\left\{\frac{1}{18}(2006.14\times10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft})(2\ \text{in.})^2\right.$$

$$\left.+(2006.14\times10^{-6}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[(2.25)^2+\left(\frac{1}{3}\times2\right)^2\right]\ \text{in}^2\right\}\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$=[(9.744+29.232)+(3.334+10.002)$$

$$+(3.096+76.720)]\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$=(38.976+13.336+79.816)\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_y = 132.1\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.139 (Continued)

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3$$

$$= \left\{\frac{1}{12}(3325.97\times 10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})[(2.25)^2 + (3.5)^2]\text{ in}^2\right.$$

$$\left. + (3325.97\times 10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2.25}{2}\right)^2 + \left(\frac{3.5}{2}\right)^2\right]\text{ in}^2\right\}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+ \left\{\frac{1}{12}(950.28\times 10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})(2.25\text{ in.})^2\right.$$

$$\left. + (950.28\times 10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2.25}{2}\right)^2 + (3.5)^2\right]\text{ in}^2\right\}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+ \left\{\frac{1}{18}(2006.14\times 10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})(4.75\text{ in.})^2\right.$$

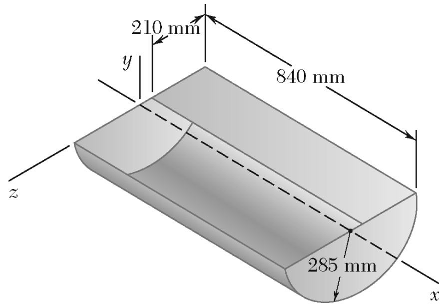
$$\left. + (2006.14\times 10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft})\left[(2.25)^2 + \left(\frac{2}{3}\times 4.75\right)^2\right]\text{ in}^2\right\}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= [(33.322 + 99.967) + (2.784 + 89.192)$$

$$+ (17.463 + 210.231)]\times 10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= (133.289 + 91.976 + 227.694)\times 10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

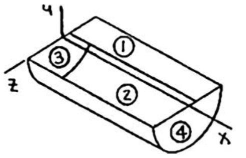
or $I_z = 453\times 10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.140*

A farmer constructs a trough by welding a rectangular piece of 2-mm-thick sheet steel to half of a steel drum. Knowing that the density of steel is $7850\ \text{kg/m}^3$ and that the thickness of the walls of the drum is 1.8 mm, determine the mass moment of inertia of the trough with respect to each of the coordinate axes. Neglect the mass of the welds.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} t A$$

Then

$$\begin{aligned} m_1 &= (7850\ \text{kg/m}^3)(0.002\ \text{m})(0.84 \times 0.21)\ \text{m}^2 \\ &= 2.76948\ \text{kg} \end{aligned}$$

$$\begin{aligned} m_2 &= (7850\ \text{kg/m}^3)(0.0018\ \text{m})(\pi \times 0.285 \times 0.84)\ \text{m}^2 \\ &= 10.62713\ \text{kg} \end{aligned}$$

$$\begin{aligned} m_3 = m_4 &= (7850\ \text{kg/m}^3)(0.0018\ \text{m})\left(\frac{\pi}{2} \times 0.285^2\right)\text{m}^2 \\ &= 1.80282\ \text{kg} \end{aligned}$$

Using Figure 9.28 for component 1 and the equations derived above for components 2 through 4, we have

$$\begin{aligned} (I_x)_3 &= (I_x)_4 \\ I_x &= (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 \\ &= \left[\frac{1}{12}(2.76948\ \text{kg})(0.21\ \text{m})^2 + (2.76948\ \text{kg})\left(0.285 - \frac{0.21}{2}\right)^2 \text{m}^2\right] \\ &\quad + [(10.62713\ \text{kg})(0.285\ \text{m})^2] + 2\left[\frac{1}{2}(1.80282\ \text{kg})(0.285\ \text{m})^2\right] \\ &= [(0.01018 + 0.08973) + (0.86319) + 2(0.07322)]\ \text{kg}\cdot\text{m}^2 \\ &= [(0.09991 + 0.86319 + 2(0.07322)]\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

or $I_x = 1.110\ \text{kg}\cdot\text{m}^2$ ◀

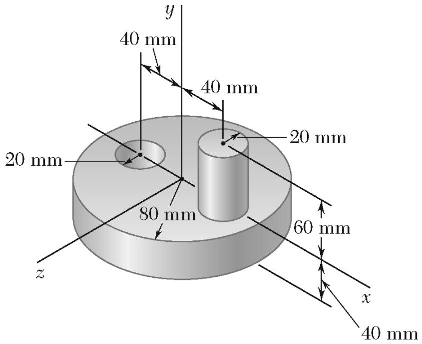
PROBLEM 9.140* (Continued)

$$
\begin{aligned}
I_y &= (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 \\
&= \left\{ \frac{1}{12}(2.76948 \text{ kg})[(0.84)^2 + (0.21)^2]\text{ m}^2 \right. \\
&\quad \left. + (2.76948 \text{ kg})\left[\left(\frac{0.84}{2}\right)^2 + \left(0.285 - \frac{0.21}{2}\right)^2\right]\text{m}^2 \right\} \\
&\quad + \left\{ \frac{1}{12}(10.62713 \text{ kg})[(0.84)^2 + 6(0.285)^2]\text{ m}^2 + (10.62713 \text{ kg})\left(\frac{0.84}{2}\text{ m}\right)^2 \right\} \\
&\quad + \left[\frac{1}{4}(1.80282 \text{ kg})(0.285 \text{ m})^2\right] \\
&\quad + \left[\frac{1}{4}(1.80282 \text{ kg})(0.285 \text{ m})^2 + (1.80282 \text{ kg})(0.84 \text{ m})^2\right] \\
&= [(0.17302 + 0.57827) + (1.05647 + 1.87463) \\
&\quad + (0.03661) + (0.03661 + 1.27207)]\text{ kg}\cdot\text{m}^2 \\
&= (0.75129 + 2.93110 + 0.03661 + 1.30868)\text{ kg}\cdot\text{m}^2
\end{aligned}
$$

or $I_y = 5.03 \text{ kg}\cdot\text{m}^2$ ◀

$$
\begin{aligned}
I_z &= (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4 \\
&= \left[\frac{1}{12}(2.76948 \text{ kg})(0.84 \text{ m})^2 + (2.76948 \text{ kg})\left(\frac{0.84}{2}\text{ m}\right)^2\right] \\
&\quad + \left\{ \frac{1}{12}(10.62713 \text{ kg})[(0.84)^2 + 6(0.285)^2]\text{ m}^2 + (10.62713 \text{ kg})\left(\frac{0.84}{2}\text{ m}\right)^2 \right\} \\
&\quad + \left[\frac{1}{4}(1.80282 \text{ kg})(0.285 \text{ m})^2\right] \\
&\quad + \left[\frac{1}{4}(1.80282 \text{ kg})(0.285 \text{ m})^2 + (1.80282 \text{ kg})(0.84 \text{ m})^2\right] \\
&= [(0.16285 + 0.48854) + (1.05647 + 1.87463) \\
&\quad + (0.03661) + (0.03661 + 1.27207)]\text{ kg}\cdot\text{m}^2 \\
&= (0.65139 + 2.93110 + 0.03661 + 1.30868)\text{ kg}\cdot\text{m}^2
\end{aligned}
$$

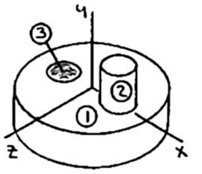
or $I_z = 4.93 \text{ kg}\cdot\text{m}^2$ ◀



PROBLEM 9.141

The machine element shown is fabricated from steel. Determine the mass moment of inertia of the assembly with respect to (*a*) the *x* axis, (*b*) the *y* axis, (*c*) the *z* axis. (The density of steel is 7850 kg/m^3.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V$$

Then

$$m_1 = (7850 \text{ kg/m}^3)(\pi(0.08 \text{ m})^2(0.04 \text{ m})]$$
$$= 6.31334 \text{ kg}$$

$$m_2 = (7850 \text{ kg/m}^3)[\pi(0.02 \text{ m})^2(0.06 \text{ m})] = 0.59188 \text{ kg}$$

$$m_3 = (7850 \text{ kg/m}^3)[\pi(0.02 \text{ m})^2(0.04 \text{ m})] = 0.39458 \text{ kg}$$

Using Figure 9.28 and the parallel-axis theorem, we have

(*a*)

$$I_x = (I_x)_1 + (I_x)_2 - (I_x)_3$$
$$= \left\{\frac{1}{12}(6.31334 \text{ kg})[3(0.08)^2 + (0.04)^2] \text{ m}^2 + (6.31334 \text{ kg})(0.02 \text{ m})^2\right\}$$
$$+ \left\{\frac{1}{12}(0.59188 \text{ kg})[3(0.02)^2 + (0.06)^2] \text{ m}^2 + (0.59188 \text{ kg})(0.03 \text{ m})^2\right\}$$
$$- \left\{\frac{1}{12}(0.39458 \text{ kg})[3(0.02)^2 + (0.04)^2] \text{ m}^2 + (0.39458 \text{ kg})(0.02 \text{ m})^2\right\}$$
$$= [(10.94312 + 2.52534) + (0.23675 + 0.53269)$$
$$- (0.09207 + 0.15783)] \times 10^{-3} \text{ kg} \cdot \text{m}^2$$
$$= (13.46846 + 0.76944 - 0.24990) \times 10^{-3} \text{ kg} \cdot \text{m}^2$$
$$= 13.98800 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

or $I_x = 13.99 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀

PROBLEM 9.141 (Continued)

(*b*)

$$\begin{aligned}I_y &= (I_y)_1 + (I_y)_2 - (I_y)_3\\
&= \left[\frac{1}{2}(6.31334\text{ kg})(0.08\text{ m})^2\right]\\
&\quad + \left[\frac{1}{2}(0.59188\text{ kg})(0.02\text{ m})^2 + (0.59188\text{ kg})(0.04\text{ m})^2\right]\\
&\quad - \left[\frac{1}{2}(0.39458\text{ kg})(0.02\text{ m}^2) + (0.39458\text{ kg})(0.04\text{ m})^2\right]\\
&= [(20.20269) + (0.11838 + 0.94701)\\
&\quad - (0.07892 + 0.63133)] \times 10^{-3}\text{ kg}\cdot\text{m}^2\\
&= (20.20269 + 1.06539 - 0.71025) \times 10^{-3}\text{ kg}\cdot\text{m}^2\\
&= 20.55783 \times 10^{-3}\text{ kg}\cdot\text{m}^2\end{aligned}$$

or $I_y = 20.6 \times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

(*c*)

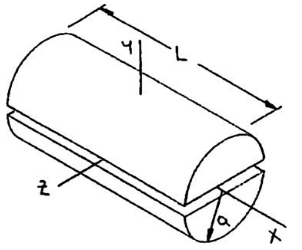
$$\begin{aligned}I_z &= (I_z)_1 + (I_z)_2 - (I_z)_3\\
&= \left\{\frac{1}{12}(6.31334\text{ kg})[3(0.08)^2 + (0.04)^2]\text{ m}^2 + (6.31334\text{ kg})(0.02\text{ m})^2\right\}\\
&\quad + \left\{\frac{1}{12}(0.59188\text{ kg})[3(0.02)^2 + (0.06)^2]\text{ m}^2 + (0.59188\text{ kg})[(0.04)^2 + (0.03)^2]\text{ m}^2\right\}\\
&\quad - \left\{\frac{1}{12}(0.39458\text{ kg})[3(0.02)^2 + (0.04)^2]\text{ m}^2 + (0.03958\text{ kg})[(0.04)^2 + (0.02)^2]\text{ m}^2\right\}\\
&= [(10.94312 + 2.52534) + (0.23675 + 1.47970)\\
&\quad - (0.09207 + 0.78916)] \times 10^{-3}\text{ kg}\cdot\text{m}^2\\
&= (13.46846 + 1.71645 - 0.88123) \times 10^{-3}\text{ kg}\cdot\text{m}^2\\
&= 14.30368 \times 10^{-3}\text{ kg}\cdot\text{m}^2\end{aligned}$$

or $I_z = 14.30 \times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

To the Instructor:

The following formulas for the mass of inertia of a semicylinder are derived at this time for use in the solutions of Problems 9.142 through 9.145.

From Figure 9.28:



Cylinder
$$(I_x)_{\text{cyl}} = \frac{1}{2}m_{\text{cyl}}a^2$$

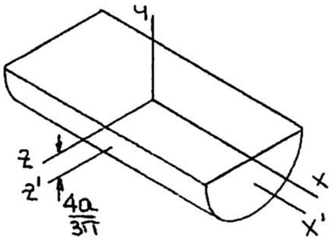
$$(I_y)_{\text{cyl}} = (I_z)_{\text{cyl}} = \frac{1}{12}m_{\text{cyl}}(3a^2 + L^2)$$

Symmetry and the definition of the mass moment of inertia $(I = \int r^2 dm)$ imply

$$(I)_{\text{semicylinder}} = \frac{1}{2}(I)_{\text{cylinder}}$$

$$(I_x)_{\text{sc}} = \frac{1}{2}\left(\frac{1}{2}m_{\text{cyl}}a^2\right)$$

and
$$(I_y)_{\text{sc}} = (I_z)_{\text{sc}} = \frac{1}{2}\left[\frac{1}{12}m_{\text{cyl}}(3a^2 + L^2)\right]$$



However,
$$m_{\text{sc}} = \frac{1}{2}m_{\text{cyl}}$$

Thus,
$$(I_x)_{\text{sc}} = \frac{1}{2}m_{\text{sc}}a^2$$

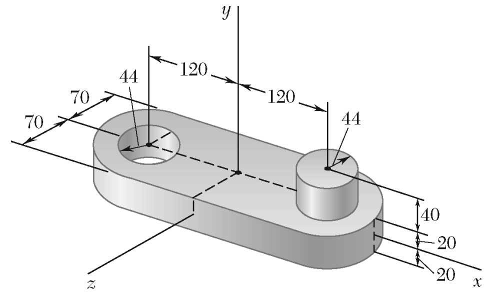
and
$$(I_y)_{\text{sc}} = (I_z)_{\text{sc}} = \frac{1}{12}m_{\text{sc}}(3a^2 + L^2)$$

Also, using the parallel axis theorem find

$$\bar{I}_{x'} = m_{\text{sc}}\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)a^2$$

$$\bar{I}_{z'} = m_{\text{sc}}\left[\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2 + \frac{1}{12}L^2\right]$$

where x' and z' are centroidal axes.

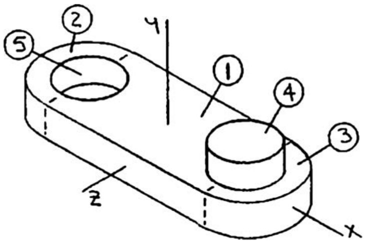


Dimensions in mm

PROBLEM 9.142

Determine the mass moments of inertia and the radii of gyration of the steel machine element shown with respect to the *x* and *y* axes. (The density of steel is 7850 kg/m^3.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\mathrm{ST}} V$$

Then

$$m_1 = (7850\ \mathrm{kg/m^3})(0.24\times 0.04\times 0.14)\ \mathrm{m^3}$$
$$= 10.5504\ \mathrm{kg}$$

$$m_2 = m_3 = (7850\ \mathrm{kg/m^3})\left[\frac{\pi}{2}(0.07)^2\times 0.04\right]\mathrm{m^3} = 2.41683\ \mathrm{kg}$$

$$m_4 = m_5 = (7850\ \mathrm{kg/m^3})[\pi(0.044)^2\times(0.04)]\ \mathrm{m^3} = 1.90979\ \mathrm{kg}$$

Using Figure 9.28 for components 1, 4, and 5 and the equations derived above (before the solution to Problem 9.144) for a semicylinder, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 - (I_x)_5 \quad \text{where} \quad (I_x)_2 = (I_x)_3$$
$$= \left[\frac{1}{12}(10.5504\ \mathrm{kg})(0.04^2 + 0.14^2)\ \mathrm{m^2}\right] + 2\left\{\frac{1}{12}(2.41683\ \mathrm{kg})[3(0.07\ \mathrm{m})^2 + (0.04\ \mathrm{m})^2]\right\}$$
$$+ \left\{\frac{1}{12}(1.90979\ \mathrm{kg})[3(0.044\ \mathrm{m})^2 + (0.04\ \mathrm{m})^2] + (1.90979\ \mathrm{kg})(0.04\ \mathrm{m})^2\right\}$$
$$- \left\{\frac{1}{12}(1.90979\ \mathrm{kg})[3(0.044\ \mathrm{m})^2 + (0.04\ \mathrm{m})^2]\right\}$$
$$= [(0.0186390) + 2(0.0032829) + (0.0011790 + 0.0030557) - (0.0011790)]\ \mathrm{kg\cdot m^2}$$
$$= 0.0282605\ \mathrm{kg\cdot m^2}$$

or $I_x = 28.3\times 10^{-3}\ \mathrm{kg\cdot m^2}$ ◀

PROBLEM 9.142 (Continued)

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 - (I_y)_5$$

where $(I_y)_2 = (I_y)_3 \quad (I_y)_4 = |(I_y)_5|$

Then
$$I_y = \left[\frac{1}{12}(10.5504 \text{ kg})(0.24^2 + 0.14^2) \text{ m}^2\right]$$
$$+2\left[(2.41683 \text{ kg})\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)(0.07 \text{ m}^2) + (2.41683 \text{ kg})\left(0.12 + \frac{4\times 0.07}{3\pi}\right)^2 \text{ m}^2\right]$$
$$= [(0.0678742) + 2(0.0037881 + 0.0541678)] \text{ kg}\cdot\text{m}^2$$
$$= 0.1837860 \text{ kg}\cdot\text{m}^2$$

or $I_y = 183.8\times 10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

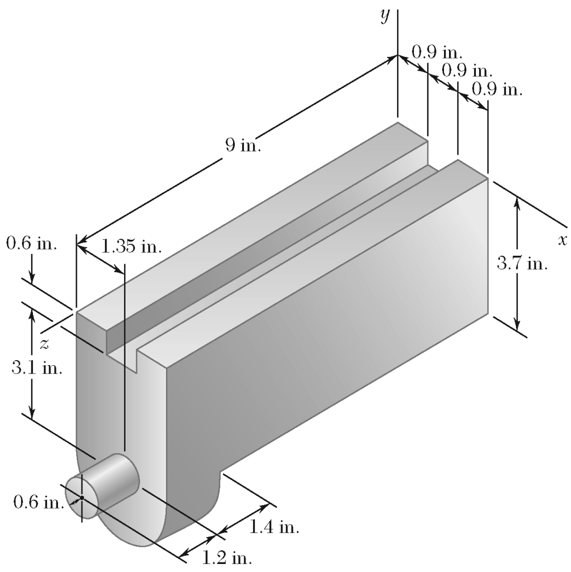
Also $m = m_1 + m_2 + m_3 + m_4 - m_5$ where $m_2 = m_3, \ m_4 = |m_5|$
$$= (10.5504 + 2\times 2.41683) \text{ kg} = 15.38406 \text{ kg}$$

Then
$$k_x^2 = \frac{I_x}{m} = \frac{0.0282605 \text{ kg}\cdot\text{m}^2}{15.38406 \text{ kg}}$$

or $k_x = 42.9$ mm ◀

and
$$k_y^2 = \frac{I_y}{m} = \frac{0.1837860 \text{ kg}\cdot\text{m}^2}{15.38406 \text{ kg}}$$

or $k_y = 109.3$ mm ◀

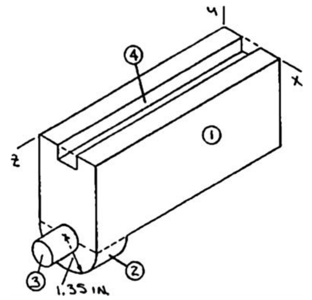


PROBLEM 9.143

Determine the mass moment of inertia of the steel machine element shown with respect to the y axis. (The specific weight of steel is $490\ \text{lb/ft}^3$.)

SOLUTION

First compute the mass of each component. We have



$$m = \rho_{\text{ST}} V = \frac{\gamma_{\text{ST}}}{g} V$$

Then

$$m_1 = \frac{490\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2} \times (2.7 \times 3.7 \times 9)\ \text{in}^3 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^3$$
$$= 791.780 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \frac{490\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2} \times \left(\frac{\pi}{2} \times 1.35^2 \times 1.4\right)\ \text{in}^3 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^3$$
$$= 35.295 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \frac{490\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2} \times (\pi \times 0.6^2 \times 1.2)\ \text{in}^3 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^3$$
$$= 11.952 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_4 = \frac{490\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2} \times (0.9 \times 0.6 \times 9)\ \text{in}^3 \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^3 = 42.799 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

The mass moments of inertia are now computed using Figure 9.28 (components 1, 3, and 4) and the equations derived above (component 2).

PROBLEM 9.143 (Continued)

Find: I_y

We have
$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 - (I_y)_4$$

$$= \left\{\frac{1}{12}(791.780\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[(2.7)^2+(9)^2]\ \text{in}^2\right.$$

$$\left.+(791.780\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2.7}{2}\right)^2+\left(\frac{9}{2}\right)^2\right]\text{in}^2\right\}\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+\left\{\frac{1}{12}(35.295\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[3(1.35)^2+(1.4)^2]\ \text{in}^2\right.$$

$$\left.+(35.295\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[(1.35)^2+\left(9-\frac{1.4}{2}\right)^2\right]\text{in}^2\right\}\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+\left\{\frac{1}{12}(11.952\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[3(0.6)^2+(1.2)^2]\ \text{in}^2\right.$$

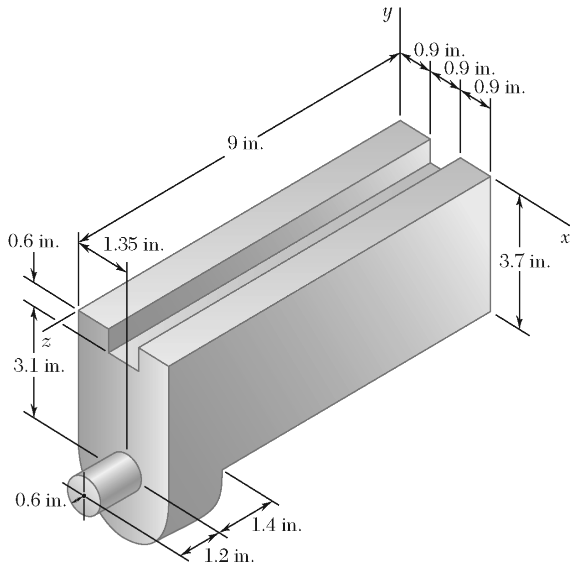
$$\left.+(11.952\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[(1.35)^2+\left(9+\frac{1.2}{2}\right)^2\ \text{in}^2\right\}\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$-\left\{\frac{1}{12}(42.799\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})[(0.9)^2+(9)^2]\ \text{in}^2\right.$$

$$\left.+(42.799\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[(1.35)^2+\left(\frac{9}{2}\right)^2\right]\text{in}^2\right\}\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$=[(40.4550+121.3650)+(0.1517+17.3319)$$
$$+(0.0174+7.8005)-(2.0263+6.5603)]\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$
$$=(161.8200+17.4836+7.8179-8.5866)\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

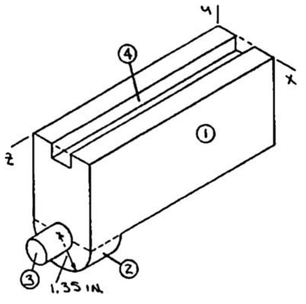
or $I_y = 0.1785\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.144

Determine the mass moment of inertia of the steel machine element shown with respect to the z axis. (The specific weight of steel is 490 lb/ft^3.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \frac{\gamma_{\text{ST}}}{g} V$$

Then

$$m_1 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (2.7 \times 3.7 \times 9) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$
$$= 791.780 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left(\frac{\pi}{2} \times 1.35^2 \times 1.4\right) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$
$$= 35.295 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (\pi \times 0.6^2 \times 1.2) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$
$$= 11.952 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_4 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times (0.9 \times 0.6 \times 9) \text{ in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3 = 42.799 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

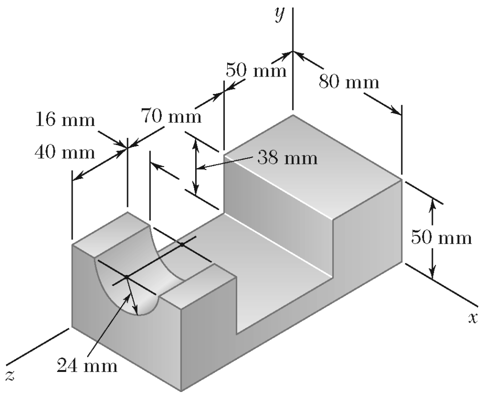
The mass moments of inertia are now computed using Figure 9.28 (components 1, 3, and 4) and the equations derived above (component 2).

PROBLEM 9.144 (Continued)

Find: I_z

We have
$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 - (I_z)_4$$
$$= \left\{ \frac{1}{12}(791.780\times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})[(2.7)^2 + (3.7)^2]\text{ in}^2 \right.$$
$$\left. + (791.780\text{ lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2.7}{2}\right)^2 + \left(\frac{3.7}{2}\right)^2\right]\text{ in}^2 \right\} \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$
$$+ \left\{ (35.295\times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)(1.35\text{ in.})^2 \right.$$
$$\left. + (35.295\times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})\left[(1.35)^2 + \left(3.7 + \frac{4\times 1.35}{3\pi}\right)^2\right]\text{ in}^2 \right\} \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$
$$+ \left\{ \frac{1}{12}(11.952\times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})(0.6\text{ in.})^2 \right.$$
$$\left. + (11.952\times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})[(1.35)^2 + (3.7)^2]\text{ in}^2 \right\} \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$
$$- \left\{ \frac{1}{12}(42.799\times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})[(0.9)^2 + (0.6)^2]\text{ in}^2 \right.$$
$$\left. + (42.799\times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})\left[(1.35)^2 + \left(\frac{0.6}{2}\right)^2\right]\text{ in}^2 \right\} \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$
$$I_z = [(9.6132 + 28.8395) + (0.1429 + 4.9219)$$
$$+ (0.0149 + 1.2875) - (0.0290 + 0.5684)]\times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$
$$= (38.4527 + 5.0648 + 1.3024 - 0.5974)\times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

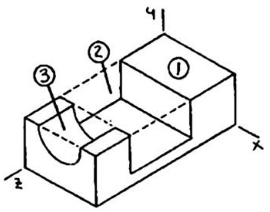
or $I_z = 0.0442\text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.145

Determine the mass moment of inertia of the steel fixture shown with respect to (*a*) the *x* axis, (*b*) the *y* axis, (*c*) the *z* axis. (The density of steel is 7850 kg/m^3.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\mathrm{ST}} V$$

Then

$$m_1 = 7850\ \mathrm{kg/m^3} \times (0.08 \times 0.05 \times 0.160)\ \mathrm{m^3}$$
$$= 5.02400\ \mathrm{kg}$$

$$m_2 = 7850\ \mathrm{kg/m^3} \times (0.08 \times 0.038 \times 0.07)\ \mathrm{m^3} = 1.67048\ \mathrm{kg}$$

$$m_3 = 7850\ \mathrm{kg/m^3} \times \left(\frac{\pi}{2} \times 0.024^2 \times 0.04\right) \mathrm{m^3} = 0.28410\ \mathrm{kg}$$

Using Figure 9.28 for components 1 and 2 and the equations derived above for component 3, we have

(*a*) $I_x = (I_x)_1 - (I_x)_2 - (I_x)_3$

$$= \left\{\frac{1}{12}(5.02400\ \mathrm{kg})[(0.05)^2 + (0.16)^2]\ \mathrm{m^2} + (5.02400\ \mathrm{kg})\left[\left(\frac{0.05}{2}\right)^2 + \left(\frac{0.16}{2}\right)^2\right] \mathrm{m^2}\right\}$$

$$- \left\{\frac{1}{12}(1.67048\ \mathrm{kg})[(0.038)^2 + (0.07)^2]\ \mathrm{m^2} + (1.67048\ \mathrm{kg})\left[\left(0.05 - \frac{0.038}{2}\right)^2 + \left(0.05 + \frac{0.07}{2}\right)^2\right] \mathrm{m^2}\right\}$$

$$- \left\{(0.28410\ \mathrm{kg})\left[\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)(0.024)^2 + \frac{1}{12}(0.04)^2\right] \mathrm{m^2}\right.$$

$$\left. + (0.28410\ \mathrm{kg})\left[\left(0.05 - \frac{4 \times 0.024}{3\pi}\right)^2 + \left(0.16 - \frac{0.04}{2}\right)^2\right] \mathrm{m^2}\right\}$$

$$= [(11.7645 + 35.2936) - (0.8831 + 13.6745) - (0.0493 + 6.0187)] \times 10^{-3}\ \mathrm{kg \cdot m^2}$$

$$= (47.0581 - 14.5576 - 6.0680) \times 10^{-3}\ \mathrm{kg \cdot m^2}$$

$$= 26.4325 \times 10^{-3}\ \mathrm{kg \cdot m^2}$$

or $I_x = 26.4 \times 10^{-3}\ \mathrm{kg \cdot m^2}$ ◀

PROBLEM 9.145 (Continued)

(*b*) $I_y = (I_y)_1 - (I_y)_2 - (I_y)_3$

$$= \left\{ \frac{1}{12}(5.02400\text{ kg})[(0.08)^2 + (0.16)^2]\text{ m}^2 + (5.02400\text{ kg})\left[\left(\frac{0.08}{2}\right)^2 + \left(\frac{0.16}{2}\right)^2 \right]\text{m}^2 \right\}$$

$$-\left\{ \frac{1}{12}(1.67048\text{ kg})[(0.08)^2 + (0.07)^2]\text{ m}^2 + (1.67048\text{ kg})\left[\left(\frac{0.08}{2}\right)^2 + \left(0.05 + \frac{0.07}{2}\right)^2 \right]\text{m}^2 \right\}$$

$$-\left\{ \frac{1}{12}(0.28410\text{ kg})[3(0.024)^2 + (0.04)^2]\text{ m}^2 + (0.28410\text{ kg})\left[\left(\frac{0.08}{2}\right)^2 + \left(0.16 - \frac{0.04}{2}\right)^2 \right]\text{m}^2 \right\}$$

$$= [(13.3973 + 40.1920) - (1.5730 + 14.7420) - (0.0788 + 6.0229)]\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= (53.5893 - 16.3150 - 6.1017)\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= 31.1726\times 10^{-3}\text{ kg}\cdot\text{m}^2 \qquad \text{or} \quad I_y = 31.2\times 10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

(*c*) $I_z = (I_z)_1 - (I_z)_2 - (I_z)_3$

$$= \left\{ \frac{1}{12}(5.02400\text{ kg})[(0.08)^2 + (0.05)^2]\text{ m}^2 + (5.02400\text{ kg})\left[\left(\frac{0.08}{2}\right)^2 + \left(\frac{0.05}{2}\right)^2 \right]\text{m}^2 \right\}$$

$$-\left\{ \frac{1}{12}(1.67048\text{ kg})[(0.08)^2 + (0.038)^2]\text{ m}^2 + (1.67048\text{ kg})\left[\left(\frac{0.08}{2}\right)^2 + \left(0.05 - \frac{0.038}{2}\right)^2 \right]\text{m}^2 \right\}$$

$$-\left\{ (0.28410\text{ kg})\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)(0.024\text{ m})^2 + (0.28410\text{ kg})\left[\left(\frac{0.08}{2}\right)^2 + \left(0.05 - \frac{4\times 0.024}{3\pi}\right)^2 \right]\text{m}^2 \right\}$$

$$= [(3.7261 + 11.1784) - (1.0919 + 4.2781) - (0.0523 + 0.9049)]\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

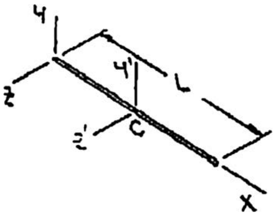
$$= (14.9045 - 5.3700 - 0.9572)\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= 8.5773\times 10^{-3}\text{ kg}\cdot\text{m}^2 \qquad \text{or} \quad I_z = 8.58\times 10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

To the instructor:

The following formulas for the mass moment of inertia of wires are derived or summarized at this time for use in the solutions of Problems 9.146 through 9.148.

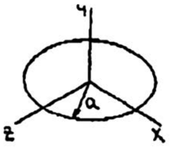
Slender Rod

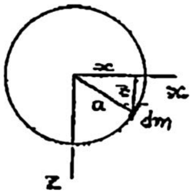


$$I_x = 0 \qquad \bar{I}_{y'} = \bar{I}_{z'} = \frac{1}{12}mL^2 \text{ (Figure 9.28)}$$

$$I_y = I_z = \frac{1}{3}mL^2 \text{ (Sample Problem 9.9)}$$

Circle





We have $\bar{I}_y = \int r^2 dm = ma^2$

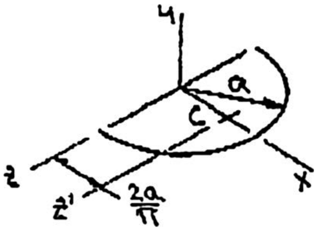
Now $\bar{I}_y = \bar{I}_x + \bar{I}_z$

And symmetry implies $\bar{I}_x = \bar{I}_z$

$$\bar{I}_x = \bar{I}_z = \frac{1}{2}ma^2$$

Semicircle

Following the above arguments for a circle, We have

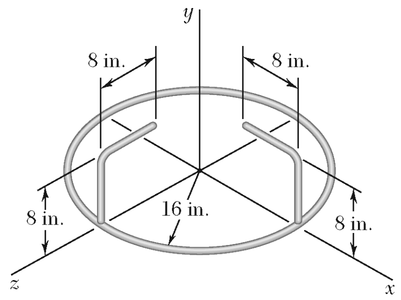


$$\bar{I}_x = I_z = \frac{1}{2}ma^2 \qquad I_y = ma^2$$

Using the parallel-axis theorem

$$I_z = \bar{I}_{z'} + m\bar{x}^2 \qquad x = \frac{2a}{\pi}$$

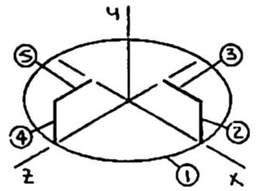
or $I_{z'} = m\left(\frac{1}{2} - \frac{4}{\pi^2}\right)a^2$



PROBLEM 9.146

Aluminum wire with a weight per unit length of 0.033 lb/ft is used to form the circle and the straight members of the figure shown. Determine the mass moment of inertia of the assembly with respect to each of the coordinate axes.

SOLUTION



First compute the mass of each component. We have

$$m = \frac{W}{g} = \frac{1}{g}\left(\frac{W}{L}\right)_{\text{AL}} L$$

Then

$$m_1 = \frac{1}{32.2\ \text{ft/s}^2} \times 0.033\ \text{lb/ft} \times (2\pi \times 16\ \text{in.}) \times \frac{1\ \text{ft}}{12\ \text{in.}}$$

$$= 8.5857 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = m_3 = m_4 = m_5 = \frac{1}{32.2\ \text{ft/s}^2} \times 0.033\ \text{lb/ft} \times 8\ \text{in.} \times \frac{1\ \text{ft}}{12\ \text{in.}}$$

$$= 0.6832\ \text{lb}\cdot\text{s}^2/\text{ft}$$

Using the equations given above and the parallel-axis theorem, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 + (I_x)_5$$

$$= \left[\frac{1}{2}(8.5857 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(16\ \text{in.})^2\right] \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+ \left[\frac{1}{3}(0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(8\ \text{in.})^2\right] \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+ [0 + (0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(8\ \text{in.})^2] \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+ \left[\frac{1}{12}(0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(8\ \text{in.})^2 + (0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(4^2 + 16^2)\ \text{in}^2\right] \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+ \left[\frac{1}{12}(0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(8\ \text{in.})^2 + (0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(8^2 + 12^2)\ \text{in}^2\right] \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= [(7.6315) + (0.1012) + (0.3036) + (0.0253 + 1.2905) + (0.0253 + 0.9868)] \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= (7.6315 + 0.1012 + 0.3036) + 1.3158 + 1.0121) \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 10.3642 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \text{or} \quad I_x = 10.36 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \ ◀$$

PROBLEM 9.146 (Continued)

$$(I_y)_2 = (I_y)_4, \qquad (I_y)_3 = (I_y)_5$$

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 + (I_y)_5$$

$$= [(8.5857\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(16\ \text{in.})^2]\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+2[0+(0.6832\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(16\ \text{in.})^2]\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

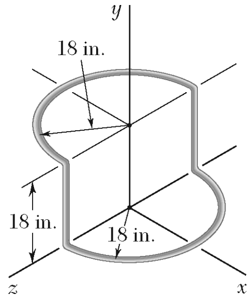
$$+2\left[\frac{1}{12}(0.6832\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})(8\ \text{in.})^2+(0.6832\ \text{lb}\cdot\text{s}^2/\text{ft})(12\ \text{in.})^2\right]\times\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= [(15.2635)+2(1.2146)+2(0.0253+0.6832)]\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= [15.2635+2(1.2146)+2(0.7085)]\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 19.1097\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \text{or} \quad I_y = 19.11\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \ ◀$$

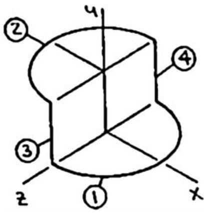
Symmetry implies $I_x = I_z$ $\qquad I_z = 10.36\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.147

The figure shown is formed of $\frac{1}{8}$-in.-diameter steel wire. Knowing that the specific weight of the steel is 490 lb/ft^3, determine the mass moment of inertia of the wire with respect to each of the coordinate axes.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \frac{\gamma_{\text{ST}}}{g} AL$$

Then

$$m_1 = m_2 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} \times \left[\frac{\pi}{4}\left(\frac{1}{8}\text{in.}\right)^2\right] \times (\pi \times 18\text{ in.}) \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$

$$= 6.1112 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = m_4 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} = \left[\frac{\pi}{4}\left(\frac{1}{8}\text{in.}\right)^2\right] \times 18\text{ in.} \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$

$$= 1.9453 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}$$

Using the equations given above and the parallel-axis theorem, we have

$$(I_x)_3 = (I_x)_4$$

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4$$

$$= \left[\frac{1}{2}(6.1112 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})(18\text{ in.})^2\right] \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+ \left[\frac{1}{2}(6.1112 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})(18\text{ in.})^2 + (6.1112 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})(18\text{ in.})^2\right] \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+ 2\left[\frac{1}{12}(1.9453 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})(18\text{ in.})^2 + (6.1112 \times 10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})(9^2 + 18^2)\text{ in}^2\right] \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= [(6.8751) + (6.8751 + 13.1502) + 2(0.3647 + 5.4712)] \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= [6.8751 + 20.6252 + 2(5.8359)] \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 39.1721 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \text{or} \quad I_x = 0.0392\text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

PROBLEM 9.147 (Continued)

$$(I_y)_1 = (I_y)_2, \quad (I_y)_3 = (I_y)_4$$

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4$$

$$= 2[(6.1112\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})(18\text{ in.})^2]\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+ 2[(0 + 1.9453\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})(18\text{ in.})^2]\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= [2(13.7502) + 2(4.3769)]\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 36.2542\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \text{or} \quad I_y = 0.0363\text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

$$(I_z)_3 = (I_z)_4$$

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4$$

$$= \left[\frac{1}{2}(6.1112\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})(18\text{ in.})^2\right]\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+ \left\{(6.1112\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{1}{2} - \frac{4}{\pi^2}\right)(18\text{ in.})^2\right.$$

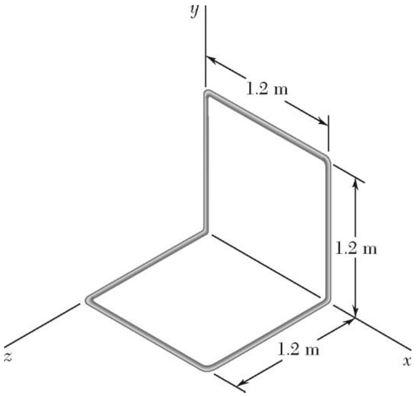
$$\left. + (6.1112\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{2\times18}{\pi}\right)^2 + (18)^2\right]\text{ in}^2\right\}\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$+ 2\left[\frac{1}{3}(1.9453\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft})(18\text{ in.})^2\right]\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= [(6.8751) + (1.3024 + 19.3229) + 2(1.4590)]\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= [6.8751 + 20.6253 + 2(1.4590)]\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

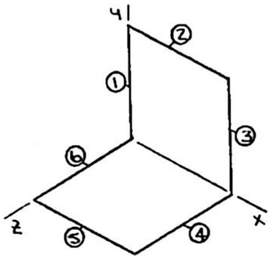
$$= 30.4184\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \text{or} \quad I_z = 0.0304\text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$



PROBLEM 9.148

A homogeneous wire with a mass per unit length of 0.056 kg/m is used to form the figure shown. Determine the mass moment of inertia of the wire with respect to each of the coordinate axes.

SOLUTION



First compute the mass m of each component. We have

$$\begin{aligned}m &= (m/L)L\\ &= 0.056\ \text{kg/m}\times 1.2\ \text{m}\\ &= 0.0672\ \text{kg}\end{aligned}$$

Using the equations given above and the parallel-axis theorem, we have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 + (I_x)_5 + (I_x)_6$$

Now $\quad (I_x)_1 = (I_x)_3 = (I_x)_4 = (I_x)_6 \quad \text{and} \quad (I_x)_2 = (I_x)_5$

Then

$$\begin{aligned}I_x &= 4\left[\frac{1}{3}(0.0672\ \text{kg})(1.2\ \text{m})^2\right] + 2[0 + (0.0672\ \text{kg})(1.2\ \text{m})^2]\\ &= [4(0.03226) + 2(0.09677)]\ \text{kg}\cdot\text{m}^2\\ &= 0.32258\ \text{kg}\cdot\text{m}^2\end{aligned}$$

or $I_x = 0.323\ \text{kg}\cdot\text{m}^2$ ◀

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 + (I_y)_5 + (I_y)_6$$

Now $\quad (I_y)_1 = 0, \quad (I_y)_2 = (I_y)_6, \quad \text{and} \quad (I_y)_4 = (I_y)_5$

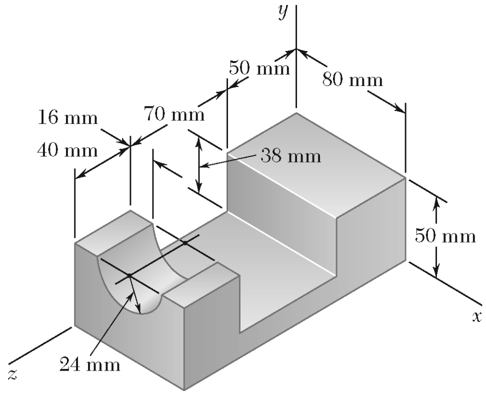
Then

$$\begin{aligned}I_y &= 2\left[\frac{1}{3}(0.0672\ \text{kg})(1.2\ \text{m})^2\right] + [0 + (0.0672\ \text{kg})(1.2\ \text{m})^2]\\ &\quad + 2\left[\frac{1}{12}(0.0672\ \text{kg})(1.2\ \text{m})^2 + (0.0672\ \text{kg})(1.2^2 + 0.6^2)\ \text{m}^2\right]\\ &= [2(0.03226) + (0.09677) + 2(0.00806 + 0.12096)]\ \text{kg}\cdot\text{m}^2\\ &= [2(0.03226) + (0.09677) + 2(0.12902)]\ \text{kg}\cdot\text{m}^2\\ &= 0.41933\ \text{kg}\cdot\text{m}^2\end{aligned}$$

or $I_y = 0.419\ \text{kg}\cdot\text{m}^2$ ◀

Symmetry implies $\quad I_y = I_z$

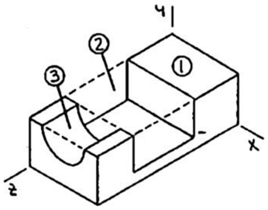
$I_z = 0.419\ \text{kg}\cdot\text{m}^2$ ◀



PROBLEM 9.149

Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the steel fixture shown. (The density of steel is 7850 kg/m^3.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho V$$

Then

$$m_1 = 7850 \text{ kg/m}^3 \times (0.08 \times 0.05 \times 0.16) \text{ m}^3 = 5.02400 \text{ kg}$$

$$m_2 = 7850 \text{ kg/m}^3 \times (0.08 \times 0.038 \times 0.07) \text{ m}^3 = 1.67048 \text{ kg}$$

$$m_3 = 7850 \text{ kg/m}^3 \times \left(\frac{\pi}{2} \times 0.024^2 \times 0.04\right) \text{m}^3 = 0.28410 \text{ kg}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry. Now

$$\bar{y}_2 = \left(0.05 - \frac{0.038}{2}\right) \text{m} = 0.031 \text{ m}$$

$$\bar{y}_3 = \left(0.05 - \frac{4 \times 0.024}{3\pi}\right) \text{m} = 0.039814 \text{ m}$$

and then

	m, kg	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$m\bar{x}\bar{y}$, kg·m^2	$m\bar{y}\bar{z}$, kg·m^2	$m\bar{z}\bar{x}$, kg·m^2
1	5.02400	0.04	0.025	0.08	5.0240×10^{-3}	10.0480×10^{-3}	16.0768×10^{-3}
2	1.67048	0.04	0.031	0.085	2.0714×10^{-3}	4.4017×10^{-3}	5.6796×10^{-3}
3	0.28410	0.04	0.039814	0.14	0.4524×10^{-3}	1.5836×10^{-3}	1.5910×10^{-3}

Finally,

$$I_{xy} = (I_{xy})_1 - (I_{xy})_2 - (I_{xy})_3 = [(0 + 5.0240) - (0 + 2.0714) - (0 + 0.4524)] \times 10^{-3}$$

$$= 2.5002 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \qquad \text{or} \quad I_{xy} = 2.50 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

PROBLEM 9.149 (Continued)

$$I_{yz} = (I_{yz})_1 - (I_{yz})_2 - (I_{yz})_3 = [(0 + 10.0480) - (0 + 4.4017) - (0 + 1.5836)] \times 10^{-3}$$

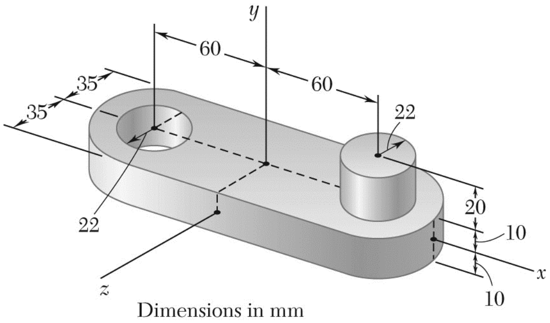
$$= 4.0627 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

or $\quad I_{yz} = 4.06 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀

$$I_{zx} = (I_{zx})_1 - (I_{zx})_2 - (I_{zx})_3 = [(0 + 16.0768) - (0 + 5.6796) - (0 + 1.5910)] \times 10^{-3}$$

$$= 8.8062 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

or $\quad I_{zx} = 8.81 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀



Dimensions in mm

PROBLEM 9.150

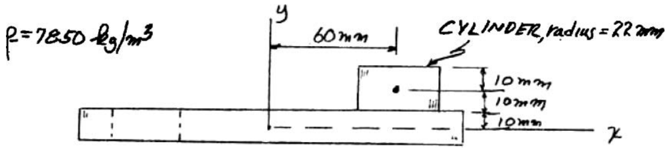
Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the steel machine element shown. (The density of steel is 7850 kg/m^3.)

SOLUTION

Since the machine element is symmetrical with respect to the xy plane, $I_{yz} = I_{zx} = 0$ ◀

Also, $I_{xy} = 0$

for all components except the cylinder, since these are symmetrical with respect to the xz plane.

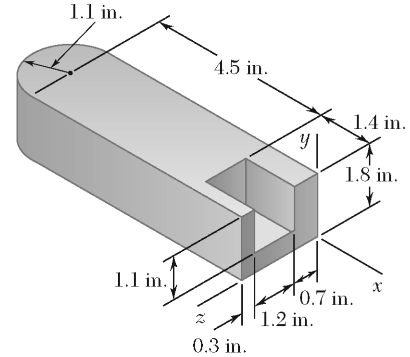


For the cylinder

$$m = \rho V = (7850\ \text{kg/m}^3)\pi(0.022\ \text{m})^2(0.02\ \text{m}) = 0.2387\ \text{kg}$$

$$I'_{xy} = \bar{I}_{x'y'} + m\bar{x}\,\bar{y} = 0 + (0.2387\ \text{kg})(0.06\ \text{m})(0.02\ \text{m}) = +286.4\times10^{-6}\ \text{kg}\cdot\text{m}^2$$

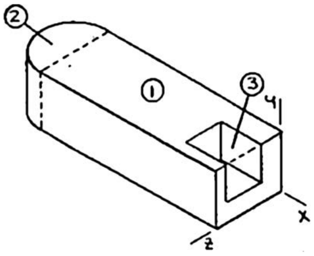
$$I_{xy} = 286\times10^{-6}\ \text{kg}\cdot\text{m}^2$$ ◀



PROBLEM 9.151

Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the cast aluminum machine component shown. (The specific weight of aluminum is 0.100 lb/in^3)

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{AL}} V = \frac{\gamma_{\text{AL}}}{g} V$$

Then

$$m_1 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times (5.9 \times 1.8 \times 2.2) \text{ in}^3$$

$$= 72.5590 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{2} (1.1)^2 \times 1.8 \right] \text{ in}^3$$

$$= 10.6248 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times (1.4 \times 1.1 \times 1.2) \text{ in}^3$$

$$= 5.7391 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

Now observe that the centroidal products of inertia, $\overline{I}_{x'y'}, \overline{I}_{y'z'}$, and $\overline{I}_{z'x'}$, of each component are zero because of symmetry. Now

$$\overline{x}_2 = -\left(5.9 + \frac{4 \times 1.1}{3\pi} \right) \text{ in.}$$

$$= -6.36685 \text{ in.}$$

$$\overline{y}_3 = \left(1.8 - \frac{1.1}{2} \right) \text{ in.}$$

$$= 1.25 \text{ in.}$$

and then

	m, lb · s²/ft	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$m\bar{x}\,\bar{y}$, lb · ft · s²	$m\bar{y}\,\bar{z}$, lb · ft · s²	$m\bar{z}\,\bar{x}$, lb · ft · s²
1	72.5590×10^{-3}	$-\frac{2.95}{12}$	$\frac{0.9}{12}$	$\frac{1.1}{12}$	-1.33781×10^{-3}	0.49884×10^{-3}	-1.63510×10^{-3}
2	10.6248×10^{-3}	$-\frac{6.36685}{12}$	$\frac{0.9}{12}$	$\frac{1.1}{12}$	-0.42279×10^{-3}	0.07305×10^{-3}	-0.51674×10^{-3}
3	5.7391×10^{-3}	$-\frac{0.7}{12}$	$\frac{1.25}{12}$	$\frac{1.3}{12}$	-0.03487×10^{-3}	0.06476×10^{-3}	-0.03627×10^{-3}

Finally,

$$I_{xy} = (I_{xy})_1 + (I_{xy})_2 - (I_{xy})_3$$
$$= [(0 - 1.33781) + (0 - 0.42279) - (0 - 0.03487)]\times10^{-3}$$

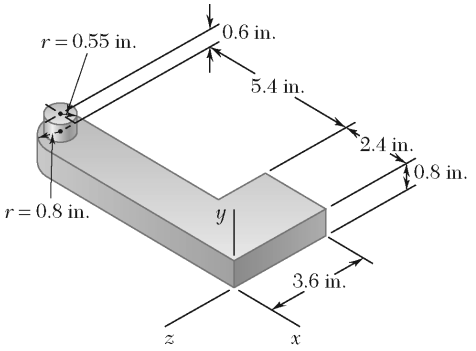
or $I_{xy} = -1.726\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_{yz} = (I_{yz})_1 + (I_{yz})_2 - (I_{yz})_3$$
$$= [(0 + 0.49884) + (0 + 0.07305) - (0 + 0.06476)]\times10^{-3}$$

or $I_{yz} = 0.507\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_{zx} = (I_{zx})_1 + (I_{zx})_2 - (I_{zx})_3$$
$$= [(0 - 1.63510) + (0 - 0.51674) - (0 - 0.3627)]\times10^{-3}$$

or $I_{zx} = -2.12\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



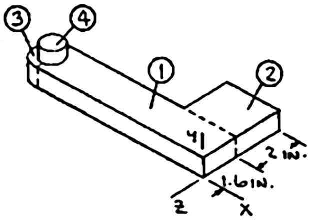
PROBLEM 9.152

Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the cast aluminum machine component shown. (The specific weight of aluminum is 0.100 lb/in^3)

SOLUTION

First compute the mass of each component. We have

$$m = \rho_{\text{AL}} V = \frac{\gamma_{\text{AL}}}{g} V$$



Then

$$m_1 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times (7.8 \times 0.8 \times 1.6) \text{ in}^3$$

$$= 31.0062 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_2 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times (2.4 \times 0.8 \times 2) \text{ in}^3$$

$$= 11.9255 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{2} (0.8)^2 \times 0.8 \right] \text{ in}^3 = 2.4977 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_4 = \frac{0.100 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times [\pi (0.55)^2 \times 0.6] \text{ in}^3 = 1.7708 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'}, \text{and } \bar{I}_{z'x'},$ of each component are zero because of symmetry. Now

$$\bar{x}_3 = -\left(7.8 + \frac{4 \times 0.8}{3\pi} \right) \text{in.} = -8.13953 \text{ in.}$$

PROBLEM 9.152 (Continued)

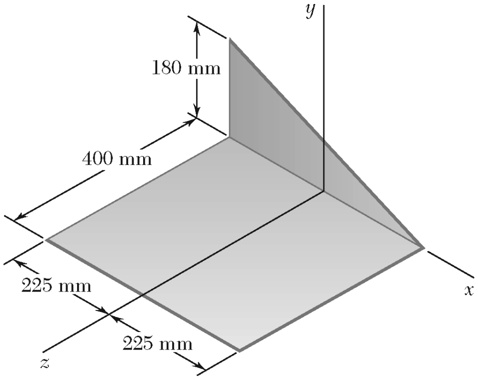
and then

	m, $\text{lb}\cdot\text{s}^2/\text{ft}$	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$m\bar{x}\,\bar{y}$, $\text{lb}\cdot\text{ft}\cdot\text{s}^2$	$m\bar{y}\,\bar{z}$, $\text{lb}\cdot\text{ft}\cdot\text{s}^2$	$m\bar{z}\,\bar{x}$, $\text{lb}\cdot\text{ft}\cdot\text{s}^2$
1	31.0062×10^{-3}	$-\frac{3.9}{12}$	$\frac{0.4}{12}$	$-\frac{0.8}{12}$	-335.901×10^{-6}	-68.903×10^{-6}	671.801×10^{-6}
2	11.9255×10^{-3}	$-\frac{1.2}{12}$	$\frac{0.4}{12}$	$-\frac{2.6}{12}$	-39.752×10^{-6}	-86.129×10^{-6}	258.386×10^{-6}
3	2.4977×10^{-3}	$-\frac{8.13953}{12}$	$\frac{0.4}{12}$	$-\frac{0.8}{12}$	-56.473×10^{-6}	-5.550×10^{-6}	112.945×10^{-6}
4	1.7708×10^{-3}	$-\frac{7.8}{12}$	$\frac{1.1}{12}$	$-\frac{0.8}{12}$	-105.511×10^{-6}	-10.822×10^{-6}	76.735×10^{-6}
Σ					-537.637×10^{-6}	-171.404×10^{-6}	1119.867×10^{-6}

Then $I_{xy} = \Sigma(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y})$ or $I_{xy} = -538\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$I_{yz} = \Sigma(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z})$ or $I_{yz} = -171.4\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

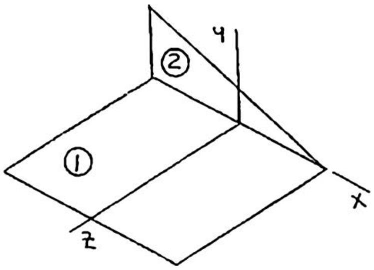
$I_{zx} = \Sigma(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x})$ or $I_{zx} = 1120\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.153

A section of sheet steel 2 mm thick is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



First compute the mass of each component.

We have $m = \rho_{\text{ST}} V = \rho_{\text{ST}} tA$

Then
$$m_1 = (7850\text{ kg/m}^3)[(0.002)(0.4)(0.45)]\text{m}^3$$
$$= 2.8260\text{ kg}$$
$$m_2 = 7850\text{ kg/m}^3\left[(0.002)\left(\frac{1}{2}\times 0.45\times 0.18\right)\right]\text{m}^3$$
$$= 0.63585\text{ kg}$$

Now observe that

$$(\bar{I}_{x'y'})_1 = (\bar{I}_{y'z'})_1 = (\bar{I}_{z'x'})_1 = 0$$
$$(\bar{I}_{y'z'})_2 = (\bar{I}_{z'x'})_2 = 0$$

From Sample Problem 9.6: $(\bar{I}_{x'y'})_{2,\text{area}} = -\dfrac{1}{72}b_2^2h_2^2$

Then $(\bar{I}_{x'y'})_2 = \rho_{\text{ST}}t(I_{x'y'})_{2,\text{area}} = \rho_{\text{ST}}t\left(-\dfrac{1}{72}b_2^2h_2^2\right) = -\dfrac{1}{36}m_2b_2h_2$

Also $\bar{x}_1 = \bar{y}_1 = \bar{z}_2 = 0 \qquad \bar{x}_2 = \left(-0.225 + \dfrac{0.45}{3}\right)\text{m} = -0.075\text{ m}$

PROBLEM 9.153 (Continued)

Finally,
$$I_{xy} = \Sigma(\overline{I}_{xy} + m\overline{x}\,\overline{y}) = (0+0) + \left[-\frac{1}{36}(0.63585\text{ kg})(0.45\text{ m})(0.18\text{ m})\right.$$
$$\left. + (0.63585\text{ kg})(-0.075\text{ m})\left(\frac{0.18\text{ m}}{3}\right)\right]$$
$$= (-1.43066\times 10^{-3} - 2.8613\times 10^{-3})\text{ kg}\cdot\text{m}^2$$

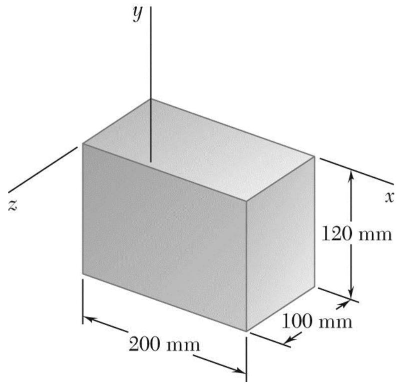
or $I_{xy} = -4.29\times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

and
$$I_{yz} = \Sigma(\overline{I}_{y'z'} + m\,\overline{y}\,\overline{z}) = (0+0) + (0+0) = 0$$

or $I_{yz} = 0$ ◀

$$I_{zx} = \Sigma(\overline{I}_{z'x'} + m\,\overline{z}\,\overline{x}) = (0+0) + (0+0) = 0$$

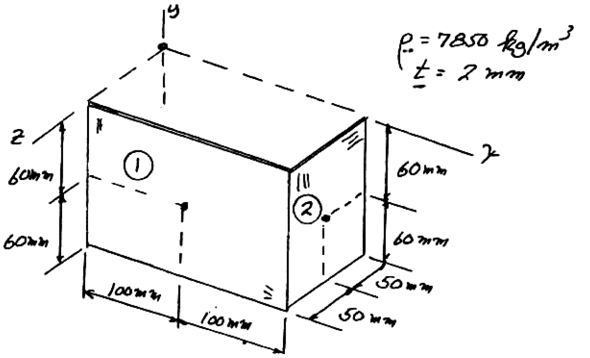
or $I_{zx} = 0$ ◀



PROBLEM 9.154

A section of sheet steel 2 mm thick is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



$$m_1 = \rho V = 7850 \text{ kg/m}^3 (0.120 \text{ m})(0.200 \text{ m})(0.002 \text{ m}) = 0.3768 \text{ kg}$$

$$m_2 = \rho V = 7850 \text{ kg/m}^3 (0.120 \text{ m})(0.100 \text{ m})(0.002 \text{ m}) = 0.1884 \text{ kg}$$

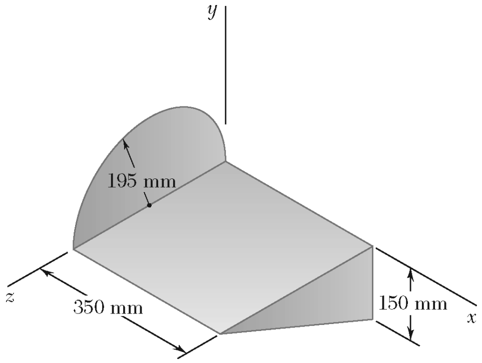
For each panel the centroidal product of inertia is zero with respect to each pair of coordinate axes.

	m, kg	$\bar{x}, m$	$\bar{y}, m$	$\bar{z}, m$	$m\bar{x}\bar{y}$	$m\bar{y}\bar{z}$	$m\bar{z}\bar{x}$
①	0.3768	0.1	–0.06	0.1	-2.261×10^{-3}	-2.261×10^{-3}	$+3.768\times10^{-3}$
②	0.1884	0.2	–0.06	0.05	-2.261×10^{-3}	-0.565×10^{-3}	$+1.884\times10^{-3}$
Σ					-4.522×10^{-3}	-2.826×10^{-3}	$+5.652\times10^{-3}$

$$I_{xy} = -4.52\times10^{-3} \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$I_{yz} = -2.83\times10^{-3} \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

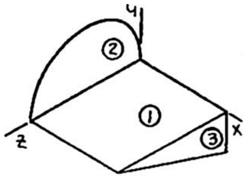
$$I_{zx} = +5.65\times10^{-3} \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$



PROBLEM 9.155

A section of sheet steel 2 mm thick is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} t A$$

Then
$$m_1 = (7850\ \text{kg/m}^3)(0.002\ \text{m})(0.35 \times 0.39)\ \text{m}^2$$
$$= 2.14305\ \text{kg}$$
$$m_2 = (7850\ \text{kg/m}^3)(0.002\ \text{m})\left(\frac{\pi}{2} \times 0.195^2\right)\ \text{m}^2$$
$$= 0.93775\ \text{kg}$$
$$m_3 = (7850\ \text{kg/m}^3)(0.002\ \text{m})\left(\frac{1}{2} \times 0.39 \times 0.15\right)\ \text{m}^2$$
$$= 0.45923\ \text{kg}$$

Now observe that because of symmetry the centroidal products of inertia of components 1 and 2 are zero and

$$(\bar{I}_{x'y'})_3 = (\bar{I}_{z'x'})_3 = 0$$

Also
$$(\bar{I}_{y'z'})_{3,\text{mass}} = \rho_{\text{ST}} t (\bar{I}_{y'z'})_{3,\text{area}}$$

Using the results of Sample Problem 9.6 and noting that the orientation of the axes corresponds to a 90° rotation, we have

$$(\bar{I}_{y'z'})_{3,\text{area}} = \frac{1}{72} b_3^2 h_3^2$$

Then
$$(I_{y'z'})_3 = \rho_{\text{ST}} t \left(\frac{1}{72} b_3^2 h_3^2\right) = \frac{1}{36} m_3 b_3 h_3$$

Also
$$\bar{y}_1 = \bar{x}_2 = 0$$
$$\bar{y}_2 = \frac{4 \times 0.195}{3\pi}\ \text{m} = 0.082761\ \text{m}$$

PROBLEM 9.155 (Continued)

Finally,

$$I_{xy} = \Sigma(\overline{I}_{x'y'} + m\overline{x}\,\overline{y})$$

$$= (0+0) + (0+0) + \left[0 + (0.45923\text{ kg})(0.35\text{ m})\left(-\frac{0.15}{3}\text{ m}\right)\right]$$

$$= -8.0365\times10^{-3}\text{ kg}\cdot\text{m}^2 \qquad \text{or } I_{xy} = -8.04\times10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$I_{yz} = \Sigma(\overline{I}_{y'z'} + m\overline{y}\,\overline{z})$$

$$= (0+0) + [0 + (0.93775\text{ kg})(0.082761\text{ m})(0.195\text{ m})]$$

$$+\left[\frac{1}{36}(0.45923\text{ kg})(0.39\text{ m})(0.15\text{ m}) + (0.45923\text{ kg})\left(-\frac{0.15}{3}\text{ m}\right)\left(\frac{0.39}{3}\text{ m}\right)\right]$$

$$= [(15.1338) + (0.7462 - 2.9850)]\times10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= (15.1338 - 2.2388)\times10^{-3}\text{ kg}\cdot\text{m}^2$$

$$= 12.8950\times10^{-3}\text{ kg}\cdot\text{m}^2 \qquad \text{or } I_{yz} = 12.90\times10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

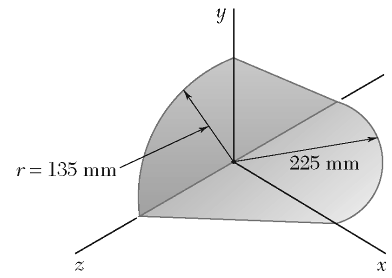
$$I_{zx} = \Sigma(\overline{I}_{z'x'} + m\overline{z}\,\overline{x})$$

$$= [0 + (2.14305\text{ kg})(0.175\text{ m})(0.195\text{ m})] + (0+0)$$

$$+\left[0 + (0.45923\text{ kg})\left(\frac{0.39}{3}\text{ m}\right)(0.35\text{ m})\right]$$

$$= (73.1316 + 20.8950)\times10^{-3}\text{ kg}\cdot\text{m}^2$$

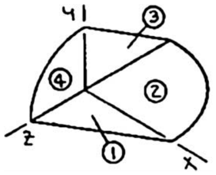
$$= 94.0266\times10^{-3}\text{ kg}\cdot\text{m}^2 \qquad \text{or } I_{zx} = 94.0\times10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$



PROBLEM 9.156

A section of sheet steel 2 mm thick is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} t A$$

Then

$$m_1 = m_3 = (7850\ \text{kg/m}^3)(0.002\ \text{m})\left(\frac{1}{2}\times 0.225\times 0.135\right)\text{m}^2 = 0.23844\ \text{kg}$$

$$m_2 = (7850\ \text{kg/m}^3)(0.002\ \text{m})\left(\frac{\pi}{4}\times 0.225^2\right)\text{m}^2 = 0.62424\ \text{kg}$$

$$m_4 = (7850\ \text{kg/m}^3)(0.002\ \text{m})\left(\frac{\pi}{4}\times 0.135^2\right)\text{m}^2 = 0.22473\ \text{kg}$$

Now observe that the following centroidal products of inertia are zero because of symmetry.

$$(\bar{I}_{x'y'})_1 = (\bar{I}_{y'z'})_1 = 0 \qquad (\bar{I}_{x'y'})_2 = (\bar{I}_{y'z'})_2 = 0$$

$$(\bar{I}_{x'y'})_3 = (\bar{I}_{z'x'})_3 = 0 \qquad (\bar{I}_{x'y'})_4 = (\bar{I}_{z'x'}) = 0$$

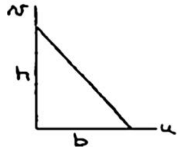
Also $\qquad \bar{y}_1 = \bar{y}_2 = 0 \qquad \bar{x}_3 = \bar{x}_4 = 0$

Now $\qquad I_{xy} = \Sigma(\bar{I}_{x'y'} + m\bar{x}\,\bar{y})$

so that $\qquad I_{xy} = 0$ ◀

Using the results of Sample Problem 9.6, we have

$$I_{uv,\text{area}} = \frac{1}{24}b^2h^2$$



Now

$$I_{uv,\text{mass}} = \rho_{\text{ST}} t\, I_{uv,\text{area}}$$

$$= \rho_{\text{ST}} t\left(\frac{1}{24}b^2h^2\right)$$

$$= \frac{1}{12}mbh$$

Thus,

$$(I_{zx})_1 = \frac{1}{12}m_1 b_1 h_1$$

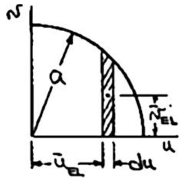
PROBLEM 9.156 (Continued)

While $$(I_{yz})_3 = -\frac{1}{12}m_3b_3h_3$$

because of a 90° rotation of the coordinate axes.

To determine I_{uv} for a quarter circle, we have

$$dI_{uv} = d\cancelto{0}{\bar{I}_{u'v'}} + u_{\text{EL}}v_{\text{EL}}dm$$



Where $$\bar{u}_{\text{EL}} = u \quad \bar{v}_{\text{EL}} = \frac{1}{2}v = \frac{1}{2}\sqrt{a^2-u^2}$$

$$dm = \rho_{\text{ST}}t\,dA = \rho_{\text{ST}}tv\,du = \rho_{\text{ST}}t\sqrt{a^2-u^2}\,du$$

Then $$I_{uv} = \int dI_{uv} = \int_0^a (u)\left(\frac{1}{2}\sqrt{a^2-u^2}\right)\left(\rho_{\text{ST}}t\sqrt{a^2-u^2}\,du\right)$$

$$= \frac{1}{2}\rho_{\text{ST}}t\int_0^a u(a^2-u^2)\,du$$

$$= \frac{1}{2}\rho_{\text{ST}}t\left[\frac{1}{2}a^2u^2 - \frac{1}{4}u^4\right]_0^a = \frac{1}{8}\rho_{\text{ST}}t\,a^4 = \frac{1}{2\pi}ma^4$$

Thus $$(I_{zx})_2 = -\frac{1}{2\pi}m_2a_2^2$$

because of a 90° rotation of the coordinate axes. Also

$$(I_{yz})_4 = \frac{1}{2\pi}m_4a_4^2$$

Finally,

$$I_{yz} = \Sigma(I_{yz}) = [(\cancelto{0}{\bar{I}_{y'z'}}) + m_1\bar{y}_1\bar{z}_1] + [(\cancelto{0}{\bar{I}_{y'z'}}) + m_2\bar{y}_2\bar{z}_2] + (I_{yz})_3 + (I_{yz})_4$$

$$= \left[-\frac{1}{12}(0.23844\text{ kg})(0.225\text{ m})(0.135\text{ m})\right] + \left[\frac{1}{2\pi}(0.22473\text{ kg})(0.135\text{ m})^2\right]$$

$$= (-0.60355 + 0.65185)\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

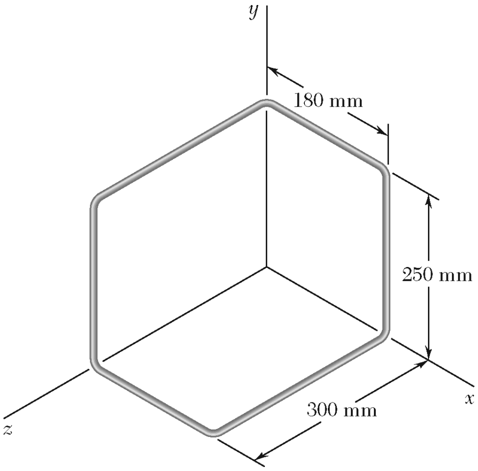
or $I_{yz} = 48.3\times 10^{-6}\text{ kg}\cdot\text{m}^2$ ◀

$$I_{zx} = \Sigma(I_{zx}) = (I_{zx})_1 + (I_{zx})_2 + [(\cancelto{0}{\bar{I}_{z'x'}})_3 + m_3\bar{z}_3\bar{x}_3] + [(\cancelto{0}{\bar{I}_{z'x'}})_4 + m_4\bar{z}_4\bar{x}_4]$$

$$= \left[\frac{1}{12}(0.23844\text{ kg})(0.225\text{ m})(0.135\text{ m})\right] + \left[-\frac{1}{2\pi}(0.62424\text{ kg})(0.225\text{ m})^2\right]$$

$$= (0.60355 - 5.02964)\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

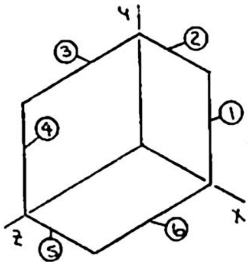
or $I_{zx} = -4.43\times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀



PROBLEM 9.157

The figure shown is formed of 1.5-mm-diameter aluminum wire. Knowing that the density of aluminum is 2800 kg/m^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{AL}} V = \rho_{\text{AL}} AL$$

Then

$$m_1 = m_4 = (2800\ \text{kg/m}^3)\left[\frac{\pi}{4}(0.0015\ \text{m})^2\right](0.25\ \text{m})$$

$$= 1.23700 \times 10^{-3}\ \text{kg}$$

$$m_2 = m_5 = (2800\ \text{kg/m}^3)\left[\frac{\pi}{4}(0.0015\ \text{m})^2\right](0.18\ \text{m})$$

$$= 0.89064 \times 10^{-3}\ \text{kg}$$

$$m_3 = m_6 = (2800\ \text{kg/m}^3)\left[\frac{\pi}{4}(0.0015\ \text{m})^2\right](0.3\ \text{m})$$

$$= 1.48440 \times 10^{-3}\ \text{kg}$$

PROBLEM 9.157 (Continued)

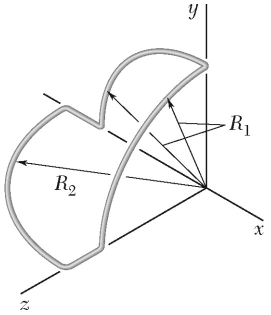
Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry.

	m, kg	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$m\bar{x}\,\bar{y}$, $\text{kg}\cdot\text{m}^2$	$m\bar{y}\,\bar{z}$, $\text{kg}\cdot\text{m}^2$	$m\bar{z}\,\bar{x}$, $\text{kg}\cdot\text{m}^2$
1	1.23700×10^{-3}	0.18	0.125	0	27.8325×10^{-6}	0	0
2	0.89064×10^{-3}	0.09	0.25	0	20.0394×10^{-6}	0	0
3	1.48440×10^{-3}	0	0.25	0.15	0	55.6650×10^{-6}	0
4	1.23700×10^{-3}	0	0.125	0.3	0	46.3875×10^{-6}	0
5	0.89064×10^{-3}	0.09	0	0.3	0	0	24.0473×10^{-6}
6	1.48440×10^{-3}	0.18	0	0.15	0	0	40.0788×10^{-6}
Σ					47.8719×10^{-6}	102.0525×10^{-6}	64.1261×10^{-6}

Then $$I_{xy} = \Sigma(\overset{0}{\cancel{\bar{I}_{x'y'}}} + m\bar{x}\,\bar{y})$$ or $I_{xy} = 47.9\times10^{-6}\ \text{kg}\cdot\text{m}^2$ ◀

$$I_{yz} = \Sigma(\overset{0}{\cancel{\bar{I}_{y'z'}}} + m\bar{y}\,\bar{z})$$ or $I_{yz} = 102.1\times10^{-6}\ \text{kg}\cdot\text{m}^2$ ◀

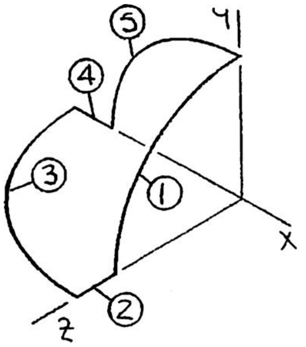
$$I_{zx} = \Sigma(\overset{0}{\cancel{\bar{I}_{z'x'}}} + m\bar{z}\,\bar{x})$$ or $I_{zx} = 64.1\times10^{-6}\ \text{kg}\cdot\text{m}^2$ ◀



PROBLEM 9.158

Thin aluminum wire of uniform diameter is used to form the figure shown. Denoting by m' the mass per unit length of the wire, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION



First compute the mass of each component. We have

$$m = \left(\frac{m}{L}\right)L = m'L$$

Then $$m_1 = m_5 = m'\left(\frac{\pi}{2}R_1\right) = \frac{\pi}{2}m'R_1$$

$$m_2 = m_4 = m'(R_2 - R_1)$$

$$m_3 = m'\left(\frac{\pi}{2}R_2\right) = \frac{\pi}{2}m'R_2$$

Now observe that because of symmetry the centroidal products of inertia, $\overline{I}_{x'y'}, \overline{I}_{y'z'},$ and $\overline{I}_{z'x'},$ of components 2 and 4 are zero and

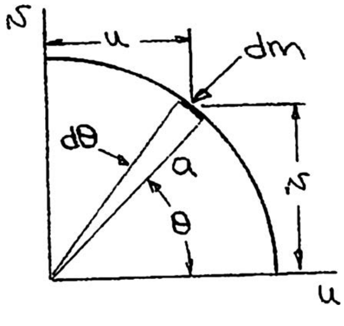
$$(\overline{I}_{x'y'})_1 = (\overline{I}_{z'x'})_1 = 0 \qquad (\overline{I}_{x'y'})_3 = (\overline{I}_{y'z'})_3 = 0$$

$$(\overline{I}_{y'z'})_5 = (\overline{I}_{z'x'})_5 = 0$$

Also $$\overline{x}_1 = \overline{x}_2 = 0 \qquad \overline{y}_2 = \overline{y}_3 = \overline{y}_4 = 0 \qquad \overline{z}_4 = \overline{z}_5 = 0$$

Using the parallel-axis theorem [Equations (9.47)], it follows that $I_{xy} = I_{yz} = I_{zx}$ for components 2 and 4.

To determine I_{uv} for one quarter of a circular arc, we have $dI_{uv} = uvdm$



where $$u = a\cos\theta \qquad v = a\sin\theta$$

and $$dm = \rho dV = \rho[A(ad\theta)]$$

where A is the cross-sectional area of the wire. Now

$$m = m'\left(\frac{\pi}{2}a\right) = \rho A\left(\frac{\pi}{2}a\right)$$

so that $$dm = m'ad\theta$$

and $$dI_{uv} = (a\cos\theta)(a\sin\theta)(m'ad\theta)$$

$$= m'a^3\sin\theta\cos\theta d\theta$$

PROBLEM 9.158 (Continued)

Then
$$I_{uv} = \int dI_{uv} = \int_0^{\pi/2} m'a^3 \sin\theta\cos\theta d\theta$$
$$= m'a^3\left[\frac{1}{2}\sin^2\theta\right]_0^{\pi/2} = \frac{1}{2}m'a^3$$

Thus,
$$(I_{yz})_1 = \frac{1}{2}m'R_1^3$$

and
$$(I_{zx})_3 = -\frac{1}{2}m'R_2^3 \quad (I_{xy})_5 = -\frac{1}{2}m'R_1^3$$

because of 90° rotations of the coordinate axes. Finally,

$$I_{xy} = \Sigma(I_{xy}) = [(\cancelto{0}{\bar{I}_{x'y'}})_1 + m_1\bar{x}_1\bar{y}_1] + [(\cancelto{0}{\bar{I}_{x'y'}})_3 + m_3\bar{x}_3\bar{y}_3] + (I_{xy})_5$$

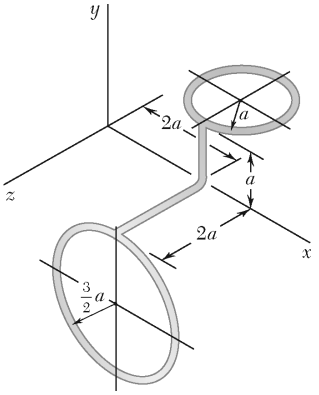
or $I_{xy} = -\frac{1}{2}m'R_1^3$ ◀

$$I_{yz} = \Sigma(I_{yz}) = (I_{yz})_1 + [(\cancelto{0}{\bar{I}_{y'z'}})_3 + m_3\bar{y}_3\bar{z}_3] + [(\cancelto{0}{\bar{I}_{y'z'}})_5 + m_5\bar{y}_5\bar{z}_5]$$

or $I_{yz} = \frac{1}{2}m'R_1^3$ ◀

$$I_{zx} = \Sigma(I_{zx}) = [(\cancelto{0}{\bar{I}_{z'x'}})_1 + m_1\bar{z}_1\bar{x}_1] + (I_{zx})_3 + [(\cancelto{0}{\bar{I}_{z'x'}})_5 + m_5\bar{z}_5\bar{x}_5]$$

or $I_{zx} = -\frac{1}{2}m'R_2^3$ ◀

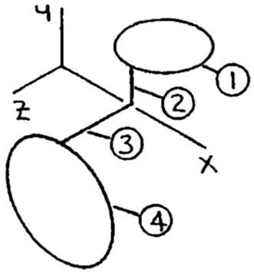


PROBLEM 9.159

Brass wire with a weight per unit length w is used to form the figure shown. Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION

First compute the mass of each component. We have



$$m = \frac{W}{g} = \frac{1}{g} wL$$

Then

$$m_1 = \frac{W}{g}(2\pi \times a) = 2\pi \frac{w}{g} a$$

$$m_2 = \frac{w}{g}(a) = \frac{w}{g} a$$

$$m_3 = \frac{w}{g}(2a) = 2\frac{w}{g} a$$

$$m_4 = \frac{w}{g}\left(2\pi \times \frac{3}{2} a\right) = 3\pi \frac{w}{g} a$$

Now observe that the centroidal products of inertia, $\overline{I}_{x'y'}$, $\overline{I}_{y'z'}$, and $\overline{I}_{z'x'}$, of each component are zero because of symmetry.

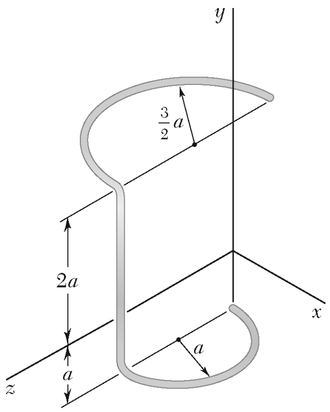
	m	$\overline{x}$	$\overline{y}$	$\overline{z}$	$m\overline{x}\,\overline{y}$	$m\overline{y}\,\overline{z}$	$m\overline{z}\,\overline{x}$
1	$2\pi \frac{w}{g} a$	$2a$	a	$-a$	$4\pi \frac{w}{g} a^3$	$-2\pi \frac{w}{g} a^3$	$-4\pi \frac{w}{g} a^3$
2	$\frac{w}{g} a$	$2a$	$\frac{1}{2} a$	0	$\frac{w}{g} a^3$	0	0
3	$2\frac{w}{g} a$	$2a$	0	a	0	0	$4\frac{w}{g} a^3$
4	$3\pi \frac{w}{g} a$	$2a$	$-\frac{3}{2} a$	$2a$	$-9\pi \frac{w}{g} a^3$	$-9\pi \frac{w}{g} a^3$	$12\pi \frac{w}{g} a^3$
Σ					$\frac{w}{g}(1-5\pi)a^3$	$-11\pi \frac{w}{g} a^3$	$4\frac{w}{g}(1+2\pi)a^3$

PROBLEM 9.159 (Continued)

Then $I_{xy} = \Sigma(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y})$ or $I_{xy} = \dfrac{w}{g}a^3(1-5\pi)$ ◀

$I_{yz} = \Sigma(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z})$ or $I_{yz} = -11\pi\dfrac{w}{g}a^3$ ◀

$I_{zx} = \Sigma(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x})$ or $I_{zx} = 4\dfrac{w}{g}a^3(1+2\pi)$ ◀



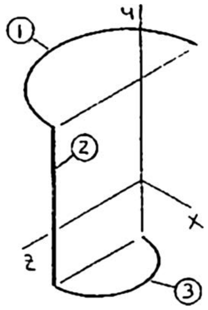
PROBLEM 9.160

Brass wire with a weight per unit length w is used to form the figure shown. Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION

First compute the mass of each component. We have

$$m = \frac{W}{g} = \frac{1}{g}wL$$



Then
$$m_1 = \frac{W}{g}\left(\pi \times \frac{3}{2}a\right) = \frac{3}{2}\pi\frac{w}{g}a$$

$$m_2 = \frac{w}{g}(3a) = 3\frac{w}{g}a$$

$$m_3 = \frac{w}{g}(\pi \times a) = \pi\frac{w}{g}a$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry.

	m	$\bar{x}$	$\bar{y}$	$\bar{z}$	$m\bar{x}\,\bar{y}$	$m\bar{y}\,\bar{z}$	$m\bar{z}\,\bar{x}$
1	$\frac{3}{2}\pi\frac{w}{g}a$	$-\frac{2}{\pi}\left(\frac{3}{2}a\right)$	$2a$	$\frac{1}{2}a$	$-9\frac{w}{g}a^3$	$\frac{3}{2}\pi\frac{w}{g}a^3$	$-\frac{9}{4}\frac{w}{g}a^3$
2	$3\frac{w}{g}a$	0	$\frac{1}{2}a$	$2a$	0	$3\frac{w}{g}a^3$	0
3	$\pi\frac{w}{g}a$	$\frac{2}{\pi}(a)$	$-a$	a	$-2\frac{w}{g}a^3$	$-\pi\frac{w}{g}a^3$	$2\frac{w}{g}a^3$
Σ					$-11\frac{w}{g}a^3$	$\frac{w}{g}\left(\frac{\pi}{2}+3\right)a^3$	$-\frac{1}{4}\frac{w}{g}a^3$

PROBLEM 9.160 (Continued)

Then $$I_{xy} = \Sigma(\overset{0}{\cancel{\bar{I}_{x'y'}}} + m\bar{x}\,\bar{y}) \qquad \text{or} \qquad I_{xy} = -11\frac{w}{g}a^3 \blacktriangleleft$$

$$I_{yz} = \Sigma(\overset{0}{\cancel{\bar{I}_{y'z'}}} + m\bar{y}\,\bar{z}) \qquad \text{or} \qquad I_{yz} = \frac{1}{2}\frac{w}{g}a^3(\pi + 6) \blacktriangleleft$$

$$I_{zx} = \Sigma(\overset{0}{\cancel{\bar{I}_{z'x'}}} + m\bar{z}\,\bar{x}) \qquad \text{or} \qquad I_{zx} = -\frac{1}{4}\frac{w}{g}a^3 \blacktriangleleft$$

PROBLEM 9.161

Complete the derivation of Eqs. (9.47), which express the parallel-axis theorem for mass products of inertia.

SOLUTION

We have $I_{xy} = \int xydm \quad I_{yz} = \int yzdm \quad I_{zx} = \int zxdm$ (9.45)

and $x = x' + \overline{x} \quad y = y' + \overline{y} \quad z = z' + \overline{z}$ (9.31)

Consider $I_{xy} = \int xydm$

Substituting for *x* and for *y*

$$I_{xy} = \int (x' + \overline{x})(y' + \overline{y})\, dm$$
$$= \int x'y'dm + \overline{y}\int x'dm + \overline{x}\int y'dm + \overline{x}\,\overline{y}\int dm$$

By definition $\overline{I}_{x'y'} = \int x'y'dm$

and $\int x'dm = m\overline{x}'$

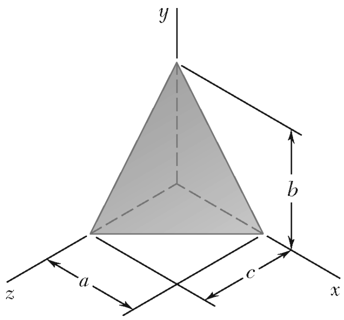
$\int y'dm = m\overline{y}'$

However, the origin of the primed coordinate system coincides with the mass center *G*, so that

$$\overline{x}' = \overline{y}' = 0$$

$I_{xy} = \overline{I}_{x'y'} + m\overline{x}\,\overline{y}$ Q.E.D. ◀

The expressions for I_{yz} and I_{zx} are obtained in a similar manner.

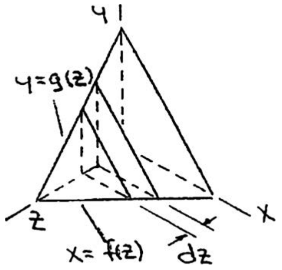


PROBLEM 9.162

For the homogeneous tetrahedron of mass m shown, (*a*) determine by direct integration the mass product of inertia I_{zx}, (*b*) deduce I_{yz} and I_{xy} from the result obtained in part *a*.

SOLUTION

(*a*) First divide the tetrahedron into a series of thin vertical slices of thickness dz as shown.



Now $$x = -\frac{a}{c}z + a = a\left(1 - \frac{z}{c}\right)$$

and $$y = -\frac{b}{c}z + b = b\left(1 - \frac{z}{c}\right)$$

The mass dm of the slab is

$$dm = \rho dV = \rho\left(\frac{1}{2}xydz\right) = \frac{1}{2}\rho ab\left(1 - \frac{z}{c}\right)^2 dz$$

Then $$m = \int dm = \int_0^c \frac{1}{2}\rho ab\left(1 - \frac{z}{c}\right)^2 dz$$

$$= \frac{1}{2}\rho ab\left[\left(-\frac{c}{3}\right)\left(1 - \frac{z}{c}\right)^3\right]_0^c = \frac{1}{6}\rho abc$$

Now $$dI_{zx} = d\bar{I}_{z'x'} + \bar{z}_{\text{EL}}\bar{x}_{\text{EL}}dm$$

where $$d\bar{I}_{z'x'} = 0 \quad \text{(symmetry)}$$

and $$\bar{z}_{\text{EL}} = z \quad \bar{x}_{\text{EL}} = \frac{1}{3}x = \frac{1}{3}a\left(1 - \frac{z}{c}\right)$$

Then $$I_{zx} = \int dI_{zx} = \int_0^c z\left[\frac{1}{3}a\left(1 - \frac{z}{c}\right)\right]\left[\frac{1}{2}\rho ab\left(1 - \frac{z}{c}\right)^2 dz\right]$$

$$= \frac{1}{6}\rho a^2 b\int_0^c \left(z - 3\frac{z^2}{c} + 3\frac{z^3}{c^2} - \frac{z^4}{c^3}\right)dz$$

$$= \frac{m}{c}a\left[\frac{1}{2}z^2 - \frac{z^3}{c} + \frac{3}{4}\frac{z^4}{c^2} - \frac{1}{5}\frac{z^5}{c^3}\right]_0^c$$

or $$I_{zx} = \frac{1}{20}mac \blacktriangleleft$$

PROBLEM 9.162 (Continued)

(*b*) Because of the symmetry of the body, I_{xy} and I_{yz} can be deduced by considering the circular permutation of (x, y, z) and (a, b, c).

Thus,

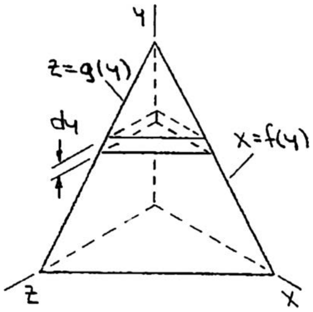
$$I_{xy} = \frac{1}{20}mab \blacktriangleleft$$

$$I_{yz} = \frac{1}{20}mbc \blacktriangleleft$$

Alternative solution for part *a*:

First divide the tetrahedron into a series of thin horizontal slices of thickness dy as shown.

Now $$x = -\frac{a}{b}y + a = a\left(1 - \frac{y}{b}\right)$$



and $$z = -\frac{c}{b}y + c = c\left(1 - \frac{y}{b}\right)$$

The mass dm of the slab is

$$dm = \rho dV = \rho\left(\frac{1}{2}xzdy\right) = \frac{1}{2}\rho ac\left(1 - \frac{y}{b}\right)^2 dy$$

Now $dI_{zx} = \rho t dI_{zx,\text{area}}$

where $t = dy$

and $dI_{zx,\text{area}} = \frac{1}{24}x^2z^2$ from the results of Sample Problem 9.6.

Then $$dIzx = \rho(dy)\left\{\frac{1}{24}\left[a\left(1 - \frac{y}{b}\right)^2\right]\left[c\left(1 - \frac{y}{b}\right)\right]^2\right\}$$

$$= \frac{1}{24}\rho a^2c^2\left(1 - \frac{y}{b}\right)^4 dy = \frac{1}{4}\frac{m}{b}ac\left(1 - \frac{y}{b}\right)^4 dy$$

Finally, $$I_{zx} = \int dI_{zx} = \int_0^b \frac{1}{4}\frac{m}{b}ac\left(1 - \frac{y}{b}\right)^4 dy$$

$$= \frac{1}{4}\frac{m}{b}ac\left[\left(-\frac{b}{5}\right)\left(1 - \frac{y}{b}\right)^5\right]_0^b$$

or $$I_{zx} = \frac{1}{20}mac \blacktriangleleft$$

Alternative solution for part *a*:

The equation of the included face of the tetrahedron is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

so that

$$y=b\left(1-\frac{x}{a}-\frac{z}{c}\right)$$

For an infinitesimal element of sides dx, dy, and dz:

$$dm=\rho dV=\rho dydxdz$$

From part *a*

$$x=a\left(1-\frac{z}{c}\right)$$

Now

$$I_{zx}=\int zxdm=\int_0^c\int_0^{a(1-z/c)}\int_0^{b(1-x/a-z/c)}zx(\rho dydxdz)$$

$$=\rho\int_0^c\int_0^{a(1-z/c)}zx\left[b\left(1-\tfrac{x}{a}-\tfrac{z}{c}\right)\right]dxdz$$

$$=\rho b\int_0^c z\left[\frac{1}{2}x^2-\frac{1}{3}\frac{x^3}{a}-\frac{1}{2}\frac{z}{c}x^2\right]_0^{a(1-z/c)}dz$$

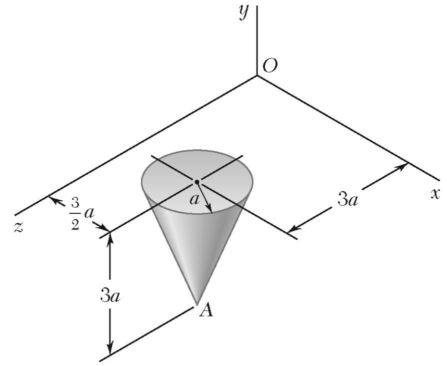
$$=\rho b\int_0^c z\left[\frac{1}{2}a^2\left(1-\frac{z}{c}\right)^2-\frac{1}{3a}a^3\left(1-\frac{z}{c}\right)^3-\frac{1}{2}\frac{z}{c}a^2\left(1-\frac{z}{c}\right)^2\right]dz$$

$$=\rho b\int_0^c\frac{1}{6}a^2z\left(1-\frac{z}{c}\right)^3dz$$

$$=\frac{1}{6}\rho a^2b\int_0^c\left(z-3\frac{z^2}{c}+3\frac{z^3}{c^2}-\frac{z^4}{c^3}\right)dz$$

$$=\frac{m}{c}a\left[\frac{1}{2}z^2-\frac{z^3}{c}+\frac{3}{4}\frac{z^4}{c^2}-\frac{1}{5}\frac{z^5}{c^3}\right]_0^c$$

or $I_{zx}=\dfrac{1}{20}mac$ ◀



PROBLEM 9.163

The homogeneous circular cone shown has a mass *m*. Determine the mass moment of inertia of the cone with respect to the line joining the origin *O* and Point *A*.

SOLUTION

First note that

$$d_{OA} = \sqrt{\left(\frac{3}{2}a\right)^2 + (-3a)^2 + (3a)^2} = \frac{9}{2}a$$

Then

$$\lambda_{OA} = \frac{1}{\frac{9}{2}a}\left(\frac{3}{2}a\mathbf{i} - 3a\mathbf{j} + 3a\mathbf{k}\right) = \frac{1}{3}(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

For a rectangular coordinate system with origin at Point *A* and axes aligned with the given x, y, z axes, we have (using Figure 9.28)

$$I_x = I_z = \frac{3}{5}m\left[\frac{1}{4}a^2 + (3a)^2\right] \qquad I_y = \frac{3}{10}ma^2$$

$$= \frac{111}{20}ma^2$$

Also, symmetry implies $\quad I_{xy} = I_{yz} = I_{zx} = 0$

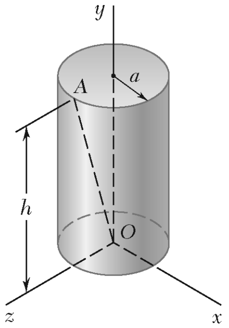
With the mass products of inertia equal to zero, Equation (9.46) reduces to

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2$$

$$= \frac{111}{20}ma^2\left(\frac{1}{3}\right)^2 + \frac{3}{10}ma^2\left(-\frac{2}{3}\right)^2 + \frac{111}{20}ma^2\left(\frac{2}{3}\right)^2$$

$$= \frac{193}{60}ma^2$$

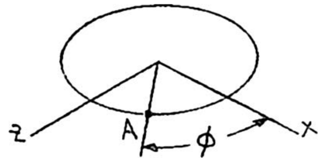
or $\quad I_{OA} = 3.22ma^2$ ◀



PROBLEM 9.164

The homogeneous circular cylinder shown has a mass m. Determine the mass moment of inertia of the cylinder with respect to the line joining the origin O and Point A that is located on the perimeter of the top surface of the cylinder.

SOLUTION



From Figure 9.28: $I_y = \frac{1}{2}ma^2$

and using the parallel-axis theorem

$$I_x = I_z = \frac{1}{12}m(3a^2 + h^2) + m\left(\frac{h}{2}\right)^2 = \frac{1}{12}m(3a^2 + 4h^2)$$

Symmetry implies $I_{xy} = I_{yz} = I_{zx} = 0$

For convenience, let Point A lie in the yz plane. Then

$$\lambda_{OA} = \frac{1}{\sqrt{h^2 + a^2}}(h\mathbf{j} + a\mathbf{k})$$

With the mass products of inertia equal to zero, Equation (9.46) reduces to

$$I_{OA} = \cancelto{0}{I_x\lambda_x^2} + I_y\lambda_y^2 + I_z\lambda_z^2$$

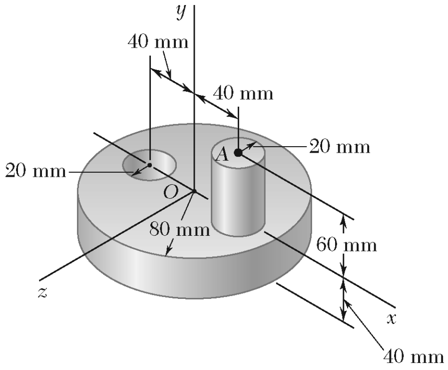
$$= \frac{1}{2}ma^2\left(\frac{h}{\sqrt{h^2 + a^2}}\right)^2 + \frac{1}{12}m(3a^2 + 4h^2)\left(\frac{a}{\sqrt{h^2 + a^2}}\right)^2$$

or $I_{OA} = \frac{1}{12}ma^2\frac{10h^2 + 3a^2}{h^2 + a^2}$ ◀

Note: For Point A located at an arbitrary point on the perimeter of the top surface, λ_{OA} is given by

$$\lambda_{OA} = \frac{1}{\sqrt{h^2 + a^2}}(a\cos\phi\mathbf{i} + h\mathbf{j} + a\sin\phi\mathbf{k})$$

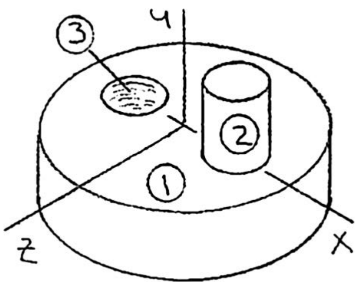
which results in the same expression for I_{OA}.



PROBLEM 9.165

Shown is the machine element of Problem 9.141. Determine its mass moment of inertia with respect to the line joining the origin O and Point A.

SOLUTION



First compute the mass of each component.

We have $$m = \rho_{\text{ST}} V = \frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2} V = (0.008819\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3)V$$

Then

$$m_1 = (7850\ \text{kg/m}^3)[\pi(0.08\ \text{m})^2(0.04\ \text{m})] = 6.31334\ \text{kg}$$
$$m_2 = (7850\ \text{kg/m}^3)[\pi(0.02\ \text{m})^2(0.06\ \text{m})] = 0.59188\ \text{kg}$$
$$m_3 = (7850\ \text{kg/m}^3)[\pi(0.02\ \text{m})^2(0.04\ \text{m})] = 0.39458\ \text{kg}$$

Symmetry implies $I_{yz} = I_{zx} = 0 \qquad (I_{xy})_1 = 0$

and $(\bar{I}_{x'y'})_2 = (\bar{I}_{x'y'})_3 = 0$

Now
$$\begin{aligned} I_{xy} &= \Sigma(\bar{I}_{x'y'} + m\bar{x}\,\bar{y}) = m_2\bar{x}_2\bar{y}_2 - m_3\bar{x}_3\bar{y}_3 \\ &= [0.59188\ \text{kg}\ (0.04\ \text{m})(0.03\ \text{m})] - [0.39458\ \text{kg}\ (-0.04\ \text{m})(-0.02\ \text{m})] \\ &= (0.71026 - 0.31566)\times 10^{-3}\ \text{kg}\cdot\text{m}^2 \\ &= 0.39460\times 10^{-3}\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

PROBLEM 9.165 (Continued)

From the solution to Problem 9.141, we have

$$I_x = 13.98800 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$
$$I_y = 20.55783 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$
$$I_z = 14.30368 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

By observation
$$\boldsymbol{\lambda}_{OA} = \frac{1}{\sqrt{13}}(2\mathbf{i} + 3\mathbf{j})$$

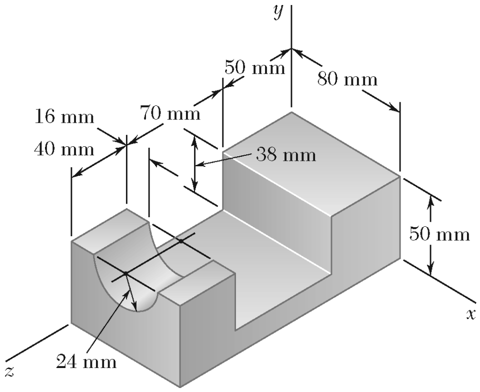
Substituting into Eq. (9.46)

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + \cancelto{0}{I_z\lambda_z^2} - 2I_{xy}\lambda_x\lambda_y - 2\cancelto{0}{I_{yz}\lambda_y\lambda_z} - 2\cancelto{0}{I_{zx}\lambda_z\lambda_x}$$

$$= \left[(13.98800)\left(\frac{2}{\sqrt{13}}\right)^2 + (20.55783)\left(\frac{3}{\sqrt{13}}\right)^2 - 2(0.39460)\left(\frac{2}{\sqrt{13}}\right)\left(\frac{2}{\sqrt{13}}\right)\right] \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$= (4.30400 + 14.23234 - 0.36425) \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

or $I_{OA} = 18.17 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀



PROBLEM 9.166

Determine the mass moment of inertia of the steel fixture of Problems 9.145 and 9.149 with respect to the axis through the origin that forms equal angles with the x, y, and z axes.

SOLUTION

From the solutions to Problems 9.145 and 9.149, we have

Problem 9.145: $I_x = 26.4325 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_y = 31.1726 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_z = 8.5773 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$

Problem 9.149: $I_{xy} = 2.5002 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_{yz} = 4.0627 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_{zx} = 8.8062 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$

From the problem statement it follows that

$$\lambda_x = \lambda_y = \lambda_z$$

Now $\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1 \Rightarrow 3\lambda_x^2 = 1$

or $\lambda_x = \lambda_y = \lambda_z = \dfrac{1}{\sqrt{3}}$

Substituting into Eq. (9.46)

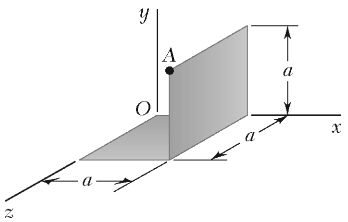
$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

Noting that $\lambda_x^2 = \lambda_y^2 = \lambda_z^2 = \lambda_x\lambda_y = \lambda_y\lambda_z = \lambda_z\lambda_x = \dfrac{1}{3}$

We have

$$I_{OL} = \frac{1}{3}[26.4325 + 31.1726 + 8.5773 - 2(2.5002 + 4.0627 + 8.8062)] \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

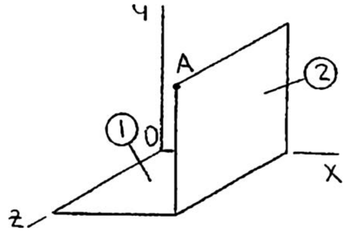
or $I_{OL} = 11.81 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀



PROBLEM 9.167

The thin bent plate shown is of uniform density and weight W. Determine its mass moment of inertia with respect to the line joining the origin O and Point A.

SOLUTION



First note that
$$m_1 = m_2 = \frac{1}{2}\frac{W}{g}$$

and that
$$\lambda_{OA} = \frac{1}{\sqrt{3}}(\mathbf{i}+\mathbf{j}+\mathbf{k})$$

Using Figure 9.28 and the parallel-axis theorem, we have

$$\begin{aligned}
I_x &= (I_x)_1 + (I_x)_2 \\
&= \left[\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)^2\right] \\
&\quad + \left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)(a^2+a^2) + \frac{1}{2}\frac{W}{g}\left[\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]\right\} \\
&= \frac{1}{2}\frac{W}{g}\left[\left(\frac{1}{12}+\frac{1}{4}\right)a^2 + \left(\frac{1}{6}+\frac{1}{2}\right)a^2\right] = \frac{1}{2}\frac{W}{g}a^2
\end{aligned}$$

$$\begin{aligned}
I_y &= (I_y)_1 + (I_y)_2 \\
&= \left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)(a^2+a^2) + \frac{1}{2}\frac{W}{g}\left[\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]\right\} \\
&\quad + \left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left[(a)^2 + \left(\frac{a}{2}\right)^2\right]\right\} \\
&= \frac{1}{2}\frac{W}{g}\left[\left(\frac{1}{6}+\frac{1}{2}\right)a^2 + \left(\frac{1}{12}+\frac{5}{4}\right)a^2\right] = \frac{W}{g}a^2
\end{aligned}$$

$$\begin{aligned}
I_z &= (I_z)_1 + (I_z)_2 \\
&= \left[\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)^2\right] \\
&\quad + \left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left[(a)^2 + \left(\frac{a}{2}\right)^2\right]\right\} \\
&= \frac{1}{2}\frac{W}{g}\left[\left(\frac{1}{12}+\frac{1}{4}\right)a^2 + \left(\frac{1}{12}+\frac{5}{4}\right)a^2\right] = \frac{5}{6}\frac{W}{g}a^2
\end{aligned}$$

PROBLEM 9.167 (Continued)

Now observe that the centroidal products of inertia, $\overline{I}_{x'y'}, \overline{I}_{y'z'}$, and $\overline{I}_{z'x'}$, of both components are zero because of symmetry. Also, $\overline{y}_1 = 0$

Then

$$I_{xy} = \Sigma(\cancelto{0}{\overline{I}_{x'y'}} + m\overline{x}\,\overline{y}) = m_2\overline{x}_2\overline{y}_2 = \frac{1}{2}\frac{W}{g}(a)\left(\frac{a}{2}\right) = \frac{1}{4}\frac{W}{g}a^2$$

$$I_{yz} = \Sigma(\cancelto{0}{\overline{I}_{y'z'}} + m\overline{y}\,\overline{z}) = m_2\overline{y}_2\overline{z}_2 = \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) = \frac{1}{8}\frac{W}{g}a^2$$

$$I_{zx} = \Sigma(\cancelto{0}{\overline{I}_{z'x'}} + m\overline{z}\,\overline{x}) = m_1\overline{z}_1\overline{x}_1 + m_2\overline{z}_2\overline{x}_2$$

$$= \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) + \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)(a) = \frac{3}{8}\frac{W}{g}a^2$$

Substituting into Equation (9.46)

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

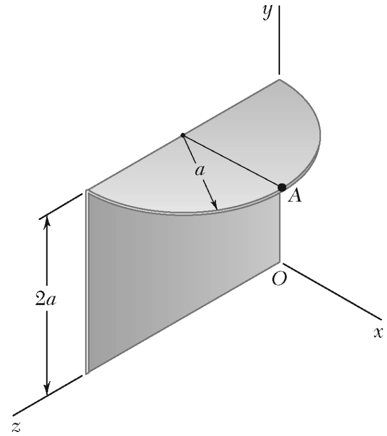
Noting that

$$\lambda_x^2 = \lambda_y^2 = \lambda_z^2 = \lambda_x\lambda_y = \lambda_y\lambda_z = \lambda_z\lambda_x = \frac{1}{3}$$

We have

$$I_{OA} = \frac{1}{3}\left[\frac{1}{2}\frac{W}{g}a^2 + \frac{W}{g}a^2 + \frac{5}{6}\frac{W}{g}a^2 - 2\left(\frac{1}{4}\frac{W}{g}a^2 + \frac{1}{8}\frac{W}{g}a^2 + \frac{3}{8}\frac{W}{g}a^2\right)\right]$$

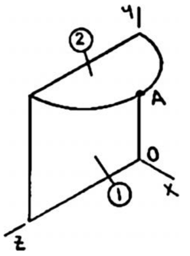
$$= \frac{1}{3}\left[\frac{14}{6} - 2\left(\frac{3}{4}\right)\right]\frac{W}{g}a^2 \qquad \text{or} \quad I_{OA} = \frac{5}{18}\frac{W}{g}a^2 \blacktriangleleft$$



PROBLEM 9.168

A piece of sheet steel of thickness t and specific weight γ is cut and bent into the machine component shown. Determine the mass moment of inertia of the component with respect to the joining the origin O and Point A.

SOLUTION



First note that

$$\boldsymbol{\lambda}_{OA} = \frac{1}{\sqrt{6}}(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

Next compute the mass of each component. We have

$$m = \rho V = \frac{\gamma}{g}(tA)$$

Then

$$m_1 = \frac{\gamma}{g}t(2a \times 2a) = 4\frac{\gamma t}{g}a^2$$

$$m_2 = \frac{\gamma}{g}t\left(\frac{\pi}{2} \times a^2\right) = \frac{\pi}{2}\frac{\gamma t}{g}a^2$$

Using Figure 9.28 for component 1 and the equations derived above (following the solution to Problem 9.134) for a semicircular plate for component 2, we have

$$\begin{aligned} I_x &= (I_x)_1 + (I_x)_2 \\ &= \left\{\frac{1}{12}\left(4\frac{\gamma t}{g}a^2\right)[(2a)^2 + (2a)^2] + 4\frac{\gamma t}{g}a^2(a^2 + a^2)\right\} \\ &\quad + \left\{\frac{1}{4}\left(\frac{\pi}{2}\frac{\gamma t}{g}a^2\right)a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2[(2a)^2 + (a)^2]\right\} \\ &= 4\frac{\gamma t}{g}a^2\left(\frac{2}{3} + 2\right)a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2\left(\frac{1}{4} + 5\right)a^2 \\ &= 18.91335\frac{\gamma t}{g}a^4 \end{aligned}$$

PROBLEM 9.168 (Continued)

$$\begin{aligned}
I_y &= (I_y)_1 + (I_y)_2 \\
&= \left[\frac{1}{12}\left(4\frac{\gamma t}{g}a^2\right)(2a)^2 + 4\frac{\gamma t}{g}a^2(a^2)\right] \\
&\quad + \left\{\frac{\pi}{2}\frac{\gamma t}{g}a^2\left(\frac{1}{2}-\frac{16}{9\pi^2}\right)a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2\left[\left(\frac{4a}{3\pi}\right)^2 + (a)^2\right]\right\} \\
&= 4\frac{\gamma t}{g}a^2\left(\frac{1}{3}+1\right)a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2\left(\frac{1}{2}-\frac{16}{9\pi^2}+\frac{16}{9\pi^2}+1\right)a^2 \\
&= 7.68953\frac{\gamma t}{g}a^4
\end{aligned}$$

$$\begin{aligned}
I_z &= (I_z)_1 + (I_z)_2 \\
&= \left[\frac{1}{12}\left(4\frac{\gamma t}{g}a^2\right)(2a)^2 + 4\frac{\gamma t}{g}a^2(a^2)\right] \\
&\quad + \left\{\frac{\pi}{2}\frac{\gamma t}{g}a^2\left(\frac{1}{4}-\frac{16}{9\pi^2}\right)a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2\left[\left(\frac{4a}{3\pi}\right)^2 + (2a)^2\right]\right\} \\
&= 4\frac{\gamma t}{g}a^2\left(\frac{1}{3}+1\right)a^2 + \frac{\pi}{2}\frac{\gamma t}{g}a^2\left(\frac{1}{4}-\frac{16}{9\pi^2}+\frac{16}{9\pi^2}+4\right)a^2 \\
&= 12.00922\frac{\gamma t}{g}a^4
\end{aligned}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of both components are zero because of symmetry. Also $\bar{x}_1 = 0$.

Then

$$\begin{aligned}
I_{xy} &= \Sigma(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}) = m_2\bar{x}_2\bar{y}_2 = \frac{\pi}{2}\frac{\gamma t}{g}a^2\left(\frac{4a}{3\pi}\right)(2a) \\
&= 1.33333\frac{\gamma t}{g}a^4
\end{aligned}$$

$$\begin{aligned}
I_{yz} &= \Sigma(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z}) = m_1\bar{y}_1\bar{z}_1 + m_2\bar{y}_2\bar{z}_2 \\
&= 4\frac{\gamma t}{g}a^2(a)(a) + \frac{\pi}{2}\frac{\gamma t}{g}a^2(2a)(a) \\
&= 7.14159\frac{\gamma t}{g}a^4
\end{aligned}$$

$$\begin{aligned}
I_{zx} &= \Sigma(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x}) = m_2\bar{z}_2\bar{x}_2 = \frac{\pi}{2}\frac{\gamma t}{g}a^2(a)\left(\frac{4a}{3\pi}\right) \\
&= 0.66667\frac{\gamma t}{g}a^4
\end{aligned}$$

PROBLEM 9.168 (Continued)

Substituting into Eq. (9.46)

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

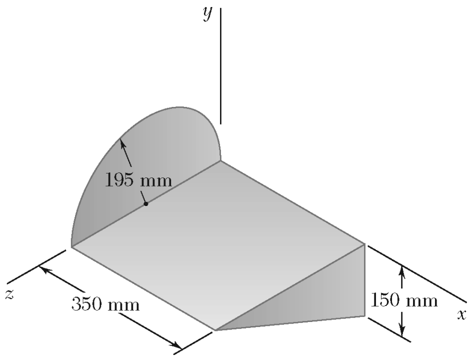
$$= 18.91335\frac{\gamma t}{g}a^4\left(\frac{1}{\sqrt{6}}\right)^2 + 7.68953\frac{\gamma t}{g}a^4\left(\frac{2}{\sqrt{6}}\right)^2$$

$$+12.00922\frac{\gamma t}{g}a^4\left(\frac{1}{\sqrt{6}}\right)^2 - 2\left(1.33333\frac{\gamma t}{g}a^4\right)\left(\frac{1}{\sqrt{6}}\right)\left(\frac{2}{\sqrt{6}}\right)$$

$$-2\left(7.14159\frac{\gamma t}{g}a^4\right)\left(\frac{2}{\sqrt{6}}\right)\left(\frac{2}{\sqrt{6}}\right) - 2\left(0.66667\frac{\gamma t}{g}a^4\right)\left(\frac{1}{\sqrt{6}}\right)\left(\frac{1}{\sqrt{6}}\right)$$

$$= (3.15223 + 5.12635 + 2.00154 - 0.88889 - 4.76106 - 0.22222)\frac{\gamma t}{g}a^4$$

or $$I_{OA} = 4.41\frac{\gamma t}{g}a^4 \blacktriangleleft$$



PROBLEM 9.169

Determine the mass moment of inertia of the machine component of Problems 9.136 and 9.155 with respect to the axis through the origin characterized by the unit vector $\boldsymbol{\lambda} = (-4\mathbf{i} + 8\mathbf{j} + \mathbf{k})/9$.

SOLUTION

From the solutions to Problems 9.136 and 9.155. We have

Problem 9.136: $I_x = 175.503 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

$I_y = 308.629 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

$I_z = 154.400 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

Problem 9.155: $I_{xy} = -8.0365 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

$I_{yz} = 12.8950 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

$I_{zx} = 94.0266 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

Substituting into Eq. (9.46)

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

$$= \left[175.503\left(-\frac{4}{9}\right)^2 + 308.629\left(\frac{8}{9}\right)^2 + 154.400\left(\frac{1}{9}\right)^2 \right.$$

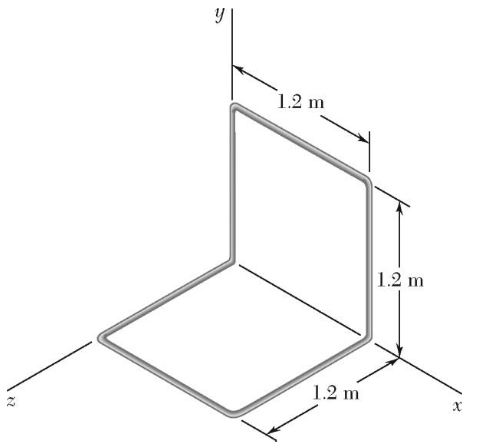
$$-2(-8.0365)\left(-\frac{4}{9}\right)\left(\frac{8}{9}\right) - 2(12.8950)\left(\frac{8}{9}\right)\left(\frac{1}{9}\right)$$

$$\left. -2(94.0266)\left(\frac{1}{9}\right)\left(-\frac{4}{9}\right)\right] \times 10^{-3}\ \text{kg} \cdot \text{m}^2$$

$$= (34.6673 + 243.855 + 1.906 - 6.350$$

$$-2.547 + 9.287) \times 10^{-3}\ \text{kg} \cdot \text{m}^2$$

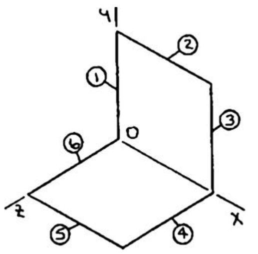
or $I_{OL} = 281 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$ ◀



PROBLEM 9.170

For the wire figure of Problem 9.148, determine the mass moment of inertia of the figure with respect to the axis through the origin characterized by the unit vector $\boldsymbol{\lambda} = (-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})/7$.

SOLUTION



First compute the mass of each component. We have

$$m = \left(\frac{m}{L}\right)L = 0.056\ \text{kg/m} \times 1.2\ \text{m}$$
$$= 0.0672\ \text{kg}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, for each component are zero because of symmetry.

Also

$$\bar{x}_1 = \bar{x}_6 = 0 \qquad \bar{y}_4 = \bar{y}_5 = \bar{y}_6 = 0 \qquad \bar{z}_1 = \bar{z}_2 = \bar{z}_3 = 0$$

Then

$$I_{xy} = \Sigma(\overset{0}{\cancel{\bar{I}_{x'y'}}} + m\bar{x}\,\bar{y}) = m_2\bar{x}_2\bar{y}_2 + m_3\bar{x}_3\bar{y}_3$$
$$= (0.0672\ \text{kg})(0.6\ \text{m})(1.2\ \text{m}) + (0.0672\ \text{kg})(1.2\ \text{m})(0.6\ \text{m})$$
$$= 0.096768\ \text{kg}\cdot\text{m}^2$$

$$I_{yz} = \Sigma(\overset{0}{\cancel{\bar{I}_{y'z'}}} + m\bar{y}\,\bar{z}) = 0$$

$$I_{zx} = \Sigma(\overset{0}{\cancel{\bar{I}_{z'x'}}} + m\bar{z}\,\bar{x}) = m_4\bar{z}_4\bar{x}_4 + m_5\bar{z}_5\bar{x}_5$$
$$= (0.0672\ \text{kg})(0.6\ \text{m})(1.2\ \text{m}) + (0.0672\ \text{kg})(1.2\ \text{m})(0.6\ \text{m})$$
$$= 0.096768\ \text{kg}\cdot\text{m}^2$$

From the solution to Problem 9.148, we have

$$I_x = 0.32258\ \text{kg}\cdot\text{m}^2$$
$$I_y = I_z = 0.41933\ \text{kg}\cdot\text{m}^2$$

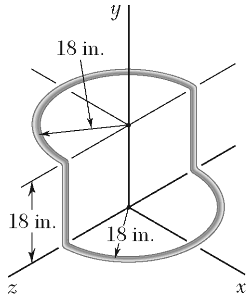
Substituting into Eq. (9.46)

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2\overset{0}{\cancel{I_{yz}}}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

$$= \left[0.32258\left(-\frac{3}{7}\right)^2 + 0.41933\left(-\frac{6}{7}\right)^2 + 0.41933\left(\frac{2}{7}\right)^2 - 2(0.096768)\left(-\frac{3}{7}\right)\left(-\frac{6}{7}\right) - 2(0.096768)\left(\frac{2}{7}\right)\left(-\frac{3}{7}\right)\right]\text{kg}\cdot\text{m}^2$$

$$I_{OL} = (0.059249 + 0.30808 + 0.034231 - 0.071095 + 0.023698)\ \text{kg}\cdot\text{m}^2$$

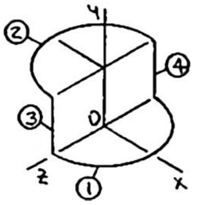
or $I_{OL} = 0.354\ \text{kg}\cdot\text{m}^2$ ◀



PROBLEM 9.171

For the wire figure of Problem 9.147, determine the mass moment of inertia of the figure with respect to the axis through the origin characterized by the unit vector $\boldsymbol{\lambda} = (-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})/7$.

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V = \frac{\gamma_{\text{ST}}}{g} AL$$

Then

$$m_1 = m_2 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{4}\left(\frac{1}{8} \text{ in.}\right)^2\right] \times (\pi \times 18 \text{ in.}) \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_3 = m_4 = \frac{490 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times \left[\frac{\pi}{4}\left(\frac{1}{8} \text{ in.}\right)^2\right] \times 18 \text{ in.} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 1.9453 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, for each component are zero because of symmetry.

Also $\quad \bar{x}_3 = \bar{x}_4 = 0 \qquad \bar{y}_1 = 0 \qquad \bar{z}_1 = \bar{z}_2 = 0$

Then

$$I_{xy} = \Sigma(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}) = m_2 \bar{x}_2 \bar{y}_2$$

$$= (6.1112 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft})\left(-\frac{2 \times 18}{\pi} \text{ in.}\right)(18 \text{ in.})\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= -8.75480 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$I_{yz} = \Sigma(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z}) = m_3 \bar{y}_3 \bar{z}_3 + m_4 \bar{y}_4 \bar{z}_4$$

Now $\quad m_3 = m_4, \qquad \bar{y}_3 = \bar{y}_4, \qquad \bar{z}_4 = -\bar{z}_3 \qquad I_{yz} = 0$

$$I_{zx} = \Sigma(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x}) \quad \text{or} \quad I_{zx} = 0$$

PROBLEM 9.171 (Continued)

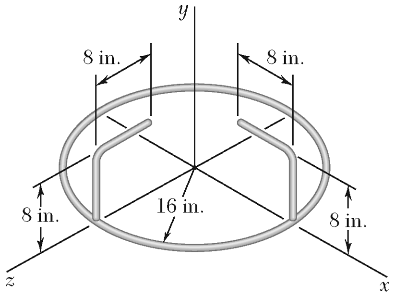
From the solution to Problem 9.147, we have

$$I_x = 39.1721\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$
$$I_y = 36.2542\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$
$$I_z = 30.4184\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Substituting into Eq. (9.46)

$$\begin{aligned}I_{OL} &= I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2\overset{0}{\cancel{I_{yz}}}\lambda_y\lambda_z - 2\overset{0}{\cancel{I_{zx}}}\lambda_z\lambda_x\\ &= \left[39.1721\left(-\frac{3}{7}\right)^2 + 36.2542\left(-\frac{6}{7}\right)^2 + 30.4184\left(\frac{2}{7}\right)^2\right.\\ &\qquad \left. - 2(-8.75480)\left(-\frac{3}{7}\right)\left(-\frac{6}{7}\right)\right]\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2\\ &= (7.19488 + 26.6357 + 2.48313 + 6.43210)\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2\end{aligned}$$

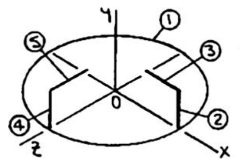
or $I_{OL} = 0.0427\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.172

For the wire figure of Problem 9.146, determine the mass moment of inertia of the figure with respect to the axis through the origin characterized by the unit vector $\boldsymbol{\lambda} = (-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})/7$.

SOLUTION



First compute the mass of each component. We have

$$m = \frac{W}{g} = \frac{1}{g}(W/L)_{\text{AL}}\,L$$

Then

$$m_1 = \frac{1}{32.2\text{ ft/s}^2}(0.033\text{ lb/ft})(2\pi \times 16\text{ in.}) \times \frac{1\text{ ft}}{12\text{ in.}}$$
$$= 8.5857 \times 10^{-3}\text{ lb}\cdot\text{s}^2\text{/ft}$$

$$m_2 = m_3 = m_4 = m_5 = \frac{1}{32.2\text{ ft/s}^2}(0.033\text{ lb/ft})(8\text{ in.}) \times \frac{1\text{ ft}}{12\text{ in.}}$$
$$= 0.6832 \times 10^{-3}\text{ lb}\cdot\text{ft/s}^2$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry. Also

$$\bar{x}_1 = \bar{x}_4 = \bar{x}_5 = 0 \qquad \bar{y}_1 = 0 \qquad \bar{z}_1 = \bar{z}_2 = \bar{z}_3 = 0$$

Then

$$I_{xy} = \Sigma(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}) = m_2\bar{x}_2\bar{y}_2 + m_3\bar{x}_3\bar{y}_3$$
$$= 0.6832 \times 10^{-3}\text{ lb}\cdot\text{s}^2\text{/ft}\left(\frac{16}{12}\text{ ft}\right)\left(\frac{4}{12}\text{ ft}\right)$$
$$+ 0.6832 \times 10^{-3}\text{ lb}\cdot\text{s}^2\text{/ft}\left(\frac{12}{12}\text{ ft}\right)\left(\frac{8}{12}\text{ ft}\right)$$
$$= (0.30364 + 0.45547) \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$
$$= 0.75911 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

Symmetry implies

$$I_{yz} = I_{xy} \qquad I_{yz} = 0.75911 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{zx} = \Sigma(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x}) = 0$$

PROBLEM 9.172 (Continued)

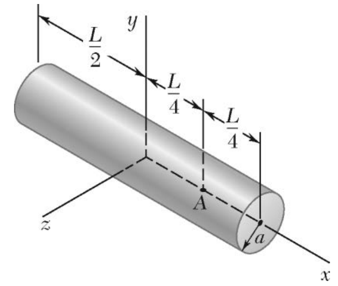
From the solution to Problem 9.146, we have

$$I_x = I_z = 10.3642 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$
$$I_y = 19.1097 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Substituting into Eq. (9.46)

$$\begin{aligned}
I_{OL} &= I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2\cancelto{0}{I_{zx}}\lambda_z\lambda_x \\
&= \left[10.3642\left(-\frac{3}{7}\right)^2 + 19.1097\left(-\frac{6}{7}\right)^2 + 10.3642\left(\frac{2}{7}\right)^2\right. \\
&\qquad \left. - 2(0.75911)\left(-\frac{3}{7}\right)\left(-\frac{6}{7}\right) - 2(0.75911)\left(-\frac{6}{7}\right)\left(\frac{2}{7}\right)\right] \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \\
&= (1.90663 + 14.03978 + 0.84606 - 0.55771 + 0.37181) \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2
\end{aligned}$$

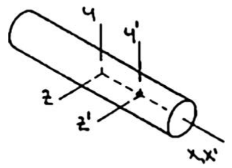
or $I_{OL} = 16.61 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.173

For the homogeneous circular cylinder shown, of radius a and length L, determine the value of the ratio a/L for which the ellipsoid of inertia of the cylinder is a sphere when computed (*a*) at the centroid of the cylinder, (*b*) at Point A.

SOLUTION



(*a*) From Figure 9.28:

$$\bar{I}_x = \frac{1}{2}ma^2$$

$$\bar{I}_y = \bar{I}_z = \frac{1}{12}m(3a^2 + L^2)$$

Now observe that symmetry implies $I_{xy} = I_{yz} = I_{zx} = 0$

Using Eq. (9.48), the equation of the ellipsoid of inertia is then

$$I_x x^2 + I_y y^2 + I_z z^2 = 1: \quad \frac{1}{2}ma^2x^2 + \frac{1}{12}m(3a^2 + L^2)y^2 + \frac{1}{12}m(3a^2 + L^2) = 1$$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore

$$\frac{1}{2}ma^2 = \frac{1}{12}m(3a^2 + L^2) \qquad \text{or} \qquad \frac{a}{L} = \frac{1}{\sqrt{3}} \blacktriangleleft$$

(*b*) Using Figure 9.28 and the parallel-axis theorem, we have

$$I_{x'} = \frac{1}{2}ma^2$$

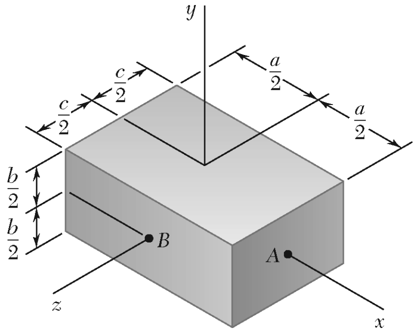
$$I_{y'} = I_{z'} = \frac{1}{12}m(3a^2 + L^2) + m\left(\frac{L}{4}\right)^2 = m\left(\frac{1}{4}a^2 + \frac{7}{48}L^2\right)$$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

From Part *a* it then immediately follows that

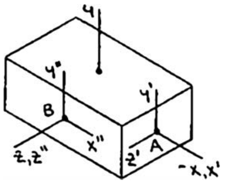
$$\frac{1}{2}ma^2 = m\left(\frac{1}{4}a^2 + \frac{7}{48}L^2\right) \qquad \text{or} \qquad \frac{a}{L} = \sqrt{\frac{7}{12}} \blacktriangleleft$$



PROBLEM 9.174

For the rectangular prism shown, determine the values of the ratios b/a and c/a so that the ellipsoid of inertia of the prism is a sphere when computed (*a*) at Point *A*, (*b*) at Point *B*.

SOLUTION



(*a*) Using Figure 9.28 and the parallel-axis theorem, we have at Point *A*

$$I_{x'} = \frac{1}{12}m(b^2 + c^2)$$

$$I_{y'} = \frac{1}{12}m(a^2 + c^2) + m\left(\frac{a}{2}\right)^2$$

$$= \frac{1}{12}m(4a^2 + c^2)$$

$$I_{z'} = \frac{1}{12}m(a^2 + b^2) + m\left(\frac{a}{2}\right)^2 = \frac{1}{12}m(4a^2 + b^2)$$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

Using Eq. (9.48), the equation of the ellipsoid of inertia is then

$$I_{x'}x^2 + I_{y'}y^2 + I_{z'}z^2 = 1: \quad \frac{1}{12}m(b^2 + c^2)x^2 + \frac{1}{12}m(4a^2 + c^2)y^2 + \frac{1}{12}m(4a^2 + b^2)z^2 = 1$$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore

$$\frac{1}{12}m(b^2 + c^2) = \frac{1}{12}m(4a^2 + c^2)$$

$$= \frac{1}{12}m(4a^2 + b^2)$$

Then $\quad b^2 + c^2 = 4a^2 + c^2 \quad$ or $\quad \frac{b}{a} = 2$ ◀

and $\quad b^2 + c^2 = 4a^2 + b^2 \quad$ or $\quad \frac{c}{a} = 2$ ◀

PROBLEM 9.174 (Continued)

(*b*) Using Figure 9.28 and the parallel-axis theorem, we have at Point *B*

$$I_{x''} = \frac{1}{12}m(b^2 + c^2) + m\left(\frac{c}{2}\right)^2 = \frac{1}{12}m(b^2 + 4c^2)$$

$$I_{y''} = \frac{1}{12}m(a^2 + c^2) + m\left(\frac{c}{2}\right)^2 = \frac{1}{12}m(a^2 + 4c^2)$$

$$I_{z''} = \frac{1}{12}m(a^2 + b^2)$$

Now observe that symmetry implies

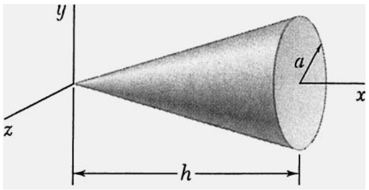
$$I_{x''y''} = I_{y''z''} = I_{z''x''} = 0$$

From part *a* it then immediately follows that

$$\frac{1}{12}m(b^2 + 4c^2) = \frac{1}{12}m(a^2 + 4c^2) = \frac{1}{12}m(a^2 + b^2)$$

Then $b^2 + 4c^2 = a^2 + 4c^2$ or $\frac{b}{a} = 1$ ◀

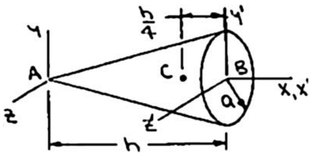
and $b^2 + 4c^2 = a^2 + b^2$ or $\frac{c}{a} = \frac{1}{2}$ ◀



PROBLEM 9.175

For the right circular cone of Sample Problem 9.11, determine the value of the ratio *a/h* for which the ellipsoid of inertia of the cone is a sphere when computed (*a*) at the apex of the cone, (*b*) at the center of the base of the cone.

SOLUTION



(*a*) From Sample Problem 9.11, we have at the apex *A*

$$I_x = \frac{3}{10}ma^2$$

$$I_y = I_z = \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right)$$

Now observe that symmetry implies $I_{xy} = I_{yz} = I_{zx} = 0$

Using Eq. (9.48), the equation of the ellipsoid of inertia is then

$$I_x x^2 + I_y y^2 + I_z z^2 = 1: \quad \frac{3}{10}ma^2x^2 + \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right)y^2 + \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right)z^2 = 1$$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore,

$$\frac{3}{10}ma^2 = \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right) \qquad \text{or} \qquad \frac{a}{h} = 2 \blacktriangleleft$$

(*b*) From Sample Problem 9.11, we have

$$I_{x'} = \frac{3}{10}ma^2$$

and at the centroid *C* $\quad I_{y''} = \frac{3}{20}m\left(a^2 + \frac{1}{4}h^2\right)$

Then $\quad I_{y'} = I_{z'} = \frac{3}{20}m\left(a^2 + \frac{1}{4}h^2\right) + m\left(\frac{h}{4}\right)^2 = \frac{1}{20}m(3a^2 + 2h^2)$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

From Part *a* it then immediately follows that

$$\frac{3}{10}ma^2 = \frac{1}{20}m(3a^2 + 2h^2) \qquad \text{or} \qquad \frac{a}{h} = \sqrt{\frac{2}{3}} \blacktriangleleft$$

PROBLEM 9.176

Given an arbitrary body and three rectangular axes x, y, and z, prove that the mass moment of inertia of the body with respect to any one of the three axes cannot be larger than the sum of the mass moments of inertia of the body with respect to the other two axes. That is, prove that the inequality $I_x \le I_y + I_z$ and the two similar inequalities are satisfied. Further, prove that $I_y \ge \frac{1}{2} I_x$ if the body is a homogeneous solid of revolution, where x is the axis of revolution and y is a transverse axis.

SOLUTION

(i) To prove $\quad I_y + I_z \ge I_x$

By definition
$$I_y = \int (z^2 + x^2)dm$$
$$I_z = \int (x^2 + y^2)dm$$

Then
$$I_y + I_z = \int (z^2 + x^2)dm + \int (x^2 + y^2)dm$$
$$= \int (y^2 + z^2)dm + 2\int x^2 dm$$

Now $\quad \int (y^2 + z^2)dm = I_x \quad$ and $\quad \int x^2 dm \ge 0$

$$I_y + I_z \ge I_x \qquad \text{Q.E.D.}$$

The proofs of the other two inequalities follow similar steps.

(ii) If the x axis is the axis of revolution, then

$$I_y = I_z$$

and from part (i) $\quad I_y + I_z \ge I_x$

or $\quad 2I_y \ge I_x$

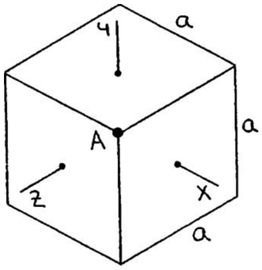
or
$$I_y \ge \frac{1}{2} I_x \qquad \text{Q.E.D.}$$

PROBLEM 9.177

Consider a cube of mass m and side a. (*a*) Show that the ellipsoid of inertia at the center of the cube is a sphere, and use this property to determine the moment of inertia of the cube with respect to one of its diagonals. (*b*) Show that the ellipsoid of inertia at one of the corners of the cube is an ellipsoid of revolution, and determine the principal moments of inertia of the cube at that point.

SOLUTION

(*a*) At the center of the cube have (using Figure 9.28)



$$I_x = I_y = I_z = \frac{1}{12}m(a^2 + a^2) = \frac{1}{6}ma^2$$

Now observe that symmetry implies

$$I_{xy} = I_{yz} = I_{zx} = 0$$

Using Equation (9.48), the equation of the ellipsoid of inertia is

$$\left(\frac{1}{6}ma^2\right)x^2 + \left(\frac{1}{6}ma^2\right)y^2 + \left(\frac{1}{6}ma^2\right)z^2 = 1$$

or $$x^2 + y^2 + z^2 = \frac{6}{ma^2} \quad (= R^2) \blacktriangleleft$$

which is the equation of a sphere.

Since the ellipsoid of inertia is a sphere, the moment of inertia with respect to any axis OL through the center O of the cube must always

be the same $\left(R = \dfrac{1}{\sqrt{I_{OL}}}\right)$. $$I_{OL} = \frac{1}{6}ma^2 \blacktriangleleft$$

(*b*) The above sketch of the cube is the view seen if the line of sight is along the diagonal that passes through corner A. For a rectangular coordinate system at A and with one of the coordinate axes aligned with the diagonal, an ellipsoid of inertia at A could be constructed. If the cube is then rotated 120° about the diagonal, the mass distribution will remain unchanged. Thus, the ellipsoid will also remain unchanged after it is rotated. As noted at the end of Section 9.17, this is possible only if the ellipsoid is an ellipsoid of revolution, where the diagonal is both the axis of revolution and a principal axis.

It then follows that $$I_{x'} = I_{OL} = \frac{1}{6}ma^2 \blacktriangleleft$$

In addition, for an ellipsoid of revolution, the two transverse principal moments of inertia are equal and any axis perpendicular to the axis of revolution is a principal axis. Then, applying the parallel-axis theorem between the center of the cube and corner A for any perpendicular axis

$$I_{y'} = I_{z'} = \frac{1}{6}ma^2 + m\left(\frac{\sqrt{3}}{2}a\right)^2$$

or $$I_{y'} = I_{z'} = \frac{11}{12}ma^2 \blacktriangleleft$$

(*Note:* Part *b* can also be solved using the method of Section 9.18.)

PROBLEM 9.177 (Continued)

First note that at corner A

$$I_x = I_y = I_z = \frac{2}{3}ma^2$$

$$I_{xy} = I_{yz} = I_{zx} = \frac{1}{4}ma^2$$

Substituting into Equation (9.56) yields

$$k^3 - 2ma^2k^2 + \frac{55}{48}m^2a^6k - \frac{121}{864}m^3a^9 = 0$$

For which the roots are

$$k_1 = \frac{1}{6}ma^2$$

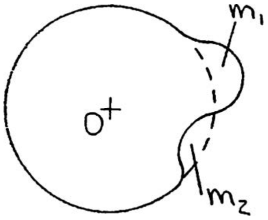
$$k_2 = k_3 = \frac{11}{12}ma^2$$

PROBLEM 9.178

Given a homogeneous body of mass m and of arbitrary shape and three rectangular axes x, y, and z with origin at O, prove that the sum $I_x + I_y + I_z$ of the mass moments of inertia of the body cannot be smaller than the similar sum computed for a sphere of the same mass and the same material centered at O. Further, using the result of Problem 9.176, prove that if the body is a solid of revolution, where x is the axis of revolution, its mass moment of inertia I_y about a transverse axis y cannot be smaller than $3ma^2/10$, where a is the radius of the sphere of the same mass and the same material.

SOLUTION

(i) Using Equation (9.30), we have



$$I_x + I_y + I_z = \int (y^2+z^2)dm + \int (z^2+x^2)\,dm + \int (x^2+y^2)dm$$

$$= 2\int (x^2+y^2+z^2)dm$$

$$= 2\int r^2 dm$$

where r is the distance from the origin O to the element of mass dm. Now assume that the given body can be formed by adding and subtracting appropriate volumes V_1 and V_2 from a sphere of mass m and radius a which is centered at O; it then follows that

$$m_1 = m_2 \ (m_{\text{body}} = m_{\text{sphere}} = m)$$

Then

$$(I_x + I_y + I_z)_{\text{body}} = (I_x + I_y + I_z)_{\text{sphere}} + (I_x + I_y + I_z)_{V_1} - (I_x + I_y + I_z)_{V_2}$$

or $$(I_x + I_y + I_z)_{\text{body}} = (I_x + I_y + I_z)_{\text{sphere}} + 2\int_{m_1} r^2 dm - 2\int_{m_2} r^2 dm$$

Now, $m_1 = m_2$ and $r_1 \ge r_2$ for all elements of mass dm in volumes 1 and 2.

$$\int_{m_1} r^2 dm - \int_{m_2} r^2 dm \ge 0$$

so that $$(I_x + I_y + I_z)_{\text{body}} \ge (I_x + I_y + I_z)_{\text{sphere}} \quad \text{Q.E.D.}$$

PROBLEM 9.178 (Continued)

(ii) First note from Figure 9.28 that for a sphere

$$I_x = I_y = I_z = \frac{2}{5}ma^2$$

Thus $$(I_x + I_y + I_z)_{\text{sphere}} = \frac{6}{5}ma^2$$

For a solid of revolution, where the *x* axis is the axis of revolution, we have

$$I_y = I_z$$

Then, using the results of part (i)

$$(I_x + 2I_y)_{\text{body}} \geq \frac{6}{5}ma^2$$

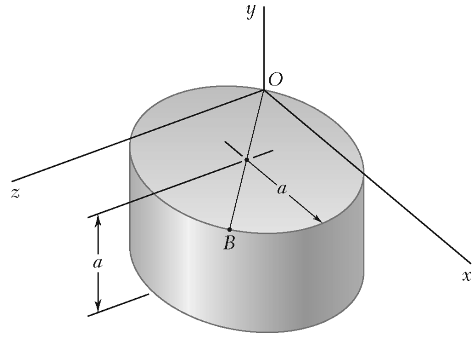
From Problem 9.178 we have $$I_y \geq \frac{1}{2}I_x$$

or $$(2I_y - I_x)_{\text{body}} \geq 0$$

Adding the last two inequalities yields

$$(4I_y)_{\text{body}} \geq \frac{6}{5}ma^2$$

or $$(I_y)_{\text{body}} \geq \frac{3}{10}ma^2 \qquad \text{Q.E.D.}$$

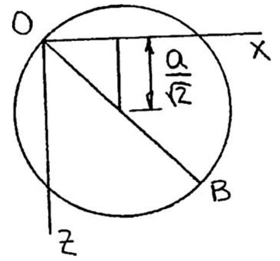


PROBLEM 9.179*

The homogeneous circular cylinder shown has a mass m, and the diameter OB of its top surface forms 45° angles with the x and z axes. (*a*) Determine the principal mass moments of inertia of the cylinder at the origin O. (*b*) Compute the angles that the principal axes of inertia at O form with the coordinate axes. (*c*) Sketch the cylinder, and show the orientation of the principal axes of inertia relative to the x, y, and z axes.

SOLUTION

(*a*) First compute the moments of inertia using Figure 9.28 and the parallel-axis theorem.



$$I_x = I_z = \frac{1}{12}m(3a^2 + a^2) + m\left[\left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{a}{2}\right)^2\right] = \frac{13}{12}ma^2$$

$$I_y = \frac{1}{2}ma^2 + m(a)^2 = \frac{3}{2}ma^2$$

Next observe that the centroidal products of inertia are zero because of symmetry. Then

$$I_{xy} = \cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y} = m\left(\frac{a}{\sqrt{2}}\right)\left(-\frac{a}{2}\right) = -\frac{1}{2\sqrt{2}}ma^2$$

$$I_{yz} = \cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z} = m\left(-\frac{a}{2}\right)\left(\frac{a}{\sqrt{2}}\right) = -\frac{1}{2\sqrt{2}}ma^2$$

$$I_{zx} = \cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x} = m\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right) = \frac{1}{2}ma^2$$

Substituting into Equation (9.56)

$$\begin{aligned}
&K^3 - \left(\frac{13}{12} + \frac{3}{2} + \frac{13}{12}\right)ma^2K^2 \\
&\quad + \left[\left(\frac{13}{12}\times\frac{3}{2}\right) + \left(\frac{3}{2}\times\frac{13}{12}\right) + \left(\frac{13}{12}\times\frac{13}{12}\right) - \left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(\frac{1}{2}\right)^2\right](ma^2)^2K \\
&\quad - \left[\left(\frac{13}{12}\times\frac{3}{2}\times\frac{13}{12}\right) - \left(\frac{13}{12}\right)\left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)^2\right. \\
&\quad \left. - \left(\frac{13}{12}\right)\left(-\frac{1}{2\sqrt{2}}\right)^2 - 2\left(-\frac{1}{2\sqrt{2}}\right)\left(-\frac{1}{2\sqrt{2}}\right)\left(\frac{1}{2}\right)\right](ma^2)^3 = 0
\end{aligned}$$

PROBLEM 9.179* (Continued)

Simplifying and letting $K = ma^2\zeta$ yields

$$\zeta^3 - \frac{11}{3}\zeta^2 + \frac{565}{144}\zeta - \frac{95}{96} = 0$$

Solving yields

$$\zeta_1 = 0.363383 \qquad \zeta_2 = \frac{19}{12} \qquad \zeta_3 = 1.71995$$

The principal moments of inertia are then

$K_1 = 0.363ma^2$ ◀

$K_2 = 1.583ma^2$ ◀

$K_3 = 1.720ma^2$ ◀

(*b*) To determine the direction cosines $\lambda_x, \lambda_y, \lambda_z$ of each principal axis, we use two of the equations of Equations (9.54) and (9.57).

Thus,

$$(I_x - K)\lambda_x - I_{xy}\lambda_y - I_{zx}\lambda_z = 0 \tag{9.54a}$$

$$-I_{zx}\lambda_x - I_{yz}\lambda_y + (I_z - K)\lambda_z = 0 \tag{9.54c}$$

$$\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1 \tag{9.57}$$

(*Note:* Since $I_{xy} = I_{yz}$, Equations (9.54a) and (9.54c) were chosen to simplify the "elimination" of λ_y during the solution process.)

Substituting for the moments and products of inertia in Equations (9.54a) and (9.54c)

$$\left(\frac{13}{12}ma^2 - K\right)\lambda_x - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_y - \left(\frac{1}{2}ma^2\right)\lambda_z = 0$$

$$-\left(\frac{1}{2}ma^2\right)\lambda_x - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_y + \left(\frac{13}{12}ma^2 - K\right)\lambda_z = 0$$

or

$$\left(\frac{13}{12} - \zeta\right)\lambda_x + \frac{1}{2\sqrt{2}}\lambda_y - \frac{1}{2}\lambda_z = 0 \tag{i}$$

and

$$-\frac{1}{2}\lambda_x + \frac{1}{2\sqrt{2}}\lambda_y + \left(\frac{13}{12} - \zeta\right)\lambda_z = 0 \tag{ii}$$

Observe that these equations will be identical, so that one will need to be replaced, if

$$\frac{13}{12} - \zeta = -\frac{1}{2} \qquad \text{or} \qquad \zeta = \frac{19}{12}$$

Thus, a third independent equation will be needed when the direction cosines associated with K_2 are determined. Then for K_1 and K_3

Eq. (i) through Eq. (ii): $\left[\frac{13}{12}-\zeta-\left(-\frac{1}{2}\right)\right]\lambda_x+\left[-\frac{1}{2}-\left(\frac{13}{12}-\zeta\right)\right]\lambda_z=0$

or $$\lambda_z=\lambda_x$$

Substituting into Eq. (i): $\left(\frac{13}{12}-\zeta\right)\lambda_x+\frac{1}{2\sqrt{2}}\lambda_y-\frac{1}{2}\lambda_x=0$

or $$\lambda_y=2\sqrt{2}\left(\zeta-\frac{7}{12}\right)\lambda_x$$

Substituting into Equation (9.57):

$$\lambda_x^2+\left[2\sqrt{2}\left(\zeta-\frac{7}{12}\right)\lambda_x\right]^2+(\lambda_x)^2=1$$

or $$\left[2+8\left(\zeta-\frac{7}{12}\right)^2\right]\lambda_x^2=1 \qquad \text{(iii)}$$

K$_1$: Substituting the value of ζ_1 into Eq. (iii):

$$\left[2+8\left(0.363383-\frac{7}{12}\right)^2\right](\lambda_x)_1^2=1$$

or $$(\lambda_x)_1=(\lambda_z)_1=0.647249$$

and then $$(\lambda_y)_1=2\sqrt{2}\left(0.363383-\frac{7}{12}\right)(0.647249)$$
$$=-0.402662$$

$$(\theta_x)_1=(\theta_z)_1=49.7° \quad (\theta_y)_1=113.7° \blacktriangleleft$$

K$_3$: Substituting the value of ζ_3 into Eq. (iii):

$$\left[2+8\left(1.71995-\frac{7}{12}\right)^2\right](\lambda_x)_3^2=1$$

or $$(\lambda_x)_3=(\lambda_z)_3=0.284726$$

and then $$(\lambda_y)_3=2\sqrt{2}\left(1.71995-\frac{7}{12}\right)(0.284726)$$
$$=0.915348$$

$$(\theta_x)_3=(\theta_z)_3=73.5° \quad (\theta_y)_3=23.7° \blacktriangleleft$$

PROBLEM 9.179* (Continued)

K$_2$: For this case, the set of equations to be solved consists of Equations (9.54a), (9.54b), and (9.57). Now

$$-I_{xy}\lambda_x + (I_y - K)\lambda_y - I_{yz}\lambda_z = 0 \tag{9.54b}$$

Substituting for the moments and products of inertia.

$$-\left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_x + \left(\frac{3}{2}ma^2 - K\right)\lambda_y - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_z = 0$$

or

$$\frac{1}{2\sqrt{2}}\lambda_x + \left(\frac{3}{2} - \xi\right)\lambda_y + \frac{1}{2\sqrt{2}}\lambda_z = 0 \tag{iv}$$

Substituting the value of ξ_2 into Eqs. (i) and (iv):

$$\left(\frac{13}{12} - \frac{19}{12}\right)(\lambda_x)_2 + \frac{1}{2\sqrt{2}}(\lambda_y)_2 - \frac{1}{2}(\lambda_z)_2 = 0$$

$$\frac{1}{2\sqrt{2}}(\lambda_x)_2 + \left(\frac{3}{2} - \frac{19}{12}\right)(\lambda_y)_2 + \frac{1}{2\sqrt{2}}(\lambda_z)_2 = 0$$

or

$$-(\lambda_x)_2 + \frac{1}{\sqrt{2}}(\lambda_y)_2 - (\lambda_z)_2 = 0$$

and

$$(\lambda_x)_2 - \frac{\sqrt{2}}{6}(\lambda_y)_2 + (\lambda_z)_2 = 0$$

Adding yields $(\lambda_y)_2 = 0$

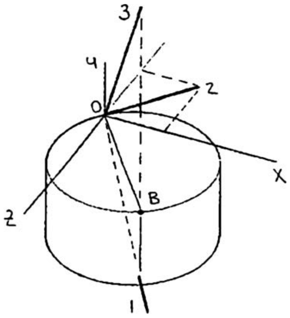
and then $(\lambda_y)_2 = -(\lambda_x)_2$

Substituting into Equation (9.57)

$$(\lambda_x)_2^2 + \overset{0}{\cancel{(\lambda_y)_2^2}} + (-\lambda_x)_2^2 = 1$$

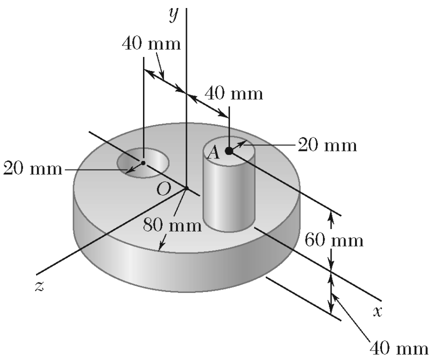
or

$$(\lambda_x)_2 = \frac{1}{\sqrt{2}} \quad \text{and} \quad (\lambda_z)_2 = -\frac{1}{\sqrt{2}}$$



$$(\theta_x)_2 = 45.0^\circ \quad (\theta_y)_2 = 90.0^\circ \quad (\theta_z)_2 = 135.0^\circ \blacktriangleleft$$

(*c*) Principal axes 1 and 3 lie in the vertical plane of symmetry passing through Points *O* and *B*. Principal axis 2 lies in the *xz* plane.



PROBLEM 9.180

For the component described in Problem 9.165, determine (*a*) the principal mass moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION

(*a*) From the solutions to Problems 9.141 and 9.165 we have

Problem 9.141:

$$I_x = 13.98800 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$$

$$I_y = 20.55783 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$$

$$I_z = 14.30368 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$$

Problem 9.165:

$$I_{yz} = I_{zx} = 0$$

$$I_{xy} = 0.39460 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$$

Eq. (9.55) then becomes

$$\begin{vmatrix} I_x - K & -I_{xy} & 0 \\ -I_{xy} & I_y - K & 0 \\ 0 & 0 & I_z - K \end{vmatrix} = 0 \quad \text{or} \quad (I_x - K)(I_y - K)(I_z - K) - (I_z - K)I_{xy}^2 = 0$$

Thus $\qquad I_z - K = 0 \qquad I_x I_y - (I_x + I_y)K + K^2 - I_{xy}^2 = 0$

Substituting: $\qquad K_1 = 14.30368 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

or $\quad K_1 = 14.30 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$ ◀

and $\quad (13.98800 \times 10^{-3})(20.55783 \times 10^{-3}) - (13.98800 + 20.55783)(10^{-3})K + K^2 - (0.39460 \times 10^{-3})^2 = 0$

or $\qquad K^2 - (34.54583 \times 10^{-3})K + 287.4072 \times 10^{-6} = 0$

Solving yields $\qquad K_2 = 13.96438 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$

or $\quad K_2 = 13.96 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$ ◀

and $\qquad K_3 = 20.58145 \times 10^{-3}\ \text{kg} \cdot \text{m}^2 \qquad$ or $\quad K_3 = 20.6 \times 10^{-3}\ \text{kg} \cdot \text{m}^2$ ◀

PROBLEM 9.180 (Continued)

(*b*) To determine the direction cosines $\lambda_x, \lambda_y, \lambda_z$ of each principal axis, use two of the equations of Eq. (9.54) and Eq. (9.57). Then

K_1: Begin with Eqs. (9.54*a*) and (9.54*b*) with $I_{yz} = I_{zx} = 0.$

$$(I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 = 0$$
$$-I_{xy}(\lambda_x)_1 + (I_y - K_1)(\lambda_y)_1 = 0$$

Substituting:

$$[(13.98800 - 14.30368)\times 10^{-3}](\lambda_x)_1 - (0.39460\times 10^{-3})(\lambda_y)_1 = 0$$
$$-(0.39460\times 10^{-3})(\lambda_x)_1 + [(20.55783 - 14.30368)\times 10^{-3}](\lambda_y)_1 = 0$$

Adding yields $(\lambda_x)_1 = (\lambda_y)_1 = 0$

Then using Eq. (9.57) $\cancelto{0}{(\lambda_x)_1^2} + \cancelto{0}{(\lambda_y)_1^2} + (\lambda_z)_1^2 = 1$

or $(\lambda_z)_1 = 1$

$(\theta_x)_1 = 90.0° \quad (\theta_y)_1 = 90.0° \quad (\theta_z)_1 = 0°$ ◀

K_2: Begin with Eqs. (9.54*b*) and (9.54*c*) with $I_{yz} = I_{zx} = 0.$

$$-I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 = 0 \qquad \text{(i)}$$
$$(I_z - K_2)(\lambda_z)_2 = 0$$

Now $I_z \neq K_2 \Rightarrow (\lambda_z)_2 = 0$

Substituting into Eq. (i):

$$-(0.39460\times 10^{-3})(\lambda_x)_z + [(20.55783 - 13.96438)\times 10^{-3}](\lambda_y)_2 = 0$$

or $(\lambda_y)_2 = 0.059847(\lambda_x)_2$

Using Eq. (9.57): $(\lambda_x)_2^2 + [0.05984](\lambda_x)_2]^2 + \cancelto{0}{(\lambda_z)_2^2} = 1$

or $(\lambda_x)_2 = 0.998214$

and $(\lambda_y)_2 = 0.059740$

$(\theta_x)_2 = 3.4° \qquad (\theta_y)_2 = 86.6° \qquad (\theta_z)_2 = 90.0°$ ◀

K_3: Begin with Eqs. (9.54*b*) and (9.54*c*) with $I_{yz} = I_{zx} = 0.$

$$-I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 = 0 \qquad \text{(ii)}$$
$$(I_z - K_3)(\lambda_z)_3 = 0$$

PROBLEM 9.180 (Continued)

Now $$I_z \neq K_3 \Rightarrow (\lambda_z)_3 = 0$$

Substituting into Eq. (ii):

$$-(0.39460\times10^{-3})(\lambda_x)_3 + [(20.55783-20.58145)\times10^{-3}](\lambda_y)_3 = 0$$

or $$(\lambda_y)_3 = -16.70618(\lambda_x)_3$$

Using Eq. (9.57): $$(\lambda_x)_3^2 + [-16.70618(\lambda_x)_3]^2 + \cancelto{0}{(\lambda_z)_3^2} = 1$$

or $$(\lambda_x)_3 = -0.059751 \quad \text{(axes right-handed set } \Rightarrow \text{"\_")}$$

and $$(\lambda_y)_3 = 0.998211$$

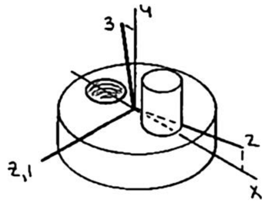
$$(\theta_x)_3 = 93.4° \qquad (\theta_y)_3 = 3.43° \qquad (\theta_z)_3 = 90.0° \blacktriangleleft$$

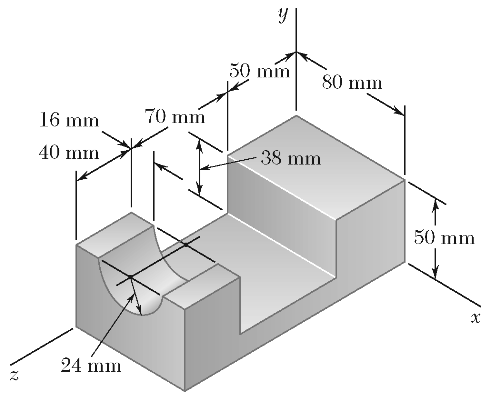
Note: Since the principal axes are orthogonal and $(\theta_z)_2 = (\theta_z)_3 = 90°$, it follows that

$$|(\lambda_x)_2| = |(\lambda_y)_3| \qquad |(\lambda_y)_2| = |(\lambda_z)_3|$$

The differences in the above values are due to round-off errors.

(*c*) Principal axis 1 coincides with the *z* axis, while principal axes 2 and 3 lie in the *xy* plane.





PROBLEM 9.181*

For the component described in Problems 9.145 and 9.149, determine (*a*) the principal mass moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION

(*a*) From the solutions to Problems 9.145 and 9.149, we have

Problem 9.145: $I_x = 26.4325\times10^{-3}\ \text{kg}\cdot\text{m}^2$ Problem 9.149: $I_{xy} = 2.5002\times10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_y = 31.1726\times10^{-3}\ \text{kg}\cdot\text{m}^2$ $I_{yz} = 4.0627\times10^{-3}\ \text{kg}\cdot\text{m}^2$

$I_z = 8.5773\times10^{-3}\ \text{kg}\cdot\text{m}^2$ $I_{zx} = 8.8062\times10^{-3}\ \text{kg}\cdot\text{m}^2$

Substituting into Eq. (9.56):

$$\begin{aligned}
&K^3 - [(26.4325 + 31.1726 + 8.5773)(10^{-3})]K^2\\
&\quad + [(26.4325)(31.1726) + (31.1726)(8.5773) + (8.5773)(26.4325)\\
&\quad - (2.5002)^2 - (4.0627)^2 - (8.8062)^2](10^{-6})K\\
&\quad - [(26.4325)(31.1726)(8.5773) - (26.4325)(4.0627)^2\\
&\quad - (31.1726)(8.8062)^2 - (8.5773)(2.5002)^2\\
&\quad - 2(2.5002)(4.0627)(8.8062)](10^{-9}) = 0
\end{aligned}$$

or $$K^3 - (66.1824\times10^{-3})K^2 + (1217.76\times10^{-6})K - (3981.23\times10^{-9}) = 0$$

Solving yields

$K_1 = 4.1443\times10^{-3}\ \text{kg}\cdot\text{m}^2$ or $K_1 = 4.14\times10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

$K_2 = 29.7840\times10^{-3}\ \text{kg}\cdot\text{m}^2$ or $K_2 = 29.8\times10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

$K_3 = 32.2541\times10^{-3}\ \text{kg}\cdot\text{m}^2$ or $K_3 = 32.3\times10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.181* (Continued)

(*b*) To determine the direction cosines λ_x, λ_y, λ_z of each principal axis, use two of the equations of Eqs. (9.54) and Eq. (9.57). Then

K_1: Begin with Eqs. (9.54*a*) and (9.54*b*).

$$(I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 - I_{zx}(\lambda_z)_1 = 0$$
$$I_{xy}(\lambda_x)_1 + (I_y - K_1)(\lambda_y)_1 - I_{yz}(\lambda_z)_1 = 0$$

Substituting:

$$[(26.4325 - 4.1443)(10^{-3})](\lambda_x)_1 - (2.5002\times10^{-3})(\lambda_y)_1 - (8.8062\times10^{-3})(\lambda_z)_1 = 0$$

$$-(2.5002\times10^{-3})(\lambda_x)_1 + [(31.1726 - 4.1443)(10^{-3})](\lambda_y)_1 - (4.0627\times10^{-3})(\lambda_z)_1 = 0$$

Simplifying

$$8.9146(\lambda_x)_1 - (\lambda_y)_1 - 3.5222(\lambda_z)_1 = 0$$
$$-0.0925(\lambda_x)_1 + (\lambda_y)_1 - 0.1503(\lambda_z)_1 = 0$$

Adding and solving for $(\lambda_z)_1$:

$$(\lambda_z)_1 = 2.4022(\lambda_x)_1$$

and then

$$(\lambda_y)_1 = [8.9146 - 3.5222(2.4022)](\lambda_x)_1$$
$$= 0.45357(\lambda_x)_1$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_1^2 + [0.45357(\lambda_x)_1]^2 + [2.4022(\lambda_x)_1]^2 = 1$$

or $(\lambda_x)_1 = 0.37861$

and $(\lambda_y)_1 = 0.17173$ $(\lambda_z)_1 = 0.90950$

$(\theta_x)_1 = 67.8°$ $(\theta_y)_1 = 80.1°$ $(\theta_z)_1 = 24.6°$ ◀

K_2: Begin with Eqs. (9.54*a*) and (9.54*b*).

$$(I_x - K_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 = 0$$
$$-I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 - I_{yz}(\lambda_z)_2 = 0$$

Substituting:

$$[(26.4325 - 29.7840)(10^{-3})](\lambda_x)_2 - (2.5002\times10^{-3})(\lambda_y)_2 - (8.8062\times10^{-3})(\lambda_z)_2 = 0$$

$$-(2.5002\times10^{-3})(\lambda_x)_2 + [(31.1726 - 29.7840)(10^{-3})](\lambda_y)_2 - (4.0627\times10^{-3})(\lambda_z)_2 = 0$$

PROBLEM 9.181* (Continued)

Simplifying

$$-1.3405(\lambda_x)_2 - (\lambda_y)_2 - 3.5222(\lambda_z)_2 = 0$$
$$-1.8005(\lambda_x)_2 + (\lambda_y)_2 - 2.9258(\lambda_z)_2 = 0$$

Adding and solving for $(\lambda_z)_2$:

$$(\lambda_z)_2 = -0.48713(\lambda_x)_2$$

and then

$$(\lambda_y)_2 = [-1.3405 - 3.5222(-0.48713)](\lambda_x)_2$$
$$= 0.37527(\lambda_x)_2$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_2^2 + [0.37527(\lambda_x)_2]^2 + [-0.48713(\lambda_x)_2]^2 = 1$$

or $(\lambda_x)_2 = 0.85184$

and $(\lambda_y)_2 = 0.31967$ $(\lambda_z)_2 = -0.41496$

$(\theta_x)_1 = 31.6°$ $(\theta_y)_2 = 71.4°$ $(\theta_z)_2 = 114.5°$ ◀

K_3: Begin with Eqs. (9.54*a*) and (9.54*b*).

$$(I_x - K_3)(\lambda_x)_3 - I_{xy}(\lambda_y)_3 - I_{zx}(\lambda_z)_3 = 0$$
$$-I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 - I_{yz}(\lambda_z)_3 = 0$$

Substituting:

$$[(26.4325 - 32.2541)(10^{-3})](\lambda_x)_3 - (2.5002 \times 10^{-3})(\lambda_y)_3 - (8.8062 \times 10^{-3})(\lambda_z)_3 = 0$$

$$-(2.5002 \times 10^{-3})(\lambda_x)_3 + [(31.1726 - 32.2541)(10^{-3})](\lambda_y)_3 - (4.0627 \times 10^{-3})(\lambda_z)_3 = 0$$

Simplifying

$$-2.3285(\lambda_x)_3 - (\lambda_y)_3 - 3.5222(\lambda_z)_3 = 0$$
$$2.3118(\lambda_x)_3 + (\lambda_y)_3 + 3.7565(\lambda_z)_3 = 0$$

Adding and solving for $(\lambda_z)_3$:

$$(\lambda_z)_3 = 0.071276(\lambda_x)_3$$

and then

$$(\lambda_y)_3 = [-2.3285 - 3.5222(0.071276)](\lambda_x)_3$$
$$= -2.5795(\lambda_x)_3$$

PROBLEM 9.181* (Continued)

Now substitute into Eq. (9.57):

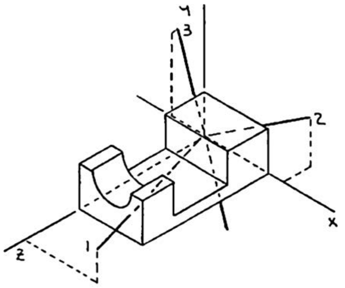
$$(\lambda_x)_3^2 + [-2.5795(\lambda_x)_3]^2 + [0.071276(\lambda_x)_3]^2 = 1 \qquad \text{(i)}$$

or $$(\lambda_x)_3 = 0.36134$$

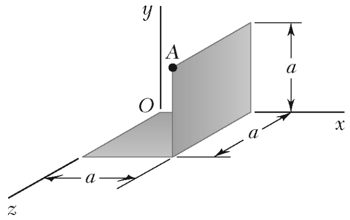
and $$(\lambda_y)_3 = 0.93208 \qquad (\lambda_z)_3 = 0.025755$$

$$(\theta_x)_3 = 68.8° \quad (\theta_y)_3 = 158.8° \quad (\theta_z)_3 = 88.5° \blacktriangleleft$$

(*c*) *Note:* Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Eq. (i) must be chosen; that is, $(\lambda_x)_3 = -0.36134$.



Then $$(\theta_x)_3 = 111.2° \quad (\theta_y)_3 = 21.2° \quad (\theta_z)_3 = 91.5° \blacktriangleleft$$



PROBLEM 9.182*

For the component described in Problem 9.167, determine (*a*) the principal mass moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION

(*a*) From the solution of Problem 9.167, we have

$$I_x = \frac{1}{2}\frac{W}{g}a^2 \qquad I_{xy} = \frac{1}{4}\frac{W}{g}a^2$$

$$I_y = \frac{W}{g}a^2 \qquad I_{yz} = \frac{1}{8}\frac{W}{g}a^2$$

$$I_z = \frac{5}{6}\frac{W}{g}a^2 \qquad I_{zx} = \frac{3}{8}\frac{W}{g}a^2$$

Substituting into Eq. (9.56):

$$K^3 - \left[\left(\frac{1}{2}+1+\frac{5}{6}\right)\left(\frac{W}{g}a^2\right)\right]K^2$$

$$+\left[\left(\frac{1}{2}\right)(1)+(1)\left(\frac{5}{6}\right)+\left(\frac{5}{6}\right)\left(\frac{1}{2}\right)-\left(\frac{1}{4}\right)^2-\left(\frac{1}{8}\right)^2-\left(\frac{3}{8}\right)^2\right]\left(\frac{W}{g}a^2\right)^2 K$$

$$-\left[\left(\frac{1}{2}\right)(1)\left(\frac{5}{6}\right)-\left(\frac{1}{2}\right)\left(\frac{1}{8}\right)^2-(1)\left(\frac{3}{8}\right)^2-\left(\frac{5}{6}\right)\left(\frac{1}{4}\right)^2-2\left(\frac{1}{4}\right)\left(\frac{1}{8}\right)\left(\frac{3}{8}\right)\right]\left(\frac{W}{g}a^2\right)^3=0$$

Simplifying and letting $K = \frac{W}{g}a^2K$ yields

$$K^3 - 2.33333K^2 + 1.53125K - 0.192708 = 0$$

Solving yields

$$K_1 = 0.163917 \quad K_2 = 1.05402 \quad K_3 = 1.11539$$

The principal moments of inertia are then $K_1 = 0.1639\frac{W}{g}a^2$ ◀

$K_2 = 1.054\frac{W}{g}a^2$ ◀

$K_3 = 1.115\frac{W}{g}a^2$ ◀

PROBLEM 9.182* (Continued)

(*b*) To determine the direction cosines λ_x, λ_y, λ_z of each principal axis, use two of the equations of Eq. (9.54) and Eq. (9.57). Then

K_1: Begin with Eqs. (9.54*a*) and (9.54*b*).

$$(I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 - I_{zx}(\lambda_z)_1 = 0$$
$$-I_{xy}(\lambda_x)_1 + (I_y - K_2)(\lambda_y)_1 - I_{yz}(\lambda_z)_1 = 0$$

Substituting

$$\left[\left(\frac{1}{2} - 0.163917\right)\left(\frac{W}{g}a^2\right)\right](\lambda_x)_1 - \left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_y)_1 - \left(\frac{3}{8}\frac{W}{g}a^2\right)(\lambda_z)_1 = 0$$

$$-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_x)_1 + \left[(1 - 0.163917)\left(\frac{W}{g}a^2\right)\right](\lambda_y)_1 - \left(\frac{1}{8}\frac{W}{g}a^2\right)(\lambda_z)_1 = 0$$

Simplifying

$$1.34433(\lambda_x)_1 - (\lambda_y)_1 - 1.5(\lambda_z)_1 = 0$$

$$-0.299013(\lambda_x)_1 + (\lambda_y)_1 - 0.149507(\lambda_z)_1 = 0$$

Adding and solving for $(\lambda_z)_1$:

$$(\lambda_z)_1 = 0.633715(\lambda_x)_1$$

and then

$$(\lambda_y)_1 = [1.34433 - 1.5(0.633715)](\lambda_x)_1$$
$$= 0.393758(\lambda_x)_1$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_1^2 + [0.393758(\lambda_x)_1]^2 + (0.633715(\lambda_x)_1]^2 = 1$$

or $(\lambda_x)_1 = 0.801504$

and $(\lambda_y)_1 = 0.315599$ $\quad (\lambda_z)_1 = 0.507925$

$(\theta_x)_1 = 36.7°$ $\quad (\theta_y)_1 = 71.6°$ $\quad (\theta_z)_1 = 59.5°$ ◀

K_2: Begin with Eqs. (9.54*a*) and (9.54*b*):

$$(I_x - k_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 = 0$$
$$-I_{xy}(\lambda_x)_2 + (I_y - k_2)(\lambda_y)_2 - I_{yz}(\lambda_z)_2 = 0$$

PROBLEM 9.182* (Continued)

Substituting

$$\left[\left(\frac{1}{2}-1.05402\right)\left(\frac{W}{g}a^2\right)\right](\lambda_x)_2-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_y)_2-\left(\frac{3}{8}\frac{W}{g}a^2\right)(\lambda_z)_2=0$$

$$-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_x)_2+\left[(1-1.05402)\left(\frac{W}{g}a^2\right)\right](\lambda_y)_2-\left(\frac{1}{8}\frac{W}{g}a^2\right)(\lambda_z)_2=0$$

Simplifying

$$-2.21608(\lambda_x)_2-(\lambda_y)_2-1.5(\lambda_z)_2=0$$

$$4.62792(\lambda_x)_2+(\lambda_y)_2+2.31396(\lambda_z)_2=0$$

Adding and solving for $(\lambda_z)_2$

$$(\lambda_z)_2=-2.96309(\lambda_x)_2$$

and then

$$(\lambda_y)_2=[-2.21608-1.5(-2.96309)](\lambda_x)_2$$
$$=2.22856(\lambda_x)_2$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_2^2+[2.22856(\lambda_x)_2]^2+[-2.96309(\lambda_x)_2]^2=1$$

or

$$(\lambda_x)_2=0.260410$$

and $(\lambda_y)_2=0.580339$ $(\lambda_z)_2=-0.771618$

$(\theta_x)_2=74.9°$ $(\theta_y)_2=54.5°$ $(\theta_z)_2=140.5°$ ◀

K_3: Begin with Eqs. (9.54*a*) and (9.54*b*):

$$(I_x-K_3)(\lambda_x)_3-I_{xy}(\lambda_y)_3-I_{zx}(\lambda_z)_3=0$$
$$-I_{xy}(\lambda_x)_3+(I_y-K_3)(\lambda_y)_3-I_{yz}(\lambda_z)_3=0$$

Substituting

$$\left[\left(\frac{1}{2}-1.11539\right)\left(\frac{W}{g}a^2\right)\right](\lambda_x)_3-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_y)_3-\left(\frac{3}{8}\frac{W}{g}a^2\right)(\lambda_z)_3=0$$

$$-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_x)_3+\left[(1-1.11539)\left(\frac{W}{g}a^2\right)\right](\lambda_y)_3-\left(\frac{1}{8}\frac{W}{g}a^2\right)(\lambda_z)_3=0$$

Simplifying

$$-2.46156(\lambda_x)_3-(\lambda_y)_3-1.5(\lambda_z)_3=0$$
$$2.16657(\lambda_x)_3+(\lambda_y)_3+1.08328(\lambda_z)_3=0$$

PROBLEM 9.182* (Continued)

Adding and solving for $(\lambda_z)_3$

$$(\lambda_z)_3 = -0.707885(\lambda_x)_3$$

and then
$$(\lambda_y)_3 = [-2.46156 - 1.5(-0.707885)](\lambda_x)_3$$
$$= -1.39973(\lambda_x)_3$$

Now substitute into Eq. (9.57):

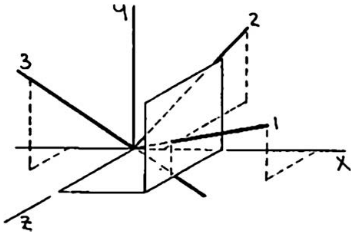
$$(\lambda_x)_3^2 + [-1.39973(\lambda_x)_3]^2 + [-0.707885(\lambda_x)_3]^2 = 1 \qquad \text{(i)}$$

or
$$(\lambda_x)_3 = 0.537577$$

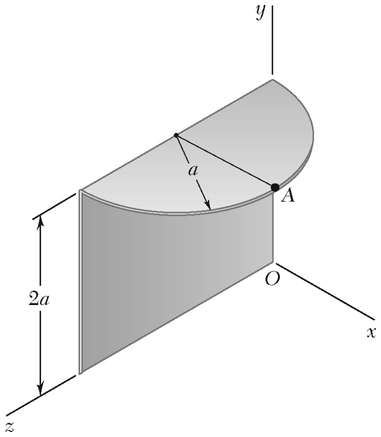
and
$$(\lambda_y)_3 = -0.752463 \qquad (\lambda_z)_3 = -0.380543$$

$$(\theta_x)_3 = 57.5° \quad (\theta_y)_3 = 138.8° \quad (\theta_z)_3 = 112.4° \blacktriangleleft$$

(*c*) *Note:* Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Eq. (i) must be chosen; that is, $(\lambda_x)_3 = -0.537577$.



Then
$$(\theta_x)_3 = 122.5° \quad (\theta_y)_3 = 41.2° \quad (\theta_z)_3 = 67.6° \blacktriangleleft$$



PROBLEM 9.183*

For the component described in Problem 9.168, determine (*a*) the principal mass moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION

(*a*) From the solution to Problem 9.168, we have

$$I_x = 18.91335\frac{\gamma t}{g}a^4 \qquad I_{xy} = 1.33333\frac{\gamma t}{g}a^4$$

$$I_y = 7.68953\frac{\gamma t}{g}a^4 \qquad I_{yz} = 7.14159\frac{\gamma t}{g}a^4$$

$$I_z = 12.00922\frac{\gamma t}{g}a^4 \qquad I_{zx} = 0.66667\frac{\gamma t}{g}a^4$$

Substituting into Eq. (9.56):

$$\begin{aligned}
&K^3 - \left[(18.91335 + 7.68953 + 12.00922)\left(\frac{\gamma t}{g}a^4\right)\right]K^2 \\
&\quad + [(18.91335)(7.68953) + (7.68953)(12.00922) + (12.00922)(18.91335) \\
&\quad - (1.33333)^2 - (7.14159)^2 + (0.66667)^2]\left(\frac{\gamma t}{g}a^4\right)^2 K \\
&\quad - [(18.91335)(7.68953)(12.00922) - (18.91335)(7.14159)^2 \\
&\quad - (7.68953)(0.66667)^2 - (12.00922)(1.33333)^2 \\
&\quad - 2(1.33333)(7.14159)(0.66667)]\left(\frac{\gamma t}{g}a^4\right)^3 = 0
\end{aligned}$$

Simplifying and letting

$$K = \frac{\gamma t}{g}a^4 K \quad \text{yields}$$

$$K^3 - 38.61210K^2 + 411.69009K - 744.47027 = 0$$

PROBLEM 9.183* (Continued)

Solving yields

$$K_1 = 2.25890 \quad K_2 = 17.27274 \quad K_3 = 19.08046$$

The principal moments of inertia are then $K_1 = 2.26\dfrac{\gamma t}{g}a^4$ ◀

$$K_2 = 17.27\frac{\gamma t}{g}a^4 \blacktriangleleft$$

$$K_3 = 19.08\frac{\gamma t}{g}a^4 \blacktriangleleft$$

(*b*) To determine the direction cosines $\lambda_x,\ \lambda_y,\ \lambda_z$ of each principal axis, use two of the equations of Eq. (9.54) and Eq. (9.57). Then

K_1: Begin with Eqs. (9.54*a*) and (9.54*b*):

$$(I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 - I_{zx}(\lambda_z)_1 = 0$$
$$-I_{xy}(\lambda_x)_1 + (I_y - K_2)(\lambda_y)_1 - I_{yz}(\lambda_z)_1 = 0$$

Substituting

$$\left[(18.91335 - 2.25890)\left(\frac{\gamma t}{g}a^4\right)\right](\lambda_x)_1 - \left(1.33333\frac{\gamma t}{a}a^4\right)(\lambda_y)_1 - \left(0.66667\frac{\gamma t}{g}a^4\right)(\lambda_z)_1 = 0$$

$$-\left(1.33333\frac{\gamma t}{g}a^4\right)(\lambda_x)_1 + \left[(7.68953 - 2.25890)\left(\frac{\gamma t}{g}a^4\right)\right](\lambda_y)_1 - \left(7.14159\frac{\gamma t}{g}a^4\right)(\lambda_z)_1 = 0$$

Simplifying

$$12.49087(\lambda_x)_1 - (\lambda_y)_1 - 0.5(\lambda_z)_1 = 0$$
$$-0.24552(\lambda_x)_1 + (\lambda_y)_1 - 1.31506(\lambda_z)_1 = 0$$

Adding and solving for $(\lambda_z)_1$

$$(\lambda_z)_1 = 6.74653(\lambda_x)_1$$

and then $(\lambda_y)_1 = [12.49087 - (0.5)(6.74653)](\lambda_x)_1$

$$= 9.11761(\lambda_x)_1$$

Now substitute into Eq. (9.57): $(\lambda_x)_1^2 + [9.11761(\lambda_x)_1]^2 + [6.74653(\lambda_x)_1]^2 = 1$

or $(\lambda_x)_1 = 0.087825$

and $(\lambda_y)_1 = 0.80075 \qquad (\lambda_z)_1 = 0.59251$

$$(\theta_x)_1 = 85.0° \quad (\theta_y)_1 = 36.8° \quad (\theta_z)_1 = 53.7° \blacktriangleleft$$

PROBLEM 9.183* (Continued)

K_2: Begin with Eqs. (9.54*a*) and (9.54*b*):

$$(I_x - K_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 = 0$$
$$-I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 - I_{yz}(\lambda_z)_2 = 0$$

Substituting

$$[(18.91335 - 17.27274)\left(\frac{\gamma t}{g}a^4\right)(\lambda_x)_2 - \left(1.33333\frac{\gamma t}{g}a^4\right)(\lambda_y)_2 - \left(0.66667\frac{\gamma t}{g}a^4\right)(\lambda_z)_2 = 0$$
$$-\left(1.33333\frac{\gamma t}{g}a^4\right)(\lambda_x)_2 + \left[(7.68953 - 17.27274)\left(\frac{\gamma t}{g}a^4\right)\right](\lambda_y)_2 - \left(7.14159\frac{\gamma t}{g}a^4\right)(\lambda_z)_2 = 0$$

Simplifying

$$1.23046(\lambda_x)_2 - (\lambda_y)_2 - 0.5(\lambda_z)_2 = 0$$
$$0.13913(\lambda_x)_2 + (\lambda_y)_2 + 0.74522(\lambda_z)_2 = 0$$

Adding and solving for $(\lambda_z)_2$

$$(\lambda_z)_2 = -5.58515(\lambda_x)_2$$

and then
$$(\lambda_y)_2 = [1.23046 - (0.5)(-5.58515)](\lambda_x)_2$$
$$= 4.02304\,(\lambda_x)_2$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_2^2 + [4.02304(\lambda_x)_2]^2 + [-5.58515(\lambda_x)_2]^2 = 1$$

or
$$(\lambda_x)_2 = 0.14377$$

and
$$(\lambda_y)_2 = 0.57839 \qquad (\lambda_z)_2 = -0.80298$$

$$(\theta_x)_2 = 81.7° \quad (\theta_y)_2 = 54.7° \quad (\theta_z)_2 = 143.4° \blacktriangleleft$$

K_3: Begin with Eqs. (9.54*a*) and (9.54*b*):

$$(I_x - K_3)(\lambda_x)_3 - I_{xy}(\lambda_y)_3 - I_{zx}(\lambda_z)_3 = 0$$
$$-I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 - I_{yz}(\lambda_y)_3 = 0$$

Substituting

$$[(18.91335 - 19.08046)\left(\frac{\gamma t}{g}a^4\right)(\lambda_x)_3 - \left(1.33333\frac{\gamma t}{g}a^4\right)(\lambda_y)_3 - \left(0.66667\frac{\gamma t}{g}a^4\right)(\lambda_z)_3 = 0$$

$$-\left(1.33333\frac{\gamma t}{g}a^4\right)(\lambda_x)_3 + \left[(7.68953 - 19.08046)\left(\frac{\gamma t}{g}a^4\right)\right](\lambda_y)_3 - \left(7.14159\frac{\gamma t}{g}a^4\right)(\lambda_z)_3 = 0$$

PROBLEM 9.183* (Continued)

Simplifying

$$-0.12533(\lambda_x)_3 - (\lambda_y)_3 - 0.5(\lambda_z)_3 = 0$$
$$0.11705(\lambda_x)_3 + (\lambda_y)_3 + 0.62695(\lambda_z)_3 = 0$$

Adding and solving for $(\lambda_z)_3$

$$(\lambda_z)_3 = 0.06522(\lambda_x)_3$$

and then

$$(\lambda_y)_3 = [-0.12533 - (0.5)(0.06522)](\lambda_x)_3$$
$$= -0.15794(\lambda_x)_3$$

Now substitute into Eq. (9.57):

$$(\lambda_x)_3^2 + [-0.15794(\lambda_x)_3]^2 + [0.06522(\lambda_x]_3]^2 = 1 \qquad \text{(i)}$$

or

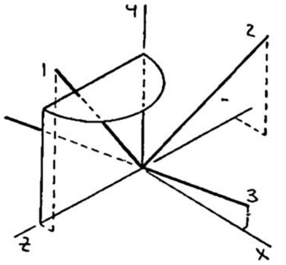
$$(\lambda_x)_3 = 0.98571$$

and

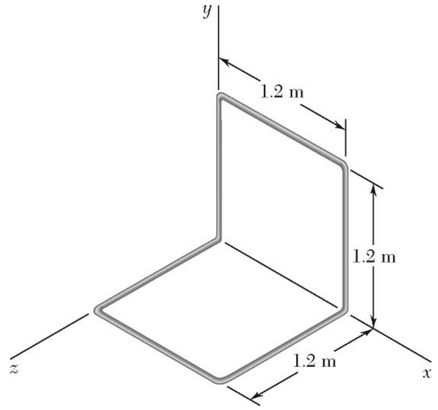
$$(\lambda_y)_3 = -0.15568 \qquad (\lambda_z)_3 = 0.06429$$

$(\theta_x)_3 = 9.7°$ $(\theta_y)_3 = 99.0°$ $(\theta_z)_3 = 86.3°$ ◀

(*c*) *Note:* Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Eq. (i) must be chosen; that is, $(\lambda_x)_3 = -0.98571$.



Then $(\theta_x)_3 = 170.3°$ $(\theta_y)_3 = 81.0°$ $(\theta_z)_3 = 93.7°$ ◀



PROBLEM 9.184*

For the component described in Problems 9.148 and 9.170, determine (*a*) the principal mass moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION

(*a*) From the solutions to Problems 9.148 and 9.170. We have

$$I_x = 0.32258 \text{ kg}\cdot\text{m}^2 \qquad I_y = I_z = 0.41933 \text{ kg}\cdot\text{m}^2$$

$$I_{xy} = I_{zx} = 0.096768 \text{ kg}\cdot\text{m}^2 \qquad I_{yz} = 0$$

Substituting into Eq. (9.56) and using

$$I_y = I_z \qquad I_{xy} = I_{zx} \qquad I_{yz} = 0$$

$$\begin{aligned} &K^3 - [0.32258 + 2(0.41933)]K^2 \\ &\quad + [2(0.32258)(0.41933) + (0.41933)^2 - 2(0.096768)^2]K \\ &\quad - [(0.32258)(0.41933)^2 - 2(0.41933)(0.096768)^2] = 0 \end{aligned}$$

Simplifying

$$K^3 - 1.16124K^2 + 0.42764K - 0.048869 = 0$$

Solving yields

$$K_1 = 0.22583 \text{ kg}\cdot\text{m}^2 \qquad \text{or} \qquad K_1 = 0.226 \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$K_2 = 0.41920 \text{ kg}\cdot\text{m}^2 \qquad \text{or} \qquad K_2 = 0.419 \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$K_3 = 0.51621 \text{ kg}\cdot\text{m}^2 \qquad \text{or} \qquad K_3 = 0.516 \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

(*b*) To determine the direction cosines $\lambda_x, \lambda_y, \lambda_z$ of each principal axis, use two of the equations of Eqs. (9.54) and (9.57). Then

K_1: Begin with Eqs. (9.54*b*) and (9.54*c*):

$$-I_{xy}(\lambda_x)_1 + (I_y - K_1)(\lambda_y)_1 - \cancelto{0}{I_{yz}}(\lambda_z)_3 = 0$$

$$-I_{zx}(\lambda_x)_1 - \cancelto{0}{I_{yz}}(\lambda_y)_1 + (I_z - K_1)(\lambda_z)_3 = 0$$

PROBLEM 9.184* (Continued)

Substituting

$$-(0.096768)(\lambda_x)_1 + (0.41933 - 0.22583)(\lambda_y)_1 = 0$$

$$-(0.096768)(\lambda_x)_1 + (0.41933 - 0.22583)(\lambda_z)_1 = 0$$

Simplifying yields

$$(\lambda_y)_1 = (\lambda_z)_1 = 0.50009(\lambda_x)_1$$

Now substitute into Eq. (9.54):

$$(\lambda_x)^2 + 2[0.50009(\lambda_x)_1]^2 = 1$$

or $$(\lambda_x)_1 = 0.81645$$

and $$(\lambda_y)_1 = (\lambda_z)_1 = 0.40830$$

$$(\theta_x)_1 = 35.3° \quad (\theta_y)_1 = (\theta_z)_1 = 65.9° \blacktriangleleft$$

K_2: Begin with Eqs. (9.54*a*) and (9.54*b*)

$$(I_x - K_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 = 0$$

$$-I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 - \cancelto{0}{I_{yz}}(\lambda_z)_2 = 0$$

Substituting

$$(0.32258 - 0.41920)(\lambda_x)_2 - (0.096768)(\lambda_y)_2 - (0.096768)(\lambda_z)_2 = 0 \tag{i}$$

$$-(0.96768)(\lambda_x)_2 + (0.41933 - 0.41920)(\lambda_y)_2 = 0 \tag{ii}$$

Eq. (ii) $\Rightarrow$ $(\lambda_x)_2 = 0$

and then Eq. (i) $\Rightarrow$ $(\lambda_z)_2 = -(\lambda_y)_2$

Now substitute into Eq. (9.57):

$$\cancelto{0}{(\lambda_x)_2^2} + (\lambda_y)_2^2 + [-(\lambda_y)_2]^2 = 1$$

or $$(\lambda_y)_2 = \frac{1}{\sqrt{2}}$$

and $$(\lambda_z)_2 = -\frac{1}{\sqrt{2}}$$

$$(\theta_x)_2 = 90.0° \quad (\theta_y)_2 = 45.0° \quad (\theta_z)_2 = 135.0° \blacktriangleleft$$

PROBLEM 9.184* (Continued)

K_3: Begin with Eqs. (9.54*b*) and (9.54*c*):

$$I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 + \cancelto{0}{I_{yz}}(\lambda_z)_3 = 0$$

$$-I_{zx}(\lambda_x)_3 - \cancelto{0}{I_{yz}}(\lambda_y)_3 + (I_z - K_3)(\lambda_z)_3 = 0$$

Substituting

$$-(0.096768)(\lambda_x)_3 + (0.41933 - 0.51621)(\lambda_y)_3 = 0$$

$$-(0.096768)(\lambda_x)_3 + (0.41933 - 0.51621)(\lambda_z)_3 = 0$$

Simplifying yields

$$(\lambda_y)_3 = (\lambda_z)_3 = -(\lambda_x)_3$$

Now substitute into Eq. (9.57):

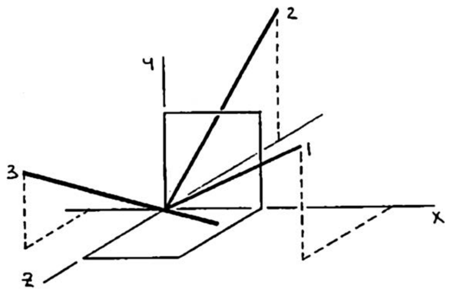
$$(\lambda_x)_3^2 + 2[-(\lambda_x)_3]^2 = 1 \qquad \text{(i)}$$

or $$(\lambda_x)_3 = \frac{1}{\sqrt{3}}$$

and $$(\lambda_y)_3 = (\lambda_z) = -\frac{1}{\sqrt{3}}$$

$(\theta_x)_3 = 54.7°$ $(\theta_y)_3 = (\theta_z)_3 = 125.3°$ ◀

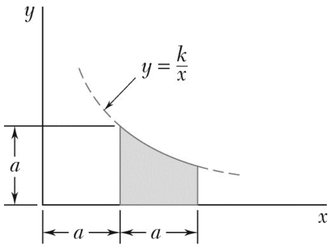
(*c*)



Note: Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Eq. (i) must be chosen;

That is, $$(\lambda_x)_3 = -\frac{1}{\sqrt{3}}$$

Then $(\theta_x)_3 = 125.3°$ $(\theta_y)_3 = (\theta_z)_3 = 54.7°$ ◀

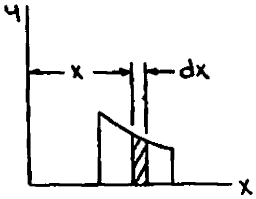


PROBLEM 9.185

Determine by direct integration the moments of inertia of the shaded area with respect to the *x* and *y* axes.

SOLUTION

At $x = a, \quad y = a$: $\quad a = \frac{k}{a}$ or $k = a^2$



Then $\quad y = \frac{a^2}{x}$

Now $\quad dI_x = \frac{1}{3}y^3 dx = \frac{1}{3}\left(\frac{a^2}{x}\right)^3 dx$

$$= \frac{1}{3}\frac{a^6}{x^3}dx$$

Then $\quad I_x = \int dI_x = \int_a^{2a} \frac{1}{3}\frac{a^6}{x^3}dx = \frac{1}{3}a^6\left[-\frac{1}{2}\frac{1}{x^2}\right]_a^{2a}$

$$= -\frac{1}{6}a^6\left[\frac{1}{(2a)^2} - \frac{1}{(a)^2}\right]$$

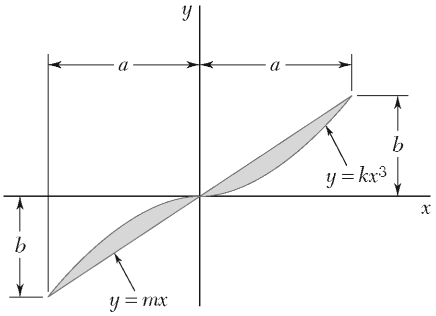
or $I_x = \frac{1}{8}a^4$ ◀

Now $\quad dI_y = x^2 \quad dA = x^2(y\,dx)$

$$= x^2\left(\frac{a^2}{x}dx\right) = a^2 x\,dx$$

Then $\quad I_y = \int dI_y = \int_a^{2a} a^2 x\,dx = a^2\left[\frac{1}{2}x^2\right]_a^{2a} = \frac{a^2}{2}[(2a)^2 - (a)^2]$

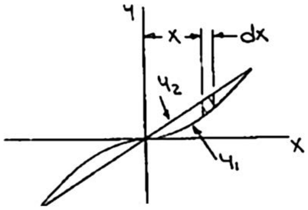
or $I_y = \frac{3}{2}a^4$ ◀



PROBLEM 9.186

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *y* axis.

SOLUTION



At $x = a, \quad y_1 = y_2 = b$:

$$y_1: \quad b = ka^3 \quad \text{or} \quad k = \frac{b}{a^3}$$

$$y_2: \quad b = ma \quad \text{or} \quad m = \frac{b}{a}$$

Then

$$y_1 = \frac{b}{a^3}x^3$$

$$y_2 = \frac{b}{a}x$$

Now

$$dA = (y_2 - y_1)dx = \left(\frac{b}{a}x - \frac{b}{a^3}x^3\right)dx$$

Then

$$A = \int dA = 2\int_0^a \frac{b}{a}\left(x - \frac{1}{a^2}x^3\right)dx$$

$$= 2\frac{b}{a}\left[\frac{1}{2}x^2 - \frac{1}{4a^2}x^4\right]_0^a = \frac{1}{2}ab$$

Now

$$dI_y = x^2 dA = x^2\left[\left(\frac{b}{a}x - \frac{b}{a^3}x^3\right)dx\right]$$

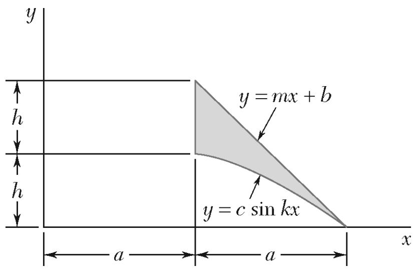
Then

$$I_y = \int dI_y = 2\int_0^a \frac{b}{a}x^2\left(x - \frac{1}{a^2}x^3\right)dx$$

$$= 2\frac{b}{a}\left[\frac{1}{4}x^4 - \frac{1}{6}\frac{1}{a^2}x^6\right]_0^a \qquad \text{or} \quad I_y = \frac{1}{6}a^3b \blacktriangleleft$$

and

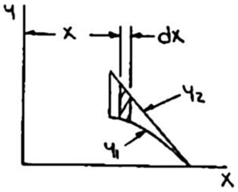
$$k_y^2 = \frac{I_y}{A} = \frac{\frac{1}{6}a^3b}{\frac{1}{2}ab} = \frac{1}{3}a^2 \qquad \text{or} \quad k_y = \frac{a}{\sqrt{3}} \blacktriangleleft$$



PROBLEM 9.187

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the x axis.

SOLUTION



y_1: At $x = 2a$, $y = 0$: $\quad 0 = c\sin k(2a)$

$$2ak = \pi \quad \text{or} \quad k = \frac{\pi}{2a}$$

At $x = a$, $y = h$: $\quad h = c\sin\frac{\pi}{2a}(a) \quad \text{or} \quad c = h$

y_2: At $x = a$, $y = 2h$: $\quad 2h = ma - b$

At $x = 2a$, $y = 0$: $\quad 0 = m(2a) + b$

Solving yields $\quad m = -\frac{2h}{a}, \quad b = 4h$

Then $\quad y_1 = h\sin\frac{\pi}{2a}x \qquad y_2 = -\frac{2h}{a}x + 4h$

$$= \frac{2h}{a}(-x + 2a)$$

Now $\quad dA = (y_2 - y_1)dx = \left[\frac{2h}{a}(-x + 2a) - h\sin\frac{\pi}{2a}x\right]dx$

Then $\quad A = \int dA = \int_a^{2a} h\left[\frac{2}{a}(-x + 2a) - \sin\frac{\pi}{2a}x\right]dx$

$$= h\left[-\frac{1}{a}(-x + 2a)^2 + \frac{2a}{\pi}\cos\frac{\pi}{2a}x\right]_a^{2a}$$

$$= h\left[\left(-\frac{2a}{\pi}\right) + \frac{1}{a}(-a + 2a)^2\right] = ah\left(1 - \frac{2}{\pi}\right)$$

$$= 0.36338ah$$

Find: I_x and k_x

We have
$$dI_x = \left(\frac{1}{3}y_2 - \frac{1}{3}y_1\right)dx = \frac{1}{3}\left\{\left[\frac{2h}{a}(-x+2a)\right]^3 - \left(h\sin\frac{\pi}{2a}x\right)^3\right\}dx$$
$$= \frac{h^3}{3}\left[\frac{8}{a^3}(-x+2a)^3 - \sin^3\frac{\pi}{2a}x\right]$$

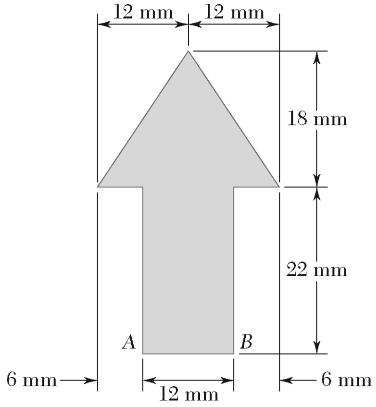
Then
$$I_x = \int dI_x = \int_a^{2a}\frac{h^3}{3}\left[\frac{8}{a^3}(-x+2a)^3 - \sin^3\frac{\pi}{2a}x\right]dx$$

Now
$$\sin^3\theta = \sin\theta(1-\cos^2\theta) = \sin\theta - \sin\theta\cos^2\theta$$

Then
$$I_x = \frac{h^3}{3}\int_a^{2a}\left[\frac{8}{a^3}(-x+2a)^3 - \left(\sin\frac{\pi}{2a}x - \sin\frac{\pi}{2a}x\cos^2\frac{\pi}{2a}x\right)\right]dx$$
$$= \frac{h^3}{3}\left[-\frac{2}{a^3}(-x+2a)^4 + \frac{2a}{\pi}\cos\frac{\pi}{2a}x - \frac{2a}{3\pi}\cos^3\frac{\pi}{2a}x\right]_a^{2a}$$
$$= \frac{h^3}{3}\left[\left(-\frac{2a}{\pi} + \frac{2a}{3\pi}\right) + \frac{2}{a^3}(-a+2a)^4\right]$$
$$= \frac{2}{3}ah^3\left(1 - \frac{2}{3\pi}\right)$$

$$I_x = 0.52520ah^3$$ or $I_x = 0.525ah^3$ ◀

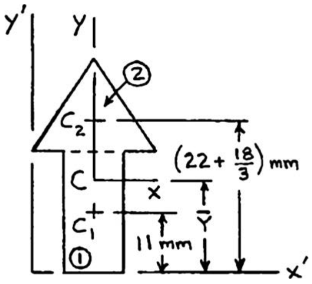
and
$$k_x^2 = \frac{I_x}{A} = \frac{0.52520ah^3}{0.36338ah}$$ or $k_x = 1.202h$ ◀



PROBLEM 9.188

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

SOLUTION



First locate *C* of the area:

Symmetry implies $\bar{X} = 12$ mm.

	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$12\times 22 = 264$	11	2904
2	$\frac{1}{2}(24)(18) = 216$	28	6048
Σ	480		8952

Then $\quad \bar{Y}\Sigma A = \Sigma\bar{y}A\text{:} \quad \bar{Y}(480\text{ mm}^2) = 8952\text{ mm}^3$

$$\bar{Y} = 18.65\text{ mm}$$

Now $\quad \bar{I}_x = (I_x)_1 + (I_x)_2$

where
$$(I_x)_1 = \frac{1}{12}(12\text{ mm})(22\text{ mm})^3 + (264\text{ mm}^2)[(18.65-11)\text{ mm}]^2$$
$$= 26{,}098\text{ mm}^4$$
$$(I_x)_2 = \frac{1}{36}(24\text{ mm})(18\text{ mm})^3 + (216\text{ mm}^2)[(28-18.65)\text{ mm}]^2$$
$$= 22{,}771\text{ mm}^4$$

Then $\quad \bar{I}_x = (26.098 + 22.771)\times 10^3\text{ mm}^4$

or $\quad \bar{I}_x = 48.9\times 10^3\text{ mm}^4$ ◀

PROBLEM 9.188 (Continued)

Also $\bar{I}_y = (I_y)_1 + (I_y)_2$

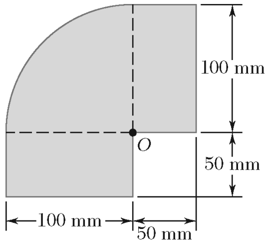
where $(I_y)_1 = \dfrac{1}{12}(22\text{ mm})(12\text{ mm})^3 = 3168\text{ mm}^4$

$$(I_y)_2 = 2\left[\frac{1}{36}(18\text{ mm})(12\text{ mm})^3 + \left(\frac{1}{2}\times 18\text{ mm}\times 12\text{ mm}\right)(4\text{ mm})^2\right]$$

$$= 5184\text{ mm}^4$$

[$(I_y)_2$ is obtained by dividing A_2 into ]

Then $\bar{I}_y = (3168 + 5184)\text{ mm}^4$ or $\bar{I}_y = 8.35\times 10^3\text{ mm}^4$ ◀

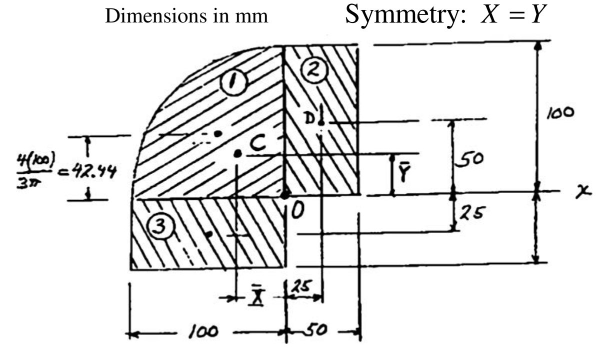


PROBLEM 9.189

Determine the polar moment of inertia of the area shown with respect to (*a*) Point *O*, (*b*) the centroid of the area.

SOLUTION

Symmetry: $\bar{X} = \bar{Y}$



Determination of centroid *C* of entire section.

Section	Area, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$\frac{\pi}{4}(100)^2 = 7.854\times10^3$	42.44	333.3×10^3
2	$(50)(100) = 5\times10^3$	50	250×10^3
3	$(100)(50) = 5\times10^3$	–25	-125×10^3
Σ	17.854×10^3		458.3×10^3

$$\bar{Y}\Sigma A = \Sigma\bar{y}A: \quad \bar{Y}(17.854\times10^3 \text{ mm}^2) = 458.3\times10^3 \text{ mm}^3$$

$$\bar{Y} = 25.67 \text{ mm} \quad \bar{X} = \bar{Y} = 25.67 \text{ mm}$$

Distance *O* to *C*: $\overline{OC} = \sqrt{2}\bar{Y} = \sqrt{2}(25.67) = 36.30 \text{ mm}$

(*a*) Section 1: $J_O = \frac{\pi}{8}(100)^4 = 39.27\times10^6 \text{ mm}^4$

Section 2: $J_O = J + A(\overline{OD})^2 = \frac{1}{12}(50)(100)[50^2 + 100^2] + (50)(100)\left[\left(\frac{50}{2}\right)^2 + \left(\frac{100}{2}\right)^2\right]$

$$J_O = 5.208\times10^6 + 15.625\times10^6 = 20.83\times10^6 \text{ mm}^4$$

Section 3: Same as Section 2; $J_O = 20.83\times10^6\ \text{mm}^4$

Entire section:

$$J_O = 39.27\times10^6 + 2(20.83\times10^6)$$

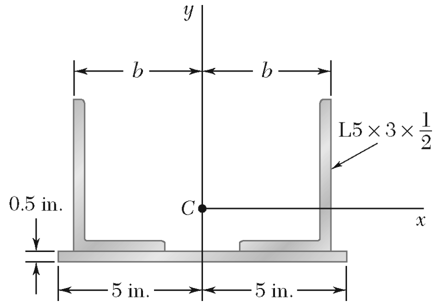
$$= 80.94\times10^6 \qquad J_O = 80.9\times10^6\ \text{mm}^4 \blacktriangleleft$$

(*b*) Recall that, $\overline{OC} = 36.30\ \text{mm}$ and $A = 17.854\times10^3\ \text{mm}^2$

$$J_O = \bar{J}_C + A(\overline{OC})^2$$

$$80.94\times10^6\ \text{mm}^4 = \bar{J}_C + (17.854\times10^3\ \text{mm}^2)(36.30\ \text{mm})^2$$

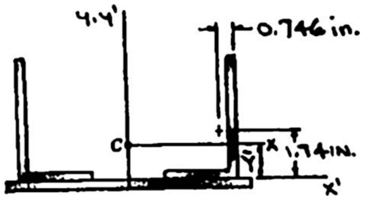
$$\bar{J}_C = 57.41\times10^6\ \text{mm}^4 \qquad \bar{J}_C = 57.4\times10^6\ \text{mm}^4 \blacktriangleleft$$



PROBLEM 9.190

Two L5 × 3 × $\frac{1}{2}$-in. angles are welded to a $\frac{1}{2}$-in. steel plate. Determine the distance b and the centroidal moments of inertia $\overline{I}_x$ and $\overline{I}_y$ of the combined section, knowing that $\overline{I}_y = 4\overline{I}_x$.

SOLUTION



Angle:

$$A = 3.75\,\text{in}^2$$

$$\overline{I}_x = 9.43\,\text{in}^4$$

$$\overline{I}_y = 2.55\,\text{in}^4$$

First locate centroid C of the section.

	Area, in^2	$\bar{y}$, in.	$\bar{y}A$, in^3
Angle	$2(3.75) = 7.50$	1.74	13.05
Plate	$(10)(0.5) = 5$	–0.25	−1.25
Σ	12.50		11.80

Then $\quad \bar{Y}\Sigma A = \Sigma \bar{y}A: \quad \bar{Y}(12.50\,\text{in}^2) = 11.80\,\text{in}^3$

or $\quad \bar{Y} = 0.944\,\text{in.}$

Now $\quad \overline{I}_x = 2(I_x)_{\text{angle}} + (I_x)_{\text{plate}}$

where

$$(I_x)_{\text{angle}} = \overline{I}_x + Ad^2 = 9.43\,\text{in}^4 + (3.75\,\text{in}^2)[(1.74 - 0.944)\,\text{in.}]^2$$

$$= 11.8061\,\text{in}^4$$

$$(I_x)_{\text{plate}} = \overline{I}_x + Ad^2 = \frac{1}{12}(10\,\text{in.})(0.5\,\text{in.})^3 + (5\,\text{in}^2)[(0.25 + 0.944)\,\text{in.}]^2$$

$$= 7.2323\,\text{in}^4$$

Then $\quad \overline{I}_x = [2(11.8061) + 7.2323]\,\text{in}^4 = 30.8445\,\text{in}^4$

or $\quad \overline{I}_x = 30.8\,\text{in}^4$ ◀

PROBLEM 9.190 (Continued)

We have $\bar{I}_y = 4\bar{I}_x = 4(30.8445\text{ in}^4) = 123.378\text{ in}^4$

or $\bar{I}_y = 123.4\text{ in}^4$ ◀

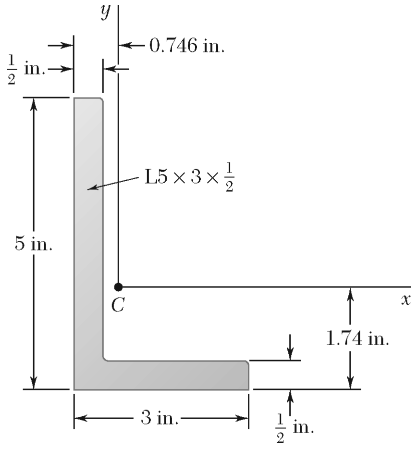
Now $\bar{I}_y = 2(I_y)_{\text{angle}} + (\bar{I}_y)_{\text{plate}}$

where $(\bar{I}_y)_{\text{angle}} = \bar{I}_y + Ad^2 = 2.55\text{ in}^4 + (3.75\text{ in}^2)[(b - 0.746)\text{ in.}]^2$

$$(\bar{I}_y)_{\text{plate}} = \frac{1}{12}(0.5\text{ in.})(10\text{ in.})^3 = 41.6667\text{ in}^4$$

Then $123.378\text{ in}^4 = 2[2.55 + 3.75(b - 0.746)^2]\text{ in}^4 + 41.6667\text{ in}^4$

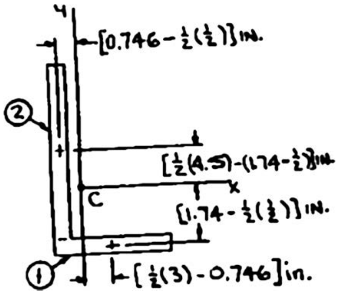
or $b = 3.94\text{ in.}$ ◀



PROBLEM 9.191

Using the parallel-axis theorem, determine the product of inertia of the L5 × 3 × $\frac{1}{2}$-in. angle cross section shown with respect to the centroidal x and y axes.

SOLUTION



We have

$$\overline{I}_{xy}=(I_{xy})_1+(I_{xy})_2$$

For each rectangle:

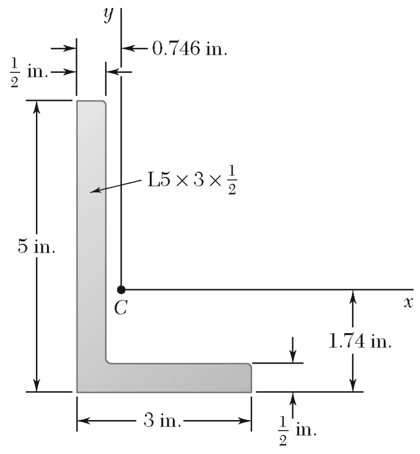
$$I_{xy}=\overline{I}_{x'y'}+\overline{x}\,\overline{y}A$$

and $\qquad \overline{I}_{x'y'}=0$ (symmetry)

Thus $\qquad I_{xy}=\Sigma\overline{x}\,\overline{y}A$

	A, in^2	$\overline{x}$, in.	$\overline{y}$, in.	$\overline{x}\,\overline{y}A$, in^4
1	$3\times\frac{1}{2}=1.5$	0.754	−1.49	−1.68519
2	$4.5\times\frac{1}{2}=2.25$	−0.496	1.01	−1.12716
Σ				−2.81235

$$\overline{I}_{xy}=-2.81\text{ in}^4 \blacktriangleleft$$

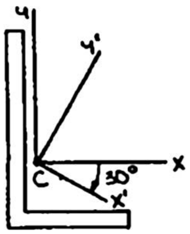


PROBLEM 9.192

For the L5 × 3 × $\frac{1}{2}$-in. angle cross section shown, use Mohr's circle to determine (*a*) the moments of inertia and the product of inertia with respect to new centroidal axes obtained by rotating the *x* and *y* axes 30° clockwise, (*b*) the orientation of the principal axes through the centroid and the corresponding values of the moments of inertia.

SOLUTION

From Figure 9.13*a*:



$$\bar{I}_x = 9.43\text{ in}^4 \qquad \bar{I}_y = 2.55\text{ in}^4$$

From the solution to Problem 9.191

$$\bar{I}_{xy} = -2.81235\text{ in}^4$$

The Mohr's circle is defined by the diameter *XY*, where

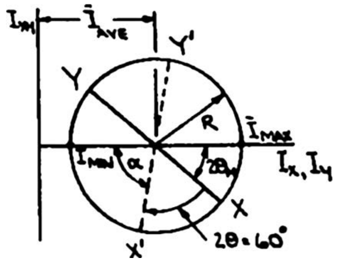
$$X(9.43 - 2.81235), \qquad Y(2.55, 2.81235)$$

Now $\qquad \bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(9.43 + 2.55) = 5.99\text{ in}^4$

and $\qquad R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{1}{2}(9.43 - 2.55)\right]^2 + (-2.81235)^2}$

$$= 4.4433\text{ in}^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}$$

$$= -\frac{2(-2.81235)}{9.43 - 2.55}$$

$$= 0.81754$$

or $\qquad 2\theta_m = 39.267°$

and $\qquad \theta_m = 19.6335°$

Then $\qquad \alpha = 180° - (39.267° + 60°)$

$$= 80.733°$$

PROBLEM 9.192 (Continued)

(*a*) We have $\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\cos\alpha = 5.99 - 4.4433\cos 80.733°$

or $\bar{I}_{x'} = 5.27 \text{ in}^4$ ◀

$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R\cos\alpha = 5.99 + 4.4433\cos 80.733°$

or $\bar{I}_{y'} = 6.71 \text{ in}^4$ ◀

$\bar{I}_{x'y'} = -R\sin\alpha = -4.4433\sin 80.733°$

or $\bar{I}_{x'y'} = -4.39 \text{ in}^4$ ◀

(*b*) First observe that the principal axes are obtained by rotating the *xy* axes through

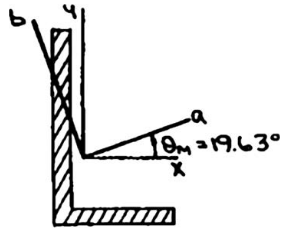
19.63° counterclockwise ◀

about *C*.

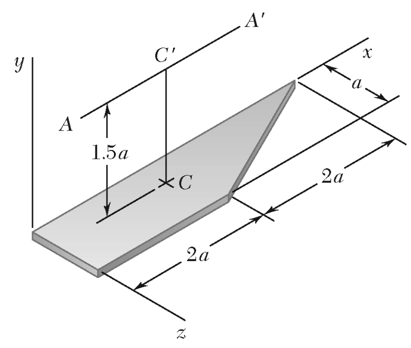
Now $\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = 5.99 \pm 4.4433$

or $\bar{I}_{\text{max}} = 10.43 \text{ in}^4$ ◀

$\bar{I}_{\text{min}} = 1.547 \text{ in}^4$ ◀



From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{max}}$ and the *b* axis corresponds to $\bar{I}_{\text{min}}$.



PROBLEM 9.193

A thin plate of mass m has the trapezoidal shape shown. Determine the mass moment of inertia of the plate with respect to (*a*) the x axis, (*b*) the y axis.

SOLUTION

First note

$$\text{mass} = m = \rho V = \rho t A$$

$$= \rho t\left[(2a)(a) + \frac{1}{2}(2a)(a)\right] = 3\rho t a^2$$

Also

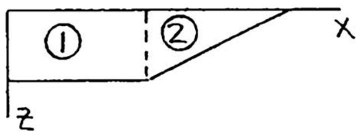
$$I_{\text{mass}} = \rho t I_{\text{area}} = \frac{m}{3a^2} I_{\text{area}}$$

(*a*) Now

$$I_{x,\text{area}} = (I_x)_{1,\text{area}} + (I_x)_{2,\text{area}}$$

$$= \frac{1}{3}(2a)(a)^3 + \frac{1}{12}(2a)(a)^3$$

$$= \frac{5}{6}a^4$$



Then

$$I_{x,\text{mass}} = \frac{m}{3a^2} \times \frac{5}{6}a^4$$

or $I_{x,\text{ mass}} = \dfrac{5}{18}ma^2$ ◀

(*b*) We have

$$I_{z,\text{area}} = (I_z)_{1,\text{area}} + (I_z)_{2,\text{ area}}$$

$$= \left[\frac{1}{3}(a)(2a)^3\right] + \left[\frac{1}{36}(a)(2a)^3 + \frac{1}{2}(2a)(a)\left(2a + \frac{1}{3} \times 2a\right)^2\right] = 10a^4$$

Then

$$I_{x,\text{mass}} = \frac{m}{3a^2} \times 10a^4 = \frac{10}{3}ma^2$$

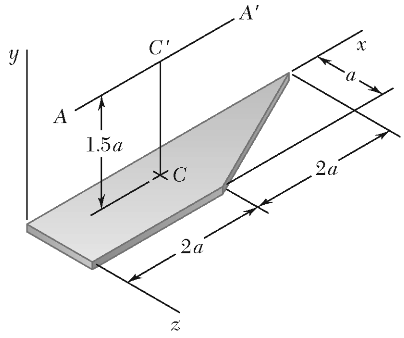
Finally,

$$I_{y,\text{mass}} = I_{x,\text{mass}} + I_{z,\text{mass}}$$

$$= \frac{5}{18}ma^2 + \frac{10}{3}ma^2$$

$$= \frac{65}{18}ma^2$$

or $I_{y,\text{mass}} = 3.61ma^2$ ◀



PROBLEM 9.194

A thin plate of mass m has the trapezoidal shape shown. Determine the mass moment of inertia of the plate with respect to (*a*) the centroidal axis CC' that is perpendicular to the plate, (*b*) the axis AA' that is parallel to the x axis and is located at a distance $1.5a$ from the plate.

SOLUTION

First locate the centroid C.

$$\bar{X}\Sigma A = \Sigma \bar{x} A: \quad \bar{X}(2a^2 + a^2) = a(2a^2) + \left(2a + \frac{1}{3}\times 2a\right)(a^2)$$

or
$$\bar{X} = \frac{14}{9}a$$

$$\bar{Z}\Sigma A = \Sigma \bar{z} A: \quad \bar{Z}(2a^2 + a^2) = \left(\frac{1}{2}a\right)(2a^2) + \left(\frac{1}{3}a\right)(a^2)$$

or
$$\bar{Z} = \frac{4}{9}a$$

(*a*) We have
$$I_{y,\,\text{mass}} = \bar{I}_{CC',\,\text{mass}} + m(\bar{X}^2 + \bar{Z}^2)$$

From the solution to Problem 9.117:

$$I_{y,\text{mass}} = \frac{65}{18}ma^2$$

Then
$$\bar{I}_{cc',\text{mass}} = \frac{65}{18}ma^2 - m\left[\left(\frac{14}{9}a\right)^2 + \left(\frac{4}{9}a\right)^2\right]$$

or $\bar{I}_{cc'} = 0.994ma^2$ ◀

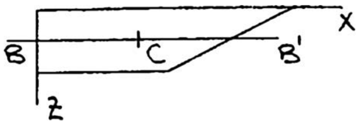
(*b*) We have
$$I_{x,\text{mass}} = \bar{I}_{BB',\text{mass}} + m(\bar{Z})^2$$

and
$$I_{AA',\,\text{mass}} = \bar{I}_{BB',\,\text{mass}} + m(1.5a)^2$$

Then
$$I_{AA',\,\text{mass}} = I_{x,\,\text{mass}} + m\left[(1.5a)^2 - \left(\frac{4}{9}a\right)^2\right]$$

PROBLEM 9.194 (Continued)

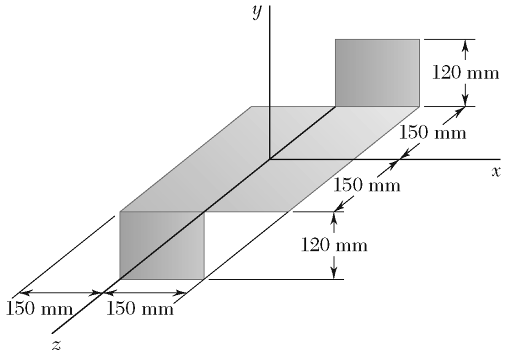
From the solution to Problem 9.193:



$$I_{x,\text{mass}} = \frac{5}{18}ma^2$$

Then $$I_{AA',\text{mass}} = \frac{5}{18}ma^2 + m\left[(1.5a)^2 - \left(\frac{4}{9}a\right)^2\right]$$

or $I_{AA'} = 2.33ma^2$ ◀

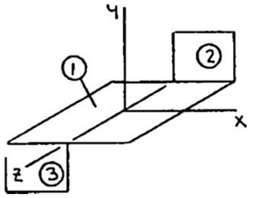


PROBLEM 9.195

A 2-mm thick piece of sheet steel is cut and bent into the machine component shown. Knowing that the density of steel is $7850\ \text{kg/m}^3$, determine the mass moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION

First compute the mass of each component. We have



$$m = \rho_{\text{ST}} V = \rho_{\text{ST}} tA$$

Then

$$m_1 = (7850\ \text{kg/m}^3)(0.002\ \text{m})(0.3\ \text{m})^2$$
$$= 1.413\ \text{kg}$$

$$m_2 = m_3 = (7850\ \text{kg/m}^3)(0.002\ \text{m}) \times (0.15 \times 0.12)\ \text{m}^2$$
$$= 0.2826\ \text{kg}$$

Using Figure 9.28 and the parallel-axis theorem, we have

$$I_x = (I_x)_1 + 2(I_x)_2$$
$$= \left[\frac{1}{12}(1.413\ \text{kg})(0.3\ \text{m})^2\right]$$
$$+ 2\left[\frac{1}{12}(0.2826\ \text{kg})(0.12\ \text{m})^2 + (0.2828\ \text{kg})(0.15^2 + 0.06^2)\ \text{m}^2\right]$$
$$= [(0.0105975) + 2(0.0003391 + 0.0073759)]\ \text{kg}\cdot\text{m}^2$$
$$= [(0.0105975) + 2(0.0077150)]\ \text{kg}\cdot\text{m}^2$$

or $I_x = 26.0 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

$$I_y = (I_y)_1 + 2(I_y)_2$$
$$= \left[\frac{1}{12}(1.413\ \text{kg})(0.3^2 + 0.3^2)\ \text{m}^2\right]$$
$$+ 2\left[\frac{1}{12}(0.2826\ \text{kg})(0.15\ \text{m})^2 + (0.2826\ \text{kg})(0.075^2 + 0.15^2)\ \text{m}^2\right]$$
$$= [(0.0211950) + 2(0.0005299 + 0.0079481)]\ \text{kg}\cdot\text{m}^2$$
$$= [(0.0211950) + 2(0.0084780)]\ \text{kg}\cdot\text{m}^2$$

or $I_y = 38.2 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.195 (Continued)

$$I_z = (I_z)_1 + 2(I_z)_2$$

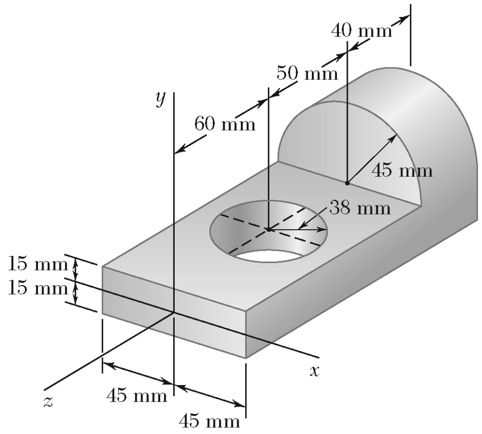
$$= \left[\frac{1}{12}(1.413 \text{ kg})(0.3 \text{ m})^2\right]$$

$$+ 2\left[\frac{1}{12}(0.2826 \text{ kg})(0.15^2 + 0.12^2) \text{ m}^2 + (0.2826 \text{ kg})(0.075^2 + 0.06^2) \text{ m}^2\right]$$

$$= [(0.0105975) + 2(0.0008690 + 0.0026070)] \text{ kg} \cdot \text{m}^2$$

$$= [(0.0105975) + 2(0.0034760)] \text{ kg} \cdot \text{m}^2$$

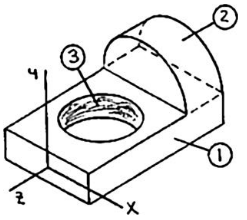
or $I_z = 17.55 \times 10^{-3} \text{ kg} \cdot \text{m}^2$ ◀



PROBLEM 9.196

Determine the mass moment of inertia and the radius of gyration of the steel machine element shown with respect to the *x* axis. (The density of steel is $7850\ \text{kg/m}^3$.)

SOLUTION



First compute the mass of each component. We have

$$m = \rho_{\text{ST}} V$$

Then

$$m_1 = (7850\ \text{kg/m}^3)(0.09 \times 0.03 \times 0.15)\ \text{m}^3$$
$$= 3.17925\ \text{kg}$$

$$m_2 = (7850\ \text{kg/m}^3)\left[\frac{\pi}{2}(0.045)^2 \times 0.04\right]\ \text{m}^3$$
$$= 0.998791\ \text{kg}$$

$$m_3 = (7850\ \text{kg/m}^3)[\pi(0.038)^2 \times 0.03]\ \text{m}^3 = 1.06834\ \text{kg}$$

Using Figure 9.28 for components 1 and 3 and the equation derived above (before the solution to Problem 9.142) for a semicylinder, we have

$$I_x = (I_x)_1 + (I_x)_2 - (I_x)_3$$
$$= \left[\frac{1}{12}(3.17925\ \text{kg})(0.03^2 + 0.15^2)\ \text{m}^2 + (3.17925\ \text{kg})(0.075\ \text{m})^2\right]$$
$$+ \left\{(0.998791\ \text{kg})\left[\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)(0.045\ \text{m})^2 + \frac{1}{12}(0.04\ \text{m})^2\right]\right.$$
$$\left. + (0.998791\ \text{kg})\left[(0.13)^2 + \left(\frac{4 \times 0.045}{3\pi} + 0.015\right)^2\right]\ \text{m}^2\right\}$$
$$- \left\{\frac{1}{12}(1.06834\ \text{kg})[3(0.038\ \text{m})^2 + (0.03\ \text{m})^2] + (1.06834\ \text{kg})(0.06\ \text{m})^2\right\}$$

PROBLEM 9.196 (Continued)

$$= [(0.0061995 + 0.0178833) + (0.0002745 + 0.0180409) - (0.0004658 + 0.0038460)]\ \text{kg}\cdot\text{m}^2$$

$$= (0.0240828 + 0.0183154 - 0.0043118)\ \text{kg}\cdot\text{m}^2$$

$$= 0.0380864\ \text{kg}\cdot\text{m}^2$$

or $I_x = 38.1\times10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

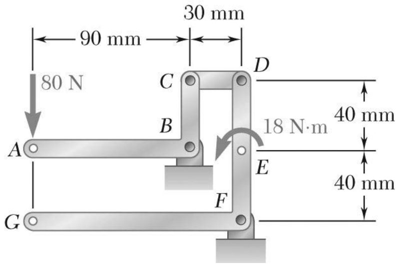
Now $m = m_1 + m_2 - m_3 = (3.17925 + 0.998791 - 1.06834)\ \text{kg}$

$= 3.10970\ \text{kg}$

and $$k_x^2 = \frac{I_x}{m} = \frac{0.0380864\ \text{kg}\cdot\text{m}^2}{3.10970\ \text{kg}}$$

or $k_x = 110.7\ \text{mm}$ ◀

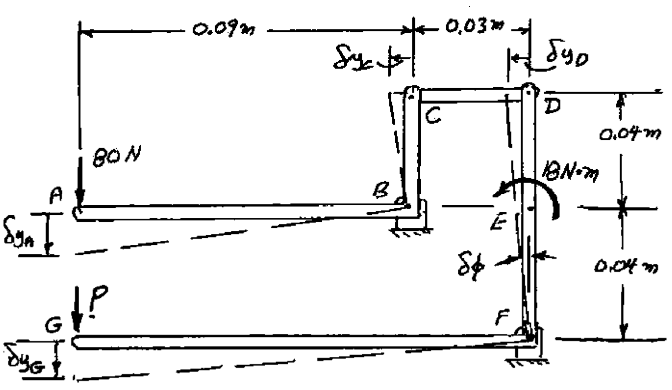
CHAPTER 10



PROBLEM 10.1

Determine the vertical force **P** that must be applied at *G* to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \downarrow$:

$$\delta y_C = \frac{0.04}{0.09}\delta y_A = \frac{4}{9}y_A \longleftarrow, \qquad \delta y_D = \delta y_C = \frac{4}{9}\delta y_A \longleftarrow$$

$$\delta y_G = \frac{0.12\text{ m}}{0.08\text{ m}}\delta y_D = 1.5\left(\frac{4}{9}\delta y_A\right) = \frac{2}{3}\delta y_A \downarrow$$

$$\delta\phi = \frac{\delta y_D}{0.08} = \frac{4}{9}\delta y_A / 0.08 = \frac{4}{0.72}\delta y_A = \frac{50}{9}\delta y_A \circlearrowleft$$

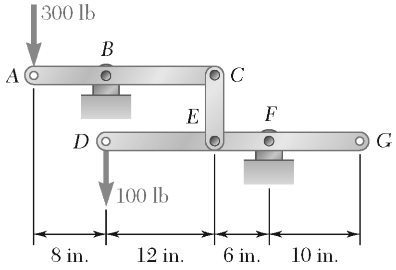
<u>Virtual Work</u>:

$$\delta U = 0: \quad (80\text{ N})\delta y_A + (18\text{ N}\cdot\text{m})\delta\phi + P\delta y_G = 0$$

$$80\delta y_A + 18\left(\frac{50}{9}\delta y_A\right) + P\left(\frac{2}{3}\delta y_A\right) = 0$$

$$80 + 100 + \frac{2}{3}P = 0$$

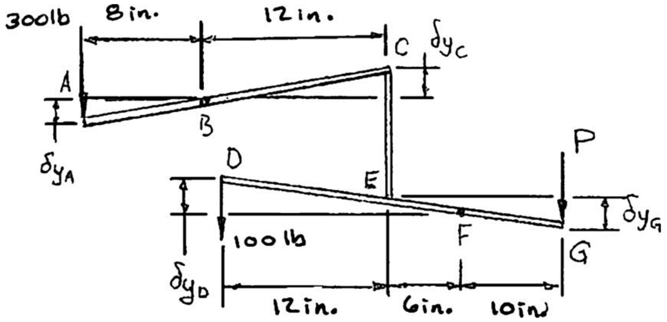
$$P = -270\text{ N} \qquad\qquad \mathbf{P} = 270\text{ N}\uparrow \blacktriangleleft$$



PROBLEM 10.2

Determine the vertical force **P** that must be applied at G to maintain the equilibrium of the linkage.

SOLUTION



Assuming $\delta y_A \downarrow$

it follows

$$\delta y_C = \frac{12}{8}\delta y_A = 1.5\delta y_A \uparrow$$

$$\delta y_E = \delta y_C = 1.5\delta y_A \uparrow$$

$$\delta y_D = \frac{18}{6}\delta y_A = 3(1.5\delta y_A) = 4.5\delta y_A \uparrow$$

$$\delta y_G = \frac{10}{6}\delta y_A = \frac{10}{6}(1.5\delta y_A) = 2.5\delta y_A \downarrow$$

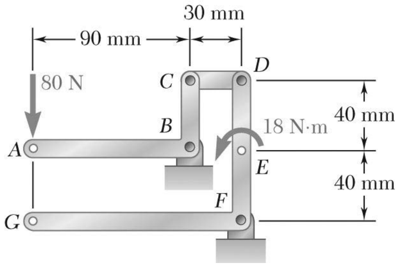
Then, by virtual work

$$\delta U = 0: \quad (300\text{ lb})\delta y_A - (100\text{ lb})\delta y_D + P\delta y_G = 0$$

$$300\delta y_A - 100(4.5\delta y_A) + P(2.5\delta y_A) = 0$$

$$300 - 450 + 2.5P = 0$$

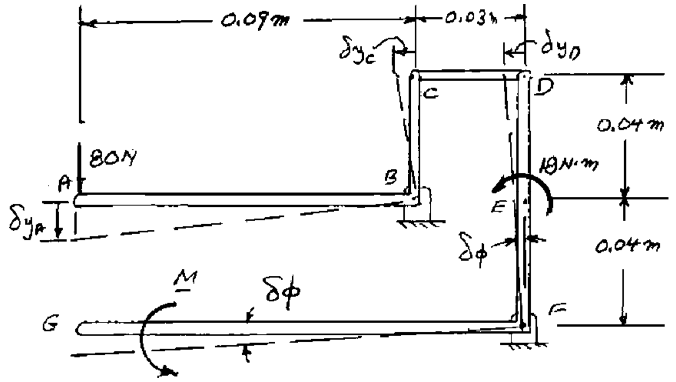
$$P = 60.0\text{ lb} \qquad \mathbf{P} = 60.0\text{ lb}\downarrow \blacktriangleleft$$



PROBLEM 10.3

Determine the couple **M** that must be applied to member *DEFG* to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \downarrow$:

$$\delta y_C = \frac{0.04}{0.09}\delta y_A = \frac{4}{9} y_A \leftarrow, \qquad \delta y_D = \delta y_C = \frac{4}{9}\delta y_A \leftarrow$$

$$\delta\phi = \frac{\delta y_C}{0.08} = \frac{4}{9}\delta y_A / 0.08 = \frac{4}{0.72}\delta y_A = \frac{50}{9}\delta y_A \circlearrowright$$

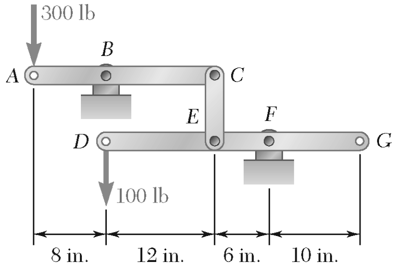
<u>Virtual Work</u>:

$$\delta U = 0: \qquad (80\ \text{N})\delta y_A + (18\ \text{N}\cdot\text{m})\delta\phi + M\,\delta\phi = 0$$

$$80\delta y_A + 18\left(\frac{50}{9}\delta y_A\right) + M\left(\frac{50}{9}\delta y_A\right) = 0$$

$$80 + 100 + \frac{50}{9}M = 0$$

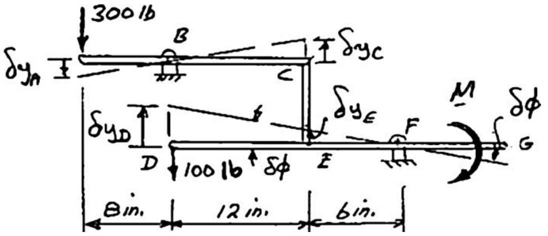
$$M = -32.4\ \text{N}\cdot\text{m} \qquad \mathbf{M} = 32.4\ \text{N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



PROBLEM 10.4

Determine the couple **M** that must be applied to member *DEFG* to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \downarrow$:

$$\delta y_C = \frac{12}{8}\delta y_A = 1.5\delta y_A \uparrow, \qquad \delta y_E = \delta y_C = 1.5\delta y_A \uparrow$$

$$\delta y_D = \frac{18}{6}\delta y_E = 3(1.5\delta y_A) = 4.5\delta y_A \uparrow$$

$$\delta\phi = \frac{\delta y_E}{6} = \frac{1.5\delta y_A}{6} = \frac{1}{4}\delta y_A \circlearrowright$$

Virtual Work:

$$\delta U = 0: \quad (300\text{ lb})\delta y_A - (100\text{ lb})\delta y_D + M\,\delta\phi = 0$$

$$300\delta y_A - 100(1.5\delta y_A) + M\left(\frac{1}{4}\delta y_A\right) = 0$$

$$300 - 450 + \frac{1}{4}M = 0$$

$$M = +600\text{ lb}\cdot\text{in.} \qquad \mathbf{M} = 600\text{ lb}\cdot\text{in.}\circlearrowright \blacktriangleleft$$

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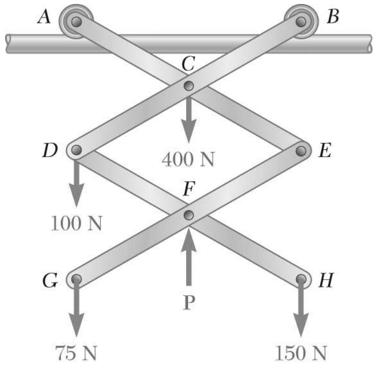


PROBLEM 10.5

Determine the force **P** required to maintain the equilibrium of the linkage shown. All members are of the same length and the wheels at *A* and *B* roll freely on the horizontal rod.

SOLUTION

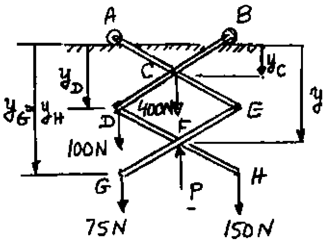
Using y_C as independent variable:

$$y_D = 2y_C \qquad \delta y_D = 2\delta y_C$$

$$y_F = 3y_C \qquad \delta y_F = 3\delta y_C$$

$$y_G = y_H = 4y_C$$

$$\delta y_G = \delta y_H = 4\delta y_C$$



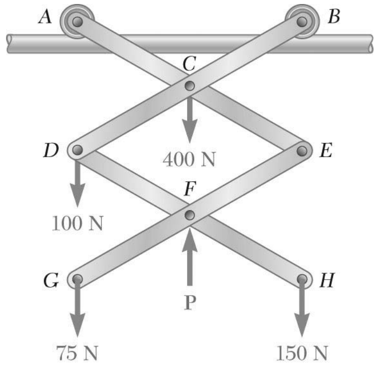
<u>Virtual Work</u>:

$$\delta U = (400\text{ N})\delta y_C + (100\text{ N})\delta y_D - P\delta y_F + (75\text{ N})\delta y_G + (150\text{ N})\delta y_H = 0$$

$$400\delta y_C + 100(2\delta y_C) - P(3\delta y_C) + (75+150)(4\delta y_C) = 0$$

$$3P = 400 + 200 + 900 \qquad P = +500\text{ N}$$

$$\mathbf{P} = 500\text{ N}\uparrow$$ ◀



PROBLEM 10.6

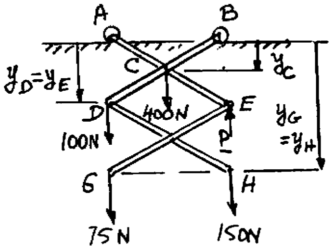
Solve Problem 10.5 assuming that the vertical force **P** is applied at Point *E*.

PROBLEM 10.5 Determine the force **P** required to maintain the equilibrium of the linkage shown. All members are of the same length and the wheels at *A* and *B* roll freely on the horizontal rod.

SOLUTION

Using y_C as independent variable:

$$y_D = y_E = 2y_C$$
$$\delta y_D = \delta y_E = 2\delta y_C$$
$$y_G = y_H = 4y_C$$
$$\delta y_G = \delta y_A = 4\delta y_C$$

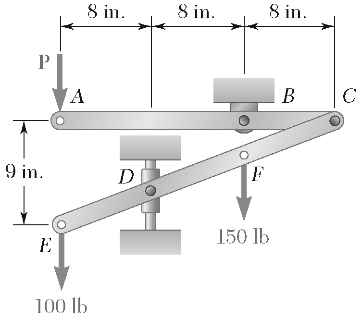


Virtual Work:

$$\delta U = (400\text{ N})\delta y_C + (100\text{ N})\delta y_D - P\delta y_E + (75\text{ N})\delta y_G + (150\text{ N})\delta y_H = 0$$
$$400\delta y_C + 100(2\delta y_C) - P(2\delta y_C) + (75+150)(4\delta y_C) = 0$$

$$2P = 400 + 200 + 900 \qquad P = +750\text{ N}$$

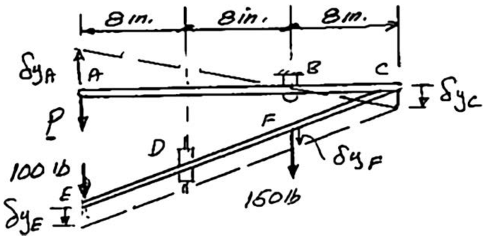
$$\mathbf{P} = 750\text{ N}\uparrow \blacktriangleleft$$



PROBLEM 10.7

The two-bar linkage shown is supported by a pin and bracket at *B* and a collar at *D* that slides freely on a vertical rod. Determine the force **P** required to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \uparrow$:

$$\delta y_C = \frac{8 \text{ in.}}{16 \text{ in.}} \delta y_A; \quad \delta y_C = \frac{1}{2} \delta y_A \downarrow$$

Since bar *CD* move in translation

$$\delta y_E = \delta y_F = \delta y_C$$

or

$$\delta y_E = \delta y_F = \frac{1}{2} y_A \downarrow$$

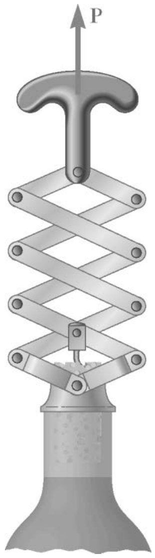
Virtual Work:

$$\delta U = 0: \quad -P\delta y_A + (100 \text{ lb})\delta y_E + (150 \text{ lb})\delta y_F = 0$$

$$-P\delta y_A + 100\left(\frac{1}{2}\delta y_A\right) + 150\left(\frac{1}{2}\delta y_A\right) = 0$$

$$P = 125 \text{ lb}$$

$$\mathbf{P} = 125.0 \text{ lb} \downarrow \blacktriangleleft$$



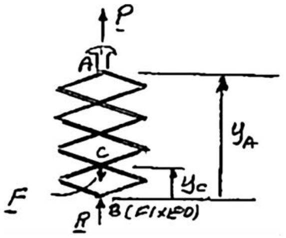
PROBLEM 10.8

Knowing that the maximum friction force exerted by the bottle on the cork is 60 lb, determine (*a*) the force **P** that must be applied to the corkscrew to open the bottle, (*b*) the maximum force exerted by the base of the corkscrew on the top of the bottle.

SOLUTION

From sketch $y_A = 4y_C$

Thus $\delta y_A = 4\delta y_C$



(*a*) Virtual Work: $\delta U = 0$: $P\delta y_A - F\delta y_C = 0$

$$P(4\delta y_C) - F\delta y_C = 0$$

$$P = \frac{1}{4}F$$

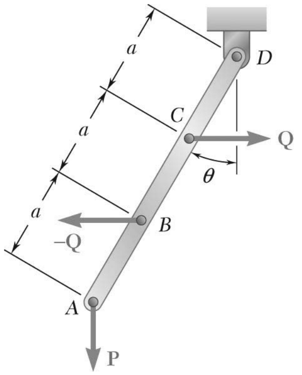
$$F = 60 \text{ lb}: \quad P = \frac{1}{4}(60 \text{ lb}) = 15 \text{ lb}$$

$$\mathbf{P} = 15.00 \text{ lb}\uparrow \blacktriangleleft$$

(*b*) Free body: Corkscrew

$$+\uparrow \Sigma F_y = 0 \quad R + P - F = 0; \quad R + 15 \text{ lb} - 60 \text{ lb} = 0$$

On corkscrew: $\mathbf{R} = 45 \text{ lb}\uparrow$, On bottle: $\mathbf{R} = 45.0 \text{ lb}\downarrow \blacktriangleleft$



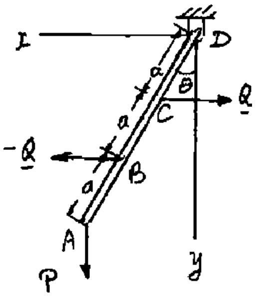
PROBLEM 10.9

Rod AD is acted upon by a vertical force **P** at end A, and by two equal and opposite horizontal forces of magnitude Q at points B and C. Derive an expression for the magnitude Q of the horizontal forces required for equilibrium.

SOLUTION

We have

$$x_C = a\sin\theta$$
$$\delta x_C = a\cos\theta\delta\theta$$
$$x_B = 2a\sin\theta$$
$$\delta x_B = 2a\cos\theta\delta\theta$$
$$y_A = 3a\cos\theta$$
$$\delta y_A = -3a\sin\theta\delta\theta$$

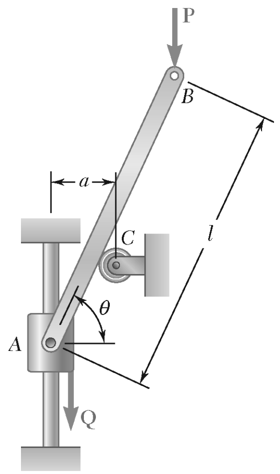


Virtual Work: We note that **P** tends to increase y_A and **−Q** tends to increase x_B, while **Q** tends to decrease x_C. Therefore

$$\delta U = P\delta y_A + Q\delta x_B - Q\delta x_C = 0$$
$$= P(-3a\sin\theta\delta\theta) + Q(2a\cos\theta\delta\theta) - Q(a\cos\theta\delta\theta) = 0$$

$$Q\cos\theta = 3P\sin\theta'$$

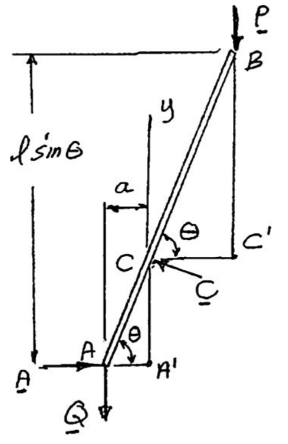
$$Q = 3P\tan\theta \blacktriangleleft$$



PROBLEM 10.10

The slender rod *AB* is attached to a collar *A* and rests on a small wheel at *C*. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$:

$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta CC'B$:

$$BC' = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$y_B = BC' = l\sin\theta - a\tan\theta$$

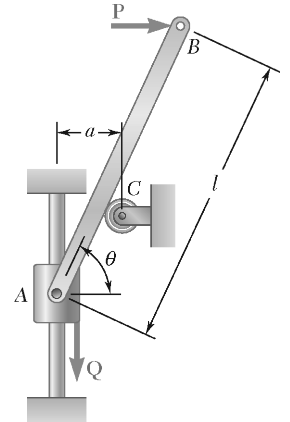
$$\delta y_B = l\cos\theta\delta\theta - \frac{a}{\cos^2\theta}\delta\theta$$

Virtual Work: $\delta U = 0{:}\quad Q\delta y_A - P\delta y_B = 0$

$$-Q\left(-\frac{a}{\cos^2\theta}\right)\delta\theta - P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)\delta\theta = 0$$

$$Q\left(\frac{a}{\cos^2\theta}\right) = P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)$$

$$Q = P\left(\frac{l}{a}\cos^3\theta - 1\right) \blacktriangleleft$$

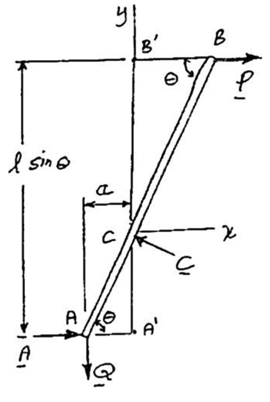


PROBLEM 10.11

The slender rod *AB* is attached to a collar *A* and rests on a small wheel at *C*. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION

For $\Delta AA'C$:



$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta BB'C$:

$$B'C = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$BB' = \frac{B'C}{\tan\theta} = \frac{l\sin\theta - a\tan\theta}{\tan\theta}$$

$$x_B = BB' = l\cos\theta - a$$

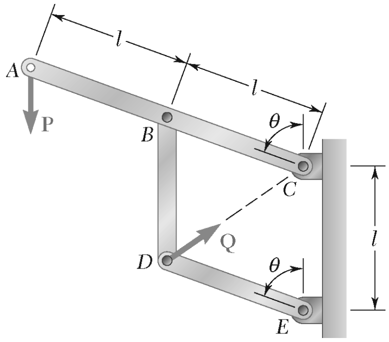
$$\delta x_B = -l\sin\theta\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = 0:\quad P\delta x_B - Q\delta y_A = 0$$

$$P(-l\sin\theta\delta\theta) - Q\left(-\frac{a}{\cos^2\theta}\delta\theta\right) = 0$$

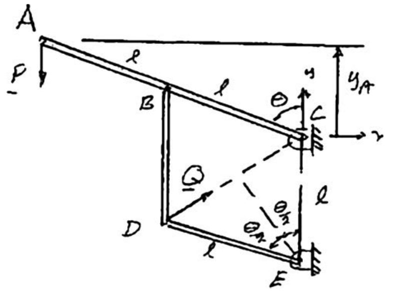
or $$Pl\sin\theta\cos^2\theta = Qa \qquad Q = P\frac{l}{a}\sin\theta\cos^2\theta \blacktriangleleft$$



PROBLEM 10.12

Knowing that the line of action of the force **Q** passes through Point *C*, derive an expression for the magnitude of **Q** required to maintain equilibrium.

SOLUTION



We have

$$y_A = 2l\cos\theta; \quad \delta y_A = -2l\sin\theta\delta\theta$$

$$CD = 2l\sin\frac{\theta}{2}; \quad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A - Q\delta(CD) = 0$$

$$-P(-2l\sin\theta\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0 \qquad Q = 2P\frac{\sin\theta}{\cos(\theta/2)} \blacktriangleleft$$

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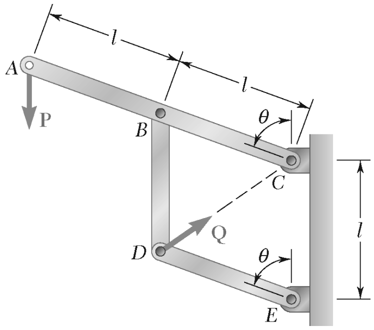
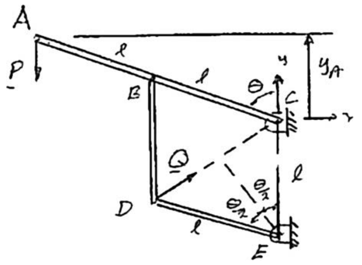


PROBLEM 10.13

Solve Problem 10.12 assuming that the force **P** applied at Point *A* acts horizontally to the left.

PROBLEM 10.12 Knowing that the line of action of the force **Q** passes through Point *C*, derive an expression for the magnitude of **Q** required to maintain equilibrium.

SOLUTION



We have

$$x_A = 2l\cos\theta;\quad \delta x_A = 2l\cos\theta\delta\theta$$

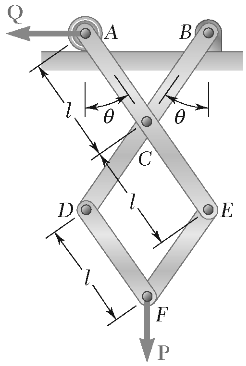
$$CD = 2l\sin\frac{\theta}{2};\quad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = 0:\quad P\delta x_A - Q\delta(CD) = 0$$

$$P(2l\cos\theta\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

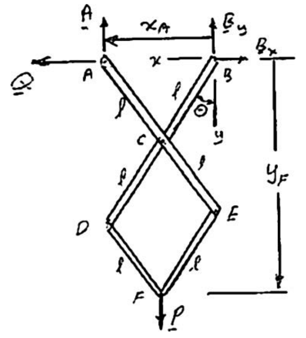
$$Q = 2P\frac{\cos\theta}{\cos(\theta/2)} \blacktriangleleft$$



PROBLEM 10.14

The mechanism shown is acted upon by the force **P**; derive an expression for the magnitude of the force **Q** required to maintain equilibrium.

SOLUTION



Virtual Work:

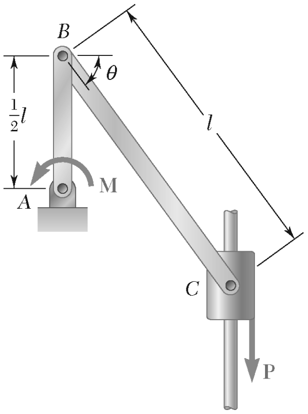
We have
$$x_A = 2l\sin\theta$$
$$\delta x_A = 2l\cos\theta\delta\theta$$

and
$$y_F = 3l\cos\theta$$
$$\delta y_F = -3l\sin\theta\delta\theta$$

Virtual Work:
$$\delta U = 0:\quad Q\delta x_A + P\delta y_F = 0$$

$$Q(2l\cos\theta\delta\theta) + P(-3l\sin\theta\delta\theta) = 0$$

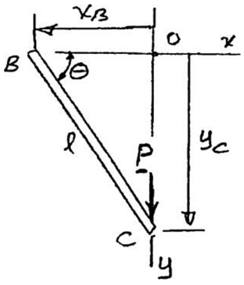
$$Q = \frac{3}{2}P\tan\theta \blacktriangleleft$$



PROBLEM 10.15

Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION



We have $x_B = l\cos\theta$

$$\delta x_B = -l\sin\theta\delta\theta \qquad (1)$$

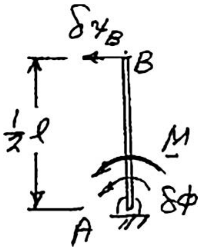
$$y_C = l\sin\theta$$

$$\delta y_C = l\cos\theta\delta\theta$$

Now $\delta x_B = \frac{1}{2}l\delta\theta$

Substituting from Equation (1)

$$-l\sin\theta\delta\theta = \frac{1}{2}l\delta\phi$$

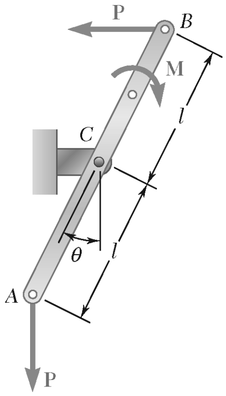


or $\delta\phi = -2\sin\theta\delta\theta \circlearrowleft$

Virtual Work: $\delta U = 0$: $M\,\delta\varphi + P\delta y_C = 0$

$$M(-2\sin\theta\delta\theta) + P(l\cos\theta\delta\theta) = 0$$

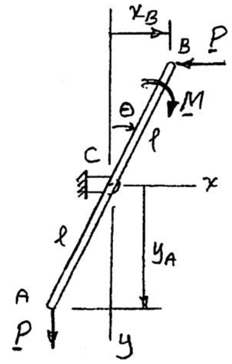
or $M = \frac{1}{2}Pl\frac{\cos\theta}{\sin\theta}$ $\qquad M = \frac{Pl}{2\tan\theta}$ ◀



PROBLEM 10.16

Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION



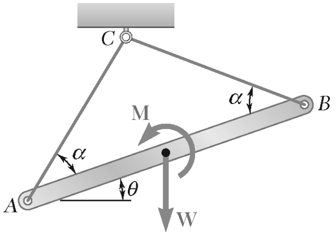
We have

$$x_B = l\sin\theta$$
$$\delta x_B = l\cos\theta\delta\theta$$
$$y_A = l\cos\theta$$
$$\delta y_A = -l\sin\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad M\delta\theta - P\delta x_B + P\delta y_A = 0$$
$$M\delta\theta - P(l\cos\theta\delta\theta) + P(-l\sin\theta\delta\theta) = 0$$

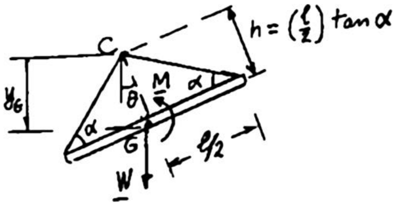
$$M = Pl(\sin\theta + \cos\theta) \blacktriangleleft$$



PROBLEM 10.17

A uniform rod *AB* of length *l* and weight *W* is suspended from two cords *AC* and *BC* of equal length. Derive an expression for the magnitude of the couple **M** required to maintain equilibrium of the rod in the position shown.

SOLUTION



$$y_G = h\cos\theta = \frac{1}{2}l\tan\alpha\cos\theta$$

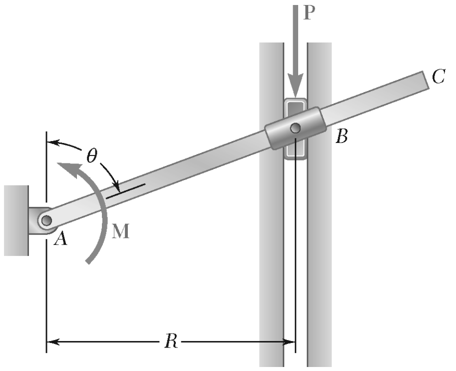
$$\delta y_G = -\frac{1}{2}l\tan\alpha\,\sin\theta\delta\theta$$

Virtual Work:

$$\delta U = W\delta y_G + M\,\delta\theta = 0$$

$$W\left(-\frac{1}{2}l\tan\alpha\sin\theta\delta\theta\right) + M\,\delta\theta = 0$$

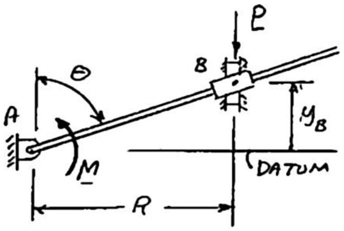
$$M = \frac{1}{2}Wl\tan\alpha\sin\theta$$ ◀



PROBLEM 10.18

Collar B can slide along rod AC and is attached by a pin to a block that can slide in the vertical slot shown. Derive an expression for the magnitude of the couple **M** required to maintain equilibrium.

SOLUTION



$$y_B = \frac{R}{\tan(90° - \theta)}$$

$$\delta y_B = \frac{-R\delta\theta}{\cos^2(90° - \theta)}$$

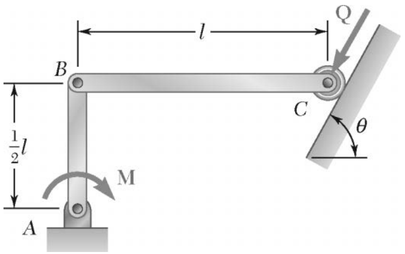
$$\delta y_B = \frac{-R\delta\theta}{\sin^2\theta}$$

Virtual Work: $\delta U = 0$: $\delta U = -M\,\delta\theta - P\delta y_B = 0$

$$-M\,\delta\theta + PR\frac{1}{\sin^2\theta}\delta\theta = 0$$

$$M = \frac{PR}{\sin^2\theta}$$

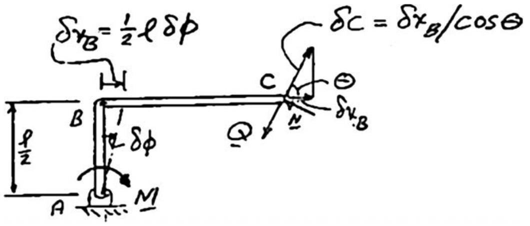
$M = PR\csc^2\theta$ ◀



PROBLEM 10.19

For the linkage shown, determine the couple **M** required for equilibrium when $l = 1.8$ ft, $Q = 40$ lb, and $\theta = 65°$.

SOLUTION



$$\delta C = \frac{\frac{1}{2} l\,\delta\phi}{\cos\theta}$$

Virtual Work:

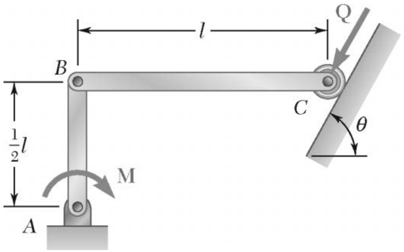
$$\delta U = 0: \quad M\,\delta\phi - Q\,\delta C = 0$$

$$M\,\delta\phi - Q\left(\frac{1}{2}\frac{l}{\cos\theta}\right)\delta\phi = 0$$

$$M = \frac{1}{2}\frac{Ql}{\cos\theta}$$

Data: $$M = \frac{1}{2}\frac{(40\text{ lb})(1.8\text{ ft})}{\cos 65°} = 85.18\text{ lb}\cdot\text{ft}$$

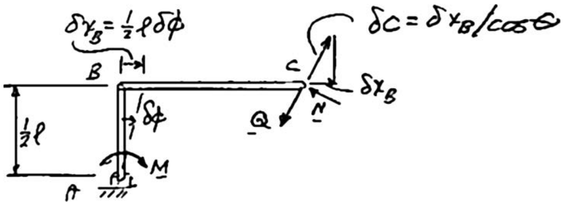
$\mathbf{M} = 85.2\text{ lb}\cdot\text{ft} \curvearrowright$ ◀



PROBLEM 10.20

For the linkage shown, determine the force **Q** required for equilibrium when $l = 18$ in., $M = 600$ lb · in., and $\theta = 70°$.

SOLUTION



$$\delta C = \frac{1}{2}\frac{l\delta\phi}{\cos\theta}$$

Virtual Work:

$$\delta U = 0: \qquad M\,\delta\phi - Q\delta C = 0$$

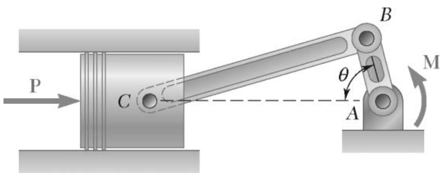
$$M\,\delta\phi - Q\left(\frac{1}{2}\frac{l}{\cos\theta}\right)\delta\phi = 0$$

$$Q = \frac{2M\cos\theta}{l}$$

Data:

$$Q = \frac{2(600\text{ lb}\cdot\text{in.})\cos 70°}{18\text{ in.}} = 22.801\text{ lb}$$

$\mathbf{Q} = 22.8$ lb ↙ 70.0° ◀

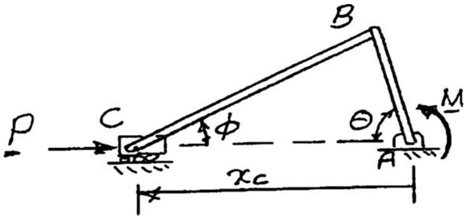


PROBLEM 10.21

A 4-kN force **P** is applied as shown to the piston of the engine system. Knowing that $AB = 50$ mm and $BC = 200$ mm, determine the couple **M** required to maintain the equilibrium of the system when (*a*) $\theta = 30°$, (*b*) $\theta = 150°$.

SOLUTION

Analysis of the geometry:



Law of sines

$$\frac{\sin\phi}{AB} = \frac{\sin\theta}{BC}$$

$$\sin\phi = \frac{AB}{BC}\sin\theta \qquad (1)$$

Now

$$x_C = AB\cos\theta + BC\cos\phi$$

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\delta\phi \qquad (2)$$

Now, from Equation (1) $$\cos\phi\delta\phi = \frac{AB}{BC}\cos\theta\delta\theta$$

or $$\delta\phi = \frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta \qquad (3)$$

From Equation (2)

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\left(\frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta\right)$$

or $$\delta x_C = -\frac{AB}{\cos\phi}(\sin\theta\cos\phi + \sin\phi\cos\theta)\delta\theta$$

Then $$\delta x_C = -\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta$$

PROBLEM 10.21 (Continued)

<u>Virtual Work</u>: $\delta U = 0{:}\quad -P\delta x_C - M\,\delta\theta = 0$

$$-P\left[-\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta\right] - M\,\delta\theta = 0$$

Thus, $$M = AB\frac{\sin(\theta+\phi)}{\cos\phi}P \qquad (4)$$

(*a*) $P = 4$ kN, $\theta = 30°$

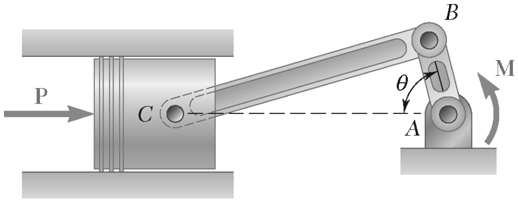
Eq. (1): $\sin\phi = \dfrac{50\text{ mm}}{200\text{ mm}}\sin 30° \qquad \phi = 7.181°$

Eq. (4): $M = (0.05\text{ m})\dfrac{\sin\,(30°+7.181°)}{\cos 7.818°}(4\text{ kN})$ $\qquad \mathbf{M} = 121.8\text{ N}\cdot\text{m}$ ↺ ◀

(*b*) $P = 4$ kN, $\theta = 150°$

Eq. (1): $\sin\phi = \dfrac{50\text{ mm}}{200\text{ mm}}\sin 160° \qquad \phi = 7.181°$

Eq. (4): $M = (0.05\text{ m})\dfrac{\sin\,(150°+7.181°)}{\cos\,7.181°}(4\text{ kN})$ $\qquad \mathbf{M} = 78.2\text{ N}\cdot\text{m}$ ↺ ◀

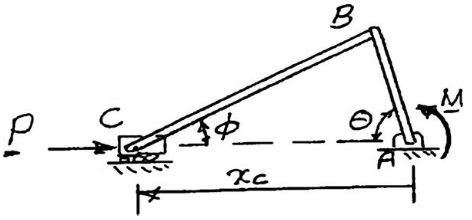


PROBLEM 10.22

A couple **M** of magnitude 100 N · m is applied as shown to the crank of the engine system. Knowing that $AB = 50$ mm and $BC = 200$ mm, determine the force **P** required to maintain the equilibrium of the system when (*a*) $\theta = 60°$, (*b*) $\theta = 120°$.

SOLUTION

Analysis of the geometry:



Law of sines

$$\frac{\sin\phi}{AB} = \frac{\sin\theta}{BC}$$

$$\sin\phi = \frac{AB}{BC}\sin\theta \tag{1}$$

Now

$$x_C = AB\cos\theta + BC\cos\phi$$

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\delta\phi \tag{2}$$

Now, from Equation (1) $$\cos\phi\delta\phi = \frac{AB}{BC}\cos\theta\delta\theta$$

or $$\delta\phi = \frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta \tag{3}$$

From Equation (2)

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\left(\frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta\right)$$

or $$\delta x_C = -\frac{AB}{\cos\phi}(\sin\theta\cos\phi + \sin\phi\cos\theta)\delta\theta$$

Then $$\delta x_C = -\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta$$

Virtual Work: $\delta U = 0{:}\quad -P\delta x_C - M\,\delta\theta = 0$

$$-P\left[-\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta\right] - M\,\delta\theta = 0$$

Thus, $$M = AB\frac{\sin(\theta+\phi)}{\cos\phi}P \tag{4}$$

(*a*) $M = 100\ \text{N}\cdot\text{m},\quad \theta = 60°$

Eq. (1): $\sin\phi = \dfrac{50\ \text{mm}}{200\ \text{mm}}\sin 60° \qquad \phi = 12.504°$

Eq. (4): $100\ \text{N}\cdot\text{m} = (0.05\ \text{m})\dfrac{\sin\,(60° + 12.504°)}{\cos 12.504°}P$

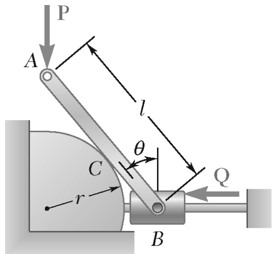
$P = 2047\ \text{N}$ $\qquad$ $\mathbf{P} = 2.05\ \text{kN} \longrightarrow$ ◀

(*b*) $M = 100\ \text{N}\cdot\text{m},\quad \theta = 120°$

Eq. (1): $\sin\phi = \dfrac{50\ \text{mm}}{200\ \text{mm}}\sin 120° \quad \phi = 12.504°$

Eq. (4): $100\ \text{N}\cdot\text{m} = (0.05\ \text{m})\dfrac{\sin\,(120° + 12.504°)}{\cos 12.504°}P$

$P = 2649\ \text{N}$ $\qquad$ $\mathbf{P} = 2.65\ \text{kN} \longrightarrow$ ◀



PROBLEM 10.23

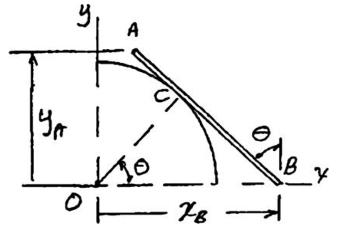
A slender rod of length l is attached to a collar at B and rests on a portion of a circular cylinder of radius r. Neglecting the effect of friction, determine the value of θ corresponding to the equilibrium position of the mechanism when $l = 200$ mm, $r = 60$ mm, $P = 40$ N, and $Q = 80$ N.

SOLUTION

Geometry

$$OC = r$$

$$\cos\theta = \frac{OC}{OB} = \frac{r}{x_B}$$

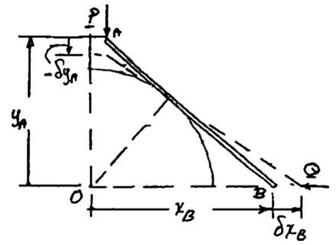


$$x_B = \frac{r}{\cos\theta}$$

$$\delta x_B = \frac{r\sin\theta}{\cos^2\theta}\delta\theta$$

$$y_A = l\cos\theta$$

$$\delta y_A = -l\sin\theta\delta\theta$$



Virtual Work:

$$\delta U = 0: \quad P(-\delta y_A) - Q\delta x_B = 0$$

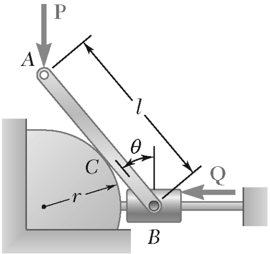
$$Pl\sin\theta\delta\theta - Q\frac{r\sin\theta}{\cos^2\theta}\delta\theta = 0$$

$$\cos^2\theta = \frac{Qr}{Pl} \qquad (1)$$

Then, with $l = 200$ mm, $r = 60$ mm, $P = 40$ N, and $Q = 80$ N

$$\cos^2\theta = \frac{(80\text{ N})(60\text{ mm})}{(40\text{ N})(200\text{ mm})} = 0.6$$

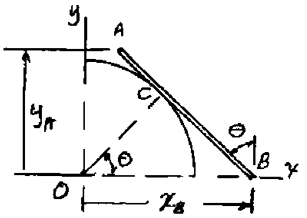
or $\theta = 39.231°$ $\qquad \theta = 39.2°$ ◀



PROBLEM 10.24

A slender rod of length l is attached to a collar at B and rests on a portion of a circular cylinder of radius r. Neglecting the effect of friction, determine the value of θ corresponding to the equilibrium position of the mechanism when $l = 14$ in., $r = 5$ in., $P = 75$ lb, and $Q = 150$ lb.

SOLUTION



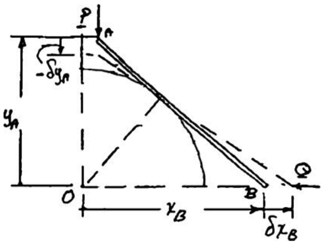
Geometry $$OC = r$$

$$\cos\theta = \frac{OC}{OB} = \frac{r}{x_B}$$

$$x_B = \frac{r}{\cos\theta}$$

$$\delta x_B = \frac{r\sin\theta}{\cos^2\theta}\delta\theta$$

$$y_A = l\cos\theta;\quad \delta y_A = -l\sin\theta\delta\theta$$



Virtual Work:

$$\delta U = 0:\quad P(-\delta y_A) - Q\delta x_B = 0$$

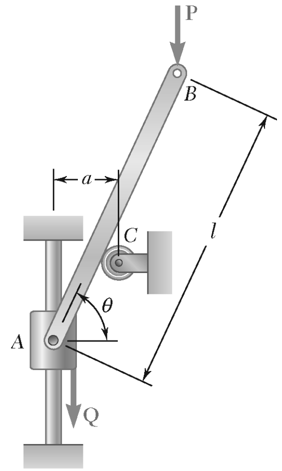
$$Pl\sin\theta\delta\theta - Q\frac{r\sin\theta}{\cos^2\theta}\delta\theta = 0$$

$$\cos^2\theta = \frac{Qr}{Pl} \tag{1}$$

Then, with $l = 14$ in., $r = 5$ in., $P = 75$ lb, and $Q = 150$ lb

Eq. (1) becomes: $$\cos^2\theta = \frac{(150\text{ lb})(5\text{ in.})}{(75\text{ lb})(14\text{ in.})} = 0.71429$$

$\theta = 32.3°$ ◀

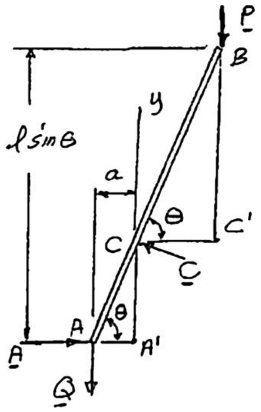


PROBLEM 10.25

Determine the value of θ corresponding to the equilibrium position of the rod of Problem 10.10 when $l = 30$ in., $a = 5$ in., $P = 25$ lb, and $Q = 40$ lb.

PROBLEM 10.10 The slender rod *AB* is attached to a collar *A* and rests on a small wheel at *C*. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$:

$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta CC'B$:

$$BC' = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$y_B = BC' = l\sin\theta - a\tan\theta$$

$$\delta y_B = l\cos\theta\delta\theta - \frac{a}{\cos^2\theta}\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = 0: \quad -Q\delta y_A - P\delta y_B = 0$$

$$-Q\left(-\frac{a}{\cos^2\theta}\right)\delta\theta - P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)\delta\theta = 0$$

$$Q\left(\frac{a}{\cos^2\theta}\right) = P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)$$

or

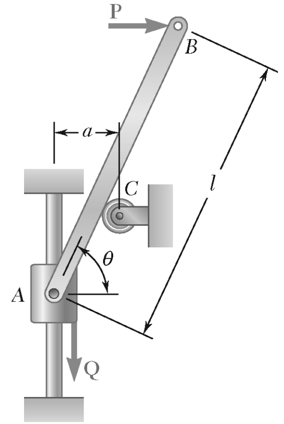
$$Q = P\left(\frac{l}{a}\cos^3\theta - 1\right)$$

PROBLEM 10.25 (Continued)

with $l = 30 \text{ in.}, a = 5 \text{ in.}, P = 25 \text{ lb, and } Q = 40 \text{ lb}$

$$(40 \text{ lb}) = (25 \text{ lb})\left(\frac{30 \text{ in.}}{5 \text{ in.}}\cos^3\theta - 1\right)$$

or $\cos^3\theta = 0.4333$ $\theta = 40.8°$ ◀

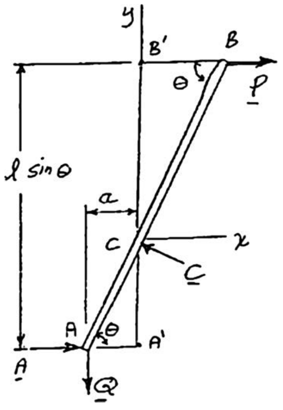


PROBLEM 10.26

Determine the values of θ corresponding to the equilibrium position of the rod of Problem 10.11 when $l = 600$ mm, $a = 100$ mm, $P = 50$ N, and $Q = 90$ N.

PROBLEM 10.11 The slender rod *AB* is attached to a collar *A* and rests on a small wheel at *C*. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$:

$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta BB'C$:

$$B'C = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$BB' = \frac{B'C}{\tan\theta} = \frac{l\sin\theta - a\tan\theta}{\tan\theta}$$

$$x_B = BB' = l\cos\theta - a$$

$$\delta x_B = -l\sin\theta\delta\theta$$

<u>Virtual Work</u>:

$$\delta U = 0: \quad P\delta x_B - Q\delta y_A = 0$$

$$P(-l\sin\theta\delta\theta) - Q\left(-\frac{a}{\cos^2\theta}\delta\theta\right) = 0$$

$$Pl\sin\theta\cos^2\theta = Qa$$

or

$$Q = P\frac{l}{a}\sin\theta\cos^2\theta$$

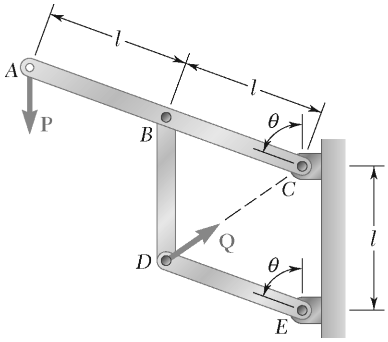
PROBLEM 10.26 (Continued)

with $l = 600\text{ mm},\ a = 100\text{ mm},\ P = 50\text{ N, and } Q = 90\text{ N}$

$$90\text{ N} = (50\text{ N})\frac{600\text{ mm}}{100\text{ mm}}\sin\theta\cos^2\theta$$

or $\sin\theta\cos^2\theta = 0.300$

Solving numerically $\theta = 19.81°\text{ and }51.9°$ ◀

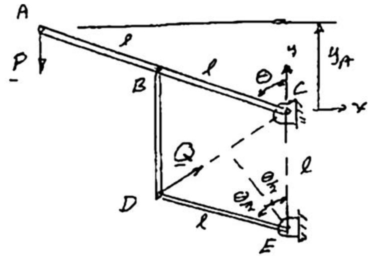


PROBLEM 10.27

Determine the value of θ corresponding to the equilibrium position of the mechanism of Problem 10.12 when $P = 80$ N and $Q = 100$ N.

PROBLEM 10.12 Knowing that the line of action of the force **Q** passes through Point C, derive an expression for the magnitude of **Q** required to maintain equilibrium.

SOLUTION



From geometry

$$y_A = 2l\cos\theta, \quad \delta y_A = -2l\sin\theta\delta\theta$$

$$CD = 2l\sin\frac{\theta}{2}, \quad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A - Q\delta(CD) = 0$$

$$-P(-2l\sin\theta\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

or

$$Q = 2P\frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)}$$

with

$$P = 80 \text{ N}, \quad Q = 100 \text{ N}$$

$$(100 \text{ N}) = 2(80 \text{ N})\frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)}$$

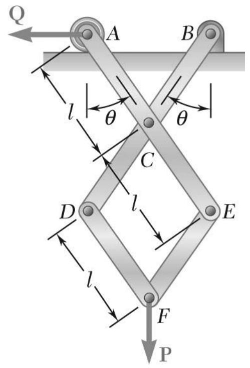
$$\frac{\sin\theta}{\cos\left(\frac{\theta}{2}\right)} = 0.625$$

or

$$\frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = 0.625$$

$$\theta = 36.42° \qquad\qquad \theta = 36.4° \blacktriangleleft$$

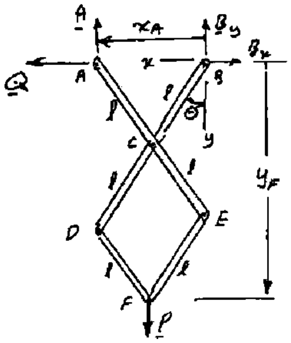
(Additional solutions discarded as not applicable are $\theta = \pm 180°$)



PROBLEM 10.28

Determine the value of θ corresponding to the equilibrium position of the mechanism of Prob. 10.14 when $P = 270$ N and $Q = 960$ N.

SOLUTION



Virtual Work:

$$x_A = 2l\sin\theta, \quad \delta x_A = 2l\cos\theta\delta\theta$$

$$y_F = 3l\cos\theta, \quad \delta y_F = -3l\sin\theta\delta\theta$$

$$\delta U = 0: \qquad Q\delta x_A + P\delta y_F = 0$$

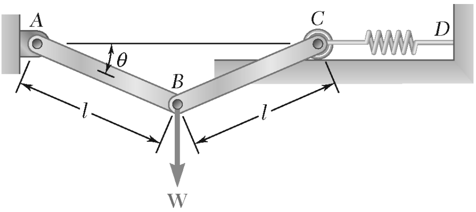
$$\delta U = 0: \qquad Q\delta\theta x_A + P\delta y_F = 0$$

$$Q(2l\cos\theta\delta\theta) + P(-3l\sin\theta\delta\theta) = 0$$

$$Q = \frac{3}{2}P\tan\theta$$

Data: $P = 270 \text{ N}, \quad Q = 960 \text{ N}$

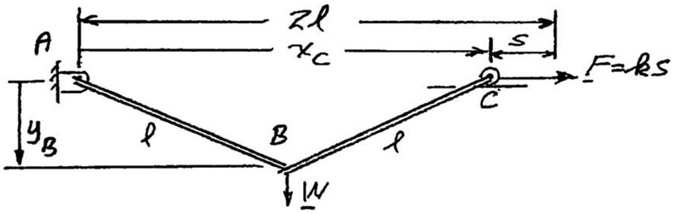
$$(960 \text{ N}) = \frac{3}{2}(270 \text{ N})\tan\theta \qquad \theta = 67.1° \blacktriangleleft$$



PROBLEM 10.29

A load **W** of magnitude 600 N is applied to the linkage at *B*. The constant of the spring is $k = 2.5$ kN/m, and the spring is unstretched when *AB* and *BC* are horizontal. Neglecting the weight of the linkage and knowing that $l = 300$ mm, determine the value of θ corresponding to equilibrium.

SOLUTION



$$x_C = 2l\cos\theta \quad \delta x_C = -2l\sin\theta\delta\theta$$

$$y_B = l\sin\theta \quad \delta y_B = l\cos\theta\delta\theta$$

$$F = ks = k(2l - x_C) = 2kl(1-\cos\theta)$$

Virtual Work: $\delta U = 0$: $F\delta x_C + W\delta y_B = 0$

$$2kl(1-\cos\theta)(-2l\sin\theta\delta\theta) + W(l\cos\theta\delta\theta) = 0$$

$$4kl^2(1-\cos\theta)\sin\theta = Wl\cos\theta$$

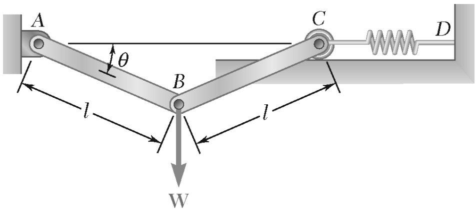
or $$(1-\cos\theta)\tan\theta = \frac{W}{4kl}$$

Given: $l = 0.3$ m, $W = 600$ N, $k = 2500$ N/m

Then $$(1-\cos\theta)\tan\theta = \frac{600 \text{ N}}{4(2500 \text{ N/m})(0.3 \text{ m})}$$

or $(1-\cos\theta)\tan\theta = 0.2$

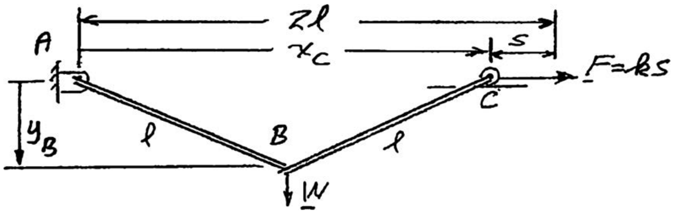
Solving numerically $\theta = 40.22°$ $\theta = 40.2°$ ◀



PROBLEM 10.30

A vertical load **W** is applied to the linkage at *B*. The constant of the spring is *k*, and the spring is unstretched when *AB* and *BC* are horizontal. Neglecting the weight of the linkage, derive an equation in *θ*, *W*, *l*, and *k* that must be satisfied when the linkage is in equilibrium.

SOLUTION



$$x_C = 2l\cos\theta \quad \delta x_C = -2l\sin\theta\delta\theta$$
$$y_B = l\sin\theta \quad \delta y_B = l\cos\theta\delta\theta$$
$$F = ks = k(2l - x_C) = 2kl(1-\cos\theta)$$

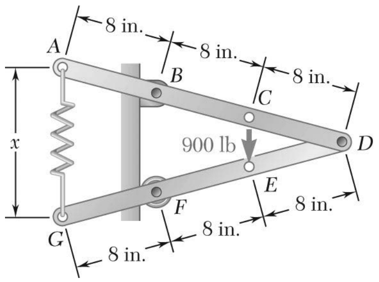
Virtual Work: $\delta U = 0$: $F\delta x_C + W\delta y_B = 0$

$$2kl(1-\cos\theta)(-2l\sin\theta\delta\theta) + W(l\cos\theta\delta\theta) = 0$$

$$4kl^2(1-\cos\theta)\sin\theta = Wl\cos\theta$$

or $$(1-\cos\theta)\tan\theta = \frac{W}{4kl}$$

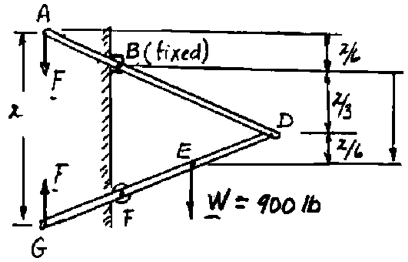
From above $$(1-\cos\theta)\tan\theta = \frac{W}{4kl} \blacktriangleleft$$



PROBLEM 10.31

Two bars AD and DG are connected by a pin at D and by a spring AG. Knowing that the spring is 12 in. long when unstretched and that the constant of the spring is 125 lb/in., determine the value of x corresponding to equilibrium when a 900-lb load is applied at E as shown.

SOLUTION



$$y_E = \frac{x}{3} + \frac{x}{6} = \frac{x}{2} \qquad \delta y_E = \frac{1}{2}\delta x$$

$$F = ks = (125\text{ lb/in.})(x - 12\text{ in.})$$

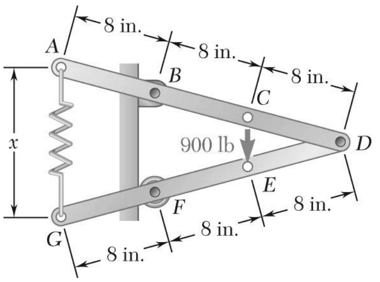
Virtual Work:

$$\delta U = 0: \quad F\delta x + W\delta y_E = 0$$

$$-(125)(x-12)\delta x + (900)\left(\frac{1}{2}\delta x\right) = 0$$

$$-125x + 1500 + 450 = 0$$

$x = 15.60$ in. ◀

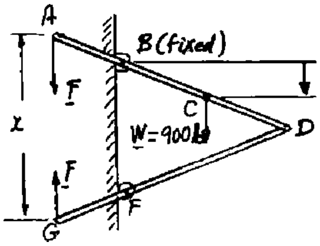


PROBLEM 10.32

Solve Problem 10.31 assuming that the 900-lb vertical force is applied at *C* instead of *E*.

PROBLEM 10.31 Two bars *AD* and *DG* are connected by a pin at *D* and by a spring *AG*. Knowing that the spring is 12 in. long when unstretched and that the constant of the spring is 125 lb/in., determine the value of *x* corresponding to equilibrium when a 900-lb load is applied at *E* as shown.

SOLUTION



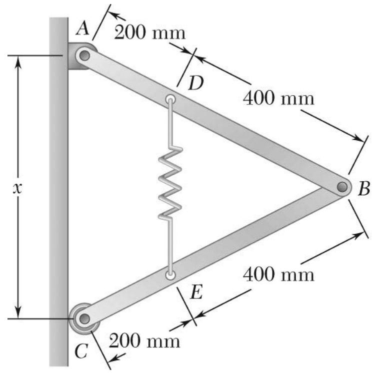
$$y_C = \frac{1}{6}x \quad \delta y_C = \frac{1}{6}\delta x$$

$$F = ks = (125\text{ lb/in.})(x - 12\text{ in.})$$

Virtual Work:

$$\delta U = -F\delta x + W\delta y_C = 125(x-12)\delta x + 900\left(\frac{1}{6}\delta x\right) = 0$$

$$-125x + 1500 + 150 = 0 \qquad x = 13.20\text{ in.} \blacktriangleleft$$

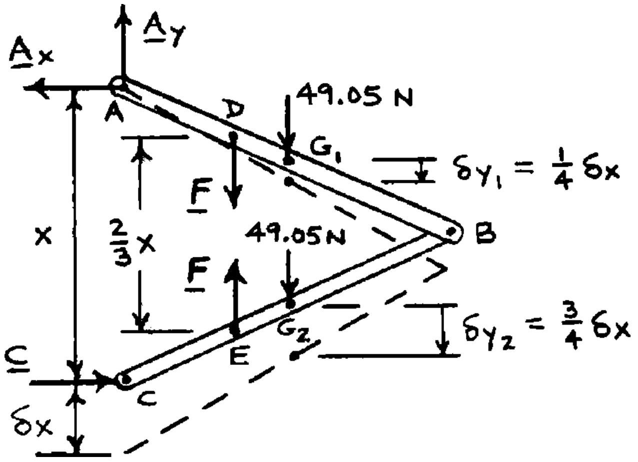


PROBLEM 10.33

Two 5-kg bars *AB* and *BC* are connected by a pin at *B* and by a spring *DE*. Knowing that the spring is 150 mm long when unstretched and that the constant of the spring is 1 kN/m, determine the value of *x* corresponding to equilibrium.

SOLUTION

First note: $W_{\text{bar}} = (5\text{ kg})(9.81\text{ m/s}^2) = 49.05\text{ N}$



During the virtual displacement, Points *D* and *E* move apart a distance $\delta(DE) = \frac{2}{3}\delta x$ and the total work done by the forces exerted at *D* and *E* is $-F\left(\frac{2}{3}\delta x\right)$

$$\delta U = 0:\quad -F\left(\frac{2}{3}\delta x\right) + 49.05\text{ N}\left(\frac{1}{4}\delta x\right) + 49.05\text{ N}\left(\frac{3}{4}\delta x\right) = 0$$

$$F = 73.575\text{ N}$$

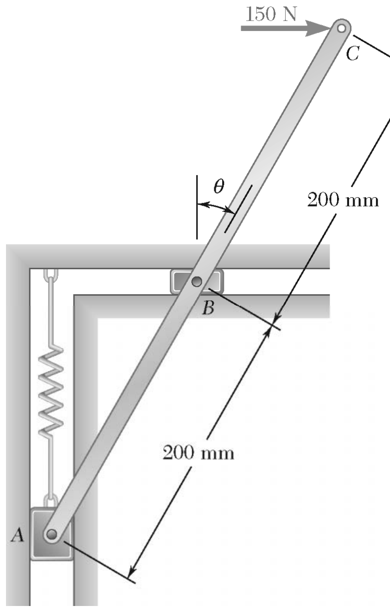
For $F = 73.575$ N, elongation of spring is

$$\frac{F}{k} = \frac{73.575\text{ N}}{1000\text{ N/m}} = 73.575\times10^{-3}\text{ m}$$

$$= 73.575\text{ mm}$$

Since undeformed length of spring is 150 mm, total length is

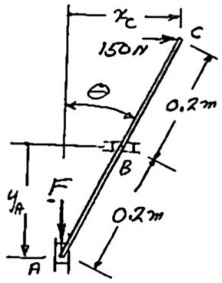
$$DE = \frac{2}{3}x = 150\text{ mm} + 73.575\text{ mm} \qquad x = 355\text{ mm} ◀$$



PROBLEM 10.34

Rod *ABC* is attached to blocks *A* and *B* that can move freely in the guides shown. The constant of the spring attached at *A* is $k = 3\text{ kN/m}$, and the spring is unstretched when the rod is vertical. For the loading shown, determine the value of θ corresponding to equilibrium.

SOLUTION



$$x_C = (0.4\text{ m})\sin\theta$$
$$\delta x_C = 0.4\cos\theta\delta\theta$$
$$y_A = (0.2\text{ m})\cos\theta$$
$$\delta y_A = -0.2\sin\theta\delta\theta$$

<u>Spring</u>:

Unstretched length $= 0.2\text{ m}$

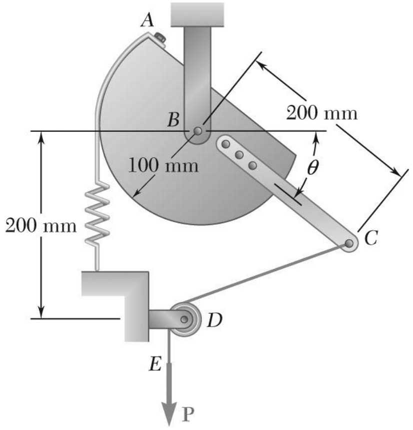
$$F = k(0.2\text{ m} - y_A) = k(0.2 - 0.2\cos\theta)$$
$$= (300\text{ N/m})(0.2)(1-\cos\theta)$$
$$F = 600(1-\cos\theta)$$

<u>Virtual Work</u>: $\delta U = 0$: $(150\text{ N})\delta x_C + F\delta y_A = 0$

$$150(0.4\cos\theta\delta\theta) + 600(1-\cos\theta)(-0.2\sin\theta\delta\theta) = 0$$

$$\frac{150(0.4)}{600(0.2)} = \frac{1}{2}: \quad \frac{1}{2} = (1-\cos\theta)\tan\theta$$

Solve by trial and error: $\theta = 57.2°$ ◀



PROBLEM 10.35

A vertical force **P** of magnitude 150 N is applied to end *E* of cable *CDE*, which passes over a small pulley *D* and is attached to the mechanism at *C*. The constant of the spring is $k = 4$ kN/m, and the spring is unstretched when $\theta = 0$. Neglecting the weight of the mechanism and the radius of the pulley, determine the value of θ corresponding to equilibrium.

SOLUTION

$$l = BC = 0.2\text{ m}$$

$$r = 0.1\text{ m}$$

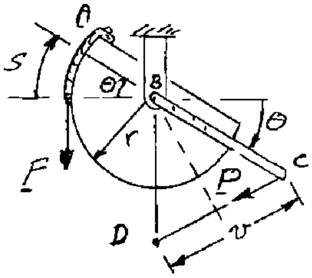
$< CBD = 90° - \theta$

$$v = 2l\sin\left(45° - \frac{\theta}{2}\right)$$

$$\delta v = -l\cos\left(45° - \frac{\theta}{2}\right)\delta\theta$$

$$s = r\theta \quad \delta s = r\delta\theta$$

$$F = ks = kr\theta$$



Virtual Work: $\delta U = 0$: $-P\delta v - F\delta s = 0$

$$-P\left[-l\cos\left(45° - \frac{\theta}{2}\right)\right]\delta\theta - kr\theta(r\delta\theta) = 0$$

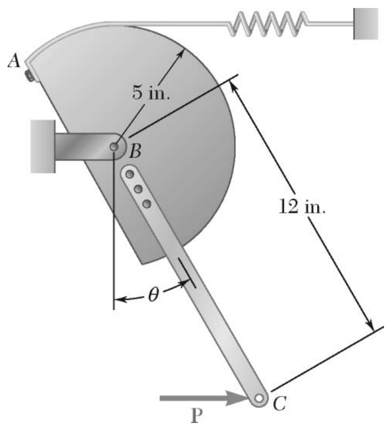
$$\frac{Pl}{kr^2} = \frac{\theta}{\cos\left(45° - \frac{\theta}{2}\right)}$$

$$\frac{Pl}{kr^2} = \frac{(150\text{ N})(0.2\text{ m})}{(4000\text{ N/m})(0.1\text{ m})^2} = 0.75$$

$$0.75 = \frac{\theta}{\cos\left(45° - \frac{\theta}{2}\right)}$$

$$\theta = 0.67623\text{ rad} = 38.745°$$

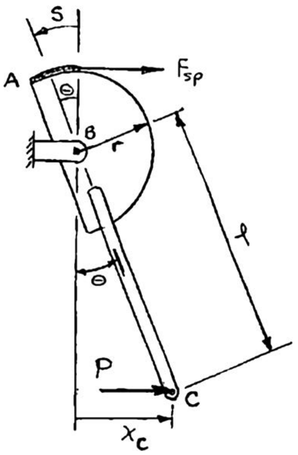
$\theta = 38.7°$ ◀



PROBLEM 10.36

A horizontal force **P** of magnitude 40 lb is applied to the mechanism at C. The constant of the spring is $k = 9$ lb/in., and the spring is unstretched when $\theta = 0$. Neglecting the weight of the mechanism, determine the value of θ corresponding to equilibrium.

SOLUTION



$$s = r\theta$$

$$\delta s = r\delta\theta$$

Spring is unstretched at $\theta = 0°$

$$F_{SP} = ks = kr\theta$$

$$x_C = l\sin\theta$$

$$\delta x_C = l\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta x_C - F_{SP}\delta s = 0$$

$$P(l\cos\theta\delta\theta) - kr\theta(r\delta\theta) = 0$$

or $$\frac{Pl}{kr^2} = \frac{\theta}{\cos\theta}$$

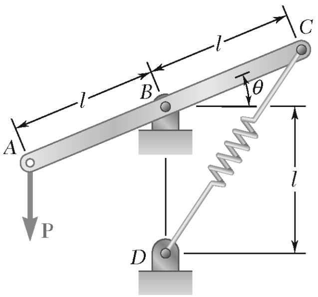
Thus $$\frac{(40\text{ lb})(12\text{ in.})}{(9\text{ lb/in.})(5\text{ in.})^2} = \frac{\theta}{\cos\theta}$$

or $$\frac{\theta}{\cos\theta} = 2.1333$$

$$\theta = 1.054\text{ rad} = 60.39°$$

$\theta = 60.4°$ ◀

PROPRIETARY MATERIAL.



PROBLEM 10.37

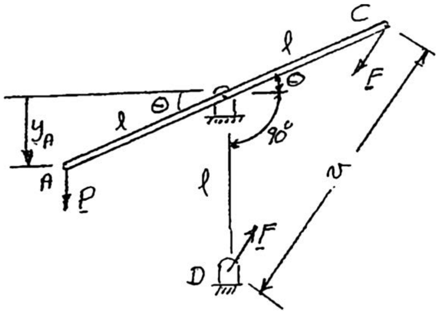
Knowing that the constant of spring *CD* is *k* and that the spring is unstretched when rod *ABC* is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

$$P = 300 \text{ N}, \quad l = 400 \text{ mm}, \quad k = 5 \text{ kN/m}.$$

SOLUTION

$$y_A = l\sin\theta$$

$$\delta y_A = l\cos\theta\delta\theta$$



Spring: $v = CD$

Unstretched when $\theta = 0$

so that $v_0 = \sqrt{2}l$

For θ: $v = 2l\sin\left(\dfrac{90° + \theta}{2}\right)$

$$\delta v = l\cos\left(45° + \frac{\theta}{2}\right)\delta\theta$$

Stretched length: $s = v - v_0 = 2l\sin\left(45° + \dfrac{\theta}{2}\right) - \sqrt{2}l$

Then $F = ks = kl\left[2\sin\left(45° + \dfrac{\theta}{2}\right) - \sqrt{2}\right]$

<u>Virtual Work</u>:

$$\delta U = 0: \quad P\delta y_A - F\delta v = 0$$

$$Pl\cos\theta\delta\theta - kl\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]l\cos\left(45° + \frac{\theta}{2}\right)\delta\theta = 0$$

or $\dfrac{P}{kl} = \dfrac{1}{\cos\theta}\left[2\sin\left(45° + \dfrac{\theta}{2}\right)\cos\left(45° + \dfrac{\theta}{2}\right) - \sqrt{2}\cos\left(45° + \dfrac{\theta}{2}\right)\right]$

$$= \frac{1}{\cos\theta}\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right)\cos\theta - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right]$$

$$= 1 - \sqrt{2}\frac{\cos\left(45° \frac{\theta}{2}\right)}{\cos\theta}$$

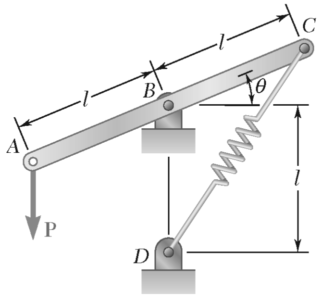
Now, with $P = 300\text{ N}, \quad l = 400\text{ mm}, \quad \text{and} \quad k = 5\text{ kN/m}$

$$\frac{(300\text{ N})}{(5000\text{ N/m})(0.4\text{ m})} = 1 - \sqrt{2}\,\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta}$$

or

$$\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = 0.60104$$

Solving numerically $\theta = 22.6°$ ◀



PROBLEM 10.38

Knowing that the constant of spring CD is k and that the spring is unstretched when rod ABC is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

$$P = 75\text{ lb},\quad l = 15\text{ in.},\quad k = 20\text{ lb/in.}$$

SOLUTION

From the analysis of Problem 10.37, we have

$$\frac{P}{kl} = 1 - \sqrt{2}\,\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta}$$

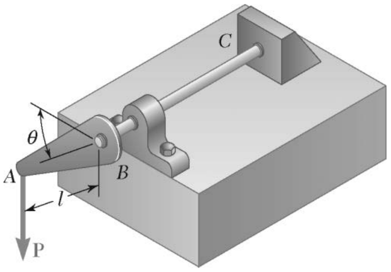
with $\quad P = 75\text{ lb},\quad l = 15\text{ in.}\quad\text{and}\quad k = 20\text{ lb/in.}$

$$\frac{(75\text{ lb})}{(20\text{ lb/in.})(15\text{ in.})} = 1 - \sqrt{2}\,\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta}$$

or

$$\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = 0.53033$$

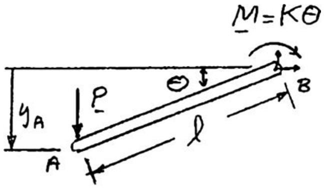
Solving numerically $\quad \theta = 51.1° \blacktriangleleft$



PROBLEM 10.39

The lever AB is attached to the horizontal shaft BC that passes through a bearing and is welded to a fixed support at C. The torsional spring constant of the shaft BC is K; that is, a couple of magnitude K is required to rotate end B through 1 rad. Knowing that the shaft is untwisted when AB is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 100$ N, $l = 250$ mm, and $K = 12.5$ N · m/rad.

SOLUTION



We have

$$y_A = l\sin\theta$$

$$\delta y_A = l\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta y_A - M\delta\theta = 0$$

$$Pl\cos\theta\delta\theta - K\theta\delta\theta = 0$$

or

$$\frac{\theta}{\cos\theta} = \frac{Pl}{K} \qquad (1)$$

with

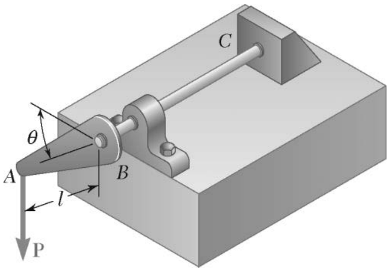
$$P = 100 \text{ N}, \quad l = 250 \text{ mm} \quad \text{and} \quad K = 12.5 \text{ N}\cdot\text{m/rad}$$

$$\frac{\theta}{\cos\theta} = \frac{(100 \text{ N})(0.250 \text{ m})}{12.5 \text{ N}\cdot\text{m/rad}}$$

or

$$\frac{\theta}{\cos\theta} = 2.0000$$

Solving numerically $\theta = 59.0°$ ◀



PROBLEM 10.40

Solve Problem 10.39 assuming that $P = 350\text{ N}$, $l = 250\text{ mm}$, and $K = 12.5\text{ N}\cdot\text{m/rad}$. Obtain answers in each of the following quadrants: $0 < \theta < 90°$, $270° < \theta < 360°$, $360° < \theta < 450°$.

PROBLEM 10.39 The lever *AB* is attached to the horizontal shaft *BC* that passes through a bearing and is welded to a fixed support at *C*. The torsional spring constant of the shaft *BC* is *K*; that is, a couple of magnitude *K* is required to rotate end *B* through 1 rad. Knowing that the shaft is untwisted when *AB* is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 100\text{ N}$, $l = 250\text{ mm}$, and $K = 12.5\text{ N}\cdot\text{m/rad}$.

SOLUTION

Using Equation (1) of Problem 10.39 and

$$P = 350\text{ N},\quad l = 250\text{ mm}\quad\text{and}\quad K = 12.5\text{ N}\cdot\text{m/rad}$$

We have

$$\frac{\theta}{\cos\theta} = \frac{(350\text{ N})(0.250\text{ m})}{12.5\text{ N}\cdot\text{m/rad}}$$

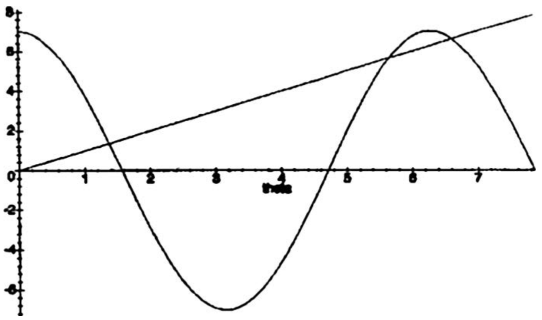
or

$$\frac{\theta}{\cos\theta} = 7\quad\text{or}\quad \theta = 7\cos\theta \qquad (1)$$

The solutions to this equation can be shown graphically using any appropriate graphing tool, such as Maple, with the command: plot ({theta, 7 * cos(theta)}, $t = 0.5 * \text{Pi}/2$);

Thus, we plot $y = \theta$ and $y = 7\cos\theta$ in the range

$$0 \le \theta \le \frac{5\pi}{2}$$



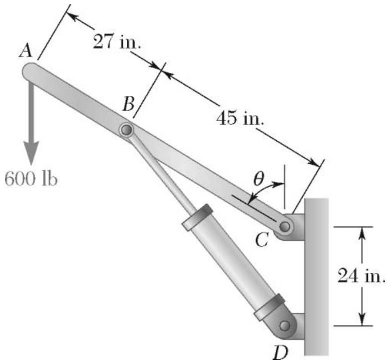
PROBLEM 10.40 (Continued)

We observe that there are three points of intersection, which implies that Equation (1) has three roots in the specified range of θ.

$$0 \le \theta \le 90^\circ \left(\frac{\pi}{2}\right); \quad \theta = 1.37333 \text{ rad}, \quad \theta = 78.69^\circ \qquad \theta = 78.7^\circ \blacktriangleleft$$

$$270 \le \theta \le 360^\circ \left(\frac{3\pi}{2} \le \theta \le 2\pi\right); \quad \theta = 5.65222 \text{ rad}, \quad \theta = 323.85^\circ \qquad \theta = 324^\circ \blacktriangleleft$$

$$360 \le \theta \le 450^\circ \left(2\pi \le \theta \le \frac{5\pi}{2}\right); \quad \theta = 6.61597 \text{ rad}, \quad \theta = 379.07 \qquad \theta = 379^\circ \blacktriangleleft$$

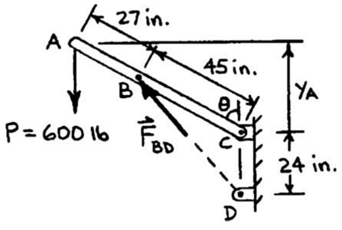


PROBLEM 10.41

The position of boom *ABC* is controlled by the hydraulic cylinder *BD*. For the loading shown, determine the force exerted by the hydraulic cylinder on pin *B* when $\theta = 65°$.

SOLUTION

We have



$$y_A = (72 \text{ in.})\cos\theta \quad \delta y_A = -72\sin\theta\delta\theta$$

$$(BD)^2 = (BC)^2 + (CD)^2 - 2(BC)(CD)\cos(180° - \theta)$$

$$= (45)^2 + (24)^2 + 2(45)(24)\cos\theta$$

$$(BD)^2 = 2601 + 2160\cos\theta \qquad (1)$$

Differentiating:

$$2(BD)\delta(BD) = -2160\sin\theta\delta\theta$$

$$\delta(BD) = -\frac{1080}{BD}\sin\theta\delta\theta \qquad (2)$$

<u>Virtual Work</u>: Noting that **P** tends to decrease y_A and $\mathbf{F}_{BD}$ tends to increase *BD*, write

$$\delta U = -P\delta y_A + F_{BD}\delta(BD) = 0$$

$$-P(-72\sin\theta\delta\theta) + F_{BD}\left(-\frac{1080}{BD}\sin\theta\delta\theta\right) = 0$$

$$F_{BD} = \frac{1}{15}(BD)P$$

or, since

$$P = 600 \text{ lb:} \quad F_{BD} = \frac{600}{15}(BD) = (40 \text{ lb})(BD) \qquad (3)$$

Making $\theta = 65°$ in Eq. (1), we have

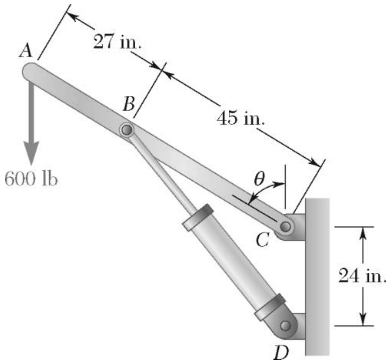
$$(BD)^2 = 2601 + 2160\cos 65° = 3513.9$$

$$BD = 59.278$$

Carrying into Eq. (3):

$$F_{BD} = (40 \text{ lb})(59.278) = 2371.1 \text{ lb}$$

$F_{BD} = 2370$ lb ◀



PROBLEM 10.42

The position of boom *ABC* is controlled by the hydraulic cylinder *BD*. For the loading shown, (*a*) express the force exerted by the hydraulic cylinder on pin *B* as a function of the length *BD*, (*b*) determine the smallest possible value of the angle θ if the maximum force that the cylinder can exert on pin *B* is 2.5 kips.

SOLUTION

(*a*) See solution of Problem 10.41 for the derivation of Eq. (3):

$$F_{BD} = (40\text{ lb})(BD) \blacktriangleleft$$

(*b*) For $(F_{BD})_{\max} = 2.5\text{ kips} = 2500\text{ lb}$, we have

$$2500\text{ lb} = (40\text{ lb})(BD)$$
$$BD = 62.5$$

Carrying this value into Eq. (1) of Problem 10.41, write

$$(BD)^2 = 2601 + 2160\cos\theta$$
$$(62.5)^2 = 2601 + 2160\cos\theta$$
$$\cos\theta = 0.60428 \qquad \theta = 52.8° \blacktriangleleft$$

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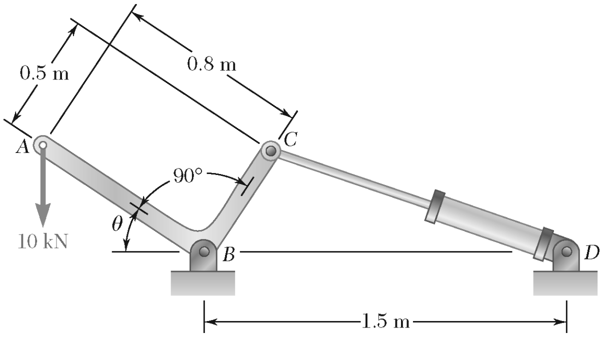
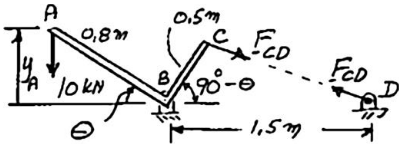


PROBLEM 10.43

The position of member *ABC* is controlled by the hydraulic cylinder *CD*. For the loading shown, determine the force exerted by the hydraulic cylinder on pin *C* when $\theta = 55°$.

SOLUTION



$$y_A = (0.8\text{ m})\sin\theta$$

$$\delta y_A = 0.8\cos\theta\delta\theta$$

$$CD^2 = BC^2 + BD^2 - 2(BC)(BD)\cos(90° - \theta)$$

$$CD^2 = 0.5^2 + 1.5^2 - 2(0.5)(1.5)\sin\theta$$

$$CD^2 = 2.5 - 1.5\sin\theta \qquad (1)$$

$$2(CD)(\delta_{CD}) = -1.5\cos\theta\delta\theta \quad \delta_{CD} = -\frac{3\cos\theta}{4CD}\delta\theta$$

Virtual Work: $\delta U = 0$: $-(10\text{ kN})\delta y_A - F_{CD}\delta_{CD} = 0$

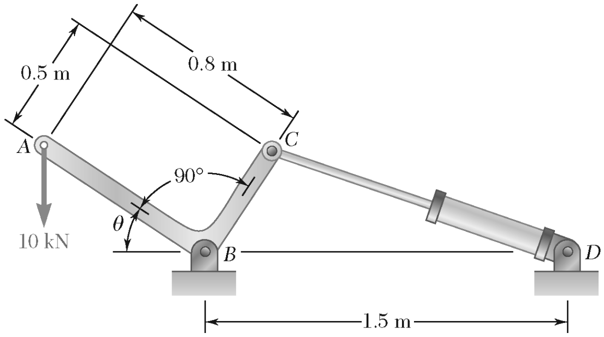
$$-10(0.8\cos\theta\delta\theta) - F_{CD}\left(-\frac{3\cos\theta}{4CD}\delta\theta\right) = 0$$

$$F_{CD} = \frac{32}{3}CD \qquad (2)$$

For $\theta = 55°$:

Eq. (1): $CD^2 = 2.5 - 1.5\sin 55° = 1.2713$; $CD = 1.1275\text{ m}$

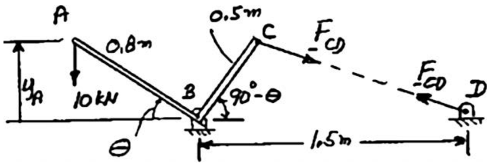
Eq. (2): $F_{CD} = \frac{32}{3}CD = \frac{32}{3}(1.1275) = 12.027\text{ kN}$ $\mathbf{F}_{CD} = 12.03\text{ kN} \searrow$ ◀



PROBLEM 10.44

The position of member *ABC* is controlled by the hydraulic cylinder *CD*. Determine the angle θ knowing that the hydraulic cylinder exerts a 15-kN force on pin *C*.

SOLUTION



$$y_A = (0.8\text{ m})\sin\theta$$

$$\delta y_A = 0.8\cos\theta\delta\theta$$

$$CD^2 = BC^2 + BD^2 - 2(BC)(BD)\cos(90° - \theta)$$

$$CD^2 = 0.5^2 + 1.5^2 - 2(0.5)(1.5)\sin\theta$$

$$CD^2 = 2.5 - 1.5\sin\theta \qquad (1)$$

$$2(CD)(\delta_{CD}) = -1.5\cos\theta\delta\theta; \qquad \delta_{CD} = -\frac{3\cos\theta}{4CD}\delta\theta$$

<u>Virtual Work</u>: $\delta U = 0$: $\quad -(10\text{ kN})\delta y_A - F_{CD}\delta_{CD} = 0$

$$-10(0.8\cos\theta\delta\theta) - F_{CD}\left(-\frac{3\cos\theta}{4CD}\delta\theta\right) = 0$$

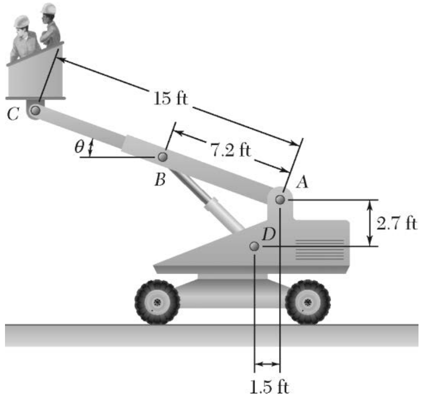
$$F_{CD} = \frac{32}{3}CD \qquad (2)$$

For $F_{CD} = 15\text{ kN}$:

Eq. (2): $\quad 15\text{ kN} = \frac{32}{3}CD$: $\qquad CD = \frac{45}{32} = 1.40625\text{ m}$

Eq. (1): $\quad (1.40625)^2 = 2.5 - 1.5\sin\theta$: $\qquad \sin\theta = 0.34831$

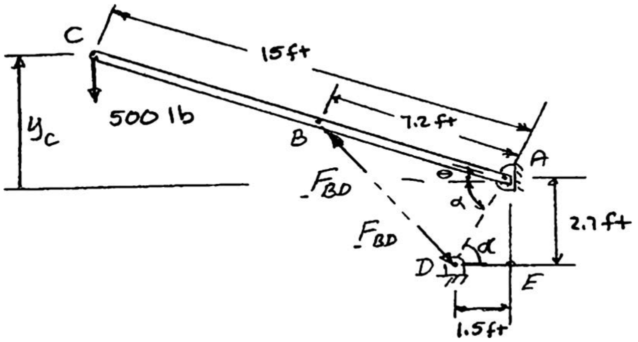
$\theta = 20.38°$ $\qquad \theta = 20.4°$ ◀



PROBLEM 10.45

The telescoping arm ABC is used to provide an elevated platform for construction workers. The workers and the platform together weigh 500 lb and their combined center of gravity is located directly above C. For the position when $\theta = 20°$, determine the force exerted on pin B by the single hydraulic cylinder BD.

SOLUTION



In ΔADE:

$$\tan\alpha = \frac{AE}{DE} = \frac{2.7 \text{ ft}}{1.5 \text{ ft}}$$

$$\alpha = 60.945°$$

$$AD = \frac{2.7 \text{ ft}}{\sin 60.945°} = 3.0887 \text{ m}$$

From the geometry:

$$y_C = (15 \text{ ft})\sin\theta$$

$$\delta y_C = (15 \text{ ft})\cos\theta\delta\theta$$

Then, in triangle BAD: Angle $BAD = \alpha + \theta$

Law of cosines:

$$BD^2 = AB^2 + AD^2 - 2(AB)(AD)\cos(\alpha + \theta)$$

or

$$BD^2 = (7.2 \text{ ft})^2 + (3.0887 \text{ ft})^2 - 2(7.2 \text{ ft})(3.0887 \text{ ft})\cos(\alpha + \theta)$$

$$BD^2 = 61.380 \text{ ft}^2 - (44.477\cos(\alpha + \theta)) \text{ ft}^2 \qquad (1)$$

PROBLEM 10.45 (Continued)

And then

$$2(BD)(\delta BD) = (44.477\sin(\alpha+\theta))\delta\theta$$

$$\delta BD = \frac{44.477\sin(\alpha+\theta)}{2(BD)}\delta\theta$$

Virtual Work: $\delta U = 0\text{:}\quad -P\delta y_C + F_{BD}\delta BD = 0$

Substituting $$-(500\text{ lb})(15\text{ ft})\cos\theta\delta\theta + F_{BD}\left[\frac{(44.477\text{ ft}^2)\sin(\alpha+\theta)}{2(BD)}\delta\theta\right] = 0$$

or $$F_{BD} = \left[337.25\frac{\cos\theta}{\sin(\alpha+\theta)}BD\right]\text{ lb/ft} \qquad (2)$$

Now, with $\theta = 20°$ and $\alpha = 60.945°$

Equation (1):

$$BD^2 = 61.380 - 44.477\cos(60.945° + 20°)$$

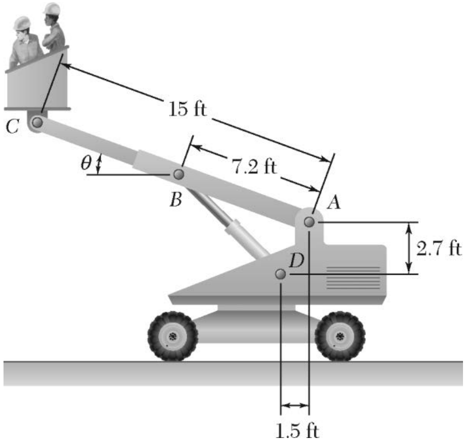
$$BD^2 = 54.380$$

$$BD = 7.3743\text{ ft}$$

Equation (2):

$$F_{BD} = \left[337.25\frac{\cos 20°}{\sin(60.945° + 20°)}(7.3743\text{ ft})\right]\text{ lb/ft}$$

or $F_{BD} = 2366\text{ lb}$ $\qquad \mathbf{F}_{BD} = 2370\text{ lb}$ ↖ ◀



PROBLEM 10.46

Solve Problem 10.45 assuming that the workers are lowered to a point near the ground so that $\theta = -20°$.

PROBLEM 10.45 The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together weigh 500 lb and their combined center of gravity is located directly above *C*. For the position when $\theta = 20°$, determine the force exerted on pin *B* by the single hydraulic cylinder *BD*.

SOLUTION

Using the figure and analysis of Problem 10.45, including Equations (1) and (2), and with $\theta = -20°$, we have

Equation (1):

$$BD^2 = 61.380 - 44.477\cos(60.945° - 20°)$$

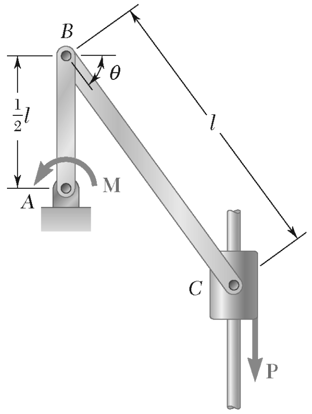
$$BD^2 = 27.785$$

$$BD = 5.2711 \text{ ft}$$

Equation (2):

$$F_{BD} = 337.25\frac{\cos(-20°)}{\sin(60.945° - 20°)}(5.2711)$$

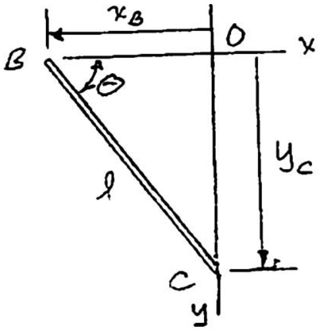
$$F_{BD} = 2549 \text{ lb} \qquad \text{or} \quad \mathbf{F}_{BD} = 2550 \text{ lb} \nwarrow \blacktriangleleft$$



PROBLEM 10.47

Denoting by μ_s the coefficient of static friction between collar C and the vertical rod, derive an expression for the magnitude of the largest couple **M** for which equilibrium is maintained in the position shown. Explain what happens if $\mu_s \geq \tan\theta$.

SOLUTION



Member BC: We have

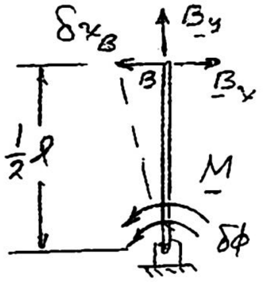
$$x_B = l\cos\theta$$

$$\delta x_B = -l\sin\theta\delta\theta \quad (1)$$

and

$$y_C = l\sin\theta$$

$$\delta y_C = l\cos\theta\delta\theta \quad (2)$$



Member AB: We have

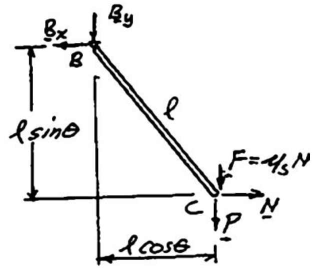
$$\delta x_B = \frac{1}{2}l\delta\phi$$

Substituting from Equation (1),

$$-l\sin\theta\delta\theta = \frac{1}{2}l\delta\phi$$

or

$$\delta\phi = -2\sin\theta\delta\theta \quad (3)$$



Free body of rod BC

For $M_{\max}$, motion of collar C impends upward

$$+\circlearrowleft \Sigma M_B = 0: \quad Nl\sin\theta - (P + \mu_s N)(l\cos\theta) = 0$$

$$N\tan\theta - \mu_s N = P$$

$$N = \frac{P}{\tan\theta - \mu_s}$$

PROBLEM 10.47 (Continued)

Virtual Work:

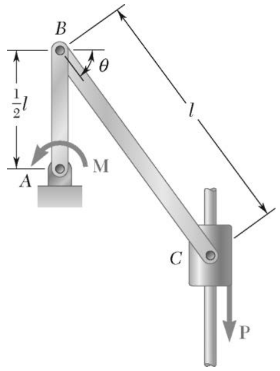
$$\delta U = 0: \quad M\,\delta\phi + (P + \mu_s N)\delta y_C = 0$$

$$M(-2\sin\theta\delta\theta) + (P + \mu_s N) l \cos\theta\delta\theta = 0$$

$$M_{\max} = \frac{(P + \mu_s N)}{2\tan\theta} l = \frac{P + \mu_s \frac{P}{\tan\theta - \mu_s}}{2\tan\theta} l$$

$$\text{or} \quad M_{\max} = \frac{Pl}{2(\tan\theta - \mu_s)} \blacktriangleleft$$

If $\mu_s = \tan\theta$, $M = \infty$, system becomes *self-locking*.



PROBLEM 10.48

Knowing that the coefficient of static friction between collar C and the vertical rod is 0.40, determine the magnitude of the largest and smallest couple **M** for which equilibrium is maintained in the position shown, when $\theta = 35°$, $l = 600$ mm, and $P = 300$ N.

SOLUTION

From the analysis of Problem 10.50, we have

$$M_{\max} = \frac{Pl}{2(\tan\theta + \mu_s)}$$

With $\theta = 35°, \quad l = 0.6 \text{ m}, \quad P = 300 \text{ N}$

$$M_{\max} = \frac{(300 \text{ N})(0.6 \text{ m})}{2(\tan 35° - 0.4)}$$

$$= 299.80 \text{ N}\cdot\text{m} \qquad M_{\max} = 300 \text{ N}\cdot\text{m} \blacktriangleleft$$

For $M_{\min}$, motion of C impends downward and F acts upward. The equations of Problem 10.50 can still be used if we replace μ_s by $-\mu_s$. Then

$$M_{\min} = \frac{Pl}{2(\tan\theta + \mu_s)}$$

Substituting,

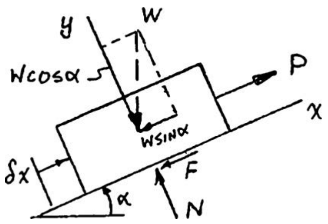
$$M_{\min} = \frac{(300 \text{ N})(0.6 \text{ m})}{2(\tan 35° + 0.4)}$$

$$= 81.803 \text{ N}\cdot\text{m} \qquad M_{\min} = 81.8 \text{ N}\cdot\text{m} \blacktriangleleft$$

PROBLEM 10.49

A block of weight W is pulled up a plane forming an angle α with the horizontal by a force **P** directed along the plane. If μ is the coefficient of friction between the block and the plane, derive an expression for the mechanical efficiency of the system. Show that the mechanical efficiency cannot exceed $\frac{1}{2}$ if the block is to remain in place when the force **P** is removed.

SOLUTION



$$\text{Input work} = P\delta x$$

$$\text{Output work} = (W\sin\alpha)\delta x$$

Efficiency:

$$\eta = \frac{W\sin\alpha\delta x}{P\delta x} \quad \text{or} \quad \eta = \frac{W\sin\alpha}{P} \tag{1}$$

$$+\nearrow \Sigma F_x = 0: \quad P - F - W\sin\alpha = 0 \quad \text{or} \quad P = W\sin\alpha + F \tag{2}$$

$$+\nwarrow \Sigma F_y = 0: \quad N - W\cos\alpha = 0 \quad \text{or} \quad N = W\cos\alpha$$

$$F = \mu N = \mu W\cos\alpha$$

Equation (2):

$$P = W\sin\alpha + \mu W\cos\alpha = W(\sin\alpha + \mu\cos\alpha)$$

Equation (1):

$$\eta = \frac{W\sin\alpha}{W(\sin\alpha + \mu\cos\alpha)} \quad \text{or} \quad \eta = \frac{1}{1+\mu\cot\alpha} \quad ◀$$

If block is to remain in place when $P = 0$, we know (see Chapter 8) that $\phi_s \geq \alpha$ or, since

$$\mu = \tan\phi_s, \quad \mu \geq \tan\alpha$$

Multiply by $\cot\alpha$:

$$\mu\cot\alpha \geq \tan\alpha\cot\alpha = 1$$

Add 1 to each side:

$$1 + \mu\cot\alpha \geq 2$$

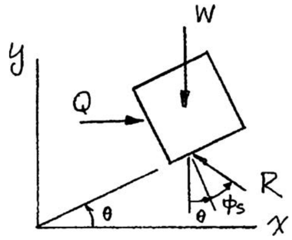
Recalling the expression for η, we find $\eta \geq \dfrac{1}{2}$ ◀

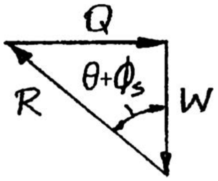
PROBLEM 10.50

Derive an expression for the mechanical efficiency of the jack discussed in Section 8.6. Show that if the jack is to be self-locking, the mechanical efficiency cannot exceed $\frac{1}{2}$.

SOLUTION

Recall Figure 8.9*a*. Draw force triangle





$$Q = W\tan(\theta + \phi_s)$$
$$y = x\tan\theta \text{ so that } \delta y = \delta x \tan\theta$$
$$\text{Input work} = Q\delta x = W\tan(\theta + \phi_s)\delta x$$
$$\text{Output work} = W\delta y = W(\delta x)\tan\theta$$

Efficiency: $$\eta = \frac{W\tan\theta\delta x}{W\tan(\theta + \phi_s)\delta x}; \qquad \eta = \frac{\tan\theta}{\tan(\theta + \phi_s)} \blacktriangleleft$$

From Page 432, we know the jack is self-locking if

$$\phi \geq \theta$$

Then $$\theta + \phi_s \geq 2\theta$$

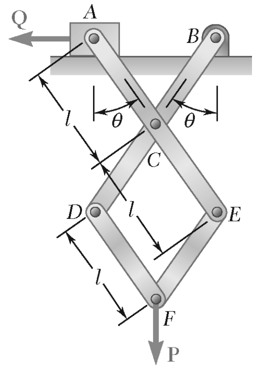
so that $$\tan(\theta + \phi_s) \geq \tan 2\theta$$

From above $$\eta = \frac{\tan\theta}{\tan(\theta + \phi_s)}$$

It then follows that $$\eta \leq \frac{\tan\theta}{\tan 2\theta}$$

But $$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Then $$\eta \leq \frac{\tan\theta(1 - \tan^2\theta)}{2\tan\theta} = \frac{1 - \tan^2\theta}{2} \qquad \eta \leq \frac{1}{2} \blacktriangleleft$$



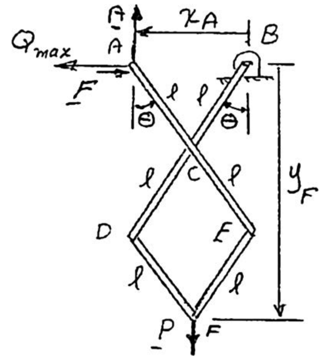
PROBLEM 10.51

Denoting by μ_s the coefficient of static friction between the block attached to rod ACE and the horizontal surface, derive expressions in terms of P, μ_s, and θ for the largest and smallest magnitude of the force **Q** for which equilibrium is maintained.

SOLUTION

For the linkage:

$$+\circlearrowleft \Sigma M_B = 0: \quad -x_A + \frac{x_A}{2}P = 0 \quad \text{or} \quad \mathbf{A} = \frac{P}{2}\uparrow$$



Then: $$F = \mu_s A = \mu_s \frac{P}{2} = \frac{1}{2}\mu_s P$$

Now $$x_A = 2l\sin\theta$$
$$\delta x_A = 2l\cos\theta\delta\theta$$

and $$y_F = 3l\cos\theta$$
$$\delta y_F = -3l\sin\theta\delta\theta$$

<u>Virtual Work</u>: $$\delta U = 0: \quad (Q_{\max} - F)\delta x_A + P\delta y_F = 0$$

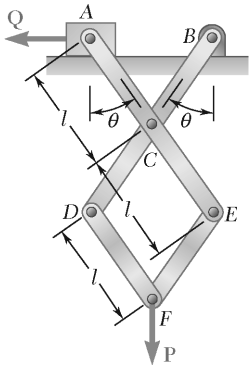
$$\left(Q_{\max} - \frac{1}{2}\mu_s P\right)(2l\cos\theta\delta\theta) + P(-3l\sin\theta\delta\theta) = 0$$

or $$Q_{\max} = \frac{3}{2}P\tan\theta + \frac{1}{2}\mu_s P$$

$$Q_{\max} = \frac{P}{2}(3\tan\theta + \mu_s) \blacktriangleleft$$

For $Q_{\min}$, motion of A impends to the right and **F** acts to the left. We change μ_s to $-\mu_s$ and find

$$Q_{\min} = \frac{P}{2}(3\tan\theta - \mu_s) \blacktriangleleft$$



PROBLEM 10.52

Knowing that the coefficient of static friction between the block attached to rod *ACE* and the horizontal surface is 0.15, determine the magnitude of the largest and smallest force **Q** for which equilibrium is maintained when $\theta = 30°$, $l = 0.2$ m, and $P = 40$ N.

SOLUTION

Using the results of Problem 10.48 with

$$\theta = 30°$$
$$l = 0.2 \text{ m}$$
$$P = 40 \text{ N}, \quad \text{and} \quad \mu_s = 0.15$$

We have

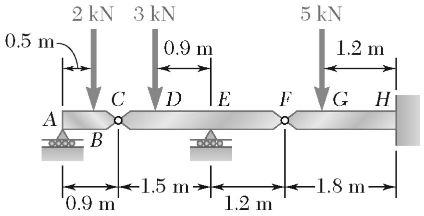
$$Q_{\max} = \frac{P}{2}(3\tan\theta + \mu_s)$$
$$= \frac{(40 \text{ N})}{2}(3\tan 30° + 0.15)$$
$$= 37.64 \text{ N}$$

$Q_{\max} = 37.6$ N ◀

and

$$Q_{\min} = \frac{P}{2}(3\tan\theta - \mu_s)$$
$$= \frac{(40 \text{ N})}{2}(3\tan 30° - 0.15)$$
$$= 31.64 \text{ N}$$

$Q_{\min} = 31.6$ N ◀

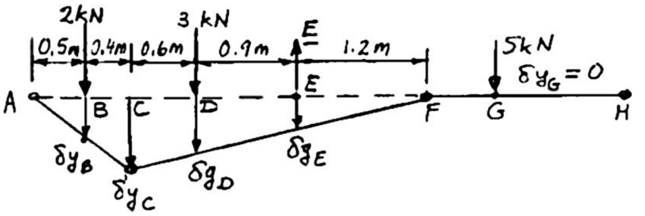


PROBLEM 10.53

Using the method of virtual work, determine the reaction at *E*.

SOLUTION

We release the support at *E* and assume a virtual displacement δy_E for Point *E*.



From similar triangles:

$$\delta y_D = \frac{2.1}{1.2}\delta y_E = 1.75\delta y_E$$

$$\delta y_C = \frac{2.7}{1.2}\delta y_E = 2.25\delta y_E$$

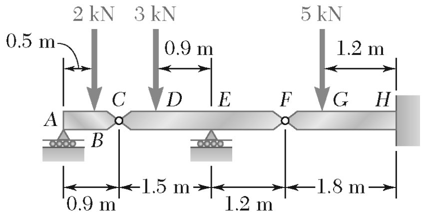
$$\delta y_B = \frac{0.5}{0.9}\delta y_C = \frac{0.5}{0.9}(2.25\delta y_E) = 1.25\delta y_E$$

Virtual Work:

$$\delta U = (2\text{ kN})\delta y_B + (3\text{ kN})\delta y_D - E\delta y_E = 0$$

$$2(1.25\delta y_E) + 3(1.75\delta y_E) - E\delta y_E = 0$$

$E = +7.75$ kN **E** = 7.75 kN ↑ ◀

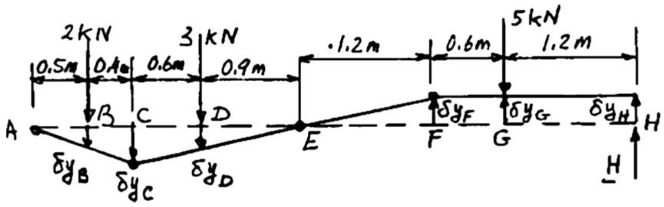


PROBLEM 10.54

Using the method of virtual work, determine separately the force and couple representing the reaction at H.

SOLUTION

Force at H. We give a vertical virtual displacement δy_H to Point H, keeping member FH horizontal.



From the geometry of the diagram:

$$\delta y_F = \delta y_G = \delta y_H$$

$$\delta y_D = \frac{0.9}{1.2}\delta y_F = \frac{0.9}{1.2}\delta y_H = 0.75\delta y_H$$

$$\delta y_C = \frac{1.5}{0.9}\delta y_D = \frac{1.5}{0.9}(0.75\delta y_H) = 1.25\delta y_H$$

$$\delta y_B = \frac{0.5}{0.9}\delta y_C = \frac{0.5}{0.9}(1.25\delta y_H) = 0.69444\delta y_H$$

Virtual Work:

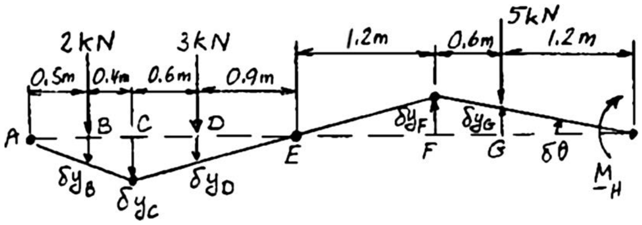
$$\delta U = (2\text{ kN})\delta y_B + (3\text{ kN})\delta y_D - (5\text{ kN})\delta y_G + H\delta y_H = 0$$

$$2(0.69444\delta y_H) + 3(0.75\delta y_H) - 5\delta y_H + H\delta y_H = 0$$

$$H = +1.3611\text{ kN}$$

$$\mathbf{H} = 1.361\text{ kN}\uparrow ◀$$

Couple at H. We rotate beam FH through $\delta\theta$ about Point H.



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PROBLEM 10.54 (Continued)

From the geometry of the diagram:

$$\delta y_G = 1.2\delta\theta \qquad \delta y_F = 1.8\delta\theta$$

$$\delta y_D = \frac{0.9}{1.2}\delta y_F = \frac{0.9}{1.2}(1.8\delta\theta) = 1.35\delta\theta$$

$$\delta y_C = \frac{1.5}{0.9}\delta y_D = \frac{1.5}{0.9}(1.35\delta\theta) = 2.25\delta\theta$$

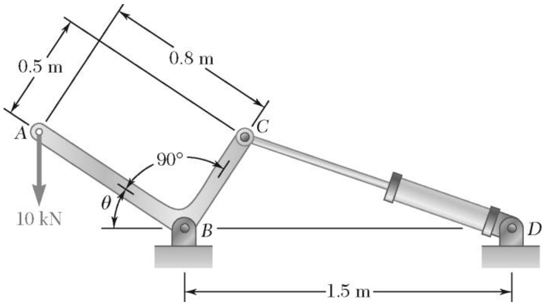
$$\delta y_B = \frac{0.5}{0.9}\delta y_C = \frac{0.5}{0.9}(2.25\delta\theta) = 1.25\delta\theta$$

Virtual Work:

$$\delta U = (2\text{ kN})\delta y_B + (3\text{ kN})\delta y_D - (5\text{ kN})\delta y_G + M_H\delta\theta = 0$$

$$2(1.25\delta\theta) + 3(1.35\delta\theta) - 5(1.2\delta\theta) + M_H\delta\theta = 0$$

$$M_H = -0.550\text{ kN}\cdot\text{m} \qquad \mathbf{M}_H = 550\text{ N}\cdot\text{m}\circlearrowleft \blacktriangleleft$$

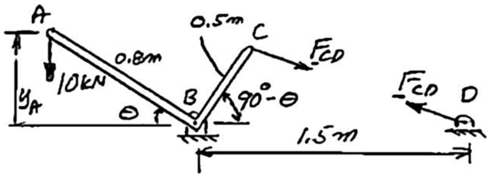


PROBLEM 10.55

Referring to Problem 10.43 and using the value found for the force exerted by the hydraulic cylinder *CD*, determine the change in the length of *CD* required to raise the 10-kN load by 15 mm.

PROBLEM 10.43 The position of member *ABC* is controlled by the hydraulic cylinder *CD*. For the loading shown, determine the force exerted by the hydraulic cylinder on pin *C* when $\theta = 55°$.

SOLUTION



Virtual Work: Assume both δy_A and δ_{CD} increase

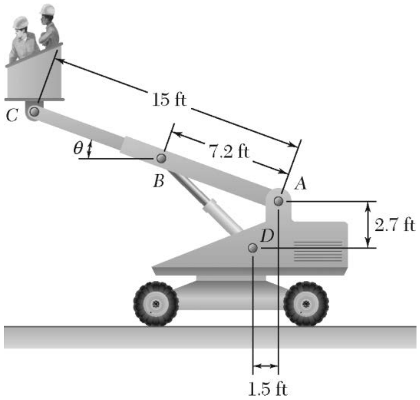
$$\delta U = 0: \quad -(10\text{ kN})\delta y_A - F_{CD}\delta_{CD} = 0$$

Substitute: $\delta y_A = 15\text{ mm}$ and $F_{CD} = 12.03\text{ kN}$

$$-(10\text{ kN})(15\text{ mm}) - (12.03\text{ kN})\delta_{CD} = 0$$

$$\delta_{CD} = -12.47\text{ mm}$$

The negative sign indicates that *CD* shortened $\delta_{CD} = 12.47\text{ mm shorter}$ ◀

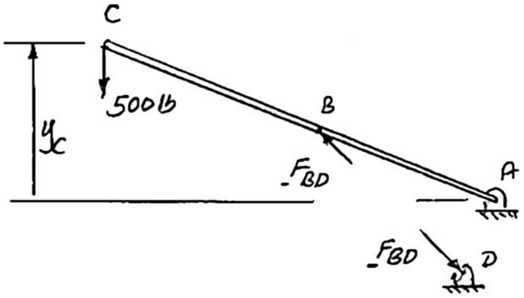


PROBLEM 10.56

Referring to Problem 10.45 and using the value found for the force exerted by the hydraulic cylinder *BD*, determine the change in the length of *BD* required to raise the platform attached at *C* by 2.5 in.

PROBLEM 10.45 The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together weigh 500 lb and their combined center of gravity is located directly above *C*. For the position when $\theta = 20°$, determine the force exerted on pin *B* by the single hydraulic cylinder *BD*.

SOLUTION



Virtual Work: Assume both δy_C and δ_{BD} increase

$$\delta U = 0: \quad -(500 \text{ lb})\delta y_C + F_{BD}\delta_{BD} = 0$$

Substitute: $\delta y_C = 2.5 \text{ in.}$ and $F_{BD} = 2370 \text{ lb}$

$$-(500 \text{ lb})(2.5 \text{ in.}) + (2370 \text{ lb})\delta_{BD} = 0$$

$$\delta_{BD} = +0.527 \text{ in.}$$

The positive sign indicates that cylinder *BD* increases in length $\delta_{BD} = 0.527$ in. longer ◀

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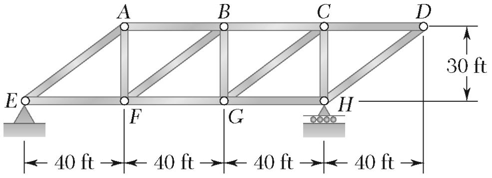
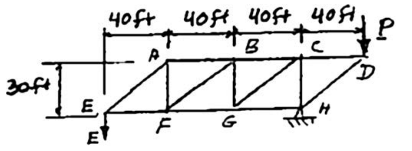


PROBLEM 10.57

Determine the vertical movement of joint *D* if the length of member *BF* is increased by 1.5 in. (*Hint:* Apply a vertical load at joint *D*, and, using the methods of Chapter 6, compute the force exerted by member *BF* on joints *B* and *F*. Then apply the method of virtual work for a virtual displacement resulting in the specified increase in length of member *BF*. This method should be used only for small changes in the lengths of members.)

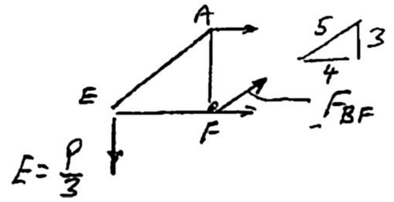
SOLUTION

Apply vertical load *P* at *D*.



$$+\circlearrowleft \Sigma M_H = 0: \quad -P(40 \text{ ft}) + E(120 \text{ ft}) = 0$$

$$\mathbf{E} = \frac{P}{3}\downarrow$$

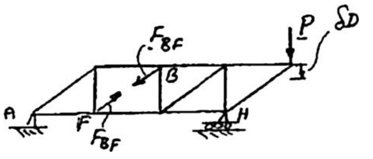


$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BF} - \frac{P}{3} = 0$$

$$F_{BF} = \frac{5}{9}P$$

Virtual Work:

We remove member *BF* and replace it with forces $\mathbf{F}_{BF}$ and $-\mathbf{F}_{BF}$ at pins *F* and *B*, respectively. Denoting the virtual displacements of Points *B* and *F* as $\delta\mathbf{r}_B$ and $\delta\mathbf{r}_F$, respectively, and noting that **P** and $\overrightarrow{\delta D}$ have the same direction, we have

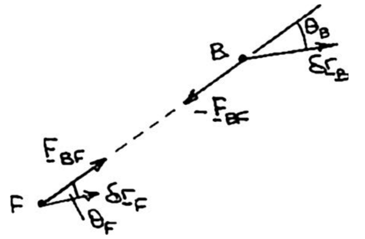


Virtual Work:

$$\delta U = 0: \qquad P\delta D + \mathbf{F}_{BF}\cdot\delta\mathbf{r}_F + (-\mathbf{F}_{BF})\cdot\delta\mathbf{r}_B = 0$$

$$P\delta D + F_{BF}\delta r_F\cos\theta_F - F_{BF}\delta r_B\cos\theta_B = 0$$

$$P\delta D - F_{BF}(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = 0$$



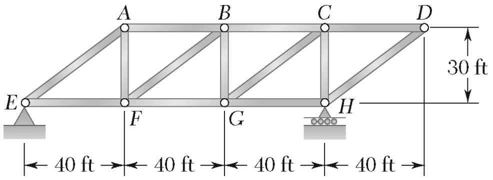
where $(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = \delta_{BF}$, which is the change in length of member *BF*. Thus,

$$P\delta D - F_{BF}\delta_{BF} = 0$$

$$P\delta D - \left(\frac{5}{9}P\right)(1.5 \text{ in.}) = 0$$

$$\delta D = 0.833 \text{ in.}$$

$$\delta D = 0.833 \text{ in.}\downarrow \blacktriangleleft$$

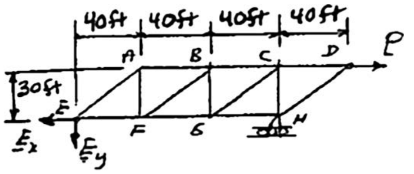


PROBLEM 10.58

Determine the horizontal movement of joint *D* if the length of member *BF* is increased by 1.5 in. (See the hint for Problem 10.57.)

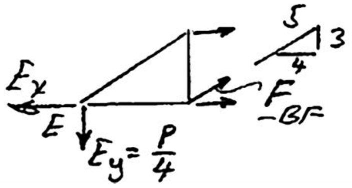
SOLUTION

Apply horizontal load *P* at *D*.



$$\overset{+}{\curvearrowright}\Sigma M_H = 0: \quad P(30\text{ ft}) - E_y(120\text{ ft}) = 0$$

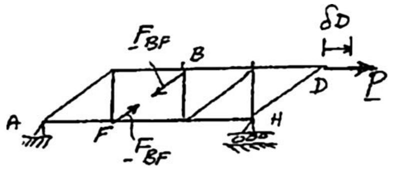
$$\mathbf{E}_y = \frac{P}{4}\downarrow$$



$$+\uparrow\Sigma F_y = 0: \quad \frac{3}{5}F_{BF} - \frac{P}{4} = 0$$

$$F_{BF} = \frac{5}{12}P$$

We remove member *BF* and replace it with forces $\mathbf{F}_{BF}$ and $-\mathbf{F}_{BF}$ at pins *F* and *B*, respectively. Denoting the virtual displacements of Points *B* and *F* as $\delta\mathbf{r}_B$ and $\delta\mathbf{r}_F$, respectively, and noting that **P** and $\overrightarrow{\delta D}$ have the same direction, we have

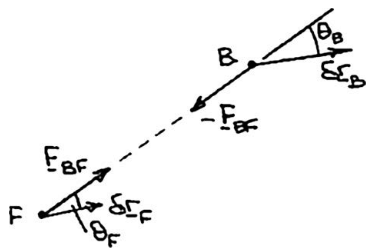


Virtual Work:

$$\delta U = 0: \qquad P\delta D + \mathbf{F}_{BF}\cdot\delta\mathbf{r}_F + (-\mathbf{F}_{BF})\cdot\delta\mathbf{r}_B = 0$$

$$P\delta D + F_{BF}\delta r_F\cos\theta_F - F_{BF}\delta r_B\cos\theta_B = 0$$

$$P\delta D - F_{BF}(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = 0$$



where $(\delta r_B\cos\theta_B - \delta r_F\cos\theta_F) = \delta_{BF}$, which is the change in length of member *BF*. Thus,

$$P\delta D - F_{BF}\delta_{BF} = 0$$

$$P\delta D - \left(\frac{5}{9}P\right)(1.5\text{ in.}) = 0$$

$$\delta D = 0.625\text{ in.}$$

$\delta D = 0.625$ in. ⟶ ◀

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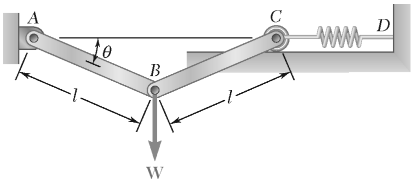
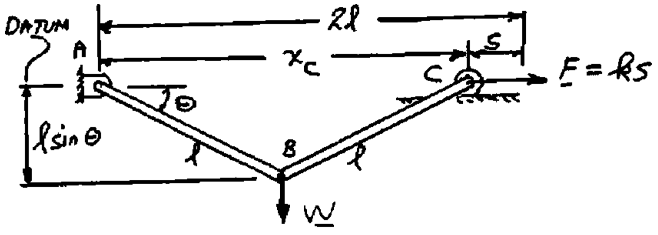


PROBLEM 10.59

Using the method of Section 10.8, solve Problem 10.29.

PROBLEM 10.29 A load **W** of magnitude 600 N is applied to the linkage at B. The constant of the spring is $k = 2.5$ kN/m, and the spring is unstretched when AB and BC are horizontal. Neglecting the weight of the linkage and knowing that $l = 300$ mm, determine the value of θ corresponding to equilibrium.

SOLUTION



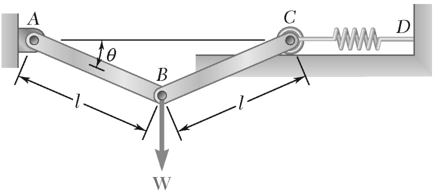
$$W = 600 \text{ N}$$

$$l = 0.3 \text{ m}, \quad \text{and} \quad k = 2500 \text{ N} \cdot \text{m}$$

We have

$$(l - \cos\theta)\tan\theta = \frac{600 \text{ N}}{4(2500 \text{ N/m})(0.3 \text{ m})} = 0.2$$

Solving numerically $\theta = 40.2°$ ◀

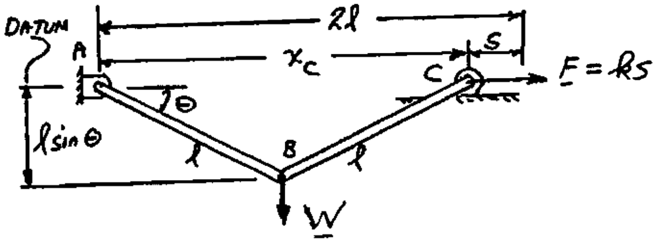


PROBLEM 10.60

Using the method of Section 10.8, solve Problem 10.30.

PROBLEM 10.30 A vertical load **W** is applied to the linkage at B. The constant of the spring is k, and the spring is unstretched when AB and BC are horizontal. Neglecting the weight of the linkage, derive an equation in θ, W, l, and k that must be satisfied when the linkage is in equilibrium.

SOLUTION



$$V = \frac{1}{2}ks^2 + Wy_B$$

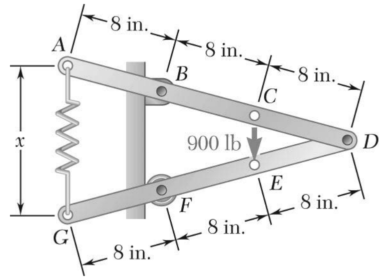
$$V = \frac{1}{2}k(2i - x_C)^2 + Wy_B$$

$$x_C = 2l\cos\theta \quad \text{and} \quad y_B = -l\sin\theta$$

Thus
$$V = \frac{1}{2}k(2l - 2l\cos\theta)^2 - Wl\sin\theta$$
$$= 2kl^2(1-\cos\theta)^2 - Wl\sin\theta$$

$$\frac{dV}{d\theta} = 2kl^2 2(1-\cos\theta)\sin\theta - Wl\cos\theta = 0$$

$$(1-\cos\theta)\tan\theta = \frac{W}{4kl} \blacktriangleleft$$

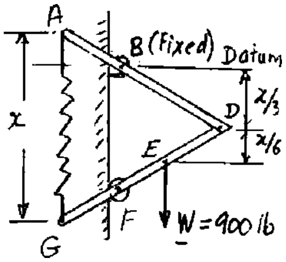


PROBLEM 10.61

Using the method of Section 10.8, solve Problem 10.31.

PROBLEM 10.31 Two bars *AD* and *DG* are connected by a pin at *D* and by a spring *AG*. Knowing that the spring is 12 in. long when unstretched and that the constant of the spring is 125 lb/in., determine the value of *x* corresponding to equilibrium when a 900-lb load is applied at *E* as shown.

SOLUTION



$$V = \frac{1}{2}ks^2 + Wy_E$$

But $$s = x - 12\text{ in.}$$

and $$y_E = -\frac{x}{3} - \frac{x}{6}$$

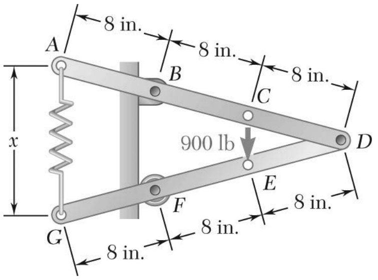
$$= -\frac{x}{2}$$

Thus $$V = \frac{1}{2}k(x-12)^2 - W\left(\frac{x}{2}\right)$$

$$\frac{dV}{dx} = k(x-12) - \frac{1}{2}W = 0$$

$$x = 12 + \frac{W}{2k} = 12\text{ in.} + \frac{900\text{ lb}}{2(125\text{ lb/in.})}$$

$x = 15.60$ in. ◀



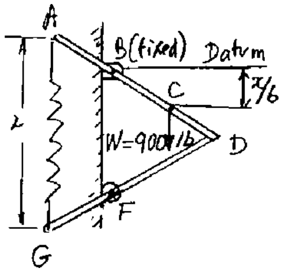
PROBLEM 10.62

Using the method of Section 10.8, solve Problem 10.32.

PROBLEM 10.32 Solve Problem 10.31 assuming that the 900-lb vertical force is applied at *C* instead of *E*.

PROBLEM 10.31 Two bars *AD* and *DG* are connected by a pin at *D* and by a spring *AG*. Knowing that the spring is 12 in. long when unstretched and that the constant of the spring is 125 lb/in., determine the value of *x* corresponding to equilibrium when a 900-lb load is applied at *E* as shown.

SOLUTION



$$V = \frac{1}{2}ks^2 + Wy_C$$

But $$s = x - 12\text{ in.}$$

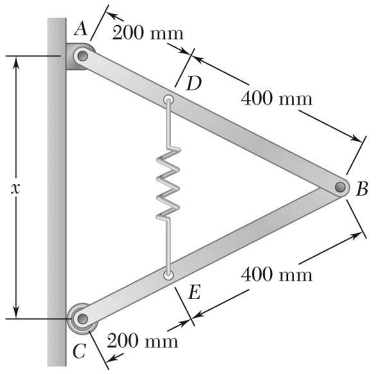
and $$y_E = -\frac{x}{6}$$

Thus $$V = \frac{1}{2}k(x-12)^2 - \frac{1}{6}Wx$$

$$\frac{dV}{dx} = k(x-12) - \frac{1}{6}W = 0$$

$$x = 12 + \frac{W}{6k} = 12\text{ in.} + \frac{900\text{ lb}}{6(125\text{ lb/in.})}$$

$x = 13.20$ in. ◀



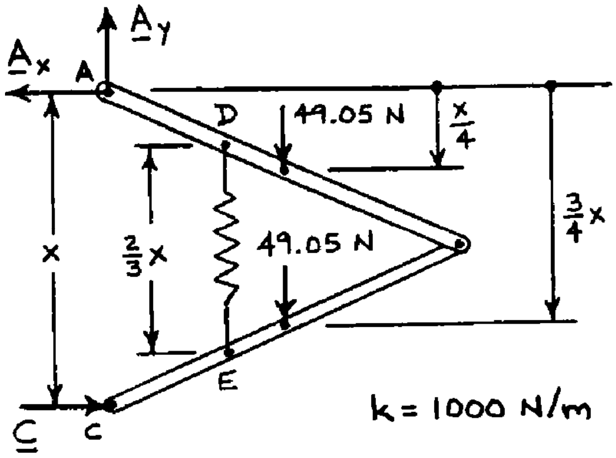
PROBLEM 10.63

Using the method of Section 10.8, solve Problem 10.33.

PROBLEM 10.33 Two 5-kg bars *AB* and *BC* are connected by a pin at *B* and by a spring *DE*. Knowing that the spring is 150 mm long when unstretched and that the constant of the spring is 1 kN/m, determine the value of *x* corresponding to equilibrium.

SOLUTION

First note: $W_{\text{bar}} = (5\text{ kg})(9.81\text{ m/s}^2) = 49.05\text{ N}$



Since unstretched length of spring is 150 mm, or 0.15 m, we have

$$\Delta_S = \frac{2}{3}x - 0.15$$

$$\mathbf{V} = \frac{1}{2}k\Delta_S^2 - (49.05\text{ N})\frac{x}{4} - (49.05\text{ N})\frac{3x}{4}$$

$$V = \frac{1}{2}(1000\text{ N/m})\left(\frac{2}{3}x - 0.15\right)^2 - 12.2625x - 36.7875x$$

$$\frac{dV}{dx} = 1000\left(\frac{2}{3}x - 0.15\right)\frac{2}{3} - 12.2625 - 36.7875 = 0$$

$x = 0.335$ m

$x = 335$ mm ◀

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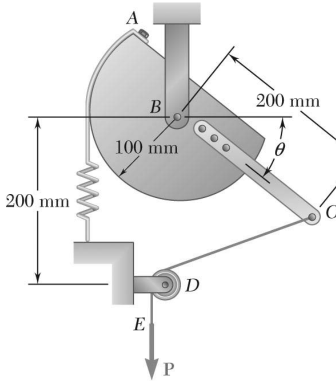
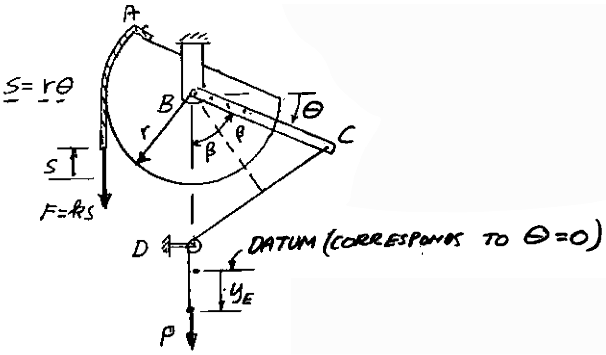


PROBLEM 10.64

Using the method of Section 10.8, solve Problem 10.35.

PROBLEM 10.35 A vertical force **P** of magnitude 150 N is applied to end *E* of cable *CDE*, which passes over a small pulley *D* and is attached to the mechanism at *C*. The constant of the spring is $k = 4$ kN/m, and the spring is unstretched when $\theta = 0$. Neglecting the weight of the mechanism and the radius of the pulley, determine the value of θ corresponding to equilibrium.

SOLUTION



$$\beta = \frac{1}{2}(90° - \theta) = 45° - \frac{\theta}{2}$$

$$BC = BD = l$$

$$CD = 2l\sin\beta = 2l\sin\left(45 - \frac{\theta}{2}\right)$$

For $\theta = 0$: $(CD)_0 = 2l\sin 45 = \sqrt{2}\,l$

$$y_E = (CD)_0 - CD$$

$$= \sqrt{2}\,l - 2l\sin\left(45° - \frac{\theta}{2}\right)$$

Potential energy:

$$V = \frac{1}{2}ks^2 - Py_E$$

$$V = \frac{1}{2}k(r\theta)^2 - P\left[\sqrt{2}l - 2l\sin\left(45° - \frac{\theta}{2}\right)\right]$$

$$\frac{dV}{d\theta} = kr^2\theta + 2Pl\cos\left(45° - \frac{\theta}{2}\right)\left(-\frac{1}{2}\right) = 0$$

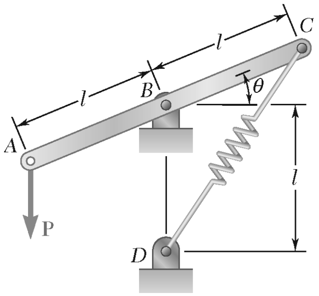
$$\frac{Pl}{kr^2} = \frac{\theta}{\cos\left(45° - \frac{\theta}{2}\right)}$$

$$\frac{Pl}{kr^2} = \frac{(150 \text{ N})(0.2 \text{ m})}{(4000 \text{ N/m})(0.1 \text{ m})^2} = 0.75$$

$$0.75 = \frac{\theta}{\cos\left(45° - \frac{\theta}{2}\right)}$$

Solve by trial and error: $\theta = 0.67623$ rad

$\theta = 38.745°$ $\theta = 38.7°$ ◀



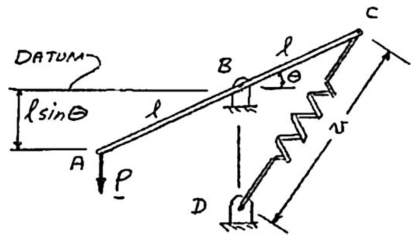
PROBLEM 10.65

Using the method of Section 10.8, solve Problem 10.37.

PROBLEM 10.37 and 10.38 Knowing that the constant of spring *CD* is k and that the spring is unstretched when rod *ABC* is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

PROBLEM 10.37 $P = 300$ N, $l = 400$ mm, $k = 5$ kN/m.

SOLUTION



Spring $$v = 2l\sin\left(\frac{90^\circ + \theta}{2}\right)$$

$$v = 2l\sin\left(45^\circ + \frac{\theta}{2}\right)$$

Unstretched $(\theta = 0)$ $$v_0 = 2l\sin 45^\circ = \sqrt{2}l$$

Deflection of spring $$s = v - v_0 = 2l\sin\left(45^\circ + \frac{\theta}{2}\right) - \sqrt{2}l$$

$$V = \frac{1}{2}ks^2 + Py_A = \frac{1}{2}kl^2\left[2\sin\left(45^\circ + \frac{\theta}{2}\right) - \sqrt{2}\right]^2 + P(-l\sin\theta)$$

$$\frac{dV}{d\theta} = kl^2\left[2\sin\left(45^\circ + \frac{\theta}{2}\right) - \sqrt{2}\right]\cos\left(45^\circ + \frac{\theta}{2}\right) - Pl\cos\theta = 0$$

$$\left[2\sin\left(45^\circ + \frac{\theta}{2}\right)\cos\left(45^\circ + \frac{\theta}{2}\right) - \sqrt{2}\cos\left(45^\circ + \frac{\theta}{2}\right)\right] = \frac{P}{kl}\cos\theta$$

$$\cos\theta - \sqrt{2}\cos\left(45^\circ + \frac{\theta}{2}\right) = \frac{P}{kl}\cos\theta$$

Divide each member by cos θ $$1 - \sqrt{2}\frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos\theta} = \frac{P}{kl}$$

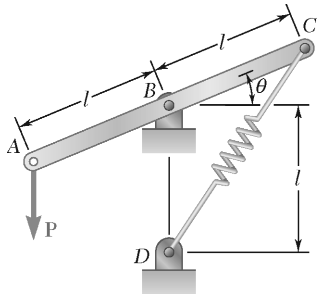
Then with $P = 300$ N, $l = 0.4$ m and $k = 5000$ N/m

$$1 - \sqrt{2}\frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos\theta} = \frac{300\text{ N}}{(5000\text{ N/m})(0.4\text{ m})}$$

$$= 0.15$$

or $$\frac{\cos\left(45^\circ + \frac{\theta}{2}\right)}{\cos\theta} = 0.60104$$

Solving numerically $\theta = 22.6^\circ$ ◀



PROBLEM 10.66

Using the method of Section 10.8, solve Problem 10.38.

PROBLEM 10.37 and 10.38 Knowing that the constant of spring *CD* is *k* and that the spring is unstretched when rod *ABC* is horizontal, determine the value of θ corresponding to equilibrium for the data indicated.

PROBLEM 10.38 $P = 75$ lb, $l = 15$ in., $k = 20$ lb/in.

SOLUTION

Using the results of Problem 10.65 with $P = 75$ lb, $l = 15$ in. and $k = 20$ lb/in., we have

$$1-\sqrt{2}\,\frac{\cos\left(45^\circ+\frac{\theta}{2}\right)}{\cos\theta}=\frac{P}{kl}$$

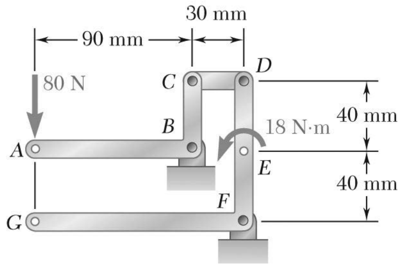
$$=\frac{75\text{ lb}}{(20\text{ lb/in.})(15\text{ in.})}$$

$$=0.25$$

or

$$\frac{\cos\left(45^\circ+\frac{\theta}{2}\right)}{\cos\theta}=0.53033$$

Solving numerically $\theta = 51.058^\circ$ $\theta = 51.1^\circ$ ◀

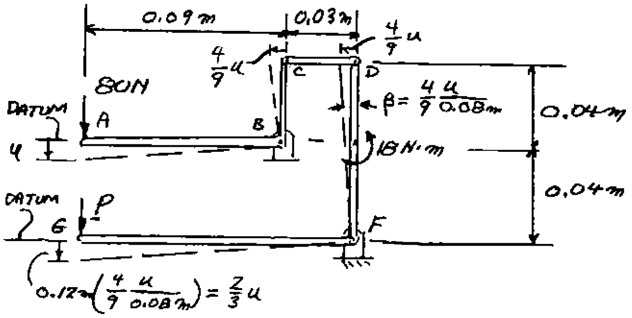


PROBLEM 10.67

Show that equilibrium is neutral in Problem 10.1.

PROBLEM 10.1 Determine the vertical force **P** that must be applied at G to maintain the equilibrium of the linkage.

SOLUTION



We have $$y_A = -u, \quad y_G = -\frac{2}{3}u, \quad \beta = \frac{4u}{0.72}$$

$$V = (80\text{ N})y_A + P(y_G) - (18\text{ N}\cdot\text{m})\beta$$

$$V = 80(-u) + P\left(-\frac{2}{3}u\right) - (18)\frac{4u}{0.72}$$

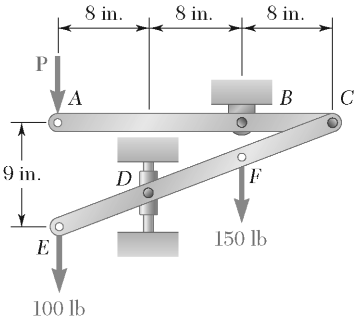
$$\frac{dV}{du} = -80 - \frac{2}{3}P - 100 = 0$$

$$P = 270\text{ N}$$

P $= 270$ N↑ ◀

Substituting $P = 270$ N in the expression for V, we have $V = 0$. Thus V is constant

and equilibrium is neutral ◀

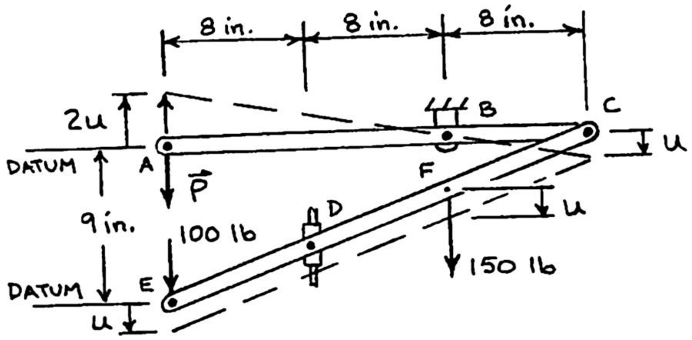


PROBLEM 10.68

Show that equilibrium is neutral in Problem 10.7.

PROBLEM 10.7 The two-bar linkage shown is supported by a pin and bracket at B and a collar at D that slides freely on a vertical rod. Determine the force **P** required to maintain the equilibrium of the linkage.

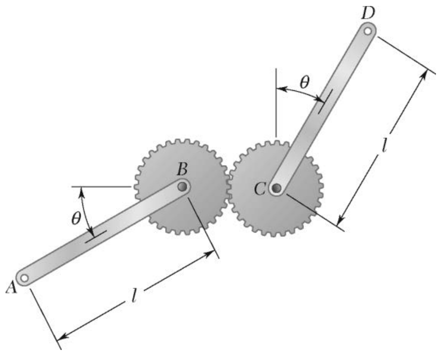
SOLUTION



$$y_A = 2u$$
$$y_E = -u$$
$$y_F = -u$$
$$V = Py_A + (100\text{ lb})y_E + (150\text{ lb})y_F$$
$$V = P(2u) + (100\text{ lb})(-u) + (150\text{ lb})(-u)$$
$$\frac{dV}{du} = 2P - 100 - 150 = 0$$
$$P = 125\text{ lb}$$

Now, substitute $P = 125$ lb in expression for V, making $V = 0$. Thus, V is constant and

<u>equilibrium is neutral.</u> ◀



PROBLEM 10.69

Two uniform rods, each of mass m, are attached to gears of equal radii as shown. Determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

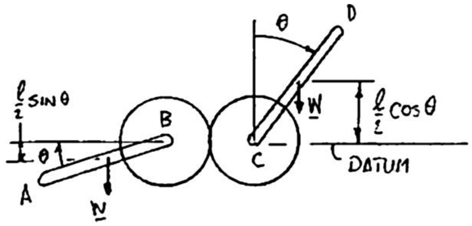
Potential energy

$$V = W\left(-\frac{l}{2}\sin\theta\right) + W\left(\frac{l}{2}\cos\theta\right) \quad W = mg$$

$$= W\frac{l}{2}(\cos\theta - \sin\theta)$$

$$\frac{dV}{d\theta} = \frac{Wl}{2}(-\sin\theta - \cos\theta)$$

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}(\sin\theta - \cos\theta)$$



For equilibrium: $\frac{dV}{d\theta} = 0$: $\sin\theta = -\cos\theta$

or $\tan\theta = -1$

Thus $\theta = -45.0°$ and $\theta = 135.0°$

Stability:

At $\theta = -45.0°$:

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}[\sin(-45°) - \cos 45°]$$

$$= \frac{Wl}{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) < 0$$

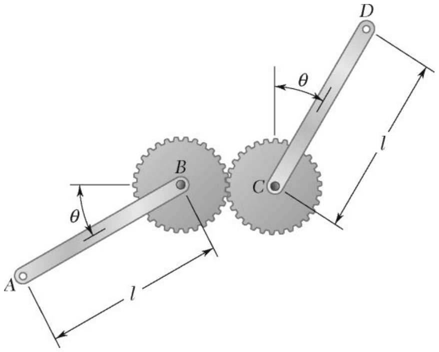
$\theta = -45.0°$, Unstable ◀

At $\theta = 135.0°$:

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}(\sin 135° - \cos 135°)$$

$$= \frac{Wl}{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) > 0$$

$\theta = 135.0°$, Stable ◀



PROBLEM 10.70

Two uniform rods, *AB* and *CD*, are attached to gears of equal radii as shown. Knowing that $W_{AB} = 8$ lb and $W_{CD} = 4$ lb, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

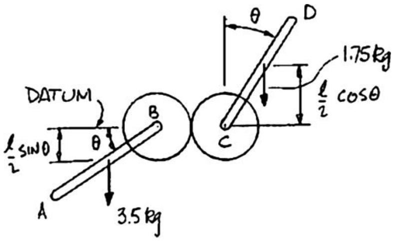
Potential energy

$$V = (3.5\text{ kg}\times 9.81\text{ m/s}^2)\left(-\frac{l}{2}\sin\theta\right) + (1.75\text{ kg}\times 9.81\text{ m/s}^2)\left(\frac{l}{2}\cos\theta\right)$$

$$= (8.5838\text{ N})l(-2\sin\theta + \cos\theta)$$

$$\frac{dV}{d\theta} = (8.5838\text{ N})l(-2\cos\theta - \sin\theta)$$

$$\frac{d^2V}{d\theta^2} = (8.5838\text{ N})l(2\sin\theta - \cos\theta)$$



Equilibrium: $\dfrac{dV}{d\theta} = 0$: $\quad -2\cos\theta - \sin\theta = 0$

or $\tan\theta = -2$

Thus $\theta = -63.4°$ and $116.6°$

Stability:

At $\theta = -63.4°$:

$$\frac{d^2V}{d\theta^2} = (8.5838\text{ N})l[2\sin(-63.4°) - \cos(-63.4°)]$$

$$= (8.5838\text{ N})l(-1.788 - 0.448) < 0$$

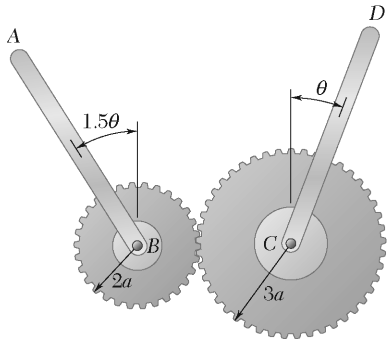
$\theta = -63.4°$, Unstable ◀

At $\theta = 116.6°$:

$$\frac{d^2V}{d\theta^2} = (8.5838\text{ N})l[2\sin(116.6°) - \cos(116.6°)]$$

$$= (8.5838\text{ N})l(1.788 + 0.447) > 0$$

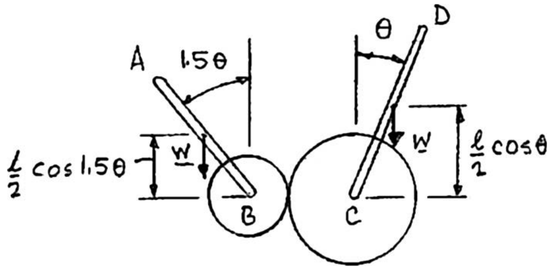
$\theta = 116.6°$, Stable ◀



PROBLEM 10.71

Two uniform rods, each of mass m and length l, are attached to gears as shown. For the range $0 \le \theta \le 180°$, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Potential energy $$V = W\left(\frac{l}{2}\cos 1.5\theta\right) + W\left(\frac{l}{2}\cos\theta\right) \quad W = mg$$

$$\frac{dV}{d\theta} = \frac{Wl}{2}(-1.5\sin 1.5\theta) + \frac{Wl}{2}(-\sin\theta)$$

$$= -\frac{Wl}{2}(1.5\sin 1.5\theta + \sin\theta)$$

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}(2.25\cos 1.5\theta + \cos\theta)$$

For equilibrium $$\frac{dV}{d\theta} = 0: \quad 1.5\sin 1.5\theta + \sin\theta = 0$$

Solutions: One solution, by inspection, is $\theta = 0$, and a second angle less than 180° can be found numerically:

$$\theta = 2.4042 \text{ rad} = 137.8°$$

Now $$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}(2.25\cos 1.5\theta + \cos\theta)$$

At $\theta = 0$: $$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}(2.25\cos 0° + \cos 0°)$$

$$= -\frac{Wl}{2}(3.25)(< 0)$$

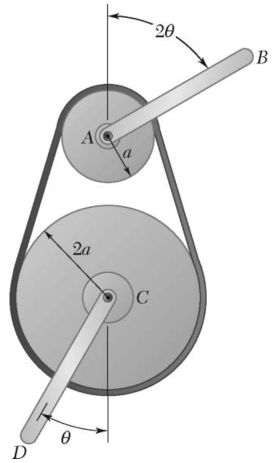
$\theta = 0$, Unstable ◀

PROBLEM 10.71 (Continued)

At $\theta = 137.8°$: $$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}[2.25\cos(1.5\times 137.8°) + \cos 137.8°]$$

$$= \frac{Wl}{2}(2.75)(> 0)$$

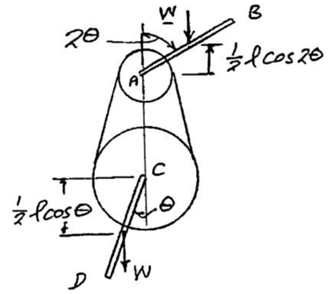
$\theta = 137.8°$, Stable ◀



PROBLEM 10.72

Two uniform rods, each of mass *m* and length *l*, are attached to drums that are connected by a belt as shown. Assuming that no slipping occurs between the belt and the drums, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



$$W = mg$$

$$V = W\left(\frac{l}{2}\cos 2\theta\right) - W\left(\frac{l}{2}\cos\theta\right)$$

$$\frac{dV}{d\theta} = W\frac{l}{2}(-2\sin 2 + \sin\theta)$$

$$\frac{d^2V}{d\theta^2} = W\frac{1}{2}(-4\cos 2\theta - \cos\theta)$$

Equilibrium: $$\frac{dV}{d\theta} = 0: \quad \frac{Wl}{2}(-2\sin 2\theta + \sin\theta) = 0$$

or $$\sin\theta(-4\cos\theta + 1) = 0$$

Solving $$\theta = 0,\ 75.5°,\ 180°,\ \text{and } 284°$$

Stability: $$\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4\cos 2\theta - \cos\theta)$$

At $\theta = 0$: $$\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4 - 1) < 0$$

$\theta = 0$, Unstable ◀

At $\theta = 75.5°$: $$\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4(-.874) - .25) > 0$$

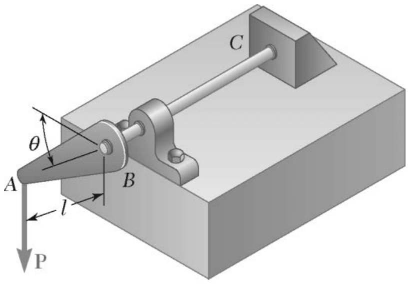
$\theta = 75.5°$, Stable ◀

At $\theta = 180°$: $$\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4 + 1) < 0$$

$\theta = 180.0°$, Unstable ◀

At $\theta = 284°$: $$\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4(-.874) - .25) > 0$$

$\theta = 284°$, Stable ◀

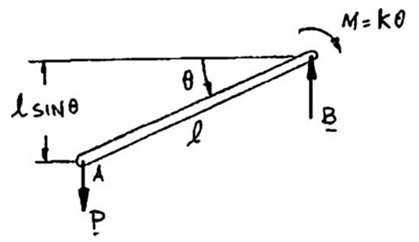


PROBLEM 10.73

Using the method of Section 10.8, solve Problem 10.39. Determine whether the equilibrium is stable, unstable, or neutral. (*Hint:* The potential energy corresponding to the couple exerted by a torsion spring is $\frac{1}{2}K\theta^2$, where K is the torsional spring constant and θ is the angle of twist.)

PROBLEM 10.39 The lever AB is attached to the horizontal shaft BC that passes through a bearing and is welded to a fixed support at C. The torsional spring constant of the shaft BC is K; that is, a couple of magnitude K is required to rotate end B through 1 rad. Knowing that the shaft is untwisted when AB is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 100$ N, $l = 250$ mm, and $K = 12.5\ \text{N}\cdot\text{m/rad}$.

SOLUTION



Potential energy $$V = \frac{1}{2}K\theta^2 - Pl\sin\theta$$

$$\frac{dV}{d\theta} = K\theta - Pl\cos\theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl\sin\theta$$

Equilibrium: $$\frac{dV}{d\theta} = 0: \quad \cos\theta = \frac{K}{Pl}\theta$$

For $$P = 100\ \text{N}, \quad l = 0.25\ \text{m.}, \quad K = 12.5\ \text{N}\cdot\text{m/rad}$$

$$\cos\theta = \frac{12.5\ \text{N}\cdot\text{m/rad}}{(100)(0.25\ \text{m})}\theta$$

$$= 0.500\theta$$

Solving numerically, we obtain

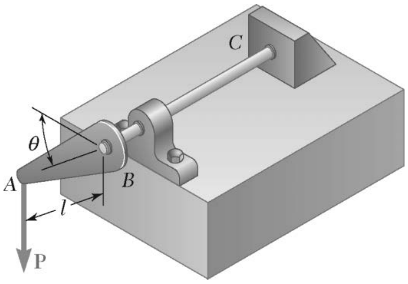
$$\theta = 1.02967\ \text{rad} = 59.000°$$

$\theta = 59.0°$ ◀

Stability

$$\frac{d^2V}{d\theta^2} = (12.5\ \text{N}\cdot\text{m/rad}) + (100\ \text{N})(0.25\ \text{m})\sin 59.0° > 0$$

Stable ◀

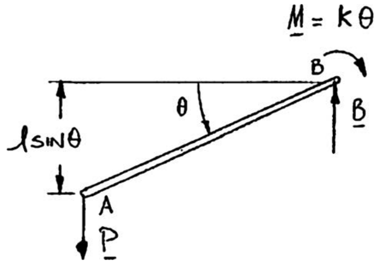


PROBLEM 10.74

In Problem 10.40, determine whether each of the positions of equilibrium is stable, unstable, or neutral. (See hint for Problem 10.73.)

PROBLEM 10.40 Solve Problem 10.39 assuming that $P = 350$ N, $l = 250$ mm, and $K = 12.5\ \text{N}\cdot\text{m/rad}$. Obtain answers in each of the following quadrants: $0 < \theta < 90°$, $270° < \theta < 360°$, $360° < \theta < 450°$.

SOLUTION



Potential energy $$V = \frac{1}{2}K\theta^2 - Pl\sin\theta$$

$$\frac{dV}{d\theta} = K\theta - Pl\cos\theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl\sin\theta$$

Equilibrium $$\frac{dV}{d\theta} = 0: \quad \cos\theta = \frac{K}{Pl}\theta$$

For $P = 350$ N, $l = 0.250$ m and $K = 12.5\ \text{N}\cdot\text{m/rad}$

$$\cos\theta = \frac{12.5\ \text{N}\cdot\text{m/rad}}{(350\ \text{N})(0.250\ \text{m})}\theta$$

or $$\cos\theta = \frac{\theta}{7}$$

Solving numerically $\theta = 1.37333$ rad, 5.652 rad, and 6.616 rad

or $\theta = 78.7°,\ 323.8°,\ 379.1°$

Stability at $\theta = 78.7°$: $$\frac{d^2V}{d\theta^2} = (12.5\ \text{N}\cdot\text{m/rad}) + (350\ \text{N})(0.250\ \text{m})\sin 78.7°$$

$$= 98.304 > 0$$

$\theta = 78.7°$, Stable ◀

At $\theta = 323.8°$: $$\frac{d^2V}{d\theta^2} = (12.5\ \text{N}\cdot\text{m/rad}) + (350\ \text{N})(0.250\ \text{m})\sin 323.8°$$

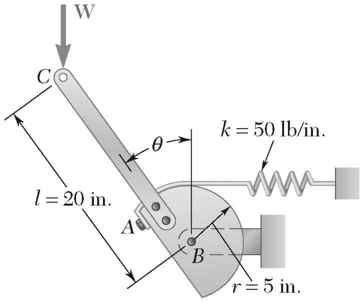
$$= -39.178\ \text{N}\cdot\text{m} < 0$$

$\theta = 324°$, Unstable ◀

At At $\theta = 379.1°$: $$\frac{d^2V}{d\theta^2} = (12.5\ \text{N}\cdot\text{m/rad}) + (350\ \text{N})(0.250\ \text{m})\sin 379.1°$$

$$= 44.132\ \text{N}\cdot\text{m} > 0$$

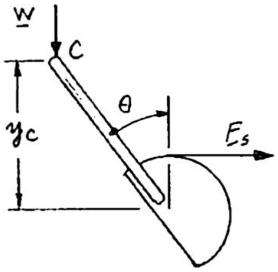
$\theta = 379°$, Stable ◀



PROBLEM 10.75

A load **W** of magnitude 100 lb is applied to the mechanism at *C*. Knowing that the spring is unstretched when $\theta = 15°$, determine that value of θ corresponding to equilibrium and check that the equilibrium is stable.

SOLUTION



We have $y_C = l\cos\theta$

$$V = \frac{1}{2}k[r(\theta - \theta_0)]^2 + Wy_C \quad \theta_0 = 15° = \frac{\pi}{12}\text{rad}$$

$$= \frac{1}{2}kr^2(\theta - \theta_0)^2 + Wl\cos\theta$$

$$\frac{dV}{d\theta} = kr^2(\theta - \theta_0) - Wl\sin\theta$$

Equilibrium $\frac{dV}{d\theta} = 0: \quad kr^2(\theta - \theta_0) - wl\sin\theta = 0$ (1)

with $W = 100$ lb, $R = 50$ lb./in., $l = 20$ in., and $r = 5$ in.

$$(50 \text{ lb./in.})(25 \text{ in.}^2)\left(\theta - \frac{\pi}{12}\right) - (100 \text{ lb})(20 \text{ in.})\sin\,\theta = 0$$

or $0.625\theta - \sin\theta = 0.16362$

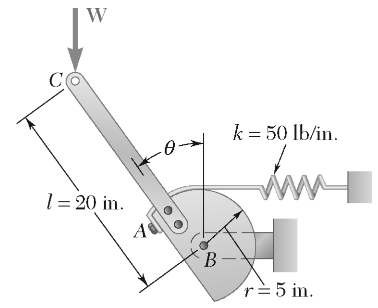
Solving numerically $\theta = 1.8145 \text{ rad} = 103.97°$

$\theta = 104.0°$ ◀

Stability $\frac{d^2V}{d\theta^2} = kr^2 - Wl\cos\theta$ (2)

or $= 1250 - 2000\cos\theta$

For $\theta = 104.0°$: $= 1734 \text{ in.} \cdot \text{lb} > 0$ Stable ◀



PROBLEM 10.76

A load **W** of magnitude 100 lb is applied to the mechanism at C. Knowing that the spring is unstretched when $\theta = 30°$, determine that value of θ corresponding to equilibrium and check that the equilibrium is stable.

SOLUTION

Using the solution of Problem 10.75, particularly Equation (1), with 15° replaced by $30°\left(\frac{\pi}{6}\text{ rad}\right)$:

For equilibrium $$kr^2\left(\theta - \frac{\pi}{6}\right) - Wl\sin\theta = 0$$

With $k = 50$ lb/in., $W = 100$ lb, $r = 5$ in., and $l = 20$ in.

$$(50 \text{ lb/in.})(25 \text{ in.}^2)\left(\theta - \frac{\pi}{6}\right) - (100 \text{ lb})(20 \text{ in.})\sin\theta = 0$$

or $$1250\theta - 654.5 - 200\sin\theta = 0$$

Solving numerically, $$\theta = 1.9870 \text{ rad } = 113.8°$$

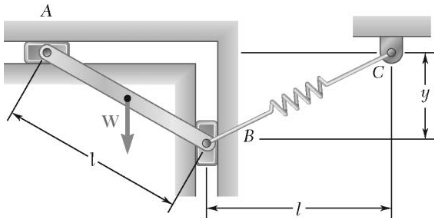
$\theta = 113.8°$ ◀

Stability: Equation (2), Problem 75:

$$\frac{d^2V}{d\theta^2} = kr^2 - Wl\cos\theta$$

or $$= 1250 - 2000\cos\theta$$

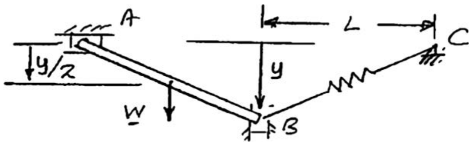
For $\theta = 113.8°$: $$= 2057 \text{ in.}\cdot\text{lb} > 0$$ Stable ◀



PROBLEM 10.77

A slender rod *AB*, of weight *W*, is attached to two blocks *A* and *B* that can move freely in the guides shown. Knowing that the spring is unstretched when $y = 0$, determine the value of *y* corresponding to equilibrium when $W = 80$ N, $l = 500$ mm, and $k = 600$ N/m.

SOLUTION



Deflection of spring = *s*, where

$$s = \sqrt{l^2 + y^2} - l$$

$$\frac{ds}{dy} = \frac{y}{\sqrt{l^2 - y^2}}$$

Potential energy:

$$V = \frac{1}{2}ks^2 - W\frac{y}{2}$$

$$\frac{dV}{dy} = ks\frac{ds}{dy} - \frac{1}{2}W$$

$$\frac{dV}{dy} = k\left(\sqrt{l^2 + y^2} - l\right)\frac{y}{\sqrt{l^2 + y^2}} - \frac{1}{2}W$$

$$= k\left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y - \frac{1}{2}W$$

Equilibrium

$$\frac{dV}{dy} = 0: \quad \left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y = \frac{1}{2}\frac{W}{k}$$

Now $\quad W = 80\text{ N}, \quad l = 0.500\text{ m}, \quad \text{and} \quad k = 600\text{ N/m}$

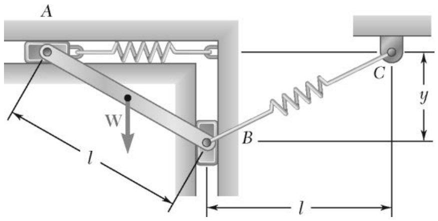
Then

$$\left(1 - \frac{0.500\text{ m}}{\sqrt{(0.500)^2 + y^2}}\right)y = \frac{1}{2}\frac{(80\text{ N})}{(600\text{ N/m})}$$

or

$$\left(1 - \frac{0.500}{\sqrt{0.25 + y^2}}\right)y = 0.066667$$

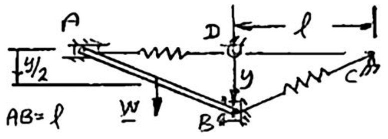
Solving numerically, $\quad y = 0.357$ m $\qquad y = 357$ mm ◀



PROBLEM 10.78

A slender rod AB, of weight W, is attached to two blocks A and B that can move freely in the guides shown. Knowing that both springs are unstretched when $y = 0$, determine the value of y corresponding to equilibrium when $W = 80$ N, l = 550 mm, and $k = 600$ N/m.

SOLUTION



Spring deflections

$$S_{AD} = l - \sqrt{l^2 - y^2}$$

$$S_{BC} = \sqrt{l^2 + y^2} - l$$

$$V = \frac{1}{2}kS_{AD}^2 + \frac{1}{2}kS_{BC}^2 - W\frac{y}{2}$$

$$V = \frac{1}{2}k\left(l - \sqrt{l^2 - y^2}\right)^2 + \frac{1}{2}k\left(\sqrt{l^2 + y^2} - l\right)^2 - W\frac{y}{2}$$

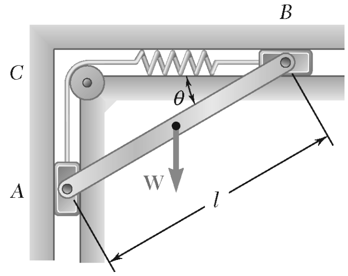
$$\frac{dV}{dy} = k\left(l - \sqrt{l^2 - y^2}\right)\left(\frac{y}{\sqrt{l^2 - y^2}}\right) + k\left(\sqrt{l^2 + y^2} - l\right)\left(\frac{y}{\sqrt{l^2 + y^2}}\right) - \frac{W}{2}$$

$$\frac{dV}{dy} = 0: \quad \left[\left(\frac{l}{\sqrt{l^2 - y^2}} - 1\right) + \left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)\right]y = \frac{W}{2k}$$

Data: $W = 80$ N, $l = 0.5$ m, $k = 600$ N/m

$$\left[\frac{0.5}{\sqrt{(0.5)^2 - y^2}} - \frac{0.5}{\sqrt{(0.5)^2 + y^2}}\right]y = \frac{80}{2(1200)} = 0.066667$$

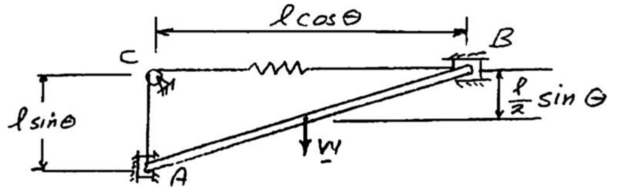
Solve by trial and error: $y = 0.252$ m $\qquad y = 252$ mm ◀



PROBLEM 10.79

A slender rod *AB*, of weight *W*, is attached to two blocks *A* and *B* that can move freely in the guides shown. The constant of the spring is *k*, and the spring is unstretched when *AB* is horizontal. Neglecting the weight of the blocks, derive an equation in *θ*, *W*, *l*, and *k* that must be satisfied when the rod is in equilibrium.

SOLUTION



Elongation of spring: $s = l\sin\theta + l\cos\theta - l$

$$s = l(\sin\theta + \cos\theta - 1)$$

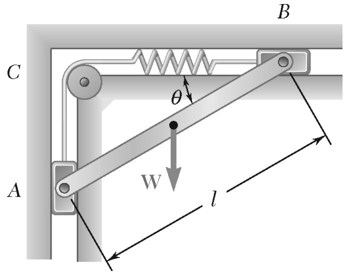
Potential energy: $V = \frac{1}{2}ks^2 - W\frac{1}{2}\sin\theta \quad W = mg$

$$= \frac{1}{2}kl^2(\sin\theta + \cos\theta - 1)^2 - mg\frac{l}{2}\sin\theta$$

$$\frac{dV}{d\theta} = kl^2(\sin\theta + \cos\theta - 1)(\cos\theta - \sin\theta) - \frac{1}{2}mgl\cos\theta \qquad (1)$$

Equilibrium: $\frac{dV}{d\theta} = 0: \quad (\sin\theta + \cos\theta - 1)(\cos\theta - \sin\theta) - \frac{mg}{2kl}\cos\theta = 0$

$$\text{or} \quad \cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - \frac{mg}{2kl}\right] = 0 \blacktriangleleft$$



PROBLEM 10.80

A slender rod *AB*, of weight *W*, is attached to two blocks *A* and *B* that can move freely in the guides shown. Knowing that the spring is unstretched when *AB* is horizontal, determine three values of θ corresponding to equilibrium when $W = 300$ lb, $l = 16$ in., and $k = 75$ lb/in. State in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Using the results of Problem 10.79, particularly the condition of equilibrium

$$\cos\theta\left[(\sin\theta+\cos\theta-1)(1-\tan\theta)-\frac{mg}{2kl}\right]=0$$

Now, with $W = 300$ lb, $l = 16$ in., and $k = 75$ lb/in.

$$\frac{W}{2kl}=\frac{300\text{ lb}}{(16\text{ in.})(75\text{ lb/in.})}=0.25$$

Thus: $$\cos\theta\left[(\sin\theta+\cos\theta-1)(1-\tan\theta)-0.25\right]=0$$

$$\cos\theta = 0 \quad\text{and}\quad (\sin\theta+\cos\theta-1)(1-\tan\theta)=0.25$$

First equation yields $\theta = 90°$. Solving the second equation by trial, we find $\theta = 9.39°$ and 34.16°
Values of θ for equilibrium are

$$\theta = 9.39°,\ 34.2°,\text{ and }90.0°$$

<u>Stability</u>: we differentiate Eq. (1).

$$\frac{d^2y}{ds^2}=kl^2[(\cos\theta-\sin\theta)(\cos\theta-\sin\theta)+(\sin\theta+\cos\theta-1)(-\sin\theta-\cos\theta)]+\frac{1}{2}wl\sin\theta$$

$$=kl^2\left[\cos^2\theta+\sin^2\theta-2\cos\theta\sin\theta-\sin^2\theta-\cos^2\theta-2\cos\theta\sin\theta+\sin\theta+\cos\theta+\frac{W}{2kl}\sin\theta\right]$$

$$=kl^2\left[\left(1+\frac{W}{2kl}\right)\sin\theta+\cos\theta-2\sin 2\theta\right]$$

$$\frac{d^2V}{d\theta^2}=kl^2(1.25\sin\theta+\cos\theta-2\sin 2\theta)$$

$\theta = 9.39°$: $$\frac{d^2V}{d\theta^2}=kl^2(1.25\sin 9.4+\cos 9.4-2\sin 18.8)$$

$$=kl^2(+0.55)<0$$

Stable ◀

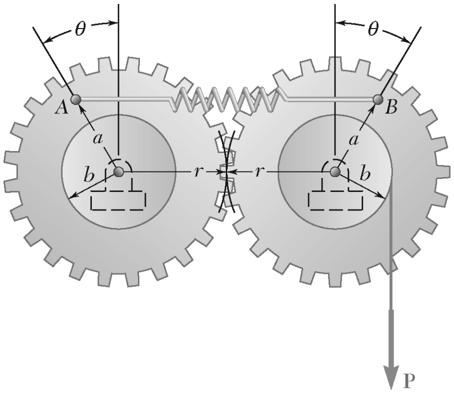
PROBLEM 10.80 (Continued)

$\theta = 34.2°$: $$\frac{d^2v}{d\theta^2} = kl^2(1.25\sin 34.2° + \cos 34.3° - 2\sin 68.4°)$$

$$= kl^2(-0.33) < 0$$ Unstable ◀

$\theta = 90.0°$: $$\frac{d^2V}{d\theta^2} = kl^2(1.25\sin 90° + \cos 90° - 2\sin 180°)$$

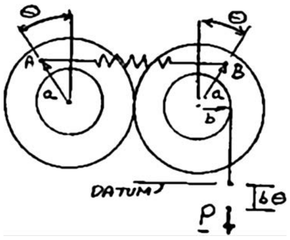
$$= kl^2(1.25) > 0$$ Stable ◀



PROBLEM 10.81

A spring AB of constant k is attached to two identical gears as shown. Knowing that the spring is undeformed when $\theta = 0$, determine two values of the angle θ corresponding to equilibrium when $P = 30$ lb, $a = 4$ in., $b = 3$ in., $r = 6$ in., and $k = 5$ lb/in. State in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Elongation of spring

$$s = 2(a\sin\theta) = 2a\sin\theta$$

$$V = \frac{1}{2}ks^2 - Pb\theta$$

$$= \frac{1}{2}k(2a\sin\theta)^2 - Pb\theta$$

$$\frac{dV}{d\theta} = 4ka^2\sin\theta\cos\theta - Pb$$

$$= 2ka^2\sin 2\theta - Pb \qquad (1)$$

Equilibrium $\qquad \dfrac{dV}{d\theta} = 0: \quad \sin 2\theta = \dfrac{Pb}{2ka^2}$

$$\sin 2\theta = \frac{(30\text{ lb})(3\text{ in.})}{2(5\text{ lb/in.})(4\text{ in.})^2}; \quad \sin 2\theta = 0.5625$$

$$2\theta = 34.229° \quad \text{and} \quad 145.771°$$

$$\theta = 17.11° \quad \text{and} \quad 72.9° \blacktriangleleft$$

Stability: We differentiate Eq. (1)

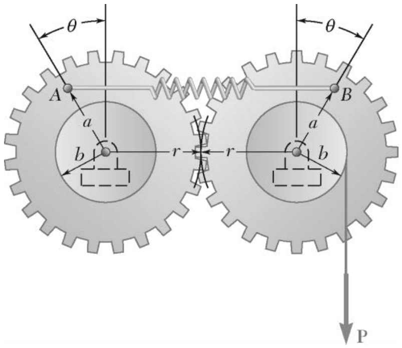
$$\frac{d^2V}{d\theta^2} = 4ka^2\cos 2\theta$$

$$\theta = 17.11°: \quad \frac{d^2v}{d\theta^2} = 4ka^2\cos 34.2° = 4ka^2(0.83) > 0$$

Stable ◀

$$\theta = 72.9°: \quad \frac{d^2v}{d\theta^2} = 4ka^2\cos 145.8° = 4ka^2(-0.83) < 0$$

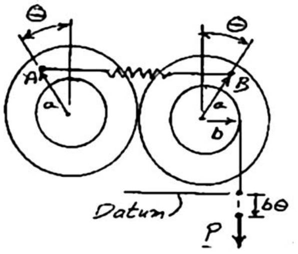
Unstable ◀



PROBLEM 10.82

A spring AB of constant k is attached to two identical gears as shown. Knowing that the spring is undeformed when $\theta = 0$, and given that $a = 60$ mm, $b = 45$ mm, $r = 90$ mm, and $k = 6$ kN/m, determine (*a*) the range of values of P for which a position of equilibrium exists, (*b*) two values of θ corresponding to equilibrium if the value of P is equal to half the upper limit of the range found in part *a*.

SOLUTION



Elongation of spring

$$s = 2(a\sin\theta) = 2a\sin\theta$$

Potential energy

$$V = \frac{1}{2}ks^2 - Pb\theta = \frac{1}{2}k(2a\sin\theta)^2 - Pb\theta$$

$$V = 2ka^2\sin^2\theta - Pb\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \quad \frac{dV}{d\theta} = 4ka^2\sin\theta\cos\theta - Pb \tag{1}$$

$$= 2ka^2\sin 2\theta - Pb = 0$$

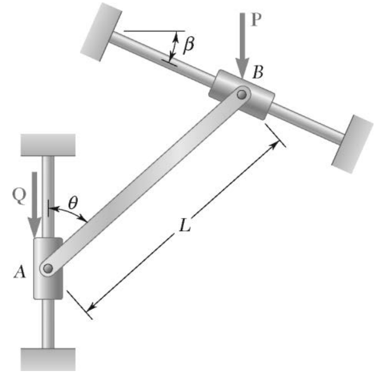
$$\sin 2\theta = \frac{Pb}{2ka^2}; \quad \text{For} \quad P_{\max}; \ \frac{P_{\max}b}{2ka^2} = 1$$

(*a*) $$\frac{P_{\max}(0.045\text{ m})}{2(6000\text{ N/m})(0.06\text{ m})^2} = 1$$ $P_{\max} = 960$ N ◀

(*b*) For $P = \frac{1}{2}P_{\max}$, $\sin 2\theta = \frac{1}{2}; 2\theta = 30°$ and $150°$

$\theta = 15.00°$ and $75.0°$ ◀

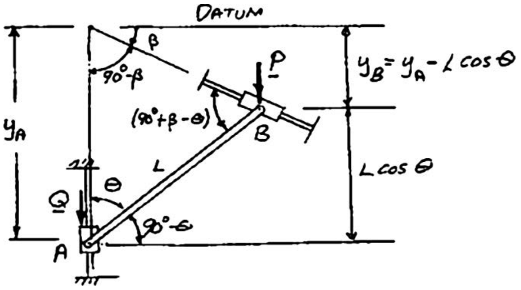
PROPRIETARY MATERIAL.



PROBLEM 10.83

A slender rod *AB* is attached to two collars *A* and *B* that can move freely along the guide rods shown. Knowing that $\beta = 30°$ and *P* = *Q* = 400 N, determine the value of the angle θ corresponding to equilibrium.

SOLUTION



Law of Sines
$$\frac{y_A}{\sin(90° + \beta - \theta)} = \frac{L}{\sin(90 - \beta)}$$

$$\frac{y_A}{\cos(\theta - \beta)} = \frac{L}{\cos\beta}$$

or
$$y_A = L\frac{\cos(\theta - \beta)}{\cos\beta}$$

From the figure:
$$y_B = L\frac{\cos(\theta - \beta)}{\cos\beta} - L\cos\theta$$

Potential Energy:
$$V = -Py_B - Qy_A = -P\left[L\frac{\cos(\theta - \beta)}{\cos\beta} - L\cos\theta\right] - QL\frac{\cos(\theta - \beta)}{\cos\beta}$$

$$\frac{dV}{d\theta} = -PL\left[-\frac{\sin(\theta - \beta)}{\cos\beta} + \sin\theta\right] + QL\frac{\sin(\theta - \beta)}{\cos\beta}$$

$$= L(P + Q)\frac{\sin(\theta - \beta)}{\cos\beta} - PL\sin\theta$$

PROBLEM 10.83 (Continued)

Equilibrium $$\frac{dV}{d\theta}=0: \quad L(P+Q)\frac{\sin(\theta-\beta)}{\cos\beta}-PL\sin\theta=0$$

or $$(P+Q)\sin(\theta-\beta)=P\sin\theta\cos\beta$$

$$(P+Q)(\sin\theta\cos\beta-\cos\theta\sin\beta)=P\sin\theta\cos\beta$$

or $$-(P+Q)\cos\theta\sin\beta+Q\sin\theta\cos\beta=0$$

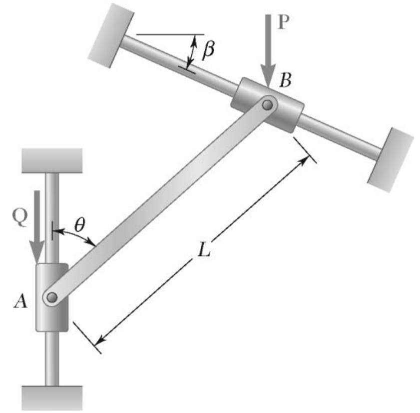
$$-\frac{P+Q}{Q}\frac{\sin\beta}{\cos\beta}+\frac{\sin\theta}{\cos\theta}=0$$

$$\tan\theta=\frac{P+Q}{Q}\tan\beta \qquad (2)$$

With $$P=Q=400\text{ N}, \quad \beta=30°$$

$$\tan\theta=\frac{800\text{ N}}{400\text{ N}}\tan 30°=1.1547$$

$\theta=49.1°$ ◀



PROBLEM 10.84

A slender rod AB is attached to two collars A and B that can move freely along the guide rods shown. Knowing that $\beta = 30°$, $P = 100$ N, and $Q = 25$ N, determine the value of the angle θ corresponding to equilibrium.

SOLUTION

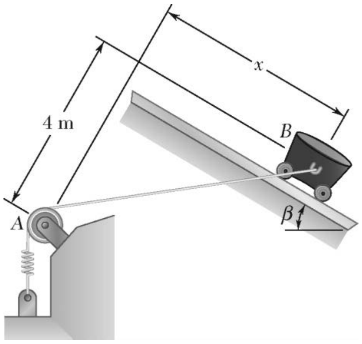
Using Equation (2) of Problem 10.83, with $P = 100$ N, $Q = 25$ N, and $\beta = 30°$, we have

$$\tan\theta = \frac{(100\text{ N})(25\text{ N})}{(25\text{ N})}\tan 30°$$

$$= 57.735$$

$$\theta = 89.007°$$

$$\theta = 89.0° \blacktriangleleft$$

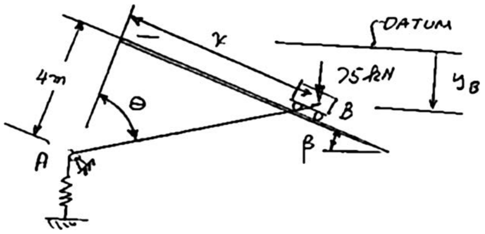


PROBLEM 10.85

Cart B, which weighs 75 kN, rolls along a sloping track that forms an angle β with the horizontal. The spring constant is 5 kN/m, and the spring is unstretched when $x = 0$. Determine the distance x corresponding to equilibrium for the angle β indicated.

Angle $\beta = 30°$.

SOLUTION



$$x = (4\text{ m})\tan\theta \tag{1}$$

$$y_B = x\sin\beta = 4\tan\theta\sin\beta$$

$$AC = (4\text{ m})\cos\theta$$

For $x = 0$, $\qquad (AC)_0 = 4\text{ m}$

Stretch of spring. $\qquad s = AC - (AC)_0 = \dfrac{4}{\cos\theta} - 4 = 4\left(\dfrac{1}{\cos\theta} - 1\right)$

$$V = \frac{1}{2}ks^2 - (75\text{ kN})y_B$$

$$= \frac{1}{2}(5\text{ kN/m})\,16\left(\frac{1}{\cos\theta} - 1\right)^2 - (75\text{ kN})4\tan\theta\sin\beta$$

$$\frac{dV}{d\theta} = 80\left(\frac{1}{\cos\theta} - 1\right)\frac{\sin\theta}{\cos^2\theta} - 300\frac{\sin\beta}{\cos^2\theta}$$

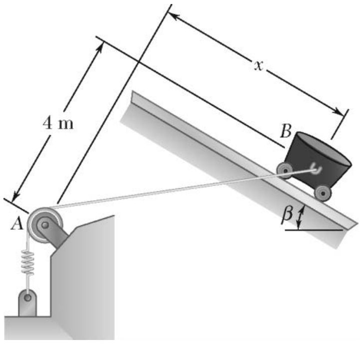
Equilibrium $\qquad \dfrac{dV}{d\theta} = 0{:}\quad \left(\dfrac{1}{\cos\theta} - 1\right)\sin\theta = 3.75\sin\beta \qquad (2)$

Given: $\qquad \beta = 30°, \quad \sin\theta = 0.5$

Eq. (2): $\qquad \left(\dfrac{1}{\cos\theta} - 1\right)\sin\theta = 3.75(0.5) = 1.875$

Solve by trial and error: $\qquad \theta = 70.46°$

Eq. (1): $\qquad x = (4\text{ m})\tan 70.46° \qquad x = 11.27\text{ m}$ ◀

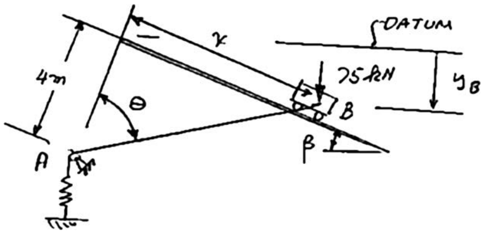


PROBLEM 10.86

Cart B, which weighs 75 kN, rolls along a sloping track that forms an angle β with the horizontal. The spring constant is 5 kN/m, and the spring is unstretched when $x = 0$. Determine the distance x corresponding to equilibrium for the angle β indicated.

Angle $\beta = 60°$.

SOLUTION



$$x = (4\text{ m})\tan\theta \qquad (1)$$

$$y_B = x\sin\beta = 4\tan\theta\sin\beta$$

$$AC = (4\text{ m})\cos\theta$$

For $x = 0$, $\qquad (AC)_0 = 4\text{ m}$

Stretch of spring: $\qquad s = AC - (AC)_0 = \dfrac{4}{\cos\theta} - 4 = 4\left(\dfrac{1}{\cos\theta} - 1\right)$

$$V = \frac{1}{2}ks^2 - (75\text{ kN})y_B$$

$$= \frac{1}{2}(5\text{ kN/m})\,16\left(\frac{1}{\cos\theta} - 1\right)^2 - (75\text{ kN})4\tan\theta\sin\beta$$

$$\frac{dV}{d\theta} = 80\left(\frac{1}{\cos\theta} - 1\right)\frac{\sin\theta}{\cos^2\theta} - 300\frac{\sin\beta}{\cos^2\theta}$$

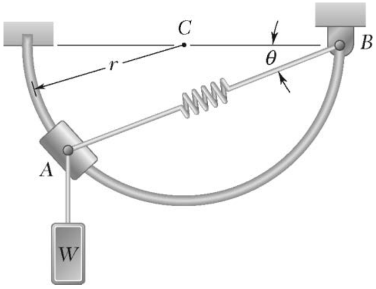
Equilibrium $\qquad \dfrac{dV}{d\theta} = 0\text{:} \quad \left(\dfrac{1}{\cos\theta} - 1\right)\sin\theta = 3.75\sin\beta \qquad (2)$

Given: $\qquad \beta = 60°,\ \sin\theta = 0.86603$

Eq. (2): $\qquad \left(\dfrac{1}{\cos\theta} - 1\right)\sin\theta = 3.75(0.86603) = 3.2476$

Solve by trial and error: $\qquad \theta = 76.67°$

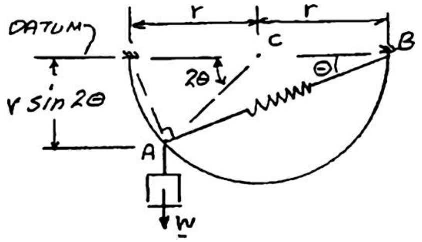
Eq. (1): $\qquad x = (4\text{ m})\tan 26.67° \qquad x = 16.88\text{ m}$ ◀



PROBLEM 10.87

Collar A can slide freely on the semicircular rod shown. Knowing that the constant of the spring is k and that the unstretched length of the spring is equal to the radius r, determine the value of θ corresponding to equilibrium when $W = 50$ lb, $r = 9$ in., and $k = 15$ lb/in.

SOLUTION



Stretch of spring $s = AB - r$

$$s = 2(r\cos\theta) - r$$

$$s = r(2\cos\theta - 1)$$

Potential energy: $V = \frac{1}{2}ks^2 - Wr\sin 2\theta \qquad W = mg$

$$V = \frac{1}{2}kr^2(2\cos\theta - 1)^2 - Wr\sin 2\theta$$

$$\frac{dV}{d\theta} = -kr^2(2\cos\theta - 1)2\sin\theta - 2Wr\cos 2\theta$$

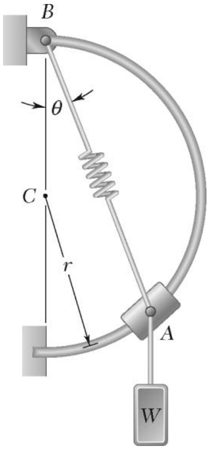
Equilibrium $\frac{dV}{d\theta} = 0: \quad -kr^2(2\cos\theta - 1)\sin\theta - Wr\cos 2\theta = 0$

$$\frac{(2\cos\theta - 1)\sin\theta}{\cos 2\theta} = -\frac{W}{kr}$$

Now $\frac{W}{kr} = \frac{(50 \text{ lb})}{(15 \text{ lb/in.})(9 \text{ in.})} = 0.37037$

Then $\frac{(2\cos\theta - 1)\sin\theta}{\cos 2\theta} = -0.37037$

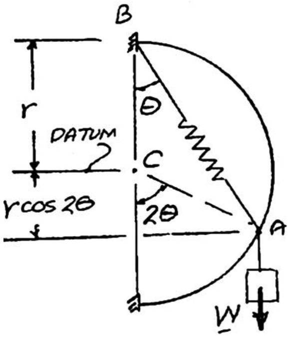
Solving numerically, $\theta = 0.95637 \text{ rad} = 54.8°$ $\qquad \theta = 54.8°$ ◀



PROBLEM 10.88

Collar A can slide freely on the semicircular rod shown. Knowing that the constant of the spring is k and that the unstretched length of the spring is equal to the radius r, determine the value of θ corresponding to equilibrium when $W = 50$ lb, $r = 9$ in., and $k = 15$ lb/in.

SOLUTION



Stretch of spring

$$s = AB - r = 2(r\cos\theta) - r$$

$$s = r(2\cos\theta - 1)$$

$$V = \frac{1}{2}ks^2 - Wr\cos 2\theta$$

$$= \frac{1}{2}kr^2(2\cos\theta - 1)^2 - Wr\cos 2\theta$$

$$\frac{dV}{d\theta} = -kr^2(2\cos\theta - 1)2\sin\theta + 2Wr\sin 2\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \quad -kr^2(2\cos\theta - 1)\sin\theta + Wr\sin 2\theta = 0$$

$$-kr^2(2\cos\theta - 1)\sin\theta + Wr(2\sin\theta\cos\theta) = 0$$

or

$$\frac{(2\cos\theta - 1)\sin\theta}{2\cos\theta} = \frac{W}{kr}$$

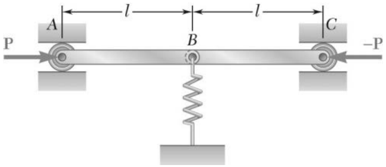
Now

$$\frac{W}{kr} = \frac{(50\text{ lb})}{(15\text{ lb/in.})(9\text{ in.})} = 0.37037$$

Then

$$\frac{2\cos\theta - 1}{2\cos\theta} = 0.37037$$

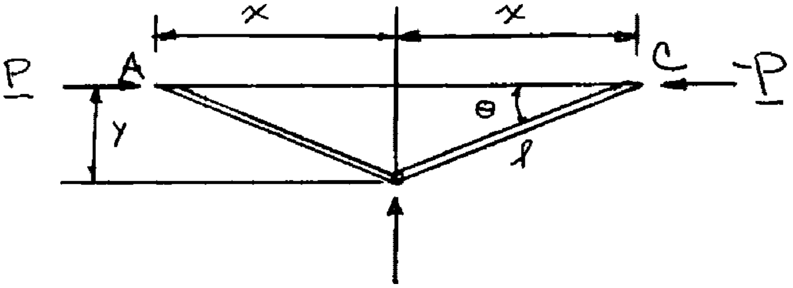
Solving $\theta = 37.4°$ ◀



PROBLEM 10.89

Two bars *AB* and *BC* of negligible weight are attached to a single spring of constant *k* that is unstretched when the bars are horizontal. Determine the range of values of the magnitude *P* of two equal and opposite forces **P** and **–P** for which the equilibrium of the system is stable in the position shown.

SOLUTION



$$x = l\cos\theta$$
$$y = l\sin\theta$$

$$V = 2Px + \frac{1}{2}ky^2$$

$$v = 2Pl\cos\theta + \frac{1}{2}kl^2\sin^2\theta$$

$$\frac{dV}{d\theta} = -2Pl\sin\theta + kl^2\sin\theta\cos\theta$$

$$= -2Pl\sin\theta + \frac{1}{2}kl^2\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -2Pl\cos\theta + kl^2\cos 2\theta \qquad (1)$$

For equilibrium position $\theta = 0$ to be stable

$$\frac{d^2V}{d\theta^2} = -2Pl + kl^2 > 0 \qquad (2)$$

$$P < \frac{1}{2}kl \blacktriangleleft$$

Note: For $P = \frac{1}{2}kl,$ we have $\frac{d^2V}{d\theta^2} = 0$ and we must determine which is the first derivative to be $\neq 0$.

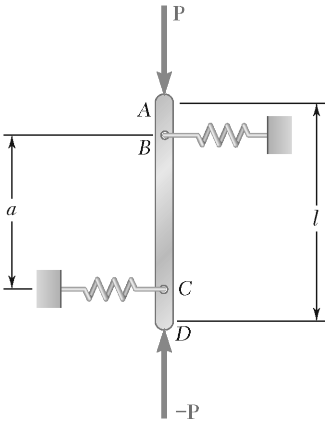
Differentiating Eq. (1):

$$\frac{d^3V}{d\theta^3} = +2Pl\sin\theta - 2kl^2\sin 2\theta = 0 \text{ for } \theta = 0$$

$$\frac{d^4V}{d\theta^4} = 2Pl\cos\theta - 4kl^2\cos 2\theta = 2Pl - 4kl^2 \text{ for } \theta = 0$$

But $P = \frac{1}{2}kl.$ Thus $\frac{d^4V}{d\theta^4} = kl^2 - 4kl^2 < 0$ and we conclude that the equilibrium is unstable for $P = \frac{1}{2}kl.$

The sign < in Eq. (2) is thus correct.



PROBLEM 10.90

A vertical bar *AD* is attached to two springs of constant *k* and is in equilibrium in the position shown. Determine the range of values of the magnitude *P* of two equal and opposite vertical forces **P** and **−P** for which the equilibrium position is stable if (*a*) $AB = CD$, (*b*) $AB = 2CD$.

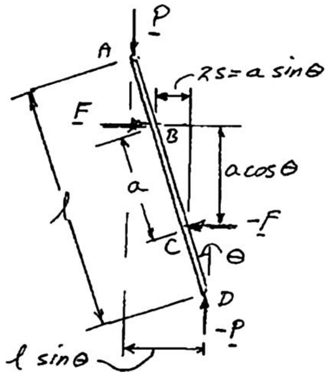
SOLUTION

For both (*a*) and (*b*): Since **P** and **−P** are vertical, they form a couple of moment

$$M_P = +Pl\sin\theta$$

The forces **F** and **−F** exerted by springs must, therefore, also form a couple, with moment

$$M_F = -Fa\cos\theta$$



We have
$$dU = M_P d\theta + M_F d\theta$$
$$= (Pl\sin\theta - Fa\cos\theta)d\theta$$

but
$$F = ks = k\left(\frac{1}{2}a\sin\theta\right)$$

Thus,
$$dU = \left(Pl\sin\theta - \frac{1}{2}ka^2\sin\theta\cos\theta\right)d\theta$$

From Equation (10.19), page 580, we have

$$dV = -dU = -Pl\sin\theta d\theta + \frac{1}{4}ka^2\sin 2\theta d\theta$$

or
$$\frac{dV}{d\theta} = -Pl\sin\theta + \frac{1}{4}ka^2\sin 2\theta$$

and
$$\frac{d^2V}{d\theta^2} = -Pl\cos\theta + \frac{1}{2}ka^2\cos 2\theta \qquad (1)$$

For $\theta = 0$:
$$\frac{d^2V}{d\theta^2} = -Pl + \frac{1}{2}ka^2$$

PROBLEM 10.90 (Continued)

For Stability: $$\frac{d^2V}{d\theta^2} > 0, \quad -Pl + \frac{1}{2}ka^2 > 0$$

or (for Parts *a* and *b*) $$P < \frac{ka^2}{2l} \blacktriangleleft$$

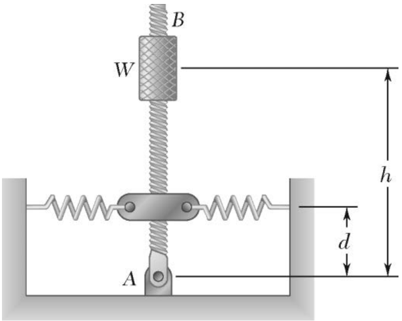
Note: To check that equilibrium is unstable for $P = \frac{ka^2}{2l}$, we differentiate (1) twice:

$$\frac{d^3V}{d\theta^3} = +Pl\sin\theta - ka^2\sin 2\theta = 0, \quad \text{for} \quad \theta = 0,$$

$$\frac{d^4V}{d\theta^4} = Pl\cos\theta - 2ka^2\cos 2\theta$$

For $\theta = 0$ $$\frac{d^4V}{d\theta^4} = Pl - 2ka^2 = \frac{ka^2}{2} - 2ka^2 < 0$$

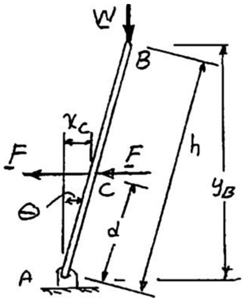
Thus, equilibrium is unstable when $$P = \frac{ka^2}{2l}$$



PROBLEM 10.91

Rod AB is attached to a hinge at A and to two springs, each of constant k. If $h = 25$ in., $d = 12$ in., and $W = 80$ lb, determine the range of values of k for which the equilibrium of the rod is stable in the position shown. Each spring can act in either tension or compression.

SOLUTION



We have $x_C = d\sin\theta \qquad y_B = h\cos\theta$

Potential Energy:
$$V = 2\left(\frac{1}{2}kx_C^2 + Wy_B\right)$$
$$= kd^2\sin^2\theta + Wh\cos\theta$$

Then
$$\frac{dV}{d\theta} = 2kd^2\sin\theta\cos\theta - Wh\sin\theta$$
$$= kd^2\sin 2\theta - Wh\sin\theta$$

and
$$\frac{d^2V}{d\theta^2} = 2kd^2\cos 2\theta - Wh\cos\theta \qquad (1)$$

For equilibrium position $\theta = 0$ to be stable, we must have
$$\frac{d^2V}{d\theta^2} = 2kd^2 - Wh > 0$$

or
$$kd^2 > \frac{1}{2}Wh \qquad (2)$$

Note: For $kd^2 = \frac{1}{2}Wh$, we have $\frac{d^2V}{d\theta^2} = 0$, so that we must determine which is the first derivative that is not equal to zero. Differentiating Equation (1), we write
$$\frac{d^3V}{d\theta^3} = -4kd^2\sin 2\theta + Wh\sin\theta = 0 \qquad \text{for } \theta = 0$$
$$\frac{d^4V}{d\theta^2} = -8kd^2\cos 2\theta + Wh\cos\theta$$

For $\theta = 0$:
$$\frac{d^4V}{d\theta^4} = -8kd^2 + Wh$$

PROBLEM 10.91 (Continued)

Since $kd^2 = \frac{1}{2}Wh$, $\frac{d^4V}{d\theta^4} = -4Wh + Wh < 0$, we conclude that the equilibrium is unstable for $kd^2 = \frac{1}{2}Wh$ and the $>$ sign in Equation (2) is correct.

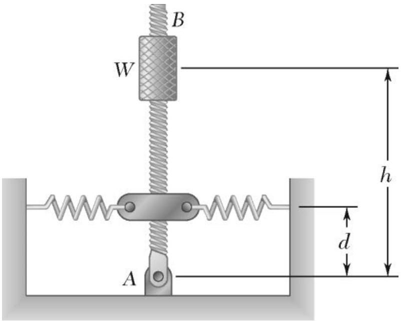
With $\quad W = 80 \text{ lb}, \quad h = 25 \text{ in.}, \quad \text{and} \quad d = 12 \text{ in.}$

Equation (2) gives

$$k(12 \text{ in.})^2 > \frac{1}{2}(80 \text{ lb})(25 \text{ in.})$$

or $\quad k > 6.944 \text{ lb/in.}$

$k > 6.94$ lb/in. ◀



PROBLEM 10.92

Rod *AB* is attached to a hinge at *A* and to two springs, each of constant *k*. If $h = 45$ in., $k = 6$ lb/in., and $W = 60$ lb, determine the smallest distance *d* for which the equilibrium of the rod is stable in the position shown. Each spring can act in either tension or compression.

SOLUTION

Using Equation (2) of Problem 10.91 with

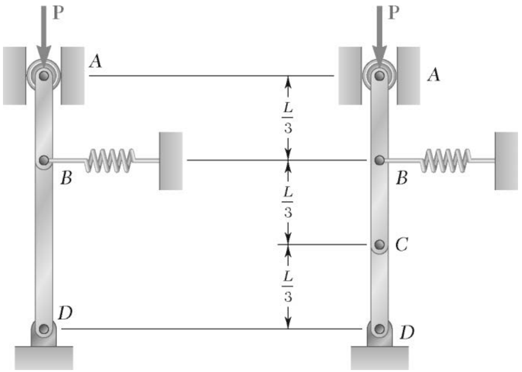
$$h = 45 \text{ in.,} \quad k = 6 \text{ lb/in.,} \quad \text{and} \quad W = 60 \text{ lb}$$

$$(6 \text{ lb/in.})d^2 > \frac{1}{2}(60 \text{ lb})(45 \text{ in.})$$

or $$d^2 > 225 \text{ in.}^2$$

$$d > 15.0000 \text{ in.}$$

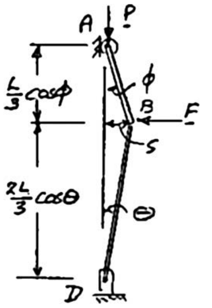
smallest $d = 15.00$ in. ◀



PROBLEM 10.93

Two bars are attached to a single spring of constant k that is unstretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.

SOLUTION



$$s = \frac{L}{3}\sin\phi = \frac{2L}{3}\sin\theta$$

For small values of ϕ and θ

$$\phi = 2\theta$$

$$V = P\left(\frac{L}{3}\cos\phi + \frac{2L}{3}\cos\theta\right) + \frac{1}{2}ks^2$$

$$V = \frac{PL}{3}(\cos 2\theta + 2\cos\theta) + \frac{1}{2}k\left(\frac{2L}{3}\sin\theta\right)^2$$

$$\frac{dV}{d\theta} = \frac{PL}{3}(-2\sin 2\theta - 2\sin\theta) + \frac{2}{9}kL^2\sin\theta\cos\theta$$

$$= -\frac{PL}{3}(2\sin 2\theta + 2\sin\theta) + \frac{2}{9}kL^2\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{PL}{3}(4\cos 2\theta + 2\cos\theta) + \frac{4}{9}kL^2\cos 2\theta$$

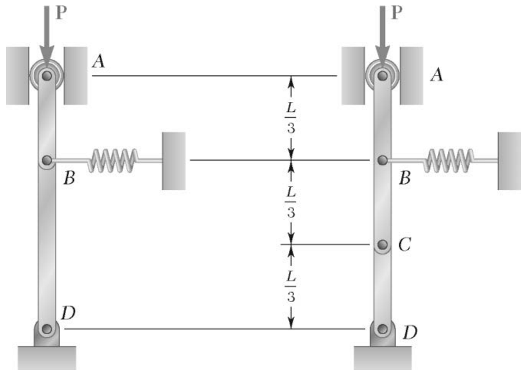
when $\theta = 0$:

$$\frac{d^2V}{d\theta^2} = -\frac{6PL}{3} + \frac{4}{9}kL^2$$

<u>For stability</u>:

$$\frac{d^2V}{d\theta^2} > 0, \quad -2PL + \frac{4}{9}kL^2 > 0$$

$$P < \frac{2}{9}kL \quad ◀$$

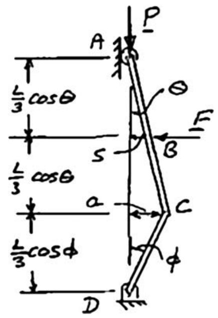


PROBLEM 10.94

Two bars are attached to a single spring of constant k that is unstretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.

SOLUTION

$$a = \frac{2L}{3}\sin\theta = \frac{L}{3}\sin\phi$$



For small values of ϕ and θ

$$\phi = 2\theta$$

$$s = \frac{L}{3}\sin\theta$$

$$V = P\left(\frac{2L}{3}\cos\theta + \frac{L}{3}\cos\phi\right) + \frac{1}{2}ks^2$$

$$= \frac{PL}{3}(2\cos\theta + \cos 2\theta) + \frac{1}{2}k\left(\frac{L}{3}\sin\theta\right)^2$$

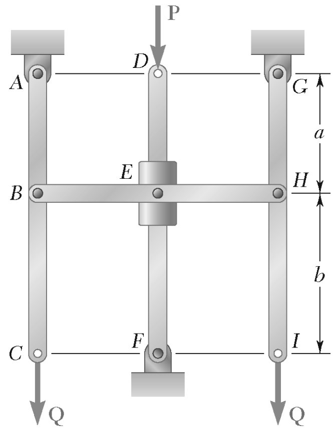
$$\frac{dV}{d\theta} = \frac{PL}{3}(-2\sin\theta - 2\sin 2\theta) + \frac{kL^2}{9}\sin\theta\cos\theta$$

$$\frac{dV}{d\theta} = -\frac{2PL}{3}(\sin\theta + \sin 2\theta) + \frac{kL^2}{18}\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{2PL}{3}(\cos\theta + 2\cos 2\theta) + \frac{kL^2}{9}\cos 2\theta$$

when $\theta = 0$: $$\frac{dV^2}{d\theta^2} = -2PL + \frac{kL^2}{9}$$

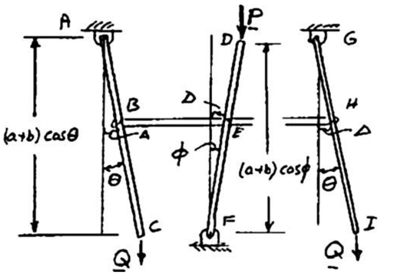
For stability: $$\frac{d^2V}{d\theta^2} > 0, \quad -2PL + \frac{kL^2}{9} > 0$$ $$P < \frac{1}{18}kL$$ ◀



PROBLEM 10.95

The horizontal bar *BEH* is connected to three vertical bars. The collar at *E* can slide freely on bar *DF*. Determine the range of values of *Q* for which the equilibrium of the system is stable in the position shown when $a = 24$ in., $b = 20$ in., and $P = 150$ lb.

SOLUTION



First note $A = a\sin\theta = b\sin\phi$

For small values of θ and ϕ: $a\theta = b\phi$

or $\phi = \dfrac{a}{b}\theta$

$$V = P(a+b)\cos\phi - 2Q(a+b)\cos\theta$$

$$= (a+b)\left[P\cos\left(\frac{a}{b}\theta\right) - 2Q\cos\theta\right]$$

$$\frac{dV}{d\theta} = (a+b)\left[-\frac{a}{b}P\sin\left(\frac{a}{b}\theta\right) + 2Q\sin\theta\right]$$

$$\frac{d^2V}{d\theta^2} = (a+b)\left[-\frac{a^2}{b^2}P\cos\left(\frac{a}{b}\theta\right) + 2Q\cos\theta\right]$$

when $\theta = 0$: $$\frac{d^2V}{d\theta^2} = (a+b)\left(-\frac{a^2}{b^2}P + 2Q\right)$$

Stability: $$\frac{d^2V}{d\theta^2} > 0: \quad -\frac{a^2}{b^2}P + 2Q > 0$$

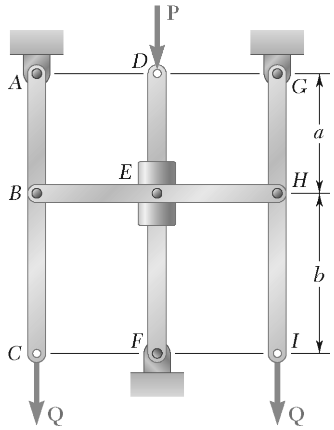
$$P < 2\frac{b^2}{a^2}Q \qquad (1)$$

or $$Q > \frac{a^2}{2b^2}P \qquad (2)$$

with $P = 150$ lb, $a = 24$ in., and $b = 20$ in.

Equation (1): $$Q > \frac{(24 \text{ in.})^2}{2(20 \text{ in.})^2}(150 \text{ lb}) = 108.000 \text{ lb}$$

For stability $Q > 108.0$ lb ◀



PROBLEM 10.96

The horizontal bar *BEH* is connected to three vertical bars. The collar at *E* can slide freely on bar *DF*. Determine the range of values of *P* for which the equilibrium of the system is stable in the position shown when $a = 150$ mm, $b = 200$ mm, and $Q = 45$ N.

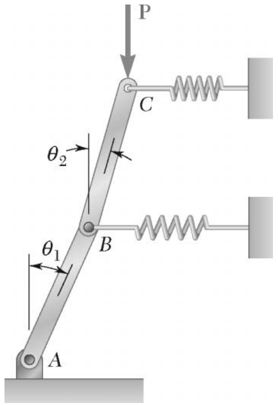
SOLUTION

Using Equation (2) of Problem 10.95 with

$$Q = 45\text{ N},\quad a = 150\text{ mm},\quad \text{and}\quad b = 200\text{ mm}$$

Equation (2)
$$P < 2\frac{(200\text{ mm})^2}{(150\text{ mm})^2}(45\text{ N})$$
$$= 160.000\text{ N}$$

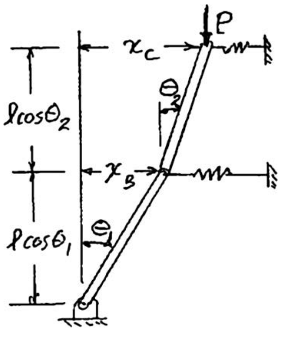
For stability $P < 160.0$ N ◀



PROBLEM 10.97*

Bars *AB* and *BC*, each of length *l* and of negligible weight, are attached to two springs, each of constant *k*. The springs are undeformed, and the system is in equilibrium when $\theta_1 = \theta_2 = 0$. Determine the range of values of *P* for which the equilibrium position is stable.

SOLUTION



We have
$$x_B = l\sin\theta$$
$$x_C = l\sin\theta_1 + l\sin\theta_2$$
$$y_C = l\cos\theta_1 + l\cos\theta_2$$
$$V = Py_C + \frac{1}{2}kx_B^2 + \frac{1}{2}kx_C^2$$

or
$$V = Pl(\cos\theta_1 + \cos\theta_2) + \frac{1}{2}kl^2\left[\sin^2\theta_1 + (\sin\theta_1 + \sin\theta_2)^2\right]$$

For small values of θ_1 and θ_2:

$$\sin\theta_1 \approx \theta_1, \quad \sin\theta_2 \approx \theta_2, \quad \cos\theta_1 \approx 1 - \frac{1}{2}\theta_1^2, \quad \cos\theta_2 \approx 1 - \frac{1}{2}\theta_2^2$$

Then
$$V = Pl\left(1 - \frac{\theta_1^2}{2} + 1 - \frac{\theta_2^2}{2}\right) + \frac{1}{2}kl^2\left[\theta_1^2 + \left(\theta_1 + \theta_2\right)^2\right]$$

and
$$\frac{\partial V}{\partial\theta_1} = -Pl\theta_1 + kl^2[\theta_1 + (\theta_1 + \theta_2)]$$
$$\frac{\partial V}{\partial\theta_2} = -Pl\theta_2 + kl^2(\theta_1 + \theta_2)$$
$$\frac{\partial^2 V}{\partial\theta_1^2} = -Pl + 2kl^2 \quad \frac{\partial^2 V}{\partial\theta_2^2} = -Pl + kl^2$$
$$\frac{\partial^2 V}{\partial\theta_1\partial\theta_2} = kl^2$$

Stability: Conditions for stability (see Page 583).

For $\theta_1 = \theta_2 = 0$: $\dfrac{\partial V}{\partial \theta_1} = \dfrac{\partial V}{\partial \theta_2} = 0$ (condition satisfied)

$$\left(\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2}\right)^2 - \frac{\partial^2 V}{\partial \theta_1^2}\frac{\partial^2 V}{\partial \theta_2^2} < 0$$

Substituting $(kl^2)^2 - (-Pl + 2kl^2)(-Pl + kl) < 0$

$$k^2l^4 - P^2l^2 + 3Pkl^3 - 2k^2l^4 < 0$$

$$P^2 - 3klP + k^2l^2 > 0$$

Solving $P < \dfrac{3-\sqrt{5}}{2}kl$ or $P > \dfrac{3+\sqrt{5}}{2}kl$

or $P < 0.382kl$ or $P > 2.62kl$

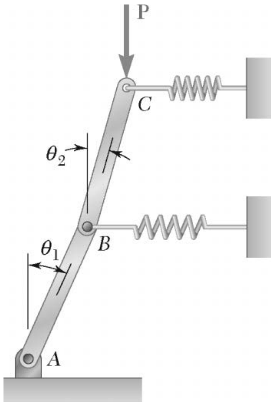
$$\frac{\partial^2 V}{\partial \theta_1^2} > 0: \quad -Pl + 2kl^2 > 0$$

or $P < \dfrac{1}{2}kl$

$$\frac{\partial^2 V}{\partial \theta_2^2} > 0: \quad -Pl + kl^2 > 0$$

or $P < kl$

Therefore, all conditions for stable equilibrium are satisfied when $0 \le P < 0.382kl$ ◀



PROBLEM 10.98*

Solve Problem 10.97 knowing that $l = 800$ mm and $k = 2.5$ kN/m.

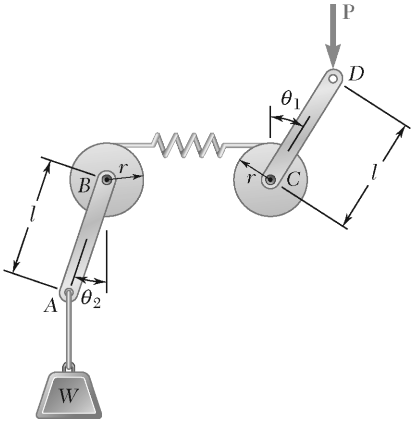
PROBLEM 10.97* Bars *AB* and *BC*, each of length *l* and of negligible weight, are attached to two springs, each of constant *k*. The springs are undeformed, and the system is in equilibrium when $\theta_1 = \theta_2 = 0$. Determine the range of values of *P* for which the equilibrium position is stable.

SOLUTION

From the analysis of Problem 10.97 with

$$l = 800 \text{ mm} \quad \text{and} \quad k = 2.5 \text{ kN/m}$$

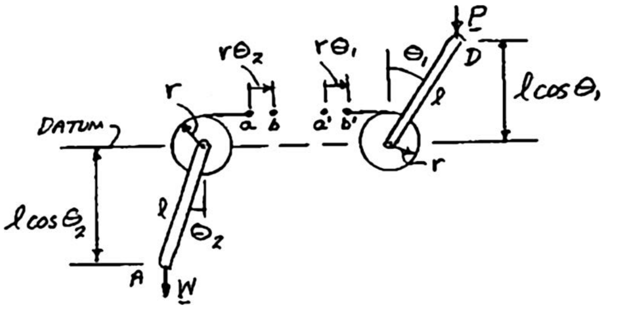
$$P < 0.382kl = 0.382(2500 \text{ N/m})(0.8 \text{ m}) = 764 \text{ N} \qquad P < 764 \text{ N} \blacktriangleleft$$



PROBLEM 10.99*

Two rods of negligible weight are attached to drums of radius r that are connected by a belt and spring of constant k. Knowing that the spring is undeformed when the rods are vertical, determine the range of values of P for which the equilibrium position $\theta_1 = \theta_2 = 0$ is stable.

SOLUTION



Left end of spring moves from a to b. Right end of spring moves from a' to b'. Elongation of spring

$$s = a'b' - ab = r\theta_1 - r\theta_2 = r(\theta_1 - \theta_2)$$

$$V = \frac{1}{2}ks^2 + pl\cos\theta_1 - wl\cos\theta_2$$

$$= \frac{1}{2}kr^2(\theta_1 - \theta_2)^2 + pl\cos\theta_1 - wl\cos\theta_2$$

$$\frac{\partial v}{\partial \theta_1} = kr^2(\theta_1 - \theta_2) - pl\sin\theta_1$$

$$\frac{\partial v}{\partial \theta_2} = -kr^2(\theta_1 - \theta_2) + wl\sin\theta_2$$

$$\frac{\partial^2 v}{\partial \theta_1^2} = kr^2 - pl\cos\theta_1$$

$$\frac{\partial^2 v}{\partial \theta_2^2} = kr^2 + wl\cos\theta_2$$

$$\frac{\partial^2 v}{\partial \theta_1 \partial \theta_2} = -kr^2$$

PROBLEM 10.99* (Continued)

For $\theta_1 = \theta_2 = 0$: $\dfrac{\partial^2 v}{\partial \theta_1^2} = kr^2 - pl$, $\dfrac{\partial^2 v}{\partial \theta_2^2} = +kr^2 + wl$, $\dfrac{r^2 v}{\partial \theta, \partial \theta_2} = -kr^2$

Conditions for stability (see Page 583)

$$\left(\frac{\partial^2 v}{\partial \theta_1 \partial \theta_2}\right)^2 - \frac{\partial^2 v}{\partial \theta_1^2} \cdot \frac{\partial^2 v}{\partial \theta_2^2} < 0$$

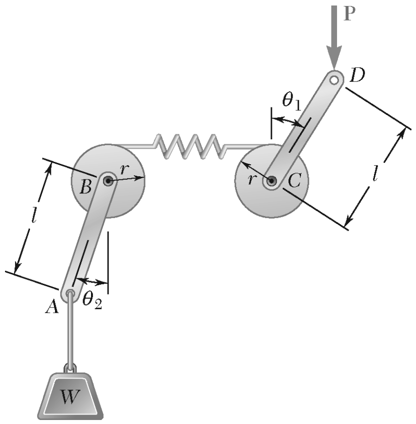
$$(kr^2)^2 - (kr^2 - pl)(kr^2 + wl) < 0$$

$$pl(kr^2 + wl) - kr^2 wl < 0$$

$$P < \frac{wkr^2}{kr^2 + wl}; \quad P < \frac{kr^2}{l}\left(\frac{W}{\frac{kr^2}{l} + w}\right) \qquad \triangleleft$$

$$\frac{\partial^2 v}{\partial \theta_1^2} >: \quad kr^2 - pl > 0; \quad P < \frac{kr^2}{l} \qquad \triangleleft$$

We choose: $P < \dfrac{kr^2}{l}\left(\dfrac{W}{\frac{kr^2}{l} + W}\right)$ ◀



PROBLEM 10.100*

Solve Problem 10.99 knowing that $k = 20$ lb/in., $r = 3$ in., $l = 6$ in., and (*a*) $W = 15$ lb, (*b*) $W = 60$ lb.

PROBLEM 10.99* Two rods of negligible weight are attached to drums of radius r that are connected by a belt and spring of constant k. Knowing that the spring is undeformed when the rods are vertical, determine the range of values of P for which the equilibrium position $\theta_1 = \theta_2 = 0$ is stable.

SOLUTION

$$k = 20 \text{ lb/in.}$$

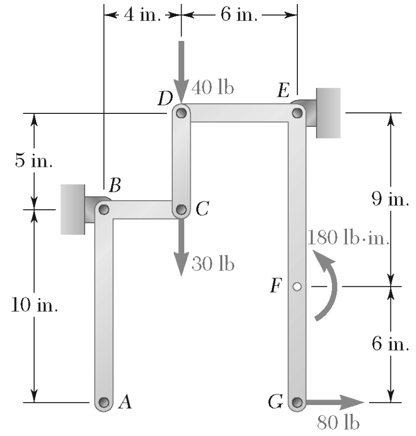
$$r = 3 \text{ in.}$$

$$l = 6 \text{ in.}$$

$$\frac{kr^2}{l} = \frac{(20 \text{ lb/in.})(3 \text{ in.})^2}{6 \text{ in.}} = 30 \text{ lb}$$

(*a*) $W = 15 \text{ lb}: \quad P < (30 \text{ lb})\dfrac{15 \text{ lb}}{(30 \text{ lb}) + (15 \text{ lb})}$ $\quad P < 10.00 \text{ lb}$ ◀

(*b*) $W = 60 \text{ lb}: \quad P < (30 \text{ lb})\dfrac{60 \text{ lb}}{(30 \text{ lb}) + (60 \text{ lb})}$ $\quad P < 20.0 \text{ lb}$ ◀



PROBLEM 10.101

Determine the horizontal force **P** that must be applied at *A* to maintain the equilibrium of the linkage.

SOLUTION

Assume $\delta\theta \circlearrowright$

$$\delta x_A = 10\delta\theta \longleftarrow$$

$$\delta y_C = 4\delta\theta \downarrow$$

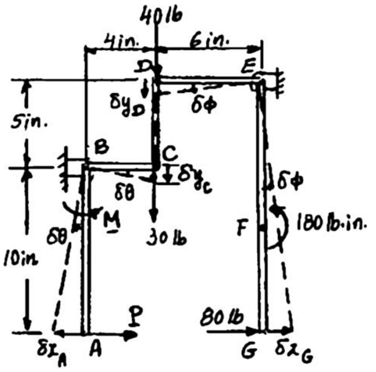
$$\delta y_D = \delta y_C = 4\delta\theta \downarrow$$

$$\delta\phi = \frac{\delta y_D}{6} = \frac{2}{3}\delta\theta \circlearrowleft$$

$$\delta x_G = 15\delta\phi$$

$$= 15\left(\frac{2}{3}\delta\theta\right)$$

$$= 10\delta\theta \longrightarrow$$



Virtual Work: We shall assume that a force **P** and a couple **M** are applied to member *ABC* as shown.

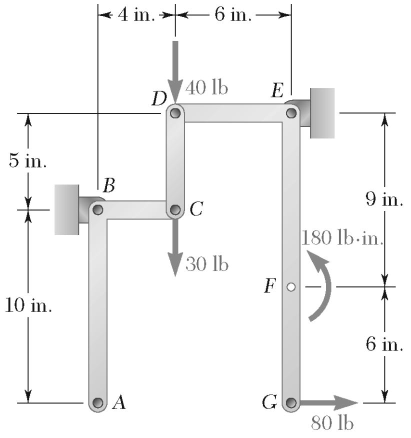
$$\delta U = -P\delta x_A - M\,\delta\theta + 30\delta y_C + 40\delta y_D + 180\delta\phi + 80\delta x_G = 0$$

$$-P(10\delta\theta) - M\,\delta\theta + 30(4\delta\theta) + 40(4\delta\theta) + 180\left(\frac{2}{3}\delta\theta\right) + 80(10\delta\theta) = 0$$

$$-10P - M + 120 + 160 + 120 + 800 = 0$$

$$(10\text{ in.})P + M = 1200\text{ lb}\cdot\text{in.} \qquad (1)$$

Making $M = 0$ in Eq. (1): $P = +120.0\text{ lb}$ $\quad$ $\mathbf{P} = 120.0\text{ lb} \longrightarrow$ ◀



PROBLEM 10.102

Determine the couple **M** that must be applied to member *ABC* to maintain the equilibrium of the linkage.

SOLUTION

Assume $\delta\theta \circlearrowright$

$$\delta x_A = 10\delta\theta \longleftarrow$$

$$\delta y_C = 4\delta\theta \downarrow$$

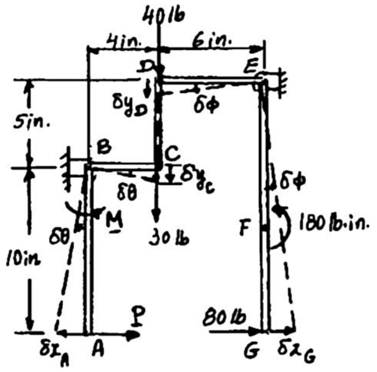
$$\delta y_D = \delta y_C = 4\delta\theta \downarrow$$

$$\delta\phi = \frac{\delta y_D}{6} = \frac{2}{3}\delta\theta \circlearrowright$$

$$\delta x_G = 15\delta\phi$$

$$= 15\left(\frac{2}{3}\delta\theta\right)$$

$$= 10\delta\theta \longrightarrow$$



Virtual Work: We shall assume that a force **P** and a couple **M** are applied to member *ABC* as shown.

$$\delta U = -P\delta x_A - M\delta\theta + 30\delta y_C + 40\delta y_D + 180\delta\phi + 80\delta x_G = 0$$

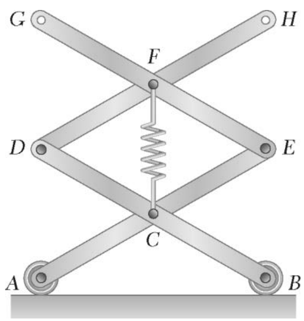
$$-P(10\delta\theta) - M\delta\theta + 30(4\delta\theta) + 40(4\delta\theta) + 180\left(\frac{2}{3}\delta\theta\right) + 80(10\delta\theta) = 0$$

$$-10P - M + 120 + 160 + 120 + 800 = 0$$

$$(10\text{ in.})P + M = 1200\text{ lb}\cdot\text{in.} \qquad (1)$$

Now from Eq. (1) for $P = 0$ $\mathbf{M} = 1200\text{ lb}\cdot\text{in.}$ ◀

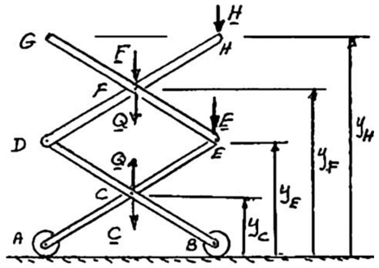
PROPRIETARY MATERIAL.



PROBLEM 10.103

A spring of constant 15 kN/m connects Points *C* and *F* of the linkage shown. Neglecting the weight of the spring and linkage, determine the force in the spring and the vertical motion of Point *G* when a vertical downward 120-N force is applied (*a*) at Point *C*, (*b*) at Points *C* and *H*.

SOLUTION



$$y_G = 4y_C$$

$$y_H = 4y_C \quad \delta y_H = 4\delta y_C$$

$$y_F = 3y_C \quad \delta y_F = 3\delta y_C$$

$$y_E = 2y_C \quad \delta y_E = 2\delta y_C$$

For spring:

$$\Delta = y_F - y_C$$

Q = Force in spring (assumed in tension)

$$Q = +k\Delta = k(y_F - y_C) = k(3y_C - y_C) = 2ky_C \qquad (1)$$

(*a*) $\mathbf{C} = 120\text{ N}, \quad \mathbf{E} = \mathbf{F} = \mathbf{H} = 0$

Virtual Work:

$$\delta U = 0: \quad -(120\text{ N})\delta y_C + Q\delta y_C - Q\delta y_F = 0$$

$$-120\delta y_C + Q\delta y_C - Q(3\delta y_C) = 0$$

$$Q = -60\text{ N} \qquad Q = 60.0\text{ N} \quad \text{C} \blacktriangleleft$$

Eq. (1): $Q = 2ky_C, \quad -60\text{ N} = 2(15\text{ kN/m})y_C, \quad y_C = -2\text{ mm}$

At Point *G*: $y_G = 4y_C = 4(-2\text{ mm}) = -8\text{ mm} \qquad \mathbf{y}_G = 8.00\text{ mm}\downarrow \blacktriangleleft$

(*b*) $\mathbf{C} = \mathbf{H} = 120\text{N}, \mathbf{E} = \mathbf{F} = 0$

Virtual Work:

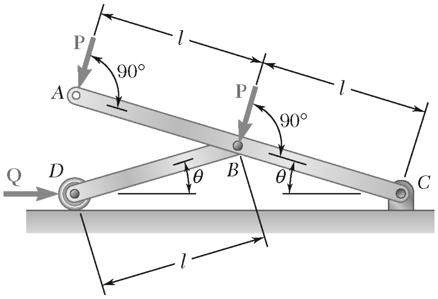
$$\delta U = 0: \quad -(120\text{ N})\delta y_C - (120\text{ N})y_H + Q\delta y_C - Q\delta y_F = 0$$

$$-120\delta y_C - 120(4\delta y_C) + Q\delta y_C - Q(3\delta y_C) = 0$$

$$Q = -300\text{ N} \qquad Q = 300\text{ N} \quad \text{C} \blacktriangleleft$$

Eq. (1): $Q = 2ky_C \quad -300\text{ N} = 2(15\text{ kN/m})y_C, \ y_C = -10\text{ mm}$

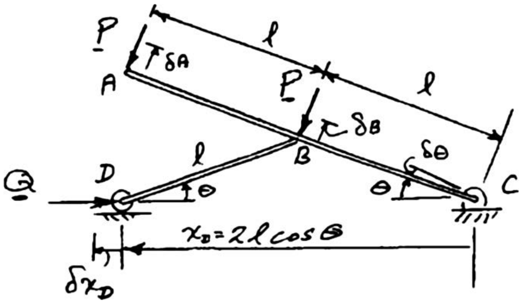
At Point *G*: $y_G = 4y_C = 4(-10\text{ mm}) = -40\text{ mm} \qquad \mathbf{y}_G = 40.0\text{ mm}\downarrow \blacktriangleleft$



PROBLEM 10.104

Derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the mechanism shown.

SOLUTION



We have $x_D = 2l\cos\theta$ so that $\delta x_D = -2l\sin\theta\delta\theta$

$\delta A = 2l\delta\theta$

$\delta B = l\delta\theta$

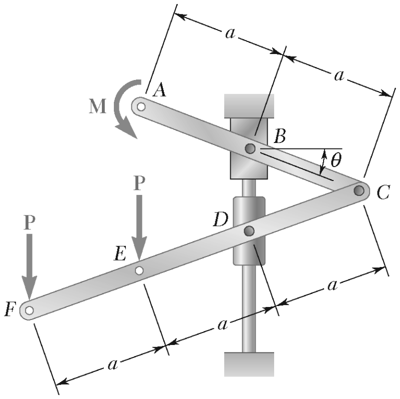
Virtual Work:

$$\delta U = 0: \quad -Q\delta x_D - P\delta A - P\delta B = 0$$

$$-Q(-2l\sin\theta\delta\theta) - P(2l\delta\theta) - P(l\delta\theta) = 0$$

$$2Ql\sin\theta - 3Pl = 0$$

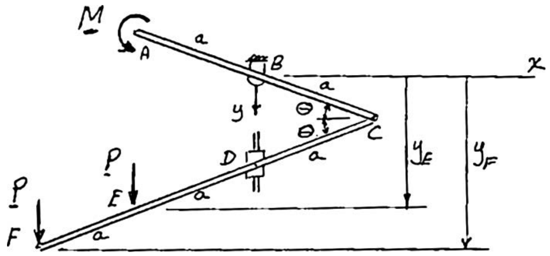
$$Q = \frac{3}{2}\frac{P}{\sin\theta} \quad ◀$$



PROBLEM 10.105

Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION



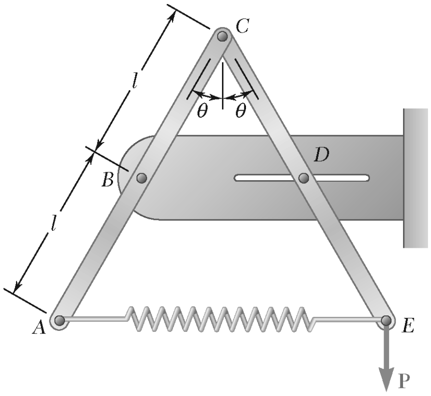
$$y_E = 3a\sin\theta \quad \delta y_E = 3a\cos\theta\delta\theta$$

$$y_F = 4a\sin\theta \quad \delta y_F = 4a\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -M\,\delta\theta + P\delta y_E + P\delta y_F = 0$$

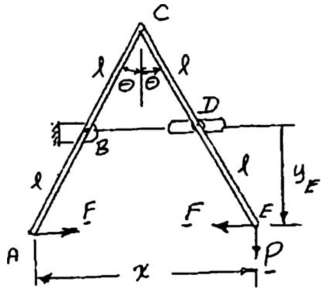
$$-M\,\delta\theta + P(3a\cos\theta\delta\theta) + P(4a\cos\theta\delta\theta) = 0 \qquad M = 7Pa\cos\theta \blacktriangleleft$$



PROBLEM 10.106

Two rods AC and CE are connected by a pin at C and by a spring AE. The constant of the spring is k, and the spring is unstretched when $\theta = 30°$. For the loading shown, derive an equation in P, θ, l, and k that must be satisfied when the system is in equilibrium.

SOLUTION



$$y_E = l\cos\theta$$

$$\delta y_E = -l\sin\theta\,\delta\theta$$

Spring:

Unstretched length = $2l$

$$x = 2(2l\sin\theta) = 4l\sin\theta$$

$$\delta x = 4l\cos\theta\delta\theta$$

$$F = k(x - 2l)$$

$$F = k(4l\sin\theta - 2l)$$

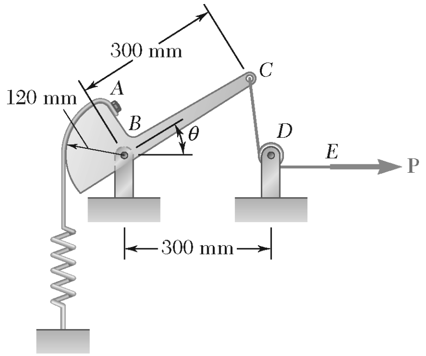
Virtual Work:

$$\delta U = 0: \quad P\delta y_E - F\delta x = 0$$

$$P(-l\sin\theta\,\delta\theta) - k(4l\sin\theta - 2l)(4l\cos\theta\,\delta\theta) = 0$$

$$-P\sin\theta - 8kl(2\sin\theta - 1)\cos\theta = 0$$

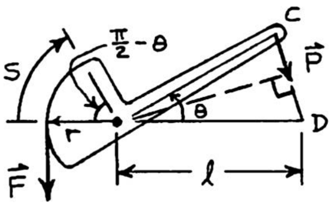
or $$\frac{P}{8kl} = (1 - 2\sin\theta)\frac{\cos\theta}{\sin\theta}$$ $$\frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta}$$ ◀



PROBLEM 10.107

A force **P** of magnitude 240 N is applied to end *E* of cable *CDE*, which passes under pulley *D* and is attached to the mechanism at *C*. Neglecting the weight of the mechanism and the radius of the pulley, determine the value of θ corresponding to equilibrium. The constant of the spring is $k = 4$ kN/m, and the spring is unstretched when $\theta = 90°$.

SOLUTION



$$s = r\left(\frac{\pi}{2} - \theta\right)$$

$$\delta s = -r\delta\theta$$

$$F = ks = kr\left(\frac{\pi}{2} - \theta\right)$$

$$CD = 2l\sin\frac{\theta}{2}$$

$$\delta(CD) = 2l\cos\frac{\theta}{2}\left(\frac{1}{2}\delta\theta\right)$$

$$= l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

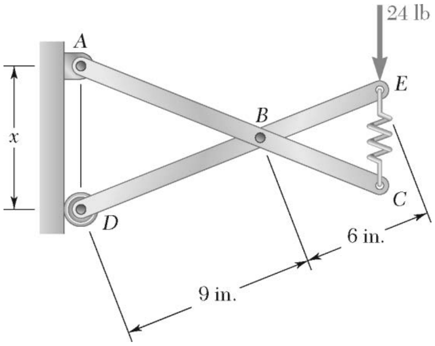
Since **F** tends to decrease *s* and **P** tends to decrease *CD*, we have

$$\delta U = -F\delta s - P\delta(CD) = 0$$

$$-kr\left(\frac{\pi}{2} - \theta\right)(-r\delta\theta) - P\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

$$\frac{\frac{\pi}{2} - \theta}{\cos\frac{\theta}{2}} = \frac{pl}{kr^2} = \frac{(240\text{ N})(0.3\text{ m})}{(4000\text{ N/m})(0.12\text{ m})^2} = 1.25$$

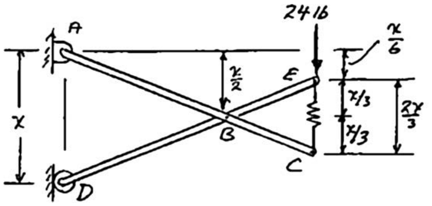
Solving by trial and error: $\theta = 0.33868$ rad $\theta = 19.40°$ ◀



PROBLEM 10.108

Two identical rods *ABC* and *DBE* are connected by a pin at *B* and by a spring *CE*. Knowing that the spring is 4 in. long when unstretched and that the constant of the spring is 8 lb/in., determine the distance *x* corresponding to equilibrium when a 24-lb load is applied at *E* as shown.

SOLUTION



Deformation of spring

$$s = EC - 4\text{ in.} = \frac{2x}{3} - 4$$

$$V = \frac{1}{2}ks^2 - (24\text{ lb})\frac{x}{6} = \frac{1}{2}(8\text{ lb/in.})\left(\frac{2x}{3} - 4\right)^2 - 4x$$

$$\frac{dV}{dx} = 8\left(\frac{2x}{3} - 4\right)\frac{2}{3} - 4$$

Equilibrium: $\frac{dV}{dx} = 0 \qquad \frac{16}{3}\left(\frac{2x}{3} - 4\right) - 4 = 0$

$$\frac{2x}{3} - 4 = 4\left(\frac{3}{16}\right)$$

$$\frac{2x}{3} = 4 + \frac{3}{4}$$

$x = 7.13$ in. ◀

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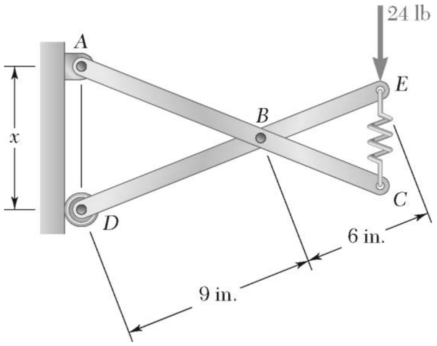
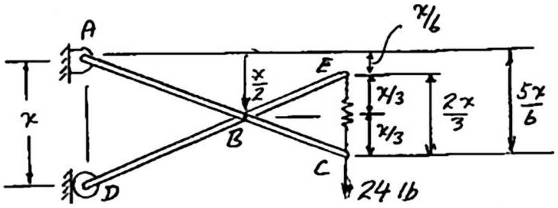


PROBLEM 10.109

Solve Problem 10.108 assuming that the 24-lb load is applied at *C* instead of *E*.

PROBLEM 10.108 Two identical rods *ABC* and *DBE* are connected by a pin at *B* and by a spring *CE*. Knowing that the spring is 4 in. long when unstretched and that the constant of the spring is 8 lb/in., determine the distance *x* corresponding to equilibrium when a 24-lb load is applied at *E* as shown.

SOLUTION



Deformation of spring

$$s = EC - 4\text{ in.} = \frac{2x}{3} - 4$$

$$V = \frac{1}{2}ks^2 - (24\text{ lb})\frac{5x}{6} = \frac{1}{2}(8\text{ lb/in.})\left(\frac{2x}{3} - 4\right)^2 - 20x$$

$$\frac{dV}{dx} = 8\left(\frac{2x}{3} - 4\right)\frac{2}{3} - 20$$

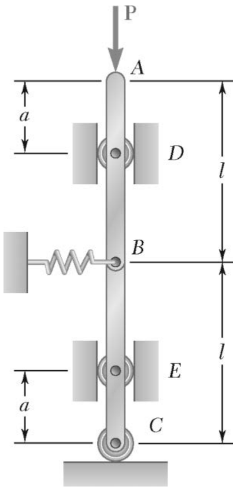
Equilibrium: $\frac{dV}{dx} = 0 \qquad \frac{16}{3}\left(\frac{2x}{3} - 4\right) - 20 = 0$

$$\frac{2x}{3} - 4 = 20\left(\frac{3}{16}\right)$$

$$\frac{2x}{3} = 4 + 3.75$$

$x = 11.63$ in. ◀

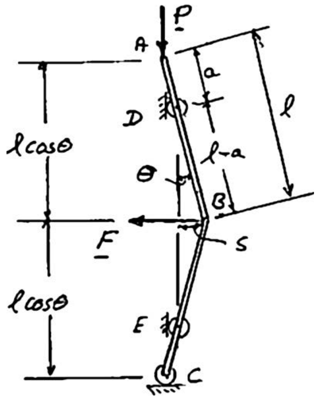
PROPRIETARY MATERIAL.



PROBLEM 10.110

Two bars *AB* and *BC* are attached to a single spring of constant *k* that is unstretched when the bars are vertical. Determine the range of values of *P* for which the equilibrium of the system is stable in the position shown.

SOLUTION



$$s = (l-a)\sin\theta$$

$$V = P(2l\cos\theta) + \frac{1}{2}ks^2$$

$$= 2Pl\cos\theta + \frac{1}{2}k(l-a)^2\sin^2\theta$$

$$\frac{dV}{d\theta} = -2Pl\sin\theta + k(l-a)^2\sin\theta\cos\theta$$

$$= -2Pl\sin\theta + \frac{1}{2}k(l-a)^2\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -2Pl\cos\theta + k(l-a)^2\cos 2\theta \qquad (1)$$

when $\theta = 0$: $\dfrac{d^2V}{d\theta^2} = -2Pl + k(l-a)^2$

<u>Stability</u>: $\dfrac{d^2V}{d\theta^2} > 0$: $-2Pl + k(l-a)^2 > 0$ $\qquad P < \dfrac{k(l-a)^2}{2l}$ ◀

To check whether equilibrium is unstable for $P = \frac{k(l-a)^2}{2l}$, we differentiate

Eq. (1) twice:

$$\frac{d^3V}{d\theta^3} = 2Pl\sin\theta - 2k(l-a)^2\sin 2\theta = 0, \quad \text{For} \quad \theta = 0$$

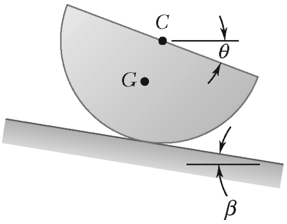
$$\frac{d^4V}{d\theta^4} = 2Pl\cos\theta - 4k(l-a)^2\cos 2\theta$$

PROBLEM 10.110 (Continued)

For $G = 0$ and $$P = \frac{k(l-a)^2}{2l}$$

$$\frac{d^4V}{d\theta^4} = 2Pl - 4k(l-a)^2$$
$$= k(l-a)^3 - 4k(l-a)^2 < 0$$

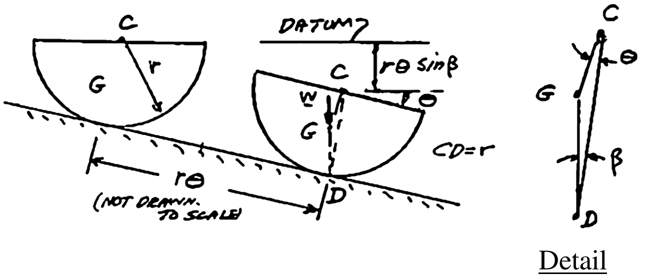
Thus equilibrium is unstable for $$P = \frac{k(l-a)^2}{2l}$$



PROBLEM 10.111

A homogeneous hemisphere of radius r is placed on an incline as shown. Assuming that friction is sufficient to prevent slipping between the hemisphere and the incline, determine the angle θ corresponding to equilibrium when $\beta = 10°$.

SOLUTION



Detail

$$CG = \frac{3}{8}r$$

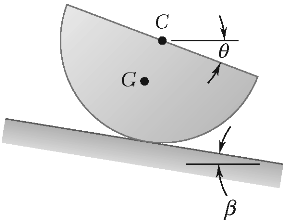
$$V = W(-r\theta \sin\beta - (CG)\cos\theta)$$

$$\frac{dV}{d\theta} = W\left(-r\sin\beta + \frac{3}{8}r\sin\theta\right)$$

Equilibrium: $$\frac{dV}{d\theta} = 0, \quad -\sin\beta + \frac{3}{8}\sin\theta = 0$$

$$\sin\beta = \frac{3}{8}\sin\theta \qquad (1)$$

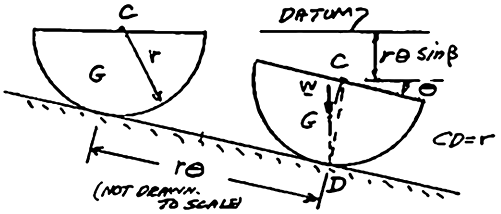
For $\beta = 10°$ $$\sin 10° = \frac{3}{8}\sin\theta$$ $$\sin\theta = 0.46306, \quad \theta = 27.6° \blacktriangleleft$$

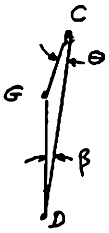


PROBLEM 10.112

A homogeneous hemisphere of radius r is placed on an incline as shown. Assuming that friction is sufficient to prevent slipping between the hemisphere and the incline, determine (*a*) the largest angle β for which a position of equilibrium exists, (*b*) the angle θ corresponding to equilibrium when the angle β is equal to half the value found in part *a*.

SOLUTION





Detail

$$CG = \frac{3}{8}r$$

$$V = W(-r\theta\sin\beta - (CG)\cos\theta)$$

$$\frac{dV}{d\theta} = W\left(-r\sin\beta + \frac{3}{8}r\sin\theta\right)$$

Equilibrium: $\quad \dfrac{dV}{d\theta} = 0, \quad -\sin\beta + \dfrac{3}{8}\sin\theta = 0$

$$\sin\beta = \frac{3}{8}\sin\theta \qquad (1)$$

(*a*) For $\beta_{\max}$, $\theta = 90°$

Eq. (1) $\quad \sin\beta_{\max} = \dfrac{3}{8}\sin 90°, \quad \sin\beta_{\max} = \dfrac{3}{8} = 22.02°$ $\quad \beta_{\max} = 22.0°$ ◀

(*b*) When $\beta = \frac{1}{2}\beta_{\max} = 11.01°$

Eq. (1) $\quad \sin 11.01° = \dfrac{3}{8}\sin\theta; \quad \sin\theta = 0.5093$ $\quad \theta = 30.6°$ ◀

Note: We can also use ΔCGD and law of sines to derive Eq. (1).

$$\frac{\sin\beta}{CG} = \frac{\sin\theta}{CD}; \quad \sin\beta = \frac{CG}{CD}\sin\theta; \quad \sin\beta = \frac{3}{8}\sin\theta$$