VECTOR MECHANICS for ENGINEERS

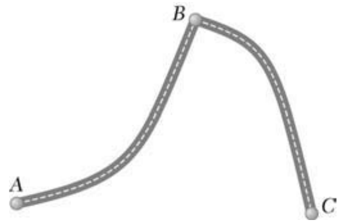
TENTH EDITION

SOLUTION MANUAL

DYNAMICS

Beer | Johnston | Cornwell

CHAPTER 11



PROBLEM 11.CQ1

A bus travels the 100 miles between *A* and *B* at 50 mi/h and then another 100 miles between *B* and *C* at 70 mi/h. The average speed of the bus for the entire 200-mile trip is:

(*a*) more than 60 mi/h

(*b*) equal to 60 mi/h

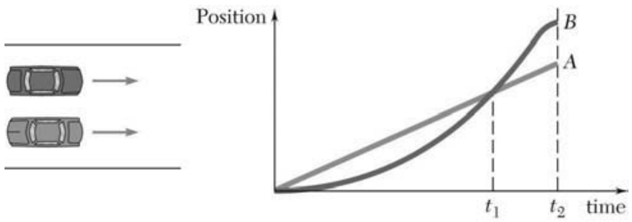
(*c*) less than 60 mi/h

SOLUTION

The time required for the bus to travel from *A* to *B* is 2 h and from *B* to *C* is 100/70 = 1.43 h, so the total time is 3.43 h and the average speed is 200/3.43 = 58 mph.

Answer: (*c*) ◀

PROBLEM 11CQ2



Two cars A and B race each other down a straight road. The position of each car as a function of time is shown. Which of the following statements are true (more than one answer can be correct)?

(*a*) At time t_2 both cars have traveled the same distance

(*b*) At time t_1 both cars have the same speed

(*c*) Both cars have the same speed at some time $t < t_1$

(*d*) Both cars have the same acceleration at some time $t < t_1$

(*e*) Both cars have the same acceleration at some time $t_1 < t < t_2$

SOLUTION

The speed is the slope of the curve, so answer c) is true.

The acceleration is the second derivative of the position. Since A's position increases linearly the second derivative will always be zero. The second derivative of curve B is zero at the pont of inflection which occurs between t_1 and t_2.

Answers: (*c*) and (*e*) ◀

PROBLEM 11.1

The motion of a particle is defined by the relation $x = t^4 - 10t^2 + 8t + 12$, where x and t are expressed in inches and seconds, respectively. Determine the position, the velocity, and the acceleration of the particle when $t = 1$ s.

SOLUTION

$$x = t^4 - 10t^2 + 8t + 12$$

$$v = \frac{dx}{dt} = 4t^3 - 20t + 8$$

$$a = \frac{dv}{dt} = 12t^2 - 20$$

At $t = 1\text{ s}$,

$$x = 1 - 10 + 8 + 12 = 11 \qquad x = 11.00\text{ in.} \blacktriangleleft$$

$$v = 4 - 20 + 8 = -8 \qquad v = -8.00\text{ in./s} \blacktriangleleft$$

$$a = 12 - 20 = -8 \qquad a = -8.00\text{ in./s}^2 \blacktriangleleft$$

PROBLEM 11.2

The motion of a particle is defined by the relation $x = 2t^3 - 9t^2 + 12t + 10$, where x and t are expressed in feet and seconds, respectively. Determine the time, the position, and the acceleration of the particle when $v = 0$.

SOLUTION

$$x = 2t^3 - 9t^2 + 12t + 10$$

Differentiating,

$$v = \frac{dx}{dt} = 6t^2 - 18t + 12 = 6(t^2 - 3t + 2)$$

$$= 6(t-2)(t-1)$$

$$a = \frac{dv}{dt} = 12t - 18$$

So $v = 0$ at $t = 1\text{ s}$ and $t = 2\text{ s}$.

At $t = 1\text{ s}$, $x_1 = 2 - 9 + 12 + 10 = 15$ $t = 1.000\text{ s}$ ◀

$a_1 = 12 - 18 = -6$ $x_1 = 15.00\text{ ft}$ ◀

$a_1 = -6.00\text{ ft/s}^2$ ◀

At $t = 2\text{ s}$,

$x_2 = 2(2)^3 - 9(2)^2 + 12(2) + 10 = 14$ $t = 2.00\text{ s}$ ◀

$x_2 = 14.00\text{ ft}$ ◀

$a_2 = (12)(2) - 18 = 6$ $a_2 = 6.00\text{ ft/s}^2$ ◀

PROBLEM 11.3

The vertical motion of mass A is defined by the relation $x = 10\sin 2t + 15\cos 2t + 100$, where x and t are expressed in mm and seconds, respectively. Determine (a) the position, velocity and acceleration of A when $t = 1$ s, (b) the maximum velocity and acceleration of A.

SOLUTION

$$x = 10\sin 2t + 15\cos 2t + 100$$

$$v = \frac{dx}{dt} = 20\cos 2t - 30\sin 2t$$

$$a = \frac{dv}{dt} = -40\sin 2t - 60\cos 2t$$

For trigonometric functions set calculator to radians:

(a) At $t = 1\text{ s}$. $\quad x_1 = 10\sin 2 + 15\cos 2 + 100 = 102.9$ $\qquad x_1 = 102.9$ mm ◀

$v_1 = 20\cos 2 - 30\sin 2 = -35.6$ $\qquad v_1 = -35.6$ mm/s ◀

$a_1 = -40\sin 2 - 60\cos 2 = -11.40$ $\qquad a_1 = -11.40$ mm/s^2 ◀

(b) Maximum velocity occurs when $a = 0$.

$$-40\sin 2t - 60\cos 2t = 0$$

$$\tan 2t = -\frac{60}{40} = -1.5$$

$$2t = \tan^{-1}(-1.5) = -0.9828 \text{ and } -0.9828 + \pi$$

Reject the negative value. $2t = 2.1588$

$$t = 1.0794 \text{ s}$$

$$t = 1.0794 \text{ s for } v_{max}$$

so $\quad v_{max} = 20\cos(2.1588) - 30\sin(2.1588)$

$= -36.056$ $\qquad v_{max} = -36.1$ mm/s ◀

Note that we could have also used

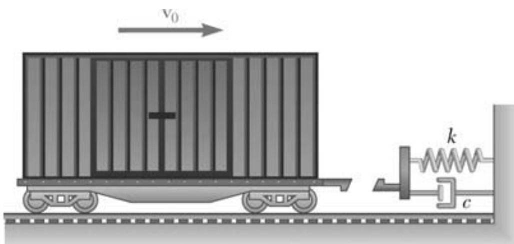
$$v_{max} = \sqrt{20^2 + 30^2} = 36.056$$

by combining the sine and cosine terms.

For a_{max} we can take the derivative and set equal to zero or just combine the sine and cosine terms.

$a_{max} = \sqrt{40^2 + 60^2} = 72.1$ mm/s^2 $\qquad a_{max} = 72.1$ mm/s^2 ◀

PROBLEM 11.4



A loaded railroad car is rolling at a constant velocity when it couples with a spring and dashpot bumper system. After the coupling, the motion of the car is defined by the relation $x = 60e^{-4.8t}\sin 16t$ where x and t are expressed in mm and seconds, respectively. Determine the position, the velocity and the acceleration of the railroad car when (*a*) $t = 0$, (*b*) $t = 0.3$ s.

SOLUTION

$$x = 60e^{-4.8t}\sin 16t$$

$$v = \frac{dx}{dt} = 60(-4.8)e^{-4.8t}\sin 16t + 60(16)e^{-4.8t}\cos 16t$$

$$v = -288e^{-4.8t}\sin 16t + 960e^{-4.8t}\cos 16t$$

$$a = \frac{dv}{dt} = 1382.4e^{-4.8t}\sin 16t - 4608e^{-4.8t}\cos 16t$$
$$\quad - 4608e^{-4.8t}\cos 16t - 15360e^{-4.8t}\sin 16t$$

$$a = -13977.6e^{-4.8t}\sin 16t - 9216e^{-4.8}\cos 16t$$

(*a*) At $t = 0$,

$x_0 = 0$ — $x_0 = 0$ mm ◀

$v_0 = 960$ mm/s — $v_0 = 960$ mm/s → ◀

$a_0 = -9216$ mm/s^2 — $a_0 = 9220$ mm/s^2 ← ◀

(*b*) At $t = 0.3$ s,

$$e^{-4.8t} = e^{-1.44} = 0.23692$$
$$\sin 16t = \sin 4.8 = -0.99616$$
$$\cos 16t = \cos 4.8 = 0.08750$$

$x_{0.3} = (60)(0.23692)(-0.99616) = -14.16$ — $x_{0.3} = 14.16$ mm ← ◀

$v_{0.3} = -(288)(0.23692)(-0.99616) + (960)(0.23692)(0.08750) = 87.9$ — $v_{0.3} = 87.9$ mm/s → ◀

$a_{0.3} = -(13977.6)(0.23692)(-0.99616) - (9216)(0.23692)(0.08750) = 3108$ — $a_{0.3} = 3110$ mm/s^2 ◀

or 3.11 m/s^2 → ◀

PROBLEM 11.5

The motion of a particle is defined by the relation $x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$, where x and t are expressed in meters and seconds, respectively. Determine the time, the position, and the velocity when $a = 0$.

SOLUTION

We have $$x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$$

Then $$v = \frac{dx}{dt} = 24t^3 - 6t^2 - 24t + 3$$

and $$a = \frac{dv}{dt} = 72t^2 - 12t - 24$$

When $a = 0$: $$72t^2 - 12t - 24 = 12(6t^2 - t - 2) = 0$$

or $$(3t - 2)(2t + 1) = 0$$

or $$t = \frac{2}{3} \text{ s} \quad \text{and} \quad t = -\frac{1}{2} \text{ s} \quad \text{(Reject)}$$

$t = 0.667$ s ◀

At $t = \frac{2}{3}$ s: $$x_{2/3} = 6\left(\frac{2}{3}\right)^4 - 2\left(\frac{2}{3}\right)^3 - 12\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right) + 3$$

or $x_{2/3} = 0.259$ m ◀

$$v_{2/3} = 24\left(\frac{2}{3}\right)^3 - 6\left(\frac{2}{3}\right)^2 - 24\left(\frac{2}{3}\right) + 3$$

or $v_{2/3} = -8.56$ m/s ◀

PROBLEM 11.6

The motion of a particle is defined by the relation $x = t^3 - 9t^2 + 24t - 8$, where x and t are expressed in inches and seconds, respectively. Determine (a) when the velocity is zero, (b) the position and the total distance traveled when the acceleration is zero.

SOLUTION

We have $\quad x = t^3 - 9t^2 + 24t - 8$

Then $\quad v = \frac{dx}{dt} = 3t^2 - 18t + 24$

and $\quad a = \frac{dv}{dt} = 6t - 18$

(a) When $v = 0$: $\quad 3t^2 - 18t + 24 = 3(t^2 - 6t + 8) = 0$

$$(t-2)(t-4) = 0$$

$t = 2.00$ s and $t = 4.00$ s ◀

(b) When $a = 0$: $\quad 6t - 18 = 0 \quad$ or $\quad t = 3$ s

At $t = 3$ s: $\quad x_3 = (3)^3 - 9(3)^2 + 24(3) - 8 \quad$ or $\quad x_3 = 10.00$ in. ◀

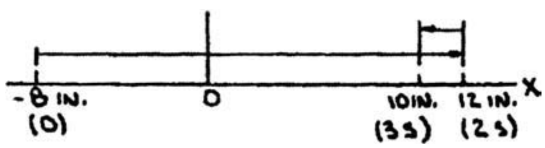
First observe that $0 \le t < 2$ s: $\quad v > 0$

$2 \text{ s} < t \le 3$ s: $\quad v < 0$

Now

At $t = 0$: $\quad x_0 = -8$ in.

At $t = 2$ s: $\quad x_2 = (2)^3 - 9(2)^2 + 24(2) - 8 = 12$ in.



Then $\quad x_2 - x_0 = 12 - (-8) = 20$ in.

$$|x_3 - x_2| = |10 - 12| = 2 \text{ in.}$$

Total distance traveled $= (20 + 2)$ in. $\quad$ Total distance $= 22.0$ in. ◀

PROBLEM 11.7

The motion of a particle is defined by the relation $x = 2t^3 - 15t^2 + 24t + 4$, where x is expressed in meters and t in seconds. Determine (a) when the velocity is zero, (b) the position and the total distance traveled when the acceleration is zero.

SOLUTION

$$x = 2t^3 - 15t^2 + 24t + 4$$

$$v = \frac{dx}{dt} = 6t^2 - 30t + 24$$

$$a = \frac{dv}{dt} = 12t - 30$$

(a) $\underline{v = 0 \text{ when}}$ $\quad 6t^2 - 30t + 24 = 0$

$6(t-1)(t-4) = 0$ $\quad$ $t = 1.000 \text{ s}$ and $t = 4.00 \text{ s}$ ◀

(b) $\underline{a = 0 \text{ when}}$ $\quad 12t - 30 = 0 \quad t = 2.5 \text{ s}$

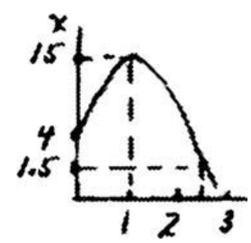
For $t = 2.5$ s: $\quad x_{2.5} = 2(2.5)^3 - 15(2.5)^2 + 24(2.5) + 4$

$x_{2.5} = +1.500 \text{ m}$ ◀

To find total distance traveled, we note that

$\underline{v = 0 \text{ when } t = 1 \text{ s}}$: $\quad x_1 = 2(1)^3 - 15(1)^2 + 24(1) + 4$

$x_1 = +15 \text{ m}$



For $t = 0$, $\quad x_0 = +4 \text{ m}$

Distance traveled

From $t = 0$ to $t = 1$ s: $\quad x_1 - x_0 = 15 - 4 = 11 \text{ m} \longrightarrow$

From $t = 1$ s to $t = 2.5$ s: $\quad x_{2.5} - x_1 = 1.5 - 15 = 13.5 \text{ m} \longleftarrow$

Total distance traveled $= 11 \text{ m} + 13.5 \text{ m}$ $\quad$ Total distance $= 24.5 \text{ m}$ ◀

PROBLEM 11.8

The motion of a particle is defined by the relation $x = t^3 - 6t^2 - 36t - 40$, where x and t are expressed in feet and seconds, respectively. Determine (a) when the velocity is zero, (b) the velocity, the acceleration, and the total distance traveled when $x = 0$.

SOLUTION

We have $x = t^3 - 6t^2 - 36t - 40$

Then $v = \dfrac{dx}{dt} = 3t^2 - 12t - 36$

and $a = \dfrac{dv}{dt} = 6t - 12$

(a) When $v = 0$: $3t^2 - 12t - 36 = 3(t^2 - 4t - 12) = 0$

or $(t+2)(t-6) = 0$

or $t = -2$ s (Reject) and $t = 6$ s $\quad t = 6.00$ s ◀

(b) When $x = 0$: $t^3 - 6t^2 - 36t - 40 = 0$

Factoring $(t-10)(t+2)(t+2) = 0$ or $t = 10$ s

Now observe that $0 \le t < 6$ s: $v < 0$

$6\text{ s} < t \le 10$ s: $v > 0$

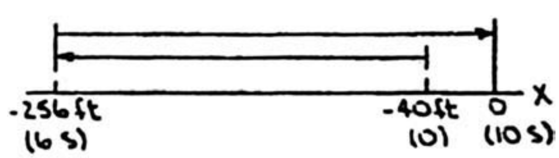
and at $t = 0$: $x_0 = -40$ ft

$t = 6$ s: $x_6 = (6)^3 - 6(6)^2 - 36(6) - 40$

$= -256$ ft

$t = 10$ s: $v_{10} = 3(10)^2 - 12(10) - 36$ $\quad$ or $v_{10} = 144.0$ ft/s ◀

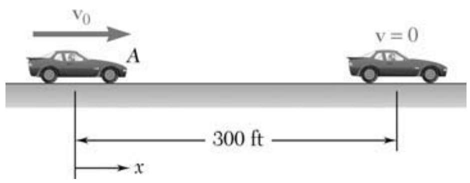
$a_{10} = 6(10) - 12$ $\quad$ or $a_{10} = 48.0\text{ ft/s}^2$ ◀



Then $|x_6 - x_0| = |-256 - (-40)| = 216$ ft

$x_{10} - x_6 = 0 - (-256) = 256$ ft

Total distance traveled $= (216 + 256)$ ft $\quad$ Total distance $= 472$ ft ◀



PROBLEM 11.9

The brakes of a car are applied, causing it to slow down at a rate of 10 m/s². Knowing that the car stops in 100 m, determine (*a*) how fast the car was traveling immediately before the brakes were applied, (*b*) the time required for the car to stop.

SOLUTION

$$a = -10 \text{ ft/s}^2$$

(*a*) Velocity at $x = 0$.

$$v\frac{dv}{dx} = a = -10$$

$$\int_{v_0}^{0} v\,dv = -\int_0^{x_f} (-10)\,dx$$

$$0 - \frac{v_0^2}{2} = -10x_f = -(10)(300)$$

$$v_0^2 = 6000$$

$v_0 = 77.5 \text{ ft/s}^2$ ◀

(*b*) Time to stop.

$$\frac{dv}{dx} = a = -10$$

$$\int_{v_0}^{0} dv = -\int_0^{t_f} -10\,dt$$

$$0 - v_0 = -10t_f$$

$$t_f = \frac{v_0}{10} = \frac{77.5}{10}$$

$t_f = 7.75 \text{ s}$ ◀

PROBLEM 11.10

The acceleration of a particle is directly proportional to the time t. At $t = 0$, the velocity of the particle is $v = 16$ in./s. Knowing that $v = 15$ in./s and that $x = 20$ in. when $t = 1$ s, determine the velocity, the position, and the total distance traveled when $t = 7$ s.

SOLUTION

We have $$a = kt \quad k = \text{constant}$$

Now $$\frac{dv}{dt} = a = kt$$

At $t = 0, v = 16$ in./s: $$\int_{16}^{v} dv = \int_{0}^{t} kt\, dt$$

or $$v - 16 = \frac{1}{2}kt^2$$

or $$v = 16 + \frac{1}{2}kt^2 \text{(in./s)}$$

At $t = 1$ s, $v = 15$ in./s: $$15 \text{ in./s} = 16 \text{ in./s} + \frac{1}{2}k(1 \text{ s})^2$$

or $$k = -2 \text{ in./s}^3 \quad \text{and} \quad v = 16 - t^2$$

Also $$\frac{dx}{dt} = v = 16 - t^2$$

At $t = 1$ s, $x = 20$ in.: $$\int_{20}^{x} dx = \int_{1}^{t} (16 - t^2)dt$$

or $$x - 20 = \left[16t - \frac{1}{3}t^3\right]_1^t$$

or $$x = -\frac{1}{3}t^3 + 16t + \frac{13}{3} \text{(in.)}$$

Then

At $t = 7$ s: $$v_7 = 16 - (7)^2$$ or $v_7 = -33.0$ in./s ◀

$$x_7 = -\frac{1}{3}(7)^3 + 16(7) + \frac{13}{3}$$ or $x_7 = 2.00$ in. ◀

When $v = 0$: $$16 - t^2 = 0 \quad \text{or} \quad t = 4 \text{ s}$$

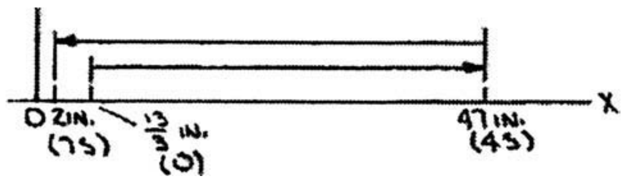
At $t = 0$: $x_0 = \dfrac{13}{3}$

$t = 4$ s: $x_4 = -\dfrac{1}{3}(4)^3 + 16(4) + \dfrac{13}{3} = 47$ in.

Now observe that

$0 \le t < 4$ s: $v > 0$

4 s $< t \le 7$ s: $v < 0$



Then

$$x_4 - x_0 = 47 - \frac{13}{3} = 42.67 \text{ in.}$$

$$|x_7 - x_4| = |2 - 47| = 45 \text{ in.}$$

Total distance traveled $= (42.67 + 45)$ in. Total distance $= 87.7$ in. ◀

PROBLEM 11.11

The acceleration of a particle is directly proportional to the square of the time t. When $t = 0$, the particle is at $x = 24$ m. Knowing that at $t = 6$ s, $x = 96$ m and $v = 18$ m/s, express x and v in terms of t.

SOLUTION

We have $$a = kt^2 \quad k = \text{constant}$$

Now $$\frac{dv}{dt} = a = kt^2$$

At $t = 6$ s, $v = 18$ m/s: $$\int_{18}^{v} dv = \int_{6}^{t} kt^2 dt$$

or $$v - 18 = \frac{1}{3}k(t^3 - 216)$$

or $$v = 18 + \frac{1}{3}k(t^3 - 216)(\text{m/s})$$

Also $$\frac{dx}{dt} = v = 18 + \frac{1}{3}k(t^3 - 216)$$

At $t = 0$, $x = 24$ m: $$\int_{24}^{x} dx = \int_{0}^{t}\left[18 + \frac{1}{3}k(t^3 - 216)\right]dt$$

or $$x - 24 = 18t + \frac{1}{3}k\left(\frac{1}{4}t^4 - 216t\right)$$

Now

At $t = 6$ s, $x = 96$ m: $$96 - 24 = 18(6) + \frac{1}{3}k\left[\frac{1}{4}(6)^4 - 216(6)\right]$$

or $$k = \frac{1}{9}\ \text{m/s}^4$$

Then $$x - 24 = 18t + \frac{1}{3}\left(\frac{1}{9}\right)\left(\frac{1}{4}t^4 - 216t\right)$$

or $$x(t) = \frac{1}{108}t^4 + 10t + 24 \quad ◀$$

and $$v = 18 + \frac{1}{3}\left(\frac{1}{9}\right)(t^3 - 216)$$

or $$v(t) = \frac{1}{27}t^3 + 10 \quad ◀$$

PROBLEM 11.12

The acceleration of a particle is defined by the relation $a = kt^2$. (*a*) Knowing that $v = -8$ m/s when $t = 0$ and that $v = +8$ m/s when $t = 2$ s, determine the constant *k*. (*b*) Write the equations of motion, knowing also that $x = 0$ when $t = 2$ s.

SOLUTION

$$a = kt^2 \qquad (1)$$

$$\frac{dv}{dt} = a = kt^2$$

$t = 0,\ v = -8$ m/s and $t = 2$ s, $v = +8$ ft/s

(*a*)

$$\int_{-8}^{8} dv = \int_0^2 kt^2\, dt$$

$$8 - (-8) = \frac{1}{3}k(2)^3$$

$k = 6.00$ m/s^4 ◀

(*b*) Substituting $k = 6$ m/s^4 into (1)

$$\frac{dv}{dt} = a = 6t^2$$

$a = 6t^2$ ◀

$t = 0,\ v = -8$ m/s:

$$\int_{-8}^{v} dv = \int_0^t 6t^2\, dt$$

$$v - (-8) = \frac{1}{3}6(t)^3$$

$v = 2t^3 - 8$ ◀

$$\frac{dx}{dt} = v = 2t^3 - 8$$

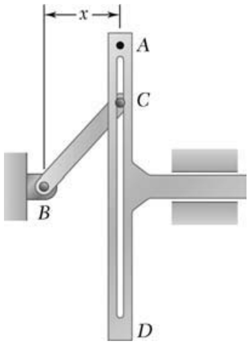
$t = 2$ s, $x = 0$:

$$\int_0^x dx = \int_2^t (2t^3 - 8)dt; \quad x = \left|\frac{1}{2}t^4 - 8t\right|_2^t$$

$$x = \left[\frac{1}{2}t^4 - 8t\right] - \left[\frac{1}{2}(2)^4 - 8(2)\right]$$

$$x = \frac{1}{2}t^4 - 8t - 8 + 16$$

$x = \frac{1}{2}t^4 - 8t + 8$ ◀



PROBLEM 11.13

The acceleration of Point A is defined by the relation $a = -1.8 \sin kt$, where a and t are expressed in m/s^2 and seconds, respectively, and $k = 3$ rad/s. Knowing that $x = 0$ and $v = 0.6$ m/s when $t = 0$, determine the velocity and position of Point A when $t = 0.5$ s.

SOLUTION

Given: $a = -1.8 \sin kt \text{ m/s}^2, \quad v_0 = 0.6 \text{ m/s}, \quad x_0 = 0, \quad k = 3 \text{ rad/s}$

$$v - v_0 = \int_0^t a\,dt = -1.8 \int_0^t \sin kt\,dt = \frac{1.8}{k} \cos kt \Big|_0^t$$

$$v - 0.6 = \frac{1.8}{3}(\cos kt - 1) = 0.6 \cos kt - 0.6$$

Velocity: $v = 0.6 \cos kt \text{ m/s}$

$$x - x_0 = \int_0^t v\,dt = 0.6 \int_0^t \cos kt\,dt = \frac{0.6}{k} \sin kt \Big|_0^t$$

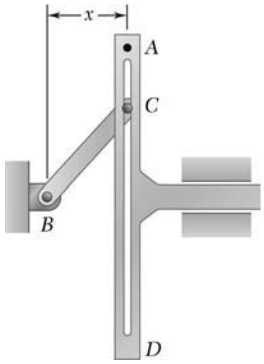
$$x - 0 = \frac{0.6}{3}(\sin kt - 0) = 0.2 \sin kt$$

Position: $x = 0.2 \sin kt \text{ m}$

When $t = 0.5$ s, $kt = (3)(0.5) = 1.5 \text{ rad}$

$v = 0.6 \cos 1.5 = 0.0424 \text{ m/s}$ $\qquad v = 42.4 \text{ mm/s}$ ◀

$x = 0.2 \sin 1.5 = 0.1995 \text{ m}$ $\qquad x = 199.5 \text{ mm}$ ◀



PROBLEM 11.14

The acceleration of Point A is defined by the relation $a = -1.08\sin kt - 1.44\cos kt$, where a and t are expressed in m/s^2 and seconds, respectively, and $k = 3$ rad/s. Knowing that $x = 0.16$ m and $v = 0.36$ m/s when $t = 0$, determine the velocity and position of Point A when $t = 0.5$ s.

SOLUTION

Given: $a = -1.08\sin kt - 1.44\cos kt \text{ m/s}^2, \qquad k = 3 \text{ rad/s}$

$$x_0 = 0.16 \text{ m}, \qquad v_0 = 0.36 \text{ m/s}$$

$$v - v_0 = \int_0^t a\,dt = -1.08\int_0^t \sin kt\,dt - 1.44\int_0^t \cos kt\,dt$$

$$v - 0.36 = \frac{1.08}{k}\cos kt\Big|_0^t - \frac{1.44}{k}\sin kt\Big|_0^t$$

$$= \frac{1.08}{3}(\cos kt - 1) - \frac{1.44}{3}(\sin kt - 0)$$

$$= 0.36\cos kt - 0.36 - 0.48\sin kt$$

Velocity: $v = 0.36\cos kt - 0.48\sin kt \text{ m/s}$

$$x - x_0 = \int_0^t v\,dt = 0.36\int_0^t \cos kt\,dt - 0.48\int_0^t \sin kt\,dt$$

$$x - 0.16 = \frac{0.36}{k}\sin kt\Big|_0^t + \frac{0.48}{k}\cos kt\Big|_0^t$$

$$= \frac{0.36}{3}(\sin kt - 0) + \frac{0.48}{3}(\cos kt - 1)$$

$$= 0.12\sin kt + 0.16\cos kt - 0.16$$

Position: $x = 0.12\sin kt + 0.16\cos kt \text{ m}$

When $t = 0.5$ s, $kt = (3)(0.5) = 1.5$ rad

$v = 0.36\cos 1.5 - 0.48\sin 1.5 = -0.453$ m/s $\qquad v = -453$ mm/s ◀

$x = 0.12\sin 1.5 + 0.16\cos 1.5 = 0.1310$ m $\qquad x = 131.0$ mm ◀

PROBLEM 11.15

A piece of electronic equipment that is surrounded by packing material is dropped so that it hits the ground with a speed of 4 m/s. After contact the equipment experiences an acceleration of $a = -kx$, where k is a constant and x is the compression of the packing material. If the packing material experiences a maximum compression of 20 mm, determine the maximum acceleration of the equipment.

SOLUTION

$$a = \frac{v\,dv}{dx} = -kx$$

Separate and integrate.

$$\int_{v_0}^{v_f} v\,dv = -\int_0^{x_f} kx\,dx$$

$$\frac{1}{2}v_f^2 - \frac{1}{2}v_0^2 = -\frac{1}{2}kx^2\Big|_0^{x_f} = -\frac{1}{2}kx_f^2$$

Use $v_0 = 4$ m/s, $x_f = 0.02$ m, and $v_f = 0$. Solve for k.

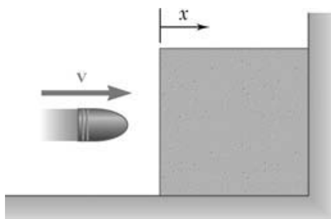
$$0 - \frac{1}{2}(4)^2 = -\frac{1}{2}k(0.02)^2 \qquad k = 40{,}000 \text{ s}^{-2}$$

Maximum acceleration.

$$a_{\max} = -kx_{\max}: \quad (-40{,}000)(0.02) = -800 \text{ m/s}^2$$

$a = 800 \text{ m/s}^2 \uparrow$ ◀

PROBLEM 11.16



A projectile enters a resisting medium at $x = 0$ with an initial velocity $v_0 = 900$ ft/s and travels 4 in. before coming to rest. Assuming that the velocity of the projectile is defined by the relation $v = v_0 - kx$, where v is expressed in ft/s and x is in feet, determine (*a*) the initial acceleration of the projectile, (*b*) the time required for the projectile to penetrate 3.9 in. into the resisting medium.

SOLUTION

First note

When $x = \frac{4}{12}$ ft, $v = 0$: $$0 = (900 \text{ ft/s}) - k\left(\frac{4}{12}\text{ ft}\right)$$

or $$k = 2700\,\frac{1}{\text{s}}$$

(*a*) We have $$v = v_0 - kx$$

Then $$a = \frac{dv}{dt} = \frac{d}{dt}(v_0 - kx) = -kv$$

or $$a = -k(v_0 - kx)$$

At $t = 0$: $$a = 2700\,\frac{1}{\text{s}}(900 \text{ ft/s} - 0)$$

or $$a_0 = -2.43\times 10^6 \text{ ft/s}^2 \blacktriangleleft$$

(*b*) We have $$\frac{dx}{dt} = v = v_0 - kx$$

At $t = 0, x = 0$: $$\int_0^x \frac{dx}{v_0 - kx} = \int_0^t dt$$

or $$-\frac{1}{k}[\ln(v_0 - kx)]_0^x = t$$

or $$t = \frac{1}{k}\ln\left(\frac{v_0}{v_0 - kx}\right) = \frac{1}{k}\ln\left(\frac{1}{1 - \frac{k}{v_0}x}\right)$$

When $x = 3.9$ in.: $$t = \frac{1}{2700\,\frac{1}{\text{s}}}\ln\left[\frac{1}{1 - \frac{2700\text{ 1/s}}{900\text{ ft/s}}\left(\frac{3.9}{12}\text{ ft}\right)}\right]$$

or $$t = 1.366\times 10^{-3}\text{ s} \blacktriangleleft$$

PROBLEM 11.17

The acceleration of a particle is defined by the relation $a = -k/x$. It has been experimentally determined that $v = 15$ ft/s when $x = 0.6$ ft and that $v = 9$ ft/s when $x = 1.2$ ft. Determine (*a*) the velocity of the particle when $x = 1.5$ ft, (*b*) the position of the particle at which its velocity is zero.

SOLUTION

$$a = \frac{v\,dv}{dx} = \frac{-k}{x}$$

Separate and integrate using $x = 0.6$ ft, $v = 15$ ft/s.

$$\int_{15}^{v} v\,dv = -k\int_{0.6}^{x} \frac{dx}{x}$$

$$\frac{1}{2}v^2\Big|_{15}^{v} = -k\ln x\Big|_{0.6}^{x}$$

$$\frac{1}{2}v^2 - \frac{1}{2}(15)^2 = -k\ln\left(\frac{x}{0.6}\right) \qquad (1)$$

When $v = 9$ ft/s, $x = 1.2$ ft

$$\frac{1}{2}(9)^2 - \frac{1}{2}(15)^2 = -k\ln\left(\frac{1.2}{0.6}\right)$$

Solve for *k*.

$$k = 103.874\ \text{ft}^2/\text{s}^2$$

(*a*) Velocity when $x = 65$ ft.

Substitute $k = 103.874\ \text{ft}^2/\text{s}^2$ and $x = 1.5$ ft into (1).

$$\frac{1}{2}v^2 - \frac{1}{2}(15)^2 = -103.874\ln\left(\frac{1.5}{0.6}\right)$$

$v = 5.89$ ft/s ◀

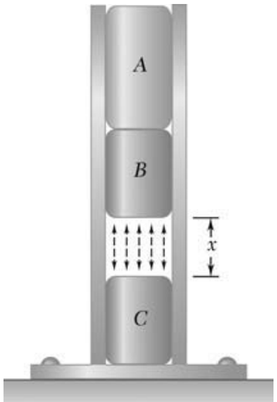
(*b*) Position when for $v = 0$,

$$0 - \frac{1}{2}(15)^2 = -103.874\ln\left(\frac{x}{0.6}\right)$$

$$\ln\left(\frac{x}{0.6}\right) = 1.083$$

$x = 1.772$ ft ◀

PROBLEM 11.18



A brass (nonmagnetic) block A and a steel magnet B are in equilibrium in a brass tube under the magnetic repelling force of another steel magnet C located at a distance $x = 0.004$ m from B. The force is inversely proportional to the square of the distance between B and C. If block A is suddenly removed, the acceleration of block B is $a = -9.81 + k/x^2$, where a and x are expressed in m/s^2 and m, respectively, and $k = 4\times10^{-4}\,\text{m}^3/\text{s}^2$. Determine the maximum velocity and acceleration of B.

SOLUTION

The maximum veolocity occurs when $a = 0$. $\qquad 0 = -9.81 + \dfrac{k}{x_m^2}$

$$x_m^2 = \frac{k}{9.81} = \frac{4\times10^{-4}}{9.81} = 40.775\times10^{-6}\,\text{m}^2 \qquad x_m = 0.0063855\ \text{m}$$

The acceleration is given as a function of x.

$$v\frac{dv}{dx} = a = -9.81 + \frac{k}{x^2}$$

Separate variables and integrate:

$$vdv = -9.81dx + \frac{k\,dx}{x^2}$$

$$\int_0^v vdv = -9.81\int_{x_0}^{x} dx + k\int_{x_0}^{x}\frac{dx}{x^2}$$

$$\frac{1}{2}v^2 = -9.81(x - x_0) - k\left(\frac{1}{x} - \frac{1}{x_0}\right)$$

$$\frac{1}{2}v_m^2 = -9.81(x_m - x_0) - k\left(\frac{1}{x_m} - \frac{1}{x_0}\right)$$

$$= -9.81(0.0063855 - 0.004) - (4\times10^{-4})\left(\frac{1}{0.0063855} - \frac{1}{0.004}\right)$$

$$= -0.023402 + 0.037358 = 0.013956\ \text{m}^2/\text{s}^2$$

Maximum velocity: $\qquad v_m = 0.1671\ \text{m/s} \qquad\qquad v_m = 167.1\ \text{mm/s}\uparrow$ ◀

The maximum acceleration occurs when x is smallest, that is, $x = 0.004$ m.

$$a_m = -9.81 + \frac{4\times10^{-4}}{(0.004)^2} \qquad\qquad a_m = 15.19\ \text{m/s}^2\uparrow \ ◀$$

PROBLEM 11.19

Based on experimental observations, the acceleration of a particle is defined by the relation $a = -(0.1 + \sin x/b)$, where a and x are expressed in m/s^2 and meters, respectively. Knowing that $b = 0.8$ m and that $v = 1$ m/s when $x = 0$, determine (*a*) the velocity of the particle when $x = -1$ m, (*b*) the position where the velocity is maximum, (*c*) the maximum velocity.

SOLUTION

We have $$v\frac{dv}{dx} = a = -\left(0.1 + \sin\frac{x}{0.8}\right)$$

When $x = 0,\ v = 1$ m/s: $$\int_1^v v\,dv = \int_0^x -\left(0.1 + \sin\frac{x}{0.8}\right)dx$$

or $$\frac{1}{2}(v^2 - 1) = -\left[0.1x - 0.8\cos\frac{x}{0.8}\right]_0^x$$

or $$\frac{1}{2}v^2 = -0.1x + 0.8\cos\frac{x}{0.8} - 0.3$$

(*a*) When $x = -1$ m: $$\frac{1}{2}v^2 = -0.1(-1) + 0.8\cos\frac{-1}{0.8} - 0.3$$

or $v = \pm 0.323$ m/s ◀

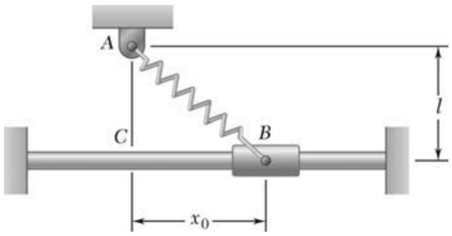
(*b*) When $v = v_{\text{max}},\ a = 0$: $$-\left(0.1 + \sin\frac{x}{0.8}\right) = 0$$

or $x = -0.080\,134$ m $x = -0.0801$ m ◀

(*c*) When $x = -0.080\,134$ m:

$$\frac{1}{2}v_{\text{max}}^2 = -0.1(-0.080\,134) + 0.8\cos\frac{-0.080\,134}{0.8} - 0.3$$
$$= 0.504\ \text{m}^2/\text{s}^2$$

or $v_{\text{max}} = 1.004$ m/s ◀



PROBLEM 11.20

A spring AB is attached to a support at A and to a collar. The unstretched length of the spring is l. Knowing that the collar is released from rest at $x = x_0$ and has an acceleration defined by the relation $a = -100(x - lx/\sqrt{l^2 + x^2})$, determine the velocity of the collar as it passes through Point C.

SOLUTION

Since a is function of x,
$$a = v\frac{dv}{dx} = -100\left(x - \frac{lx}{\sqrt{l^2 + x^2}}\right)$$

Separate variables and integrate:

$$\int_{v_0}^{v_f} v\,dv = -100\int_{x_0}^{0}\left(x - \frac{lx}{\sqrt{l^2 + x^2}}\right)dx$$

$$\frac{1}{2}v_f^2 - \frac{1}{2}v_0^2 = -100\left(\frac{x^2}{2} - l\sqrt{l^2 + x^2}\right)\Bigg|_{x_0}^{0}$$

$$\frac{1}{2}v_f^2 - 0 = -100\left(-\frac{x_0^2}{2} - l^2 + l\sqrt{l^2 + x_0^2}\right)$$

$$\frac{1}{2}v_f^2 = \frac{100}{2}(-l^2 + x_0^2 - l^2 - 2l\sqrt{l^2 + x_0^2})$$
$$= \frac{100}{2}(\sqrt{l^2 + x_0^2} - l)^2$$

$$v_f = 10(\sqrt{l^2 + x_0^2} - l) \blacktriangleleft$$

PROBLEM 11.21

The acceleration of a particle is defined by the relation $a = -0.8v$ where a is expressed in m/s^2 and v in m/s. Knowing that at $t = 0$ the velocity is 1 m/s, determine (*a*) the distance the particle will travel before coming to rest, (*b*) the time required for the particle's velocity to be reduced by 50 percent of its initial value.

SOLUTION

(*a*) Determine relationship between x and v.

$$a = \frac{vdv}{dx} = -0.8v \qquad dv = -0.8dx$$

Separate and integrate with $v = 1$ m/s when $x = 0$.

$$\int_1^v dv = -0.8\int_0^x dx$$

$$v - 1 = -0.8x$$

Distance traveled.

For $v = 0$, $\quad x = \frac{-1}{-0.8} \Rightarrow \quad x = 1.25 \text{ m}$ ◀

(*b*) Determine realtionship between v and t.

$$a = \frac{dv}{dt} = 0.8v$$

$$\int_1^v \frac{dv}{v} = -\int_0^x 0.8dt$$

$$\ln\left(\frac{v}{1}\right) = -0.8t \qquad t = 1.25\ln\left(\frac{1}{v}\right)$$

For $v = 0.5(1 \text{ m/s}) = 0.5$ m/s,

$$t = 1.25\ln\left(\frac{1}{0.5}\right) \qquad t = 0.866 \text{ s}$$ ◀

PROBLEM 11.22

Starting from $x = 0$ with no initial velocity, a particle is given an acceleration $a = 0.1\sqrt{v^2 + 16}$, where a and v are expressed in ft/s^2 and ft/s, respectively. Determine (a) the position of the particle when $v = 3$ft/s, (b) the speed and acceleration of the particle when $x = 4$ ft.

SOLUTION

$$a = \frac{vdv}{dx} = 0.1(v^2 + 16)^{1/2} \quad (1)$$

Separate and integrate.

$$\int_0^v \frac{vdv}{\sqrt{v^2 + 16}} = \int_0^x 0.1\, dx$$

$$(v^2 + 16)^{1/2}\Big|_0^v = 0.1x$$

$$(v^2 + 16)^{1/2} - 4 = 0.1x$$

$$x = 10[(v^2 + 16)^{1/2} - 4] \quad (2)$$

(a) $v = 3$ ft/s.

$$x = 10[(3^2 + 16)^{1/2} - 4] \qquad x = 10.00 \text{ ft} \blacktriangleleft$$

(b) $x = 4$ ft.

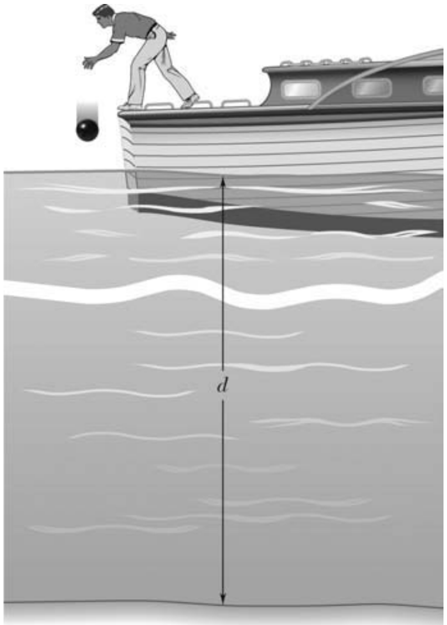
From (2), $$(v^2 + 16)^{1/2} = 4 + 0.1x = 4 + (0.1)(4) = 4.4$$

$$v^2 + 16 = 19.36$$

$$v^2 = 3.36 \text{ ft}^2/\text{s}^2 \qquad v = 1.833 \text{ ft/s} \blacktriangleleft$$

From (1), $$a = 0.1(1.833^2 + 16)^{1/2} \qquad a = 0.440 \text{ ft/s}^2 \blacktriangleleft$$

PROBLEM 11.23



A ball is dropped from a boat so that it strikes the surface of a lake with a speed of 16.5 ft/s. While in the water the ball experiences an acceleration of $a = 10 - 0.8v,$ where a and v are expressed in ft/s^2 and ft/s, respectively. Knowing the ball takes 3 s to reach the bottom of the lake, determine (*a*) the depth of the lake, (*b*) the speed of the ball when it hits the bottom of the lake.

SOLUTION

$$a = \frac{dv}{dt} = 10 - 0.8v$$

Separate and integrate:

$$\int_{v_0}^{v} \frac{dv}{10 - 0.8v} = \int_0^t dt$$

$$-\frac{1}{0.8}\ln(10 - 0.8v)\Big|_{v_0}^{v} = t$$

$$\ln\left(\frac{10 - 0.8v}{10 - 0.8v_0}\right) = -0.8t$$

$$10 - 0.8v = (10 - 0.8v_0)e^{-0.8t}$$

or

$$0.8v = 10 - (10 - 0.8v_0)e^{-0.8t}$$

$$v = 12.5 - (12.5 - v_0)e^{-0.8t}$$

With $v_0 = 16.5\,\text{ft/s}$

$$v = 12.5 + 4e^{-0.8t}$$

PROBLEM 11.23 (Continued)

Integrate to determine x as a function of t.

$$v = \frac{dx}{dt} = 12.5 + 4e^{-0.8t}$$

$$\int_0^x dx = \int_0^t (12.5 + 4e^{-0.8t})dt$$

$$x = 12.5t - 5e^{-0.8t}\Big|_0^t = 12.5t - 5e^{-0.8t} + 5$$

(a) At $t = 35$ s,

$$x = 12.5(3) - 5e^{-2.4} + 5 = 42.046 \text{ ft}$$

$x = 42.0$ ft ◀

(b) $v = 12.5 + 4e^{-2.4} = 12.863$ ft/s

$v = 12.86$ ft/s ◀

PROBLEM 11.24

The acceleration of a particle is defined by the relation $a = -k\sqrt{v}$, where k is a constant. Knowing that $x = 0$ and $v = 81$ m/s at $t = 0$ and that $v = 36$ m/s when $x = 18$ m, determine (a) the velocity of the particle when $x = 20$ m, (b) the time required for the particle to come to rest.

SOLUTION

(a) We have $$v\frac{dv}{dx} = a = -k\sqrt{v}$$

so that $$\sqrt{v}\; dv = -k\,dx$$

When $x = 0,\ v = 81\text{ m/s}$: $$\int_{81}^{v} \sqrt{v}\; dv = \int_0^x -k\,dx$$

or $$\frac{2}{3}[v^{3/2}]_{81}^{v} = -kx$$

or $$\frac{2}{3}[v^{3/2} - 729] = -kx$$

When $x = 18\text{ m},\ v = 36\text{ m/s}$: $$\frac{2}{3}(36^{3/2} - 729) = -k(18)$$

or $$k = 19\sqrt{\text{m/s}^2}$$

Finally

When $x = 20\text{ m}$: $$\frac{2}{3}(v^{3/2} - 729) = -19(20)$$

or $$v^{3/2} = 159$$ $v = 29.3$ m/s ◀

(b) We have $$\frac{dv}{dt} = a = -19\sqrt{v}$$

At $t = 0,\ v = 81\text{ m/s}$: $$\int_{81}^{v} \frac{dv}{\sqrt{v}} = \int_0^t -19dt$$

or $$2[\sqrt{v}]_{81}^{v} = -19t$$

or $$2(\sqrt{v} - 9) = -19t$$

When $v = 0$: $$2(-9) = -19t$$

or $t = 0.947$ s ◀

PROBLEM 11.25

A particle is projected to the right from the position $x = 0$ with an initial velocity of 9 m/s. If the acceleration of the particle is defined by the relation $a = -0.6v^{3/2}$, where a and v are expressed in m/s^2 and m/s, respectively, determine (*a*) the distance the particle will have traveled when its velocity is 4 m/s, (*b*) the time when $v = 1$ m/s, (*c*) the time required for the particle to travel 6 m.

SOLUTION

(*a*) We have $$v\frac{dv}{dx} = a = -0.6v^{3/2}$$

When $x = 0,\ v = 9$ m/s: $$\int_9^v v^{-(1/2)}dv = \int_0^x -0.6dx$$

or $$2[v^{1/2}]_9^v = -0.6x$$

or $$x = \frac{1}{0.3}(3 - v^{1/2}) \qquad (1)$$

When $v = 4$ m/s: $$x = \frac{1}{0.3}(3 - 4^{1/2})$$

or $x = 3.33$ m ◀

(*b*) We have $$\frac{dv}{dt} = a = -0.6v^{3/2}$$

When $t = 0,\ v = 9$ m/s: $$\int_9^v v^{-(3/2)}dv = \int_0^t -0.6dt$$

or $$-2[v^{-(1/2)}]_9^v = -0.6t$$

or $$\frac{1}{\sqrt{v}} - \frac{1}{3} = 0.3t$$

When $v = 1$ m/s: $$\frac{1}{\sqrt{1}} - \frac{1}{3} = 0.3t$$

or $t = 2.22$ s ◀

(*c*) We have $$\frac{1}{\sqrt{v}} - \frac{1}{3} = 0.3t$$

or $$v = \left(\frac{3}{1 + 0.9t}\right)^2 = \frac{9}{(1 + 0.9t)^2}$$

Now $$\frac{dx}{dt} = v = \frac{9}{(1 + 0.9t)^2}$$

PROBLEM 11.25 (Continued)

At $t = 0,\ x = 0$: $$\int_0^x dx = \int_0^t \frac{9}{(1+0.9t)^2}\,dt$$

or $$x = 9\left[-\frac{1}{0.9}\frac{1}{1+0.9t}\right]_0^t$$

$$= 10\left(1 - \frac{1}{1+0.9t}\right)$$

$$= \frac{9t}{1+0.9t}$$

When $x = 6$ m: $$6 = \frac{9t}{1+0.9t}$$

or $t = 1.667$ s ◀

An alternative solution is to begin with Eq. (1).

$$x = \frac{1}{0.3}(3 - v^{1/2})$$

Then $$\frac{dx}{dt} = v = (3 - 0.3x)^2$$

Now

At $t = 0,\ x = 0$: $$\int_0^x \frac{dx}{(3-0.3x)^2} = \int_0^t dt$$

or $$t = \frac{1}{0.3}\left[\frac{1}{3-0.3x}\right]_0^x = \frac{x}{9-0.9x}$$

which leads to the same equation as above.

PROBLEM 11.26

The acceleration of a particle is defined by the relation $a = 0.4(1 - kv)$, where k is a constant. Knowing that at $t = 0$ the particle starts from rest at $x = 4$ m and that when $t = 15$ s, $v = 4$ m/s, determine (*a*) the constant k, (*b*) the position of the particle when $v = 6$ m/s, (*c*) the maximum velocity of the particle.

SOLUTION

(*a*) We have $$\frac{dv}{dt} = a = 0.4(1 - kv)$$

At $t = 0$, $v = 0$: $$\int_0^v \frac{dv}{1 - kv} = \int_0^t 0.4dt$$

or $$-\frac{1}{k}[\ln(1 - kv)]_0^v = 0.4t$$

or $$\ln(1 - kv) = -0.4kt \qquad (1)$$

At $t = 15$ s, $v = 4$ m/s: $$\ln(1 - 4k) = -0.4k(15)$$
$$= -6k$$

Solving yields $$k = 0.145703 \text{ s/m}$$

or $k = 0.1457$ s/m ◄

(*b*) We have $$v\frac{dv}{dx} = a = 0.4(1 - kv)$$

When $x = 4$ m, $v = 0$: $$\int_0^v \frac{vdv}{1 - kv} = \int_4^x 0.4dx$$

Now $$\frac{v}{1 - kv} = -\frac{1}{k} + \frac{1/k}{1 - kv}$$

Then $$\int_0^v \left[-\frac{1}{k} + \frac{1}{k(1 - kv)}\right] dv = \int_4^x 0.4dx$$

or $$\left[-\frac{v}{k} - \frac{1}{k^2}\ln(1 - kv)\right]_0^v = 0.4[x]_4^x$$

or $$-\left[\frac{v}{k} + \frac{1}{k^2}\ln(1 - kv)\right] = 0.4(x - 4)$$

When $v = 6$ m/s: $$-\left[\frac{6}{0.145\,703} + \frac{1}{(0.145\,703)^2}\ln(1 - 0.145\,703 \times 6)\right] = 0.4(x - 4)$$

or $$0.4(x - 4) = 56.4778$$

or $x = 145.2$ m ◄

(*c*) The maximum velocity occurs when $a = 0$.

$$a = 0: \quad 0.4(1 - k v_{\text{max}}) = 0$$

or

$$v_{\text{max}} = \frac{1}{0.145\,703}$$

or

$$v_{\text{max}} = 6.86 \text{ m/s} \blacktriangleleft$$

An alternative solution is to begin with Eq. (1).

$$\ln(1 - kv) = -0.4kt$$

Then

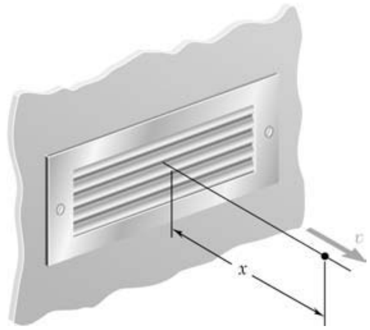
$$v = \frac{1}{k}(1 - k^{-0.4kt})$$

Thus, v_{max} is attained as $t \longrightarrow \infty$

$$v_{\text{max}} = \frac{1}{k}$$

as above.

PROBLEM 11.27



Experimental data indicate that in a region downstream of a given louvered supply vent the velocity of the emitted air is defined by $v = 0.18v_0/x$, where v and x are expressed in m/s and meters, respectively, and v_0 is the initial discharge velocity of the air. For $v_0 = 3.6$ m/s, determine (a) the acceleration of the air at $x = 2$ m, (b) the time required for the air to flow from $x = 1$ to $x = 3$ m.

SOLUTION

(a) We have

$$a = v\frac{dv}{dx}$$

$$= \frac{0.18v_0}{x}\frac{d}{dx}\left(\frac{0.18v_0}{x}\right)$$

$$= -\frac{0.0324v_0^2}{x^3}$$

When $x = 2$ m:

$$a = -\frac{0.0324(3.6)^2}{(2)^3}$$

or

$$a = -0.0525 \text{ m/s}^2 \blacktriangleleft$$

(b) We have

$$\frac{dx}{dt} = v = \frac{0.18v_0}{x}$$

From $x = 1$ m to $x = 3$ m:

$$\int_1^3 x dx = \int_{t_1}^{t_3} 0.18v_0\, dt$$

or

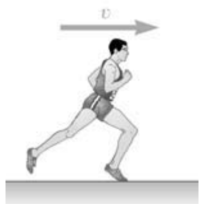
$$\left[\frac{1}{2}x^2\right]_1^3 = 0.18v_0(t_3 - t_1)$$

or

$$(t_3 - t_1) = \frac{\frac{1}{2}(9-1)}{0.18(3.6)}$$

or

$$t_3 - t_1 = 6.17 \text{ s} \blacktriangleleft$$



PROBLEM 11.28

Based on observations, the speed of a jogger can be approximated by the relation $v = 7.5(1-0.04x)^{0.3}$, where v and x are expressed in mi/h and miles, respectively. Knowing that $x = 0$ at $t = 0$, determine (*a*) the distance the jogger has run when $t = 1$ h, (*b*) the jogger's acceleration in ft/s^2 at $t = 0$, (*c*) the time required for the jogger to run 6 mi.

SOLUTION

(*a*) We have $$\frac{dx}{dt} = v = 7.5(1-0.04x)^{0.3}$$

At $t = 0,\ x = 0$: $$\int_0^x \frac{dx}{(1-0.04x)^{0.3}} = \int_0^t 7.5dt$$

or $$\frac{1}{0.7}\left(-\frac{1}{0.04}\right)[(1-0.04x)^{0.7}]_0^x = 7.5t$$

or $$1-(1-0.04x)^{0.7} = 0.21t \qquad (1)$$

or $$x = \frac{1}{0.04}[1-(1-0.21t)^{1/0.7}]$$

At $t = 1$ h: $$x = \frac{1}{0.04}\{1-[1-0.21(1)]^{1/0.7}\}$$

or $x = 7.15$ mi ◀

(*b*) We have $$a = v\frac{dv}{dx}$$
$$= 7.5(1-0.04x)^{0.3}\frac{d}{dx}[7.5(1-0.04x)^{0.3}]$$
$$= 7.5^2(1-0.04x)^{0.3}[(0.3)(-0.04)(1-0.04x)^{-0.7}]$$
$$= -0.675(1-0.04x)^{-0.4}$$

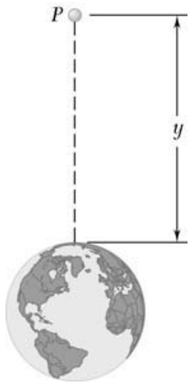
At $t = 0,\ x = 0$: $$a_0 = -0.675 \text{ mi/h}^2 \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2$$

or $a_0 = -275 \times 10^{-6}$ ft/s^2 ◀

(*c*) From Eq. (1) $$t = \frac{1}{0.21}[1-(1-0.04x)^{0.7}]$$

When $x = 6$ mi: $$t = \frac{1}{0.21}\{1-[1-0.04(6)]^{0.7}\}$$
$$= 0.83229 \text{ h}$$

or $t = 49.9$ min ◀



PROBLEM 11.29

The acceleration due to gravity at an altitude y above the surface of the earth can be expressed as

$$a = \frac{-32.2}{[1 + (y/20.9 \times 10^6)]^2}$$

where a and y are expressed in ft/s^2 and feet, respectively. Using this expression, compute the height reached by a projectile fired vertically upward from the surface of the earth if its initial velocity is (*a*) 1800 ft/s, (*b*) 3000 ft/s, (*c*) 36,700 ft/s.

SOLUTION

We have $$v\frac{dv}{dy} = a = -\frac{32.2}{\left(1 + \frac{y}{20.9\times10^6}\right)^2}$$

When $$y = 0, \quad v = v_0$$

provided that v does reduce to zero, $$y = y_{max}, \quad v = 0$$

Then $$\int_{v_0}^{0} v\,dv = \int_0^{y_{max}} \frac{-32.2}{\left(1+\frac{y}{20.9\times10^6}\right)^2}dy$$

or $$-\frac{1}{2}v_0^2 = -32.2\left[-20.9\times10^6 \frac{1}{1+\frac{y}{20.9\times10^6}}\right]_0^{y_{max}}$$

or $$v_0^2 = 1345.96\times10^6\left(1 - \frac{1}{1+\frac{y_{max}}{20.9\times10^6}}\right)$$

or $$y_{max} = \frac{v_0^2}{64.4 - \frac{v_0^2}{20.9\times10^6}}$$

(*a*) $v_0 = 1800$ ft/s: $$y_{max} = \frac{(1800)^2}{64.4 - \frac{(1800)^2}{20.9\times10^6}}$$

or $$y_{max} = 50.4\times10^3 \text{ ft} \blacktriangleleft$$

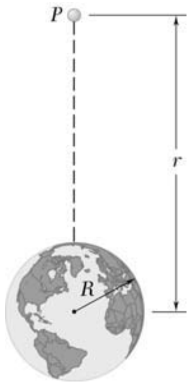
(*b*) $v_0 = 3000$ ft/s: $$y_{max} = \frac{(3000)^2}{64.4 - \frac{(3000)^2}{20.9\times10^6}}$$

or $$y_{max} = 140.7\times10^3 \text{ ft} \blacktriangleleft$$

(*c*) $v_0 = 36,700$ ft/s: $$y_{max} = \frac{(36,700)^2}{64.4 - \frac{(36,700)^2}{20.9\times10^6}} = -3.03\times10^{10} \text{ ft}$$

This solution is invalid since the velocity does not reduce to zero. The velocity 36,700 ft/s is above the escape velocity v_R from the earth. For v_R and above.

$y_{max} \longrightarrow \infty$ ◀



PROBLEM 11.30

The acceleration due to gravity of a particle falling toward the earth is $a = -gR^2/r^2$, where r is the distance from the *center* of the earth to the particle, R is the radius of the earth, and g is the acceleration due to gravity at the surface of the earth. If $R = 3960$ mi, calculate the *escape velocity,* that is, the minimum velocity with which a particle must be projected vertically upward from the surface of the earth if it is not to return to the earth. (*Hint:* $v = 0$ for $r = \infty$.)

SOLUTION

We have $$v\frac{dv}{dr} = a = -\frac{gR^2}{r^2}$$

When $$r = R, \quad v = v_e$$
$$r = \infty, \quad v = 0$$

then $$\int_{v_e}^{0} v\,dv = \int_R^{\infty} -\frac{gR^2}{r^2}\,dr$$

or $$-\frac{1}{2}v_e^2 = gR^2\left[\frac{1}{r}\right]_R^{\infty}$$

or $$v_e = \sqrt{2gR}$$
$$= \left(2 \times 32.2 \text{ ft/s}^2 \times 3960 \text{ mi} \times \frac{5280 \text{ ft}}{1 \text{ mi}}\right)^{1/2}$$

or $$v_e = 36.7 \times 10^3 \text{ ft/s} \blacktriangleleft$$

PROBLEM 11.31

The velocity of a particle is $v = v_0[1 - \sin(\pi t/T)]$. Knowing that the particle starts from the origin with an initial velocity v_0, determine (*a*) its position and its acceleration at $t = 3T$, (*b*) its average velocity during the interval $t = 0$ to $t = T$.

SOLUTION

(*a*) We have

$$\frac{dx}{dt} = v = v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]$$

At $t = 0, x = 0$:

$$\int_0^x dx = \int_0^t v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]dt$$

$$x = v_0\left[t + \frac{T}{\pi}\cos\left(\frac{\pi t}{T}\right)\right]_0^t = v_0\left[t + \frac{T}{\pi}\cos\left(\frac{\pi t}{T}\right) - \frac{T}{\pi}\right] \qquad (1)$$

At $t = 3T$:

$$x_{3T} = v_0\left[3T + \frac{T}{\pi}\cos\left(\frac{\pi \times 3T}{T}\right) - \frac{T}{\pi}\right] = v_0\left(3T - \frac{2T}{\pi}\right)$$

$x_{3T} = 2.36\, v_0 T$ ◀

$$a = \frac{dv}{dt} = \frac{d}{dt}\left\{v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]\right\} = -v_0\frac{\pi}{T}\cos\frac{\pi t}{T}$$

At $t = 3T$:

$$a_{3T} = -v_0\frac{\pi}{T}\cos\frac{\pi \times 3T}{T}$$

$a_{3T} = \dfrac{\pi v_0}{T}$ ◀

(*b*) Using Eq. (1)

At $t = 0$:

$$x_0 = v_0\left[0 + \frac{T}{\pi}\cos(0) - \frac{T}{\pi}\right] = 0$$

At $t = T$:

$$x_T = v_0\left[T + \frac{T}{\pi}\cos\left(\frac{\pi T}{T}\right) - \frac{T}{\pi}\right] = v_0\left(T - \frac{2T}{\pi}\right) = 0.363 v_0 T$$

Now

$$v_{\text{ave}} = \frac{x_T - x_0}{\Delta t} = \frac{0.363 v_0 T - 0}{T - 0}$$

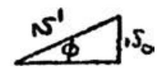
$v_{\text{ave}} = 0.363 v_0$ ◀

PROBLEM 11.32

The velocity of a slider is defined by the relation $v = v' \sin(\omega_n t + \phi)$. Denoting the velocity and the position of the slider at $t = 0$ by v_0 and x_0, respectively, and knowing that the maximum displacement of the slider is $2x_0$, show that (*a*) $v' = (v_0^2 + x_0^2\omega_n^2)/2x_0\omega_n$, (*b*) the maximum value of the velocity occurs when $x = x_0[3 - (v_0/x_0\omega_n)^2]/2$.

SOLUTION

(*a*) At $t = 0, v = v_0$: $\quad v_0 = v' \sin(0 + \phi) = v' \sin\phi$



Then $\quad \cos\phi = \sqrt{v'^2 - v_0^2} \Big/ v'$

Now $\quad \dfrac{dx}{dt} = v = v' \sin(\omega_n t + \phi)$

At $t = 0, x = x_0$: $\quad \int_{x0}^{x} dx = \int_0^t v' \sin(\omega_n t + \phi) dt$

or $\quad x - x_0 = v' \left[-\dfrac{1}{\omega_n} \cos(\omega_n t + \phi) \right]_0^t$

or $\quad x = x_0 + \dfrac{v'}{\omega_n} \left[\cos\phi - \cos(\omega_n t + \phi) \right]$

Now observe that x_{max} occurs when $\cos(\omega_n t + \phi) = -1$. Then

$$x_{max} = 2x_0 = x_0 + \frac{v'}{\omega_n}[\cos\phi - (-1)]$$

Substituting for $\cos\phi$ $\quad x_0 = \dfrac{v'}{\omega_n} \left(\dfrac{\sqrt{v'^2 - v_0^2}}{v^1} + 1 \right)$

or $\quad x_0\omega_n - v' = \sqrt{v'^2 - v_0^2}$

Squaring both sides of this equation

$$x_0^2\omega_n^2 - 2x_0\omega_n + v'^2 = v'^2 - v_0^2$$

or $\quad v' = \dfrac{v_0^2 + x_0^2\omega_n^2}{2x_0\omega_n}$ $\quad$ Q. E. D.

PROBLEM 11.32 (Continued)

(*b*) First observe that v_{max} occurs when $\omega_n t + \phi = \frac{\pi}{2}$. The corresponding value of x is

$$x_{v_{max}} = x_0 + \frac{v'}{\omega_n}\left[\cos\phi - \cos\left(\frac{\pi}{2}\right)\right]$$
$$= x_0 + \frac{v'}{\omega_n}\cos\phi$$

Substituting first for $\cos\phi$ and then for v'

$$x_{v_{max}} = x_0 + \frac{v'}{\omega_n}\frac{\sqrt{v'^2 - v_0^2}}{v'}$$
$$= x_0 + \frac{1}{\omega_n}\left[\left(\frac{v_0^2 + x_0^2\omega_n^2}{2x_0\omega_n}\right)^2 - v_0^2\right]^{1/2}$$
$$= x_0 + \frac{1}{2x_0\omega_n^2}\left(v_0^4 + 2v_0^2x_0^2\omega_n^2 + x_0^4\omega_n^4 - 4x_0^2\omega_n^2v_0^2\right)^{1/2}$$
$$= x_0 + \frac{1}{2x_0\omega_n^2}\left[\left(x_0^2\omega_n^2 - v_0^2\right)^2\right]^{1/2}$$
$$= x_0 + \frac{x_0^2\omega_n^2 - v_0^2}{2x_0\omega_n^2}$$
$$= \frac{x_0}{2}\left[3 - \left(\frac{v_0}{x_0\omega_n}\right)^2\right]$$

Q. E. D.

PROBLEM 11.33

A stone is thrown vertically upward from a point on a bridge located 40 m above the water. Knowing that it strikes the water 4 s after release, determine (*a*) the speed with which the stone was thrown upward, (*b*) the speed with which the stone strikes the water.

SOLUTION

Uniformly accelerated motion. Origin at water. $+\uparrow$

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + a t$$

where $y_0 = 40 \text{ m}$ and $a = -9.81 \text{ m/s}^2$.

(*a*) *Initial speed.*

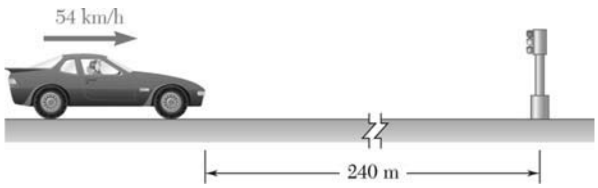
$$y = 0 \text{ when } t = 4 \text{ s.}$$

$$0 = 40 + v_0(4) - \frac{1}{2}(9.81)(4)^2$$

$$v_0 = 9.62 \text{ m/s} \qquad \mathbf{v}_0 = 9.62 \text{ m/s} \uparrow \blacktriangleleft$$

(*b*) *Speed when striking the water.* (v at $t = 4$ s)

$$v = 9.62 - (9.81)(4) = -29.62 \text{ m/s} \qquad \mathbf{v} = 29.6 \text{ m/s} \downarrow \blacktriangleleft$$



PROBLEM 11.34

A motorist is traveling at 54 km/h when she observes that a traffic light 240 m ahead of her turns red. The traffic light is timed to stay red for 24 s. If the motorist wishes to pass the light without stopping just as it turns green again, determine (*a*) the required uniform deceleration of the car, (*b*) the speed of the car as it passes the light.

SOLUTION

Uniformly accelerated motion:

$$x_0 = 0 \quad v_0 = 54 \text{ km/h} = 15 \text{ m/s}$$

(a) $x = x_0 + v_0 t + \frac{1}{2} a t^2$

when $t = 24 \text{ s}$, $x = 240 \text{ m}$:

$$240 \text{ m} = 0 + (15 \text{ m/s})(24 \text{ s}) + \frac{1}{2} a (24 \text{ s})^2$$

$$a = -0.4167 \text{ m/s}^2 \qquad a = -0.417 \text{ m/s}^2 \blacktriangleleft$$

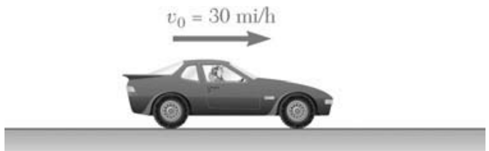
(*b*) $v = v_0 + a t$

when $t = 24 \text{ s}$:

$$v = (15 \text{ m/s}) + (-0.4167 \text{ m/s})(24 \text{ s})$$

$$v = 5.00 \text{ m/s}$$

$$v = 18.00 \text{ km/h} \qquad v = 18.00 \text{ km/h} \blacktriangleleft$$



PROBLEM 11.35

A motorist enters a freeway at 30 mi/h and accelerates uniformly to 60 mi/h. From the odometer in the car, the motorist knows that she traveled 550 ft while accelerating. Determine (*a*) the acceleration of the car, (*b*) the time required to reach 60 mi/h.

SOLUTION

(*a*) Acceleration of the car.

$$v_1^2 = v_0^2 + 2a(x_1 - x_0)$$

$$a = \frac{v_1^2 - v_0^2}{2(x_1 - x_0)}$$

Data:

$$v_0 = 30 \text{ mi/h} = 44 \text{ ft/s}$$
$$v_1 = 60 \text{ mi/h} = 88 \text{ ft/s}$$

$$x_0 = 0$$
$$x_1 = 550 \text{ ft}$$

$$a = \frac{(88)^2 - (44)^2}{(2)(55 - 0)}$$

$a = 5.28 \text{ ft/s}^2$ ◀

(*b*) Time to reach 60 mi/h.

$$v_1 = v_0 + a(t_1 - t_0)$$

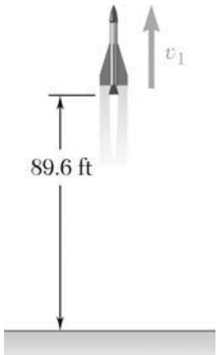
$$t_1 - t_0 = \frac{v_1 - v_0}{a}$$

$$= \frac{88 - 44}{5.28}$$

$$= 8.333 \text{ s}$$

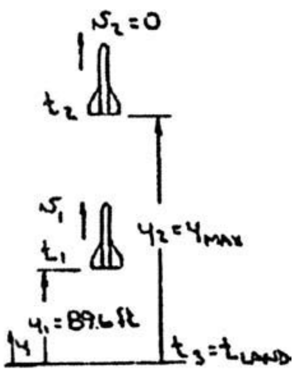
$t_1 - t_0 = 8.33 \text{ s}$ ◀

PROBLEM 11.36



A group of students launches a model rocket in the vertical direction. Based on tracking data, they determine that the altitude of the rocket was 89.6 ft at the end of the powered portion of the flight and that the rocket landed 16 s later. Knowing that the descent parachute failed to deploy so that the rocket fell freely to the ground after reaching its maximum altitude and assuming that $g = 32.2 \text{ ft/s}^2$, determine (*a*) the speed v_1 of the rocket at the end of powered flight, (*b*) the maximum altitude reached by the rocket.

SOLUTION



(*a*) We have $y = y_1 + v_1 t + \frac{1}{2}at^2$

At t_{land}, $y = 0$

Then $0 = 89.6 \text{ ft} + v_1(16 \text{ s}) + \frac{1}{2}(-32.2 \text{ ft/s}^2)(16 \text{ s})^2$

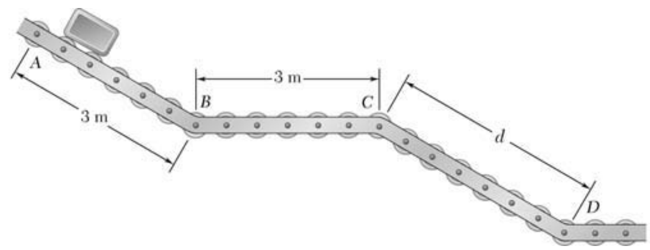
or $v_1 = 252 \text{ ft/s}$ ◀

(*b*) We have $v^2 = v_1^2 + 2a(y - y_1)$

At $y = y_{max}, \quad v = 0$

Then $0 = (252 \text{ ft/s})^2 + 2(-32.2 \text{ ft/s}^2)(y_{max} - 89.6) \text{ ft}$

or $y_{max} = 1076 \text{ ft}$ ◀



PROBLEM 11.37

A small package is released from rest at A and moves along the skate wheel conveyor $ABCD$. The package has a uniform acceleration of 4.8 m/s^2 as it moves down sections AB and CD, and its velocity is constant between B and C. If the velocity of the package at D is 7.2 m/s, determine (*a*) the distance d between C and D, (*b*) the time required for the package to reach D.

SOLUTION

(*a*) For $A \longrightarrow B$ and $C \longrightarrow D$ we have

$$v^2 = v_0^2 + 2a(x - x_0)$$

Then, at B $\quad v_{BC}^2 = 0 + 2(4.8\text{ m/s}^2)(3-0)\text{ m}$

$$= 28.8\text{ m}^2/\text{s}^2 \qquad (v_{BC} = 5.3666\text{ m/s})$$

and at D $\quad v_D^2 = v_{BC}^2 + 2a_{CD}(x_D - x_C) \qquad d = x_D - x_C$

or $\quad (7.2\text{ m/s})^2 = (28.8\text{ m}^2/\text{s}^2) + 2(4.8\text{ m/s}^2)d$

or $\quad d = 2.40\text{ m}$ ◀

(*b*) For $A \longrightarrow B$ and $C \longrightarrow D$ we have

$$v = v_0 + at$$

Then $A \longrightarrow B$ $\quad 5.3666\text{ m/s} = 0 + (4.8\text{ m/s}^2)t_{AB}$

or $\quad t_{AB} = 1.11804\text{ s}$

and $C \longrightarrow D$ $\quad 7.2\text{ m/s} = 5.3666\text{ m/s} + (4.8\text{ m/s}^2)t_{CD}$

or $\quad t_{CD} = 0.38196\text{ s}$

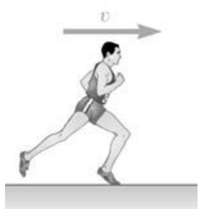
Now, for $B \longrightarrow C$, we have $\quad x_C = x_B + v_{BC}t_{BC}$

or $\quad 3\text{ m} = (5.3666\text{ m/s})t_{BC}$

or $\quad t_{BC} = 0.55901\text{ s}$

Finally, $\quad t_D = t_{AB} + t_{BC} + t_{CD} = (1.11804 + 0.55901 + 0.38196)\text{ s}$

or $\quad t_D = 2.06\text{ s}$ ◀



PROBLEM 11.38

A sprinter in a 100-m race accelerates uniformly for the first 35 m and then runs with constant velocity. If the sprinter's time for the first 35 m is 5.4 s, determine (*a*) his acceleration, (*b*) his final velocity, (*c*) his time for the race.

SOLUTION

Given: $0 \le x \le 35$ m, $a = \text{constant}$

$35 \text{ m} < x \le 100 \text{ m}, \quad v = \text{constant}$

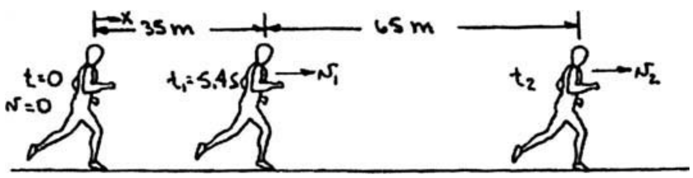
At $t = 0$, $v = 0$ when $x = 35$ m, $t = 5.4$ s

Find:

(*a*) a

(*b*) v when $x = 100$ m

(*c*) t when $x = 100$ m



(*a*) We have $x = 0 + 0t + \frac{1}{2}at^2$ for $0 \le x \le 35$ m

At $t = 5.4$ s: $35 \text{ m} = \frac{1}{2}a(5.4 \text{ s})^2$

or $a = 2.4005 \text{ m/s}^2$

$a = 2.40 \text{ m/s}^2$ ◀

(*b*) First note that $v = v_{max}$ for $35 \text{ m} \le x \le 100$ m.

Now $v^2 = 0 + 2a(x - 0)$ for $0 \le x \le 35$ m

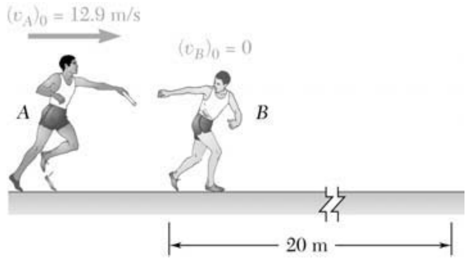
When $x = 35$ m: $v_{max}^2 = 2(2.4005 \text{ m/s}^2)(35 \text{ m})$

or $v_{max} = 12.9628$ m/s $\qquad v_{max} = 12.96$ m/s ◀

(*c*) We have $x = x_1 + v_0(t - t_1)$ for $35 \text{ m} < x \le 100$ m

When $x = 100$ m: $100 \text{ m} = 35 \text{ m} + (12.9628 \text{ m/s})(t_2 - 5.4) \text{ s}$

or $t_2 = 10.41$ s ◀



PROBLEM 11.39

As relay runner A enters the 20-m-long exchange zone with a speed of 12.9 m/s, he begins to slow down. He hands the baton to runner B 1.82 s later as they leave the exchange zone with the same velocity. Determine (*a*) the uniform acceleration of each of the runners, (*b*) when runner B should begin to run.

SOLUTION

(*a*) For runner A: $$x_A = 0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

At $t = 1.82$ s: $$20 \text{ m} = (12.9 \text{ m/s})(1.82 \text{ s}) + \frac{1}{2} a_A (1.82 \text{ s})^2$$

or $$a_A = -2.10 \text{ m/s}^2 \blacktriangleleft$$

Also $$v_A = (v_A)_0 + a_A t$$

At $t = 1.82$ s: $$(v_A)_{1.82} = (12.9 \text{ m/s}) + (-2.10 \text{ m/s}^2)(1.82 \text{ s})$$
$$= 9.078 \text{ m/s}$$

For runner B: $$v_B^2 = 0 + 2a_B [x_B - 0]$$

When $$x_B = 20 \text{ m}, \quad v_B = v_A: \quad (9.078 \text{ m/s})^2 = 2a_B (20 \text{ m})$$

or $$a_B = 2.0603 \text{ m/s}^2$$

$$a_B = 2.06 \text{ m/s}^2 \blacktriangleleft$$

(*b*) For runner B: $$v_B = 0 + a_B (t - t_B)$$

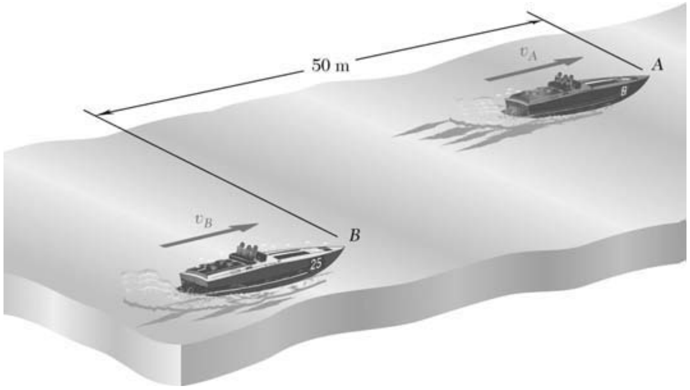
where t_B is the time at which he begins to run.

At $t = 1.82$ s: $$9.078 \text{ m/s} = (2.0603 \text{ m/s}^2)(1.82 - t_B) \text{ s}$$

or $$t_B = -2.59 \text{ s}$$

Runner B should start to run 2.59 s before A reaches the exchange zone. ◀

PROBLEM 11.40



In a boat race, boat A is leading boat B by 50 m and both boats are traveling at a constant speed of 180 km/h. At $t = 0$, the boats accelerate at constant rates. Knowing that when B passes A, $t = 8$ s and $v_A = 225$ km/h, determine (a) the acceleration of A, (b) the acceleration of B.

SOLUTION

(a) We have $v_A = (v_A)_0 + a_A t$

$$(v_A)_0 = 180 \text{ km/h} = 50 \text{ m/s}$$

At $t = 8$ s: $v_A = 225 \text{ km/h} = 62.5 \text{ m/s}$

Then $62.5 \text{ m/s} = 50 \text{ m/s} + a_A (8 \text{ s})$

or $a_A = 1.563 \text{ m/s}^2$ ◀

(b) We have

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2 = 50 \text{ m} + (50 \text{ m/s})(8 \text{ s}) + \frac{1}{2}(1.5625 \text{ m/s}^2)(8 \text{ s})^2 = 500 \text{ m}$$

and $x_B = 0 + (v_B)_0 t + \frac{1}{2} a_B t^2 \qquad (v_B)_0 = 50 \text{ m/s}$

At $t = 8$ s: $x_A = x_B$

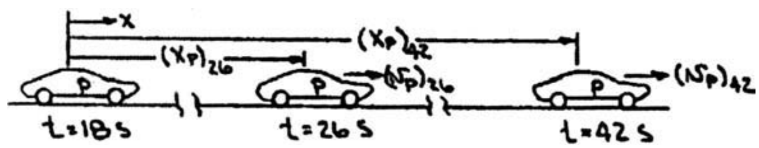
$$500 \text{ m} = (50 \text{ m/s})(8 \text{ s}) + \frac{1}{2} a_B (8 \text{ s})^2$$

or $a_B = 3.13 \text{ m/s}^2$ ◀

PROBLEM 11.41

A police officer in a patrol car parked in a 45 mi/h speed zone observes a passing automobile traveling at a slow, constant speed. Believing that the driver of the automobile might be intoxicated, the officer starts his car, accelerates uniformly to 60 mi/h in 8 s, and, maintaining a constant velocity of 60 mi/h, overtakes the motorist 42 s after the automobile passed him. Knowing that 18 s elapsed before the officer began pursuing the motorist, determine (*a*) the distance the officer traveled before overtaking the motorist, (*b*) the motorist's speed.

SOLUTION



$(v_P)_{18} = 0 \qquad (v_P)_{26} = 60 \text{ mi/h} = 88 \text{ ft/s} \qquad (v_P)_{42} = 90 \text{ mi/h} = 88 \text{ ft/s}$

(*a*) Patrol car:

For $18 \text{ s} < t \leq 26 \text{ s}$: $\quad v_P = 0 + a_P(t - 18)$

At $t = 26$ s: $\quad 88 \text{ ft/s} = a_P(26 - 18) \text{ s}$

or $\quad a_P = 11 \text{ ft/s}^2$

Also, $\quad x_P = 0 + 0(t - 18) - \dfrac{1}{2}a_P(t - 18)^2$

At $t = 26$ s: $\quad (x_P)_{26} = \dfrac{1}{2}(11 \text{ ft/s}^2)(26 - 18)^2 = 352 \text{ ft}$

For $26 \text{ s} < t \leq 42 \text{ s}$: $\quad x_P = (x_P)_{26} + (v_P)_{26}(t - 26)$

At $t = 42$ s:

$$(x_P)_{42} = 352 \text{ m} + (88 \text{ ft/s})(42 - 26) \text{ s} = 1760 \text{ ft}$$

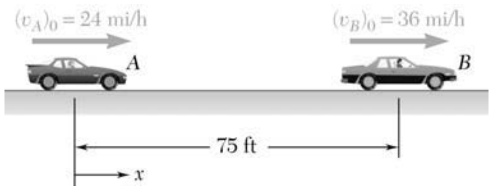
$(x_P)_{42} = 1760 \text{ ft}$ ◀

(*b*) For the motorist's car: $\quad x_M = 0 + v_M t$

At $t = 42$ s, $x_M = x_P$: $\quad 1760 \text{ ft} = v_M (42 \text{ s})$

or $\quad v_M = 41.9048 \text{ ft/s}$

or $\quad v_M = 28.6 \text{ mi/h}$ ◀



PROBLEM 11.42

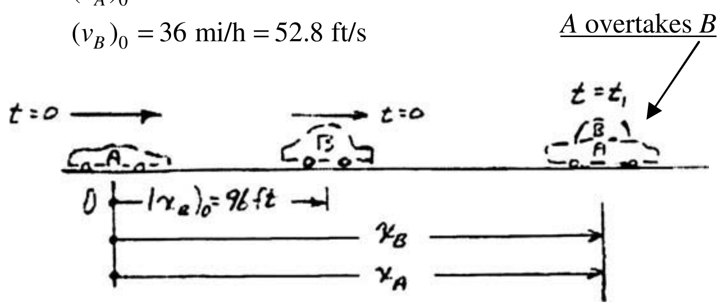
Automobiles A and B are traveling in adjacent highway lanes and at $t = 0$ have the positions and speeds shown. Knowing that automobile A has a constant acceleration of 1.8 ft/s^2 and that B has a constant deceleration of 1.2 ft/s^2, determine (a) when and where A will overtake B, (b) the speed of each automobile at that time.

SOLUTION

$$a_A = +1.8 \text{ ft/s}^2 \quad a_B = -1.2 \text{ ft/s}^2$$

$$(v_A)_0 = 24 \text{ mi/h} = 35.2 \text{ ft/s}$$

$$(v_B)_0 = 36 \text{ mi/h} = 52.8 \text{ ft/s}$$



Motion of auto A:

$$v_A = (v_A)_0 + a_A t = 35.2 + 1.8t \tag{1}$$

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2 = 0 + 35.2t + \frac{1}{2}(1.8)t^2 \tag{2}$$

Motion of auto B:

$$v_B = (v_B)_0 + a_B t = 52.8 - 1.2t \tag{3}$$

$$x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2 = 75 + 52.8t + \frac{1}{2}(-1.2)t^2 \tag{4}$$

(a) A overtakes B at $t = t_1$.

$$x_A = x_B: \quad 35.2t + 0.9t_1^2 = 75 + 52.8t_1 - 0.6t_1^2$$

$$1.5t_1^2 - 17.6t_1 - 75 = 0$$

$$t_1 = -3.22 \text{ s} \quad \text{and} \quad t_1 = 15.0546 \qquad t_1 = 15.05 \text{ s} \blacktriangleleft$$

Eq. (2): $$x_A = 35.2(15.05) + 0.9(15.05)^2 \qquad x_A = 734 \text{ ft} \blacktriangleleft$$

PROBLEM 11.42 (Continued)

(*b*) Velocities when $t_1 = 15.05$ s

Eq. (1): $v_A = 35.2 + 1.8(15.05)$

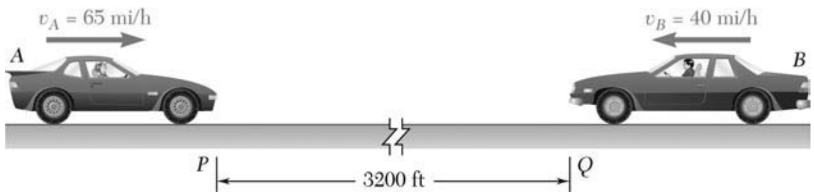
$v_A = 62.29$ ft/s $\qquad v_A = 42.5$ mi/h $\longrightarrow$ ◀

Eq. (3): $v_B = 52.8 - 1.2(15.05)$

$v_B = 34.74$ ft/s $\qquad v_B = 23.7$ mi/h $\longrightarrow$ ◀

PROBLEM 11.43

Two automobiles A and B are approaching each other in adjacent highway lanes. At $t = 0$, A and B are 3200 ft apart, their speeds are $v_A = 65$ mi/h and $v_B = 40$ mi/h, and they are at Points P and Q, respectively. Knowing that A passes Point Q 40 s after B was there and that B passes Point P 42 s after A was there, determine (a) the uniform accelerations of A and B, (b) when the vehicles pass each other, (c) the speed of B at that time.



SOLUTION

(a) We have $$x_A = 0 + (v_A)_0 t + \frac{1}{2} a_A t^2 \quad (v_A)_0 = 65 \text{ mi/h} = 95.33 \text{ ft/s}$$

(x is positive $\longrightarrow$; origin at P.)

At $t = 40$ s: $$3200 \text{ m} = (95.333 \text{ m/s})(40 \text{ s}) + \frac{1}{2} a_A (40 \text{ s})^2$$ $$a_A = -0.767 \text{ ft/s}^2 \blacktriangleleft$$

Also, $x_B = 0 + (v_B)_0 t + \frac{1}{2} a_B t^2 \quad (v_B)_0 = 40 \text{ mi/h} = 58.667 \text{ ft/s}$

(x_B is positive $\longleftarrow$; origin at Q.)

At $t = 42$ s: $$3200 \text{ ft} = (58.667 \text{ ft/s})(42 \text{ s}) + \frac{1}{2} a_B (42 \text{ s})^2$$

or $$a_B = 0.83447 \text{ ft/s}^2 \qquad a_B = 0.834 \text{ ft/s}^2 \blacktriangleleft$$

(b) When the cars pass each other $\quad x_A + x_B = 3200$ ft

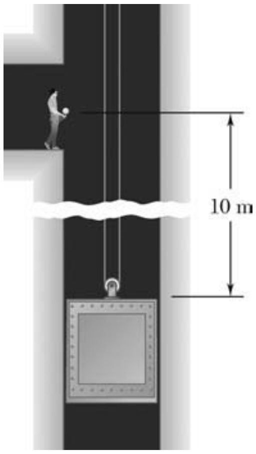
Then $$(95.333 \text{ ft/s})t_{AB} + \frac{1}{2}(-0.76667 \text{ ft/s})t_{AB}^2 + (58.667 \text{ ft/s})t_{AB} + \frac{1}{2}(0.83447 \text{ ft/s}^2)t_{AB}^2 = 3200 \text{ ft}$$

or $$0.03390 t_{AB}^2 + 154 t_{AB} - 3200 = 0$$

Solving $\quad t = 20.685$ s $\quad$ and $\quad t = -4563$ s $\qquad t > 0 \Rightarrow \qquad t_{AB} = 20.7 \text{ s} \blacktriangleleft$

(c) We have $$v_B = (v_B)_0 + a_B t$$

At $t = t_{AB}$: $$v_B = 58.667 \text{ ft/s} + (0.83447 \text{ ft/s}^2)(20.685 \text{ s})$$ $$= 75.927 \text{ ft/s} \qquad v_B = 51.8 \text{ mi/h} \blacktriangleleft$$



PROBLEM 11.44

An elevator is moving upward at a constant speed of 4 m/s. A man standing 10 m above the top of the elevator throws a ball upward with a speed of 3 m/s. Determine (*a*) when the ball will hit the elevator, (*b*) where the ball will hit the elevator with respect to the location of the man.

SOLUTION

Place the origin of the position coordinate at the level of the standing man, the positive direction being up. The ball undergoes uniformly accelerated motion.

$$y_B = (y_B)_0 + (v_B)_0 t - \frac{1}{2} g t^2$$

with $(y_B)_0 = 0$, $(v_B)_0 = 3$ m/s, and $g = 9.81$ m/s^2.

$$y_B = 3t - 4.905t^2$$

The elevator undergoes uniform motion.

$$y_E = (y_E)_0 + v_E t$$

with $(y_E)_0 = -10$ m and $v_E = 4$ m/s.

(*a*) Time of impact. Set $y_B = y_E$

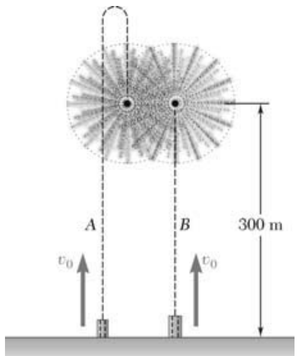
$$3t - 4.905t^2 = -10 + 4t$$
$$4.905t^2 + t - 10 = 0$$

$t = 1.3295$ and -1.5334 $\qquad t = 1.330$ s ◀

(*b*) Location of impact.

$$y_B = (3)(1.3295) - (4.905)(1.3295)^2 = -4.68 \text{ m}$$
$$y_E = -10 + (4)(1.3295) = -4.68 \text{ m} \qquad \text{(checks)}$$

4.68 m below the man ◀



PROBLEM 11.45

Two rockets are launched at a fireworks display. Rocket A is launched with an initial velocity $v_0 = 100$ m/s and rocket B is launched t_1 seconds later with the same initial velocity. The two rockets are timed to explode simultaneously at a height of 300 m as A is falling and B is rising. Assuming a constant acceleration $g = 9.81\ \text{m/s}^2$, determine (a) the time t_1, (b) the velocity of B relative to A at the time of the explosion.

SOLUTION

Place origin at ground level. The motion of rockets A and B is

Rocket A:
$$v_A = (v_A)_0 - gt = 100 - 9.81t \quad (1)$$

$$y_A = (y_A)_0 + (v_A)_0 t - \frac{1}{2}gt^2 = 100t - 4.905t^2 \quad (2)$$

Rocket B:
$$v_B = (v_B)_0 - g(t - t_1) = 100 - 9.81(t - t_1) \quad (3)$$

$$y_B = (y_B)_0 + (v_B)_0(t - t_1) - \frac{1}{2}g(t - t_1)^2$$
$$= 100(t - t_1) - 4.905(t - t_1)^2 \quad (4)$$

Time of explosion of rockets A and B. $y_A = y_B = 300$ ft

From (2),
$$300 = 100t - 4.905t^2$$
$$4.905t^2 - 100t + 300 = 0$$
$$t = 16.732 \text{ s} \quad \text{and} \quad 3.655 \text{ s}$$

From (4),
$$300 = 100(t - t_1) - 4.905(t - t_1^2)$$
$$t - t_1 = 16.732 \text{ s} \quad \text{and} \quad 3.655 \text{ s}$$

Since rocket A is falling, $t = 16.732$ s

Since rocket B is rising, $t - t_1 = 3.655$ s

(a) Time t_1: $t_1 = t - (t - t_1)$ $\quad t_1 = 13.08 \text{ s}$ ◀

(b) Relative velocity at explosion.

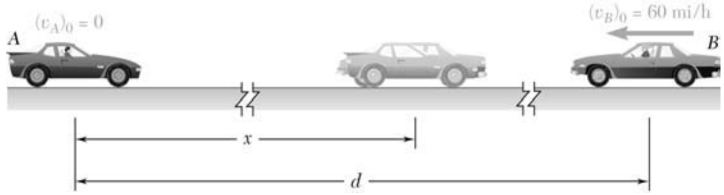
From (1), $v_A = 100 - (9.81)(16.732) = -64.15$ m/s

From (3), $v_B = 100 - (9.81)(16.732 - 13.08) = 64.15$ m/s

Relative velocity: $v_{B/A} = v_B - v_A$ $\quad v_{B/A} = 128.3 \text{ m/s} \uparrow$ ◀

PROBLEM 11.46

Car A is parked along the northbound lane of a highway, and car B is traveling in the southbound lane at a constant speed of 60 mi/h. At $t = 0$, A starts and accelerates at a constant rate a_A, while at $t = 5$ s, B begins to slow down with a constant deceleration of magnitude $a_A/6$. Knowing that when the cars pass each other $x = 294$ ft and $v_A = v_B$, determine (*a*) the acceleration a_A, (*b*) when the vehicles pass each other, (*c*) the distance *d* between the vehicles at $t = 0$.



SOLUTION

For $t \geq 0$: $v_A = 0 + a_A t$

$$x_A = 0 + 0 + \frac{1}{2}a_A t^2$$

$0 \leq t < 5$ s: $x_B = 0 + (v_B)_0 t \quad (v_B)_0 = 60 \text{ mi/h} = 88 \text{ ft/s}$

At $t = 5$ s: $x_B = (88 \text{ ft/s})(5 \text{ s}) = 440 \text{ ft}$

For $t \geq 5$ s: $v_B = (v_B)_0 + a_B(t-5) \qquad a_B = -\frac{1}{6}a_A$

$$x_B = (x_B)_S + (v_B)_0(t-5) + \frac{1}{2}a_B(t-5)^2$$

Assume $t > 5$ s when the cars pass each other.

At that time (t_{AB}),

$v_A = v_B$: $a_A t_{AB} = (88 \text{ ft/s}) - \frac{a_A}{6}(t_{AB} - 5)$

$x_A = 294$ ft: $294 \text{ ft} = \frac{1}{2}a_A t_{AB}^2$

Then $$\frac{a_A\left(\frac{7}{6}t_{AB} - \frac{5}{6}\right)}{\frac{1}{2}a_A t_{AB}^2} = \frac{88}{294}$$

or $$44t_{AB}^2 - 343t_{AB} + 245 = 0$$

PROBLEM 11.46 (Continued)

Solving $t_{AB} = 0.795 \text{ s}$ and $t_{AB} = 7.00 \text{ s}$

(a) With $t_{AB} > 5 \text{ s}$, $$294 \text{ ft} = \frac{1}{2} a_A (7.00 \text{ s})^2$$

or $$a_A = 12.00 \text{ ft/s}^2 \blacktriangleleft$$

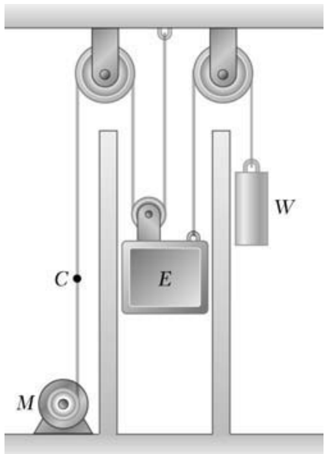
(b) From above $$t_{AB} = 7.00 \text{ s} \blacktriangleleft$$

Note: An acceptable solution cannot be found if it is assumed that $t_{AB} \leq 5$ s.

(c) We have
$$\begin{aligned} d &= x + (x_B)_{t_{AB}} \\ &= 294 \text{ ft} + 440 \text{ ft} + (88 \text{ ft/s})(2.00 \text{ s}) \\ &\quad + \frac{1}{2}\left(-\frac{1}{6} \times 12.00 \text{ ft/s}^2\right)(2.00 \text{ s})^2 \end{aligned}$$

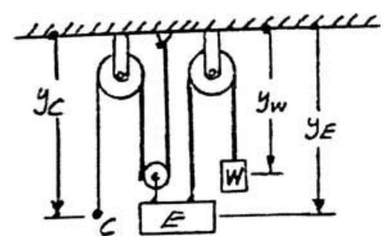
or $$d = 906 \text{ ft} \blacktriangleleft$$

PROBLEM 11.47



The elevator shown in the figure moves downward with a constant velocity of 4 m/s. Determine (*a*) the velocity of the cable *C*, (*b*) the velocity of the counterweight *W*, (*c*) the relative velocity of the cable *C* with respect to the elevator, (*d*) the relative velocity of the counterweight *W* with respect to the elevator.

SOLUTION



Choose the positive direction downward.

(*a*) Velocity of cable *C*.

$$y_C + 2y_E = \text{constant}$$

$$v_C + 2v_E = 0$$

But, $v_E = 4 \text{ m/s}$

or $v_C = -2v_E = -8 \text{ m/s}$ $\mathbf{v}_C = 8.00 \text{ m/s} \uparrow$ ◀

(*b*) Velocity of counterweight *W*.

$$y_W + y_E = \text{constant}$$

$$v_W + v_E = 0 \quad v_W = -v_E = -4 \text{ m/s}$$ $\mathbf{v}_W = 4.00 \text{ m/s} \uparrow$ ◀

(*c*) Relative velocity of *C* with respect to *E*.

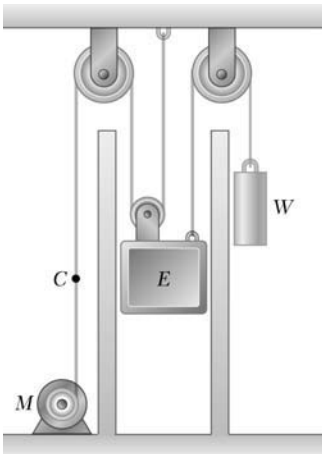
$$v_{C/E} = v_C - v_E = (-8 \text{ m/s}) - (+4 \text{ m/s}) = -12 \text{ m/s}$$

$\mathbf{v}_{C/E} = 12.00 \text{ m/s} \uparrow$ ◀

(*d*) Relative velocity of *W* with respect to *E*.

$$v_{W/E} = v_W - v_E = (-4 \text{ m/s}) - (4 \text{ m/s}) = -8 \text{ m/s}$$

$\mathbf{v}_{W/E} = 8.00 \text{ m/s} \uparrow$ ◀

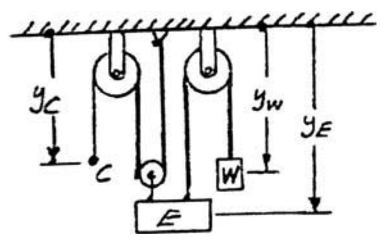


PROBLEM 11.48

The elevator shown starts from rest and moves upward with a constant acceleration. If the counterweight W moves through 30 ft in 5 s, determine (*a*) the acceleration of the elevator and the cable C, (*b*) the velocity of the elevator after 5 s.

SOLUTION

We choose positive direction downward for motion of counterweight.



$$y_W = \frac{1}{2}a_W t^2$$

At $t = 5$ s,

$$y_W = 30 \text{ ft}$$

$$30 \text{ ft} = \frac{1}{2}a_W (5 \text{ s})^2$$

$$a_W = 2.4 \text{ ft/s}^2 \qquad \mathbf{a}_W = 2.4 \text{ ft/s}^2 \downarrow$$

(*a*) Accelerations of E and C.

Since $y_W + y_E = \text{constant}$ $\quad v_W + v_E = 0,$ and $\quad a_W + a_E = 0$

Thus: $a_E = -a_W = -(2.4 \text{ ft/s}^2),$ $\qquad \mathbf{a}_E = 2.40 \text{ ft/s}^2 \uparrow$ ◀

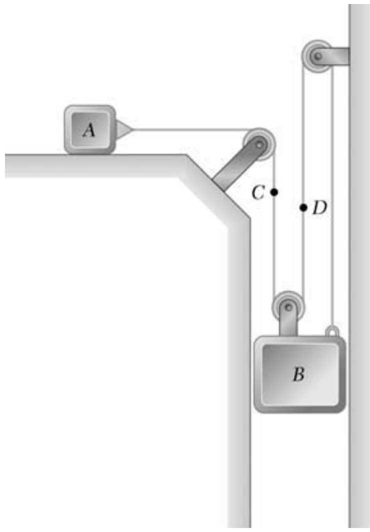
Also, $y_C + 2y_E = \text{ constant},$ $\quad v_C + 2v_E = 0,$ and $\quad a_C + 2a_E = 0$

Thus: $a_C = -2a_E = -2(-2.4 \text{ ft/s}^2) = +4.8 \text{ ft/s}^2,$ $\qquad \mathbf{a}_C = 4.80 \text{ ft/s}^2 \downarrow$ ◀

(*b*) Velocity of elevator after 5 s.

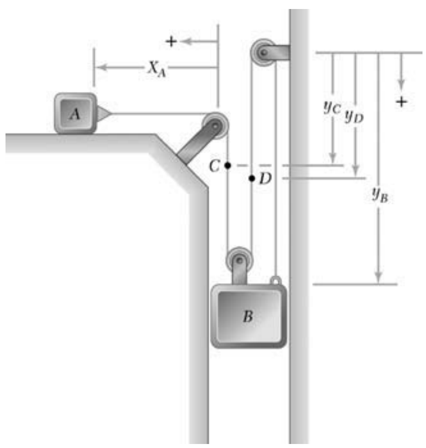
$$v_E = (v_E)_0 + a_E t = 0 + (-2.4 \text{ ft/s}^2)(5 \text{ s}) = -12 \text{ ft/s} \qquad (\mathbf{v}_E)_5 = 12.00 \text{ ft/s} \uparrow$$ ◀

PROBLEM 11.49



Slider block A moves to the left with a constant velocity of 6 m/s. Determine (a) the velocity of block B, (b) the velocity of portion D of the cable, (c) the relative velocity of portion C of the cable with respect to portion D.

SOLUTION



From the diagram, we have

$$x_A + 3y_B = \text{constant}$$

Then $\quad v_A + 3v_B = 0 \qquad (1)$

and $\quad a_A + 3a_B = 0 \qquad (2)$

(a) Substituting into Eq. (1) $\quad 6 \text{ m/s} + 3v_B = 0$

or $\quad \mathbf{v}_B = 2.00 \text{ m/s} \uparrow$ ◀

(b) From the diagram $\quad y_B + y_D = \text{constant}$

Then $\quad v_B + v_D = 0$

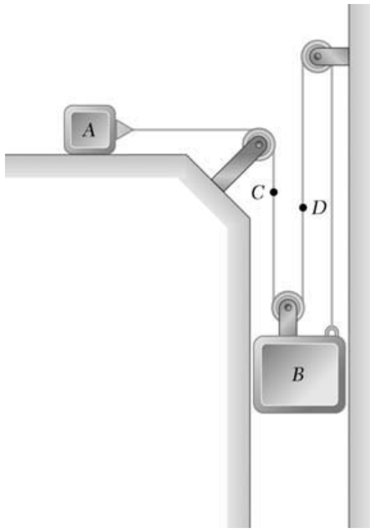
$\mathbf{v}_D = 2.00 \text{ m/s} \downarrow$ ◀

(c) From the diagram $\quad x_A + y_C = \text{constant}$

Then $\quad v_A + v_C = 0 \qquad v_C = -6 \text{ m/s}$

Now $\quad v_{C/D} = v_C - v_D = (-6 \text{ m/s}) - (2 \text{ m/s}) = -8 \text{ m/s}$

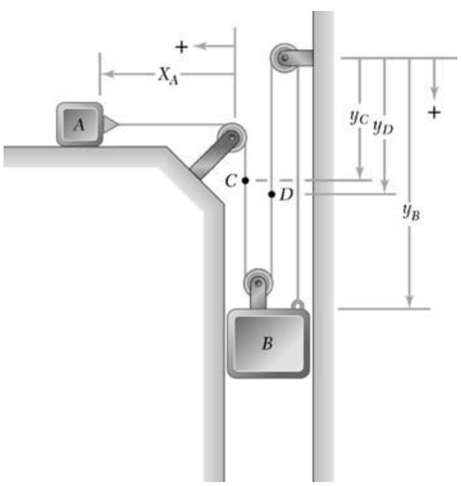
$\mathbf{v}_{C/D} = 8.00 \text{ m/s} \uparrow$ ◀



PROBLEM 11.50

Block B starts from rest and moves downward with a constant acceleration. Knowing that after slider block A has moved 9 in. its velocity is 6 ft/s, determine (*a*) the accelerations of A and B, (*b*) the velocity and the change in position of B after 2 s.

SOLUTION



From the diagram, we have

$$x_A + 3y_B = \text{constant}$$

Then $\quad v_A + 3v_B = 0 \quad (1)$

and $\quad a_A + 3a_B = 0 \quad (2)$

(*a*) Eq. (2): $a_A + 3a_B = 0$ and $\mathbf{a}_B$ is constant and positive $\Rightarrow \mathbf{a}_A$ is constant and negative

Also, Eq. (1) and $(v_B)_0 = 0 \Rightarrow (v_A)_0 = 0$

Then $\quad v_A^2 = 0 + 2a_A[x_A - (x_A)_0]$

When $|\Delta x_A| = 0.4$ m: $\quad (6 \text{ ft/s})^2 = 2a_A(9/12 \text{ ft})$

or $\quad \mathbf{a}_A = 24.0 \text{ ft/s}^2 \longrightarrow$ ◀

Then, substituting into Eq. (2):

$$-24 \text{ ft/s}^2 + 3a_B = 0$$

or $\quad a_B = \frac{24}{3} \text{ ft/s}^2 \qquad \mathbf{a}_B = 8.00 \text{ ft/s}^2 \downarrow$ ◀

PROBLEM 11.50 (Continued)

(*b*) We have $v_B = 0 + a_B t$

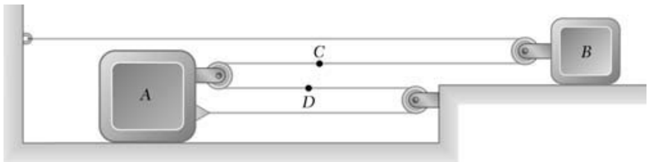
At $t = 2$ s: $v_B = \left(\frac{24}{3} \text{ ft/s}^2\right)(2 \text{ s})$

or $\mathbf{v}_B = 16.00 \text{ ft/s} \downarrow$ ◀

Also $y_B = (y_B)_0 + 0 + \frac{1}{2} a_B t^2$

At $t = 2$ s: $y_B - (y_B)_0 = \frac{1}{2}\left(\frac{24}{3} \text{ ft/s}^2\right)(2 \text{ s})^2$

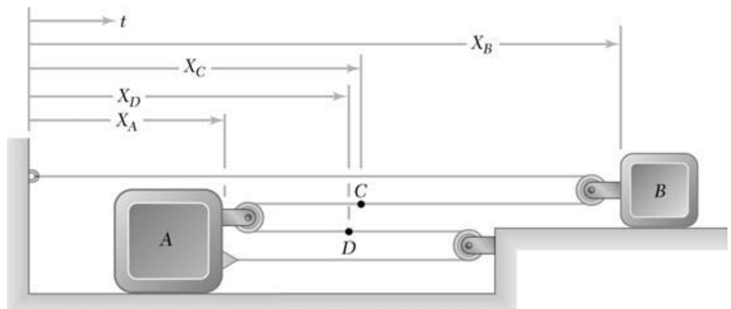
or $\mathbf{y}_B - (\mathbf{y}_B)_0 = 16.00 \text{ ft} \downarrow$ ◀



PROBLEM 11.51

Slider block B moves to the right with a constant velocity of 300 mm/s. Determine (a) the velocity of slider block A, (b) the velocity of portion C of the cable, (c) the velocity of portion D of the cable, (d) the relative velocity of portion C of the cable with respect to slider block A.

SOLUTION



From the diagram $$x_B + (x_B - x_A) - 2x_A = \text{constant}$$

Then $$2v_B - 3v_A = 0 \quad (1)$$

and $$2a_B - 3a_A = 0 \quad (2)$$

Also, we have $$-x_D - x_A = \text{constant}$$

Then $$v_D + v_A = 0 \quad (3)$$

(a) Substituting into Eq. (1) $$2(300 \text{ mm/s}) - 3v_A = 0$$

or $$\mathbf{v}_A = 200 \text{ mm/s} \longrightarrow \blacktriangleleft$$

(b) From the diagram $$x_B + (x_B - x_C) = \text{constant}$$

Then $$2v_B - v_C = 0$$

Substituting $$2(300 \text{ mm/s}) - v_C = 0$$

or $$\mathbf{v}_C = 600 \text{ mm/s} \longrightarrow \blacktriangleleft$$

PROBLEM 11.51 (Continued)

(*c*) From the diagram $(x_C - x_A) + (x_D - x_A) = \text{constant}$

Then $v_C - 2v_A + v_D = 0$

Substituting $600 \text{ mm/s} - 2(200 \text{ mm/s}) + v_D = 0$

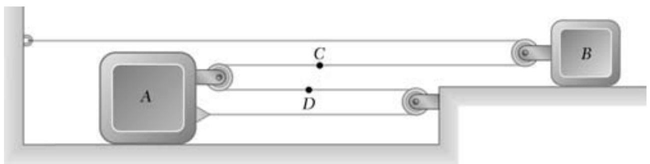
or $\mathbf{v}_D = 200 \text{ mm/s} \longleftarrow$ ◀

(*d*) We have

$$v_{C/A} = v_C - v_A$$
$$= 600 \text{ mm/s} - 200 \text{ mm/s}$$

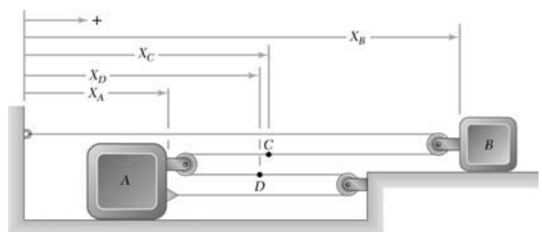
or $\mathbf{v}_{C/A} = 400 \text{ mm/s} \longrightarrow$ ◀

PROBLEM 11.52



At the instant shown, slider block B is moving with a constant acceleration, and its speed is 150 mm/s. Knowing that after slider block A has moved 240 mm to the right its velocity is 60 mm/s, determine (*a*) the accelerations of A and B, (*b*) the acceleration of portion D of the cable, (*c*) the velocity and change in position of slider block B after 4 s.

SOLUTION



From the diagram $\quad x_B + (x_B - x_A) - 2x_A = \text{constant}$

Then
$$2v_B - 3v_A = 0 \qquad (1)$$

and
$$2a_B - 3a_A = 0 \qquad (2)$$

(*a*) First observe that if block A moves to the right, $\mathbf{v}_A \rightarrow$ and Eq. (1) $\Rightarrow \mathbf{v}_B \rightarrow$. Then, using Eq. (1) at $t = 0$

$$2(150 \text{ mm/s}) - 3(v_A)_0 = 0$$

or
$$(v_A)_0 = 100 \text{ mm/s}$$

Also, Eq. (2) and $a_B =$ constant $\Rightarrow a_A = \text{constant}$

Then
$$v_A^2 = (v_A)_0^2 + 2a_A[x_A - (x_A)_0]$$

When $x_A - (x_A)_0 = 240$ mm:

$$(60 \text{ mm/s})^2 = (100 \text{ mm/s})^2 + 2a_A(240 \text{ mm})$$

or
$$a_A = -\frac{40}{3} \text{ mm/s}^2$$

or
$$\mathbf{a}_A = 13.33 \text{ mm/s}^2 \leftarrow \blacktriangleleft$$

PROBLEM 11.52 (Continued)

Then, substituting into Eq. (2)

$$2a_B - 3\left(-\frac{40}{3}\ \text{mm/s}^2\right) = 0$$

or $a_B = -20\ \text{mm/s}^2$ $\mathbf{a}_B = 20.0\ \text{mm/s}^2 \longleftarrow$ ◀

(*b*) From the diagram, $-x_D - x_A = \text{constant}$

$$v_D + v_A = 0$$

Then $a_D + a_A = 0$

Substituting $a_D + \left(-\frac{40}{3}\ \text{mm/s}^2\right) = 0$

or $\mathbf{a}_D = 13.33\ \text{mm/s}^2 \longrightarrow$ ◀

(*c*) We have $v_B = (v_B)_0 + a_B t$

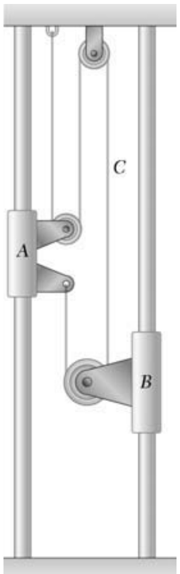
At $t = 4$ s: $v_B = 150\ \text{mm/s} + (-20.0\ \text{mm/s}^2)(4\ \text{s})$

or $\mathbf{v}_B = 70.0\ \text{mm/s} \longrightarrow$ ◀

Also $x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$

At $t = 4$ s: $x_B - (x_B)_0 = (150\ \text{mm/s})(4\ \text{s}) + \frac{1}{2}(-20.0\ \text{mm/s}^2)(4\ \text{s})^2$

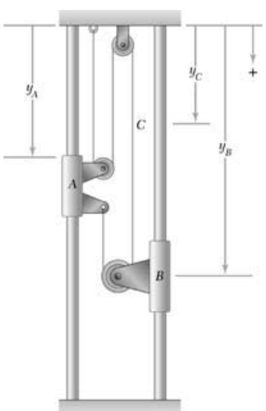
or $\mathbf{x}_B - (\mathbf{x}_B)_0 = 440\ \text{mm} \longrightarrow$ ◀



PROBLEM 11.53

Collar A starts from rest and moves upward with a constant acceleration. Knowing that after 8 s the relative velocity of collar B with respect to collar A is 24 in./s, determine (*a*) the accelerations of A and B, (*b*) the velocity and the change in position of B after 6 s.

SOLUTION



From the diagram

$$2y_A + y_B + (y_B - y_A) = \text{constant}$$

Then $$v_A + 2v_B = 0 \qquad (1)$$

and $$a_A + 2a_B = 0 \qquad (2)$$

(*a*) Eq. (1) and $(v_A)_0 = 0 \Rightarrow (v_B)_0 = 0$

Also, Eq. (2) and $\mathbf{a}_A$ is constant and negative $\Rightarrow \mathbf{a}_B$ is constant and positive.

Then $$v_A = 0 + a_A t \qquad v_B = 0 + a_B t$$

Now $$v_{B/A} = v_B - v_A = (a_B - a_A)t$$

From Eq. (2) $$a_B = -\frac{1}{2}a_A$$

So that $$v_{B/A} = -\frac{3}{2}a_A t$$

PROBLEM 11.53 (Continued)

At $t = 8$ s: $$24 \text{ in./s} = -\frac{3}{2}a_A(8 \text{ s})$$

or $$\mathbf{a}_A = 2.00 \text{ in./s}^2 \uparrow \blacktriangleleft$$

and then $$a_B = -\frac{1}{2}(-2 \text{ in./s}^2)$$

or $$\mathbf{a}_B = 1.000 \text{ in./s}^2 \downarrow \blacktriangleleft$$

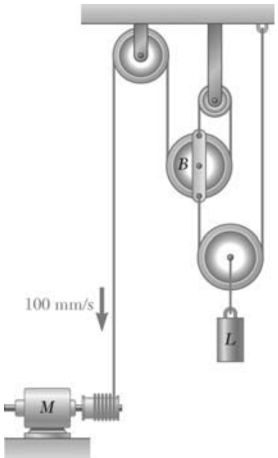
(*b*) At $t = 6$ s: $$v_B = (1 \text{ in./s}^2)(6 \text{ s})$$

or $$\mathbf{v}_B = 6.00 \text{ in./s} \downarrow \blacktriangleleft$$

Now $$y_B = (y_B)_0 + 0 + \frac{1}{2}a_B t^2$$

At $t = 6$ s: $$y_B - (y_B)_0 = \frac{1}{2}(1 \text{ in./s}^2)(6 \text{ s})^2$$

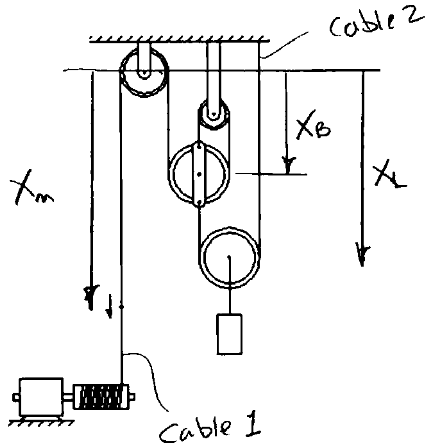
or $$\mathbf{y}_B - (\mathbf{y}_B)_0 = 18.00 \text{ in.} \downarrow \blacktriangleleft$$



PROBLEM 11.54

The motor M reels in the cable at a constant rate of 100 mm/s. Determine (*a*) the velocity of load L, (*b*) the velocity of pulley B with respect to load L.

SOLUTION



Let x_B and x_L be the positions, respectively, of pulley B and load L measured downward from a fixed elevation above both. Let x_M be the position of a point on the cable about to enter the reel driven by the motor. Then, considering the lengths of the two cables,

$$x_M + 3x_B = \text{constant} \qquad v_M + 3v_B = 0$$

$$x_L + (x_L - x_B) = \text{constant} \qquad 2v_L + v_B = 0$$

with

$$v_M = 100 \text{ mm/s}$$

$$v_B = -\frac{v_M}{3} = -33.333 \text{ m/s}$$

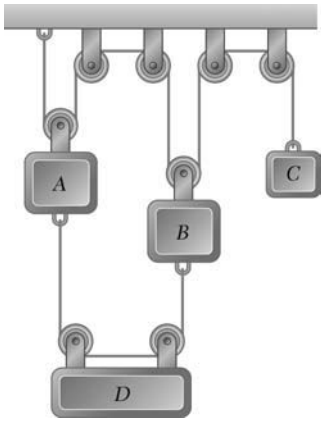
$$v_L = \frac{v_B}{2} = -16.667 \text{ mm/s}$$

(*a*) Velocity of load L. $\mathbf{v}_L = 16.67 \text{ mm/s} \uparrow$ ◀

(*b*) Velocity of pulley B with respect to load L. $v_{B/L} = v_B - v_L = -33.333 - (-16.667) = -16.667$

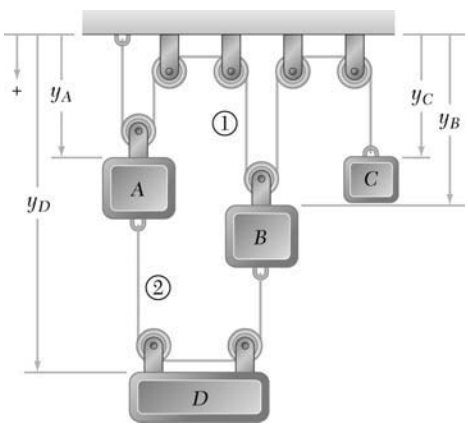
$\mathbf{v}_{B/L} = 16.67 \text{ mm/s} \uparrow$ ◀

PROBLEM 11.55



Block C starts from rest at $t = 0$ and moves downward with a constant acceleration of 4 in./s^2. Knowing that block B has a constant velocity of 3 in./s upward, determine (*a*) the time when the velocity of block A is zero, (*b*) the time when the velocity of block A is equal to the velocity of block D, (*c*) the change in position of block A after 5 s.

SOLUTION



From the diagram:

Cord 1: $$2y_A + 2y_B + y_C = \text{constant}$$

Then $$2v_A + 2v_B + v_C = 0$$

and $$2a_A + 2a_B + a_C = 0 \qquad (1)$$

Cord 2: $$(y_D - y_A) + (y_D - y_B) = \text{constant}$$

Then $$2v_D - v_A - v_B = 0$$

and $$2a_D - a_A - a_B = 0 \qquad (2)$$

Use units of inches and seconds.

Motion of block C:
$$v_C = v_{C0} + a_C t$$
$$= 0 + 4t \quad \text{where} \quad a_C = -4 \text{ in./s}^2$$

Motion of block B:
$$v_B = -3 \text{ in./s}; \qquad a_B = 0$$

Motion of block A: From (1) and (2),

$$v_A = -v_B - \frac{1}{2}v_C = 3 - \frac{1}{2}(4t) = 3 - 2t \text{ in./s}$$

$$a_A = -a_B - \frac{1}{2}a_C = 0 - \frac{1}{2}(4) = -2 \text{ in./s}^2$$

PROBLEM 11.55 (Continued)

(*a*) Time when v_B is zero.

$$3-2t=0 \qquad t=1.500 \text{ s} \blacktriangleleft$$

Motion of block *D*: From (3),

$$v_D=\frac{1}{2}v_A+\frac{1}{2}v_B=\frac{1}{2}(3-2t)-\frac{1}{2}(3)=-1t$$

(*b*) Time when v_A is equal to v_0.

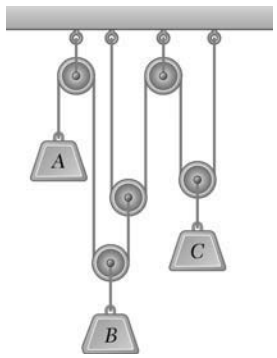
$$3-2t=-t \qquad t=3.00 \text{ s} \blacktriangleleft$$

(*c*) Change in position of block *A* ($t=5$ s).

$$\Delta y_A=(v_A)_0t+\frac{1}{2}a_At^2$$

$$=(3)(5)+\frac{1}{2}(-2)(5)^2=-10 \text{ in.}$$

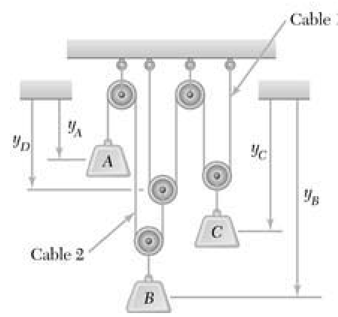
$$\text{Change in position } =10.00 \text{ in.}\uparrow \blacktriangleleft$$



PROBLEM 11.56

Block A starts from rest at $t = 0$ and moves downward with a constant acceleration of 6 in./s^2. Knowing that block B moves up with a constant velocity of 3 in./s, determine (*a*) the time when the velocity of block C is zero, (*b*) the corresponding position of block C.

SOLUTION



The cable lengths are constant.

$$L_1 = 2y_C + 2y_D + \text{constant}$$

$$L_2 = y_A + y_B + (y_B - y_D) + \text{constant}$$

Eliminate y_D.

$$L_1 + 2L_2 = 2y_C + 2y_D + 2y_A + 2y_B + 2(y_B - y_D) + \text{constant}$$

$$2(y_C + y_A + 2y_B) = \text{constant}$$

Differentiate to obtain relationships for velocities and accelerations, positive downward.

$$v_C + v_A + 2v_B = 0 \qquad (1)$$

$$a_C + a_A + 2a_B = 0 \qquad (2)$$

Use units of inches and seconds.

Motion of block A: $\quad v_A = a_A t + 6t$

$$\Delta y_A = \frac{1}{2}a_A t^2 = \frac{1}{2}(6)t^2 = 3t^2$$

Motion of block B: $\quad \mathbf{v}_B = 3\text{ in./s}\uparrow \quad v_B = -3\text{ in./s}$

$$\Delta y_B = v_B t = -3t$$

Motion of block C: From (1),

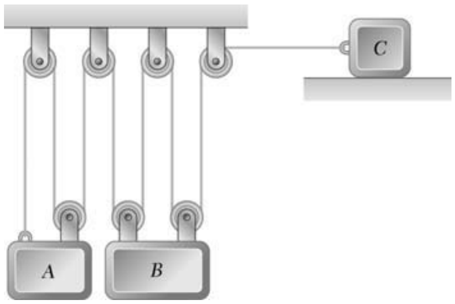
$$v_C = -v_A - 2v_B = -6t - 2(-3) = 6 - 6t$$

$$\Delta y_C = \int_0^t v_C\,dt = 6t - 3t^2$$

(*a*) Time when v_C is zero. $\quad 6 - 6t = 0 \qquad t = 1.000\text{ s}$ ◀

(*b*) Corresponding position.

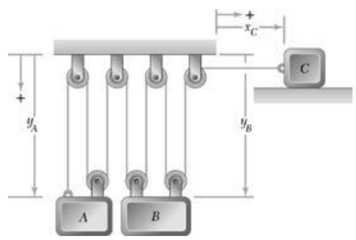
$$\Delta y_C = (6)(1) - (3)(1)^2 = 3\text{ in.} \qquad \Delta y_C = 3.00\text{ in.}\downarrow$$ ◀



PROBLEM 11.57

Block B starts from rest, block A moves with a constant acceleration, and slider block C moves to the right with a constant acceleration of 75 mm/s^2. Knowing that at $t = 2\text{ s}$ the velocities of B and C are 480 mm/s downward and 280 mm/s to the right, respectively, determine (*a*) the accelerations of A and B, (*b*) the initial velocities of A and C, (*c*) the change in position of slider block C after 3 s.

SOLUTION



From the diagram

$$3y_A + 4y_B + x_C = \text{constant}$$

Then $$3v_A + 4v_B + v_C = 0 \qquad (1)$$

and $$3a_A + 4a_B + a_C = 0 \qquad (2)$$

Given: $$(v_B) = 0,$$

$$a_A = \text{constent}$$

$$(\mathbf{a}_C) = 75\text{ mm/s}^2 \longrightarrow$$

At $t = 2\text{ s}$, $$\mathbf{v}_B = 480\text{ mm/s}\downarrow$$

$$\mathbf{v}_C = 280\text{ mm/s} \longrightarrow$$

(*a*) Eq. (2) and $a_A = \text{constant}$ and $a_C = \text{constant} \Rightarrow a_B = \text{constant}$

Then $$v_B = 0 + a_B t$$

At $t = 2\text{ s}$: $$480\text{ mm/s} = a_B(2\text{ s})$$

$$a_B = 240\text{ mm/s}^2 \qquad \text{or} \qquad \mathbf{a}_B = 240\text{ mm/s}^2\downarrow \blacktriangleleft$$

Substituting into Eq. (2)

$$3a_A + 4(240\text{ mm/s}^2) + (75\text{ mm/s}^2) = 0$$

$$a_A = -345\text{ mm/s} \qquad \text{or} \qquad \mathbf{a}_A = 345\text{ mm/s}^2\uparrow \blacktriangleleft$$

PROBLEM 11.57 (Continued)

(*b*) We have $v_C = (v_C)_0 + a_C t$

At $t = 2$ s: $280 \text{ mm/s} = (v_C)_0 + (75 \text{ mm/s})(2 \text{ s})$

$v_C = 130 \text{ mm/s}$ or $(\mathbf{v}_C)_0 = 130.0 \text{ mm/s} \longrightarrow$ ◀

Then, substituting into Eq. (1) at $t = 0$

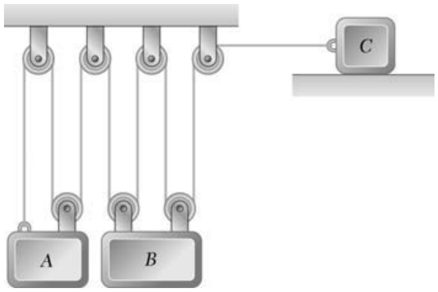
$$3(v_A)_0 + 4(0) + (130 \text{ mm/s}) = 0$$

$v_A = -43.3 \text{ mm/s}$ or $(\mathbf{v}_A)_0 = 43.3 \text{ mm/s} \uparrow$ ◀

(*c*) We have $x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2$

At $t = 3$ s: $x_C - (x_C)_0 = (130 \text{ mm/s})(3 \text{ s}) + \frac{1}{2}(75 \text{ mm/s}^2)(3 \text{ s})^2$

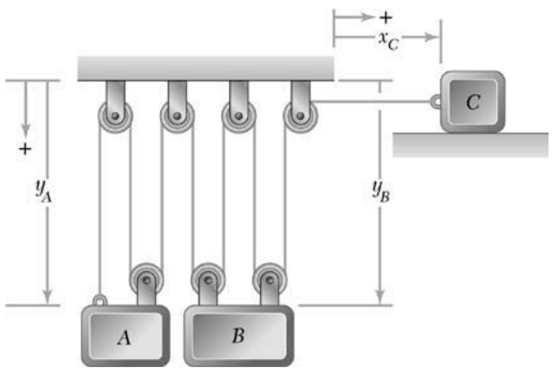
$= 728 \text{ mm}$ or $\mathbf{x}_C - (\mathbf{x}_C)_0 = 728 \text{ mm} \longrightarrow$ ◀



PROBLEM 11.58

Block B moves downward with a constant velocity of 20 mm/s. At $t = 0$, block A is moving upward with a constant acceleration, and its velocity is 30 mm/s. Knowing that at $t = 3$ s slider block C has moved 57 mm to the right, determine (*a*) the velocity of slider block C at $t = 0$, (*b*) the accelerations of A and C, (*c*) the change in position of block A after 5 s.

SOLUTION



From the diagram

$$3y_A + 4y_B + x_C = \text{constant}$$

Then $$3v_A + 4v_B + v_C = 0 \quad (1)$$

and $$3a_A + 4a_B + a_C = 0 \quad (2)$$

Given: $$\mathbf{v}_B = 20 \text{ mm/s} \downarrow;$$

$$(\mathbf{v}_A)_0 = 30 \text{ mm/s} \uparrow$$

(*a*) Substituting into Eq. (1) at $t = 0$

$$3(-30 \text{ mm/s}) + 4(20 \text{ mm/s}) + (v_C)_0 = 0$$

$$(v_C)_0 = 10 \text{ mm/s} \qquad \text{or} \qquad (\mathbf{v}_C)_0 = 10.00 \text{ mm/s} \longrightarrow \blacktriangleleft$$

(*b*) We have $$x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2$$

At $t = 3$ s: $$57 \text{ mm} = (10 \text{ mm/s})(3 \text{ s}) + \frac{1}{2} a_C (3 \text{ s})^2$$

$$a_C = 6 \text{ mm/s}^2 \qquad \text{or} \qquad \mathbf{a}_C = 6.00 \text{ mm/s}^2 \longrightarrow \blacktriangleleft$$

Now $$\mathbf{v}_B = \text{constant} \rightarrow a_B = 0$$

PROBLEM 11.58 (Continued)

Then, substituting into Eq. (2)

$$3a_A + 4(0) + (6 \text{ mm/s}^2) = 0$$

$$a_A = -2 \text{ mm/s}^2 \qquad \text{or} \qquad \mathbf{a}_A = 2.00 \text{ mm/s}^2 \uparrow \blacktriangleleft$$

(*c*) We have

$$y_A = (y_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

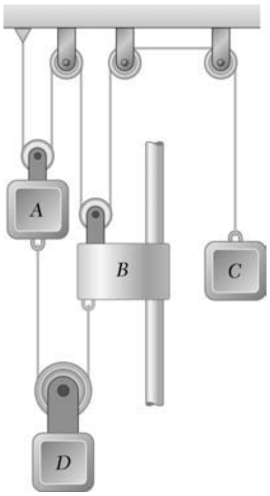
At $t = 5$ s:

$$y_A - (y_A)_0 = (-30 \text{ mm/s})(5 \text{ s}) + \frac{1}{2}(-2 \text{ mm/s}^2)(5 \text{ s})^2$$

$$= -175 \text{ mm}$$

or

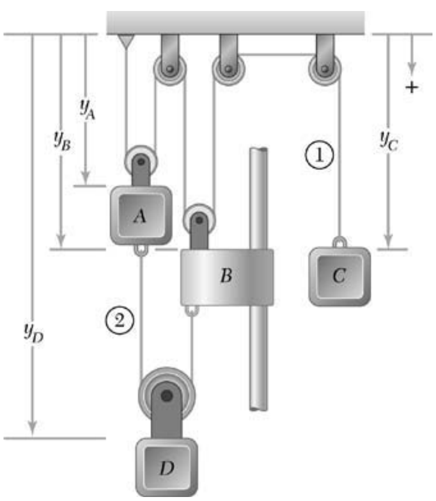
$$\mathbf{y}_A - (\mathbf{y}_A)_0 = 175.0 \text{ mm} \uparrow \blacktriangleleft$$



PROBLEM 11.59

The system shown starts from rest, and each component moves with a constant acceleration. If the relative acceleration of block *C* with respect to collar *B* is $60\,\text{mm/s}^2$ upward and the relative acceleration of block *D* with respect to block *A* is $110\ \text{mm/s}^2$ downward, determine (*a*) the velocity of block *C* after 3 s, (*b*) the change in position of block *D* after 5 s.

SOLUTION



From the diagram

Cable 1: $$2y_A + 2y_B + y_C = \text{constant}$$

Then $$2v_A + 2v_B + v_C = 0 \quad (1)$$

and $$2a_A + 2a_B + a_C = 0 \quad (2)$$

Cable 2: $$(y_D - y_A) + (y_D - y_B) = \text{constant}$$

Then $$-v_A - v_B + 2v_D = 0 \quad (3)$$

and $$-a_A - a_B + 2a_D = 0 \quad (4)$$

Given: At $t = 0,\ v = 0;$ all accelerations constant;
$a_{C/B} = 60\ \text{mm/s}^2 \uparrow,\ a_{D/A} = 110\ \text{mm/s}^2 \downarrow$

(a) We have $a_{C/B} = a_C - a_B = -60$ or $a_B = a_C + 60$

and $a_{D/A} = a_D - a_A = 110$ or $a_A = a_D - 110$

Substituting into Eqs. (2) and (4)

Eq. (2): $$2(a_D - 110) + 2(a_C + 60) + a_C = 0$$

or $$3a_C + 2a_D = 100 \quad (5)$$

Eq. (4): $$-(a_D - 110) - (a_C + 60) + 2a_D = 0$$

or $$-a_C + a_D = -50 \quad (6)$$

PROBLEM 11.59 (Continued)

Solving Eqs. (5) and (6) for a_C and a_D

$$a_C = 40 \text{ mm/s}^2$$

$$a_D = -10 \text{ mm/s}^2$$

Now $$v_C = 0 + a_C t$$

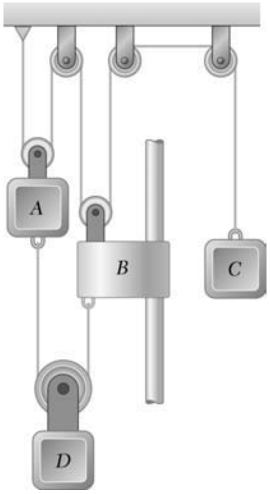
At $t = 3$ s: $$v_C = (40 \text{ mm/s}^2)(3 \text{ s})$$

or $$\mathbf{v}_C = 120.0 \text{ mm/s}\downarrow \blacktriangleleft$$

(*b*) We have $$y_D = (y_D)_0 + (0)t + \frac{1}{2}a_D t^2$$

At $t = 5$ s: $$y_D - (y_D)_0 = \frac{1}{2}(-10 \text{ mm/s}^2)(5 \text{ s})^2$$

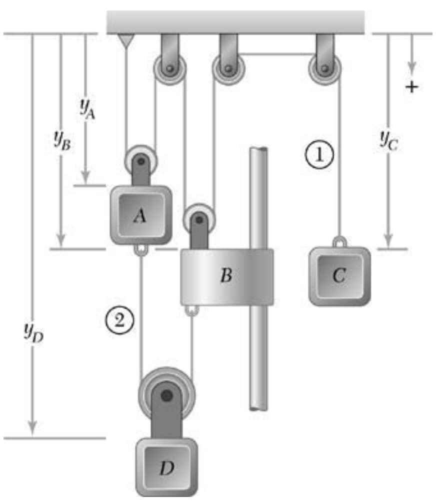
or $$\mathbf{y}_D - (\mathbf{y}_D)_0 = 125.0 \text{ mm}\uparrow \blacktriangleleft$$



PROBLEM 11.60*

The system shown starts from rest, and the length of the upper cord is adjusted so that A, B, and C are initially at the same level. Each component moves with a constant acceleration, and after 2 s the relative change in position of block C with respect to block A is 280 mm upward. Knowing that when the relative velocity of collar B with respect to block A is 80 mm/s downward, the displacements of A and B are 160 mm downward and 320 mm downward, respectively, determine (*a*) the accelerations of A and B if $a_B > 10\text{ mm/s}^2$, (*b*) the change in position of block D when the velocity of block C is 600 mm/s upward.

SOLUTION



From the diagram

Cable 1: $2y_A + 2y_B + y_C = \text{constant}$

Then $2v_A + 2v_B + v_C = 0$ (1)

and $2a_A + 2a_B + a_C = 0$ (2)

Cable 2: $(y_D - y_A) + (y_D - y_B) = \text{constant}$

Then $-v_A - v_B - 2v_D = 0$ (3)

and $-a_A - a_B + 2a_D = 0$ (4)

Given: At $t = 0$

$$v = 0$$

$$(y_A)_0 = (y_B)_0 = (y_C)_0$$

All accelerations constant.

At $t = 2$ s

$$y_{C/A} = 280\text{ mm}\uparrow$$

When $v_{B/A} = 80\text{ mm/s}\downarrow$

$$y_A - (y_A)_0 = 160\text{ mm}\uparrow$$

$$y_B - (y_B)_0 = 320\text{ mm}\downarrow$$

$$a_B > 10\text{ mm/s}^2$$

(*a*) We have $$y_A = (y_A)_0 + (0)t + \frac{1}{2}a_A t^2$$

and $$y_C = (y_C)_0 + (0)t + \frac{1}{2}a_C t^2$$

Then $$y_{C/A} = y_C - y_A = \frac{1}{2}(a_C - a_A)t^2$$

At $t = 2$ s, $y_{C/A} = -280$ mm:

$$-280 \text{ mm} = \frac{1}{2}(a_C - a_A)(2 \text{ s})^2$$

or $$a_C = a_A - 140 \qquad (5)$$

Substituting into Eq. (2)

$$2a_A + 2a_B + (a_A - 140) = 0$$

or $$a_A = \frac{1}{3}(140 - 2a_B) \qquad (6)$$

Now $$v_B = 0 + a_B t$$

$$v_A = 0 + a_A t$$

$$v_{B/A} = v_B - v_A = (a_B - a_A)t$$

Also $$y_B = (y_B)_0 + (0)t + \frac{1}{2}a_B t^2$$

When $$\mathbf{v}_{B/A} = 80 \text{ mm/s}\downarrow: \quad 80 = (a_B - a_A)t \qquad (7)$$

$$\Delta y_A = 160 \text{ mm}\downarrow: \quad 160 = \frac{1}{2}a_A t^2$$

$$\Delta y_B = 320 \text{ mm}\downarrow: \quad 320 = \frac{1}{2}a_B t^2$$

Then $$160 = \frac{1}{2}(a_B - a_A)t^2$$

Using Eq. (7) $$320 = (80)t \quad \text{or} \quad t = 4 \text{ s}$$

Then $$160 = \frac{1}{2}a_A(4)^2 \quad \text{or} \quad \mathbf{a}_A = 20.0 \text{ mm/s}^2\downarrow \blacktriangleleft$$

and $$320 = \frac{1}{2}a_B(4)^2 \quad \text{or} \quad \mathbf{a}_B = 40.0 \text{ mm/s}^2\downarrow \blacktriangleleft$$

Note that Eq. (6) is not used; thus, the problem is over-determined.

PROBLEM 11.60* (Continued)

(*b*) Substituting into Eq. (5)

$$a_C = 20 - 140 = -120 \text{ mm/s}^2$$

and into Eq. (4)

$$-(20 \text{ mm/s}^2) - (40 \text{ mm/s}^2) + 2a_D = 0$$

or $$a_D = 30 \text{ mm/s}^2$$

Now $$v_C = 0 + a_C t$$

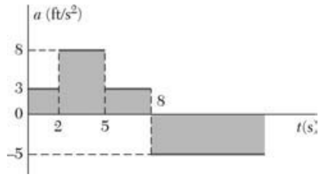
When $v_C = -600$ mm/s: $$-600 \text{ mm/s} = (-120 \text{ mm/s}^2)t$$

or $$t = 5 \text{ s}$$

Also $$y_D = (y_D)_0 + (0)t + \frac{1}{2}a_D t^2$$

At $t = 5$ s: $$y_D - (y_D)_0 = \frac{1}{2}(30 \text{ mm/s}^2)(5 \text{ s})^2$$

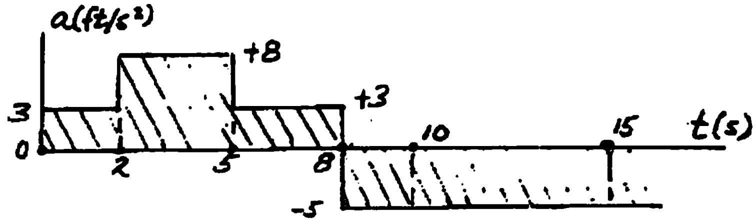
or $$\mathbf{y}_D - (\mathbf{y}_D)_0 = 375 \text{ mm} \downarrow \blacktriangleleft$$



PROBLEM 11.61

A particle moves in a straight line with the acceleration shown in the figure. Knowing that it starts from the origin with $v_0 = -14$ ft/s, plot the v–t and x–t curves for $0 < t < 15$ s and determine (*a*) the maximum value of the velocity of the particle, (*b*) the maximum value of its position coordinate.

SOLUTION



Change in v = area under a–t curve. $\qquad v_0 = -14$ ft/s

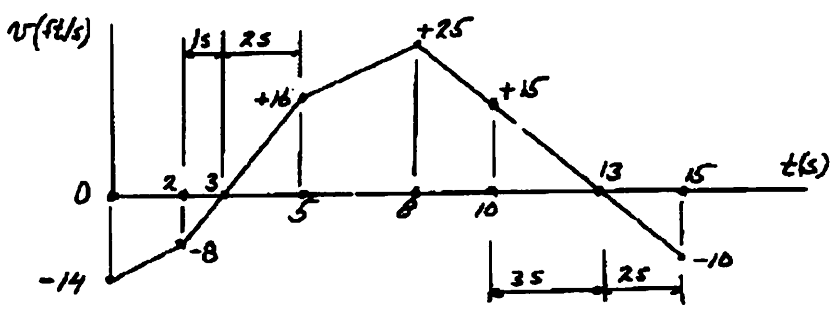
$t = 0$ to $t = 2$ s: $\quad v_2 - v_0 = (3\text{ ft/s}^2)(2\text{ s}) = +6$ ft/s $\quad v_2 = -8$ ft/s

$t = 2$ s to $t = 5$ s: $\quad v_5 - v_2 = (8\text{ ft/s}^2)(3\text{ s}) = +24$ ft/s $\quad v_5 = +16$ ft/s

$t = 5$ s to $t = 8$ s: $\quad v_8 - v_5 = (3\text{ ft/s}^2)(3\text{ s}) = +9$ ft/s $\quad v_8 = +25$ ft/s

$t = 8$ s to $t = 10$ s: $\quad v_{10} - v_8 = (-5\text{ ft/s}^2)(2\text{ s}) = -10$ ft/s $\quad v_{10} = +15$ ft/s

$t = 10$ s to $t = 15$ s: $\quad v_{15} - v_{10} = (-5\text{ ft/s}^2)(5\text{ s}) = -25$ ft/s $\quad v_{15} = -10$ ft/s



PROBLEM 11.61 (Continued)

Plot v–t curve. Then by similar triangles Δ's find t for $v = 0$.

Change in x = area under v–t curve $\qquad x_0 = 0$

$t = 0$ to $t = 2$ s: $\quad x_2 - x_0 = \frac{1}{2}(-14-8)(2) = -22 \text{ ft} \qquad x_2 = -22 \text{ ft}$

$t = 2$ s to $t = 3$ s: $\quad x_3 - x_2 = \frac{1}{2}(-8)(1) = -4 \text{ ft} \qquad x_8 = -26 \text{ ft}$

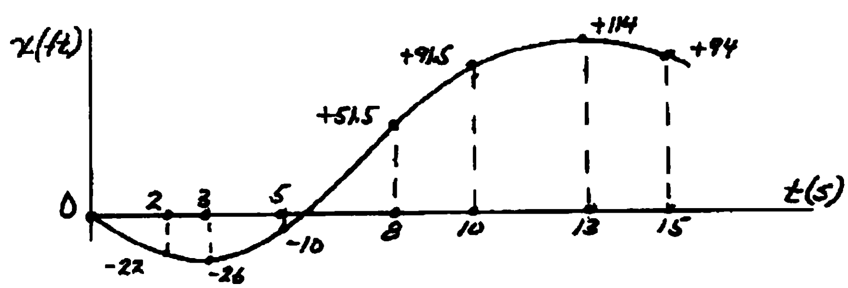
$t = 3$ s to $t = 5$ s: $\quad x_5 - x_3 = \frac{1}{2}(+16)(2) = +16 \text{ ft} \qquad x_5 = -10 \text{ ft}$

$t = 5$ s to $t = 8$ s: $\quad x_8 - x_5 = \frac{1}{2}(+16+25)(3) = +61.5 \text{ ft} \qquad x_8 = +51.6 \text{ ft}$

$t = 8$ s to $t = 10$ s: $\quad x_{10} - x_8 = \frac{1}{2}(+25+15)(2) = +40 \text{ ft} \qquad x_{10} = +91.6 \text{ ft}$

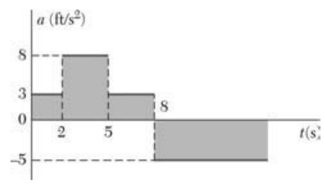
$t = 10$ s to $t = 13$ s: $\quad x_{13} - x_{10} = \frac{1}{2}(+15)(3) = +22.5 \text{ ft} \qquad x_{13} = +114 \text{ ft}$

$t = 13$ s to $t = 15$ s: $\quad x_{15} - x_{13} = \frac{1}{2}(-10)(2) = -10 \text{ ft} \qquad x_{15} = +94 \text{ ft}$



(a) Maximum velocity: When $t = 8$ s, $\qquad v_m = 25.0 \text{ ft/s}$ ◀

(b) Maximum x: When $t = 13$ s, $\qquad x_m = 114.0 \text{ ft}$ ◀

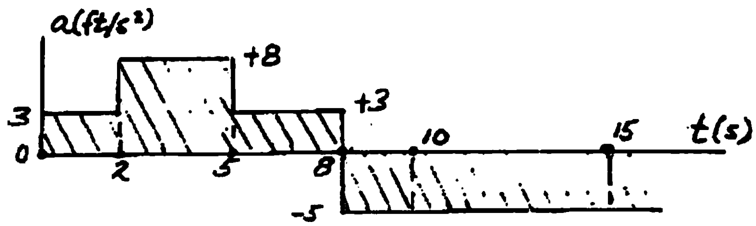


PROBLEM 11.62

For the particle and motion of Problem 11.61, plot the v–t and x–t curves for $0 < t < 15$ s and determine the velocity of the particle, its position, and the total distance traveled after 10 s.

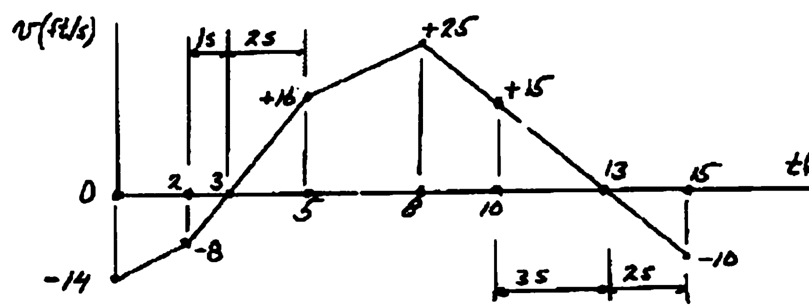
PROBLEM 11.61 A particle moves in a straight line with the acceleration shown in the figure. Knowing that it starts from the origin with $v_0 = -14$ ft/s, plot the v–t and x–t curves for $0 < t < 15$ s and determine (*a*) the maximum value of the velocity of the particle, (*b*) the maximum value of its position coordinate.

SOLUTION



Change in v = area under a–t curve. $\qquad v_0 = -14$ ft/s

$t = 0$ to $t = 2$ s:	$v_2 - v_0 = (3\text{ ft/s}^2)(2\text{ s}) = +6$ ft/s	$v_2 = -8$ ft/s
$t = 2$ s to $t = 5$ s:	$v_5 - v_2 = (8\text{ ft/s}^2)(3\text{ s}) = +24$ ft/s	$v_5 = +16$ ft/s
$t = 5$ s to $t = 8$ s:	$v_8 - v_5 = (3\text{ ft/s}^2)(3\text{ s}) = +9$ ft/s	$v_8 = +25$ ft/s
$t = 8$ s to $t = 10$ s:	$v_{10} - v_8 = (-5\text{ ft/s}^2)(2\text{ s}) = -10$ ft/s	$v_{10} = +15$ ft/s
$t = 10$ s to $t = 15$ s:	$v_{15} - v_{10} = (-5\text{ ft/s}^2)(5\text{ s}) = -25$ ft/s	$v_{15} = -10$ ft/s



PROBLEM 11.62 (Continued)

Plot v–t curve. Then by similar triangles Δ's find t for $v = 0$.

Change in x = area under v–t curve $\qquad x_0 = 0$

$t = 0$ to $t = 2$ s: $\quad x_2 - x_0 = \frac{1}{2}(-14-8)(2) = -22$ ft $\qquad x_2 = -22$ ft

$t = 2$ s to $t = 3$ s: $\quad x_3 - x_2 = \frac{1}{2}(-8)(1) = -4$ ft $\qquad x_8 = -26$ ft

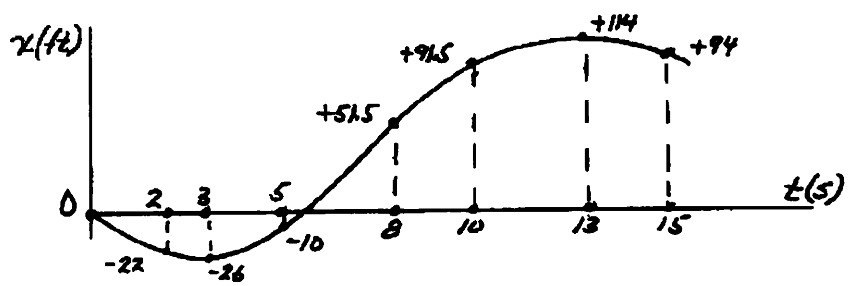
$t = 3$ s to $t = 5$ s: $\quad x_5 - x_3 = \frac{1}{2}(+16)(2) = +16$ ft $\qquad x_5 = -10$ ft

$t = 5$ s to $t = 8$ s: $\quad x_8 - x_5 = \frac{1}{2}(+16+25)(3) = +61.5$ ft $\qquad x_8 = +51.6$ ft

$t = 8$ s to $t = 10$ s: $\quad x_{10} - x_8 = \frac{1}{2}(+25+15)(2) = +40$ ft $\qquad x_{10} = +91.6$ ft

$t = 10$ s to $t = 13$ s: $\quad x_{13} - x_{10} = \frac{1}{2}(+15)(3) = +22.5$ ft $\qquad x_{13} = +114$ ft

$t = 13$ s to $t = 15$ s: $\quad x_{15} - x_{13} = \frac{1}{2}(-10)(2) = -10$ ft $\qquad x_{15} = +94$ ft



when $t = 10$ s: $\qquad v_{10} = +15$ ft/s ◀

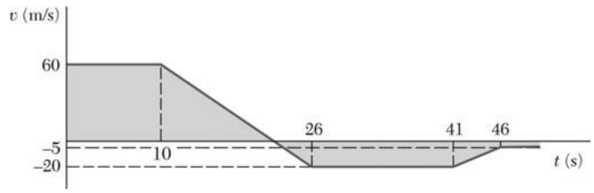
$x_{10} = +91.5$ ft/s ◀

Distance traveled: $t = 0$ to $t = 105$

$t = 0$ to $t = 3$ s: $\qquad$ Distance traveled = 26 ft

$t = 3$ s to $t = 10$ s $\qquad$ Distance traveled = 26 ft + 91.5 ft = 117.5 ft

Total distance traveled = 26 + 117.5 = 143.5 ft ◀

	**PROBLEM 11.63** A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -540$ m at $t = 0$, (*a*) construct the a–t and x–t curves for $0 < t < 50$ s, and determine (*b*) the total distance traveled by the particle when $t = 50$ s, (*c*) the two times at which $x = 0$.

SOLUTION

(*a*) $a_t =$ slope of v–t curve at time t

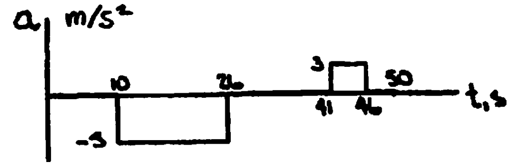
From $t = 0$ to $t = 10$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 10$ s to $t = 26$ s: $a = \dfrac{-20-60}{26-10} = -5 \text{ m/s}^2$

$t = 26$ s to $t = 41$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 41$ s to $t = 46$ s: $a = \dfrac{-5-(-20)}{46-41} = 3 \text{ m/s}^2$

$t = 46$ s: $v = \text{constant} \Rightarrow a = 0$



$x_2 = x_1 +$ (area under v–t curve from t_1 to t_2)

At $t = 10$ s: $x_{10} = -540 + 10(60) + 60$ m

Next, find time at which $v = 0$. Using similar triangles

$$\frac{t_{v=0} - 10}{60} = \frac{26-10}{80} \quad \text{or} \quad t_{v=0} = 22 \text{ s}$$

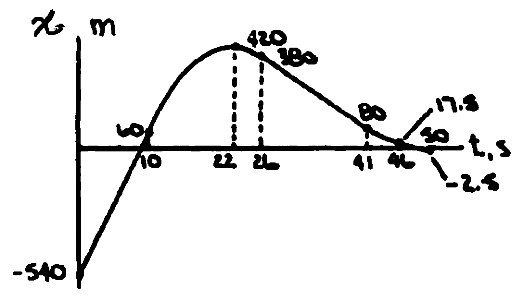
At $t = 22$ s: $x_{22} = 60 + \frac{1}{2}(12)(60) = 420$ m

$t = 26$ s: $x_{26} = 420 - \frac{1}{2}(4)(20) = 380$ m

$t = 41$ s: $x_{41} = 380 - 15(20) = 80$ m

$t = 46$ s: $x_{46} = 80 - 5\left(\dfrac{20+5}{2}\right) = 17.5$ m

$t = 50$ s: $x_{50} = 17.5 - 4(5) = -2.5$ m



PROBLEM 11.63 (Continued)

(*b*) From $t = 0$ to $t = 22$ s: Distance traveled $= 420 - (-540)$

$$= 960 \text{ m}$$

$t = 22$ s to $t = 50$ s: Distance traveled $= |-2.5 - 420|$

$$= 422.5 \text{ m}$$

Total distance traveled $= (960 + 422.5)$ ft $= 1382.5$ m

Total distance traveled $= 1383$ m ◀

(*c*) Using similar triangles

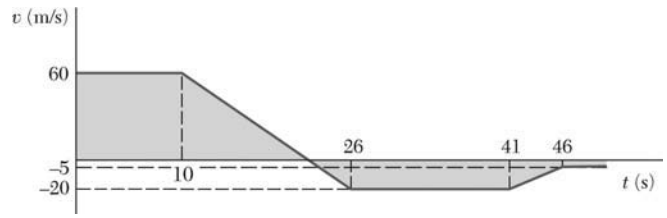
Between 0 and 10 s: $\dfrac{(t_{x=0})_1 - 0}{540} = \dfrac{10}{600}$

$(t_{x=0})_1 = 9.00$ s ◀

Between 46 s and 50 s: $\dfrac{(t_{x=0})_2 - 46}{17.5} = \dfrac{4}{20}$

$(t_{x=0})_2 = 49.5$ s ◀

PROBLEM 11.64



A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -540$ m at $t = 0$, (*a*) construct the a–t and x–t curves for $0 < t < 50$ s, and determine (*b*) the maximum value of the position coordinate of the particle, (*c*) the values of t for which the particle is at $x = 100$ m.

SOLUTION

(*a*) $a_t =$ slope of v–t curve at time t

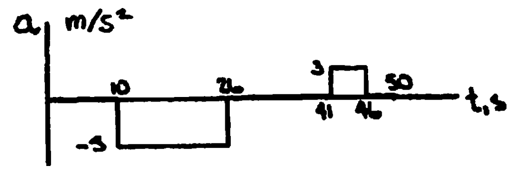
From $t = 0$ to $t = 10$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 10$ s to $t = 26$ s: $a = \dfrac{-20-60}{26-10} = -5 \text{ m/s}^2$

$t = 26$ s to $t = 41$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 41$ s to $t = 46$ s: $a = \dfrac{-5-(-20)}{46-41} = 3 \text{ m/s}^2$

$t = 46$ s: $v = \text{constant} \Rightarrow a = 0$



$x_2 = x_1 +$ (area under v–t curve from t_1 to t_2)

At $t = 10$ s: $x_{10} = -540 + 10(60) = 60$ m

Next, find time at which $v = 0$. Using similar triangles

$$\frac{t_{v=0} - 10}{60} = \frac{26-10}{80} \qquad \text{or} \qquad t_{v=0} = 22 \text{ s}$$

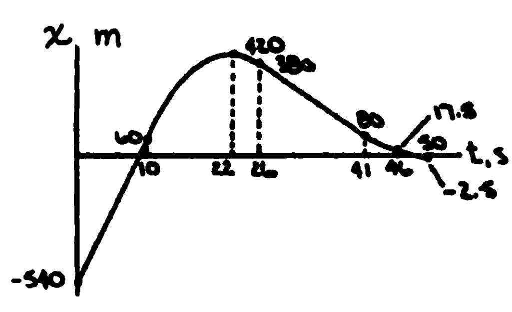
At $t = 22$ s: $x_{22} = 60 + \dfrac{1}{2}(12)(60) = 420$ m

$t = 26$ s: $x_{26} = 420 - \dfrac{1}{2}(4)(20) = 380$ m

$t = 41$ s: $x_{41} = 380 - 15(20) = 80$ m

$t = 46$ s: $x_{46} = 80 - 5\left(\dfrac{20+5}{2}\right) = 17.5$ m

$t = 50$ s: $x_{50} = 17.5 - 4(5) = -2.5$ m

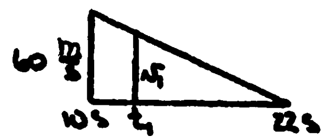


PROBLEM 11.64 (Continued)

(*b*) Reading from the x–t curve $\qquad x_{max} = 420 \text{ m}$ ◀

(*c*) Between 10 s and 22 s

$100 \text{ m} = 420 \text{ m} - (\text{area under } v\text{–}t \text{ curve from } t, \text{ to } 22 \text{ s}) \text{ m}$



$$100 = 420 - \frac{1}{2}(22 - t_1)(v_1)$$

$$(22 - t_1)(v_1) = 640$$

Using similar triangles

$$\frac{v_1}{22 - t_1} = \frac{60}{12} \qquad \text{or} \qquad v_1 = 5(22 - t_1)$$

Then

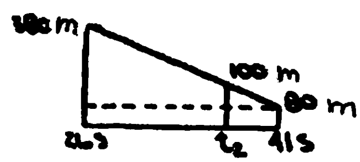
$$(22 - t_1)[5(22 - t_1)] = 640$$

$$t_1 = 10.69 \text{ s} \quad \text{and} \quad t_1 = 33.3 \text{ s}$$

We have $\qquad 10 \text{ s} < t_1 < 22 \text{ s} \Rightarrow \qquad t_1 = 10.69 \text{ s}$ ◀

Between 26 s and 41 s:

Using similar triangles



$$\frac{41 - t_2}{20} = \frac{15}{300}$$

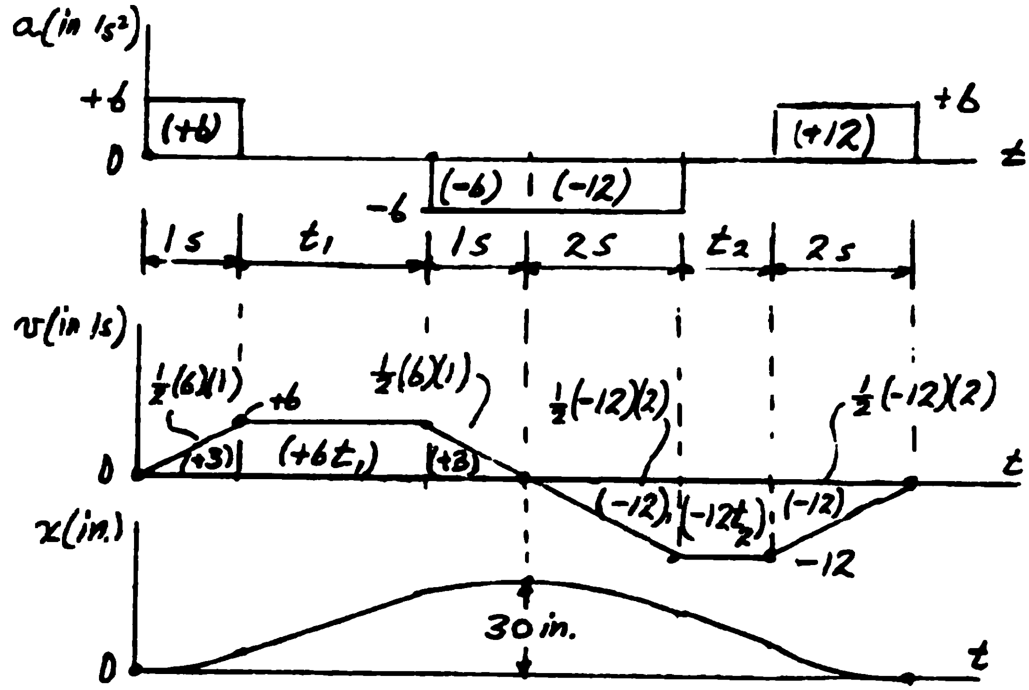
$t_2 = 40.0 \text{ s}$ ◀

PROBLEM 11.65

During a finishing operation the bed of an industrial planer moves alternately 30 in. to the right and 30 in. to the left. The velocity of the bed is limited to a maximum value of 6 in./s to the right and 12 in./s to the left; the acceleration is successively equal to 6 in./s^2 to the right, zero 6 in./s^2 to the left, zero, etc. Determine the time required for the bed to complete a full cycle, and draw the v–t and x–t curves.

SOLUTION

We choose positive to the right, thus the range of permissible velocities is $-12 \text{ in./s} < v < 6 \text{ in./s}$ since acceleration is $-6 \text{ in./s}^2, 0, \text{ or } +6 \text{ in./s}^2$. The slope the v–t curve must also be –6 in./s^2, 0, or +6 in./s^2.

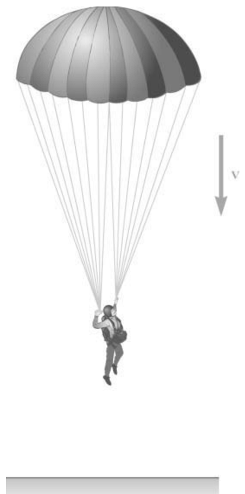


Planer moves = 30 in. to right: $+30 \text{ in.} = 3 + 6t_1 + 3$ $\qquad t_1 = 4.00 \text{ s}$

Planer moves = 30 in. to left: $-30 \text{ in.} = -12 - 12t_2 + 12$ $\qquad t_2 = 0.50 \text{ s}$

Total time $= 1 \text{ s} + 4 \text{ s} + 1 \text{ s} + 2 \text{ s} + 0.5 \text{ s} + 2 \text{ s} = 10.5 \text{ s}$ $\qquad t_{total} = 10.50 \text{ s}$ ◀

PROBLEM 11.66



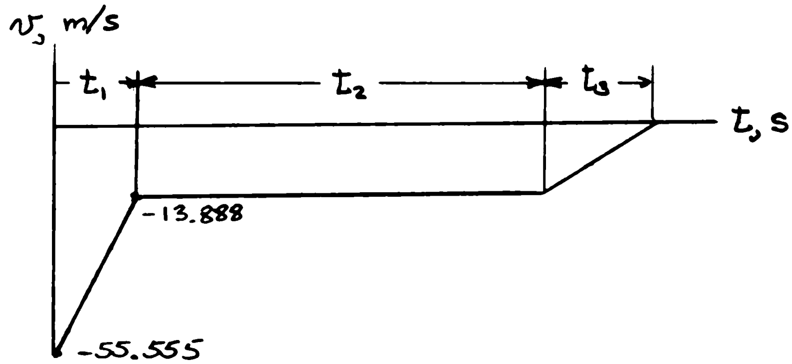
A parachutist is in free fall at a rate of 200 km/h when he opens his parachute at an altitude of 600 m. Following a rapid and constant deceleration, he then descends at a constant rate of 50 km/h from 586 m to 30 m, where he maneuvers the parachute into the wind to further slow his descent. Knowing that the parachutist lands with a negligible downward velocity, determine (*a*) the time required for the parachutist to land after opening his parachute, (*b*) the initial deceleration.

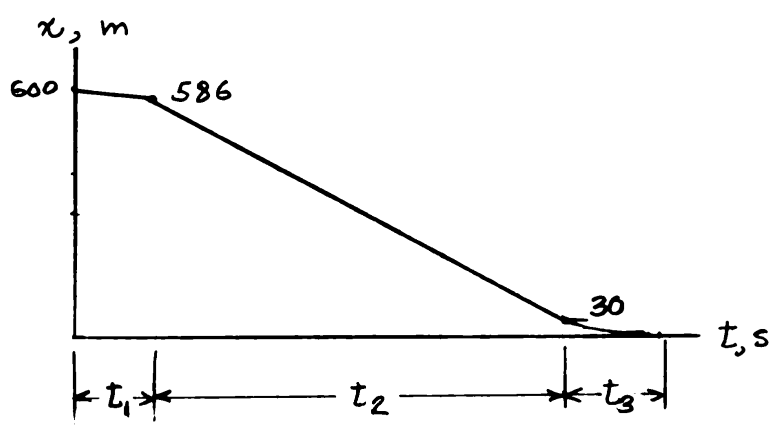
SOLUTION

Assume second deceleration is constant. Also, note that

$200 \text{ km/h} = 55.555 \text{ m/s},$

$50 \text{ km/h} = 13.888 \text{ m/s}$





(*a*) Now $\Delta x =$ area under v–t curve for given time interval

Then

$$(586-600)\text{ m} = -t_1\left(\frac{55.555+13.888}{2}\right)\text{ m/s}$$

$$t_1 = 0.4032\text{ s}$$

$$(30-586)\text{ m} = -t_2(13.888\text{ m/s})$$

$$t_2 = 40.0346\text{ s}$$

$$(0-30)\text{ m} = -\frac{1}{2}(t_3)(13.888\text{ m/s})$$

$$t_3 = 4.3203\text{ s}$$

$$t_{\text{total}} = (0.4032+40.0346+4.3203)\text{ s}$$

$t_{\text{total}} = 44.8\text{ s}$ ◀

(*b*) We have

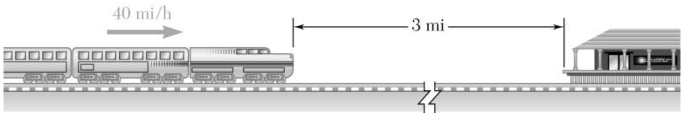
$$a_{\text{initial}} = \frac{\Delta v_{\text{initial}}}{t_1}$$

$$= \frac{[-13.888-(-55.555)]\text{ m/s}}{0.4032\text{ s}}$$

$$= 103.3\text{ m/s}^2$$

$\mathbf{a}_{\text{initial}} = 103.3\text{ m/s}^2 \uparrow$ ◀

PROBLEM 11.67

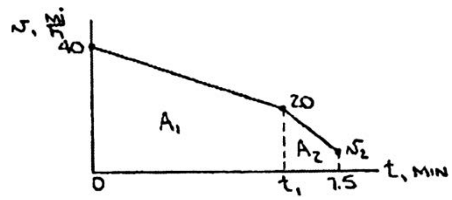


A commuter train traveling at 40 mi/h is 3 mi from a station. The train then decelerates so that its speed is 20 mi/h when it is 0.5 mi from the station. Knowing that the train arrives at the station 7.5 min after beginning to decelerate and assuming constant decelerations, determine (*a*) the time required for the train to travel the first 2.5 mi, (*b*) the speed of the train as it arrives at the station, (*c*) the final constant deceleration of the train.

SOLUTION

Given: At $t = 0$, $v = 40$ mi/h, $x = 0$; when $x = 2.5$ mi, $v = 20$ mi/h;
at $t = 7.5$ min, $x = 3$ mi; constant decelerations.

The $v-t$ curve is first drawn as shown.



(*a*) We have $A_1 = 2.5$ mi

$$(t_1 \text{ min})\left(\frac{40+20}{2}\right) \text{mi/h} \times \frac{1 \text{ h}}{60 \text{ min}} = 2.5 \text{ mi}$$

$t_1 = 5.00$ min ◀

(*b*) We have $A_2 = 0.5$ mi

$$(7.5-5) \text{ min} \times \left(\frac{20+v_2}{2}\right) \text{mi/h} \times \frac{1 \text{ h}}{60 \text{ min}} = 0.5 \text{ mi}$$

$v_2 = 4.00$ mi/h ◀

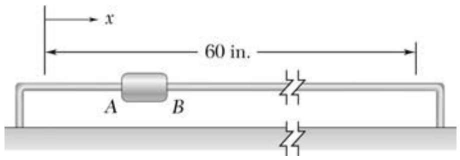
(*c*) We have

$$a_{\text{final}} = a_{12}$$

$$= \frac{(4-20) \text{ mi/h}}{(7.5-5) \text{ min}} \times \frac{5280 \text{ ft}}{\text{mi}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ h}}{3600 \text{ s}}$$

$a_{\text{final}} = -0.1564$ ft/s^2 ◀

PROBLEM 11.68



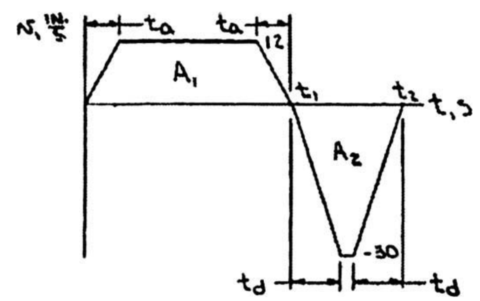
A temperature sensor is attached to slider *AB* which moves back and forth through 60 in. The maximum velocities of the slider are 12 in./s to the right and 30 in./s to the left. When the slider is moving to the right, it accelerates and decelerates at a constant rate of 6 in./s^2; when moving to the left, the slider accelerates and decelerates at a constant rate of 20 in./s^2. Determine the time required for the slider to complete a full cycle, and construct the $v-t$ and $x-t$ curves of its motion.

SOLUTION

The $v-t$ curve is first drawn as shown. Then

$$t_a = \frac{v_{\text{right}}}{a_{\text{right}}} = \frac{12 \text{ in./s}}{6 \text{ in./s}^2} = 2 \text{ s}$$

$$t_d = \frac{v_{\text{left}}}{a_{\text{left}}} = \frac{30 \text{ in./s}}{20 \text{ in./s}} = 1.5 \text{ s}$$



Now $A_1 = 60 \text{ in.}$

or $[(t_1 - 2) \text{ s}](12 \text{ in./s}) = 60 \text{ in.}$

or $t_1 = 7 \text{ s}$

and $A_2 = 60 \text{ in.}$

or $\{[(t_2 - 7) - 1.5] \text{ s}\}(30 \text{ in./s}) = 60 \text{ in.}$

or $t_2 = 10.5 \text{ s}$

Now $t_{\text{cycle}} = t_2$ $\qquad t_{\text{cycle}} = 10.5 \text{ s}$ ◀

We have $x_{ii} = x_i + (\text{area under } v\text{–}t \text{ curve from } t_i \text{ to } t_{ii})$

At $t = 2 \text{ s}$: $x_2 = \frac{1}{2}(2)(12) = 12 \text{ in.}$

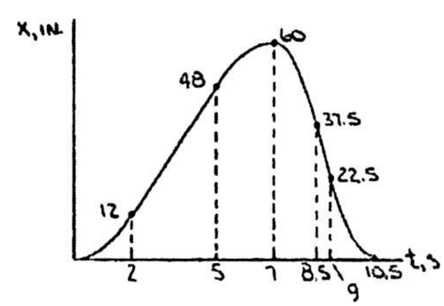
$t = 5 \text{ s}$: $x_5 = 12 + (5-2)(12) = 48 \text{ in.}$

$t = 7 \text{ s}$: $x_7 = 60 \text{ in.}$

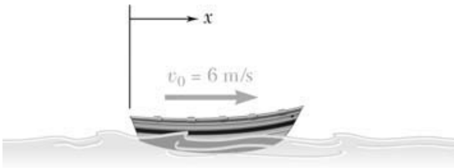
$t = 8.5 \text{ s}$: $x_{8.5} = 60 - \frac{1}{2}(1.5)(30) = 37.5 \text{ in.}$

$t = 9 \text{ s}$: $x_9 = 37.5 - (0.5)(30) = 22.5 \text{ in.}$

$t = 10.5 \text{ s}$: $x_{10.5} = 0$



PROBLEM 11.69



In a water-tank test involving the launching of a small model boat, the model's initial horizontal velocity is 6 m/s, and its horizontal acceleration varies linearly from -12 m/s^2 at $t = 0$ to -2 m/s^2 at $t = t_1$ and then remains equal to -2 m/s^2 until $t = 1.4$ s. Knowing that $v = 1.8$ m/s when $t = t_1$, determine (*a*) the value of t_1, (*b*) the velocity and the position of the model at $t = 1.4$ s.

SOLUTION

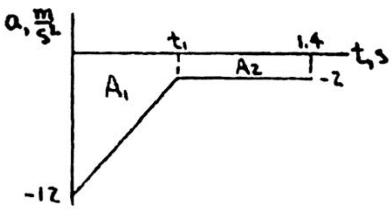
Given: $v_0 = 6\text{ m/s};\ \text{for } 0 < t < t_1,$

for $t_1 < t < 1.4\text{ s} \quad a = -2\text{ m/s}^2;$

at $t = 0 \quad a = -12\text{ m/s}^2;$

at $t = t_1$ $\quad a = -2\text{ m/s}^2,\ v = 1.8\text{ m/s}^2$

The a–t and v–t curves are first drawn as shown. The time axis is not drawn to scale.



(*a*) We have $v_{t_1} = v_0 + A_1$

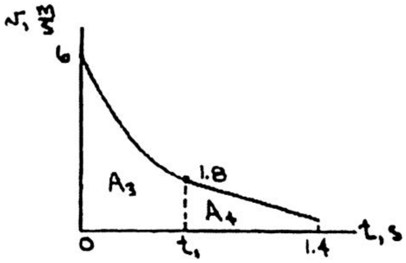
$$1.8\text{ m/s} = 6\text{ m/s} - (t_1\text{ s})\left(\frac{12+2}{2}\right)\text{m/s}^2$$

$$t_1 = 0.6\text{ s} \blacktriangleleft$$

(*b*) We have $v_{1.4} = v_{t_1} + A_2$

$$v_{1.4} = 1.8\text{ m/s} - (1.4 - 0.6)\text{ s} \times 2\text{ m/s}^2$$

$$v_{1.4} = 0.20\text{ m/s} \blacktriangleleft$$



Now $x_{1.4} = A_3 + A_4$, where A_3 is most easily determined using integration. Thus,

for $0 < t < t_1$: $\quad a = \dfrac{-2-(-12)}{0.6}t - 12 = \dfrac{50}{3}t - 12$

Now $\quad \dfrac{dv}{dt} = a = \dfrac{50}{3}t - 12$

PROBLEM 11.69 (Continued)

At $t = 0,\ v = 6$ m/s: $$\int_6^v dv = \int_0^t \left(\frac{50}{3}t - 12\right) dt$$

or $$v = 6 + \frac{25}{3}t^2 - 12t$$

We have $$\frac{dx}{dt} = v = 6 - 12t + \frac{25}{3}t^2$$

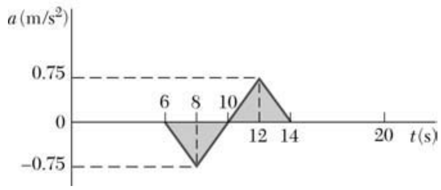
Then $$A_3 = \int_0^{x_{t_1}} dx = \int_0^{0.6} (6 - 12t + \frac{25}{3}t^2) dt$$

$$= \left[6t - 6t^2 + \frac{25}{9}t^3\right]_0^{0.6} = 2.04 \text{ m}$$

Also $$A_4 = (1.4 - 0.6)\left(\frac{1.8 + 0.2}{2}\right) = 0.8 \text{ m}$$

Then $$x_{1.4} = (2.04 + 0.8) \text{ m}$$

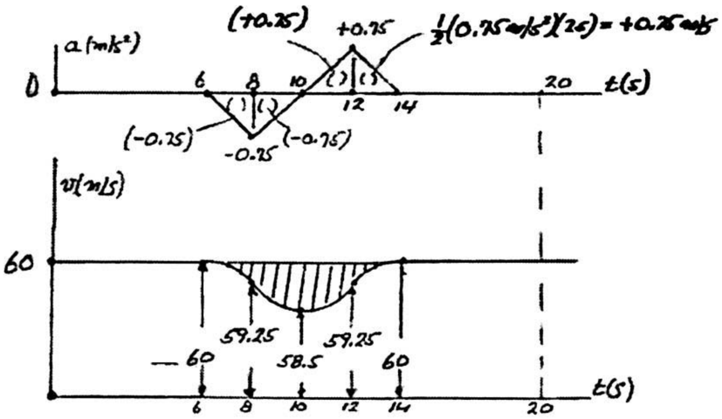
or $x_{1.4} = 2.84$ m ◀



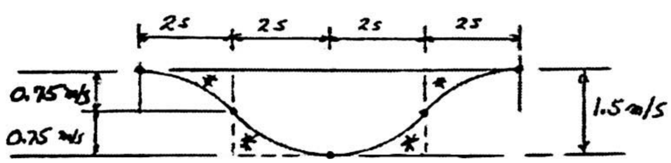
PROBLEM 11.70

The acceleration record shown was obtained for a small airplane traveling along a straight course. Knowing that $x = 0$ and $v = 60$ m/s when $t = 0$, determine (*a*) the velocity and position of the plane at $t = 20$ s, (*b*) its average velocity during the interval $6 \text{ s} < t < 14$ s.

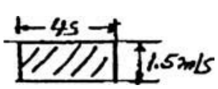
SOLUTION



Geometry of "bell-shaped" portion of $v-t$ curve



The parabolic spandrels marked by * are of equal area. Thus, total area of shaded portion of $v-t$ diagram is:



$= \Delta x = 6$ m

(*a*) When $t = 20$ s: $v_{20} = 60$ m/s ◀

$$x_{20} = (60 \text{ m/s})(20 \text{ s}) - (\text{shaded area})$$

$$= 1200 \text{ m} - 6 \text{ m}$$

$x_{20} = 1194$ m ◀

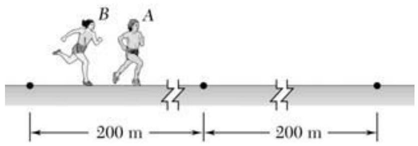
(*b*) From $t = 6$ s to $t = 14$ s: $\Delta t = 8$ s

$$\Delta x = (60 \text{ m/s})(14 \text{ s} - 6 \text{ s}) - (\text{shaded area})$$
$$= (60 \text{ m/s})(8 \text{ s}) - 6 \text{ m} = 480 \text{ m} - 6 \text{ m} = 474 \text{ m}$$

$$v_{\text{average}} = \frac{\Delta x}{\Delta t} = \frac{474 \text{ m}}{8 \text{ s}}$$

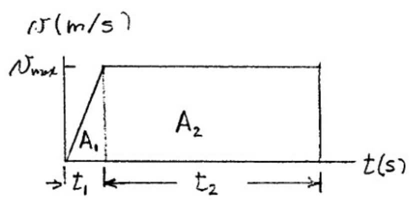
$v_{\text{average}} = 59.25$ m/s ◀

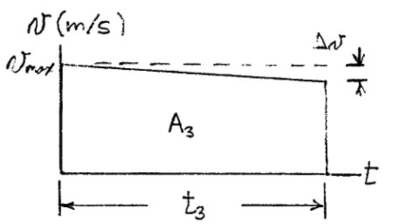
PROBLEM 11.71



In a 400-m race, runner *A* reaches her maximum velocity v_A in 4 s with constant acceleration and maintains that velocity until she reaches the half-way point with a split time of 25 s. Runner *B* reaches her maximum velocity v_B in 5 s with constant acceleration and maintains that velocity until she reaches the half-way point with a split time of 25.2 s. Both runners then run the second half of the race with the same constant deceleration of 0.1 m/s^2. Determine (*a*) the race times for both runners, (*b*) the position of the winner relative to the loser when the winner reaches the finish line.

SOLUTION





Sketch v–t curves for first 200 m.

Runner *A*: $t_1 = 4$ s, $t_2 = 25 - 4 = 21$ s

$$A_1 = \frac{1}{2}(4)(v_A)_{\max} = 2(v_A)_{\max}$$

$$A_2 = 21(v_A)_{\max}$$

$$A_1 + A_2 = \Delta x = 200 \text{ m}$$

$$23(v_A)_{\max} = 200 \qquad \text{or} \qquad (v_A)_{\max} = 8.6957 \text{ m/s}$$

Runner *B*: $t_1 = 5$ s, $t_2 = 25.2 - 5 = 20.2$ s

$$A_1 = \frac{1}{2}(5)(v_B)_{\max} = 2.5(v_B)_{\max}$$

$$A_2 = 20.2(v_B)_{\max}$$

$$A_1 + A_2 = \Delta x = 200 \text{ m}$$

$$22.7(v_B)_{\max} = 200 \qquad \text{or} \qquad (v_B)_{\max} = 8.8106 \text{ m/s}$$

Sketch v–t curve for second 200 m. $\Delta v = |a|t_3 = 0.1t_3$

$$A_3 = v_{\max}t_3 - \frac{1}{2}\Delta v t_3 = 200 \qquad \text{or} \qquad 0.05t_3^2 - v_{\max}t_3 + 200 = 0$$

$$t_3 = \frac{v_{\max} \pm \sqrt{(v_{\max})^2 - (4)(0.05)(200)}}{(2)(0.05)} = 10\left(v_{\max} \pm \sqrt{(v_{\max})^2 - 40}\right)$$

Runner *A*: $(v_{\max})_A = 8.6957$, $(t_3)_A = 146.64$ s and 27.279 s

Reject the larger root. Then total time $t_A = 25 + 27.279 = 52.279$ s $\qquad t_A = 52.2$ s ◀

PROBLEM 11.71 (Continued)

Runner B: $(v_{max})_B = 8.8106$, $(t_3)_B = 149.45$ s and 26.765 s

Reject the larger root. Then total time $t_B = 25.2 + 26.765 = 51.965$ s

$t_B = 52.0$ s ◀

Velocity of A at $t = 51.965$ s:

$$v_1 = 8.6957 - (0.1)(51.965 - 25) = 5.999 \text{ m/s}$$

Velocity of A at $t = 51.279$ s:

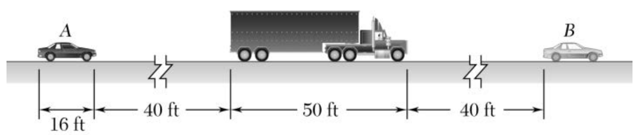
$$v_2 = 8.6957 - (0.1)(52.279 - 25) = 5.968 \text{ m/s}$$

Over $51.965 \text{ s} \le t \le 52.965$ s, runner A covers a distance Δx

$$\Delta x = v_{ave}(\Delta t) = \frac{1}{2}(5.999 + 5.968)(52.279 - 51.965)$$

$\Delta x = 1.879$ m ◀

PROBLEM 11.72

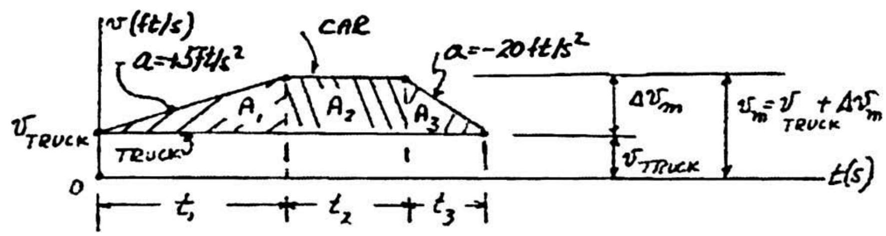


A car and a truck are both traveling at the constant speed of 35 mi/h; the car is 40 ft behind the truck. The driver of the car wants to pass the truck, i.e., he wishes to place his car at B, 40 ft in front of the truck, and then resume the speed of 35 mi/h. The maximum acceleration of the car is $5\ \text{ft/s}^2$ and the maximum deceleration obtained by applying the brakes is $20\ \text{ft/s}^2$. What is the shortest time in which the driver of the car can complete the passing operation if he does not at any time exceed a speed of 50 mi/h? Draw the $v-t$ curve.

SOLUTION

Relative to truck, car must move a distance: $\Delta x = 16 + 40 + 50 + 40 = 146$ ft

Allowable increase in speed: $\Delta v_m = 50 - 35 = 15$ mi/h $= 22$ ft/s



<u>Acceleration Phase</u>: $t_1 = 22/5 = 4.4$ s $\quad A_1 = \frac{1}{2}(22)(4.4) = 48.4$ ft

<u>Deceleration Phase</u>: $t_3 = 22/20 = 1.1$ s $\quad A_3 = \frac{1}{2}(22)(1.1) = 12.1$ ft

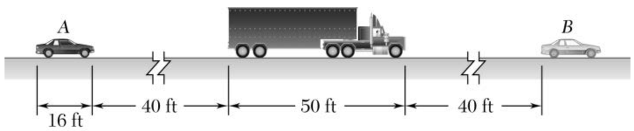
But: $\Delta x = A_1 + A_2 + A_3$: $\quad 146\ \text{ft} = 48.4 + (22)t_2 + 12.1 \quad t_2 = 3.89$ s

$t_{\text{total}} = t_1 + t_2 + t_3 = 4.4\ \text{s} + 3.89\ \text{s} + 1.1\ \text{s} = 9.39\ \text{s} \quad t_B = 9.39\ \text{s}$ ◀

PROBLEM 11.73

Solve Problem 11.72, assuming that the driver of the car does not pay any attention to the speed limit while passing and concentrates on reaching position *B* and resuming a speed of 35 mi/h in the shortest possible time. What is the maximum speed reached? Draw the $v\text{--}t$ curve.

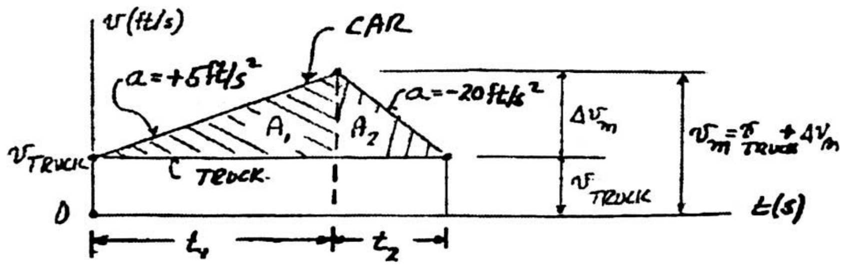
PROBLEM 11.72 A car and a truck are both traveling at the constant speed of 35 mi/h; the car is 40 ft behind the truck. The driver of the car wants to pass the truck, i.e., he wishes to place his car at *B*, 40 ft in front of the truck, and then resume the speed of 35 mi/h. The maximum acceleration of the car is 5 ft/s^2 and the maximum deceleration obtained by applying the brakes is 20 ft/s^2. What is the shortest time in which the driver of the car can complete the passing operation if he does not at any time exceed a speed of 50 mi/h? Draw the $v\text{--}t$ curve.



SOLUTION

Relative to truck, car must move a distance:

$$\Delta x = 16 + 40 + 50 + 40 = 146 \text{ ft}$$



$$\Delta v_m = 5t_1 = 20t_2; \quad t_2 = \frac{1}{4}t_1$$

$\Delta x = A_1 + A_2$: $$146 \text{ ft} = \frac{1}{2}(\Delta v_m)(t_1 + t_2)$$

$$146 \text{ ft} = \frac{1}{2}(5t_1)\left(t_1 + \frac{1}{4}t_1\right)$$

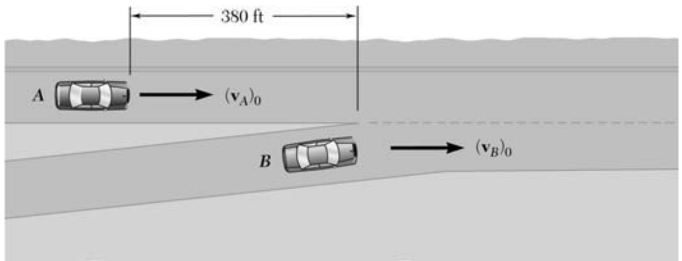
$$t_1^2 = 46.72 \qquad t_1 = 6.835 \text{ s} \qquad t_2 = \frac{1}{4}t_1 = 1.709$$

$$t_{\text{total}} = t_1 + t_2 = 6.835 + 1.709$$ $t_B = 8.54 \text{ s}$ ◀

$$\Delta v_m = 5t_1 = 5(6.835) = 34.18 \text{ ft/s} = 23.3 \text{ mi/h}$$

Speed $v_{\text{total}} = 35 \text{ mi/h}$, $v_m = 35 \text{ mi/h} + 23.3 \text{ mi/h}$ $v_m = 58.3 \text{ mi/h}$ ◀

PROBLEM 11.74

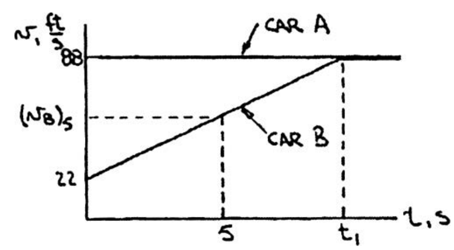


Car A is traveling on a highway at a constant speed $(v_A)_0 = 60$ mi/h, and is 380 ft from the entrance of an access ramp when car B enters the acceleration lane at that point at a speed $(v_B)_0 = 15$ mi/h. Car B accelerates uniformly and enters the main traffic lane after traveling 200 ft in 5 s. It then continues to accelerate at the same rate until it reaches a speed of 60 mi/h, which it then maintains. Determine the final distance between the two cars.

SOLUTION

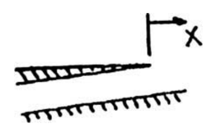
<u>Given</u>: $(v_A)_0 = 60$ mi/h, $(v_B)_0 = 1.5$ mi/h; at $t = 0$,
$(x_A)_0 = -380$ ft, $(x_B)_0 = 0$; at $t = 5$ s,
$x_B = 200$ ft; for 15 mi/h $< v_B \leq 60$ mi/h,
$a_B =$ constant; for $v_B = 60$ mi/h,
$a_B = 0$

First note 60 mi/h = 88 ft/s
15 mi/h = 22 ft/s



The v–t curves of the two cars are then drawn as shown.

Using the coordinate system shown, we have



at $t = 5$ s, $x_B = 200$ ft: $$(5\text{ s})\left[\frac{22 + (v_B)_5}{2}\right]\text{ft/s} = 200\text{ ft}$$

or $$(v_B)_5 = 58\text{ ft/s}$$

Then, using similar triangles, we have

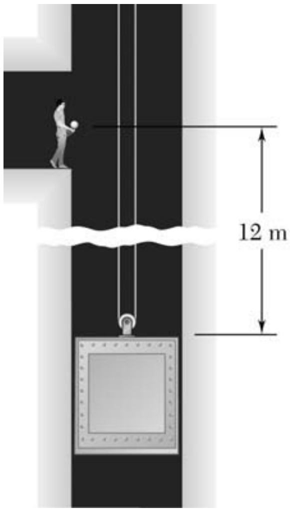
$$\frac{(88-22)\text{ ft/s}}{t_1} = \frac{(58-22)\text{ ft/s}}{5\text{ s}} (= a_B)$$

or $$t_1 = 9.1667\text{ s}$$

Finally, at $t = t_1$

$$x_{B/A} = x_B - x_A = \left[(9.1667\text{ s})\left(\frac{22+88}{2}\right)\text{ft/s}\right] - [-380\text{ ft} + (9.1667\text{ s})(88\text{ ft/s})]$$

or $$x_{B/A} = 77.5\text{ ft} \blacktriangleleft$$



PROBLEM 11.75

An elevator starts from rest and moves upward, accelerating at a rate of 1.2 m/s^2 until it reaches a speed of 7.8 m/s, which it then maintains. Two seconds after the elevator begins to move, a man standing 12 m above the initial position of the top of the elevator throws a ball upward with an initial velocity of 20 m/s. Determine when the ball will hit the elevator.

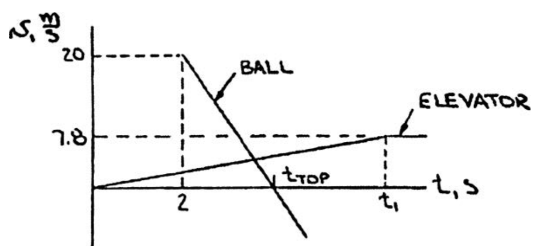
SOLUTION

Given: At $t = 0 \quad v_E = 0$; For $0 < v_E \le 7.8$ m/s, $a_E = 1.2 \text{ m/s}^2 \uparrow$;

For $v_E = 7.8$ m/s, $\quad a_E = 0$;

At $t = 2$ s, $\quad v_B = 20\text{m/s} \uparrow$

The $v-t$ curves of the ball and the elevator are first drawn as shown. Note that the initial slope of the curve for the elevator is 1.2 m/s^2, while the slope of the curve for the ball is $-g \, (-9.81 \text{ m/s}^2)$.



The time t_1 is the time when v_E reaches 7.8 m/s.

Thus, $$v_E = (0) + a_E t$$

or $$7.8 \text{ m/s} = (1.2 \text{ m/s}^2) t_1$$

or $$t_1 = 6.5 \text{ s}$$

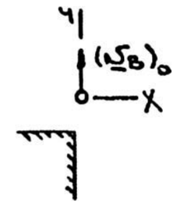
The time t_{top} is the time at which the ball reaches the top of its trajectory.

Thus, $$v_B = (v_B)_0 - g(t - 2)$$

or $$0 = 20 \text{ m/s} - (9.81 \text{ m/s}^2)(t_{\text{top}} - 2) \text{ s}$$

or $$t_{\text{top}} = 4.0387 \text{ s}$$

Using the coordinate system shown, we have



$0 < t < t_1$: $\quad y_E = -12\text{ m} + \left(\frac{1}{2}a_E t^2\right)\text{ m}$

At $t = t_{\text{top}}$: $\quad y_B = \frac{1}{2}(4.0387 - 2)\text{ s} \times (20\text{ m/s})$

$$= 20.387\text{ m}$$

and $\quad y_E = -12\text{ m} + \frac{1}{2}(1.2\text{ m/s}^2)(4.0387\text{ s})^2$

$$= -2.213\text{ m}$$

At $\quad t = [2 + 2(4.0387 - 2)]\text{ s} = 6.0774\text{ s}, \quad y_B = 0$

and at $t = t_1$, $\quad y_E = -12\text{ m} + \frac{1}{2}(6.5\text{ s})(7.8\text{ m/s}) = 13.35\text{ m}$

The ball hits the elevator $(y_B = y_E)$ when $t_{\text{top}} \le t \le t_1$.

For $t \ge t_{\text{top}}$: $\quad y_B = 20.387\text{ m} - \left[\frac{1}{2}\text{ g }(t - t_{\text{top}})^2\right]\text{ m}$

Then,

when $\quad y_B = y_E$

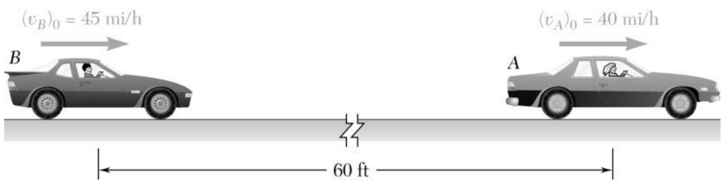
$$20.387\text{ m} - \frac{1}{2}(9.81\text{ m/s}^2)(t - 4.0387)^2$$
$$= -12\text{ m} + \frac{1}{2}(1.2\text{ m/s}^2)(t\text{ s})^2$$

or $\quad 5.505t^2 - 39.6196t + 47.619 = 0$

Solving $\quad t = 1.525\text{ s} \quad \text{and} \quad t = 5.67\text{ s}$

Since 1.525 s is less than 2 s, $\quad t = 5.67\text{ s}$ ◀

PROBLEM 11.76



Car *A* is traveling at 40 mi/h when it enters a 30 mi/h speed zone. The driver of car *A* decelerates at a rate of 16 ft/s^2 until reaching a speed of 30 mi/h, which she then maintains. When car *B*, which was initially 60 ft behind car *A* and traveling at a constant speed of 45 mi/h, enters the speed zone, its driver decelerates at a rate of 20 ft/s^2 until reaching a speed of 28 mi/h. Knowing that the driver of car *B* maintains a speed of 28 mi/h, determine (*a*) the closest that car *B* comes to car *A*, (*b*) the time at which car *A* is 70 ft in front of car *B*.

SOLUTION

Given: $(v_A)_0 = 40$ mi/h; For 30 mi/h $< v_A \le 40$ mi/h, $a_A = -16$ ft/s^2; For $v_A = 30$ mi/h, $a_A = 0$;

$(x_{A/B})_0 = 60$ ft; $(v_B)_0 = 45$ mi/h;

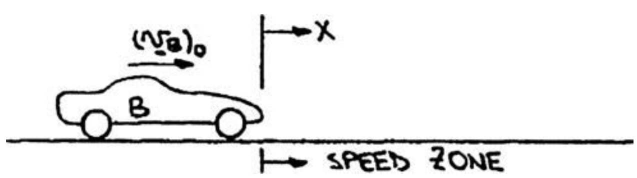
When $x_B = 0$, $a_B = -20$ ft/s^2;

For $v_B = 28$ mi/h, $a_B = 0$

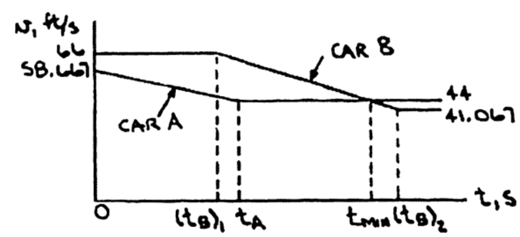
First note 40 mi/h = 58.667 ft/s 30 mi/h = 44 ft/s

45 mi/h = 66 ft/s 28 mi/h = 41.067 ft/s

At $t = 0$



The $v\text{–}t$ curves of the two cars are as shown.



At $t = 0$: Car *A* enters the speed zone.

$t = (t_B)_1$: Car *B* enters the speed zone.

$t = t_A$: Car *A* reaches its final speed.

$t = t_{\min}$: $v_A = v_B$

$t = (t_B)_2$: Car *B* reaches its final speed.

PROBLEM 11.76 (Continued)

(*a*) We have $$a_A = \frac{(v_A)_{\text{final}} - (v_A)_0}{t_A}$$

or $$-16 \text{ ft/s}^2 = \frac{(44 - 58.667) \text{ ft/s}}{t_A}$$

or $$t_A = 0.91669 \text{ s}$$

Also $$60 \text{ ft} = (t_B)_1 (v_B)_0$$

or $$60 \text{ ft} = (t_B)_1 (66 \text{ ft/s}) \qquad \text{or} \qquad (t_B)_1 = 0.90909 \text{ s}$$

and $$a_B = \frac{(v_B)_{\text{final}} - (v_B)_0}{(t_B)_2 - (t_B)_1}$$

or $$-20 \text{ ft/s}^2 = \frac{(41.067 - 66) \text{ ft/s}}{[(t_B)_2 - 0.90909] \text{ s}}$$

Car *B* will continue to overtake car *A* while $v_B > v_A$. Therefore, $(x_{A/B})_{\min}$ will occur when $v_A = v_B$, which occurs for

$$(t_B)_1 < t_{\min} < (t_B)_2$$

For this time interval

$$v_A = 44 \text{ ft/s}$$
$$v_B = (v_B)_0 + a_B[t - (t_B)_1]$$

Then at $t = t_{\min}$: $$44 \text{ ft/s} = 66 \text{ ft/s} + (-20 \text{ ft/s}^2)(t_{\min} - 0.90909) \text{ s}$$

or $$t_{\min} = 2.00909 \text{ s}$$

Finally $$(x_{A/B})_{\min} = (x_A)_{t_{\min}} - (x_B)_{t_{\min}}$$
$$= \left\{ t_A \left[\frac{(v_A)_0 + (v_A)_{\text{final}}}{2} \right] + (t_{\min} - t_A)(v_A)_{\text{final}} \right\}$$
$$- \left\{ (x_B)_0 + (t_B)_1 (v_B)_0 + [t_{\min} - (t_B)_1] \left[\frac{(v_B)_0 + (v_A)_{\text{final}}}{2} \right] \right\}$$
$$= \left[(0.91669 \text{ s}) \left(\frac{58.667 + 44}{2} \right) \text{ft/s} + (2.00909 - 0.91669) \text{ s} \times (44 \text{ ft/s}) \right]$$
$$- \left[-60 \text{ ft} + (0.90909 \text{ s})(66 \text{ ft/s}) + (2.00909 - 0.90909) \text{ s} \times \left(\frac{66 + 44}{2} \right) \text{ft/s} \right]$$
$$= (47.057 + 48.066) \text{ ft} - (-60 + 60.000 + 60.500) \text{ ft}$$
$$= 34.623 \text{ ft}$$

or $(x_{A/B})_{\min} = 34.6 \text{ ft}$ ◀

PROBLEM 11.76 (Continued)

(*b*) Since $(x_{A/B}) \le 60$ ft for $t \le t_{\min}$, it follows that $x_{A/B} = 70$ ft for $t > (t_B)_2$

[Note $(t_B)_2 \simeq t_{\min}$]. Then, for $t > (t_B)_2$

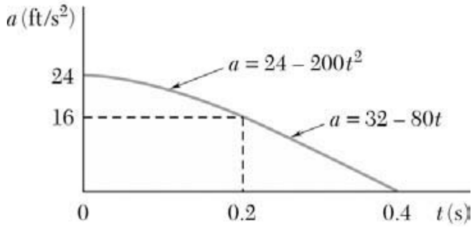
$$x_{A/B} = (x_{A/B})_{\min} + [(t - t_{\min})(v_A)_{\text{final}}] - \left\{[(t_B)_2 - (t_{\min})]\left[\frac{(v_A)_{\text{final}} + (v_B)_{\text{final}}}{2}\right] + [t - (t_B)_2](v_B)_{\text{final}}\right\}$$

or

$$70 \text{ ft} = 34.623 \text{ ft} + [(t - 2.00909) \text{ s} \times (44 \text{ ft/s})] - \left[(2.15574 - 2.00909) \text{ s} \times \left(\frac{44 + 41.06}{2}\right) \text{ft/s} + (t - 2.15574) \text{ s} \times (41.067) \text{ ft/s}\right]$$

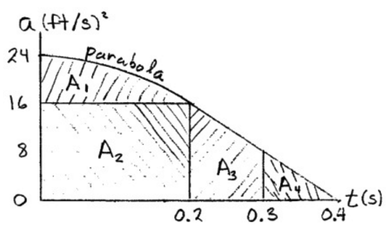
or $t = 14.14$ s ◀

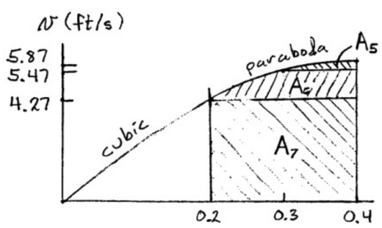
PROBLEM 11.77



An accelerometer record for the motion of a given part of a mechanism is approximated by an arc of a parabola for 0.2 s and a straight line for the next 0.2 s as shown in the figure. Knowing that $v = 0$ when $t = 0$ and $x = 0.8$ ft when $t = 0.4$ s, (*a*) construct the $v\text{–}t$ curve for $0 \le t \le 0.4$ s, (*b*) determine the position of the part at $t = 0.3$ s and $t = 0.2$ s.

SOLUTION





Divide the area of the $a\text{–}t$ curve into the four areas A_1, A_2, A_3 and A_4.

$$A_1 = \frac{2}{3}(8)(0.2) = 1.0667 \text{ ft/s}$$

$$A_2 = (16)(0.2) = 3.2 \text{ ft/s}$$

$$A_3 = \frac{1}{2}(16 + 8)(0.1) = 1.2 \text{ ft/s}$$

$$A_4 = \frac{1}{2}(8)(0.1) = 0.4 \text{ ft/s}$$

Velocities: $v_0 = 0$

$v_{0.2} = v_0 + A_1 + A_2$ $\qquad v_{0.2} = 4.27 \text{ ft/s}$ ◀

$v_{0.3} = v_{0.2} + A_3$ $\qquad v_{0.3} = 5.47 \text{ ft/s}$ ◀

$v_{0.4} = v_{0.3} + A_4$ $\qquad v_{0.4} = 5.87 \text{ ft/s}$ ◀

Sketch the $v\text{–}t$ curve and divide its area into A_5, A_6, and A_7 as shown.

$$\int_x^{0.8} dx = 0.8 - x = \int_t^{0.4} v\,dt \qquad \text{or} \qquad x = 0.8 - \int_t^{0.4} v\,dt$$

At $t = 0.3$ s, $\qquad x_{0.3} = 0.8 - A_5 - (5.47)(0.1)$

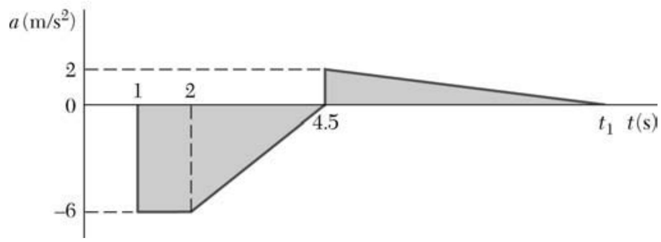
With $A_5 = \frac{2}{3}(0.4)(0.1) = 0.0267$ ft, $\qquad x_{0.3} = 0.227$ ft ◀

At $t = 0.2$ s, $\qquad x_{0.2} = 0.8 - (A_5 + A_6) - A_7$

With $A_5 + A_6 = \frac{2}{3}(1.6)(0.2) = 0.2133$ ft,

and $\quad A_7 = (4.27)(0.2) = 0.8533$ ft

$x_{0.2} = 0.8 - 0.2133 - 0.8533$ $\qquad x_{0.2} = -0.267$ ft ◀



PROBLEM 11.78

A car is traveling at a constant speed of 54 km/h when its driver sees a child run into the road. The driver applies her brakes until the child returns to the sidewalk and then accelerates to resume her original speed of 54 km/h; the acceleration record of the car is shown in the figure. Assuming $x = 0$ when $t = 0$, determine (*a*) the time t_1 at which the velocity is again 54 km/h, (*b*) the position of the car at that time, (*c*) the average velocity of the car during the interval $1 \text{ s} \le t \le t_1$.

SOLUTION

Given: At $t = 0, \quad x = 0, \quad v = 54$ km/h;

For $t = t_1, \quad v = 54$ km/h

First note 54 km/h = 15 m/s

(*a*) We have $v_b = v_a + (\text{area under } a\text{–}t \text{ curve from } t_a \text{ to } t_b)$

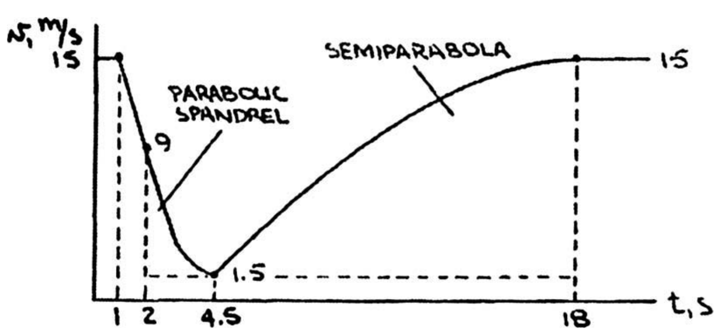
Then at $t = 2$ s: $v = 15 - (1)(6) = 9$ m/s

$$t = 4.5 \text{ s:} \quad v = 9 - \frac{1}{2}(2.5)(6) = 1.5 \text{ m/s}$$

$$t = t_1: \quad 15 = 1.5 + \frac{1}{2}(t_1 - 4.5)(2)$$

or $t_1 = 18.00$ s ◀

(*b*) Using the above values of the velocities, the $v\text{–}t$ curve is drawn as shown.



Now x at $t = 18$ s

$$x_{18} = 0 + \Sigma\,(\text{area under the } v\text{–}t \text{ curve from } t = 0 \text{ to } t = 18 \text{ s})$$

$$= (1\text{ s})(15\text{ m/s}) + (1\text{ s})\left(\frac{15+9}{2}\right)\text{m/s}$$

$$+\left[(2.5\text{ s})(1.5\text{ m/s}) + \frac{1}{3}(2.5\text{ s})(7.5\text{ m/s})\right]$$

$$+\left[(13.5\text{ s})(1.5\text{ m/s}) + \frac{2}{3}(13.5\text{ s})(13.5\text{ m/s})\right]$$

$$= [15 + 12 + (3.75 + 6.25) + (20.25 + 121.50)]\text{ m}$$

$$= 178.75\text{ m}$$

or $x_{18} = 178.8$ m ◀

(*c*) First note

$$x_1 = 15\text{ m}$$

$$x_{18} = 178.75\text{ m}$$

Now

$$v_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{(178.75 - 15)\text{ m}}{(18 - 1)\text{ s}} = 9.6324\text{ m/s}$$

or $v_{\text{ave}} = 34.7$ km/h ◀

PROBLEM 11.79

An airport shuttle train travels between two terminals that are 1.6 mi apart. To maintain passenger comfort, the acceleration of the train is limited to ±4 ft/s^2, and the jerk, or rate of change of acceleration, is limited to ±0.8 ft/s^2 per second. If the shuttle has a maximum speed of 20 mi/h, determine (*a*) the shortest time for the shuttle to travel between the two terminals, (*b*) the corresponding average velocity of the shuttle.

SOLUTION

Given: $x_{max} = 1.6 \text{ mi}; \quad |a_{max}| = 4 \text{ ft/s}^2$

$$\left|\left(\frac{da}{dt}\right)_{max}\right| = 0.8 \text{ ft/s}^2/\text{s}; \quad v_{max} = 20 \text{ mi/h}$$

First note $20 \text{ mi/h} = 29.333 \text{ ft/s}$

$$1.6 \text{ mi} = 8448 \text{ ft}$$

(*a*) To obtain t_{min}, the train must accelerate and decelerate at the maximum rate to maximize the time for which $v = v_{max}$. The time Δt required for the train to have an acceleration of 4 ft/s^2 is found from

$$\left(\frac{da}{dt}\right)_{max} = \frac{a_{max}}{\Delta t}$$

or $$\Delta t = \frac{4 \text{ ft/s}^2}{0.8 \text{ ft/s}^2/\text{s}}$$

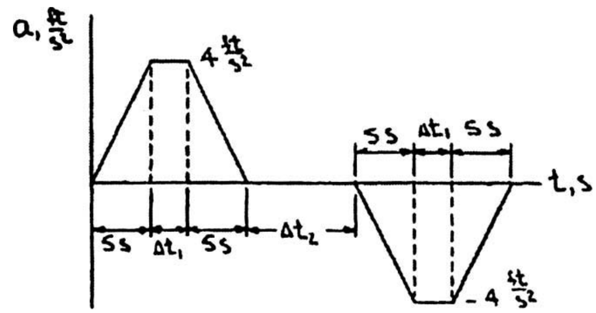
or $$\Delta t = 5 \text{ s}$$

Now,

after 5 s, the speed of the train is $v_5 = \frac{1}{2}(\Delta t)(a_{max}) \quad \left(\text{since } \frac{da}{dt} = \text{constant}\right)$

or $$v_5 = \frac{1}{2}(5 \text{ s})(4 \text{ ft/s}^2) = 10 \text{ ft/s}$$

Then, since $v_5 < v_{max}$, the train will continue to accelerate at 4 ft/s^2 until $v = v_{max}$. The $a\text{–}t$ curve must then have the shape shown. Note that the magnitude of the slope of each inclined portion of the curve is 0.8 ft/s^2/s.



Now $\quad$ at $t = (10 + \Delta t_1)$ s, $v = v_{max}$:

$$2\left[\frac{1}{2}(5\text{ s})(4\text{ ft/s}^2)\right] + (\Delta t_1)(4\text{ ft/s}^2) = 29.333\text{ ft/s}$$

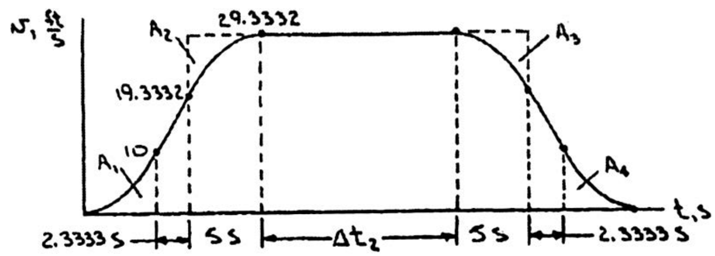
or $\quad$ $\Delta t_1 = 2.3333$ s

Then $\quad$ at $t = 5$ s: $\quad v = 0 + \frac{1}{2}(5)(4) = 10$ ft/s

$t = 7.3333$ s: $\quad v = 10 + (2.3333)(4) = 19.3332$ ft/s

$t = 12.3333$ s: $\quad v = 19.3332 + \frac{1}{2}(5)(4) = 29.3332$ ft/s

Using symmetry, the v–t curve is then drawn as shown.



Noting that $A_1 = A_2 = A_3 = A_4$ and that the area under the v–t curve is equal to x_{max}, we have

$$2\left[(2.3333\text{ s})\left(\frac{10 + 19.3332}{2}\right)\text{ft/s}\right]$$
$$+ (10 + \Delta t_2)\text{ s} \times (29.3332\text{ ft/s}) = 8448\text{ ft}$$

or $\quad$ $\Delta t_2 = 275.67$ s

Then $\quad$ $t_{min} = 4(5\text{ s}) + 2(2.3333\text{ s}) + 275.67\text{ s}$

$= 300.34$ s

or $\quad$ $t_{min} = 5.01$ min ◀

(*b*) $\quad$ We have $\quad$ $v_{ave} = \frac{\Delta x}{\Delta t} = \frac{1.6\text{ mi}}{300.34\text{ s}} \times \frac{3600\text{ s}}{1\text{ h}}$

or $\quad$ $v_{ave} = 19.18$ mi/h ◀

PROBLEM 11.80

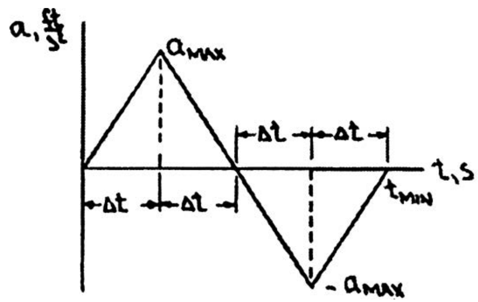
During a manufacturing process, a conveyor belt starts from rest and travels a total of 1.2 ft before temporarily coming to rest. Knowing that the jerk, or rate of change of acceleration, is limited to ±4.8 ft/s² per second, determine (*a*) the shortest time required for the belt to move 1.2 ft, (*b*) the maximum and average values of the velocity of the belt during that time.

SOLUTION

Given: At $t = 0, \quad x = 0, \quad v = 0; \quad x_{max} = 1.2 \text{ ft};$

when $x = x_{max}, \quad v = 0; \quad \left| \left(\frac{da}{dt} \right)_{max} \right| = 4.8 \text{ ft/s}^2$

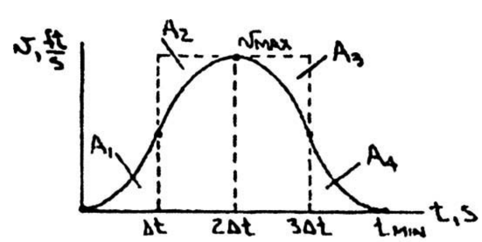
(*a*) Observing that v_{max} must occur at $t = \frac{1}{2} t_{min}$, the $a-t$ curve must have the shape shown. Note that the magnitude of the slope of each portion of the curve is 4.8 ft/s²/s.



We have at $t = \Delta t$: $v = 0 + \frac{1}{2}(\Delta t)(a_{max}) = \frac{1}{2} a_{max} \Delta t$

$t = 2\Delta t$: $v_{max} = \frac{1}{2} a_{max} \Delta t + \frac{1}{2}(\Delta t)(a_{max}) = a_{max} \Delta t$

Using symmetry, the $v-t$ is then drawn as shown.



Noting that $A_1 = A_2 = A_3 = A_4$ and that the area under the $v-t$ curve is equal to x_{max}, we have

$$(2\Delta t)(v_{max}) = x_{max}$$

$$v_{max} = a_{max} \Delta t \Rightarrow 2a_{max} \Delta t^2 = x_{max}$$

PROBLEM 11.80 (Continued)

Now $\dfrac{a_{max}}{\Delta t} = 4.8 \text{ ft/s}^2/\text{s}$ so that

$$2(4.8\Delta t \text{ ft/s}^3)\Delta t^2 = 1.2 \text{ ft}$$

or $\Delta t = 0.5 \text{ s}$

Then $t_{min} = 4\Delta t$

or $t_{min} = 2.00 \text{ s}$ ◀

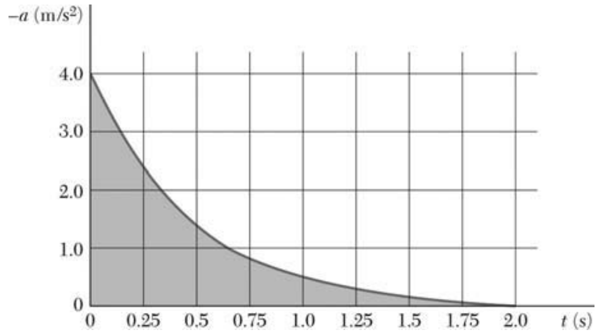
(*b*) We have

$$v_{max} = a_{max}\Delta t$$
$$= (4.8 \text{ ft/s}^2/\text{s} \times \Delta\text{t})\Delta t$$
$$= 4.8 \text{ ft/s}^2/\text{s} \times (0.5 \text{ s})^2$$

or $v_{max} = 1.2 \text{ ft/s}$ ◀

Also $v_{ave} = \dfrac{\Delta x}{\Delta t_{total}} = \dfrac{1.2 \text{ ft}}{2.00 \text{ s}}$

or $v_{ave} = 0.6 \text{ ft/s}$ ◀



PROBLEM 11.81

Two seconds are required to bring the piston rod of an air cylinder to rest; the acceleration record of the piston rod during the 2 s is as shown. Determine by approximate means (*a*) the initial velocity of the piston rod, (*b*) the distance traveled by the piston rod as it is brought to rest.

SOLUTION

Given: $a\text{–}t$ curve; at $t = 2\text{ s}$, $v = 0$

1. The $a\text{–}t$ curve is first approximated with a series of rectangles, each of width $\Delta t = 0.25$ s. The area $(\Delta t)(a_{\text{ave}})$ of each rectangle is approximately equal to the change in velocity Δv for the specified interval of time. Thus,

$$\Delta v \cong a_{\text{ave}}\, \Delta t$$

where the values of a_{ave} and Δv are given in columns 1 and 2, respectively, of the following table.

2. Now $$v(2) = v_0 + \int_0^2 a\, dt = 0$$

and approximating the area $\int_0^2 a\, dt$ under the $a\text{–}t$ curve by $\Sigma a_{\text{ave}}\Delta t \approx \Sigma \Delta v$, the initial velocity is then equal to

$$v_0 = -\Sigma \Delta v$$

Finally, using

$$v_2 = v_1 + \Delta v_{12}$$

where Δv_{12} is the change in velocity between times t_1 and t_2, the velocity at the end of each 0.25 interval can be computed; see column 3 of the table and the $v\text{–}t$ curve.

3. The $v\text{–}t$ curve is then approximated with a series of rectangles, each of width 0.25 s. The area $(\Delta t)(v_{\text{ave}})$ of each rectangle is approximately equal to the change in position Δx for the specified interval of time. Thus

$$\Delta x \approx v_{\text{ave}}\Delta t$$

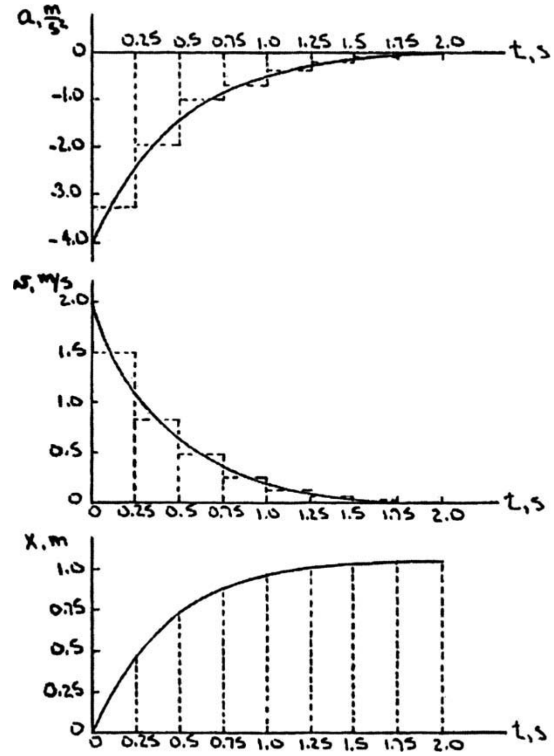
where v_{ave} and Δx are given in columns 4 and 5, respectively, of the table.

4. With $x_0 = 0$ and noting that

$$x_2 = x_1 + \Delta x_{12}$$

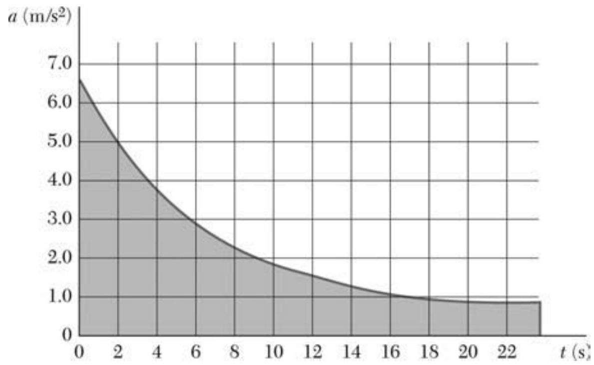
where Δx_{12} is the change in position between times t_1 and t_2, the position at the end of each 0.25 s interval can be computed; see column 6 of the table and the $x–t$ curve.

		1	2	3	4	5	6
t, s	a, m/s²	a_{ave}, m/s²	Δv, m/s	v, m/s	v_{ave}, m/s	Δx, m	x, m
0	−4.00			1.914			0
		−3.215	−0.804		1.512	0.378	
0.25	−2.43			1.110			0.378
		−1.915	−0.479		0.871	0.218	
0.50	−1.40			0.631			0.596
		−1.125	−0.281		0.491	0.123	
0.75	−0.85			0.350			0.719
		−0.675	−0.169		0.266	0.067	
1.00	−0.50			0.181			0.786
		−0.390	−0.098		0.132	0.033	
1.25	−0.28			0.083			0.819
		−0.205	−0.051		0.058	0.015	
1.50	−0.13			0.032			0.834
		−0.095	−0.024		0.020	0.005	
1.75	−0.06			0.008			0.839
		−0.030	−0.008		0.004	0.001	
2.00	0			0			0.840



(*a*) We had found $v_0 = 1.914$ m/s ◀

(*b*) At $t = 2$ s $x = 0.840$ m ◀



PROBLEM 11.82

The acceleration record shown was obtained during the speed trials of a sports car. Knowing that the car starts from rest, determine by approximate means (*a*) the velocity of the car at $t = 8$ s, (*b*) the distance the car has traveled at $t = 20$ s.

SOLUTION

Given: $a\text{–}t$ curve; at $t = 0,\ x = 0,\ v = 0$

1. The $a\text{–}t$ curve is first approximated with a series of rectangles, each of width $\Delta t = 2$ s. The area $(\Delta t)(a_{\text{ave}})$ of each rectangle is approximately equal to the change in velocity Δv for the specified interval of time. Thus,

$$\Delta v \cong a_{\text{ave}}\ \Delta t$$

where the values of a_{ave} and Δv are given in columns 1 and 2, respectively, of the following table.

2. Noting that $v_0 = 0$ and that

$$v_2 = v_1 + \Delta v_{12}$$

where Δv_{12} is the change in velocity between times t_1 and t_2, the velocity at the end of each 2 s interval can be computed; see column 3 of the table and the $v\text{–}t$ curve.

3. The $v\text{–}t$ curve is next approximated with a series of rectangles, each of width $\Delta t = 2$ s. The area $(\Delta t)(v_{\text{ave}})$ of each rectangle is approximately equal to the change in position Δx for the specified interval of time.

Thus, $$\Delta x \cong v_{\text{ave}}\ \Delta t$$

where v_{ave} and Δx are given in columns 4 and 5, respectively, of the table.

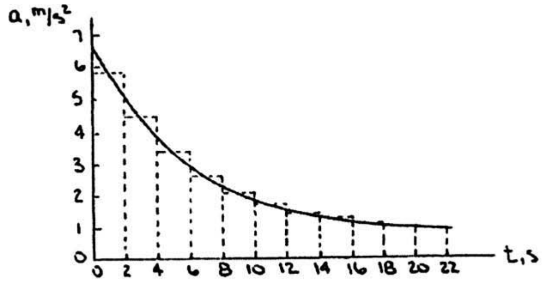
4. With $x_0 = 0$ and noting that

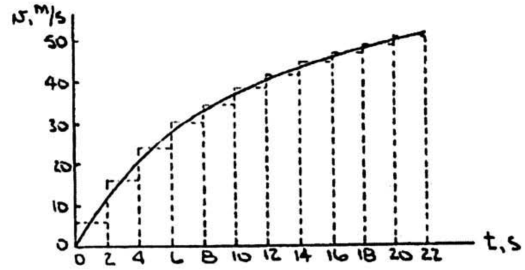
$$x_2 = x_1 + \Delta x_{12}$$

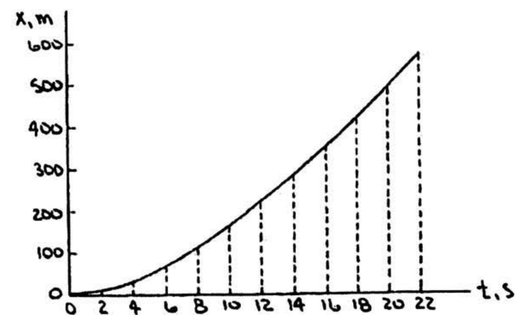
where Δx_{12} is the change in position between times t_1 and t_2, the position at the end of each 2 s interval can be computed; see column 6 of the table and the $x\text{–}t$ curve.

PROBLEM 11.82 (Continued)

t, s	a, m/s²	1: a_{ave}, m/s²	2: Δv, m/s	3: v, m/s	4: v_{ave}, m/s	5: Δx, m	6: x, m
0	6.63			0			0
		5.86	11.72		5.86	11.72	
2	5.08			11.72			11.72
		4.47	8.94		16.19	32.38	
4	3.86			20.66			44.10
		3.38	6.76		24.04	48.08	
6	2.90			27.42			92.18
		2.58	5.16		30.00	60.00	
8	2.25			32.58			152.18
		2.06	4.12		34.64	69.28	
10	1.87			36.70			221.46
		1.71	3.42		38.41	76.82	
12	1.54			40.12			298.28
		1.42	2.84		41.54	83.08	
14	1.29			42.96			381.36
		1.23	2.46		44.19	88.38	
16	1.16			45.42			469.74
		1.10	2.20		46.52	93.04	
18	1.03			47.62			562.78
		1.00	2.00		48.62	97.24	
20	0.97			49.62			660.02
		0.94	1.88		50.56	101.12	
22	0.90			51.50			761.14

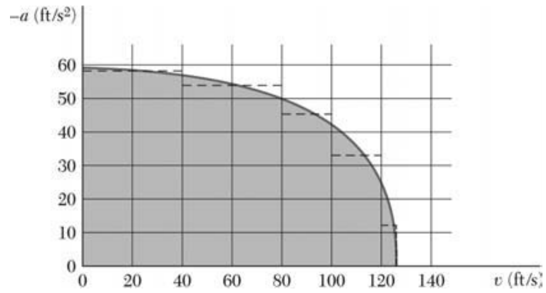






(*a*) At $t = 8$ s, $v = 32.58$ m/s or $v = 117.3$ km/h ◀

(*b*) At $t = 20$ s $x = 660$ m ◀



PROBLEM 11.83

A training airplane has a velocity of 126 ft/s when it lands on an aircraft carrier. As the arresting gear of the carrier brings the airplane to rest, the velocity and the acceleration of the airplane are recorded; the results are shown (solid curve) in the figure. Determine by approximate means (*a*) the time required for the airplane to come to rest, (*b*) the distance traveled in that time.

SOLUTION

Given: $a\!-\!v$ curve:

$$v_0 = 126 \text{ ft/s}$$

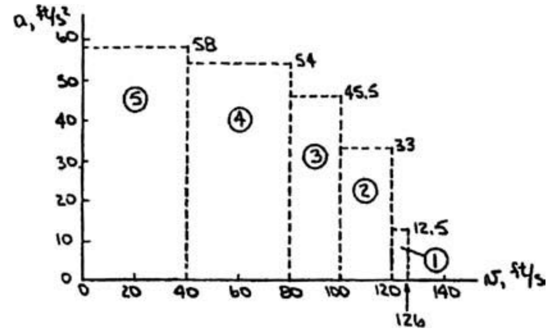
The given curve is approximated by a series of uniformly accelerated motions (the horizontal dashed lines on the figure).

For uniformly accelerated motion

$$v_2^2 = v_1^2 + 2a(x_2 - x_1)$$
$$v_2 = v_1 + a(t_2 - t_1)$$

or

$$\Delta x = \frac{v_2^2 - v_1^2}{2a}$$
$$\Delta t = \frac{v_2 - v_1}{a}$$

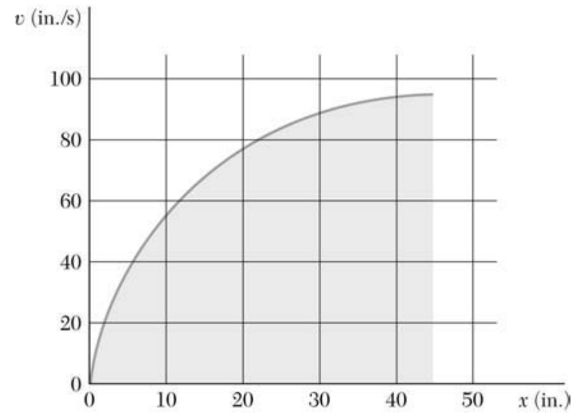


For the five regions shown above, we have

Region	v_1, ft/s	v_2, ft/s	a, ft/s^2	Δx, ft	Δt, s
1	126	120	−12.5	59.0	0.480
2	120	100	−33	66.7	0.606
3	100	80	−45.5	39.6	0.440
4	80	40	−54	44.4	0.741
5	40	0	−58	13.8	0.690
Σ				223.5	2.957

(*a*) From the table, when $v = 0$ $\quad t = 2.96 \text{ s}$ ◀

(*b*) From the table and assuming $x_0 = 0$, when $v = 0$ $\quad x = 224 \text{ ft}$ ◀

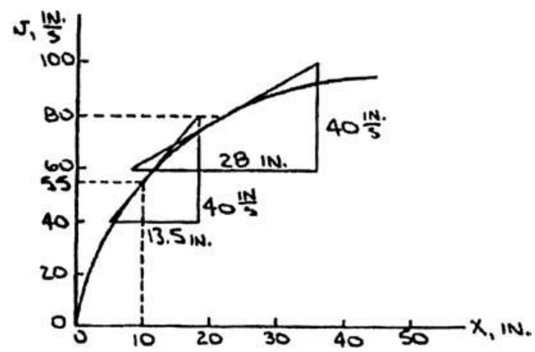


PROBLEM 11.84

Shown in the figure is a portion of the experimentally determined $v-x$ curve for a shuttle cart. Determine by approximate means the acceleration of the cart (a) when $x = 10$ in., (b) when $v = 80$ in./s.

SOLUTION

Given: $v-x$ curve



First note that the slope of the above curve is $\frac{dv}{dx}$. Now

$$a = v\frac{dv}{dx}$$

(a) When $x = 10$ in., $v = 55$ in./s

Then $$a = 55 \text{ in./s}\left(\frac{40 \text{ in./s}}{13.5 \text{ in.}}\right)$$

or $a = 163.0 \text{ in./s}^2$ ◀

(b) When $v = 80$ in./s, we have

$$a = 80 \text{ in./s}\left(\frac{40 \text{ in./s}}{28 \text{ in.}}\right)$$

or $a = 114.3 \text{ in./s}^2$ ◀

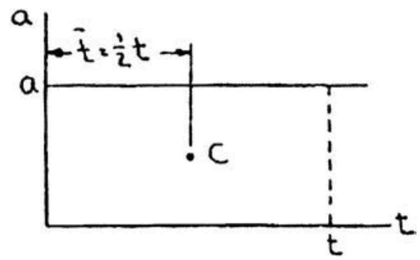
Note: To use the method of measuring the subnormal outlined at the end of Section 11.8, it is necessary that the same scale be used for the x and v axes (e.g., 1 in. = 50 in., 1 in. = 50 in./s). In the above solution, Δv and Δx were measured directly, so different scales could be used.

PROBLEM 11.85

Using the method of Section 11.8, derive the formula $x = x_0 + v_0 t + \frac{1}{2}at^2$ for the position coordinate of a particle in uniformly accelerated rectilinear motion.

SOLUTION

The $a-t$ curve for uniformly accelerated motion is as shown.



Using Eq. (11.13), we have

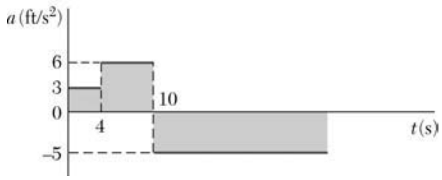
$$x = x_0 + v_0 t + (\text{area under } a\text{–}t \text{ curve})\,(t - \bar{t})$$

$$= x_0 + v_0 t + (t \times a)\left(t - \frac{1}{2}t\right)$$

$$= x_0 + v_0 t + \frac{1}{2}at^2 \qquad \text{Q.E.D.}$$

◀

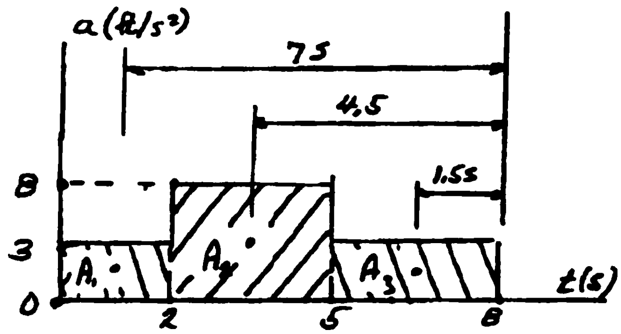
PROBLEM 11.86



Using the method of Section 11.8 determine the position of the particle of Problem 11.61 when $t = 8$ s.

PROBLEM 11.61 A particle moves in a straight line with the acceleration shown in the figure. Knowing that it starts from the origin with $v_0 = -14$ ft/s, plot the v–t and x–t curves for $0 < t < 15$ s and determine (a) the maximum value of the velocity of the particle, (b) the maximum value of its position coordinate.

SOLUTION



$x_0 = 0$

$v_0 = -14$ ft/s

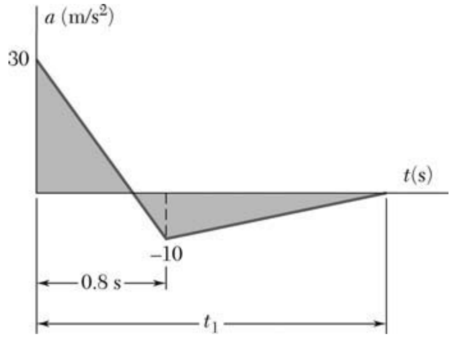
when $t = 8$ s:

$$x = x_0 + v_0 t + \Sigma A(t_1 - t)$$

$$= 0 - (14 \text{ ft/s})(8 \text{ s}) + [(3 \text{ ft/s}^2)(2 \text{ s})](7 \text{ s}) + [(8 \text{ ft/s}^2)(3 \text{ s})](4.5 \text{ s}) + [(3 \text{ ft/s})(3 \text{ s})](1.5 \text{ s})$$

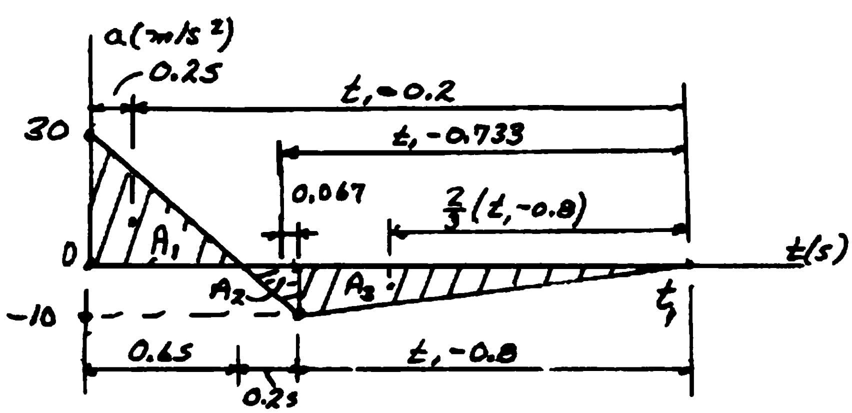
$x_8 = -112 \text{ ft} + 42 \text{ ft} + 108 \text{ ft} + 13.5 \text{ ft}$ $\qquad$ $x_8 = 51.5$ ft ◀

PROBLEM 11.87



The acceleration of an object subjected to the pressure wave of a large explosion is defined approximately by the curve shown. The object is initially at rest and is again at rest at time t_1. Using the method of section 11.8, determine (*a*) the time t_1, (b) the distance through which the object is moved by the pressure wave.

SOLUTION



(*a*) Since $v = 0$ when $t = 0$ and when $t = t_1$ the change in *v* between $t = 0$ and $t = t_1$ is zero.

Thus, area under *a*–*t* curve is zero

$$A_1 + A_2 + A_3 = 0$$

$$\frac{1}{2}(30)(0.6) + \frac{1}{2}(-10)(0.2) + \frac{1}{2}(-10)(t_1 - 0.8) = 0$$

$$9 - 1 - 5t_1 + 4 = 0$$

$t_1 = 2.40$ s ◀

(*b*) <u>Position when $t = t_1 = 2.4$ s</u>

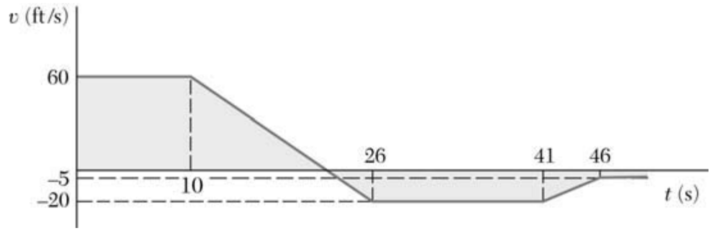
$$x = x_0 + v_0 t_1 + A_1(t_1 - 0.2) + A_2(t_1 - 0.733) + A_3\left(\frac{2}{3}\right)(t_1 - 0.8)$$

$$= 0 + 0 + (9)(2.4 - 0.2) + (-1)(2.4 - 0.733) + \left[\frac{1}{2}(-10)(2.4 - 0.8)\right]\frac{2}{3}(2.4 - 0.8)$$

$$= 19.8 \text{ m} - 1.667 \text{ m} - 8.533 \text{ m}$$

$x = 9.60$ m ◀

PROBLEM 11.88



For the particle of Problem 11.63, draw the $a\text{–}t$ curve and determine, using the method of Section 11.8, (*a*) the position of the particle when $t = 52$ s, (*b*) the maximum value of its position coordinate.

PROBLEM 11.63 A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -540$ m at $t = 0$, (*a*) construct the $a\text{–}t$ and $x\text{–}t$ curves for $0 < t < 50$ s, and determine (*b*) the total distance traveled by the particle when $t = 50$ s, (*c*) the two times at which $x = 0$.

SOLUTION

We have $$a = \frac{dv}{dt}$$

where $\frac{dv}{dt}$ is the slope of the $v\text{–}t$ curve. Then

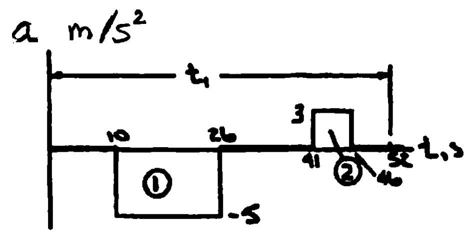
from $t = 0$ to $t = 10$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 10$ s to $t = 26$ s: $a = \dfrac{-20-60}{26-10} = -5 \text{ m/s}^2$

$t = 26$ s to $t = 41$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 41$ s to $t = 46$ s: $a = \dfrac{-5-(-20)}{46-41} = 3 \text{ m/s}^2$

$t > 46$ s: $v = \text{constant} \Rightarrow a = 0$



The $a\text{–}t$ curve is then drawn as shown.

(*a*) From the discussion following Eq. (11.13),

we have $$x = x_0 + v_0 t_1 + \Sigma A(t_1 - \bar{t})$$

where A is the area of a region and $\bar{t}$ is the distance to its centroid. Then, for $t_1 = 52$ s

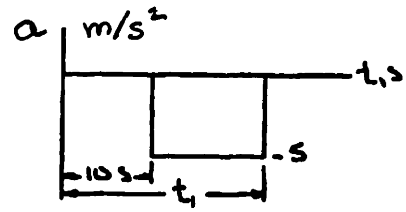
$$\begin{aligned} x &= -540 \text{ m} + (60 \text{ m/s})(52 \text{ s}) + \{-[(16 \text{ s})(5 \text{ m/s}^2)](52-18) \text{ s} \\ &\quad + [(5 \text{ s})(3 \text{ m/s}^2)](52-43.5) \text{ s}\} \\ &= [-540 + (3120) + (-2720 + 127.5)] \text{ m} \end{aligned}$$

or $x = -12.50$ m ◀

PROBLEM 11.88 (Continued)

(*b*) Noting that x_{max} occurs when $v = 0\left(\frac{dx}{dt} = 0\right)$, it is seen from the v–t curve that x_{max} occurs for $10 \text{ s} < t < 26 \text{ s}$. Although similar triangles could be used to determine the time at which $x = x_{max}$ (see the solution to Problem 11.63), the following method will be used.

For $10 \text{ s} < t_1 < 26 \text{ s}$, we have



$$x = -540 + 60t_1 - [(t_1 - 10)(5)]\left[\frac{1}{2}(t_1 - 10)\right] \text{ m}$$

$$= -540 + 60t_1 - \frac{5}{2}(t_1 - 10)^2$$

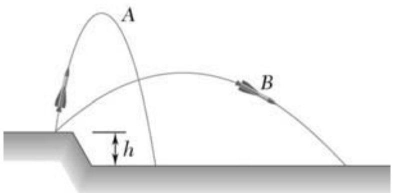
When $x = x_{max}$: $\quad \frac{dx}{dt} = 60 - 5(t_1 - 10) = 0$

or $\quad (t_1)_{x_{max}} = 22 \text{ s}$

Then $\quad x_{max} = -540 + 60(22) - \frac{5}{2}(22 - 10)^2$

or $\quad x_{max} = 420 \text{ m}$ ◀

PROBLEM 11.CQ3



Two model rockets are fired simultaneously from a ledge and follow the trajectories shown. Neglecting air resistance, which of the rockets will hit the ground first?

(a) A

(b) B

(c) They hit at the same time.

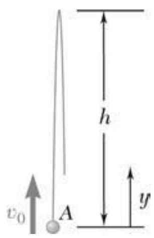
(d) The answer depends on h.

SOLUTION

The motion in the vertical direction depends on the initial velocity in the y-direction. Since A has a larger initial velocity in this direction it will take longer to hit the ground.

Answer: (b) ◀

PROBLEM 11.CQ4



Ball A is thrown straight up. Which of the following statements about the ball are true at the highest point in its path?

(*a*) The velocity and acceleration are both zero.

(*b*) The velocity is zero, but the acceleration is not zero.

(*c*) The velocity is not zero, but the acceleration is zero.

(*d*) Neither the velocity nor the acceleration are zero.

SOLUTION

At the highest point the velocity is zero. The acceleration is never zero.

Answer: (*b*) ◀

PROBLEM 11.CQ5

Ball A is thrown straight up with an initial speed v_0 and reaches a maximum elevation h before falling back down. When A reaches its maximum elevation, a second ball is thrown straight upward with the same initial speed v_0. At what height, y, will the balls cross paths?

(*a*) $y = h$

(*b*) $y > h/2$

(*c*) $y = h/2$

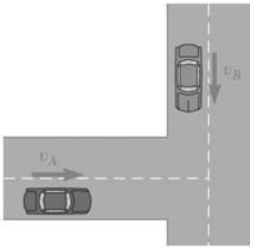
(*d*) $y < h/2$

(*e*) $y = 0$

SOLUTION

When the ball is thrown up in the air it will be constantly slowing down until it reaches its apex, at which point it will have a speed of zero. So, the time it will take to travel the last half of the distance to the apex will be longer than the time it takes for the first half. This same argument can be made for the ball falling from the maximum elevation. It will be speeding up, so the first half of the distance will take longer than the second half. Therefore, the balls should cross above the half-way point.

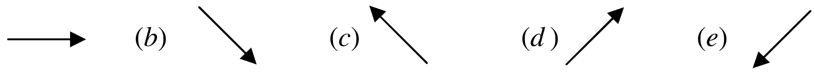
Answer: (*b*) ◀



PROBLEM 11.CQ6

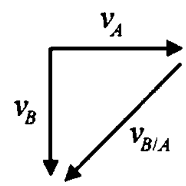
Two cars are approaching an intersection at constant speeds as shown. What velocity will car B appear to have to an observer in car A?

(*a*) (*b*) (*c*) (*d*) (*e*)



SOLUTION

Since $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$ we can draw the vector triangle and see

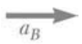


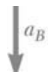
$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

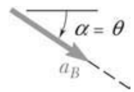
Answer: (*e*) ◀

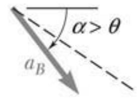
PROBLEM 11.CQ7

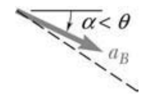
Blocks A and B are released from rest in the positions shown. Neglecting friction between all surfaces, which figure below best indicates the direction α of the acceleration of block B?

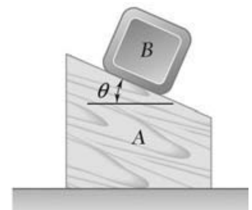
(a) 

(b) 

(c) 

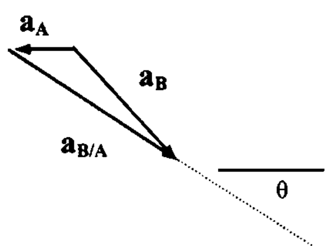
(d) 

(e) 

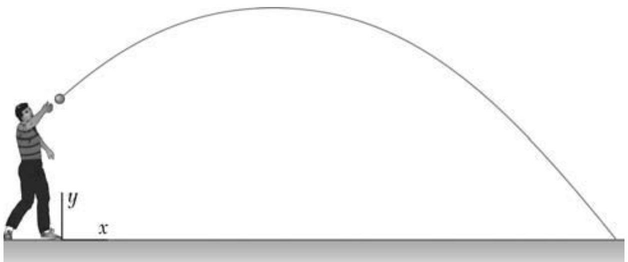


SOLUTION

Since $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$ we get



Answer: (d) ◀



PROBLEM 11.89

A ball is thrown so that the motion is defined by the equations $x = 5t$ and $y = 2 + 6t - 4.9t^2$, where x and y are expressed in meters and t is expressed in seconds. Determine (*a*) the velocity at $t = 1$ s, (*b*) the horizontal distance the ball travels before hitting the ground.

SOLUTION

Units are meters and seconds.

Horizontal motion: $v_x = \dfrac{dx}{dt} = 5$

Vertical motion: $v_y = \dfrac{dy}{dt} = 6 - 9.8t$

(*a*) Velocity at $t = 1$ s. $v_x = 5$

$v_y = 6 - 9.8 = -3.8$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{5^2 + 3.8^2} = 6.28 \text{ m/s}$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{-3.8}{5} \qquad \theta = -37.2°$$

$v = 6.28$ m/s ↘ 37.2° ◀

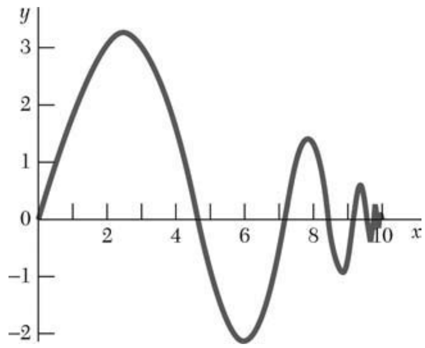
(*b*) Horizontal distance: $(y = 0)$

$y = 2 + 6t - 4.9t^2$

$t = 1.4971$ s

$x = (5)(1.4971) = 7.4856$ m

$x = 7.49$ m ◀



PROBLEM 11.90

The motion of a vibrating particle is defined by the position vector $\mathbf{r} = 10(1 - e^{-3t})\mathbf{i} + (4e^{-2t}\sin 15t)\mathbf{j}$, where $\mathbf{r}$ and t are expressed in millimeters and seconds, respectively. Determine the velocity and acceleration when (*a*) $t = 0$, (*b*) $t = 0.5$ s.

SOLUTION

$$\mathbf{r} = 10(1 - e^{-3t})\mathbf{i} + (4e^{-2t}\sin 15t)\mathbf{j}$$

Then $$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 30e^{-3t}\mathbf{i} + [60e^{-2t}\cos 15t - 8e^{-2t}\sin 15t]\mathbf{j}$$

and $$\mathbf{a} = \frac{d\mathbf{v}}{dt} = -90e^{-3t}\mathbf{i} + [-120e^{-2t}\cos 15t - 900e^{-2t}\sin 15t - 120e^{-2t}\cos 15t + 16e^{-2t}\sin 15t]\mathbf{j}$$

$$= -90e^{-3t}\mathbf{i} + [-240e^{-2t}\cos 15t - 884e^{-2t}\sin 15t]\mathbf{j}$$

(*a*) When $t = 0$:

$\mathbf{v} = 30\mathbf{i} + 60\mathbf{j}$ mm/s $\qquad$ $\mathbf{v} = 67.1$ mm/s $\measuredangle$ 63.4° ◀

$\mathbf{a} = -90\mathbf{i} - 240\mathbf{j}$ mm/s^2 $\qquad$ $\mathbf{a} = 256$ mm/s^2 $\measuredangle$ 69.4° ◀

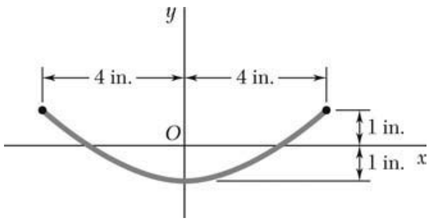
When $t = 0.5$ s:

$$\mathbf{v} = 30e^{-1.5}\mathbf{i} + [60e^{-1}\cos 7.5 - 8e^{-1}\sin 7.5]$$
$$= 6.694\mathbf{i} + 4.8906\mathbf{j} \text{ mm/s}$$

$\mathbf{v} = 8.29$ mm/s $\measuredangle$ 36.2° ◀

$$\mathbf{a} = 90e^{-1.5}\mathbf{i} + [-240e^{-1}\cos 7.5 - 884e^{-1}\sin 7.5\mathbf{j}]$$
$$= -20.08\mathbf{i} - 335.65\mathbf{j} \text{ mm/s}^2$$

$\mathbf{a} = 336$ mm/s^2 $\measuredangle$ 86.6° ◀



PROBLEM 11.91

The motion of a vibrating particle is defined by the position vector $\mathbf{r} = (4\sin \pi t)\mathbf{i} - (\cos 2\pi t)\mathbf{j}$, where r is expressed in inches and t in seconds. (a) Determine the velocity and acceleration when $t = 1$ s. (b) Show that the path of the particle is parabolic.

SOLUTION

$$\mathbf{r} = (4\sin \pi t)\mathbf{i} - (\cos 2\pi t)\mathbf{j}$$
$$\mathbf{v} = (4\pi \cos \pi t)\mathbf{i} + (2\pi \sin 2\pi t)\mathbf{j}$$
$$\mathbf{a} = -(4\pi^2 \sin \pi t)\mathbf{i} + (4\pi^2 \cos 2\pi t)\mathbf{j}$$

(a) When $t = 1$ s:

$$\mathbf{v} = (4\pi \cos \pi)\mathbf{i} + (2\pi \sin 2\pi)\mathbf{j}$$ $\mathbf{v} = -(4\pi \text{ in/s})\mathbf{i}$ ◀

$$\mathbf{a} = -(4\pi^2 \sin \pi)\mathbf{i} - (4\pi^2 \cos \pi)\mathbf{j}$$ $\mathbf{a} = -(4\pi^2 \text{in/s}^2)\mathbf{j}$ ◀

(b) Path of particle:

Since $\mathbf{r} = x\mathbf{i} + y\,\mathbf{j}$; $x = 4\sin \pi t$, $y = -\cos 2\pi t$

Recall that $\cos 2\theta = 1 - 2\sin^2\theta$ and write

$$y = -\cos 2\pi t = -(1 - 2\sin^2 \pi t) \qquad (1)$$

But since $x = 4\sin \pi t$ or $\sin \pi t = \frac{1}{4}x$, Eq.(1) yields

$$y = -\left[1 - 2\left(\frac{1}{4}x\right)^2\right]$$ $y = \frac{1}{8}x^2 - 1$ (Parabola) ◀

PROBLEM 11.92

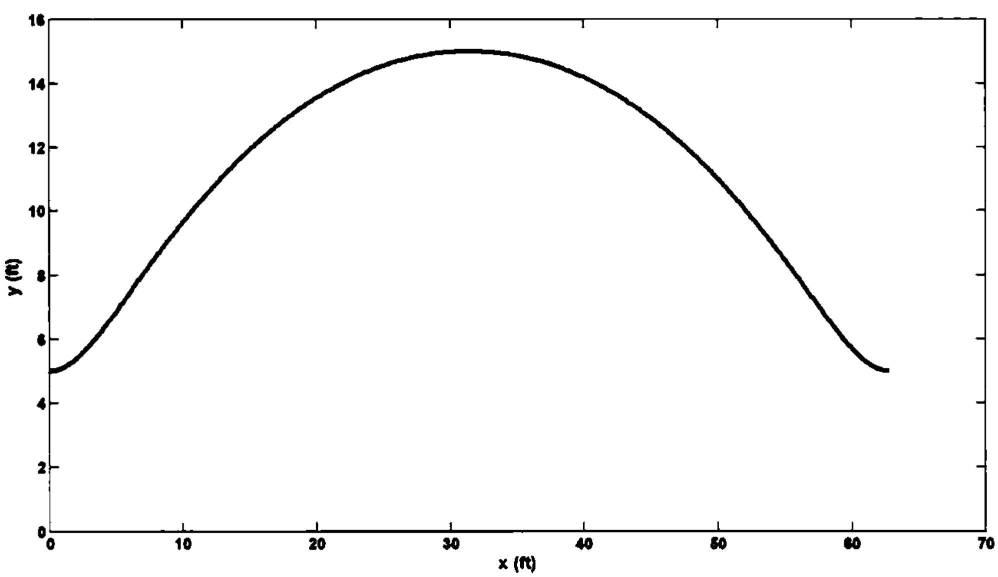
The motion of a particle is defined by the equations $x = 10t - 5\sin t$ and $y = 10 - 5\cos t$, where x and y are expresed in feet and t is expressed in seconds. Sketch the path of the particle for the time interval $0 \le t \le 2\pi$, and determine (*a*) the magnitudes of the smallest and largest velocities reached by the particle, (*b*) the corresponding times, positions, and directions of the velocities.

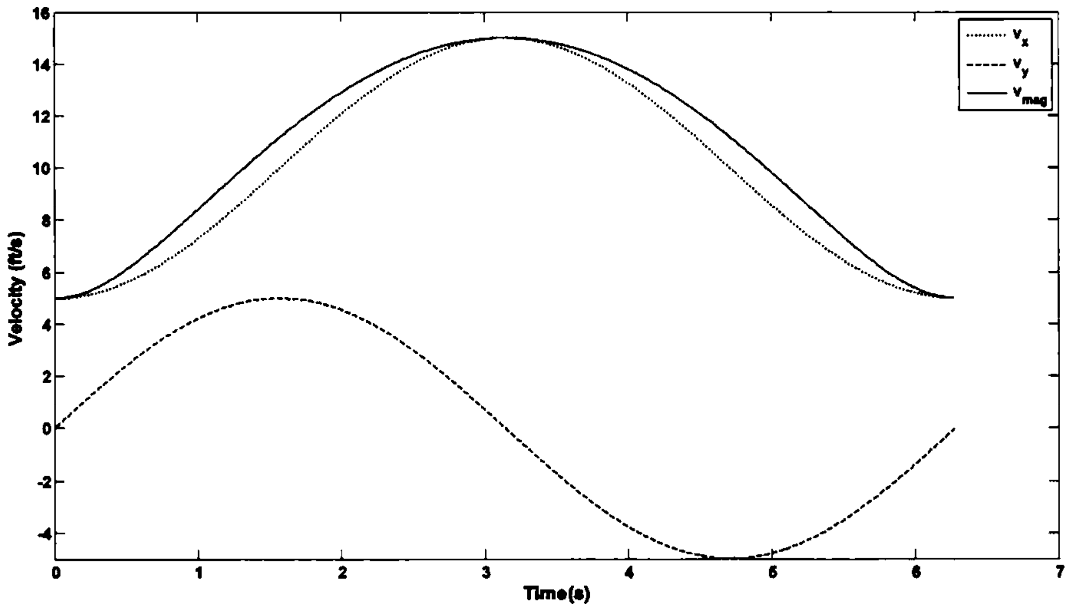
SOLUTION

Sketch the path of the particle, i.e., plot of y versus x.

Using $x = 10t - 5\sin t$, and $y = 10 - 5\cos t$ obtain the values in the table below. Plot as shown.

$t(s)$	x(ft)	y(ft)
0	0.00	5
$\frac{\pi}{2}$	10.71	10
π	31.41	15
$3\frac{\pi}{2}$	52.12	10
2π	62.83	5





(*a*) Differentiate with respect to *t* to obtain velocity components.

$$v_x = \frac{dx}{dt} = 10 - 5\cos t \quad \text{and} \quad v_y = 5\sin t$$

$$v^2 = v_x^2 + v_y^2 = (10 - 5\cos t)^2 + 25\sin^2 t = 125 - 100\cos t$$

$$\frac{d(v)^2}{dt} = 100\sin t = 0 \quad t = 0.\ \pm\pi.\ \pm 2\pi \ldots \pm N\pi$$

When $t = 2N\pi$. $\cos t = 1$. and v^2 is minimum.

When $t = (2N+1)\pi$. $\cos t = -1$. and v^2 is maximum.

$$(v^2)_{\min} = 125 - 100 = 25 (\text{ft/s})^2 \qquad v_{\min} = 5 \text{ ft/s} \blacktriangleleft$$

$$(v^2)_{\max} = 125 + 100 = 225 (\text{ft/s})^2 \qquad v_{\max} = 15 \text{ ft/s} \blacktriangleleft$$

(*b*) When $v = v_{\min}$.

When $N = 0, 1, 2, \ldots$ $\quad x = 10(2\pi N) - 5\sin(2\pi N) \qquad x = 20\pi N \text{ ft} \blacktriangleleft$

$$y = 10 - 5\cos(2\pi N) \qquad y = 5 \text{ ft} \blacktriangleleft$$

$$v_x = 10 - 5\cos(2\pi N) \qquad v_x = 5 \text{ ft/s} \blacktriangleleft$$

$$v_y = 5\sin(2\pi N) \qquad v_y = 0 \blacktriangleleft$$

$$\tan\theta = \frac{v_y}{v_x} = 0, \qquad \theta = 0 \blacktriangleleft$$

PROBLEM 11.92 (Continued)

When $v = v_{\max}$. $t = (2N+1)\pi$ s ◀

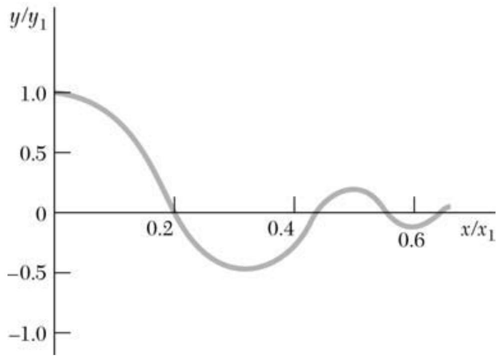
$$x = 10[2\pi(N-1)] - 5\sin[2\pi(N+1)] \qquad x = 20\pi(N+1) \text{ ft} \;◀$$

$$y = 10 - 5\cos[2\pi(N+1)] \qquad y = 15 \text{ ft} \;◀$$

$$v_x = 10 - 5\cos[2\pi(N+1)] \qquad v_x = 15 \text{ ft/s} \;◀$$

$$v_y = 5\sin[2\pi(N+1)] \qquad v_y = 0 \;◀$$

$$\tan\theta = \frac{v_y}{v_x} = 0, \qquad \theta = 0 \;◀$$



PROBLEM 11.93

The damped motion of a vibrating particle is defined by the position vector $\mathbf{r} = x_1[1-1/(t+1)]\mathbf{i} + (y_1 e^{-\pi t/2}\cos 2\pi t)\mathbf{j}$, where t is expressed in seconds. For $x_1 = 30$ mm and $y_1 = 20$ mm, determine the position, the velocity, and the acceleration of the particle when (*a*) $t = 0$, (*b*) $t = 1.5$ s.

SOLUTION

We have $\mathbf{r} = 30\left(1-\dfrac{1}{t+1}\right)\mathbf{i} + 20(e^{-\pi t/2}\cos 2\pi t)\mathbf{j}$

Then $\mathbf{v} = \dfrac{d\mathbf{r}}{dt}$

$$= 30\frac{1}{(t+1)^2}\mathbf{i} + 20\left(-\frac{\pi}{2}e^{-\pi t/2}\cos 2\pi t - 2\pi e^{-\pi t/2}\sin 2\pi t\right)\mathbf{j}$$

$$= 30\frac{1}{(t+1)^2}\mathbf{i} - 20\pi\left[e^{-\pi t/2}\left(\frac{1}{2}\cos 2\pi t + 2\sin 2\pi t\right)\right]\mathbf{j}$$

and $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$

$$= -30\frac{2}{(t+1)^3}\mathbf{i} - 20\pi\left[-\frac{\pi}{2}e^{-\pi t/2}\left(\frac{1}{2}\cos 2\pi t + 2\sin 2\pi t\right) + e^{-\pi t/2}(-\pi\sin 2\pi t + 4\cos 2\pi t)\right]\mathbf{j}$$

$$= -\frac{60}{(t+1)^3}\mathbf{i} + 10\pi^2 e^{-\pi t/2}(4\sin 2\pi t - 7.5\cos 2\pi t)\mathbf{j}$$

(*a*) At $t = 0$: $\mathbf{r} = 30\left(1-\dfrac{1}{1}\right)\mathbf{i} + 20(1)\mathbf{j}$

or $\mathbf{r} = 20$ mm ↑ ◀

$$\mathbf{v} = 30\left(\frac{1}{1}\right)\mathbf{i} - 20\pi\left[(1)\left(\frac{1}{2}+0\right)\right]\mathbf{j}$$

or $\mathbf{v} = 43.4$ mm/s ◿ 46.3° ◀

$$\mathbf{a} = -\frac{60}{(1)}\mathbf{i} + 10\pi^2(1)(0-7.5)\mathbf{j}$$

or $\mathbf{a} = 743$ mm/s^2 ◺ 85.4° ◀

PROBLEM 11.93 (Continued)

(*b*) At $t = 1.5$ s: $\mathbf{r} = 30\left(1 - \frac{1}{2.5}\right)\mathbf{i} + 20e^{-0.75\pi}(\cos 3\pi)\mathbf{j}$

$$= (18 \text{ mm})\mathbf{i} + (-1.8956 \text{ mm})\mathbf{j}$$

or $\mathbf{r} = 18.10 \text{ mm} \searrow 6.01° \blacktriangleleft$

$$\mathbf{v} = \frac{30}{(2.5)^2}\mathbf{i} - 20\pi e^{-0.75\pi}\left(\frac{1}{2}\cos 3\pi + 0\right)\mathbf{j}$$

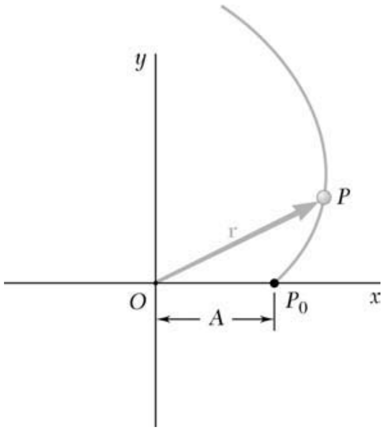
$$= (4.80 \text{ mm/s})\mathbf{i} + (2.9778 \text{ mm/s})\mathbf{j}$$

or $\mathbf{v} = 5.65 \text{ mm/s} \nearrow 31.8° \blacktriangleleft$

$$\mathbf{a} = -\frac{60}{(2.5)^3}\mathbf{i} + 10\pi^2 e^{-0.75\pi}(0 - 7.5\cos 3\pi)\mathbf{j}$$

$$= (-3.84 \text{ mm/s}^2)\mathbf{i} + (70.1582 \text{ mm/s}^2)\mathbf{j}$$

or $\mathbf{a} = 70.3 \text{ mm/s}^2 \nwarrow 86.9° \blacktriangleleft$



PROBLEM 11.94

The motion of a particle is defined by the position vector $\mathbf{r} = A(\cos t + t \sin t)\mathbf{i} + A(\sin t - t \cos t)\mathbf{j}$, where t is expressed in seconds. Determine the values of t for which the position vector and the acceleration are (*a*) perpendicular, (*b*) parallel.

SOLUTION

We have $$\mathbf{r} = A(\cos t + t \sin t)\mathbf{i} + A(\sin t - t \cos t)\mathbf{j}$$

Then $$\mathbf{v} = \frac{d\mathbf{r}}{dt} = A(-\sin t + \sin t + t \cos t)\mathbf{i} + A(\cos t - \cos t + t \sin t)\mathbf{j} = A(t \cos t)\mathbf{i} + A(t \sin t)\mathbf{j}$$

and $$\mathbf{a} = \frac{d\mathbf{v}}{dt} = A(\cos t - t \sin t)\mathbf{i} + A(\sin t + t \cos t)\mathbf{j}$$

(*a*) When **r** and **a** are perpendicular, $\mathbf{r} \cdot \mathbf{a} = 0$

$$A[(\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j}] \cdot A[(\cos t - t \sin t)\mathbf{i} + (\sin t + t \cos t)\mathbf{j}] = 0$$

or $$(\cos t + t \sin t)(\cos t - t \sin t) + (\sin t - t \cos t)(\sin t + t \cos t) = 0$$

or $$(\cos^2 t - t^2 \sin^2 t) + (\sin^2 t - t^2 \cos^2 t) = 0$$

or $$1 - t^2 = 0 \quad \text{or} \quad t = 1 \text{ s} \blacktriangleleft$$

(*b*) When **r** and **a** are parallel, $\mathbf{r} \times \mathbf{a} = 0$

$$A[(\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j}] \times A[(\cos t - t \sin t)\mathbf{i} + (\sin t + t \cos t)\mathbf{j}] = 0$$

or $$[(\cos t + t \sin t)(\sin t + t \cos t) - (\sin t - t \cos t)(\cos t - t \sin t)]\mathbf{k} = 0$$

Expanding $$(\sin t \cos t + t + t^2 \sin t \cos t) - (\sin t \cos t - t + t^2 \sin t \cos t) = 0$$

or $$2t = 0 \quad \text{or} \quad t = 0 \blacktriangleleft$$

PROBLEM 11.95

The three-dimensional motion of a particle is defined by the position vector $\mathbf{r} = (Rt \cos \omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt \sin \omega_n t)\mathbf{k}$. Determine the magnitudes of the velocity and acceleration of the particle. (The space curve described by the particle is a conic helix.)

SOLUTION

We have
$$\mathbf{r} = (Rt \cos \omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt \sin \omega_n t)\mathbf{k}$$

Then
$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = R(\cos \omega_n t - \omega_n t \sin \omega_n t)\mathbf{i} + c\mathbf{j} + R(\sin \omega_n t + \omega_n t \cos \omega_n t)\mathbf{k}$$

and
$$\begin{aligned}\mathbf{a} &= \frac{d\mathbf{v}}{dt} \\ &= R(-\omega_n \sin \omega_n t - \omega_n \sin \omega_n t - \omega_n^2 t \cos \omega_n t)\mathbf{i} \\ &\quad + R(\omega_n \cos \omega_n t + \omega_n \cos \omega_n t - \omega_n^2 t \sin \omega_n t)\mathbf{k} \\ &= R(-2\omega_n \sin \omega_n t - \omega_n^2 t \cos \omega_n t)\mathbf{i} + R(2\omega_n \cos \omega_n t - \omega_n^2 t \sin \omega_n t)\mathbf{k}\end{aligned}$$

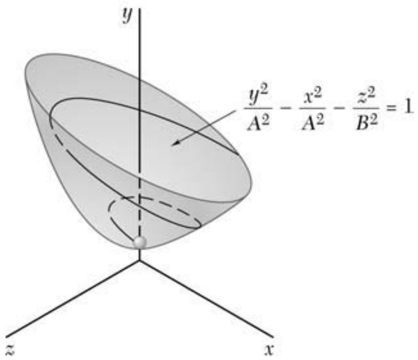
Now
$$\begin{aligned}v^2 &= v_x^2 + v_y^2 + v_z^2 \\ &= [R(\cos \omega_n t - \omega_n t \sin \omega_n t)]^2 + (c)^2 + [R(\sin \omega_n t + \omega_n t \cos \omega_n t)]^2 \\ &= R^2\left[\left(\cos^2 \omega_n t - 2\omega_n t \sin \omega_n t \cos \omega_n t + \omega_n^2 t^2 \sin^2 \omega_n t\right)\right. \\ &\quad \left.+ \left(\sin^2 \omega_n t + 2\omega_n t \sin \omega_n t \cos \omega_n t + \omega_n^2 t^2 \cos^2 \omega_n t\right)\right] + c^2 \\ &= R^2\left(1 + \omega_n^2 t^2\right) + c^2\end{aligned}$$

or
$$v = \sqrt{R^2\left(1 + \omega_n^2 t^2\right) + c^2} \quad ◀$$

Also,
$$\begin{aligned}a^2 &= a_x^2 + a_y^2 + a_z^2 \\ &= \left[R\left(-2\omega_n \sin \omega_n t - \omega_n^2 t \cos \omega_n t\right)\right]^2 + (0)^2 \\ &\quad + \left[R\left(2\omega_n \cos \omega_n t - \omega_n^2 t \sin \omega_n t\right)\right]^2 \\ &= R^2\left[\left(4\omega_n^2 \sin^2 \omega_n t + 4\omega_n^3 t \sin \omega_n t \cos \omega_n t + \omega_n^4 t^2 \cos^2 \omega_n t\right)\right. \\ &\quad \left.+ \left(4\omega_n^2 \cos^2 \omega_n t - 4\omega_n^3 t \sin \omega_n t \cos \omega_n t + \omega_n^4 t^2 \sin^2 \omega_n t\right)\right] \\ &= R^2\left(4\omega_n^2 + \omega_n^4 t^2\right)\end{aligned}$$

or
$$a = R\omega_n \sqrt{4 + \omega_n^2 t^2} \quad ◀$$

PROBLEM 11.96



The three-dimensional motion of a particle is defined by the position vector $\mathbf{r} = (At\cos t)\mathbf{i} + (A\sqrt{t^2+1})\mathbf{j} + (Bt\sin t)\mathbf{k}$, where r and t are expressed in feet and seconds, respectively. Show that the curve described by the particle lies on the hyperboloid $(y/A)^2 - (x/A)^2 - (z/B)^2 = 1$. For $A = 3$ and $B = 1$, determine (*a*) the magnitudes of the velocity and acceleration when $t = 0$, (*b*) the smallest nonzero value of t for which the position vector and the velocity are perpendicular to each other.

SOLUTION

We have
$$\mathbf{r} = (At\cos t)\mathbf{i} + (A\sqrt{t^2+1})\mathbf{j} + (Bt\sin t)\mathbf{k}$$

or
$$x = At\cos t \quad y = A\sqrt{t^2+1} \quad z = Bt\sin t$$

Then
$$\cos t = \frac{x}{At} \qquad \sin t = \frac{z}{Bt} \qquad t^2 = \left(\frac{y}{A}\right)^2 - 1$$

Now
$$\cos^2 t + \sin^2 t = 1 \quad \Rightarrow \quad \left(\frac{x}{At}\right)^2 + \left(\frac{z}{Bt}\right)^2 = 1$$

or
$$t^2 = \left(\frac{x}{A}\right)^2 + \left(\frac{z}{B}\right)^2$$

Then
$$\left(\frac{y}{A}\right)^2 - 1 = \left(\frac{x}{A}\right)^2 + \left(\frac{z}{B}\right)^2$$

or
$$\left(\frac{y}{A}\right)^2 - \left(\frac{x}{A}\right)^2 - \left(\frac{z}{B}\right)^2 = 1 \qquad \text{Q.E.D.}$$ ◀

(*a*) With $A = 3$ and $B = 1$, we have

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 3(\cos t - t\sin t)\mathbf{i} + 3\frac{t}{\sqrt{t^2+1}}\mathbf{j} + (\sin t + t\cos t)\mathbf{k}$$

and
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t\cos t)\mathbf{i} + 3\frac{\sqrt{t^2+1} - t\left(\frac{t}{\sqrt{t^2+1}}\right)}{(t^2+1)}\mathbf{j}$$
$$+ (\cos t + \cos t - t\sin t)\mathbf{k}$$
$$= -3(2\sin t + t\cos t)\mathbf{i} + 3\frac{1}{(t^2+1)^{3/2}}\mathbf{j} + (2\cos t - t\sin t)\mathbf{k}$$

PROBLEM 11.96 (Continued)

At $t = 0$: $\mathbf{v} = 3(1-0)\mathbf{i} + (0)\mathbf{j} + (0)\mathbf{k}$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

or $v = 3 \text{ ft/s}$ ◀

and $\mathbf{a} = -3(0)\mathbf{i} + 3(1)\mathbf{j} + (2-0)\mathbf{k}$

Then $a^2 = (0)^2 + (3)^2 + (2)^2 = 13$

or $a = 3.61 \text{ ft/s}^2$ ◀

(*b*) If $\mathbf{r}$ and $\mathbf{v}$ are perpendicular, $\mathbf{r} \cdot \mathbf{v} = 0$

$$[(3t \cos t)\mathbf{i} + (3\sqrt{t^2+1})\mathbf{j} + (t \sin t)\mathbf{k}] \cdot [3(\cos t - t \sin t)\mathbf{i} + \left(3\frac{t}{\sqrt{t^2+1}}\right)\mathbf{j} + (\sin t + t \cos t)\mathbf{k}] = 0$$

or $$(3t \cos t)[3(\cos t - t \sin t)] + (3\sqrt{t^2+1})\left(3\frac{t}{\sqrt{t^2+1}}\right) + (t \sin t)(\sin t + t \cos t) = 0$$

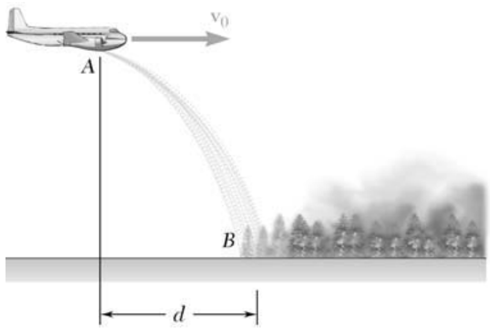
Expanding $$(9t \cos^2 t - 9t^2 \sin t \cos t) + (9t) + (t \sin^2 t + t^2 \sin t \cos t) = 0$$

or (with $t \neq 0$) $$10 + 8 \cos^2 t - 8t \sin t \cos t = 0$$

or $$7 + 2 \cos 2t - 2t \sin 2t = 0$$

Using "trial and error" or numerical methods, the smallest root is $t = 3.82 \text{ s}$ ◀

Note: The next root is $t = 4.38 \text{ s}$.



PROBLEM 11.97

An airplane used to drop water on brushfires is flying horizontally in a straight line at 180 mi/h at an altitude of 300 ft. Determine the distance d at which the pilot should release the water so that it will hit the fire at B.

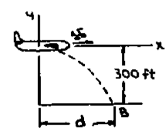
SOLUTION

First note $\qquad v_0 = 180 \text{ km/h} = 264 \text{ ft/s}$

Place origin of coordinates at Point A.

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (0)t - \frac{1}{2}gt^2$$



At B: $$-300 \text{ ft} = -\frac{1}{2}(32.2 \text{ ft/s}^2)t^2$$

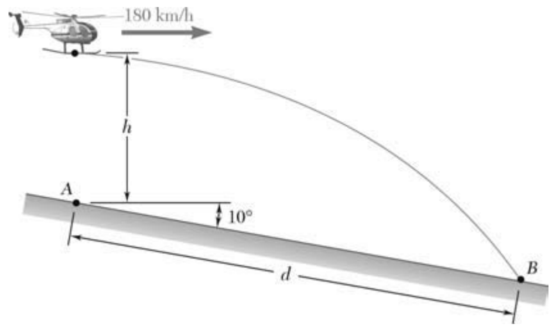
or $$t_B = 4.31666 \text{ s}$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At B: $$d = (264 \text{ ft/s})(4.31666 \text{ s})$$

or $$d = 1140 \text{ ft} \blacktriangleleft$$



PROBLEM 11.98

A helicopter is flying with a constant horizontal velocity of 180 km/h and is directly above Point *A* when a loose part begins to fall. The part lands 6.5 s later at Point *B* on an inclined surface. Determine (*a*) the distance *d* between Points *A* and *B*, (*b*) the initial height *h*.

SOLUTION

Place origin of coordinates at Point *A*.

Horizontal motion: $(v_x)_0 = 180 \text{ km/h} = 50 \text{ m/s}$

$$x = x_0 + (v_x)_0 t = 0 + 50t \text{ m}$$

At Point *B* where $t_B = 6.5$ s, $x_B = (50)(6.5) = 325$ m

(*a*) Distance *AB*.

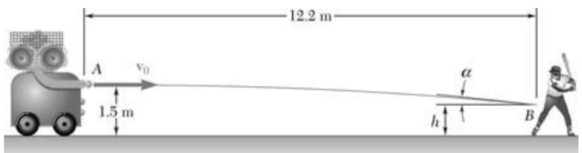
From geometry $d = \dfrac{325}{\cos 10°}$ $\qquad d = 330 \text{ m}$ ◀

Vertical motion: $y = y_0 + (v_y)_0 t - \dfrac{1}{2} g t^2$

At Point *B* $-x_B \tan 10° = h + 0 - \dfrac{1}{2}(9.81)(6.5)^2$

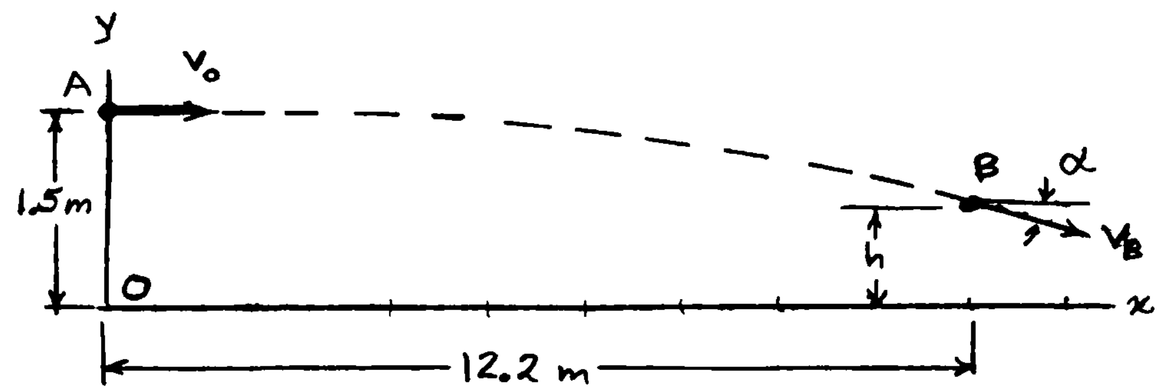
(*b*) Initial height. $\qquad h = 149.9 \text{ m}$ ◀

PROBLEM 11.99



A baseball pitching machine "throws" baseballs with a horizontal velocity $\mathbf{v}_0$. Knowing that height h varies between 788 mm and 1068 mm, determine (*a*) the range of values of v_0, (*b*) the values of α corresponding to $h = 788$ mm and $h = 1068$ mm.

SOLUTION



(*a*) Vertical motion: $y_0 = 1.5\text{ m},\ (v_y)_0 = 0$

$$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2 \qquad \text{or} \qquad t = \sqrt{\frac{2(y_0 - y)}{g}}$$

At Point B, $\quad y = h \quad$ or $\quad t_B = \sqrt{\dfrac{2(y_0 - h)}{g}}$

When $h = 788\text{ mm} = 0.788\text{ m}$, $\quad t_B = \sqrt{\dfrac{(2)(1.5 - 0.788)}{9.81}} = 0.3810\text{ s}$

When $h = 1068\text{ mm} = 1.068\text{ m}$, $\quad t_B = \sqrt{\dfrac{(2)(1.5 - 1.068)}{9.81}} = 0.2968\text{ s}$

Horizontal motion: $x_0 = 0,\ (v_x)_0 = v_0,$

$$x = v_0 t \qquad \text{or} \qquad v_0 = \frac{x}{t} = \frac{x_B}{t_B}$$

PROBLEM 11.99 (Continued)

With $x_B = 12.2$ m, we get $v_0 = \dfrac{12.2}{0.3810} = 32.02$ m/s

and $v_0 = \dfrac{12.2}{0.2968} = 41.11$ m/s

$32.02 \text{ m/s} \le v_0 \le 41.11 \text{ m/s}$ or $115.3 \text{ km/h} \le v_0 \le 148.0 \text{ km/h}$ ◀

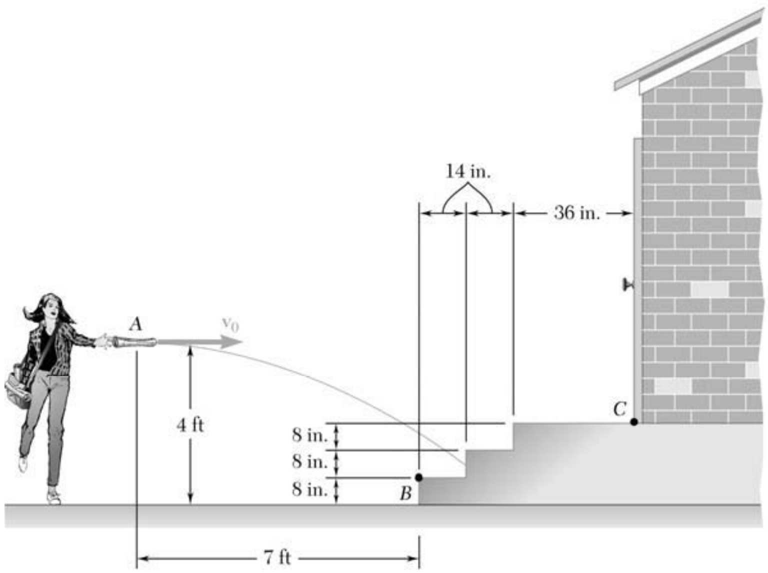
(*b*) Vertical motion: $v_y = (v_y)_0 - gt = -gt$

Horizontal motion: $v_x = v_0$

$$\tan\alpha = -\frac{dy}{dx} = -\frac{(v_y)_B}{(v_x)_B} = \frac{gt_B}{v_0}$$

For $h = 0.788$ m, $\tan\alpha = \dfrac{(9.81)(0.3810)}{32.02} = 0.11673,$ $\alpha = 6.66°$ ◀

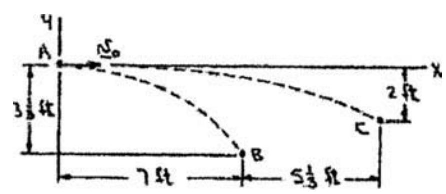
For $h = 1.068$ m, $\tan\alpha = \dfrac{(9.81)(0.2968)}{41.11} = 0.07082,$ $\alpha = 4.05°$ ◀



PROBLEM 11.100

While delivering newspapers, a girl throws a newspaper with a horizontal velocity $\mathbf{v}_0$. Determine the range of values of v_0 if the newspaper is to land between Points B and C.

SOLUTION



Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (0)t - \frac{1}{2}gt^2$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = v_0 t$$

At B: y: $$-3\frac{1}{3}\text{ ft} = -\frac{1}{2}(32.2\text{ ft/s}^2)t^2$$

or $$t_B = 0.455016\text{ s}$$

Then x: $$7\text{ ft} = (v_0)_B(0.455016\text{ s})$$

or $$(v_0)_B = 15.38\text{ ft/s}$$

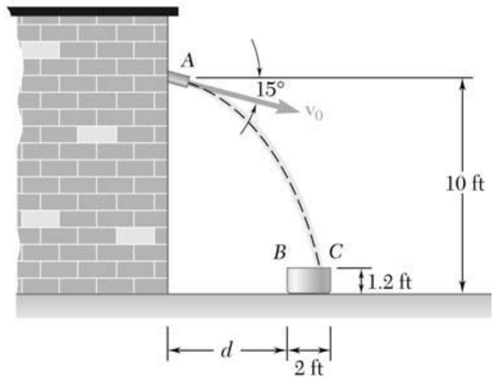
At C: y: $$-2\text{ ft} = -\frac{1}{2}(32.2\text{ ft/s}^2)t^2$$

or $$t_C = 0.352454\text{ s}$$

Then x: $$12\frac{1}{3}\text{ ft} = (v_0)_C(0.352454\text{ s})$$

or $$(v_0)_C = 35.0\text{ ft/s}$$

$$15.38\text{ ft/s} < v_0 < 35.0\text{ ft/s} \blacktriangleleft$$



PROBLEM 11.101

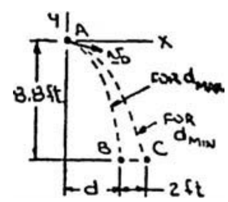
Water flows from a drain spout with an initial velocity of 2.5 ft/s at an angle of 15° with the horizontal. Determine the range of values of the distance *d* for which the water will enter the trough *BC*.

SOLUTION

First note $(v_x)_0 = (2.5 \text{ ft/s}) \cos 15° = 2.4148 \text{ ft/s}$

$(v_y)_0 = -(2.5 \text{ ft/s}) \sin 15° = -0.64705 \text{ ft/s}$

Vertical motion. (Uniformly accelerated motion)



$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2$$

At the top of the trough

$$-8.8 \text{ ft} = (-0.64705 \text{ ft/s})t - \frac{1}{2}(32.2 \text{ ft/s}^2)t^2$$

or $t_{BC} = 0.719491 \text{ s}$ (the other root is negative)

Horizontal motion. (Uniform)

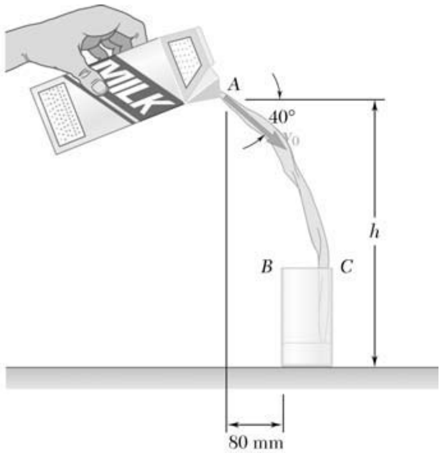
$$x = 0 + (v_x)_0 t$$

In time t_{BC} $x_{BC} = (2.4148 \text{ ft/s})(0.719491 \text{ s}) = 1.737 \text{ ft}$

Thus, the trough must be placed so that

$$x_B < 1.737 \text{ ft or } x_C \geq 1.737 \text{ ft}$$

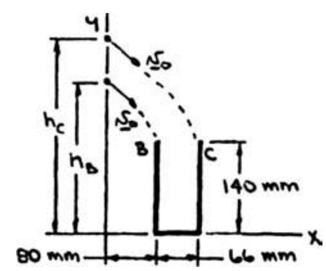
Since the trough is 2 ft wide, it then follows that $0 < d < 1.737 \text{ ft}$ ◀



PROBLEM 11.102

Milk is poured into a glass of height 140 mm and inside diameter 66 mm. If the initial velocity of the milk is 1.2 m/s at an angle of 40° with the horizontal, determine the range of values of the height h for which the milk will enter the glass.

SOLUTION



First note

$$(v_x)_0 = (1.2 \text{ m/s}) \cos 40° = 0.91925 \text{ m/s}$$
$$(v_y)_0 = -(1.2 \text{ m/s}) \sin 40° = -0.77135 \text{ m/s}$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

Vertical motion. (Uniformly accelerated motion)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

Milk enters glass at B.

$$x: \quad 0.08 \text{ m} = (0.91925 \text{ m/s})t \quad \text{or} \quad t_B = 0.087028 \text{ s}$$
$$y: \quad 0.140 \text{ m} = h_B + (-0.77135 \text{ m/s})(0.087028 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.087028 \text{ s})^2$$

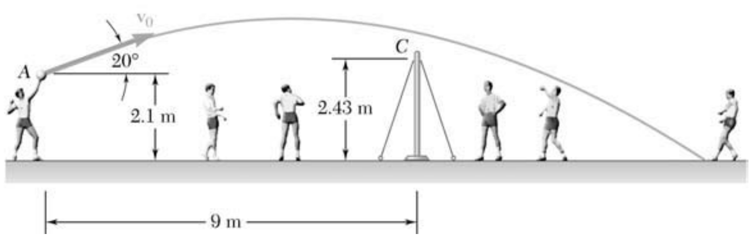
or $$h_B = 0.244 \text{ m}$$

Milk enters glass at C.

$$x: \quad 0.146 \text{ m} = (0.91925 \text{ m/s})t \quad \text{or} \quad t_C = 0.158825 \text{ s}$$
$$y: \quad 0.140 \text{ m} = h_C + (-0.77135 \text{ m/s})(0.158825 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.158825 \text{ s})^2$$

or $$h_C = 0.386 \text{ m}$$

$$0.244 \text{ m} < h < 0.386 \text{ m} \blacktriangleleft$$



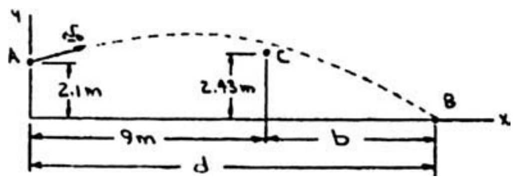
PROBLEM 11.103

A volleyball player serves the ball with an initial velocity $\mathbf{v}_0$ of magnitude 13.40 m/s at an angle of 20° with the horizontal. Determine (*a*) if the ball will clear the top of the net, (*b*) how far from the net the ball will land.

SOLUTION

First note

$$(v_x)_0 = (13.40\text{ m/s})\cos 20° = 12.5919\text{ m/s}$$
$$(v_y)_0 = (13.40\text{ m/s})\sin 20° = 4.5831\text{ m/s}$$



(*a*) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At C $\quad 9\text{ m} = (12.5919\text{ m/s})t \quad \text{or} \quad t_C = 0.71475\text{ s}$

Vertical motion. (Uniformly accelerated motion)

$$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$$

At C:

$$y_C = 2.1\text{ m} + (4.5831\text{ m/s})(0.71475\text{ s}) - \frac{1}{2}(9.81\text{ m/s}^2)(0.71475\text{ s})^2 = 2.87\text{ m}$$

$y_C > 2.43$ m (height of net) $\Rightarrow$ ball clears net ◀

(*b*) At B, $y = 0$: $\quad 0 = 2.1\text{ m} + (4.5831\text{ m/s})t - \frac{1}{2}(9.81\text{ m/s}^2)t^2$

Solving $\quad t_B = 1.271175$ s (the other root is negative)

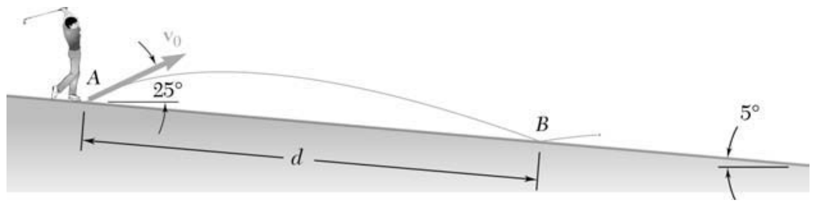
Then

$$d = (v_x)_0 t_B = (12.5919\text{ m/s})(1.271175\text{ s}) = 16.01\text{ m}$$

The ball lands $\quad b = (16.01 - 9.00)\text{ m} = 7.01\text{ m}$ from the net ◀

PROBLEM 11.104

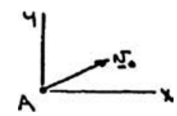
A golfer hits a golf ball with an initial velocity of 160 ft/s at an angle of 25° with the horizontal. Knowing that the fairway slopes downward at an average angle of 5°, determine the distance *d* between the golfer and Point *B* where the ball first lands.



SOLUTION

First note

$$(v_x)_0 = (160 \text{ ft/s})\cos 25°$$
$$(v_y)_0 = (160 \text{ ft/s})\sin 25°$$



and at *B*

$$x_B = d\cos 5° \quad y_B = -d\sin 5°$$

Now Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At *B*

$$d\cos 5° = (160\cos 25°)t \quad \text{or} \quad t_B = \frac{\cos 5°}{160\cos 25°}d$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2 \quad (g = 32.2 \text{ ft/s}^2)$$

At *B*:

$$-d\sin 5° = (160\sin 25°)t_B - \frac{1}{2}gt_B^2$$

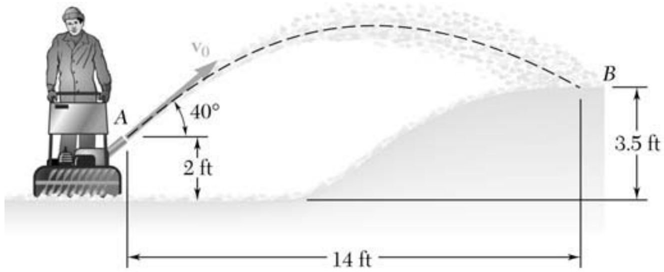
Substituting for t_B

$$-d\sin 5° = (160\sin 25°)\left(\frac{\cos 5°}{160\cos 25°}\right)d - \frac{1}{2}g\left(\frac{\cos 5°}{160\cos 25°}\right)^2 d^2$$

or

$$d = \frac{2}{32.2\cos 5°}(160\cos 25°)^2(\tan 5° + \tan 25°)$$
$$= 726.06 \text{ ft}$$

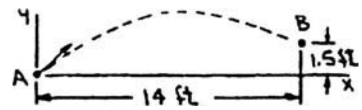
or $d = 242$ yd ◀



PROBLEM 11.105

A homeowner uses a snowblower to clear his driveway. Knowing that the snow is discharged at an average angle of 40° with the horizontal, determine the initial velocity v_0 of the snow.

SOLUTION



First note $$(v_x)_0 = v_0 \cos 40°$$ $$(v_y)_0 = v_0 \sin 40°$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At B: $$14 = (v_0 \cos 40°)t \quad \text{or} \quad t_B = \frac{14}{v_0 \cos 40°}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2 \qquad (g = 32.2 \text{ ft/s}^2)$$

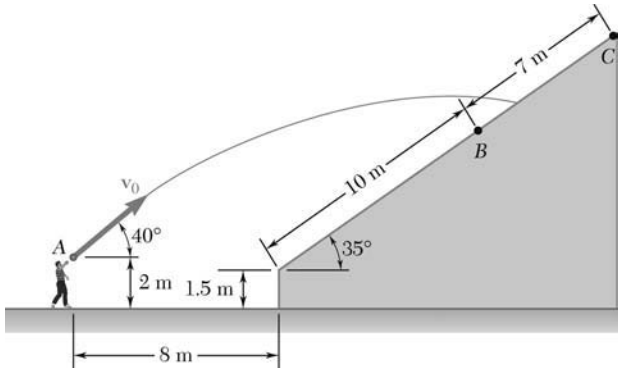
At B: $$1.5 = (v_0 \sin 40°)t_B - \frac{1}{2}gt_B^2$$

Substituting for t_B

$$1.5 = (v_0 \sin 40°)\left(\frac{14}{v_0 \cos 40°}\right) - \frac{1}{2}g\left(\frac{14}{v_0 \cos 40°}\right)^2$$

or $$v_0^2 = \frac{\frac{1}{2}(32.2)(196)/\cos^2 40°}{-1.5 + 14 \tan 40°}$$

or $$v_0 = 22.9 \text{ ft/s} \blacktriangleleft$$



PROBLEM 11.106

At halftime of a football game souvenir balls are thrown to the spectators with a velocity $\mathbf{v}_0$. Determine the range of values of v_0 if the balls are to land between Points *B* and *C*.

SOLUTION

The motion is projectile motion. Place the origin of the *xy*-coordinate system at ground level just below Point *A*. The coordinates of Point *A* are $x_0 = 0,\ y_0 = 2\text{m}.$ The components of initial velocity are $(v_x)_0 = v_0 \cos 40°$ m/s and $(v_y)_0 = v_0 \sin 40°.$

Horizontal motion: $$x = x_0 + (v_x)_0 t = (v_0 \cos 40°)t \quad (1)$$

Vertical motion: $$y = y_0 + (v_y)_0 t = \frac{1}{2} g t^2$$

$$= 2 + (v_0 \sin 40°) = -\frac{1}{2}(9.81)t^2 \quad (2)$$

From (1), $$v_0 t = \frac{x}{\cos 40°} \quad (3)$$

Then $$y = 2 + x \tan 40° - 4.905t^2$$

$$t^2 = \frac{2 + x \tan 40° - y}{4.905} \quad (4)$$

Point *B*: $$x = 8 + 10\cos 35° = 16.1915 \text{ m}$$

$$y = 1.5 + 10\sin 35° = 7.2358 \text{ m}$$

$$v_0 t = \frac{16.1915}{\cos 40°} = 21.1365 \text{ m}$$

$$t^2 = \frac{2 + 16.1915 \tan 40° - 7.2358}{4.905} \qquad t = 1.3048 \text{ s}$$

$$v_0 = \frac{21.1365}{1.3048} \qquad v_0 = 16.199 \text{ m/s}$$

PROBLEM 11.106 (Continued)

Point C:

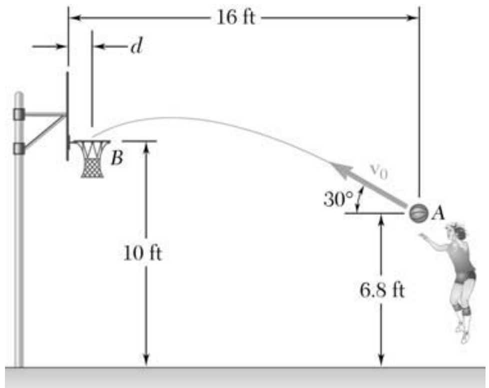
$$x = 8 + (10 + 7)\cos 35° = 21.9256 \text{ m}$$
$$y = 1.5 + (10 + 7)\sin 35° = 11.2508 \text{ m}$$

$$v_0 t = \frac{21.9256}{\cos 40°} = 28.622 \text{ m}$$

$$t^2 = \frac{2 + 21.9256\tan 40° - 11.2508}{4.905} \qquad t = 1.3656 \text{ s}$$

$$v_0 = \frac{28.622}{1.3656} \qquad v_0 = 20.96 \text{ m/s}$$

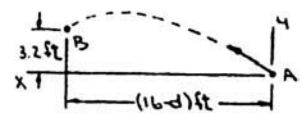
Range of values of v_0. $16.20 \text{ m/s} < v_0 < 21.0 \text{ m/s}$ ◀



PROBLEM 11.107

A basketball player shoots when she is 16 ft from the backboard. Knowing that the ball has an initial velocity $\mathbf{v}_0$ at an angle of 30° with the horizontal, determine the value of v_0 when d is equal to (*a*) 9 in., (*b*) 17 in.

SOLUTION



First note $\quad (v_x)_0 = v_0 \cos 30° \qquad (v_y)_0 = v_0 \sin 30°$

Horizontal motion. (Uniform) $\quad x = 0 + (v_x)_0 t$

At B: $\quad (16-d) = (v_0 \cos 30°)t \quad \text{or} \quad t_B = \dfrac{16-d}{v_0 \cos 30°}$

Vertical motion. (Uniformly accelerated motion) $\quad y = 0 + (v_y)_0 t - \frac{1}{2}gt^2 \quad (g = 32.2 \text{ ft/s}^2)$

At B: $\quad 3.2 = (v_0 \sin 30°)t_B - \frac{1}{2}gt_B^2$

Substituting for t_B

$$3.2 = (v_0 \sin 30°)\left(\frac{16-d}{v_0 \cos 30°}\right) - \frac{1}{2}g\left(\frac{16-d}{v_0 \cos 30°}\right)^2$$

or

$$v_0^2 = \frac{2g(16-d)^2}{3\left[\frac{1}{\sqrt{3}}(16-d) - 3.2\right]}$$

(*a*) $\quad d = 9$ in.:

$$v_0^2 = \frac{2(32.2)\left(16 - \frac{9}{12}\right)^2}{3\left[\frac{1}{\sqrt{3}}\left(16 - \frac{9}{12}\right) - 3.2\right]}$$

$v_0 = 29.8$ ft/s ◀

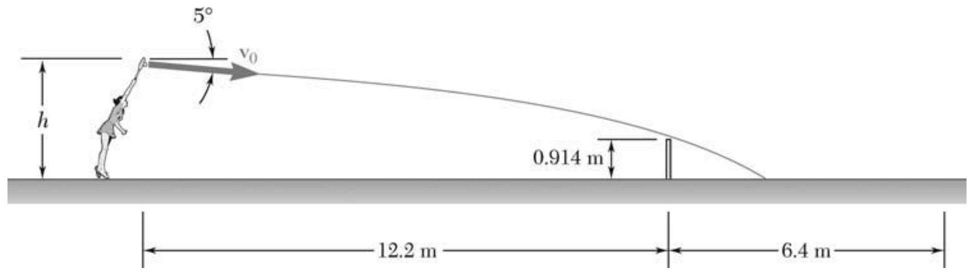
(*b*) $\quad d = 17$ in.:

$$v_0^2 = \frac{2(32.2)\left(16 - \frac{17}{12}\right)^2}{3\left[\frac{1}{\sqrt{3}}\left(16 - \frac{17}{12}\right) - 3.2\right]}$$

$v_0 = 29.6$ ft/s ◀

PROBLEM 11.108

A tennis player serves the ball at a height h = 2.5 m with an initial velocity of $\mathbf{v}_0$ at an angle of 5° with the horizontal. Determine the range for which of v_0 for which the ball will land in the service area which extends to 6.4 m beyond the net.



SOLUTION

The motion is projectile motion. Place the origin of the xy-coordinate system at ground level just below the point where the racket impacts the ball. The coordinates of this impact point are $x_0 = 0,\ y_0 = h = 2.5$ m. The components of initial velocity are $(v_x)_0 = v_0 \cos 5°$ and $(v_y)_0 = v_0 \sin 5°$.

Horizontal motion:
$$x = x_0 + (v_x)_0 t = (v_0 \cos 5°)t \qquad (1)$$

Vertical motion:
$$y = y_0 + (v_y)_0 t = \frac{1}{2} g t^2$$
$$= 2.5 - (v_0 \sin 5°)t = -\frac{1}{2}(9.81)t^2 \qquad (2)$$

From (1),
$$v_0 t = \frac{x}{\cos 5°} \qquad (3)$$

Then
$$y = 2.5 - x \tan 5° - 4.905 t^2$$
$$t^2 = \frac{2.5 - x \tan 5° - y}{4.905} \qquad (4)$$

At the minimum speed the ball just clears the net.

$$x = 12.2 \text{ m}, \quad y = 0.914 \text{ m}$$
$$v_0 t = \frac{12.2}{\cos 5°} = 12.2466 \text{ m}$$
$$t^2 = \frac{2.5 - 12.2 \tan 5° - 0.914}{4.905} \qquad t = 0.32517 \text{ s}$$
$$v_0 = \frac{12.2466}{0.32517} \qquad v_0 = 37.66 \text{ m/s}$$

PROBLEM 11.108 (Continued)

At the maximum speed the ball lands 6.4 m beyond the net.

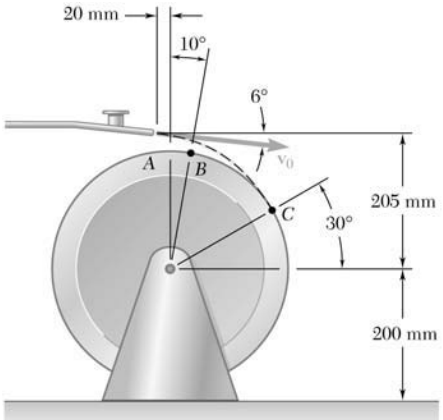
$$x = 12.2 + 6.4 = 18.6 \text{ m} \qquad y = 0$$

$$v_0 t = \frac{18.6}{\cos 5^\circ} = 18.6710 \text{ m}$$

$$t^2 = \frac{2.5 - 18.6 \tan 5^\circ - 0}{4.905} \qquad t = 0.42181 \text{ s}$$

$$v_0 = \frac{18.6710}{0.42181} \qquad v_0 = 44.26 \text{ m/s}$$

Range for v_0. $37.7 \text{ m/s} < v_0 < 44.3 \text{ m/s}$ ◀

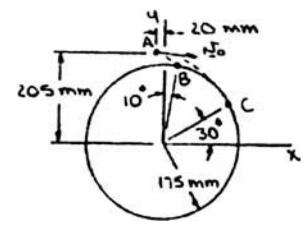


PROBLEM 11.109

The nozzle at *A* discharges cooling water with an initial velocity $\mathbf{v}_0$ at an angle of 6° with the horizontal onto a grinding wheel 350 mm in diameter. Determine the range of values of the initial velocity for which the water will land on the grinding wheel between Points *B* and *C*.

SOLUTION

First note



$$(v_x)_0 = v_0 \cos 6°$$
$$(v_y)_0 = -v_0 \sin 6°$$

Horizontal motion. (Uniform)

$$x = x_0 + (v_x)_0 t$$

Vertical motion. (Uniformly accelerated motion)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 \quad (g = 9.81 \text{ m/s}^2)$$

At Point *B*: $x = (0.175 \text{ m}) \sin 10°$

$$y = (0.175 \text{ m}) \cos 10°$$

$$x: \quad 0.175 \sin 10° = -0.020 + (v_0 \cos 6°)t$$

or $$t_B = \frac{0.050388}{v_0 \cos 6°}$$

$$y: \quad 0.175 \cos 10° = 0.205 + (-v_0 \sin 6°)t_B - \frac{1}{2} g t_B^2$$

Substituting for t_B

$$-0.032659 = (-v_0 \sin 6°)\left(\frac{0.050388}{v_0 \cos 6°}\right) - \frac{1}{2}(9.81)\left(\frac{0.050388}{v_0 \cos 6°}\right)^2$$

PROBLEM 11.109 (Continued)

or $$v_0^2 = \frac{\frac{1}{2}(9.81)(0.050388)^2}{\cos^2 6°(0.032659 - 0.050388 \tan 6°)}$$

or $$(v_0)_B = 0.678 \text{ m/s}$$

At Point C: $$x = (0.175 \text{ m}) \cos 30°$$
$$y = (0.175 \text{ m}) \sin 30°$$

$$x: \quad 0.175 \cos 30° = -0.020 + (v_0 \cos 6°)t$$

or $$t_C = \frac{0.171554}{v_0 \cos 6°}$$

$$y: \quad 0.175 \sin 30° = 0.205 + (-v_0 \sin 6°)t_C - \frac{1}{2}gt_C^2$$

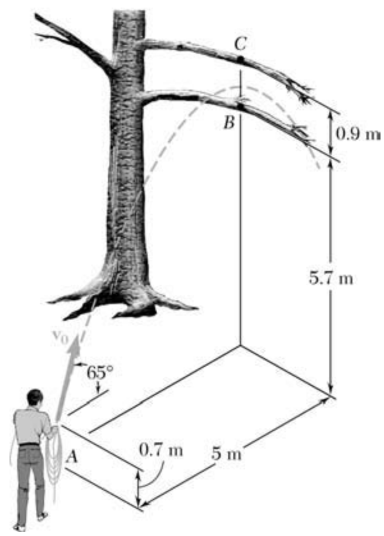
Substituting for t_C

$$-0.117500 = (-v_0 \sin 6°)\left(\frac{0.171554}{v_0 \cos 6°}\right) - \frac{1}{2}(9.81)\left(\frac{0.171554}{v_0 \cos 6°}\right)^2$$

or $$v_0^2 = \frac{\frac{1}{2}(9.81)(0.171554)^2}{\cos^2 6°(0.117500 - 0.171554 \tan 6°)}$$

or $$(v_0)_C = 1.211 \text{ m/s}$$

$$0.678 \text{ m/s} < v_0 < 1.211 \text{ m/s} \quad ◀$$



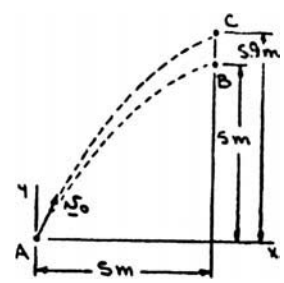
PROBLEM 11.110

While holding one of its ends, a worker lobs a coil of rope over the lowest limb of a tree. If he throws the rope with an initial velocity $\mathbf{v}_0$ at an angle of 65° with the horizontal, determine the range of values of v_0 for which the rope will go over only the lowest limb.

SOLUTION

First note

$$(v_x)_0 = v_0 \cos 65^\circ$$
$$(v_y)_0 = v_0 \sin 65^\circ$$



Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At either B or C, $x = 5$ m

$$s = (v_0 \cos 65^\circ) t_{B,C}$$

or

$$t_{B,C} = \frac{5}{(v_0 \cos 65^\circ)}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2 \qquad (g = 9.81 \text{ m/s}^2)$$

At the tree limbs, $t = t_{B,C}$

$$y_{B,C} = (v_0 \sin 65^\circ)\left(\frac{5}{v_0 \cos 65^\circ}\right) - \frac{1}{2} g \left(\frac{5}{v_0 \cos 65^\circ}\right)^2$$

PROBLEM 11.110 (Continued)

or $$v_0^2 = \frac{\frac{1}{2}(9.81)(25)}{\cos^2 65°(5\tan 65° - y_{B,C})}$$

$$= \frac{686.566}{5\tan 65° - y_{B,C}}$$

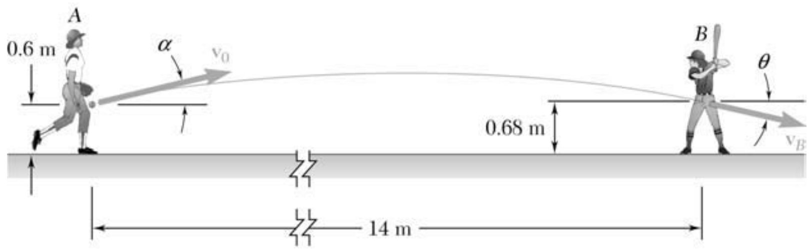
At Point B: $$v_0^2 = \frac{686.566}{5\tan 65° - 5} \quad \text{or} \quad (v_0)_B = 10.95 \text{ m/s}$$

At Point C: $$v_0^2 = \frac{686.566}{5\tan 65° - 5.9} \quad \text{or} \quad (v_0)_C = 11.93 \text{ m/s}$$

$10.95 \text{ m/s} < v_0 < 11.93 \text{ m/s}$ ◀

PROBLEM 11.111

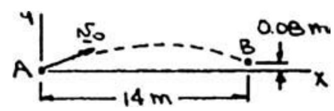
The pitcher in a softball game throws a ball with an initial velocity $\mathbf{v}_0$ of 72 km/h at an angle α with the horizontal. If the height of the ball at Point B is 0.68 m, determine (*a*) the angle α, (*b*) the angle θ that the velocity of the ball at Point B forms with the horizontal.



SOLUTION

First note

$$v_0 = 72 \text{ km/h} = 20 \text{ m/s}$$



and

$$(v_x)_0 = v_0 \cos\alpha = (20 \text{ m/s}) \cos\alpha$$
$$(v_y)_0 = v_0 \sin\alpha = (20 \text{ m/s}) \sin\alpha$$

(*a*) <u>Horizontal motion</u>. (Uniform)

$$x = 0 + (v_x)_0 t = (20 \cos\alpha) t$$

At Point B:

$$14 = (20 \cos\alpha) t \quad \text{or} \quad t_B = \frac{7}{10 \cos\alpha}$$

<u>Vertical motion</u>. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2 = (20 \sin\alpha) t - \frac{1}{2} g t^2 \quad (g = 9.81 \text{ m/s}^2)$$

At Point B:

$$0.08 = (20 \sin\alpha) t_B - \frac{1}{2} g t_B^2$$

Substituting for t_B

$$0.08 = (20 \sin\alpha)\left(\frac{7}{10 \cos\alpha}\right) - \frac{1}{2} g \left(\frac{7}{10 \cos\alpha}\right)^2$$

or

$$8 = 1400 \tan\alpha - \frac{1}{2} g \frac{49}{\cos^2\alpha}$$

Now

$$\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$$

PROBLEM 11.111 (Continued)

Then $$8 = 1400 \tan\alpha - 24.5g(1 + \tan^2\alpha)$$

or $$240.345 \tan^2\alpha - 1400 \tan\alpha + 248.345 = 0$$

Solving $$\alpha = 10.3786° \quad \text{and} \quad \alpha = 79.949°$$

Rejecting the second root because it is not physically reasonable, we have

$$\alpha = 10.38° \blacktriangleleft$$

(*b*) We have $$v_x = (v_x)_0 = 20 \cos\alpha$$

and $$v_y = (v_y)_0 - gt = 20 \sin\alpha - gt$$

At Point *B*: $$(v_y)_B = 20 \sin\alpha - gt_B$$

$$= 20 \sin\alpha - \frac{7g}{10 \cos\alpha}$$

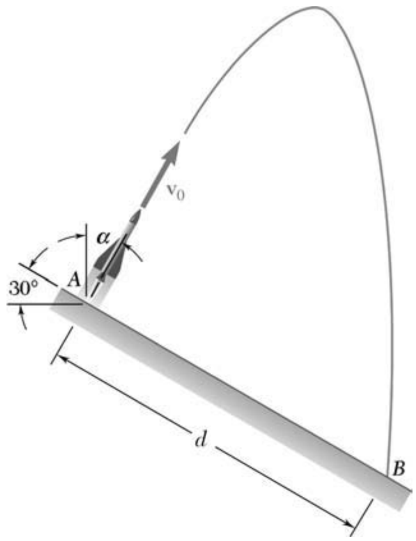
Noting that at Point *B*, $v_y < 0$, we have

$$\tan\theta = \frac{|(v_y)_B|}{v_x}$$

$$= \frac{\frac{7g}{10\cos\alpha} - 20\sin\alpha}{20\cos\alpha}$$

$$= \frac{\frac{7}{200}\frac{9.81}{\cos 10.3786°} - \sin 10.3786°}{\cos 10.3786°}$$

or $$\theta = 9.74° \blacktriangleleft$$



PROBLEM 11.112

A model rocket is launched from Point A with an initial velocity $\mathbf{v}_0$ of 75 m/s. If the rocket's descent parachute does not deploy and the rocket lands a distance $d = 100$ m from A, determine (a) the angle α that $\mathbf{v}_0$ forms with the vertical, (b) the maximum height above Point A reached by the rocket, and (c) the duration of the flight.

SOLUTION

Set the origin at Point A. $\quad x_0 = 0, \quad y_0 = 0$

Horizontal motion: $\quad x = v_0 t \sin\alpha \qquad \sin\alpha = \dfrac{x}{v_0 t} \qquad (1)$

Vertical motion: $\quad y = v_0 t \cos\alpha - \dfrac{1}{2} g t^2$

$$\cos\alpha = \frac{1}{v_0 t}\left(y + \frac{1}{2} g t^2\right) \qquad (2)$$

$$\sin^2\alpha + \cos^2\alpha = \frac{1}{(v_0 t)^2}\left[x^2 + \left(y + \frac{1}{2} g t^2\right)^2\right] = 1$$

$$x^2 + y^2 + g y t^2 + \frac{1}{4} g^2 t^4 = v_0^2 t^2$$

$$\frac{1}{4} g^2 t^4 - \left(v_0^2 - g y\right) t^2 + (x^2 + y^2) = 0 \qquad (3)$$

At Point B, $\quad \sqrt{x^2 + y^2} = 100 \text{ m}, \quad x = 100 \cos 30° \text{ m}$

$$y = -100 \sin 30° = -50 \text{ m}$$

$$\frac{1}{4}(9.81)^2 t^4 - [75^2 - (9.81)(-50)] t^2 + 100^2 = 0$$

$$24.0590\, t^4 - 6115.5\, t^2 + 10000 = 0$$

$$t^2 = 252.54 \text{ s}^2 \quad \text{and} \quad 1.6458 \text{ s}^2$$

$$t = 15.8916 \text{ s} \quad \text{and} \quad 1.2829 \text{ s}$$

PROBLEM 11.112 (Continued)

Restrictions on α: $0 < \alpha < 120°$

$$\tan\alpha = \frac{x}{y + \frac{1}{2}gt^2} = \frac{100\cos 30°}{-50 + (4.905)(15.8916)^2} = 0.0729$$

$$\alpha = 4.1669°$$

and $$\frac{100\cos 30°}{-50 + (4.905)(1.2829)^2} = -2.0655$$

$$\alpha = 115.8331°$$

Use $\alpha = 4.1669°$ corresponding to the steeper possible trajectory.

(*a*) Angle α. $\alpha = 4.17°$ ◀

(*b*) Maximum height. $v_y = 0$ at $y = y_{max}$

$$v_y = v_0 \cos\alpha - gt = 0$$

$$t = \frac{v_0 \cos\alpha}{g}$$

$$y_{max} = v_0 t \cos\alpha - \frac{1}{2}gt = \frac{v_0^2 \cos^2\alpha}{2g}$$

$$= \frac{(75)^2 \cos^2 4.1669°}{(2)(9.81)}$$

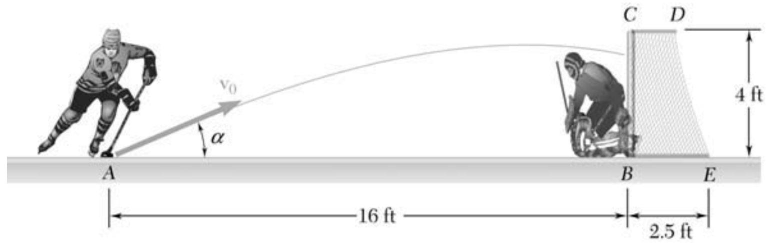
$y_{max} = 285$ m ◀

(*c*) Duration of the flight. (time to reach *B*)

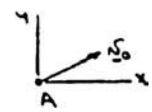
$t = 15.89$ s ◀

PROBLEM 11.113

The initial velocity $\mathbf{v}_0$ of a hockey puck is 105 mi/h. Determine (*a*) the largest value (less than 45°) of the angle α for which the puck will enter the net, (*b*) the corresponding time required for the puck to reach the net.



SOLUTION



First note $$v_0 = 105 \text{ mi/h} = 154 \text{ ft/s}$$

and $$(v_x)_0 = v_0 \cos\alpha = (154 \text{ ft/s}) \cos\alpha$$
$$(v_y)_0 = v_0 \sin\alpha = (154 \text{ ft/s}) \sin\alpha$$

(*a*) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (154 \cos\alpha)t$$

At the front of the net, $x = 16$ ft

Then $$16 = (154 \cos\alpha)t$$

or $$t_{\text{enter}} = \frac{8}{77\cos\alpha}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2$$
$$= (154 \sin\alpha)t - \frac{1}{2}gt^2 \quad (g = 32.2 \text{ ft/s}^2)$$

At the front of the net,

$$y_{\text{front}} = (154 \sin\alpha)t_{\text{enter}} - \frac{1}{2}gt_{\text{enter}}^2$$
$$= (154 \sin\alpha)\left(\frac{8}{77\cos\alpha}\right) - \frac{1}{2}g\left(\frac{8}{77\cos\alpha}\right)^2$$
$$= 16\tan\alpha - \frac{32g}{5929\cos^2\alpha}$$

Now $\dfrac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$

Then $y_{\text{front}} = 16\tan\alpha - \dfrac{32g}{5929}(1 + \tan^2\alpha)$

or $\tan^2\alpha - \dfrac{5929}{2g}\tan\alpha + \left(1 + \dfrac{5929}{32g}y_{\text{front}}\right) = 0$

Then $$\tan\alpha = \frac{\frac{5929}{2g} \pm \left[\left(-\frac{5929}{2g}\right)^2 - 4\left(1 + \frac{5929}{32g}y_{\text{front}}\right)\right]^{1/2}}{2}$$

or $$\tan\alpha = \frac{5929}{4\times 32.2} \pm \left[\left(-\frac{5929}{4\times 32.2}\right)^2 - \left(1 + \frac{5929}{32\times 32.2}y_{\text{front}}\right)\right]^{1/2}$$

or $\tan\alpha = 46.0326 \pm [(46.0326)^2 - (1 + 5.7541\, y_{\text{front}})]^{1/2}$

Now $0 < y_{\text{front}} < 4$ ft so that the positive root will yield values of $\alpha > 45°$ for all values of y_{front}. When the negative root is selected, α increases as y_{front} is increased. Therefore, for $\alpha_{\max}$, set

$$y_{\text{front}} = y_C = 4 \text{ ft}$$

Then $\tan\alpha = 46.0326 - [(46.0326)^2 - (1 + 5.7541 + 4)]^{1/2}$

or $\alpha_{\max} = 14.6604°$ $\qquad \alpha_{\max} = 14.66°$ ◀

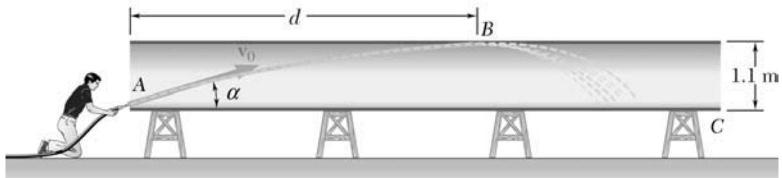
(*b*) We had found $t_{\text{enter}} = \dfrac{8}{77\cos\alpha}$

$$= \frac{8}{77\cos 14.6604°}$$

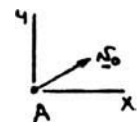
or $\qquad t_{\text{enter}} = 0.1074$ s ◀

PROBLEM 11.114

A worker uses high-pressure water to clean the inside of a long drainpipe. If the water is discharged with an initial velocity $\mathbf{v}_0$ of 11.5 m/s, determine (*a*) the distance d to the farthest Point B on the top of the pipe that the worker can wash from his position at A, (*b*) the corresponding angle α.



SOLUTION



First note $\quad (v_x)_0 = v_0 \cos\alpha = (11.5 \text{ m/s}) \cos\alpha$

$$(v_y)_0 = v_0 \sin\alpha = (11.5 \text{ m/s}) \sin\alpha$$

By observation, d_{max} occurs when $\quad y_{max} = 1.1$ m.

<u>Vertical motion</u>. (Uniformly accelerated motion)

$$v_y = (v_y)_0 - gt \qquad y = 0 + (v_y)_0 t - \frac{1}{2}gt^2$$

$$= (11.5 \sin\alpha) - gt \qquad = (11.5 \sin\alpha)t - \frac{1}{2}gt^2$$

When $\quad y = y_{max} \quad \text{at} \quad B, \quad (v_y)_B = 0$

Then $\quad (v_y)_B = 0 = (11.5 \sin\alpha) - gt$

or $\quad t_B = \dfrac{11.5 \sin\alpha}{g} \quad (g = 9.81 \text{ m/s}^2)$

and $\quad y_B = (11.5 \sin\alpha)t_B - \dfrac{1}{2}gt_B^2$

Substituting for t_B and noting $y_B = 1.1$ m

$$1.1 = (11.5 \sin\alpha)\left(\frac{11.5 \sin\alpha}{g}\right) - \frac{1}{2}g\left(\frac{11.5 \sin\alpha}{g}\right)^2$$

$$= \frac{1}{2g}(11.5)^2 \sin^2\alpha$$

or $\quad \sin^2\alpha = \dfrac{2.2 \times 9.81}{11.5^2} \qquad \alpha = 23.8265°$

PROBLEM 11.114 (Continued)

(*a*) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (11.5 \cos \alpha) t$$

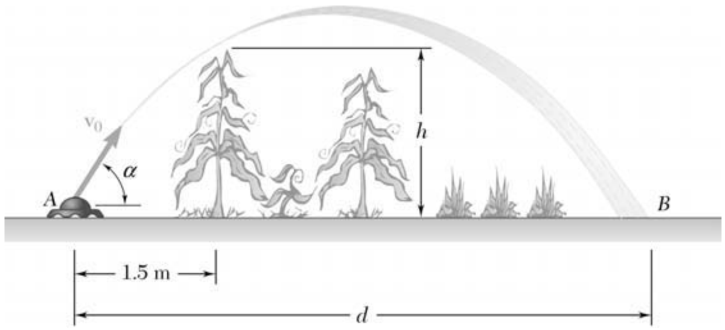
At Point B: $x = d_{\max}$ and $t = t_B$

where $t_B = \dfrac{11.5}{9.81} \sin 23.8265° = 0.47356 \text{ s}$

Then $d_{\max} = (11.5)(\cos 23.8265°)(0.47356)$

or $d_{\max} = 4.98 \text{ m}$ ◀

(*b*) From above $\alpha = 23.8°$ ◀



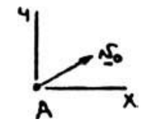
PROBLEM 11.115

An oscillating garden sprinkler which discharges water with an initial velocity $\mathbf{v}_0$ of 8 m/s is used to water a vegetable garden. Determine the distance d to the farthest Point B that will be watered and the corresponding angle α when (*a*) the vegetables are just beginning to grow, (*b*) the height h of the corn is 1.8 m.

SOLUTION

First note $\quad (v_x)_0 = v_0 \cos\alpha = (8 \text{ m/s}) \cos\alpha$

$$(v_y)_0 = v_0 \sin\alpha = (8 \text{ m/s}) \sin\alpha$$



Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (8\cos\alpha)t$$

At Point B: $\quad x = d: \quad d = (8\cos\alpha)t$

or $\quad t_B = \dfrac{d}{8\cos\alpha}$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2$$

$$= (8\sin\alpha)t - \frac{1}{2}gt^2 \quad (g = 9.81 \text{ m/s}^2)$$

At Point B: $\quad 0 = (8\sin\alpha)t_B - \dfrac{1}{2}gt_B^2$

Simplifying and substituting for t_B

$$0 = 8\sin\alpha - \frac{1}{2}g\left(\frac{d}{8\cos\alpha}\right)$$

or $\quad d = \dfrac{64}{g}\sin 2\alpha \qquad (1)$

(*a*) When $h = 0$, the water can follow any physically possible trajectory. It then follows from Eq. (1) that d is maximum when $2\alpha = 90°$

or $\quad \alpha = 45°$ ◀

Then $\quad d = \dfrac{64}{9.81}\sin(2\times 45°)$

or $\quad d_{max} = 6.52$ m ◀

(*b*) Based on Eq. (1) and the results of Part *a*, it can be concluded that *d* increases in value as α increases in value from 0 to 45° and then *d* decreases as α is further increased. Thus, $d_{\max}$ occurs for the value of α closest to 45° and for which the water just passes over the first row of corn plants. At this row, $x_{\text{com}} = 1.5$ m

so that
$$t_{\text{corn}} = \frac{1.5}{8 \cos \alpha}$$

Also, with $y_{\text{corn}} = h$, we have

$$h = (8 \sin \alpha) t_{\text{corn}} - \frac{1}{2} g t_{\text{corn}}^2$$

Substituting for t_{corn} and noting $h = 1.8$ m,

$$1.8 = (8 \sin \alpha)\left(\frac{1.5}{8 \cos \alpha}\right) - \frac{1}{2} g \left(\frac{1.5}{8 \cos \alpha}\right)^2$$

or
$$1.8 = 1.5 \tan \alpha - \frac{2.25 g}{128 \cos^2 \alpha}$$

Now
$$\frac{1}{\cos^2 \alpha} = \sec^2 \alpha = 1 + \tan^2 \alpha$$

Then
$$1.8 = 1.5 \tan \alpha - \frac{2.25(9.81)}{128}(1 + \tan^2 \alpha)$$

or
$$0.172441 \tan^2 \alpha - 1.5 \tan \alpha + 1.972441 = 0$$

Solving
$$\alpha = 58.229° \quad \text{and} \quad \alpha = 81.965°$$

From the above discussion, it follows that $d = d_{\max}$ when

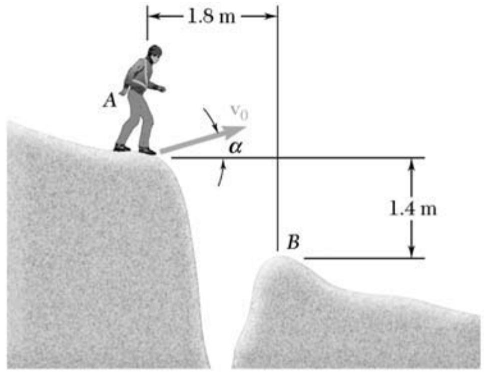
$$\alpha = 58.2° \blacktriangleleft$$

Finally, using Eq. (1)

$$d = \frac{64}{9.81} \sin (2 \times 58.229°)$$

or
$$d_{\max} = 5.84 \text{ m} \blacktriangleleft$$

PROBLEM 11.116*

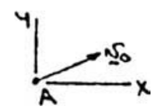


A mountain climber plans to jump from A to B over a crevasse. Determine the smallest value of the climber's initial velocity $\mathbf{v}_0$ and the corresponding value of angle α so that he lands at B.

SOLUTION

First note

$$(v_x)_0 = v_0 \cos \alpha$$
$$(v_y)_0 = v_0 \sin \alpha$$



Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (v_0 \cos \alpha) t$$

At Point B:

$$1.8 = (v_0 \cos \alpha) t$$

or

$$t_B = \frac{1.8}{v_0 \cos \alpha}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2$$
$$= (v_0 \sin \alpha) t - \frac{1}{2} g t^2 \quad (g = 9.81 \text{ m/s}^2)$$

At Point B:

$$-1.4 = (v_0 \sin \alpha) t_B - \frac{1}{2} g t_B^2$$

Substituting for t_B

$$-1.4 = (v_0 \sin \alpha)\left(\frac{1.8}{v_0 \cos \alpha}\right) - \frac{1}{2} g \left(\frac{1.8}{v_0 \cos \alpha}\right)^2$$

or

$$v_0^2 = \frac{1.62 g}{\cos^2 \alpha (1.8 \tan \alpha + 1.4)}$$
$$= \frac{1.62 g}{0.9 \sin 2\alpha + 1.4 \cos^2 \alpha}$$

PROBLEM 11.116* (Continued)

Now minimize v_0^2 with respect to α.

We have
$$\frac{dv_0^2}{d\alpha} = 1.62g\frac{-(1.8\cos 2\alpha - 2.8\cos\alpha\sin\alpha)}{(0.9\sin 2\alpha + 1.4\cos^2\alpha)^2} = 0$$

or
$$1.8\cos 2\alpha - 1.4\sin 2\alpha = 0$$

or
$$\tan 2\alpha = \frac{18}{14}$$

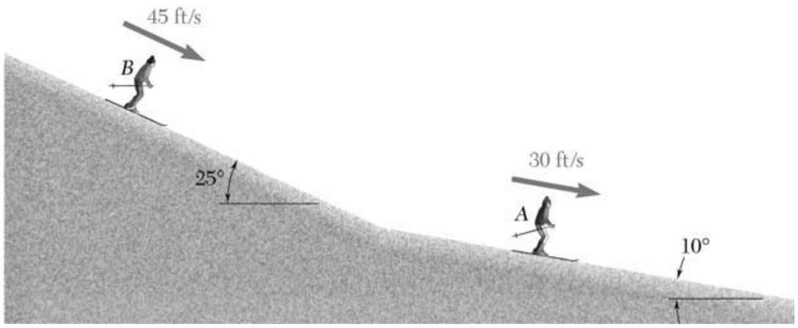
or
$$\alpha = 26.0625° \quad \text{and} \quad \alpha = 206.06°$$

Rejecting the second value because it is not physically possible, we have

$$\alpha = 26.1° \blacktriangleleft$$

Finally,
$$v_0^2 = \frac{1.62 \times 9.81}{\cos^2 26.0625°(1.8\tan 26.0625° + 1.4)}$$

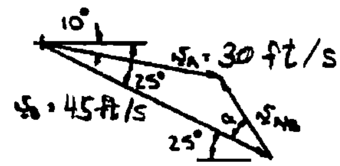
or
$$(v_0)_{\min} = 2.94 \text{ m/s} \blacktriangleleft$$



PROBLEM 11.117

The velocities of skiers A and B are as shown. Determine the velocity of A with respect to B.

SOLUTION



We have $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

The graphical representation of this equation is then as shown.

Then $$\mathbf{v}_{A/B}^2 = 30^2 + 45^2 - 2(30)(45)\cos 15°$$

or $$v_{A/B} = 17.80450 \text{ ft/s}$$

and $$\frac{30}{\sin\alpha} = \frac{17.80450}{\sin 15°}$$

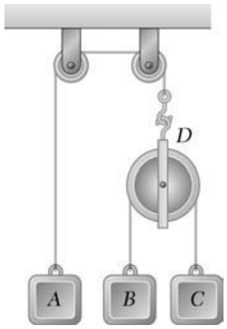
or $$\alpha = 25.8554°$$

$$\alpha + 25° = 50.8554°$$

$$\mathbf{v}_{A/B} = 17.8 \text{ ft/s} \measuredangle\, 50.9° \blacktriangleleft$$

Alternative solution.

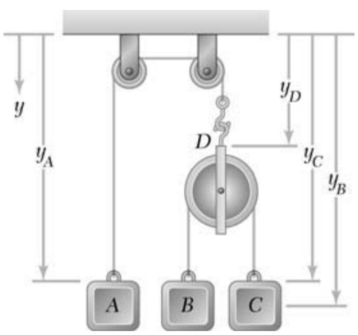
$$\begin{aligned}\mathbf{v}_{A/B} &= \mathbf{v}_A - \mathbf{v}_B \\ &= 30\cos 10°\,\mathbf{i} - 30\sin 10°\mathbf{j} - (45\cos 25°\mathbf{i} - 45°\sin 25°\mathbf{j}) \\ &= 11.2396\mathbf{i} + 13.8084\,\mathbf{j} \\ &= 5.05 \text{ m/s} = 17.8 \text{ ft/s} \measuredangle\, 50.9°\end{aligned}$$



PROBLEM 11.118

The three blocks shown move with constant velocities. Find the velocity of each block, knowing that the relative velocity of A with respect to C is 300 mm/s upward and that the relative velocity of B with respect to A is 200 mm/s downward.

SOLUTION



From the diagram

Cable 1: $y_A + y_D = \text{constant}$

Then $v_A + v_D = 0$ (1)

Cable 2: $(y_B - y_D) + (y_C - y_D) = \text{constant}$

Then $v_B + v_C - 2v_D = 0$ (2)

Combining Eqs. (1) and (2) to eliminate v_D,

$$2v_A + v_B + v_C = 0 \quad (3)$$

Now $v_{A/C} = v_A - v_C = -300 \text{ mm/s}$ (4)

and $v_{B/A} = v_B - v_A = 200 \text{ mm/s}$ (5)

Then $(3) + (4) - (5) \Rightarrow$

$$(2v_A + v_B + v_C) + (v_A - v_C) - (v_B - v_A) = (-300) - (200)$$

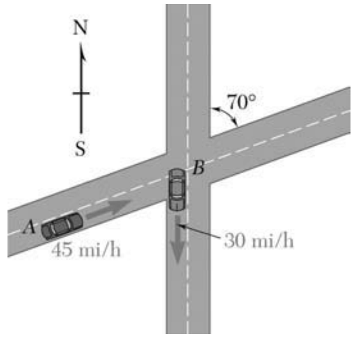
or $\mathbf{v}_A = 125 \text{ mm/s} \uparrow$ ◀

and using Eq. (5) $v_B - (-125) = 200$

or $\mathbf{v}_B = 75 \text{ mm/s} \downarrow$ ◀

Eq. (4) $-125 - v_C = -300$

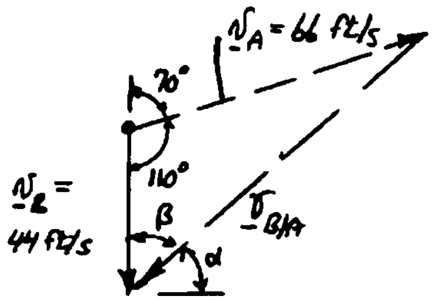
or $\mathbf{v}_C = 175 \text{ mm/s} \downarrow$ ◀



PROBLEM 11.119

Three seconds after automobile *B* passes through the intersection shown, automobile *A* passes through the same intersection. Knowing that the speed of each automobile is constant, determine (*a*) the relative velocity of *B* with respect to *A*, (*b*) the change in position of *B* with respect to *A* during a 4-s interval, (*c*) the distance between the two automobiles 2 s after *A* has passed through the intersection.

SOLUTION



$$v_A = 45 \text{ mi/h} = 66 \text{ ft/s}$$
$$v_B = 30 \text{ mi/h} = 44 \text{ ft/s}$$

Law of cosines

$$v_{B/A}^2 = 66^2 + 44^2 - 2(66)(44)\cos 110°$$
$$v_{B/A} = 90.99 \text{ ft/s}$$

$$v_B = v_A + v_{B/A}$$

Law of sines

$$\frac{\sin\beta}{66} = \frac{\sin 110°}{90.99} \qquad \beta = 42.97°$$

$$\alpha = 90° - \beta = 90° - 42.97° = 47.03°$$

(*a*) Relative velocity: $\mathbf{v}_{B/A} = 91.0 \text{ ft/s} \nearrow 47.0°$ ◀

(*b*) Change in position for $\Delta t = 4$ s.

$$\Delta r_{B/A} = v_{B/A}\Delta t = (91.0 \text{ ft/s})(4 \text{ s})$$

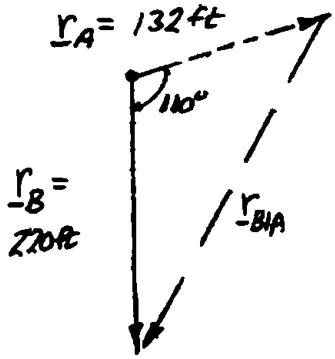
$\mathbf{r}_{B/A} = 364 \text{ ft} \nearrow 47.0°$ ◀

(*c*) Distance between autos 2 seconds after auto *A* has passed intersection.

Auto *A* travels for 2 s.

$$v_A = 66 \text{ ft/s} \measuredangle 20°$$
$$r_A = v_A t = (66 \text{ ft/s})(2 \text{ s}) = 132 \text{ ft}$$
$$\mathbf{r}_A = 132 \text{ ft} \measuredangle 20°$$

PROBLEM 11.119 (Continued)



Auto *B*

$$\mathbf{v}_B = 44 \text{ ft/s} \downarrow$$

$$\mathbf{r}_B = \mathbf{v}_B t = (44 \text{ ft/s})(5 \text{ s}) = 220 \text{ ft} \downarrow$$

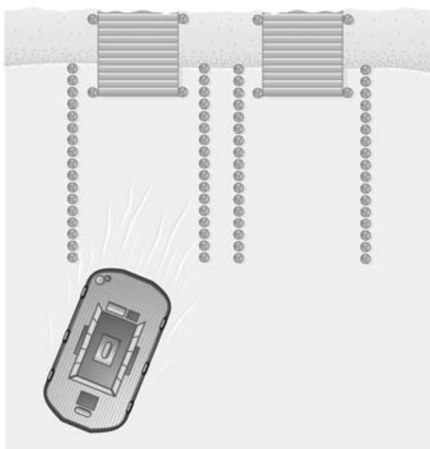
$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

Law of cosines

$$r_{B/A}^2 = (132)^2 + (220)^2 - 2(132)(220)\cos 110°$$

$$\mathbf{r}_{B/A} = 292.7 \text{ ft}$$

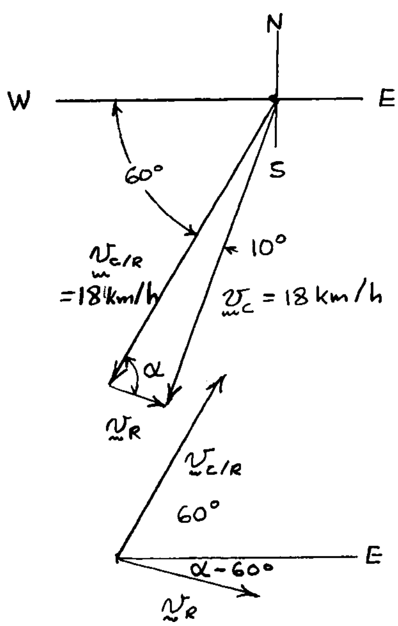
Distance between autos = 293 ft ◀



PROBLEM 11.120

Shore-based radar indicates that a ferry leaves its slip with a velocity $\mathbf{v} = 18$ km/h ⦨ 70°, while instruments aboard the ferry indicate a speed of 18.4 km/h and a heading of 30° west of south relative to the river. Determine the velocity of the river.

SOLUTION



We have $\mathbf{v}_F = \mathbf{v}_R + \mathbf{v}_{F/R}$ or $\mathbf{v}_F = \mathbf{v}_{F/R} + \mathbf{v}_R$

The graphical representation of the second equation is then as shown.

We have $v_R^2 = 18^2 + 18.4^2 - 2(18)(18.4)\cos 10°$

or $v_R = 3.1974$ km/h

and $\dfrac{18}{\sin\alpha} = \dfrac{3.1974}{\sin 10°}$

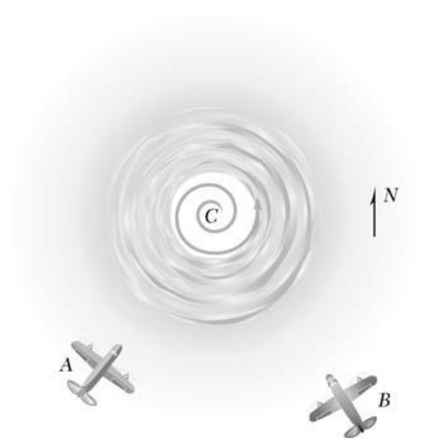
or $\alpha = 77.84°$

Noting that

$\mathbf{v}_R = 3.20$ km/h ⦪ 17.8° ◀

Alternatively one could use vector algebra.

PROBLEM 11.121



Airplanes A and B are flying at the same altitude and are tracking the eye of hurricane C. The relative velocity of C with respect to A is $\mathbf{v}_{C/A} = 350$ km/h $\measuredangle$ 75°, and the relative velocity of C with respect to B is $\mathbf{v}_{C/B} = 400$ km/h $\measuredangle$ 40°. Determine (*a*) the relative velocity of B with respect to A, (*b*) the velocity of A if ground-based radar indicates that the hurricane is moving at a speed of 30 km/h due north, (*c*) the change in position of C with respect to B during a 15-min interval.

SOLUTION

(*a*) We have $\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$

and $\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

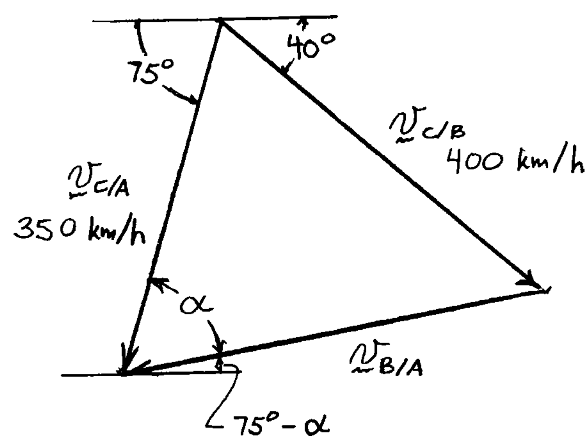
Then $\mathbf{v}_A + \mathbf{v}_{C/A} = \mathbf{v}_B + \mathbf{v}_{C/B}$

or $\mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_{C/A} - \mathbf{v}_{C/B}$

Now $\mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_{B/A}$

so that $\mathbf{v}_{B/A} = \mathbf{v}_{C/A} - \mathbf{v}_{C/B}$

or $\mathbf{v}_{C/A} = \mathbf{v}_{C/B} + \mathbf{v}_{B/A}$



The graphical representation of the last equation is then as shown.

We have $v_{B/A}^2 = 350^2 + 400^2 - 2(350)(400)\cos 65°$

or $v_{B/A} = 405.175$ km/h

and $\dfrac{400}{\sin\alpha} = \dfrac{405.175}{\sin 65°}$

or $\alpha = 63.474°$

$75° - \alpha = 11.526°$

$\mathbf{v}_{B/A} = 405$ km/h $\angle$ 11.53° ◀

PROBLEM 11.121 (Continued)

(*b*) We have $\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$

or $\mathbf{v}_A = (30\text{ km/h})\mathbf{j} - (350\text{ km/h})(-\cos 75°\mathbf{i} - \sin 75°\mathbf{j})$

$$\mathbf{v}_A = (90.587\text{ km/h})\mathbf{i} + (368.07\text{ km/h})\mathbf{j}$$

or $\mathbf{v}_A = 379\text{ km/h} \measuredangle 76.17°$ ◀

(*c*) Noting that the velocities of *B* and *C* are constant, we have

$$\mathbf{r}_B = (\mathbf{r}_B)_0 + \mathbf{v}_B t \qquad \mathbf{r}_C = (\mathbf{r}_C)_0 + \mathbf{v}_C t$$

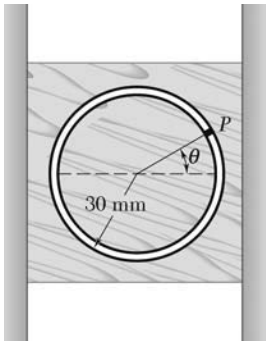
Now $\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = [(\mathbf{r}_C)_0 - (\mathbf{r}_B)_0] + (\mathbf{v}_C - \mathbf{v}_B)t$

$$= [(\mathbf{r}_C)_0 - (\mathbf{r}_B)_0] + \mathbf{v}_{C/B}t$$

Then $\Delta\mathbf{r}_{C/B} = (\mathbf{r}_{C/B})_{t_2} - (\mathbf{r}_{C/B})_{t_1} = \mathbf{v}_{C/B}(t_2 - t_1) = \mathbf{v}_{C/B}\Delta t$

For $\Delta t = 15$ min: $\Delta r_{C/B} = (400\text{ km/h})\left(\frac{1}{4}\text{h}\right) = 100\text{ km}$

$\Delta\mathbf{r}_{C/B} = 100\text{ km} \searrow 40°$ ◀



PROBLEM 11.122

Pin P moves at a constant speed of 150 mm/s in a counterclockwise sense along a circular slot which has been milled in the slider block A shown. Knowing that the block moves downward at a constant speed 100 mm/s determine the velocity of pin P when (*a*) $\theta = 30°$, (*b*) $\theta = 120°$.

SOLUTION

$$\mathbf{v}_P = \mathbf{v}_A + \mathbf{v}_{P/A}$$

$$\mathbf{v}_P = 100 \text{ ms } (-\mathbf{j}) + 150(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})\text{mm/s}$$

(*a*) For $\theta = 30°$

$$\mathbf{v}_P = -100 \text{ mm/s } (\mathbf{j}) + 150(-\cos(30°)\mathbf{i} + \sin(30°)\mathbf{j})\text{mm/s}$$

$$\mathbf{v}_P = (-75\mathbf{i} + 29.9038\mathbf{j})\text{mm/s}$$

$\mathbf{v}_P = 80.7$ mm/s ⦣ 21.7° ◀

(*b*) For $\theta = 120°$

$$\mathbf{v}_P = -100 \text{ mm/s } (\mathbf{j}) + 150(-\cos(120°)\mathbf{i} + \sin(120°)\mathbf{j})\text{mm/s}$$

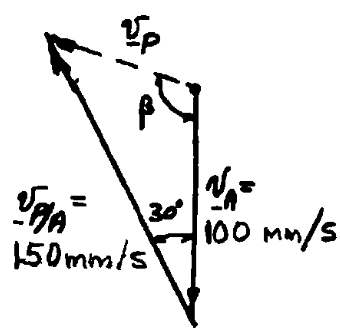
$$\mathbf{v}_P = (-129.9038\mathbf{i} + -175\mathbf{j}) \text{ mm/s}$$

$\mathbf{v}_P = 218$ mm/s ⦣ 53.4° ◀

Alternative Solution

(*a*) For $\theta = 30°$, $v_{P/A} = 7.5$ in./s ⦣ 30°

$$v_P = v_A + v_{P/A}$$



Law of cosines

$$v_P^2 = (150)^2 + (100)^2 - 2(100)(150)\cos 30°$$

$$v_P = 80.7418 \text{ mm/s}$$

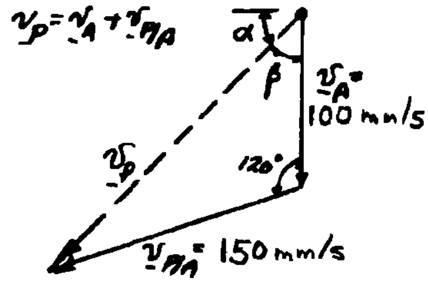
Law of sines

$$\frac{\sin\beta}{150} = \frac{\sin 30°}{80.7418} \qquad \beta = 111.7°$$

$v_P = 80.7$ mm/s ⦣ 21.7° ◀

PROBLEM 11.122 (Continued)

(*b*) For $\theta = 120°$, $v_{P/A} = 150$ mm/s $\searrow$ 30°



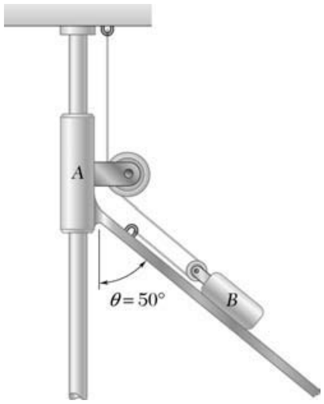
Law of cosines

$$v_P^2 = (150)^2 + (100)^2 - 2(100)(150)\cos 120°$$
$$v_P = 217.9449 \text{ mm/s}$$

Law of sines

$$\frac{\sin\beta}{150} = \frac{\sin 120°}{217.9449} \quad \beta = 36.6°$$
$$\alpha = 90 - \beta = 90° - 36.6 = 53.4°$$

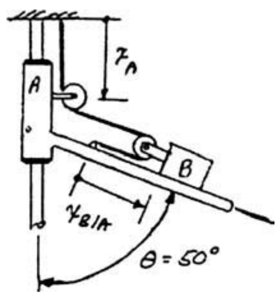
$v_P = 218$ mm/s $\searrow$ 53.4° ◀



PROBLEM 11.123

Knowing that at the instant shown assembly A has a velocity of 9 in./s and an acceleration of 15 in./s^2 both directed downward, determine (a) the velocity of block B, (b) the acceleration of block B.

SOLUTION



Length of cable = constant

$$L = x_A + 2x_{B/A} = \text{constant}$$

$$v_A + 2v_{B/A} = 0 \qquad (1)$$

$$a_A + 2a_{B/A} = 0 \qquad (2)$$

Data:

$$\mathbf{a}_A = 15 \text{ in./s}^2 \downarrow$$

$$\mathbf{v}_A = 9 \text{ in./s} \downarrow$$

Eqs. (1) and (2)

$$a_A = -2a_{B/A} \qquad v_A = -2v_{B/A}$$

$$15 = -2a_{B/A} \qquad 9 = -2v_{B/A}$$

$$a_{B/A} = -7.5 \text{ in./s}^2 \qquad v_{B/A} = -4.5 \text{ in./s}$$

$$\mathbf{a}_{B/A} = 7.5 \text{ in./s}^2 \searrow 40° \qquad \mathbf{v}_{B/A} = -4.5 \text{ in./s} \searrow 40°$$

(a) Velocity of B.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

Law of cosines:

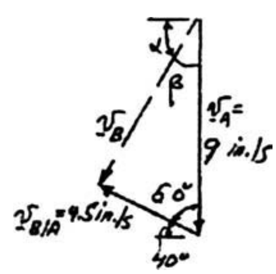
$$v_B^2 = (9)^2 + (4.5)^2 - 2(9)(4.5)\cos 50°$$

$$v_B = 7.013 \text{ in./s}$$

Law of sines:

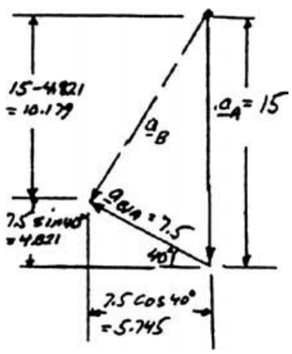
$$\frac{\sin\beta}{4.5} = \frac{\sin 50°}{7.013} \qquad \beta = 29.44°$$

$$\alpha = 90° - \beta = 90° - 29.44° = 60.56°$$



$$\mathbf{v}_B = 7.01 \text{ in./s} \nearrow 60.6° \blacktriangleleft$$

(*b*) Acceleration of *B*. $\mathbf{a}_B$ may be found by using analysis similar to that used above for v_B. An alternate method is



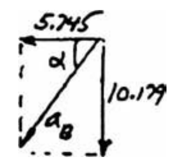
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\mathbf{a}_B = 15 \text{ in./s}^2 \downarrow + 7.5 \text{ in./s}^2 \measuredangle 40^\circ$$

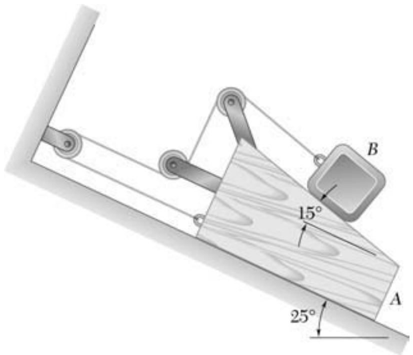
$$= -15\mathbf{j} - (7.5\cos 40^\circ)\mathbf{i} + (7.5\sin 40^\circ)\mathbf{j}$$

$$= -15\mathbf{j} - 5.745\mathbf{i} + 4.821\mathbf{j}$$

$$\mathbf{a}_B = -5.745\mathbf{i} - 10.179\mathbf{j}$$



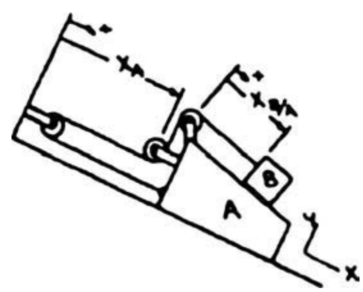
$$\mathbf{a}_B = 11.69 \text{ in./s}^2 \measuredangle 60.6^\circ \blacktriangleleft$$



PROBLEM 11.124

Knowing that at the instant shown block A has a velocity of 8 in./s and an acceleration of 6 in./s^2 both directed down the incline, determine (*a*) the velocity of block B, (*b*) the acceleration of block B.

SOLUTION



From the diagram $\quad 2x_A + x_{B/A} = \text{constant}$

Then $\quad 2v_A + v_{B/A} = 0$

or $\quad |v_{B/A}| = 16 \text{ in./s}$

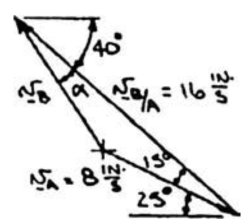
and $\quad 2a_A + a_{B/A} = 0$

or $\quad |a_{B/A}| = 12 \text{ in./s}^2$

Note that $\mathbf{v}_{B/A}$ and $\mathbf{a}_{B/A}$ must be parallel to the top surface of block A.

(*a*) We have $\quad \mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

The graphical representation of this equation is then as shown. Note that because A is moving downward, B must be moving upward relative to A.



We have $\quad v_B^2 = 8^2 + 16^2 - 2(8)(16)\cos 15°$

or $\quad v_B = 8.5278 \text{ in./s}$

and $\quad \dfrac{8}{\sin\alpha} = \dfrac{8.5278}{\sin 15°}$

or $\quad \alpha = 14.05°$

$\mathbf{v}_B = 8.53 \text{ in./s} \measuredangle 54.1°$ ◀

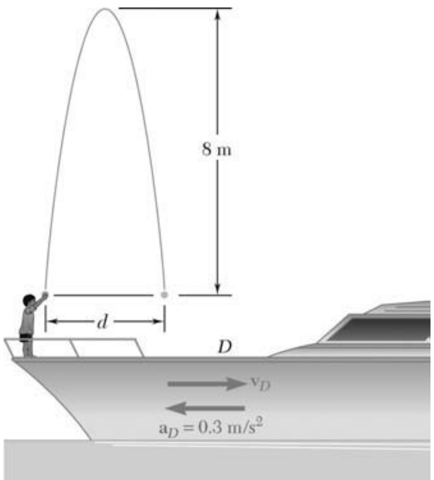
(*b*) The same technique that was used to determine $\mathbf{v}_B$ can be used to determine $\mathbf{a}_B$. An alternative method is as follows.

We have

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$
$$= (6\mathbf{i}) + 12(-\cos 15°\mathbf{i} + \sin 15°\mathbf{j})^*$$
$$= -(5.5911 \text{ in./s}^2)\mathbf{i} + (3.1058 \text{ in./s}^2)\mathbf{j}$$

or $\quad \mathbf{a}_B = 6.40 \text{ in./s}^2 \measuredangle 54.1°$ ◀

* Note the orientation of the coordinate axes on the sketch of the system.



PROBLEM 11.125

A boat is moving to the right with a constant deceleration of 0.3 m/s^2 when a boy standing on the deck D throws a ball with an initial velocity relative to the deck which is vertical. The ball rises to a maximum height of 8 m above the release point and the boy must step forward a distance d to catch it at the same height as the release point. Determine (a) the distance d, (b) the relative velocity of the ball with respect to the deck when the ball is caught.

SOLUTION

Horizontal motion of the ball: $v_x = (v_x)_0, \quad x_{\text{ball}} = (v_x)_0 t$

Vertical motion of the ball: $v_y = (v_y)_0 - gt$

$$y_B = (v_y)_0 t - \frac{1}{2}gt^2, \quad (v_y)^2 - (v_y)_0^2 = -2gy$$

At maximum height, $v_y = 0$ and $y = y_{\max}$

$$(v_y)^2 = 2gy_{\max} = (2)(9.81)(8) = 156.96 \text{ m}^2/\text{s}^2$$

$$(v_y)_0 = 12.528 \text{ m/s}$$

At time of catch, $y = 0 = 12.528 - \frac{1}{2}(9.81)t^2$

or $t_{\text{catch}} = 2.554$ s and $v_y = 12.528$ m/s $\downarrow$

Motion of the deck: $v_x = (v_x)_0 + a_D t, \quad x_{\text{deck}} = (v_x)_0 t + \frac{1}{2}a_D t^2$

Motion of the ball relative to the deck:

$$(v_{B/D})_x = (v_x)_0 - [(v_x)_0 + a_D t] = -a_D t$$

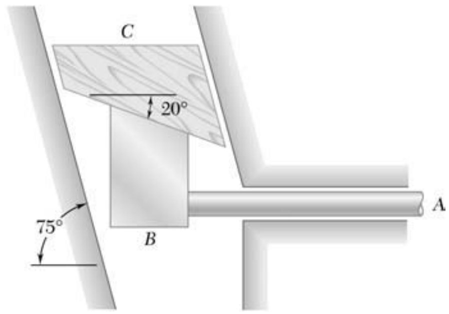
$$x_{B/D} = (v_x)_0 t - \left[(v_x)_0 t + \frac{1}{2}a_D t^2\right] = -\frac{1}{2}a_D t^2$$

$$(v_{B/D})_y = (v_y)_0 - gt, \quad y_{B/D} = y_B$$

(a) At time of catch, $d = x_{D/B} = -\frac{1}{2}(-0.3)(2.554)^2$ $\quad d = 0.979$ m ◀

(b) $(v_{B/D})_x = -(-0.3)(2.554) = +0.766$ m/s or 0.766 m/s $\longrightarrow$

$(v_{B/D})_y = 12.528$ m/s $\downarrow$ $\quad \mathbf{v}_{B/D} = 12.55$ m/s ⦪ 86.5° ◀



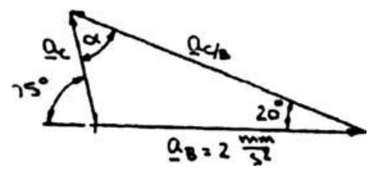
PROBLEM 11.126

The assembly of rod A and wedge B starts from rest and moves to the right with a constant acceleration of 2 mm/s^2. Determine (a) the acceleration of wedge C, (b) the velocity of wedge C when $t = 10$ s.

SOLUTION

(a) We have $\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$

The graphical representation of this equation is then as shown.



First note
$$\alpha = 180° - (20° + 105°)$$
$$= 55°$$

Then
$$\frac{a_C}{\sin 20°} = \frac{2}{\sin 55°}$$
$$a_C = 0.83506 \text{ mm/s}^2$$

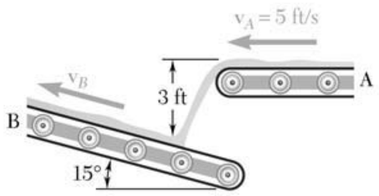
$$\mathbf{a}_C = 0.835 \text{ mm/s}^2 \searrow 75° \blacktriangleleft$$

(b) For uniformly accelerated motion

$$v_C = 0 + a_C t$$

At $t = 10$ s:
$$v_C = (0.83506 \text{ mm/s}^2)(10 \text{ s})$$
$$= 8.3506 \text{ mm/s}$$

or
$$\mathbf{v}_C = 8.35 \text{ mm/s} \searrow 75° \blacktriangleleft$$



PROBLEM 11.127

Determine the required velocity of the belt B if the relative velocity with which the sand hits belt B is to be (*a*) vertical, (*b*) as small as possible.

SOLUTION

A grain of sand will undergo projectile motion.

$$v_{s_x} = v_{s_{x_0}} = \text{constant} = -5 \text{ ft/s}$$

<u>y-direction</u>. $$v_{s_y} = \sqrt{2gh} = \sqrt{(2)(32.2 \text{ ft/s}^2)(3 \text{ ft})} = 13.90 \text{ ft/s} \downarrow$$

<u>Relative velocity</u>. $$\mathbf{v}_{S/B} = \mathbf{v}_S - \mathbf{v}_B \qquad (1)$$

(*a*) If $v_{S/B}$ is vertical,

$$-v_{S/B}\mathbf{j} = -5\mathbf{i} - 13.9\mathbf{j} - (-v_B \cos 15°\mathbf{i} + v_B \sin 15°\mathbf{j})$$
$$= -5\mathbf{i} - 13.9\mathbf{j} + v_B \cos 15°\mathbf{i} - v_B \sin 15°\mathbf{j}$$

<u>Equate components</u>. $\mathbf{i}$: $0 = -5 + v_B \cos 15°$ $\qquad v_B = \dfrac{5}{\cos 15°} = 5.176$ ft/s

$$\mathbf{v}_B = 5.18 \text{ ft/s} \measuredangle 15° \blacktriangleleft$$

(*b*) $v_{S/C}$ is as small as possible, so make $v_{S/B} \perp$ to v_B into (1).

$$-v_{S/B} \sin 15°\mathbf{i} - v_{S/B} \cos 15°\mathbf{j} = -5\mathbf{i} - 13.9\mathbf{j} + v_B \cos 15°\mathbf{i} - v_B \sin 15°\mathbf{j}$$

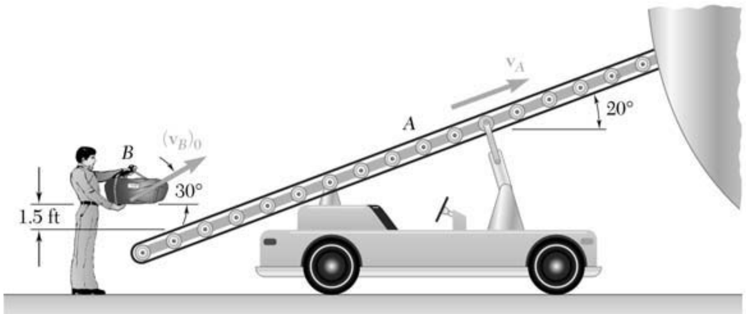
Equate components and transpose terms.

$$(\sin 15°)\, v_{S/B} + (\cos 15°)\, v_B = 5$$
$$(\cos 15°)\, v_{S/B} - (\sin 15°)\, v_B = 13.90$$

Solving, $\qquad v_{S/B} = 14.72$ ft/s $\qquad v_B = 1.232$ ft/s

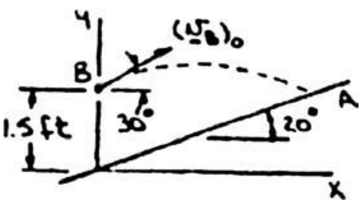
$$\mathbf{v}_B = 1.232 \text{ ft/s} \measuredangle 15° \blacktriangleleft$$

PROBLEM 11.128



Conveyor belt A, which forms a 20° angle with the horizontal, moves at a constant speed of 4 ft/s and is used to load an airplane. Knowing that a worker tosses duffel bag B with an initial velocity of 2.5 ft/s at an angle of 30° with the horizontal, determine the velocity of the bag relative to the belt as it lands on the belt.

SOLUTION



First determine the velocity of the bag as it lands on the belt. Now

$$[(v_B)_x]_0 = (v_B)_0 \cos 30^\circ$$
$$= (2.5 \text{ ft/s}) \cos 30^\circ$$
$$[(v_B)_y]_0 = (v_B)_0 \sin 30^\circ$$
$$= (2.5 \text{ ft/s}) \sin 30^\circ$$

<u>Horizontal motion</u>. (Uniform)

$$x = 0 + [(v_B)_x]_0 t \qquad (v_B)_x = [(v_B)_x]_0$$
$$= (2.5 \cos 30^\circ)t \qquad = 2.5 \cos 30^\circ$$

<u>Vertical motion</u>. (Uniformly accelerated motion)

$$y = y_0 + [(v_B)_y]_0 t - \frac{1}{2}gt^2 \qquad (v_B)_y = [(v_B)_y]_0 - gt$$
$$= 1.5 + (2.5 \sin 30^\circ)t - \frac{1}{2}gt^2 \qquad = 2.5 \sin 30^\circ - gt$$

The equation of the line collinear with the top surface of the belt is

$$y = x \tan 20^\circ$$

Thus, when the bag reaches the belt

$$1.5 + (2.5 \sin 30^\circ)t - \frac{1}{2}gt^2 = [(2.5 \cos 30^\circ)t] \tan 20^\circ$$

or $$\frac{1}{2}(32.2)t^2 + 2.5(\cos 30^\circ \tan 20^\circ - \sin 30^\circ)t - 1.5 = 0$$

or $$16.1t^2 - 0.46198t - 1.5 = 0$$

Solving $t = 0.31992$ s and $t = -0.29122$ s (Reject)

PROBLEM 11.128 (Continued)

The velocity $\mathbf{v}_B$ of the bag as it lands on the belt is then

$$\mathbf{v}_B = (2.5\cos 30°)\mathbf{i} + [2.5\sin 30° - 32.2(0.319\,92)]\mathbf{j}$$
$$= (2.1651 \text{ ft/s})\mathbf{i} - (9.0514 \text{ ft/s})\mathbf{j}$$

Finally $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

or $\mathbf{v}_{B/A} = (2.1651\mathbf{i} - 9.0514\mathbf{j}) - 4(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$

$$= -(1.59367 \text{ ft/s})\mathbf{i} - (10.4195 \text{ ft/s})\mathbf{j}$$

or $\mathbf{v}_{B/A} = 10.54 \text{ ft/s} \measuredangle 81.3°$ ◀

PROBLEM 11.129

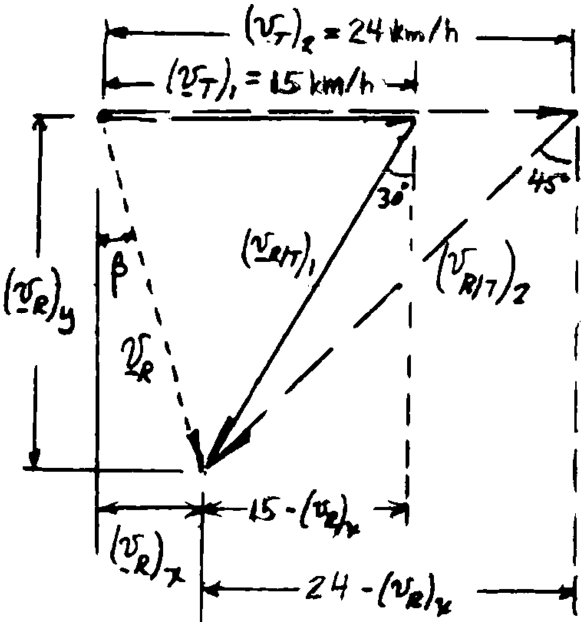
During a rainstorm the paths of the raindrops appear to form an angle of 30° with the vertical and to be directed to the left when observed from a side window of a train moving at a speed of 15 km/h. A short time later, after the speed of the train has increased to 24 km/h, the angle between the vertical and the paths of the drops appears to be 45°. If the train were stopped, at what angle and with what velocity would the drops be observed to fall?

SOLUTION

$$v_{\text{rain}} = v_{\text{train}} + v_{\text{rain/train}}$$

Case ①: $v_T = 15 \text{ km/h} \longrightarrow; \quad v_{R/T} \swarrow 30°$

Case ②: $v_T = 24 \text{ km/h} \longrightarrow; \quad v_{R/T} \swarrow 45°$



Case ①: $(v_R)_y \tan 30 = 15 - (v_R)_x$ (1)

Case ②: $(v_R)_y \tan 45° = 24 - (v_R)_x$ (2)

Substract (1) from (2) $(v_R)_y (\tan 45° - \tan 30°) = 9$

$$(v_R)_y = 21.294 \text{ km/h}$$

Eq. (2): $21.294 \tan 45° = 25 - (v_R)_x$

$$(v_R)_x = 2.706 \text{ km/h}$$

$$\tan\beta = \frac{3.706}{21.294}$$

$$\beta = 7.24°$$

$$v_R = \frac{21.294}{\cos 7.24°} = 21.47 \text{ km/h} = 5.96 \text{ m/s}$$

$v_R = 5.96$ m/s ◣ 82.8° ◀

Alternate solution

Alternate, vector equation $\mathbf{v}_R = \mathbf{v}_T + \mathbf{v}_{R/T}$

For first case, $\mathbf{v}_R = 15\mathbf{i} + v_{R/T-1}(-\sin 30°\mathbf{i} - \cos 30°\mathbf{j})$

For second case, $\mathbf{v}_R = 24\mathbf{i} + v_{R/T-2}(-\sin 45°\mathbf{i} - \cos 45°\mathbf{j})$

Set equal

$$15\mathbf{i} + v_{R/T-1}(-\sin 30°\mathbf{i} - \cos 30°\mathbf{j}) = 24\mathbf{i} + v_{R/T-2}(-\sin 45°\mathbf{i} - \cos 45°\mathbf{j})$$

Separate into components:

i: $$15 - v_{R/T-1}\sin 30° = 24 - v_{R/T-2}\sin 45°$$

$$-v_{R/T-1}\sin 30° + v_{R/T-2}\sin 45° = 9 \quad (3)$$

j: $$-v_{R/T-1}\cos 30° = -v_{R/T-2}\cos 45°$$

$$v_{R/T-1}\cos 30° + v_{R/T-2}\cos 45° = 0 \quad (4)$$

Solving Eqs. (3) and (4) simultaneously,

$v_{R/T-1} = 24.5885$ km/h $\qquad v_{R/T-2} = 30.1146$ km/h

Substitute $\mathbf{v}_{R/T-2}$ back into equation for $\mathbf{v}_R$.

$$\mathbf{v}_R = 24\mathbf{i} + 30.1146(-\sin 45°\mathbf{i} - \cos 45°\mathbf{j})$$

$\mathbf{v}_R = 2.71\mathbf{i} - 21.29\mathbf{j} \qquad \mathbf{v}_R = 21.4654$ km/hr $= 5.96$ m/s

$\theta = \tan^{-1}\left(\frac{-21.29}{2.71}\right) = -82.7585° \qquad \mathbf{v}_R = 5.96$ m/s ◣ 82.8° ◀

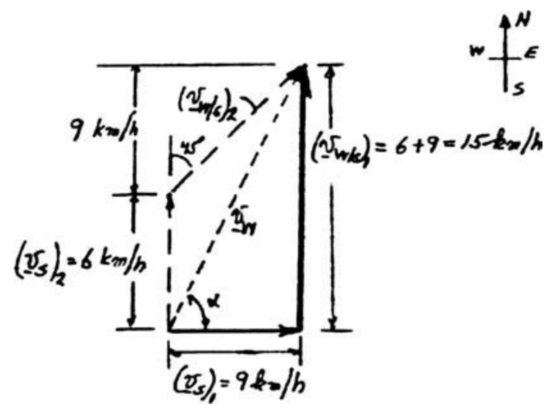
PROBLEM 11.130

As observed from a ship moving due east at 9 km/h, the wind appears to blow from the south. After the ship has changed course and speed, and as it is moving north at 6 km/h, the wind appears to blow from the southwest. Assuming that the wind velocity is constant during the period of observation, determine the magnitude and direction of the true wind velocity.

SOLUTION

$$\mathbf{v}_{\text{wind}} = \mathbf{v}_{\text{ship}} + \mathbf{v}_{\text{wind/ship}}$$

$$\mathbf{v}_w = \mathbf{v}_s + \mathbf{v}_{w/s}$$



Case ①

$$\mathbf{v}_s = 9 \text{ km/h} \rightarrow; \quad \mathbf{v}_{w/s} \uparrow$$

Case ②

$$\mathbf{v}_s = 6 \text{ km/h} \uparrow; \quad \mathbf{v}_{w/s} \nearrow$$

$$\tan \alpha = \frac{15}{9} = 1.6667$$

$$\alpha = 59.0°$$

$$v_w = \sqrt{9^2 + 15^2} = 17.49 \text{ km/h}$$

$$\mathbf{v}_w = 17.49 \text{ km/h} \measuredangle 59.0° \blacktriangleleft$$

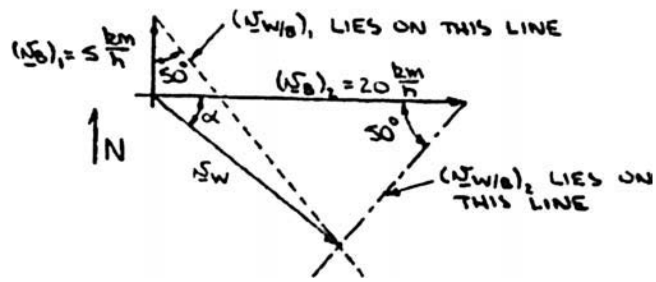
PROBLEM 11.131

When a small boat travels north at 5 km/h, a flag mounted on its stern forms an angle $\theta = 50°$ with the centerline of the boat as shown. A short time later, when the boat travels east at 20 km/h, angle θ is again 50°. Determine the speed and the direction of the wind.

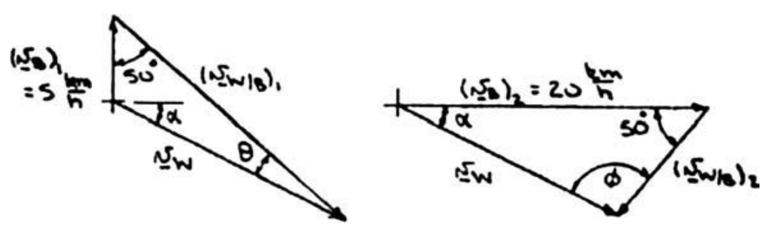
SOLUTION

We have
$$\mathbf{v}_W = \mathbf{v}_B + \mathbf{v}_{W/B}$$

Using this equation, the two cases are then graphically represented as shown.



With $\mathbf{v}_W$ now defined, the above diagram is redrawn for the two cases for clarity.



Noting that

$$\theta = 180° - (50° + 90° + \alpha) \qquad \phi = 180° - (50° + \alpha)$$
$$= 40° - \alpha \qquad\qquad = 130° - \alpha$$

We have
$$\frac{v_W}{\sin 50°} = \frac{5}{\sin(40° - \alpha)} \qquad \frac{v_W}{\sin 50°} = \frac{20}{\sin(130° - \alpha)}$$

Therefore
$$\frac{5}{\sin(40^\circ - \alpha)} = \frac{20}{\sin(130^\circ - \alpha)}$$

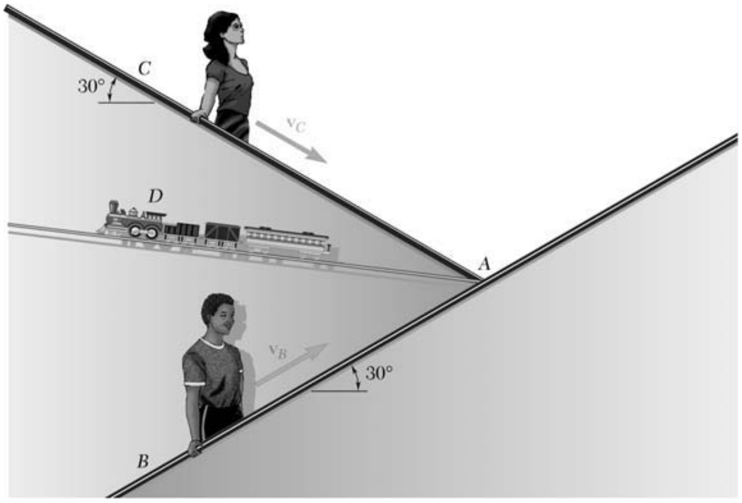
or
$$\sin 130^\circ \cos\alpha - \cos 130^\circ \sin\alpha = 4(\sin 40^\circ \cos\alpha - \cos 40^\circ \sin\alpha)$$

or
$$\tan\alpha = \frac{\sin 130^\circ - 4\sin 40^\circ}{\cos 130^\circ - 4\cos 40^\circ}$$

or
$$\alpha = 25.964^\circ$$

Then
$$v_W = \frac{5\sin 50^\circ}{\sin(40^\circ - 25.964^\circ)} = 15.79 \text{ km/h}$$

$$\mathbf{v}_W = 15.79 \text{ km/h} \;\; 26.0^\circ \blacktriangleleft$$



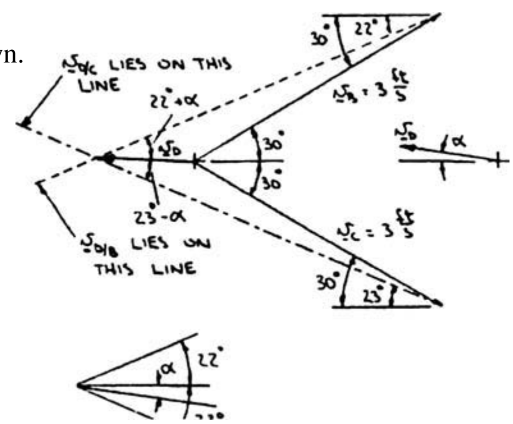
PROBLEM 11.132

As part of a department store display, a model train D runs on a slight incline between the store's up and down escalators. When the train and shoppers pass Point A, the train appears to a shopper on the up escalator B to move downward at an angle of 22° with the horizontal, and to a shopper on the down escalator C to move upward at an angle of 23° with the horizontal and to travel to the left. Knowing that the speed of the escalators is 3 ft/s, determine the speed and the direction of the train.

SOLUTION

We have
$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$
$$\mathbf{v}_D = \mathbf{v}_C + \mathbf{v}_{D/C}$$

The graphical representations of these equations are then as shown.



Then $\dfrac{v_D}{\sin 8°} = \dfrac{3}{\sin (22° + \alpha)}$ $\qquad \dfrac{v_D}{\sin 7°} = \dfrac{3}{\sin (23° - \alpha)}$

Equating the expressions for $\dfrac{v_D}{3}$

$$\frac{\sin 8°}{\sin (22° + \alpha)} = \frac{\sin 7°}{\sin (23° - \alpha)}$$

or $\quad \sin 8°(\sin 23° \cos \alpha - \cos 23° \sin \alpha)$
$$= \sin 7°(\sin 22° \cos \alpha + \cos 22° \sin \alpha)$$

or
$$\tan \alpha = \frac{\sin 8° \sin 23° - \sin 7° \sin 22°}{\sin 8° \cos 23° + \sin 7° \cos 22°}$$

or
$$\alpha = 2.0728°$$

Then
$$v_D = \frac{3 \sin 8°}{\sin (22° + 2.0728°)} = 1.024 \text{ ft/s}$$

$\mathbf{v}_D = 1.024$ ft/s ⦪ 2.07° ◀

PROBLEM 11.132 (Continued)

Alternate solution using components.

$$\mathbf{v}_B = (3\text{ ft/s}) \angle 30° = (2.5981\text{ ft/s})\mathbf{i} + (1.5\text{ ft/s})\mathbf{j}$$

$$\mathbf{v}_C = (3\text{ ft/s}) \angle 30° = (2.5981\text{ ft/s})\mathbf{i} - (1.5\text{ ft/s})\mathbf{j}$$

$$\mathbf{v}_{D/B} = u_1 \angle 22° = -(u_1 \cos 22°)\mathbf{i} - (u_1 \sin 22°)\mathbf{j}$$

$$\mathbf{v}_{D/C} = u_2 \angle 23° = -(u_2 \cos 23°)\mathbf{i} + (u_2 \sin 23°)\mathbf{j}$$

$$\mathbf{v}_D = v_D \angle \alpha = -(v_D \cos \alpha)\mathbf{i} + (v_D \sin \alpha)\mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = \mathbf{v}_C + \mathbf{v}_{D/C}$$

$$2.5981\mathbf{i} + 1.5\mathbf{j} - (u_1 \cos 22°)\mathbf{i} - (u_1 \sin 22°)\mathbf{j} = 2.5981\mathbf{i} - 1.5\mathbf{j} - (u_2 \cos 23°)\mathbf{i} + (u_2 \sin 23°)\mathbf{j}$$

Separate into components, transpose, and change signs.

$$u_1 \cos 22° - u_2 \cos 23° = 0$$

$$u_1 \sin 22° + u_1 \sin 23° = 3$$

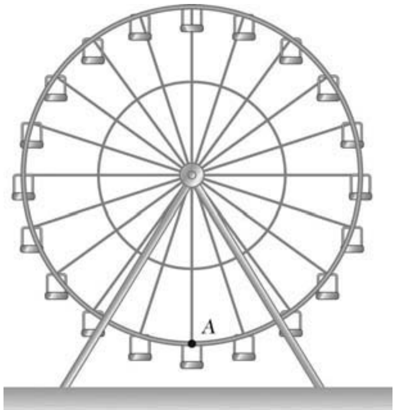
Solving for u_1 and u_2, $\quad u_1 = 3.9054\text{ ft/s} \qquad u_2 = 3.9337\text{ ft/s}$

$$\mathbf{v}_D = 2.5981\mathbf{i} + 1.5\mathbf{j} - (3.9054 \cos 22°)\mathbf{i} - (3.9054 \sin 22°)\mathbf{j}$$
$$= -(1.0229\text{ ft/s})\mathbf{i} + (0.0370\text{ ft/s})\mathbf{j}$$

or

$$\mathbf{v}_D = 2.5981\mathbf{i} - 1.5\mathbf{j} - (3.9337 \cos 23°)\mathbf{i} + (3.9337 \sin 23°)\mathbf{j}$$
$$= -(1.0229\text{ ft/s})\mathbf{i} + (0.0370\text{ ft/s})\mathbf{j}$$

$$\mathbf{v}_D = 1.024\text{ ft/s} \angle 2.07° \blacktriangleleft$$



PROBLEM 11.CQ8

The Ferris wheel is rotating with a constant angular velocity ω. What is the direction of the acceleration of Point A?

(*a*) →

(*b*) ↑

(*c*) ↓

(*d*) ←

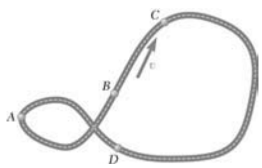
(*e*) The acceleration is zero.

SOLUTION

The tangential acceleration is zero since the speed is constant, so there will only be normal acceleration pointed upwards.

Answer: (*b*) ◀

PROBLEM 11.CQ9



A racecar travels around the track shown at a constant speed. At which point will the racecar have the largest acceleration?

(*a*) A

(*b*) B

(*c*) C

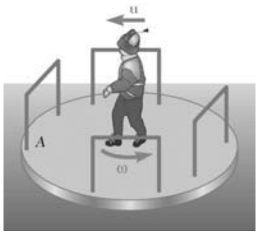
(*d*) The acceleration will be zero at all the points.

SOLUTION

The tangential acceleration is zero since the speed is constant, so there will only be normal acceleration. The normal acceleration will be maximum where the radius of curvature is a minimum, that is at Point A.

Answer: (*a*) ◀

PROBLEM 11.CQ10



A child walks across merry-go-round A with a constant speed u relative to A. The merry-go-round undergoes fixed axis rotation about its center with a constant angular velocity ω counterclockwise.When the child is at the center of A, as shown, what is the direction of his acceleration when viewed from above.

(*a*) →

(*b*) ←

(*c*) ↑

(*d*) ↓

(*e*) The acceleration is zero.

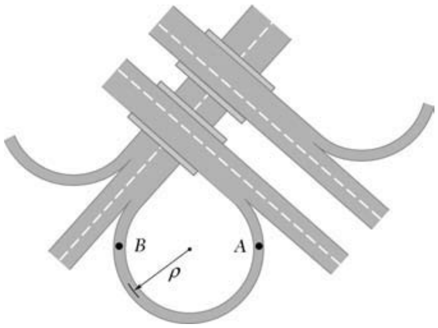
SOLUTION

Polar coordinates are most natural for this problem, that is,

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta \tag{1}$$

From the information given, we know $\ddot{r} = 0,\ \ddot{\theta} = 0,\ r = 0,\ \dot{\theta} = \omega,\ \dot{r} = -u.$ When we substitute these values into (1), we will only have a term in the $-\theta$ direction.

Answer: (*d*) ◀



PROBLEM 11.133

Determine the smallest radius that should be used for a highway if the normal component of the acceleration of a car traveling at 72 km/h is not to exceed $0.8\ \text{m/s}^2$.

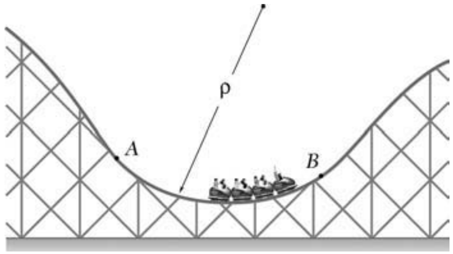
SOLUTION

$$a_n = \frac{v^2}{\rho} \qquad a_n = 0.8\ \text{m/s}^2$$

$$v = 72\ \text{km/h} = 20\ \text{m/s}$$

$$0.8\ \text{m/s}^2 = \frac{(20\ \text{m/s})^2}{\rho}$$

$\rho = 500\ \text{m}$ ◀



PROBLEM 11.134

Determine the maximum speed that the cars of the roller-coaster can reach along the circular portion AB of the track if ρ is 25 m and the normal component of their acceleration cannot exceed 3 g.

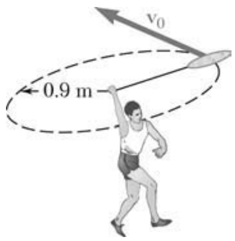
SOLUTION

We have $$a_n = \frac{v^2}{\rho}$$

Then $$(v_{\max})_{AB}^2 = (3 \times 9.81 \text{ m/s}^2)(25 \text{ m})$$

or $$(v_{\max})_{AB} = 27.124 \text{ m/s}$$

or $$(v_{\max})_{AB} = 97.6 \text{ km/h} \blacktriangleleft$$



PROBLEM 11.135

A bull-roarer is a piece of wood that produces a roaring sound when attached to the end of a string and whirled around in a circle. Determine the magnitude of the normal acceleration of a bull-roarer when it is spun in a circle of radius 0.9 m at a speed of 20 m/s.

SOLUTION

$$a_n = \frac{v^2}{\rho} = \frac{(20 \text{ m/s})^2}{0.9 \text{ m}} = 444.4 \text{ m/s}^2$$

$$a_n = 444 \text{ m/s}^2 \blacktriangleleft$$

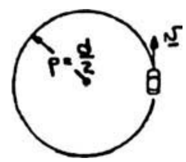
PROBLEM 11.136

To test its performance, an automobile is driven around a circular test track of diameter *d*. Determine (*a*) the value of *d* if when the speed of the automobile is 45 mi/h, the normal component of the acceleration is 11 ft/s^2, (*b*) the speed of the automobile if $d = 600 \text{ ft}$ and the normal component of the acceleration is measured to be 0.6 g.

SOLUTION

(*a*) First note $v = 45 \text{ mi/h} = 66 \text{ ft/s}$

Now $a_n = \dfrac{v^2}{\rho}$



$$\rho = \frac{(66 \text{ ft/s})^2}{11 \text{ ft/s}^2} = 396 \text{ ft}$$

$$d = 2\rho$$

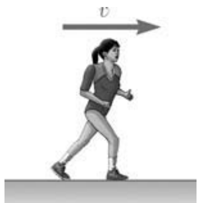
$d = 792 \text{ ft}$ ◀

(*b*) We have $a_n = \dfrac{v^2}{\rho}$

Then $v^2 = (0.6 \times 32.2 \text{ ft/s}^2)\left(\dfrac{1}{2} \times 600 \text{ ft}\right)$

$$v = 76.131 \text{ ft/s}$$

$v = 51.9 \text{ mi/h}$ ◀

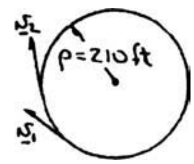


PROBLEM 11.137

An outdoor track is 420 ft in diameter. A runner increases her speed at a constant rate from 14 to 24 ft/s over a distance of 95 ft. Determine the magnitude of the total acceleration of the runner 2 s after she begins to increase her speed.

SOLUTION

We have uniformly accelerated motion



$$v_2^2 = v_1^2 + 2a_t \Delta s_{12}$$

Substituting $$(24 \text{ ft/s})^2 = (14 \text{ ft/s})^2 + 2a_t (95 \text{ ft})$$

or $$a_t = 2 \text{ ft/s}^2$$

Also $$v = v_1 + a_t t$$

At $t = 2$ s: $$v = 14 \text{ ft/s} + (2 \text{ ft/s}^2)(2 \text{ s}) = 18 \text{ ft/s}$$

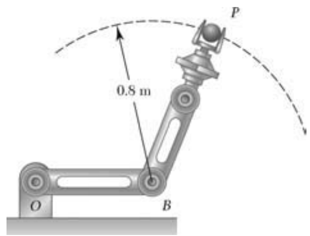
Now $$a_n = \frac{v^2}{\rho}$$

At $t = 2$ s: $$a_n = \frac{(18 \text{ ft/s})^2}{210 \text{ ft}} = 1.54286 \text{ ft/s}^2$$

Finally $$a^2 = a_t^2 + a_n^2$$

At $t = 2$ s: $$a^2 = 2^2 + 1.54286^2$$

or $$a = 2.53 \text{ ft/s}^2 \blacktriangleleft$$



PROBLEM 11.138

A robot arm moves so that P travels in a circle about Point B, which is not moving. Knowing that P starts from rest, and its speed increases at a constant rate of 10 mm/s^2, determine (*a*) the magnitude of the acceleration when $t = 4$ s, (*b*) the time for the magnitude of the acceleration to be 80 mm/s^2.

SOLUTION

Tangential acceleration: $a_t = 10 \text{ mm/s}^2$

Speed: $v = a_t t$

Normal acceleration: $a_n = \dfrac{v^2}{\rho} = \dfrac{a_t^2 t^2}{\rho}$

where $\rho = 0.8 \text{ m} = 800 \text{ mm}$

(*a*) When $t = 4$ s $\quad v = (10)(4) = 40 \text{ mm/s}$

$$a_n = \frac{(40)^2}{800} = 2 \text{ mm/s}^2$$

Acceleration: $a = \sqrt{a_t^2 + a_n^2} = \sqrt{(10)^2 + (2)^2}$

$$a = 10.20 \text{ mm/s}^2 \blacktriangleleft$$

(*b*) Time when $a = 80 \text{ mm/s}^2$

$$a^2 = a_n^2 + a_t^2$$

$$(80)^2 = \left[\frac{(10)^2 t^2}{800}\right]^2 + 10^2 \qquad t^4 = 403200 \text{ s}^4$$

$$t = 25.2 \text{ s} \blacktriangleleft$$

PROBLEM 11.139

A monorail train starts from rest on a curve of radius 400 m and accelerates at the constant rate a_t. If the maximum total acceleration of the train must not exceed 1.5 m/s^2, determine (*a*) the shortest distance in which the train can reach a speed of 72 km/h, (*b*) the corresponding constant rate of acceleration a_t.

SOLUTION

When $v = 72 \text{ km/h} = 20 \text{ m/s}$ and $\rho = 400 \text{ m}$,

$$a_n = \frac{v^2}{\rho} = \frac{(20)^2}{400} = 1.000 \text{ m/s}^2$$

But
$$a = \sqrt{a_n^2 + a_t^2}$$

$$a_t = \sqrt{a^2 - a_n^2} = \sqrt{(1.5)^2 - (1.000)^2} = \pm 1.11803 \text{ m/s}^2$$

Since the train is accelerating, reject the negative value.

(*a*) Distance to reach the speed.

$$v_0 = 0$$

Let
$$x_0 = 0$$

$$v_1^2 = v_0^2 + 2a_t(x_1 - x_0) = 2a_t x_1$$

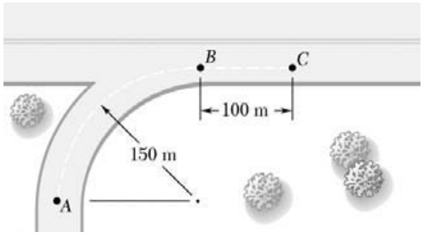
$$x_1 = \frac{v_1^2}{2a_t} = \frac{(20)^2}{(2)(1.11803)}$$

$x_1 = 178.9 \text{ m}$ ◀

(*b*) Corresponding tangential acceleration.

$a_t = 1.118 \text{ m/s}^2$ ◀

PROBLEM 11.140



A motorist starts from rest at Point A on a circular entrance ramp when $t = 0$, increases the speed of her automobile at a constant rate and enters the highway at Point B. Knowing that her speed continues to increase at the same rate until it reaches 100 km/h at Point C, determine (a) the speed at Point B, (b) the magnitude of the total acceleration when $t = 20$ s.

SOLUTION

Speeds: $v_0 = 0 \qquad v_1 = 100 \text{ km/h} = 27.78 \text{ m/s}$

Distance: $$s = \frac{\pi}{2}(150) + 100 = 335.6 \text{ m}$$

Tangential component of acceleration: $$v_1^2 = v_0^2 + 2a_t s$$

$$a_t = \frac{v_1^2 - v_0^2}{2s} = \frac{(27.78)^2 - 0}{(2)(335.6)} = 1.1495 \text{ m/s}^2$$

At Point B, $\quad v_B^2 = v_0^2 + 2a_t s_B \quad$ where $\quad s_B = \dfrac{\pi}{2}(150) = 235.6 \text{ m}$

$$v_B^2 = 0 + (2)(1.1495)(235.6) = 541.69 \text{ m}^2/\text{s}^2$$

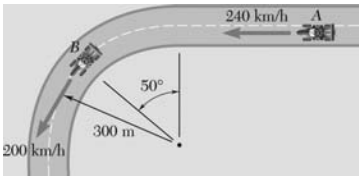
$$v_B = 23.27 \text{ m/s}$$ $\qquad v_B = 83.8 \text{ km/h}$ ◀

(a) At $t = 20$ s, $\quad v = v_0 + a_t t = 0 + (1.1495)(20) = 22.99 \text{ m/s}$

Since $v < v_B$, the car is still on the curve. $\quad \rho = 150 \text{ m}$

Normal component of acceleration: $$a_n = \frac{v^2}{\rho} = \frac{(22.99)^2}{150} = 3.524 \text{ m/s}^2$$

(b) Magnitude of total acceleration: $\quad |a| = \sqrt{a_t^2 + a_n^2} = \sqrt{(1.1495)^2 + (3.524)^2} \qquad |a| = 3.71 \text{ m/s}^2$ ◀



PROBLEM 11.141

Racecar A is traveling on a straight portion of the track while racecar B is traveling on a circular portion of the track. At the instant shown, the speed of A is increasing at the rate of 10 m/s^2, and the speed of B is decreasing at the rate of 6 m/s^2. For the position shown, determine (*a*) the velocity of B relative to A, (*b*) the acceleration of B relative to A.

SOLUTION

Speeds:
$$v_A = 240 \text{ km/h} = 66.67 \text{ m/s}$$
$$v_B = 200 \text{ km/h} = 55.56 \text{ m/s}$$

Velocities:
$$\mathbf{v}_A = 66.67 \text{ m/s} \longleftarrow$$
$$\mathbf{v}_B = 55.56 \text{ m/s} \measuredangle 50^\circ$$

(*a*) Relative velocity:
$$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A$$
$$\mathbf{v}_{B/A} = (55.56\cos 50^\circ) \leftarrow + 55.56\sin 50^\circ \downarrow + 66.67 \longrightarrow$$
$$= 30.96 \rightarrow + 42.56 \downarrow$$
$$= 52.63 \text{ m/s} \searrow 53.96^\circ$$

$$\mathbf{v}_{B/A} = 189.5 \text{ km/h} \searrow 54.0^\circ \blacktriangleleft$$

Tangential accelerations:
$$(\mathbf{a}_A)_t = 10 \text{ m/s}^2 \longleftarrow$$
$$(\mathbf{a}_B)_t = 6 \text{ m/s}^2 \measuredangle 50^\circ$$

Normal accelerations:
$$a_n = \frac{v^2}{\rho}$$

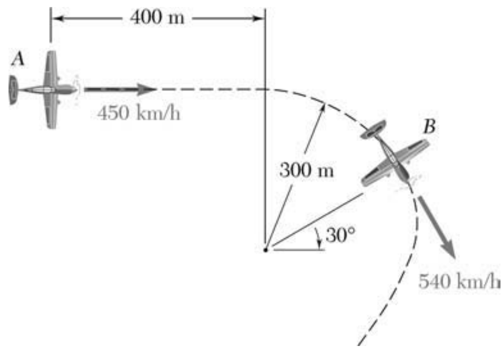
Car A: $(\rho = \infty)$ $\quad (\mathbf{a}_A)_n = 0$

Car B: $(\rho = 300 \text{ m})$

$$(\mathbf{a}_B)_n = \frac{(55.56)^2}{300} = 10.288 \qquad (\mathbf{a}_B)_n = 10.288 \text{ m/s}^2 \searrow 40^\circ$$

(*b*) Acceleration of B relative to A:
$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$$
$$\mathbf{a}_{B/A} = (\mathbf{a}_B)_t + (\mathbf{a}_B)_n - (\mathbf{a}_A)_t - (\mathbf{a}_A)_n$$
$$= 6 \measuredangle 50^\circ + 10.288 \searrow 40^\circ + 10 \rightarrow + 0$$
$$= (6\cos 50^\circ + 10.288\cos 40^\circ + 10) \longrightarrow$$
$$+ (6\sin 50^\circ - 10.288\sin 40^\circ) \uparrow$$
$$= 21.738 \rightarrow + 2.017 \downarrow \qquad \mathbf{a}_{B/A} = 21.8 \text{ m/s}^2 \searrow 5.3^\circ \blacktriangleleft$$

PROBLEM 11.142



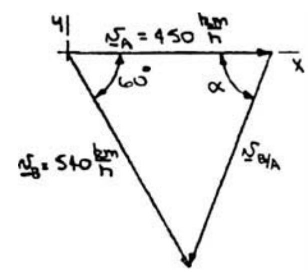
At a given instant in an airplane race, airplane A is flying horizontally in a straight line, and its speed is being increased at the rate of 8 m/s^2. Airplane B is flying at the same altitude as airplane A and, as it rounds a pylon, is following a circular path of 300-m radius. Knowing that at the given instant the speed of B is being decreased at the rate of 3 m/s^2, determine, for the positions shown, (a) the velocity of B relative to A, (b) the acceleration of B relative to A.

SOLUTION

First note $\quad v_A = 450 \text{ km/h} \quad v_B = 540 \text{ km/h} = 150 \text{ m/s}$

(a) We have $\quad \mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

The graphical representation of this equation is then as shown.



We have $\quad v_{B/A}^2 = 450^2 + 540^2 - 2(450)(540)\cos 60°$

$$v_{B/A} = 501.10 \text{ km/h}$$

and $\quad \dfrac{540}{\sin\alpha} = \dfrac{501.10}{\sin 60°}$

$$\alpha = 68.9°$$

$$\mathbf{v}_{B/A} = 501 \text{ km/h} \nearrow 68.9° \blacktriangleleft$$

(b) First note $\quad \mathbf{a}_A = 8 \text{ m/s}^2 \longrightarrow \quad (\mathbf{a}_B)_t = 3 \text{ m/s}^2 \searrow 60°$

Now $\quad (a_B)_n = \dfrac{v_B^2}{\rho_B} = \dfrac{(150 \text{ m/s})^2}{300 \text{ m}}$

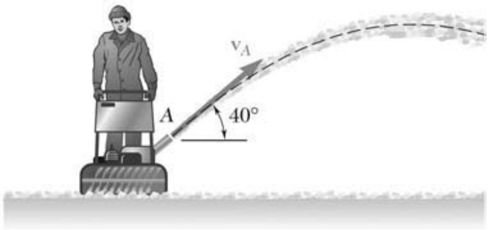
$$(\mathbf{a}_B)_n = 75 \text{ m/s}^2 \nearrow 30°$$

Then
$$\mathbf{a}_B = (\mathbf{a}_B)_t + (\mathbf{a}_B)_n$$
$$= 3(-\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j}) + 75(-\cos 30°\,\mathbf{i} - \sin 30°\,\mathbf{j})$$
$$= -(66.452 \text{ m/s}^2)\mathbf{i} - (34.902 \text{ m/s}^2)\mathbf{j}$$

Finally $\quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

$$\mathbf{a}_{B/A} = (-66.452\mathbf{i} - 34.902\mathbf{j}) - (8\mathbf{i})$$
$$= -(74.452 \text{ m/s}^2)\mathbf{i} - (34.902 \text{ m/s}^2)\mathbf{j}$$

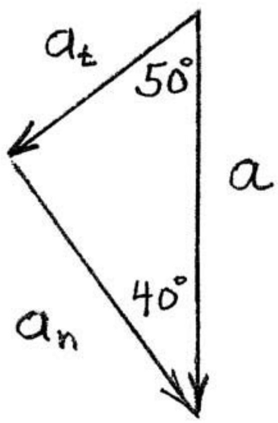
$$\mathbf{a}_{B/A} = 82.2 \text{ m/s}^2 \nearrow 25.1° \blacktriangleleft$$



PROBLEM 11.143

From a photograph of a homeowner using a snowblower, it is determined that the radius of curvature of the trajectory of the snow was 30 ft as the snow left the discharge chute at A. Determine (*a*) the discharge velocity $\mathbf{v}_A$ of the snow, (*b*) the radius of curvature of the trajectory at its maximum height.

SOLUTION



(*a*) The acceleration vector is $32.2 \text{ ft/s}\downarrow$.

At Point A, tangential and normal components of **a** are as shown in the sketch.

$$a_n = a\cos 40° = 32.2\cos 40° = 24.67 \text{ ft/s}^2$$

$$v_A^2 = \rho_A (a_A)_n = (30)(24.67) = 740.0 \text{ ft}^2/\text{s}^2$$

$$\mathbf{v}_A = 27.2 \text{ ft/s} \measuredangle\ 40° \blacktriangleleft$$

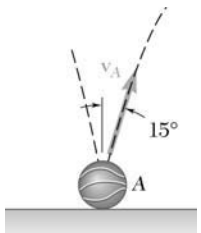
$$v_x = 27.20\cos 40° = 20.84 \text{ ft/s}$$

(*b*) At maximum height, $v = v_x = 20.84$ ft/s

$$a_n = g = 32.2 \text{ ft/s}^2,$$

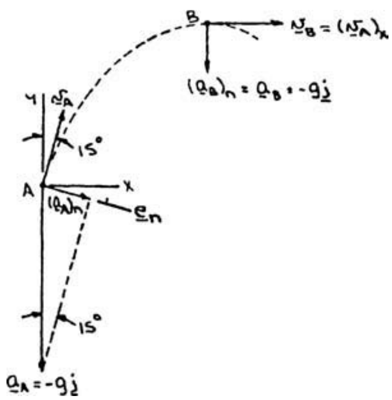
$$\rho = \frac{v^2}{a_n} = \frac{(20.84)^2}{32.2} \qquad \rho = 13.48 \text{ ft} \blacktriangleleft$$

PROBLEM 11.144



A basketball is bounced on the ground at Point A and rebounds with a velocity $\mathbf{v}_A$ of magnitude 2.5 m/s as shown. Determine the radius of curvature of the trajectory described by the ball (*a*) at Point A, (*b*) at the highest point of the trajectory.

SOLUTION



(*a*) We have $(a_A)_n = \dfrac{v_A^2}{\rho_A}$

or $\rho_A = \dfrac{(2.5 \text{ m/s})^2}{(9.81 \text{ m/s}^2) \sin 15°}$

or $\rho_A = 2.46 \text{ m}$ ◀

(*b*) We have $(a_B)_n = \dfrac{v_B^2}{\rho_B}$

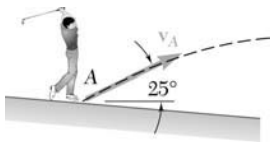
where Point B is the highest point of the trajectory, so that

$$v_B = (v_A)_x = v_A \sin 15°$$

Then $\rho_B = \dfrac{[(2.5 \text{ m/s}) \sin 15°]^2}{9.81 \text{ m/s}^2} = 0.0427 \text{ m}$

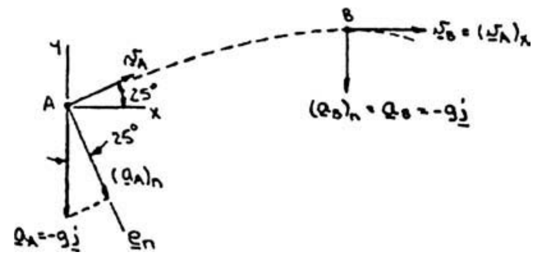
or $\rho_B = 42.7 \text{ mm}$ ◀

PROBLEM 11.145



A golfer hits a golf ball from Point A with an initial velocity of 50 m/s at an angle of 25° with the horizontal. Determine the radius of curvature of the trajectory described by the ball (a) at Point A, (b) at the highest point of the trajectory.

SOLUTION



(a) We have $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$\rho_A = \frac{(50 \text{ m/s})^2}{(9.81 \text{ m/s}^2)\cos 25°}$$

or $$\rho_A = 281 \text{ m} \blacktriangleleft$$

(b) We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

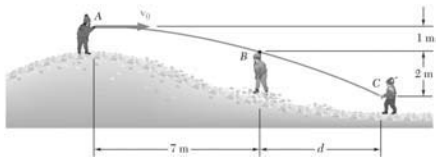
where Point B is the highest point of the trajectory, so that

$$v_B = (v_A)_x = v_A \cos 25°$$

Then $$\rho_B = \frac{[(50 \text{ m/s}) \cos 25°]^2}{9.81 \text{ m/s}^2}$$

or $$\rho_B = 209 \text{ m} \blacktriangleleft$$

PROBLEM 11.146



Three children are throwing snowballs at each other. Child *A* throws a snowball with a horizontal velocity $\mathbf{v}_0$. If the snowball just passes over the head of child *B* and hits child C, determine the radius of curvature of the trajectory described by the snowball (*a*) at Point *B*, (*b*) at Point *C*.

SOLUTION

The motion is projectile motion. Place the origin at Point *A*.

Horizontal motion: $\mathbf{v}_x = v_0 \qquad x = v_0 t$

Vertical motion: $y_0 = 0, \qquad (v_y) = 0$

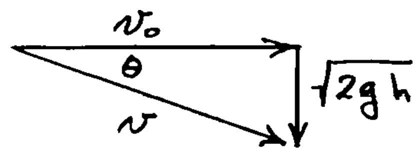
$$v_y = -gt \qquad y = -\frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}}, \qquad \text{where } h \text{ is the vertical distance fallen.}$$

$$|v_y| = \sqrt{2gh}$$

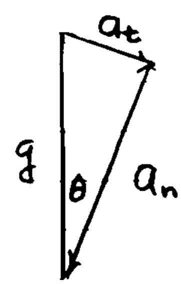
Speed: $v^2 = v_x^2 + v_y^2 = v_0^2 + 2gh$

Direction of velocity.



$$\cos\theta = \frac{v_0}{v}$$

Direction of normal acceleration.



$$a_n = g\cos\theta = \frac{gv_0}{v} = \frac{v^2}{\rho}$$

Radius of curvature: $\rho = \dfrac{v^3}{gv_0}$

At Point *B*, $h_B = 1 \text{ m}; \quad x_B = 7 \text{ m}$

$$t_B = \sqrt{\frac{(2)(1 \text{ m})}{9.81 \text{ m/s}^2}} = 0.45152 \text{ s}$$

$$x_B = v_0 t_B \qquad v_0 = \frac{x_B}{t_B} = \frac{7 \text{ m}}{0.45152 \text{ s}} = 15.504 \text{ m/s}$$

$$v_B^2 = (15.504)^2 + (2)(9.81)(1) = 259.97 \text{ m}^2/\text{s}^2$$

PROBLEM 11.146 (Continued)

(*a*) <u>Radius of curvature at Point *B*.</u>

$$\rho_B = \frac{(259.97\ \text{m}^2/\text{s}^2)^{3/2}}{(9.81\ \text{m/s}^2)(15.504\ \text{m/s})}$$

$\rho_B = 27.6\ \text{m}$ ◀

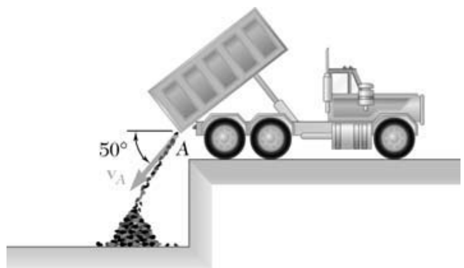
At Point *C* $\quad h_C = 1\ \text{m} + 2\ \text{m} = 3\ \text{m}$

$$v_C^2 = (15.504)^2 + (2)(9.81)(3) = 299.23\ \text{m}^2/\text{s}^2$$

(*b*) <u>Radius of curvature at Point *C*.</u>

$$\rho_C = \frac{(299.23\ \text{m}^2/\text{s}^2)^{3/2}}{(9.81\ \text{m/s}^2)(15.504\ \text{m/s})}$$

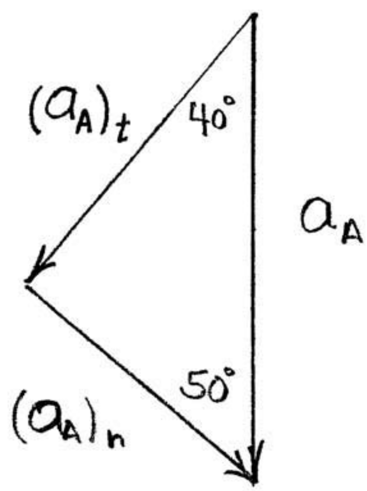
$\rho_C = 34.0\ \text{m}$ ◀



PROBLEM 11.147

Coal is discharged from the tailgate A of a dump truck with an initial velocity $\mathbf{v}_A = 2$ m/s ↙ 50°. Determine the radius of curvature of the trajectory described by the coal (*a*) at Point A, (*b*) at the point of the trajectory 1 m below Point A.

SOLUTION



(*a*) At Point A. $a_A = g \downarrow = 9.81 \text{ m/s}^2 \downarrow$

Sketch tangential and normal components of acceleration at A.

$$(a_A)_n = g\cos 50°$$

$$\rho_A = \frac{v_A{}^2}{(a_A)_n} = \frac{(2)^2}{9.81\cos 50°} \qquad \rho_A = 0.634 \text{ m} \blacktriangleleft$$

(*b*) At Point B, 1 meter below Point A.

Horizontal motion: $(v_B)_x = (v_A)_x = 2\cos 50° = 1.286 \text{ m/s} \leftarrow$

Vertical motion: $(v_B)_y^2 = (v_A)_y^2 + 2a_y(y_B - y_A)$

$$= (2\cos 40°)^2 + (2)(-9.81)(-1)$$
$$= 21.97 \text{ m}^2/\text{s}^2$$

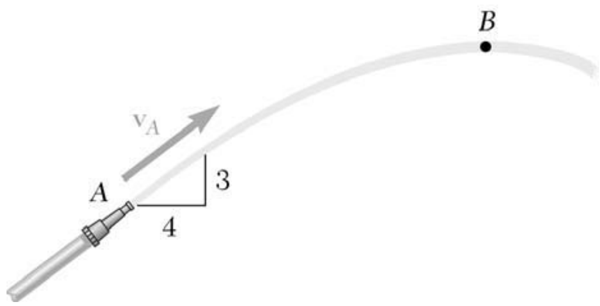
$$(v_B)_y = 4.687 \text{ m/s} \downarrow$$

$$\tan\theta = \frac{(v_B)_y}{(v_B)_x} = \frac{4.687}{1.286}, \qquad \text{or} \qquad \theta = 74.6°$$

$$a_B = g\cos 74.6°$$

$$\rho_B = \frac{v_B{}^2}{(a_B)_n} = \frac{(v_B)_x^2 + (v_B)_y^2}{g\cos 74.6°}$$

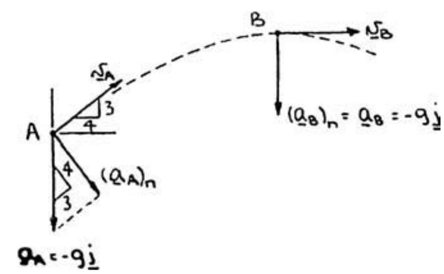
$$= \frac{(1.286)^2 + 21.97}{9.81\cos 74.6°} \qquad \rho_B = 9.07 \text{ m} \blacktriangleleft$$



PROBLEM 11.148

From measurements of a photograph, it has been found that as the stream of water shown left the nozzle at A, it had a radius of curvature of 25 m. Determine (a) the initial velocity $\mathbf{v}_A$ of the stream, (b) the radius of curvature of the stream as it reaches its maximum height at B.

SOLUTION



(a) We have $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$v_A^2 = \left[\frac{4}{5}(9.81 \text{ m/s}^2)\right](25 \text{ m})$$

or $$v_A = 14.0071 \text{ m/s}$$

$$\mathbf{v}_A = 14.01 \text{ m/s} \angle 36.9° \blacktriangleleft$$

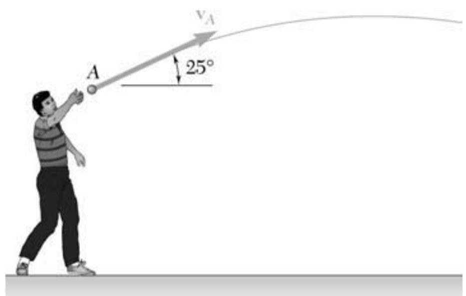
(b) We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

Where $$v_B = (v_A)_x = \frac{4}{5}v_A$$

Then $$\rho_B = \frac{\left(\frac{4}{5}\times 14.0071 \text{ m/s}\right)^2}{9.81 \text{ m/s}^2}$$

or $$\rho_B = 12.80 \text{ m} \blacktriangleleft$$

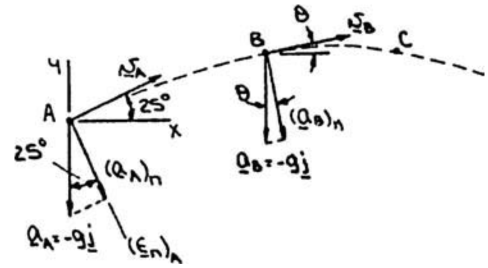
PROBLEM 11.149



A child throws a ball from Point A with an initial velocity $\mathbf{v}_A$ of 20 m/s at an angle of 25° with the horizontal. Determine the velocity of the ball at the points of the trajectory described by the ball where the radius of curvature is equal to three-quarters of its value at A.

SOLUTION

Assume that Points B and C are the points of interest, where $y_B = y_C$ and $v_B = v_C$.



Now $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$\rho_A = \frac{v_A^2}{g\cos 25°}$$

Then $$\rho_B = \frac{3}{4}\rho_A = \frac{3}{4}\frac{v_A^2}{g\cos 25°}$$

We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

where $$(a_B)_n = g\cos\theta$$

so that $$\frac{3}{4}\frac{v_A^2}{g\cos 25°} = \frac{v_B^2}{g\cos\theta}$$

or $$v_B^2 = \frac{3}{4}\frac{\cos\theta}{\cos 25°}v_A^2 \qquad (1)$$

Noting that the horizontal motion is uniform, we have

$$(v_A)_x = (v_B)_x$$

where $$(v_A)_x = v_A\cos 25° \qquad (v_B)_x = v_B\cos\theta$$

Then $$v_A\cos 25° = v_B\cos\theta$$

or $$\cos\theta = \frac{v_A}{v_B}\cos 25°$$

PROBLEM 11.149 (Continued)

Substituting for $\cos\theta$ in Eq. (1), we have

$$v_B^2 = \frac{3}{4}\left(\frac{v_A}{v_B}\cos 25°\right)\frac{v_A^2}{\cos 25°}$$

or $$v_B^3 = \frac{3}{4}v_A^3$$

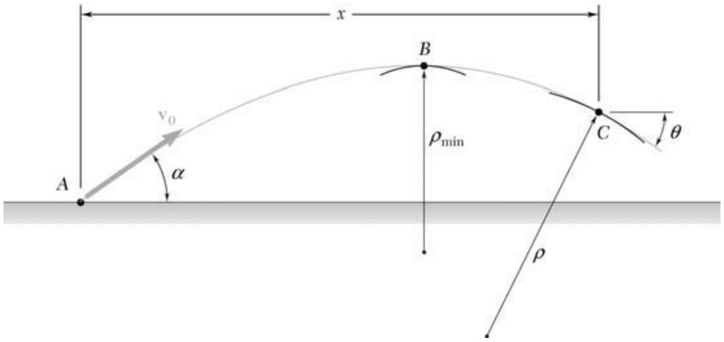
$$v_B = \sqrt[3]{\frac{3}{4}}v_A = 18.17 \text{ m/s}$$

$$\cos\theta = \sqrt[3]{\frac{4}{3}}\cos 25°$$

$$\theta = \pm 4.04°$$

$\mathbf{v}_B = 18.17$ m/s ∠ 4.04° ◀

and $\mathbf{v}_B = 18.17$ m/s ∖ 4.04° ◀

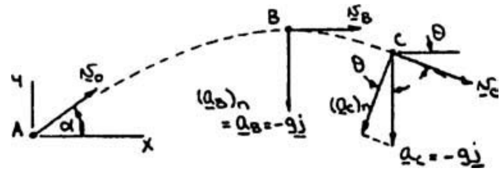


PROBLEM 11.150

A projectile is fired from Point A with an initial velocity $\mathbf{v}_0$. (a) Show that the radius of curvature of the trajectory of the projectile reaches its minimum value at the highest Point B of the trajectory. (b) Denoting by θ the angle formed by the trajectory and the horizontal at a given Point C, show that the radius of curvature of the trajectory at C is $\rho = \rho_{min}/\cos^3\theta$.

SOLUTION

For the arbitrary Point C, we have



$$(a_C)_n = \frac{v_C^2}{\rho_C}$$

or
$$\rho_C = \frac{v_C^2}{g\cos\theta}$$

Noting that the horizontal motion is uniform, we have

$$(v_A)_x = (v_C)_x$$

where
$$(v_A)_x = v_0\cos\alpha \qquad (v_C)_x = v_C\cos\theta$$

Then
$$v_0\cos\alpha = v_C\cos\theta$$

or
$$v_C = \frac{\cos\alpha}{\cos\theta}v_0$$

so that
$$\rho_C = \frac{1}{g\cos\theta}\left(\frac{\cos\alpha}{\cos\theta}v_0\right)^2 = \frac{v_0^2\cos^2\alpha}{g\cos^3\theta}$$

(a) In the expression for ρ_C, v_0, α, and g are constants, so that ρ_C is minimum where $\cos\theta$ is maximum. By observation, this occurs at Point B where $\theta = 0$.

$$\rho_{min} = \rho_B = \frac{v_0^2\cos^2\alpha}{g}$$ Q.E.D. ◀

(b)
$$\rho_C = \frac{1}{\cos^3\theta}\left(\frac{v_0^2\cos^2\alpha}{g}\right)$$

$$\rho_C = \frac{\rho_{min}}{\cos^3\theta}$$ Q.E.D. ◀

PROBLEM 11.151*

Determine the radius of curvature of the path described by the particle of Problem 11.95 when $t = 0$.

PROBLEM 11.95 The three-dimensional motion of a particle is defined by the position vector $\mathbf{r} = (Rt\cos\omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt\sin\omega_n t)\mathbf{k}$. Determine the magnitudes of the velocity and acceleration of the particle. (The space curve described by the particle is a conic helix.)

SOLUTION

We have $$\mathbf{v} = \frac{d\mathbf{r}}{dt} = R(\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} + c\mathbf{j} + R(\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}$$

and $$\mathbf{a} = \frac{d\mathbf{v}}{dt} = R\left(-\omega_n \sin\omega_n t - \omega_n \sin\omega_n t - \omega_n^2 t\cos\omega_n t\right)\mathbf{i}$$

$$+ R\left(\omega_n \cos\omega_n t + \omega_n \cos\omega_n t - \omega_n^2 t\sin\omega_n t\right)\mathbf{k}$$

or $$\mathbf{a} = \omega_n R[-(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{i} + (2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{k}]$$

Now $$v^2 = R^2(\cos\omega_n t - \omega_n t\sin\omega_n t)^2 + c^2 + R^2(\sin\omega_n t + \omega_n t\cos\omega_n t)^2$$
$$= R^2\left(1 + \omega_n^2 t^2\right) + c^2$$

Then $$v = \left[R^2\left(1 + \omega_n^2 t^2\right) + c^2\right]^{1/2}$$

and $$\frac{dv}{dt} = \frac{R^2\omega_n^2 t}{\left[R^2\left(1 + \omega_n^2 t^2\right) + c^2\right]^{1/2}}$$

Now $$a^2 = a_t^2 + a_n^2 = \left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2$$

At $t = 0$: $$\frac{dv}{dt} = 0$$
$$\mathbf{a} = \omega_n R(2\mathbf{k}) \quad \text{or} \quad a = 2\omega_n R$$
$$v^2 = R^2 + c^2$$

Then, with $$\frac{dv}{dt} = 0,$$

we have $$a = \frac{v^2}{\rho}$$

or $$2\omega_n R = \frac{R^2 + c^2}{\rho}$$ $$\rho = \frac{R^2 + c^2}{2\omega_n R} \blacktriangleleft$$

PROBLEM 11.152*

Determine the radius of curvature of the path described by the particle of Problem 11.96 when $t = 0$, $A = 3$, and $B = 1$.

SOLUTION

With $\quad A = 3, \quad B = 1$

we have $\quad \mathbf{r} = (3t\cos t)\mathbf{i} + \left(3\sqrt{t^2+1}\right)\mathbf{j} + (t\sin t)\mathbf{k}$

Now
$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 3(\cos t - t\sin t)\mathbf{i} + \left(\frac{3t}{\sqrt{t^2+1}}\right)\mathbf{j} + (\sin t + t\cos t)\mathbf{k}$$

and
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t\cos t)\mathbf{i} + 3\left[\frac{\sqrt{t^2+1} - t\left(\frac{t}{\sqrt{t^2+1}}\right)}{t^2+1}\right]\mathbf{j}$$
$$+ (\cos t + \cos t - t\sin t)\mathbf{k}$$
$$= -3(2\sin t + t\cos t)\mathbf{i} + 3\frac{1}{(t^2+1)^{1/2}}\mathbf{j}$$
$$+ (2\cos t - t\sin t)\mathbf{k}$$

Then
$$v^2 = 9(\cos t - t\sin t)^2 + 9\frac{t^2}{t^2+1} + (\sin t + t\cos t)^2$$

Expanding and simplifying yields
$$v^2 = t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3 + t)\sin 2t$$

Then
$$v = [t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3 + t)\sin 2t]^{1/2}$$

and
$$\frac{dv}{dt} = \frac{4t^3 + 38t + 8(-2\cos t\sin t + 4t^3\sin^2 t + 2t^4\sin t\cos t) - 8[(3t^2+1)\sin 2t + 2(t^3+t)\cos 2t]}{2[t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3+t)\sin 2t]^{1/2}}$$

Now
$$a^2 = a_t^2 + a_n^2 = \left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2$$

PROBLEM 11.152* (Continued)

At $t = 0$: $\mathbf{a} = 3\mathbf{j} + 2\mathbf{k}$

or $a = \sqrt{13}\ \text{ft/s}^2$

$$\frac{dv}{dt} = 0$$

$$v^2 = 9\ (\text{ft/s})^2$$

Then, with $\dfrac{dv}{dt} = 0,$

we have $a = \dfrac{v^2}{\rho}$

or $\rho = \dfrac{9\ \text{ft}^2/\text{s}^2}{\sqrt{13}\ \text{ft/s}^2}$ $\rho = 2.50\ \text{ft}$ ◀

PROBLEM 11.153

A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to $g(R/r)^2$, where g is the acceleration of gravity at the surface of the planet, R is the radius of the planet, and r is the distance from the center of the planet to the satellite. Knowing that the diameter of the sun is 1.39 Gm and that the acceleration of gravity at its surface is $274\ \text{m/s}^2$, determine the radius of the orbit of the indicated planet around the sun assuming that the orbit is circular.

Earth: $(\upsilon_{\text{mean}})_{\text{orbit}} = 107\ \text{Mm/h}$.

SOLUTION

For the sun, $$g = 274\ \text{m/s}^2,$$

and $$R = \frac{1}{2}D = \left(\frac{1}{2}\right)(1.39\times 10^9) = 0.695\times 10^9\ \text{m}$$

Given that $a_n = \dfrac{gR^2}{r^2}$ and that for a circular orbit $a_n = \dfrac{v^2}{r}$

Eliminating a_n and solving for r, $$r = \frac{gR^2}{v^2}$$

For the planet Earth, $$v = 107\times 10^6\ \text{m/h} = 29.72\times 10^3\ \text{m/s}$$

Then $$r = \frac{(274)(0.695\times 10^9)^2}{(29.72\times 10^3)^2} = 149.8\times 10^9\ \text{m}$$

$r = 149.8\ \text{Gm}$ ◀

PROBLEM 11.154

A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to $g(R/r)^2$, where g is the acceleration of gravity at the surface of the planet, R is the radius of the planet, and r is the distance from the center of the planet to the satellite. Knowing that the diameter of the sun is 1.39 Gm and that the acceleration of gravity at its surface is 274 m/s^2, determine the radius of the orbit of the indicated planet around the sun assuming that the orbit is circular.

Saturn: $(v_{\text{mean}})_{\text{orbit}} = 34.7\text{ Mm/h}$.

SOLUTION

For the sun, $g = 274\text{ m/s}^2$

and $R = \frac{1}{2}D = \left(\frac{1}{2}\right)(1.39\times 10^9) = 0.695\times 10^9\text{ m}$

Given that $a_n = \frac{gR^2}{r^2}$ .and that for a circular orbit: $a_n = \frac{v^2}{r}$

Eliminating a_n and solving for r, $r = \frac{gR^2}{v^2}$

For the planet Saturn, $v = 34.7\times 10^6\text{m/h} = 9.639\times 10^3\text{m/s}$

Then, $r = \frac{(274)(0.695\times 10^9)^2}{(9.639\times 10^3)^2} = 1.425\times 10^{12}\text{ m}$ $r = 1425\text{ Gm}$ ◀

PROBLEM 11.155

Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 100 mi above the surface of the planet. (See information given in Problems 11.153–11.154).

Venus: $g = 29.20 \text{ ft/s}^2$, $R = 3761$ mi.

SOLUTION

From Problems 11.153 and 11.154, $$a_n = \frac{gR^2}{r^2}$$

For a circular orbit, $$a_n = \frac{v^2}{r}$$

Eliminating a_n and solving for v, $$v = R\sqrt{\frac{g}{r}}$$

For Venus, $$g = 29.20 \text{ ft/s}^2$$

$$R = 3761 \text{ mi} = 19.858 \times 10^6 \text{ ft.}$$

$$r = 3761 + 100 = 3861 \text{ mi} = 20.386 \times 10^6 \text{ ft}$$

Then, $$v = 19.858 \times 10^6 \sqrt{\frac{29.20}{20.386 \times 10^6}} = 23.766 \times 10^3 \text{ ft/s}$$

$$v = 16200 \text{ mi/h} \blacktriangleleft$$

PROBLEM 11.156

Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 100 mi above the surface of the planet. (See information given in Problems 11.153–11.154).

Mars: $g = 12.17 \text{ ft/s}^2$, $R = 2102 \text{ mi}$.

SOLUTION

From Problems 11.153 and 11.154,
$$a_n = \frac{gR^2}{r^2}$$

For a circular orbit,
$$a_n = \frac{v^2}{r}$$

Eliminating a_n and solving for v,
$$v = R\sqrt{\frac{g}{r}}$$

For Mars,
$$g = 12.17 \text{ ft/s}^2$$
$$R = 2102 \text{ mi} = 11.0986 \times 10^6 \text{ ft}$$
$$r = 2102 + 100 = 2202 \text{ mi} = 11.6266 \times 10^6 \text{ ft}$$

Then,
$$v = 11.0986 \times 10^6 \sqrt{\frac{12.17}{11.6266 \times 10^6}} = 11.35 \times 10^3 \text{ ft/s}$$

$v = 7740$ mi/h ◀

PROBLEM 11.157

Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 100 mi above the surface of the planet. (See information given in Problems 11.153–11.154).

Jupiter: $g = 75.35 \text{ ft/s}^2$, $R = 44,432$ mi.

SOLUTION

From Problems 11.153 and 11.154, $$a_n = \frac{gR^2}{r^2}$$

For a circular orbit, $$a_n = \frac{v^2}{r}$$

Eliminating a_n and solving for v, $$v = R\sqrt{\frac{g}{r}}$$

For Jupiter, $$g = 75.35 \text{ ft/s}^2$$

$$R = 44432 \text{ mi} = 234.60 \times 10^6 \text{ ft}$$

$$r = 44432 + 100 = 44532 \text{ mi} = 235.13 \times 10^6 \text{ ft}$$

Then, $$v = (234.60 \times 10^6)\sqrt{\frac{75.35}{235.13 \times 10^6}} = 132.8 \times 10^3 \text{ ft/s}$$

$v = 90600$ mi/h ◀

PROBLEM 11.158

A satellite is traveling in a circular orbit around Mars at an altitude of 300 km. After the altitude of the satellite is adjusted, it is found that the time of one orbit has increased by 10 percent. Knowing that the radius of Mars is 3382 km, determine the new altitude of the satellite. (See information given in Problems 11.153–11.155.)

SOLUTION

We have $a_n = g\dfrac{R^2}{r^2}$ and $a_n = \dfrac{v^2}{r}$

Then $g\dfrac{R^2}{r^2} = \dfrac{v^2}{r}$

$$v = R\sqrt{\frac{g}{r}} \quad \text{where} \quad r = R + h$$

The circumference s of a circular orbit is equal to

$$s = 2\pi r$$

Assuming that the speed of the satellite in each orbit is constant, we have

$$s = vt_{\text{orbit}}$$

Substituting for s and v

$$2\pi r = R\sqrt{\frac{g}{r}}\, t_{\text{orbit}}$$

$$t_{\text{orbit}} = \frac{2\pi}{R}\frac{r^{3/2}}{\sqrt{g}}$$

$$= \frac{2\pi}{R}\frac{(R+h)^{3/2}}{\sqrt{g}}$$

Now $(t_{\text{orbit}})_2 = 1.1(t_{\text{orbit}})_1$

$$\frac{2\pi}{R}\frac{(R+h_2)^{3/2}}{\sqrt{g}} = 1.1\frac{2\pi}{R}\frac{(R+h_1)^{3/2}}{\sqrt{g}}$$

$$h_2 = (1.1)^{2/3}(R+h_1) - R$$

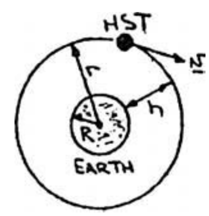
$$= (1.1)^{2/3}(3382 + 300)\text{ km} - (3382\text{ km})$$

$h_2 = 542$ km ◀

PROBLEM 11.159

Knowing that the radius of the earth is 6370 km, determine the time of one orbit of the Hubble Space Telescope, knowing that the telescope travels in a circular orbit 590 km above the surface of the earth. (See information given in Problems 11.153–11.155.)

SOLUTION



We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$

Then $$g\frac{R^2}{r^2} = \frac{v^2}{r}$$

or $$v = R\sqrt{\frac{g}{r}} \quad \text{where} \quad r = R + h$$

The circumference s of the circular orbit is equal to

$$s = 2\pi r$$

Assuming that the speed of the telescope is constant, we have

$$s = vt_{\text{orbit}}$$

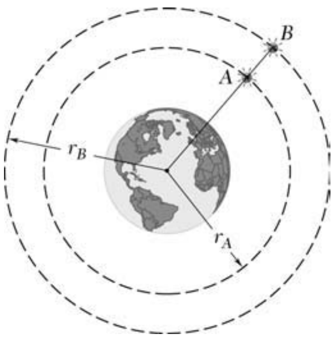
Substituting for s and v

$$2\pi r = R\sqrt{\frac{g}{r}}\, t_{\text{orbit}}$$

or $$t_{\text{orbit}} = \frac{2\pi}{R}\frac{r^{3/2}}{\sqrt{g}}$$

$$= \frac{2\pi}{6370 \text{ km}}\frac{[(6370 + 590) \text{ km}]^{3/2}}{[9.81\times10^{-3} \text{ km/s}^2]^{1/2}} \times \frac{1 \text{ h}}{3600 \text{ s}}$$

or $$t_{\text{orbit}} = 1.606 \text{ h} \blacktriangleleft$$



PROBLEM 11.160

Satellites A and B are traveling in the same plane in circular orbits around the earth at altitudes of 120 and 200 mi, respectively. If at $t = 0$ the satellites are aligned as shown and knowing that the radius of the earth is $R = 3960$ mi, determine when the satellites will next be radially aligned. (See information given in Problems 11.153–11.155.)

SOLUTION

We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$

Then $$g\frac{R^2}{r^2} = \frac{v^2}{r} \quad \text{or} \quad v = R\sqrt{\frac{g}{r}}$$

where $$r = R + h$$

The circumference s of a circular orbit is

equal to $$s = 2\pi r$$

Assuming that the speeds of the satellites are constant, we have

$$s = vT$$

Substituting for s and v

$$2\pi r = R\sqrt{\frac{g}{r}}T$$

or $$T = \frac{2\pi}{R}\frac{r^{3/2}}{\sqrt{g}} = \frac{2\pi}{R}\frac{(R+h)^{3/2}}{\sqrt{g}}$$

Now $$h_B > h_A \Rightarrow (T)_B > (T)_A$$

Next let time T_C be the time at which the satellites are next radially aligned. Then, if in time T_C satellite B completes N orbits, satellite A must complete $(N+1)$ orbits.

Thus,

$$T_C = N(T)_B = (N+1)(T)_A$$

or $$N\left[\frac{2\pi}{R}\frac{(R+h_B)^{3/2}}{\sqrt{g}}\right] = (N+1)\left[\frac{2\pi}{R}\frac{(R+h_A)^{3/2}}{\sqrt{g}}\right]$$

or
$$N = \frac{(R+h_A)^{3/2}}{(R+h_B)^{3/2} - (R+h_A)^{3/2}} = \frac{1}{\left(\frac{R+h_B}{R+h_A}\right)^{3/2} - 1}$$

$$= \frac{1}{\left(\frac{3960+200}{3960+120}\right)^{3/2} - 1} = 33.835 \text{ orbits}$$

Then
$$T_C = N(T)_B = N\frac{2\pi}{R}\frac{(R+h_B)^{3/2}}{\sqrt{g}}$$

$$= 33.835\frac{2\pi}{3960 \text{ mi}}\frac{[(3960+200) \text{ mi}]^{3/2}}{\left(32.2 \text{ ft/s}^2 \times \frac{1 \text{ mi}}{5280 \text{ ft}}\right)^{1/2}} \times \frac{1 \text{ h}}{3600 \text{ s}}$$

or $T_C = 51.2 \text{ h}$ ◀

Alternative solution

From above, we have $(T)_B > (T)_A$. Thus, when the satellites are next radially aligned, the angles θ_A and θ_B swept out by radial lines drawn to the satellites must differ by 2π. That is,

$$\theta_A = \theta_B + 2\pi$$

For a circular orbit $s = r\theta$

From above $s = vt$ and $v = R\sqrt{\frac{g}{r}}$

Then
$$\theta = \frac{s}{r} = \frac{vt}{r} = \frac{1}{r}\left(R\sqrt{\frac{g}{r}}\right)t = \frac{R\sqrt{g}}{r^{3/2}}t = \frac{R\sqrt{g}}{(R+h)^{3/2}}t$$

At time T_C:
$$\frac{R\sqrt{g}}{(R+h_A)^{3/2}}T_C = \frac{R\sqrt{g}}{(R+h_B)^{3/2}}T_C + 2\pi$$

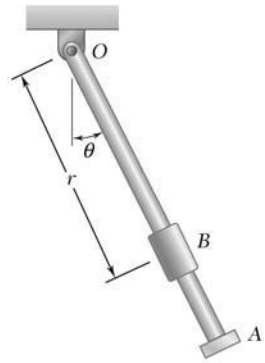
or
$$T_C = \frac{2\pi}{R\sqrt{g}\left[\frac{1}{(R+h_A)^{3/2}} - \frac{1}{(R+h_B)^{3/2}}\right]}$$

$$= \frac{2\pi}{(3960 \text{ mi})\left(32.2 \text{ ft/s}^2 \times \frac{1 \text{ mi}}{5280 \text{ ft}}\right)^{1/2}}$$

$$\times \frac{1}{\frac{1}{[(3960+120) \text{ mi}]^{3/2}} - \frac{1}{[(3960+200) \text{ mi}]^{3/2}}}$$

$$\times \frac{1 \text{ h}}{3600 \text{ s}}$$

or $T_C = 51.2 \text{ h}$ ◀



PROBLEM 11.161

The oscillation of rod OA about O is defined by the relation $\theta = (3/\pi)(\sin \pi t)$, where θ and t are expressed in radians and seconds, respectively. Collar B slides along the rod so that its distance from O is $r = 6(1 - e^{-2t})$ where r and t are expressed in inches and seconds, respectively. When $t = 1$ s, determine (a) the velocity of the collar, (b) the acceleration of the collar, (c) the acceleration of the collar relative to the rod.

SOLUTION

Calculate the derivatives with respect to time.

$$r = 6 - 6e^{-2t} \text{ in.} \qquad \theta = \frac{3}{\pi}\sin \pi t \text{ rad}$$

$$\dot{r} = 12e^{-2t} \text{ in/s} \qquad \dot{\theta} = 3\cos \pi t \text{ rad/s}$$

$$\ddot{r} = -24e^{-2t} \text{ in/s}^2 \qquad \ddot{\theta} = -3\pi \sin \pi t \text{ rad/s}^2$$

At $t = 1$ s,

$$r = 6 - 6e^{-2} = 5.1880 \text{ in.} \qquad \theta = \frac{3}{\pi}\sin \pi = 0$$

$$\dot{r} = 12e^{-2} = 1.6240 \text{ in/s} \qquad \dot{\theta} = 3\cos \pi = -3 \text{ rad/s}$$

$$\ddot{r} = -24e^{-2} = -3.2480 \text{ in/s}^2 \qquad \ddot{\theta} = -3\pi \sin \pi = 0$$

(a) *Velocity of the collar.*

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = 1.6240\,\mathbf{e}_r + (5.1880)(-3)\mathbf{e}_\theta$$

$$\mathbf{v} = (1.624 \text{ in/s})\mathbf{e}_r + (15.56 \text{ in/s})\mathbf{e}_\theta \blacktriangleleft$$

(b) *Acceleration of the collar.*

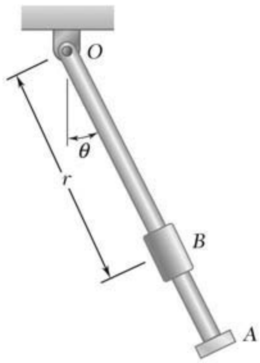
$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

$$= [-3.2480 - (5.1880)(-3)^2]\mathbf{e}_r + (5.1880)(0) + (2)(1.6240)(-3)]\mathbf{e}_\theta$$

$$(-49.9 \text{ in/s}^2)\mathbf{e}_r + (-9.74 \text{ in/s}^2)\mathbf{e}_\theta \blacktriangleleft$$

(c) *Acceleration of the collar relative to the rod.*

$$\mathbf{a}_{B/OA} = \ddot{r}\mathbf{e}_r \qquad \mathbf{a}_{B/OA} = (-3.25 \text{ in/s}^2)\mathbf{e}_r \blacktriangleleft$$



PROBLEM 11.162

The rotation of rod OA about O is defined by the relation $\theta = t^3 - 4t$, where θ and t are expressed in radians and seconds, respectively. Collar B slides along the rod so that its distance from O is $r = 2.5t^3 - 5t^2$, where r and t are expressed in inches and seconds, respectively. When $t = 1$ s, determine (a) the velocity of the collar, (b) the acceleration of the collar, (c) the radius of curvature of the path of the collar.

SOLUTION

Calculate the derivatives with respect to time.

$$r = 2.5t^3 - 5t^2 \qquad \theta = t^3 - 4t$$

$$\dot{r} = 7.5t^2 - 10t \qquad \dot{\theta} = 3t^2 - 4$$

$$\ddot{r} = 15t - 10 \qquad \ddot{\theta} = 6t$$

At $t = 1$ s,

$$r = 2.5 - 5 = -2.5 \text{ in.} \qquad \theta = 1 - 4 = -3 \text{ rad}$$

$$\dot{r} = 7.5 - 10 = -2.5 \text{ in./s} \qquad \dot{\theta} = 3 - 4 = -1 \text{ rad/s}$$

$$\ddot{r} = 15 - 10 = 5 \text{ in./s}^2 \qquad \ddot{\theta} = 6 \text{ rad/s}^2$$

(a) Velocity of the collar.

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = -2.5\mathbf{e}_r + (-2.5)(-1)\mathbf{e}_\theta$$

$$\mathbf{v} = (-2.50 \text{ in./s})\mathbf{e}_r + (2.50 \text{ in./s})\mathbf{e}_\theta \blacktriangleleft$$

$$v = \sqrt{(2.50)^2 + (2.50)^2} = 3.5355 \text{ in./s}$$

Unit vector tangent to the path.

$$\mathbf{e}_t = \frac{\mathbf{v}}{v} = -0.70711\mathbf{e}_r + 0.70711\mathbf{e}_\theta$$

(b) Acceleration of the collar.

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

$$= [5 - (-2.5)(-1)^2]\mathbf{e}_r + [(-2.5)(6) + (2)(-2.5)(-1)]\mathbf{e}_\theta$$

$$\mathbf{a} = (7.50 \text{ in/s}^2)\mathbf{e}_r + (-10.00 \text{ in/s}^2)\mathbf{e}_\theta \blacktriangleleft$$

PROBLEM 11.162 (Continued)

Magnitude: $a = \sqrt{(7.50)^2 + (10.00)^2} = 12.50 \text{ in./s}^2$

Tangential component: $a_t = \mathbf{a}\mathbf{e}_t$

$$a_t = (7.50)(-0.70711) + (-10.00)(0.70711) = -12.374 \text{ in./s}^2$$

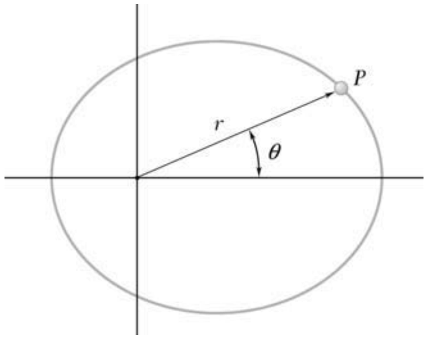
Normal component: $a_n = \sqrt{a^2 - a_t^2} = 1.7674 \text{ in./s}^2$

(*c*) <u>*Radius of curvature of path*</u>.

$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{(3.5355 \text{ in./s})^2}{1.7674 \text{ in./s}^2}$$

$\rho = 7.07$ in. ◀



PROBLEM 11.163

The path of particle P is the ellipse defined by the relations $r = 2/(2-\cos \pi t)$ and $\theta = \pi t$, where r is expressed in meters, t is in seconds, and θ is in radians. Determine the velocity and the acceleration of the particle when (*a*) $t = 0$, (*b*) $t = 0.5$ s.

SOLUTION

We have $$r = \frac{2}{2-\cos \pi t} \qquad \theta = \pi t$$

Then $$\dot{r} = \frac{-2\pi \sin \pi t}{(2-\cos \pi t)^2} \qquad \dot{\theta} = \pi$$

and $$\ddot{r} = -2\pi \frac{\pi \cos \pi t(2-\cos \pi t) - \sin \pi t(2\pi \sin \pi t)}{(2-\cos \pi t)^3} \qquad \ddot{\theta} = 0$$

$$= -2\pi^2 \frac{2\cos \pi t - 1 - \sin^2 \pi t}{(2-\cos \pi t)^3}$$

(*a*) At $t = 0$: $\quad r = 2$ m $\qquad \theta = 0$

$\dot{r} = 0 \qquad \dot{\theta} = \pi$ rad/s

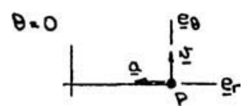
$\ddot{r} = -2\pi^2 \text{m/s}^2 \qquad \ddot{\theta} = 0$

Now $\quad \mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = (2)(\pi)\mathbf{e}_\theta$

or $\quad \mathbf{v} = (2\pi \text{ m/s})\mathbf{e}_\theta$ ◀

and $\quad \mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$

$= [-2\pi^2 - (2)(\pi)^2]\mathbf{e}_r$



or $\quad \mathbf{a} = -(4\pi^2 \text{m/s}^2)\mathbf{e}_r$ ◀

(*b*) At $t = 0.5$ s: $\quad r = 1$ m $\qquad \theta = \frac{\pi}{2}$ rad

$\dot{r} = \frac{-2\pi}{(2)^2} = -\frac{\pi}{2}$ m/s $\qquad \dot{\theta} = \pi$ rad/s

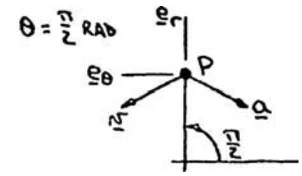
$\ddot{r} = -2\pi^2 \frac{-1-1}{(2)^3} = \frac{\pi^2}{2}$ m/s^2 $\qquad \ddot{\theta} = 0$

Now $\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = \left(-\dfrac{\pi}{2}\right)\mathbf{e}_r + (1)(\pi)\mathbf{e}_\theta$

or $\mathbf{v} = -\left(\dfrac{\pi}{2}\text{ m/s}\right)\mathbf{e}_r + (\pi\text{ m/s})\mathbf{e}_\theta$ ◀

and $\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$

$$= \left[\frac{\pi^2}{2} - (1)(\pi)^2\right]\mathbf{e}_r + \left[2\left(-\frac{\pi}{2}\right)(\pi)\right]\mathbf{e}_\theta$$



or $\mathbf{a} = -\left(\dfrac{\pi^2}{2}\text{ m/s}^2\right)\mathbf{e}_r - (\pi^2\text{ m/s}^2)\mathbf{e}_\theta$ ◀

PROBLEM 11.164

The two-dimensional motion of a particle is defined by the relations $r = 2a\cos\theta$ and $\theta = bt^2/2$, where a and b are constants. Determine (*a*) the magnitudes of the velocity and acceleration at any instant, (*b*) the radius of curvature of the path. What conclusion can you draw regarding the path of the particle?

SOLUTION

(*a*) We have $r = 2a\cos\theta \qquad \theta = \frac{1}{2}bt^2$

Then $\dot{r} = -2a\dot{\theta}\sin\theta \qquad \dot{\theta} = bt$

and $\ddot{r} = -2a(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \qquad \ddot{\theta} = b$

Substituting for $\dot{\theta}$ and $\ddot{\theta}$

$$\dot{r} = -2abt\sin\theta$$

$$\ddot{r} = -2ab(\sin\theta + bt^2\cos\theta)$$

Now $v_r = \dot{r} = -2abt\sin\theta \qquad v_\theta = r\dot{\theta} = 2abt\cos\theta$

Then $v = \sqrt{v_r^2 + v_\theta^2} = 2abt[(-\sin\theta)^2 + (\cos\theta)^2]^{1/2}$

or $$v = 2abt \blacktriangleleft$$

Also $a_r = \ddot{r} - r\dot{\theta}^2 = -2ab(\sin\theta + bt^2\cos\theta) - 2ab^2t^2\cos\theta$

$= -2ab(\sin\theta + 2bt^2\cos\theta)$

and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2ab\cos\theta - 4ab^2t^2\sin\theta$

$= -2ab(\cos\theta - 2bt^2\sin\theta)$

Then $a = \sqrt{a_r^2 + a_B^2} = 2ab[(\sin\theta + 2bt^2\cos\theta)^2 + (\cos\theta - 2bt^2\sin\theta)^2]^{1/2}$

or $$a = 2ab\sqrt{1 + 4b^2t^4} \blacktriangleleft$$

(*b*) Now $a^2 = a_t^2 + a_n^2 = \left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2$

Then $\frac{dv}{dt} = \frac{d}{dt}(2abt) = 2ab$

PROBLEM 11.164 (Continued)

so that $\left(2ab\sqrt{1+4b^2t^4}\right)^2 = (2ab)^2 + a_n^2$

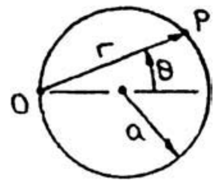
or $4a^2b^2(1+4b^2t^4) = 4a^2b^2 + a_n^2$

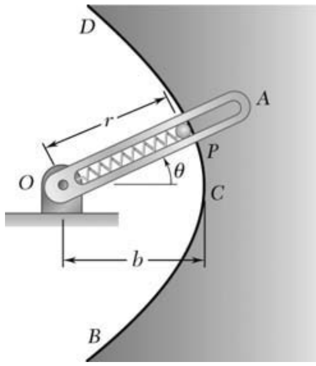
or $a_n = 4ab^2t^2$

Finally $a_n = \dfrac{v^2}{\rho} \Rightarrow \rho = \dfrac{(2abt)^2}{4ab^2t^2}$

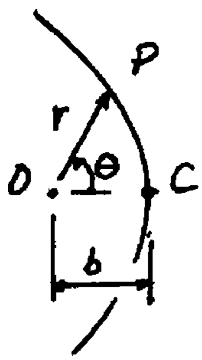
or $\rho = a$ ◀

Since the radius of curvature is a constant, the path is a circle of radius a. ◀



	**PROBLEM 11.165** As rod OA rotates, pin P moves along the parabola BCD. Knowing that the equation of this parabola is $r = 2b/(1+\cos\theta)$ and that $\theta = kt$, determine the velocity and acceleration of P when (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

SOLUTION



$$r = \frac{2b}{1+\cos kt} \qquad \theta = kt$$

$$\dot{r} = \frac{2bk\sin kt}{(1+\cos kt)^2} \qquad \dot{\theta} = k \qquad \ddot{\theta} = 0$$

$$\ddot{r} = \frac{2bk}{(1+\cos kt)^4}[(1+\cos kt)^2 k\cos kt + (\sin kt)2(1+\cos kt)(k\sin kt)]$$

(*a*) <u>When $\theta = kt = 0$</u>:

$$r = b \qquad \dot{r} = 0 \qquad \ddot{r} = \frac{2bk}{(2)^4}[(2)^2 k(1) + 0] = \frac{1}{2}bk^2$$

$$\theta = 0 \qquad \dot{\theta} = k \qquad \ddot{\theta} = 0$$

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = bk$$

$$\mathbf{v} = bk\,\mathbf{e}_\theta \blacktriangleleft$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{1}{2}bk^2 - bk^2 = -\frac{1}{2}bk^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = b(0) + 2(0) = 0$$

$$\mathbf{a} = -\frac{1}{2}bk^2\mathbf{e}_r \blacktriangleleft$$

(*b*) <u>When $\theta = kt = 90°$</u>:

$$r = 2b \qquad \dot{r} = 2bk \qquad \ddot{r} = \frac{2bk}{19}[0 + 2k] = 4bk^2$$

$$\theta = 90° \qquad \dot{\theta} = k \qquad \ddot{\theta} = 0$$

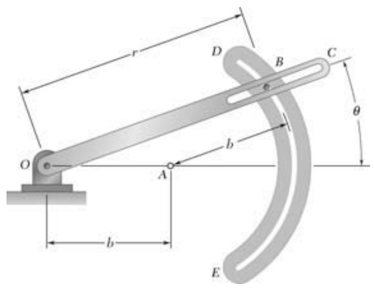
$$v_r = \dot{r} = 2bk \qquad v_\theta = r\dot{\theta} = 2bk$$

$$\mathbf{v} = 2bk\,\mathbf{e}_r + 2bk\,\mathbf{e}_\theta \blacktriangleleft$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4bk^2 - 2bk^2 = 2bk^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2b(0) + 2(2bk)k = 4bk^2$$

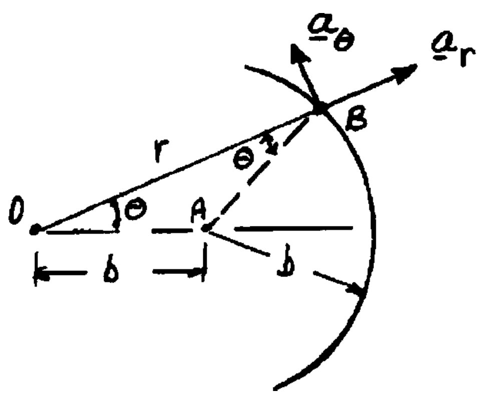
$$\mathbf{a} = 2bk^2\mathbf{e}_r + 4bk^2\mathbf{e}_\theta \blacktriangleleft$$



PROBLEM 11.166

The pin at B is free to slide along the circular slot DE and along the rotating rod OC. Assuming that the rod OC rotates at a constant rate $\dot{\theta}$, (*a*) show that the acceleration of pin B is of constant magnitude, (*b*) determine the direction of the acceleration of pin B.

SOLUTION



From the sketch:

$$r = 2b\cos\theta$$

$$\dot{r} = -2b\sin\theta\,\dot{\theta}$$

Since $\dot{\theta} = \text{constant}$, $\quad \ddot{\theta} = 0$

$$\ddot{r} = -2b\cos\theta\,\dot{\theta}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2b\cos\theta\,\dot{\theta}^2 - (2b\cos\theta)\dot{\theta}^2$$

$$a_r = -4b\cos\theta\,\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (2b\cos\theta)(0) + 2(-2b\sin\theta)\dot{\theta}^2$$

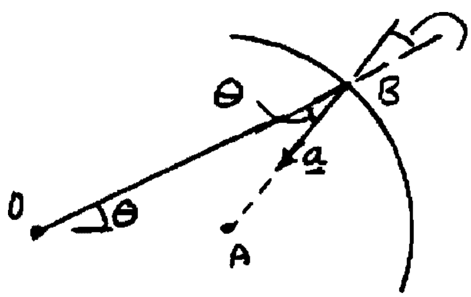
$$a_\theta = -4b\sin\theta\,\dot{\theta}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = 4b\dot{\theta}^2\sqrt{(-\cos\theta)^2 + (-\sin\theta)^2}$$

$$a = 4b\dot{\theta}^2$$

Since both b and $\dot{\theta}$ are constant, we find that

$a = \text{constant}$ ◀

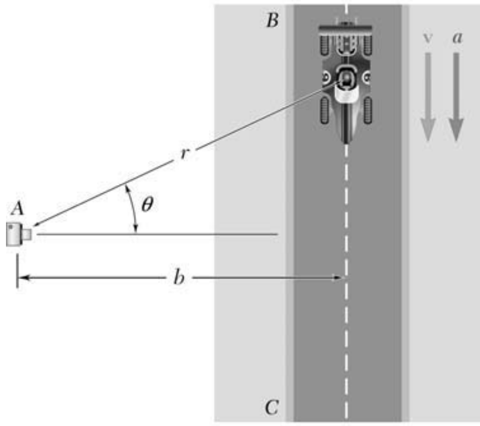


$$\gamma = \tan^{-1}\frac{a_\theta}{a_r} = \tan^{-1}\left(\frac{-4b\sin\theta\,\dot{\theta}^2}{-4b\cos\theta\,\dot{\theta}^2}\right)$$

$$\gamma = \tan^{-1}(\tan\theta)$$

$$\gamma = \theta$$

Thus, **a** is directed toward A ◀

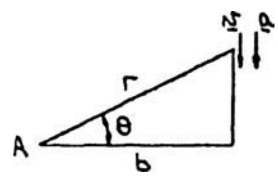


PROBLEM 11.167

To study the performance of a racecar, a high-speed camera is positioned at Point *A*. The camera is mounted on a mechanism which permits it to record the motion of the car as the car travels on straightway *BC*. Determine (*a*) the speed of the car in terms of b, θ, and $\dot{\theta}$, (*b*) the magnitude of the acceleration in terms of b, θ, $\dot{\theta}$, and $\ddot{\theta}$.

SOLUTION

(*a*) We have
$$r = \frac{b}{\cos\theta}$$



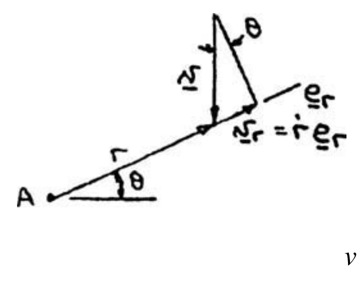
Then
$$\dot{r} = \frac{b\dot{\theta}\sin\theta}{\cos^2\theta}$$

We have
$$v^2 = v_r^2 + v_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$$
$$= \left(\frac{b\theta\sin\theta}{\cos^2\theta}\right)^2 + \left(\frac{b\theta}{\cos\theta}\right)^2$$
$$= \frac{b^2\dot{\theta}^2}{\cos^2\theta}\left(\frac{\sin^2\theta}{\cos^2\theta} + 1\right) = \frac{b^2\dot{\theta}^2}{\cos^4\theta}$$

or
$$v = \pm\frac{b\dot{\theta}}{\cos^2\theta}$$

For the position of the car shown, θ is decreasing; thus, the negative root is chosen. $v = -\dfrac{b\dot{\theta}}{\cos^2\theta}$ ◀

Alternative solution.



From the diagram
$$\dot{r} = -v\sin\theta$$

or
$$\frac{b\dot{\theta}\sin\theta}{\cos^2\theta} = -v\sin\theta$$

or
$$v = -\frac{b\dot{\theta}}{\cos^2\theta}$$ ◀

PROBLEM 11.167 (Continued)

(*b*) For rectilinear motion $a = \dfrac{dv}{dt}$

Using the answer from Part *a*

$$v = -\frac{b\dot{\theta}}{\cos^2\theta}$$

Then
$$a = \frac{d}{dt}\left(-\frac{b\dot{\theta}}{\cos^2\theta}\right)$$
$$= -b\frac{\ddot{\theta}\cos^2\theta - \dot{\theta}(-2\dot{\theta}\cos\theta\sin\theta)}{\cos^4\theta}$$

or
$$a = -\frac{b}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta) \blacktriangleleft$$

Alternative solution

From above
$$r = \frac{b}{\cos\theta} \quad \dot{r} = \frac{b\dot{\theta}\sin\theta}{\cos^2\theta}$$

Then
$$\ddot{r} = b\frac{(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)(\cos^2\theta) - (\dot{\theta}\sin\theta)(-2\dot{\theta}\cos\theta\sin\theta)}{\cos^4\theta}$$
$$= b\left[\frac{\ddot{\theta}\sin\theta}{\cos^2\theta} + \frac{\dot{\theta}^2(1+\sin^2\theta)}{\cos^3\theta}\right]$$

Now
$$a^2 = a_r^2 + a_\theta^2$$

where
$$a_r = \ddot{r} - r\dot{\theta}^2 = b\left[\frac{\ddot{\theta}\sin\theta}{\cos^2\theta} + \frac{\dot{\theta}^2(1+\sin^2\theta)}{\cos^2\theta}\right] - \frac{b\dot{\theta}^2}{\cos\theta}$$
$$= \frac{b}{\cos^2\theta}\left(\ddot{\theta}\sin\theta + \frac{2\dot{\theta}^2\sin^2\theta}{\cos\theta}\right)$$
$$a_r = \frac{b\sin\theta}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)$$

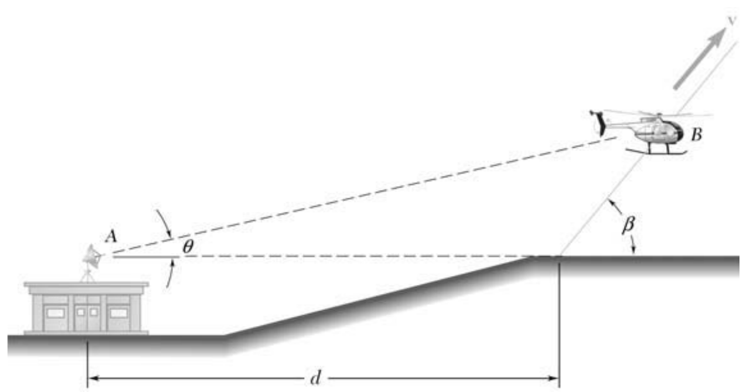
and
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{b\ddot{\theta}}{\cos\theta} + 2\frac{b\dot{\theta}^2\sin\theta}{\cos^2\theta}$$
$$= \frac{b\cos\theta}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}\tan\theta)$$

Then
$$a = \pm\frac{b}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)[(\sin\theta)^2 + (\cos\theta)^2]^{1/2}$$

For the position of the car shown, $\ddot{\theta}$ is negative; for *a* to be positive, the negative root is chosen.

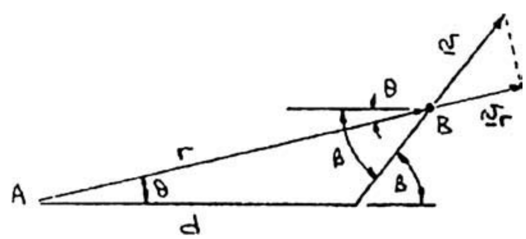
$$a = -\frac{b}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta) \blacktriangleleft$$

PROBLEM 11.168



After taking off, a helicopter climbs in a straight line at a constant angle β. Its flight is tracked by radar from Point A. Determine the speed of the helicopter in terms of d, β, θ, and $\dot{\theta}$.

SOLUTION



From the diagram

$$\frac{r}{\sin(180° - \beta)} = \frac{d}{\sin(\beta - \theta)}$$

or $$d \sin\beta = r(\sin\beta\cos\theta - \cos\beta\sin\theta)$$

or $$r = d\frac{\tan\beta}{\tan\beta\cos\theta - \sin\theta}$$

Then $$\dot{r} = d\tan\beta\frac{-(-\tan\beta\sin\theta - \cos\theta)}{(\tan\beta\cos\theta - \sin\theta)^2}\dot{\theta}$$

$$= d\dot{\theta}\tan\beta\frac{\tan\beta\sin\theta + \cos\theta}{(\tan\beta\cos\theta - \sin\theta)^2}$$

From the diagram

$$v_r = v\cos(\beta - \theta) \qquad \text{where} \qquad v_r = \dot{r}$$

Then

$$d\dot{\theta}\tan\beta\frac{\tan\beta\sin\theta + \cos\theta}{(\tan\beta\cos\theta - \sin\theta)^2} = v(\cos\beta\cos\theta + \sin\beta\sin\theta)$$

$$= v\cos\beta(\tan\beta\sin\theta + \cos\theta)$$

or $$v = \frac{d\dot{\theta}\tan\beta\sec\beta}{(\tan\beta\cos\theta - \sin\theta)^2} \blacktriangleleft$$

Alternative solution.

We have $$v^2 = v_r^2 + v_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$$

PROBLEM 11.168 (Continued)

Using the expressions for r and $\dot{r}$ from above

$$v = \left[d\dot{\theta} \tan\beta \frac{\tan\beta \sin\theta + \cos\theta}{(\tan\beta \cos\theta - \sin\theta)^2} \right]^2$$

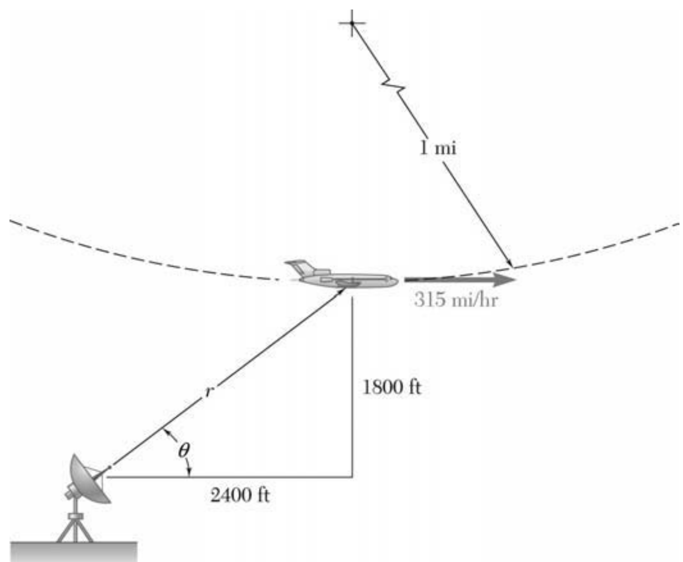
or

$$v = \pm \frac{d\dot{\theta} \tan\beta}{(\tan\beta \cos\theta - \sin\theta)} \left[\frac{(\tan\beta \sin\theta + \cos\theta)^2}{(\tan\beta \cos\theta - \sin\theta)^2} + 1 \right]^{1/2}$$

$$= \pm \frac{d\dot{\theta} \tan\beta}{(\tan\beta \cos\theta - \sin\theta)} \left[\frac{\tan^2\beta + 1}{(\tan\beta \cos\theta - \sin\theta)^2} \right]^{1/2}$$

Note that as θ increases, the helicopter moves in the indicated direction. Thus, the positive root is chosen.

$$v = \frac{d\dot{\theta} \tan\beta \sec\beta}{(\tan\beta \cos\theta - \sin\theta)^2} \blacktriangleleft$$



PROBLEM 11.169

At the bottom of a loop in the vertical plane, an airplane has a horizontal velocity of 315 mi/h and is speeding up at a rate of 10 ft/s^2. The radius of curvature of the loop is 1 mi. The plane is being tracked by radar at O. What are the recorded values of $\dot{r}$, $\ddot{r}$, $\dot{\theta}$ and $\ddot{\theta}$ for this instant?

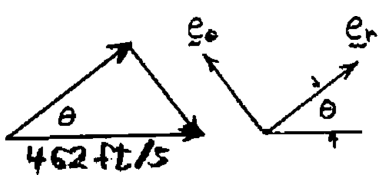
SOLUTION

Geometry. The polar coordinates are

$$r = \sqrt{(2400)^2 + (1800)^2} = 3000 \text{ ft} \qquad \theta = \tan^{-1}\left(\frac{1800}{2400}\right) = 36.87°$$

Velocity Analysis.

$$\mathbf{v} = 315 \text{ mi/h} = 462 \text{ ft/s} \longrightarrow$$



$$v_r = 462\cos\theta = 369.6 \text{ ft/s}$$

$$v_\theta = -462\sin\theta = -277.2 \text{ ft/s}$$

$$v_r = \dot{r} \qquad \dot{r} = 370 \text{ ft/s} \blacktriangleleft$$

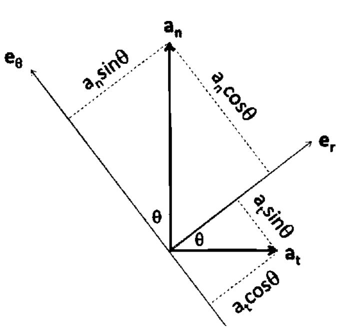
$$v_\theta = r\dot{\theta} \qquad \dot{\theta} = \frac{v_\theta}{r} = -\frac{277.2}{3000}$$

$$\dot{\theta} = -0.0924 \text{ rad/s} \blacktriangleleft$$

Acceleration analysis.

$$a_t = 10 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(462)^2}{5280} = 40.425 \text{ ft/s}^2$$



PROBLEM 11.169 (Continued)

$$a_r = a_t \cos\theta + a_n \sin\theta = 10 \cos 36.87° + 40.425 \sin 36.87° = 32.255 \text{ ft/s}^2$$

$$a_\theta = -a_t \sin\theta + a_n \cos\theta = -10 \sin 36.87° + 40.425 \cos 36.87° = 26.34 \text{ ft/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \quad \ddot{r} = a_r + r\dot{\theta}^2$$

$$\ddot{r} = 32.255 + (3000)(-0.0924)^2$$

$\ddot{r} = 57.9 \text{ ft/s}^2$ ◀

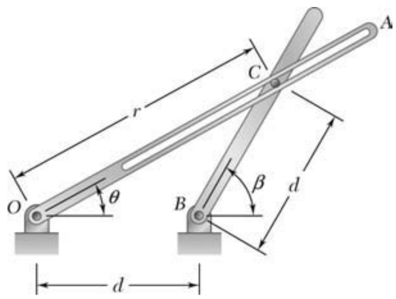
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\ddot{\theta} = \frac{a_\theta}{r} - \frac{2\dot{r}\dot{\theta}}{r}$$

$$= \frac{26.34}{3000} - \frac{(2)(369.6)(-0.0924)}{3000}$$

$\ddot{\theta} = 0.0315 \text{ rad/s}^2$ ◀

PROBLEM 11.170



Pin C is attached to rod BC and slides freely in the slot of rod OA which rotates at the constant rate ω. At the instant when $\beta = 60°$, determine (*a*) $\dot{r}$ and $\dot{\theta}$, (*b*) $\ddot{r}$ and $\ddot{\theta}$. Express your answers in terms of d and ω.

SOLUTION

Looking at d and β as polar coordinates with $\dot{d} = 0$,

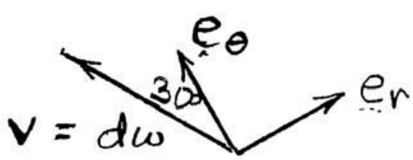
$$v_\beta = d\dot{\beta} = d\omega, \qquad v_d = \dot{d} = 0$$

$$a_\beta = d\ddot{\beta} + 2\dot{d}\dot{\beta} = 0, \qquad a_d = \ddot{d} - d\dot{\beta}^2 = -d\omega^2$$

Geometry analysis: $r = d\sqrt{3}$ for angles shown.

(*a*) Velocity analysis:

Sketch the directions of $\mathbf{v}$, $\mathbf{e}_r$ and $\mathbf{e}_\theta$.



$$v_r = \dot{r} = \mathbf{v} \cdot \mathbf{e}_r = d\omega\cos 120°$$

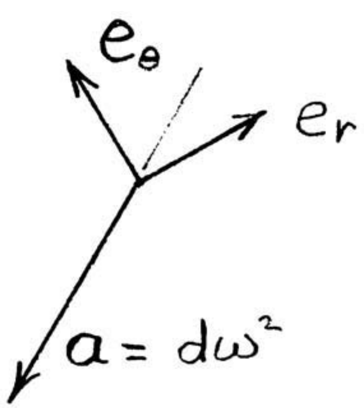
$$\dot{r} = -\frac{1}{2}d\omega \ \blacktriangleleft$$

$$v_\theta = r\dot{\theta} = \mathbf{v} \cdot \mathbf{e}_\theta = d\omega\cos 30°$$

$$\dot{\theta} = \frac{d\omega\cos 30°}{r} = \frac{d\omega\frac{\sqrt{3}}{2}}{d\sqrt{3}} \qquad \dot{\theta} = \frac{1}{2}\omega \ \blacktriangleleft$$

(*b*) Acceleration analysis:

Sketch the directions of $\mathbf{a}$, $\mathbf{e}_r$ and $\mathbf{e}_\theta$.



$$a_r = \mathbf{a} \cdot \mathbf{e}_r = a\cos 150° = -\frac{\sqrt{3}}{2}d\omega^2$$

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\sqrt{3}}{2}d\omega^2$$

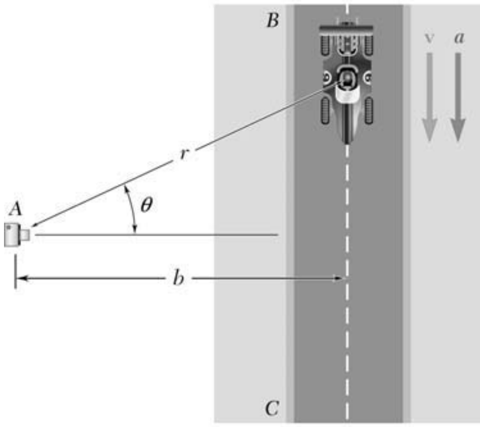
$$\ddot{r} = -\frac{\sqrt{3}}{2}d\omega^2 + r\dot{\theta}^2 = -\frac{\sqrt{3}}{2}d\omega^2 + d\sqrt{3}\left(\frac{1}{2}\omega\right)^2$$

$$\ddot{r} = -\frac{\sqrt{3}}{4}d\omega^2 \ \blacktriangleleft$$

$$a_\theta = \mathbf{a} \cdot \mathbf{e}_\theta = d\omega^2\cos 120° = -\frac{1}{2}d\omega^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\ddot{\theta} = \frac{1}{r}(a_\theta - 2\dot{r}\dot{\theta}) = \frac{1}{\sqrt{3}d}\left[-\frac{1}{2}d\omega^2 - (2)\left(-\frac{1}{2}d\omega\right)\left(\frac{1}{2}\omega\right)\right] \qquad \ddot{\theta} = 0 \ \blacktriangleleft$$



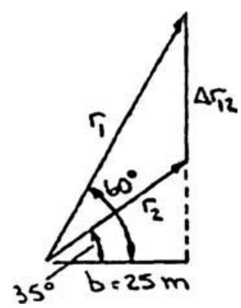
PROBLEM 11.171

For the racecar of Problem 11.167, it was found that it took 0.5 s for the car to travel from the position $\theta = 60°$ to the position $\theta = 35°$. Knowing that $b = 25$ m, determine the average speed of the car during the 0.5-s interval.

PROBLEM 11.167 To study the performance of a racecar, a high-speed camera is positioned at Point A. The camera is mounted on a mechanism which permits it to record the motion of the car as the car travels on straightway BC. Determine (a) the speed of the car in terms of b, θ, and $\dot{\theta}$, (b) the magnitude of the acceleration in terms of b, θ, $\dot{\theta}$, and $\ddot{\theta}$.

SOLUTION

From the diagram:



$$\Delta r_{12} = 25 \tan 60° - 25 \tan 35°$$
$$= 25.796 \text{ m}$$

Now

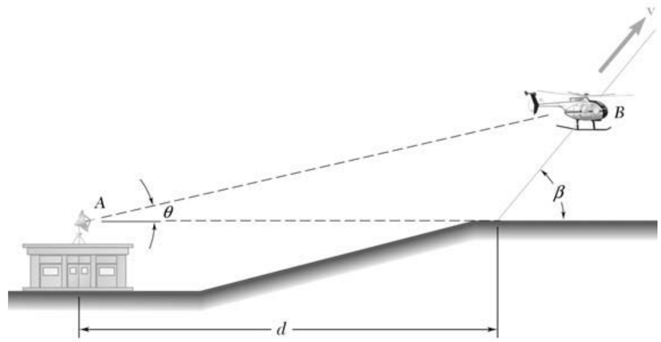
$$v_{\text{ave}} = \frac{\Delta r_{12}}{\Delta t_{12}}$$
$$= \frac{25.796 \text{ m}}{0.5 \text{ s}}$$
$$= 51.592 \text{ m/s}$$

or $v_{\text{ave}} = 185.7$ km/h ◀

PROBLEM 11.172

For the helicopter of Problem 11.168, it was found that when the helicopter was at B, the distance and the angle of elevation of the helicopter were $r = 3000$ ft and $\theta = 20°$, respectively. Four seconds later, the radar station sighted the helicopter at $r = 3320$ ft and $\theta = 23.1°$. Determine the average speed and the angle of climb β of the helicopter during the 4-s interval.

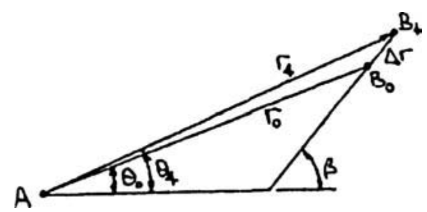
PROBLEM 11.168 After taking off, a helicopter climbs in a straight line at a constant angle β. Its flight is tracked by radar from Point A. Determine the speed of the helicopter in terms of d, β, θ, and $\dot{\theta}$.



SOLUTION

We have $r_0 = 3000$ ft $\quad \theta_0 = 20°$

$r_4 = 3320$ ft $\quad \theta_4 = 23.1°$



From the diagram:

$$\Delta r^2 = 3000^2 + 3320^2 - 2(3000)(3320)\cos(23.1° - 20°)$$

or $$\Delta r = 362.70 \text{ ft}$$

Now $$v_{\text{ave}} = \frac{\Delta r}{\Delta t} = \frac{362.70 \text{ ft}}{4 \text{ s}} = 90.675 \text{ ft/s}$$

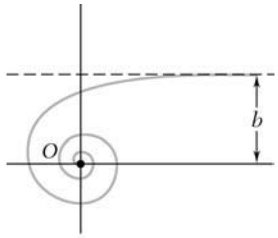
or $$v_{\text{ave}} = 61.8 \text{ mi/h} \blacktriangleleft$$

Also, $$\Delta r \cos\beta = r_4 \cos\theta_4 - r_0 \cos\theta_0$$

or $$\cos\beta = \frac{3320\cos 23.1° - 3000\cos 20°}{362.70}$$

or $$\beta = 49.7° \blacktriangleleft$$

PROBLEM 11.173



Hyperbolic spiral $r\theta = b$

A particle moves along the spiral shown; determine the magnitude of the velocity of the particle in terms of b, θ, and $\dot{\theta}$.

SOLUTION

Hyperbolic spiral.

$$r = \frac{b}{\theta}$$

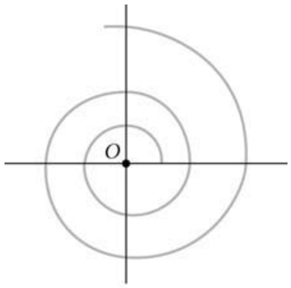
$$\dot{r} = \frac{dr}{dt} = -\frac{b}{\theta^2}\frac{d\theta}{dt} = -\frac{b}{\theta^2}\dot{\theta}$$

$$v_r = \dot{r} = -\frac{b}{\theta^2}\dot{\theta} \qquad v_\theta = r\dot{\theta} = \frac{b}{\theta}\dot{\theta}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = b\dot{\theta}\sqrt{\left(-\frac{1}{\theta^2}\right)^2 + \left(\frac{1}{\theta}\right)^2}$$

$$= \frac{b\dot{\theta}}{\theta^2}\sqrt{1+\theta^2}$$

$$v = \frac{b}{\theta^2}\sqrt{1+\theta^2}\,\dot{\theta} \blacktriangleleft$$



PROBLEM 11.174

A particle moves along the spiral shown; determine the magnitude of the velocity of the particle in terms of b, θ, and $\dot{\theta}$.

SOLUTION

Logarithmic spiral.

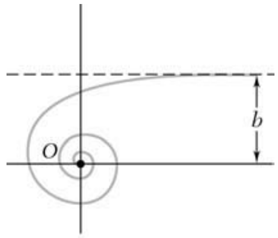
$$r = e^{b\theta}$$

$$\dot{r} = \frac{dr}{dt} = be^{b\theta}\frac{d\theta}{dt} = be^{b\theta}\dot{\theta}$$

$$v_r = \dot{r} = be^{b\theta}\dot{\theta} \quad v_\theta = r\dot{\theta} = e^{b\theta}\dot{\theta}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = e^{b\theta}\dot{\theta}\sqrt{b^2 + 1}$$

$$v = e^{b\theta}\sqrt{1 + b^2}\ \dot{\theta} \blacktriangleleft$$



Hyperbolic spiral $r\theta = b$

PROBLEM 11.175

A particle moves along the spiral shown. Knowing that $\dot{\theta}$ is constant and denoting this constant by ω, determine the magnitude of the acceleration of the particle in terms of b, θ, and ω.

SOLUTION

Hyperbolic spiral. $$r = \frac{b}{\theta}$$

From Problem 11.173 $$\dot{r} = -\frac{b}{\theta^2}\dot{\theta}$$

$$\ddot{r} = -\frac{b}{\theta^2}\ddot{\theta} + \frac{2b}{\theta^3}\dot{\theta}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{b}{\theta^2}\ddot{\theta} + \frac{2b}{\theta^3}\dot{\theta}^2 - \frac{b}{\theta}\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{b}{\theta}\ddot{\theta} + 2\left(-\frac{b}{\theta^2}\dot{\theta}\right)\dot{\theta} = \frac{b}{\theta}\ddot{\theta} - 2\frac{b}{\theta^2}\dot{\theta}^2$$

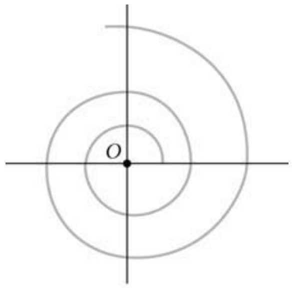
Since $\dot{\theta} = \omega = \text{constant}$, $\ddot{\theta} = 0$, and we write:

$$a_r = +\frac{2b}{\theta^3}\omega^2 - \frac{b}{\theta}\omega^2 = \frac{b\omega^2}{\theta^3}(2 - \theta^2)$$

$$a_\theta = -2\frac{b}{\theta^2}\omega^2 = -\frac{b\omega^2}{\theta^3}(2\theta)$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \frac{b\omega^2}{\theta^3}\sqrt{(2-\theta^2)^2 + (2\theta)^2} = \frac{b\omega^2}{\theta^3}\sqrt{4 - 4\theta^2 + \theta^4 + 4\theta^2}$$

$$a = \frac{b\omega^2}{\theta^3}\sqrt{4 + \theta^4} \blacktriangleleft$$



PROBLEM 11.176

A particle moves along the spiral shown. Knowing that $\dot{\theta}$ is constant and denoting this constant by ω, determine the magnitude of the acceleration of the particle in terms of b, θ, and ω.

SOLUTION

Logarithmic spiral. $r = e^{b\theta}$

$$\dot{r} = \frac{dr}{dt} = be^{b\theta}\dot{\theta}$$

$$\ddot{r} = be^{b\theta}\ddot{\theta} + b^2 e^{b\theta}\dot{\theta}^2 = be^{b\theta}(\ddot{\theta} + b\dot{\theta}^2)$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = be^{b\theta}(\ddot{\theta} + b\dot{\theta}^2) - e^{b\theta}\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = e^{b\theta}\ddot{\theta} + 2(be^{b\theta}\dot{\theta})\dot{\theta}$$

Since $\dot{\theta} = \omega =$ constant, $\ddot{\theta} = \theta$, and we write

$$a_r = be^{b\theta}(b\omega^2) - e^{b\theta}\omega^2 = e^{b\theta}(b^2 - 1)\omega^2$$

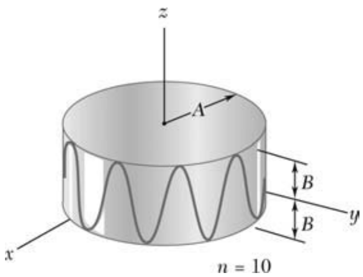
$$a_\theta = 2be^{b\theta}\omega^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = e^{b\theta}\omega^2\sqrt{(b^2-1)^2 + (2b)^2}$$

$$= e^{b\theta}\omega^2\sqrt{b^4 - 2b^2 + 1 + 4b^2} = e^{b\theta}\omega^2\sqrt{b^4 + 2b^2 + 1}$$

$$= e^{b\theta}\omega^2\sqrt{(b^2+1)^2} = e^{b\theta}\omega^2(b^2+1)$$

$$a = (1 + b^2)\omega^2 e^{b\theta} \blacktriangleleft$$



PROBLEM 11.177

The motion of a particle on the surface of a right circular cylinder is defined by the relations $R = A$, $\theta = 2\pi t$, and $z = B\sin 2\pi nt$, where A and B are constants and n is an integer. Determine the magnitudes of the velocity and acceleration of the particle at any time t.

SOLUTION

$$R = A \qquad \theta = 2\pi t \qquad z = B\sin 2\pi nt$$
$$\dot{R} = 0 \qquad \dot{\theta} = 2\pi \qquad \dot{z} = 2\pi n\, B\cos 2\pi nt$$
$$\ddot{R} = 0 \qquad \ddot{\theta} = 0 \qquad \ddot{z} = -4\pi^2 n^2\, B\sin 2\pi nt$$

Velocity (Eq. 11.49)

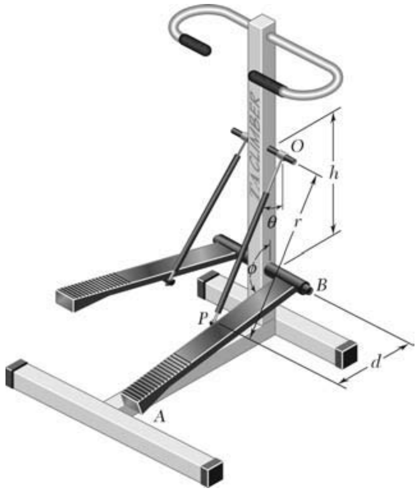
$$\mathbf{v} = \dot{R}\mathbf{e}_R + R\dot{\theta}\mathbf{e}_\theta + \dot{z}\mathbf{k}$$
$$\mathbf{v} = \quad + A(2\pi)\mathbf{e}_\theta + 2\pi n\, B\cos 2\pi nt\,\mathbf{k}$$

$$v = 2\pi\sqrt{A^2 + n^2B^2\cos^2 2\pi nt} \blacktriangleleft$$

Acceleration (Eq. 11.50)

$$\mathbf{a} = (\ddot{R} - R\dot{\theta}^2)\mathbf{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\mathbf{e}_\theta + \ddot{z}\mathbf{k}$$
$$\mathbf{a} = -4\pi^2 A\mathbf{e}_k - 4\pi^2 n^2\; B\sin 2\pi nt\,\mathbf{k}$$

$$a = 4\pi^2\sqrt{A^2 + n^4B^2\sin^2 2\pi nt} \blacktriangleleft$$

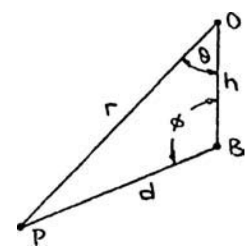


PROBLEM 11.178

Show that $\dot{r} = h\dot{\phi}\sin\theta$ knowing that at the instant shown, step *AB* of the step exerciser is rotating counterclockwise at a constant rate $\dot{\phi}$.

SOLUTION

From the diagram



$$r^2 = d^2 + h^2 - 2dh\cos\phi$$

Then $$2r\dot{r} = 2dh\dot{\phi}\sin\phi$$

Now $$\frac{r}{\sin\phi} = \frac{d}{\sin\theta}$$

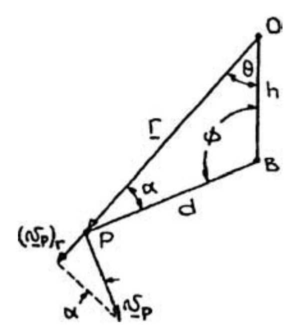
or $$r = \frac{d\sin\phi}{\sin\theta}$$

Substituting for *r* in the expression for $\dot{r}$

$$\left(\frac{d\sin\phi}{\sin\theta}\right)\dot{r} = dh\dot{\phi}\sin\phi$$

or $$\dot{r} = h\dot{\phi}\sin\theta \qquad \text{Q.E.D.}$$ ◀

Alternative solution.



First note $$\alpha = 180° - (\phi + \theta)$$

Now $$\mathbf{v} = \mathbf{v}_r + \mathbf{v}_\theta = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

With *B* as the origin

$$v_P = d\dot{\phi} \qquad (d = \text{constant} \Rightarrow \dot{d} = 0)$$

PROBLEM 11.178 (Continued)

With O as the origin $(v_P)_r = \dot{r}$

where $(v_P)_r = v_P \sin\alpha$

Then $\dot{r} = d\dot{\phi}\sin\alpha$

Now $\dfrac{h}{\sin\alpha} = \dfrac{d}{\sin\theta}$

or $d\sin\alpha = h\sin\theta$

substituting $\dot{r} = h\dot{\phi}\sin\theta$ Q.E.D.

PROBLEM 11.179

The three-dimensional motion of a particle is defined by the relations $R = A(1-e^{-t}), \theta = 2\pi t,$ and $z = B(1-e^{-t})$. Determine the magnitudes of the velocity and acceleration when (*a*) $t = 0$, (*b*) $t = \infty$.

SOLUTION

$$R = A(1-e^{-t}) \qquad \theta = 2\pi t \qquad z = B(1-e^{-t})$$
$$\dot{R} = Ae^{-t} \qquad \dot{\theta} = 2\pi \qquad \dot{z} = Be^{-t}$$
$$\ddot{R} = -Ae^{-t} \qquad \ddot{\theta} = 0 \qquad \ddot{z} = -Be^{-t}$$

Velocity (Eq. 11.49)

$$\mathbf{v} = \dot{R}\mathbf{e}_R + R\dot{\theta}\mathbf{e}_\theta + \dot{z}\mathbf{k}$$
$$\mathbf{v} = Ae^{-t}\mathbf{e}_R + 2\pi A(1-e^{-t})\mathbf{e}_\theta + Be^{-t}\mathbf{k}$$

(*a*) When $t = 0$: $e^{-t} = e^0 = 1$; $\mathbf{v} = A\mathbf{e}_R + B\mathbf{k}$ $\qquad v = \sqrt{A^2 + B^2}$ ◀

(*b*) When $t = \infty$: $e^{-t} = e^{-\infty} = 0$ $\quad \mathbf{v} = 2\pi A\mathbf{e}_\theta$ $\qquad v = 2\pi A$ ◀

Acceleration (Eq. 11.50)

$$\mathbf{a} = (\ddot{R} - R\dot{\theta}^1)\mathbf{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\mathbf{e}_\theta + \ddot{z}\mathbf{k}$$
$$= [-Ae^{-t} - A(1-e^{-t})4\pi^2]\mathbf{e}_R + [0 + 2Ae^{-t}(2\pi)]\mathbf{e}_\theta - Be^{-t}\mathbf{k}$$

(*a*) When $t = 0$: $e^{-t} = e^0 = 1$

$$\mathbf{a} = -A\mathbf{e}_R + 4\pi A\mathbf{e}_\theta - B\,\mathbf{k}$$

$$a = \sqrt{A^2 + (4\pi A)^2 + B^2} \qquad a = \sqrt{(1+16\pi^2)A^2 + B^2}$$ ◀

(*b*) When $t = \infty$: $e^{-t} = e^{-\infty} = 0$

$$\mathbf{a} = -4\pi^2 A\mathbf{e}_R \qquad a = 4\pi^2 A$$ ◀

PROBLEM 11.180*

For the conic helix of Problem 11.95, determine the angle that the osculating plane forms with the y axis.

PROBLEM 11.95 The three-dimensional motion of a particle is defined by the position vector $\mathbf{r} = (Rt\cos\omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt\sin\omega_n t)\mathbf{k}$. Determine the magnitudes of the velocity and acceleration of the particle. (The space curve described by the particle is a conic helix.)

SOLUTION

First note that the vectors **v** and **a** lie in the osculating plane.

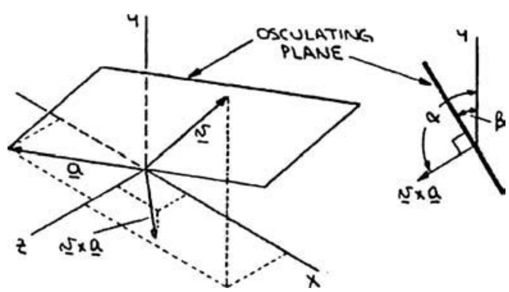
Now $\qquad \mathbf{r} = (Rt\cos\omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt\sin\omega_n t)\mathbf{k}$

Then $\qquad \mathbf{v} = \dfrac{dr}{dt} = R(\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} + c\mathbf{j} + R(\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}$

and

$$\begin{aligned}\mathbf{a} &= \frac{dv}{dt}\\ &= R\left(-\omega_n\sin\omega_n t - \omega_n\sin\omega_n t - \omega_n^2 t\cos\omega_n t\right)\mathbf{i}\\ &\quad + R\left(\omega_n\cos\omega_n t + \omega_n\cos\omega_n t - \omega_n^2 t\sin\omega_n t\right)\mathbf{k}\\ &= \omega_n R[-(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{i} + (2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{k}]\end{aligned}$$

It then follows that the vector $(\mathbf{v}\times\mathbf{a})$ is perpendicular to the osculating plane.



$$(\mathbf{v}\times\mathbf{a}) = \omega_n R\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ R(\cos\omega_n t - \omega_n t\sin\omega_n t) & c & R(\sin\omega_n t + \omega_n t\cos\omega_n t)\\ -(2\sin\omega_n t + \omega_n t\cos\omega_n t) & 0 & (2\cos\omega_n t - \omega_n t\sin\omega_n t)\end{vmatrix}$$

$$\begin{aligned}&= \omega_n R\{c(2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} + R[-(\sin\omega_n t + \omega_n t\cos\omega_n t)(2\sin\omega_n t + \omega_n t\cos\omega_n t)\\ &\quad - (\cos\omega_n t - \omega_n t\sin\omega_n t)(2\cos\omega_n t - \omega_n t\sin\omega_n t)]\mathbf{j} + c(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}\\ &= \omega_n R\left[c(2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} - R\left(2 + \omega_n^2 t^2\right)\mathbf{j} + c(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}\right]\end{aligned}$$

The angle α formed by the vector $(\mathbf{v}\times\mathbf{a})$ and the y axis is found from

$$\cos\alpha = \frac{(\mathbf{v}\times\mathbf{a})\cdot\mathbf{j}}{|(\mathbf{v}\times\mathbf{a})||\mathbf{j}|}$$

Where $|\mathbf{j}| = 1$

$$(\mathbf{v}\times\mathbf{a})\cdot\mathbf{j} = -\omega_n R^2\left(2+\omega_n^2 t^2\right)$$

$$|(\mathbf{v}\times\mathbf{a})| = \omega_n R\left[c^2(2\cos\omega_n t - \omega_n t\sin\omega_n t)^2 + R^2\left(2+\omega_n^2 t^2\right)^2 + c^2(2\sin\omega_n t + \omega_n t\cos\omega_n t)^2\right]^{1/2}$$

$$= \omega_n R\left[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\right]^{1/2}$$

Then

$$\cos\alpha = \frac{-\omega_n R^2\left(2+\omega_n^2 t^2\right)}{\omega_n R\left[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\right]^{1/2}}$$

$$= \frac{-R\left(2+\omega_n^2 t^2\right)}{\left[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\right]^{1/2}}$$

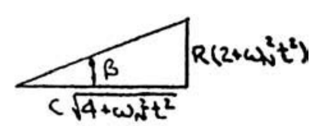
The angle β that the osculating plane forms with y axis (see the above diagram) is equal to

$$\beta = \alpha - 90°$$

Then

$$\cos\alpha = \cos(\beta + 90°) = -\sin\beta$$

$$-\sin\beta = \frac{-R\left(2+\omega_n^2 t^2\right)}{\left[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\right]^{1/2}}$$



Then

$$\tan\beta = \frac{R\left(2+\omega_n^2 t^2\right)}{c\sqrt{4+\omega_n^2 t^2}}$$

or

$$\beta = \tan^{-1}\left[\frac{R\left(2+\omega_n^2 t^2\right)}{c\sqrt{4+\omega_n^2 t^2}}\right] \blacktriangleleft$$

PROBLEM 11.181*

Determine the direction of the binormal of the path described by the particle of Problem 11.96 when (*a*) $t = 0$, (*b*) $t = \pi/2$ s.

SOLUTION

Given: $\mathbf{r} = (At\cos t)\mathbf{i} + \left(A\sqrt{t^2+1}\right)\mathbf{j} + (Bt\sin t)\mathbf{k}$

$r - \text{ft}, \quad t - \text{s}; \quad A = 3, \quad B - 1$

First note that $\mathbf{e}_b$ is given by

$$\mathbf{e}_b = \frac{\mathbf{v}\times\mathbf{a}}{|\mathbf{v}\times\mathbf{a}|}$$

Now $\mathbf{r} = (3t\cos t)\mathbf{i} + \left(3\sqrt{t^2+1}\right)\mathbf{j} + (t\sin t)\mathbf{k}$

Then
$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$
$$= 3(\cos t - t\sin t)\mathbf{i} + \frac{3t}{\sqrt{t^2+1}}\mathbf{j} + (\sin t + t\cos t)\mathbf{k}$$

and
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t\cos t)\mathbf{i} + 3\frac{\sqrt{t^2+1} - t\left(\frac{t}{\sqrt{t^2+1}}\right)}{t^2+1}\mathbf{j}$$
$$+ (\cos t + \cos t - t\sin t)\mathbf{k}$$
$$= -3(2\sin t + t\cos t)\mathbf{i} + \frac{3}{(t^2+1)^{3/2}}\mathbf{j} + (2\cos t - t\sin t)\mathbf{k}$$

(*a*) At $t = 0$: $\mathbf{v} = (3\text{ ft/s})\mathbf{i}$

$$\mathbf{a} = (3\text{ ft/s}^2)\mathbf{j} + (2\text{ ft/s}^2)\mathbf{k}$$

Then
$$\mathbf{v}\times\mathbf{a} = 3\mathbf{i}\times(3\mathbf{j} + 2\mathbf{k})$$
$$= 3(-2\mathbf{j} + 3\mathbf{k})$$

and
$$|\mathbf{v}\times\mathbf{a}| = 3\sqrt{(-2)^2 + (3)^2} = 3\sqrt{13}$$

Then
$$\mathbf{e}_b = \frac{3(-2\mathbf{j}+3\mathbf{k})}{3\sqrt{13}} = \frac{1}{\sqrt{13}}(-2\mathbf{j}+3\mathbf{k})$$

$$\cos\theta_x = 0 \qquad \cos\theta_y = -\frac{2}{\sqrt{13}} \qquad \cos\theta_z = \frac{3}{\sqrt{13}}$$

or $\theta_x = 90° \qquad \theta_y = 123.7° \qquad \theta_z = 33.7°$ ◀

PROBLEM 11.181* (Continued)

(*b*) At $t = \dfrac{\pi}{2}$ s:

$$\mathbf{v} = -\left(\frac{3\pi}{2}\ \text{ft/s}\right)\mathbf{i} + \left(\frac{3\pi}{\sqrt{\pi^2+4}}\ \text{ft/s}\right)\mathbf{j} + (1\ \text{ft/s})\mathbf{k}$$

$$\mathbf{a} = -(6\ \text{ft/s}^2)\mathbf{i} + \left[\frac{24}{(\pi^2+4)^{3/2}}\ \text{ft/s}^2\right]\mathbf{j} - \left(\frac{\pi}{2}\ \text{ft/s}^2\right)\mathbf{k}$$

Then

$$\mathbf{v}\times\mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\dfrac{3\pi}{2} & \dfrac{3\pi}{(\pi^2+4)^{1/2}} & 1 \\ -6 & \dfrac{24}{(\pi^2+4)^{3/2}} & -\dfrac{\pi}{2} \end{vmatrix}$$

$$= -\left[\frac{3\pi^2}{2(\pi^2+4)^{1/2}} + \frac{24}{(\pi^2+4)^{3/2}}\right]\mathbf{i} - \left(6 + \frac{3\pi^2}{4}\right)\mathbf{j}$$

$$+\left[-\frac{36\pi}{(\pi^2+4)^{3/2}} + \frac{18\pi}{(\pi^2+4)^{1/2}}\right]\mathbf{k}$$

$$= -4.43984\mathbf{i} - 13.40220\mathbf{j} + 12.99459\mathbf{k}$$

and

$$|\mathbf{v}\times\mathbf{a}| = [(-4.43984)^2 + (-13.40220)^2 + (12.99459)^2]^{1/2}$$

$$= 19.18829$$

Then

$$\mathbf{e}_b = \frac{1}{19.1829}(-4.43984\mathbf{i} - 13.40220\mathbf{j} + 12.99459\mathbf{k})$$

$$\cos\theta_x = -\frac{4.43984}{19.18829} \quad \cos\theta_y = -\frac{13.40220}{19.18829} \quad \cos\theta_z = \frac{12.99459}{19.18829}$$

or $\theta_x = 103.4°$ $\quad \theta_y = 134.3°$ $\quad \theta_z = 47.4°$ ◀

PROBLEM 11.182

The motion of a particle is defined by the relation $x = 2t^3 - 15t^2 + 24t + 4,$ where x and t are expressed in meters and seconds, respectively. Determine (a) when the velocity is zero, (b) the position and the total distance traveled when the acceleration is zero.

SOLUTION

$$x = 2t^3 - 15t^2 + 24t + 4$$

so $$v = \frac{dx}{dt} = 6t^2 - 30t + 24$$

$$a = \frac{dv}{dt} = 12t - 30$$

(a) Times when $v = 0$. $\quad 0 = 6t^2 - 30t + 24 = 6\,(t^2 - 5t + 4)$

$$(t-4)(t-1) = 0$$

$t = 1.00$ s, $\quad t = 4.00$ s ◀

(b) Position and distance traveled when $a = 0$.

$$a = 12t - 30 = 0 \qquad t = 2.5 \text{ s}$$

so $$x_2 = 2(2.5)^3 - 15(2.5)^2 + 24(2.5) + 4$$

Final position $\quad x = 1.50$ m ◀

For $\quad 0 \le t \le 1\,\text{s}, \quad v > 0.$

For $\quad 1\,\text{s} \le t \le 2.5\text{ s}, \quad v \le 0.$

At $\quad t = 0, \quad x_0 = 4$ m.

At $\quad t = 1\,\text{s}, \quad x_1 = (2)(1)^3 - (15)(1)^2 + (24)(1) + 4 = 15$ m

Distance traveled over interval: $x_1 - x_0 = 11$ m

For $\quad 1\,\text{s} \le t \le 2.5\text{ s}, \quad v \le 0$

Distance traveled over interval

$$|x_2 - x_1| = |1.5 - 15| = 13.5 \text{ m}$$

Total distance: $\quad d = 11 + 13.5$ $\quad d = 24.5$ m ◀

PROBLEM 11.183

A particle starting from rest at $x = 1$ m is accelerated so that its velocity doubles in magnitude between $x = 2$ m and $x = 8$ m. Knowing that the acceleration of the particle is defined by the relation $a = k[x - (A/x)]$, determine the values of the constants A and k if the particle has a velocity of 29 m/s when $x = 16$ m.

SOLUTION

We have $$v\frac{dv}{dx} = a = k\left(x - \frac{A}{x}\right)$$

When $x = 1$ ft, $v = 0$: $$\int_0^v v\,dv = \int_1^x k\left(x - \frac{A}{x}\right)dx$$

or $$\frac{1}{2}v^2 = k\left[\frac{1}{2}x^2 - A\ln x\right]_1^x$$

$$= k\left(\frac{1}{2}x^2 - A\ln x - \frac{1}{2}\right)$$

At $x = 2$ ft: $$\frac{1}{2}v_2^2 = k\left[\frac{1}{2}(2)^2 - A\ln 2 - \frac{1}{2}\right] = k\left(\frac{3}{2} - A\ln 2\right)$$

$x = 8$ ft: $$\frac{1}{2}v_8^2 = k\left[\frac{1}{2}(8)^2 - A\ln 8 - \frac{1}{2}\right] = k(31.5 - A\ln 8)$$

Now $\frac{v_8}{v_2} = 2$: $$\frac{\frac{1}{2}v_8^2}{\frac{1}{2}v_2^2} = (2)^2 = \frac{k(31.5 - A\ln 8)}{k\left(\frac{3}{2} - A\ln 2\right)}$$

$$6 - 4A\ln 2 = 31.5 - A\ln 8$$

$$25.5 = A(\ln 8 - 4\ln 2) = A(\ln 8 - \ln 2^4) = A\ln\left(\frac{1}{2}\right)$$

$$A = \frac{25.5}{\ln\frac{1}{2}}$$

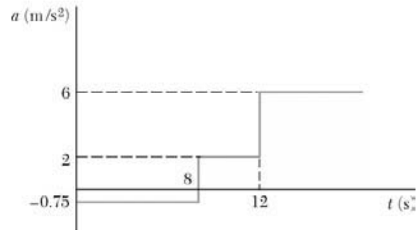
$A = -36.8\ \text{m}^2$ ◀

When $x = 16$ m, $v = 29$ m/s: $$\frac{1}{2}(29)^2 = k\left[\frac{1}{2}(16)^2 - \frac{25.5}{\ln\left(\frac{1}{2}\right)}\ln(16) - \frac{1}{2}\right]$$

$$420.5k = k\left[128 + 102 - \frac{1}{2}\right] = 230.5k$$

$$= 230.5k$$

$k = 1.832\ \text{s}^{-2}$ ◀

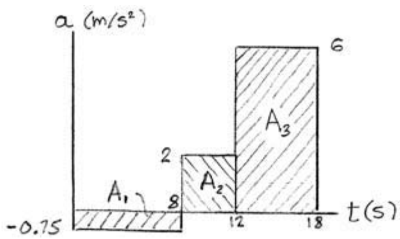


PROBLEM 11.184

A particle moves in a straight line with the acceleration shown in the figure. Knowing that the particle starts from the origin with $v_0 = -2$ m/s, (*a*) construct the $v{-}t$ and $x{-}t$ curves for $0 < t < 18$ s, (*b*) determine the position and the velocity of the particle and the total distance traveled when $t = 18$ s.

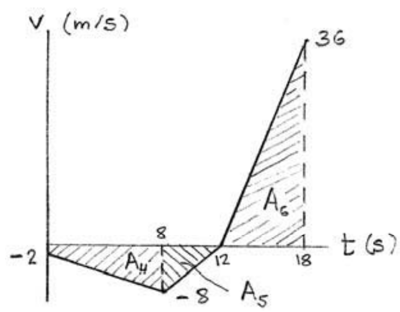
SOLUTION

Compute areas under $a - t$ curve.



$$A_1 = (-0.75)(8) = -6 \text{ m/s}$$
$$A_2 = (2)(4) = 8 \text{ m/s}$$
$$A_3 = (6)(6) = 36 \text{ m/s}$$

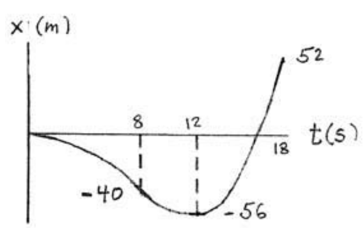
$$v_0 = -2 \text{ m/s}$$
$$v_8 = v_0 + A_1 = -8 \text{ m/s}$$
$$v_{12} = v_8 + A_2 = 0$$
$$v_{18} = v_{12} + A_3 \qquad v_{18} = 36 \text{ m/s} \blacktriangleleft$$



Sketch $v{-}t$ curve using straight line portions over the constant acceleration periods.

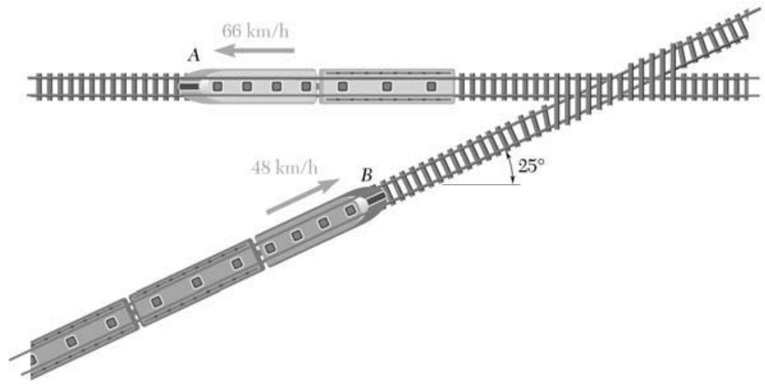
Compute areas under the $v{-}t$ curve.

$$A_4 = \frac{1}{2}(-2 - 8)(8) = -40 \text{ m}$$
$$A_5 = \frac{1}{2}(-8)(4) = -16 \text{ m}$$
$$A_6 = \frac{1}{2}(36)(6) = 108 \text{ m}$$



$$x_0 = 0$$
$$x_8 = x_0 + A_4 = -40 \text{ m}$$
$$x_{12} = x_8 + A_5 = -56 \text{ m}$$
$$x_{18} = x_{12} + A_6 \qquad x_{18} = 52 \text{ m} \blacktriangleleft$$

Total distance traveled $= 56 + 108$ $\qquad d = 164$ m $\blacktriangleleft$



PROBLEM 11.185

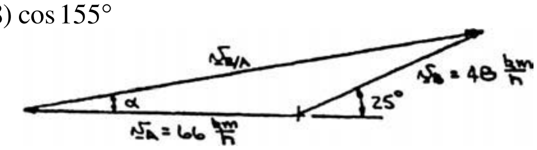
The velocities of commuter trains A and B are as shown. Knowing that the speed of each train is constant and that B reaches the crossing 10 min after A passed through the same crossing, determine (*a*) the relative velocity of B with respect to A, (*b*) the distance between the fronts of the engines 3 min after A passed through the crossing.

SOLUTION

(*a*) We have $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

The graphical representation of this equation is then as shown.

Then $$v_{B/A}^2 = 66^2 + 48^2 - 2(66)(48)\cos 155°$$



or $$v_{B/A} = 111.366 \text{ km/h}$$

and $$\frac{48}{\sin\alpha} = \frac{111.366}{\sin 155°}$$

or $$\alpha = 10.50°$$

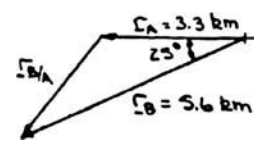
$$\mathbf{v}_{B/A} = 111.4 \text{ km/h} \measuredangle\, 10.50° \blacktriangleleft$$

(*b*) First note that

at $t = 3$ min, A is $(66 \text{ km/h})\left(\frac{3}{60}\right) = 3.3$ km west of the crossing.

at $t = 3$ min, B is $(48 \text{ km/h})\left(\frac{7}{60}\right) = 5.6$ km southwest of the crossing.

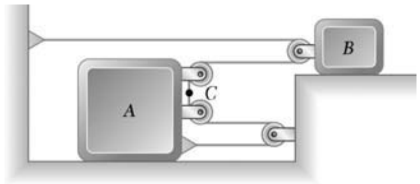
Now $$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$



Then at $t = 3$ min, we have

$$r_{B/A}^2 = 3.3^2 + 5.6^2 - 2(3.3)(5.6)\cos 25°$$

or $$r_{B/A} = 2.96 \text{ km} \blacktriangleleft$$



PROBLEM 11.186

Slider block B starts from rest and moves to the right with a constant acceleration of $1\ \text{ft/s}^2$. Determine (a) the relative acceleration of portion C of the cable with respect to slider block A, (b) the velocity of portion C of the cable after 2 s.

SOLUTION

Let d be the distance between the left and right supports.

Constraint of entire cable: $x_B + (x_B - x_A) + 2(d - x_A) = \text{constant}$

$$2v_B - 3v_A = 0 \qquad \text{and} \qquad 2a_B - 3a_A = 0$$

$$a_A = \frac{2}{3}a_B = \frac{2}{3}(1) = 0.667\ \text{ft/s}^2 \qquad \text{or} \qquad a_A = 0.667\ \text{ft/s}^2 \longrightarrow$$

Constraint of Point C: $\quad 2(d - x_A) + y_{C/A} = \text{constant}$

$$-2v_A + v_{C/A} = 0 \qquad \text{and} \qquad -2a_A + a_{C/A} = 0$$

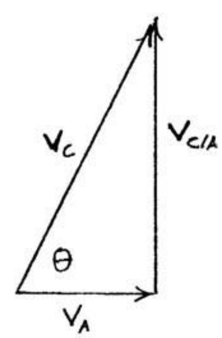
(a) $\quad a_{C/A} = 2a_A = 2(0.667) = 1.333\ \text{ft/s}^2$

$$\mathbf{a}_{C/A} = 1.333\ \text{ft/s}^2 \uparrow \blacktriangleleft$$

Velocity vectors after 2s: $\mathbf{v}_A = (0.667)(2) = 1.333\ \text{ft/s} \longrightarrow$

$$\mathbf{v}_{C/A} = (1.333)(2) = 2.666\ \text{ft/s} \uparrow$$

$$\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$$



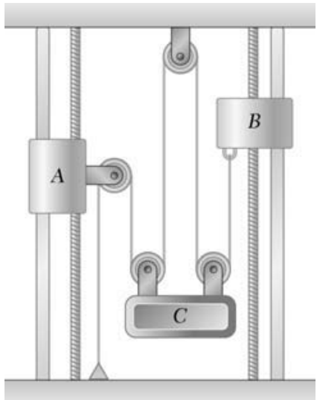
Sketch the vector addition.

$$v_C^2 = v_A^2 + v_{C/A}^2 = (1.333)^2 + (2.666)^2 = 8.8889(\text{ft/s})^2$$

$$v_C = 2.981\ \text{ft/s}$$

$$\tan\theta = \frac{v_{C/A}}{v_A} = \frac{2.666}{1.333} = 2, \quad \theta = 63.4°$$

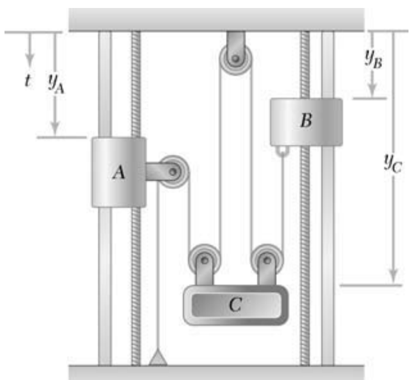
(b) $$\mathbf{v}_C = 2.98\ \text{ft/s} \angle 63.4° \blacktriangleleft$$



PROBLEM 11.187

Collar A starts from rest at $t = 0$ and moves downward with a constant acceleration of 7 in./s^2. Collar B moves upward with a constant acceleration, and its initial velocity is 8 in./s. Knowing that collar B moves through 20 in. between $t = 0$ and $t = 2$ s, determine (*a*) the accelerations of collar B and block C, (*b*) the time at which the velocity of block C is zero, (*c*) the distance through which block C will have moved at that time.

SOLUTION



From the diagram

$$-y_A + (y_C - y_A) + 2y_C + (y_C - y_B) = \text{constant}$$

Then $$-2v_A - v_B + 4v_C = 0 \quad (1)$$

and $$-2a_A - a_B + 4a_C = 0 \quad (2)$$

Given: $$(v_A)_0 = 0$$

$$(\mathbf{a}_A) = 7 \text{ in./s}^2 \downarrow$$

$$(\mathbf{v}_B)_0 = 8 \text{ in./s} \uparrow$$

$$\mathbf{a}_B = \text{constant} \uparrow$$

At $t = 2$ s $$y - (y_B)_0 = 20 \text{ in.} \uparrow$$

(*a*) We have $$y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

At $t = 2$ s: $$-20 \text{ in.} = (-8 \text{ in./s})(2 \text{ s}) + \frac{1}{2} a_B (2 \text{ s})^2$$

$$a_B = -4 \text{ in./s}^2 \quad \text{or} \quad \mathbf{a}_B = 2 \text{ in./s}^2 \uparrow \blacktriangleleft$$

Then, substituting into Eq. (2)

$$-2(7 \text{ in./s}^2) - (-2 \text{ in./s}^2) + 4a_C = 0$$

$$a_C = 3 \text{ in./s}^2 \quad \text{or} \quad \mathbf{a}_C = 3 \text{ in./s}^2 \downarrow \blacktriangleleft$$

PROBLEM 11.187 (Continued)

(*b*) Substituting into Eq. (1) at $t = 0$

$$-2(0) - (-8 \text{ in./s}) + 4(v_C)_0 = 0 \quad \text{or} \quad (v_C)_0 = -2 \text{ in./s}$$

Now $$v_C = (v_C)_0 + a_C t$$

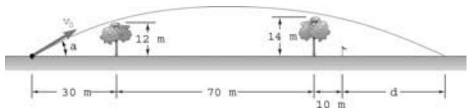
When $v_C = 0$: $$0 = (-2 \text{ in./s}) + (3 \text{ in./s}^2)t$$

or $$t = \frac{2}{3} \text{ s}$$ $t = 0.667$ s ◀

(*c*) We have $$y_C = (y_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2$$

At $t = \frac{2}{3}$ s: $$y_C - (y_C)_0 = (-2 \text{ in./s})\left(\frac{2}{3} \text{ s}\right) + \frac{1}{2}(3 \text{ in./s}^2)\left(\frac{2}{3} \text{ s}\right)^2$$

$$= -0.667 \text{ in.}$$ or $\mathbf{y}_C - (\mathbf{y}_C)_0 = 0.667 \text{ in.} \uparrow$ ◀



PROBLEM 11.188

A golfer hits a ball with an initial velocity of magnitude v_0 at an angle α with the horizontal. Knowing that the ball must clear the tops of two trees and land as close as possible to the flag, determine v_0 and the distance d when the golfer uses (*a*) a six-iron with $\alpha = 31°$, (*b*) a five-iron with $\alpha = 27°$.

SOLUTION

The horizontal and vertical motions are

$$x = (v_0 \cos\alpha)t \qquad \text{or} \qquad v_0 = \frac{x}{t\cos\alpha} \tag{1}$$

$$y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2 = x\tan\alpha - \frac{1}{2}gt^2$$

or
$$t^2 = \frac{2(x\tan\alpha - y)}{g} \tag{2}$$

At the landing Point *C*: $\quad y_C = 0, \qquad t = \dfrac{2v_0 \sin\alpha}{g}$

And
$$x_C = (v_0\cos\alpha)t = \frac{2v_0^2 \sin\alpha\cos\alpha}{g} \tag{3}$$

(*a*) $\alpha = 31°$

To clear tree *A*: $\quad x_A = 30 \text{ m}, \; y_A = 12 \text{ m}$

From (2), $\quad t_A^2 = \dfrac{2(30\tan 31° - 12)}{9.81} = 1.22851 \text{ s}^2, \qquad t_A = 1.1084 \text{ s}$

From (1), $\quad (v_0)_A = \dfrac{30}{1.1084\cos 31°} = 31.58 \text{ m/s}$

To clear tree *B*: $\quad x_B = 100 \text{ m}, \quad y_B = 14 \text{ m}$

From (2), $\quad (t_B)^2 = \dfrac{2(100\tan 31° - 14)}{9.81} = 9.3957 \text{ s}^2, \qquad t_B = 3.0652 \text{ s}$

From (1), $\quad (v_0)_B = \dfrac{100}{3.0652\cos 31°} = 38.06 \text{ m/s}$

The larger value governs, $\quad v_0 = 38.06 \text{ m/s}$ $\qquad v_0 = 38.1 \text{ m/s}$ ◀

From (3), $\quad x_C = \dfrac{(2)(38.06)^2 \sin 31° \cos 31°}{9.81} = 130.38 \text{ m}$

$d = x_C - 110$ $\qquad d = 20.4 \text{ m}$ ◀

PROBLEM 11.188 (Continued)

(*b*) $\alpha = 27°$

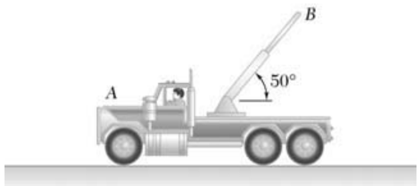
By a similar calculation, $t_A = 0.81846$ s, $(v_0)_A = 41.138$ m/s,

$t_B = 2.7447$ s, $(v_0)_B = 40.890$ m/s,

$v_0 = 41.138$ m/s $\qquad v_0 = 41.1$ m/s ◀

$x_C = 139.56$ m, $\qquad d = 29.6$ m ◀

PROBLEM 11.189



As the truck shown begins to back up with a constant acceleration of 4 ft/s^2, the outer section B of its boom starts to retract with a constant acceleration of 1.6 ft/s^2 relative to the truck. Determine (*a*) the acceleration of section B, (*b*) the velocity of section B when $t = 2$ s.

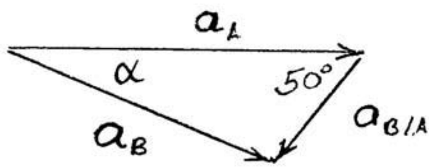
SOLUTION

For the truck, $\mathbf{a}_A = 4 \text{ ft/s}^2 \longrightarrow$

For the boom, $\mathbf{a}_{B/A} = 1.6 \text{ ft/s}^2 \swarrow 50°$

(*a*) $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Sketch the vector addition.



By law of cosines:

$$a_B^2 = a_A^2 + a_{B/A}^2 - 2a_A a_{B/A} \cos 50°$$

$$= 4^2 + 1.6^2 - 2(4)(1.6)\cos 50°$$

$$a_B = 3.214 \text{ ft/s}^2$$

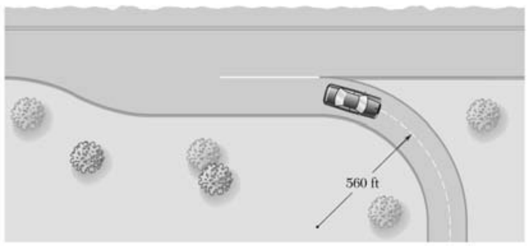
Law of sines: $\sin\alpha = \dfrac{a_{B/A}\sin 50°}{a_B} = \dfrac{1.6\sin 50°}{3.214} = 0.38131$

$\alpha = 22.4°$, $\mathbf{a}_B = 3.21 \text{ ft/s}^2 \nwarrow 22.4°$ ◀

(*b*) $\mathbf{v}_B = (v_B)_0 + a_B t = 0 + (3.214)(2)$

$\mathbf{v}_B = 6.43 \text{ ft/s}^2 \nwarrow 22.4°$ ◀

PROBLEM 11.190



A motorist traveling along a straight portion of a highway is decreasing the speed of his automobile at a constant rate before exiting from the highway onto a circular exit ramp with a radius of 560-ft. He continues to decelerate at the same constant rate so that 10 s after entering the ramp, his speed has decreased to 20 mi/h, a speed which he then maintains. Knowing that at this constant speed the total acceleration of the automobile is equal to one-quarter of its value prior to entering the ramp, determine the maximum value of the total acceleration of the automobile.

SOLUTION

First note $$v_{10} = 20 \text{ mi/h} = \frac{88}{3} \text{ ft/s}$$

While the car is on the straight portion of the highway.

$$a = a_{\text{straight}} = a_t$$

and for the circular exit ramp

$$a = \sqrt{a_t^2 + a_n^2}$$

where $$a_n = \frac{v^2}{\rho}$$

By observation, $a_{\max}$ occurs when v is maximum, which is at $t = 0$ when the car first enters the ramp.

For uniformly decelerated motion

$$v = v_0 + a_t t$$

and at $t = 10$ s: $$v = \text{constant} \Rightarrow a = a_n = \frac{v_{10}^2}{\rho}$$

$$a = \frac{1}{4} a_{\text{st.}}$$

Then $$a_{\text{straight}} = a_t \Rightarrow \frac{1}{4} a_t = \frac{v_{10}^2}{\rho} = \frac{\left(\frac{88}{3} \text{ ft/s}\right)^2}{560 \text{ ft}}$$

or $$a_t = -6.1460 \text{ ft/s}^2$$

(The car is decelerating; hence the minus sign.)

PROBLEM 11.190 (Continued)

Then at $t = 10$ s: $\frac{88}{3}$ ft/s $= v_0 + (-6.1460 \text{ ft/s}^2)(10 \text{ s})$

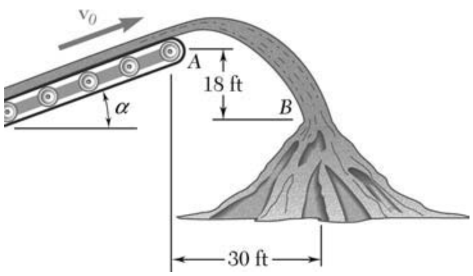
or $v_0 = 90.793$ ft/s

Then at $t = 0$:

$$a_{\max} = \sqrt{a_t^2 + \left(\frac{v_0^2}{\rho}\right)^2}$$

$$= \left\{(-6.1460 \text{ ft/s}^2)^2 + \left[\frac{(90.793 \text{ ft/s})^2}{560 \text{ ft}}\right]^2\right\}^{1/2}$$

or $a_{\max} = 15.95 \text{ ft/s}^2$ ◀



PROBLEM 11.191

Sand is discharged at A from a conveyor belt and falls onto the top of a stockpile at B. Knowing that the conveyor belt forms an angle $\alpha = 25°$ with the horizontal, determine (*a*) the speed v_0 of the belt, (*b*) the radius of curvature of the trajectory described by the sand at Point B.

SOLUTION

The motion is projectile motion. Place the origin at Point A. Then $x_0 = 0$ and $y_0 = 0$.
The coordinates of Point B are $x_B = 30$ ft and $y_B = -18$ ft.

Horizontal motion:

$$v_x = v_0 \cos 25° \quad (1)$$

$$x = v_0 t \cos 25° \quad (2)$$

Vertical motion:

$$v_y = v_0 \sin 25° - gt \quad (3)$$

$$y = v_0 t \sin 25° - \frac{1}{2} g t^2 \quad (4)$$

At Point B, Eq. (2) gives

$$v_0 t_B = \frac{x_B}{\cos 25°} = \frac{30}{\cos 25°} = 33.101 \text{ ft}$$

Substituting into Eq. (4),

$$-18 = (33.101)(\sin 25°) - \frac{1}{2}(32.2) t_B^2$$

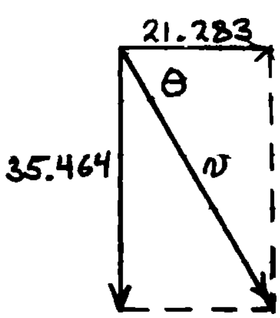
$$t_B = 1.40958 \text{ s}$$

(*a*) Speed of the belt.

$$v_0 = \frac{v_0 t_B}{t_B} = \frac{33.101}{1.40958} = 23.483$$

$v_0 = 23.4$ ft/s ◀

Eqs. (1) and (3) give



$$v_x = 23.483 \cos 25° = 21.283 \text{ ft/s}$$

$$v_y = (23.483) \sin 25° - (32.2)(1.40958) = -35.464 \text{ ft/s}$$

$$\tan\theta \frac{-v_y}{v_x} = 1.66632 \qquad \theta = 59.03°$$

$$v = 41.36 \text{ ft/s}$$

Components of acceleration.

$$\mathbf{a} = 32.2 \text{ ft/s}^2 \downarrow \qquad a_t = 32.2 \sin\theta$$

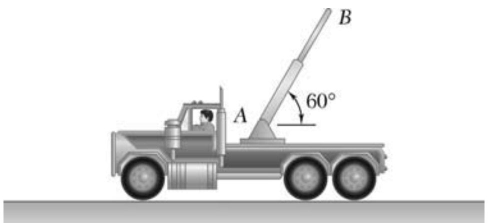
$$a_n = 32.2\cos\theta = 32.2\cos 59.03° = 16.57 \text{ ft/s}^2$$

(*b*) Radius of curvature at *B*.

$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{(41.36)^2}{16.57}$$

$\rho = 103.2$ ft ◀



PROBLEM 11.192

The end Point B of a boom is originally 5 m from fixed Point A when the driver starts to retract the boom with a constant radial acceleration of $\ddot{r} = -1.0\ \text{m/s}^2$ and lower it with a constant angular acceleration $\ddot{\theta} = -0.5\ \text{rad/s}^2$. At $t = 2$ s, determine (*a*) the velocity of Point B, (*b*) the acceleration of Point B, (*c*) the radius of curvature of the path.

SOLUTION

Radial motion. $r_0 = 5\ \text{m}, \quad \dot{r}_0 = 0, \quad \ddot{r} = -1.0\ \text{m/s}^2$

$$r = r_0 + \dot{r}_0 t + \frac{1}{2}\ddot{r}t^2 = 5 + 0 - 0.5t^2$$

$$\dot{r} = \dot{r}_0 + \ddot{r}t = 0 - 1.0t$$

At $t = 2$ s,

$$r = 5 - (0.5)(2)^2 = 3\ \text{m}$$

$$\dot{r} = (-1.0)(2) = -2\ \text{m/s}$$

Angular motion. $\theta_0 = 60° = \frac{\pi}{3}\text{rad}, \quad \dot{\theta}_0 = 0, \quad \ddot{\theta} = -0.5\ \text{rad/s}^2$

$$\theta = \theta_0 + \dot{\theta}_0 + \frac{1}{2}\ddot{\theta}t^2 = \frac{\pi}{3} + 0 - 0.25t^2$$

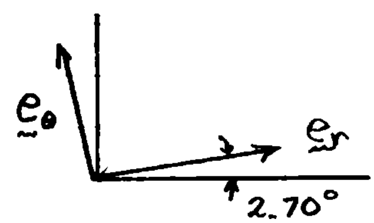
$$\dot{\theta} = \dot{\theta}_0 + \ddot{\theta}t = 0 - 0.5t$$

At $t = 2$ s,

$$\theta = \frac{\pi}{3} + 0 - (0.25)(2)^2 = 0.047198\ \text{rad} = 2.70°$$

$$\dot{\theta} = -(0.5)(2) = -1.0\ \text{rad/s}$$

Unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$.



(*a*) Velocity of Point B at $t = 2$ s.

$$\mathbf{v}_B = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

$$= -(2\ \text{m/s})\mathbf{e}_r + (3\ \text{m})(-1.0\ \text{rad/s})\mathbf{e}_\theta$$

$$\mathbf{v}_B = (-2.00\ \text{m/s})\mathbf{e}_r + (-3.00\ \text{m/s})\mathbf{e}_\theta \ ◀$$

$$\tan\alpha = \frac{v_\theta}{v_r} = \frac{-3.0}{-2.0} = 1.5 \qquad \alpha = 56.31°$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-2)^2 + (-3)^2} = 3.6055\ \text{m/s}$$

PROBLEM 11.192 (Continued)

Direction of velocity.

$$\mathbf{e}_t = \frac{\mathbf{v}}{v} = \frac{-2\mathbf{e}_r - 3\mathbf{e}_\theta}{3.6055} = -0.55470\mathbf{e}_r - 0.83205\mathbf{e}_\theta$$

$$\theta + \alpha = 2.70 + 56.31° = 59.01°$$

$$\mathbf{v}_B = 3.61 \text{ m/s} \nearrow 59.0° \blacktriangleleft$$

(*b*) Acceleration of Point *B* at $t = 2$ s.

$$\mathbf{a}_B = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$
$$= [-1.0 - (3)(-1)^2]\mathbf{e}_r + [(3)(-0.5) + (2)(-1.0)(-0.5)]\mathbf{e}_\theta$$

$$\mathbf{a}_B = (-4.00 \text{ m/s}^2)\mathbf{e}_r + (2.50 \text{ m/s}^2)\mathbf{e}_\theta \blacktriangleleft$$

$$\tan\beta = \frac{a_\theta}{a_r} = \frac{2.50}{-4.00} = -0.625 \qquad \beta = -32.00°$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-4)^2 + (2.5)^2} = 4.7170 \text{ m/s}^2$$

$$\theta + \beta = 2.70° - 32.00° = -29.30°$$

$$\mathbf{a}_B = 4.72 \text{ m/s}^2 \nearrow 29.3° \blacktriangleleft$$

Tangential component: $\mathbf{a}_t = (\mathbf{a} \cdot \mathbf{e}_t)\mathbf{e}_t$

$$\mathbf{a}_t = (-4\mathbf{e}_r + 2.5\mathbf{e}_\theta) \cdot (-0.55470\mathbf{e}_r - 0.83205\mathbf{e}_\theta)\mathbf{e}_t$$
$$= [(-4)(-0.55470) + (2.5)(-0.83205)]\mathbf{e}_t$$

$$= (0.138675 \text{ m/s}^2)\mathbf{e}_t = 0.1389 \text{ m/s}^2 \nearrow 59.0°$$

Normal component: $\mathbf{a}_n = \mathbf{a} - \mathbf{a}_t$

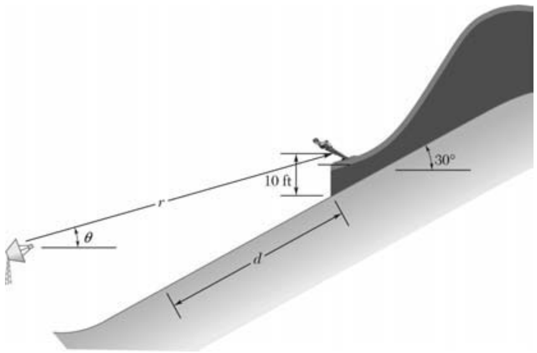
$$\mathbf{a}_n = -4\mathbf{e}_r + 2.5\mathbf{e}_\theta - (0.138675)(-0.55470\mathbf{e}_r - 0.83205\mathbf{e}_\theta)$$
$$= (-3.9231 \text{ m/s}^2)\mathbf{e}_r + (2.6154 \text{ m/s}^2)\mathbf{e}_\theta$$

$$a_n = \sqrt{(3.9231)^2 + (2.6154)^2} = 4.7149 \text{ m/s}^2$$

(*c*) Radius of curvature of the path.

$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{(3.6055 \text{ m/s})^2}{4.7149 \text{ m/s}} \qquad \rho = 2.76 \text{ m} \blacktriangleleft$$



PROBLEM 11.193

A telemetry system is used to quantify kinematic values of a ski jumper immediately before she leaves the ramp. According to the system $r = 500$ ft, $\dot{r} = -105$ ft/s, $\ddot{r} = -10$ ft/s^2, $\theta = 25°$, $\dot{\theta} = 0.07$ rad/s, $\ddot{\theta} = 0.06$ rad/s^2. Determine (*a*) the velocity of the skier immediately before she leaves the jump, (*b*) the acceleration of the skier at this instant, (*c*) the distance of the jump *d* neglecting lift and air resistance.

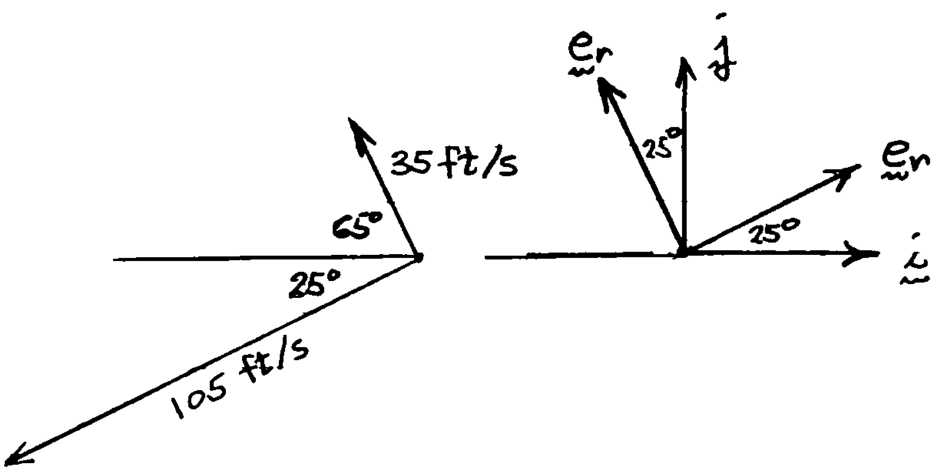
SOLUTION

(*a*) Velocity of the skier. $(r = 500 \text{ ft}, \quad \theta = 25°)$

$$\mathbf{v} = v_r\mathbf{e}_r + v_\theta\mathbf{e}_\theta = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$
$$= (-105 \text{ ft/s})\mathbf{e}_r + (500 \text{ ft})(0.07 \text{ rad/s})\mathbf{e}_\theta$$

$$\mathbf{v} = (-105 \text{ ft/s})\mathbf{e}_r + (35 \text{ ft/s})\mathbf{e}_\theta \blacktriangleleft$$

Direction of velocity:



$$\mathbf{v} = (-105\cos 25° - 35\cos 65°)\mathbf{i} + (35\sin 65° - 105\sin 25°)\mathbf{j}$$
$$= (-109.95 \text{ ft/s})\mathbf{i} + (-12.654 \text{ ft/s})\mathbf{j}$$

$$\tan\alpha = \frac{v_y}{v_x} = \frac{-12.654}{-109.95} \qquad \alpha = 6.565°$$

$$v = \sqrt{(105)^2 + (35)^2} = 110.68 \text{ ft/s}$$

$$v = 110.7 \text{ ft/s} \nearrow 6.57° \blacktriangleleft$$

(*b*) Acceleration of the skier.

$$\mathbf{a} = a_r\mathbf{e}_r + a_\theta\mathbf{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

$$a_r = -10 - (500)(0.07)^2 = -12.45 \text{ ft/s}^2$$

$$a_\theta = (500)(0.06) + (2)(-105)(0.07) = 15.30 \text{ ft/s}^2$$

$$\mathbf{a} = (-12.45 \text{ ft/s}^2)\mathbf{e}_r + (15.30 \text{ ft/s}^2)\mathbf{e}_\theta \blacktriangleleft$$

$$\mathbf{a} = (-12.45)(\mathbf{i}\cos 25° + \mathbf{j}\sin 25°) + (15.30)(-\mathbf{i}\cos 65° + \mathbf{j}\sin 65°)$$

$$= (-17.750 \text{ ft/s}^2)\mathbf{i} + (8.6049 \text{ ft/s}^2)\mathbf{j}$$

$$\tan\beta = \frac{a_y}{a_x} = \frac{8.6049}{-17.750} \qquad \beta = -25.9°$$

$$a = \sqrt{(12.45)^2 + (15.30)^2} = 19.725 \text{ ft/s}^2$$

$$a = 19.73 \text{ ft/s}^2 \measuredangle 25.9° \blacktriangleleft$$

(*c*) Distance of the jump *d*.

Projectile motion. Place the origin of the *xy*-coordinate system at the end of the ramp with the *x*-coordinate horizontal and positive to the left and the *y*-coordinate vertical and positive downward.

Horizontal motion: (Uniform motion)

$$x_0 = 0$$

$$\dot{x}_0 = 109.95 \text{ ft/s} \qquad \text{(from Part } a\text{)}$$

$$x = x_0 + \dot{x}_0 t = 109.95t$$

Vertical motion: (Uniformly accelerated motion)

$$y_0 = 0$$

$$\dot{y}_0 = 12.654 \text{ ft/s} \qquad \text{(from Part } a\text{)}$$

$$\ddot{y} = 32.2 \text{ ft/s}^2$$

$$y = y_0 + \dot{y}_0 t + \frac{1}{2}\ddot{y}t^2 = 12.654t - 16.1t^2$$

At the landing point,

$$x = d\cos 30° \qquad (1)$$

$$y = 10 + d\sin 30° \quad \text{or} \quad y - 10 = d\sin 30° \qquad (2)$$

Multiply Eq. (1) by sin 30° and Eq. (2) by cos 30° and subtract

$$x\sin 30° - (y - 10)\cos 30° = 0$$

$$(109.95t)\sin 30° - (12.654t + 16.1t^2 - 10)\cos 30° = 0$$

$$-13.943t^2 + 44.016t + 8.6603 = 0$$

$$t = -0.1858 \text{ s} \quad \text{and} \quad 3.3427 \text{ s}$$

Reject the negative root.

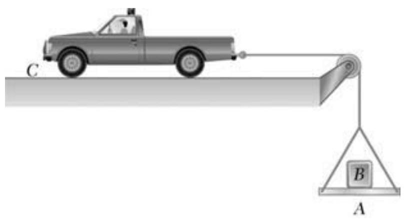
$$x = (109.95 \text{ ft/s})(3.3427 \text{ s}) = 367.53 \text{ ft}$$

$$d = \frac{x}{\cos 30°}$$

$d = 424$ ft ◀

CHAPTER 12

PROBLEM 12.CQ1



A 1000 lb boulder *B* is resting on a 200 lb platform *A* when truck *C* accelerates to the left with a constant acceleration. Which of the following statements are true (more than one may be true)?

(*a*) The tension in the cord connected to the truck is 200 lb

(*b*) The tension in the cord connected to the truck is 1200 lb

(*c*) The tension in the cord connected to the truck is greater than 1200 lb

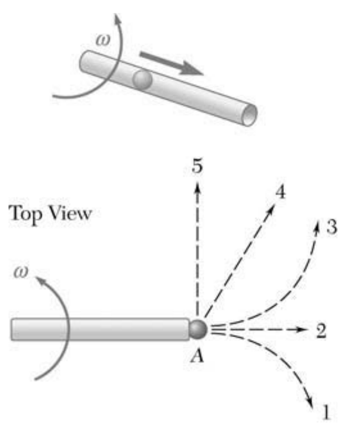
(*d*) The normal force between *A* and *B* is 1000 lb

(*e*) The normal force between *A* and *B* is 1200 lb

(*f*) None of the above

SOLUTION

Answer: (*c*) The tension will be greater than 1200 lb and the normal force will be greater than 1000 lb.



PROBLEM 12.CQ2

Marble A is placed in a hollow tube, and the tube is swung in a horizontal plane causing the marble to be thrown out. As viewed from the top, which of the following choices best describes the path of the marble after leaving the tube?

(*a*) 1

(*b*) 2

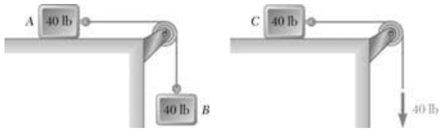
(*c*) 3

(*d*) 4

(*e*) 5

SOLUTION

Answer: (*d*) The particle will have velocity components along the tube and perpendicular to the tube. After it leaves, it will travel in a straight line.



PROBLEM 12.CQ3

The two systems shown start from rest. On the left, two 40 lb weights are connected by an inextensible cord, and on the right, a constant 40 lb force pulls on the cord. Neglecting all frictional forces, which of the following statements is true?

(*a*) Blocks *A* and *C* will have the same acceleration

(*b*) Block *C* will have a larger acceleration than block *A*

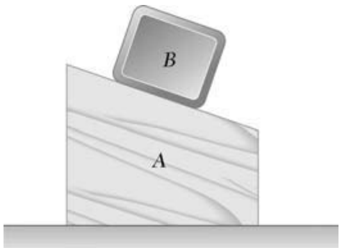
(*c*) Block *A* will have a larger acceleration than block *C*

(*d*) Block *A* will not move

(*e*) None of the above

SOLUTION

Answer: (*b*) If you draw a FBD of *B*, you will see that since it is accelerating downward, the tension in the cable will be less than 40 lb, so the acceleration of *A* will be less than the acceleration of *C*. Also, the system on the left has more inertia, so it is harder to accelerate than the system on the right.



PROBLEM 12.CQ4

The system shown is released from rest in the position shown. Neglecting friction, the normal force between block *A* and the ground is

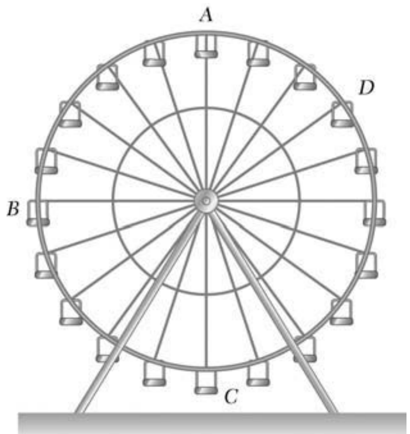
(*a*) less than the weight of *A* plus the weight of *B*

(*b*) equal to the weight of *A* plus the weight of *B*

(*c*) greater than the weight of *A* plus the weight of *B*

SOLUTION

Answer: (*a*) Since *B* has an acceleration component downward the normal force between *A* and the ground will be less than the sum of the weights.



PROBLEM 12.CQ5

People sit on a Ferris wheel at Points *A*, *B*, *C* and *D*. The Ferris wheel travels at a constant angular velocity. At the instant shown, which person experiences the largest force from his or her chair (back and seat)? Assume you can neglect the size of the chairs, that is, the people are located the same distance from the axis of rotation.

(*a*) *A*

(*b*) *B*

(*c*) *C*

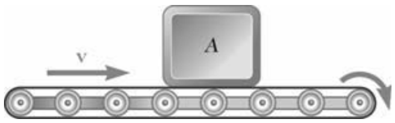
(*d*) *D*

(*e*) The force is the same for all the passengers.

SOLUTION

Answer: (*c*) Draw a FBD and KD at each location and it will be clear that the maximum force will be experiences by the person at Point *C*.

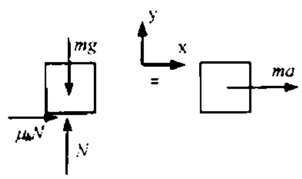
PROBLEM 12.F1



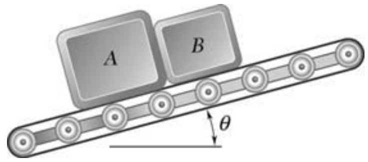
Crate A is gently placed with zero initial velocity onto a moving conveyor belt. The coefficient of kinetic friction between the crate and the belt is μ_k. Draw the FBD and KD for A immediately after it contacts the belt.

SOLUTION

Answer:



PROBLEM 12.F2

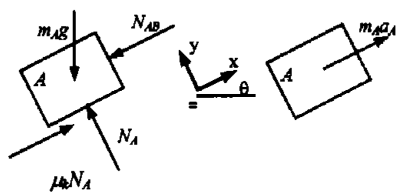


Two blocks weighing W_A and W_B are at rest on a conveyor that is initially at rest. The belt is suddenly started in an upward direction so that slipping occurs between the belt and the boxes. Assuming the coefficient of friction between the boxes and the belt is μ_k, draw the FBDs and KDs for blocks A and B. How would you determine if A and B remain in contact?

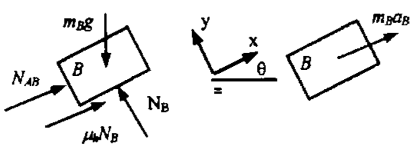
SOLUTION

Answer:

Block A

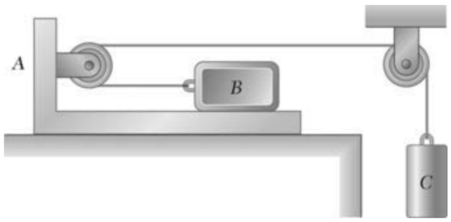


Block B



To see if they remain in contact assume $a_A = a_B$ and then check to see if N_{AB} is greater than zero.

PROBLEM 12.F3

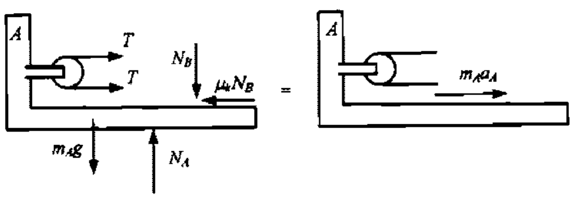


Objects *A*, *B*, and *C* have masses m_A, m_B, and m_C respectively. The coefficient of kinetic friction between *A* and *B* is μ_k, and the friction between *A* and the ground is negligible and the pulleys are massless and frictionless. Assuming *B* slides on *A* draw the FBD and KD for each of the three masses *A*, *B* and *C*.

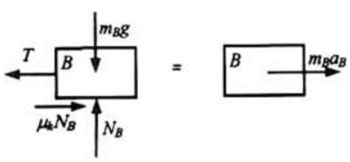
SOLUTION

Answer:

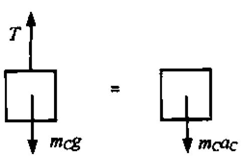
Block *A*

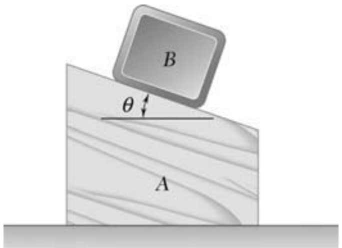


Block *B*



Block *C*



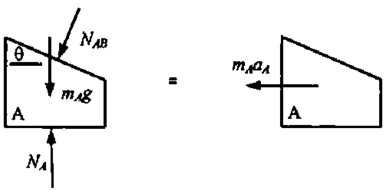


PROBLEM 12.F4

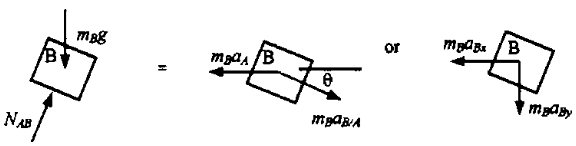
Blocks A and B have masses m_A and m_B respectively. Neglecting friction between all surfaces, draw the FBD and KD for each mass.

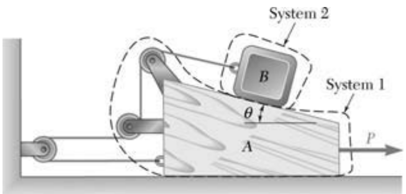
SOLUTION

Block A



Block B



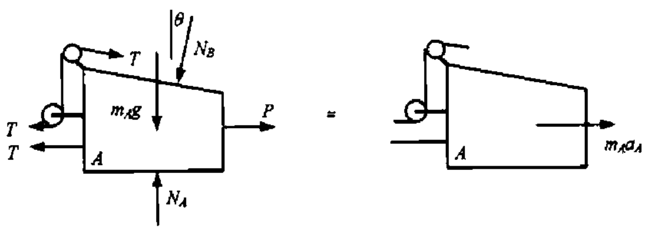


PROBLEM 12.F5

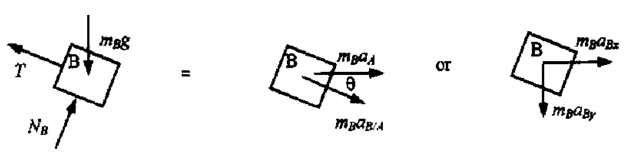
Blocks A and B have masses m_A and m_B respectively. Neglecting friction between all surfaces, draw the FBD and KD for the two systems shown.

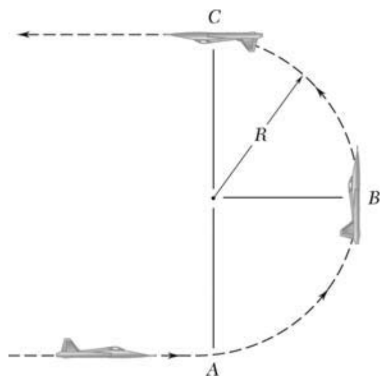
SOLUTION

System 1



System 2

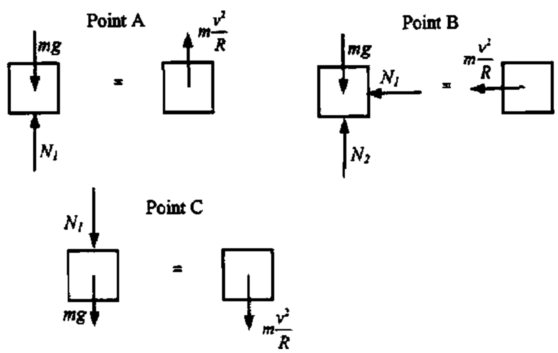


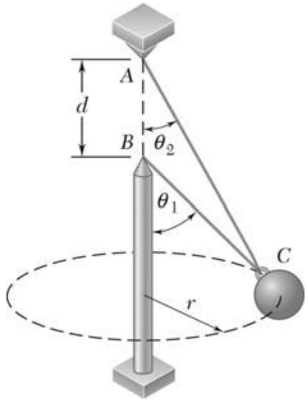


PROBLEM 12.F6

A pilot of mass m flies a jet in a half vertical loop of radius R so that the speed of the jet, v, remains constant. Draw a FBD and KD of the pilot at Points A, B and C.

SOLUTION

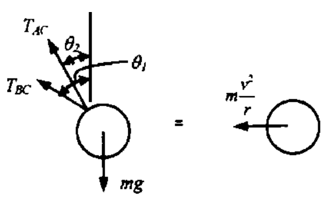


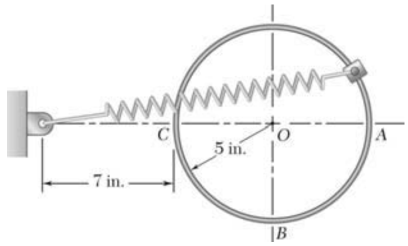


PROBLEM 12.F7

Wires AC and BC are attached to a sphere which revolves at a constant speed v in the horizontal circle of radius r as shown. Draw a FBD and KD of C.

SOLUTION

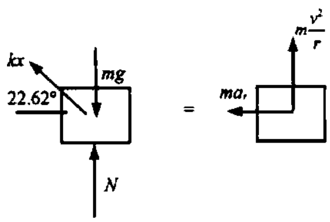




PROBLEM 12.F8

A collar of mass m is attached to a spring and slides without friction along a circular rod in a vertical plane. The spring has an undeformed length of 5 in. and a constant k. Knowing that the collar has a speed v at Point B, draw the FBD and KD of the collar at this point.

SOLUTION



where $x = 7/12$ ft and $r = 5/12$ ft.

PROBLEM 12.1

Astronauts who landed on the moon during the Apollo 15, 16 and 17 missions brought back a large collection of rocks to the earth. Knowing the rocks weighed 139 lb when they were on the moon, determine (*a*) the weight of the rocks on the earth, (*b*) the mass of the rocks in slugs. The acceleration due to gravity on the moon is 5.30 ft/s^2.

SOLUTION

Since the rocks weighed 139 lb on the moon, their mass is

$$m = \frac{W_{\text{moon}}}{g_{\text{moon}}} = \frac{139 \text{ lb}}{5.30 \text{ ft/s}^2} = 26.226 \text{ lb} \cdot \text{s}^2/\text{ft}$$

(*a*) On the earth, $W_{\text{earth}} = mg_{\text{earth}}$

$$w = (26.226 \text{ lb} \cdot \text{s}^2/\text{ft})(32.2 \text{ ft/s}^2)$$

$w = 844$ lb ◀

(*b*) Since $1 \text{ slug} = 1 \text{ lb} \cdot \text{s}^2/\text{ft}$,

$m = 26.2$ slugs ◀

PROBLEM 12.2

The value of g at any latitude ϕ may be obtained from the formula

$$g = 32.09(1 + 0.0053 \sin^2\phi) \text{ ft/s}^2$$

which takes into account the effect of the rotation of the earth, as well as the fact that the earth is not truly spherical. Determine to four significant figures (a) the weight in pounds, (b) the mass in pounds, (c) the mass in $\text{lb}\cdot\text{s}^2/\text{ft}$, at the latitudes of 0°, 45°, and 60°, of a silver bar, the mass of which has been officially designated as 5 lb.

SOLUTION

$$g = 32.09(1 + 0.0053 \sin^2\phi) \text{ ft/s}^2$$

$\phi = 0°$: $\quad g = 32.09 \text{ ft/s}^2$

$\phi = 45°$: $\quad g = 32.175 \text{ ft/s}^2$

$\phi = 90°$: $\quad g = 32.26 \text{ ft/s}^2$

(a) Weight: $\quad W = mg$

$\phi = 0°$: $\quad W = (0.1554 \text{ lb}\cdot\text{s}^2/\text{ft})(32.09 \text{ ft/s}^2) = 4.987 \text{ lb}$ ◀

$\phi = 45°$: $\quad W = (0.1554 \text{ lb}\cdot\text{s}^2/\text{ft})(32.175 \text{ ft/s}^2) = 5.000 \text{ lb}$ ◀

$\phi = 90°$: $\quad W = (0.1554 \text{ lb}\cdot\text{s}^2/\text{ft})(32.26 \text{ ft/s}^2) = 5.013 \text{ lb}$ ◀

(b) Mass: At all latitudes: $\quad m = 5.000 \text{ lb}$ ◀

(c) or $\quad m = \dfrac{5.00 \text{ lb}}{32.175 \text{ ft/s}^2} \quad m = 0.1554 \text{ lb}\cdot\text{s}^2/\text{ft}$ ◀

PROBLEM 12.3

A 400-kg satellite has been placed in a circular orbit 1500 km above the surface of the earth. The acceleration of gravity at this elevation is 6.43 m/s^2. Determine the linear momentum of the satellite, knowing that its orbital speed is 25.6×10^3 km/h.

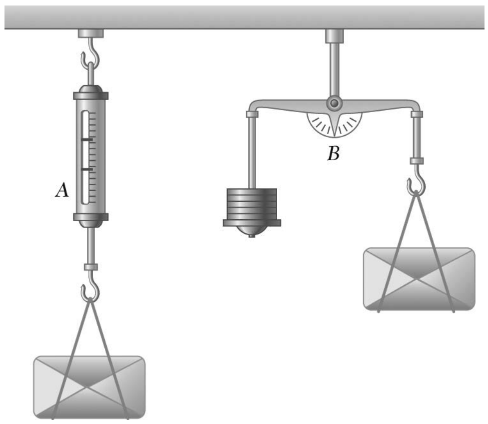
SOLUTION

Mass of satellite is independent of gravity: $m = 400$ kg

$$v = 25.6\times10^3 \text{ km/h}$$
$$= (25.6\times10^6 \text{ m/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 7.111\times10^3 \text{ m/s}$$

$$L = mv = (400 \text{ kg})(7.111\times10^3 \text{ m/s})$$

$L = 2.84\times10^6 \text{ kg}\cdot\text{m/s}$ ◀

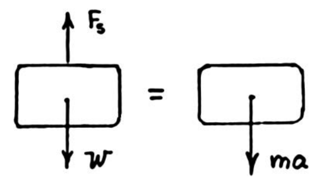


PROBLEM 12.4

A spring scale A and a lever scale B having equal lever arms are fastened to the roof of an elevator, and identical packages are attached to the scales as shown. Knowing that when the elevator moves downward with an acceleration of 1 m/s^2 the spring scale indicates a load of 60 N, determine (a) the weight of the packages, (b) the load indicated by the spring scale and the mass needed to balance the lever scale when the elevator moves upward with an acceleration of 1 m/s^2.

SOLUTION

Assume $\quad g = 9.81 \text{ m/s}^2$



$$m = \frac{W}{g}$$

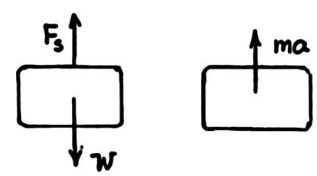
$$+\uparrow \Sigma F = ma: \quad F_s - W = -\frac{W}{g}a$$

$$W\left(1 - \frac{a}{g}\right) = F_s$$

or $\quad W = \dfrac{F_s}{1 - \dfrac{a}{g}} = \dfrac{60}{1 - \dfrac{1}{9.81}}$

$W = 66.8 \text{ N}$ ◀

(b)



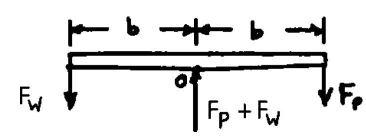
$$\Sigma F = ma: \quad F_s - W = \frac{W}{g}a$$

$$F_s = W\left(1 + \frac{a}{g}\right)$$

$$= 66.81\left(1 + \frac{1}{9.81}\right)$$

$F_s = 73.6 \text{ N}$ ◀

For the balance system B,



$$\Sigma M_0 = 0: \quad bF_w - bF_p = 0$$

$$F_w = F_p$$

PROBLEM 12.4 (Continued)

But $$F_w = W_w \left(1 + \frac{a}{g}\right)$$

and $$F_p = W_p \left(1 + \frac{a}{g}\right)$$

so that $$W_w = W_p$$

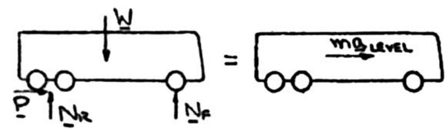
and $$m_w = \frac{W_p}{g} = \frac{66.81}{9.81}$$ $m_w = 6.81 \text{ kg}$ ◀

PROBLEM 12.5

In anticipation of a long 7° upgrade, a bus driver accelerates at a constant rate of 3 ft/s^2 while still on a level section of the highway. Knowing that the speed of the bus is 60 mi/h as it begins to climb the grade and that the driver does not change the setting of his throttle or shift gears, determine the distance traveled by the bus up the grade when its speed has decreased to 50 mi/h.

SOLUTION

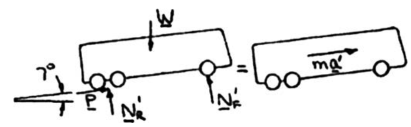
First consider when the bus is on the level section of the highway.



$$a_{\text{level}} = 3 \text{ ft/s}^2$$

We have $\xrightarrow{+} \Sigma F_x = ma: \quad P = \dfrac{W}{g} a_{\text{level}}$

Now consider when the bus is on the upgrade.



We have $+\!\nearrow \Sigma F_x = ma: \quad P - W \sin 7° = \dfrac{W}{g} a'$

Substituting for P $\quad \dfrac{W}{g} a_{\text{level}} - W \sin 7° = \dfrac{W}{g} a'$

or

$$a' = a_{\text{level}} - g \sin 7°$$
$$= (3 - 32.2 \sin 7°) \text{ ft/s}^2$$
$$= -0.92419 \text{ ft/s}^2$$

For the uniformly decelerated motion

$$v^2 = (v_0)^2_{\text{upgrade}} + 2a'(x_{\text{upgrade}} - 0)$$

Noting that $60 \text{ mi/h} = 88 \text{ ft/s}$, then when $v = 50 \text{ mi/h} \left(= \dfrac{5}{6} v_0\right)$, we have

$$\left(\frac{5}{6} \times 88 \text{ ft/s}\right)^2 = (88 \text{ ft/s})^2 + 2(-0.92419 \text{ ft/s}^2) x_{\text{upgrade}}$$

or

$$x_{\text{upgrade}} = 1280.16 \text{ ft}$$

or $\quad x_{\text{upgrade}} = 0.242 \text{ mi}$ ◀

PROBLEM 12.6

A hockey player hits a puck so that it comes to rest 10 s after sliding 100 ft on the ice. Determine (*a*) the initial velocity of the puck, (*b*) the coefficient of friction between the puck and the ice.

SOLUTION

(*a*) Assume uniformly decelerated motion.

Then $v = v_0 + at$

At $t = 10$ s: $0 = v_0 + a(10)$

$$a = -\frac{v_0}{10}$$

Also $v^2 = v_0^2 + 2a(x - 0)$

At $t = 10$ s: $0 = v_0^2 + 2a(100)$

Substituting for a $\quad 0 = v_0^2 + 2\left(-\frac{v_0}{10}\right)(100) = 0$

$v_0 = 20.0$ ft/s $\qquad$ or $\quad v_0 = 20.0$ ft/s ◀

and $\quad a = -\frac{20}{10} = -2$ ft/s^2

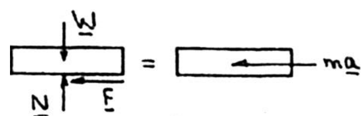
Alternate solution to part (*a*) $\quad d = d_0 + v_0 t + \frac{1}{2}at^2$

$$d = v_0 t + \frac{1}{2}\left(-\frac{v_0}{t}\right)t^2$$

$$d = \frac{1}{2}v_0 t$$

$$v_0 = \frac{2d}{t}$$

(*b*) We have

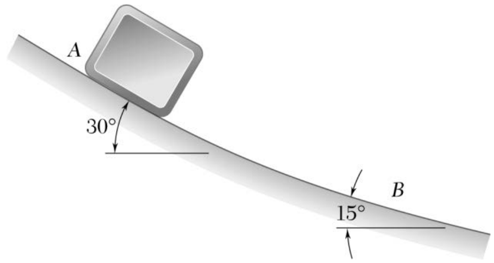


$+\uparrow \Sigma F_y = 0$: $\quad N - W = 0 \qquad N = W = mg$

Sliding: $\quad F = \mu_k N = \mu_k mg$

$\xrightarrow{+} \Sigma F_x = ma$: $\quad -F = ma \qquad -\mu_k mg = ma$

$\mu_k = -\frac{a}{g} = -\frac{-2.0 \text{ ft/s}^2}{32.2 \text{ ft/s}^2} \qquad \mu_k = 0.0621$ ◀



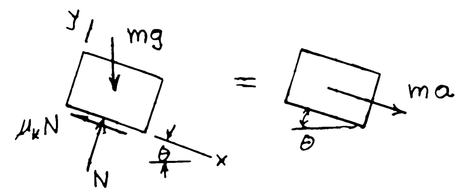
PROBLEM 12.7

The acceleration of a package sliding at Point A is 3 m/s^2. Assuming that the coefficient of kinetic friction is the same for each section, determine the acceleration of the package at Point B.

SOLUTION

For any angle θ.

Use x and y coordinates as shown.



$$a_y = 0$$

$$+\nearrow \Sigma F_y = ma_y: \quad N - mg\cos\theta = 0$$

$$N = mg\cos\theta$$

$$+\searrow \Sigma F_x = ma_x: \quad mg\sin\theta - \mu_k N = ma_x$$

$$a_x = g(\sin\theta - \mu_k\cos\theta)$$

At Point A. $\quad \theta = 30°, \quad a_x = 3 \text{ m/s}^2$

$$\mu_k = \frac{g\sin 30° - a_x}{g\cos 30°}$$

$$= \frac{9.81\sin 30° - 3}{9.81\cos 30°}$$

$$= 0.22423$$

At Point B. $\quad \theta = 15°, \quad \mu_k = 0.22423$

$$a_x = 9.81(\sin 15° - 0.22423\cos 15°)$$

$$= 0.414 \text{ m/s}$$

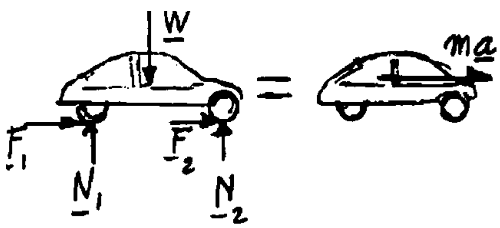
$\mathbf{a} = 0.414 \text{ m/s}^2 \searrow 15°$ ◀

PROBLEM 12.8

Determine the maximum theoretical speed that may be achieved over a distance of 60 m by a car starting from rest, knowing that the coefficient of static friction is 0.80 between the tires and the pavement and that 60 percent of the weight of the car is distributed over its front wheels and 40 percent over its rear wheels. Assume (*a*) four-wheel drive, (*b*) front-wheel drive, (*c*) rear-wheel drive.

SOLUTION

(*a*) Four-wheel drive



$$+\uparrow \Sigma F_y = 0: \quad N_1 + N_2 - W = 0$$

$$F_1 + F_2 = \mu N_1 + \mu N_2$$
$$= \mu(N_1 + N_2) = \mu w$$

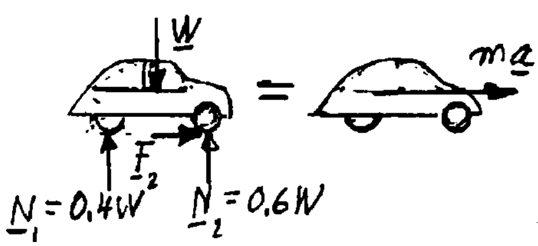
$$\xrightarrow{+} \Sigma F_x = ma: \quad F_1 + F_2 = ma$$
$$\mu w = ma$$

$$a = \frac{\mu W}{m} = \mu \frac{mg}{m} = \mu g = 0.80(9.81)$$
$$a = 7.848 \text{ m/s}^2$$

$$v^2 = 2ax = 2(7.848 \text{ m/s}^2)(60 \text{ m}) = 941.76 \text{ m}^2/\text{s}^2$$

$v = 30.69$ m/s $\qquad v = 110.5$ km/h ◀

(*b*) Front-wheel drive



$$F_2 = ma$$
$$\mu(0.6\ W) = ma$$

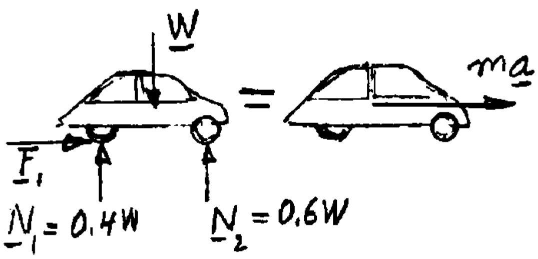
$$a = \frac{0.6\mu W}{m} = \frac{0.6\mu mg}{m}$$
$$= 0.6\mu g = 0.6(0.80)(9.81)$$
$$a = 4.709 \text{ m/s}^2$$

$$v^2 = 2ax = 2(4.709 \text{ m/s}^2)(60 \text{ m}) = 565.1 \text{ m}^2/\text{s}^2$$

$v = 23.77$ m/s $\qquad v = 85.6$ km/h ◀

PROBLEM 12.8 (Continued)

(*c*) Rear-wheel drive



$$F_1 = ma$$

$$\mu(0.4\,W) = ma$$

$$a = \frac{0.4\mu W}{m} = \frac{0.4\mu mg}{m}$$

$$= 0.4\mu g = 0.4(0.80)(9.81)$$

$$a = 3.139 \text{ m/s}^2$$

$$v^2 = 2ax = 2(3.139 \text{ m/s}^2)(60 \text{ m}) = 376.7 \text{ m}^2/\text{s}^2$$

$$v = 19.41 \text{ m/s}$$

$v = 69.9$ km/h ◀

PROBLEM 12.9

If an automobile's braking distance from 90 km/h is 45 m on level pavement, determine the automobile's braking distance from 90 km/h when it is (*a*) going up a 5° incline, (*b*) going down a 3-percent incline. Assume the braking force is independent of grade.

SOLUTION

Assume uniformly decelerated motion in all cases.

For braking on the level surface,

$$v_0 = 90 \text{ km/h} = 25 \text{ m/s}, \quad v_f = 0$$

$$x_f - x_0 = 45 \text{ m}$$

$$v_f^2 = v_0^2 + 2a(x_f - x_0)$$

$$a = \frac{v_f^2 - v_0^2}{2(x_f - x_0)}$$

$$= \frac{0 - (25)^2}{(2)(45)}$$

$$= -6.9444 \text{ m/s}^2$$

Braking force.

$$F_b = ma$$

$$= \frac{W}{g}a$$

$$= -\frac{6.944}{9.81}W$$

$$= -0.70789W$$

(*a*) Going up a 5° incline.

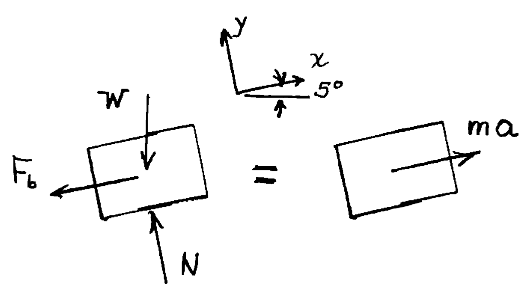
$$\nearrow \Sigma F = ma$$

$$-F_b - W\sin 5° = \frac{W}{g}a$$

$$a = -\frac{F_b + W\sin 5°}{W}g$$

$$= -(0.70789 + \sin 5°)(9.81)$$

$$= -7.79944 \text{ m/s}^2$$



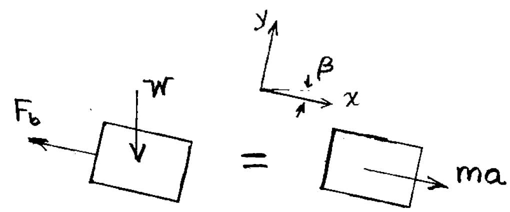
$$x_f - x_0 = \frac{v_f^2 - v_0^2}{2a}$$

$$= \frac{0 - (25)^2}{(2)(-7.79944)}$$

$$x_f - x_0 = 40.1 \text{ m} \blacktriangleleft$$

PROBLEM 12.9 (Continued)

(*b*) Going down a 3 percent incline.



$$\tan\beta = \frac{3}{100} \qquad \beta = 1.71835°$$

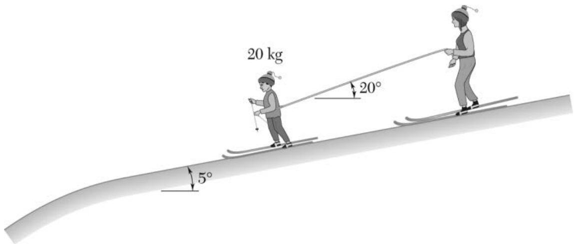
$$-F_b + W\sin\beta = \frac{W}{g}a$$

$$a = -(0.70789 - \sin\beta)(9.81)$$

$$= -6.65028 \text{ m/s}$$

$$x_f = x_0 = \frac{0-(25)^2}{(2)(-6.65028)}$$

$x_f - x_0 = 47.0$ m ◀

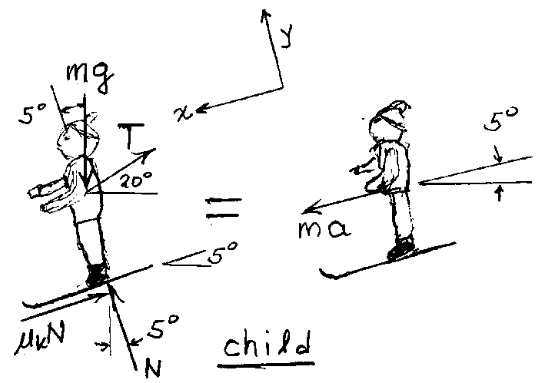


PROBLEM 12.10

A mother and her child are skiing together, and the mother is holding the end of a rope tied to the child's waist. They are moving at a speed of 7.2 km/h on a gently sloping portion of the ski slope when the mother observes that they are approaching a steep descent. She pulls on the rope with an average force of 7 N. Knowing the coefficient of friction between the child and the ground is 0.1 and the angle of the rope does not change, determine (*a*) the time required for the child's speed to be cut in half, (*b*) the distance traveled in this time.

SOLUTION

Draw free body diagram of child.



$\Sigma \mathbf{F} = m\mathbf{a}$:

x-direction: $mg \sin 5° - \mu_k N - T \cos 15° = ma$

y-direction: $N - mg \cos 5° + T \sin 15° = 0$

From *y*-direction,

$$N = mg \cos 5° - T \sin 15° = (20 \text{ kg})(9.81 \text{ m/s}^2)\cos 5° - (7 \text{ N})\sin 15° = 193.64 \text{ N}$$

From *x*-direction,

$$a = g \sin 5° - \frac{\mu_k N}{m} - \frac{T \cos 15°}{m}$$

$$= (9.81 \text{ m/s}^2)\sin 5° - \frac{(0.1)(193.64 \text{ N})}{20 \text{ kg}} - \frac{(7 \text{ N}) \cos 15°}{20 \text{ kg}}$$

$$= -0.45128 \text{ m/s}^2 \qquad \text{(in } x\text{-direction.)}$$

$$v_0 = 7.2 \text{ km/h} = 2 \text{ m/s} \qquad x_0 = 0$$

$$v_f = \frac{1}{2}\, v_0 = 1 \text{ m/s}$$

$$v_f = v_0 + at \qquad t = \frac{v_f - v_0}{a} = \frac{-1 \text{ m/s}}{-0.45128 \text{ m/s}^2} = 2.2159 \text{ s}$$

PROBLEM 12.10 (Continued)

(*a*) *Time elapsed.* $t = 2.22 \text{ s}$ ◀

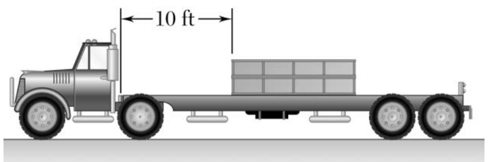
(*b*) *Corresponding distance.*

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$= 0 + (2 \text{ m/s})(2.2159 \text{ s}) + \frac{1}{2}(-0.45128 \text{ m/s}^2)(2.2159 \text{ s})^2$$

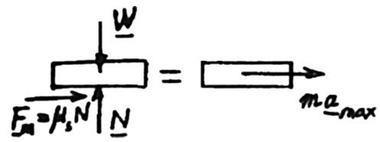
$x = 3.32 \text{ m}$ ◀

PROBLEM 12.11



The coefficients of friction between the load and the flat-bed trailer shown are $\mu_s = 0.40$ and $\mu_k = 0.30$. Knowing that the speed of the rig is 72 km/h, determine the shortest distance in which the rig can be brought to a stop if the load is not to shift.

SOLUTION



Load: We assume that sliding of load relative to trailer is impending:

$$F = F_m$$
$$= \mu_s N$$

Deceleration of load is same as deceleration of trailer, which is the maximum allowable deceleration $\mathbf{a}_{max}$.

$$+\uparrow \Sigma F_y = 0: \quad N - W = 0 \quad N = W$$

$$F_m = \mu_s N = 0.40\,W$$

$$\xrightarrow{+} \Sigma F_x = ma: \quad F_m = ma_{max}$$

$$0.40\ W = \frac{W}{g} a_{max} \qquad a_{max} = 3.924 \text{ m/s}^2$$

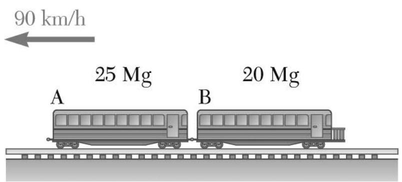
$$\mathbf{a}_{max} = 3.92 \text{ m/s}^2 \longrightarrow$$

Uniformly accelerated motion.

$$v^2 = v_0^2 + 2ax \quad \text{with} \quad v = 0 \qquad v_0 = 72 \text{ km/h} = 20 \text{ m/s}$$

$$a = -a_{max} = 3.924 \text{ m/s}^2$$

$$0 = (20)^2 + 2(-3.924)x \qquad x = 51.0 \text{ m} \blacktriangleleft$$



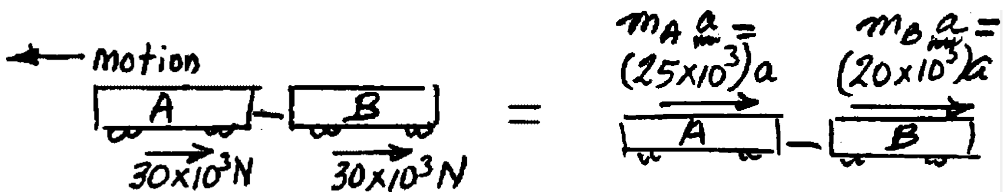
PROBLEM 12.12

A light train made up of two cars is traveling at 90 km/h when the brakes are applied to both cars. Knowing that car *A* has a mass of 25 Mg and car *B* a mass of 20 Mg, and that the braking force is 30 kN on each car, determine (*a*) the distance traveled by the train before it comes to a stop, (*b*) the force in the coupling between the cars while the train is showing down.

SOLUTION

$v_0 = 90 \text{ km/h} = 90/3.6 = 25 \text{ m/s}$

(*a*) Both cars:

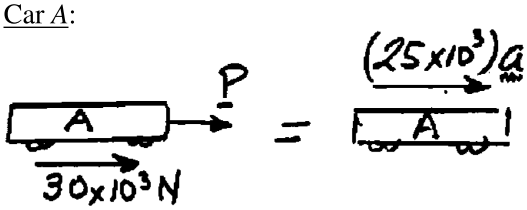


$\xrightarrow{+} \Sigma F_x = \Sigma ma: \quad 60\times10^3 \text{ N} = (45\times10^3 \text{ kg})a \qquad a = 1.333 \text{ m/s}^2 \longrightarrow$

$v^2 = v_0^1 + 2ax: \quad 0 = (25)^2 + 2(-1.333)x$

Stopping distance: $x = 234 \text{ m}$ ◀

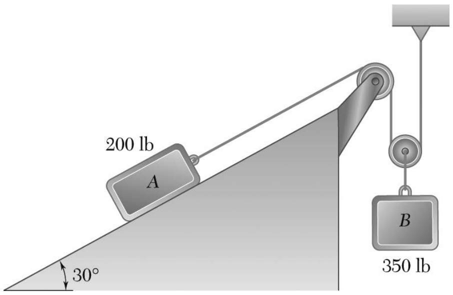
(*b*) Car *A*:



$\xrightarrow{+} \Sigma F_x = ma: \quad 30\times10^3 + P = (25\times10^3)a$

$P = (25\times10^3)(1.333) - 30\times10^3$

Coupling force: $P = +3332 \text{ N}$ $\qquad P = 3.33 \text{ kN (tension)}$ ◀



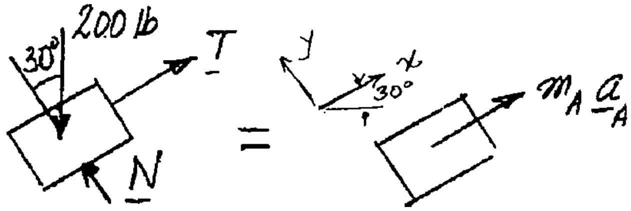
PROBLEM 12.13

The two blocks shown are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and between block A and the incline, determine (*a*) the acceleration of each block, (*b*) the tension in the cable.

SOLUTION

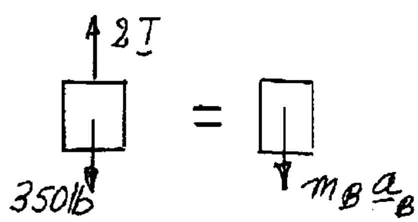
(*a*) We note that $a_B = \frac{1}{2} a_A$.

Block *A*



$$\nearrow + \Sigma F_x = m_A a_A: \quad T - (200 \text{ lb}) \sin 30° = \frac{200}{32.2} a_A \tag{1}$$

Block *B*



$$+\downarrow \Sigma F_y = m_B a_B: \quad 350 \text{ lb} - 2T = \frac{350}{32.2}\left(\frac{1}{2}\, a_A\right) \tag{2}$$

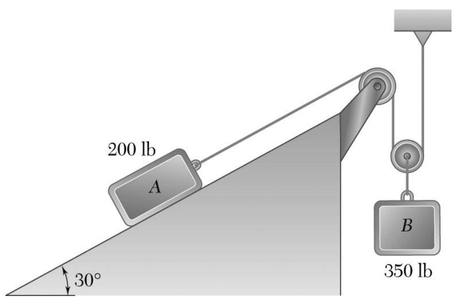
(*a*) Multiply Eq. (1) by 2 and add Eq. (2) in order to eliminate *T*:

$$-2(200)\sin 30° + 350 = 2\frac{200}{32.2} a_A + \frac{350}{32.2}\left(\frac{1}{2} a_A\right)$$

$$150 = \frac{575}{32.2} a_A \qquad \mathbf{a}_A = 8.40 \text{ ft/s}^2 \angle 30° \blacktriangleleft$$

$$a_B = \frac{1}{2} a_A = \frac{1}{2}(8.40 \text{ ft/s}^2), \qquad \mathbf{a}_B = 4.20 \text{ ft/s}^2 \downarrow \blacktriangleleft$$

(*b*) From Eq. (1), $T - (200)\sin 30° = \frac{200}{32.2}(8.40)$ $\qquad T = 152.2 \text{ lb} \blacktriangleleft$



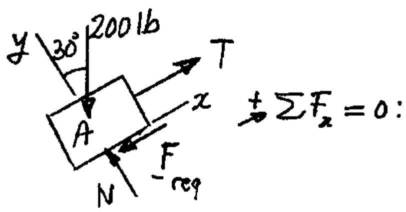
PROBLEM 12.14

Solve Problem 12.13, assuming that the coefficients of friction between block *A* and the incline are $\mu_s = 0.25$ and $\mu_k = 0.20$.

PROBLEM 12.13 The two blocks shown are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and between block *A* and the incline, determine (*a*) the acceleration of each block, (*b*) the tension in the cable.

SOLUTION

We first determine whether the blocks move by computing the friction force required to maintain block A in equilibrium. $T = 175$ lb. When B in equilibrium,



$$+\nearrow \Sigma F_x = 0: \quad 175 - 200\sin 30° - F_{\text{req}} = 0$$

$$F_{\text{req}} = 75.0 \text{ lb}$$

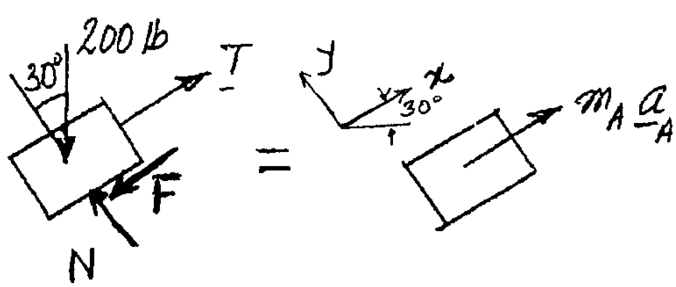
$$+\nwarrow \Sigma F_y = 0: \quad N - 200\cos 30° = 0 \quad N = 173.2 \text{ lb}$$

$$F_M = \mu_s N = 0.25(173.2 \text{ lb}) = 43.3 \text{ lb}$$

Since $F_{\text{req}} > \mathbf{F}_m$, blocks will move ($A$ up and B down).

We note that $a_B = \frac{1}{2} a_A$.

<u>Block *A*</u>

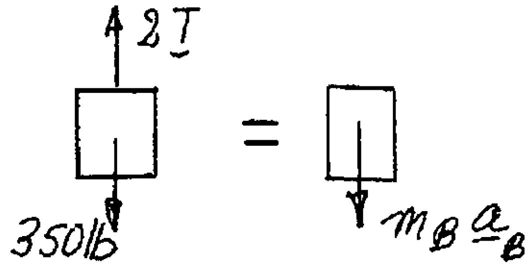


$$F = \mu_k N = (0.20)(173.2) = 34.64 \text{ lb.}$$

$$+\nearrow \Sigma F_x = m_A 0_A: \quad -200\sin 30° - 34.64 + T = \frac{200}{32.2} a_A \qquad (1)$$

PROBLEM 12.14 (Continued)

Block B



$$+\downarrow \Sigma F_y = m_B a_B: \quad 350\,\text{lb} - 2T = \frac{350}{32.2}\left(\frac{1}{2}a_A\right) \qquad (2)$$

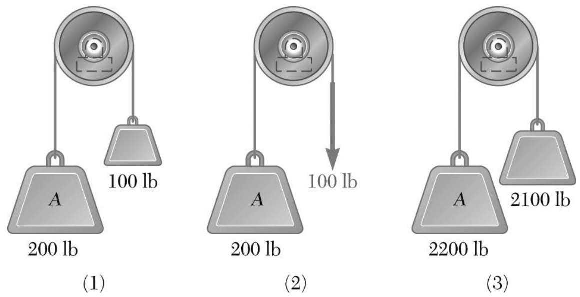
(*a*) Multiply Eq. (1) by 2 and add Eq. (2) in order to eliminate T:

$$-2(200)\sin 30° - 2(34.64) + 350 = 2\frac{200}{32.2}a_A + \frac{350}{32.2}\left(\frac{1}{2}a_A\right)$$

$$81.32 = \frac{575}{32.2}a_A \qquad a_A = 4.55\text{ ft/s}^2 \measuredangle 30° \blacktriangleleft$$

$$a_B = \frac{1}{2}a_A = \frac{1}{2}(4.52\text{ ft/s}^2), \qquad a_B = 2.28\text{ ft/s}^2\downarrow \blacktriangleleft$$

(*b*) From Eq. (1), $T - (200)\sin 30° - 34.64 = \frac{200}{32.2}(4.52)$ $\qquad T = 162.9\text{ lb} \blacktriangleleft$



PROBLEM 12.15

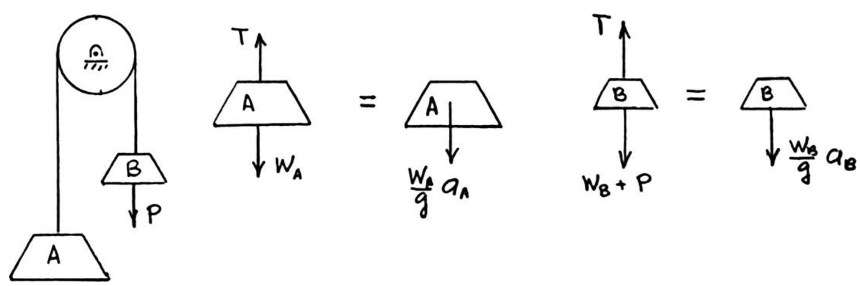
Each of the systems shown is initially at rest. Neglecting axle friction and the masses of the pulleys, determine for each system (*a*) the acceleration of block *A*, (*b*) the velocity of block *A* after it has moved through 10 ft, (*c*) the time required for block *A* to reach a velocity of 20 ft/s.

SOLUTION

Let y be positive downward for both blocks.

Constraint of cable: $y_A + y_B = \text{constant}$

$$a_A + a_B = 0 \qquad \text{or} \qquad a_B = -a_A$$



For blocks *A* and *B*, $+\!\downarrow \Sigma F = ma$:

Block *A*: $$W_A - T = \frac{W_A}{g}a_A \qquad \text{or} \qquad T = W_A - \frac{W_A}{g}a_A$$

Block *B*: $$P + W_B - T = \frac{W_B}{g}a_B = -\frac{W_B}{g}a_A$$

$$P + W_B - W_A + \frac{W_A}{g}a_A = -\frac{W_B}{g}a_A$$

Solving for a_A, $$a_A = \frac{W_A - W_B - P}{W_A + W_B}g \qquad (1)$$

$$v_A^2 - (v_A)_0^2 = 2a_A[y_A - (y_A)_0] \quad \text{with} \quad (v_A)_0 = 0$$

$$v_A = \sqrt{2a_A[y_A - (y_A)_0]} \qquad (2)$$

$$v_A - (v_A)_0 = a_A t \quad \text{with} \quad (v_A)_0 = 0$$

$$t = \frac{v_A}{a_A} \qquad (3)$$

(*a*) Acceleration of block *A*.

System (1): $W_A = 200 \text{ lb}, \quad W_B = 100 \text{ lb}, \quad P = 0$

By formula (1), $(a_A)_1 = \dfrac{200-100}{200+100}(32.2)$ $\qquad (\mathbf{a}_A)_1 = 10.73 \text{ ft/s}^2 \downarrow$ ◀

System (2): $W_A = 200 \text{ lb}, \quad W_B = 0, \quad P = 50 \text{ lb}$

By formula (1), $(a_A)_2 = \dfrac{200-100}{200}(32.2)$ $\qquad (\mathbf{a}_A)_2 = 16.10 \text{ ft/s}^2 \downarrow$ ◀

System (3): $W_A = 2200 \text{ lb}, \quad W_B = 2100 \text{ lb}, \quad P = 0$

By formula (1), $(a_A)_3 = \dfrac{2200-2100}{2200+2100}(32.2)$ $\qquad (\mathbf{a}_A)_3 = 0.749 \text{ ft/s}^2 \downarrow$ ◀

(*b*) v_A at $y_A - (y_A)_0 = 10$ ft. Use formula (2).

System (1): $(v_A)_1 = \sqrt{(2)(10.73)(10)}$ $\qquad (v_A)_1 = 14.65 \text{ ft/s} \downarrow$ ◀

System (2): $(v_A)_2 = \sqrt{(2)(16.10)(10)}$ $\qquad (v_A)_2 = 17.94 \text{ ft/s} \downarrow$ ◀

System (3): $(v_A)_3 = \sqrt{(2)(0.749)(10)}$ $\qquad (v_A)_3 = 3.87 \text{ ft/s} \downarrow$ ◀

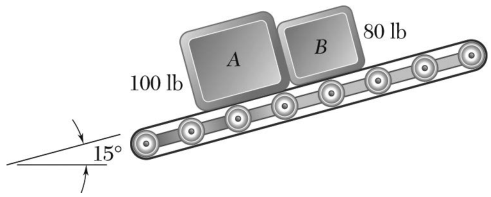
(*c*) Time at $v_A = 20$ ft/s. Use formula (3).

System (1): $t_1 = \dfrac{20}{10.73}$ $\qquad t_1 = 1.864 \text{ s}$ ◀

System (2): $t_2 = \dfrac{20}{16.10}$ $\qquad t_2 = 1.242 \text{ s}$ ◀

System (3): $t_3 = \dfrac{20}{0.749}$ $\qquad t_3 = 26.7 \text{ s}$ ◀

PROBLEM 12.16



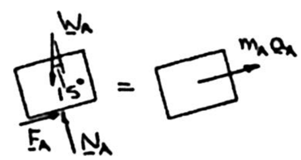
Boxes A and B are at rest on a conveyor belt that is initially at rest. The belt is suddenly started in an upward direction so that slipping occurs between the belt and the boxes. Knowing that the coefficients of kinetic friction between the belt and the boxes are $(\mu_k)_A = 0.30$ and $(\mu_k)_B = 0.32$, determine the initial acceleration of each box.

SOLUTION

Assume that $a_B > a_A$ so that the normal force N_{AB} between the boxes is zero.

A: $+\nwarrow \Sigma F_y = 0: \quad N_A - W_A \cos 15° = 0$

A:



or $N_A = W_A \cos 15°$

Slipping: $F_A = (\mu_k)_A N_A$

$= 0.3W_A \cos 15°$

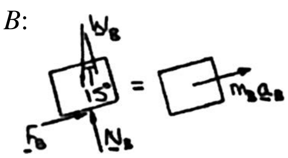
$+\nearrow \Sigma F_x = m_A a_A: \quad F_A - W_A \sin 15° = m_A a_A$

or $$0.3W_A \cos 15° - W_A \sin 15° = \frac{W_A}{g} a_A$$

or $$a_A = (32.2 \text{ ft/s}^2)(0.3 \cos 15° - \sin 15°)$$

$$= 0.997 \text{ ft/s}^2$$

B:



B: $+\nwarrow \Sigma F_y = 0: \quad N_B - W_B \cos 15° = 0$

or $N_B = W_B \cos 15°$

Slipping: $F_B = (\mu_k)_B N_B$

$= 0.32W_B \cos 15°$

$+\nearrow \Sigma F_x = m_B a_B: \quad F_B - W_B \sin 15° = m_B a_B$

or $$0.32W_B \cos 15° - W_B \sin 15° = \frac{W_B}{g} a_B$$

or $$a_B = (32.2 \text{ ft/s}^2)(0.32 \cos 15° - \sin 15°) = 1.619 \text{ ft/s}^2$$

$a_B > a_A \Rightarrow$ assumption is correct

$\mathbf{a}_A = 0.997 \text{ ft/s}^2 \;\angle\!\nearrow 15°$ ◀

$\mathbf{a}_B = 1.619 \text{ ft/s}^2 \;\angle\!\nearrow 15°$ ◀

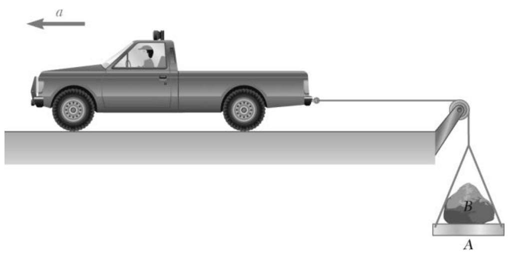
Note: If it is assumed that the boxes remain in contact $(N_{AB} \neq 0)$, then assuming N_{AB} to be compression,

$$a_A = a_B \quad \text{and find} \quad (\Sigma F_x = ma) \text{ for each box.}$$

A:
$$0.3W_A \cos 15° - W_A \sin 15° - N_{AB} = \frac{W_A}{g} a$$

B:
$$0.32W_B \cos 15° - W_B \sin 15° + N_{AB} = \frac{W_B}{g} a$$

Solving yields $a = 1.273 \text{ ft/s}^2$ and $N_{AB} = -0.859$ lb, which contradicts the assumption.



PROBLEM 12.17

A 5000-lb truck is being used to lift a 1000 lb boulder *B* that is on a 200 lb pallet *A*. Knowing the acceleration of the truck is 1 ft/s^2, determine (*a*) the horizontal force between the tires and the ground, (*b*) the force between the boulder and the pallet.

SOLUTION

Kinematics: $\mathbf{a}_T = 1\ \text{m/s}^2 \longleftarrow$

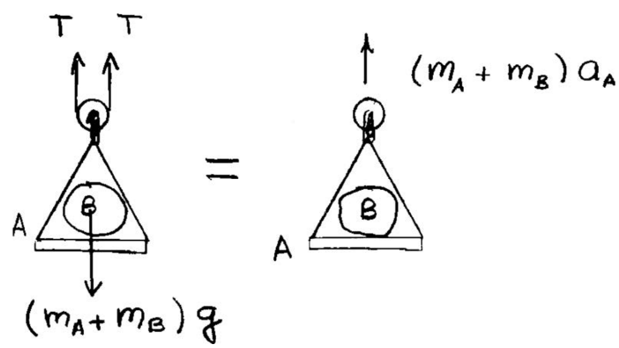
$$\mathbf{a}_A = \mathbf{a}_B = 0.5\ \text{m/s}^2 \uparrow$$

Masses: $m_T = \dfrac{5000}{32.2} = 155.28$ slugs

$$m_A = \frac{200}{32.2} = 6.211 \text{ slugs}$$

$$m_B = \frac{1000}{32.2} = 31.056 \text{ slugs}$$

Let *T* be the tension in the cable. Apply Newton's second law to the lower pulley, pallet and boulder.

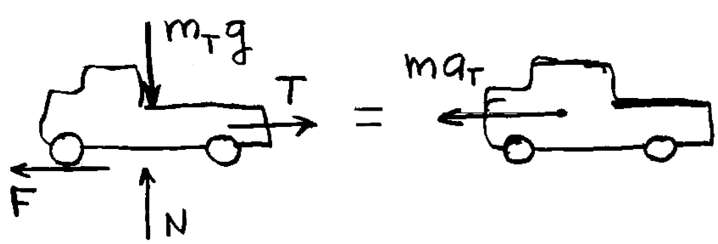


Vertical components $+\uparrow$:

$$2T - (m_A + m_B)g = (m_A + m_B)a_A$$
$$2T - (37.267)(32.2) = (37.267)(0.5)$$

$$T = 609.32 \text{ lb}$$

Apply Newton's second law to the truck.



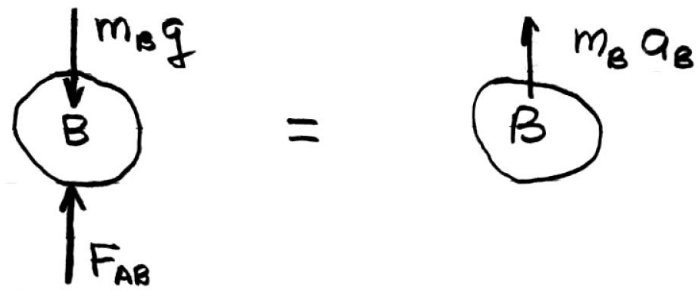
PROBLEM 12.17 (Continued)

Horizontal components $\overset{+}{\longleftarrow}$: $F - T = m_T a_T$

(*a*) Horizontal force between lines and ground.

$F = T + m_T a_T = 609.32 + (155.28)(1.0)$ $F = 765 \text{ lb}$ ◀

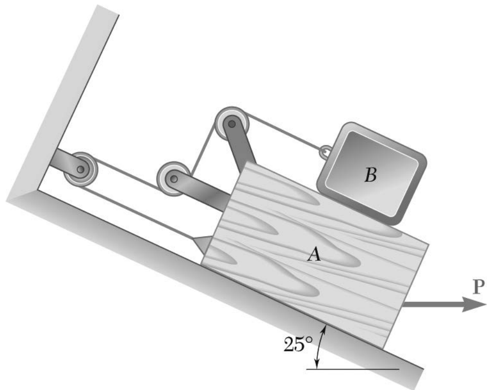
Apply Newton's second law to the boulder.



Vertical components $+\uparrow$: $F_{AB} - m_B g = m_B a_B$

$$F_{AB} = m_B(g + a) = 31.056(32.2 + 0.5)$$

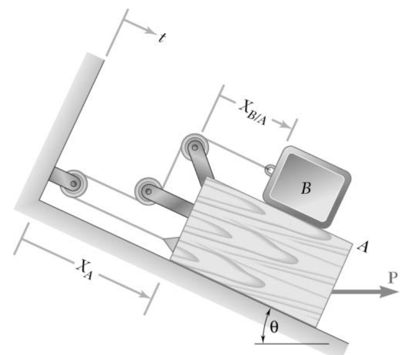
(*b*) Contact force: $F_{AB} = 1016 \text{ lb}$ ◀



PROBLEM 12.18

Block A has a mass of 40 kg, and block B has a mass of 8 kg. The coefficients of friction between all surfaces of contact are $\mu_s = 0.20$ and $\mu_k = 0.15$. If $P = 0$, determine (*a*) the acceleration of block B, (*b*) the tension in the cord.

SOLUTION



From the constraint of the cord:

$$2x_A + x_{B/A} = \text{constant}$$

Then $\quad 2v_A + v_{B/A} = 0$

and $\quad 2a_A + a_{B/A} = 0$

Now $\quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Then $\quad a_B = a_A + (-2a_A)$

or $\quad a_B = -a_A \qquad (1)$

First we determine if the blocks will move for the given value of θ. Thus, we seek the value of θ for which the blocks are in impending motion, with the impending motion of A down the incline.

B: $\quad +\nearrow \Sigma F_y = 0: \quad N_{AB} - W_B \cos\theta = 0$

B:

or $\quad N_{AB} = m_B g \cos\theta$

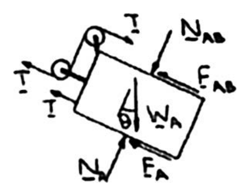
Now $\quad F_{AB} = \mu_s N_{AB}$

$$= 0.2 m_B g \cos\theta$$

$$\overset{+}{\searrow} \Sigma F_x = 0: \quad -T + F_{AB} + W_B \sin\theta = 0$$

or $\quad T = m_B g(0.2\cos\theta + \sin\theta)$

A:



A: $\quad +\nearrow \Sigma F_y = 0: \quad N_A - N_{AB} - W_A \cos\theta = 0$

or $\quad N_A = (m_A + m_B) g \cos\theta$

Now $\quad F_A = \mu_s N_A$

$$= 0.2(m_A + m_B) g \cos\theta$$

$$\overset{+}{\searrow} \Sigma F_x = 0: \quad -T - F_A - F_{AB} + W_A \sin\theta = 0$$

or $$T = m_A g\sin\theta - 0.2(m_A + m_B)g\cos\theta - 0.2m_B g\cos\theta$$
$$= g[m_A \sin\theta - 0.2(m_A + 2m_B)\cos\theta]$$

Equating the two expressions for T

$$m_B g(0.2\cos\theta + \sin\theta) = g[m_A \sin\theta - 0.2(m_A + 2m_B)\cos\theta]$$

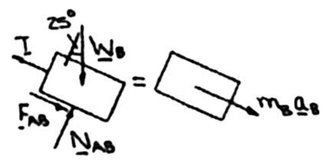
or $$8(0.2 + \tan\theta) = [40\tan\theta - 0.2(40 + 2\times 8)]$$

or $$\tan\theta = 0.4$$

or $\theta = 21.8°$ for impending motion. Since $\theta < 25°$, the blocks will move. Now consider the motion of the blocks.

(*a*) $+\nearrow \Sigma F_y = 0$: $N_{AB} - W_B\cos 25° = 0$

B:



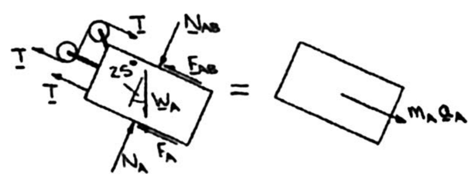
or $$N_{AB} = m_B g\cos 25°$$

Sliding: $$F_{AB} = \mu_k N_{AB} = 0.15 m_B g\cos 25°$$

$$\overset{+}{\searrow}\Sigma F_x = m_B a_B:\quad -T + F_{AB} + W_B\sin 25° = m_B a_B$$

or $$T = m_B[g(0.15\cos 25° + \sin 25°) - a_B]$$
$$= 8[9.81(0.15\cos 25° + \sin 25°) - a_B]$$
$$= 8(5.47952 - a_B) \quad \text{(N)}$$

A:



$$+\nearrow \Sigma F_y = 0:\quad N_A - N_{AB} - W_A\cos 25° = 0$$

or $$N_A = (m_A + m_B)g\cos 25°$$

Sliding: $$F_A = \mu_k N_A = 0.15(m_A + m_B)g\cos 25°$$

$$\overset{+}{\searrow}\Sigma F_x = m_A a_A:\quad -T - F_A - F_{AB} + W_A\sin 25° = m_A a_A$$

Substituting and using Eq. (1)

$$T = m_A g\ \sin 25° - 0.15(m_A + m_B)g\ \cos 25°$$
$$-0.15 m_B g\ \cos 25° - m_A(-a_B)$$
$$= g[m_A\ \sin 25° - 0.15(m_A + 2m_B)\cos 25°] + m_A a_B$$
$$= 9.81[40\ \sin 25° - 0.15(40 + 2\times 8)\cos 25°] + 40a_B$$
$$= 91.15202 + 40a_B \quad \text{(N)}$$

Equating the two expressions for T

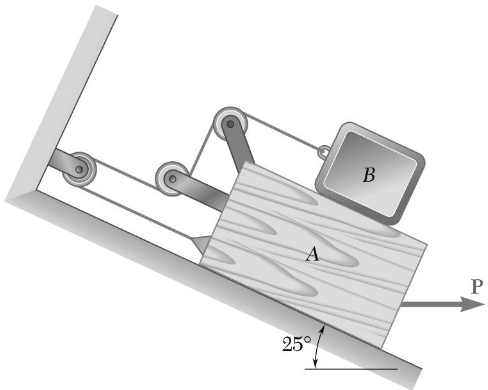
$$8(5.47952 - a_B) = 91.15202 + 40a_B$$

or $$a_B = -0.98575\ \text{m/s}^2$$

$$\mathbf{a}_B = 0.986\ \text{m/s}^2 \searrow 25° \blacktriangleleft$$

(*b*) We have $$T = 8[5.47952 - (-0.98575)]$$

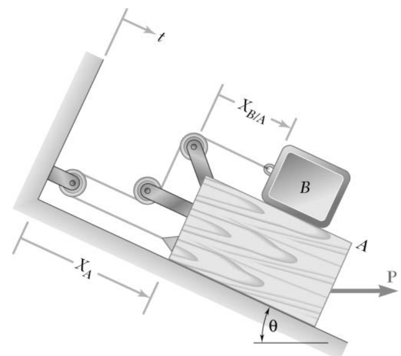
or $$T = 51.7\ \text{N} \blacktriangleleft$$



PROBLEM 12.19

Block A has a mass of 40 kg, and block B has a mass of 8 kg. The coefficients of friction between all surfaces of contact are $\mu_s = 0.20$ and $\mu_k = 0.15$. If $P = 40\text{ N} \longrightarrow$, determine (*a*) the acceleration of block B, (*b*) the tension in the cord.

SOLUTION



From the constraint of the cord.

$$2x_A + x_{B/A} = \text{constant}$$

Then $$2v_A + v_{B/A} = 0$$

and $$2a_A + a_{B/A} = 0$$

Now $$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Then $$a_B = a_A + (-2a_A)$$

or $$a_B = -a_A \qquad (1)$$

First we determine if the blocks will move for the given value of P. Thus, we seek the value of P for which the blocks are in impending motion, with the impending motion of a down the incline.

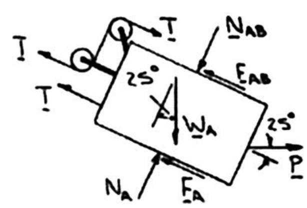
B:

B: $$+\nearrow \Sigma F_y = 0: \quad N_{AB} - W_B \cos 25° = 0$$

or $$N_{AB} = m_B g \cos 25°$$

Now $$F_{AB} = \mu_s N_{AB}$$
$$= 0.2\, m_B g \cos 25°$$

A:



$$\overset{+}{\searrow} \Sigma F_x = 0: \quad -T + F_{AB} + W_B \sin 25° = 0$$

or $$T = 0.2\, m_B g \cos 25° + m_B g \sin 25°$$
$$= (8\text{ kg})(9.81\text{ m/s}^2)(0.2 \cos 25° + \sin 25°)$$
$$= 47.39249\text{ N}$$

A: $$+\nearrow \Sigma F_y = 0: \quad N_A - N_{AB} - W_A \cos 25° + P \sin 25° = 0$$

or $$N_A = (m_A + m_B) g \cos 25° - P \sin 25°$$

PROBLEM 12.19 (Continued)

Now $$F_A = \mu_s N_A$$

or $$F_A = 0.2[(m_A + m_B)g \cos 25° - P \sin 25°]$$

$$\searrow^{+} \Sigma F_x = 0: \quad -T - F_A - F_{AB} + W_A \sin 25° + P \cos 25° = 0$$

or $$-T - 0.2[(m_A + m_B)g \cos 25° - P \sin 25°] - 0.2 m_B g \cos 25° + m_A g \sin 25° + P \cos 25° = 0$$

or $$P(0.2 \sin 25° + \cos 25°) = T + 0.2[(m_A + 2m_B)g \cos 25°] - m_A g \sin 25°$$

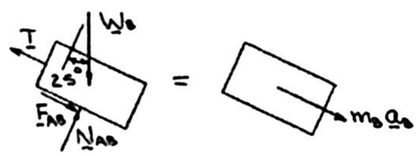
Then $$P(0.2 \sin 25° + \cos 25°) = 47.39249 \text{ N} + 9.81 \text{ m/s}^2 \{0.2[(40 + 2 \times 8)\cos 25° - 40 \sin 25°] \text{ kg}\}$$

or $$P = -19.04 \text{ N for impending motion.}$$

Since $P, < 40$ N, the blocks will move. Now consider the motion of the blocks.

(*a*) $$+\nearrow \Sigma F_y = 0: \quad N_{AB} - W_B \cos 25° = 0$$

B:



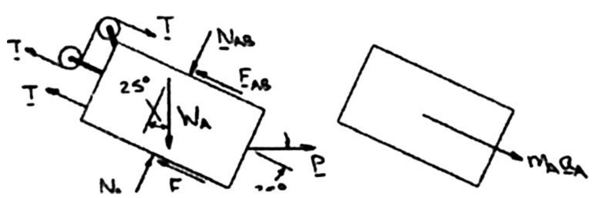
or $$N_{AB} = m_B g \cos 25°$$

Sliding: $$F_{AB} = \mu_k N_{AB} = 0.15\, m_B g \cos 25°$$

$$\searrow^{+} \Sigma F_x = m_B a_B: \quad -T + F_{AB} + W_B \sin 25° = m_B a_B$$

or $$T = m_B[g(0.15 \cos 25° + \sin 25°) - a_B] = 8[9.81(0.15 \cos 25° + \sin 25°) - a_B] = 8(5.47952 - a_B) \quad \text{(N)}$$

A:



$$+\nearrow \Sigma F_y = 0: \quad N_A - N_{AB} - W_A \cos 25° + P \sin 25° = 0$$

or $$N_A = (m_A + m_B)g \cos 25° - P \sin 25°$$

Sliding: $$F_A = \mu_k N_A = 0.15[(m_A + m_B)g \cos 25° - P \sin 25°]$$

$$\searrow^{+} \Sigma F_x = m_A a_A: \quad -T - F_A - F_{AB} + W_A \sin 25° + P \cos 25° = m_A a_A$$

Substituting and using Eq. (1)

$$T = m_A g \sin 25° - 0.15[(m_A + m_B)g \cos 25° - P \sin 25°]$$
$$- 0.15\, m_B g \cos 25° + P \cos 25° - m_A(-a_B)$$
$$= g[m_A \sin 25° - 0.15(m_A + 2m_B)\cos 25°]$$
$$+ P(0.15 \sin 25° + \cos 25°) + m_A a_B$$
$$= 9.81[40 \sin 25° - 0.15(40 + 2 \times 8)\cos 25°]$$
$$+ 40(0.15 \sin 25° + \cos 25°) + 40 a_B$$
$$= 129.94004 + 40 a_B$$

Equating the two expressions for T

$$8(5.47952 - a_B) = 129.94004 + 40 a_B$$

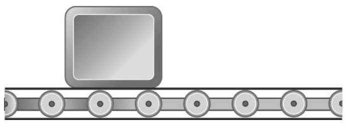
or $$a_B = -1.79383 \text{ m/s}^2$$

$$\mathbf{a}_B = 1.794 \text{ m/s}^2 \searrow 25° \blacktriangleleft$$

(*b*) We have $$T = 8[5.47952 - (-1.79383)]$$

or $$T = 58.2 \text{ N} \blacktriangleleft$$

PROBLEM 12.20



A package is at rest on a conveyor belt which is initially at rest. The belt is started and moves to the right for 1.3 s with a constant acceleration of 2 m/s^2. The belt then moves with a constant deceleration $\mathbf{a}_2$ and comes to a stop after a total displacement of 2.2 m. Knowing that the coefficients of friction between the package and the belt are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine (*a*) the deceleration $\mathbf{a}_2$ of the belt, (*b*) the displacement of the package relative to the belt as the belt comes to a stop.

SOLUTION

(*a*) Kinematics of the belt. $v_o = 0$

1. Acceleration phase with $\mathbf{a}_1 = 2 \text{ m/s}^2 \longrightarrow$

$$v_1 = v_o + a_1 t_1 = 0 + (2)(1.3) = 2.6 \text{ m/s}$$

$$x_1 = x_o + v_o t_1 + \frac{1}{2} a_1 t_1^2 = 0 + 0 + \frac{1}{2}(2)(1.3)^2 = 1.69 \text{ m}$$

2. Deceleration phase: $v_2 = 0$ since the belt stops.

$$v_2^2 - v_1^2 = 2a_2(x_2 - x_1)$$

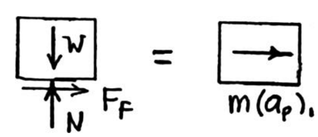
$$a_2 = \frac{v_2^2 - v_1^2}{2(x_2 - x_1)} = \frac{0 - (2.6)^2}{2(2.2 - 1.69)} = -6.63$$

$\mathbf{a}_2 = 6.63 \text{ m/s}^2 \longleftarrow$ ◀

$$t_2 - t_1 = \frac{v_2 - v_1}{a_2} = \frac{0 - 2.6}{-6.63} = 0.3923 \text{ s}$$

(*b*) Motion of the package.

1. Acceleration phase. Assume no slip. $(\mathbf{a}_p)_1 = 2 \text{ m/s}^2 \longrightarrow$



$$\Sigma F_y = 0: \quad N - W = 0 \quad \text{or} \quad N = W = mg$$

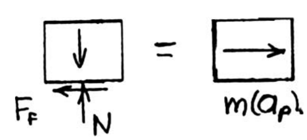
$$\xrightarrow{+} \Sigma F_x = ma: \quad F_f = m(a_p)_1$$

The required friction force is F_f.

The available friction force is $\mu_s N = 0.35W = 0.35mg$

$$\frac{F_f}{m} = (a_p)_1, < \frac{\mu_s N}{m} = \mu_s g = (0.35)(9.81) = 3.43 \text{ m/s}^2$$

Since $2.0 \text{ m/s}^2 < 3.43 \text{ m/s}^2$, the package does not slip.



$$(v_p)_1 = v_1 = 2.6 \text{ m/s} \quad \text{and} \quad (x_p)_1 = 1.69 \text{ m.}$$

PROBLEM 12.20 (Continued)

2. Deceleration phase. Assume no slip. $(a_p)_2 = -11.52 \text{ m/s}^2$

$$\xrightarrow{+} \Sigma F_x = ma: \quad -F_f = m(a_p)_2$$

$$\frac{F_f}{m} = (a_p)_2 = -6.63 \text{ m/s}^2$$

$$\frac{\mu_s N}{m} = \frac{\mu_s mg}{m} = \mu_s g = 3.43 \text{ m/s}^2 < 6.63 \text{ m/s}^2$$

Since the available friction force $\mu_s N$ is less than the required friction force F_f for no slip, the package does slip.

$$(a_p)_2 < 6.63 \text{ m/s}^2, \quad F_f = \mu_k N$$

$$\xrightarrow{+} \Sigma F_x = m(a_p)_2: \quad -\mu_k N = m(a_p)_2$$

$$\begin{aligned}(a_p)_2 &= -\frac{\mu_k N}{m} = -\mu_k g \\ &= -(0.25)(9.81) \\ &= -2.4525 \text{ m/s}^2\end{aligned}$$

$$\begin{aligned}(v_p)_2 &= (v_p)_1 + (a_p)_2(t_2 - t_1) \\ &= 2.6 + (-2.4525)(0.3923) \\ &= 1.638 \text{ m/s}^2\end{aligned}$$

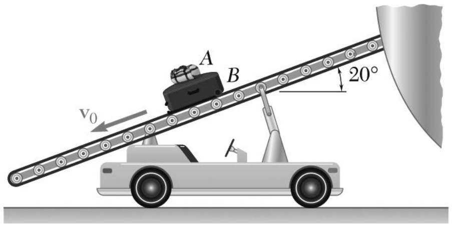
$$\begin{aligned}(x_p)_2 &= (x_p)_1 + (v_p)_1(t_2 - t_1) + \frac{1}{2}(a_p)_2(t_2 - t_1)^2 \\ &= 1.69 + (2.6)(0.3923) + \frac{1}{2}(-2.4525)(0.3923)^2 \\ &= 2.521 \text{ m}\end{aligned}$$

Position of package relative to the belt

$$(x_p)_2 - x_2 = 2.521 - 2.2 = 0.321$$

$x_{p/\text{belt}} = 0.321 \text{ m} \longrightarrow$ ◀

PROBLEM 12.21



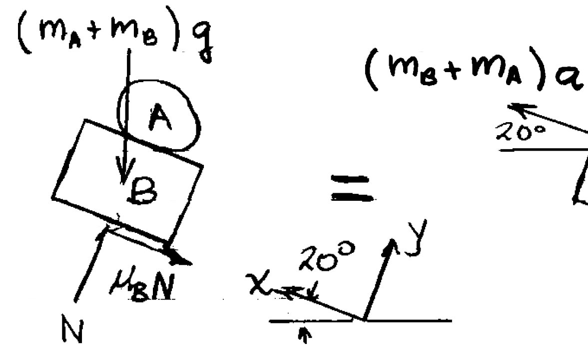
A baggage conveyor is used to unload luggage from an airplane. The 10-kg duffel bag *A* is sitting on top of the 20-kg suitcase *B*. The conveyor is moving the bags down at a constant speed of 0.5 m/s when the belt suddenly stops. Knowing that the coefficient of friction between the belt and *B* is 0.3 and that bag *A* does not slip on suitcase *B*, determine the smallest allowable coefficient of static friction between the bags.

SOLUTION

Since bag *A* does not slide on suitcase *B*, both have the same acceleration.

$$\mathbf{a} = a \searrow 20°$$

Apply Newton's second law to the bag *A* – suitcase *B* combination treated as a single particle.



$$+\nearrow \Sigma F_y = ma_y: \quad -(m_B + m_A) g \cos 20° + N = 0$$

$$N = (m_A + m_B) g \cos 30° = (30)(9.81)\cos 20° = 276.55 \text{ N}$$

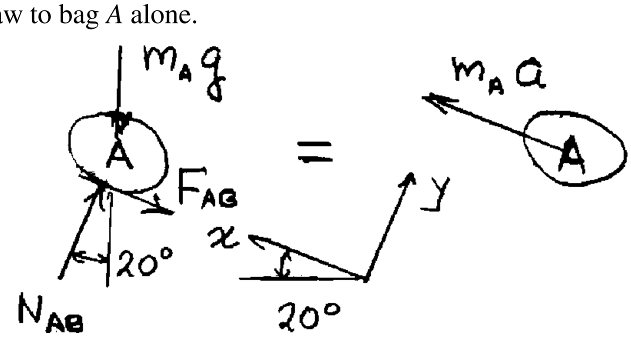
$$\mu_B N = (0.3)(276.55) = 82.965 \text{ N}$$

$$+\nwarrow \Sigma F_x = ma_x: \quad \mu_B N + (m_A + m_B) g \sin 20° = (m_A + m_B) a$$

$$a = g \sin 20° + \frac{\mu_B N}{m_A m_B} = 9.81 \sin 20° - \frac{82.965}{30}$$

$$a = 0.58972 \text{ m/s}^2 \qquad \mathbf{a} = 0.58972 \text{ m/s}^2 \nearrow 20°$$

Apply Newton's second law to bag *A* alone.



PROBLEM 12.21 (Continued)

$$+\nearrow \Sigma F_y = ma_y: \quad N_{AB} - m_A g \cos 20° = 0$$

$$N_{AB} = m_a g \sin 20° = (10)(9.81)\cos 20° = 92.184 \text{ N}$$

$$+\nwarrow ZF_x = ma_x: \quad M_A g \sin 20° - F_{AB} = m_A a$$

$$F_{AB} = m_A (g \sin 20° - a) = (10)(9.81 \sin 20° - 0.58972)$$

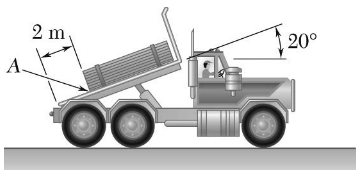
$$= 27.655 \text{ N}$$

Since bag A does not slide on suitcase B,

$$\mu_s > \frac{F_{AB}}{N_{AB}} = \frac{27.655}{92.184} = 0.300$$

$\mu_s > 0.300$ ◀

PROBLEM 12.22



To unload a bound stack of plywood from a truck, the driver first tilts the bed of the truck and then accelerates from rest. Knowing that the coefficients of friction between the bottom sheet of plywood and the bed are $\mu_s = 0.40$ and $\mu_k = 0.30$, determine (*a*) the smallest acceleration of the truck which will cause the stack of plywood to slide, (*b*) the acceleration of the truck which causes corner *A* of the stack to reach the end of the bed in 0.9 s.

SOLUTION

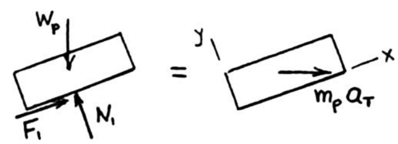
Let $\mathbf{a}_P$ be the acceleration of the plywood, $\mathbf{a}_T$ be the acceleration of the truck, and $\mathbf{a}_{P/T}$ be the acceleration of the plywood relative to the truck.

(*a*) Find the value of $\mathbf{a}_T$ so that the relative motion of the plywood with respect to the truck is impending.

$$a_P = a_T \text{ and } F_1 = \mu_s N_1 = 0.40 N_1$$

$$+\nwarrow \Sigma F_y = m_P a_y: \quad N_1 - W_P \cos 20° = -m_P a_T \sin 20°$$

$$N_1 = m_P (g \cos 20° - a_T \sin 20°)$$



$$+\nearrow \Sigma F_x = m a_x: \quad F_1 - W_P \sin 20° = m_P a_T \cos 20°$$

$$F_1 = m_P (g \sin 20° + a_T \cos 20°)$$

$$m_P (g \sin 20° + a_T \cos 20°) = 0.40\, m_P (g \cos 20° - a_T \sin 20°)$$

$$a_T = \frac{(0.40 \cos 20° - \sin 20°)}{\cos 20° + 0.40 \sin 20°} g$$

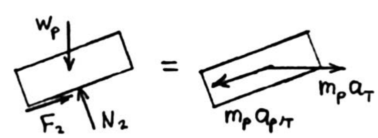
$$= (0.03145)(9.81)$$

$$= 0.309$$

$$\mathbf{a}_T = 0.309 \text{ m/s}^2 \longrightarrow \blacktriangleleft$$

(*b*) $x_{P/T} = (x_{P/T})_o + (v_{P/T})t + \frac{1}{2} a_{P/T} t^2 = 0 + 0 + \frac{1}{2} a_{P/T} t^2$

$$a_{P/T} = \frac{2 x_{P/T}}{t^2} = \frac{(2)(2)}{(0.9)^2} = 4.94 \text{ m/s}^2$$



$$\mathbf{a}_{P/T} = 4.94 \text{ m/s}^2 \swarrow 20°$$

$$\mathbf{a}_P = \mathbf{a}_T + \mathbf{a}_{P/T} = (a_T \rightarrow) + (4.94 \text{ m/s}^2 \swarrow 20°)$$

$$+\nwarrow F_y = m_P a_y: \quad N_2 - W_P \cos 20° = -m_P a_T \sin 20°$$

$$N_2 = m_P (g \cos 20° - a_T \sin 20°)$$

PROBLEM 12.22 (Continued)

$$+\nearrow \Sigma F_x = \Sigma m a_x: \quad F_2 - W_P \sin 20° = m_P a_T \cos 20° - m_P a_{P/T}$$

$$F_2 = m_P (g \sin 20° + a_T \cos 20° - a_{P/T})$$

For sliding with friction $\quad F_2 = \mu_k N_2 = 0.30 N_2$

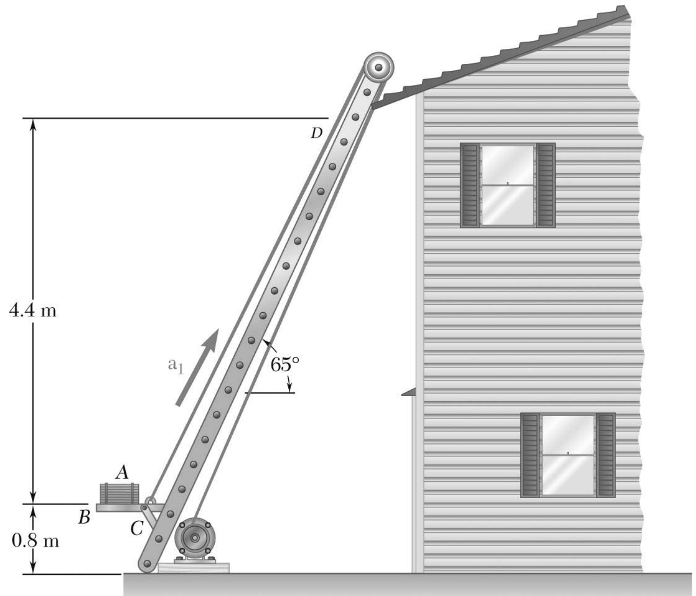
$$m_P (g \sin 20° + a_T \cos 20° - a_{P/T}) = 0.30 m_P (g \cos 20° + a_T \sin 20°)$$

$$a_T = \frac{(0.30 \cos 20° - \sin 20°) g + a_{P/T}}{\cos 20° + 0.30 \sin 20°}$$

$$= (-0.05767)(9.81) + (0.9594)(4.94)$$

$$= 4.17$$

$\mathbf{a}_T = 4.17 \text{ m/s}^2 \longrightarrow$ ◀



PROBLEM 12.23

To transport a series of bundles of shingles *A* to a roof, a contractor uses a motor-driven lift consisting of a horizontal platform *BC* which rides on rails attached to the sides of a ladder. The lift starts from rest and initially moves with a constant acceleration $\mathbf{a}_1$ as shown. The lift then decelerates at a constant rate $\mathbf{a}_2$ and comes to rest at *D*, near the top of the ladder. Knowing that the coefficient of static friction between a bundle of shingles and the horizontal platform is 0.30, determine the largest allowable acceleration $\mathbf{a}_1$ and the largest allowable deceleration $\mathbf{a}_2$ if the bundle is not to slide on the platform.

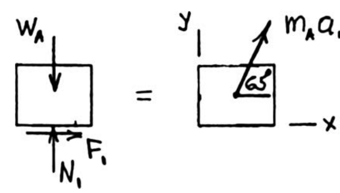
SOLUTION

Acceleration $\mathbf{a}_1$: Impending slip. $F_1 = \mu_s N_1 = 0.30 N_1$

$\Sigma F_y = m_A a_y$: $N_1 - W_A = m_A a_1 \sin 65°$

$N_1 = W_A + m_A a_1 \sin 65°$

$= m_A (g + a_1 \sin 65°)$



$\xrightarrow{+} \Sigma F_x = m_A a_x$: $F_1 = m_A a_1 \cos 65°$

$F_1 = \mu_s N$

or $m_A a_1 \cos 65° = 0.30 m_A (g + a_1 \sin 65°)$

$$a_1 = \frac{0.30 g}{\cos 65° - 0.30 \sin 65°}$$

$= (1.990)(9.81)$

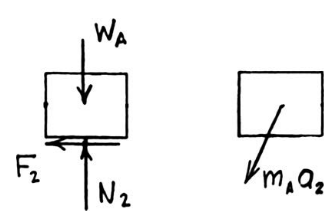
$= 19.53 \text{ m/s}^2$

$\mathbf{a}_1 = 19.53 \text{ m/s}^2 \measuredangle 65°$ ◀

Deceleration $\mathbf{a}_2$: Impending slip. $F_2 = \mu_s N_2 = 0.30 N_2$

$\Sigma F_y = m a_y$: $N_1 - W_A = -m_A a_2 \sin 65°$

$N_1 = W_A - m_A a_2 \sin 65°$



$\xleftarrow{+} \Sigma F_x = m a_x$: $F_2 = m_A a_2 \cos 65°$

$F_2 = \mu_s N_2$

or $$m_A a_2 \cos 65° = 0.30\, m_A (g - a_2 \cos 65°)$$

$$a_2 = \frac{0.30g}{\cos 65° + 0.30 \sin 65°}$$

$$= (0.432)(9.81)$$

$$= 4.24 \text{ m/s}^2$$

$\mathbf{a}_2 = 4.24 \text{ m/s}^2$ ↗ 65° ◀

PROBLEM 12.24

An airplane has a mass of 25 Mg and its engines develop a total thrust of 40 kN during take-off. If the drag **D** exerted on the plane has a magnitude $D = 2.25v^2$, where v is expressed in meters per second and D in newtons, and if the plane becomes airborne at a speed of 240 km/h, determine the length of runway required for the plane to take off.

SOLUTION

$$F = ma: \quad 40\times10^3 \text{ N} - 2.25v^2 = (25\times10^3 \text{ kg})a$$

Substituting

$$a = v\frac{dv}{dx}: \quad 40\times10^3 - 2.25\ v^2 = (25\times10^3)\ v\frac{dv}{dx}$$

$$\int_0^{x_1} dx = \int_0^{v_1} \frac{(25\times10^3)vdv}{40\times10^3 - 2.25v^2}$$

$$x_1 = -\frac{25\times10^3}{2(2.25)}[\ln(40\times10^3 - 2.25v^2)]_0^{v_1}$$

$$= \frac{25\times10^3}{4.5}\ln\frac{40\times10^3}{40\times10^3 - 2.25v_1^2}$$

For $v_1 = 240 \text{ km/h} = 66.67 \text{ m/s}$

$$x_1 = \frac{25\times10^3}{4.5}\ln\frac{40\times10^3}{40\times10^3 - 2.25(66.67)^2} = 5.556\ln 1.333$$

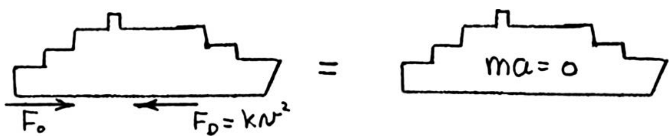
$$= 1.5982\times10^3 \text{ m}$$

$x_1 = 1.598 \text{ km}$ ◀

PROBLEM 12.25

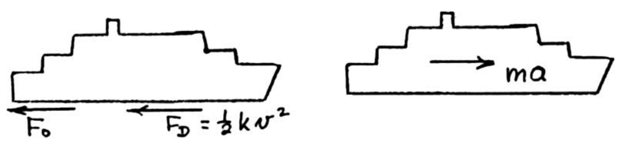
The propellers of a ship of weight W can produce a propulsive force $\mathbf{F}_0$; they produce a force of the same magnitude but of opposite direction when the engines are reversed. Knowing that the ship was proceeding forward at its maximum speed v_0 when the engines were put into reverse, determine the distance the ship travels before coming to a stop. Assume that the frictional resistance of the water varies directly with the square of the velocity.

SOLUTION



At maximum speed $a = 0$. $\quad F_0 = kv_0^2 = 0 \qquad k = \dfrac{F_0}{v_0^2}$

When the propellers are reversed, F_0 is reversed.



$$\xrightarrow{+} \Sigma F_x = ma: \quad -F_0 - kv^2 = ma$$

$$-F_0 - F_0 \frac{v^2}{v_0^2} = ma \qquad a - \frac{F_0}{mv_0^2}\left(v_0^2 + v^2\right)$$

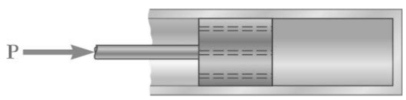
$$dx = \frac{vdv}{a} = \frac{mv_0^2 vdv}{F_0\left(v_0^2 + v^2\right)}$$

$$\int_0^x dx = -\frac{mv_0^2}{F_0}\int_{v_0}^{0} \frac{vdv}{v_0^2 + v^2}$$

$$x = -\frac{mv_0^2}{F_0}\frac{1}{2}\ln\left(v_0^2 + v^2\right)\Big|_{v_0}^{0}$$

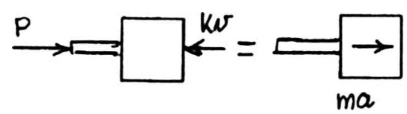
$$= -\frac{mv_0^2}{2F_0}\left[\ln v_0^2 - \ln\left(2v_0^2\right)\right] = \frac{mv_0^2}{2F_0}\ln 2 \qquad x = 0.347\frac{m_0 v_0^2}{F_0} \blacktriangleleft$$

PROBLEM 12.26



A constant force **P** is applied to a piston and rod of total mass m to make them move in a cylinder filled with oil. As the piston moves, the oil is forced through orifices in the piston and exerts on the piston a force of magnitude kv in a direction opposite to the motion of the piston. Knowing that the piston starts from rest at $t = 0$ and $x = 0$, show that the equation relating x, v, and t, where x is the distance traveled by the piston and v is the speed of the piston, is linear in each of these variables.

SOLUTION



$$\xrightarrow{+} \Sigma F = ma: \quad P - kv = ma$$

$$\frac{dv}{dt} = a = \frac{P - kv}{m}$$

$$\int_0^t dt = \int_0^v \frac{m\, dv}{P - kv}$$

$$= -\frac{m}{k} \ln (P - kv)\Big|_0^v$$

$$= -\frac{m}{k} [\ln (P - kv) - \ln P]$$

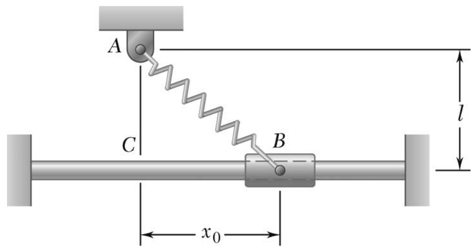
$$t = -\frac{m}{k} \ln \frac{P - kv}{P} \qquad \text{or} \qquad \ln \frac{P - kv}{m} = -\frac{kt}{m}$$

$$\frac{P - kv}{m} = e^{-kt/m} \qquad \text{or} \qquad v = \frac{P}{k}(1 - e^{-kt/m})$$

$$x = \int_0^t v\, dt = \frac{Pt}{k}\Bigg|_0^t - \frac{P}{k}\left(-\frac{k}{m} e^{-kt/m}\right)\Bigg|_0^t$$

$$= \frac{Pt}{k} + \frac{P}{m}(e^{-kt/m} - 1) = \frac{Pt}{k} - \frac{P}{m}(1 - e^{-kt/m})$$

$$x = \frac{Pt}{k} - \frac{kv}{m}, \quad \text{which is linear.} \qquad \blacktriangleleft$$



PROBLEM 12.27

A spring AB of constant k is attached to a support at A and to a collar of mass m. The unstretched length of the spring is ℓ. Knowing that the collar is released from rest at $x = x_0$ and neglecting friction between the collar and the horizontal rod, determine the magnitude of the velocity of the collar as it passes through Point C.

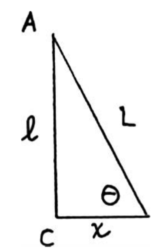
SOLUTION

Choose the origin at Point C and let x be positive to the right. Then x is a position coordinate of the slider B and x_0 is its initial value. Let L be the stretched length of the spring. Then, from the right triangle

$$L = \sqrt{\ell^2 + x^2}$$

The elongation of the spring is $e = L - \ell$, and the magnitude of the force exerted by the spring is

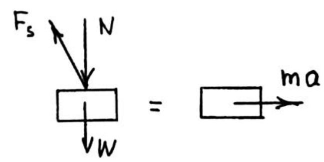
$$F_s = ke = k(\sqrt{\ell^2 + x^2} - \ell)$$



By geometry, $$\cos\theta = \frac{x}{\sqrt{\ell^2 + x^2}}$$

$$\xrightarrow{+} \Sigma F_x = ma_x: \quad -F_s \cos\theta = ma$$

$$-k(\sqrt{\ell^2 + x^2} - \ell)\frac{x}{\sqrt{\ell^2 + x^2}} = ma$$



$$a = -\frac{k}{m}\left(x - \frac{\ell x}{\sqrt{\ell^2 + x^2}}\right)$$

$$\int_0^v v\,dv = \int_{x_0}^0 a\,dx$$

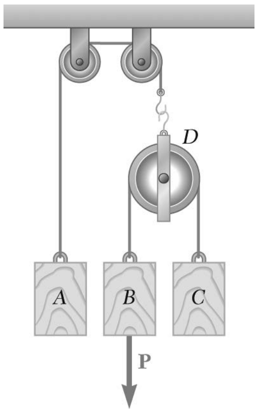
$$\frac{1}{2}v^2\Big|_0^v = -\frac{k}{m}\int_{x_0}^0 \left(x - \frac{\ell x}{\sqrt{\ell^2 + x^2}}\right)dx = -\frac{k}{m}\left(\frac{1}{2}x^2 - \ell\sqrt{\ell^2 + x^2}\right)\Bigg|_{x_0}^0$$

$$\frac{1}{2}v^2 = -\frac{k}{m}\left(0 - \ell^2 - \frac{1}{2}x_0^2 + \ell\sqrt{\ell^2 + x_0^2}\right)$$

$$v^2 = \frac{k}{m}\left(2\ell^2 + x_0^2 - 2\ell\sqrt{\ell^2 + x_0^2}\right)$$

$$= \frac{k}{m}\left[\left(\ell^2 + x_0^2\right) - 2\ell\sqrt{\ell^2 + x_0^2} + \ell^2\right]$$

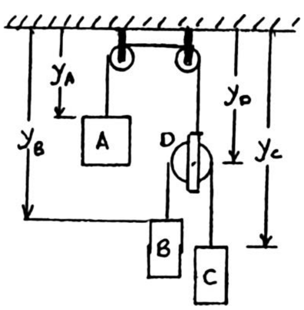
answer: $v = \sqrt{\frac{k}{m}}\left(\sqrt{\ell^2 + x_0^2} - \ell\right)$ ◀

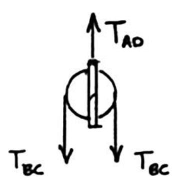


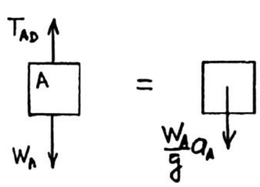
PROBLEM 12.28

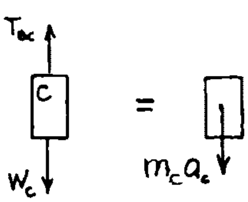
Block *A* has a mass of 10 kg, and blocks *B* and *C* have masses of 5 kg each. Knowing that the blocks are initially at rest and that *B* moves through 3 m in 2 s, determine (*a*) the magnitude of the force **P**, (*b*) the tension in the cord *AD*. Neglect the masses of the pulleys and axle friction.

SOLUTION









Let the position coordinate *y* be positive downward.

Constraint of cord *AD*: $y_A + y_D = \text{constant}$

$$v_A + v_D = 0, \quad a_A + a_D = 0$$

Constraint of cord *BC*: $(y_B - y_D) + (y_C - y_D) = \text{constant}$

$$v_B + v_C - 2v_D = 0, \quad a_B + a_C - 2a_D = 0$$

Eliminate a_D. $\quad 2a_A + a_B + a_C = 0 \quad (1)$

We have uniformly accelerated motion because all of the forces are constant.

$$y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2}a_B t^2, \quad (v_B)_0 = 0$$

$$a_B = \frac{2[y_B - (y_B)_0]}{t^2} = \frac{(2)(3)}{(2)^2} = 1.5 \text{ m/s}^2$$

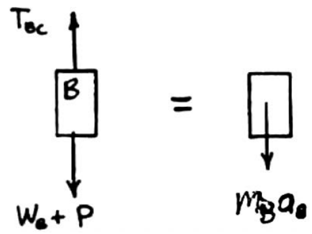
Pulley *D*: $+\!\downarrow \Sigma F_y = 0: \quad 2T_{BC} - T_{AD} = 0$

$$T_{AD} = 2T_{BC}$$

Block *A*: $+\!\downarrow \Sigma F_y = ma_y: \quad W_A - T_{AD} = m_A a_A$

or $\quad a_A = \dfrac{W_A - T_{AD}}{m_A} = \dfrac{W_A - 2T_{BC}}{m_A} \quad (2)$

PROBLEM 12.28 (Continued)



Block C: $\quad +\downarrow \Sigma F_y = ma_y\text{:} \quad W_C - T_{BC} = m_C a_C$

or $$a_C = \frac{W_C - T_{BC}}{m_C} \qquad (3)$$

Substituting the value for a_B and Eqs. (2) and (3) into Eq. (1), and solving for T_{BC},

$$2\left(\frac{W_A - 2T_{BC}}{m_A}\right) + a_B + \left(\frac{W_C - T_{BC}}{m_C}\right) = 0$$

$$2\left(\frac{m_A g - 2T_{BC}}{m_A}\right) + a_B + \left(\frac{m_C g - T_{BC}}{m_C}\right) = 0$$

$$\left(\frac{4}{m_A} + \frac{1}{m_C}\right) T_{BC} = 3g + a_B$$

$$\left(\frac{4}{10} + \frac{1}{5}\right) T_{BC} = 3(9.81) + 1.5 \quad \text{or} \quad T_{BC} = 51.55 \text{ N}$$

Block B: $\quad +\downarrow \Sigma F_y = ma_y\text{:} \quad P + W_B - T_{BC} = m_B a_B$

(a) <u>Magnitude of P.</u>

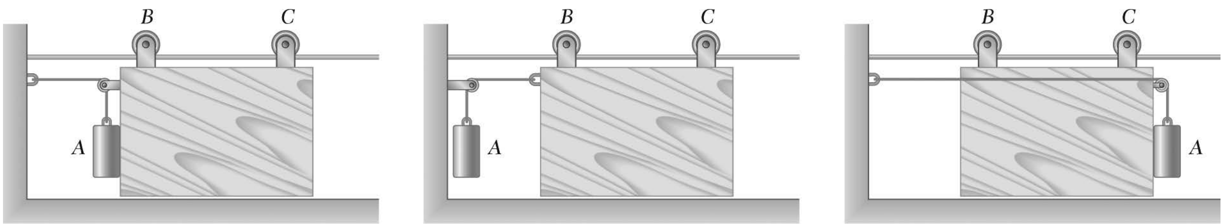
$$P = T_{BC} - W_B + m_B a_B$$
$$= 51.55 - 5(9.81) + 5(1.5) \qquad P = 10.00 \text{ N} \blacktriangleleft$$

(b) <u>Tension in cord AD.</u>

$$T_{AD} = 2T_{BC} = (2)(51.55) \qquad T_{AD} = 103.1 \text{ N} \blacktriangleleft$$

PROBLEM 12.29

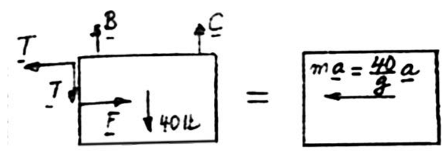
A 40-lb sliding panel is supported by rollers at B and C. A 25-lb counterweight A is attached to a cable as shown and, in cases a and c, is initially in contact with a vertical edge of the panel. Neglecting friction, determine in each case shown the acceleration of the panel and the tension in the cord immediately after the system is released from rest.



(a) (b) (c)

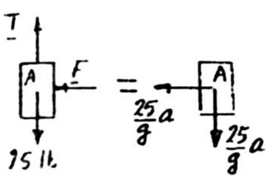
SOLUTION

(a) Panel: F = Force exerted by counterweight



$\overset{+}{\leftarrow} \Sigma F_x = ma$: $$T - F = \frac{40}{g} a \qquad (1)$$

Counterweight A: Its acceleration has two components



$$\mathbf{a}_A = \mathbf{a}_P + \mathbf{a}_{A/P} = a \rightarrow + \, a \downarrow$$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x$: $$F = \frac{25}{g} a \qquad (2)$$

$+\downarrow \Sigma F_g = ma_g$: $$25 - T = \frac{25}{g} a \qquad (3)$$

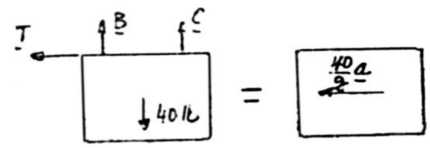
Adding (1), (2), and (3):

$$\cancel{T} - F + F + 25 - \cancel{T} = \frac{40 + 25 + 25}{g} a$$

$$a = \frac{25}{90} g = \frac{25}{90}(32.2) \qquad \mathbf{a} = 8.94 \text{ ft/s}^2 \longleftarrow \blacktriangleleft$$

Substituting for a into (3):

$$25 - T = \frac{25}{g}\left(\frac{25}{90} g\right) \qquad T = 25 - \frac{625}{90} \qquad T = 18.06 \text{ lb} \blacktriangleleft$$

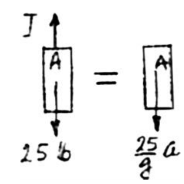


(*b*) Panel:

$\overset{+}{\longleftarrow} \Sigma F_y = ma$: $\quad T = \dfrac{40}{g}a$ (1)

Counterweight *A*:

$+\downarrow \Sigma F_y = ma$: $\quad 25 - T = \dfrac{25}{g}a$ (2)



Adding (1) and (2): $\quad \not{T} + 25 - \not{T} = \dfrac{40+25}{g}a$

$$a = \frac{25}{65}g$$

$\mathbf{a} = 12.38 \text{ ft/s}^2 \longleftarrow$ ◀

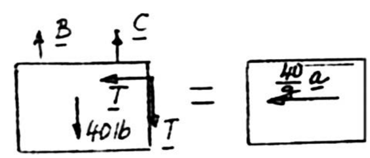
Substituting for *a* into (1):

$$T = \frac{40}{g}\left(\frac{25}{65}g\right) = \frac{1000}{65}$$

$T = 15.38$ lb ◀

(*c*) Since panel is accelerated to the left, there is no force exerted by panel on counterweight and vice versa.

Panel:



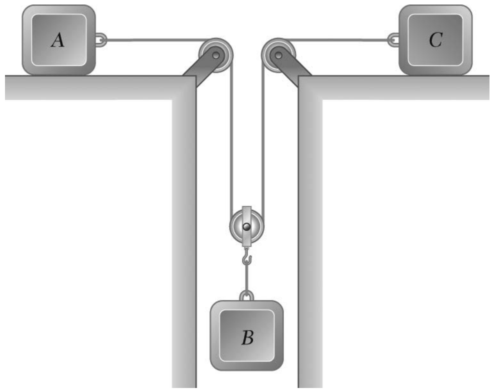
$\overset{+}{\longleftarrow} \Sigma F_x = ma$: $\quad T = \dfrac{40}{g}a$ (1)

Counterweight *A*: Same free body as in Part (*b*):

$+\downarrow \Sigma F_y = ma$: $\quad 25 - T = \dfrac{25}{g}a$ (2)

Since Eqs. (1) and (2) are the same as in (*b*), we get the same answers:

$\mathbf{a} = 12.38 \text{ ft/s}^2 \longleftarrow$; $T = 15.38$ lb ◀



PROBLEM 12.30

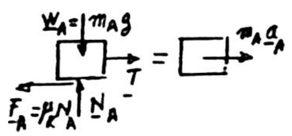
The coefficients of friction between blocks A and C and the horizontal surfaces are $\mu_s = 0.24$ and $\mu_k = 0.20$. Knowing that $m_A = 5$ kg, $m_B = 10$ kg, and $m_C = 10$ kg, determine (*a*) the tension in the cord, (*b*) the acceleration of each block.

SOLUTION

We first check that static equilibrium is not maintained:

$$(F_A)_m + (F_C)_m = \mu_s(m_A + m_C)g$$
$$= 0.24(5+10)g$$
$$= 3.6g$$

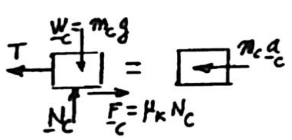
Since $W_B = m_B g = 10\text{g} > 3.6\text{g}$, equilibrium is *not* maintained.



Block A: $\quad \Sigma F_y: \quad N_A = m_A g$

$$F_A = \mu_k N_A = 0.2 m_A g$$

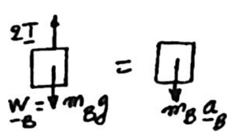
$$\xrightarrow{+} \Sigma F_\lambda = m_A a_A: \quad T - 0.2 m_A g = m_A a_A \qquad (1)$$



Block C: $\quad \Sigma F_y: \quad N_C = m_C g$

$$F_C = \mu_k N_C = 0.2 m_C g$$

$$\xleftarrow{+} \Sigma F_x = m_C a_C: \quad T - 0.2 m_C g = m_C a_C \qquad (2)$$



Block B: $\quad +\!\downarrow \Sigma F_y = m_B a_B$

$$m_B g - 2T = m_B a_B \qquad (3)$$

From kinematics: $\quad a_B = \frac{1}{2}(a_A + a_C) \qquad (4)$

(*a*) Tension in cord. Given data: $m_A = 5$ kg, $m_B = m_C = 10$ kg

Eq. (1): $T - 0.2(5)g = 5a_A \qquad a_A = 0.2T - 0.2g \qquad (5)$

Eq. (2): $T - 0.2(10)g = 10a_C \qquad a_C = 0.1T - 0.2g \qquad (6)$

Eq. (3): $10g - 2T = 10a_B \qquad a_B = g - 0.2T \qquad (7)$

PROBLEM 12.30 (Continued)

Substitute into (4):

$$g - 0.2T = \frac{1}{2}(0.2T - 0.2g + 0.1T - 0.2g)$$

$$1.2g = 0.35T \qquad T = \frac{24}{7}g = \frac{24}{7}(9.81\text{ m/s}^2) \qquad T = 33.6\text{ N} \blacktriangleleft$$

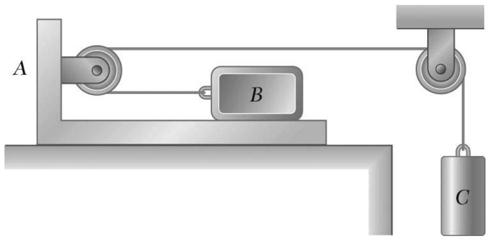
(*b*) Substitute for T into (5), (7), and (6):

$$a_A = 0.2\left(\frac{24}{7}g\right) - 0.2g = 0.4857(9.81\text{ m/s}^2) \qquad \mathbf{a}_A = 4.76\text{ m/s}^2 \longrightarrow \blacktriangleleft$$

$$a_B = g - 0.2\left(\frac{24}{7}g\right) = 0.3143(9.81\text{ m/s}^2) \qquad \mathbf{a}_B = 3.08\text{ m/s}^2 \downarrow \blacktriangleleft$$

$$a_C = 0.1\left(\frac{24}{7}g\right) - 0.2g = 0.14286(9.81\text{ m/s}^2) \qquad \mathbf{a}_C = 1.401\text{ m/s}^2 \longleftarrow \blacktriangleleft$$

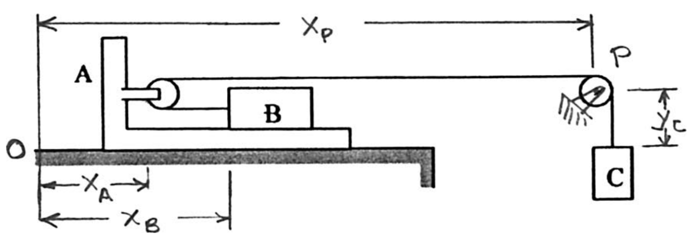
PROBLEM 12.31



A 10-lb block B rests as shown on a 20-lb bracket A. The coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.25$ between block B and bracket A, and there is no friction in the pulley or between the bracket and the horizontal surface. (a) Determine the maximum weight of block C if block B is not to slide on bracket A. (b) If the weight of block C is 10% larger than the answer found in a determine the accelerations of A, B and C.

SOLUTION

<u>Kinematics</u>. Let x_A and x_B be horizontal coordinates of A and B measured from a fixed vertical line to the left of A and B. Let y_C be the distance that block C is below the pulley. Note that y_C increases when C moves downward. See figure.



The cable length L is fixed.

$$L = (x_B - x_A) + (x_P - x_A) + y_C + \text{constant}$$

Differentiating and noting that $\dot{x}_P = 0$,

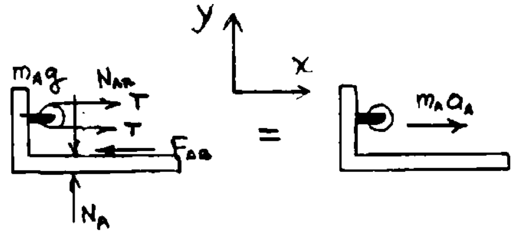
$$v_B - 2v_A + v_C = 0$$

$$-2a_A + a_B + a_C = 0 \tag{1}$$

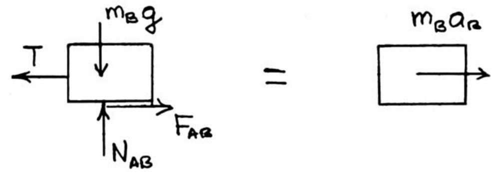
Here, a_A and a_B are positive to the right, and a_C is positive downward.

<u>Kinetics</u>. Let T be the tension in the cable and F_{AB} be the friction force between blocks A and B. The free body diagrams are:

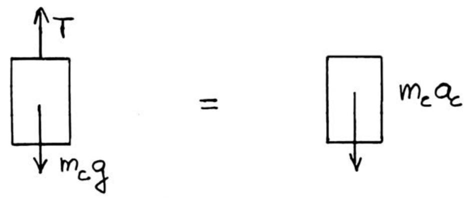
Bracket A:



Block *B*:



Block *C*:



Bracket *A*: $\xrightarrow{+} \Sigma F_x = ma_x:\quad 2T - F_{AB} = \frac{W_A}{g}a_A$ (2)

Block *B*: $\xrightarrow{+} \Sigma F_x = ma_x:\quad F_{AB} - T = \frac{W_B}{g}a_B$ (3)

$+\uparrow \Sigma F_y = ma_y:\quad N_{AB} - W_B = 0$

or $N_{AB} = W_B$

Block *C*: $+\downarrow \Sigma F_y = ma_y:\quad m_C - T = \frac{W_C}{g}a_C$ (4)

Adding Eqs. (2), (3), and (4), and transposing,

$$\frac{W_A}{g}a_A + \frac{W_B}{g}a_B + \frac{W_C}{g}a_C = W_C \tag{5}$$

Subtracting Eq. (4) from Eq. (3) and transposing,

$$\frac{W_B}{g}a_B - \frac{W_C}{g}a_C = F_{AB} - W_C \tag{6}$$

(*a*) No slip between *A* and *B*. $a_B = a_A$

From Eq. (1), $a_A = a_B = a_C = a$

From Eq. (5), $a = \frac{W_C g}{W_A + W_B + W_C}$

For impending slip, $F_{AB} = \mu_s N_{AB} = \mu_s W_B$

PROBLEM 12.31 (Continued)

Substituting into Eq. (6),

$$\frac{(W_B - W_C)(W_C g)}{W_A + W_B + W_C} = \mu_s W_B - W_C$$

Solving for W_C,

$$W_C = \frac{\mu_s W_B (W_A + W_B)}{W_A + 2W_B - \mu_s W_B}$$

$$= \frac{(0.30)(10)(20+10)}{20+(2)(10)-(0.30)(10)}$$

$W_C = 2.43$ lbs ◀

(*b*) W_C increased by 10%. $W_C = 2.6757$ lbs

Since slip is occurring, $F_{AB} = \mu_k N_{AB} = \mu_k W_B$

Eq. (6) becomes $\frac{W_B}{g} a_B - \frac{W_C}{g} a_C = \mu_k W_B - W_C$

or $10a_B - 2.6757a_C = [(0.25)(10) - 2.6757](32.2)$ (7)

With numerical data, Eq. (5) becomes

$$20a_A + 10a_B + 2.6757a_C = (2.6757)(32.2) \quad (8)$$

Solving Eqs. (1), (7), and (8) gives

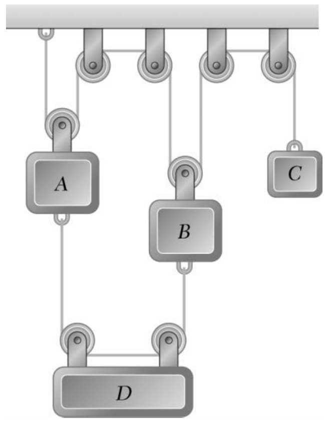
$$a_A = 3.144 \text{ ft/s}^2, \quad a_B = 0.881 \text{ ft/s}^2, \quad a_C = 5.407 \text{ ft/s}^2$$

$\mathbf{a}_A = 3.14 \text{ ft/s}^2 \longrightarrow$ ◀

$\mathbf{a}_B = 0.881 \text{ ft/s}^2 \longrightarrow$ ◀

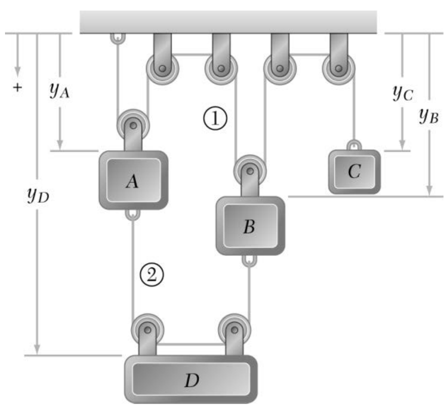
$\mathbf{a}_C = 5.41 \text{ ft/s}^2 \downarrow$ ◀

PROBLEM 12.32



The masses of blocks A, B, C and D are 9 kg, 9 kg, 6 kg and 7 kg, respectively. Knowing that a downward force of magnitude 120 N is applied to block D, determine (a) the acceleration of each block, (b) the tension in cord ABC. Neglect the weights of the pulleys and the effect of friction.

SOLUTION



Note: As shown, the system is in equilibrium.

From the diagram:

Cord 1: $$2y_A + 2y_B + y_C = \text{constant}$$

Then $$2v_A + 2v_B + v_C = 0$$

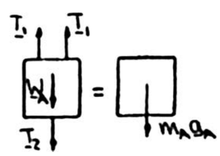
and $$2a_A + 2a_B + a_C = 0 \qquad (1)$$

Cord 2: $$(y_D - y_A) + (y_D - y_B) = \text{constant}$$

Then $$2v_D - v_A - v_B = 0$$

and $$2a_D - a_A - a_B = 0 \qquad (2)$$

A:



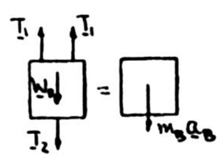
(a) $+\downarrow \Sigma F_y = m_A a_A: \quad m_A g - 2T_1 + T_2 = m_A a_A$

or $$9(9.81) - 2T_1 + T_2 = 9a_A \qquad (3)$$

$+\downarrow \Sigma F_y = m_B a_B: \quad m_B g - 2T_1 + T_2 = m_B a_B$

or $$9(9.81) - 2T_1 + T_2 = 9a_B \qquad (4)$$

B:



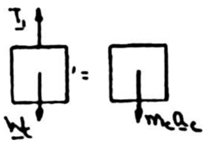
Note: Eqs. (3) and (4) $\Rightarrow \mathbf{a}_A = \mathbf{a}_B$

Then Eq. (1) $\Rightarrow a_C = -4a_A$

Eq. (2) $\Rightarrow a_D = a_A$

PROBLEM 12.32 (Continued)

C:



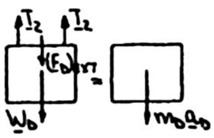
$$+\downarrow \Sigma F_y = m_C a_C: \quad m_C g - T_1 = m_C a_C$$

or $$T_1 = m_C(g - a_C) = 6(g + 4a_A) \qquad (5)$$

$$+\downarrow \Sigma F_y = m_D a_D: \quad m_D g - 2T_2 + (F_D)_{\text{ext}} = m_D a_D$$

or $$T_2 = \frac{1}{2}[m_D(g - a_D) + 120] = 94.335 - \frac{1}{2}(7a_A) \qquad (6)$$

D:



Substituting for T_1 [Eq. (5)] and T_2 [Eq. (6)] in Eq. (3)

$$9(9.81) - 2\times 6(g + 4a_A) + 94.335 - \frac{1}{2}(7a_A) = 9a_A$$

or $$a_A = \frac{9(9.81) - 2\times 6(9.81) + 94.335}{48 + 3.5 + 9} = 1.0728 \text{ m/s}^2$$

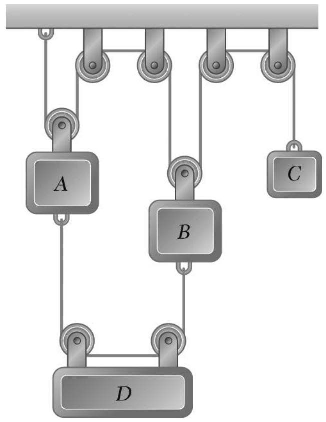
$$\mathbf{a}_A = \mathbf{a}_B = \mathbf{a}_D = 1.073 \text{ m/s}^2 \downarrow \blacktriangleleft$$

and $a_C = -4(1.0728 \text{ m/s}^2)$ or $\mathbf{a}_C = 4.29 \text{ m/s}^2 \uparrow \blacktriangleleft$

(*b*) Substituting into Eq. (5)

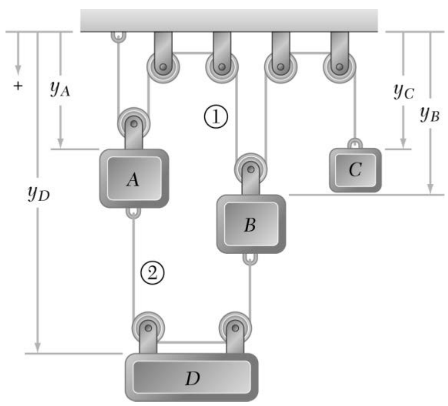
$$T_1 = 6\big(9.81 + 4(1.0728)\big) \qquad \text{or} \quad T_1 = 84.6 \text{ N} \blacktriangleleft$$

PROBLEM 12.33



The masses of blocks A, B, C and D are 9 kg, 9 kg, 6 kg and 7 kg, respectively. Knowing that a downward force of magnitude 50 N is applied to block B and that the system starts from rest, determine at $t = 3$ s the velocity (a) of D relative to A, (b) of C relative to D. Neglect the weights of the pulleys and the effect of friction.

SOLUTION



Note: As shown, the system is in equilibrium.

From the constraint of the two cords,

Cord 1: $$2y_A + 2y_B + y_C = \text{constant}$$

Then $$2v_A + 2v_B + v_C = 0$$

and $$2a_A + 2a_B + a_C = 0 \qquad (1)$$

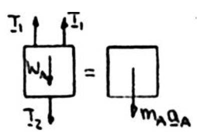
Cord 2: $$(y_D - y_A) + (y_D - y_B) = \text{constant}$$

Then $$2v_D - v_A - v_B = 0$$

and $$2a_D - a_A - a_B = 0 \qquad (2)$$

We determine the accelerations of blocks A, C, and D using the blocks as free bodies.

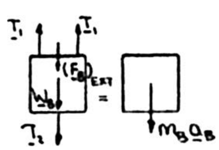
A:



$$+\!\downarrow \Sigma F_y = m_A a_A: \quad W_A - 2T_1 + T_2 = \frac{W_A}{g} a_A$$

or $$m_A g - 2T_1 + T_2 = m_A a_A \qquad (3)$$

B:



$$+\!\downarrow \Sigma F_y = m_B a_B: \quad W_B - 2T_1 + T_2 + (F_B)_{\text{ext}} = \frac{W_B}{g} a_B$$

or $$m_B g - 2T_1 + T_2 + (F_B)_{\text{ext}} = m_B a_B \qquad (4)$$

Forming $$(3) - (4) \Rightarrow -(F_B)_{\text{ext}} = 9(a_A - a_B)$$

or $$a_B = a_A + \frac{(F_B)_{\text{ext}}}{m_B}$$

PROBLEM 12.33 (Continued)

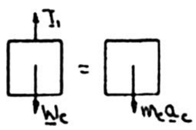
Then Eq. (1): $$2a_A + 2\left(a_A + \frac{(F_B)_{\text{ext}}}{m_g}\right) + a_C = 0$$

or $$a_C = -4a_A - \frac{2(F_B)_{\text{ext}}}{m_B}$$

Eq. (2): $$2a_D - a_A - \left(a_A + \frac{(F_B)_{\text{ext}}}{m_B}\right) = 0$$

or $$a_D = a_A + \frac{(F_B)_{\text{ext}}}{2m_B}$$

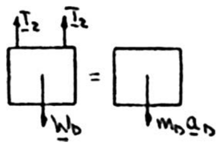
C:



$$+\downarrow \Sigma F_y = m_C a_C: \quad W_C - T_1 = m_C a_C = m_C\left(-4a_A - \frac{2(F_B)_{\text{ext}}}{m_B}\right)$$

or $$T_1 = m_C g + 4m_C a_A + \frac{2m_C(F_B)_{\text{ext}}}{m_B} \qquad (5)$$

D:



$$+\downarrow \Sigma F_y = m_D a_D: \quad W_D - 2T_2 = m_D a_D$$

or $$T_2 = \frac{1}{2} \times m_D\left[g - a_A - \frac{(F_B)_{\text{ext}}}{2m_B}\right] \qquad (6)$$

Substituting for T_1 [Eq. (5)] and T_2 [Eq. (6)] in Eq. (3)

$$m_A g - 2\left[m_C g + 4m_C a_A + \frac{2m_C(F_B)_{\text{ext}}}{m_B}\right] + \frac{1}{2} \times m_D\left[g - a_A - \frac{(F_B)_{\text{ext}}}{2m_B}\right] = m_A a_A$$

or $$a_A = \frac{m_A g - 2m_C g - \frac{4m_C(F_B)_{\text{ext}}}{m_B} - \frac{m_D(F_B)_{\text{ext}}}{4m_B} + \frac{m_D g}{2}}{m_A + 8m_C + \frac{m_D}{2}} = -2.2835 \text{ m/s}^2$$

Then $$a_C = -4(-2.2835 \text{ m/s}^2) - \frac{2(50)}{9} = -1.9771 \text{ m/s}^2$$

$$a_D = -2.2835 \text{ m/s}^2 + \frac{(50)}{2(9)} = 0.4943 \text{ m/s}^2$$

Note: We have uniformly accelerated motion, so that

$$v = 0 + at$$

(*a*) We have $$\mathbf{v}_{D/A} = \mathbf{v}_D - \mathbf{v}_A$$

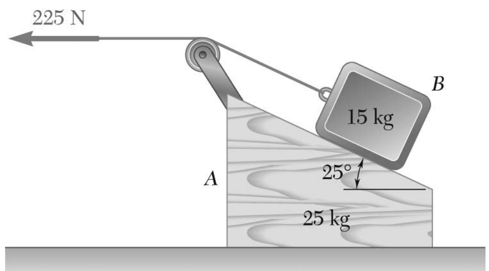
or $$\mathbf{v}_{D/A} = a_D t - a_A t = [0.4943 - (-2.2835)] \text{ m/s}^2 \times 3 \text{ s}$$

or $$\mathbf{v}_{D/A} = 8.33 \text{ m/s} \downarrow \blacktriangleleft$$

(*b*) And $$\mathbf{v}_{C/D} = \mathbf{v}_C = \mathbf{v}_D$$

or $$\mathbf{v}_{C/D} = a_C t - a_D t = (-1.9771 - 0.4943) \text{ m/s}^2 \times 3 \text{ s}$$

or $$\mathbf{v}_{C/D} = 7.41 \text{ m/s} \uparrow \blacktriangleleft$$



PROBLEM 12.34

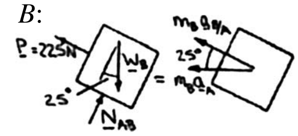
The 15-kg block B is supported by the 25-kg block A and is attached to a cord to which a 225-N horizontal force is applied as shown. Neglecting friction, determine (a) the acceleration of block A, (b) the acceleration of block B relative to A.

SOLUTION

(a) First we note $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$, where $\mathbf{a}_{B/A}$ is directed along the inclined surface of A.

B: $\nwarrow^+ \Sigma F_x = m_B a_x$: $P - W_B \sin 25° = m_B a_A \cos 25° + m_B a_{B/A}$

B:



or $$225 - 15g \sin 25° = 15(a_A \cos 25° + a_{B/A})$$

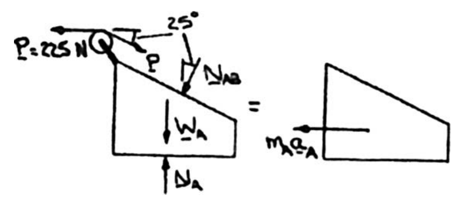
or $$15 - g \sin 25° = a_A \cos 25° + a_{B/A} \qquad (1)$$

$+\nearrow \Sigma F_y = m_B a_y$: $N_{AB} - W_B \cos 25° = -m_B a_A \sin 25°$

or $$N_{AB} = 15(g \cos 25° - a_A \sin 25°)$$

A: $\overset{+}{\leftarrow} \Sigma F_{x'} = m_A a_A$: $P - P\cos 25° + N_{AB} \sin 25° = m_A a_A$

A:



or $$N_{AB} = [25a_A - 225(1 - \cos 25°)]/\sin 25°$$

Equating the two expressions for N_{AB}

$$15(g \cos 25° - a_A \sin 25°) = \frac{25a_A - 225(1 - \cos 25°)}{\sin 25°}$$

or $$a_A = \frac{3(9.81)\cos 25° \sin 25° + 45(1 - \cos 25°)}{5 + 3\sin^2 25°}$$

$$= 2.7979 \text{ m/s}^2$$

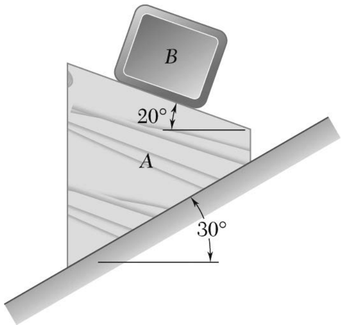
$\mathbf{a}_A = 2.80 \text{ m/s}^2 \longleftarrow$ ◀

(b) From Eq. (1)

$$a_{B/A} = 15 - (9.81)\sin 25° - 2.7979 \cos 25°$$

or $\mathbf{a}_{B/A} = 8.32 \text{ m/s}^2 \searrow 25°$ ◀

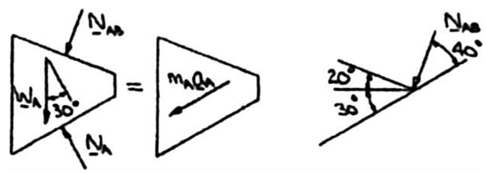
PROBLEM 12.35



Block *B* of mass 10-kg rests as shown on the upper surface of a 22-kg wedge *A*. Knowing that the system is released from rest and neglecting friction, determine (*a*) the acceleration of *B*, (*b*) the velocity of *B* relative to *A* at $t = 0.5$ s.

SOLUTION

A:

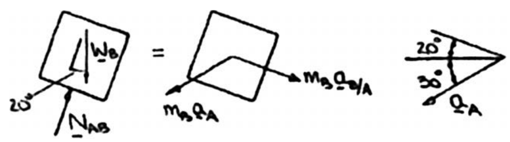


(*a*) $+\swarrow \Sigma F_x = m_A a_A: \quad W_A \sin 30° + N_{AB} \cos 40° = m_A a_A$

or
$$N_{AB} = \frac{22\left(a_A - \frac{1}{2}g\right)}{\cos 40°}$$

Now we note: $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$, where $\mathbf{a}_{B/A}$ is directed along the top surface of *A*.

B:



$+\nearrow \Sigma F_{y'} = m_B a_{y'}: \quad N_{AB} - W_B \cos 20° = -m_B a_A \sin 50°$

or
$$N_{AB} = 10\,(g \cos 20° - a_A \sin 50°)$$

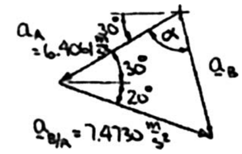
Equating the two expressions for N_{AB}

$$\frac{22\left(a_A - \dfrac{1}{2}g\right)}{\cos 40°} = 10(g \cos 20° - a_A \sin 50°)$$

or
$$a_A = \frac{(9.81)(1.1 + \cos 20° \cos 40°)}{2.2 + \cos 40° \sin 50°} = 6.4061 \text{ m/s}^2$$

$\overset{+}{\searrow} \Sigma F_{x'} = m_B a_{x'}: \quad W_B \sin 20° = m_B a_{B/A} - m_B a_A \cos 50°$

or
$$\begin{aligned} a_{B/A} &= g \sin 20° + a_A \cos 50° \\ &= (9.81 \sin 20° + 6.4061 \cos 50°) \text{ m/s}^2 \\ &= 7.4730 \text{ m/s}^2 \end{aligned}$$



Finally $\quad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

We have $\quad a_B^2 = 6.4061^2 + 7.4730^2 - 2(6.4061 \times 7.4730)\cos 50°$

or $\quad a_B = 5.9447 \text{ m/s}^2$

and $\quad \dfrac{7.4730}{\sin\alpha} = \dfrac{5.9447}{\sin 50°}$

or $\quad \alpha = 74.4°$

$\mathbf{a}_B = 5.94 \text{ m/s}^2 \;\searrow\; 75.6°$ ◀

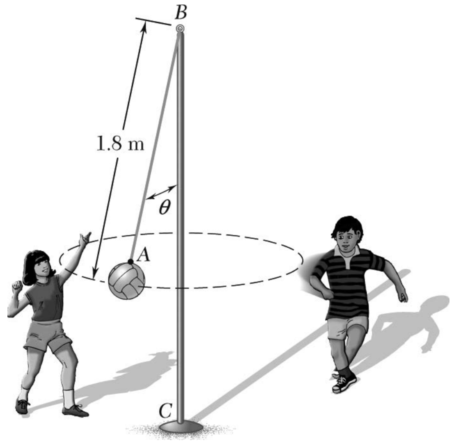
(*b*) *Note:* We have uniformly accelerated motion, so that

$$v = 0 + at$$

Now $\quad \mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A = \mathbf{a}_B t - \mathbf{a}_A t = \mathbf{a}_{B/A} t$

At $t = 0.5$ s: $\quad v_{B/A} = 7.4730 \text{ m/s}^2 \times 0.5 \text{ s}$

or $\quad \mathbf{v}_{B/A} = 3.74 \text{ m/s} \;\searrow\; 20°$ ◀

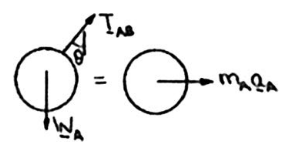


PROBLEM 12.36

A 450-g tetherball A is moving along a horizontal circular path at a constant speed of 4 m/s. Determine (a) the angle θ that the cord forms with pole BC, (b) the tension in the cord.

SOLUTION

First we note $$a_A = a_n = \frac{v_A^2}{\rho}$$



where $$\rho = l_{AB}\sin\theta$$

(a) $+\uparrow \Sigma F_y = 0$: $T_{AB}\cos\theta - W_A = 0$

or $$T_{AB} = \frac{m_A g}{\cos\theta}$$

$\xrightarrow{+} \Sigma F_x = m_A a_A$: $T_{AB}\sin\theta = m_A \dfrac{v_A^2}{\rho}$

Substituting for T_{AB} and ρ

$$\frac{m_A g}{\cos\theta}\sin\theta = m_A \frac{v_A^2}{l_{AB}\sin\theta} \qquad \sin^2\theta = 1 - \cos^2\theta$$

$$1 - \cos^2\theta = \frac{(4\text{ m/s})^2}{1.8\text{ m}\times 9.81\text{ m/s}^2}\cos\theta$$

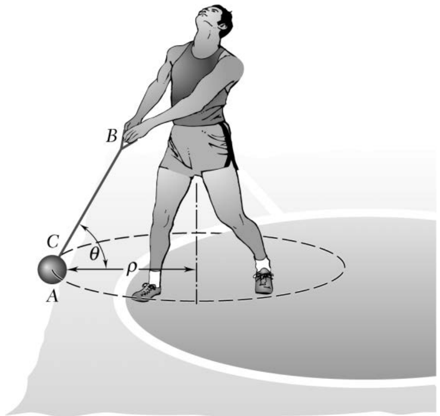
or $$\cos^2\theta + 0.906105\cos\theta - 1 = 0$$

Solving $$\cos\theta = 0.64479$$

or $$\theta = 49.9° \blacktriangleleft$$

(b) From above $$T_{AB} = \frac{m_A g}{\cos\theta} = \frac{0.450\text{ kg}\times 9.81\text{ m/s}^2}{0.64479}$$

or $$T_{AB} = 6.85\text{ N} \blacktriangleleft$$

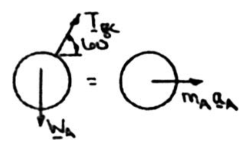


PROBLEM 12.37

During a hammer thrower's practice swings, the 7.1-kg head A of the hammer revolves at a constant speed v in a horizontal circle as shown. If $\rho = 0.93$ m and $\theta = 60°$, determine (a) the tension in wire BC, (b) the speed of the hammer's head.

SOLUTION

First we note $$a_A = a_n = \frac{v_A^2}{\rho}$$



(a) $+\uparrow \Sigma F_y = 0$: $T_{BC} \sin 60° - W_A = 0$

or $$T_{BC} = \frac{7.1 \text{ kg} \times 9.81 \text{ m/s}^2}{\sin 60°}$$
$$= 80.426 \text{ N}$$

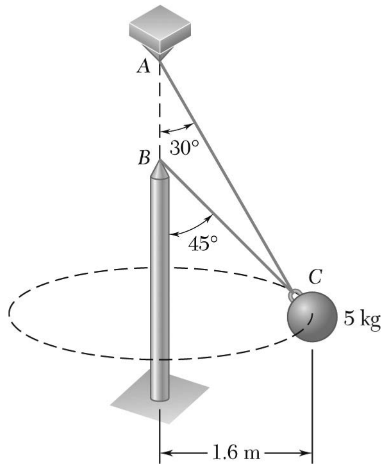
$T_{BC} = 80.4$ N ◀

(b) $\xrightarrow{+} \Sigma F_x = m_A a_A$: $T_{BC} \cos 60° = m_A \dfrac{v_A^2}{\rho}$

or $$v_A^2 = \frac{(80.426 \text{ N})\cos 60° \times 0.93 \text{ m}}{7.1 \text{ kg}}$$

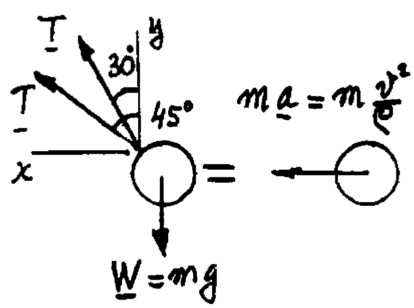
or $v_A = 2.30$ m/s ◀

PROBLEM 12.38



A single wire ACB passes through a ring at C attached to a sphere which revolves at a constant speed v in the horizontal circle shown. Knowing that the tension is the same in both portions of the wire, determine the speed v.

SOLUTION



$$\overset{+}{\leftarrow}\ \Sigma F_x = ma: \quad T(\sin 30° + \sin 45°) = \frac{mv^2}{\rho} \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad T(\cos 30° + \cos 45°) - mg = 0$$

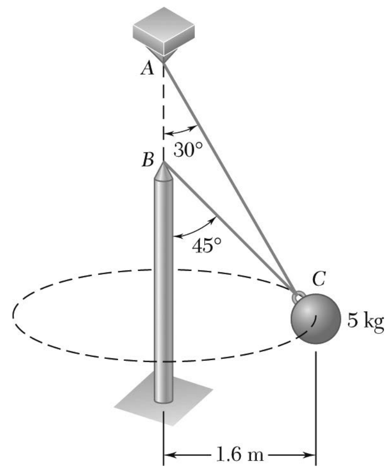
$$T(\cos 30° + \cos 45°) = mg \qquad (2)$$

Divide Eq. (1) by Eq. (2):

$$\frac{\sin 30° + \sin 45°}{\cos 30° + \cos 45°} = \frac{v^2}{\rho g}$$

$$v^2 = 0.76733 \rho g = 0.76733\,(1.6 \text{ m})(9.81 \text{ m/s}^2) = 12.044 \text{ m}^2/\text{s}^2$$

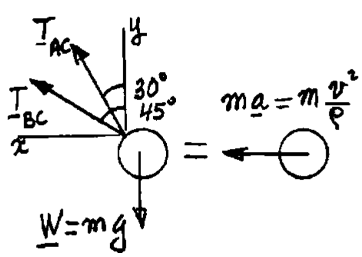
$v = 3.47$ m/s ◀



PROBLEM 12.39

Two wires AC and BC are tied at C to a sphere which revolves at a constant speed v in the horizontal circle shown. Determine the range of values of v for which both wires remain taut.

SOLUTION



$$\overset{+}{\leftarrow} \Sigma F_x = ma: \quad T_{AC} \sin 30° + T_{BC} \sin 45° = \frac{mv^2}{\rho} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad T_{AC} \cos 30° + T_{BC} \cos 45° - mg = 0$$

$$T_{AC} \cos 30° + T_{BC} \cos 45° = mg \qquad (2)$$

Divide Eq (1) by Eq. (2):

$$\frac{T_{AC} \sin 30° + T_{BC} \sin 45°}{T_{AC} \cos 30° + T_{BC} \cos 45°} = \frac{v^2}{\rho g} \qquad (3)$$

When AC is slack, $T_{AC} = 0$.

Eq. (3) yields $\quad v_1^2 = \rho g \tan 45° = (1.6 \text{ m})\ (9.81 \text{ m/s}^2) \tan 45° = 15.696 \text{ m}^2/\text{s}^2$

Wire AC will remain taut if $v \le v_1$, that is, if $\quad v_1 = 3.96$ m/s $\quad v \le 3.96$ m/s ◁

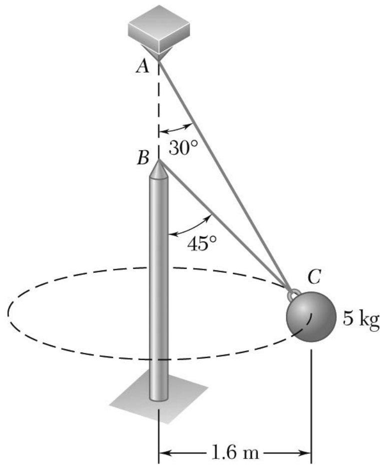
When BC is slack, $T_{BC} = 0$.

Eq. (3) yields $\quad v_2^2 = \rho g \tan 30° = (1.6 \text{ m})(9.81 \text{ m/s}^2) \tan 30° = 9.0621 \text{ m}^2/\text{s}^2$

Wire BC will remain taut if $v \ge v_2$, that is, if $\quad v_2 = 3.01$ m/s $\quad v \ge 3.01$ m/s ◀

Combining the results obtained, we conclude that both wires remain taut for

$3.01 \text{ m/s} \le v \le 3.96 \text{ m/s}$ ◀



PROBLEM 12.40

Two wires AC and BC are tied at C to a sphere which revolves at a constant speed v in the horizontal circle shown. Determine the range of the allowable values of v if both wires are to remain taut and if the tension in either of the wires is not to exceed 60 N.

SOLUTION

From the solution of Problem 12.39, we find that both wires remain taut for $3.01 \text{ m/s} \le v \le 3.96 \text{ m/s}$ ◁

To determine the values of v for which the tension in either wire will not exceed 60 N, we recall Eqs. (1) and (2) from Problem 12.39:

$$T_{AC}\sin 30° + T_{BC}\sin 45° = \frac{mv^2}{\rho} \tag{1}$$

$$T_{AC}\cos 30° + T_{BC}\cos 45° = mg \tag{2}$$

Subtract Eq. (1) from Eq. (2). Since sin 45° = cos 45°, we obtain

$$T_{AC}(\cos 30° - \sin 30°) = mg - \frac{mv^2}{\rho} \tag{3}$$

Multiply Eq. (1) by cos 30°, Eq. (2) by sin 30°, and subtract:

$$T_{BC}(\sin 45°\cos 30° - \cos 45°\sin 30°) = \frac{mv^2}{\rho}\cos 30° - mg\sin 30°$$

$$T_{BC}\sin 15° = \frac{mv^2}{\rho}\cos 30° - mg\sin 30° \tag{4}$$

Making $T_{AC} = 60$ N, $m = 5$ kg, $\rho = 1.6$ m, $g = 9.81 \text{ m/s}^2$ in Eq. (3), we find the value v_1 of v for which

$T_{AC} = 60$ N: $\quad 60(\cos 30° - \sin 30°) = 5(9.81) - \dfrac{5v_1^2}{1.6}$

$$21.962 = 49.05 - \frac{v_1^2}{0.32} \qquad v_1^2 = 8.668, \qquad v_1 = 2.94 \text{ m/s}$$

We have $T_{AC} \le 60$ N for $v \ge v_1$, that is, for $v \ge 2.94$ m/s ◁

PROBLEM 12.40 (Continued)

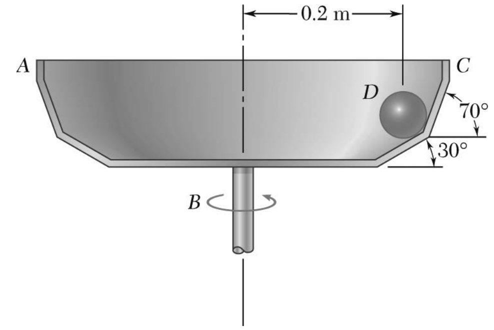
Making $T_{BC} = 60$ N, $m = 5$ kg, $\rho = 1.6$ m, $g = 9.81$ m/s^2 in Eq. (4), we find the value v_2 of v for which

$T_{BC} = 60$ N: $$60 \sin 15° = \frac{5v_2^2}{1.6} \cos 30° - 5(9.81) \sin 30°$$

$$15.529 = 2.7063 v_2^2 - 24.523 \qquad v_2^2 = 14.80, \qquad v_2 = 3.85 \text{ m/s}$$

We have $T_{BC} \leq 60$ N for $v \leq v_2$, that is, for $v \leq 3.85$ m/s ◁

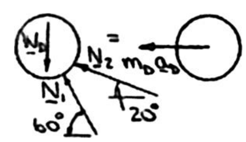
Combining the results obtained, we conclude that the range of allowable value is 3.01 m/s $\leq v \leq$ 3.85 m/s ◀



PROBLEM 12.41

A 100-g sphere D is at rest relative to drum ABC which rotates at a constant rate. Neglecting friction, determine the range of the allowable values of the velocity v of the sphere if neither of the normal forces exerted by the sphere on the inclined surfaces of the drum is to exceed 1.1 N.

SOLUTION



First we note $$a_D = a_n = \frac{v_D^2}{\rho}$$

where $$\rho = 0.2 \text{ m}$$

$$\overset{+}{\longleftarrow} \Sigma F_x = m_D a_D: \quad N_1 \cos 60° + N_2 \cos 20° = m_D \frac{v_D^2}{\rho} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_1 \sin 60° + N_2 \sin 20° - W_D = 0$$

or $$N_1 \sin 60° + N_2 \sin 20° = m_D g \qquad (2)$$

<u>Case 1: N_1 is maximum.</u>

Let $$N_1 = 1.1 \text{ N}$$

Eq. (2) $$(1.1 \text{ N}) \sin 60° + N_2 \sin 20° = (0.1 \text{ kg})\ (9.81 \text{ m/s}^2)$$

or $$N_2 = 0.082954 \text{ N}$$

$$(N_2)_{(N_1)_{\max}} < 1.1 \text{ N} \qquad \text{OK}$$

Eq. (1) $$(v_D^2)_{(N_1)_{\max}} = \frac{0.2 \text{ m}}{0.1 \text{ kg}} (1.1 \cos 60° + 0.082954 \cos 20°) \text{ N}$$

or $$(v_D)_{(N_1)_{\max}} = 1.121 \text{ m/s}$$

Now we form $$(\sin 20°) \times [\text{Eq. (1)}] - (\cos 20°) \times [\text{Eq. (2)}]$$

$$N_1 \cos 60° \sin 20° - N_1 \sin 60° \cos 20° = m_D \frac{v_D^2}{\rho} \sin 20° - m_D g \cos 20°$$

or $$-N_1 \sin 40° = m_D \frac{v_D^2}{\rho} \sin 20° - m_D g \cos 20°$$

$(v_D)_{\min}$ occurs when $N_1 = (N_1)_{\max}$

$(v_D)_{\min} = 1.121$ m/s

Case 2: N_2 is maximum.

Let $$N_2 = 1.1 \text{ N}$$

Eq. (2) $$N_1 \sin 60° + (1.1 \text{ N}) \sin 20° = (0.1 \text{ kg})(9.81 \text{ m/s}^2)$$

or $$N_1 = 0.69834 \text{ N}$$

$$(N_1)_{(N_2)_{\max}} \leq 1.1 \text{ N} \qquad \text{OK}$$

Eq. (1) $$(v_D^2)_{(N_2)_{\max}} = \frac{0.2 \text{ m}}{0.1 \text{ kg}}(0.69834 \cos 60° + 1.1 \cos 20°) \text{ N}$$

or $$(v_D)_{(N_2)_{\max}} = 1.663 \text{ m/s}$$

Now we form $$(\sin 60°) \times [\text{Eq. (1)}] - (\cos 60°) \times [\text{Eq. (2)}]$$

$$N_2 \cos 20° \sin 60° - N_2 \sin 20° \cos 60° = m_D \frac{v_D^2}{\rho} \sin 60° - m_D g \cos 60°$$

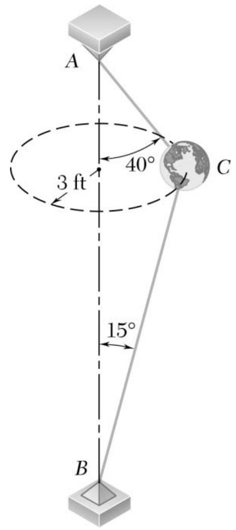
or $$N_2 \cos 40° = m_D \frac{v_D^2}{\rho} \sin 60° - m_D g \cos 60°$$

$(v_D)_{\max}$ occurs when $N_2 = (N_2)_{\max}$

$(v_D)_{\max} = 1.663 \text{ m/s}$

For $N_1 \leq N_2 < 1.1 \text{ N}$

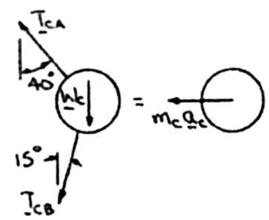
$$1.121 \text{ m/s} < v_D < 1.663 \text{ m/s} \blacktriangleleft$$



PROBLEM 12.42*

As part of an outdoor display, a 12-lb model C of the earth is attached to wires AC and BC and revolves at a constant speed v in the horizontal circle shown. Determine the range of the allowable values of v if both wires are to remain taut and if the tension in either of the wires is not to exceed 26 lb.

SOLUTION



First note $$a_C = a_n = \frac{v_C^2}{\rho}$$

where $$\rho = 3 \text{ ft}$$

$$\xleftarrow{+} \Sigma F_x = m_C a_C: \quad T_{CA}\sin 40° + T_{CB}\sin 15° = \frac{W_C}{g}\frac{v_C^2}{\rho} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad T_{CA}\cos 40° - T_{CB}\cos 15° - W_C = 0 \qquad (2)$$

Note that Eq. (2) implies that

(*a*) when $T_{CB} = (T_{CB})_{\max}, \quad T_{CA} = (T_{CA})_{\max}$

(*b*) when $T_{CB} = (T_{CB})_{\min}, \quad T_{CA} = (T_{CA})_{\min}$

Case 1: T_{CA} is maximum.

Let $$T_{CA} = 26 \text{ lb}$$

Eq. (2) $$(26 \text{ lb})\cos 40° - T_{CB}\cos 15° - (12 \text{ lb}) = 0$$

or $$T_{CB} = 8.1964 \text{ lb}$$

$$(T_{CB})_{(T_{CA})_{\max}} < 26 \text{ lb} \qquad \text{OK}$$

$$[(T_{CB})_{\max} = 8.1964 \text{ lb}]$$

PROBLEM 12.42* (Continued)

Eq. (1)

$$(v_C^2)_{(T_{CA})_{\max}} = \frac{(32.2 \text{ ft/s}^2)(3 \text{ ft})}{12 \text{ lb}}(26\sin 40° + 8.1964 \sin 15°) \text{ lb}$$

or $$(v_C)_{(T_{CA})_{\max}} = 12.31 \text{ ft/s}$$

Now we form $$(\cos 15°)(\text{Eq. 1}) + (\sin 15°)(\text{Eq. 2})$$

$$T_{CA}\sin 40° \cos 15° + T_{CA}\cos 40° \sin 15° = \frac{W_C}{g}\frac{v_C^2}{\rho}\cos 15° + W_C \sin 15°$$

or $$T_{CA}\sin 55° = \frac{W_C}{g}\frac{v_C^2}{\rho}\cos 15° + W_C \sin 15° \quad (3)$$

$(v_c)_{\max}$ occurs when $T_{CA} = (T_{CA})_{\max}$

$(v_C)_{\max} = 12.31$ ft/s

Case 2: T_{CA} is minimum.

Because $(T_{CA})_{\min}$ occurs when $T_{CB} = (T_{CB})_{\min}$,

let $T_{CB} = 0$ (note that wire BC will not be taut).

Eq. (2) $$T_{CA}\cos 40° - (12 \text{ lb}) = 0$$

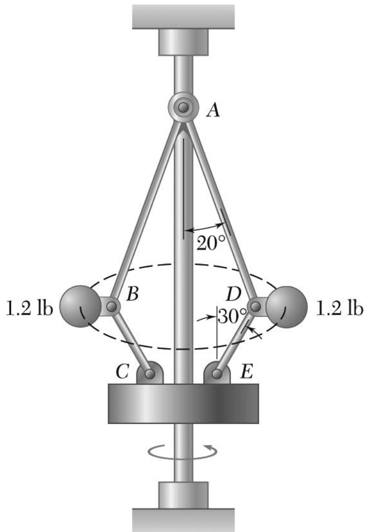
or $$T_{CA} = 15.6649 \text{ lb}, \quad 26 \text{ lb} \quad \text{OK}$$

Note: Eq. (3) implies that when $T_{CA} = (T_{CA})_{\min}$, $v_C = (v_C)_{\min}$. Then

Eq. (1) $$(v_C^2)_{\min} = \frac{(32.2 \text{ ft/s}^2)(3 \text{ ft})}{12 \text{ lb}}(15.6649 \text{ lb})\sin 40°$$

or $$(v_C)_{\min} = 9.00 \text{ ft/s}$$

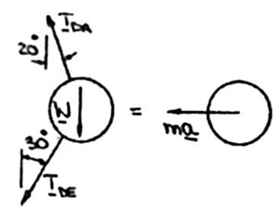
$0 < T_{CA} \leq T_{CB} < 6$ lb when $9.00 \text{ ft/s} < v_C < 12.31 \text{ ft/s}$ ◀



PROBLEM 12.43*

The 1.2-lb flyballs of a centrifugal governor revolve at a constant speed v in the horizontal circle of 6-in. radius shown. Neglecting the weights of links AB, BC, AD, and DE and requiring that the links support only tensile forces, determine the range of the allowable values of v so that the magnitudes of the forces in the links do not exceed 17 lb.

SOLUTION



First note $\qquad a = a_n = \dfrac{v^2}{\rho}$

where $\qquad \rho = 0.5$ ft

$$\overset{+}{\leftarrow} \Sigma F_x = ma: \quad T_{DA}\sin 20° + T_{DE}\sin 30° = \frac{W}{g}\frac{v^2}{\rho} \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad T_{DA}\cos 20° - T_{DE}\cos 30° - W = 0 \tag{2}$$

Note that Eq. (2) implies that

(*a*) when $\qquad T_{DE} = (T_{DE})_{\max}, \quad T_{DA} = (T_{DA})_{\max}$

(*b*) when $\qquad T_{DE} = (T_{DE})_{\min}, \quad T_{DA} = (T_{DA})_{\min}$

Case 1: T_{DA} is maximum.

Let $\qquad T_{DA} = 17$ lb

Eq. (2) $\qquad (17\text{ lb})\cos 20° - T_{DE}\cos 30° - (1.2\text{ lb}) = 0$

or $\qquad T_{DE} = 17.06$ lb $\quad$ unacceptable (>17 lb)

Now let $\qquad T_{DE} = 17$ lb

Eq. (2) $\qquad T_{DA}\cos 20° - (17\text{ lb})\cos 30° - (1.2\text{ lb}) = 0$

or $\qquad T_{DA} = 16.9443$ lb $\quad$ OK $\quad$ (≤ 17 lb)

PROBLEM 12.43* (Continued)

$$(T_{DA})_{\max} = 16.9443 \text{ lb}$$
$$(T_{DE})_{\max} = 17 \text{ lb}$$

Eq. (1) $$(v^2)_{(T_{DA})_{\max}} = \frac{(32.2 \text{ ft/s}^2)(0.5 \text{ ft})}{1.2 \text{ lb}}(16.9443 \sin 20° + 17 \sin 30°) \text{ lb}$$

or $$v_{(T_{DA})_{\max}} = 13.85 \text{ ft/s}$$

Now form $$(\cos 30°) \times [\text{Eq. (1)}] + (\sin 30°) \times [\text{Eq. (2)}]$$

$$T_{DA} \sin 20° \cos 30° + T_{DA} \cos 20° \sin 30° = \frac{W}{g}\frac{v^2}{\rho}\cos 30° + W \sin 30°$$

or $$T_{DA} \sin 50° = \frac{W}{g}\frac{v^2}{\rho}\cos 30° + W \sin 30° \qquad (3)$$

$v_{\max}$ occurs when $T_{DA} = (T_{DA})_{\max}$

$v_{\max} = 13.85$ ft/s

Case 2: T_{DA} is minimum.

Because $(T_{DA})_{\min}$ occurs when $T_{DE} = (T_{DE})_{\min}$,

let $T_{DE} = 0$.

Eq. (2) $$T_{DA} \cos 20° - (1.2 \text{ lb}) = 0$$

or $$T_{DA} = 1.27701 \text{ lb}, \ 17 \text{ lb} \qquad \text{OK}$$

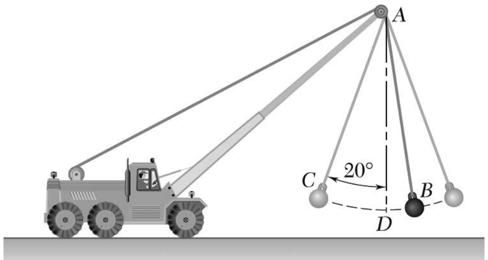
Note: Eq. (3) implies that when $T_{DA} = (T_{DA})_{\min}$, $v = v_{\min}$. Then

Eq. (1) $$(v^2)_{\min} = \frac{(32.2 \text{ ft/s}^2)(0.5 \text{ ft})}{1.2 \text{ lb}}(1.27701 \text{ lb}) \sin 20°$$

or $$v_{\min} = 2.42 \text{ ft/s}$$

$0 < T_{AB}, T_{BC}, T_{AD}, T_{DE} < 17$ lb

when $2.42 \text{ ft/s} < v < 13.85 \text{ ft/s}$ ◀

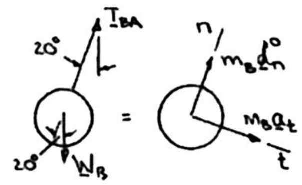


PROBLEM 12.44

A 130-lb wrecking ball B is attached to a 45-ft-long steel cable AB and swings in the vertical arc shown. Determine the tension in the cable (a) at the top C of the swing, (b) at the bottom D of the swing, where the speed of B is 13.2 ft/s.

SOLUTION

(a) At C, the top of the swing, $v_B = 0$; thus



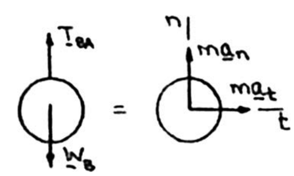
$$a_n = \frac{v_B^2}{L_{AB}} = 0$$

$+\nearrow \; \Sigma F_n = 0: \quad T_{BA} - W_B \cos 20° = 0$

or $$T_{BA} = (130 \text{ lb}) \times \cos 20°$$

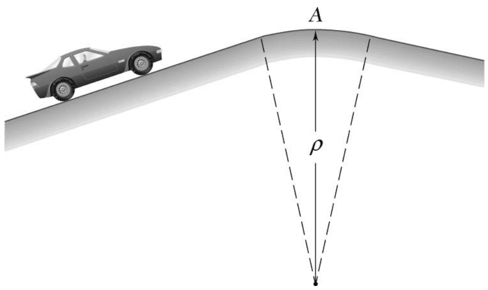
or $T_{BA} = 122.2$ lb ◀

(b) $+\uparrow \; \Sigma F_n = ma_n: \quad T_{BA} - W_B = m_B \dfrac{(v_B)_D^2}{L_{AB}}$



or $$T_{BA} = (130 \text{ lb}) + \left[\left(\frac{130 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\left(\frac{(13.2 \text{ ft/s})^2}{45 \text{ ft}}\right)\right]$$

or $T_{BA} = 145.6$ lb ◀



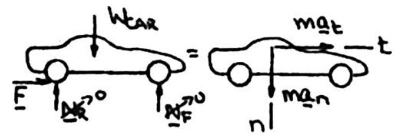
PROBLEM 12.45

During a high-speed chase, a 2400-lb sports car traveling at a speed of 100 mi/h just loses contact with the road as it reaches the crest A of a hill. (*a*) Determine the radius of curvature ρ of the vertical profile of the road at A. (*b*) Using the value of ρ found in part *a*, determine the force exerted on a 160-lb driver by the seat of his 3100-lb car as the car, traveling at a constant speed of 50 mi/h, passes through A.

SOLUTION

(*a*) *Note:* 100 mi/h = 146.667 ft/s

$$+\downarrow \Sigma F_n = ma_n: \quad W_{\text{car}} = \frac{W_{\text{car}}}{g}\frac{v_A^2}{\rho}$$

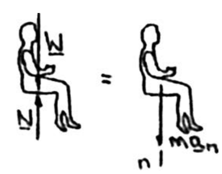


or
$$\rho = \frac{(146.667 \text{ ft/s})^2}{32.2 \text{ ft/s}^2}$$
$$= 668.05 \text{ ft}$$

or $\rho = 668 \text{ ft}$ ◀

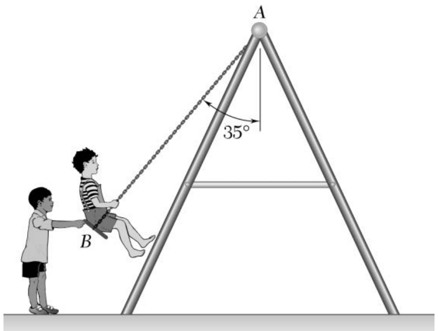
(*b*) *Note:* v is constant $\Rightarrow a_t = 0$; 50 mi/h = 73.333 ft/s

$$+\downarrow \Sigma F_n = ma_n: \quad W - N = \frac{W}{g}\frac{v_A^2}{\rho}$$



or
$$N = (160 \text{ lb})\left[1 - \frac{(73.333 \text{ ft/s})^2}{(32.2 \text{ ft/s}^2)(668.05 \text{ ft})}\right]$$

or $\mathbf{N} = 120.0 \text{ lb}\uparrow$ ◀



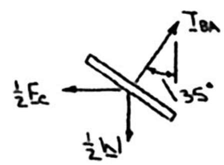
PROBLEM 12.46

A child having a mass of 22 kg sits on a swing and is held in the position shown by a second child. Neglecting the mass of the swing, determine the tension in rope *AB* (*a*) while the second child holds the swing with his arms outstretched horizontally, (*b*) immediately after the swing is released.

SOLUTION

Note: The factors of "$\frac{1}{2}$" are included in the following free-body diagrams because there are two ropes and only one is considered.

(*a*) For the swing at rest

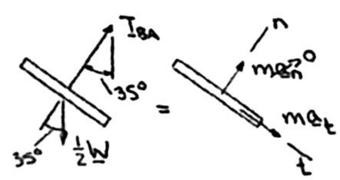


$$\Sigma F_y = 0: \quad T_{BA} \cos 35° - \frac{1}{2} W = 0$$

or $$T_{BA} = \frac{22 \text{ kg} \times 9.81 \text{ m/s}^2}{2 \cos 35°}$$

or $$T_{BA} = 131.7 \text{ N} \blacktriangleleft$$

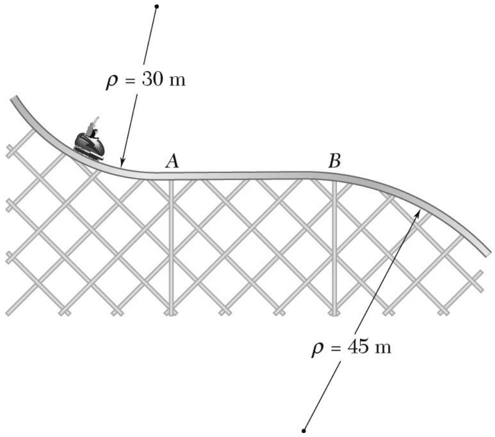
(*b*) At $t = 0$, $v = 0$, so that $$a_n = \frac{v^2}{\rho} = 0$$



$$+\nearrow \Sigma F_n = 0: \quad T_{BA} - \frac{1}{2} W \cos 35° = 0$$

or $$T_{BA} = \frac{1}{2}(22 \text{ kg})(9.81 \text{ m/s}^2) \cos 35°$$

or $$T_{BA} = 88.4 \text{ N} \blacktriangleleft$$



PROBLEM 12.47

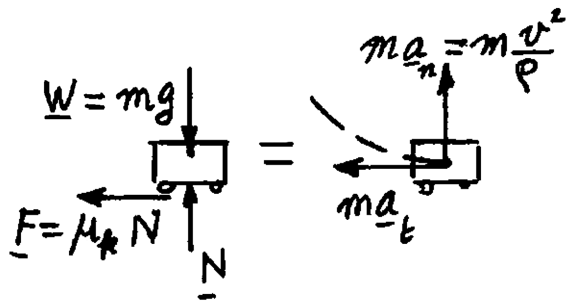
The roller-coaster track shown is contained in a vertical plane. The portion of track between *A* and *B* is straight and horizontal, while the portions to the left of *A* and to the right of *B* have radii of curvature as indicated. A car is traveling at a speed of 72 km/h when the brakes are suddenly applied, causing the wheels of the car to slide on the track ($\mu_k = 0.20$). Determine the initial deceleration of the car if the brakes are applied as the car (*a*) has almost reached *A*, (*b*) is traveling between *A* and *B*, (*c*) has just passed *B*.

SOLUTION

(*a*) $+\uparrow \Sigma F_n = ma_n: \quad N - mg = m\dfrac{v^2}{\rho}$

$$N = m\left(g + \frac{v^2}{\rho}\right)$$

$$F = \mu_k N = \mu_k m\left(g + \frac{v^2}{\rho}\right)$$



$$\overset{+}{\longleftarrow} \Sigma F_t = ma_t: \quad F = ma_t$$

$$a_t = \frac{F}{m} = \mu_k\left(g + \frac{v^2}{\rho}\right)$$

Given data: $\mu_k = 0.20,\ v = 72 \text{ km/h} = 20 \text{ m/s}$

$$g = 9.81 \text{ m/s}^2, \quad \rho = 30 \text{ m}$$

$$a_t = 0.20\left[9.81 + \frac{(20)^2}{30}\right]$$

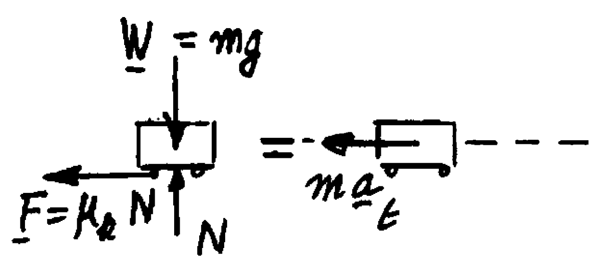
$a_t = 4.63 \text{ m/s}^2$ ◀

(*b*) $a_n = 0$ $\quad +\uparrow \Sigma F_n = ma_n = 0: \quad N - mg = 0$

$$N = mg$$

$$F = \mu_k N = \mu_k mg$$

$$\overset{+}{\longleftarrow} \Sigma F_t = ma_t: \quad F = ma_t$$



$$a_t = \frac{F}{m} = \mu_k g = 0.20(9.81)$$

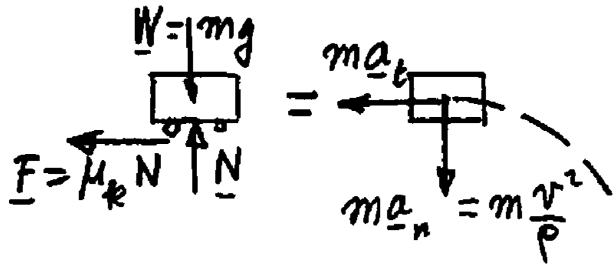
$a_t = 1.962 \text{ m/s}^2$ ◀

PROBLEM 12.47 (Continued)

(c) $+\downarrow \Sigma F_n = ma_n$: $mg - N = \dfrac{mv^2}{\rho}$

$$N = m\left(g - \frac{v^2}{\rho}\right)$$

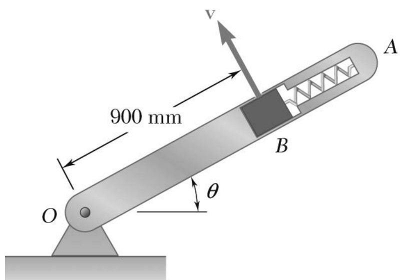
$$F = \mu_k N = \mu_k m\left(g - \frac{v^2}{\rho}\right)$$



$\overset{+}{\leftarrow} \Sigma F_t = ma_t$: $F = ma_t$

$$a_t = \frac{F}{m} = \mu_k\left(g - \frac{v^2}{\rho}\right) = 0.20\left[9.81 - \frac{(20)^2}{45}\right]$$

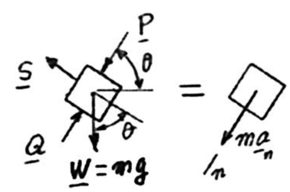
$a_t = 0.1842 \text{ m/s}^2$ ◀



PROBLEM 12.48

A 250-g block fits inside a small cavity cut in arm OA, which rotates in the vertical plane at a constant rate such that $v = 3$ m/s. Knowing that the spring exerts on block B a force of magnitude $P = 1.5$ N and neglecting the effect of friction, determine the range of values of θ for which block B is in contact with the face of the cavity closest to the axis of rotation O.

SOLUTION



$$+\swarrow \Sigma F_n = ma_n: \quad P + mg\sin\theta - Q = m\frac{v^2}{\rho}$$

To have contact with the specified surface, we need $Q \geq 0$,

or $$Q = P + mg\sin\theta - \frac{mv^2}{\rho} > 0$$

$$\sin\theta > \frac{1}{g}\left(\frac{v^2}{\rho} - \frac{P}{m}\right) \qquad (1)$$

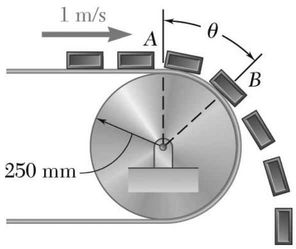
Data: $$m = 0.250 \text{ kg}, \quad v = 3 \text{ m/s}, \quad P = 1.5 \text{ N}, \quad \rho = 0.9 \text{ m}$$

Substituting into (1):

$$\sin\theta > \frac{1}{9.81}\left[\frac{(3)^2}{0.9} - \frac{1.5}{0.25}\right]$$

$$\sin\theta > 0.40775$$

$$24.1° < \theta < 155.9° \blacktriangleleft$$



PROBLEM 12.49

A series of small packages, each with a mass of 0.5 kg, are discharged from a conveyor belt as shown. Knowing that the coefficient of static friction between each package and the conveyor belt is 0.4, determine (*a*) the force exerted by the belt on a package just after it has passed Point *A*, (*b*) the angle θ defining the Point *B* where the packages first *slip* relative to the belt.

SOLUTION

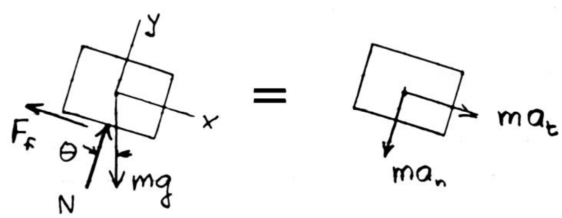
Assume package does not slip.

$$a_t = 0, \quad F_f \leq \mu_s N$$

On the curved portion of the belt

$$a_n = \frac{v^2}{\rho} = \frac{(1\text{ m/s})^2}{0.250\text{ m}} = 4\text{ m/s}^2$$

For any angle θ



$$+\nearrow \Sigma F_y = ma_y: \quad N - mg\cos\theta = -ma_n = -\frac{mv^2}{\rho}$$

$$N = mg\cos\theta - \frac{mv^2}{\rho} \tag{1}$$

$$+\searrow \Sigma F_x = ma_x: \quad -F_f + mg\sin\theta = ma_t = 0$$

$$F_f = mg\sin\theta \tag{2}$$

(*a*) At Point *A*,

$$\theta = 0°$$

$$N = (0.5)(9.81)(1.000) - (0.5)(4)$$

$N = 2.905\text{ N}$ ◀

(*b*) At Point *B*,

$$F_f = \mu_s N$$

$$mg\sin\theta = \mu_s(mg\cos\theta - ma_n)$$

$$\sin\theta = \mu_s\left(\cos\theta - \frac{a_n}{g}\right) = 0.40\left[\cos\theta - \frac{4}{9.81}\right]$$

Squaring and using trigonometic identities,

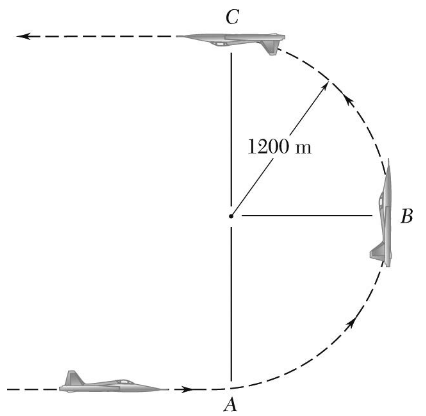
$$1-\cos^2\theta = 0.16\cos^2\theta - 0.130479\cos\theta + 0.026601$$

$$1.16\cos^2\theta - 0.130479\cos\theta - 0.97340 = 0$$

$$\cos\theta = 0.97402 \qquad \theta = 13.09° \blacktriangleleft$$

Check that package does not separate from the belt.

$$N = \frac{F_f}{\mu_s} = \frac{mg\sin\theta}{\mu_s} \qquad N > 0.$$



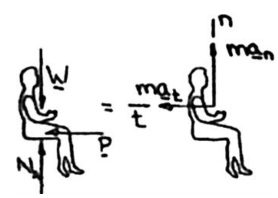
PROBLEM 12.50

A 54-kg pilot flies a jet trainer in a half vertical loop of 1200-m radius so that the speed of the trainer decreases at a constant rate. Knowing that the pilot's apparent weights at Points A and C are 1680 N and 350 N, respectively, determine the force exerted on her by the seat of the trainer when the trainer is at Point B.

SOLUTION

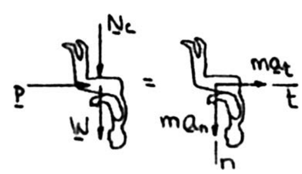
First we note that the pilot's apparent weight is equal to the vertical force that she exerts on the seat of the jet trainer.

At A: $+\uparrow \Sigma F_n = ma_n$: $\quad N_A - W = m\dfrac{v_A^2}{\rho}$



or
$$v_A^2 = (1200\text{ m})\left(\frac{1680\text{ N}}{54\text{ kg}} - 9.81\text{ m/s}^2\right)$$
$$= 25{,}561.3\text{ m}^2/\text{s}^2$$

At C: $+\downarrow \Sigma F_n = ma_n$: $\quad N_C + W = m\dfrac{v_C^2}{\rho}$

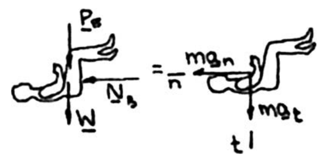


or
$$v_C^2 = (1200\text{ m})\left(\frac{350\text{ N}}{54\text{ kg}} + 9.81\text{ m/s}^2\right)$$
$$= 19{,}549.8\text{ m}^2/\text{s}^2$$

Since $a_t = \text{constant}$, we have from A to C
$$v_C^2 = v_A^2 + 2a_t\ \Delta s_{AC}$$
or
$$19{,}549.8\text{ m}^2/\text{s}^2 = 25{,}561.3\text{ m}^2/\text{s}^2 + 2a_t(\pi \times 1200\text{ m})$$
or
$$a_t = -0.79730\text{ m/s}^2$$

Then from A to B
$$v_B^2 = v_A^2 + 2a_t\ \Delta s_{AB}$$
$$= 25{,}561.3\text{ m}^2/\text{s}^2 + 2(-0.79730\text{ m/s}^2)\left(\frac{\pi}{2}\times 1200\text{ m}\right)$$
$$= 22{,}555\text{ m}^2/\text{s}^2$$

PROBLEM 12.50 (Continued)



At B: $\overset{+}{\leftarrow}\ \Sigma F_n = ma_n$: $N_B = m\dfrac{v_B^2}{\rho}$

or $$N_B = 54 \text{ kg}\,\frac{22{,}555 \text{ m}^2/\text{s}^2}{1200 \text{ m}}$$

or $$\mathbf{N}_B = 1014.98 \text{ N} \longleftarrow$$

$+\!\downarrow \Sigma F_t = ma_t$: $W + P_B = m\,|a_t|$

or $$P_B = (54 \text{ kg})(0.79730 - 9.81) \text{ m/s}^2$$

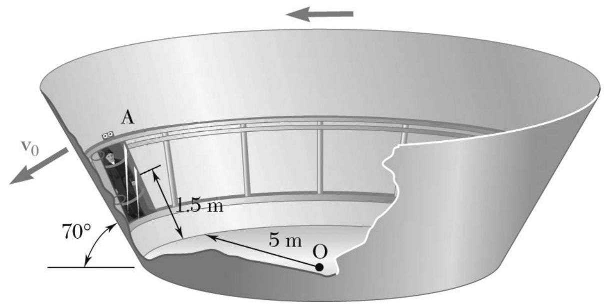
or $$\mathbf{P}_B = 486.69 \text{ N} \uparrow$$

Finally, $$(F_{\text{pilot}})_B = \sqrt{N_B^2 + P_B^2} = \sqrt{(1014.98)^2 + (486.69)^2}$$
$$= 1126 \text{ N}$$

or $(\mathbf{F}_{\text{pilot}})_B = 1126 \text{ N} \searrow 25.6° \blacktriangleleft$

PROBLEM 12.51

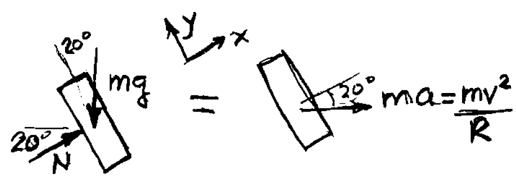
A carnival ride is designed to allow the general public to experience high acceleration motion. The ride rotates about Point O in a horizontal circle such that the rider has a speed v_0. The rider reclines on a platform A which rides on rollers such that friction is negligible. A mechanical stop prevents the platform from rolling down the incline. Determine (*a*) the speed v_0 at which the platform A begins to roll upwards, (*b*) the normal force experienced by an 80-kg rider at this speed.



SOLUTION

Radius of circle: $R = 5 + 1.5\cos 70° = 5.513 \text{ m}$

$\Sigma \mathbf{F} = m\mathbf{a}$:



Components up the incline, 70°:

$$-m_A g\cos 20° = -\frac{mv_0^2}{R}\sin 20°$$

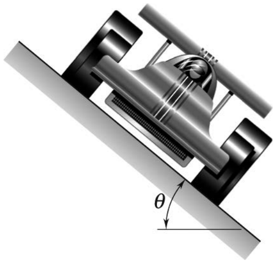
(*a*) Speed v_0: $v_0 = \left[\frac{g\,R}{\tan 20°}\right]^{\frac{1}{2}} = \left[\frac{(9.81 \text{ m/s})(5.513 \text{ m})}{\tan 20°}\right]^{\frac{1}{2}} = 12.1898 \text{ m/s}$ $\qquad v_0 = 12.19 \text{ m/s}$ ◀

Components normal to the incline, 20°.

$$N - mg\sin 20° = \frac{mv_0^2}{R}\cos 20°.$$

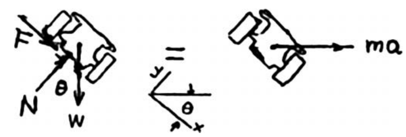
(*b*) Normal force: $N = (80)(9.81)\sin 20° + \frac{80(12.1898)^2}{5.513}\cos 20° = 2294 \text{ N}$ $\qquad N = 2290 \text{ N}$ ◀

PROBLEM 12.52



A curve in a speed track has a radius of 1000 ft and a rated speed of 120 mi/h. (See Sample Problem 12.6 for the definition of rated speed). Knowing that a racing car starts skidding on the curve when traveling at a speed of 180 mi/h, determine (*a*) the banking angle θ, (*b*) the coefficient of static friction between the tires and the track under the prevailing conditions, (*c*) the minimum speed at which the same car could negotiate that curve.

SOLUTION



Weight $W = mg$

Acceleration $a = \dfrac{v^2}{\rho}$

$\Sigma F_x = ma_x$: $F + W \sin\theta = ma\cos\theta$

$$F = \frac{mv^2}{\rho}\cos\theta - mg\sin\theta \qquad (1)$$

$\Sigma F_y = ma_y$: $N - W\cos\theta = ma\sin\theta$

$$N = \frac{mv^2}{\rho}\sin\theta + mg\cos\theta \qquad (2)$$

(*a*) <u>Banking angle</u>. Rated speed $v = 120$ mi/h $= 176$ ft/s. $F = 0$ at rated speed.

$$0 = \frac{mv^2}{\rho}\cos\theta - mg\sin\theta$$

$$\tan\theta = \frac{v^2}{\rho g} = \frac{(176)^2}{(1000)(32.2)} = 0.96199$$

$$\theta = 43.89°$$

$\theta = 43.9°$ ◀

(*b*) <u>Slipping outward</u>. $v = 180$ mi/h $= 264$ ft/s

$$F = \mu N \quad \mu = \frac{F}{N} = \frac{v^2\cos\theta - \rho g\sin\theta}{v^2\sin\theta + \rho g\cos\theta}$$

$$\mu = \frac{(264)^2\cos 43.89° - (1000)(32.2)\sin 43.89°}{(264)^2\sin 43.89° + (1000)(32.2)\cos 43.89°}$$

$$= 0.39009$$

$\mu = 0.390$ ◀

(*c*) Minimum speed. $F = -\mu N$

$$-\mu = \frac{v^2 \cos\theta - \rho g \sin\theta}{v^2 \sin\theta + \rho g \cos\theta}$$

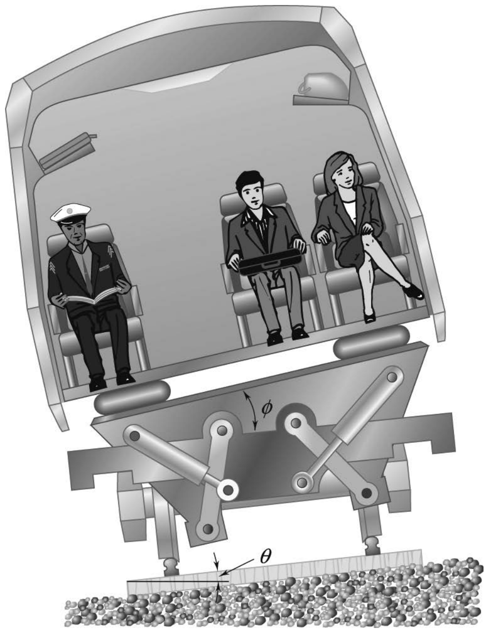
$$v^2 = \frac{\rho g (\sin\theta - \mu \cos\theta)}{\cos\theta + \mu \sin\theta}$$

$$= \frac{(1000)(32.2)(\sin 43.89° - 0.39009 \cos 43.89°)}{\cos 43.89° + 0.39009 \sin 43.89°}$$

$$= 13.369 \text{ ft}^2/\text{s}^2$$

$$v = 115.62 \text{ ft/s}$$

$v = 78.8$ mi/h ◀



PROBLEM 12.53

Tilting trains, such as the *American Flyer* which will run from Washington to New York and Boston, are designed to travel safely at high speeds on curved sections of track which were built for slower, conventional trains. As it enters a curve, each car is tilted by hydraulic actuators mounted on its trucks. The tilting feature of the cars also increases passenger comfort by eliminating or greatly reducing the side force $\mathbf{F}_s$ (parallel to the floor of the car) to which passengers feel subjected. For a train traveling at 100 mi/h on a curved section of track banked through an angle $\theta = 6°$ and with a rated speed of 60 mi/h, determine (*a*) the magnitude of the side force felt by a passenger of weight W in a standard car with no tilt $(\phi = 0)$, (*b*) the required angle of tilt ϕ if the passenger is to feel no side force. (See Sample Problem 12.6 for the definition of rated speed.)

SOLUTION

Rated speed: $v_R = 60 \text{ mi/h} = 88 \text{ ft/s}, \; 100 \text{ mi/h} = 146.67 \text{ ft/s}$

From Sample Problem 12.6,

$$v_R^2 = g\rho \tan\theta$$

or

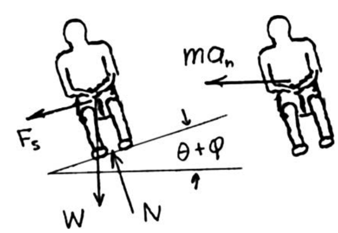
$$\rho = \frac{v_R^2}{g\tan\theta} = \frac{(88)^2}{32.2\tan 6°} = 2288 \text{ ft}$$

Let the *x*-axis be parallel to the floor of the car.

$$+\swarrow \Sigma F_x = ma_x: \quad F_s + W\sin(\theta+\phi) = ma_n\cos(\theta+\phi)$$

$$= \frac{mv^2}{\rho}\cos(\theta+\phi)$$

(*a*) $\phi = 0.$



$$F_s = W\left[\frac{v^2}{g\rho}\cos(\theta+\phi) - \sin(\theta+\phi)\right]$$

$$= W\left[\frac{(146.67)^2}{(32.2)(2288)}\cos 6° - \sin 6°\right]$$

$$= 0.1858W$$

$F_s = 0.1858W$ ◀

PROBLEM 12.53 (Continued)

(*b*) For $F_s = 0$,

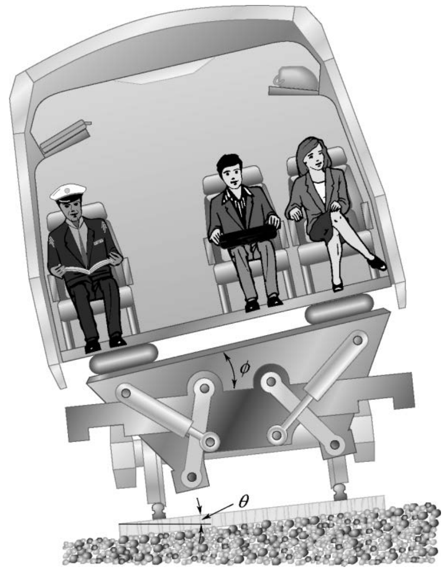
$$\frac{v^2}{g\rho}\cos(\theta+\phi) - \sin(\theta+\phi) = 0$$

$$\tan(\theta+\phi) = \frac{v^2}{g\rho} = \frac{(146.67)^2}{(32.2)(2288)} = 0.29199$$

$$\theta+\phi = 16.28°$$

$$\phi = 16.28° - 6°$$

$\phi = 10.28°$ ◀



PROBLEM 12.54

Tests carried out with the tilting trains described in Problem 12.53 revealed that passengers feel queasy when they see through the car windows that the train is rounding a curve at high speed, yet do not feel any side force. Designers, therefore, prefer to reduce, but not eliminate, that force. For the train of Problem 12.53, determine the required angle of tilt ϕ if passengers are to feel side forces equal to 10% of their weights.

SOLUTION

Rated speed: $v_R = 60 \text{ mi/h} = 88 \text{ ft/s}, \;\; 100 \text{ mi/h} = 146.67 \text{ ft/s}$

From Sample Problem 12.6,

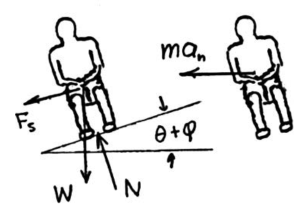
$$v_R^2 = g\rho \tan\theta$$

or $$\rho = \frac{v_R^2}{g\tan\theta} = \frac{(88)^2}{32.2\tan 6°} = 2288 \text{ ft}$$

Let the x-axis be parallel to the floor of the car.

$$+\swarrow \Sigma F_x = ma_x: \quad F_s + W\sin(\theta+\phi) = ma_n\cos(\theta+\phi)$$

$$= \frac{mv^2}{\rho}\cos(\theta+\phi)$$



Solving for F_s, $$F_s = W\left[\frac{v^2}{g\rho}\cos(\theta+\phi) - \sin(\theta+\phi)\right]$$

Now $$\frac{v^2}{g\rho} = \frac{(146.67)^2}{(32.2)(2288)} = 0.29199 \quad \text{and} \quad F_s = 0.10W$$

So that $$0.10W = W[0.29199\cos(\theta+\phi) - \sin(\theta+\phi)]$$

Let $$u = \sin(\theta+\phi)$$

Then $$\cos(\theta+\phi) = \sqrt{1-u^2}$$

$$0.10 = 0.29199\sqrt{1-u^2} - u \quad \text{or} \quad 0.29199\sqrt{1-u^2} = 0.10 + u$$

PROBLEM 12.54 (Continued)

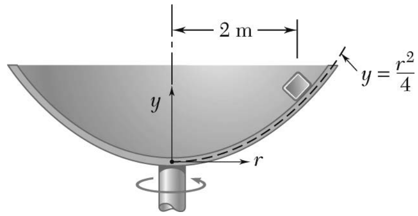
Squaring both sides, $0.08526(1-u^2) = 0.01 + 0.2u + u^2$

or $1.08526u^2 + 0.2u - 0.07526 = 0$

The positive root of the quadratic equation is $u = 0.18685$

Then, $\theta + \phi = \sin^{-1} u = 10.77°$

$\phi = 10.77° - 6°$ $\phi = 4.77°$ ◀



PROBLEM 12.55

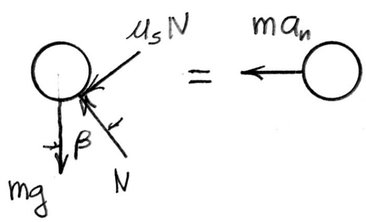
A 3-kg block is at rest relative to a parabolic dish which rotates at a constant rate about a vertical axis. Knowing that the coefficient of static friction is 0.5 and that $r = 2$ m, determine the maximum allowable velocity v of the block.

SOLUTION

Let β be the slope angle of the dish. $\tan\beta = \frac{dy}{dr} = \frac{1}{2}r$

At $r = 2$ m, $\tan\beta = 1$ or $\beta = 45°$

Draw free body sketches of the sphere.



$$\Sigma F_y = 0: \quad N\cos\beta - \mu_S N\sin\beta - mg = 0$$

$$N = \frac{mg}{\cos\beta - \mu_S \sin\beta}$$

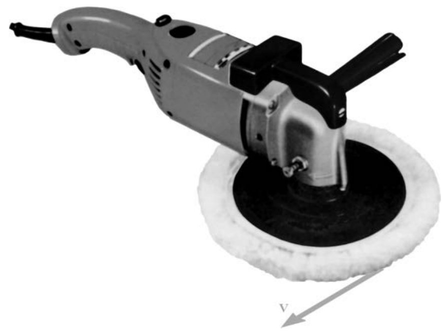
$$\overset{+}{\leftarrow} \Sigma F_n = ma_n: \quad N\sin\beta + \mu_S N\cos\beta = \frac{mv^2}{\rho}$$

$$\frac{mg(\sin\beta + \mu_S N\cos\beta)}{\cos\beta - \mu_S\sin\beta} = \frac{mv^2}{\rho}$$

$$v^2 = \rho g \frac{\sin\beta + \mu_S\cos\beta}{\cos\beta - \mu_S\sin\beta} = (2)(9.81)\frac{\sin 45° + 0.5\cos 45°}{\cos 45° - 0.5\sin 45°} = 58.86 \text{ m}^2/\text{s}^2$$

$v = 7.67$ m/s ◀

PROBLEM 12.56



Three seconds after a polisher is started from rest, small tufts of fleece from along the circumference of the 225-mm-diameter polishing pad are observed to fly free of the pad. If the polisher is started so that the fleece along the circumference undergoes a constant tangential acceleration of 4 m/s^2, determine (*a*) the speed *v* of a tuft as it leaves the pad, (*b*) the magnitude of the force required to free a tuft if the average mass of a tuft is 1.6 mg.

SOLUTION

(*a*) $a_t = \text{constant} \Rightarrow$ uniformly acceleration motion

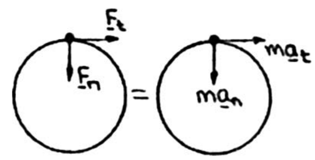
Then $$v = 0 + a_t t$$

At $t = 3$ s: $$v = (4 \text{ m/s}^2)(3 \text{ s})$$

or $$v = 12.00 \text{ m/s} \blacktriangleleft$$

(*b*) $\Sigma F_t = ma_t: \quad F_t = ma_t$

or $$F_t = (1.6\times10^{-6} \text{kg})(4 \text{ m/s}^2) = 6.4\times10^{-6} \text{N}$$



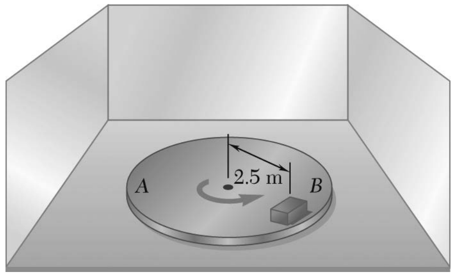
$$\Sigma F_n = ma_n: \quad F_n = m\frac{v^2}{\rho}$$

At $t = 3$ s: $$F_n = (1.6\times10^{-6} \text{kg})\frac{(12 \text{ m/s})^2}{\left(\frac{0.225}{2} \text{m}\right)} = 2.048\times10^{-3} \text{N}$$

Finally, $$F_{\text{tuft}} = \sqrt{F_t^2 + F_n^2} = \sqrt{(6.4\times10^{-6} \text{N})^2 + (2.048\times10^{-3} \text{N})^2}$$

or $$F_{\text{tuft}} = 2.05\times10^{-3} \text{N} \blacktriangleleft$$

PROBLEM 12.57



A turntable A is built into a stage for use in a theatrical production. It is observed during a rehearsal that a trunk B starts to slide on the turntable 10 s after the turntable begins to rotate. Knowing that the trunk undergoes a constant tangential acceleration of $0.24\ \text{m/s}^2$, determine the coefficient of static friction between the trunk and the turntable.

SOLUTION

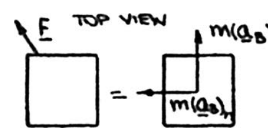
First we note that $(a_B)_t =$ constant implies uniformly accelerated motion.

$$v_B = 0 + (a_B)_t t$$

At $t = 10$ s:

$$v_B = (0.24\ \text{m/s}^2)(10\ \text{s}) = 2.4\ \text{m/s}$$

In the plane of the turntable

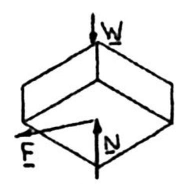


$$\Sigma \mathbf{F} = m_B \mathbf{a}_B: \quad \mathbf{F} = m_B(\mathbf{a}_B)_t + m_B(\mathbf{a}_B)_n$$

Then

$$F = m_B\sqrt{(a_B)_t^2 + (a_B)_n^2}$$

$$= m_B\sqrt{(a_B)_t^2 + \left(\frac{v_B^2}{\rho}\right)^2}$$



$$+\uparrow \Sigma F_y = 0: \quad N - W = 0$$

or

$$N = m_B g$$

At $t = 10$ s:

$$F = \mu_s N = \mu_s m_B g$$

Then

$$\mu_s m_B g = m_B\sqrt{(a_B)_t^2 + \left(\frac{v_B^2}{\rho}\right)^2}$$

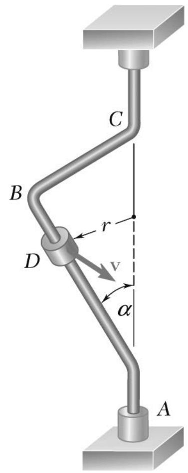
or

$$\mu_s = \frac{1}{9.81\ \text{m/s}^2}\left\{(0.24\ \text{m/s}^2)^2 + \left[\frac{(2.4\ \text{m/s})^2}{2.5\ \text{m}}\right]^2\right\}^{1/2}$$

or

$$\mu_s = 0.236 \blacktriangleleft$$

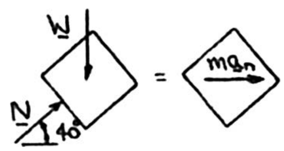
PROBLEM 12.58



A small, 300-g collar D can slide on portion AB of a rod which is bent as shown. Knowing that $\alpha = 40°$ and that the rod rotates about the vertical AC at a constant rate of 5 rad/s, determine the value of r for which the collar will not slide on the rod if the effect of friction between the rod and the collar is neglected.

SOLUTION

First note $$v_D = r\dot{\theta}_{ABC}$$



$$+\uparrow \Sigma F_y = 0: \quad N\sin 40° - W = 0$$

or $$N = \frac{mg}{\sin 40°}$$

$$\xrightarrow{+} \Sigma F_n = ma_n: \quad N\cos 40° = m\frac{v_D^2}{r}$$

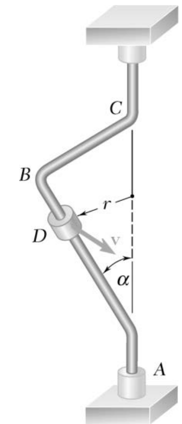
or $$\frac{mg}{\sin 40°}\cos 40° = m\frac{(r\dot{\theta}_{ABC})^2}{r}$$

or $$r = \frac{g}{\dot{\theta}_{ABC}^2}\frac{1}{\tan 40°}$$

$$= \frac{9.81 \text{ m/s}^2}{(5 \text{ rad/s})^2}\frac{1}{\tan 40°}$$

$$= 0.468 \text{ m}$$

or $r = 468$ mm ◀

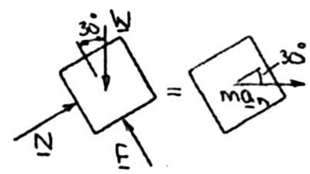


PROBLEM 12.59

A small, 200-g collar D can slide on portion AB of a rod which is bent as shown. Knowing that the rod rotates about the vertical AC at a constant rate and that $\alpha = 30°$ and $r = 600$ mm, determine the range of values of the speed v for which the collar will not slide on the rod if the coefficient of static friction between the rod and the collar is 0.30.

SOLUTION

Case 1: $v = v_{\min}$, impending motion downward



$$+\nearrow \Sigma F_x = ma_x: \quad N - W\sin 30° = m\frac{v^2}{r}\cos 30°$$

or
$$N = m\left(g\sin 30° + \frac{v^2}{r}\cos 30°\right)$$

$$+\nwarrow \Sigma F_y = ma_y: \quad F - W\cos 30° = -m\frac{v^2}{r}\sin 30°$$

or
$$F = m\left(g\cos 30° - \frac{v^2}{r}\sin 30°\right)$$

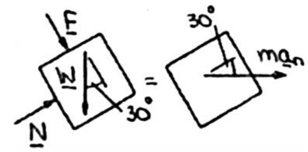
Now
$$F = \mu_s N$$

Then
$$m\left(g\cos 30° - \frac{v^2}{r}\sin 30°\right) = \mu_s \times m\left(g\sin 30° + \frac{v^2}{r}\cos 30°\right)$$

or
$$v^2 = gr\frac{1 - \mu_s \tan 30°}{\mu_s + \tan 30°}$$

$$= (9.81\ \text{m/s}^2)(0.6\ \text{m})\frac{1 - 0.3\tan 30°}{0.3 + \tan 30°}$$

or
$$v_{\min} = 2.36\ \text{m/s}$$



Case 2: $v = v_{max}$, impending motion upward

$$+\nearrow \Sigma F_x = ma_x: \quad N - W\sin 30° = m\frac{v^2}{r}\cos 30°$$

or
$$N = m\left(g\sin 30° + \frac{v^2}{r}\cos 30°\right)$$

$$+\searrow \Sigma F_y = ma_y: \quad F + W\cos 30° = m\frac{v^2}{r}\sin 30°$$

or
$$F = m\left(-g\cos 30° + \frac{v^2}{r}\sin 30°\right)$$

Now
$$F = \mu_s N$$

Then
$$m\left(-g\cos 30° + \frac{v^2}{r}\sin 30°\right) = \mu_s \times m\left(g\sin 30° + \frac{v^2}{r}\cos 30°\right)$$

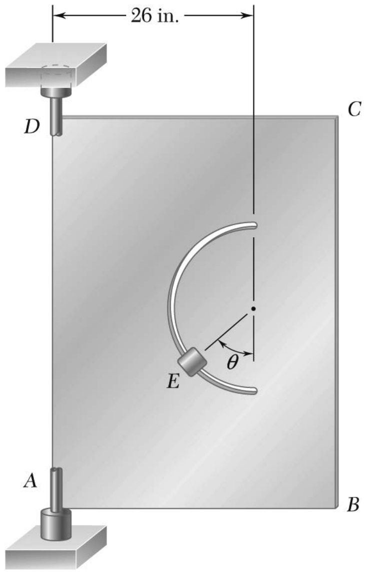
or
$$v^2 = gr\frac{1+\mu_s \tan 30°}{\tan 30° - \mu_s}$$

$$= (9.81 \text{ m/s}^2)(0.6 \text{ m})\frac{1+0.3\tan 30°}{\tan 30° - 0.3}$$

or
$$v_{max} = 4.99 \text{ m/s}$$

For the collar not to slide

$$2.36 \text{ m/s} < v < 4.99 \text{ m/s} \blacktriangleleft$$



PROBLEM 12.60

A semicircular slot of 10-in. radius is cut in a flat plate which rotates about the vertical *AD* at a constant rate of 14 rad/s. A small, 0.8-lb block *E* is designed to slide in the slot as the plate rotates. Knowing that the coefficients of friction are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine whether the block will slide in the slot if it is released in the position corresponding to (*a*) $\theta = 80°$, (*b*) $\theta = 40°$. Also determine the magnitude and the direction of the friction force exerted on the block immediately after it is released.

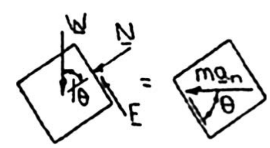
SOLUTION

First note
$$\rho = \frac{1}{12}(26 - 10\sin\theta)\text{ ft}$$
$$v_E = \rho\dot{\phi}_{ABCD}$$

Then
$$a_n = \frac{v_E^2}{\rho} = \rho(\dot{\phi}_{ABCD})^2$$
$$= \left[\frac{1}{12}(26 - 10\sin\theta)\text{ ft}\right](14\text{ rad/s})^2$$
$$= \frac{98}{3}(13 - 5\sin\theta)\text{ ft/s}^2$$

Assume that the block is at rest with respect to the plate.

$$+\swarrow \Sigma F_x = ma_x: \quad N + W\cos\theta = m\frac{v_E^2}{\rho}\sin\theta$$



or
$$N = W\left(-\cos\theta + \frac{v_E^2}{g\rho}\sin\theta\right)$$

$$+\searrow \Sigma F_y = ma_y: \quad -F + W\sin\theta = -m\frac{v_E^2}{\rho}\cos\theta$$

or
$$F = W\left(\sin\theta + \frac{v_E^2}{g\rho}\cos\theta\right)$$

(*a*) We have $\theta = 80°$

Then

$$N = (0.8 \text{ lb})\left[-\cos 80° + \frac{1}{32.2 \text{ ft/s}^2} \times \frac{98}{3}(13 - 5\sin 80°) \text{ ft/s}^2 \times \sin 80°\right]$$
$$= 6.3159 \text{ lb}$$

$$F = (0.8 \text{ lb})\left[\sin 80° + \frac{1}{32.2 \text{ ft/s}^2} \times \frac{98}{3}(13 - 5\sin 80°) \text{ ft/s}^2 \times \cos 80°\right]$$
$$= 1.92601 \text{ lb}$$

Now $F_{\max} = \mu_s N = 0.35(6.3159 \text{ lb}) = 2.2106 \text{ lb}$

The <u>block does not slide</u> in the slot, and

$\mathbf{F} = 1.926 \text{ lb} \measuredangle 80°$ ◀

(*b*) We have $\theta = 40°$

Then

$$N = (0.8 \text{ lb})\left[-\cos 40° + \frac{1}{32.2 \text{ ft/s}^2} \times \frac{98}{3}(13 - 5\sin 40°) \text{ ft/s}^2 \times \sin 40°\right]$$
$$= 4.4924 \text{ lb}$$

$$F = (0.8 \text{ lb})\left[\sin 40° + \frac{1}{32.2 \text{ ft/s}^2} \times \frac{98}{3}(13 - 5\sin 40°) \text{ ft/s}^2 \times \cos 40°\right]$$
$$= 6.5984 \text{ lb}$$

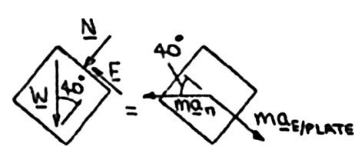
Now $F_{\max} = \mu_s N$, from which it follows that

$F > F_{\max}$

Block E will slide in the slot

and
$$\mathbf{a}_E = \mathbf{a}_n + \mathbf{a}_{E/\text{plate}}$$
$$= \mathbf{a}_n + (\mathbf{a}_{E/\text{plate}})_t + (\mathbf{a}_{E/\text{plate}})_n$$

At $t = 0$, the block is at rest relative to the plate, thus $(\mathbf{a}_{E/\text{plate}})_n = 0$ at $t = 0$, so that $\mathbf{a}_{E/\text{plate}}$ must be directed tangentially to the slot.



$$+\swarrow \Sigma F_x = ma_x: \quad N + W\cos 40° = m\frac{v_E^2}{\rho}\sin 40°$$

or
$$N = W\left(-\cos 40° + \frac{v_E^2}{g\rho}\sin 40°\right) \quad \text{(as above)}$$
$$= 4.4924 \text{ lb}$$

Sliding: $F = \mu_k N$

$= 0.25(4.4924 \text{ lb})$

$= 1.123 \text{ lb}$

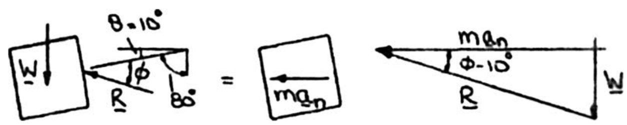
Noting that **F** and $\mathbf{a}_{E/\text{plane}}$ must be directed as shown (if their directions are reversed, then $\Sigma \mathbf{F}_x$ is ↘ while $m\mathbf{a}_x$ is ↖), we have

the <u>block slides downward</u> in the slot and $\mathbf{F} = 1.123 \text{ lb} \searrow 40°$ ◀

<u>Alternative solutions</u>.

(*a*) Assume that the block is at rest with respect to the plate.

$$\Sigma \mathbf{F} = m\mathbf{a}: \quad \mathbf{W} + \mathbf{R} = m\mathbf{a}_n$$



Then
$$\tan(\phi - 10°) = \frac{W}{ma_n} = \frac{W}{\frac{W}{g}\frac{v_E^2}{\rho}} = \frac{g}{\rho(\dot{\phi}_{ABCD})^2}$$

$$= \frac{32.2 \text{ ft/s}^2}{\frac{98}{3}(13 - 5\sin 80°) \text{ ft/s}^2} \qquad \text{(from above)}$$

or $\phi - 10° = 6.9588°$

and $\phi = 16.9588°$

Now $\tan\phi_s = \mu_s \quad \mu_s = 0.35$

so that $\phi_s = 19.29°$

$0 < \phi < \phi_s \Rightarrow$ Block does not slide and **R** is directed as shown.

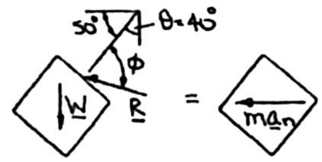
Now
$$F = R\sin\phi \quad \text{and} \quad R = \frac{W}{\sin(\phi - 10°)}$$

Then
$$F = (0.8 \text{ lb})\frac{\sin 16.9588°}{\sin 6.9588°}$$

$= 1.926 \text{ lb}$

The <u>block does not slide</u> in the slot and $\mathbf{F} = 1.926 \text{ lb} \searrow 80°$ ◀

(*b*) Assume that the block is at rest with respect to the plate.



$$\Sigma \mathbf{F} = m\mathbf{a}: \quad \mathbf{W} + \mathbf{R} = m\mathbf{a}_n$$

From Part *a* (above), it then follows that

$$\tan(\phi - 50°) = \frac{g}{\rho(\dot{\phi}_{ABCD})^2} = \frac{32.2 \text{ ft/s}^2}{\frac{98}{3}(13 - 5\sin 40°) \text{ ft/s}^2}$$

or $\quad \phi - 50° = 5.752°$

and $\quad \phi = 55.752°$

Now $\quad \phi_s = 19.29°$

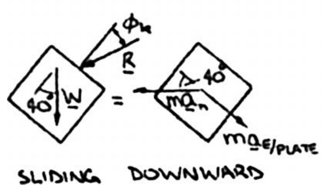
so that $\quad \phi > \phi_s$

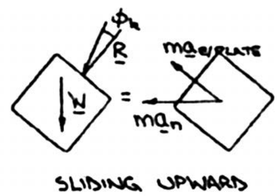
The block will slide in the slot and then

$$\phi = \phi_k, \quad \text{where} \quad \tan\phi_k = \mu_k \quad \mu_k = 0.25$$

or $\quad \phi_k = 14.0362°$

To determine in which direction the block will slide, consider the free-body diagrams for the two possible cases.





Now $\quad \Sigma \mathbf{F} = m\mathbf{a}: \quad \mathbf{W} + \mathbf{R} = m\mathbf{a}_n + m\mathbf{a}_{E/\text{plate}}$

From the diagrams it can be concluded that this equation can be satisfied only if the block is sliding downward. Then

$$+\swarrow \Sigma F_x = ma_x: \quad W\cos 40° + R\cos\phi_k = m\frac{v_E^2}{\rho}\sin 40°$$

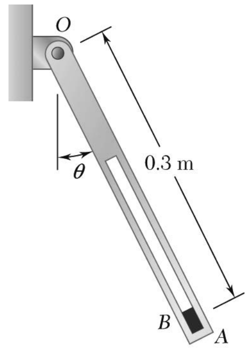
Now $\quad F = R\sin\phi_k$

Then $\quad W\cos 40° + \dfrac{F}{\tan\phi_k} = \dfrac{W}{g}\dfrac{v_E^2}{\rho}\sin 40°$

or $\quad F = \mu_k W\left(-\cos 40° + \dfrac{v_E^2}{g\rho}\sin 40°\right)$

$\quad = 1.123 \text{ lb} \qquad$ (see the first solution)

The block slides downward in the slot and $\quad \mathbf{F} = 1.123 \text{ lb} \searrow 40°$ ◀

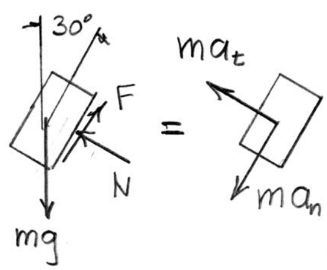


PROBLEM 12.61

A small block B fits inside a lot cut in arm OA which rotates in a vertical plane at a constant rate. The block remains in contact with the end of the slot closest to A and its speed is 1.4 m/s for $0 \le \theta \le 150°$. Knowing that the block begins to slide when $\theta = 150°$, determine the coefficient of static friction between the block and the slot.

SOLUTION

Draw the free body diagrams of the block B when the arm is at $\theta = 150°$.



$$\dot{v} = a_t = 0, \quad g = 9.81 \text{ m/s}^2$$

$$+\nwarrow \Sigma F_t = ma_t: \quad -mg \sin 30° + N = 0$$

$$N = mg \sin 30°$$

$$+\swarrow \Sigma F_n = ma_n: \quad mg \cos 30° - F = m\frac{v^2}{\rho}$$

$$F = mg \cos 30° - \frac{mv^2}{\rho}$$

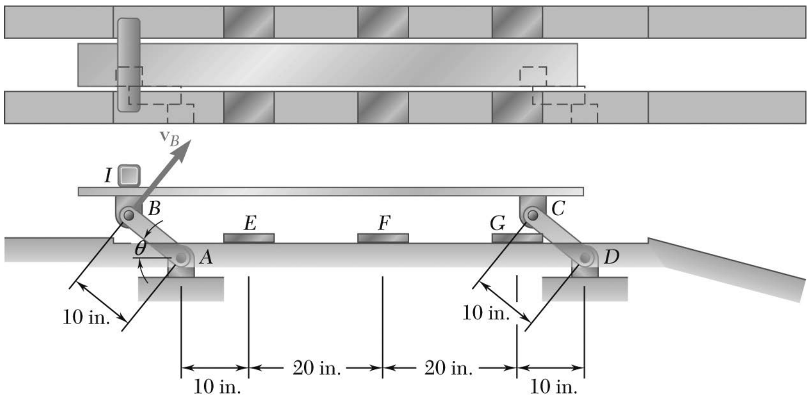
Form the ratio $\frac{F}{N}$, and set it equal to μ_s for impending slip.

$$\mu_s = \frac{F}{N} = \frac{g \cos 30° - v^2/\rho}{g \sin 30°} = \frac{9.81 \cos 30° - (1.4)^2/0.3}{9.81 \sin 30°}$$

$$\mu_s = 0.400 \blacktriangleleft$$

PROBLEM 12.62

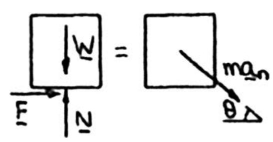
The parallel-link mechanism *ABCD* is used to transport a component *I* between manufacturing processes at stations *E*, *F*, and *G* by picking it up at a station when $\theta = 0$ and depositing it at the next station when $\theta = 180°$. Knowing that member *BC* remains horizontal throughout its motion and that links *AB* and *CD* rotate at a constant rate in a vertical plane in such a way that $v_B = 2.2$ ft/s, determine (*a*) the minimum value of the coefficient of static friction between the component and *BC* if the component is not to slide on *BC* while being transferred, (*b*) the values of θ for which sliding is impending.



SOLUTION

$$\xrightarrow{+} \Sigma F_x = ma_x: \quad F = \frac{W}{g}\frac{v_B^2}{\rho}\cos\theta$$

$$+\uparrow \Sigma F_y = ma_y: \quad N - W = -\frac{W}{g}\frac{v_B^2}{\rho}\sin\theta$$



or

$$N = W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$$

Now

$$F_{\max} = \mu_s N = \mu_s W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$$

and for the component not to slide

$$F < F_{\max}$$

or

$$\frac{W}{g}\frac{v_B^2}{\rho}\cos\theta < \mu_s W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$$

or

$$\mu_s > \frac{\cos\theta}{\frac{g\rho}{v_B^2} - \sin\theta}$$

PROBLEM 12.62 (Continued)

We must determine the values of θ which maximize the above expression. Thus

$$\frac{d}{d\theta}\left(\frac{\cos\theta}{\frac{g\rho}{v_B^2}-\sin\theta}\right)=\frac{-\sin\theta\left(\frac{g\rho}{v_B^2}-\sin\theta\right)-(\cos\theta)(-\cos\theta)}{\left(\frac{g\rho}{v_B^2}-\sin\theta\right)^2}=0$$

or $$\sin\theta=\frac{v_B^2}{g\rho}\qquad\text{for}\qquad \mu_s=(\mu_s)_{\min}$$

Now $$\sin\theta=\frac{(2.2\text{ ft/s})^2}{(32.2\text{ ft/s}^2)\left(\frac{10}{12}\text{ ft}\right)}=0.180373$$

or $$\theta=10.3915^\circ\quad\text{and}\quad\theta=169.609^\circ$$

(*a*) From above,

$$(\mu_s)_{\min}=\frac{\cos\theta}{\frac{g\rho}{v_B^2}-\sin\theta}\quad\text{where}\quad\sin\theta=\frac{v_B^2}{g\rho}$$

$$(\mu_s)_{\min}=\frac{\cos\theta}{\frac{1}{\sin\theta}-\sin\theta}=\frac{\cos\theta\sin\theta}{1-\sin^2\theta}=\tan\theta$$

$$=\tan 10.3915^\circ$$

or $(\mu_s)_{\min}=0.1834$ ◀

(*b*) We have impending motion

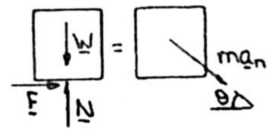
to the left for $\theta=10.39^\circ$ ◀

to the right for $\theta=169.6^\circ$ ◀

PROBLEM 12.63

Knowing that the coefficients of friction between the component I and member BC of the mechanism of Problem 12.62 are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine (*a*) the maximum allowable constant speed v_B if the component is not to slide on BC while being transferred, (*b*) the values of θ for which sliding is impending.

SOLUTION



$$\xrightarrow{+} \Sigma F_x = ma_x: \quad F = \frac{W}{g}\frac{v_B^2}{\rho}\cos\theta$$

$$+\uparrow \Sigma F_y = ma_y: \quad N - W = -\frac{W}{g}\frac{v_B^2}{\rho}\sin\theta$$

or

$$N = W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$$

Now

$$F_{\max} = \mu_s N$$

$$= \mu_s W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$$

and for the component not to slide

$$F < F_{\max}$$

or

$$\frac{W}{g}\frac{v_B^2}{\rho}\cos\theta < \mu_s W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$$

or

$$v_B^2 < \mu_s \frac{g\rho}{\cos\theta + \mu_s \sin\theta} \tag{1}$$

To ensure that this inequality is satisfied, $\left(v_B^2\right)_{\max}$ must be less than or equal to the minimum value of $\mu_s g\rho/(\cos\theta + \mu_s \sin\theta)$, which occurs when $(\cos\theta + \mu_s \sin\theta)$ is maximum. Thus

$$\frac{d}{d\theta}(\cos\theta + \mu_s \sin\theta) = -\sin\theta + \mu_s \cos\theta = 0$$

or

$$\tan\theta = \mu_s$$

$$\mu_s = 0.35$$

or

$$\theta = 19.2900°$$

(*a*) The maximum allowed value of v_B is then

$$\left(v_B^2\right)_{\max} = \mu_s \frac{g\rho}{\cos\theta + \mu_s \sin\theta} \quad \text{where} \quad \tan\theta = \mu_s$$

$$= g\rho \frac{\tan\theta}{\cos\theta + (\tan\theta)\sin\theta} = g\rho\sin\theta$$

$$= (32.2 \text{ ft/s}^2)\left(\frac{10}{12}\text{ ft}\right)\sin 19.2900°$$

or $(v_B)_{\max} = 2.98$ ft/s ◀

(*b*) First note that for $90° < \theta < 180°$, Eq. (1) becomes

$$v_B^2 < \mu_s \frac{g\rho}{\cos\alpha + \mu_s \sin\alpha}$$

where $\alpha = 180° - \theta.$ It then follows that the second value of θ for which motion is impending is

$$\theta = 180° - 19.2900°$$
$$= 160.7100°$$

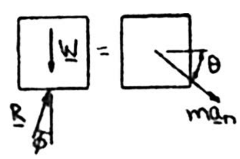
we have impending motion

to the left for $\theta = 19.29°$ ◀

to the right for $\theta = 160.7°$ ◀

<u>Alternative solution</u>.

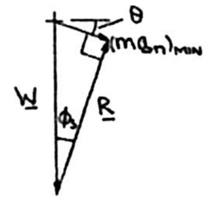
$$\Sigma\mathbf{F} = m\mathbf{a}: \quad \mathbf{W} + \mathbf{R} = m\mathbf{a}_n$$



For impending motion, $\phi = \phi_s$. Also, as shown above, the values of θ for which motion is impending minimize the value of v_B, and thus the value of a_n is $\left(a_n = \frac{v_B^2}{\rho}\right)$. From the above diagram, it can be concluded that a_n is minimum when $m\mathbf{a}_n$ and $\mathbf{R}$ are perpendicular.

PROBLEM 12.63 (Continued)

Therefore, from the diagram



$$\theta = \phi_s = \tan^{-1}\mu_s \qquad \text{(as above)}$$

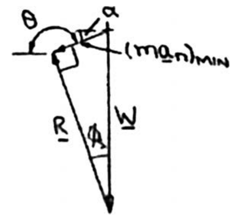
and $$ma_n = W\sin\phi_s$$

or $$m\frac{v_B^2}{\rho} = mg\sin\theta$$

or $$v_B^2 = g\rho\sin\theta \qquad \text{(as above)}$$

For $90° \le \theta \le 180°$, we have

from the diagram

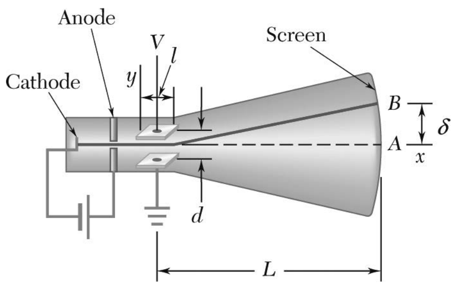


$$\alpha = 180° - \theta \qquad \text{(as above)}$$

$$\alpha = \phi_s$$

and $$ma_n = W\sin\phi_s$$

or $$v_B^2 = g\rho\sin\theta \qquad \text{(as above)}$$



PROBLEM 12.64

In the cathode-ray tube shown, electrons emitted by the cathode and attracted by the anode pass through a small hole in the anode and then travel in a straight line with a speed v_0 until they strike the screen at A. However, if a difference of potential V is established between the two parallel plates, the electrons will be subjected to a force **F** perpendicular to the plates while they travel between the plates and will strike the screen at Point B, which is at a distance δ from A. The magnitude of the force **F** is $F = eV/d$, where $-e$ is the charge of an electron and d is the distance between the plates. Derive an expression for the deflection d in terms of V, v_0, the charge $-e$ and the mass m of an electron, and the dimensions d, ℓ, and L.

SOLUTION

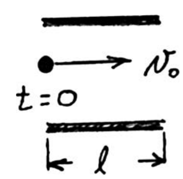
Consider the motion of one electron. For the horizontal motion, let $x = 0$ at the left edge of the plate and $x = \ell$ at the right edge of the plate. At the screen,

$$x = \frac{\ell}{2} + L$$

Horizontal motion: There are no horizontal forces acting on the electron so that $a_x = 0$.

Let $t_1 = 0$ when the electron passes the left edge of the plate, $t = t_1$ when it passes the right edge, and $t = t_2$ when it impacts on the screen. For uniform horizontal motion,

$$x = v_0 t,$$



so that $$t_1 = \frac{\ell}{v_0}$$

and $$t_2 = \frac{\ell}{2v_0} + \frac{L}{v_0}.$$

Vertical motion: The gravity force acting on the electron is neglected since we are interested in the deflection produced by the electric force. While the electron is between plates $(0 < t < t_1)$, the vertical force on the electron is $F_y = eV/d$. After it passes the plates $(t_1 < t < t_2)$, it is zero.

For $0 < t < t_1$, $\quad \Sigma F_y = ma_y: \quad a_y = \frac{F_y}{m} = \frac{eV}{md}$

$$v_y = (v_y)_0 + a_y t = 0 + \frac{eVt}{md}$$

$$y = y_0 + (v_y)_0 t + \frac{1}{2}a_y t^2 = 0 + 0 + \frac{eVt^2}{2md}$$

PROBLEM 12.64 (Continued)

At $t = t_1,$ $\qquad (v_y)_1 = \dfrac{eVt_1}{md}$ and $y_1 = \dfrac{eVt_1^2}{2md}$

For $t_1 < t < t_2,\ a_y = 0$

$$y = y_1 + (v_y)_1(t - t_1)$$

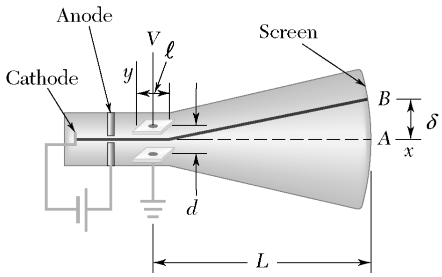
At $t = t_2,$

$$y_2 = \delta = y_1 + (v_y)_1(t_2 - t_1)$$

$$\delta = \frac{eVt_1^2}{2md} + \frac{eVt_1}{md}(t_2 - t_1) = \frac{eVt_1}{md}\left(t_2 - \frac{1}{2}t_1\right)$$

$$= \frac{eV\ell}{mdv_0}\left(\frac{\ell}{2v_0} + \frac{L}{v_0} - \frac{1}{2}\frac{\ell}{v_0}\right) \qquad \text{or} \qquad \delta = \frac{eV\ell L}{mdv_0^2} \blacktriangleleft$$

PROBLEM 12.65



In Problem 12.64, determine the smallest allowable value of the ratio d/ℓ in terms of e, m, v_0, and V if at $x = \ell$ the minimum permissible distance between the path of the electrons and the positive plate is $0.05d$.

Problem 12.64 In the cathode-ray tube shown, electrons emitted by the cathode and attracted by the anode pass through a small hole in the anode and then travel in a straight line with a speed v_0 until they strike the screen at A. However, if a difference of potential V is established between the two parallel plates, the electrons will be subjected to a force **F** perpendicular to the plates while they travel between the plates and will strike the screen at point B, which is at a distance δ from A. The magnitude of the force **F** is $F = eV/d$, where $-e$ is the charge of an electron and d is the distance between the plates. Derive an expression for the deflection d in terms of V, v_0, the charge $-e$ and the mass m of an electron, and the dimensions d, ℓ, and L.

SOLUTION

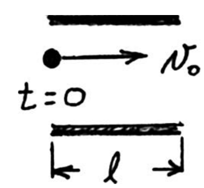
Consider the motion of one electron. For the horizontal motion, let $x = 0$ at the left edge of the plate and $x = \ell$ at the right edge of the plate. At the screen,

$$x = \frac{\ell}{2} + L$$

Horizontal motion: There are no horizontal forces acting on the electron so that $a_x = 0$.

Let $t_1 = 0$ when the electron passes the left edge of the plate, $t = t_1$ when it passes the right edge, and $t = t_2$ when it impacts on the screen. For uniform horizontal motion,

$$x = v_0 t,$$



so that

$$t_1 = \frac{\ell}{v_0}$$

and

$$t_2 = \frac{\ell}{2v_0} + \frac{L}{v_0}.$$

Vertical motion: The gravity force acting on the electron is neglected since we are interested in the deflection produced by the electric force. While the electron is between the plates $(0 < t < t_1)$, the vertical force on the electron is $F_y = eV/d$. After it passes the plates $(t_1 < t < t_2)$, it is zero.

For $0 < t < t_1$, $\qquad \Sigma F_y = ma_y$: $\quad a_y = \frac{F_y}{m} = \frac{eV}{md}$

$$v_y = (v_y)_0 + a_y t = 0 + \frac{eVt}{md}$$

$$y = y_0 + (v_y)_0 t + \frac{1}{2}a_y t^2 = 0 + 0 + \frac{eVt^2}{2md}$$

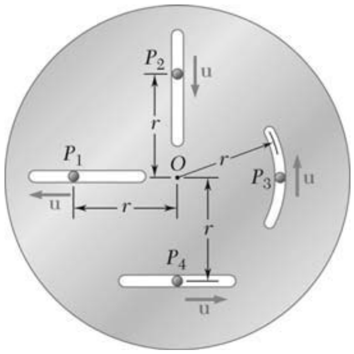
At $t = t_1$, $\quad \dfrac{\ell}{v_0}, \quad y = \dfrac{eV\ell^2}{2mdv_0^2}$

But $\quad y < \dfrac{d}{2} - 0.05d = 0.450d$

so that $\quad \dfrac{eV\ell^2}{2mdv_0^2} < 0.450\,d$

$$\frac{d^2}{\ell^2} > \frac{1}{0.450}\frac{eV}{2mv_0^2} = 1.111\frac{eV}{mv_0^2}$$

$$\frac{d}{\ell} > 1.054\sqrt{\frac{eV}{mv_0^2}} \quad \blacktriangleleft$$

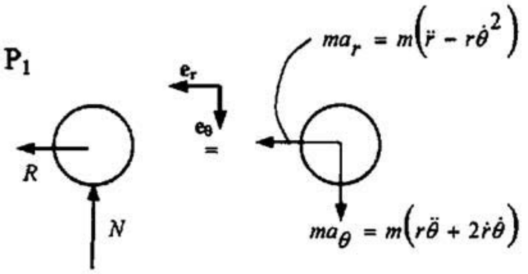


PROBLEM 12.F9

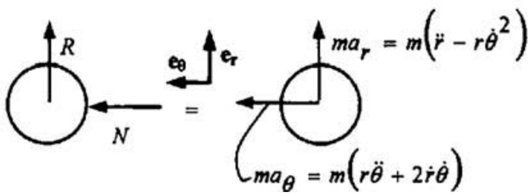
Four pins slide in four separate slots cut in a horizontal circular plate as shown. When the plate is at rest, each pin has a velocity directed as shown and of the same constant magnitude u. Each pin has a mass m and maintains the same velocity relative to the plate when the plate rotates about O with a constant counterclockwise angular velocity ω. Draw the FBDs and KDs to determine the forces on pins P_1 and P_2.

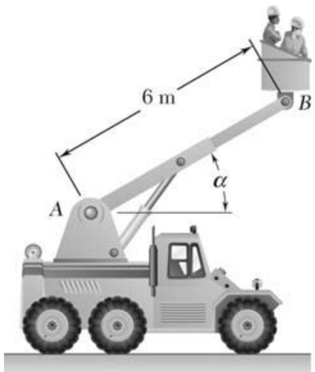
SOLUTION

Pin P_1



Pin P_2

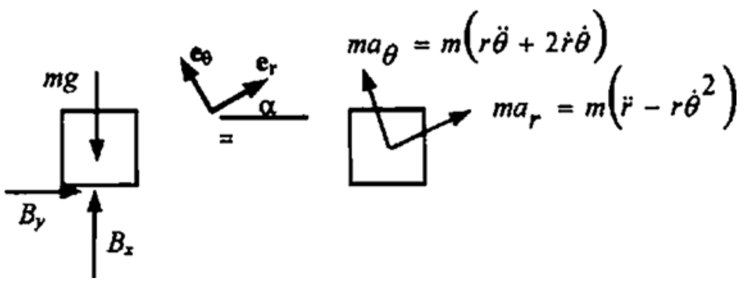




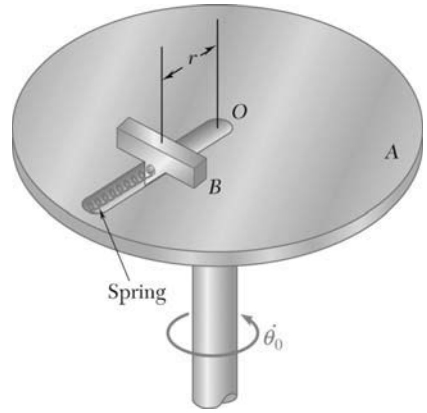
PROBLEM 12.F10

At the instant shown, the length of the boom AB is being *decreased* at the constant rate of 0.2 m/s, and the boom is being lowered at the constant rate of 0.08 rad/s. If the mass of the men and lift connected to the boom at Point B is m, draw the FBD and KD that could be used to determine the horizontal and vertical forces at B.

SOLUTION



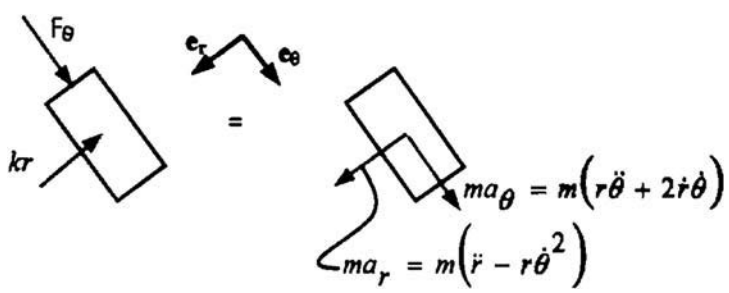
Where $r = 6$ m, $\dot{r} = -0.2$ m/s, $\ddot{r} = 0$, $\dot{\theta} = -0.08$ rad/s, $\ddot{\theta} = 0$

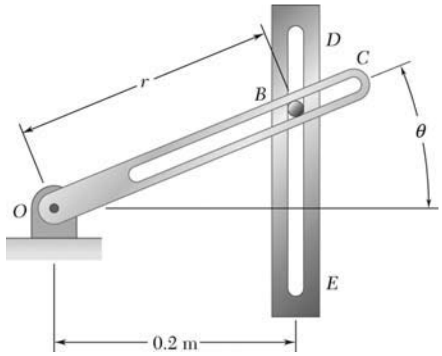


PROBLEM 12.F11

Disk A rotates in a horizontal plane about a vertical axis at the constant rate $\dot{\theta}_0$. Slider B has a mass m and moves in a frictionless slot cut in the disk. The slider is attached to a spring of constant k, which is undeformed when $r = 0$. Knowing that the slider is released with no radial velocity in the position $r = r_0$, draw a FBD and KD at an arbitrary distance r from O.

SOLUTION

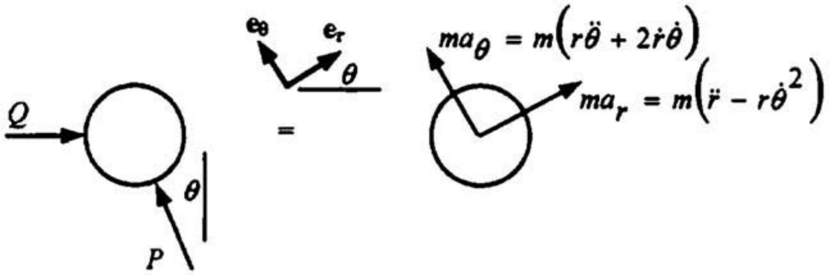


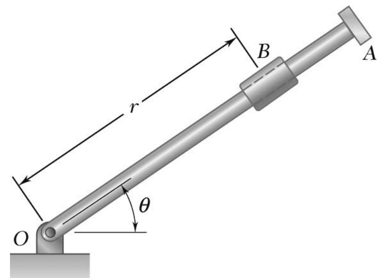


PROBLEM 12.F12

Pin *B* has a mass *m* and slides along the slot in the rotating arm *OC* and along the slot *DE* which is cut in a fixed horizontal plate. Neglecting friction and knowing that rod *OC* rotates at the constant rate $\dot{\theta}_0$, draw a FBD and KD that can be used to determine the forces **P** and **Q** exerted on pin *B* by rod OC and the wall of slot *DE*, respectively.

SOLUTION





PROBLEM 12.66

Rod OA rotates about O in a horizontal plane. The motion of the 0.5-lb collar B is defined by the relations $r = 10 + 6\cos\pi t$ and $\theta = \pi(4t^2 - 8t)$, where r is expressed in inches, t in seconds, and θ in radians. Determine the radial and transverse components of the force exerted on the collar when (a) $t = 0$, (b) $t = 0.5$ s.

SOLUTION

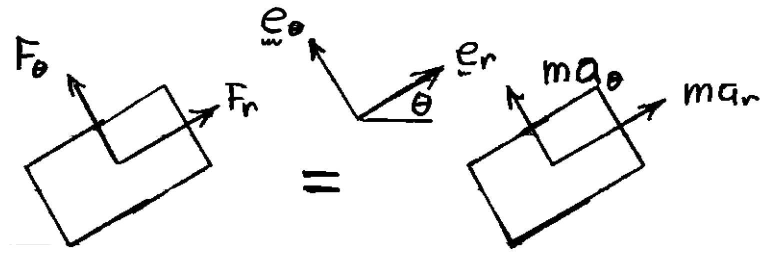
Use polar coordinates and calculate the derivatives of the coordinates r and θ with respect to time.

$$r = 10 + 6\cos\pi t \text{ in.} \qquad \theta = \pi(4t^2 - 8t) \text{ rad}$$

$$\dot{r} = -6\pi\sin\pi t \text{ in./s} \qquad \dot{\theta} = \pi(8t - 8) \text{ rad/s}$$

$$\ddot{r} = -6\pi^2\cos\pi t \text{ in./s}^2 \qquad \ddot{\theta} = 8\pi \text{ rad/s}^2$$

Mass of collar: $$m = \frac{0.5 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.015528 \text{ lb s}^2/\text{ft} = 1.294\times10^{-3} \text{ lb}\cdot\text{s}^2/\text{in.}$$



(a) $t = 0$:

$r = 16$ in. $\qquad \theta = 0$

$\dot{r} = 0 \qquad \dot{\theta} = -8\pi = -25.1327$ rad/s

$\ddot{r} = -6\pi^2 = -59.218 \text{ in./s}^2 \qquad \ddot{\theta} = 8\pi = -25.1327 \text{ rad/s}^2$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -59.218 - (16)(-25.1327)^2 = -10165.6 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (16)(25.1327) + 0 = 402.12 \text{ in./s}^2$$

$$F_r = ma_r = (1.294\times10^{-3} \text{ lb}\cdot\text{s}^2/\text{in.})(-10165.6 \text{ in./s}^2)$$ $F_r = -13.15$ lb ◀

$$F_\theta = ma_\theta = (1.294\times10^{-3} \text{ lb}\cdot\text{s}^2/\text{in.})(402.12 \text{ in./s}^2)$$ $F_\theta = 0.520$ lb ◀

$\theta = 0$ ◀

PROBLEM 12.66 (Continued)

(*b*) $t = 0.5$ s:

$r = 10 + 6\cos(0.5\pi) = 10$ in. $\qquad \theta = \pi[(4)(0.25) - (8)(0.5)] = -9.4248$ rad $= -540° = 180°$

$\dot{r} = -6\pi\sin(0.5\pi) = -18.8496$ in./s $\qquad \dot{\theta} = \pi[(8)(0.5) - 8] = -12.5664$ rad/s

$\ddot{r} = -6\pi^2\cos(0.5\pi) = 0 \qquad \ddot{\theta} = 8\pi = 25.1327$ rad/s^2

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (10)(-12.5664)^2 = -1579.14 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (10)(25.1327) + (2)(-18.8496)(-12.5664) = 725.07 \text{ in./s}^2$$

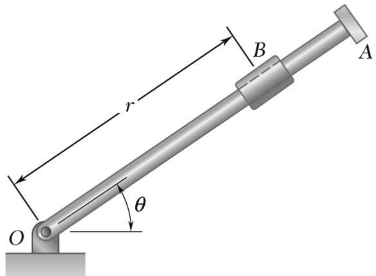
$$F_r = ma_r = (1.294\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{in.})(-1579.14\text{ in./s}^2)$$

$F_r = -2.04$ lb ◀

$$F_\theta = ma_\theta = (1.294\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{in.})(725.07\text{ in./s}^2)$$

$F_\theta = 0.938$ lb ◀

$\theta = 180°$ ◀



PROBLEM 12.67

Rod OA oscillates about O in a horizontal plane. The motion of the 2-lb collar B is defined by the relations $r = 6(1 - e^{-2t})$ and $\theta = (3/\pi)(\sin \pi t)$, where r is expressed in inches, t in seconds, and θ in radians. Determine the radial and transverse components of the force exerted on the collar when (*a*) $t = 1$ s, (*b*) $t = 1.5$ s.

SOLUTION

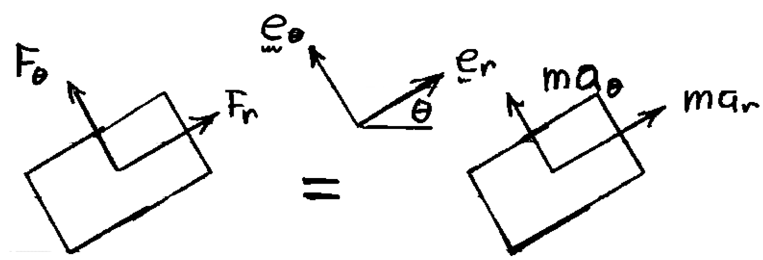
Use polar coordinates and calculate the derivatives of the coordinates r and θ with respect to time.

$$r = 6(1 - e^{-2t}) \text{ in.} \qquad \theta = (3/\pi)\sin \pi t \text{ radians}$$

$$\dot{r} = 12e^{-2t} \text{ in./s} \qquad \dot{\theta} = 3\cos \pi t \text{ rad/s}$$

$$\ddot{r} = -24e^{-2t} \text{ in./s}^2 \qquad \ddot{\theta} = -3\pi \sin \pi t \text{ rad/s}^2$$

Mass of collar: $$m = \frac{2 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.06211 \text{ lb} \cdot \text{s}^2/\text{ft} = 5.176 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{in.}$$



(*a*) $t = 1$ s: $e^{-2t} = 0.13534, \quad \sin \pi t = 0, \quad \cos \pi t = -1$

$$r = 6(1 - 0.13534) = 5.188 \text{ in.} \qquad \theta = 0$$

$$\dot{r} = (12)(0.13534) = 1.62402 \text{ in./s} \qquad \dot{\theta} = -3.0 \text{ rad/s}$$

$$\ddot{r} = (-24)(0.13534) = -3.2480 \text{ in./s}^2 \qquad \ddot{\theta} = 0$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -3.2480 - (5.188)(-3.0)^2 = -49.94 \text{ in./s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + (2)(1.62402)(-3) = -9.744 \text{ in./s}^2$$

$$F_r = ma_r = (5.176 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{in.})(-49.94 \text{ in./s}^2)$$

$F_r = -0.258$ lb ◀

$$F_\theta = ma_\theta = (5.176 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{in.})(-9.744 \text{ in./s}^2)$$

$F_\theta = -0.0504$ lb ◀

$\theta = 0$ ◀

PROBLEM 12.67 (Continued)

(*b*) $t = 1.5$ s: $e^{-2t} = 0.049787$, $\sin \pi t = -1$, $\cos \pi t = 0$

$r = 6(1 - 0.049787) = 5.7013$ in. $\theta = (3/\pi)(-1) = -0.9549$ rad $= -54.7°$

$\dot{r} = (12)(0.049787) = 0.59744$ in./s^2 $\dot{\theta} = 0$

$\ddot{r} = -(24)(0.049787) = -1.19489$ in./s^2 $\ddot{\theta} = -(3\pi)(-1) = 9.4248$ rad/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -1.19489 - 0 = -1.19489$ in./s^2

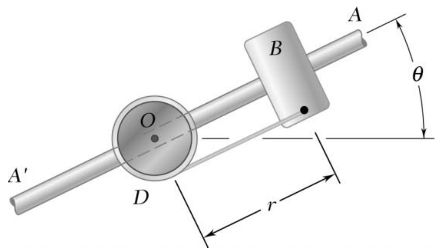
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (5.7013)(9.4248) + 0 = 53.733$ in./s^2

$F_r = ma_r = (5.176 \times 10^{-3}$ lb$\cdot$s^2/in.$)(-1.19489$ in./s$^2)$ $F_r = -0.00618$ lb ◀

$F_\theta = ma_\theta = (5.176 \times 10^{-3}$ lb$\cdot$s^2/in.$)(53.733$ in./s$^2)$ $F_\theta = 0.278$ lb ◀

$\theta = -54.7°$ ◀

PROBLEM 12.68



The 3-kg collar B slides on the frictionless arm AA'. The arm is attached to drum D and rotates about O in a horizontal plane at the rate $\dot{\theta} = 0.75t$, where $\dot{\theta}$ and t are expressed in rad/s and seconds, respectively. As the arm-drum assembly rotates, a mechanism within the drum releases cord so that the collar moves outward from O with a constant speed of 0.5 m/s. Knowing that at t = 0, r = 0, determine the time at which the tension in the cord is equal to the magnitude of the horizontal force exerted on B by arm AA'.

SOLUTION

Kinematics

We have $\dfrac{dr}{dt} = \dot{r} = 0.5 \text{ m/s}$

At $t = 0, r = 0$: $\int_0^r dr = \int_0^t 0.5\, dt$

or $r = (0.5t) \text{ m}$

Also, $\ddot{r} = 0 \qquad \dot{\theta} = (0.75t) \text{ rad/s}$

$\ddot{\theta} = 0.75 \text{ rad/s}^2$

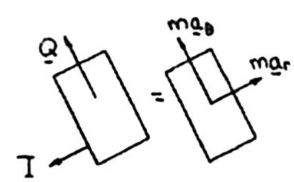
Now $a_r = \ddot{r} - r\dot{\theta}^2 = 0 - [(0.5t)\text{ m}][(0.75t)\text{ rad/s}]^2 = -(0.28125t^3)\text{ m/s}^2$

and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$= [(0.5t)\text{ m}][0.75\text{ rad/s}^2] + 2(0.5\text{ m/s})[(0.75t)\text{ rad/s}]$

$= (1.125t)\text{ m/s}^2$

Kinetics



$+\nearrow \Sigma F_r = ma_r$: $-T = (3\text{ kg})(-0.28125t^3)\text{ m/s}^2$

or $T = (0.84375t^3)\text{ N}$

$+\nwarrow \Sigma F_\theta = m_B a_\theta$: $Q = (3\text{ kg})(1.125t)\text{ m/s}^2$

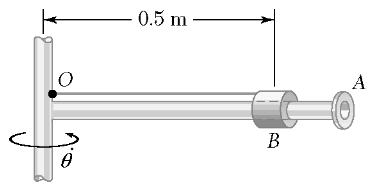
or $Q = (3.375t)\text{ N}$

Now require that $T = Q$

or $(0.84375t^3)\text{ N} = (3.375t)\text{ N}$

or $t^2 = 4.000$

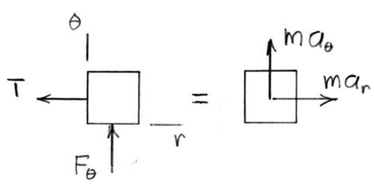
or $t = 2.00 \text{ s}$ ◀



PROBLEM 12.69

The horizontal rod *OA* rotates about a vertical shaft according to the relation $\dot{\theta} = 10t$, where $\dot{\theta}$ and t are expressed in rad/s and seconds, respectively. A 250-g collar *B* is held by a cord with a breaking strength of 18 N. Neglecting friction, determine, immediately after the cord breaks, (*a*) the relative acceleration of the collar with respect to the rod, (*b*) the magnitude of the horizontal force exerted on the collar by the rod.

SOLUTION



$$\dot{\theta} = 10t \text{ rad/s}, \quad \ddot{\theta} = 10 \text{ rad/s}^2$$

$$m = 250 \text{ g} = 0.250 \text{ kg}$$

Before cable breaks: $F_r = -T$ and $\ddot{r} = 0$.

$$F_r = ma_r: \quad -T = m(\ddot{r} - r\dot{\theta}^2)$$

$$mr\dot{\theta}^2 = m\ddot{r} + T \qquad \text{or} \qquad \dot{\theta}^2 = \frac{m\ddot{r} + T}{mr} = \frac{0 - 18}{(0.25)(0.5)} = 144 \text{ rad}^2/\text{s}^2$$

$$\dot{\theta} = 12 \text{ rad/s}$$

Immediately after the cable breaks: $F_r = 0, \ \dot{r} = 0$

(*a*) Acceleration of *B* relative to the rod.

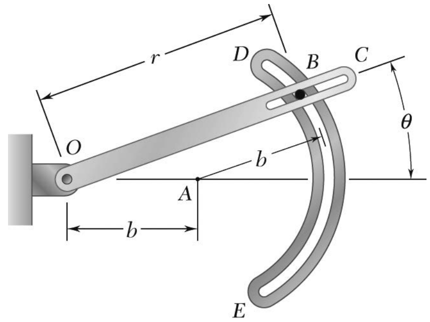
$$m(\ddot{r} - r\dot{\theta}^2) = 0 \qquad \text{or} \qquad \ddot{r} = r\dot{\theta}^2 = (0.5)(12)^2 = 72 \text{ m/s}^2$$

$\mathbf{a}_{B/\text{rod}} = 72 \text{ m/s}^2$ radially outward ◀

(*b*) Transverse component of the force.

$$F_\theta = ma_\theta: \quad F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$F_\theta = (0.250)[(0.5)(10) + (2)(0)(12)] = 1.25 \qquad F_\theta = 1.25 \text{ N} ◀$$



PROBLEM 12.70

Pin B weighs 4 oz and is free to slide in a horizontal plane along the rotating arm OC and along the circular slot DE of radius $b = 20$ in. Neglecting friction and assuming that $\dot{\theta} = 15$ rad/s and $\ddot{\theta} = 250\text{ rad/s}^2$ for the position $\theta = 20°$, determine for that position (*a*) the radial and transverse components of the resultant force exerted on pin B, (*b*) the forces **P** and **Q** exerted on pin B, respectively, by rod OC and the wall of slot DE.

SOLUTION

<u>Kinematics</u>.

From the geometry of the system, we have

Then $\quad r = 2b\cos\theta \qquad \dot{r} = -(2b\sin\theta)\dot{\theta} \qquad \ddot{r} = -2b(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)$

and $\quad a_r = \ddot{r} - r\dot{\theta}^2 = -2b(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) - (2b\cos\theta)\dot{\theta}^2 = -2b(\ddot{\theta}\sin\theta + 2\dot{\theta}^2\cos\theta)$

Now $\quad = -2\left(\dfrac{20}{12}\text{ ft}\right)[(250\text{ rad/s}^2)\sin 20° + 2(15\text{ rad/s})^2\cos 20°] = -1694.56\text{ ft/s}^2$

and $\quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (2b\cos\theta)\ddot{\theta} + 2(-2b\dot{\theta}\sin\theta)\dot{\theta} = 2b(\ddot{\theta}\cos\theta - 2\dot{\theta}^2\sin\theta)$

$$= 2\left(\frac{20}{12}\text{ ft}\right)[(250\text{ rad/s}^2)\cos 20° - 2(15\text{ rad/s})^2\sin 20°] = 270.05\text{ ft/s}^2$$

<u>Kinetics</u>.

(*a*) We have $\quad F_r = ma_r = \dfrac{\frac{1}{4}\text{ lb}}{32.2\text{ ft/s}^2} \times (-1694.56\text{ ft/s}^2) = -13.1565\text{ lb} \qquad F_r = -13.16\text{ lb}$ ◀

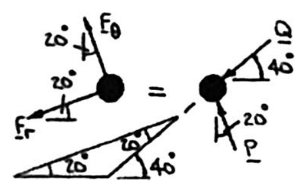
and $\quad F_\theta = ma_\theta = \dfrac{\frac{1}{4}\text{ lb}}{32.2\text{ ft/s}^2} \times (270.05\text{ ft/s}^2) = 2.0967\text{ lb} \qquad F_\theta = 2.10\text{ lb}$ ◀

(*b*) $\quad +\nearrow\ \Sigma F_r:\quad -F_r = -Q\cos 20°$

or $\quad Q = \dfrac{1}{\cos 20°}(13.1565\text{ lb}) = 14.0009\text{ lb}$

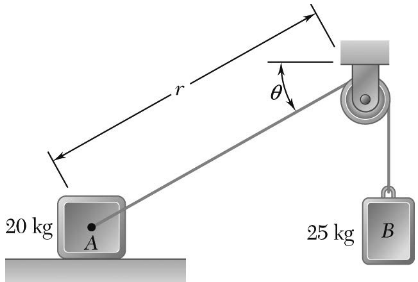
$+\nwarrow\ \Sigma F_\theta:\quad F_\theta = P - Q\sin 20°$

or $\quad P = (2.0967 + 14.0009\sin 20°)\text{ lb} = 6.89\text{ lb}$



$\mathbf{P} = 6.89\text{ lb} \searrow 70°$ ◀

$\mathbf{Q} = 14.00\text{ lb} \nearrow 40°$ ◀



PROBLEM 12.71

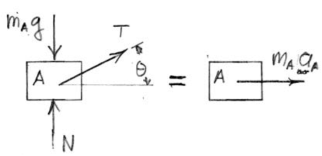
The two blocks are released from rest when $r = 0.8$ m and $\theta = 30°$. Neglecting the mass of the pulley and the effect of friction in the pulley and between block A and the horizontal surface, determine (*a*) the initial tension in the cable, (*b*) the initial acceleration of block A, (*c*) the initial acceleration of block B.

SOLUTION

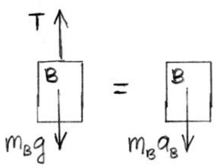
Let r and θ be polar coordinates of block A as shown, and let y_B be the position coordinate (positive downward, origin at the pulley) for the rectilinear motion of block B.

Constraint of cable: $r + y_B = \text{constant},$

$$\dot{r} + v_B = 0, \quad \ddot{r} + a_B = 0 \qquad \text{or} \qquad \ddot{r} = -a_B \qquad (1)$$



For block A, $\xrightarrow{+} \Sigma F_x = m_A a_A$: $T\cos\theta = m_A a_A$ or $T = m_A a_A \sec\theta \quad (2)$



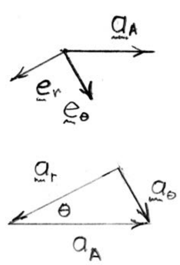
For block B, $+\!\downarrow \Sigma F_y = m_B a_B$: $m_B g - T = m_B a_B \quad (3)$

Adding Eq. (1) to Eq. (2) to eliminate T, $m_B g = m_A a_A \sec\theta + m_B a_B \quad (4)$

Radial and transverse components of $\mathbf{a}_A$.

Use either the scalar product of vectors or the triangle construction shown, being careful to note the positive directions of the components.

$$\ddot{r} - r\dot{\theta}^2 = a_r = \mathbf{a}_A \cdot \mathbf{e}_r = -a_A \cos\theta \qquad (5)$$



Noting that initially $\dot{\theta} = 0$, using Eq. (1) to eliminate $\ddot{r}$, and changing signs gives

$$a_B = a_A \cos\theta \qquad (6)$$

Substituting Eq. (6) into Eq. (4) and solving for a_A,

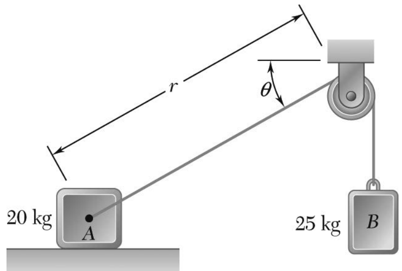
$$a_A = \frac{m_B g}{m_A \sec\theta + m_B \cos\theta} = \frac{(25)(9.81)}{20\sec 30° + 25\cos 30°} = 5.48 \text{ m/s}^2$$

From Eq. (6), $a_B = 5.48\cos 30° = 4.75 \text{ m/s}^2$

(*a*) From Eq. (2), $T = (20)(5.48)\sec 30° = 126.6$ $\quad T = 126.6 \text{ N}$ ◀

(*b*) Acceleration of block A. $\quad \mathbf{a}_A = 5.48 \text{ m/s}^2 \longrightarrow$ ◀

(*c*) Acceleration of block B. $\quad \mathbf{a}_B = 4.75 \text{ m/s}^2 \downarrow$ ◀



PROBLEM 12.72

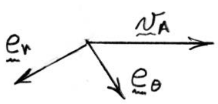
The velocity of block A is 2 m/s to the right at the instant when $r = 0.8$ m and $\theta = 30°$. Neglecting the mass of the pulley and the effect of friction in the pulley and between block A and the horizontal surface, determine, at this instant, (a) the tension in the cable, (b) the acceleration of block A, (c) the acceleration of block B.

SOLUTION

Let r and θ be polar coordinates of block A as shown, and let y_B be the position coordinate (positive downward, origin at the pulley) for the rectilinear motion of block B.

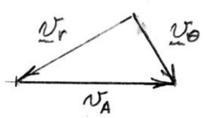
Radial and transverse components of $\mathbf{v}_A$.

Use either the scalar product of vectors or the triangle construction shown, being careful to note the positive directions of the components.



$$\dot{r} = v_r = \mathbf{v}_A \cdot \mathbf{e}_r = -v_A \cos 30°$$

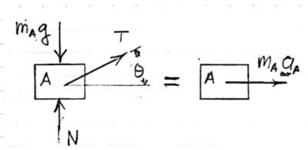
$$= -2\cos 30° = -1.73205 \text{ m/s}$$



$$r\dot{\theta} = v_\theta = \mathbf{v}_A \cdot \mathbf{e}_\theta = -v_A \sin 30°$$

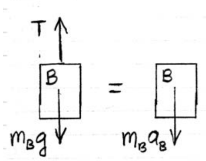
$$= 2\sin 30° = 1.000 \text{ m/s}^2$$

$$\dot{\theta} = \frac{v_\theta}{r} = \frac{1.000}{0.8} = 1.25 \text{ rad/s}$$



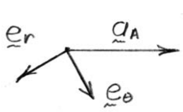
Constraint of cable: $r + y_B = \text{constant}$,

$$\dot{r} + v_B = 0, \quad \ddot{r} + a_B = 0 \quad \text{or} \quad \ddot{r} = -a_B \tag{1}$$



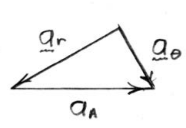
For block A, $\xrightarrow{+} \Sigma F_x = m_A a_A : T\cos\theta = m_A a_A$ or $T = m_A a_A \sec\theta$ (2)

For block B, $+\!\downarrow \Sigma F_y = m_B a_B$: $m_B g - T = m_B a_B$ (3)



Adding Eq. (1) to Eq. (2) to eliminate T, $m_B g = m_A a_A \sec\theta + m_B a_B$ (4)

Radial and transverse components of $\mathbf{a}_A$.



Use a method similar to that used for the components of velocity.

$$\ddot{r} - r\dot{\theta}^2 = a_r = \mathbf{a}_A \cdot \mathbf{e}_r = -a_A \cos\theta \tag{5}$$

Using Eq. (1) to eliminate $\ddot{r}$ and changing signs gives

$$a_B = a_A \cos\theta - r\dot{\theta}^2 \tag{6}$$

Substituting Eq. (6) into Eq. (4) and solving for a_A,

$$a_A = \frac{m_B\left(g + r\dot{\theta}^2\right)}{m_A \sec\theta + m_B \cos\theta} = \frac{(25)[9.81 + (0.8)(1.25)^2]}{20\sec 30° + 25\cos 30°} = 6.18 \text{ m/s}^2$$

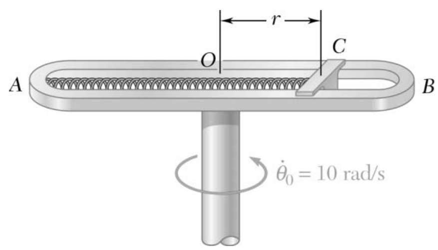
From Eq. (6), $a_B = 6.18\cos 30° - (0.8)(1.25)^2 = 4.10 \text{ m/s}^2$

(*a*) From Eq. (2), $T = (20)(6.18)\sec 30° = 142.7$ $\qquad T = 142.7 \text{ N}$ ◀

(*b*) Acceleration of block *A*. $\qquad \mathbf{a}_A = 6.18 \text{ m/s}^2 \longrightarrow$ ◀

(*c*) Acceleration of block *B*. $\qquad \mathbf{a}_B = 4.10 \text{ m/s}^2 \downarrow$ ◀

PROBLEM 12.73*



Slider C has a weight of 0.5 lb and may move in a slot cut in arm AB, which rotates at the constant rate $\dot{\theta}_0 = 10$ rad/s in a horizontal plane. The slider is attached to a spring of constant k = 2.5 lb/ft, which is unstretched when r = 0. Knowing that the slider is released from rest with no radial velocity in the position r = 18 in. and neglecting friction, determine for the position r = 12 in. (a) the radial and transverse components of the velocity of the slider, (b) the radial and transverse components of its acceleration, (c) the horizontal force exerted on the slider by arm AB.

SOLUTION

Let l_0 be the radial coordinate when the spring is unstretched. Force exerted by the spring.

$$F_r = -k(r - l_0)$$

$$\Sigma F_r = ma_r: \quad -k(r - l_0) = m(\ddot{r} - r\dot{\theta}^2)$$

$$\ddot{r} = \left(\dot{\theta}^2 - \frac{k}{m}\right)r + \frac{kl_0}{m} \tag{1}$$

But

$$\ddot{r} = \frac{d}{dt}(\dot{r}) = \frac{d\dot{r}}{dr}\frac{dr}{dt} = \dot{r}\frac{d\dot{r}}{dr}$$

$$\dot{r}d\dot{r} = \ddot{r}dr = \left[\left(\dot{\theta}^2 - \frac{k}{m}\right)r + \frac{kl_0}{m}\right]dr$$

Integrate using the condition $\dot{r} = \dot{r}_0$ when $r = r_0$.

$$\frac{1}{2}\dot{r}^2\Big|_{\dot{r}_0}^{\dot{r}} = \left[\frac{1}{2}\left(\dot{\theta}^2 - \frac{k}{m}\right)r^2 + \frac{kl_0}{m}r\right]\Bigg|_{r_0}^{r}$$

$$\frac{1}{2}\dot{r}^2 - \frac{1}{2}\dot{r}_0^2 = \frac{1}{2}\left(\dot{\theta}^2 - \frac{k}{m}\right)\left(r^2 - r_0^2\right) + \frac{kl_0}{m}(r - r_0)$$

$$\dot{r}^2 = \dot{r}_0^2 + \left(\dot{\theta}^2 - \frac{k}{m}\right)\left(r^2 - r_0^2\right) + \frac{2kl_0}{m}(r - r_0)$$

<u>Data</u>:

$$m = \frac{W}{g} = \frac{0.5 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.01553 \text{ lb} \cdot \text{s}^2\text{/ft}$$

$$\dot{\theta} = 10 \text{ rad/s}, \quad k = 2.5 \text{ lb/ft}, \quad l_0 = 0$$

$$\dot{r}_0 = (v_r)_0 = 0, \quad r_0 = 18 \text{ in.} = 1.5 \text{ ft}, \quad r = 12 \text{ in.} = 1.0 \text{ ft}$$

PROBLEM 12.73* (Continued)

(*a*) Components of velocity when $r = 12$ in.

$$\dot{r}^2 = 0 + \left(10^2 - \frac{2.5}{0.01553}\right)(1.0^2 - 1.5^2) + 0$$

$$= 76.223 \text{ ft}^2/\text{s}^2$$

$$v_r = \dot{r} = \pm 8.7306 \text{ ft/s}$$

Since r is decreasing, v_r is negative

$$\dot{r} = -8.7306 \text{ ft/s} \qquad v_r = -8.73 \text{ ft/s} \blacktriangleleft$$

$$v_\theta = r\dot{\theta} = (1.0)(10) \qquad v_\theta = 10.00 \text{ ft/s} \blacktriangleleft$$

(*b*) Components of acceleration.

$$F_r = -kr + kl_0 = -(2.5)(1.0) + 0 = -2.5 \text{ lb}$$

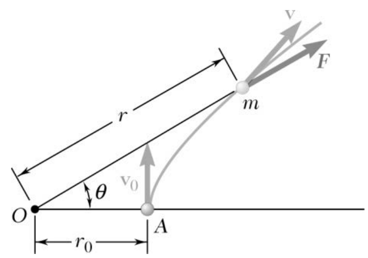
$$a_r = \frac{F_r}{m} = -\frac{2.5}{0.01553} \qquad a_r = 161.0 \text{ ft/s}^2 \blacktriangleleft$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + (2)(-8.7306)(10)$$

$$a_\theta = -174.6 \text{ ft/s}^2 \blacktriangleleft$$

(*c*) Transverse component of force.

$$F_\theta = ma_\theta = (0.01553)(-174.6) \qquad F_\theta = -2.71 \text{ lb} \blacktriangleleft$$



PROBLEM 12.74

A particle of mass m is projected from Point A with an initial velocity $\mathbf{v}_0$ perpendicular to line OA and moves under a central force $\mathbf{F}$ directed away from the center of force O. Knowing that the particle follows a path defined by the equation $r = r_0/\sqrt{\cos 2\theta}$ and using Eq. (12.27), express the radial and transverse components of the velocity $\mathbf{v}$ of the particle as functions of θ.

SOLUTION

Since the particle moves under a central force, $h = \text{constant}$.

Using Eq. (12.27),
$$h = r^2\dot{\theta} = h_0 = r_0 v_0$$

or
$$\dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0 \cos 2\theta}{r_0^2} = \frac{v_0}{r_0}\cos 2\theta$$

Radial component of velocity.

$$v_r = \dot{r} = \frac{dr}{d\theta}\dot{\theta} = \frac{d}{d\theta}\left(\frac{r_0}{\sqrt{\cos 2\theta}}\right)\dot{\theta} = r_0\frac{\sin 2\theta}{(\cos 2\theta)^{3/2}}\dot{\theta}$$

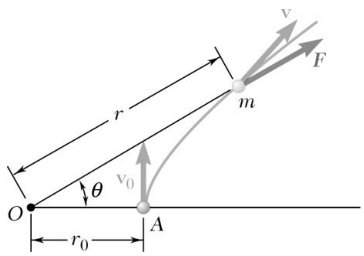
$$= r_0\frac{\sin 2\theta}{(\cos 2\theta)^{3/2}}\frac{v_0}{r}\cos 2\theta$$

$$v_r = v_0\frac{\sin 2\theta}{\sqrt{\cos 2\theta}} \blacktriangleleft$$

Transverse component of velocity.

$$v_\theta = \frac{h}{r} = \frac{r_0 v_0}{r_0}\sqrt{\cos 2\theta}$$

$$v_\theta = v_0\sqrt{\cos 2\theta} \blacktriangleleft$$



PROBLEM 12.75

For the particle of Problem 12.74, show (*a*) that the velocity of the particle and the central force **F** are proportional to the distance *r* from the particle to the center of force *O*, (*b*) that the radius of curvature of the path is proportional to r^3.

PROBLEM 12.74 A particle of mass *m* is projected from Point *A* with an initial velocity $\mathbf{v}_0$ perpendicular to line *OA* and moves under a central force **F** directed away from the center of force *O*. Knowing that the particle follows a path defined by the equation $r = r_0/\sqrt{\cos 2\theta}$ and using Eq. (12.27), express the radial and transverse components of the velocity **v** of the particle as functions of θ.

SOLUTION

Since the particle moves under a central force, $h = \text{constant}$.

Using Eq. (12.27),

$$h = r^2\dot{\theta} = h_0 = r_0 v_0 \quad \text{or} \quad \dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0 \cos 2\theta}{r_0^2} = \frac{v_0}{r_0}\cos 2\theta$$

Differentiating the expression for *r* with respect to time,

$$\dot{r} = \frac{dr}{d\theta}\dot{\theta} = \frac{d}{d\theta}\left(\frac{r_0}{\sqrt{\cos 2\theta}}\right)\dot{\theta} = r_0\frac{\sin 2\theta}{(\cos 2\theta)^{3/2}}\dot{\theta} = r_0\frac{\sin 2\theta}{(\cos 2\theta)^{3/2}}\frac{v_0}{r_0}\cos 2\theta = v_0\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}$$

Differentiating again,

$$\ddot{r} = \frac{d\dot{r}}{d\theta}\dot{\theta} = \frac{d}{d\theta}\left(v_0\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}\right)\dot{\theta} = v_0\frac{2\cos^2 2\theta + \sin^2 2\theta}{(\cos 2\theta)^{3/2}}\dot{\theta} = \frac{v_0^2}{r_0}\frac{2\cos^2 2\theta + \sin^2 2\theta}{\sqrt{\cos 2\theta}}$$

(*a*) $\quad v_r = \dot{r} = v_0\dfrac{\sin 2\theta}{\sqrt{\cos 2\theta}} = \dfrac{v_0 r}{r_0}\sin 2\theta \qquad v_\theta = r\dot{\theta} = \dfrac{v_0 r}{r_0}\cos 2\theta$

$$v = \sqrt{(v_r)^2 + (v_\theta)^2} = \frac{v_0 r}{r_0}\sqrt{\sin^2 2\theta + \cos^2 2\theta}$$

$$v = \frac{v_0 r}{r_0} \blacktriangleleft$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{v_0^2}{r_0}\frac{2\cos^2 2\theta + \sin^2 2\theta}{\sqrt{\cos 2\theta}} - \frac{r_0}{\sqrt{\cos 2\theta}}\frac{v_0^2}{r_0^2}\cos^2 2\theta$$

$$= \frac{v_0^2}{r_0}\frac{\cos^2 2\theta + \sin^2 2\theta}{\sqrt{\cos 2\theta}} = \frac{v_0}{r_0\sqrt{\cos 2\theta}} = \frac{v_0^2 r}{r_0^2}$$

$$F_r = ma_r = \frac{mv_0^2 r}{r_0^2}:$$

$$F_r = \frac{mv_0^2 r}{r_0^2} \blacktriangleleft$$

PROBLEM 12.75 (Continued)

Since the particle moves under a central force, $a_\theta = 0$.

Magnitude of acceleration.

$$a = \sqrt{a_r^{\,2} + a_\theta^{\,2}} = \frac{v_0^{\,2} r}{r_0^2}$$

Tangential component of acceleration.

$$a_t = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{v_0 r}{r_0}\right) = \frac{v_0}{r_0}\dot{r} = \frac{v_0^{\,2} r}{r_0^2}\sin 2\theta$$

Normal component of acceleration.

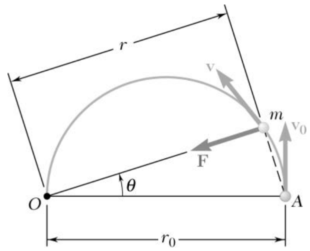
$$a_t = \sqrt{a^2 - a_t^{\,2}} = \frac{v_0^{\,2} r}{r_0^2}\sqrt{1 - \sin^2 2\theta} = \frac{v_0^{\,2} r \cos 2\theta}{r_0^2}$$

But $\qquad \cos 2\theta = \left(\dfrac{r_0}{r}\right)^2$

Hence, $\qquad a_n = \dfrac{v_0^{\,2}}{r}$

(*b*) But $a_n = \dfrac{v^2}{\rho}$ or $\rho = \dfrac{v^2}{a_n} = \dfrac{v_0^{\,2} r^2}{r_0^{\,2}} \cdot \dfrac{r}{v_0^{\,2}}$ $\qquad \rho = \dfrac{r^3}{r_0^2}$ ◀

PROBLEM 12.76



A particle of mass m is projected from Point A with an initial velocity $\mathbf{v}_0$ perpendicular to line OA and moves under a central force $\mathbf{F}$ along a semicircular path of diameter OA. Observing that $r = r_0 \cos\theta$ and using Eq. (12.27), show that the speed of the particle is $v = v_0/\cos^2\theta$.

SOLUTION

Since the particle moves under a central force, $h = \text{constant}$.

Using Eq. (12.27),

$$h = r^2\dot{\theta} = h_0 = r_0 v_0$$

or

$$\dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0}{r_0^2 \cos^2\theta} = \frac{v_0}{r_0 \cos^2\theta}$$

Radial component of velocity.

$$v_r = \dot{r} = \frac{d}{dt}(r_0 \cos\theta) = -(r_0 \sin\theta)\dot{\theta}$$

Transverse component of velocity.

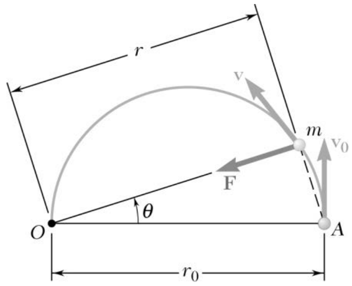
$$v_\theta = r\dot{\theta} = (r_0 \cos\theta)\dot{\theta}$$

Speed.

$$v = \sqrt{v_r^2 + v_\theta^2} = r_0\dot{\theta} = \frac{r_0 v_0}{r_0 \cos^2\theta}$$

$$v = \frac{v_0}{\cos^2\theta} \blacktriangleleft$$

PROBLEM 12.77



For the particle of Problem 12.76, determine the tangential component F_t of the central force **F** along the tangent to the path of the particle for (*a*) $\theta = 0$, (*b*) $\theta = 45°$.

PROBLEM 12.76 A particle of mass *m* is projected from Point *A* with an initial velocity $\mathbf{v}_0$ perpendicular to line *OA* and moves under a central force **F** along a semicircular path of diameter *OA*. Observing that $r = r_0 \cos\theta$ and using Eq. (12.27), show that the speed of the particle is $v = v_0/\cos^2\theta$.

SOLUTION

Since the particle moves under a central force, $h = \text{constant}$

Using Eq. (12.27),

$$h = r^2\dot{\theta} = h_0 = r_0 v_0$$

$$\dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0}{r_0^2 \cos^2\theta} = \frac{v_0}{r_0 \cos^2\theta}$$

Radial component of velocity.

$$v_r = \dot{r} = \frac{d}{dt}(r_0 \cos\theta) = -(r_0 \sin\theta)\dot{\theta}$$

Transverse component of velocity.

$$v_\theta = r\dot{\theta} = (r_0 \cos\theta)\dot{\theta}$$

Speed.

$$v = \sqrt{v_r^2 + v_\theta^2} = r_0\dot{\theta} = \frac{r_0 v_0}{r_0 \cos^2\theta} = \frac{v_0}{\cos^2\theta}$$

Tangential component of acceleration.

$$a_t = \frac{dv}{dt} = v_0 \frac{(-2)(-\sin\theta)\dot{\theta}}{\cos^3\theta} = \frac{2v_0 \sin\theta}{\cos^3\theta} \cdot \frac{v_0}{r_0 \cos^2\theta}$$

$$= \frac{2v_0^2 \sin\theta}{r_0 \cos^5\theta}$$

Tangential component of force.

$$F_t = ma_t: \quad F_t = \frac{2mv_0^2 \sin\theta}{r_0 \cos^5\theta}$$

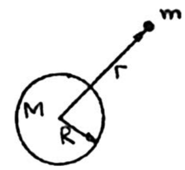
(*a*) $\theta = 0, \quad F_t = 0$ $\qquad F_t = 0$ ◀

(*b*) $\theta = 45°, \quad F_t = \dfrac{2mv_0 \sin 45°}{\cos^5 45°}$ $\qquad F_t = \dfrac{8mv_0^2}{r_0}$ ◀

PROBLEM 12.78

Determine the mass of the earth knowing that the mean radius of the moon's orbit about the earth is 238,910 mi and that the moon requires 27.32 days to complete one full revolution about the earth.

SOLUTION



We have $$F = G\frac{Mm}{r^2} \quad \text{[Eq. (12.28)]}$$

and $$F = F_n = ma_n = m\frac{v^2}{r}$$

Then $$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

or $$M = \frac{r}{G}v^2$$

Now $$v = \frac{2\pi r}{\tau}$$

so that $$M = \frac{r}{G}\left(\frac{2\pi r}{\tau}\right)^2 = \frac{1}{G}\left(\frac{2\pi}{\tau}\right)^2 r^3$$

Noting that $$\tau = 27.32\,\text{days} = 2.3604\times 10^6\,\text{s}$$

and $$r = 238{,}910\text{ mi} = 1.26144\times 10^9\,\text{ft}$$

we have $$M = \frac{1}{34.4\times 10^{-9}\,\text{ft}^4/\text{lb}\cdot\text{s}^4}\left(\frac{2\pi}{2.3604\times 10^6\,\text{s}}\right)^2(1.26144\times 10^9\,\text{ft})^3$$

or $$M = 413\times 10^{21}\,\text{lb}\cdot\text{s}^2/\text{ft} \blacktriangleleft$$

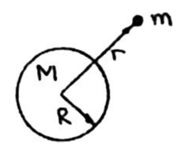
PROBLEM 12.79

Show that the radius r of the moon's orbit can be determined from the radius R of the earth, the acceleration of gravity g at the surface of the earth, and the time τ required for the moon to complete one full revolution about the earth. Compute r knowing that $\tau = 27.3$ days, giving the answer in both SI and U.S. customary units.

SOLUTION

We have $$F = G\frac{Mm}{r^2} \qquad \text{[Eq. (12.28)]}$$

and $$F = F_n = ma_n = m\frac{v^2}{r}$$



Then $$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

or $$v^2 = \frac{GM}{r}$$

Now $$GM = gR^2 \qquad \text{[Eq. (12.30)]}$$

so that $$v^2 = \frac{gR^2}{r} \quad \text{or} \quad v = R\sqrt{\frac{g}{r}}$$

For one orbit, $$\tau = \frac{2\pi r}{v} = \frac{2\pi r}{R\sqrt{\frac{g}{r}}}$$

or $$r = \left(\frac{g\tau^2R^2}{4\pi^2}\right)^{1/3} \qquad \text{Q.E.D.}$$ ◀

Now $$\tau = 27.3 \text{ days} = 2.35872\times10^6 \text{ s}$$
$$R = 3960 \text{ mi} = 20.9088\times10^6 \text{ ft}$$

SI: $$r = \left[\frac{9.81 \text{ m/s}^2 \times (2.35872\times10^6 \text{ s})^2 \times (6.37\times10^6 \text{ m})^2}{4\pi^2}\right]^{1/3} = 382.81\times10^6 \text{ m}$$

or $$r = 383\times10^3 \text{ km}$$ ◀

U.S. customary units:

$$r = \left[\frac{32.2 \text{ ft/s}^2 \times (2.35872\times10^6 \text{ s})^2 \times (20.9088\times10^6 \text{ ft})^2}{4\pi^2}\right]^{1/3} = 1256.52\times10^6 \text{ ft}$$

or $$r = 238\times10^3 \text{ mi}$$ ◀

PROBLEM 12.80

Communication satellites are placed in a geosynchronous orbit, i.e., in a circular orbit such that they complete one full revolution about the earth in one sidereal day (23.934 h), and thus appear stationary with respect to the ground. Determine (*a*) the altitude of these satellites above the surface of the earth, (*b*) the velocity with which they describe their orbit. Give the answers in both SI and U.S. customary units.

SOLUTION

For gravitational force and a circular orbit,

$$|F_r| = \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \text{or} \qquad v = \sqrt{\frac{GM}{r}}$$

Let τ be the period time to complete one orbit.

But
$$v\tau = 2\pi r \qquad \text{or} \qquad v^2\tau^2 = \frac{GM\tau^2}{r} = 4\pi^2 r^2$$

Then
$$r^3 = \frac{GM\tau^2}{4\pi^2} \qquad \text{or} \qquad r = \left(\frac{GM\tau^2}{4\pi^2}\right)^{1/3}$$

<u>Data</u>:
$$\tau = 23.934 \text{ h} = 86.1624\times10^3 \text{ s}$$

(*a*) In SI units:
$$g = 9.81 \text{ m/s}^2, \quad R = 6.37\times10^6 \text{ m}$$

$$GM = gR^2 = (9.81)(6.37\times10^6)^2 = 398.06\times10^{12} \text{ m}^3/\text{s}^2$$

$$r = \left[\frac{(398.06\times10^{12})(86.1624\times10^3)^2}{4\pi^2}\right]^{1/3} = 42.145\times10^6 \text{ m}$$

altitude $h = r - R = 35.775\times10^6$ m $\qquad h = 35{,}800$ km ◀

In U.S. units:
$$g = 32.2 \text{ ft/s}^2, \quad R = 3960 \text{ mi} = 20.909\times10^6 \text{ ft}$$

$$GM = gR^2 = (32.2)(20.909\times10^6)^2 = 14.077\times10^{15} \text{ ft}^3/\text{s}^2$$

$$r = \left[\frac{(14.077\times10^{15})(86.1624\times10^3)^2}{4\pi^2}\right]^{1/3} = 138.334\times10^6 \text{ ft}$$

altitude $h = r - R = 117.425\times10^6$ ft $\qquad h = 22{,}200$ mi ◀

PROBLEM 12.80 (Continued)

(*b*) In SI units:

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{398.06\times 10^{12}}{42.145\times 10^{6}}} = 3.07\times 10^{3} \text{ m/s} \qquad v = 3.07 \text{ km/s} ◀$$

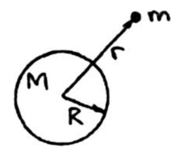
In U.S. units:

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{14.077\times 10^{15}}{138.334\times 10^{6}}} = 10.09\times 10^{3} \text{ ft/s} \qquad v = 10.09\times 10^{3} \text{ ft/s} ◀$$

PROBLEM 12.81

Show that the radius r of the orbit of a moon of a given planet can be determined from the radius R of the planet, the acceleration of gravity at the surface of the planet, and the time τ required by the moon to complete one full revolution about the planet. Determine the acceleration of gravity at the surface of the planet Jupiter knowing that $R = 71{,}492$ km and that $\tau = 3.551$ days and $r = 670.9 \times 10^3$ km for its moon Europa.

SOLUTION



We have $$F = G\frac{Mm}{r^2} \qquad \text{[Eq. (12.28)]}$$

and $$F = F_n = ma_n = m\frac{v^2}{r}$$

Then $$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

or $$v^2 = \frac{GM}{r}$$

Now $$GM = gR^2 \qquad \text{[Eq. (12.30)]}$$

so that $$v^2 = \frac{gR^2}{r} \qquad \text{or} \qquad v = R\sqrt{\frac{g}{r}}$$

For one orbit, $$\tau = \frac{2\pi r}{v} = \frac{2\pi r}{R\sqrt{\frac{g}{r}}}$$

or $$r = \left(\frac{g\tau^2 R^2}{4\pi^2}\right)^{1/3} \qquad \text{Q.E.D.} \; \blacktriangleleft$$

Solving for g, $$g = 4\pi^2 \frac{r^3}{\tau^2 R^2}$$

and noting that $\tau = 3.551\,\text{days} = 306{,}806$ s, then

$$g_{\text{Jupiter}} = 4\pi^2 \frac{r_{\text{Eur}}^3}{\tau_{\text{Eur}}^2 R_{\text{Jup}}}$$

$$= 4\pi^2 \frac{(670.9\times 10^6\,\text{m})^3}{(306{,}806\,\text{s})^2 (71.492\times 10^6\,\text{m})^2}$$

or $$g_{\text{Jupiter}} = 24.8\ \text{m/s}^2 \; \blacktriangleleft$$

Note: $g_{\text{Jupiter}} \approx 2.53 g_{\text{Earth}}$

PROBLEM 12.82

The orbit of the planet Venus is nearly circular with an orbital velocity of 126.5×10^3 km/h. Knowing that the mean distance from the center of the sun to the center of Venus is 108×10^6 km and that the radius of the sun is 695×10^3 km, determine (*a*) the mass of the sun, (*b*) the acceleration of gravity at the surface of the sun.

SOLUTION

Let M be the mass of the sun and m the mass of Venus.

For the circular orbit of Venus,

$$\frac{GMm}{r^2} = ma_n = \frac{mv^2}{r} \qquad GM = rv^2$$

where r is radius of the orbit.

Data:

$$r = 108 \times 10^6 \text{ km} = 108 \times 10^9 \text{ m}$$

$$v = 126.5 \times 10^3 \text{ km/hr} = 35.139 \times 10^3 \text{ m/s}$$

$$GM = (108 \times 10^9)(35.139 \times 10^3)^2 = 1.3335 \times 10^{20} \text{ m}^3/\text{s}^2$$

(*a*) Mass of sun.

$$M = \frac{GM}{G} = \frac{1.3335 \times 10^{20} \text{ m}^3/\text{s}^2}{66.73 \times 10^{-12}}$$

$M = 1.998 \times 10^{30}$ kg ◀

(*b*) At the surface of the sun,

$$R = 695.5 \times 10^3 \text{ km} = 695.5 \times 10^6 \text{ m}$$

$$\frac{GMm}{R^2} = mg$$

$$g = \frac{GM}{R^2} = \frac{1.3335 \times 10^{20}}{(695.5 \times 10^6)^2}$$

$g = 276 \text{ m/s}^2$ ◀

PROBLEM 12.83

A satellite is placed into a circular orbit about the planet Saturn at an altitude of 2100 mi. The satellite describes its orbit with a velocity of 54.7×10^3 mi/h. Knowing that the radius of the orbit about Saturn and the periodic time of Atlas, one of Saturn's moons, are 85.54×10^3 mi and 0.6017 days, respectively, determine (*a*) the radius of Saturn, (*b*) the mass of Saturn. (The *periodic time* of a satellite is the time it requires to complete one full revolution about the planet.)

SOLUTION

Velocity of Atlas. $$v_A = \frac{2\pi r_A}{\tau_A}$$

where $$v_A = 85.54\times10^3 \text{ mi} = 451.651\times10^6 \text{ ft}$$

and $$\tau_A = 0.6017 \text{ days} = 51{,}987 \text{ s}$$

$$v_A = \frac{(2\pi)(451.651\times10^6)}{51{,}987} = 54.587\times10^3 \text{ ft/s}$$

Gravitational force. $$F = \frac{GMm}{r^2} = \frac{mv^2}{r}$$

from which $$GM = rv^2 = \text{constant}$$

For the satellite, $$r_s v_s^2 = r_A v_A^2$$

$$r_s = \frac{r_A v_A^2}{v_s{}^2}$$

where $$v_s = 54.7\times10^3 \text{mi/h} = 80.227\times10^3 \text{ ft/s}$$

$$r_s = \frac{(451.651\times10^6)(54.587\times10^3)^2}{(80.227\times10^3)^2} = 209.09\times10^6 \text{ ft}$$

$$r_s = 39{,}600 \text{ mi}$$

(*a*) Radius of Saturn.

$$R = r_s - (\text{altitude}) = 39{,}600 - 2100$$

$R = 37{,}500$ mi ◀

(*b*) Mass of Saturn.

$$M = \frac{r_A v_A^2}{G} = \frac{(451.651\times10^6)(54.587\times10^3)^2}{34.4\times10^{-9}}$$

$M = 39.1\times10^{24}$ lb·s²/ft ◀

PROBLEM 12.84

The periodic times (see Problem 12.83) of the planet Uranus's moons Juliet and Titania have been observed to be 0.4931 days and 8.706 days, respectively. Knowing that the radius of Juliet's orbit is 40,000 mi, determine (*a*) the mass of Uranus, (*b*) the radius of Titania's orbit.

SOLUTION

Velocity of Juliet.

$$v_J = \frac{2\pi r_J}{\tau_J}$$

where $r_J = 40,000 \text{ mi} = 2.112\times10^8 \text{ ft}$

and $\tau_J = 0.4931 \text{ days} = 42,604 \text{ s}$

$$v_J = \frac{(2\pi)(2.112\times10^8 \text{ ft})}{42,604 \text{ s}} = 3.11476\times10^4 \text{ ft/s}$$

Gravitational force.

$$F = \frac{GMm}{r^2} = \frac{mv^2}{r}$$

from which $GM = rv^2 = \text{constant}$

(*a*) Mass of Uranus.

$$M = \frac{r_J v_J^2}{G}$$

$$M = \frac{(2.112\times10^8)(3.11476\times10^4)^2}{34.4\times10^{-9}}$$
$$= 5.95642\times10^{24} \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$M = 5.96\times10^{24} \text{ lb}\cdot\text{s}^2/\text{ft} \blacktriangleleft$$

(*b*) Radius of Titania's orbit.

$$GM = r_T v_T^2 = \frac{4\pi^2 r_T^3}{\tau_T^{\,2}} = \frac{4\pi^2 r_J^3}{\tau_J^{\,2}}$$

$$r_T^3 = r_J^3\left(\frac{\tau_T}{\tau_J}\right)^2 = (2.112\times10^8)^3\left(\frac{8.706}{0.4931}\right)^2 = 2.93663\times10^{27} \text{ ft}^3$$

$$r_T = 1.43202\times10^9 \text{ ft} = 2.71216\times10^5 \text{ mi}$$

$$r_T = 2.71\times10^5 \text{ mi} \blacktriangleleft$$

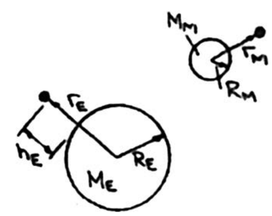
PROBLEM 12.85

A 500 kg spacecraft first is placed into a circular orbit about the earth at an altitude of 4500 km and then is transferred to a circular orbit about the moon. Knowing that the mass of the moon is 0.01230 times the mass of the earth and that the radius of the moon is 1737 km, determine (*a*) the gravitational force exerted on the spacecraft as it was orbiting the earth, (*b*) the required radius of the orbit of the spacecraft about the moon if the periodic times (see Problem 12.83) of the two orbits are to be equal, (*c*) the acceleration of gravity at the surface of the moon.

SOLUTION

First note that $R_E = 6.37\times10^6$ m

Then $r_E = R_E + h_E = (6.37\times10^6 + 4.5\times10^6)$ m $= 10.87\times10^6$ m



(*a*) We have $F = \dfrac{GMm}{r^2}$ [Eq. (12.28)]

and $GM = gR^2$ [Eq. (12.29)]

Then $F = gR^2\dfrac{m}{r^2} = W\left(\dfrac{R}{r}\right)^2$

For the earth orbit, $F = (500\text{ kg})(9.81\text{ m/s}^2)\left(\dfrac{6.37\times10^6\text{ m}}{10.87\times10^6\text{ m}}\right)^2$

or $F = 1684$ N ◀

(*b*) From the solution to Problem 12.78, we have

$$M = \frac{1}{G}\left(\frac{2\pi}{\tau}\right)^2 r^3$$

Then $\tau = \dfrac{2\pi r^{3/2}}{\sqrt{GM}}$

Now $\tau_E = \tau_M \Rightarrow \dfrac{2\pi r_E^{3/2}}{\sqrt{GM_E}} = \dfrac{2\pi r_M^{3/2}}{\sqrt{GM_M}}$ (1)

or $r_M = \left(\dfrac{M_M}{M_E}\right)^{1/3} r_E = (0.01230)^{1/3}(10.87\times10^6\text{ m})$

or $r_M = 2.509\times10^6$ m $r_M = 2510$ km ◀

(*c*) We have $GM = gR^2$ [Eq.(12.29)]

Substituting into Eq. (1)

$$\frac{2\pi r_E^{3/2}}{R_E\sqrt{g_E}} = \frac{2\pi r_M^{3/2}}{R_M\sqrt{g_M}}$$

or

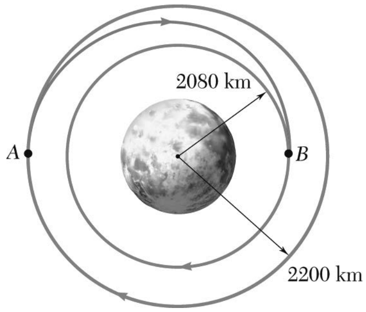
$$g_M = \left(\frac{R_E}{R_M}\right)^2\left(\frac{r_M}{r_E}\right)^3 g_E = \left(\frac{R_E}{R_M}\right)^2\left(\frac{M_M}{M_E}\right) g_E$$

using the results of Part (b). Then

$$g_M = \left(\frac{6370\text{ km}}{1737\text{ km}}\right)^2 (0.01230)(9.81\text{ m/s}^2)$$

or $g_{\text{moon}} = 1.62\text{ m/s}^2$ ◀

Note: $g_{\text{moon}} \approx \dfrac{1}{6} g_{\text{earth}}$



PROBLEM 12.86

A space vehicle is in a circular orbit of 2200-km radius around the moon. To transfer it to a smaller circular orbit of 2080-km radius, the vehicle is first placed on an elliptic path *AB* by reducing its speed by 26.3 m/s as it passes through *A*. Knowing that the mass of the moon is 73.49×10^{21} kg, determine (*a*) the speed of the vehicle as it approaches *B* on the elliptic path, (*b*) the amount by which its speed should be reduced as it approaches *B* to insert it into the smaller circular orbit.

SOLUTION

For a circular orbit, $\Sigma F_n = ma_n: \quad F = m\dfrac{v^2}{r}$

Eq. (12.28): $$F = G\frac{Mm}{r^2}$$

Then $$G\frac{Mm}{r^2} = m\frac{v^2}{r}$$

or $$v^2 = \frac{GM}{r}$$

Then $$(v_A)^2_{\text{circ}} = \frac{66.73\times10^{-12}\ \text{m}^3/\text{kg}\cdot\text{s}^2 \times 73.49\times10^{21}\ \text{kg}}{2200\times10^3\ \text{m}}$$

or $$(v_A)_{\text{circ}} = 1493.0\ \text{m/s}$$

and $$(v_B)^2_{\text{circ}} = \frac{66.73\times10^{-12}\ \text{m}^3/\text{kg}\cdot\text{s}^2 \times 73.49\times10^{21}\ \text{kg}}{2080\times10^3\ \text{m}}$$

or $$(v_B)_{\text{circ}} = 1535.5\ \text{m/s}$$

(*a*) We have
$$(v_A)_{TR} = (v_A)_{\text{circ}} + \Delta v_A$$
$$= (1493.0 - 26.3)\ \text{m/s}$$
$$= 1466.7\ \text{m/s}$$

Conservation of angular momentum requires that

$$r_A m(v_A)_{TR} = r_B m(v_B)_{TR}$$

or $$(v_B)_{TR} = \frac{2200\ \text{km}}{2080\ \text{km}} \times 1466.7\ \text{m/s}$$
$$= 1551.3\ \text{m/s}$$

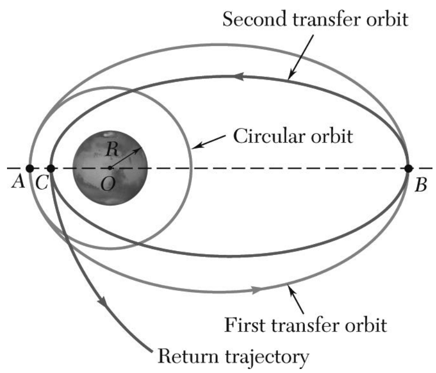
or $$(v_B)_{TR} = 1551\ \text{m/s} \blacktriangleleft$$

(*b*) Now $$(v_B)_{\text{circ}} = (v_B)_{TR} + \Delta v_B$$

or $$\Delta v_B = (1535.5 - 1551.3)\ \text{m/s}$$

or $$\Delta v_B = -15.8\ \text{m/s} \blacktriangleleft$$

PROBLEM 12.87



Plans for an unmanned landing mission on the planet Mars called for the earth-return vehicle to first describe a circular orbit at an altitude d_A = 2200 km above the surface of the planet with a velocity of 2771 m/s. As it passed through Point A, the vehicle was to be inserted into an elliptic transfer orbit by firing its engine and increasing its speed by $\Delta v_A = 1046$ m/s. As it passed through Point B, at an altitude d_B = 100,000 km, the vehicle was to be inserted into a second transfer orbit located in a slightly different plane, by changing the direction of its velocity and reducing its speed by $\Delta v_B = -22.0$ m/s. Finally, as the vehicle passed through Point C, at an altitude d_C = 1000 km, its speed was to be increased by $\Delta v_C = 660$ m/s to insert it into its return trajectory. Knowing that the radius of the planet Mars is $R = 3400$ km, determine the velocity of the vehicle after completion of the last maneuver.

SOLUTION

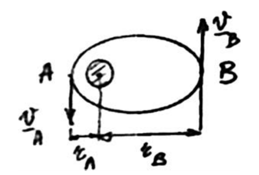
$$r_A = 3400 + 2200 = 5600 \text{ km} = 5.60 \times 10^6 \text{ m}$$

$$r_B = 3400 + 100{,}000 = 103{,}400 \text{ km} = 103.4 \times 10^6 \text{ m}$$

$$r_C = 3400 + 1000 = 4400 \text{ km} = 4.40 \times 10^6 \text{ m}$$

First transfer orbit.

$$v_A = 2771 \text{ m/s} + 1046 \text{ m/s} = 3817 \text{ m/s}$$



Conservation of angular momentum:

$$r_A m v_A = r_B m v_B$$

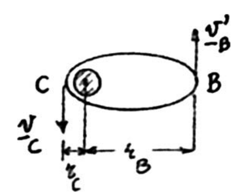
$$(5.60 \times 10^6)(3817) = (103.4 \times 10^6) v_B$$

$$v_B = 206.7 \text{ m/s}$$

Second transfer orbit.

$$v'_B = v_B + \Delta v_B$$

$$= 206.7 - 22.0 = 184.7 \text{ m/s}$$



Conservation of angular momentum:

$$r_B \, m v'_B = r_C \, m v_C$$

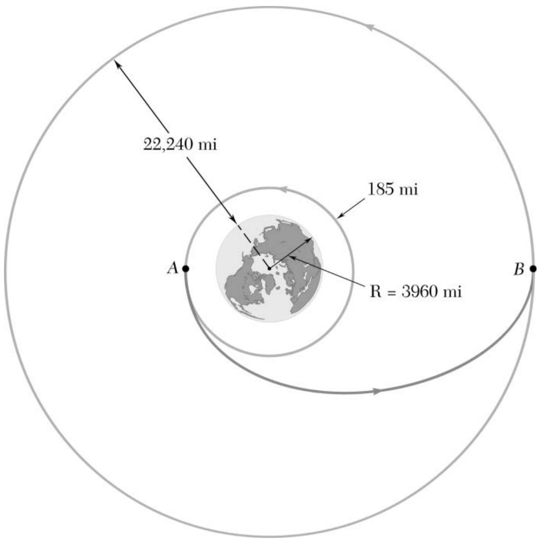
$$(103.4 \times 10^6)(184.7) = (4.40 \times 10^6) v_C$$

$$v_C = 4340 \text{ m/s}$$

After last maneuver.

$$v = v_C + \Delta v_C = 4340 + 660$$

$v = 5000$ m/s ◀



PROBLEM 12.88

To place a communications satellite into a geosynchronous orbit (see Problem 12.80) at an altitude of 22,240 mi above the surface of the earth, the satellite first is released from a space shuttle, which is in a circular orbit at an altitude of 185 mi, and then is propelled by an upper-stage booster to its final altitude. As the satellite passes through *A*, the booster's motor is fired to insert the satellite into an elliptic transfer orbit. The booster is again fired at *B* to insert the satellite into a geosynchronous orbit. Knowing that the second firing increases the speed of the satellite by 4810 ft/s, determine (*a*) the speed of the satellite as it approaches *B* on the elliptic transfer orbit, (*b*) the increase in speed resulting from the first firing at *A*.

SOLUTION

For earth, $R = 3960 \text{ mi} = 20.909\times10^6 \text{ ft}$

$$GM = gR^2 = (32.2)(20.909\times10^6)^2 = 14.077\times10^{15} \text{ ft}^3/\text{s}^2$$

$$r_A = 3960 + 185 = 4145 \text{ mi} = 21.8856\times10^6 \text{ ft}$$

$$r_B = 3960 + 22,240 = 26,200 \text{ mi} = 138.336\times10^6 \text{ ft}$$

Speed on circular orbit through *A*.

$$(v_A)_{\text{circ}} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.077\times10^{15}}{21.8856\times10^6}} = 25.362\times10^3 \text{ ft/s}$$

Speed on circular orbit through *B*.

$$(v_B)_{\text{circ}} = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{14.077\times10^{15}}{138.336\times10^6}} = 10.088\times10^3 \text{ ft/s}$$

PROBLEM 12.88 (Continued)

(*a*) Speed on transfer trajectory at *B*.

$$(v_B)_{\text{tr}} = 10.088\times10^3 - 4810$$
$$= 5.278\times10^3$$

5280 ft/s ◀

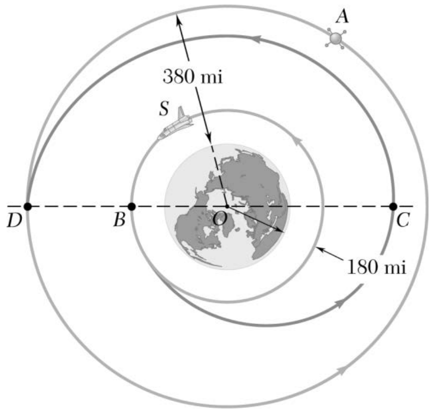
Conservation of angular momentum for transfer trajectory.

$$r_A(v_A)_{\text{tr}} = r_B(v_B)_{\text{tr}}$$
$$(v_A)_{\text{tr}} = \frac{r_B(v_B)_{\text{tr}}}{r_A}$$
$$= \frac{(138.336\times10^6)(5278)}{21.8856\times10^6}$$
$$= 33.362\times10^3 \text{ ft/s}$$

(*b*) Change in speed at *A*.

$$\Delta v_A = (v_A)_{\text{tr}} - (v_A)_{\text{circ}}$$
$$= 33.362\times10^3 - 25.362\times10^3$$
$$= 8.000\times10^3$$

$\Delta v_A = 8000$ ft/s ◀



PROBLEM 12.89

A space shuttle S and a satellite A are in the circular orbits shown. In order for the shuttle to recover the satellite, the shuttle is first placed in an elliptic path BC by increasing its speed by $\Delta v_B = 280$ ft/s as it passes through B. As the shuttle approaches C, its speed is increased by $\Delta v_C = 260$ ft/s to insert it into a second elliptic transfer orbit CD. Knowing that the distance from O to C is 4289 mi, determine the amount by which the speed of the shuttle should be increased as it approaches D to insert it into the circular orbit of the satellite.

SOLUTION

First note $\quad R = 3960 \text{ mi} = 20.9088\times10^6 \text{ ft}$

$$r_A = (3960+380) \text{ mi} = 4340 \text{ mi} = 22.9152\times10^6 \text{ ft}$$

$$r_B = (3960+180) \text{ mi} = 4140 \text{ mi} = 21.8592\times10^6 \text{ ft}$$

For a circular orbit, $\quad \Sigma F_n = ma_n: \qquad F = m\dfrac{v^2}{r}$

Eq. (12.28): $\quad F = G\dfrac{Mm}{r^2}$

Then $\quad G\dfrac{Mm}{r^2} = m\dfrac{v^2}{r}$

or $\quad v^2 = \dfrac{GM}{r} = \dfrac{gR^2}{r} \qquad$ using Eq. (12.29).

Then $\quad (v_A)^2_{\text{circ}} = \dfrac{32.2 \text{ ft/s}^2 \times (20.9088\times10^6 \text{ ft})^2}{22.9152\times10^6 \text{ ft}}$

or $\quad (v_A)_{\text{circ}} = 24,785 \text{ ft/s}$

and $\quad (v_B)^2_{\text{circ}} = \dfrac{32.2 \text{ ft/s}^2 \times (20.9088\times10^6 \text{ ft})^2}{21.8592\times10^6 \text{ ft}}$

or $\quad (v_B)_{\text{circ}} = 25,377 \text{ ft/s}$

We have $\quad (v_B)_{TR_{BC}} = (v_B)_{\text{circ}} + \Delta v_B = (25,377 + 280) \text{ ft/s}$

$$= 25,657 \text{ ft/s}$$

PROBLEM 12.89 (Continued)

Conservation of angular momentum requires that

$$BC: \quad r_B\, m(v_B)_{TR_{BC}} = r_C\, m(v_C)_{TR_{BC}} \tag{1}$$

$$CD: \quad r_C\, m(v_C)_{TR_{CD}} = r_A\, m(v_D)_{TR_{CD}} \tag{2}$$

From Eq. (1)

$$(v_C)_{TR_{BC}} = \frac{r_B}{r_C}(v_B)_{TR_{BC}} = \frac{4140 \text{ mi}}{4289 \text{ mi}} \times 25{,}657 \text{ ft/s}$$
$$= 24{,}766 \text{ ft/s}$$

Now

$$(v_C)_{TR_{CD}} = (v_C)_{TR_{BC}} + \Delta v_C = (24{,}766 + 260) \text{ ft/s}$$
$$= 25{,}026 \text{ ft/s}$$

From Eq. (2)

$$(v_D)_{TR_{CD}} = \frac{r_C}{r_A}(v_C)_{TR_{CD}} = \frac{4289 \text{ mi}}{4340 \text{ mi}} \times 25{,}026 \text{ ft/s}$$
$$= 24{,}732 \text{ ft/s}$$

Finally,

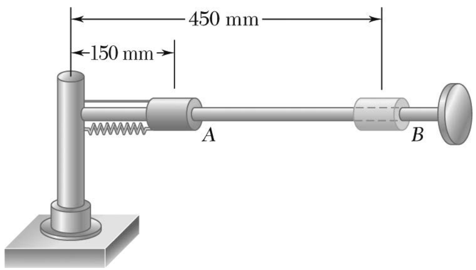
$$(v_A)_{\text{circ}} = (v_D)_{TR_{CD}} + \Delta v_D$$

or

$$\Delta v_D = (24{,}785 - 24{,}732) \text{ ft/s}$$

or

$$\Delta v_D = 53 \text{ ft/s} \quad ◀$$

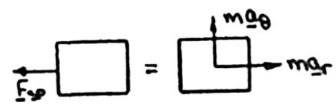


PROBLEM 12.90

A 1 kg collar can slide on a horizontal rod, which is free to rotate about a vertical shaft. The collar is initially held at A by a cord attached to the shaft. A spring of constant 30 N/m is attached to the collar and to the shaft and is undeformed when the collar is at A. As the rod rotates at the rate $\dot{\theta} = 16$ rad/s, the cord is cut and the collar moves out along the rod. Neglecting friction and the mass of the rod, determine (a) the radial and transverse components of the acceleration of the collar at A, (b) the acceleration of the collar relative to the rod at A, (c) the transverse component of the velocity of the collar at B.

SOLUTION

First note $\quad F_{sp} = k(r - r_A)$



(a) $F_\theta = 0$ and at A, $\quad F_r = -F_{sp} = 0$

$(a_A)_r = 0$ ◀

$(a_A)_\theta = 0$ ◀

(b) $\xrightarrow{+} \Sigma F_r = ma_r$: $\quad -F_{sp} = m(\ddot{r} - r\dot{\theta}^2)$

Noting that $\quad a_{\text{collar/rod}} = \ddot{r}$, we have at A

$$0 = m[a_{\text{collar/rod}} - (150 \text{ mm})(16 \text{ rad/s})^2]$$

$$a_{\text{collar/rod}} = 38400 \text{ mm/s}^2$$

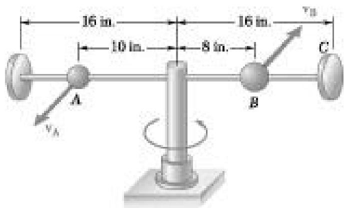
or $\quad (a_{\text{collar/rod}})_A = 38.4 \text{ m/s}^2$ ◀

(c) After the cord is cut, the only horizontal force acting on the collar is due to the spring. Thus, angular momentum about the shaft is conserved.

$$r_A\, m(v_A)_\theta = r_B\, m(v_B)_\theta \quad \text{where} \quad (v_A)_\theta = r_A\, \dot{\theta}_0$$

Then $\quad (v_B)_\theta = \dfrac{150 \text{ mm}}{450 \text{ mm}}[(150 \text{ mm})(16 \text{ rad/s})] = 800 \text{ mm/s}$

or $\quad (v_B)_\theta = 0.800 \text{ m/s}$ ◀



PROBLEM 12.91

A 1-lb ball A and a 2-lb ball B are mounted on a horizontal rod which rotates freely about a vertical shaft. The balls are held in the positions shown by pins. The pin holding B is suddenly removed and the ball moves to position C as the rod rotates. Neglecting friction and the mass of the rod and knowing that the initial speed of A is $v_A = 8$ ft/s, determine (*a*) the radial and transverse components of the acceleration of ball B immediately after the pin is removed, (*b*) the acceleration of ball B relative to the rod at that instant, (*c*) the speed of ball A after ball B has reached the stop at C.

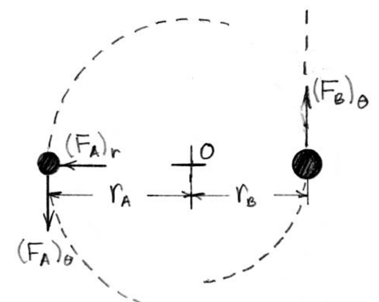
SOLUTION

Let r and θ be polar coordinates with the origin lying at the shaft.

Constraint of rod: $\theta_B = \theta_A + \pi$ radians; $\dot{\theta}_B = \dot{\theta}_A = \dot{\theta}$; $\ddot{\theta}_B = \ddot{\theta}_A = \ddot{\theta}$.

(*a*) Components of acceleration

Sketch the free body diagrams of the balls showing the radial and transverse components of the forces acting on them. Owing to frictionless sliding of B along the rod, $(F_B)_r = 0$.



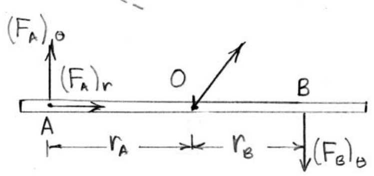
Radial component of acceleration of B.

$$F_r = m_B(a_B)_r: \qquad (a_B)_r = 0 \blacktriangleleft$$

Transverse components of acceleration.

$$(a_A)_\theta = r_A\ddot{\theta} + 2\dot{r}_A\dot{\theta} = ra\ddot{\theta}$$

$$(a_B)_\theta = r_B\ddot{\theta} + 2\dot{r}_B\dot{\theta} \qquad (1)$$



Since the rod is massless, it must be in equilibrium. Draw its free body diagram, applying Newton's 3rd Law.

$$+\circlearrowleft \Sigma M_0 = 0: \quad r_A(F_A)_\theta + r_B(F_B)_\theta = r_A m_A (a_A)_\theta + r_B m_B (a_B)_\theta = 0$$

$$r_A m_A r_A \ddot{\theta} + r_B m_B (r_B\ddot{\theta} + 2\dot{r}_B\dot{\theta}) = 0$$

$$\ddot{\theta} = \frac{-2m_B\dot{r}_B\dot{\theta}}{m_A r_A^{\ 2} + m_B r_B^{\ 2}}$$

At $t = 0$, $\quad \dot{r}_B = 0 \quad$ so that $\quad \ddot{\theta} = 0.$

From Eq. (1), $\qquad (a_B)_\theta = 0 \blacktriangleleft$

(*b*) Acceleration of *B* relative to the rod.

At $t = 0$, $(v_A)_\theta = 8$ ft/s $= 96$ in./s, $\dot{\theta} = \dfrac{(v_A)_\theta}{r_A} = \dfrac{96}{10} = 9.6$ rad/s

$$\ddot{r}_B - r_B\dot{\theta}^2 = (a_B)_r = 0$$

$$\ddot{r}_B = r_B\dot{\theta}^2 = (8)(9.6)^2 = 737.28 \text{ in./s}^2$$

$$\ddot{r}_B = 61.4 \text{ ft/s}^2 \blacktriangleleft$$

(*c*) Speed of *A*.

Substituting $\dfrac{d}{dt}(mr^2\dot{\theta})$ for rF_θ in each term of the moment equation gives

$$\frac{d}{dt}\left(m_A r_A{}^2\dot{\theta}\right) + \frac{d}{dt}\left(m_B r_B{}^2\dot{\theta}\right) = 0$$

Integrating with respect to time,

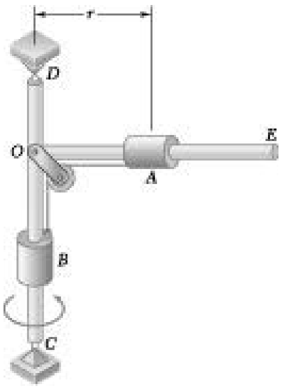
$$m_A r_A{}^2\dot{\theta} + m_B r_B{}^2\dot{\theta} = \left(m_A r_A{}^2\dot{\theta}\right)_0 + \left(m_B r_B{}^2\dot{\theta}\right)_0$$

Applying to the final state with ball *B* moved to the stop at *C*,

$$\left(\frac{W_A}{g} r_A^2 + \frac{W_B}{g} r_C^2\right)\dot{\theta}_f = \left[\frac{W_A}{g} r_A^2 + \frac{W_B}{g}(r_B)_0^2\right]\dot{\theta}_0$$

$$\dot{\theta}_f = \frac{W_A r_A{}^2 + W_B (r_B)_0^2}{W_A r_A{}^2 + W_B r_C{}^2}\dot{\theta}_0 = \frac{(1)(10)^2 + (2)(8)^2}{(1)(10)^2 + (2)(16)^2}(9.6) = 3.5765 \text{ rad/s}$$

$$(v_A)_f = r_A\dot{\theta}_f = (10)(3.5765) = 35.765 \text{ in./s} \qquad (v_A)_f = 2.98 \text{ ft/s} \blacktriangleleft$$



PROBLEM 12.92

Two 2.6-lb collars *A* and *B* can slide without friction on a frame, consisting of the horizontal rod *OE* and the vertical rod *CD*, which is free to rotate about *CD*. The two collars are connected by a cord running over a pulley that is attached to the frame at *O* and a stop prevents collar *B* from moving. The frame is rotating at the rate $\dot{\theta} = 12$ rad/s and $r = 0.6$ ft when the stop is removed allowing collar *A* to move out along rod *OE*. Neglecting friction and the mass of the frame, determine, for the position $r = 1.2$ ft, (*a*) the transverse component of the velocity of collar *A*, (*b*) the tension in the cord and the acceleration of collar *A* relative to the rod *OE*.

SOLUTION

Masses: $m_A = m_B = \dfrac{2.6}{32.2} = 0.08075 \text{ lb}\cdot\text{s}^2/\text{ft}$

(*a*) Conservation of angular momentum of collar *A*: $(H_0)_2 = (H_0)_1$

$$m_A r_1 (v_\theta)_1 = m_A r_2 (v_\theta)_2$$

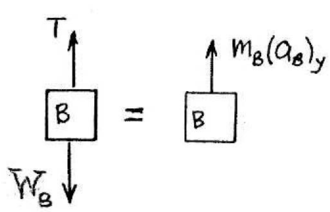
$$(v_\theta)_2 = \frac{r_1 (v_\theta)_1}{r_2} = \frac{r_1^2 \dot{\theta}_1}{r_2} = \frac{(0.6)^2(12)}{1.2} = 3.6$$

$(v_\theta)_2 = 3.60$ ft/s ◀

$$\dot{\theta}_2 = \frac{(v_\theta)_2}{r_A} = \frac{3.6}{1.2} = 3.00 \text{ rad/s}$$

(*b*) Let *y* be the position coordinate of *B*, positive upward with origin at *O*.

Constraint of the cord: $r - y = \text{constant}$ or $\ddot{y} = \ddot{r}$



Kinematics:

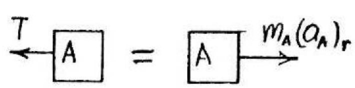
$$(a_B)_y = \ddot{y} = \ddot{r} \qquad \text{and} \qquad (a_A)_r = \ddot{r} - r\dot{\theta}^2$$

Collar *B*: $\Sigma F_y = m_B a_B: \quad T - W_B = m_B \ddot{y} = m_B \ddot{r}$ (1)

Collar *A*: $\xrightarrow{+} \Sigma F_r = m_A (a_A)_r: \quad -T = m_A(\ddot{r} - r\dot{\theta}^2)$ (2)

Adding (1) and (2) to eliminate *T*,

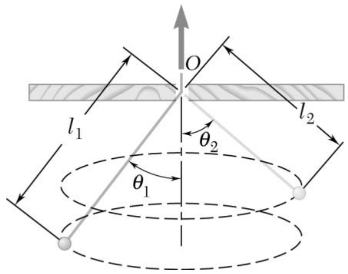
$$-W_B = (m_A + m_B)\ddot{r} + m_A r\dot{\theta}^2$$



$$a_{A/\text{rod}} = \ddot{r} = \frac{m_A r\dot{\theta}^2 - W_B}{m_A + m_B} = \frac{(0.08075)(1.2)(3.00)^2 - (2.6)}{0.08075 + 0.08075} = -10.70 \text{ ft/s}^2$$

$T = m_B(\ddot{r} + g) = (0.08075)(-10.70 + 32.2)$ $\qquad T = 1.736$ lb ◀

$a_{A/\text{rod}} = 10.70$ ft/s² radially inward. ◀



PROBLEM 12.93

A small ball swings in a horizontal circle at the end of a cord of length l_1, which forms an angle θ_1 with the vertical. The cord is then slowly drawn through the support at O until the length of the free end is l_2. (*a*) Derive a relation among l_1, l_2, θ_1, and θ_2. (*b*) If the ball is set in motion so that initially $l_1 = 0.8$ m and $\theta_1 = 35°$, determine the angle θ_2 when $l_2 = 0.6$ m.

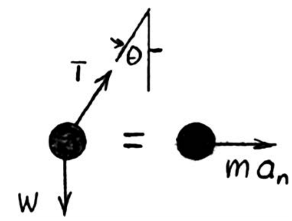
SOLUTION

(*a*) For state 1 or 2, neglecting the vertical component of acceleration,

$$+\uparrow \Sigma F_y = 0: \quad T\cos\theta - W = 0$$

$$T = W\cos\theta$$

$$\xrightarrow{+} \Sigma F_x = ma_n: \quad T\sin\theta = W\sin\theta\cos\theta = \frac{mv^2}{\rho}$$



But $\rho = \ell\sin\theta$ so that

$$v^2 = \frac{\rho W}{m}\sin^2\theta\cos\theta = \ell\, g\,\sin\theta\tan\theta$$

$$v_1 = \sqrt{\ell_1 g\,\sin\theta_1\tan\theta_1}$$

and

$$v_2 = \sqrt{\ell_2\, g\,\sin\theta_2\tan\theta_2}$$

$$\Sigma M_y = 0: \quad H_y = \text{constant}$$

$$r_1 m v_1 = r_2 m v_2 \qquad \text{or} \qquad v_1 \ell_1 \sin\theta_1 = v_2 \ell_2 \sin\theta_2$$

$$\ell_1^{3/2} g\,\sin\theta_1\sqrt{\sin\theta_1\tan\theta_1} = \ell_2^{3/2}\,\sin\theta_2\sqrt{\sin\theta_2\tan\theta_2}$$

$$\ell_1^3\sin^3\theta_1\tan\theta_1 = \ell_2^3\sin^3\theta_2\tan\theta_2 \blacktriangleleft$$

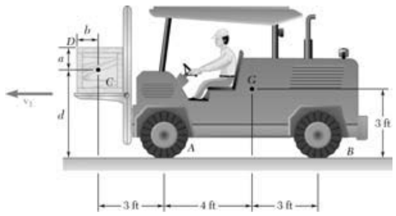
(*b*) With $\theta_1 = 35°$, $\ell_1 = 0.8$ m, and $\ell_2 = 0.6$ m

$$(0.8)^3\sin^3 35°\tan 35° = (0.6)^3\sin^3\theta_2\tan\theta_2$$

$$\sin^3\theta_2\tan\theta_2 - 0.31320 = 0$$

$$\theta_2 = 43.6° \blacktriangleleft$$

PROBLEM 12.CQ6



A uniform crate C with mass m_C is being transported to the left by a forklift with a constant speed v_1. What is the magnitude of the angular momentum of the crate about Point D, that is, the upper left corner of the crate?

(*a*) 0

(*b*) mv_1a

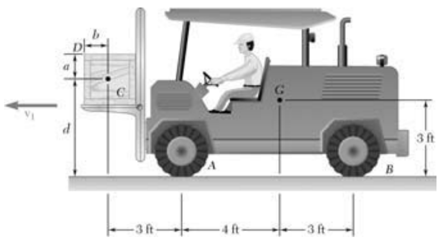
(*c*) mv_1b

(*d*) $mv_1\sqrt{a^2+b^2}$

SOLUTION

Answers: (*b*) The angular momentum is the moment of the momentum, so simply take the linear momentum, mv_1, and multiply it by the perpendicular distance from the line of action of mv_1 and Point D.

PROBLEM 12.CQ7



A uniform crate C with mass m_C is being transported to the left by a forklift with a constant speed v_1. What is the magnitude of the angular momentum of the crate about Point A, that is, the point of contact between the front tire of the forklift and the ground?

(*a*) 0

(*b*) mv_1d

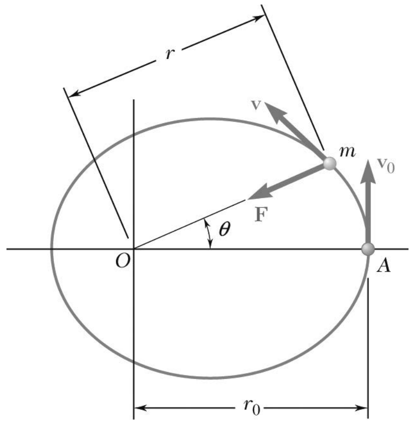
(*c*) $3mv_1$

(*d*) $mv_1\sqrt{3^2 + d^2}$

SOLUTION

Answer: (*b*)

PROBLEM 12.94



A particle of mass m is projected from Point A with an initial velocity $\mathbf{v}_0$ perpendicular to OA and moves under a central force $\mathbf{F}$ along an elliptic path defined by the equation $r = r_0/(2-\cos\theta)$. Using Eq. (12.37), show that $\mathbf{F}$ is inversely proportional to the square of the distance r from the particle to the center of force O.

SOLUTION

$$u = \frac{1}{r} = \frac{2-\cos\theta}{r_0}, \quad \frac{du}{d\theta} = \frac{\sin\theta}{r_0}, \quad \frac{d^2u}{d\theta^2} = \frac{\cos\theta}{r_0}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{2}{r_0} = \frac{F}{mh^2u^2} \quad \text{by Eq. (12.37).}$$

Solving for F,

$$F = \frac{2mh^2u^2}{r_0} = \frac{2mh^2}{r_0 r^2}$$

Since m, h, and r_0 are constants, F is proportional to $\frac{1}{r^2}$, or inversely proportional to r^2.

PROBLEM 12.95

A particle of mass m describes the logarithmic spiral $r = r_0 e^{b\theta}$ under a central force **F** directed toward the center of force O. Using Eq. (12.37) show that **F** is inversely proportional to the cube of the distance r from the particle to O.

SOLUTION

$$u = \frac{1}{r} = \frac{1}{r_0} e^{-b\theta}$$

$$\frac{du}{d\theta} = -\frac{b}{r_0} e^{-b\theta}$$

$$\frac{d^2u}{d\theta^2} = \frac{b^2}{r_0} e^{-b\theta}$$

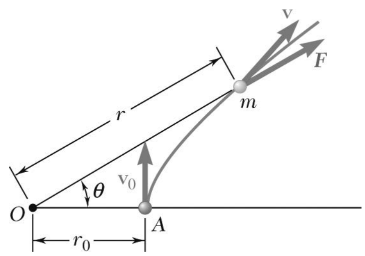
$$\frac{d^2u}{d\theta^2} + u = \frac{b^2 + 1}{r_0} e^{-b\theta} = \frac{F}{mh^2u^2}$$

$$F = \frac{(b^2 + 1)mh^2u^2}{r_0} e^{-b\theta}$$

$$= \frac{(b^2 + 1)mh^2u^2}{r} = \frac{(b^2 + 1)mh^2}{r^3}$$

Since b, m, and h are constants, **F** is proportional to $\frac{1}{r^3}$, or inversely proportional to r^3.

PROBLEM 12.96



For the particle of Problem 12.74, and using Eq. (12.37), show that the central force **F** is proportional to the distance r from the particle to the center of force O.

PROBLEM 12.74 A particle of mass m is projected from Point A with an initial velocity $\mathbf{v}_0$ perpendicular to line OA and moves under a central force **F** directed away from the center of force O. Knowing that the particle follows a path defined by the equation $r = r_0/\sqrt{\cos 2\theta}$ and using Eq. (12.27), express the radial and transverse components of the velocity **v** of the particle as functions of θ.

SOLUTION

$$u = \frac{1}{r} = \frac{\sqrt{\cos 2\theta}}{r_0}, \quad \frac{du}{d\theta} = -\frac{\sin 2\theta}{r_0\sqrt{\cos 2\theta}}$$

$$\frac{d^2u}{d\theta} = -\frac{\sqrt{\cos 2\theta}(2\cos 2\theta) - \sin 2\theta(-\sin 2\theta/\sqrt{\cos 2\theta})}{r_0 \cos 2\theta}$$

$$= -\frac{2\cos^2 2\theta + \sin^2 2\theta}{r_0(\cos 2\theta)^{3/2}} = -\frac{(1+\cos^2 2\theta)}{r_0(\cos 2\theta)^{3/2}}$$

Eq. (12.37): $\dfrac{d^2u}{d\theta^2} + u = \dfrac{F}{mh^2u^2}$

Solving for F,

$$F = mh^2u^2\left(\frac{d^2u}{d\theta} + u\right)$$

$$= mh^2\frac{\cos 2\theta}{r_0^2}\left[-\frac{1+\cos^2 2\theta}{r_0(\cos 2\theta)^{3/2}} + \frac{\sqrt{\cos 2\theta}}{r_0}\right]$$

$$= mh^2\frac{\cos 2\theta}{r_0^2}\left[-\frac{1}{r_0(\cos 2\theta)^{3/2}} - \frac{\sqrt{\cos 2\theta}}{r_0} + \frac{\sqrt{\cos 2\theta}}{r_0}\right]$$

$$= -\frac{mh^2}{r_0^3\sqrt{\cos 2\theta}} = -\frac{mh^2}{r_0^4}\frac{r_0}{\sqrt{\cos 2\theta}}$$

$$F = -\frac{mh^2 r}{r_0^4} \blacktriangleleft$$

The force F is proportional to r. The minus sign indicates that it is repulsive.

PROBLEM 12.97

A particle of mass m describes the path defined by the equation $r = r_0 \sin\theta$ under a central force **F** directed toward the center of force O. Using Eq. (12.37), show that **F** is inversely proportional to the fifth power of the distance r from the particle to O.

SOLUTION

We have $$\frac{d^2u}{d\theta^2} + u = \frac{F}{mh^2u^2} \qquad \text{Eq. (12.37)}$$

where $$u = \frac{1}{r} \quad \text{and} \quad mh^2 = \text{constant}$$

$$F \times u^2\left(\frac{d^2u}{d\theta^2} + u\right)$$

Now $$u = \frac{1}{r} = \frac{1}{r_0 \sin\theta}$$

Then $$\frac{du}{d\theta} = \frac{1}{d\theta}\left(\frac{1}{r_0 \sin\theta}\right) = -\frac{1}{r_0}\frac{\cos\theta}{\sin^2\theta}$$

and $$\frac{d^2u}{d\theta^2} = -\frac{1}{r_0}\left[\frac{-\sin\theta(\sin^2\theta) - \cos\theta(2\sin\theta\cos\theta)}{\sin^4\theta}\right]$$

$$= \frac{1}{r_0}\frac{1+\cos^2\theta}{\sin^3\theta}$$

Then $$F \times \left(\frac{1}{r^2}\right)\left(\frac{1}{r_0}\frac{1+\cos^2\theta}{\sin^3\theta} + \frac{1}{r_0\sin\theta}\right)$$

$$= mh^2\frac{1}{r_0}\frac{1}{r^2}\left(\frac{1+\cos^2\theta}{\sin^3\theta} + \frac{\sin^2\theta}{\sin^3\theta}\right)$$

$$= mh^2\frac{2}{r_0}\frac{1}{r^2}\frac{1}{\sin^3\theta} \qquad \sin^3\theta = \left(\frac{r}{r_0}\right)^3$$

$$= mh^2\frac{2r_0^2}{r^5}$$

F is proportional to $\frac{1}{r^5}$ $\quad F \times \frac{1}{r^5}$ Q.E.D. ◀

Note: $F > 0$ implies that **F** is attractive.

PROBLEM 12.98

It was observed that during its second flyby of the earth, the Galileo spacecraft had a velocity of 14.1 km/s as it reached its minimum altitude of 303 km above the surface of the earth. Determine the eccentricity of the trajectory of the spacecraft during this portion of its flight.

SOLUTION

For earth, $R = 6.37 \times 10^6$ m

$$r_0 = 6.37 \times 10^6 + 303. \times 10^3 = 6.673 \times 10^6 \text{ m}$$

$$h = r_0 v_0 = (6.673 \times 10^6)(14.1 \times 10^3) = 94.09 \times 10^9 \text{ m}^2/\text{s}$$

$$GM = gR^2 = (9.81)(6.37 \times 10^6)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

$$\frac{1}{r_0} = \frac{GM}{h^2}(1 + \varepsilon)$$

$$1 + \varepsilon = \frac{h^2}{r_0 GM} = \frac{(94.09 \times 10^9)^2}{(6.673 \times 10^6)(398.06 \times 10^{12})} = 3.33$$

$$\varepsilon = 3.33 - 1$$

$\varepsilon = 2.33$ ◀

PROBLEM 12.99

It was observed that during the Galileo spacecraft's first flyby of the earth, its maximum altitude was 600 mi above the surface of the earth. Assuming that the trajectory of the spacecraft was parabolic, determine the maximum velocity of Galileo during its first flyby of the earth.

SOLUTION

For the earth: $R = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$

$$GM = gR^2 = (32.2)(20.909 \times 10^6)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

For a parabolic trajectory, $\varepsilon = 1.$

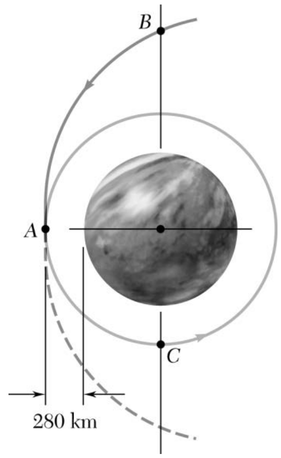
Eq. (12.39′): $\dfrac{1}{r} = \dfrac{GM}{h^2}(1 + \cos\theta)$

At $\theta = 0$, $\dfrac{1}{r_0} = \dfrac{2GM}{h^2} = \dfrac{2GM}{r_0^2 v_0^2}$ or $v_0 = \sqrt{\dfrac{2GM}{r_0}}$

At $r_0 = 3960 + 600 = 4560 \text{ mi} = 24.077 \times 10^6 \text{ ft},$

$$v_0 = \sqrt{\frac{(2)(14.077 \times 10^{15})}{24.077 \times 10^6}} = 34.196 \times 10^3 \text{ ft/s}$$

$v_0 = 6.48 \text{ mi/s}$ ◀



PROBLEM 12.100

As a space probe approaching the planet Venus on a parabolic trajectory reaches Point A closest to the planet, its velocity is decreased to insert it into a circular orbit. Knowing that the mass and the radius of Venus are 4.87×10^{24} kg and 6052 km, respectively, determine (*a*) the velocity of the probe as it approaches A, (*b*) the decrease in velocity required to insert it into the circular orbit.

SOLUTION

First note $$r_A = (6052 + 280)\text{ km} = 6332\text{ km}$$

(*a*) From the textbook, the velocity at the point of closest approach on a parabolic trajectory is given by

$$v_0 = \sqrt{\frac{2GM}{r_0}}$$

Thus, $$(v_A)_{\text{par}} = \left[\frac{2\times 66.73\times10^{-12}\text{ m}^3/\text{kg}\cdot\text{s}^2\times 4.87\times10^{24}\text{kg}}{6332\times10^3\text{ m}}\right]^{1/2}$$

$$= 10{,}131.4\text{ m/s}$$

or $$(v_A)_{\text{par}} = 10.13\text{ km/s} \blacktriangleleft$$

(*b*) We have $$(v_A)_{\text{circ}} = (v_A)_{\text{par}} + \Delta v_A$$

Now $$(v_A)_{\text{circ}} = \sqrt{\frac{GM}{r_0}} \qquad \text{Eq. (12.44)}$$

$$= \frac{1}{\sqrt{2}}(v_A)_{\text{par}}$$

Then $$\Delta v_A = \frac{1}{\sqrt{2}}(v_A)_{\text{par}} - (v_A)_{\text{par}}$$

$$= \left(\frac{1}{\sqrt{2}} - 1\right)(10.1314\text{ km/s})$$

$$= -2.97\text{ km/s}$$

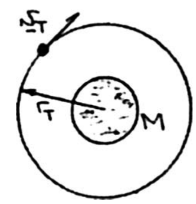
$$|\Delta v_A| = 2.97\text{ km/s} \blacktriangleleft$$

PROBLEM 12.101

It was observed that as the Voyager I spacecraft reached the point of its trajectory closest to the planet Saturn, it was at a distance of 185×10^3 km from the center of the planet and had a velocity of 21.0 km/s. Knowing that Tethys, one of Saturn's moons, describes a circular orbit of radius 295×10^3 km at a speed of 11.35 km/s, determine the eccentricity of the trajectory of Voyager I on its approach to Saturn.

SOLUTION

For a circular orbit,



Eq. (12.44) $$v = \sqrt{\frac{GM}{r}}$$

For the orbit of Tethys,

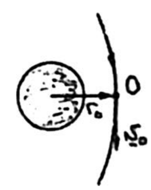
$$GM = r_T v_T^2$$

For Voyager's trajectory, we have

$$\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta)$$

where $h = r_0 v_0$

At O, $$r = r_0,\ \theta = 0$$

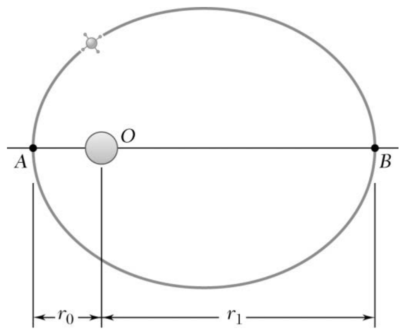


Then $$\frac{1}{r_0} = \frac{GM}{(r_0 v_0)^2}(1+\varepsilon)$$

or $$\varepsilon = \frac{r_0 v_0^2}{GM} - 1 = \frac{r_0 v_0^2}{r_T v_T^2} - 1$$

$$= \frac{185\times10^3\text{ km}}{295\times10^3\text{ km}} \times \left(\frac{21.0\text{ km/s}}{11.35\text{ km/s}}\right)^2 - 1$$

or $\varepsilon = 1.147$ ◀



PROBLEM 12.102

A satellite describes an elliptic orbit about a planet of mass M. Denoting by r_0 and r_1, respectively, the minimum and maximum values of the distance r from the satellite to the center of the planet, derive the relation

$$\frac{1}{r_0}+\frac{1}{r_1}=\frac{2GM}{h^2}$$

where h is the angular momentum per unit mass of the satellite.

SOLUTION

Using Eq. (12.39), $$\frac{1}{r_A}=\frac{GM}{h^2}+C\cos\theta_A$$

and $$\frac{1}{r_B}=\frac{GM}{h^2}+C\cos\theta_B.$$

But $$\theta_B=\theta_A+180°,$$

so that $$\cos\theta_A=-\cos\theta_B.$$

Adding, $$\frac{1}{r_A}+\frac{1}{r_B}=\frac{1}{r_0}+\frac{1}{r_1}=\frac{2GM}{h^2}$$

PROBLEM 12.103

A space probe is describing a circular orbit about a planet of radius R. The altitude of the probe above the surface of the planet is αR and its speed is v_0. To place the probe in an elliptic orbit which will bring it closer to the planet, its speed is reduced from v_0 to βv_0, where $\beta < 1$, by firing its engine for a short interval of time. Determine the smallest permissible value of β if the probe is not to crash on the surface of the planet.

SOLUTION

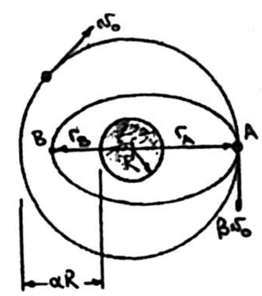
For the circular orbit, $$v_0 = \sqrt{\frac{GM}{r_A}} \qquad \text{Eq. (12.44)},$$

where $$r_A = R + \alpha R = R(1+\alpha)$$

Then $$GM = v_0^2 R(1+\alpha)$$

From the solution to Problem 12.102, we have for the elliptic orbit,

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM}{h^2}$$



Now $$h = h_A = r_A (v_A)_{AB}$$
$$= [R(1+\alpha)](\beta v_0)$$

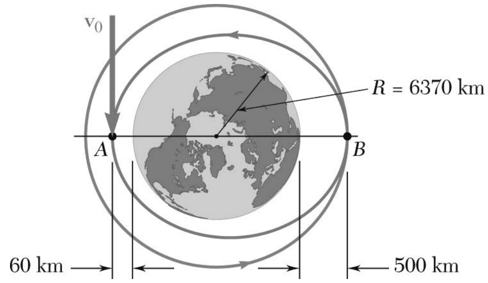
Then $$\frac{1}{R(1+\alpha)} + \frac{1}{r_B} = \frac{2v_0^2 R(1+\alpha)}{[R(1+\alpha)\beta v_0]^2}$$
$$= \frac{2}{\beta^2 R(1+\alpha)}$$

Now $\beta_{\min}$ corresponds to $r_B \to R$.

Then $$\frac{1}{R(1+\alpha)} + \frac{1}{R} = \frac{2}{\beta_{\min}^2 R(1+\alpha)}$$

or $$\beta_{\min} = \sqrt{\frac{2}{2+\alpha}} \blacktriangleleft$$

PROBLEM 12.104



At main engine cutoff of its thirteenth flight, the space shuttle Discovery was in an elliptic orbit of minimum altitude 60 km and maximum altitude 500 km above the surface of the earth. Knowing that at Point A the shuttle had a velocity $\mathbf{v}_0$ parallel to the surface of the earth and that the shuttle was transferred to a circular orbit as it passed through Point B, determine (a) the speed v_0 of the shuttle at A, (b) the increase in speed required at B to insert the shuttle into the circular orbit.

SOLUTION

For earth, $R = 6370 \text{ km} = 6370\times10^3 \text{ m}$

$$GM = gR^2 = (9.81)(6370\times10^3)^2 = 3.9806\times10^{14} \text{ m}^3/\text{s}^2$$
$$r_A = 6370 + 60 = 6430 \text{ km} = 6430\times10^3 \text{ m}$$
$$r_B = 6370 + 500 = 6870 \text{ km} = 6870\times10^3 \text{ m}$$

Elliptic trajectory.

Using Eq. (12.39), $\quad \dfrac{1}{r_A} = \dfrac{GM}{h^2} + C\cos\theta_A \quad$ and $\quad \dfrac{1}{r_B} = \dfrac{GM}{h^2} + C\cos\theta_B.$

But $\quad \theta_B = \theta_A + 180°, \quad$ so that $\quad \cos\theta_A = -\cos\theta_B$

Adding, $\quad \dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_A + r_B}{r_A r_B} = \dfrac{2GM}{h^2}$

$$h = \sqrt{\frac{2GMr_Ar_B}{r_A + r_B}} = \sqrt{\frac{(2)(3.9806\times10^{14})(6430\times10^3)(6870\times10^3)}{6430\times10^3 + 6870\times10^3}} = 51.422\times10^9 \text{ m}^2/\text{s}$$

(a) Speed v_0 at A.

$$v_0 = v_A = \frac{h}{r_A} = \frac{51.422\times10^9}{6430\times10^3}$$

$v_0 = 8.00\times10^3$ m/s ◀

$$(v_B)_1 = \frac{h}{r_A} = \frac{51.422\times10^9}{6870\times10^3} = 7.48497\times10^3 \text{ m/s}$$

For a circular orbit through Point B,

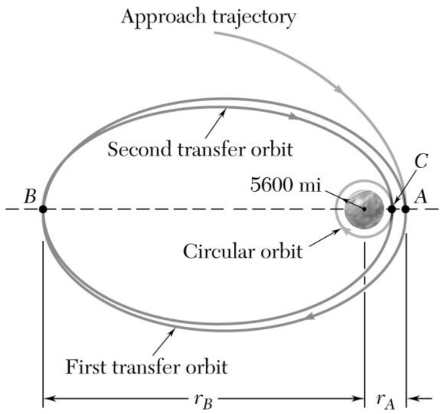
$$(v_B)_{\text{circ}} = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{3.9806\times10^{14}}{6870\times10^3}} = 7.6119\times10^3 \text{ m/s}$$

(b) Increase in speed at Point B.

$$\Delta v_B = (v_B)_{\text{circ}} - (v_B)_1 = 126.97 \text{ m/s}$$

$\Delta v_B = 127$ m/s ◀

PROBLEM 12.105



A space probe is to be placed in a circular orbit of 5600 mi radius about the planet Venus in a specified plane. As the probe reaches *A*, the point of its original trajectory closest to Venus, it is inserted in a first elliptic transfer orbit by reducing its speed by Δv_A. This orbit brings it to Point *B* with a much reduced velocity. There the probe is inserted in a second transfer orbit located in the specified plane by changing the direction of its velocity and further reducing its speed by Δv_B. Finally, as the probe reaches Point *C*, it is inserted in the desired circular orbit by reducing its speed by Δv_C. Knowing that the mass of Venus is 0.82 times the mass of the earth, that $r_A = 9.3 \times 10^3$ mi and $r_B = 190 \times 10^3$ mi, and that the probe approaches *A* on a parabolic trajectory, determine by how much the velocity of the probe should be reduced (*a*) at *A*, (*b*) at *B*, (*c*) at *C*.

SOLUTION

For Earth, $R = 3690 \text{ mi} = 20.9088 \times 10^6 \text{ ft}, \quad g = 32.2 \text{ ft/s}^2$

$$GM_{\text{earth}} = gR^2 = (32.2)(20.9088 \times 10^6)^2 = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

For Venus, $GM = 0.82 GM_{\text{earth}} = 11.543 \times 10^{15} \text{ ft}^3/\text{s}^2$

For a parabolic trajectory with $r_A = 9.3 \times 10^3 \text{ mi} = 49.104 \times 10^6 \text{ ft}$

$$(v_A)_1 = v_{\text{esc}} = \sqrt{\frac{2GM}{r_A}} = \sqrt{\frac{(2)(11.543 \times 10^{15})}{49.104 \times 10^6}} = 21.683 \times 10^3 \text{ ft/s}$$

First transfer orbit *AB*. $r_B = 190 \times 10^3 \text{ mi} = 1003.2 \times 10^6 \text{ ft}$

At Point *A*, where $\theta = 180°$

$$\frac{1}{r_A} = \frac{GM}{h_{AB}^2} + C \cos 180° = \frac{GM}{h_{AB}^2} - C \qquad (1)$$

At Point *B*, where $\theta = 0°$

$$\frac{1}{r_B} = \frac{GM}{h_{AB}^2} + C \cos 0 = \frac{GM}{h_{AB}^2} + C \qquad (2)$$

Adding, $$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_B + r_A}{r_A r_B} = \frac{2GM}{h_{AB}^2}$$

PROBLEM 12.105 (Continued)

Solving for h_{AB},

$$h_{AB} = \sqrt{\frac{2GMr_A r_B}{r_B + r_A}} = \sqrt{\frac{(2)(11.543\times 10^{15})(49.104\times 10^6)(1003.2\times 10^6)}{1052.3\times 10^6}} = 1.039575\times 10^{12}\ \text{ft}^2/\text{s}$$

$$(v_A)_2 = \frac{h_{AB}}{r_A} = \frac{1.039575\times 10^{12}}{49.104\times 10^6} = 21.174\times 10^3\ \text{ft/s}$$

$$(v_B)_1 = \frac{h_{AB}}{r_B} = \frac{1.039575\times 10^{12}}{1003.2\times 10^6} = 1.03626\times 10^3\ \text{ft/s}$$

<u>Second transfer orbit *BC*</u>. $r_C = 5600\ \text{mi} = 29.568\times 10^6\ \text{ft}$

At Point *B*, where $\theta = 0$

$$\frac{1}{r_B} = \frac{GM}{h_{BC}^2} + C\cos 0 = \frac{GM}{h_{BC}^2} + C$$

At Point *C*, where $\theta = 180°$

$$\frac{1}{r_C} = \frac{GM}{h_{BC}^2} + C\cos 180° = \frac{GM}{h_{BC}^2} - C$$

Adding, $$\frac{1}{r_B} + \frac{1}{r_C} = \frac{r_B + r_C}{r_B r_C} = \frac{2GM}{h_{BC}^2}$$

$$h_{BC} = \sqrt{\frac{2GMr_B r_C}{r_B + r_C}} = \sqrt{\frac{(2)(11.543\times 10^{15})(1003.2\times 10^6)(29.568\times 10^6)}{1032.768\times 10^6}} = 814.278\times 10^9\ \text{ft}^2/\text{s}$$

$$(v_B)_2 = \frac{h_{BC}}{r_B} = \frac{814.278\times 10^9}{1003.2\times 10^6} = 811.69\ \text{ft/s}$$

$$(v_C)_1 = \frac{h_{BC}}{r_C} = \frac{814.278\times 10^9}{29.568\times 10^6} = 27.539\times 10^3\ \text{ft/s}$$

<u>Final circular orbit</u>. $r_C = 29.568\times 10^6\ \text{ft}$

$$(v_C)_2 = \sqrt{\frac{GM}{r_C}} = \sqrt{\frac{11.543\times 10^{15}}{29.568\times 10^6}} = 19.758\times 10^3\ \text{ft/s}$$

<u>Speed reductions</u>.

(*a*) At *A*: $(v_A)_1 - (v_A)_2 = 21.683\times 10^3 - 21.174\times 10^3$ $\Delta v_A = 509\ \text{ft/s}$ ◀

(*b*) At *B*: $(v_B)_1 - (v_B)_2 = 1.036\times 10^3 - 811.69$ $\Delta v_B = 224\ \text{ft/s}$ ◀

(*c*) At *C*: $(v_C)_1 - (v_C)_2 = 27.539\times 10^3 - 19.758\times 10^3$ $\Delta v_C = 7.78\times 10^3\ \text{ft/s}$ ◀

PROBLEM 12.106

For the space probe of Problem 12.105, it is known that $r_A = 9.3\times 10^3$ mi and that the velocity of the probe is reduced to $20,000$ ft/s as it passes through A. Determine (a) the distance from the center of Venus to Point B, (b) the amounts by which the velocity of the probe should be reduced at B and C, respectively.

SOLUTION

Data from Problem 12.105: $r_C = 29.568\times 10^6$ ft, $M = 0.82\, M_{\text{earth}}$

For Earth, $R = 3960 \text{ mi} = 20.9088\times 10^6 \text{ ft}, \quad g = 32.2 \text{ ft/s}^2$

$$GM_{\text{earth}} = gR^2 = (32.2)(20.9088\times 10^6)^2 = 14.077\times 10^{15} \text{ m}^3/\text{s}^2$$

For Venus, $GM = 0.82GM_{\text{earth}} = 11.543\times 10^{15} \text{ ft}^3/\text{s}^2$

Transfer orbit AB: $v_A = 20,000 \text{ ft/s}, \quad r_A = 9.3\times 10^3 \text{ mi} = 49.104\times 10^6 \text{ ft}$

$$h_{AB} = r_A v_A = (49.104\times 10^6)(20,000) = 982.08\times 10^9 \text{ ft}^2/\text{s}$$

At Point A, where $\theta = 180°$

$$\frac{1}{r_A} = \frac{GM}{h_{AB}^2} + C\cos 180° = \frac{GM}{h_{AB}^2} - C$$

At Point B, where $\theta = 0°$

$$\frac{1}{r_B} = \frac{GM}{h_{AB}^2} + C\cos 0 = \frac{GM}{h_{AB}^2} + C$$

Adding, $\dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{2GM}{h_{AB}^2}$

$$\frac{1}{r_B} = \frac{2GM}{h_{AB}^2} - \frac{1}{r_A}$$

$$= \frac{(2)(11.543\times 10^{15})}{(982.08\times 10^9)^2} - \frac{1}{49.104\times 10^6}$$

$$= 3.57125\times 10^{-9} \text{ ft}^{-1}$$

(a) Radial coordinate r_B. $r_B = 280.01\times 10^6$ ft $\qquad r_B = 53.0\times 10^3$ mi ◀

$$(v_B)_1 = \frac{h_{AB}}{r_B} = \frac{982.08\times 10^9}{280.01\times 10^6} = 3.5073\times 10^3 \text{ ft/s}$$

Second transfer orbit BC. $r_C = 5600 \text{ mi} = 29.568\times 10^6 \text{ ft}$

At Point B, where $\theta = 0$

$$\frac{1}{r_B} = \frac{GM}{h_{BC}^2} + C\cos 0 = \frac{GM}{h_{BC}^2} + C$$

At Point C, where $\theta = 180°$

$$\frac{1}{r_C} = \frac{GM}{h_{BC}^2} + C\cos 180° = \frac{GM}{h_{BC}^2} - C$$

Adding,
$$\frac{1}{r_B} + \frac{1}{r_C} = \frac{r_B + r_C}{r_B r_C} = \frac{2GM}{h_{BC}^2}$$

$$h_{BC} = \sqrt{\frac{2GM r_B r_C}{r_B + r_C}} = \sqrt{\frac{(2)(11.543\times 10^{15})(280.01\times 10^6)(29.568\times 10^6)}{309.578\times 10^6}}$$

$$= 785.755\times 10^9 \text{ ft}^2/\text{s}$$

$$(v_B)_2 = \frac{h_{BC}}{r_B} = \frac{785.755\times 10^9}{280.01\times 10^6} = 2.8062\times 10^3 \text{ ft/s}$$

$$(v_C)_1 = \frac{h_{BC}}{r_C} = \frac{785.755\times 10^9}{29.568\times 10^6} = 26.575\times 10^3 \text{ ft/s}$$

Circular orbit with $r_C = 29.568\times 10^6 \text{ ft}$

$$(v_C)_2 = \sqrt{\frac{GM}{r_C}} = \sqrt{\frac{11.543\times 10^{15}}{29.568\times 10^6}} = 19.758\times 10^3 \text{ ft/s}$$

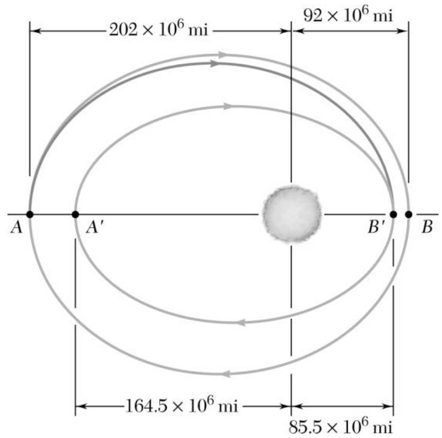
(*b*) Speed reductions at B and C.

At B: $(v_B)_1 - (v_B)_2 = 3.5073\times 10^3 - 2.8062\times 10^3$

$\Delta v_B = 701 \text{ ft/s}$ ◀

At C: $(v_C)_1 - (v_C)_2 = 26.575\times 10^3 - 19.758\times 10^3$

$\Delta v_C = 6.82\times 10^3 \text{ ft/s}$ ◀



PROBLEM 12.107

As it describes an elliptic orbit about the sun, a spacecraft reaches a maximum distance of 202×10^6 mi from the center of the sun at Point A (called the aphelion) and a minimum distance of 92×10^6 mi at Point B (called the perihelion). To place the spacecraft in a smaller elliptic orbit with aphelion at A' and perihelion at B', where A' and B' are located 164.5×10^6 mi and 85.5×10^6 mi, respectively, from the center of the sun, the speed of the spacecraft is first reduced as it passes through A and then is further reduced as it passes through B'. Knowing that the mass of the sun is 332.8×10^3 times the mass of the earth, determine (*a*) the speed of the spacecraft at A, (*b*) the amounts by which the speed of the spacecraft should be reduced at A and B' to insert it into the desired elliptic orbit.

SOLUTION

First note

$$R_{\text{earth}} = 3960 \text{ mi} = 20.9088\times10^6 \text{ ft}$$

$$r_A = 202\times10^6 \text{ mi} = 1066.56\times10^9 \text{ ft}$$

$$r_B = 92\times10^6 \text{ mi} = 485.76\times10^9 \text{ ft}$$

From the solution to Problem 12.102, we have for any elliptic orbit about the sun

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2GM_{\text{sun}}}{h^2}$$

(*a*) For the elliptic orbit AB, we have

$$r_1 = r_A, \quad r_2 = r_B, \quad h = h_A = r_A v_A$$

Also,

$$GM_{\text{sun}} = G[(332.8\times10^3)M_{\text{earth}}]$$

$$= gR_{\text{earth}}^2(332.8\times10^3) \qquad \text{using Eq. (12.30).}$$

Then

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2gR_{\text{earth}}^2(332.8\times10^3)}{(r_A v_A)^2}$$

or

$$v_A = \frac{R_{\text{earth}}}{r_A}\left(\frac{665.6g\times10^3}{\frac{1}{r_A}+\frac{1}{r_B}}\right)^{1/2}$$

$$= \frac{3960 \text{ mi}}{202\times10^6 \text{ mi}}\left(\frac{665.6\times10^3\times32.2 \text{ ft/s}^2}{\frac{1}{1066.56\times10^9 \text{ ft}}+\frac{1}{485.76\times10^9 \text{ ft}}}\right)^{1/2}$$

$$= 52,431 \text{ ft/s}$$

or $$v_A = 52.4\times10^3 \text{ ft/s} \blacktriangleleft$$

(*b*) From Part (*a*), we have

$$2GM_{\text{sun}} = (r_A v_A)^2 \left(\frac{1}{r_A} + \frac{1}{r_B} \right)$$

Then, for any other elliptic orbit about the sun, we have

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{(r_A v_A)^2 \left(\frac{1}{r_A} + \frac{1}{r_B} \right)}{h^2}$$

For the elliptic transfer orbit AB', we have

$$r_1 = r_A, \quad r_2 = r_{B'}, \quad h = h_{\text{tr}} = r_A (v_A)_{\text{tr}}$$

Then
$$\frac{1}{r_A} + \frac{1}{r_{B'}} = \frac{(r_A v_A)^2 \left(\frac{1}{r_A} + \frac{1}{r_B} \right)}{[r_A (v_A)_{\text{tr}}]^2}$$

or
$$(v_A)_{\text{tr}} = v_A \left(\frac{\frac{1}{r_A} + \frac{1}{r_B}}{\frac{1}{r_A} + \frac{1}{r_{B'}}} \right)^{1/2} = v_A \left(\frac{1 + \frac{r_A}{r_B}}{1 + \frac{r_A}{r_{B'}}} \right)^{1/2}$$

$$= (52,431 \text{ ft/s}) \left(\frac{1 + \frac{202}{92}}{1 + \frac{202}{85.5}} \right)^{1/2}$$

$$= 51,113 \text{ ft/s}$$

Now
$$h_{\text{tr}} = (h_A)_{\text{tr}} = (h_{B'})_{\text{tr}}: \quad r_A (v_A)_{\text{tr}} = r_{B'} (v_{B'})_{\text{tr}}$$

Then
$$(v_{B'})_{\text{tr}} = \frac{202 \times 10^6 \text{ mi}}{85.5 \times 10^6 \text{ mi}} \times 51,113 \text{ ft/s} = 120,758 \text{ ft/s}$$

For the elliptic orbit $A'B'$, we have

$$r_1 = r_{A'}, \quad r_2 = r_{B'}, \quad h = r_{B'} v_{B'}$$

Then
$$\frac{1}{r_{A'}} + \frac{1}{r_{B'}} = \frac{(r_A v_A)^2 \left(\frac{1}{r_A} + \frac{1}{r_B} \right)}{(r_{B'} v_{B'})^2}$$

or
$$v_{B'} = v_A \frac{r_A}{r_{B'}} \left(\frac{\frac{1}{r_A} + \frac{1}{r_B}}{\frac{1}{r_{A'}} + \frac{1}{r_{B'}}} \right)^{1/2}$$

$$= (52,431 \text{ ft/s}) \frac{202 \times 10^6 \text{ mi}}{85.5 \times 10^6 \text{ mi}} \left(\frac{\frac{1}{202 \times 10^6} + \frac{1}{92 \times 10^6}}{\frac{1}{164.5 \times 10^6} + \frac{1}{85.5 \times 10^6}} \right)^{1/2}$$

$$= 116,862 \text{ ft/s}$$

PROBLEM 12.107 (Continued)

Finally, $(v_A)_{\text{tr}} = v_A + \Delta v_A$

or $\Delta v_A = (51{,}113 - 52{,}431)$ ft/s

or $|\Delta v_A| = 1318$ ft/s ◀

and $v_{B'} = (v_{B'})_{\text{tr}} + \Delta v_B$

or $\Delta v_{B'} = (116{,}862 - 120{,}758)$ ft/s

$= -3896$ ft/s

or $|\Delta v_B| = 3900$ ft/s ◀

PROBLEM 12.108

Halley's comet travels in an elongated elliptic orbit for which the minimum distance from the sun is approximately $\frac{1}{2}r_E$, where $r_E = 150\times10^6$ km is the mean distance from the sun to the earth. Knowing that the periodic time of Halley's comet is about 76 years, determine the maximum distance from the sun reached by the comet.

SOLUTION

We apply Kepler's Third Law to the orbits and periodic times of earth and Halley's comet:

$$\left(\frac{\tau_H}{\tau_E}\right)^2 = \left(\frac{a_H}{a_E}\right)^3$$

Thus

$$a_H = a_E\left(\frac{\tau_H}{\tau_E}\right)^{2/3}$$

$$= r_E\left(\frac{76 \text{ years}}{1 \text{ year}}\right)^{2/3}$$

$$= 17.94 r_E$$

But

$$a_H = \frac{1}{2}(r_{\min} + r_{\max})$$

$$17.94 r_E = \frac{1}{2}\left(\frac{1}{2}r_E + r_{\max}\right)$$

$$r_{\max} = 2(17.94 r_E) - \frac{1}{2}r_E$$

$$= (35.88 - 0.5) r_E$$

$$= 35.38\, r_E$$

$$r_{\max} = (35.38)(150\times10^6 \text{ km})$$

$$r_{\max} = 5.31\times10^9 \text{ km} \blacktriangleleft$$

PROBLEM 12.109

Based on observations made during the 1996 sighting of comet Hyakutake, it was concluded that the trajectory of the comet is a highly elongated ellipse for which the eccentricity is approximately $\varepsilon = 0.999887$. Knowing that for the 1996 sighting the minimum distance between the comet and the sun was $0.230R_E$, where R_E is the mean distance from the sun to the earth, determine the periodic time of the comet.

SOLUTION

For Earth's orbit about the sun,

$$v_0 = \sqrt{\frac{GM}{R_E}}, \quad \tau_0 = \frac{2\pi R_E}{v_0} = \frac{2\pi R_E^{3/2}}{\sqrt{GM}} \quad \text{or} \quad \sqrt{GM} = \frac{2\pi R_E^{3/2}}{\tau_0} \tag{1}$$

For the comet Hyakutake,

$$\frac{1}{r_0} = \frac{GM}{h^2} = (1+\varepsilon), \quad \frac{1}{r_1} = \frac{GM}{h^2}(1+\varepsilon), \quad r_1 = \frac{1+\varepsilon}{1-\varepsilon} r_0$$

$$a = \frac{1}{2}(r_0 + r_1) = \frac{r_0}{1-\varepsilon}, \quad b = \sqrt{r_0 r_1} = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}}\, r_0$$

$$h = \sqrt{GMr_0(1+\varepsilon)}$$

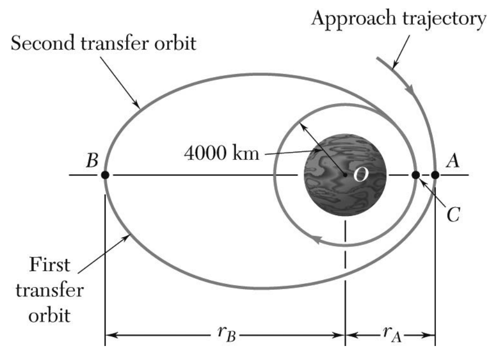
$$\tau = \frac{2\pi ab}{h} = \frac{2\pi r_0^2 (1+\varepsilon)^{1/2}}{(1-\varepsilon)^{3/2}\sqrt{GMr_0(1+\varepsilon)}}$$

$$= \frac{2\pi r_0^{3/2}}{\sqrt{GM}\,(1-\varepsilon)^{3/2}} = \frac{2\pi r_0^{3/2}\tau_0}{2\pi R_E^3 (1-\varepsilon)^{3/2}}$$

$$= \left(\frac{r_0}{R_E}\right)^{3/2} \frac{1}{(1-\varepsilon)^{3/2}}\ \tau_0$$

$$= (0.230)^{3/2} \frac{1}{(1-0.999887)^{3/2}}\ \tau_0 = 91.8\times 10^3 \tau_0$$

Since $\tau_0 = 1 \text{ yr}, \quad \tau = (91.8\times 10^3)(1.000)$ $\qquad \tau = 91.8\times 10^3 \text{ yr}$ ◀



PROBLEM 12.110

A space probe is to be placed in a circular orbit of radius 4000 km about the planet Mars. As the probe reaches A, the point of its original trajectory closest to Mars, it is inserted into a first elliptic transfer orbit by reducing its speed. This orbit brings it to Point B with a much reduced velocity. There the probe is inserted into a second transfer orbit by further reducing its speed. Knowing that the mass of Mars is 0.1074 times the mass of the earth, that $r_A = 9000$ km and $r_B = 180{,}000$ km, and that the probe approaches A on a parabolic trajectory, determine the time needed for the space probe to travel from A to B on its first transfer orbit.

SOLUTION

For earth, $R = 6373 \text{ km} = 6.373 \times 10^6 \text{ m}$

$$GM = gR^2 = (9.81)(6.373 \times 10^6)^2 = 398.43 \times 10^{12} \text{ m}^3/\text{s}^2$$

For Mars, $GM = (0.1074)(398.43 \times 10^{12}) = 42.792 \times 10^{12} \text{ m}^3/\text{s}^2$

$$r_A = 9000 \text{ km} = 9.0 \times 10^6 \text{ m} \qquad r_B = 180000 \text{ km} = 180 \times 10^6 \text{ m}$$

For the parabolic approach trajectory at A,

$$(v_A)_1 = \sqrt{\frac{2GM}{r_A}} = \sqrt{\frac{(2)(42.792 \times 10^{12})}{9.0 \times 10^6}} = 3.0837 \times 10^3 \text{ m/s}$$

First elliptic transfer orbit AB.

Using Eq. (12.39), $\dfrac{1}{r_A} = \dfrac{GM}{h_{AB}^2} + C\cos\theta_A$ and $\dfrac{1}{r_B} = \dfrac{GM}{h_{AB}^2} + C\cos\theta_B$.

But $\theta_B = \theta_A + 180°$, so that $\cos\theta_A = -\cos\theta_B$.

Adding, $\dfrac{1}{r_A} + \dfrac{1}{r_B} = \dfrac{r_A + r_B}{r_A r_B} = \dfrac{2GM}{h_{AB}^2}$

$$h_{AB} = \sqrt{\frac{2GMr_Ar_B}{r_A + r_B}} = \sqrt{\frac{(2)(42.792 \times 10^{12})(9.0 \times 10^6)(180 \times 10^6)}{189.0 \times 10^6}}$$

$$h_{AB} = 27.085 \times 10^9 \text{ m}^2/\text{s}$$

$$a = \frac{1}{2}(r_A + r_B) = \frac{1}{2}(9.0 \times 10^6 + 180 \times 10^6) = 94.5 \times 10^6 \text{ m}$$

$$b = \sqrt{r_A r_B} = \sqrt{(9.0 \times 10^6)(180 \times 10^6)} = 40.249 \times 10^6 \text{ m}$$

Periodic time for full ellipse: $\tau = \dfrac{2\pi ab}{h}$

For half ellipse AB, $\tau_{AB} = \dfrac{1}{2}\tau = \dfrac{\pi ab}{h}$

$$\tau_{AB} = \frac{\pi(94.5\times10^6)(40.249\times10^6)}{27.085\times10^9} = 444.81\times10^3 \text{ s}$$

$\tau_{AB} = 122.6$ h ◀

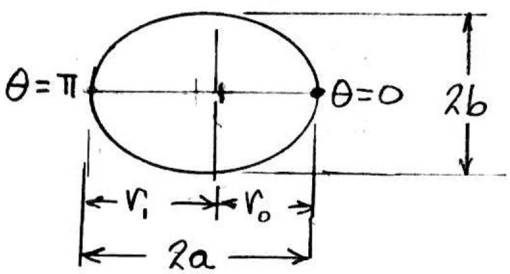
PROBLEM 12.111

A space shuttle is in an elliptic orbit of eccentricity 0.0356 and a minimum altitude of 300 km above the surface of the earth. Knowing that the radius of the earth is 6370 km, determine the periodic time for the orbit.

SOLUTION

For earth, $g = 9.81 \text{ m/s}^2, \quad R = 6370 \text{ km} = 6.370 \times 10^6 \text{ m}$

$$GM = gR^2 = (9.81)(6.370 \times 10^6)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$



For the orbit, $r_0 = 6370 + 300 = 6670 \text{ km} = 6.670 \times 10^6 \text{ m}$

$$\frac{1}{r_0} = \frac{GM}{h^2}(1+\varepsilon) \qquad \frac{1}{r_1} = \frac{GM}{h^2}(1-\varepsilon)$$

$$r_1 = r_0 \frac{1+\varepsilon}{1-\varepsilon} - (6.670 \times 10^6)\frac{1.0356}{0.9644} = 7.1624 \times 10^6 \text{ m}$$

$$a = \frac{1}{2}(r_0 + r_1) = 6.9162 \times 10^6 \text{ m}$$

$$b = \sqrt{r_0 r_1} = 6.9118 \times 10^6 \text{ m}$$

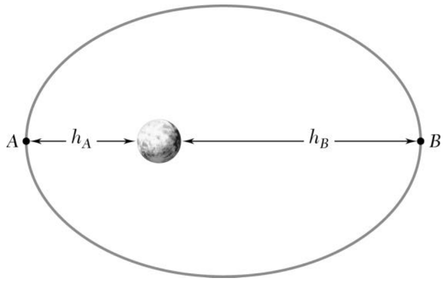
$$h = \sqrt{(1+\varepsilon)GMr_0} = \sqrt{(1.0356)(398.06 \times 10^{12})(6.670 \times 10^6)}$$

$$= 52.436400 \times 10^9 \text{ m}^2/\text{s}$$

$$\tau = \frac{2\pi ab}{h} = \frac{2\pi(6.9118 \times 10^6)(6.9162 \times 10^6)}{52.436400 \times 10^9 \text{ m}^2/\text{s}}$$

$$= 5.7281 \times 10^3 \text{ s}$$

$\tau = 95.5 \text{ min}$ ◀



PROBLEM 12.112

The Clementine spacecraft described an elliptic orbit of minimum altitude $h_A = 400$ km and a maximum altitude of $h_B = 2940$ km above the surface of the moon. Knowing that the radius of the moon is 1737 km and that the mass of the moon is 0.01230 times the mass of the earth, determine the periodic time of the spacecraft.

SOLUTION

For earth, $R = 6370 \text{ km} = 6.370 \times 10^6 \text{ m}$

$$GM = gR^2 = (9.81)(6.370 \times 10^6)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

For moon, $GM = (0.01230)(398.06 \times 10^{12}) = 4.896 \times 10^{12} \text{ m}^3/\text{s}^2$

$$r_A = 1737 + 400 = 2137 \text{ km} = 2.137 \times 10^6 \text{ m}$$

$$r_B = 1737 + 2940 = 4677 \text{ km} = 4.677 \times 10^6 \text{ m}$$

Using Eq. (12.39), $\frac{1}{r_A} = \frac{GM}{h^2} + C\cos\theta_A \quad \text{and} \quad \frac{1}{r_B} = \frac{GM}{h^2} + C\cos\theta_B.$

But $\theta_B = \theta_A + 180°, \quad \text{so that} \quad \cos\theta_A = -\cos\theta_B.$

Adding, $\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A + r_B}{r_A r_B} = \frac{2GM}{h_{AB}^2}$

$$h_{AB} = \sqrt{\frac{2GMr_A r_B}{r_A + r_B}} = \sqrt{\frac{(2)(4.896 \times 10^{12})(2.137 \times 10^6)(4.677 \times 10^6)}{6.814 \times 10^6}}$$

$$= 3.78983 \times 10^9 \text{ m}^2/\text{s}$$

$$a = \frac{1}{2}(r_A + r_B) = 3.402 \times 10^6 \text{ m}$$

$$b = \sqrt{r_A r_B} = 3.16145 \times 10^6 \text{ m}$$

<u>Periodic time</u>.

$$\tau = \frac{2\pi ab}{h_{AB}} = \frac{2\pi(3.402 \times 10^6)(3.16145 \times 10^6)}{3.78983 \times 10^9} = 17.831 \times 10^3 \text{ s}$$

$\tau = 4.95$ h ◀

PROBLEM 12.113

Determine the time needed for the space probe of Problem 12.100 to travel from B to C.

SOLUTION

From the solution to Problem 12.100, we have

$$(v_A)_{\text{par}} = 10{,}131.4 \text{ m/s}$$

and
$$(v_A)_{\text{circ}} = \frac{1}{\sqrt{2}}(v_A)_{\text{par}} = 7164.0 \text{ m/s}$$

Also,
$$r_A = (6052 + 280) \text{ km} = 6332 \text{ km}$$

For the parabolic trajectory BA, we have

$$\frac{1}{r} = \frac{GM_v}{h_{BA}^2}(1 + \varepsilon \cos\theta) \qquad \text{[Eq. (12.39')]}$$

where $\varepsilon = 1$. Now

at A, $\theta = 0$:
$$\frac{1}{r_A} = \frac{GM_v}{h_{BA}^2}(1+1)$$

or
$$r_A = \frac{h_{BA}^2}{2GM_v}$$

at B, $\theta = -90°$:
$$\frac{1}{r_B} = \frac{GM_v}{h_{BA}^2}(1+0)$$

or
$$r_B = \frac{h_{BA}^2}{GM_v}$$

$$r_B = 2r_A$$

As the probe travels from B to A, the area swept out is the semiparabolic area defined by Vertex A and Point B. Thus,

$$(\text{Area swept out})_{BA} = A_{BA} = \frac{2}{3} r_A r_B = \frac{4}{3} r_A^2$$

Now
$$\frac{dA}{dt} = \frac{1}{2} h$$

where $h = \text{constant}$

PROBLEM 12.113 (Continued)

Then $$A = \frac{1}{2}ht \quad \text{or} \quad t_{BA} = \frac{2A_{BA}}{h_{BA}} \qquad h_{BA} = r_A v_A$$

$$t_{BA} = \frac{2 \times \frac{4}{3} r_A^2}{r_A v_A} = \frac{8}{3}\frac{r_A}{v_A}$$

$$= \frac{8}{3}\frac{6332 \times 10^3 \text{ m}}{10,131.4 \text{ m/s}}$$

$$= 1666.63 \text{ s}$$

For the circular trajectory AC,

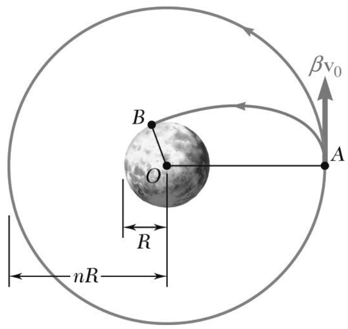
$$t_{AC} = \frac{\frac{\pi}{2} r_A}{(v_A)_{\text{circ}}} = \frac{\pi}{2}\frac{6332 \times 10^3 \text{ m}}{7164.0 \text{ m/s}} = 1388.37 \text{ s}$$

Finally, $$t_{BC} = t_{BA} + t_{AC}$$

$$= (1666.63 + 1388.37) \text{ s}$$

$$= 3055.0 \text{ s}$$

or $$t_{BC} = 50 \text{ min } 55 \text{ s} \blacktriangleleft$$



PROBLEM 12.114

A space probe is describing a circular orbit of radius nR with a velocity v_0 about a planet of radius R and center O. As the probe passes through Point A, its velocity is reduced from v_0 to βv_0, where $\beta < 1$, to place the probe on a crash trajectory. Express in terms of n and β the angle AOB, where B denotes the point of impact of the probe on the planet.

SOLUTION

For the circular orbit, $r_0 = r_A = nR$

$$v_0 = \sqrt{\frac{GM}{r_0}} = \sqrt{\frac{GM}{nR}}$$

The crash trajectory is elliptic.

$$v_A = \beta v_0 = \sqrt{\frac{\beta^2 GM}{nR}}$$

$$h = r_A v_A = nRv_A = \sqrt{\beta^2 nGMR}$$

$$\frac{GM}{h^2} = \frac{1}{\beta^2 nR}$$

$$\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta) = \frac{1+\varepsilon\cos\theta}{\beta^2 nR}$$

At Point A, $\theta = 180°$

$$\frac{1}{r_A} = \frac{1}{nR} = \frac{1-\varepsilon}{\beta^2 nR} \quad \text{or} \quad \beta^2 = 1-\varepsilon \quad \text{or} \quad \varepsilon = 1-\beta^2$$

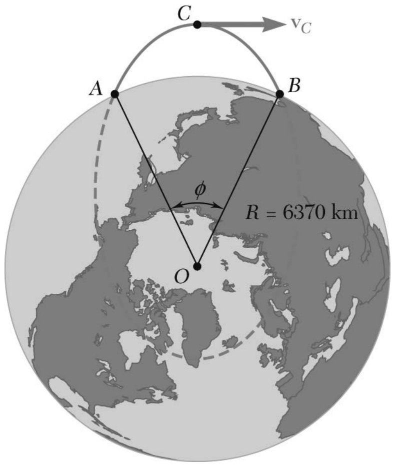
At impact Point B, $\theta = \pi - \phi$

$$\frac{1}{r_B} = \frac{1}{R}$$

$$\frac{1}{R} = \frac{1+\varepsilon\cos(\pi-\phi)}{\beta^2 nR} = \frac{1-\varepsilon\cos\phi}{\beta^2 nR}$$

$$\varepsilon\cos\phi = 1-n\beta^2 \quad \text{or} \quad \cos\phi = \frac{1-n\beta^2}{\varepsilon} = \frac{1-n\beta^2}{1-\beta^2}$$

$$\phi = \cos^{-1}[(1-n\beta^2)/(1-\beta^2)] \blacktriangleleft$$



PROBLEM 12.115

A long-range ballistic trajectory between Points A and B on the earth's surface consists of a portion of an ellipse with the apogee at Point C. Knowing that Point C is 1500 km above the surface of the earth and the range $R\phi$ of the trajectory is 6000 km, determine (*a*) the velocity of the projectile at C, (*b*) the eccentricity ε of the trajectory.

SOLUTION

For earth, $R = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}$

$$GM = gR^2 = (9.81)(6.37 \times 10^6)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

For the trajectory, $r_C = 6370 + 1500 = 7870 \text{ km} = 7.87 \times 10^6 \text{ m}$

$$r_A = r_B = R = 6.37 \times 10^6 \text{ m}, \qquad \frac{r_C}{r_A} = \frac{7870}{6370} = 1.23548$$

Range A to B: $s_{AB} = 6000 \text{ km} = 6.00 \times 10^6 \text{ m}$

$$\varphi = \frac{s_{AB}}{R} = \frac{6.00 \times 10^6}{6.37 \times 10^6} = 0.94192 \text{ rad} = 53.968°$$

For an elliptic trajectory, $\dfrac{1}{r} = \dfrac{GM}{h^2}(1 + \varepsilon \cos\theta)$

At A, $\theta = 180° - \dfrac{\varphi}{2} = 153.016°$, $\dfrac{1}{r_A} = \dfrac{GM}{h^2}(1 + \varepsilon \cos 153.016°)$ (1)

At C, $\theta = 180°$, $\dfrac{1}{r_C} = \dfrac{GM}{h^2}(1 - \varepsilon)$ (2)

Dividing Eq. (1) by Eq. (2),

$$\frac{r_C}{r_A} = \frac{1 + \varepsilon \cos 153.016°}{1 - \varepsilon} = 1.23548$$

$$\varepsilon = \frac{1.23548 - 1}{1.23548 + \cos 153.016°} = 0.68384$$

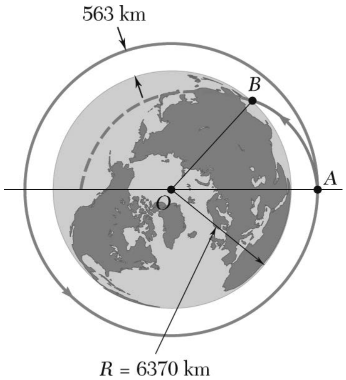
PROBLEM 12.115 (Continued)

From Eq. (2), $h = \sqrt{GM(1-\varepsilon)r_C}$

$$h = \sqrt{(398.06\times 10^{12})(0.31616)(7.87\times 10^{6})} = 31.471\times 10^{9}\ \text{m}^2/\text{s}$$

(*a*) Velocity at *C*. $v_C = \dfrac{h}{r_C} = \dfrac{31.471\times 10^{9}}{7.87\times 10^{6}} = 4.00\times 10^{3}\ \text{m/s}$ $v_C = 4\ \text{km/s}$ ◀

(*b*) Eccentricity of trajectory. $\varepsilon = 0.684$ ◀



PROBLEM 12.116

A space shuttle is describing a circular orbit at an altitude of 563 km above the surface of the earth. As it passes through Point *A*, it fires its engine for a short interval of time to reduce its speed by 152 m/s and begin its descent toward the earth. Determine the angle *AOB* so that the altitude of the shuttle at Point *B* is 121 km. (*Hint:* Point *A* is the apogee of the elliptic descent trajectory.)

SOLUTION

$$GM = gR^2 = (9.81)(6.37\times10^6)^2 = 398.06\times10^{12}\ \text{m}^3/\text{s}^2$$

$$r_A = 6370 + 563 = 6933\ \text{km} = 6.933\times10^6\ \text{m}$$

$$r_B = 6370 + 121 = 6491\ \text{km} = 6.491\times10^6\ \text{m}$$

For the circular orbit through Point *A*,

$$v_{\text{circ}} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{398.06\times10^{12}}{6.933\times10^6}} = 7.5773\times10^3\ \text{m/s}$$

For the descent trajectory,

$$v_A = v_{\text{circ}} + \Delta v = 7.5773\times10^3 - 152 = 7.4253\times10^3\ \text{m/s}$$

$$h = r_A v_A = (6.933\times10^6)(7.4253\times10^3) = 51.4795\times10^9\ \text{m}^2/\text{s}$$

$$\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta)$$

At Point *A*, $\theta = 180°$, $r = r_A$

$$\frac{1}{r_A} = \frac{GM}{h^2}(1-\varepsilon)$$

$$1-\varepsilon = \frac{h^2}{GM\ r_A} = \frac{(51.4795\times10^9)^2}{(398.06\times10^{12})(6.933\times10^6)} = 0.96028$$

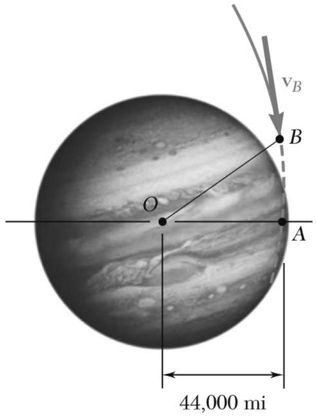
$$\varepsilon = 0.03972$$

$$\frac{1}{r_B} = \frac{GM}{h^2}(1+\varepsilon\cos\theta_B)$$

$$1+\varepsilon\cos\theta_B = \frac{h^2}{GM\ r_B} = \frac{(51.4795\times10^9)^2}{(398.06\times10^{12})(6.491\times10^6)} = 1.02567$$

$$\cos\theta_B = \frac{1.02567-1}{\varepsilon} = 0.6463$$

$\theta_B = 49.7°$ $\quad\measuredangle AOB = 180° - \theta_B = 130.3°$ $\quad\measuredangle AOB = 130.3°$ ◀



PROBLEM 12.117

As a spacecraft approaches the planet Jupiter, it releases a probe which is to enter the planet's atmosphere at Point B at an altitude of 280 mi above the surface of the planet. The trajectory of the probe is a hyperbola of eccentricity $\varepsilon = 1.031$. Knowing that the radius and the mass of Jupiter are 44423 mi and 1.30×10^{26} slug, respectively, and that the velocity $\mathbf{v}_B$ of the probe at B forms an angle of 82.9° with the direction of OA, determine (*a*) the angle AOB, (*b*) the speed v_B of the probe at B.

SOLUTION

First we note $\quad r_B = (44.423\times10^3 + 280)\text{ mi} = 44.703\times10^3\text{ mi}$

(*a*) We have $\quad \dfrac{1}{r} = \dfrac{GM_j}{h^2}(1+\varepsilon\cos\theta) \qquad$ [Eq. (12.39′)]

At A, $\theta = 0$: $\quad \dfrac{1}{r_A} = \dfrac{GM_j}{h^2}(1+\varepsilon)$

or $\quad \dfrac{h^2}{GM_j} = r_A(1+\varepsilon)$

At B, $\theta = \theta_B = \measuredangle AOB$: $\quad \dfrac{1}{r_B} = \dfrac{GM_j}{h^2}(1+\varepsilon\cos\theta_B)$

or $\quad \dfrac{h^2}{GM_j} = r_B(1+\varepsilon\cos\theta_B)$

Then $\quad r_A(1+\varepsilon) = r_B(1+\varepsilon\cos\theta_B)$

or
$$\cos\theta_B = \frac{1}{\varepsilon}\left[\frac{r_A}{r_B}(1+\varepsilon)-1\right]$$
$$= \frac{1}{1.031}\left[\frac{44.0\times10^3\text{ mi}}{44.703\times10^3\text{ mi}}(1+1.031)-1\right]$$
$$= 0.96902$$

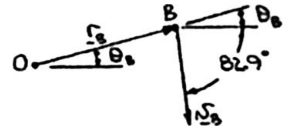
or $\quad \theta_B = 14.2988°$ $\qquad \measuredangle AOB = 14.30°$ ◀

PROBLEM 12.117 (Continued)

(*b*) From above $h^2 = GM_j r_B (1+\varepsilon \cos\theta_B)$

where $h = \frac{1}{m}|\mathbf{r}_B \times m\mathbf{v}_B| = r_B v_B \sin\phi$

$\phi = (\theta_B + 82.9°) = 97.1988°$



Then $(r_B v_B \sin\phi)^2 = GM_j r_B (1+\varepsilon \cos\theta_B)$

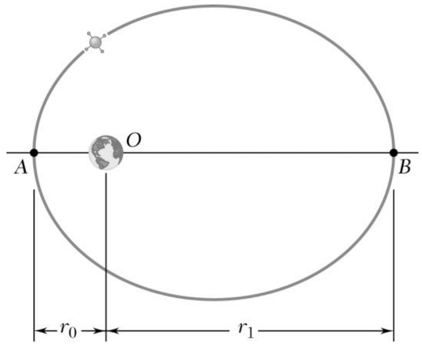
or

$$v_B = \frac{1}{\sin\phi}\left[\frac{GM_j}{r_B}(1+\varepsilon\cos\theta_B)\right]^{1/2}$$

$$= \frac{1}{\sin 97.1988°}\left\{\frac{34.4\times 10^{-9}\ \text{ft}^4/\text{lb}\cdot\text{s}^4 \times (1.30\times 10^{26}\ \text{slug})}{236.03\times 10^6\ \text{ft}}\times[1+(1.031)(0.96902)]\right\}^{1/2}$$

or $v_B = 196.2$ ft/s ◀

PROBLEM 12.118



A satellite describes an elliptic orbit about a planet. Denoting by r_0 and r_1 the distances corresponding, respectively, to the perigee and apogee of the orbit, show that the curvature of the orbit at each of these two points can be expressed as

$$\frac{1}{\rho} = \frac{1}{2}\left(\frac{1}{r_0} + \frac{1}{r_1}\right)$$

SOLUTION

Using Eq. (12.39), $$\frac{1}{r_A} = \frac{GM}{h^2} + C\cos\theta_A$$

and $$\frac{1}{r_B} = \frac{GM}{h^2} + C\cos\theta_B.$$

But $$\theta_B = \theta_A + 180°,$$

so that $$\cos\theta_A = -\cos\theta_B$$

Adding, $$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM}{h^2}$$

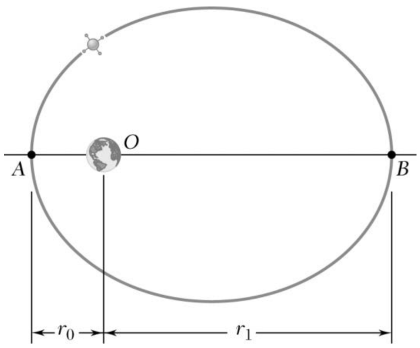
At Points A and B, the radial direction is normal to the path.

$$a_n = \frac{v^2}{\rho} = \frac{h^2}{r^2\rho}$$

But $$F_n = \frac{GMm}{r^2} = ma_n = \frac{mh^2}{r^2\rho}$$

$$\frac{1}{\rho} = \frac{GM}{h^2} = \frac{1}{2}\left(\frac{1}{r_A} + \frac{1}{r_B}\right)$$

$$\frac{1}{\rho} = \frac{1}{2}\left(\frac{1}{r_0} + \frac{1}{r_1}\right) \blacktriangleleft$$



PROBLEM 12.119

(*a*) Express the eccentricity ε of the elliptic orbit described by a satellite about a planet in terms of the distances r_0 and r_1 corresponding, respectively, to the perigee and apogee of the orbit. (*b*) Use the result obtained in Part *a* and the data given in Problem 12.109, where $R_E = 149.6\times10^6$ km, to determine the approximate maximum distance from the sun reached by comet Hyakutake.

SOLUTION

(*a*) We have $$\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta) \qquad \text{Eq. (12.39}'\text{)}$$

At A, $\theta = 0$: $$\frac{1}{r_0} = \frac{GM}{h^2}(1+\varepsilon)$$

or $$\frac{h^2}{GM} = r_0(1+\varepsilon)$$

At B, $\theta = 180°$: $$\frac{1}{r_1} = \frac{GM}{h^2}(1-\varepsilon)$$

or $$\frac{h^2}{GM} = r_1(1-\varepsilon)$$

Then $$r_0(1+\varepsilon) = r_1(1-\varepsilon)$$

or $$\varepsilon = \frac{r_1 - r_0}{r_1 + r_0} \blacktriangleleft$$

(*b*) From above, $$r_1 = \frac{1+\varepsilon}{1-\varepsilon} r_0$$

where $$r_0 = 0.230R_E$$

Then $$r_1 = \frac{1+0.999887}{1-0.999887}\times 0.230(149.6\times10^9 \text{ m})$$

or $$r_1 = 609\times10^{12} \text{ m} \blacktriangleleft$$

Note: $r_1 = 4070R_E$ or $r_1 = 0.064$ lightyears.

PROBLEM 12.120

Derive Kepler's third law of planetary motion from Eqs. (12.39) and (12.45).

SOLUTION

For an ellipse, $2a = r_A + r_B \quad \text{and} \quad b = \sqrt{r_A r_B}$

Using Eq. (12.39),

$$\frac{1}{r_A} = \frac{GM}{h^2} + C\cos\theta_A$$

and

$$\frac{1}{r_B} = \frac{GM}{h^2} + C\cos\theta_B.$$

But $\theta_B = \theta_A + 180°,$

so that $\cos\theta_A = -\cos\theta_B.$

Adding,

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A + r_B}{r_A r_B} = \frac{2a}{b^2} = \frac{2GM}{h^2}$$

$$h = b\sqrt{\frac{GM}{a}}$$

By Eq. (12.45),

$$\tau = \frac{2\pi ab}{h} = \frac{2\pi ab\sqrt{a}}{b\sqrt{GM}} = \frac{2\pi a^{3/2}}{\sqrt{GM}}$$

$$\tau^2 = \frac{4\pi^2 a^3}{GM}$$

For Orbits 1 and 2 about the same large mass,

$$\tau_1^2 = \frac{4\pi^2 a_1^3}{GM}$$

and

$$\tau_2^2 = \frac{4\pi^2 a_2^3}{GM}$$

Forming the ratio,

$$\left(\frac{\tau_1}{\tau_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3 \blacktriangleleft$$

PROBLEM 12.121

Show that the angular momentum per unit mass h of a satellite describing an elliptic orbit of semimajor axis a and eccentricity ε about a planet of mass M can be expressed as

$$h = \sqrt{GMa(1-\varepsilon^2)}$$

SOLUTION

By Eq. (12.39′), $$\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta)$$

At A, $\theta = 0°$: $$\frac{1}{r_A} = \frac{GM}{h^2} = (1+\varepsilon) \qquad \text{or} \qquad r_A = \frac{h^2}{GM(1+\varepsilon)}$$

At B, $\theta = 180°$: $$\frac{1}{r_B} = \frac{GM}{h^2} = (1-\varepsilon) \qquad \text{or} \qquad r_B = \frac{h^2}{GM(1-\varepsilon)}$$

Adding, $$r_A + r_B = \frac{h^2}{GM} = \left(\frac{1}{1+\varepsilon} + \frac{1}{1-\varepsilon}\right) = \frac{2h^2}{GM(1-\varepsilon^2)}$$

But for an ellipse, $$r_A + r_B = 2a$$

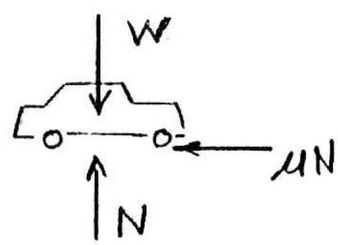
$$2a = \frac{2h^2}{GM(1-\varepsilon^2)}$$

$$h = \sqrt{GMa(1-\varepsilon^2)} \quad ◀$$

PROBLEM 12.122

In the braking test of a sports car its velocity is reduced from 70 mi/h to zero in a distance of 170 ft with slipping impending. Knowing that the coefficient of kinetic friction is 80 percent of the coefficient of static friction, determine (*a*) the coefficient of static friction, (*b*) the stopping distance for the same initial velocity if the car skids. Ignore air resistance and rolling resistance.

SOLUTION



(*a*) Coefficient of static friction.

$$\Sigma F_y = 0: \quad N - W = 0 \qquad N = W$$

$$v_0 = 70 \text{ mi/h} = 102.667 \text{ ft/s}$$

$$\frac{v^2}{2} - \frac{v_0^2}{2} = a_t(s - s_0)$$

$$a_t = \frac{v^2 - v_0^2}{2(s - s_0)} = \frac{0 - (102.667)^2}{(2)(170)} = -31.001 \text{ ft/s}^2$$

For braking without skidding $\mu = \mu_s$, so that $\mu_s N = m|a_t|$

$$\xrightarrow{+} \Sigma F_t = ma_t: \quad -\mu_s N = ma_t$$

$$\mu_s = -\frac{ma_t}{W} = -\frac{a_t}{g} = \frac{31.001}{32.2} \qquad \mu_s = 0.963 \blacktriangleleft$$

(*b*) Stopping distance with skidding.

Use $\mu = \mu_k = (0.80)(0.963) = 0.770$

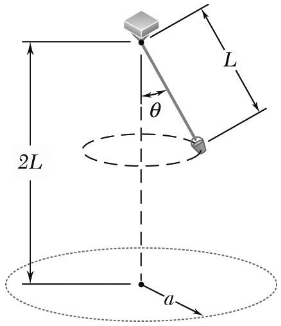
$$\xleftarrow{+} \Sigma F = ma_t: \quad \mu_k N = -ma_t$$

$$a_t = -\frac{\mu_k N}{m} = -\mu_k g = -24.801 \text{ ft/s}^2$$

Since acceleration is constant,

$$(s - s_0) = \frac{v^2 - v_0^2}{2a_t} = \frac{0 - (102.667)^2}{(2)(-24.801)}$$

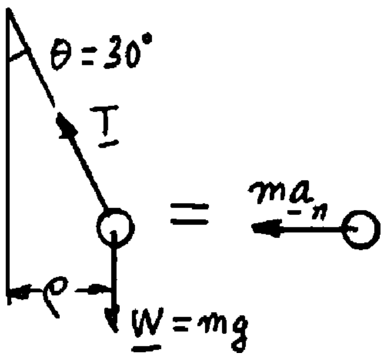
$$s - s_0 = 212 \text{ ft} \blacktriangleleft$$



PROBLEM 12.123

A bucket is attached to a rope of length $L = 1.2$ m and is made to revolve in a horizontal circle. Drops of water leaking from the bucket fall and strike the floor along the perimeter of a circle of radius a. Determine the radius a when $\theta = 30°$.

SOLUTION



Initial velocity of drop = velocity of bucket

$$\Sigma F_y = 0: \qquad T\cos 30° = mg \qquad (1)$$

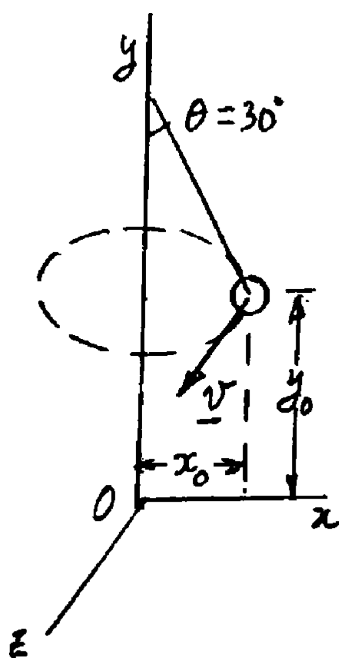
$$\overset{+}{\leftarrow} \Sigma F_x = ma_x: \qquad T\sin 30° = ma_n \qquad (2)$$

Divide (2) by (1): $\quad \tan 30° = \dfrac{a_n}{g} = \dfrac{v^2}{\rho g}$

Thus $\quad v^2 = \rho g \tan 30°$

But $\quad \rho = L\sin 30° = (1.2 \text{ m})\sin 30° = 0.6 \text{ m}$

Thus $\quad v^2 = 0.6(9.81)\tan 30° = 3.398 \text{ m}^2/\text{s}^2 \qquad v = 1.843 \text{ m/s}$



Assuming the bucket to rotate clockwise (when viewed from above), and using the axes shown, we find that the components of the initial velocity of the drop are

$$(v_0)_x = 0,\ (v_0)_y = 0,\ (v_0)_2 = 1.843 \text{ m/s}$$

<u>Free fall of drop</u>

$$y = y_0 + (v_0)_y t - \frac{1}{2}gt^2 \qquad y = y_0 - \frac{1}{2}gt^2$$

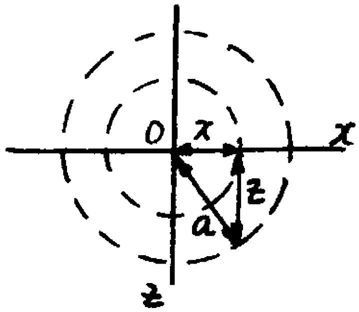
When drop strikes floor:

$$y = 0 \qquad y_0 - \frac{1}{2}gt^2 = 0$$

But $\quad y_0 = 2L - L\cos 30° = 2(1.2) - 1.2\cos 30° = 1.361 \text{ m}$

Thus $\quad 1.361 - \dfrac{1}{2}(9.81)t^2 = 0 \qquad t = 0.5275$

PROBLEM 12.123 (Continued)



<u>Projection on horizontal floor (uniform motion)</u>

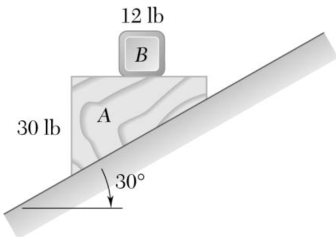
$$x = x_0 + (v_0)_z t = L\sin 30° + 0, \qquad x = 0.6 \text{ m}$$

$$z = z_0 + (v_0)_z t = 0 + 1.843(0.527) = 0.971 \text{ m}$$

Radius of circle: $a = \sqrt{x^2 + z^2}$

$$a = \sqrt{(0.6)^2 + (0.971)^2} \qquad a = 1.141 \text{ m} \blacktriangleleft$$

Note: The drop travels in a vertical plane parallel to the yz plane.



PROBLEM 12.124

A 12-lb block *B* rests as shown on the upper surface of a 30-lb wedge *A*. Neglecting friction, determine immediately after the system is released from rest (*a*) the acceleration of *A*, (*b*) the acceleration of *B* relative to *A*.

SOLUTION

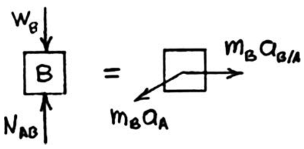
Acceleration vectors:

$$\mathbf{a}_A = a_A \swarrow 30^\circ, \quad \mathbf{a}_{B/A} = a_{B/A} \longrightarrow$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Block *B*:

$$\xrightarrow{+} \Sigma F_x = ma_x: \quad m_B a_{B/A} - m_B a_A \cos 30^\circ = 0$$



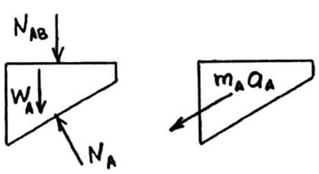
$$a_{B/A} = a_A \cos 30^\circ \qquad (1)$$

$$+\!\downarrow \Sigma F_y = ma_y: \quad N_{AB} - W_B = -m_B a_A \sin 30^\circ$$

$$N_{AB} = W_B - (W_B \sin 30^\circ)\frac{a_A}{g} \qquad (2)$$

Block *A*:

$$+\swarrow \Sigma F = ma: \quad W_A \sin 30^\circ + N_{AB} \sin 30^\circ = W_A \frac{a_A}{g}$$



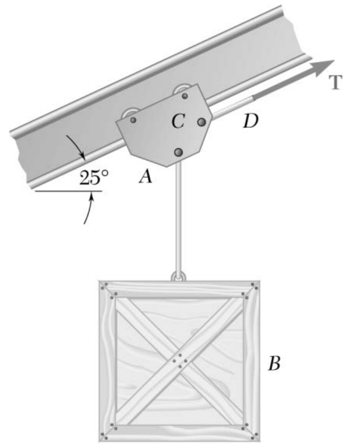
$$W_A \sin 30^\circ + W_B \sin 30^\circ - (W_B \sin^2 30^\circ)\frac{a_A}{g} = W_A \frac{a_A}{g}$$

$$a_A = \frac{(W_A + W_B)\sin 30^\circ}{W_A + W_B \sin^2 30^\circ} g = \frac{(30+12)\sin 30^\circ}{30 + 12\sin^2 30^\circ}(32.2) = 20.49 \text{ ft/s}^2$$

(*a*) $\mathbf{a}_A = 20.49 \text{ ft/s}^2 \swarrow 30^\circ$ ◀

$$a_{B/A} = (20.49)\cos 30^\circ = 17.75 \text{ ft/s}^2$$

(*b*) $\mathbf{a}_{B/A} = 17.75 \text{ ft/s}^2 \longrightarrow$ ◀

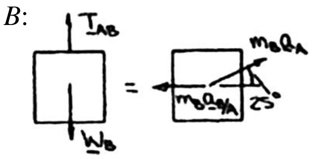
	**PROBLEM 12.125** A 500-lb crate B is suspended from a cable attached to a 40-lb trolley A which rides on an inclined I-beam as shown. Knowing that at the instant shown the trolley has an acceleration of 1.2 ft/s^2 up and to the right, determine (a) the acceleration of B relative to A, (b) the tension in cable CD.

SOLUTION

(a) First we note: $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$, where $\mathbf{a}_{B/A}$ is directed perpendicular to cable AB.

$$\xrightarrow{+} \Sigma F_x = m_B a_x: \quad 0 = -m_B a_x + m_B a_A \cos 25°$$

B:



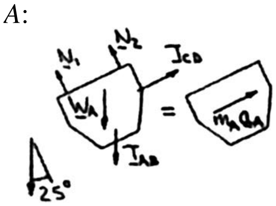
or
$$a_{B/A} = (1.2 \text{ ft/s}^2)\cos 25°$$

or
$$\mathbf{a}_{B/A} = 1.088 \text{ ft/s}^2 \longleftarrow \blacktriangleleft$$

(b) For crate B

$$+\!\uparrow \Sigma F_y = m_B a_y: \quad T_{AB} - W_B = \frac{W_B}{g} a_A \sin 25°$$

A:



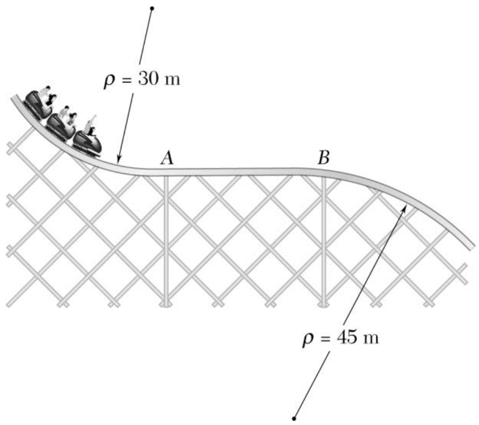
or
$$T_{AB} = (500 \text{ lb})\left[1 + \frac{(1.2 \text{ ft/s}^2)\sin 25°}{32.2 \text{ ft/s}^2}\right]$$
$$= 507.87 \text{ lb}$$

For trolley A

$$+\!\nearrow \Sigma F_x = m_A a_A: \quad T_{CD} - T_{AB}\sin 25° - W_A \sin 25° = \frac{W_A}{g} a_A$$

or
$$T_{CD} = (507.87 \text{ lb})\sin 25° + (40 \text{ lb})\left(\sin 25° + \frac{1.2 \text{ ft/s}^2}{32.2 \text{ ft/s}^2}\right)$$

or
$$T_{CD} = 233 \text{ lb} \blacktriangleleft$$

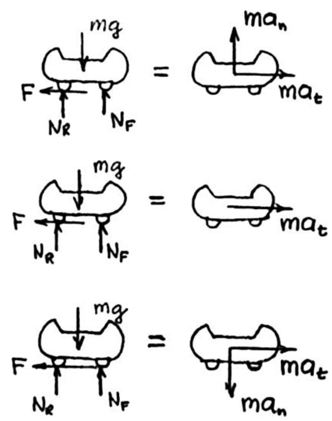


PROBLEM 12.126

The roller-coaster track shown is contained in a vertical plane. The portion of track between *A* and *B* is straight and horizontal, while the portions to the left of *A* and to the right of *B* have radii of curvature as indicated. A car is traveling at a speed of 72 km/h when the brakes are suddenly applied, causing the wheels of the car to slide on the track ($\mu_k = 0.25$). Determine the initial deceleration of the car if the brakes are applied as the car (*a*) has almost reached *A*, (*b*) is traveling between *A* and *B*, (*c*) has just passed *B*.

SOLUTION

$$v = 72 \text{ km/h} = 20 \text{ m/s}$$



(*a*) Almost reached Point *A*. $\rho = 30$ m

$$a_n = \frac{v^2}{\rho} = \frac{(20)^2}{30} = 13.333 \text{ m/s}^2 \uparrow$$

$$\Sigma F_y = ma_y: \quad N_R + N_F - mg = ma_n$$

$$N_R + N_F = m(g + a_n)$$

$$F = \mu_k (N_R + N_F) = \mu_k m(g + a_n)$$

$$\xrightarrow{+} \Sigma F_x = ma_x: \quad -F = ma_t$$

$$a_t = -\frac{F}{m} = -\mu_k (g + a_n)$$

$$|a_t| = \mu_k (g + a_n) = 0.25(9.81 + 13.33) \qquad |a_t| = 5.79 \text{ m/s}^2 \blacktriangleleft$$

(*b*) Between *A* and *B*. $\rho = \infty$

$$a_n = 0$$

$$|a_t| = \mu_k g = (0.25)(9.81) \qquad |a_t| = 2.45 \text{ m/s}^2 \blacktriangleleft$$

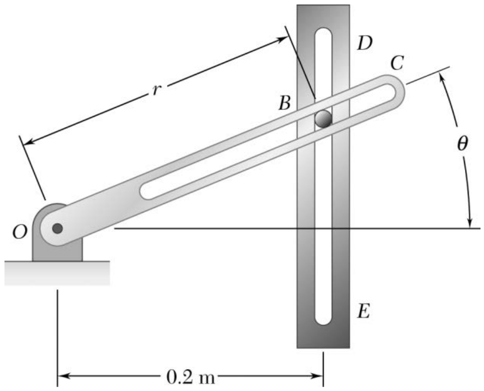
(*c*) Just passed Point *B*. $\rho = 45$ m

$$a_n = \frac{v^2}{\rho} = \frac{(20)^2}{45} = 8.8889 \text{ m/s}^2 \downarrow$$

$$\Sigma F_y = ma_y: \quad N_R + N_F - mg = -ma_n$$

PROBLEM 12.126 (Continued)

or
$$N_R + N_F = m(g - a_n)$$
$$F = \mu_k (N_R + N_F) = \mu_k m(g - a_n)$$
$$\xrightarrow{+} \Sigma F_x = ma_x: \quad -F = ma_t$$
$$a_t = -\frac{F}{m} = -\mu_k (g - a_n)$$
$$|a_t| = \mu_k (g - a_n) = (0.25)(9.81 - 8.8889)$$
$$|a_t| = 0.230 \text{ m/s}^2 \blacktriangleleft$$



PROBLEM 12.127

The 100-g pin *B* slides along the slot in the rotating arm *OC* and along the slot *DE* which is cut in a fixed horizontal plate. Neglecting friction and knowing that rod *OC* rotates at the constant rate $\dot{\theta}_0 = 12$ rad/s, determine for any given value of θ (*a*) the radial and transverse components of the resultant force **F** exerted on pin *B*, (*b*) the forces **P** and **Q** exerted on pin *B* by rod *OC* and the wall of slot *DE*, respectively.

SOLUTION

<u>Kinematics</u>

From the drawing of the system, we have

$$r = \frac{0.2}{\cos\theta} \text{ m}$$

Then $$\dot{r} = \left(0.2\frac{\sin\theta}{\cos^2\theta}\dot{\theta}\right) \text{ m/s} \qquad \dot{\theta} = 12 \text{ rad/s}$$

and $$\ddot{\theta} = 0$$

$$\ddot{r} = 0.2\,\frac{\cos\theta(\cos^2\theta) - \sin\theta(-2\cos\theta\sin\theta)}{\cos^4\theta}\,\dot{\theta}^2$$

$$= \left(0.2\frac{1+\sin^2\theta}{\cos^3\theta}\dot{\theta}^2\right) \text{ m/s}^2$$

Substituting for $\dot{\theta}$

$$\dot{r} = 0.2\frac{\sin\theta}{\cos^2\theta}(12) = \left(2.4\frac{\sin\theta}{\cos^2\theta}\right) \text{ m/s}$$

$$\ddot{r} = 0.2\frac{1+\sin^2\theta}{\cos^3\theta}(12)^2 = \left(28.8\frac{1+\sin^2\theta}{\cos^3\theta}\right) \text{ m/s}^2$$

Now $$a_r = \ddot{r} - r\dot{\theta}^2$$

$$= \left(28.8\frac{1+\sin^2\theta}{\cos^3\theta}\right) - \left(\frac{0.2}{\cos\theta}\right)(12)^2$$

$$= \left(57.6\frac{\sin^2\theta}{\cos^3\theta}\right) \text{ m/s}^2$$

and $$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$= 0 + 2\left(2.4\frac{\sin\theta}{\cos^2\theta}\right)(12)$$

$$= \left(57.6\frac{\sin\theta}{\cos^2\theta}\right)\text{m/s}^2$$

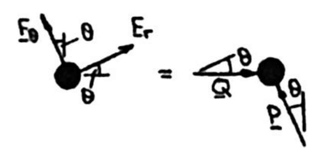
<u>Kinetics</u>

(*a*) We have $$F_r = m_B a_r = (0.1\text{ kg})\left[\left(57.6\frac{\sin^2\theta}{\cos^3\theta}\right)\text{m/s}^2\right]$$

or $$F_r = (5.76\text{ N})\tan^2\theta\sec\theta \blacktriangleleft$$

and $$F_\theta = m_B a_\theta = (0.1\text{ kg})\left[\left(57.6\frac{\sin\theta}{\cos^2\theta}\right)\text{m/s}^2\right]$$

or $$F_\theta = (5.76\text{ N})\tan\theta\sec\theta \blacktriangleleft$$



(*b*) Now $$+\uparrow \Sigma F_y: \quad F_\theta\cos\theta + F_r\sin\theta = P\cos\theta$$

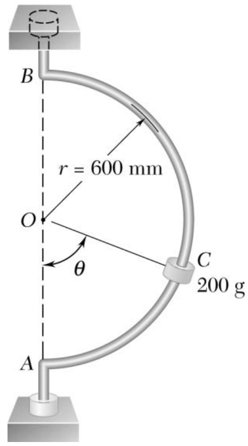
or $$P = 5.76\tan\theta\sec\theta + (5.76\tan^2\theta\sec\theta)\tan\theta$$

or $$\mathbf{P} = (5.76\text{ N})\tan\theta\sec^2\theta \;\nwarrow\!|\,\theta \blacktriangleleft$$

$$+\nearrow \Sigma F_r: \quad F_r = Q\cos\theta$$

or $$Q = (5.76\tan^2\theta\sec\theta)\frac{1}{\cos\theta}$$

or $$\mathbf{Q} = (5.76\text{ N})\tan^2\theta\sec^2\theta \longrightarrow \blacktriangleleft$$



PROBLEM 12.128

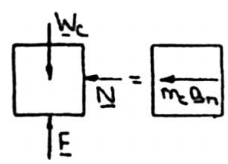
A small 200-g collar C can slide on a semicircular rod which is made to rotate about the vertical AB at the constant rate of 6 rad/s. Determine the minimum required value of the coefficient of static friction between the collar and the rod if the collar is not to slide when (*a*) $\theta = 90°$, (*b*) $\theta = 75°$, (*c*) $\theta = 45°$. Indicate in each case the direction of the impending motion.

SOLUTION

First note

$$v_C = (r \sin\theta)\dot{\phi}_{AB}$$
$$= (0.6 \text{ m})(6 \text{ rad/s})\sin\theta$$
$$= (3.6 \text{ m/s})\sin\theta$$

(*a*) With $\theta = 90°$, $v_C = 3.6 \text{ m/s}$



$$+\uparrow \Sigma F_y = 0: \quad F - W_C = 0$$

or $F = m_C g$

Now $F = \mu_s N$

or $N = \dfrac{1}{\mu_s} m_C g$

$$\overset{+}{\longleftarrow} \Sigma F_n = m_C a_n: \quad N = m_C \frac{v_C^2}{r}$$

or $\dfrac{1}{\mu_s} m_C g = m_C \dfrac{v_C^2}{r}$

or $\mu_s = \dfrac{gr}{v_C^2} = \dfrac{(9.81 \text{ m/s}^2)(0.6 \text{ m})}{(3.6 \text{ m/s})^2}$

or $(\mu_s)_{\min} = 0.454$ ◀

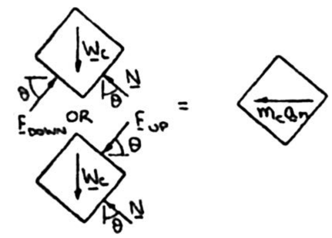
The direction of the impending motion is downward. ◀

PROBLEM 12.128 (Continued)

(*b*) and (*c*)

First observe that for an arbitrary value of θ, it is not known whether the impending motion will be upward or downward. To consider both possibilities for each value of θ, let F_{down} correspond to impending motion downward, F_{up} correspond to impending motion upward, then with the "top sign" corresponding to F_{down}, we have

$$+\uparrow \Sigma F_y = 0: \quad N\cos\theta \pm F\sin\theta - W_C = 0$$



Now $$F = \mu_s N$$

Then $$N\cos\theta \pm \mu_s N\sin\theta - m_C g = 0$$

or $$N = \frac{m_C g}{\cos\theta \pm \mu_s \sin\theta}$$

and $$F = \frac{\mu_s m_C g}{\cos\theta \pm \mu_s \sin\theta}$$

$$\overset{+}{\longleftarrow} \Sigma F_n = m_C a_n: \quad N\sin\theta \mp F\cos\theta = m_C \frac{v_C^2}{\rho} \qquad \rho = r\sin\theta$$

Substituting for N and F

$$\frac{m_C g}{\cos\theta \pm \mu_s \sin\theta}\sin\theta \mp \frac{\mu_s m_C g}{\cos\theta \pm \mu_s \sin\theta}\cos\theta = m_C \frac{v_C^2}{r\sin\theta}$$

or $$\frac{\tan\theta}{1 \pm \mu_s \tan\theta} \mp \frac{\mu_s}{1 \pm \mu_s \tan\theta} = \frac{v_C^2}{gr\sin\theta}$$

or $$\mu_s = \pm \frac{\tan\theta - \frac{v_C^2}{gr\sin\theta}}{1 + \frac{v_C^2}{gr\sin\theta}\tan\theta}$$

Now $$\frac{v_C^2}{gr\sin\theta} = \frac{[(3.6 \text{ m/s})\sin\theta]^2}{(9.81 \text{ m/s}^2)(0.6 \text{ m})\sin\theta} = 2.2018\sin\theta$$

Then $$\mu_s = \pm \frac{\tan\theta - 2.2018\sin\theta}{1 + 2.2018\sin\theta\tan\theta}$$

(*b*) $\theta = 75°$

$$\mu_s = \pm \frac{\tan 75° - 2.2018\sin 75°}{1 + 2.2018\sin 75°\tan 75°} = \pm 0.1796$$

PROBLEM 12.128 (Continued)

Then

downward: $\mu_s = +0.1796$

upward: $\mu_s < 0$ not possible

$(\mu_s)_{\min} = 0.1796$ ◀

The direction of the impending motion is downward. ◀

(*c*) $\theta = 45°$

$$\mu_s = \pm \frac{\tan 45° - 2.2018 \sin 45°}{1 + 2.2018 \sin 45° \tan 45°} = \pm(-0.218)$$

Then

downward: $\mu_s < 0$ not possible

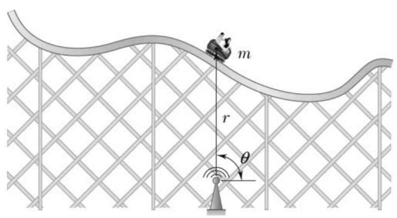
upward: $\mu_s = 0.218$ $(\mu_s)_{\min} = 0.218$ ◀

The direction of the impending motion is upward. ◀

Note: When $\tan\theta - 2.2018 \sin\theta = 0$

or $\theta = 62.988°$,

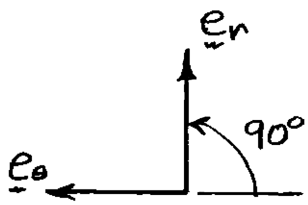
$\mu_s = 0$. Thus, for this value of θ, friction is not necessary to prevent the collar from sliding on the rod.

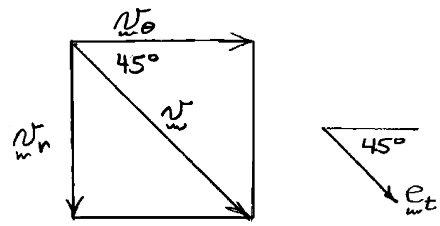


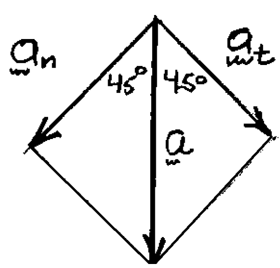
PROBLEM 12.129

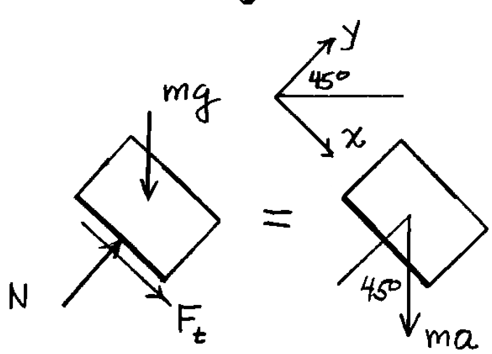
Telemetry technology is used to quantify kinematic values of a 200-kg roller coaster cart as it passes overhead. According to the system, $r = 25$ m, $\dot{r} = -10$ m/s, $\ddot{r} = -2$ m/s^2, $\theta = 90°$, $\dot{\theta} = -0.4$ rad/s, $\dot{\theta} = -0.32$ rad/s^2. At this instant, determine (*a*) the normal force between the cart and the track, (*b*) the radius of curvature of the track.

SOLUTION









Find the acceleration and velocity using polar coordinates.

$$v_r = \dot{r} = -10 \text{ m/s}$$

$$v_\theta = r\dot{\theta} = (25 \text{ m})(-0.4 \text{ rad/s}) = -10 \text{ m/s}$$

So the tangential direction is ↘ 45° and $v = 10\sqrt{2}$ m/s.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2 \text{ m/s} - (25 \text{ m})(-0.4 \text{ rad/s})^2$$
$$= -6 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$
$$= (25 \text{ m})(-0.32) \text{ rad/s}^2| + 2(-10 \text{ m/s})(-0.4 \text{ rad/s})$$
$$= 0$$

So the acceleration is vertical and downward.

(*a*) To find the normal force use Newton's second law.

y-direction

$$N - mg\sin 45° = -ma\cos 45°$$
$$N = m(g\sin 45° - a\cos 45°)$$
$$= (200 \text{ kg})(9.81) \text{ m/s}^2 - 6 \text{ m/s}^2)(0.70711)$$
$$= 538.815 \text{ N}$$

$N = 539$ N ◀

(*b*) Radius l curvature of the track.

$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{(10\sqrt{2})^2}{6\cos 45°}$$

$\rho = 47.1$ m ◀

PROBLEM 12.130

The radius of the orbit of a moon of a given planet is equal to twice the radius of that planet. Denoting by ρ the mean density of the planet, show that the time required by the moon to complete one full revolution about the planet is $(24\,\pi/G\rho)^{1/2}$, where G is the constant of gravitation.

SOLUTION

For gravitational force and a circular orbit,

$$|F_r| = \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \text{or} \qquad v = \sqrt{\frac{GM}{r}}$$

Let τ be the periodic time to complete one orbit.

$$v\tau = 2\pi r \qquad \text{or} \qquad \tau\sqrt{\frac{GM}{r}} = 2\pi r$$

Solving for τ,

$$\tau = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

But

$$M = \frac{4}{3}\pi R^3 \rho, \quad \text{hence,} \quad \sqrt{GM} = 2\sqrt{\frac{\pi}{3}G\rho}\; R^{3/2}$$

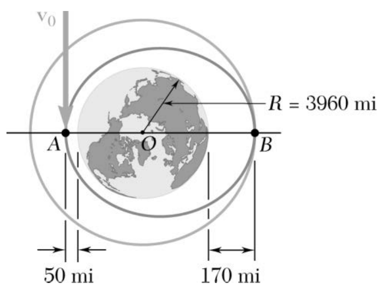
Then

$$\tau = \sqrt{\frac{3\pi}{G\rho}}\left(\frac{r}{R}\right)^{3/2}$$

Using $r = 2R$ as a given leads to

$$\tau = 2^{3/2}\sqrt{\frac{3\pi}{G\rho}} = \sqrt{\frac{24\pi}{G\rho}}$$

$\tau = (24\pi/G\rho)^{1/2}$ ◀



PROBLEM 12.131

At engine burnout on a mission, a shuttle had reached Point A at an altitude of 40 mi above the surface of the earth and had a horizontal velocity $\mathbf{v}_0$. Knowing that its first orbit was elliptic and that the shuttle was transferred to a circular orbit as it passed through Point B at an altitude of 170 mi, determine (a) the time needed for the shuttle to travel from A to B on its original elliptic orbit, (b) the periodic time of the shuttle on its final circular orbit.

SOLUTION

For Earth, $R = 3960 \text{ mi} = 20.909\times 10^6 \text{ ft}, \quad g = 32.2 \text{ ft/s}^2$

$$GM = gR^2 = (32.2)(20.909\times 10^6)^2 = 14.077\times 10^{15} \text{ ft}^3/\text{s}^2$$

(a) For the elliptic orbit,

$$r_A = 3960 + 40 = 4000 \text{ mi} = 21.12\times 10^6 \text{ ft}$$

$$r_B = 3960 + 170 = 4130 \text{ mi} = 21.8064\times 10^6 \text{ ft}$$

$$a = \frac{1}{2}(r_A + r_B) = 21.5032\times 10^6 \text{ ft}$$

$$b = \sqrt{r_A r_B} = 21.4605\times 10^6 \text{ ft}$$

Using Eq. 12.39, $$\frac{1}{r_A} = \frac{GM}{h^2} + C\cos\theta_A$$

and $$\frac{1}{r_B} = \frac{GM}{r_B^2} + C\cos\theta_B$$

But $\theta_B = \theta_A + 180°,$ so that $\cos\theta_A = -\cos\theta_B$

Adding, $$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A + r_B}{r_A r_B} = \frac{2a}{b^2} = \frac{2GM}{h^2}$$

or $$h = \sqrt{\frac{GMb^2}{a}}$$

<u>Periodic time</u>. $$\tau = \frac{2\pi ab}{h} = \frac{2\pi ab\sqrt{a}}{\sqrt{GMb^2}} = \frac{2\pi a^{3/2}}{\sqrt{GM}}$$

$$\tau = \frac{2\pi(21.5032\times 10^6)^{3/2}}{\sqrt{14.077\times 10^{15}}} = 5280.6 \text{ s} = 1.4668 \text{ h}$$

The time to travel from A to B is one half the periodic time

$$\tau_{AB} = 0.7334 \text{ h}$$

$\tau_{AB} = 44.0 \text{ min}$ ◀

PROBLEM 12.131 (Continued)

(*b*) For the circular orbit, $a = b = r_B = 21.8064 \times 10^6 \text{ ft}$

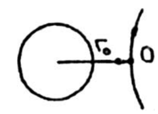
$$\tau_{\text{circ}} = \frac{2\pi a^{3/2}}{\sqrt{GM}} = \frac{2\pi(21.8064 \times 10^6)^{3/2}}{\sqrt{14.077 \times 10^{15}}} = 5393 \text{ s}$$

$\tau_{\text{circ}} = 1.498 \text{ h}$ $\tau_{\text{circ}} = 89.9 \text{ min}$ ◀

PROBLEM 12.132

It was observed that as the Galileo spacecraft reached the point on its trajectory closest to Io, a moon of the planet Jupiter, it was at a distance of 1750 mi from the center of Io and had a velocity of 49.4×10^3 ft/s. Knowing that the mass of Io is 0.01496 times the mass of the earth, determine the eccentricity of the trajectory of the spacecraft as it approached Io.

SOLUTION



First note $$r_0 = 1750 \text{ mi } = 9.24\times10^6 \text{ ft}$$

$$R_{\text{earth}} = 3960 \text{ mi} = 20.9088\times10^6 \text{ ft}$$

We have $$\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta) \qquad \text{Eq. (12.39)}$$

At Point O, $$r = r_0, \quad \theta = 0, \quad h = h_0 = r_0 v_0$$

Also, $$GM_{Io} = G(0.01496 M_{\text{earth}})$$

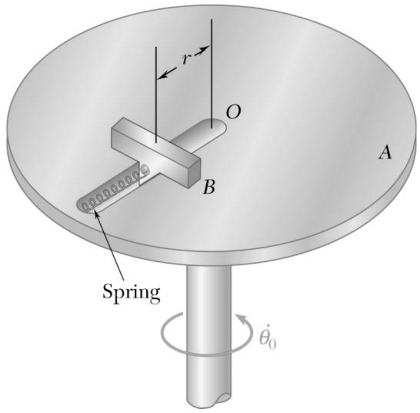
$$= 0.01496 g R_{\text{earth}}^2 \qquad \text{using Eq. (12.30).}$$

Then $$\frac{1}{r_0} = \frac{0.01496 g R_{\text{earth}}^2}{(r_0 v_0)^2}(1+\varepsilon)$$

or $$\varepsilon = \frac{r_0 v_0^2}{0.01496 g R_{\text{earth}}^2} - 1$$

$$= \frac{(9.24\times10^6 \text{ ft})(49.4\times10^3 \text{ ft/s})^2}{0.01496(32.2 \text{ ft/s}^2)(20.9088\times10^6 \text{ ft})^2} - 1$$

or $$\varepsilon = 106.1 \blacktriangleleft$$



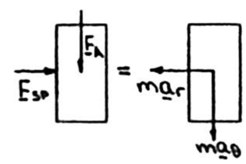
PROBLEM 12.133*

Disk A rotates in a horizontal plane about a vertical axis at the constant rate $\dot{\theta}_0 = 10$ rad/s. Slider B has mass 1 kg and moves in a frictionless slot cut in the disk. The slider is attached to a spring of constant k, which is undeformed when $r = 0$. Knowing that the slider is released with no radial velocity in the position $r = 500$ mm, determine the position of the slider and the horizontal force exerted on it by the disk at $t = 0.1$ s for (a) $k = 100$ N/m, (b) $k = 200$ N/m.

SOLUTION

First we note when $r = 0, \quad x_{sp} = 0 \Rightarrow F_{sp} = kr$

$$r_0 = 500 \text{ mm} = 0.5 \text{ m}$$



and $\dot{\theta} = \dot{\theta}_0 = 12$ rad/s

then $\ddot{\theta} = 0$

$$\overset{+}{\longleftarrow} \Sigma F_r = m_B a_r: \quad -F_{sp} = m_B(\ddot{r} - r\dot{\theta}_0^2)$$

$$\ddot{r} + \left(\frac{k}{m_B} - \dot{\theta}_0^2\right) r = 0 \qquad (1)$$

$$+\downarrow \Sigma F_\theta = m_B a_\theta: \quad F_A = m_B(0 + 2\dot{r}\dot{\theta}_0) \qquad (2)$$

(a) $k = 100$ N/m

Substituting the given values into Eq. (1)

$$\ddot{r} + \left[\frac{100 \text{ N/m}}{1 \text{ kg}} - (10 \text{ rad/s})^2\right] r = 0$$

$$\ddot{r} = 0$$

Then $\dfrac{d\dot{r}}{dt} = \ddot{r} = 0$ and at $t = 0$, $\dot{r} = 0$:

$$\int_0^{\dot{r}} d\dot{r} = \int_0^{0.1} (0)\, dt$$

$$\dot{r} = 0$$

and $$\frac{dr}{dt} = \dot{r} = 0 \text{ and at } t = 0,\ r_0 = 0.5 \text{ m}$$

$$\int_{r_0}^{r} dr = \int_0^{0.1} (0)\, dt$$

$$r = r_0$$

$$r = 0.5 \text{ m} \blacktriangleleft$$

Note: $\dot{r} = 0$ implies that the slider remains at its initial radial position.

With $\dot{r} = 0$, Eq. (2) implies

$$F_H = 0 \blacktriangleleft$$

(*b*) $k = 200$ N/m

Substituting the given values into Eq. (1)

$$\ddot{r} + \left[\frac{200}{1 \text{ kg}} - (10 \text{ rad/s})^2\right] r = 0$$

$$\ddot{r} + 100\, r = 0$$

Now $$\ddot{r} = \frac{d}{dt}(\dot{r}) \qquad \dot{r} = v_r \qquad \frac{d}{dt} = \frac{dr}{dt}\frac{d}{dr} = v_r \frac{d}{dr}$$

Then $$\ddot{r} = v_r \frac{dv_r}{dr}$$

so that $$v_r \frac{dv_r}{dr} + 100r = 0$$

At $t = 0,\ v_r = 0,\ r = r_0$: $$\int_0^{v_r} v_r d\, v_r = -100 \int_{r_0}^{r} r\, dr$$

$$v_r^2 = -100(r^2 - r_0^2)$$

$$v_r = 10\sqrt{r_0^2 - r^2}$$

Now $$v_r = \frac{dr}{dt} = 10\sqrt{r_0^2 - r^2}$$

At $t = 0,\ r = r_0$: $$\int_{r_0}^{r} \frac{dr}{\sqrt{r_0^2 - r^2}} = \int_0^t 10\, dt = 10t$$

Let $r = r_0 \sin\phi, \qquad dr = r_0 \cos\phi d\phi$

Then
$$\int_{\pi/2}^{\sin^{-1}(r/r_0)} \frac{r_0 \cos\phi d\phi}{\sqrt{r_0^2 - r_0^2 \sin^2\phi}} = 10t$$

$$\int_{\pi/2}^{\sin^{-1}(r/r_0)} d\phi = 10t$$

$$\sin^{-1}\left(\frac{r}{r_0}\right) - \frac{\pi}{2} = 10t$$

$$r = r_0 \sin\left(10t + \frac{\pi}{2}\right) = r_0 \cos 10t = (0.5 \text{ ft})\cos 10t$$

Then $\dot{r} = -(5 \text{ m/s})\sin 10t$

Finally, at $t = 0.1$ s:

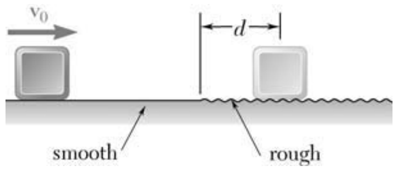
$$r = (0.5 \text{ ft})\cos(10\times 0.1)$$

$r = 0.270$ m ◀

Eq. (2) $F_H = 1 \text{ kg}\times 2\times[-(5 \text{ ft/s})\sin(10\times 0.1)]\,(10 \text{ rad/s})$

$F_H = -84.1$ N ◀

CHAPTER 13



PROBLEM 13.CQ1

Block A is traveling with a speed v_0 on a smooth surface when the surface suddenly becomes rough with a coefficient of friction of μ causing the block to stop after a distance d. If block A were traveling twice as fast, that is, at a speed $2v_0$, how far will it travel on the rough surface before stopping?

(*a*) $d/2$

(*b*) d

(*c*) $\sqrt{2}d$

(*d*) $2d$

(*e*) $4d$

SOLUTION

Answer: (*e*)

PROBLEM 13.1

A 400-kg satellite was placed in a circular orbit 1500 km above the surface of the earth. At this elevation the acceleration of gravity is 6.43 m/s^2. Determine the kinetic energy of the satellite, knowing that its orbital speed is 25.6×10^3 km/h.

SOLUTION

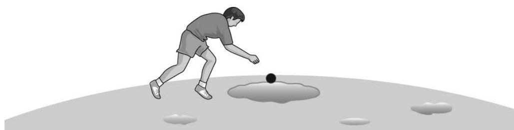
Mass of satellite: $m = 400$ kg

Velocity: $v = 25.6\times10^3 \text{ km/h} = 7.111\times10^3 \text{ m/s}$

Kinetic energy: $T = \frac{1}{2}mv^2 = \frac{1}{2}(400 \text{ kg})(7.111\times10^3 \text{ m/s})^2$

$T = 10.113\times10^9$ J $T = 10.11$ GJ ◀

Note: Acceleration of gravity has no effect on the mass of the satellite.



PROBLEM 13.2

A 1-lb stone is dropped down the "bottomless pit" at Carlsbad Caverns and strikes the ground with a speed of 95ft/s. Neglecting air resistance, determine (*a*) the kinetic energy of the stone as it strikes the ground and the height h from which it was dropped, (*b*) Solve Part *a* assuming that the same stone is dropped down a hole on the moon. (Acceleration of gravity on the moon = 5.31 ft/s^2.)

SOLUTION

Mass of stone: $m = \dfrac{W \text{ lb}}{g} = \dfrac{1 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.031056 \text{ lb} \cdot \text{s}^2/\text{ft}$

Initial kinetic energy: $T_1 = 0$ (rest)

(*a*) Kinetic energy at ground strike:

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}(0.031056)(95)^2 = 140.14 \text{ ft} \cdot \text{lb}$$

$T_2 = 140.1 \text{ ft} \cdot \text{lb}$ ◀

Use work and energy: $T_1 + U_{1 \to 2} = T_2$

where $U_{1 \to 2} = wh = mgh$

$$0 + mgh = \frac{1}{2} m v_2^2$$

$$h = \frac{v_2^2}{2g} = \frac{(95)^2}{(2)(32.2)}$$

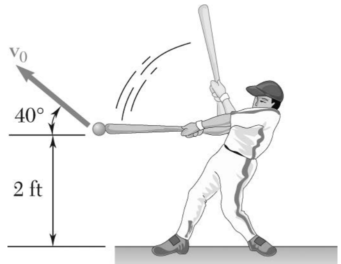
$h = 140.1 \text{ ft}$ ◀

(*b*) On the moon: $g = 5.31 \text{ ft/s}^2$

T_1 and T_2 will be the same, hence $T_2 = 140.1 \text{ ft} \cdot \text{lb}$ ◀

$$h = \frac{v_2^2}{2g} = \frac{(95)^2}{(2)(5.31)}$$

$h = 850 \text{ ft}$ ◀



PROBLEM 13.3

A baseball player hits a 5.1-oz baseball with an initial velocity of 140 ft/s at an angle of 40° with the horizontal as shown. Determine (*a*) the kinetic energy of the ball immediately after it is hit, (*b*) the kinetic energy of the ball when it reaches its maximum height, (*c*) the maximum height above the ground reached by the ball.

SOLUTION

Mass of baseball:

$$W = (5.1 \text{ oz})\left(\frac{1 \text{ lb}}{16 \text{ oz}}\right) = 0.31875 \text{ lb}$$

$$m = \frac{W}{g} = \frac{0.31875 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.009899 \text{ lb} \cdot \text{s}^2/\text{ft}$$

(*a*) Kinetic energy immediately after hit.

$$v = v_0 = 140 \text{ ft/s}$$

$$T_1 = \frac{1}{2}mv^2 = \frac{1}{2}(0.009899)(140)^2 \qquad T_1 = 97.0 \text{ ft} \cdot \text{lb} \blacktriangleleft$$

(*b*) Kinetic energy at maximum height:

$$v = v_0 \cos 40° = 140 \cos 40° = 107.246 \text{ ft/s}$$

$$T_2 = \frac{1}{2}mv^2 = \frac{1}{2}(0.009899)(107.246)^2 \qquad T_2 = 56.9 \text{ ft} \cdot \text{lb} \blacktriangleleft$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$U_{1\to 2} = T_2 - T_1 = -40.082 \text{ ft} \cdot \text{lb}$$

Work of weight: $U_{1\to 2} = -Wd$

Maximum height above impact point.

$$d = \frac{T_2 - T_1}{-W} = \frac{-40.082 \text{ ft} \cdot \text{lb}}{-0.31875 \text{ lb}} = 125.7 \text{ ft} \qquad 125.7 \text{ ft} \blacktriangleleft$$

(*c*) Maximum height above ground:

$$h = 125.7 \text{ ft} + 2 \text{ ft} \qquad h = 127.7 \text{ ft} \blacktriangleleft$$

PROBLEM 13.4

A 500-kg communications satellite is in a circular geosynchronous orbit and completes one revolution about the earth in 23 h and 56 min at an altitude of 35800 km above the surface of the earth. Knowing that the radius of the earth is 6370 km, determine the kinetic energy of the satellite.

SOLUTION

Radius of earth: $R = 6370 \text{ km}$

Radius of orbit: $r = R + h = 6370 + 35800 = 42170 \text{ km} = 42.170 \times 10^6 \text{ m}$

Time one revolution: $t = 23 \text{ h} + 56 \text{ min}$

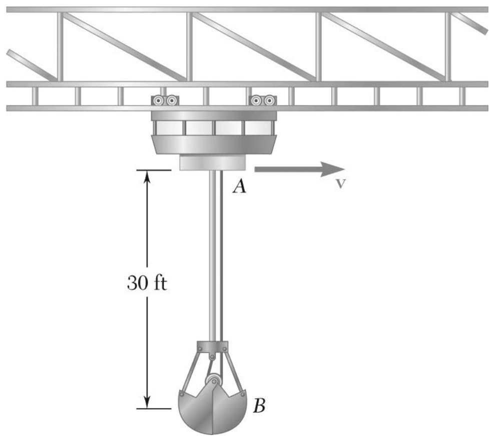
$$t = (23 \text{ h})(3600 \text{ s/h}) + (56 \text{ min})(60 \text{ s/min}) = 86.160 \times 10^3 \text{ s}$$

Speed: $v = \dfrac{2\pi r}{t} = \dfrac{2\pi(42.170 \times 10^6)}{86.160 \times 10^3} = 3075.2 \text{ m/s}$

Kinetic energy: $T = \dfrac{1}{2} m v^2$

$$T = \frac{1}{2}(500 \text{ kg})(3075.2 \text{ m/s})^2 = 2.3643 \times 10^9 \text{ J}$$

$T = 2.36 \text{ GJ}$ ◄



PROBLEM 13.5

In an ore-mixing operation, a bucket full of ore is suspended from a traveling crane which moves along a stationary bridge. The bucket is to swing no more than 10 ft horizontally when the crane is brought to a sudden stop. Determine the maximum allowable speed v of the crane.

SOLUTION

Let position ① be the position with bucket B directly below A, and position ② be that of maximum swing where $d = 10$ ft. Let L be the length AB.

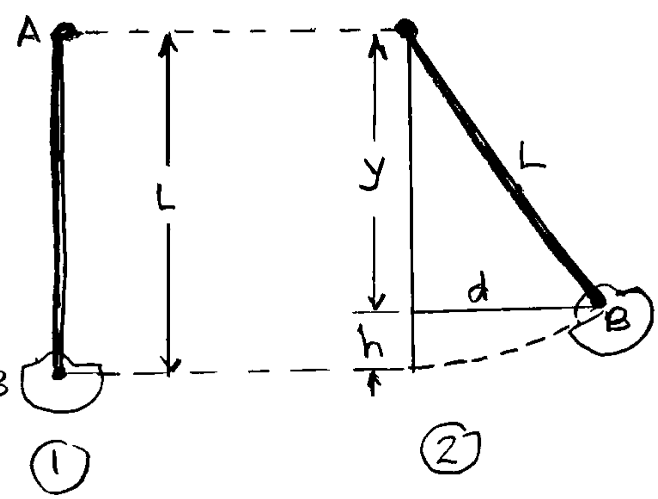
Kinetic energies: $T_1 = \frac{1}{2}mv^2, \; T_2 = 0$

Work of the weight: $U_{1\to 2} = -Wh = -mgh$

where h is the vertical projection of position ② above position ①

From geometry (see figure),

$$y = \sqrt{L^2 - d^2}$$
$$h = L - y$$
$$= L - \sqrt{L^2 - d^2}$$
$$= 30 - \sqrt{(30)^2 - (10)^2}$$
$$= 1.7157 \text{ ft}$$

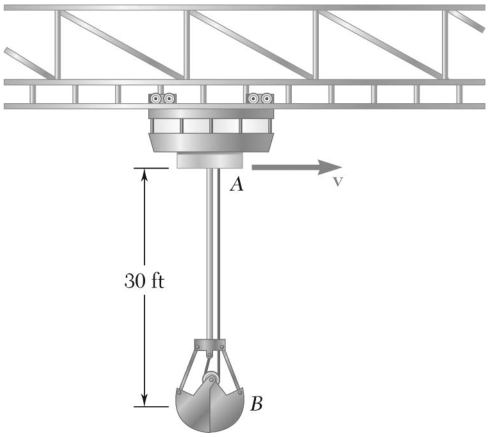


Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$\frac{1}{2}mv^2 - mgh = 0$$

$$v^2 = 2gh = (2)(32.2 \text{ ft/s}^2)(1.7157 \text{ ft}) = 110.49 \text{ ft}^2/\text{s}^2$$

$v = 10.51$ ft/s ◀



PROBLEM 13.6

In an ore-mixing operation, a bucket full of ore is suspended from a traveling crane which moves along a stationary bridge. The crane is traveling at a speed of 10 ft/s when it is brought to a sudden stop. Determine the maximum horizontal distance through which the bucket will swing.

SOLUTION

Let position ① be the position with bucket B directly below A, and position ② be that of maximum swing where the horizontal distance is d. Let L be the length AB.

Kinetic energies: $T_1 = \frac{1}{2}mv^2,\ T_2 = 0$

Work of the weight: $U_{1\to 2} = -Wh = -mgh$

where h is the vertical projection of position ② above position ①.

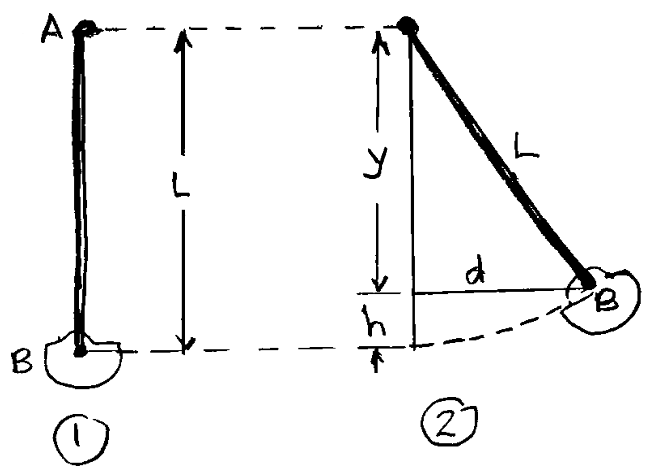
Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$\frac{1}{2}mv^2 - mgh = 0$$

$$h = \frac{v^2}{2g} = \frac{(10\text{ ft/s})^2}{(2)(32.2\text{ ft/s}^2)} = 1.5528\text{ ft}$$

From geometry (see figure),

$$d = \sqrt{L^2 - y^2}$$
$$= \sqrt{L^2 - (L-h)^2}$$
$$= \sqrt{(30)^2 - (30 - 1.5528)^2}$$
$$= 9.53\text{ ft}$$



$d = 9.53$ ft ◀

PROBLEM 13.7

Determine the maximum theoretical speed that may be achieved over a distance of 110 m by a car starting from rest assuming there is no slipping. The coefficient of static friction between the tires and pavement is 0.75, and 60 percent of the weight of the car is distributed over its front wheels and 40 percent over its rear wheels. Assume (*a*) front-wheel drive, (*b*) rear-wheel drive.

SOLUTION

Let W be the weight and m the mass. $W = mg$

(*a*) *Front wheel drive*: $N = 0.60W = 0.60mg$

$\mu_s = 0.75$

Maximum friction force without slipping:

$$F = \mu_s N = (0.75)(0.60W) = 0.45mg$$

$$U_{1\to 2} = Fd = 0.45mgd$$

$$T_1 = 0, \qquad T_2 = \frac{1}{2}mv_2^2$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$0 + 0.45mgd = \frac{1}{2}mv_2^2$$

$$v_2^2 = (2)(0.45gd) = (2)(0.45)(9.81 \text{ m/s}^2)(110 \text{ m}) = 971.19 \text{ m}^2/\text{s}^2$$

$$v_2 = 31.164 \text{ m/s}$$

$v_2 = 112.2 \text{ km/h}$ ◀

(*b*) *Rear wheel drive*: $N = 0.40W = 0.40mg$

$\mu_s = 0.75$

Maximum friction force without slipping:

$$F = \mu_s N = (0.75)(0.40W) = 0.30mg$$

$$U_{1\to 2} = Fd = 0.30mgd$$

$$T_1 = 0, \qquad T_2 = \frac{1}{2}mv_2^2$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$0 + 0.30mgd = \frac{1}{2}mv_2^2$$

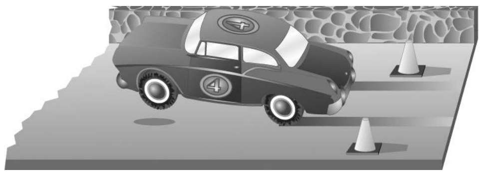
$$v_2^2 = (2)(0.30)gd = (2)(0.30)(9.81 \text{ m/s}^2)(110 \text{ m}) = 647.46 \text{ m}^2/\text{s}^2$$

$$v_2 = 25.445 \text{ m/s}$$

$v_2 = 91.6 \text{ km/h}$ ◀

Note: The car is treated as a particle in this problem. The weight distribution is assumed to be the same for static and dynamic conditions. Compare with sample Problem 16.1 where the vehicle is treated as a rigid body.

PROBLEM 13.8



Skid marks on a drag racetrack indicate that the rear (drive) wheels of a car slip for the first 20 m of the 400-m track. (*a*) Knowing that the coefficient of kinetic friction is 0.60, determine the speed of the car at the end of the first 20-m portion of the track if it starts from rest and the front wheels are just off the ground. (*b*) What is the maximum theoretical speed for the car at the finish line if, after skidding for 20 m, it is driven without the wheels slipping for the remainder of the race? Assume that while the car is rolling without slipping, 60 percent of the weight of the car is on the rear wheels and the coefficient of static friction is 0.75. Ignore air resistance and rolling resistance.

SOLUTION

(*a*) For the first 20 m, the normal force at the real wheels is equal to the weight of the car. Since the wheels are skidding, the friction force is

$$F = \mu_k N = \mu_k W = \mu_k mg$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$0 + Fd = \frac{1}{2}mv_2^2$$

$$0 + \mu_k mgd = \frac{1}{2}mv_2^2$$

$$v_2^2 = 2\mu_k gd = (2)(0.6)(9.81 \text{ m/s}^2)(20 \text{ m}) = 235.44 \text{ m}^2/\text{s}^2$$

$v_2 = 15.34$ m/s ◀

(*b*) Assume that for the remainder of the race, sliding is impending and $N = 0.6\ W$

$$F = \mu_s N = \mu_s (0.6W) = (0.75)(0.6mg) = 0.45mg$$

Principle of work and energy: $T_2 + U_{2\to 3} = T_3$

$$\frac{1}{2}mv_2^2 + (0.45mg)d' = \frac{1}{2}mv_3^2$$

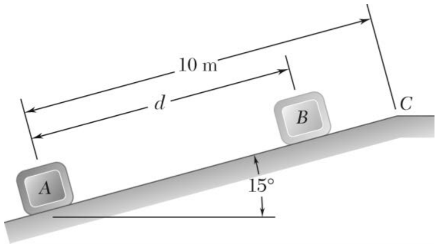
$$v_3^2 = v_2^2(2)(0.45)gd'$$

$$= 235.44 \text{ m}^2/\text{s}^2 + (2)(0.45)(9.81 \text{ m/s}^2)(400 \text{ m} - 20 \text{ m})$$

$$= 3590.5 \text{ m}^2/\text{s}^2$$

$v_3 = 59.9$ m/s ◀

PROBLEM 13.9



A package is projected up a 15° incline at A with an initial velocity of 8 m/s. Knowing that the coefficient of kinetic friction between the package and the incline is 0.12, determine (a) the maximum distance d that the package will move up the incline, (b) the velocity of the package as it returns to its original position.

SOLUTION

(a) Up the plane from A to B:

$$T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}\frac{W}{g}(8 \text{ m/s})^2 = 32\frac{W}{g} \qquad T_B = 0$$

$$U_{A-B} = (-W \sin 15° - F)d \qquad F = \mu_k N = 0.12\,\text{N}$$

$$\nwarrow \Sigma F = 0 \quad N - W\cos 15° = 0 \quad N = W\cos 15°$$

$$U_{A-B} = -W(\sin 15° + 0.12\cos 15°)d = -Wd(0.3747)$$

$$T_A + U_{A-B} = T_B: \quad 32\frac{W}{g} - Wd(0.3743) = 0$$

$$d = \frac{32}{(9.81)(0.3747)} \qquad d = 8.71 \text{ m} \blacktriangleleft$$

(b) Down the plane from B to A: (F reverses direction)

$$T_A = \frac{1}{2}\frac{W}{g}v_A^2 \qquad T_B = 0 \qquad d = 8.71 \text{ m/s}$$

$$U_{B-A} = (W \sin 15° - F)d$$

$$= W(\sin 15° - 0.12\cos 15°)(8.70 \text{ m/s})$$

$$U_{B-A} = 1.245W$$

$$T_B + U_{B-A} = T_A \quad 0 + 1.245W = \frac{1}{2}\frac{W}{g}v_A^2$$

$$v_A^2 = (2)(9.81)(1.245)$$

$$= 24.43$$

$$v_A = 4.94 \text{ m/s} \qquad \mathbf{v}_A = 4.94 \text{ m/s} \swarrow 15° \blacktriangleleft$$

PROBLEM 13.10

A 1.4 kg model rocket is launched vertically from rest with a constant thrust of 25 N until the rocket reaches an altitude of 15 m and the thrust ends. Neglecting air resistance, determine (*a*) the speed of the rocket when the thrust ends, (*b*) the maximum height reached by the rocket, (*c*) the speed of the rocket when it returns to the ground.

SOLUTION

Weight: $W = mg = (1.4)(9.81) = 13.734 \text{ N}$

(*a*) First stage: $T_1 = 0$

$$U_{1\to 2} = (25 - 13.734)(15) = 169.0 \text{ N}\cdot\text{m}$$

$$T_1 + U_{1\to 2} = T_2$$

$$T_2 = \frac{1}{2}mv^2 = U_{1\to 2} = 169.0 \text{ N}\cdot\text{m}$$

$$v_2 = \sqrt{\frac{2U_{1\to 2}}{m}} = \sqrt{\frac{(2)(169.0)}{1.4}}$$

$v_2 = 15.54 \text{ m/s}$ ◀

(*b*) Unpowered flight to maximum height *h*:

$$T_2 = 169.0 \text{ N}\cdot\text{m} \qquad T_3 = 0$$

$$U_{2\to 3} = -W(h - 15)$$

$$T_2 + U_{2\to 3} = T_3$$

$$W(h - 15) = T_2$$

$$h - 15 = \frac{T_2}{W} = \frac{169.0}{13.734}$$

$h = 27.3 \text{ m}$ ◀

(*c*) Falling from maximum height:

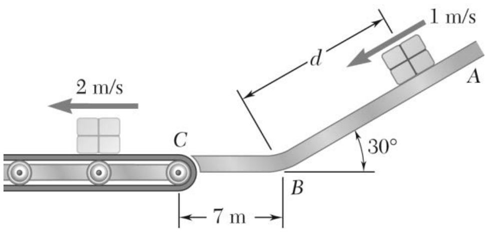
$$T_3 = 0 \qquad T_4 = \frac{1}{2}mv_4^2$$

$$U_{3\to 4} = Wh = mgh$$

$$T_3 = U_{3\to 4} = T_4: \quad 0 + mgh = \frac{1}{2}mv_4^2$$

$$v_4^2 = 2\,gh = (2)(9.81 \text{ m/s}^2)(27.3 \text{ m}) = 535.6 \text{ m}^2/\text{s}^2$$

$v_4 = 23.1 \text{ m/s}$ ◀



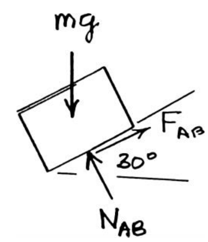
PROBLEM 13.11

Packages are thrown down an incline at A with a velocity of 1 m/s. The packages slide along the surface ABC to a conveyor belt which moves with a velocity of 2 m/s. Knowing that $\mu_k = 0.25$ between the packages and the surface ABC, determine the distance d if the packages are to arrive at C with a velocity of 2 m/s.

SOLUTION

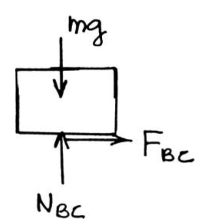
On incline AB:

$$N_{AB} = mg\cos 30°$$
$$F_{AB} = \mu_k N_{AB} = 0.25mg\ \cos 30°$$
$$U_{A\to B} = mgd\sin 30° - F_{AB}\,d$$
$$= mgd(\sin 30° - \mu_k\cos 30°)$$



On level surface BC:

$$N_{BC} = mg \qquad x_{BC} = 7\text{ m}$$
$$F_{BC} = \mu_k mg$$
$$U_{B\to C} = -\mu_k mg\ x_{BC}$$



At A, $$T_A = \frac{1}{2}mv_A^2 \quad\text{and}\quad v_A = 1\text{ m/s}$$

At C, $$T_C = \frac{1}{2}mv_C^2 \quad\text{and}\quad v_C = 2\text{ m/s}$$

Assume that no energy is lost at the corner B.

Work and energy. $$T_A + U_{A\to B} + U_{B\to C} = T_C$$

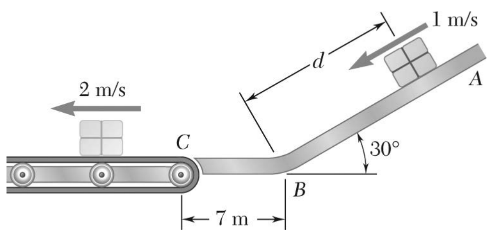
$$\frac{1}{2}mv_A^2 + mgd(\sin 30° - \mu_k\cos 30°) - \mu_k mg\ x_{BC} = \frac{1}{2}mv_0^2$$

Dividing by m and solving for d,

$$d = \frac{\left[v_C^2/2g + \mu_k x_{BC} - v_A^2/2g\right]}{(\sin 30° - \mu_k\cos 30°)}$$

$$= \frac{(2)^2/(2)(9.81) + (0.25)(7) - (1)^2/(2)(9.81)}{\sin 30° - 0.25\cos 30°}$$

$d = 6.71$ m ◀



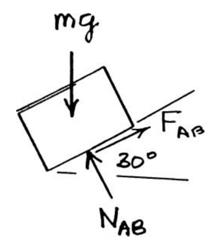
PROBLEM 13.12

Packages are thrown down an incline at A with a velocity of 1 m/s. The packages slide along the surface ABC to a conveyor belt which moves with a velocity of 2 m/s. Knowing that $d = 7.5$ m and $\mu_k = 0.25$ between the packages and all surfaces, determine (a) the speed of the package at C, (b) the distance a package will slide on the conveyor belt before it comes to rest relative to the belt.

SOLUTION

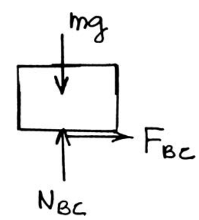
(a) On incline AB:

$$N_{AB} = mg\cos 30°$$
$$F_{AB} = \mu_k N_{AB} = 0.25mg\cos 30°$$
$$U_{A\to B} = mgd\sin 30° - F_{AB}d$$
$$= mgd(\sin 30° - \mu_k\cos 30°)$$



On level surface BC:

$$N_{BC} = mg \quad x_{BC} = 7 \text{ m}$$
$$F_{BC} = \mu_k mg$$
$$U_{B\to C} = -\mu_k mg\, x_{BC}$$



At A, $\quad T_A = \frac{1}{2}mv_A^2 \quad$ and $\quad v_A = 1$ m/s

At C, $\quad T_C = \frac{1}{2}mv_C^2 \quad$ and $\quad v_C = 2$ m/s

Assume that no energy is lost at the corner B.

Work and energy. $\quad T_A + U_{A\to B} + U_{B\to C} = T_C$

$$\frac{1}{2}mv_A^2 + mgd(\sin 30° - \mu_k\cos 30°) - \mu_k mg\, x_{BC} = \frac{1}{2}mv_0^2$$

Solving for v_C^2,

$$v_C^2 = v_A^2 + 2gd(\sin 30° - \mu_k\cos 30°) - 2\mu_k g\, x_{BC}$$
$$= (1)^2 + (2)(9.81)(7.5)(\sin 30° - 0.25\cos 30°) - (2)(0.25)(9.81)(7)$$
$$= 8.3811 \text{ m}^2/\text{s}^2$$

$v_C = 2.90$ m/s ◀

(b) Box on belt: Let x_{belt} be the distance moves by a package as it slides on the belt.

$$+\!\uparrow \Sigma F_y = ma_y \quad N - mg = 0 \quad N = mg$$
$$F_x = \mu_k N = \mu_k mg$$

At the end of sliding, $\quad v = v_{\text{belt}} = 2$ m/s

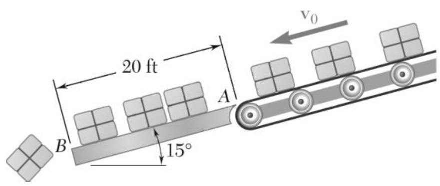
Principle of work and energy:

$$\frac{1}{2}mv_C^2 - \mu_k mg\, x_{\text{belt}} = \frac{1}{2}mv_{\text{belt}}^2$$

$$x_{\text{belt}} = \frac{v_C^2 - v_{\text{belt}}^2}{2\mu_k g}$$

$$= \frac{8.3811 - (2)^2}{(2)(0.25)(9.81)}$$

$x_{\text{belt}} = 0.893 \text{ m}$ ◀

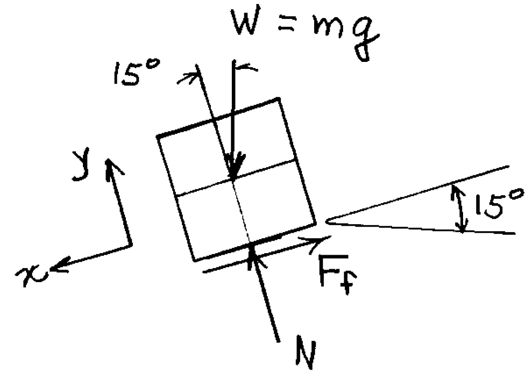


PROBLEM 13.13

Boxes are transported by a conveyor belt with a velocity $\mathbf{v}_0$ to a fixed incline at A where they slide and eventually fall off at B. Knowing that $\mu_k = 0.40$, determine the velocity of the conveyor belt if the boxes leave the incline at B with a velocity of 8 ft/s.

SOLUTION

Forces when box is on AB.



$$\Sigma F_y = 0: \quad N - W\cos 15° = 0$$

$$N = W\cos 15°$$

Box is sliding on AB.

$$F_f = \mu_k N = \mu_k W\cos 15°$$

Distance $\quad AB = d = 20 \text{ ft}$

Work of gravity force: $\quad (U_{A-B})_g = Wd\sin 15°$

Work of friction force: $\quad -F_f d = -\mu_k Wd\cos 15°$

Total work $\quad U_{A\to B} = Wd(\sin 15° - \mu_k \sin 15°)$

Kinetic energy: $\quad T_A = \frac{1}{2}\frac{W}{g}v_0^2$

$$T_B = \frac{1}{2}\frac{W}{g}v_B^2$$

Principle of work and energy: $\quad T_A + U_{A\to B} = T_B$

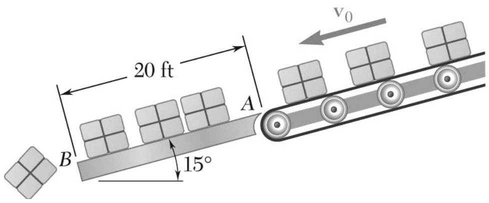
$$\frac{1}{2}\frac{W}{g}v_0^2 + Wd(\sin 15° - \mu_k\cos 15°) = \frac{1}{2}\frac{W}{g}v_B^2$$

$$v_0^2 = v_B^2 - 2gd(\sin 15° - \mu_k\cos 15°)$$

$$= (8)^2 - (2)(32.2)(20)[\sin 15° - (0.40)(\cos 15°)]$$

$$= 228.29 \text{ ft}^2/\text{s}^2$$

$\mathbf{v}_0 = 15.11$ ft/s ∡ 15° ◀



PROBLEM 13.14

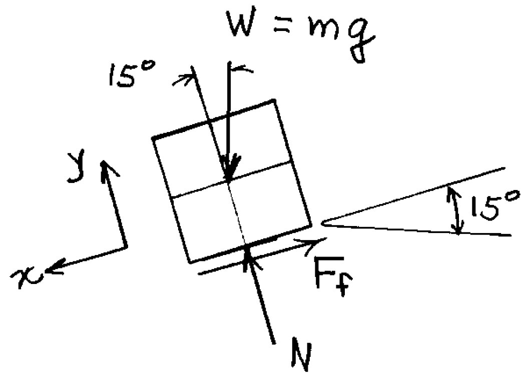
Boxes are transported by a conveyor belt with a velocity $\mathbf{v}_0$ to a fixed incline at A where they slide and eventually fall off at B. Knowing that $\mu_k = 0.40$, determine the velocity of the conveyor belt if the boxes are to have zero velocity at B.

SOLUTION

Forces when box is on AB.

$$\Sigma F_y = 0: \quad N - W\cos 15° = 0$$

$$N = W\cos 15°$$



Box is sliding on AB.

$$F_f = \mu_k N = \mu_k W\cos 15°$$

Distance $\quad AB = d = 20 \text{ ft}$

Work of gravity force: $\quad (U_{A-B})_g = Wd\sin 15°$

Work of friction force: $\quad -F_f d = -\mu_k Wd\cos 15°$

Total work $\quad U_{A-B} = Wd(\sin 15° - \mu_k \cos 15°)$

Kinetic energy:

$$T_A = \frac{1}{2}\frac{W}{g}v_0^2$$

$$T_B = \frac{1}{2}\frac{W}{g}v_B^2$$

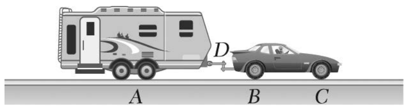
Principle of work and energy: $\quad T_A + U_{A-B} = T_B$

$$\frac{1}{2}\frac{W}{g}v_0^2 + Wd(\sin 15° - \mu_k \cos 15°) = \frac{1}{2}\frac{W}{g}v_B^2$$

$$\begin{aligned} v_0^2 &= v_B^2 - 2gd(\sin 15° - \mu_k \cos 15°) \\ &= 0 - (2)(32.2)(20)[\sin 15° - (0.40)(\cos 15°)] \\ &= 164.29 \text{ ft}^2/\text{s}^2 \end{aligned}$$

$\mathbf{v}_0 = 12.81 \text{ ft/s} \measuredangle 15°$ ◀

PROBLEM 13.15



A 1200-kg trailer is hitched to a 1400-kg car. The car and trailer are traveling at 72 km/h when the driver applies the brakes on both the car and the trailer. Knowing that the braking forces exerted on the car and the trailer are 5000 N and 4000 N, respectively, determine (*a*) the distance traveled by the car and trailer before they come to a stop, (*b*) the horizontal component of the force exerted by the trailer hitch on the car.

SOLUTION

Let position 1 be the initial state at velocity $v_1 = 72$ km/h $= 20$ m/s and position 2 be at the end of braking $(v_2 = 0)$. The braking forces and $F_C = 5000$ N for the car and 4000 N for the trailer.

(*a*) Car and trailer system. (d = braking distance)

$$T_1 = \frac{1}{2}(m_C + m_T)v_1^2 \qquad T_2 = 0$$

$$U_{1\to 2} = -(F_C + F_T)d$$

$$T_1 + U_{1\to 2} = T_2$$

$$\frac{1}{2}(m_C + m_T)v_1^2 - (F_C + F_T)d = 0$$

$$d = \frac{(m_C + m_T)v_1^2}{2(F_C + F_T)} = \frac{(2600)(20)^2}{(2)(9000)} = 57.778$$

$d = 57.8$ m ◀

(*b*) Car considered separately.

Let H be the horizontal pushing force that the trailer exerts on the car through the hitch.

$$T_1 = \frac{1}{2}m_C v_1^2 \qquad T_2 = 0$$

$$U_{1\to 2} = (H - F_C)d$$

$$T_1 + U_{1\to 2} = T_2$$

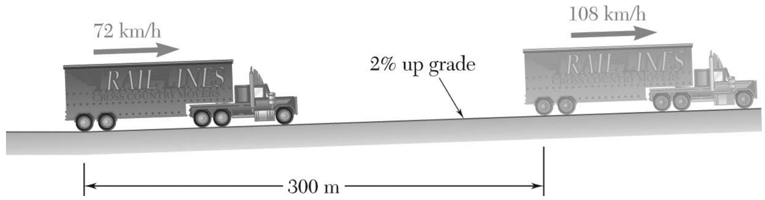
$$\frac{1}{2}m_C v_1^2 + (H - F_C)d = 0$$

$$H = F_C - \frac{m_C v_1^2}{2d} = 5000 - \frac{(1400)(20)^2}{(2)(57.778)}$$

Trailer hitch force on car: $\mathbf{H} = 154 \text{ N} \longrightarrow$ ◀

PROBLEM 13.16

A trailer truck enters a 2 percent uphill grade traveling at 72 km/h and reaches a speed of 108 km/h in 300 m. The cab has a mass of 1800 kg and the trailer 5400 kg. Determine (*a*) the average force at the wheels of the cab, (*b*) the average force in the coupling between the cab and the trailer.



SOLUTION

Initial speed: $v_1 = 72 \text{ km/h} = 20 \text{ m/s}$

Final speed: $v_2 = 108 \text{ km/h} = 30 \text{ m/s}$

Vertical rise: $h = (0.02)(300) = 6.00 \text{ m}$

Distance traveled: $d = 300 \text{ m}$

(*a*) Traction force. Use cab and trailer as a free body.

$m = 1800 + 5400 = 7200 \text{ kg}$ $\quad$ $W = mg = (7200)(9.81) = 70.632 \times 10^3 \text{ N}$

Work and energy: $T_1 + U_{1\to 2} = T_2$ $\quad$ $\frac{1}{2}mv_1^2 - Wh + F_t d = \frac{1}{2}mv_2^2$

$$F_t = \frac{1}{d}\left[\frac{1}{2}mv_1^2 + Wh - mv_1^2\right] = \frac{1}{300}\left[\frac{1}{2}(7200)(30)^2 + (70.632\times 10^3)(6.00) - \frac{1}{2}(7200)(20)^2\right]$$

$= 7.4126 \times 10^3 \text{ N}$ $\quad$ $F_t = 7.41 \text{ kN}$ ◀

(*b*) Coupling force F_c. Use the trailer alone as a free body.

$m = 5400 \text{ kg}$ $\quad$ $W = mg = (5400)(9.81) = 52.974 \times 10^3 \text{ N}$

Assume that the tangential force at the trailer wheels is zero.

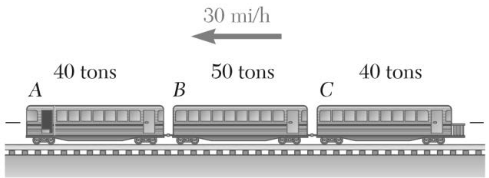
Work and energy: $T_1 + U_{1\to 2} = T_2$ $\quad$ $\frac{1}{2}mv_1^2 - Wh + F_c d = \frac{1}{2}mv_2^2$

The plus sign before F_c means that we have assumed that the coupling is in tension.

$$F_c = \frac{1}{d}\left[\frac{1}{2}mv_2^2 + Wh - \frac{1}{2}mv_1^2\right] = \frac{1}{300}\left[\frac{1}{2}(5400)(30)^2 + (52.974\times 10^3)(6.00) - \frac{1}{2}(5400)(20)^2\right]$$

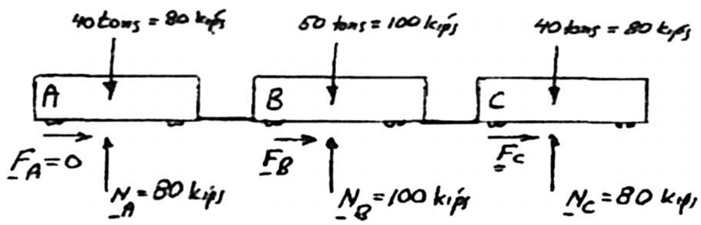
$= 5.5595 \times 10^3 \text{ N}$ $\quad$ $F_c = 5.56 \text{ kN (tension)}$ ◀

PROBLEM 13.17



The subway train shown is traveling at a speed of 30 mi/h when the brakes are fully applied on the wheels of cars *B* and *C*, causing them to slide on the track, but are not applied on the wheels of car *A*. Knowing that the coefficient of kinetic friction is 0.35 between the wheels and the track, determine (*a*) the distance required to bring the train to a stop, (*b*) the force in each coupling.

SOLUTION



$$\mu_k = 0.35 \quad F_B = (0.35)(100 \text{ kips}) = 35 \text{ kips}$$

$$F_C = (0.35)(80 \text{ kips}) = 28 \text{ kips}$$

$$v_1 = 30 \text{ mi/h} = 44 \text{ ft/s} \longleftarrow \qquad v_2 = 0 \quad T_2 = 0$$

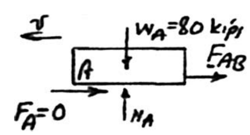
(*a*) Entire train: $T_1 + U_{1-2} = T_2$

$$\frac{1}{2}\frac{(80 \text{ kips} + 100 \text{ kips } + 80 \text{ kips})}{32.2 \text{ ft/s}^2}(44 \text{ ft/s})^2 - (28 \text{ kips} + 35 \text{ kips})\, x = 0$$

$$x = 124.07 \text{ ft} \qquad x = 124.1 \text{ ft} \blacktriangleleft$$

(*b*) Force in each coupling: Recall that $x = 124.07$ ft

Car *A*: Assume F_{AB} to be in tension



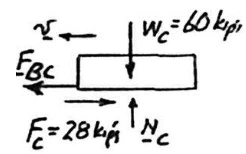
$$T_1 + V_{1-2} = T_2$$

$$\frac{1}{2}\frac{80 \text{ kips}}{32.2}(44)^2 - F_{AB}(124.07 \text{ ft}) = 0$$

$$F_{AB} = +19.38 \text{ kips}$$

$$F_{AB} = 19.38 \text{ kips (tension)} \blacktriangleleft$$

Car *C*: $T_1 + U_{1-2} = T_2$

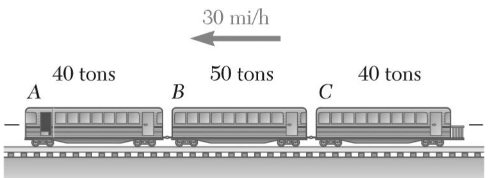


$$\frac{1}{2}\frac{80 \text{ kips}}{32.2}(44)^2 + (F_{BC} - 28 \text{ kips})(124.07 \text{ ft}) = 0$$

$$F_{BC} - 28 \text{ kips} = -19.38 \text{ kips}$$

$$F_{BC} = +8.62 \text{ kips} \qquad F_{BC} = 8.62 \text{ kips (tension)} \blacktriangleleft$$

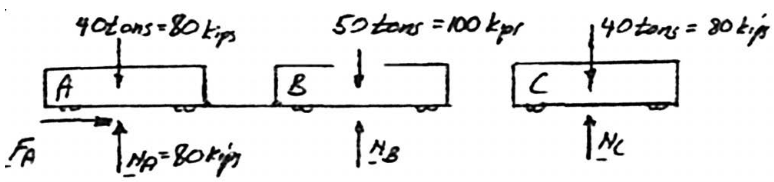
PROBLEM 13.18



The subway train shown is traveling at a speed of 30 mi/h when the brakes are fully applied on the wheels of cars *A*, causing it to slide on the track, but are not applied on the wheels of cars *A* or *B*. Knowing that the coefficient of kinetic friction is 0.35 between the wheels and the track, determine (*a*) the distance required to bring the train to a stop, (*b*) the force in each coupling.

SOLUTION

(*a*) Entire train:



$$F_A = \mu N_A = (0.35)(80 \text{ kips}) = 28 \text{ kips}$$

$$v_1 = 30 \text{ mi/h} = 44 \text{ ft/s} \leftarrow \qquad v_2 = 0 \quad T_2 = 0$$

$$T_1 + V_{1-2} = T_2$$

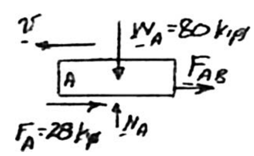
$$\frac{1}{2}\frac{(80 \text{ kips} + 100 \text{ kips} + 80 \text{ kips})}{32.2 \text{ ft/s}^2}(44 \text{ ft/s})^2 - (28 \text{ kips})\, x = 0$$

$$x = 279.1 \text{ ft}$$

$x = 279$ ft ◀

(*b*) Force in each coupling:

Car *A*: Assume F_{AB} to be in tension



$$T_1 + V_{1-2} = T_2$$

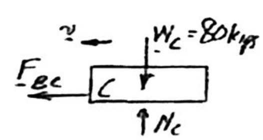
$$\frac{1}{2}\frac{80 \text{ kips}}{32.2 \text{ ft/s}^2}(44 \text{ ft/s})^2 - (28 \text{ kips} + F_{AB})(279.1 \text{ ft}) = 0$$

$$28 \text{ kips} + F_{AB} = +8.62 \text{ kips}$$

$$F_{AB} = -19.38 \text{ kips}$$

$F_{AB} = 19.38$ kips (compression) ◀

Car *C*:

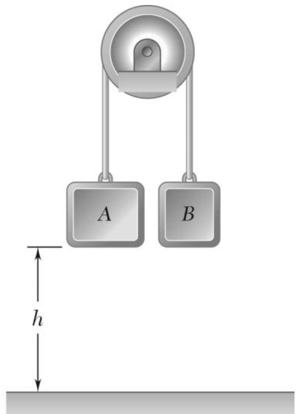


$$T_1 + V_{1-2} = T_2$$

$$\frac{1}{2}\frac{80 \text{ kips}}{32.2 \text{ ft/s}^2}(44 \text{ ft/s})^2 + F_{BC}(279.1 \text{ ft}) = 0$$

$$F_{BC} = -8.617 \text{ kips}$$

$F_{BC} = 8.62$ kips (compression) ◀



PROBLEM 13.19

Blocks A and B weigh 25 lbs and 10 lbs, respectively, and they are both at a height 6 ft above the ground when the system is released from rest. Just before hitting the ground block A is moving at a speed of 9 ft/s. Determine (a) the amount of energy dissipated in friction by the pulley, (b) the tension in each portion of the cord during the motion.

SOLUTION

By constraint of the cable block B moves up a distance h when block A moves down a distance h. ($h = 6$ ft) Their speeds are equal.

Let F_A and F_B be the tensions on the A and B sides, respectively, of the pulley.

Masses:
$$M_A = \frac{W_A}{g} = \frac{25}{32.2} = 0.7764 \text{ lb}\cdot\text{s}^2/\text{ft}$$
$$M_B = \frac{W_B}{g} = \frac{10}{32.2} = 0.31056 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Let position 1 be the initial position with both blocks a distance h above the ground and position 2 be just before block A hits the ground.

Kinetic energies:
$$(T_1)_A = 0, \qquad (T_1)_B = 0$$
$$(T_2)_A = \frac{1}{2}m_A v^2 = \frac{1}{2}(0.7764)(9)^2 = 31.444 \text{ ft}\cdot\text{lb}$$
$$(T_2)_B = \frac{1}{2}m_B v^2 = \frac{1}{2}(0.31056)(9)^2 = 12.578 \text{ ft}\cdot\text{lb}$$

Principle of work and energy:
$$T_1 + U_{1\to 2} = T_2$$

Block A:
$$U_{1\to 2} = (W_A - F_A)h$$
$$0 + (25 - F_A)(6) = 31.444 \qquad F_A = 19.759 \text{ lb}$$

Block B:
$$U_{1\to 2} = (F_B - W_B)h$$
$$0 + (F_B - 10)(6) = 12.578 \qquad F_B = 12.096 \text{ lb}$$

At the pulley F_A moves a distance h down, and F_B moves a distance h up. The work done is
$$U_{1\to 2} = (F_A - F_B)h = (19.759 - 12.096)(6) = 46.0 \text{ ft}\cdot\text{lb}$$

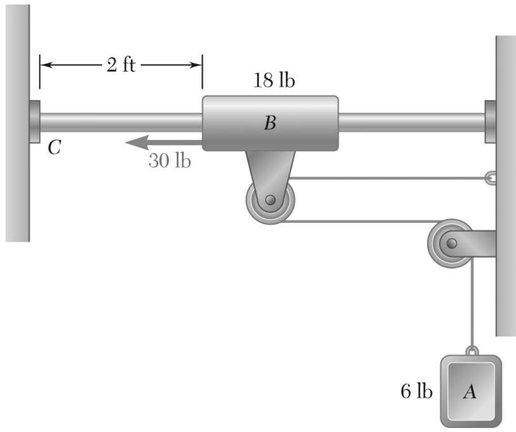
PROBLEM 13.19 (Continued)

Since the pulley is assumed to be massless, it cannot acquire kinetic energy; hence,

(*a*) Energy dissipated by the pulley: $E_p = 46.0 \text{ ft} \cdot \text{lb}$ ◀

(b) Tension in each portion of the cord: $A: 19.76 \text{ lb}$ ◀

$B: 12.10 \text{ lb}$ ◀



PROBLEM 13.20

The system shown is at rest when a constant 30 lb force is applied to collar *B*. (*a*) If the force acts through the entire motion, determine the speed of collar *B* as it strikes the support at *C*. (*b*) After what distance *d* should the 30 lb force be removed if the collar is to reach support *C* with zero velocity?

SOLUTION

Let *F* be the cable tension and v_B be the velocity of collar *B* when it strikes the support. Consider the collar *B*. Its movement is horizontal so only horizontal forces acting on *B* do work. Let *d* be the distance through which the 30 lb applied force moves.

$$(T_1)_B + (U_{1\to 2})_B = (T_2)_B$$

$$0 + 30d - (2F)(2) = \frac{1}{2}\frac{18}{32.2}v_B^2$$

$$30d - 4F = 0.27950 v_B^2 \qquad (1)$$

Now consider the weight *A*. When the collar moves 2 ft to the left, the weight moves 4 ft up, since the cable length is constant. Also, $v_A = 2v_B$.

$$(T_1)_A + (U_{1-2})_A = (T_2)_B$$

$$0 + (F - W_A)(4) = \frac{1}{2}\frac{W_A}{g}v_A^2$$

$$4F - (6)(4) = \frac{1}{2}\frac{6}{32.2}(2v_B)^2$$

$$4F - 24 = 0.37267\, v_B^2 \qquad (2)$$

Add Eqs. (1) and (2) to eliminate *F*.

$$30d - 24 = 0.65217 v_B^2 \qquad (3)$$

(*a*) Case *a*: $d = 2$ ft, $v_B = ?$

$$(30)(2) - (24) = 0.65217 v_B^2$$

$$v_B^2 = 55.2 \text{ ft}^2/\text{s}^2$$

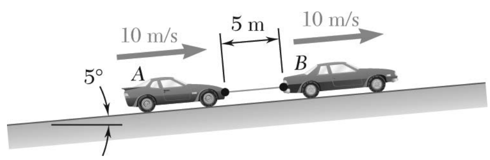
$v_B = 7.43$ ft/s ◀

(*b*) Case *b*: $d = ?$, $v_B = 0$.

$$30d - 24 = 0$$

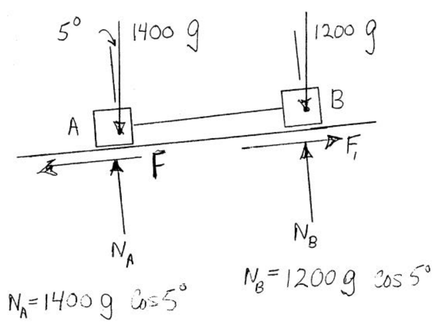
$d = 0.800$ ft ◀

PROBLEM 13.21



Car B is towing car A at a constant speed of 10 m/s on an uphill grade when the brakes of car A are fully applied causing all four wheels to skid. The driver of car B does not change the throttle setting or change gears. The masses of the cars A and B are 1400 kg and 1200 kg, respectively, and the coefficient of kinetic friction is 0.8. Neglecting air resistance and rolling resistance, determine (*a*) the distance traveled by the cars before they come to a stop, (*b*) the tension in the cable.

SOLUTION



$F = 0.8N_A$

Given: Car B tows car A at 10 m/s uphill.

Car A brakes so 4 wheels skid. $\mu_k = 0.8$

Car B continues in same gear and throttle setting.

Find: (*a*) Distance d, traveled to stop

(*b*) Tension in cable

(*a*) F_1 = traction force (from equilibrium)

$$F_1 = (1400g)\sin 5° + (1200g)\sin 5°$$
$$= 2600(9.81)\sin 5°$$

For system: $A + B$

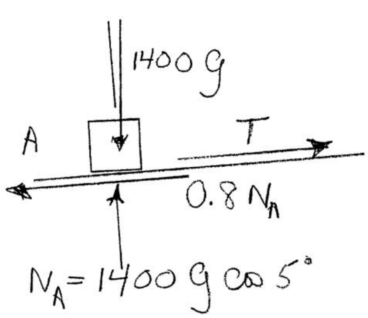
$$U_{1-2} = [(F_1 - 1400g\sin 5° - 1200g\sin 5°) - F]d$$
$$= T_2 - T_1 = 0 - \frac{1}{2}m_{A+B}v^2 = -\frac{1}{2}(2600)(10)^2$$

Since $(F_1 - 1400g\sin 5° - 1200g\sin 5°) = 0$

$$-Fd = -0.8[1400(9.81)\cos 5°]d = -130{,}000 \text{ N}\cdot\text{m}$$

$d = 11.88$ m ◀

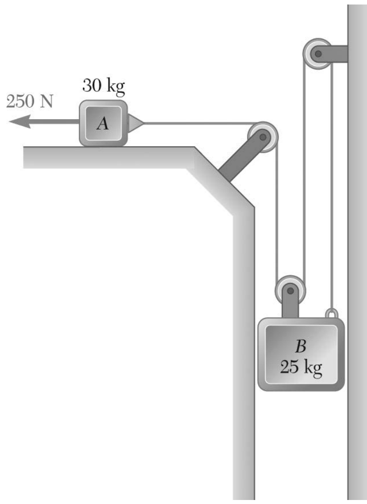
(*b*) Cable tension, T



$$U_{1-2} = [T - 0.8N_A](11.88) = T_2 - T_1$$
$$(T - 0.8(1400)(9.81)\cos 5°)11.88 = -\frac{1400}{2}(10)^2$$
$$(T - 10945) = -5892$$
$$= 5.053 \text{ kN}$$

$T = 5.05$ kN ◀

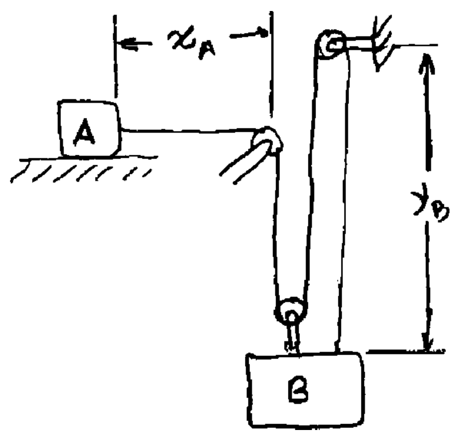
PROBLEM 13.22



The system shown is at rest when a constant 250-N force is applied to block A. Neglecting the masses of the pulleys and the effect of friction in the pulleys and between block A and the horizontal surface, determine (*a*) the velocity of block B after block A has moved 2 m, (*b*) the tension in the cable.

SOLUTION

Constraint of cable:



$$x_A + 3y_B = \text{constant}$$
$$\Delta x_A + 3\Delta y_B = 0$$
$$v_A + 3v_B = 0$$

Let F be the tension in the cable.

Block A: $\quad m_A = 30 \text{ kg}, \quad P = 250 \text{ N}, \quad (T_1)_A = 0$

$$(T_1)_A + (U_{1\to 2})_A = (T_2)_A$$
$$0 + (P - F)(\Delta x_A) = \frac{1}{2} m_A v_A^2$$
$$0 + (250 - F)(2) = \frac{1}{2}(30)(3v_B)^2$$
$$500 - 2F = 135 v_B^2 \qquad (1)$$

Block B: $\quad m_B = 25 \text{ kg}, \quad W_B = m_B g = 245.25 \text{ N}$

$$(T_1)_B + (U_{1\to 2})_B = (T_2)_B$$
$$0 + (3F - W_B)(-\Delta y_B) = \frac{1}{2} m_B v_B^2$$
$$(3F) - 245.25)\left(\frac{2}{3}\right) = \frac{1}{2}(25)\, v_B^2$$
$$2F - 163.5 = 12.5\, v_B^2 \qquad (2)$$

Add Eqs. (1) and (2) to eliminate F.

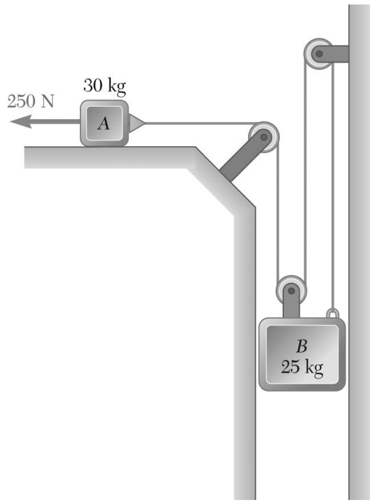
$$500 - 163.5 = 147.5 v_B^2$$

$$v_B^2 = 2.2814 \text{ m}^2/\text{s}^2$$

(*a*) Velocity of B. $\mathbf{v}_B = 1.510 \text{ m/s} \longleftarrow$ ◀

(*b*) Tension in the cable.

From Eq. (2), $2F - 163.5 = (12.5)(2.2814)$ $F = 96.0 \text{ N}$ ◀



PROBLEM 13.23

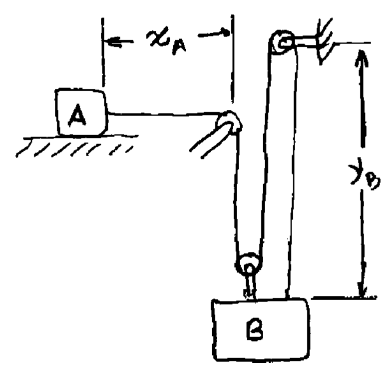
The system shown is at rest when a constant 250-N force is applied to block *A*. Neglecting the masses of the pulleys and the effect of friction in the pulleys and assuming that the coefficients of friction between block *A* and the horizontal surface are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (*a*) the velocity of block *B* after block *A* has moved 2 m, (*b*) the tension in the cable.

SOLUTION

Check the equilibrium position to see if the blocks move. Let *F* be the tension in the cable.

Block *B*:
$$3F - m_B g = 0$$
$$F = \frac{m_B g}{3} = \frac{(25)(9.81)}{3} = 81.75 \text{ N}$$

Block *A*:
$$+\uparrow \Sigma F_y = 0: \quad N_A - m_A g = 0$$
$$N_A = m_A g = (30)(9.81) = 294.3 \text{ N}$$
$$\overset{+}{\leftarrow} \Sigma F_x = 0: \quad 250 - F_A - F = 0$$
$$F_A = 250 - 81.75 = 168.25 \text{ N}$$



Available static friction force: $\mu_s N_A = (0.25)(294.3) = 73.57$ N

Since $F_A > \mu_s N_A$, the blocks move.

The friction force, F_A, during sliding is
$$F_A = \mu_k N_A = (0.20)(294.3) = 58.86 \text{ N}$$

Constraint of cable:
$$x_A + 3y_B = \text{constant}$$
$$\Delta x_A + 3\Delta y_B = 0$$
$$v_A + 3v_B = 0$$

PROBLEM 13.23 (Continued)

Block A: $m_A = 30 \text{ kg}, \quad P = 250 \text{ N}, \quad (T_1)_A = 0.$

$$(T_1)_A + (U_{1\to 2})_A = (T_2)_A$$

$$0 + (P - F_A - F)(\Delta x_A) = \frac{1}{2} m_A v_A^2$$

$$0 + (250 - 58.86 - F)(2) = \frac{1}{2}(30)(3v_B)^2$$

$$382.28 - 2F = 135 v_B^2 \qquad (1)$$

Block B: $M_B = 25 \text{ kg}, \quad W_B = m_B g = 245.25 \text{ N}$

$$(T_1)_B + (U_{1\to 2})_B = (T_2)_B$$

$$0 + (3F - W_B)(-\Delta y_B) = \frac{1}{2} m_B v_B^2$$

$$(3F - 245.25)\left(\frac{2}{3}\right) = \frac{1}{2}(25)v_B^2$$

$$2F - 163.5 = 12.5 v_B^2 \qquad (2)$$

Add Eqs. (1) and (2) to eliminate F.

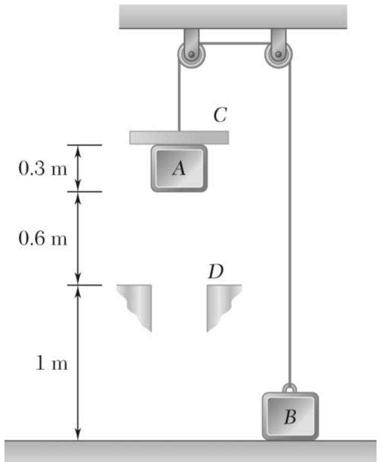
$$382.28 - 163.5 = 147.5 v_B^2$$

$$v_B^2 = 1.48325 \text{ m}^2/\text{s}^2$$

(a) Velocity of B: $\mathbf{v}_B = 1.218 \text{ m/s} \longleftarrow$ ◀

(b) Tension in the cable:

From Eq. (2), $2F - 163.5 = (12.5)(1.48325)$ $F = 91.0 \text{ N}$ ◀



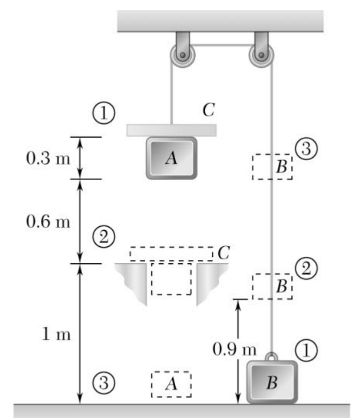
PROBLEM 13.24

Two blocks A and B, of mass 4 kg and 5 kg, respectively, are connected by a cord which passes over pulleys as shown. A 3 kg collar C is placed on block A and the system is released from rest. After the blocks have moved 0.9 m, collar C is removed and blocks A and B continue to move. Determine the speed of block A just before it strikes the ground.

SOLUTION

Position ① to Position ②. $v_1 = 0 \quad T_1 = 0$

At ② before C is removed from the system



$$T_2 = \frac{1}{2}(m_A + m_B + m_C)v_2^2 = \frac{1}{2}(12 \text{ kg})v_2^2 = 6v_2^2$$

$$U_{1-2} = (m_A + m_C - m_B)g(0.9 \text{ m})$$

$$U_{1-2} = (4+3-5)(g)(0.9 \text{ m}) = (2 \text{ kg})(9.81 \text{ m/s}^2)(0.9 \text{ m})$$

$$U_{1-2} = 17.658 \text{ J}$$

$$T_1 + U_{1-2} = T_2:$$

$$0 + 17.658 = 6v_2^2 \qquad v_2^2 = 2.943$$

At Position ②, collar C is removed from the system.

Position ② to Position ③. $T_2' = \frac{1}{2}(m_A + m_B)v_2^2 = \left(\frac{9}{2}\text{ kg}\right)(2.943) = 13.244 \text{ J}$

$$T_3 = \frac{1}{2}(m_A + m_B)(v_3)^2 = \frac{9}{2}v_3^2$$

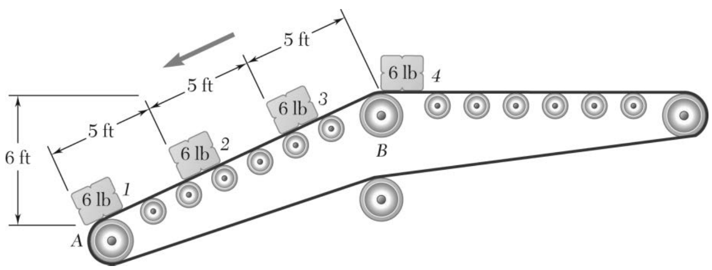
$$U_{2'-3} = (m_A - m_B)(g)(0.7 \text{ m}) = (-1 \text{ kg})(9.81 \text{ m/s}^2)(0.7 \text{ m}) = -6.867 \text{ J}$$

$$T_2' + U_{2-3} = T_3$$

$$13.244 - 6.867 = 4.5v_3^2 \qquad v_3^2 = 1.417$$

$$v_A = v_3 = 1.190 \text{ m/s}$$

$v_A = 1.190$ m/s ◀



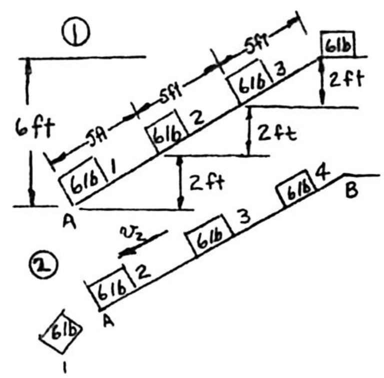
PROBLEM 13.25

Four packages, each weighing 6 lb, are held in place by friction on a conveyor which is disengaged from its drive motor. When the system is released from rest, package 1 leaves the belt at *A* just as package 4 comes onto the inclined portion of the belt at *B*. Determine (*a*) the speed of package 2 as it leaves the belt at *A*, (*b*) the speed of package 3 as it leaves the belt at *A*. Neglect the mass of the belt and rollers.

SOLUTION

Slope angle: $\sin\ \beta = \dfrac{6 \text{ ft}}{15 \text{ ft}} \qquad \beta = 23.6°$

(*a*) Package falls off the belt and 2, 3, 4 move down



$$\frac{6}{3} = 2 \text{ ft.}$$

$$T_2 = 3\left[\frac{1}{2}mv_2^2\right] = \frac{3}{2}\left(\frac{6 \text{ lb}}{32.2 \text{ ft/s}^2}\right)v_2^2 = 0.2795v_2^2$$

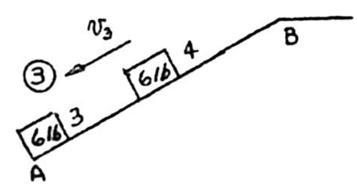
$$U_{1-2} = (3)(W)(R) = (3)(6 \text{ lb})(2 \text{ ft}) = 36 \text{ lb}\cdot\text{ft}$$

$$T_1 + U_{1-2} = T_2$$

$$0 + 36 = 0.2795v_2^2 \qquad v_2^2 = 128.8$$

$\mathbf{v}_2 = 11.35 \text{ ft/s} \measuredangle\ 23.6°$ ◀

(*b*) Package 2 falls off the belt and its energy is lost to the system and 3 and 4 move down 2 ft.



$$T_2' = (2)\left[\frac{1}{2}mv_2^2\right] = \left(\frac{6 \text{ lb}}{32 \text{ ft/s}^2}\right)(128.8)$$

$$T_2' = 24 \text{ lb}\cdot\text{ft}$$

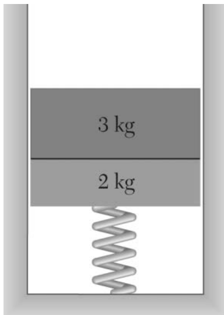
$$T_3 = (2)\left[\frac{1}{2}mv_3^2\right] = \left(\frac{6 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(v_3^2) = 0.18634v_3^2$$

$$U_{2-3} = (2)(W)(2) = (2)(6 \text{ lb})(2 \text{ ft}) = 24 \text{ lb}\cdot\text{ft}$$

$$T_2 + U_{2-3} = T_3$$

$$24 + 24 = 0.18634v_3^2 \qquad v_3^2 = 257.6$$

$\mathbf{v}_3 = 16.05 \text{ ft/s} \measuredangle\ 23.6°$ ◀



PROBLEM 13.26

A 3-kg block rests on top of a 2-kg block supported by, but not attached to, a spring of constant 40 N/m. The upper block is suddenly removed. Determine (*a*) the maximum speed reached by the 2-kg block, (*b*) the maximum height reached by the 2-kg block.

SOLUTION

Call blocks *A* and *B*. $m_A = 2\text{ kg}, \quad m_B = 3\text{ kg}$

(*a*) Position 1: Block *B* has just been removed.

Spring force: $F_S = -(m_A + m_B)g = -kx \uparrow$

Spring stretch: $x_1 = -\dfrac{(m_A+m_B)g}{k} = -\dfrac{(5\text{ kg})(9.81\text{ m/s}^2)}{40\text{ N/m}} = -1.22625\text{ m}$

Let position 2 be a later position while the spring still contacts block *A*.

Work of the force exerted by the spring: $(U_{1\to2})_e = -\int_{x_1}^{x_2} kx\,dx$

$$= -\frac{1}{2}kx^2\Big|_{x_1}^{x_2} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

$$= \frac{1}{2}(40)(-1.22625)^2 - \frac{1}{2}(40)x_2^2 = 30.074 - 20x_2^2$$

Work of the gravitational force: $(U_{1\to2})_g = -m_A g(x_2 - x_1)$

$$= -(2)(9.81)(x_2 + 1.22625) = -19.62x_2 - 24.059$$

Total work: $U_{1\to2} = -20x_2^2 + 19.62x_2 + 6.015$

Kinetic energies: $T_1 = 0$

$$T_2 = \frac{1}{2}m_A v_2^2 = \frac{1}{2}(2)v_2^2 = v_2^2$$

Principle of work and energy: $T_1 + U_{1\to2} = T_2$

$$0 + 20x_2^2 - 19.62x_2 + 6.015 = v_2^2$$

Speed squared: $v_2^2 = -20x_2^2 - 19.62x_2 + 6.015$ (1)

At maximum speed, $\dfrac{dv_2}{dx_2} = 0$

PROBLEM 13.26 (Continued)

Differentiating Eq. (1), and setting equal to zero,

$$2v_2 \frac{dv_2}{dx} = -40x_2 = -19.62 = 0$$

$$x_2 = -\frac{19.62}{40} = -0.4905 \text{ m}$$

Substituting into Eq. (1), $v_2^2 = -(20)(-0.4905)^2 - (19.62)(-0.4905) + 6.015 = 10.827 \text{ m}^2/\text{s}^2$

Maximum speed: $v^2 = 3.29 \text{ m/s}$ ◀

(*b*) Position 3: Block *A* reaches maximum height. Assume that the block has separated from the spring. Spring force is zero at separation.

Work of the force exerted by the spring:

$$(U_{1\to 3})_e = -\int_{x_1}^{0} kx dx = \frac{1}{2}kx_1^2 = \frac{1}{2}(40)(1.22625)^2 = 30.074 \text{ J}$$

Work of the gravitational force:

$$(U_{1\to 3})_g = -m_A gh = -(2)(9.81)h = -19.62\ h$$

Total work: $U_{1\to 3} = 30.074 - 19.62\ h$

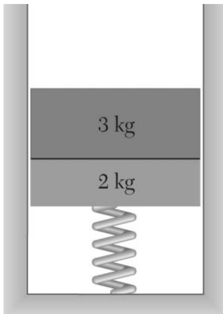
At maximum height, $v_3 = 0, \quad T_3 = 0$

Principle of work and energy: $T_1 + U_{1\to 3} = T_3$

$$0 + 30.074 - 19.62\ h = 0$$

Maximum height: $h = 1.533 \text{ m}$ ◀

PROBLEM 13.27



Solve Problem 13.26, assuming that the 2-kg block is attached to the spring.

PROBLEM 13.26 A 3-kg block rests on top of a 2-kg block supported by, but not attached to, a spring of constant 40 N/m. The upper block is suddenly removed. Determine (*a*) the maximum speed reached by the 2-kg block, (*b*) the maximum height reached by the 2-kg block.

SOLUTION

Call blocks *A* and *B*. $m_A = 2\text{ kg},\quad m_B = 3\text{ kg}$

(*a*) Position 1: Block *B* has just been removed.

Spring force: $F_S = -(m_A + m_B)g = -kx_1$

Spring stretch: $x_1 = -\dfrac{(m_A + m_B)g}{k} = -\dfrac{(5\text{ kg})(9.81\text{ m/s}^2)}{40\text{ N/m}} = -1.22625\text{ m}$

Let position 2 be a later position. Note that the spring remains attached to block *A*.

Work of the force exerted by the spring: $(U_{1\to 2})_e = -\int_{x_1}^{x_2} kx\,dx$

$$= -\frac{1}{2}kx^2\Big|_{x_1}^{x_2} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

$$= \frac{1}{2}(40)(-1.22625)^2 - \frac{1}{2}(40)x_2^2 = 30.074 - 20x_2^2$$

Work of the gravitational force: $(U_{1\to 2})_g = -m_A g(x_2 - x_1)$

$$= -(2)(9.81)(x_2 + 1.22625) = -19.62x_2 - 24.059$$

Total work: $U_{1\to 2} = -20x_2^2 - 19.62x_2 + 6.015$

Kinetic energies: $T_1 = 0$

$$T_2 = \frac{1}{2}m_A v_2^2 = \frac{1}{2}(2)v_2^2 = v_2^2$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$0 + 20x_2^2 - 19.62x_2 + 6.015 = v_2^2$$

Speed squared: $v_2^2 = -20x_2^2 - 19.62x_2 + 6.015$ (1)

At maximum speed, $\dfrac{dv_2}{dx_2} = 0$

Differentiating Eq. (1) and setting equal to zero,

$$2v_2 \frac{dv_2}{dx_2} = -40x_2 = -19.62 = 0$$

$$x_2 = -\frac{19.62}{40} = -0.4905 \text{ m}$$

Substituting into Eq. (1), $v_2^2 = -(20)(-0.4905)^2 - (19.62)(-0.4905) + 6.015 = 10.827 \text{ m}^1/\text{s}^2$

Maximum speed: $v_2 = 3.29 \text{ m/s}$ ◀

(*b*) Maximum height occurs when $v_2 = 0$.

Substituting into Eq. (1), $0 = -20x_2^2 - 19.62x_2 + 6.015$

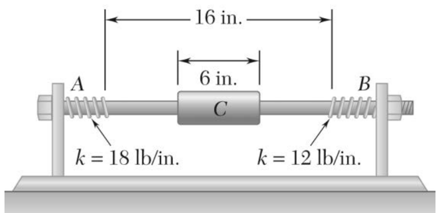
Solving the quadratic equation

$$x_2 = -1.22625 \text{ m} \quad \text{and} \quad 0.24525 \text{ m}$$

Using the larger value, $x_2 = 0.24525 \text{ m}$

Maximum height: $h = x_2 - x_1 = 0.24525 + 1.22625$ $h = 1.472 \text{ m}$ ◀

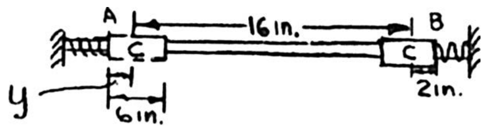
PROBLEM 13.28



An 8-lb collar C slides on a horizontal rod between springs A and B. If the collar is pushed to the right until spring B is compressed 2 in. and released, determine the distance through which the collar will travel assuming (a) no friction between the collar and the rod, (b) a coefficient of friction $\mu_k = 0.35$.

SOLUTION

(a)



$$k_B = 144 \text{ lb/ft}$$
$$k_A = 216 \text{ lb/ft}$$

Since the collar C leaves the spring at B and there is no friction, it must engage the spring at A.

$$T_A = 0 \qquad T_B = 0$$

$$U_{A-B} = \int_0^{2/12} k_B x dx - \int_0^y k_A x dx$$

$$U_{A-B} = \left(\frac{144 \text{ lb/ft}}{2}\right)\left(\frac{2}{12}\text{ ft}\right)^2 - \left(\frac{216 \text{ lb/ft}}{2}\right)(y)^2$$

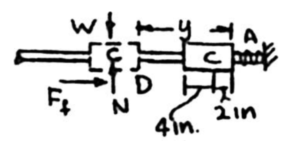
$$T_A + U_{A-B} = T_B: \qquad 0 + 2 - 108y^2 = 0$$
$$y = 0.1361 \text{ ft} = 1.633 \text{ in.}$$

Total distance $\qquad d = 2 + 16 - (6 - 1.633)$

$d = 13.63$ in. ◀

(b) Assume that C does not reach the spring at B because of friction.

$$N = W = 6 \text{ lb}$$
$$F_f = (0.35)(8 \text{ lb}) = 2.80 \text{ lb}$$
$$T_A = T_D = 0$$

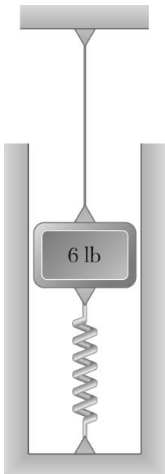


$$U_{A-D} = \int_0^{2/12} 144 \times dx - F_f(y) = 2 - 2.80y$$

$$T_A + U_{A-D} = T_D \quad 0 + 2 - 2.80y = 0$$
$$y = 0.714 \text{ ft} = 8.57 \text{ in.}$$

The collar must travel $16 - 6 + 2 = 12$ in. before it engages the spring at B. Since $y = 8.57$ in., it stops before engaging the spring at B.

Total distance $d = 8.57$ in. ◀

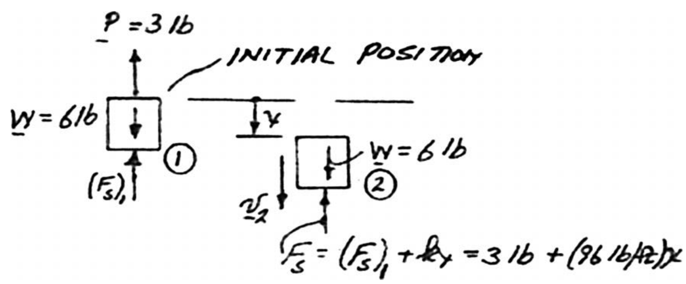


PROBLEM 13.29

A 6-lb block is attached to a cable and to a spring as shown. The constant of the spring is $k = 8$ lb/in. and the tension in the cable is 3 lb. If the cable is cut, determine (*a*) the maximum displacement of the block, (*b*) the maximum speed of the block.

SOLUTION

$$k = 8 \text{ lb/in.} = 96 \text{ lb/ft}$$



$$\Sigma F_y = 0: \quad (F_s)_1 = 6 - 3 = 3 \text{ lb } C$$

$$v_1 = 0 \quad T_1 = 0: \quad T_2 = \frac{1}{2}\left(\frac{6 \text{ lb}}{32.2}\right)v_2^2 = 0.09317 v_2^2$$

For weight: $\quad U_{1-2} = (6 \text{ lb})x = 6x$

For spring: $\quad U_{1-2} = -\int_0^x (3 + 96x)dx = -3x - 48x^2$

$$T_1 + U_{1-2} = T_2: \quad 0 + 6x - 3x - 48x^2 = 0.09317 v_2^2$$

$$3x - 48x^2 = 0.09317 v_2^2 \qquad (1)$$

(*a*) For x_m, $v_2 = 0$: $\quad 3x - 48x^2 = 0$

$$x = 0, \quad x_m = \frac{3}{48} = \frac{1}{16} \text{ ft}$$

$\mathbf{x}_m = 0.75$ in.↓ ◀

(*b*) For v_m we see maximum of $U_{1-2} = 3x - 48x^2$

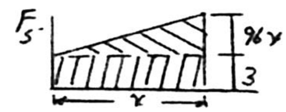
$$\frac{dU_{1-2}}{dx} = 3 - 96x = 0 \qquad x = \frac{3}{96}\text{ ft} = \frac{1}{32}\text{ ft}$$

Eq. (1): $$3\left(\frac{1}{32}\text{ ft}\right) - 48\left(\frac{1}{32}\text{ ft}\right)^2 = 0.09317v_m^2$$

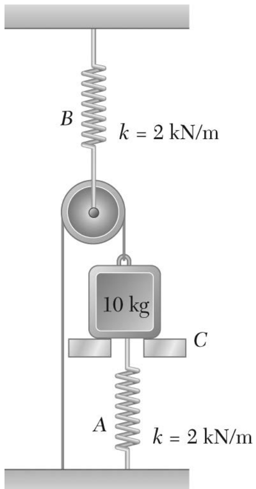
$$v_m^2 = 0.5031 \qquad v_m = 0.7093\text{ ft/s}$$

$\mathbf{v}_m = 8.51$ in./s ↕ ◀

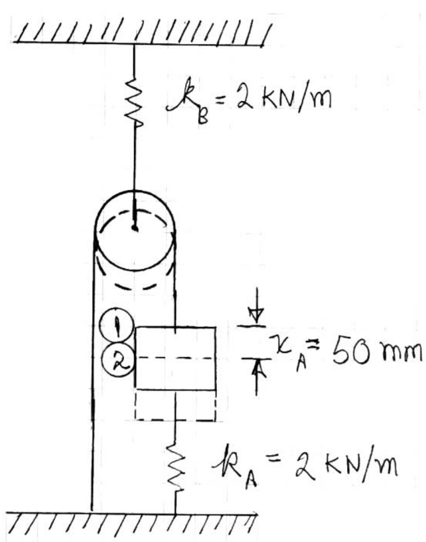
Note: U_{1-2} for the spring may be computed using $F_6 - x$ curve



$$U_{1-2} = \text{area}$$
$$= 3x + \frac{1}{2}96x^2$$

	**PROBLEM 13.30** A 10-kg block is attached to spring *A* and connected to spring *B* by a cord and pulley. The block is held in the position shown with both springs unstretched when the support is removed and the block is released with no initial velocity. Knowing that the constant of each spring is 2 kN/m, determine (*a*) the velocity of the block after it has moved down 50 mm, (*b*) the maximum velocity achieved by the block.

SOLUTION



(*a*) W = weight of the block $= 10\,(9.81) = 98.1$ N

$$x_B = \frac{1}{2}x_A$$

$$U_{1-2} = W(x_A) - \frac{1}{2}k_A(x_A)^2 - \frac{1}{2}k_B(x_B)^2$$

(Gravity) (Spring *A*) (Spring *B*)

$$U_{1-2} = (98.1\text{ N})(0.05\text{ m}) - \frac{1}{2}(2000\text{ N/m})(0.05\text{ m})^2$$

$$-\frac{1}{2}(2000\text{ N/m})(0.025\text{ m})^2$$

$$U_{1-2} = \frac{1}{2}(m)v^2 = \frac{1}{2}(10\text{ kg})v^2$$

$$4.905 - 2.5 - 0.625 = \frac{1}{2}(10)v^2$$

$v = 0.597$ m/s ◀

(*b*) Let x = distance moved down by the 10 kg block

$$U_{1-2} = W(x) - \frac{1}{2}k_A(x)^2 - \frac{1}{2}k_B\left(\frac{x}{2}\right)^2 = \frac{1}{2}(m)v^2$$

$$\frac{d}{dx}\left[\frac{1}{2}(m)v^2\right] = 0 = W - k_A(x) - \frac{k_B}{8}(2x)$$

PROBLEM 13.30 (Continued)

$$0 = 98.1 - 2000\,(x) - \frac{2000}{8}\,(2x) = 98.1 - (2000 + 250)x$$

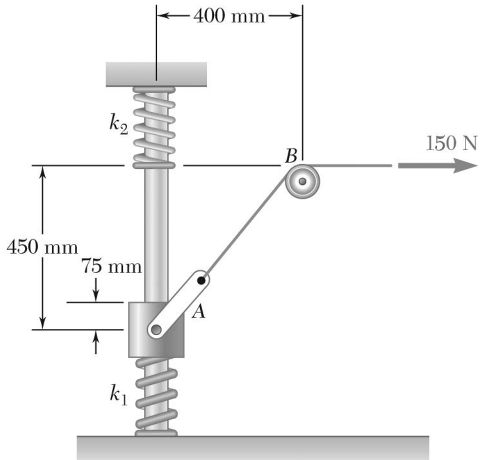
$$x = 0.0436 \text{ m } (43.6 \text{ mm})$$

For $\quad x = 0.0436, \;\; U = 4.2772 - 1.9010 - 0.4752 = \frac{1}{2}(10)v^2$

$$v_{\max} = 0.6166 \text{ m/s}$$

$v_{\max} = 0.617$ m/s ◀

PROBLEM 13.31



A 5-kg collar *A* is at rest on top of, but not attached to, a spring with stiffness k_1 = 400 N/m; when a constant 150-N force is applied to the cable. Knowing *A* has a speed of 1 m/s when the upper spring is compressed 75 mm, determine the spring stiffness k_2. Ignore friction and the mass of the pulley.

SOLUTION

Use the method of work and energy applied to the collar *A*.

$$T_1 + U_{1\to 2} = T_2$$

Since collar is initially at rest, $T_1 = 0.$

In position 2, where the upper spring is compressed 75 mm and $v_2 = 1.00$ m/s, the kinetic energy is

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(5\text{ kg})(1.00\text{ m/s})^2 = 2.5\text{ J}$$

As the collar is raised from level *A* to level *B*, the work of the weight force is

$$(U_{1\to 2})_g = -mgh$$

where $m = 5$ kg, $g = 9.81$ m/s^2 and $h = 450$ mm $= 0.450$ m

Thus, $(U_{1\to 2})_g = -(5)(9.81)(0.450) = -22.0725$ J

In position 1, the force exerted by the lower spring is equal to the weight of collar *A*.

$$F_1 = mg = -(5\text{ kg})(9.81\text{ m/s}) = -49.05\text{ N}$$

As the collar moves up a distance x_1, the spring force is

$$F = F_1 - k_1 x_2$$

until the collar separates from the spring at

$$x_f = \frac{F_1}{k_1} = \frac{49.05\text{ N}}{400\text{ N/m}} = 0.122625\text{ m} = 122.625\text{ mm}$$

Work of the force exerted by the lower spring:

$$(U_{1\to 2})_1 = \int_0^{x_f} (F_1 - k_1 x)dx$$

$$= F_1 x_f - \frac{1}{2}kx_f^2 = k_1 x_f^2 - \frac{1}{2}k_1 x_f^2 = \frac{1}{2}k_1 x_f^2$$

$$= \frac{1}{2}(400 \text{ N/m})(0.122625)^2 = 3.0074 \text{ J}$$

In position 2, the upper spring is compressed by $y = 75 \text{ mm} = 0.075 \text{ m}$. The work of the force exerted by this spring is

$$(U_{1\to 2})_2 = -\frac{1}{2}k_2 y^2 = -\frac{1}{2}k_2 (0.075)^2 = -0.0028125 \text{ k}_2$$

Finally, we must calculate the work of the 150 N force applied to the cable. In position 1, the length *AB* is

$$(l_{AB})_1 = \sqrt{(450)^2 + (400)^2} = 602.08 \text{ mm}$$

In position 2, the length *AB* is $(l_{AB})_2 = 400$ mm.

The displacement *d* of the 150 N force is

$$d = (l_{AB})_1 - (l_{AB})_2 = 202.08 \text{ mm} = 0.20208 \text{ m}$$

The work of the 150 N force *P* is

$$(U_{1\to 2})_P = Pd = (150 \text{ N})(0.20208 \text{ m}) = 30.312 \text{ J}$$

Total work:

$$U_{1\to 2} = -22.0725 + 3.0074 - 0.0028125k_2 + 30.312$$
$$= 11.247 - 0.0028125k_2$$

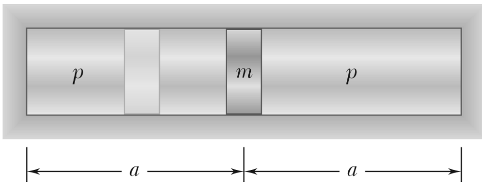
Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2$$

$$0 + 11.247 - 0.0028125k_2 = 2.5$$

$$k_2 = 3110 \text{ N/m}$$

$k_2 = 3110$ N/m ◀

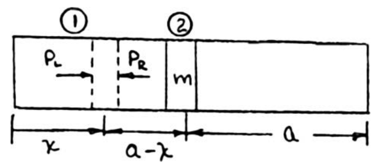


PROBLEM 13.32

A piston of mass m and cross-sectional area A is equilibrium under the pressure p at the center of a cylinder closed at both ends. Assuming that the piston is moved to the left a distance $a/2$ and released, and knowing that the pressure on each side of the piston varies inversely with the volume, determine the velocity of the piston as it again reaches the center of the cylinder. Neglect friction between the piston and the cylinder and express your answer in terms of m, a, p, and A.

SOLUTION

Pressures vary inversely as the volume



$$\frac{p_L}{P} = \frac{Aa}{Ax} \qquad p_L = \frac{pa}{x}$$

$$\frac{p_R}{P} = \frac{Aa}{A(2a-x)} \qquad p_R = \frac{pa}{(2a-x)}$$

Initially at ①, $\quad v = 0 \quad x = \frac{a}{2}$

$$T_1 = 0$$

At ②, $\quad x = a, \quad T_2 = \frac{1}{2}mv^2$

$$U_{1-2} = \int_{a/2}^{a} (p_L - p_R)A\,dx = \int_{a/2}^{a} paA\left[\frac{1}{x} - \frac{1}{2a-x}\right]dx$$

$$U_{1-2} = paA[\ln x + \ln(2a-x)]_{a/2}^{a}$$

$$U_{1-2} = paA\left[\ln a + \ln a - \ln\left(\frac{a}{2}\right) - \ln\left(\frac{3a}{2}\right)\right]$$

$$U_{1-2} = paA\left[\ln a^2 - \ln\frac{3a^2}{4}\right] = paA\,\ln\left(\frac{4}{3}\right)$$

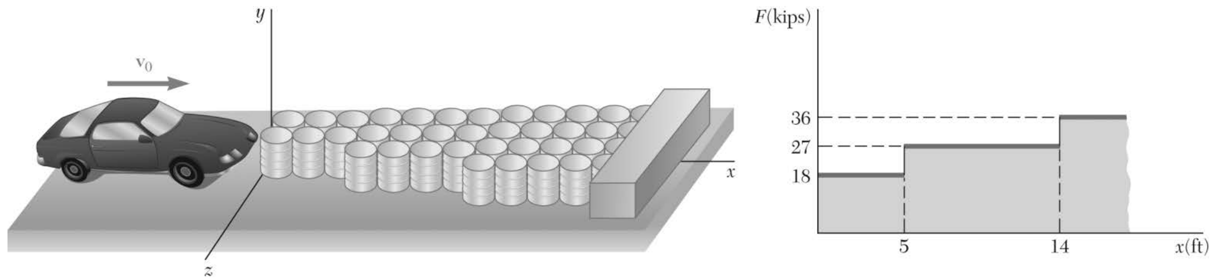
$$T_1 + U_{1-2} = T_2 \qquad 0 + paA\,\ln\left(\frac{4}{3}\right) = \frac{1}{2}mv^2$$

$$v^2 = \frac{2paA\,\ln\left(\frac{4}{3}\right)}{m} = 0.5754\frac{paA}{m}$$

$$v = 0.759\sqrt{\frac{paA}{m}} \blacktriangleleft$$

PROBLEM 13.33

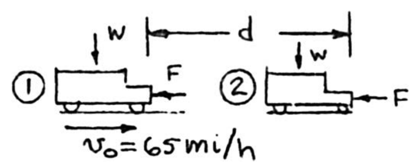
An uncontrolled automobile traveling at 65 mph strikes squarely a highway crash cushion of the type shown in which the automobile is brought to rest by successively crushing steel barrels. The magnitude F of the force required to crush the barrels is shown as a function of the distance x the automobile has moved into the cushion. Knowing that the weight of the automobile is 2250 lb and neglecting the effect of friction, determine (a) the distance the automobile will move into the cushion before it comes to rest, (b) the maximum deceleration of the automobile.



SOLUTION

(a) 65 mi/h = 95.3 ft/s

Assume auto stops in $5 \le d \le 14$ ft.

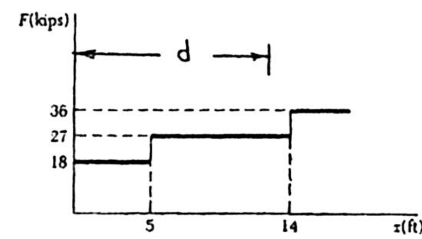


$$v_1 = 95.33 \text{ ft/s}$$

$$T_1 = \frac{1}{2}mv_1^2 \frac{1}{2}\left(\frac{2250 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(95.3 \text{ ft/s})^2$$

$$T_1 = 317{,}530 \text{ lb} \cdot \text{ft}$$

$$= 317.63 \text{ k} \cdot \text{ft}$$



$$v_2 = 0$$

$$T_2 = 0$$

$$U_{1-2} = (18 \text{ k})(5 \text{ ft}) + (27 \text{ k})(d - 5)$$

$$= 90 + 27d - 135$$

$$= 27d - 45 \text{ k} \cdot \text{ft}$$

$$T_1 + U_{1-2} = T_2$$

$$317.53 = 27d - 45$$

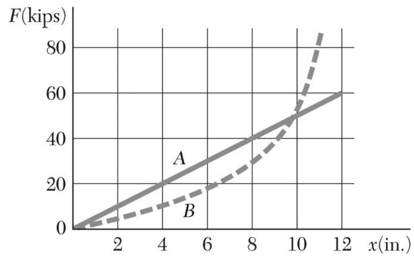
$d = 13.43$ ft ◀

Assumption that $d \le 14$ ft is ok.

(b) Maximum deceleration occurs when F is largest. For $d = 13.43$ ft, $F = 27$ k. Thus, $F = ma_D$

$$(27{,}000 \text{ lb}) = \left(\frac{2250 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(a_D)$$

$a_D = 386 \text{ ft/s}^2$ ◀



PROBLEM 13.34

Two types of energy-absorbing fenders designed to be used on a pier are statically loaded. The force-deflection curve for each type of fender is given in the graph. Determine the maximum deflection of each fender when a 90-ton ship moving at 1 mi/h strikes the fender and is brought to rest.

SOLUTION

Weight: $W_1 = (90 \text{ ton})(2000 \text{ lb/ton}) = 180 \times 10^3 \text{ lb}$

Mass: $m = \dfrac{W}{g} = \dfrac{180 \times 10^3}{32.2} = 5590 \text{ lb} \cdot \text{s}^2/\text{ft}$

Speed: $v_1 = 1 \text{ mi/h} = \dfrac{5280 \text{ ft}}{3600 \text{ s}} = 1.4667 \text{ ft/s}$

Kinetic energy: $T_1 = \dfrac{1}{2} m v_1^2 = \dfrac{1}{2}(5590)(1.4667)^2$

$= 6012 \text{ ft} \cdot \text{lb}$

$T_2 = 0 \quad \text{(rest)}$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$6012 + U_{1\to 2} = 0$

$U_{1\to 2} = -6012 \text{ ft} \cdot \text{lb} = -72.15 \text{ kip} \cdot \text{in.}$

The area under the force-deflection curve up to the maximum deflection is equal to $72.15 \text{ kip} \cdot \text{in.}$

Fender *A*: From the force-deflection curve $F = kx \qquad k = \dfrac{F_{max}}{x_{max}} = \dfrac{60}{12} = 5 \text{ kip/in.}$

$$\text{Area} = \int_0^x f dx = \int_0^x kx\, dx = \frac{1}{2} kx^2$$

$\dfrac{1}{2}(5)x^2 = 72.51$

$x^2 = 28.86 \text{ in.}^2 \qquad\qquad x = 5.37 \text{ in.}$ ◀

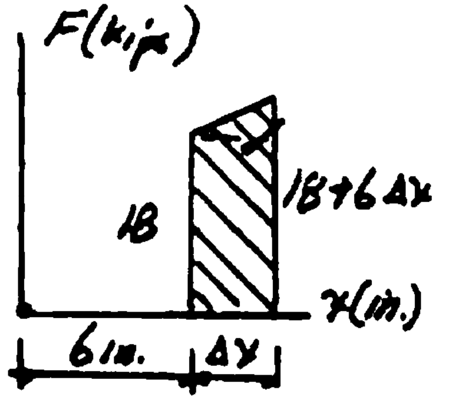
Fender *B*: We divide area under curve *B* into trapezoids

		Partial area	Total Area
From $x = 0$	to $x = 2$ in.:	$\frac{1}{2}(2 \text{ in.})(4 \text{ kips}) = 4 \text{ kip} \cdot \text{in.}$	$4 \text{ kip} \cdot \text{in.}$
From $x = 2$ in.	to $x = 4$ in.:	$\frac{1}{2}(2 \text{ in.})(4 + 10) = 14 \text{ kip} \cdot \text{in.}$	$18 \text{ kip} \cdot \text{in.}$
From $x = 4$ in.	to $x = 6$ in.:	$\frac{1}{2}(2 \text{ in.})(10 + 18) = 28 \text{ kip} \cdot \text{in.}$	$46 \text{ kip} \cdot \text{in.}$

PROBLEM 13.34 (Continued)

We still need $\Delta U = 72.15 - 46 = 26.15 \text{ kip} \cdot \text{in.}$

Equation of straight line approximating curve B from $x = 6$ in. to $x = 8$ in. is



$$\frac{\Delta x}{2} = \frac{F - 18}{30 - 18} \quad F = 18 + 6\Delta x$$

$$\Delta U = 18\Delta x + \frac{1}{2}(6\Delta x)\Delta x = 26.15 \text{ kip} \cdot \text{in.}$$

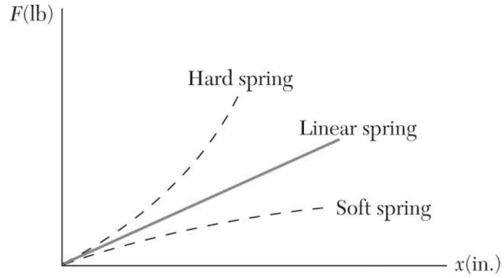
$$(\Delta x)^2 + 6\Delta x - 8.716 = 0$$

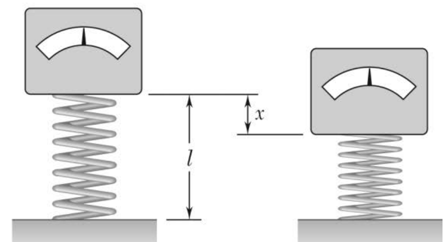
$$\Delta x = 1.209 \text{ in.}$$

Thus: $x = 6 \text{ in.} + 1.209 \text{ in.} = 7.209 \text{ in.}$ $\qquad x = 7.21 \text{ in.}$ ◀

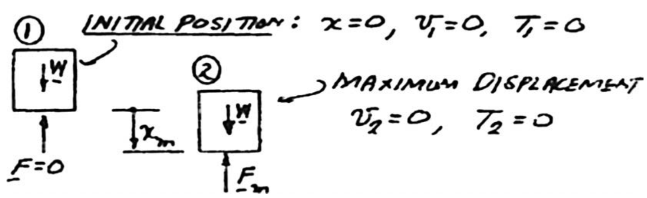
PROBLEM 13.35

Nonlinear springs are classified as hard or soft, depending upon the curvature of their force-deflection curve (see figure). If a delicate instrument having a mass of 5 kg is placed on a spring of length l so that its base is just touching the undeformed spring and then inadvertently released from that position, determine the maximum deflection x_m of the spring and the maximum force F_m exerted by the spring, assuming (*a*) a linear spring of constant $k = 3$ kN/m, (*b*) a hard, nonlinear spring, for which $F = (3\text{ kN/m})(x + 160x^2)$.





SOLUTION



$W = mg = (5\text{ kg})g$
$W = 49.05$ N

Since $$T_1 = T_2 = 0, \quad T_1 + U_{1-2} = T_2 \text{ yields } U_{1-2} = 0$$

$$U_{1-2} = Wx_m - \int_0^{x_m} F dx = 49.05x_m - \int_0^{x_m} F dx = 0 \qquad (1)$$

(*a*) For $F = kx = (300\text{ N/m})x$

Eq. (1): $$49.05x_m - \int_0^{x_m} 3000x\, dx = 0$$

$$49.05x_m - 1500x_m^2 = 0 \qquad x_m = 32.7 \times 10^{-3}\text{ m} = 32.7\text{ mm} \blacktriangleleft$$

$$F_m = 3000x_m = 3000(32.7 \times 10^{-3}) \qquad \mathbf{F}_m = 98.1\text{ N} \uparrow \blacktriangleleft$$

(*b*) For $$F = (3000\text{ N/m})x(1 + 160x^2)$$

Eq. (1) $$49.05x_m - \int_0^{x_m} 3000(x + 160x^3)dx = 0$$

$$49.05x_m - 3000\left(\frac{1}{2}x_m^2 + 40x_m^4\right) = 0 \qquad (2)$$

Solve by trial: $$x_m = 30.44 \times 10^{-3}\text{ m} \qquad x_m = 30.4\text{ mm} \blacktriangleleft$$

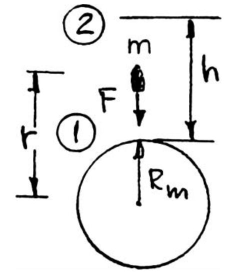
$$F_m = (3000)(30.44 \times 10^{-3})[1 + 160(30.44 \times 10^{-3})^2] \qquad \mathbf{F}_m = 104.9\text{ N} \uparrow \blacktriangleleft$$

PROBLEM 13.36

A rocket is fired vertically from the surface of the moon with a speed v_0. Derive a formula for the ratio h_n/h_u of heights reached with a speed v, if Newton's law of gravitation is used to calculate h_n and a uniform gravitational field is used to calculate h_u. Express your answer in terms of the acceleration of gravity g_m on the surface of the moon, the radius R_m of the moon, and the speeds v and v_0.

SOLUTION

Newton's law of gravitation



$$T_1 = \frac{1}{2}mv_0^2 \qquad T_2 = \frac{1}{2}mv^2$$

$$U_{1-2} = \int_{R_m}^{R_m+h_n} (-F_n)dr \qquad F_n = \frac{mg_m R_m^2}{r^2}$$

$$U_{1-2} = -mg_m R_m^2 \int_{R_m}^{R_m+h_n} \frac{dr}{r^2}$$

$$U_{1-2} = mg_m R_m^2 \left(\frac{1}{R_m} - \frac{1}{R_m + h_n} \right)$$

$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2}mv_0^2 + mg_m \left(R_m - \frac{R_m}{R_m + h_n} \right) = \frac{1}{2}mv^2$$

$$h_n = \frac{\left(v_0^2 - v^2\right)}{2g_m} \left[\frac{R_m}{R_m - \frac{\left(v_0^2 - v^2\right)}{2_{gm}}} \right] \qquad (1)$$

Uniform gravitational field

$$T_1 = \frac{1}{2}mv_0^2 \quad T_2 = mv^2$$

$$U_{1-2} = \int_{R_m}^{R_m+h_n} (-F_u)dr = -mg_m(R_m + h_u - R_m) = -mgh_u$$

$$T_1 + U_{1-2} = T_2 \quad \frac{1}{2}mv_0^2 - mg_m h_u = \frac{1}{2}mv^2$$

$$h_u = \frac{\left(v_0^2 - v^2\right)}{2g_m} \qquad (2)$$

Dividing (1) *by* (2)

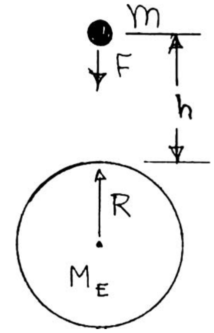
$$\frac{h_n}{h_u} = \frac{1}{1 - \frac{\left(v_0^2 - v^2\right)}{(2g_m R_m)}} \quad ◀$$

PROBLEM 13.37

Express the acceleration of gravity g_h at an altitude h above the surface of the earth in terms of the acceleration of gravity g_0 at the surface of the earth, the altitude h and the radius R of the earth. Determine the percent error if the weight that an object has on the surface of earth is used as its weight at an altitude of (*a*) 1 km, (*b*) 1000 km.

SOLUTION

$$F = \frac{GM_E m}{(h+R)^2} = \frac{GM_E m/R^2}{\left(\frac{h}{R}+1\right)^2} mg_h$$



At earth's surface, ($h = 0$) $$\frac{GM_E m}{R^2} = mg_0$$

$$\frac{GM_E}{R^2} = g_0 \qquad g_h = \frac{\frac{GM_E}{R^2}}{\left(\frac{h}{R}+1\right)^2}$$

Thus, $$g_h = \frac{g_0}{\left(\frac{h}{R}+1\right)^2}$$

$$R = 6370 \text{ km}$$

At altitude h, "true" weight $F = mg_h = W_T$

Assumed weight $W_0 = mg_0$

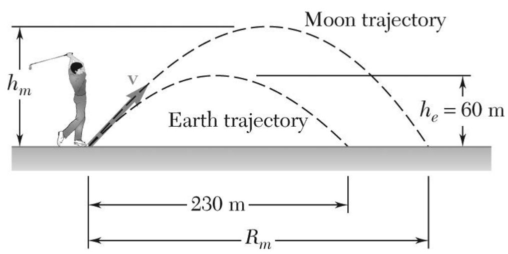
$$\text{Error} = E = \frac{W_0 - W_T}{W_0} = \frac{mg_0 - mg_h}{mg_0} = \frac{g_0 - g_h}{g_0}$$

$$g_h = \frac{g_0}{\left(\frac{h}{R}+1\right)^2} \qquad E = \frac{g_0 - \frac{g_0}{\left(1+\frac{h}{R}\right)^2}}{g_0} = \left[1 - \frac{1}{\left(1+\frac{h}{R}\right)^2}\right]$$

(*a*) $h = 1$ km: $$P = 100E = 100\left[1 - \frac{1}{\left(1+\frac{1}{6370}\right)^2}\right]$$ $P = 0.0314\%$ ◀

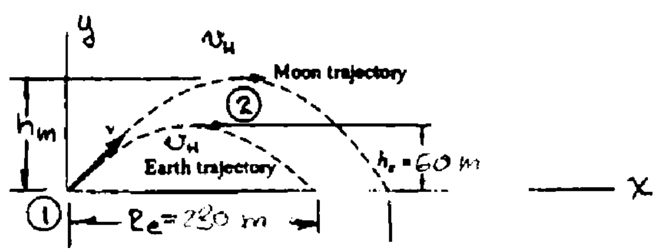
(*b*) $h = 1000$ km: $$P = 100E = 100\left[1 - \frac{1}{\left(1+\frac{1000}{6370}\right)^2}\right]$$ $P = 25.3\%$ ◀

PROBLEM 13.38



A golf ball struck on earth rises to a maximum height of 60 m and hits the ground 230 m away. How high will the same golf ball travel on the moon if the magnitude and direction of its velocity are the same as they were on earth immediately after the ball was hit? Assume that the ball is hit and lands at the same elevation in both cases and that the effect of the atmosphere on the earth is neglected, so that the trajectory in both cases is a parabola. The acceleration of gravity on the moon is 0.165 times that on earth.

SOLUTION



Solve for h_m.

At maximum height, the total velocity is the horizontal component of the velocity, which is constant and the same in both cases.

$$T_1 = \frac{1}{2}mv^2 \qquad T_2 = \frac{1}{2}mv_H^2$$

$$U_{1-2} = -mg_e h_e \quad \text{Earth}$$

$$U_{1-2} = -mg_m h_m \quad \text{Moon}$$

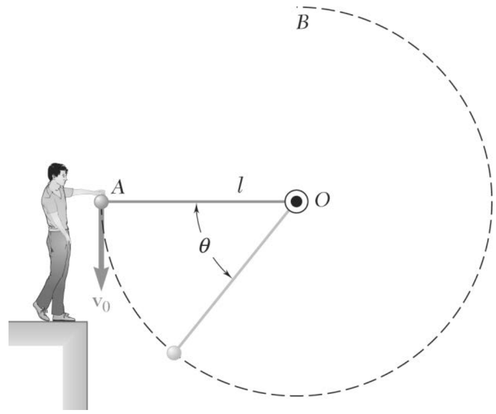
Earth $$\frac{1}{2}mv^2 - mg_e h_e = \frac{1}{2}mv_H^2$$

Moon $$\frac{1}{2}mv^2 - mg_m h_m = \frac{1}{2}mv_H^2$$

$$-g_e h_e + g_m h_m = 0 \qquad \frac{h_m}{h_e} = \frac{g_e}{g_m}$$

Subtracting $$h_m = (60\text{ m})\left(\frac{g_e}{0.165 g_e}\right)$$ $h_m = 364$ m ◀

PROBLEM 13.39



The sphere at A is given a downward velocity $\mathbf{v}_0$ of magnitude 5 m/s and swings in a vertical plane at the end of a rope of length $l = 2$ m attached to a support at O. Determine the angle θ at which the rope will break, knowing that it can withstand a maximum tension equal to twice the weight of the sphere.

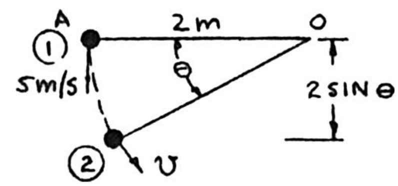
SOLUTION

$$T_1 = \frac{1}{2}mv_0^2 = \frac{1}{2}m\,(5)^2$$

$$T_1 = 12.5\text{ m}$$

$$T_2 = \frac{1}{2}mv^2$$

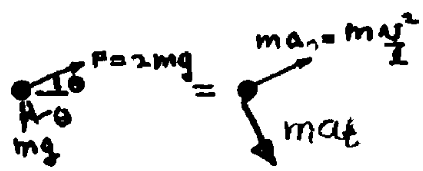
$$U_{1-2} = mg(l)\sin\theta$$



$$T_1 + U_{1-2} = T_2 \qquad 12.5m + 2mg\sin\theta = \frac{1}{2}mv^2$$

$$25 + 4g\sin\theta = v^2 \tag{1}$$

Newton's law at ②.



$$+\nearrow\; 2mg - mg\sin\theta = m\frac{v^2}{\ell} = m\frac{v^2}{2} \tag{2}$$

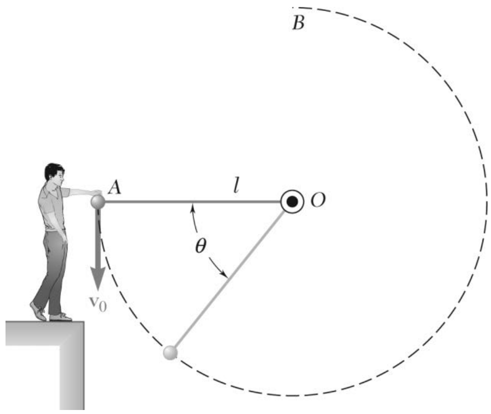
$$v^2 = 4g - 2g\sin\theta$$

Substitute for v^2 from Eq. (2) into Eq. (1)

$$25 + 4g\sin\theta = 4g - 2g\sin\theta$$

$$\sin\theta = \frac{(4)(9.81) - 25}{(6)(9.81)} = 0.2419$$

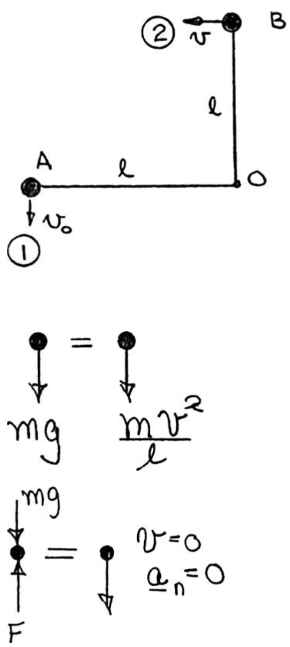
$\theta = 14.00°$ ◀



PROBLEM 13.40

The sphere at A is given a downward velocity $\mathbf{v}_0$ and swings in a vertical circle of radius l and center O. Determine the smallest velocity $\mathbf{v}_0$ for which the sphere will reach Point B as it swings about Point O (a) if AO is a rope, (b) if AO is a slender rod of negligible mass.

SOLUTION



$$T_1 = \frac{1}{2}mv_0^2$$

$$T_2 = \frac{1}{2}mv^2$$

$$U_{1-2} = -mgl$$

$$T_1 + U_{1-2} = T_2 \qquad \frac{1}{2}mv_0^2 - mgl = \frac{1}{2}mv^2$$

$$v_0^2 = v^2 + 2gl$$

Newton's law at ②

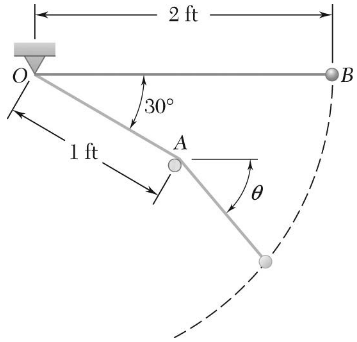
(a) For minimum v, tension in the cord must be zero.

Thus, $v^2 = gl$

$$v_0^2 = v^2 + 2gl = 3gl \qquad v_0 = \sqrt{3gl} \blacktriangleleft$$

(b) Force in the rod can support the weight so that v can be zero.

Thus, $$v_0^2 = 0 + 2gl \qquad v_0 = \sqrt{2gl} \blacktriangleleft$$



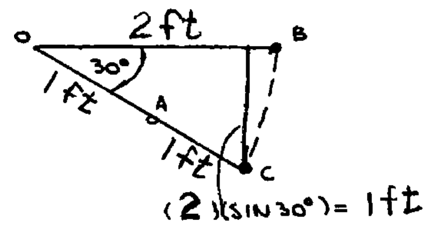
PROBLEM 13.41

A small sphere B of weight W is released from rest in the position shown and swings freely in a vertical plane, first about O and then about the peg A after the cord comes in contact with the peg. Determine the tension in the cord (a) just before the sphere comes in contact with the peg, (b) just after it comes in contact with the peg.

SOLUTION

Velocity of the sphere as the cord contacts A

$$v_B = 0 \qquad T_B = 0$$



$$T_C = \frac{1}{2}mv_C^2$$

$$U_{B-C} = (mg)(1)$$

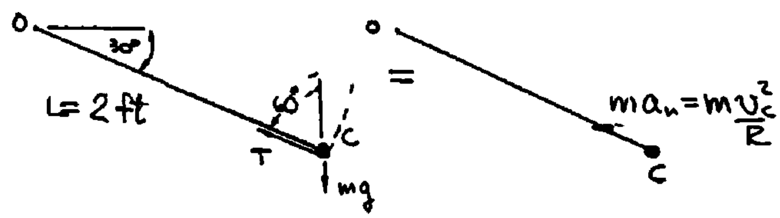
$$T_B + U_{B-C} = T_C$$

$$0 + 1mg = \frac{1}{2}mv_C^2$$

$$v_C^2 = (2)(g)$$

Newton's law

(a) Cord rotates about Point $O\,(R = L)$

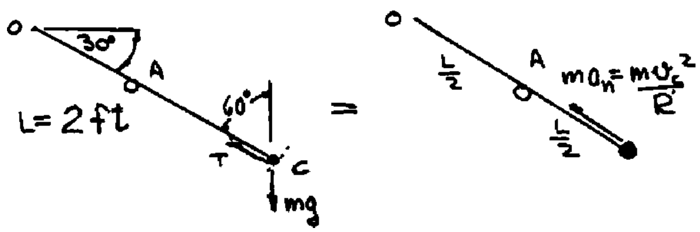


$$+\nwarrow T - mg\,(\cos 60°) = m\frac{v_C^2}{L}$$

$$T = mg(\cos 60°) + \frac{m(2)g}{2}$$

$$T = \frac{3}{2}mg \qquad\qquad T = 1.5\ W \blacktriangleleft$$

(*b*) Cord rotates about $A\left(R = \frac{L}{2}\right)$

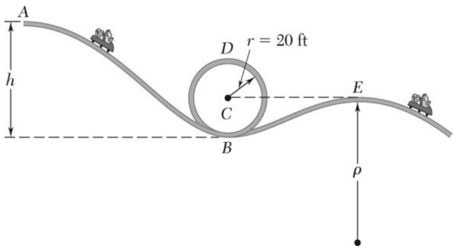


$$T - mg(\cos 60°) = \frac{mv_C^2}{\frac{L}{2}}$$

$$T = \frac{mg}{2} + \frac{m(2)(g)}{1}$$

$$T = \left(\frac{1}{2} + 2\right)mg = \frac{5}{2}mg$$

$T = 2.5W$ ◀



PROBLEM 13.42

A roller coaster starts from rest at A, rolls down the track to B, describes a circular loop of 40-ft diameter, and moves up and down past Point E. Knowing that h = 60 ft and assuming no energy loss due to friction, determine (a) the force exerted by his seat on a 160-lb rider at B and D, (b) the minimum value of the radius of curvature at E if the roller coaster is not to leave the track at that point.

SOLUTION

Let y_p be the vertical distance from Point A to any Point P on the track. Let position 1 be at A and position 2 be at P. Apply the principle of work and energy.

$$T_1 = 0 \qquad T_2 = \frac{1}{2}mv_P^2$$

$$U_{1\to 2} = mgy_P$$

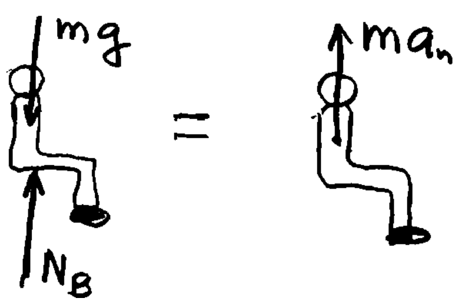
$$T_1 + U_{1\to 2} = T_2: \quad 0 + mgy_P = \frac{1}{2}mv_P^2$$

$$v_P^2 = 2gy_P$$

Magnitude of normal acceleration at P:

$$(a_P)_n = \frac{v_P^2}{\rho_P} = \frac{2gy_P}{\rho_P}$$

(a) Rider at Point B.



$$y_B = h = 60 \text{ ft}$$
$$\rho_B = r = 20 \text{ ft}$$
$$a_n = \frac{(2g)(60)}{20} = 6g$$

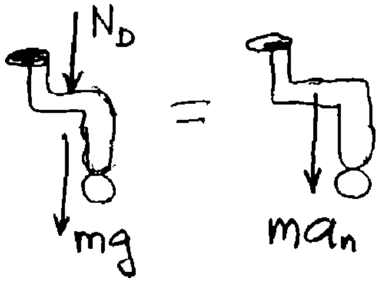
$+\uparrow \Sigma F = ma$:

$$N_B - mg = m(6g)$$
$$N_B = 7mg = 7W = (7)(160 \text{ lb})$$

$$\mathbf{N}_B = 1120 \text{ lb} \uparrow \blacktriangleleft$$

PROBLEM 13.42 (Continued)

Rider at Point D.



$$y_D = h - 2r = 20 \text{ ft}$$

$$\rho_D = 20 \text{ ft}$$

$$a_n = \frac{(2g)(20)}{20} = 2g$$

$+\downarrow \Sigma F = ma$:

$$N_D + mg = m(2g)$$

$$N_D = mg = W = 160 \text{ lb}$$

$\mathbf{N}_D = 160 \text{ lb} \downarrow$ ◀

(*b*) Car at Point E.

$$y_E = h - r = 40 \text{ ft}$$

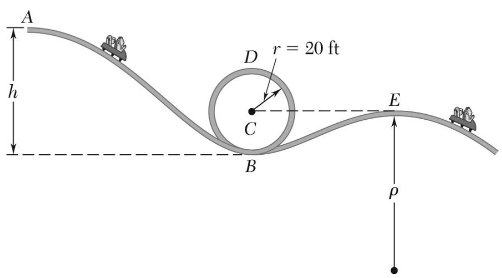
$$N_E = 0$$

$+\uparrow \Sigma F = ma_n$:

$$mg = m \cdot \frac{2gy_E}{\rho_E}$$

$$\rho_E = 2y_E$$

$\rho = 80.0 \text{ ft}$ ◀



PROBLEM 13.43

In Problem 13.42, determine the range of values of h for which the roller coaster will not leave the track at D or E, knowing that the radius of curvature at E is $\rho = 75$ ft. Assume no energy loss due to friction.

PROBLEM 13.42 A roller coaster starts from rest at A, rolls down the track to B, describes a circular loop of 40-ft diameter, and moves up and down past Point E. Knowing that $h = 60$ ft and assuming no energy loss due to friction, determine (*a*) the force exerted by his seat on a 160-lb rider at B and D, (*b*) the minimum value of the radius of curvature at E if the roller coaster is not to leave the track at that point.

SOLUTION

Let y_p be the vertical distance from Point A to any Point P on the track. Let position 1 be at A and position 2 be at P. Apply the principle of work and energy.

$$T_1 = 0 \qquad T_2 = \frac{1}{2}mv_P^2$$

$$U_{1\to 2} = mg\,y_P$$

$$T_1 + U_{1\to 2} = T_2: \quad 0 + mgy_p = \frac{1}{2}mv_P^2$$

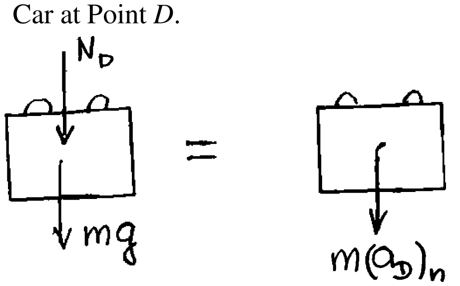
$$v_P^2 = 2g\,y_P$$

Magnitude of normal acceleration of P:

$$(a_P)_n = \frac{v_P^2}{\rho_P} = \frac{2g\,y_P}{\rho_P}$$

The condition of loss of contact with the track at P is that the curvature of the path is equal to ρ_p and the normal contact force $N_P = 0$.

Car at Point D.



$$\rho_D = r = 20 \text{ ft}$$

$$y_D = h - 2r$$

$$(a_D)_n = \frac{2\,g\,(h-2r)}{r}$$

$$+\!\downarrow \Sigma F = ma$$

$$N_D + mg = m\frac{2g(h-2r)}{r}$$

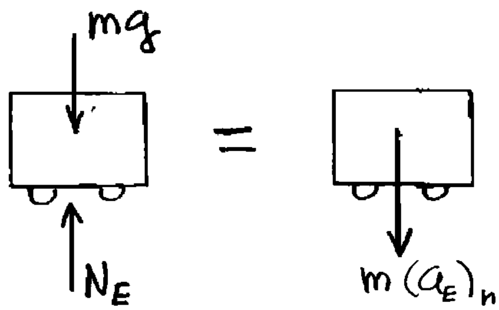
$$N_D = mg\,\frac{2h-5r}{r}$$

For $N_D > 0$

$$2h - 5r > 0$$

$$h > \frac{5}{2} r = 50 \text{ ft}$$

Car at Point *E*.



$$\rho_E = \rho = 75 \text{ ft}$$

$$y_E = h - r = h - 20 \text{ ft}$$

$$(a_E)_n = \frac{2g(h-20)}{75}$$

$$+\uparrow \Sigma F = ma$$

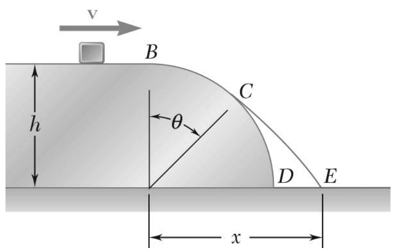
$$N_E - mg = -\frac{2mg(h-20)}{75}$$

$$N_E = mg \frac{115 - 2h}{75}$$

For $\qquad N_E > 0, \qquad 115 - 2h > 0$

$$h < 57.5 \text{ ft}$$

Range of values for *h*: $\qquad 50.0 \text{ ft} \le h \le 57.5 \text{ ft}$ ◀

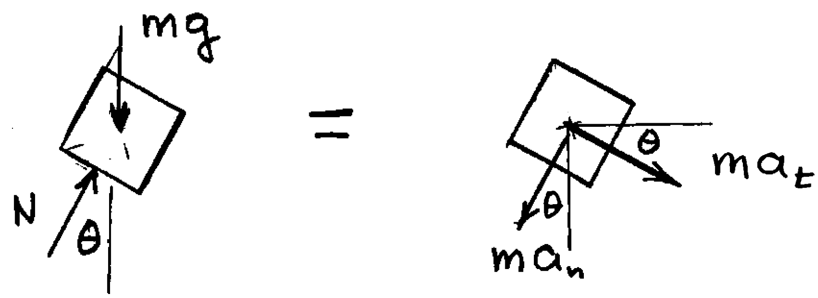


PROBLEM 13.44

A small block slides at a speed v on a horizontal surface. Knowing that $h = 0.9$ m, determine the required speed of the block if it is to leave the cylindrical surface BCD when $\theta = 30°$.

SOLUTION

At Point C where the block leaves the surface BCD the contact force is reduced to zero. Apply Newton's second law at Point C.



n-direction: $$N - mg\cos\theta = -ma_n = -\frac{mv_C^2}{h}$$

With $N = 0$, we get $$v_c^2 = gh\cos\theta$$

Apply the work-energy principle to the block sliding over the path BC. Let position 1 correspond to Point B and position 2 to C.

$$T_1 = \frac{1}{2}mv_B^2 \qquad T_2 = \frac{1}{2}\,mv_C^2 = \frac{1}{2}mgh\cos\theta$$

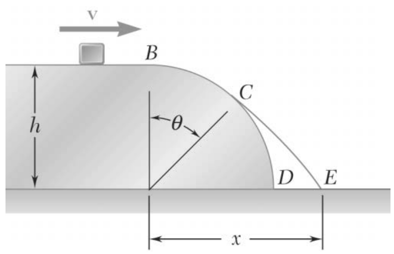
$$U_{1\to 2} = \text{ weight} \times \text{change in vertical distance}$$
$$= mgh(1-\cos\theta)$$

$$T_1 + U_{1\to 2} = T_2:\ \frac{1}{2}mv_B^2 + mg(1-\cos\theta) = \frac{1}{2}mgh\cos\theta$$
$$v_B^2 = gh\cos\theta - 2gh(1-\cos\theta) = gh(3\cos\theta - 2)$$

Data: $g = 9.81\text{ m/s}^2,\ h = 0.9\text{ m},\ \theta = 30°.$

$$v_B^2 = (9.81)(0.9)(3\cos 30° - 2) = 5.2804\text{ m}^2/\text{s}^2$$

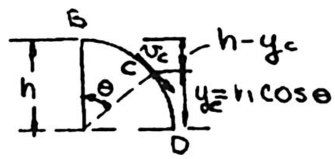
$$v_B = 2.30\text{ m/s} \blacktriangleleft$$

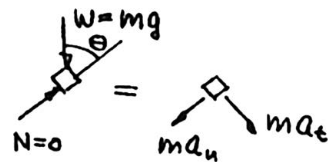


PROBLEM 13.45

A small block slides at a speed $v = 8$ ft/s on a horizontal surface at a height $h = 3$ ft above the ground. Determine (*a*) the angle θ at which it will leave the cylindrical surface *BCD*, (*b*) the distance *x* at which it will hit the ground. Neglect friction and air resistance.

SOLUTION





Block leaves surface at *C* when the normal force $N = 0$.

$$+\swarrow \; mg\cos\theta = ma_n$$

$$g\cos\theta = \frac{v_C^2}{h}$$

$$v_C^2 = gh\cos\theta = gy \qquad (1)$$

Work-energy principle.

(*a*)

$$T_B = \frac{1}{2}mv^2 = \frac{1}{2}m(8)^2 = 32m$$

$$T_C = \frac{1}{2}mv_C^2 \qquad U_{B-C} = W(h-g) = mg(h-y_C)$$

$$T_B + U_{B-C} = T_C$$

$$32m + mg(h-y) = \frac{1}{2}mv_C^2$$

Use Eq. (1)

$$32 + g(h-y_C) = \frac{1}{2}gy_C \qquad (2)$$

$$32 + gh = \frac{3}{2}gy_C$$

$$y_C = \frac{(32+gh)}{\left(\frac{3}{2}g\right)}$$

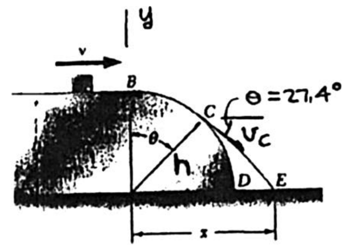
$$y_C = \frac{(32+(32.2)(3))}{\frac{3}{2}(32.2)}$$

$$y_C = 2.6625\text{ ft} \qquad (3)$$

$$y_C = h\cos\theta \qquad \cos\theta = \frac{y_C}{h} = \frac{2.6625}{3} = 0.8875 \qquad \theta = 27.4° \blacktriangleleft$$

PROBLEM 13.45 (Continued)

(*b*) From (1) and (3)



$$v_C = \sqrt{gy}$$

$$v_C = \sqrt{(32.2)(2.6625)}$$

$$v_C = 9.259 \text{ ft/s}$$

At *C*: $(v_C)_x = v_C \cos\theta = (9.259)(\cos 27.4°) = 8.220 \text{ ft/s}$

$$(v_C)_y = -v_C \sin\theta = -(9.259)(\sin 27.4°) = 4.261 \text{ ft/s}$$

$$y = y_C + (v_C)_y t - \frac{1}{2}gt^2 = 2.6625 - 4.261t - 16.1t^2$$

At *E*: $y_E = 0$: $\quad t^2 + 0.2647t - 0.1654 = 0$

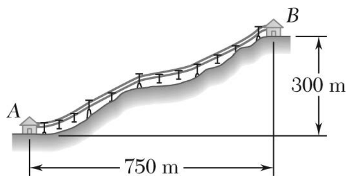
$$t = 0.2953 \text{ s}$$

At *E*: $x = h(\sin\theta) + (v_C)_x t = (3)(\sin 27.4°) + (8.220)(0.2953)$

$$x = 1.381 + 2.427 = 3.808 \text{ ft}$$

$x = 3.81$ ft ◀

PROBLEM 13.46



A chair-lift is designed to transport 1000 skiers per hour from the base A to the summit B. The average mass of a skier is 70 kg and the average speed of the lift is 75 m/min. Determine (a) the average power required, (b) the required capacity of the motor if the mechanical efficiency is 85 percent and if a 300 percent overload is to be allowed.

SOLUTION

Note: Solution is independent of speed.

(a)

$$\text{Average power } = \frac{\Delta U}{\Delta t} = \frac{(1000)(70 \text{ kg})(9.81 \text{ m/s}^2)(300 \text{ m})}{3600 \text{ s}} = 57,225 \, \frac{\text{N} \cdot \text{m}}{\text{s}}$$

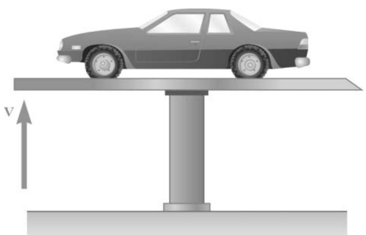
Average power $= 57.2$ kW ◀

(b) Maximum power required with 300% over load

$$= \frac{100 + 300}{100}(57.225 \text{ kW}) = 229 \text{ kW}$$

Required motor capacity (85% efficient)

$$\text{Motor capacity } = \frac{229 \text{ kW}}{0.85} = 269 \text{ kW}$$ ◀



PROBLEM 13.47

It takes 15 s to raise a 1200-kg car and the supporting 300-kg hydraulic car-lift platform to a height of 2.8 m. Determine (*a*) the average output power delivered by the hydraulic pump to lift the system, (*b*) the average power electric required, knowing that the overall conversion efficiency from electric to mechanical power for the system is 82 percent.

SOLUTION

(*a*) $(P_P)_A = (F)(v_A) = (m_C + m_L)(g)(v_A)$

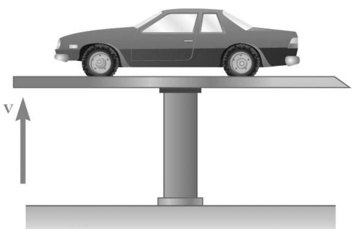
$v_A = \text{s/t} = (2.8 \text{ m})/(15 \text{ s}) = 0.18667 \text{ m/s}$

$(P_P)_A = [(1200 \text{ kg}) + (300 \text{ kg})](9.81 \text{ m/s}^2)(0.18667 \text{ m/s})^3$

$(P_P)_A = 2.747 \text{ kJ/s}$ $\qquad (P_P)_A = 2.75 \text{ kW}$ ◀

(*b*) $(P_E)_A = (P_P)/\eta = (2.75 \text{ kW})/(0.82)$ $\qquad (P_E)_A = 3.35 \text{ kW}$ ◀

PROBLEM 13.48

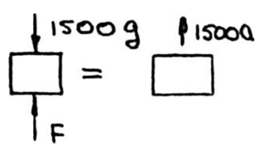


The velocity of the lift of Problem 13.47 increases uniformly from zero to its maximum value at mid-height 7.5 s and then decreases uniformly to zero in 7.5 s. Knowing that the peak power output of the hydraulic pump is 6 kW when the velocity is maximum, determine the maximum life force provided by the pump.

PROBLEM 13.47 It takes 15 s to raise a 1200-kg car and the supporting 300-kg hydraulic car-lift platform to a height of 2.8 m. Determine (*a*) the average output power delivered by the hydraulic pump to lift the system, (*b*) the average power electric required, knowing that the overall conversion efficiency from electric to mechanical power for the system is 82 percent.

SOLUTION

Newton's law



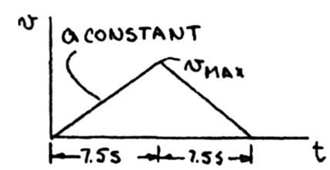
$$Mg = (M_C + M_L)g = (1200 + 300)g$$
$$Mg = 1500g$$
$$+\uparrow \Sigma F = F - 1500g = 1500a \qquad (1)$$

Since motion is uniformly accelerated, a = constant

Thus, from (1), F is constant and peak power occurs when the velocity is a maximum at 7.5 s.

$$a = \frac{v_{max}}{7.5 \text{ s}}$$

$$P = (6000 \text{ W}) = (F)(v_{max})$$
$$v_{max} = (6000)/F$$



Thus,
$$a = (6000)/(7.5)(F) \qquad (2)$$

Substitute (2) into (1)

$$F - 1500g = (1500)(6000)/(7.5)(F)$$

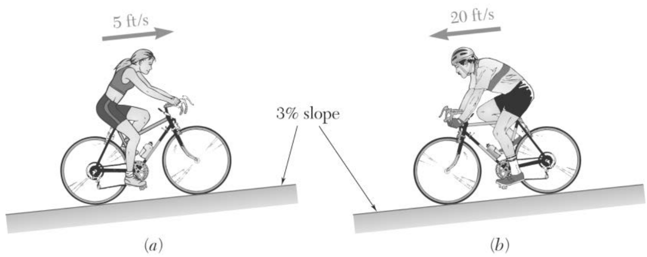
$$F^2 - (1500 \text{ kg})(9.81 \text{ m/s}^2)F - \frac{(1500 \text{ kg})(6000 \text{ N}\cdot\text{m/s})}{(7.5 \text{ s})} = 0$$

$$F^2 - 14{,}715F - 1.2\times10^6 = 0$$
$$F = 14{,}800 \text{ N}$$

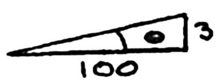
$F = 14.8$ kN ◀

PROBLEM 13.49

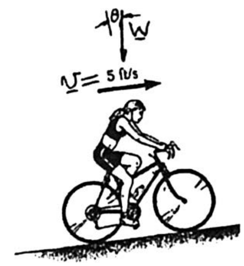
(a) A 120-lb woman rides a 15-lb bicycle up a 3-percent slope at a constant speed of 5 ft/s. How much power must be developed by the woman? (*b*) A 180-lb man on an 18-lb bicycle starts down the same slope and maintains a constant speed of 20 ft/s by braking. How much power is dissipated by the brakes? Ignore air resistance and rolling resistance.



SOLUTION



$$\tan\theta = \frac{3}{100} \qquad \theta = 1.718°$$



(*a*)

$$W = W_B + W_W = 15 + 120$$

$$W = 135 \text{ lb}$$

$$P_W = \mathbf{W}\cdot\mathbf{v} = (W\sin\theta)(v)$$

$$P_W = (135)(\sin 1.718°)(5)$$

$$P_W = 20.24 \text{ ft}\cdot\text{lb/s}$$

(*a*)

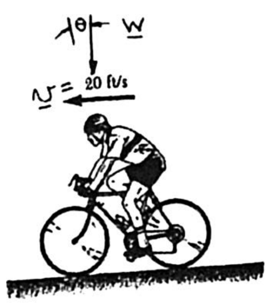
$$P_W = 20.2 \text{ ft}\cdot\text{lb/s} \blacktriangleleft$$

(*b*)

$$W = W_B + W_m = 18 + 180$$

$$W = 198 \text{ lb}$$

Brakes must dissipate the power generated by the bike and the man going down the slope at 20 ft/s.



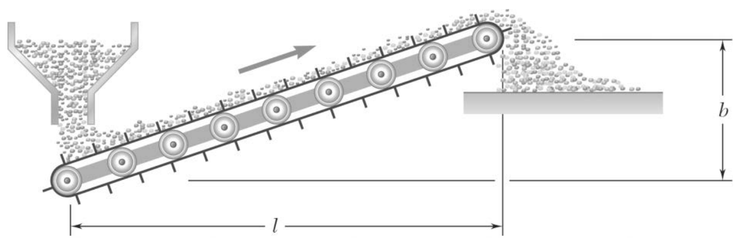
$$P_B = \mathbf{W}\cdot\mathbf{v} = (W\sin\theta)(v)$$

$$P_B = (198)(\sin 1.718)(20)$$

$$P_B = 118.7 \text{ ft}\cdot\text{lb/s} \blacktriangleleft$$

(*b*)

PROBLEM 13.50



A power specification formula is to be derived for electric motors which drive conveyor belts moving solid material at different rates to different heights and distances. Denoting the efficiency of the motors by η and neglecting the power needed to drive the belt itself, derive a formula (*a*) in the SI system of units for the power *P* in kW, in terms of the mass flow rate *m* in kg/h, the height *b* and horizontal distance *l* in meters, and (*b*) in U.S. customary units, for the power in hp, in terms of the material flow rate *w* in tons/h, and the height *b* and horizontal distance *l* in feet.

SOLUTION

(*a*) Material is lifted to a height *b* at a rate, $(m \text{ kg/h})(g \text{ m/s}^2) = [mg(\text{N/h})]$

Thus,

$$\frac{\Delta U}{\Delta t} = \frac{[mg(\text{N/h})][b(m)]}{(3600 \text{ s/h})} = \left(\frac{mgb}{3600}\right) \text{N}\cdot\text{m/s}$$

$$1000 \text{ N}\cdot\text{m/s} = 1 \text{ kW}$$

Thus, including motor efficiency, η

$$P(\text{kw}) = \frac{mgb(\text{N}\cdot\text{m/s})}{(3600)\left(\frac{1000 \text{ N}\cdot\text{m/s}}{\text{kW}}\right)(\eta)}$$

$$P(\text{kW}) = 0.278\times 10^{-6}\frac{mgb}{\eta} \blacktriangleleft$$

(*b*)

$$\frac{\Delta U}{\Delta t} = \frac{[W(\text{tons/h})(2000 \text{ lb/ton})][b(\text{ft})]}{3600 \text{ s/h}}$$

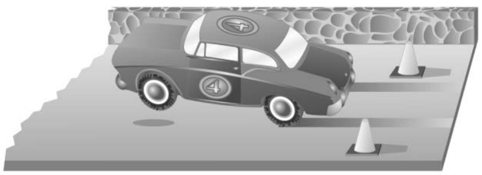
$$= \frac{Wb}{1.8} \text{ ft}\cdot\text{lb/s}; \quad 1 \text{hp} = 550 \text{ ft}\cdot\text{lb/s}$$

With η,

$$hp = \left[\frac{Wb}{1.8}(\text{ft}\cdot\text{lb/s})\right]\left[\frac{1 \text{ hp}}{550 \text{ ft}\cdot\text{lb/s}}\right]\left[\frac{1}{\eta}\right]$$

$$hp = \frac{1.010\times 10^{-3} Wb}{\eta} \blacktriangleleft$$

PROBLEM 13.51



In an automobile drag race, the rear (drive) wheels of a 1000 kg car skid for the first 20 m and roll with sliding impending during the remaining 380 m. The front wheels of the car are just off the ground for the first 20 m, and for the remainder of the race 80 percent of the weight is on the rear wheels. Knowing that the coefficients of friction are $\mu_s = 0.90$ and $\mu_k = 0.68$, determine the power developed by the car at the drive wheels (*a*) at the end of the 20-m portion of the race, (*b*) at the end of the race: Give your answer in kW and in hp. Ignore the effect of air resistance and rolling friction.

SOLUTION

(*a*) First 20 m. (Calculate velocity at 20 m.) Force generated by rear wheels $= \mu_k W$, since car skids.

Thus,

$$F_s = (0.68)(1000)(g)$$

$$F_s = (0.68)(1000 \text{ kg})(9.81 \text{ m/s}^2) = 6670.8 \text{ N}$$

Work and energy.

$$T_1 = 0, \quad T_2 = \frac{1}{2}mv_{20}^2 = 500v_{20}^2$$

$$T_1 + U_{1-2} = T_2$$

$$U_{1-2} = (20 \text{ m})(F_s) = (20 \text{ m})(6670.8 \text{ N})$$

$$U_{1-2} = 133,420 \text{ J}$$

$$0 + 133,420 = 500v_{20}^2$$

$$v_{20}^2 = \frac{133,420}{500} = 266.83$$

$$v_{20} = 16.335 \text{ m/s}$$

$$\text{Power} = (F_s)(v_{20}) = (6670.8 \text{ N})(16.335 \text{ m/s})$$

$$\text{Power} = 108,970 \text{ J/s} = 108.97 \text{ kJ/s}$$

$$1 \text{ kJ/s} = 1 \text{ kW}$$

$$1 \text{ hp} = 0.7457 \text{ kW} \qquad \text{Power} = 109.0 \text{ kJ/s} = 109.0 \text{ kW} \blacktriangleleft$$

$$\text{Power} = \frac{(109.0 \text{ kW})}{(0.7457 \text{ kW/hp})} = 146.2 \text{ hp} \blacktriangleleft$$

PROBLEM 13.51 (Continued)

(*b*) End of race. (Calculate velocity at 400 m.) For remaining 380 m, with 80% of weight on rear wheels, the force generated at impending sliding is $(\mu_s)(0.80)(mg)$

$$F_I = (0.90)(0.80)(1000 \text{ kg})(9.81 \text{ m/s}^2)$$

$$F_I = 7063.2 \text{ N}$$

Work and energy, from 20 m ② to 28 m ③.

$$v_2 = 16.335 \text{ m/s} \ \ [\text{from part } (a)]$$

$$T_2 = \frac{1}{2}(1000 \text{ kg})(16.335 \text{ m/s})^2$$

$$T_2 = 133,420 \text{ J}$$

$$T_3 = \frac{1}{2}mv_{380}^2 = 500v_{380}^2$$

$$U_{2-3} = (F_I)(380 \text{ m}) = (7063.2 \text{ N})(380 \text{ m})$$

$$U_{2-3} = 2,684,000 \text{ J}$$

$$T_2 + U_{2-3} = T_3$$

$$(133,420 \text{ J}) + (2,684,000 \text{ J}) = 500\,v_{30}^2$$

$$v_{30} = 75.066 \text{ m/s}$$

$$\text{Power} = (F_I)(v_{30}) = (7063.2 \text{ N})(75.066 \text{ m/s})$$

$$= 530,200 \text{ J/s}$$

$$\text{kW Power} = 530,200 \text{ J} = 530 \text{ kW} \quad \blacktriangleleft$$

$$\text{hp Power} = \frac{530 \text{ kW}}{(0.7457 \text{ kW/hp})} = 711 \text{ hp} \quad \blacktriangleleft$$

PROBLEM 13.52

The frictional resistance of a ship is known to vary directly as the 1.75 power of the speed v of the ship. A single tugboat at full power can tow the ship at a constant speed of 4.5 km/h by exerting a constant force of 300 kN. Determine (*a*) the power developed by the tugboat, (*b*) the maximum speed at which two tugboats, capable of delivering the same power, can tow the ship.

SOLUTION

(*a*) Power developed by tugboat at 4.5 km/h.

$$v_0 = 4.5 \text{ km/h} = 1.25 \text{ m/s}$$
$$F_0 = 300 \text{ kN}$$

$$P_0 = F_0 v_0 = (300 \text{ kN})(1.25 \text{ m/s}) \qquad P_0 = 375 \text{ kW} \blacktriangleleft$$

(*b*) Maximum speed.

Power required to tow ship at speed v:

$$F = F_0\left(\frac{v}{v_0}\right)^{1.75} \qquad P = Fv = F_0 v\left(\frac{v}{v_0}\right)^{1.75} = F_0 v_0\left(\frac{v}{v_0}\right)^{2.75} \qquad (1)$$

Since we have two tugboats, the available power is twice maximum power $F_0 v_0$ developed by one tugboat.

$$2F_0 v_0 = F_0 v_0\left(\frac{v}{v_0}\right)^{2.75}$$

$$\left(\frac{v}{v_0}\right)^{2.75} = 2 \qquad v = v_0(2)^{1/2.75} = v_0(1.2867)$$

Recalling that $v_0 = 4.5$ km/h

$$v = (4.5 \text{ km/h})(1.2867) = 5.7902 \text{ km/h} \qquad v = 5.79 \text{ km/h} \blacktriangleleft$$

PROBLEM 13.53

A train of total mass equal to 500 Mg starts from rest and accelerates uniformly to a speed of 90 km/h in 50 s. After reaching this speed, the train travels with a constant velocity. The track is horizontal and axle friction and rolling resistance result in a total force of 15 kN in a direction opposite to the direction of motion. Determine the power required as a function of time.

SOLUTION

Let F_P be the driving force and F_R be the resisting force due to axle friction and rolling resistance.

Uniformly accelerated motion. $(t < 50 \text{ s})$:

$$v = v_0 + at \qquad v_0 = 0$$

At $\quad t = 50s, \qquad v = 90 \text{ km/h} = 25 \text{ m/s}$

$$25 \text{ m/s} = 0 + a(50)$$
$$a = 0.5 \text{ m/s}^2$$
$$v = (0.5 \text{ m/s}^2)t$$

Newton's second law: $\quad F_P - F_R = ma$

where $\quad F_R = 15 \text{ kN} = 15\times 10^3 \text{ N}$

$$m = 500 \text{ Mg} = 500\times 10^3 \text{ kg}$$
$$a = 0.5 \text{ m/s}^2$$

$$F_P = F_R + ma = 15\times 10^3 + (500\times 10^3)(0.5)$$
$$= 265\times 10^3 \text{ N} = 265 \text{ kN}$$

Power: $\quad F_P v = (265\times 10^3)(0.5t)$

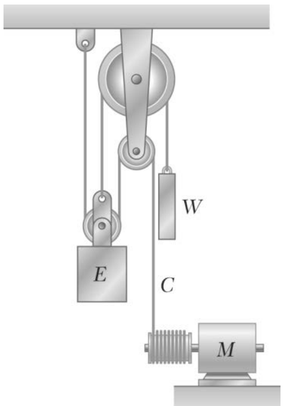
$(0 < 50\text{s}) \qquad$ Power = (132.5 kW/s)t ◄

Uniform motion. $\quad (t > 50 \text{ s})$: $\quad a = 0$

$$F_P = F_R = 15\times 10^3 \text{ N}; \; v = 25 \text{ m/s}$$

Power: $\quad F_P v = (15\times 10^3)(25 \text{ m/s}) = 375\times 10^3 W$

$(t > 50 \text{ s}) \qquad$ Power = 375 kW ◄



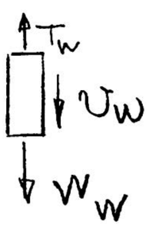
PROBLEM 13.54

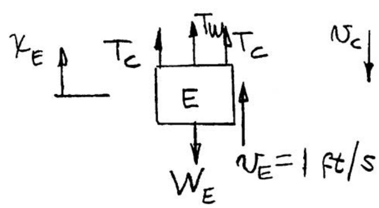
The elevator E has a weight of 6600 lbs when fully loaded and is connected as shown to a counterweight W of weight of 2200 lb. Determine the power in hp delivered by the motor (a) when the elevator is moving down at a constant speed of 1 ft/s, (b) when it has an upward velocity of 1 ft/s and a deceleration of 0.18 ft/s^2.

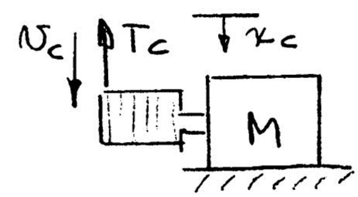
SOLUTION

(a) Acceleration = 0

Counterweight *Elevator* *Motor*







$$+\uparrow \Sigma F_y = 0: \quad T_W - W_W = 0 \qquad\qquad +\uparrow \Sigma F = 0: \quad 2T_C + T_W - 6600 = 0$$

$$T_W = 2200 \text{ lb} \qquad\qquad T_C = 2200 \text{ lb}$$

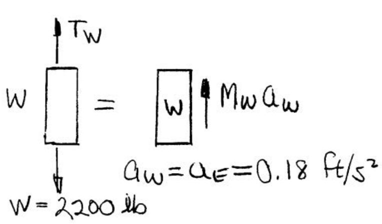
Kinematics: $2x_E = x_C,\ 2\dot{x}_E = \dot{x}_C,\ v_C = 2v_E = 2 \text{ ft/s}$

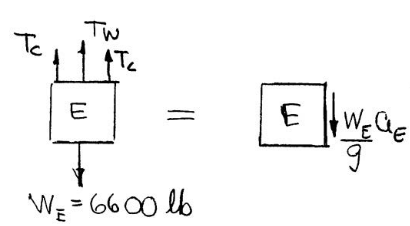
$$P = T_C \cdot v_C = (2200 \text{ lb})(2 \text{ ft/s}) = 4400 \text{ lb}\cdot\text{ft/s} = 8.00 \text{ hp}$$

$P = 8.00$ hp ◀

(b) $a_E = 0.18 \text{ ft/s}^2 \uparrow,\ v_E = 1 \text{ ft/s} \downarrow$

Counterweight *Elevator*





Counterweight: $+\uparrow \Sigma F = Ma: \quad T_W - W = \dfrac{W}{g}(a_W)$

PROBLEM 13.54 (Continued)

$$T_W = (2200 \text{ lb}) + \frac{(2200 \text{ lb})(0.18 \text{ ft/s}^2)}{(32.2 \text{ ft/s}^2)}$$

$$T_W = 2212 \text{ lb}$$

Elevator

$$+\uparrow \Sigma F = ma \qquad 2T_C + T_W - W_E = \frac{-W_E}{g}(a_E)$$

$$2T_C = (-2212 \text{ lb}) + (6600 \text{ lb}) - \frac{(6600 \text{ lb})(0.18 \text{ ft/s}^2)}{(32.2 \text{ ft/s}^2)}$$

$$2T_C = 4351 \text{ lb}$$

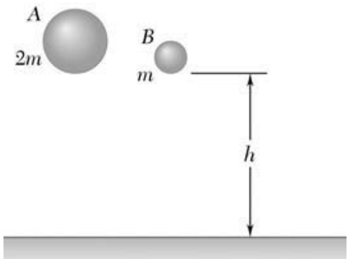
$$T_C = 2175.6 \text{ lb}$$

$$v_C = 2 \text{ ft/s (see part}(a))$$

$$P = T_C \cdot v_C = (2175.6 \text{ lb})(2 \text{ ft/s}) = 4351.2 \text{ lb}\cdot\text{ft/s}$$

$$= 7.911 \text{ hp}$$

$P = 7.91$ hp ◀



PROBLEM 13.CQ2

Two small balls A and B with masses $2m$ and m respectively are released from rest at a height h above the ground. Neglecting air resistance, which of the following statements are true when the two balls hit the ground?

(*a*) The kinetic energy of A is the same as the kinetic energy of B.

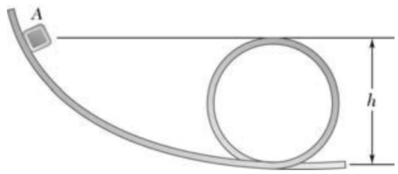
(*b*) The kinetic energy of A is half the kinetic energy of B.

(*c*) The kinetic energy of A is twice the kinetic energy of B.

(*d*) The kinetic energy of A is four times the kinetic energy of B.

SOLUTION

Answer: (*c*)



PROBLEM 13.CQ3

Block A is released from rest and slides down the frictionless ramp to the loop. The maximum height h of the loop is the same as the initial height of the block. Will A make it completely around the loop without losing contact with the track?

(*a*) Yes

(*b*) No

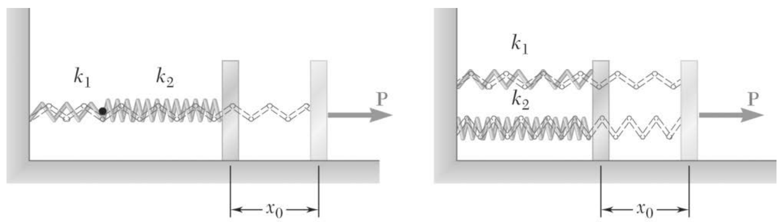
(*c*) need more information

SOLUTION

Answer: (*b*) In order for A to not maintain contact with the track, the normal force must remain greater than zero, which requires a non-zero speed at the top of the loop.

PROBLEM 13.55

A force **P** is slowly applied to a plate that is attached to two springs and causes a deflection x_0. In each of the two cases shown, derive an expression for the constant k_e, in terms of k_1 and k_2, of the single spring equivalent to the given system, that is, of the single spring which will undergo the same deflection x_0 when subjected to the same force **P**.



(*a*) (*b*)

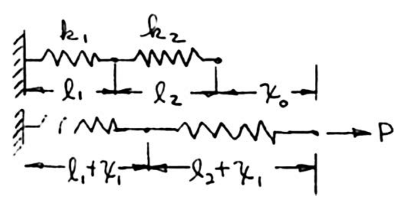
SOLUTION

System is in equilibrium in deflected x_0 position.

Case (*a*) Force in both springs is the same = P

$$x_0 = x_1 + x_2$$

$$x_0 = \frac{P}{k_e}$$



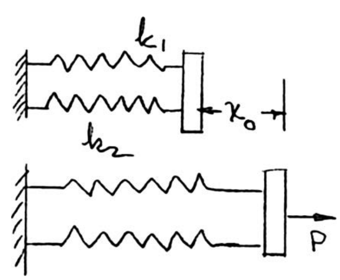
$$x_1 = \frac{P}{k_1} \qquad x_2 = \frac{P}{k_2}$$

Thus,

$$\frac{P}{k_e} = \frac{P}{k_1} + \frac{P}{k_2}$$

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_e = \frac{k_1 k_2}{k_1 + k_2} \blacktriangleleft$$



Case (*b*) Deflection in both springs is the same = x_0

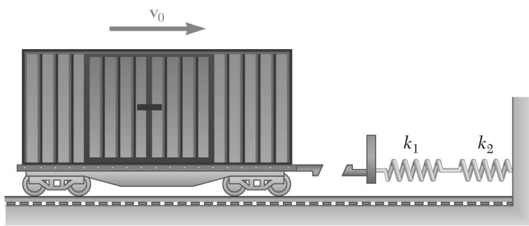
$$P = k_1 x_0 + k_2 x_0$$

$$P = (k_1 + k_2)x_0$$

$$P = k_e x_0$$

Equating the two expressions for

$$P = (k_1 + k_2)x_0 = k_e x_0 \qquad k_e = k_1 + k_2 \blacktriangleleft$$



PROBLEM 13.56

A loaded railroad car of mass m is rolling at a constant velocity $\mathbf{v_0}$ when it couples with a massless bumper system. Determine the maximum deflection of the bumper assuming the two springs are (*a*) in series (as shown), (*b*) in parallel.

SOLUTION

Let position A be at the beginning of contact and position B be at maximum deflection.

$$T_A = \frac{1}{2} m v_0^2$$

$$V_A = 0 \qquad \text{(zero force in springs)}$$

$$T_B = 0 \qquad (v = 0 \text{ at maximum deflection})$$

$$V_B = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2$$

where x_1 is deflection of spring k_1 and x_2 is that of spring k_2.

Conservation of energy: $T_A + V_A = T_B + V_B$

$$\frac{1}{2} m v_0^2 + 0 = 0 + \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2$$

$$k_1 x_1^2 + k_2 x_2^2 = m v_0^2 \qquad (1)$$

(*a*) *Springs are in series.*

Let F be the force carried by the two springs.

Then, $x_1 = \dfrac{F}{k_1}$ and $x_2 = \dfrac{F}{k_2}$

Eq. (1) becomes $F^2 \left(\dfrac{1}{k_1} + \dfrac{1}{k_2} \right) = m v_0^2$

so that $F = v_0 \sqrt{m / \left(\dfrac{1}{k_1} + \dfrac{1}{k_2} \right)}$

The maximum deflection is

$$\delta = x_1 + x_2 = \left(\frac{1}{k_1} + \frac{1}{k_2} \right) F$$

$$= \left(\frac{1}{k_1} + \frac{1}{k_2} \right) v_0 \sqrt{m / \left(\frac{1}{k_1} + \frac{1}{k_2} \right)}$$

$$= v_0 \sqrt{m \left(\frac{1}{k_1} + \frac{1}{k_2} \right)}$$

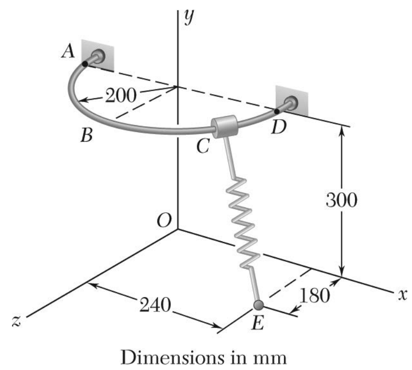
$$\delta = v_0 \sqrt{m(k_1 + k_2)/k_1 k_2} \blacktriangleleft$$

PROBLEM 13.56 (Continued)

(*b*) *Springs are in parallel.*

$$x_1 = x_2 = \delta$$

Eq. (1) becomes $(k_1 + k_2)\delta^2 = mv_0^2$ $\qquad \delta = v_0\sqrt{\dfrac{m}{k_1 + k_2}}$ ◀



PROBLEM 13.57

A 600-g collar C may slide along a horizontal, semicircular rod ABD. The spring CE has an undeformed length of 250 mm and a spring constant of 135 N/m. Knowing that the collar is released from rest at A and neglecting friction, determine the speed of the collar (*a*) at B, (*b*) at D.

SOLUTION

First calculate the lengths of the spring when the collar is at positions A, B, and D.

$$l_A = \sqrt{440^2 + 300^2 + 180^2} = 562.14 \text{ mm}$$

$$l_B = \sqrt{240^2 + 300^2 + 20^2} = 384.71 \text{ mm}$$

$$l_D = \sqrt{40^2 + 300^2 + 180^2} = 352.14 \text{ mm}$$

The elongations of springs are given by $e = l - l_0$.

$$e_A = 562.14 - 250 = 312.14 \text{ mm} = 0.31214 \text{ m}$$

$$e_B = 384.71 - 250 = 134.71 \text{ mm} = 0.13471 \text{ m}$$

$$e_D = 352.14 - 250 = 102.14 \text{ mm} = 0.10214 \text{ m}$$

Potential energies: $V = \frac{1}{2}ke^2$

$$V_A = \frac{1}{2}(135 \text{ N/m})(0.31214 \text{ m})^2 = 6.5767 \text{ J}$$

$$V_B = \frac{1}{2}(135 \text{ N/m})(0.13471 \text{ m})^2 = 1.2249 \text{ J}$$

$$V_D = \frac{1}{2}(135 \text{ N/m})(0.10214 \text{ m})^2 = 0.7042 \text{ J}$$

Since the semicircular rod ABC is horizontal, there is no change in gravitational potential energy.

Mass of collar: $m = 600 \text{ g} = 0.600 \text{ kg}$

Kinetic energies:

$$T_A = \frac{1}{2}mv_A^2 = 0.300\,v_A^2 = 0$$

$$T_B = \frac{1}{2}mv_B^2 = 0.300\,v_B^2$$

$$T_D = \frac{1}{2}mv_D^2 = 0.300\,v_D^2$$

(*a*) *Speed of collar at B.*

Conservation of energy: $T_A + V_A = T_B + V_B$

$$0 + 6.5767 = 0.300 v_B^2 + 1.2249$$

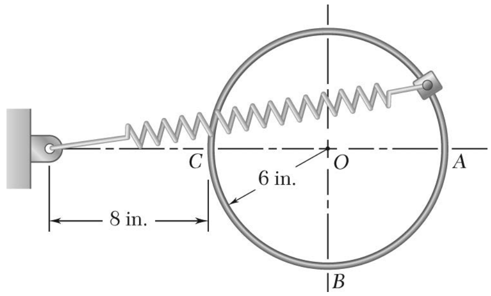
$$v_B^2 = 17.839 \text{ m}^2/\text{s}^2 \qquad v_B = 4.22 \text{ m/s} \blacktriangleleft$$

(*b*) *Speed of collar at D.*

Conservation of energy: $T_A + V_A = T_D + V_D$

$$0 + 6.5767 = 0.300 v_0^2 + 0.7042$$

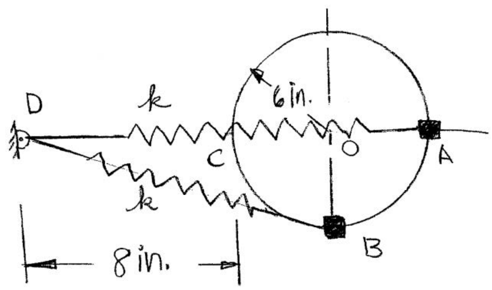
$$v_D^2 = 19.575 \text{ m}^2/\text{s}^2 \qquad v_D = 4.42 \text{ m/s} \blacktriangleleft$$



PROBLEM 13.58

A 3-lb collar is attached to a spring and slides without friction along a circular rod in a *horizontal* plane. The spring has an undeformed length of 7 in. and a constant $k = 1.5$ lb/in. Knowing that the collar is in equilibrium at A and is given a slight push to get it moving, determine the velocity of the collar (*a*) as it passes through B, (*b*) as it passes through C.

SOLUTION



$$L_0 = 7 \text{ in.}, L_{DA} = 20 \text{ in.}$$

$$L_{DB} = \sqrt{(8+6)^2 + 6^2} = 15.23 \text{ in.}$$

$$L_{DC} = 8 \text{ in.}$$

$$\Delta L_{DA} = 20 - 7 = 13 \text{ in.}$$

$$\Delta L_{DB} = 15.23 - 7 = 8.23 \text{ in.}$$

$$\Delta L_{DC} = 8 - 7 = 1 \text{ in.}$$

(*a*)

$$T_A = 0,\ V_A = \frac{1}{2}k(\Delta L_{DA})^2 = \frac{1}{2}(1.5)(13)^2 = 126.75 \text{ lb}\cdot\text{in.}$$

$$= 10.5625 \text{ lb}\cdot\text{ft}$$

$$T_B = \frac{1}{2}mv_B^2 = \frac{1.5}{g}\ v_B^2$$

$$V_B = \frac{1}{2}(1.5)(8.23)^2 = 50.8 \text{ lb}\cdot\text{in.} = 4.233 \text{ lb}\cdot\text{ft}$$

$$T_A + V_A = T_B + V_B:\quad 0 + 10.5625 = \frac{1.5v_B^2}{32.2} + 4.233$$

$v_B = 11.66$ ft/s ◀

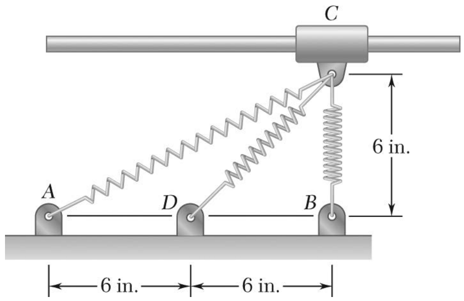
(*b*)

$$T_A = 0,\ V_A = 10.5625 \text{ lb}\cdot\text{ft},\ T_C = \frac{1.5}{32.2}\ v_C^2$$

$$V_C = \frac{1}{2}(1.5)(1)^2 = 0.75 \text{ lb}\cdot\text{in.} = 0.0625 \text{ lb}\cdot\text{ft}$$

$$T_A + V_A = T_C + V_C: \quad 0 + 10.5625 = \frac{1.5}{32.2} v_c^2 + 0.0625$$

$v_C = 15.01$ ft/s ◀



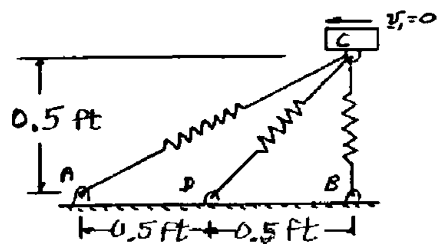
PROBLEM 13.59

A 3-lb collar C may slide without friction along a horizontal rod. It is attached to three springs, each of constant $k = 2$ lb/in. and 6 in. undeformed length. Knowing that the collar is released from rest in the position shown, determine the maximum speed it will reach in the ensuing motion.

SOLUTION

Maximum velocity occurs at E where collar is passing through position of equilibrium.

Position ①



$$T_1 = 0$$

Note: Undeformed length of springs is 6 in. = 0.5 ft.

Spring AC: $L = \sqrt{(1\text{ ft})^2 + (0.5\text{ ft})^2} = 1.1180\text{ ft}$

$\Delta = 1.1180 - 0.50 = 0.6180\text{ ft}$

Spring CD: $L = \sqrt{(0.5\text{ ft})^2 + (0.5\text{ ft})^2} = 0.70711\text{ ft}$

$\Delta = 0.70711 - 0.50 = 0.20711\text{ ft}$

Spring BD: $L = 0.50\text{ ft},\quad \Delta = 0$

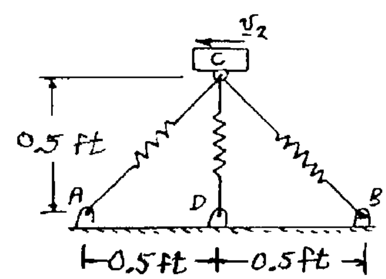
Potential energy. ($k = 2$ lb/in. $= 24$ lb/ft for each spring)

$$V_1 = \Sigma\frac{1}{2}k\Delta^2 = \frac{1}{2}k\Sigma\Delta^2 = \frac{1}{2}(24\text{ lb/ft})[(0.6180\text{ ft})^2 + (0.20711\text{ ft})^2 + 0]$$

$$V_1 = 5.0983\text{ lb}\cdot\text{ft}$$

Position ②

$$m = \frac{3.0\text{ lb}}{32.2\text{ ft/s}^2} = 0.093168\text{ slug};\quad T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.093168\text{ slug})\,v_2^2$$



Spring AC: $L = \sqrt{(0.5\text{ ft})^2 + (0.5\text{ ft})^2} = 0.7071067\text{ ft}$

$\Delta = 0.70711 - 0.50 = 0.20711\text{ ft}$

Spring CD: $L = 0.50\text{ ft}$

$\Delta = 0$

Spring BC: $L = \sqrt{(0.5\text{ ft})^2 + (0.5\text{ ft})^2} = 0.7071067\text{ ft}$

$\Delta = 0.70711 - 0.50 = 0.20711\text{ ft}$

PROBLEM 13.59 (Continued)

Potential energy. $V_2 = \Sigma \frac{1}{2} k\Delta^2 = \frac{1}{2} k\Sigma\Delta^2$

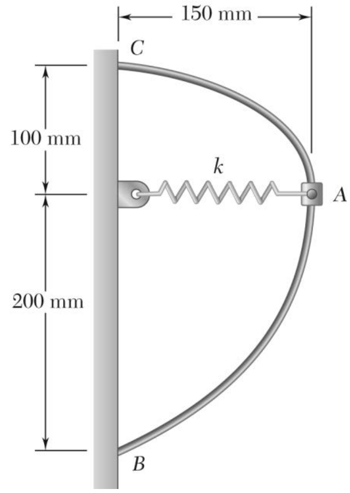
$$V_2 = \frac{1}{2}(24 \text{ lb/ft})[(0.20711 \text{ ft})^2 + 0 + (0.20711 \text{ ft})^2] = 1.0294 \text{ lb} \cdot \text{ft}$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2$: $0 + 5.0983 \text{ lb} \cdot \text{ft} = \frac{1}{2}(0.093168 \text{ slug})v_2^2 + 1.0294 \text{ lb} \cdot \text{ft}$

$$v_2^2 = 87.345$$

$\mathbf{v}_2 = 9.35 \text{ ft/s} \leftrightarrow$ ◀

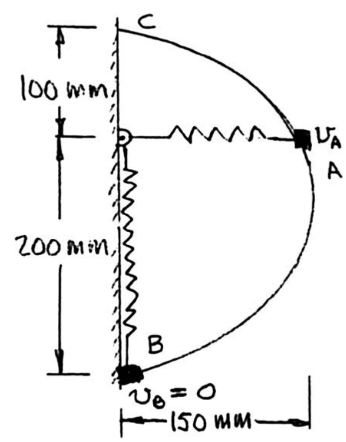
PROBLEM 13.60



A 500-g collar can slide without friction on the curved rod *BC* in a *horizontal* plane. Knowing that the undeformed length of the spring is 80 mm and that $k = 400\,\text{kN/m}$, determine (*a*) the velocity that the collar should be given at *A* to reach *B* with zero velocity, (*b*) the velocity of the collar when it eventually reaches *C*.

SOLUTION

(*a*) Velocity at *A*:



$$T_A = \frac{1}{2}mv_A^2 = \left(\frac{0.5}{2}\text{ kg}\right)v_A^2$$

$$T_A = (0.25)v_A^2$$

$$\Delta L_A = 0.150\text{ m} - 0.080\text{ m}$$

$$\Delta L_A = 0.070\text{ m}$$

$$V_A = \frac{1}{2}k(\Delta L_A)^2$$

$$V_A = \frac{1}{2}(400\times 10^3\text{ N/m})(0.070\text{ m})^2$$

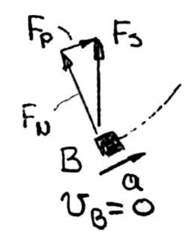
$$V_A = 980\text{ J}$$

$$v_B = 0 \qquad T_B = 0$$

$$\Delta L_B = 0.200\text{ m} - 0.080\text{ m} = 0.120\text{ m}$$

$$V_B = \frac{1}{2}k(\Delta L_B)^2 = \frac{1}{2}(400\times 10^3\text{ N/m})(0.120\text{ m})^2$$

$$V_B = 2880\text{ J}$$



Substitute into conservation of energy.

$$T_A + V_A = T_B + V_B \qquad 0.25v_A^2 + 980 = 0 + 2880$$

$$v_A^2 = \frac{(2880 - 980)}{(0.25)}$$

$$v_A^2 = 7600\text{ m}^2/\text{s}^2$$

$v_A = 87.2\text{ m/s}$ ◀

(*b*) Velocity at *C*:

Since slope at *B* is positive, the component of the spring force F_P, parallel to the rod, causes the block to move back toward *A*.

$$T_B = 0, \quad V_B = 2880 \text{ J} \quad \text{[from part } (a)]$$

$$T_C = \frac{1}{2} m v_C^2 = \frac{(0.5 \text{ kg})}{2} v_C^2 = 0.25 v_C^2$$

$$\Delta L_C = 0.100 \text{ m} - 0.080 \text{ m} = 0.020 \text{ m}$$

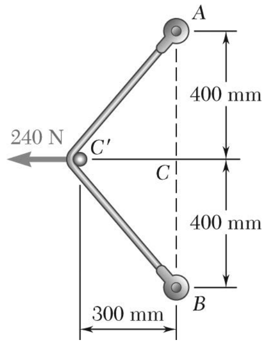
$$V_C = \frac{1}{2} k (\Delta L_C)^2 = \frac{1}{2}(400 \times 10^3 \text{ N/m})(0.020 \text{ m})^2 = 80.0 \text{ J}$$

Substitute into conservation of energy.

$$T_B + V_B = T_C + V_C \quad 0 + 2880 = 0.25 v_C^2 + 80.0$$

$$v_C^2 = 11,200 \text{ m}^2/\text{s}^2$$

$v_C = 105.8$ m/s ◀



PROBLEM 13.61

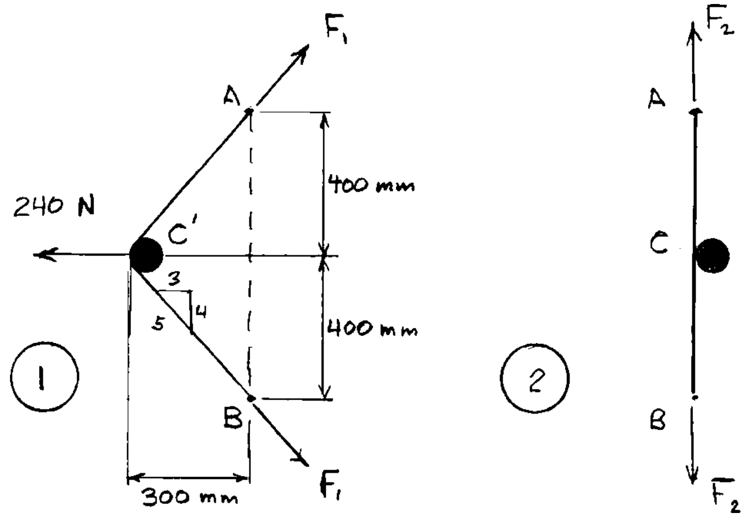
An elastic cord is stretched between two Points *A* and *B*, located 800 mm apart in the same horizontal plane. When stretched directly between *A* and *B*, the tension is 40 N. The cord is then stretched as shown until its midpoint *C* has moved through 300 mm to C'; a force of 240 N is required to hold the cord at C'. A 0.1 kg pellet is placed at C', and the cord is released. Determine the speed of the pellet as it passes through *C*.

SOLUTION

Let ℓ = undeformed length of cord.

Position 1. Length $AC'B = 1.0$ m; Elongation $= x_1 = 1.0 - \ell$

$$\Sigma F_x = 0: \quad 2\left(\frac{3}{5}F_1\right) - 240 \text{ N} = 0 \quad F_1 = 200 \text{ N}$$



Position 2. Length $ACB = 0.8$ m; Elongation $= x_2 = 0.8 - \ell$

Given $F_2 = 40$ N

$$F_1 = kx, \quad F_2 = kx_2$$

$$F_1 - F_2 = k(x_1 - x_2)$$

$$200 - 40 = k[(1.0 - \ell) - (0.8 - \ell)] = 0.2k$$

$$k = \frac{160}{0.2} = 800 \text{ N/m}$$

$$x_1 = \frac{F_1}{k} = \frac{200\text{ N}}{800\text{ N/m}} = 0.25\text{ m}$$

$$x_2 = \frac{F_2}{k} = \frac{40\text{ N}}{800\text{ N/m}} = 0.05\text{ m}$$

Position ①: $T_1 = 0 \qquad V_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(800\text{ N/m})(0.25\text{ m})^2 = 25.0\text{ N}\cdot\text{m}$

Position②: $m = 0.10\text{ kg}$

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.1\text{ kg})v_2^2 = 0.05v_2^2$$

$$V_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(800\text{ N/m})(0.05\text{ m})^2 = 1\text{ N}\cdot\text{m}$$

Conservation of energy:

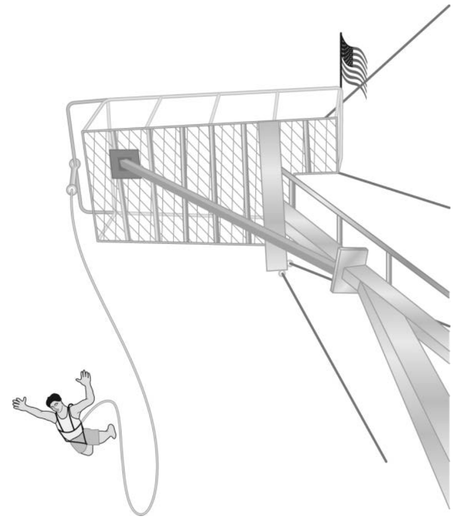
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 25.0\text{ N}\cdot\text{m} = 0.05v_2^2 + 1.0\text{ N}\cdot\text{m}$$

$$24.0 = 0.05v_2^2 \qquad v_2 = 21.909\text{ m/s}$$

$v_2 = 21.9\text{ m/s}$ ◀

Note: The horizontal force applied at the midpoint of the cord is not proportional to the horizontal distance $C'C$. A solution based on the work of the horizontal force would be rather involved.



PROBLEM 13.62

An elastic cable is to be designed for bungee jumping from a tower 130 ft high. The specifications call for the cable to be 85 ft long when unstretched, and to stretch to a total length of 100 ft when a 600-lb weight is attached to it and dropped from the tower. Determine (*a*) the required spring constant k of the cable, (*b*) how close to the ground a 186-lb man will come if he uses this cable to jump from the tower.

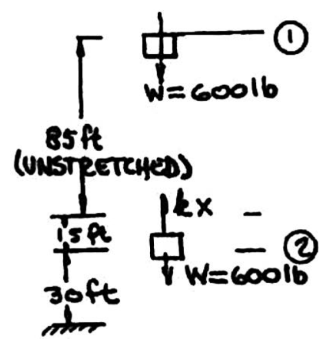
SOLUTION

(*a*) Conservation of energy:

$$V_1 = 0 \quad T_1 = 0 \quad V_1 = 100W$$

Datum at ②:

$$V_1 = (100 \text{ ft})(600 \text{ lb})$$
$$= 6 \times 10^4 \text{ ft} \cdot \text{lb}$$



$$V_2 = 0 \quad T_2 = 0$$

$$V_2 = V_g + V_e = 0 + \frac{1}{2}k\,(15 \text{ ft})^2$$

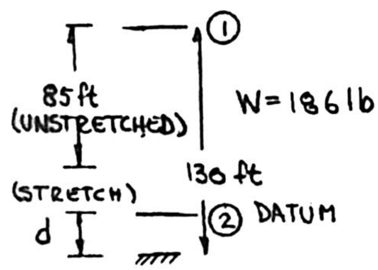
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 6 \times 10^4 = 0 + (112.5)k$$

$k = 533$ lb/ft ◀

(*b*) From (*a*), $k = 533$ lb/ft

$$T_1 = 0$$
$$W = 186 \text{ lb}$$
$$V_1 = (186)(130 - d)$$
$$T_2 = 0$$



Datum:

$$V_2 = V_g + V_e = 0 + \frac{1}{2}(533)(130 - 85 - d)^2$$

$$V_2 = (266.67)(45 - d)^2$$

PROBLEM 13.62 (Continued)

$d =$ distance from the ground $\qquad T_1 + V_1 = T_2 + V_2$

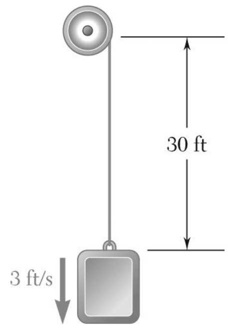
$$0 + (186)(130 - d) = 0 + (266.67)(45 - d)^2$$

$$266.7d^2 - 23815d + 515827 = 0$$

$$d = \frac{23815 \mp \sqrt{(23815)^2 - 4(266.7)(515827)}}{(2)(266.7)} = \begin{matrix} 36.99 \text{ ft} \\ 52.3 \text{ ft} \end{matrix}$$

Discard 52.3 ft (since the cord acts in compression when rebound occurs).

$d = 37.0$ ft ◀



PROBLEM 13.63

It is shown in mechanics of materials that the stiffness of an elastic cable is $k = AE/L$ where A is the cross sectional area of the cable, E is the modulus of elasticity and L is the length of the cable. A winch is lowering a 4000-lb piece of machinery using at a constant speed of 3ft/s when the winch suddenly stops. Knowing that the steel cable has a diameter of 0.4 in., $E = 29 \times 10^6$ lb/in^2, and when the winch stops $L = 30$ ft, determine the maximum downward displacement of the piece of machinery from the point it was when the winch stopped.

SOLUTION

Mass of machinery: $$m = \frac{W}{g} = \frac{4000}{32.2} = 124.22 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Let position 1 be the state just before the winch stops and the gravitational potential V_g be equal to zero at this state.

For the cable, $$A = \frac{\pi}{4}(\text{diameter})^2 = \frac{\pi}{4}(0.4 \text{ in.})^2 = 0.12566 \text{ in}^2$$

$$AE = (0.12556 \text{ in.}^2)(29\times10^6 \text{ lb/in}^2) = 3.6442\times10^6 \text{ lb}$$

For $$L = 30 \text{ ft}, \quad k = \frac{AE}{L} = \frac{3.6442\times10^6 \text{ lb}}{30 \text{ ft}} = 121.47\times10^3 \text{ lb/ft}$$

Initial force in cable (equilibrium): $$F_1 = W = 4000 \text{ lb.}$$

Elongation in position 1: $$x_1 = \frac{F_1}{k} = \frac{4000}{121.47\times10^3} = 0.03293 \text{ ft}$$

Potential energy: $$V_1 = \frac{1}{2}kx_1^2 = \frac{F_1^2}{2k}$$

$$V_1 = \frac{(4000 \text{ lb})^2}{(2)(121.47\times10^3 \text{ lb/ft})} = 65.860 \text{ ft}\cdot\text{lb}$$

Kinetic energy: $$T_1 = \frac{1}{2}mv_1^2$$

$$T_1 = \frac{1}{2}(124.22 \text{ lb}\cdot\text{s}^2/\text{ft})(3 \text{ ft/s}^2) = 558.99 \text{ ft}\cdot\text{lb}$$

PROBLEM 13.63 (Continued)

Let position 2 be the position of maximum downward displacement. Let x_2 be the elongation in this position.

Potential energy: $V_2 = \frac{1}{2}kx_2^2 - W(x_2 - x_1)$

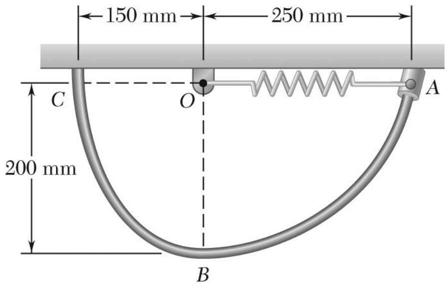
$$V_2 = \frac{1}{2}(121.47\times10^3)x_2^2 - (4000)(x_2 - 0.03293)$$
$$= 60.735\times10^3 x^2 - 4000x_2 + 131.72$$

Kinetic energy: $T_2 = 0 \qquad (\text{since } v_2 = 0)$

Principle of work and energy: $T_1 + V_1 = T_2 + V_2$

$$558.99 + 65.860 = 60.735\times10^3 x_2^2 - 4000x_2 + 131.72$$
$$60.735\times10^3 x_2^2 - 4000x_2 - 493.13 = 0$$
$$x_2 = 0.12887 \text{ ft}$$

Maximum displacement: $\delta = x_2 - x_1 = 0.09594 \text{ ft}$ $\qquad \delta = 1.151 \text{ in.}$



PROBLEM 13.64

A 2-kg collar is attached to a spring and slides without friction in a vertical plane along the curved rod *ABC*. The spring is undeformed when the collar is at *C* and its constant is 600 N/m. If the collar is released at *A* with no initial velocity, determine its velocity (*a*) as it passes through *B*, (*b*) as it reaches *C*.

SOLUTION

Spring elongations:

At *A*, $x_A = 250 \text{ mm} - 150 \text{ mm} = 100 \text{ mm} = 0.100 \text{ m}$

At *B*, $x_B = 200 \text{ mm} - 150 \text{ mm} = 50 \text{ mm} = 0.050 \text{ m}$

At *C*, $x_C = 0$

Potential energies for springs.

$$(V_A)_e = \frac{1}{2}kx_A^2 = \frac{1}{2}(600)(0.100)^2 = 3.00 \text{ J}$$

$$(V_B)_e = \frac{1}{2}kx_B^2 = \frac{1}{2}(600)(0.050)^2 = 0.75 \text{ J}$$

$$(V_C)_e = 0$$

Gravitational potential energies: Choose the datum at level *AOC*.

$$(V_A)_g = (V_C)_g = 0$$

$$(V_B)_g = -mg\,y = -(2)(9.81)(0.200) = -3.924 \text{ J}$$

Kinetic energies: $T_A = 0$

$$T_B = \frac{1}{2}mv_B^2 = 1.00\,v_B^2$$

$$T_C = \frac{1}{2}mv_C^2 = 1.00\,v_C^2$$

(*a*) *Velocity as the collar passes through B.*

Conservation of energy: $T_A + V_A = T_B + V_B$

$$0 + 3.00 + 0 = 1.00\,v_B^2 + 0.75 - 3.924$$

$v_B^2 = 6.174 \text{ m}^2/\text{s}^2$ $\mathbf{v}_B = 2.48 \text{ m/s} \longleftarrow$ ◀

PROBLEM 13.64 (Continued)

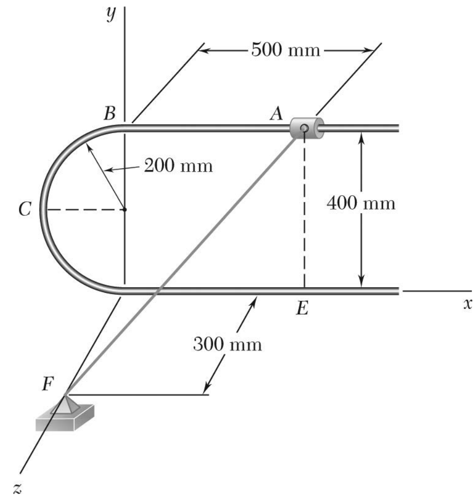
(*b*) *Velocity as the collar reaches C.*

Conservation of energy: $T_A + V_A = T_C + V_C$

$$0 + 3.00 + 0 = 1.00 v_C^2 + 0 + 0$$

$$v_C^2 = 3.00 \text{ m}^2/\text{s}^2$$

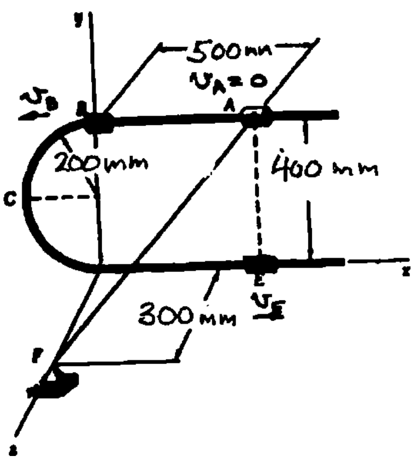
$\mathbf{v}_C = 1.732 \text{ m/s} \uparrow$ ◀



PROBLEM 13.65

A 1-kg collar can slide along the rod shown. It is attached to an elastic cord anchored at *F*, which has an undeformed length of 250 mm and a spring constant of 75 N/m. Knowing that the collar is released from rest at *A* and neglecting friction, determine the speed of the collar (*a*) at *B*, (*b*) at *E*.

SOLUTION



$$L_{AF} = \sqrt{(0.5)^2 + (0.4)^2 + (0.3)^2}$$

$$L_{AF} = 0.70711 \text{ m}$$

$$L_{BF} = \sqrt{(0.4)^2 + (0.3)^2}$$

$$L_{BF} = 0.5 \text{ m}$$

$$L_{FE} = \sqrt{(0.5)^2 + (0.3)^2}$$

$$L_{FE} = 0.58309 \text{ m}$$

$$V = V_e + V_g$$

(*a*) Speed at *B*: $v_A = 0, \qquad T_A = 0$

Point *A*:

$$(V_A)_e = \frac{1}{2}k(\Delta L_{AF})^2 \qquad \Delta L_{AF} = L_{AF} - L_0 = 0.70711 - 0.25$$

$$\Delta L_{AF} = 0.45711 \text{ m}$$

$$(V_A)_e = \frac{1}{2}(75 \text{ N/m})(0.45711 \text{ m})^2$$

$$(V_A)_e = 7.8355 \text{ N}\cdot\text{m}$$

$$(V_A)_g = (mg)(0.4)$$

$$= (1.0 \text{ kg})(9.81 \text{ m/s}^2)(0.4 \text{ m})$$

$$= 3.9240 \text{ N}\cdot\text{m}$$

$$V_A = (V_A)_e + (V_A)_g$$

$$= 7.8355 + 3.9240$$

$$= 11.7595 \text{ N}\cdot\text{m}$$

Point B: $T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(1.0 \text{ kg})v_B^2$

$$T_B = 0.5v_B^2$$

$$(V_B)_e = \frac{1}{2}k(\Delta L_{BF})^2 \qquad \Delta L_{BF} = L_{BF} - L_0 = 0.5 - 0.25$$

$$\Delta L_{BF} = 0.25 \text{ m}$$

$$(V_B)_e = \frac{1}{2}(75 \text{ N/m})(0.25 \text{ m})^2 = 2.3438 \text{ N}\cdot\text{m}$$

$$(V_B)_g = (mg)(0.4) = (1.0 \text{ kg})(9.81 \text{ m/s}^2)(0.4 \text{ m}) = 3.9240 \text{ N}\cdot\text{m}$$

$$V_B = (v_B)_e + (V_B)_g = 2.3438 + 3.9240 = 6.2678 \text{ N}\cdot\text{m}$$

$$T_A + V_A = T_B + V_B$$

$$0 + 11.7595 = 0.5v_B^2 + 6.2678$$

$$v_B^2 = (5.49169)/(0.5)$$

$$v_B^2 = 10.983 \text{ m}^2/\text{s}^2$$

$v_B = 3.31$ m/s ◀

(*b*) Speed at E:

Point A: $T_A = 0 \qquad V_A = 11.7595 \text{ N}\cdot\text{m}$ (from part (*a*))

Point E:

$$T_E = \frac{1}{2}mv_E^2 = \frac{1}{2}(1.0 \text{ kg})v_E^2 = 0.5v_E^2$$

$$(V_E)_e = \frac{1}{2}k(\Delta L_{FE})^2 \qquad \Delta L_{FE} = L_{FE} - L_0 = 0.5831 - 0.25$$

$$\Delta L_{FE} = 0.3331 \text{ m}$$

$$(V_E)_e = \frac{1}{2}(75 \text{ N/m})(0.3331 \text{ m})^2 = 4.1607 \text{ N}\cdot\text{m}$$

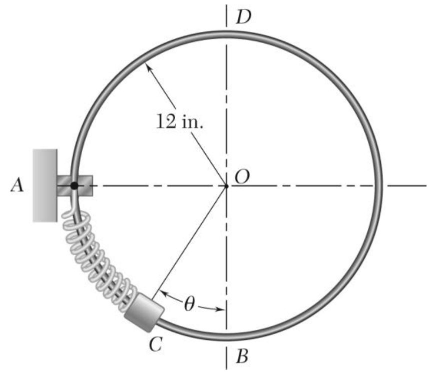
$$(V_E)_g = 0 \quad V_E = 4.1607 \text{ N}\cdot\text{m}$$

$$T_A + V_A = T_E + V_E \quad 0 + 11.7595 = 0.5v_E^2 + 4.1607$$

$$v_E^2 = 7.5988/0.5$$

$$v_E^2 = 15.1976 \text{ m}^2/\text{s}^2$$

$v_E = 3.90$ m/s ◀

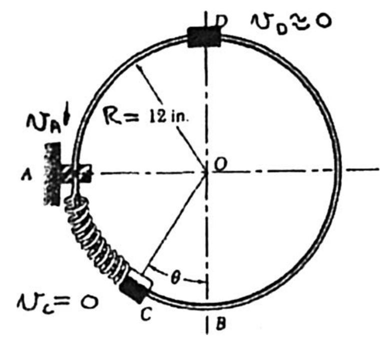


PROBLEM 13.66

A thin circular rod is supported in a *vertical plane* by a bracket at A. Attached to the bracket and loosely wound around the rod is a spring of constant $k = 3$ lb/ft and undeformed length equal to the arc of circle AB. An 8-oz collar C, not attached to the spring, can slide without friction along the rod. Knowing that the collar is released from rest at an angle θ with the vertical, determine (a) the smallest value of θ for which the collar will pass through D and reach Point A, (b) the velocity of the collar as it reaches Point A.

SOLUTION

(a) Smallest angle θ occurs when the velocity at D is close to zero.



$$v_C = 0 \qquad v_D = 0$$
$$T_C = 0 \qquad T_D = 0$$
$$V = V_e + V_g$$

Point C:

$R = 12 \text{ in.} = 1 \text{ ft}$

$$\Delta L_{BC} = (1 \text{ ft})(\theta) = \theta \text{ ft}$$
$$(V_C)_e = \frac{1}{2}k(\Delta L_{BC})^2$$
$$(V_C)_e = \frac{3}{2}\theta^2$$
$$(V_C)_g = WR(1-\cos\theta)$$
$$(V_C)_g = \left(\frac{8 \text{ oz}}{16 \text{ oz/lb}}\right)(1 \text{ ft})(1-\cos\theta)$$
$$(V_C)_g = \frac{1}{2}(1-\cos\theta)$$
$$V_C = (V_C)_e + (V_C)_g = \frac{3}{2}\theta^2 + \frac{1}{2}(1-\cos\theta)$$

Point D:

$$(V_D)_e = 0 \quad \text{(spring is unattached)}$$
$$(V_D)_g = W(2R) = (2)(0.5 \text{ lb})(1 \text{ ft}) = 1 \text{ lb}\cdot\text{ft}$$
$$T_C + V_C = T_D + V_D \quad 0 + \frac{3}{2}\theta^2 + \frac{1}{2}(1-\cos\theta) = 1$$
$$(1.5)\theta^2 - (0.5)\cos\theta = 0.5$$

By trial, $\theta = 0.7592$ rad $\qquad \theta = 43.5°$ ◀

PROBLEM 13.66 (Continued)

(*b*) Velocity at *A*:

Point *D*:

$$V_D = 0 \quad T_D = 0 \quad V_D = 1 \text{ lb} \cdot \text{ft} \text{[see Part } (a)]$$

Point *A*:

$$T_A = \frac{1}{2} m v_A^2 = \frac{1}{2} \frac{(0.5 \text{ lb})}{(32.2 \text{ ft/s}^2)} v_A^2$$

$$T_A = 0.0077640 v_A^2$$

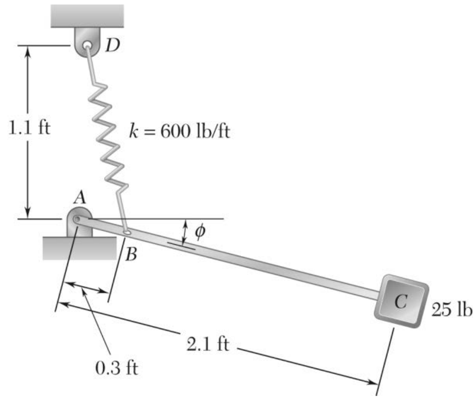
$$V_A = (V_A)_g = W(R) = (0.5 \text{ lb})(1 \text{ ft}) = 0.5 \text{ lb} \cdot \text{ft}$$

$$T_A + V_A = T_D + V_D$$

$$0.0077640 v_A^2 + 0.5 = 0 + 1$$

$$v_A^2 = 64.4 \text{ ft}^2/\text{s}^2$$

$\mathbf{v}_A = 8.02 \text{ ft/s} \downarrow$ ◀

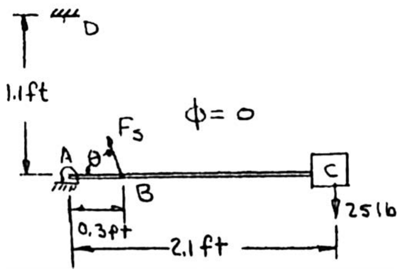


PROBLEM 13.67

The system shown is in equilibrium when $\phi = 0$. Knowing that initially $\phi = 90°$ and that block C is given a slight nudge when the system is in that position, determine the velocity of the block as it passes through the equilibrium position $\phi = 0$. Neglect the weight of the rod.

SOLUTION

Find the unstretched length of the spring.



$$\theta = \tan^{-1}\frac{1.1}{0.3}$$
$$= 1.3045 \text{ rad}$$
$$\theta = 74.745°$$
$$L_{BD} = \sqrt{(1.1)^2 + .3^2}$$
$$L_{BD} = 1.140 \text{ ft}$$

Equilibrium $$\Sigma M_A = (0.3)(F_s \sin\theta) - (25)(2.1) = 0$$

$$F_s = \frac{(25 \text{ lb})(2.1 \text{ ft})}{(0.3 \text{ ft})(\sin 74.745°)}$$
$$= 181.39 \text{ lb}$$
$$F_s = k\Delta L_{BD}$$

$$181.39 \text{ lb} = (600 \text{ lb/ft})(\Delta L_{BD})$$
$$\Delta L_{BD} = 0.30232 \text{ ft}$$

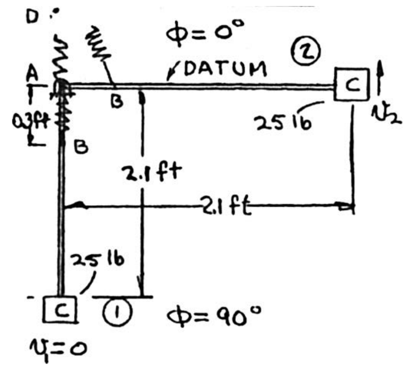
Unstretched length $$L_0 = L_{BD} - \Delta L_{BD}$$

$$L_0 = 1.140 - 0.3023$$
$$= 0.83768 \text{ ft}$$

Spring elongation, $\Delta L'_{BD}$, when $\phi = 90°$.

$$\Delta L'_{BD} = (1.1 \text{ ft} + 0.3 \text{ ft}) - L_0$$
$$\Delta L'_{BD} = 1.4 \text{ ft} - 0.8377 \text{ ft}$$
$$= 0.56232 \text{ ft}$$

PROBLEM 13.67 (Continued)



At ①, $(\phi = 90°)$

$$v_1 = 0, \quad T_1 = 0$$

$$V_1 = (V_1)_e + (V_1)_g$$

$$(V_1)_e = \frac{1}{2}k(\Delta L'_{BD})^2$$

$$(V_1)_e = \frac{1}{2}(600 \text{ lb/ft})(0.5623 \text{ ft})^2$$

$$(V_1)_e = 94.86 \text{ lb} \cdot \text{ft}$$

$$(V_1)_g = -(25 \text{ lb})(2.1 \text{ ft}) = -52.5 \text{ ft} \cdot \text{lb}$$

$$V_1 = 94.86 - 52.5 = 42.36 \text{ ft} \cdot \text{lb}$$

At ②, $(\phi = 0°)$

$$(V_2)_e = \frac{1}{2}k(\Delta L_{BD})^2 = \frac{1}{2}(600 \text{ lb/ft})(0.3023 \text{ ft})^2$$

$$(V_2)_e = 27.42 \text{ lb} \cdot \text{ft}$$

$$(V_2)_g = 0 \qquad V_2 = 27.42 \text{ ft} \cdot \text{lb}$$

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}\left(\frac{25 \text{ lb}}{32.2 \text{ ft/s}^2}\right)v_2^2 = 0.3882 v_2^2$$

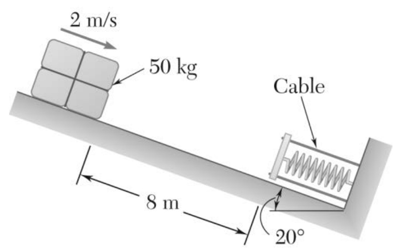
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 42.36 = 0.3882 v_2^2 + 27.42$$

$$v_2^2 = (14.941)/(0.3882)$$

$$v_2^2 = 38.48 \text{ ft}^2/\text{s}^2$$

$v_2 = 6.20$ ft/s ◀



PROBLEM 13.68

A spring is used to stop a 50-kg package which is moving down a 20º incline. The spring has a constant $k = 30$ kN/m and is held by cables so that it is initially compressed 50 mm. Knowing that the velocity of the package is 2 m/s when it is 8 m from the spring and neglecting friction, determine the maximum additional deformation of the spring in bringing the package to rest.

SOLUTION

Let position 1 be the starting position 8 m from the end of the spring when it is compressed 50 mm by the cable. Let position 2 be the position of maximum compression. Let x be the additional compression of the spring. Use the principle of conservation of energy. $T_1 + V_1 = T_2 + V_2$.

Position 1:

$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(50)(2)^2 = 100 \text{ J}$$

$$V_{1g} = mgh_1 = (50)(9.81)(8\sin 20°) = 1342.09 \text{ J}$$

$$V_{1e} = \frac{1}{2}ke_1^2 = \frac{1}{2}(30\times 10^3)(0.050)^2 = 37.5 \text{ J}$$

Position 2:

$$T_2 = \frac{1}{2}mv_2^2 = 0 \quad \text{since} \quad v_2 = 0.$$

$$V_{2g} = mgh_2 = (50)(9.81)(-x\sin 20°) = -167.76x$$

$$V_{2e} = \frac{1}{2}ke_2^2 = \frac{1}{2}(30\times 10^3)(0.05 + x)^2 = 37.5 + 1500x + 15000x^2$$

Principle of conservation of energy:

$$100 + 1342.09 + 37.5 = -167.61x + 37.5 + 1500x + 15000x^2$$

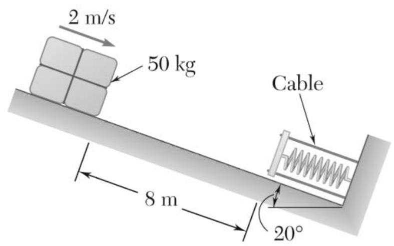
$$15{,}000x^2 + 1332.24x - 1442.09 = 0$$

Solving for x,

$$x = 0.26882 \quad \text{and} \quad -0.357\,64$$

$x = 0.269$ m ◀

PROBLEM 13.69



Solve Problem 13.68 assuming the kinetic coefficient of friction between the package and the incline is 0.2.

PROBLEM 13.68 A spring is used to stop a 50-kg package which is moving down a 20° incline. The spring has a constant $k = 30$ kN/m and is held by cables so that it is initially compressed 50 mm. Knowing that the velocity of the package is 2 m/s when it is 8 m from the spring and neglecting friction, determine the maximum additional deformation of the spring in bringing the package to rest.

SOLUTION

Let position 1 be the starting position 8 m from the end of the spring when it is compressed 50 mm by the cable. Let position 2 be the position of maximum compression. Let x be the additional compression of the spring. Use the principle of work and energy. $T_1 + V_1 + U_{1\to 2} = T_2 + V_2$

Position 1.

$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(50)(2)^2 = 100 \text{ J}$$

$$V_{1g} = mgh_1 = (50)(9.81)(8\sin 20°) = 1342.09 \text{ J}$$

$$V_{1e} = \frac{1}{2}ke_1^2 = \frac{1}{2}(30\times 10^3)(0.05)^2 = 37.5 \text{ J}$$

Position 2.

$$T_2 = \frac{1}{2}mv_2^2 = 0 \quad \text{since} \quad v_2 = 0.$$

$$V_{2g} = mgh_2 = (50)(9.81)(-x\sin 20°) = -167.76x$$

$$V_{2e} = \frac{1}{2}ke_2^2 = \frac{1}{2}(30\times 10^3)(0.05 + x)^2 = 37.5 + 1500x + 15{,}000x^2$$

Work of the friction force.

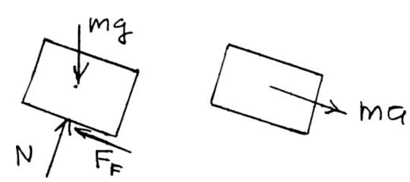
$$+\nearrow \; \Sigma F_n = 0$$

$$N - mg\cos 20° = 0$$

$$N = mg\cos 20°$$

$$= (50)(9.81)\cos 20°$$

$$= 460.92 \text{ N}$$



$$F_f = \mu_k N$$

$$= (0.2)(460.92)$$

$$= 92.184$$

$$U_{1\to 2} = -F_f d$$

$$= -92.184(8 + x)$$

$$= -737.47 - 92.184x$$

Principle of work and energy: $T_1 + V_1 + U_{1-2} = T_2 + V_2$

$$100 + 1342.09 + 37.5 - 737.47 - 92.184x$$

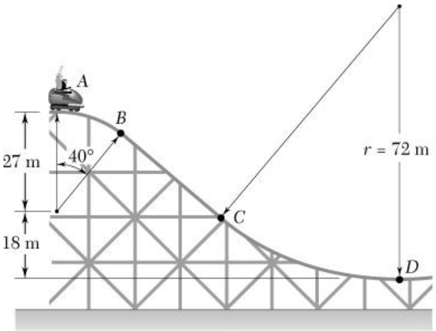
$$= -167.76x + 37.5 + 1500x + 15{,}000x^2$$

$$15{,}000\,x^2 + 1424.42x - 704.62 = 0$$

Solving for x,

$$x = 0.17440 \quad \text{and} \quad -0.26936$$

$x = 0.1744$ m ◀



PROBLEM 13.70

A section of track for a roller coaster consists of two circular arcs AB and CD joined by a straight portion BC. The radius of AB is 27 m and the radius of CD is 72 m. The car and its occupants, of total mass 250 kg, reach Point A with practically no velocity and then drop freely along the track. Determine the normal force exerted by the track on the car as the car reaches point B. Ignore air resistance and rolling resistance.

SOLUTION

Calculate the speed of the car as it reaches Point B using the principle of conservation of energy as the car travels from position A to position B.

Position A: $v_A = 0, \quad T_A = \frac{1}{2}mv_A^2 = 0, \quad V_A = 0$ (datum)

Position B: $V_B = -mgh$

where h is the decrease in elevation between A and B.

$$T_B = \frac{1}{2}mv_B^2$$

Conservation of energy: $T_A + V_A = T_B + V_B$:

$$0 + 0 = \frac{1}{2}mv_B^2 - mgh$$

$$v_B^2 = 2gh$$

$$= (2)(9.81\text{ m/s}^2)(27\text{ m})(1 - \cos 40°)$$

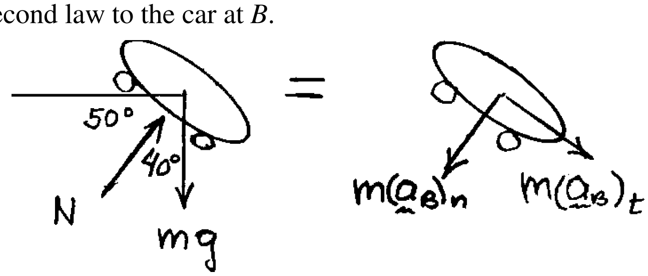
$$= 123.94\text{ m}^2/\text{s}^2$$

Normal acceleration at B:

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{123.94\text{ m}^2/\text{s}^2}{27\text{ m}} = 4.59\text{ m/s}^2$$

$$(\mathbf{a}_B)_n = 4.59\text{ m/s}^2 \measuredangle 50°$$

Apply Newton's second law to the car at B.



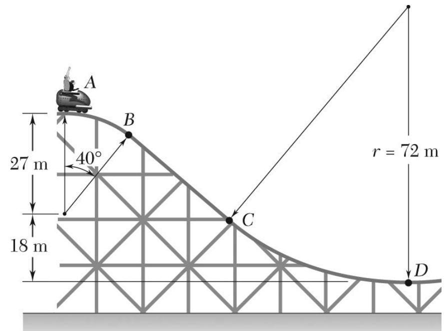
PROBLEM 13.70 (Continued)

$$\measuredangle 50° \Sigma F_n = ma_n: \quad N - mg\cos 40° = -ma_n$$

$$N = mg\cos 40° - ma_n = m(g\cos 40° - a_n)$$

$$= (250\text{ kg})[(9.81\text{ m/s}^2)\cos 40° - 4.59\text{ m/s}^2]$$

$$= 1878.7 - 1147.5 \qquad N = 731\text{ N} \blacktriangleleft$$



PROBLEM 13.71

A section of track for a roller coaster consists of two circular arcs *AB* and *CD* joined by a straight portion *BC*. The radius of *AB* is 27 m and the radius of *CD* is 72 m. The car and its occupants, of total mass 250 kg, reach Point *A* with practically no velocity and then drop freely along the track. Determine the maximum and minimum values of the normal force exerted by the track on the car as the car travels from *A* to *D*. Ignore air resistance and rolling resistance.

SOLUTION

Calculate the speed of the car as it reaches Point *P*, any point on the roller coaster track. Apply the principle of conservation of energy.

Position *A*: $v_A = 0, \quad T_A = \frac{1}{2}mv_A^2 = 0, \quad V_A = 0$ (datum)

Position *P*: $V_P = -mgh$

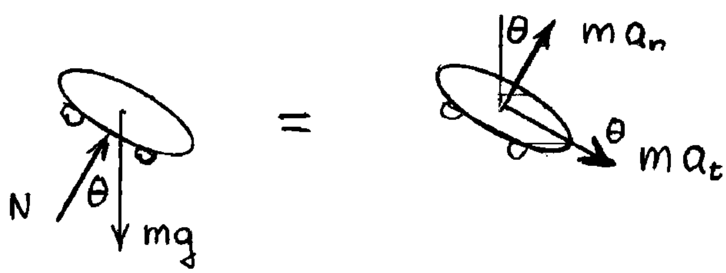
where *h* is the decrease in elevation along the track.

$$T_P = \frac{1}{2}\,mv^2$$

Conservation of energy: $T_A + V_A = T_P + V_P$

$$0 + 0 = \frac{1}{2}mv^2 - mgh \qquad v^2 = 2gh \tag{1}$$

Calculate the normal force using Newton's second law. Let θ be the slope angle of the track.



$$\Sigma F_n = ma_n: \quad N - mg\cos\theta = ma_n$$

$$N = mg\cos\theta + ma_n \tag{2}$$

Over portion *AB* of the track, $h = \rho(1 - \cos\theta)$

and $a_n = -\dfrac{mv^2}{\rho}$

where ρ is the radius of curvature. ($\rho = 27$ m)

$$N = mg\cos\theta - \frac{2mg\rho(1-\cos\theta)}{\rho} = mg(3\cos\theta - 2)$$

At Point A ($\theta = 0$) $\quad N_A = mg = (250)(9.81) = 2452.5$ N

At Point B ($\theta = 40°$) $\quad N_B = (2452.5)(3\cos 40° - 2)$

$$N_B = 731 \text{ N}$$

Over portion BC, $\quad \theta = 40°, \quad a_n = 0 \quad$ (straight track)

$$N_{BC} = mg\cos 40° = 2452.5\cos 40°$$
$$N_{BC} = 1879 \text{ N}$$

Over portion CD, $\quad h = h_{\max} - r(1-\cos\theta)$

and $\quad a_n = \dfrac{mv^2}{r}$

where r is the radius of curvature. ($r = 72$ m)

$$\begin{aligned} N &= mg\cos\theta + \frac{2mgh}{r} \\ &= mg\cos\theta + 2mg\left(\frac{h_{\max}}{r} - 1 - \cos\theta\right) \\ &= mg\left(3\cos\theta - 2 + \frac{2h_{\max}}{r}\right) \end{aligned}$$

which is maximum at Point D, where

$$N_D = mg\left(1 + \frac{2h_{\max}}{r}\right)$$

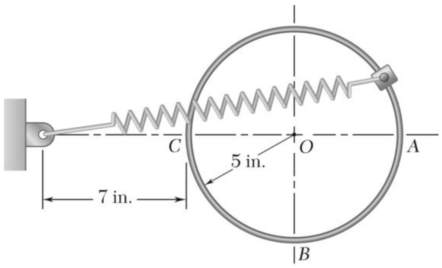
Data: $\quad h_{\max} = 27 + 18 = 45$ m, $\quad r = 72$ m

$$N_D = (2452.5)\left[1 + \frac{(2)(45)}{72}\right] = 5520 \text{ N}$$

Summary: minimum (just above B): 731 N ◀

maximum (at D): 5520 N ◀

PROBLEM 13.72



A 1-lb collar is attached to a spring and slides without friction along a circular rod in a *vertical* plane. The spring has an undeformed length of 5 in. and a constant $k = 10$ lb/ft. Knowing that the collar is released from being held at A determine the speed of the collar and the normal force between the collar and the rod as the collar passes through B.

SOLUTION

For the collar, $$m = \frac{W}{g} = \frac{1}{32.2} = 0.031056 \text{ lb} \cdot \text{s}^2/\text{ft}$$

For the spring, $$k = 10 \text{ lb/ft} \quad l_0 = 5 \text{ in.}$$

At A: $$\ell_A = 7 + 5 + 5 = 17 \text{ in.}$$

$$\ell_\Delta - \ell_0 = 12 \text{ in.} = 1 \text{ ft}$$

At B: $$\ell_B = \sqrt{(7+5)^2 + 5^2} = 13 \text{ in.}$$

$$\ell_B - \ell_0 = 1.8 \text{ in.} = \frac{2}{3} \text{ ft}$$

Velocity of the collar at B.

Use the principle of conservation of energy.

$$T_A + V_A = T_B + V_B$$

Where $$T_A = \frac{1}{2} m v_A^2 = 0$$

$$V_A = \frac{1}{2} k (\ell_A - \ell_0)^2 + W(0)$$

$$= \frac{1}{2}(10)(1)^2 + 0 = 5 \text{ ft} \cdot \text{lb}$$

$$T_B = \frac{1}{2} m v_B^2 = \frac{1}{2}(0.031056) v_B^2 = 0.015528 v_B^2$$

$$V_B = \frac{1}{2} k (\ell_B - \ell_0)^2 + Wh$$

$$= \frac{1}{2}(10)\left(\frac{2}{3}\right)^2 + (1)\left(-\frac{5}{12}\right)$$

$$= 1.80556 \text{ ft} \cdot \text{lb}$$

$$0 + 5 = 0.015528 v_B^2 = 1.80556$$

$$v_B^2 = 205.72 \text{ ft}^2/\text{s}^2$$

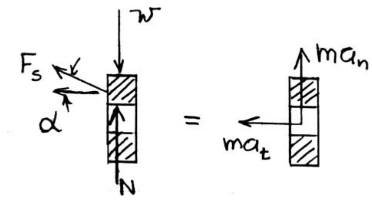
$v_B = 14.34$ ft/s ◀

Forces at B.

$$F_s = k(\ell_B - \ell_0) = (10)\left(\frac{2}{3}\right) = 6.6667 \text{ lb.}$$

$$\sin\alpha = \frac{5}{13}$$

$$\rho = 5 \text{ in.} = \frac{5}{12}\text{ft}$$



$$ma_n = \frac{mv_B^2}{\rho}$$

$$= \frac{(0.031056)(205.72)}{5/12}$$

$$= 15.3332 \text{ lb}$$

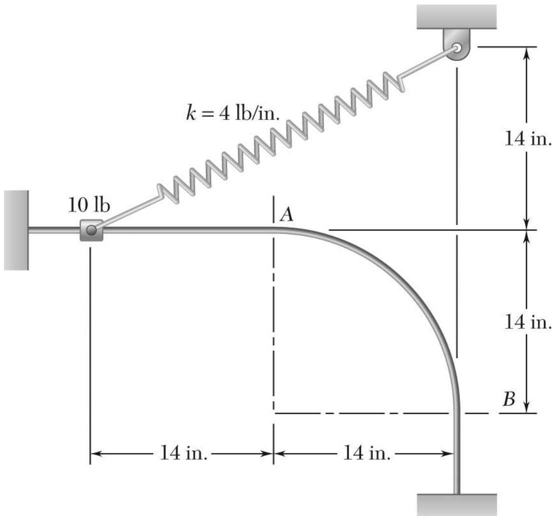
$$+\uparrow \Sigma F_y = ma_y: \quad F_s \sin\alpha - W + N = ma_n$$

$$N = ma_n + W - F_s \sin\alpha$$

$$= 15.3332 + 1 - (6.6667)\left(\frac{5}{13}\right)$$

$$N = 13.769 \text{ lb}$$

$\mathbf{N} = 13.77 \text{ lb} \uparrow$ ◀



PROBLEM 13.73

A 10-lb collar is attached to a spring and slides without friction along a fixed rod in a vertical plane. The spring has an undeformed length of 14 in. and a constant $k = 4$ lb/in. Knowing that the collar is released from rest in the position shown, determine the force exerted by the rod on the collar at (*a*) Point *A*, (*b*) Point *B*. Both these points are on the curved portion of the rod.

SOLUTION

Mass of collar: $m = \frac{W}{g} = \frac{10}{32.2} = 0.31056 \text{ lb} \cdot \text{s}^2/\text{ft}$

Let position 1 be the initial position shown, and calculate the potential energies of the spring for positions 1, *A*, and *B*. $l_0 = 14$ in.

$$l_1 = \sqrt{(14+14)^2 + (14)^2} = 31.305 \text{ in.}$$
$$x_1 = l_1 - l_0 = 31.305 - 14 = 17.305 \text{ in.}$$
$$(V_1)_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(4)(17.305)^2 = 598.92 \text{ in} \cdot \text{lb} = 49.910 \text{ ft} \cdot \text{lb}$$

$$l_A = \sqrt{(14)^2 + (14)^2} = 19.799 \text{ in.}$$
$$x_A = l_A - l_0 = 19.799 - 14 = 5.799 \text{ in.}$$
$$(V_A)_e = \frac{1}{2}kx_A^2 = \frac{1}{2}(4)(5.799)^2 = 67.257 \text{ in} \cdot \text{lb} = 5.605 \text{ ft} \cdot \text{lb}$$

$$l_B = 14 + 14 = 28 \text{ in.}$$
$$x_B = l_B - l_0 = 28 - 14 = 14 \text{ in.}$$
$$(V_B)_e = \frac{1}{2}kx_B^2 = \frac{1}{2}(4)(14)^2 = 392 \text{ in} \cdot \text{lb} = 32.667 \text{ ft} \cdot \text{lb}$$

Gravitational potential energies: Datum at level *A*.

$$(V_1)_g = 0 \quad (V_A)_g = 0$$

$$(V_B)g = Wy = (10 \text{ lb})(-14 \text{ in.}) = -140 \text{ in} \cdot \text{lb} = -11.667 \text{ ft} \cdot \text{lb}$$

Total potential energies: $V = V_e + V_g$

$$V_1 = 49.910 \text{ ft}\cdot\text{lb}, \quad V_A = 5.605 \text{ ft}\cdot\text{lb}, \quad V_B = 21.0 \text{ ft}\cdot\text{lb}$$

Kinetic energies: $T_1 = 0$

$$T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}(0.31056)v_A^2 = 0.15528v_A^2$$

$$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.31056)v_B^2 = 0.15528v_B^2$$

Conservation of energy: $T_1 + V_1 = T_A + V_A$:

$$0 + 49.910 = 0.15528v_A^2 + 5.605 \qquad v_A^2 = 285.32 \text{ ft}^2/\text{s}^2$$

Conservation of energy: $T_1 + V_1 = T_B + V_B$

$$0 + 49.910 = 0.15528v_B^2 + 21.0 \qquad v_B^2 = 186.18 \text{ ft}^2/\text{s}^2$$

Normal accelerations at A and B. $a_n = v^2/\rho$

$$\rho = 14 \text{ in.} = 1.16667 \text{ ft}$$

$$(a_A)_n = \frac{285.32 \text{ ft}^2/\text{s}^2}{1.16667 \text{ ft}} \qquad (\mathbf{a}_A)_n = 244.56 \text{ ft/s}^2 \downarrow$$

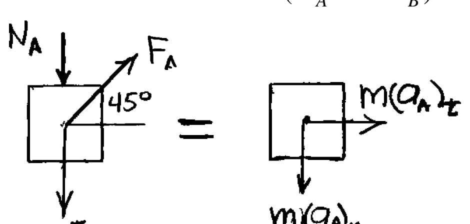
$$(a_B)_n = \frac{189.10 \text{ ft}^2/\text{s}^2}{1.16667 \text{ ft}} \qquad (\mathbf{a}_B)_n = 159.58 \text{ ft/s}^2 \longleftarrow$$

Spring forces at A and B: $F = kx$

$$F_A = (4 \text{ lb/in.})(5.799 \text{ in.}) \qquad \mathbf{F}_A = 23.196 \text{ lb} \measuredangle 45^\circ$$

$$F_B = (4 \text{ lb/in.})(14 \text{ in.}) \qquad \mathbf{F}_B = 56.0 \text{ lb} \uparrow$$

To determine the forces ($\mathbf{N}_A$ and $\mathbf{N}_B$) exerted by the rod on the collar, apply Newton's second law.



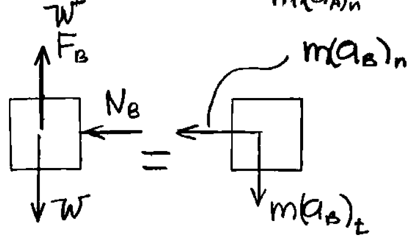
(*a*) At Point *A*:

$$\downarrow \Sigma F = m(a_A)_n:$$

$$W + N_A - F_A \sin 45^\circ = m(a_A)_n$$

$$10 + N_A - 23.196 \sin 45^\circ = (0.31056)(244.56)$$

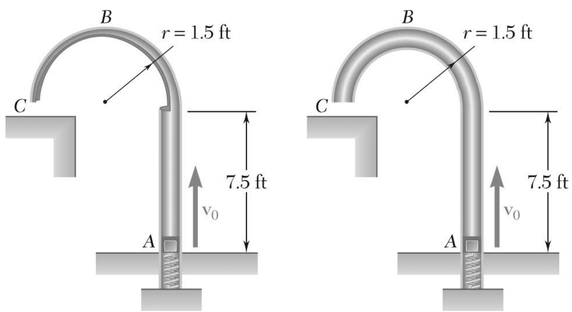
$$\mathbf{N}_A = 82.4 \text{ lb} \downarrow \blacktriangleleft$$



(*b*) At Point *B*:

$$\overset{+}{\longleftarrow} \Sigma F = m(a_B)_n:$$

$$N_B = (0.31056)(159.58) \qquad \mathbf{N}_B = 49.6 \text{ lb} \longleftarrow \blacktriangleleft$$



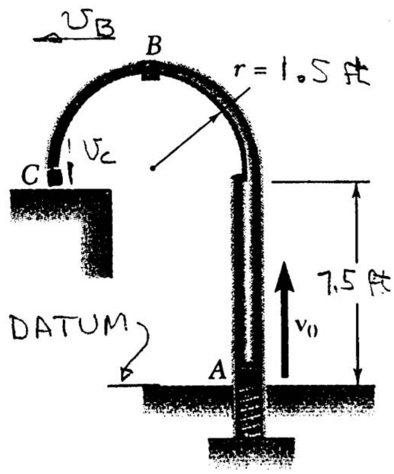
PROBLEM 13.74

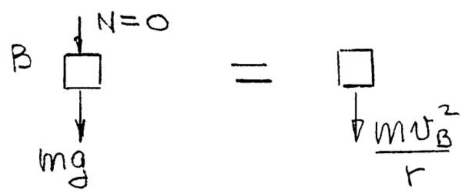
An 8-oz package is projected upward with a velocity $\mathbf{v}_0$ by a spring at A; it moves around a frictionless loop and is deposited at C. For each of the two loops shown, determine (*a*) the smallest velocity $\mathbf{v}_0$ for which the package will reach C, (*b*) the corresponding force exerted by the package on the loop just before the package leaves the loop at C.

SOLUTION

Loop 1

(*a*) The smallest velocity at B will occur when the force exerted by the tube on the package is zero.





$$+\downarrow \Sigma F = 0 + mg = \frac{mv_B^2}{r}$$

$$v_B^2 = rg = 1.5 \text{ ft}(32.2 \text{ ft/s}^2)$$

$$v_B^2 = 48.30$$

At A $\quad T_A = \frac{1}{2}mv_0^2$

$$V_A = 0 \quad \left(8 \text{ oz} = 0.5 \text{ lb} \Rightarrow = \frac{0.5}{32.2} = 0.01553\right)$$

At B $\quad T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}m(48.30) = 24.15 \text{ m}$

$$V_B = mg(7.5 + 1.5) = 9mg = 9(0.5) = 4.5 \text{ lb} \cdot \text{ft}$$

$$T_A + V_A = T_B + V_B: \quad \frac{1}{2}(0.01553)v_0^2 = 24.15(0.01553) + 4.5$$

$$v_0^2 = 627.82 \qquad v_0 = 25.056 \qquad v_0 = 25.1 \text{ ft/s} \blacktriangleleft$$

At C

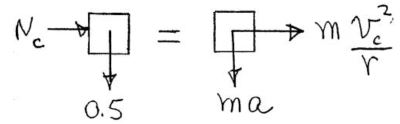
$$T_C = \frac{1}{2}mv_C^2 = 0.007765v_C^2 \qquad V_C = 7.5mg = 7.5(0.5) = 3.75$$

$$T_A + V_A = T_C + V_C: \quad 0.007765v_0^2 = 0.007765v_C^2 + 3.75$$

$$0.007765(25.056)^2 - 3.75 = 0.007765v_C^2$$

$$v_C^2 = 144.87$$

(*b*)

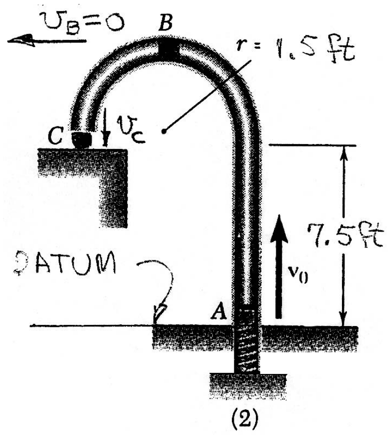


$$\xrightarrow{+} \Sigma F = ma_n: \quad N = 0.01553\frac{(144.87)}{1.5}$$

$$N = 1.49989$$

{Package in tube} $N_C = 1.500 \text{ lb} \longleftarrow$ ◀

Loop 2



(*a*) At *B*, tube supports the package so,

$$v_B \approx 0$$

$$v_B = 0, \; T_B = 0 \qquad V_B = mg(7.5 + 1.5)$$

$$= 4.5 \text{ lb}\cdot\text{ft}$$

$$T_A + V_A = T_B + V_B$$

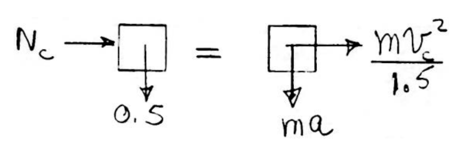
$$\frac{1}{2}(0.01553)v_A^2 = 4.5 \Rightarrow v_A = 24.073$$

$v_A = 24.1$ ft/s ◀

(*b*) At *C* $\qquad T_C = 0.007765v_C^2, \; V_C = 7.5mg = 3.75$

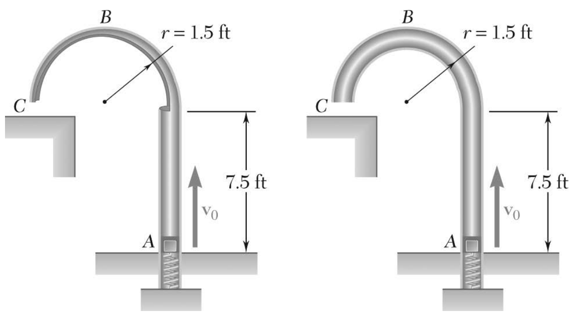
$$T_A + V_A = T_C + V_C: \quad 0.007765(24.073)^2 = 0.007765v_C^2 + 3.75$$

$$v_C^2 = 96.573$$



$$N_C = 0.01553\left(\frac{96.573}{1.5}\right) = 0.99985$$

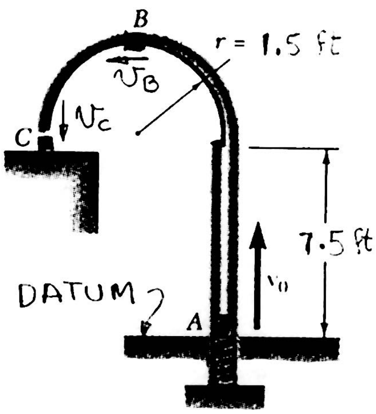
{Package on tube} $N_C = 1.000$ lb ◀



PROBLEM 13.75

If the package of Problem 13.74 is not to hit the horizontal surface at C with a speed greater than 10 ft/s, (*a*) show that this requirement can be satisfied only by the second loop, (*b*) determine the largest allowable initial velocity $\mathbf{v}_0$ when the second loop is used.

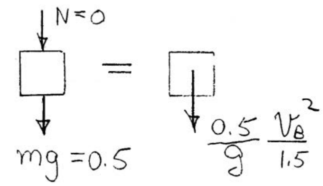
SOLUTION



(1)

(*a*) Loop 1

From Problem 13.74, at B



$$v_B^2 = gr = 48.3 \text{ ft}^2/\text{s}^2 \Rightarrow v_B = 6.9498 \text{ ft/s}$$

$$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.01553)(48.3) = 0.37505$$

$$V_B = mg(7.5 + 1.5) = (0.5)(9) = 4.5 \text{ lb}\cdot\text{ft}$$

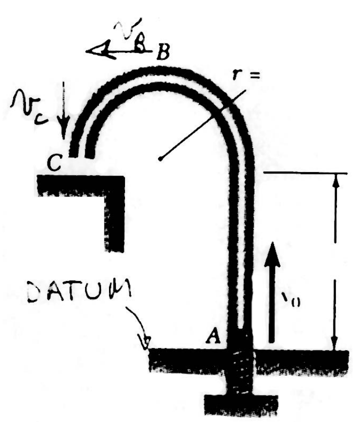
$$T_C = \frac{1}{2}mv_C^2 = \frac{1}{2}(0.01553)v_C^2 = 0.007765v_C^2$$

$$V_C = 7.5(0.5) = 3.75 \text{ lb}\cdot\text{ft}$$

$$T_B + V_B = T_C + V_C: \quad 0.37505 + 4.5 = 0.007765v_C^2 + 3.75$$

$$v_C^2 = 144.887 \Rightarrow v_C = 12.039 \text{ ft/s}$$

12.04 ft/s > 10 ft/s ⇒ Loop (1) does not work ◀



(2)

(*b*) Loop 2 at A $\quad T_A = \frac{1}{2}mv_0^2 = 0.007765v_0^2$

$V_A = 0$

At C assume $\quad v_C = 10$ ft/s

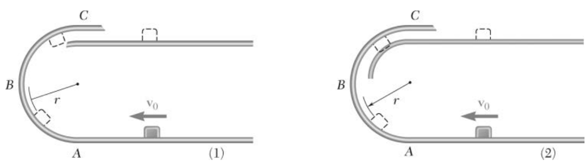
$$T_C = \frac{1}{2}mv_C^2 = 0.007765(10)^2 = 0.7765$$

$$v_C = 7.5(0.5) = 3.75$$

$$T_A + V_A = T_C + V_C: \quad 0.007765v_0^2 = 0.7765 + 3.75$$

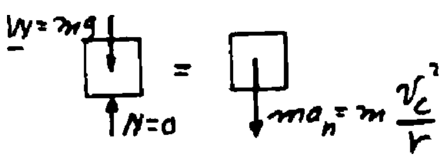
$v_0 = 24.144 \qquad v_0 = 24.1$ ft/s ◀

PROBLEM 13.76



A small package of weight W is projected into a vertical return loop at A with a velocity $\boldsymbol{v}_0$. The package travels without friction along a circle of radius r and is deposited on a horizontal surface at C. For each of the two loops shown, determine (*a*) the smallest velocity $\mathbf{v}_0$ for which the package will reach the horizontal surface at C, (*b*) the corresponding force exerted by the loop on the package as it passes Point B.

SOLUTION



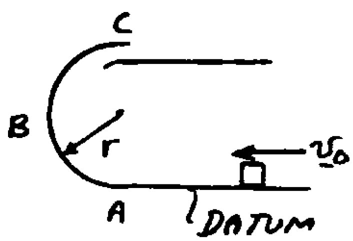
Loop 1:

(*a*) Newton's second law at position C:

$$+\downarrow \Sigma F = \text{ma}:$$

$$mg = m\frac{v_c^2}{r} \qquad v_c^2 = gr$$

Conservation of energy between position A and B.



$$T_A = \frac{1}{2}mv_0^2$$

$$V_A = 0$$

$$T_C = \frac{1}{2}mv_C^2 = \frac{1}{2}mgr$$

$$V_C = mg(2r) = 2mgr$$

$$T_A + V_A = T_C + V_C: \quad \frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mgr + 2mgr$$

$$v_0^2 = 5gr$$

Smallest velocity $\mathbf{v}_0$: $\qquad v_0 = \sqrt{5gr} \longleftarrow$ ◀

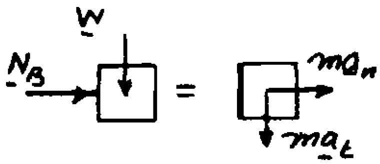
(*b*) Conservation of energy between positions A and B.

$$(b)\ T_B = \frac{1}{2}mv_B^2; \quad V_B = mg(r)$$

$$T_A + V_A = T_B + V_B: \quad \frac{1}{2}mv_A^2 + 0 = \frac{1}{2}mv_B^2 + mgr$$

$$\frac{1}{2}m(5\,gr) + 0 = \frac{1}{2}mv_B^2 + mgr \qquad v_B^2 = 3gr$$

PROBLEM 13.76 (Continued)



Newton's second law at position B.

$$ma_n = m\frac{v_B^2}{r} = m\frac{3gr}{r} = 3\ mg$$

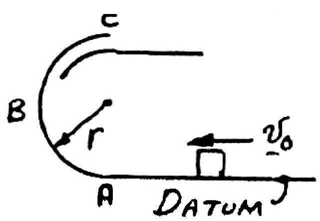
$$\xrightarrow{+} \Sigma F = \Sigma F_{eff}: \qquad N_B = 3\ mg$$

Force exerted by the loop: $N_B = 3W \longrightarrow$ ◀

Loop 2:

(a) At point C, $v_c = 0$

Conservation of energy between positions A and C.



$$T_C = \frac{1}{2}mv_C^2 = 0$$

$$V_C = mg(2r) = 2mgr$$

$$T_A + V_A = T_C + V_C:$$

$$\frac{1}{2}mv_0^2 + 0 = 0 + 2mgr$$

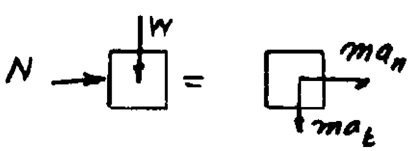
$$v_0^2 = 4gr$$

Smallest velocity $\mathbf{v}_0$: $v_0 = \sqrt{4gr} \longleftarrow$ ◀

(b) Conservation of energy between positions A and B.

$$T_A + V_A - T_B + V_B: \ \frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_B^2 + mgr$$

$$\frac{1}{2}m(4gr) = \frac{1}{2}mv_B^2 + mgr \qquad v_B^2 = 2gr$$

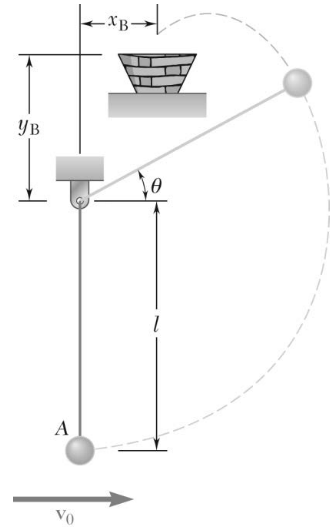


Newton's second law at position B.

$$ma_n = m\frac{v_B^2}{r} = m\frac{2gr}{r} = 2\ mg$$

$$\xrightarrow{+} \Sigma F = \Sigma_{\text{eff}}: \ N = 2\ mg$$

Force exerted by loop: $\mathbf{N} = 2W \longrightarrow$ ◀



PROBLEM 13.77

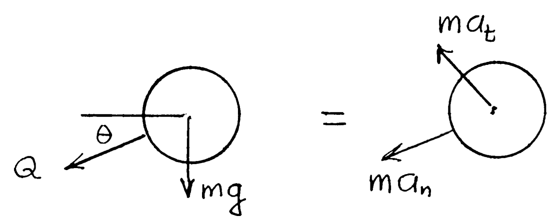
The 1 kg ball at A is suspended by an inextensible cord and given an initial horizontal velocity of 5 m/s. If $l = 0.6$ m and $x_B = 0,$ determine y_B so that the ball will enter the basket.

SOLUTION

Let position 1 be at A. $\quad v_1 = v_0$

Let position 2 be the point described by the angle where the path of the ball changes from circular to parabolic. At position 2, the tension Q in the cord is zero.

Relationship between v_2 and θ based on $Q = 0.$ Draw the free body diagram.



$$+\swarrow \Sigma F = 0: \quad Q + mg\sin\theta = ma_n = \frac{mv_2^2}{l}$$

With $\quad Q = 0, \quad v_2^2 = gl\sin\theta \quad$ or $\quad v_2 = \sqrt{gl\sin\theta} \qquad (1)$

Relationship among v_0, v_2 and θ based on conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_0^2 - mgl = \frac{1}{2}mv_2^2 + mgl\sin\theta$$

$$v_0^2 - v_2^2 = 2gl(1 + \sin\theta) \qquad (2)$$

Eliminating v_2 from Eqs. (1) and (2),

$$v_0^2 - gl\sin\theta = 2gl(1+\sin\theta)$$

$$\sin\theta = \frac{1}{3}\left[\frac{v_0^2}{gl} - 2\right] = \frac{1}{3}\left[\frac{(5)^2}{(9.81)(0.6)} - 2\right] = 0.74912$$

$$\theta = 48.514°$$

From Eq. (1), $$v_2^2 = (9.81)(0.6)\sin 48.514° = 4.4093 \text{ m}^2/\text{s}^2$$

$$v_2 = 2.0998 \text{ m/s}$$

x and y coordinates at position 2.

$$x_2 = l\cos\theta = 0.6\cos 48.514° = 0.39746 \text{ m}$$

$$y_2 = l\sin\theta = 0.6\sin 48.514° = 0.44947 \text{ m}$$

Let t_2 be the time when the ball is a position 2.

Motion on the parabolic path. The horizontal motion is

$$\dot{x} = -v_2 \sin\theta = -2.0998 \sin 48.514°$$

$$= -1.5730 \text{ m/s}$$

$$x = x_2 - 1.5730(t - t_2)$$

At Point B, $$x_B = 0$$

$$0 = 0.39746 - 1.5730(t_B - t_2) \quad t_B - t_2 = 0.25267 \text{ s}$$

The vertical motion is

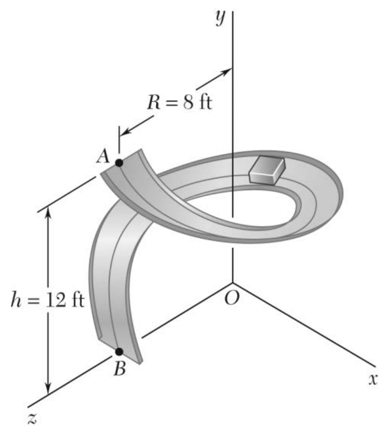
$$y = y_2 + v_2\cos\theta(t - t_2) - \frac{1}{2}g(t - t_2)^2$$

At Point B, $$y_B = y_2 + v_2\cos\theta(t_B - t_2) - \frac{1}{2}g(t_B - t_2)^2$$

$$y_B = 0.44947 + (2.0998\cos 48.514°)(0.25267) - \frac{1}{2}(9.81)(0.25267)^2$$

$$= 0.48779 \text{ m}$$

$y_B = 0.448$ m ◀



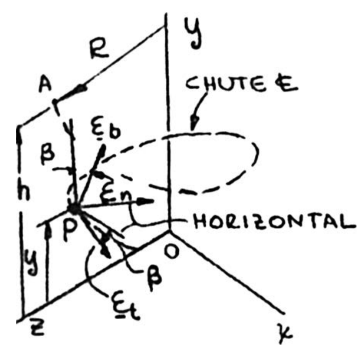
PROBLEM 13.78*

Packages are moved from Point A on the upper floor of a warehouse to Point B on the lower floor, 12 ft directly below A, by means of a chute, the centerline of which is in the shape of a helix of vertical axis y and radius $R = 8$ ft. The cross section of the chute is to be banked in such a way that each package, after being released at A with no velocity, will slide along the centerline of the chute without ever touching its edges. Neglecting friction, (*a*) express as a function of the elevation y of a given Point P of the centerline the angle ϕ formed by the normal to the surface of the chute at P and the principal normal of the centerline at that point, (*b*) determine the magnitude and direction of the force exerted by the chute on a 20-lb package as it reaches Point B. *Hint:* The principal normal to the helix at any Point P is horizontal and directed toward the y axis, and the radius of curvature of the helix is $\rho = R[1-(h/2\pi R)^2]$.

SOLUTION

(*a*) At Point A: $\quad v_A = 0 \quad T_A = 0$

$$V_A = mgh$$



At any Point P: $\quad T_P = \frac{1}{2}mv^2$

$$V_P = Wy = mgy$$

$$T_A + V_A = T_P + V_P$$

$$0 + mgh = \frac{1}{2}mv^2 + mgy$$

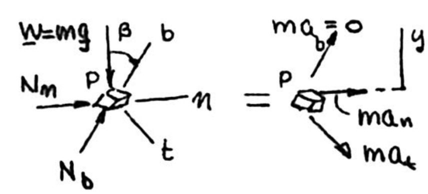
$$v^2 = 2g(h-y)$$

$\mathbf{e}_n$ along principal normal, horizontal and directed toward y axis

$\mathbf{e}_t$ tangent to centerline of the chute

$\mathbf{e}_D$ along binormal

$$\beta = \tan^{-1}\frac{h}{2\pi R} = \tan^{-1}\frac{(12\text{ ft})}{2\pi(8\text{ ft})} = 13.427°$$



$$ma_b = 0$$

since $\quad a_b = 0$

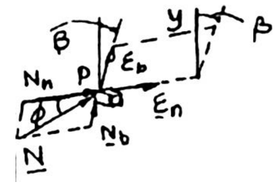
Note: Friction is zero,

$$\Sigma F_t = ma_t: \quad mg\sin\beta = ma_t \quad a_t = g\sin\beta$$

$$\Sigma F_b = ma_b: \quad N_b - W\cos\beta = 0 \quad N_B = W\cos\beta$$

$$\Sigma F_n = ma_n: \quad N_n = \frac{mv^2}{e} = \frac{m2g(h-y)}{e} = 2W\frac{(h-y)}{e}$$

The total normal force is the resultant of $\mathbf{N}_b$ and $\mathbf{N}_m$, it lies in the b–m plane, and forms angle ϕ with m axis.



$$\tan\phi = N_b / N_n$$

$$\tan\phi = W\cos\beta \Big/ \frac{2(w(h-y)}{e}$$

$$\tan\phi = (e/2(h-y))\cos\beta$$

Given: $$e = R\left[1+\left(\frac{h}{2\pi R}\right)^2\right] = R(1+\tan^2\beta) = \frac{R}{\cos^2\beta}$$

Thus, $$\tan\phi = \frac{e}{2(h-y)}\cos\beta = \frac{R}{2(h-y)\cos\beta}$$

$$\tan\phi = \frac{8\text{ ft}}{2(12-y)\cos 13.427^\circ} = \frac{4.112}{12-y}$$

or $$\cot\phi = 0.243(12-y) \blacktriangleleft$$

(*b*) At Point *B*: $y = 0$ for x, y, z axes, we write, with $W = 20$ lb,

$$N_x = N_b\sin\beta = W\cos\beta\sin\beta = (20\text{ lb})\cos 13.427^\circ\sin 13.427^\circ \quad N_x = 4.517\text{ lb}$$

$$N_y = N_b\cos\beta = W\cos^2\beta = (20\text{ lb})\cos^2 13.427^\circ \quad N_y = 18.922\text{ lb}$$

$$N_z = -N_n = -2w\frac{h-y}{e} = -2W\frac{h-y}{R/\cos^2\beta}$$

$$N_z = -2(20\text{ lb})\frac{(12\text{ ft-}0)}{8\text{ ft}}\cos^2 13.427^\circ \quad N_z = -56.765\text{ lb}$$

$$N = \sqrt{(4.517)^2 + (18.922)^2 + (-56.765)^2} \qquad N = 60.0\text{ lb} \blacktriangleleft$$

$$\cos\theta_x = \frac{N_x}{N} = \frac{4.517}{60} \qquad \theta_x = 85.7^\circ \blacktriangleleft$$

$$\cos\theta_y = \frac{N_y}{N} = \frac{18.922}{60} \qquad \theta_y = 71.6^\circ \blacktriangleleft$$

$$\cos\theta_z = \frac{N_z}{N} = -\frac{56.742}{60} \qquad \theta_z = 161.1^\circ \blacktriangleleft$$

PROBLEM 13.79*

Prove that a force $F(x, y, z)$ is conservative if, and only if, the following relations are satisfied:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \quad \frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \quad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z}$$

SOLUTION

For a conservative force, Equation (13.22) must be satisfied.

$$F_x = -\frac{\partial V}{\partial x} \quad F_y = -\frac{\partial V}{\partial y} \quad F_z = -\frac{\partial V}{\partial z}$$

We now write

$$\frac{\partial F_x}{\partial y} = -\frac{\partial^2 V}{\partial x \partial y} \quad \frac{\partial F_y}{\partial x} = -\frac{\partial^2 V}{\partial y \partial x}$$

Since $\frac{\partial^2 V}{\partial x \partial y} = \frac{\partial^2 V}{\partial y \partial x}$:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \blacktriangleleft$$

We obtain in a similar way

$$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \quad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z} \blacktriangleleft$$

PROBLEM 13.80

The force $\mathbf{F} = (yz\mathbf{i} + zx\mathbf{j} + xy\mathbf{k})/xyz$ acts on the particle $P(x, y, z)$ which moves in space. (*a*) Using the relation derived in Problem 13.79, show that this force is a conservative force. (*b*) Determine the potential function associated with **F**.

SOLUTION

(*a*)

$$F_x = \frac{yz}{xyz} \quad F_y = \frac{zx}{xyz} \quad F_z = \frac{xy}{xyz}$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial\left(\frac{1}{x}\right)}{\partial y} = 0 \quad \frac{\partial F_y}{\partial x} = \frac{\partial\left(\frac{1}{y}\right)}{\partial x} = 0$$

Thus,

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

The other two equations derived in Problem 13.79 are checked in a similar way.

(*b*) Recall that

$$F_x = -\frac{\partial V}{\partial x}, \qquad F_y = -\frac{\partial V}{\partial y}, \qquad F_z = -\frac{\partial V}{\partial z}$$

$$F_x = \frac{1}{x} = -\frac{\partial V}{\partial x} \qquad V = -\ln x + f(y, z) \tag{1}$$

$$F_y = \frac{1}{y} = -\frac{\partial V}{\partial y} \qquad V = -\ln y + g(z, x) \tag{2}$$

$$F_z = \frac{1}{z} = -\frac{\partial V}{\partial z} \qquad V = -\ln z + h(x, y) \tag{3}$$

Equating (1) and (2)

$$-\ln x + f(y, z) = -\ln y + g(z, x)$$

Thus,

$$f(y, z) = -\ln y + k(z) \tag{4}$$

$$g(z, x) = -\ln x + k(z) \tag{5}$$

Equating (2) and (3)

$$-\ln z + h(x, y) = -\ln y + g(z, x)$$

$$g(z, x) = -\ln z + l(x)$$

From (5),

$$g(z, x) = -\ln x + k(z)$$

PROBLEM 13.80 (Continued)

Thus,

$$k(z) = -\ln z$$

$$l(x) = -\ln x$$

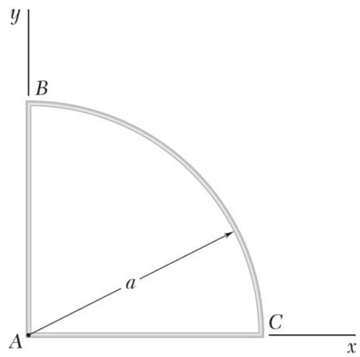
From (4),

$$f(y,z) = -\ln y - \ln z$$

Substitute for $f(y,z)$ in (1)

$$V = -\ln x - \ln y - \ln z$$

$V = -\ln xyz$ ◀



PROBLEM 13.81*

A force **F** acts on a particle $P(x, y)$ which moves in the xy plane. Determine whether **F** is a conservative force and compute the work of **F** when P describes in a clockwise sense the path A, B, C, A including the quarter circle $x^2 + y^2 = a^2$, if (*a*) $\mathbf{F} = ky\mathbf{i}$, (*b*) $\mathbf{F} = k(y\mathbf{i} + x\mathbf{j})$.

SOLUTION

(*a*) $F_x = ky \quad F_y = 0 \quad \dfrac{\partial F_x}{\partial y} = k \quad \dfrac{\partial F_y}{\partial x} = 0$

Thus, $\dfrac{\partial F_x}{\partial y} \neq \dfrac{\partial F_y}{\partial x}$ **F** is not conservative.

$$U_{ABCA} = \int_{ABCA} \mathbf{F} \cdot d\mathbf{r} = \int_A^B ky\mathbf{i} \cdot dy\mathbf{j} + \int_B^C ky\mathbf{i} \cdot (dx\mathbf{i} + dy\mathbf{j}) + \int_B^A ky\mathbf{i} \cdot dx\mathbf{j}$$

$\int_A^B = 0$, **F** is perpendicular to the path.

$$\int_B^C ky\mathbf{i} \cdot (dx\mathbf{i} + dy\mathbf{j}) = \int_B^C ky\,dx$$

From B to C, the path is a quarter circle with origin at A.

Thus,
$$x^2 + y^2 = a^2$$
$$y = \sqrt{a^2 - x^2}$$

Along BC, $\displaystyle\int_B^C ky dx = \int_0^a k\sqrt{a^2 - x^2}\,dx = \frac{\pi k a^2}{4}$

$$\int_C^A ky\mathbf{i} \cdot dx\mathbf{j} = 0 \quad (y = 0 \text{ on } CA)$$

$$U_{ABCA} = \int_A^B + \int_B^C + \int_C^A = 0 + \frac{\pi k a^2}{4} + 0$$

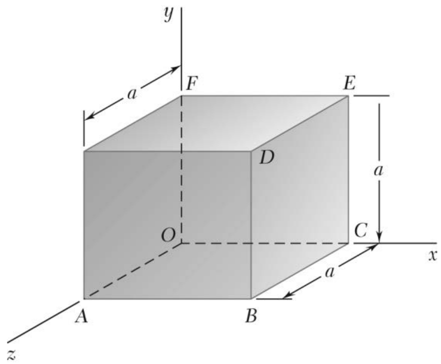
$U_{ABCA} = \dfrac{\pi k a^2}{4}$ ◀

(*b*) $F_x = ky \quad F_y = kx \quad \dfrac{\partial F_x}{\partial y} = k \quad \dfrac{\partial F_y}{\partial x} = k$

$\dfrac{\partial F_x}{\partial y} = \dfrac{\partial F_y}{\partial x}$, **F** is conservative.

Since $ABCA$ is a closed loop and **F** is conservative, $U_{ABCA} = 0$ ◀

PROBLEM 13.82*



The potential function associated with a force **P** in space is known to be $V(x, y, z) = -(x^2 + y^2 + z^2)^{1/2}$. (*a*) Determine the *x*, *y*, and *z* components of **P**. (*b*) Calculate the work done by **P** from *O* to *D* by integrating along the path *OABD*, and show that it is equal to the negative of the change in potential from *O* to *D*.

SOLUTION

(*a*) $$P_x = -\frac{\partial V}{\partial x} = -\frac{\partial[-(x^2 + y^2 + z^2)^{1/2}]}{\partial x} = x(x^2 + y^2 + z^2)^{-1/2} \quad ◀$$

$$P_y = -\frac{\partial V}{\partial y} = -\frac{\partial[-(x^2 + y^2 + z^2)^{1/2}]}{\partial y} = y(x^2 + y^2 + z^2)^{-1/2} \quad ◀$$

$$P_z = -\frac{\partial V}{\partial z} = -\frac{\partial[-(x^2 + y^2 + z^2)^{1/2}]}{\partial z} = z(x^2 + y^2 + z^2)^{-1/2} \quad ◀$$

(*b*) $$U_{OABD} = U_{OA} + U_{AB} + U_{BD}$$

O–A: P_y and P_x are perpendicular to *O–A* and do no work.

Also, on *O–A* $\quad x = y = 0 \quad \text{and} \quad P_z = 1$

Thus, $$U_{O-A} = \int_0^a P_z dz = \int_0^a dz = a$$

A–B: P_z and P_y are perpendicular to *A–B* and do no work.

Also, on *A–B* $\quad y = 0, \quad z = a \quad \text{and} \quad P_x = \dfrac{x}{(x^2 + a^2)^{1/2}}$

Thus, $$U_{A-B} = \int_0^a \frac{xdx}{(x^2 + a^2)^{1/2}}$$
$$= a(\sqrt{2} - 1)$$

B–D: P_x and P_z are perpendicular to *B–D* and do no work.

On *B–D*, $$k = a$$
$$z = a$$
$$P_y = \frac{y}{(y^2 + 2a^2)^{1/2}}$$

PROBLEM 13.82* (Continued)

Thus, $$U_{BD} = \int_0^a \frac{y}{(y^2+2a^2)^{1/2}}\,dy = (y^2+2a^2)^{1/2}\Big|_0^a$$

$$U_{BD} = (a^2+2a^2)^{1/2} - (2a^2)^{1/2} = a(\sqrt{3}-\sqrt{2})$$

$$U_{OABD} = U_{O-A} + U_{A-B} + U_{B-D}$$

$$= a + a(\sqrt{2}-1) + a(\sqrt{3}-\sqrt{2})$$

$U_{OABD} = a\sqrt{3}$ ◀

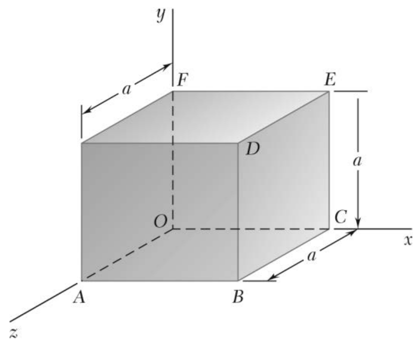
$$\Delta V_{OD} = V(a,a,a) - V(0,0,0)$$

$$= -(a^2+a^2+a^2)^{1/2} - 0$$

$\Delta V_{OD} = -a\sqrt{3}$ ◀

Thus, $$U_{OABD} = -\Delta V_{OD}$$

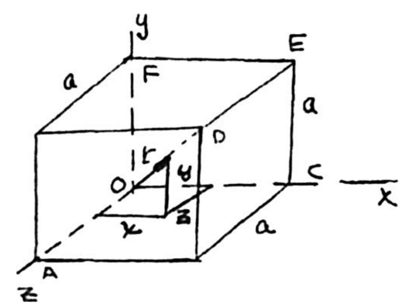
PROBLEM 13.83*



(*a*) Calculate the work done from D to O by the force $\mathbf{P}$ of Problem 13.82 by integrating along the diagonal of the cube. (*b*) Using the result obtained and the answer to part *b* of Problem 13.82, verify that the work done by a conservative force around the closed path *OABDO* is zero.

PROBLEM 13.82 The potential function associated with a force $\mathbf{P}$ in space is known to be $V(x, y, z) = -(x^2 + y^2 + z^2)^{1/2}$. (*a*) Determine the x, y, and z components of $\mathbf{P}$. (*b*) Calculate the work done by $\mathbf{P}$ from O to D by integrating along the path *OABD*, and show that it is equal to the negative of the change in potential from O to D.

SOLUTION



From solution to (*a*) of Problem 13.82

$$\mathbf{P} = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{1/2}}$$

(*a*) $\quad U_{OD} = \int_O^D \mathbf{P} \cdot d\mathbf{r}$

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$$

$$\mathbf{P} = \frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{(x^2 + y^2 + z^2)^{1/2}}$$

Along the diagonal. $\quad x = y = z$

Thus, $\quad \mathbf{P} \cdot dr = \dfrac{3x}{(3x^2)^{1/2}} = \sqrt{3}$

$$U_{O-D} = \int_0^a \sqrt{3}\, dx = \sqrt{3}a$$

$U_{OD} = \sqrt{3}a$ ◀

(*b*) $$U_{OABDO} = U_{OABD} + U_{DO}$$

From Problem 13.82

$$U_{OABD} = \sqrt{3}a \quad \text{at left}$$

The work done from D to O along the diagonal is the negative of the work done from O to D.

$$U_{DO} = -U_{OD} = -\sqrt{3}a \qquad \text{[see part (a)]}$$

Thus,

$$U_{OABDO} = \sqrt{3}a - \sqrt{3}a = 0$$ ◀

PROBLEM 13.84*

The force $\mathbf{F} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})/(x^2 + y^2 + z^2)^{3/2}$ acts on the particle $P(x, y, z)$ which moves in space. (*a*) Using the relations derived in Problem 13.79, prove that **F** is a conservative force. (*b*) Determine the potential function $V(x, y, z)$ associated with **F**.

SOLUTION

(*a*)

$$F_x = \frac{x}{(x^2 + y^2 + z^2)^{3/2}}$$

$$F_y = \frac{y}{(x^2 + y^2 + z^2)^{1/2}}$$

$$\frac{\partial F_x}{\partial y} = \frac{x\left(-\frac{3}{2}\right)(2y)}{(x^2 + y^2 + z^2)^{5/2}}$$

$$\frac{\partial F_y}{\partial x} = \frac{y\left(-\frac{3}{2}\right)2y}{(x^2 + y^2 + z^2)^{5/2}}$$

Thus,

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

The other two equations derived in Problem 13.79 are checked in a similar fashion.

(*b*) Recalling that

$$F_x = -\frac{\partial V}{\partial x}, \quad F_y = -\frac{\partial V}{\partial y}, \quad F_z = -\frac{\partial V}{\partial z}$$

$$F_x = -\frac{\partial V}{\partial x} \quad V = -\int \frac{x}{(x^2 + y^2 + z^2)^{3/2}} dx$$

$$V = (x^2 + y^2 + z^2)^{-1/2} + f(y, z)$$

Similarly integrating $\partial V/\partial y$ and $\partial V/\partial z$ shows that the unknown function $f(x, y)$ is a constant.

$$V = \frac{1}{(x^2 + y^2 + z^2)^{1/2}} \blacktriangleleft$$

PROBLEM 13.85

(*a*) Determine the kinetic energy per unit mass which a missile must have after being fired from the surface of the earth if it is to to reach an infinite distance from the earth. (*b*) What is the initial velocity of the missile (called the *escape velocity*)? Give your answers in SI units and show that the answer to part *b* is independent of the firing angle.

SOLUTION

At the surface of the earth, $g = 9.81 \text{ m/s}^2$

$$r_1 = R = 6370 \text{ km} = 6.37 \times 10^6 \text{ m}$$

Centric force at the surface of the earth,

$$F = mg = \frac{GMm}{R^2}$$

$$GM = gR^2 = (9.81)(6.37 \times 10^6)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

Let position 1 be on the surface of the earth $(r_1 = R)$ and position 2 be at $r_2 = OD.$ Apply the conservation of energy principle.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 + \frac{GMm}{r_2}$$

$$T_1 = T_2 + \frac{GMm}{R} - \frac{GMm}{\infty}$$

$$\frac{T_1}{m} = \frac{T_2}{m} + \frac{GM}{R} = \frac{T_2}{m} + gR$$

For the escape condition set $\frac{T_2}{m} = 0$

$$\frac{T_1}{m} = gR = (9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m}) = 62.49 \times 10^6 \text{ m}^2/\text{s}^2$$

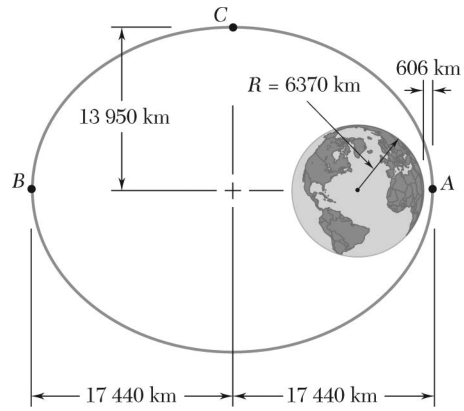
(*a*) $\frac{T_1}{m} = 62.5 \text{ MJ/kg}$ ◀

$$\frac{1}{2}mv_{\text{esc}}^2 = gr$$

$$v_{\text{esc}} = \sqrt{2gR}$$

(*b*) $v_{\text{esc}} = \sqrt{(2)(9.81)(6.37 \times 10^6)} = 11.18 \times 10^3 \text{ m/s}$ $\quad v_{\text{esc}} = 11.18 \text{ km/s}$ ◀

Note that the escape condition depends only on the speed in position 1 and is independent of the direction of the velocity (firing angle).



PROBLEM 13.86

A satellite describes an elliptic orbit of minimum altitude 606 km above the surface of the earth. The semimajor and semiminor axes are 17,440 km and 13,950 km, respectively. Knowing that the speed of the satellite at Point C is 4.78 km/s, determine (*a*) the speed at Point A, the perigee, (*b*) the speed at Point B, the apogee.

SOLUTION

$$r_A = 6370 + 606 = 6976 \text{ km} = 6.976\times10^6 \text{ m}$$

$$r_C = \sqrt{(17440 - 6976)^2 + (13950)^2} = 17438.4 \text{ km} = 17.4384\times10^6 \text{ m}$$

$$r_B = (2)(17440) - 6976 = 27904 \text{ km} = 27.904\times10^6 \text{ m}$$

For earth, $\quad R = 6370 \text{ km} = 6.37\times10^6 \text{ m}$

$$GM = gR^2 = (9.81 \text{ m/s}^2)(6.370\times10^6)^2 = 398.06\times10^{12} \text{ m}^3/\text{s}^2$$

$$v_C = 4.78 \text{ km/s} = 4780 \text{ m/s}$$

(*a*) *Speed at Point A*: Use conservation of energy.

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}mv_A^2 - \frac{GMm}{r_A} = \frac{1}{2}mv_C^2 - \frac{GMm}{r_C}$$

$$v_A^2 = v_C^2 + 2GM\left(\frac{1}{r_A} - \frac{1}{r_C}\right)$$

$$= (4780)^2 + (2)(398.06\times10^{12})\left[\frac{1}{6.976\times10^6} - \frac{1}{17.4384\times10^6}\right]$$

$$= 91.318\times10^6 \text{ m}^2/\text{s}^2$$

$$V_A = 9.556\times10^3 \text{ m/s}$$

$v_A = 9.56 \text{ km/s}$ ◀

(*b*) *Speed at Point B*: Use conservation of energy.

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B^2 - \frac{GMm}{r_B} = \frac{1}{2}mv_C^2 - \frac{GMm}{r_C}$$

$$v_B^2 = v_C^2 + 2GM\left(\frac{1}{r_B} - \frac{1}{r_C}\right)$$

$$= (4780)^2 + (2)(398.06\times10^{12})\left[\frac{1}{27.904\times10^6} - \frac{1}{17.4384\times10^6}\right]$$

$$= 5.7258\times10^6\ \text{m}^2/\text{s}^2$$

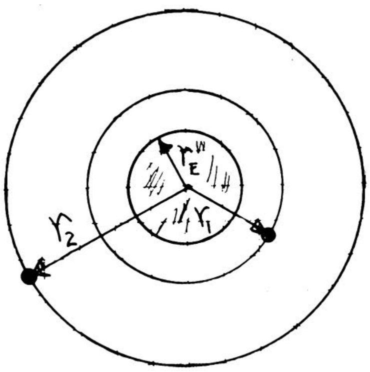
$$v_B = 2.39\times10^3\ \text{m/s}$$

$v_B = 2.39$ km/s ◀

PROBLEM 13.87

While describing a circular orbit 200 mi above the earth a space vehicle launches a 6000-lb communications satellite. Determine (*a*) the additional energy required to place the satellite in a geosynchronous orbit at an altitude of 22,000 mi above the surface of the earth, (*b*) the energy required to place the satellite in the same orbit by launching it from the surface of the earth, excluding the energy needed to overcome air resistance. (A *geosynchronous orbit* is a circular orbit in which the satellite appears stationary with respect to the ground).

SOLUTION



Geosynchronous orbit

$$r_1 = 3960 + 200 = 4160 \text{ mi} = 21.965 \times 10^6 \text{ ft}$$

$$r_2 = 3960 + 22,000 = 25,960 \text{ mi} = 137.07 \times 10^6 \text{ ft}$$

Total energy $\quad E = T + V = \frac{1}{2}mv^2 - \frac{GMm}{r}$

M = mass of earth

m = mass of satellite

Newton's second law $\quad F = ma_n: \quad \frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v^2 = \frac{GM}{r}$

$$T = \frac{1}{2}mv^2 = m\frac{GM}{2r} \qquad V = -\frac{GMm}{r}$$

$$E = T + V = \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r}$$

$$GM = gR_E^2 \qquad E = -\frac{1}{2}\frac{gR_E^2 m}{r} = -\frac{1}{2}\frac{R_E^2 W}{r} \text{ where } (W = mg)$$

$$E = -\frac{1}{2}\frac{(6000)(20.9088 \times 10^6 \text{ ft})^2}{r} = -\frac{1.3115 \times 10^{18}}{r} \text{ ft} \cdot \text{lb}$$

Geosynchronous orbit at $r_2 = 137.07 \times 10^6$ ft

$$E_{Gs} = \frac{-1.3115 \times 10^{18}}{137.07 \times 10^6} = -9.5681 \times 10^9 \text{ ft} \cdot \text{lb}$$

(*a*) At 200 mi, $\quad r_1 = 21.965 \times 10^6$ ft

$$E_{200} = -\frac{1.3115 \times 10^{18}}{21.965 \times 10^6} = -5.9709 \times 10^{10}$$

$$\Delta E_{300} = E_{Gs} - E_{200} = 5.0141 \times 10^{10}$$

$$\Delta E_{300} = 50.1 \times 10^9 \text{ ft} \cdot \text{lb} \blacktriangleleft$$

PROBLEM 13.87 (Continued)

(*b*) Launch from earth

At launch pad $E_E = -\dfrac{GMm}{R_E} = \dfrac{-gR_E^2 m}{R_E} = -WR_E$

$$E_E = -6000(3960 \times 5280) = -1.25453 \times 10^{11}$$

$$\Delta E_E = E_{Gs} - E_E = -9.5681 \times 10^9 + 125.453 \times 10^9$$

$$\Delta E_E = 115.9 \times 10^9 \text{ ft} \cdot \text{lb} \blacktriangleleft$$

PROBLEM 13.88

A lunar excursion module (LEM) was used in the Apollo moon-landing missions to save fuel by making it unnecessary to launch the entire Apollo spacecraft from the moon's surface on its return trip to earth. Check the effectiveness of this approach by computing the energy per pound required for a spacecraft (as weighed on the earth) to escape the moon's gravitational field if the spacecraft starts from (*a*) the moon's surface, (*b*) a circular orbit 50 mi above the moon's surface. Neglect the effect of the earth's gravitational field. (The radius of the moon is 1081 mi and its mass is 0.0123 times the mass of the earth.)

SOLUTION

Note: $$GM_{\text{moon}} = 0.0123\, GM_{\text{earth}}$$

By Eq. 12.30, $$GM_{\text{moon}} = 0.0123\, gR_E^2$$

At ∞ distance from moon: $r_2 = \infty$, Assume $v_2 = 0$

$$\begin{aligned} E_2 &= T_2 + V_2 \\ &= 0 - \frac{GM_M m}{\infty} \\ &= 0 - 0 \\ &= 0 \end{aligned}$$

(*a*) On surface of moon: $R_M = 1081 \text{ mi} = 5.7077 \times 10^6 \text{ ft}$

$v_1 = 0 \quad T_1 = 0$ $\qquad R_E = 3960 \text{ mi} = 20.909 \times 10^6 \text{ ft}$

$V_1 = -\dfrac{GM_M m}{R_M}$ $\qquad E_1 = T_1 + V_1 = 0 - \dfrac{0.0123\, gR_E^2 m}{R_M}$

$$E_1 = -\frac{(0.0123)(32.2 \text{ ft/s}^2)(20.909 \times 10^6 \text{ ft})^2 m}{(5.7077 \times 10^6 \text{ ft})}$$

W_E = Weight of LEM on the earth

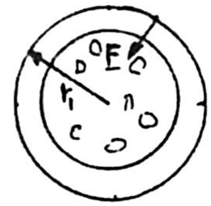
$$E_1 = (-30.336 \times 10^6 \text{ ft}^2/\text{s}^2)m \qquad m = \frac{W_E}{g}$$

$$E_1 = \left(-\frac{30.336 \times 10^6}{32.2 \text{ ft/s}^2} \text{ft}^2/\text{s}^2\right) W_E$$

$$\begin{aligned} \Delta E &= E_2 - E_1 \\ &= 0 + (942.1 \times 10^3 \text{ ft} \cdot \text{lb/lb}) W_E \end{aligned}$$

Energy per pound: $$\frac{\Delta E}{W_E} = 942 \times 10^3 \text{ ft} \cdot \text{lb/lb} \quad ◀$$

PROBLEM 13.88 (Continued)



(*b*) $r_1 = R_M + 50 \text{ mi}$

$r_1 = (1081 \text{ mi} + 50 \text{ mi}) = 1131 \text{ mi} = 5.9717 \times 10^6 \text{ ft}$

Newton's second law:

$$F = ma_n: \quad \frac{GM_M m}{r_1^2} = m\frac{v_1^2}{r_1}$$

$$v_1^2 = \frac{GMm}{r_1} \qquad T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m\frac{GM_M}{r_1}$$

$$V_1 = -\frac{GM_M m}{r_1}$$

$$E_1 = T_1 + V_1 = \frac{1}{2}\frac{GM_M m}{r_1} - \frac{GM_M m}{r_1}$$

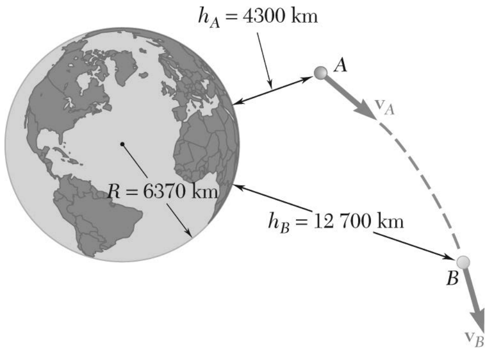
$$E_1 = -\frac{1}{2}\frac{GM_M m}{r_1} = -\frac{1}{2}\frac{0.0123\, gR_E^2 m}{r_1}$$

$$E_1 = -\frac{1}{2}\frac{(0.0123)(32.2 \text{ ft/s}^2)(20.909 \times 10^6 \text{ ft})^2 m}{5.9717 \times 10^6 \text{ ft}}$$

$$E_1 = \frac{(14.498 \times 10^6 \text{ ft}^2/\text{s}^2) W_E}{(32.2 \text{ ft/s}^2)} = 450.2 \times 10^3 \text{ ft} \cdot \text{lb/lb } W_E$$

$$\Delta E = E_2 - E_1 = 0 + 450.2 \times 10^3 \text{ ft} \cdot \text{lb/lb } W_E$$

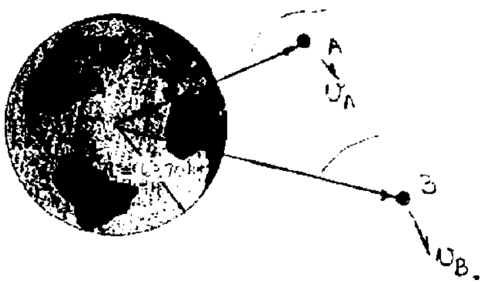
Energy per pound: $\dfrac{\Delta E}{W_E} = 450 \times 10^3 \text{ ft} \cdot \text{lb/lb}$ ◀



PROBLEM 13.89

Knowing that the velocity of an experimental space probe fired from the earth has a magnitude $v_A = 32.5$ Mm/h at Point A, determine the speed of the probe as it passes through Point B.

SOLUTION



$$r_A = R + h_A = 6370 + 4300 = 10740 \text{ km} = 10.670 \times 10^6 \text{ m}$$

$$r_B = 6370 + 12700 = 19070 \text{ km} = 19.070 \times 10^6 \text{ m}$$

$$GM = gR^2 = (9.81)(6.370 \times 10^6)^2 = 398.06 \times 10^{12} \text{ m}^3/\text{s}^2$$

$$v_A = 32.5 \text{ Mm/h} = 9.0278 \times 10^3 \text{ m/s}$$

Use conservation of energy.

$$T_B + V_B = T_A + V_A$$

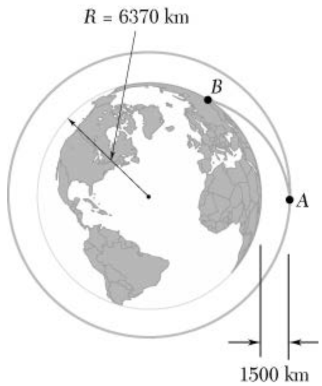
$$\frac{1}{2}mv_B^2 - \frac{GMm}{r_B} = \frac{1}{2}mv_A^2 - \frac{GMm}{r_A}$$

$$v_B^2 = v_A^2 + 2GM\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$= (9.0278 \times 10^3)^2 + (2)(398.06 \times 10^{12})\left[\frac{1}{19.070 \times 10^6} - \frac{1}{10.670 \times 10^6}\right]$$

$$= 48.635 \times 10^6 \text{ m}^2/\text{s}^2$$

$v_B = 6.97 \times 10^3$ m/s $\qquad$ $v_B = 25.1$ Mm/h ◀



PROBLEM 13.90

A spacecraft is describing a circular orbit at an altitude of 1500 km above the surface of the earth. As it passes through Point A, its speed is reduced by 40 percent and it enters an elliptic crash trajectory with the apogee at Point A. Neglecting air resistance, determine the speed of the spacecraft when it reaches the earth's surface at Point B.

SOLUTION

Circular orbit velocity

$$\frac{v_C^2}{r} = \frac{GM}{r^2}, \quad GM = gR^2$$

$$v_C^2 = \frac{GM}{r} = \frac{gR^2}{r} = \frac{(9.81\text{ m/s}^2)(6.370\times 10^6\text{ m})^2}{(6.370\times 10^6\text{ m} + 1.500\times 10^6\text{ m})}$$

$$v_C^2 = 50.579\times 10^6\text{ m}^2/\text{s}^2$$

$$v_C = 7112\text{ m/s}$$

Velocity reduced to 60% of v_C gives $v_A = 4267$ m/s.

Conservation of energy:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\cancel{m}v_A^2 - \frac{GM\cancel{m}}{r_A} = \frac{1}{2}\cancel{m}v_B^2 - \frac{GM\cancel{m}}{r_B}$$

$$\frac{1}{2}(4.267\times 10^3)^2 - \frac{9.81(6.370\times 10^6)^2}{(7.870\times 10^6)} = \frac{v_B^2}{2} - \frac{9.81(6.370\times 10^6)^2}{(6.370\times 10^6)}$$

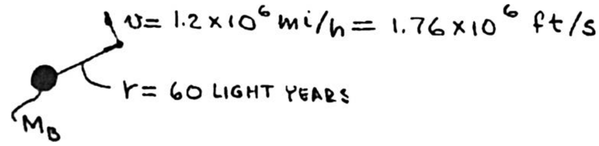
$$v_B = 6.48\times 10^3\text{ m/s}$$

$v_B = 6.48$ km/s ◀

PROBLEM 13.91

Observations show that a celestial body traveling at 1.2×10^6 mi/h appears to be describing about Point B a circle of radius equal to 60 light years. Point B is suspected of being a very dense concentration of mass called a black hole. Determine the ratio M_B/M_S of the mass at B to the mass of the sun. (The mass of the sun is 330,000 times the mass of the earth, and a light year is the distance traveled by light in one year at a velocity of 186,300 mi/s.)

SOLUTION



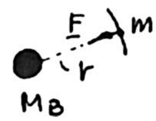
One light year is the distance traveled by light in one year.

Speed of light = 186,300 mi/s

$$r = (60 \text{ yr})(186{,}300 \text{ mi/s})(5280 \text{ ft/mi})(365 \text{ days/yr})(24 \text{ h/day})(3600 \text{ s/h})$$

$$r = 1.8612\times10^{18} \text{ ft}$$

Newton's second law



$$F = \frac{GM_B m}{r^2} = m\frac{v^2}{r}$$

$$M_B = \frac{rv^2}{G}$$

$$GM_{\text{earth}} = gR_{\text{earth}}^2$$

$$= (32.2 \text{ ft/s}^2)(3960 \text{ mi}\times5280 \text{ ft/mi})^2$$

$$= 14.077\times10^{15} (\text{ft}^3/\text{s}^2)$$

$$M_{\text{sun}} = 330{,}000 M_E: \quad GM_{\text{sun}} = 330{,}000\, GM_{\text{earth}}$$

$$GM_{\text{sun}} = (330{,}000)(14.077\times10^{15})$$

$$= 4.645\times10^{21} \text{ ft}^3/\text{s}^2$$

$$G = \frac{4.645\times10^{21}}{M_{\text{sun}}}$$

$$M_B = \frac{rv^2}{G} = \frac{rv^2 M_{\text{sun}}}{4.645\times10^{21}}$$

$$\frac{M_B}{M_{\text{sun}}} = \frac{(1.8612\times10^{18})(1.76\times10^6)^2}{4.645\times10^{21}}$$

$$\frac{M_B}{M_{\text{sun}}} = 1.241\times10^9 \blacktriangleleft$$

PROBLEM 13.92

(*a*) Show that, by setting $r = R + y$ in the right-hand member of Eq. (13.17′) and expanding that member in a power series in *y/R*, the expression in Eq. (13.16) for the potential energy V_g due to gravity is a first-order approximation for the expression given in Eq. (13.17′). (*b*) Using the same expansion, derive a second-order approximation for V_g.

SOLUTION

$$V_g = -\frac{WR^2}{r} \quad \text{setting } r = R + y: \quad V_g = -\frac{WR^2}{R+y} = -\frac{WR}{1+\frac{y}{R}}$$

$$V_g = -WR\left(1+\frac{y}{R}\right)^{-1} = -WR\left[1+\frac{(-1)}{1}\frac{y}{R}+\frac{(-1)(-2)}{1\cdot 2}\left(\frac{y}{R}\right)^2+\cdots\right]$$

We add the constant *WR*, which is equivalent to changing the datum from $r = \infty$ to $r = R$:

$$V_g = WR\left[\frac{y}{R}-\left(\frac{y}{R}\right)^2+\cdots\right]$$

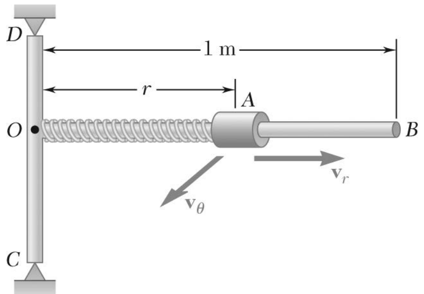
(*a*) First order approximation:

$$V_g = WR\left(\frac{y}{R}\right) = Wy \quad \blacktriangleleft$$

[Equation 13.16]

(*b*) Second order approximation: $$V_g = WR\left[\frac{y}{R}-\left(\frac{y}{R}\right)^2\right]$$

$$V_g = Wy - \frac{Wy^2}{R} \quad \blacktriangleleft$$



PROBLEM 13.93

Collar A has a mass of 3 kg and is attached to a spring of constant 1200 N/m and of undeformed length equal to 0.5 m. The system is set in motion with $r = 0.3$ m, $v_\theta = 2$ m/s, and $v_r = 0$. Neglecting the mass of the rod and the effect of friction, determine the radial and transverse components of the velocity of the collar when $r = 0.6$ m.

SOLUTION

Let position 1 be the initial position.

$$r_1 = 0.3 \text{ m}$$

$$(v_r)_1 = 0, \qquad (v_\theta)_1 = 2 \text{ m/s}, \qquad v_1 = 2 \text{ m/s}$$

$$x_1 = r_1 - l_0 = (0.3 - 0.5) = -0.2 \text{ m}$$

Let position 2 be when $r = 0.6$ m.

$$r_2 = 0.6 \text{ m}$$

$$(v_r)_2 = ?, \qquad (v_\theta)_2 = ?, \qquad v_2 = ?$$

$$x_2 = r_2 - l_0 = (0.6 - 0.5) = 0.1 \text{ m}$$

Conservation of angular momentum: $\quad r_1 m(v_\theta)_1 = v_2 m(v_\theta)_2$

$$(v_\theta)_2 = \frac{r_1 (v_\theta)_1}{r_2} = \frac{(0.3)(2)}{0.6} = 1.000 \text{ m/s}$$

Conservation of energy: $\quad T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2} m v_1^2 + \frac{1}{2} k x_1^2 = \frac{1}{2} m v_2^2 + \frac{1}{2} k x_2^2$$

$$v_2^2 = v_1^2 + \frac{k}{m}\left(x_1^2 - x_2^2\right)$$

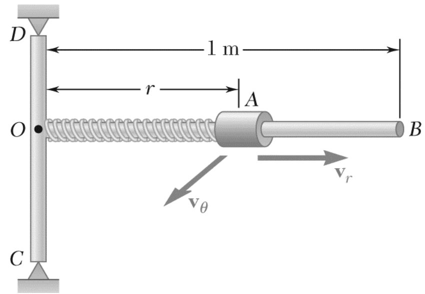
$$= (2)^2 + \frac{1200}{3}\left[(0.2)^2 - (0.1)^2\right] = 16 \text{ m}^2/\text{s}^2$$

$$(v_r)_2^2 = v_2^2 - (v_\theta)^2 = 16 - 1 = 15 \text{ m}^2/\text{s}^2$$

$$v_r = \pm 3.87 \text{ m/s}$$

$v_r = \pm 3.87$ m/s ◀

$v_\theta = 1.000$ m/s ◀



PROBLEM 13.94

Collar A has a mass of 3 kg and is attached to a spring of constant 1200 N/m and of undeformed length equal to 0.5 m. The system is set in motion with $r = 0.3$ m, $v_\theta = 2$ m/s, and $v_r = 0$. Neglecting the mass of the rod and the effect of friction, determine (*a*) the maximum distance between the origin and the collar, (*b*) the corresponding speed. (*Hint:* Solve the equation obtained for r by trial and error.)

SOLUTION

Let position 1 be the initial position.

$$r_1 = 0.3 \text{ m}$$

$$(v_r)_1 = 0, \qquad (v_\theta)_1 = 2 \text{ m/s}, \qquad v_1 = 2 \text{ m/s}$$

$$x_1 = r_1 - l_0 = 0.3 - 0.5 = -0.2 \text{ m}$$

$$T_1 = \frac{1}{2}mv^2 = \frac{1}{2}(3)(2)^2 = 6 \text{ J}$$

$$V_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(1200)(-0.2)^2 = 24 \text{ J}$$

Let position 2 be when r is maximum. $(v_r)_2 = 0$

$$r_2 = r_m$$

$$x_2 = (r_m - 0.5)$$

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(3)(v_\theta)_2^2 = 1.5(v_\theta)_2^2$$

$$V_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(1200)(r_m - 0.5)^2$$

$$= 600(r_m - 0.5)^2$$

Conservation of angular momentum: $\quad r_1 m(v_\theta)_1 = v_2 m(v_\theta)_2$

$$(v_\theta)_2 = \frac{r_1}{r_2}(v_\theta)_, = \frac{(0.3)}{r_m}(2) = \frac{0.6}{r_m}$$

Conservation of energy: $\quad T_1 + V_1 = T_2 + V_2$

$$6 + 24 = 1.5(v_\theta)_2^2 + 600(r_m - 0.5)^2$$

$$30 = (1.5)\left(\frac{0.6}{r_m}\right)^2 + 600(r_m - 0.5)^2$$

$$f(r_m) = \frac{0.54}{r_m^2} + 600(r_m - 0.5)^2 - 30 = 0$$

PROBLEM 13.94 (Continued)

Solve for r_m by trial and error.

r_m (m)	0.5	1.0	0.8	0.7	0.72	0.71
$f(r_m)$	–27.8	120.5	24.8	–4.9	0.080	–2.469

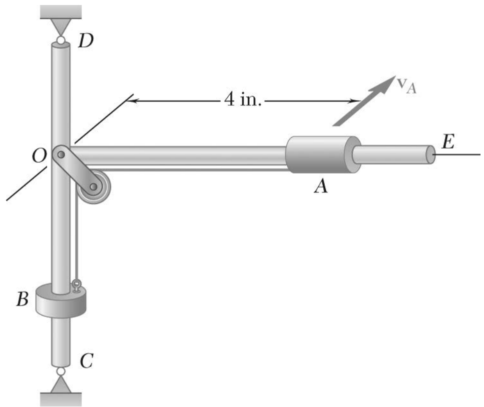
$$r_m = 0.72 - \frac{(0.01)(0.08)}{2.467 + 0.08} = 0.7197 \text{ m}$$

(*a*) *Maximum distance.* $r_m = 0.720 \text{ m}$ ◀

(*b*) *Corresponding speed.*

$$(v_\theta)_2 = \frac{0.6}{0.7197} = 0.8337 \text{ m/s}$$

$$(v_r)_2 = 0$$ $v_2 = 0.834 \text{ m/s}$ ◀



PROBLEM 13.95

A 4-lb collar A and a 1.5-lb collar B can slide without friction on a frame, consisting of the horizontal rod OE and the vertical rod CD, which is free to rotate about CD. The two collars are connected by a cord running over a pulley that is attached to the frame at O. At the instant shown, the velocity $\mathbf{v}_A$ of collar A has a magnitude of 6 ft/s and a stop prevents collar B from moving. If the stop is suddenly removed, determine (*a*) the velocity of collar A when it is 8 in. from O, (*b*) the velocity of collar A when collar B comes to rest. (Assume that collar B does not hit O, that collar A does not come off rod OE, and that the mass of the frame is negligible.)

SOLUTION

Masses:

$$m_A = \frac{4}{32.2} = 0.12422 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_B = \frac{1.5}{32.2} = 0.04658 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Constraint of the cord. Let r be the radial distance to the center of collar A and y be the distance that collar B moves up from its initial level. $y = \Delta r$; $\dot{y} = v_r$

(*a*) Let position 1 be the initial position just after the stop at B is removed and position 2 be when the collar is 8 in. (0.66667 ft) from O.

$$r_1 = 4 \text{ in.} = 0.33333 \text{ ft} \qquad (v_r)_1 = 0$$

$$r_2 = 8 \text{ in.} = 0.66667 \text{ ft}$$

$$\Delta r = y_2 = 8 - 4 = 4 \text{ in.} = 0.33333 \text{ ft}$$

Potential energy: $V_1 = 0,$

$$V_2 = W_B y_2 = (1.5)(0.33333) = 0.5 \text{ ft} \cdot \text{lb}$$

Conservation of angular momentum of collar A:

$$m_A r_1 (v_\theta)_1 = m_A r_2 (v_\theta)_2$$

$$(v_\theta)_2 = \frac{r_1 (v_\theta)_1}{r_2} = \frac{(0.33333)(6)}{0.66667} = 3 \text{ ft/s}$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2} m_A [(v_r)_1^2 + (v_\theta)_1^2] + \frac{1}{2} m_B \dot{y}_1^2 = \frac{1}{2} m_A [(v_r)_2^2 + (v_\theta)_2^2] + \frac{1}{2} m_B \dot{y}^2 + V_2$$

$$\frac{1}{2} m_A [0 + (v_\theta)_1^2] + 0 + 0 = \frac{1}{2} m_A [(v_r)_2^2 + (v_\theta)_2^2] + \frac{1}{2} m_B (v_r)_2^2 + 0.5$$

$$\frac{1}{2}(0.12422)(6)^2 = \frac{1}{2}(0.12422)[(v_r)_2^2 + (3)^2] + \frac{1}{2}(0.04658)(v_r)_2^2 + 0.5$$

$$2.236 = 0.06211(v_r)_2^2 + 0.559 + 0.02329(v_r)_2^2 + 0.5$$

$$0.0854(v_r)_2^2 = 1.177$$

$$(v_r)_2 = 13.78 \text{ ft}^2/\text{s}^2$$

$(v_r)_2 = 3.71$ ft/s ◀

$(v_\theta)_2 = 3.00$ ft/s ◀

$v = 4.77$ ft/s ◀

(*b*) Let position 3 be when collar *B* comes to rest.

$$y_3 = r_3 - 0.33333, \quad (v_r)_3 = 0, \quad \dot{y}_3 = 0$$

Conservation of angular momentum of collar *A*.

$$m_A r_1 (v_\theta)_1 = m_A r_3 (v_\theta)_3$$

$$(v_\theta)_3 = \frac{r_1 (v_\theta)_1}{r_3} = \frac{(0.33333)(6)}{r_3} = \frac{2}{r_3}$$

Conservation of energy: $T_1 + V_1 = T_3 + V_3$

$$\frac{1}{2}m_A[(v_r)_1^2 + (v_\theta)_1^2] + \frac{1}{2}m_B \dot{y}_1^2 = \frac{1}{2}m_A[(v_r)_3^2 + (v_\theta)_3^2] + \frac{1}{2}m_B \dot{y}_3^2 + w_B y_3$$

$$\frac{1}{2}(0.12422)[0 + (6)^2] + 0 = \frac{1}{2}(0.12422)\left[0 + \left(\frac{2}{r_3}\right)^2\right] + 0 + (1.5)(r_3 - 0.33333)$$

$$2.236 = \frac{0.24844}{r_3^2} + 1.5r_3 - 0.5$$

$$1.5r_3^3 - 2.736r_3^2 + 0.24844 = 0$$

Solving the cubic equation for r_3,

$$r_3 = 1.7712 \text{ ft}, \quad -0.2805 \text{ ft}, \quad 0.33333 \text{ ft}$$

Since $r_3 > r_1 = 0.33333$ ft, the required root is

$$r_3 = 1.7712 \text{ ft}$$

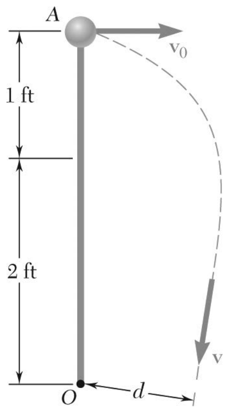
Corresponding velocity of collar *A*:

$(v_r)_3 = 0$ ◀

$$(v_\theta)_3 = \frac{2}{r_3} = \frac{2}{1.7712}$$

$(v_\theta)_3 = 1.129$ ft/s ◀

$v_3 = 1.129$ ft/s ◀



PROBLEM 13.96

A 1.5-lb ball that can slide on a *horizontal* frictionless surface is attached to a fixed Point O by means of an elastic cord of constant $k = 1$ lb/in. and undeformed length 2 ft. The ball is placed at Point A, 3 ft from O, and given an initial velocity $\mathbf{v}_0$ perpendicular to OA. Determine (a) the smallest allowable value of the initial speed v_0 if the cord is not to become slack, (b) the closest distance d that the ball will come to Point O if it is given half the initial speed found in part a.

SOLUTION

Let L_1 be the initial stretched length of the cord and L_2 the length of the closest approach to Point O if the cord does not become slack. Let position 1 be the initial state and position 2 be that of closest approach to Point O. The only horizontal force acting on the ball is the conservative central force due to the elastic cord. At the point of closest approach the velocity of the ball is perpendicular to the cord.

Conservation of angular momentum:

$$r_1 m v_1 = r_2 m v_2$$

$$L_1 m v_0 = L_2 m v_2 \quad \text{or} \quad v_2 = \frac{L_1 v_0}{L_2}$$

Conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m v_1^2 + \frac{1}{2} k (L_1 - L_0)^2 = \frac{1}{2} m v_2^2 + \frac{1}{2} k (L_2 - L_0)^2$$

$$v_1^2 - v_2^2 = -\frac{k}{m}[(L_1 - L_0)^2 + (L_2 - L_0)^2]$$

$$v_0^2 - \frac{L_1^2}{L_2^2} v_0^2 = -\frac{k}{m}[(L_1 - L_0)^2 + (L_2 - L_0)^2]$$

Data:

$$L_0 = 2 \text{ ft}, \quad L_1 = 3 \text{ ft}$$

$L_2 = L_0 = 2$ ft for zero tension in the cord at the point of closest approach.

$$k = 1 \text{ lb/in.} = 12 \text{ lb/ft}$$

$$m = W/g = 1.5/32.2 = 0.04658 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$v_0^2 - \frac{(3)^2}{(2)^2} v_0 = -\frac{12}{0.04658}[(3-2)^2 + (2-2)^2]$$

$$-1.25 \, v_0^2 = -257.6$$

(a) $\quad v_0^2 = 206.1 \text{ ft}^2/\text{s}^2 \qquad v_0 = 14.36$ ft/s ◀

(*b*) Let $v_0 = \frac{1}{2}(14.36 \text{ ft/s}) = 7.18 \text{ ft/s}^2$ so that the cord is slack in the position of closest approach to Point *O*.

Let position 1 be the initial position and position 2 be position of closest approach with the cord being slack.

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}mv_0^2 + \frac{1}{2}k(L_1 - L_0)^2 = \frac{1}{2}mv_2^2$$

$$v_2^2 = v_0^2 + \frac{k}{m}(L - L_0)^2$$

$$= (7.18)^2 + \frac{12}{0.04658}(3 - 2)^2 = 309.17 \text{ ft}^2/\text{s}^2$$

$$v_2 = 17.583 \text{ ft/s}$$

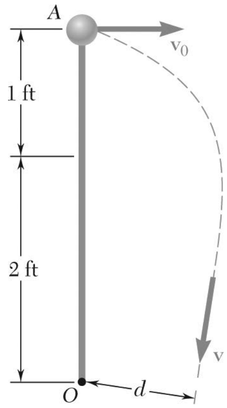
Conservation of angular momentum:

$$r_1 m v_1 = r_2 m v_2 \sin\phi$$

$$r_2 \sin\phi = d = \frac{r_1 v_1}{v_2} = \frac{L_1 v_0}{v_2}$$

$$d = \frac{(3)(7.18)}{17.583}$$

$d = 1.225$ ft ◀



PROBLEM 13.97

A 1.5-lb ball that can slide on a *horizontal* frictionless surface is attached to a fixed Point O by means of an elastic cord of constant $k = 1$ lb/in. and undeformed length 2 ft. The ball is placed at Point A, 3 ft from O, and given an initial velocity $\mathbf{v}_0$ perpendicular to OA, allowing the ball to come within a distance $d = 9$ in. of Point O after the cord has become slack. Determine (*a*) the initial speed v_0 of the ball, (*b*) its maximum speed.

SOLUTION

Let L_1 be the initial stretched length of the cord. Let position 1 be the initial position. Let position 2 be the position of closest approach to point after the cord has become slack. While the cord is slack there are no horizontal forces acting on the ball, so the velocity remains constant. While the cord is stretched, the only horizontal force acting on the ball is the conservative central force due to the elastic cord. At the point of closest approach the velocity of the ball is perpendicular to the radius vector.

Conservation of angular momentum: $r_1 m v_1 = r_2 m v_2$

$$L_1 v_0 = d\, v_2 \qquad \text{or} \qquad v_2 = \frac{L_1}{d} v_0$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2} m v_0^2 + \frac{1}{2} k (L_1 - L_0)^2 = \frac{1}{2} m v_2^2 + 0$$

$$v_0^2 - v_2^2 = -\frac{k}{m}(L_1 - L_0)^2$$

$$v_0^2 - \left(\frac{L_1}{d^2} v_0\right)^2 = -\frac{k}{m}(L_1 - L_0)$$

Data: $L_0 = 2$ ft, $L_1 = 3$ ft, $d = 9$ in. $= 0.75$ ft

$k = 1$ lb/in. $= 12$ lb/ft

$m = W/g = 1.5/32.2 = 0.04658\ \text{lb}\cdot\text{s}^2/\text{ft}$

$$v_0^2 - \left(\frac{3\, v_0}{0.75}\right)^2 = -\frac{12}{0.04658}(3-2)^2$$

$$-15 v_0^2 = -257.6$$

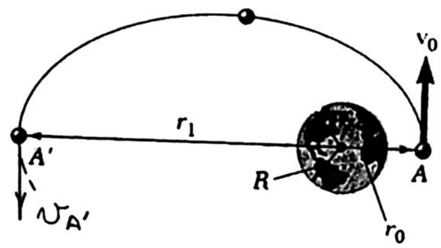
(*a*) $v_0^2 = 17.17\ \text{ft}^2/\text{s}^2$ $\quad v_0 = 4.14$ ft/s ◀

(*b*) Maximum speed. $v_m = v_2 = \dfrac{3v_0}{0.75}$ $\quad v_m = 16.58$ ft/s ◀

PROBLEM 13.98

Using the principles of conservation of energy and conservation of angular momentum, solve part *a* of Sample Problem 12.9.

SOLUTION



$$R = 6370 \text{ km}$$
$$r_0 = 500 \text{ km} + 6370 \text{ km}$$
$$r_0 = 6870 \text{ km} = 6.87\times10^6 \text{ m}$$
$$v_0 = 36,900 \text{ km/h}$$
$$= \frac{36.9\times10^6 \text{ m}}{3.6\times10^3 \text{ s}}$$
$$= 10.25\times10^3 \text{ m/s}$$

Conservation of angular momentum:

$$r_0 m v_0 = r_1 m v_A, \qquad r_0 = r_{\min}, \;\; r_1 = r_{\max}$$
$$v_{A'} = \left(\frac{r_0}{r_1}\right) v_0 = \left(\frac{6.870\times10^6}{r_1}\right)(10.25\times10^3)$$
$$v_{A'} = \frac{70.418\times10^9}{r_1} \tag{1}$$

Conservation of energy:

Point A:

$$v_0 = 10.25\times10^3 \text{ m/s}$$
$$T_A = \frac{1}{2} m v_0^2 = \frac{1}{2} m (10.25\times10^3)^2$$
$$T_A = (m)(52.53\times10^6)(\text{J})$$
$$V_A = -\frac{GMm}{r_0}$$
$$GM = gR^2 = (9.81 \text{ m/s}^2)(6.37\times10^6 \text{ m})^2$$
$$GM = 398\times10^{12} \text{ m}^3/\text{s}^2$$
$$r_0 = 6.87\times10^6 \text{ m}$$
$$V_A = -\frac{(398\times10^{12} \text{ m}^3/\text{s}^2)m}{(6.87\times10^6 \text{ m})}$$
$$= -57.93\times10^6 \, m \text{ (J)}$$

PROBLEM 13.98 (Continued)

Point A':

$$T_{A'} = \frac{1}{2} m v_{A'}^2$$

$$V_{A'} = -\frac{GMm}{r_1}$$

$$= -\frac{398 \times 10^{12}\ m}{r_1} \text{(J)}$$

$$T_A + V_A = T_{A'} + V_{A'}$$

$$52.53 \times 10^6\ \not{m} - 57.93 \times 10^6\ \not{m} = \frac{1}{2} \not{m} v_{A'}^2 - \frac{398 \times 10^{12}\ \not{m}}{r_1}$$

Substituting for $v_{A'}$ from (1)

$$-5.402 \times 10^6 = \frac{(70.418 \times 10^9)^2}{(2)(r_1)^2} - \frac{398 \times 10^{12}}{r_1}$$

$$-5.402 \times 10^6 = \frac{(2.4793 \times 10^{21})}{r_1^2} - \frac{398 \times 10^{12}}{r_1}$$

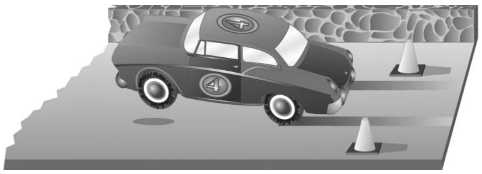
$$(5.402 \times 10^6) r_1^2 - (398 \times 10^{12}) r_1 + 2.4793 \times 10^{21} = 0$$

$$r_1 = 66.7 \times 10^6 \text{ m},\ 6.87 \times 10^6 \text{ m}$$

$r_{max} = 66{,}700$ km ◀

PROBLEM 13.99

Solve sample Problem 13.8, assuming that the elastic cord is replaced by a central force **F** of magnitude $(80/r^2)$ N directed toward *O*.



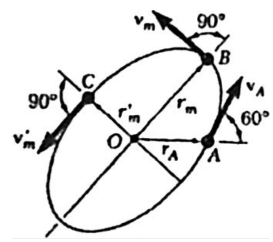
PROBLEM 13.8 Skid marks on a drag racetrack indicate that the rear (drive) wheels of a car slip for the first 20 m of the 400-m track.(*a*) Knowing that the coefficient of kinetic friction is 0.60, determine the speed of the car at the end of the first 20-m portion of the track if it starts from rest and the front wheels are just off the ground. (*b*) What is the maximum theoretical speed for the car at the finish line if, after skidding for 20 m, it is driven without the wheels slipping for the remainder of the race? Assume that while the car is rolling without slipping, 60 percent of the weight of the car is on the rear wheels and the coefficient of static friction is 0.75. Ignore air resistance and rolling resistance.

SOLUTION

(*a*) The force exerted on the sphere passes through *O*. Angular momentum about *O* is conserved.

Minimum velocity is at *B*, where the distance from *O* is maximum.

Maximum velocity is at *C*, where distance from *O* is minimum.



$$r_A m v_A \sin 60° = r_m m v_m$$

$$(0.5 \text{ m})(0.6 \text{ kg})(20 \text{ m/s})\sin 60° = r_m (0.6 \text{ kg}) v_m$$

$$v_m = \frac{8.66}{r_m} \qquad (1)$$

Conservation of energy:

At Point *A*,
$$T_A = \frac{1}{2} m v_A^2 = \frac{1}{2}(0.6 \text{ kg})(20 \text{ m/s})^2 = 120 \text{ J}$$

$$V = \int F dr = \int \frac{80}{r^2} dr = \frac{-80}{r},$$

$$V_A = \frac{-80}{0.5} = -160 \text{ J}$$

At Point *B*,
$$T_B = \frac{1}{2} m v_m^2 = \frac{1}{2}(0.6 \text{ kg}) v_m^2 = 0.3 v_m^2$$

and Point C: $$V_B = \frac{-80}{r_m}$$

$$T_A + V_A = T_B + V_B$$

$$120 - 160 = 0.3v_m^2 - \frac{80}{r_m} \qquad (2)$$

Substitute (1) into (2)

$$-40 = (0.3)\left(\frac{8.66}{r_m}\right)^2 - \frac{80}{r_m}$$

$$r_m^2 - 2\,r_m + 0.5625 = 0$$

$$r_m' = 0.339 \text{ m} \quad \text{and} \quad r_m = 1.661 \text{ m}$$

$r_{max} = 1.661$ m ◀

$r_{min} = 0.339$ m ◀

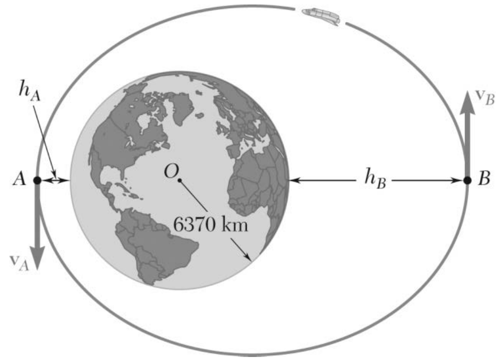
(*b*) Substitute r_m' and r_m from results of part (*a*) into (1) to get corresponding maximum and minimum values of the speed.

$$v_m' = \frac{8.66}{0.339} = 25.6 \text{ m/s}$$

$v_{max} = 25.6$ m/s ◀

$$v_m = \frac{8.66}{1.661} = 5.21 \text{ m/s}$$

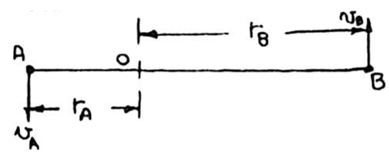
$v_{min} = 5.21$ m/s ◀



PROBLEM 13.100

A spacecraft is describing an elliptic orbit of minimum altitude $h_A = 2400$ km and maximum altitude $h_B = 9600$ km above the surface of the earth. Determine the speed of the spacecraft at A.

SOLUTION



$$r_A = 6370 \text{ km} + 2400 \text{ km}$$
$$r_A = 8770 \text{ km}$$
$$r_B = 6370 \text{ km} + 9600 \text{ km}$$
$$= 15{,}970 \text{ km}$$

Conservation of momentum: $r_A m v_A = r_B m v_B$

$$v_B = \frac{r_A}{r_B} v_A = \frac{8770}{15{,}970} v_A = 0.5492 v_A \quad (1)$$

Conservation of energy: $T_A = \frac{1}{2} m v_A^2 \quad V_A = \frac{-GMm}{r_A} \quad T_B = \frac{1}{2} m v_B^2 \quad V_B = \frac{-GMm}{r_B}$

$$GM = gR^2 = (9.81 \text{ m/s}^2)(6370 \times 10^3 \text{ m})^2 = 398.1 \times 10^{12} \text{ m}^3/\text{s}^2$$
$$V_A = \frac{-(398.1 \times 10^{12})m}{8770 \times 10^3} = -45.39 \times 10^6\ m$$
$$V_B = \frac{-(398.1 \times 10^{12})m}{(15{,}970 \times 10^3)} = -24.93\ m$$

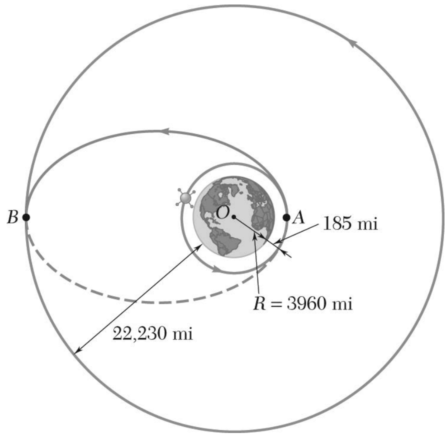
$$T_A + V_A = T_B + V_B:$$
$$\frac{1}{2} \cancel{m} v_A^2 - 45.39 \times 10^6 \cancel{m} = \frac{1}{2} \cancel{m} v_B^2 - 24.93 \times 10^6\ m \quad (2)$$

Substituting for v_B in (2) from (1)

$$v_A^2[1 - (0.5492)^2] = 40.92 \times 10^6$$
$$v_A^2 = 58.59 \times 10^6 \text{ m}^2/\text{s}^2$$
$$v_A = 7.65 \times 10^3 \text{ m/s}$$

$v_A = 27.6 \times 10^3$ km/h ◀

PROBLEM 13.101



While describing a circular orbit, 185 mi above the surface of the earth, a space shuttle ejects at Point A an inertial upper stage (IUS) carrying a communication satellite to be placed in a geosynchronous orbit (see Problem 13.87) at an altitude of 22,230 mi above the surface of the earth. Determine (a) the velocity of the IUS relative to the shuttle after its engine has been fired at A, (b) the increase in velocity required at B to plae the satellite in its final orbit.

SOLUTION

For earth, $R = 3960 \text{ mi} = 20.909\times10^6 \text{ ft}$

$$g = 32.2 \text{ ft}^2$$

$$GM = gR^2 = (32.2)(20.909\times10^6)^2 = 14.077\times10^{15} \text{ ft}^3/\text{s}^2$$

Speed on a circular orbits of radius r, r_A, and r_B.

$$F = ma_n$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v^2 = \frac{GM}{r} \qquad v = \sqrt{\frac{GM}{r}}$$

$$r_A = 3960 + 185 = 4145 \text{ mi} = 21.886\times10^6 \text{ ft}$$

$$(v_A)_{\text{circ}} = \sqrt{\frac{14.077\times10^{15}}{21.886\times10^6}} = 25.362\times10^3 \text{ ft/s}$$

$$r_B = 3960 + 22230 = 26190 \text{ mi} = 138.283\times10^6 \text{ ft}$$

$$(v_B)_{\text{circ}} = \sqrt{\frac{14.077\times10^{15}}{138.283\times10^6}} = 10.089\times10^3 \text{ ft/s}$$

Calculate speeds at A and B for path AB.

Conservation of angular momentum: $mr_Av_A\sin\phi_A = mr_Bv_B\sin\phi_A$

$$v_B = \frac{r_Av_A\sin 90°}{r_B\sin 90°} = \frac{21.886\times10^6 v_A}{138.283\times10^6} = 0.15816 v_A$$

PROBLEM 13.101 (Continued)

Conservation of energy: $T_A + V_A = T_B + V_B$

$$\frac{1}{2}mv_A^2 + \frac{GMm}{r_A} = \frac{1}{2}mv_B^2 - \frac{GMm}{r_B}$$

$$v_A^2 - v_B^2 = 2GM\left(\frac{1}{r_A} - \frac{1}{r_B}\right) = \frac{2GM(r_B - r_A)}{r_A r_B}$$

$$v_A^2 - (0.15816v_A)^2 = \frac{(2)(14.077\times10^{15})(116.397\times10^6)}{(21.886\times10^6)(138.283\times10^6)}$$

$$0.97499v_A^2 = 1.082796\times10^9$$

$$v_A = 33.325\times10^3 \text{ ft/s}$$

$$v_B = (0.15816)(33.325\times10^6) = 5.271\times10^3 \text{ ft/s}$$

(*a*) Increase in speed at *A*:

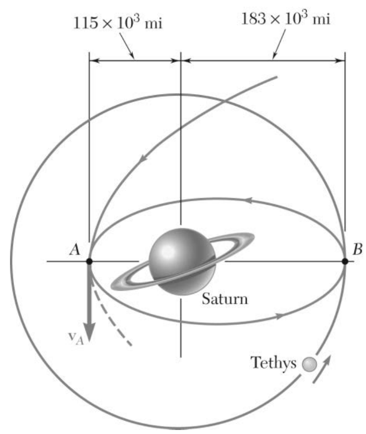
$$\Delta v_A = 33.325\times10^3 - 25.362\times10^3 = 7.963\times10^3 \text{ ft/s}$$

$\Delta v_A = 7960$ ft/s ◀

(*b*) Increase in speed at *B*:

$$\Delta v_B = 10.089\times10^3 - 5.271\times10^3 = 4.818\times10^3 \text{ ft/s}$$

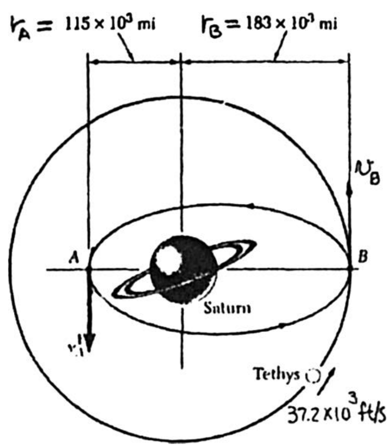
$\Delta v_B = 4820$ ft/s ◀



PROBLEM 13.102

A spacecraft approaching the planet Saturn reaches Point *A* with a velocity $\mathbf{v}_A$ of magnitude 68.8×10^3 ft/s. It is to be placed in an elliptic orbit about Saturn so that it will be able to periodically examine Tethys, one of Saturn's moons. Tethys is in a circular orbit of radius 183×10^3 mi about the center of Saturn, traveling at a speed of 37.2×10^3 ft/s. Determine (*a*) the decrease in speed required by the spacecraft at *A* to achieve the desired orbit, (*b*) the speed of the spacecraft when it reaches the orbit of Tethys at *B*.

SOLUTION



(*a*)

$$r_A = 607.2\times10^6 \text{ ft}$$
$$r_B = 966.2\times10^6 \text{ ft}$$

$v'_A =$ speed of spacecraft in the elliptical orbit after its speed has been decreased.

Elliptical orbit between *A* and *B*.

Conservation of energy

Point *A*: $\quad T_A = \frac{1}{2}mv_A'^2$

$$V_A = \frac{-GM_{\text{sat}}m}{r_A}$$

$M_{\text{sa}} =$ Mass of Saturn, determine GM_{sa} from the speed of Tethys in its circular orbit.

(Eq. 12.44) $\quad v_{\text{circ}} = \sqrt{\frac{GM_{\text{sat}}}{r}} \quad GM_{\text{sat}} = r_B v_{\text{circ}}^2$

$$GM_{\text{sat}} = (966.2\times10^6 \text{ ft}^2)(37.2\times10^3 \text{ ft/s})^2$$
$$= 1.337\times10^{18} \text{ ft}^3/\text{s}^2$$
$$V_A = -\frac{(1.337\times10^{18} \text{ ft}^3/\text{s}^2)\, m}{(607.2\times10^6 \text{ ft})}$$
$$= -2.202\times10^9\, m$$

PROBLEM 13.102 (Continued)

Point B: $$T_B = \frac{1}{2}mv_B^2 \qquad V_B = \frac{-GM_{\text{sat}}m}{r_B} = -\frac{(1.337\times 10^{18}\ \text{ft}^3/\text{s}^2)\,m}{(966.2\times 10^6\ \text{ft})}$$

$$V_B = 1.384\times 10^9$$

$$T_A + V_A = T_B + V_B;$$

$$\frac{1}{2}\cancel{m}v_A'^2 - 2.202\times 10^9\,\cancel{m} = \frac{1}{2}\cancel{m}v_B^2 - 1.384\times 10^9\,\cancel{m}$$

$$v_A'^2 - v_B^2 = 1.636\times 10^9$$

Conservation of angular momentum:

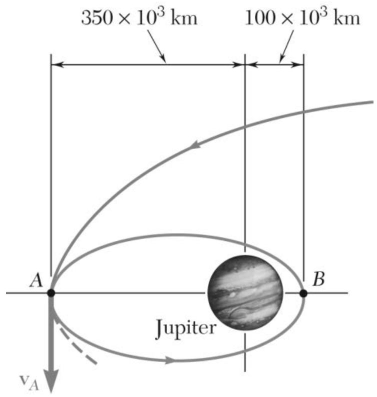
$$r_A m v_A' = r_B m v_B \qquad v_B = \frac{r_A}{r_B}v_A' = \frac{607.2\times 10^6}{966.2\times 10^6}v_A' = 0.6284v_A'$$

$$v_A'^2[1-(0.6284)^2] = 1.636\times 10^9$$

$$v_A' = 52{,}005\ \text{ft/s}$$

(a) $\Delta v_A = v_A - v_A' = 68{,}800 - 52{,}005$ $\qquad \Delta v_A = 16{,}795\ \text{ft/s}$ ◀

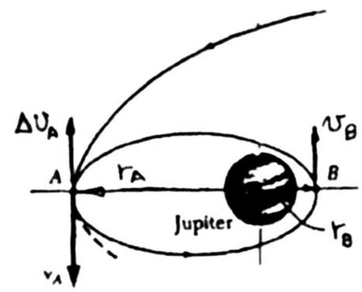
(b) $v_B = \frac{r_A}{r_B}v_A' = (0.6284)(52{,}005)$ $\qquad v_B = 32{,}700\ \text{ft/s}$ ◀



PROBLEM 13.103

A spacecraft traveling along a parabolic path toward the planet Jupiter is expected to reach Point A with a velocity $\mathbf{v}_A$ of magnitude 26.9 km/s. Its engines will then be fired to slow it down, placing it into an elliptic orbit which will bring it to within 100×10^3 km of Jupiter. Determine the decrease in speed Δv at Point A which will place the spacecraft into the required orbit. The mass of Jupiter is 319 times the mass of the earth.

SOLUTION



Conservation of energy.

Point A:

$$T_A = \frac{1}{2}m(v_A - \Delta v_A)^2$$

$$V_A = \frac{-GM_J m}{r_A}$$

$$GM_J = 319GM_E = 319gR_E^2$$

$$R_E = 6.37 \times 10^6 \text{ m}$$

$$GM_J = (319)(9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2$$

$$GM_J = 126.98 \times 10^{15} \text{ m}^3/\text{s}^2$$

$$r_A = 350 \times 10^6 \text{ m}$$

$$V_A = \frac{-(126.98 \times 10^{15} \text{ m}^3/\text{s}^2)m}{(350 \times 10^6 \text{ m})}$$

$$V_A = -(362.8 \times 10^6)m$$

Point B:

$$T_B = \frac{1}{2}mv_B^2$$

$$V_B = \frac{-GM_J m}{r_B} = \frac{-(126.98 \times 10^{15} \text{ m}^3/\text{s}^2)m}{(100 \times 10^6 m)}$$

$$V_B = -(1269.8 \times 10^6)m$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m(v_A - \Delta v_A)^2 - 362.8 \times 10^6 m = \frac{1}{2}mv_B^2 - 1269.8 \times 10^6 \text{ m}$$

$$(v_A - \Delta v_A)^2 - v_B^2 = -1814 \times 10^6 \qquad (1)$$

Conservation of angular momentum.

$$r_A = 350\times10^6 \text{ m}$$
$$r_B = 100\times10^6 \text{ m}$$
$$r_A m(v_A - \Delta v_A) = r_B m v_B$$
$$v_B = \left(\frac{r_A}{r_B}\right)(v_A - \Delta v_A)$$
$$= \left(\frac{350}{100}\right)(v_A - \Delta v_A) \qquad (2)$$

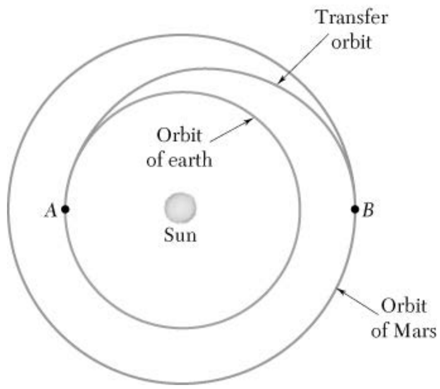
Substitute v_B in (2) into (1)

$$(v_A - \Delta v_A)^2[1-(3.5)^2] = -1814\times10^6$$
$$(v_A - \Delta v_A)^2 = 161.24\times10^6$$
$$(v_A - \Delta v_A) = \mp 12.698\times10^3 \text{ m/s}$$

(Take positive root; negative root reverses flight direction.)

$$v_A = 26.9\times10^3 \text{ m/s} \qquad \text{(given)}$$
$$\Delta v_A = (26.9\times10^3 \text{ m/s} - 12.698\times10^3 \text{ m/s})$$

$$\Delta v_A = 14.20 \text{ km/s} \blacktriangleleft$$



PROBLEM 13.104

As a first approximation to the analysis of a space flight from the earth to Mars, it is assumed that the orbits of the earth and Mars are circular and coplanar. The mean distances from the sun to the earth and to Mars are 149.6×10^6 km and 227.8×10^6 km, respectively. To place the spacecraft into an elliptical transfer orbit at Point A, its speed is increased over a short interval of time to v_A which is faster than the earth's orbital speed. When the spacecraft reaches Point B on the elliptical transfer orbit, its speed v_B is increased to the orbital speed of Mars. Knowing that the mass of the sun is 332.8×10^3 times the mass of the earth, determine the increase in velocity required (*a*) at A, (*b*) at B.

SOLUTION

M = mass of the sun

$$GM = 332.8(10)^3(9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2 = 1.3247(10)^{20} \text{ m}^3/\text{s}^2$$

Circular orbits

$$\text{Earth } v_E = \sqrt{\frac{GM}{149.6(10)^9}} = 29.758 \text{ m/s}$$

$$\text{Mars } v_M = \sqrt{\frac{GM}{227.8(10)^9}} = 24.115 \text{ m/s}$$

Conservation of angular momentum

Elliptical orbit

$$v_A(149.6) = v_B(227.8)$$

Conservation of energy

$$\frac{1}{2}v_A^2 - \frac{GM}{149.6(10)^9} = \frac{1}{2}v_B^2 - \frac{GM}{227.8(10)^9}$$

$$v_A = v_B\frac{(227.8)}{(149.6)} = 1.52273 v_B$$

$$\frac{1}{2}(1.52273)^2 v_B^2 - \frac{1.3247(10)^{20}}{149.6(10)^9} = \frac{1}{2}v_B^2 - \frac{1.3247(10)^{20}}{227.8(10)^9}$$

$$0.65935 v_B^2 = 3.0398(10)^8$$

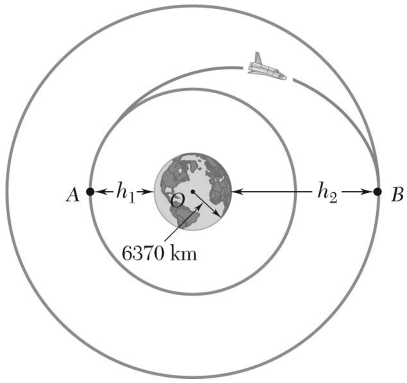
$$v_B^2 = 4.6102(10)^8$$

$$v_B = 21{,}471 \text{ m/s}, \; v_A = 32{,}695 \text{ m/s}$$

(*a*) Increase at A, $v_A - v_E = 32.695 - 29.758 = 2.94$ km/s ◀

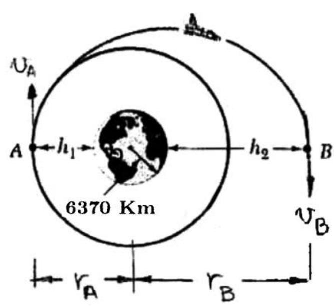
(*b*) Increase at B, $v_B - v_M = 24.115 - 21.471 = 2.64$ km/s ◀

PROBLEM 13.105



The optimal way of transferring a space vehicle from an inner circular orbit to an outer coplanar circular orbit is to fire its engines as it passes through A to increase its speed and place it in an elliptic transfer orbit. Another increase in speed as it passes through B will place it in the desired circular orbit. For a vehicle in a circular orbit about the earth at an altitude $h_1 = 200$ mi, which is to be transferred to a circular orbit at an altitude $h_2 = 500$ mi, determine (a) the required increases in speed at A and at B, (b) the total energy per unit mass required to execute the transfer.

SOLUTION



Elliptical orbit between A and B

Conservation of angular momentum

$$mr_A v_A = mr_B v_B$$

$$v_A = \frac{r_B}{r_A} v_B = \frac{7.170}{6.690} v_B$$

$$r_A = 6370 \text{ km} + 320 \text{ km} = 6690 \text{ km}, \qquad r_A = 6.690 \times 10^6 \text{ m}$$

$$v_A = 1.0718 v_B \tag{1}$$

$$r_B = 6370 \text{ km} + 800 \text{ km} = 7170 \text{ km}, \qquad r_B = 7.170 \times 10^6 \text{ m}$$

$$R = (6370 \text{ km}) = 6.37 \times 10^6 \text{ m}$$

Conservation of energy

$$GM = gR^2 = (9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2 = 398.060 \times 10^{12} \text{ m}^3/\text{s}^2$$

Point A:

$$T_A = \frac{1}{2} m v_A^2 \qquad V_A = -\frac{GMm}{r_A} = -\frac{(398.060 \times 10^{12})m}{(6.690 \times 10^6)}$$

$$V_A = 59.501 \times 10^6 \text{ m}$$

Point B:

$$T_B = \frac{1}{2} m v_B^2 \qquad V_B = -\frac{GMm}{r_B} = -\frac{(398.060 \times 10^{12})m}{(7.170 \times 10^6)}$$

$$V_B = 55.5 \times 10^6 \text{ m}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 - 59.501\times 10^6\text{ m} = \frac{1}{2}mv_B^2 - 55.5\times 10^6\text{ m}$$

$$v_A^2 - v_B^2 = 8.002\times 10^6$$

From (1) $\quad v_A = 1.0718v_B \qquad v_B^2[(1.0718)^2 - 1] = 8.002\times 10^6$

$$v_B^2 = 53.79\times 10^6\text{ m}^2/\text{s}^2, \qquad v_B = 7334\text{ m/s}$$

$$v_A = (1.0718)(7334\text{ m/s}) = 7861\text{ m/s}$$

Circular orbit at A and B

(Equation 12.44)

$$(v_A)_C = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{398.060\times 10^{12}}{6.690\times 10^6}} = 7714\text{ m/s}$$

$$(v_B)_C = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{398.060\times 10^{12}}{7.170\times 10^6}} = 7451\text{ m/s}$$

(*a*) Increases in speed at A and B

$$\Delta v_A = v_A - (v_A)_C = 7861 - 7714 = 147\text{ m/s} \blacktriangleleft$$

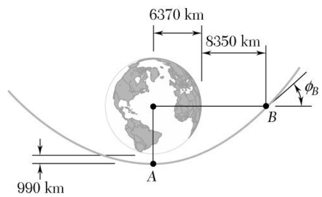
$$\Delta v_B = (v_B)_C - v_B = 7451 - 7334 = 117\text{ m/s} \blacktriangleleft$$

(*b*) Total energy per unit mass

$$E/m = \frac{1}{2}[(v_A)^2 - (v_A)_C^2 + (v_B)_C^2 - (v_B)^2]$$

$$E/m = \frac{1}{2}[(7861)^2 - (7714)^2 + (7451)^2 - (7334)^2]$$

$$E/m = 2.01\times 10^6\text{ J/kg} \blacktriangleleft$$



PROBLEM 13.106

During a flyby of the earth, the velocity of a spacecraft is 10.4 km/s as it reaches its minimum altitude of 990 km above the surface at Point A. At Point B the spacecraft is observed to have an altitude of 8350 km. Determine (a) the magnitude of the velocity at Point B, (b) the angle ϕ_B.

SOLUTION

At A:

$$h_A = vr = [1.04(10)^4 \text{ m/s}][6.37(10)^6 \text{ m} + 0.990(10)^6 \text{ m}]$$

$$h_A = 76.544(10)^9 \text{ m}^2/\text{s}$$

$$\frac{1}{m}(T_A + V_A) = \frac{1}{2}v^2 - \frac{GM}{r}$$

$$= \frac{1}{2}[1.04(10)^4]^2 - \frac{(9.81)[6.37(10)^6]^2}{[6.37(10)^6 + 0.990(10)^6]} \cong 0$$

(Parabolic orbit)

At B:

$$\frac{1}{m}(T_B + V_B) = \frac{1}{2}v_B^2 - \frac{GM}{r_B} = 0$$

$$\frac{1}{2}v_B^2 = \frac{(9.81)[6.37(10^6)]^2}{[6.37(10)^6 + 8.35(10)^6]}$$

$$v_B^2 = 54.084(10)^6$$

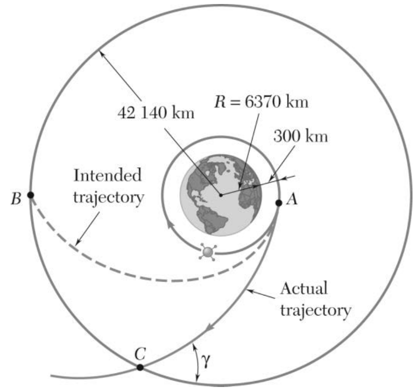
(a) $v_B = 7.35$ km/s ◀

$$h_B = v_B \sin\phi_B r_B = 76.544(10)^9$$

$$\sin\phi_B = \frac{76.544(10)^9}{7.35(10^6)[6.37(10)^6 + 8.35(10)^6]}$$

$$= 0.707483$$

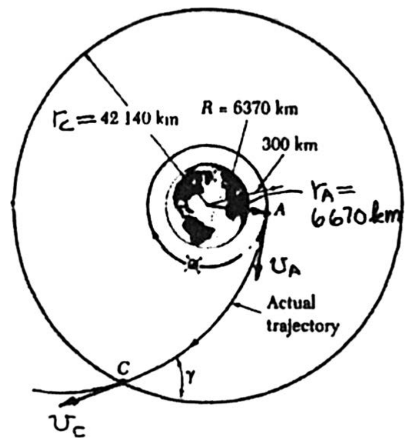
(b) $\phi_B = 45.0°$ ◀



PROBLEM 13.107

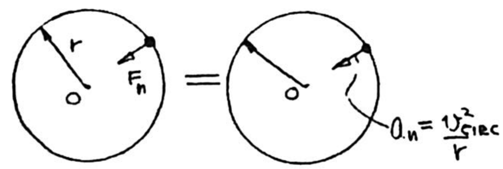
A space platform is in a circular orbit about the earth at an altitude of 300 km. As the platform passes through A, a rocket carrying a communications satellite is launched from the platform with a relative velocity of magnitude 3.44 km/s in a direction tangent to the orbit of the platform. This was intended to place the rocket in an elliptic transfer orbit bringing it to Point B, where the rocket would again be fired to place the satellite in a geosynchronous orbit of radius 42,140 km. After launching, it was discovered that the relative velocity imparted to the rocket was too large. Determine the angle γ at which the rocket will cross the intended orbit at Point C.

SOLUTION



$$R = 6370 \text{ km}$$
$$r_A = 6370 \text{ km} + 300 \text{ km}$$
$$r_A = 6.67 \times 10^6 \text{ m}$$
$$r_C = 42.14 \times 10^6 \text{ m}$$
$$GM = gR^2$$
$$GM = (9.81 \text{ m/s})(6.37 \times 10^6 \text{ m})^2$$
$$GM = 398.1 \times 10^{12} \text{ m}^3/\text{s}^2$$

For any circular orbit:



$$F_n = ma_n = \frac{mv_{\text{circ}}^2}{r}$$

$$F_n = \frac{GMm}{r^2} = m\frac{v_{\text{circ}}^2}{r}$$

$$v_{\text{circ}} = \sqrt{\frac{GM}{r}}$$

Velocity at A:

$$(v_A)_{\text{circ}} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{(398.1 \times 10^{12} \text{ m}^3/\text{s}^3)}{(6.67 \times 10^6 \text{ m})}} = 7.726 \times 10^3 \text{ m/s}$$

$$v_A = (v_A)_{\text{circ}} + (v_A)_R = 7.726 \times 10^3 + 3.44 \times 10^3 = 11.165 \times 10^3 \text{ m/s}$$

Velocity at C:

Conservation of energy: $T_A + V_A = T_C + V_C$

$$\frac{1}{2}\not{m}v_A^2 - \frac{GM\not{m}}{r_A} = \frac{1}{2}\not{m}v_C^2 - \frac{GM\not{m}}{r_C}$$

$$v_C^2 = v_A^2 + 2GM\left(\frac{1}{r_C} - \frac{1}{r_A}\right)$$

$$= (11.165\times10^3)^2 + 2(398.1\times10^{12})\left(\frac{1}{42.14\times10^6} - \frac{1}{6.67\times10^6}\right)$$

$$v_C^2 = 124.67\times10^6 - 100.48\times10^6$$

$$= 24.19\times10^6 \text{ m}^2/\text{s}^2$$

$$v_C = 4.919\times10^3 \text{ m/s}$$

Conservation of angular momentum:

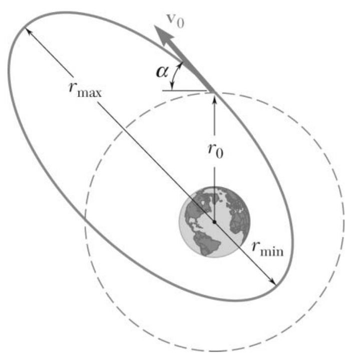
$$r_A m v_A = r_C m v_C \cos\gamma$$

$$\cos\gamma = \frac{r_A v_A}{r_C v_C}$$

$$= \frac{(6.67\times10^6)(11.165\times10^3)}{(42.14\times10^6)(4.919\times10^3)}$$

$$\cos\gamma = 0.35926$$

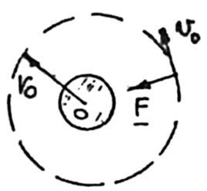
$\gamma = 68.9°$ ◀



PROBLEM 13.108

A satellite is projected into space with a velocity $\mathbf{v}_0$ at a distance r_0 from the center of the earth by the last stage of its launching rocket. The velocity $\mathbf{v}_0$ was designed to send the satellite into a circular orbit of radius r_0. However, owing to a malfunction of control, the satellite is not projected horizontally but at an angle α with the horizontal and, as a result, is propelled into an elliptic orbit. Determine the maximum and minimum values of the distance from the center of the earth to the satellite.

SOLUTION



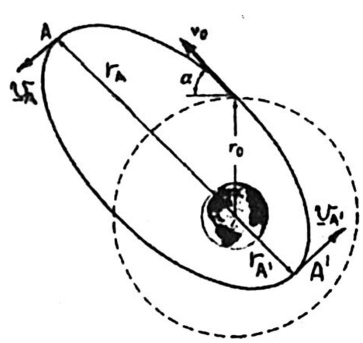
For circular orbit of radius r_0

$$F = ma_n \qquad \frac{GMm}{r_0^2} = m\frac{v_0^2}{r_0}$$

$$v_0^2 = \frac{GM}{r_0}$$

But v_0 forms an angle α with the intended circular path.

For elliptic orbit.



Conservation of angular momentum:

$$r_0 m v_0 \cos\alpha = r_A m v_A$$

$$v_A = \left(\frac{r_0}{r_A}\cos\alpha\right)v_0 \qquad (1)$$

Conservation of energy:

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \frac{1}{2}mv_A^2 - \frac{GMm}{r_A}$$

$$v_0^2 - v_A^2 = \frac{2GM}{r_0}\left(1 - \frac{r_0}{r_A}\right)$$

Substitute for v_A from (1)

$$v_0^2\left[1 - \left(\frac{r_0}{r_A}\right)^2\cos^2\alpha\right] = \frac{2GM}{r_0}\left(1 - \frac{r_0}{r_A}\right)$$

But $\quad v_0^2 = \dfrac{GM}{r_0}, \qquad$ thus $\quad 1 - \left(\dfrac{r_0}{r_A}\right)^2\cos^2\alpha = 2\left(1 - \dfrac{r_0}{r_A}\right)$

$$\cos^2\alpha\left(\frac{r_0}{r_A}\right)^2 - 2\left(\frac{r_0}{r_A}\right) + 1 = 0$$

PROBLEM 13.108 (Continued)

Solving for $\frac{r_0}{r_A}$

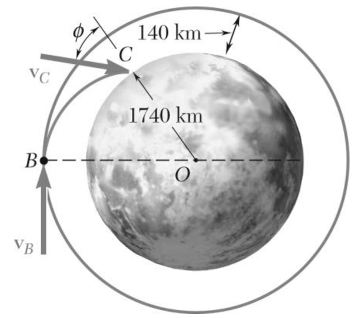
$$\frac{r_0}{r_A} = \frac{+2 \pm \sqrt{4 - 4\cos^2\alpha}}{2\cos^2\alpha} = \frac{1 \pm \sin\alpha}{1 - \sin^2\alpha}$$

$$r_A = \frac{(1+\sin\alpha)(1-\sin\alpha)}{1 \pm \sin\alpha} r_0 = (1 \mp \sin\alpha) r_0$$

↑ also valid for Point A'

Thus,

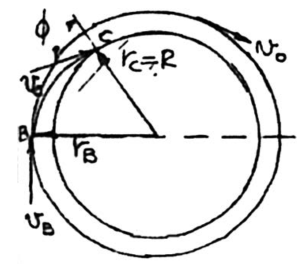
$r_{\max} = (1 + \sin\alpha) r_0$ $r_{\min} = (1 - \sin\alpha) r_0$ ◀



PROBLEM 13.109

Upon the LEM's return to the command module, the Apollo spacecraft of Problem 13.88 was turned around so that the LEM faced to the rear. The LEM was then cast adrift with a velocity of 200 m/s relative to the command module. Determine the magnitude and direction (angle ϕ formed with the vertical OC) of the velocity $\mathbf{v}_C$ of the LEM just before it crashed at C on the moon's surface.

SOLUTION



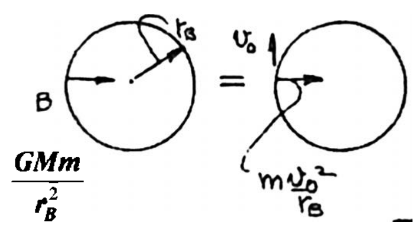
$R = 1740$ km

Command module in circular orbit

$$r_B = 1740 + 140 = 1880 \text{ km} = 1.88\times 10^6 \text{ m}$$

$$GM_{\text{moon}} = 0.0123\, GM_{\text{earth}} = 0.0123\, gR^2$$

$$= 0.0123(9.81)(6.37\times 10^6)^2 = 4.896\times 10^{12} \text{ m}^3/\text{s}^2$$



$$\Sigma F = ma_n \quad \frac{GM_m m}{r_B^2} = \frac{mv_0^2}{r_B}$$

$$v_0 = \sqrt{\frac{GMm}{r_B}} = \sqrt{\frac{4.896\times 10^{12}}{1.88\times 10^6}}$$

$$v_0 = 1614 \text{ m/s} \qquad v_B = 1614 - 200 = 1414 \text{ m/s}$$

Conservation of energy between B and C:

$$\frac{1}{2}mv_B^2 - \frac{GM_m m}{r_B} = \frac{1}{2}mv_C^2 - \frac{GM_m m}{r_C} \qquad r_C = R$$

$$v_C^2 = v_B^2 + \frac{2GMm}{r_B}\left(\frac{r_B}{R} - 1\right)$$

$$v_C^2 = (1414 \text{ m/s})^2 + 2\frac{(4.896\times 10^{12} \text{ m}^3/\text{s}^2)}{(1.88\times 10^6 \text{ m})}\left(\frac{1.88\times 10^6}{1.74\times 10^6} - 1\right)$$

$$v_C^2 = 1.999\times 10^6 + 0.4191\times 10^6 = 2.418\times 10^6 \text{ m}^2/\text{s}^2$$

$v_C = 1555$ m/s ◀

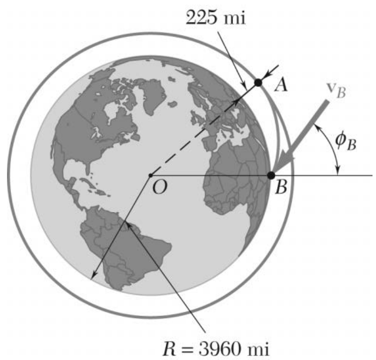
Conservation of angular momentum:

$$r_B m v_B = R m v_C \sin\phi$$

$$\sin\phi = \frac{r_B v_B}{r_C v_C} = \frac{(1.88\times 10^6 \text{ m})(1414 \text{ m/s})}{(1.74\times 10^6 \text{ m})(1555 \text{ m/s})} = 0.98249$$

$$\phi = 79.26°$$

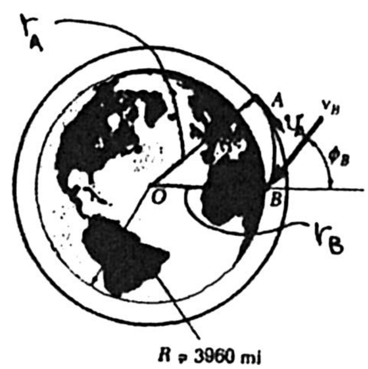
$\phi = 79.3°$ ◀



PROBLEM 13.110

A space vehicle is in a circular orbit at an altitude of 225 mi above the earth. To return to earth, it decreases its speed as it passes through A by firing its engine for a short interval of time in a direction opposite to the direction of its motion. Knowing that the velocity of the space vehicle should form an angle $\phi_B = 60°$ with the vertical as it reaches Point B at an altitude of 40 mi, determine (*a*) the required speed of the vehicle as it leaves its circular orbit at A, (*b*) its speed at Point B.

SOLUTION



(*a*) $$r_A = 3960 \text{ mi} + 225 \text{ mi} = 4185 \text{ mi}$$

$$r_A = 4185 \text{ mi} \times 5280 \text{ ft/mi} = 22,097 \times 10^3 \text{ ft}$$

$$r_B = 3960 \text{ mi} + 40 \text{ mi} = 4000 \text{ mi}$$

$$r_B = 4000 \times 5280 = 21,120 \times 10^3 \text{ ft}$$

$$R = 3960 \text{ mi} = 20,909 \times 10^3 \text{ ft}$$

$$GM = gR^2 = (32.2 \text{ ft/s}^2)(20,909 \times 10^3 \text{ ft})^2$$

$$GM = 14.077 \times 10^{15} \text{ ft}^3/\text{s}^2$$

Conservation of energy:

$$T_A = \frac{1}{2} m v_A^2$$

$$V_A = \frac{-GMm}{r_A}$$

$$= \frac{-14.077 \times 10^{15} m}{22,097 \times 10^3}$$

$$= -637.1 \times 10^6 m$$

$$T_B = \frac{1}{2} m v_B^2$$

$$V_B = \frac{-GMm}{r_B}$$

$$= \frac{-14.077 \times 10^{15} m}{21,120 \times 10^3}$$

$$= -666.5 \times 10^6 m$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} \not{m} v_A^2 - 637.1 \times 10^6 \not{m} = \frac{1}{2} \not{m} v_B^2 - 666.5 \times 10^6 \not{m}$$

$$v_A^2 = v_B^2 - 58.94 \times 10^6 \quad (1)$$

PROBLEM 13.110 (Continued)

Conservation of angular momentum:

$$r_A m v_A = r_B m v_B \sin\phi_B$$

$$v_B = \frac{(r_A)v_A}{(r_B)(\sin\phi_B)} = \frac{4185}{4000}\left(\frac{1}{\sin 60^\circ}\right)v_A$$

$$v_B = 1.208\, v_A \qquad (2)$$

Substitute v_B from (2) in (1)

$$v_A^2 = (1.208 v_A)^2 - 58.94\times 10^6$$

$$v_A^2[(1.208)^2 - 1] = 58.94\times 10^6$$

$$v_A^2 = 128.27\times 10^6 \text{ ft}^2/\text{s}^2$$

(*a*) $v_A = 11.32\times 10^3$ ft/s ◀

(*b*) From (2)

$$v_B = 1.208 v_A$$

$$= 1.208(11.32\times 10^6)$$

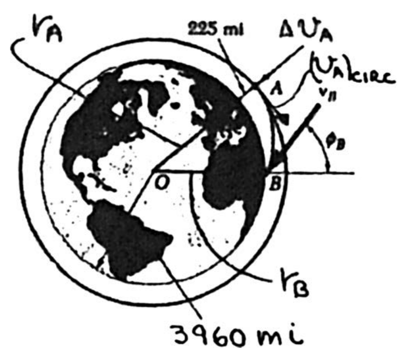
$$= 13.68\times 10^3 \text{ ft/s}$$

$v_B = 13.68\times 10^3$ ft/s ◀

PROBLEM 13.111*

In Problem 13.110, the speed of the space vehicle was decreased as it passed through A by firing its engine in a direction opposite to the direction of motion. An alternative strategy for taking the space vehicle out of its circular orbit would be to turn it around so that its engine would point away from the earth and then give it an incremental velocity $\Delta\mathbf{v}_A$ toward the center O of the earth. This would likely require a smaller expenditure of energy when firing the engine at A, but might result in too fast a descent at B. Assuming this strategy is used with only 50 percent of the energy expenditure used in Problem 13.109, determine the resulting values of ϕ_B and v_B.

SOLUTION



$$r_A = 3960 \text{ mi} + 225 \text{ mi}$$

$$r_A = 4185 \text{ mi} = 22.097\times 10^6 \text{ ft}$$

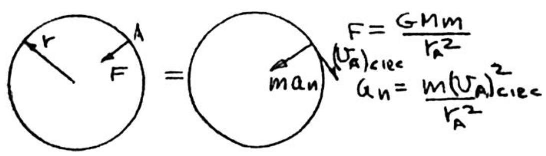
$$r_B = 3960 \text{ mi} + 40 \text{ mi} = 4000 \text{ mi}$$

$$r_B = 21.120\times 10^6 \text{ ft}$$

$$GM = gR^2 = (32.2 \text{ ft/s}^2)[(3960)(5280) \text{ ft}^2]$$

$$GM = 14.077\times 10^{15} \text{ ft}^3/\text{s}^2$$

Velocity in circular orbit at 225 m altitude:



Newton's second law

$$F = ma_n: \quad \frac{GMm}{r_A^2} = \frac{m(v_A)^2_{\text{circ}}}{r_A}$$

$$(v_A)_{\text{circ}} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.077\times 10^{15}}{22.097\times 10^6}}$$

$$= 25.24\times 10^3 \text{ ft/s}$$

Energy expenditure:

From Problem 13.110, $\quad v_A = 11.32\times 10^3$ ft/s

Energy, $\quad \Delta E_{109} = \frac{1}{2}m(v_A)^2_{\text{circ}} - \frac{1}{2}mv_A^2$

$$\Delta E_{109} = \frac{1}{2}m(25.24\times 10^3)^2 - \frac{1}{2}m(11.32\times 10^3)^2$$

$$\Delta E_{109} = 254.46\times 10^6\, m \text{ ft}\cdot\text{lb}$$

$$\Delta E_{110} = (0.50)\Delta E_{109} = \frac{(254.46\times 10^6\, m)}{2} \text{ ft}\cdot\text{lb}$$

PROBLEM 13.111* (Continued)

Thus, additional kinetic energy at A is

$$\frac{1}{2}m(\Delta v_A)^2 = \Delta E_{110} = \frac{(254.46\times 10^6 m)}{2} \text{ ft}\cdot\text{lb} \qquad (1)$$

Conservation of energy between A and B:

$$T_A = \frac{1}{2}m[(v_A)_{\text{circ}}^2 + (\Delta v_A)^2] \qquad V_A = \frac{-GMm}{r_A}$$

$$T_B = \frac{1}{2}mv_B^2 \qquad V_B = \frac{-GMm}{r_A}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m(25.24\times 10^3)^2 + \frac{254.46\times 10^6 m}{2} - \frac{14.077\times 10^{15} m}{22.097\times 10^6} = \frac{1}{2}mv_B^2 - \frac{14.077\times 10^{15} m}{21.120\times 10^6}$$

$$v_B^2 = 637.06\times 10^6 + 254.46\times 10^6 - 1274.1\times 10^6 + 1333\times 10^6$$

$$v_B^2 = 950.4\times 10^3$$

$$v_B = 30.88\times 10^3 \text{ ft/s} \blacktriangleleft$$

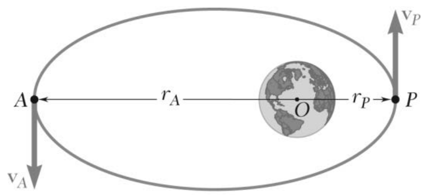
Conservation of angular momentum between A and B:

$$r_A m(v_A)_{\text{circ}} = r_B m v_B \sin\phi_B$$

$$\sin\phi_B = \left(\frac{r_A}{r_B}\right)\frac{(v_A)_{\text{circ}}}{(v_B)} = \frac{(4185)}{(4000)}\frac{(25.24\times 10^3)}{(30.88\times 10^3)} = 0.8565$$

$$\phi_B = 58.9° \blacktriangleleft$$

PROBLEM 13.112



Show that the values v_A and v_P of the speed of an earth satellite at the apogee A and the perigee P of an elliptic orbit are defined by the relations

$$v_A^2 = \frac{2GM}{r_A + r_P}\frac{r_P}{r_A} \qquad v_P^2 = \frac{2GM}{r_A + r_P}\frac{r_A}{r_P}$$

where M is the mass of the earth, and r_A and r_P represent, respectively, the maximum and minimum distances of the orbit to the center of the earth.

SOLUTION

Conservation of angular momentum:

$$r_A m v_A = r_P m v_P$$

$$v_A = \frac{r_P}{r_A} v_P \tag{1}$$

Conservation of energy:

$$\frac{1}{2}mv_P^2 - \frac{GMm}{r_P} = \frac{1}{2}mv_A^2 - \frac{GMm}{r_A} \tag{2}$$

Substituting for v_A from (1) into (2)

$$v_P^2 - \frac{2GM}{r_P} = \left(\frac{r_P}{r_A}\right)^2 v_P^2 - \frac{2GM}{r_A}$$

$$\left(1 - \left(\frac{r_P}{r_A}\right)^2\right) v_P^2 = 2GM\left(\frac{1}{r_P} - \frac{1}{r_A}\right)$$

$$\frac{r_A^2 - r_P^2}{r_A^2} v_P^2 = 2GM\,\frac{r_A - r_P}{r_A r_P}$$

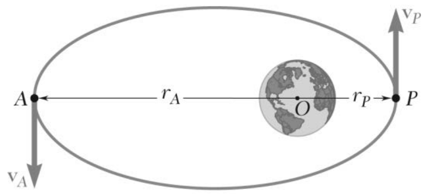
with $$r_A^2 - r_P^2 = (r_A - r_P)(r_A + r_P)$$

$$v_P^2 = \frac{2GM}{r_A + r_P}\left(\frac{r_A}{r_P}\right) \quad (3) \blacktriangleleft$$

Exchanging subscripts P and A

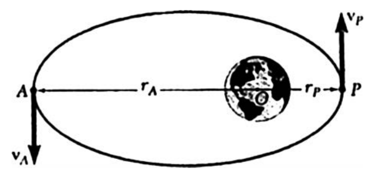
$$v_A^2 = \frac{2GM}{r_A + r_P}\left(\frac{r_P}{r_A}\right) \qquad \text{Q.E.D.} \quad \blacktriangleleft$$

PROBLEM 13.113



Show that the total energy E of an earth satellite of mass m describing an elliptic orbit is $E = -GMm/(r_A + r_P)$, where M is the mass of the earth, and r_A and r_P represent, respectively, the maximum and minimum distances of the orbit to the center of the earth. (Recall that the gravitational potential energy of a satellite was defined as being zero at an infinite distance from the earth.)

SOLUTION



See solution to Problem 13.112 (above) for derivation of Equation (3).

$$v_P^2 = \frac{2GM}{(r_A + r_P)} \frac{r_A}{r_P}$$

Total energy at Point P is

$$E = T_P + V_P = \frac{1}{2} m v_P^2 - \frac{GMm}{r_P}$$

$$= \frac{1}{2}\left[\frac{2GMm}{(r_A + r_0)} \frac{r_A}{r_P}\right] - \frac{GMm}{r_P}$$

$$= GMm\left[\frac{r_A}{r_P(r_A + r_P)} - \frac{1}{r_P}\right]$$

$$= GMm\frac{(r_A - r_A - r_P)}{r_P(r_A + r_P)}$$

$$E = -\frac{GMm}{r_A + r_P} \blacktriangleleft$$

Note: Recall that gravitational potential of a satellite is defined as being zero at an infinite distance from the earth.

PROBLEM 13.114*

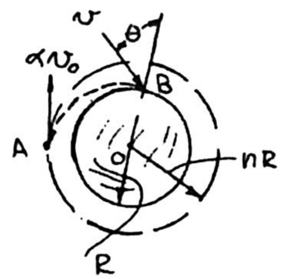
A space probe describes a circular orbit of radius nR with a velocity $\mathbf{v}_0$ about a planet of radius R and center O. Show that (a) in order for the probe to leave its orbit and hit the planet at an angle θ with the vertical, its velocity must be reduced to $\alpha\mathbf{v}_0$, where

$$\alpha = \sin\theta\sqrt{\frac{2(n-1)}{n^2-\sin^2\theta}}$$

(b) the probe will not hit the planet if α is larger than $\sqrt{2/(1+n)}$.

SOLUTION

(a) Conservation of energy:



At A: $$T_A = \frac{1}{2}m(\alpha v_0)^2$$

$$V_A = -\frac{GMm}{nR}$$

At B: $$T_B = \frac{1}{2}mv^2$$

$$V_B = -\frac{GMm}{R}$$

M = mass of planet

m = mass of probe

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\cancel{m}(\alpha v_0)^2 - \frac{GM\cancel{m}}{nR} = \frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{R} \quad (1)$$

Conservation of angular momentum:

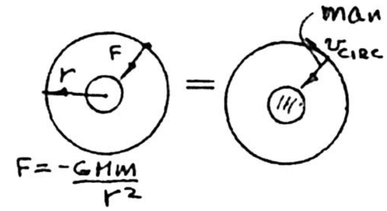
$$nRm\alpha v_0 = Rmv\sin\theta$$

$$v = \frac{n\alpha v_0}{\sin\theta} \quad (2)$$

Replacing v in (1) by (2)

$$(\alpha v_0)^2 - \frac{2GM}{nR} = \left(\frac{n\alpha v_0}{\sin\theta}\right)^2 - \frac{2GM}{R} \quad (3)$$

For any circular orbit.



$$a_n = \frac{v^2}{r}$$

Newton's second law

$$\frac{-GMm}{r^2} = \frac{m(v)_{\text{circ}}^2}{r}$$

$$v_{\text{circ}} = \sqrt{\frac{GM}{r}}$$

For $r = nR$, $$v_0 = v_{\text{circ}} = \sqrt{\frac{GM}{nR}}$$

Substitute for v_0 in (3)

$$\alpha^2 \frac{GM}{nR} - \frac{2GM}{nR} = \frac{n^2\alpha^2}{\sin^2\theta}\left(\frac{GM}{nR}\right) - \frac{2GM}{R}$$

$$\alpha^2\left[1 - \frac{n^2}{\sin^2\theta}\right] = 2(1-n)$$

$$\alpha^2 = \frac{2(1-n)(\sin^2\theta)}{(\sin^2\theta - n^2)} = \frac{2(n-1)\sin^2\theta}{(n^2 - \sin^2\theta)}$$

$$\alpha = \sin\theta\sqrt{\frac{2(n-1)}{n^2 - \sin^2\theta}} \qquad \text{Q.E.D.}$$ ◀

(*b*) Probe will just miss the planet if $\theta > 90°$,

$$\alpha = \sin 90°\sqrt{\frac{2(n-1)}{n^2 - \sin^2 90°}} = \sqrt{\frac{2}{n+1}}$$ ◀

Note: $n^2 - 1 = (n-1)(n+1)$

PROBLEM 13.115

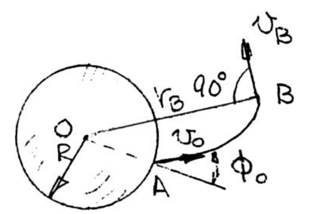
A missile is fired from the ground with an initial velocity $\mathbf{v}_0$ forming an angle ϕ_0 with the vertical. If the missile is to reach a maximum altitude equal to αR, where R is the radius of the earth, (*a*) show that the required angle ϕ_0 is defined by the relation

$$\sin\phi_0 = (1+\alpha)\sqrt{1-\frac{\alpha}{1+\alpha}\left(\frac{v_{\text{esc}}}{v_0}\right)^2}$$

where v_{esc} is the escape velocity, (*b*) determine the range of allowable values of v_0.

SOLUTION

(*a*) $$r_A = R$$



Conservation of angular momentum:

$$Rmv_0\sin\phi_0 = r_B m v_B$$

$$r_B = R + \alpha R = (1+\alpha)R$$

$$v_B = \frac{Rv_0\sin\phi_0}{(1+\alpha)R} = \frac{v_0\sin\phi_0}{(1+\alpha)} \qquad (1)$$

Conservation of energy:

$$T_A + V_A = T_B + V_B \qquad \frac{1}{2}mv_0^2 - \frac{GMm}{R} = \frac{1}{2}mv_B^2 - \frac{GMm}{(1+\alpha)R}$$

$$v_0^2 - v_B^2 = \frac{2GMm}{R}\left(1-\frac{1}{1+\alpha}\right) = \frac{2GMm}{R}\left(\frac{\alpha}{1+\alpha}\right)$$

Substitute for v_B from (1)

$$v_0^2\left(1-\frac{\sin^2\phi_0}{(1+\alpha)^2}\right) = \frac{2GMm}{R}\left(\frac{\alpha}{1+\alpha}\right)$$

From Equation (12.43): $$v_{\text{esc}}^2 = \frac{2GM}{R}$$

$$v_0^2\left(1-\frac{\sin^2\phi_0}{(1+\alpha)^2}\right) = v_{\text{esc}}^2\left(\frac{\alpha}{1+\alpha}\right)$$

$$\frac{\sin^2\phi_0}{(1+\alpha)^2} = 1-\left(\frac{v_{\text{esc}}}{v_0}\right)^2\frac{\alpha}{1+\alpha} \qquad (2)$$

$$\sin\phi_0 = (1+\alpha)\sqrt{1-\frac{\alpha}{1+\alpha}\left(\frac{v_{\text{esc}}}{v_0}\right)^2} \qquad \text{Q.E.D.}$$

PROBLEM 13.115 (Continued)

(*b*) Allowable values of v_0 (for which maximum altitude = αR)

$$0 < \sin^2 \phi_0 < 1$$

For $\sin \phi_0 = 0,$ from (2)

$$0 = 1 - \left(\frac{v_{esc}}{v_0}\right)^2 \frac{\alpha}{1+\alpha}$$

$$v_0 = v_{esc}\sqrt{\frac{\alpha}{1+\alpha}}$$

For $\sin \phi_0 = 1,$ from (2)

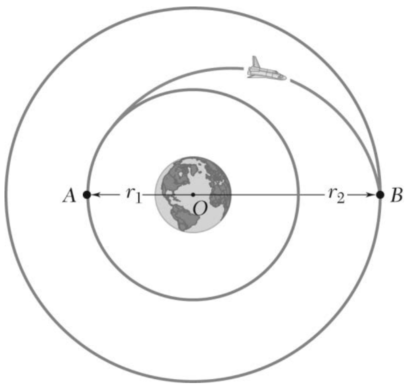
$$\frac{1}{(1+\alpha)^2} = 1 - \left(\frac{v_{esc}}{v_0}\right)^2 \frac{\alpha}{1+\alpha}$$

$$\left(\frac{v_{esc}}{v_0}\right)^2 = \frac{1}{\alpha}\left(1+\alpha - \frac{1}{1+\alpha}\right) = \frac{1+2\alpha+\alpha^2-1}{\alpha(1+\alpha)} = \frac{2+\alpha}{1+\alpha}$$

$$v_0 = v_{esc}\sqrt{\frac{1+\alpha}{2+\alpha}}$$

$$v_{esc}\sqrt{\frac{\alpha}{1+\alpha}} < v_0 < v_{esc}\sqrt{\frac{1+\alpha}{2+\alpha}} \blacktriangleleft$$

PROBLEM 13.116



A spacecraft of mass m describes a circular orbit of radius r_1 around the earth. (a) Show that the additional energy ΔE which must be imparted to the spacecraft to transfer it to a circular orbit of larger radius r_2 is

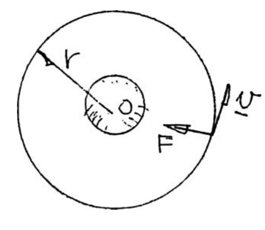
$$\Delta E = \frac{GMm(r_2 - r_1)}{2r_1 r_2}$$

where M is the mass of the earth. (b) Further show that if the transfer from one circular orbit to the other is executed by placing the spacecraft on a transitional semielliptic path AB, the amounts of energy ΔE_A and ΔE_B which must be imparted at A and B are, respectively, proportional to r_2 and r_1:

$$\Delta E_A = \frac{r_2}{r_1 + r_2}\Delta E \qquad \Delta E_B = \frac{r_1}{r_1 + r_2}\Delta E$$

SOLUTION

(a) For a *circular* orbit of radius r



$$F = ma_n: \quad \frac{GMm}{r^2} = m\frac{v^2}{r}$$

$$v^2 = \frac{GM}{r}$$

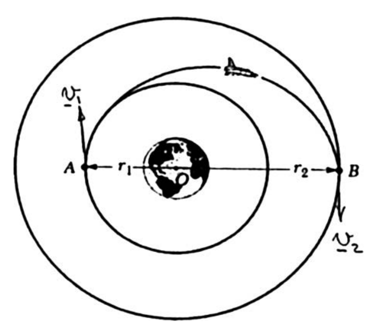
$$E = T + V = \frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r} \qquad (1)$$

Thus ΔE required to pass from circular orbit of radius r_1 to circular orbit of radius r_2 is

$$\Delta E = E_1 - E_2 = -\frac{1}{2}\frac{GMm}{r_1} + \frac{1}{2}\frac{GMm}{r_2}$$

$$\Delta E = \frac{GMm(r_2 - r_1)}{2r_1 r_2} \qquad \text{Q.E.D.} \qquad (2)$$

(b) For an elliptic orbit, we recall Equation (3) derived in Problem 13.113 (with $v_P = v_1$)



$$v_1^2 = \frac{2Gm}{(r_1 + r_2)}\frac{r_2}{r_1}$$

At Point A: Initially spacecraft is in a circular orbit of radius r_1.

$$v_{\text{circ}}^2 = \frac{GM}{r_1}$$

$$T_{\text{circ}} = \frac{1}{2}mv_{\text{circ}}^2 = \frac{1}{2}m\frac{GM}{r_1}$$

After the spacecraft engines are fired and it is placed on a semi-elliptic path AB, we recall

$$v_1^2 = \frac{2GM}{(r_1 + r_2)} \cdot \frac{r_2}{r_1}$$

and
$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m\frac{2GMr_2}{r_1(r_1 + r_2)}$$

At Point A, the increase in energy is

$$\Delta E_A = T_1 - T_{\text{circ}} = \frac{1}{2}m\frac{2GMr_2}{r_1(r_1 + r_2)} - \frac{1}{2}m\frac{GM}{r_1}$$

$$\Delta E_A = \frac{GMm(2r_2 - r_1 - r_2)}{2r_1(r_1 + r_2)} = \frac{GMm(r_2 - r_1)}{2r_1(r_1 + r_2)}$$

$$\Delta E_A = \frac{r_2}{r_1 + r_2}\left[\frac{GMm(r_2 - r_1)}{2r_1 r_2}\right]$$

Recall Equation (2):
$$\Delta E_A = \frac{r_2}{(r_1 + r_2)}\Delta E \qquad \text{Q.E.D.}$$

A similar derivation at Point B yields,

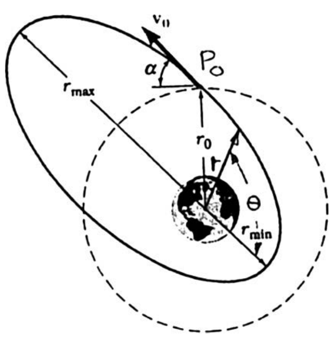
$$\Delta E_B = \frac{r_1}{(r_1 + r_2)}\Delta E \qquad \text{Q.E.D.}$$

PROBLEM 13.117*

Using the answers obtained in Problem 13.108, show that the intended circular orbit and the resulting elliptic orbit intersect at the ends of the minor axis of the elliptic orbit.

PROBLEM 13.108 A satellite is projected into space with a velocity $\mathbf{v}_0$ at a distance r_0 from the center of the earth by the last stage of its launching rocket. The velocity $\mathbf{v}_0$ was designed to send the satellite into a circular orbit of radius r_0. However, owing to a malfunction of control, the satellite is not projected horizontally but at an angle α with the horizontal and, as a result, is propelled into an elliptic orbit. Determine the maximum and minimum values of the distance from the center of the earth to the satellite.

SOLUTION



If the point of intersection P_0 of the circular and elliptic orbits is at an end of the minor axis, then v_0 is parallel to the major axis. This will be the case only if $\alpha + 90° = \theta_0$, that is if $\cos\theta_0 = -\sin\alpha$. We must therefore prove that

$$\cos\theta_0 = -\sin\alpha \qquad (1)$$

We recall from Equation (12.39):

$$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta \qquad (2)$$

When $\theta = 0$, $\quad r = r_{min}$ and $r_{min} = r_0(1-\sin\alpha)$

$$\frac{1}{r_0(1-\sin\alpha)} = \frac{GM}{h^2} + C \qquad (3)$$

For $\theta = 180°$, $\quad r = r_{max} = r_0(1+\sin\alpha)$

$$\frac{1}{r_0(1+\sin\alpha)} = \frac{GM}{h^2} - C \qquad (4)$$

Adding (3) and (4) and dividing by 2:

$$\frac{GM}{h^2} = \frac{1}{2r_0}\left(\frac{1}{1-\sin\alpha} + \frac{1}{1+\sin\alpha}\right)$$
$$= \frac{1}{r_0\cos^2\alpha}$$

Subtracting (4) from (3) and dividing by 2:

$$C = \frac{1}{2r_0}\left(\frac{1}{1-\sin\alpha} - \frac{1}{1+\sin\alpha}\right) = \left(\frac{1}{2r_0}\right)\frac{2\sin\alpha}{1-\sin^2\alpha}$$
$$C = \frac{\sin\alpha}{r_0\cos^2\alpha}$$

PROBLEM 13.117* (Continued)

Substituting for $\frac{GM}{h^2}$ and C into Equation (2)

$$\frac{1}{r} = \frac{1}{r_0 \cos^2 \alpha}(1 + \sin \alpha \cos \theta) \qquad (5)$$

Letting $r = r_0$ and $\theta = \theta_0$ in Equation (5), we have

$$\cos^2 \alpha = 1 + \sin \alpha \cos \theta_0$$

$$\cos \theta_0 = \frac{\cos^2 \alpha - 1}{\sin \alpha}$$

$$= -\frac{\sin^2 \alpha}{\sin \alpha}$$

$$= -\sin \alpha$$

This proves the validity of Equation (1) and thus P_0 is an end of the minor axis of the elliptic orbit.

PROBLEM 13.118*

(*a*) Express in terms of $r_{\min}$ and $v_{\max}$ the angular momentum per unit mass, *h*, and the total energy per unit mass, *E*/*m*, of a space vehicle moving under the gravitational attraction of a planet of mass *M* (Figure 13.15). (*b*) Eliminating $v_{\max}$ between the equations obtained, derive the formula

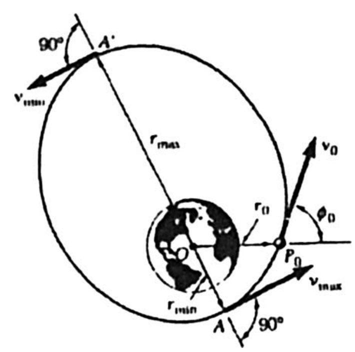
$$\frac{1}{r_{\min}} = \frac{GM}{h^2}\left[1 + \sqrt{1 + \frac{2E}{m}\left(\frac{h}{GM}\right)^2}\right]$$

(*c*) Show that the eccentricity ε of the trajectory of the vehicle can be expressed as

$$\varepsilon = \sqrt{1 + \frac{2E}{m}\left(\frac{h}{GM}\right)^2}$$

(*d*) Further show that the trajectory of the vehicle is a hyperbola, an ellipse, or a parabola, depending on whether *E* is positive, negative, or zero.

SOLUTION



(*a*) Point *A*:

Angular momentum per unit mass.

$$h = \frac{H_0}{m} = \frac{r_{\min} m v_{\max}}{m}$$

$$h = r_{\min} v_{\max} \qquad (1) \blacktriangleleft$$

Energy per unit mass

$$\frac{E}{m} = \frac{1}{m}(T + V)$$

$$\frac{E}{m} = \frac{1}{m}\left(\frac{1}{2}mv_{\max}^2 - \frac{GMm}{r_{\min}}\right) = \frac{1}{2}v_{\max}^2 - \frac{GM}{r_{\min}} \qquad (2) \blacktriangleleft$$

(*b*) From Eq. (1): $v_{\max} = h/r_{\min}$ substituting into (2)

$$\frac{E}{m} = \frac{1}{2}\frac{h^2}{r_{\min}^2} - \frac{GM}{r_{\min}}$$

$$\left(\frac{1}{r_{\min}}\right)^2 - \frac{2GM}{h^2}\cdot\frac{1}{r_{\min}} - \frac{2\left(\frac{E}{m}\right)}{h^2} = 0$$

Solving the quadratic: $$\frac{1}{r_{\min}} = \frac{GM}{h^2} + \sqrt{\left(\frac{GM}{h^2}\right)^2 + \frac{2\left(\frac{E}{m}\right)}{h^2}}$$

Rearranging

$$\frac{1}{r_{\min}} = \frac{GM}{h^2}\left[1 + \sqrt{1 + \frac{2E}{m}\left(\frac{h}{GM}\right)^2}\right] \qquad (3) ◀$$

(*c*) Eccentricity of the trajectory:

Eq. (12.39′) $\frac{1}{r} = \frac{GM}{h^2}(1 + \varepsilon\cos\theta)$

When $\theta = 0$, $\cos\theta = 1$ and $r = r_{\min}$

Thus,

$$\frac{1}{r_{\min}} = \frac{GM}{h^2}(1 + \varepsilon) \qquad (4)$$

Comparing (3) and (4), $$\varepsilon = \sqrt{1 + \frac{2E}{m}\left(\frac{h}{GM}\right)^2} \qquad (5)$$

(*d*) Recalling discussion in section 12.12 and in view of Eq. (5)

1. Hyperbola if $\varepsilon > 1$, that is, if $E > 0$ ◀
2. Parabola if $\varepsilon = 1$, that is, if $E = 0$ ◀
3. Ellipse if $\varepsilon < 1$, that is, if $E < 0$ ◀

Note: For circular orbit $\varepsilon = 0$ and

$$1 + \frac{2E}{m}\left(\frac{h}{GM}\right)^2 = 0 \qquad \text{or} \qquad E = -\left(\frac{GM}{h}\right)^2\frac{m}{2},$$

but for circular orbit $v^2 = \frac{GM}{r}$ and $h^2 = v^2r^2 = GMr$,

thus $$E = -\frac{1}{2}m\frac{(GM)^2}{GMr} = -\frac{1}{2}\frac{GMm}{r}$$

PROBLEM 13.CQ4

A large insect impacts the front windshield of a sports car traveling down a road. Which of the following statements is true during the collision?

(*a*) The car exerts a greater force on the insect than the insect exerts on the car.

(*b*) The insect exerts a greater force on the car than the car exerts on the insect.

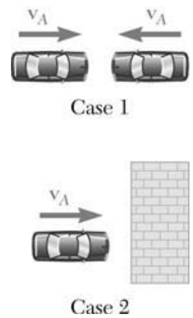
(*c*) The car exerts a force on the insect, but the insect does not exert a force on the car.

(*d*) The car exerts the same force on the insect as the insect exerts on the car.

(*e*) Neither exerts a force on the other; the insect gets smashed simply because it gets in the way of the car.

SOLUTION

Answer: (*d*) This is Newton's 3rd Law.



PROBLEM 13.CQ5

The expected damages associated with two types of perfectly plastic collisions are to be compared. In the first case, two identical cars traveling at the same speed impact each other head on. In the second case, the car impacts a massive concrete wall. In which case would you expect the car to be more damaged?

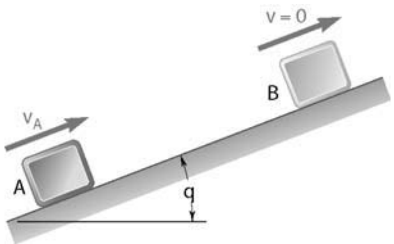
(*a*) Case 1

(*b*) Case 2

(*c*) The same damage in each case

SOLUTION

Answer: (*c*) In both cases the car will come to a complete stop, so the applied impulse will be the same.

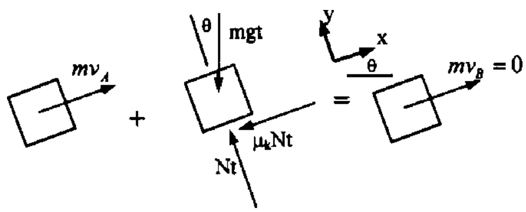


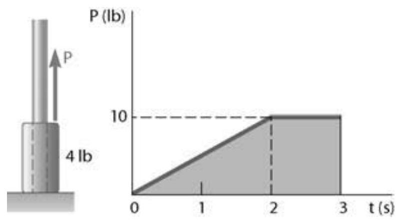
PROBLEM 13.F1

The initial velocity of the block in position *A* is 30 ft/s. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.30$. Draw impulse-momentum diagrams that could be used to determine the time it takes for the block to reach *B* with zero velocity, if $\theta = 20°$.

SOLUTION

Answer:



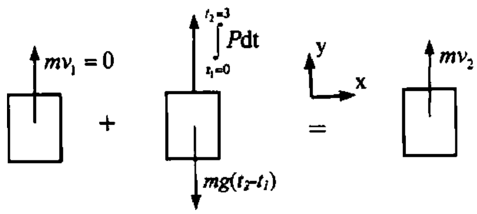


PROBLEM 13.F2

A 4-lb collar which can slide on a frictionless vertical rod is acted upon by a force **P** which varies in magnitude as shown. Knowing that the collar is initially at rest, draw impulse-momentum diagrams that could be used to determine its velocity at $t = 3$ s.

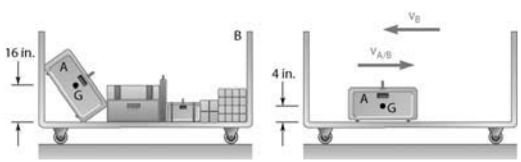
SOLUTION

Answer:



Where $\int_{t_1=0}^{t_2=3} Pdt$ is the area under the curve.

PROBLEM 13.F3

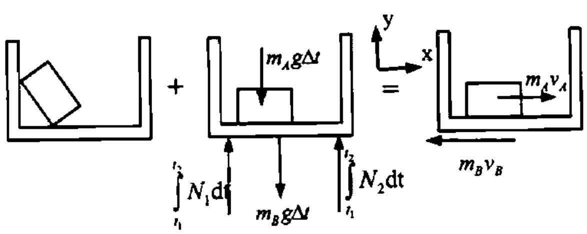


The 15-kg suitcase *A* has been propped up against one end of a 40-kg luggage carrier *B* and is prevented from sliding down by other luggage. When the luggage is unloaded and the last heavy trunk is removed from the carrier, the suitcase is free to slide down, causing the 40-kg carrier to move to the left with a velocity v_B of magnitude 0.8 m/s. Neglecting friction, draw impulse-momentum diagrams that could be used to determine (*a*) the velocity of *A* as it rolls on the carrier and (*b*) the velocity of the carrier after the suitcase hits the right side of the carrier without bouncing back.

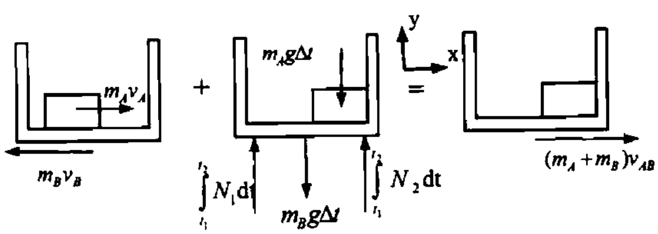
SOLUTION

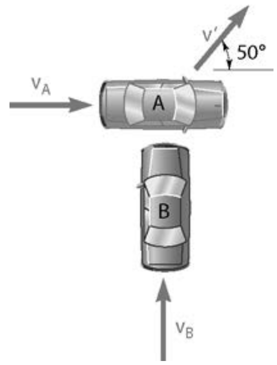
Answer:

(*a*)



(*b*)



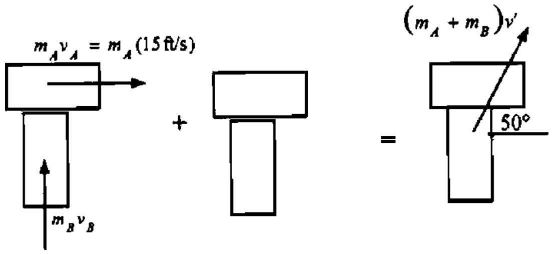


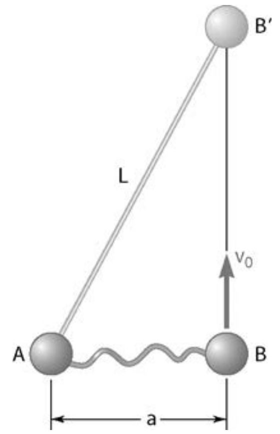
PROBLEM 13.F4

Car *A* was traveling west at a speed of 15 m/s and car *B* was traveling north at an unknown speed when they slammed into each other at an intersection. Upon investigation it was found that after the crash the two cars got stuck and skidded off at an angle of 50° north of east. Knowing the masses of *A* and *B* are m_A and m_B respectively, draw impulse-momentum diagrams that could be used to determine the velocity of *B* before impact.

SOLUTION

Answer:



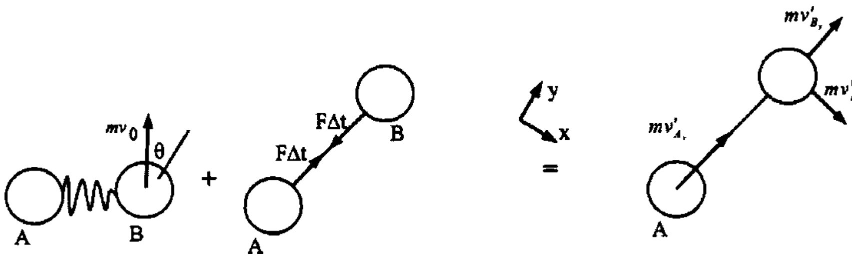


PROBLEM 13.F5

Two identical spheres A and B, each of mass m, are attached to an inextensible inelastic cord of length L and are resting at a distance a from each other on a frictionless horizontal surface. Sphere B is given a velocity $\mathbf{v}_0$ in a direction perpendicular to line AB and moves it without friction until it reaches B' where the cord becomes taut. Draw impulse-momentum diagrams that could be used to determine the magnitude of the velocity of each sphere immediately after the cord has become taut.

SOLUTION

Answer:



Where $v'_{A_y} = v'_{B_y}$ since the cord is inextensible.

PROBLEM 13.119

A 35,000 Mg ocean liner has an initial velocity of 4 km/h. Neglecting the frictional resistance of the water, determine the time required to bring the liner to rest by using a single tugboat which exerts a constant force of 150 kN.

SOLUTION

$$m = 35,000\ \ Mg = 35\times10^6 \text{ kg}$$

$$F = 150\times10^3 \text{ N}$$

$$v_1 = 4 \text{ km/hr} = 1.1111 \text{ m/s}$$

$$mv_1 - Ft = 0$$

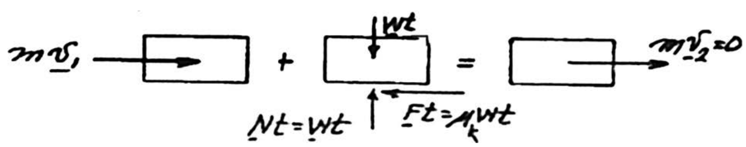
$$(35\times10^6 \text{ kg})(1.1111 \text{ m/s}) - (150\times10^3 \text{ N})t = 0$$

$$t = 259.26 \text{ s} \qquad t = 4 \text{ min} 19 \text{ s} \blacktriangleleft$$

PROBLEM 13.120

A 2500-lb automobile is moving at a speed of 60 mi/h when the brakes are fully applied, causing all four wheels to skid. Determine the time required to stop the automobile (*a*) on dry pavement ($\mu_k = 0.75$), (*b*) on an icy road ($\mu_k = 0.10$).

SOLUTION



$$v_1 = 60 \text{ mph} = 88 \text{ ft/s}$$

$$mv_1 - \mu_k Wt = 0$$

$$t = \frac{mv_1}{\mu_k W} = \frac{mv_1}{\mu_k mg} = \frac{v_1}{\mu_k g}$$

(*a*) For $\mu_k = 0.75$

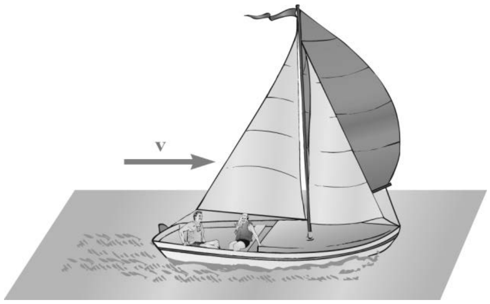
$$t = \frac{88 \text{ ft/s}}{(0.75)(32.2 \text{ ft/s}^2)}$$

$t = 3.64$ s ◀

(*b*) For $\mu_k = 0.10$

$$t = \frac{88 \text{ ft/s}}{(0.10)(32.2 \text{ ft/s}^2)}$$

$t = 27.3$ s ◀

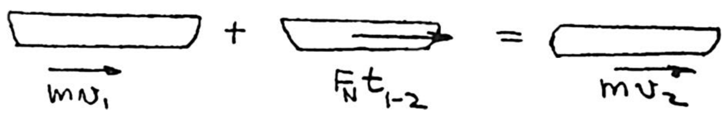


PROBLEM 13.121

A sailboat weighing 980 lb with its occupants is running down wind at 8 mi/h when its spinnaker is raised to increase its speed. Determine the net force provided by the spinnaker over the 10-s interval that it takes for the boat to reach a speed of 12 mi/h.

SOLUTION

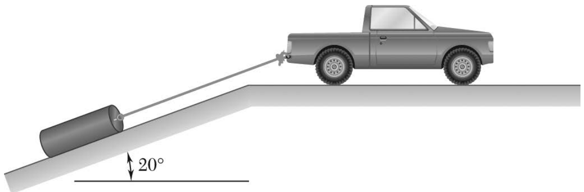
$$v_1 = 8 \text{ mi/h} = 11.73 \text{ ft/s} \quad t_{1-2} = 10 \text{ sec}$$
$$v_2 = 12 \text{ mi/h} = 17.60 \text{ ft/s}$$



$$m \cdot v_1 + \text{imp}_{1-2} = mv_2$$
$$m(11.73 \text{ ft/s}) + F_n(10 \text{ s}) = m(17.60 \text{ ft/s})$$

$$F_n = \frac{(980 \text{ lb})(17.60 \text{ ft/s} - 11.73 \text{ ft/s})}{(32.2 \text{ ft/s}^2)(10 \text{ s})} \qquad F_n = 178.6 \text{ lb} \blacktriangleleft$$

Note: F_n is the *net* force *provided* by the sails. The force on the sails is actually greater and includes the force needed to overcome the water resistance on the hull.

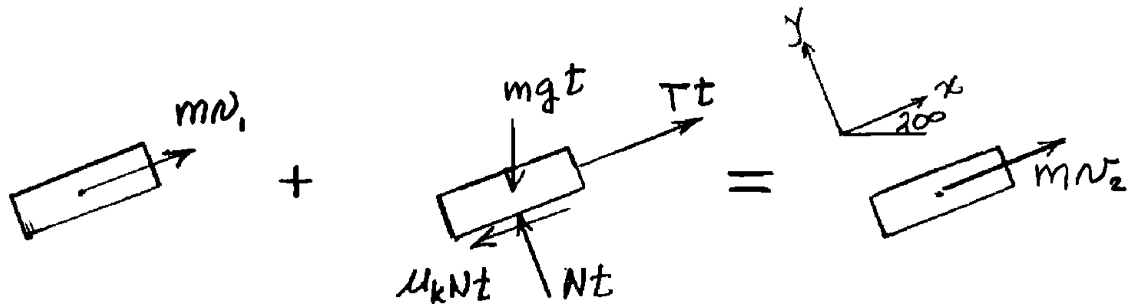


PROBLEM 13.122

A truck is hauling a 300-kg log out of a ditch using a winch attached to the back of the truck. Knowing the winch applies a constant force of 2500 N and the coefficient of kinetic friction between the ground and the log is 0.45, determine the time for the log to reach a speed of 0.5m/s.

SOLUTION

Apply the principle of impulse and momentum to the log.



$$m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = m\mathbf{v}_2$$

Components in y-direction:

$$0 + Nt - mgt\cos 20° = 0$$

$$N = mg\cos 20°$$

Components in x-direction:

$$0 + Tt - mgt\sin 20° - \mu_k Nt = mv_2$$

$$(T - mg\sin 20° - \mu_k mg\cos 20°)t = mv_2$$

$$[T - mg(\sin 20° + \mu_k \cos 20°)]t = mv_2$$

Data: $T = 2500$ N, $m = 300$ kg, $\mu_k = 0.45$,

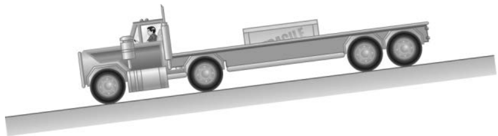
$g = 9.81$ m/s^2, $v_2 = 0.5$ m/s

$$[2500 - (300)(9.81)(\sin 20° + 0.45\cos 20°)]t = (300)(0.5)$$

$$248.95\, t = 150$$

$t = 0.603$ s ◀

PROBLEM 13.123

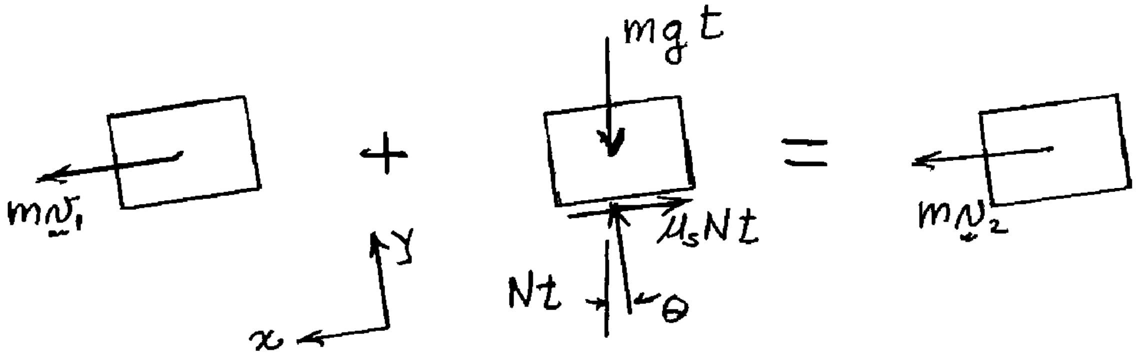


A truck is traveling down a road with a 3-percent grade at a speed of 55 mi/h when the brakes are applied. Knowing the coefficients of friction between the load and the flatbed trailer shown are $\mu_s = 0.40$ and $\mu_k = 0.35$, determine the shortest time in which the rig can be brought to a stop if the load is not to shift.

SOLUTION

Apply the principle impulse-momentum to the crate, knowing that, if the crate does not shift, the velocity of the crate matches that of the truck. For impending slip the friction and normal components of the contact force between the crate and the flatbed trailer satisfy the following equation:

$$F_f = \mu_s N$$



$$m\mathbf{v}_1 + \Sigma\mathbf{Imp}_{1\to 2} = m\mathbf{v}_2$$

Components in y-direction: $0 + Nt - mgt\cos\theta = 0$

$$N = mg\cos\theta$$

Components in x-direction: $mv_1 + mgt\sin\theta - \mu_s Nt = mv_2$

$$mv_1 + mgt(\sin\theta - \mu_s\cos\theta) = 0$$

$$t = \frac{v_1}{g(\mu_s\cos\theta - \sin\theta)}$$

Data: $v_1 = 55 \text{ mi/h} = 80.667 \text{ ft/s}, \quad v_2 = 0,$

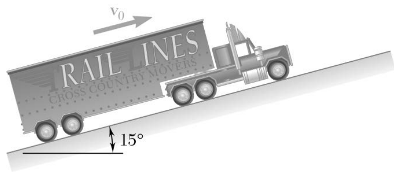
$$g = 32.2 \text{ ft/s}^2, \quad \mu_s = 0.40, \quad \tan\theta = 3/100$$

$$\theta = 1.71835°$$

$$\mu_s\cos\theta - \sin\theta = 0.36983$$

$$t = \frac{80.667}{(32.2)(0.36983)}$$

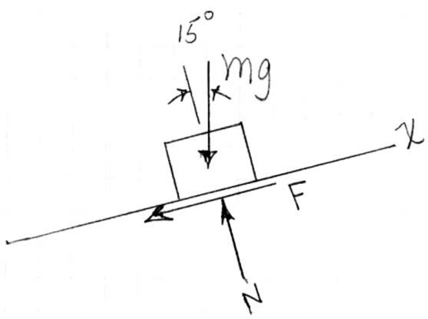
$t = 6.77 \text{ s}$ ◀



PROBLEM 13.124

Steep safety ramps are built beside mountain highways to enable vehicles with defective brakes to stop. A 10-ton truck enters a 15° ramp at a high speed v_0 = 108 ft/s and travels for 6 s before its speed is reduced to 36 ft/s. Assuming constant deceleration, determine (*a*) the magnitude of the braking force, (*b*) the additional time required for the truck to stop. Neglect air resistance and rolling resistance.

SOLUTION



$$W = 20{,}000 \text{ lb}$$

$$m = \frac{20{,}000}{32.2} = 621.118 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Momentum in the *x* direction

$$x\colon mv_0 - (F + mg\sin 15°)t = mv_1$$

$$621.118(108) - (F + mg\sin 15°)6 = (621.118)(36)$$

$$F + mg\sin 15° = 7453.4$$

(*a*) $F = 7453.4 - 20{,}000\sin 15° = 2277 \text{ lb}$

$F = 2280$ lb ◀

(*b*) $mv_0 - (F + mg\sin 15°)t = 0$ $\quad$ t = total time

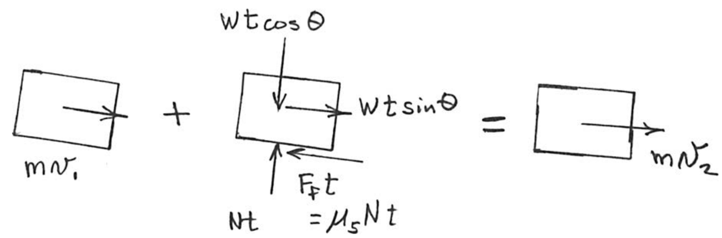
$621.118(108) - 7453.4t = 0;$ $\quad$ $t = 9.00$ s

Additional time = 9 – 6 $\quad$ $t = 3.00$ s ◀

PROBLEM 13.125

Baggage on the floor of the baggage car of a high-speed train is not prevented from moving other than by friction. The train is travelling down a 5 percent grade when it decreases its speed at a constant rate from 120 mi/h to 60 mi/h in a time interval of 12 s. Determine the smallest allowable value of the coefficient of static friction between a trunk and the floor of the baggage car if the trunk is not to slide.

SOLUTION



$$v_1 = 120 \text{ mi/h} = 176 \text{ ft/s}$$

$$v_2 = 60 \text{ mi/h} = 88 \text{ ft/s}$$

$$t_{1-2} = 12 \text{ s}$$

$$Nt_{1-2} = Wt_{1-2}\cos\theta$$

$$\overset{+}{\searrow}\ \cancel{m}v_1 - \mu_s \cancel{m} g t_{1-2}\cos\theta + \cancel{m} g t_{1-2}\sin\theta = \cancel{m}v_2$$

$$(176 \text{ ft/s}) - \mu_s(32.2 \text{ ft/s}^2)(12 \text{ s})(\cos 2.86°) + (32.2 \text{ ft/s}^2)(12 \text{ s})(\sin 2.86°) = 88 \text{ ft/s}$$

$$\mu_s = \frac{176 - 88 + (32.2)(12)(\sin 2.86°)}{(32.2)(12)(\cos 2.86°)}$$

$\mu_s = 0.278$ ◀

PROBLEM 13.126

A 2-kg particle is acted upon by the force, expressed in newtons, $\mathbf{F} = (8-6t)\mathbf{i} + (4-t^2)\mathbf{j} + (4+t)\mathbf{k}$. Knowing that the velocity of the particle is $\mathbf{v} = (150 \text{ m/s})\mathbf{i} + (100 \text{ m/s})\mathbf{j} - (250 \text{ m/s})\mathbf{k}$ at $t = 0$, determine (*a*) the time at which the velocity of the particle is parallel to the *yz* plane, (*b*) the corresponding velocity of the particle.

SOLUTION

$$m\mathbf{v}_0 + \int \mathbf{F}dt = m\mathbf{v} \qquad (1)$$

Where

$$\int \mathbf{F}dt = \int_0^t [(8-6t)\mathbf{i} + (4-t^2)\mathbf{j} + (4+t)\mathbf{k}]dt$$

$$= (8t-3t^2)\mathbf{i} + \left(4t - \frac{1}{3}t^3\right)\mathbf{j} + \left(4t + \frac{1}{2}t^2\right)\mathbf{k}$$

Substituting $m = 2$ kg, $v_0 = 150\mathbf{i} + 100\mathbf{j} - 250\mathbf{k}$ into (1):

$$(2 \text{ kg})(150\mathbf{i} + 100\mathbf{j} - 250\mathbf{k}) + (8t-3t^2)\mathbf{i} + \left(4t - \frac{1}{3}t^3\right)\mathbf{j} + \left(4t + \frac{1}{2}t^2\right)\mathbf{k} = (2 \text{ kg})\mathbf{v}$$

$$\mathbf{v} = \left(150 + 4t - \frac{3}{2}t^2\right)\mathbf{i} + \left(100 + 2t - \frac{1}{6}t^3\right)\mathbf{j} + \left(-250 + 2t + \frac{1}{4}t^2\right)\mathbf{k}$$

(*a*) **v** is parallel to *yz* plane when $v_x = 0$, that is, when

$$150 + 4t - \frac{3}{2}t^2 = 0 \quad t = 11.422 \text{ s}$$

$t = 11.42$ s ◀

(*b*)

$$\mathbf{v} = \left[100 + 2(11.422) - \frac{1}{6}(11.422)^3\right]\mathbf{j}$$

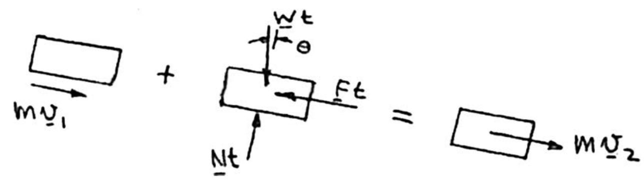
$$+ \left[-250 + 2(11.422) + \frac{1}{4}(11.422)^2\right]\mathbf{k}$$

$\mathbf{v} = -(125.5 \text{ m/s})\mathbf{j} - (194.5 \text{ m/s})\mathbf{k}$ ◀

PROBLEM 13.127

A truck is traveling down a road with a 4-percent grade at a speed of 60 mi/h when its brakes are applied to slow it down to 20 mi/h. An antiskid braking system limits the braking force to a value at which the wheels of the truck are just about to slide. Knowing that the coefficient of static friction between the road and the wheels is 0.60, determine the shortest time needed for the truck to slow down.

SOLUTION



$$\theta = \tan^{-1}\frac{4}{100} = 2.29°$$

$$m\mathbf{v}_1 + \Sigma\,\mathbf{imp}_{1-2} = m\mathbf{v}_2$$

$$+\searrow \quad mv_1 + Wt\sin\theta - Ft = mv_2$$

$$v_1 = 60 \text{ mi/h} = 88 \text{ ft/s} \qquad N = W\cos\theta \quad W = mg$$

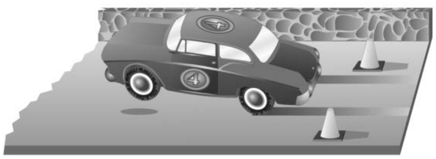
$$v_2 = 20 \text{ mi/h} = 29.33 \text{ ft/s} \qquad F = \mu_s N = \mu_s W\cos\theta$$

$$(\not{m})(88 \text{ ft/s}) + (\not{m})(32.2 \text{ ft/s}^2)(t)(\sin 2.29°) - (0.60)(\not{m})(32.2 \text{ ft/s}^2)(\cos 2.29°)(t) = (\not{m})(29.33 \text{ ft/s})$$

$$t = \frac{88 - 29.33}{32.2[(0.60)\cos 2.29° - \sin 2.29°]}$$

$t = 3.26$ s ◀

PROBLEM 13.128



Skid marks on a drag race track indicate that the rear (drive) wheels of a car slip for the first 20 m of the 400-m track. (*a*) Knowing that the coefficient of kinetic friction is 0.60, determine the shortest possible time for the car to travel the initial 20-m portion of the track if it starts from rest with its front wheels just off the ground. (*b*) Determine the minimum time for the car to run the whole race if, after skidding for 20 m, the wheels roll without sliding for the remainder of the race. Assume for the rolling portion of the race that 65 percent of the weight is on the rear wheels and that the coefficient of static friction is 0.85. Ignore air resistance and rolling resistance.

SOLUTION

(*a*) First 20 m

Velocity at 20 m. Rear wheels skid to generate the maximum force resulting in maximum velocity and minimum time since all the weight is on the rear wheel: This force is $F = \mu_k N = 0.60W$.

Work and energy. $T_0 + U_{0-20} = T_{20}$

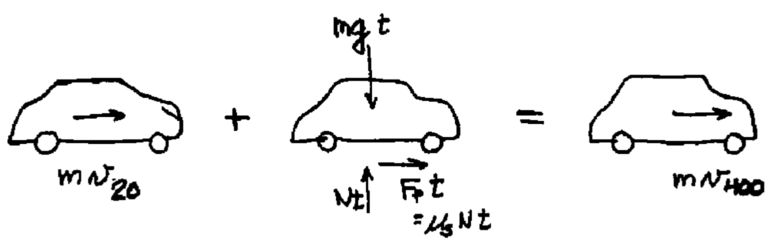
$$T_0 = 0 \qquad U_{0-20} = (F)(20) \qquad T_{20} = \frac{1}{2}mv_{20}^2$$

$$0 + \mu_k mg(20) = \frac{1}{2}m v_{20}^2$$

$$v_{20}^2 = (2)(0.60)(20\text{ m})(9.81\text{ m/s}^2)$$

$$v_{20} = 15.344\text{ m/s}$$

Impulse-momentum.



$$\xrightarrow{+}\ 0 + \mu_k mgt_{0-20} = mv_{20} \qquad v_{20} = 15.344\text{ m/s}$$

$$t_{0-20} = \frac{15.344\text{ m/s}}{(0.60)(9.81\text{ m/s}^2)} \qquad t_{0-20} = 2.61\text{ s} \blacktriangleleft$$

(*b*) For the whole race:

The maximum force on the wheels for the first 20 m is $F = \mu_k mg = 0.60mg$. For remaining 360 m, the maximum force, if there is no sliding and 65 percent of the weight is on the rear (drive) wheels, is

$$F = \mu_s (0.65)\, mg = (0.85)(0.65)\, mg = 0.5525\, mg$$

Velocity at 400 m.

Work and energy. $$T_0 + U_{0-20} + U_{20-400} = T_{400}$$

$$T_0 = 0 \quad U_{0-20} = (0.60mg)(20 \text{ m}), \quad U_{60-400} = (0.5525mg)(380 \text{ m})$$

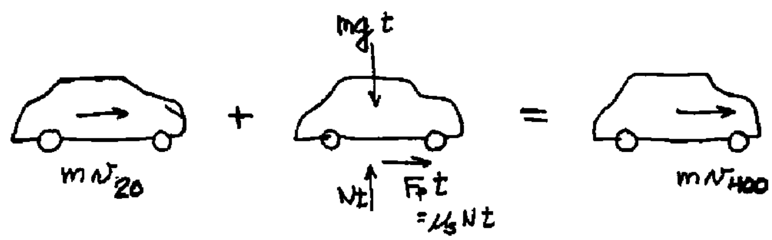
$$T_{400} = \frac{1}{2} m v_{400}^2$$

$$0 + 12\, mg + (0.5525)(380) mg = \frac{1}{2} m v_{400}^2$$

$$v_{400} = 65.990 \text{ m/s}$$

Impulse–momentum.

From 20 m to 400 m



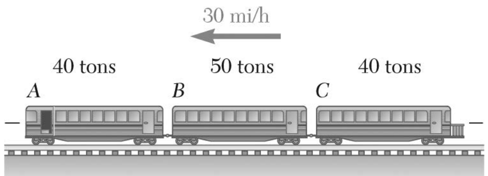
$$F = \mu_s N = 0.510mg$$

$$v_{20} = 15.344 \text{ m/s}$$

$$v_{400} = 65.990 \text{ m/s}$$

$$m(15.344) + 0.5525\, mgt_{20-400} = m(65.990); \quad t_{20-400} = 9.3442 \text{ s}$$

$$t_{0-400} = t_{0-20} + t_{20-400} = 2.61 + 9.34 \qquad t_{0-400} = 11.95 \text{ s} \blacktriangleleft$$



PROBLEM 13.129

The subway train shown is traveling at a speed of 30 mi/h when the brakes are fully applied on the wheels of cars B and C, causing them to slide on the track, but are not applied on the wheels of car A. Knowing that the coefficient of kinetic friction is 0.35 between the wheels and the track, determine (a) the time required to bring the train to a stop, (b) the force in each coupling.

SOLUTION

Weights of cars: $W_A = W_C = 80,000 \text{ lb}, \ W_B = 100,000 \text{ lb}$

Masses of cars: $m_A = m_C = 2484 \text{ lb} \cdot \text{s}^2/\text{ft}, \ m_B = 3106 \text{ lb} \cdot \text{s}^2/\text{ft}$

For each car the normal force (upward) is equal in magnitude to the weight of the car.

$$N_A = N_C = 80,000 \text{ lb} \qquad N_B = 100,000 \text{ lb}$$

Friction forces: $F_A = 0$ (brakes not applied)

$$F_B = (0.35)(100,000) = 35000 \text{ lb}$$

$$F_C = (0.35)(80,000) = 28,000 \text{ lb}$$

Stopping data: $v_1 = 30 \text{ mi/h} = 44 \text{ ft/s}, \ v_2 = 0.$

(a) Apply the principle of impulse-momentum to the entire train.

$$m = m_A + m_B + m_C = 8074 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$F = F_A + F_B + F_C = 63,000 \text{ lb}$$

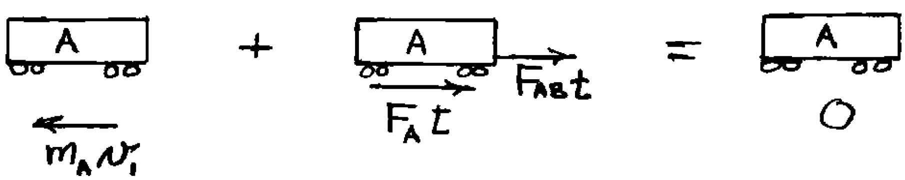
$$-mv_1 + Ft = mv_2$$

$$t = \frac{m(v_1 - v_2)}{F} = \frac{(8074)(44)}{63,000} = 5.639 \text{ s}$$

$t = 5.64 \text{ s}$ ◀

(b) Coupling force F_{AB}:

Apply the principle of impulse-momentum to car A alone.



$$-m_A v_1 + F_A t + F_{AB} t = 0$$

$$-(2484)(44) + 0 + F_{AB}(5.639) = 0$$

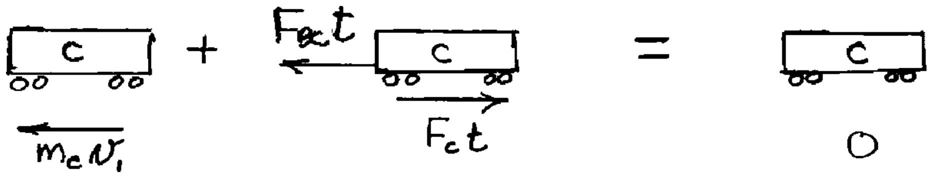
$$F_{AB} = 19,390 \text{ lb}$$

$F_{AB} = 19,390 \text{ lb (tension)}$ ◀

PROBLEM 13.129 (Continued)

Coupling force F_{BC}:

Apply the principle of impulse-momentum to car C alone.



$$-m_C v_1 + F_C t - F_{BC} t = 0$$

$$-(2484)(44) + (28000)(5.639) - F_{BC}(5.639) = 0$$

$$F_{BC} = 8620 \text{ lb}$$

$F_{BC} = 8620$ lb (tension) ◀

PROBLEM 13.130

Solve Problem 13.129 assuming that the brakes are applied only on the wheels of car *A*.

PROBLEM 13.129 The subway train shown is traveling at a speed of 30 mi/h when the brakes are fully applied on the wheels of cars *B* and *C*, causing them to slide on the track, but are not applied on the wheels of car *A*. Knowing that the coefficient of kinetic friction is 0.35 between the wheels and the track, determine (*a*) the time required to bring the train to a stop, (*b*) the force in each coupling.

SOLUTION

Weights of cars: $W_A = W_C = 80{,}000 \text{ lb}, \ W_B = 100{,}000 \text{ lb}$

Masses of cars: $m_A = m_C = 2484 \text{ lb}\cdot\text{s}^2/\text{ft}, \ m_B = 3106 \text{ lb}\cdot\text{s}^2/\text{ft}$

For each car the normal force (upward) is equal in magnitude to the weight of the car.

$$N_A = N_C = 80{,}000 \text{ lb} \qquad N_B = 100{,}000 \text{ lb}$$

Friction forces: $F_A = (0.35)(80{,}000) = 28{,}000 \text{ lb}$

$$\left.\begin{aligned} F_B &= 0 \\ F_C &= 0 \end{aligned}\right\} \text{(brakes not applied)}$$

Stopping data: $v_1 = 30 \text{ mi/h} = 44 \text{ ft/s}, \ v_2 = 0.$

(*a*) Apply the principle of impulse-momentum to the entire train.

$$m = m_A + m_B + m_C = 8074 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$F = F_A + F_B + F_C = 28{,}000 \text{ lb}$$

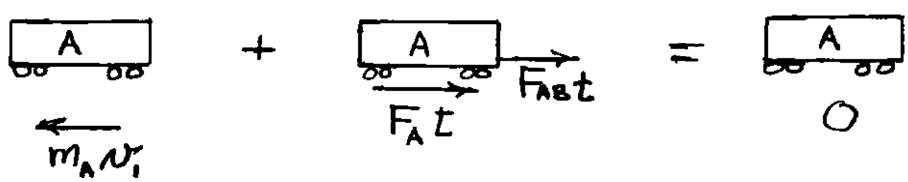
$$-mv_1 + \text{Ft} = mv_2$$

$$t = \frac{m(v_1 - v_2)}{F} = \frac{(8074)(44)}{28{,}000} = 12.688 \text{ s}$$

$t = 12.69 \text{ s}$ ◀

(*b*) Coupling force F_{AB}:

Apply the principle of impulse-momentum to car *A* alone.



$$-m_A v_1 + F_A t + F_{AB} t = 0$$

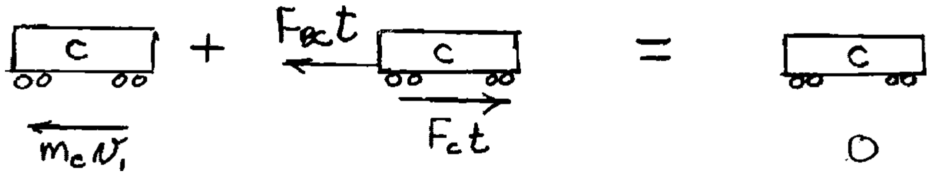
$$-(2484)(44) + (28{,}000)(12.688) + F_{AB}(12.688) = 0$$

$F_{AB} = -19{,}390 \text{ lb}$

$F_{AB} = 19{,}390 \text{ lb (compression)}$ ◀

Coupling force F_{BC}:

Apply the principle of impulse-momentum to car *C* alone.

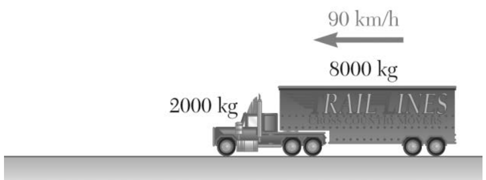


$$-m_C v_1 + F_C t - F_{BC} t = 0$$

$$-(2484)(44) + (0) - F_{BC}(12.688) = 0$$

$$F_{BC} = -8620 \text{ lb} \qquad F_{BC} = 8620 \text{ lb (compression)} \blacktriangleleft$$

PROBLEM 13.131

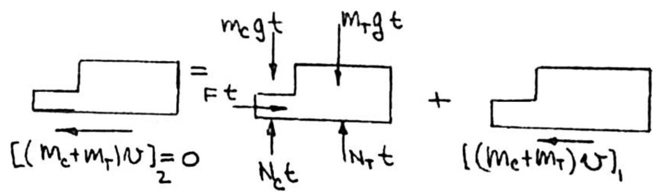


A trailer truck with a 2000-kg cab and an 8000-kg trailer is traveling on a level road at 90 km/h. The brakes on the trailer fail and the antiskid system of the cab provides the largest possible force which will not cause the wheels of the cab to slide. Knowing that the coefficient of static friction is 0.65, determine (*a*) the shortest time for the rig to come to a stop, (*b*) the force in the coupling during that time.

SOLUTION

$$v = 90 \text{ km/h} = 25 \text{ m/s}$$

(*a*) The shortest time for the rig to come to a stop will be when the friction force on the wheels is maximum. The downward force exerted by the trailer on the cab is assumed to be zero. Since the trailer brakes fail, all of the braking force is supplied by the wheels of the cab, which is maximum when the wheels of the cab are at impending sliding.



$$Ft_{1-2} = \mu_s N_C t_{1-2} \quad N_C = m_C g = (2000)g$$

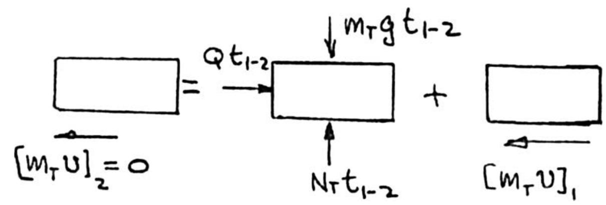
$$Ft_{1-2} = (0.65)(2000)gt$$

$$[(m_C + m_T)v]_2 = -Ft + [(m_C + m_T)v]_1$$

$$\overset{+}{\longleftarrow} \quad 0 = -(0.65)(2000 \text{ kg})(9.81 \text{ m/s}^2)(t_{1-2}) = 10{,}000 \text{ kg}(25 \text{ m/s})$$

$t_{1-2} = 19.60$ s ◀

(*b*) For the trailer:



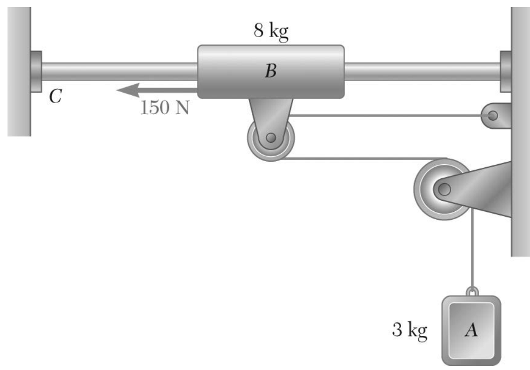
$$\overset{+}{\longleftarrow} \quad [m_T v]_2 = -Qt_{1-2} + [m_T v]_1$$

From (*a*), $t_{1-2} = 19.60$ s

$$0 = -Q(19.60 \text{ s}) + (8000 \text{ kg})(25 \text{ m/s})$$

$$Q = 10{,}204 \text{ N}$$

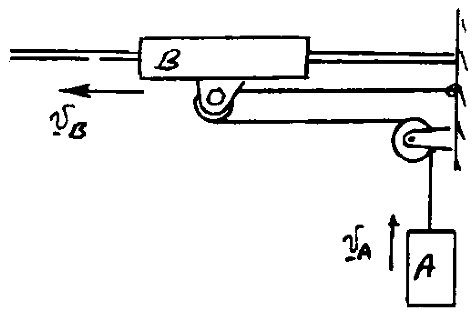
$Q = 10.20$ kN (compression) ◀



PROBLEM 13.132

The system shown is at rest when a constant 150-N force is applied to collar B. Neglecting the effect of friction, determine (a) the time at which the velocity of collar B will be 2.5 m/s to the left, (b) the corresponding tension in the cable.

SOLUTION



Constraint of cord. When the collar B moves 1 unit to the left, the weight A moves up 2 units. Thus

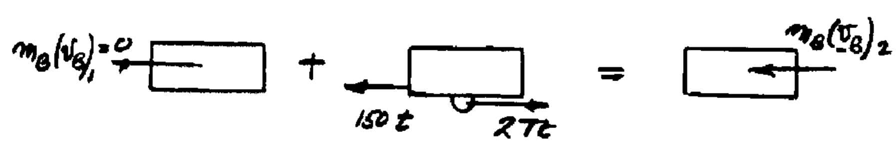
$$v_A = 2v_B \qquad v_B = \frac{1}{2}v_A$$

Masses and weights: $m_A = 3\text{ kg}$ $\qquad W_A = 29.43\text{ N}$

$m_B = 8\text{ kg}$

Let T be the tension in the cable.

Principle of impulse and momentum applied to collar B.

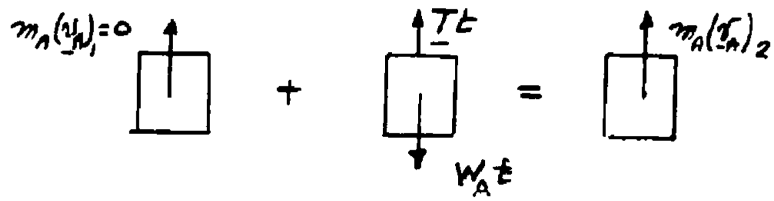


$$\overset{+}{\longleftarrow}: \; 0 + 150t - 2Tt = m_B(v_B)_2$$

For $(v_B)_2 = 2.5\text{ m/s}$ $\qquad 150t - 2Tt = (8\text{ kg})(2.5\text{ m/s})$

$$150t - 2Tt = 20 \qquad (1)$$

Principle of impulse and momentum applied to weight *A*.



$$+\uparrow: \quad 0 + Tt - W_A t = m_A (v_A)_2$$

$$Tt + W_A t = m_A (2V_{B2})$$

$$Tt - 29.43t = (3 \text{ kg})(2)(2.5 \text{ m/s})$$

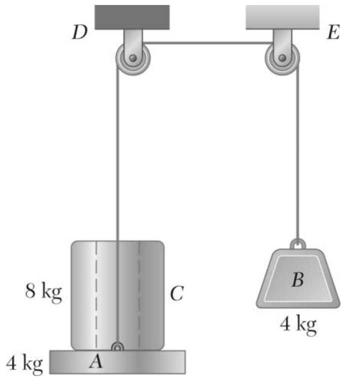
$$Tt - 29.43t = 15 \qquad (2)$$

To eliminate *T* multiply Eq. (2) by 2 and add to Eq. (1).

(*a*) Time: $91.14t = 50$ $\qquad t = 0.549 \text{ s}$ ◀

From Eq. (2), $T = \dfrac{15}{t} + 29.43$

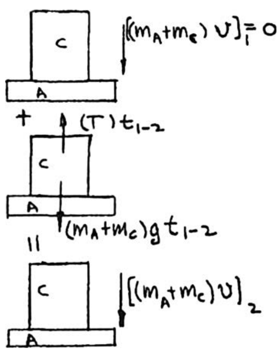
(*b*) Tension in the cable. $\qquad T = 56.8 \text{ N}$ ◀



PROBLEM 13.133

An 8-kg cylinder *C* rests on a 4-kg platform *A* supported by a cord which passes over the pulleys *D* and *E* and is attached to a 4-kg block *B*. Knowing that the system is released from rest, determine (*a*) the velocity of block *B* after 0.8 s, (*b*) the force exerted by the cylinder on the platform.

SOLUTION



(*a*) Blocks *A* and *C*:

$$\downarrow +$$

$$[(m_A + m_C)v]_1 - T(t_{1-2}) + (m_A + m_C)gt_{1-2} = [(m_A + m_C)v]_2$$

$$0 + (12g - T)(0.8) = 12v \qquad (1)$$

Block *B*:

$$\uparrow +$$

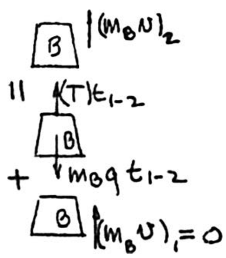
$$[m_B v]_1 + (T)t_{1-2} - m_B g t_{1-2} = (m_B v)_2$$

$$0 + (T - 4g)(0.8) = 4v \qquad (2)$$

Adding (1) and (2), (eliminating *T*)

$$(12g - 4g)(0.8) = (12 + 4)$$

$$v = \frac{(8\text{ kg})(9.81\text{ m/s}^2)(0.8\text{ s})}{16\text{ kg}} \qquad v = 3.92\text{ m/s} \blacktriangleleft$$



(*b*) Collar *A*:

$$\downarrow +$$

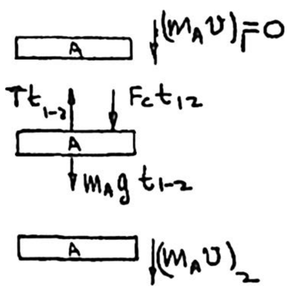
$$(m_A v)_1 = 0 \quad 0 + (F_C + m_A g) \qquad (3)$$

From Eq. (2) with $v = 3.92$ m/s

$$T = \frac{4v}{0.8} + 4g$$

$$T = \frac{(4\text{ kg})(3.92\text{ m/s})}{(0.8\text{ s})} + (4\text{ kg})(9.81\text{ m/s}^2)$$

$$T = 58.84\text{ N}$$

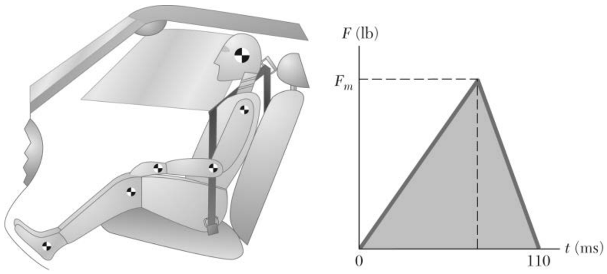


Solving for F_C in (3)

$$F_C = \frac{(4\text{ kg})(3.92\text{ m/s})}{(0.8\text{ s})} - (4\text{ kg})(9.81\text{ m/s}^2) + 58.84\text{ N}$$

$$F_C = 39.2\text{ N} \blacktriangleleft$$

PROBLEM 13.134

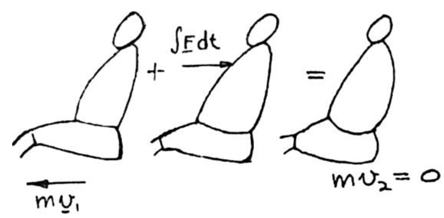


An estimate of the expected load on over-the-shoulder seat belts is to be made before designing prototype belts that will be evaluated in automobile crash tests. Assuming that an automobile traveling at 45 mi/h is brought to a stop in 110 ms, determine (*a*) the average impulsive force exerted by a 200-lb man on the belt, (*b*) the maximum force F_m exerted on the belt if the force-time diagram has the shape shown.

SOLUTION

(*a*) Force on the belt is opposite to the direction shown.

$$v_1 = 45 \text{ mi/h} = 66 \text{ ft/s},$$
$$W = 200 \text{ lb}$$



$$m\mathbf{v}_1 - \int \mathbf{F}dt = m\mathbf{v}_2$$

$$\int F dt = F_{\text{ave}}\Delta t$$

$$\Delta t = 0.110 \text{ s}$$

$$\frac{(200 \text{ lb})(66 \text{ ft/s})}{(32.2 \text{ ft/s}^2)} - F_{\text{ave}}(0.110 \text{ s}) = 0$$

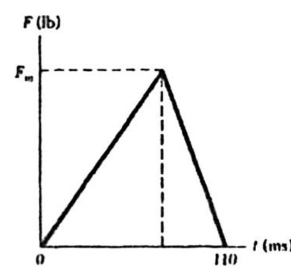
$$F_{\text{ave}} = \frac{(200)(66)}{(32.2)(0.110)} = 3727 \text{ lb}$$

$F_{\text{ave}} = 3730 \text{ lb}$ ◀

(*b*) Impulse = area under $F-t$ diagram $= \frac{1}{2}F_m(0.110 \text{ s})$

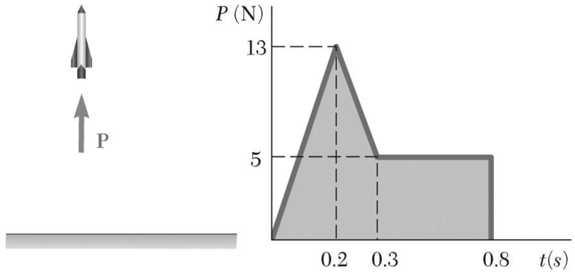
From (*a*), impulse $= F_{\text{ave}}\Delta t = (3727 \text{ lb})(0.110 \text{ s})$

$$\frac{1}{2}F_m(0.110) = (3727)(0.110)$$



$F_m = 7450 \text{ lb}$ ◀

PROBLEM 13.135



A 60-g model rocket is fired vertically. The engine applies a thrust **P** which varies in magnitude as shown. Neglecting air resistance and the change in mass of the rocket, determine (*a*) the maximum speed of the rocket as it goes up, (*b*) the time for the rocket to reach its maximum elevation.

SOLUTION

Mass: $m = 0.060$ kg

Weight: $mg = (0.060)(9.81) = 0.5886$ N

Forces acting on the model rocket:

Thrust: $P(t)$(given function of t)$\uparrow$

Weight: W (constant)$\downarrow$

Support: S (acts until $P > W$)$\uparrow$

Over $0 < t < 0.2$ s:

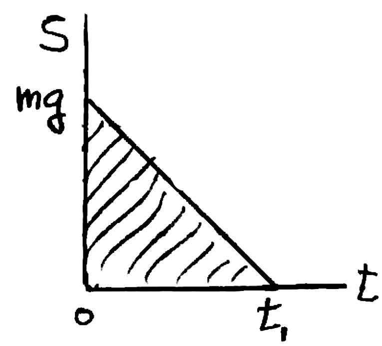
$$P = \frac{13}{0.2}t = 65t$$
$$W = 0.5886 \text{ N}$$

Before the rocket lifts off, $S = W - P = 0.5886 - 65t$

S become zero when $t = t_1$.

$$0 = 0.5886 - 65t_1 \qquad t_1 = 0.009055 \text{ s.}$$

Impulse due to S: $(t > t_1)$



$$\int_0^t Sdt = \int_0^{t_1} Sdt$$
$$= \frac{1}{2}\ mgt_1$$
$$= (0.5)(0.5886)(0.009055)$$
$$= 0.00266 \text{ N} \cdot \text{s}$$

The maximum speed occurs when $\frac{dv}{dt} = a = 0.$

At this time, $W - P = 0,$ which occurs at $t_2 = 0.8$ s.

(*a*) Maximum speed (upward motion):

Apply the principle of impulse-momentum to the rocket over $0 \le t \le t_2$.

$$\int_0^{0.8} Pdt = \text{area under the given thrust-time plot.}$$

$$= \frac{1}{2}(0.2)(13) + \frac{1}{2}(0.1)(13+5) + (0.8-0.3)(5)$$
$$= 4.7 \text{ N} \cdot \text{s}$$

$$\int_0^{0.8} Wdt = (0.5886)(0.8) = 0.47088 \text{ N} \cdot \text{S}$$

$$m_1 v_1 + \int_0^{0.8} Pdt + \int_0^{0.8} Sdt - \int_0^{0.8} Wdt = mv_2$$
$$0 + 4.7 + 0.00266 - 0.47088 = 0.060\ v_2$$

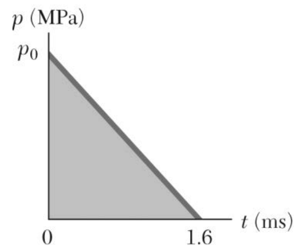
$v_2 = 70.5$ m/s ◀

(*b*) Time t_3 to reach maximum height: $(v_3 = 0)$

$$mv_1 + \int_0^{t_3} Pdt + \int_0^{t_3} Sdt - Wt_3 = mv_3$$

$$0 + 4.7 + 0.00266 - 0.5886 t_3 = 0$$

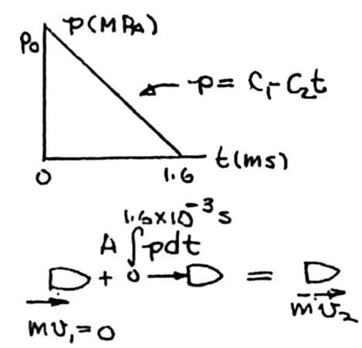
$t_3 = 7.99$ s ◀



PROBLEM 13.136

A simplified model consisting of a single straight line is to be obtained for the variation of pressure inside the 10-mm-diameter barrel of a rifle as a 20-g bullet is fired. Knowing that it takes 1.6 ms for the bullet to travel the length of the barrel and that the velocity of the bullet upon exit is 700 m/s, determine the value of p_0.

SOLUTION



At $t = 0$, $p = p_0 = c_1 - c_2 t$

$$c_1 = p_0$$

At $t = 1.6\times10^{-3}\,\text{s}$, $p = 0$

$$0 = c_1 - c_2(1.6\times10^{-3}\,\text{s})$$

$$c_2 = \frac{p_0}{1.6\times10^{-3}\,\text{s}}$$

$$m = 20\times10^{-3}\,\text{kg}$$

$$0 + A\int_0^{1.6\times10^{-3}\,\text{s}} p\,dt = mv_2$$

$$A = \frac{\pi(10\times10^{-3})^2}{4}$$

$$A = 78.54\times10^{-6}\,\text{m}^2$$

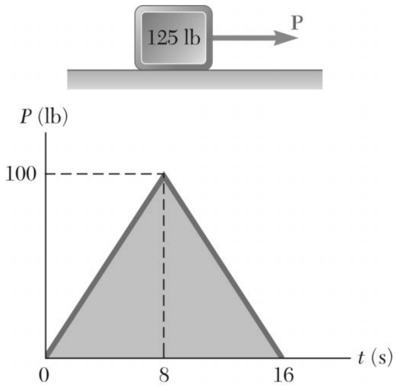
$$0 + A\int_0^{1.6\times10^{-3}\,\text{s}} (c_1 - c_2 t)\,dt = \frac{20\times10^{-3}}{g}$$

$$(78.54\times10^{-6}\,\text{m}^2)\left[(c_1)(1.6\times10^{-3}\,\text{s}) - \frac{(c_2)(1.6\times10^{-3}\,\text{s})^2}{2}\right] = (20\times10^{-3}\,\text{kg})(700\,\text{m/s})$$

$$1.6\times10^{-3}c_1 - 1.280\times10^{-6}c_2 = 178.25\times10^3$$

$$(1.6\times10^{-3}\,\text{m}^2\cdot\text{s})\,p_0 - \frac{(1.280\times10^{-6}\,\text{m}^2\cdot\text{s}^2)}{(1.6\times10^{-3}\,\text{s})}\,p_0 = 178.25\times10^3\,\text{kg}\cdot\text{m/s}$$

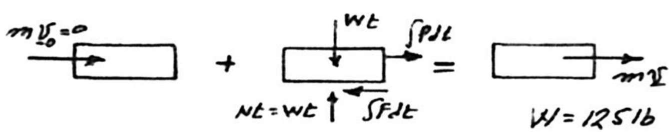
$$p_0 = 222.8\times10^6\,\text{N/m}^2 \qquad p_0 = 223\text{ MPa} \blacktriangleleft$$



PROBLEM 13.137

A 125-lb block initially at rest is acted upon by a force **P** which varies as shown. Knowing that the coefficients of friction between the block and the horizontal surface are $\mu_s = 0.50$ and $\mu_k = 0.40$, determine (*a*) the time at which the block will start moving, (*b*) the maximum speed reached by the block, (*c*) the time at which the block will stop moving.

SOLUTION

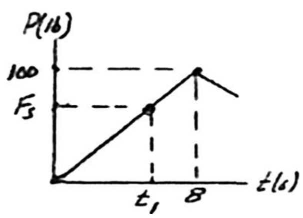


$$\xrightarrow{+} 0 + \int P dt - \int F dt = mv$$

At any time:

$$v = \frac{1}{m}\left[\int P dt - \int F dt\right] \qquad (1)$$

(*a*) Block starts moving at *t*.



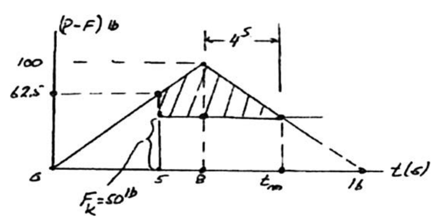
$$P = F_s = \mu_s W = (0.50)(125 \text{ lb}) = 62.5 \text{ lb}$$

$$\frac{t_1}{F_s} = \frac{8 \text{ s}}{100 \text{ lb}}; \quad \frac{t_1}{62.5 \text{ lb}} = \frac{8 \text{ s}}{100 \text{ lb}}$$

$t_1 = 5.00$ s ◀

(*b*) Maximum velocity: At $t = t_m$

where $P = F_k = \mu_k W = 0.4(125) = 50$ lb



Block moves at $t = 5$ s.

Shaded area is maximum net impulse $\int P dt - \int F_R dt$

when $t = t_{m1}$ $v = v_m$

Eq. (1):
$$v_m = \frac{1}{m}\begin{bmatrix}\text{shaded}\\\text{area}\end{bmatrix} = \frac{1}{m}\left[\frac{1}{2}(12.5+50)(3) + \frac{1}{2}(50)(4)\right] = \frac{1}{m}(193.75)$$

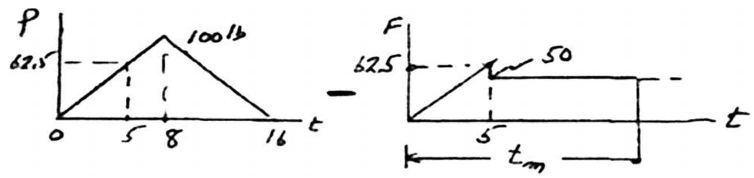
$$v_m = \frac{1}{\frac{125 \text{ lb}}{32.2}}[193.75] = 49.91 \text{ ft/s}$$

$\mathbf{v}_m = 49.9$ ft/s ⟶ ◀

PROBLEM 13.137 (Continued)

(*c*) Block stops moving when $\left[\int P dt - \int F dt\right] = 0$; or $\int Q dt = \int F dt$

Assume $t_m > 16$ s.

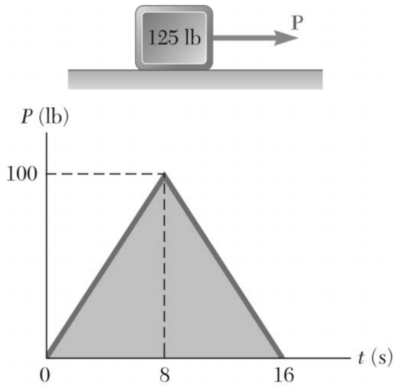


$$\int P dt = \frac{1}{2}(100)(16) = 800 \text{ lb} \cdot \text{s}$$

$$\int F dt = \frac{1}{2}(62.5)(5) + (50)(t_m - 5)$$

$$\int P dt - \int F dt = 800 - [156.25 + 50(t_m - 5)] = 0 \qquad t_m = 17.875 \text{ s}$$

$t_m > 16$ s OK $\qquad t_m = 17.88$ s ◀



PROBLEM 13.138

Solve Problem 13.137, assuming that the weight of the block is 175 lb.

PROBLEM 13.137 A 125-lb block initially at rest is acted upon by a force **P** which varies as shown. Knowing that the coefficients of friction between the block and the horizontal surface are $\mu_s = 0.50$ and $\mu_k = 0.40$, determine (*a*) the time at which the block will start moving, (*b*) the maximum speed reached by the block, (*c*) the time at which the block will stop moving.

SOLUTION

See solution of Problem 13.137.

$$W = 175 \text{ lb} \qquad v = \frac{1}{m}\left[\int P\,dt - \int F\,dt\right] \tag{1}$$

(*a*) Block starts moving: $P = F_s = \mu_s W = (0.50)(175) = 87.5 \text{ lb}$

See first figure of Problem 13.137.

$$\frac{t_1}{F_s} = \frac{8 \text{ s}}{100 \text{ lb}}; \quad \frac{t_1}{87.5 \text{ lb}} = \frac{8 \text{ s}}{100 \text{ s}}$$

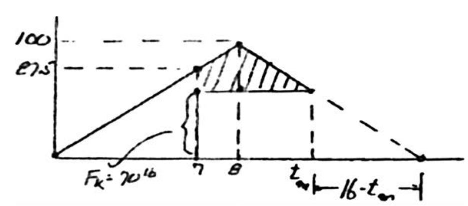
$t_1 = 7.00 \text{ s}$ ◀

(*b*) Maximum velocity: $P = F_k = \mu_k W = 0.4(175) = 70 \text{ lb}$

$$\frac{16 - t_m}{70 \text{ lb}} = \frac{8 \text{ s}}{100 \text{ lb}}$$

$$16 - t_m = 70\left(\frac{8}{100}\right) = 5.6$$

$$t_m = 10.40 \text{ s}$$



Eq. (1):

$$v_m = \frac{1}{m}\begin{bmatrix}\text{shaded}\\ \text{area}\end{bmatrix}$$

$$= \frac{1}{m}\left[\frac{1}{2}(17.5 + 30)(1.0) + \frac{1}{2}(30)(10.4 - 8)\right]$$

$$= \frac{1}{m}(59.75)$$

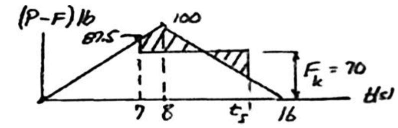
$$v_m = \frac{1}{\frac{175 \text{ lb}}{32.2}}[59.75]$$

$$= 10.994 \text{ ft/s}$$

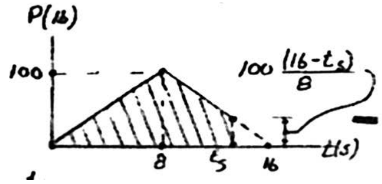
$\mathbf{v}_m = 10.99 \text{ ft/s} \longrightarrow$ ◀

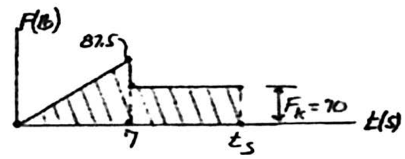
PROBLEM 13.138 (Continued)

(*c*) Block stops moving when net impulse $\left[\int (P-F)dt\right]=0$



Assume $t_s, < 16$ s.





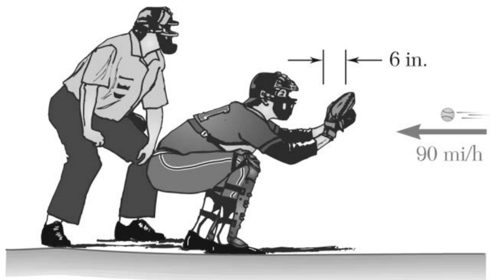
$$\int_0^{t_s} P dt = \frac{1}{2}(100)(8) + \frac{1}{2}\left[100 + 100\frac{(16-t_s)}{8}\right](t_s - 8)$$

$$= \frac{1}{2}(100)(16) - \frac{1}{2}\left(\frac{100}{8}\right)(16-t_s)^2$$

$$\int_0^{t_s} F dt = \frac{1}{2}(87.5)(7) + (70)(t_s - 7)$$

$$\int P dt - \int F dt = 800 - \frac{100}{16}(16-t_s)^2 - 306.25 - 70(t_s - 7) = 0$$

Solving for t_s, $t_s = 13.492$ s $t_s = 13.49$ s ◀



PROBLEM 13.139

A baseball player catching a ball can soften the impact by pulling his hand back. Assuming that a 5-oz ball reaches his glove at 90 mi/h and that the player pulls his hand back during the impact at an average speed of 30 ft/s over a distance of 6 in., bringing the ball to a stop, determine the average impulsive force exerted on the player's hand.

SOLUTION

$$v = 90 \text{ mi/h} = 132 \text{ ft/s}$$

$$m = \frac{5}{16}/\text{g}$$

$$t = \frac{d}{v_{av}} = \frac{\left(\frac{6}{12}\right)}{30} = \left(\frac{1}{60}\right) \text{ s}$$

$$\overset{+}{\longrightarrow} \quad 0 = F_{av}t + mv \qquad F_{av} = \frac{Wv}{gt}$$

$$F_{av} = \frac{mv}{t}$$

$$= \frac{\left(\frac{5}{16}\text{ lb}\right)(132 \text{ ft/s})}{(32.2 \text{ ft/s}^2)\left(\frac{1}{60}\text{ s}\right)}$$

$F_{av} = 76.9$ lb ◀

PROBLEM 13.140

A 1.62 ounce golf ball is hit with a golf club and leaves it with a velocity of 100 mi/h. We assume that for $0 \le t \le t_0$, where t_0 is the duration of the impact, the magnitude F of the force exerted on the ball can be expressed as $F = F_m \sin(\pi t/t_0)$. Knowing that $t_0 = 0.5$ ms, determine the maximum value F_m of the force exerted on the ball.

SOLUTION

$$W = 1.62 \text{ ounces} = 0.10125 \text{ lb} \quad m = 3.1444\times10^{-3} \text{ slug}$$

$$t = 0.5 \text{ ms} = 0.5\times10^{-3}\text{s}$$

$$v = 100 \text{ mi/h} = 146.67 \text{ ft/s}$$

The impulse applied to the ball is

$$\int_0^{t_0} F dt = \int_0^{t_0} F_m \sin\frac{\pi t}{t_0} dt = -\frac{F_m t_0}{\pi}\cos\frac{\pi t}{t_0}\Big|_0^{t_0}$$

$$= -\frac{F_m t_0}{\pi}(\cos\pi - \cos 0) = \frac{2F_m t_0}{\pi}$$

Principle of impulse and momentum.

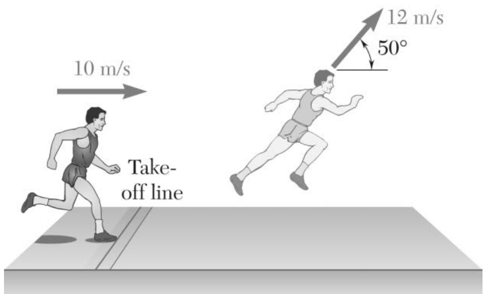
$$m\mathbf{v}_1 + \int_0^{t_0} \mathbf{F}\, dt = m\mathbf{v}_2$$

with $\mathbf{v}_1 = 0$,

$$0 + \frac{2F_m t_0}{\pi} = mv_2$$

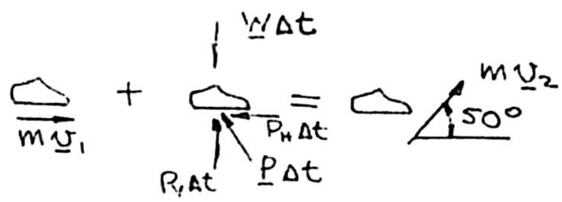
Solving for F_m, $\quad F_m = \dfrac{\pi m v_2}{2t_0} = \dfrac{\pi(3.1444\times10^{-3})(146.67)}{(2)(0.5\times10^{-3})} = 1.4488\times10^3 \text{lb}$ $\qquad F_m = 1.45$ kip ◀

PROBLEM 13.141



The triple jump is a track-and-field event in which an athlete gets a running start and tries to leap as far as he can with a hop, step, and jump. Shown in the figure is the initial hop of the athlete. Assuming that he approaches the takeoff line from the left with a horizontal velocity of 10 m/s, remains in contact with the ground for 0.18 s, and takes off at a 50° angle with a velocity of 12 m/s, determine the vertical component of the average impulsive force exerted by the ground on his foot. Give your answer in terms of the weight *W* of the athlete.

SOLUTION



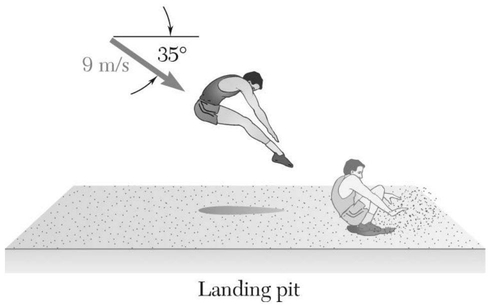
$$m\mathbf{v}_1 + (\mathbf{P} - \mathbf{W})\Delta t = m\mathbf{v}_2 \qquad \Delta t = 0.18 \text{ s}$$

Vertical components

$$0 + (P_v - W)(0.18) = \frac{W}{g}(12)(\sin 50°)$$

$$P_v = W + \frac{(12)(\sin 50°)}{(9.81)(0.18)} W$$

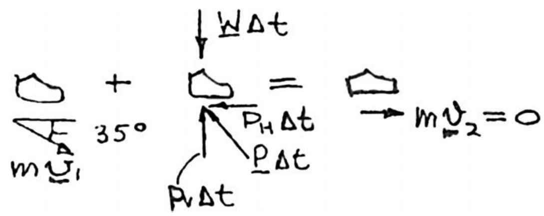
$$P_v = 6.21W \blacktriangleleft$$



PROBLEM 13.142

The last segment of the triple jump track-and-field event is the jump, in which the athlete makes a final leap, landing in a sand-filled pit. Assuming that the velocity of a 80-kg athlete just before landing is 9 m/s at an angle of 35° with the horizontal and that the athlete comes to a complete stop in 0.22 s after landing, determine the horizontal component of the average impulsive force exerted on his feet during landing.

SOLUTION



$$m = 80 \text{ kg}$$
$$\Delta t = 0.22 \text{ s}$$

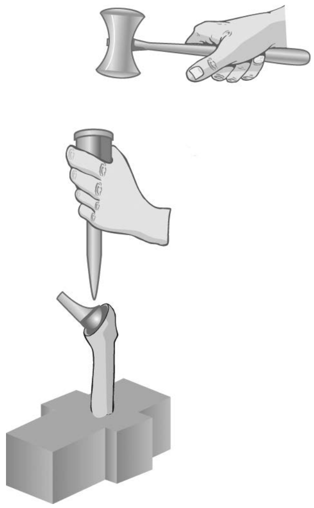
$$m\mathbf{v}_1 + (\mathbf{P} - \mathbf{W})\Delta t = m\mathbf{v}_2$$

Horizontal components

$$m(9)(\cos 35°) - P_H(0.22) = 0$$

$$P_H = \frac{(80 \text{ kg})(9 \text{ m/s})(\cos 35°)}{(0.22 \text{ s})} = 2.6809 \text{ kN}$$

$P_H = 2.68$ kN ◀



PROBLEM 13.143

The design for a new cementless hip implant is to be studied using an instrumented implant and a fixed simulated femur. Assuming the punch applies an average force of 2 kN over a time of 2 ms to the 200 g implant determine (*a*) the velocity of the implant immediately after impact, (*b*) the average resistance of the implant to penetration if the implant moves 1 mm before coming to rest.

SOLUTION

$$m = 200 \text{ g} = 0.200 \text{ kg}$$

$$F_{\text{ave}} = 2 \text{ kN} = 2000 \text{ N}$$

$$\Delta t = 2 \text{ ms} = 0.002 \text{ s}$$

(*a*) Velocity immediately after impact:

Use principle of impulse and momentum:

$$v_1 = 0 \qquad v_2 = ? \qquad \text{Imp}_{1\to 2} = F_{\text{ave}}(\Delta t)$$

$$m\mathbf{v}_1 + \mathbf{Imp}_{1\to 2} = m\mathbf{v}_2$$

$$0 + F_{\text{ave}}(\Delta t) = mv_2$$

$$v_2 = \frac{F_{\text{ave}}(\Delta t)}{m} = \frac{(2000)(0.002)}{0.200}$$

$v_2 = 20.0$ m/s ◀

(*b*) Average resistance to penetration:

$$\Delta x = 1 \text{ mm} = 0.001 \text{ m}$$

$$v_2 = 20.0 \text{ ft/s}$$

$$v_3 = 0$$

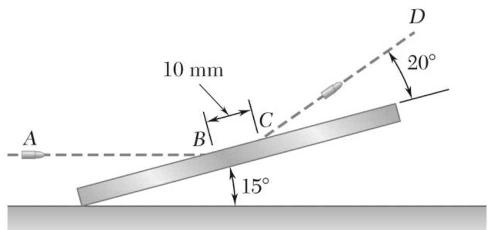
Use principle of work and energy.

$$T_2 + U_{2\to 3} = T_3 \qquad \text{or} \qquad \frac{1}{2}mv_2^2 - R_{\text{ave}}(\Delta x) = 0$$

$$R_{\text{ave}} = \frac{mv_2^2}{2(\Delta x)} = \frac{(0.200)(20.0)^2}{(2)(0.001)} = 40\times 10^3 \text{ N}$$

$R_{\text{ave}} = 40.0$ kN ◀

PROBLEM 13.144

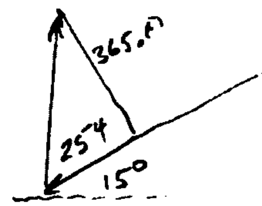


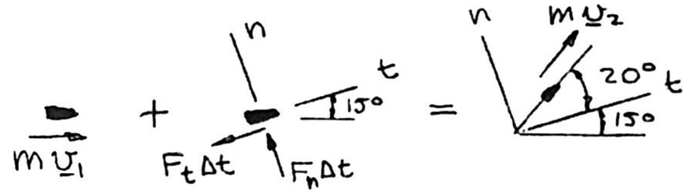
A 25-g steel-jacketed bullet is fired horizontally with a velocity of 600 m/s and ricochets off a steel plate along the path *CD* with a velocity of 400 m/s. Knowing that the bullet leaves a 10-mm scratch on the plate and assuming that its average speed is 500 m/s while it is in contact with the plate, determine the magnitude and direction of the average impulsive force exerted by the bullet on the plate.

SOLUTION

Impulse and momentum.

Bullet alone:





$$m\mathbf{v}_1 + \mathbf{F}\,\Delta t = m\mathbf{v}_2$$

t direction: $mv_1\cos 15° - F_t\Delta t = mv_2\cos 20°$

$$\nearrow + \mathbf{F}_t\Delta t = (0.025\text{ kg})[600\text{ m/s}\cos 15° - 400\text{ m/s}\cos 20°] = 5.092\text{ kg}\cdot\text{m/s}$$

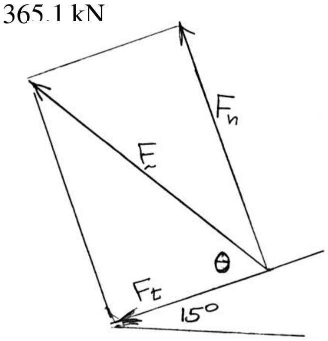
$$\Delta t = \frac{S_{BC}}{v_{AV}} = \frac{0.010\text{ m}}{500\text{ m/s}} = 20\times10^{-6}\text{ s}$$

$$\mathbf{F}_t = (5.092\text{ kg}\cdot\text{m/s})/(20\times10^{-6}\text{ s}) = 254.6\times10^3\text{ kg}\cdot\text{m/s}^2 = 254.6\text{ kN}$$

n direction: $\nwarrow^{+} -mv_1\sin 15° + F_n\Delta t = mv_2\sin 20°$

$$\mathbf{F}_n\Delta t = (0.025\text{ kg})[600\text{ m/s}\sin 15° + 400\text{ m/s}\sin 20°] = 7.3025\text{ kg}\cdot\text{m/s}$$

$$\mathbf{F}_n = (43025\text{ kg}\cdot\text{m/s})/(20\times10^{-6}) = 365.1\times10^3\text{ kg}\cdot\text{m/s}^2 = 365.1\text{ kN}$$



Force on bullet: $F = \sqrt{F_n^2 + F_t^2} = \sqrt{365.1^2 + 254.6^2} = 445\text{ kN}$

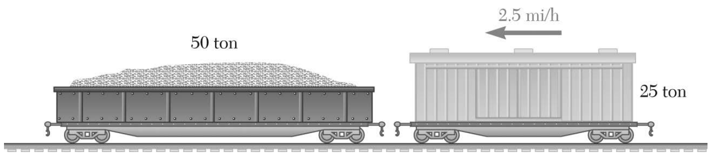
$$\tan\theta = \frac{F_n}{F_t} = \frac{365.1}{254.6} \qquad \theta = 55.1°$$

$$\theta - 15° = 40.1°$$

$$\mathbf{F} = 445\text{ kN} \searrow 40.1°$$

Force on plate: $\mathbf{F}' = -\mathbf{F}$

$\mathbf{F}' = 445\text{ kN} \nwarrow 40.1°$ ◀



PROBLEM 13.145

A 25-ton railroad car moving at 2.5 mi/h is to be coupled to a 50 ton car which is at rest with locked wheels ($\mu_k = 0.30$). Determine (*a*) the velocity of both cars after the coupling is completed, (*b*) the time it takes for both cars to come to rest.

SOLUTION

Weight and mass: (Label cars *A* and *B*.)

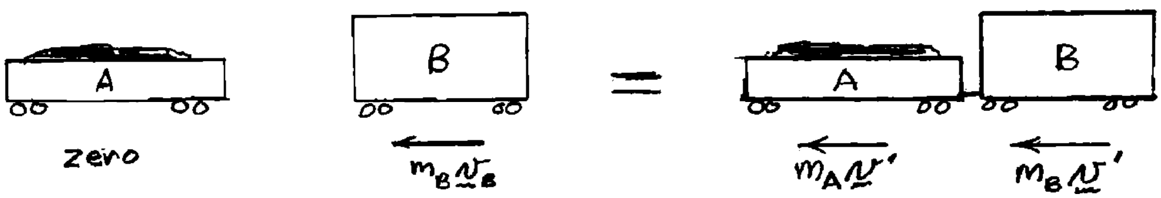
$$\text{Car } A: \quad W_A = 50 \text{ tons} = 100{,}000 \text{ lb}, \quad m_A = 3106 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\text{Car } B: \quad W_B = 25 \text{ tons} = 50{,}000 \text{ lb}, \quad m_B = 1553 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Initital velocities: $\mathbf{v}_A = 0$

$$v_B = 2.5 \text{ mi/h} = 3.6667 \text{ ft/s} \qquad \mathbf{v}_B = 3.6667 \text{ ft/s} \longleftarrow$$

(*a*) The momentum of the system consisting of the two cars is conserved immediately before and after coupling.



Let $\mathbf{v}'$ be the common velocity of that cars immediately after coupling. Apply conservation of momentum.

$$+\longleftarrow: \; m_B v_B = m_A v' + m_B v'$$

$$v' = \frac{m_B v_B}{m_A + m_B} = \frac{(3106)(3.6667)}{4569} = 2.444 \text{ ft/s} \qquad \mathbf{v}' = 1.667 \text{ mi/h} \longleftarrow \blacktriangleleft$$

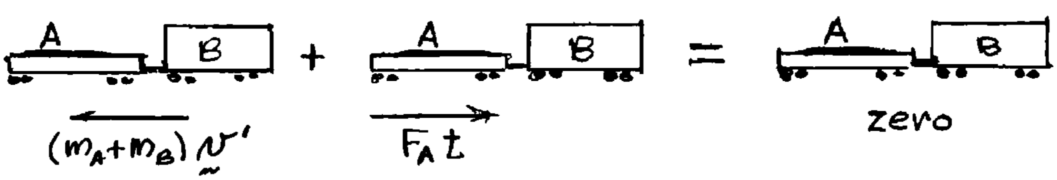
(*b*) After coupling: The friction force acts only on car *A*.

$$+\uparrow \Sigma F = 0_A: \quad N_A - W_A = 0 \qquad N_A = W_A$$

$$F_A = \mu_k N_A = \mu_k W_A \quad \text{(sliding)}$$

$$F_B = 0 \quad (\text{Car } B \text{ is rolling.})$$

Apply impuslse-momentum to the coupled cars.



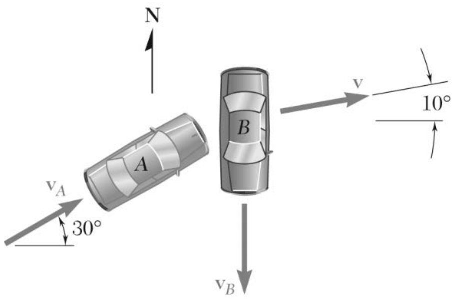
PROBLEM 13.145 (Continued)

$$\xrightarrow{+}: -(m_A + m_B)v' + F_A t = 0$$

$$t = \frac{(m_A + m_B)v_1^1}{F_A} = \frac{m_B v_B}{\mu_k W_A}$$

$$t = \frac{(1553)(3.6667)}{(0.30)(100,000)} = 0.1898$$

$t = 0.190 \text{ s}$ ◀

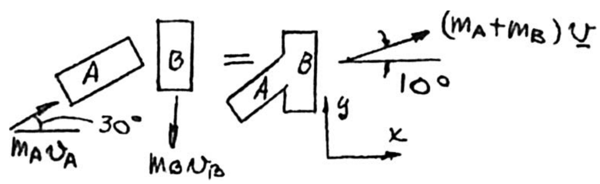


PROBLEM 13.146

At an intersection car B was traveling south and car A was traveling 30° north of east when they slammed into each other. Upon investigation it was found that after the crash the two cars got stuck and skidded off at an angle of 10° north of east. Each driver claimed that he was going at the speed limit of 50 km/h and that he tried to slow down but couldn't avoid the crash because the other driver was going a lot faster. Knowing that the masses of cars A and B were 1500 kg and 1200 kg, respectively, determine (a) which car was going faster, (b) the speed of the faster of the two cars if the slower car was traveling at the speed limit.

SOLUTION

(a) Total momentum of the two cars is conserved.



$$\Sigma mv, x: \quad m_A v_A \cos 30° = (m_A + m_B) v \cos 10° \tag{1}$$

$$\Sigma mv, y: \quad m_A v_A \sin 30° - m_B v_B = (m_A + m_B) v \sin 10° \tag{2}$$

Dividing (1) into (2),

$$\frac{\sin 30°}{\cos 30°} - \frac{m_B v_B}{m_A v_A \cos 30°} = \frac{\sin 10°}{\cos 10°}$$

$$\frac{v_B}{v_A} = \frac{(\tan 30° - \tan 10°)(m_A \cos 30°)}{m_B}$$

$$\frac{v_B}{v_A} = (0.4010)\frac{(1500)}{(1200)}\cos 30°$$

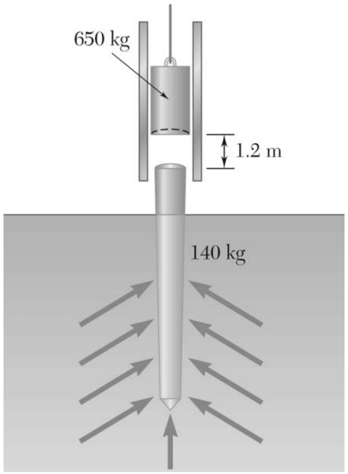
$$\frac{v_B}{v_A} = 0.434 \qquad v_A = 2.30\, v_B$$

Thus, A was going faster. ◀

(b) Since v_B was the slower car.

$$v_B = 50 \text{ km/h}$$

$$v_A = (2.30)(50) \qquad v_A = 115.2 \text{ km/h} ◀$$



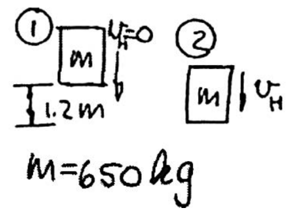
PROBLEM 13.147

The 650-kg hammer of a drop-hammer pile driver falls from a height of 1.2 m onto the top of a 140-kg pile, driving it 110 mm into the ground. Assuming perfectly plastic impact $(e = 0)$, determine the average resistance of the ground to penetration.

SOLUTION

Velocity of the hammer at impact:

Conservation of energy.



$$T_1 = 0$$

$$V_H = mg\ (1.2\text{ m})$$

$$V_H = (650\text{ kg})(9.81\text{ m/s}^2)(1.2\text{ m})$$

$$V_1 = 7652\text{ J}$$

$$T_2 = \frac{1}{2}m$$

$$V_H^2 = \frac{650}{2}v^2 = 325v_H^2$$

$$V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 7652 = 325v^2$$

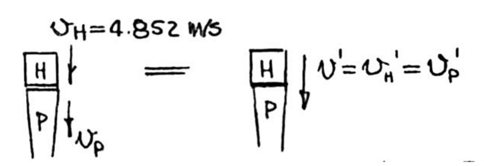
$$v^2 = 23.54\text{ m}^2/\text{s}^2$$

$$v = 4.852\text{ m/s}$$

Velocity of pile after impact:

Since the impact is plastic $(e = 0)$, the velocity of the pile and hammer are the same after impact.

Conservation of momentum:



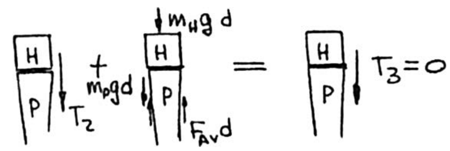
The ground reaction and the weights are non-impulsive.

PROBLEM 13.147 (Continued)

Thus, $m_H v_H = (m_H + m_p)v'$

$$v' = \frac{m_H v_H}{(m_H + m_p)} = \frac{(650)}{(650+140)}(4.852 \text{ m/s}) = 3.992 \text{ m/s}$$

Work and energy: $d = 0.110 \text{ m}$



$$T_2 + U_{2-3} = T_3$$

$$T_2 = \frac{1}{2}(m_H + m_H)(v')^2$$

$$T_3 = 0$$

$$T_2 = \frac{1}{2}(650+140)(3.992)^2$$

$$T_2 = 6.295\times10^3 \text{ J}$$

$$U_{2-3} = (m_H + m_p)gd - F_{AV}d$$

$$= (650+140)(9.81)(0.110) - F_{AV}(0.110)$$

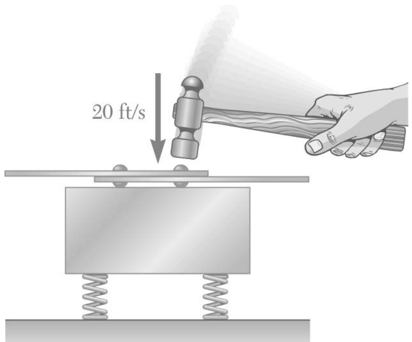
$$U_{2-3} = 852.49 - (0.110)F_{AV}$$

$$T_2 + U_{2-3} = T_3$$

$$6.295\times10^3 + 852.49 - (0.110)F_{AV} = 0$$

$$F_{AV} = (7147.5)/(0.110) = 64.98\times10^3 \text{ N}$$

$F_{AV} = 65.0 \text{ kN}$ ◀



PROBLEM 13.148

A small rivet connecting two pieces of sheet metal is being clinched by hammering. Determine the impulse exerted on the rivet and the energy absorbed by the rivet under each blow, knowing that the head of the hammer has a weight of 1.5 lbs and that it strikes the rivet with a velocity of 20 ft/s. Assume that the hammer does not rebound and that the anvil is supported by springs and (*a*) has an infinite mass (rigid support), (*b*) has a weight of 9 lb.

SOLUTION

Weight and mass:

$$\text{Hammer: } W_H = 1.5 \text{ lb} \qquad m_H = 0.04658 \text{ lb}\cdot\text{s}^2/\text{ft}$$

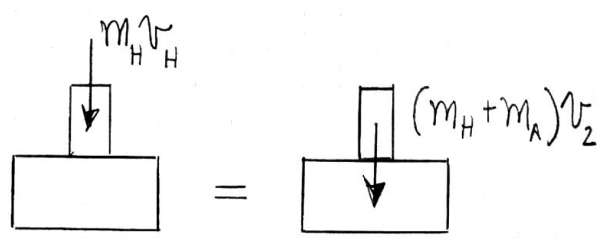
$$\text{Anvil: Part } a\text{: } W_A = \infty \qquad m_A = \infty$$

$$\text{Part } b\text{: } W_A = 9 \text{ lb} \qquad m_A = 0.2795 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Kinetic energy before impact:

$$T_1 = \frac{1}{2}m_H v_H^2 = \frac{1}{2}(0.04658)(20)^2 = 9.316 \text{ ft}\cdot\text{lb}$$

Let v_2 be the velocity common to the hammer and anvil immediately after impact. Apply the principle of conservation of momentum to the hammer and anvil over the duration of the impact.



$$+\downarrow: \Sigma m v_1 = \Sigma m v_2$$

$$m_H v_H = (m_H + m_A)v_2$$

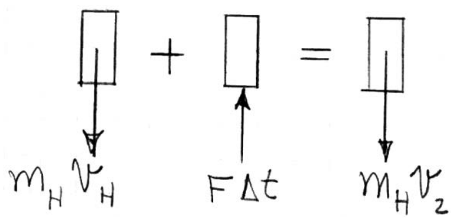
$$v_2 = \frac{m_H v_H}{m_H + m_A} \qquad (1)$$

Kinetic energy after impact:

$$T_A = \frac{1}{2}(m_H + m_A)v_2^2 = \frac{1}{2}\frac{m_H^2 v_H^2}{m_H + m_A}$$

$$T_2 = \frac{m_H}{m_H + m_A}T_1 \qquad (2)$$

Impulse exerted on the hammer:



$$+\downarrow: m_H v_H - F(\Delta t) = m_H v_2$$

$$F\Delta t = m_H(v_H - v_2) \qquad (3)$$

(*a*) $W_A = \infty$:

By Eq. (1), $v_2 = 0$

By Eq. (2), $T_2 = 0$

Energy absorbed: $T_1 - T_2 = 9.32 \text{ ft}\cdot\text{lb}$ ◀

By Eq. (3), $F(\Delta t) = (0.04658)(20 - 0) = 0.932 \text{ lb}\cdot\text{s}$

The impulse exerted on the rivet the same magnitude but opposite to direction.

$F\Delta t = 0.932 \text{ lb}\cdot\text{s}$ ◀

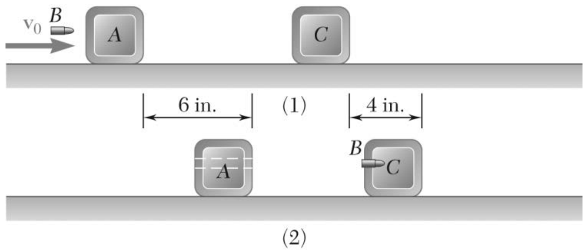
(*b*) $W_A = 9$ lb:

By Eq. (1), $v_2 = \dfrac{(0.04658)(20)}{0.32608} = 2.857 \text{ ft/s}$

By Eq. (2), $T_2 = \dfrac{(0.04658)(9.316)}{0.32608} = 1.331 \text{ ft}\cdot\text{lb}$

Energy absorbed: $T_1 - T_2 = 7.99 \text{ ft}\cdot\text{lb}$ ◀

By Eq. (3), $F(\Delta t) = (0.04658)(20 - 2.857)$ $F(\Delta t) = 0.799 \text{ lb}\cdot\text{s}$ ◀



PROBLEM 13.149

Bullet B weighs 0.5 oz and blocks A and C both weigh 3 lb. The coefficient of friction between the blocks and the plane is $\mu_k = 0.25$. Initially the bullet is moving at v_0 and blocks A and C are at rest (Figure 1). After the bullet passes through A it becomes embedded in block C and all three objects come to stop in the positions shown (Figure 2). Determine the initial speed of the bullet v_0.

SOLUTION

Masses:

Bullet: $$m_B = \frac{0.5}{(16)(32.2)} = 970.5 \times 10^{-6}\,\text{lb}\cdot\text{s}^2/\text{ft}$$

Blocks A and C: $$m_A = m_C = \frac{3}{32.2} = 93.168 \times 10^{-3}\,\text{lb}\cdot\text{s}^2/\text{ft}$$

Block C + bullet: $$m_C + m_B = 94.138 \times 10^{-3}\,\text{lb}\cdot\text{s}^2/\text{ft}$$

Normal forces for sliding blocks from $N - mg = 0$

Block A: $$N_A = m_A\,g = 3.00\text{ lb.}$$

Block C + bullet: $$N_C = (m_C + m_B)g = 3.03125\text{ lb.}$$

Let v_0 be the initial speed of the bullet;

v_1 be the speed of the bullet after it passes through block A;

v_A be the speed of block A immediately after the bullet passes through it;

v_C be the speed block C immediately after the bullet becomes embedded in it.

Four separate processes and their governing equations are described below.

1. The bullet hits block A and passes through it. Use the principle of conservation of momentum.

$$(v_A)_0 = 0$$

$$m_B v_0 + m_A (v_A)_0 = m_B v_1 + m_A v_A$$

$$v_0 = v_1 + \frac{m_A v_A}{m_B} \qquad (1)$$

2. The bullet hits block C and becomes embedded in it. Use the principle of conservation of momentum.

$$(v_C)_0 = 0$$

$$m_B v_1 + m_C (v_C)_0 = (m_B + m_C) v_C$$

$$v_1 = \frac{(m_B + m_C) v_C}{m_B} \qquad (2)$$

3. Block A slides on the plane. Use principle of work and energy.

$$T_1 + U_{1\to 2} = T_2$$

$$\frac{1}{2}m_A v_A^2 - \mu_k N_A d_A = 0 \quad \text{or} \quad v_A = \sqrt{\frac{2\mu_k N_A d_A}{m_A}} \tag{3}$$

4. Block C with embedded bullet slides on the plane. Use principle of work and energy.

$$d_C = 4 \text{ in.} = 0.33333 \text{ ft}$$

$$T_1 + U_{1\to 2} = T_2$$

$$\frac{1}{2}(m_C + m_B)v_C^2 - \mu_k N_C d_C = 0 \quad \text{or} \quad v_C = \sqrt{\frac{2\mu_k N_C d_C}{m_C + m_B}} \tag{4}$$

Applying the numerical data:

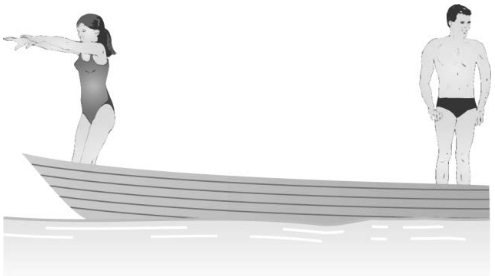
From Eq. (4),
$$v_C = \sqrt{\frac{(2)(0.25)(3.03125)(0.33333)}{94.138\times 10^{-3}}}$$
$$= 2.3166 \text{ ft/s}$$

From Eq. (3),
$$v_A = \sqrt{\frac{(2)(0.25)(3.00)(0.5)}{93.168\times 10^{-3}}}$$
$$= 2.8372 \text{ ft/s}$$

From Eq. (2),
$$v_1 = \frac{(94.138\times 10^{-3})(2.3166)}{970.5\times 10^{-6}}$$
$$= 224.71 \text{ ft/s}$$

From Eq. (1),
$$v_0 = 224.71 + \frac{(93.138\times 10^{-3})(2.8372)}{970.5\times 10^{-6}}$$

$v_0 = 497$ ft/s ◀



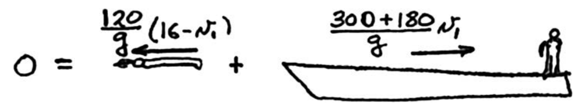
PROBLEM 13.150

A 180-lb man and a 120-lb woman stand at opposite ends of a 300-lb boat, ready to dive, each with a 16-ft/s velocity relative to the boat. Determine the velocity of the boat after they have both dived, if (*a*) the woman dives first, (*b*) the man dives first.

SOLUTION

(*a*) Woman dives first:

Conservation of momentum:



$$-\frac{120}{g}(16 - v_1) + \frac{300+180}{g}v_1 = 0$$

$$v_1 = \frac{(120)(16)}{600} = 3.20 \text{ ft/s} \longrightarrow$$

Man dives next. Conservation of momentum:

$$\frac{300+180}{g}v_1 = -\frac{300}{g}v_2 + \frac{180}{g}(16 - v_2)$$

$$v_2 = \frac{480v_1 - (180)(16)}{480} = 2.80 \text{ ft/s} \qquad \mathbf{v}_2 = 2.80 \text{ ft/s} \longleftarrow \blacktriangleleft$$

(*b*) Man dives first:

Conservation of momentum:

$$\frac{180}{g}(16 - v_1') - \frac{300+120}{g}v_1' = 0$$

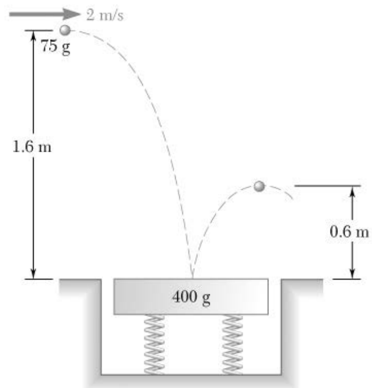
$$v_1' = \frac{(180)(16)}{600} = 4.80 \text{ ft/s} \longleftarrow$$

Woman dives next. Conservation of momentum:

$$-\frac{300+120}{g}v_1' = \frac{300}{g}v_2' + \frac{120}{g}(16 - v_2')$$

$$v_2' = \frac{-420v_1' + (120)(16)}{420} = -0.229 \text{ ft/s}$$

$$\mathbf{v}_2' = 0.229 \text{ ft/s} \longleftarrow \blacktriangleleft$$

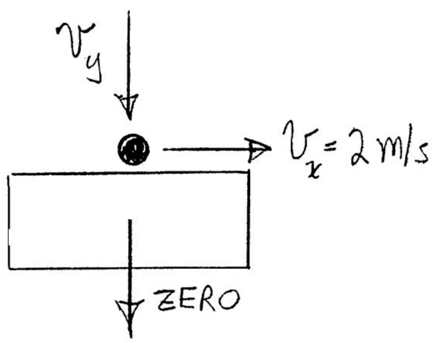


PROBLEM 13.151

A 75-g ball is projected from a height of 1.6 m with a horizontal velocity of 2 m/s and bounces from a 400-g smooth plate supported by springs. Knowing that the height of the rebound is 0.6 m, determine (*a*) the velocity of the plate immediately after the impact, (*b*) the energy lost due to the impact.

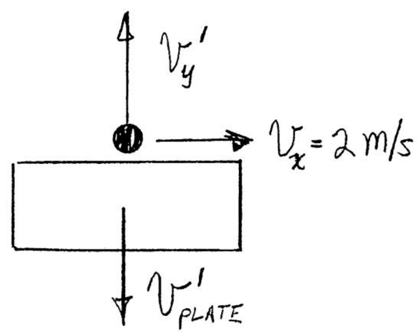
SOLUTION

Just before impact



$$v_y = \sqrt{2g(1.6)} = 5.603 \text{ m/s}$$

Just after impact



$$v_y = \sqrt{2g(0.6)} = 3.431 \text{ m/s}$$

(*a*) Conservation of momentum: $(+y\downarrow)$

$$m_{\text{ball}}v_y + 0 = -m_{\text{ball}}v_y' + m_{\text{plate}}v_{\text{plate}}'$$

$$(0.075)(5.603) + 0 = -0.075(3.431) + 0.4v_{\text{plate}}'$$

$$v_{\text{plate}}' = 1.694 \text{ m/s} \downarrow \blacktriangleleft$$

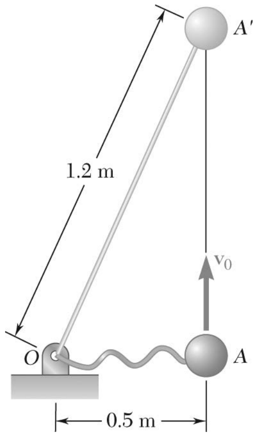
(*b*) Energy loss

Initial energy $\quad (T+V)_1 = \frac{1}{2}(0.075)(2)^2 + 0.075g(1.6)$

Final energy $\quad (T+V)_2 = \frac{1}{2}(0.075)(2)^2 + 0.075g(0.6) + \frac{1}{2}(0.4)(1.694)^2$

$$\text{Energy lost} = (1.3272 - 1.1653)\,\text{J} = 0.1619\,\text{J} \blacktriangleleft$$

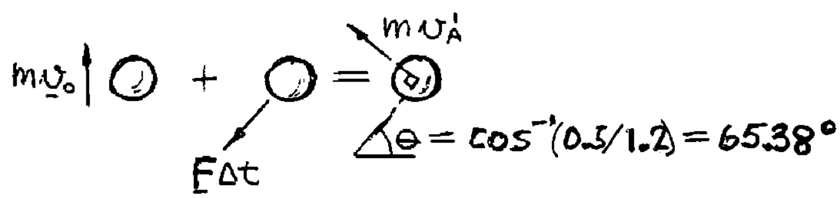
PROBLEM 13.152



A 2-kg sphere A is connected to a fixed Point O by an inextensible cord of length 1.2 m. The sphere is resting on a frictionless horizontal surface at a distance of 1.5 ft from O when it is given a velocity $\mathbf{v}_0$ in a direction perpendicular to line OA. It moves freely until it reaches position A', when the cord becomes taut. Determine the maximum allowable velocity $\mathbf{v}_0$ if the impulse of the force exerted on the cord is not to exceed 3 N·s.

SOLUTION

For the sphere at A' immediately before and after the cord becomes taut



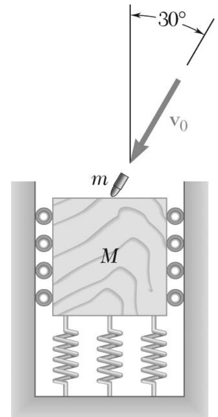
$$mv_0 + F\Delta t = mv_{A'}$$

$$\nearrow^{+}_{\theta} \quad mv_0 \sin\theta - F\Delta t = 0 \qquad F\Delta t = 3 \text{ N} \cdot \text{s}$$

$$m = 2 \text{ kg}$$

$$2(\sin 65.38°)v_0 = 3$$

$$v_0 = 1.650 \text{ m/s} \blacktriangleleft$$



PROBLEM 13.153

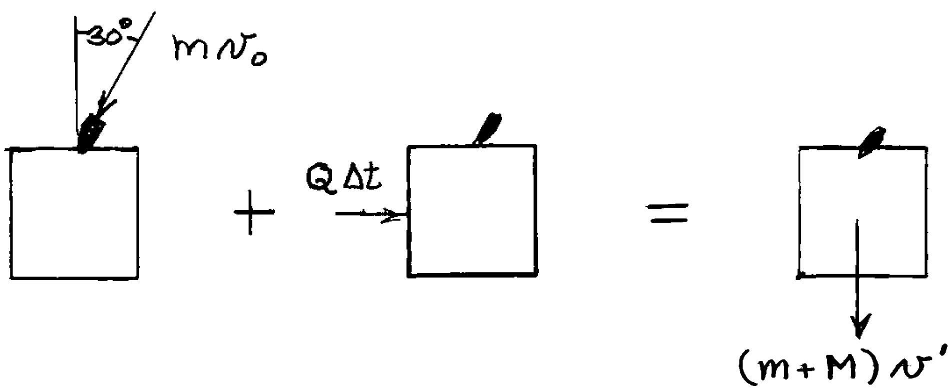
A 1-oz bullet is traveling with a velocity of 1400 ft/s when it impacts and becomes embedded in a 5-lb wooden block. The block can move vertically without friction. Determine (*a*) the velocity of the bullet and block immediately after the impact, (*b*) the horizontal and vertical components of the impulse exerted by the block on the bullet.

SOLUTION

Weight and mass. Bullet: $w = 1 \text{ oz} = \dfrac{1}{16} \text{ lb} \qquad m = 0.001941 \text{ lb} \cdot \text{s}^2/\text{ft.}$

Block: $W = 5 \text{ lb} \qquad M = 0.15528 \text{ lb} \cdot \text{s}^2/\text{ft.}$

(*a*) Use the principle of impulse and momentum applied to the bullet and the block together.



$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = m\mathbf{v}_2$$

Components $\downarrow$: $\quad mv_0 \cos 30° + 0 = (m + M)v'$

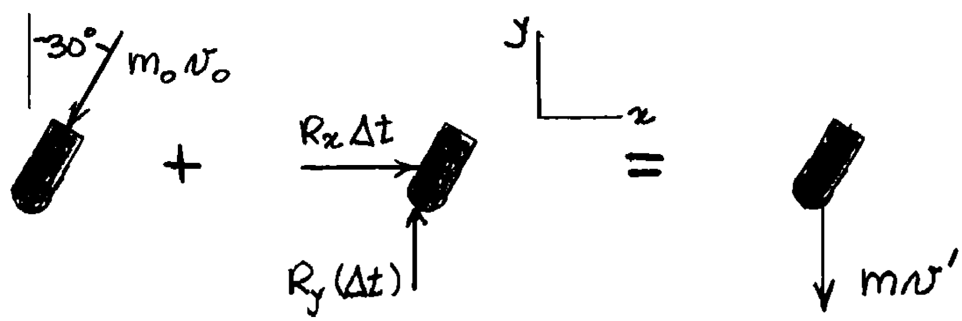
$$v' = \frac{mv_0 \cos 30°}{m + M} = \frac{(0.001941)(1400)\cos 30°}{0.157221}$$

$$v' = 14.968 \text{ ft/s}$$

$\mathbf{v}' = 14.97 \text{ ft/s} \downarrow$ ◀

PROBLEM 13.153 (Continued)

(*b*) Use the principle of impulse and momentum applied to the bullet alone.



x-components: $-mv_0 \sin 30° + R_x \Delta t = 0$

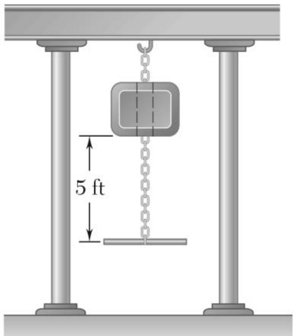
$$R_x \Delta t = mv_0 \sin 30° = (0.001941)(1400) \sin 30°$$

$$= 1.3587 \text{ lb} \cdot \text{s} \qquad R_x \Delta t = 1.359 \text{ lb} \cdot \text{s} \blacktriangleleft$$

y-components: $-mv_0 \cos 30° + R_y \Delta t = -mv'$

$$R_y \Delta t = m(v_0 \cos 30° - v')$$

$$= (0.001941)(1400 \cos 30° - 14.968) \qquad R_y \Delta t = 2.32 \text{ lb} \cdot \text{s} \blacktriangleleft$$



PROBLEM 13.154

In order to test the resistance of a chain to impact, the chain is suspended from a 240-lb rigid beam supported by two columns. A rod attached to the last link is then hit by a 60-lb block dropped from a 5-ft height. Determine the initial impulse exerted on the chain and the energy absorbed by the chain, assuming that the block does not rebound from the rod and that the columns supporting the beam are (*a*) perfectly rigid, (*b*) equivalent to two perfectly elastic springs.

SOLUTION

Velocity of the block just before impact:

$$T_1 = 0 \qquad V_1 = Wh = (60 \text{ lb})(5 \text{ ft}) = 300 \text{ lb} \cdot \text{ft}$$

$$T_2 = \frac{1}{2}mv^2 \quad V_2 = 0$$

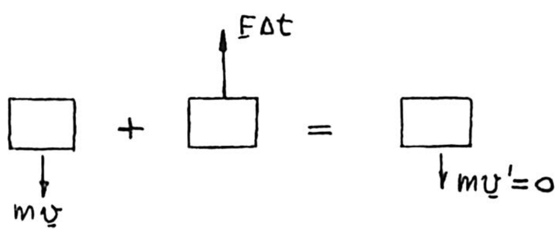
$$T_1 + V_1 = T_2 + V_2$$

$$0 + 300 = \frac{1}{2}\left(\frac{60}{g}\right)v^2$$

$$v = \sqrt{\frac{(600)(32.2)}{60}}$$

$$= 17.94 \text{ ft/s}$$

(*a*) Rigid columns:



$$+\!\uparrow \; -mv + F\Delta t = 0 \qquad \left(\frac{60}{g}\right)(17.94) = F\Delta t$$

$F\Delta t = 33.43 \text{ lb} \cdot \text{s} \uparrow$ on the block. $\qquad F\Delta t = 33.4 \text{ lb} \cdot \text{s}$ ◀

All of the kinetic energy of the block is absorbed by the chain.

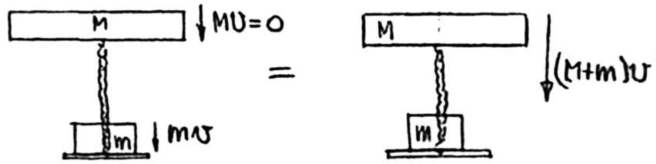
$$T = \frac{1}{2}\left(\frac{60}{g}\right)(17.94)^2$$

$= 300 \text{ ft} \cdot \text{lb}$ $\qquad E = 300 \text{ ft} \cdot \text{lb}$ ◀

PROBLEM 13.154 (Continued)

(*b*) Elastic columns:

Momentum of system of block and beam is conserved.



$$mv = (M + m)v'$$

$$v' = \frac{m}{(m+M)} v = \frac{60}{300}(17.94 \text{ ft/s}) \qquad v' = 3.59 \text{ ft/s}$$

Referring to figure in part (*a*),

$$-mv + F\Delta t = -mv'$$

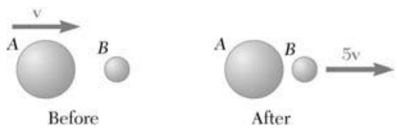
$$F\Delta t = m(v - v')$$

$$= \left(\frac{60}{g}\right)(17.94 - 3.59) \qquad F\Delta t = 26.7 \text{ lb} \cdot \text{s} \blacktriangleleft$$

$$E = \frac{1}{2}mv^2 - \frac{1}{2}mv'^2 - \frac{1}{2}Mv'^2$$

$$= \frac{60}{2g}[(17.94)^2 - (3.59)^2] - \frac{240}{2g}(3.59)^2 \qquad E = 240 \text{ ft} \cdot \text{lb} \blacktriangleleft$$

PROBLEM 13.CQ6



A 5 kg ball A strikes a 1 kg ball B that is initially at rest. Is it possible that after the impact A is not moving and B has a speed of $5v$?

(*a*) Yes

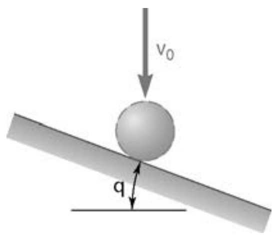
(*b*) No

Explain your answer.

SOLUTION

Answer: (*b*) No.

Conservation of momentum is satisfied, but the coefficient of restitution equation is not. The coefficient of restitution must be less than 1.

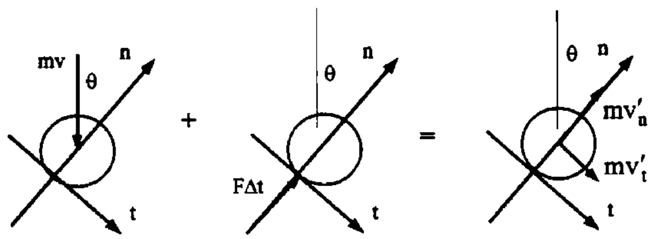


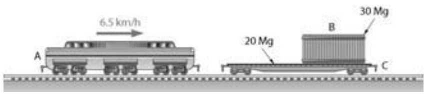
PROBLEM 13.F6

A sphere with a speed v_0 rebounds after striking a frictionless inclined plane as shown. Draw impulse-momentum diagrams that could be used to find the velocity of the sphere after the impact.

SOLUTION

Answer:





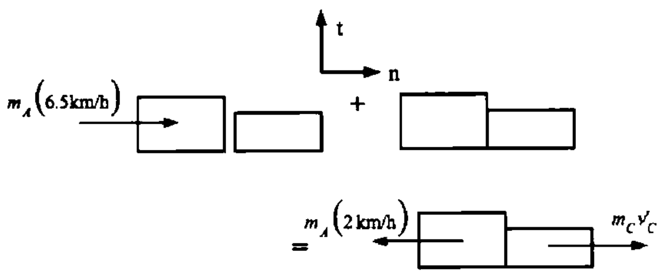
PROBLEM 13.F7

An 80-Mg railroad engine *A* coasting at 6.5 km/h strikes a 20-Mg flatcar *C* carrying a 30-Mg load *B* which can slide along the floor of the car ($\mu_k = 0.25$). The flatcar was at rest with its brakes released. Instead of *A* and *C* coupling as expected, it is observed that *A* rebounds with a speed of 2 km/h after the impact. Draw impulse-momentum diagrams that could be used to determine (*a*) the coefficient of restitution and the speed of the flatcar immediately after impact, and (*b*) the time it takes the load to slide to a stop relative to the car.

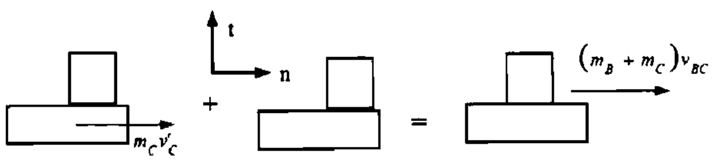
SOLUTION

Answer:

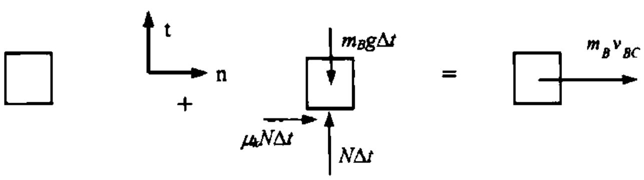
(*a*) Look at *A* and *C* (the friction force between *B* and *C* is not impulsive) to find the velocity after impact.

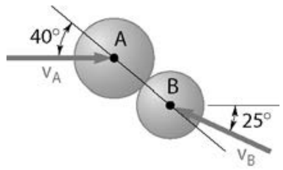


(*b*) Consider just *B* and *C* to find their final velocity.



Consider just *B* to find the time.



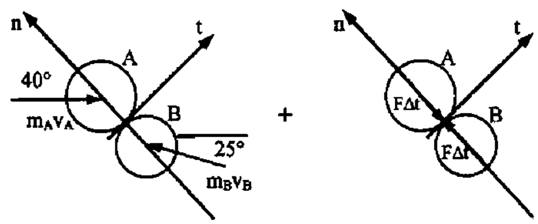


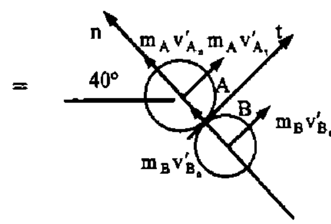
PROBLEM 13.F8

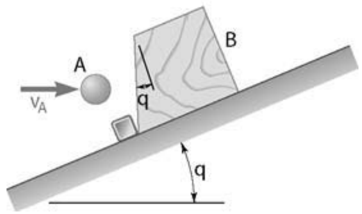
Two frictionless balls strike each other as shown. The coefficient of restitution between the balls is *e*. Draw the impulse-momentum diagrams that could be used to find the velocities of *A* and *B* after the impact.

SOLUTION

Answer:





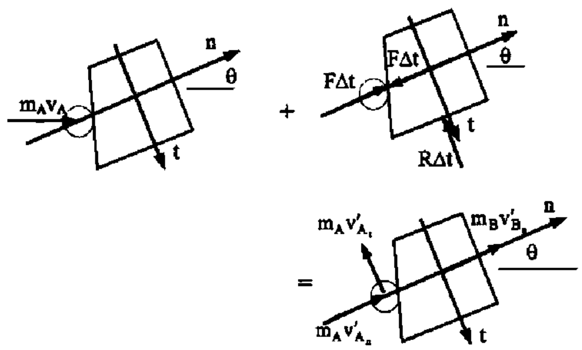


PROBLEM 13.F9

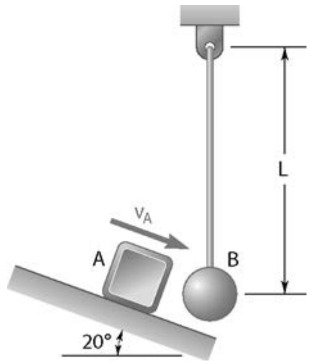
A 10-kg ball *A* moving horizontally at 12 m/s strikes a 10-kg block *B*. The coefficient of restitution of the impact is 0.4 and the coefficient of kinetic friction between the block and the inclined surface is 0.5. Draw impulse-momentum diagrams that could be used to determine the speeds of *A* and *B* after the impact.

SOLUTION

Answer:



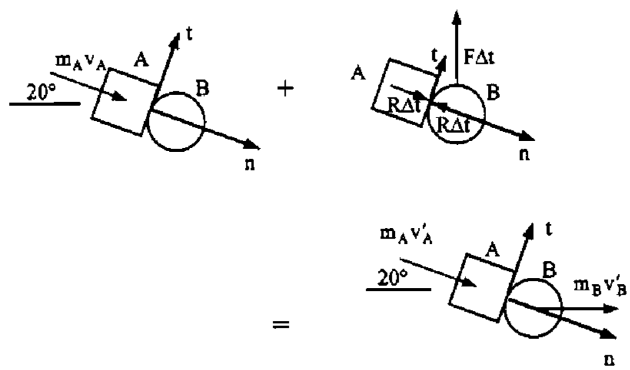
PROBLEM 13.F10

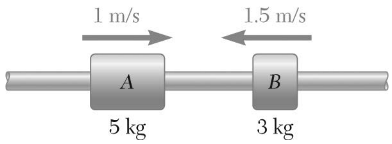


Block A of mass m_A strikes ball B of mass m_B with a speed of v_A as shown. Draw impulse-momentum diagrams that could be used to determine the speeds of A and B after the impact and the impulse during the impact.

SOLUTION

Answer:





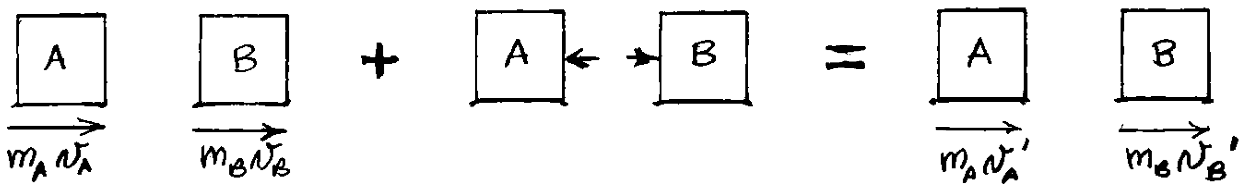
PROBLEM 13.155

The coefficient of restitution between the two collars is known to be 0.70. Determine (*a*) their velocities after impact, (*b*) the energy loss during impact.

SOLUTION

Impulse-momentum principle (collars *A* and *B*):

$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$



Horizontal components $\xrightarrow{+}$: $m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

Using data, $(5)(1) + (3)(-1.5) = 5v_A' + 3v_B'$

or $5v_A' + 3v_B' = 0.5$ (1)

Apply coefficient of restitution.

$$v_B' - v_A' = e(v_A - v_B)$$
$$v_B' - v_A' = 0.70[1 - (-0.5)]$$
$$v_B' - v_A' = 1.75 \qquad (2)$$

(*a*) Solving Eqs. (1) and (2) simultaneously for the velocities,

$v_A' = -0.59375$ m/s $\qquad \mathbf{v}_A = 0.594$ m/s $\longleftarrow$ ◀

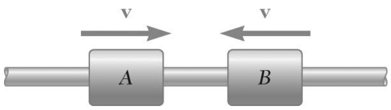
$v_B' = 1.15625$ m/s $\qquad \mathbf{v}_B = 1.156$ m/s $\longrightarrow$ ◀

Kinetic energies: $T_1 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(5)(1)^2 + \frac{1}{2}(3)(-1.5)^2 = 5.875 \text{ J}$

$$T_2 = \frac{1}{2}m_A(v_A')^2 + \frac{1}{2}m_B(v_B')^2 = \frac{1}{2}(5)(-0.59375)^2 + \frac{1}{2}(3)(1.15625)^2 = 2.8867 \text{ J}$$

(*b*) Energy loss: $T_1 - T_2 = 2.99$ J ◀

PROBLEM 13.156

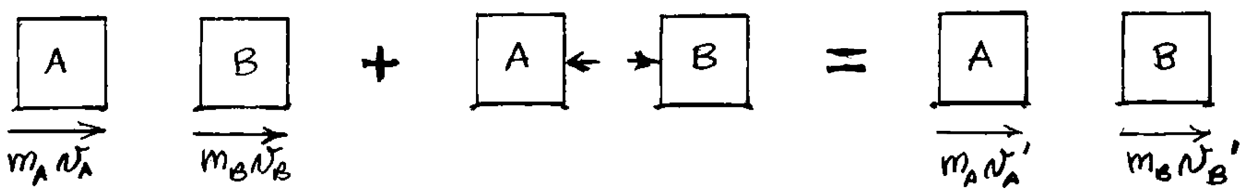


Collars A and B, of the same mass m, are moving toward each other with identical speeds as shown. Knowing that the coefficient of restitution between the collars is e, determine the energy lost in the impact as a function of m, e and v.

SOLUTION

Impulse-momentum principle (collars A and B):

$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$



Horizontal components $\xrightarrow{+}$: $m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

Using data, $mv + m(-v) = mv_A' + mv_B'$

or $v_A' + v_B' = 0$ (1)

Apply coefficient of restitution.

$$v_B' - v_A' = e(v_A - v_B)$$
$$v_B' - v_A' = e[v - (-v)]$$
$$v_B' - v_A' = 2ev \quad (2)$$

Subtracting Eq. (1) from Eq. (2), $-2v_A = 2ev$

$v_A = -ev$ $\qquad \mathbf{v}_A = ev \longleftarrow$

Adding Eqs. (1) and (2), $2v_B = 2ev$

$v_B = ev$ $\qquad \mathbf{v}_B = ev \longrightarrow$

Kinetic energies: $T_1 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}mv^2 + \frac{1}{2}m(-v)^2 = mv^2$

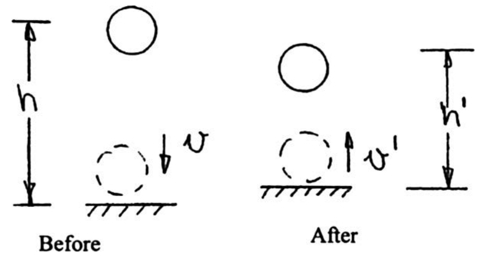
$$T_2 = \frac{1}{2}m_A(v_A')^2 + \frac{1}{2}m_B(v_B')^2 = \frac{1}{2}m(ev)^2 + \frac{1}{2}m(ev)^2 = e^2mv^2$$

Energy loss: $T_1 - T_2 = (1 - e^2)\, mv^2$ ◀

PROBLEM 13.157

One of the requirements for tennis balls to be used in official competition is that, when dropped onto a rigid surface from a height of 100 in., the height of the first bounce of the ball must be in the range 53 in. $\leq h \leq 58$ in. Determine the range of the coefficient of restitution of the tennis balls satisfying this requirement.

SOLUTION



Uniform accelerated motion:

$$v = \sqrt{2gh}$$

$$v' = \sqrt{2gh'}$$

Coefficient of restitution: $e = \dfrac{v'}{v}$

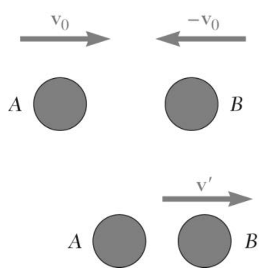
$$e = \sqrt{\frac{h'}{h}}$$

Height of drop $h = 100 \text{ in.}$

Height of bounce $53 \text{ in.} \leq h' \leq 58 \text{ in.}$

Thus, $\sqrt{\dfrac{53}{100}} \leq e \leq \sqrt{\dfrac{58}{100}}$

$0.728 \leq e \leq 0.762$ ◀

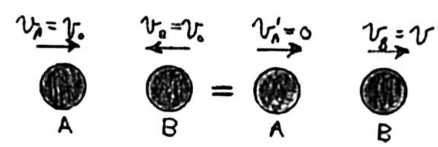


PROBLEM 13.158

Two disks sliding on a frictionless horizontal plane with opposite velocities of the same magnitude v_0 hit each other squarely. Disk *A* is known to have a weight of 6-lb and is observed to have zero velocity after impact. Determine (*a*) the weight of disk *B*, knowing that the coefficient of restitution between the two disks is 0.5, (*b*) the range of possible values of the weight of disk *B* if the coefficient of restitution between the two disks is unknown.

SOLUTION

Total momentum conserved:



$$\xrightarrow{+} m_A v_A + m_B v_B = m_A v_A' + m_B v'$$

$$(m_A)v_0 + m_B(-v_0) = 0 + m_B v'$$

$$v' = \left(\frac{m_A}{m_B} - 1\right)v_0 \qquad (1)$$

Relative velocities:

$$v_B' - v_A' = e(v_A - v_B)$$
$$v' = 2ev_0 \qquad (2)$$

Subtracting Eq. (2) from Eq. (1) and dividing by v_0,

$$\frac{m_A}{m_B} - 1 - 2e = 0 \qquad \frac{m_A}{m_B} = 1 + 2e \qquad m_B = \frac{m_A}{1+2e}$$

Since weight is proportional to mass, $W_B = \dfrac{W_A}{1+2e}$ (3)

(*a*) With $W_A = 6$ lb and $e = 0.5$,

$$W_B = \frac{6}{1+(2)(0.5)}$$

$W_B = 3.00$ lb ◀

(*b*) With $W_A = 6$ lb and $e = 1$,

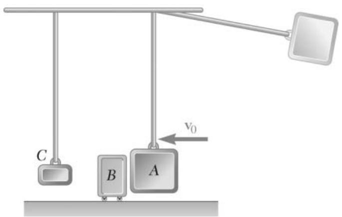
$$W_B = \frac{6}{1+(2)(1)} = 2 \text{ lb}$$

With $W_A = 6$ lb and $e = 0$,

$$W_B = \frac{6}{1+(2)(0)} = 6 \text{ lb}$$

Range: $2.00 \text{ lb} \le W_B \le 6.00 \text{ lb}$ ◀

PROBLEM 13.159

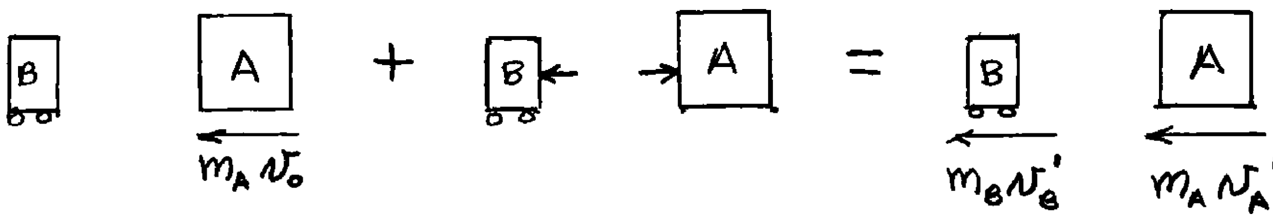


To apply shock loading to an artillery shell, a 20-kg pendulum *A* is released from a known height and strikes impactor *B* at a known velocity $\mathbf{v_0}$. Impactor *B* then strikes the 1-kg artillery shell *C*. Knowing the coefficient of restitution between all objects is *e*, determine the mass of *B* to maximize the impulse applied to the artillery shell *C*.

SOLUTION

First impact: *A* impacts *B*. $m_A = 20 \text{ kg}, \ m_B = ?$

Impulse-momentum: $\Sigma m\mathbf{v} + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$



Components directed left: $m_A v_0 = m_A v_A' + m_B v_B'$

$$20v_0 = 20v_A' + m_B v_B' \tag{1}$$

Coefficient of restitution: $v_B' - v_A' = e(v_A - v_B)$

$$v_B' - v_A' = ev_0$$

$$v_A' = v_B' - ev_0 \tag{2}$$

Substituting Eq. (2) into Eq. (1) yields

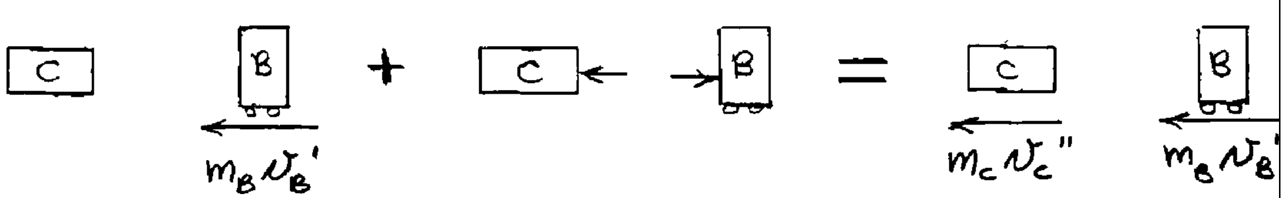
$$20v_0 = 20(v_B' - ev_0) + m_B v_B'$$

$$20v_0(1+e) = (+m_B)v_B'$$

$$v_B' = \frac{20v_0(1+e)}{20+m_B} \tag{3}$$

Second impact: *B* impacts *C*. $m_B = ?, \ m_C = 1 \text{ kg}$

Impulse-momentum: $\Sigma m\mathbf{v}_2 + \Sigma \mathbf{Imp}_{2\to 3} = \Sigma m\mathbf{v}_3$



Components directed left:

$$m_B v_B' = m_B v_B'' + m_C v_C''$$

$$m_B v_B' = m_B v_B'' + v_C'' \qquad (4)$$

Coefficient of restitution:

$$v_C'' - v_B'' = e(v_B' - v_C')$$

$$v_C'' - v_B'' = e v_B'$$

$$v_B'' - v_C'' = e v_C' \qquad (5)$$

Substituting Eq. (4) into Eq. (5) yields

$$m_B v_B' = m_B (v_C'' - e v_B') + m_C v_C''$$

$$m_B v_B' (1+e) = (1+m_B) v_C''$$

$$v_C'' = \frac{m_B v_B' (1+e)}{1+m_B} \qquad (6)$$

Substituting Eq. (3) for v_B' in Eq. (6) yields

$$v_C'' = \frac{20 m_B v_0 (1+e)^2}{(20+m_B)(1+m_B)}$$

The impulse applied to the shell C is

$$m_C v_C'' = \frac{(1)(20)\, m_B v_0 (1+e)^2}{(20+m_B)(1+m_B)}$$

To maximize this impulse choose m_B such that

$$Z = \frac{m_B}{(20+m_B)(1+m_B)}$$

is maximum. Set dZ/dm_B equal to zero.

$$\frac{dZ}{dm_B} = \frac{(20+m_B)(1+m_B) - m_B[(20+m_B)+(1+m_B)]}{(20+m_B)^2 (1+m_B)^2} = 0$$

$$20 + 21 m_B + m_B^2 - m_B(21 + 2m_B) = 0$$

$$20 - m_B^2 = 0$$

$m_B = 4.47$ kg ◀

PROBLEM 13.160

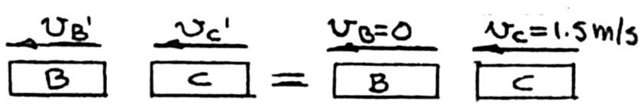
Two identical cars A and B are at rest on a loading dock with brakes released. Car C, of a slightly different style but of the same weight, has been pushed by dockworkers and hits car B with a velocity of 1.5 m/s. Knowing that the coefficient of restitution is 0.8 between B and C and 0.5 between A and B, determine the velocity of each car after all collisions have taken place.

SOLUTION

$$m_A = m_B = m_C = m$$

Collision between B and C:

The total momentum is conserved:



$$\overset{+}{\leftarrow}\quad mv'_B + mv'_C = mv_B + mv_C$$

$$v'_B + v'_C = 0 + 1.5 \qquad (1)$$

Relative velocities:

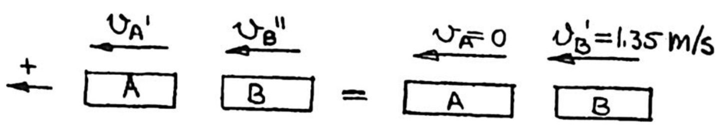
$$(v_B - v_C)(e_{BC}) = (v'_C - v'_B)$$
$$(-1.5)(0.8) = (v'_C - v'_B)$$
$$-1.2 = v'_C - v'_B \qquad (2)$$

Solving (1) and (2) simultaneously,

$$v'_B = 1.35 \text{ m/s}$$
$$v'_C = 0.15 \text{ m/s} \qquad \mathbf{v}'_C = 0.150 \text{ m/s} \leftarrow \blacktriangleleft$$

Since $v'_B > v'_C$, car B collides with car A.

Collision between A and B:



$$mv'_A + mv''_B = mv_A + mv'_B$$
$$v'_A + v''_B = 0 + 1.35 \qquad (3)$$

PROBLEM 13.160 (Continued)

Relative velocities:

$$(v_A - v_B')e_{AB} = (v_B'' - v_A')$$
$$(0 - 1.35)(0.5) = v_B'' - v_A'$$
$$v_A' - v_B'' = 0.675 \tag{4}$$

Solving (3) and (4) simultaneously,

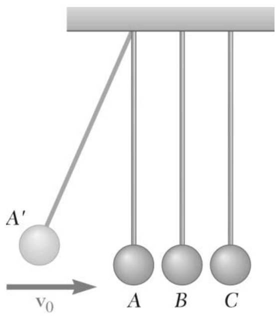
$$2v_A' = 1.35 + 0.675$$

$\mathbf{v}_A' = 1.013 \text{ m/s} \longleftarrow$ ◀

$\mathbf{v}_B'' = 0.338 \text{ m/s} \longleftarrow$ ◀

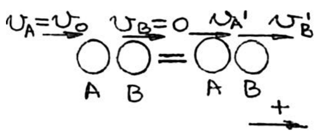
Since $v_C' < v_B'' < v_A'$, there are no further collisions.

PROBLEM 13.161



Three steel spheres of equal weight are suspended from the ceiling by cords of equal length which are spaced at a distance slightly greater than the diameter of the spheres. After being pulled back and released, sphere *A* hits sphere *B*, which then hits sphere *C*. Denoting by *e* the coefficient of restitution between the spheres and by $\mathbf{v}_0$ the velocity of *A* just before it hits *B*, determine (*a*) the velocities of *A* and *B* immediately after the first collision, (*b*) the velocities of *B* and *C* immediately after the second collision. (*c*) Assuming now that *n* spheres are suspended from the ceiling and that the first sphere is pulled back and released as described above, determine the velocity of the last sphere after it is hit for the first time. (*d*) Use the result of Part *c* to obtain the velocity of the last sphere when $n = 5$ and $e = 0.9$.

SOLUTION



(*a*) First collision (between *A* and *B*):

The total momentum is conserved:

$$mv_A + mv_B = mv_A' + mv_B'$$

$$v_0 = v_A' + v_B' \qquad (1)$$

Relative velocities:

$$(v_A - v_B)e = (v_B' - v_A')$$

$$v_0 e = v_B' - v_A' \qquad (2)$$

Solving Equations (1) and (2) simultaneously,

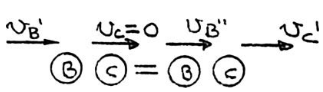
$$v_A' = \frac{v_0(1-e)}{2} \blacktriangleleft$$

$$v_B' = \frac{v_0(1+e)}{2} \blacktriangleleft$$

(*b*) Second collision (between *B* and *C*):

The total momentum is conserved.

$$mv_B' + mv_C = mv_B'' + mv_C'$$



Using the result from (*a*) for v_B'

$$\frac{v_0(1+e)}{2} + 0 = v_B'' + v_C' \qquad (3)$$

Relative velocities:

$$(v_B' - 0)e = v_C' - v_B''$$

PROBLEM 13.161 (Continued)

Substituting again for v'_B from (a)

$$v_0 \frac{(1+e)}{2}(e) = v'_C - v''_B \tag{4}$$

Solving equations (3) and (4) simultaneously,

$$v'_C = \frac{1}{2}\left[\frac{v_0(1+e)}{2} + v_0(1+e)\frac{(e)}{2}\right]$$

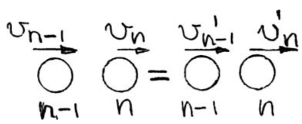
$$v'_C = \frac{v_0(1+e)^2}{4} \blacktriangleleft$$

$$v''_B = \frac{v_0(1-e^2)}{4} \blacktriangleleft$$

(c) For n spheres

n balls

$(n-1)$th collision,



we note from the answer to part (b) with $n = 3$

$$v'_n = v'_3 = v'_C = \frac{v_0(1+e)^2}{4}$$

or $$v'_3 = \frac{v_0(1+e)^{(3-1)}}{2^{(3-1)}}$$

Thus, for n balls

$$v'_n = \frac{v_0(1+e)^{(n-1)}}{2^{(n-1)}} \blacktriangleleft$$

(d) For $n = 5$, $e = 0.90$,

from the answer to part (c) with $n = 5$

$$v'_B = \frac{v_0(1+0.9)^{(5-1)}}{2^{(5-1)}}$$

$$= \frac{v_0(1.9)^4}{(2)^4}$$

$$v'_B = 0.815 v_0 \blacktriangleleft$$

PROBLEM 13.162

At an amusement park there are 200-kg bumper cars A, B, and C that have riders with masses of 40 kg, 60 kg, and 35 kg respectively. Car A is moving to the right with a velocity $\mathbf{v}_A = 2$ m/s and car C has a velocity $\mathbf{v}_B = 1.5$ m/s to the left, but car B is initially at rest. The coefficient of restitution between each car is 0.8. Determine the final velocity of each car, after all impacts, assuming (a) cars A and C hit car B at the same time, (b) car A hits car B before car C does.

SOLUTION

Assume that each car with its rider may be treated as a particle. The masses are:

$$m_A = 200 + 40 = 240 \text{ kg},$$
$$m_B = 200 + 60 = 260 \text{ kg},$$
$$m_C = 200 + 35 = 235 \text{ kg}.$$

Assume velocities are positive to the right. The initial velocities are:

$$v_A = 2 \text{ m/s} \quad v_B = 0 \quad v_C = -1.5 \text{ m/s}$$

Let v'_A, v'_B, and v'_C be the final velocities.

(a) Cars A and C hit B at the same time. Conservation of momentum for all three cars.

$$m_A v_A + m_B v_B + m_C v_C = m_A v'_A + m_B v'_B + m_C v'_C$$
$$(240)(2) + 0 + (235)(-1.5) = 240 v'_A + 260 v'_B + 235 v'_C \qquad (1)$$

Coefficient of restition for cars A and B.

$$v'_B - v'_A = e(v_A - v_B) = (0.8)(2 - 0) = 1.6 \qquad (2)$$

Coefficient of restitution for cars B and C.

$$v'_C - v'_B = e(v_B - v_C) = (0.8)[0 - (-1.5)] = 1.2 \qquad (3)$$

Solving Eqs. (1), (2), and (3) simultaneously,

$$v'_A = -1.288 \text{ m/s} \quad v'_B = 0.312 \text{ m/s} \quad v'_C = 1.512 \text{ m/s}$$

$\mathbf{v}'_A = 1.288$ m/s ⟵ ◀

$\mathbf{v}'_B = 0.312$ m/s ⟶ ◀

$\mathbf{v}'_C = 1.512$ m/s ⟶ ◀

(*b*) Car *A* hits car *B* before *C* does.

First impact. Car *A* hits car *B*. Let v'_A and v'_B be the velocities after this impact. Conservation of momentum for cars *A* and *B*.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

$$(240)(2) + 0 = 240 v'_A + 260 v'_B \tag{4}$$

Coefficient of restitution for cars *A* and *B*.

$$v'_B - v'_A = e(v_A - v_B) = (0.8)(2 - 0) = 1.6 \tag{5}$$

Solving Eqs. (4) and (5) simultaneously,

$$v'_A = 0.128 \text{ m/s}, \quad v'_B = 1.728 \text{ m/s}$$

$$\mathbf{v}'_A = 0.128 \text{ m/s} \longrightarrow$$

$$\mathbf{v}'_B = 1.728 \text{ m/s} \longrightarrow$$

Second impact. Cars *B* and *C* hit. Let v''_B and v''_C be the velocities after this impact. Conservation of momentum for cars *B* and *C*.

$$m_B v'_B + m_C v_C = m_B v''_B + m_C v''_C$$

$$(260)(1.728) + (235)(-1.5) = 260 v''_B + 235 v''_C \tag{6}$$

Coefficient of restitution for cars *B* and *C*.

$$v''_C - v''_B = e(v'_B - v_C) = (0.8)[1.728 - (-1.5)] = 2.5824 \tag{7}$$

Solving Eqs. (6) and (7) simultaneously,

$$v''_B = -1.03047 \text{ m/s} \quad v''_C = 1.55193 \text{ m/s}$$

$$\mathbf{v}''_B = 1.03047 \text{ m/s} \longleftarrow$$

$$\mathbf{v}''_C = 1.55193 \text{ m/s} \longrightarrow$$

Third impact. Cars *A* and *B* hit again. Let v'''_A and v'''_B be the velocities after this impact. Conservation of momentum for cars *A* and *B*.

$$m_A v'_A + m_B v''_B = m_A v'''_A + m_B v'''_B$$

$$(240)(0.128) + (260)(-1.03047) = 240 v'''_A + 260 v'''_B \tag{8}$$

Coefficient of restitution for cars *A* and *B*.

$$v'''_B - v'''_A = e(v'_A - v''_B) = (0.8)[0.128 - (-1.03047)] = 0.926776 \tag{9}$$

Solving Eqs. (8) and (9) simultaneously,

$$v'''_A = -0.95633 \text{ m/s}$$

$$v'''_B = -0.02955 \text{ m/s}$$

$$\mathbf{v}'''_A = 0.95633 \text{ m/s} \longleftarrow$$

$$\mathbf{v}'''_B = 0.02955 \text{ m/s} \longleftarrow$$

There are no more impacts. The final velocities are:

$$\mathbf{v}_A''' = 0.956 \text{ m/s} \longleftarrow \blacktriangleleft$$

$$\mathbf{v}_B''' = 0.0296 \text{ m/s} \longleftarrow \blacktriangleleft$$

$$\mathbf{v}_C'' = 1.552 \text{ m/s} \longrightarrow \blacktriangleleft$$

We may check our results by considering conservation of momentum of all three cars over all three impacts.

$$\begin{aligned} m_A v_A + m_B v_B + m_C v_C &= (240)(2) + 0 + (235)(-1.5) \\ &= 127.5 \text{ kg} \cdot \text{m/s} \end{aligned}$$

$$\begin{aligned} m_A v_A''' + m_B v_B''' + m_C v_C'' &= (240)(-0.95633) + (260)(-0.02955) + (235)(1.55193) \\ &= 127.50 \text{ kg} \cdot \text{m/s.} \end{aligned}$$

PROBLEM 13.163

At an amusement park there are 200-kg bumper cars *A*, *B*, and *C* that have riders with masses of 40 kg, 60 kg, and 35 kg respectively. Car *A* is moving to the right with a velocity $\mathbf{v}_A = 2$ m/s when it hits stationary car *B*. The coefficient of restitution between each car is 0.8. Determine the velocity of car *C* so that after car *B* collides with car *C* the velocity of car *B* is zero.

SOLUTION

Assume that each car with its rider may be treated as a particle. The masses are:

$$m_A = 200 + 40 = 240 \text{ kg}$$
$$m_B = 200 + 60 = 260 \text{ kg}$$
$$m_C = 200 + 35 = 235 \text{ kg}$$

Assume velocities are positive to the right. The initial velocities are:

$$v_A = 2 \text{ m/s}, \quad v_B = 0, \quad v_C = ?$$

First impact. Car *A* hits car *B*. Let v'_A and v'_B be the velocities after this impact. Conservation of momentum for cars *A* and *B*.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$
$$(240)(2) + 0 = 240 v'_A + 260 v'_B \qquad (1)$$

Coefficient of restitution for cars *A* and *B*.

$$v'_B - v'_A = e(v_A - v_B) = (0.8)(2 - 0) = 1.6 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$v'_A = 0.128 \text{ m/s}$$
$$v'_B = 1.728 \text{ m/s}$$

$$\mathbf{v}'_A = 0.128 \text{ m/s} \longrightarrow$$

$$\mathbf{v}'_B = 1.728 \text{ m/s} \longrightarrow$$

Second impact. Cars *B* and *C* hit. Let v''_B and v''_C be the velocities after this impact. $v''_B = 0$. Coefficient of restitution for cars *B* and *C*.

$$v''_C - v''_B = e(v'_B - v_C) = (0.8)(1.728 - v_C)$$
$$v''_C = 1.3824 - 0.8 v_C$$

PROBLEM 13.163 (Continued)

Conservation of momentum for cars *B* and *C*.

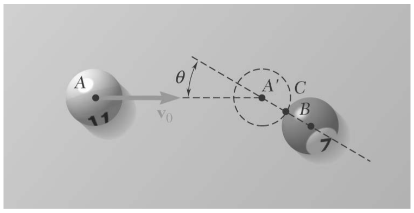
$$m_B v_B' + m_C v_C = m_B v_B'' + m_C v_C''$$

$$(260)(1.728) + 235 v_C = (260)(0) + (235)(1.3824 - 0.8 v_C)$$

$$(235)(1.8) v_C = (235)(1.3824) - (260)(1.728)$$

$$v_C = -0.294 \text{ m/s} \qquad \mathbf{v}_C = 0.294 \text{ m/s} \longleftarrow \blacktriangleleft$$

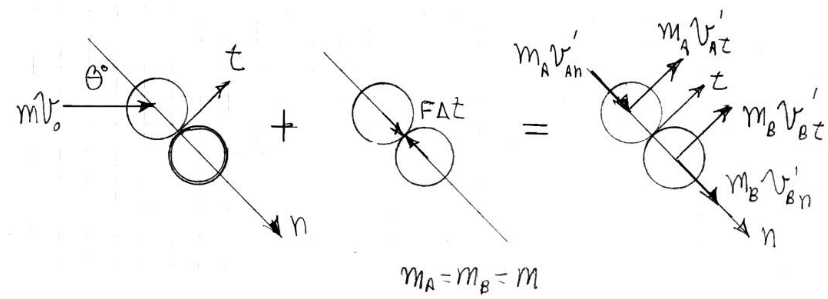
Note: There will be another impact between cars *A* and *B*.



PROBLEM 13.164

Two identical billiard balls can move freely on a horizontal table. Ball A has a velocity $\mathbf{v}_0$ as shown and hits ball B, which is at rest, at a Point C defined by $\theta = 45°$. Knowing that the coefficient of restitution between the two balls is $e = 0.8$ and assuming no friction, determine the velocity of each ball after impact.

SOLUTION



Ball A: *t-dir*
$$\cancel{m}v_0 \sin\theta = \cancel{m}v'_{At} \;\Rightarrow\; v'_{At} = v_0 \sin\theta$$

Ball B: *t-dir*
$$0 = m_B v'_{Bt} \;\Rightarrow\; v'_{Bt} = 0$$

Balls $A + B$: *n-dir*
$$\cancel{m}v_0 \cos\theta + 0 = \cancel{m}\, v'_{An} + \cancel{m}\, v'_{Bn} \tag{1}$$

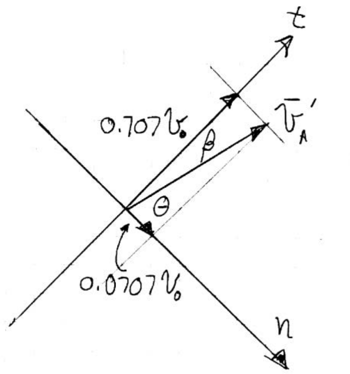
Coefficient of restitution
$$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$$
$$v'_{Bn} - v'_{An} = e(v_0 \cos\theta - 0) \tag{2}$$

Solve (1) and (2)
$$v'_{An} = v_0\left(\frac{1-e}{2}\cos\theta\right); \quad v'_{Bn} = v_0\left(\frac{1+e}{2}\right)\cos\theta$$

With numbers
$$e = 0.8; \quad \theta = 45°$$
$$v'_{At} = v_0 \sin 45° = 0.707\, v_0$$
$$v'_{An} = v_0\left(\frac{1-0.8}{2}\cos 45°\right) = 0.0707\ v_0$$
$$v'_{Bt} = 0$$
$$v'_{Bn} = v_0\left(\frac{1+0.8}{2}\right)\cos 45° = 0.6364\, v_0$$

PROBLEM 13.164 (Continued)

(*A*)

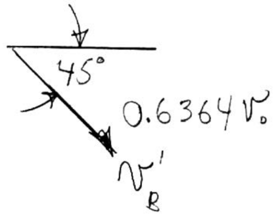


$$|v''_A| = [(0.707\,v_0)^2 + (0.0707 v_0)^2]^{\frac{1}{2}} = 0.711 v_0$$

$$\beta = \tan^{-1}\left(\frac{0.0707}{0.707}\right) = 5.7106°$$

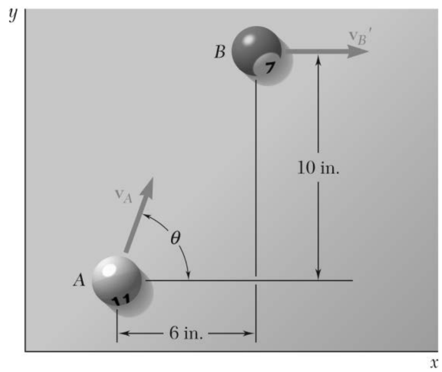
So $\theta = 45 - 5.7106 = 39.3°$

(*B*)



$\vec{v}'_A = 0.711 v_0$ ∠39.3° ◀

$\vec{v}'_B = 0.636 v_0$ ∠45° ◀

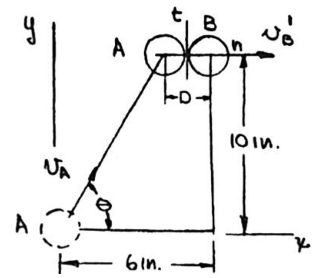


PROBLEM 13.165

The coefficient of restitution is 0.9 between the two 2.37-in. diameter billiard balls *A* and *B*. Ball *A* is moving in the direction shown with a velocity of 3 ft/s when it strikes ball *B*, which is at rest. Knowing that after impact *B* is moving in the *x* direction, determine (*a*) the angle θ, (*b*) the velocity of *B* after impact.

SOLUTION

(*a*) Since v_B' is in the *x*-direction and (assuming no friction), the common tangent between *A* and *B* at impact must be parallel to the *y*-axis,



$$\tan\theta = \frac{10}{6-D}$$

$$\theta = \tan^{-1}\frac{10}{6-2.37}$$

$$= 70.04^\circ$$

$\theta = 70.0^\circ$ ◀

(*b*) Conservation of momentum in *x*(*n*) direction:

$$mv_A\cos\theta + m(v_B)_n = m(v_A')_n + mv_B'$$

$$(3)(\cos 70.04) + 0 = (v_A')_n + v_B'$$

$$1.0241 = (v_A')_n + (v_B') \qquad (1)$$

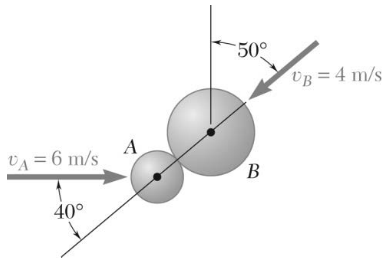
Relative velocities in the *n* direction:

$e = 0.9$ $\qquad (v_A\cos\theta - (v_B)_n)e = v_B' - (v_A')_n$

$$(1.0241 - 0)(0.9) = v_B' - (v_A')_n \qquad (2)$$

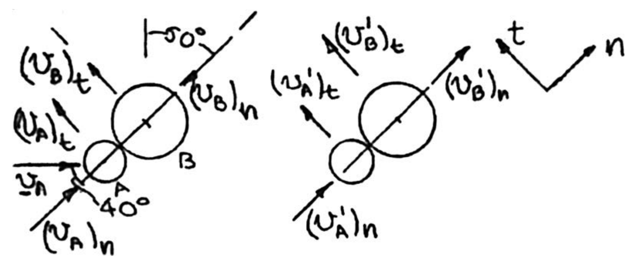
$(1)+(2)$ $\qquad 2v_B' = 1.0241(1.9)$ $\qquad \mathbf{v}_B' = 0.972 \text{ ft/s} \longrightarrow$ ◀

PROBLEM 13.166



A 600-g ball *A* is moving with a velocity of magnitude 6 m/s when it is hit as shown by a 1-kg ball *B* which has a velocity of magnitude 4 m/s. Knowing that the coefficient of restitution is 0.8 and assuming no friction, determine the velocity of each ball after impact.

SOLUTION



Before After

$$v_A = 6 \text{ m/s}$$
$$(v_A)_n = (6)(\cos 40°) = 4.596 \text{ m/s}$$
$$(v_A)_t = -6(\sin 40°) = -3.857 \text{ m/s}$$
$$v_B = (v_B)_n = -4 \text{ m/s}$$
$$(v_B)_t = 0$$

t-direction:

Total momentum conserved:

$$m_A(v_A)_t + m_B(v_B)_t = m_A(v'_B)_t + m_B(v'_B)_t$$
$$(0.6 \text{ kg})(-3.857 \text{ m/s}) + 0 = (0.6 \text{ kg})(v'_A)_t + (1 \text{ kg})(v'_B)t$$
$$-2.314 \text{ m/s} = 0.6\,(v'_A)_t + (v'_B)_t \qquad (1)$$

Ball *A* alone:

Momentum conserved:

$$m_A(v_A)_t = m_A(v'_A)_t \qquad -3.857 = (v'_A)_t$$
$$(v'_A)_t = -3.857 \text{ m/s} \qquad (2)$$

Replacing $(v'_A)_t$ in (2) in Eq. (1)

$$-2.314 = (0.6)(-3.857) + (v'_B)_t$$
$$-2.314 = -2.314 + (v'_B)_t$$
$$(v'_B)_t = 0$$

n-direction:

Relative velocities:

$$[(v_A)_n - (v_B)_n]e = (v'_B)_n - (v'_A)_n$$
$$[(4.596) - (-4)](0.8) = (v'_B)_n - (v'_A)_n$$
$$6.877 = (v'_B)_n - (v'_A)_n \qquad (3)$$

Total momentum conserved:

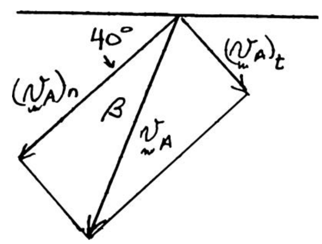
$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$
$$(0.6 \text{ kg})(4.596 \text{ m/s}) + (1 \text{ kg})(-4 \text{ m/s}) = (1 \text{ kg})(v'_B)_n + (0.6 \text{ kg})(v'_A)_n$$
$$-1.2424 = (v'_B)_n + 0.6(v'_A)_n \qquad (4)$$

Solving Eqs. (4) and (3) simultaneously,

$$(v'_A)_n = 5.075 \text{ m/s}$$
$$(v'_B)_n = 1.802 \text{ m/s}$$

Velocity of *A*:

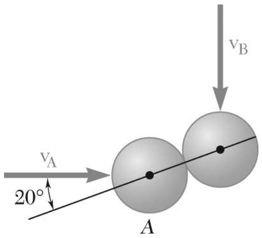
$$\tan\beta = \frac{|(v_A)_t|}{|(v_A)_n|} = \frac{3.857}{5.075}$$
$$\beta = 37.2° \qquad \beta + 40° = 77.2°$$
$$v'_A = \sqrt{(3.857)^2 + (5.075)^2} = 6.37 \text{ m/s}$$



$\mathbf{v}'_A = 6.37$ m/s ↙ 77.2° ◀

Velocity of *B*: $\mathbf{v}'_B = 1.802$ m/s ∠ 40° ◀

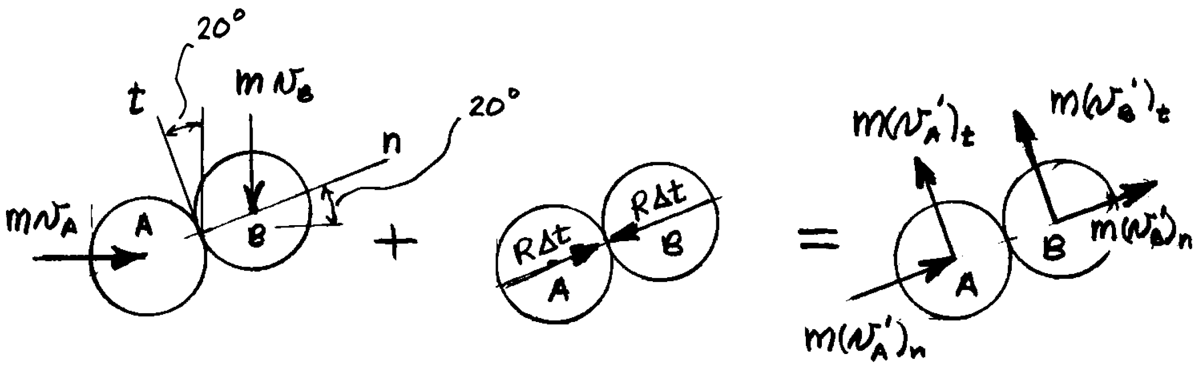
PROBLEM 13.167



Two identical hockey pucks are moving on a hockey rink at the same speed of 3 m/s and in perpendicular directions when they strike each other as shown. Assuming a coefficient of restitution $e = 0.9$, determine the magnitude and direction of the velocity of each puck after impact.

SOLUTION

Use principle of impulse-momentum: $\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$



t-direction for puck A:

$$-mv_A \sin 20° + 0 = m(v'_A)_t$$
$$(v'_A)_t = v_A \sin 20° = 3\sin 20° = 1.0261 \text{ m/s}$$

t-direction for puck B:

$$-mv_B \cos 20° + 0 = m(v'_B)_t$$
$$(v'_B)_t = v_B \cos 20° = -3\cos 20° = -2.8191 \text{ m/s}$$

n-direction for both pucks:

$$mv_A \cos 20° - mv_B \sin 20° = m(v'_A)_n + m(v'_B)_n$$
$$(v'_A)_n + (v'_B)_n = v_A \cos 20° - v_B \sin 20°$$
$$= 3\cos 20° - 3\sin 20° \qquad (1)$$

Coefficient of restitution: $e = 0.9$

$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$$
$$= 0.9[3\cos 20° - (-3)\sin 20°] \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$(v'_A)_n = -0.8338 \text{ m/s} \qquad (v'_B)_n = 2.6268 \text{ m/s}$$

PROBLEM 13.167 (Continued)

Summary:

$$(\mathbf{v}'_A)_n = 0.8338 \text{ m/s} \swarrow 20°$$

$$(\mathbf{v}'_A)_t = 1.0261 \text{ m/s} \searrow 70°$$

$$(\mathbf{v}'_B)_n = 2.6268 \text{ m/s} \nearrow 20°$$

$$(\mathbf{v}'_B)_t = 2.8191 \text{ m/s} \searrow 70°$$

$$v_A = \sqrt{(0.8338)^2 + (1.0261)^2} = 1.322 \text{ m/s}$$

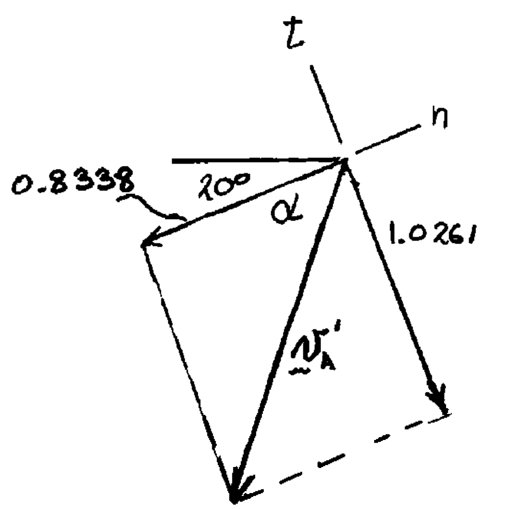
$$\tan\alpha = \frac{1.0261}{0.8338} \qquad \alpha = 50.9° \qquad \alpha + 20° = 70.9°$$

$$\mathbf{v}'_A = 1.322 \text{ m/s} \swarrow 70.9° \blacktriangleleft$$

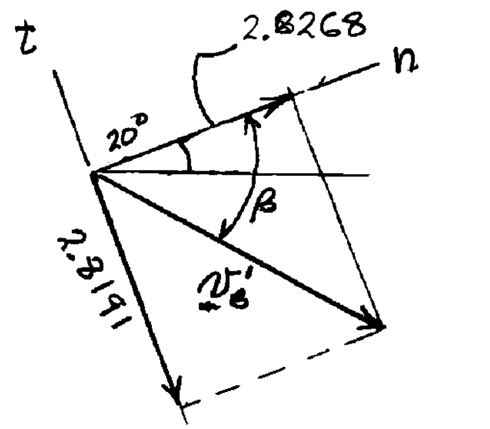
$$v'_B = \sqrt{(2.6268)^2 + (2.8191)^2} = 3.85 \text{ m/s}$$

$$\tan\beta = \frac{2.8191}{2.6268} \qquad \beta = 47.0° \qquad \beta - 20° = 27.0°$$

$$\mathbf{v}'_B = 3.85 \text{ m/s} \swarrow 27.0° \blacktriangleleft$$

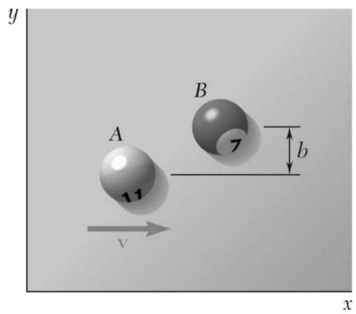


Velocity of Puck A



Velocity of Puck B

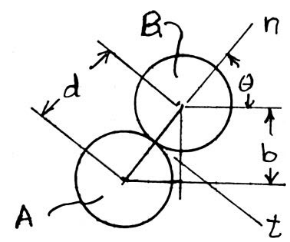
PROBLEM 13.168



Two identical pool balls of 57.15-mm diameter, may move freely on a pool table. Ball B is at rest and ball A has an initial velocity $\mathbf{v} = v_0\mathbf{i}$. ($a$) Knowing that $b = 50$ mm and $e = 0.7$, determine the velocity of each ball after impact. (b) Show that if $e = 1$, the final velocities of the balls from a right angle for all values of b.

SOLUTION

Geometry at instant of impact:

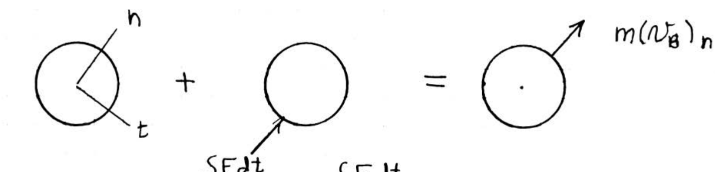


$$\sin\theta = \frac{b}{d} = \frac{50}{57.15}$$
$$\theta = 61.032°$$

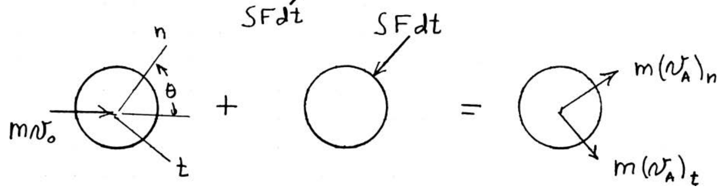
Directions n and t are shown in the figure.

Principle of impulse and momentum:

Ball B:



Ball A:



Ball A, t-direction: $\quad mv_0 \sin\theta + 0 = m(v_A)_t \quad (v_A)_t = v_0 \sin\theta \qquad (1)$

Ball B, t-direction: $\quad 0 + 0 = m(v_B)_t \quad (v_B)_t = 0 \qquad (2)$

Balls A and B, n-direction: $\quad mv_0 \cos\theta + 0 + m(v_A)_n + m(v_B)_n$

$$(v_A)_n + (v_B)_n = v_0 \cos\theta \qquad (3)$$

Coefficient of restitution: $\quad (v_B)_n - (v_A)_n = e[v_0 \cos\theta] \qquad (4)$

(a) $\quad e = 0.7.$ From Eqs. (1) and (2),

$$(v_A)_t = 0.87489 v_0 \qquad (1)'$$

$$(v_B)_t = 0 \qquad (2)'$$

From Eqs. (3) and (4),

$$(v_A)_n + (v_B)_n = 0.48432v_0 \tag{3$'$}$$

$$(v_B)_n - (v_A)_n = (0.7)(0.48432v_0) \tag{4$'$}$$

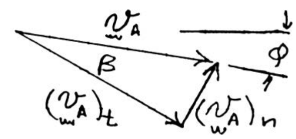
Solving Eqs. (5) and (6) simultaneously,

$$(v_A)_n = 0.072648v_0 \qquad (v_B)_n = 0.41167v_0$$

$$v_A = \sqrt{(v_A)_n^2 + (v_A)_t^2}$$

$$= \sqrt{(0.072648v_0)^2 + (0.87489v_0^2}$$

$$= 0.87790v_0$$



$$\tan\beta = \frac{(v_A)_n}{(v_A)_t} = \frac{0.072648v_0}{0.87489v_0} = 0.083037$$

$$\beta = 4.7468^\circ$$

$$\varphi = 90^\circ - \theta - \beta$$

$$= 90^\circ - 61.032^\circ - 4.7468^\circ$$

$$= 24.221^\circ$$

$\mathbf{v}_A = 0.878v_0 \searrow 24.2^\circ$ ◀

$\mathbf{v}_B = 0.412v_0 \measuredangle 61.0^\circ$ ◀

(*b*) $e = 1$. Eqs. (3) and (4) become

$$(v_A)_n + (v_B)_n = v_0 \cos\theta \tag{3$''$}$$

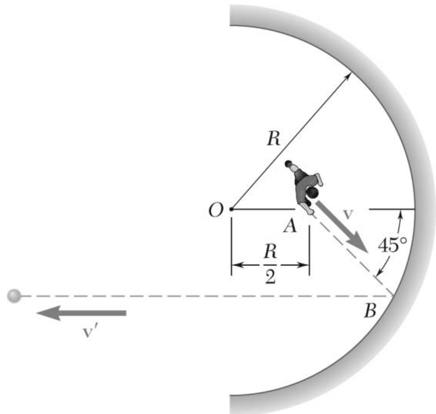
$$(v_B)_n - (v_A)_n = v_0 \cos\theta \tag{4$''$}$$

Solving Eqs. (3)$''$ and (4)$''$ simultaneously,

$$(v_A)_n = 0, \quad (v_B)_t = v_0 \cos\theta$$

But $\quad (v_A)_t = v_0 \sin\theta, \quad \text{and} \quad (v_B)_t = 0$

$\mathbf{v}_A$ is in the *t*-direction and $\mathbf{v}_B$ is in the *n*-direction; therefore, the velocity vectors form a right angle.

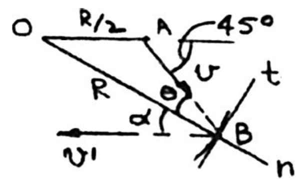


PROBLEM 13.169

A boy located at Point A halfway between the center O of a semicircular wall and the wall itself throws a ball at the wall in a direction forming an angle of 45° with OA. Knowing that after hitting the wall the ball rebounds in a direction parallel to OA, determine the coefficient of restitution between the ball and the wall.

SOLUTION

Law of sines:



$$\frac{\sin\theta}{\frac{R}{2}} = \frac{\sin 135°}{R}$$

$$\theta = 20.705°$$

$$\alpha = 45° - 20.705°$$

$$= 24.295°$$

Conservation of momentum for ball in t-direction:

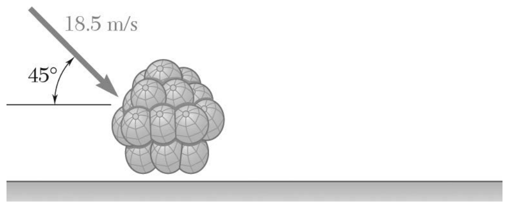
$$-v \sin\theta = -v' \sin\alpha$$

Coefficient of restitution in n: $\quad v(\cos\theta)e = v' \cos\alpha$

Dividing, $\quad \dfrac{\tan\theta}{e} = \tan\alpha$

$$e = \frac{\tan 20.705°}{\tan 24.295°}$$

$e = 0.837$ ◀



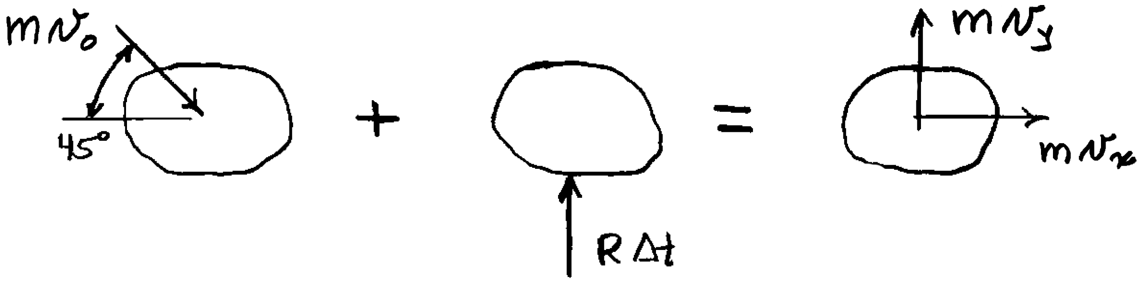
PROBLEM 13.170

The Mars Pathfinder spacecraft used large airbags to cushion its impact with the planet's surface when landing. Assuming the spacecraft had an impact velocity of 18 m/s at an angle of 45° with respect to the horizontal, the coefficient of restitution is 0.85 and neglecting friction, determine (*a*) the height of the first bounce, (*b*) the length of the first bounce. (Acceleration of gravity on the Mars = 3.73 m/s^2.)

SOLUTION

Use impulse-momentum principle.

$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$



The horizontal direction (x to the right) is the tangential direction and the vertical direction (y upward) is the normal direction.

Horizontal components: $mv_0 \sin 45° = 0 = mv_x$

$$v_x = v_0 \sin 45°. \qquad \mathbf{v}_x = 12.728 \text{ m/s} \longrightarrow$$

Vertical components, using coefficient of restitution $e = 0.85$

$$v_y - 0 = e[0 - (-v_0 \cos 45°)]$$

$$v_y = (0.85)(18 \cos 45°) \qquad \mathbf{v}_y = 10.819 \text{ m/s} \uparrow$$

The motion during the first bounce is projectile motion.

Vertical motion: $y = (v_y)_0 t - \frac{1}{2} g t^2$

$$v_y = (v_y)_0 - gt$$

Horizontal motion: $x = v_x t$

PROBLEM 13.170 (Continued)

(*a*) Height of first bounce:

$$v_y = 0: \qquad 0 = (v_y)_0 = gt$$

$$t = \frac{(v_y)_0}{g} = \frac{10.819 \text{ m/s}}{3.73 \text{ m/s}^2} = 2.901 \text{ s}$$

$$y = (10.819)(2.901) - \frac{1}{2}(3.73)(2.901)^2$$

$y = 15.69$ m ◀

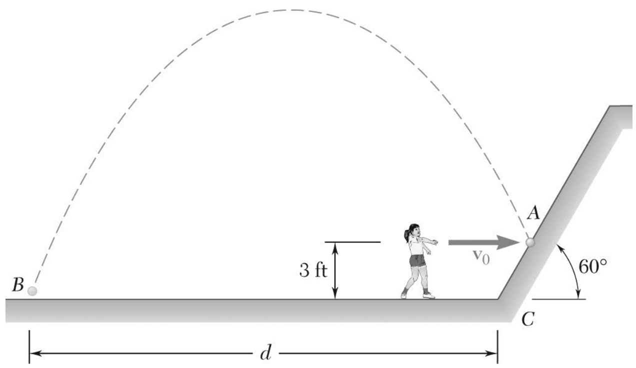
(*b*) Length of first bounce:

$$y = 0: \qquad 10.819t - \frac{1}{2}(3.73)t^2 = 0$$

$$t = 5.801 \text{ s}$$

$$x = (12.728)(5.801)$$

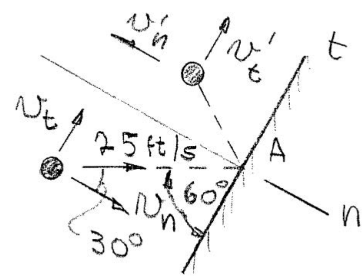
$x = 73.8$ m ◀



PROBLEM 13.171

A girl throws a ball at an inclined wall from a height of 3 ft, hitting the wall at A with a horizontal velocity $\mathbf{v}_0$ of magnitude 25 ft/s. Knowing that the coefficient of restitution between the ball and the wall is 0.9 and neglecting friction, determine the distance d from the foot of the wall to the Point B where the ball will hit the ground after bouncing off the wall.

SOLUTION



Momentum in t direction is conserved

$$mv\sin 30^\circ = mv_t'$$

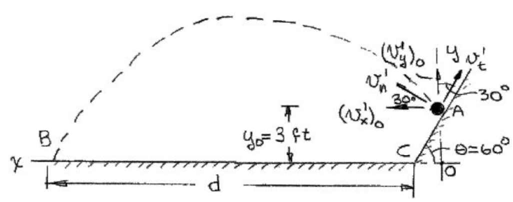
$$(25)(\sin 30^\circ) = v_t'$$

$$v_t' = 12.5 \text{ ft/s}$$

Coefficient of restitution in n-direction

$$(v\cos 30^\circ)e = v_n'$$

$$(25)(\cos 30^\circ)(0.9) = v_n' \qquad v_n' = 19.49 \text{ ft/s}$$



Write v' in terms of x and y components

$$(v_x')_0 = v_n'(\cos 30^\circ) - v_t'(\sin 30^\circ) = 19.49(\cos 30^\circ) - 12.5(\sin 30^\circ)$$
$$= 10.63 \text{ ft/s}$$

$$(v_y')_0 = v_n'(\sin 30^\circ) + v_t'(\cos 30^\circ) = 19.49(\sin 30^\circ) + 12.5(\cos 30^\circ)$$
$$= 20.57 \text{ ft/s}$$

PROBLEM 13.171 (Continued)

Projectile motion

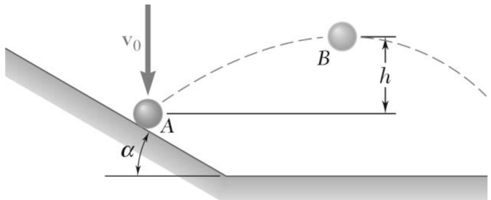
$$y = y_0 + (v'_y)_0 t - \frac{1}{2}gt^2 = 3\text{ ft} + (20.57\text{ ft/s})t - (32.2\text{ ft/s}^2)\frac{t^2}{2}$$

At B, $$y = 0 = 3 + 20.57t_B - 16.1t_B^2; \qquad t_B = 1.4098\text{ s}$$

$$x_B = x_0 + (v'_x)_0 t_B = 0 + 10.63(1.4098); \qquad x_B = 14.986\text{ ft}$$

$$d = x_B - 3\cot 60° = (14.986\text{ ft}) - (3\text{ ft})\cot 60° = 13.254\text{ ft}$$

$d = 13.25$ ft ◀



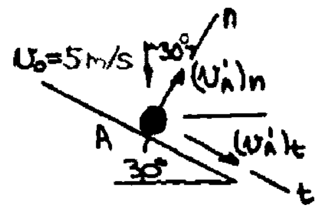
PROBLEM 13.172

A sphere rebounds as shown after striking an inclined plane with a vertical velocity $\mathbf{v}_0$ of magnitude $v_0 = 5$ m/s. Knowing that $\alpha = 30°$ and $e = 0.8$ between the sphere and the plane, determine the height h reached by the sphere.

SOLUTION

Rebound at A

Conservation of momentum in the t-direction:



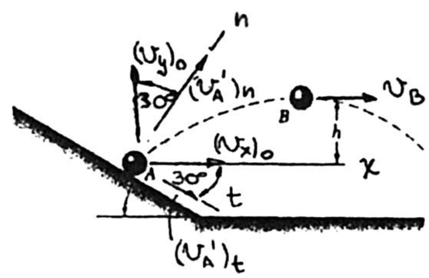
$$mv_0 \sin 30° = m(v_A')_t$$
$$(v_A')_t = (5 \text{ m/s})(\sin 30°) = 2.5 \text{ m/s}$$

Relative velocities in the n-direction:

$$(-v_0 \cos 30° - 0)e = 0 - (v_A')_n$$
$$(v_A')_n = (0.8)(5 \text{ m/s})(\cos 30°) = 3.4641 \text{ m/s}$$

Projectile motion between A and B:

After rebound



$$(v_x)_0 = (v_A')_t \cos 30° + (v_A')_n \sin 30°$$
$$(v_x)_0 = (2.5)(\cos 30°) + (3.4641)\sin 30 = 3.8971 \text{ m/s}$$
$$(v_y)_0 = -(v_A')_t \sin 30° + (v_A')_n \cos 30°$$
$$(v_y)_0 = -(2.5)(\sin 30°) + (3.4641)\cos 30° = 1.750 \text{ m/s}$$

x-direction:

$$x = (v_x)_0 t \qquad v_x = (v_x)_0$$
$$x = 3.8971t \quad v_x = 3.8971 \text{ m/s} = v_B$$

y-direction:

$$y = (v_y)_0 t - \frac{1}{2} g t^2$$
$$v_y = (v_y)_0 - gt$$

At A:

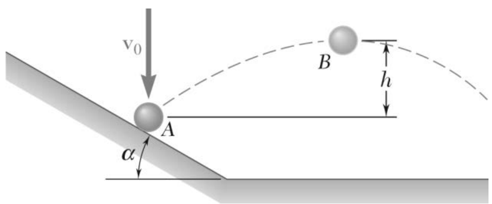
$$v_y = 0 = (v_y)_0 - g t_{AB}$$
$$t_{AB} = (v_y)_0 / g = \frac{1.75 \text{ m/s}}{9.81 \text{ m/s}^2}$$
$$t_{A-B} = 0.17839 \text{ s}$$

At B:

$$y = h = (v_y)_0 t_{A-B} - \frac{g t_{A-B}^2}{2}$$
$$h = (1.75)(0.17839) - \frac{9.81}{2}(0.17839)^2$$

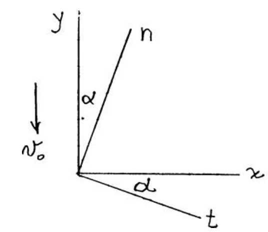
$h = 0.156$ m ◀

PROBLEM 13.173



A sphere rebounds as shown after striking an inclined plane with a vertical velocity $\mathbf{v}_0$ of magnitude $v_0 = 6$ m/s. Determine the value of α that will maximize the horizontal distance the ball travels before reaching its maximum height h assuming the coefficient of restitution between the ball and the ground is (*a*) $e = 1$, (*b*) $e = 0.8$.

SOLUTION



Directions *x*, *y*, *n*, and *t* are shown in the sketch.

Analysis of the impact: Use the principle of impulse and momentum for components in the *t*-direction.

$$mv_0 \sin\alpha + 0 = m(v_t')_1$$

$$(v_t)_1 = v_0 \sin\alpha \qquad (1)$$

Coefficient of restitution: $(\mathbf{v}_n)_1 = -e(\mathbf{v}_n)_0$

$$(v_n)_1 = ev_0 \cos\alpha \qquad (2)$$

x and *y* components of velocity immediately after impact:

$$(v_x)_1 = (v_n)_1 \sin\alpha + (v_t)_1 \cos\alpha = v_0(1+e)\sin\alpha\cos\alpha$$

$$= \frac{1}{2}v_0(1+e)\sin 2\alpha \qquad (3)$$

$$(v_y)_1 = (v_n)_1 \cos\alpha - (v_t)_1 \sin\alpha = v_0(e\cos^2\alpha - \sin^2\alpha)$$

$$= \frac{1}{2}v_0[e(1+\cos 2\alpha) - (1-\cos 2\alpha)]$$

$$= \frac{1}{2}v_0[(1+e)\cos 2\alpha - (1-e)] \qquad (4)$$

Projectile motion: Use coordinates *x* and *y* with the origin at the point of impact.

$$x_0 = 0$$

$$y_0 = 0$$

Vertical motion: $v_y = (v_y)_1 - gt$

$$v_y = \frac{1}{2}v_0[(1+e)\cos 2\alpha - (1-e)] - gt$$

$v_y = 0$ at the position of maximum height where

$$t_2 = \frac{(v_y)_1}{g} = \frac{v_0}{2g}[(1+e)\cos\alpha - (1-e)] \qquad (5)$$

Horizontal motion: $v_x = (v_x)_1 = \frac{1}{2} v_0 (1+e) \sin 2\alpha$

$$x = (v_x)_1 t$$

At the point of maximum height,

$$x_2 = (v_x)_1 t_2 = \frac{v_0^2}{4g}(1+e)\sin 2\alpha[(1+e)\cos 2\alpha - (1-e)]$$

Let $\theta = 2\alpha$ and $Z = 4gx_2/v_0^2(1+e)$. To determine the value of θ that maximizes x_2 (or Z), differentiate Z with respect to θ and set the derivative equal to zero.

$$Z = \sin\theta[(1+e)\cos\theta - (1-e)]$$

$$\frac{dZ}{d\theta} = \cos\theta[(1+e)\cos\theta - (1-e)] - (1+e)\sin^2\theta$$

$$= (1+e)\cos^2\theta - (1-e)\cos\theta - (1+e)(1-\cos^2\theta) = 0$$

$$2(1+e)\cos^2\theta - (1-e)\cos\theta - (1+e) = 0$$

This is a quadratic equation for $\cos\theta$.

(*a*) $e = 1$ $4\cos^2\theta - 2 = 0$

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \pm\frac{\sqrt{2}}{2}$$

$$\theta = \pm 45^\circ \quad \text{and} \quad \pm 135^\circ$$

$$\alpha = 22.5^\circ \quad \text{and} \quad 67.5^\circ$$

Reject the negative values of θ which make x_2 negative.

Reject $\alpha = 67.5^\circ$ since it makes a smaller maximum height.

$\alpha = 22.5^\circ$ ◀

(*b*) $e = 0.8$ $3.6\cos^2\theta - 0.2\cos\theta - 1.8 = 0$

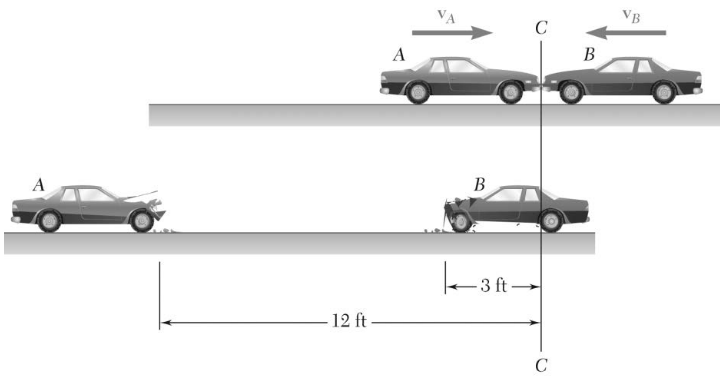
$$\cos\theta = 0.73543 \quad \text{and} \quad -0.67987$$

$$\theta = \pm 42.656^\circ \quad \text{and} \quad \pm 132.833^\circ$$

$$\alpha = \pm 21.328^\circ \quad \text{and} \quad \pm 66.417^\circ$$

$\alpha = 21.3^\circ$ ◀

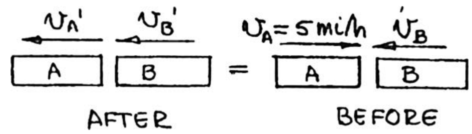
PROBLEM 13.174



Two cars of the same mass run head-on into each other at C. After the collision, the cars skid with their brakes locked and come to a stop in the positions shown in the lower part of the figure. Knowing that the speed of car A just before impact was 5 mi/h and that the coefficient of kinetic friction between the pavement and the tires of both cars is 0.30, determine (*a*) the speed of car B just before impact, (*b*) the effective coefficient of restitution between the two cars.

SOLUTION

(*a*) At C:



Conservation of total momentum:

$$m_A = m_B = m$$

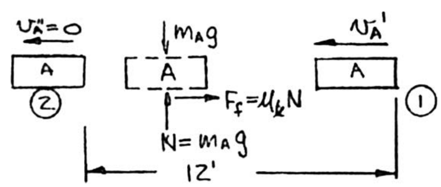
$$5 \text{ mi/h} = 7.333 \text{ ft/s}$$

$$\overset{+}{\leftarrow} \quad m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$-7.333 + v_B = v_A' + v_B' \tag{1}$$

Work and energy.

Care A (after impact):



$$T_1 = \frac{1}{2} m_A (v_A')^2$$

$$T_2 = 0$$

$$U_{1-2} = F_f (12)$$

$$U_{1-2} = \mu_k m_A g (12 \text{ ft})$$

$$T_1 + U_{1-2} = T_2$$

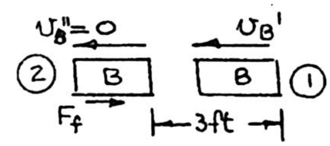
$$\frac{1}{2} m_A (v_A')^2 - m_k m_A g (12) = 0$$

$$(v_A')^2 = (2)(12 \text{ ft})(0.3)(32.2 \text{ ft/s}^2)$$

$$= 231.84 \text{ ft/s}^2$$

$$v_A' = 15.226 \text{ ft/s}$$

Car B (after impact):



$$T_1 = \frac{1}{2} m_B (v_B')^2$$
$$T_2 = 0$$
$$U_{1-2} = \mu_k\, m_B\, g(3)$$
$$T_1 + U_{1-2} = T_2$$
$$\frac{1}{2} m_B (v_B')^2 - \mu_k m_B g(3)$$
$$v_B'^2 = (2)(3\text{ ft})(0.3)(32.2\text{ ft/s}^2)$$
$$(v_B')^2 = 57.96\text{ ft/s}^2$$
$$v_B' = 7.613\text{ ft/s}$$

From (1)
$$v_B = 7.333 + v_A' + v_B'$$
$$= 7.333 + 15.226 + 7.613$$

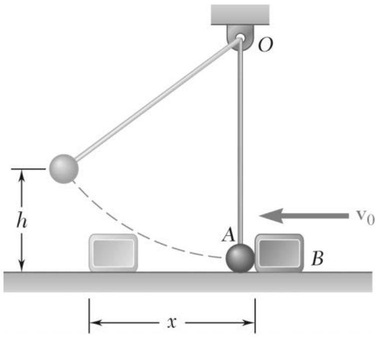
$$v_B = 30.2\text{ ft/s} = 20.6\text{ mi/h} \blacktriangleleft$$

(b) Relative velocities:

$$\overset{+}{\leftarrow} \quad (-v_A - v_B)e = v_B' - v_A'$$
$$(-7.333 - 30.2)e = 7.613 - 15.226$$
$$e = \frac{-(7.613)}{-(37.53)} = 0.2028$$

$$e = 0.203 \blacktriangleleft$$

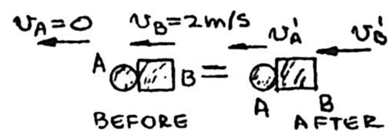
PROBLEM 13.175



A 1-kg block B is moving with a velocity $\mathbf{v}_0$ of magnitude $v_0 = 2$ m/s as it hits the 0.5-kg sphere A, which is at rest and hanging from a cord attached at O. Knowing that $\mu_k = 0.6$ between the block and the horizontal surface and $e = 0.8$ between the block and the sphere, determine after impact (*a*) the maximum height h reached by the sphere, (*b*) the distance x traveled by the block.

SOLUTION

Velocities just after impact



Total momentum in the horizontal direction is conserved:

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$0 + (1\text{ kg})(2\text{ m/s}) = (0.5\text{ kg})(v_A') + (1\text{ kg})(v_B')$$

$$4 = v_A' + 2v_B' \qquad (1)$$

Relative velocities:

$$(v_A - v_B)e = (v_B' - v_A')$$

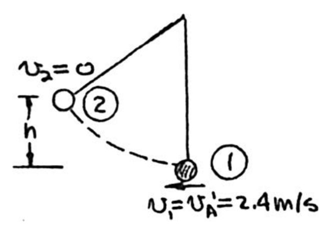
$$(0-2)(0.8) = v_B' - v_A'$$

$$-1.6 = v_B' - v_A' \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously:

$$v_B' = 0.8\text{ m/s}$$

$$v_A' = 2.4\text{ m/s}$$



(*a*) Conservation of energy:

$$T_1 = \frac{1}{2}m_A v_1^2 \quad V_1 = 0$$

$$T_1 = \frac{1}{2}m_A(2.4\text{ m/s})^2 = 2.88 m_A$$

$$T_2 = 0$$

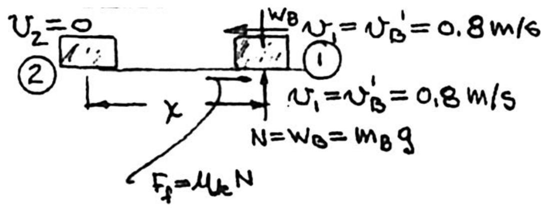
$$V_2 = m_A g h$$

$$T_1 + V_1 = T_2 + V_2 \qquad 2.88 \not{m}_A + 0 = 0 + \not{m}_A(9.81)h$$

$h = 0.294$ m ◀

PROBLEM 13.175 (Continued)

(*b*) Work and energy:



$$T_1 = \frac{1}{2} m_B v_1^2 = \frac{1}{2} m_B (0.8 \text{ m/s})^2 = 0.32 m_B \qquad T_2 = 0$$

$$U_{1-2} = -F_f x = -\mu_k N x = -\mu_x m_B g x = -(0.6)(m_B)(9.81)x$$

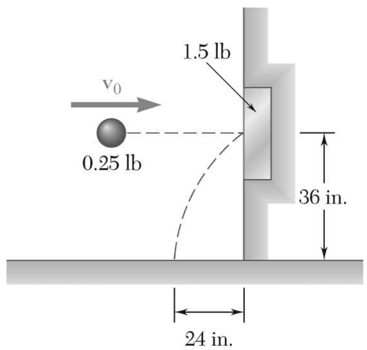
$$U_{1-2} = -5.886 m_B x$$

$$T_1 + U_{1-2} = T_2: \qquad 0.32 m_B - 5.886 m_B x = 0$$

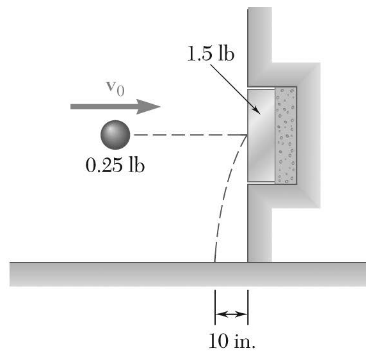
$$x = 0.0544 \text{ m} \qquad\qquad x = 54.4 \text{ mm} \blacktriangleleft$$

PROBLEM 13.176

A 0.25-lb ball thrown with a horizontal velocity $\mathbf{v}_0$ strikes a 1.5-lb plate attached to a vertical wall at a height of 36 in. above the ground. It is observed that after rebounding, the ball hits the ground at a distance of 24 in. from the wall when the plate is rigidly attached to the wall (Figure 1) and at a distance of 10 in. when a foam-rubber mat is placed between the plate and the wall (Figure 2). Determine (*a*) the coefficient of restitution e between the ball and the plate, (*b*) the initial velocity $\mathbf{v}_0$ of the ball.

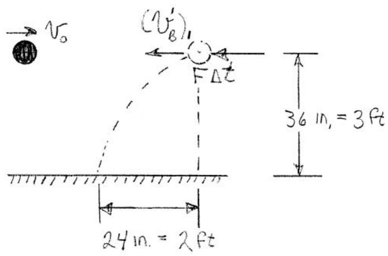


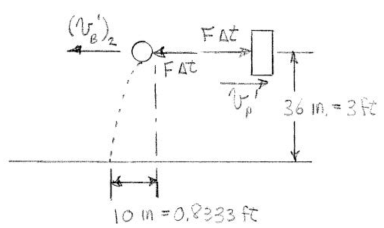
(1)



(2)

SOLUTION





(*a*) Figure (1), ball alone relative velocities

$$v_0 e = (v'_B)_1$$

Projectile motion

t = time for the ball to hit ground

$$2 \text{ ft} = v_0 e t \qquad (1)$$

Figure (2), ball and plate relative velocities

$$(v_B - v_A)e = v'_P + (v'_B)_2$$

$$v_B = v_0, \qquad v_P = 0$$

$$v_0 e = v'_P + (v'_B)_2 \qquad (2)$$

Conservation of momentum

$$\xrightarrow{+} m_B v_B + m_P v_P = m_B v'_B + m_P v'_P$$

$$\frac{0.25}{g} v_0 + 0 = \frac{0.25}{g}(-v'_B)_2 + \frac{1.5}{g} v'_P$$

PROBLEM 13.176 (Continued)

$$0.25v_0 = -0.25(v_B')_2 + 1.5v_p' \Rightarrow v_0 = -(v_B')_2 - 6v_p' \quad (3)$$

Solving (2) and (3) for $(v_B')_2$, $\quad (v_B')_2 = \frac{(6e-1)}{7}v_0$

Projectile motion

$$0.8333 = \frac{(6e-1)}{7}v_0 t \quad (4)$$

Dividing Equation (4) by Equation (1)

$$\frac{0.8333}{2} = \frac{6e-1}{7e}; \ 2.91655e = 6e - 1$$

$$e = 0.324 \blacktriangleleft$$

(*b*) From Figure (1)

Projectile motion, $\quad h = \frac{1}{2}gt^2; \ 3 = \frac{1}{2}(32.2)t^2$

$$6 = 32.2t^2 \quad (5)$$

From Equation (1),

$$2 = v_0 e t \Rightarrow t = \frac{2}{0.324v_0} = \frac{6.1728}{v_0}$$

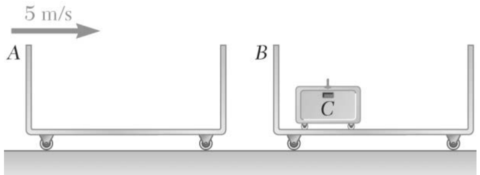
Using Equation (5)

$$6 = 32.2\left(\frac{6.1728}{v_0}\right)^2 \Rightarrow 6v_0^2 = 1226.947$$

$$v_0^2 = 204.49$$

$$v_0 = 14.30 \text{ ft/s} \blacktriangleleft$$

PROBLEM 13.177

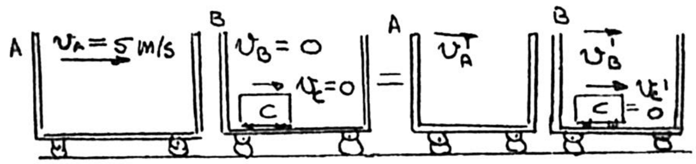


After having been pushed by an airline employee, an empty 40-kg luggage carrier *A* hits with a velocity of 5 m/s an identical carrier *B* containing a 15-kg suitcase equipped with rollers. The impact causes the suitcase to roll into the left wall of carrier *B*. Knowing that the coefficient of restitution between the two carriers is 0.80 and that the coefficient of restitution between the suitcase and the wall of carrier *B* is 0.30, determine (*a*) the velocity of carrier *B* after the suitcase hits its wall for the first time, (*b*) the total energy lost in that impact.

SOLUTION

(*a*) Impact between *A* and *B*:

Total momentum conserved:



$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \qquad m_A = m_B = 40 \text{ kg}$$

$$\xrightarrow{+} 5 \text{ m/s} + 0 = v_A' + v_B' \tag{1}$$

Relative velocities:

$$(v_A - v_B)e_{AB} = v_B' - v_A'$$

$$(5-0)(0.80) = v_B' - v_A' \tag{2}$$

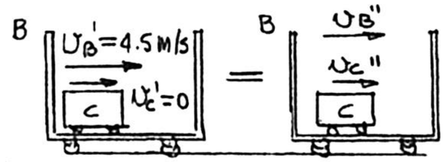
Adding Eqs. (1) and (2)

$$(5 \text{ m/s})(1+0.80) = 2v_B'$$

$$v_B' = 4.5 \text{ m/s} \longrightarrow$$

Impact between *B* and *C* (after *A* hits *B*)

Total momentum conserved:



$$\xrightarrow{+} m_B v_B' + m_C v_C' = m_B v_B'' + m_C v_C''$$

$$(40 \text{ kg})(4.5 \text{ m/s}) + 0 = (40 \text{ kg})\, v_B'' + (15 \text{ kg})\, v_C''$$

$$4.5 = v_B'' + 0.375 v_C'' \tag{3}$$

Relative velocities:

$$(v_B' - v_C')\, e_{BC} = v_C'' - v_B''$$

$$(4.5-0)(0.30) = v_C'' - v_B'' \tag{4}$$

PROBLEM 13.177 (Continued)

Adding Eqs. (4) and (3)

$$(4.5)(1+0.3) = (1.375)v_C''$$

$$v_C'' = 4.2545 \text{ m/s}$$

$$v_B'' = 4.5 - 0.375(4.2545)\, v_B'' = 2.90 \text{ m/s} \qquad v_B' = 2.90 \text{ m/s} \blacktriangleleft$$

(*b*)

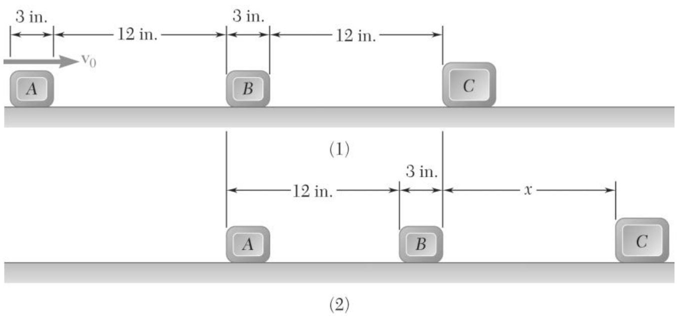
$$\Delta T_L = (T_B' + T_C') - (T_B'' + T_C'')$$

$$T_B' = \frac{1}{2} m_B (v_B')^2 = \left(\frac{40}{2} \text{ kg}\right)(4.5 \text{ m/s})^2 = 405 \text{ J}$$

$$T_C' = 0 \qquad T_B'' = \frac{1}{2} m_B (v_B'')^2 = \left(\frac{40}{2} \text{ kg}\right)(2.90)^2 = 168.72 \text{ J}$$

$$T_C'' = \frac{1}{2} m_C (v_C'')^2 = \left(\frac{15}{2} \text{ kg}\right)(4.2545 \text{ m/s})^2 = 135.76 \text{ J}$$

$$\Delta T_L = (405 + 0) - (168.72 + 135.76) = 100.5 \text{ J} \qquad \Delta T_L = 100.5 \text{ J} \blacktriangleleft$$



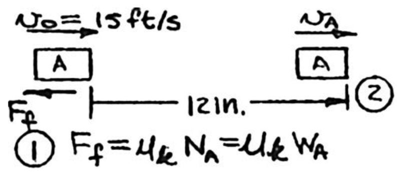
PROBLEM 13.178

Blocks A and B each weigh 0.8 lb and block C weighs 2.4 lb. The coefficient of friction between the blocks and the plane is $\mu_k = 0.30$. Initially block A is moving at a speed $v_0 = 15$ ft/s and blocks B and C are at rest (Fig. 1). After A strikes B and B strikes C, all three blocks come to a stop in the positions shown (Fig. 2). Determine (*a*) the coefficients of restitution between A and B and between B and C, (*b*) the displacement x of block C.

SOLUTION

(*a*) Work and energy

Velocity of A just before impact with B:



$$T_1 = \frac{1}{2}\frac{W_A}{g}v_0^2 \quad T_2 = \frac{1}{2}\frac{W_A}{g}\left(v_A^2\right)_2$$

$$U_{1-2} = -\mu_k W_A\,(1\text{ ft})$$

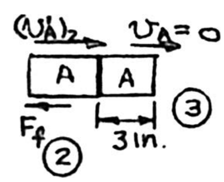
$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2}\frac{W_A}{g}v_0^2 - \mu_k W_A(1) = \frac{1}{2}\frac{W_A}{g}\left(v_A^2\right)_2$$

$$(v_A^2)_2 = v_0^2 - 2\mu_k g = (15\text{ ft/s})^2 - 2(0.3)(32.2\text{ ft/s}^2)(1\text{ ft})$$

$$(v_A^2)_2 = 205.68\text{ ft/s}^2, \quad (v_A)_2 = 14.342\text{ ft/s}$$

Velocity of A after impact with B: $(v'_A)_2$



$$T'_2 = \frac{1}{2}\frac{W_A}{g}(v'_A)_2^2 \quad T_3 = 0$$

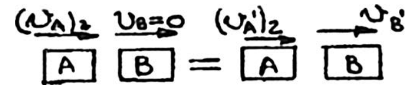
$$U_{2-3} = -\mu_k W_A(3/12)$$

$$T'_2 + U_{2-3} = T_3, \quad \frac{1}{2}\frac{W_A}{g}(v'_A)_2^2 - (\mu_k)(W_A/4) = 0$$

$$(v'_A)_2^2 = 2(0.3)(32.2\text{ ft/s}^2)\left(\frac{1}{4}\text{ft}\right) = 4.83\text{ ft}^2/\text{s}^2$$

$$(v'_A)_2 = 2.198\text{ ft/s}$$

Conservation of momentum as *A* hits *B*:



$$(v_A)_2 = 14.342 \text{ ft/s}$$
$$(v'_A)_2 = 2.198 \text{ ft/s}$$

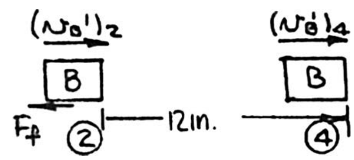
$$\xrightarrow{+} m_A(v_A)_2 + m_B v_B = m_B(v'_A)_2 + m_B v'_B \quad m_A = m_B$$
$$14.342 + 0 = 2.198 + v'_B \qquad v'_B = 12.144 \text{ ft/s}$$

Relative velocities *A* and *B*:

$$\xrightarrow{+} [(v_A)_2 - v_B]e_{AB} = v'_B - (v'_A)_2$$
$$(14.342 - 0)e_{AB} = 12.144 - 2.198 \qquad e_{AB} = 0.694 \blacktriangleleft$$

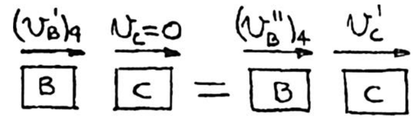
Work and energy.

Velocity of *B* just before impact with *C*:



$$T_2 = \frac{1}{2}\frac{W_B}{g}(v'_B)_2^2 = \frac{W_B}{2g}(12.144)^2$$
$$T_4 = \frac{1}{2}\frac{W_B}{g}(v'_B)_4^2 = \frac{W_B}{2g}(v'_B)_4^2$$
$$(v'_B) = 12.144 \text{ ft/s} \qquad U_{2-4} = -\mu_k W_B(1 \text{ ft}) = (0.3)W_B$$
$$F_f = \mu_k W_B \qquad T_2 + U_{2-4} = T_4, \qquad \frac{(12.144)^2}{2g} - 0.3 = \frac{(v'_B)_4^2}{2g}$$
$$(v'_B)_4 = 11.321 \text{ ft/s}$$

Conservation of momentum as *B* hits *C*:



$$m_B = \frac{0.8}{g}$$
$$m_C = \frac{2.4}{g}$$
$$(v'_B)_4 = 11.321 \text{ ft/s}$$

$$\xrightarrow{+} m_B(v'_B)_4 + m_C v_C = m_B(v''_B)_4 + m_C v'_C$$
$$\frac{0.8}{g}(11.321) + 0 = \frac{0.8}{g}(v''_B)_4 + \frac{(2.4)}{g}(v'_C)$$
$$11.321 = (v''_B)_4 + 3v'_C$$

PROBLEM 13.178 (Continued)

Velocity of B after B hits $C, (v_B'')_4 = 0.$

(Compare Figures (1) and (2).)

$$v_C' = 3.774 \text{ ft/s}$$

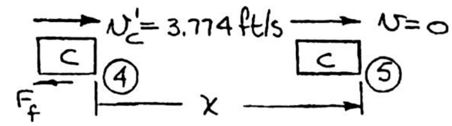
Relative velocities B and C:

$$((v_B')_4 - v_C)e_{BC} = v_C' - (v_B'')_4$$
$$(11.321 - 0)e_{BC} = 3.774 - 0$$

$e_{BC} = 0.333$ ◀

(*b*) Work and energy, Block C:

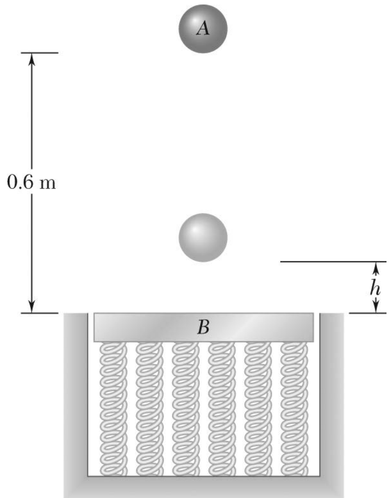
$$T_4 = \frac{1}{2}\frac{W_C}{g}(v_C)^2 \qquad T_5 = 0 \qquad U_{4-5} = -\mu_k W_C(x)$$



$$T_4 + U_{4-5} = T_5 \qquad \frac{1}{2}\frac{W_C}{g}(3.774)^2 - (0.3)W_C(x) = 0$$

$$x = \frac{(3.774)^2}{2(32.2)(0.3)} = 0.737 \text{ ft}$$

$x = 8.84$ in. ◀

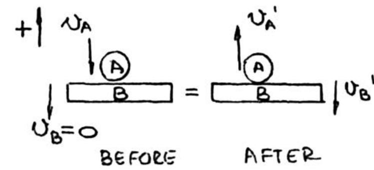


PROBLEM 13.179

A 0.5-kg sphere A is dropped from a height of 0.6 m onto a 1.0 kg plate B, which is supported by a nested set of springs and is initially at rest. Knowing that the coefficient of restitution between the sphere and the plate is $e = 0.8$, determine (*a*) the height h reached by the sphere after rebound, (*b*) the constant k of the single spring equivalent to the given set if the maximum deflection of the plate is observed to be equal to $3h$.

SOLUTION

Velocity of A and B after impact.



$$m_A = 0.5 \text{ kg}$$
$$m_B = 1.0 \text{ kg}$$

Sphere A falls. Use conservation of energy to find v_A, the speed just before impact. Use the plate surface as the datum.

$$T_1 = 0, \quad V_1 = m_A g h_0, \quad T_2 = \frac{1}{2} m_A v_A^2, \quad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + m_A g h_0 = \frac{1}{2} m_A v_A^2 + 0$$

With $h_0 = 0.6$ m,

$$v_A = \sqrt{2 g h_0} = \sqrt{(2)(9.81)(0.6)}$$

$$\mathbf{v}_A = 3.4310 \text{ m/s} \downarrow$$

Analysis of the impact. Conservation of momentum.

$$m_A \mathbf{v}_A + m_B \mathbf{v}_B = m_A \mathbf{v}_A' + m_B \mathbf{v}_B' \quad \text{with} \quad \mathbf{v}_B = 0$$

Dividing by m_A and using y-components $+\uparrow$ with $(m_B/m_A = 2)$

$$-3.4310 + 0 = (v_A')_y + 2(v_B')_y \tag{1}$$

Coefficient of restitution. $(v_B')_y - (v_A')_y = e[(v_A)_y - (v_B)_y]$

$$(v_B')_y - (v_A')_y = e(v_A)_y = -3.4310e \tag{2}$$

PROBLEM 13.179 (Continued)

Solving Eqs. (1) and (2) simultaneously with $e = 0.8$ gives

$$(v'_A)_y = 0.68621 \text{ m/s}$$
$$(v'_B)_y = -2.0586 \text{ m/s}$$
$$\mathbf{v}'_A = 0.68621 \text{ m/s} \uparrow$$
$$\mathbf{v}'_B = 2.0586 \text{ m/s} \downarrow$$

(*a*) Sphere *A* rises. Use conservation of energy to find *h*.

$$T_1 = \frac{1}{2}m_A(v'_A)^2, \quad V_1 = 0, \quad T_2 = 0, \quad V_2 = m_A gh$$

$$T_1 + V_1 = T_2 + V_2: \qquad \frac{1}{2}m_A(v'_A)^2 + 0 = 0 + m_A gh$$

$$h = \frac{(v'_A)^2}{2g} = \frac{(0.68621)^2}{(2)(9.81)} \qquad h = 0.0240 \text{ m} \blacktriangleleft$$

(*b*) Plate *B* falls and compresses the spring. Use conservation of energy.

Let δ_0 be the initial compression of the spring and Δ be the additional compression of the spring after impact. In the initial equilibrium state,

$$+\uparrow \Sigma F_y = 0: \quad k\delta_0 - W_B = 0 \quad \text{or} \quad k\delta_0 = W_B \qquad (3)$$

Just after impact: $T_1 = \frac{1}{2}m_B(v'_B)^2, \quad V_1 = \frac{1}{2}k\delta_0^2$

At maximum deflection of the plate, $T_2 = 0$

$$V_2 = (V_2)_g + (V_2)_e = -W_B\Delta + \frac{1}{2}k(\delta_0 + \Delta)^2$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}m_B(v'_B)^2 + \frac{1}{2}k\delta_0^2 = 0 - W_B\Delta + \frac{1}{2}k\delta_0^2 + k\delta_0\Delta + \frac{1}{2}k\Delta^2$$

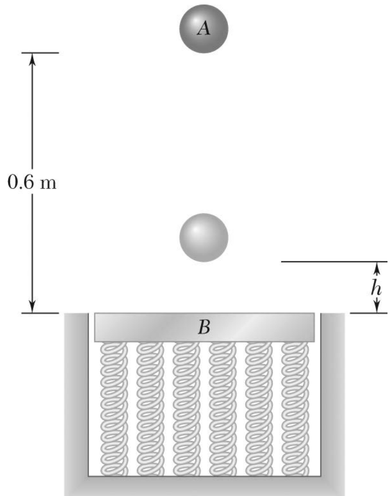
Invoking the result of Eq. (3) gives

$$\frac{1}{2}m_B(v'_B)^2 = \frac{1}{2}k\Delta^2 \qquad (4)$$

Data: $m_B = 1.0 \text{ kg}, \quad v'_B = 2.0586 \text{ m/s}$

$$\Delta = 3h = (3)(0.024) = 0.072 \text{ m}$$

$$k = \frac{m_B(v'_B)^2}{\Delta^2} = \frac{(1.0)(2.0586)^2}{(0.072)^2} \qquad k = 817 \text{ N/m} \blacktriangleleft$$



PROBLEM 13.180

A 0.5-kg sphere A is dropped from a height of 0.6 m onto a 1.0-kg plate B, which is supported by a nested set of springs and is initially at rest. Knowing that the set of springs is equivalent to a single spring of constant $k = 900$ N/m, determine (a) the value of the coefficient of restitution between the sphere and the plate for which the height h reached by the sphere after rebound is maximum, (b) the corresponding value of h, (c) the corresponding value of the maximum deflection of the plate.

SOLUTION

$$m_A = 0.5 \text{ kg}$$
$$m_B = 1.0 \text{ kg}$$
$$k = 900 \text{ N/m}$$

<u>Sphere A falls</u>. Use conservation of energy to find v_A, the speed just before impact. Use the plate surface as the datum.

$$T_1 = 0 \qquad V_1 = m_A g h_0$$
$$T_2 = \frac{1}{2} m_A v_A^2, \qquad V_2 = 0$$

With $h_0 = 0.6$ m, $\quad v_A = \sqrt{2gh_0} = \sqrt{(2)(9.81)(0.6)}$

$$\mathbf{v}_A = 3.4310 \text{ m/s} \downarrow$$

<u>Analysis of impact</u>. Conservation of momentum.

$$m_A \mathbf{v}_A + m_B \mathbf{v}_B = m_A \mathbf{v}'_A + m_B v'_B \text{ with } \mathbf{v}_B = 0$$

Dividing by m_A and using y components $+\uparrow$ with $(m_B/m_A = 2)$

$$-3.4310 = (v'_A)_y + 2(v'_B)_y \qquad (1)$$

<u>Coefficient of restitution</u>. $\quad (v'_B)_y - (v'_A)_y = e[(v_A)_y - (v_B)_y]$

$$(v'_B)_y - (v'_A)_y = e(v_A)_y = -3.4310e$$
$$(v'_B)_y = -3.4310 + (v'_A)_y \qquad (2)$$

PROBLEM 13.180 (Continued)

Substituting into Eq. (1),

$$-3.4310 = (v'_A)_y + (2)[-3.4310e + (v'_A)_y]$$

$$(v'_A)_y = 1.1437(2e-1) \quad (3)$$

From Eq. (2), $$(v'_B)_y = -1.1437(1+e) \quad (4)$$

(*a*) Sphere *A* rises. Use conservation of energy to find *h*.

$$T_1 = \frac{1}{2}m_A(v'_A)^2, \qquad V_1 = 0$$

$$T_2 = 0, \qquad V_2 = m_A gh$$

$$T_1 + V_1 = T_2 + V_2: \quad \frac{1}{2}m_A(v'_A)^2 + 0 = 0 + m_A gh$$

$$h = \frac{(v'_A)^2}{2g} = \frac{(1.1437)^2(2e-1)^2}{(2)(9.81)}$$

Since h is to be maximum, e must be as large as possible.

Coefficient of restitution for maximum h: $e = 1.000$ ◀

(*b*) Corresponding value of *h*. $(v'_A) = 1.1437[(2)(1)-1] = 1.1437 \text{ m/s}$

$$h = \frac{(v'_A)^2}{2g} = \frac{(1.1437)^2}{(2)(9.81)}$$

$h = 0.0667$ m ◀

(*c*) Plate *B* falls and compresses the spring. Use conservation of energy.

Let δ_0 be the initial compression of the spring and Δ be the additional compression of the spring after impact. In the initial equilibrium state,

$$+\uparrow \Sigma F_y = 0 \quad k\delta_0 - W_B = 0 \quad \text{or} \quad k\delta_0 = W_B \quad (3)$$

Just after impact: $$T_1 = \frac{1}{2}m_B(v'_B)^2, \qquad V_1 = \frac{1}{2}k\delta_0^2$$

At maximum deflection of the plate, $T_2 = 0$

$$V_2 = (V_2)_g + (V_2)_e = -W_B\Delta + \frac{1}{2}k(\delta_0 + \Delta)^2$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}m_B(v'_B)^2 + \frac{1}{2}k\delta_0^2 = 0 - W_B\Delta + \frac{1}{2}k\delta_0^2 + k\delta_0\Delta + \frac{1}{2}k\Delta^2$$

Invoking the result of Eq. (3) gives

$$\frac{1}{2}m_B(v'_B)^2 = \frac{1}{2}k\Delta^2$$

Data: $m_B = 1.0 \text{ kg},$

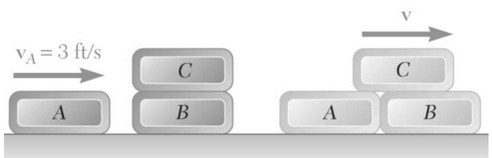
$$(v'_B)_y = -1.1437(1+1) = -2.2874 \text{ m/s.}$$

$$\mathbf{v}'_B = 2.2874 \text{ m/s} \downarrow, \quad k = 900 \text{ N/m}$$

$$\Delta^2 = \frac{m_B(v'_B)^2}{k} = \frac{(1.0)(2.2874)^2}{900} = 0.0058133 \text{ m}^2$$

$\Delta = 0.0762 \text{ m}$ ◀

PROBLEM 13.181



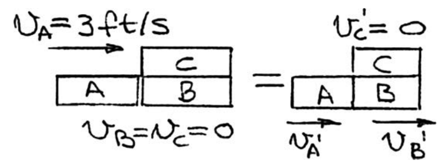
The three blocks shown are identical. Blocks B and C are at rest when block B is hit by block A, which is moving with a velocity $\mathbf{v}_A$ of 3 ft/s. After the impact, which is assumed to be perfectly plastic $(e = 0)$, the velocity of blocks A and B decreases due to friction, while block C picks up speed, until all three blocks are moving with the same velocity $\mathbf{v}$. Knowing that the coefficient of kinetic friction between all surfaces is $\mu_k = 0.20$, determine (a) the time required for the three blocks to reach the same velocity, (b) the total distance traveled by each block during that time.

SOLUTION

(a) Impact between A and B, conservation of momentum

$$mv_A + mv_B + mv_C = mv'_A + mv'_B + mv'_C$$

$$3 + 0 = v'_A + v'_B + 0$$



Relative velocities $(e = 0)$

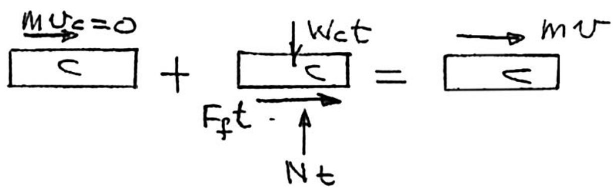
$$(v_A - v_B)e = v'_B - v'_A \qquad 3 = 2v'_B$$

$$0 = v'_B - v'_A \qquad v'_B = 1.5 \text{ ft/s}$$

$$v'_A = v'_B$$

$v =$ Final (common) velocity

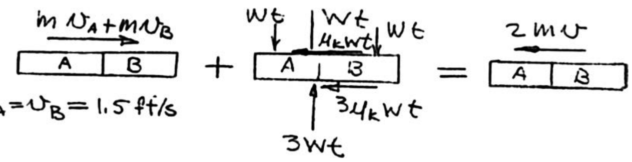
Block C: Impulse and momentum



$$\xrightarrow{+} W_C v_C + F_f t = \frac{W_C}{g} v \qquad F_f = \mu_k W_C$$

$$0 + (0.2)t = \frac{v}{g} \qquad v = (0.2)gt \qquad (1)$$

Blocks *A* and *B*: Impulse and momentum



$$2\frac{W}{g}(1.5) - 4(0.2)Wt = 2\frac{W}{g}v$$

$$1.5 - 0.4gt = v \qquad (2)$$

Substitute *v* from Eq. (1) into Eq. (2)

$$1.5 - 0.4gt = 0.2gt$$

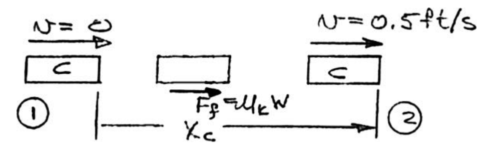
$$t = \frac{(1.5 \text{ ft/s})}{0.6(32.2 \text{ ft/s}^2)} \qquad t = 0.0776 \text{ s} \blacktriangleleft$$

(*b*) Work and energy:

From Eq. (1) $\quad v = (0.2)(32.2)(0.0776) = 0.5$ ft/s

Block *C*:

$$T_1 = 0 \qquad T_2 = \frac{1}{2}\frac{W}{g}(v)^2 = \frac{W}{2g}(0.5)^2$$



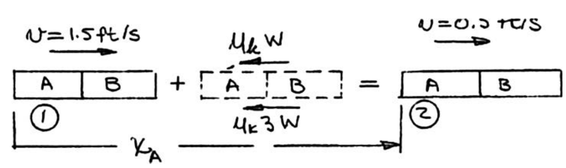
$$U_{1-2} = F_f x_C = \mu_k W x_C = 0.2W x_C$$

$$T_1 + U_{1-2} = T_2 \qquad 0 + (0.2)(W)x_C = \frac{1}{2}\frac{W}{g}v^2$$

$$x_C = \frac{(0.5 \text{ ft/s})^2}{0.2(2)(32.2 \text{ ft/s}^2)} = 0.01941 \text{ ft}$$

$$x_C = 0.01941 \text{ ft} \blacktriangleleft$$

Blocks *A* and *B*:



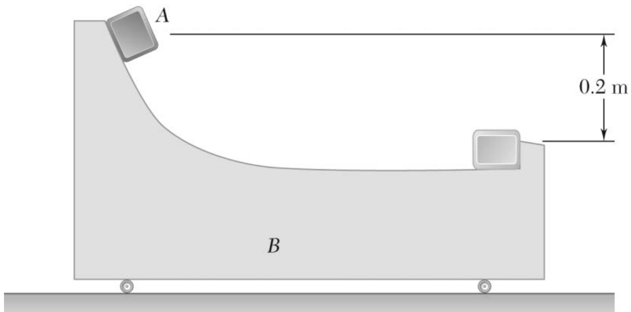
$$T_1 = \frac{1}{2}\left(2\frac{W}{g}\right)(1.5)^2 = 2.25W \qquad T_2 = \frac{1}{2}\left(2\frac{W}{g}\right)(0.5)^2 = 0.25W$$

$$U_{1-2} = -4\mu_k W g x_A = -0.8W g x_A$$

$$T_1 + U_{1-2} = T_2$$

$$2.25W - 4(0.2)W(32.2)x_A = 0.25W$$

$$x_A = 0.07764 \text{ ft} \qquad x_A = 0.0776 \text{ ft} \blacktriangleleft$$



PROBLEM 13.182

Block A is released from rest and slides down the frictionless surface of B until it hits a bumper on the right end of B. Block A has a mass of 10 kg and object B has a mass of 30 kg and B can roll freely on the ground. Determine the velocities of A and B immediately after impact when (a) $e = 0$, (b) $e = 0.7$.

SOLUTION

Let the x-direction be positive to the right and the y-direction vertically upward.

Let $(v_A)_x$, $(v_A)_y$, $(v_A)_x$ and $(v_B)_y$ be velocity components just before impact and $(v'_A)_x, (v'_A)'_y, (v'_B)_x$, and $(v'_B)_y$ those just after impact. By inspection,

$$(v_A)_y = (v_B)_y = (v'_A)_y = (v'_B)_y = 0$$

Conservation of momentum for x-direction:

While block is sliding down: $\quad 0 + 0 = m_A(v_A)_x + m_B(v_B)_x \quad (v_B)_x = -\beta(v_A)_x \qquad (1)$

Impact: $\quad 0 + 0 = m_A(v'_A)_x + m_B(v'_B)_x \quad (v'_B)_x = -\beta(v'_A)_x \qquad (2)$

where $\quad \beta = m_A/m_B$

Conservation of energy during frictionless sliding:

Initial potential energies: $m_A gh$ for A, $\quad$ 0 for B.

Potential energy just before impact: $\quad V_1 = 0$

Initial kinetic energy: $\quad T_0 = 0$ (rest)

Kinetic energy just before impact: $\quad T_1 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$

$$T_0 + V_0 = T_1 + V_1$$

$$m_A gh = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(m_A + m_B\beta^2)v_A^2$$

$$= \frac{1}{2}m_A(1+\beta)v_A^2$$

$$v_A^2 = (v_A)_x^2 = \frac{2gh}{1+\beta} \qquad v_A = \sqrt{\frac{2gh}{1+\beta}} \qquad (3)$$

PROBLEM 13.182 (Continued)

Velocities just before impact:

$$\mathbf{v}_A = \sqrt{\frac{2gh}{1+\beta}} \longrightarrow$$

$$\mathbf{v}_B = \beta\sqrt{\frac{2gh}{1+\beta}} \longleftarrow$$

Analysis of impact. Use Eq. (2) together with coefficient of restitution.

$$(v_B')_x - (v_A')_x = e[(v_A)_x - (v_B)_x]$$

$$-\beta(v_A')_x - (v_A')_x = e[(v_A)_x + \beta(v_A)_x]$$

$$(v_A')_x = -e(v_A)_x \tag{4}$$

Data:

$$m_A = 10 \text{ kg}$$

$$m_B = 30 \text{ kg}$$

$$h = 0.2 \text{ m}$$

$$g = 9.81 \text{ m/s}^2$$

$$\beta = \frac{10\,\text{kg}}{30\,\text{kg}} = 0.33333$$

From Eq. (3),

$$v_A = \sqrt{\frac{(2)(9.81)(0.2)}{1.33333}} = 1.71552 \text{ m/s}$$

(*a*) $e = 0$:

$$(v_A')_x = 0 \qquad (v_B')_x = 0$$

$\mathbf{v}_A' = 0$ ◀

$\mathbf{v}_B' = 0$ ◀

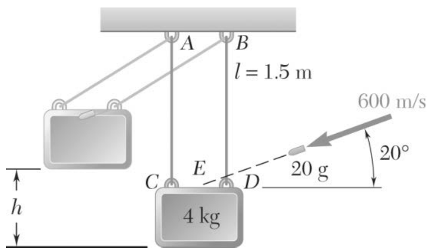
(*b*) $e = 0.7$:

$$(v_A')_x = -(0.7)(1.71552) = -1.20086 \text{ m/s}$$

$$(v_B')_x = -(0.33333)(1.20086) = 0.40029 \text{ m/s}$$

$\mathbf{v}_A' = 1.201 \text{ m/s} \longleftarrow$ ◀

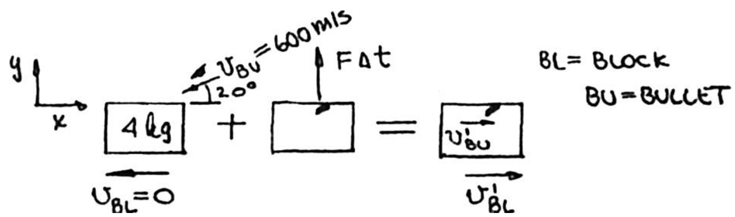
$\mathbf{v}_B' = 0.400 \text{ m/s} \longrightarrow$ ◀



PROBLEM 13.183

A 20-g bullet fired into a 4-kg wooden block suspended from cords *AC* and *BD* penetrates the block at Point *E*, halfway between *C* and *D*, without hitting cord *BD*. Determine (*a*) the maximum height *h* to which the block and the embedded bullet will swing after impact, (*b*) the total impulse exerted on the block by the two cords during the impact.

SOLUTION



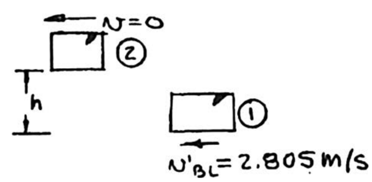
Total momentum in *x* is conserved:

$$\xrightarrow{+}\ m_{bl}v_{bl} + m_{bu}v_{bu}\cos 20° = m_{bl}v'_{bl} + m_{bu}v'_{bu} \qquad (v'_{bl} = v'_{bu})$$

$$0 + (0.02\text{ kg})(-600\text{ m/s})(\cos 20) = (4.02\text{ kg})(v'_{bl})$$

$$v'_{bl} = -2.805\text{ m/s}$$

Conservation of energy:



$$T_1 = \frac{1}{2}(m_{bl} + m_{bu})(v'_{bl})^2$$

$$T_1 = \left(\frac{4.02\text{ kg}}{2}\right)(2.805\text{ m/s})^2$$

$$T_1 = 15.815\text{ J}$$

$$V_1 = 0$$

$$T_2 = 0 \qquad V_2 = (m_{bl} + m_{bu})gh$$

$$V_2 = (4.02\text{ kg})(9.81\text{ m/s}^2)(h) = 39.44h$$

$$T_1 + V_1 = T_2 + V_2$$

$$15.815 + 0 = 0 + 39.44h$$

$$h = 0.401\text{ m} \qquad\qquad h = 401\text{ mm} \blacktriangleleft$$

(*b*) Refer to figure in part (*a*).

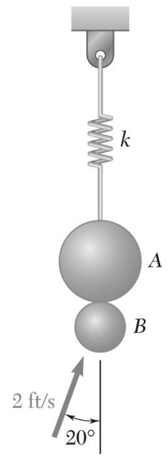
Impulse-momentum in *y*-direction:

$$+\!\uparrow\ m_{bu}v_{bu}\sin 20 + F\Delta t = (m_{bl} + m_{bu})(v'_{bl})_y$$

$$(v_{bl})_y = 0$$

$$(0.02\text{ kg})(-600\text{ m/s})(\sin 20°) + F\Delta t = 0 \qquad\qquad F\Delta t = 4.10\text{ N}\cdot\text{s} \blacktriangleleft$$

PROBLEM 13.184



A 2-lb ball A is suspended from a spring of constant 10 lb/in and is initially at rest when it is struck by 1-lb ball B as shown. Neglecting friction and knowing the coefficient of restitution between the balls is 0.6, determine (*a*) the velocities of A and B after the impact, (*b*) the maximum height reached by A.

SOLUTION

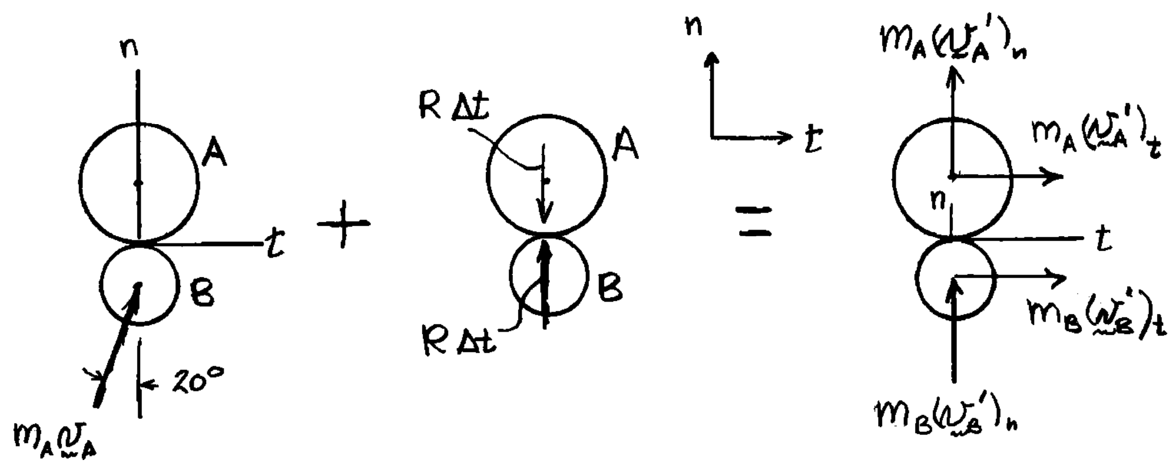
Masses: $m_A = \dfrac{2\text{ lb}}{32.2\text{ ft/s}^2} = 0.062112\text{ lb}\cdot\text{s}^2/\text{ft} \qquad m_B = \dfrac{1\text{ lb}}{32.2\text{ ft/s}^2} = 0.031056\text{ lb}\cdot\text{s}^2/\text{ft}$

Other data: $k = (10\text{ lb/in.})(12\text{ in./ft.}) = 120\text{ lb/ft}, \quad e = 0.6$

$v_A = 0, \quad v_B = 2\text{ ft/s}$

For analysis of the impact use the principle of impulse and momentum.

$$\Sigma m\mathbf{v}_1 + \Sigma\mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$



t-direction for ball A:

$$0 + 0 = m_A(v'_A)_t \qquad (v'_A)_t = 0$$

t-direction for ball B: $m_B v_B \sin 20° + 0 = m_B(v'_B)_t$

$$(v'_B)_t = v_B \sin 20° = (2)(\sin 20°) = 0.6840\text{ ft/s}$$

n-direction for balls A and B:

$$m_B v_B \cos 20° + 0 = m_B (v'_B)_n + m_A (v'_A)_n$$

$$(v'_B)_n + \frac{m_A}{m_B}(v'_A)_n = v_B \cos 20°$$

$$(v'_B)_n + 2(v'_A)_n = (2)\cos 20° \qquad (1)$$

Coefficient of restitution:

$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$$
$$= e[0 - (v_B \cos 20°)]$$
$$= -(0.6)(2)\cos 20° \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$(v'_A)_n = 1.00234 \text{ ft/s} \qquad (v'_B)_n = -0.12529 \text{ ft/s}$$

(*a*) Velocities after the impact:

$$\mathbf{v}'_A = 1.00234 \text{ ft/s}\uparrow \qquad \mathbf{v}'_A = 1.002 \text{ ft/s}\uparrow \blacktriangleleft$$

$$\mathbf{v}'_B = (0.6840 \text{ ft/s} \rightarrow) + (0.12529 \text{ ft/s}\downarrow)$$

$$v_B = \sqrt{(0.6840)^2 + (0.12529)^2} = 0.695 \text{ ft/s}$$

$$\tan\beta = \frac{0.12529}{0.6840} \qquad \beta = 10.4°$$

$$\mathbf{v}'_B = 0.695 \text{ ft/s} \searrow 10.4° \blacktriangleleft$$

(*b*) Maximum height reached by A:

Use conservation of energy for ball A after the impact.

Position 1: Just after impact.

$$T_1 = \frac{1}{2}m_A(v'_A)^2 = \frac{1}{2}(0.062112)(1.00234)^2 = 0.0312021 \text{ ft}\cdot\text{lb}$$

Force in spring = weight of A

$$x_1 = -\frac{F}{k} = -\frac{W_A}{k} = -\frac{2 \text{ lb}}{120 \text{ lb/ft}} = -0.016667 \text{ ft}$$

$$(V_1)_e = \frac{1}{2}kx_1^2 = \frac{1}{2}k\left(\frac{W_B}{k}\right)^2 = \frac{W_A^2}{2k}$$

$$= \frac{(2 \text{ lb})^2}{(2)(120)} = 0.016667 \text{ ft}\cdot\text{lb}$$

$$(V_1)_g = 0 \qquad \text{(datum)}$$

PROBLEM 13.184 (Continued)

Position 2: Maximum height h.

$$V_2 = 0$$
$$T_2 = 0$$
$$(V_2)_e = \frac{1}{2}k(h + x_1)^2 = \frac{1}{2}(120)(h - 0.016667)^2$$
$$= 60h^2 - 2h + 0.016667$$
$$(V_2)_g = W_A h = (2\text{ lb})h = 2h$$

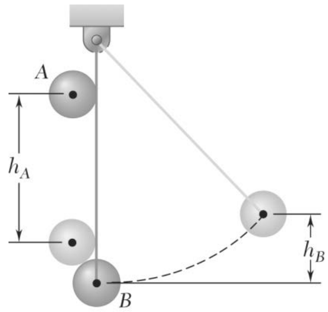
Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$0.031202 + 0.016667 = 0 + 60h^2 - 2h + 0.016667 + 2h$$

$60h^2 = 0.031202 \qquad h = \pm 0.022804\text{ ft}$

Using the positive root, $h = 0.274\text{ in.}$ ◀

PROBLEM 13.185



Ball B is hanging from an inextensible cord. An identical ball A is released from rest when it is just touching the cord and drops through the vertical distance $h_A = 8$ in. before striking ball B. Assuming perfectly elastic impact $(e = 0.9)$ and no friction, determine the resulting maximum vertical displacement h_B of ball B.

SOLUTION

Ball A falls

$$T_1 = 0 \quad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2 \qquad \text{(Put datum at 2)}$$

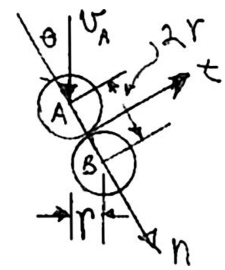
$$h = 8 \text{ in.} = 0.66667 \text{ ft}$$

$$mgh = \frac{1}{2} m v_A^2$$

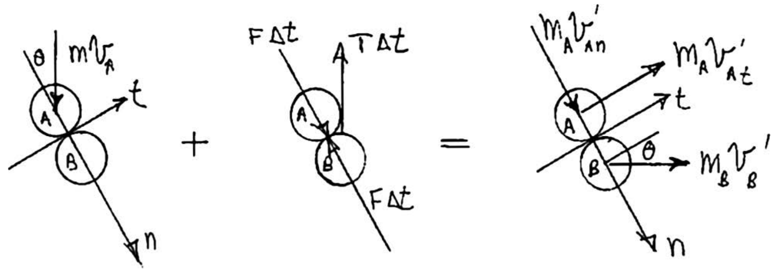
$$v_A = \sqrt{2gh} = \sqrt{(2)(32.2)(0.66667)} = 6.5524 \text{ ft/s}$$

Impact

$$\theta = \sin^{-1} \frac{r}{2r} = 30°$$



Impulse-Momentum



Unknowns: v_B', v_{At}', v_{An}'

x-dir $\longrightarrow$

$$0 + 0 = m_B v_B' + m_A v_{An}' \sin 30° + m_A v_{At}' \cos 30° \qquad (1)$$

Noting that $m_A = m_B$ and dividing by m_A

$$v_B' + v_{An}' \sin 30° + v_{At}' \cos 30° = 0 \qquad (1)$$

Ball A alone:

Momentum in t-direction:

$$-m_A v_A \sin 30° + 0 = m_A v_{At}$$

$$v'_{At} = -v_A \sin 30° = -6.5524 \sin 30° = -3.2762 \text{ ft/s} \qquad (2)$$

Coefficient of restitution:

$$v'_{Bn} - v'_{An} = e(v_{An} - v_{en})$$

$$v'_B \sin 30° - v'_{An} = 0.9(v_A \cos 30° - 0) \qquad (3)$$

With known value for v_{At}, Eqs. (1) and (3) become

$$v'_B + v'_{An} \sin 30° = 3.2762 \cos 30°$$
$$v'_B \sin 30° - v'_{An} = (0.9)(6.5524)\cos 30°$$

Solving the two equations simultaneously,

$$v'_B = 4.31265 \text{ ft/s}$$
$$v'_{An} = -2.9508 \text{ ft/s}$$

After the impact, ball B swings upward. Using B as a free body

$$T' + V' = T_B + V_B$$

where $T' = \frac{1}{2} m_B (v'_B)^2,$

$V' = 0,$

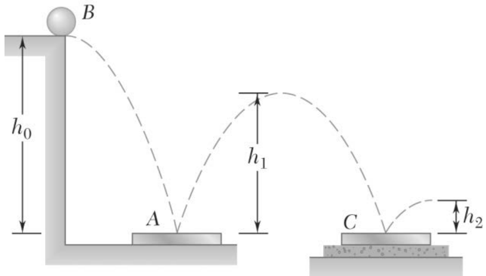
$T_B = 0$

and $V_B = m_B g h_B$

$$\frac{1}{2} m_B (v'_B)^2 = m_B g h_B$$

$$h_B = \frac{1}{2}\frac{(v'_B)^2}{g}$$
$$= \frac{1}{2}\frac{(4.31265)^2}{32.2}$$
$$= 0.2888 \text{ ft}$$

$h_B = 3.47$ in. ◀



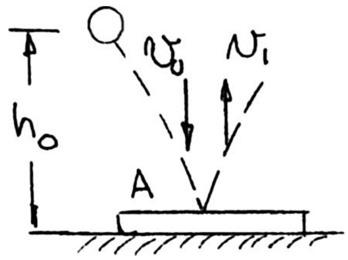
PROBLEM 13.186

A 70 g ball B dropped from a height $h_0 = 1.5$ m reaches a height $h_2 = 0.25$ m after bouncing twice from identical 210-g plates. Plate A rests directly on hard ground, while plate C rests on a foam-rubber mat. Determine (a) the coefficient of restitution between the ball and the plates, (b) the height h_1 of the ball's first bounce.

SOLUTION

(a) Plate on hard ground (first rebound):

Conservation of energy:



$$\cancel{\frac{1}{2}m_B v_y^2} + \frac{1}{2}m_B v_0^2 = m_B g h_0 + \cancel{\frac{1}{2}m_B v_x^2}$$

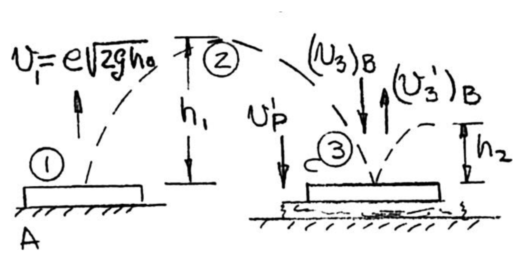
$$v_0 = \sqrt{2gh_0}$$

Relative velocities., n-direction:

$$v_0 e = v_1 \qquad v_1 = e\sqrt{2gh_0}$$

t-direction $\qquad v'_{Bx} = v_{Bx}$

Plate on foam rubber support at C.



Conservation of energy:

Points ① and ③: $\qquad V_1 = V_3 = 0$

$$\cancel{\frac{1}{2}m_B(v'_{Bx})^2} \times \frac{1}{2}m_B v_1^2 = \frac{1}{2}m_B(v_3)_B^2 + \cancel{\frac{1}{2}m_B(v'_{Bx})^2}$$

$$(v_3)_B = e\sqrt{2gh_0}$$

Conservation of momentum:

$\uparrow +$ At ③: $\qquad m_B(-v_3)_B + m_P v_P = m_B(v'_3)_B - m_P v'_P$

$$\frac{m_P}{m_B} = \frac{210}{70} = 3 \qquad -e\sqrt{2gh_0} = (v'_3)_B - 3v'_P \qquad (1)$$

PROBLEM 13.186 (Continued)

Relative velocities:

$$[(-v_3)_B - (v_P)]e = -v'_P - (v'_3)_B$$

$$e^2\sqrt{2gh_0} + 0 = v'_P + (v'_3)_B \quad (2)$$

Multiplying (2) by 3 and adding to (1)

$$4(v'_3)_B = \sqrt{2gh_0}(3e^2 - e)$$

Conservation of energy at ③, $(v'_3)_B = \sqrt{2gh_2}$

Thus, $4\sqrt{2gh_2} = \sqrt{2gh_0}(3e^2 - e)$

$$3e^2 - e = 4\sqrt{\frac{h_2}{h_0}} = 4\sqrt{\frac{0.25}{1.5}} = 1.63299$$

$$3e^2 - e - 1.633 = 0 \qquad e = 0.923 \blacktriangleleft$$

(*b*) Points ① and ②:

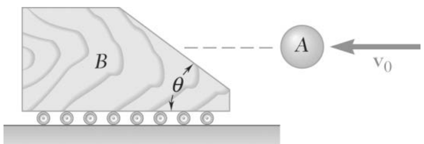
Conservation of energy.

$$\cancel{\frac{1}{2}m_B(v'_{Bx})^2} + \frac{1}{2}m_B v_1^2 = \cancel{\frac{1}{2}m_B(v'_{Bx})^2}; \quad \frac{1}{2}e^2(2gh_0) = gh_1$$

$$h_1 = e^2 h_0 = (0.923)^2(1.5)$$

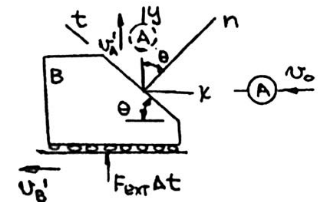
$$h_1 = 1.278 \text{ m} \blacktriangleleft$$

PROBLEM 13.187



A 700-g sphere A moving with a velocity $\mathbf{v}_0$ parallel to the ground strikes the inclined face of a 2.1-kg wedge B which can roll freely on the ground and is initially at rest. After impact the sphere is observed from the ground to be moving straight up. Knowing that the coefficient of restitution between the sphere and the wedge is $e = 0.6$, determine (a) the angle θ that the inclined face of the wedge makes with the horizontal, (b) the energy lost due to the impact.

SOLUTION



(a) Momentum of sphere A alone is conserved in the t-direction:

$$m_A v_0 \cos\theta = m_A v_A' \sin\theta$$

$$v_0 = v_A' \tan\theta \qquad (1)$$

Total momentum is conserved in the x-direction:

$$m_B v_B + m_A v_0 = m_B v_B' + (v_A')_x$$

$$v_B = 0, \quad (v_A')_x = 0$$

$$0 + 0.700\, v_0 = 2.1 v_B' + 0$$

$$v_B' = v_0/3 \qquad (2)$$

Relative velocities in the n-direction:

$$(-v_0 \sin\theta - 0)e = -v_B' \sin\theta - v_A' \cos\theta$$

$$(v_0)(0.6) = v_B' + v_A' \cot\theta \qquad (3)$$

Substituting v_B' from Eq. (2) into Eq. (3)

$$0.6 v_0 = 0.333\, v_0 + v_A' \cot\theta$$

$$0.267 v_0 = v_A' \cot\theta \qquad (4)$$

Divide (4) into (1)

$$\frac{1}{0.267} = \frac{\tan\theta}{\cot\theta} = \tan^2\theta$$

$$\tan\theta = 1.935 \qquad \theta = 62.7° ◀$$

(b) From (1)

$$v_0 = v_A' \tan\theta = v_A'(1.935)$$

$$v_A' = 0.5168 v_0, \quad v_B' = v_0/3 \qquad (2)$$

$$T_{\text{lost}} = \frac{1}{2} m_A v_A^2 - \frac{1}{2}\left(m_A (v_A')^2 + m_B v_B^2\right)$$

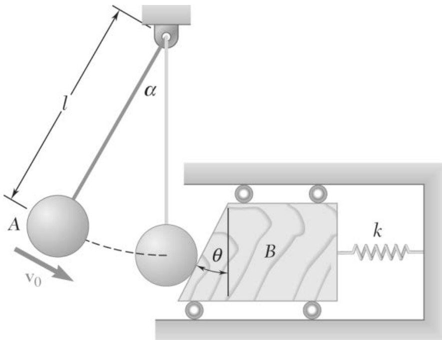
PROBLEM 13.187 (Continued)

$$T_{\text{lost}} = \frac{1}{2}(0.7)(v_0)^2 - \frac{1}{2}[(0.7)(0.5168v_0)^2 + (2.1)(v_0/3)^2]$$

$$T_{\text{lost}} = \frac{1}{2}[0.7 - 0.1870 - 0.2333]v_0^2$$

$$T_{\text{lost}} = 0.1400v_0^2 \text{ J}$$

$T_{\text{lost}} = 0.1400v_0^2$ ◀



PROBLEM 13.188

When the rope is at an angle of $\alpha = 30°$ the 1-lb sphere A has a speed $v_0 = 4$ ft/s. The coefficient of restitution between A and the 2-lb wedge B is 0.7 and the length of rope $l = 2.6$ ft. The spring constant has a value of 2 lb/in. and $\theta = 20°$. Determine (*a*) the velocities of A and B immediately after the impact, (*b*) the maximum deflection of the spring assuming A does not strike B again before this point.

SOLUTION

Masses: $m_A = (1/32.2)\ \text{lb}\cdot\text{s}^2/\text{ft} \qquad m_B = (2/32.2)\ \text{lb}\cdot\text{s}^2/\text{ft}$

<u>Analysis of sphere A as it swings down</u>:

Initial state: $\alpha = 30°, \quad h_0 = l(1-\cos\alpha) = (2.6)(1-\cos 30°) = 0.34833$ ft

$$V_0 = m_A g h_0 = (1)(0.34833) = 0.34833\ \text{lb}\cdot\text{ft}$$

$$T_0 = \frac{1}{2}m_A v_0^2 = \frac{1}{2}\left(\frac{1.0}{32.2}\right)(4)^2 = 0.24845\ \text{lb}\cdot\text{ft}$$

Just before impact: $\alpha = 0, \quad h_1 = 0, \quad V_1 = 0$

$$T_1 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}\left(\frac{1.0}{32.2}\right)v_A^2 = \left(\frac{1.0}{64.4}\right)v_A^2$$

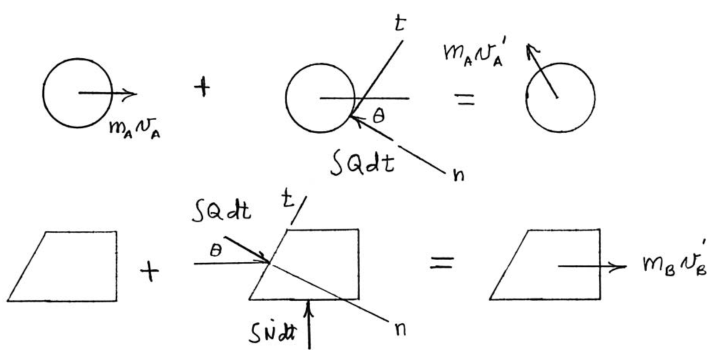
Conservation of energy: $T_0 + V_0 = T_1 + V_1$

$$0.24845 + 0.34833 = \frac{1}{64.4}v_A^2 + 0$$

$$v_A^2 = 38.433\ \text{ft}^2/\text{s}^2$$

$$\mathbf{v}_A = 6.1994\ \text{ft/s} \longrightarrow$$

<u>Analysis of the impact</u>. Use conservation of momentum together with the coefficient of restitution. $e = 0.7$.



Note that the rope does not apply an impulse since it becomes slack.

PROBLEM 13.188 (Continued)

Sphere A: Momentum in t-direction:

$$m_A v_A \sin\theta + 0 = m_A (v'_A)_t$$
$$(v'_A)_t = v_A \sin\theta = 6.1994 \sin 20° = 2.1203 \text{ m/s}$$
$$(\mathbf{v}_A)_t = 2.1203 \text{ m/s} \ \angle 70°$$

Both A and B: Momentum in x-direction:

$$m_A v_A + 0 = m_A (v'_A)_n \cos\theta + m_A (v'_A)_t \sin\theta + m_B v'_B$$
$$(1/32.2)(6.1994) = (1/32.2)(v_A)_n \cos 20° + (1/32.2)(2.120323) \sin 20° + (2/32.2) v'_B$$
$$(1/32.2)(v'_A)_n \cos 20° + (2/32.2) v'_B = 0.17001 \quad (1)$$

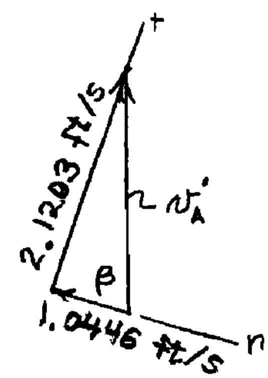
Coefficient of restitution:

$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$$
$$v'_B \cos\theta - (v'_A)_n = e[v_A \cos\theta - 0]$$
$$v'_B \cos 20° - (v'_A)_n = (0.7)(6.1994) \cos 20° \quad (2)$$

Solving Eqs. (1) and (2) simultaneously for $(v'_A)_n$ and v'_B,

$$(v'_A)_n = -1.0446 \text{ ft/s}$$
$$v'_B = 3.2279 \text{ ft/s}$$

Resolve $\mathbf{v}_A$ into horizontal and vertical components.



$$\tan\beta = \frac{(v'_A)_t}{-(v'_A)_n} = \frac{2.1203}{1.0446}$$
$$\beta = 63.77° \qquad \beta + 20° = 83.8°$$
$$v'_A = \sqrt{(2.1203)^2 + (1.0446)^2} = 2.3637 \text{ ft/s}$$

(*a*) *Velocities immediately after impact.*

$\mathbf{v}'_A = 2.36$ ft/s ⦣ 83.8° ◀

$\mathbf{v}'_B = 3.23$ ft/s ⟶ ◀

(*b*) *Maximum deflection of wedge B.*

Use conservation of energy: $T_{B1} + V_{B1} = T_{B2} + V_{B2}$

$$T_{B1} = \frac{1}{2} m_B v_B^2$$
$$V_{B1} = 0$$
$$T_{B2} = 0$$
$$V_{B2} = \frac{1}{2} k(\Delta x)^2$$

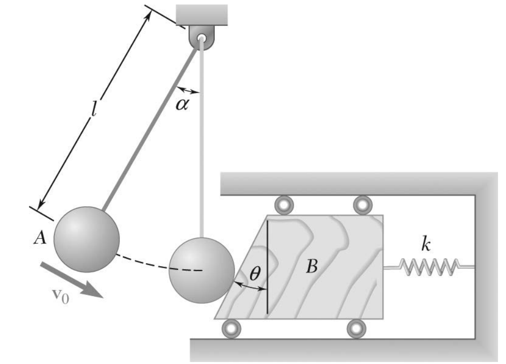
PROBLEM 13.188 (Continued)

The maximum deflection will occur when the block comes to rest (ie, no kinetic energy)

$$\frac{1}{2}m_B v_B^2 = \frac{1}{2}k(\Delta x)^2$$

$$(\Delta x)^2 = \frac{m_B v_B^2}{k} = \frac{\left(\frac{2\text{ lb}}{32.2\text{ ft/s}^2}\right)(3.2279\text{ ft/s})^2}{2\text{ lb/in (12 in/ft)}}$$

$$(\Delta x) = 0.1642118\text{ ft} \qquad (\Delta x) = 1.971\text{ in.} \blacktriangleleft$$



PROBLEM 13.189

When the rope is at an angle of $\alpha = 30°$ the 1-kg sphere A has a speed $v_0 = 0.6$ m/s. The coefficient of restitution between A and the 2-kg wedge B is 0.8 and the length of rope $l = 0.9$ m The spring constant has a value of 1500 N/m and $\theta = 20°$. Determine, (*a*) the velocities of A and B immediately after the impact (*b*) the maximum deflection of the spring assuming A does not strike B again before this point.

SOLUTION

Masses: $m_A = 1$ kg

$m_B = 2$ kg

Analysis of sphere A as it swings down:

Initial state: $\alpha = 30°, \quad h_0 = l(1-\cos\alpha) = (0.9)(1-\cos 30°) = 0.12058$ m

$$V_0 = m_A g h_0 = (1)(9.81)(0.12058) = 1.1829 \text{ N}\cdot\text{m}$$

$$T_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(1)(0.6)^2 = 0.180 \text{ N}\cdot\text{m}$$

Just before impact: $\alpha = 0, \quad h_1 = 0, \quad V_1 = 0$

$$T_1 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(1)v_A^2 = 0.5v_A^2$$

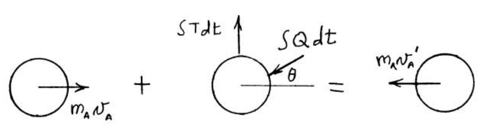
Conservation of energy: $T_0 + V_0 = T_1 + V_1$

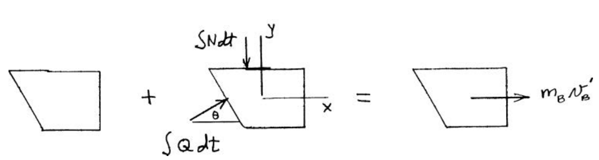
$$0.180 + 1.1829 = 0.5v_A^2 + 0$$

$$v_A^2 = 2.7257 \text{ m}^2/\text{s}^2$$

$$\mathbf{v}_A = 1.6510 \text{ m/s} \longrightarrow$$

Analysis of the impact: Use conservation of momentum together with the coefficient of restitution. $e = 0.8$.





Note that the ball rebounds horizontally and that an impulse $\int T dt$ is applied by the rope. Also, an impulse $\int N dt$ is applied to B through its supports.

Both A and B:

Momentum in x-direction:

$$m_A(v_A)_x + 0 = m_A(v'_A)_x + m_B(v'_B)_x$$
$$(1)(1.6510) = (1)(v'_A)_x + (2)(v'_B)_x \qquad (1)$$

Coefficient of restitution: $(v_A)_n = (v_A)_x \cos\theta$

$$(v_B)_n = 0, \quad (v'_A)_n = (v'_A)_x \cos\theta, \quad (v'_B)_x \cos 30°$$
$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$$
$$(v'_B)_x \cos\theta - (v'_A)_x \cos\theta = e[(v_A)_x \cos\theta]$$

Dividing by $\cos\theta$ and applying $e = 0.8$ gives

$$(v'_B)_x - (v'_A)_x = (0.8)(1.6510) \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$(v'_A)_x = -0.33020 \text{ m/s}$$
$$(v'_B)_x = 0.99059 \text{ m/s}$$

$\mathbf{v}'_A = 0.330$ m/s ← ◀

(*a*) *Velocities immediately after impact.* $\mathbf{v}'_B = 0.991$ m/s → ◀

(*b*) Maximum deflection of wedge B.

Use conservation of energy: $T_{B1} + V_{B1} = T_{B2} + V_{B2}$

$$T_{B1} = \frac{1}{2} m_B v_B^2$$
$$V_{B1} = 0$$
$$T_{B2} = 0$$
$$V_{B2} = \frac{1}{2} k(\Delta x)^2$$

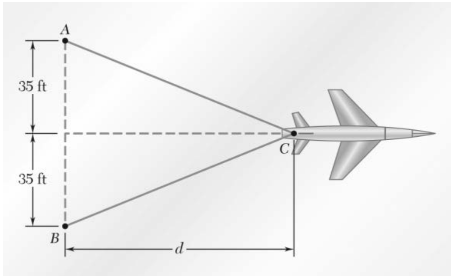
The maximum deflection will occur when the block comes to rest (ie, no kinetic energy)

$$\frac{1}{2} m_B v_B^2 = \frac{1}{2} k(\Delta x)^2$$

$$(\Delta x)^2 = \frac{m_B v_B^2}{k} = \frac{(2)(0.99059 \text{ m/s})^2}{1500 \text{ N/m})}$$

$$(\Delta x) = 0.0362 \text{ m}$$

$\Delta x = 36.2$ mm ◀

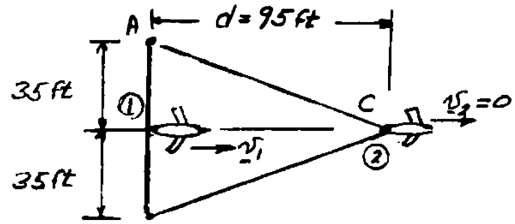


PROBLEM 13.190

A 32,000-lb airplane lands on an aircraft carrier and is caught by an arresting cable. The cable is inextensible and is paid out at *A* and *B* from mechanisms located below dock and consisting of pistons moving in long oil-filled cylinders. Knowing that the piston-cylinder system maintains a constant tension of 85 kips in the cable during the entire landing, determine the landing speed of the airplane if it travels a distance $d = 95$ ft after being caught by the cable.

SOLUTION

Mass of airplaine: $m = \dfrac{W}{g} = \dfrac{32000 \text{ lb}}{32.2 \text{ ft/s}^2} = 993.79 \text{ lb} \cdot \text{s}^2/\text{ft}$



Work of arresting cable force.

$$Q = 85 \text{ kips} = 85000 \text{ lb.}$$

As the cable is pulled out, the cable tension acts paralled to the cable at the airplane hook. For a small displacement

$$\Delta U = -Q(\Delta l_{AC}) - Q(\Delta l_{BC})$$

Since Q is constant,

$$U_{1\to 2} = -Q\left[\overline{AC} + \overline{BC} - \overline{AB}\right]$$

For $\quad d = 95 \text{ ft}, \quad \overline{AC} = \overline{BC} = \sqrt{(35)^2 + (95)^2} = 101.24 \text{ ft}$

$$U_{1\to 2} = -(85000)(101.24 + 101.24 - 70) = -11.261 \text{ ft} \cdot \text{lb}$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$\frac{1}{2}mv_1^2 + U_{1\to 2} = \frac{1}{2}mv_2^2$$

Since $v_2 = 0,$ we get

$$v_1^2 = -\frac{2U_{1\to 2}}{m} = -\frac{(2)(-11.261)}{993.79} = 22.663 \times 10^3 \text{ ft}^2/\text{s}^2$$

Initial speed: $v_1 = 150.54$ ft/s $\qquad v_1 = 102.6$ mi/h ◀

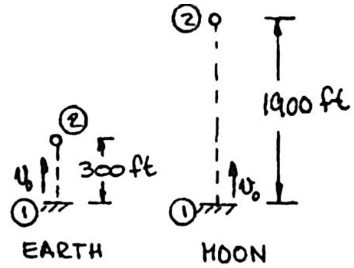
PROBLEM 13.191

A 2-oz pellet shot vertically from a spring-loaded pistol on the surface of the earth rises to a height of 300 ft. The same pellet shot from the same pistol on the surface of the moon rises to a height of 1900 ft. Determine the energy dissipated by aerodynamic drag when the pellet is shot on the surface of the earth. (The acceleration of gravity on the surface of the moon is 0.165 times that on the surface of the earth.)

SOLUTION

Since the pellet is shot from the same pistol the initial velocity v_0 is the same on the moon and on the earth.

Work and energy.



Earth: $T_1 = \frac{1}{2}mv_0^2$

$$U_{1-2} = -mg_E(300 \text{ ft}) - E_L$$

($E_L =$ Loss of energy due to drag)

Moon: $T_1 = \frac{1}{2}mv_0^2 \qquad T_2 = 0$

$$U_{1-2} = -mg_M(1900) \qquad T_1 - 300mg_E - E_L = 0 \qquad (1)$$

$$T_2 = 0$$

$$T_1 - 1900mg_M = 0 \qquad (2)$$

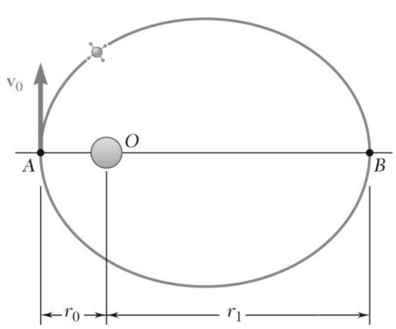
Subtracting (1) from (2) $\quad -1900mg_M + 300mg_E + E_L = 0$

$$g_M = 0.165g_E$$

$$m = \frac{(2/16)}{g_E}$$

$$E_L = (1900)\frac{(2/16)}{g_E}(0.165g_E) - 300\frac{(2/16)}{g_E}g_E$$

$E_L = 1.688 \text{ ft} \cdot \text{lb}$ ◀



PROBLEM 13.192

A satellite describes an elliptic orbit about a planet of mass M. The minimum and maximum values of the distance r from the satellite to the center of the planet are, respectively, r_0 and r_1. Use the principles of conservation of energy and conservation of angular momentum to derive the relation

$$\frac{1}{r_0}+\frac{1}{r_1}=\frac{2GM}{h^2}$$

where h is the angular momentum per unit mass of the satellite and G is the constant of gravitation.

SOLUTION

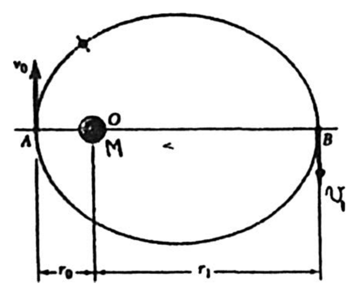
Angular momentum:

$$h = r_0, \quad v_0 = r_1 v_1$$

$$b = r_0 v_0 = r_1 v_1$$

$$v_0 = \frac{h}{r_0} \quad v_1 = \frac{h}{r_1} \qquad (1)$$

Conservation of energy:



$$T_A = \frac{1}{2}mv_0^2$$

$$V_A = -\frac{GMm}{r_0}$$

$$T_B = \frac{1}{2}mv_1^2$$

$$V_B = -\frac{GMm}{r_1}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \frac{1}{2}mv_1^2 - \frac{GMm}{r_1}$$

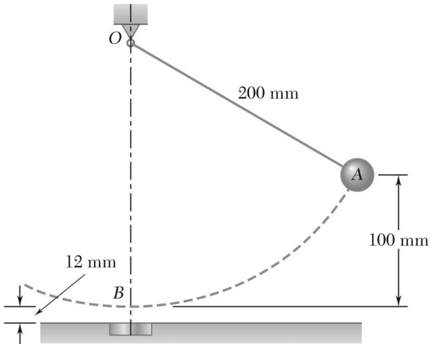
$$v_0^2 - v_1^2 = 2GM\left[\frac{1}{r_0}-\frac{1}{r_1}\right] = 2GM\left[\frac{r_1-r_0}{r_1 r_0}\right]$$

Substituting for v_0 and v_1 from Eq. (1)

$$h^2\left[\frac{1}{r_0^2}-\frac{1}{r_1^2}\right] = 2GM\left[\frac{r_1-r_0}{r_1 r_0}\right]$$

$$h^2\left[\frac{r_1^2-r_0^2}{r_1^2 r_0^2}\right] = \frac{h^2}{r_1^2 r_0^2}(r_1-r_0)(r_1+r_0) = 2GM\left[\frac{r_1-r_0}{r_1 r_0}\right]$$

$$h^2\left(\frac{1}{r_0}+\frac{1}{r_1}\right) = 2GM \qquad \left(\frac{1}{r_0}+\frac{1}{r_1}\right) = \frac{2GM}{h^2} \quad \text{Q.E.D.}$$



PROBLEM 13.193

A 60-g steel sphere attached to a 200-mm cord can swing about Point O in a vertical plane. It is subjected to its own weight and to a force $\mathbf{F}$ exerted by a small magnet embedded in the ground. The magnitude of that force expressed in newtons is $F = 3000/r^2$ where r is the distance from the magnet to the sphere expressed in millimeters. Knowing that the sphere is released from rest at A, determine its speed as it passes through Point B.

SOLUTION

Mass and weight: $m = 0.060$ kg

$$W = mg = (0.060)(9.81) = 0.5886 \text{ N}$$

Gravitational potential energy: $V_g = Wh$

where h is the elevation above level at B.

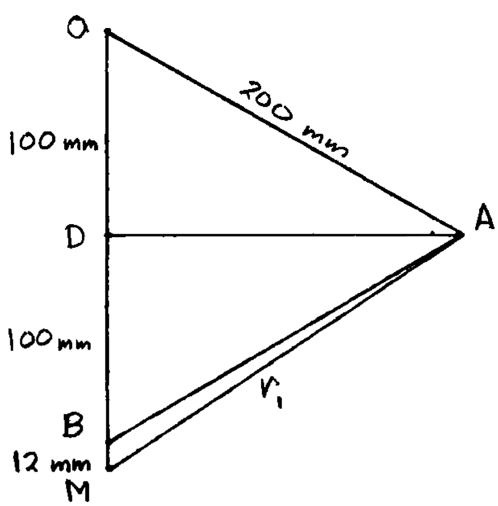
Potential energy of magnetic force:

$$F = \frac{3000}{r^2} = -\frac{dV}{dr} \quad (F\text{, in newtons, } r \text{ in mm})$$

$$V_m = -\int_{\infty}^{r} \frac{3000}{r^2} = \frac{3000}{r} \text{ N} \cdot \text{mm}$$

Use conservation of energy: $T_1 + V_1 = T_2 + V_2$

Position 1: (Rest at A.)



$$v_1 = 0 \qquad T_1 = 0$$

$$h_1 = 100 \text{ mm}$$

$$(V_g)_1 = (0.5886 \text{ N})(100 \text{ mm}) = 58.86 \text{ N} \cdot \text{mm}$$

From the figure, $\overline{AD}^2 = 200^2 - 100^2 \text{ (mm}^2)$

$$\overline{MD} = 100 + 12 = 112 \text{ mm}$$

$$r_1^2 = \overline{AD}^2 + \overline{MD}^2$$

$$= 200^2 - 100^2 + 112^2$$

$$= 42544 \text{ mm}^2$$

$$r_1 = 206.26 \text{ mm}$$

$$(V_r)_1 = -\frac{3000}{r_1} = -14.545 \text{ N} \cdot \text{mm}$$

$$V_1 = 58.86 - 14.545 = 44.3015 \text{ N} \cdot \text{mm} = 44.315 \times 10^{-3} \text{ N} \cdot \text{m}$$

PROBLEM 13.193 (Continued)

Position 2. (Sphere at Point B.)

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.060)v_2^2 = 0.030\ v_2^2$$

$$(V_g)_2 = 0 \qquad \text{(since } h_2 = 0\text{)}$$

$$r_2 = \overline{MB} = 12 \text{ mm} \qquad \text{(See figure.)}$$

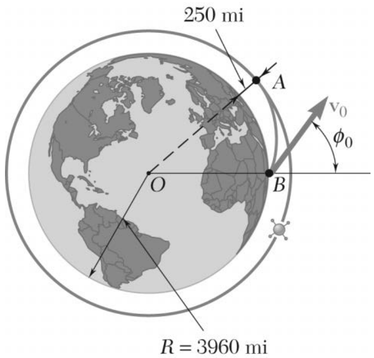
$$(V_m)_2 = -\frac{3000}{12} = -250 \text{ N}\cdot\text{mm} = -250\times10^{-3} \text{ N}\cdot\text{mm}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 44.315\times10^{-3} = 0.030v_2^2 - 250\times10^{-3}$$

$$v_2^2 = 9.8105 \text{ m}^2/\text{s}^2$$

$v_2 = 3.13$ m/s ◀

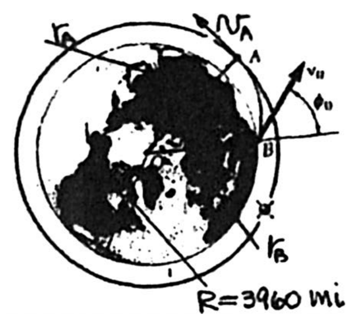


PROBLEM 13.194

A shuttle is to rendezvous with a space station which is in a circular orbit at an altitude of 250 mi above the surface of the earth. The shuttle has reached an altitude of 40 mi when its engine is turned off at Point *B*. Knowing that at that time the velocity $\mathbf{v}_0$ of the shuttle forms an angle $\phi_0 = 55°$ with the vertical, determine the required magnitude of $\mathbf{v}_0$ if the trajectory of the shuttle is to be tangent at *A* to the orbit of the space station.

SOLUTION

Conservation of energy:



$$T_B = \frac{1}{2}mv_0^2 \qquad V_B = -\frac{GMm}{r_B}$$

$$T_A = \frac{1}{2}mv_A^2 \qquad V_A = -\frac{GMm}{r_A}$$

$$GM = gR^2 \quad \text{(Eq. 12.30)}$$

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\not{m}v_0^2 - \frac{gR^2}{r_B}\not{m} = \frac{1}{2}\not{m}v_A^2\frac{gR^2}{r_A}\not{m}$$

$$r_A = 3960 + 250 = 4210 \text{ mi}$$

$$v_A^2 = v_0^2 - \frac{2gR^2}{r_B}\left(1 - \frac{r_B}{r_A}\right)$$

$$r_B = 3960 + 40 = 4000 \text{ mi}$$

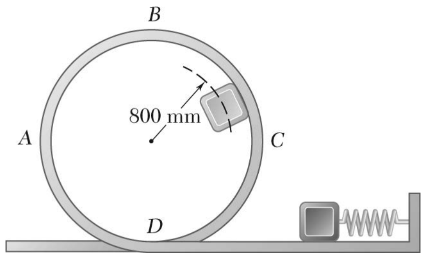
$$v_A^2 = v_0^2 - \frac{2(32.2)(3960\times 528)^3}{(4000\times 5280)}\left(1 - \frac{4000}{4210}\right)$$

$$v_A^2 = v_0^2 - 66.495\times 10^6 \qquad (1)$$

Conservation of angular momomentum: $\quad r_A v_A = r_B v_B \sin\phi_0;$

$$v_A = (4000/4210)v_0 \sin 55° = 0.77829\, v_0 \qquad (2)$$

Eqs. (2) and (1) $\quad [1 - (0.77829)^2]\, v_0^2 = 66.495\times 10^6 \qquad v_0 = 12{,}990 \text{ ft/s}$ ◄

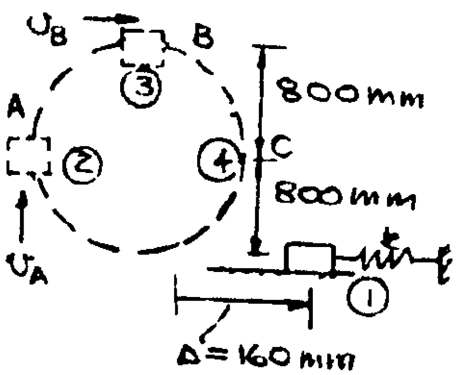


PROBLEM 13.195

A 300-g block is released from rest after a spring of constant $k = 600$ N/m has been compressed 160 mm. Determine the force exerted by the loop *ABCD* on the block as the block passes through (*a*) Point *A*, (*b*) Point *B*, (*c*) Point *C*. Assume no friction.

SOLUTION

Conservation of energy to determine speeds at locations *A*, *B*, and *C*.



Mass: $m = 0.300$ kg

Initial compression in spring: $x_1 = 0.160$ m

Place datum for gravitational potential energy at position 1.

Position 1: $v_1 = 0 \qquad T_1 = \frac{1}{2}mv_1^2 = 0$

$$V_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(600 \text{ N/m})(0.160 \text{ m})^2 = 7.68 \text{ J}$$

Position 2: $T_2 = \frac{1}{2}mv_A^2 = \frac{1}{2}(0.3)v_A^2 = 0.15v_A^2$

$$V_2 = mgh_2 = (0.3 \text{ kg})(9.81 \text{ m/s}^2)(0.800 \text{ m}) = 2.3544 \text{ J}$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 7.68 = 0.15v_A^2 + 2.3544$$

$$v_A^2 = 35.504 \text{ m}^2/\text{s}^2$$

Position 3: $T_3 = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.3)v_B^2 = 0.15v_B^2$

$$V_3 = mgh_3 = (0.3 \text{ kg})(9.81 \text{ m/s}^2)(1.600 \text{ m}) = 4.7088 \text{ J}$$

$$T_1 + V_1 = T_3 + V_3: \quad 0 + 7.68 = 0.15v_B^2 + 4.7088$$

$$v_B^2 = 19.808 \text{ m}^2/\text{s}^2$$

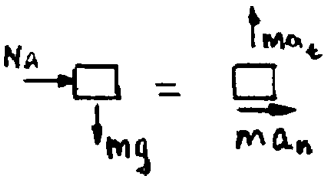
Position 4: $T_2 = \frac{1}{2}mv_C^2 = \frac{1}{2}(0.3)v_C^2 = 0.15v_C^2$

$$V_4 = mgh_4 = (0.3 \text{ kg})(9.81 \text{ m/s})(0.800 \text{ m}) = 2.3544 \text{ J}$$

$$T_1 + V_1 = T_4 + V_4: \quad 0 + 7.68 = 0.15v_C^2 + 2.3544$$

$$v_C^2 = 35.504 \text{ m}^2/\text{s}^2$$

(*a*) Newton's second law at *A*:



$$a_n = \frac{v_A^2}{\rho} = \frac{35.504 \text{ m}^2/\text{s}^2}{0.800 \text{ m}} = 44.38 \text{ m/s}^2$$

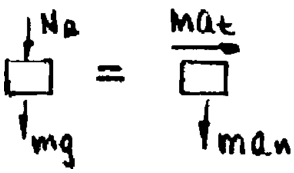
$$\mathbf{a}_n = 44.38 \text{ m/s}^2 \longrightarrow$$

$$\xrightarrow{+} \Sigma F = ma_n: \quad N_A = ma_n$$

$$N_A = (0.3 \text{ kg})(44.38 \text{ m/s}^2)$$

$\mathbf{N}_A = 13.31 \text{ N} \longrightarrow$ ◀

(*b*) Newton's second law at *B*:



$$a_n = \frac{v_B^2}{\rho} = \frac{19.808 \text{ m}^2/\text{s}^2}{0.800 \text{ m}} = 24.76 \text{ m/s}^2$$

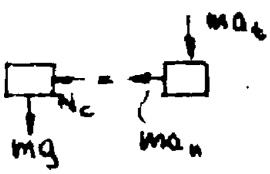
$$\mathbf{a}_n = 24.76 \text{ m/s}^2 \downarrow$$

$$+\!\downarrow \Sigma F = ma_n: \quad N_B = mg = ma_n$$

$$N_B = m(a_n - g) = (0.3 \text{ kg})(24.76 \text{ m/s}^2 - 9.81 \text{ m/s}^2)$$

$\mathbf{N}_B = 4.49 \text{ N} \downarrow$ ◀

(*c*) Newton's second law at *C*:



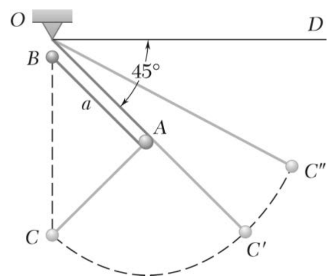
$$a_n = \frac{v_C^2}{\rho} = \frac{35.504 \text{ m}^2/\text{s}^2}{0.800 \text{ m}} = 44.38 \text{ m/s}^2$$

$$\mathbf{a}_n = 44.38 \text{ m/s}^2 \longleftarrow$$

$$\xleftarrow{+} \Sigma F = ma_n: \quad N_C = ma_n$$

$$N_C = (0.3 \text{ kg})(44.38 \text{ m/s}^2)$$

$\mathbf{N}_C = 13.31 \text{ N} \longleftarrow$ ◀



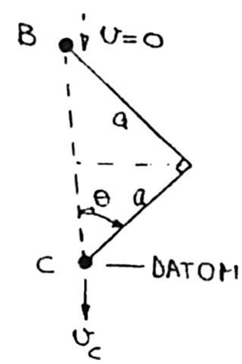
PROBLEM 13.196

A small sphere B of mass m is attached to an inextensible cord of length $2a$, which passes around the fixed peg A and is attached to a fixed support at O. The sphere is held close to the support at O and released with no initial velocity. It drops freely to Point C, where the cord becomes taut, and swings in a vertical plane, first about A and then about O. Determine the vertical distance from line OD to the highest Point C'' that the sphere will reach.

SOLUTION

Velocity at Point C (before the cord is taut).

Conservation of energy from B to C:



$$T_B = 0$$

$$V_B = mg(2)\left(\frac{\sqrt{2}}{2}\right)a = mga\sqrt{2}$$

$$T_C = \frac{1}{2}mv_C^2 \qquad V_C = 0$$

$$T_B + V_B = T_C + V_C$$

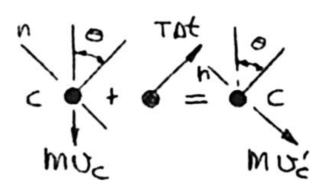
$$0 + mga\sqrt{2} = \frac{1}{2}mv_C^2 + 0$$

$$v_C = \sqrt{2\sqrt{2}\,ga}$$

Velocity at C (after the cord becomes taut).

Linear momentum perpendicular to the cord is conserved:

$$\theta = 45°$$



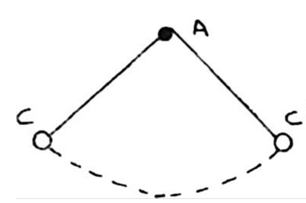
$$\nwarrow^{+} \quad -mv_C \sin\theta = mv'_C$$

$$v'_C = \left(\sqrt{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{2}\right)\sqrt{ga}$$

$$v'_C = \sqrt{\sqrt{2}\,ga} = 2^{\frac{1}{4}}\sqrt{ga}$$

Note: The weight of the sphere is a non-impulsive force.

Velocity at C:



C to C' (conservation of energy):

$$T_C = \frac{1}{2}m(v'_C)^2 \qquad V_C = 0$$

$$T_{C'} = \frac{1}{2}m(v'_{C'})^2 \qquad V_{C'} = 0$$

Datum: $T_C + V_C = T_{C'} + V_{C'}$

$$\frac{1}{2}m(v_C')^2 + 0 = \frac{1}{2}m(v_C')^2 + 0$$

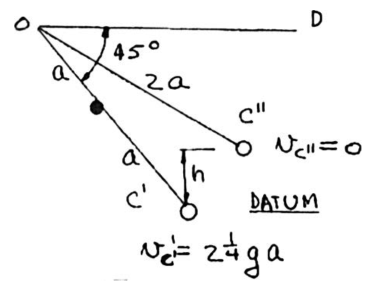
$$v_C' = v_{C'}'$$

$\underline{C' \text{ to } C''}$ (conservation of energy):

$$T_{C'} = \frac{1}{2}m(v_{C'}')^2$$

$$T_{C'} = \frac{1}{2}m\left(2^{1/4}\sqrt{ga}\right)^2$$

$$T_{C'} = \frac{\sqrt{2}}{2}mga$$



Datum: $T_{C'} + V_{C'} = T_{C''} + V_{C''}$

$$V_{C'} = 0$$

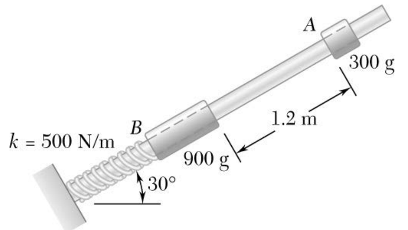
$$T_{C''} = 0$$

$$V_{C''} = mgh$$

$$\frac{\sqrt{2}}{2}mga + 0 = 0 + mgh$$

$$h = \frac{\sqrt{2}}{2}a$$

$h = 0.707\ a$ ◀

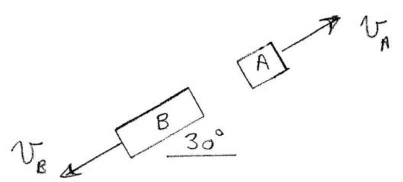


PROBLEM 13.197

A 300-g collar A is released from rest, slides down a frictionless rod, and strikes a 900-g collar B which is at rest and supported by a spring of constant 500 N/m. Knowing that the coefficient of restitution between the two collars is 0.9, determine (*a*) the maximum distance collar A moves up the rod after impact, (*b*) the maximum distance collar B moves down the rod after impact.

SOLUTION

After impact



Velocity of A just before impact, v_0

$$v_0 = \sqrt{2gh} = \sqrt{2(9.81\text{ m/s}^2)(1.2\text{ m})\sin 30°}$$

$$= \sqrt{2(9.81)(1.2)(0.5)} = 3.431\text{ m/s}$$

Conservation of momentum

$$\overset{+}{\swarrow} 30°\quad m_A v_0 = m_B v_B - m_A v_A:\quad 0.3v_0 = 0.9v_B - 0.3v_A \quad (1)$$

Restitution

$$(v_A + v_B) = e(v_0 + 0) = 0.9v_0 \,(2)$$

Substituting for v_B from (2) in (1)

$$0.3v_0 = 0.9(0.9v_0 - v_A) - 0.3v_A \qquad 1.2v_A = 0.51v_0$$

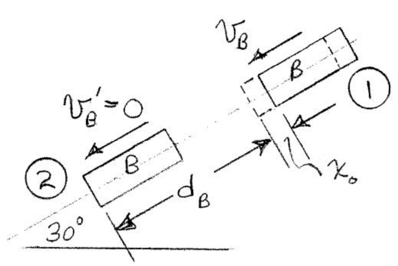
$$v_A = 1.4582\text{ m/s},\; v_B = 1.6297\text{ m/s}$$

(*a*) A moves up the distance d where:

$$\frac{1}{2}\cancel{m_A}v_A^2 = \cancel{m_A}gd\sin 30°;\quad \frac{1}{2}(1.4582\text{ m/s})^2 = (9.81\text{ m/s}^2)d(0.5)$$

$$\nearrow 30°\; d_A = 0.21675\text{ m} = 217\text{ mm} \blacktriangleleft$$

(*b*) Static deflection = x_0, B moves down $d_B \swarrow 30°$



Conservation of energy (1) to (2)

Position (1) – spring deflected, x_0

$$k x_0 = m_B g \sin 30°$$

$$T_1 + V_1 = T_2 + V_2:\quad T_1 = \frac{1}{2}m_B v_B^2,\; T_2 = 0$$

PROBLEM 13.197 (Continued)

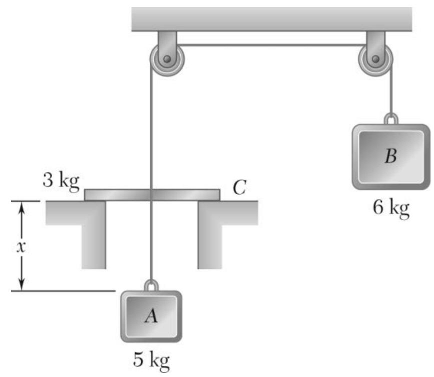
$$V_1 = V_e + V_g = \frac{1}{2}kx_0^2 + m_B g d_B \sin 30°$$

$$V_2 = V_e' + V_g' = \int_0^{x_0+d_B} kx dx = \frac{1}{2}k\left(d_B^2 + 2d_B x_0 + x_0^2\right)$$

$$\frac{1}{2}kx_0^2 + mgd_B \sin 30° + \frac{1}{2}m_B v_B^2 = \frac{1}{2}k\left(d_B^2 + 2d_B x_0 + x_0^2\right) + 0 + 0$$

$$\therefore \quad kd_B^2 = m_B v_B^2; \quad 500d_B^2 = 0.9(1.6297)^2 \quad d_B = 0.0691 \text{ m}$$

$d_B = 69.1 \text{ mm}$ ◀



PROBLEM 13.198

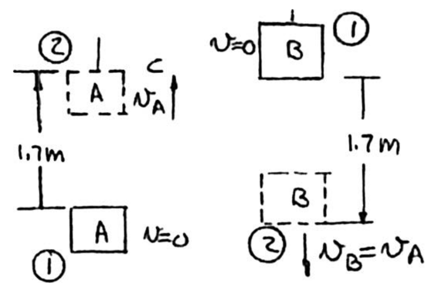
Blocks *A* and *B* are connected by a cord which passes over pulleys and through a collar *C*. The system is released from rest when $x = 1.7$ m. As block *A* rises, it strikes collar *C* with perfectly plastic impact $(e = 0)$. After impact, the two blocks and the collar keep moving until they come to a stop and reverse their motion. As *A* and *C* move down, *C* hits the ledge and blocks *A* and *B* keep moving until they come to another stop. Determine (*a*) the velocity of the blocks and collar immediately after *A* hits *C*, (*b*) the distance the blocks and collar move after the impact before coming to a stop, (*c*) the value of *x* at the end of one compete cycle.

SOLUTION

(*a*) Velocity of *A* just before it hits *C*:

Conservations of energy:

Datum at ①:



Position ①:

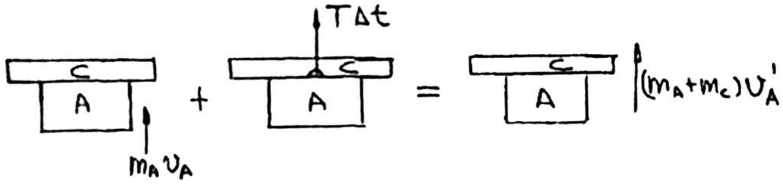
$$(v_A)_1 = (v_B)_1 = 0$$
$$T_1 = 0$$
$$v_1 = 0$$

Position ②:

$$T_2 = \frac{1}{2}m_A(v_A)^2 + \frac{1}{2}m_B v_B^2$$
$$v_A = v_B \quad \text{(kinematics)}$$
$$T_2 = \frac{1}{2}(5+6)v_A^2 = \frac{11}{2}v_A^2$$
$$V_2 = m_A g(1.7) - m_B g(1.7)$$
$$= (5-6)(g)(1.7)$$
$$V_2 = -1.7g$$

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 0 = \frac{11}{2}v_A^2 - 1.7g$$
$$v_A^2 = \left(\frac{3.4}{11}\right)(9.81)$$
$$= 3.032 \text{ m}^2/\text{s}^2$$
$$v_A = 1.741 \text{ m/s}$$

Velocity of A and C after A hits C:



$v_A' = v_C'$ (plastic impact)

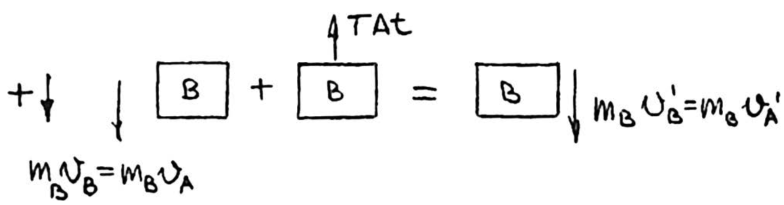
Impulse-momentum A and C:

$$+\uparrow \quad m_A v_A + T\Delta t = (m_A + m_C) v_A'$$

$$(5)(1.741) + T\Delta t = 8v_A' \qquad (1)$$

$$v_B = v_A; \; v_B' = v_A' \text{ (cord remains taut)}$$

B alone:



$$m_B v_A - T\Delta t = m_B v_A'$$

$$(6)(1.741) - T\Delta t = 6v_A' \qquad (2)$$

Adding Equations (1) and (2), $\quad 11(1.741) = 14v_A'$

$$v_A' = 1.3679 \text{ m/s}$$

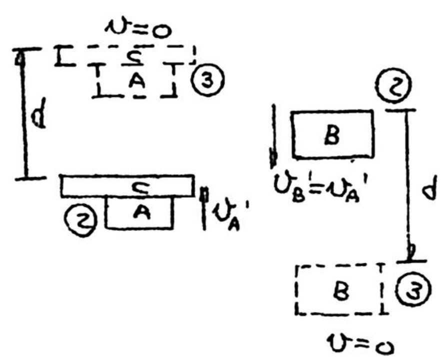
$$v_A' = v_B' = v_C' = 1.368 \text{ m/s} \blacktriangleleft$$

(*b*) Distance A and C move before stopping:

Conservations of energy:

Datum at ②:

Position ②:



$$T_2 = \frac{1}{2}(m_A + m_B + m_C)(v_A')$$

$$T_2 = \left(\frac{14}{2}\right)(1.3681)^2$$

$$T_2 = 13.103 \text{ J}$$

$$V_2 = 0$$

Position ③:

$T_3 = 0$

$$V_3 = (m_A + m_C)gd - m_B gd$$
$$V_3 = (8-6)gd = 2gd$$

$T_2 + V_2 = T_3 + V_3$

$$13.103 + 0 = 0 + 2gd$$

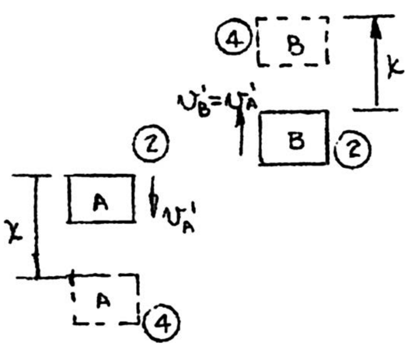
$$d = (13.103)/(2)(9.81) = 0.6679 \text{ m}$$

$d = 0.668$ m ◀

(*c*) As the system returns to position ② after stopping in position ③, energy is conserved, and the velocities of *A*, *B*, and *C* before the collar at *C* is removed are the same as they were in Part (*a*) above with the directions reversed. Thus, $v'_A = v'_C = v'_B = 1.3679$ m/s. After the collar *C* is removed, the velocities of *A* and *B* remain the same since there is no impulsive force acting on either.

<u>Conversation of energy</u>:

Datum at ②:



$$T_2 = \frac{1}{2}(m_A + m_B)(v'_A)^2$$
$$T_2 = \frac{1}{2}(5+6)(1.3679)^2$$
$$T_2 = 10.291 \text{ J}$$

$V_2 = 0$

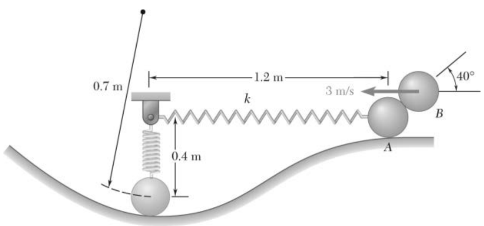
$$T_4 = 0 \quad V_4 = m_B gx - m_A gx$$
$$V_4 = (6-5)gx$$

$T_2 + V_2 = T_4 + V_4$

$$10.291 + 0 = (1)(9.81)x$$

$x = 1.049$ m ◀

PROBLEM 13.199

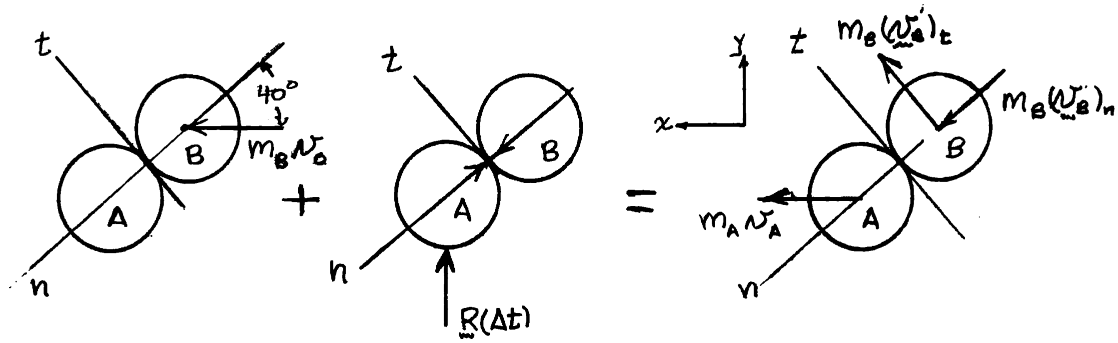


A 2-kg ball B is traveling horizontally at 10 m/s when it strikes 2-kg ball A. Ball A is initially at rest and is attached to a spring with constant 100 N/m and an unstretched length of 1.2 m. Knowing the coefficient of restitution between A and B is 0.8 and friction between all surfaces is negligible, determine the normal force between A and the ground when it is at the bottom of the hill.

SOLUTION

Ball B impacts on ball A. Use the principle of impulse and momentum.

$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$



Velocity components: $v_0 = 10$ m/s

$$(v_0)_x = v_0 \quad (v_0)_n = v_0\cos 40° \quad (v_0)_t = v_0 \sin 40°$$
$$(v_A)_x = v_A \quad (v_A)_n = v_A \cos 40°$$
$$(v_B)_x = (v_B)_n \cos 40° + (v_B)_t \sin 40°$$

Impulse-momentum for ball B alone.

t-direction:

$$m_B(v_0)_t = m_B(v_B)_t$$
$$(v_B)_t = (v_0)_t = 10\sin 40° = 6.4279 \text{ m/s} \qquad (1)$$

Impulse-momentum for balls A and B.

x-direction ←

$$m_B v_0 + 0 = m_A v_A + m_B(v_B)_x + m_B(v_B)_t$$
$$(2)(10) + 0 = 2v_A + 2[(v_B)_n \cos 40° + 6.4279\sin 40°]$$
$$2v_A + 2(v_B)_n \cos 40° = 11.7365 \qquad (1)$$

Coefficient of restitution. $(e = 0.8)$

$$(v_B)_n = (v_A)_n = e[0 - (v_0)_n]$$

$$(v_B)_n - v_A \cos 40° = -(0.8)(10)\cos 40° \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$v_A = 6.6566 \text{ m/s} \qquad (v_B)_n = -1.0291 \text{ m/s}$$

As ball A moves from the impact location to the lowest point on the path, the spring compresses and the elevation decreases. Since friction is negligible, energy is conserved.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_A v_A^2 + (V_e)_1 + (V_g)_1 = \frac{1}{2}m_A v_2^2 + (V_e)_2 + (V_g)_2$$

Position 1: (Just after impact.)

$$T_1 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(2)(6.6566)^2 = 44.3101 \text{ J}$$

$(V_e)_1 = 0$ (The spring is unstretched.)

$(V_g)_1 = 0$ (Datum)

Position 2: (Lowest point on path.)

$$T_2 = \frac{1}{2}m_A v_2^2 = \frac{1}{2}(2)v_2^2 = v_2^2$$

For the spring, $x_2 = l_2 - l_0 = 0.4 \text{ m} - 1.2 \text{ m} = 0.8 \text{ m}$

$$F_e = kx_2 = (100)(0.8) = 80 \text{ N}$$

$$(V_2)_e = \frac{1}{2}kx_2^2 = \frac{1}{2}(100)(0.8)^2 = 32 \text{ J}$$

Elevation above datum: $h_2 = -0.4 \text{ m}$

$$(V_2)_g = m_A g\, h_2 = (2)(9.81)(-0.4) = -7.848$$

Conservation of energy:

$$44.310 + 0 + 0 = v_2^2 + 32 - 7.848$$

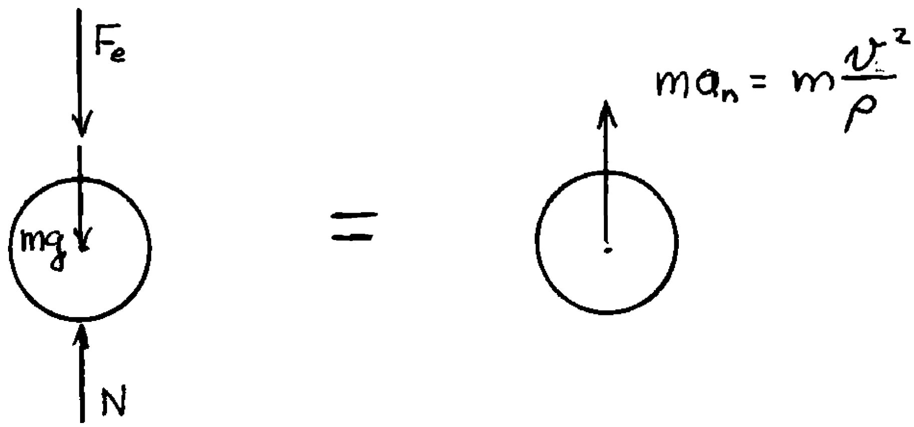
$$v_2^2 = 20.158 \text{ m}^2/\text{s}^2 \qquad v_2 = 4.489 \text{ m/s}$$

Normal acceleration at lowest point on path:

$$a_n = \frac{v_2^2}{\rho} = \frac{20.158}{0.7} = 28.798 \text{ m/s}^2$$

$\mathbf{a}_n = 28.8 \text{ m/s}^2 \uparrow$

Apply Newton's second law to the ball.



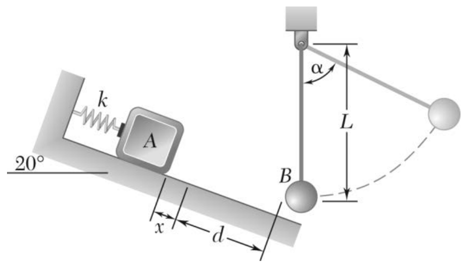
$$+\uparrow \Sigma F = ma_n: N - mg - F_e = ma_n$$

$$N = mg + F_e + ma_n$$

$$= (2)(9.81) + 80 + (2)(28.798)$$

$N = 157.2$ N ◀

PROBLEM 13.200



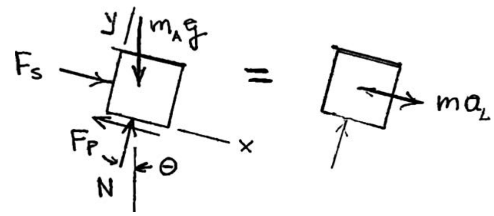
A 2-kg block A is pushed up against a spring compressing it a distance $x = 0.1$ m. The block is then released from rest and slides down the 20° incline until it strikes a 1-kg sphere B which is suspended from a 1 m inextensible rope. The spring constant $k = 800$ N/m, the coefficient of friction between A and the ground is 0.2, the distance A slides from the unstretched length of the spring $d = 1.5$ m and the coefficient of restitution between A and B is 0.8. When $\alpha = 40°$, determine (a) the speed of B (b) the tension in the rope.

SOLUTION

Data: $m_A = 2$ kg, $m_B = 1$ kg, $k = 800$ N/m, $x = 0.1$ m, $d = 1.5$ m

$\mu_k = 0.2$, $e = 0.8$, $\theta = 20°$, $\alpha = 40°$, $l = 1.0$ m

Block slides down the incline:



$$+\nearrow \Sigma F_y = 0$$

$$N - m_A g\cos\theta = 0$$

$$\begin{aligned} N &= m_A g \cos\theta \\ &= (2)(9.81)\cos 20° \\ &= 18.4368 \text{ N} \\ F_f &= \mu_k N = (0.2)(18.4368) \\ &= 3.6874 \text{ N} \end{aligned}$$

Use work and energy. Datum for V_g is the impact point near B.

$$T_1 = 0, \quad (V_1)_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(800)(0.1)^2 = 4.00 \text{ J}$$

$$(V_1)_g = m_A g h_1 = m_A g(x+d)\sin\theta = (2)(9.81)(1.6)\sin 20° = 10.7367 \text{ J}$$

$$U_{1\to 2} = -F_f(x+d) = -(3.6874)(1.6) = -5.8998 \text{ J}$$

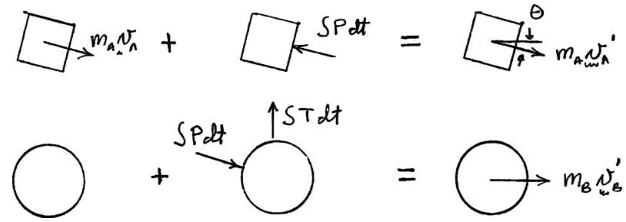
$$T_2 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(1)(v_A^2) = 1.000\, v_A^2 \qquad V_2 = 0$$

$$T_1 + V_1 + U_{1\to 2} = T_2 + V_2: \quad 0 + 4.00 + 10.7367 - 5.8998 = 1.000\, v_A^2 + 0$$

$$v_A^2 = 8.8369 \text{ m}^2/\text{s}^2$$

$$\mathbf{v}_A = 2.9727 \text{ m/s} \searrow 20°$$

Impact: Conservation of momentum.



Both A and B, horizontal components $\xrightarrow{+}$:

$$m_A v_A \cos\theta + 0 = m_A v_A' \cos\theta + m_B v_B$$

$$(2)(2.9727)\cos 20^\circ = 2v_A' \cos 20^\circ + (1.00)v_B \qquad (1)$$

Relative velocities: $(v_B')_n - (v_A')_n = e[(v_B)_n - (v_A)_n]$

$$v_B' \cos\theta - v_A' = e[v_A - 0]$$

$$v_B' \cos 20^\circ - v_A' = (0.8)(2.9727) \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$v_A' = 1.0382 \text{ m/s}$$

$$v_B' = 3.6356 \text{ m/s}$$

Sphere B rises: Use conservation of energy.

$$T_1 = \frac{1}{2}m_B(v_B')^2 \quad V_1 = 0$$

$$T_2 = \frac{1}{2}m_B v_2^2 \qquad V_2 = m_B g h_2 = m_B g l(1-\cos\alpha)$$

$$T_1 + V_1 = T_2 + V_2: \quad \frac{1}{2}m_B(v_B')^2 + 0 = \frac{1}{2}m_B v_2^2 + m_B g(1-\cos)$$

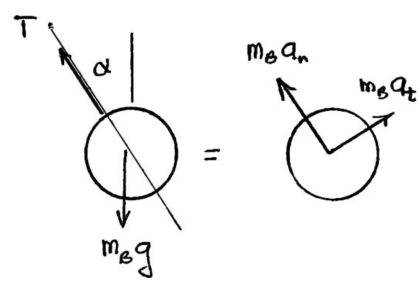
$$v_2^2 = (v_B')^2 - 2gl(1-\cos\alpha)$$

$$= (3.6356)^2 - (2)(9.81)(1-\cos 40^\circ)$$

$$= 8.6274 \text{ m}^2/\text{s}^2$$

(*a*) Speed of B: $v_2 = 2.94$ m/s ◀

(*b*) Tension in the rope: $\rho = 1.00$ m



$$a_n = \frac{v_2^2}{\rho} = \frac{8.6274}{1.00} = 8.6274 \text{ m/s}^2$$

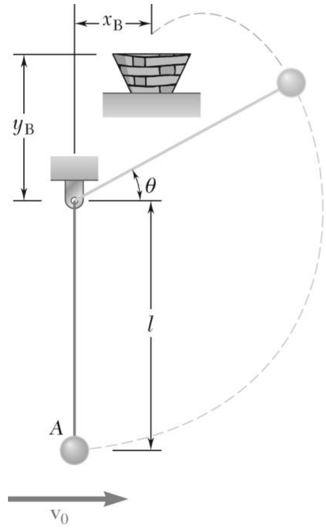
$$+\nwarrow \Sigma F_n = m_B a_n:$$

$$T - m_B g\cos\alpha = m_B a_n$$

$$T = m_B(a_n + g\cos\alpha)$$

$$= (1.0)(8.6274 + 9.81\cos 40^\circ)$$

$T = 16.14$ N ◀



PROBLEM 13.201*

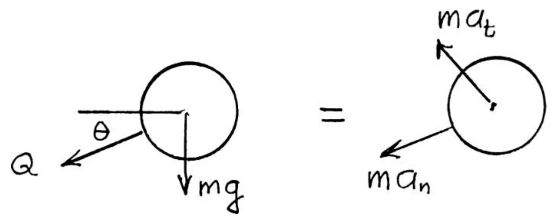
The 2-lb ball at A is suspended by an inextensible cord and given an initial horizontal velocity of $\mathbf{v}_0$. If l = 2 ft, x_B = 0.3 ft and y_B = 0.4 ft determine the initial velocity $\mathbf{v}$ so that the ball will enter in the basket. *Hint*: use a computer to solve the resulting set of equations.

SOLUTION

Let position 1 be at A. $\qquad v_1 = v_0$

Let position 2 be the point described by the angle θ where the path of the ball changes from circular to parabolic. At position 2 the tension Q in the cord is zero.

Relationship between v_2 and θ based on $Q = 0$. Draw the free body diagram.



$$\Sigma F = 0: \quad Q + mg\sin\theta = ma_n = \frac{mv_2^2}{\ell}$$

With $Q = 0$, $\qquad v_2^2 = g\ell\sin\theta \quad \text{or} \quad v_2 = \sqrt{g\ell\sin\theta} \qquad (1)$

Relationship among $v_0, v_2,$ and θ based on conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_0^2 - mg\ell = \frac{1}{2}mv_2^2 + mg\ell\sin\theta$$

$$v_0^2 = v_2^2 + 2g\ell(1+\sin\theta) \qquad (2)$$

PROBLEM 13.201* (Continued)

$\underline{x \text{ and } y \text{ coordinates at position 2}}$:

$$x_2 = \ell \cos\theta \quad (3)$$

$$y_2 = \ell \sin\theta \quad (4)$$

Let t_2 be the time when the ball is in position 2.

Motion on the parabolic path. The horizontal motion is

$$\dot{x} = -v_2 \sin\theta$$

$$x = x_2 - (v_2 \sin\theta)(t - t_2) \quad (5)$$

At Point B, $x = x_B$ and $t = t_B$. From Eq. (5),

$$(t_B - t_2) = \frac{\ell \cos\theta - x_B}{v_\theta \sin\theta} \quad (6)$$

Vertical motion: $\dot{y} = v_2 \cos\theta - g(t - t_2)$

$$y = y_2 + (v_2 \cos\theta)(t - t_2) - \frac{1}{2} g(t - t_2)^2$$

At Point B,

$$y_B = \ell \sin\theta + (v_2 \cos\theta)(t_B - t_2) - \frac{1}{2} g(t_B - t_2)^2 \quad (7)$$

Data: $\ell = 2 \text{ ft}, \quad x_B = 0.3 \text{ ft}, \quad y_B = 0.4 \text{ ft}, \quad g = 32.2 \text{ ft/s}^2$

With the numerical data,

Eq. (1) becomes $v_2 = \sqrt{64.4 \sin\theta}$ (1)′

Eq. (6) becomes $t_B - t_2 = \dfrac{2\cos\theta - 0.3}{v_2 \sin\theta}$ (6)′

Eq. (7) becomes $y_B = 2\sin\theta + (v_2 \cos\theta)(t_B - t_2) - 16.1(t_B - t_2)^2$ (7)′

Method of solution. From a trial value of θ, calculate v_2 from Eq. (1)′, $t_B - t_2$ from Eq. (6)′, and y_B from Eq. (7)′. Repeat until $y_B = 0.4$ ft as required.

Try $\theta = 30°$. $v_2 = \sqrt{64.4 \sin 30°} = 5.6745 \text{ ft/s}$

$$t_B - t_2 = \frac{2\cos 30° - 0.3}{5.6745 \sin 30°} = 0.50473 \text{ s}$$

$$y_B = 2\sin 30° + (5.6745 \cos 30°)(0.50473) - (16.1)(0.50473)^2$$
$$= -0.62116 \text{ ft}$$

Try $\theta = 45°$.

$$v_2 = \sqrt{64.4 \sin 45°} = 6.7482$$

$$t_B - t_2 = \frac{2\cos 45° - 0.3}{6.7482 \sin 45°} = 0.23351 \text{ s}$$

$$y_B = 2\sin 45° + (6.7482 \cos 45°)(0.23351) - (16.1)(0.23351)^2$$
$$= 1.65060 \text{ ft}$$

Try $\theta = 37.5°$.

$$v_2 = \sqrt{64.4 \sin 37.5°} = 6.2613 \text{ ft/s}$$

$$t_B - t_2 = \frac{2\cos 37.5° - 0.3}{6.2613 \sin 37.5°} = 0.33757 \text{ s}$$

$$y_B = 2\sin 37.5° + (6.2613 \cos 37.5°)(0.33757) - (16.1)(0.33757)^2$$
$$= 1.05972 \text{ ft}$$

Let $u = \theta - 30°$. The following sets of data points have be determined:

$$(u,\ y_B) = (0°, -0.62114 \text{ ft}), (7.5°, 1.05972 \text{ ft}), (15°, 1.65060 \text{ ft})$$

The quadratic curve fit of this data gives

$$y_B = -0.62114 + 0.29678\,u - 0.009688711\,u^2$$

Setting $y_B = 0.4$ ft gives the quadratic equation

$$-0.009688711\,u^2 + 0.29678\,u - 1.02114 = 0$$

Solving for u, $u = 3.95°$ and $26.68°$

Rejecting the second value gives $\theta = 30° + u = 33.95°$.

Try $\theta = 33.95°$.

$$v_2 = \sqrt{64.4 \sin 33.95°} = 5.997 \text{ ft/s}$$

$$t_B - t_2 = \frac{2\cos 33.95° - 0.3}{5.9971 \sin 33.95°} = 0.40578 \text{ s}$$

$$y_B = 2\sin 33.95° + (5.997 \cos 33.95°)(0.40578) - (16.1)(0.40578)^2$$
$$= 0.48462 \text{ ft}$$

The new quadratic curve-fit is based on the data points

$$(u,\ y_B) = (0°, -0.62114 \text{ ft}), (3.95°, 0.48462 \text{ ft}), (7.5°, 1.05972 \text{ ft}).$$

The quadratic curve fit of this data is

$$y_B = -0.62114 + 0.342053907\,u - 0.015725232\,u^2$$

Setting $y_B = 0.4$ ft gives

$$-0.015725232\,u^2 + 0.342053907\,u - 1.02114 = 0$$

PROBLEM 13.201* (Continued)

Solving for u,

$$u = 3.572° \qquad \theta = 30° + 3.572° = 33.572°$$

Try $\theta = 33.572°$.

$$v_2 = \sqrt{64.4 \sin 33.572°} = 5.9676 \text{ ft/s}$$

$$t_B - t_2 = \frac{2 \cos 33.572° - 0.3}{5.9676 \sin 33.572°} = 0.41406 \text{ s}$$

$$y_B = 2 \sin 33.572° + (5.9676 \cos 33.572°)(0.41406) - (16.1)(0.41406)^2$$
$$= 0.40445 \text{ ft}$$

which is close enough to 0.4 ft.

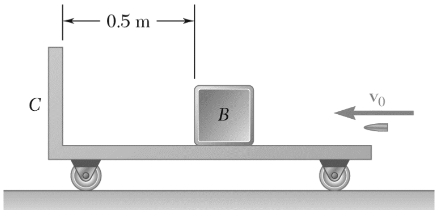
Substituting $\theta = 33.572°$ and $v_2 = 5.9676$ ft/s into Eq. (2) along with other data gives

$$v_0^2 = (5.9676)^2 + (2)(32.2)(2)(1 + \sin 33.572°) = 235.64 \text{ ft}^2/\text{s}^2$$

$$\mathbf{v}_0 = 15.35 \text{ ft/s} \longrightarrow \blacktriangleleft$$

CHAPTER 14

PROBLEM 14.1



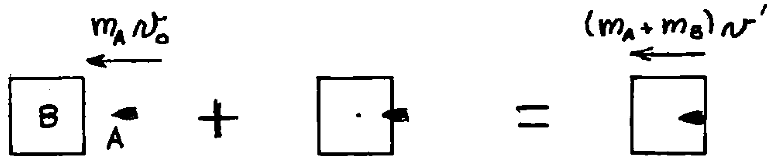
A 30-g bullet is fired with a horizontal velocity of 450 m/s and becomes embedded in block *B* which has a mass of 3 kg. After the impact, block *B* slides on 30-kg carrier *C* until it impacts the end of the carrier. Knowing the impact between *B* and *C* is perfectly plastic and the coefficient of kinetic friction between *B* and *C* is 0.2, determine (*a*) the velocity of the bullet and *B* after the first impact, (*b*) the final velocity of the carrier.

SOLUTION

For convenience, label the bullet as particle *A* of the system of three particles *A*, *B*, and *C*.

(*a*) *Impact between A and B*: Use conservation of linear momentum of *A* and *B*. Assume that the time period is so short that any impulse due to the friction force between *B* and *C* may be neglected.

$$\Sigma m\bar{\mathbf{v}}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\bar{\mathbf{v}}_2$$



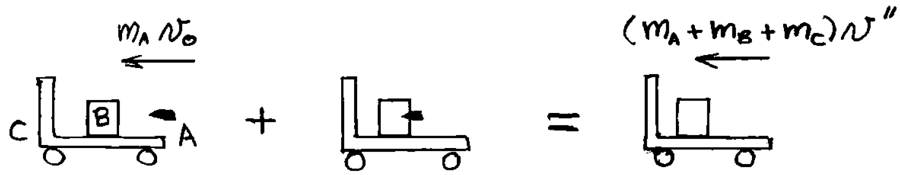
Components $\longleftarrow$: $\quad m_A v_0 + 0 = (m_A + m_B)v'$

$$v' = \frac{m_A v_0}{m_A + m_B} = \frac{(30\times 10^{-3}\text{ kg})(450\text{ m/s})}{(30\times 10^{-3}\text{ kg} + 3\text{ kg}} = 4.4554\text{ m/s}$$

$$\mathbf{v}' = 4.46\text{ m/s} \longleftarrow \blacktriangleleft$$

(*b*) *Final velocity of the carrier*: Particles *A*, *B*, and *C* have the same velocity v'' to the left. Use conservation of linear momentum of all three particles. The friction forces between *B* and *C* are internal forces. Neglect friction at the wheels of the carrier.

$$\Sigma m\bar{\mathbf{v}}_2 + \Sigma \mathbf{Imp}_{2\to 3} = \Sigma m\bar{\mathbf{v}}_3$$



Components $\longleftarrow$: $\quad (m_A + m_B)v' + 0 = (m_A + m_B + m_C)v$

$$v'' = \frac{(m_A + m_B)v'}{m_A + m_B + m_C} = \frac{m_A v_0}{m_A + m_B + m_C}$$

$$= \frac{(30\times 10^{-3}\text{ kg})(450\text{ m/s})}{30\times 10^{-3}\text{ kg} + 3\text{ kg} + 30\text{ kg}} = 0.4087\text{ m/s}$$

$$\mathbf{v}'' = 0.409\text{ m/s} \longleftarrow \blacktriangleleft$$

PROBLEM 14.2

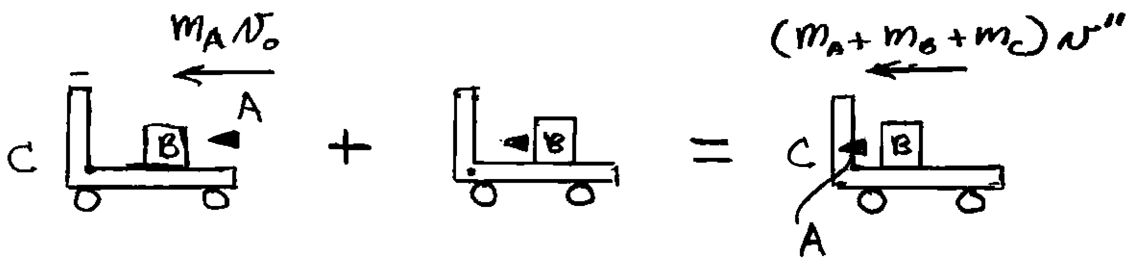
A 30-g bullet is fired with a horizontal velocity of 450 m/s through 3-kg block *B* and becomes embedded in carrier *C* which has a mass of 30 kg. After the impact, block *B* slides 0.3 m on *C* before coming to rest relative to the carrier. Knowing the coefficient of kinetic friction between *B* and *C* is 0.2, determine (*a*) the velocity of the bullet immediately after passing through *B*, (*b*) the final velocity of the carrier.

SOLUTION

For convenience, label the bullet as particle *A* of the system of three particles *A*, *B*, and *C*.

(*b*) *Final velocity of carrier*: Use conservation momentum for all three particles, since the impact forces and the friction force between *B* and *C* are internal forces of the system.

$$\Sigma m\bar{\mathbf{v}}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\bar{\mathbf{v}}_2$$



Components $\longleftarrow$: $\quad m_A v_0 + 0 = (m_A + m_B + m_C)v''$

$$v'' = \frac{m_A v_0}{m_A + m_B + m_C} = \frac{(0.030\text{ kg})(450\text{ m/s})}{33.03\text{ kg}} = 0.40872\text{ m/s}$$

$$\mathbf{v}'' = 0.409\text{ m/s} \longleftarrow \blacktriangleleft$$

(*a*) *Velocity v_A of the bullet*:

The sequence of events described is broken into the following states and processes. The symbols for velocities of *A*, *B*, and *C* at the various states are given in the following table:

	Symbol for velocity			
State	**A**	**B**	**C**	**Process**
(1)	v_0	0	0	Initial state
(2)	v_A	v_B	0	1 $\longrightarrow$ 2: Bullet passes through block
(3)	v_{AC}	v_B	v_{AC}	2 $\longrightarrow$ 3: Bullet impacts end of carrier
(4)	v''	v''	v''	3 $\longrightarrow$ 4: Block slides to rest relative to carrier

PROBLEM 14.2 (Continued)

For process 1 ⟶ 2 apply conservation of momentum.

$$m_A v_0 = m_A v_A + m_B v_B \tag{1}$$

For process 2 ⟶ 3 apply conservation of momentum to *A* and *C*.

$$m_A v_A = (m_A + m_C) v_{AC} \tag{2}$$

For process 3 ⟶ 4 apply conservation of momentum to *A*, *B*, and *C*.

$$(m_A + m_C) v_{AC} + m_B v_B = (m_A + m_B + m_C) v'' \tag{3}$$

For process 3 ⟶ 4 apply the principle of work and energy, since the work $U_{3\to4}$ of the friction force may be calculated.

Normal force: $$N = W_B = m_B g = (3 \text{ kg})(9.81 \text{ m/s}) = 29.43 \text{ N}$$

Friction force: $$F_f = \mu_k N = (0.2)(29.43) = 5.886 \text{ N}$$

Work: $$U_{3\to4} = -F_f d = -(5.886 \text{ N})(0.3 \text{ m}) = -1.7658 \text{ J}$$

Principle of work and energy: $$T_{AC} + T_B + U_{3\to4} = T'' \tag{4}$$

where $$T_{AC} = \frac{1}{2}(m_A + m_C) v_{AC}^2$$

$$T_B = \frac{1}{2} m_B v_B^2$$

$$T'' = \frac{1}{2}(m_A + m_B + m_C)(v'')^2$$

Applying the numerical data gives

$$(0.030)(450) = 0.030 v_A + 3 v_B \tag{1$'$}$$

$$0.030 v_A = 30.03 v_{AC} \tag{2$'$}$$

$$30.03 v_{AC} + 3 v_B = (33.03)(0.40872) \tag{3$'$}$$

$$\frac{1}{2}(30.03) v_{AC}^2 + \frac{1}{2}(3) v_B^2 - 1.7658 = \frac{1}{2}(33.03)(0.40872)^2 \tag{4$'$}$$

From Eq. (3)′, $$v_{AC} = \frac{(33.03)(0.40872) - 3 v_B}{30.03} = 0.44955 - 0.0999\, v_B$$

Substituting into Eq. (4)′ gives

$$(15.015)(0.44955 - 0.0999 v_B)^2 + 1.5 v_B^2 - 1.7658 = 2.7586$$

which reduces to the quadratic equation

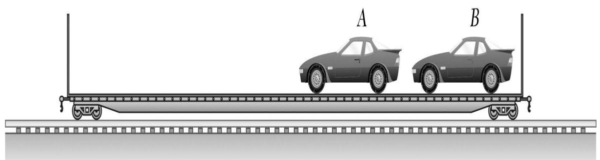
$$1.64985 v_B^2 - 1.34865 v_B - 1.48995 = 0$$

Solving, $v_B = 1.44319$ and -0.62575

$v_B = 1.44319$ m/s

Using Eq. (1)′ with numerical data, $13.5 = 0.030v_A + (3)(1.44319)$

$\mathbf{v}_A = 306$ m/s ← ◀



PROBLEM 14.3

Car A weighing 4000 lb and car B weighing 3700 lb are at rest on a 22-ton flatcar which is also at rest. Cars A and B then accelerate and quickly reach constant speeds relative to the flatcar of 7 ft/s and 3.5 ft/s, respectively, before decelerating to a stop at the opposite end of the flatcar. Neglecting friction and rolling resistance, determine the velocity of the flatcar when the cars are moving at constant speeds.

SOLUTION

The masses are $m_A = \dfrac{4000}{32.2} = 124.2$ slugs, $m_B = \dfrac{3700}{32.2} = 114.9$ slugs, and $m_F = \dfrac{(22)(2000)}{32.2} = 1366.5$ slugs

Let v_A, v_B, and v_F be the sought after velocities in ft/s, positive to the right.

Initial values: $(v_A)_0 = (v_B)_0 = (v_F)_0 = 0.$

Initial momentum of system: $m_A(v_A)_0 + m_B(v_B)_0 + m_F(v_F)_0 = 0.$

There are no horizontal external forces acting during the time period under consideration. Momentum is conserved.

$$0 = m_A v_A + m_B v_B + m_F v_F$$

$$124.2v_A + 114.9v_B + 1366.5v_F = 0 \qquad (1)$$

The relative velocities are given as

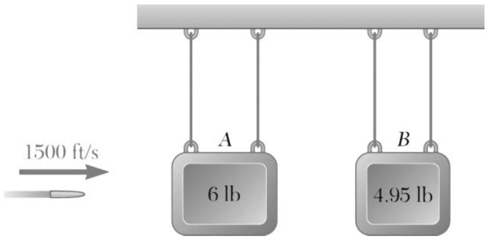
$$v_{A/F} = v_A - v_F = -7 \text{ ft/s} \qquad (2)$$

$$v_{B/F} = v_B - v_F = -3.5 \text{ ft/s} \qquad (3)$$

Solving (1), (2), and (3) simultaneously,

$$v_A = -6.208 \text{ ft/s}, \ v_B = -2.708 \text{ ft/s}, \ v_F = 0.7919 \text{ ft/s}$$

$\mathbf{v}_F = 0.792 \text{ ft/s} \longrightarrow$ ◀



PROBLEM 14.4

A bullet is fired with a horizontal velocity of 1500 ft/s through a 6-lb block *A* and becomes embedded in a 4.95-lb block *B*. Knowing that blocks *A* and *B* start moving with velocities of 5 ft/s and 9 ft/s, respectively, determine (*a*) the weight of the bullet, (*b*) its velocity as it travels from block *A* to block *B*.

SOLUTION

The masses are m for the bullet and m_A and m_B for the blocks.

(*a*) The bullet passes through block *A* and embeds in block *B*. Momentum is conserved.

Initial momentum: $mv_0 + m_A(0) + m_B(0) = mv_0$

Final momentum: $mv_B + m_A v_A + m_B v_B$

Equating, $mv_0 = mv_B + m_A v_A + m_B v_B$

$$m = \frac{m_A v_A + m_B v_B}{v_0 - v_B} = \frac{(6)(5) + (4.95)(9)}{1500 - 9} = 0.0500 \text{ lb}$$

$m = 0.800$ oz ◀

(*b*) The bullet passes through block *A*. Momentum is conserved.

Initial momentum: $mv_0 + m_A(0) = mv_0$

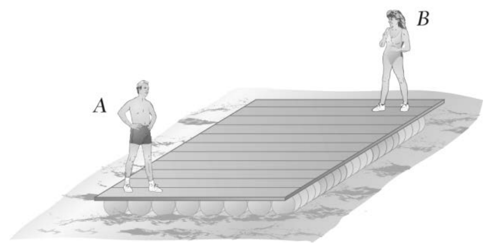
Final momentum: $mv_1 + m_A v_A$

Equating, $mv_0 = mv_1 + m_A v_A$

$$v_1 = \frac{mv_0 - m_A v_A}{m} = \frac{(0.0500)(1500) - (6)(5)}{0.0500} = 900 \text{ ft/s}$$

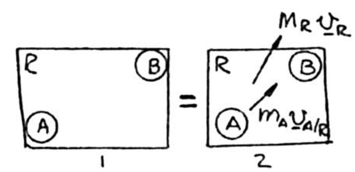
$\mathbf{v}_1 = 900 \text{ ft/s} \longrightarrow$ ◀

PROBLEM 14.5



Two swimmers A and B, of weight 190 lb and 125 lb, respectively, are at diagonally opposite corners of a floating raft when they realize that the raft has broken away from its anchor. Swimmer A immediately starts walking toward B at a speed of 2 ft/s relative to the raft. Knowing that the raft weighs 300 lb, determine (a) the speed of the raft if B does not move, (b) the speed with which B must walk toward A if the raft is not to move.

SOLUTION



(a) The system consists of A and B and the raft R.

Momentum is conserved.

$$(\Sigma m\mathbf{v})_1 = (\Sigma m\mathbf{v})_2$$

$$0 = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_R\mathbf{v}_R \qquad (1)$$

$$\mathbf{v}_A = \mathbf{v}_{A/R} + \mathbf{v}_R \qquad \mathbf{v}_B - \mathbf{v}_{B/R} + \mathbf{v}_R \qquad v_{B/R} = 0$$

$$\mathbf{v}_A = 2 \text{ ft/s} \underset{A}{\nearrow}^{B} + \mathbf{v}_R \qquad \mathbf{v}_B = \mathbf{v}_R$$

$$0 = m_A[2 \underset{A}{\nearrow}^{B} + \mathbf{v}_R] + m_B\mathbf{v}_R + m_R\mathbf{v}_e$$

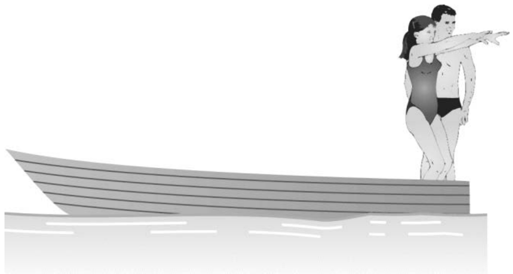
$$\mathbf{v}_R = \frac{-2m_A}{(m_A + m_B + m_R)} = \frac{-(2 \text{ ft/s})(190 \text{ lb})}{(190 \text{ lb} + 125 \text{ lb} + 300 \text{ lb})} \qquad v_R = 0.618 \text{ ft/s} \blacktriangleleft$$

(b) From Eq. (1),

$$0 = m_A v_A + m_B v_B + 0 \qquad (v_R = 0)$$

$$v_B = -\frac{m_A v_A}{m_B} \qquad v_A = v_{A/R} + \cancel{v_R}^{0} = 2 \text{ ft/s}$$

$$v_B = -\frac{(2 \text{ ft/s})(190 \text{ lb})}{(125 \text{ lb})} = 3.04 \text{ ft/s} \qquad v_B = 3.04 \text{ ft/s} \blacktriangleleft$$



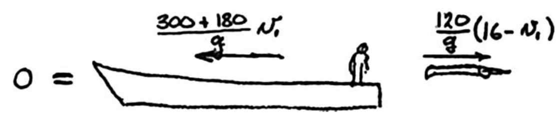
PROBLEM 14.6

A 180-lb man and a 120-lb woman stand side by side at the same end of a 300-lb boat, ready to dive, each with a 16-ft/s velocity relative to the boat. Determine the velocity of the boat after they have both dived, if (*a*) the woman dives first, (*b*) the man dives first.

SOLUTION

(*a*) Woman dives first.

Conservation of momentum:



$$\frac{120}{g}(16 - v_1) - \frac{300+180}{g}v_1 = 0$$

$$v_1 = \frac{(120)(16)}{600} = 3.20 \text{ ft/s} \longleftarrow$$

Man dives next. Conservation of momentum:

$$-\frac{300+180}{g}v_1 = -\frac{300}{g}v_2 + \frac{180}{g}(16 - v_2)$$

$$v_2 = \frac{480v_1 + (180)(16)}{480} = 9.20 \text{ ft/s}$$

$\mathbf{v}_2 = 9.20 \text{ ft/s} \longleftarrow$ ◀

(*b*) Man dives first.

Conservation of momentum:

$$\frac{180}{g}(16 - v_1') - \frac{300+120}{g}v_1' = 0$$

$$v_1' = \frac{(180)(16)}{600} = 4.80 \text{ ft/s}$$

Woman dives next. Conservation of momentum:

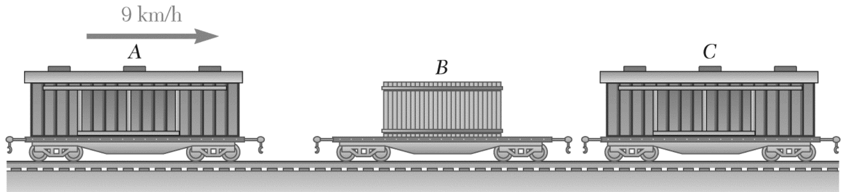
$$-\frac{300+120}{g}v_1' = -\frac{300}{g}v_2' + \frac{120}{g}(16 - v_2')$$

$$v_2' = \frac{420v_1' + (120)(16)}{420} = 9.37 \text{ ft/s}$$

$\mathbf{v}_2' = 9.37 \text{ ft/s} \longleftarrow$ ◀

PROBLEM 14.7

A 40-Mg boxcar *A* is moving in a railroad switchyard with a velocity of 9 km/h toward cars *B* and *C*, which are both at rest with their brakes off at a short distance from each other. Car *B* is a 25-Mg flatcar supporting a 30-Mg container, and car *C* is a 35-Mg boxcar. As the cars hit each other they get automatically and tightly coupled. Determine the velocity of car *A* immediately after each of the two couplings, assuming that the container (*a*) does not slide on the flatcar, (*b*) slides after the first coupling but hits a stop before the second coupling occurs, (*c*) slides and hits the stop only after the second coupling has occurred.



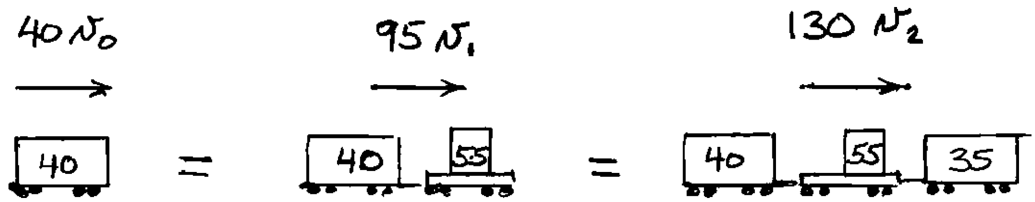
SOLUTION

Each term of the conservation of momentum equation is mass times velocity. As long as the same units are used in all terms, any unit may be used for mass and for velocity. We use Mg for mass and km/h for velocity and apply conservation of momentum.

Note: Only moving masses are shown in the diagrams.

Initial momentum: $m_A v_0 = (40)(9) = 360$

(*a*) Container does not slide

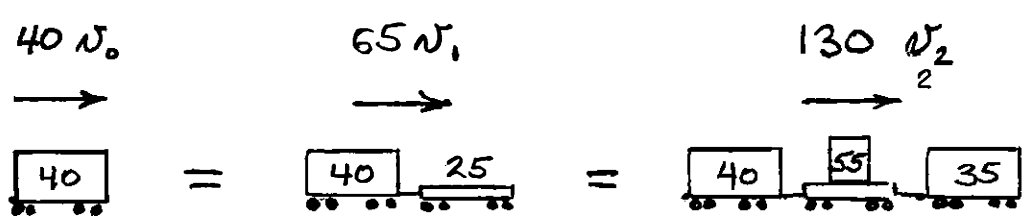


$$360 = 95v_1 = 130v_2$$

$\mathbf{v}_1 = 3.79$ km/h ⟶ ◀

$\mathbf{v}_2 = 2.77$ km/h ⟶ ◀

(*b*) Container slides after 1st coupling, stops before 2nd



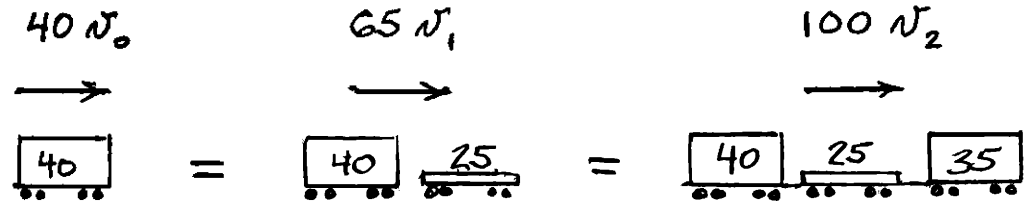
$$360 = 65v_1 = 130v_2$$

$\mathbf{v}_1 = 5.54$ km/h ⟶ ◀

$\mathbf{v}_2 = 2.77$ km/h ⟶ ◀

PROBLEM 14.7 (Continued)

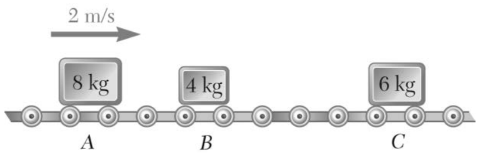
(*c*) Container slides and stops only after 2nd coupling



$$360 = 65v_1 = 100v_2$$

$\mathbf{v}_1 = 5.54 \text{ km/h} \longrightarrow$ ◀

$\mathbf{v}_2 = 3.60 \text{ km/h} \longrightarrow$ ◀

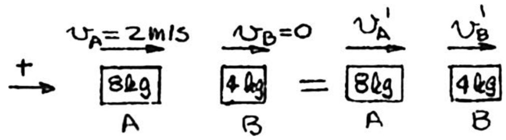


PROBLEM 14.8

Packages in an automobile parts supply house are transported to the loading dock by pushing them along on a roller track with very little friction. At the instant shown, packages *B* and *C* are at rest and package *A* has a velocity of 2 m/s. Knowing that the coefficient of restitution between the packages is 0.3, determine (*a*) the velocity of package *C* after *A* hits *B* and *B* hits *C*, (*b*) the velocity of *A* after it hits *B* for the second time.

SOLUTION

(*a*) Packages *A* and *B*:



Total momentum conserved:

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$
$$(8\text{ kg})(2\text{ m/s}) + 0 = (8\text{ kg})v_A' + (4\text{ kg})v_B'$$
$$4 = 2v_A' + v_B' \qquad (1)$$

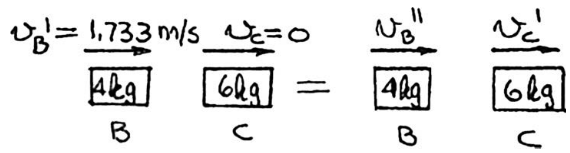
Relative velocities.

$$(v_A - v_B)e = (v_B' - v_A')$$
$$(2)(0.3) = v_B' - v_A' \qquad (2)$$

Solving Equations (1) and (2) simultaneously,

$$v_A' = 1.133\text{ m/s} \longrightarrow$$
$$v_B' = 1.733\text{ m/s} \longrightarrow$$

Packages *B* and *C*:



$$\xrightarrow{+} m_B v_B' + m_C v_C = m_B v_B'' + m_C v_C'$$
$$(4\text{ kg})(1.733\text{ m/s}) + 0 = 4v_B'' + 6v_c'$$
$$6.932 = 4v_B'' + 6v_C' \qquad (3)$$

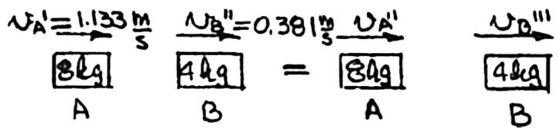
Relative velocities:

$$(v_B' - v_C)e = v_C' - v_B''$$
$$(1.733)(0.3) = 0.5199 = v_C' - v_B'' \qquad (4)$$

Solving equations (3) and (4) simultaneously,

$$\mathbf{v}_C' = 0.901 \text{ m/s} \longrightarrow \blacktriangleleft$$

(*b*) Packages *A* and *B* (second time),



Total momentum conserved:

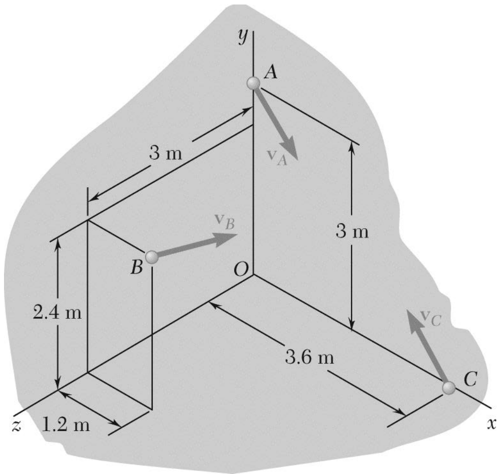
$$(8)(1.133) + (4)(0.381) = 8v_A'' + 4v_B'''$$
$$10.588 = 8v_A'' + 4v_B'' \qquad (5)$$

Relative velocities:

$$(v_A' - v_B'')e = v_B''' - v_A''$$
$$(1.133 - 0.381)(0.3) = 0.2256 = v_B''' - v_A'' \qquad (6)$$

Solving (5) and (6) simultaneously,

$$v_A'' = 0.807 \text{ m/s} \qquad \mathbf{v}_A'' = 0.807 \text{ m/s} \longrightarrow \blacktriangleleft$$

	**PROBLEM 14.9** A system consists of three particles A, B, and C. We know that $m_A = 3$ kg, $m_B = 2$ kg, and $m_C = 4$ kg and that the velocities of the particles expressed in m/s are, respectively, $\mathbf{v}_A = 4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{v}_B = 4\mathbf{i} + 3\mathbf{j}$, and $\mathbf{v}_C = -2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$. Determine the angular momentum $\mathbf{H}_O$ of the system about O.

SOLUTION

Linear momentum of each particle expressed in kg·m/s.

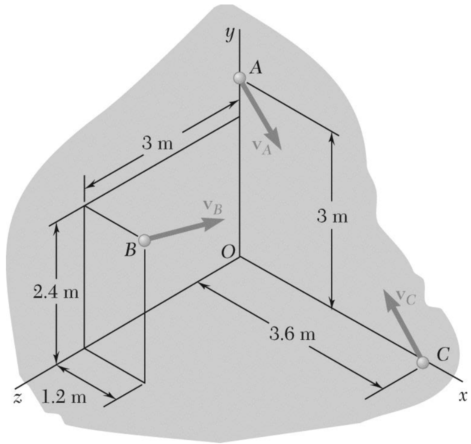
$$m_A\mathbf{v}_A = 12\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$$
$$m_B\mathbf{v}_B = 8\mathbf{i} + 6\mathbf{j}$$
$$m_C\mathbf{v}_C = -8\mathbf{i} + 16\mathbf{j} + 8\mathbf{k}$$

Position vectors, (meters): $\mathbf{r}_A = 3\mathbf{j}$, $\mathbf{r}_B = 1.2\mathbf{i} + 2.4\mathbf{j} + 3\mathbf{k}$, $\mathbf{r}_C = 3.6\mathbf{i}$

Angular momentum about O, (kg·m²/s).

$$\mathbf{H}_O = \mathbf{r}_A \times (m_A\mathbf{v}_A) + \mathbf{r}_B \times (m_B\mathbf{v}_B) + \mathbf{r}_C \times (m_C\mathbf{v}_C)$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 12 & 6 & 6 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 2.4 & 3 \\ 8 & 6 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.6 & 0 & 0 \\ -8 & 16 & 8 \end{vmatrix}$$
$$= (18\mathbf{i} - 36\mathbf{k}) + (-18\mathbf{i} + 24\mathbf{j} - 12\mathbf{k}) + (-28.8\mathbf{j} + 57.6\mathbf{k})$$
$$= 0\mathbf{i} - 4.8\mathbf{j} + 9.6\mathbf{k}$$

$$\mathbf{H}_O = -(4.80\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (9.60\text{ kg}\cdot\text{m}^2/\text{s})\,\mathbf{k} \blacktriangleleft$$



PROBLEM 14.10

For the system of particles of Problem 14.9, determine (*a*) the position vector $\bar{\mathbf{r}}$ of the mass center G of the system, (*b*) the linear momentum $m\bar{\mathbf{v}}$ of the system, (*c*) the angular momentum $\mathbf{H}_G$ of the system about G. Also verify that the answers to this problem and to problem 14.9 satisfy the equation given in Problem 14.27.

PROBLEM 14.9 A system consists of three particles A, B, and C. We know that $m_A = 3$ kg, $m_B = 2$ kg, and $m_C = 4$ kg and that the velocities of the particles expressed in m/s are, respectively, $\mathbf{v}_A = 4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{v}_B = 4\mathbf{i} + 3\mathbf{j}$, and $\mathbf{v}_C = -2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$. Determine the angular momentum $\mathbf{H}_O$ of the system about O.

SOLUTION

Position vectors, (meters): $\mathbf{r}_A = 3\mathbf{j}, \quad \mathbf{r}_B = 1.2\mathbf{i} + 2.4\mathbf{j} + 3\mathbf{k}, \quad \mathbf{r}_C = 3.6\mathbf{i}$

(*a*) Mass center: $(m_A + m_B + m_C)\bar{\mathbf{r}} = m_A\mathbf{r}_A + m_B\mathbf{r}_B + m_C\mathbf{r}_C$

$$9\bar{\mathbf{r}} = (3)(3\mathbf{j}) + (2)(1.2\mathbf{i} + 2.4\mathbf{j} + 3\mathbf{k}) + (4)(3.6\mathbf{i})$$
$$\bar{\mathbf{r}} = 1.86667\mathbf{i} + 1.53333\mathbf{j} + 0.66667\mathbf{k}$$

$$\bar{\mathbf{r}} = (1.867\text{ m})\mathbf{i} + (1.533\text{ m})\mathbf{j} + (0.667\text{ m})\mathbf{k} \blacktriangleleft$$

Linear momentum of each particle, (kg·m²/s).

$$m_A\mathbf{v}_A = 12\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$$
$$m_B\mathbf{v}_B = 8\mathbf{i} + 6\mathbf{j}$$
$$m_C\mathbf{v}_C = -8\mathbf{i} + 16\mathbf{j} + 8\mathbf{k}$$

(*b*) Linear momentum of the system, (kg·m/s.)

$$m\bar{\mathbf{v}} = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C = 12\mathbf{i} + 28\mathbf{j} + 14\mathbf{k}$$

$$m\bar{\mathbf{v}} = (12.00\text{ kg}\cdot\text{m/s})\mathbf{i} + (28.0\text{ kg}\cdot\text{m/s})\mathbf{j} + (14.00\text{ kg}\cdot\text{m/s})\mathbf{k} \blacktriangleleft$$

Position vectors relative to the mass center, (meters).

$$\mathbf{r}'_A = \mathbf{r}_A - \bar{\mathbf{r}} = -1.86667\mathbf{i} + 1.46667\mathbf{j} - 0.66667\mathbf{k}$$
$$\mathbf{r}'_B = \mathbf{r}_B - \bar{\mathbf{r}} = -0.66667\mathbf{i} + 0.86667\mathbf{j} + 2.33333\mathbf{k}$$
$$\mathbf{r}'_C = \mathbf{r}_C - \bar{\mathbf{r}} = 1.73333\mathbf{i} - 1.53333\mathbf{j} - 0.66667\mathbf{k}$$

(*c*) Angular momentum about *G*, (kg·m²/s).

$$\mathbf{H}_G = \mathbf{r}'_A \times m_A\mathbf{v}_A + \mathbf{r}'_B \times m_B\mathbf{v}_B + \mathbf{r}'_C \times m_C\mathbf{v}_C$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.86667 & 1.46667 & -0.66667 \\ 12 & 6 & 6 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.66667 & 0.86667 & 2.33333 \\ 8 & 6 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.73333 & -1.53333 & -0.66667 \\ -8 & 16 & 8 \end{vmatrix}$$

$$= (12.8\mathbf{i} + 3.2\mathbf{j} - 28.8\mathbf{k}) + (-14\mathbf{i} + 18.6667\mathbf{j} - 10.9333\mathbf{k})$$
$$+ (-1.6\mathbf{i} - 8.5333\mathbf{j} + 15.4667\mathbf{k})$$
$$= -2.8\mathbf{i} + 13.3333\mathbf{j} - 24.2667\mathbf{k}$$

$$\mathbf{H}_G = -(2.80\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{i} + (13.33\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{j} - (24.3\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$ ◀

$$\bar{\mathbf{r}} \times m\bar{\mathbf{v}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.86667 & 1.53333 & 0.66667 \\ 12 & 28 & 14 \end{vmatrix}$$

$$= (2.8\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (18.1333\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (33.8667\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

$$\mathbf{H}_G + \bar{\mathbf{r}} \times m\bar{\mathbf{v}} = -(4.8\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (9.6\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

Angular momentum about *O*.

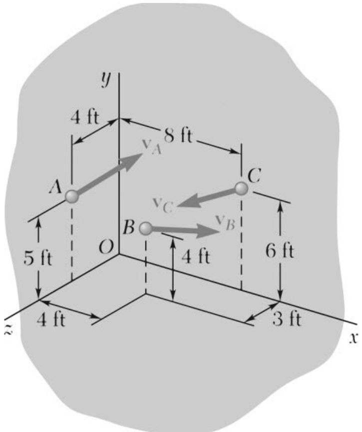
$$\mathbf{H}_O = \mathbf{r}_A \times (m_A\mathbf{v}_A) + \mathbf{r}_B \times (m_B\mathbf{v}_B) + \mathbf{r}_C \times (m_C\mathbf{v}_C)$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 12 & 6 & 6 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 2.4 & 3 \\ 8 & 6 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.6 & 0 & 0 \\ -8 & 16 & 8 \end{vmatrix}$$

$$= (18\mathbf{i} - 36\mathbf{k}) + (-18\mathbf{i} + 24\mathbf{j} - 12\mathbf{k}) + (-28.8\mathbf{j} + 57.6\mathbf{k})$$
$$= -(4.8\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (9.6\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

Note that

$$\mathbf{H}_O = \mathbf{H}_G + \bar{\mathbf{r}} \times m\bar{\mathbf{v}}$$



PROBLEM 14.11

A system consists of three particles A, B, and C. We know that $W_A = 5\text{ lb}$, $W_B = 4\text{ lb}$, and $W_C = 3\text{ lb}$, and that the velocities of the particles expressed in ft/s are, respectively, $\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $\mathbf{v}_B = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$, and $\mathbf{v}_C = -3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. Determine ($a$) the components v_x and v_z of the velocity of particle B for which the angular momentum $\mathbf{H}_O$ of the system about O is parallel to the x axis, (b) the value of $\mathbf{H}_O$.

SOLUTION

$$\mathbf{H}_O = \Sigma\mathbf{r}_i \times m\mathbf{v}_i = \Sigma m_i \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_i & y_i & z_i \\ (v_i)_x & (v_i)_y & (v_i)_z \end{vmatrix}$$

$$= \frac{5}{g}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 4 \\ 2 & 3 & -2 \end{vmatrix} + \frac{4}{g}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 4 & 3 \\ v_x & 2 & v_z \end{vmatrix} + \frac{3}{g}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 6 & 0 \\ -3 & -2 & 1 \end{vmatrix}$$

$$= \frac{1}{g}[5(-10-12) + 4(4v_z - 6) + 3(6-0)]\mathbf{i}$$

$$+ \frac{1}{g}[5(8-0) + 4(3v_x - 4v_z) + 3(0-8)]\mathbf{j}$$

$$+ \frac{1}{g}[5(0-10) + 4(8-4v_x) + 3(-16+18)]\mathbf{k}$$

$$\mathbf{H}_O = \frac{1}{g}[(16v_z - 116)\mathbf{i} + (12v_z - 16v_z + 16)\mathbf{j} + (-16v_x - 12)\mathbf{k}] \qquad (1)$$

(a) For $\mathbf{H}_O$ to be parallel to the x axis, we must have $H_y = H_z = 0$:

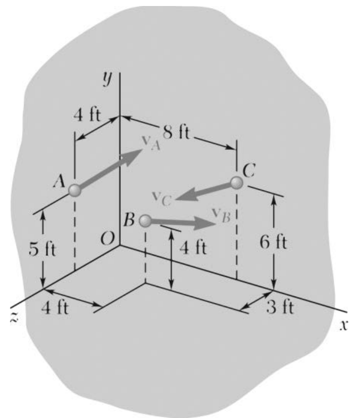
$H_z = 0$: $-16v_x - 12 = 0$ $\qquad v_x = -0.75\text{ ft/s}$ ◀

$H_y = 0$: $12(-0.75) - 16v_z + 16 = 0$ $\qquad v_z = 0.4375\text{ ft/s}$ ◀

(b) Substitute into Eq. (1):

$$\mathbf{H}_O = \frac{1}{g}(16v_z - 116)\mathbf{i} = \frac{1}{g}[16(0.4375) - 116]\mathbf{i} = -\frac{109.0}{32.2}\mathbf{i}$$

$\mathbf{H}_O = -(3.39\text{ ft}\cdot\text{lb}\cdot\text{s})\mathbf{i}$ ◀



PROBLEM 14.12

For the system of particles of Problem 14.11, determine (*a*) the components v_x and v_z of the velocity of particle *B* for which the angular momentum $\mathbf{H}_O$ of the system about *O* is parallel to the *z* axis, (*b*) the value of $\mathbf{H}_O$.

PROBLEM 14.11 A system consists of three particles *A*, *B*, and *C*. We know that $W_A = 5\text{ lb}$, $W_B = 4\text{ lb}$, and $W_C = 3\text{ lb}$ and that the velocities of the particles expressed in ft/s are, respectively, $\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $\mathbf{v}_B = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$, and $\mathbf{v}_C = -3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. Determine (*a*) the components v_x and v_z of the velocity of particle *B* for which the angular momentum $\mathbf{H}_O$ of the system about *O* is parallel to the *x* axis, (*b*) the value of $\mathbf{H}_O$.

SOLUTION

$$\mathbf{H}_O = \Sigma \mathbf{r}_i \times m\mathbf{v}_i = \Sigma m_i \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_i & y_i & z_i \\ (v_i)_x & (v_i)_y & (v_i)_z \end{vmatrix}$$

$$= \frac{5}{g}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 4 \\ 2 & 3 & -2 \end{vmatrix} + \frac{4}{g}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 4 & 3 \\ v_x & 2 & v_z \end{vmatrix} + \frac{3}{g}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 6 & 0 \\ -3 & -2 & 1 \end{vmatrix}$$

$$= \frac{1}{g}[5(-10-12) + 4(4v_z - 6) + 3(6-0)]\mathbf{i}$$

$$+ \frac{1}{g}[5(8-0) + 4(3v_x - 4v_z) + 3(0-8)]\mathbf{j}$$

$$+ \frac{1}{g}[5(0-10) + 4(8-4v_x) + 3(-16+18)]\mathbf{k}$$

$$\mathbf{H}_O = \frac{1}{g}[(16v_z - 116)\mathbf{i} + (12v_z - 16v_z + 16)\mathbf{j} + (-16v_x - 12)\mathbf{k}] \quad (1)$$

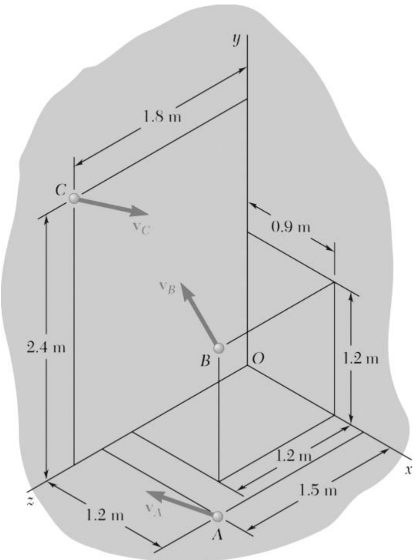
(*a*) For $\mathbf{H}_O$ to be parallel to the *z* axis, we must have $H_x = H_y = 0$:

$H_x = 0$: $16v_z - 116 = 0$ $\qquad v_z = 7.25 \text{ ft/s}$ ◀

$H_y = 0$: $12v_x - 16(7.25) + 16 = 0$ $\qquad v_x = 8.33 \text{ ft/s}$ ◀

(*b*) Substituting into Eq. (1):

$\mathbf{H}_O = \dfrac{1}{32.2}[-16(8.33) - 12]\mathbf{k}$ $\qquad \mathbf{H}_O = -(4.51 \text{ ft}\cdot\text{lb}\cdot\text{s})\mathbf{k}$ ◀



PROBLEM 14.13

A system consists of three particles A, B, and C. We know that $m_A = 3$ kg, $m_B = 4$ kg, and $m_C = 5$ kg, and that the velocities of the particles expressed in m/s are, respectively, $\mathbf{v}_A = -4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$, $\mathbf{v}_B = -6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$, and $\mathbf{v}_C = 2\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}$. Determine the angular momentum $\mathbf{H}_O$ of the system about O.

SOLUTION

Linear momentum of each particle, $(\text{kg}\cdot\text{m/s})$:

$$m_A\mathbf{v}_A = -12\mathbf{i} + 12\mathbf{j} + 18\mathbf{k}$$
$$m_B\mathbf{v}_B = -24\mathbf{i} + 32\mathbf{j} + 16\mathbf{k}$$
$$m_C\mathbf{v}_C = 10\mathbf{i} - 30\mathbf{j} - 20\mathbf{k}$$

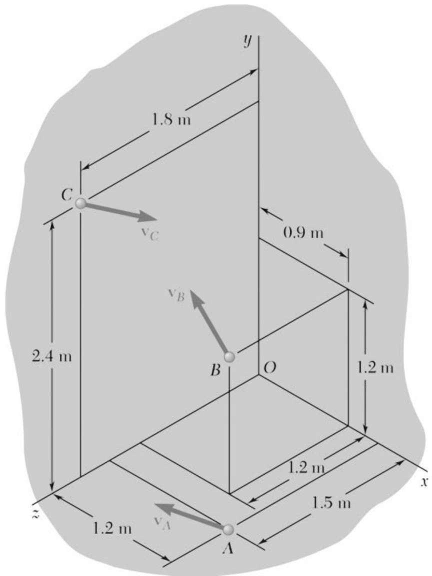
Position vectors, (meters):

$$\mathbf{r}_A = 1.2\mathbf{i} + 1.5\mathbf{k}, \quad \mathbf{r}_B = 0.9\mathbf{i} + 1.2\mathbf{j} + 1.2\mathbf{k}, \quad \mathbf{r}_C = 2.4\mathbf{j} + 1.8\mathbf{k}$$

Angular momentum about O, $(\text{kg}\cdot\text{m}^2/\text{s})$:

$$\mathbf{H}_O = \mathbf{r}_A \times m_A\mathbf{v}_A + \mathbf{r}_B \times m_B\mathbf{v}_B + \mathbf{r}_C \times m_C\mathbf{v}_C$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 0 & 1.5 \\ -12 & 12 & 18 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 1.2 & 1.2 \\ -24 & 32 & 16 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.4 & 1.8 \\ 10 & -30 & -20 \end{vmatrix}$$
$$= (-18\mathbf{i} - 39.6\mathbf{j} + 14.4\mathbf{k}) + (-19.2\mathbf{i} - 43.2\mathbf{j} + 57.6\mathbf{k}) + (6\mathbf{i} + 18\mathbf{j} - 24\mathbf{k})$$
$$= -31.2\mathbf{i} - 64.8\mathbf{j} + 48.0\mathbf{k}$$

$$\mathbf{H}_O = -(31.2 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (64.8 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (48.0 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k} \blacktriangleleft$$



PROBLEM 14.14

For the system of particles of Problem 14.13, determine (*a*) the position vector $\bar{\mathbf{r}}$ of the mass center G of the system, (*b*) the linear momentum $m\bar{\mathbf{v}}$ of the system, (*c*) the angular momentum $\mathbf{H}_G$ of the system about G. Also verify that the answers to this problem and to Problem 14.13 satisfy the equation given in Problem 14.27.

PROBLEM 14.13 A system consists of three particles A, B, and C. We know that $m_A = 3$ kg, $m_B = 4$ kg, and $m_C = 5$ kg and that the velocities of the particles expressed in m/s are, respectively, $\mathbf{v}_A = -4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$, $\mathbf{v}_B = -6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$, and $\mathbf{v}_C = 2\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}$. Determine the angular momentum $\mathbf{H}_O$ of the system about O.

SOLUTION

Position vectors, (meters):

$$\mathbf{r}_A = 1.2\mathbf{i} + 1.5\mathbf{k}, \quad \mathbf{r}_B = 0.9\mathbf{i} + 1.2\mathbf{j} + 1.2\mathbf{k}, \quad \mathbf{r}_C = 2.4\mathbf{j} + 1.8\mathbf{k}$$

(*a*) Mass center:

$$(m_A + m_B + m_C)\bar{\mathbf{r}} = m_A\mathbf{r}_A + m_B\mathbf{r}_B + m_C\mathbf{r}_C$$

$$12\bar{\mathbf{r}} = (3)(1.2\mathbf{i} + 1.5\mathbf{k}) + (4)(0.9\mathbf{i} + 1.2\mathbf{j} + 1.2\mathbf{k}) + (5)(2.4\mathbf{j} + 1.8\mathbf{k})$$

$$\bar{\mathbf{r}} = 0.6\mathbf{i} + 1.4\mathbf{j} + 1.525\mathbf{k}$$

$$\bar{\mathbf{r}} = (0.600 \text{ m})\mathbf{i} + (1.400 \text{ m})\mathbf{j} + (1.525 \text{ m})\mathbf{k} \blacktriangleleft$$

Linear momentum of each particle, (kg · m/s):

$$m_A\mathbf{v}_A = -12\mathbf{i} + 12\mathbf{j} + 18\mathbf{k}$$
$$m_B\mathbf{v}_B = -24\mathbf{i} + 32\mathbf{j} + 16\mathbf{k}$$
$$m_C\mathbf{v}_C = 10\mathbf{i} - 30\mathbf{j} - 20\mathbf{k}$$

(*b*) Linear momentum of the system, (kg · m/s):

$$m\bar{\mathbf{v}} = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C = -26\mathbf{i} + 14\mathbf{j} + 14\mathbf{k}$$

$$m\bar{\mathbf{v}} = -(26.0 \text{ kg}\cdot\text{m/s})\mathbf{i} + (14.00 \text{ kg}\cdot\text{m/s})\mathbf{j} + (14.00 \text{ kg}\cdot\text{m/s})\mathbf{k} \blacktriangleleft$$

Position vectors relative to the mass center, (meters).

$$\mathbf{r}'_A = \mathbf{r}_A - \bar{\mathbf{r}} = 0.6\mathbf{i} - 1.4\mathbf{j} - 0.025\mathbf{k}$$
$$\mathbf{r}'_B = \mathbf{r}_B - \bar{\mathbf{r}} = 0.3\mathbf{i} - 0.2\mathbf{j} - 0.325\mathbf{k}$$
$$\mathbf{r}'_C = \mathbf{r}_C - \bar{\mathbf{r}} = -0.6\mathbf{i} + 1.0\mathbf{j} + 0.275\mathbf{k}$$

(*c*) Angular momentum about G, $(\text{kg}\cdot\text{m}^2/\text{s})$:

$$\mathbf{H}_G = \mathbf{r}'_A \times m_A \mathbf{v}_A + \mathbf{r}'_B \times m_B \mathbf{v}_B + \mathbf{r}'_C \times m_C \mathbf{v}_C$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & -1.4 & -0.025 \\ -12 & 12 & 18 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & -0.2 & -0.325 \\ -24 & 32 & 16 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.6 & 1.0 & 0.275 \\ 10 & -30 & -20 \end{vmatrix}$$

$$= (-24.9\mathbf{i} - 10.5\mathbf{j} - 9.6\mathbf{k}) + (7.2\mathbf{i} + 3.0\mathbf{j} + 4.8\mathbf{k}) + (-11.75\mathbf{i} - 9.25\mathbf{j} + 8.0\mathbf{k})$$

$$= -29.45\mathbf{i} - 16.75\mathbf{j} + 3.2\mathbf{k}$$

$$\mathbf{H}_G = -(29.5 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (16.75 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (3.20 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k} \quad \blacktriangleleft$$

$$\bar{\mathbf{r}} \times m\bar{\mathbf{v}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 1.4 & 1.525 \\ -26 & 14 & 14 \end{vmatrix} = -1.75\mathbf{i} - 48.05\mathbf{j} + 44.8\mathbf{k}$$

$$\mathbf{H}_G + \bar{\mathbf{r}} \times m\mathbf{v} = -(31.2 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (64.8 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (48.0 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

Angular momentum about O, $(\text{kg}\cdot\text{m}^2/\text{s})$:

$$\mathbf{H}_O = \mathbf{r}_A \times m_A \mathbf{v}_A + \mathbf{r}_B \times m_B \mathbf{v}_B + \mathbf{r}_C \times m_C \mathbf{v}_C$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 0 & 1.5 \\ -12 & 12 & 18 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 1.2 & 1.2 \\ -24 & 32 & 16 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.4 & 1.8 \\ 10 & -30 & -20 \end{vmatrix}$$

$$= (-18\mathbf{i} - 39.6\mathbf{j} + 14.4\mathbf{k}) + (-19.2\mathbf{i} - 43.2\mathbf{j} + 57.6\mathbf{k}) + (6\mathbf{i} + 18\mathbf{j} - 24\mathbf{k})$$

$$= -(31.2 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (64.8 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (48.0 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

Note that $\mathbf{H}_O = \mathbf{H}_G + \bar{\mathbf{r}} \times m\bar{\mathbf{v}}$.

PROBLEM 14.15

A 13-kg projectile is passing through the origin O with a velocity $\mathbf{v}_0 = (35 \text{ m/s})\mathbf{i}$ when it explodes into two fragments A and B, of mass 5 kg and 8 kg, respectively. Knowing that 3 s later the position of fragment A is (90 m, 7 m, –14 m), determine the position of fragment B at the same instant. Assume $a_y = -g = -9.81 \text{ m/s}^2$ and neglect air resistance.

SOLUTION

Motion of mass center:

It moves as if projectile had not exploded.

$$\begin{aligned}\overline{\mathbf{r}} &= v_0 t\mathbf{i} - \frac{1}{2}gt^2\mathbf{j} \\ &= (35 \text{ m/s})(3 \text{ s})\mathbf{i} - \frac{1}{2}(9.81 \text{ m/s}^2)(3 \text{ s})^2\mathbf{j} \\ &= (105 \text{ m})\mathbf{i} - (44.145 \text{ m})\mathbf{j}\end{aligned}$$

Equation (14.12):

$$m\overline{\mathbf{r}} = \Sigma m_i \mathbf{r}_i:$$

$$m\overline{\mathbf{r}} = m_A\mathbf{r}_A + m_B\mathbf{r}_B$$

$$13(105\mathbf{i} - 44.145\mathbf{j}) = 5(90\mathbf{i} + 7\mathbf{j} - 14\mathbf{k}) + 8\mathbf{r}_B$$

$$\begin{aligned}8\mathbf{r}_B &= (13\times 105 - 5\times 90)\mathbf{i} \\ &\quad + (-13\times 44.145 - 5\times 7)\mathbf{j} + (5\times 14)\mathbf{k} \\ &= 915\mathbf{i} - 608.89\mathbf{j} + 70\mathbf{k}\end{aligned}$$

$$\mathbf{r}_B = (114.4 \text{ m})\mathbf{i} - (76.1 \text{ m})\mathbf{j} + (8.75 \text{ m})\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 14.16

A 300-kg space vehicle traveling with a velocity $\mathbf{v}_0$ = (360 m/s)**i** passes through the origin O at t = 0. Explosive charges then separate the vehicle into three parts A, B, and C, with mass, respectively, 150 kg, 100 kg, and 50 kg. Knowing that at t = 4 s, the positions of parts A and B are observed to be A (1170 m, –290 m, –585 m) and B (1975 m, 365 m, 800 m), determine the corresponding position of part C. Neglect the effect of gravity.

SOLUTION

Motion of mass center:

Since there is no external force,

$$\bar{\mathbf{r}} = \mathbf{v}_0 t = (360 \text{ m/s})\mathbf{i}\,(4 \text{ s}) = (1440 \text{ m})\mathbf{i}$$

Equation (14.12):

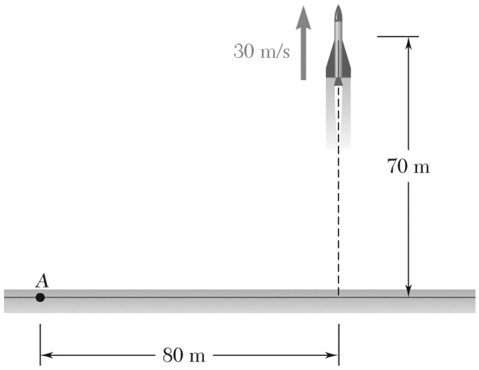
$$m\bar{\mathbf{r}} = \Sigma m_i \mathbf{r}_i:$$

$$(300)(1440\mathbf{i}) = (150)(1170\mathbf{i} - 290\mathbf{j} - 585\mathbf{k}) + (100)(1975\mathbf{i} + 365\mathbf{j} + 800\mathbf{k}) + (50)\mathbf{r}_C$$

$$50\mathbf{r}_C = (300\times 1440 - 150\times 1170 - 100\times 1975)\mathbf{i} + (150\times 290 - 100\times 365)\mathbf{j} + (150\times 585 - 100\times 800)\mathbf{k}$$

$$= 59{,}000\mathbf{i} + 7{,}000\mathbf{j} + 7{,}750\mathbf{k}$$

$$\mathbf{r}_C = (1180 \text{ m})\mathbf{i} + (140 \text{ m})\mathbf{j} + (155 \text{ m})\mathbf{k} \blacktriangleleft$$



PROBLEM 14.17

A 2-kg model rocket is launched vertically and reaches an altitude of 70 m with a speed of 30 m/s at the end of powered flight, time $t = 0$. As the rocket approaches its maximum altitude it explodes into two parts of masses $m_A = 0.7\text{ kg}$ and $m_B = 1.3\text{ kg}$. Part A is observed to strike the ground 80 m west of the launch point at $t = 6$ s. Determine the position of part B at that time.

SOLUTION

Choose a planar coordinate system having coordinates x and y with the origin at the launch point on the ground and the x-axis pointing east and the y-axis vertically upward.

Let subscript E refer to the point where the explosion occurs, and A and B refer to the fragments A and B.
Let t be the time elapsed after the explosition.

Motion of the mass center:

$$\overline{x} = x_E + (\overline{v}_x)t = 0$$

$$\overline{y} = y_E + (\overline{v}_y)_0^t - \frac{1}{2}gt^2$$

where $y_E = 70\text{ m}$ and $(\overline{v}_y)_0 = 30\text{ m/s}$

At $t = 6\text{ s}$, $\overline{x} = 0$

$$\overline{y} = 70 + (30)(6) - \frac{1}{2}(9.81)(6)^2 = 73.42\text{ m}$$

Definition of mass center:

$$m_A\overline{x} = m_A x_A + m_B x_B$$

$$0 = (0.7\text{ kg})(-80\text{ m}) + (1.3\text{ kg})x_B$$

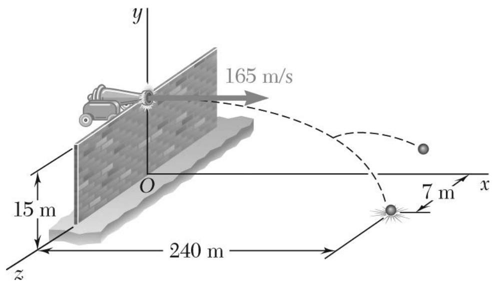
$$x_B = 43.1\text{ m}$$

$$m\overline{y} = m_A y_A + m_B y_B$$

$$(2\text{ kg})(73.42\text{ m}) = (0.7\text{ kg})(0) + (1.3\text{ kg})y_B$$

$$y_B = 113.0\text{ m}$$

Position of part B: 43.1 m (east), 113.0 m (up) ◀



PROBLEM 14.18

An 18-kg cannonball and a 12-kg cannonball are chained together and fired horizontally with a velocity of 165 m/s from the top of a 15-m wall. The chain breaks during the flight of the cannonballs and the 12-kg cannonball strikes the ground at $t = 1.5$ s, at a distance of 240 m from the foot of the wall, and 7 m to the right of the line of fire. Determine the position of the other cannonball at that instant. Neglect the resistance of the air.

SOLUTION

Let subscript A refer to the 12-kg cannonball and B to the 18-kg cannonball.

The motion of the mass center of A and B is uniform in the x-direction, uniformly accelerated with acceleration $-g = -9.81\text{ m/s}^2$ in the y-direction, and zero in the z-direction.

$$\bar{x} = (v_0)_x t = (165\text{ m/s})(1.5\text{ s}) = 247.5\text{ m}$$

$$\bar{y} = \bar{y}_0 + (\bar{v}_0)_y t - \frac{1}{2}gt^2$$

$$= 15\text{ m} + 0 - \frac{1}{2}(9.81\text{ m/s}^2)(1.55)^2 = 3.964\text{ m}$$

$$\bar{z} = 0$$

$$\bar{\mathbf{r}} = (247.5\text{ m})\mathbf{i} + (3.964\text{ m})\mathbf{j}$$

Definition of mass center:

$$m\bar{\mathbf{r}} = m_A\mathbf{r}_A + m_B\mathbf{r}_B$$

Data: $m_A = 12\text{ kg}, \quad m_B = 18\text{ kg}, \quad m = m_A + m_B = 30\text{ kg}$

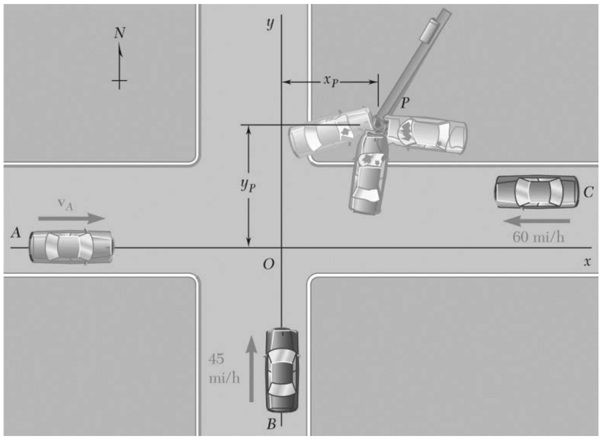
$$t = 1.5\text{ s}, \quad x_A = 240\text{ m}, \quad y_A = 0, \quad z_A = 7\text{ m}$$

$$(30)(247.5\mathbf{i} + 3.964\,\mathbf{j}) = (12)(240\mathbf{i} + 7\mathbf{k}) + (18)(x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k})$$

$\mathbf{i}$: $(30)(247.5) = (12)(240) + 18\,x_B$ $\qquad x_B = 253\text{ m}$ ◀

$\mathbf{j}$: $(30)(3.964) = (12)(0) + 18\,y_B$ $\qquad y_B = 6.61\text{ m}$ ◀

$\mathbf{k}$: $(30)(0) = (12)(7) + 18\,z_B$ $\qquad z_B = -4.67\text{ m}$ ◀



PROBLEM 14.19

Car A was traveling east at high speed when it collided at Point O with car B, which was traveling north at 45 mi/h. Car C, which was traveling west at 60 mi/h, was 32 ft east and 10 ft north of Point O at the time of the collision. Because the pavement was wet, the driver of car C could not prevent his car from sliding into the other two cars, and the three cars, stuck together, kept sliding until they hit the utility pole P. Knowing that the weights of cars A, B, and C are, respectively, 3000 lb, 2600 lb, and 2400 lb, and neglecting the forces exerted on the cars by the wet pavement, solve the problems indicated.

Knowing that the speed of car A was 75 mi/h and that the time elapsed from the first collision to the stop at P was 2.4 s, determine the coordinates of the utility pole P.

SOLUTION

Let t be the time elapsed since the first collision. No external forces in the xy plane act on the system consisting of cars A, B, and C during the impacts with one another. The mass center of the system moves at the velocity it had before the collision.

Setting the origin at O, we can find the initial mass center $\bar{\mathbf{r}}_0$: at the moment of the first collision:

$$(m_A + m_B + m_C)(\bar{x}_0\mathbf{i} + \bar{y}_0\mathbf{j}) = m_A(0) + m_B(0) + m_C(x_C\mathbf{i} + y_C\mathbf{j})$$

$$\bar{x}_0 = 0.3x_C = (0.3)(32) = 9.6 \text{ ft}, \quad \bar{y}_0 = 0.3y_C = (0.3)(10) = 3 \text{ ft}$$

Given velocities:

$$\mathbf{v}_A = (75 \text{ mi/h})\mathbf{i} = (110 \text{ ft/s})\mathbf{i}, \quad \mathbf{v}_B = (45 \text{ mi/h})\mathbf{i} = (66 \text{ ft/s})\mathbf{j}, \quad \mathbf{v}_C = (60 \text{ mi/h})\mathbf{i} = (88 \text{ ft/s})\mathbf{i}$$

Velocity of mass center:

$$(m_A + m_B + m_C)\bar{\mathbf{v}} = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C$$

$$\bar{\mathbf{v}} = 0.375\mathbf{v}_A + 0.325\mathbf{v}_B + 0.3\mathbf{v}_C$$

Since the collided cars hit the pole at

$$\mathbf{r}_P = x_P\mathbf{i} + y_P\mathbf{j}$$

$$\mathbf{x}_P\mathbf{i} + y_P\mathbf{j} = \bar{x}_0\mathbf{i} + \bar{y}_0\mathbf{j} + \bar{\mathbf{v}}t \qquad \text{Resolve into components.}$$

$$x: \quad x_P = \bar{x}_0 + 0.375v_At_P - 0.3v_Ct_P \tag{1}$$

$$y: \quad y_P = \bar{y}_0 + 0.325v_Bt_P \tag{2}$$

PROBLEM 14.19 (Continued)

Data: $t_P = 2.4$ s

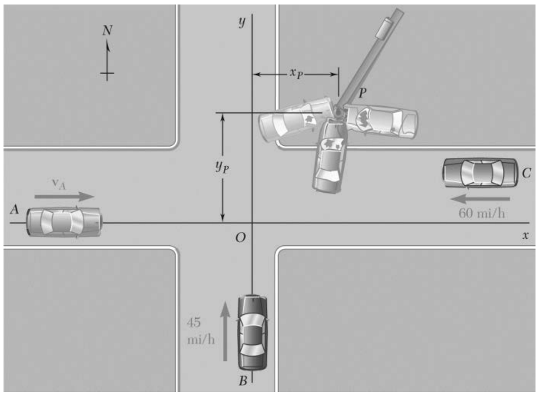
From (1), $x_P = 9.6 + (0.375)(110)(2.4) - (0.3)(88)(2.4) = 45.240$

From (2), $y_P = 3.0 + (0.325)(66)(2.4) = 54.480$ ft

$x_P = 45.2$ ft ◀

$y_P = 54.5$ ft ◀

PROBLEM 14.20



Car A was traveling east at high speed when it collided at Point O with car B, which was traveling north at 45 mi/h. Car C, which was traveling west at 60 mi/h, was 32 ft east and 10 ft north of Point O at the time of the collision. Because the pavement was wet, the driver of car C could not prevent his car from sliding into the other two cars, and the three cars, stuck together, kept sliding until they hit the utility pole P. Knowing that the weights of cars A, B, and C are, respectively, 3000 lb, 2600 lb, and 2400 lb, and neglecting the forces exerted on the cars by the wet pavement, solve the problems indicated. Knowing that the coordinates of the utility pole are $x_P = 46$ ft and $y_P = 59$ ft, determine (a) the time elapsed from the first collision to the stop at P, (b) the speed of car A.

SOLUTION

Let t be the time elapsed since the first collision. No external forces in the xy plane act on the system consisting of cars A, B, and C during the impacts with one another. The mass center of the system moves at the velocity it had before the collision.

Setting the origin at O, we can find the initial mass center $\overline{\mathbf{r}}_0$: at the moment of the first collision:

$$(m_A + m_B + m_C)(\overline{x}_0\mathbf{i} + \overline{y}_0\mathbf{j}) = m_A(0) + m_B(0) + m_C(x_C\mathbf{i} + y_C\mathbf{j})$$

$$\overline{x}_0 = 0.3x_C = (0.3)(32) = 9.6 \text{ ft}, \quad \overline{y}_0 = 0.3y_C = (0.3)(10) = 3 \text{ ft}$$

Given velocities:

$$\mathbf{v}_A = v_A\mathbf{i}, \quad \mathbf{v}_B = (45 \text{ mi/h})\mathbf{i} = (66 \text{ ft/s})\mathbf{j}, \quad \mathbf{v}_C = (60 \text{ mi/h})\mathbf{i} = (88 \text{ ft/s})\mathbf{i}$$

Velocity of mass center:

$$(m_A + m_B + m_C)\mathbf{v} = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C$$

$$\overline{\mathbf{v}} = 0.375\mathbf{v}_A + 0.325\mathbf{v}_B + 0.3\mathbf{v}_C$$

Since the collided cars hit the pole at

$$\mathbf{r}_P = x_P\mathbf{i} + y_P\mathbf{j}$$

$$x_P\mathbf{i} + y_P\mathbf{j} = \overline{x}_0\mathbf{i} + \overline{y}_0\mathbf{j} + \overline{\mathbf{v}}t \qquad \text{Resolve into components.}$$

$$x: \quad x_P = \overline{x}_0 + 0.375v_At_P - 0.3v_Ct_P \tag{1}$$

$$y: \quad y_P = \overline{y}_0 + 0.325v_Bt_P \tag{2}$$

PROBLEM 14.20 (Continued)

Data: $x_P = 59 \text{ ft}, \;\; y_P = 46 \text{ ft}$

(*a*) From (2), $46 = 3 + (0.325)(66)t_P$

$t_P = 2.0047 \text{ s}$ $\qquad t_P = 2.00 \text{ s}$ ◀

(*b*) From (1), $59 = 9.6 + (0.375)v_A(2.0047) - (0.3)(88)(2.0047)$

$v_A = 136.11 \text{ ft/s}$ $\qquad v_A = 92.8 \text{ mi/h}$ ◀

PROBLEM 14.21

An expert archer demonstrates his ability by hitting tennis balls thrown by an assistant. A 2-oz tennis ball has a velocity of (32 ft/s)**i** – (7 ft/s)**j** and is 33 ft above the ground when it is hit by a 1.2-oz arrow traveling with a velocity of (165 ft/s)**j** + (230 ft/s)**k** where **j** is directed upwards. Determine the position *P* where the ball and arrow will hit the ground, relative to Point *O* located directly under the point of impact.

SOLUTION

Assume that the ball and arrow move together after the hit.

Conservation of momentum of ball and arrow during the hit.

$$m_A = \frac{1.2/16}{32.2} = 2.3292\times 10^{-3} \text{ slug} \qquad m_B = \frac{2/16}{32.2} = 3.8820\times 10^{-3} \text{ slug}$$

$$m_A\mathbf{v}_A + m_B\mathbf{v}_B = (m_A + m_B)\overline{\mathbf{v}}$$

$$(2.3292\times 10^{-3})(165\mathbf{j} + 230\mathbf{k}) + (3.8820\times 10^{-3})(32\mathbf{i} - 7\mathbf{j}) = (2.3292\times 10^{-3} + 3.8820\times 10^{-3})\overline{\mathbf{v}}$$

$$\overline{\mathbf{v}} = (20.0 \text{ ft/s})\mathbf{i} + (57.5 \text{ ft/s})\mathbf{j} + (86.25 \text{ ft/s})\mathbf{k}$$

After the hit, the ball and arrow move as a projectile.

<u>Vertical motion</u>: $y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$

$$y = 33 + 57.5t - \frac{1}{2}(32.2)t^2$$

$y = 0$ at ground.

$$-16.1t^2 + 57.5t + 33 = 0 \qquad \text{Solve for } t.$$

After rejecting the negative root, $t = 4.0745$ s

<u>Horizontal motion</u>: $x = x_0 + (v_x)_0 t$

$$z = z_0 + (v_2)_0 t$$

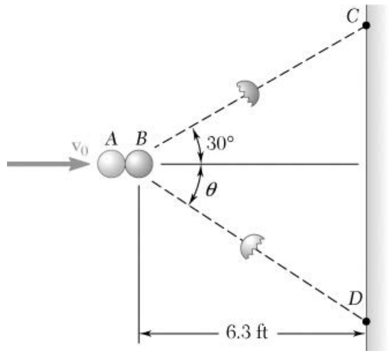
$$x = 0 + (20)(4.07448)$$

$$= 81.490 \text{ ft}$$

$$z = 0 + (86.25)(4.07448)$$

$$= 351.42 \text{ ft}$$

$$\mathbf{r}_P = (81.5 \text{ ft})\mathbf{i} + (351 \text{ ft})\mathbf{k} \blacktriangleleft$$



PROBLEM 14.22

Two spheres, each of mass m, can slide freely on a frictionless, horizontal surface. Sphere A is moving at a speed $v_0 = 16$ ft/s when it strikes sphere B which is at rest and the impact causes sphere B to break into two pieces, each of mass $m/2$. Knowing that 0.7 s after the collision one piece reaches Point C and 0.9 s after the collision the other piece reaches Point D, determine (a) the velocity of sphere A after the collision, (b) the angle θ and the speeds of the two pieces after the collision.

SOLUTION

Velocities of pieces C and D after impact and fracture.

$$(v'_C)_x = \frac{x_C}{t_C} = \frac{6.3}{0.7} = 9 \text{ ft/s}, \quad (v'_C)_y = 9\tan 30° \text{ ft/s}$$

$$(v'_D)_x = \frac{x_D}{t_D} = \frac{6.3}{0.9} = 7 \text{ ft/s}, \quad (v'_D)_y = -7\tan\theta \text{ ft/s}$$

Assume that during the impact the impulse between spheres A and B is directed along the x-axis. Then, the y component of momentum of sphere A is conserved.

$$0 = m(v'_A)_y$$

Conservation of momentum of system:

$$\xrightarrow{+}: \quad m_A v_0 + m_B(0) = m_A v'_A + m_C (v'_C)_x + m_D (v'_D)_x$$

$$m(16) + 0 = m v'_A + \frac{m}{2}(9) + \frac{m}{2}(7)$$

(a) $\mathbf{v}'_A = 8.00$ ft/s $\longrightarrow$ ◀

$$+\uparrow: \quad m_A(0) + m_B(0) = m_A(v'_A)_y + m_C(v'_C)_y + m_D(v'_D)_y$$

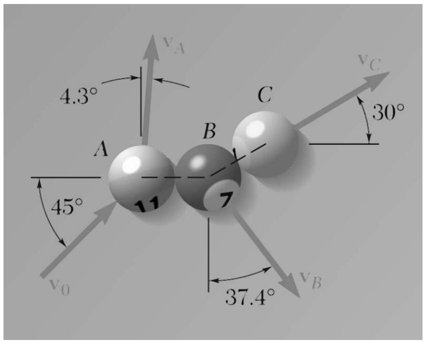
$$0 + 0 = 0 + \frac{m}{2}(9\tan 30°) - \frac{m}{2}(7\tan\theta)$$

(b) $\tan\theta = \frac{9}{7}\tan 30° = 0.7423$ $\qquad \theta = 36.6°$ ◀

$$v_C = \sqrt{(v_C)_x^2 + (v_C)_y^2} = \sqrt{(9)^2 + (9\tan 30°)^2}$$ $v_C = 10.39$ ft/s ◀

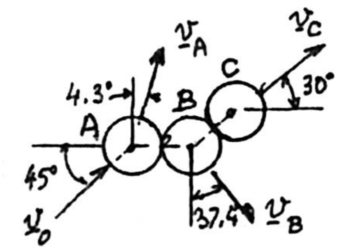
$$v_D = \sqrt{(v_D)_x^2 + (v_D)_y^2} = \sqrt{(7)^2 + (7\tan 36.6°)^2}$$ $v_D = 8.72$ ft/s ◀

PROBLEM 14.23



In a game of pool, ball A is moving with a velocity $\mathbf{v}_0$ when it strikes balls B and C which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated, and that $v_0 = 12$ ft/s and $v_C = 6.29$ ft/s, determine the magnitude of the velocity of (*a*) ball A, (*b*) ball B.

SOLUTION



Conservation of linear momentum. In x direction:

$$m(12 \text{ ft/s})\cos 45° = mv_A \sin 4.3° + mv_B \sin 37.4° + m(6.29)\cos 30°$$

$$0.07498v_A + 0.60738v_B = 3.0380 \qquad (1)$$

In y direction:

$$m(12 \text{ ft/s})\sin 45° = mv_A \cos 4.3° - mv_B \cos 37.4° + m(6.29)\sin 30°$$

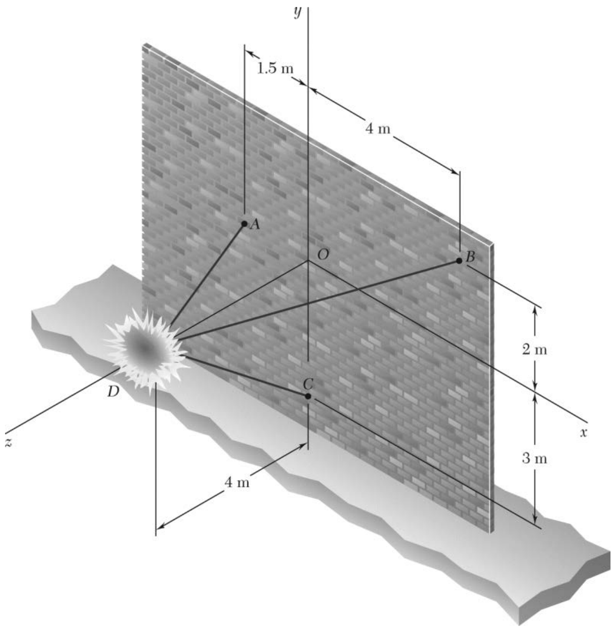
$$0.99719v_A - 0.79441v_B = 5.3403 \qquad (2)$$

(*a*) Multiply (1) by 0.79441, (2) by 0.60738, and add:

$$0.66524v_A = 5.6570 \qquad v_A = 8.50 \text{ ft/s} ◀$$

(*b*) Multiply (1) by 0.99719, (2) by –0.07498, and add:

$$0.66524v_B = 2.6290 \qquad v_B = 3.95 \text{ ft/s} ◀$$



PROBLEM 14.24

A 6-kg shell moving with a velocity $\mathbf{v}_0 = (12 \text{ m/s})\mathbf{i} - (9 \text{ m/s})\mathbf{j} - (360 \text{ m/s})\mathbf{k}$ explodes at Point D into three fragments A, B, and C of mass, respectively, 3 kg, 2 kg, and 1 kg. Knowing that the fragments hit the vertical wall at the points indicated, determine the speed of each fragment immediately after the explosion. Assume that elevation changes due to gravity may be neglected.

SOLUTION

Position vectors (m): $\mathbf{r}_D = 4\mathbf{k}$

$$\mathbf{r}_A = -1.5\mathbf{i} \qquad \mathbf{r}_{A/D} = -1.5\mathbf{i} - 4\mathbf{k} \qquad r_{A/D} = 4.272$$

$$\mathbf{r}_B = 4\mathbf{i} + 2\mathbf{j} \qquad \mathbf{r}_{B/D} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k} \qquad r_{B/D} = 6$$

$$\mathbf{r}_C = -3\mathbf{j} \qquad \mathbf{r}_{C/D} = -3\mathbf{j} - 4\mathbf{k} \qquad r_{C/D} = 5$$

Unit vectors:

Along $\mathbf{r}_{A/D}$, $\boldsymbol{\lambda}_A = \dfrac{1}{4.272}(-1.5\mathbf{i} - 4\mathbf{k})$

Along $\mathbf{r}_{B/D}$, $\boldsymbol{\lambda}_B = \dfrac{1}{6}(4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$

Along $\mathbf{r}_{C/D}$, $\boldsymbol{\lambda}_C = \dfrac{1}{5}(-3\mathbf{j} - 4\mathbf{k})$

Assume that elevation changes due to gravity may be neglected. Then, the velocity vectors after the explosion have the directions of the unit vectors.

$$\mathbf{v}_A = v_A \boldsymbol{\lambda}_A$$

$$\mathbf{v}_B = v_B \boldsymbol{\lambda}_B$$

$$\mathbf{v}_C = v_C \boldsymbol{\lambda}_C$$

Conservation of momentum: $m\mathbf{v}_0 = m_A \mathbf{v}_A + m_B \mathbf{v}_B + m_C \mathbf{v}_C$

$$6(12\mathbf{i} - 9\mathbf{j} - 360\mathbf{k}) = 3\left(\frac{v_A}{4.272}\right)(-1.5\mathbf{i} - 4\mathbf{k}) + 2\left(\frac{v_B}{6}\right)(4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$

$$+1\left(\frac{v_C}{5}\right)(-3\mathbf{j} - 4\mathbf{k})$$

PROBLEM 14.24 (Continued)

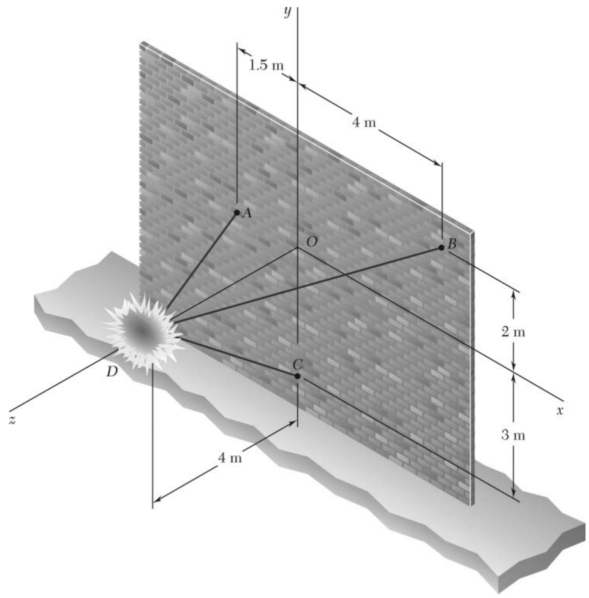
Resolve into components.

$$72 = -1.0534v_A + 1.3333v_B$$
$$-54 = 0.66667v_B - 0.60000v_C$$
$$-2160 = -2.8090v_A - 1.3333v_B - 0.80000v_C$$

Solving, $v_A = 431\text{ m/s}$ ◀

$v_B = 395\text{ m/s}$ ◀

$v_C = 528\text{ m/s}$ ◀



PROBLEM 14.25

A 6-kg shell moving with a velocity $\mathbf{v}_0 = (12\text{ m/s})\mathbf{i} - (9\text{ m/s})\mathbf{j} - (360\text{ m/s})\mathbf{k}$ explodes at Point D into three fragments A, B, and C of mass, respectively, 2 kg, 1 kg, and 3 kg. Knowing that the fragments hit the vertical wall at the points indicated, determine the speed of each fragment immediately after the explosion. Assume that elevation changes due to gravity may be neglected.

SOLUTION

Position vectors (m): $\mathbf{r}_D = 4\mathbf{k}$

$$\mathbf{r}_A = -1.5\mathbf{i} \qquad \mathbf{r}_{A/D} = -1.5\mathbf{i} - 4\mathbf{k} \qquad r_{A/D} = 4.272$$

$$\mathbf{r}_B = 4\mathbf{i} + 2\mathbf{j} \qquad \mathbf{r}_{B/D} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k} \qquad r_{B/D} = 6$$

$$\mathbf{r}_C = -3\mathbf{j} \qquad \mathbf{r}_{C/D} = -3\mathbf{j} - 4\mathbf{k} \qquad r_{C/D} = 5$$

Unit vectors: Along $\mathbf{r}_{A/D}$, $\boldsymbol{\lambda}_A = \dfrac{1}{4.272}(-1.5\mathbf{i} - 4\mathbf{k})$

Along $\mathbf{r}_{B/D}$, $\boldsymbol{\lambda}_B = \dfrac{1}{6}(4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$

Along $\mathbf{r}_{C/D}$, $\boldsymbol{\lambda}_C = \dfrac{1}{5}(-3\mathbf{j} - 4\mathbf{k})$

Assume that elevation changes due to gravity may be neglected. Then the velocity vectors after the explosion have the directions of the unit vectors.

$$\mathbf{v}_A = v_A \boldsymbol{\lambda}_A$$

$$\mathbf{v}_B = v_B \boldsymbol{\lambda}_B$$

$$\mathbf{v}_C = v_C \boldsymbol{\lambda}_C$$

Conservation of momentum: $m\mathbf{v}_0 = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C$

Resolve into components.

$$6(12\mathbf{i} - 9\mathbf{j} - 360\mathbf{k}) = 2\left(\frac{v_A}{4.272}\right)(-1.5\mathbf{i} - 4\mathbf{k}) + 1\left(\frac{v_B}{6}\right)(4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$

$$+3\left(\frac{v_C}{5}\right)(3\mathbf{j} - 4\mathbf{k})$$

PROBLEM 14.25 (Continued)

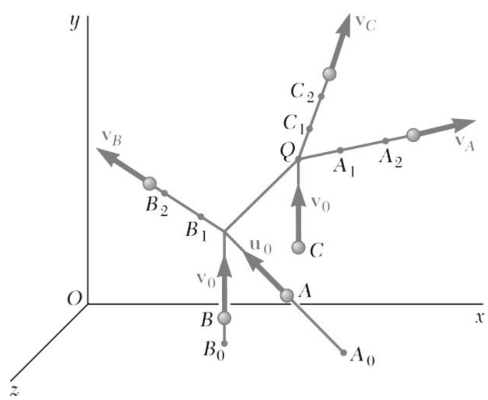
$$72 = -0.70225 v_A + 0.66667 v_B$$
$$-54 = 0.33333 v_B - 1.8000 v_C$$
$$-2160 = -1.8727 v_A - 0.66667 v_B - 2.40000 v_C$$

Solving, $v_A = 646 \text{ m/s}$ ◀

$v_B = 789 \text{ m/s}$ ◀

$v_C = 176 \text{ m/s}$ ◀

PROBLEM 14.26



In a scattering experiment, an alpha particle A is projected with the velocity $\mathbf{u}_0 = -(600 \text{ m/s})\mathbf{i} + (750 \text{ m/s})\mathbf{j} - (800 \text{ m/s})\mathbf{k}$ into a stream of oxygen nuclei moving with a common velocity $\mathbf{v}_0 = (600 \text{ m/s})\mathbf{j}$. After colliding successively with nuclei B and C, particle A is observed to move along the path defined by the Points $A_1(280, 240, 120)$and $A_2(360, 320, 160)$, while nuclei B and C are observed to move along paths defined, respectively, by $B_1(147, 220, 130)$, $B_2(114, 290, 120)$,and by $C_1(240, 232, 90)$ and $C_2(240, 280, 75)$. All paths are along straight lines and all coordinates are expressed in millimeters. Knowing that the mass of an oxygen nucleus is four times that of an alpha particle, determine the speed of each of the three particles after the collisions.

SOLUTION

Position vectors (mm): $\overrightarrow{A_1A_2} = 80\mathbf{i} + 80\mathbf{j} + 40\mathbf{k}$ $\quad (A_1A_2) = 120$

$\overrightarrow{B_1B_2} = -33\mathbf{i} + 70\mathbf{j} - 10\mathbf{k}$ $\quad (B_1B_2) = 78.032$

$\overrightarrow{C_1C_2} = 48\mathbf{j} - 15\mathbf{k}$ $\quad (C_1C_2) = 50.289$

Unit vectors: Along A_1A_2, $\quad \lambda_A = 0.66667\mathbf{i} + 0.66667\mathbf{j} + 0.33333\mathbf{k}$

Along B_1B_2, $\quad \lambda_B = -0.42290\mathbf{i} + 0.89707\mathbf{j} - 0.12815\mathbf{k}$

Along C_1C_2, $\quad \lambda_C = 0.95448\mathbf{j} - 0.29828\mathbf{k}$

Velocity vectors after the collisions:

$$\mathbf{v}_A = v_A\lambda_A$$
$$\mathbf{v}_B = v_B\lambda_B$$
$$\mathbf{v}_C = v_C\lambda_C$$

Conservation of momentum:

$$m\mathbf{u}_0 + 4m\mathbf{v}_0 + 4m\mathbf{v}_0 = m\mathbf{v}_A + 4m\mathbf{v}_B + 4m\mathbf{v}_C$$

Divide by m and substitute data.

$$(-600\mathbf{i} + 750\mathbf{j} - 800\mathbf{k}) + 2400\mathbf{j} + 2400\mathbf{j} = v_A\lambda_A + 4v_B\lambda_B + 4v_C\lambda_C$$

Resolving into components,

$$\mathbf{i}: \ -600 = 0.66667v_A - 1.69160v_B$$
$$\mathbf{j}: \ 5550 = 0.66667v_A + 3.58828v_B + 3.81792v_C$$
$$\mathbf{k}: \ -800 = 0.33333v_A - 0.51260v_B - 1.19312v_C$$

PROBLEM 14.26 (Continued)

Solving the three equations simultaneously,

$$v_A = 919.26 \text{ m/s}$$
$$v_B = 716.98 \text{ m/s}$$
$$v_C = 619.30 \text{ m/s}$$

$v_A = 919 \text{ m/s}$ ◀

$v_B = 717 \text{ m/s}$ ◀

$v_C = 619 \text{ m/s}$ ◀

PROBLEM 14.27

Derive the relation

$$\mathbf{H}_O = \overline{\mathbf{r}} \times m\overline{\mathbf{v}} + \mathbf{H}_G$$

between the angular momenta $\mathbf{H}_O$ and $\mathbf{H}_G$ defined in Eqs. (14.7) and (14.24), respectively. The vectors $\overline{\mathbf{r}}$ and $\overline{\mathbf{v}}$ define, respectively, the position and velocity of the mass center *G* of the system of particles relative to the newtonian frame of reference *Oxyz*, and *m* represents the total mass of the system.

SOLUTION

From Eq. (14.7),

$$\begin{aligned}\mathbf{H}_O &= \sum_{i=1}^{n}(\mathbf{r}_i \times m_i\mathbf{v}_i)\\ &= \sum_{i=1}^{n}\left[(\overline{\mathbf{r}} + \mathbf{r}_i') \times m_i\mathbf{v}_i\right]\\ &= \overline{\mathbf{r}} \times \sum_{i=1}^{n}(m_i v_i) + \sum_{i=1}^{n}(r_i' \times m_i\mathbf{v}_i)\\ &= \overline{\mathbf{r}} \times m\overline{\mathbf{v}} + \mathbf{H}_G\end{aligned}$$

PROBLEM 14.28

Show that Eq. (14.23) may be derived directly from Eq. (14.11) by substituting for $\mathbf{H}_O$ the expression given in Problem 14.27.

SOLUTION

From Eq. (14.7),

$$\mathbf{H}_O = \sum_{i=1}^{n}(\mathbf{r}_i \times m_i\mathbf{v}_i)$$

$$= \sum_{i=1}^{n}\left[\left(\overline{\mathbf{r}} + \mathbf{r}_i'\right)\times m_i\mathbf{v}_i\right]$$

$$= \overline{\mathbf{r}} \times \sum_{i=1}^{n}(m_i v_i) + \sum_{i=1}^{n}\left(r_i' \times m_i\mathbf{v}_i\right)$$

$$= \overline{\mathbf{r}} \times m\overline{\mathbf{v}} + \mathbf{H}_G$$

Differentiating,

$$\dot{\mathbf{H}}_O = \dot{\overline{\mathbf{r}}} \times m\overline{\mathbf{v}} + \overline{\mathbf{r}} \times m\dot{\mathbf{v}} + \dot{\mathbf{H}}_G$$

Using Eq. (14.11),

$$\Sigma\mathbf{M}_O = \dot{\overline{\mathbf{r}}} \times m\overline{\mathbf{v}} + \overline{\mathbf{r}} \times m\dot{\mathbf{v}} + \dot{\mathbf{H}}_G$$

$$= \overline{\mathbf{v}} \times m\overline{\mathbf{v}} + \overline{\mathbf{r}} \times m\overline{\mathbf{a}} + \dot{\mathbf{H}}_G$$

$$= 0 + \overline{\mathbf{r}} \times \left(\sum_{i=1}^{n}\mathbf{F}_i\right) + \dot{\mathbf{H}}_G \qquad (1)$$

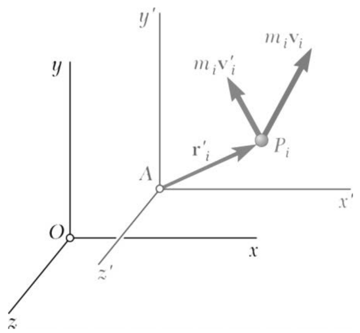
But

$$\sum_{i=1}^{n}\mathbf{M}_O = \sum_{i=1}^{n}\mathbf{M}_G + \sum_{i=1}^{n}\overline{\mathbf{r}} \times \left(\sum_{i=1}^{n}F_i\right)$$

Subtracting $\overline{\mathbf{r}} \times \left(\Sigma_{i=1}^{n} F_i\right)$ from each side of Eq. (1) gives

$$\Sigma\mathbf{M}_G = \dot{\mathbf{H}}_G$$

PROBLEM 14.29



Consider the frame of reference $Ax'y'z'$ in translation with respect to the newtonian frame of reference *Oxyz*. We define the angular momentum $\mathbf{H}'_A$ of a system of *n* particles about *A* as the sum

$$\mathbf{H}'_A = \sum_{i=1}^{n} \mathbf{r}'_i \times m_i \mathbf{v}'_i \qquad (1)$$

of the moments about *A* of the momenta $m_i\mathbf{v}'_i$ of the particles in their motion relative to the frame $Ax'y'z'$. Denoting by $\mathbf{H}_A$ the sum

$$\mathbf{H}_A = \sum_{i=1}^{n} \mathbf{r}'_i \times m_i \mathbf{v}_i \qquad (2)$$

of the moments about *A* of the momenta $m_i\mathbf{v}_i$ of the particles in their motion relative to the newtonian frame *Oxyz*, show that $\mathbf{H}_A = \mathbf{H}'_A$ at a given instant if, and only if, one of the following conditions is satisfied at that instant: (*a*) *A* has zero velocity with respect to the frame *Oxyz*, (*b*) *A* coincides with the mass center *G* of the system, (*c*) the velocity $\mathbf{v}_A$ relative to *Oxyz* is directed along the line *AG*.

SOLUTION

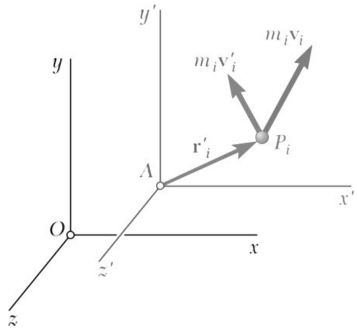
$$\mathbf{v}_i = \mathbf{v}_A + \mathbf{v}'_i$$

$$\begin{aligned}
\mathbf{H}_A &= \sum_{i=1}^{n} \mathbf{r}'_i \times m_i \mathbf{v}_i \\
&= \sum_{i=1}^{n} \mathbf{r}'_i \times m_i (\mathbf{v}_A + \mathbf{v}'_i) \\
&= \sum_{i=1}^{n} (\mathbf{r}'_i \times m_i \mathbf{v}_A) + \sum_{i=1}^{n} \mathbf{r}'_i \times m_i \mathbf{v}_i \\
&= \sum_{i=1}^{n} (m_i \mathbf{r}'_i) \times \mathbf{v}_A + \mathbf{H}'_A \\
&= \sum_{i=1}^{n} m_i (\mathbf{r}_i - \mathbf{r}_A) \times \mathbf{v}_A + \mathbf{H}'_A \\
&= m(\overline{\mathbf{r}} - \mathbf{r}_A) \times \mathbf{v}_A + \mathbf{H}'_A
\end{aligned}$$

$$\mathbf{H}_A = \mathbf{H}'_A \quad \text{if, and only if,} \qquad m(\mathbf{r} - \mathbf{r}_A) \times \mathbf{v}_A = 0$$

This condition is satisfied if

	(*a*)	$\mathbf{v}_A = 0$	Point *A* has zero velocity.
or	(*b*)	$\overline{\mathbf{r}} = \mathbf{r}_A$	Point *A* coincides with the mass center.
or	(*c*)	$\mathbf{v}_A$ is parallel to $\overline{\mathbf{r}} - \mathbf{r}_A$.	Velocity $\mathbf{v}_A$ is directed along line *AG*.



PROBLEM 14.30

Show that the relation $\Sigma\mathbf{M}_A = \dot{\mathbf{H}}'_A$, where $\mathbf{H}'_A$ is defined by Eq. (1) of Problem 14.29 and where $\Sigma\mathbf{M}_A$ represents the sum of the moments about A of the external forces acting on the system of particles, is valid if, and only if, one of the following conditions is satisfied: (*a*) the frame $Ax'y'z'$ is itself a newtonian frame of reference, (*b*) A coincides with the mass center G, (*c*) the acceleration $\mathbf{a}_A$ of A relative to $Oxyz$ is directed along the line AG.

SOLUTION

From equation (1),

$$\mathbf{H}'_A = \sum_{i=1}^{n}\left(\mathbf{r}'_i \times m_i\mathbf{v}'_i\right)$$

$$\mathbf{H}'_A = \sum_{i=1}^{n}[(\mathbf{r}_i - \mathbf{r}_A)\times m_i(\mathbf{v}_i - \mathbf{v}_A)]$$

Differentiate with respect to time.

$$\dot{\mathbf{H}}'_A = \sum_{i=1}^{n}[(\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_A)\times m_i(\mathbf{v}_i - \mathbf{v}_A)] + \sum_{i=1}^{n}[(\mathbf{r}_i - \mathbf{r}_A)\times m_i(\dot{\mathbf{v}}_i - \dot{\mathbf{v}}_A)]$$

But

$$\dot{\mathbf{r}}_i = \mathbf{v}_i$$
$$\dot{\mathbf{v}}_i = \mathbf{a}_i$$
$$\dot{\mathbf{r}}_A = \mathbf{v}_A$$

and

$$\dot{\mathbf{v}}_A = \mathbf{a}_A$$

Hence,

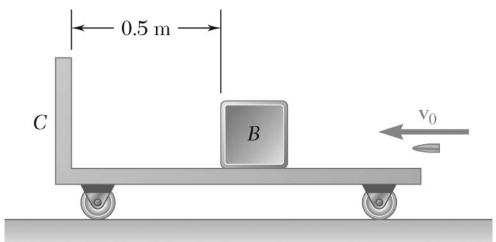
$$\begin{aligned}\dot{\mathbf{H}}'_A &= 0 + \sum_{i=1}^{n}[(\mathbf{r}_i - \mathbf{r}_A)\times m_i(\mathbf{a}_i - \mathbf{a}_A)]\\ &= \sum_{i=1}^{n}[(\mathbf{r}_i - \mathbf{r}_A)\times(\mathbf{F}_i - m_i\mathbf{a}_A)]\\ &= \sum_{i=1}^{n}[(\mathbf{r}_i - \mathbf{r}_A)\times\mathbf{F}_i] - \sum_{i=1}^{n}[m_i(\mathbf{r}_i - \mathbf{r}_A)]\times\mathbf{a}_A\\ &= \mathbf{M}_A - m(\overline{\mathbf{r}} - r_A)\times\mathbf{a}_A\end{aligned}$$

$$\dot{\mathbf{H}}'_A = M_A \quad \text{if, and only if,} \quad m(\overline{\mathbf{r}} - \mathbf{r}_A)\times\mathbf{a}_A = 0$$

This condition is satisfied if

	(*a*)	$\mathbf{a}_A = 0$	The frame is newtonian.
or	(*b*)	$\overline{\mathbf{r}} = \mathbf{r}_A$	Point A coincides with the mass center.
or	(*c*)	$\mathbf{a}_A$ is parallel to $\overline{\mathbf{r}} - \mathbf{r}_A$.	Acceleration $\mathbf{a}_A$ is directed along line AG.

PROBLEM 14.31



Determine the energy lost due to friction and the impacts for Problem 14.1.

PROBLEM 14.1 A 30-g bullet is fired with a horizontal velocity of 450 m/s and becomes embedded in block *B* which has a mass of 3 kg. After the impact, block *B* slides on 30-kg carrier *C* until it impacts the end of the carrier. Knowing the impact between *B* and *C* is perfectly plastic and the coefficient of kinetic friction between *B* and *C* is 0.2, determine (*a*) the velocity of the bullet and *B* after the first impact, (*b*) the final velocity of the carrier.

SOLUTION

From the solution to Problem 4.1 the velocity of *A* and *B* after the first impact is $v' = 4.4554$ m/s and the velocity common to *A*, *B*, and *C* after the sliding of block *B* and bullet *A* relative to the carrier *C* has ceased in $v'' = 0.4087$ m/s.

Friction loss due to sliding:

Normal force:
$$N = W_A + W_B = (m_A + m_B)\,g$$
$$= (0.030 \text{ kg} + 3 \text{ kg})(9.81 \text{ m/s}) = 29.724 \text{ N}$$

Friction force:
$$F_f = \mu_k N = (0.2)(29.724) = 5.945 \text{ N}$$

Relative sliding distance: Assume $d = 0.5$ m.

Energy loss due to friction: $F_f d = (5.945)(0.5)$ $\qquad F_f d = 2.97 \text{ J}$ ◄

Kinetic energy of block with embedded bullet immediately after first impact:

$$T'_{AB} = \frac{1}{2}(m_A + m_B)(v')^2 = \frac{1}{2}(3.03 \text{ kg})(4.4554 \text{ m/s})^2 = 30.07 \text{ J}$$

Final kinetic energy of *A*, *B*, and *C* together

$$T''_{ABC} = \frac{1}{2}(m_A + m_B + m_C)(v'')^2 = \frac{1}{2}(33.03 \text{ kg})(0.4087 \text{ m/s})^2 = 2.76 \text{ J}$$

Loss due to friction and stopping impact: $T'_{AB} - T''_{ABC} = 30.07 - 2.76 = 27.31$ J

Since $27.31 \text{ J} \geq 2.97 \text{ J},$ the block slides 0.5 m relative to the carrier as assumed above.

PROBLEM 14.31 (Continued)

Impact loss due to AB impacting the carrier:

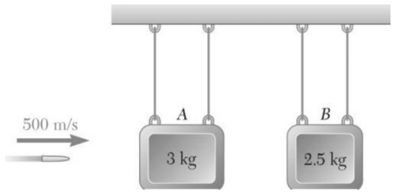
$$27.31 - 2.97 = 24.34 \qquad \text{Loss} = 24.3 \text{ J} \blacktriangleleft$$

Initial kinetic energy of system ABC.

$$T_0 = \frac{1}{2} m_A v_0^2 = \frac{1}{2}(0.030 \text{ kg})(450 \text{ m/s})^2 = 3037.5 \text{ J}$$

Impact loss at first impact:

$$T_0 - T'_{AB} = 3037.5 - 30.07 \qquad \text{Loss} = 3007 \text{ J} \blacktriangleleft$$



PROBLEM 14.32

In Problem 14.4, determine the energy lost as the bullet (*a*) passes through block *A*, (*b*) becomes embedded in block *B*.

SOLUTION

The masses are *m* for the bullet and m_A and m_B for the blocks.

The bullet passes through block *A* and embeds in block *B*. Momentum is conserved.

Initial momentum: $mv_0 + m_A(0) + m_B(0) = mv_0$

Final momentum: $mv_B + m_A v_A + m_B v_B$

Equating, $mv_0 = mv_B + m_A v_A + m_B v_B$

$$m = \frac{m_A v_A + m_B v_B}{v_0 - v_B} = \frac{(6)(5) + (4.95)(9)}{1500 - 9} = 0.0500 \text{ lb}$$

The bullet passes through block *A*. Momentum is conserved.

Initial momentum: $mv_0 + m_A(0) = mv_0$

Final momentum: $mv_1 + m_A v_A$

Equating, $mv_0 = mv_1 + m_A v_A$

$$v_1 = \frac{mv_0 - m_A v_A}{m} = \frac{(0.0500)(1500) - (6)(5)}{0.0500} = 900 \text{ ft/s}$$

The masses are:

$$m = \frac{0.05}{32.2} = 1.5528 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_A = \frac{6}{32.2} = 0.18633 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_B = \frac{4.95}{32.2} = 0.153727 \text{ lb} \cdot \text{s}^2/\text{ft}$$

(*a*) Bullet passes through block *A*. Kinetic energies:

Before: $T_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(1.5528 \times 10^{-3})(1500)^2 = 1746.9 \text{ ft} \cdot \text{lb}$

After: $T_1 = \frac{1}{2}mv_1^2 + \frac{1}{2}m_A v_A^2 = \frac{1}{2}(1.5528 \times 10^{-3})(900)^2 + \frac{1}{2}(0.18633)(5)^2 = 631.2 \text{ ft} \cdot \text{lb}$

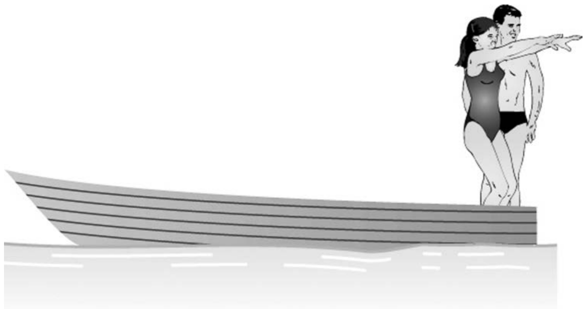
Lost: $T_0 - T_1 = 1746.9 - 631.2 = 1115.7 \text{ ft} \cdot \text{lb}$ energy lost $= 1116 \text{ ft} \cdot \text{lb}$ ◀

(*b*) Bullet becomes embedded in block *B*. Kinetic energies:

Before: $T_2 = \frac{1}{2}mv_1^2 = \frac{1}{2}(1.5528 \times 10^{-3})(900)^2 = 628.9 \text{ ft} \cdot \text{lb}$

After: $T_3 = \frac{1}{2}(m + m_B)v_B^2 = \frac{1}{2}(0.15528)(9)^2 = 6.29 \text{ ft} \cdot \text{lb}$

Lost: $T_2 - T_3 = 628.9 - 6.29 = 622.6 \text{ ft} \cdot \text{lb}$ energy lost $= 623 \text{ ft} \cdot \text{lb}$ ◀



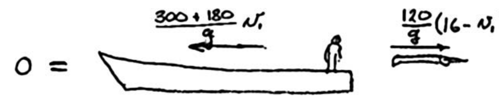
PROBLEM 14.33

In Problem 14.6, determine the work done by the woman and by the man as each dives from the boat, assuming that the woman dives first.

SOLUTION

Woman dives first.

Conservation of momentum:



$$\frac{120}{g}(16-v_1)-\frac{300+180}{g}v_1=0$$

$$v_1=\frac{(120)(16)}{600}=3.20\text{ ft/s} \longleftarrow$$

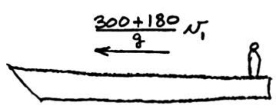
$$16-v_1=12.80\text{ ft/s} \longrightarrow$$

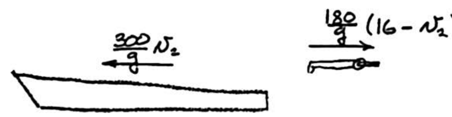
Kinetic energy before dive: $T_0=0$

Kinetic energy after dive:
$$T_1=\frac{1}{2}\frac{300+180}{32.2}(3.20)^2+\frac{1}{2}\frac{120}{32.2}(12.80)^2$$
$$=381.61\text{ ft}\cdot\text{lb}$$

Work of woman: $T_1-T_0=381.61\text{ ft}\cdot\text{lb}$ $\qquad T_1-T_0=382\text{ ft}\cdot\text{lb}$ ◀

Man dives next. Conservation of momentum:





$$-\frac{300+180}{g}v_1=-\frac{300}{g}v_2+\frac{180}{g}(16-v_2)$$

$$v_2=\frac{480v_1+(180)(16)}{480}=9.20\text{ ft/s} \longleftarrow$$

$$16-9.20=6.80\text{ ft/s} \longrightarrow$$

PROBLEM 14.33 (Continued)

Kinetic energy before dive:

$$T_1' = \frac{1}{2}\frac{300+180}{32.2}(3.20)^2$$
$$= 76.323 \text{ ft}\cdot\text{lb}$$

Kinetic energy after dive:

$$T_2' = \frac{1}{2}\frac{300}{32.2}(9.20)^2 + \frac{1}{2}\frac{180}{32.2}(6.80)^2$$
$$= 523.53 \text{ ft}\cdot\text{lb}$$

Work of man: $T_2' - T_1' = 447.2 \text{ ft}\cdot\text{lb}$

$T_2' - T_1' = 447 \text{ ft}\cdot\text{lb}$ ◀

PROBLEM 14.34

Determine the energy lost as a result of the series of collisions described in Problem 14.8.

PROBLEM 14.8 Packages in an automobile parts supply house are transported to the loading dock by pushing them along on a roller track with very little friction. At the instant shown, packages *B* and *C* are at rest and package *A* has a velocity of 2 m/s. Knowing that the coefficient of restitution between the packages is 0.3, determine (*a*) the velocity of package *C* after *A* hits *B* and *B* hits *C*, (*b*) the velocity of *A* after it hits *B* for the second time.

SOLUTION

From the solution to Problem 14.8

$$v_A = 2 \text{ m/s}, \quad v_B = v_C = 0,$$

$$v'_A = 1.133 \text{ m/s}, \quad v'_B = 1.733 \text{ m/s}, \quad v''_B = 0.382 \text{ m/s}$$

$$v'_B = 0.901 \text{ m/s} \quad v''_A = 0.807 \text{ m/s}, \quad v''_B = 1.033 \text{ m/s}$$

$$m_A = 8 \text{ kg}, \quad m_B = 4 \text{ kg}, \quad m_C = 6 \text{ kg}$$

A hits *B*:

$$T_1 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(8 \text{ kg})(2 \text{ m/s})^2 = 16 \text{ J}$$

$$T_2 = \frac{1}{2}m_A(v'_A)^2 + \frac{1}{2}m_B(v'_B)^2$$

$$T = \frac{1}{2}(8 \text{ kg})(1.133 \text{ m/s})^2 + \frac{1}{2}(4 \text{ kg})(1.733 \text{ m/s})^2 = 11.14 \text{ J}$$

Loss $= T_1 - T_2$: Loss $= 4.86$ J ◀

B hits *C*:

$$T_3 = \frac{1}{2}m_B(v'_B)^2 = \frac{1}{2}(4 \text{ kg})(1.733)^2 = 6.007 \text{ J}$$

$$T_4 = \frac{1}{2}m_B(v''_B)^2 + \frac{1}{2}m_C(v'_C)^2$$

$$= \frac{1}{2}(4 \text{ kg})(0.382 \text{ m/s})^2 + \frac{1}{2}(6 \text{ kg})(0.901 \text{ m/s})^2 = 2.727 \text{ J}$$

Loss $= T_3 - T_4$: Loss $= 3.28$ J ◀

A hits *B* again:

$$T_5 = \frac{1}{2}m_A(v'_A)^2 + \frac{1}{2}m_B(v''_B)^2$$

$$= \frac{1}{2}(8 \text{ kg})(1.33 \text{ m/s})^2 + \frac{1}{2}(4 \text{ kg})(0.382)^2 = 5.427 \text{ J}$$

$$T_6 = \frac{1}{2}m_A(v''_A)^2 + \frac{1}{2}m_B(v'''_B)^2$$

$$= \frac{1}{2}(8 \text{ kg})(0.807 \text{ m/s})^2 + \frac{1}{2}(4 \text{ kg})(1.033 \text{ m/s})^2 = 4.739 \text{ J}$$

Loss $= T_5 - T_6$: Loss $= 0.688$ J

PROBLEM 14.35

Two automobiles A and B, of mass m_A and m_B, respectively, are traveling in opposite directions when they collide head on. The impact is assumed perfectly plastic, and it is further assumed that the energy absorbed by each automobile is equal to its loss of kinetic energy with respect to a moving frame of reference attached to the mass center of the two-vehicle system. Denoting by E_A and E_B, respectively, the energy absorbed by automobile A and by automobile B, (a) show that $E_A/E_B = m_B/m_A$, that is, the amount of energy absorbed by each vehicle is inversely proportional to its mass, (b) compute E_A and E_B, knowing that $m_A = 1600$ kg and $m_B = 900$ kg and that the speeds of A and B are, respectively, 90 km/h and 60 km/h.

SOLUTION

Velocity of mass center: $(m_A + m_B)\overline{\mathbf{v}} = m_A\mathbf{v}_A + m_B\mathbf{v}_B$

$$\overline{\mathbf{v}} = \frac{m_A\mathbf{v}_A + m_B\mathbf{v}_B}{m_A + m_B}$$

Velocities relative to the mass center:

$$\mathbf{v}'_A = \mathbf{v}_A - \overline{\mathbf{v}} = \mathbf{v}_A - \frac{m_A\mathbf{v}_A + m_B\mathbf{v}_B}{m_A + m_B} = \frac{m_B(\mathbf{v}_A + \mathbf{v}_B)}{m_A + m_B}$$

$$\mathbf{v}'_B = \mathbf{v}_B - \overline{\mathbf{v}} = \mathbf{v}_B - \frac{m_A\mathbf{v}_A + m_B\mathbf{v}_B}{m_A + m_B} = \frac{m_A(\mathbf{v}_A + \mathbf{v}_B)}{m_A + m_B}$$

Energies:

$$E_A = \frac{1}{2}m_A\mathbf{v}'_A \cdot \mathbf{v}'_A = \frac{m_A m_B^2(\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{2(m_A + m_B)^2}$$

$$E_B = \frac{1}{2}m_B\mathbf{v}'_B \cdot \mathbf{v}'_B = \frac{m_A^2 m_B(\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{2(m_A + m_B)^2}$$

(a) Ratio: $\dfrac{E_A}{E_B} = \dfrac{m_B}{m_A}$ ◀

(b)

$$\mathbf{v}_A = 90 \text{ km/h} = 25 \text{ m/s} \longrightarrow$$

$$\mathbf{v}_B = 60 \text{ km/h} = 16.667 \text{ m/s} \longleftarrow$$

$$\mathbf{v}_A + \mathbf{v}_B = 41.667 \text{ m/s} \longrightarrow$$

$$E_A = \frac{(1600)(900)^2(41.667)^2}{(2)(2500)^2} = 180.0\times 10^3 \text{ J}$$

$E_A = 180.0$ kJ ◀

$$E_B = \frac{(1600)^2(900)(41.667)^2}{(2)(2500)^2} = 320\times 10^3 \text{ J}$$

$E_B = 320$ kJ ◀

PROBLEM 14.36

It is assumed that each of the two automobiles involved in the collision described in Problem 14.35 had been designed to safely withstand a test in which it crashed into a solid, immovable wall at the speed v_0. The severity of the collision of Problem 14.35 may then be measured for each vehicle by the ratio of the energy it absorbed in the collision to the energy it absorbed in the test. On that basis, show that the collision described in Problem 14.35 is $(m_A/m_B)^2$ times more severe for automobile B than for automobile A.

SOLUTION

Velocity of mass center: $$(m_A + m_B)\overline{\mathbf{v}} = m_A\mathbf{v}_A + m_B\mathbf{v}_B$$

$$\overline{\mathbf{v}} = \frac{m_A\mathbf{v}_A + m_B\mathbf{v}_B}{m_A + m_B}$$

Velocities relative to the mass center:

$$\mathbf{v}'_A = \mathbf{v}_A - \overline{\mathbf{v}} = \mathbf{v}_A - \frac{m_A\mathbf{v}_A + m_B\mathbf{v}_B}{m_A + m_B} = \frac{m_B(\mathbf{v}_A + \mathbf{v}_B)}{m_A + m_B}$$

$$\mathbf{v}'_B = \mathbf{v}_B - \overline{\mathbf{v}} = \mathbf{v}_B - \frac{m_A\mathbf{v}_A + m_B\mathbf{v}_B}{m_A + m_B} = \frac{m_A(\mathbf{v}_A + \mathbf{v}_B)}{m_A + m_B}$$

Energies:

$$E_A = \frac{1}{2}m_A\mathbf{v}'_A \cdot \mathbf{v}'_A = \frac{m_A m_B^2(\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{2(m_A + m_B)^2}$$

$$E_B = \frac{1}{2}m_B\mathbf{v}'_B \cdot \mathbf{v}'_B = \frac{m_A^2 m_B(\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{2(m_A + m_B)^2}$$

Energies from tests: $$(E_A)_0 = \frac{1}{2}m_A v_0^2, \quad (E_B)_0 = \frac{1}{2}m_B v_0^2$$

Severities: $$S_A = \frac{E_A}{(E_A)_0} = \frac{m_B^2(\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{(m_A + m_B)^2 v_0^2}$$

$$S_B = \frac{E_B}{(E_B)_0} = \frac{m_A^2(\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{(m_A + m_B)^2 v_0^2}$$

Ratio: $$\frac{S_A}{S_B} = \frac{m_B^2}{m_A^2} \blacktriangleleft$$

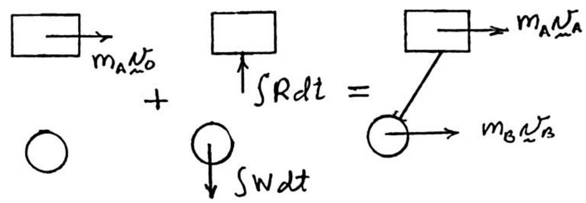
PROBLEM 14.37

Solve Sample Problem 14.4, assuming that cart A is given an initial horizontal velocity $\mathbf{v}_0$ while ball B is at rest.

SOLUTION

(a) Velocity of B at maximum elevation: At maximum elevation, ball B is at rest relative to cart A. $\mathbf{v}_B = \mathbf{v}_A$

Use impulse-momentum principle.



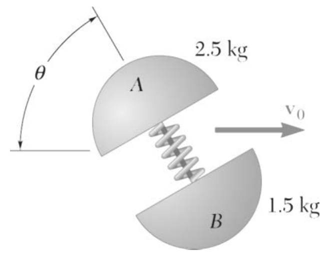
x components:
$$m_A v_0 + 0 = m_A v_A + m_B v_B$$
$$= (m_A + m_B)v_B$$

$$v_B = \frac{m_A v_0}{m_A + m_B} \longrightarrow \blacktriangleleft$$

(b) Conservation of energy:

$$T_1 = \frac{1}{2}m_A v_0^2, \quad V_1 = 0$$
$$T_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$
$$= \frac{1}{2}(m_A + m_B)v_B^2$$
$$= \frac{m_A^2 v_0^2}{2(m_A + m_B)}$$
$$V_2 = m_B gh$$
$$T_2 + V_2 = T_1 + V_1$$
$$\frac{m_A^2 v_0^2}{2(m_A + m_B)} + m_B gh = \frac{1}{2}m_A v_0^2$$
$$h = \frac{1}{2m_B g}\left[m_A v_0^2 - \frac{m_A^2 v_0^2}{m_A + m_B}\right]$$

$$h = \frac{m_A}{m_A + m_B}\frac{v_0^2}{2g} \blacktriangleleft$$

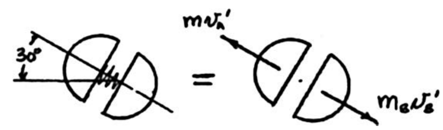


PROBLEM 14.38

Two hemispheres are held together by a cord which maintains a spring under compression (the spring is not attached to the hemispheres). The potential energy of the compressed spring is 120 J and the assembly has an initial velocity $\mathbf{v}_0$ of magnitude $v_0 = 8$ m/s. Knowing that the cord is severed when $\theta = 30°$, causing the hemispheres to fly apart, determine the resulting velocity of each hemisphere.

SOLUTION

Use a frame of reference moving with the mass center.



Conservation of momentum:

$$0 = -m_A v_A' + m_B v_B'$$

$$v_A' = \frac{m_B}{m_A} v_B'$$

Conservation of energy:

$$V = \frac{1}{2} m_A (v_A')^2 + \frac{1}{2} m_B (v_B')^2$$

$$= \frac{1}{2} m_A \left(\frac{m_B}{m_A} v_B' \right)^2 + \frac{1}{2} m_B (v_B')^2$$

$$= \frac{m_B (m_A + m_B)}{2m_A} (v_B')^2$$

$$v_B' = \sqrt{\frac{2m_A V}{m_B (m_A + m_B)}}$$

Data: $m_A = 2.5$ kg $\quad m_B = 1.5$ kg

$$V = 120 \text{ J}$$

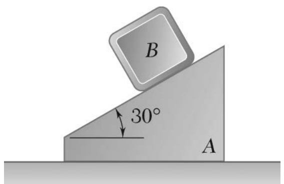
$$v_B' = \sqrt{\frac{(2)(2.5)(120)}{(1.5)(4.0)}} = 10 \qquad v_B' = 10 \text{ m/s} \searrow 30°$$

$$v_A' = \frac{1.5}{2.5}(10) = 6 \qquad v_A' = 6 \text{ m/s} \nwarrow 30°$$

Velocities of *A* and *B*.

$$\mathbf{v}_A = [8 \text{ m/s} \longrightarrow] + [6 \text{ m/s} \nwarrow 30°] \qquad \mathbf{v}_A = 4.11 \text{ m/s} \measuredangle 46.9° \blacktriangleleft$$

$$\mathbf{v}_B = [8 \text{ m/s} \longrightarrow] + [10 \text{ m/s} \searrow 30°] \qquad \mathbf{v}_B = 17.39 \text{ m/s} \searrow 16.7° \blacktriangleleft$$

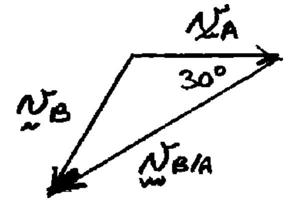


PROBLEM 14.39

A 15-lb block B starts from rest and slides on the 25-lb wedge A, which is supported by a horizontal surface. Neglecting friction, determine (*a*) the velocity of B relative to A after it has slid 3 ft down the inclined surface of the wedge, (*b*) the corresponding velocity of A.

SOLUTION

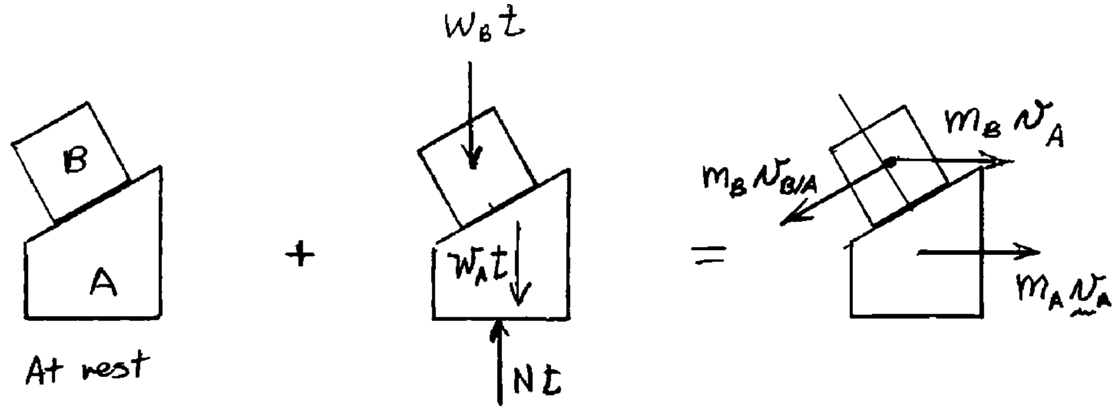
Kinematics: $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$



Law of cosines: $v_B^2 = v_A^2 + v_{B/A}^2 - 2v_A v_{B/A}\cos 30°$ (1)

Principle of impulse and momentum:

$$\Sigma m\mathbf{v}_0 + \Sigma \mathbf{F}t = \Sigma m\mathbf{v}$$



Components $\longrightarrow$: $0 + 0 = m_A v_A + m_B(v_A - v_{B/A}\cos 30°)$

$$v_A = \frac{m_B v_{B/A}\cos 30°}{m_A + m_B} = \frac{15\cos 30°}{25+15}v_{B/A}$$
$$= 0.32476\, v_{B/A}$$

From Eq. (1) $v_B^2 = (0.32476)^2\, v_{B/A}^2 + v_{B/A}^2 - (2)(0.32476)\cos 30° v_{B/A}^2$

$$= 0.54297\ v_{B/A}^2$$

Principle of conservation of energy: $T_0 + V_0 = T_1 + V_1$

$$0 + 0 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 - W_B d\sin 30°$$

$$\frac{1}{2}\frac{W_A}{g}(0.32476 v_{B/A})^2 + \frac{1}{2}\frac{W_B}{g}(0.54297)\, v_{B/A}^2 = W_B d\sin 30°$$

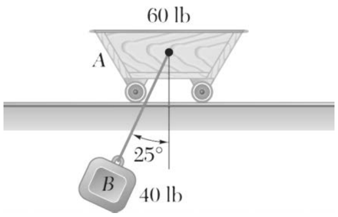
PROBLEM 14.39 (Continued)

$$\left[\frac{1}{2}\frac{25}{32.2}(0.32476)^2 + \frac{1}{2}\frac{15}{32.2}(0.54297)\right] v_{B/A}^2 = (15)(3)\sin 30°$$

$$0.16741 v_{B/A}^2 = 22.5$$

(*a*) $\mathbf{v}_{B/A} = 11.59 \text{ ft/s} \measuredangle 30°$ ◀

(b) $v_A = (0.32476)(11.59)$ $\mathbf{v}_A = 3.76 \text{ ft/s} \longrightarrow$ ◀



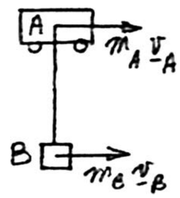
PROBLEM 14.40

A 40-lb block B is suspended from a 6-ft cord attached to a 60-lb cart A, which may roll freely on a frictionless, horizontal track. If the system is released from rest in the position shown, determine the velocities of A and B as B passes directly under A.

SOLUTION

Conservation of linear momentum:

Since block and cart are initially at rest,



$$\mathbf{L}_0 = 0$$

Thus, as B passes under A,

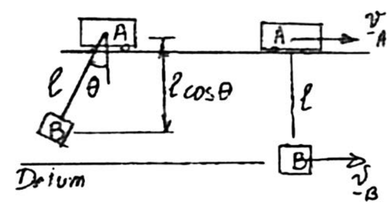
$$\mathbf{L} = m_A \mathbf{v}_A + m_B \mathbf{v}_B = 0$$

$$\xrightarrow{+} m_A v_A + m_B v_B = 0$$

$$v_A = -\frac{m_B}{m_A} v_B \qquad (1)$$

Conservation of energy:

Initially,



$$T_0 = 0$$
$$V_0 = m_B g l (1 - \cos\theta)$$

As B passes under A,

$$T = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$
$$V = 0$$

Thus, $$T_0 + V_0 = T + V: \quad m_B g l (1 - \cos\theta) = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

Substituting for v_A from (1) and multiplying by 2:

$$2 m_B g l (1 - \cos\theta) = m_A \left(\frac{m_B^2}{m_A^2} v_B^2 \right) + m_B v_B^2$$

$$= \left(\frac{m_B^2}{m_A} + m_B \right) v_B^2 = m_B \frac{m_B + m_A}{m_A} v_B^2$$

$$v_B = \sqrt{\frac{2 m_A}{m_A + m_B} g l (1 - \cos\theta)} \qquad (2)$$

PROBLEM 14.40 (Continued)

Given data:

$$w_A = 60 \text{ lb} \quad w_B = 40 \text{ lb}, \quad l = 6 \text{ ft}$$

$$g = 32.2 \text{ ft/s}^2 \quad \theta = 25°$$

$$\frac{2m_A}{m_A + m_B} = \frac{2w_A}{w_A + w_B} = \frac{(2)(60)}{60+40} = 1.2$$

From Eq. (2),

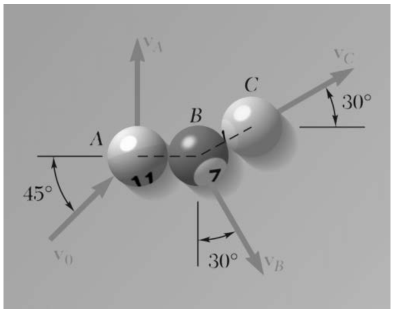
$$v_B = \sqrt{(1.2)(32.2)(6)(1-\cos 25°)}$$

$\mathbf{v}_B = 4.66 \text{ ft/s} \longrightarrow$ ◀

From Eq. (1),

$$v_A = -\frac{w_B}{w_A} v_B = -\frac{40}{60}(4.66)$$

$\mathbf{v}_A = 3.11 \text{ ft/s} \longleftarrow$ ◀



PROBLEM 14.41

In a game of pool, ball A is moving with a velocity $\mathbf{v}_0$ of magnitude $v_0 = 15$ ft/s when it strikes balls B and C, which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated and assuming frictionless surfaces and perfectly elastic impact (i.e., conservation of energy), determine the magnitudes of the velocities $\mathbf{v}_A$, $\mathbf{v}_B$, and $\mathbf{v}_C$.

SOLUTION

Velocity vectors: $\mathbf{v}_0 = v_0(\cos 45° \mathbf{i} + \sin 45° \mathbf{j})$ $v_0 = 15$ ft/s

$$\mathbf{v}_A = v_A \mathbf{j}$$
$$\mathbf{v}_B = v_B(\sin 30° \mathbf{i} - \cos 30° \mathbf{j})$$
$$\mathbf{v}_C = v_C(\cos 30° \mathbf{i} + \sin 30° \mathbf{j})$$

Conservation of momentum:

$$m\mathbf{v}_0 = m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C$$

Divide by m and resolve into components.

$$\mathbf{i}: \quad v_0 \cos 45° = v_B \sin 30° + v_C \cos 30°$$

$$\mathbf{j}: \quad v_0 \sin 45° = v_A - v_B \cos 30° + v_C \sin 30°$$

Solving for v_B and v_C,

$$v_B = -0.25882 v_0 + 0.86603 v_A$$
$$v_C = 0.96593 v_0 - 0.5 v_A$$

Conservation of energy: $\frac{1}{2} m v_0^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2$

Divide by $\frac{1}{2} m$ and substitute for v_B and v_C.

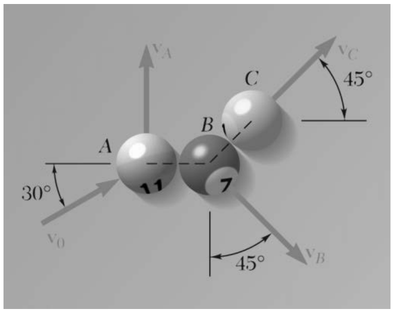
$$v_0^2 = v_A^2 + (-0.25882 v_0 + 0.86603 v_A)^2 + (0.96593 v_0 - 0.5 v_A)^2$$
$$= 2v_A^2 + v_0^2 - 1.41422 v_0 v_A$$

$$v_A = 0.70711 v_0 = 10.61 \text{ ft/s}$$ $v_A = 10.61$ ft/s ◀

$$v_B = 0.35355 v_0 = 5.30 \text{ ft/s}$$ $v_B = 5.30$ ft/s ◀

$$v_C = 0.61237 v_0 = 9.19 \text{ ft/s}$$ $v_C = 9.19$ ft/s ◀

PROBLEM 14.42



In a game of pool, ball A is moving with a velocity $\mathbf{v}_0$ of magnitude $v_0 = 15$ ft/s when it strikes balls B and C, which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated and assuming frictionless surfaces and perfectly elastic impact (i.e., conservation of energy), determine the magnitudes of the velocities $\mathbf{v}_A$, $\mathbf{v}_B$, and $\mathbf{v}_C$.

SOLUTION

Velocity vectors: $\mathbf{v}_0 = v_0(\cos 30°\,\mathbf{i} + \sin 30°\,\mathbf{j})$ $\quad v_0 = 15$ ft/s

$$\mathbf{v}_A = v_A\mathbf{j}$$
$$\mathbf{v}_B = v_B(\sin 45°\,\mathbf{i} - \cos 45°\,\mathbf{j})$$
$$\mathbf{v}_C = v_C(\cos 45°\,\mathbf{i} + \sin 45°\,\mathbf{j})$$

Conservation of momentum:

$$m\mathbf{v}_0 = m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C$$

Divide by m and resolve into components.

$$\mathbf{i}:\quad v_0\cos 30° = v_B\sin 45° + v_C\cos 45°$$

$$\mathbf{j}:\quad v_0\sin 30° = v_A - v_B\cos 45° + v_C\sin 45°$$

Solving for v_B and v_C,

$$v_B = 0.25882v_0 + 0.70711v_A$$
$$v_C = 0.96593v_0 - 0.70711v_A$$

Conservation of energy: $\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$

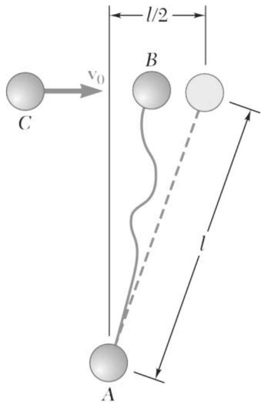
Divide by m and substitute for v_B and v_C.

$$v_0^2 = v_A^2 + (0.25882v_0 + 0.70711v_A)^2$$
$$+ (0.96593v_0 - 0.70711v_A)^2$$
$$= v_0^2 - v_0v_A + 2v_A^2$$

$v_A = 0.5v_0 = 7.500$ ft/s $\qquad v_A = 7.50$ ft/s ◀

$v_B = 0.61237v_0 = 9.1856$ ft/s $\qquad v_B = 9.19$ ft/s ◀

$v_C = 0.61237v_0 = 9.1856$ ft/s $\qquad v_C = 9.19$ ft/s ◀



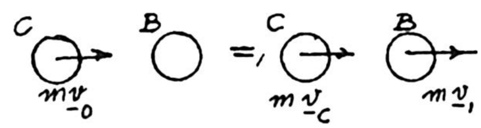
PROBLEM 14.43

Three spheres, each of mass m, can slide freely on a frictionless, horizontal surface. Spheres A and B are attached to an inextensible, inelastic cord of length l and are at rest in the position shown when sphere B is struck squarely by sphere C, which is moving to the right with a velocity $\mathbf{v}_0$. Knowing that the cord is slack when sphere B is struck by sphere C and assuming perfectly elastic impact between B and C, determine (a) the velocity of each sphere immediately after the cord becomes taut, (b) the fraction of the initial kinetic energy of the system which is dissipated when the cord becomes taut.

SOLUTION

(a) Determination of velocities.

Impact of C and B.



Conservation of momentum:

$$m\mathbf{v}_0 = m\mathbf{v}_C + m\mathbf{v}_1$$

$$v_C + v_1 = v_0 \tag{1}$$

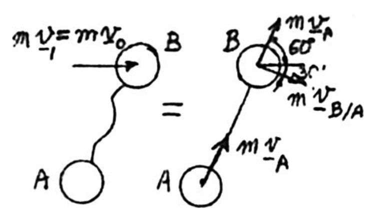
Conservation of energy (perfectly elastic impact):

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_C^2 + \frac{1}{2}mv_1^2 \quad v_C^2 + v_1^2 = v_0^2 \tag{2}$$

Square Eq. (1): $\quad v_C^2 + 2v_Cv_1 + v_1^2 = v_0^2$

Subtract Eq. (2): $\quad 2v_Cv_1 = 0$

$v_1 = 0$ corresponds to initial conditions and should be eliminated. Therefore, $\quad \mathbf{v}_C = 0$ ◀



From Eq. (1): $\quad v_1 = v_0$

Cord AB becomes taut:

Because cord is inextensible, component of $\mathbf{v}_B$ along AB must be equal to $\mathbf{v}_A$.

Conservation of momentum:

$$m\mathbf{v}_0 = 2m\mathbf{v}_A + m\mathbf{v}_{B/A}$$

$+\uparrow$ y comp: $\quad 0 = 2mv_A \sin 60° - mv_{B/A} \sin 30°$

$$v_{B/A} = 2\sqrt{3}v_A \tag{3}$$

$\xrightarrow{+}$ x comp: $\quad mv_0 = 2mv_A \cos 60° + mv_{B/A} \cos 30°$

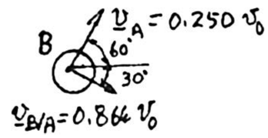
PROBLEM 14.43 (Continued)

Dividing by m and substituting for $v_{B/A}$ from Eq. (3):

$$v_0 = 2v_A(0.5) + (2\sqrt{3}v_A)(\sqrt{3/2})$$

$$v_0 = 4v_A \quad v_A = 0.250v_0 \qquad \mathbf{v}_A = 0.250v_0 \measuredangle 60° \blacktriangleleft$$

Carrying into Eq. (3): $v_{B/A} = 2\sqrt{3}(0.250v_0) = 0.866v_0$



Thus, $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$$= 0.250v_0 \measuredangle 60° + 0.866v_0 \searrow 30°$$

$$\mathbf{v}_B = (0.250v_0 \cos 60° + 0.866v_0 \cos 30°)\mathbf{i} + (0.250v_0 \sin 60° - 0.866v_0 \sin 30°)\mathbf{j}$$

$$\mathbf{v}_B = 0.875v_0\mathbf{i} - 0.2165\mathbf{j}$$

$$\mathbf{v}_B = 0.90139v_0 \searrow 13.90° \qquad \mathbf{v}_B = 0.901v_0 \searrow 13.9° \blacktriangleleft$$

(*b*) Fraction of kinetic energy lost:

$$T_0 = \frac{1}{2}mv_0^2$$

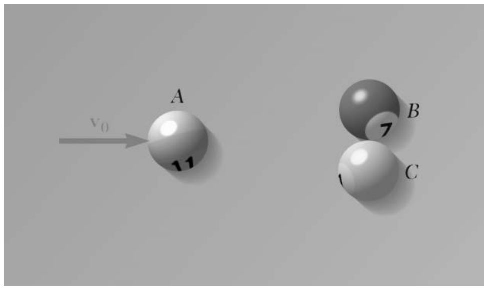
$$T_{\text{final}} = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$

$$= \frac{1}{2}m(0.250v_0)^2 + \frac{1}{2}(0.90139v_0)^2 + \frac{1}{2}m(0)$$

$$= \frac{1}{2}m(0.875)v_0^2$$

$$\text{Kinetic energy lost} = T_0 - T_{\text{final}} = \frac{1}{2}m(1-0.875)v_0^2 = \frac{1}{2}\cdot\frac{1}{8}mv_0^2$$

$$\text{Fraction of kinetic energy lost} = \frac{1}{8} \blacktriangleleft$$



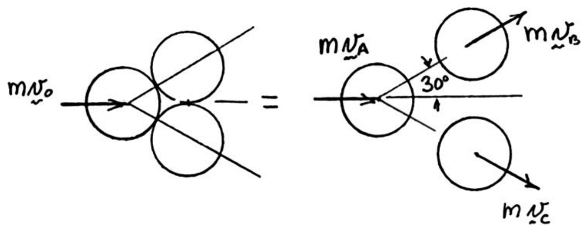
PROBLEM 14.44

In a game of pool, ball A is moving with the velocity $\mathbf{v}_0 = v_0\mathbf{i}$ when it strikes balls B and C, which are at rest side by side. Assuming frictionless surfaces and perfectly elastic impact (i.e., conservation of energy), determine the final velocity of each ball, assuming that the path of A is (a) perfectly centered and that A strikes B and C simultaneously, (b) not perfectly centered and that A strikes B slightly before it strikes C.

SOLUTION

(a) A strikes B and C simultaneously:

During the impact, the contact impulses make $30°$ angles with the velocity $\mathbf{v}_0$.



Thus, $\mathbf{v}_B = v_B(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$

$\mathbf{v}_C = v_C(\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$

By symmetry, $\mathbf{v}_A = v_A\mathbf{i}$

Conservation of momentum: $m\mathbf{v}_0 = m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C$

y component: $0 = 0 + mv_B \sin 30° - mv_C \sin 30° \qquad v_C = v_B$

x component: $mv_0 = mv_A + mv_B \cos 30° + mv_C \cos 30°$

$$v_B + v_C = \frac{v_0 - v_A}{\cos 30°} = \frac{2}{\sqrt{3}}(v_0 - v_A)$$

$$v_B = v_C = \frac{v_0 - v_A}{\sqrt{3}}$$

Conservation of energy: $\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$

$$v_0^2 = v_A^2 + \frac{2}{3}(v_0 - v_A)^2$$

$$v_0^2 - v_A^2 = (v_0 - v_A)(v_0 + v_A) = \frac{2}{3}(v_0 - v_A)^2$$

$$v_0 + v_A = \frac{2}{3}(v_0 - v_A) \qquad \frac{1}{3}v_0 = -\frac{5}{3}v_A \qquad v_A = -\frac{1}{5}v_0$$

$$v_B = v_C = \frac{6}{5\sqrt{3}}v_0 = \frac{2\sqrt{3}}{5}v_0$$

$\mathbf{v}_A = 0.200v_0 \longleftarrow$ ◀

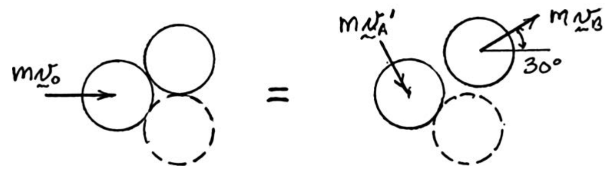
$\mathbf{v}_B = 0.693v_0 \measuredangle\ 30°$ ◀

$\mathbf{v}_C = 0.693v_0 \searrow 30°$ ◀

(*b*) *A* strikes *B* before it strikes *C*:

First impact: *A* strikes *B*.

During the impact, the contact impulse makes a 30° angle with the velocity $\mathbf{v}_0$.



Thus, $\mathbf{v}_B = v_B(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$

Conservation of momentum: $m\mathbf{v}_0 = m\mathbf{v}_A + m\mathbf{v}_B$

y component: $0 = m(v_A')_y + mv_B \sin 30° \qquad (v_A')_y = -v_B \sin 30°$

x component: $v_0 = m(v_A')_x + mv_B \cos 30° \qquad (v_A')_x = v_0 - v_B \cos 30°$

Conservation of energy:

$$\frac{1}{2}mv_0^2 = \frac{1}{2}m(v_A')_x^2 + \frac{1}{2}m(v_A')_y^2 + \frac{1}{2}mv_B^2$$

$$= \frac{1}{2}m(v_0 - v_B\cos 30°)^2 + \frac{1}{2}(v_B \sin 30°)^2 + \frac{1}{2}v_B^2$$

$$= \frac{1}{2}m\left(v_0^2 - 2v_0v_B + v_B^2\cos^2 30° + v_B^2\sin^2 30° + v_B^2\right)$$

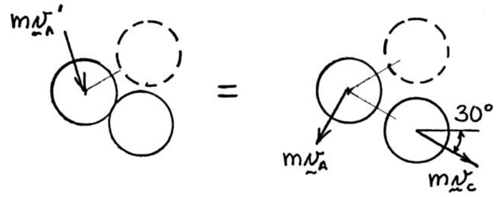
$$v_B = v_0\cos 30° = \frac{\sqrt{3}}{2}v_0, \quad (v_A')_x = v_0\sin^2 30° = \frac{1}{4}v_0,$$

$$(v_A')_y = -v_0\cos 30°\sin 30° = -\frac{\sqrt{3}}{4}v_0$$

PROBLEM 14.44 (Continued)

Second impact: A strikes C.

During the impact, the contact impulse makes a $30°$ angle with the velocity $\mathbf{v}_0$.



Thus, $\mathbf{v}_C = v_C(\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$

Conservation of momentum: $m\mathbf{v}'_A = m\mathbf{v}_A + m\mathbf{v}_C$

x component: $m(v'_A)_x = m(v_A)_x + mv_C \cos 30°,$

$$(v_A)_x = (v'_A)_x - v_C \cos 30° = \frac{1}{4}v_0 - v_C \cos 30°$$

y component: $m(v'_A)_y = m(v_A)_y - mv_C \sin 30°$

$$(v_A)_y = (v'_A)_y + v_C \sin 30° = -\frac{\sqrt{3}}{4}v_0 + v_C \sin 30°$$

Conservation of energy:

$$\frac{1}{2}m(v'_A)_x^2 + \frac{1}{2}m(v'_A)_y^2 = \frac{1}{2}m(v_A)_x^2 + \frac{1}{2}m(v_A)_y^2 + \frac{1}{2}mv_C^2$$

$$\frac{1}{2}m\left[\frac{1}{16}v_0^2 + \frac{3}{16}v_0^2\right] = \frac{1}{2}m\left[\left(\frac{1}{4}v_0 - v_C \cos 30°\right)^2 + \left(-\frac{\sqrt{3}}{4}v_0 + v_C \sin 30°\right)^2 + v_C^2\right]$$

$$= \frac{1}{2}m\left[\frac{1}{16}v_0^2 - \frac{1}{2}v_0 v_C \cos 30° + v_C^2 \cos^2 30° + \frac{3}{16}v_0^2 - \frac{\sqrt{3}}{2}v_0 v_C \sin 30° + v_C^2 \sin^2 30° + v_C^2\right]$$

$$0 = -v_0 v_C\left(\frac{1}{2}\cos 30° + \frac{\sqrt{3}}{2}\sin 30°\right) + 2v_C^2$$

$$v_C = v_0\left(\frac{1}{4}\cos 30° + \frac{\sqrt{3}}{4}\sin 30°\right) = \frac{\sqrt{3}}{4}v_0$$

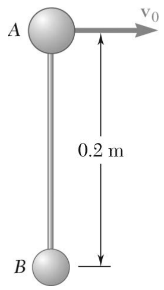
$$(v_A)_x = \frac{1}{4}v_0 - \frac{\sqrt{3}}{4}v_0 \cos 30° = -\frac{1}{8}v_0$$

$$(v_A)_y = -\frac{\sqrt{3}}{4}v_0 + \frac{\sqrt{3}}{4}v_0 \sin 30° = -\frac{\sqrt{3}}{8}v_0$$

$\mathbf{v}_A = 0.250v_0$ ↙ $60°$ ◀

$\mathbf{v}_B = 0.866v_0$ ↗ $30°$ ◀

$\mathbf{v}_C = 0.433v_0$ ↘ $30°$ ◀



PROBLEM 14.45

Two small spheres *A* and *B*, of mass 2.5 kg and 1 kg, respectively, are connected by a rigid rod of negligible weight. The two spheres are resting on a horizontal, frictionless surface when *A* is suddenly given the velocity $\mathbf{v}_0 = (3.5 \text{ m/s})\mathbf{i}$. Determine (*a*) the linear momentum of the system and its angular momentum about its mass center *G*, (*b*) the velocities of *A* and *B* after the rod *AB* has rotated through 180°.

SOLUTION

Position of mass center:

$$\overline{y} = \sum \frac{m_i y_i}{m_i} = \frac{2.5(0) + 1(0.2)}{2.5 + 1} = 0.057143 \text{ m}$$

(*a*) *Linear and angular momentum*:

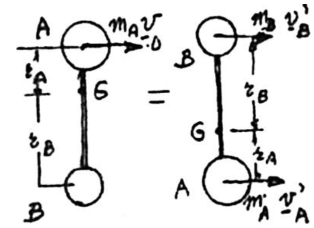
$$L = m_A \mathbf{v}_0 = 2.5 \text{ kg}(3.5 \text{ m/s})\,\mathbf{i} = (8.75 \text{ kg}\cdot\text{m/s})\,\mathbf{i}$$

$$\mathbf{L} = (8.75 \text{ kg}\cdot\text{m/s})\mathbf{i} \blacktriangleleft$$

$$\mathbf{H}_G = \overrightarrow{GA} \times m_A \mathbf{v}_0 = (0.05714285 \text{ m})\mathbf{j} \times (8.75 \text{ kg}\cdot\text{m/s})\,\mathbf{i}$$
$$= -(0.50000 \text{ kg}\cdot\text{m}^2/\text{s})\,\mathbf{k}$$

$$\mathbf{H}_G = -(0.500 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k} \blacktriangleleft$$

(*b*) *Velocities of A and B after 180° rotation*



Conservation of linear momentum:

$$m_A v_0 = m_A v'_A + m_B v'_B$$
$$(2.5)(3.5) = (2.5)v'_A + (1.0)v'_B$$

$$2.55v'_A + v'_B = 8.75 \quad (1)$$

Conservation of angular momentum about G':

$$+\circlearrowright: \quad r_A m_A v_0 = -r_A m_A v'_A + r_B m_B v'_B$$

$$r_B = 0.20 - r_A = 0.14286 \text{ m}$$
$$(0.057143)(2.5)(3.5) = -(0.057143)(2.5)v'_A + (0.14286)(1.0)v'_B$$

Dividing by 0.057143: $$-2.5v'_A + \frac{0.14286}{0.057143}v'_B = 8.75 \quad (2)$$

Add Eqs. (1) and (2): $3.5v'_B = 17.5 \qquad v'_B = +5.00 \text{ m/s}$

From Eq. (1): $2.5v'_A + (5) = 8.75 \qquad v'_A = +1.50 \text{ m/s}$

$$\mathbf{v}'_A = (1.50 \text{ m/s})\mathbf{i}; \quad \mathbf{v}'_B = (5.00 \text{ m/s})\mathbf{i} \blacktriangleleft$$

PROBLEM 14.46

A 900-lb space vehicle traveling with a velocity $\mathbf{v}_0 = (1500 \text{ ft/s})\mathbf{k}$ passes through the origin O. Explosive charges then separate the vehicle into three parts A, B, and C, with masses of 150 lb, 300 lb, and 450 lb, respectively. Knowing that shortly thereafter the positions of the three parts are, respectively, A(250, 250, 2250), B (600, 1300, 3200), and C (–475, –950, 1900), where the coordinates are expressed in ft, that the velocity of B is $\mathbf{v}_B = (500 \text{ ft/s})\mathbf{i} + (1100 \text{ ft/s})\mathbf{j} + (2100 \text{ ft/s})\mathbf{k}$, and that the x component of the velocity of C is -400 ft/s, determine the velocity of part A.

SOLUTION

Position vectors (ft):

$$\mathbf{r}_A = 250\mathbf{i} + 250\mathbf{j} + 2250\mathbf{k}$$
$$\mathbf{r}_B = 600\mathbf{i} + 1300\mathbf{j} + 3200\mathbf{k}$$
$$\mathbf{r}_C = -475\mathbf{i} - 950\mathbf{j} + 1900\mathbf{k}$$

Since there are no external forces, linear momentum is conserved.

$$(m_A + m_B + m_C)\mathbf{v}_0 = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C$$

$$\mathbf{v}_A = \frac{m_A + m_B + m_C}{m_A}\mathbf{v}_0 - \frac{m_B}{m_A}\mathbf{v}_B - \frac{m_C}{m_A}\mathbf{v}_C = 6\mathbf{v}_0 - 2\mathbf{v}_B - 3\mathbf{v}_C \quad (1)$$

$$= (6)(1500\mathbf{k}) - (2)(500\mathbf{i} + 1100\mathbf{j} + 2100\mathbf{k}) - (3)[-400\mathbf{i} + (v_C)_y\mathbf{j} + (v_C)_z\mathbf{k}]$$

$$= -3(v_C)_y\mathbf{j} - 3(v_C)_z\mathbf{k} + 200\mathbf{i} - 2200\mathbf{j} + 4800\mathbf{k}$$

$$(v_A)_x = 200, \quad (v_C)_y = -3(v_C)_y - 2200, \quad (v_A)_z = -3(v_C)_z + 4800$$

Conservation of angular momentum about O:

$$(\mathbf{H}_O)_2 = (\mathbf{H}_O)_1$$

Since the vehicle passes through the origin, $(\mathbf{H}_O)_1 = 0.$

$$(\mathbf{H}_O)_2 = \mathbf{r}_A \times (m_A\mathbf{v}_A) + \mathbf{r}_B \times (m_B\mathbf{v}_B) + \mathbf{r}_C \times (m_C\mathbf{v}_C) = 0$$

Divide by m_A.

$$\mathbf{r}_A \times \mathbf{v}_A + \frac{m_B}{m_A}\mathbf{r}_B \times \mathbf{v}_B + \frac{m_C}{m_A}\mathbf{r}_C \times \mathbf{v}_C = \mathbf{r}_A \times \mathbf{v}_A + 2\mathbf{r}_B \times \mathbf{v}_B + 3\mathbf{r}_C \times \mathbf{v}_C$$
$$= \mathbf{r}_A \times (6\mathbf{v}_0 - 2\mathbf{v}_B - 3\mathbf{v}_C) + 2\mathbf{r}_B \times \mathbf{v}_B + 3\mathbf{r}_C \times \mathbf{v}_C$$
$$= 3(\mathbf{r}_C - \mathbf{r}_A) \times \mathbf{v}_C + 6\mathbf{r}_A \times \mathbf{v}_0 + 2(\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{v}_B$$
$$= (-2175\mathbf{i} - 3600\mathbf{j} - 1050\mathbf{k}) \times \mathbf{v}_C + (1500\mathbf{i} + 1500\mathbf{j} + 13500\mathbf{k}) \times \mathbf{v}_0$$
$$+ (700\mathbf{i} + 2100\mathbf{j} + 1900\mathbf{k}) \times \mathbf{v}_B = 0$$

PROBLEM 14.46 (Continued)

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2175 & -3600 & -1050 \\ (v_C)_x & (v_C)_y & (v_C)_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1500 & 1500 & 13500 \\ 0 & 0 & 1500 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 700 & 2100 & 1900 \\ 500 & 1100 & 2100 \end{vmatrix} = 0$$

Resolve into components.

i: $1050(v_C)_y - 3600(v_C)_z + 2,250,000 + 2,320,000 = 0$ (2)

j: $2175(v_C)_z - 1050(v_C)_x - 2,250,000 - 520,000 = 0$ (3)

k: $3600(v_C)_x - 2175(v_C)_y + 0 - 280,000 = 0$ (4)

Set $(v_C)_x = -400$ ft/s

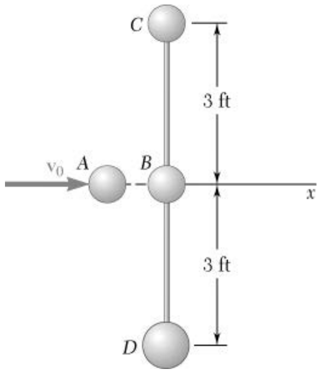
From Eq. (4), $(v_C)_y = -790.80$ ft/s

From Eq. (3), $(v_C)_z = 1080.5$ ft/s

From Eq. (1), $\mathbf{v}_A = (6)(1500\mathbf{k}) - (2)(500\mathbf{i} + 1100\mathbf{j} + 2100\mathbf{k}) - (3)[-400\mathbf{i} + (790.80)\mathbf{j} + (1080.5)\mathbf{k}]$
$= 200\mathbf{i} + 172.4\mathbf{j} + 1558.6\mathbf{k}$

$\mathbf{v}_A = (200 \text{ ft/s})\mathbf{i} + (172 \text{ ft/s})\mathbf{j} + (1560 \text{ ft/s})\mathbf{k}$ ◀

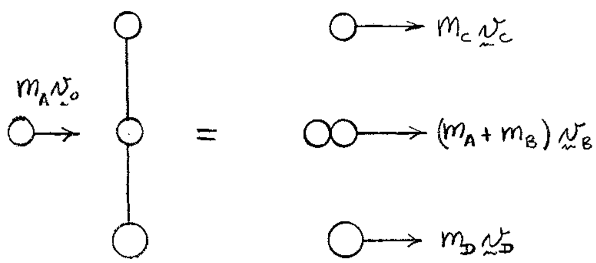
PROBLEM 14.47



Four small disks A, B, C, and D can slide freely on a frictionless horizontal surface. Disks B, C, and D are connected by light rods and are at rest in the position shown when disk B is struck squarely by disk A, which is moving to the right with a velocity $\mathbf{v}_0 = (38.5 \text{ ft/s})\mathbf{i}$. The weights of the disks are $W_A = W_B = W_C = 15$ lb, and $W_D = 30$ lb. Knowing that the velocities of the disks immediately after the impact are $\mathbf{v}_A = \mathbf{v}_B = (8.25 \text{ ft/s})\mathbf{i}$, $\mathbf{v}_C = v_C\,\mathbf{i}$, and $\mathbf{v}_D = v_D\,\mathbf{i}$, determine ($a$) the speeds v_C and v_D, (b) the fraction of the initial kinetic energy of the system which is dissipated during the collision.

SOLUTION

There are no external forces. Momentum is conserved.



(a) Moments about D +↻: $\quad 3m_A v_0 = 6m_C v_C + 3(m_A + m_B)v_B$

$$v_C = \frac{3m_A}{6m_C}v_0 - \frac{3(m_A + m_B)}{6m_C}v_B = (0.5)(38.5) - (8.25) = 11 \qquad v_C = 11.00 \text{ ft/s} \blacktriangleleft$$

Moments about C +↺: $\quad 3m_A v_0 = 3(m_A + m_B)v_B + 6m_D v_D$

$$v_D = \frac{3m_A v_0}{6m_D} - \frac{3(m_A + m_B)}{6m_D}v_B = (0.25)(38.5) - (0.5)(8.25) = 5.5 \text{ ft/s} \qquad v_D = 5.50 \text{ ft/s} \blacktriangleleft$$

(b) Initial kinetic energy:

$$T_1 = \frac{1}{2}\frac{W_A}{g}v_0^2 = \frac{1}{2}\frac{15}{32.2}(38.5)^2 = 345.24 \text{ ft}\cdot\text{lb}$$

Final kinetic energy:

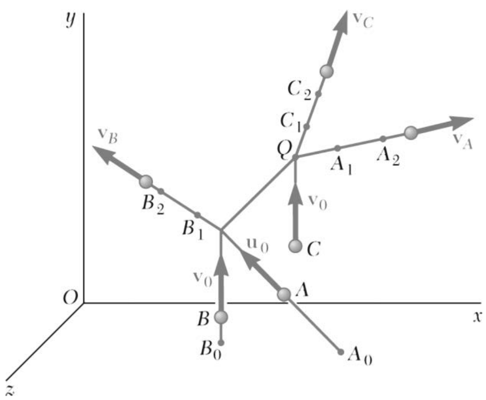
$$T_2 = \frac{1}{2}\frac{W_A + W_B}{g}v_B^2 + \frac{1}{2}\frac{W_C}{g}v_C^2 + \frac{1}{2}\frac{W_D}{g}v_D^2$$

$$= \frac{1}{2}\frac{30}{32.2}(8.25)^2 + \frac{1}{2}\frac{15}{32.2}(11.00)^2 + \frac{1}{2}\frac{30}{32.2}(5.50)^2 = 73.98 \text{ ft}\cdot\text{lb}$$

Energy lost: $\quad 345.24 - 73.98 = 271.26 \text{ ft}\cdot\text{lb}$

Fraction of energy lost $= \dfrac{271.26}{345.24} = 0.786 \qquad \dfrac{(T_1 - T_2)}{T_1} = 0.786 \blacktriangleleft$

PROBLEM 14.48



In the scattering experiment of Problem 14.26, it is known that the alpha particle is projected from $A_0(300, 0, 300)$ and that it collides with the oxygen nucleus C at $Q(240, 200, 100)$, where all coordinates are expressed in millimeters. Determine the coordinates of Point B_0 where the original path of nucleus B intersects the zx plane. (*Hint:* Express that the angular momentum of the three particles about Q is conserved.)

PROBLEM 14.26 In a scattering experiment, an alpha particle A is projected with the velocity $\mathbf{u}_0 = -(600 \text{ m/s})\mathbf{i} + (750 \text{ m/s})\mathbf{j} - (800 \text{ m/s})\mathbf{k}$ into a stream of oxygen nuclei moving with a common velocity $\mathbf{v}_0 = (600 \text{ m/s})\mathbf{j}$. After colliding successively with nuclei B and C, particle A is observed to move along the path defined by the Points $A_1(280, 240, 120)$ and $A_2(360, 320, 160)$, while nuclei B and C are observed to move along paths defined, respectively, by $B_1(147, 220, 130)$, $B_2(114, 290, 120)$, and by $C_1(240, 232, 90)$ and $C_2(240, 280, 75)$. All paths are along straight lines and all coordinates are expressed in millimeters. Knowing that the mass of an oxygen nucleus is four times that of an alpha particle, determine the speed of each of the three particles after the collisions.

SOLUTION

Conservation of angular momentum about Q:

$$\overrightarrow{QA}_0 \times (m\mathbf{u}_0) + \overrightarrow{QB}_0 \times (4m\mathbf{v}_0) + \overrightarrow{QC}_0 \times (4m\mathbf{v}_0) = \overrightarrow{QA}_1 \times (m\mathbf{v}_A) + \overrightarrow{QB}_1 \times (4m\mathbf{v}_B) + \overrightarrow{QC}_1 \times (4m\mathbf{v}_C)$$

$$\overrightarrow{QA}_0 \times (m\mathbf{u}_0) + \overrightarrow{QB}_0 \times (4m\mathbf{v}_0) + 0 = 0 + \overrightarrow{QB}_1 \times (4m\mathbf{v}_B) + 0 \qquad (1)$$

where

$$\overrightarrow{QA}_0 = \mathbf{r}_{A_0} - \mathbf{r}_Q = (300\mathbf{i} + 300\mathbf{k}) - (240\mathbf{i} + 200\mathbf{j} + 100\mathbf{k})$$
$$= (60 \text{ mm})\mathbf{i} - (200 \text{ mm})\mathbf{j} + (200 \text{ mm})\mathbf{k}$$

$$\overrightarrow{QB}_0 = (\Delta x)\mathbf{i} + (\Delta y)\mathbf{j} + (\Delta z)\mathbf{k}$$

$$\overrightarrow{QB}_1 = \mathbf{r}_{B_1} - \mathbf{r}_Q = (147\mathbf{i} + 220\mathbf{j} + 130\mathbf{k}) - (240\mathbf{i} + 200\mathbf{j} + 100\mathbf{k})$$
$$-(93 \text{ mm})\mathbf{i} + (20 \text{ mm})\mathbf{j} + (30 \text{ mm})\mathbf{k}$$

$$\mathbf{u}_0 = -(600 \text{ m/s})\mathbf{i} + (750 \text{ m/s})\mathbf{j} - (800 \text{ m/s})\mathbf{k} \qquad \mathbf{v}_0 = (600 \text{ m/s})\mathbf{j}$$

and from the solution to Problem 14.26,

$$\mathbf{v}_B = v_B\boldsymbol{\lambda}_B = (716.98)(-0.42290\mathbf{i} + 0.89707\mathbf{j} - 0.12815\mathbf{k})$$
$$= -(303.21 \text{ m/s})\,\mathbf{i} + (643.18 \text{ m/s})\,\mathbf{j} - (91.88 \text{ m/s})\mathbf{k}$$

Calculating each term and dividing by *m*,

$$\overrightarrow{QA}_0 \times \mathbf{u}_0 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 60 & -200 & 200 \\ -600 & 750 & -800 \end{vmatrix} = 10,000\mathbf{i} - 72,000\mathbf{j} - 75,000\mathbf{k}$$

$$\overrightarrow{QB}_0 \times (4\mathbf{v}_0) = [(\Delta x)\mathbf{i} + (\Delta y)\mathbf{j} + (\Delta z)\mathbf{k}] \times (2400\mathbf{j})$$
$$= -2400(\Delta z)\mathbf{i} + 2400(\Delta x)\mathbf{k}$$

$$\overrightarrow{QB}_1 \times (4\mathbf{v}_B) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -93 & 20 & 30 \\ -1212.84 & 2572.72 & -367.52 \end{vmatrix} = -84,532\mathbf{i} - 70,565\mathbf{j} - 215,006\mathbf{k}$$

Collect terms and resolve into components.

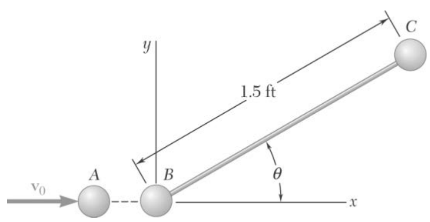
$\mathbf{i}$: $\quad 10,000 - 2400(\Delta z) = -84,532 \qquad \Delta z = 39.388 \text{ mm}$

$\mathbf{k}$: $\quad -75,000 + 2400(\Delta x) = -215,006 \qquad \Delta x = -58.336 \text{ mm}$

Coordinates: $\quad x_{B_0} = x_Q + \Delta x = 240 - 58.336 \qquad x_{B_0} = 181.7 \text{ mm}$ ◀

$y_{B_0} = 0$ ◀

$z_{B_0} = z_Q + \Delta z = 100 + 39.388 \qquad z_{B_0} = 139.4 \text{ mm}$ ◀



PROBLEM 14.49

Three identical small spheres, each of weight 2 lb, can slide freely on a horizontal frictionless surface. Spheres B and C are connected by a light rod and are at rest in the position shown when sphere B is struck squarely by sphere A which is moving to the right with a velocity $\mathbf{v}_0 = (8\text{ ft/s})\mathbf{i}$. Knowing that $\theta = 45°$ and that the velocities of spheres A and B immediately after the impact are $\mathbf{v}_A = 0$ and $\mathbf{v}_B = (6\text{ ft/s})\mathbf{i} + (\mathbf{v}_B)_y\ \mathbf{j}$, determine $(v_B)_y$ and the velocity of C immediately after impact.

SOLUTION

Let m be the mass of one ball.

Conservation of linear momentum: $(\Sigma m\mathbf{v}) = (\Sigma m\mathbf{v})_0$

$$m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C = m(\mathbf{v}_A)_0 + (m\mathbf{v}_B)_0 + (m\mathbf{v}_C)_0$$

Dividing by m and applying numerical data,

$$0 + [(6\text{ ft/s})\mathbf{i} + (v_B)_y\mathbf{j}] + [(v_C)_x\mathbf{i} + (v_C)_y\mathbf{j}] = (8\text{ ft/s})\mathbf{i} + 0 + 0$$

Components:

x: $6 + (v_C)_x = 8$ $\qquad (v_C)_x = 2\text{ ft/s}$

y: $(v_B)_y + (v_C)_y = 0$ $\qquad (1)$

Conservation of angular momentum about O:

$$\Sigma[\mathbf{r} \times (m\mathbf{v})] = \Sigma[\mathbf{r} \times (m\mathbf{v}_0)]$$

where $\mathbf{r}_A = 0,\qquad \mathbf{r}_B = 0,\qquad \mathbf{r}_C = (1.5\text{ ft})(\cos 45°\mathbf{i} + \sin 45°\mathbf{j})$

$$(1.5)(\cos 45°\mathbf{i} + \sin 45°\mathbf{j}) \times [m(v_C)_x\mathbf{i} + m(v_C)_y\mathbf{j}] = 0$$

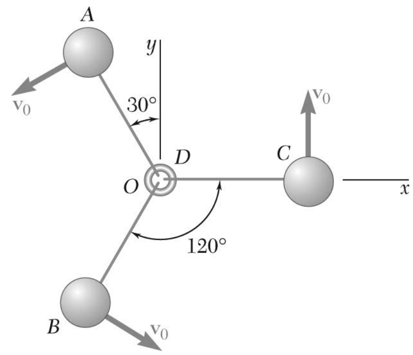
Since their cross product is zero, the two vectors are parallel.

$$(v_C)_y = (v_C)_x \tan 45° = 2\tan 45° = 2\text{ ft/s}$$

From (1), $(v_B)_y = -2\text{ ft/s}$

$(v_B)_y = -2.00\text{ ft/s}$ ◀

$\mathbf{v}_C = (2.00\text{ ft/s})\mathbf{i} + (2.00\text{ ft/s})\mathbf{j}$ ◀



PROBLEM 14.50

Three small spheres A, B, and C, each of mass m, are connected to a small ring D of negligible mass by means of three inextensible, inelastic cords of length l. The spheres can slide freely on a frictionless horizontal surface and are rotating initially at a speed v_0 about ring D which is at rest. Suddenly the cord CD breaks. After the other two cords have again become taut, determine (a) the speed of ring D, (b) the relative speed at which spheres A and B rotate about D, (c) the fraction of the original energy of spheres A and B which is dissipated when cords AD and BD again became taut.

SOLUTION

Let the system consist of spheres A and B.

State 1: Instant cord DC breaks.

$$m(\mathbf{v}_A)_1 = mv_0\left(-\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$$

$$m(\mathbf{v}_B)_1 = mv_0\left(\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$$

$$\mathbf{L}_1 = m(\mathbf{v}_A)_1 + m(\mathbf{v}_B)_1 = -mv_0\mathbf{j}$$

$$\overline{\mathbf{v}} = \frac{\mathbf{L}_1}{2m} = -\frac{1}{2}v_0\mathbf{j}$$

Mass center lies at Point G midway between balls A and B.

$$(\mathbf{H}_G)_1 = \frac{\sqrt{3}}{2}l\mathbf{j}\times(m\mathbf{v}_A)_1 + -\frac{\sqrt{3}}{2}l\mathbf{j}\times(m\mathbf{v}_B)_1$$

$$= \frac{3}{2}lmv_0\mathbf{k}$$

$$T_1 = \frac{1}{2}mv_0^2 + \frac{1}{2}mv_0^2 = mv_0^2$$

State 2: The cord is taut. Conservation of linear momentum:

(a) $$\mathbf{v}_D = \overline{\mathbf{v}} = -\frac{1}{2}v_0\mathbf{j}$$ $v_D = 0.500v_0$ ◀

Let $$(\mathbf{v}_A)_2 = \overline{\mathbf{v}} + \mathbf{u}_A \quad \text{and} \quad \mathbf{v}_B = \overline{\mathbf{v}} + \mathbf{u}_B$$

$$\mathbf{L}_2 = 2m\overline{\mathbf{v}} + m\mathbf{u}_A + m\mathbf{u}_B = \mathbf{L}_1$$

$$\mathbf{u}_B = -\mathbf{u}_A \qquad u_B = u_A$$

$$(\mathbf{H}_G)_2 = lmu_A\mathbf{k} + lmu_B\mathbf{k} = 2lmu_A\mathbf{k}$$

PROBLEM 14.50 (Continued)

(*b*) Conservation of angular momentum:

$$(\mathbf{H}_G)_2 = (\mathbf{H}_G)_1$$

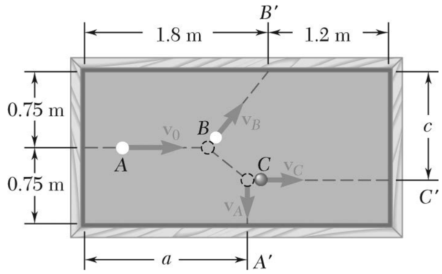
$$2lmu_A\mathbf{k} = \frac{3}{2}lmv_0\mathbf{k} \qquad u_A = u_B = \frac{3}{4}v_0 \qquad u = 0.750v_0 \blacktriangleleft$$

$$T_2 = \frac{1}{2}(2m)\overline{v}^2 + \frac{1}{2}mu_A^2 + \frac{1}{2}mu_B^2$$

$$= \frac{1}{2}mv_0^2\left(\frac{1}{2} + \frac{9}{16} + \frac{9}{16}\right) = \frac{13}{16}mv_0^2$$

(*c*) Fraction of energy lost: $$\frac{T_1 - T_2}{T_1} = \frac{1 - \frac{13}{16}}{1} = \frac{3}{16} \qquad \frac{T_1 - T_2}{T_1} = 0.1875 \blacktriangleleft$$

PROBLEM 14.51



In a game of billiards, ball A is given an initial velocity $\mathbf{v}_0$ along the longitudinal axis of the table. It hits ball B and then ball C, which are both at rest. Balls A and C are observed to hit the sides of the table squarely at A' and C', respectively, and ball B is observed to hit the side obliquely at B'. Knowing that $v_0 = 4$ m/s, $v_A = 1.92$ m/s, and $a = 1.65$ m, determine (a) the velocities $\mathbf{v}_B$ and $\mathbf{v}_C$ of balls B and C, (b) the Point C' where ball C hits the side of the table. Assume frictionless surfaces and perfectly elastic impacts (that is, conservation of energy).

SOLUTION

Velocities in m/s. Lengths in meters. Assume masses are 1.0 for each ball.

Before impacts: $(\mathbf{v}_A)_0 = v_0\mathbf{i} = 4\mathbf{i}, \qquad (\mathbf{v}_B)_0 = (\mathbf{v}_C)_0 = 0$

After impacts: $\mathbf{v}_A = -1.92\mathbf{j}, \qquad \mathbf{v}_B = (v_B)_x\mathbf{i} + (v_B)_y\mathbf{j}, \qquad \mathbf{v}_C = v_C\mathbf{i}$

Conservation of linear momentum: $\mathbf{v}_0 = \mathbf{v}_A + \mathbf{v}_B + \mathbf{v}_C$

$$\mathbf{i}: \quad 4 = 0 + (v_B)_x + v_C \qquad (v_B)_x = 4 - v_C$$

$$\mathbf{j}: \quad 0 = -1.92 + (v_B)_y + 0 \qquad (v_B)_y = 1.92$$

Conservation of energy: $\frac{1}{2}v_0^2 = \frac{1}{2}v_A^2 + \frac{1}{2}v_B^2 + \frac{1}{2}v_C^2$

$$\frac{1}{2}(4)^2 = \frac{1}{2}(1.92)^2 + \frac{1}{2}(1.92)^2 + \frac{1}{2}(4 - v_C)^2 + \frac{1}{2}v_C^2$$

$$v_C^2 - 4v_C + 3.6864 = 0$$

$$v_C = \frac{4 \pm \sqrt{(4)^2 - (4)(3.6864)}}{2} = 2 \pm 0.56 = 2.56 \qquad \text{or} \qquad 1.44$$

Conservation of angular momentum about B':

$$0.75v_0 = (1.8 - a)v_A + cv_C$$

$$cv_C = (0.75)(4) - (1.8 - 1.65)(1.92) = 2.712$$

$$c = \frac{2.712}{v_C}$$

If $v_C = 1.44$, $\qquad c = 1.8833$ off the table. Reject.

If $v_C = 2.56$, $\qquad c = 1.059$

PROBLEM 14.51 (Continued)

Then, $(v_B)_x = 4 - 2.56 = 1.44, \qquad \mathbf{v}_B = 1.44\mathbf{i} + 1.92\mathbf{j}$

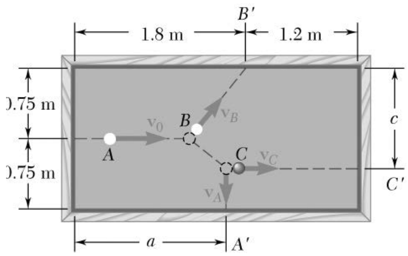
Summary.

(*a*) $\mathbf{v}_B = 2.40 \text{ m/s} \measuredangle 53.1° \blacktriangleleft$

$\mathbf{v}_C = 2.56 \text{ m/s} \longrightarrow \blacktriangleleft$

(*b*) $c = 1.059 \text{ m} \blacktriangleleft$

PROBLEM 14.52



For the game of billiards of Problem 14.51, it is now assumed that $v_0 = 5$ m/s, $v_C = 3.2$ m/s, and $c = 1.22$ m. Determine (*a*) the velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ of balls *A* and *B*, (*b*) the Point A' where ball *A* hits the side of the table.

PROBLEM 14.51 In a game of billiards, ball *A* is given an initial velocity $\mathbf{v}_0$ along the longitudinal axis of the table. It hits ball *B* and then ball *C*, which are both at rest. Balls *A* and *C* are observed to hit the sides of the table squarely at A' and C', respectively, and ball *B* is observed to hit the side obliquely at B'. Knowing that $v_0 = 4$ m/s, $v_A = 1.92$ m/s, and $a = 1.65$ m, determine (*a*) the velocities $\mathbf{v}_B$ and $\mathbf{v}_C$ of balls *B* and *C*, (*b*) the Point C' where ball *C* hits the side of the table. Assume frictionless surfaces and perfectly elastic impacts (that is, conservation of energy).

SOLUTION

Velocities in m/s. Lengths in meters. Assume masses are 1.0 for each ball.

Before impacts: $(\mathbf{v}_A)_0 = v_0\mathbf{i} = 5\mathbf{i}, \quad (\mathbf{v}_B)_0 = (\mathbf{v}_C)_0 = 0$

After impacts: $\mathbf{v}_A = -v_A\mathbf{j}, \quad \mathbf{v}_B = (v_B)_x\mathbf{i} + (v_B)_y\mathbf{j}, \quad \mathbf{v}_C = 3.2\mathbf{i}$

Conservation of linear momentum: $\mathbf{v}_0 = \mathbf{v}_A + \mathbf{v}_B + \mathbf{v}_C$

$$\mathbf{i}: \quad 5 = 0 + (v_B)_x + 3.2 \qquad (v_B)_x = 1.8$$

$$\mathbf{j}: \quad 0 = -v_A + (v_B)_y + 0 \qquad (v_B)_y = v_A$$

Conservation of energy: $\frac{1}{2}v_0^2 = \frac{1}{2}v_A^2 + \frac{1}{2}v_B^2 + \frac{1}{2}v_C^2$

$$\frac{1}{2}(5)^2 = \frac{1}{2}(v_A)^2 + \frac{1}{2}(1.8)^2 + \frac{1}{2}(v_A)^2 + \frac{1}{2}(3.2)^2$$

(*a*) $v_A^2 = 11.52 \qquad v_A = 2.4$ $\qquad \mathbf{v}_A = 2.40 \text{ m/s}\downarrow$ ◀

$(v_B)_y = 2.4 \qquad \mathbf{v}_B = 1.8\mathbf{i} + 2.4\mathbf{j}$ $\qquad \mathbf{v}_B = 3.00 \text{ m/s} \measuredangle 53.1°$ ◀

Conservation of angular momentum about B':

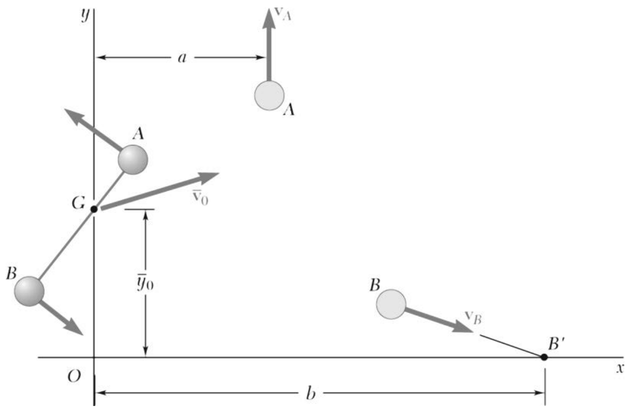
$$0.75v_0 = (1.8 - a)v_A + cv_C$$

$$av_A = 1.8v_A + cv_C - 0.75v_0$$

$$= (1.8)(2.4) + (1.22)(3.2) - (0.75)(5) = 4.474$$

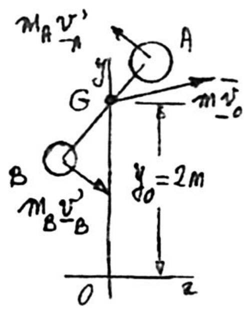
(*b*) $a = \frac{4.474}{v_A} = \frac{4.474}{2.4}$ $\qquad a = 1.864 \text{ m}$ ◀

PROBLEM 14.53



Two small disks A and B, of mass 3 kg and 1.5 kg, respectively, may slide on a horizontal, frictionless surface. They are connected by a cord, 600 mm, long, and spin counterclockwise about their mass center G at the rate of 10 rad/s. At $t = 0$, the coordinates of G are $\bar{x}_0 = 0$, $\bar{y}_0 = 2$ m, and its velocity is $\bar{\mathbf{v}}_0 = (1.2 \text{ m/s})\mathbf{i} + (0.96 \text{ m/s})\mathbf{j}$. Shortly thereafter, the cord breaks; disk A is then observed to move along a path parallel to the y axis and disk B along a path which intersects the x axis at a distance $b = 7.5$ m from O. Determine (a) the velocities of A and B after the cord breaks, (b) the distance a from the y-axis to the path of A.

SOLUTION



Initial conditions.

Location of G:

$$\frac{AG}{m_B} = \frac{BG}{m_A} = \frac{AG + GB}{m_B + m_A} = \frac{AB}{m} = \frac{0.6 \text{ m}}{4.5 \text{ kg}}$$

$$AG = 1.5\left(\frac{0.6}{4.5}\right) = 0.2 \text{ m}$$

$$BG = 0.4 \text{ m}$$

Linear momentum:

$$\mathbf{L}_0 = m\bar{\mathbf{v}}_0 = (4.5 \text{ kg})(1.2\mathbf{i} + 0.96\mathbf{j})$$
$$= 5.4\mathbf{i} + 4.32\mathbf{j}$$

Angular momentum:

About G:

$$(\mathbf{H}_G)_0 = \overrightarrow{GA} \times m_A\mathbf{v}'_A + \overrightarrow{GB} \times m_B\mathbf{v}'_B$$
$$= (0.2 \text{ m})(3 \text{ kg})(0.2 \text{ m} \times 10 \text{ rad/s})\mathbf{k}$$
$$+ (0.4 \text{ m})(1.5 \text{ kg})(0.4 \text{ m} \times 10 \text{ rad/s})\mathbf{k}$$
$$(\mathbf{H}_G)_0 = (3.6 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k}$$

About O: Using formula derived in Problem 14.27,

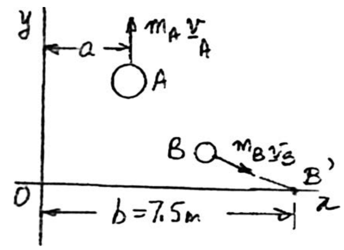
$$(\mathbf{H}_O)_0 = \bar{\mathbf{r}} \times m\bar{\mathbf{v}}_0 + (\mathbf{H}_G)_0$$
$$= 2\mathbf{j} \times (5.4\mathbf{i} + 4.32\mathbf{j}) + 3.6\mathbf{k}$$
$$= -10.8\mathbf{k} + 3.6\mathbf{k} = -(7.2 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k}$$

Kinetic energy: Using Eq. (14.29),

$$T_0 = \frac{1}{2}m\bar{v}_0^2 + \frac{1}{2}\sum_i m: \quad v_i'^2 = \frac{1}{2}m\bar{v}_0^2 + \frac{1}{2}m_A v_A'^2 + \frac{1}{2}m_B v_B'^2$$

$$= \frac{1}{2}(4.5)[(1.2)^2 + (0.96)^2] + \frac{1}{2}(3)(0.2\times 10)^2 + \frac{1}{2}(1.5)(0.4\times 10)^2$$

$$= (5.3136 + 6 + 12) = 23.314 \text{ J}$$



(*a*) Conservation of linear momentum:

$$\mathbf{L}_0 = \mathbf{L}$$

$$5.4\mathbf{i} + 4.32\mathbf{j} = m_A\mathbf{v}_A + m_B\mathbf{v}_B$$

$$= 3(v_A\mathbf{j}) + 1.5[(v_B)_z\mathbf{i} + (v_B)_y\mathbf{j}]$$

Equating coefficients of **i**: $5.4 = 1.5(v_B)_x$

$$(v_B)_x = 3.6 \text{ m/s} \qquad (1)$$

Equating coefficients of **j**: $4.32 = 3v_A + 1.5(v_B)_y$

$$(v_B)_y = 2.88 - 2v_A \qquad (2)$$

Conservation of energy:

$$T_0 = T: \quad T_0 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$23.314 \text{ J} = \frac{1}{2}(3)\left(v_A^2\right) + \frac{1}{2}(1.5)[(v_B)_x^2 + (v_B)_y^2]$$

Substituting from Eqs. (1) and (2):

$$23.314 = 1.5v_A^2 + 0.75(3.6)^2 + 0.75(2.88 - 2v_A)^2$$

$$4.5v_A^2 - 8.64v_A - 7.373 = 0$$

$$v_A^2 - 1.92v_A - 1.6389 = 0$$

$$v_A = 0.96 + 1.60 = 2.56 \text{ m/s}$$

$\mathbf{v}_A = 2.56 \text{ m/s} \uparrow$ ◀

and $v_A = 0.96 - 1.60 = -0.64$ m/s (rejected, since $\mathbf{v}_A$ is shown directed up)

From Eqs. (1) and (2): $(v_B)_x = 3.6$ m/s

$$(v_B)_y = 2.88 - 2(2.56) = -2.24 \text{ m/s}$$

$$\mathbf{v}_B = 3.6\mathbf{i} - 2.24\mathbf{j}$$

$\mathbf{v}_B = 4.24 \text{ m/s} \searrow 31.9°$ ◀

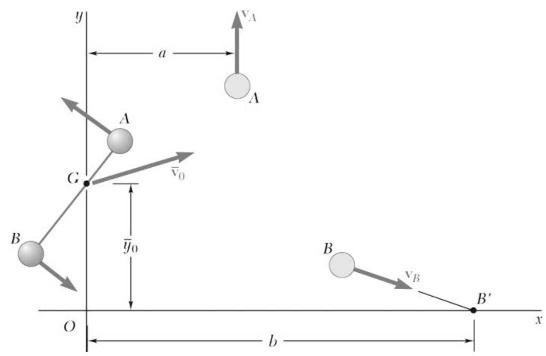
(*b*) Conservation of angular momentum about O:

$$(\mathbf{H}_O)_0 = \mathbf{H}_O: \quad -7.2\mathbf{k} = a\mathbf{i}\times m_A\mathbf{v}_A + b\mathbf{i}\times m_B\mathbf{v}_B$$

$$-7.2\mathbf{k} = a\mathbf{i}\times 3(2.56\mathbf{j}) + 7.5\mathbf{i}\times 1.5(3.6\mathbf{i} - 2.24\mathbf{j})$$

$$-7.2\mathbf{k} = 7.68a\mathbf{k} - 25.2\mathbf{k}$$

$a = 2.34$ m ◀



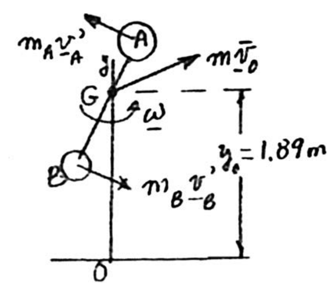
PROBLEM 14.54

Two small disks A and B, of mass 2 kg and 1 kg, respectively, may slide on a horizontal and frictionless surface. They are connected by a cord of negligible mass and spin about their mass center G. At $t = 0$, G is moving with the velocity $\overline{\mathbf{v}}_0$ and its coordinates are $\bar{x}_0 = 0$, $\bar{y}_0 = 1.89$ m. Shortly thereafter, the cord breaks and disk A is observed to move with a velocity $\mathbf{v}_A = (5 \text{ m/s})\mathbf{j}$ in a straight line and at a distance $a = 2.56$ m from the y-axis, while B moves with a velocity $\mathbf{v}_B = (7.2 \text{ m/s})\mathbf{i} - (4.6 \text{ m/s})\mathbf{j}$ along a path intersecting the x-axis at a distance $b = 7.48$ m from the origin O. Determine (*a*) the initial velocity $\overline{\mathbf{v}}_0$ of the mass center G of the two disks, (*b*) the length of the cord initially connecting the two disks, (*c*) the rate in rad/s at which the disks were spinning about G.

SOLUTION

Initial conditions.

Location of G:



$$\frac{AG}{m_B} = \frac{BG}{m_A} = \frac{AG + GB}{m_B + m_A} = \frac{l}{m}$$

$$AG = \frac{m_B}{m} l = \frac{1}{3} l$$

$$BG = \frac{m_A}{m} l = \frac{2}{3} l$$

Linear momentum:

$$\mathbf{L}_0 = m\overline{\mathbf{v}}_0 = 3\overline{\mathbf{v}}_0$$

Angular momentum about G:

$$(\mathbf{H}_G)_0 = \overrightarrow{GA} \times m_A \mathbf{v}'_A + \overrightarrow{GB} \times m_B \mathbf{v}'_B$$

$$= \left(\frac{1}{3} l\right)(2 \text{ kg})\left(\frac{1}{3} l\omega\right)\mathbf{k} + \left(\frac{2}{3} l\right)(1 \text{ kg})\left(\frac{2}{3} l\omega\right)\mathbf{k}$$

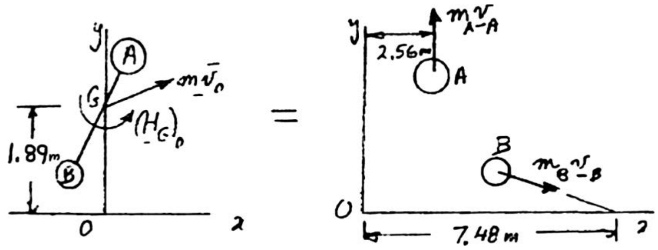
$$= \frac{2}{3} l^2 \omega \mathbf{k}^2$$

Kinetic energy: Using Eq. (14.29),

$$T_0 = \frac{1}{2} m\bar{v}_0^2 + \frac{1}{2}\sum_i m_i v_i^2 = \frac{1}{2} m\bar{v}_0^2 + \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2$$

$$= \frac{1}{2}(3)\bar{v}_0^2 + \frac{1}{2}(2)\left(\frac{1}{3} l\omega\right)^2 + \frac{1}{2}(1)\left(\frac{2}{3} l\omega\right)^2$$

$$T_0 = \frac{3}{2}\bar{v}_0^2 + \frac{1}{3} l^2 \omega^2$$



Conservation of linear momentum:

$$m\bar{\mathbf{v}}_0 = m_A\mathbf{v}_A + m_B\mathbf{v}_B$$
$$3\bar{\mathbf{v}}_0 = (2)(5\mathbf{j}) + (1)(7.2\mathbf{i} - 4.6\mathbf{j}) = 7.2\mathbf{i} + 5.4\mathbf{j}$$

(*a*) $\bar{\mathbf{v}}_0 = (2.4 \text{ m/s})\mathbf{i} + (1.8 \text{ m/s})\mathbf{j}$ ◀

Conservation of angular momentum about *O*:

$$+\circlearrowleft: \quad (1.89\mathbf{j})\times m\bar{\mathbf{v}}_0 + (\mathbf{H}_G)_0 = (2.56\mathbf{i})\times m_A\mathbf{v}_A + (7.48\mathbf{i})\times m_B\mathbf{v}_A$$

Substituting for $\bar{\mathbf{v}}_0$, $(\mathbf{H}_G)_0$, $\mathbf{v}_A$, $\mathbf{v}_B$ and masses:

$$(1.89\mathbf{j})\times 3(2.4\mathbf{i} + 1.8\mathbf{j}) + \frac{2}{3}l^2\omega\mathbf{k} = (2.56\mathbf{i})\times 2(5\mathbf{j}) + (7.48\mathbf{i})\times(7.2\mathbf{i} - 4.6\mathbf{j})$$

$$-13.608\mathbf{k} + \frac{2}{3}l^2\omega\mathbf{k} = 25.6\mathbf{k} - 34.408\mathbf{k}$$

$$\frac{2}{3}l^2\omega = 4.80 \qquad l^2\omega = 7.20 \tag{1}$$

Conservation of energy:

$$T_0 = T: \quad \frac{3}{2}\bar{v}_0^2 + \frac{1}{3}l^2\omega^2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$\frac{3}{2}[(2.4)^2 + (1.8)^2] + \frac{1}{3}l^2\omega^2 = \frac{1}{2}(2)(5)^2 + \frac{1}{2}(1)[(7.2)^2 + (4.6)^2]$$

$$13.5 + \frac{1}{3}l^2\omega^2 = 25 + 36.5 \qquad l^2\omega^2 = 144.0 \tag{2}$$

Dividing Eq. (2) by Eq. (1), member by member:

$$\omega = \frac{144.0}{7.20} = 20.0 \text{ rad/s}$$

(*c*) Original rate of spin = 20.0 rad/s ◀

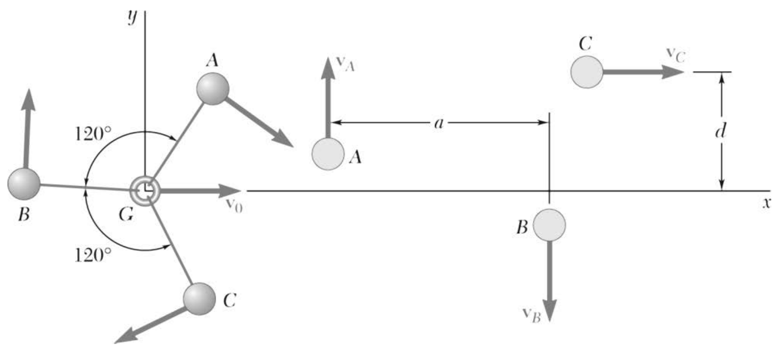
Substituting for ω into Eq. (1):

$$l^2(20.0) = 7.20 \qquad l^2 = 0.360 \qquad l = 0.600 \text{ m}$$

(*b*) Length of cord = 600 mm ◀

PROBLEM 14.55

Three small identical spheres A, B, and C, which can slide on a horizontal, frictionless surface, are attached to three 9-in-long strings, which are tied to a ring G. Initially, the spheres rotate clockwise about the ring with a relative velocity of 2.6 ft/s and the ring moves along the x-axis with a velocity $\mathbf{v}_0 = (1.3 \text{ ft/s})\mathbf{i}$. Suddenly, the ring breaks and the three spheres move freely in the xy plane with A and B, following paths parallel to the y-axis at a distance $a = 1.0$ ft from each other and C following a path parallel to the x-axis. Determine (a) the velocity of each sphere, (b) the distance d.



SOLUTION

Conservation of linear momentum:

Before break: $\mathbf{L}_0 = (3m)\overline{\mathbf{v}} = 3m(1.3\mathbf{i}) = m(3.9 \text{ ft/s})\mathbf{i}$

After break: $\mathbf{L} = mv_A\mathbf{j} - mv_B\mathbf{j} + mv_C\mathbf{i}$

$\mathbf{L} = \mathbf{L}_0$: $mv_C\mathbf{i} + m(v_A - v_B)\mathbf{j} = m(3.9 \text{ ft/s})\mathbf{i}$

Therefore, $v_A = v_B$ (1)

$v_C = 3.9000 \text{ ft/s} \quad \mathbf{v}_C = 3.90 \text{ ft/s} \longrightarrow$ (2)

Conservation of angular momentum:

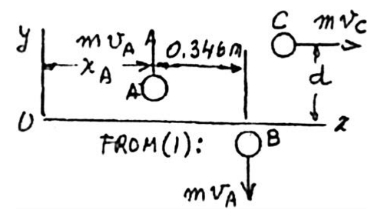
Before break: $+\circlearrowright (H_O)_0 = 3mlv' = 3m(0.75\,\text{ft})(2.6 \text{ ft/s})$

$= 5.85m$

After break: $H_O = -mv_Ax_A$

$+ mv_A(x_A + 0.346)$

$+ mv_Cd$



$+\circlearrowright H_O = -m_Ax_A + mv_A(x_A + 1.0) + mv_Cd$

$H_O = (H_O)_0$: $1.0mv_A + mv_Cd = 5.85m$

Recalling Eq. (2):

$v_A + 3.9d = 5.85$

$d = 1.5 - 0.25641v_A$ (3)

Conservation of energy.

Before break:

$$T_0 = \frac{1}{2}(3m)\bar{v}^2 + 3\left(\frac{1}{2}mv'^2\right)$$
$$= \frac{3}{2}m\left(v_0^2 + v'^2\right) = \frac{3}{2}[(1.3)^2 + (2.6)^2]m = 12.675m$$

After break:

$$T = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$

$T = T_0$: Substituting for v_B from Eq. (1) and v_C from Eq. (2),

$$\frac{1}{2}\left[v_A^2 + v_A^2 + (3.900)^2\right] = 12.675$$
$$v_A^2 = 5.0700$$
$$v_A = v_B = 2.2517 \text{ ft/s}$$

(*a*) *Velocities*:

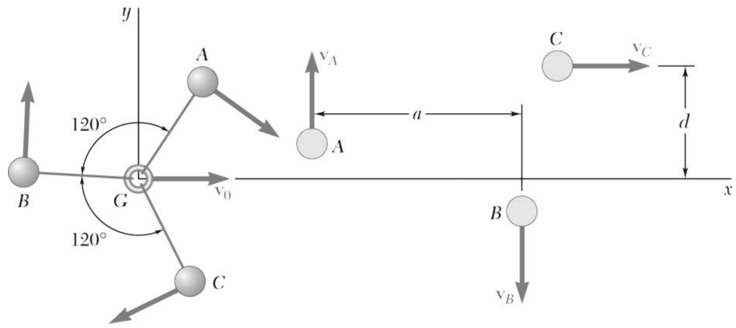
$$\mathbf{v}_A = 2.25 \text{ ft/s} \uparrow; \quad \mathbf{v}_B = 2.25 \text{ ft/s} \downarrow; \quad \mathbf{v}_C = 3.9 \text{ ft/s} \longrightarrow \blacktriangleleft$$

(*b*) *Distance d*:

From Eq. (3): $d = 1.5 - 0.25641(2.2517) = 0.92265$ ft $\qquad d = 11.1$ in. ◀

PROBLEM 14.56

Three small identical spheres A, B, and C, which can slide on a horizontal, frictionless surface, are attached to three strings of length l which are tied to a ring G. Initially, the spheres rotate clockwise about the ring which moves along the x axis with a velocity $\mathbf{v}_0$. Suddenly the ring breaks and the three spheres move freely in the xy plane. Knowing that $\mathbf{v}_A = (3.5 \text{ ft/s})\mathbf{j}$, $\mathbf{v}_C = (6.0 \text{ ft/s})\mathbf{i}$, $a = 16$ in. and $d = 9$ in., determine (a) the initial velocity of the ring, (b) the length l of the strings, (c) the rate in rad/s at which the spheres were rotating about G.



SOLUTION

Conservation of linear momentum:

$$(3m)\overline{\mathbf{v}} = m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C$$
$$3mv_0\mathbf{i} = m(3.5 \text{ ft/s})\mathbf{j} - mv_B\mathbf{j} + m(6.0 \text{ ft/s})\mathbf{i}$$

Equating coefficients of unit vectors:

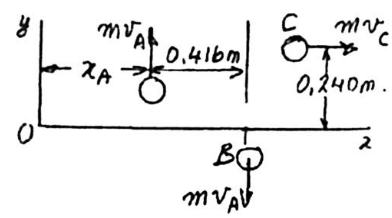
$$3v_0 = 6.00 \text{ ft/s}$$
$$0 = 3.5 \text{ ft/s} - \mathbf{v}_B \qquad v_B = 3.5 \text{ ft/s} \qquad (1)$$

$$v_0 = 2.00 \text{ ft/s} \longrightarrow \blacktriangleleft$$

(a) Conservation of angular momentum:

Before break: $+\circlearrowright (H_O)_0 = 3ml^2\dot{\theta}$

After break:
$$H_O = -mv_Ax_A + mv_A(x_A + 16/12) + mv_C(9/12)$$
$$= m(3.5)(16/12) + m(6.0)(9/12)$$
$$= m(9.1667)$$



$(H_O)_0 = H_0$: $\quad 3ml^2\dot{\theta} = m(9.1667) \qquad l^2\dot{\theta} = 3.0556 \qquad (2)$

PROBLEM 14.56 (Continued)

Conservation of energy:

Before break:

$$T_0 = \frac{1}{2}(3m)\overline{v}^2 + 3\left(\frac{1}{2}mv'^2\right) = \frac{3}{2}mv_0^2 + \frac{3}{2}m(l\dot{\theta})^2$$

$$= \frac{3}{2}m(2.0)^2 + \frac{3}{2}ml^2\dot{\theta}^2$$

After break:

$$T = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$

$$= \frac{1}{2}m[(3.5)^2 + (6.0)^2] = \frac{1}{2}m(60.5)$$

$T = T_0$: $\quad \frac{1}{2}m(60.5) = \frac{3}{2}m(2.0)^2 + \frac{3}{2}ml^2\dot{\theta}^2$

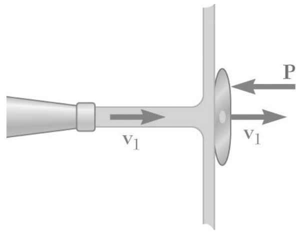
$$l^2\dot{\theta}^2 = 16.167 \qquad (3)$$

Dividing Eq. (3) by Eq. (2): $\quad \dot{\theta} = \frac{16.167}{3.0556} = 5.2909$

(*b*) From Eq. (2): $\quad l^2 = \frac{3.0556}{5.2909} \qquad l = 0.75994 \text{ ft}$

$l = 0.76 \text{ ft}$ ◀

(*c*) Rate of rotation: $\quad \dot{\theta} = 5.29 \text{ rad/s}$ ↻ ◀



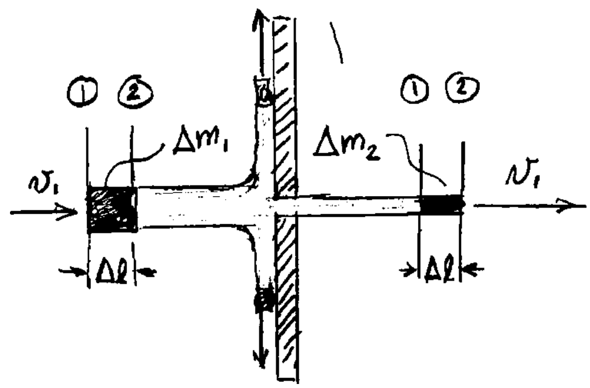
PROBLEM 14.57

A stream of water of cross-sectional area A_1 and velocity $\mathbf{v}_1$ strikes a circular plate which is held motionless by a force **P**. A hole in the circular plate of area A_2 results in a discharge jet having a velocity $\mathbf{v}_1$. Determine the magnitude of **P**.

SOLUTION

<u>Mass flow rates</u>. As the fluid ahead of the plate moves from section 1 to section 2 Δt, the mass Δm_1 moved is

$$\Delta m_1 = \rho A_1(\Delta t) = \rho A_1 v_1(\Delta t)$$



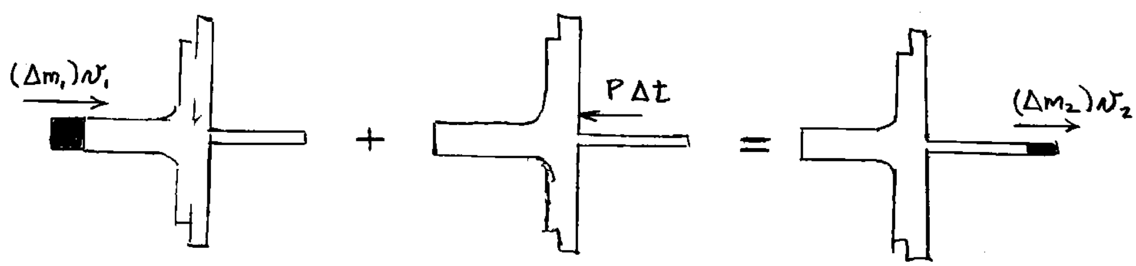
so that
$$\frac{dm_1}{dt} = \frac{\Delta m_1}{\Delta t} = \rho A_1 v_1$$

Likewise, for the fluid that has passed through the hole

$$\Delta m_2 = \rho A_2(\Delta l) = \rho A_2 v_1(\Delta t)$$

so that
$$\frac{dm_2}{dt} = \rho A_2 v_1$$

Apply the impulse-momentum principle.



$$\Sigma m\mathbf{v}_1 + \int \mathbf{F}\, dt = \Sigma m\mathbf{v}_2$$

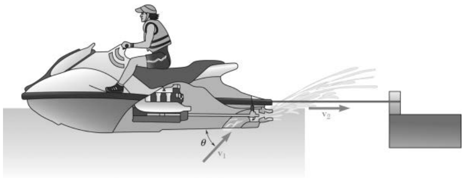
Components in the direction of the flow.

$$(\Delta m_1)v_1 - P\Delta t = (\Delta m_2)v_1$$

$$P = \frac{\Delta m_1}{\Delta t}v_1 - \frac{\Delta m_2}{\Delta t}v_1 = \rho A_1 v_1^2 - \rho A_2 v_1^2$$

$$P = \rho(A_1 - A_2)v_1^2 \blacktriangleleft$$

PROBLEM 14.58



A jet ski is placed in a channel and is tethered so that it is stationary. Water enters the jet ski with velocity $\mathbf{v}_1$ and exits with velocity $\mathbf{v}_2$. Knowing the inlet area is A_1 and the exit area is A_2, determine the tension in the tether.

SOLUTION

Mass flow rates. Consider a cylindrical portion of the fluid lying in a section of pipe of cross sectional area A and length Δl.

The volume and mass are

$$\Delta m = \rho A(\Delta l)$$

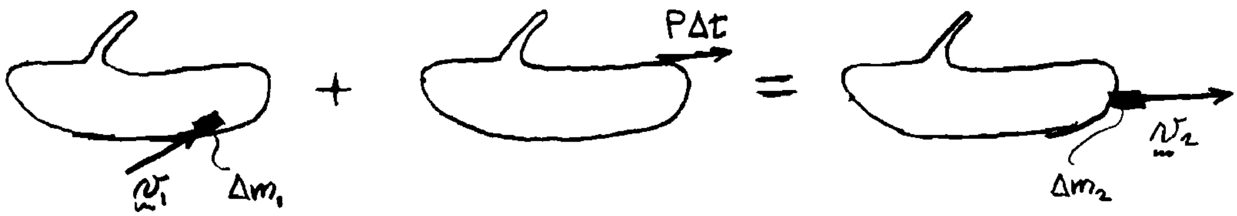
Then

$$\frac{\Delta m}{\Delta t} = \rho A \frac{\Delta l}{\Delta t} = \rho A v$$

At the pipe inlet and outlet, we get

$$\frac{\Delta m_1}{\Delta t} = \rho A_1 v_1, \qquad \frac{\Delta m_2}{\Delta t} = \rho A_2 v_2$$

Impulse and momentum principle:



$$\Sigma m\mathbf{v}_1 + \text{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$

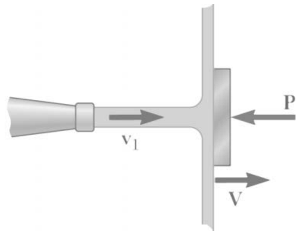
Using horizontal components $(+\rightarrow)$,

$$(\Delta m_1)v_1\cos\theta + P(\Delta t) = (\Delta m_2)v_2$$

$$P = \frac{\Delta m_1}{\Delta t}v_2 - \frac{\Delta m_1}{\Delta t}v_1\cos\theta$$

$$= \rho A_2 v_2^2 - \rho A_1 v_1^2\cos\theta$$

$$P = \rho A_2 v_2^2 - \rho A_1 v_1^2\cos\theta \blacktriangleleft$$

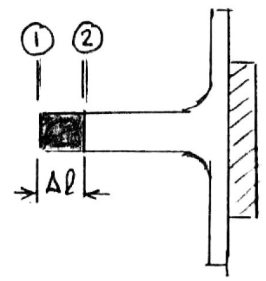


PROBLEM 14.59

A stream of water of cross-sectional area A and velocity $\mathbf{v}_1$ strikes a plate which is held motionless by a force **P**. Determine the magnitude of **P**, knowing that $A = 0.75 \text{ in}^2$, $v_1 = 80$ ft/s, and $V = 0$.

SOLUTION

Mass flow rate. As the fluid moves from section 1 to section 2 in time Δt, the mass Δm moved is

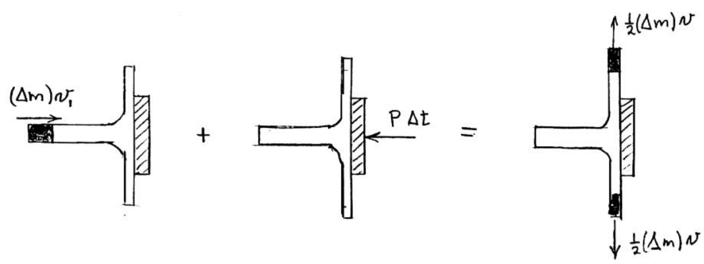


$$\Delta m = \rho A(\Delta l)$$

Then
$$\frac{dm}{dt} = \frac{\Delta m}{\Delta t} = \frac{\rho A(\Delta l)}{\Delta t} = \rho A v_1$$

Data: $\gamma = 62.4 \text{ lb/ft}^3$, $A = 0.75 \text{ in.}^2 = 0.0052083 \text{ ft}^2$, $v_1 = 80$ ft/s

$$\frac{dm}{dt} = \frac{(62.4)}{32.2}(0.0052083)(80) = 0.80745 \text{ slug/s}$$



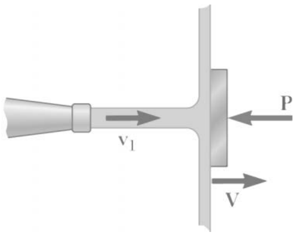
Principle of impulse and momentum:

$$\xrightarrow{+}: \quad (\Delta m)v_1 - P\Delta t = 0$$

$$P = \frac{\Delta m}{\Delta t}v = \frac{dm}{dt}v$$

$$P = (0.80745)(80) = 64.596 \text{ lb}$$

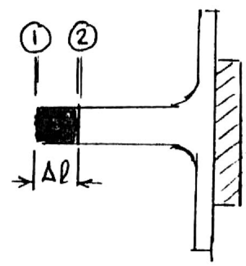
$P = 64.6$ lb ◀



PROBLEM 14.60

A stream of water of cross-sectional area A and velocity $\mathbf{v}_1$ strikes a plate which moves to the right with a velocity $\mathbf{V}$. Determine the magnitude of $\mathbf{V}$, knowing that $A = 1\text{ in}^2$, $v_1 = 100$ ft/s, and $P = 90$ lb.

SOLUTION



Consider velocities measured with respect to the plate, which is moving with velocity V. The velocity of the stream relative to the plate is

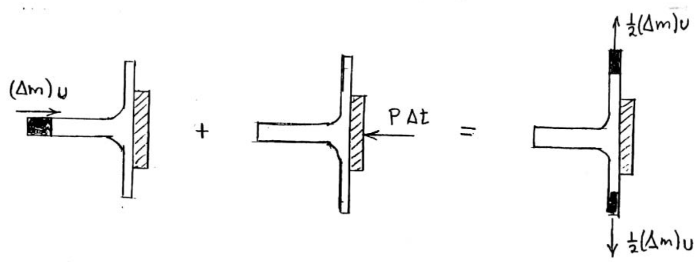
$$\mathbf{u} = \mathbf{v}_1 - \mathbf{V} \qquad (1)$$

Mass flow rate. As the fluid moves from section 1 to section 2 in time Δt, the mass Δm moved is

$$\Delta m = \rho A(\Delta l)$$

Then

$$\frac{dm}{dt} = \frac{\Delta m}{\Delta t} = \frac{\rho A(\Delta l)}{\Delta t} = \rho A u \qquad (2)$$



Principle of impulse and momentum:

$$\xrightarrow{+}\ (\Delta m)u - P(\Delta t) = 0$$

$$P = \frac{\Delta m}{\Delta t}u = \frac{dm}{dt}u = \rho A u^2$$

$$u = \sqrt{\frac{P}{\rho A}}$$

PROBLEM 14.60 (Continued)

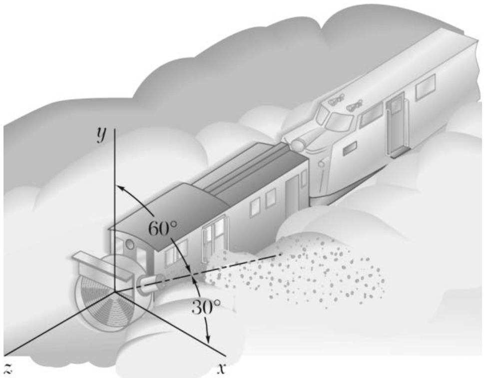
From Eq. (1), $V = v_1 - u = v_1 - \sqrt{\dfrac{P}{\rho A}}$

Data: $P = 90 \text{ lb}, \qquad A = 1 \text{ in}^2 = 0.0069444 \text{ ft}^2$

$V_1 = 100 \text{ ft/s}, \quad \gamma = 62.4 \text{ lb/ft}^3$

$$v = 100 - \sqrt{\frac{90}{(62.4/32.2)(0.0069444)}}$$

$V = 18.2 \text{ ft/s}$ ◀



PROBLEM 14.61

A rotary power plow is used to remove snow from a level section of railroad track. The plow car is placed ahead of an engine which propels it at a constant speed of 20 km/h. The plow car clears 160 Mg of snow per minute, projecting it in the direction shown with a velocity of 12 m/s relative to the plow car. Neglecting friction, determine (*a*) the force exerted by the engine on the plow car, (*b*) the lateral force exerted by the track on the plow.

SOLUTION

Velocity of the plow: $v_P = 20 \text{ km/h} = 5.5556 \text{ m/s}$

Velocity of thrown snow:

$$\mathbf{v}_s = (12 \text{ m/s})(\cos 30° \mathbf{i} + \sin 30° \mathbf{j}) + (5.5556 \text{ m/s})\mathbf{k}$$

Mass flow rate:

$$\frac{dm}{dt} = \frac{(160000 \text{ kg/min})}{(60 \text{ s/min})} = 2666.7 \text{ kg/s}$$

Let F be the force exerted on the plow and the snow.

Apply impulse-momentum, noting that the snow is initially at rest and that the velocity of the plow is constant. Neglect gravity.

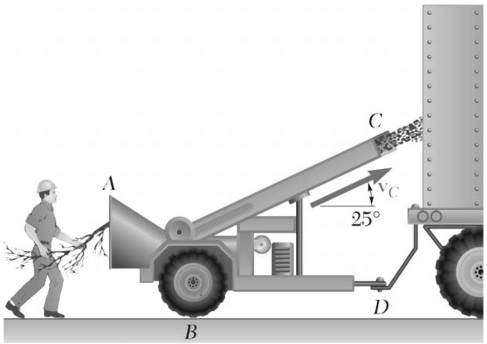
$$\mathbf{F}(\Delta t) = (\Delta m)\mathbf{v}_s$$

$$\mathbf{F} = \left(\frac{dm}{dt}\right)v_s = (2.666.7)(12\cos 30° \mathbf{i} + 12 \sin 30° \mathbf{j} + 5.5556\mathbf{k})$$

$$= (27713 \text{ N})\mathbf{i} + (16000 \text{ N})\mathbf{j} + (14815 \text{ N})\mathbf{k}$$

(*a*) Force exerted by engine. $F_z = 14.8 \text{ kN}$ ◀

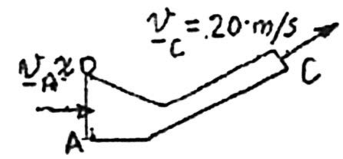
(*b*) Lateral force exerted by track. $F_x = 27.7 \text{ kN}$ ◀



PROBLEM 14.62

Tree limbs and branches are being fed at *A* at the rate of 5 kg/s into a shredder which spews the resulting wood chips at *C* with a velocity of 20 m/s. Determine the horizontal component of the force exerted by the shredder on the truck hitch at *D*.

SOLUTION

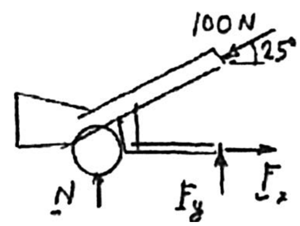


Eq. (14.38):

$$(\Delta m)\,\mathbf{v}_A + \Sigma\mathbf{F}\Delta t = (\Delta m)\,\mathbf{v}_C$$

$$\Sigma\mathbf{F} = \frac{\Delta m}{\Delta t}\mathbf{v}_C = (5 \text{ kg/s})(20 \text{ m/s} \measuredangle 25°)$$

Force exerted on chips $= \Sigma\mathbf{F} = 100 \text{ N} \measuredangle 25°$

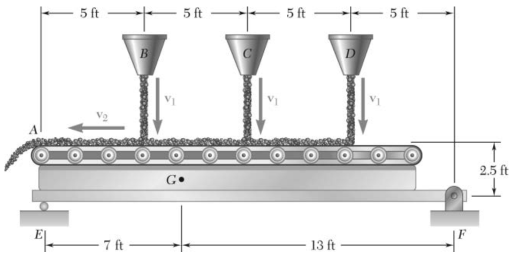


Free body: shredder:

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_x - (100 \text{ N})\cos 25° = 0$$

$$\mathbf{F}_x = 90.6 \text{ N} \longrightarrow$$

On hitch: $\mathbf{F}_x = 90.6 \text{ N} \longleftarrow$ ◀

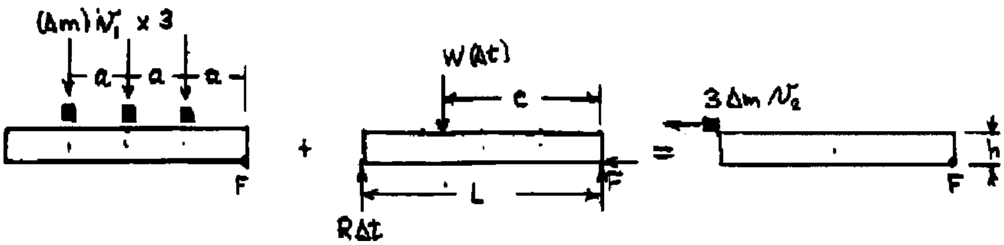


PROBLEM 14.63

Sand falls from three hoppers onto a conveyor belt at a rate of 90 lb/s for each hopper. The sand hits the belt with a vertical velocity v_1 = 10 ft/s and is discharged at A with a horizontal velocity v_2 = 13 ft/s. Knowing that the combined mass of the beam, belt system, and the sand it supports is 1300 lb with a mass center at G, determine the reaction at E.

SOLUTION

Principle of impulse and momentum:



+↻ Moments about F:

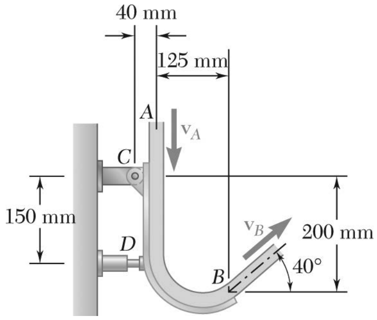
$$(\Delta m)v_1(3a) + \Delta m v_1(2a) + (\Delta m)v_1 a + (W\Delta t)c - (R\Delta t)L = 3(\Delta m)v_2 h$$

$$R = \frac{1}{L}\left[cW + 6av_1\frac{\Delta m}{\Delta t} - 3hv_2\frac{\Delta m}{\Delta t}\right]$$

Data: $L = 20$ ft, $c = 13$ ft, $a = 5$ ft, $h = 2.5$ ft, $\frac{\Delta m}{\Delta t} = \frac{\Delta W/g}{dt} = \frac{1}{g}\frac{dW}{dt} = 2.7950$ slug/s

$$R = \frac{1}{20}\left[13(1300) + (6)(5)(10)(2.7950) - (3)(2.5)(13)(2.7950)\right]$$

R = 873 lb ↑ ◀



PROBLEM 14.64

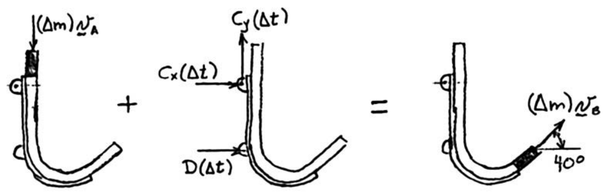
The stream of water shown flows at a rate of 550 liters/min and moves with a velocity of magnitude 18 m/s at both A and B. The vane is supported by a pin and bracket at C and by a load cell at D which can exert only a horizontal force. Neglecting the weight of the vane, determine the components of the reactions at C and D.

SOLUTION

Mass flow rate:
$$\frac{dm}{dt} = \rho Q = \frac{(1000\text{ kg/m}^3)(550\text{ liters/min})(1\text{ min})}{(1000\text{ liters/m}^3)(60\text{ sec})}$$
$$\frac{dm}{dt} = 9.1667\text{ kg/s}$$

Velocity vectors: $\mathbf{v}_A = 18\text{ m/s} \downarrow \qquad \mathbf{v}_B = 18\text{ m/s} \measuredangle 40°$

Apply the impulse-momentum principle.



$+\circlearrowleft$ Moments about C: $-0.040(\Delta m)v_A + 0.150D(\Delta t) = 0.200(\Delta m)v_B\cos 40° + 0.165(\Delta m)v_B\sin 40°$

$$D = \frac{1}{0.150}\left(\frac{\Delta m}{\Delta t}\right)[0.200v_B\cos 40° + 0.165v_B\sin 40° + 0.040v_A]$$
$$= \frac{1}{0.150}(9.1667)[(0.200)(18)\cos 40° + 0.165(18)\sin 40° + 0.040(18)]$$
$$= 329.20\text{ N}$$

$D_x = 329\text{ N}$ ◀

$D_y = 0$ ◀

$\xrightarrow{+}$ x components: $C_x(\Delta t) + D(\Delta t) = (\Delta m)v_B\cos 40°$

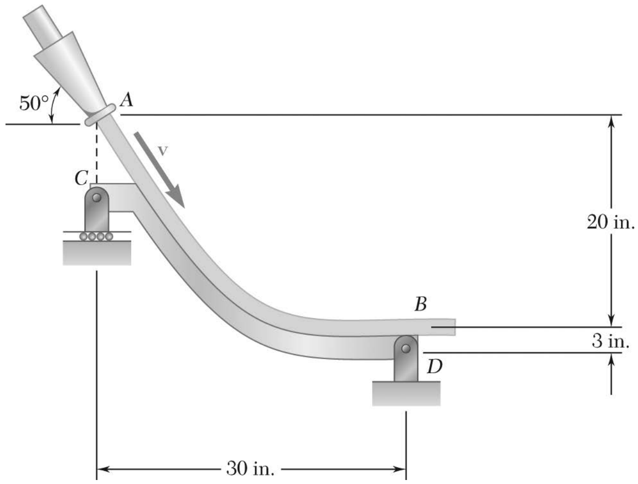
$$C_x = \left(\frac{\Delta m}{\Delta t}\right)v_B\cos 40° - D = (9.1667)(18\cos 40°) - 329.20 = -202.79\text{ N}$$

$C_x = -203\text{ N}$ ◀

$+\uparrow$ y components: $-(\Delta m)v_A + C_y(\Delta t) = (\Delta m)v_B\sin 40°$

$$C_y = \left(\frac{\Delta m}{\Delta t}\right)v_A + \frac{\Delta m}{\Delta t}v_B\sin 40° = (9.1667)(18 + 18\sin 40°) = 271.06\text{ N}$$

$C_y = 271\text{ N}$ ◀



PROBLEM 14.65

The nozzle discharges water at the rate of 340 gal/min. Knowing the velocity of the water at both A and B has a magnitude of 65 ft/s and neglecting the weight of the vane, determine the components of the reactions at C and D. (1 ft^3 = 7.48 gallons)

SOLUTION

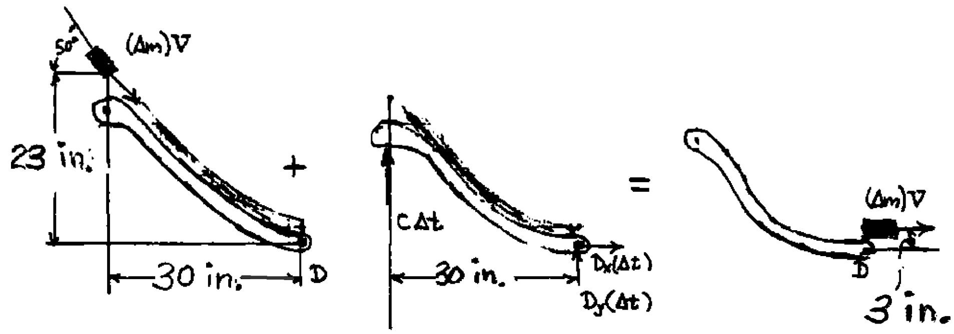
Volumetric flow rate: $Q = 340 \text{ gal/min} \times (1 \text{ ft}^3/7.48 \text{ gal}) \times (1 \text{ min}/60 \text{ sec}) = 0.75758 \text{ ft}^3/\text{s}$

Mass density of water: $\dfrac{\gamma}{g} = \dfrac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}$

Mass flow rate: $\dfrac{dm}{dt} = \dfrac{\gamma}{g}Q = \dfrac{62.4}{32.2}(0.75758) = 1.4681 \text{ lb}\cdot\text{s/ft}$

Assume that the flow speed remains constant.

Principle of impulse and momentum.



↺+ Moments about D:

$$C = \frac{((30/12)\sin 50° - (23/12)\cos 50° + (3/12)}{(30/12)}\frac{\Delta m}{\Delta t}V$$

$$= 0.37324\frac{dm}{dt}V$$

$$= (0.37324)(1.4681 \text{ lb}\cdot\text{s/ft})(65 \text{ ft/s}) = 35.617 \text{ lb}$$

$\mathbf{C} = 35.6 \text{ lb}\uparrow$ ◀

PROBLEM 14.65 (Continued)

$\xrightarrow{+}$ Horizontal components:

$$(\Delta m)V\cos 50° + D_x(\Delta t) = (\Delta m)V$$

$$D_x = (1-\cos 50°)\frac{\Delta m}{\Delta t}V$$

$$= 0.35721\frac{dm}{dt}V$$

$$= (0.35721)(1.4681\text{ lb}\cdot\text{s/ft})(65\text{ ft/s})$$

$$= 34.087\text{ lb}$$

$\mathbf{D}_x = 34.1\text{ N} \longrightarrow$ ◀

$+\uparrow$ Vertical components: $-(\Delta m)V\sin 50° + C(\Delta t) + D_y(\Delta t) = 0$

$$D_y = (\sin 50°)\frac{\Delta m}{\Delta t}V - C$$

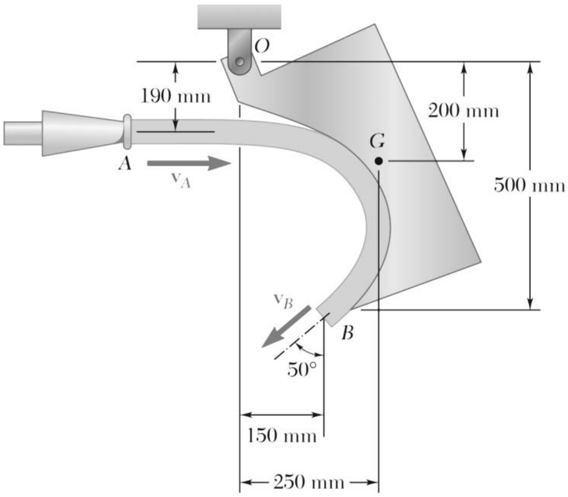
$$= 0.76604\frac{dm}{dt}V - C$$

$$= (0.76604)(1.4681\text{ lb}\cdot\text{s/ft})(65\text{ ft/s}) - 35.617\text{ N}$$

$$= 37.484\text{ lb}$$

$\mathbf{D}_y = 37.5\text{ lb}\uparrow$ ◀

PROBLEM 14.66



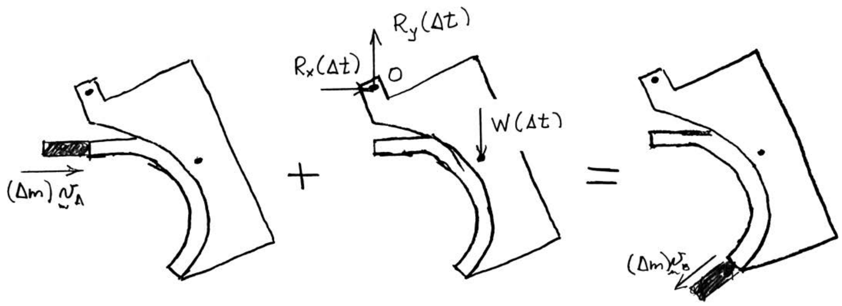
A high speed jet of air issues from the nozzle A with a velocity of $\mathbf{v}_A$ and mass flow rate of 0.36 kg/s. The air impinges on a vane causing it to rotate to the position shown. The vane has a mass of 6-kg. Knowing that the magnitude of the air velocity is equal at A and B determine (a) the magnitude of the velocity at A, (b) the components of the reactions at O.

SOLUTION

Assume that the speed of the air jet is the same at A and B.

$$v_A = v_B = v$$

Apply the principle of impulse and momentum.



(a) $\quad$ +↻ Moments about O: $\quad (0.190)(\Delta m)v - (0.250)W(\Delta t) = -(0.250)(\Delta m)v\cos 50° - (0.500)(\Delta m)v\sin 50°$

$$v = \frac{\Delta t}{\Delta m} \cdot \frac{0.250W}{0.150\cos 50° + 0.500\sin 50° + 0.190}$$

$$= \frac{0.250W}{0.66944}\frac{dm}{dt}$$

$$= \frac{(0.250)(6)(9.81)}{(0.66944)(0.36)}$$

$$= 61.058 \text{ m/s}$$

$v_A = 61.1$ m/s ◀

PROBLEM 14.66 (Continued)

(*b*) $\xrightarrow{+}$ x components: $(\Delta m)v + R_x(\Delta t) = -(\Delta m)v \sin 50°$

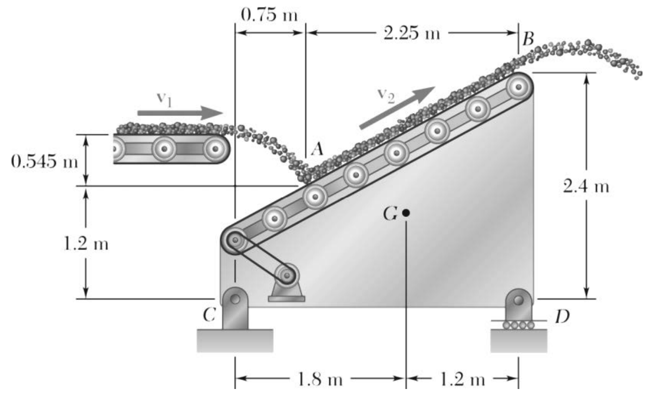
$$R_x = -\frac{\Delta m}{\Delta t} v(1 + \sin 50°)$$
$$= -(0.36)(61.058)(1 + \sin 50°)$$
$$= -38.82 \text{ N}$$

$+\uparrow$ y components: $0 + R_y(\Delta t) - W(\Delta t) = -(\Delta m)v \cos 50°$

$$R_y = W + \frac{\Delta m}{\Delta t} v \cos 50°$$
$$= (6)(9.81) - (0.36)(61.058)\cos 50°$$
$$= 44.73 \text{ N}$$

$$R = \sqrt{(38.82)^2 + (44.73)^2}$$
$$= 59.2 \text{ N}$$

$\tan\alpha = \dfrac{44.73}{38.82}$ $\alpha = 49.0°$ $\mathbf{R} = 59.2 \text{ N} \measuredangle 49.0°$ ◀



PROBLEM 14.67

Coal is being discharged from a first conveyor belt at the rate of 120 kg/s. It is received at A by a second belt which discharges it again at B. Knowing that $v_1 = 3$ m/s and $v_2 = 4.25$ m/s and that the second belt assembly and the coal it supports have a total mass of 472 kg, determine the components of the reactions at C and D.

SOLUTION

Velocity before impact at A:

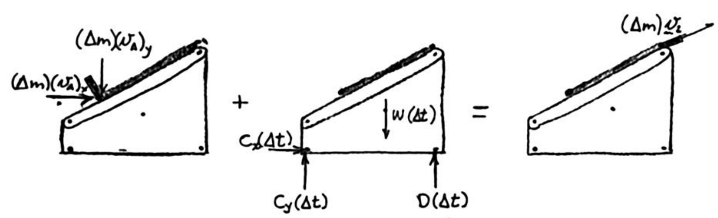
$$(v_A)_x = v_1 = 3 \text{ m/s} \longrightarrow$$

$$(v_A)_y^2 = 2g(\Delta y) = (2)(9.81)(0.545) = 10.693 \text{ m}^2/\text{s}^2 \qquad (v_A)_y = 3.270 \text{ m/s} \downarrow$$

Slope of belt: $\tan\theta = \dfrac{2.4 - 1.2}{2.25}, \qquad \theta = 28.07°$

Velocity of coal leaving at B: $\mathbf{v}_2 = 4.25\,(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$

Apply the impulse-momentum principle.



$\xrightarrow{+}$ x components: $(\Delta m)(v_A)_x + C_x(\Delta t) = (\Delta m)v_2\cos\theta$

$$C_x = \frac{\Delta m}{\Delta t}[v_2\cos\theta - (v_A)_x] = (120)(4.25\cos 28.07° - 3)$$

$$C_x = 90.0 \text{ N} \longrightarrow$$

$+\circlearrowleft$ moments about C: $(\Delta m)[-1.2(v_A)_x - 0.75(v_A)_y] + 3.00D(\Delta t) - 1.8W(\Delta t) = (\Delta m)[-2.4\ v_2\cos\theta + 3v_2\sin\theta]$

$$\frac{\Delta m}{\Delta t}[-(1.2)(3) - (0.75)(3.270)] + 3D - (1.8)(472)(9.81)$$

$$= \frac{\Delta m}{\Delta t}[-(2.4)(4.25\cos\theta) + (3)(4.25\sin\theta)]$$

$$D = 2775 + 1.0168\frac{dm}{dt} = 2775 + (1.0168)(120) = 2897 \text{ N} \uparrow$$

PROBLEM 14.67 (Continued)

$+\uparrow$ y components: $(\Delta m)(-v_A)_y + (C_y + D - W)(\Delta t) = (\Delta m)v_2 \sin\theta$

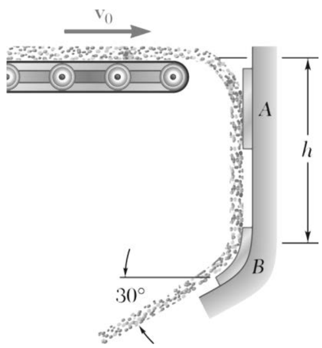
$$C_y + D - W = \frac{\Delta m}{\Delta t}(3.270 + 4.25\sin\theta)$$

$$= (120)(5.268) = 632.2 \text{ N}$$

$$C_y = 4625.6 - 2897 + 632.2 \qquad C_y = 2361 \text{ N}\uparrow$$

$$C_x = 90.0 \text{ N}, \quad C_y = 2360 \text{ N} \blacktriangleleft$$

$$D_x = 0, \qquad D_y = 2900 \text{ N} \blacktriangleleft$$

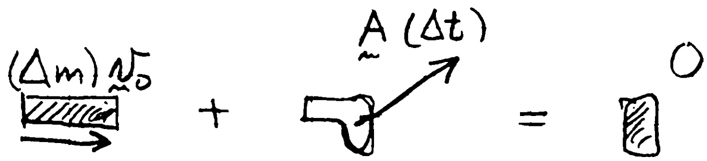


PROBLEM 14.68

A mass q of sand is discharged per unit time from a conveyor belt moving with a velocity $\mathbf{v}_0$. The sand is deflected by a plate at A so that it falls in a vertical stream. After falling a distance h the sand is again deflected by a curved plate at B. Neglecting the friction between the sand and the plates, determine the force required to hold in the position shown (a) plate A, (b) plate B.

SOLUTION

(a) When the sand impacts on plate A, it is momentarily brought to rest. Apply the principle of impulse and momentum to find the force on the sand.



$\xrightarrow{+}$ x component: $(\Delta m)v_0 + A_x(\Delta t) = 0$

$$A_x = -\frac{\Delta m}{\Delta t}v_0 = -qv_0$$

$+\uparrow$ y component: $0 + A_y(\Delta t) = 0 \qquad A_y = 0$

$$\mathbf{A} = qv_0 \longleftarrow \blacktriangleleft$$

The sand falls vertically. Use conservation of energy for mass element Δm. Let v be the speed at the curved portion of plate B.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + (\Delta m)gh = \frac{1}{2}(\Delta m)v^2 + 0$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

Over the curved portion of plate B, there is negligible change of elevation. Hence, by conservation of energy, v is both the entrance speed and exit speed of the curved portion of plate B.

(b) Force exerted through plate B:

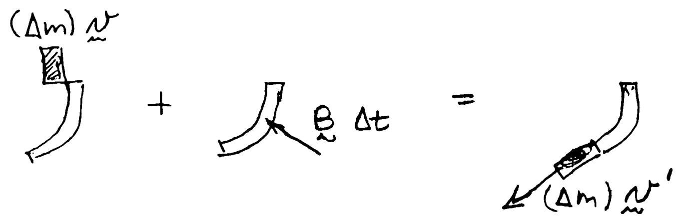
Entrance velocity: $\mathbf{v} = -\sqrt{2gh}\,\mathbf{j}$

Exit velocity: $\mathbf{v}' = \sqrt{2gh}\,(-\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$

Mass flow rate: $\dfrac{dm}{dt} = \dfrac{\Delta m}{\Delta t} = q$

PROBLEM 14.68 (Continued)

Principle of impulse and momentum:



$$(\Delta m)\mathbf{v} + \mathbf{B}(\Delta t) = (\Delta m)\mathbf{v}'$$

$$\mathbf{B} = \left(\frac{\Delta m}{\Delta t}\right)(\mathbf{v}' - \mathbf{v}) = q\sqrt{2gh}(-\cos 30°\mathbf{i} - \sin 30°\mathbf{j} + \mathbf{j})$$

$$B_x = -\sqrt{2gh}\cos 30° = -\frac{\sqrt{3}}{2}\sqrt{2gh}$$

$$B_y = \sqrt{2gh}(1 - \sin 30°) = \frac{1}{2}\sqrt{2gh}$$

$\mathbf{B} = \sqrt{2gh}$ ⦪ 30° ◀

PROBLEM 14.69

The total drag due to air friction on a jet airplane traveling at 900 km/h is 35 kN. Knowing that the exhaust velocity is 600 m/s relative to the airplane, determine the mass of air which must pass through the engine per second to maintain the speed of 900 km/h in level flight.

SOLUTION

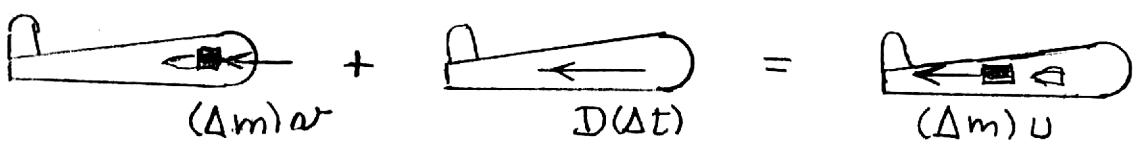
Symbols: $\frac{dm}{dt}$ = mass flow rate

u = exhaust relative to the airplane

v = speed of airplane

D = drag force

Principle of impulse and momentum:



$\overset{+}{\leftarrow}$ $(\Delta m)v + D(\Delta t) = (\Delta m)u$

$$\frac{\Delta m}{\Delta t} = \frac{dm}{dt} = \frac{D}{u - v}$$

Data: $v = 900$ km/h $= 250$ m/s

$u = 600$ m/s

$D = 35$ kN $= 35000$ N

$$\frac{dm}{dt} = \frac{35000}{600 - 250}$$

$\frac{dm}{dt} = 100$ kg/s ◀

PROBLEM 14.70

While cruising in level flight at a speed of 600 mi/h, a jet plane scoops in air at the rate of 200 lb/s and discharges it with a velocity of 2100 ft/s relative to the airplane. Determine the total drag due to air friction on the airplane.

SOLUTION

Flight speed: $v = 600 \text{ mi/h} = 880 \text{ ft/s}$

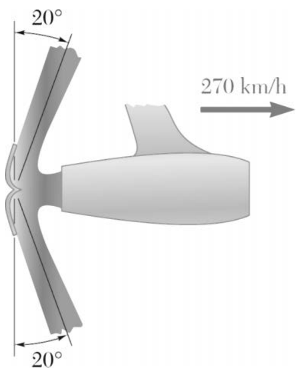
Mass flow rate: $\dfrac{dm}{dt} = \dfrac{200 \text{ lb/s}}{32.2 \text{ ft/s}^2} = 6.2112 \text{ slug/s}$

$$\Sigma \mathbf{F} = \mathbf{D} = \frac{dm}{dt}(\mathbf{u} - \mathbf{v}) \qquad \text{or} \qquad D = \frac{dm}{dt}(v - u)$$

where, for a frame of reference moving with the plane, v is the free stream velocity (equal to the air speed) and u is the relative exhaust velocity.

$$D = (6.2112)(2100 - 880) = 7577.6 \text{ lb}$$

$D = 7580 \text{ lb}$ ◀



PROBLEM 14.71

In order to shorten the distance required for landing, a jet airplane is equipped with movable vanes, which partially reverse the direction of the air discharged by each of its engines. Each engine scoops in the air at a rate of 120 kg/s and discharges it with a velocity of 600 m/s relative to the engine. At an instant when the speed of the airplane is 270 km/h, determine the reverse thrust provided by each of the engines.

SOLUTION

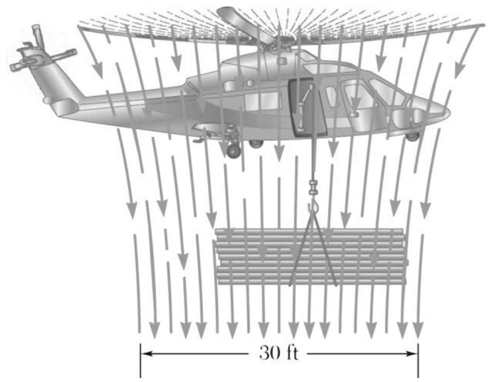
Apply the impulse-momentum principle to the moving air. Use a frame of reference that is moving with the airplane. Let **F** be the force on the air.

$$v = 270 \text{ km/h} = 75 \text{ m/s}$$
$$u = 600 \text{ m/s}$$

$$-(\Delta m)v + F(\Delta t) = 2\frac{(\Delta m)}{2}u\sin 20°$$

$$F = \frac{\Delta m}{\Delta t}(v + u\sin 20°) = \frac{dm}{dt}(v + u\sin 20°)$$
$$F = (120)(75 + 600\sin 20°) = 33.6\times 10^3 \text{ N}$$

Force on airplane is $-\mathbf{F}$. $\mathbf{F} = 33.6 \text{ kN} \longleftarrow$ ◀



PROBLEM 14.72

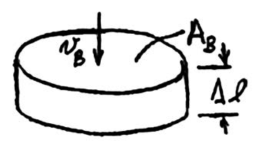
The helicopter shown can produce a maximum downward air speed of 80 ft/s in a 30-ft-diameter slipstream. Knowing that the weight of the helicopter and its crew is 3500 lb and assuming $\gamma = 0.076 \text{ lb/ft}^3$ for air, determine the maximum load that the helicopter can lift while hovering in midair.

SOLUTION

The thrust is

$$F = \frac{dm}{dt}(v_B - v_A)$$

Calculation of $\frac{dm}{dt}$.   mass = density × volume = density × area × length



$$\Delta m = \rho A_B(\Delta l) = \rho A_B v_B(\Delta t)$$

$$\frac{\Delta m}{\Delta t} = \rho A_B v_B = \frac{\gamma}{g} A_B v_B = \frac{dm}{dt}$$

where A_B is the area of the slipstream well below the helicopter and v_B is the corresponding velocity in the slipstream. Well above the blade, $v_A \approx 0$.

Hence,

$$F = \frac{\gamma}{g} A_B v_B^2$$

$$= \left(\frac{0.076 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}\right)\left(\frac{\pi}{4}\right)(30 \text{ ft})^2(80 \text{ ft/s})^2$$

$$= 10,678 \text{ lb}$$

$$F = 10,678 \text{ lb} \downarrow$$

The force on the helicopter is 10,678 lb $\uparrow$.

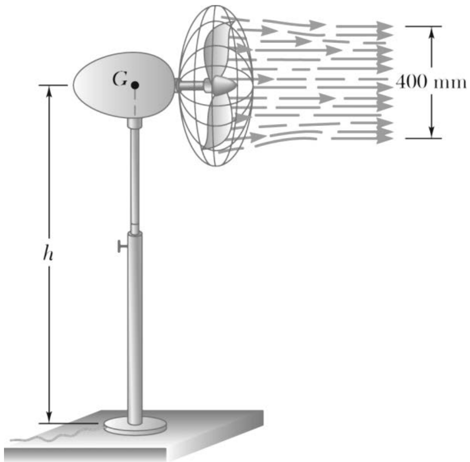
Weight of helicopter: $\mathbf{W}_H = 3500 \text{ lb} \downarrow$

Weight of payload: $\mathbf{W}_P = W_P \downarrow$

Statics: $+\uparrow \Sigma F_y = F - W_H - W_P = 0$

$$W_P = F - W_H = 10,678 - 3500 = 7178 \text{ lb}$$

$W = 7180$ lb ◀

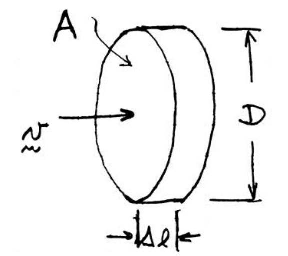


PROBLEM 14.73

A floor fan designed to deliver air at a maximum velocity of 6 m/s in a 400-mm-diameter slipstream is supported by a 200-mm-diameter circular base plate. Knowing that the total weight of the assembly is 60 N and that its center of gravity is located directly above the center of the base plate, determine the maximum height h at which the fan may be operated if it is not to tip over. Assume $\rho = 1.21\ \text{kg/m}^3$ for air and neglect the approach velocity of the air.

SOLUTION

Calculation of $\frac{dm}{dt}$ at a section in the airstream:



$$\text{mass} = \text{density} \times \text{volume} = \text{density} \times \text{area} \times \text{length}$$

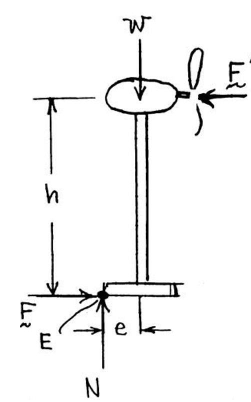
$$\Delta m = \rho A(\Delta l) = \rho A v(\Delta t)$$

$$\frac{\Delta m}{\Delta t} = \frac{dm}{dt} = \rho A v$$

Thrust on the airstream:

$$\mathbf{F} = \frac{dm}{dt}(\mathbf{v}_B - \mathbf{v}_A)$$

where $\mathbf{v}_B$ is the velocity just downstream of the fan and $\mathbf{v}_A$ is the velocity for upstream. Assume that $\mathbf{v}_A$ is negligible.



$$F = (\rho A v)v = \rho\left(\frac{\pi}{4}D^2\right)v^2$$

$$F = (1.21\ \text{kg/m}^3)\left(\frac{\pi}{4}\right)(0.400\ \text{m})^2(6\ \text{m/s})^2 = 5.474\ \text{N}$$

$$\mathbf{F} = 5.474\ \text{N} \longrightarrow$$

Force on fan: $\mathbf{F}' = -\mathbf{F} = 5.474\ \text{N} \longleftarrow$

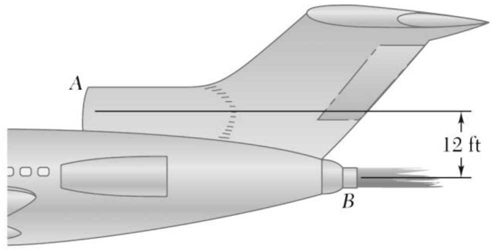
Maximum height h: $\left(e = \frac{1}{2}d = 100\ \text{mm} = 0.1\ \text{m}\right)$

$$+\circlearrowleft \Sigma M_E = 0$$

$$F'h - We = 0$$

$$h = \frac{We}{F'} = \frac{(60)(0.1)}{5.474}$$

$h = 1.096\ \text{m}$ ◀

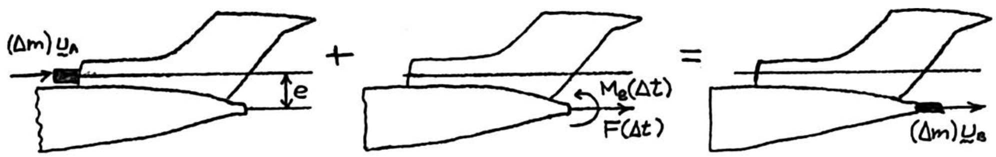


PROBLEM 14.74

The jet engine shown scoops in air at A at a rate of 200 lb/s and discharges it at B with a velocity of 2000 ft/s relative to the airplane. Determine the magnitude and line of action of the propulsive thrust developed by the engine when the speed of the airplane is (*a*) 300 mi/h, (*b*) 600 mi/h.

SOLUTION

Use a frame of reference moving with the plane. Apply the impulse-momentum principle. Let F be the force that the plane exerts on the air.



$\xrightarrow{+}$ x components: $$(\Delta m)u_A + F(\Delta t) = (\Delta m)u_B$$

$$F = \frac{\Delta m}{\Delta t}(u_B - u_A) = \frac{dm}{dt}(u_B - u_A) \qquad (1)$$

$+\circlearrowleft$ moments about B: $$-e(\Delta m)u_A + M_B(\Delta t) = 0$$

$$M_B = e\frac{dm}{dt}u_A \qquad (2)$$

Let d be the distance that the line of action is below B.

$$Fd = M_B \qquad d = \frac{M_B}{F} = \frac{eu_A}{u_B - u_A} \qquad (3)$$

Data: $$\frac{dm}{dt} = 200 \text{ lb/s} = \frac{200}{32.2} = 6.2112 \text{ slugs/s}, \quad u_B = 2000 \text{ ft/s}, \quad e = 12 \text{ ft}$$

(*a*) $$u_A = 300 \text{ mi/h} = 440 \text{ ft/s}$$

From Eq. (1), $$F = (6.2112)(2000 - 440)$$ $F = 9690$ lb ◀

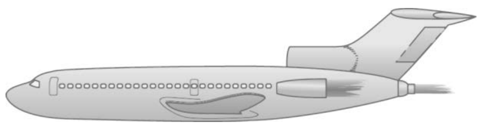
From Eq. (3), $$d = \frac{(12)(440)}{2000 - 440}$$ $d = 3.38$ ft ◀

(*b*) $$u_A = 600 \text{ mi/h} = 880 \text{ ft/s}$$

From Eq. (1), $$F = (6.2112)(2000 - 880)$$ $F = 6960$ lb ◀

From Eq. (3), $$d = \frac{(12)(880)}{2000 - 880}$$ $d = 9.43$ ft ◀

PROBLEM 14.75



A jet airliner is cruising at a speed of 900 km/h with each of its three engines discharging air with a velocity of 800 m/s relative to the plane. Determine the speed of the airliner after it has lost the use of (*a*) one of its engines, (*b*) two of its engines. Assume that the drag due to air friction is proportional to the square of the speed and that the remaining engines keep operating at the same rate.

SOLUTION

Let v be the airliner speed and u be the discharge relative velocity.

$$u = 800 \text{ m/s.}$$

Thrust formula for one engine: $$F = \frac{dm}{dt}(u - v)$$

Drag formula: $$D = kv^2$$

Three engines working. Cruising speed $= v_0 = 900$ km/h $= 250$ m/s

$$3F - D = 3\frac{dm}{dt}(u - v_0) - kv_0^2 = 0$$

$$\frac{dm}{dt} = \frac{kv_0^2}{3(u - v_0)} = \frac{k(250)^2}{3(800 - 250)} = 37.879k$$

(*a*) *One engine fails*. Two engines working. Cruising speed $= v_1$

$$2F - D = 2\frac{dm}{dt}(u - v_1) - kv_1^2 = 0$$

$$(2)(37.879k)(800 - v_1) - kv_1^2 = 0$$

$$v_1^2 + 75.758v_1 - 60.606 \times 10^3 = 0$$

$v_1 = 211.20$ m/s $\qquad v_1 = 760$ km/h ◀

(*b*) *Two engines fail*. One engine working. Cruising speed $= v_2$

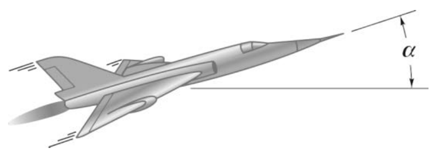
$$F - D = \frac{dm}{dt}(u - v_2) - kv_2^2 = 0$$

$$(37.879k)(800 - v_2) - kv_2^2 = 0$$

$$v_2^2 + 37.879v_2 - 30.303 \times 10^3 = 0$$

$v_2 = 156.17$ m/s $\qquad v_2 = 562$ km/h ◀

PROBLEM 14.76



A 16-Mg jet airplane maintains a constant speed of 774 km/h while climbing at an angle $\alpha = 18°$. The airplane scoops in air at a rate of 300 kg/s and discharges it with a velocity of 665 m/s relative to the airplane. If the pilot changes to a horizontal flight while maintaining the same engine setting, determine (*a*) the initial acceleration of the plane, (*b*) the maximum horizontal speed that will be attained. Assume that the drag due to air friction is proportional to the square of the speed.

SOLUTION

Calculate the propulsive force using velocities relative to the airplane.

$$F = \frac{dm}{dt}(v_B - v_A)$$

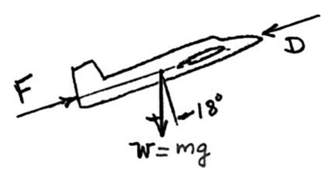
Data:

$$\frac{dm}{dt} = 300 \text{ kg/s}$$

$$v_A = 774 \text{ km/h} = 215 \text{ m/s}$$

$$v_B = 665 \text{ m/s}$$

$$F = (300)(665 - 215) = 135{,}000 \text{ N}$$



Since there is no acceleration while the airplane is climbing, the forces are in equilibrium.

$$+\measuredangle 18° \Sigma F = 0: \quad F - D - mg\sin\alpha = 0$$

$$F - D = mg\sin\alpha = (16{,}000)(9.8)\sin 18° = 48{,}454 \text{ N}$$

(*a*) *Initial acceleration of airplane in horizontal flight*:

$$ma = F - D: \quad 16{,}000a = 48.454 \times 10^3 \qquad \mathbf{a} = 3.03 \text{ m/s}^2 \measuredangle 18° ◀$$

Corresponding drag force:

$$D = 135{,}000 - 48{,}454 = 86{,}546 \text{ N}$$

Drag force factor:

$$D = kv_A^2$$

or

$$k = \frac{D}{v_A^2} = \frac{86{,}546}{(215)^2} = 1.87228 \text{ N}\cdot\text{s}^2/\text{m}^2$$

PROBLEM 14.76 (Continued)

(*b*) *Maximum speed in horizontal flight*:

Since the acceleration is zero, the forces are in equilibrium.

$$F - D = 0$$

$$\frac{dm}{dt}(v_B - v_A) - kv_A^2 = 0 \quad kv_A^2 + \frac{dm}{dt}v_A - \frac{dm}{dt}v_B = 0$$

$$1.87228v_A^2 + 300v_A - (300)(665) = 0$$

$$v_A = 256.0 \text{ m/s}$$

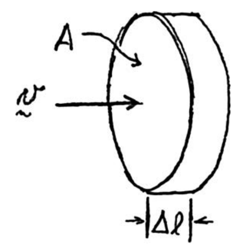
$v_A = 922$ km/h ◀

PROBLEM 14.77

The propeller of a small airplane has a 2-m-diameter slipstream and produces a thrust of 3600 N when the airplane is at rest on the ground. Assuming $\rho = 1.225 \text{ kg/m}^3$ for air, determine (*a*) the speed of the air in the slipstream, (*b*) the volume of air passing through the propeller per second, (*c*) the kinetic energy imparted per second to the air in the slipstream.

SOLUTION

Calculation of $\frac{dm}{dt}$ at a section in the airstream:



$$\text{mass} = \text{density} \times \text{volume}$$
$$= \text{density} \times \text{area} \times \text{length}$$
$$\Delta m = \rho A(\Delta l) = \rho A v\, \Delta t$$
$$\frac{\Delta m}{\Delta t} = \frac{dm}{dt} = \rho A v$$

(*a*) Thrust $= \frac{dm}{dt}(\mathbf{v}_B - \mathbf{v}_A)$ where $\mathbf{v}_B$ is the velocity just downstream of propeller and $\mathbf{v}_A$ is the velocity far upstream. Assume $\mathbf{v}_A$ is negligible.

$$\text{Thrust} = (\rho A v)v = \rho\left(\frac{\pi}{4}D^2\right)v^2$$
$$3600 = 1.225\left(\frac{\pi}{4}\right)(2)^2 v^2$$
$$v^2 = 935.44$$
$$v = 30.585 \text{ m/s}$$

$v = 30.6 \text{ m/s}$ ◀

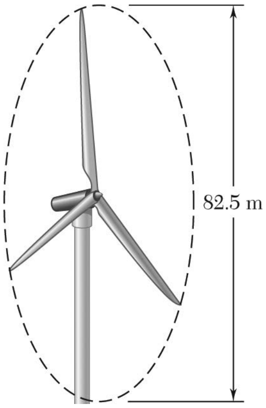
(*b*) $$Q = \frac{1}{\rho}\frac{dm}{dt} = Av = \left(\frac{\pi}{4}D^2\right)v = \frac{\pi}{4}(2)^2(30.585) = 96.086$$

$Q = 96.1 \text{ m}^3\text{/s}$ ◀

(*c*) Kinetic energy of mass Δm:

$$\Delta T = \frac{1}{2}(\Delta m)v^2 = \frac{1}{2}\rho A(\Delta l)v^2 = \frac{1}{2}\rho A\, v(\Delta t)v^3$$
$$\frac{\Delta T}{\Delta t} = \frac{dT}{dt}$$
$$= \frac{1}{2}\rho A v^3$$
$$= \frac{1}{2}\rho\left(\frac{\pi}{4}D^2\right)v^3$$
$$= \frac{1}{2}(1.225)\left(\frac{\pi}{4}\right)(2)^2(30.585)^3$$
$$= 55{,}053 \text{ N}\cdot\text{m/s}$$

$\frac{dT}{dt} = 55{,}100 \text{ N}\cdot\text{m/s}$ ◀



PROBLEM 14.78

The wind turbine-generator shown has an output-power rating of 1.5 MW for a wind speed of 36 km/h. For the given wind speed, determine (*a*) the kinetic energy of the air particles entering the 82.5-m-diameter circle per second, (*b*) the efficiency of this energy conversion system. Assume $\rho = 1.21\ \text{kg/m}^3$ for air.

SOLUTION

(*a*) Rate of kinetic energy in the slipstream.

Let Δm be the mass moving through the slipstream of area A in the time Δt. Then,

$$\Delta m = \rho A(\Delta l) = \rho A v(\Delta t)$$

The kinetic energy carried by this mass is

$$\Delta t = \frac{1}{2}(\Delta m)v^2 = \frac{1}{2}\rho A v^3(\Delta t)$$

$$\frac{dT}{dt} = \frac{\Delta T}{\Delta t} = \frac{1}{2}\rho A v^3$$

Data:

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(82.5\ \text{m})^2 = 5345.6\ \text{m}^2$$

$$v = 36\ \text{km/h} = 10\ \text{m/s}$$

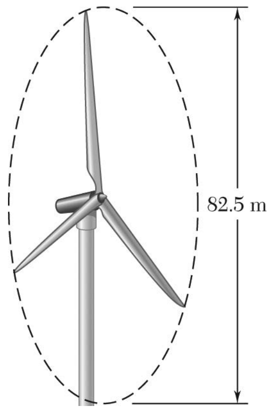
$$\frac{dT}{dt} = \frac{1}{2}(1.21\ \text{kg/m}^3)(5345.6\ \text{m}^2)(10\ \text{m/s})^3$$

$$= 3.234 \times 10^6\ \text{kg} \cdot \text{m}^2/\text{s}^3$$

$$\frac{dT}{dt} = 3.234\ \text{MW} \ ◀$$

(*b*) Efficiency *n*:

$$\eta = \frac{\text{output power}}{\text{available input power}} = \frac{1.5\ \text{MW}}{3.234\ \text{MW}} \qquad \eta = 0.464 \ ◀$$



PROBLEM 14.79

A wind turbine-generator system having a diameter of 82.5 m produces 1.5 MW at a wind speed of 12 m/s. Determine the diameter of blade necessary to produce 10 MW of power assuming the efficiency is the same for both designs and $\rho = 1.21\ \text{kg/m}^3$ for air.

SOLUTION

Rate of kinetic energy in the slipstream.

Let Δm be the mass moving through the slipstream of area A in time Δt. Then

$$\Delta m = \rho A(\Delta l) = \rho A v(\Delta t)$$

The kinetic energy carried by this mass is

$$\Delta T = \frac{1}{2}(\Delta m)v^2 = \frac{1}{2}\rho A v^3(\Delta t)$$

$$\frac{dT}{dt} = \frac{\Delta T}{\Delta t} = \frac{1}{2}\rho A v^3$$

This is the available input power for the wind turbine. For a wind turbine of efficiency η, the output power P is

$$P = \eta\frac{dT}{dt} = \frac{\eta}{2}\rho A v^3$$

We want to compare two turbines having $P_1 = 1.5$ MW and $P_2 = 10$ MW, respectively. Then

$$\frac{P_2}{P_1} = \frac{\eta_2\rho_2 A_2 v_2^3}{\eta_1\rho_1 A_1 v_1^3}$$

Since $\eta_2 = \eta_1$, $\rho_2 = \rho_1$, and $v_2 = v_1$, we get

$$\frac{P_2}{P_1} = \frac{A_2}{A_1} = \frac{d_2^2}{d_1^2} = \frac{10}{1.5} = 6.6667$$

$$d_2^2 = 6.6667 d_1^2 = (6.6667)(82.5\ \text{in})^2$$

$d_2 = 213$ m ◀

PROBLEM 14.80

While cruising in level flight at a speed of 570 mi/h, a jet airplane scoops in air at a rate of 240 lb/s and discharges it with a velocity of 2200 ft/s relative to the airplane. Determine (*a*) the power actually used to propel the airplane, (*b*) the total power developed by the engine, (*c*) the mechanical efficiency of the airplane.

SOLUTION

Data:
$$\frac{dm}{dt} = \frac{240}{32.2} = 7.4534 \text{ slugs/s}$$
$$u = 2200 \text{ ft/s}$$
$$v = 570 \text{ mi/h} = 836 \text{ ft/s}$$
$$F = \frac{dm}{dt}(u - v)$$
$$= (7.4534)(2200 - 836)$$
$$= 10{,}166 \text{ lb}$$

(*a*) Power used to propel airplane:

$$P_1 = Fv$$
$$= (10{,}166)(836)$$
$$= 8.499 \times 10^6 \text{ ft} \cdot \text{lb/s}$$

Propulsion power = 15,450 hp ◀

Power of kinetic energy of exhaust:

$$P_2(\Delta t) = \frac{1}{2}(\Delta m)(u - v)^2$$
$$P_2 = \frac{1}{2}\frac{dm}{dt}(u - v)^2$$
$$= \frac{1}{2}(7.4534)(2200 - 836)^2$$
$$= 6.934 \times 10^6 \text{ ft} \cdot \text{lb/s}$$

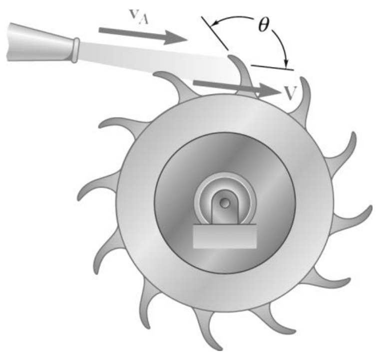
(*b*) Total power:
$$P = P_1 + P_2$$
$$= 15.433 \times 10^6 \text{ ft} \cdot \text{lb/s}$$

Total power = 28,060 hp ◀

(*c*) Mechanical efficiency:
$$\frac{P_1}{P} = \frac{8.499 \times 10^6}{15.433 \times 10^6}$$
$$= 0.551$$

Mechanical efficiency = 0.551 ◀

PROBLEM 14.81



In a Pelton-wheel turbine, a stream of water is deflected by a series of blades so that the rate at which water is deflected by the blades is equal to the rate at which water issues from the nozzle $(\Delta m/\Delta t = A\rho v_A)$. Using the same notation as in Sample Problem 14.7, (*a*) determine the velocity **V** of the blades for which maximum power is developed, (*b*) derive an expression for the maximum power, (*c*) derive an expression for the mechanical efficiency.

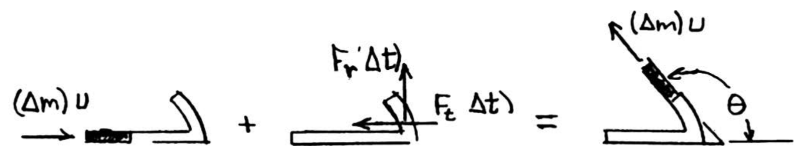
SOLUTION

Let **u** be the velocity of the stream relative to the velocity of the blade.

$$u = (v - V)$$

Mass flow rate:

$$\frac{\Delta m}{\Delta t} = \rho A v_A$$



Principle of impulse and momentum:

$$\xrightarrow{+} (\Delta m)u - F_t(\Delta t) = (\Delta m)u\cos\theta$$

$$F_t = \frac{\Delta m}{\Delta t}u(1-\cos\theta)$$

$$= \rho A v_A(v_A - V)(1-\cos\theta)$$

where F_t is the tangential force on the fluid.

The force F_t on the fluid is directed to the left as shown. By Newton's law of action and reaction, the tangential force on the blade is F_t to the right.

Output power:

$$P_{\text{out}} = F_t V$$

$$= \rho A v_A(v_A - V)V(1-\cos\theta)$$

(*a*) V for maximum power output:

$$\frac{dP_{\text{out}}}{dV} = \rho A(v_A - 2V)(1-\cos\theta) = 0 \qquad v_A = \frac{1}{2}V \blacktriangleleft$$

(*b*) Maximum power:

$$(P_{\text{out}})_{\max} = \rho A v_A\left(v_A - \frac{1}{2}v_A\right)\left(\frac{1}{2}v_A\right)(1-\cos\theta)$$

$$(P_{\text{out}})_{\max} = \frac{1}{4}\rho A v_A^3(1-\cos\theta) \blacktriangleleft$$

PROBLEM 14.81 (Continued)

Input power = rate of supply of kinetic energy of the stream

$$P_{\text{in}} = \frac{1}{\Delta t}\left[\frac{1}{2}(\Delta m)v_A^2\right]$$

$$= \frac{1}{2}\frac{\Delta m}{\Delta t}v_A^2$$

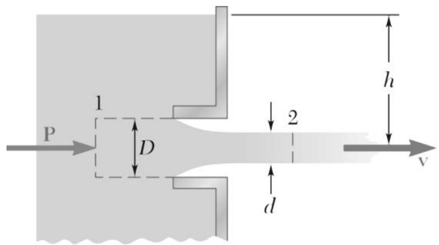
$$= \frac{1}{2}\rho A v_A^3$$

(*c*) Efficiency:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}}$$

$$\eta = \frac{\rho A v_A(v_A - V)V(1-\cos\theta)}{\frac{1}{2}\rho A v_A^3}$$

$$\eta = 2\left(1 - \frac{V}{v_A}\right)\frac{V}{v_A}(1-\cos\theta) \blacktriangleleft$$



PROBLEM 14.82

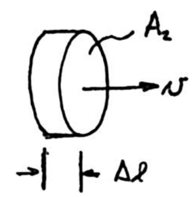
A circular reentrant orifice (also called Borda's mouthpiece) of diameter D is placed at a depth h below the surface of a tank. Knowing that the speed of the issuing stream is $v = \sqrt{2gh}$ and assuming that the speed of approach v_1 is zero, show that the diameter of the stream is $d = D/\sqrt{2}$. (*Hint*: Consider the section of water indicated, and note that P is equal to the pressure at a depth h multiplied by the area of the orifice).

SOLUTION

From hydrostatics, the pressure at section 1 is $p_1 = \gamma h = \rho g h$.

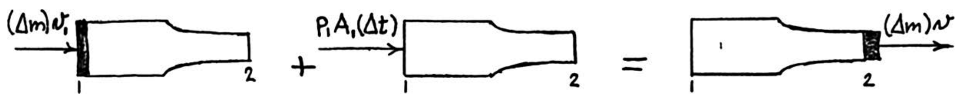
The pressure at section 2 is $p_2 = 0$.

Calculate the mass flow rate using section 2.



$$\text{mass} = \text{density} \times \text{volume}$$
$$= \text{density} \times \text{area} \times \text{length}$$
$$\Delta m = \rho A_2(\Delta l) = \rho A_2 v(\Delta t)$$
$$\frac{dm}{dt} = \frac{\Delta m}{\Delta t} = \rho A_2 v$$

Apply the impulse-momentum principle to fluid between sections 1 and 2.

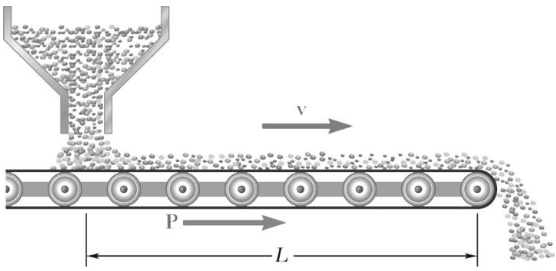


$$(\Delta m)v_1 + p_1 A_1(\Delta t) = (\Delta m)v$$
$$\frac{dm}{dt}v_1 + p_1 A_1 = \frac{dm}{dt}v$$
$$p_1 A_1 = \frac{dm}{dt}(v - v_1)$$
$$= \rho A_2 v(v - v_1)$$

But v_1 is negligible, $\quad p_1 = \rho g h \quad \text{and} \quad v = \sqrt{2gh}$

$$\rho g h A_1 = \rho A_2(2gh) \quad \text{or} \quad A_1 = 2A_2$$
$$\frac{\pi}{4}D^2 = 2\left(\frac{\pi}{4}d^2\right)$$

$d = \dfrac{D}{\sqrt{2}}$ ◀

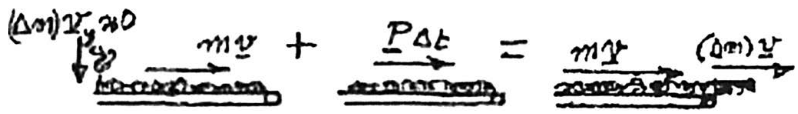


PROBLEM 14.83

Gravel falls with practically zero velocity onto a conveyor belt at the constant rate $q = dm/dt$. (*a*) Determine the magnitude of the force **P** required to maintain a constant belt speed v. (*b*) Show that the kinetic energy acquired by the gravel in a given time interval is equal to half the work done in that interval by the force **P**. Explain what happens to the other half of the work done by **P**.

SOLUTION

(*a*) We apply the impulse-momentum principle to the gravel on the belt and to the mass Δm of gravel hitting and leaving belt in interval Δt.



$\xrightarrow{+}$ x comp.: $mv + P\Delta t = mv + (\Delta m)v$

$$P = \frac{\Delta m}{\Delta t}v = qv$$

$P = qv$ ◀

(*b*) Kinetic energy acquired for unit time:

$$\Delta T = \frac{1}{2}(\Delta m)v^2$$

$$\frac{\Delta T}{\Delta t} = \frac{1}{2}\frac{\Delta m}{\Delta t}v^2 = \frac{1}{2}qv^2 \quad (1)$$

Work done per unit time:

$$\frac{\Delta U}{\Delta t} = \frac{P\Delta x}{\Delta t} = Pv$$

Recalling the result of part *a*:

$$\Delta U = P(\Delta x)$$

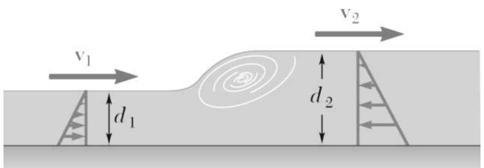
$$\frac{\Delta U}{\Delta t} = (qv)v = qv^2 \quad (2)$$

Comparing Eqs. (1) and (2), we conclude that

$$\frac{\Delta T}{\Delta t} = \frac{1}{2}\frac{\Delta U}{\Delta t}$$

Q.E.D. ◀

The other half of the work of **P** is dissipated into heat by friction as the gravel slips on the belt before reaching the speed v. ◀



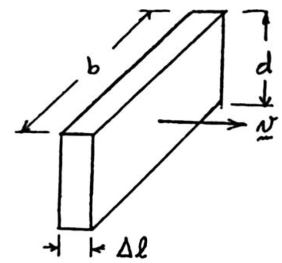
PROBLEM 14.84*

The depth of water flowing in a rectangular channel of width b at a speed v_1 and a depth d_1 increases to a depth d_2 at a *hydraulic jump*. Express the rate of flow Q in terms of b, d_1, and d_2.

SOLUTION

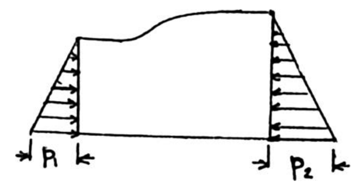
Mass flow rate:

$$\text{mass} = \text{density} \times \text{volume}$$
$$= \text{density} \times \text{area} \times \text{length}$$
$$\Delta m = \rho bd(\Delta l) = \rho bdv(\Delta t)$$
$$\frac{dm}{dt} = \frac{\Delta m}{\Delta t} = \rho bdv$$
$$Q = \frac{1}{\rho}\frac{dm}{dt} = bdv$$



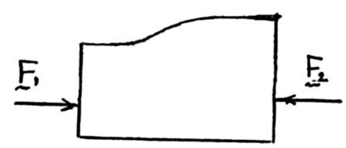
Continuity of flow:

$$Q_1 = Q_2 = Q$$
$$v_1 = \frac{Q}{bd_1} \qquad v_2 \frac{Q}{bd_2}$$

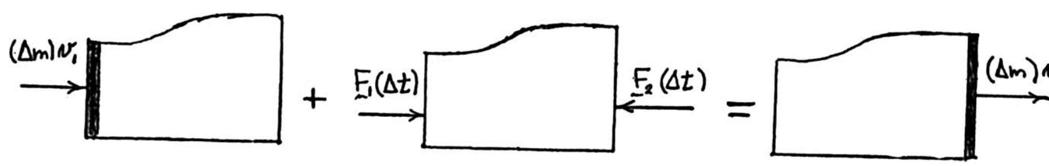


Resultant pressure forces:

$$p_1 = \gamma d_1 \qquad p_2 = \gamma d_2$$
$$F_1 = \frac{1}{2}p_1 bd_1 = \frac{1}{2}\gamma bd_1^2$$
$$F_2 = \frac{1}{2}p_2 bd_2 = \frac{1}{2}\gamma bd_2^2$$



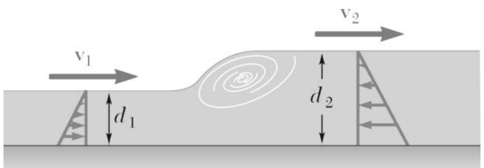
Apply impulse-momentum principle to water between sections 1 and 2.



$$(\Delta m)v_1 + F_1(\Delta t) - F_2(\Delta t) = (\Delta m)v_2$$
$$\frac{\Delta m}{\Delta t}(v_1 - v_2) = F_2 - F_1 \qquad \rho Q \cdot \left(\frac{Q}{bd_1} - \frac{Q}{bd_2}\right) = \frac{1}{2}\gamma b\left(d_2^2 - d_1^2\right)$$
$$\frac{\rho Q^2(d_2 - d_1)}{bd_1 d_2} = \frac{1}{2}\gamma b(d_1 + d_2)(d_2 - d_1)$$

Noting that $\gamma = \rho g$,

$$Q = b\sqrt{\frac{1}{2}gd_1 d_2(d_1 + d_2)} \quad ◀$$

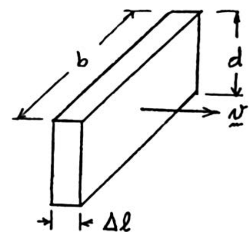


PROBLEM 14.85*

Determine the rate of flow in the channel of Problem 14.84, knowing that $b = 12$ ft, $d_1 = 4$ ft, and $d_2 = 5$ ft.

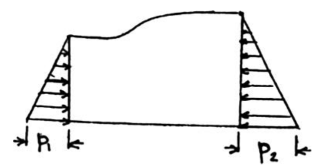
PROBLEM 14.84 The depth of water flowing in a rectangular channel of width b at a speed v_1 and a depth d_1 increases to a depth d_2 at a *hydraulic jump*. Express the rate of flow Q in terms of b, d_1, and d_2.

SOLUTION



Mass flow rate:

$$\text{mass} = \text{density} \times \text{volume}$$
$$= \text{density} \times \text{area} \times \text{length}$$
$$\Delta m = \rho bd(\Delta l) = \rho bdv(\Delta t)$$
$$\frac{dm}{dt} = \frac{\Delta m}{\Delta t} = \rho bdv$$
$$Q = \frac{1}{\rho}\frac{dm}{dt} = bdv$$

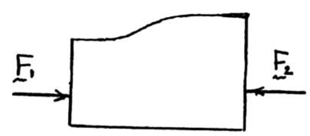


Continuity of flow:

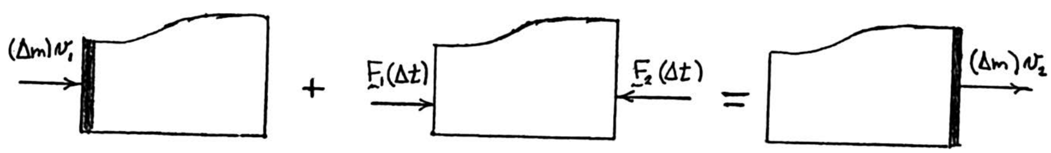
$$Q_1 = Q_2 = Q$$
$$v_1 = \frac{Q}{bd_1} \qquad v_2 \frac{Q}{bd_2}$$

Resultant pressure forces:

$$p_1 = \gamma d_1 \qquad p_2 = \gamma d_2$$
$$F_1 = \frac{1}{2} p_1 bd_1 = \frac{1}{2}\gamma bd_1^2$$
$$F_2 = \frac{1}{2} p_2 bd_2 = \frac{1}{2}\gamma bd_2^2$$



Apply impulse-momentum principle to water between sections 1 and 2.



$$(\Delta m)v_1 + F_1(\Delta t) - F_2(\Delta t) = (\Delta m)v_2$$
$$\frac{\Delta m}{\Delta t}(v_1 - v_2) = F_2 - F_1 \qquad \rho Q \cdot \left(\frac{Q}{bd_1} - \frac{Q}{bd_2}\right) = \frac{1}{2}\gamma b\left(d_2^2 - d_1^2\right)$$
$$\frac{\rho Q^2 (d_2 - d_1)}{bd_1 d_2} = \frac{1}{2}\gamma b(d_1 + d_2)(d_2 - d_1)$$

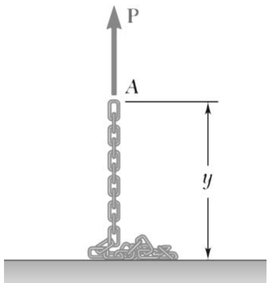
PROBLEM 14.85* (Continued)

Noting that $\gamma = \rho g$, $$Q = b\sqrt{\frac{1}{2} g d_1 d_2 (d_1 + d_2)}$$

Data: $g = 32.2 \text{ ft/s,}^2 \quad b = 12 \text{ ft}, \quad d_1 = 4 \text{ ft}, \quad d_2 = 5 \text{ ft}$

$$Q = 12\sqrt{\frac{1}{2}(32.2)(4)(5)(9)}$$

$Q = 646 \text{ ft}^3/\text{s}$ ◀



PROBLEM 14.86

A chain of length l and mass m lies in a pile on the floor. If its end A is raised vertically at a constant speed v, express in terms of the length y of chain which is off the floor at any given instant (a) the magnitude of the force **P** applied at A, (b) the reaction of the floor.

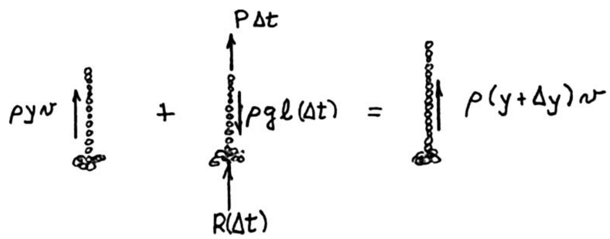
SOLUTION

Let ρ be the mass per unit length of chain. Apply the impulse-momentum to the entire chain. Assume that the reaction from the floor is equal to the weight of chain still in contact with the floor.

Calculate the floor reaction.

$$R = \rho g(l - y)$$

$$R = mg\left(1 - \frac{y}{l}\right)$$



Apply the impulse-momentum principle.

$$\rho yv + P(\Delta t) + R(\Delta t) - \rho gL(\Delta t) = \rho(y + \Delta y)v$$

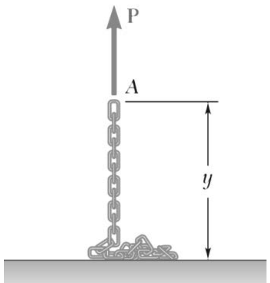
$$P\Delta t = \rho(\Delta y)v + \rho gL(\Delta t) - R(\Delta t)$$

(a)
$$P = \rho\frac{\Delta y}{\Delta t}v + \rho gL - \rho(L - y)g$$

$$= \rho v^2 + \rho gy \qquad P = \frac{m}{l}(v^2 + gy) \blacktriangleleft$$

Let
$$\frac{\Delta y}{\Delta t} = \frac{dy}{dt} = v$$

(b) From above, $\mathbf{R} = mg\left(1 - \frac{y}{l}\right)\uparrow \blacktriangleleft$



PROBLEM 14.87

Solve Problem 14.86, assuming that the chain is being *lowered* to the floor at a constant speed v.

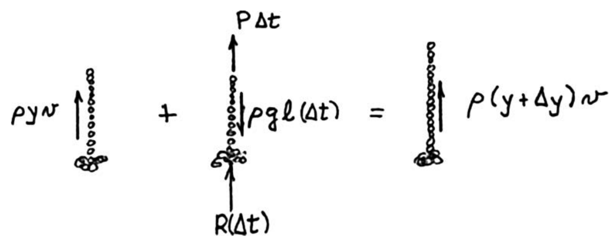
PROBLEM 14.86 A chain of length l and mass m lies in a pile on the floor. If its end A is raised vertically at a constant speed v, express in terms of the length y of chain which is off the floor at any given instant (a) the magnitude of the force **P** applied at A, (b) the reaction of the floor.

SOLUTION

(a) Let ρ be the mass per unit length of chain. The force P supports the weight of chain still off the floor.

$$P = \rho g y \qquad\qquad P = \frac{mgy}{l} \blacktriangleleft$$

(b) Apply the impulse-momentum principle to the entire chain.



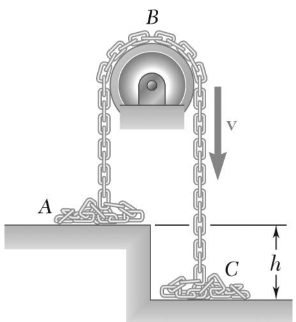
$$-\rho y v + P(\Delta t) + R(\Delta t) - \rho g L(\Delta t) = -\rho g(y + \Delta y)v$$

$$R(\Delta t) = \rho g L(\Delta t) - P(\Delta t) - \rho g(\Delta y)v$$

$$R = \rho g L - \rho g y - \rho \frac{\Delta y}{\Delta t} v$$

Let $\Delta t \to 0$. Then $\quad \dfrac{\Delta y}{\Delta t} = \dfrac{dy}{dt} = -v$

$$R = \rho g(L - y) + \rho v^2 \qquad\qquad \mathbf{R} = \frac{m}{l}[g(L - y) + v^2] \uparrow \blacktriangleleft$$



PROBLEM 14.88

The ends of a chain lie in piles at *A* and *C*. When released from rest at time $t = 0$, the chain moves over the pulley at *B*, which has a negligible mass. Denoting by *L* the length of chain connecting the two piles and neglecting friction, determine the speed *v* of the chain at time *t*.

SOLUTION

Let *m* be the mass of the portion of the chain between the two piles. This is the portion of the chain that is moving with speed *v*. The remainder of the chain lies in either of the two piles. Consider the time period between *t* and $t + \Delta t$ and apply the principle impulse and momentum. Let Δm be the amount of chain that is picket up at *A* and deposited at *C* during the time period Δt.

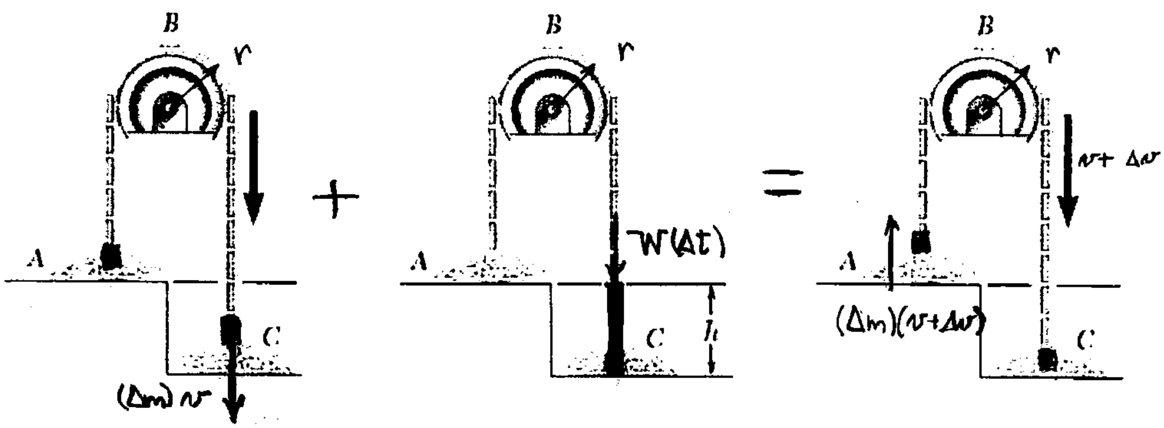
At time *t*, Δm is still in pile *A* while Δm has a downward at speed *v* just above pile *C*. The remaining mass $(m - \Delta m)$ is moving with speed *v*.

At time $t + \Delta t$, Δm is moving with speed $v + \Delta v$ just above pile *A* and Δm is at rest in pile *C*.

Over the time period an unbalanced weight of chain acts on the system. The weight is

$$W = \frac{mgh}{L}$$

Apply the impulse-momentum principle to the system.



Consider moments about the pulley axle.

$$r[(\Delta m)v + (m - \Delta m)v] + rW(\Delta t)$$
$$= r[(\Delta m)(v + \Delta v) + (m - \Delta m)(v + \Delta v)]$$

PROBLEM 14.88 (Continued)

Dividing by r and canceling the terms $(\Delta m)v$ and $(m-\Delta m)v$

$$0+\frac{mgh}{L}(\Delta t)=(\Delta m)(v+\Delta v)+(m-\Delta m)(\Delta v)$$
$$=v(\Delta m)+m(\Delta v)$$

But $$\Delta m=\frac{m}{L}v(\Delta t)$$

Hence, $$\frac{mgh}{L}(\Delta t)=\frac{mv^2}{L}(\Delta t)+m(\Delta v)$$

Solving for Δt, $$\Delta t=\frac{L(\Delta v)}{gh-v^2}$$

Letting $c^2=gh,$ and considering the limit as Δt and Δv become infinitesimal, gives

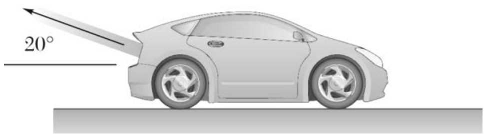
$$dt=\frac{L\,dv}{c^2-v^2}$$

Integrate, noting that $v=0$ when $t=0$

$$t=L\int_0^v\frac{dv}{c^2-v^2}=\frac{L}{c}\tanh^{-1}\frac{v}{c}\bigg|_0^v$$

$$\tanh\frac{ct}{L}=\frac{v}{c}$$

$$v=\sqrt{gh}\tanh\left(\frac{\sqrt{gh}}{L}t\right) \blacktriangleleft$$

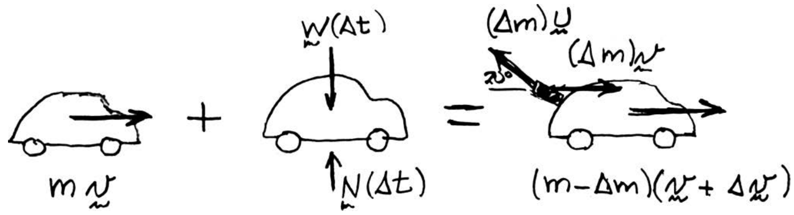


PROBLEM 14.89

A toy car is propelled by water that squirts from an internal tank at a constant 6 ft/s relative to the car. The weight of the empty car is 0.4 lb and it holds 2 lb of water. Neglecting other tangential forces, determine the top speed of the car.

SOLUTION

Consider a time interval Δt. Let m be the mass of the car plus the water in the tank at the beginning of the interval and $(m - \Delta m)$ the corresponding mass at the end of the interval. m_0 is the initial value of m. Let **v** be the velocity of the car. Apply the impulse and momentum principle over the time interval.



Horizontal components $\xrightarrow{+}$:

$$mv + 0 = (\Delta m)(\cancel{v} - u\cos 20°) + (\cancel{m} - \Delta m)(\cancel{v} + \Delta v)$$

$$\Delta v = u\cos 20° \frac{\Delta m}{m - \Delta m}$$

Let Δv be replaced by differential dv and Δm be replaced by the small differential $-dm$, the minus sign meaning that dm is the infinitesimal increase in m.

$$dv = -u\cos 20° \frac{dm}{m}$$

Integrating, $$v = v_0 - u\cos 20° \ln\frac{m}{m_0}$$

Since $v_0 = 0$, $$v = u\cos 20° \ln\frac{m_0}{m}$$

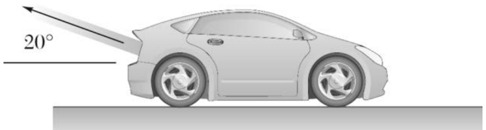
The velocity is maximum when $m = m_f$, the value of m when all of the water is expelled.

$$v_{max} = u\cos 20° \ln\frac{m_0}{m_f}$$

$$v_{max} = (6\text{ ft/s})\cos 20° \ln\frac{0.4+2}{0.4}$$

$v_{max} = 10.10$ ft/s ◀

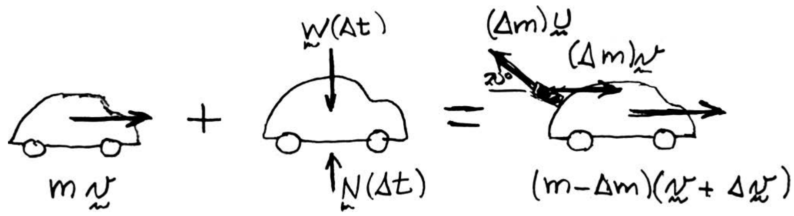
PROBLEM 14.90



A toy car is propelled by water that squirts from an internal tank. The weight of the empty car is 0.4 lb and it holds 2 lb of water. Knowing the top speed of the car is 8 ft/s, determine the relative velocity of the water that is being ejected.

SOLUTION

Consider a time interval Δt. Let m be the mass of the car plus the water in the tank at the beginning of the interval and $(m - \Delta m)$ the corresponding mass at the end of the interval. m_0 is the initial value of m. Let **v** be the velocity of the car. Apply the impulse and momentum principle over the time interval.



Horizontal components $\xrightarrow{+}$:

$$mv + 0 = (\Delta m)(\cancel{v} - u\cos 20°) + (\cancel{m} - \Delta m)(\cancel{v} + \Delta v)$$

$$\Delta v = u\cos 20° \frac{\Delta m}{m - \Delta m}$$

Let Δv be replaced by differential dv and Δm be replaced by the small differential $-dm$, the minus sign meaning that dm is the infinitesimal increase in m.

$$dv = -u\cos 20° \frac{dm}{m}$$

Integrating, $$v = v_0 - u\cos 20° \ln\frac{m}{m_0}$$

Since $v_0 = 0$, $$v = u\cos 20° \ln\frac{m_0}{m}$$

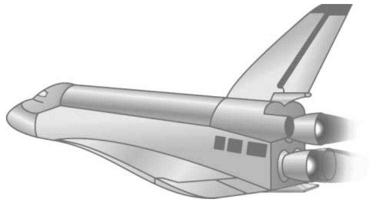
The velocity is maximum when $m = m_f$, the value of m when all of the water is expelled.

$$v_{\max} = u\cos 20° \ln\frac{m_0}{m_f}$$

$$8 \text{ ft/s} = u\cos 20° \ln\frac{0.4 + 2}{0.4}$$

$u = 4.75$ ft/s ◀

PROBLEM 14.91



The main propulsion system of a space shuttle consists of three identical rocket engines which provide a total thrust of 6 MN. Determine the rate at which the hydrogen-oxygen propellant is burned by each of the three engines, knowing that it is ejected with a relative velocity of 3750 m/s.

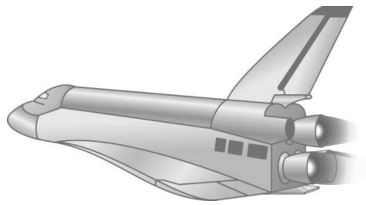
SOLUTION

Thrust of each engine: $P = \frac{1}{3}(6 \text{ MN}) = 2\times 10^6 \text{ N}$

Eq. (14.44): $$P = \frac{dm}{dt}u$$

$$2\times 10^6 \text{ N} = \frac{dm}{dt}(3750 \text{ m/s})$$

$$\frac{dm}{dt} = \frac{2\times 10^6 \text{ N}}{3750 \text{ m/s}} \qquad \frac{dm}{dt} = 533 \text{ kg/s} \blacktriangleleft$$



PROBLEM 14.92

The main propulsion system of a space shuttle consists of three identical rocket engines, each of which burns the hydrogen-oxygen propellant at the rate of 750 lb/s and ejects it with a relative velocity of 12000 ft/s. Determine the total thrust provided by the three engines.

SOLUTION

From Eq. (14.44) for each engine:

$$\begin{aligned} P &= \frac{dm}{dt}u \\ &= \frac{(750 \text{ lb/s})}{32.2 \text{ ft/s}^2}(12000 \text{ ft/s}) \\ &= 279.50 \times 10^3 \text{ lb} \end{aligned}$$

For the 3 engines:

$$\text{Total thrust} = 3(279.50 \times 10^3 \text{ lb})$$

Total thrust = 839,000 lb ◀

PROBLEM 14.93

A rocket weighs 2600 lb, including 2200 lb of fuel, which is consumed at the rate of 25 lb/s and ejected with a relative velocity of 13000 ft/s. Knowing that the rocket is fired vertically from the ground, determine its acceleration (*a*) as it is fired, (*b*) as the last particle of fuel is being consumed.

SOLUTION

From Eq. (14.44) of the textbook, the thrust is

$$P = \frac{dm}{dt}u$$

$$= \frac{(25 \text{ lb/s})}{32.2 \text{ ft/s}^2}(13000 \text{ ft/s})$$

$$= 10.093 \times 10^3 \text{ lb}$$

$$\Sigma F = ma$$

$$P - mg = ma \qquad a = \frac{P}{m} - g \tag{1}$$

(*a*) *At the start of firing,* $W = W_0 = 2600$ lb

$$g = 32.2 \text{ ft/s}^2 \qquad m = \frac{2600}{32.2} = 80.745 \text{ slug}$$

From Eq. (1), $a = \dfrac{10.093 \times 10^3 \text{ lb}}{80.745 \text{ slug}} - 32.2 = 92.80 \text{ ft/s}^2$ $\qquad \mathbf{a} = 92.8 \text{ ft/s}^2 \uparrow$ ◀

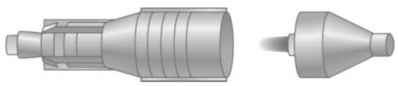
(*b*) *As the last particle of fuel is consumed,*

$$W = 2600 - 2200 = 400 \text{ lb}$$

$$g = 32.2 \text{ ft/s}^2 \text{ (assumed)} \qquad m = \frac{400}{32.2} = 12.422 \text{ slug}$$

From Eq. (1), $a = \dfrac{10.093 \times 10^3 \text{ lb}}{12.422} - 32.2 = 780.30 \text{ ft/s}^2$ $\qquad \mathbf{a} = 780 \text{ ft/s}^2 \uparrow$ ◀

PROBLEM 14.94



A space vehicle describing a circular orbit at a speed of 24×10^3 km/h releases its front end, a capsule which has a gross mass of 600 kg, including 400 kg of fuel. If the fuel is consumed at the rate of 18 kg/s and ejected with a relative velocity of 3000 m/s, determine (*a*) the tangential acceleration of the capsule as the engine is fired, (*b*) the maximum speed attained by the capsule.

SOLUTION

Thrust:

$$P = \left|\frac{dm}{dt}\right| u$$
$$= (18 \text{ kg/s})(3000 \text{ m/s})$$
$$= 54\times10^3 \text{ N}$$

(*a*)

$$(a_t)_0 = \frac{P}{m_0} = \frac{54\times10^3}{600} = 90 \text{ m/s}^2$$

$(a_t)_0 = 90.0 \text{ m/s}^2$ ◀

(*b*) Maximum speed is attained when all the fuel is used up:

$$v_1 = v_0 + \int_0^{t_1} a_t dt = v_0 + \int_0^{t_1} \frac{P}{m} dt$$
$$= v_0 + \int_0^{t_1} \frac{u\left(\frac{dm}{dt}\right)}{m} dt = v_0 + u\int_{m_0}^{m_1}\left(-\frac{dm}{m}\right)$$
$$v_1 = v_0 + u\left(-\ln\frac{m_1}{m_0}\right) = v_0 + u\ln\frac{m_0}{m_1}$$

Data:

$$v_0 = 24\times10^3 \text{ km/h}$$
$$= 6.6667\times10^3 \text{ m/s}$$
$$u = 3000 \text{ m/s}$$
$$m_0 = 600 \text{ kg}$$
$$m_1 = 600 - 400 = 200 \text{ kg}$$
$$v_1 = 6.6667\times10^3 + 3000\ln\frac{600}{200}$$
$$= 9.9625\times10^3 \text{ m/s}$$

$v_1 = 35.9\times10^3$ km/h ◀

PROBLEM 14.95

A 540-kg spacecraft is mounted on top of a rocket with a mass of 19 Mg, including 17.8 Mg of fuel. Knowing that the fuel is consumed at a rate of 225 kg/s and ejected with a relative velocity of 3600 m/s, determine the maximum speed imparted to the spacecraft if the rocket is fired vertically from the ground.

SOLUTION

See sample Problem 14.8 for derivation of

$$v = u \ln \frac{m_0}{m_0 - qt} - gt \qquad (1)$$

Data: $u = 3600 \text{ m/s} \quad q = 225 \text{ kg/s}, \quad m_{\text{fuel}} = 17{,}800 \text{ kg}$

$$m_0 = 19{,}000 \text{ kg} + 540 \text{ kg} = 19{,}540 \text{ kg}$$

We have $m_{\text{fuel}} = qt, \quad 17{,}800 \text{ kg} = (225 \text{ kg/s})t$

$$t = \frac{17{,}800 \text{ kg}}{225 \text{ kg/s}} = 79.111 \text{ s}$$

Maximum velocity is reached when all fuel has been consumed, that is, when $qt = m_{\text{fuel}}$. Eq. (1) yields

$$v_m = u \ln \frac{m_0}{m_0 - m_{\text{fuel}}} - gt$$

$$= (3600 \text{ m/s}) \ln \frac{19{,}540}{19{,}540 - 17{,}800} - (9.81 \text{ m/s}^2)(79.111 \text{ s})$$

$$= (3600 \text{ m/s}) \ln 11.230 - 776.1 \text{ m/s}$$

$$= 7930.8 \text{ m/s}$$

$v_m = 7930 \text{ m/s}$ ◀

PROBLEM 14.96

The rocket used to launch the 540-kg spacecraft of Problem 14.95 is redesigned to include two stages A and B, each of mass 9.5 Mg, including 8.9 Mg of fuel. The fuel is again consumed at a rate of 225 kg/s and ejected with a relative velocity of 3600 m/s. Knowing that when stage A expels its last particle of fuel its casing is released and jettisoned, determine (a) the speed of the rocket at that instant, (b) the maximum speed imparted to the spacecraft.

SOLUTION

Thrust force: $P = u\dfrac{dm}{dt} = uq$

Mass of rocket + unspent fuel: $m = m_0 - qt$

Corresponding weight force: $W = mg$

Acceleration: $a = \dfrac{F}{m} = \dfrac{P-W}{m} = \dfrac{P}{m} - g = \dfrac{uq}{m_0 - qt} - g$

Integrating with respect to time to obtain the velocity,

$$v = v_0 + \int_0^t a\,dt = v_0 + u\int_0^t \frac{q\,dt}{m_0 - qt} - gt$$

$$= v_0 - u\ln\frac{m_0 - qt}{m_0} - gt \qquad (1)$$

For each stage, $m_{\text{fuel}} = 8900$ kg $\quad u = 3600$ m/s

$q = 225$ kg/s $\quad t = \dfrac{m_{\text{fuel}}}{q} = \dfrac{8900}{225} = 39.556$ s

For the first stage, $v_0 = 0 \quad m_0 = 540 + (2)(9500) = 19{,}540$ kg

(a) $v_1 = 0 - 3600\ln\dfrac{19{,}540 - 8900}{19{,}540} - (9.81)(39.556) = 1800.1$ m/s

$v_1 = 1800$ m/s ◀

For the second stage, $v_0 = 1800.1$ m/s, $\quad m_0 = 540 + 9500 = 10{,}040$ kg

(b) $v_2 = 1800.1 - 3600\ln\dfrac{10{,}040 - 8900}{10{,}040} - (9.81)(39.556) = 9244$ m/s

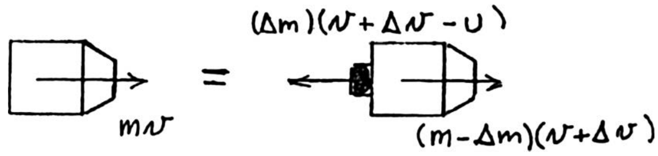
$v_2 = 9240$ m/s ◀

PROBLEM 14.97

A communication satellite weighing 10,000 lb, including fuel, has been ejected from a space shuttle describing a low circular orbit around the earth. After the satellite has slowly drifted to a safe distance from the shuttle, its engine is fired to increase its velocity by 8000 ft/s as a first step to its transfer to a geosynchronous orbit. Knowing that the fuel is ejected with a relative velocity of 13,750 ft/s, determine the weight of fuel consumed in this maneuver.

SOLUTION

Apply the *principle of impulse and momentum* to the satellite plus the fuel expelled in time Δt.



$$\xrightarrow{+}: \quad mv = (m-\Delta m)(v+\Delta v) + (\Delta m)(v+\Delta v - v)$$

$$= mv + m(\Delta v) - (\Delta m)v - (\Delta m)(\Delta v) + (\Delta m)v + (\Delta m)(\Delta v) - (\Delta m)v$$

$$m(\Delta v) - u(\Delta m) = 0$$

$$\Delta m = -\frac{dm}{dt}(\Delta t)$$

$$\frac{\Delta v}{\Delta t} = \frac{dv}{dt} = -\frac{u}{m}\frac{dm}{dt}$$

$$\int_{v_0}^{v_1} dv = -\int_0^{t_1} \frac{u}{m}\frac{dm}{dt}dt = -\int_{m_0}^{m_1} u\frac{dm}{m}$$

$$v_1 - v_0 = -u\ln\frac{m_1}{m_0} = u\ln\frac{m_0}{m_1}$$

$$\frac{m_0}{m_1} = \exp\left(\frac{v_1 - v_0}{u}\right)$$

PROBLEM 14.97 (Continued)

Data:

$$v_1 - v_0 = 8000 \text{ ft/s}$$

$$u = 13,750 \text{ ft/s}$$

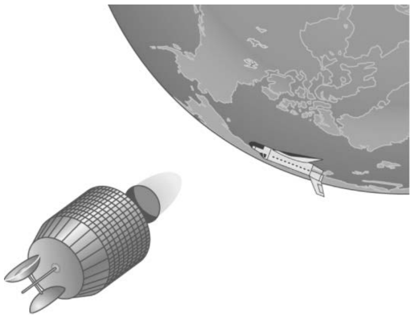
$$m_0 = 10,000 \text{ lb}$$

$$\frac{10,000}{m_1} = \exp\frac{8000}{13,750}$$

$$= 1.7893$$

$$m_1 = 5589 \text{ kg}$$

$$m_{\text{fuel}} = m_0 - m_1 = 10,000 - 5589 \qquad m_{\text{fuel}} = 4410 \text{ lb} \blacktriangleleft$$



PROBLEM 14.98

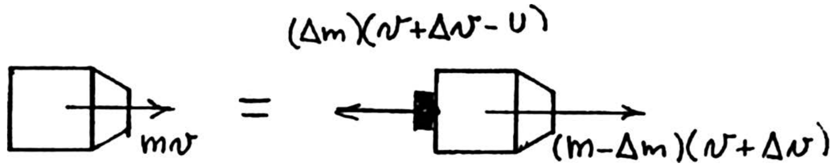
Determine the increase in velocity of the communication satellite of Problem 14.97 after 2500 lb of fuel has been consumed.

SOLUTION

Data from Problem 14.95: $m_0 = 10,000 \text{ lb} \quad u = 13,750 \text{ ft/s}$

$$m_1 = m_0 - m_{\text{fuel}} = 10,000 - 2500 = 7500 \text{ lb.}$$

Apply the *principle of impulse and momentum* to the satellite plus the fuel expelled in time Δt.



$$\xrightarrow{+}: \quad mv = (m - \Delta m)(v + \Delta v) + (\Delta m)(v + \Delta v - v)$$

$$= mv + m(\Delta v) - (\Delta m)v - (\Delta m)(\Delta v)$$

$$+ (\Delta m)v + (\Delta m)(\Delta v) - (\Delta m)v$$

$$m(\Delta v) - u(\Delta m) = 0$$

$$\Delta m = -\frac{dm}{dt}(\Delta t)$$

$$\frac{\Delta v}{\Delta t} = \frac{dv}{dt} = -\frac{u}{m}\frac{dm}{dt}$$

$$\int_{v_0}^{v_1} dv = -\int_0^{t_1} \frac{u}{m}\frac{dm}{dt}\,dt = -\int_{m_0}^{m_1} u\frac{dm}{m}$$

$$v_1 - v_0 = -u\ln\frac{m_1}{m_0} = u\ln\frac{m_0}{m_1}$$

$$\Delta v = v_1 - v_0 = 13,750 \ln\frac{10,000}{7500}$$

$\Delta v = 3960 \text{ ft/s}$ ◀

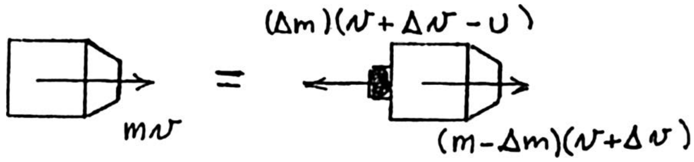
PROBLEM 14.99

Determine the distance separating the communication satellite of Problem 14.97 from the space shuttle 60 s after its engine has been fired, knowing that the fuel is consumed at a rate of 37.5 lb/s.

PROBLEM 14.97 A communication satellite weighing 10,000 lb, including fuel, has been ejected from a space shuttle describing a low circular orbit around the earth. After the satellite has slowly drifted to a safe distance from the shuttle, its engine is fired to increase its velocity by 8000 ft/s as a first step to its transfer to a geosynchronous orbit. Knowing that the fuel is ejected with a relative velocity of 13,750 ft/s, determine the weight of fuel consumed in this maneuver.

SOLUTION

Apply the *principle of impulse and momentum* to the satellite plus the fuel expelled in time Δt.



$$\xrightarrow{+}: \quad mv = (m - \Delta m)(v + \Delta v) + (\Delta m)(v + \Delta v - v)$$

$$= mv + m(\Delta v) - (\Delta m)v - (\Delta m)(\Delta v) + (\Delta m)v + (\Delta m)(\Delta v) - (\Delta m)v$$

$$m(\Delta v) - u(\Delta m) = 0$$

$$\Delta m = -\frac{dm}{dt}(\Delta t)$$

$$\frac{\Delta v}{\Delta t} = \frac{dv}{dt} = -\frac{u}{m}\frac{dm}{dt} = -\frac{uq}{m} = -\frac{uq}{m_0 - qt}$$

$$v = v_0 + \int_0^t \frac{uq}{m_0 - qt}\,dt = v_0 - u\ln(m_0 - qt)\Big|_0^t$$

$$= v_0 + u\ln(m_0 - qt) + u\ln m_0$$

$$= v_0 - u\ln\left(\frac{m_0 - qt}{m_0}\right) \qquad (1)$$

Set $\frac{dx}{dt} = v$ in Eq. (1) and integrate with respect to time.

$$x = x_0 + v_0 t + u\int_0^t \ln\left(\frac{m_0 - qt}{m_0}\right)dt$$

PROBLEM 14.99 (Continued)

Call the last term x' and let

$$z = \frac{m_0 - qt}{m_0} \qquad dz = -\frac{q}{m_0}dt \qquad \text{or} \qquad dt = -\frac{m_0}{q}dz$$

$$x' = \frac{m_0 u}{q}\int_{z_0}^{z} \ln z\, dz = \frac{m_0 u}{q}[(z\ln z + z)]_{z_0}^{z}$$

$$= \frac{m_0 u}{q}\left[\frac{m_0 - qt}{m_0}\left(\ln\frac{m_0 - qt}{m_0} - 1\right) - \frac{m_0}{m_0}\left(\ln\frac{m_0}{m_0} - 1\right)\right]$$

$$= \frac{m_0 u}{q}\left[\left(1 - \frac{qt}{m_0}\right)\left(\ln\frac{m_0 - qt}{m_0} - 1\right) + 1\right]$$

$$= \frac{m_0 u}{q}\left[\ln\frac{m_0 - qt}{m_0} - 1 + 1\right] - ut\left[\ln\frac{m_0 - qt}{m_0} - 1\right]$$

$$= ut + \left(\frac{m_0 u}{q} - ut\right)\ln\frac{m_0 - qt}{m_0}$$

$$= u\left[t - \left(\frac{m_0}{q} - t\right)\ln\frac{m_0}{m_0 - qt}\right]$$

$$x = x_0 + v_0 t + u\left[t - \left(\frac{m_0}{q} - t\right)\ln\frac{m_0}{m_0 - qt}\right] \qquad (2)$$

Data: $x_0 = 0 \qquad v_0 = 0 \qquad q = 37.5$ lb/s.

$m_0 = 10{,}000$ lb, $\quad t = 60$ sec $\quad u = 13{,}750$ ft/s

$$x = 0 + 0 + (13{,}750)\left[60 - \left(\frac{10{,}000}{37.5} - 60\right)\ln\frac{10{,}000}{10{,}000 - (37.5)(60)}\right]$$

$= 100{,}681$ ft $\qquad\qquad x = 19.07$ mi ◀

PROBLEM 14.100

For the rocket of Problem 14.93, determine (*a*) the altitude at which all the fuel has been consumed, (*b*) the velocity of the rocket at that time.

PROBLEM 14.93 A rocket weighs 2600 lb, including 2200 lb of fuel, which is consumed at the rate of 25 lb/s and ejected with a relative velocity of 13000 ft/s. Knowing that the rocket is fired vertically from the ground, determine its acceleration (*a*) as it is fired, (*b*) as the last particle of fuel is being consumed.

SOLUTION

See Sample Problem 14.8 for derivation of

$$v = u\ln\frac{m_0}{m_0 - qt} - gt = -u\ln\frac{m_0 - qt}{m_0} - gt \qquad (1)$$

Note that g is assumed to be constant.

Set $\frac{dy}{dt} = v$ in Eq. (1) and integrate with respect to time.

$$h = \int_0^h dy = \int_0^t v\,dt = \int_0^t \left(u\ln\frac{m_0}{m_0 - qt} - gt \right) dt$$

$$= -u\int_0^t \ln\frac{m_0 - qt}{m_0}\,dt - \frac{1}{2}gt^2$$

Let $\qquad z = \dfrac{m_0 - qt}{m_0} \qquad dz = -\dfrac{q}{m_0}dt \qquad \text{or} \qquad dt = -\dfrac{m_0}{q}dz$

$$h = \frac{m_0 u}{q}\int_{z_0}^{z} \ln z\,dz - \frac{1}{2}gt^2 = \frac{m_0 u}{q}[(z\ln z + z)]_{z_0}^{z} - \frac{1}{2}gt^2$$

$$= \frac{m_0 u}{q}\left[\frac{m_0 - qt}{m_0}\left(\ln\frac{m_0 - qt}{m_0} - 1\right) - \frac{m_0}{m_0}\left(\ln\frac{m_0}{m_0} - 1\right)\right] - \frac{1}{2}gt^2$$

$$= \frac{m_0 u}{q}\left[\left(1 - \frac{qt}{m_0}\right)\left(\ln\frac{m_0 - qt}{m_0} - 1\right) + 1\right] - \frac{1}{2}gt^2$$

$$= \frac{m_0 u}{q}\left[\ln\frac{m_0 - qt}{m_0} - 1 + 1\right] - ut\left[\ln\frac{m_0 - qt}{m_0} - 1\right] - \frac{1}{2}gt^2$$

$$= ut + \left(\frac{m_0 u}{q} - ut\right)\ln\frac{m_0 - qt}{m_0} - \frac{1}{2}gt^2$$

$$h = u\left[t - \left(\frac{m_0}{q} - t\right)\ln\frac{m_0}{m_0 - qt}\right] - \frac{1}{2}gt^2 \qquad (2)$$

PROBLEM 14.100 (Continued)

Data: $W_0 = 2600 \text{ lb} \qquad qt = W_{\text{fuel}} = 2200 \text{ lb} \qquad q = 25 \text{ lb/s}$

$u = 13000 \text{ ft/s} \qquad g = 32.2 \text{ ft/s}^2$

$$t = \frac{W_{\text{fuel}}}{q} = \frac{2200}{25} = 88 \text{ s}$$

(*a*) From Eq. (2),

$$h = (13000)\left[88 - \left(\frac{2600}{25} - 88\right)\ln\frac{2600}{2600-2200}\right] - \frac{1}{2}(32.2)(88)^2$$

$$= (13000)(88 - 16 \ln 6.5) - 124680$$

$$= 630{,}000 \text{ ft}$$

$h = 119.3 \text{ mi}$ ◀

(*b*) From Eq. (1),

$$v = -13000 \ln\frac{2600-2200}{2600} - (32.2)(88)$$

$$= 13000 \ln 6.5 - 2834$$

$$= 21500 \text{ ft/s}$$

$v = 14{,}660 \text{ mi/h}$ ◀

PROBLEM 14.101

Determine the altitude reached by the spacecraft of Problem 14.95 when all the fuel of its launching rocket has been consumed.

SOLUTION

See Sample Problem 14.8 for derivation of

$$v = u\ln\frac{m_0}{m_0 - qt} - gt = -u\ln\frac{m_0 - qt}{m_0} - gt \qquad (1)$$

Note that g is assumed to be constant.

Set $\frac{dy}{dt} = v$ in Eq. (1) and integrate with respect to time.

$$h = \int_0^h dy = \int_0^t v\,dt = \int_0^t \left(u\ln\frac{m_0}{m_0 - qt} - gt\right)dt$$

$$= -u\int_0^t \ln\frac{m_0 - qt}{m_0}\,dt - \frac{1}{2}gt^2$$

Let $\qquad z = \frac{m_0 - qt}{m_0} \qquad dz = -\frac{q}{m_0}dt \qquad \text{or} \qquad dt = -\frac{m_0}{q}dz$

$$h = \frac{m_0 u}{q}\int_{z_0}^{z} \ln z\,dz - \frac{1}{2}gt^2 = \frac{m_0 u}{q}\left[(z\ln z + z)\right]_{z_0}^{z} - \frac{1}{2}gt^2$$

$$= \frac{m_0 u}{q}\left[\frac{m_0 - qt}{m_0}\left(\ln\frac{m_0 - qt}{m_0} - 1\right) - \frac{m_0}{m_0}\left(\ln\frac{m_0}{m_0} - 1\right)\right] - \frac{1}{2}gt^2$$

$$= \frac{m_0 u}{q}\left[\left(1 - \frac{qt}{m_0}\right)\left(\ln\frac{m_0 - qt}{m_0} - 1\right) + 1\right] - \frac{1}{2}gt^2$$

$$= \frac{m_0 u}{q}\left[\ln\frac{m_0 - qt}{m_0} - 1 + 1\right] - ut\left[\ln\frac{m_0 - qt}{m_0} - 1\right] - \frac{1}{2}gt^2$$

$$= ut + \left(\frac{m_0 u}{q} - ut\right)\ln\frac{m_0 - qt}{m_0} - \frac{1}{2}gt^2$$

$$h = u\left[t - \left(\frac{m_0}{q} - t\right)\ln\frac{m_0}{m_0 - qt}\right] - \frac{1}{2}gt^2 \qquad (2)$$

Data:

$$u = 3600 \text{ m/s} \qquad m_0 = 19{,}000 + 540 = 19{,}540 \text{ kg}$$

$$q = 225 \text{ kg/s} \qquad m_{\text{fuel}} = 17{,}800 \text{ kg}$$

$$t = \frac{m_{\text{fuel}}}{q} = \frac{17{,}800}{225} = 79.111 \text{ s} \qquad g = 9.81 \text{ m/s}^2$$

$$m_0 - qt = 1740 \text{ kg}$$

From Eq. (2),

$$h = (3600)\left[79.111 - \left(\frac{19{,}540}{225} - 79.111\right)\ln\frac{19{,}540}{1740}\right] - \frac{1}{2}(9.81)(79.111)^2$$

$$= 186{,}766 \text{ m}$$

$h = 186.8$ km ◀

PROBLEM 14.102

For the spacecraft and the two-stage launching rocket of Problem 14.96, determine the altitude at which (*a*) stage *A* of the rocket is released, (*b*) the fuel of both stages has been consumed.

SOLUTION

Thrust force: $P = u\dfrac{dm}{dt} = uq$

Mass of rocket + unspent fuel: $m = m_0 - qt$

Corresponding weight force: $W = mg$

Acceleration: $a = \dfrac{F}{m} = \dfrac{P-W}{m} = \dfrac{P}{m} - g = \dfrac{uq}{m_0 - qt} - g$

Integrating with respect to time to obtain velocity,

$$v = v_0 + \int_0^t a dt = v_0 + u\int_0^t \frac{q dt}{m_0 - qt} - gt$$

$$= v_0 - u\ln\frac{m_0 - qt}{m_0} - gt \qquad (1)$$

Integrating again to obtain the displacement,

$$s = s_0 + v_0 t - u\int_0^t \ln\frac{m_0 - qt}{m_0} dt - \frac{1}{2}gt^2$$

Let $z = \dfrac{m_0 - qt}{m_0} \qquad dz = -\dfrac{q}{m_0}\, dt \qquad dt = -\dfrac{m_0}{q}\, dz$

Then

$$s = s_0 + v_0 t + \frac{m_0 u}{q}\int_{z_0}^{z} \ln z\, dz - \frac{1}{2}gt^2$$

$$= s_0 + v_0 t + \frac{m_0 u}{q}(z\ln z + z)\Big|_{z_0}^{z} - \frac{1}{2}gt^2$$

$$= s_0 + v_0 t + \frac{m_0 u}{q}\left[\frac{m_0 - qt}{m_0}\ln\frac{m_0 - qt}{m_0} + \frac{m_0 - qt}{m_0} - \frac{m_0}{m_0}\ln\frac{m_0}{m_0} + \frac{m_0}{m_0}\right] - \frac{1}{2}gt^2$$

$$= s_0 + v_0 t + u\left[t + \left(\frac{m_0}{q} - t\right)\ln\frac{m_0 - qt}{m_0}\right] - \frac{1}{2}gt^2 \qquad (2)$$

For each stage, $m_{\text{fuel}} = 8900 \text{ kg} \qquad u = 3600 \text{ m/s}$

$$q = 225 \text{ kg/s} \qquad t = \frac{m_{\text{fuel}}}{q} = \frac{8900}{225} = 39.556 \text{ s}$$

For the first stage, $v_0 = 0 \qquad s_0 = 0$

$$m_0 = 540 + (2)(9500) = 19{,}540 \text{ kg}$$

From Eq. (1),

$$v_1 = 0 - 3600 \ln \frac{19{,}540 - 8900}{19{,}540} - (9.81)(39.556)$$

$$= 1800.1 \text{ m/s}$$

From Eq. (2),

(*a*) $$s_1 = 0 + 0 + 3600\left[39.556 + \left(\frac{19{,}540}{225} - 39.556\right)\ln\frac{19{,}540 - 8900}{19{,}540}\right] - \frac{1}{2}(9.81)(39.556)^2$$

$$= 31{,}249 \text{ m}$$

$h_1 = 31.2 \text{ km}$ ◀

For the second stage, $v_0 = 1800.1 \text{ m/s} \quad s_0 = 31{,}249 \text{ m}$

$$m_0 = 540 + 9500 = 10{,}040 \text{ kg}$$

From Eq. (2),

(*b*) $$s_2 = 31{,}249 + (1800.1)(39.556) + 3600\left[39.556 + \left(\frac{10{,}040}{225} - 39.556\right)\ln\frac{10{,}040 - 8900}{10{,}040}\right]$$

$$- \frac{1}{2}(9.81)(39.556)^2$$

$$= 197{,}502 \text{ m}$$

$h_2 = 197.5 \text{ km}$ ◀

PROBLEM 14.103

In a jet airplane, the kinetic energy imparted to the exhaust gases is wasted as far as propelling the airplane is concerned. The useful power is equal to the product of the force available to propel the airplane and the speed of the airplane. If v is the speed of the airplane and u is the relative speed of the expelled gases, show that the mechanical efficiency of the airplane is $\eta = 2v/(u+v)$. Explain why $\eta = 1$ when $u = v$.

SOLUTION

Let F be the thrust force, and $\frac{dm}{dt}$ be the mass flow rate.

Absolute velocity of exhaust: $v_e = u - v$

Thrust force: $F = \dfrac{dm}{dt}(u - v)$

Power of thrust force: $P_1 = Fv = \dfrac{dm}{dt}(u - v)v$

Power associated with exhaust:
$$P_2(\Delta t) = \frac{1}{2}(\Delta m)v_e^2 = \frac{1}{2}(\Delta m)(u - v)^2$$
$$P_2 = \frac{1}{2}\frac{dm}{dt}(u - v)^2$$

Total power supplied by engine:
$$P = P_1 + P_2$$
$$P = \frac{dm}{dt}\left[(u - v)v - \frac{1}{2}(u - v)^2\right]$$
$$= \frac{1}{2}\frac{dm}{dt}(u^2 - v^2)$$

Mechanical efficiency:
$$\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_1}{P}$$
$$\eta = \frac{2(u - v)v}{u^2 - v^2}$$

$$\eta = \frac{2v}{(u + v)} \blacktriangleleft$$

$\eta = 1$ when $u = v$. The exhaust, having zero velocity, carries no power away.

PROBLEM 14.104

In a rocket, the kinetic energy imparted to the consumed and ejected fuel is wasted as far as propelling the rocket is concerned. The useful power is equal to the product of the force available to propel the rocket and the speed of the rocket. If v is the speed of the rocket and u is the relative speed of the expelled fuel, show that the mechanical efficiency of the rocket is $\eta = 2uv/(u^2 + v^2)$. Explain why $\eta = 1$ when $u = v$.

SOLUTION

Let F be the thrust force and $\frac{dm}{dt}$ be the mass flow rate.

Absolute velocity of exhaust: $v_e = u - v$

Thrust force: $F = \frac{dm}{dt}u$

Power of thrust force: $P_1 = Fv = \frac{dm}{dt}uv$

Power associated with exhaust:

$$P_2(\Delta t) = \frac{1}{2}(\Delta m)v_e^2 = \frac{1}{2}(\Delta m)(u - v)^2$$

$$P_2 = \frac{1}{2}\frac{dm}{dt}(u - v)^2$$

Total power supplied by engine:

$$P = P_1 + P_2$$

$$P = \frac{dm}{dt}\left[uv - \frac{1}{2}(u - v)^2\right] = \frac{1}{2}\frac{dm}{dt}(u^2 - v^2)$$

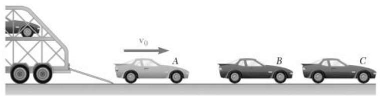
Mechanical efficiency:

$$\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_1}{P}$$

$$\eta = \frac{2uv}{(u^2 + v^2)} \blacktriangleleft$$

$\eta = 1$ when $u = v$. The exhaust, having zero velocity, carries no power away.

PROBLEM 14.105



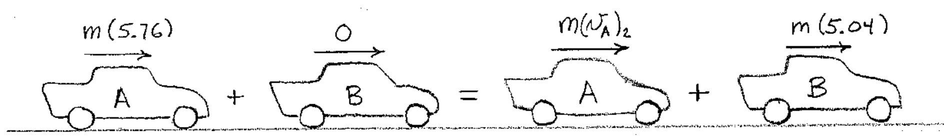
Three identical cars are being unloaded from an automobile carrier. Cars *B* and *C* have just been unloaded and are at rest with their brakes off when car *A* leaves the unloading ramp with a velocity of 5.76 ft/s and hits car *B*, which hits car *C*. Car *A* then again hits car *B*. Knowing that the velocity of car *B* is 5.04 ft/s after the first collision, 0.630 ft/s after the second collision, and 0.709 ft/s after the third collision, determine (*a*) the final velocities of cars *A* and *C*, (*b*) the coefficient of restitution for each of the collisions.

SOLUTION

There are no horizontal forces acting. Horizontal momentum is conserved.

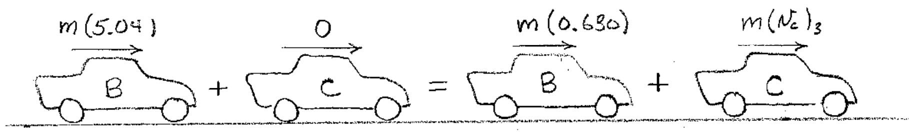
(a) *Velocities*:

Event 1⟶2: Car *A* hits car *B*.



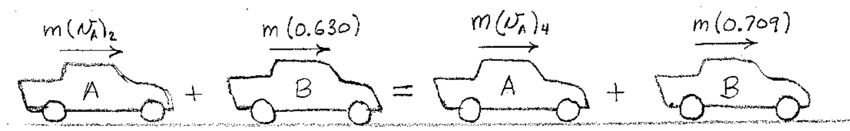
$$m(5.76) + 0 = m(v_A)_2 + m(5.04) \qquad (\mathbf{v}_A)_2 = 0.720 \text{ ft/s} \longrightarrow$$

Event 2⟶3: Car *B* hits car *C*.



$$m(5.04) + 0 = m(0.630) + m(v_C)_3 \qquad (\mathbf{v}_C)_3 = 4.41 \text{ ft/s} \longrightarrow \blacktriangleleft$$

Event 3⟶4: Car *A* hits car *B* again.



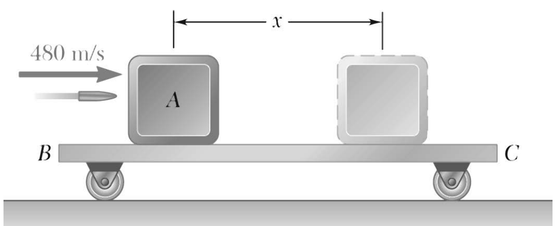
$$m(0.720) + m(0.630) = m(v_A)_4 + m(0.709) \qquad (\mathbf{v}_A)_4 = 0.641 \text{ ft/s} \longrightarrow \blacktriangleleft$$

(*b*) *Coefficients of restitution*:

Event 1⟶2: $$e_{1\to 2} = \left| \frac{(v_A)_2 - (v_B)_2}{(v_A)_1 - (v_B)_1} \right| = \frac{5.04 - 0.720}{5.76 - 0} \qquad e_{1\to 2} = 0.750 \blacktriangleleft$$

Event 2⟶3: $$e_{2\to 3} = \left| \frac{(v_B)_3 - (v_C)_3}{(v_B)_2 - (v_C)_2} \right| = \frac{4.41 - 0.630}{5.04 - 0} \qquad e_{2\to 3} = 0.750 \blacktriangleleft$$

Event 3⟶4: $$e_{3\to 4} = \left| \frac{(v_A)_4 - (v_B)_4}{(v_A)_3 - (v_B)_3} \right| = \frac{0.709 - 0.641}{0.720 - 0.630} \qquad e_{3\to 4} = 0.756 \blacktriangleleft$$

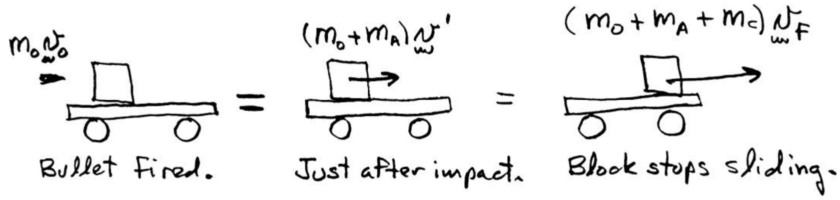


PROBLEM 14.106

A 30-g bullet is fired with a velocity of 480 m/s into block *A*, which has a mass of 5 kg. The coefficient of kinetic friction between block *A* and cart *BC* is 0.50. Knowing that the cart has a mass of 4 kg and can roll freely, determine (*a*) the final velocity of the cart and block, (*b*) the final position of the block on the cart.

SOLUTION

(*a*) Conservation of linear momentum:



$$m_0 v_0 = (m_0 + m_A)v' = (m_0 + m_A + m_C)v_f$$

$$(0.030 \text{ kg})(480 \text{ m/s}) = (5.030 \text{ kg})v' = (9.030 \text{ kg})v_f$$

$$v' = \frac{0.030}{5.030}(480 \text{ m/s}) = 2.863 \text{ m/s}$$

$$v_f = \frac{0.030}{9.030}(480 \text{ m/s}) = 1.5947 \text{ m/s}$$

$v_f = 1.595$ m/s ◀

(*b*) Work-energy principle:

Just after impact:

$$T' = \frac{1}{2}(m_0 + m_A)v'^2$$
$$= \frac{1}{2}(5.030 \text{ kg})(2.863 \text{ m/s})^2$$
$$= 20.615 \text{ J}$$

Final kinetic energy:

$$T_f = \frac{1}{2}(m_0 + m_A + m_C)\frac{1}{2}v_f^2$$
$$= \frac{1}{2}(9.030 \text{ kg})(1.5947 \text{ m/s})^2$$
$$= 11.482 \text{ J}$$

Work of friction force:

$$F = \mu_k N$$
$$= \mu_k (m_0 + m_A)g$$
$$= 0.50(5.030)(9.81)$$
$$= 24.672 \text{ N}$$

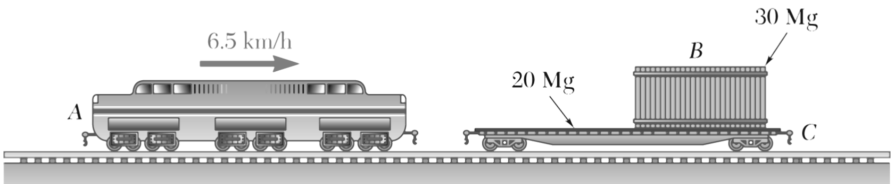
Work $= U = -Fx = -24.672x$

$$T' + U = T_f: \quad 20.615 - 24.672x = 11.482$$

$x = 0.370$ m ◀

PROBLEM 14.107

An 80-Mg railroad engine *A* coasting at 6.5 km/h strikes a 20-Mg flatcar *C* carrying a 30-Mg load *B* which can slide along the floor of the car $(\mu_k = 0.25)$. Knowing that the car was at rest with its brakes released and that it automatically coupled with the engine upon impact, determine the velocity of the car (*a*) immediately after impact, (*b*) after the load has slid to a stop relative to the car.



SOLUTION

The masses are the engine $(m_A = 80\times10^3 \text{ kg})$, the load $(m_B = 30\times10^3 \text{ kg})$, and the flat car $(m_C = 20\times10^3 \text{ kg})$.

Initial velocities:
$$(v_A)_0 = 6.5 \text{ km/h} = 1.80556 \text{ m/s}$$
$$(v_B)_0 = (v_C)_0 = 0.$$

No horizontal external forces act on the system during the impact and while the load is sliding relative to the flat car. Momentum is conserved.

Initial momentum:
$$m_A(v_A)_0 + m_B(0) + m_C(0) = m_A(v_A)_0 \quad (1)$$

(*a*) Let v' be the common velocity of the engine and flat car immediately after impact. Assume that the impact takes place before the load has time to acquire velocity.

Momentum immediately after impact:

$$m_A v' + m_B(0) + m_C v' = (m_A + m_C)v' \quad (2)$$

Equating (1) and (2) and solving for v',

$$v' = \frac{m_A(v_A)_0}{m_A + m_C} = \frac{(80\times10^3)(1.80556)}{(100\times10^3)} = 1.44444 \text{ m/s}$$

$\mathbf{v}' = 5.20 \text{ km/h} \longrightarrow$ ◀

(*b*) Let v_f be the common velocity of all three masses after the load has slid to a stop relative to the car.

Corresponding momentum:

$$m_A v_f + m_B v_f + m_C v_f = (m_A + m_B + m_C)v_f \quad (3)$$

PROBLEM 14.107 (Continued)

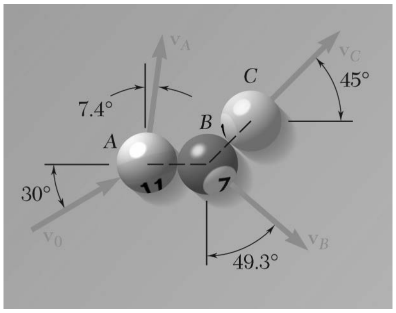
Equating (1) and (3) and solving for v_f,

$$v_f = \frac{m_A (v_A)_0}{m_A + m_B + m_C}$$

$$= \frac{(80\times10^3)(1.80556)}{(130\times10^3)}$$

$$= 1.11111 \text{ m/s}$$

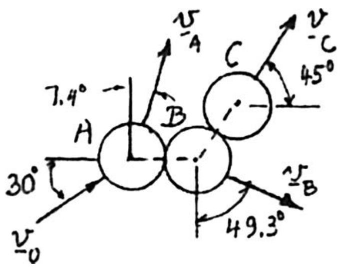
$\mathbf{v}_f = 4.00 \text{ km/h} \longrightarrow$ ◀



PROBLEM 14.108

In a game of pool, ball A is moving with a velocity $\mathbf{v}_0$ when it strikes balls B and C which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated and that $v_0 = 12$ ft/s and $v_C = 6.29$ ft/s, determine the magnitude of the velocity of (*a*) ball A, (*b*) ball B.

SOLUTION



Conservation of linear momentum. In x direction:

$$m(12 \text{ ft/s})\cos 30° = mv_A \sin 7.4° + mv_B \sin 49.3° + m(6.29)\cos 45°$$

$$0.12880v_A + 0.75813v_B = 5.9446 \quad (1)$$

In y direction:

$$m(12 \text{ ft/s})\sin 30° = mv_A \cos 7.4° - mv_B \cos 49.3° + m(6.29)\sin 45°$$

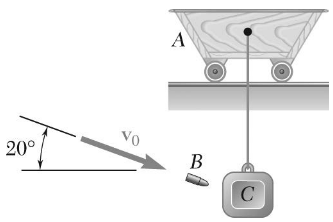
$$0.99167v_A - 0.65210v_B = 1.5523 \quad (2)$$

(*a*) Multiply (1) by 0.65210, (2) by 0.75813, and add:

$$0.83581\, v_A = 5.0533 \qquad v_A = 6.05 \text{ ft/s} \blacktriangleleft$$

(*b*) Multiply (1) by 0.99167, (2) by –0.12880, and add:

$$0.83581\, v_B = 5.6951 \qquad v_B = 6.81 \text{ ft/s} \blacktriangleleft$$

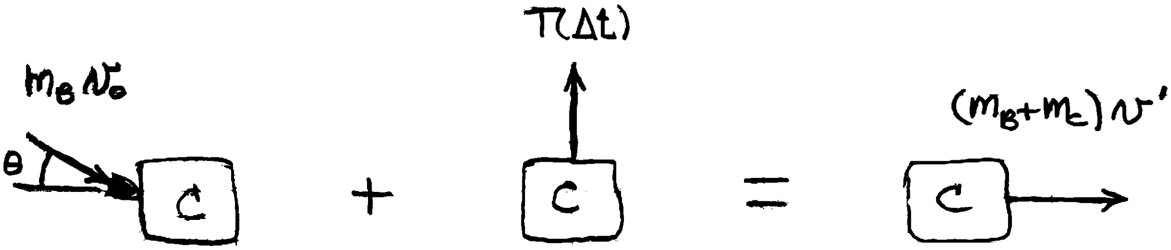


PROBLEM 14.109

Mass C, which has a mass of 4 kg, is suspended from a cord attached to cart A, which has a mass of 5 kg and can roll freely on a frictionless horizontal track. A 60-g bullet is fired with a speed v_0 = 500 m/s and gets lodged in block C. Determine (a) the velocity of C as it reaches its maximum elevation, (b) the maximum vertical distance h through which C will rise.

SOLUTION

Consider the impact as bullet B hits mass C. Apply the principle of impulse-momentum to the two particle system.



$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$

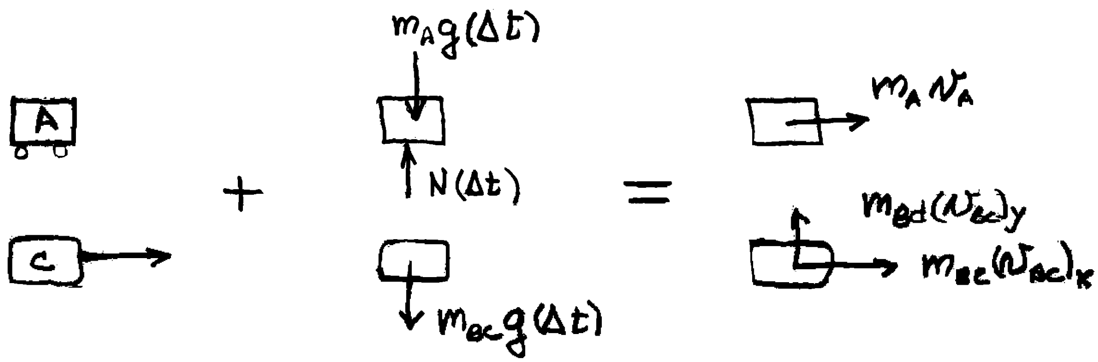
Using both B and C and taking horizontal components gives

$$m_B v_0 \cos\theta + O = (m_B + m_C)v' = m_{BC} v'$$

$$v' = \frac{m_B v_0 \cos\theta}{m_{BC}}$$

$$= \frac{(0.060\text{ kg})(500\text{ m/s})\cos 20°}{(4.06\text{ kg})} = 6.9435\text{ m/s}$$

Now consider the system of m_A and m_{BC} after the impact, and apply to impulse momentum principle.



$$\Sigma m\mathbf{v}_2 + \Sigma \mathbf{Imp}_{2\to 3} = \Sigma m\mathbf{v}_3$$

Horizontal components: $+\longrightarrow$

$$m_{BC}v' + 0 = m_A v_A + m_{BC}v_{cx}$$

$$v_A = \frac{m_{BC}}{m_A}(v' - v_{cx})$$

$$= \frac{4.06}{5}(6.9435 - v_{cx})$$

$$v_A = 5.6381 - 0.812v_{cx} \quad \text{in m/s} \qquad (1)$$

(*a*) At maximum elevation.

Both particles have the same velocity, thus

$$v_{cx} = v_A$$
$$v_A = 5.6381 - 0.812\,v_A$$

$$v_A = 3.1115 \text{ m/s} \qquad v_A = 3.11 \text{ m/s} \blacktriangleleft$$

(*b*) Conservation of energy: $T_2 + V_2 = T_3 + V_3$

$$T_2 = \frac{1}{2}m_A(0) + \frac{1}{2}m_{BC}(v')^2$$

$$= \frac{1}{2}(4.06)(6.9435)^2 = 97.871 \text{ J}$$

$$V_2 = 0 \qquad \text{(datum)}$$

$$T_3 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_{BC}(v_{Bx}^2 + v_{By}^2)^2$$

$$= \frac{1}{2}(5)(3.1115)^2 + \frac{1}{2}(4.06)[(3.1115)^2 + 0] = 43.857 \text{ J}$$

$$V_3 = m_{BC}gh = (4.06)(9.81)h = 39.829 \text{ h}$$

$$97.871 + 0 = 43.857 + 39.829 \text{ h}$$

$$h = 1.356 \text{ m} \blacktriangleleft$$

Another method: We observe that no external horizontal forces are exerted on the system consisting of *A*, *B*, and *C*. Thus the horizontal component of the velocity of the mass center remains constant.

$$m = m_A + m_B + m_C = 5 + 0.06 + 4 = 9.06 \text{ kg}$$

$$\bar{v}_x = \frac{m_B v_0 \cos\theta}{m_A + m_B + m_C} = \frac{(0.060 \text{ kg})(500 \text{ m/s})\cos 20°}{9.06 \text{ kg}} = 3.1115 \text{ m/s}$$

(*a*) At maximum elevation, v_A and v_{BC} are equal.

$$v_A = 3.1115 \text{ m/s} \qquad \mathbf{v}_A = 3.11 \text{ m/s} \longrightarrow \blacktriangleleft$$

Immediately after the impact of B on C, the velocity v_A is zero.

$$(m_B + m_C)v' = (m_A + m_B + m_C)\overline{v}_x$$

$$v' = \frac{m_A + m_B + m_C}{m_B + m_C}\overline{v}_x = \frac{9.06}{4.06}(3.1115 \text{ m/s}) = 6.9435 \text{ m/s}$$

(*b*) Principle of work and energy: $T_2 + V_2 = T_3 + V_3$

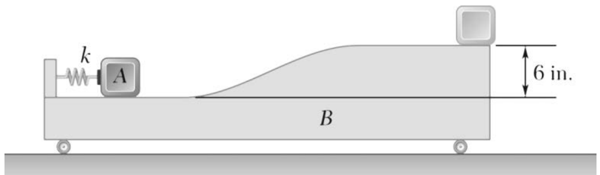
$T_2, V_2,$ and V_3 are calculated as before.

For T_3 we note that the velocities $\mathbf{v}'_A$ and $\mathbf{v}'_{BC}$ relative to the mass center are zero. Thus, T_3 is given by

$$T_3 = \frac{1}{2}m\overline{\mathbf{v}}^2 = \frac{1}{2}(9.06)(3.1115)^2 = 43.857 \text{ J}$$

As before, h is found to be $h = 1.356 \text{ m}$ ◀

PROBLEM 14.110



A 15-lb block B is at rest and a spring of constant $k = 72$ lb/in. is held compressed 3 in. by a cord. After 5-lb block A is placed against the end of the spring, the cord is cut causing A and B to move. Neglecting friction, determine the velocities of blocks A and B immediately after A leaves B.

SOLUTION

$$m_A = \frac{5}{32.2} = 0.15528 \text{ lb} \cdot \text{s}^2/\text{ft}$$

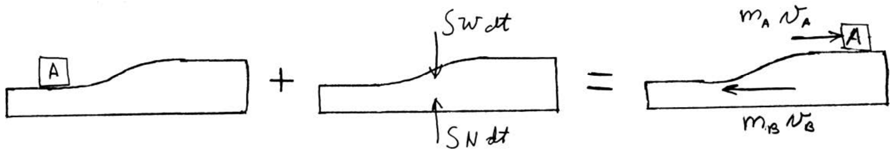
$$m_B = \frac{15}{32.2} = 0.46584 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$k = 72 \text{ lb/in} = 864 \text{ lb/ft}$$

$$e = 3 \text{ in.} = 0.25 \text{ ft}$$

$$h = 6 \text{ in.} = 0.5 \text{ ft}$$

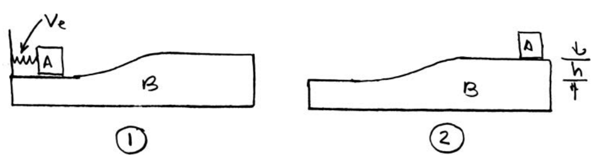
Conservation of linear momentum:



Horizontal components $\xrightarrow{+}$: $\quad 0 + 0 = m_A v_A - m_B v_B$

$$v_B = \frac{m_A}{m_B} v_A = \frac{1}{3} v_A$$

Conservation of energy:



State 1: $\quad V_{1e} = \frac{1}{2} k e^2 = \frac{1}{2}(864)(0.25)^2 = 27 \text{ ft} \cdot \text{lb}$

$$V_{1g} = 0$$

$$T_1 = 0$$

State 2: $\quad V_{2e} = 0$

$$V_{2g} = W_A h = (5)(0.5) = 2.5 \text{ ft} \cdot \text{lb}$$

$$T_2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$= \frac{1}{2}(0.15528) v_A^2 + \frac{1}{2}(0.46584)\left(\frac{v_A}{3}\right)^2 = 0.10352 v_A^2$$

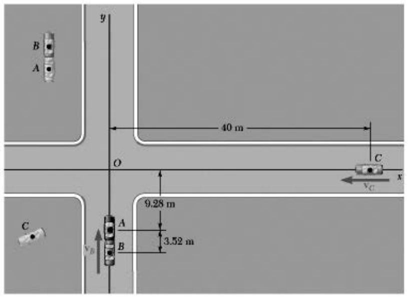
PROBLEM 14.110 (Continued)

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + 27 = 0.10352 v_A^2 + 2.5$

$$v_A^2 = 236.67 \text{ ft}^2$$

$\mathbf{v}_A = 15.38 \text{ ft/s} \longrightarrow$ ◀

$\mathbf{v}_B = 5.13 \text{ ft/s} \longleftarrow$ ◀



PROBLEM 14.111

Car A was at rest 9.28 m south of the Point O when it was struck in the rear by car B, which was traveling north at a speed v_B. Car C, which was traveling west at a speed v_C, was 40 m east of Point O at the time of the collision. Cars A and B stuck together and, because the pavement was covered with ice, they slid into the intersection and were struck by car C which had not changed its speed. Measurements based on a photograph taken from a traffic helicopter shortly after the second collision indicated that the positions of the cars, expressed in meters, were $\mathbf{r}_A = -10.1\mathbf{i} + 16.9\mathbf{j}$, $\mathbf{r}_B = -10.1\mathbf{i} + 20.4\mathbf{j}$, and $\mathbf{r}_C = -19.8\mathbf{i} - 15.2\mathbf{j}$. Knowing that the masses of cars A, B, and C are, respectively, 1400 kg, 1800 kg, and 1600 kg, and that the time elapsed between the first collision and the time the photograph was taken was 3.4 s, determine the initial speeds of cars B and C.

SOLUTION

Mass center at time of first collision.

$$(m_A + m_B + m_C)\bar{\mathbf{r}}_1 = m_A(\mathbf{r}_A)_1 + m_B(\mathbf{r}_B)_1 + m_C(\mathbf{r}_C)_1$$

$$4800\,\bar{\mathbf{r}}_1 = (1400)(-9.28\mathbf{j}) + (1800)(-12.8\mathbf{j}) + (1600)(40\mathbf{i})$$

$$\bar{\mathbf{r}}_1 = (13.3333\text{ m})\mathbf{i} - (7.5067\text{ m})\mathbf{j}$$

Mass center at time of photo.

$$(m_A + m_B + m_C)\bar{\mathbf{r}}_2 = m_A(\mathbf{r}_A)_2 + m_B(\mathbf{r}_B)_2 + m_C(\mathbf{r}_C)_2$$

$$4800\,\bar{\mathbf{r}}_2 = (1400)(-10.1\mathbf{i} + 16.9\mathbf{j}) + (1800)(-10.1\mathbf{i} + 20.4\mathbf{j}) + (1600)(-19.8\mathbf{i} - 15.2\mathbf{j})$$

$$\bar{\mathbf{r}}_2 = -(13.3333\text{ m})\mathbf{i} + (7.5125\text{ m})\mathbf{j}$$

Since no external horizontal forces act, momentum is conserved and the mass center moves at constant velocity.

$$(m_A + m_B + m_C)\bar{\mathbf{v}} = m_A(\mathbf{v}_A)_1 + m_B(\mathbf{v}_B)_1 + m_C(\mathbf{v}_C)_1 \quad (1)$$

$$\bar{\mathbf{r}}_2 - \bar{\mathbf{r}}_1 = \bar{\mathbf{v}}t \quad (2)$$

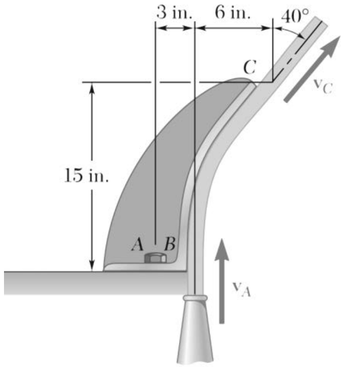
Combining (1) and (2),

$$(m_A + m_B + m_C)(\bar{\mathbf{r}}_2 - \bar{\mathbf{r}}_1) = [m_A(\mathbf{v}_A)_1 + m_B(\mathbf{v}_B)_1 + m_C(\mathbf{v}_C)_1]\,t$$

$$(4800)(-26.6667\mathbf{i} + 15.0192\mathbf{j}) = [0 + (1800)(v_B)_1\mathbf{j} - (1600)(v_C)_1\mathbf{i}](3.4)$$

Components. $\quad \mathbf{j}$: $72092 = 6120(v_B)_1, \quad (v_B)_1 = 11.78\text{ m/s}, \qquad v_B = 42.4\text{ km/h}$ ◀

$\mathbf{i}$: $-128000 = -5440\,(v_C)_1, \quad (v_C)_1 = 23.53\text{ m/s}, \qquad v_C = 84.7\text{ km/h}$ ◀



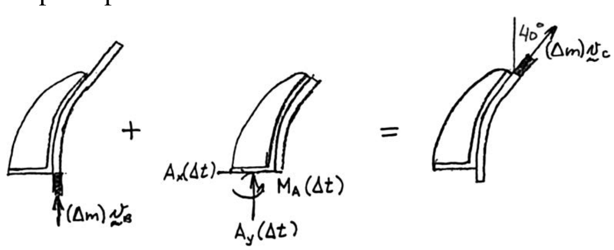
PROBLEM 14.112

The nozzle shown discharges water at the rate of 200 gal/min. Knowing that at both B and C the stream of water moves with a velocity of magnitude 100 ft/s, and neglecting the weight of the vane, determine the force-couple system which must be applied at A to hold the vane in place $(1\text{ ft}^3 = 7.48\text{ gal})$.

SOLUTION

$$Q = \frac{200\text{ gal/min}}{(7.48\text{ gal/ft}^3)(60\text{ s/min})}$$
$$= 0.44563\text{ ft}^3\text{/s}$$
$$\frac{dm}{dt} = \frac{\gamma Q}{g}$$
$$= \frac{(62.4\text{ lb/ft}^3)(0.44563\text{ ft}^3\text{/s})}{32.2\text{ ft/s}^2}$$
$$= 0.8636\text{ lb}\cdot\text{s/ft}$$
$$\mathbf{v}_B = (100\text{ ft/s})\mathbf{j}$$
$$\mathbf{v}_C = (100\text{ ft/s})(\sin 40°\,\mathbf{i} + \cos 40°\,\mathbf{j})$$

Apply the impulse-momentum principle.



$\xrightarrow{+}$ x components: $\quad 0 + A_x(\Delta t) = (\Delta m)(100\sin 40°)$

$$A_x = \frac{\Delta m}{\Delta t}(100\sin 40°)$$
$$= (0.8636)(100\sin 40°)$$

$A_x = 55.5\text{ lb} \longrightarrow$

$+\uparrow$ y components: $\quad (\Delta m)(100) + A_y(\Delta t) = (\Delta m)(100\cos 40°)$

$$A_y = \frac{\Delta m}{\Delta t}(100)(\cos 40° - 1)$$
$$= (0.8636)(100)(\cos 40° - 1)$$
$$= -20.2\text{ lb}$$

$A_y = 20.2\text{ lb} \downarrow$

PROBLEM 14.112 (Continued)

+↶ Moments about A:

$$\left(\frac{3}{12}\right)(\Delta m)(100) + M_A(\Delta t) = \left(\frac{9}{12}\right)(\Delta m)(100\cos 40^\circ) - \left(\frac{15}{12}\right)(\Delta m)(100\sin 40^\circ)$$

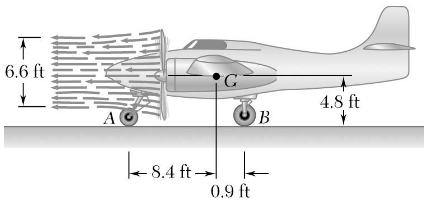
$$M_A = \left(\frac{\Delta m}{\Delta t}\right)(75\cos 40^\circ - 125\sin 40^\circ - 25)$$

$$= (0.8636)(-47.895)$$

$$= -41.36\ \text{lb}\cdot\text{ft}$$

$\mathbf{M}_A = 41.4\ \text{lb}\cdot\text{ft}$ ↻ ◀

$\mathbf{A} = 59.1\ \text{lb}$ ↘ 20.0° ◀



PROBLEM 14.113

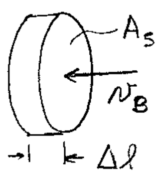
Prior to takeoff the pilot of a 6000-lb twin-engine airplane tests the reversible-pitch propellers with the brakes at Point B locked. Knowing that the velocity of the air in the two 6.6-ft-diameter slipstreams is 60 ft/s and that Point G is the center of gravity of the airplane, determine the reactions at Points A and B. Assume $\gamma = 0.075\ \text{lb/ft}^3$ and neglect the approach velocity of the air.

SOLUTION

Let F be the force exerted on the slipstream of one engine.

$$F = \frac{dm}{dt}(v_B - v_A)$$

Calculation of $\dfrac{dm}{dt}$. mass = density × volume = density × area × length



$$\Delta m = \rho A_B(\Delta l) = \rho A_B v_B(\Delta t) = \frac{\gamma A_B v_B(\Delta t)}{g}$$

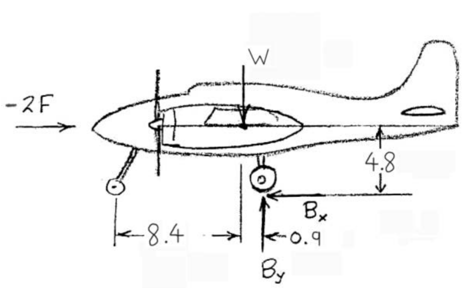
$$\frac{\Delta m}{\Delta t} = \frac{\gamma A_B v_B}{g} \quad \text{or} \quad \frac{dm}{dt} = \frac{\gamma}{g}\left(\frac{\pi}{4}D^2\right)v_B$$

Assume that v_A, the velocity far upstream, is negligible.

$$F = \frac{\gamma}{g}\left(\frac{\pi}{4}D^2\right)v_B\left(v_B - 0\right) = \left(\frac{0.075}{32.2}\right)\left(\frac{\pi}{4}\right)(6.6)^2(60)^2 = 286.87\ \text{lb} \leftarrow$$

The force exerted by two slipstreams on the airplane is $-2F$. $\quad -2F = 573.74\ \text{lb} \rightarrow$

Statics.



$$+\circlearrowleft\ \Sigma M_B = 0:$$

$$A = \frac{1}{9.3}[(0.9)(6000) - (4.8)(573.74)]$$

$$= 284.5\ \text{lb} \qquad \mathbf{A} = 285\ \text{lb} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} F_x = 0: \quad -2F - B_x = 0$$

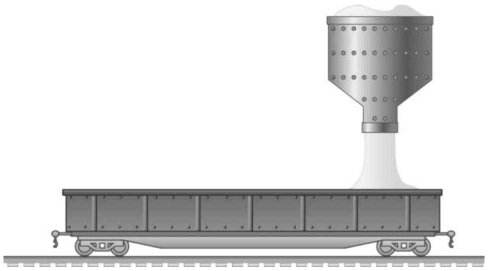
$$B_x = -2F = 573.74\ \text{lb} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A + B_y - W = 284.5 + B_y - (6000) = 0$$

$$B_y = 5715.5\ \text{lb}\uparrow$$

$$\mathbf{B} = [573.74\ \text{lb} \leftarrow] + [5715.5\ \text{lb}\uparrow]$$

$$\mathbf{B} = 5740\ \text{lb} \measuredangle\ 84.3° \blacktriangleleft$$

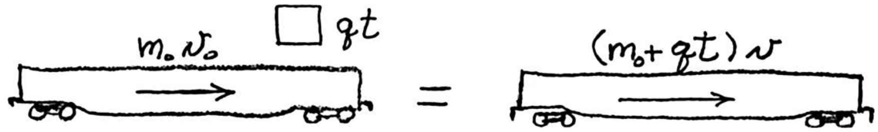


PROBLEM 14.114

A railroad car of length L and mass m_0 when empty is moving freely on a horizontal track while being loaded with sand from a stationary chute at a rate $dm/dt = q$. Knowing that the car was approaching the chute at a speed v_0, determine (*a*) the mass of the car and its load after the car has cleared the chute, (*b*) the speed of the car at that time.

SOLUTION

Consider the conservation of the horizontal component of momentum of the railroad car of mass m_0 and the sand mass qt.



$$m_0 v_0 = (m_0 + qt)v \qquad v = \frac{m_0 v_0}{m_0 + qt} \tag{1}$$

$$\frac{dx}{dt} = v = \frac{m_0 v_0}{m_0 + qt}$$

Integrating, using $\quad x_0 = 0 \quad$ and $\quad x = L$ when $t = t_L$,

$$L = \int_0^{t_L} v\,dt = \int_0^{t_L} \frac{m_0 v_0}{m_0 + qt}\,dt = \frac{m_0 v_0}{q}[\ln(m_0 + qt_L) - \ln m_0]$$

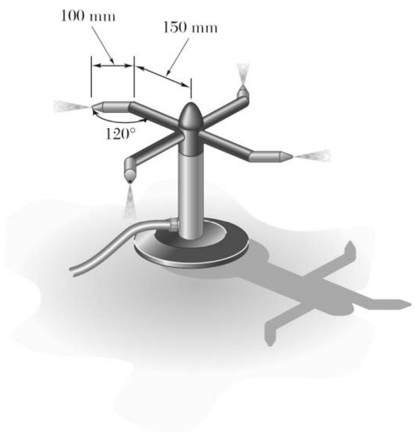
$$= \frac{m_0 v_0}{q} \ln \frac{m_0 + qt_L}{m_0}$$

$$\ln \frac{m_0 + qt_L}{m_0} = \frac{qL}{m_0 v_0} \qquad \frac{m_0 + qt_L}{m_0} = e^{qL/m_0 v_0}$$

(*a*) Final mass of railroad car and sand $\qquad m_0 + qt_L = m_0 e^{qL/m_0 v_0}$ ◀

(*b*) Using Eq. (1), $\qquad v_L = \frac{m_0 v_0}{m_0 + qt_L} = \frac{m_0 v_0}{m_0} e^{-qL/m_0 v_0}$

$$v_L = v_0 e^{-qL/m_0 v_0}$$ ◀



PROBLEM 14.115

A garden sprinkler has four rotating arms, each of which consists of two horizontal straight sections of pipe forming an angle of 120° with each other. Each arm discharges water at a rate of 20 L/min with a velocity of 18 m/s relative to the arm. Knowing that the friction between the moving and stationary parts of the sprinkler is equivalent to a couple of magnitude $M = 0.375\ \text{N}\cdot\text{m}$, determine the constant rate at which the sprinkler rotates.

SOLUTION

The flow through each arm is 20 L/min.

$$Q = \frac{20\ \text{L/min}}{1000\ \text{L/m}^3} \times \frac{1\,\text{min}}{60\ \text{s}} = 333.33\times 10^{-6}\ \text{m}^3/\text{s}$$

$$\frac{dm}{dt} = \rho Q = (1000\ \text{kg/m}^3)(333.33\times 10^{-6})$$

$$= 0.33333\ \text{kg/s}$$

Consider the moment about O exerted on the fluid stream of one arm. Apply the impulse-momentum principle. Compute moments about O. First, consider the geometry of triangle OAB. Using first the law of cosines,

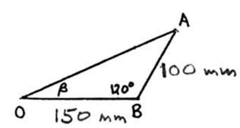
$$(OA)^2 = 150^2 + 100^2 - (2)(150)(100)\cos 120°$$

$$OA = 217.95\ \text{mm} = 0.21795\ \text{m}$$

Law of sines:

$$\frac{\sin\beta}{100} = \frac{\sin 120°}{217.95}$$

$$\beta = 23.413°, \quad \alpha = 60° - \beta = 36.587°$$



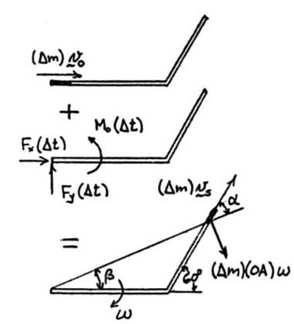
$+\circlearrowleft$ Moments about O:

$$(\Delta m)(v_O)(0) + M_O(\Delta t) = (OA)(\Delta m)v_s \sin\alpha - (OA)(\Delta m)(OA)\omega$$

$$M_O = \frac{\Delta m}{\Delta t}[(OA)v_s \sin\alpha - (OA)^2\omega]$$

$$= (0.33333)[(0.21795)(18)\sin 36.587° - (0.21795)^2\omega]$$

$$= 0.77945 - 0.015834\omega$$



Moment that the stream exerts on the arm is $-M_O$.

Balance of the friction couple and the four streams

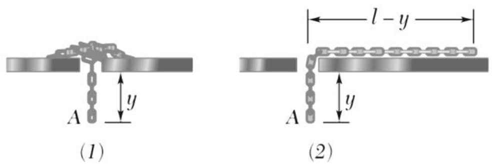
$$M_F - 4M_O = 0$$

$$0.375 - 4(0.77945 - 0.015834\omega) = 0$$

$$\omega = 43.305\ \text{rad/s}$$

$\omega = 414$ rpm ◀

PROBLEM 14.116

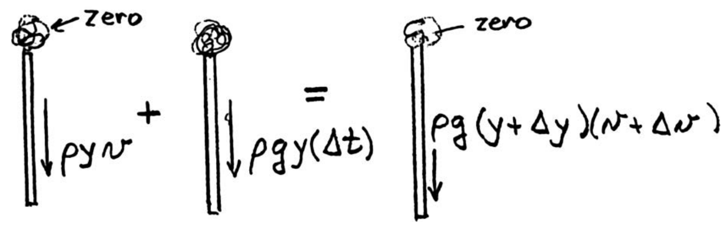


A chain of length l and mass m falls through a small hole in a plate. Initially, when y is very small, the chain is at rest. In each case shown, determine (*a*) the acceleration of the first link A as a function of y, (*b*) the velocity of the chain as the last link passes through the hole. In case 1, assume that the individual links are at rest until they fall through the hole; in case 2, assume that at any instant all links have the same speed. Ignore the effect of friction.

SOLUTION

Let ρ be the mass per unit length of chain. Assume that the weight of any chain above the hole is supported by the floor. It and the corresponding upward reaction of the floor are not shown in the diagrams.

Case 1: Apply the impulse-momentum principle to the entire chain.



$$\rho yv + \rho gy\Delta t = \rho(y+\Delta y)(v+\Delta v)$$

$$= \rho yv + \rho(\Delta y)v + \rho y(\Delta v) + \rho(\Delta y)(\Delta v)$$

$$\rho gy = \rho\frac{\Delta y}{\Delta t}v + \rho y\frac{\Delta v}{\Delta t} + \rho\frac{(\Delta y)(\Delta v)}{\Delta t}$$

Let $\Delta t \to 0$.
$$\rho gy = \rho\frac{dy}{dt}v + \rho y\frac{dv}{dt}$$

$$= \rho\frac{d}{dt}(yv)$$

Multiply both sides by yv.
$$\rho gy^2 v = \rho yv\frac{d}{dt}(yv)$$

Let $v = \frac{dy}{dt}$ on left hand side.
$$\rho gy^2\frac{dy}{dt} = \rho yv\frac{d}{dt}(yv)$$

Integrate with respect to time.
$$\rho g\int y^2 dy = \rho\int(yv)\,d(yv)$$

$$\frac{1}{3}\rho gy^3 = \frac{1}{2}\rho(yv)^2 \quad \text{or} \quad v^2 = \frac{2}{3}gy \qquad (1)$$

Differentiate with respect to time.
$$2v\frac{dv}{dt} = \frac{2}{3}g\frac{dy}{dt} = \frac{2}{3}gv$$

PROBLEM 14.116 (Continued)

(*a*) $$a = \frac{dv}{dt} = \frac{1}{3}g$$ $\mathbf{a} = 0.333g \downarrow$ ◀

(*b*) Set $y = l$ in Eq. (1). $$v^2 = \frac{2}{3}gl$$ $\mathbf{v} = 0.817\sqrt{gl} \downarrow$ ◀

Case 2: Apply conservation of energy using the floor as the level from which the potential energy is measured.

$$T_1 = 0 \qquad V_1 = 0$$

$$T_2 = \frac{1}{2}mv^2 \quad V_2 = -\rho gy\frac{y}{2}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 = \frac{1}{2}mv^2 - \frac{1}{2}\rho gy^2 \qquad v^2 = \frac{\rho gy^2}{m} = \frac{gy^2}{l} \qquad (2)$$

Differentiating with respect to *y*, $$2v\frac{dv}{dy} = \frac{2gy}{l}$$

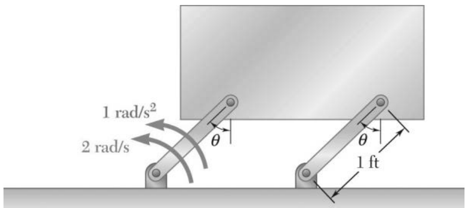
(*a*) Acceleration: $$a = v\frac{dv}{dy} = \frac{gy}{l}$$ $\mathbf{a} = \frac{gy}{l} \downarrow$ ◀

(*b*) Setting $y = l$ in Eq. (2), $$v^2 = gl$$ $\mathbf{v} = \sqrt{gl} \downarrow$ ◀

Note: The impulse-momentum principle may be used to obtain the force that the edge of the hole exerts on the chain.

CHAPTER 15

PROBLEM 15.CQ1



A rectangular plate swings from arms of equal length as shown below. What is the magnitude of the angular velocity of the plate?

(*a*) 0 rad/s

(*b*) 1 rad/s

(*c*) 2 rad/s

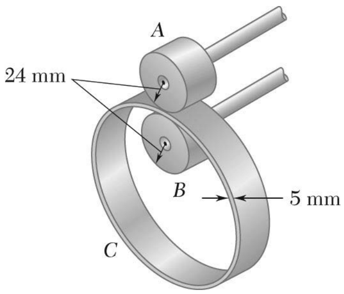
(*d*) 3 rad/s

(*e*) Need to know the location of the center of gravity

SOLUTION

Answer: (*a*) ◀

PROBLEM 15.CQ2



Knowing that wheel *A* rotates with a constant angular velocity and that no slipping occurs between ring *C* and wheel *A* and wheel *B*, which of the following statements concerning the angular speeds are true?

(*a*) $\omega_a = \omega_b$

(*b*) $\omega_a > \omega_b$

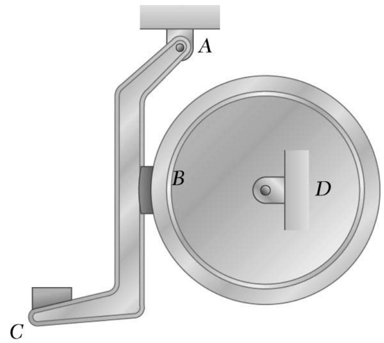
(*c*) $\omega_a < \omega_b$

(*d*) $\omega_a = \omega_c$

(*e*) the contact points between *A* and *C* have the same acceleration

SOLUTION

Answer: (*b*) ◀



PROBLEM 15.1

The brake drum is attached to a larger flywheel that is not shown. The motion of the brake drum is defined by the relation $\theta = 36t - 1.6t^2$, where θ is expressed in radians and t in seconds. Determine (*a*) the angular velocity at $t = 2$ s, (*b*) the number of revolutions executed by the brake drum before coming to rest.

SOLUTION

Given: $\theta = 36t - 1.6t^2$ radians

Differentiate to obtain the angular velocity.

$$\omega = \frac{d\theta}{dt} = 36 - 3.2t \quad \text{rad/s}$$

(*a*) At $t = 2$ s, $\omega = 36 - (3.2)(2)$ $\omega = 29.6$ rad/s ◀

(*b*) When the rotor stops, $\omega = 0$.

$0 = 36 - 3.2t$ $t = 11.25$ s

$$\theta = (36)(11.25) - (1.6)(11.25)^2 = 202.5 \text{ radians}$$

In revolutions, $\theta = \frac{202.5}{2\pi}$ $\theta = 32.2$ rev ◀

PROBLEM 15.2

The motion of an oscillating crank is defined by the relation $\theta = \theta_0 \sin(\pi t/T) - (0.5\theta_0)\sin(2\pi t/T)$, where θ is expressed in radians and t in seconds. Knowing that $\theta_0 = 6$ rad and $T = 4$ s, determine the angular coordinate, the angular velocity, and the angular acceleration of the crank when (*a*) $t = 0$, (*b*) $t = 2$ s.

SOLUTION

$$\omega = \frac{d\theta}{dt} = \theta_0 \frac{\pi}{T}\cos\left(\frac{\pi t}{T}\right) - 0.5\theta_0 \frac{2\pi}{T}\cos\left(\frac{2\pi t}{T}\right)$$

$$\alpha = \frac{d\omega}{dt} = -\theta_0\left(\frac{\pi}{T}\right)^2 \sin\left(\frac{\pi t}{T}\right) + 0.5\theta_0\left(\frac{2\pi}{T}\right)^2 \sin\left(\frac{2\pi t}{T}\right)$$

(*a*) $t = 0$:

$\theta = 0$ ◀

$$\omega = 6\frac{\pi}{4} - 0.5(6)\frac{2\pi}{4}$$

$\omega = 0$ ◀

$\alpha = 0$ ◀

(*b*) $t = 2$ s:

$$\theta = 6\sin\left(\frac{2\pi}{4}\right) - 0.5(6)\sin\left(\frac{4\pi}{4}\right) = 6 - 0$$

$\theta = 6.00$ rad ◀

$$\omega = 6\left(\frac{\pi}{4}\right)\cos\left(\frac{2\pi}{4}\right) - 0.5(6)\frac{2\pi}{4}\cos\left(\frac{4\pi}{4}\right)$$

$$= 6\frac{\pi}{4}(0) - 0.5(6)\frac{2\pi}{4}(-1)$$

$$= \frac{6\pi}{4}$$

$\omega = 4.71$ rad/s ◀

$$\alpha = -6\left(\frac{\pi}{4}\right)^2 \sin\left(\frac{2\pi}{4}\right) + 0.5(6)\left(\frac{2\pi}{4}\right)^2 \sin\left(\frac{4\pi}{4}\right)$$

$$= -6\left(\frac{\pi}{4}\right)^2 (1) + 3\left(\frac{2\pi}{4}\right)^2 (0)$$

$$= -\frac{3}{8}\pi^2$$

$\alpha = -3.70$ rad/s^2 ◀

PROBLEM 15.3

The motion of a disk rotating in an oil bath is defined by the relation $\theta = \theta_0(1 - e^{-t/4})$, where θ is expressed in radians and t in seconds. Knowing that $\theta_0 = 0.40$ rad, determine the angular coordinate, velocity, and acceleration of the disk when (*a*) $t = 0$, (*b*) $t = 3$ s, (*c*) $t = \infty$.

SOLUTION

$$\theta = 0.40(1 - e^{-t/4})$$

$$\omega = \frac{d\theta}{dt} = \frac{1}{4}(0.40)e^{-t/4} = 0.10e^{-t/4}$$

$$\alpha = \frac{d\omega}{dt} = -\frac{1}{4}(0.10)e^{-t/4} = -0.025e^{-t/4}$$

(*a*) $t = 0$:

$\theta = 0.40(1 - e^0)$ $\qquad \theta = 0$ ◀

$\omega = 0.10e^0$ $\qquad \omega = 0.1000$ rad/s ◀

$\alpha = -0.025e^0$ $\qquad \alpha = -0.0250$ rad/s^2 ◀

(*b*) $t = 3$ s:

$\theta = 0.40(1 - e^{-3/4})$

$= 0.40(1 - 0.4724)$ $\qquad \theta = 0.211$ rad ◀

$\omega = 0.10e^{-3/4}$

$= 0.10(0.4724)$ $\qquad \omega = 0.0472$ rad/s ◀

$\alpha = -0.025e^{-3/4}$

$= -0.025(0.4724)$ $\qquad \alpha = -0.01181$ rad/s^2 ◀

(*c*) $t = \infty$:

$\theta = 0.40(1 - e^{-\infty})$

$= 0.40(1 - 0)$ $\qquad \theta = 0.400$ rad ◀

$\omega = 0.10e^{-\infty}$ $\qquad \omega = 0$ ◀

$\alpha = -0.025e^{-\infty}$ $\qquad \alpha = 0$ ◀

PROBLEM 15.4

The rotor of a gas turbine is rotating at a speed of 6900 rpm when the turbine is shut down. It is observed that 4 min is required for the rotor to coast to rest. Assuming uniformly accelerated motion, determine (*a*) the angular acceleration, (*b*) the number of revolutions that the rotor executes before coming to rest.

SOLUTION

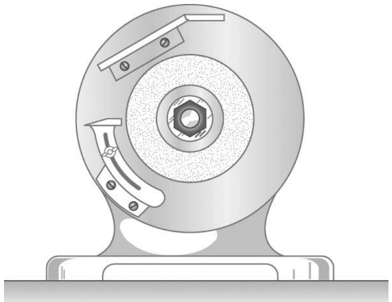
$$\omega_0 = 6900 \text{ rpm}$$
$$= 722.57 \text{ rad/s}$$
$$t = 4 \text{ min} = 240 \text{ s}$$

(*a*) $$\omega = \omega_0 + \alpha t; \quad 0 = 722.57 + \alpha(240)$$

$$\alpha = -3.0107 \text{ rad/s}$$ $\alpha = -3.01 \text{ rad/s}^2$ ◀

(*b*) $$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (722.57)(240) + \frac{1}{2}(-3.0107)(240)^2$$
$$\theta = 173,416 - 86,708 = 86,708 \text{ rad}$$

$$\theta = 86,708 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)$$ $\theta = 13,80 \text{ rev}$ ◀



PROBLEM 15.5

A small grinding wheel is attached to the shaft of an electric motor which has a rated speed of 3600 rpm. When the power is turned on, the unit reaches its rated speed in 5 s, and when the power is turned off, the unit coasts to rest in 70 s. Assuming uniformly accelerated motion, determine the number of revolutions that the motor executes (*a*) in reaching its rated speed, (*b*) in coasting to rest.

SOLUTION

For uniformly accelerated motion,

$$\omega = \omega_0 + \alpha t \qquad (1)$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \qquad (2)$$

(*a*) Data for start up: $\theta_0 = 0,\ \omega_0 = 0,$

At $t = 5$ s, $\omega = 3600 \text{ rpm} = \dfrac{2\pi(3600)}{60} = 120\pi \text{ rad/s}$

From Eq. (1), $120\pi = \alpha(5) \qquad \alpha = 24\pi \text{ rad/s}^2$

From Eq. (2), $\theta = 0 + 0 + \dfrac{1}{2}(24\pi)(5)^2 = 300\pi$ radians

In revolutions, $\theta = \dfrac{300\pi}{2\pi}$ $\qquad \theta = 150$ rev ◀

(*b*) Data for coasting to rest:

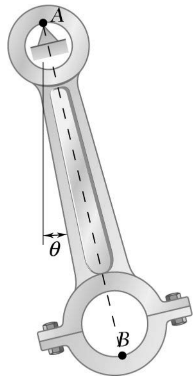
$$\theta_0 = 0, \quad \omega_0 = 120\pi \text{ rad/s}$$

At $t = 70$ s, $\omega = 0$

From Eq. (1), $0 = 120\pi - \alpha(70) \qquad \alpha = \dfrac{120\pi}{70}$ rad/s

From Eq. (2), $\theta = 0 + (120\pi)(70) - \dfrac{(120\pi)(70)^2}{2(70)} = 4200\pi$ radians

In revolutions, $\theta = \dfrac{4200\pi}{2\pi}$ $\qquad \theta = 2100$ rev ◀



PROBLEM 15.6

A connecting rod is supported by a knife-edge at Point A. For small oscillations the angular acceleration of the connecting rod is governed by the relation $\alpha = -6\theta$ where α is expressed in rad/s^2 and θ in radians. Knowing that the connecting rod is released from rest when $\theta = 20°$, determine (*a*) the maximum angular velocity, (*b*) the angular position when $t = 2$ s.

SOLUTION

Angular motion relations:

$$\alpha = \frac{d\omega}{dt} = \frac{\omega d\omega}{d\theta} = -6\theta \qquad (1)$$

Separation of variables ω and θ gives

$$\omega d\omega = -6\theta d\theta$$

Integrating, using $\omega = 0$ when $\theta = \theta_0$,

$$\int_0^{\omega} \omega d\omega = -6\int_{\theta_0}^{\theta} \theta d\theta$$

$$\frac{1}{2}\omega^2 = -3(\theta^2 - \theta_0^2) = 3(\theta_0^2 - \theta^2)$$

$$\omega^2 = 6(\theta_0^2 - \theta^2) \qquad \omega = \sqrt{6(\theta_0^2 - \theta^2)}$$

(*a*) ω is maximum when $\theta = 0$.

Data: $\theta_0 = 20° = 0.34907$ radians

$$\omega_{max}^2 = 6(0.34907^2 - 0) = 0.73108 \text{ rad}^2/\text{s}$$

$\omega_{max} = 0.855$ rad/s ◀

(*b*) From $\omega = \frac{d\theta}{dt}$ we get $dt = \frac{d\theta}{\omega} = \frac{1}{\sqrt{6}} \frac{d\theta}{\sqrt{\theta_0^2 - \theta^2}}$

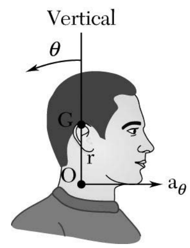
Integrating, using $t = 0$ when $\theta = \theta_0$,

$$\int_0^t dt = \frac{1}{\sqrt{6}} \int_{\theta_0}^{\theta} \frac{d\theta}{\sqrt{\theta_0^2 - \theta^2}}$$

$$t = -\frac{1}{\sqrt{6}} \cos^{2-1} \frac{\theta}{\theta_0}\Bigg|_{\theta_0}^{\theta} = -\frac{1}{\sqrt{6}}\left[0 - \cos^{-1}\frac{\theta}{\theta_0}\right] = -\frac{1}{\sqrt{6}} \cos^{-1}\frac{\theta}{\theta_0}$$

$$\theta = \theta_0 \cos(\sqrt{6}t) = 0.34907 \cos[(\sqrt{6})2] = (0.34907)(0.18551) = 0.064756 \text{ radians}$$

$\theta = 3.71°$ ◀



PROBLEM 15.7

When studying whiplash resulting from rear end collisions, the rotation of the head is of primary interest. An impact test was performed, and it was found that the angular acceleration of the head is defined by the relation $\alpha = 700\cos\theta + 70\sin\theta$ where α is expressed in rad/s^2 and θ in radians. Knowing that the head is initially at rest, determine the angular velocity of the head when $\theta = 30°$.

SOLUTION

Angular motion relations:

$$\alpha = \frac{d\omega}{dt} = \frac{\omega d\omega}{d\theta} = 700\cos\theta + 70\sin\theta$$

Separating variables ω and θ gives

$$\omega d\omega = (700\cos\theta + 70\sin\theta)d\theta$$

Integrating, using $\omega = 0$ when $\theta = 0$,

$$\int_0^\omega \omega d\omega = \int_0^\theta (700\cos\theta + 70\sin\theta)d\theta$$

$$\frac{1}{2}\omega^2 = (700\sin\theta - 70\cos\theta)\Big|_0^\theta$$

$$= 700\sin\theta + 70(1-\cos\theta)$$

$$\omega = \sqrt{1400\sin\theta + 140(1-\cos\theta)}$$

Data: $\theta = 30° = \dfrac{\pi}{6}\text{rad}$

With calculator set to "degrees" for trigonometric functions,

$$\omega = \sqrt{1400\sin 30° + 140(1-\cos 30°)} = 26.8 \text{ rad/s}$$

$\omega = 26.8$ rad/s ◀

With calculator set to "radians" for trigonometric functions,

$$\omega = \sqrt{1400\sin(\pi/6) + 140(1-\cos(\pi/6))} = 26.8 \text{ rad/s}$$

PROBLEM 15.8

The angular acceleration of an oscillating disk is defined by the relation $\alpha = -k\theta$. Determine (*a*) the value of *k* for which $\omega = 8$ rad/s when $\theta = 0$ and $\theta = 4$ rad when $\omega = 0$, (*b*) the angular velocity of the disk when $\theta = 3$ rad.

SOLUTION

$$\alpha = -k\theta$$

$$\omega = \frac{d\omega}{d\theta} = -k\theta$$

$$\omega d\omega = -k\theta d\theta$$

(*a*)

$$\int_{8\text{ rad/s}}^{0} \omega d\omega = -\int_{0}^{4\text{ rad}} k\theta d\theta; \qquad \left|\frac{1}{2}\omega^2\right|_8^0 = -\left|\frac{1}{2}k\theta^2\right|_0^4$$

$$\frac{1}{2}(0 - 8^2) = -\frac{1}{2}k(4^2 - 0)$$

$k = 4.00\ \text{s}^{-2}$ ◀

(*b*)

$$\int_{8\text{ rad/s}}^{\omega} \omega d\omega = -\int_{0}^{3\text{ rad}} k\theta d\theta; \qquad \left|\frac{1}{2}\omega^2\right|_0^\omega = -\left|\frac{1}{2}(4\ \text{s}^{-1})\theta^2\right|_0^3$$

$$\frac{1}{2}(\omega^2 - 8^2) = -\frac{1}{2}(4)(3^2 - 0)$$

$$\omega^2 - 64 = -36; \quad \omega^2 = 64 - 36 = 28$$

$\omega = 5.29$ rad/s ◀

PROBLEM 15.9

The angular acceleration of a shaft is defined by the relation $\alpha = -0.25\omega$, where α is expressed in rad/s^2 and ω in rad/s. Knowing that at $t = 0$ the angular velocity of the shaft is 20 rad/s, determine (*a*) the number of revolutions the shaft will execute before coming to rest, (*b*) the time required for the shaft to come to rest, (*c*) the time required for the angular velocity of the shaft to be reduced to 1 percent of its initial value.

SOLUTION

$$\alpha = -0.25\omega$$

$$\omega\frac{d\omega}{d\theta} = -0.25\omega$$

$$d\omega = -0.25 d\theta$$

(*a*) $$\int_{20\text{ rad/s}}^{0} d\omega = -0.25\int_0^{\theta} d\theta; \quad (0-20) = -0.25\theta; \quad \theta = 80 \text{ rad}$$

$$\theta = (80\text{ rad})\frac{\text{rev}}{2\pi\text{ rad}} \qquad \theta = 12.73 \text{ rev} ◀$$

(*b*) $$\alpha = -0.25\omega; \quad \frac{d\omega}{dt} = -0.25\omega; \quad \frac{d\omega}{\omega} = -0.25dt$$

$$\int_{20\text{ rad/s}}^{\omega}\frac{d\omega}{\omega} = -0.25\int_0^t dt \qquad |\ln\omega|_{20}^{\omega} = -0.25t$$

$$t = -\frac{1}{0.25}(\ln\omega - \ln 20) = 4(\ln 20 - \ln\omega)$$

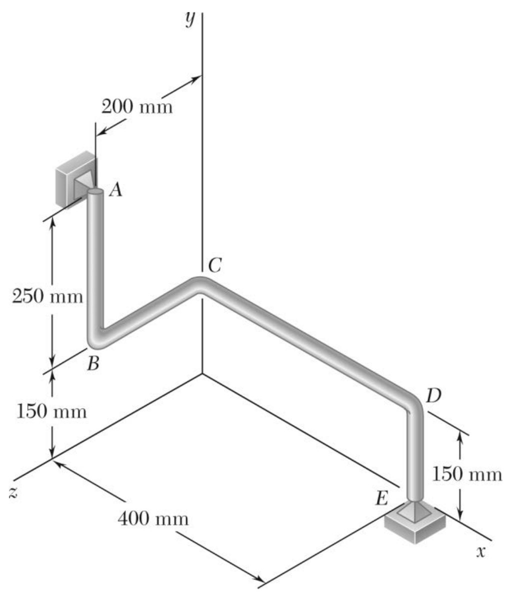
$$t = 4\ln\frac{20}{\omega} \qquad (1)$$

For $\omega = 0$ $$t = 4\ln\frac{20}{0} = 4\ln\infty \qquad t = \infty ◀$$

(*c*) For $$\omega = 0.01\omega_0 = 0.01(20) = 0.2 \text{ rad}$$

Use Eq. (1): $$t = 4\ln\left(\frac{20}{0.2}\right) = 4\ln 100 = 4(4.605) \qquad t = 18.42 \text{ s} ◀$$

PROBLEM 15.10



The bent rod *ABCDE* rotates about a line joining Points *A* and *E* with a constant angular velocity of 9 rad/s. Knowing that the rotation is clockwise as viewed from *E*, determine the velocity and acceleration of corner *C*.

SOLUTION

$$EA^2 = 0.4^2 + 0.4^2 + 0.2^2$$

$$EA = 0.6 \text{ m}$$

$$\mathbf{r}_{C/E} = -(0.4 \text{ m})\mathbf{i} + (0.15 \text{ m})\mathbf{j}$$

$$\overrightarrow{EA} = -(0.4 \text{ m})\mathbf{i} + (0.4 \text{ m})\mathbf{j} + (0.2 \text{ m})\mathbf{k}$$

$$\boldsymbol{\lambda}_{EA} = \frac{\overrightarrow{EA}}{EA} = \frac{1}{0.6}(-0.4\mathbf{i} + 0.4\mathbf{j} + 0.2\mathbf{k}) = \frac{1}{3}(-2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

$$\boldsymbol{\omega} = \omega_{AE}\boldsymbol{\lambda}_{EA} = (9 \text{ rad/s})\frac{1}{3}(-2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

$$\boldsymbol{\omega} = -(6 \text{ rad/s})\mathbf{i} + (6 \text{ rad/s})\mathbf{j} + (3 \text{ rad/s})\mathbf{k}$$

$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_{C/E} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 6 & 3 \\ -0.4 & 0.15 & 0 \end{vmatrix} = -0.45\mathbf{i} - 1.2\mathbf{j} + (-0.9 + 2.4)\mathbf{k}$$

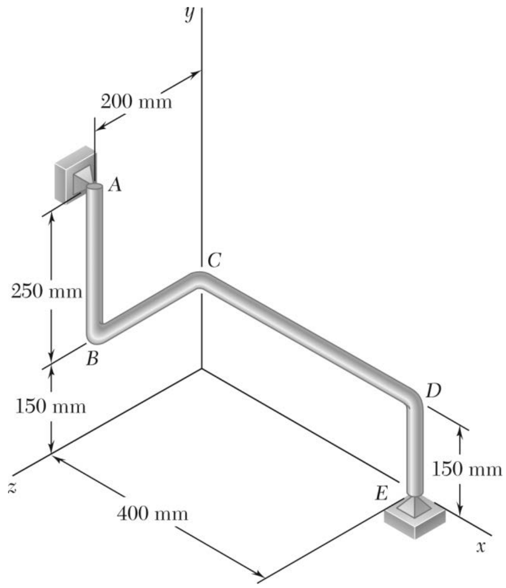
$$\mathbf{v}_C = -(0.45 \text{ m/s})\mathbf{i} - (1.2 \text{ m/s})\mathbf{j} + (1.5 \text{ m/s})\mathbf{k} \blacktriangleleft$$

$$\mathbf{a}_C = \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{C/E} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{C/E}) = \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{C/E} + \boldsymbol{\omega} \times \mathbf{v}_C$$

$$\mathbf{a}_C = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 6 & 3 \\ -0.45 & -1.2 & 1.5 \end{vmatrix}$$

$$= (9 + 3.6)\mathbf{i} + (-1.35 + 9)\mathbf{j} + (7.2 + 2.7)\mathbf{k}$$

$$\mathbf{a}_C = (12.60 \text{ m/s}^2)\mathbf{i} + (7.65 \text{ m/s}^2)\mathbf{j} + (9.90 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.11

In Problem 15.10, determine the velocity and acceleration of corner B, assuming that the angular velocity is 9 rad/s and increases at the rate of 45 rad/s^2.

PROBLEM 15.10 The bent rod *ABCDE* rotates about a line joining Points A and E with a constant angular velocity of 9 rad/s. Knowing that the rotation is clockwise as viewed from E, determine the velocity and acceleration of corner C.

SOLUTION

$$EA^2 = 0.4^2 + 0.4^2 + 0.2^2$$

$$EA = 0.6 \text{ m}$$

$$\mathbf{r}_{B/A} = -(0.25 \text{ m})\mathbf{j}$$

$$\overrightarrow{EA} = -(0.4 \text{ m})\mathbf{i} + (0.4 \text{ m})\mathbf{j} + (0.2 \text{ m})\mathbf{k}$$

$$\boldsymbol{\lambda}_{EA} = \frac{\overrightarrow{EA}}{EA} = \left(\frac{-0.4\mathbf{i} + 0.4\mathbf{j} + 0.2\mathbf{k}}{0.6}\right) = \frac{1}{3}(-2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

$$\boldsymbol{\omega} = \omega_{AE}\boldsymbol{\lambda}_{EA} = (9 \text{ rad/s})\frac{1}{3}(-2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

$$\boldsymbol{\omega} = -(6 \text{ rad/s})\mathbf{i} + (6 \text{ rad/s})\mathbf{j} + (3 \text{ rad/s})\mathbf{k}$$

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A} = (-6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}) \times (-0.25)\mathbf{j} = 1.5\mathbf{k} + 0.75\mathbf{i}$$

$$\mathbf{v}_B = (0.75 \text{ m/s})\mathbf{i} + (1.5 \text{ m/s})\mathbf{k} \blacktriangleleft$$

$$\boldsymbol{\alpha} = \alpha_{AE}\boldsymbol{\lambda}_{EA} = (45 \text{ rad/s}^2)\frac{1}{3}(-2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

$$\boldsymbol{\alpha} = -(30 \text{ rad/s}^2)\mathbf{i} + (30 \text{ rad/s}^2)\mathbf{j} + (15 \text{ rad/s}^2)\mathbf{k}$$

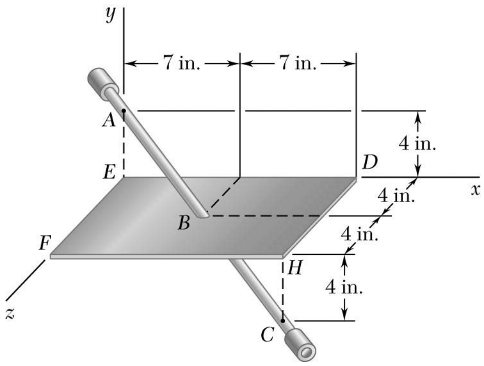
$$\mathbf{a}_B = \boldsymbol{\alpha} \times \mathbf{r}_{B/A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{B/A}) = \boldsymbol{\alpha} \times \mathbf{r}_{B/A} + \boldsymbol{\omega} \times \mathbf{v}_B$$

$$\mathbf{a}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -30 & 30 & 15 \\ 0 & -0.25 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 6 & 3 \\ 0.75 & 0 & 1.5 \end{vmatrix}$$

$$= 3.75\mathbf{i} + 7.5\mathbf{k} + 9\mathbf{i} + (2.25 + 9)\mathbf{j} - 4.5\mathbf{k}$$

$$\mathbf{a}_B = (12.75 \text{ m/s}^2)\mathbf{i} + (11.25 \text{ m/s}^2)\mathbf{j} + (3 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$

PROBLEM 15.12



The assembly shown consists of the straight rod *ABC* which passes through and is welded to the rectangular plate *DEFH*. The assembly rotates about the axis *AC* with a constant angular velocity of 9 rad/s. Knowing that the motion when viewed from *C* is counterclockwise, determine the velocity and acceleration of corner *F*.

SOLUTION

$$\overrightarrow{AC} = (14 \text{ in.})\mathbf{i} - (8 \text{ in.})\mathbf{j} + (8 \text{ in.})\mathbf{k} \quad AC = 18 \text{ in.}$$

$$\lambda_{AC} = \frac{\overrightarrow{AC}}{AC} = \frac{14\mathbf{i} - 8\mathbf{j} + 8\mathbf{k}}{18} = (0.77778\mathbf{i} - 0.44444\mathbf{j} + 0.44444\mathbf{k})$$

$$\boldsymbol{\omega} = \omega\lambda_{AC} = (9 \text{ rad/s})(0.77778\mathbf{i} - 0.44444\mathbf{j} + 0.44444\mathbf{k})$$

$$\boldsymbol{\omega} = (7 \text{ rad/s})\mathbf{i} - (4 \text{ rad/s})\mathbf{j} + (4 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\alpha} = 0$$

Corner *F*:

$$\mathbf{r}_{F/B} = (-7 \text{ in.})\mathbf{i} + (4 \text{ in.})\mathbf{k}$$

$$= -(0.58333 \text{ ft})\mathbf{i} + (0.33333 \text{ ft})\mathbf{k}$$

$$\mathbf{v}_F = \boldsymbol{\omega} \times \mathbf{r}_{F/B}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & 4 \\ -0.58333 & 0 & 0.33333 \end{vmatrix}$$

$$= -1.3333\mathbf{i} - 4.6667\mathbf{j} - 2.3333\mathbf{k}$$

$$\mathbf{v}_F = -(1.333 \text{ ft/s})\mathbf{i} - (4.67 \text{ ft/s})\mathbf{j} - (2.33 \text{ ft/s})\mathbf{k} \blacktriangleleft$$

$$\alpha = 0$$

$$\mathbf{a}_F = \boldsymbol{\alpha} \times \mathbf{r}_{F/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{F/B})$$

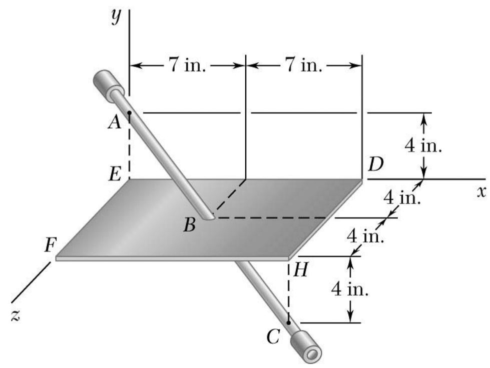
$$= 0 + \boldsymbol{\omega} \times \mathbf{v}_F$$

$$\mathbf{a}_F = \boldsymbol{\omega} \times \mathbf{v}_F$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & 4 \\ -1.3333 & -4.6667 & -2.3333 \end{vmatrix}$$

$$= (28.0)\mathbf{i} + (11.0)\mathbf{j} + (-38.0)\mathbf{k}$$

$$\mathbf{a}_F = (28.0 \text{ ft/s}^2)\mathbf{i} + (11.00 \text{ ft/s}^2)\mathbf{j} - (38.0 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.13

In Problem 15.12, determine the acceleration of corner H, assuming that the angular velocity is 9 rad/s and decreases at a rate of 18 rad/s^2.

PROBLEM 15.12 The assembly shown consists of the straight rod ABC which passes through and is welded to the rectangular plate $DEFH$. The assembly rotates about the axis AC with a constant angular velocity of 9 rad/s. Knowing that the motion when viewed from C is counterclockwise, determine the velocity and acceleration of corner F.

SOLUTION

$$\overrightarrow{AC} = (14 \text{ in.})\mathbf{i} - (8 \text{ in.})\mathbf{j} + (8 \text{ in.})\mathbf{k} \quad AC = 18 \text{ in.}$$

$$\lambda_{AC} = \frac{\overrightarrow{AC}}{AC} = \frac{14\mathbf{i} - 8\mathbf{j} + 8\mathbf{k}}{18} = (0.77778\mathbf{i} - 0.44444\mathbf{j} + 0.44444\mathbf{k})$$

$$\boldsymbol{\omega} = \omega\lambda_{AC} = (9 \text{ rad/s})(0.77778\mathbf{i} - 0.44444\mathbf{j} + 0.44444\mathbf{k})$$

$$\boldsymbol{\omega} = (7 \text{ rad/s})\mathbf{i} - (4 \text{ rad/s})\mathbf{j} + (4 \text{ rad/s})\mathbf{k}$$

$$\alpha = -18 \text{ rad/s}^2; \quad \boldsymbol{\alpha} = \alpha\lambda_{AC} = (-18 \text{ rad/s}^2)(0.77778\mathbf{i} - 0.44444\mathbf{j} + 0.44444\mathbf{k})$$

$$\boldsymbol{\alpha} = -(14 \text{ rad/s}^2)\mathbf{i} + (8 \text{ rad/s}^2)\mathbf{j} - (8 \text{ rad/s}^2)\mathbf{k}$$

Corner H:

$$\mathbf{r}_{H/B} = (7 \text{ in.})\mathbf{i} + (4 \text{ in.})\mathbf{k}$$

$$= (0.58333 \text{ ft})\mathbf{i} + (0.33333 \text{ ft})\mathbf{k}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & 4 \\ 0.58333 & 0 & 0.33333 \end{vmatrix}$$

$$= -1.3333\mathbf{i} + 2.3333\mathbf{k}$$

$$\mathbf{v}_H = -(1.333 \text{ ft/s})\mathbf{i} + (2.33 \text{ ft/s})\mathbf{k} \blacktriangleleft$$

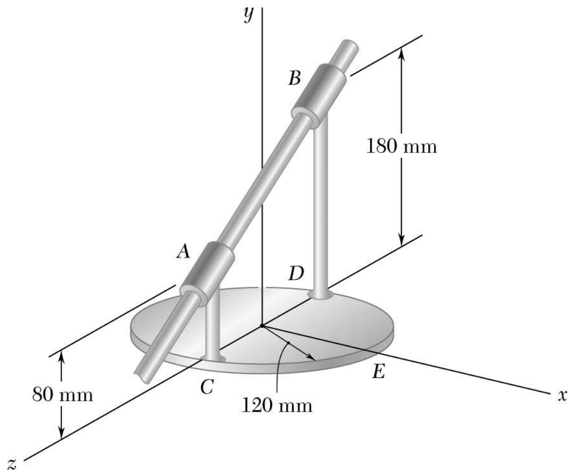
$$\mathbf{a}_H = \boldsymbol{\alpha} \times \mathbf{r}_{H/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{H/B})$$

$$= \boldsymbol{\alpha} \times \mathbf{r}_{H/B} + \boldsymbol{\omega} \times \mathbf{v}_H$$

$$\mathbf{a}_H = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -14 & 8 & -8 \\ 0.58333 & 0 & 0.33333 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & +4 \\ -1.3333 & 0 & 2.3333 \end{vmatrix}$$

$$= 2.6667\mathbf{i} + 0\mathbf{j} - 4.46667\mathbf{k} - 9.3333\mathbf{i} - 21.667\mathbf{j} - 5.3333\mathbf{k}$$

$$\mathbf{a}_H = -(6.67 \text{ ft/s}^2)\mathbf{i} - (21.7 \text{ ft/s}^2)\mathbf{j} - (10.00 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.14

A circular plate of 120 mm radius is supported by two bearings A and B as shown. The plate rotates about the rod joining A and B with a constant angular velocity of 26 rad/s. Knowing that, at the instant considered, the velocity of Point C is directed to the right, determine the velocity and acceleration of Point E.

SOLUTION

$$\overrightarrow{BA} = -(100\text{ mm})\mathbf{j} + (240\text{ mm})\mathbf{k} \qquad BA = 260\text{ mm}$$

$$\lambda_{BA} = \frac{\overrightarrow{BA}}{BA} = \frac{-(100)\mathbf{j} + (240)\mathbf{k}}{260}, \qquad \alpha = 0$$

$$\boldsymbol{\omega} = \omega\lambda_{BA} = 26\left(\frac{1}{260}\right)(-(100)\mathbf{j} + (240)\mathbf{k})$$

Point E:

$$\mathbf{r}_{E/A} = (120\text{ mm})\mathbf{i} - (80\text{ mm})\mathbf{j} - (120\text{ mm})\mathbf{k}$$

$$\mathbf{v}_E = \boldsymbol{\omega} \times \mathbf{r}_{E/A}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -10 & 24 \\ 120 & -80 & -120 \end{vmatrix}$$

$$= (3120\text{ mm/s})\mathbf{i} + (2880\text{ mm/s})\mathbf{j} + (1200\text{ mm/s})\mathbf{k}$$

$$\mathbf{v}_E = (3.12\text{ m/s})\mathbf{i} + (2.88\text{ m/s})\mathbf{j} + (1.200\text{ m/s})\mathbf{k} \blacktriangleleft$$

$$\mathbf{a}_E = \boldsymbol{\alpha} \times \mathbf{r}_{E/A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{E/A}) = 0 + \boldsymbol{\omega} \times \mathbf{v}_B$$

$$\mathbf{a}_E = \boldsymbol{\omega} \times \mathbf{v}_B$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -10 & 24 \\ 3120 & 2880 & 1200 \end{vmatrix}$$

$$= -(81120\text{ mm/s}^2)\mathbf{i} + (74880\text{ mm/s}^2)\mathbf{j} + (31200\text{ mm/s}^2)$$

$$\mathbf{a}_E = -(81.1\text{ m/s}^2)\mathbf{i} + (74.9\text{ m/s}^2)\mathbf{j} + (31.2\text{ m/s}^2)\mathbf{k} \blacktriangleleft$$

PROBLEM 15.15

In Problem 15.14, determine the velocity and acceleration of Point E, assuming that the angular velocity is 26 rad/s and increases at the rate of 65 rad/s^2.

SOLUTION

See Problem 15.14 for $\boldsymbol{\lambda}_{BA}$ and $\boldsymbol{\omega}$

$$\boldsymbol{\lambda}_{BA} = \frac{-(100)\mathbf{j} + (240)\mathbf{k}}{260}$$

$$\omega = (10 \text{ rad/s})\mathbf{j} = (24 \text{ rad/s})\mathbf{k}$$

$$\boldsymbol{\alpha} = +65 \text{ rad/s}^2; \qquad \boldsymbol{\alpha} = \boldsymbol{\alpha}\lambda_{BA} = (65 \text{ rad/s}^2)\left(\frac{1}{260}\right) - (100)\mathbf{j} + (240)\mathbf{k}$$

$$\boldsymbol{\alpha} = -(25 \text{ rad/s}^2)\mathbf{j} + (60 \text{ rad/s}^2)\mathbf{k}$$

Point E:

$$\mathbf{r}_{E/A} = (120 \text{ mm})\mathbf{i} - (80)\mathbf{j} - (120)\mathbf{k}$$

$$\mathbf{v}_E = \boldsymbol{\omega} \times \mathbf{r}_{E/A}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -10 & 24 \\ 120 & -80 & -120 \end{vmatrix}$$

$$= (3120 \text{ mm/s})\mathbf{i} + (2880 \text{ mm/s})\mathbf{j} + (1200 \text{ mm/s})\mathbf{k}$$

$$\mathbf{v}_E = (3.12 \text{ m/s})\mathbf{i} + (2.88 \text{ m/s})\mathbf{j} + (1.200 \text{ m/s})\mathbf{k} \quad \blacktriangleleft$$

$$\mathbf{a}_D = \boldsymbol{\alpha} \times \mathbf{r}_{E/A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{E/A}) = \boldsymbol{\alpha} \times \mathbf{r}_{E/A} + \boldsymbol{\omega} \times \mathbf{v}_E$$

$$\mathbf{a}_D = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -25 & 60 \\ 120 & -80 & -120 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -10 & 24 \\ 3120 & 2880 & 1200 \end{vmatrix}$$

$$= -(7800 \text{ mm/s}^2)\mathbf{i} + (7200 \text{ mm/s}^2)\mathbf{j} + (3000 \text{ mm/s}^2)\mathbf{k}$$

$$-(81120 \text{ mm/s}^2)\mathbf{i} + (74880 \text{ mm/s}^2)\mathbf{j} + (31200 \text{ mm/s}^2)\mathbf{k}$$

$$\mathbf{a}_B = -(73320 \text{ mm/s}^2)\mathbf{i} + (82080 \text{ mm/s}^2)\mathbf{j} + (34200 \text{ mm/s}^2)\mathbf{k}$$

$$= -(73.3 \text{ m/s}^2)\mathbf{i} + (82.1 \text{ m/s}^2)\mathbf{j} + (34.2 \text{ m/s}^2)\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 15.16

The earth makes one complete revolution around the sun in 365.24 days. Assuming that the orbit of the earth is circular and has a radius of 93,000,000 mi, determine the velocity and acceleration of the earth.

SOLUTION

$$\omega = \frac{2\pi \text{ rad}}{(365.24 \text{ days})\left(\frac{24 \text{ h}}{\text{day}}\right)\left(\frac{3600 \text{ s}}{\text{h}}\right)}$$

$$= 199.11 \times 10^{-9} \text{ rad/s}$$

$$v = r\omega$$

$$= (93 \times 10^6 \text{ mi})\left(\frac{5280 \text{ ft}}{\text{mi}}\right)(199.11 \times 10^{-9} \text{ rad/s})$$

$$v = 97,770 \text{ ft/s}$$

$v = 66,700$ mi/h ◀

$$a = r\omega^2$$

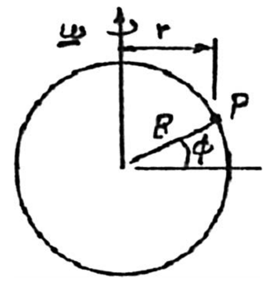
$$= (93 \times 10^6)(5280)(199.11 \times 10^{-9} \text{ rad/s})^2$$

$a = 19.47 \times 10^{-3}$ ft/s^2 ◀

PROBLEM 15.17

The earth makes one complete revolution on its axis in 23 h 56 min. Knowing that the mean radius of the earth is 3960 mi, determine the linear velocity and acceleration of a point on the surface of the earth (*a*) at the equator, (*b*) at Philadelphia, latitude 40° north, (*c*) at the North Pole.

SOLUTION



$$23 \text{ h } 56 \text{ m} = 23.933 \text{ h}$$

$$\omega = \frac{2\pi \text{ rad}}{(23.933 \text{ h})\left(\frac{3600 \text{ s}}{\text{h}}\right)}$$

$$= 72.925\times 10^{-6} \text{ rad/s}$$

$$R = (3960 \text{ mi})\left(\frac{5280 \text{ ft}}{\text{mi}}\right)$$

$$= 20.91\times 10^{6} \text{ ft}$$

$$r = \text{radius of path}$$

$$= R\cos\phi$$

(*a*) Equator: Latitude $= \phi = 0$

$$v = r\omega$$

$$= R(\cos 0)\omega$$

$$= (20.91\times 10^{6} \text{ ft})(1)(72.925\times 10^{-6} \text{ rad/s})$$

$v = 1525$ ft/s ◀

$$a = r\omega^2$$

$$= R(\cos 0)\omega^2$$

$$= (20.91\times 10^{6} \text{ ft})(1)(72.925\times 10^{-6} \text{ rad/s})^2$$

$a = 0.1112$ ft/s^2 ◀

(*b*) Philadelphia: Latitude $= \phi = 40°$

$$v = r\omega$$

$$= R(\cos 40°)\omega$$

$$= (20.91\times 10^{6} \text{ ft})(\cos 40°)(72.925\times 10^{-6} \text{ rad/s})$$

$v = 1168$ ft/s ◀

$$a = r\omega^2$$

$$= R(\cos 40°)\omega^2$$

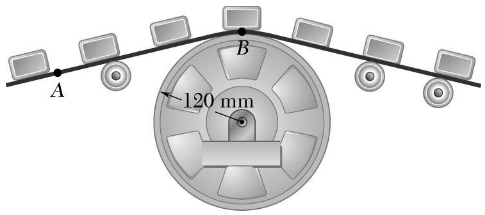
$$= (20.91\times 10^{6} \text{ ft})(\cos 40°)(72.925\times 10^{-6} \text{ rad/s})^2$$

$a = 0.0852$ ft/s^2 ◀

(*c*) North Pole: Latitude $= \phi = 0$

$$r = R\cos 0 = 0$$

$v = a = 0$ ◀



PROBLEM 15.18

A series of small machine components being moved by a conveyor belt pass over a 120 mm radius idler pulley. At the instant shown, the velocity of Point A is 300 mm/s to the left and its acceleration is 180 mm/s^2 to the right. Determine (*a*) the angular velocity and angular acceleration of the idler pulley, (*b*) the total acceleration of the machine component at B.

SOLUTION

$$v_B = v_A = 300 \text{ mm/s} \longleftarrow \qquad r_B = 120 \text{ mm}$$

$$(a_B)_t = a_A = 180 \text{ mm/s} \longrightarrow$$

(*a*) $v_B = \omega r_B,$ $\quad \omega = \dfrac{v_B}{r_B} = \dfrac{300}{120} = 2.5 \text{ rad/s} \circlearrowleft$ $\qquad \boldsymbol{\omega} = 2.50 \text{ rad/s} \circlearrowleft$ ◀

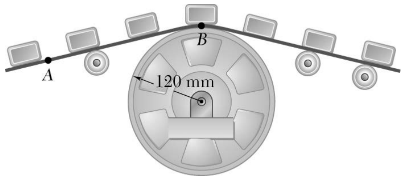
$(a_B)_t = \alpha r_B,$ $\quad \alpha = \dfrac{(a_B)_t}{r_B} = \dfrac{180}{120} = 1.5 \text{ rad/s} \circlearrowright$ $\qquad \boldsymbol{\alpha} = 1.500 \text{ rad/s}^2 \circlearrowright$ ◀

(*b*) $(a_B)_n = r_B\omega^2 = (120)(2.5)^2 = 750 \text{ mm/s}^2 \downarrow$

$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{(180)^2 + (750)^2} = 771 \text{ mm/s}^2$$

$$\tan\beta = \frac{750}{180}, \qquad \beta = 76.5°$$

$\mathbf{a}_B = 771 \text{ mm/s}^2 \searrow 76.5°$ ◀



PROBLEM 15.19

A series of small machine components being moved by a conveyor belt pass over a 120-mm-radius idler pulley. At the instant shown, the angular velocity of the idler pulley is 4 rad/s clockwise. Determine the angular acceleration of the pulley for which the magnitude of the total acceleration of the machine component at *B* is $2400\ \text{mm/s}^2$.

SOLUTION

$$\omega_B = 4\ \text{rad/s} \circlearrowright, \quad r_B = 120\ \text{mm}$$

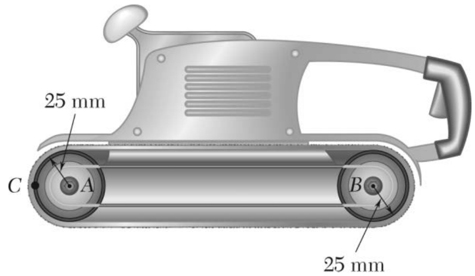
$$(a_B)_n = r_B\omega_B^2 = (120)(4)^2 = 1920\ \text{mm/s}^2$$

$$a_B = 2400\ \text{mm/s}^2$$

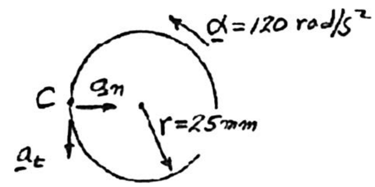
$$(a_B)_t = \sqrt{a_B^2 - (a_B)_n^2} = \sqrt{2400^2 - 1920^2} = \pm 1440\ \text{mm/s}^2$$

$$(a_B)_t = r_B\alpha, \qquad \alpha = \frac{(a_B)_t}{r_B} = \frac{\pm 1440}{120} = \pm 12\ \text{rad/s}^2$$

$$12.00\ \text{rad/s}^2 \circlearrowleft \text{ or } \circlearrowright \blacktriangleleft$$

	**PROBLEM 15.20** The belt sander shown is initially at rest. If the driving drum B has a constant angular acceleration of 120 rad/s^2 counterclockwise, determine the magnitude of the acceleration of the belt at Point C when (*a*) $t = 0.5$ s, (*b*) $t = 2$ s.

SOLUTION



$$a_t = r\alpha = (0.025\text{ m})(120\text{ rad/s}^2)$$

$$\mathbf{a}_t = 3\text{ m/s}^2 \downarrow$$

(*a*) $t = 0.5$ s:

$$\omega = \alpha t = (120\text{ rad/s}^2)(0.5\text{ s}) = 60\text{ rad/s}$$

$$a_n = r\omega^2 = (0.025\text{ m})(60\text{ rad/s})^2$$

$$\mathbf{a}_n = 90\text{ m/s}^2 \longrightarrow$$

$$a_B^2 = a_t^2 + a_n^2 = 3^2 + 90^2$$

$a_B = 90.05\text{ m/s}^2$ ◀

(*b*) $t = 2$ s:

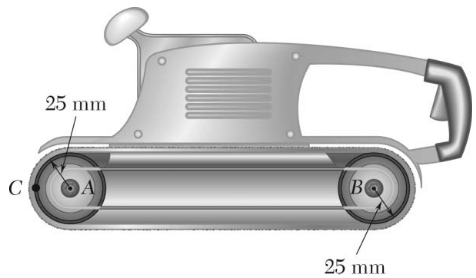
$$\omega = \alpha t = (120\text{ rad/s}^2)(2\text{ s}) = 240\text{ rad/s}$$

$$a_n = r\omega^2 = (0.025\text{ m})(240\text{ rad/s})^2$$

$$a_n = 1440\text{ m/s}^2$$

$$a_B^2 = a_t^2 + a_n^2 = 3^2 + 1440^2$$

$a_B = 1440\text{ m/s}^2$ ◀



PROBLEM 15.21

The rated speed of drum *B* of the belt sander shown is 2400 rpm. When the power is turned off, it is observed that the sander coasts from its rated speed to rest in 10 s. Assuming uniformly decelerated motion, determine the velocity and acceleration of Point *C* of the belt, (*a*) immediately before the power is turned off, (*b*) 9 s later.

SOLUTION

$$\omega_0 = 2400 \text{ rpm}$$
$$= 251.3 \text{ rad/s}$$
$$r = 0.025 \text{ m}$$

(*a*)

$$v_C = r\omega = (0.025 \text{ m})(251.3 \text{ rad/s}) \qquad v_C = 6.28 \text{ m/s} \blacktriangleleft$$

$$a_C = r\omega^2 = (0.025 \text{ m})(251.3 \text{ rad/s})^2 \qquad a_C = 1579 \text{ m/s}^2 \blacktriangleleft$$

(*b*) When $t = 10$ s:

$$\omega = 0.$$
$$\omega = \omega_0 + \alpha t$$
$$0 = 251.3 \text{ rad/s} + \alpha(10 \text{ s})$$
$$\alpha = -25.13 \text{ rad/s}^2$$

When $t = 9$ s:

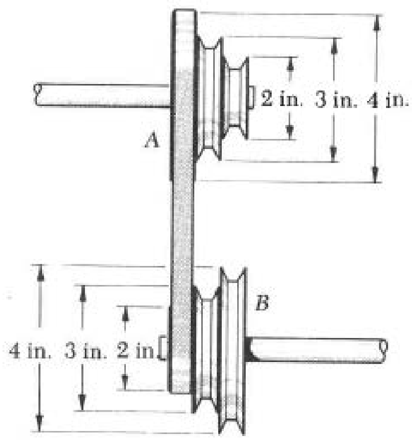
$$\omega = \omega_0 + \alpha t$$
$$\omega_9 = 251.3 \text{ rad/s} - (25.13 \text{ rad/s}^2)(9 \text{ s})$$
$$= 25.13 \text{ rad/s}$$

$$v_C = r\omega_9$$
$$= (0.025 \text{ m})(25.13 \text{ rad/s}) \qquad v_9 = 0.628 \text{ m/s} \blacktriangleleft$$

$$(a_C)_t = r\alpha_n$$
$$= (0.025 \text{ m})(-25.13 \text{ rad/s}^2)$$
$$(a_C)_t = 0.628 \text{ m/s}^2$$
$$(a_C)_n = r\omega_9^2$$
$$= (0.025 \text{ m})(25.13 \text{ rad/s})^2$$
$$(a_C)_n = 15.79 \text{ m/s}^2$$

$$a_C^2 = (a_C)_t^2 + (a_C)_n^2$$
$$= (0.628 \text{ m/s}^2)^2 + (15.79 \text{ m/s}^2)^2 \qquad a_C = 15.80 \text{ m/s}^2 \blacktriangleleft$$

PROBLEM 15.22



The two pulleys shown may be operated with the V belt in any of three positions. If the angular acceleration of shaft A is 6 rad/s^2 and if the system is initially at rest, determine the time required for shaft B to reach a speed of 400 rpm with the belt in each of the three positions.

SOLUTION

Angular velocity of shaft A: $\omega_A = \alpha_A t$

Belt speed: $v = r_A\omega_A = r_B\omega_B$

Angular speed of shaft B: $\omega_B = \dfrac{v}{r_B} = \dfrac{r_A\alpha_A t}{r_B}$

Solving for t, $t = \dfrac{r_B\omega_B}{r_A\alpha_A}$

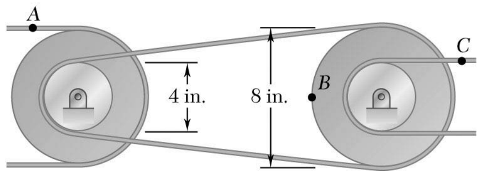
Data: $\alpha_A = 6$ rad/s, $\omega_B = 400 \text{ rpm} = 41.889 \text{ rad/s}$

$$t = \frac{r_B}{r_A}\cdot\frac{41.889}{6} = 6.9813\frac{r_B}{r_A} = 6.9813\frac{d_B}{d_A}$$

Belt at left: $\dfrac{d_B}{d_A} = \dfrac{2 \text{ in.}}{4 \text{ in.}}$ $t = 3.49$ s ◀

Belt in middle: $\dfrac{d_B}{d_A} = \dfrac{3 \text{ in.}}{3 \text{ in.}}$ $t = 6.98$ s ◀

Belt at right: $\dfrac{d_B}{d_A} = \dfrac{4 \text{ in.}}{2 \text{ in.}}$ $t = 13.96$ s ◀

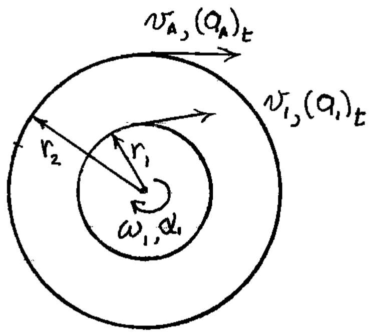


PROBLEM 15.23

Three belts move over two pulleys without slipping in the speed reduction system shown. At the instant shown the velocity of Point *A* on the input belt is 2 ft/s to the right, decreasing at the rate of 6 ft/s². Determine, at this instant, (*a*) the velocity and acceleration of Point *C* on the output belt, (*b*) the acceleration of Point *B* on the output pulley.

SOLUTION

Left pulley.



Inner radius $r_1 = 2$ in.

Outer radius $r_2 = 4$ in.

$v_A = 2 \text{ ft/s} \longrightarrow$

$(a_A)_t = -6 \text{ ft/s}^2 = 6 \text{ ft/s}^2 \longleftarrow$

$$\omega_1 = \frac{v_A}{r_2} = \frac{2}{\frac{4}{12}} = 6 \text{ rad/s} \circlearrowright$$

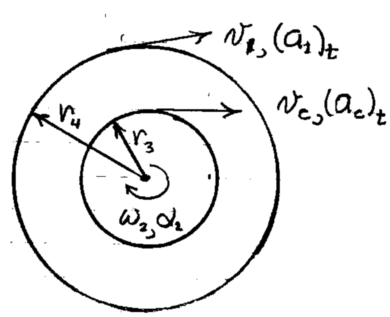
$$\alpha_1 = \frac{(a_A)_t}{r_2} = \frac{6}{\frac{4}{12}} = 18 \text{ rad/s}^2 \circlearrowleft$$

Intermediate belt.

$$v_1 = r_1\omega_1 = \left(\frac{2}{12}\right)(6) = 1 \text{ ft/s}$$

$$(a_1)_t = r_1\alpha_1 = \left(\frac{2}{12}\right)(18) = 3 \text{ ft/s}^2$$

Right pulley.



Inner radius $r_3 = 2$ in.

Outer radius $r_4 = 4$ in.

$$\omega_2 = \frac{v_1}{r_4} = \frac{1}{\left(\frac{4}{12}\right)} = 3 \text{ rad/s} \circlearrowright$$

$$\alpha_2 = \frac{(a_1)_t}{r_4} = \frac{3}{\left(\frac{4}{12}\right)} = 9 \text{ rad/s}^2 \circlearrowleft$$

PROBLEM 15.23 (Continued)

(*a*) *Velocity and acceleration of Point C.*

$$v_C = r_3\omega_2 = \left(\frac{2}{12}\right)(3) = 0.5 \text{ ft/s}$$

$\mathbf{v}_C = 0.5 \text{ ft/s} \longrightarrow$ ◀

$$(a_C)_t = r_3\alpha_2 = \left(\frac{2}{12}\right)(9) = 1.5 \text{ ft/s}^2$$

$\mathbf{a}_C = 1.5 \text{ ft/s}^2 \longleftarrow$ ◀

(*b*) *Acceleration of Point B.*

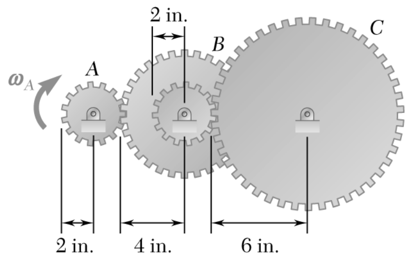
$$(a_B)_n = r_4\omega_2^2 = \left(\frac{4}{12}\right)(3)^2 = 3 \text{ ft/s}^2$$

$(\mathbf{a}_B)_n = 3 \text{ ft/s}^2 \longrightarrow$

$$(a_B)_t = r_4\alpha_2 = \left(\frac{4}{12}\right)(9) = 3 \text{ ft/s}^2$$

$(\mathbf{a}_B)_t = 3 \text{ ft/s}^2 \downarrow$

$\mathbf{a}_B = 4.24 \text{ ft/s}^2 \searrow 45°$ ◀



PROBLEM 15.24

A gear reduction system consists of three gears *A*, *B*, and *C*. Knowing that gear *A* rotates clockwise with a constant angular velocity $\omega_A = 600$ rpm, determine (*a*) the angular velocities of gears *B* and *C*, (*b*) the accelerations of the points on gears *B* and *C* which are in contact.

SOLUTION

(*a*)

$$\omega_A = 600 \text{ rpm} = \frac{(600)(2\pi)}{60} = 20\pi \text{ rad/s.}$$

Let Points *A*, *B*, and *C* lie at the axles of gears *A*, *B*, and *C*, respectively.

Let *D* be the contact point between gears *A* and *B*.

$$v_D = r_{D/A}\omega_A = (2)(20\pi) = 40\pi \text{ in./s} \downarrow$$

$$\omega_B = \frac{v_D}{r_{D/B}} = \frac{40\pi}{4} = 10\pi \text{ rad/s} = 10\pi \cdot \frac{60}{2\pi} = 300 \text{ rpm} \circlearrowleft$$

$$\boldsymbol{\omega}_B = 300 \text{ rpm} \circlearrowleft \blacktriangleleft$$

Let *E* be the contact point between gears *B* and *C*.

$$v_E = r_{E/B}\omega_B = (2)(10\pi) = 20\pi \text{ in./s} \uparrow$$

$$\omega_C = \frac{v_E}{r_{E/C}} = \frac{20\pi}{6} = 3.333\pi \text{ rad/s} = (3.333\pi)\frac{60}{2\pi} = 100 \text{ rpm} \circlearrowright$$

$$\boldsymbol{\omega}_C = 100 \text{ rpm} \circlearrowright \blacktriangleleft$$

(*b*) *Accelerations at Point E.*

On gear *B*:

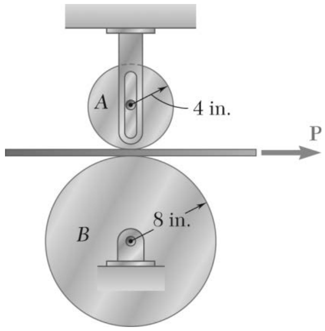
$$a_B = \frac{v_E^2}{r_{E/B}} = \frac{(20\pi)^2}{2} = 1973.9 \text{ in./s}^2 \leftarrow$$

$$\mathbf{a}_B = 1974 \text{ in./s}^2 \leftarrow \blacktriangleleft$$

On gear *C*:

$$a_C = \frac{v_E^2}{r_{E/C}} = \frac{(20\pi)^2}{6} = 658 \text{ in./s}^2 \rightarrow$$

$$\mathbf{a}_C = 658 \text{ in./s}^2 \rightarrow \blacktriangleleft$$



PROBLEM 15.25

A belt is pulled to the right between cylinders A and B. Knowing that the speed of the belt is a constant 5 ft/s and no slippage occurs, determine (a) the angular velocities of A and B, (b) the accelerations of the points which are in contact with the belt.

SOLUTION

(a) Angular velocities.

Disk A: $\omega_A = \dfrac{v_P}{r_A} = \dfrac{5 \text{ ft/s}}{(4/12) \text{ ft}}$ $\boldsymbol{\omega}_A = 15.00 \text{ rad/s} \circlearrowright$ ◀

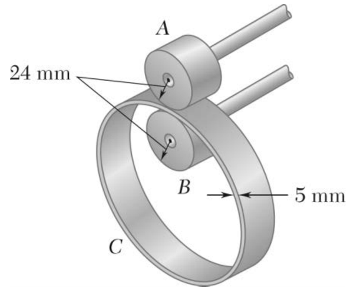
Disk B: $\omega_B = \dfrac{v_P}{r_B} = \dfrac{5 \text{ ft/s}}{(8/12) \text{ ft}}$ $\boldsymbol{\omega}_B = 7.50 \text{ rad/s} \circlearrowright$ ◀

(b) Accelerations of contact points.

Disk A: $a_A = \omega_A^2 r_A = (15.00 \text{ rad/s})^2((4/12) \text{ ft})$ $\mathbf{a}_A = 75.00 \text{ rad/s}^2 \uparrow$ ◀

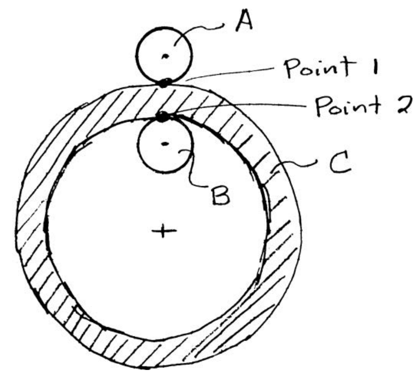
Disk B: $a_B = \omega_B^2 r_B = (7.50 \text{ rad/s})^2((8/12) \text{ ft})$ $\mathbf{a}_B = 37.5 \text{ rad/s}^2 \downarrow$ ◀

PROBLEM 15.26



Ring C has an inside radius of 55 mm and an outside radius of 60 mm and is positioned between two wheels A and B, each of 24-mm outside radius. Knowing that wheel A rotates with a constant angular velocity of 300 rpm and that no slipping occurs, determine (*a*) the angular velocity of the ring C and of wheel B, (*b*) the acceleration of the Points of A and B which are in contact with C.

SOLUTION



$$\omega_A = 300 \text{ rpm}\left(\frac{2\pi}{60}\right)$$

$$= 31.416 \text{ rad/s}$$

$$r_A = 24 \text{ mm}$$

$$r_B = 24 \text{ mm}$$

$$r_1 = 60 \text{ mm}$$

$$r_2 = 55 \text{ mm}$$

[We assume senses of rotation shown for our computations.]

(*a*) Velocities:

<u>Point</u> 1 (Point of contact of A and C)

$$v_1 = r_A\omega_A = r_1\omega_C$$

$$\omega_C = \frac{r_A}{r_1}\omega_A$$

$$= \frac{24 \text{ mm}}{60 \text{ mm}}(300 \text{ rpm})$$

$$= 120 \text{ rpm}$$

$\omega_C = 120 \text{ rpm}$ ◀

<u>Point</u> 2 (Point of contact of B and C)

$$v_2 = r_B\omega_B = r_2\omega_C$$

$$\omega_B = \frac{r_2}{r_B}\omega_C$$

$$= \frac{r_2}{r_B}\left(\frac{r_A}{r_1}\right)\omega_A$$

$$= \frac{55 \text{ mm}}{24 \text{ mm}}\left(\frac{24 \text{ mm}}{60 \text{ mm}}\right)300 \text{ rpm}$$

$$\omega_B = 275 \text{ mm}$$

$\omega_B = 275 \text{ rpm}$ ◀

PROBLEM 15.26 (Continued)

(*b*) Accelerations:

Point on rim of *A*: $r_A = 24\text{ mm} = 0.024\text{ m}$

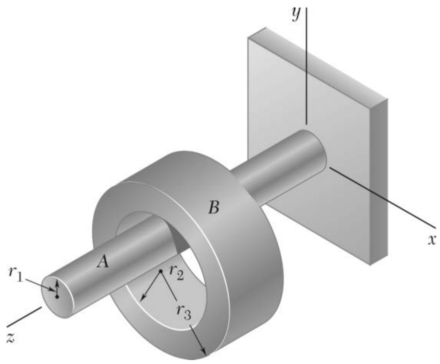
$$a_A = r_A\omega_A^2$$
$$= (0.024\text{ m})(31.416\text{ rad/s})^2$$
$$= 23.687\text{ m/s}^2$$

$\mathbf{a}_A = 23.7\text{ m/s}^2\uparrow$ ◀

Point on rim of *B*: $\omega_B = 275\text{ rpm}\left(\dfrac{2\pi}{60}\right)$

$$= 28.798\text{ rad/s}$$
$$a_B = r_B\omega_B^2$$
$$= (0.024\text{ m})(28.798\text{ rad/s})^2$$
$$= 19.904\text{ m/s}^2$$

$\mathbf{a}_B = 19.90\text{ m/s}^2\downarrow$ ◀



PROBLEM 15.27

Ring B has an inside radius r_2 and hangs from the horizontal shaft A as shown. Shaft A rotates with a constant angular velocity of 25 rad/s and no slipping occurs. Knowing that r_1 = 12 mm, r_2 = 30 mm, and r_3 = 40 mm, determine (*a*) the angular velocity of ring B, (*b*) the accelerations of the points of shaft A and ring B which are in contact, (*c*) the magnitude of the acceleration of a point on the outside surface of ring B.

SOLUTION

Let Point C be the point of contact between the shaft and the ring.

$$v_C = r_1\omega_A$$

$$\omega_B = \frac{v_C}{r_2}$$

$$= \frac{r_1\omega_A}{r_2} \qquad \omega_B = \frac{r_1\omega_A}{r_2} \downarrow$$

On shaft A: $a_A = r_1\omega_A^2 \qquad \mathbf{a}_A = r_1\omega_A^2 \downarrow$

On ring B: $a_B = r_2\omega_B^2 = r_2\left(\frac{r_1\omega_A}{r_2}\right)^2 \qquad \mathbf{a}_B = \frac{r_1^2\omega_A^2}{r_2} \downarrow$

<u>Acceleration of Point *D* on outside of ring</u>.

$$a_D = r_3\omega_B^2 = r_3\left(\frac{r_1}{r_2}\omega_A\right)^2$$

$$a_D = r_3\left(\frac{r_1}{r_2}\right)^2\omega_A^2 \downarrow$$

<u>Data</u>:

$$\omega_A = 25 \text{ rad/s}$$
$$r_1 = 12 \text{ mm}$$
$$r_2 = 30 \text{ mm}$$
$$r_3 = 40 \text{ mm}$$

(*a*)

$$\omega_B = \frac{r_1}{r_2}\omega_A$$

$$= \frac{12 \text{ mm}}{30 \text{ mm}}(25 \text{ rad/s}) \qquad \boldsymbol{\omega}_B = 10 \text{ rad/s} \circlearrowright ◀$$

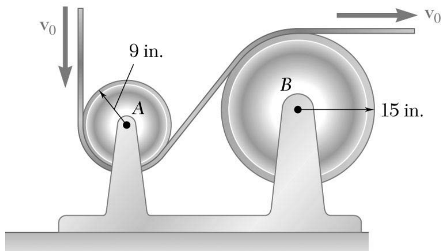
(*b*)

$$a_A = r_1\omega_A^2$$
$$= (12 \text{ mm})(25 \text{ rad/s})^2$$
$$= 7.5\times10^3 \text{ mm/s}^2 \qquad \mathbf{a}_A = 7.50 \text{ m/s}^2 \downarrow \blacktriangleleft$$

$$a_B = \frac{r_1^2}{r_2}\omega_A^2$$
$$= \frac{(12 \text{ mm})^2}{(30 \text{ mm})}(25 \text{ rad/s})^2$$
$$= 3\times10^3 \text{ mm/s}^2 \qquad \mathbf{a}_B = 3.00 \text{ m/s}^2 \downarrow \blacktriangleleft$$

(*c*)

$$a_D = r_3\left(\frac{r_1}{r_2}\right)^2 \omega_A^2$$
$$= (40 \text{ mm})\left(\frac{12 \text{ mm}}{30 \text{ mm}}\right)^2 (25 \text{ rad/s})^2$$
$$a_D = 4\times10^3 \text{ mm/s}^2 \qquad \mathbf{a}_D = 4.00 \text{ m/s}^2 \downarrow \blacktriangleleft$$



PROBLEM 15.28

A plastic film moves over two drums. During a 4-s interval the speed of the tape is increased uniformly from $v_0 = 2$ ft/s to $v_1 = 4$ ft/s. Knowing that the tape does not slip on the drums, determine (*a*) the angular acceleration of drum *B*, (*b*) the number of revolutions executed by drum *B* during the 4-s interval.

SOLUTION

Belt motion:

$$v = v_0 + at$$

$$4 \text{ ft/s} = 2 \text{ ft/s} + a(4 \text{ s})$$

$$a = \frac{4 \text{ ft/s} - 2 \text{ ft/s}}{4 \text{ s}} = 0.5 \text{ ft/s}^2 = 6 \text{ in./s}^2$$

Since the belt does not slip relative to the periphery of the drum, the tangential accelation at the periphery of the drum is

$$a_t = 6 \text{ in./s}^2$$

(*a*) *Angular acceleration of drum B.*

$$\alpha_B = \frac{a_t}{r_B} = \frac{6 \text{ in./s}^2}{15 \text{ in.}}$$

$\boldsymbol{\alpha}_B = 0.400 \text{ rad/s}^2$ ↻ ◀

(*b*) *Angular displacement of drum B.*

At $t = 0$, $$\omega_0 = \frac{v_0}{r_B} = \frac{24 \text{ in./s}}{15 \text{ in.}} = 1.6 \text{ rad/s}$$

At $t = 4$ s, $$\omega_1 = \frac{v_1}{r_B} = \frac{48 \text{ in./s}}{15 \text{ in.}} = 3.2 \text{ rad/s}$$

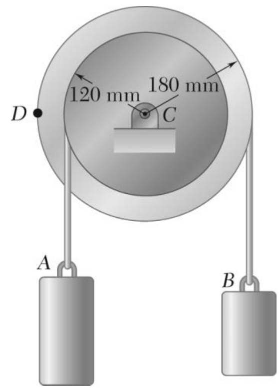
$$\omega_1^2 = \omega_0^2 + 2\alpha_B\theta_B$$

$$\theta_B = \frac{\omega_1^2 - \omega_0^2}{2\alpha_B} = \frac{(3.2)^2 - (1.6)^2}{(2)(0.400)} = 9.6 \text{ radians}$$

In revolutions, $$\theta_B = \frac{9.6}{2\pi}$$

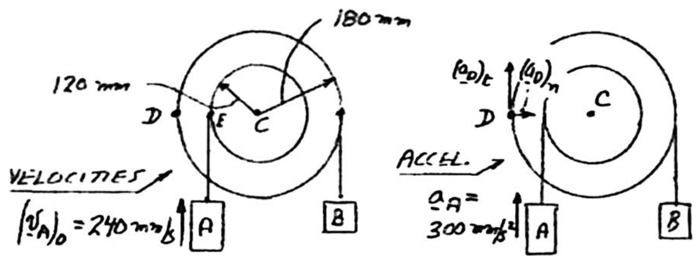
$\theta_B = 1.528$ rev ◀

PROBLEM 15.29



A pulley and two loads are connected by inextensible cords as shown. Load A has a constant acceleration of 300 mm/s^2 and an initial velocity of 240 mm/s, both directed upward. Determine (a) the number of revolutions executed by the pulley in 3 s, (b) the velocity and position of load B after 3 s, (c) the acceleration of Point D on the rim of the pulley at $t = 0$.

SOLUTION



(a) Motion of pulley:

$$(\mathbf{v}_E)_0 = (v_A)_0 = 240 \text{ mm/s} \uparrow$$

$$(\mathbf{a}_E)_t = \mathbf{a}_A = 300 \text{ mm/s}^2 \uparrow$$

$$(v_E)_0 = r\omega_0: \quad 240 \text{ mm/s} = (120 \text{ mm})\omega_0 \qquad \boldsymbol{\omega}_0 = 2 \text{ rad/s} \circlearrowright$$

$$(a_E)_t = r\alpha: \quad 300 \text{ mm}^2\text{/s} = (120 \text{ mm})\alpha \qquad \boldsymbol{\alpha} = 2.5 \text{ rad/s}^2 \circlearrowright$$

For $t = 3$ s:

$$\omega = \omega_0 + \alpha t$$
$$= 2 \text{ rad/s} + (2.5 \text{ rad/s}^2)(3 \text{ s})$$
$$= 9.5 \text{ rad/s}$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$= (2 \text{ rad/s})(3 \text{ s}) + \frac{1}{2}(2.5 \text{ rad/s}^2)(3 \text{ s})^2$$

$$\theta = 17.25 \text{ rad} \qquad \theta = 17.25\left(\frac{1}{2\pi}\right) \qquad \theta = 2.75 \text{ rev} \blacktriangleleft$$

(b) Load B:

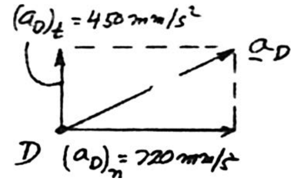
$$r = 180 \text{ mm}$$
$$t = 3 \text{ s}$$

$$v_B = r\omega = (0.180 \text{ m})(9.5 \text{ rad/s}) = 1.710 \text{ m/s} \qquad \mathbf{v}_B = 1.710 \text{ m/s} \downarrow \blacktriangleleft$$

$$\Delta y_B = r\theta = (0.180 \text{ m})(17.25 \text{ rad}) = 3.105 \text{ m} \qquad \Delta y_B = 3.11 \text{ m} \downarrow \blacktriangleleft$$

PROBLEM 15.29 (Continued)

(*c*) Point *D*: $r = 180 \text{ mm} \qquad t = 0$

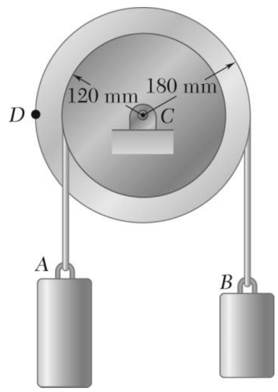


$$(\mathbf{a}_D)_t = r\alpha = (180 \text{ mm})(2.5 \text{ rad/s}^2) = 450 \text{ mm/s}^2$$

$$(a_D)_n = r\omega_0^2 = (180 \text{ mm})(2 \text{ rad/s})^2 = 720 \text{ mm/s}^2$$

$$(\mathbf{a}_D)_n = 720 \text{ mm/s}^2 \longrightarrow$$

$$\mathbf{a}_D = 849 \text{ mm/s}^2 \measuredangle 32.0° \blacktriangleleft$$



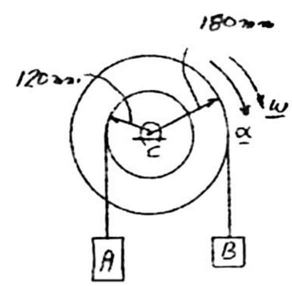
PROBLEM 15.30

A pulley and two loads are connected by inextensible cords as shown. The pulley starts from rest at $t = 0$ and is accelerated at the uniform rate of 2.4 rad/s^2 clockwise. At $t = 4$ s, determine the velocity and position (*a*) of load *A*, (*b*) of load *B*.

SOLUTION

Uniformly accelerated motion. $\omega_0 = 0$

$\alpha = 2.4 \text{ rad/s}^2 \circlearrowright$



At $t = 4$ s: $\omega = \omega_0 + \alpha t = 0 + (2.4 \text{ rad/s}^2)(4 \text{ s})$

$\boldsymbol{\omega} = 9.6 \text{ rad/s}^2 \circlearrowright$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 0 + 0 + \frac{1}{2}(2.4 \text{ rad/s}^2)(4 \text{ s})^2$$

$\theta = 19.20 \text{ rad} \circlearrowright$

(*a*) Load *A*. At $t = 4$ s: $r_A = 120$ mm

$v_A = r_A \omega$

$= (120 \text{ mm})(9.6 \text{ rad/s})$

$= 1152 \text{ mm/s}$ $\mathbf{v}_A = 1.152 \text{ m/s} \uparrow$ ◀

$y_A = r_A \theta$

$= (120 \text{ mm})(19.2 \text{ rad})$

$= 2304 \text{ mm}$ $\mathbf{y}_A = 2.30 \text{ m} \uparrow$ ◀

(*b*) Load *B*. At $t = 4$ s: $r_B = 180$ mm

$v_B = r_B \omega$

$= (180 \text{ mm})(9.6 \text{ rad/s})$

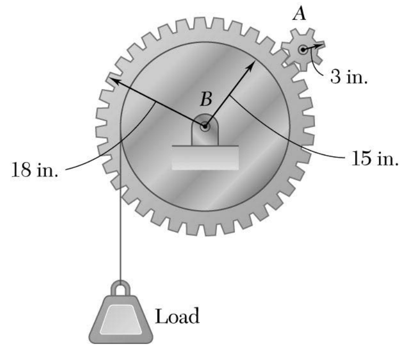
$= 1728 \text{ mm/s}$ $\mathbf{v}_B = 1.728 \text{ m/s} \downarrow$ ◀

$y_B = r_B \theta$

$= (180 \text{ mm})(19.2 \text{ rad})$

$= 3456 \text{ mm}$ $\mathbf{y}_B = 3.46 \text{ m} \downarrow$ ◀

PROBLEM 15.31



A load is to be raised 20 ft by the hoisting system shown. Assuming gear *A* is initially at rest, accelerates uniformly to a speed of 120 rpm in 5 s, and then maintains a constant speed of 120 rpm, determine (*a*) the number of revolutions executed by gear *A* in raising the load, (*b*) the time required to raise the load.

SOLUTION

The load is raised a distance $h = 20$ ft $= 240$ in.

For gear-pulley *B*, radius to rope groove is $r_1 = 15$ in.

Required angle change for *B*: $$\theta_B = \frac{h}{r_1} = \frac{240}{15} = 16 \text{ radians}$$

Circumferential travel of gears *A* and *B*:

$$s = r_2\theta_B = r_A\theta_A \qquad \text{where } r_2 = 18 \text{ in. and } r_A = 3 \text{ in.}$$

$$s = (18 \text{ in.})(16 \text{ radians}) = 288 \text{ in.}$$

(*a*) *Angle change of gear A*: $$\theta_A = \frac{s}{r_A} = \frac{288}{3} = 96 \text{ radians}$$

In revolutions, $$\theta_A = \frac{96}{2\pi}$$ $\theta_A = 15.28$ rev ◀

(*b*) *Motion of gear A*. $\omega_0 = 0, \quad \omega_f = 120 \text{ rpm} = 4\pi \text{ rad/s}$

Gear *A* is uniformly accelerated over the first 5 seconds.

$$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{4\pi \text{ rad/s}}{5 \text{ s}} = 2.5133 \text{ rad/s}^2$$

$$\theta_A = \frac{1}{2}\alpha t^2 = \frac{1}{2}(2.5133)(5)^2 = 31.416 \text{ radians}$$

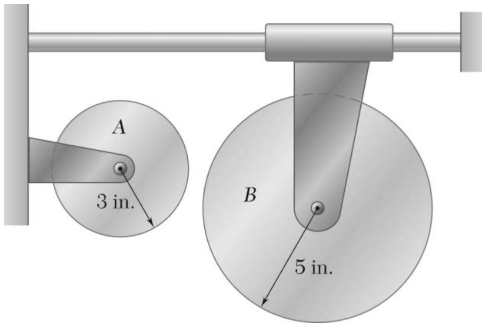
The angle change over the constant speed phase is

$$\Delta\theta = \theta_A - \theta = 96 - 31.416 = 64.584 \text{ radians}$$

For uniform motion, $$\Delta\theta = \omega_f(\Delta t)$$

$$\Delta t = \frac{\Delta\theta}{\omega_f} = \frac{64.584}{4\pi} = 5.139 \text{ s}$$

Total time elapsed: $t_f = 5 \text{ s} + \Delta t$ $t_f = 10.14$ s ◀



PROBLEM 15.32

Disk B is at rest when it is brought into contact with disk A which is rotating freely at 450 rpm clockwise. After 6 s of slippage, during which each disk has a constant angular acceleration, disk A reaches a final angular velocity of 140 rpm clockwise. Determine the angular acceleration of each disk during the period of slippage.

SOLUTION

Disk A: $(\omega_A)_0 = 450 \text{ rpm} = 47.124 \text{ rad/s} \circlearrowright$

When $t = 6$ s: $\boldsymbol{\omega}_A = 140 \text{ rpm} = 14.661 \text{ rad/s} \circlearrowright$

$$\omega_A = (\omega_A)_0 + \alpha_A t$$

$$14.661 \text{ rad/s} = 47.124 \text{ rad/s} + \alpha_A (6 \text{ s})$$

$$\alpha_A = -5.41 \text{ rad/s}$$

$\boldsymbol{\alpha}_A = 5.41 \text{ rad/s}^2 \circlearrowleft$ ◀

Disk B: $\omega_0 = 0$

When $t = 6$ s: (end of slippage)

$+\downarrow\ r_A\omega_A = r_B\omega_B$: $(3 \text{ in.})(14.661 \text{ rad/s}) = (5 \text{ in.})(\omega_B)$

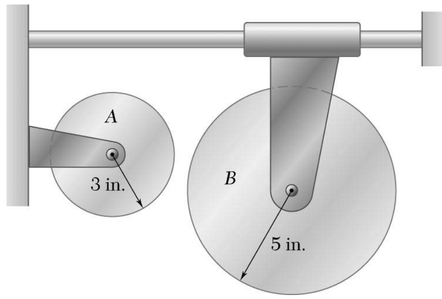
$$\omega_B = 8.796 \text{ rad/s} \circlearrowleft$$

$$\omega_B = (\omega_B)_0 + \alpha_B t$$

$$8.796 \text{ rad/s} = 0 + \alpha_B (6 \text{ s})$$

$$\alpha_B = 1.466 \text{ rad/s}^2$$

$\boldsymbol{\alpha}_B = 1.466 \text{ rad/s}^2 \circlearrowleft$ ◀



PROBLEM 15.33

A simple friction drive consists of two disks *A* and *B*. Initially, disk *A* has a clockwise angular velocity of 500 rpm and disk *B* is at rest. It is known that disk *A* will coast to rest in 60 s. However, rather than waiting until both disks are at rest to bring them together, disk *B* is given a constant angular acceleration of 2.5 rad/s^2 counterclockwise. Determine (*a*) at what time the disks can be brought together if they are not to slip, (*b*) the angular velocity of each disk as contact is made.

SOLUTION

<u>Disk *A*</u>: $(\omega_A)_0 = 500 \text{ rpm} = 52.36 \text{ rad/s} \curvearrowright$

Disk *A* will coast to rest in 60 s.

$$\omega_A = (\omega_A)_0 + \alpha_A t; \quad 0 = 52.36 \text{ rad/s} + \alpha_A (60 \text{ s})$$

$$\alpha_A = -0.87266 \text{ rad/s}^2$$

At time *t*:

$$\omega_A = (\omega_A)_0 + \alpha_A t$$

$$\omega_A = 52.36 - 0.87266t \quad (1)$$

<u>Disk *B*</u>: $\alpha_B = 2.5 \text{ rad/s}^2 \quad (\omega_B)_0 = 0$

At time *t*: $\omega_B = (\omega_B)_0 + \alpha_B t; \quad \omega_B = 2.5t \quad (2)$

(*a*) Bring disks together when: $r_A \omega_A = r_B \omega_B$

$$(3 \text{ in.})(52.36 - 0.87266t) = (5 \text{in.})(2.5t)$$

$$157.08 - 2.618t = 12.5t$$

$$157.08 = 15.118t$$

$t = 10.39$ s ◀

(*b*) When contact is made $(t = 10.39 \text{ s})$

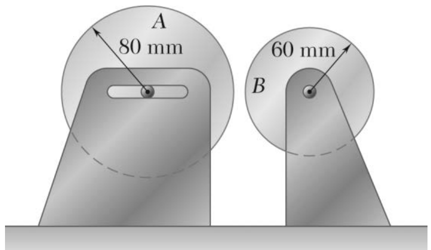
Eq. (1): $\omega_A = 52.36 - 0.87266(10.39)$

$\omega_A = 43.29 \text{ rad/s}$ $\qquad \boldsymbol{\omega}_A = 413 \text{ rpm} \curvearrowright$ ◀

Eq. (2): $\omega_B = 2.5(10.39)$

$\omega_B = 25.975 \text{ rad/s}$ $\qquad \boldsymbol{\omega}_B = 248 \text{ rpm} \curvearrowleft$ ◀

PROBLEM 15.34



A simple friction drive consists of two disks A and B. Initially, disk A has a clockwise angular velocity of 500 rpm and disk B is at rest. It is known that disk A will coast to rest in 60 s. However, rather than waiting until both disks are at rest to bring them together, disk B is given a constant angular acceleration of 2.5 rad/s^2 counterclockwise. Determine (a) at what time the disks can be brought together if they are not to slip, (b) the angular velocity of each disk as contact is made.

SOLUTION

Disk A: $(\boldsymbol{\omega}_A)_0 = 500 \text{ rpm} = 52.36 \text{ rad/s} \circlearrowright$

Disk a will coast to rest in 60 s.

$$\omega_A = (\omega_A)_0 + \alpha_A t; \quad 0 = 52.36 + \alpha_A (60 \text{ s})$$
$$\alpha_A = -0.87266 \text{ rad/s}^2$$

At time t:

$$\omega_A = (\omega_A)_0 + \alpha_A t; \quad \omega_A = 52.36 - 0.87266t \qquad (1)$$

Disk B: $\alpha_B = 2.5 \text{ rad/s}^2 \quad (\omega_B)_0 = 0$

At time t: $\omega_B = (\omega_B)_0 + \alpha_B t; \quad \omega_B = 2.5t \qquad (2)$

(a) Bring disks together when: $r_A \omega_A = r_B \omega_B$

$$(80 \text{ mm})(52.36 - 0.87266t) = (60 \text{ mm})(2.5t)$$
$$4188.8 - 69.813t = 150t$$
$$4188.8 = 219.813t$$
$$t = 19.056 \text{ s}$$

$t = 19.06 \text{ s}$ ◀

(b) Contact is made:

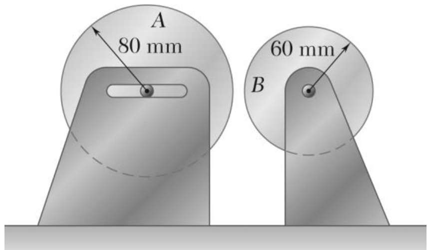
Eq. (1): $\omega_A = 52.36 - 0.87266(19.056)$

$\omega_A = 35.73 \text{ rad/s}$ $\qquad \boldsymbol{\omega}_A = 341 \text{ rpm} \circlearrowright$ ◀

Eq. (2): $\omega_B = 2.5(19.056)$

$\omega_B = 47.64 \text{ rad/s}$ $\qquad \boldsymbol{\omega}_B = 455 \text{ rpm} \circlearrowleft$ ◀

PROBLEM 15.35



Two friction disks A and B are both rotating freely at 240 rpm counterclockwise when they are brought into contact. After 8 s of slippage, during which each disk has a constant angular acceleration, disk A reaches a final angular velocity of 60 rpm counterclockwise. Determine (*a*) the angular acceleration of each disk during the period of slippage, (*b*) the time at which the angular velocity of disk B is equal to zero.

SOLUTION

(*a*) Disk A: $(\boldsymbol{\omega}_A)_0 = 240 \text{ rpm} = 25.133 \text{ rad/s} \circlearrowleft$

When $t = 8$ s, $\boldsymbol{\omega}_A = 60 \text{ rpm} = 6.283 \text{ rad/s} \circlearrowleft$

$$\omega_A = (\omega_A)_0 + \alpha_A t; \quad 6.283 \text{ rad/s} = 25.133 \text{ rad/s} + \alpha_A (8 \text{ s})$$

$$\alpha_A = -2.356 \text{ rad/s}^2$$

$$\boldsymbol{\alpha}_A = 2.36 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

Disk B: $(\omega_B)_0 = 240 \text{ rpm} = 25.123 \text{ rad/s} \circlearrowleft$

When $t = 8$ s: (slippage stops)

$$r_A \omega_A = r_B \omega_B$$

$$(80 \text{ mm})(6.283 \text{ rad/s}) = (60 \text{ mm})\omega_B$$

$$\omega_B = 8.378 \text{ rad/s} \qquad \boldsymbol{\omega}_B = 8.38 \text{ rad/s} \circlearrowright$$

For $+\circlearrowright$: $\omega_B = (\omega_B)_0 + \alpha_B t$

$$8.375 \text{ rad/s} = -25.133 \text{ rad/s} + \alpha_B (8 \text{ s})$$

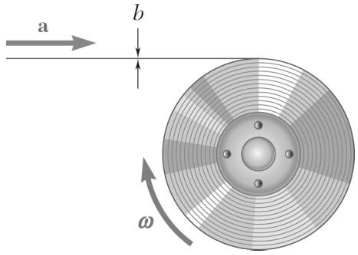
$$\alpha_B = 4.188 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_B = 4.19 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

(*b*) Time when $\omega_B = 0$

For $+\circlearrowright$: $\omega_B = (\omega_B)_0 + \alpha_B t$

$$0 = -25.133 \text{ rad/s} + (4.188 \text{ rad/s}^2)t$$

$$t = 6.00 \text{ s} \qquad t = 6.00 \text{ s} \blacktriangleleft$$



PROBLEM 15.36*

Steel tape is being wound onto a spool which rotates with a constant angular velocity ω_0. Denoting by r the radius of the spool and tape at any given time and by b the thickness of the tape, derive an expression for the acceleration of the tape as it approaches the spool.

SOLUTION

Let one layer of tape be wound and let v be the tape speed.

$$v\Delta t = 2\pi r \quad \text{and} \quad \Delta r = b$$

$$\frac{\Delta r}{\Delta t} = \frac{bv}{2\pi r} = \frac{b\omega}{2\pi}$$

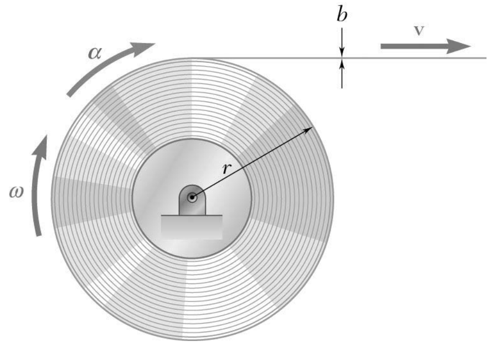
For the spool:

$$\frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{v}{r}\right) = \frac{1}{r}\frac{dv}{dt} + v\frac{d}{dt}\left(\frac{1}{r}\right)$$

$$= \frac{a}{r} - \frac{v}{r^2}\frac{dr}{dt} = \frac{a}{r} - \frac{v}{r^2}\frac{b\omega}{2\pi}$$

$$= \frac{1}{r}\left[a - \frac{b\omega^2}{2\pi}\right] = 0$$

$$\mathbf{a} = \frac{b\omega_0^2}{2\pi} \longrightarrow \blacktriangleleft$$



PROBLEM 15.37*

In a continuous printing process, paper is drawn into the presses at a constant speed *v*. Denoting by *r* the radius of the paper roll at any given time and by *b* the thickness of the paper, derive an expression for the angular acceleration of the paper roll.

SOLUTION

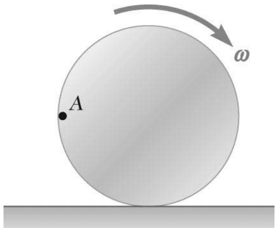
Let one layer of paper be unrolled.

$$v\Delta t = 2\pi r \quad \text{and} \quad \Delta r = -b$$

$$\frac{\Delta r}{\Delta t} = \frac{-bv}{2\pi r} = \frac{dr}{dt}$$

$$\begin{aligned}\alpha &= \frac{d\omega}{dt} \\ &= \frac{d}{dt}\left(\frac{v}{r}\right) \\ &= \frac{1}{r}\frac{dv}{dt} + v\frac{d}{dt}\left(\frac{1}{r}\right) \\ &= 0 - \frac{v}{r^2}\frac{dr}{dt} \\ &= \left(-\frac{v}{r^2}\right)\left(\frac{-bv}{2\pi r}\right) \\ &= \frac{bv^2}{2\pi r^3}\end{aligned}$$

$$\alpha = \frac{bv^2}{2\pi r^3} \circlearrowright \blacktriangleleft$$



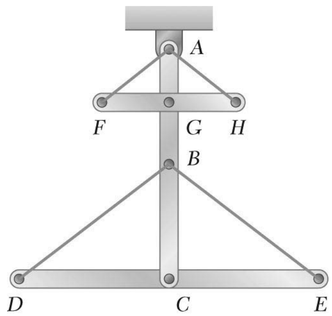
PROBLEM 15.CQ3

The ball rolls without slipping on the fixed surface as shown. What is the direction of the velocity of Point A?

(a) → (b) ↗ (c) ↑ (d) ↓ (e) ↘

SOLUTION

Answer: (b) ◀



PROBLEM 15.CQ4

Three uniform rods, *ABC*, *DCE* and *FGH* are connected as shown. Which of the following statements are true?

(*a*) $\omega_{ABC} = \omega_{DCE} = \omega_{FGH}$

(*b*) $\omega_{DCE} > \omega_{ABC} > \omega_{FGH}$

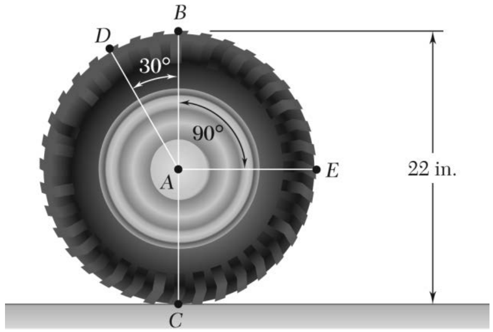
(*c*) $\omega_{DCE} < \omega_{ABC} < \omega_{FGH}$

(*d*) $\omega_{ABC} > \omega_{DCE} > \omega_{FGH}$

(*e*) $\omega_{FGH} = \omega_{DCE} < \omega_{ABC}$

SOLUTION

Answer: (*a*) ◀



PROBLEM 15.38

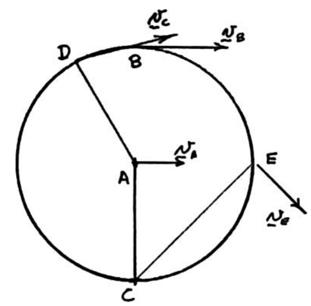
An automobile travels to the right at a constant speed of 48 mi/h. If the diameter of a wheel is 22 in., determine the velocities of Points B, C, D, and E on the rim of the wheel.

SOLUTION

$$\mathbf{v}_A = 48 \text{ mi/h} = 70.4 \text{ ft/s}$$

$$\mathbf{v}_C = 0 \blacktriangleleft$$

$$d = 22 \text{ in.} \quad r = \frac{d}{2} = 11 \text{ in.} = 0.91667 \text{ ft}$$



$$\omega = \frac{v_A}{r} = \frac{70.4}{0.91667} = 76.8 \text{ rad/s} \circlearrowright$$

$$v_{B/A} = v_{D/A} = v_{E/A} = r\omega$$

$$= (0.91667)(76.8) = 70.4 \text{ ft/s}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = [70.4 \text{ ft/s} \longrightarrow] + [70.4 \text{ ft/s} \longrightarrow]$$

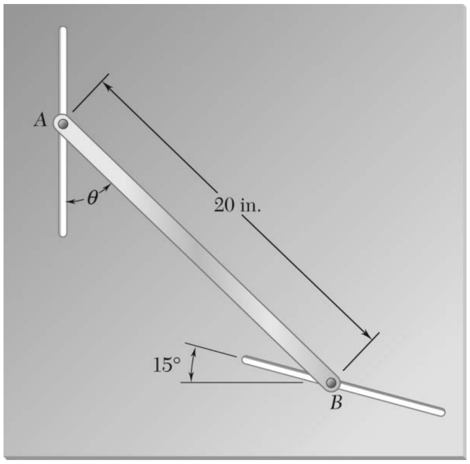
$$\mathbf{v}_B = 140.8 \text{ ft/s} \longrightarrow \blacktriangleleft$$

$$\mathbf{v}_D = \mathbf{v}_A + \mathbf{v}_{D/A} = [70.4 \text{ ft/s} \longrightarrow] + [70.4 \text{ ft/s} \measuredangle 30°]$$

$$\mathbf{v}_D = 136.0 \text{ ft/s} \measuredangle 15.0° \blacktriangleleft$$

$$\mathbf{v}_E = \mathbf{v}_A + \mathbf{v}_{E/A} = [70.4 \text{ ft/s} \longrightarrow] + [70.4 \text{ ft/s} \downarrow]$$

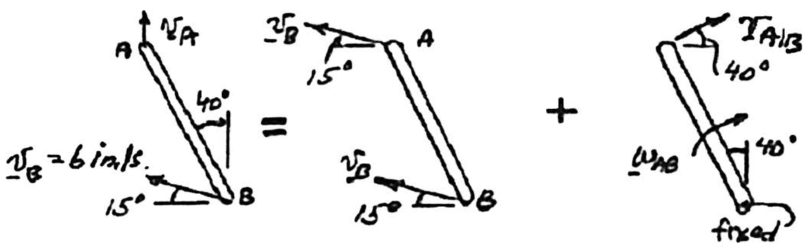
$$\mathbf{v}_E = 99.6 \text{ ft/s} \searrow 45.0° \blacktriangleleft$$



PROBLEM 15.39

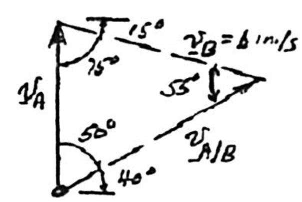
The motion of rod *AB* is guided by pins attached at *A* and *B* which slide in the slots shown. At the instant shown, $\theta = 40°$ and the pin at *B* moves upward to the left with a constant velocity of 6 in./s. Determine (*a*) the angular velocity of the rod, (*b*) the velocity of the pin at end *A*.

SOLUTION



$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$[v_A \uparrow] = [6 \text{ in./s} \measuredangle 15°] + [v_{A/B} \measuredangle 40°]$$



Law of sines.

$$\frac{v_A}{\sin 55°} = \frac{v_{A/B}}{\sin 75°} = \frac{6 \text{ in./s}}{\sin 50°}$$

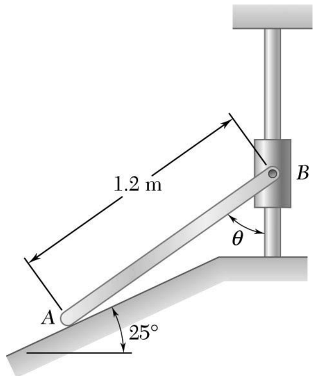
(*b*) $\mathbf{v}_A = 6.42 \text{ in./s} \uparrow$ ◀

$$\mathbf{v}_{A/B} = 7.566 \text{ in./s} \measuredangle 40°$$

$$v_{A/B} = (AB)\omega_{AB} \quad AB = 20 \text{ in.}$$

$$7.566 \text{ in./s} = (20 \text{ in.})\omega_{AB}$$

(*a*) $\omega_{AB} = 0.3783 \text{ rad/s}$ $\boldsymbol{\omega}_{AB} = 0.378 \text{ rad/s} \circlearrowright$ ◀

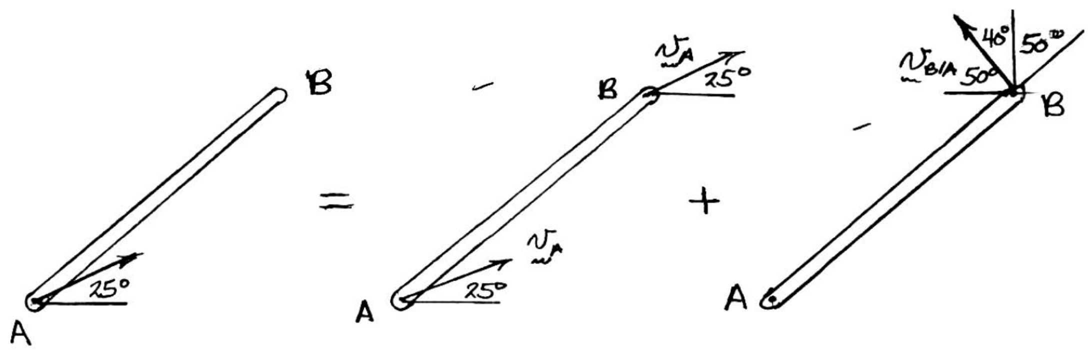


PROBLEM 15.40

Collar B moves upward with a constant velocity of 1.5 m/s. At the instant when $\theta = 50°$, determine (*a*) the angular velocity of rod AB, (*b*) the velocity of end A of the rod.

SOLUTION

Draw a diagram showing the motion of rod AB.



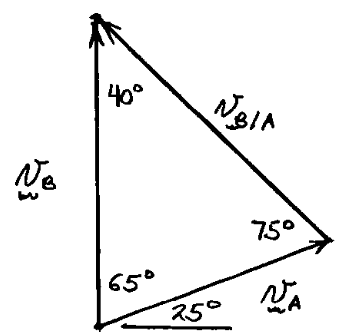
Plane motion = Translation + Rotation

$\mathbf{v}_A = v_A \measuredangle 25°$ $\quad$ $\mathbf{v}_B = 1.5 \text{ m/s}\uparrow$ $\quad$ $\mathbf{v}_{B/A} = v_{B/A} \searrow 50°$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$[1.5 \text{ m/s}\uparrow] = [v_A \measuredangle 25°] + [v_{B/A} \searrow 50°]$$

Draw the velocity vector diagram.



Interior angles of the triangle.

$$90° - 25° = 65°$$
$$90° - 50° = 40°$$
$$25° + 50° = 75°$$

Law of sines.

$v_B = 1.5 \text{ m/s}$

$$\frac{v_B}{\sin 75°} = \frac{v_A}{\sin 40°} = \frac{v_{B/A}}{\sin 65°}$$

PROBLEM 15.40 (Continued)

(*a*) *Angular velocity of AB.*

$$v_{B/A} = \frac{\sin 65°}{\sin 75°}(1.5 \text{ m/s}) = 1.4074 \text{ m/s}$$

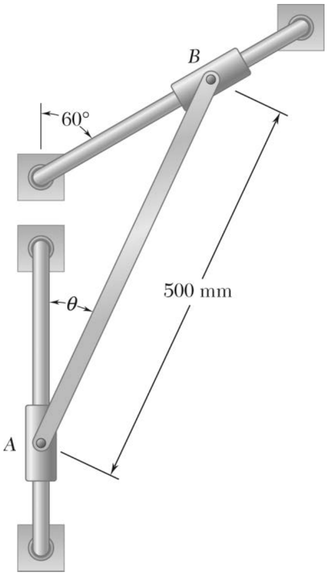
$$\omega_{AB} = \frac{v_{B/A}}{l_{AB}} = \frac{1.4074 \text{ m/s}}{1.2 \text{ m}}$$

$\boldsymbol{\omega}_{AB} = 1.173 \text{ rad/s}$ ↺ ◀

(*b*) *Velocity of end A.*

$$v_A = \frac{\sin 40°}{\sin 75°}(1.5 \text{ m/s})$$

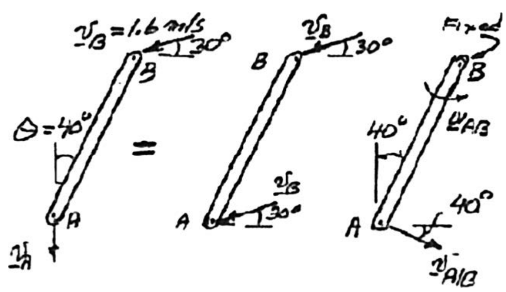
$\mathbf{v}_A = 0.998 \text{ m/s} \measuredangle\ 25°$ ◀



PROBLEM 15.41

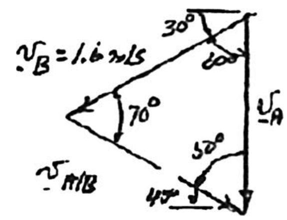
Collar B moves downward to the left with a constant velocity of 1.6 m/s. At the instant shown when $\theta = 40°$, determine (a) the angular velocity of rod AB, (b) the velocity of collar A.

SOLUTION



$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$[v_A \downarrow] = [1.6 \text{ m/s} \swarrow 30°] + [v_{A/B} \searrow 40°]$$



Law of sines.

$$\frac{v_A}{\sin 70°} = \frac{v_{A/B}}{\sin 60°} = \frac{1.6 \text{ m/s}}{\sin 50°}$$

(b) $\mathbf{v}_A = 1.963 \text{ m/s} \downarrow$ ◀

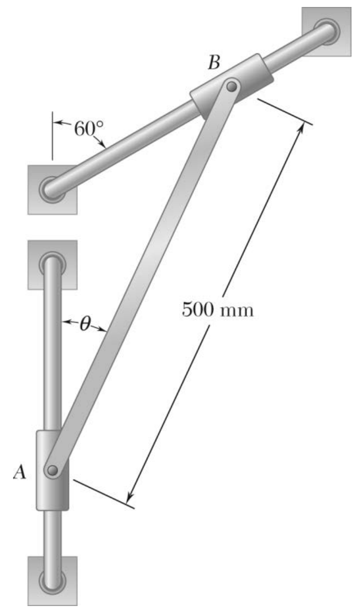
$$\mathbf{v}_{A/B} = 1.809 \text{ m/s}^2 \searrow 40°$$

$$AB = 0.5 \text{ m}$$

$$v_{A/B} = (AB)\omega_{AB}$$

$$1.809 \text{ m/s} = (0.5 \text{ m})\omega_{AB}$$

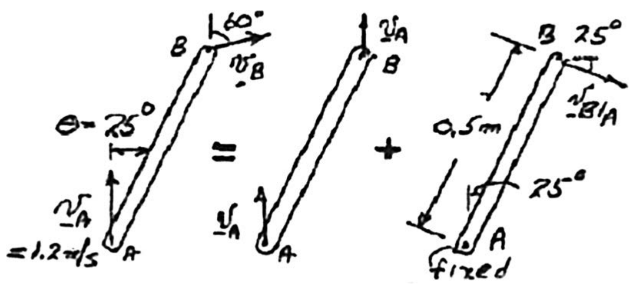
(a) $\omega_{AB} = 3.618$ rad/s $\boldsymbol{\omega}_{AB} = 3.62 \text{ rad/s} \circlearrowright$ ◀

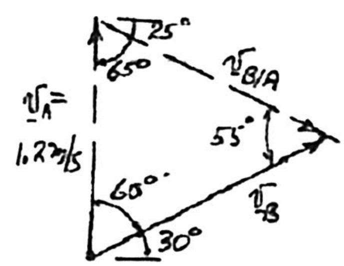


PROBLEM 15.42

Collar A moves upward with a constant velocity of 1.2 m/s. At the instant shown when $\theta = 25°$, determine (*a*) the angular velocity of rod AB, (*b*) the velocity of collar B.

SOLUTION





$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$[v_B \measuredangle 30°] = [1.2 \text{ m/s} \uparrow] + [v_{B/A} \searrow 25°]$$

Law of sines.

$$\frac{v_B}{\sin 65°} = \frac{v_{B/A}}{\sin 60°} = \frac{1.2 \text{ m/s}}{\sin 55°}$$

(*b*) $\mathbf{v}_B = 1.328 \text{ m/s} \measuredangle 30°$ ◀

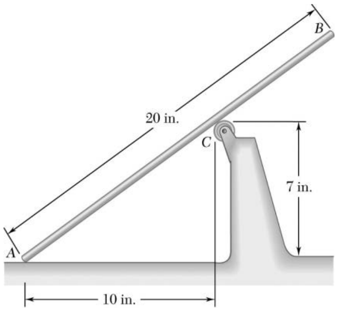
$$\mathbf{v}_{B/A} = 1.269 \text{ m/s} \searrow 65°$$

$$v_{B/A} = (AB)\omega_{AB}$$

$$1.269 \text{ m/s} = (0.5 \text{ m})\omega_{AB}$$

$$\omega_{AB} = 2.538 \text{ rad/s}$$

(*a*) $\boldsymbol{\omega}_{AB} = 2.54 \text{ rad/s} \circlearrowright$ ◀



PROBLEM 15.43

Rod *AB* moves over a small wheel at *C* while end *A* moves to the right with a constant velocity of 25 in./s. At the instant shown, determine (*a*) the angular velocity of the rod, (*b*) the velocity of end *B* of the rod.

SOLUTION

Slope angle of rod. $\tan\theta = \frac{7}{10} = 0.7, \qquad \theta = 35^\circ$

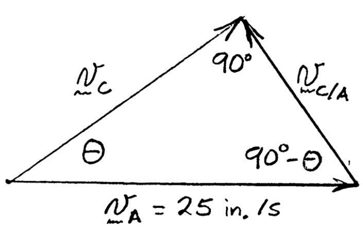
$$\overline{AC} = \frac{10}{\cos\theta} = 12.2066 \text{ in.} \qquad \overline{CB} = 20 - \overline{AC} = 7.7934 \text{ in.}$$

Velocity analysis.

$$\mathbf{v}_A = 25 \text{ in./s} \longrightarrow, \qquad \mathbf{v}_C = v_C \measuredangle \theta$$

$$\mathbf{v}_{C/A} = \overline{AC}\omega_{AB} \searrow \theta$$

$$\mathbf{v}_C = \mathbf{v}_A + v_{C/A}$$



Draw corresponding vector diagram.

$$v_{C/A} = v_A \sin\theta = 25\sin 35^\circ = 14.34 \text{ in./s}$$

(*a*) $\omega_{AB} = \frac{v_{C/A}}{AC} = \frac{14.34}{12.2066} = 1.175 \text{ rad/s}$

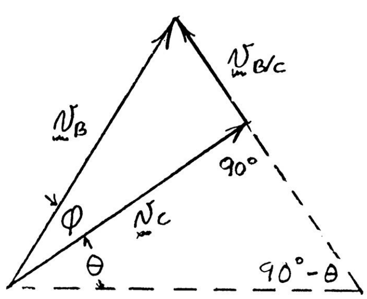
$$\boldsymbol{\omega}_{AB} = 1.175 \text{ rad/s} \circlearrowleft \blacktriangleleft$$

$$v_C = v_A \cos\theta = 25\cos\theta = 20.479 \text{ in./s}$$

$$v_{B/C} = \overline{CB}\omega_{AB} = (7.7934)(1.175) = 9.1551 \text{ in./s}$$

$v_{B/C}$ has same direction as $v_{C/A}$.

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$



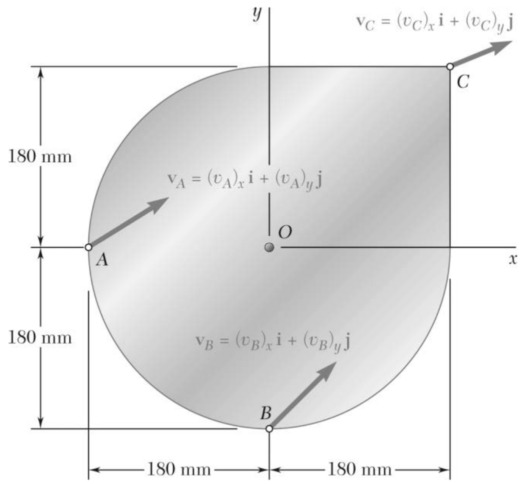
Draw corresponding vector diagram.

$$\tan\phi = \frac{v_{B/C}}{v_C} = \frac{9.1551}{20.479}, \quad \phi = 24.09^\circ$$

(*b*) $v_B = \frac{v_C}{\cos\phi} = \frac{20.479}{\cos 24.09^\circ} = 22.4 \text{ in./s} = 1.869 \text{ ft/s}$

$$\phi + \theta = 59.1^\circ$$

$$\mathbf{v}_B = 1.869 \text{ ft/s} \measuredangle 59.1^\circ \blacktriangleleft$$



PROBLEM 15.44

The plate shown moves in the xy plane. Knowing that $(v_A)_x = 120$ mm/s, $(v_B)_y = 300$ mm/s, and $(v_C)_y = -60$ mm/s, determine (*a*) the angular velocity of the plate, (*b*) the velocity of Point A.

SOLUTION

$$\mathbf{r}_{C/B} = (180 \text{ mm})\mathbf{i} + (360 \text{ mm})\mathbf{j}$$

$$\boldsymbol{\omega} = \omega\mathbf{k}$$

$$\mathbf{v}_B = (v_B)_x\mathbf{i} + (300 \text{ mm/s})\mathbf{j}$$

$$\mathbf{v}_C = (v_C)_x\mathbf{i} - (60 \text{ mm/s})\mathbf{j}$$

(*a*) $$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$(v_C)_x\mathbf{i} - (60 \text{ mm/s})\mathbf{j} = (v_B)_x\mathbf{i} + (300 \text{ mm/s})\mathbf{j} + \boldsymbol{\omega}\times\mathbf{r}_{C/B}$$

$$(v_C)_x\mathbf{i} - 60\mathbf{j} = (v_B)_x\mathbf{i} + 300\mathbf{j} + \omega\mathbf{k}\times(180\mathbf{i} + 360\mathbf{j})$$

$$(v_C)_x\mathbf{i} - 60\mathbf{j} = (v_B)_x\mathbf{i} + 300\mathbf{j} + 180\omega\mathbf{j} - 360\omega\mathbf{i}$$

Coefficients of $\mathbf{j}$: $$-60 = 300 + 180\omega$$

$$\omega = -2 \text{ rad/s}$$

$\boldsymbol{\omega} = 2$ rad/s ↻ ◀

(*b*) Velocity of A: $$\mathbf{r}_{A/B} = -(180 \text{ mm})\mathbf{i} + (180 \text{ mm})\mathbf{j}$$

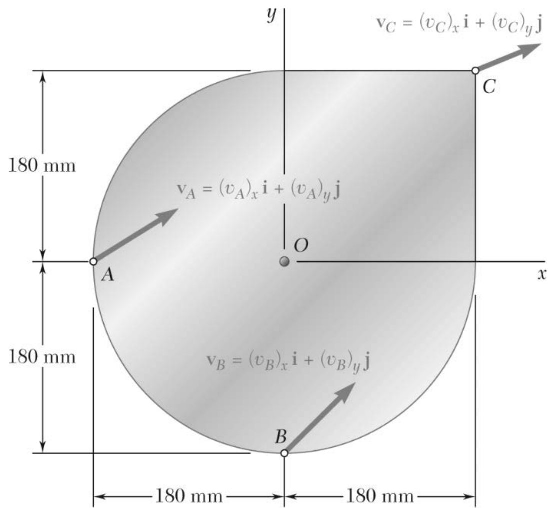
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} = \mathbf{v}_B + \boldsymbol{\omega}\times\mathbf{r}_{A/B}$$

$$120\mathbf{i} + (v_A)_y\mathbf{j} = (v_B)_x\mathbf{i} + 300\mathbf{j} + (-2\mathbf{k})\times(-180\mathbf{i} + 180\mathbf{j})$$

$$120\mathbf{i} + (v_A)_y\mathbf{j} = (v_B)_x\mathbf{i} + 300\mathbf{j} + 360\mathbf{j} + 360\mathbf{i}$$

Coefficients of $\mathbf{j}$: $$(v_A)_y = 300 + 360 = 660 \text{ mm/s}$$

$\mathbf{v}_A = (120 \text{ mm/s})\mathbf{i} + (660 \text{ mm/s})\mathbf{j}$ ◀



PROBLEM 15.45

In Problem 15.44, determine (*a*) the velocity of Point *B*, (*b*) the point of the plate with zero velocity.

PROBLEM 15.44 The plate shown moves in the *xy* plane. Knowing that $(v_A)_x = 120$ mm/s, $(v_B)_y = 300$ mm/s, and $(v_C)_y = -60$ mm/s, determine (*a*) the angular velocity of the plate, (*b*) the velocity of Point *A*.

SOLUTION

$$\mathbf{r}_{B/A} = (180 \text{ mm})\mathbf{i} - (180 \text{ mm})\mathbf{j}$$

From the answer of Problem 15.44, we have

$$\boldsymbol{\omega} = -(2 \text{ rad/s})\mathbf{k}$$
$$\mathbf{v}_A = (120 \text{ mm/s})\mathbf{i} + (660 \text{ mm/s})\mathbf{j}$$

(*a*) *Velocity of B*:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$
$$= 120\mathbf{i} + 660\mathbf{j} - 2\mathbf{k} \times (180\mathbf{i} - 180\mathbf{j})$$
$$= 120\mathbf{i} + 660\mathbf{j} - 360\mathbf{j} - 360\mathbf{i}$$

$$\mathbf{v}_B = -(240 \text{ mm/s})\mathbf{i} + (300 \text{ mm/s})\mathbf{j} \blacktriangleleft$$

(*b*) *Point with* $v = 0$:

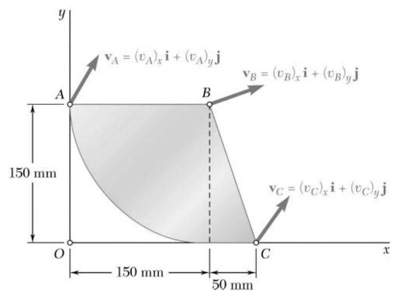
Let $\mathbf{P} = x\mathbf{i} + y\mathbf{j}$ be an arbitrary point.

Thus

$$\mathbf{r}_{P/A} = (180 + x)\mathbf{i} + y\mathbf{j}$$
$$\mathbf{v}_P = \mathbf{v}_A + \mathbf{v}_{P/A} = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{P/A}$$
$$\mathbf{v}_P = 120\mathbf{i} + 660\mathbf{j} + (-2\mathbf{k}) \times [(180 + x)\mathbf{i} + y\mathbf{j}]$$
$$\mathbf{v}_P = 120\mathbf{i} + 660\mathbf{j} - (360 + 2x)\mathbf{j} + 2y\mathbf{i}$$
$$\mathbf{v}_P = (120 + 2y)\mathbf{i} + (300 - 2x)\mathbf{j}$$

For $\mathbf{v}_P = 0$: $\quad 120 + 2y = 0 \quad$ and $\quad 300 - 2x = 0$

$\mathbf{v} = 0$ at: $\quad y = -60$ mm, $\quad x = 150$ mm $\blacktriangleleft$



PROBLEM 15.46

The plate shown moves in the xy plane. Knowing that $(v_A)_x = 250$ mm/s, $(v_B)_x = -450$ mm/s, and $(v_C)_x = -500$ mm/s, determine (*a*) the angular velocity of the plate, (*b*) the velocity of Point A.

SOLUTION

Angular velocity: $\boldsymbol{\omega} = \omega\mathbf{k}$

Relative position vectors: $\mathbf{r}_{B/A} = (150\text{ mm})\mathbf{i}$

$$\mathbf{r}_{C/A} = (200\text{ mm})\mathbf{i} + (150\text{ mm})\mathbf{j}$$

Velocity vectors: $\mathbf{v}_A = (250\text{ mm/s})\mathbf{i} + (v_A)_y\,\mathbf{j}$

$$\mathbf{v}_B = (v_B)_x\mathbf{i} - (450\text{ mm/s})\mathbf{j}$$

$$\mathbf{v}_C = -(500\text{ mm/s})\mathbf{i} + (v_C)_y\,\mathbf{j}$$

Unknowns are ω, $(v_A)_y$, $(v_B)_x$, and $(v_C)_y$.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = \mathbf{v}_A + \omega\mathbf{k}\times\mathbf{r}_{B/A}$$

$$(v_B)_x\mathbf{i} - 450\mathbf{j} = 250\mathbf{i} + (v_A)_y\,\mathbf{j} + \omega\mathbf{k}\times 150\mathbf{i}$$

$$= 250\mathbf{i} + (v_A)_y\,\mathbf{j} + 150\omega\mathbf{j}$$

i: $\quad (v_B)_x = 250 \qquad (1)$

j: $\quad -450 = (v_A)_y + 150\omega \qquad (2)$

$$\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A} = \mathbf{v}_A + \omega\mathbf{k}\times\mathbf{r}_{C/A}$$

$$-500\mathbf{i} + (v_C)_y\,\mathbf{j} = 250\mathbf{i} + (v_A)_y\,\mathbf{j} + \omega\mathbf{k}\times(200\mathbf{i} - 150\mathbf{j})$$

$$= 250\mathbf{i} + (v_A)_y\,\mathbf{j} + 200\omega\mathbf{j} + 150\omega\mathbf{i}$$

i: $\quad -500 = 250 + 150\omega \qquad (3)$

j: $\quad (v_C)_y = (v_A)_y + 150\omega \qquad (4)$

(*a*) *Angular velocity of the plate.*

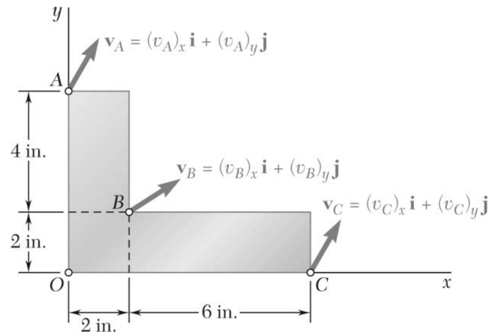
From Eq. (3), $\quad \omega = -\dfrac{750}{150} = -5$

$$\boldsymbol{\omega} = -(5.00\text{ rad/s})\mathbf{k} = 5.00\text{ rad/s} \curvearrowright \blacktriangleleft$$

(*b*) *Velocity of Point A.*

From Eq. (2), $(v_A)_y = -450 - 150\omega = -450 - (150)(-5) = 300$ mm/s

$$\mathbf{v}_A = (250\text{ mm/s})\mathbf{i} + (300\text{ mm/s})\mathbf{j} \blacktriangleleft$$



PROBLEM 15.47

The plate shown moves in the *xy* plane. Knowing that $(v_A)_x = 12$ in./s, $(v_B)_x = -4$ in./s, and $(v_C)_y = -24$ in./s, determine (*a*) the angular velocity of the plate, (*b*) the velocity of Point *B*.

SOLUTION

Angular velocity: $\boldsymbol{\omega} = \omega\mathbf{k}$

Relative position vectors:

$$\mathbf{r}_{A/B} = -(2\text{ in.})\mathbf{i} + (4\text{ in.})\mathbf{j}$$

$$\mathbf{r}_{C/B} = (6\text{ in.})\mathbf{i} - (2\text{ in.})\mathbf{j}$$

Velocity vectors:

$$\mathbf{v}_A = (12\text{ in./s})\mathbf{i} + (v_A)_y\,\mathbf{j}$$

$$\mathbf{v}_B = -(4\text{ in./s})\mathbf{i} + (v_B)_y\,\mathbf{j}$$

$$\mathbf{v}_C = (v_C)_x\,\mathbf{i} - (24\text{ in./s})\mathbf{j}$$

Unknowns are ω, $(v_A)_y$, $(v_B)_y$, and $(v_C)_x$.

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} = \mathbf{v}_B + \omega\mathbf{k}\times\mathbf{r}_{A/B}$$

$$12\mathbf{i} + (v_A)_y = -4\mathbf{i} + (v_B)_y\,\mathbf{j} + \omega\mathbf{k}\times(-2\mathbf{i} + 4\mathbf{j})$$

$$= -4\mathbf{i} + (v_B)_y\,\mathbf{j} - 2\omega\mathbf{j} - 4\omega\mathbf{i}$$

i: $$12 = -4 - 4\omega \qquad (1)$$

j: $$(v_A)_y = (v_B)_y - 2\omega \qquad (2)$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B} = \mathbf{v}_B + \omega\mathbf{k}\times\mathbf{r}_{C/B}$$

$$(v_C)_x\mathbf{i} - 24\mathbf{j} = -4\mathbf{i} + (v_B)_y\,\mathbf{j} + \omega\mathbf{k}\times(6\mathbf{i} - 2\mathbf{j})$$

$$= -4\mathbf{i} + (v_B)_y\,\mathbf{j} + 6\omega\mathbf{j} + 2\omega\mathbf{i}$$

i: $$(v_C)_x = -4 + 2\omega \qquad (3)$$

j: $$-24 = (v_B)_y + 6\omega \qquad (4)$$

PROBLEM 15.47 (Continued)

(*a*) *Angular velocity of the plate*.

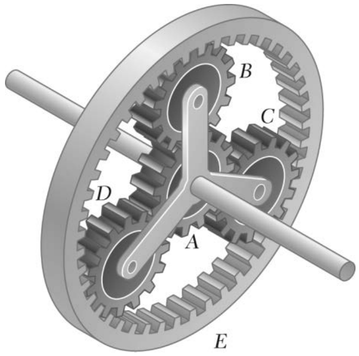
From Eq. (1), $\omega = -\dfrac{16}{4} = -4 \text{ rad/s}$

$\boldsymbol{\omega} = -(4.00 \text{ rad/s})\mathbf{k} = 4.00 \text{ rad/s} \circlearrowright$ ◀

(*b*) *Velocity of Point B*.

From Eq. (4), $(v_B)_y = -24 - (6)(-4) = 0$

$\mathbf{v}_B = -(4.00 \text{ in./s})\mathbf{i}$ ◀



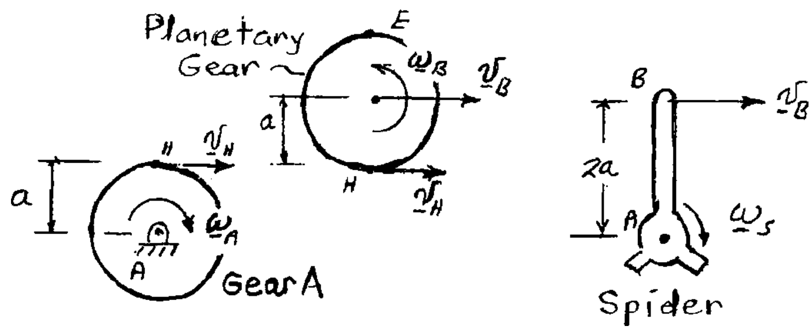
PROBLEM 15.48

In the planetary gear system shown, the radius of gears *A*, *B*, *C*, and *D* is *a* and the radius of the outer gear *E* is 3*a*. Knowing that the angular velocity of gear *A* is ω_A clockwise and that the outer gear *E* is stationary, determine (*a*) the angular velocity of each planetary gear, (*b*) the angular velocity of the spider connecting the planetary gears.

SOLUTION

Gear *E* is stationary. $v_E = 0$

Let *A* be the center of gear *A* and the spider. Since the motions of gears *B*, *C*, and *D* are similar, only gear *B* is considered. Let *H* be the effective contact point between gears *A* and *B*.



Gear *A*: $\mathbf{v}_H = a\omega_A \longrightarrow$

(*a*) *Planetary gears B, C, and D*: $\mathbf{v}_H = \mathbf{v}_E + \mathbf{v}_{H/E}$

$$+\longrightarrow: a\omega_A = 0 + (2a)\omega_B \qquad \omega_B = \frac{1}{2}\omega_A$$

$$\boldsymbol{\omega}_B = \boldsymbol{\omega}_C = \boldsymbol{\omega}_D = \frac{1}{2}\omega_A \circlearrowleft \blacktriangleleft$$

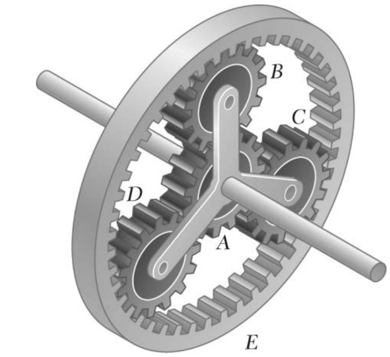
$$\mathbf{v}_B = \mathbf{v}_E + \mathbf{v}_{B/E}$$

$$+\longrightarrow: v_B = 0 + a\left(\frac{1}{2}\omega_A\right) \qquad \mathbf{v}_B = \frac{1}{2}a\omega_A \longrightarrow \qquad (1)$$

(*b*) *Spider*. $\mathbf{v}_B = (2a)\omega_s \longrightarrow$ (2)

Equating expressions (1) and (2) for $\mathbf{v}_B$,

$$\frac{1}{2}a\omega_A = (2a)\omega_s \qquad \omega_s = \frac{1}{4}\omega_A \qquad \boldsymbol{\omega}_s = \frac{1}{4}\omega_A \circlearrowright \blacktriangleleft$$



PROBLEM 15.49

In the planetary gear system shown, the radius of gears *A*, *B*, *C*, and *D* is 30 mm and the radius of the outer gear *E* is 90 mm. Knowing that gear *E* has an angular velocity of 180 rpm clockwise and that the central gear *A* has an angular velocity of 240 rpm clockwise, determine (*a*) the angular velocity of each planetary gear, (*b*) the angular velocity of the spider connecting the planetary gears.

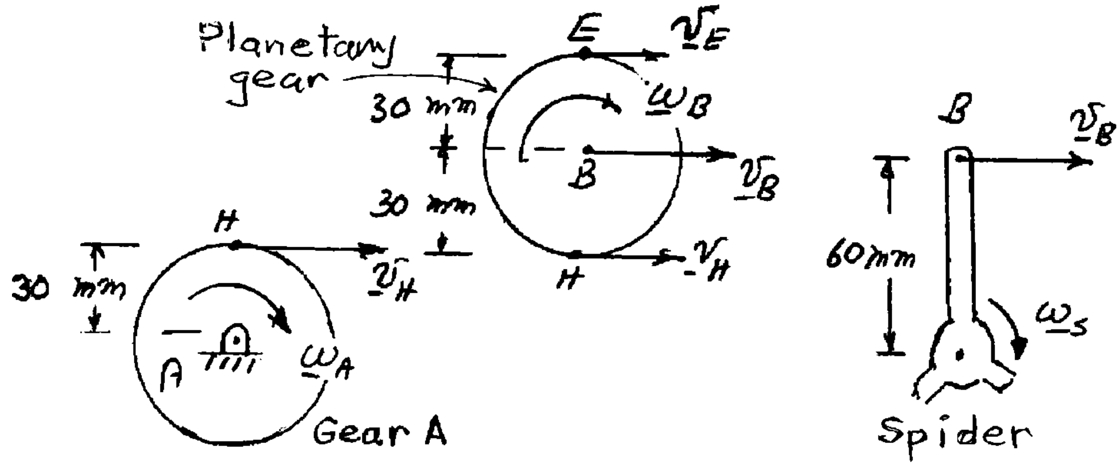
SOLUTION

Since the motions of the planetary gears *B*, *C*, and *D* are similar, only gear *B* is considered. Let Point *H* be the effect contact point between gears *A* and *B* and let Point *E* be the effective contact point between gears *B* and *E*.

Given angular velocities:

$$\boldsymbol{\omega}_E = 180 \text{ rpm} \circlearrowright = 6\pi \text{ rad/s} \circlearrowright$$

$$\boldsymbol{\omega}_A = 240 \text{ rpm} \circlearrowright = 8\pi \text{ rad/s} \circlearrowright$$



Outer gear *E*: radius $= r_E = 90$ mm

$$v_E = r_E \omega_E = (90 \text{ mm})(6\pi \text{ rad/s}) = 540\pi \text{ mm/s}$$

$$\mathbf{v}_E = 540\pi \text{ mm/s} \longrightarrow$$

Gear *A*: radius $= r_A = 30$ mm

$$v_H = r_A \omega_A = (30 \text{ mm})(8\pi \text{ rad/s}) = 240\pi \text{ mm/s}$$

$$\mathbf{v}_H = 240\pi \text{ mm/s} \longrightarrow$$

Planetary gear *B*: radius $= r_B = 30$ mm, $\boldsymbol{\omega}_B = \omega_B$ ↻

$$\mathbf{v}_H = \mathbf{v}_E + \mathbf{v}_{H/E}$$

$$[(30\text{ mm})\omega_A \longrightarrow] = [540\pi\text{ mm/s} \longrightarrow] + [(60\text{ mm})\omega_B \longleftarrow]$$

$$30\omega_A = 540\pi - 60\omega_B$$

$$\omega_B = \frac{540\pi + 30\omega_A}{60} = 9\pi + \frac{1}{2}\omega_A = 9\pi - \frac{1}{2}(8\pi) = 5\pi\text{ rad/s} \circlearrowright$$

(*a*) *Angular velocity of planetary grears*:

$$\boldsymbol{\omega}_B = \boldsymbol{\omega}_C = \boldsymbol{\omega}_D = 5\pi\text{ rad/s} \circlearrowright = 150\text{ rpm} \circlearrowright \blacktriangleleft$$

$$\mathbf{v}_B = \mathbf{v}_H + v_{B/H} = [(30\text{ mm})\omega_A \longrightarrow] + [30\text{ mm } \omega_B \longrightarrow]$$

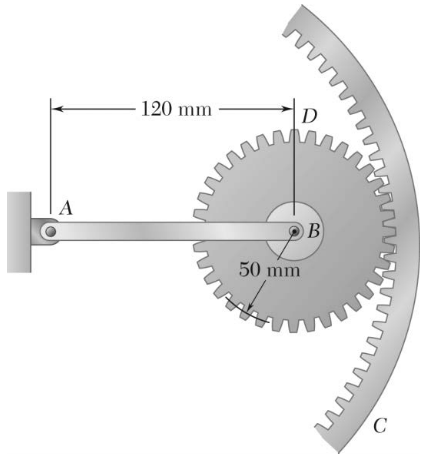
$$v_B = (30\text{ mm})(8\pi\text{ rad/s}) + (30\text{ mm})(5\pi\text{ rad/s}) = 390\pi\text{ mm/s}$$

(*b*) *Spider*: arm $= r_s = 60$ mm, $\boldsymbol{\omega}_s = \omega_s$ ↻

$$v_B = r_s\omega_s$$

$$\omega_s = \frac{v_B}{r_s} = \frac{390\pi\text{ mm/s}}{60\text{ mm}} = 6.5\pi\text{ rad/s}$$

$$\boldsymbol{\omega}_s = 6.5\pi\text{ rad/s} \circlearrowright = 195\text{ rpm} \circlearrowright \blacktriangleleft$$

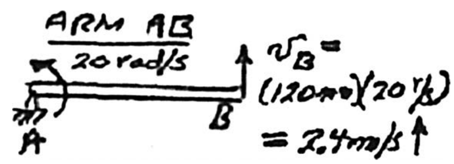


PROBLEM 15.50

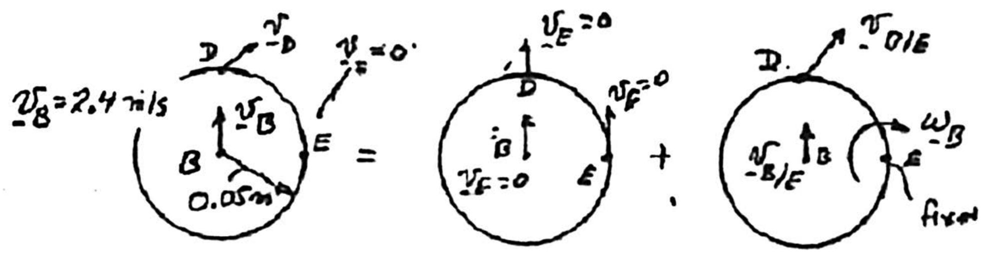
Arm AB rotates with an angular velocity of 20 rad/s counter-clockwise. Knowing that the outer gear C is stationary, determine (*a*) the angular velocity of gear B, (*b*) the velocity of the gear tooth located at Point D.

SOLUTION

<u>Arm *AB*</u>:



<u>Gear *B*</u>:



(*a*) $BE = 0.05$ m:

$$\mathbf{v}_B = \mathbf{v}_D + \mathbf{v}_{B/E} = 0 + (BE)\omega_B$$

$$2.4 \text{ m/s}\uparrow = 0 + (0.05 \text{ m})\omega_B \uparrow$$

$$\omega_B = 48 \text{ rad/s}$$

$\boldsymbol{\omega}_B = 48$ rad/s ↻ ◀

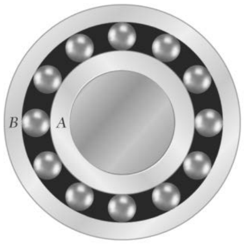
(*b*) $DE = (0.05\sqrt{2})$:

$$\mathbf{v}_D = \mathbf{v}_E + \mathbf{v}_{D/E} = 0 + (DE)\omega_B$$

$$v_D = 0 + (0.05\sqrt{2})(48)$$

$$v_D = 3.39 \text{ m/s}$$

$\mathbf{v}_D = 3.39$ m/s ∠ 45° ◀



PROBLEM 15.51

In the simplified sketch of a ball bearing shown, the diameter of the inner race A is 60 mm and the diameter of each ball is 12 mm. The outer race B is stationary while the inner race has an angular velocity of 3600 rpm. Determine (*a*) the speed of the center of each ball, (*b*) the angular velocity of each ball, (*c*) the number of times per minute each ball describes a complete circle.

SOLUTION

Data: $\omega_A = 3600 \text{ rpm} = 376.99 \text{ rad/s}, \quad \omega_B = 0$

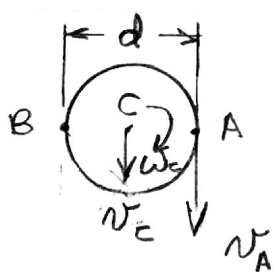
$$r_A = \frac{1}{2} d_A = 30 \text{ mm}$$

$$d = \text{diameter of ball} = 12 \text{ mm}$$

Velocity of point on inner race in contact with a ball.

$$v_A = r_A \omega_A = (30)(376.99) = 11310 \text{ mm/s}$$

Consider a ball with its center at Point C.



$$v_A = v_B + v_{A/B}$$

$$v_A = 0 + \omega_C d$$

$$\omega_C = \frac{v_A}{d} = \frac{11310}{12}$$

$$= 942.48 \text{ rad/s}$$

$$v_C = v_B + v_{C/B}$$

$$= 0 + \frac{1}{2} d\omega = (6)(942.48) = 5654.9 \text{ mm/s}$$

(*a*) $v_C = 5.65 \text{ m/s}$ ◀

(*b*) Angular velocity of ball.

$$\omega_C = 942.48 \text{ rad/s}$$

$\omega_C = 9000 \text{ rpm}$ ◀

(*c*) Distance traveled by center of ball in 1 minute.

$$l_C = v_C t = 5654.9(60) = 339290 \text{ mm}$$

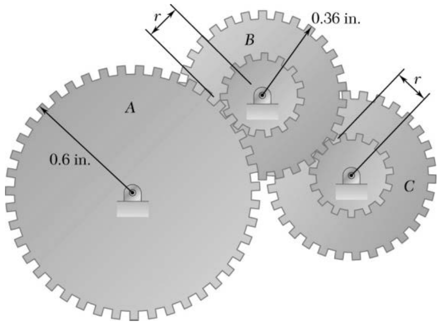
Circumference of circle: $2\pi r = 2\pi(30 + 6)$

$$= 226.19 \text{ mm}$$

Number of circles completed in 1 minute:

$$n = \frac{l}{2\pi r} = \frac{339290}{226.19}$$

$n = 1500$ ◀



PROBLEM 15.52

A simplified gear system for a mechanical watch is shown. Knowing that gear A has a constant angular velocity of 1 rev/h and gear C has a constant angular velocity of 1 rpm, determine (*a*) the radius r, (*b*) the magnitudes of the accelerations of the points on gear B that are in contact with gears A and C.

SOLUTION

Point where A contacts B:

$$v_1 = r_A\omega_A = r\omega_B \qquad \omega_B = \frac{r_A\omega_A}{r} \qquad (1)$$

Point where B contacts C:

$$v_2 = r_B\omega_B = r\omega_C \qquad \omega_C = \frac{r_B}{r}\omega_B \qquad (2)$$

From Eqs. (1) and (2),

$$\omega_C = \frac{r_A r_B}{r^2}\omega_A$$

$$r^2 = r_A r_B \frac{\omega_A}{\omega_C}$$

Data:

$$r_A = 0.6 \text{ in.}, \quad r_B = 0.36 \text{ in.} \quad \frac{\omega_A}{\omega_C} = \frac{1 \text{ rev/h}}{1 \text{ rev/m}} = \frac{1}{60}$$

$$r^2 = \frac{(0.6 \text{ in.})(0.36 \text{ in.})}{60} = 0.0036 \text{ in}^2$$

(*a*) *Radius r*: $r = 0.0600$ in. ◀

Angular velocity of B.

$$\omega_C = 1 \text{ rpm} = \frac{2\pi}{60} \text{ rad/s}$$

$$\omega_B = \frac{r}{r_B}\omega_C = \frac{0.060}{0.36}\frac{2\pi}{60} = 0.017453 \text{ rad/s}$$

(*b*) *Point where B contacts A.*

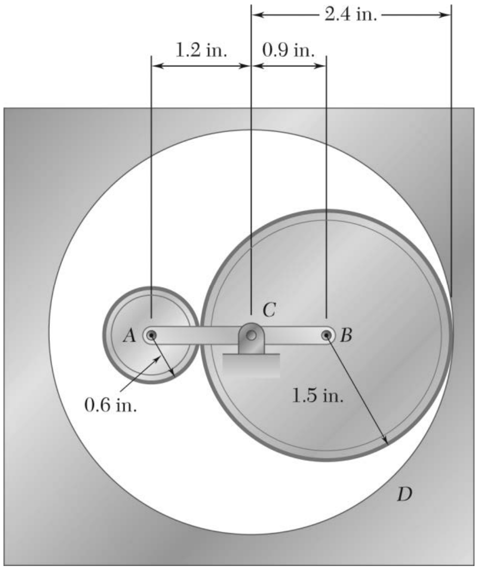
$$a_n = r\omega_B^2 = (0.0600 \text{ in.})(0.017453 \text{ rad/s})^2$$

$$a_n = 18.28\times 10^{-6} \text{ in./s}^2 \text{ ◀}$$

Point where B contacts C.

$$a_n = r_B\omega_B^2 = (0.36 \text{ in.})(0.017453 \text{ rad/s})^2$$

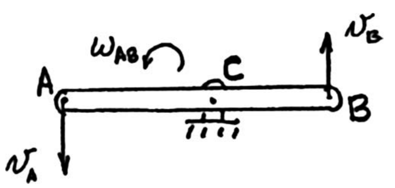
$$a_n = 109.7\times 10^{-6} \text{ in./s}^2 \text{ ◀}$$



PROBLEM 15.53

Arm ACB rotates about Point C with an angular velocity of 40 rad/s counterclockwise. Two friction disks A and B are pinned at their centers to arm ACB as shown. Knowing that the disks roll without slipping at surfaces of contact, determine the angular velocity of (*a*) disk A, (*b*) disk B.

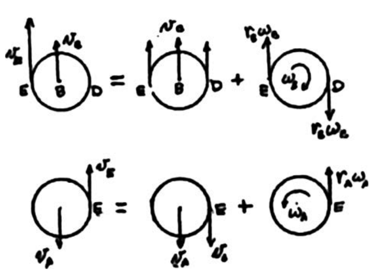
SOLUTION



Arm ACB: Fixed axis rotation.

$$r_{A/C} = 24 \text{ mm}, \quad \mathbf{v}_A = r_{A/C}\omega_{AB} = (24)(40) = 960 \text{ mm/s} \downarrow$$

$$r_{B/C} = 18 \text{ mm}, \quad \mathbf{v}_B = r_{B/C}\omega_{AB} = (18)(40) = 720 \text{ mm/s} \uparrow$$



Disk B: Plane motion = Translation with B + Rotation about B.

$$r_B = 30 \text{ mm}, \quad \mathbf{v}_D = \mathbf{v}_B - \mathbf{v}_{D/B}$$

$$0 = 720 \uparrow + 30\ \omega_B \downarrow \text{ mm/s}$$

$$\omega_B = \frac{720}{30} = 24 \text{ rad/s} \circlearrowright$$

$$\mathbf{v}_E = \mathbf{v}_B + \mathbf{v}_{E/B}$$

$$= 720 \uparrow + (30)(24) \uparrow = 1440 \text{ mm/s} \uparrow$$

Disk A: Plane motion = Translation with A + Rotation about A.

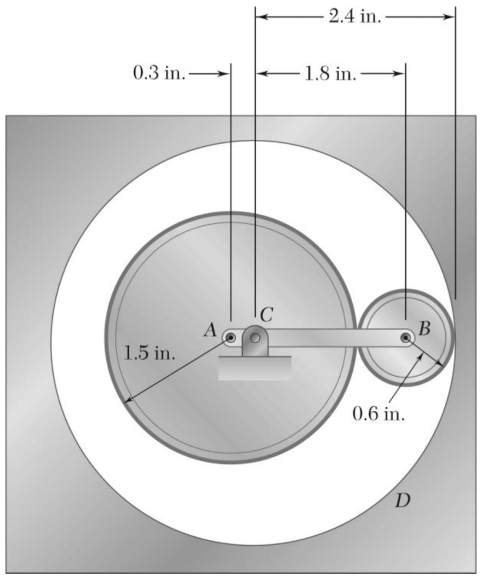
$$r_A = 12 \text{ mm}, \quad \mathbf{v}_E = \mathbf{v}_A - \mathbf{v}_{E/A}$$

$$1440 \uparrow = 960 \downarrow + 12\omega_A \uparrow$$

$$\omega_A = \frac{1440 + 960}{12} = 200 \text{ rad/s} \circlearrowleft$$

(*a*) $\boldsymbol{\omega}_A = 200 \text{ rad/s} \circlearrowleft$ ◀

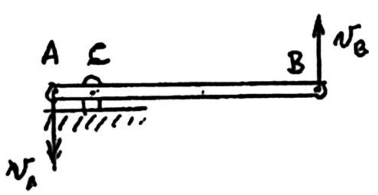
(*b*) $\boldsymbol{\omega}_B = 24.0 \text{ rad/s} \circlearrowright$ ◀



PROBLEM 15.54

Arm *ACB* rotates about Point *C* with an angular velocity of 40 rad/s counterclockwise. Two friction disks *A* and *B* are pinned at their centers to arm *ACB* as shown. Knowing that the disks roll without slipping at surfaces of contact, determine the angular velocity of (*a*) disk *A*, (*b*) disk *B*.

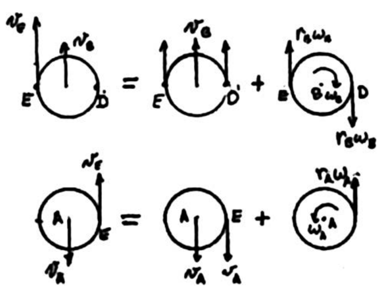
SOLUTION



Arm *ACB*: Fixed axis rotation.

$$r_{A/C} = 0.3 \text{ in.}, \quad \mathbf{v}_A = r_{A/C}\omega_{AB} \downarrow = (0.3)(40) \downarrow = 12 \text{ in./s} \downarrow$$

$$r_{B/C} = 1.8 \text{ in.}, \quad \mathbf{v}_B = r_{B/C}\omega_{AB} \uparrow = (1.8)(40) \uparrow = 72 \text{ in./s} \uparrow$$



Disk *B*: Plane motion = Translation with *B* + Rotation about *B*.

$$r_B = 0.6 \text{ in.}, \quad \mathbf{v}_D = \mathbf{v}_B - \mathbf{v}_{B/A}$$

$$0 = 72 \uparrow + 0.6\omega_B \downarrow$$

$$\omega_B = \frac{72}{0.6} = 120 \text{ rad/s} \circlearrowright$$

$$\mathbf{v}_E = \mathbf{v}_B + \mathbf{v}_{E/B}$$

$$= 72 \uparrow + (0.6)(120) \uparrow = 144 \text{ in./s} \uparrow$$

Disk *A*: Plane motion = Translation with *A* + Rotation about *A*.

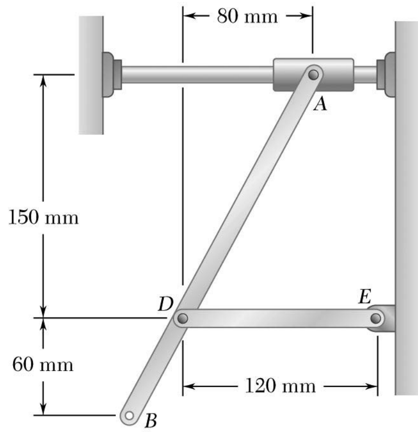
$$r_A = 1.5 \text{ in.}, \quad \mathbf{v}_E = \mathbf{v}_A - \mathbf{v}_{E/A}$$

$$144 \uparrow = 12 \downarrow \; 1.5\omega_A$$

$$\omega_A = \frac{144 + 12}{1.5} = 104 \text{ rad/s} \circlearrowleft$$

(*a*) $\boldsymbol{\omega}_A = 104.0 \text{ rad/s} \circlearrowleft$ ◀

(*b*) $\boldsymbol{\omega}_B = 120.0 \text{ rad/s} \circlearrowright$ ◀



PROBLEM 15.55

Knowing that at the instant shown the velocity of collar A is 900 mm/s to the left, determine (a) the angular velocity of rod ADB, (b) the velocity of Point B.

SOLUTION

Consider rod ADB.

$$\mathbf{v}_D = v_D\mathbf{j}, \quad \mathbf{v}_A = -(900 \text{ mm/s})\mathbf{i}$$
$$\mathbf{r}_{D/A} = -(80 \text{ mm})\mathbf{i} - (150 \text{ mm})\mathbf{j}$$
$$\mathbf{v}_{D/A} = \boldsymbol{\omega}_{AD} \times \mathbf{r}_{D/A} = \omega_{AD}\mathbf{k} \times (-80\mathbf{i} - 150\mathbf{j})$$
$$= 150\omega_{AD}\mathbf{i} - 80\omega_{AD}\mathbf{j}$$
$$\mathbf{v}_0 = \mathbf{v}_A + \mathbf{v}_{D/A}$$
$$v_D\mathbf{j} = -900\mathbf{i} + 150\omega_{AD}\mathbf{i} - 80\omega_{AD}\mathbf{j}$$

Equate components.

$$\mathbf{i}: \quad 0 = -900 + 150\omega_{AD} \qquad \omega_{AD} = 6 \text{ rad/s}$$

(a) *Angular velocity of ADB.* $\boldsymbol{\omega}_{AD} = (6.00 \text{ rad/s})\mathbf{k} = 6.00 \text{ rad/s}$ ⤾ ◀

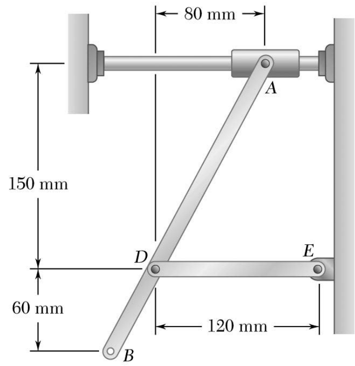
By proportions,

$$\mathbf{r}_{B/A} = \frac{150+60}{150}\mathbf{r}_{D/A} = 1.4\, \mathbf{r}_{D/A}$$
$$= -(112 \text{ mm})\mathbf{i} - (210 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AD}\mathbf{k} \times \mathbf{r}_{B/A}$$
$$= -900\mathbf{i} + 6\mathbf{k} \times (-112\mathbf{i} - 210\mathbf{j})$$
$$= -900\mathbf{i} - 672\mathbf{j} + 1260\mathbf{i}$$

(b) *Velocity of B.* $\mathbf{v}_B = (360 \text{ mm/s})\mathbf{i} - (672 \text{ mm/s})\mathbf{j} = 762 \text{ mm/s} \measuredangle 61.8°$ ◀

PROBLEM 15.56



Knowing that at the instant shown the angular velocity of rod *DE* is 2.4 rad/s clockwise, determine (*a*) the velocity *A*, (*b*) the velocity of Point *B*.

SOLUTION

Rod *DE*: Point *E* is fixed. $\boldsymbol{\omega}_{DE} = 2.4 \text{ rad} \circlearrowright$

$$v_D = \omega_{DB} r_{DE} = (2.4 \text{ rad/s})(120 \text{ mm}) = 288 \text{ mm/s}$$

$$\mathbf{v}_D = 288 \text{ mm/s} \uparrow = (288 \text{ mm/s})\mathbf{j}$$

Rod *ADB*: $\mathbf{r}_{A/D} = (80 \text{ mm})\mathbf{i} + (150 \text{ mm})\mathbf{j}, \;\; \boldsymbol{\omega}_{AD} = \boldsymbol{\omega}_{AD}\mathbf{k}, \quad \mathbf{v}_A = v_A\mathbf{i}$

$$\mathbf{v}_A = \mathbf{v}_D + \mathbf{v}_{D/A} = \mathbf{v}_D + \omega_{AD}\mathbf{k} \times \mathbf{r}_{A/D}$$

$$v_A\mathbf{i} = (288 \text{ mm/s})\mathbf{j} + \omega_{AD}\mathbf{k} \times [(80 \text{ mm})\mathbf{i} + (150 \text{ mm})\mathbf{j}]$$

$$v_A\mathbf{i} = 288\mathbf{j} + 80\omega_{AD}\mathbf{j} - 150\omega_{AD}\mathbf{i}$$

Equate components.

i: $$v_A = -150\omega_{AD} \tag{1}$$

j: $$0 = 288 + 80\omega_{AD} \tag{2}$$

From Eq. (2), $\omega_{AD} = -\dfrac{288}{80}$ $\qquad \boldsymbol{\omega}_{AD} = (-3.6 \text{ rad/s})\mathbf{k}$

From Eq. (1), $v_A = -(150)(-3.6) = 540 \text{ mm/s}$

(*a*) *Velocity of collar A*. $\mathbf{v}_A = 540 \text{ mm/s} \longrightarrow$ ◀

(*b*) *Velocity of Point B*.

By proportions $\mathbf{r}_{B/D} = -\dfrac{60}{150}\mathbf{r}_{A/D} = -(32 \text{ mm})\mathbf{i} - 60 \text{ mm } \mathbf{j}$

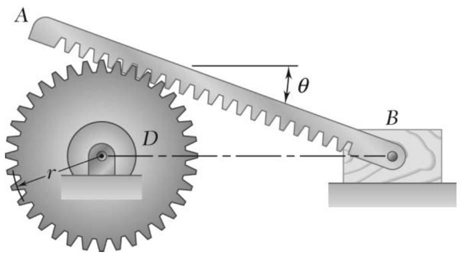
$$\mathbf{v}_B = \mathbf{v}_D + \mathbf{v}_{B/D} = \mathbf{v}_D + \boldsymbol{\omega}_{AD} \times \mathbf{r}_{B/D}$$

$$= (288 \text{ mm/s})\mathbf{j} + [-(3.6 \text{ rad/s})\mathbf{k}] \times [-(32 \text{ mm})\mathbf{i} - (60 \text{ mm})\mathbf{j}]$$

$$= (288 \text{ mm/s})\mathbf{j} + (115.2 \text{ mm/s})\mathbf{j} - (216 \text{ mm/s})\mathbf{i}$$

$$\mathbf{v}_B = -(216 \text{ mm/s})\mathbf{i} + (403.2 \text{ mm/s})\mathbf{j}$$

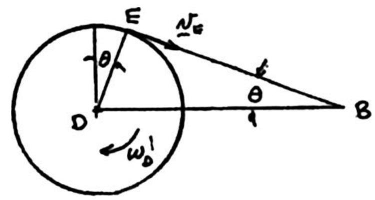
$\mathbf{v}_B = 457 \text{ mm/s} \measuredangle 61.8°$ ◀



PROBLEM 15.57

A straight rack rests on a gear of radius r and is attached to a block B as shown. Denoting by ω_D the clockwise angular velocity of gear D and by θ the angle formed by the rack and the horizontal, derive expressions for the velocity of block B and the angular velocity of the rack in terms of r, θ, and ω_D.

SOLUTION



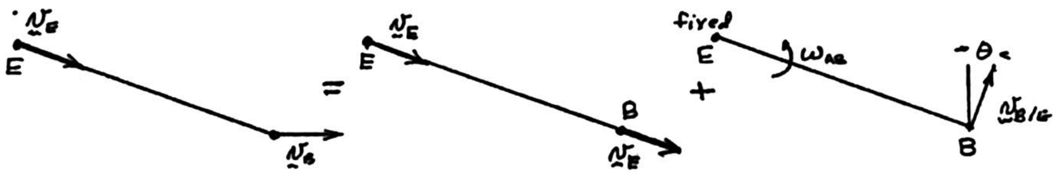
Gear D: Rotation about D. Tooth E is in contact with rack AB.

$$\mathbf{v}_E = r\omega_D \searrow \theta$$

Rack AB:

$$l_{EB} = \frac{r}{\tan\theta}$$

Plane motion = Translation with E + Rotation about E.



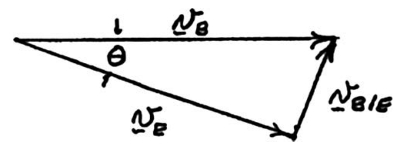
$$\mathbf{v}_B = \mathbf{v}_E + \mathbf{v}_{B/E} \qquad [v_B \longrightarrow] = [v_E \searrow \theta] + [v_{B/E} \nearrow \theta]$$

Draw velocity vector diagram.

$$v_B = \frac{v_E}{\cos\theta} = \frac{r\omega_D}{\cos\theta} \qquad \mathbf{v}_B = \frac{r\omega_D}{\cos\theta} \longrightarrow \blacktriangleleft$$

$$v_{B/E} = v_E \tan\theta$$

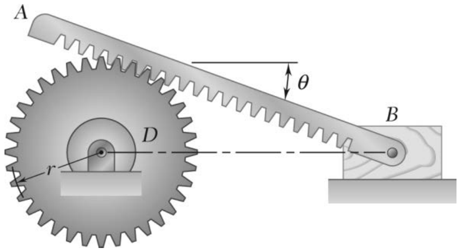
$$= r\omega_D \tan\theta$$



$$\omega_{AB} = \frac{v_{B/E}}{l_{EB}}$$

$$= \frac{r\omega_D \tan\theta}{\frac{r}{\tan\theta}}$$

$$= \omega_D \tan^2\theta \qquad \boldsymbol{\omega}_{AB} = \omega_D \tan^2\theta \circlearrowright \blacktriangleleft$$

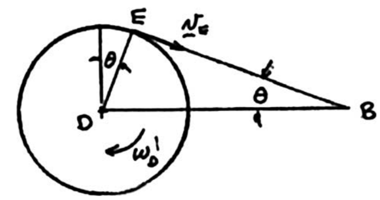


PROBLEM 15.58

A straight rack rests on a gear of radius $r = 2.5$ in. and is attached to a block B as shown. Knowing that at the instant shown the velocity of block B is 8 in./s to the right and $\theta = 25°$, determine (*a*) the angular velocity of gear D, (*b*) the angular velocity of the rack.

SOLUTION

Gear D: Rotation about D. Tooth E is in contact with rack AB.

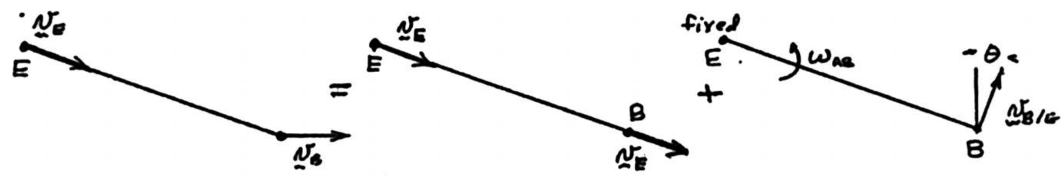


$$\mathbf{v}_E = r\omega_D \searrow \theta$$

Rack AB:

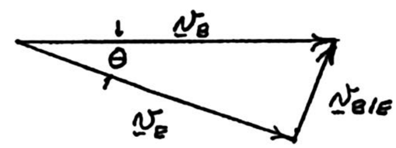
$$l_{EB} = \frac{r}{\tan\theta}$$

<u>Plane motion</u> = <u>Translation with *E*</u> + <u>Rotation about *E*</u>.



$$\mathbf{v}_B = \mathbf{v}_E + \mathbf{v}_{B/E} \quad [v_B \longrightarrow] = [v_E \searrow \theta] + [v_{B/E} \nearrow \theta]$$

Draw velocity vector diagram.



$$v_B = \frac{v_E}{\cos\theta} = \frac{r\omega_D}{\cos\theta}$$

$$\mathbf{v}_B = \frac{r\omega_D}{\cos\theta} \longrightarrow$$

$$v_{B/E} = v_E \tan\theta = r\omega_D \tan\theta$$

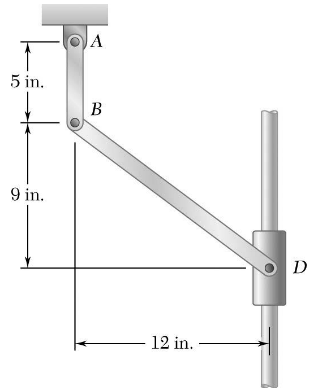
$$\omega_{AB} = \frac{v_{B/E}}{l_{EB}} = \frac{r\omega_D \tan\theta}{\frac{r}{\tan\theta}} = \omega_D \tan^2\theta$$

$$\omega_{AB} = \omega_D \tan^2\theta \circlearrowleft$$

<u>Data</u>: $r = 2.5$ in. $\quad \theta = 25° \quad \mathbf{v}_B = 8$ in./s $\longrightarrow$

(*a*) $$\omega_D = \frac{v_B \cos\theta}{r} = \frac{8\cos 25°}{2.5}$$ $\boldsymbol{\omega}_D = 2.90$ rad/s $\circlearrowright$ ◀

(*b*) $$\omega_{AB} = 2.90 \tan^2 25°$$ $\boldsymbol{\omega}_{AB} = 0.631$ rad/s $\circlearrowleft$ ◀



PROBLEM 15.59

Knowing that at the instant shown the angular velocity of crank *AB* is 2.7 rad/s clockwise, determine (*a*) the angular velocity of link *BD*, (*b*) velocity of collar *D*, (*c*) the velocity of the midpoint of link *BD*.

SOLUTION

Crank *AB*: Point *A* is fixed. $\boldsymbol{\omega}_{AB} = 2.7 \text{ rad/s} \circlearrowright$

$$v_B = \omega_{AB} r_{AB} = (2.7 \text{ rad/s})(5 \text{ in.}) = 13.5 \text{ in./s}$$

$$\mathbf{v}_B = 13.5 \text{ in./s} \longleftarrow = -(13.5 \text{ in./s})\mathbf{i}$$

Link *BD*: $\mathbf{r}_{D/B} = (12 \text{ in.})\mathbf{i} - (9 \text{ in.})\mathbf{j}, \quad \boldsymbol{\omega}_{BD} = \omega_{BD} \circlearrowleft = \omega_{AB}\mathbf{k},$

$$\mathbf{v}_D = v_D \uparrow = v_D\mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{B/D} = \mathbf{v}_B + \omega_{BD}\mathbf{k} \times \mathbf{r}_{B/D}$$

$$v_D\mathbf{j} = -(13.5 \text{ in./s})\mathbf{i} + \omega_{BD}\mathbf{k} \times [(12 \text{ in.})\mathbf{i} - (9 \text{ in.})\mathbf{j}]$$

$$= -13.5\mathbf{i} + 12\omega_{BD}\mathbf{j} + 9\omega_{BD}\mathbf{i}$$

Equate components.

$$\mathbf{i}: \quad 0 = -13.5 + 9\omega_{BD} \tag{1}$$

$$\mathbf{j}: \quad v_D = 12\omega_{BD} \tag{2}$$

(*a*) *Angular velocity of link BD.*

From Eq. (1), $\omega_{BD} = \dfrac{13.5}{9}$ $\qquad \omega_{BD} = 1.500 \text{ rad/s} \circlearrowleft$ ◀

(*b*) *Velocity of collar D.*

From Eq. (2), $v_D = (12)(1.5)$ $\qquad \mathbf{v}_D = 18.00 \text{ in./s} \uparrow$ ◀

(*c*) *Velocity of midpoint M of link BD.*

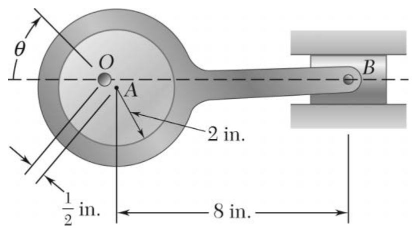
$$\mathbf{r}_{M/B} = \frac{1}{2}\mathbf{r}_{D/B} = (6 \text{ in.})\mathbf{i} - (4.5 \text{ in.})\mathbf{j}$$

$$\mathbf{v}_M = \mathbf{v}_B + \mathbf{v}_{M/B} = \mathbf{v}_B + \omega_{BD}\mathbf{k} \times \mathbf{r}_{M/D}$$

$$= -13.5\mathbf{i} + (1.500\mathbf{k}) \times (6\mathbf{i} - 4.5\mathbf{j})$$

$$= -13.5\mathbf{i} + 9\mathbf{j} + 6.75\mathbf{i}$$

$$\mathbf{v}_M = -(6.75 \text{ in./s})\mathbf{i} + (9.00 \text{ in./s})\mathbf{j} = 11.25 \text{ in./s} \measuredangle 53.1^\circ$$ ◀



PROBLEM 15.60

In the eccentric shown, a disk of 2-in.-radius revolves about shaft O that is located 0.5 in. from the center A of the disk. The distance between the center A of the disk and the pin at B is 8 in. Knowing that the angular velocity of the disk is 900 rpm clockwise, determine the velocity of the block when $\theta = 30°$.

SOLUTION

Geometry.

$$(OA)\sin\theta = (AB)\sin\beta$$

$$\sin\beta = \frac{(OA)\sin\theta}{AB}$$

$$= \frac{0.5\ \sin 30°}{8}, \quad \beta = 1.79°$$

Shaft and eccentric disk. (Rotation about O)

$$\omega_{OA} = 900\text{ rpm} = 30\pi\text{ rad/s} \circlearrowright$$

$$\mathbf{v}_A = (OA)\,\omega_{OA} = (0.5)(30\pi) = 15\pi\text{ in/s} \swarrow$$

Rod AB. (Plane motion = Translation with A + Rotation about A.)

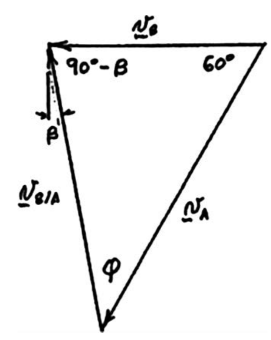
$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad [v_B \longleftarrow] = [v_A \swarrow 60°] + [v_{A/B} \searrow \beta]$$

Draw velocity vector diagram.

$$90° - \beta = 88.21°$$

$$\varphi = 180° - 60° - 88.21°$$

$$= 31.79°$$



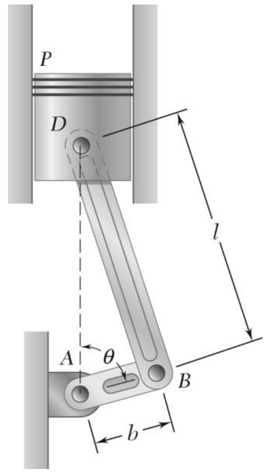
Law of sines.

$$\frac{v_B}{\sin\phi} = \frac{v_A}{\sin(90° - \beta)}$$

$$v_B = \frac{v_A \sin\varphi}{\sin(90° - \beta)}$$

$$= \frac{(15\pi)\sin 31.79°}{\sin 88.21°}$$

$$= 24.837\text{ in./s} \qquad \mathbf{v}_B = 24.8\text{ in./s} \longleftarrow \blacktriangleleft$$



PROBLEM 15.61

In the engine system shown, $l = 160$ mm and $b = 60$ mm. Knowing that the crank AB rotates with a constant angular velocity of 1000 rpm clockwise, determine the velocity of the piston P and the angular velocity of the connecting rod when (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

SOLUTION

$$\omega_{AB} = 1000 \text{ rpm} \circlearrowright = \frac{(1000)(2\pi)}{60} = 104.72 \text{ rad/s} \circlearrowright$$

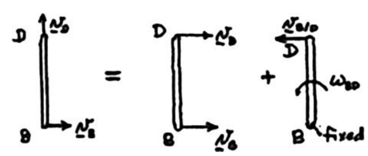
(*a*) $\theta = 0°$. *Crank AB.* (Rotation about A) $\mathbf{r}_{B/A} = 0.06 \text{ m} \uparrow$

$$\mathbf{v}_B = v_{B/A}\omega_{AB} = (0.06)(104.72) = 6.2832 \text{ m/s} \longrightarrow$$

Rod BD. (Plane motion = Translation with B + Rotation about B)

$$\mathbf{v}_D = \mathbf{v}_B + v_{D/B}$$

$$v_D \uparrow = [6.2832 \longrightarrow] + [v_{D/B} \longleftarrow]$$



$$v_D = 0$$

$$v_{D/B} = 6.2832 \text{ m/s}$$

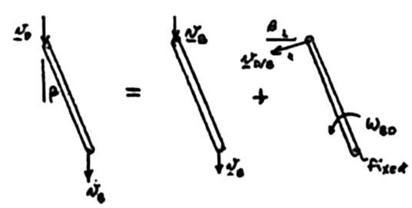
$$\text{v}_P = \text{v}_D \qquad \mathbf{v}_P = 0 \blacktriangleleft$$

$$\omega_{BD} = \frac{v_B}{l} = \frac{6.2832}{0.16} \qquad \boldsymbol{\omega}_{BD} = 39.3 \text{ rad/s} \circlearrowleft \blacktriangleleft$$

(*b*) $\theta = 90°$. *Crank AB.* (Rotation about A) $\mathbf{r}_{B/A} = 0.06 \text{ m} \longrightarrow$

$$\mathbf{v}_B = r_{B/A}\omega_{AB} = (0.06)(104.72) = 6.2832 \text{ m/s} \downarrow$$

Rod BD. (Plane motion = Translation with B + Rotation about B.)



$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

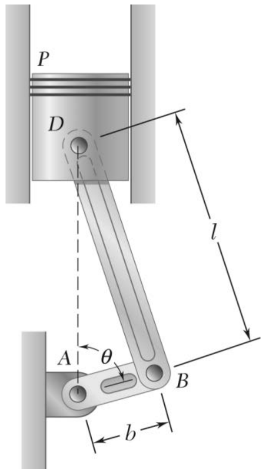
$$[v_D] \downarrow] = [6.2832] \downarrow + \left[v_{D/B} \measuredangle \beta\right]$$

PROBLEM 15.61 (Continued)

$$v_{D/B} = 0, \quad v_D = 6.2832 \text{ m/s}$$

$$\omega_{BD} = \frac{v_{D/B}}{l} \qquad \omega_{BD} = 0 \blacktriangleleft$$

$$\mathbf{v}_P = \mathbf{v}_D = 6.2832 \text{ m/s}\downarrow \qquad \mathbf{v}_P = 6.28 \text{ m/s}\downarrow \blacktriangleleft$$



PROBLEM 15.62

In the engine system shown $l = 160$ mm and $b = 60$ mm. Knowing that crank *AB* rotates with a constant angular velocity of 1000 rpm clockwise, determine the velocity of the piston *P* and the angular velocity of the connecting rod when $\theta = 60°$.

SOLUTION

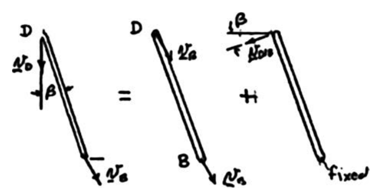
$$\omega_{AB} = 1000 \text{ rpm} = \frac{(1000)(2\pi)}{60} = 104.72 \text{ rad/s} \circlearrowright$$

$\underline{\theta = 60°}$. *Crank AB*. (Rotation about *A*) $\quad \mathbf{r}_{B/A} = 3$ in. $\measuredangle$ 30°

$$\mathbf{v}_B = r_{B/A}\omega_{AB} = (0.06)(104.72) = 6.2832 \text{ m/s} \searrow 60°$$

Rod BD. (Plane motion = Translation with *B* + Rotation about *B*.)

Geometry. $\quad l \sin\beta = r \sin\theta$



$$\sin\beta = \frac{r}{l}\sin\theta = \frac{0.06}{0.16}\sin 60°$$

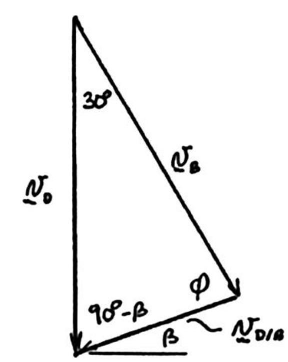
$$\beta = 18.95°$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

$$[v_D \downarrow] = [314.16 \searrow 60°] + [v_{D/B} \swarrow \beta]$$

Draw velocity vector diagram.

$$\varphi = 180° - 30° - (90° - \beta) = 78.95°$$



Law of sines.

$$\frac{v_D}{\sin\varphi} = \frac{v_{D/B}}{\sin 30°} = \frac{v_B}{\sin(90° - \beta)}$$

$$v_D = \frac{v_B \sin\varphi}{\cos\beta}$$

$$= \frac{6.2832 \ \sin 78.95°}{\cos 18.95°}$$

$$= 6.52 \text{ m/s}$$

$$v_P = v_D$$

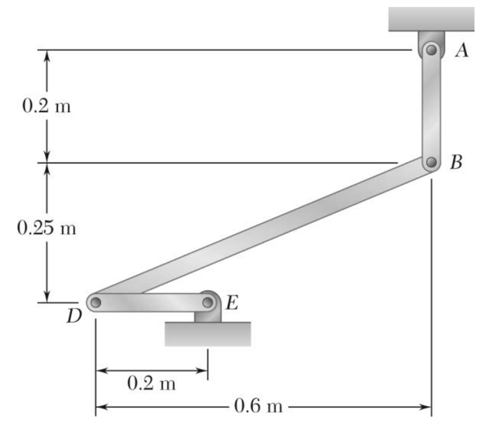
$\mathbf{v}_P = 6.52$ m/s $\downarrow$ ◀

PROBLEM 15.62 (Continued)

$$v_{D/B} = \frac{v_B \sin 30°}{\cos\beta} = \frac{6.2832 \sin 30°}{\cos 18.95°} = 3.3216 \text{ m/s}$$

$$\omega_{BD} = \frac{v_{D/B}}{l} = \frac{3.3216}{0.16}$$

$\omega_{BD} = 20.8$ rad/s ↻ ◀



PROBLEM 15.63

Knowing that at the instant shown the angular velocity of rod *AB* is 15 rad/s clockwise, determine (*a*) the angular velocity of rod *BD*, (*b*) the velocity of the midpoint of rod *BD*.

SOLUTION

Rod *AB*: $\boldsymbol{\omega}_{AB} = 15 \text{ rad/s} \circlearrowright$

$$v_B = (AB)\omega_{AB} = (0.200)(15) = 3 \text{ m/s} \qquad \mathbf{v}_B = 3 \text{ m/s} \longleftarrow$$

Rod *BD*: $\mathbf{v}_B = -(3 \text{ m/s})\mathbf{i}, \quad \mathbf{v}_D = v_D\mathbf{j}, \quad \boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k}$

$$\mathbf{r}_{B/D} = (0.6 \text{ m})\mathbf{i} + (0.25 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_D + \mathbf{v}_{B/D} = \mathbf{v}_D + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{B/D}$$

$$-3\mathbf{i} = v_D\mathbf{j} + \omega_{BD}\mathbf{k} \times (0.6\mathbf{i} + 0.25\mathbf{j})$$

$$= v_D\mathbf{j} + 0.6\omega_{BD}\mathbf{j} - 0.25\omega_{BD}\mathbf{i}$$

Equate components.

$$\mathbf{i}: \quad -3 = -0.25\omega_{BD} \qquad (1)$$

$$\mathbf{j}: \quad 0 = v_D + 0.6\omega_{BD} \qquad (2)$$

(*a*) *Angular velocity of rod BD.*

From Eq. (1), $\omega_{BD} = \dfrac{3}{0.25}$ $\qquad \boldsymbol{\omega}_{BD} = 12.00 \text{ rad/s} \circlearrowleft$ ◀

From Eq. (2), $v_D = -0.6\omega_{BD}$ $\qquad v_D = -7.2 \text{ m/s}$

(*b*) *Velocity of midpoint M of rod BD.*

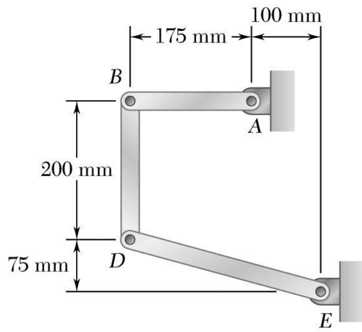
$$\mathbf{r}_{M/D} = \frac{1}{2}\mathbf{r}_{B/D} = (0.3 \text{ m})\mathbf{i} + (0.125 \text{ m})\mathbf{j}$$

$$\mathbf{v}_M = \mathbf{v}_D + \mathbf{v}_{M/D} = v_D\mathbf{j} + \omega_{BD}\mathbf{k} \times \mathbf{r}_{M/D}$$

$$= -7.2\mathbf{j} + 12.00\mathbf{k} \times (0.3\mathbf{i} + 0.125\mathbf{j})$$

$$= -(1.500 \text{ m/s})\mathbf{i} - (3.60 \text{ m/s})\mathbf{j}$$

$\mathbf{v}_M = 3.90 \text{ m/s} \measuredangle 67.4°$ ◀



PROBLEM 15.64

In the position shown, bar AB has an angular velocity of 4 rad/s clockwise. Determine the angular velocity of bars BD and DE.

SOLUTION

Bar AB: (Rotation about A) $\quad \boldsymbol{\omega}_{AB} = 4 \text{ rad/s} \circlearrowright = -(4 \text{ rad/s})\mathbf{k}$

$$\mathbf{r}_{B/A} = -(175 \text{ mm})\mathbf{i} \quad \mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (-4\mathbf{k}) \times (-175\mathbf{i})$$

$$\mathbf{v}_B = (700 \text{ mm/s})\mathbf{j}$$

Bar BD: (Plane motion = Translation with B + Rotation about B.)

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \quad \mathbf{r}_{D/B} = -(200 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B} = 700\mathbf{j} + (\omega_{BD}\mathbf{k}) \times (-200\mathbf{j})$$

$$\mathbf{v}_D = 700\mathbf{j} + 200\omega_{BD}\mathbf{i}$$

Bar DE: (Rotation about E) $\quad \boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}$

$$\mathbf{r}_{D/E} = -(275 \text{ mm})\mathbf{i} + (75 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (\omega_{DE}\mathbf{k}) \times (-275\mathbf{i} + 75\mathbf{j})$$

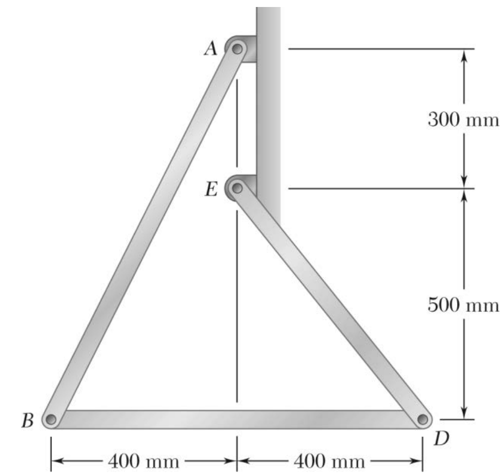
$$\mathbf{v}_D = -275\omega_{DE}\mathbf{j} - 75\omega_{DE}\mathbf{i}$$

Equating components of the two expressions for $\mathbf{v}_D$,

$\mathbf{j}$: $\quad 700 = -275\omega_{DE} \qquad \omega_{DE} = -2.5455 \text{ rad/s} \qquad \boldsymbol{\omega}_{DE} = 2.55 \text{ rad/s} \circlearrowright$ ◀

$\mathbf{i}$: $\quad 200\omega_{BD} = -75\omega_{DE} \qquad \omega_{BD} = -\frac{3}{8}\omega_{BD}$

$\omega_{BD} = -\left(\frac{3}{8}\right)(-2.5455) = 0.95455 \text{ rad/s} \qquad \boldsymbol{\omega}_{BD} = 0.955 \text{ rad/s} \circlearrowleft$ ◀

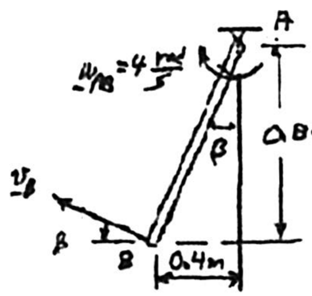


PROBLEM 15.65

In the position shown, bar AB has an angular velocity of 4 rad/s clockwise. Determine the angular velocity of bars BD and DE.

SOLUTION

Bar AB:



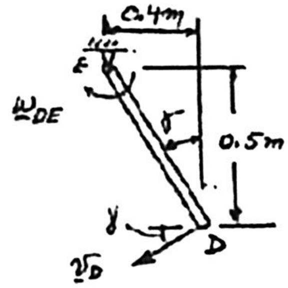
$$\beta = \tan^{-1}\frac{0.4}{0.8} = 26.56^\circ$$

$$AB = \frac{0.8}{\cos\beta} = 0.8944 \text{ m}$$

$$\mathbf{v}_B = (AB)\omega_{AB} = (0.8944 \text{ m})(4 \text{ m/s})$$

$$\mathbf{v}_B = 3.578 \text{ m/s} \searrow 26.56^\circ$$

Bar DE:



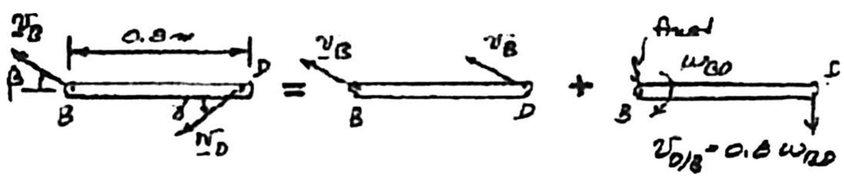
$$\gamma = \tan^{-1}\frac{0.4}{0.5} = 38.66^\circ$$

$$DE = \frac{0.5}{\cos\gamma} = 0.6403 \text{ m}$$

$$v_D = (DE)\omega_{DE}$$

$$\mathbf{v}_D = (0.6403 \text{ m})\omega_{DE} \swarrow 38.66^\circ$$

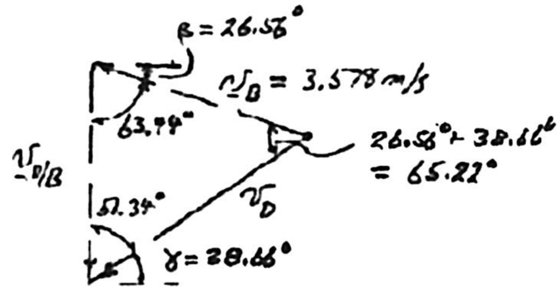
Bar BD:



$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

$$[\mathbf{v}_D \swarrow \gamma] = [\mathbf{v}_B \searrow \beta] + [\mathbf{v}_{D/B} \downarrow]$$

PROBLEM 15.65 (Continued)



Law of sines.

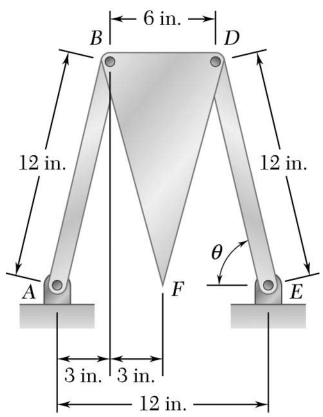
$$\frac{v_D}{\sin 63.44^\circ} = \frac{v_{D/B}}{\sin 65.22^\circ} = \frac{3.578 \text{ m/s}}{\sin 51.34^\circ}$$

$$v_D = 4.099 \text{ m/s}$$

$$(0.6403 \text{ m})\omega_{DE} = 4.099 \text{ m/s} \qquad \boldsymbol{\omega}_{DE} = 6.4 \text{ rad/s} \circlearrowright \blacktriangleleft$$

$$v_{D/B} = 4.160 \text{ m/s}$$

$$(0.8 \text{ m})v_{BD} = 4.16 \text{ m/s} \qquad \boldsymbol{\omega}_{BD} = 5.2 \text{ rad/s} \circlearrowright \blacktriangleleft$$



PROBLEM 15.66

Robert's linkage is named after Richard Robert (1789–1864) and can be used to draw a close approximation to a straight line by locating a pen at Point *F*. The distance *AB* is the same as *BF*, *DF* and *DE*. Knowing that the angular velocity of bar *AB* is 5 rad/s clockwise in the position shown, determine (*a*) the angular velocity of bar *DE*, (*b*) the velocity of Point *F*.

SOLUTION

Bar *AB*: $\boldsymbol{\omega}_{AB} = 5 \text{ rad/s} \circlearrowright = -(5 \text{ rad/s})\mathbf{k}$

In inches,

$$\mathbf{r}_{B/A} = 3\mathbf{i} + \sqrt{12^2 - 3^2}\,\mathbf{j} = 3\mathbf{i} + \sqrt{135}\,\mathbf{j}$$

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = -5\mathbf{k} \times (3\mathbf{i} + \sqrt{135}\,\mathbf{j})$$

$$= 5\sqrt{135}\,\mathbf{i} - 15\mathbf{j}$$

Object *BDF*: $\mathbf{r}_{D/B} = (6 \text{ in.})\mathbf{i}, \quad \mathbf{r}_{F/B} = 3\mathbf{i} - \sqrt{135}\,\mathbf{j} \text{ (in.)}, \quad \boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k}$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{B/D} = \mathbf{v}_B + \omega_{BD}\mathbf{k} \times \mathbf{r}_{D/B}$$

$$= 5\sqrt{135}\,\mathbf{i} - 15\mathbf{j} + \omega_{BD}\mathbf{k} \times 6\mathbf{i}$$

$$= 5\sqrt{135}\,\mathbf{i} - 15\mathbf{j} + 6\omega_{BD}\mathbf{j} \qquad (1)$$

Bar *DE*: $\boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}, \quad \mathbf{r}_{D/E} = -(3 \text{ in.})\mathbf{i} + (\sqrt{135} \text{ in.})\mathbf{j},$

Point *E* is fixed so $\mathbf{v}_E = 0$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = \omega_{DE}\mathbf{k} \times (-3\mathbf{i} + \sqrt{135}\,\mathbf{j})$$

$$= -\sqrt{135}\,\omega_{DE}\mathbf{i} - 3\omega_{DE}\mathbf{j} \qquad (2)$$

Equating like components of $\mathbf{v}_D$ from Eqs. (1) and (2),

i: $$5\sqrt{135} = -\sqrt{135}\,\omega_{DE} \qquad (3)$$

j: $$-15 + 6\omega_{BD} = -3\omega_{DE} \qquad (4)$$

(*a*) *Angular velocity of bar DE.*

From Eq. (3), $\omega_{DE} = -5 \text{ rad/s}$ $\qquad \boldsymbol{\omega}_{DE} = 5.00 \text{ rad/s} \circlearrowright$ ◀

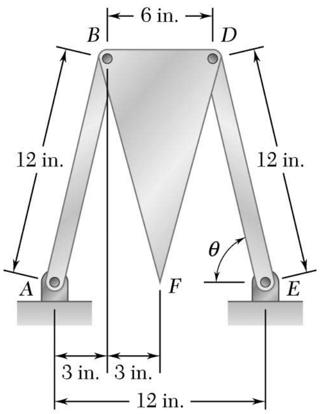
From Eq. (4), $\omega_{BD} = \frac{1}{6}(15 - 3\omega_{DE}) = \frac{1}{6}(15 + 15)$ $\qquad \boldsymbol{\omega}_{BD} = 5.00 \text{ rad/s} \circlearrowleft$

PROBLEM 15.66 (Continued)

(*b*) *Velocity of Point F.* $\mathbf{v}_{F/B} = 3i - \sqrt{135}\mathbf{j}$

$$\mathbf{v}_F = \mathbf{v}_B = \mathbf{v}_{F/B} = \mathbf{v}_B + \omega_{BD}\mathbf{k} \times \mathbf{r}_{F/B}$$
$$= 5\sqrt{135}\mathbf{i} - 15\mathbf{j} + 5\mathbf{k} \times (3\mathbf{i} - \sqrt{135}\mathbf{j})$$
$$= 5\sqrt{135}\mathbf{i} - 15\mathbf{j} + 15\mathbf{j} + 5\sqrt{135}\mathbf{i} = 10\sqrt{135}\mathbf{i}$$

$\mathbf{v}_F = 116.2$ in./s $\longrightarrow$ ◀

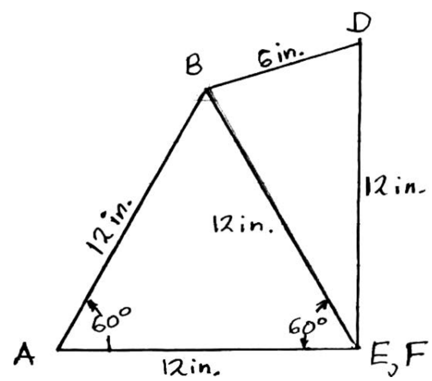


PROBLEM 15.67

Robert's linkage is named after Richard Robert (1789–1864) and can be used to draw a close approximation to a straight line by locating a pen at Point F. The distance AB is the same as BF, DF and DE. Knowing that the angular velocity of plate BDF is 2 rad/s counterclockwise when $\theta = 90°$, determine (*a*) the angular velocities of bars AB and DE, (*b*) the velocity of Point F. When $\theta = 90°$, determine (*a*) the angular velocity of bar **DE** (*b*) the velocity of Point **F**.

SOLUTION

When $\theta = 90°$, the configuration of the linkage is close to that shown at the right.



Bar AB: $\quad \boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k}$

In inches, $\quad \mathbf{r}_{B/A} = 6\mathbf{i} + 6\sqrt{3}\mathbf{j}$

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \omega_{AB}\mathbf{k} \times (6\mathbf{i} + 6\sqrt{3}\mathbf{j}) = -6\sqrt{3}\omega_{AB}\mathbf{i} + 6\omega_{AB}\mathbf{j}$$

Object BDF: $\quad \mathbf{r}_{D/B} = 6\mathbf{i} + 6(2-\sqrt{3})\mathbf{j}$

$$\mathbf{r}_{F/B} = 6\mathbf{i} - 6\sqrt{3}\mathbf{j} \qquad \boldsymbol{\omega}_{BD} = (2\text{ rad/s})\mathbf{k}$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = \mathbf{v}_B = \omega_{BD}\mathbf{k} \times \mathbf{r}_{D/B}$$
$$= -6\sqrt{3}\omega_{AB}\mathbf{i} + 6\omega_{AB}\mathbf{j} + 2\mathbf{k} \times [6\mathbf{i} + 6(2-\sqrt{3})\mathbf{j}]$$
$$= -6\sqrt{3}\omega_{AB}\mathbf{i} + 6\omega_{AB}\mathbf{j} + 24\mathbf{i} - 12\sqrt{3}\mathbf{i} + 12\mathbf{j} \tag{1}$$

Bar DE: $\quad \boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}, \qquad \mathbf{r}_{D/E} = 12\mathbf{j}$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = \omega_{DE}\mathbf{k} \times 12\mathbf{j} = -12\omega_{DE}\mathbf{i} \tag{2}$$

Equating like components of $\mathbf{v}_D$ from Eqs. (1) and (2),

i: $$-6\sqrt{3}\omega_{AB} + 12 - 6\sqrt{3} = -12\omega_{DE} \tag{3}$$

j: $$6\omega_{AB} + 12 = 0$$

$$\omega_{AB} = -2\text{ rad/s} \qquad \omega_{AB} = 2.00\text{ rad/s} \circlearrowright \blacktriangleleft$$

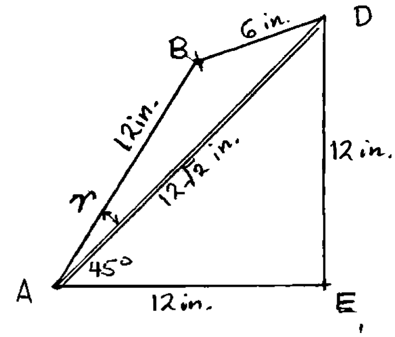
From Eq. (3), $$-12\omega_{DE} = -(6\sqrt{3})(-2) + 24 - 12\sqrt{3}$$

$$\omega_{DE} = 2 - 2\sqrt{3} = -1.4641 \qquad \boldsymbol{\omega}_{DE} = 1.464\text{ rad/s} \circlearrowright \blacktriangleleft$$

$$\mathbf{v}_D = -(12)(-1.4641)\mathbf{i} = 17.569\mathbf{i}$$
$$\mathbf{v}_F = \mathbf{v}_D + \omega_{BD}\mathbf{k} \times \mathbf{r}_{F/D}$$
$$= 17.569\mathbf{i} + 2\mathbf{k} \times (-12\mathbf{j}) = (41.569 \text{ in./s})\mathbf{i}$$

$$\mathbf{v}_F = 41.6 \text{ in./s} \longrightarrow \blacktriangleleft$$

Note: The exact configuration of the linkage when $\theta = 90°$ may be calculated from trigonometry using the figure given below.



Applying the law of cosines to triangle *ADB* gives

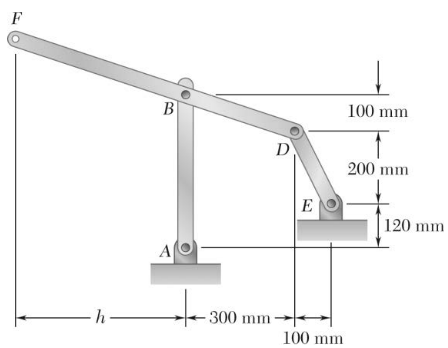
$$\gamma = 13.5°$$

so that angle *EAB* is

$$45° + 13.5° = 58.5°.$$

We used 60° in the approximate analysis.

Point *F* then lies about 0.53 in. to the right of Point *E*.



PROBLEM 15.68

In the position shown, bar *DE* has a constant angular velocity of 10 rad/s clockwise. Knowing that $h = 500$ mm, determine (*a*) the angular velocity of bar *FBD*, (*b*) the velocity of Point *F*.

SOLUTION

<u>Bar *DE*</u>: (Rotation about *E*)

$$\boldsymbol{\omega}_{DE} = 10 \text{ rad/s } \circlearrowright = -(10 \text{ rad/s})\mathbf{k}$$

$$\mathbf{r}_{D/E} = -(0.1 \text{ m})\mathbf{i} + (0.2 \text{ m})\mathbf{j}$$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (-10\mathbf{k}) \times (-0.1\mathbf{i} + 0.2\mathbf{j})$$

$$= (1 \text{ m/s})\mathbf{j} + (2 \text{ m/s})\mathbf{i}$$

<u>Bar *FBD*</u>: (Plane motion = Translation with *D* + Rotation about *D*.)

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \quad \mathbf{r}_{B/D} = -(0.3 \text{ m})\mathbf{i} + (0.1 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_D + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{B/D}$$

$$= \mathbf{j} + 2\mathbf{i} + (\omega_{BD}\mathbf{k}) \times (-0.3\mathbf{i} + 0.1\mathbf{j})$$

$$= \mathbf{j} + 2\mathbf{i} - 0.3\omega_{BD}\mathbf{j} - 0.1\omega_{BD}\mathbf{i}$$

<u>Bar *AB*</u>: (Rotation about *A*)

$$\boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k} \quad \mathbf{r}_{B/A} = (0.42 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (\omega_{AB}\mathbf{k}) \times (0.42\mathbf{j}) = -0.42\omega_{AB}\mathbf{i}$$

Equating components of the two expressions for $\mathbf{v}_B$,

(*a*) $\mathbf{j}$: $\quad 1 - 0.3\omega_{BD} = 0 \qquad \omega_{BD} = 3.3333$ rad/s $\qquad \boldsymbol{\omega}_{BD} = 3.33 \text{ rad/s } \circlearrowleft$ ◀

$\mathbf{i}$: $\quad 2 - 0.1\omega_{BD} = -0.42\omega_{AB} \qquad 2 - (0.1)(3.3333) = -0.42\omega_{AB}$

$$\omega_{AB} = -3.9683 \text{ rad/s} \qquad \omega_{AB} = 3.97 \text{ rad/s } \circlearrowright$$

<u>Bar *FBD*</u>: $\quad \mathbf{r}_{F/D} = C\mathbf{r}_{B/D} \quad \text{where} \quad C = \dfrac{h + 0.3}{0.3}$

$$\mathbf{v}_F = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{F/D}$$

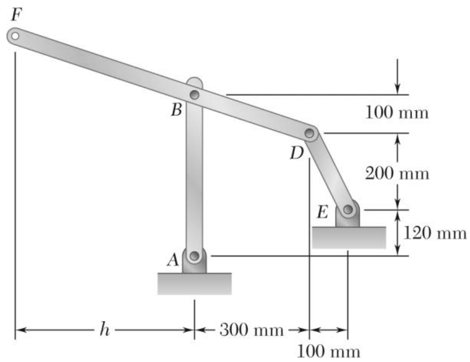
$$= \mathbf{j} + 2\mathbf{i} + C(-0.3\omega_{BD}\mathbf{j} - 0.1\omega_{BD}\mathbf{i})$$

$$= \mathbf{j} + 2\mathbf{i} + C(-\mathbf{j} - 0.33333\mathbf{i})$$

With $\quad h = 500 \text{ mm} = 0.5 \text{ m}, \quad C = \dfrac{0.8}{0.3} = 2.6667$

$$\mathbf{v}_F = \mathbf{j} + 2\mathbf{i} - 2.6667\mathbf{j} - 0.88889\mathbf{i}$$

(*b*) $\quad \mathbf{v}_F = (1.11111 \text{ m/s})\mathbf{i} - (1.66667 \text{ m/s})\mathbf{j} \qquad \mathbf{v}_F = 2.00 \text{ m/s} \searrow 56.3^\circ$ ◀



PROBLEM 15.69

In the position shown, bar DE has a constant angular velocity of 10 rad/s clockwise. Determine (a) the distance h for which the velocity of Point F is vertical, (b) the corresponding velocity of Point F.

SOLUTION

Bar DE: (Rotation about E) $\quad \boldsymbol{\omega}_{DE} = 10 \text{ rad/s} \circlearrowright = -(10 \text{ rad/s})\mathbf{k}$

$$\mathbf{r}_{D/E} = -(0.1 \text{ m})\mathbf{i} + (0.2 \text{ m})\mathbf{j}$$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (-10\mathbf{k}) \times (-0.1\mathbf{i} + 0.2\mathbf{j})$$
$$= (1 \text{ m/s})\mathbf{j} + (2 \text{ m/s})\mathbf{i}$$

Bar FBD: (Plane motion = Translation with D + Rotation about D.)

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \quad \mathbf{r}_{B/D} = -(0.3 \text{ m})\mathbf{i} + (0.1 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_D + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{B/D}$$
$$= \mathbf{j} + 2\mathbf{i} + (\omega_{BD}\mathbf{k}) \times (-0.3\mathbf{i} + 0.1\mathbf{j})$$
$$= \mathbf{j} + 2\mathbf{i} - 0.3\omega_{BD}\mathbf{j} - 0.1\omega_{BD}\mathbf{i}$$

Bar AB: (Rotation about A)

$$\boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k} \quad \mathbf{r}_{B/A} = (0.42 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (\omega_{AB}\mathbf{k}) \times (0.42\mathbf{j}) = -0.42\omega_{AB}\mathbf{i}$$

Equating components of the two expressions for $\mathbf{v}_B$,

(a) $\quad \mathbf{j}$: $\quad 1 - 0.3\omega_{BD} = 0 \qquad \omega_{BD} = 3.3333 \text{ rad/s}$

$\mathbf{i}$: $\quad 2 - 0.1\omega_{BD} = -0.42\omega_{AB} \qquad 2 - (0.1)(3.3333) = -0.42\omega_{AB}$

$$\omega_{AB} = -3.9683 \text{ rad/s} \qquad \omega_{AB} = 3.97 \text{ rad/s} \circlearrowright$$

Bar FBD: $\quad \mathbf{r}_{F/D} = C\mathbf{r}_{B/D} \quad \text{where} \quad C = \dfrac{h + 0.3}{0.3}$

$$\mathbf{v}_F = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{F/D}$$
$$= \mathbf{j} + 2\mathbf{i} + C(-0.3\omega_{BD}\mathbf{j} - 0.1\omega_{BD}\mathbf{i})$$
$$= \mathbf{j} + 2\mathbf{i} + C(-\mathbf{j} - 0.33333\mathbf{i})$$

PROBLEM 15.69 (Continued)

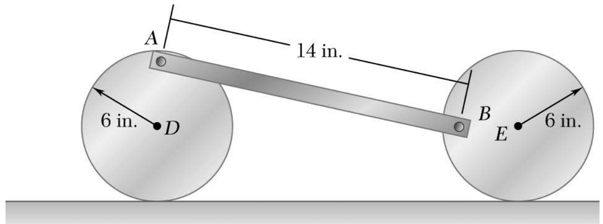
But $\mathbf{v}_F = v_F\mathbf{j}$. Equating components of the two expressions for $\mathbf{v}_F$,

$$\mathbf{i}: \quad 0 = 2 - 0.33333C \qquad C = 6$$

(*a*) $h = 0.3C - 0.3 = (0.3)(6) - 0.3$ $\qquad h = 1.500 \text{ m}$ ◀

(*b*) $\mathbf{j}: \quad v_F = \mathbf{j} - C\mathbf{j} = (1-6)\mathbf{j}$ $\qquad \mathbf{v}_F = 5.00 \text{ m/s}\downarrow$ ◀

PROBLEM 15.70



Both 6-in.-radius wheels roll without slipping on the horizontal surface. Knowing that the distance *AD* is 5 in., the distance *BE* is 4 in. and *D* has a velocity of 6 in./s to the right, determine the velocity of Point *E*.

SOLUTION

Disk *D*: Velocity at the contact Point *P* with the ground is zero.

$$\mathbf{v}_0 = 6 \text{ in./s} \longrightarrow$$

$$\omega_D = \frac{v_D}{r_{D/P}} = \frac{6 \text{ in./s}}{6 \text{ in.}} = 1 \text{ rad/s} \qquad \boldsymbol{\omega}_D = 1 \text{ rad/s} \circlearrowright$$

At Point *A*, $v_A = r_{A/P}\omega_D = (6 \text{ in.} + 5 \text{ in.})(1 \text{ rad/s}) = 11 \text{ in./s}$

$$\mathbf{v}_A = 11 \text{ in./s} \longrightarrow$$

Disk *E*: Velocity at the contact Point *Q* with the ground is zero. $\boldsymbol{\omega}_E = \omega_E \circlearrowleft = \omega_E\mathbf{k}$.

$$\mathbf{r}_{B/Q} = -(4 \text{ in.})\mathbf{i} + (6 \text{ in.})\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_{B/Q} = \boldsymbol{\omega}_E \times \mathbf{r}_{B/Q} = \omega_E\mathbf{k} \times (-4\mathbf{i} + 6\mathbf{j})$$

$$\mathbf{v}_B = -6\omega_E\mathbf{i} - 4\omega_E\mathbf{j} \qquad (1)$$

Connecting rod *AB*: $\mathbf{r}_{B/A} = (\sqrt{14^2 - 5^2})\mathbf{i} - 5\mathbf{j}$ in inches.

$$\mathbf{v}_{B/A} = \sqrt{171}\mathbf{i} - 5\mathbf{j} \qquad \boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = \mathbf{v}_A + \omega_{AB}\mathbf{k} \times (\sqrt{171}\mathbf{i} - 5\mathbf{j})$$

$$= 11\mathbf{i} + 5\omega_{AB}\mathbf{i} + \sqrt{171}\omega_{AB}\mathbf{j} \qquad (2)$$

Equating expressions (1) and (2) for $\mathbf{v}_B$ gives

$$-6\omega_E\mathbf{i} - 4\omega_E\mathbf{j} = 11\mathbf{i} + 5\omega_{AB}\mathbf{i} + \sqrt{171}\omega_{AB}\mathbf{j}$$

Equating like components and transposing terms,

$$\mathbf{i}: \qquad 5\omega_{AB} + 6\omega_E = -11 \qquad (3)$$

$$\mathbf{j}: \qquad \sqrt{171}\omega_{AB} + 4\omega_E = 0 \qquad (4)$$

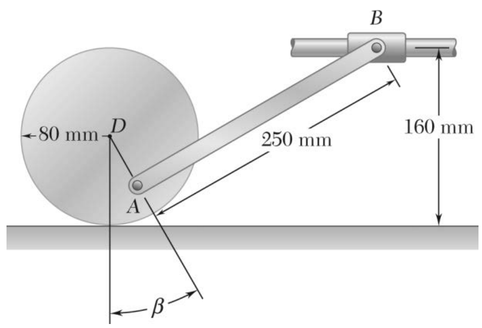
PROBLEM 15.70 (Continued)

Solving the simultaneous equaitons (3) and (4),

$$\omega_{AB} = 0.75265 \text{ rad/s}, \quad \omega_E = -2.4605 \text{ rad/s}$$

Velocity of Point E. $\mathbf{v}_E = \omega_E \mathbf{k} \times \mathbf{r}_{E/Q} = -2.4605\mathbf{k} \times 6\mathbf{j}$

$$\mathbf{v}_E = 14.76 \text{ in./s } \mathbf{i} = 14.76 \text{ in./s} \longrightarrow \blacktriangleleft$$

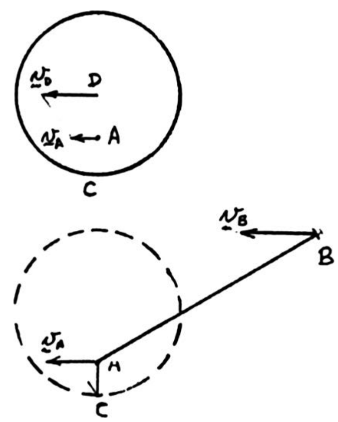


PROBLEM 15.71

The 80-mm-radius wheel shown rolls to the left with a velocity of 900 mm/s. Knowing that the distance AD is 50 mm, determine the velocity of the collar and the angular velocity of rod AB when (*a*) $\beta = 0$, (*b*) $\beta = 90°$.

SOLUTION

(*a*) $\beta = 0.$ *Wheel AD.* $\mathbf{v}_C = 0, \quad \mathbf{v}_D = 45 \text{ in./s} \longleftarrow$



$$\omega_{AD} = \frac{v_D}{CD} = \frac{900}{80} = 11.25 \text{ rad/s} \circlearrowleft$$

$$CA = (CD) - (DA) = 80 - 50 = 30 \text{ mm}$$

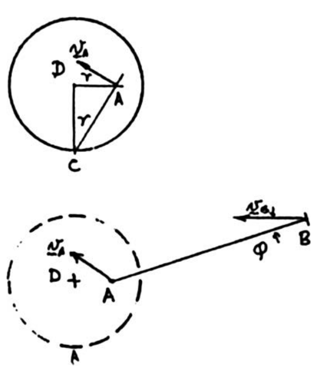
$$v_A = (CA)\omega_{AD} = (30)(11.25) = 337.5 \text{ mm/s} \longleftarrow$$

Rod AB. $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$$[v_B \longleftarrow] = [337.5 \longleftarrow] + [v_{B/A} \searrow \varphi] \qquad \mathbf{v}_B = 338 \text{ mm/s} \longleftarrow \blacktriangleleft$$

$$v_{B/A} = 0 \qquad \omega_{AB} = 0 \blacktriangleleft$$

(*b*) $\beta = 90°.$ *Wheel AD.* $\mathbf{v}_C = 0, \quad \omega_{AD} = 11.25 \text{ rad/s} \circlearrowleft$



$$\tan\gamma = \frac{DA}{DC} = \frac{50}{80}, \qquad \gamma = 32.005°$$

$$CA = \frac{DC}{\cos\gamma} = 94.34 \text{ mm}$$

$$v_A = (CA)\omega_{AD} = (94.34)(11.25) = 1061.3 \text{ mm/s}$$

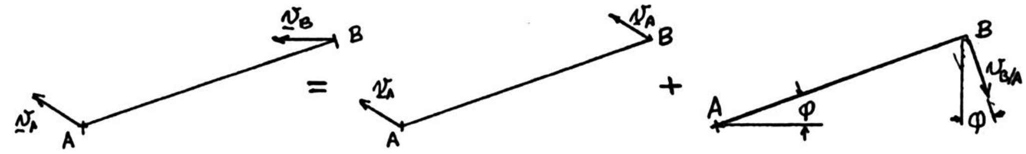
$$\mathbf{v}_A = [1061.3 \text{ mm/s} \searrow 32.005°]$$

Rod AB. $\mathbf{v}_B = v_B \longleftarrow$

$$\sin\varphi = \frac{80}{250}, \quad \varphi = 18.663°$$

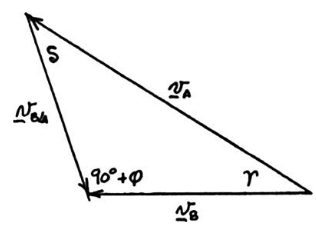
Plane motion = Translation with A + Rotation about A.

PROBLEM 15.71 (Continued)



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \qquad [v_B \longleftarrow] = [v_A \searrow \gamma] + [v_{B/A} \; \varphi]$$

Draw velocity vector diagram.



$$\delta = 180° - \gamma - (90° + \varphi)$$

$$= 90° - 32.005° - 18.663° = 39.332°$$

Law of sines.

$$\frac{v_B}{\sin\delta} = \frac{v_{B/A}}{\sin\gamma} = \frac{v_A}{\sin(90° + \varphi)}$$

$$v_B = \frac{v_A \sin\delta}{\sin(90° + \varphi)} = \frac{(1061.3)\sin 39.332°}{\sin 108.663°}$$

$$= 710 \text{ mm/s}$$

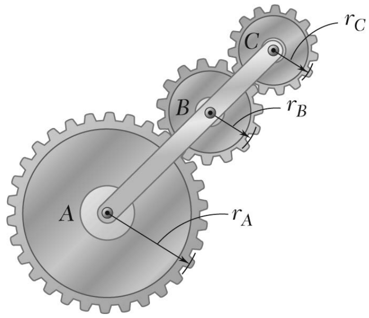
$\mathbf{v}_B = 710$ mm/s ← ◀

$$v_{B/A} = \frac{v_A \sin\gamma}{\sin(90° + \varphi)} = \frac{(1061.3)\sin 32.005°}{\sin 108.663°}$$

$$= 593.8 \text{ mm/s}$$

$$\omega_{AB} = \frac{v_{B/A}}{\overline{AB}} = \frac{593.8}{250} = 2.37 \text{ rad/s}$$

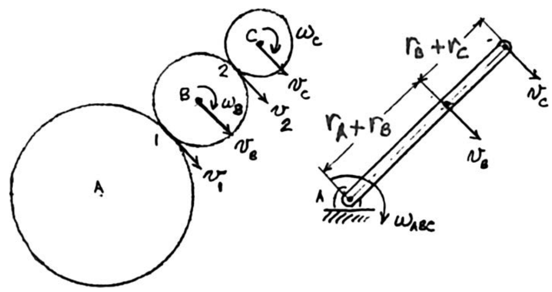
$\boldsymbol{\omega}_{AB} = 2.37$ rad/s ↻ ◀



PROBLEM 15.72*

For the gearing shown, derive an expression for the angular velocity ω_C of gear *C* and show that ω_C is independent of the radius of gear *B*. Assume that Point *A* is fixed and denote the angular velocities of rod *ABC* and gear *A* by ω_{ABC} and ω_A respectively.

SOLUTION



Label the contact point between gears *A* and *B* as 1 and that between gears *B* and *C* as 2.

Rod ABC: $\omega_{ABC} = \omega_{ABC}$ Assume ↻ for sketch.

$$v_A = 0$$

$$v_B = (r_A + r_B)\omega_{ABC} \searrow$$

$$v_C = (r_A + 2r_B + r_C)\omega_{ABC} \searrow$$

Gear A: $\omega_A = 0, \quad v_A = 0, \quad v_1 = 0$

Gear B: $v_1 = v_B - r_B\,\omega_B = 0$

$$(r_A + r_B)\omega_{ABC} - r_B\,\omega_B = 0$$

$$\omega_B = \left(\frac{r_A + r_B}{r_B}\right)\omega_{ABC} \circlearrowright$$

$$v_2 = v_B + r_B\,\omega_B$$

$$= 2(r_A + r_B)\omega_{ABC} \searrow$$

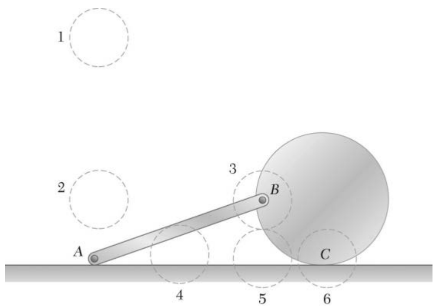
Gear C: $$v_2 = v_C - r_C\,\omega_C$$

$$2(r_A + r_B)\omega_{ABC} = (r_A + 2r_B + r_C)\omega_{ABC} - r_C\omega_C$$

$$\omega_C = (r_A - r_C)\omega_{ABC} = -r_C\,\omega_C \qquad \omega_C = \left(1 - \frac{r_A}{r_C}\right)\omega_{ABC} \blacktriangleleft$$

Note that the result is independent of r_B.

PROBLEM 15.CQ5



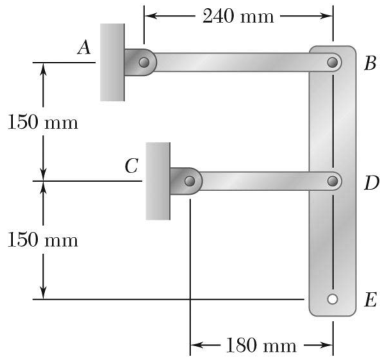
The disk rolls without sliding on the fixed horizontal surface. At the instant shown, the instantaneous center of zero velocity for rod *AB* would be located in which region?

(*a*) region 1
(*b*) region 2
(*c*) region 3
(*d*) region 4
(*e*) region 5
(*f*) region 6

SOLUTION

Answer: (*a*) ◀

PROBLEM 15.CQ6



Bar *BDE* is pinned to two links, *AB* and *CD*. At the instant shown the angular velocities of link *AB*, link *CD* and bar *BDE* are ω_{AB}, ω_{CD}, and ω_{BDE}, respectively. Which of the following statements concerning the angular speeds of the three objects is true at this instant?

(*a*) $\omega_{AB} = \omega_{CD} = \omega_{BDE}$

(*b*) $\omega_{BDE} > \omega_{AB} > \omega_{CD}$

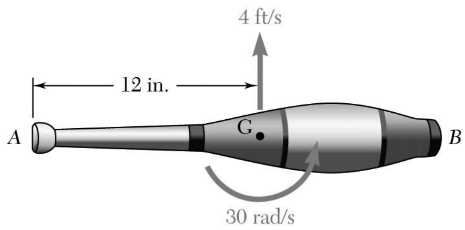
(*c*) $\omega_{AB} = \omega_{CD} > \omega_{BDE}$

(*d*) $\omega_{AB} > \omega_{CD} > \omega_{BDE}$

(*e*) $\omega_{CD} > \omega_{AB} > \omega_{BDE}$

SOLUTION

Answer: (*e*) ◀



PROBLEM 15.73

A juggling club is thrown vertically into the air. The center of gravity G of the 20 in. club is located 12 in. from the knob. Knowing that at the instant shown G has a velocity of 4 ft/s upwards and the club has an angular velocity of 30 rad/s counterclockwise, determine (*a*) the speeds of Point A and B, (*b*) the location of the instantaneous center of rotation.

SOLUTION

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

Relative positions: $\mathbf{r}_{A/G} = -(1\text{ ft})\mathbf{i}, \quad \mathbf{r}_{B/A} = \left(\frac{8}{12}\text{ ft}\right)\mathbf{i}$

Angular velocity: $\boldsymbol{\omega} = 30\text{ rad/s} \circlearrowleft = (30\text{ rad/s})\mathbf{k}$

Velocity at A:

$$\begin{aligned}\mathbf{v}_A &= \mathbf{v}_G + \mathbf{v}_{A/G} = \mathbf{v}_G + \boldsymbol{\omega}\times\mathbf{r}_{A/G}\\ &= (4\text{ ft/s})\mathbf{j} + (30\text{ rad/s})\mathbf{k}\times(-1\text{ ft})\mathbf{i}\\ &= (4\text{ ft/s})\mathbf{j} - (30\text{ ft/s})\mathbf{j} = -(26\text{ ft/s})\mathbf{j}\\ &= 26\text{ ft/s}\downarrow\end{aligned}$$

$v_A = 26.0\text{ ft/s}$ ◀

Velocity at B:

$$\begin{aligned}\mathbf{v}_B &= \mathbf{v}_G + \mathbf{v}_{B/G} = \mathbf{v}_G + \boldsymbol{\omega}\times\mathbf{r}_{B/G}\\ &= (4\text{ ft/s})\mathbf{j} + (30\text{ rad/s})\mathbf{k}\times\left(\frac{8}{12}\text{ ft}\right)\mathbf{i}\\ &= (4\text{ ft/s})\mathbf{j} + (20\text{ ft/s})\mathbf{j} = (24\text{ ft/s})\mathbf{j}\\ &= 24\text{ ft/s}\uparrow\end{aligned}$$

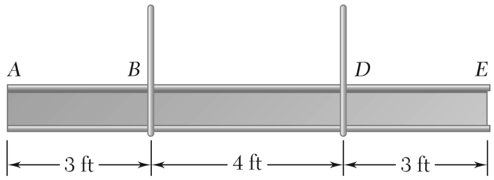
$v_B = 24.0\text{ ft/s}$ ◀

Let $\mathbf{r}_{C/G} = x\mathbf{i}$ bet the position of the instantaneous center C relative to G.

$$\begin{aligned}\mathbf{v}_C &= \mathbf{v}_G + \mathbf{v}_{C/G} = \mathbf{v}_G + \boldsymbol{\omega}\times(x\,\mathbf{i})\\ &= (4\text{ ft/s})\mathbf{j} + (30\text{ rad/s})\mathbf{k}\times(x\,\mathbf{i})\\ &= (4\text{ ft/s})\mathbf{j} + (30\text{ ft/s})\,x\,\mathbf{j} = 0\end{aligned}$$

$$x = -\frac{4\text{ ft/s}}{30\text{ rad/s}} = -\frac{4}{30}\text{ ft} = -1.6\text{ in.}$$

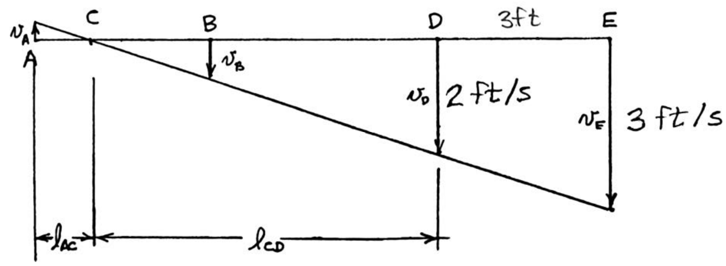
Point C lies 1.6 in. to the left of G. ◀



PROBLEM 15.74

A 10-ft beam *AE* is being lowered by means of two overhead cranes. At the instant shown, it is known that the velocity of Point *D* is 24 in./s downward and the velocity of Point *E* is 36 in./s downward. Determine (*a*) the instantaneous center of rotation of the beam, (*b*) the velocity of Point *A*.

SOLUTION



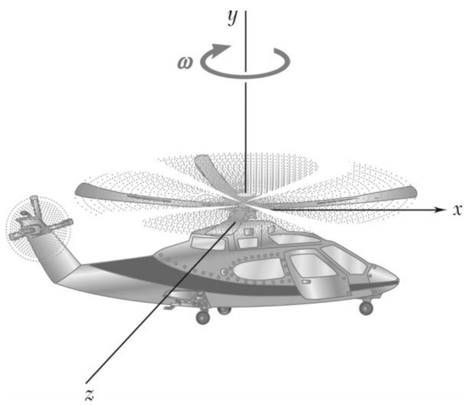
$$\omega = \frac{v_E - v_D}{l_{ED}} = \frac{3-2}{3} = \frac{1}{3} \text{ rad/s} \circlearrowright$$

$$l_{CE} = \frac{v_D}{\omega} = \frac{2}{\frac{1}{3}} = 6 \text{ ft}$$

(*a*) $l_{AC} = 3 + 4 - 6 = 1 \text{ ft}$ — *C* lies 1 ft to the right of *A*. ◀

(*b*) $v_A = l_{AC}\,\omega = (1)\left(\frac{1}{3}\right) = 0.3333 \text{ ft/s}$ — $\mathbf{v}_A = 4.00 \text{ in./s} \uparrow$ ◀

PROBLEM 15.75

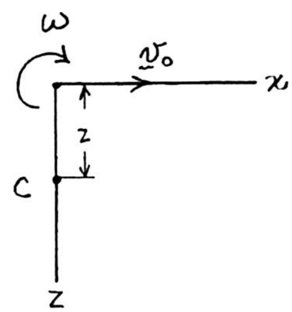


A helicopter moves horizontally in the x direction at a speed of 120 mi/h. Knowing that the main blades rotate clockwise with an angular velocity of 180 rpm, determine the instantaneous axis of rotation of the main blades.

SOLUTION

$$\mathbf{v}_0 = 120 \text{ mi/h} = 176 \text{ ft/s} \longrightarrow$$

$$\omega = 180 \text{ rpm} = \frac{(180)(2\pi)}{60} = 18.85 \text{ rad/s}$$

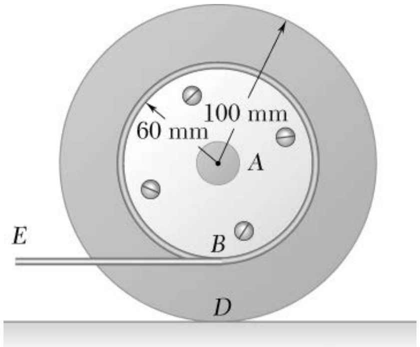


$$v_0 = z\omega$$

$$z = \frac{v_0}{\omega} = \frac{176}{18.85} = 9.34 \text{ ft}$$

Instantaneous axis is parallel to the y axis and passes through the point $x = 0$ ◀

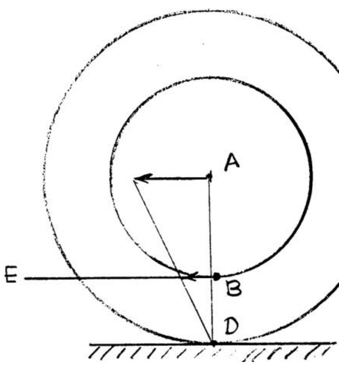
$z = 9.34$ ft ◀



PROBLEM 15.76

A 60-mm-radius drum is rigidly attached to a 100-mm-radius drum as shown. One of the drums rolls without sliding on the surface shown, and a cord is wound around the other drum. Knowing that end E of the cord is pulled to the left with a velocity of 120 mm/s, determine (*a*) the angular velocity of the drums, (*b*) the velocity of the center of the drums, (*c*) the length of cord wound or unwound per second.

SOLUTION



Since the drum rolls without sliding, its instantaneous center lies at D.

$$\mathbf{v}_E = \mathbf{v}_B = 120 \text{ mm/s} \longleftarrow$$

$$v_A = v_{A/D}\omega, \quad v_B = r_{B/D}\omega$$

(*a*) $$\omega = \frac{v_B}{v_{B/D}} = \frac{120}{100-60} = 3 \text{ rad/s}$$

$$\boldsymbol{\omega} = 3.00 \text{ rad/s} \circlearrowleft \blacktriangleleft$$

(*b*) $$v_A = (100)(3) = 300 \text{ mm/s}$$

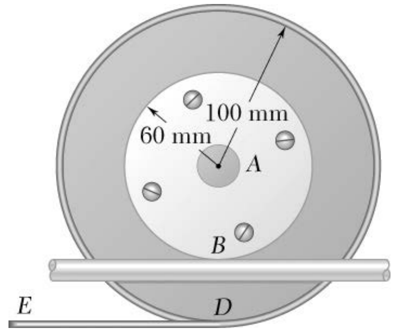
$$\mathbf{v}_A = 300 \text{ mm/s} \longleftarrow \blacktriangleleft$$

Since v_A is greater than v_B, cord is being wound.

$$v_A - v_B = 300 - 120 = 180 \text{ mm/s}$$

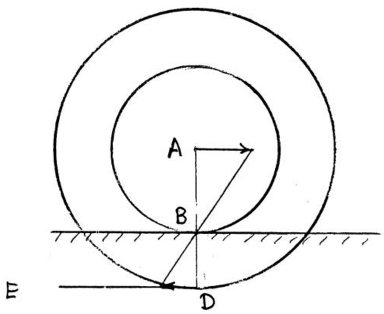
(*c*) Cord wound per second = 180.0 mm ◀

PROBLEM 15.77



A 60-mm-radius drum is rigidly attached to a 100-mm-radius drum as shown. One of the drums rolls without sliding on the surface shown, and a cord is wound around the other drum. Knowing that end E of the cord is pulled to the left with a velocity of 120 mm/s, determine (a) the angular velocity of the drums, (b) the velocity of the center of the drums, (c) the length of cord wound or unwound per second.

SOLUTION



Since the drum rolls without sliding, its instantaneous center lies at B.

$$\mathbf{v}_E = \mathbf{v}_D = 120 \text{ mm/s} \longleftarrow$$

$$v_A = r_{A/B}\omega, \; v_D = r_{D/B}\omega$$

(a)
$$\omega = \frac{v_D}{r_{D/B}} = \frac{120}{100-60} = 3 \text{ rad/s}$$

$$\boldsymbol{\omega} = 3.00 \text{ rad/s} \circlearrowright \blacktriangleleft$$

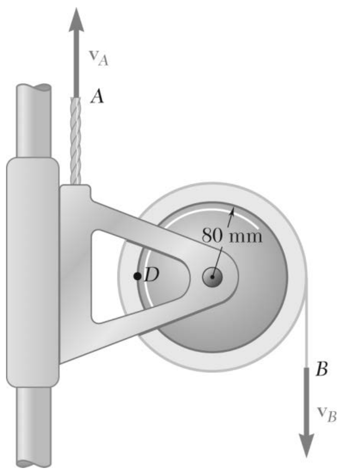
(b)
$$v_A = (60)(3.00) = 180 \text{ mm/s}$$

$$\mathbf{v}_A = 180 \text{ mm/s} \longrightarrow \blacktriangleleft$$

Since $\mathbf{v}_A$ is to the right and $\mathbf{v}_D$ is to the left, cord is being unwound.

$$v_A - v_E = 180 + 120 = 300 \text{ mm/s}$$

(c) Cord unwound per second $= 300$ mm ◀



PROBLEM 15.78

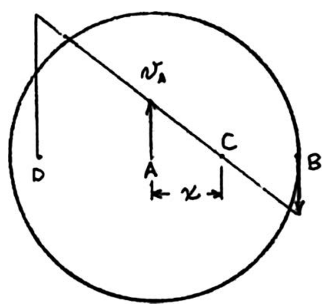
The spool of tape shown and its frame assembly are pulled upward at a speed $v_A = 750$ mm/s. Knowing that the 80-mm-radius spool has an angular velocity of 15 rad/s clockwise and that at the instant shown the total thickness of the tape on the spool is 20 mm, determine (*a*) the instantaneous center of rotation of the spool, (*b*) the velocities of Points *B* and *D*.

SOLUTION

$$\mathbf{v}_A = 750 \text{ mm/s} \uparrow$$

$$\omega = 15 \text{ rad/s} \circlearrowright$$

$$x = \frac{v_A}{\omega} = \frac{750}{15} = 50 \text{ mm}$$



(*a*) The instantaneous center lies 50 mm to the right of the axle. ◀

$$CB = 80 + 20 - 50 = 50 \text{ mm}$$

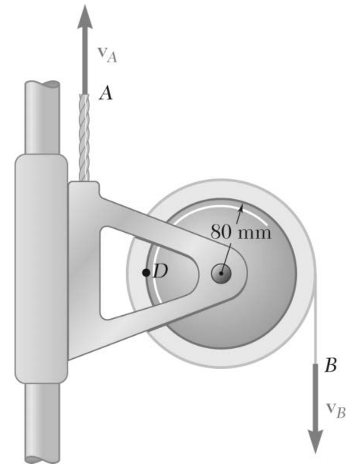
(*b*) $v_B = (CB)\omega = (50)(15) = 750$ mm/s

$$\mathbf{v}_B = 750 \text{ mm/s} \downarrow \blacktriangleleft$$

$$CD = 80 + 50 = 130 \text{ mm}$$

$$v_D = (CD)\omega = (130)(15) = 1950 \text{ mm/s}$$

$$\mathbf{v}_D = 1.950 \text{ m/s} \uparrow \blacktriangleleft$$



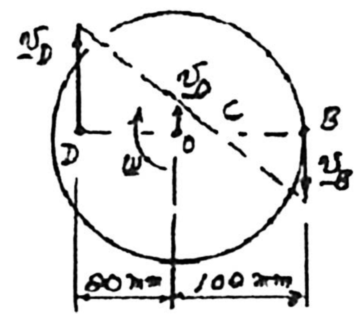
PROBLEM 15.79

The spool of tape shown and its frame assembly are pulled upward at a speed $v_A = 100$ mm/s. Knowing that end B of the tape is pulled downward with a velocity of 300 mm/s and that at the instant shown the total thickness of the tape on the spool is 20 mm, determine (a) the instantaneous center of rotation of the spool, (b) the velocity of Point D of the spool.

SOLUTION

$$v_D = v_A = 100 \text{ mm/s}$$

(a) Since $\mathbf{v}_0$ and $\mathbf{v}_B$ are parallel, instantaneous center C is located at intersection of BC and line joining end points of $\mathbf{v}_D$ and $\mathbf{v}_B$.



Similar triangles.

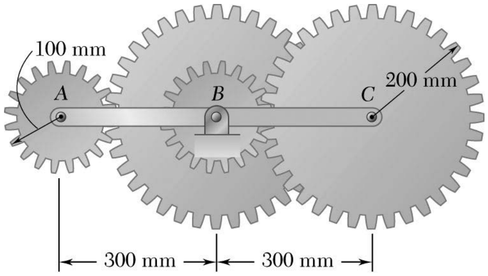
$$\frac{OC}{v_0} = \frac{BC}{v_B} = \frac{OC + BC}{v_0 + v_B}$$

$$OC = \frac{v_0}{v_0 + v_B}(OC + BC)$$

$$OC = \frac{100 \text{ mm/s}}{(100 + 300) \text{ mm/s}}(100 \text{ mm})$$

$$= 25 \text{ mm}$$ ◀

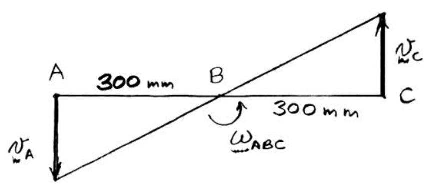
(b) $\dfrac{v_D}{(DO) + (OC)} = \dfrac{v_0}{(OC)}$; $\quad \dfrac{v_D}{(80 + 25) \text{ mm}} = \dfrac{100 \text{ mm/s}}{25 \text{ mm}}$ $\quad \mathbf{v}_D = 420 \text{ mm/s} \uparrow$ ◀



PROBLEM 15.80

The arm ABC rotates with an angular velocity of 4 rad/s counterclockwise. Knowing that the angular velocity of the intermediate gear B is 8 rad/s counterclockwise, determine (*a*) the instantaneous centers of rotation of gears A and C, (*b*) the angular velocities of gears A and C.

SOLUTION



Contact points:

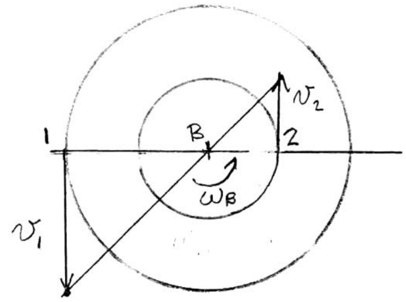
1 between gears A and B.

2 between gears B and C.

Arm ABC: $\omega_{ABC} = 4 \text{ rad/s} \circlearrowleft$

$$v_A = (0.300)(4) = 1.2 \text{ m/s} \downarrow$$

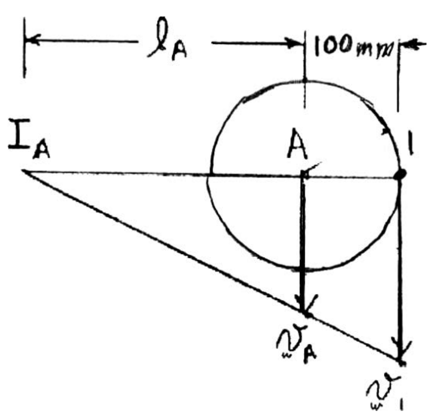
$$v_C = (0.300)(4) = 1.2 \text{ m/s} \uparrow$$



Gear B: $\omega_B = 8 \text{ rad/s} \circlearrowleft$

$$v_1 = (0.200)(8) = 1.6 \text{ m/s} \downarrow$$

$$v_2 = (0.100)(8) = 0.8 \text{ m/s} \uparrow$$



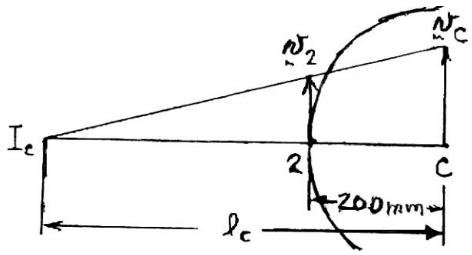
Gear A:

$$\omega_A = \frac{v_1 - v_A}{0.100} = \frac{1.6 - 1.2}{0.100}$$

$$\omega_A = 4 \text{ rad/s} \circlearrowright$$

$$\ell_A = \frac{v_A}{\omega_A} = \frac{1.2}{4} = 0.3 \text{ m} = 300 \text{ mm}$$

PROBLEM 15.80 (Continued)



Gear C:

$$\omega_C = \frac{v_C - v_2}{0.200} = \frac{1.2 - 0.8}{0.2}$$

$$\omega_C = 2 \text{ rad/s} \circlearrowleft$$

$$\ell_C = \frac{v_C}{\omega_C} = \frac{1.2}{2} = 0.6 \text{ m}$$

(*a*) *Instantaneous centers.*

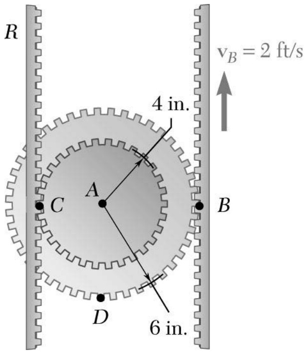
Gear *A*: 300 mm left of *A* ◀

Gear *C*: 600 mm left of *C* ◀

(*b*) *Angular velocities.*

$\boldsymbol{\omega}_A = 4.00$ rad/s ↻ ◀

$\boldsymbol{\omega}_C = 2.00$ rad/s ↺ ◀



PROBLEM 15.81

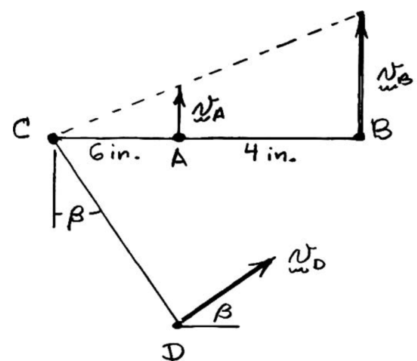
The double gear rolls on the stationary left rack R. Knowing that the rack on the right has a constant velocity of 2 ft/s, determine (*a*) the angular velocity of the gear, (*b*) the velocities of Points A and D.

SOLUTION

Since the rack R is stationary, Point C is the instantaneous center of the double gear.

Given: $\mathbf{v}_B = 2 \text{ ft/s}\uparrow = 24 \text{ in./s}\uparrow$

Make a diagram showing the locations of Points A, B, C, and D on the double gear.



$$v_B = \omega l_{CB}$$

$$\omega = \frac{v_B}{l_{CB}} = \frac{24 \text{ in./s}}{10 \text{ in.}} = 2.40 \text{ rad/s}$$

(*a*) *Angular velocity of the gear.* $\boldsymbol{\omega} = 2.40 \text{ rad/s} \circlearrowright$ ◀

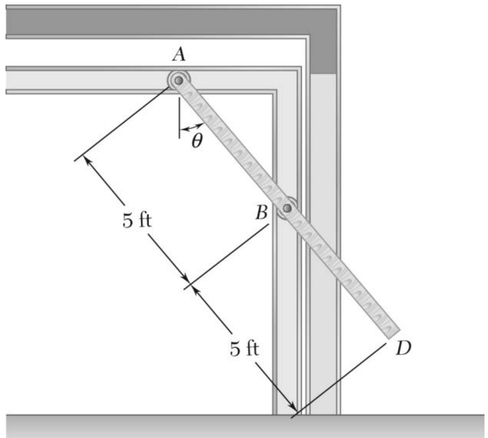
(*b*) *Velocity of Point A.* $v_A = l_{AC}\omega = (4 \text{ in.})(2.40 \text{ rad/s})$ $\mathbf{v}_A = 9.60 \text{ in./s} = 0.800 \text{ ft/s}\uparrow$ ◀

Geometry: $l_{CD} = \sqrt{(4 \text{ in.})^2 + (6 \text{ in.})^2} = \sqrt{52} \text{ in.}$

$$\tan\beta = \frac{4 \text{ in.}}{6 \text{ in.}} \qquad \beta = 33.7°$$

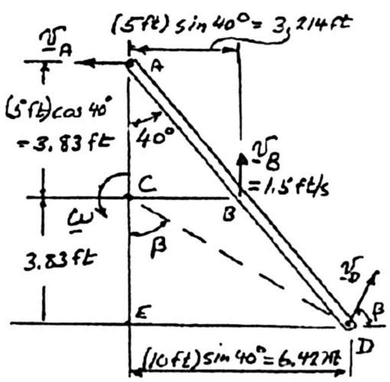
Velocity of Point D. $\mathbf{v}_D = l_{CD}\omega = \sqrt{52}(2.40) = 17.31 \text{ in./s}$ $\mathbf{v}_D = 1.442 \text{ ft/s} \measuredangle 33.7°$ ◀

PROBLEM 15.82



An overhead door is guided by wheels at A and B that roll in horizontal and vertical tracks. Knowing that when $\theta = 40°$ the velocity of wheel B is 1.5 ft/s upward, determine (*a*) the angular velocity of the door, (*b*) the velocity of end D of the door.

SOLUTION



Locate instantaneous center at intersection of lines drawn perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$.

(*a*) <u>Angular velocity</u>.

$$v_B = (BC)\omega$$

$$1.5 \text{ ft/s} = (3.214 \text{ ft})\omega$$

$$\omega = 0.4667 \text{ rad/s} \qquad \boldsymbol{\omega} = 0.467 \text{ rad/s} \circlearrowright \blacktriangleleft$$

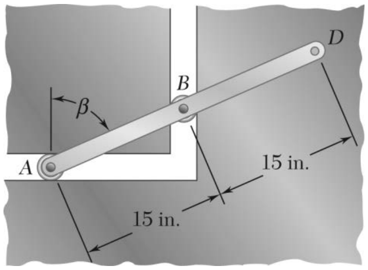
(*b*) Velocity of D:

In ΔCDE:
$$\beta = \tan^{-1}\frac{6.427}{3.83} = 59.2°$$

$$CD = \frac{6.427}{\sin\beta} = 7.482 \text{ ft}$$

$$v_D = (CD)\omega$$
$$= (7.482 \text{ ft})(0.4667 \text{ rad/s})$$
$$= 3.49 \text{ ft/s}$$

$$\mathbf{v}_D = 3.49 \text{ ft/s} \measuredangle 59.2° \blacktriangleleft$$

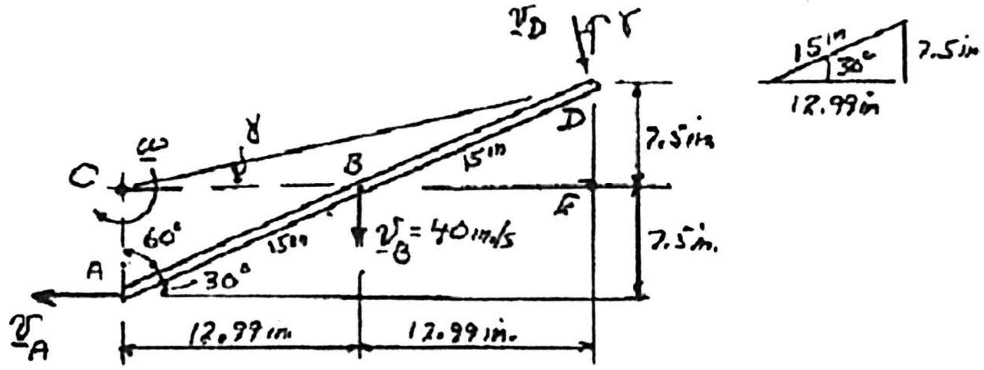


PROBLEM 15.83

Rod *ABD* is guided by wheels at *A* and *B* that roll in horizontal and vertical tracks. Knowing that at the instant shown $\beta = 60°$ and the velocity of wheel *B* is 40 in./s downward, determine (*a*) the angular velocity of the rod, (*b*) the velocity of Point *D*.

SOLUTION

Rod *ABD*:



We locate the instantaneous center by drawing lines perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_D$.

(*a*) Angular velocity.

$$v_B = (BC)\omega$$

$$40 \text{ in./s} = (12.99 \text{ in.})\omega$$

$$\omega = 3.079 \text{ rad/s}$$

$\boldsymbol{\omega} = 3.08 \text{ rad/s}$ ↻ ◀

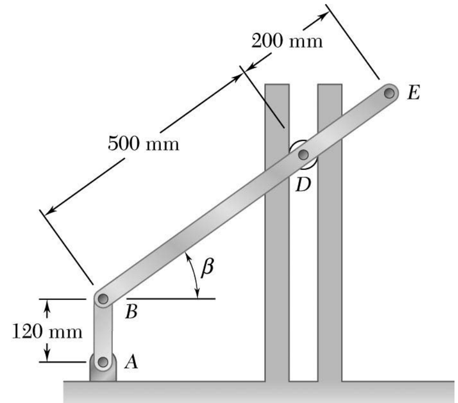
(*b*) Velocity of *D*:

In ΔCDE: $\gamma = \tan^{-1}\dfrac{7.5}{25.98} = 16.1°$; $CD = \dfrac{25.98}{\cos\gamma} = 27.04$ in.

$$v_D = (CD)\omega = (27.04 \text{ in.})(3.079 \text{ rad/s}) = 83.3 \text{ in./s}$$

$\mathbf{v}_D = 83.3$ in./s ↖ 16.1°

$\mathbf{v}_D = 83.3$ in./s ↖ 73.9° ◀



PROBLEM 15.84

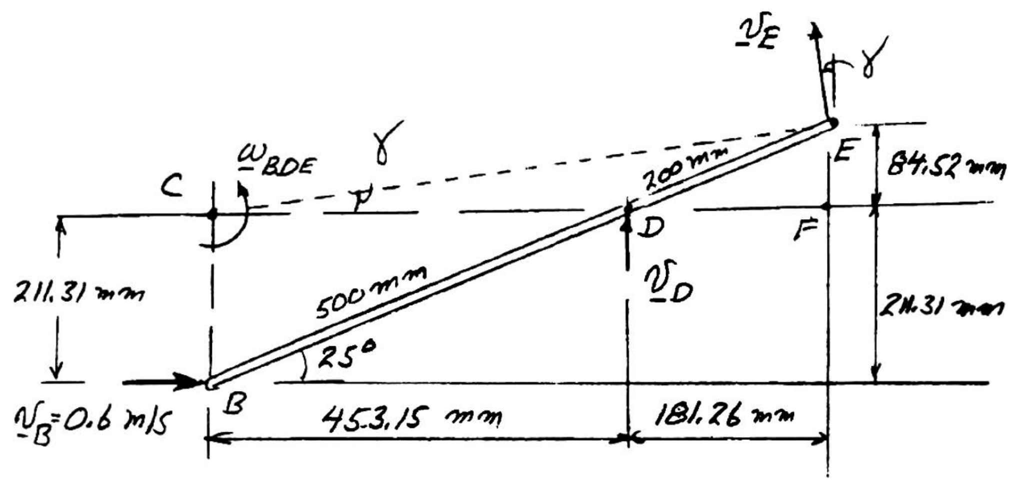
Rod *BDE* is partially guided by a roller at *D* which moves in a vertical track. Knowing that at the instant shown the angular velocity of crank *AB* is 5 rad/s clockwise and that $\beta = 25°$, determine (*a*) the angular velocity of the rod, (*b*) the velocity of Point *E*.

SOLUTION

Crank *AB*: $\boldsymbol{\omega}_{AB} = 5 \text{ rad/s} \circlearrowright \qquad \mathbf{r}_{B/A} = 120 \text{ mm} \uparrow$

$$v_B = \omega_{AB} r_{B/A} = (5)(0.120) \qquad \mathbf{v}_B = 0.6 \text{ m/s} \longrightarrow$$

Rod *BDE*: Draw a diagram of the geometry of the rod and note that $\mathbf{v}_B = 0.6 \text{ m/s} \longrightarrow$ and $\mathbf{v}_D = \mathbf{v}_D \uparrow$.



Locate Point *C*, the instantaneous center, by noting that *BC* is perpendicular to $\mathbf{v}_B$ and *DC* is perpendicular to $\mathbf{v}_D$. Calculate lengths of *BC* and *CD*.

$$l_{BC} = 500 \sin 25° = 211.31 \text{ mm}$$
$$l_{CD} = 500 \cos 25° = 453.15.$$

(*a*) *Angular velocity of the rod.*

$$\omega_{BCD} = \frac{v_B}{l_{BC}} = \frac{0.6 \text{ m/s}}{0.21131 \text{ m}} = 2.8394 \text{ rad/s}$$

$\boldsymbol{\omega}_{BCD} = 2.84 \text{ rad/s} \circlearrowleft$ ◀

PROBLEM 15.84 (Continued)

(*b*) *Velocity of Point E.*

Locate Point *F* on the diagram.

$$CF = 700\cos 25° \text{ mm} \qquad FE = 200\sin 25°$$

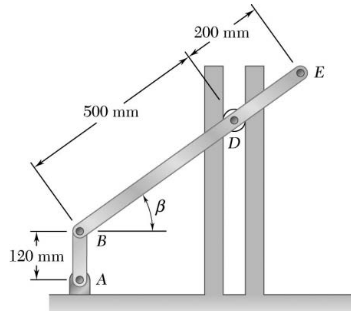
$$\tan\gamma = \frac{FE}{CF} = \frac{200\sin 25°}{700\cos 25°} = \frac{2}{7}\tan 25° = 0.13323$$

$$\gamma = 7.6° \qquad \beta = 90° - \gamma = 82.4°$$

$$l_{CE} = \sqrt{(CF)^2 + (FE)^2} = 640.02 \text{ mm} = 0.64002 \text{ m}$$

$$v_E = l_{CE}\omega = (0.64002)(2.8394)$$

$\mathbf{v}_E = 1.817$ m/s ⦪ 82.4° ◀



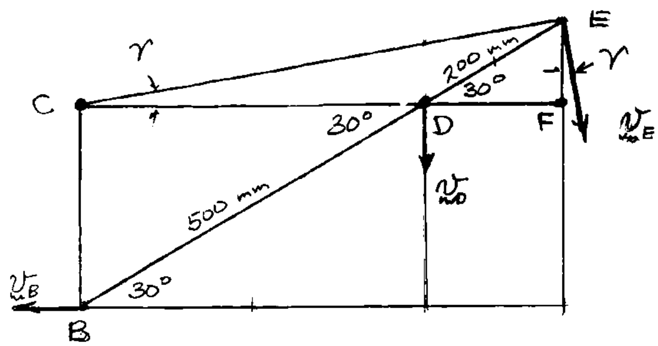
PROBLEM 15.85

Rod *BDE* is partially guided by a roller at *D* which moves in a vertical track. Knowing that at the instant shown $\beta = 30°$, Point *E* has a velocity of 2 m/s down and to the right, determine the angular velocities of rod *BDE* and crank *AB*.

SOLUTION

Crank *AB*: When *AB* is vertical, the velocity $\mathbf{v}_B$ at Point *B* is horizontal.

Rod *BDE*: Draw a diagram of the geometry of the rod and note that $\mathbf{v}_B$ is horizontal and $\mathbf{v}_D$ is vertical.



Locate Point *C*, the instantaneous center *C*, by noting that *CB* is vertical and *CD* is horizontal. From the diagram, with Point *F* added,

$$CF = 700\cos 30° \text{ mm} \qquad FE = 200\sin 30° \text{ mm}$$

$$CE = \sqrt{(CF)^2 + (FE)^2} = 614.41 \text{ mm} = 0.61441 \text{ m}$$

Angular velocity of rod BDE

$$\omega_{BDE} = \frac{v_E}{(CE)} = \frac{2 \text{ m/s}}{0.61441 \text{ m}} = 3.2552 \text{ rad/s}$$

$$\boldsymbol{\omega}_{BDE} = 3.26 \text{ rad/s} \circlearrowright \blacktriangleleft$$

Velocity of B. $\quad CB = 500\sin 30° \text{ mm} = 250 \text{ mm} = 0.250 \text{ m}$

$$v_B = (CB)\omega_{BDE} = (0.250)(3.2552)$$

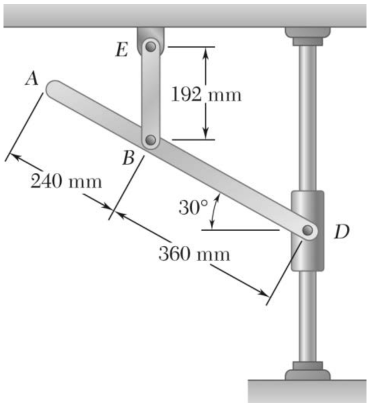
$$\mathbf{v}_B = 0.81379 \text{ m/s} \longleftarrow$$

Angular velocity of crank AB: $\quad AB = 120 \text{ mm} = 0.120 \text{ m}$

$$\omega_{AB} = \frac{v_B}{(AB)} = \frac{0.81379 \text{ m/s}}{0.120 \text{ m}}$$

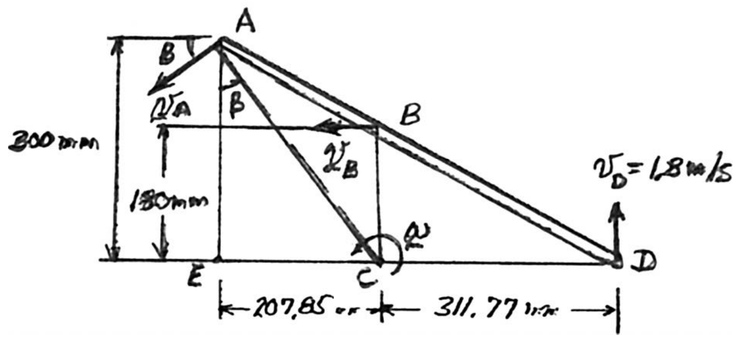
$$\boldsymbol{\omega}_{AB} = 6.78 \text{ rad/s} \circlearrowright \blacktriangleleft$$

PROBLEM 15.86



Knowing that at the instant shown the velocity of collar D is 1.6 m/s upward, determine (a) the angular velocity of rod AD, (b) the velocity of Point B, (c) the velocity of Point A.

SOLUTION



We draw perpendiculars to $\mathbf{v}_B$ and $\mathbf{v}_D$ to locate instantaneous center C.

(a) Angular velocity:

$v_D = (CD)\omega$ $\qquad$ $1.6 \text{ m/s} = (0.31177 \text{ m})\omega$

$$\omega = 5.132 \text{ rad/s}$$

$\boldsymbol{\omega} = 5.13 \text{ rad/s} \circlearrowleft$ ◀

(b) $v_B = (BC)\omega = (180 \text{ mm})(5.132 \text{ rad/s})$

$$v_D = 923.76 \text{ mm/s}$$

$\mathbf{v}_D = 0.924 \text{ m/s} \longleftarrow$ ◀

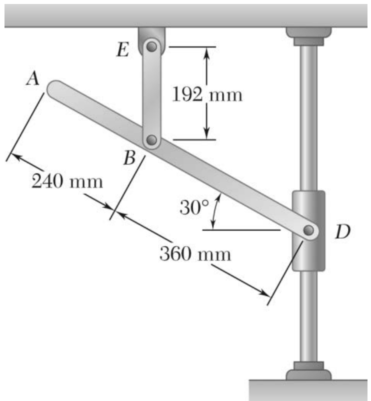
(c) $v_A = (AC)\omega$

In triangle ACE: $\qquad \tan\beta = \dfrac{207.85 \text{ mm}}{300 \text{ mm}} \qquad \beta = 34.72°$

$$AC = \sqrt{(207.85)^2 + (300)^2} \qquad AC = 364.97 \text{ mm}$$

$$v_A = (364.97 \text{ mm})(5.132 \text{ rad/s}) = 1873.0 \text{ mm/s}$$

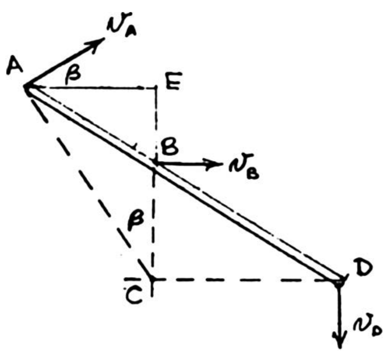
$\mathbf{v}_A = 1.870 \text{ m/s} \measuredangle 34.7°$ ◀



PROBLEM 15.87

Knowing that at the instant shown the angular velocity of rod BE is 4 rad/s counterclockwise, determine (*a*) the angular velocity of rod AD, (*b*) the velocity of collar D, (*c*) the velocity of Point A.

SOLUTION



Rod AD.

$$\mathbf{v}_B = r_{B/E}\omega_{BE} = (0.192)(4) = 0.768 \text{ m/s} \longrightarrow$$

(*a*) Instantaneous center C is located by noting that CD is perpendicular to $\mathbf{v}_D$ and CB is perpendicular to $\mathbf{v}_B$.

$$r_{B/C} = 0.360 \ \sin 30° = 0.180 \text{ m}$$

$$\omega_{AD} = \frac{v_B}{r_{B/C}} = \frac{0.768}{0.180} = 4.2667$$

$$\boldsymbol{\omega}_{AD} = 4.27 \text{ rad/s} \circlearrowright \blacktriangleleft$$

(*b*) Velocity of D.

$$r_{D/C} = 0.360 \ \cos 30° = 0.31177 \text{ m}$$

$$v_D = r_{D/C}\omega = (0.31177)(4.2667)$$

$$\mathbf{v}_D = 1.330 \text{ m/s} \downarrow \blacktriangleleft$$

(*c*) Velocity of A.

$$l_{AE} = 0.240 \cos 30° = 0.20785 \text{ m}$$

$$l_{CE} = 0.600 \sin 30° = 0.300 \text{ m}$$

$$\tan\beta = \frac{0.20785}{0.300} \qquad \beta = 34.7°$$

$$l_{CA} = \sqrt{(0.20785)^2 + (0.300)^2}$$

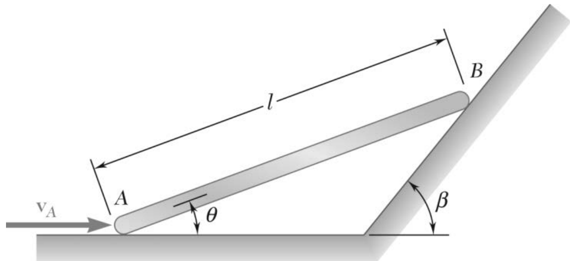
$$= 0.36497 \text{ m}$$

$$v_A = l_{CA}\omega_{AD}$$

$$= (0.36497)(4.2667)$$

$$= 1.557 \text{ m/s}$$

$$\mathbf{v}_A = 1.557 \text{ m/s} \measuredangle 34.7° \blacktriangleleft$$



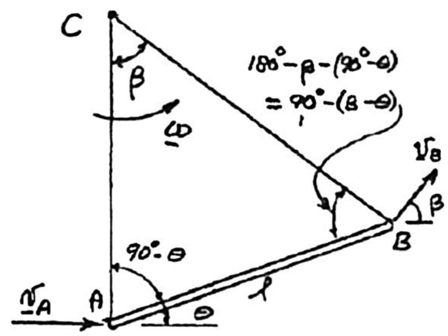
PROBLEM 15.88

Rod AB can slide freely along the floor and the inclined plane. Denoting by $\mathbf{v}_A$ the velocity of Point A, derive an expression for (a) the angular velocity of the rod, (b) the velocity of end B.

SOLUTION

Locate the instantaneous center at intersection of lines drawn perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$.

Law of sines.



$$\frac{AC}{\sin[90° - (\beta - \theta)]} = \frac{BC}{\sin(90° - \theta)} = \frac{l}{\sin\beta}$$

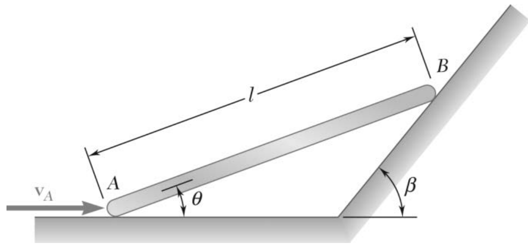
$$\frac{AC}{\cos(\beta - \theta)} = \frac{BC}{\cos\theta} = \frac{l}{\sin\beta}$$

$$AC = l\frac{\cos(\beta - \theta)}{\sin\beta}$$

$$BC = l\frac{\cos\theta}{\sin\beta}$$

(a) Angular velocity: $v_A = (AC)\omega = l\dfrac{\cos(\beta - \theta)}{\sin\beta}\omega$ $\qquad \omega = \dfrac{v_A}{l}\cdot\dfrac{\sin\beta}{\cos(\beta - \theta)}$ ◀

(b) Velocity of B: $v_B = (BC)\omega = l\dfrac{\cos\theta}{\sin\beta}\cdot\left[\dfrac{v_\theta}{l}\cdot\dfrac{\sin\beta}{\cos(\beta - \theta)}\right]$ $\qquad v_B = v_A\dfrac{\cos\theta}{\cos(\beta - \theta)}$ ◀



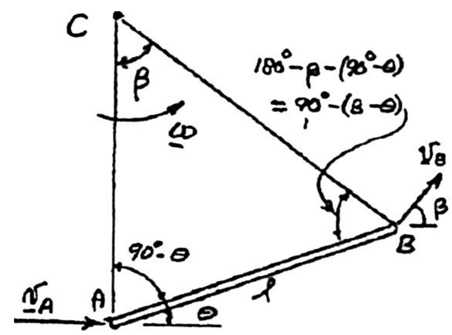
PROBLEM 15.89

Rod *AB* can slide freely along the floor and the inclined plane. Knowing that $\theta = 20°$, $\beta = 50°$, $l = 2$ ft, and $v_A = 8$ ft/s, determine (*a*) the angular velocity of the rod, (*b*) the velocity of end *B*.

SOLUTION

Locate the instantaneous center at intersection of lines draw perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$.

Law of sines.



$$\frac{AC}{\sin[90° - (\beta - \theta)]} = \frac{BC}{\sin(90° - \theta)} = \frac{l}{\sin\beta}$$

$$\frac{AC}{\cos(\beta - \theta)} = \frac{BC}{\cos\theta} = \frac{l}{\sin\beta}$$

$$AC = l\frac{\cos(\beta - \theta)}{\sin\beta}$$

$$BC = l\frac{\cos\theta}{\sin\beta}$$

Angular velocity:

$$v_A = (AC)\omega = l\frac{\cos(\beta - \theta)}{\sin\beta}\omega$$

$$\omega = \frac{v_A}{l}\cdot\frac{\sin\beta}{\cos(\beta - \theta)}$$

Velocity of *B*:

$$v_B = (BC)\omega = l\frac{\cos\theta}{\sin\beta}\cdot\left[\frac{v_A}{l}\cdot\frac{\sin\beta}{\cos(\beta - \theta)}\right]$$

$$v_B = v_A\frac{\cos\theta}{\cos(\beta - \theta)}$$

Data: $\theta = 20°$, $\beta = 50°$, $l = 2$ ft, $v_A = 8$ ft/s

(*a*)

$$\omega = \frac{v_A}{l}\frac{\sin\beta}{\cos(\beta - \theta)} = \frac{8 \text{ ft/s}}{2 \text{ ft}}\cdot\frac{\sin 50°}{\cos(50° - 20°)}$$

$$\omega = 3.5382 \text{ rad/s}$$

$\boldsymbol{\omega} = 3.54$ rad/s ↻ ◀

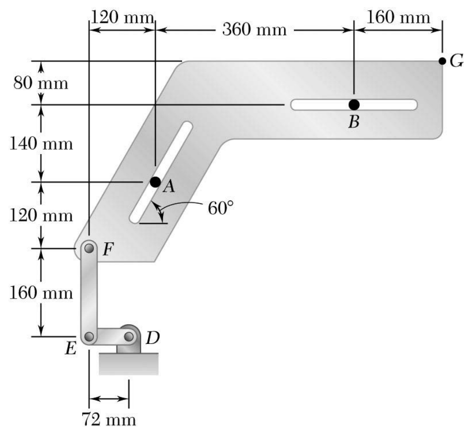
(*b*)

$$v_B = v_A \frac{\cos\theta}{\cos(\beta-\theta)}$$

$$= (8 \text{ ft/s}) \frac{\cos 20°}{\cos(50° - 20°)}$$

$$v_B = 8.6805 \text{ ft/s}$$

$\mathbf{v}_B = 8.68$ ft/s ∠ 50° ◀



PROBLEM 15.90

Two slots have been cut in plate *FG* and the plate has been placed so that the slots fit two fixed pins *A* and *B*. Knowing that at the instant shown the angular velocity of crank *DE* is 6 rad/s clockwise, determine (*a*) the velocity of Point *F*, (*b*) the velocity of Point *G*.

SOLUTION

Crank *DE*: $v_E = (DE)\omega_{DE} = (72 \text{ mm})(6 \text{ rad/s})$

$\mathbf{v}_E = 432 \text{ mm/s} \uparrow$

Rod *EF*: $(v_F)_y = v_E = 432 \text{ mm/s} \uparrow$

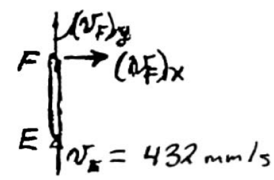
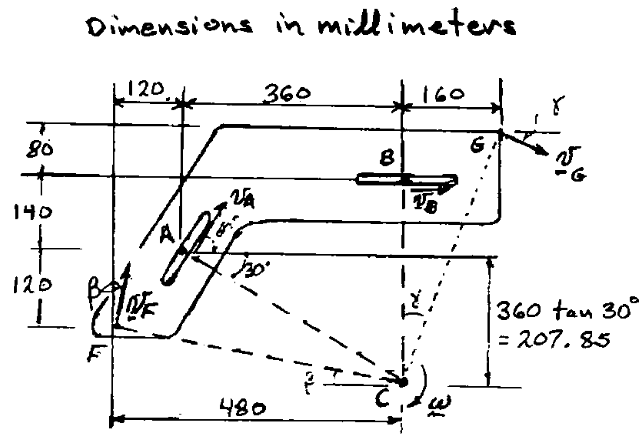


Plate *FG*:



$\mathbf{v}_A$ and $\mathbf{v}_B$ are velocities of points on the plate next to the pins *A* and *B*. We draw lines perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$ to locate the instantaneous center *C*.

(*a*) Velocity of Point *F*:

$$v_F = (CF)\omega$$
$$(v_F)_y = [(CF)\omega]\cos\beta = [(CF)\cos\beta]\omega$$

But $(v_F)_y = 432 \text{ mm/s}$ and $(CF)\cos\beta = 480 \text{ mm}$:

$$432 \text{ mm/s} = (480 \text{ mm})\omega$$
$$\omega = 0.9 \text{ rad/s}$$

$\boldsymbol{\omega} = 0.9 \text{ rad/s}$ ↻

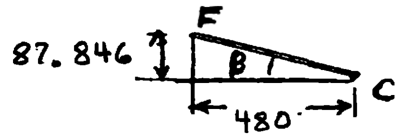
$CF = 487.97$ mm

$\beta = 10.37°$

$v_F = (CF)\omega$

$= (487.97 \text{ mm})(0.9 \text{ rad/s})$

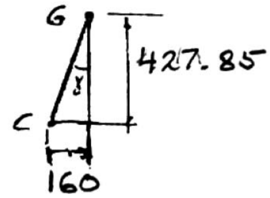
$= 439.18$ mm/s



$\mathbf{v}_F = 439$ mm/s ⦪ 10.4°

$\mathbf{v}_F = 439$ mm/s ⦨ 79.6° ◀

(*b*) Velocity of Point *G*:



$CG = 456.78$ mm

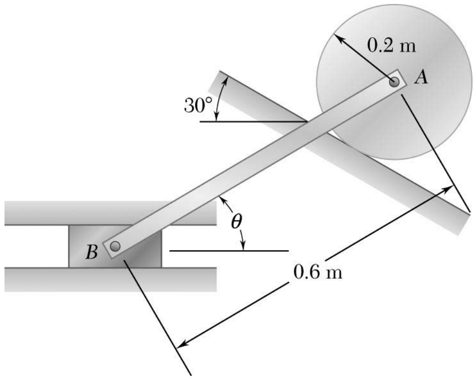
$\gamma = 20.50°$

$v_G = (CG)\omega$

$= (456.78 \text{ in.})(0.9 \text{ rad/s})$

$v_G = 411.11$ mm/s

$\mathbf{v}_G = 411$ mm/s ⦩ 20.5° ◀



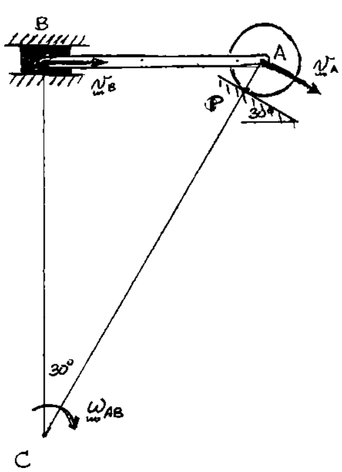
PROBLEM 15.91

The disk is released from rest and rolls down the incline. Knowing that the speed of A is 1.2 m/s when $\theta = 0°$, determine at that instant (*a*) the angular velocity of the rod, (*b*) the velocity of B. Only portions of the two tracks are shown.

SOLUTION

Draw the slider, rod, and disk at $\theta = 0°$.

Let Point P be the contact point between the disk and the incline. It is the instantaneous center of the disk. $\mathbf{v}_A$ is parallel to the incline. So that



$$\mathbf{v}_A = v_A \searrow 30°$$

Constraint of slider:

$$\mathbf{v}_B = v_B \longrightarrow$$

To locate the instantaneous center C of the rod AB, extend the line AP to meet the vertical line through P at Point C.

$$l_{AC} = l_{AB} / \sin 30°$$
$$l_{BC} = l_{AB} / \tan 30°$$

Angular velocity of rod AB.

$$\omega_{AB} = \frac{v_A}{l_{AC}} = \frac{v_A \sin 30°}{l_{AB}} = \frac{(1.2 \text{ m/s}) \sin 30°}{0.6 \text{ m}}$$

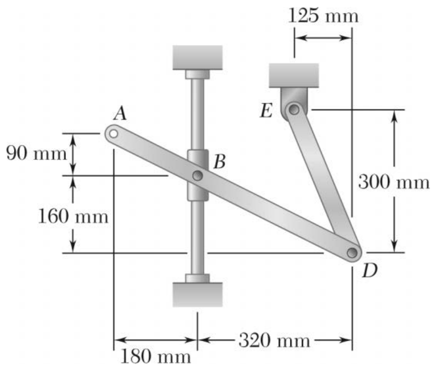
$$\boldsymbol{\omega}_{AB} = 1.000 \text{ rad/s} \circlearrowright \blacktriangleleft$$

(*b*) *Velocity of Point B.*

$$v_B = l_{BC}\,\omega_{AB}$$

$$v_B = \frac{l_{AB}}{\tan 30°} \frac{v_A \sin 30°}{l_{AB}} = v_A \cos 30° = 1.2 \cos 30°$$

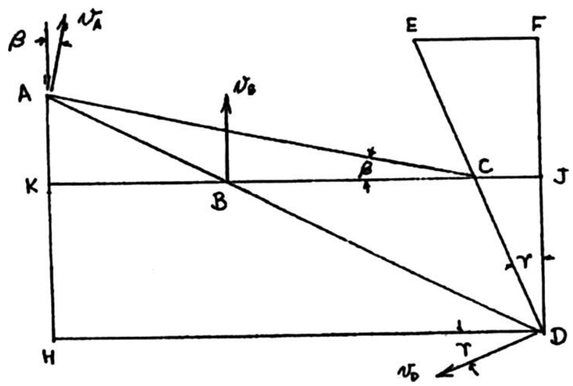
$$\mathbf{v}_B = 1.039 \text{ m/s} \longrightarrow \blacktriangleleft$$



PROBLEM 15.92

Arm ABD is connected by pins to a collar at B and to crank DE. Knowing that the velocity of collar B is 400 mm/s upward, determine (a) the angular velocity of arm ABD, (b) the velocity of Point A.

SOLUTION



$\mathbf{v}_B = 16 \text{ in./s} \uparrow \qquad \tan\gamma = \dfrac{EF}{DF} = \dfrac{125}{300} \qquad \mathbf{v}_D = v_D \nearrow \gamma$

Locate the instantaneous center (Point C) of bar ABD by noting that velocity directions at Points B and D are known. Draw BC perpendicular to $\mathbf{v}_B$ and DC perpendicular to $\mathbf{v}_D$.

$$CJ = (DJ)\tan\gamma = (160)\left(\frac{125}{300}\right) = 66.667 \text{ mm}$$

$$CB = JB - CJ = 320 - 66.667 = 253.33 \text{ mm}$$

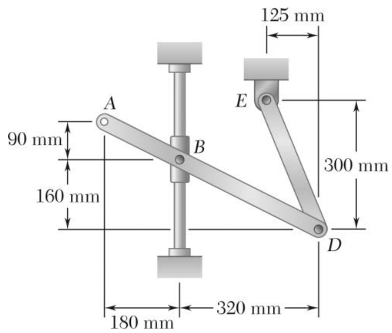
(a) $$\omega_{ABD} = \frac{v_B}{CB} = \frac{400}{253.33} = 1.57895 \text{ rad/s}$$ $\boldsymbol{\omega}_{ABD} = 1.579 \text{ rad/s} \circlearrowright$ ◀

$$CK = CB + BK = 253.33 + 180 = 433.33 \text{ mm}$$

$$\tan\beta = \frac{KA}{CK} = \frac{90}{433.33}, \quad \beta = 11.733°, \quad 90° - \beta = 78.3°$$

$$AC = \frac{CK}{\cos\beta} = \frac{433.33}{\cos 11.733°} = 442.58 \text{ mm}$$

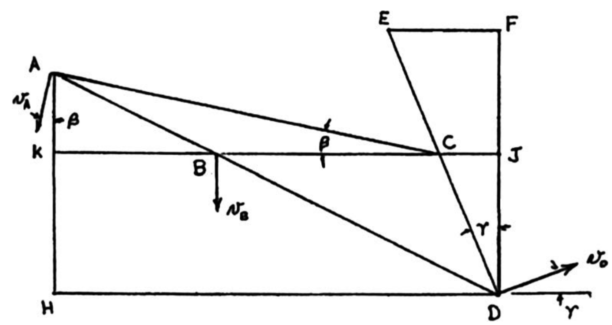
(b) $$v_A = (AC)\omega_{ABD} = (442.58)(1.57895) = 699 \text{ mm/s}$$ $\mathbf{v}_A = 699 \text{ mm/s} \measuredangle 78.3°$ ◀



PROBLEM 15.93

Arm *ABD* is connected by pins to a collar at *B* and to crank *DE*. Knowing that the angular velocity of crank *DE* is 1.2 rad/s counterclockwise, determine (*a*) the angular velocity of arm *ABD*, (*b*) the velocity of Point *A*.

SOLUTION



$$\tan\gamma = \frac{EF}{DF} = \frac{125}{300}, \quad \gamma = 22.620°, \quad ED = \frac{FD}{\cos\gamma} = \frac{300}{\cos\gamma} = 325 \text{ mm}$$

$$v_D = (ED)\omega_{DE} = (325)(1.2) = 390 \text{ mm/s} \quad \mathbf{v}_D = 390 \text{ mm/s} \measuredangle \gamma$$

$$\mathbf{v}_B = v_B \downarrow$$

Locate the instantaneous center (Point *C*) of bar *ABD* by noting that velocity directions at Points *B* and *D* are known. Draw *BC* perpendicular to $\mathbf{v}_B$ and *DC* perpendicular to $\mathbf{v}_D$.

$$CJ = (DJ)\tan\gamma = (160)\left(\frac{125}{300}\right) = 66.667 \text{ mm}, \quad CD = \frac{DJ}{\cos\gamma} = \frac{160}{\cos\gamma} = 173.33 \text{ mm}$$

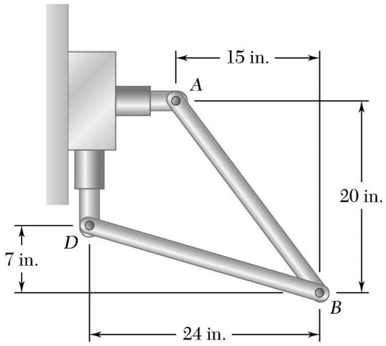
(*a*) $$\omega_{ABD} = \frac{v_D}{CD} = \frac{390}{173.33} = 2.25 \text{ rad/s} \circlearrowright$$ $\omega_{ABD} = 2.25 \text{ rad/s} \circlearrowright$ ◀

$$CK = KJ - CJ = 500 - 66.667 = 433.33 \text{ mm}$$

$$\tan\beta = \frac{AK}{CK} = \frac{90}{433.33} \qquad \beta = 11.733° \qquad 90° - \beta = 78.3°$$

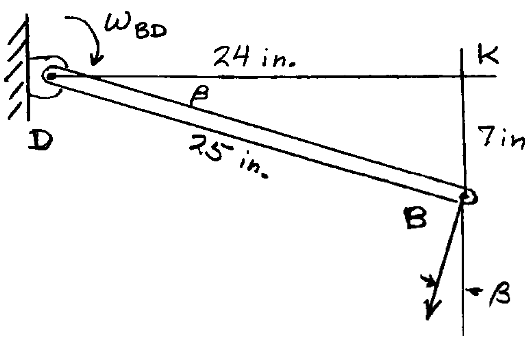
$$AC = \frac{CK}{\cos\beta} = \frac{433.33}{\cos\beta} = 442.58 \text{ mm}$$

(*b*) $$v_A = (AC)\omega_{ABD} = (442.58)(2.25) = 996 \text{ mm/s}$$ $\mathbf{v}_A = 996 \text{ mm/s} \nearrow 78.3°$ ◀

	**PROBLEM 15.94** Two links *AB* and *BD*, each 25 in. long, are connected at *B* and guided by hydraulic cylinders attached at *A* and *D*. Knowing that *D* is stationary and that the velocity of *A* is 30 in./s to the right, determine at the instant shown (*a*) the angular velocity of each link, (*b*) the velocity of *B*.

SOLUTION

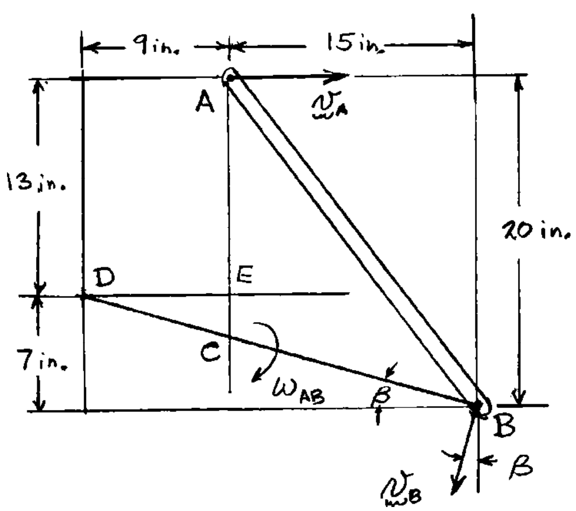
Link *DB*: Point *D* is stationary. Assume $\boldsymbol{\omega}_{BD} = \omega_{BD}$ ↻.



$v_B = (DB)\omega_{BD}$ $\mathbf{v}_B$ is perpendicular to *DB*.

$\tan\beta = \dfrac{7 \text{ in.}}{24 \text{ in.}}$ $\beta = 16.3°$ $90° - \beta = 73.7°$

Link *AB*: Draw the configuration. Locate the instantaneous center *C* of link *AB* by noting that the line *BC* is perpendicular to $\mathbf{v}_B$, i.e., along *DB*, and that *AC* is perependicular to $\mathbf{v}_A = v_A \longrightarrow$. ($v_A = 30$ in./s).



PROBLEM 15.94 (Continued)

$$\overline{AC} = \overline{AE} + \overline{EC} = 13 \text{ in.} + (7 \text{ in.})\frac{9 \text{ in.}}{24 \text{ in.}} = 15.625 \text{ in.}$$

$$\overline{BC} = \frac{15 \text{ in.}}{24 \text{ in.}}(25 \text{ in.}) = 15.625 \text{ in.}$$

$$\omega_{AB} = \frac{v_A}{(AC)} = \frac{30 \text{ in./s}}{15.625 \text{ in.}} = 1.92 \text{ rad/s}$$

$$v_B = (\overline{BC})\omega_{AB} = (15.625)(1.92) = 30 \text{ in./s}$$

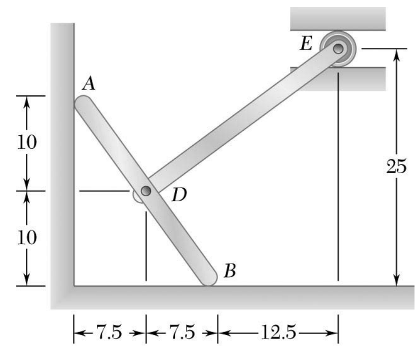
Returning to link *DB*,

$$\omega_{BD} = \frac{v_B}{(DB)} = \frac{30 \text{ in./s}}{25 \text{ in.}} = 1.20 \text{ rad/s}$$

(*a*) *Angular velocities*: $\boldsymbol{\omega}_{AB} = 1.920 \text{ rad/s}$ ↻ ◀

$\boldsymbol{\omega}_{BD} = 1.200 \text{ rad/s}$ ↻ ◀

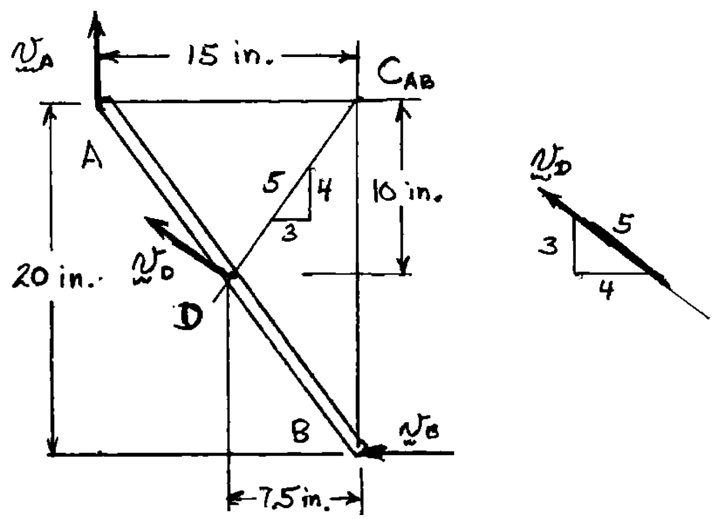
(*b*) *Velocity of Point B*: $\mathbf{v}_B = 30.0 \text{ in./s} \measuredangle 73.7°$ ◀



PROBLEM 15.95

Two 25-in. rods are pin-connected at D as shown. Knowing that B moves to the left with a constant velocity of 24 in./s, determine at the instant shown (*a*) the angular velocity of each rod, (*b*) the velocity of E.

SOLUTION



Rod AB: Draw lines perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$ to locate instantaneous center C_{AB}.

$$v_B = (BC_{AB})\omega_{AB}$$

$$24 \text{ in./s} = (20 \text{ in.})\omega_{AB}$$

$$\boldsymbol{\omega}_{AB} = 1.200 \text{ rad/s} \circlearrowright \blacktriangleleft$$

Velocity of D:

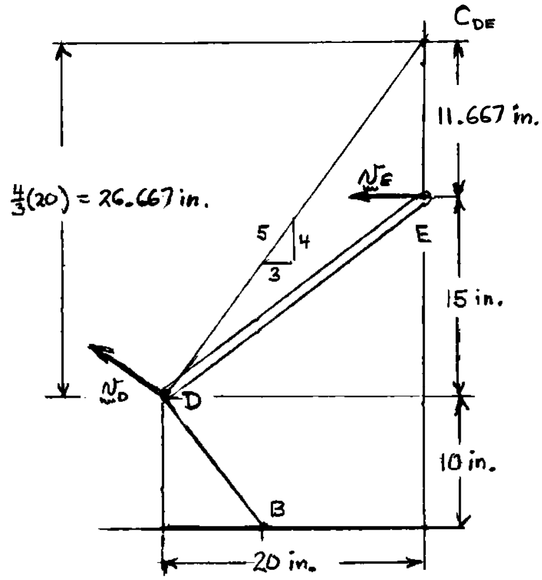
$$DC_{AB} = 12.5 \text{ in.}$$

$$v_D = (DC_{AB})\omega_{AB}$$

$$= (12.5 \text{ in.})(1.2 \text{ rad/s})$$

$$\mathbf{v}_D = 15 \text{ in./s} \nwarrow \tfrac{4}{3}$$

Rod *DE*:



Draw lines perpendicular to $\mathbf{v}_D$ and $\mathbf{v}_E$ to locate instantaneous center C_{DE}.

$$DC_{DE} = \sqrt{(20)^2 + (26.667)^2} = 33.333 \text{ in.}$$

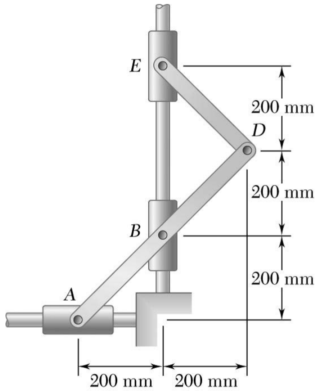
$$v_D = (DC_{DE})\omega_{DE}$$

(*a*) $15 \text{ in./s} = (33.333 \text{ in.})\omega_{DE}; \quad \omega_{DE} = 0.45 \text{ rad/s}$ $\boldsymbol{\omega}_{DE} = 0.450 \text{ rad/s}$ ↻ ◀

$$v_E = (EC_{DE})\omega_{DE} = (11.667 \text{ in.})(0.45 \text{ rad/s})$$

(*b*) $v_E = 5.25 \text{ in./s}$ $\mathbf{v}_E = 5.25 \text{ in./s}$ ← ◀

PROBLEM 15.96



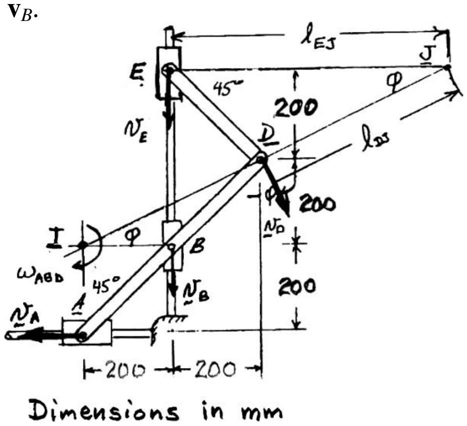
Two rods *ABD* and *DE* are connected to three collars as shown. Knowing that the angular velocity of *ABD* is 5 rad/s clockwise, determine at the instant shown (*a*) the angular velocity of *DE*, (*b*) the velocity of collar *E*.

SOLUTION

$$\omega_{ABC} = 5 \text{ rad/s} \circlearrowright \qquad \mathbf{v}_A = v_A \longleftarrow$$

$$\mathbf{v}_B = v_B \downarrow \qquad \mathbf{v}_E = v_E \downarrow$$

Locate Point *I*, the instantaneous center of rod *ABD* by drawing *IA* perepENDicular to $\mathbf{v}_A$ and *IB* perpendicular to $\mathbf{v}_B$.



$$\tan\phi = \frac{200}{400} \qquad \phi = 26.565°$$

$$I_{ID} = \frac{400}{\cos\phi} = 447.21 \text{ mm}$$

$$v_D = \omega_{ABD} l_{ID} = (5)(447.21 \text{ mm})$$

$$v_D = 2236.1 \text{ mm/s} \searrow \phi$$

Locate Point *J*, the instantaneous center of rod *DE* by drawing *JD* perpendicular to $\mathbf{v}_D$ and *JE* perpendicular to $\mathbf{v}_E$.

$$l_{JD} = \frac{400}{\cos\phi} = 447.21 \text{ mm}$$

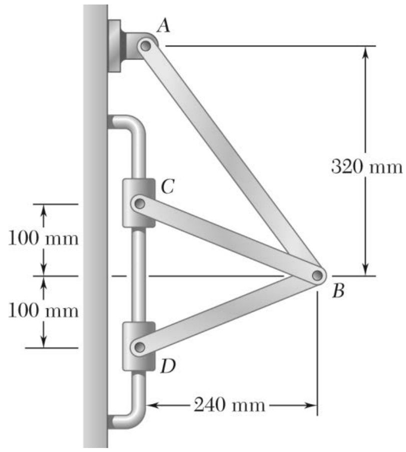
$$\omega_{DE} = \frac{v_D}{l_{JD}} = \frac{2236.1 \text{ mm/s}}{447.21} = 5 \text{ rad/s}$$

(*a*) $\boldsymbol{\omega}_{DE} = 5.00 \text{ rad/s} \circlearrowleft$ ◀

$$l_{JE} = 200 + l_{JD}\cos\phi = 600 \text{ mm}$$

$$v_E = l_{JE}\,\omega_{DE} = (600)(5) = 3000 \text{ mm/s}$$

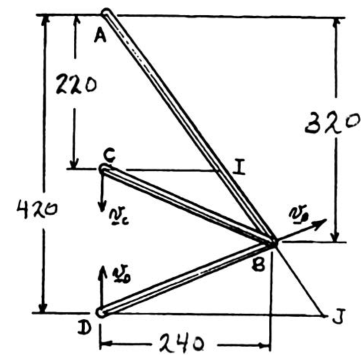
(*b*) $\mathbf{v}_E = 3.00 \text{ m/s} \downarrow$ ◀



PROBLEM 15.97

Two collars C and D move along the vertical rod shown. Knowing that the velocity of collar C is 660 mm/s downward, determine (*a*) the velocity of collar D, (*b*) the angular velocity of member AB.

SOLUTION



$$AB = 400 \text{ mm}$$

Instantaneous centers: at I for BC.

at J for BD.

Geometry.

$$IC = \left(\frac{240}{320}\right)(220) = 165 \text{ mm}$$

$$JD = \left(\frac{240}{320}\right)(420) = 315 \text{ mm}$$

$$AI = \left(\frac{220}{320}\right)(400) = 275 \text{ mm}$$

$$BI = AB - AI = 400 - 275 = 125 \text{ mm}$$

$$BJ = BI = 125 \text{ mm}$$

Member BC. $\mathbf{v}_C = 660 \text{ mm/s}\downarrow$

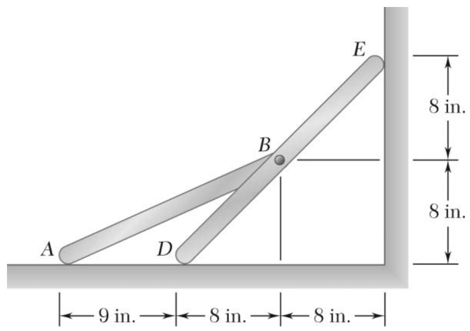
$$\omega_{BC} = \frac{v_C}{IC} = \frac{660}{165} = 4 \text{ rad/s}$$

$$v_B = (BI)\omega_{BC} = (125 \text{ mm})(4 \text{ rad/s}) = 500 \text{ mm/s}$$

Member BD. $\omega_{BD} = \frac{v_B}{BJ} = \frac{500 \text{ mm/s}}{125 \text{ mm}} = 4 \text{ rad/s}$

(*a*) $\mathbf{v}_D = (JD)\omega_{BD} = (315 \text{ mm})(4 \text{ rad/s})$ $\mathbf{v}_D = 1260 \text{ mm/s}\uparrow$ ◀

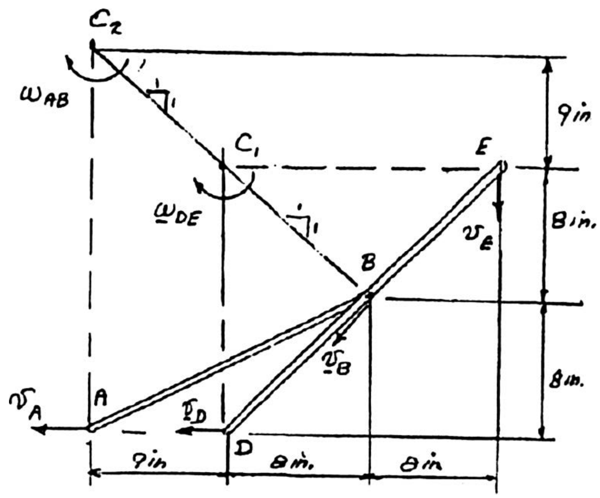
(*b*) $\omega_{AB} = \frac{v_B}{AB} = \frac{500 \text{ mm/s}}{400 \text{ mm}}$ $\boldsymbol{\omega}_{AB} = 1.250 \text{ rad/s}$ ↺ ◀



PROBLEM 15.98

Two rods AB and DE are connected as shown. Knowing that Point D moves to the left with a velocity of 40 in./s, determine (*a*) the angular velocity of each rod, (*b*) the velocity of Point A.

SOLUTION



We locate two instantaneous centers at intersections of lines drawn as follows:

C_1: For rod DE, draw lines perpendicular to $\mathbf{v}_D$ and $\mathbf{v}_E$.

C_2: For rod AB, draw lines perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$.

Geometry:

$$BC_1 = (8\text{ in.})\sqrt{2} = 8\sqrt{2}\text{ in.}$$
$$DC_1 = 16\text{ in.}$$
$$BC_2 = (9\text{ in.} + 8\text{ in.})\sqrt{2} = 17\sqrt{2}\text{ in.}$$
$$AC_2 = 25\text{ in.}$$

(*a*) Rod DE:

$$v_D = (DC_1)\omega_{DE}$$
$$40\text{ in./s} = (16\text{ in.})\omega_{DE}$$
$$\omega_{DE} = 2.5\text{ rad/s}$$

$\boldsymbol{\omega}_{DE} = 2.5\text{ rad/s}$ ↻ ◀

$$v_B = (BC)\omega_{DE}$$
$$= (8\sqrt{2}\text{ in.})(2.5\text{ rad/s})$$
$$\mathbf{v}_B = 20\sqrt{2}\text{ in./s} \nearrow 45°$$

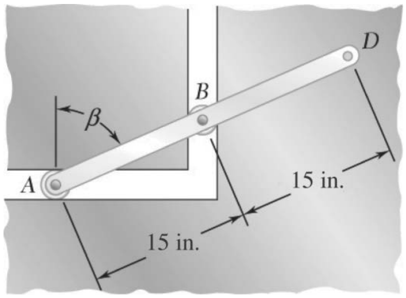
PROBLEM 15.98 (Continued)

Rod AB:
$$v_B = (BC_2)\omega_{AB}$$
$$20\sqrt{2}\text{ in./s} = (17\sqrt{2}\text{ in.})\omega_{AB}$$
$$\omega_{AB} = \frac{20}{17}\text{ rad/s} = 1.1765\text{ rad/s}$$

$\boldsymbol{\omega}_{AB} = 1.177\text{ rad/s}$ ↻ ◀

(*b*)
$$v_A = (AC_2)\omega_{AB}$$
$$= (25\text{ in.})(1.1765\text{ rad/s})$$
$$v_A = 29.41\text{ in./s}$$

$\mathbf{v}_A = 29.4\text{ in./s} \leftarrow$ ◀

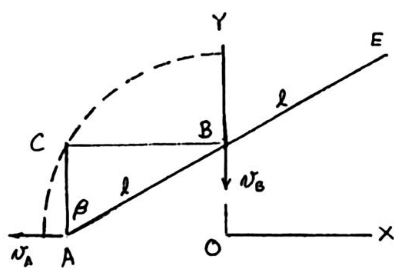


PROBLEM 15.99

Describe the space centrode and the body centrode of rod *ABD* of Problem 15.83. (*Hint:* The body centrode need not lie on a physical portion of the rod.)

PROBLEM 15.83 Rod *ABD* is guided by wheels at *A* and *B* that roll in horizontal and vertical tracks. Knowing that at the instant shown $\beta = 60°$ and the velocity of wheel *B* is 40 in./s downward, determine (*a*) the angular velocity of the rod, (*b*) the velocity of Point *D*.

SOLUTION



Draw *x* and *y* axes as shown with origin at the intersection of the two slots. These axes are fixed in space.

$$\mathbf{v}_A = v_A \longleftarrow, \qquad \mathbf{v}_B = v_B \downarrow$$

Locate the space centrode (Point *C*) by noting that velocity directions at Points *A* and *B* are known. Draw *AC* perpendicular to $\mathbf{v}_A$ and *BC* perpendicular to $\mathbf{v}_B$.

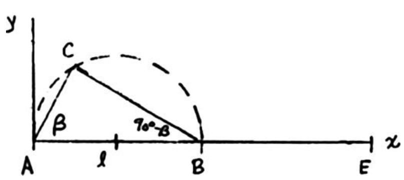
The coordinates of Point *C* are $x_C = -l\sin\beta$ and $y_C = l\cos\beta$

$$x_C^2 + y_C^2 = l^2 = (15\text{ in.})^2$$

The *space centrode* is a quarter circle of 15 in. radius centered at *O*. ◀

Redraw the figure, but use axes *x* and *y* that move with the body. Place origin at *A*.

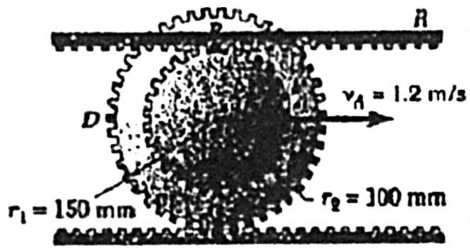
$$x_C = (AC)\cos\beta$$
$$= l\cos^2\beta = \frac{l}{2}(1+\cos 2\beta)$$
$$y_C = (AC)\sin\beta$$
$$= l\cos\beta\sin\beta = \frac{l}{2}\sin 2\beta$$



$$\left(x_C - \frac{l}{2}\right)^2 + y_C^2 = \left(\frac{l}{2}\right)^2 = (x_C - 7.5)^2 + y_C^2 = 7.5^2$$

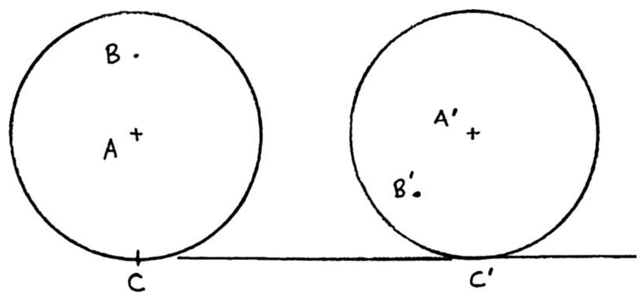
The *body centrode* is a semicircle of 7.5 in. radius centered midway between *A* and *B*. ◀

PROBLEM 15.100



Describe the space centrode and the body centrode of the gear of Sample Problem 15.2 as the gear rolls on the stationary horizontal rack.

SOLUTION



Let Points, *A, B,* and *C* move to *A*′, *B*′, and *C*′ as shown.

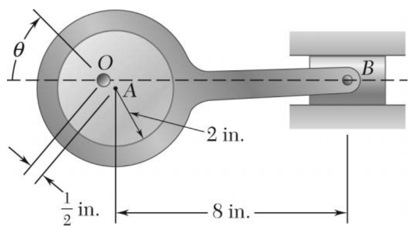
Since the instantaneous center always lies on the fixed lower rack, the space centrode is the lower rack.

space centrode: lower rack ◀

Since the point of contact of the gear with the lower rack is always a point on the circumference of the gear, the body centrode is the circumference of the gear.

body centrode: circumference of gear ◀

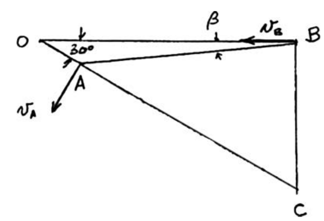
PROBLEM 15.101



Using the method of Section 15.7, solve Problem 15.60.

PROBLEM 15.60 In the eccentric shown, a disk of 2-in.-radius revolves about shaft O that is located 0.5 in. from the center A of the disk. The distance between the center A of the disk and the pin at B is 8 in. Knowing that the angular velocity of the disk is 900 rpm clockwise, determine the velocity of the block when $\theta = 30°$.

SOLUTION



$$(OA) = 0.5 \text{ in.} \quad \omega_{OA} = 900 \text{ rpm} = 30\pi \text{ rad/s}$$

$$\mathbf{v}_A = (OA)\omega_{OA} = (0.5)(30\pi) = 15\pi \text{ in./s}$$

$$\mathbf{v}_A = v_A \; \measuredangle \; 60°, \qquad \mathbf{v}_B = v_A \longleftarrow$$

Locate the instantaneous center (Point C) of bar BD by noting that velocity directions at Point B and A are known. Draw BC perpendicular to $\mathbf{v}_B$ and AC perpendicular to $\mathbf{v}_A$.

$$\sin\beta = \frac{(OA)\sin 30°}{AB} = \frac{0.5\sin 30°}{8}, \quad \beta = 1.79°$$

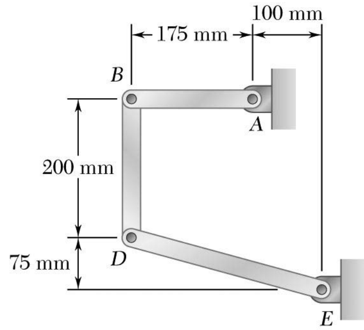
$$OB = (OA)\cos 30° + (AB)\cos\beta = 0.5\cos 30° + 8\cos\beta$$
$$= 8.4291 \text{ in.}$$

$$AC = \frac{OB}{\cos 30°} - OA = \frac{8.4291}{\cos 30°} - 0.5 = 9.2331 \text{ in.}$$

$$BC = (OB)\tan 30° = 4.8665 \text{ in.}$$

$$\omega_{AB} = \frac{v_A}{AC} = \frac{v_B}{BC}$$

$$v_B = \left(\frac{BC}{AC}\right)v_A = \frac{(4.8665)(15\pi)}{9.2331} = 24.84 \text{ in./s} \qquad \mathbf{v}_B = 24.8 \text{ in./s} \longleftarrow \blacktriangleleft$$

	**PROBLEM 15.102** Using the method of Section 15.7, solve Problem 15.64. **PROBLEM 15.64** In the position shown, bar *AB* has an angular velocity of 4 rad/s clockwise. Determine the angular velocity of bars *BD* and *DE*.

SOLUTION

Bar *AB*: (Rotation about *A*) $\boldsymbol{\omega}_{AB} = 4 \text{ rad/s} \circlearrowright$

$$\overline{AB} = 175 \text{ mm} \qquad v_B = \omega_{AB}(\overline{AB}) = (4)(175)$$

$$\mathbf{v}_B = 700 \text{ mm/s} \uparrow$$

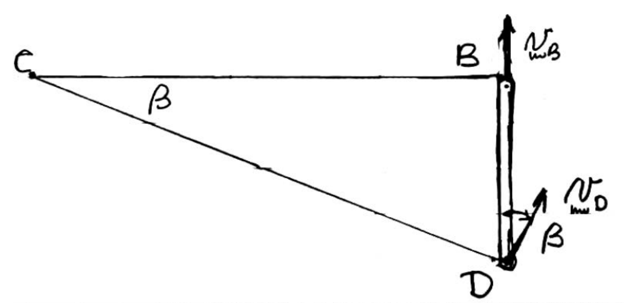
Bar *DE*: (Rotation about *E*) $\boldsymbol{\omega}_{DE} = \omega_{DE} \circlearrowright$

$$\overline{DE} = \sqrt{(275)^2 + (75)^2} = 285.04 \text{ mm}$$

$$\mathbf{v}_D = 285.04\omega_{DE} \measuredangle \beta$$

$$\tan\beta = \frac{75 \text{ mm}}{275 \text{ mm}} = 0.27273$$

Bar *BD*: $\mathbf{v}_B = 700 \text{ mm/s} \uparrow, \quad \mathbf{v}_D = 285.04\omega_{DE} \measuredangle \beta$



Locate the instantaneous center of bar *BD* by drawing line *BC* perpendicular to $\mathbf{v}_B$ and line *DC* perpendicular to $\mathbf{v}_D$.

$$\overline{BD} = 200 \text{ mm}$$

$$\overline{CB} = \frac{\overline{BD}}{\tan\beta} = \frac{(200)(275)}{75} = 733.3 \text{ mm}$$

$$\overline{CD} = \frac{\overline{BD}}{\sin\beta} = \frac{(200)(285.04)}{75} = 760.11 \text{ mm}$$

PROBLEM 15.102 (Continued)

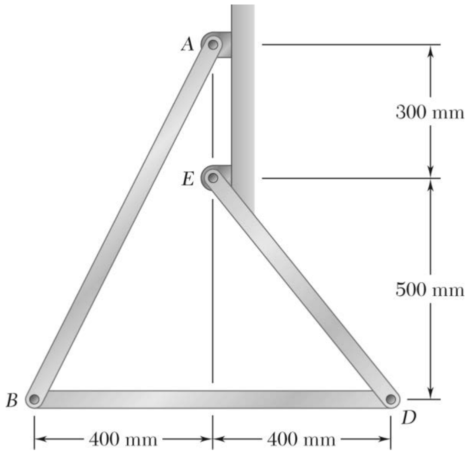
$$\omega_{BD} = \frac{v_B}{CB} = \frac{700 \text{ mm/s}}{733.33 \text{ mm}} = 0.95455 \text{ rad/s}$$

$$v_D = \omega_{BD}(CD) = (0.95455 \text{ rad/s})(760.11 \text{ mm}) = 725.56 \text{ mm/s}$$

$$\omega_{DE} = \frac{v_D}{285.04} = \frac{725.56}{285.04} = 2.5455 \text{ rad/s}$$

Angular velocities: $\boldsymbol{\omega}_{BD} = 0.955 \text{ rad/s}$ ↻ ◀

$\boldsymbol{\omega}_{DE} = 2.55 \text{ rad/s}$ ↺ ◀

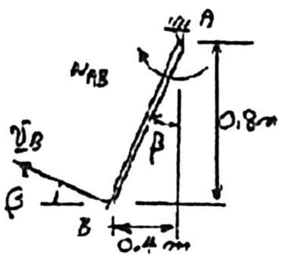


PROBLEM 15.103

Using the method of Section 15.7, solve Problem 15.65.

PROBLEM 15.65 In the position shown, bar *AB* has an angular velocity of 4 rad/s clockwise. Determine the angular velocity of bars *BD* and *DE*.

SOLUTION

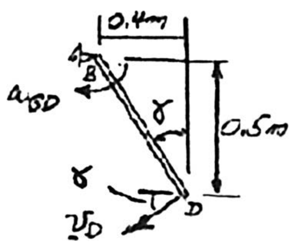


Bar *AB*: $\beta = \tan^{-1}\dfrac{0.4\text{ m}}{0.8\text{ m}} = 26.56°$

$$AB = \frac{0.8\text{ m}}{\cos\beta} = 0.8944\text{ m}$$

$$v_B = (AB)\omega_{AB} = (0.8944\text{ m})(4\text{ rad/s})$$

$$\mathbf{v}_B = 3.578\text{ m/s} \searrow 26.56°$$

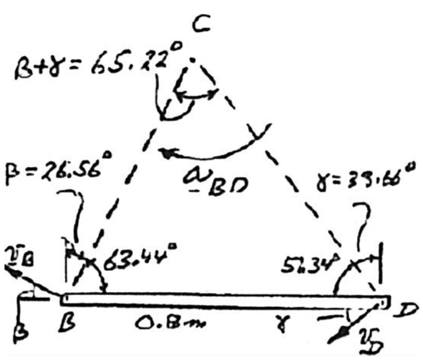


Bar *DE*: $\gamma = \tan^{-1}\dfrac{0.4\text{ m}}{0.5\text{ m}} = 38.66°$

$$DE = \frac{0.5\text{ m}}{\cos\gamma} = 0.6403\text{ m}$$

$$v_D = (DE)\omega_{DE}$$

$$\mathbf{v}_D = (0.6403\text{ m})\omega_{DE} \swarrow 38.66° \qquad (1)$$



Bar *BD*: Locate instantaneous center at intersection of lines drawn perpendicular to $\mathbf{v}_B$ and $\mathbf{v}_D$.

Law of sines. $\dfrac{BC}{\sin 51.34°} = \dfrac{CD}{\sin 63.44°} = \dfrac{0.8\text{ m}}{\sin 65.22°}$

$$BC = 0.688\text{ m}$$

$$CD = 0.7881\text{ m}$$

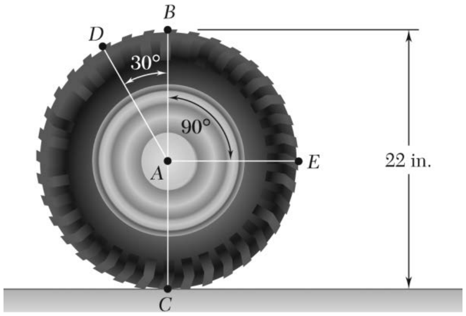
$$v_B = (BC)\omega_{BD}$$

$$3.578\text{ m/s} = (0.688\text{ m})\omega_{BD}; \qquad \boldsymbol{\omega}_{BD} = 5.2\text{ rad/s} \circlearrowright \blacktriangleleft$$

$$v_D = (CD)\omega_{BD} = (0.7881\text{ m})(5.2\text{ m/s})$$

$$= 4.098\text{ m/s}$$

Eq. (1): $4.098\text{ m/s} = (0.6403\text{ m})\omega_{DE}; \qquad \boldsymbol{\omega}_{DE} = 6.4\text{ rad/s} \circlearrowright \blacktriangleleft$



PROBLEM 15.104

Using the method of section 15.7, solve Problem 15.38.

PROBLEM 15.38 An automobile travels to the right at a constant speed of 48 mi/h. If the diameter of a wheel is 22 in., determine the velocities of Points *B*, *C*, *D*, and *E* on the rim of the wheel.

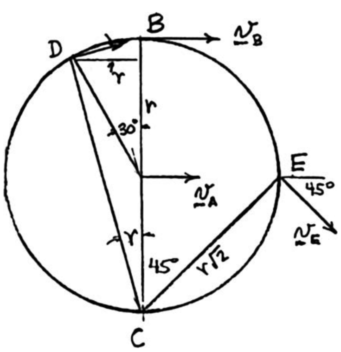
SOLUTION

$$v_A = 48 \text{ mi/h} = 70.4 \text{ ft/s}$$

$$\mathbf{v}_C = 0 \blacktriangleleft$$

$$d = 22 \text{ in.}, \quad r = \frac{1}{2}d = 11 \text{ in.} = 0.91667 \text{ ft}$$

Point *C* is the instantaneous center.



$$\omega = \frac{v_A}{r} = \frac{70.4}{9.1667} = 76.8 \text{ rad/s}$$

$$CB = 2r = 1.8333 \text{ ft}$$

$$v_B = (CB)\omega = (1.8333)(76.8) = 140.8 \text{ ft/s}$$

$$\mathbf{v}_B = 140.8 \text{ ft/s} \longrightarrow \blacktriangleleft$$

$$\gamma = \frac{1}{2}(30°) = 15°$$

$$CD = 2r\cos 15° = (2)(0.91667)\cos 15° = 1.7709 \text{ ft}$$

$$v_D = (CD)\omega = (1.7709)(76.8) = 136.0 \text{ ft/s}$$

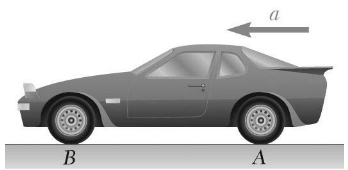
$$\mathbf{v}_D = 136.0 \text{ ft/s} \measuredangle 15.0° \blacktriangleleft$$

$$CE = r\sqrt{2} = 0.91667\sqrt{2} = 1.2964 \text{ ft}$$

$$v_E = (CE)\omega = (1.2964)(76.8) = 99.56 \text{ ft/s}$$

$$\mathbf{v}_E = 99.6 \text{ ft/s} \searrow 45.0° \blacktriangleleft$$

PROBLEM 15.CQ7



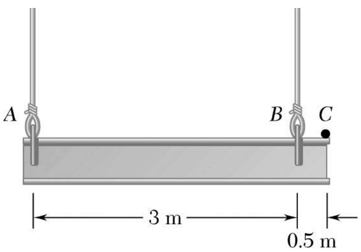
A rear wheel drive car starts from rest and accelerates to the left so that the tires do not slip on the road. What is the direction of the acceleration of the point on the tire in contact with the road, that is, Point *A*?

(*a*) ← (*b*) ↖ (*c*) ↑ (*d*) ↓ (*e*) ↙

SOLUTION

The tangential acceleration will be zero since the tires do not slip, but there will be an acceleration component perpendicular to the ground.

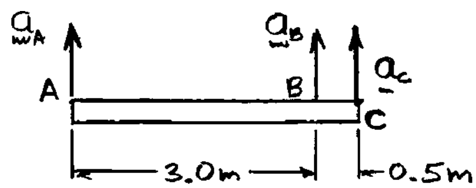
Answer: (*c*) ◀



PROBLEM 15.105

A 3.5-m steel beam is lowered by means of two cables unwinding at the same speed from overhead cranes. As the beam approaches the ground, the crane operators apply brakes to slow down the unwinding motion. At the instant considered, the deceleration of the cable attached at *A* is 4 m/s^2, while that of the cable at *B* is 1.5 m/s^2. Determine (*a*) the angular acceleration of the beam, (*b*) the acceleration of Point *C*.

SOLUTION



$$\mathbf{a}_A = 4 \text{ m/s}\uparrow$$

$$\mathbf{a}_B = 1.5 \text{ m/s}\uparrow$$

Assume $\omega = 0$.

(*a*) Angular acceleration. $\boldsymbol{\alpha} = \alpha\mathbf{k}$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$1.5\mathbf{j} = 4\mathbf{j} + \alpha\mathbf{k} \times (3\mathbf{i}) = 4\mathbf{j} + 3\alpha\mathbf{j}$$

$$\alpha = \frac{1.5 - 4}{3} = -0.83333$$

$$\boldsymbol{\alpha} = -0.833\mathbf{k}$$

$$\boldsymbol{\alpha} = 0.833 \text{ rad/s}^2 \circlearrowright$$

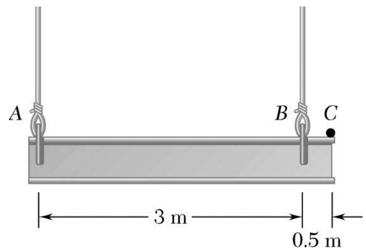
(*b*) Acceleration of Point *C*. Because the cables are unwinding at the same speed, $\omega = 0$

$$\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A} = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{C/A}$$

$$= 4\mathbf{j} + (-0.83333\mathbf{k} \times 3.5\mathbf{i})$$

$$= 4\mathbf{j} - 2.9167\mathbf{j} = 1.0833\mathbf{j}$$

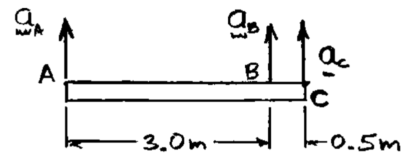
$$\mathbf{a}_C = 1.083 \text{ m/s}^2 \uparrow \blacktriangleleft$$



PROBLEM 15.106

The acceleration of Point C is $0.3\ \text{m/s}^2$ downward and the angular acceleration of the beam is $0.8\ \text{rad/s}^2$ clockwise. Knowing that the angular velocity of the beam is zero at the instant considered, determine the acceleration of each cable.

SOLUTION



$$\omega = 0$$
$$\boldsymbol{\alpha} = (-0.8\ \text{rad/s})\mathbf{k}$$
$$\mathbf{a}_C = 0.3\ \text{m/s}\downarrow$$

Acceleration of cable A.

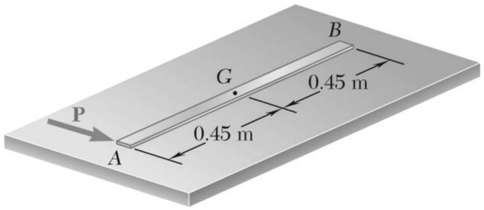
$$\mathbf{a}_A = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{A/C}$$
$$= -0.3\mathbf{j} + [-0.8\mathbf{k} \times (-3.5\mathbf{i})]$$
$$= -0.3\mathbf{j} + 2.8\mathbf{j} = (2.5\ \text{m/s}^2)\mathbf{j}\uparrow$$

$$\mathbf{a}_A = 2.50\ \text{m/s}^2\uparrow \blacktriangleleft$$

Acceleration of cable B.

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{B/C}$$
$$= -0.3\mathbf{j} + [-0.8\mathbf{k} \times (-0.5\mathbf{i})]$$
$$= -0.3\mathbf{j} + 0.4\mathbf{j} = 0.1\mathbf{j} = (0.1\ \text{m/s}^2)\mathbf{j}$$

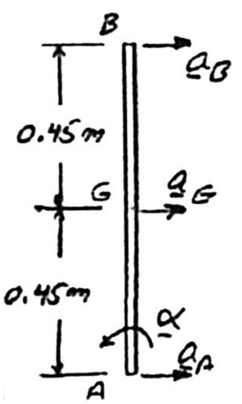
$$\mathbf{a}_B = 0.100\ \text{m/s}^2\uparrow$$



PROBLEM 15.107

A 900-mm rod rests on a horizontal table. A force **P** applied as shown produces the following accelerations: $\mathbf{a}_A = 3.6\text{ m/s}^2$ to the right, $\alpha = 6\text{ rad/s}^2$ counterclockwise as viewed from above. Determine the acceleration (*a*) of Point *G*, (*b*) of Point *B*.

SOLUTION



(*a*)

$$\mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A} = [a_A \longrightarrow] + [(AG)\alpha \longleftarrow]$$

$$\mathbf{a}_G = [3.6\text{ m/s}^2 \longrightarrow] + [(0.45\text{ m})(6\text{ rad/s}^2) \longleftarrow]$$

$$\mathbf{a}_G = [3.6\text{ m/s}^2 \longrightarrow] + [2.7\text{ m/s}^2 \longleftarrow]$$

$$\mathbf{a}_G = 0.9\text{ m/s}^2 \longrightarrow \blacktriangleleft$$

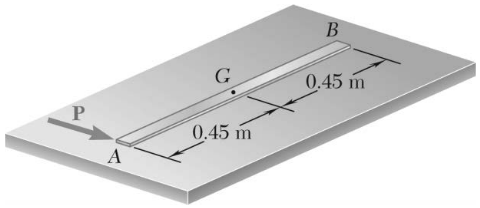
(*b*)

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = [a_A \longrightarrow] + [(AB)\alpha \longleftarrow]$$

$$\mathbf{a}_B = [3.6\text{ m/s}^2 \longrightarrow] + [(0.9\text{ m})(6\text{ rad/s}^2) \longleftarrow]$$

$$\mathbf{a}_B = [3.6\text{ m/s}^2 \longrightarrow] + [5.4\text{ m/s}^2 \longleftarrow]$$

$$\mathbf{a}_B = 1.8\text{ m/s}^2 \longleftarrow \blacktriangleleft$$

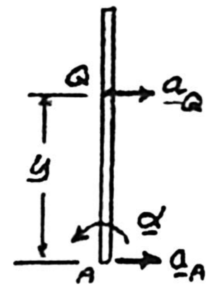


PROBLEM 15.108

In Problem 15.107, determine the point of the rod that (*a*) has no acceleration, (*b*) has an acceleration of 2.4 m/s^2 to the right.

SOLUTION

(*a*) For $a_Q = 0$:



$$\mathbf{a}_Q = \mathbf{a}_A + \mathbf{a}_{Q/A} = \mathbf{a}_A \longrightarrow + (AQ)\alpha \longleftarrow$$

$$0 = 3.6 \text{ m/s}^2 \longrightarrow + (y)(6 \text{ rad/s}^2) \longleftarrow$$

$$y = \frac{3.6 \text{ m/s}^2}{6 \text{ rad/s}} = 0.6 \text{ m}$$

$\mathbf{a} = 0$ at 0.6 m from A ◀

(*b*) For $\mathbf{a}_Q = 2.4 \text{ m/s}^2 \longrightarrow$:

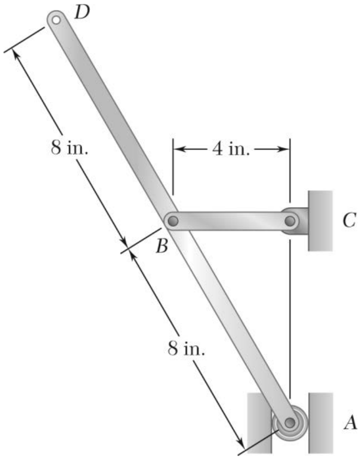
$$\mathbf{a}_Q = \mathbf{a}_A + \mathbf{a}_{Q/A} = [a_A \longrightarrow] + [(AQ)\alpha \longleftarrow]$$

$$2.4 \text{ m/s}^2 \longrightarrow = [3.6 \text{ m/s}^2 \longrightarrow] + [(y)(6 \text{ rad/s}^2) \longleftarrow]$$

$$1.2 \text{ m/s}^2 \longleftarrow = (y)(6 \text{ rad/s}^2) \longleftarrow$$

$$y = 0.2 \text{ m}$$

$\mathbf{a} = 2.4 \text{ m/s}^2 \longrightarrow$ at 0.2 m from A ◀



PROBLEM 15.109

Knowing that at the instant shown crank BC has a constant angular velocity of 45 rpm clockwise, determine the acceleration (*a*) of Point A, (*b*) of Point D.

SOLUTION

Geometry. Let β be angle BAC.

$$\sin\beta = \frac{4 \text{ in.}}{8 \text{ in.}} \quad \beta = 30°$$

Velocity analysis. $\boldsymbol{\omega}_{BC} = 45 \text{ rpm} \circlearrowright = 4.7124 \text{ rad/s} \circlearrowright \qquad \mathbf{v}_A = v_A \uparrow$

$$v_B = (BC)\omega_{BC} = (4)(4.7124) = 18.8496 \text{ in./s} \qquad \mathbf{v}_B = 18.8496 \text{ in./s} \uparrow$$

$\mathbf{v}_A$ and $\mathbf{v}_B$ are parallel; hence, the instantaneous center of rotation of rod AD lies at infinity.

$$\boldsymbol{\omega}_{AD} = 0 \quad \mathbf{v}_A = \mathbf{v}_B = 18.8496 \text{ in./s} \uparrow$$

Acceleration analysis. $\alpha_{BC} = 0$

Crank BC: $(a_B)_t = (BC)\alpha = 0$

$$(a_B)_n = (BC)\omega_{BC}^2 = (4)(4.7124)^2$$

$$\mathbf{a}_B = 88.827 \text{ in./s}^2 \longrightarrow$$

Rod ABD: $\boldsymbol{\alpha}_{AD} = \alpha_{AD} \circlearrowleft \quad \mathbf{a}_A = a_A \uparrow$

$$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_t + (\mathbf{a}_{A/B})_A$$

$$[a_A \uparrow] = [88.827 \longrightarrow] + [8\alpha_{AD} \measuredangle 30°] + [8\omega_{AD}^2 \searrow 60°]$$

Resolve into components.

$$\xrightarrow{+}: \quad 0 = 88.827 + 8\alpha_{AD}\cos 30° + 0 \qquad \alpha_{AD} = -12.821 \text{ rad/s}^2$$

(*a*) $+\uparrow: \quad a_A = 8\alpha_{AD}\sin 30° = (8)(-12.821)\sin 30° = -51.284 \text{ in./s}^2$

$$\mathbf{a}_A = 51.3 \text{ in./s}^2 \downarrow \blacktriangleleft$$

PROBLEM 15.109 (Continued)

(*b*)

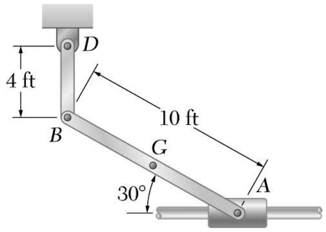
$$\mathbf{a}_D = \mathbf{a}_B + (\mathbf{a}_{D/B})_t + (\mathbf{a}_{D/B})_t$$

$$= [88.827 \longrightarrow] + [8\alpha_{BD} \swarrow 30°] + [8\omega_{BD}^2 [\searrow 60°]$$

$$= [88.827 \longrightarrow] + [(8)(-12.821) \swarrow 30°] + 0$$

$$= [88.827 \longrightarrow] + [102.568 \measuredangle 30°] = [177.653 \longrightarrow + 51.284 \uparrow]$$

$$\mathbf{a}_D = 184.9 \text{ in./s}^2 \measuredangle 16.1° \blacktriangleleft$$



PROBLEM 15.110

End A of rod AB moves to the right with a constant velocity of 6 ft/s. For the position shown, determine (*a*) the angular acceleration of rod AB, (*b*) the acceleration of the midpoint G of rod AB.

SOLUTION

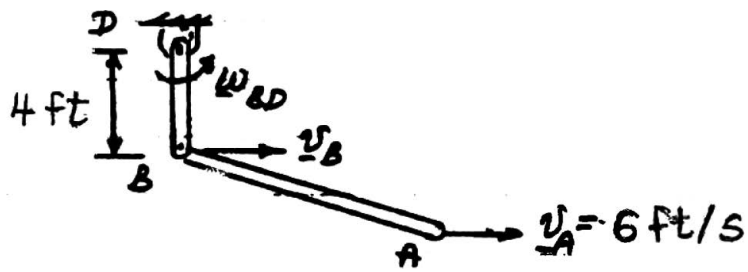
Use units of ft and seconds.

Geometry and unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1 \uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

$$\mathbf{r}_{B/A} = -(10\cos 30°)\mathbf{i} + (10\sin 30°)\mathbf{j} \qquad \mathbf{r}_{B/D} = -4\mathbf{j}$$

Velocity analysis.

Rod AB: $\mathbf{v}_A = 6 \text{ ft/s} \longrightarrow, \quad \mathbf{v}_B = v_B \longrightarrow$



Since $\mathbf{v}_A$ and $\mathbf{v}_B$ are parallel, the instantaneous center lies at infinity, so $\omega_{AB} = 0$ and $\mathbf{v}_B = \mathbf{v}_A$.

Acceleration analysis.

Rod AB: $\mathbf{a}_A = 0$ since $\mathbf{v}_A$ is constant. $\qquad \boldsymbol{\alpha}_{AB} = \alpha_{AB} \circlearrowleft = \alpha_{AB}\mathbf{k}$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_A + \mathbf{a}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$
$$= 0 + \alpha_{AB}\mathbf{k} \times (-10\cos 30°\mathbf{i} + 5\mathbf{j}) - 0$$
$$\mathbf{a}_B = -(10\cos 30°)\alpha_{AB}\mathbf{j} - 5\alpha_{AB}\mathbf{j} \tag{1}$$

Rod BD: $\mathbf{a}_D = 0, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k}$

$$\mathbf{a}_B = \mathbf{a}_D + \mathbf{a}_{B/D} = 0 + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{B/D} - \omega_{BD}^2 \mathbf{r}_{B/D}$$
$$= \alpha_{BD}\mathbf{k} \times (-4\mathbf{j}) - (1.5)^2(-4\mathbf{j})$$
$$= 4\alpha_{BD}\mathbf{i} + 9\mathbf{j} \tag{2}$$

Equating the coefficients of **j** in the expressions (1) and (2) for $\mathbf{a}_B$.

$$-(10\cos 30°)\alpha_{AB} = 9 \qquad \alpha_{AB} = -1.0392$$

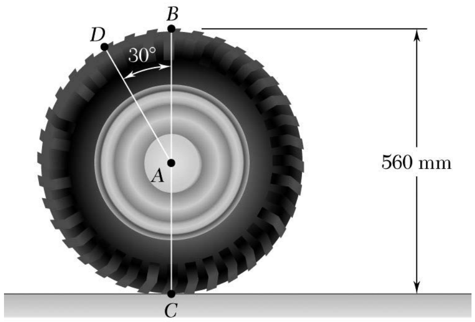
PROBLEM 15.110 (Continued)

(*a*) *Angular acceleration of rod AB*: $\boldsymbol{\alpha}_{AB} = 1.039\text{ rad/s}^2 \circlearrowright$ ◀

(*b*) *Acceleration of midpoint G of rod AB*.

$$\mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A} = \mathbf{a}_A + \boldsymbol{\alpha}_{AB}\mathbf{k} \times \mathbf{r}_{G/A} - \omega_{AB}^2 \mathbf{r}_{G/A}$$
$$= 0 - 1.0392\mathbf{k} \times (-5\cos 30°\mathbf{i} + 5\sin 30°\mathbf{j})$$

$$\mathbf{a}_G = (2.60\text{ ft/s}^2)\mathbf{i} + (4.50\text{ ft/s}^2)\mathbf{j} = 5.20\text{ ft/s}^2 \measuredangle 60°$$ ◀



PROBLEM 15.111

An automobile travels to the left at a constant speed of 72 km/h. Knowing that the diameter of the wheel is 560 mm, determine the acceleration (*a*) of Point *B*, (*b*) of Point *C*, (*c*) of Point *D*.

SOLUTION

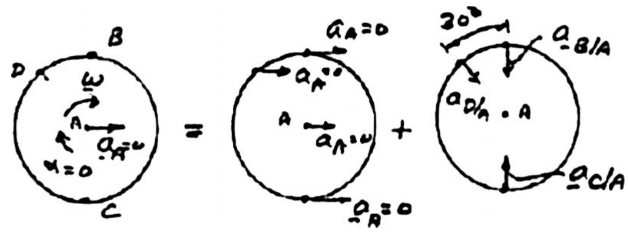
$$\mathbf{v}_A = 72\text{ km/h}\cdot\frac{\text{h}}{3600\text{ s}}\cdot\frac{1000\text{ m}}{\text{km}} = 20\text{ m/s} \longrightarrow$$

Rolling with no sliding, instantaneous center is at *C*.

$$v_A = (AC)\omega;\quad 20\text{ m/s} = (0.28\text{ m})\omega$$

$$\boldsymbol{\omega} = 71.429\text{ rad/s}\ \circlearrowright$$

Acceleration.



Plane motion = Trans. with *A* + Rotation about *A*

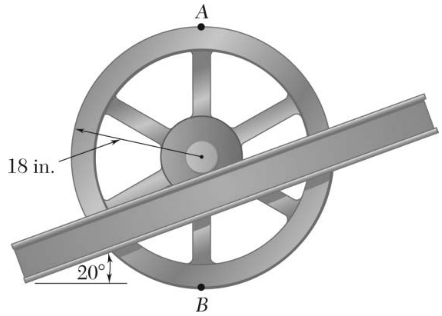
$$a_{B/A} = a_{C/A} = a_{D/A} = r\omega^2 = (0.280\text{ m})(71.429\text{ rad/s})^2 = 1428.6\text{ m/s}^2$$

(*a*) $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = 0 + 1428.6\text{ m/s}^2\downarrow$ $\qquad \mathbf{a}_B = 1430\text{ m/s}^2\downarrow$ ◀

(*b*) $\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A} = 0 + 1428.6\text{ m/s}^2\uparrow$ $\qquad \mathbf{a}_C = 1430\text{ m/s}^2\uparrow$ ◀

(*c*) $\mathbf{a}_D = \mathbf{a}_A + \mathbf{a}_{D/A} = 0 + 1428.6\text{ m/s}^2\ \searrow 60^\circ$

$$\mathbf{a}_D = 1430\text{ m/s}^2\ \searrow 60^\circ$$ ◀



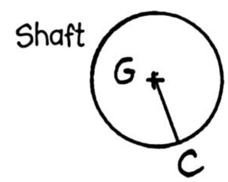
PROBLEM 15.112

The 18-in.-radius flywheel is rigidly attached to a 1.5-in.-radius shaft that can roll along parallel rails. Knowing that at the instant shown the center of the shaft has a velocity of 1.2 in./s and an acceleration of 0.5 in./s^2, both directed down to the left, determine the acceleration (*a*) of Point *A*, (*b*) of Point *B*.

SOLUTION

Velocity analysis.

Let Point *G* be the center of the shaft and Point *C* be the point of contact with the rails. Point *C* is the instantaneous center of the wheel and shaft since that point does not slip on the rails.



$$\mathbf{v}_G = r\omega, \quad \omega = \frac{v_G}{r} = \frac{1.2}{1.5} = 0.8 \text{ rad/s} \circlearrowleft$$

Acceleration analysis.

Since the shaft does not slip on the rails,

$$\mathbf{a}_C = a_C \searrow 20°$$

Also, $\mathbf{a}_G = [0.5 \text{ in./s}^2 \swarrow 20°]$

$$\mathbf{a}_C = \mathbf{a}_G + (\mathbf{a}_{C/G})_t + (\mathbf{a}_{C/G})_n$$

$$[a_C \searrow 20°] = [0.5 \text{ in./s}^2 \swarrow 20°] + [1.5\alpha \nearrow 20°] + [1.5\omega^2 \nwarrow 20°]$$

Components $\swarrow$ 20°: $\quad 0.5 = -1.5\alpha \qquad \alpha = 0.33333 \text{ rad/s}^2 \circlearrowleft$

(*a*) *Acceleration of Point A.*

$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

$$= [0.5 \swarrow 20°] + [18\alpha \leftarrow] + [18\omega^2 \downarrow]$$

$$= [0.4698 \leftarrow] + [0.1710 \downarrow] + [6 \leftarrow] + [11.52 \downarrow]$$

$$= [6.4698 \leftarrow] + [11.670 \downarrow]$$

$\mathbf{a}_A = 13.35 \text{ in./s}^2 \swarrow 61.0°$ ◀

(*b*) *Acceleration of Point B.*

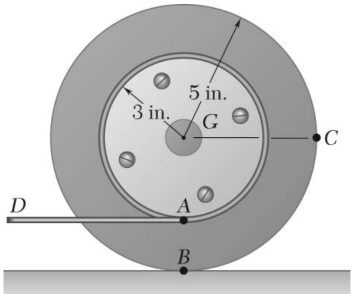
$$\mathbf{a}_B = \mathbf{a}_G + (\mathbf{a}_{B/G})_t + (\mathbf{a}_{B/G})_n$$

$$= [0.5 \swarrow 20°] + [18\alpha \rightarrow] + [18\omega^2 \uparrow]$$

$$= [0.4698 \leftarrow] + [0.1710 \downarrow] + [6 \rightarrow] + [11.52 \uparrow]$$

$$= [5.5302 \rightarrow] + [11.349 \uparrow]$$

$\mathbf{a}_B = 12.62 \text{ in./s}^2 \nearrow 64.0°$ ◀



PROBLEM 15.113

A 3-in.-radius drum is rigidly attached to a 5-in.-radius drum as shown. One of the drums rolls without sliding on the surface shown, and a cord is wound around the other drum. Knowing that at the instant shown end D of the cord has a velocity of 8 in./s and an acceleration of 30 in./s^2, both directed to the left, determine the accelerations of Points A, B, and C of the drums.

SOLUTION

Velocity analysis. $v_D = v_A = 8 \text{ in./s}$

Instantaneous center is at Point B. $v_A = (AB)\omega, \quad 8 = (5-3)\omega$

$$\omega = 4 \text{ rad/s} \circlearrowleft$$

Acceleration analysis. $\mathbf{a}_B = [a_B \uparrow]$ for no slipping.

$$\alpha = \alpha \circlearrowleft$$

$$\mathbf{a}_A = [30 \text{ in./s}^2 \longleftarrow] + [(a_A)_n \uparrow]$$

$$\mathbf{a}_G = [a_G \longleftarrow]$$

$$\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$$

$$[a_B \uparrow] = [30 \longleftarrow] + [(a_A)_n \uparrow] + [(5-3)\alpha \longrightarrow] + [5-3)\omega^2 \uparrow]$$

Components $\xrightarrow{+}$: $0 = -30 + 2\alpha \qquad \alpha = 15 \text{ rad/s}^2 \circlearrowleft$

$$\mathbf{a}_B = \mathbf{a}_G + (\mathbf{a}_{B/G})_t + (\mathbf{a}_{B/G})_n$$

$$[a_B \uparrow] = [a_G \longleftarrow] + [5\alpha \longrightarrow] + [5\omega^2 \uparrow]$$

Components $\xrightarrow{+}$: $0 = -a_G + 5\alpha \qquad a_G = 5\alpha = 75 \text{ in./s}^2$

$+\uparrow$: $a_B = (5)(4)^2 = 80 \text{ in./s}^2$ $\qquad \mathbf{a}_B = 80.0 \text{ in./s}^2 \uparrow$ ◀

$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

$$= [75 \longleftarrow] + [3\alpha \longrightarrow] + [3\omega^2 \uparrow]$$

$$= [75 \longleftarrow] + [45 \longrightarrow] + [48 \uparrow]$$

$$= [30 \text{ in./s}^2 \longleftarrow] + [48 \text{ in./s}^2 \uparrow]$$

$$\mathbf{a}_A = 56.6 \text{ in./s}^2 \measuredangle 58.0° \text{ ◀}$$

PROBLEM 15.113 (Continued)

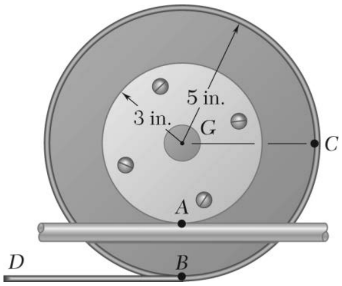
$$\mathbf{a}_C = \mathbf{a}_G + (\mathbf{a}_{C/G})_t + (\mathbf{a}_{C/G})_n$$

$$= [75 \longleftarrow] + [5\alpha \uparrow] + [5\omega^2 \longleftarrow]$$

$$= [75 \longleftarrow] + [75 \uparrow] + [80 \longleftarrow]$$

$$= [155 \text{ in./s}^2 \longleftarrow] + [75 \text{ in./s}^2 \uparrow]$$

$\mathbf{a}_C = 172.2 \text{ in./s}^2 \nwarrow 25.8°$ ◀



PROBLEM 15.114

A 3-in.-radius drum is rigidly attached to a 5-in.-radius drum as shown. One of the drums rolls without sliding on the surface shown, and a cord is wound around the other drum. Knowing that at the instant shown end D of the cord has a velocity of 8 in./s and an acceleration of 30 in./s^2, both directed to the left, determine the accelerations of Points A, B, and C of the drums.

SOLUTION

Velocity analysis. $\mathbf{v}_D = \mathbf{v}_B = 8 \text{ in./s}$

Instantaneous center is at Point A. $v_B = (AB)\omega, \quad 8 = (5-3)\omega$

$$\omega = 4 \text{ rad/s} \circlearrowright$$

Acceleration analysis. $\mathbf{a}_A = [a_A \uparrow]$ for no slipping. $\alpha = \alpha \circlearrowright$

$$\mathbf{a}_B = [30 \text{ in./s}^2 \leftarrow] + [(a_B)_n \uparrow]$$

$$\mathbf{a}_G = [a_G \rightarrow]$$

$$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_t + (\mathbf{a}_{A/B})_n$$

$$[a_A \uparrow] = [30 \leftarrow] + [(a_B)_n \uparrow] + [(5-3)\alpha \rightarrow] + [(5-3)]\omega^2 \downarrow]$$

Components $\xrightarrow{+}$: $0 = -30 + 2\alpha \qquad \alpha = 15 \text{ rad/s}^2 \circlearrowright$

$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

$$a_A \uparrow = [a_G \rightarrow] + [3\alpha \leftarrow] + [3\omega^2 \uparrow]$$

Components $\xrightarrow{+}$: $0 = a_G - 3\alpha \qquad a_G = 3\alpha = 45 \text{ in./s}^2$

$+\uparrow$: $a_A = 3\omega^2 = (3)(4)^2 = 48 \text{ in./s}^2$ $\qquad \mathbf{a}_A = 48.0 \text{ in./s}^2 \uparrow$ ◀

$$\mathbf{a}_B = \mathbf{a}_G + (\mathbf{a}_{B/G})_t + (\mathbf{a}_{B/G})_n$$

$$= [45 \rightarrow] + [5\alpha \leftarrow] + [5\omega^2 \uparrow]$$

$$= [45 \rightarrow] + [75 \leftarrow] + [80 \uparrow]$$

$$= [30 \text{ in./s}^2 \leftarrow] + [80 \text{ in./s}^2 \uparrow]$$

$$\mathbf{a}_B = 85.4 \text{ in./s}^2 \measuredangle 69.4° \text{ ◀}$$

PROBLEM 15.114 (Continued)

$$\mathbf{a}_C = \mathbf{a}_G + (\mathbf{a}_{C/G})_t + (\mathbf{a}_{C/G})_n$$

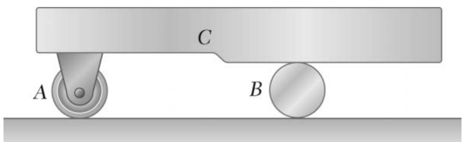
$$= [45 \longrightarrow] + [5\alpha \downarrow] + [5\omega^2 \longleftarrow]$$

$$= [45 \longrightarrow] + [75 \downarrow] + [80 \longleftarrow]$$

$$= [35 \text{ in./s}^2 \longleftarrow] + [75 \text{ in./s}^2 \downarrow]$$

$\mathbf{a}_C = 82.8 \text{ in./s}^2 \measuredangle 65.0°$ ◀

PROBLEM 15.115



A carriage C is supported by a caster A and a cylinder B, each of 50-mm diameter. Knowing that at the instant shown the carriage has an acceleration of 2.4 m/s^2 and a velocity of 1.5 m/s, both directed to the left, determine (*a*) the angular accelerations of the caster and of the cylinder, (*b*) the accelerations of the centers of the caster and of the cylinder.

SOLUTION

Rolling occurs at all surfaces of contact. Instantaneous centers are at points of contact with floor.

Caster: $r = 0.025 \text{ m}$

$$\mathbf{a}_A = \mathbf{a}_C = 2.4 \text{ m/s}^2 \longleftarrow$$

$$(a_D)_x = 0 \text{ (rolling with no sliding)}$$

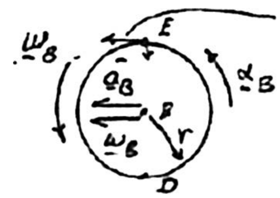
$$\mathbf{a}_A = \mathbf{a}_D + \mathbf{a}_{A/D}$$

$$[a_A \longleftarrow] = [(a_D)_x \longleftarrow] + [(a_D)_y \uparrow] + [rd_A \longleftarrow] + [r\omega_A^2 \downarrow]$$

$$\overset{+}{\longleftarrow} \quad a_A = 0 + r\alpha_A$$

$$2.4 \text{ m/s}^2 \longleftarrow = (0.025 \text{ m})\alpha_A \qquad \boldsymbol{\alpha}_A = 96 \text{ rad/s}^2 \circlearrowleft$$

Cylinder: $r = 0.025 \text{ m}$



$$(\mathbf{a}_E)_x = \mathbf{a}_C = 2.4 \text{ m/s}^2 \longleftarrow$$

$$(a_E)_x = 0$$

$$\mathbf{a}_E = \mathbf{a}_D + \mathbf{a}_{E/D}$$

$$[(a_E)_x \longleftarrow] + [(a_E)_y \downarrow] = [(a_D)_x \longleftarrow] + [(a_D)_y \downarrow] + [2r\alpha_B \longleftarrow] + 2r\omega_B^2 \downarrow]$$

$$\overset{+}{\longleftarrow}: \quad (a_E)_x = (a_D)_y + 2r\alpha_B$$

$$[2.4 \text{ m/s}^2 \longleftarrow] = 0 + 2(0.025 \text{ m})\alpha_B$$

$$\boldsymbol{\alpha}_B = 48 \text{ rad/s}^2 \circlearrowleft$$

$$[a_B \longleftarrow] = [(a_D)_x \longleftarrow] + [(a_D)_y \uparrow] + [r\alpha \longleftarrow] + [r\omega^2 \downarrow]$$

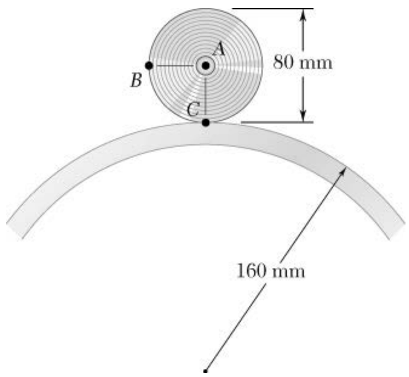
$$\overset{+}{\longleftarrow}: \quad a_B = 0 + r\alpha_B$$

$$a_B = (0.025 \text{ m})(48 \text{ rad/s}^2); \qquad \mathbf{a}_B = 1.2 \text{ m/s}^2 \longleftarrow$$

Answers:

(*a*) $\boldsymbol{\alpha}_A = 96.0 \text{ rad/s}^2 \circlearrowleft, \quad \mathbf{a}_A = 2.40 \text{ m/s}^2 \longleftarrow$ ◀

(*b*) $\boldsymbol{\alpha}_B = 48.0 \text{ rad/s}^2 \circlearrowleft, \quad \mathbf{a}_B = 1.200 \text{ m/s}^2 \longleftarrow$ ◀



PROBLEM 15.116

A wheel rolls without slipping on a fixed cylinder. Knowing that at the instant shown the angular velocity of the wheel is 10 rad/s clockwise and its angular acceleration is 30 rad/s^2 counterclockwise, determine the acceleration of (*a*) Point *A*, (*b*) Point *B*, (*c*) Point *C*.

SOLUTION

Velocity analysis. $r = 0.04$ m $\quad \boldsymbol{\omega} = 10$ rad/s ↻

Point *C* is the instantaneous center of the wheel.

$$\mathbf{v}_A = [(r\omega) \longrightarrow] = [(0.04)(10) \longrightarrow] = 0.4 \text{ m/s} \longrightarrow]$$

Acceleration analysis. $\alpha = 30$ rad/s^2 ↺

Point *A* moves on a circle of radius $\rho = R + r = 0.16 + 0.04 = 0.2$ m.

Since the wheel does not slip, $\mathbf{a}_C = a_C \uparrow$

$$\mathbf{a}_C = \mathbf{a}_A + (\mathbf{a}_{C/A})_t + (\mathbf{a}_{C/A})_n$$

$$[a_C \uparrow] = [(a_A)_t \longleftarrow] + \left[\frac{v_A^2}{\rho} \downarrow\right] + [r\alpha \longrightarrow] + [r\omega^2 \uparrow]$$

$$= [(a_A)_t \longleftarrow] + \left[\frac{(0.4)^2}{0.2} \downarrow\right] + [(0.04)(30) \longrightarrow] + [(0.04)(10)^2 \uparrow]$$

$$= [(a_A)_t \longleftarrow] + [0.8 \downarrow] + [1.2 \longrightarrow] + [4 \uparrow]$$

Components. $\xrightarrow{+}$: $\quad -(a_A)_t + 1.2 = 0 \qquad (a_A)_t = 1.2$ m/s^2

$+\uparrow$: $\quad a_C = -0.8 + 4.0 \qquad a_C = 3.2$ m/s^2

(*a*) *Acceleration of Point A.*

$$\mathbf{a}_A = [1.2 \text{ m/s}^2 \longleftarrow] + [0.8 \text{ m/s}^2 \downarrow] \qquad \mathbf{a}_A = 1.442 \text{ m/s}^2 \measuredangle\ 33.7° \blacktriangleleft$$

PROBLEM 15.116 (Continued)

(*b*) *Acceleration of Point B.* $\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$

$$\mathbf{a}_B = [1.2 \longleftarrow] + [0.8 \downarrow] + [r\alpha \downarrow] + [r\omega^2 \longrightarrow]$$

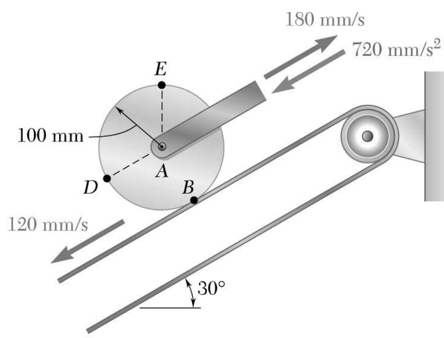
$$= [1.2 \longleftarrow] + [0.8 \downarrow] + [(0.04)(30) \downarrow] + [(0.04)(10)^2 \longrightarrow]$$

$$= [2.8 \text{ m/s}^2 \longrightarrow] + [2 \text{ m/s}^2 \downarrow]$$

$\mathbf{a}_B = 3.44 \text{ m/s}^2 \searrow 35.5°$ ◀

(*c*) *Acceleration of Point C.* $\mathbf{a}_C = a_C \uparrow$

$\mathbf{a}_C = 3.20 \text{ m/s}^2 \uparrow$ ◀



PROBLEM 15.117

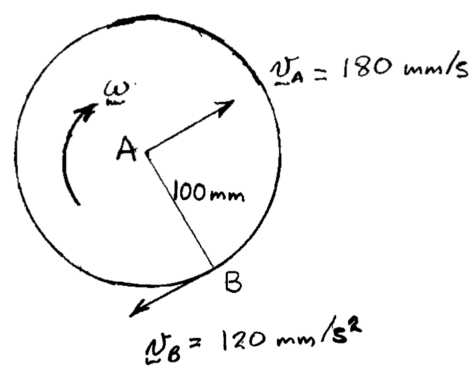
The 100 mm radius drum rolls without slipping on a portion of a belt which moves downward to the left with a constant velocity of 120 mm/s. Knowing that at a given instant the velocity and acceleration of the center A of the drum are as shown, determine the acceleration of Point D.

SOLUTION

Velocity analysis.

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$[180 \text{ mm/s} \nearrow] = [120 \text{ mm/s} \swarrow] + [(100 \text{ mm})\omega \nearrow]$$



Components $\nearrow$:

$$180 = -120 = 100\omega$$

$$\omega = 3 \text{ rad/s} \circlearrowright$$

Acceleration analysis.

Point A moves on a path parallel to the belt. The path is assumed to be straight.

$$\mathbf{a}_A = 720 \text{ mm/s}^2 \swarrow 30°$$

Since the drum rolls without slipping on the belt, the component of acceleration of Point B on the drum parallel to the belt is the same as the belt acceleration. Since the belt moves at constant velocity, this component of acceleration is zero. Thus

$$\mathbf{a}_B = a_B \nwarrow 60°$$

Let the angular acceleration of the drum be $\alpha \circlearrowleft$.

$$\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$$

$$[a_B \nwarrow] = [720 \swarrow] + [r\alpha \nearrow] + [r\omega^2 \nwarrow]$$

Components $\swarrow$: $\quad 0 = 720 - 100\alpha$

$$\alpha = 7.2 \text{ rad/s} \circlearrowleft$$

Acceleration of Point D.

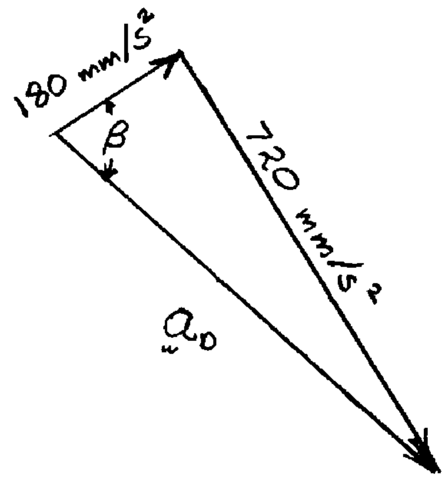
$$\mathbf{a}_D = \mathbf{a}_A + (\mathbf{a}_{D/A})_t + (\mathbf{a}_{D/A})_n$$

$$= [a_A \;\nearrow 30°] + [r\alpha \;\searrow 60°] + [r\omega^2 \;\measuredangle 30°]$$

$$= [720 \;\nearrow 30°] + [(100)(7.2) \;\searrow 60°] + [(100)(3)^2 \;\measuredangle 30°]$$

Components: $\measuredangle$ 30°: $\quad -720 + 900 = 180 \text{ mm/s}^2$

Components: $\searrow$ 60°: $\quad 720 \text{ mm/s}^2$



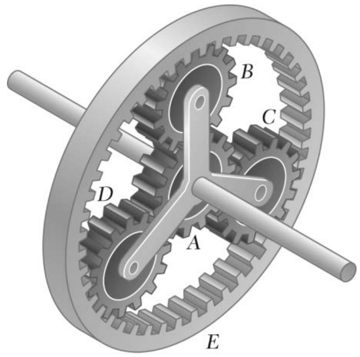
$$a_D = \sqrt{180^2 + 720^2} = 742.16 \text{ mm/s}^2$$

$$\tan\beta = \frac{720}{180} \qquad \beta = 76.0°$$

$$\beta - 30° = 46.0°$$

$\mathbf{a}_D = 742 \text{ mm/s}^2 \;\searrow 46.0°$ ◀

PROBLEM 15.118

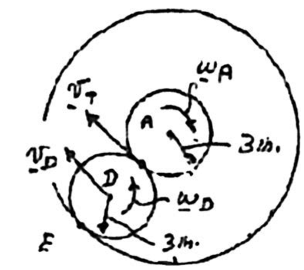


In the planetary gear system shown the radius of gears *A*, *B*, *C*, and *D* is 3 in. and the radius of the outer gear *E* is 9 in. Knowing that gear *A* has a constant angular velocity of 150 rpm clockwise and that the outer gear *E* is stationary, determine the magnitude of the acceleration of the tooth of gear *D* that is in contact with (*a*) gear *A*, (*b*) gear *E*.

SOLUTION

<u>Velocity</u>. $T =$ Tooth of gear *D* in contact with gear *A*

Gears: $v_T = r\omega_A = (3 \text{ in.})\omega_A$



Since $v_E = 0$, *E* is instantaneous center of gear *D*.

$$v_T = 2r\omega_D$$

$$(3 \text{ in.})\omega_A = 2(3 \text{ in.})\omega_D$$

$$\omega_D = \frac{1}{2}\omega_A$$

$$v_D = r\omega_D = (3 \text{ in.})\frac{1}{2}\omega_A = (1.5 \text{ in.})\omega_\text{A}$$

Spider:

$$v_D = (6 \text{ in.})\omega_S$$

$$(1.5 \text{ in.})\omega_\text{A} = (6 \text{ in.})\omega_S$$

$$\omega_S = \frac{1}{4}\omega_A$$

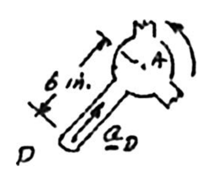
$$\boldsymbol{\omega}_A = 150 \text{ rpm} = 15.708 \text{ rad/s} \circlearrowright$$

$$\boldsymbol{\omega}_D = \frac{1}{2}\omega_A = 7.854 \text{ rad/s} \circlearrowleft$$

$$\boldsymbol{\omega}_S = \frac{1}{4}\omega_A = 3.927 \text{ rad/s} \circlearrowright$$

<u>Acceleration</u>.

Spider: $\omega_S = 3.927 \text{ rad/s}$

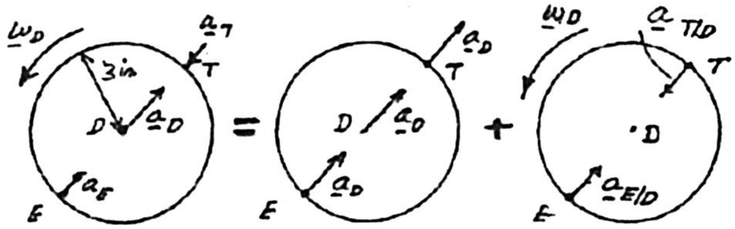


$$a_D = (AD)\omega_S^2 = (6 \text{ in.})(3.927 \text{ rad/s})^2$$

$$\mathbf{a}_D = 92.53 \text{ in./s}^2 \nearrow$$

PROBLEM 15.118 (Continued)

Gear D:



Plane motion = Trans. with D + Rotation about D

(a) Tooth T in contact with gear A.

$$\mathbf{a}_T = \mathbf{a}_D + \mathbf{a}_{T/D} = a_D + (DT)\omega_D^2$$

$$= 92.53 \text{ in./s}^2 \nearrow + (3 \text{ in.})(7.854 \text{ rad/s})^2 \swarrow$$

$$= 92.53 \text{ in./s}^2 \nearrow + 185.06 \text{ in./s}^2 \swarrow$$

$$\mathbf{a}_T = 92.53 \text{ in./s}^2 \swarrow$$

$a_T = 92.5 \text{ in./s}^2$ ◀

(b) Tooth E in contact with gear E.

$$\mathbf{a}_E = \mathbf{a}_D + \mathbf{a}_{E/D} = \mathbf{a}_D + (ED)\omega_D^2$$

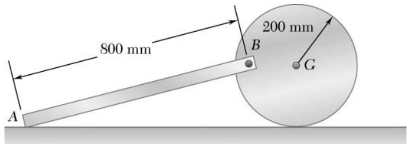
$$= 92.53 \text{ in./s}^2 \nearrow + (3 \text{ in.})(7.854 \text{ rad/s})^2 \nearrow$$

$$= 92.53 \text{ in./s}^2 \nearrow + 185.06 \text{ in./s}^2 \nearrow$$

$$\mathbf{a}_E = 277.6 \text{ in./s}^2 \nearrow$$

$a_E = 278 \text{ in./s}^2$ ◀

PROBLEM 15.119



The 200-mm-radius disk rolls without sliding on the surface shown. Knowing that the distance *BG* is 160 mm and that at the instant shown the disk has an angular velocity of 8 rad/s counterclockwise and an angular acceleration of 2 rad/s^2 clockwise, determine the acceleration of *A*.

SOLUTION

Units: m, m/s, m/s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow$, $\mathbf{j} = 1 \uparrow$, $\mathbf{k} = 1 \circlearrowleft$

Geometric analysis. Let *P* be the point where the disk contacts the flat surface.

$$\mathbf{r}_{G/A} = 0.200\mathbf{j} \qquad \mathbf{r}_{B/G} = -0.16\mathbf{i}$$

$$\mathbf{r}_{A/B} = -\sqrt{0.600}\mathbf{i} - 0.200\mathbf{j}$$

Velocity analysis. $\boldsymbol{\omega}_G = (8 \text{ rad/s})\mathbf{k}, \quad \mathbf{v}_P = 0, \ \mathbf{v}_A = v_A\mathbf{i}$

$$\mathbf{v}_B = \mathbf{v}_P + \mathbf{v}_{G/P} = \mathbf{v}_P + \boldsymbol{\omega}_G \times \mathbf{r}_{G/P}$$

$$= 0 + 8\mathbf{k} \times (-0.160\mathbf{i} + 0.200\mathbf{j}) = -1.6\mathbf{i} - 1.28\mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} = \mathbf{v}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

$$v_A\mathbf{i} = -1.6\mathbf{i} - 1.28\mathbf{j} + \omega_{AB}\mathbf{k} \times (-\sqrt{0.600}\mathbf{i} - 0.200\mathbf{j})$$

$$= -1.6\mathbf{i} - 1.28\mathbf{j} - 0.77460\omega_{AB}\mathbf{j} + 0.2\omega_{AB}\mathbf{i}$$

Resolve into components and transpose terms.

j: $\quad 0.77460\omega_{AB} = -1.28 \qquad \omega_{AB} = -1.6525 \text{ rad/s}$ ◀

Acceleration analysis: $\mathbf{a}_A = \mathbf{a}_A\mathbf{j}, \quad \mathbf{a}_G = -2 \text{ rad/s}^2\mathbf{k}$

$$\mathbf{a}_P = (\omega_G^2 r)\mathbf{j} = (8)^2(0.2)\mathbf{j} = (12.8 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_B = \mathbf{a}_P + \mathbf{a}_{B/P} = \mathbf{a}_P + \mathbf{a}_G \times \mathbf{r}_{B/P} - \omega_G^2\mathbf{r}_{B/P}$$

$$= 12.8\mathbf{j} + (-2\mathbf{k}) \times (-0.160\mathbf{i} + 0.200\mathbf{j}) - (8)^2(-0.160\mathbf{i} + 0.200\mathbf{j})$$

$$= 12.8\mathbf{j} + 0.32\mathbf{j} + 0.4\mathbf{i} + 10.24\mathbf{i} - 12.8\mathbf{j}$$

$$= 10.64\mathbf{i} + 0.32\mathbf{j}$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B} = \mathbf{a}_B + \alpha_{AB}\mathbf{k} \times \mathbf{r}_{A/B} - \omega_{AB}^2\mathbf{r}_{A/B}$$

$$= 10.64\mathbf{i} + 0.32\mathbf{j} + \alpha_{AB}\mathbf{k} \times (-\sqrt{0.600}\mathbf{i} - 0.200\mathbf{j}) - (1.6525)^2(-\sqrt{0.600}\mathbf{i} - 0.200\mathbf{j})$$

$$= 10.64\mathbf{i} + 0.32\mathbf{j} - 0.77460\alpha_{AB}\mathbf{j} + 0.2\alpha_{AB}\mathbf{i} + 2.115\mathbf{i} + 0.54615\mathbf{j}$$

$$a_A\mathbf{i} = 12.755\mathbf{i} + 0.86615\mathbf{j} + 0.2\alpha_{AB}\mathbf{i} - 0.77460\alpha_{AB}\mathbf{j}$$

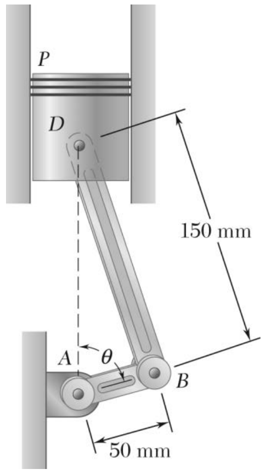
Resolve into components and transpose terms.

j: $0 = 0.86615 - 0.77460\alpha_{AB} \qquad \alpha_{AB} = 1.1182$

i: $a_A = 12.755 + 0.2\alpha_{AB} = 12.755 + (0.2)(1.1182) = 12.98$

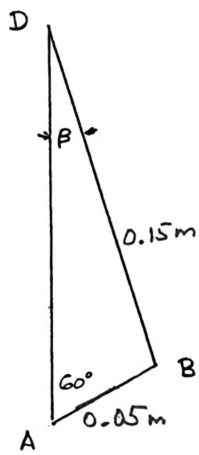
$$\mathbf{a}_A = (12.98 \text{ m/s}^2)\mathbf{i} = 12.98 \text{ m/s}^2 \longrightarrow \blacktriangleleft$$

PROBLEM 15.120



Knowing that crank AB rotates about Point A with a constant angular velocity of 900 rpm clockwise, determine the acceleration of the piston P when $\theta = 60°$.

SOLUTION



Law of sines.

$$\frac{\sin\beta}{0.05} = \frac{\sin 60°}{0.15} \qquad \beta = 16.779°$$

Velocity analysis.

$$\omega_{AB} = 900 \text{ rpm} = 30\pi \text{ rad/s} \;\circlearrowright$$

$$\mathbf{v}_B = 0.05\omega_{AB} = 1.5\pi \text{ m/s} \;\nwarrow 60°$$

$$\mathbf{v}_D = v_D \downarrow \qquad \omega_{BD} = \omega_{BD} \;\circlearrowleft$$

$$\mathbf{v}_{D/B} = 0.15\,\omega_{BD} \;\swarrow \beta$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

$$[v_D \downarrow] = [1.5\pi \;\nwarrow 60°] + [0.15\omega_{BD} \;\swarrow \beta]$$

Components $\xrightarrow{+}$:

$$0 = 1.5\pi\cos 60° - 0.15\omega_{BD}\cos\beta$$

$$\omega_{BD} = \frac{1.5\pi\cos 60°}{0.15\cos\beta} = 16.4065 \text{ rad/s} \;\circlearrowleft$$

Acceleration analysis.

$$\alpha_{AB} = 0$$

$$\mathbf{a}_B = 0.05\omega_{AB}^2 = (0.05)(30\pi)^2 = 444.13 \text{ m/s}^2 \;\swarrow 30°$$

$$\mathbf{a}_D = a_D \downarrow \qquad \alpha_{BD} = \alpha_{BD} \;\circlearrowright$$

$$\mathbf{a}_{D/B} = [0.15\alpha_{AB} \;\nearrow \beta] + [0.15\omega_{BD}^2 \;\searrow \beta]$$

$$= [0.15\alpha_{BD} \;\nearrow \beta] + [40.376 \;\searrow \beta]$$

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B}$$ Resolve into components.

PROBLEM 15.120 (Continued)

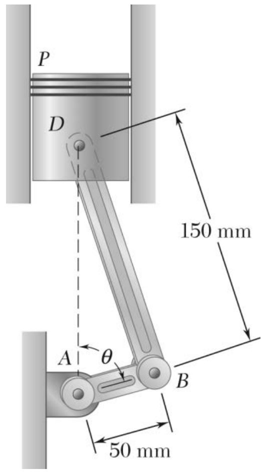
$\xrightarrow{+}$: $\quad 0 = -444.13\cos 30° + 0.15\alpha_{BD}\cos\beta + 40.376\sin\beta$

$\alpha_{BD} = 2597.0 \text{ rad/s}^2$

$+\downarrow$: $\quad a_D = 444.13\sin 30° - (0.15)(2597.0)\sin\beta + 40.376\cos\beta$

$= 148.27 \text{ m/s}^2 \qquad \mathbf{a}_P = \mathbf{a}_D$

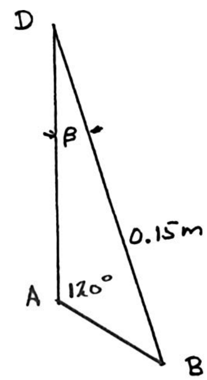
$\mathbf{a}_P = 148.3 \text{ m/s}^2 \downarrow$ ◀



PROBLEM 15.121

Knowing that crank AB rotates about Point A with a constant angular velocity of 900 rpm clockwise, determine the acceleration of the piston P when $\theta = 120°$.

SOLUTION



Law of sines. $\dfrac{\sin\beta}{0.05} = \dfrac{\sin 120°}{0.15}, \qquad \beta = 16.779°$

Velocity analysis. $\omega_{AB} = 900 \text{ rpm} = 30\pi \text{ rad/s} \circlearrowright$

$$\mathbf{v}_B = 0.05\omega_{AB} = 1.5\pi \text{ m/s} \measuredangle 60°$$

$$\mathbf{v}_D = v_D \downarrow \qquad \omega_{BD} = \omega_{BD} \circlearrowleft$$

$$\mathbf{v}_{D/B} = 0.15\omega_{BD} \measuredangle \beta$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

$$[v_D \downarrow] = [1.5\pi \measuredangle 60°] + [0.15\omega_{BD} \measuredangle \beta]$$

Components $\xrightarrow{+}$: $0 = -1.5\pi \cos 60° - 0.15\omega_{BD} \cos\beta$

$$\omega_{BD} = -\frac{1.5\pi \cos 60°}{0.15 \cos\beta} = 16.4065 \text{ rad/s} \circlearrowright$$

Acceleration analysis. $\alpha_{AB} = 0$

$$\mathbf{a}_B = 0.05\omega_{AB}^2 = (0.05)(30\pi)^2 = 444.13 \text{ m/s}^2 \measuredangle 30°$$

$$\mathbf{a}_D = a_D \downarrow \qquad \alpha_{BD} = \alpha_{BD} \circlearrowright$$

$$\mathbf{a}_{D/B} = [0.15\alpha_{AB} \measuredangle \beta] + [0.15\omega_{BD}^2 \measuredangle \beta]$$

$$= [6\alpha_{BD} \measuredangle \beta] + [40.376 \measuredangle \beta]$$

$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B}$ Resolve into components.

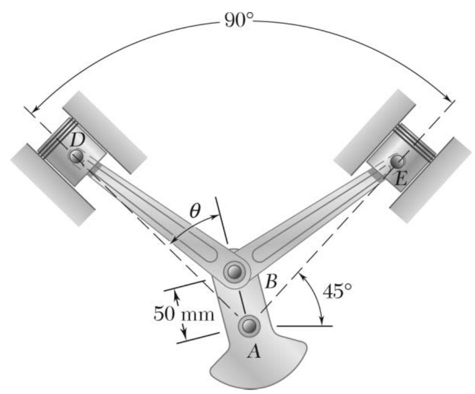
PROBLEM 15.121 (Continued)

$\xrightarrow{+}$: $0 = -444.13\cos 30° + 0.15\alpha_{BD}\cos\beta + 40.376\sin\beta$

$\alpha_{BD} = 2597.0 \text{ rad/s}^2$

$+\downarrow$: $a_D = -444.13\sin 30° - (0.15)(2597.0)\sin\beta + 40.376\cos\beta$

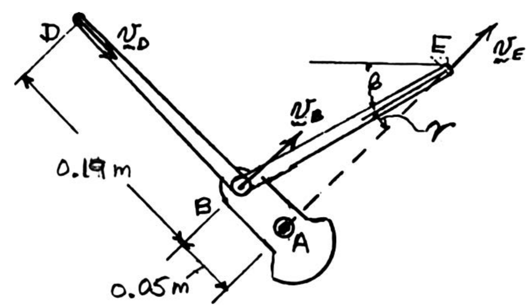
$= -296 \text{ m/s}^2 \qquad \mathbf{a}_P = \mathbf{a}_D$ $\qquad \mathbf{a}_P = 296 \text{ m/s}^2 \uparrow$ ◀



PROBLEM 15.122

In the two-cylinder air compressor shown the connecting rods *BD* and *BE* are each 190 mm long and crank *AB* rotates about the fixed Point *A* with a constant angular velocity of 1500 rpm clockwise. Determine the acceleration of each piston when $\theta = 0$.

SOLUTION



Crank AB.

$$\mathbf{v}_A = 0, \quad \mathbf{a}_A = 0, \quad \omega_{AB} = 1500 \text{ rpm} = 157.08 \text{ rad/s} \circlearrowright, \quad \alpha_{AB} = 0$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = 0 + [0.05\,\omega_{AB} \measuredangle 45°] = [7.854 \text{ m/s} \measuredangle 45°]$$

$$\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$$

$$= 0 + [0.05\alpha_{AB} \swarrow 45°] + [0.05\omega_{AB}^2 \nwarrow 45°]$$

$$= [(0.05)(157.08)^2 \nwarrow 45°] = 1233.7 \text{ m/s}^2 \nwarrow 45°$$

Rod BD.

$$\mathbf{v}_D = v_D \nwarrow 45° \qquad \boldsymbol{\omega}_{BD} = \omega_{BD} \circlearrowright$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{B/D}$$

$$v_D \nwarrow 45° = [7.854 \measuredangle 45°] + [0.19\omega_{BD} \swarrow 45°]$$

Components $\measuredangle 45°$:

$$0 = 7.854 - 0.19\omega_{BD} \qquad \omega_{BD} = 41.337 \text{ rad/s}$$

$$\mathbf{a}_D = a_D \nwarrow 45°$$

$$\mathbf{a}_D = \mathbf{a}_B + (\mathbf{a}_{D/B})_t + (\mathbf{a}_{D/B})_n$$

$$[a_D \nwarrow 45°] = [1233.7 \nwarrow 45°] + [0.19\,\alpha_{BD} \swarrow 45°] + [0.19\,\omega_{BD}^2 \nwarrow 45°]$$

PROBLEM 15.122 (Continued)

Components ⦨45°: $a_D = 1233.7 + (0.19)(41.337)^2 = 1558.4 \text{ m/s}^2$

$\mathbf{a}_D = 1558 \text{ m/s}^2$ ⦨ 45° ◀

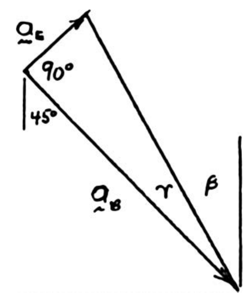
Rod BE. $\sin\gamma = \dfrac{0.05}{0.19}, \quad \gamma = 15.258°, \quad \beta = 45° - \gamma = 29.742°$

$\mathbf{v}_E = v_E$ ⦩ 45°

Since $\mathbf{v}_E$ is parallel to $\mathbf{v}_B$, $\omega_{BE} = 0.$

$\mathbf{a}_E = a_E$ ⦩ 45° $\quad (a_{B/E})_n = 0.19\,\omega_{BE}^2 = 0$

$(\mathbf{a}_{E/B})_t = (a_{E/B})_t$ ⦪ $\beta \qquad \mathbf{a}_E = \mathbf{a}_B + (\mathbf{a}_{B/E})_t$

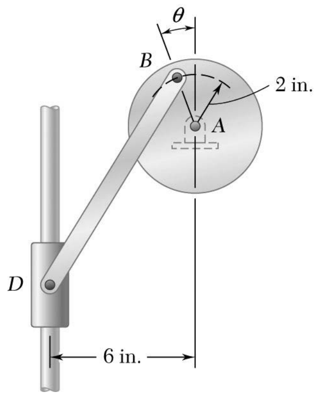


Draw vector addition diagram.

$$\gamma = 45° - \beta$$
$$= 15.258°$$
$$a_E = a_B \tan\gamma$$
$$= 1233.7 \tan\gamma$$
$$= 336.52 \text{ m/s}^2$$

$\mathbf{a}_E = 337 \text{ m/s}^2$ ⦩ 45° ◀

PROBLEM 15.123



The disk shown has a constant angular velocity of 500 rpm counter-clockwise. Knowing that rod *BD* is 10 in. long, determine the acceleration of collar *D* when (*a*) $\theta = 90°$, (*b*) $\theta = 180°$.

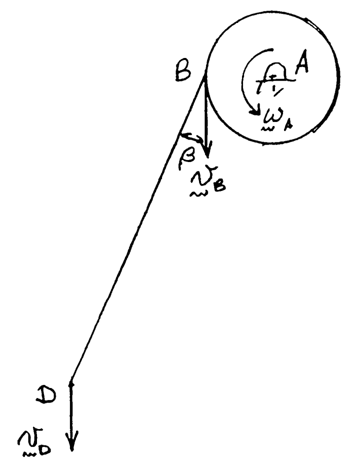
SOLUTION

Disk *A*. $\boldsymbol{\omega}_A = 500 \text{ rpm} \circlearrowleft = 52.36 \text{ rad/s} \circlearrowleft$

$\boldsymbol{\alpha}_A = 0, \quad (AB) = 2 \text{ in.}$

$v_B = (AB)\omega_A = (2)(52.36) = 104.72 \text{ in./s}$

$a_B = (AB)\omega_A^2 = (2)(52.36)^2 = 5483.1 \text{ in./s}^2$



(*a*) $\theta = 90°$. $\mathbf{v}_B = 104.72 \text{ m/s}\downarrow, \quad \mathbf{v}_D = v_D\downarrow$

$$\sin\beta = \frac{2 \text{ in.}}{5 \text{ in.}} = 0.4 \qquad \beta = 23.58°$$

$\mathbf{v}_D$ and $\mathbf{v}_B$ are parallel.

$\omega_{BD} = 0$

$\mathbf{a}_B = 5483.1 \text{ in./s}^2 \longrightarrow, \quad \mathbf{a}_D = a_D\uparrow, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD} \circlearrowleft$

$$\mathbf{a}_{D/B} = [(BD)\alpha_{BD} \nwarrow \beta] + [(BD)\omega_{BD}^2 \nearrow \beta]$$

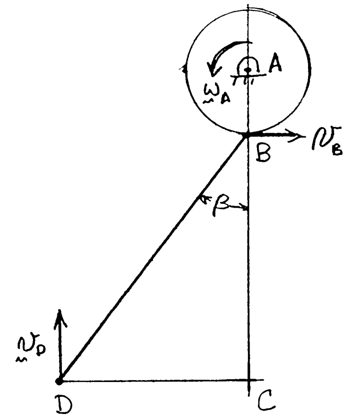
$$= [10\,\alpha_{BD} \nwarrow \beta] + 0$$

$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B}$ Resolve into components.

$\xrightarrow{+}$: $0 = 5483.1 + (10\cos\beta)\alpha_{BD} \qquad \alpha_{BD} = -598.26 \text{ rad/s}^2$

$+\uparrow$: $a_D = 0 - (10\sin\beta)(-598.26) + 0 = 2393.0 \text{ in./s}^2 \qquad \mathbf{a}_D = 199.4 \text{ ft/s}^2\uparrow$ ◀

PROBLEM 15.123 (Continued)



(*b*) $\theta = 180°$. $\qquad \mathbf{v}_B = 104.72 \text{ in./s} \longrightarrow, \qquad \mathbf{v}_D = v_D \uparrow$

$$\sin\beta = \frac{6 \text{ in.}}{10 \text{ in.}} = 0.6 \qquad \beta = 36.87$$

$$\mathbf{v}_B = 104.72 \text{ in./s} \longrightarrow, \qquad \mathbf{v}_D = v_D \uparrow$$

Instantaneous center of bar *BD* lies at Point *C*.

$$\omega_{BD} = \frac{v_B}{(BD)} = \frac{104.72}{10 \cos\beta} = 13.09 \text{ rad/s}$$

$$\mathbf{a}_B = 5483.1 \text{ in./s}^2 \uparrow, \quad \mathbf{a}_D = a_D \uparrow, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD} \circlearrowleft$$

$$\mathbf{a}_{D/B} = [(BD)\alpha_{BD} \searrow \beta] + [(BD)\omega_{BD}^2 \nearrow \beta]$$

$$= [10\alpha_{BD} \searrow \beta] + [1713.5 \nearrow \beta]$$

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} \quad \text{Resolve in components.}$$

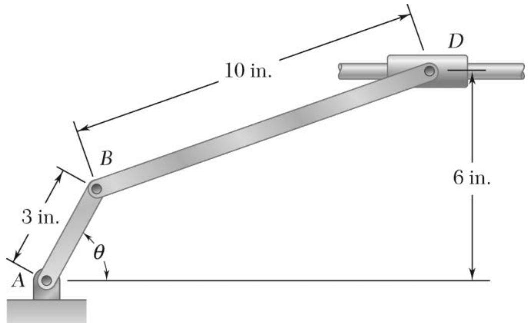
$\xrightarrow{+}$: $\qquad 0 = 0 + (10 \cos\beta)\alpha_{BD} + 1713.5 \sin\beta$

$$\alpha_{BD} = -128.51 \text{ rad/s}^2$$

$+\uparrow$: $\qquad a_D = 5483.1 - (10 \sin\beta)(-128.51) + 1713.5 \cos\beta$

$$= 7625.0 \text{ in./s}^2$$

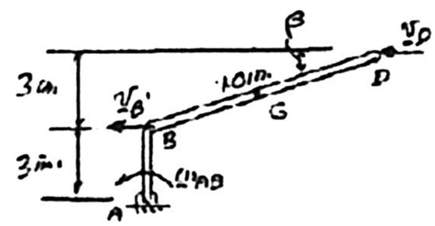
$\mathbf{a}_D = 635 \text{ ft/s}^2 \uparrow$ ◀



PROBLEM 15.124

Arm AB has a constant angular velocity of 16 rad/s counterclockwise. At the instant when $\theta = 90°$, determine the acceleration (*a*) of collar D, (*b*) of the midpoint G of bar BD.

SOLUTION



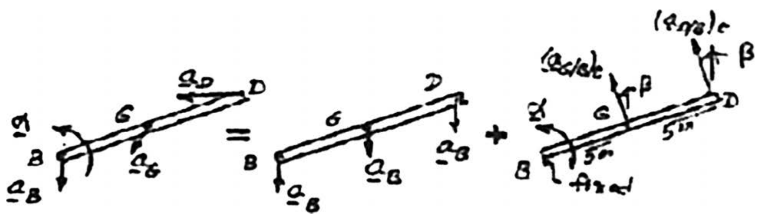
Rod AB:

$$a_B = (AB)\omega_{AB}^2$$
$$= (3\text{ in.})(16\text{ rad/s})^2$$
$$\mathbf{a}_B = 768\text{ in./s}^2 \downarrow$$

Rod BD: instantaneous center is at OD; $\boldsymbol{\omega}_{BD} = 0$

$$\sin\beta = (3\text{ in.})/(10\text{ in.}) = 0.3; \quad \beta = 17.46°$$

<u>Acceleration</u>.

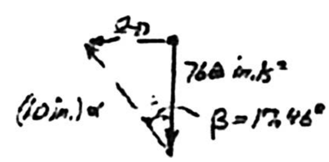


<u>Plane motion</u> = <u>Trans. with B</u> + <u>Rotation about B</u>

(*a*)

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} = a_B + (\mathbf{a}_{D/B})_t + (\mathbf{a}_{D/B})_n$$
$$= [a_B \downarrow] + [(BD)\alpha \nwarrow \beta] + [(BD)\omega_{BD}^2 \measuredangle \beta]$$
$$= [768\text{ in./s}^2 \downarrow] + [(10\text{ in.})\,\alpha \nwarrow \beta] + [(10\text{ in.})(0)^2 \measuredangle \beta]$$
$$\mathbf{a}_D \leftrightarrow = [768\text{ in./s} \downarrow] + [(10\text{ in.})\alpha \nwarrow \beta]$$

Vector diagram:



$$\mathbf{a}_D = (768\text{ in./s}^2)\tan 17.46°$$
$$= 241.62\text{ in./s}^2$$

$$\mathbf{a}_D = 242\text{ in./s}^2 \longleftarrow \blacktriangleleft$$

$$(10\text{ in.})\alpha = (768\text{ in./s}^2)/\cos 17.46°$$
$$(10\text{ in.})\alpha = 805.08\text{ in./s}^2$$
$$\boldsymbol{\alpha} = 80.5\text{ rad/s}^2 \circlearrowleft$$

PROBLEM 15.124 (Continued)

(*b*)

$$\mathbf{a}_G = \mathbf{a}_B + \mathbf{a}_{G/B} = \mathbf{a}_B + (\mathbf{a}_{G/B})_t + (\mathbf{a}_{G/B})_n$$

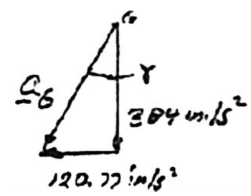
$$= [\mathbf{a}_B \downarrow] + [(BG)\alpha \searrow \beta] + [(BG)\omega_{BD}^2 \measuredangle \beta]$$

$$= [768 \text{ in./s}^2 \downarrow] + [(5 \text{ in.})(80.5 \text{ rad/s}^2) \searrow \beta] + [(BG)(0)^2]$$

$$\mathbf{a}_G = [768 \text{ in./s}^2 \downarrow] + [402.5 \text{ in./s}^2 \searrow 17.46°]$$

$\overset{+}{\leftarrow}$ components: $(a_G)_x = (402.5 \text{ in./s}^2)\sin 17.46°$

$$(\mathbf{a}_G)_x = 120.77 \text{ in./s}^2 \leftarrow$$



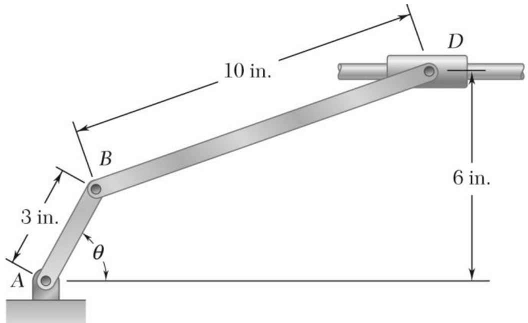
$+\downarrow$ components: $(a_G)_y = 768 \text{ in./s}^2 - (402.5 \text{ in./s}^2)\cos 17.46°$

$$= 768 \text{ in./s}^2 - 384 \text{ in./s}^2$$

$$(\mathbf{a}_G)_y = 384 \text{ in./s}^2 \downarrow$$

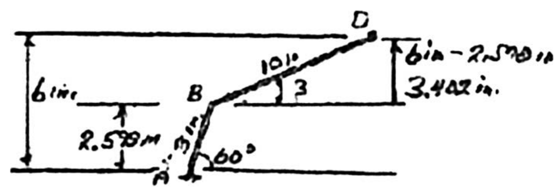
$$\mathbf{a}_G = 403 \text{ in./s}^2 \swarrow 72.5° \blacktriangleleft$$

PROBLEM 15.125



Arm *AB* has a constant angular velocity of 16 rad/s counterclockwise. At the instant when $\theta = 60°$, determine the acceleration of collar *D*.

SOLUTION



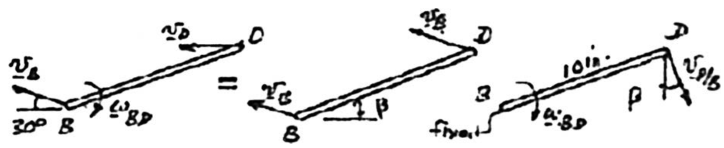
$$\beta = \sin^{-1}\frac{3.403 \text{ in.}}{10 \text{ in.}}$$

$$\beta = 19.89°$$

Velocity.

$$\mathbf{v}_B = (AB)\omega_{AB} = (3 \text{ in.})(16 \text{ rad/s}) = 48 \text{ in./s} \searrow 30°$$

Rod *BD*:



Plane motion = Trans. with *B* + Rotation about *B*

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = \mathbf{v}_B + [(BD)\omega_{BD} \searrow \beta]$$

$$\mathbf{v}_D \leftrightarrow = [48 \text{ in./s} \searrow 30°] + [(10 \text{ in.})\omega_{BD} \searrow 19.89°]$$

$+\uparrow$ components: $(48 \text{ in./s})\sin 30° - (10 \text{ in.})\omega_{BD}\cos 19.89°$

$$\boldsymbol{\omega}_{BD} = \frac{(48 \text{ in./s})\sin 30°}{(10 \text{ in.})\cos 19.890°} = 2.552 \text{ rad/s} \circlearrowright$$

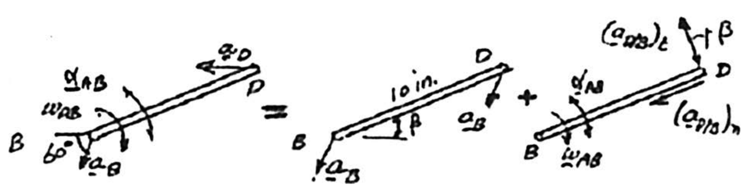
Acceleration.

Rod *AB*:

$$\mathbf{a}_B = [(AB)\omega_{AB}^2 \swarrow 60°] = [(3 \text{ in.})(16 \text{ rad/s})^2 \swarrow 60°]$$

$$\mathbf{a}_B = 768 \text{ in./s}^2 \swarrow 60°$$

Rod *BD*:



Plane motion = Trans. with *B* + Rotation about *B*

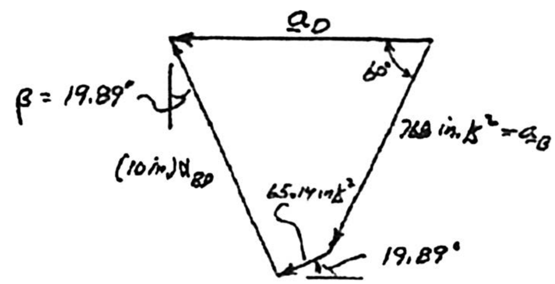
$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{B/D} = \mathbf{a}_B + (\mathbf{a}_{D/B})_t + (\mathbf{a}_{D/B})_n$$

$$a_D \leftrightarrow = [a_B \swarrow 60°] + [(BD)\alpha_{BD} \nwarrow \beta] + [(BD)\omega_{DB}^2 \swarrow \beta]$$

$$= [768 \text{ in./s}^2 \swarrow 60°] + [(10 \text{ in.})\alpha_{BD} \nwarrow \beta] + [(10 \text{ in.})(2.552 \text{ rad/s}^2) \swarrow \beta]$$

$$a_D \leftrightarrow = [768 \text{ in./s}^2 \swarrow] + [(10 \text{ in.})\alpha_{BD} \nwarrow 19.89°] + [65.14 \text{ in./s}^2 \swarrow 19.89°]$$

Vector diagram.



y components:

$+\downarrow$: $\quad 768\sin 60° + 65.14\sin 19.89° - 10\alpha_{BD}\cos 19.89° = 0$

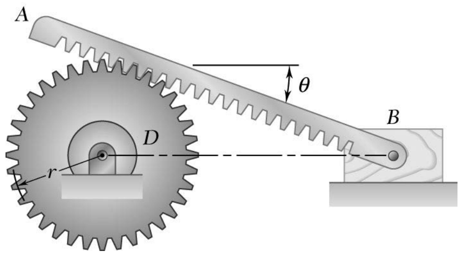
$$\alpha_{BD} = 73.09 \text{ rad/s}^2$$

x components:

$\overset{+}{\leftarrow}$: $\quad a_D = 768\cos 60° + 65.14\cos 19.89° + (10)(73.09)\sin 19.89°$

$$a_D = 693.9 \text{ in./s}^2$$

$\mathbf{a}_D = 694 \text{ in./s}^2 \longleftarrow$ ◀



PROBLEM 15.126

A straight rack rests on a gear of radius $r = 3$ in. and is attached to a block B as shown. Knowing that at the instant shown $\theta = 20°$, the angular velocity of gear D is 3 rad/s clockwise, and it is speeding up at a rate of 2 rad/s^2, determine (*a*) the angular acceleration of AB, (*b*) the acceleration of block B.

SOLUTION

Let Point P on the gear and Point Q on the rack be located at the contact point between them.

Units: inches, in./s, in./s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \ \mathbf{j} = 1 \uparrow, \ \mathbf{k} = 1 \circlearrowleft$

Geometry:

$$\mathbf{r}_{P/D} = 3(\sin\theta\mathbf{i} + \cos\theta\mathbf{j})$$

$$\mathbf{r}_{B/Q} = \frac{3}{\tan\theta}(\cos\theta\mathbf{i} - \sin\theta\mathbf{j})$$

$$\theta = 20°$$

Gear D: $\boldsymbol{\omega}_{\text{gear}} = 3 \text{ rad/s} \circlearrowright$ $\qquad \boldsymbol{\alpha}_{\text{gear}} = 2 \text{ rad/s}^2 \circlearrowright$

$$v_P = \omega_{\text{gear}} r = (3)(3) = 9 \text{ in./s} \qquad \mathbf{v}_P = 9 \text{ in./s} \searrow \theta$$

$$(a_P)_n = \omega_{\text{gear}}^2 r = (3)^2(3) = 27 \text{ in./s}^2 \qquad (\mathbf{a}_P)_n = 27 \text{ in./s}^2 \swarrow \theta$$

$$(a_P)_t = \alpha_{\text{gear}} r = (2)(3) = 6 \text{ in./s}^2 \qquad (\mathbf{a}_P)_t = 6 \text{ in./s}^2 \searrow \theta$$

Velocity analysis.

Gear to rack contact: $\mathbf{v}_Q = \mathbf{v}_P = 9 \text{ in./s} \searrow \theta$

Rack AQB: $\boldsymbol{\omega}_{AB} = \omega_{AB} \circlearrowleft, \qquad \boldsymbol{\alpha}_{AB} = \alpha_{AB} \circlearrowleft$

$$\mathbf{v}_B = v_B \longrightarrow, \qquad \boldsymbol{\alpha}_B = \alpha_B \longrightarrow$$

$$\mathbf{v}_B = \mathbf{v}_Q + \mathbf{v}_{B/Q} = \mathbf{v}_Q + \omega_{AB}\mathbf{k} \times \mathbf{r}_{B/Q}$$

$$v_B\mathbf{i} = 9(\cos\theta\mathbf{i} - \sin\theta\mathbf{j}) + \omega_{AB}\mathbf{k} \times (7.74535\mathbf{i} - 2.81908\mathbf{j})$$

$$= (9\cos\theta + 2.81907\omega_{AB})\mathbf{i} + (-9\sin\theta + 2.81907\omega_{AB})\mathbf{j}$$

Equating like components,

$\mathbf{j}$: $\qquad 0 = -9\sin\theta + 7.74535\omega_{AB}$

$$\omega_{AB} = 0.39742 \qquad \omega_{AB} = 0.39742 \text{ rad/s} \circlearrowleft$$

Acceleration analysis.

Gear to rack contact: $(\mathbf{a}_Q)_t = (\mathbf{a}_P)_t = 6 \text{ in./s}^2 \searrow \theta$

$$(\mathbf{a}_Q)_n = (\mathbf{a}_P)_n + r\omega_{rd}^2 \measuredangle \theta$$

where $\boldsymbol{\omega}_{rd} = \boldsymbol{\omega}_{AB} - \boldsymbol{\omega}_D = 3.39742 \text{ rad/s} \circlearrowright$

$$(\mathbf{a}_Q)_n = 27 \text{ in./s}^2 \measuredangle 20° + (3)(3.39742)^2 \measuredangle 20°$$

$$= 7.6274 \text{ in./s}^2 \measuredangle 20°$$

Then, $\mathbf{a}_Q = 6(\cos\theta\,\mathbf{i} - \sin\theta\,\mathbf{j}) + 7.6274(\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$

$$= (8.2469 \text{ in./s}^2)\mathbf{i} + (5.1153 \text{ in./s}^2)\mathbf{j}$$

$$\mathbf{a}_B = \mathbf{a}_Q + \mathbf{a}_{B/Q} = \mathbf{a}_Q + \alpha_{AB}\mathbf{k} \times \mathbf{r}_{B/Q} - \omega_{AB}^2 \mathbf{r}_{B/Q}$$

$$a_B\mathbf{i} = 8.2469\mathbf{i} + 5.1153\mathbf{j} + \alpha_{AB}\mathbf{k} \times (7.74535\mathbf{i} - 2.81908\mathbf{j})$$

$$- (0.39742)^2(7.74535\mathbf{i} - 2.81908\mathbf{j})$$

$$= (8.2469 + 2.81908\alpha_{AB} - 1.22332)\mathbf{i}$$

$$+ (5.1153 + 7.74535\alpha_{AB} + 0.44526)\mathbf{j}$$

Equating like components of $\mathbf{a}_B$,

$\mathbf{j}$: $0 = 5.1153 + 7.74535\alpha_{AB} + 0.44526$

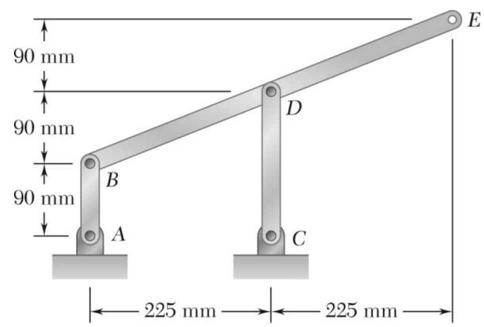
$$\alpha_{AB} = -0.71792 \text{ rad/s}^2$$

$\mathbf{i}$: $a_B = 8.2469 + (2.81908)(-0.71792) - 1.22332$

$$a_B = 5.00 \text{ in./s}^2$$

(*a*) *Angular acceleration of AB*: $\boldsymbol{\alpha}_{AB} = 0.718 \text{ rad/s}^2 \circlearrowright$ ◀

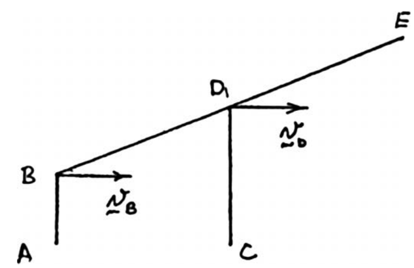
(*b*) *Acceleration of block B*: $\mathbf{a}_B = 5.00 \text{ in./s}^2 \longrightarrow$ ◀



PROBLEM 15.127

Knowing that at the instant shown rod AB has a constant angular velocity of 6 rad/s clockwise, determine the acceleration of Point D.

SOLUTION



Velocity analysis. $\omega_{AB} = 6 \text{ rad/s} \circlearrowright$

$$\mathbf{v}_B = (AB)\omega_{AB} = (90)(6) = 540 \text{ mm/s}$$

$$\mathbf{v}_B = v_B \rightarrow, \quad \mathbf{v}_D = v_D \rightarrow$$

The instantaneous center of bar BDE lies at ∞.

Then $\omega_{BD} = 0$ and $v_D = v_B = 540$ mm/s

$$\omega_{CD} = \frac{v_D}{CD} = \frac{540}{180} = 3 \text{ rad/s} \circlearrowright$$

Acceleration analysis. $\alpha_{AB} = 0$

$$\mathbf{a}_B = (AB)\omega_{AB}^2 = [(90)(6)^2 \downarrow] = 3240 \text{ mm/s}^2 \downarrow$$

$$\mathbf{a}_D = [(CD)\alpha_{CD} \leftarrow] + [(CD)\omega_{CD}^2 \downarrow] = [180\alpha_{CD} \leftarrow] + [(180)(3)^2 \downarrow]$$

$$= [180\alpha_{CD} \leftarrow] + [1620 \text{ mm/s}^2 \downarrow]$$

$$\mathbf{a}_{D/B} = [90\alpha_{BD} \leftarrow] + [225\alpha_{BD} \uparrow] + [225\omega_{BD}^2 \leftarrow] + [90\omega_{BD}^2 \downarrow]$$

$$= [90\alpha_{BD} \leftarrow] + [225\alpha_{BD} \uparrow]$$

$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B}$ Resolve into components.

$+\uparrow$: $-1620 = -3240 + 225\alpha_{BD}$, $\alpha_{BD} = 7.2 \text{ rad/s}^2 \circlearrowleft$

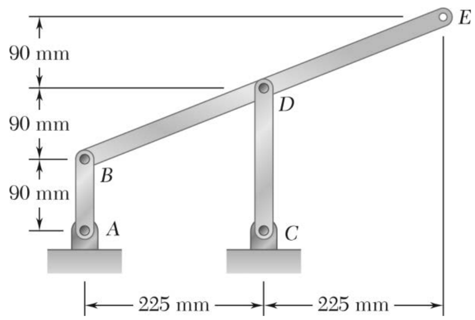
$\overset{+}{\leftarrow}$: $180\alpha_{CD} = 0 + (90)(7.2)$, $\alpha_{CD} = 3.6 \text{ rad/s}^2 \circlearrowleft$

$$\mathbf{a}_D = [3240 \downarrow] + [(90)(7.2) \leftarrow] + [(225)(7.2) \uparrow]$$

$$= [648 \leftarrow] + [1620 \text{ mm/s}^2 \downarrow]$$

$$= 1745 \text{ mm/s}^2 \swarrow 68.2^\circ$$

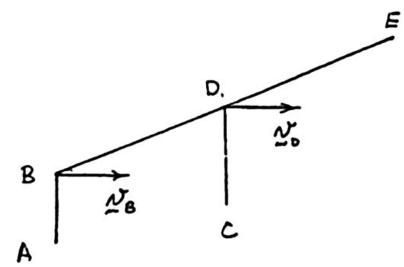
$\mathbf{a}_D = 1.745 \text{ m/s}^2 \swarrow 68.2^\circ$ ◀



PROBLEM 15.128

Knowing that at the instant shown rod AB has a constant angular velocity of 6 rad/s clockwise, determine (a) the angular acceleration of member BDE, (b) the acceleration of Point E.

SOLUTION



Velocity analysis. $\omega_{AB} = 6 \text{ rad/s} \circlearrowright$

$$\mathbf{v}_B = (AB)\omega_{AB} = (90)(6) = 540 \text{ mm/s}$$

$$\mathbf{v}_B = v_B \longrightarrow, \quad \mathbf{v}_D = v_D \longrightarrow$$

The instantaneous center of bar BDE lies at ∞.

Then $\omega_{BD} = 0$ and $v_D = v_B = 540$ mm/s

$$\omega_{CD} = \frac{v_D}{CD} = \frac{27}{9} = 3 \text{ rad/s} \circlearrowright$$

Acceleration analysis. $\alpha_{AB} = 0$

$$\mathbf{a}_B = (AB)\omega_{AB}^2 = [(90)(6)^2 \downarrow] = 3240 \text{ mm/s}^2 \downarrow$$

$$\mathbf{a}_D = [(CD)\alpha_{CD} \longleftarrow] + [(CD)\omega_{CD}^2 \downarrow] = [180\alpha_{CD} \longleftarrow] + [(180)(3)^2 \downarrow]$$

$$= [180\alpha_{CD} \longleftarrow] + [1620 \text{ mm/s}^2 \downarrow]$$

$$\mathbf{a}_{D/B} = [90\alpha_{BD} \longleftarrow] + [225\alpha_{BD} \uparrow] + [225\omega_{BD}^2 \longleftarrow] + [90\omega_{BD}^2 \downarrow]$$

$$= [90\alpha_{BD} \longleftarrow] + [225\alpha_{BD} \uparrow]$$

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} \qquad \text{Resolve into components.}$$

(a) $+\uparrow$: $\quad -1620 = -3240 + 225\alpha_{BD}, \qquad \alpha_{BD} = 7.20 \text{ rad/s}^2 \circlearrowleft$

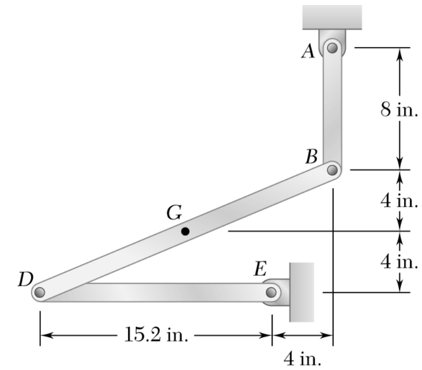
$$\mathbf{a}_{E/B} = [180\alpha_{BD} \longleftarrow] + [450\alpha_{BD} \uparrow] + [450\omega_{BD}^2 \longleftarrow] + [180\omega_{BD}^2 \downarrow]$$

$$= [(180)(7.2) \longleftarrow] + [(450)(7.2) \uparrow] + [0 \longleftarrow] + [0 \downarrow]$$

$$= [1296 \text{ mm/s}^2 \longleftarrow] + [3240 \text{ mm/s}^2 \uparrow]$$

(b) $$\mathbf{a}_E = \mathbf{a}_B + \mathbf{a}_{B/E} = [3240 \text{ mm/s}^2 \downarrow] + [1296 \text{ mm/s}^2 \longleftarrow] + [3240 \text{ mm/s}^2 \uparrow]$$

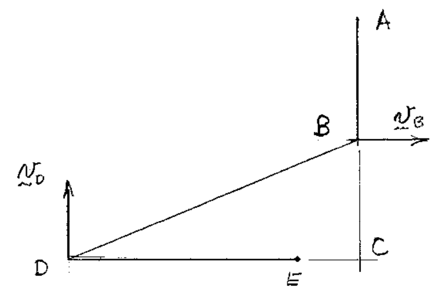
$$= 1296 \text{ mm/s}^2 \longleftarrow \qquad \mathbf{a}_E = 1.296 \text{ m/s}^2 \longleftarrow \blacktriangleleft$$



PROBLEM 15.129

Knowing that at the instant shown bar AB has a constant angular velocity of 19 rad/s clockwise, determine (*a*) the angular acceleration of bar BGD, (*b*) the angular acceleration of bar DE.

SOLUTION



Velocity analysis.

$$\omega_{AB} = 19 \text{ rad/s} \circlearrowright$$

$$v_B = (AB)\omega_{AB} = (8)(19) = 152 \text{ in./s}$$

$$\mathbf{v}_B = v_B \longrightarrow, \quad \mathbf{v}_D = v_D \uparrow$$

Instantaneous center of bar BD lies at C.

$$\omega_{BD} = \frac{v_B}{BC} = \frac{152}{8} = 19 \text{ rad/s} \circlearrowleft$$

$$v_D = (CD)\omega_{BD} = (19.2)(19) = 364.8 \text{ in./s}$$

$$\omega_{DE} = \frac{v_D}{DE} = \frac{364.8}{15.2} = 24 \text{ rad/s}^2 \circlearrowright$$

Acceleration analysis.

$$\alpha_{AB} = 0.$$

$$\mathbf{a}_B = [(AB)\omega_{AB}^2 \uparrow] = [(8)(19)^2 \uparrow] = 2888 \text{ in./s} \uparrow$$

$$\mathbf{a}_D = [(DE)\alpha_{DE} \downarrow] + [(DE)\omega_{DE}^2 \longrightarrow]$$

$$= [15.2\alpha_{DE} \downarrow] + [8755.2 \text{ in./s}^2 \longrightarrow]$$

$$(\mathbf{a}_{D/B})_t = [19.2\alpha_{BD} \downarrow] + [8\alpha_{DB} \longrightarrow]$$

$$(\mathbf{a}_{D/B})_n = [19.2\omega_{BD}^2 \longrightarrow] + [8\omega_{BD}^2 \uparrow]$$

$$= [6931.2 \text{ in./s}^2 \longrightarrow] + [2888 \text{ in./s}^2 \uparrow]$$

$$\mathbf{a}_D = \mathbf{a}_B + (\mathbf{a}_{D/B})_t + (a_{D/B})_n \qquad \text{Resolve into components.}$$

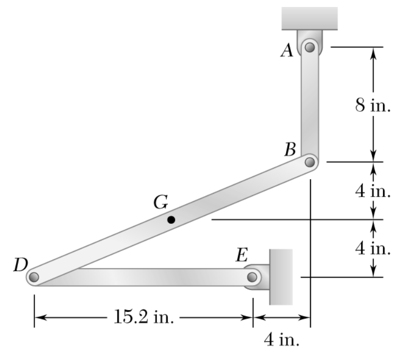
$\xrightarrow{+}$: $8755.2 = 0 + 8\alpha_{BD} + 6931.2$

(*a*) $\boldsymbol{\alpha}_{BD} = 228 \text{ rad/s}^2 \circlearrowleft$ ◀

$+\downarrow$: $15.2\alpha_{DE} = -2888 + (19.2)(228) - 2888$

(*b*) $\alpha_{DE} = -92$ rad/s $\qquad$ $\boldsymbol{\alpha}_{DE} = 92.0 \text{ rad/s}^2 \circlearrowright$ ◀

PROBLEM 15.130



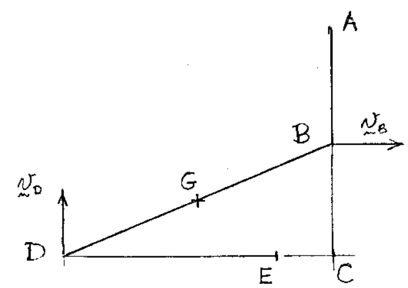
Knowing that at the instant shown bar DE has a constant angular velocity of 18 rad/s clockwise, determine (*a*) the acceleration of Point B, (*b*) the acceleration of Point G.

SOLUTION

Velocity analysis. $\omega_{DE} = 18 \text{ rad/s} \circlearrowright$

$$\mathbf{v}_D = (DE)\omega_{DE} = (15.2)(18) = 273.6 \text{ in./s}$$

$$\mathbf{v}_D = v_D \uparrow, \quad \mathbf{v}_B = v_B \longrightarrow$$



Point C is the instantaneous center of bar BD.

$$\omega_{BD} = \frac{v_D}{CD} = \frac{273.6}{19.2} = 14.25 \text{ rad/s} \circlearrowright$$

$$v_B = (CB)\omega_{BD} = (8)(14.25) = 114 \text{ in./s}$$

$$\omega_{AB} = \frac{v_B}{AB} = \frac{114}{8} = 14.25 \text{ rad/s} \circlearrowleft$$

Acceleration analysis. $\alpha_{DE} = 0$

$$\mathbf{a}_D = [(DE)\omega_{DE}^2 \longrightarrow] = [(15.2)(18)^2 \longrightarrow] = [4924.8 \text{ in./s}^2 \longrightarrow]$$

$$\mathbf{a}_B = [(AB)\alpha_{AB} \longrightarrow] + [(AB)\omega_{AB}^2 \uparrow]$$

$$= [8\alpha_{AB} \longrightarrow] + [1624.5 \text{ in./s}^2 \uparrow]$$

$$(\mathbf{a}_{D/B})_t = [19.2\alpha_{BD} \downarrow] + [8\alpha_{BD} \longrightarrow]$$

$$(\mathbf{a}_{D/B})_n = [19.2\omega_{BD}^2 \longrightarrow] + [8\omega_{BD}^2 \uparrow]$$

$$\mathbf{a}_D = \mathbf{a}_B + (\mathbf{a}_{D/B})_t + (\mathbf{a}_{D/B})_n$$ Resolve into components.

PROBLEM 15.130 (Continued)

$+\downarrow$: $0 = 1624.5 - 19.2\alpha_{BD} + 1624.5,$

$$\alpha_{BD} = 169.21875 \text{ rad/s}^2$$

$\xrightarrow{+}$: $4924.8 = 8\alpha_{AB} + (8)(169.21875) + 3898.8$

$$\alpha_{AB} = -40.96875 \text{ rad/s}^2$$

(*a*) $\mathbf{a}_B = [(8)(-40.96875) \longrightarrow] + [1624.5 \text{ in./s}^2 \uparrow]$

$= [327.75 \text{ in./s}^2 \longleftarrow] + [1624.5 \text{ in./s}^2 \uparrow],$

$$\mathbf{a}_B = 138.1 \text{ ft/s}^2 \searrow 78.6° ◀$$

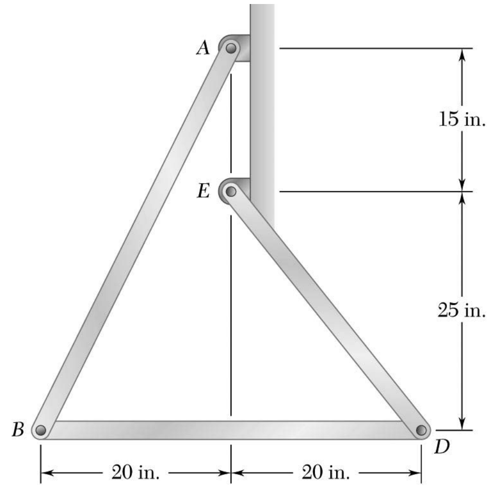
(*b*) $\mathbf{a}_G = \mathbf{a}_B + \mathbf{a}_{G/B} = \mathbf{a}_B + \frac{1}{2}\mathbf{a}_{D/B}$

$= \mathbf{a}_B + \frac{1}{2}(\mathbf{a}_D - \mathbf{a}_B) = \frac{1}{2}(\mathbf{a}_B + \mathbf{a}_D)$

$= \left[\frac{-327.75 + 4924.8}{2} \longrightarrow\right] + \left[\frac{1624.5}{2} \uparrow\right]$

$= [2298.5 \text{ in./s}^2] \longrightarrow + [812.25 \text{ in./s}^2 \uparrow]$

$$\mathbf{a}_G = 203 \text{ ft/s}^2 \nearrow 19.5° ◀$$



PROBLEM 15.131

Knowing that at the instant shown bar AB has a constant angular velocity of 4 rad/s clockwise, determine the angular acceleration (a) of bar BD, (b) of bar DE.

SOLUTION

Relative position vectors.

$$\mathbf{r}_{B/A} = -(20\text{ in.})\mathbf{i} - (40\text{ in.})\mathbf{j}$$
$$\mathbf{r}_{D/B} = (40\text{ in.})\mathbf{i}$$
$$\mathbf{r}_{D/E} = (20\text{ in.})\mathbf{i} - (25\text{ in.})\mathbf{j}$$

Velocity analysis.

Bar AB (Rotation about A):

$$\boldsymbol{\omega}_{AB} = 4\text{ rad/s} \circlearrowright = -(4\text{ rad/s})\mathbf{k}$$

$$\mathbf{r}_{B/A} = -(20\text{ in.})\mathbf{i} - (40\text{ in.})\mathbf{j} \qquad \mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (-4\mathbf{k}) \times (-20\mathbf{i} - 40\mathbf{j})$$
$$\mathbf{v}_B = -(160\text{ in./s})\mathbf{i} + (80\text{ in./s})\mathbf{j}$$

Bar BD (Plane motion = Translation with B + Rotation about B):

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \qquad \mathbf{r}_{D/B} = (40\text{ in.})\mathbf{i}$$
$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B} = \mathbf{v}_B + (\omega_{BD}\mathbf{k}) \times (40\mathbf{i})$$
$$\mathbf{v}_D = -(160\text{ in/s})\mathbf{i} + (40\omega_{BD} + 80\text{ in./s})\mathbf{j}$$

Bar DE (Rotation about E):

$$\boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}$$
$$\mathbf{r}_{D/E} = (20\text{ in.})\mathbf{i} - (25\text{ in.})\mathbf{j}$$
$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (\omega_{DE}\mathbf{k}) \times (20\mathbf{i} - 25\mathbf{j})$$
$$\mathbf{v}_D = 20\omega_{DE}\mathbf{j} + 25\omega_{DE}\mathbf{i}$$

Equating components of the two expression for $\mathbf{v}_D$,

$\mathbf{i}$: $\quad -160 = 25\omega_{DE} \qquad \omega_{DE} = -6.4\text{ rad/s}$

$\mathbf{j}$: $\quad 40\omega_{BE} + 80 = 20\omega_{DE} \qquad 40\omega_{BD} + 80 = 20(-6.4) \qquad \omega_{BD} = -5.2\text{ rad/s}$

Summary of angular velocities: $\boldsymbol{\omega}_{AB} = 4$ rad/s ↻ $\boldsymbol{\omega}_{DE} = 6.4$ rad/s ↻ $\boldsymbol{\omega}_{BD} = 5.2$ rad/s ↻

Acceleration analysis. $\boldsymbol{\alpha}_{AB} = 0, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k}, \quad \boldsymbol{\alpha}_{DE} = \alpha_{DE}\mathbf{k}$

<u>Bar *AB*</u> (Rotation about *A*):

$$\begin{aligned}\mathbf{a}_B &= \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} \\ &= 0 - (4)^2(-20\mathbf{i} - 40\mathbf{j}) \\ &= (320 \text{ in./s}^2)\mathbf{i} + (640 \text{ in./s}^2)\mathbf{j}\end{aligned}$$

<u>Bar *BD*</u> (Translation with *B* + Rotation about *B*):

$$\begin{aligned}\mathbf{a}_D &= \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2\, \mathbf{r}_{D/B} \\ &= 320\mathbf{i} + 640\mathbf{j} + \alpha_{BD}\mathbf{k} \times (40\mathbf{i}) - (5.2)^2(40)\mathbf{i} \\ &= 320\mathbf{i} + 640\mathbf{j} + 40\alpha_{BD}\mathbf{j} - 1081.6\mathbf{i} \\ &= -761.60\mathbf{i} + (640 + 40\alpha_{BD})\mathbf{j} \qquad (1)\end{aligned}$$

<u>Bar *DE*</u> (Rotation about *E*):

$$\begin{aligned}\mathbf{a}_D &= \boldsymbol{\alpha}_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2 \mathbf{r}_{D/E} \\ &= \alpha_{DE}\mathbf{k} \times (20\mathbf{i} - 25\mathbf{j}) - (6.4)^2(20\mathbf{i} - 25\mathbf{j}) \\ &= -20\alpha_{DE}\mathbf{j} + 25\alpha_{DE}\mathbf{i} - 819.20\mathbf{i} + 1024\mathbf{j} \\ &= (25\alpha_{DE} - 819.20)\mathbf{i} + (20\alpha_{DE} + 1024)\mathbf{j} \qquad (2)\end{aligned}$$

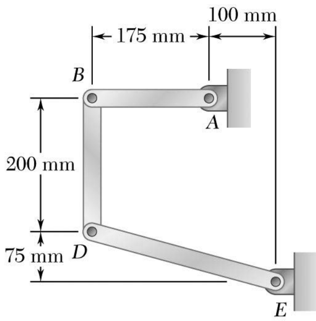
Equate like components of $\mathbf{a}_D$ expressed by Eqs. (1) and (2).

i: $-761.60 = 25\alpha_{DE} - 819.20 \qquad \alpha_{DE} = 2.3040 \text{ rad/s}^2$

j: $640 + 40\alpha_{BD} = (20)(2.304) + 1024 \qquad \alpha_{BD} = 10.752 \text{ rad/s}^2$

(*a*) <u>Angular acceleration of bar *BD*.</u> $\boldsymbol{\alpha}_{BD} = 10.75 \text{ rad/s}^2$ ↺ ◀

(*b*) <u>Angular acceleration of bar *DE*.</u> $\boldsymbol{\alpha}_{DE} = 2.30 \text{ rad/s}^2$ ↺ ◀



PROBLEM 15.132

Knowing that at the instant shown bar *AB* has a constant angular velocity of 4 rad/s clockwise, determine the angular acceleration (*a*) of bar *BD*, (*b*) of bar *DE*.

SOLUTION

Velocity analysis.

Bar *AB* (Rotation about *A*): $\boldsymbol{\omega}_{AB} = 4 \text{ rad/s} \circlearrowright = -(4 \text{ rad/s})\mathbf{k}$

$\mathbf{r}_{B/A} = -(175 \text{ mm})\mathbf{i} \qquad \mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (-4\mathbf{k}) \times (-175\mathbf{i})$

$\mathbf{v}_B = (700 \text{ mm/s})\mathbf{j}$

Bar *BD* (Plane motion = Translation with $B +$ Rotation about B):

$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \qquad \mathbf{r}_{D/B} = -(200 \text{ mm})\mathbf{j}$

$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B} = 700\mathbf{j} + (\omega_{BD}\mathbf{k}) \times (-200\mathbf{j})$

$\mathbf{v}_D = 700\mathbf{j} + 200\omega_{BD}\mathbf{i}$

Bar *DE* (Rotation about E): $\boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}$

$\mathbf{r}_{D/E} = -(275 \text{ mm})\mathbf{i} + (75 \text{ mm})\mathbf{j}$

$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (\omega_{DE}\mathbf{k}) \times (-275\mathbf{i} + 75\mathbf{j})$

$\mathbf{v}_D = -275\omega_{DE}\mathbf{j} - 75\omega_{DE}\mathbf{i}$

Equating components of the two expressions for $\mathbf{v}_D$,

$\mathbf{j}$: $\quad 700 = -275\omega_{DE} \qquad \omega_{DE} = -2.5455 \text{ rad/s} \qquad\qquad \boldsymbol{\omega}_{DE} = 2.55 \text{ rad/s} \circlearrowright$

$\mathbf{i}$: $\quad 200\omega_{BD} = -75\omega_{BD} \qquad \omega_{DE} = -\frac{3}{8}\omega_{BD}$

$$\omega_{BD} = -\left(\frac{3}{8}\right)(-2.5455) = 0.95455 \text{ rad/s} \qquad\qquad \boldsymbol{\omega}_{BD} = 0.955 \text{ rad/s} \circlearrowleft$$

Acceleration analysis. $\boldsymbol{\alpha}_{AB} = 0$

Bar *AB*: $\mathbf{a}_B = -\omega_{AB}^2\mathbf{r}_{B/A} = -(4)^2(-175\mathbf{i}) = (2800 \text{ mm/s}^2)\mathbf{i}$

Bar *BD*: $\boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k}$

$\mathbf{a}_D = \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2\mathbf{r}_{D/B}$

$= 2800\mathbf{i} + \alpha_{BD}\mathbf{k} \times (-200\mathbf{j}) - (0.95455)^2(-200\mathbf{j})$

$= (2800 + 200\,\alpha_{BD})\mathbf{i} + 182.23\mathbf{j} \qquad (1)$

PROBLEM 15.132 (Continued)

Bar DE:

$$\alpha_{DE} = \alpha_{DE}\mathbf{k}$$

$$\begin{aligned}\mathbf{a}_D &= \boldsymbol{\alpha}_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2 r_{D/E} \\ &= \alpha_{DE}\mathbf{k} \times (-275\mathbf{i} + 75\mathbf{j}) - (2.5455)^2(-275\mathbf{i} + 75\mathbf{j}) \\ &= -275\alpha_{DE}\mathbf{j} - 75\alpha_{DE}\mathbf{i} + 1781.8\mathbf{i} - 485.95\mathbf{j} \\ &= (-75\alpha_{DE} + 1781.8)\mathbf{i} - (275\alpha_{DE} + 485.95)\mathbf{j} \qquad (2)\end{aligned}$$

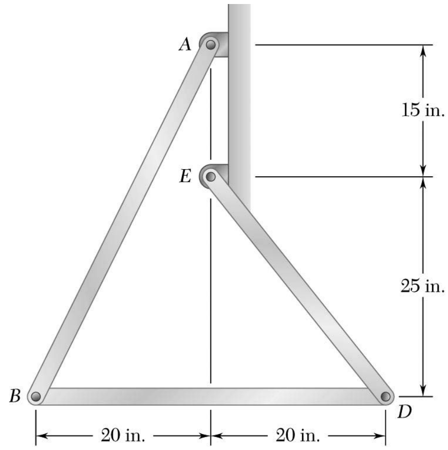
Equate like components of $\mathbf{a}_D$ expressed by Eqs. (1) and (2).

$\mathbf{j}$: $\quad 182.23 = -(275\alpha_{DE} + 485.95) \qquad \alpha_{DE} = -2.4298 \text{ rad/s}^2$

$\mathbf{i}$: $\quad (2800 + 200\alpha_{BD}) = [-(75)(-2.4298) + 1781.8] \qquad \alpha_{BD} = -4.1795 \text{ rad/s}^2$

(*a*) Angular acceleration of bar *BD*. $\qquad \boldsymbol{\alpha}_{BD} = 4.18 \text{ rad/s}^2$ ↻ ◀

(*b*) Angular acceleration of bar *DE*. $\qquad \boldsymbol{\alpha}_{DE} = 2.43 \text{ rad/s}^2$ ↻ ◀



PROBLEM 15.133

Knowing that at the instant shown bar *AB* has an angular velocity of 4 rad/s and an angular acceleration of 2 rad/s^2, both clockwise, determine the angular acceleration (*a*) of bar *BD*, (*b*) of bar *DE* by using the vector approach as is done in Sample Problem 15.8.

SOLUTION

Relative position vectors.

$$\mathbf{r}_{B/A} = -(20\text{ in.})\mathbf{i} - (40\text{ in.})\mathbf{j}$$
$$\mathbf{r}_{D/B} = (40\text{ in.})\mathbf{i}$$
$$\mathbf{r}_{D/E} = (20\text{ in.})\mathbf{i} - (25\text{ in.})\mathbf{j}$$

Velocity analysis.

Bar *AB* (Rotation about *A*):

$$\boldsymbol{\omega}_{AB} = 4\text{ rad/s} \curvearrowright = -(4\text{ rad/s})\mathbf{k}$$

$$\mathbf{r}_{B/A} = -(20\text{ in.})\mathbf{i} - (40\text{ in.})\mathbf{j} \qquad \mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (-4\mathbf{k}) \times (-20\mathbf{i} - 40\mathbf{j})$$
$$\mathbf{v}_B = -(160\text{ in./s})\mathbf{i} + (80\text{ in./s})\mathbf{j}$$

Bar *BD* (Plane motion = Translation with *B* + Rotation about *B*):

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \qquad \mathbf{r}_{D/B} = (40\text{ in.})\mathbf{i}$$
$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B} = \mathbf{v}_B + (\omega_{BD}\mathbf{k}) \times (40\mathbf{i})$$
$$\mathbf{v}_D = -(160\text{ in/s})\mathbf{i} + (40\omega_{BD} + 80\text{ in./s})\mathbf{j}$$

Bar *DE* (Rotation about *E*):

$$\boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}$$
$$\mathbf{r}_{D/E} = (20\text{ in.})\mathbf{i} - (25\text{in.})\mathbf{j}$$
$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (\omega_{DE}\mathbf{k}) \times (20\mathbf{i} - 25\mathbf{j})$$
$$\mathbf{v}_D = 20\omega_{DE}\mathbf{j} + 25\omega_{DE}\mathbf{i}$$

Equating components of the two expression for $\mathbf{v}_D$,

$\mathbf{i}$: $\quad -160 = 25\omega_{DE} \qquad \omega_{DE} = -6.4\text{ rad/s}$

$\mathbf{j}$: $\quad 40\omega_{BE} + 80 = 20\omega_{DE} \qquad 40\omega_{BD} + 80 = 20(-6.4) \qquad \omega_{BD} = -5.2\text{ rad/s}$

Summary of angular velocities: $\boldsymbol{\omega}_{AB} = 4 \text{ rad/s} \circlearrowright \quad \boldsymbol{\omega}_{DE} = 6.4 \text{ rad/s} \circlearrowright \quad \boldsymbol{\omega}_{BD} = 5.2 \text{ rad/s} \circlearrowright$

Acceleration analysis. $\boldsymbol{\alpha}_{AB} = -(2 \text{ rad/s}^2)\mathbf{k}, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k}, \quad \boldsymbol{\alpha}_{DE} = \alpha_{DE}\mathbf{k}$

Bar *AB* (Rotation about *A*)

$$\begin{aligned}\mathbf{a}_B &= \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}\\ &= (-2\mathbf{k}) \times (-20\mathbf{i} - 40\mathbf{j}) - (4)^2(-20\mathbf{i} - 40\mathbf{j})\\ &= -(80 \text{ in./s}^2)\mathbf{i} + (40 \text{ in./s}^2)\mathbf{j} + (320 \text{ in./s}^2)\mathbf{i} + (640 \text{ in./s}^2)\mathbf{j}\\ &= (240 \text{ in./s}^2)\mathbf{i} + (680 \text{ in./s}^2)\mathbf{j}\end{aligned}$$

Bar *BD* (Translation with *B* + Rotation about *B*):

$$\begin{aligned}\mathbf{a}_D &= \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2 \mathbf{r}_{D/B}\\ &= 240\mathbf{i} + 680\mathbf{j} + \alpha_{BD}\mathbf{k} \times (40\mathbf{i}) - (5.2)^2(40)\mathbf{i}\\ &= 240\mathbf{i} + 680\mathbf{j} + 40\alpha_{BD}\mathbf{j} - 1081.6\mathbf{i}\\ &= -841.60\mathbf{i} + (680 + 40\alpha_{BD})\mathbf{j} \qquad (1)\end{aligned}$$

Bar *DE* (Rotation about *E*):

$$\begin{aligned}\mathbf{a}_D &= \boldsymbol{\alpha}_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2 \mathbf{r}_{D/E}\\ &= \alpha_{DE}\mathbf{k} \times (20\mathbf{i} - 25\mathbf{j}) - (6.4)^2(20\mathbf{i} - 25\mathbf{j})\\ &= 20\alpha_{DE}\mathbf{j} + 25\alpha_{DE}\mathbf{i} - 819.20\mathbf{i} + 1024\mathbf{j}\\ &= (25\alpha_{DE} - 819.20)\mathbf{i} + (20\alpha_{DE} + 1024)\mathbf{j} \qquad (2)\end{aligned}$$

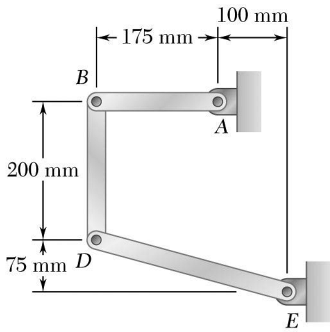
Equate like components of $\mathbf{a}_D$ expressed by Eqs. (1) and (2).

i: $\quad -841.60 = 25\alpha_{DE} - 819.20 \qquad \alpha_{DE} = -0.896 \text{ rad/s}^2$

j: $\quad 680 + 40\alpha_{BD} = (20)(-0.896) + 1024 \qquad \alpha_{BD} = 8.152 \text{ rad/s}^2$

(*a*) Angular acceleration of bar *BD*. $\qquad \boldsymbol{\alpha}_{BD} = 8.15 \text{ rad/s}^2 \circlearrowleft$ ◀

(*b*) Angular acceleration of bar *DE*. $\qquad \boldsymbol{\alpha}_{DE} = 0.896 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 15.134

Knowing that at the instant shown bar *AB* has an angular velocity of 4 rad/s and an angular acceleration of 2 rad/s^2, both clockwise, determine the angular acceleration (*a*) of bar BD, (*b*) of bar *DE* by using the vector approach as is done in Sample Problem 15.8.

SOLUTION

Velocity analysis.

Bar *AB* (Rotation about *A*): $\boldsymbol{\omega}_{AB} = 4 \text{ rad/s} \circlearrowright = -(4 \text{ rad/s})\mathbf{k}$

$$\mathbf{r}_{B/A} = -(175 \text{ mm})\mathbf{i} \qquad \mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (-4\mathbf{k}) \times (-175\mathbf{i})$$

$$\mathbf{v}_B = (700 \text{ mm/s})\mathbf{j}$$

Bar *BD* (Plane motion = Translation with $B +$ Rotation about B):

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \qquad \mathbf{r}_{D/B} = -(200 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B} = 700\mathbf{j} + (\omega_{BD}\mathbf{k}) \times (-200\mathbf{j})$$

$$\mathbf{v}_D = 700\mathbf{j} + 200\omega_{BD}\mathbf{i}$$

Bar *DE* (Rotation about *E*): $\boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}$

$$\mathbf{r}_{D/E} = -(275 \text{ mm})\mathbf{i} + (75 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (\omega_{DE}\mathbf{k}) \times (-275\mathbf{i} + 75\mathbf{j})$$

$$\mathbf{v}_D = -275\omega_{DE}\mathbf{j} - 75\omega_{DE}\mathbf{i}$$

Equating components of the two expressions for $\mathbf{v}_D$,

$$\mathbf{j}: \quad 700 = -275\omega_{DE} \quad \omega_{DE} = -2.5455 \text{ rad/s} \qquad \boldsymbol{\omega}_{DE} = 2.55 \text{ rad/s} \circlearrowright$$

$$\mathbf{i}: \quad 200\omega_{BD} = -75\omega_{DE} \quad \omega_{BD} = -\frac{3}{8}\omega_{DE}$$

$$\omega_{BD} = -\left(\frac{3}{8}\right)(-2.5455) = 0.95455 \text{ rad/s} \qquad \boldsymbol{\omega}_{BD} = 0.955 \text{ rad/s} \circlearrowleft$$

Acceleration analysis. $\boldsymbol{\alpha}_{AB} = 2 \text{ rad/s} \circlearrowright = -(2 \text{ rad/s}^2)\mathbf{k}$

Bar *AB*:

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 r_{B/A}$$

$$= (-2\mathbf{k}) \times (-175\mathbf{i}) - (4)^2(-175\mathbf{i}) = 2800 \text{ mm/s}^2\mathbf{i} + 350 \text{ mm/s}^2\mathbf{j}$$

Bar *BD*: $\boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k}$

$$\mathbf{a}_D = \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2\mathbf{r}_{D/B}$$

$$= 2800\mathbf{i} + 350\mathbf{j} + \alpha_{BD}\mathbf{k} \times (-200\mathbf{j}) - (0.95455)^2(-200\mathbf{j})$$

$$= (2800 + 200\,\alpha_{BD})\mathbf{i} + 532.23\mathbf{j} \qquad (1)$$

PROBLEM 15.134 (Continued)

Bar DE:

$$\alpha_{DE} = \alpha_{DE}\mathbf{k}$$

$$\mathbf{a}_D = \boldsymbol{\alpha}_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2 r_{D/E}$$

$$= \alpha_{DE}\mathbf{k} \times (-275\mathbf{i} + 75\mathbf{j}) - (2.5455)^2(-275\mathbf{i} + 75\mathbf{j})$$

$$= -275\alpha_{DE}\mathbf{j} - 75\alpha_{DE}\mathbf{i} + 1781.8\mathbf{i} - 485.95\mathbf{j}$$

$$= (-75\alpha_{DE} + 1781.8)\mathbf{i} - (275\alpha_{DE} + 485.95)\mathbf{j} \qquad (2)$$

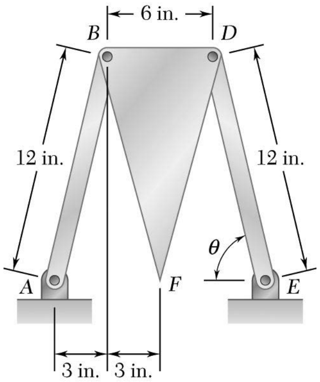
Equate like components of $\mathbf{a}_D$ expressed by Eqs. (1) and (2).

$\mathbf{j}$: $\quad 532.23 = -(275\alpha_{DE} + 485.95) \qquad \alpha_{DE} = -3.7025 \text{ rad/s}^2$

$\mathbf{i}$: $\quad (2800 + 200\alpha_{BD}) = [-(75)(-3.7025) + 1781.8] \qquad \alpha_{BD} = -3.7025 \text{ rad/s}^2$

(*a*) Angular acceleration of bar *BD*. $\quad \boldsymbol{\alpha}_{BD} = 3.70 \text{ rad/s}^2$ ↻ ◀

(*b*) Angular acceleration of bar *DE*. $\quad \boldsymbol{\alpha}_{DE} = 3.70 \text{ rad/s}^2$ ↻ ◀



PROBLEM 15.135

Robert's linkage is named after Richard Robert (1789–1864) and can be used to draw a close approximation to a straight line by locating a pen at Point *F*. The distance *AB* is the same as *BF*, *DF* and *DE*. Knowing that at the instant shown bar *AB* has a constant angular velocity of 4 rad/s clockwise, determine (*a*) the angular acceleration of bar *DE*, (*b*) the acceleration of Point *F*.

SOLUTION

Units: inches, in./s, in./s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \ \mathbf{j} = 1 \uparrow, \ \mathbf{k} = 1 \circlearrowleft.$

Geometry:

$$\mathbf{r}_{B/A} = 3\mathbf{i} + \sqrt{12^2 - 3^2}\,\mathbf{j} = 3\mathbf{i} + \sqrt{135}\,\mathbf{j}$$

$$\mathbf{r}_{D/B} = 6\mathbf{i} \qquad \mathbf{r}_{F/B} = 3\mathbf{i} - \sqrt{135}\,\mathbf{j}$$

$$\mathbf{r}_{D/E} = -3\mathbf{i} + \sqrt{135}\,\mathbf{j}$$

Velocity analysis:

$$\boldsymbol{\omega}_{AB} = 4 \text{ rad/s}^2 \circlearrowright = -4\,\mathbf{k}$$

Bar *AB*:

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = -4\mathbf{k} \times (3\mathbf{i} + \sqrt{135}\,\mathbf{j}) = 4\sqrt{135}\,\mathbf{i} - 12\mathbf{j}$$

Object *BDF*:

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B}$$

$$= 4\sqrt{135}\,\mathbf{i} - 12\mathbf{j} + \omega_{BD}\mathbf{k} \times 6\mathbf{i}$$

$$= 4\sqrt{135}\,\mathbf{i} - 12\mathbf{j} + 6\omega_{BD}\mathbf{j} \qquad (1)$$

Bar *DE*:

$$\mathbf{v}_D = \omega_{DE} \times \mathbf{r}_{D/E} = \omega_{DE}\mathbf{k} \times (-3\mathbf{i} + \sqrt{135}\,\mathbf{j})$$

$$= -\sqrt{135}\,\omega_{DE}\mathbf{i} - 3\omega_{DE}\mathbf{j} \qquad (2)$$

Equating like components of $\mathbf{v}_D$ from Eqs. (1) and (2),

$$\mathbf{i}: \quad 4\sqrt{135} = -\sqrt{135}\,\omega_{DE} \qquad (3)$$

$$\mathbf{j}: \quad -12 + 6\omega_{BD} = -3\omega_{DE} \qquad (4)$$

From Eq. (3), $\omega_{DE} = -4 \qquad \omega_{DE} = 5 \text{ rad/s} \circlearrowright$

From Eq. (4), $\omega_{BD} = \dfrac{1}{6}(12 - 3\omega_{DE}) = 5 \qquad \omega_{BD} = 4 \text{ rad/s} \circlearrowright$

PROBLEM 15.135 (Continued)

Acceleration Analysis: $\boldsymbol{\alpha}_{AB} = 0$

Bar *AB*:

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$
$$= 0 - (4)^2(3\mathbf{i} + \sqrt{135}\mathbf{j})$$
$$= -48\mathbf{i} - 16\sqrt{135}\mathbf{j}$$

Object *BDF*:

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} = \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2 \mathbf{r}_{D/B}$$

$$\mathbf{a}_D = -48\mathbf{i} - 16\sqrt{135}\mathbf{j} + \alpha_{BD}\mathbf{k} \times (6\mathbf{i}) - (4)^2(6\mathbf{i})$$
$$= -144\mathbf{i} - 16\sqrt{135}\mathbf{j} + 6\alpha_{BD}\mathbf{j} \qquad (5)$$

Bar *DE*:

$$\mathbf{a}_D = \boldsymbol{\alpha}_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2 \mathbf{r}_{D/E}$$

$$\mathbf{a}_D = \alpha_{DE}\mathbf{k} \times (-3\mathbf{i} + \sqrt{135}\mathbf{j}) - (4)^2(-3\mathbf{i} + \sqrt{135}\mathbf{j})$$
$$= -\sqrt{135}\alpha_{DE}\mathbf{i} - 3\alpha_{DE}\mathbf{j} + 48\mathbf{i} - 16\sqrt{135}\mathbf{j} \qquad (6)$$

Equating like components of $\mathbf{a}_D$ from Eqs. (5) and (6),

$$\mathbf{i}: \quad -144 = -\sqrt{135}\alpha_{DE} + 48 \qquad (7)$$

$$\mathbf{j}: \quad -16\sqrt{135} + 6\alpha_{BD} = -3\alpha_{DE} - 16\sqrt{135} \qquad (8)$$

From Eq. (7), $\alpha_{DE} = \dfrac{192}{\sqrt{135}}$

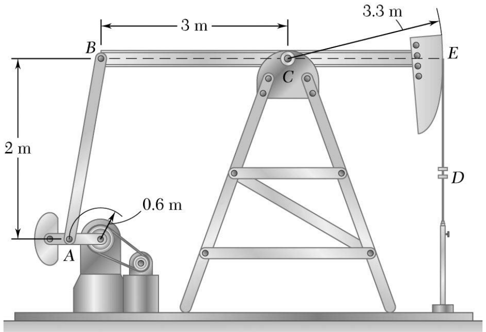
From Eq. (8), $\alpha_{BD} = -\dfrac{1}{2}\alpha_{BD} = -\dfrac{96}{\sqrt{135}}$

(*a*) Angular acceleration of bar *DE*: $\boldsymbol{\alpha}_{DE} = 16.53 \text{ rad/s}^2$ ↺ ◀

(*b*) Acceleration of Point *F*:

$$\mathbf{a}_F = \mathbf{a}_B + \mathbf{a}_{F/B} = \mathbf{a}_B + \alpha_{BD} \times \mathbf{r}_{F/B} - \omega_{BD}^2 \mathbf{r}_{F/B}$$
$$= -48\mathbf{i} - 16\sqrt{135}\mathbf{j} + \left(-\frac{96}{\sqrt{135}}\mathbf{k}\right) \times (3\mathbf{i} - \sqrt{135}\mathbf{j}) - (4)^2(3\mathbf{i} - \sqrt{135}\mathbf{j})$$
$$= -48\mathbf{i} - 16\sqrt{135}\mathbf{j} - \frac{288}{\sqrt{135}}\mathbf{j} - 96\mathbf{i} - 48\mathbf{i} + 16\sqrt{135}\mathbf{j}$$
$$= -192\mathbf{i} - \frac{288}{\sqrt{135}}\mathbf{j}$$

$$\mathbf{a}_F = -(192.0 \text{ in./s}^2)\mathbf{i} - (24.8 \text{ in./s}^2)\mathbf{j} = 193.6 \text{ in./s}^2 \measuredangle 7.36° \text{ ◀}$$



PROBLEM 15.136

For the oil pump rig shown, link AB causes the beam BCE to oscillate as the crank OA revolves. Knowing that OA has a radius of 0.6 m and a constant clockwise angular velocity of 20 rpm, determine the velocity and acceleration of Point D at the instant shown.

SOLUTION

Units: meters, m/s, m/s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1 \uparrow, \quad \mathbf{k} = 1 \circlearrowleft.$

Crank OA: $\mathbf{r}_{OA} = 0.6 \text{ m}, \quad \boldsymbol{\omega}_{OA} = 20 \text{ rpm} \circlearrowright = 2.0944 \text{ rad/s} \circlearrowright$

$$\mathbf{v}_A = \omega_{OA} r_{OA} = (2.0944)(0.6) \qquad \mathbf{v}_A = 1.25664 \text{ m/s} \uparrow$$

$$\boldsymbol{\alpha}_{OA} = 0 \qquad (a_A)_t = 0$$

$$(\mathbf{a}_A)_n = \omega_{OA}^2 r_{OA} = (2.0944)^2 (0.6) = 2.6319 \text{ m/s}^2$$

$$\mathbf{a}_A = 2.6319 \text{ m/s}^2 \longrightarrow$$

Rod AB: $\mathbf{v}_B = v_A \uparrow,$

Since $\mathbf{v}_B$ and $\mathbf{v}_A$ are parallel, $\mathbf{v}_A = \mathbf{v}_B$ and $\omega_{AB} = 0.$

$$\mathbf{v}_B = 1.25664 \text{ m/s} \uparrow$$

$$\begin{aligned}\mathbf{a}_B &= \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_A + \alpha_{AB}\mathbf{k} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} \\ &= 2.6319\mathbf{i} + \alpha_{AB}\mathbf{k} \times (0.6\mathbf{i} + 2\mathbf{j}) - 0 \\ &= (2.6319 - 2\alpha_{AB})\mathbf{i} + 0.6\alpha_{AB}\mathbf{j} \qquad (1)\end{aligned}$$

Beam BCE: Point C is a pivot.

$$v_B = \omega_{BCE} r_{BC} \qquad \omega_{BCE} = \frac{v_B}{r_{BC}} = \frac{1.25664}{3} = 0.41888$$

$$v_E = \omega_{BCE} r_{CE} = (0.41888)(3.3) = 1.38230$$

$$\boldsymbol{\omega}_{BCE} = 0.41888 \text{ rad/s} \circlearrowright \qquad \mathbf{v}_E = 1.38230 \text{ m/s} \downarrow$$

$$\begin{aligned}\mathbf{a}_B &= \boldsymbol{\alpha}_{BCE} \times \mathbf{r}_{B/C} - \omega_{BCE}^2 r_{B/C} \\ &= \alpha_{BCE}\mathbf{k} \times (-3\mathbf{i}) - (0.41888)^2(-3\mathbf{i}) \\ &= 0.52638\mathbf{i} - 3\alpha_{BCE}\mathbf{j} \qquad (2)\end{aligned}$$

Equating like components of $\boldsymbol{\alpha}_B$ expressed by Eqs. (1) and (2),

i: $2.6319 - 2\alpha_{AB} = 0.52638$ $\qquad \alpha_{AB} = 1.05276 \text{ rad/s}$

j: $0.6\alpha_{AB} = -3\alpha_{BCE}$ $\qquad \alpha_{BCE} = -0.21055 \text{ rad/s}^2$

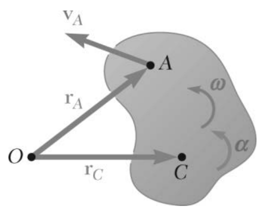
$$\boldsymbol{\alpha}_{AB} = 1.055276 \text{ rad/s} \circlearrowleft \qquad \boldsymbol{\alpha}_{BCE} = 0.21055 \text{ rad/s}^2 \circlearrowright$$

$$\mathbf{a}_E = (\mathbf{a}_{E/C})_t + (\mathbf{a}_{E/C})_n$$
$$(\mathbf{a}_{E/C})_t = \boldsymbol{\alpha}_{BCE} \times \mathbf{r}_{E/C} = (-0.21055\mathbf{k}) \times (3.3\mathbf{i})$$
$$= -(0.69482 \text{ m/s}^2)\mathbf{j}$$

String *ED*: $\mathbf{v}_D = \mathbf{v}_E$ $\qquad \mathbf{v}_D = 1.382 \text{ m/s} \downarrow$ ◀

$\mathbf{a}_D = (\mathbf{a}_{E/C})_t = -(0.69482 \text{ m/s}^2)\mathbf{j}$ $\qquad \mathbf{a}_D = 0.695 \text{ m/s}^2 \downarrow$ ◀

PROBLEM 15.137



Denoting by $\mathbf{r}_A$ the position vector of Point A of a rigid slab that is in plane motion, show that (a) the position vector $\mathbf{r}_C$ of the instantaneous center of rotation is

$$\mathbf{r}_C = \mathbf{r}_A + \frac{\boldsymbol{\omega} \times \mathbf{v}_A}{\omega^2}$$

where $\boldsymbol{\omega}$ is the angular velocity of the slab and $\mathbf{v}_A$ is the velocity of Point A, (b) the acceleration of the instantaneous center of rotation is zero if, and only if,

$$\mathbf{a}_A = \frac{\alpha}{\omega}\mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{v}_A$$

where $\boldsymbol{\alpha} = \alpha\mathbf{k}$ is the angular acceleration of the slab.

SOLUTION

(a) At the instantaneous center C, $\mathbf{v}_C = 0$

$$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{A/C} = \boldsymbol{\omega} \times \mathbf{r}_{A/C}$$

$$\boldsymbol{\omega} \times \mathbf{v}_A = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/C}) = -\omega^2 \mathbf{r}_{A/C}$$

$$\mathbf{r}_{A/C} = -\frac{\boldsymbol{\omega} \times \mathbf{v}_A}{\omega^2} = -\mathbf{r}_{C/A} \quad \text{or} \quad \mathbf{r}_{C/A} = \frac{\boldsymbol{\omega} \times \mathbf{v}_A}{\omega^2}$$

$$\mathbf{r}_C - \mathbf{r}_A = -\frac{\boldsymbol{\omega} \times \mathbf{v}_A}{\omega^2}$$

$$\mathbf{r}_C = \mathbf{r}_A + \frac{\boldsymbol{\omega} \times \mathbf{v}_A}{\omega^2} \blacktriangleleft$$

(b)

$$\mathbf{a}_A = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{A/C} + \boldsymbol{\omega} \times \mathbf{v}_{A/C}$$

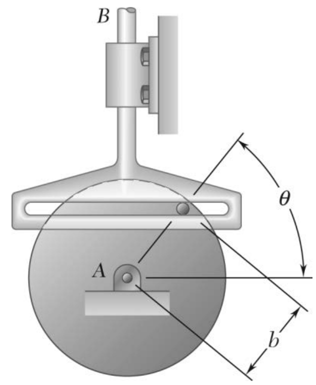
$$= \mathbf{a}_C - \alpha\mathbf{k} \times \frac{\boldsymbol{\omega} \times \mathbf{v}_A}{\omega^2} + \boldsymbol{\omega} \times (\mathbf{v}_A - \mathbf{v}_C)$$

$$= \mathbf{a}_C - \frac{\alpha\omega}{\omega^2}\mathbf{k} \times (\mathbf{k} \times \mathbf{v}_A) + \boldsymbol{\omega} \times \mathbf{v}_A$$

$$= \mathbf{a}_C + \frac{\alpha}{\omega}\mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{v}_A$$

Set $\mathbf{a}_C = 0$.

$$\mathbf{a}_A = \frac{\alpha}{\omega}\mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{v}_A \blacktriangleleft$$

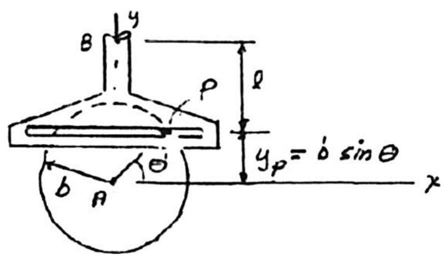


PROBLEM 15.138*

The drive disk of the scotch crosshead mechanism shown has an angular velocity ω and an angular acceleration α, both directed counterclockwise. Using the method of Section 15.9, derive expressions for the velocity and acceleration of Point *B*.

SOLUTION

Origin at *A*.



$$y_B = l + y_P = l + b\sin\theta$$

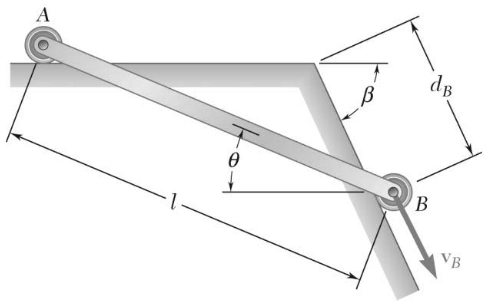
$$v_B = \dot{y}_B = b\cos\theta\dot{\theta} = b\cos\theta\,\omega \qquad v_B = b\omega\cos\theta \blacktriangleleft$$

$$a_B = \ddot{y}_B$$

$$= \frac{d}{dt}v_B$$

$$= \frac{d}{dt}(b\cos\theta\dot{\theta})$$

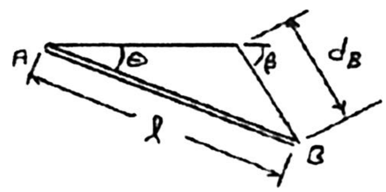
$$a_B = -b\sin\theta\dot{\theta}^2 + b\cos\theta\ddot{\theta} \qquad a_B = b\alpha\cos\theta - b\omega^2\sin\theta \blacktriangleleft$$



PROBLEM 15.139*

The wheels attached to the ends of rod AB roll along the surfaces shown. Using the method of Section 15.9, derive an expression for the angular velocity of the rod in terms of v_B, θ, l, and β.

SOLUTION



Law of sines.

$$\frac{d_B}{\sin\theta} = \frac{l}{\sin\beta}$$

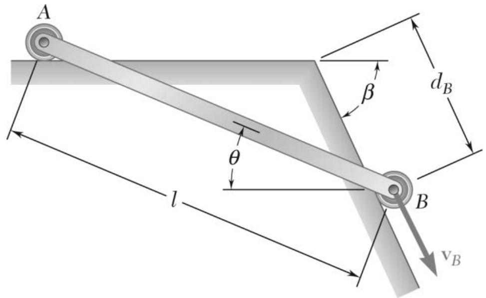
$$d_B = \frac{l}{\sin\beta}\sin\theta$$

$$v_B = \frac{d}{dt}(d_B)$$

$$= \frac{l}{\sin\beta}\cos\theta\frac{d\theta}{dt}$$

$$= \frac{l}{\sin\beta}\cos\theta\,\omega$$

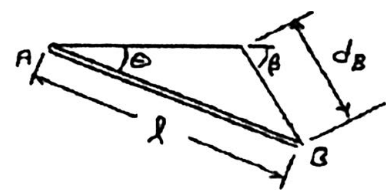
$$\omega = \frac{v_B\sin\beta}{l\cos\theta} \blacktriangleleft$$



PROBLEM 15.140*

The wheels attached to the ends of rod AB roll along the surfaces shown. Using the method of Section 15.9 and knowing that the acceleration of wheel B is zero, derive an expression for the angular acceleration of the rod in terms of $v_B, \theta, l,$ and β.

SOLUTION



Law of sines.

$$\frac{d_B}{\sin\theta} = \frac{l}{\sin\beta}$$

$$d_B = \frac{l}{\sin\beta}\sin\theta$$

$$v_B = \frac{d}{dt}(d_B)$$

$$= \frac{l}{\sin\beta}\cos\theta\frac{d\theta}{dt}$$

$$= \frac{l}{\sin\beta}\cos\theta\,\omega$$

$$\omega = \frac{v_B\sin\beta}{l\cos\theta}$$

Note that

$$a_B = \frac{dv_B}{dt} = 0.$$

$$\alpha = \frac{d\omega}{dt} = \frac{v_B\sin\beta}{l}\cdot\frac{\sin\theta}{\cos^2\theta}\cdot\frac{d\theta}{dt}$$

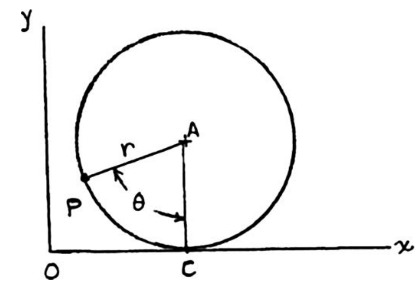
$$\alpha = \frac{v_B\sin\beta\sin\theta}{l\cos^2\theta}\cdot\frac{v_B\sin\beta}{l\cos\theta}$$

$$\alpha = \left[\frac{v_B\sin\beta}{l}\right]^2\frac{\sin\theta}{\cos^3\theta} \blacktriangleleft$$

PROBLEM 15.141*

A disk of radius r rolls to the right with a constant velocity $\mathbf{v}$. Denoting by P the point of the rim in contact with the ground at $t = 0$, derive expressions for the horizontal and vertical components of the velocity of P at any time t.

SOLUTION



$$x_A = r\theta, \quad y_A = r$$

$$x_P = x_A - r\sin\theta$$

$$= r\theta - r\sin\theta$$

$$y_P = y_A - r\cos\theta$$

$$= r - r\cos\theta$$

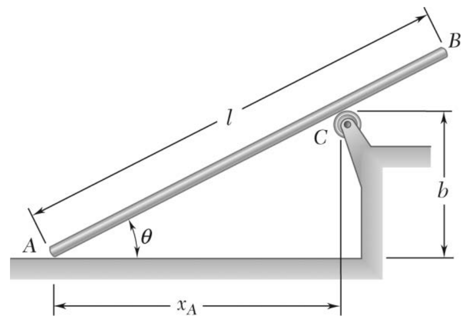
$$\theta = \frac{x_A}{r}$$

$$\dot{x}_A = v, \quad \dot{y}_A = 0, \quad \dot{\theta} = \frac{v}{r}$$

$$x_A = vt, \quad \theta = \frac{vt}{r}$$

$$\dot{x}_P = v_x = r\dot{\theta} - r\cos\theta\dot{\theta} = r\left(1 - \cos\frac{vt}{r}\right)\frac{v}{r} \qquad v_x = v\left(1 - \cos\frac{vt}{r}\right) \blacktriangleleft$$

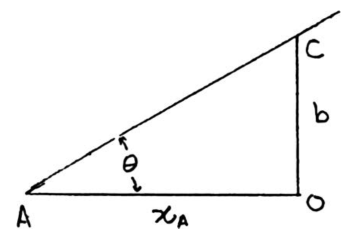
$$\dot{y}_P = v_y = r\sin\theta\dot{\theta} = r\left(\sin\frac{vt}{r}\right)\frac{v}{r} \qquad v_y = v\sin\frac{vt}{r} \blacktriangleleft$$



PROBLEM 15.142*

Rod AB moves over a small wheel at C while end A moves to the right with a constant velocity $\mathbf{v}_A$. Using the method of Section 15.9, derive expressions for the angular velocity and angular acceleration of the rod.

SOLUTION



$$\tan\theta = \frac{b}{x_A} \qquad \cot\theta = \frac{x_A}{b} = u$$

$$\theta = \cot^{-1} u$$

$$\dot{\theta} = -\frac{\dot{u}}{1+u^2}$$

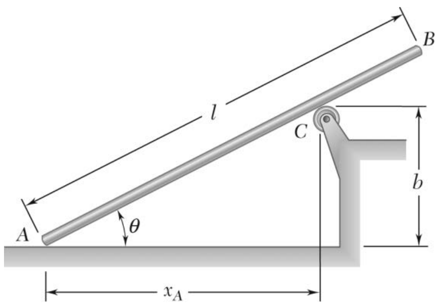
$$\ddot{\theta} = \frac{(2u\dot{u})\dot{u}}{(1+u^2)^2} - \frac{\ddot{u}}{1+u^2}$$

But $\qquad \omega = \dot{\theta} \quad \text{and} \quad \alpha = \ddot{\theta}$

$$u = \frac{x_A}{b}, \quad \dot{u} = \frac{\dot{x}_A}{b} = -\frac{v_A}{b}, \quad \ddot{u} = -\frac{\dot{v}_A}{b} = 0$$

Then $\qquad \omega = \frac{\frac{v_A}{b}}{1+\left(\frac{x_A}{b}\right)^2} = \frac{bv_A}{b^2+x_A^2},$ $\qquad \omega = \frac{bv_A}{b^2+x_A^2}$ ↺ ◀

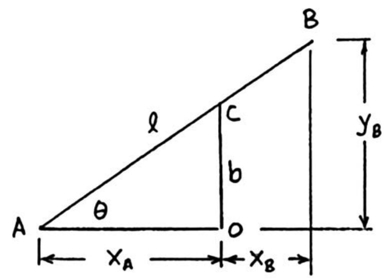
$$\alpha = \frac{2\left(\frac{x_A}{b}\right)\left(\frac{v_A}{b}\right)^2}{\left[1+\left(\frac{x_A}{b}\right)^2\right]^2} - 0 = \frac{2bx_Av_A^2}{\left(b^2+x_A^2\right)^2},$$ $\qquad \alpha = \frac{2bx_Av_A^2}{\left(b^2+x_A^2\right)^2}$ ↺ ◀



PROBLEM 15.143*

Rod AB moves over a small wheel at C while end A moves to the right with a constant velocity $\mathbf{v}_A$. Using the method of Section 15.9, derive expressions for the horizontal and vertical components of the velocity of Point B.

SOLUTION



$$\sin\theta = \frac{b}{\left(b^2 + x_A^2\right)^{1/2}}, \qquad \cos\theta = \frac{x_A}{\left(b^2 + x_A^2\right)^{1/2}}$$

$$x_B = l\cos\theta - x_A$$

$$= \frac{lx_A}{\left(b^2 + x_A^2\right)^{1/2}} - x_A$$

$$y_B = l\sin\theta = \frac{lb}{\left(b^2 + x_A^2\right)^{1/2}}$$

$$\dot{x}_B = \frac{l\dot{x}_A}{\left(b^2 + x_A^2\right)^{1/2}} - \frac{lx_A x_A \dot{x}_A}{\left(b^2 + x_A^2\right)^{3/2}} - \dot{x}_A$$

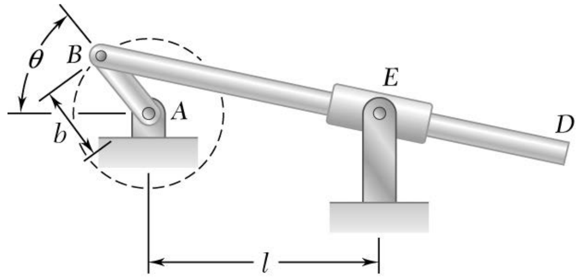
$$= \frac{lb^2\dot{x}_A}{\left(b^2 + x_A^2\right)^{3/2}} - \dot{x}_A$$

$$\dot{y}_B = -\frac{lbx_A\dot{x}_A}{\left(b^2 + x_A^2\right)^{3/2}}$$

But $\quad \dot{x}_A = -v_A, \quad \dot{x}_B = (v_B)_x, \quad \dot{y}_B = (v_B)_y$

$$(v_B)_x = v_A - \frac{lb^2 v_A}{\left(b^2 + x_A^2\right)^{3/2}} \longrightarrow \blacktriangleleft$$

$$(v_B)_y = \frac{lbx_A v_A}{\left(b^2 + x_A^2\right)^{3/2}} \uparrow \blacktriangleleft$$

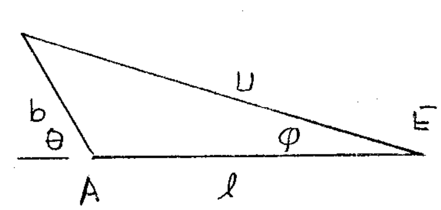


PROBLEM 15.144

Crank AB rotates with a constant clockwise angular velocity $\boldsymbol{\omega}$. Using the method of Section 15.9, derive expressions for the angular velocity of rod BD and the velocity of the point on the rod coinciding with Point E in terms of θ, ω, b, and l.

SOLUTION

Law of cosines for triangle ABE.



$$u^2 = l^2 + b^2 - 2bl\cos(180° - \theta)$$
$$= l^2 + b^2 + 2bl\cos\theta$$
$$\cos\varphi = \frac{l + b\cos\theta}{u}$$
$$\tan\varphi = \frac{b\sin\theta}{l + b\cos\theta}$$

$$\frac{d}{dt}(\tan\varphi) = \sec^2\varphi\dot{\varphi} = \frac{(l + b\cos\theta)(b\cos\theta)\dot{\theta} + (b\sin\theta)(b\cos\theta)\dot{\theta}}{(l + b\cos\theta)^2}$$

$$\dot{\varphi} = \frac{(\cos^2\varphi)[bl\cos\theta + b^2(\cos^2\theta + \sin^2\theta)]\dot{\theta}}{(l + b\cos\theta)^2}$$
$$= \frac{bl\cos\theta + b^2}{u^2}\dot{\theta} = \frac{b(b + l\cos\theta)}{l^2 + b^2 + 2bl\cos\theta}\dot{\theta}$$

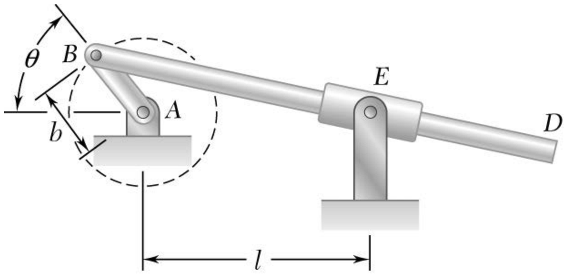
But, $\quad \dot{\theta} = \omega, \quad \dot{\varphi} = \omega_{BD}, \quad$ and $\quad v_E = -\dot{u}$

Hence,
$$\omega_{BD} = \frac{b(b + l\cos\theta)}{l^2 + b^2 + 2bl\cos\theta}\omega \circlearrowright ◀$$

Differentiate the expression for u^2.

$$2u\dot{u} = -2bl\sin\theta\dot{\theta}$$
$$v_E = -\dot{u} = \frac{bl\sin\theta}{l^2 + b^2 + 2bl\cos\theta}\omega$$

$$\mathbf{v}_E = \frac{bl\sin\theta}{l^2 + b^2 + 2bl\cos\theta}\omega \nwarrow \tan^{-1}\left(\frac{b\sin\theta}{l + b\cos\theta}\right) ◀$$

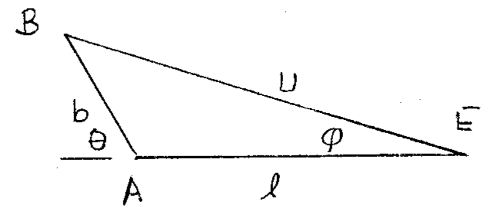


PROBLEM 15.145

Crank *AB* rotates with a constant clockwise angular velocity **ω**. Using the method of Section 15.9, derive an expression for the angular acceleration of rod *BD* in terms of θ, ω, b, and l.

SOLUTION

Law of cosines for triangle *ABE*.



$$u^2 = l^2 + b^2 - 2bl\cos(180° - \theta)$$
$$= l^2 + b^2 + 2bl\cos\theta$$
$$\cos\varphi = \frac{l + b\cos\theta}{u}$$
$$\tan\varphi = \frac{b\sin\theta}{l + b\cos\theta}$$

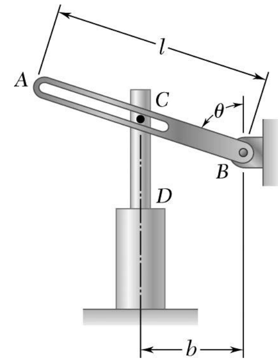
$$\frac{d}{dt}(\tan\varphi) = \sec^2\varphi\dot{\varphi} = \frac{(l + b\cos\theta)(b\cos\theta)\dot{\theta} + (b\sin\theta)(b\cos\theta)\dot{\theta}}{(l + b\cos\theta)^2}$$

$$\dot{\varphi} = \frac{(\cos^2\varphi)[bl\cos\theta + b^2(\cos^2\theta + \sin^2\theta)]\dot{\theta}}{(l + b\cos\theta)^2}$$
$$= \frac{bl\cos\theta + b^2}{u^2}\dot{\theta} = \frac{b(b + l\cos\theta)}{l^2 + b^2 + 2bl\cos\theta}\dot{\theta}$$
$$\ddot{\varphi} = \frac{b(b + l\cos\theta)}{l^2 + b^2 + 2bl\cos\theta}\ddot{\theta}$$
$$+\frac{(l^2 + b^2 + 2bl\cos\theta)(-bl\sin\theta) - b(b + l\cos\theta)(-2bl\sin\theta)}{(l^2 + b^2 + 2bl\cos\theta)^2}\dot{\theta}^2$$
$$= \frac{b(b + l\cos\theta)}{l^2 + b^2 + 2bl\cos\theta}\ddot{\theta} - \frac{bl(l^2 - b^2)\sin\theta}{(l^2 + b^2 + 2bl\cos\theta)^2}\dot{\theta}^2$$

But, $\dot{\theta} = \omega,\quad \ddot{\theta} = \dot{\omega} = 0,\quad \ddot{\varphi} = \alpha_{BD}$

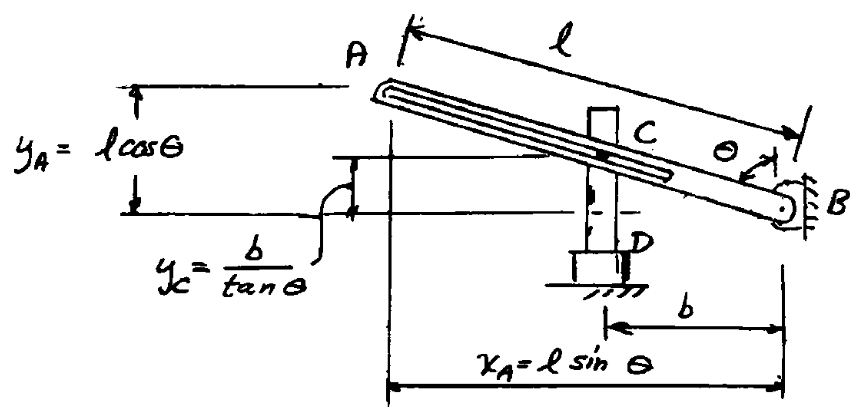
$$\alpha_{BD} = \frac{bl(l^2 - b^2)\sin\theta}{l^2 + b^2 + 2bl\cos\theta}\omega^2 ⟲ ◀$$

PROBLEM 15.146*



Pin C is attached to rod CD and slides in a slot cut in arm AB. Knowing that rod CD moves vertically upward with a constant velocity $\mathbf{v}_0$, derive an expression for (*a*) the angular velocity of arm AB, (*b*) the components of the velocity of Point A; and (*c*) an expression for the angular acceleration of arm AB.

SOLUTION



(*a*)

$$y_C = \frac{b\cos\theta}{\sin\theta}$$

$$v_C = v_0 = \frac{dy_C}{dt} = b\frac{d}{d\theta}\left(\frac{\cos\theta}{\sin\theta}\right)\frac{d\theta}{dt}$$

$$= b\frac{-\sin^2\theta - \cos^2\theta}{\sin^2\theta}\dot{\theta} = -\frac{b}{\sin^2\theta}\dot{\theta}$$

$$\dot{\theta} = -\frac{v_0\sin^2\theta}{b} \qquad (1)$$

But $\omega = \dot{\theta}$ $\qquad \boldsymbol{\omega} = \frac{v_0}{b}\sin^2\theta$ ↻ ◀

(*b*)

$$x_A = l\sin\theta$$

$$\dot{x}_A = l\cos\theta\dot{\theta} = -\frac{v_0 l}{b}(\sin^2\theta\cos\theta)$$

$$y_A = l\cos\theta$$

$$\dot{y}_A = -l\sin\theta\dot{\theta} = \frac{v_0 l}{b}\sin^3\theta \uparrow$$

PROBLEM 15.146* (Continued)

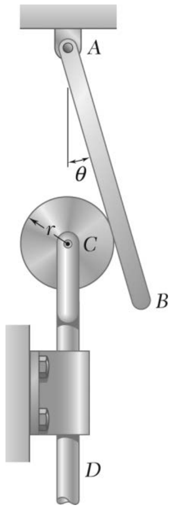
Components: $\mathbf{v}_A = \dfrac{v_0 l}{b}\sin^2\theta\cos\theta \longrightarrow + \dfrac{v_0 l}{b}\sin^3\theta \uparrow$ ◀

(*c*) Differentiating Eq. (1),

$$\ddot{\theta} = \frac{d}{dt}\left(-\frac{v_0 \sin^2\theta}{b}\right) = -\frac{2v_0 \sin\theta\cos\theta}{b}\frac{d\theta}{dt}$$

$$= -\left(\frac{2v_0 \sin\theta\cos\theta}{b}\right)\left(-\frac{v_0 \sin^2\theta}{b}\right) = \frac{2v_0^2}{b^2}\sin^3\theta\cos\theta$$

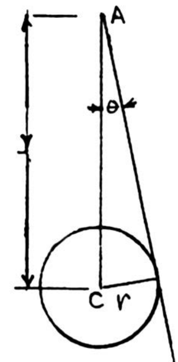
$$\boldsymbol{\alpha} = \ddot{\theta} \circlearrowleft \qquad \boldsymbol{\alpha} = \frac{2v_0^2}{b^2}\sin^3\theta\cos\theta \circlearrowleft$$ ◀



PROBLEM 15.147*

The position of rod AB is controlled by a disk of radius r which is attached to yoke CD. Knowing that the yoke moves vertically upward with a constant velocity $\mathbf{v}_0$, derive expression for the angular velocity and angular acceleration of rod AB.

SOLUTION



From geometry,

$$y = \frac{r}{\sin\theta}$$

$$\frac{dy}{dt} = -\frac{r\cos\theta}{\sin^2\theta}\frac{d\theta}{dt}$$

But,

$$\frac{dy}{dt} = -v_0 \quad \text{and} \quad \frac{d\theta}{dt} = \omega$$

$$v_0 = \frac{r\cos\theta}{\sin^2\theta}\omega$$

$$\omega = \frac{v_0}{r}\frac{\sin^2\theta}{\cos\theta}$$

$$\boldsymbol{\omega} = \frac{v_0}{r}\frac{\sin^2\theta}{\cos\theta} \circlearrowleft \blacktriangleleft$$

From geometry,

$$y = \frac{r}{\sin\theta}$$

$$\frac{dy}{dt} = -\frac{r\cos\theta}{\sin^2\theta}\frac{d\theta}{dt}$$

But,

$$\frac{dy}{dt} = -v_0 \quad \text{and} \quad \frac{d\theta}{dt} = \omega$$

$$v_0 = \frac{r\cos\theta}{\sin^2\theta}\omega$$

$$\omega = \frac{v_0}{r}\frac{\sin^2\theta}{\cos\theta}$$

PROBLEM 15.147* (Continued)

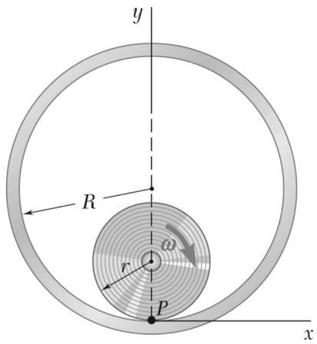
Angular acceleration.

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \frac{d\omega}{d\theta}\omega$$

$$= \frac{v_0}{r}\frac{(2\cos^2\theta\sin\theta + \sin^3\theta)}{\cos^2\theta}\left(\frac{v_0}{r}\frac{\sin^2\theta}{\cos\theta}\right)$$

$$= \left(\frac{v_0}{r}\right)^2 \frac{(1+\cos^2\theta)\sin^3\theta}{\cos^3\theta}$$

$$\boldsymbol{\alpha} = \left(\frac{v_0}{r}\right)^2 (1+\cos^2\theta)\tan^3\theta \circlearrowright \blacktriangleleft$$



PROBLEM 15.148*

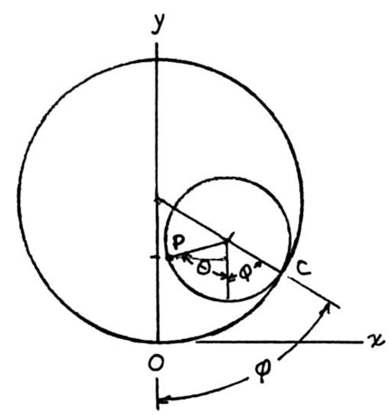
A wheel of radius r rolls without slipping along the inside of a fixed cylinder of radius R with a constant angular velocity $\boldsymbol{\omega}$. Denoting by P the point of the wheel in contact with the cylinder at $t = 0$, derive expressions for the horizontal and vertical components of the velocity of P at any time t. (The curve described by Point P is a *hypocycloid*.)

SOLUTION

Define angles θ and φ as shown.

$$\dot{\theta} = \omega, \qquad \theta = \omega t$$

Since the wheel rolls without slipping, the arc OC is equal to arc PC.



$$r(\varphi + \theta) = R\varphi$$

$$\varphi = \frac{r\theta}{R-r}$$

$$\dot{\varphi} = \frac{r\dot{\theta}}{R-r} = \frac{r\omega}{R-r}$$

$$\varphi = \frac{r\omega t}{R-r}$$

$$x_P = (R-r)\sin\varphi - r\sin\theta$$

$$(v_P)_x = \dot{x}_P$$

$$= (R-r)\cos\varphi\dot{\varphi} - r\cos\theta\dot{\theta}$$

$$= (R-r)\left(\cos\frac{r\omega t}{R-r}\right)\left(\frac{r\omega}{R-r}\right) - r(\cos\omega t)(\omega)$$

$$(v_P)_x = r\omega\left(\cos\frac{r\omega t}{R-r} - \cos\omega t\right) \blacktriangleleft$$

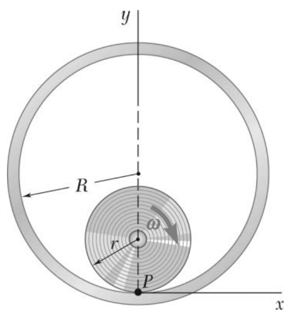
$$y_P = R - (R-r)\cos\varphi - r\cos\theta$$

$$(v_P)_y = \dot{y}_P$$

$$= (R-r)\sin\varphi\dot{\varphi} + r\sin\theta\dot{\theta}$$

$$= (R-r)\left(\sin\frac{r\omega t}{R-r}\right)\left(\frac{r\omega}{R-r}\right) + r(\sin\omega t)(\omega)$$

$$(v_P)_y = r\omega\left(\sin\frac{r\omega t}{R-r} + \sin\omega t\right) \blacktriangleleft$$

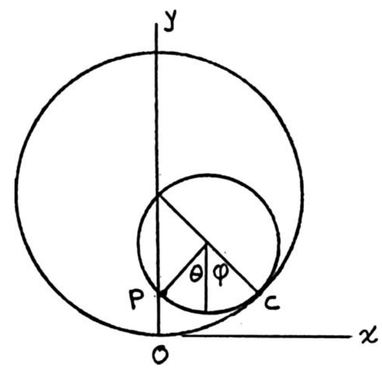


PROBLEM 15.149*

In Problem 15.148, show that the path of P is a vertical straight line when $r = R/2$. Derive expressions for the corresponding velocity and acceleration of P at any time t.

SOLUTION

Define angles θ and φ as shown.



$$\dot{\theta} = \omega, \quad \theta = \omega t, \quad \ddot{\theta} = 0$$

Since the wheel rolls without slipping, the arc OC is equal to arc PC.

$$r(\varphi + \theta) = R\theta$$
$$= 2r\theta$$
$$\varphi = 0$$
$$\dot{\varphi} = \dot{\theta} = \omega$$
$$\ddot{\varphi} = \ddot{\theta} = 0$$

$$x_P = (R - r)\sin\varphi - r\sin\theta$$
$$= r\sin\theta - r\sin\theta$$
$$= 0$$

The path is the y axis. ◀

$$y_P = R - (R - r)\cos\varphi - r\cos\theta$$
$$= R - r\cos\theta - r\cos\theta$$
$$= R(1 - \cos\theta)$$
$$v = \dot{y}_P = R\sin\theta\dot{\theta}$$

$\mathbf{v} = (R\omega \sin \omega t)\mathbf{j}$ ◀

$$a = \dot{v}$$
$$= (R\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta})$$
$$= R\omega^2\cos\theta$$

$\mathbf{a} = (R\omega^2 \cos \omega t)\mathbf{j}$ ◀

PROBLEM 15.CQ8

A person walks radially inward on a platform that is rotating counterclockwise about its center. Knowing that the platform has a constant angular velocity **ω** and the person walks with a constant speed **u** relative to the platform, what is the direction of the acceleration of the person at the instant shown?

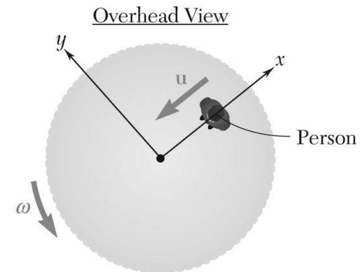
(*a*) Negative *x*

(*b*) Negative *y*

(*c*) Negative *x* and positive *y*

(*d*) Positive *x* and positive *y*

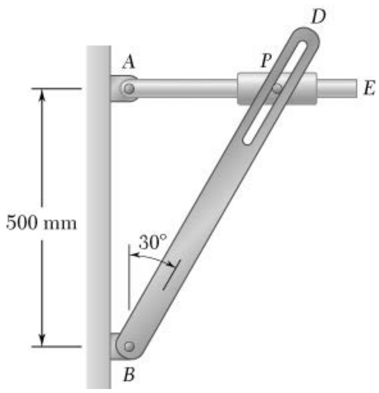
(*e*) Negative *x* and negative *y*



SOLUTION

The $\omega^2 r$ term will be in the negative *x*-direction and the Coriolis acceleration will be in the negative *y*-direction.

Answer: (*e*) ◀



PROBLEM 15.150

Pin P is attached to the collar shown; the motion of the pin is guided by a slot cut in rod BD and by the collar that slides on rod AE. Knowing that at the instant considered the rods rotate clockwise with constant angular velocities, determine for the given data the velocity of pin P.

$\omega_{AE} = 8$ rad/s, $\omega_{BD} = 3$ rad/s

SOLUTION

$$AB = 500 \text{ mm} = 0.5 \text{ m}, \quad AP = 0.5\tan 30°, \quad BP = \frac{0.5}{\cos 30°}$$

$$\boldsymbol{\omega}_{AE} = 8 \text{ rad/s} \circlearrowright, \qquad \boldsymbol{\omega}_{BD} = 3 \text{ rad/s} \circlearrowright$$

Let P' be the coinciding point on AE and u_1 be the outward velocity of the collar along the rod AE.

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/AE} = [(AP)\omega_{AE} \downarrow] + [u_1 \longrightarrow]$$

Let P'' be the coinciding point on BD and u_2 be the outward speed along the slot in rod BD.

$$\mathbf{v}_P = \mathbf{v}_{P''} + \mathbf{v}_{P/BD} = [(BP)\omega_{BD} \nwarrow 30°] + [u_2 \nearrow 60°]$$

Equate the two expressions for $\mathbf{v}_P$ and resolve into components.

$$\xrightarrow{+}: \quad u_1 = \left(\frac{0.5}{\cos 30°}\right)(3)(\cos 30°) + u_2 \cos 60°$$

or

$$u_1 = 1.5 + 0.5u_2 \qquad (1)$$

$$+\uparrow: \quad -(0.5\tan 30°)(8) = -\left(\frac{0.5}{\cos 30°}\right)(3)\sin 30° + u_2 \sin 60°$$

$$u_2 = \frac{1}{\sin 60°}[1.5\tan 30° - 4\tan 30°] = -1.66667 \text{ m/s}$$

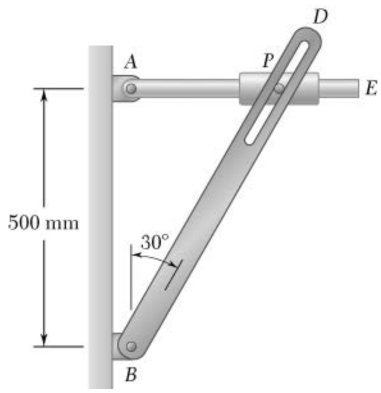
From (1), $\quad u_1 = 1.5 + (0.5)(-1.66667) = 0.66667$ m/s

$$\mathbf{v}_P = [(0.5\tan 30°)(8) \downarrow] + [0.66667 \longrightarrow] = [2.3094 \text{ m/s} \downarrow] + [0.66667 \text{ m/s} \longrightarrow]$$

$$v_P = -\sqrt{2.3094^2 + 0.66667^2} = 2.4037 \text{ m/s}$$

$$\tan\beta = \frac{2.3094}{0.66667} \qquad \beta = 73.9°$$

$$\mathbf{v}_P = 2.40 \text{ m/s} \searrow 73.9° \blacktriangleleft$$



PROBLEM 15.151

Pin P is attached to the collar shown; the motion of the pin is guided by a slot cut in rod BD and by the collar that slides on rod AE. Knowing that at the instant considered the rods rotate clockwise with constant angular velocities, determine for the given data the velocity of pin P.

$\omega_{AE} = 7$ rad/s, $\omega_{BD} = 4.8$ rad/s

SOLUTION

$$AB = 500 \text{ mm} = 0.5 \text{ m}, \quad AP = 0.5\tan 30°, \quad BP = \frac{0.5}{\cos 30°}$$

$$\boldsymbol{\omega}_{AE} = 7 \text{ rad/s} \circlearrowright, \qquad \boldsymbol{\omega}_{BD} = 4.8 \text{ rad/s} \circlearrowright$$

Let P' be the coinciding point on AE and u_1 be the outward velocity of the collar along the rod AE.

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/AE} = [(AP)\omega_{AE} \downarrow] + [u_1 \longrightarrow]$$

Let P'' be the coinciding point on BD and u_2 be the outward speed along the slot in rod BD.

$$\mathbf{v}_P = \mathbf{v}_{P''} + \mathbf{v}_{P/BD} = [(BP)\omega_{BD} \searrow 30°] + [u_2 \swarrow 60°]$$

Equate the two expressions for $\mathbf{v}_P$ and resolve into components.

$$\xrightarrow{+}: \quad u_1 = \left(\frac{0.5}{\cos 30°}\right)(4.8)(\cos 30°) + u_2 \cos 60°$$

or

$$u_1 = 2.4 + 0.5u_2 \tag{1}$$

$$+\uparrow: \quad -(0.5\tan 30°)(7) = -\left(\frac{0.5}{\cos 30°}\right)(4.8)\sin 30° + u_2 \sin 60°$$

$$u_2 = \frac{1}{\sin 60°}[2.4\tan 30° - 3.5\tan 30°] = -0.73333 \text{ m/s}$$

From (1), $\quad u_1 = 2.4 + (0.5)(-0.73333) = 2.0333$ m/s

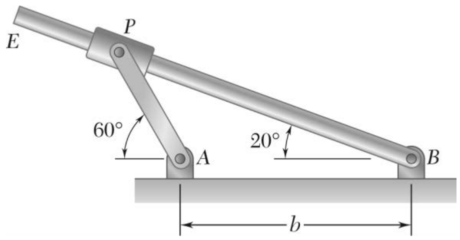
$$\mathbf{v}_P = [(0.5\tan 30°)(7) \downarrow] + [2.0333 \longrightarrow] = [2.0207 \text{ m/s} \downarrow] + [2.0333 \text{ m/s} \longrightarrow]$$

$$v_P = \sqrt{(2.0333)^2 + (2.0207)^2} = 2.87 \text{ m/s}$$

$$\tan\beta = -\frac{2.0207}{2.0333}, \qquad \beta = -44.8°$$

$$\mathbf{v}_P = 2.87 \text{ m/s} \searrow 44.8° \blacktriangleleft$$

PROBLEM 15.152



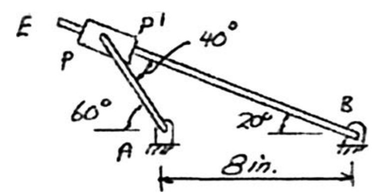
Two rotating rods are connected by slider block P. The rod attached at A rotates with a constant angular velocity ω_A. For the given data, determine for the position shown (*a*) the angular velocity of the rod attached at B, (*b*) the relative velocity of slider block P with respect to the rod on which it slides.

$b = 8$ in., $\omega_A = 6$ rad/s.

SOLUTION

Dimensions:

Law of sines.



$$\frac{AP}{\sin 20^\circ} = \frac{BP}{\sin 120^\circ} = \frac{8 \text{ in.}}{\sin 40^\circ}$$

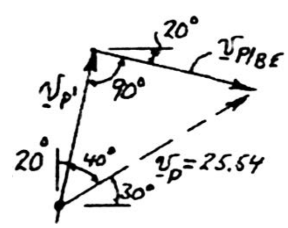
$$AP = 4.2567 \text{ in.}$$

$$BP = 10.7784 \text{ in.}$$

$$\boldsymbol{\omega}_{AP} = 6 \text{ rad/s} \circlearrowright$$

Velocities.

Note: $P' =$ Point of BE coinciding with P.



$$v_P = (AP)\omega_{AP}$$
$$= (4.2567 \text{ in.})(6 \text{ rad/s})$$
$$= 25.540 \text{ in./s} \measuredangle 30^\circ$$

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/BE}$$

$$[25.540 \measuredangle 30^\circ] = [v_{P'} \measuredangle 70^\circ] + [v_{P/BE} \searrow 30^\circ]$$

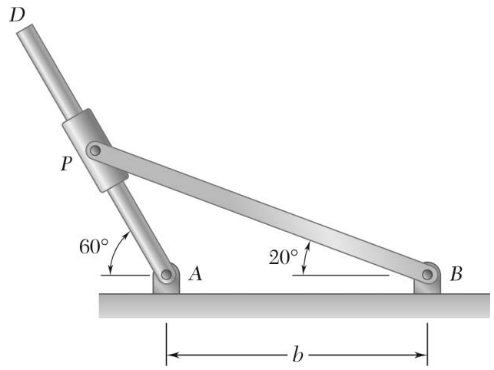
(*a*)
$$v_{P'} = (25.54)\cos 40^\circ$$
$$= 19.565 \text{ in./s}$$

$$\omega_{BE} = \frac{v_{P'}}{BP}$$
$$= \frac{19.565 \text{ in./s}}{10.7784 \text{ in.}}$$
$$= 1.8152 \text{ rad/s}$$

$\boldsymbol{\omega}_{BE} = 1.815 \text{ rad/s} \circlearrowright$ ◀

(*b*)
$$v_{P/BE} = (25.54)\sin 40^\circ$$
$$= 16.417 \text{ in./s}$$

$\mathbf{v}_{P/BE} = 16.42 \text{ in./s} \searrow 20^\circ$ ◀



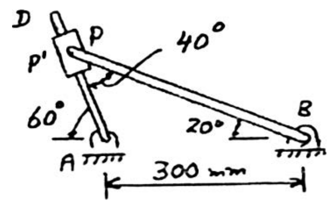
PROBLEM 15.153

Two rotating rods are connected by slider block *P*. The rod attached at *A* rotates with a constant angular velocity ω_A. For the given data, determine for the position shown (*a*) the angular velocity of the rod attached at *B*, (*b*) the relative velocity of slider block *P* with respect to the rod on which it slides.

$b = 300$ mm, $\omega_A = 10$ rad/s.

SOLUTION

Dimensions:



Law of sines.

$$\frac{AP}{\sin 20°} = \frac{BP}{\sin 120°} = \frac{300 \text{ mm}}{\sin 40°}$$

$$AP = 159.63 \text{ mm}$$

$$BP = 404.19 \text{ mm}$$

$$\boldsymbol{\omega}_{AD} = 10 \text{ rad/s} \circlearrowright$$

Velocities.

Note: $P' =$ Point of AD coinciding with P.

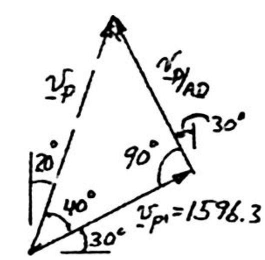
$$\mathbf{v}_{P'} = (AP')\omega_{AD}$$

$$= (159.63 \text{ mm})(10 \text{ rad/s})$$

$$= 1596.3 \text{ mm/s} \angle 30°$$

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/AD}$$

$$[v_P \angle 70°] = [1596.3 \angle 30°] + [v_{P/AD} \measuredangle 60°]$$



(*a*)

$$v_P = (1596.3)/\cos 40°$$

$$= 2083.8 \text{ mm/s}$$

$$\omega_{BP} = \frac{v_P}{BP}$$

$$= \frac{2083.8 \text{ mm/s}}{404.19 \text{ mm}}$$

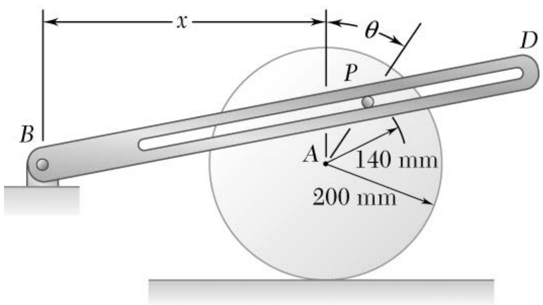
$$= 5.155 \text{ rad/s}$$

$\boldsymbol{\omega}_{BD} = 5.16$ rad/s $\circlearrowright$ ◀

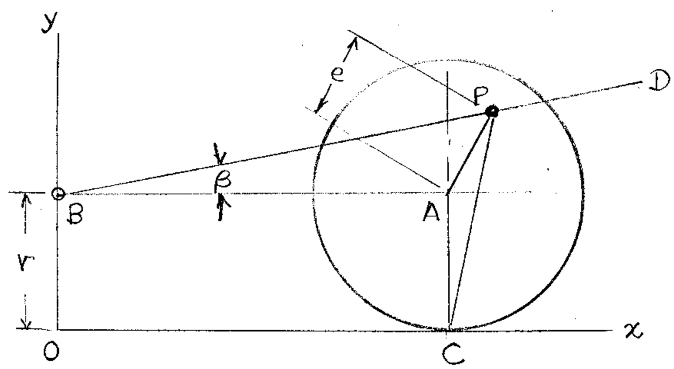
(*b*)

$$v_{P/AD} = (1596.3)\tan 40° = 1339.5 \text{ mm/s}$$

$\mathbf{v}_{P/AD} = 1.339$ m/s $\measuredangle$ 60° ◀

	**PROBLEM 15.154** Pin P is attached to the wheel shown and slides in a slot cut in bar BD. The wheel rolls to the right without slipping with a constant angular velocity of 20 rad/s. Knowing that $x = 480$ mm when $\theta = 0$, determine the angular velocity of the bar and the relative velocity of pin P with respect to the rod for the given data. (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

SOLUTION



Coordinates.

$$x_A = (x_A)_0 + r\theta, \quad y_A = r$$

$$x_B = 0, \quad y_B = r$$

$$x_C = x_A, \quad y_C = 0$$

$$x_P = x_A + e\sin\theta$$

$$y_P = r + e\cos\theta$$

Data:

$$(x_A)_0 = 480 \text{ mm} = 0.48 \text{ m}$$

$$r = 200 \text{ mm} = 0.20 \text{ m}$$

$$e = 140 \text{ mm} = 0.14 \text{ m}$$

Velocity analysis.

$$\boldsymbol{\omega}_{AC} = \omega_{AC} \circlearrowright, \quad \boldsymbol{\omega}_{BD} = \omega_{BD} \circlearrowright,$$

$$\mathbf{v}_P = \mathbf{v}_A + \mathbf{v}_{P/A} = [r\omega_{AC} \longrightarrow] + [e\omega_{AC} \searrow \theta]$$

$$\mathbf{v}_{P'} = [x_P\omega_{BD} \downarrow] + [(e\cos\theta)]\omega_{BD} \longrightarrow]$$

$$\mathbf{v}_{P/F} = [u\cos\beta \longrightarrow] + [u\sin\beta \uparrow]$$

Use $\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/F}$ and resolve into components.

$$\xrightarrow{+}: \quad (r + e\cos\theta)\omega_{AC} = (e\cos\theta)\omega_{BD} + (\cos\beta)u \qquad (1)$$

$$+\big\downarrow: \quad (e\sin\theta)\omega_{AC} = x_P\omega_{BD} - (\sin\beta)u \qquad (2)$$

(*a*) $\theta = 0.$ $\quad x_A = 0.48 \text{ m}, \quad x_P = 0.48 \text{ m}, \quad \omega_{AC} = 20 \text{ rad/s}$

$$\tan\beta = \frac{e\cos\theta}{x_P} = \frac{0.14}{0.48}, \quad \beta = 16.26°$$

Substituting into Eqs. (1) and (2),

$$(0.20 + 0.14)(20) = 0.14\omega_{BD} + (\cos 16.26°)u \qquad (1)$$

$$0 = 0.48\omega_{BD} - (\sin 16.26°)u \qquad (2)$$

Solving simultaneously, $\omega_{BD} = 3.81$ rad/s, $\qquad \boldsymbol{\omega}_{BD} = 3.81 \text{ rad/s} \circlearrowright$ ◀

$u = 6.53$ m/s, $\qquad \mathbf{v}_{P/F} = 6.53 \text{ m/s} \measuredangle 16.26°$ ◀

(*b*) $\theta = 90°.$ $\quad x_P = 0.48 + (0.20)\left(\frac{\pi}{2}\right) + 0.14 = 0.93416 \text{ m}$

$$\beta = 0$$

Substituting into Eqs. (1) and (2),

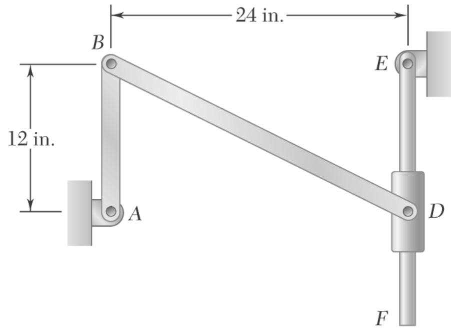
$$(0.20)(20) = u \qquad (1)$$

$$u = 4 \text{ m/s}$$

$$(0.14)(20) = 0.93416\omega_{BD} \qquad (2)$$

$\omega_{BD} = 2.9973$ rad/s, $\qquad \boldsymbol{\omega}_{BD} = 3.00 \text{ rad/s} \circlearrowright$ ◀

$\mathbf{v}_{P/F} = 4.00 \text{ m/s} \longrightarrow$ ◀



PROBLEM 15.155

Bar AB rotates clockwise with a constant angular velocity of 8 rad/s and rod EF rotates clockwise with a constant angular velocity of 6 rad/s. Determine at the instant shown (*a*) the angular velocity of bar BD, (*b*) the relative velocity of collar D with respect to rod EF.

SOLUTION

Bar AB. (Rotation about A) $\omega_{AB} = 8$ rad/s ↻

$$\mathbf{v}_B = (12)(8) = 96 \text{ in./s} \longrightarrow$$

Rod EF. (Rotation about E) $\omega_{EF} = 6$ rad/s. ↻

$$\mathbf{v}_{D'} = (12)(6) = 72 \text{ in./s} \longleftarrow$$

Bar BD. Assume angular velocity is ω_{BD} ↺.

Plane motion = Translation with B + Rotation about B.

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = [96 \longrightarrow] + [24\omega_{BD} \uparrow] + [12\omega_{BD} \longrightarrow] \tag{1}$$

Collar D. Sliding on rotating rod EF with relative velocity $u \uparrow$.

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/EF} = [72 \longleftarrow] + [u \uparrow] \tag{2}$$

Matching the expressions (1) and (2) for $\mathbf{v}_D$,

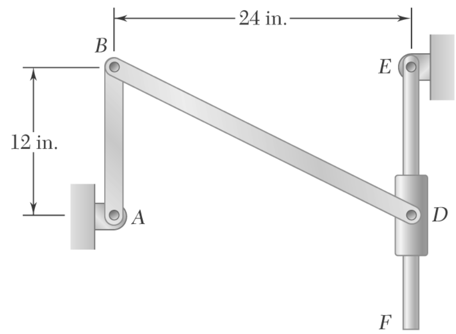
Components $\longrightarrow$: $96 + 12\omega_{BD} = -72$ $\omega_{BD} = -14$

(*a*) $\boldsymbol{\omega}_{BD} = 14.00$ rad/s ↻ ◀

Components $\uparrow$: $24\omega_{BD} = u$ $u = (24)(-14) = -336$ in./s

(*b*) $\mathbf{v}_{D/EF} = 28.0$ ft/s $\downarrow$ ◀

PROBLEM 15.156



Bar *AB* rotates clockwise with a constant angular velocity of 4 rad/s. Knowing that the magnitude of the velocity of collar *D* is 20 ft/s and that the angular velocity of bar *BD* is counterclockwise at the instant shown, determine (*a*) the angular velocity of bar *EF*, (*b*) the relative velocity of collar *D* with respect to rod *EF*.

SOLUTION

Bar AB. (Rotation about *A*) $\boldsymbol{\omega}_{AB} = 4 \text{ rad/s} \circlearrowright$

$$\mathbf{v}_B = (1 \text{ ft})(4 \text{ rad/s}) = 4 \text{ ft/s} \longrightarrow$$

Bar BD. Angular velocity is $\omega_{BD} \circlearrowleft$.

Plane motion = Translation with *B* + Rotation about *B*.

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = [4 \longrightarrow] + [2\omega_{BD} \uparrow] + [1\omega_{BD} \longrightarrow]$$

Magnitude of $\mathbf{v}_D$: $v_D = 20$ ft/s

$$v_D^2 = (4 + \omega_{BD})^2 + (2\omega_{BD})^2 = (20)^2$$

$$5\omega_{BD}^2 + 8\omega_{BD} - 384 = 0$$

$$\omega_{BD} = \frac{-8 \pm 88}{10}$$

Positive root $\omega_{BD} = 8 \text{ rad/s} \circlearrowleft$

$$\mathbf{v}_D = [4 \longrightarrow] + [(2)(8) \uparrow] + [(1)(8) \longrightarrow] = [12 \longrightarrow] + [16 \uparrow] \quad (1)$$

Rod EF. (Rotation about *E*) Angular velocity = $\omega_{EF} \circlearrowleft$

$$\mathbf{v}_{D'} = [(1)\omega_{EF} \longrightarrow]$$

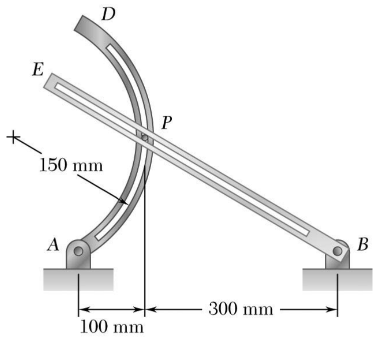
Collar D. Slides on rotating rod *EF* with relative velocity $u \uparrow$.

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/EF} = [1\omega_{EF} \longrightarrow] + [u \uparrow] \quad (2)$$

Matching the expressions (1) and (2) for $\mathbf{v}_D$,

(*a*) Component $\longrightarrow$: $12 = 1\omega_{EF}$ $\boldsymbol{\omega}_{EF} = 12.00 \text{ rad/s} \circlearrowleft$ ◀

(*b*) Component $\uparrow$: $16 = u$ $\mathbf{v}_{D/EF} = 16 \text{ ft/s} \uparrow$ ◀



PROBLEM 15.157

The motion of pin P is guided by slots cut in rods AD and BE. Knowing that bar AD has a constant angular velocity of 4 rad/s clockwise and bar BE has an angular velocity of 5 rad/s counterclockwise and is slowing down at a rate of 2 rad/s^2, determine the velocity of P for the position shown.

SOLUTION

Units: meters, m/s, m/s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1 \uparrow, \quad \mathbf{k} = 1 \circlearrowleft.$

Geometry: Slope angle θ of rod BE.

$$\tan\theta = \frac{0.15}{0.3} = 0.5 \qquad \theta = 26.565° \measuredangle$$

$$\mathbf{r}_{P/A} = 0.1\mathbf{i} + 0.15\mathbf{j} \qquad \mathbf{r}_{P/B} = -0.3\mathbf{i} + 0.15\mathbf{j}$$

Angular velocities: $\boldsymbol{\omega}_{AD} = -(4 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\omega}_{BE} = (5 \text{ rad/s})\mathbf{k}$

Angular accelerations: $\boldsymbol{\alpha}_{AD} = 0 \qquad \boldsymbol{\alpha}_{BE} = -(2 \text{ rad/s}^2)\mathbf{k}$

Velocity of Point P' on rod AD coinciding with the pin:

$$\mathbf{v}_{P'} = \boldsymbol{\omega}_{AD} \times \mathbf{r}_{P/A} = (-4\mathbf{k}) \times (0.1\mathbf{i} + 0.15\mathbf{j}) = 0.6\mathbf{i} - 0.4\mathbf{j}$$

Velocity of the pin relative to rod AD:

$$\mathbf{v}_{P/AD} = u_1 \uparrow = u_1\mathbf{j}$$

Velocity of P: $\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/AD} = 0.6\mathbf{i} - 0.4\mathbf{j} + u_1\mathbf{j}$

Velocity of Point P'' on rod BE coinciding with the pin:

$$\mathbf{v}_{P''} = \boldsymbol{\omega}_{BE} \times \mathbf{r}_{P/B} = 5\mathbf{k} \times (-0.3\mathbf{i} + 0.15\mathbf{j}) = -0.75\mathbf{i} - 1.5\mathbf{j}$$

Velocity of the pin relative to rod BE:

$$\mathbf{v}_{P/BE} = u_2 \searrow \theta = -u_2 \cos\theta\,\mathbf{i} + u_2 \sin\theta\,\mathbf{j}$$

Velocity of P:

$$\mathbf{v}_P = \mathbf{v}_{P''} + \mathbf{v}_{P/BE}$$
$$= -0.75\mathbf{i} - 1.5\mathbf{j} - u_2 \cos\theta\,\mathbf{i} + u_2 \sin\theta\,\mathbf{j}$$

PROBLEM 15.157 (Continued)

Equating the two expressions for $\mathbf{v}_P$ and resolving into components,

i: $$0.6 = -0.75 - u_2 \cos\theta$$

$$u_2 = -\frac{1.35}{\cos 26.565°} = -1.50965$$

j: $$-0.4 + u_1 = -1.5 + u_2 \sin\theta$$

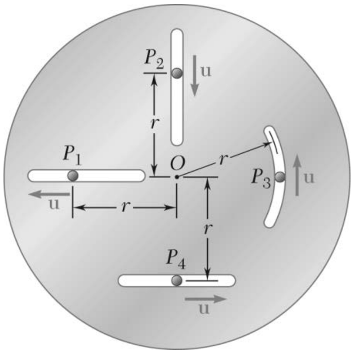
$$u_1 = -1.1 + (-1.50935)\sin 26.535° = -1.77500$$

Velocity of P:

$$\mathbf{v}_P = 0.6\mathbf{i} - 0.4\mathbf{j} - 1.775\mathbf{j} = 0.6\mathbf{i} - 2.175\mathbf{j}$$

$\mathbf{v}_P = 2.26$ m/s $\measuredangle$ 74.6° ◀

PROBLEM 15.158



Four pins slide in four separate slots cut in a circular plate as shown. When the plate is at rest, each pin has a velocity directed as shown and of the same constant magnitude u. If each pin maintains the same velocity relative to the plate when the plate rotates about O with a constant counterclockwise angular velocity ω, determine the acceleration of each pin.

SOLUTION

For each pin: $\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + \mathbf{a}_C$

Acceleration of the coinciding Point P′ of the plate.

For each pin $\mathbf{a}_{P'} = r\omega^2$ towards the center O.

Acceleration of the pin relative to the plate.

For pins P_1, P_2 and P_4, $\mathbf{a}_{P/F} = 0$

For pin P_3, $\mathbf{a}_{P/F} = \dfrac{u^2}{r} \leftarrow$

Coriolis acceleration $\mathbf{a}_C$.

For each pin $a_C = 2\omega u$ with $\mathbf{a}_C$ in a direction obtained by rotating $\mathbf{u}$ through $90°$ in the sense of $\boldsymbol{\omega}$, i.e., ⤴.

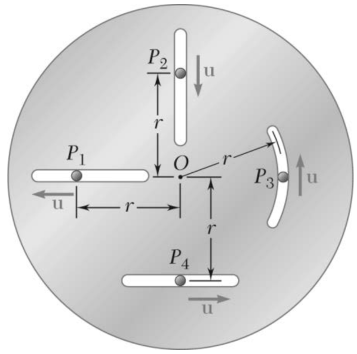
Then

$$\mathbf{a}_1 = [r\omega^2 \rightarrow] + [2\omega u \downarrow] \qquad \mathbf{a}_1 = r\omega^2\mathbf{i} - 2\omega u\mathbf{j} \blacktriangleleft$$

$$\mathbf{a}_2 = [r\omega^2 \downarrow] + [2\omega u \rightarrow] \qquad \mathbf{a}_2 = 2\omega u\mathbf{i} - r\omega^2\mathbf{j} \blacktriangleleft$$

$$\mathbf{a}_3 = [r\omega^2 \leftarrow] + \left[\frac{u^2}{r} \leftarrow\right] + [2\omega u \leftarrow] \qquad \mathbf{a}_3 = -\left(r\omega^2 + \frac{u^2}{r} + 2\omega u\right)\mathbf{i} \blacktriangleleft$$

$$\mathbf{a}_4 = [r\omega^2 \uparrow] + [2\omega u \uparrow] \qquad \mathbf{a}_4 = (r\omega^2 + 2\omega u)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.159

Solve Problem 15.158, assuming that the plate rotates about O with a constant clockwise angular velocity $\boldsymbol{\omega}$.

PROBLEM 15.158 Four pins slide in four separate slots cut in a circular plate as shown. When the plate is at rest, each pin has a velocity directed as shown and of the same constant magnitude u. If each pin maintains the same velocity relative to the plate when the plate rotates about O with a constant counterclockwise angular velocity ω, determine the acceleration of each pin.

SOLUTION

For each pin: $\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + \mathbf{a}_C$

Acceleration of the coinciding Point P′ of the plate.

For each pin $\mathbf{a}_{P'} = r\omega^2$ towards the center O.

Acceleration of the pin relative to the plate.

For pins P_1, P_2 and P_4, $\mathbf{a}_{P/F} = 0$

For pin P_3, $\mathbf{a}_{P/F} = \dfrac{u^2}{r} \leftarrow$

Coriolis acceleration $\mathbf{a}_C$.

For each pin $a_C = 2\omega u$ with $\mathbf{a}_C$ in a direction obtained by rotating $\mathbf{u}$ through $90°$ in the sense of $\boldsymbol{\omega}$.

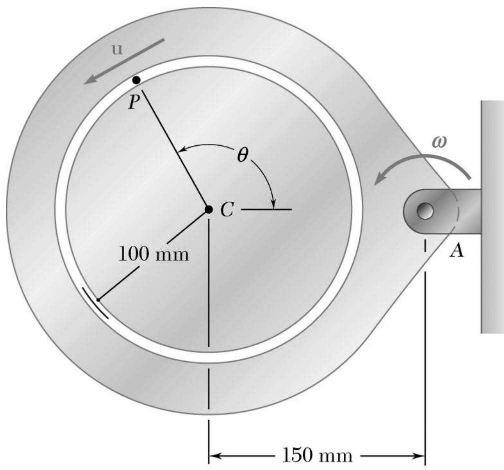
Then

$$\mathbf{a}_1 = [r\omega^2 \rightarrow] + [2\omega u \uparrow] \qquad \mathbf{a}_1 = r\omega^2\mathbf{i} + 2\omega u\mathbf{j} \blacktriangleleft$$

$$\mathbf{a}_2 = [r\omega^2 \downarrow] + [2\omega u \leftarrow] \qquad \mathbf{a}_2 = -2\omega u\mathbf{i} - r\omega^2\mathbf{j} \blacktriangleleft$$

$$\mathbf{a}_3 = [r\omega^2 \leftarrow] + \left[\frac{u^2}{r} \leftarrow\right] + [2\omega u \rightarrow] \qquad \mathbf{a}_3 = \left(2\omega u - r\omega^2 - \frac{u^2}{r}\right)\mathbf{i} \blacktriangleleft$$

$$\mathbf{a}_4 = [r\omega^2 \uparrow] + [2\omega u \downarrow] \qquad \mathbf{a}_4 = (r\omega^2 - 2\omega u)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.160

Pin P slides in the circular slot cut in the plate shown at a constant relative speed u = 500 mm/s. Assuming that at the instant shown the angular velocity of the plate is 6 rad/s and is increasing at the rate of 20 rad/s^2, determine the acceleration of pin P when $\theta = 90°$.

SOLUTION

$\theta = 90°$ Units: meters, m/s, m/s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1 \uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

$$\mathbf{r}_{P/A} = (0.15\mathbf{i} + 0.1\mathbf{j}) \qquad \mathbf{r}_{P/C} = 0.1\mathbf{j}$$

Motion of Point P' on the plate coinciding with P.

$$\boldsymbol{\omega} = (6 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\alpha} = (20 \text{ rad/s}^2)\mathbf{k}$$

$$\mathbf{v}_{P'} = \boldsymbol{\omega} \times \mathbf{r}_{P/A} = 6\mathbf{k} \times (-0.15\mathbf{i} + 0.1\mathbf{j}) = -0.6\mathbf{i} - 0.9\mathbf{j}$$

$$\begin{aligned}\mathbf{a}_{P'} &= \boldsymbol{\alpha} \times r_{P/A} - \omega^2 r_{B/A} \\ &= 20\mathbf{k} \times (-0.15\mathbf{i} + 0.1\mathbf{j}) - (6)^2(-0.15\mathbf{i} + 0.1\mathbf{j}) \\ &= -2\mathbf{i} - 3\mathbf{j} + 5.4\mathbf{i} - 3.6\mathbf{j} = 3.4\mathbf{i} - 6.6\mathbf{j}\end{aligned}$$

Motion of P relative to the plate AC.

$$u = 500 \text{ mm/s} = 0.5 \text{ m/s} \qquad \dot{u} = 0$$

$$\mathbf{v}_{P/AC} = -u\mathbf{i} = -0.5\mathbf{i}$$

$$\mathbf{a}_{P/AC} = -\dot{u}\mathbf{i} - \frac{u}{R}\mathbf{j} = 0 - \frac{(0.5)^2}{0.1}\mathbf{j} = -2.5\mathbf{j}$$

Coriolis acceleration: $2\boldsymbol{\omega} \times \mathbf{v}_{P/AC} = (2)(6\mathbf{k}) \times (-0.5\mathbf{i}) = -6\mathbf{j}$

Acceleration of P.

$$\begin{aligned}\mathbf{a}_P &= \mathbf{a}_{P'} + \mathbf{a}_{P/AC} + 2\boldsymbol{\omega} \times \mathbf{v}_{P/AC} \\ &= 3.4\mathbf{i} - 6.6\mathbf{j} - 2.5\mathbf{j} - 6\mathbf{j} \\ &= (3.4 \text{ m/s}^2)\mathbf{i} - (15.1 \text{ m/s})\mathbf{j}\end{aligned}$$

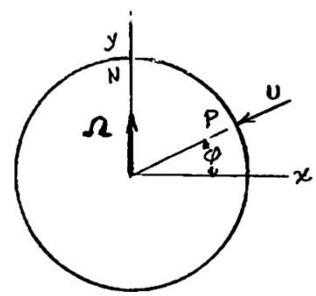
$\mathbf{a}_P = 15.47 \text{ m/s}^2 \measuredangle 77.3°$ ◀

PROBLEM 15.161

The cage of a mine elevator moves downward at a constant speed of 40 ft/s. Determine the magnitude and direction of the Coriolis acceleration of the cage if the elevator is located (*a*) at the equator, (*b*) at latitude 40° north, (*c*) at latitude 40° south.

SOLUTION

Earth makes one revolution $(2\pi \text{ radians})$ in 23.933 h (86,160 s).



$$\boldsymbol{\Omega} = \frac{2\pi}{86{,}160}\mathbf{j}$$

$$= (72.926 \times 10^{-6} \text{ rad/s})\mathbf{j}$$

Velocity relative to the Earth at latitude angle φ.

$$\mathbf{v}_{P/\text{earth}} = 40(-\cos\varphi\mathbf{i} - \sin\varphi\mathbf{j})$$

Coriolis acceleration $\mathbf{a}_C$.

$$\mathbf{a}_C = 2\boldsymbol{\Omega} \times \mathbf{v}_{P/\text{earth}}$$

$$= (2)(72.926 \times 10^{-6}\,\mathbf{j}) \times [40(-\cos\varphi\mathbf{i} - \sin\varphi\mathbf{j})]$$

$$= (5.8341 \times 10^{-3} \cos\varphi)\mathbf{k}$$

(*a*) $\varphi = 0°$, $\cos\varphi = 1.000$ $\mathbf{a}_C = 5.83 \times 10^{-3}\,\text{ft/s}^2$ west ◀

(*b*) $\varphi = 40°$, $\cos\varphi = 0.76604$ $\mathbf{a}_C = 4.47 \times 10^{-3}\,\text{ft/s}^2$ west ◀

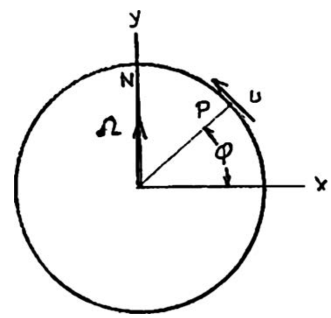
(*c*) $\varphi = -40°$, $\cos\varphi = 0.76604$ $\mathbf{a}_C = 4.47 \times 10^{-3}\,\text{ft/s}^2$ west ◀

PROBLEM 15.162

A rocket sled is tested on a straight track that is built along a meridian. Knowing that the track is located at latitude 40° north, determine the Coriolis acceleration of the sled when it is moving north at a speed of 900 km/h.

SOLUTION

Earth makes one revolution $(2\pi \text{ radians})$ in $23.933 \text{ h} = 86{,}160 \text{ s}$.



$$\Omega = \frac{2\pi}{86{,}160} = (72.926\times 10^{-6} \text{ rad/s})\mathbf{j}$$

Speed of sled.

$$u = 900 \text{ km/h} = 250 \text{ m/s}$$

Velocity of sled relative to the Earth.

$$\mathbf{v}_{P/\text{earth}} = 250(-\sin\varphi\mathbf{i} + \cos\varphi\mathbf{j})$$

Coriolis acceleration.

$$\mathbf{a}_C = 2\mathbf{\Omega}\times\mathbf{v}_{P/\text{earth}}$$

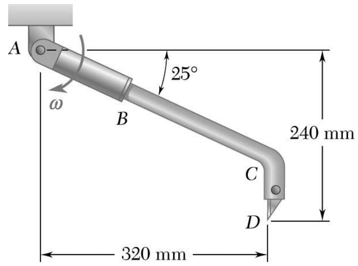
$$\mathbf{a}_C = (2)(72.926\times 10^{-6}\mathbf{j})\times[250(-\sin\varphi\mathbf{i} + \cos\varphi\mathbf{j})] = 0.036463\sin\varphi\mathbf{k}$$

At latitude $\varphi = 40°$,

$$\mathbf{a}_C = 0.036463 \sin 40°\mathbf{k} = (0.0234 \text{ m/s}^2)\mathbf{k}$$

$\mathbf{a}_C = 0.0234 \text{ m/s}^2$ west ◀

PROBLEM 15.163



The motion of blade D is controlled by the robot arm ABC. At the instant shown, the arm is rotating clockwise at the constant rate $\omega = 1.8$ rad/s and the length of portion BC of the arm is being decreased at the constant rate of 250 mm/s. Determine (*a*) the velocity of D, (*b*) the acceleration of D.

SOLUTION

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \mathbf{k} = \circlearrowleft$

Units: meters, m/s, m/s^2

$$\mathbf{r}_{D/A} = (0.32 \text{ m})\mathbf{i} - (0.24 \text{ m})\mathbf{j}$$

Motion of Point D' of extended frame AB.

$$\boldsymbol{\omega} = -(1.8 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\alpha} = 0$$

$$\mathbf{v}_{D'} = \boldsymbol{\omega} \times \mathbf{r}_{D/A} = (-1.8\mathbf{k}) \times (0.32\mathbf{i} - 0.24\mathbf{j})$$
$$= -0.432\mathbf{i} - 0.576\mathbf{j}$$

$$\mathbf{a}_{D'} = \boldsymbol{\alpha} \times \mathbf{r}_{D/A} - \omega^2(\mathbf{r}_{D/A})$$
$$= 0 - (1.8)^2(0.32\mathbf{i} - 0.24\mathbf{j})$$
$$= -1.0368\mathbf{i} + 0.7776\mathbf{j}$$

Motion of Point D relative to frame AB.

$$\mathbf{v}_{D/AB} = 250 \text{ mm/s} \nwarrow 25°$$
$$= -(0.25\cos 25°)\mathbf{i} + (0.25\sin 25°)\mathbf{j}$$
$$= -0.22658\mathbf{i} + 0.10565\mathbf{j}$$

$$\mathbf{a}_{D/AB} = 0$$

Coriolis acceleration $\quad 2\boldsymbol{\omega} \times \mathbf{v}_{D/AB} = (2)(-1.8\mathbf{k}) \times (-0.22658\mathbf{i} + 0.10565\mathbf{j})$

$$= 0.38034\mathbf{i} + 0.81569\mathbf{j}$$

(*a*) *Velocity of Point D*.

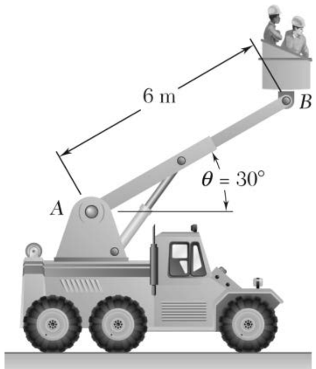
$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/AB}$$
$$= -0.432\mathbf{i} - 0.576\mathbf{j} - 0.22658\mathbf{i} + 0.10565\mathbf{j}$$
$$= -0.65858\mathbf{i} - 0.47035\mathbf{j}$$

$$\mathbf{v}_D = (0.659 \text{ m/s})\mathbf{i} - (0.470 \text{ m/s})\mathbf{j} = 0.809 \text{ m/s} \swarrow 35.5° \blacktriangleleft$$

(*b*) *Acceleration of Point D.*

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/AB} + 2\boldsymbol{\omega} \times \mathbf{v}_{D/AB}$$
$$= -1.0368\mathbf{i} + 0.7776\mathbf{j} + 0 + 0.38034\mathbf{i} + 0.81569\mathbf{j}$$
$$= -0.6565\mathbf{i} + 1.5933\mathbf{j}$$

$$\mathbf{a}_D = -(0.657 \text{ m/s}^2)\mathbf{i} + (1.593 \text{ m/s}^2)\mathbf{j} = 1.723 \text{ m/s}^2 \;\measuredangle\; 67.6° \blacktriangleleft$$



PROBLEM 15.164

At the instant shown the length of the boom *AB* is being *decreased* at the constant rate of 0.2 m/s and the boom is being lowered at the constant rate of 0.08 rad/s. Determine (*a*) the velocity of Point *B*, (*b*) the acceleration of Point *B*.

SOLUTION

Velocity of coinciding Point B′ on boom.

$$\mathbf{v}_{B'} = r\omega = (6)(0.08) = 0.48 \text{ m/s} \searrow 60^\circ$$

Velocity of Point B relative to the boom.

$$\mathbf{v}_{B/\text{boom}} = 0.2 \text{ m/s} \swarrow 30^\circ$$

(*a*) *Velocity of Point B.*

$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/\text{boom}}$$

$$\xrightarrow{+}: \quad (v_B)_x = 0.48\cos 60^\circ - 0.2\cos 30^\circ = 0.06680 \text{ m/s}$$

$$+\uparrow: \quad (v_B)_y = -0.48\sin 60^\circ - 0.2\sin 30^\circ = -0.51569 \text{ m/s}$$

$$v_B = \sqrt{0.06680^2 + 0.51569^2}$$

$$= 0.520 \text{ m/s}$$

$$\tan\beta = \frac{0.51569}{0.06680}, \quad \beta = 82.6^\circ \qquad \mathbf{v}_B = 0.520 \text{ m/s} \searrow 82.6^\circ \blacktriangleleft$$

Acceleration of coinciding Point B′ on boom.

$$\mathbf{a}_{B'} = r\omega^2 = (6)(0.08)^2 = 0.0384 \text{ m/s}^2 \swarrow 30^\circ$$

Acceleration of B relative to the boom.

$$\mathbf{a}_{B/\text{boom}} = 0$$

Coriolis acceleration.

$$2\omega u = (2)(0.08)(0.2) = 0.032 \text{ m/s}^2 \searrow 60^\circ$$

(*b*) *Acceleration of Point B.*

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/\text{boom}} + 2\omega u$$

$$\xrightarrow{+}: \quad (a_B)_x = -0.0384\cos 30° + 0 - 0.032\cos 60° = -0.04926 \text{ m/s}^2$$

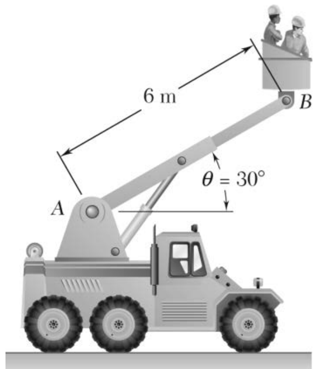
$$+\uparrow: \quad (a_B)_y = -0.0384\sin 30° + 0 + 0.032\sin 60° = 0.008513 \text{ m/s}^2$$

$$a_B = \sqrt{(0.04926)^2 + (0.008513)^2}$$

$$= 0.0500 \text{ m/s}^2$$

$$\tan\beta = \frac{0.008513}{0.04926}, \quad \beta = 9.8°$$

$$\mathbf{a}_B = 50.0 \text{ mm/s}^2 \nwarrow 9.8° \blacktriangleleft$$



PROBLEM 15.165

At the instant shown the length of the boom *AB* is being *increased* at the constant rate of 0.2 m/s and the boom is being lowered at the constant rate of 0.08 rad/s. Determine (*a*) the velocity of Point *B*, (*b*) the acceleration of Point *B*.

SOLUTION

Velocity of coinciding Point B′ on boom.

$$\mathbf{v}_{B'} = r\omega = (6)(0.08) = 0.48 \text{ m/s} \searrow 60°$$

Velocity of Point B relative to the boom.

$$\mathbf{v}_{B/\text{boom}} = 0.2 \text{ m/s} \nearrow 30°$$

(*a*) *Velocity of Point B.*

$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/\text{boom}}$$

$$\xrightarrow{+}: \quad (v_B)_x = 0.48\cos 60° + 0.2\cos 30° = 0.4132 \text{ m/s}$$

$$+\uparrow: \quad (v_B)_y = -0.48\sin 60° + 0.2\sin 30° = -0.3157 \text{ m/s}$$

$$v_B = \sqrt{(0.4132)^2 + (0.3157)^2}$$
$$= 0.520 \text{ m/s}$$

$$\tan\beta = -\frac{0.3157}{0.4132}, \quad \beta = -37.4°$$

$$\mathbf{v}_B = 0.520 \text{ m/s} \searrow 37.4° \blacktriangleleft$$

Acceleration of coinciding Point B′ on boom.

$$\mathbf{a}_{B'} = r\omega^2 = (6)(0.08)^2 = 0.0384 \text{ m/s}^2 \swarrow 30°$$

Acceleration of B relative to the boom.

$$\mathbf{a}_{B/\text{boom}} = 0$$

Coriolis acceleration.

$$2\omega u = (2)(0.08)(2) = 0.032 \text{ m/s}^2 \searrow 60°$$

PROBLEM 15.165 (Continued)

(*b*) *Acceleration of Point B.*

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/\text{boom}} + 2\omega u$$

$$\xrightarrow{+} : \quad (a_B)_x = -0.0384 \cos 30° + 0.032 \cos 60° = -0.017255 \text{ m/s}^2$$

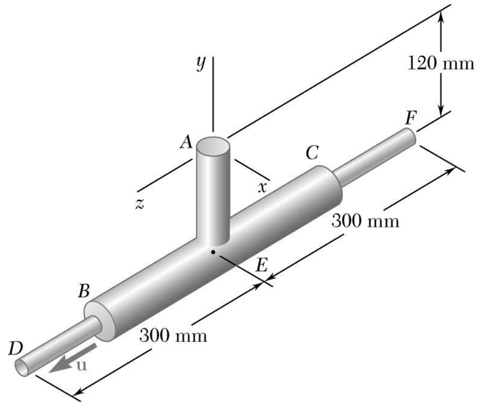
$$+\uparrow : \quad (a_B)_y = -0.0384 \sin 30° - 0.032 \sin 60° = -0.04691 \text{ m/s}^2$$

$$a_B = \sqrt{(0.017255)^2 + (0.04691)^2}$$

$$= 0.0500 \text{ m/s}^2$$

$$\tan\beta = \frac{0.04691}{0.017255}, \quad \beta = 69.8°$$

$$\mathbf{a}_B = 50.0 \text{ mm/s}^2 \measuredangle 69.8° \blacktriangleleft$$



PROBLEM 15.166

The sleeve *BC* is welded to an arm that rotates about *A* with a constant angular velocity ω. In the position shown rod *DF* is being moved to the left at a constant speed $u = 400$ mm/s relative to the sleeve. For the given angular velocity ω, determine the acceleration (*a*) of Point *D*, (*b*) of the point of rod *DF* that coincides with Point *E*.

$\omega = (3 \text{ rad/s})\mathbf{i}$.

SOLUTION

(*a*) Point *D*.

$$\mathbf{v}_{D/F} = \mathbf{v}_{D/BC} = (0.4 \text{ m/s})\mathbf{k}; \quad a_{D/F} = 0$$

$$\overrightarrow{AD} = -(0.12 \text{ m})\mathbf{j} + (0.3 \text{ m})\mathbf{k}$$

$$\mathbf{a}_{D'} = \boldsymbol{\omega} \times (\omega \times \overrightarrow{AD})$$

$$= -\omega^2 (\overline{AD})$$

$$= -(3 \text{ rad/s})^2 \overline{AD}$$

$$= +(1.08 \text{ m/s}^2)\mathbf{j} - (2.70 \text{ m/s}^2)\mathbf{k}$$

$$\mathbf{a}_c = 2\boldsymbol{\omega} \times \mathbf{v}_{D/F}$$

$$= 2[(3 \text{ rad/s})\mathbf{i}] \times (0.40 \text{ m/s})\mathbf{k}$$

$$= -(2.4 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + \mathbf{a}_c$$

$$= [+(1.08 \text{ m/s}^2)\mathbf{j} - (2.70 \text{ m/s}^2)\mathbf{k}] + 0 + [-(2.4 \text{ m/s})\mathbf{j}]$$

$$\mathbf{a}_D = -(1.32 \text{ m/s})\mathbf{j} - (2.70 \text{ m/s})\mathbf{k} \; ◀$$

(*b*) Point *P* of *DF* that coincides with *E*.

$$\mathbf{v}_{P/F} = \mathbf{v}_{P/BC} = (0.40 \text{ m/s})\mathbf{k}; \quad \mathbf{a}_{P/F} = 0$$

$$\overrightarrow{AE} = -(0.120 \text{ m})\mathbf{j}$$

$$\mathbf{a}_{P'} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \overrightarrow{AE}) = -\omega^2 AE = -(3 \text{ rad/s})^2 AE = (1.08 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_c = 2\boldsymbol{\omega} \times \mathbf{v}_{P/F}$$

$$= 2[(3 \text{ rad/s})\mathbf{i}] \times (0.40 \text{ m/s})\mathbf{k}$$

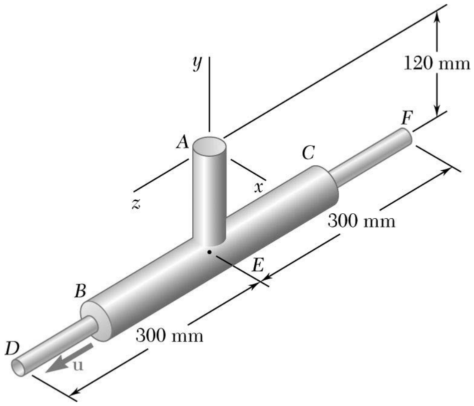
$$= -(2.40 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + \mathbf{a}_c$$

$$= [(1.08 \text{ m/s}^2)\mathbf{j}] + 0 + [-(2.4 \text{ m/s}^2)\mathbf{j}]$$

$$\mathbf{a}_P = -(1.32 \text{ m/s}^2)\mathbf{j} \; ◀$$

PROBLEM 15.167



The sleeve BC is welded to an arm that rotates about A with a constant angular velocity ω. In the position shown rod DF is being moved to the left at a constant speed $u = 400$ mm/s relative to the sleeve. For the given angular velocity ω, determine the acceleration (*a*) of Point D, (*b*) of the point of rod DF that coincides with Point E.

$\omega = (3 \text{ rad/s})\mathbf{j}$.

SOLUTION

(*a*) Point D.

$$\mathbf{v}_{D/F} = \mathbf{v}_{D/BC} = (0.4 \text{ m/s})\mathbf{k}; \quad a_{D/F} = 0$$

$$\overrightarrow{AD} = -(0.12 \text{ m})\mathbf{j} + (0.3 \text{ m})\mathbf{k}$$

$$\mathbf{a}_{D'} = \boldsymbol{\omega} \times (\omega \times \overrightarrow{AD})$$
$$= 3\mathbf{j} \times (3\mathbf{j} \times (-0.12\mathbf{j} + 0.3)\mathbf{k})$$
$$= -(2.70 \text{ m/s}^2)\mathbf{k}$$

$$\mathbf{a}_c = 2\boldsymbol{\omega} \times \mathbf{v}_{D/F}$$
$$= 2[(3 \text{ rad/s})\mathbf{j}] \times (0.40 \text{ m/s})\mathbf{k}$$
$$= (2.4 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + \mathbf{a}_c$$
$$= [-(2.70 \text{ m/s}^2)\mathbf{k}] + 0 + [(2.4 \text{ m/s})\mathbf{i}]$$

$$\mathbf{a}_D = (2.4 \text{ m/s})\mathbf{i} - (2.70 \text{ m/s})\mathbf{k} \blacktriangleleft$$

(*b*) Point P of DF that coincides with E.

$$\mathbf{v}_{P/F} = \mathbf{v}_{P/BC} = (0.40 \text{ m/s})\mathbf{k}; \quad \mathbf{a}_{P/F} = 0$$

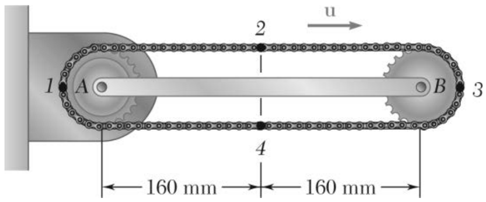
$$\overrightarrow{AE} = -(0.120 \text{ m})\mathbf{j}$$

$$\mathbf{a}_{P'} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \overrightarrow{AE})$$
$$= 3\mathbf{j} \times (3\mathbf{j} \times (-0.12\mathbf{j}) = 0$$

$$\mathbf{a}_c = 2\boldsymbol{\omega} \times \mathbf{v}_{P/F}$$
$$= 2[(3 \text{ rad/s})\mathbf{j}] \times (0.40 \text{ m/s})\mathbf{k}$$
$$= (2.40 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + \mathbf{a}_c$$
$$= 0 + 0 + (2.4 \text{ m/s}^2)\mathbf{i} \qquad \mathbf{a}_P = (2.4 \text{ m/s}^2)\mathbf{i} \blacktriangleleft$$

PROBLEM 15.168



A chain is looped around two gears of radius 40 mm that can rotate freely with respect to the 320-mm arm *AB*. The chain moves about arm *AB* in a clockwise direction at the constant rate of 80 mm/s relative to the arm. Knowing that in the position shown arm *AB* rotates clockwise about *A* at the constant rate $\omega = 0.75$ rad/s, determine the acceleration of each of the chain links indicated.

Links 1 and 2.

SOLUTION

Let the arm *AB* be a rotating frame of reference. $\Omega = 0.75 \text{ rad/s} \curvearrowright = -(0.75 \text{ rad/s})\mathbf{k}$:

Link 1:

$$\mathbf{r}_1 = -(40 \text{ mm})\mathbf{i}, \quad \mathbf{v}_{1/AB} = u \uparrow = (80 \text{ mm/s})\mathbf{j}$$

$$\mathbf{a}_1' = -\Omega^2 \mathbf{r}_1 = -(0.75)^2(-40) = (22.5 \text{ mm/s})\mathbf{i}$$

$$\mathbf{a}_{1/AB} = \frac{u^2}{\rho} = \frac{80^2}{40} 160 \text{ mm/s} \rightarrow = (160 \text{ mm/s}^2)\mathbf{i}$$

$$2\Omega \times \mathbf{v}_{P/AB} = (2)(-0.75\mathbf{k}) \times (80\mathbf{j})$$

$$= (120 \text{ mm/s})\mathbf{i}$$

$$\mathbf{a}_1 = \mathbf{a}_1' + \mathbf{a}_{1/AB} + 2\Omega \times \mathbf{v}_{1/AB}$$

$$= (302.5 \text{ mm/s}^2)\mathbf{i}$$

$\mathbf{a}_1 = 303 \text{ mm/s}^2 \longrightarrow$ ◀

Link 2:

$$\mathbf{r}_2 = (160 \text{ mm})\mathbf{i} + (40 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_{2/AB} = u \rightarrow = (80 \text{ mm/s})\mathbf{i}$$

$$\mathbf{a}_2' = -\Omega^2 \mathbf{r}_2$$

$$= -(0.75)^2(160\mathbf{i} + 40\mathbf{j})$$

$$= -(90 \text{ mm/s}^2)\mathbf{i} - (22.5 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}_{2/AB} = 0$$

$$2\Omega \times \mathbf{v}_{2/AB} = (2)(-0.75\mathbf{k}) \times (80\mathbf{i})$$

$$= -(120 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}^2 = \mathbf{a}_2' + \mathbf{a}_{2/AB} + 2\Omega \times \mathbf{v}_{2/AB}$$

$$= -90\mathbf{i} - 22.5\mathbf{j} - 120\mathbf{j}$$

$$= -(90 \text{ mm/s}^2)\mathbf{i} - (142.5 \text{ mm/s}^2)\mathbf{j}$$

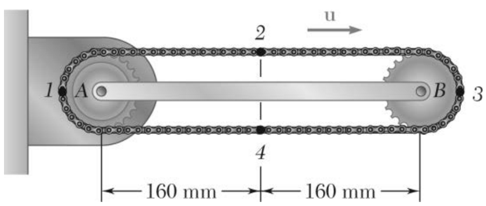
$$\mathbf{a}_2 = \sqrt{(90)^2 + (142.5)^2}$$

$$= 168.5 \text{ mm/s}^2$$

$$\tan \beta = \frac{142.5}{90}, \quad \beta = 57.7°$$

$\mathbf{a}_2 = 168.5 \text{ mm/s}^2 \swarrow 57.7°$ ◀

PROBLEM 15.169



A chain is looped around two gears of radius 40 mm that can rotate freely with respect to the 320-mm arm *AB*. The chain moves about arm *AB* in a clockwise direction at the constant rate 80 mm/s relative to the arm. Knowing that in the position shown arm *AB* rotates clockwise about *A* at the constant rate $\omega = 0.75$ rad/s, determine the acceleration of each of the chain links indicated.

Links 3 and 4.

SOLUTION

Let arm *AB* be a rotating frame of reference. $\quad \mathbf{\Omega} = 0.75 \text{ rad/s} \circlearrowright = -(0.75 \text{ rad/s})\mathbf{k}$

Link 3:

$$\mathbf{r}_3 = (360 \text{ mm})\mathbf{i} \quad \mathbf{v}_{3/AB} = u \downarrow = -(80 \text{ mm/s})\mathbf{j}$$

$$\mathbf{a}_{3'} = -\Omega^2 \mathbf{r}_3 = -(0.75)^2(360) = -(202.5 \text{ mm/s}^2)\mathbf{i}$$

$$\mathbf{a}_{3/AB} = \frac{u^2}{\rho} = \frac{(80)^2}{40} = 160 \text{ mm/s}^2\mathbf{i} \leftarrow = -(160 \text{ mm/s}^2)\mathbf{i}$$

$$2\Omega \times \mathbf{v}_{3/AB} = (2)(-0.75\mathbf{k}) \times (-80\mathbf{j}) = -(120 \text{ mm/s}^2)\mathbf{i}$$

$$\mathbf{a}_3 = \mathbf{a}_{3'} + \mathbf{a}_{3/AB} + 2\Omega \times \mathbf{v}_{3/AB}$$

$$= -(482.5 \text{ mm/s}^2)\mathbf{i}$$

$\mathbf{a}_3 = 483 \text{ mm/s}^2 \longleftarrow$ ◀

Link 4:

$$\mathbf{r}_4 = (160 \text{ mm})\mathbf{i} - (40 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_{4/AB} = u \leftarrow = -(80 \text{ mm/s}^2)\mathbf{i}$$

$$\mathbf{a}_{4'} = -\Omega^2 \mathbf{r}_4$$

$$= -(0.75)^2(160\mathbf{i} - 40\mathbf{j})$$

$$= -(90 \text{ mm/s}^2)\mathbf{i} + (22.5 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}_{4/AB} = 0$$

$$2\Omega \times \mathbf{v}_{4/AB} = (2)(-0.75\mathbf{k}) \times (-80\mathbf{i})$$

$$= (120 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}_4 = \mathbf{a}_{4'} + \mathbf{a}_{4/AB} + 2\Omega \times \mathbf{v}_{4/AB}$$

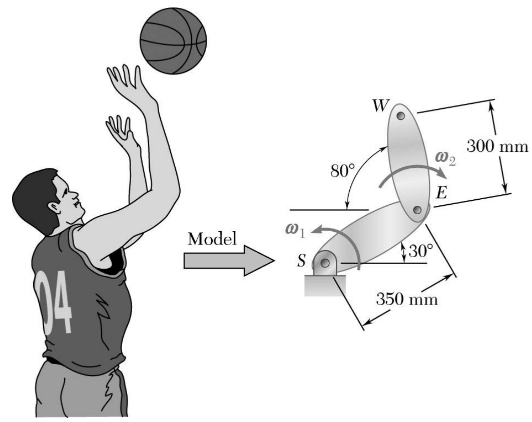
$$= -(90 \text{ mm/s}^2)\mathbf{i} + (142.5 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}_4 = \sqrt{(90)^2 + (142.5)^2}$$

$$= 168.5 \text{ mm/s}^2$$

$$\tan\beta = \frac{142.5}{90}, \quad \beta = 57.7°$$

$\mathbf{a}_4 = 168.5 \text{ mm/s}^2 \measuredangle 57.7°$ ◀



PROBLEM 15.170

A basketball player shoots a free throw in such a way that his shoulder can be considered a pin joint at the moment of release as shown. Knowing that at the instant shown the upper arm *SE* has a constant angular velocity of 2 rad/s counterclockwise and the forearm *EW* has a constant clockwise angular velocity of 4 rad/s with respect to *SE*, determine the velocity and acceleration of the wrist *W*.

SOLUTION

Units: meters, m/s, m/s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1 \uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

Relative positions:

$$\mathbf{r}_{E/S} = (0.35\cos 30°)\mathbf{i} + (0.35\sin 30°)\mathbf{j} = 0.30311\mathbf{i} + 0.175\mathbf{j}$$

$$\mathbf{r}_{W/E} = -(0.3\cos 80°)\mathbf{i} + (0.3\sin 80°)\mathbf{j} = -0.05209\mathbf{i} + 0.29544\mathbf{j}$$

$$\mathbf{r}_{W/S} = \mathbf{r}_{E/S} + \mathbf{r}_{W/E} = 0.25101\mathbf{i} + 0.47044\mathbf{j}$$

Use a frame of reference rotating with the upper arm *SE* with angular velocity

$$\mathbf{\Omega} = (2 \text{ rad/s})\mathbf{k} \quad (\dot{\mathbf{\Omega}} = 0)$$

The motion of the wrist *W* relative to this frame is a rotation about the elbow *E* with angular velocity

$$\boldsymbol{\omega} = -(4 \text{ rad/s})\mathbf{k} \quad (\dot{\boldsymbol{\omega}} = 0)$$

Motion of Point *W′* in the frame coinciding with *W*.

$$\mathbf{v}_{W'} = \mathbf{\Omega} \times \mathbf{r}_{W/S} = (2\mathbf{k}) \times (0.25101\mathbf{i} + 0.47044\mathbf{j})$$

$$= -0.94088\mathbf{i} + 0.50204\mathbf{j}$$

$$\mathbf{a}_{W'} = -\Omega^2 \mathbf{r}_{W/S} = -(2)^2(0.25101\mathbf{i} + 0.47044\mathbf{j})$$

$$= -1.00408\mathbf{i} - 1.88176\mathbf{j}$$

Motion of *W* relative to the frame.

$$\mathbf{v}_{W/SE} = \boldsymbol{\omega} \times \mathbf{r}_{W/E} = (-4\mathbf{k}) \times (-0.05210\mathbf{i} + 0.29544\mathbf{j})$$

$$= 1.18176\mathbf{i} + 0.2084\mathbf{j}$$

$$\mathbf{a}_{W/SE} = -\omega^2 \mathbf{r}_{W/E} = -(4)^2(-0.05210\mathbf{i} + 0.29544\mathbf{j})$$

$$= 0.8336\mathbf{i} - 4.72708\mathbf{j}$$

Velocity of *W*: $\mathbf{v}_W = \mathbf{v}_{W'} + \mathbf{v}_{W/SE} = 0.24088\mathbf{i} + 0.71044\mathbf{j}$

$$\mathbf{v}_W = 0.750 \text{ m/s} \measuredangle 71.3° \blacktriangleleft$$

PROBLEM 15.170 (Continued)

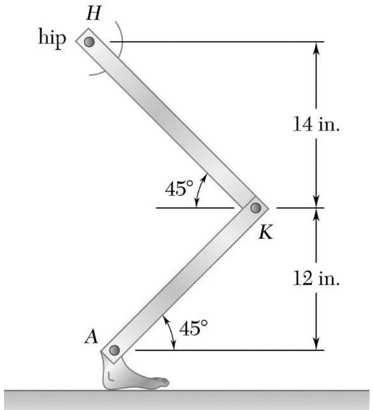
Coriolis acceleration:

$$2\mathbf{\Omega}\times\mathbf{v}_{W/SE} = (2)(2\mathbf{k})\times(1.18176\mathbf{i}+0.2084\mathbf{j})$$
$$= -0.8336\mathbf{i}+4.72704\mathbf{j}$$

Acceleration of *W*:

$$\mathbf{a}_W = \mathbf{a}_{W'}+\mathbf{a}_{W/SE}+2\mathbf{\Omega}\times\mathbf{v}_{W/SE}$$
$$= -1.00408\mathbf{i}-1.88176\mathbf{j}$$

$\mathbf{a}_W = 2.13\text{ m/s}^2 \nearrow 61.9°$ ◀



PROBLEM 15.171

The human leg can be crudely approximated as two rigid bars (the femur and the tibia) connected with a pin joint. At the instant shown the veolcity and acceleration of the ankle is zero. During a jump, the velocity of the ankle *A* is zero, the tibia *AK* has an angular velocity of 1.5 rad/s counterclockwise and an angular acceleration of 1 rad/s^2 counterclockwise. Determine the relative angular velocity and angular acceleration of the femur *KH* with respect to *AK* so that the velocity and acceleration of *H* are both straight up at the instant shown.

SOLUTION

Units: inches, in./s, in./s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

Relative positions:
$$\mathbf{r}_{K/A} = (12\text{ in.})\mathbf{i} + (12\text{ in.})\mathbf{j}$$
$$\mathbf{r}_{H/K} = -(14\text{ in.})\mathbf{i} + (14\text{ in.})\mathbf{j}$$
$$\mathbf{r}_{H/A} = \mathbf{r}_{K/A} + \mathbf{r}_{H/K} = -(2\text{ in.})\mathbf{i} + (26\text{ in.})\mathbf{j}$$

Use a frame of reference moving with the lower leg *AK* with angular velocity
$$\boldsymbol{\Omega} = 1.5\text{ rad/s} \circlearrowleft = (1.5\text{ rad/s})\mathbf{k}$$

and angular acceleration
$$\dot{\boldsymbol{\Omega}} = 1.0\text{ rad/s} \circlearrowleft = (1.0\text{ rad/s}^2)\mathbf{k}$$

The motion of the hip *H* relative to this frame is a rotation about the knee *K* with angular velocity
$$\boldsymbol{\omega} = \omega\mathbf{k}$$

and angular acceleration
$$\boldsymbol{\alpha} = \alpha\mathbf{k}$$

Both $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ are measured relative to the lower leg *AK*.

Motion of Point H' in the frame coinciding with *H*.
$$\mathbf{v}_{H'} = \boldsymbol{\Omega}\times\mathbf{r}_{H/A} = 1.5\mathbf{k}\times(-2\mathbf{i} + 26\mathbf{j})$$
$$= -(39\text{ in./s})\mathbf{i} - (3\text{ in./s})\mathbf{j}$$
$$\mathbf{a}_{H'} = \dot{\boldsymbol{\Omega}}\times\mathbf{r}_{H/A} - \Omega^2\mathbf{r}_{H/A}$$
$$= (1.0\mathbf{k})\times(-2\mathbf{i} + 26\mathbf{j}) - (1.5)^2(-2\mathbf{i} + 26\mathbf{j})$$
$$= -26\mathbf{i} - 2\mathbf{j} + 4.5\mathbf{i} - 58.5\mathbf{j}$$
$$= -(21.5\text{ in./s}^2)\mathbf{i} - (60.5\text{ in./s}^2)\mathbf{j}$$

Motion of H relative to the frame.

$$\mathbf{v}_{H/AK} = \omega\mathbf{k}\times\mathbf{r}_{H/K} = \omega\mathbf{k}\times(-14\mathbf{i}+14\mathbf{j})$$
$$= -14\omega\mathbf{i} - 14\omega\mathbf{j}$$
$$\mathbf{a}_{H/AK} = \alpha\mathbf{k}\times\mathbf{r}_{H/K} - \omega^2\mathbf{r}_{H/K}$$
$$= \alpha\mathbf{k}\times(-14\mathbf{i}+14\mathbf{j}) - \omega^2(-14\mathbf{i}+14\mathbf{j})$$
$$= -14\alpha\mathbf{i} - 14\alpha\mathbf{j} + 14\omega^2\mathbf{i} - 14\omega^2\mathbf{j}$$

Velocity of H. $$\mathbf{v}_H = \mathbf{v}_H \uparrow = \mathbf{v}_H\mathbf{j}$$

$$\mathbf{v}_H = \mathbf{v}_{H'} + \mathbf{v}_{H/AK}$$
$$v_H\mathbf{j} = -39\mathbf{i} - 3\mathbf{j} - 14\omega\mathbf{i} - 14\omega\mathbf{j}$$

Resolve into components.

$\mathbf{i}$: $\quad 0 = -39 - 14\omega \qquad \omega = -\dfrac{39}{14} = -2.7857 \text{ rad/s}$

$\mathbf{j}$: $\quad v_H = -3 - 14\omega \qquad v_H = -36 \text{ in./s}$

Relative angular velocity: $\boldsymbol{\omega} = -(2.79 \text{ rad/s})\mathbf{k} = 2.79 \text{ rad/s} \circlearrowright$ ◀

Coriolis acceleration: $$2\boldsymbol{\Omega}\times\mathbf{v}_{H/AK} = (2)(1.5\mathbf{k})\times(-14\omega\mathbf{i} - 14\omega\mathbf{j})$$
$$= 42\omega\mathbf{i} - 42\omega\mathbf{j} = -(117 \text{ in./s}^2)\mathbf{i} + (117 \text{ in./s}^2)\mathbf{j}$$

Acceleration of H. $$\mathbf{a}_H = a_H \uparrow = a_H\mathbf{j}$$

$$\mathbf{a}_H = \mathbf{a}_{H'} + \mathbf{a}_{H/AK} + 2\boldsymbol{\Omega}\times\mathbf{v}_{H/AK}$$
$$a_H\mathbf{j} = -21.5\mathbf{i} - 60.5\mathbf{j} - 14\alpha\mathbf{i} - 14\alpha\mathbf{j} + 14\omega^2\mathbf{i} - 14\omega^2\mathbf{j} - 117\mathbf{i} + 117\mathbf{j}$$

Resolve into components.

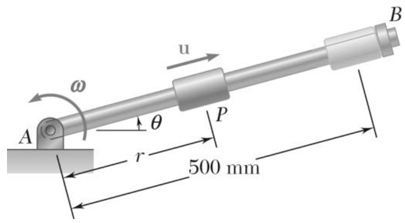
$\mathbf{i}$: $\quad 0 = -21.5 - 14\alpha + (14)(-2.7857)^2 - 117$

$\quad \alpha = -2.1327 \text{ rad/s}^2$

$\mathbf{j}$: $\quad a_H = -60.5 - (14)(-2.1327) - (14)(-2.7857)^2 + 117$

$\quad = -22.284 \text{ in./s}^2$

Relative angular acceleration: $\boldsymbol{\alpha} = -(2.13 \text{ rad/s}^2)\mathbf{k} = 2.13 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 15.172

The collar P slides outward at a constant relative speed u along rod AB, which rotates counterclockwise with a constant angular velocity of 20 rpm. Knowing that $r = 250$ mm when $\theta = 0$ and that the collar reaches B when $\theta = 90°$, determine the magnitude of the acceleration of the collar P just as it reaches B.

SOLUTION

$$\omega = 20 \text{ rpm} = \frac{(20)(2\pi)}{60} = \frac{2\pi}{3} \text{ rad/s}$$

$$\alpha = 0$$

$$\theta = 90° = \frac{\pi}{2} \text{ radians}$$

Uniform rotational motion. $\theta = \theta_0 + \omega t$

$$t = \frac{\theta - \theta_0}{\omega} = \frac{\frac{\pi}{2}}{\frac{2\pi}{3}} = 0.75 \text{ s}$$

Uniform motion along rod. $r = r_0 + ut$

$$u = \frac{r - r_0}{t} = \frac{0.5 - 0.25}{0.75} = \frac{1}{3} \text{ m/s},$$

$$\mathbf{v}_{P/AB} = \frac{1}{3} \text{ m/s} \uparrow$$

Acceleration of coinciding Point P′ on the rod. $(r = 0.5 \text{ m})$

$$\mathbf{a}_{P'} = r\omega^2$$

$$= (0.5)\left(\frac{2\pi}{3}\right)^2$$

$$= \frac{2\pi^2}{9} \text{ m/s}^2 \downarrow$$

$$= 2.1932 \text{ m/s}^2 \downarrow$$

Acceleration of collar P relative to the rod. $\mathbf{a}_{P/AB} = 0$

Coriolis acceleration. $2\boldsymbol{\omega} \times \mathbf{v}_{P/AB} = 2\omega u = (2)\left(\frac{2\pi}{3}\right)\left(\frac{1}{3}\right) = 1.3963 \text{ m/s}^2 \longleftarrow$

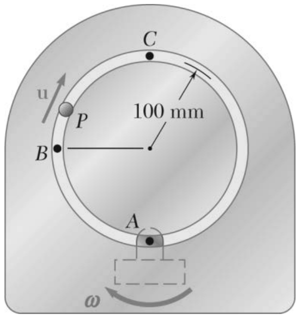
Acceleration of collar P. $\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/AB} + 2\boldsymbol{\omega} \times \mathbf{v}_{P/AB}$

$$\mathbf{a}_P = [2.1932 \text{ m/s}^2 \downarrow] + [1.3963 \text{ m/s}^2 \longleftarrow]$$

$$\mathbf{a}_P = 2.60 \text{ m/s}^2 \nearrow 57.5°$$

$a_P = 2.60 \text{ m/s}^2$ ◀

PROBLEM 15.173



Pin P slides in a circular slot cut in the plate shown at a constant relative speed $u = 90$ mm/s. Knowing that at the instant shown the plate rotates clockwise about A at the constant rate $\omega = 3$ rad/s, determine the acceleration of the pin if it is located at (*a*) Point A, (*b*) Point B, (*c*) Point C.

SOLUTION

$$\omega = 3 \text{ rad/s} \circlearrowright, \quad \alpha = 0, \quad u = 90 \text{ mm/s} = 0.09 \text{ m/s}, \quad \dot{u} = 0$$

$$\rho = 100 \text{ mm}$$

$$\frac{u^2}{\rho} = \frac{(90)^2}{100} = 81 \text{ mm/s}^2 = 0.081 \text{ m/s}^2$$

$$\omega^2 = 36 \text{ rad}^2/\text{s}^2$$

$$2\omega u = (2)(3)(90) = 540 \text{ mm/s}^2 = 0.54 \text{ m/s}^2$$

(*a*) *Point A.*

$$\mathbf{r}_A = 0, \qquad \mathbf{v}_{A/F} = 0.09 \text{ m/s} \leftarrow$$

$$\mathbf{a}_{A'} = 0, \qquad \mathbf{a}_{A/F} = \frac{u^2}{\rho} \uparrow = 0.081 \text{ m/s}^2 \uparrow$$

Coriolis acceleration. $2\omega u \uparrow = 0.54 \text{ m/s}^2 \uparrow$

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + [2\omega u \uparrow] = 0.621 \text{ m/s}^2 \uparrow \qquad \mathbf{a}_A = 0.621 \text{ m/s}^2 \uparrow \blacktriangleleft$$

(*b*) *Point B.*

$$\mathbf{r}_B = 0.1\sqrt{2} \text{ m} \searrow 45°, \qquad \mathbf{v}_{B/F} = 0.09 \text{ m/s} \uparrow$$

$$\mathbf{a}_{B'} = -\omega^2 \mathbf{r}_B = -(9)(0.1\sqrt{2}) \searrow 45° = 0.9\sqrt{2} \text{ m/s}^2 \nwarrow 45°$$

$$\mathbf{a}_{B/F} = \frac{u^2}{\rho}$$

$$= 0.081 \text{ m/s}^2 \rightarrow$$

Coriolis acceleration. $2\omega u = 0.54 \text{ m/s}^2 \rightarrow$

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + [2\omega u \rightarrow]$$

$$= [1.521 \text{ m/s}^2 \rightarrow] + [0.9 \text{ m/s}^2 \downarrow] \qquad \mathbf{a}_B = 1.767 \text{ m/s}^2 \nwarrow 30.6° \blacktriangleleft$$

PROBLEM 15.173 (Continued)

(*c*) *Point C.*

$$\mathbf{r}_C = 0.2 \text{ m} \uparrow$$

$$\mathbf{v}_{C/F} = 0.09 \text{ m/s} \rightarrow$$

$$\mathbf{a}_{C'} = -\omega^2 \mathbf{r}_C = -(9)(0.2 \uparrow) = 1.8 \text{ m/s}^2 \downarrow$$

$$\mathbf{a}_{C/F} = \frac{u^2}{\rho} = 0.081 \text{ m/s}^2 \downarrow$$

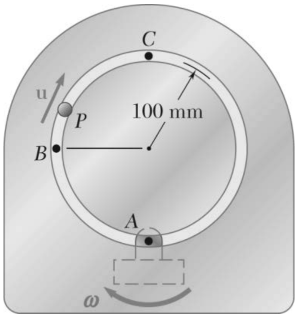
Coriolis acceleration. $2\omega u = 0.54 \text{ m/s}^2 \downarrow$

$$\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/F} + [2\omega u \downarrow]$$

$$= 2.421 \text{ m/s}^2 \downarrow$$

$\mathbf{a}_C = 2.42 \text{ m/s}^2 \downarrow$ ◀

PROBLEM 15.174



Pin P slides in a circular slot cut in the plate shown at a constant relative speed $u = 90$ mm/s. Knowing that at the instant shown the angular velocity ω of the plate is 3 rad/s clockwise and is decreasing at the rate of 5 rad/s^2, determine the acceleration of the pin if it is located at (*a*) Point *A*, (*b*) Point *B*, (*c*) Point *C*.

SOLUTION

$$\omega = 3\text{ rad/s} \circlearrowright, \quad \alpha = 5\text{ rad/s} \circlearrowleft, \quad u = 90\text{ mm/s} = 0.09\text{ m/s}, \quad \dot{u} = 0$$

$$\rho = 100\text{ mm}$$

$$\frac{u^2}{\rho} = \frac{(90)^2}{100} = 81\text{ mm/s}^2 = 0.081\text{ m/s}^2$$

$$\omega^2 = 36\text{ rad}^2/\text{s}^2$$

$$2\omega u = (2)(3)(90) = 540\text{ mm/s}^2 = 0.54\text{ m/s}^2$$

(*a*) *Point A.*

$$\mathbf{r}_A = 0$$

$$\mathbf{v}_{A/F} = 0.09\text{ m/s} \leftarrow$$

$$\mathbf{a}_{A'} = 0$$

$$\mathbf{a}_{A/F} = \frac{u^2}{\rho}\uparrow = 0.081\text{ m/s}^2 \uparrow$$

Coriolis acceleration. $2\omega u \uparrow = 0.54\text{ m/s}^2 \uparrow$

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + [2\omega u \uparrow]$$

$$= 0.621\text{ m/s}^2 \uparrow$$

$\mathbf{a}_A = 0.621\text{ m/s}^2 \uparrow$ ◀

(*b*) *Point B.*

$$\mathbf{r}_B = 0.1\sqrt{2}\text{ m} \searrow 45°, \qquad \mathbf{v}_{B/F} = 0.09\text{ m/s} \uparrow$$

$$\mathbf{a}_{B'} = \alpha\mathbf{k}\times\mathbf{r}_B - \omega^2\mathbf{r}_B = [(0.1\sqrt{2})(5) \swarrow 45°] - [(9)(0.1\sqrt{2}) \searrow 45°]$$

$$= [0.5\sqrt{2}\text{ m/s}^2 \swarrow 45°] + [0.9\sqrt{2}\text{ m/s}^2 \nwarrow 45°]$$

$$\mathbf{a}_{B/F} = \frac{u^2}{\rho} = 0.081\text{ m/s}^2 \rightarrow$$

Coriolis acceleration. $2\omega u = 0.54\text{ m/s}^2 \rightarrow$

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + [2\omega u \rightarrow]$$

$$= [1.021\text{ m/s}^2 \rightarrow] + [1.4\text{ m/s}^2 \downarrow]$$

$\mathbf{a}_B = 1.733\text{ m/s}^2 \nwarrow 53.9°$ ◀

PROBLEM 15.174 (Continued)

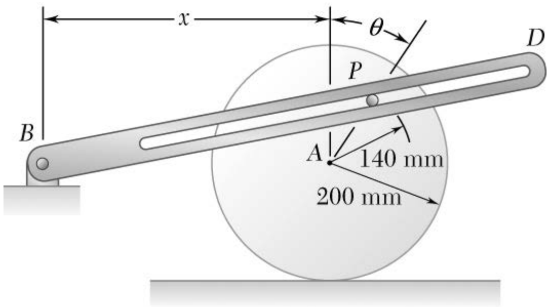
(*c*) *Point C.*

$$\mathbf{r}_C = 0.2 \text{ m} \uparrow$$
$$\mathbf{v}_{C/F} = 0.09 \text{ m/s} \rightarrow$$
$$\mathbf{a}_{C'} = \alpha\mathbf{k}\times\mathbf{r}_C - \omega^2\mathbf{r}_C$$
$$= [(0.2)(5) \leftarrow] - [(9)(0.2 \uparrow)]$$
$$= [1 \text{ m/s}^2 \leftarrow] + [1.8 \text{ m/s}^2 \downarrow]$$
$$\mathbf{a}_{C/F} = \frac{u^2}{\rho}$$
$$= 0.081 \text{ m/s}^2 \downarrow$$

Coriolis acceleration. $2\omega u = 0.54 \text{ m/s}^2 \downarrow$

$$\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/F} + 2\omega u \downarrow$$
$$= [1 \text{ m/s}^2 \leftarrow] + [2.421 \text{ m/s}^2 \downarrow]$$

$\mathbf{a}_C = 2.62 \text{ m/s}^2 \measuredangle\ 67.6°$ ◀

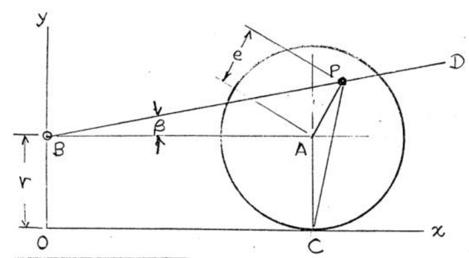


PROBLEM 15.175

Pin P is attached to the wheel shown and slides in a slot cut in bar BD. The wheel rolls to the right without slipping with a constant angular velocity of 20 rad/s. Knowing that x = 480 mm when $\theta = 0$, determine (*a*) the angular acceleration of the bar and (*b*) the relative acceleration of pin P with respect to the bar when $\theta = 0$.

SOLUTION

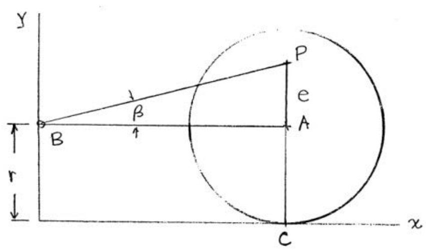
Coordinates.



$$x_A = (x_A)_0 + r\theta, \qquad y_A = r$$
$$x_B = 0, \qquad y_B = r$$
$$x_C = x_A, \qquad y_C = 0$$
$$x_P = x_A + e\sin\theta, \qquad y_P = r + e\cos\theta$$

Data:

$$(x_A)_0 = 480 \text{ mm} = 0.48 \text{ m}$$
$$r = 200 \text{ mm} = 0.20 \text{ m}$$
$$e = 140 \text{ mm} = 0.14 \text{ m}$$
$$\theta = 0 \qquad x_P = 480 \text{ mm} = 0.48 \text{ m}$$



Velocity analysis.

$$\boldsymbol{\omega}_{AC} = 20 \text{ rad/s} \circlearrowright, \qquad \boldsymbol{\omega}_{BD} = \omega_{BD} \circlearrowright$$

$$\mathbf{v}_P = (r+e)\omega_{AC} \longrightarrow$$
$$= (0.20 + 0.14)(20)$$
$$= 6.8 \text{ m/s} \longrightarrow$$

$$\mathbf{v}_{P'} = [x_P\omega_{BD} \downarrow] + [e\omega_{BD} \longrightarrow]$$

$$\mathbf{v}_{P/F} = [u\cos\beta \longrightarrow] + [u\sin\beta \uparrow]$$

$$\tan\beta = \frac{e}{x_P} = \frac{0.14}{0.48}$$
$$\beta = 16.260°$$

Use $\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/F}$ and resolve into components.

$$\xrightarrow{+}: \quad 6.8 = 0.14\omega_{BD} + u\cos\beta \tag{1}$$

$$+\downarrow: \quad 0 = 0.48\omega_{BD} - u\sin\beta \tag{2}$$

Solving (1) and (2), $\omega_{BD} = 3.8080$ rad/s, $u = 6.528$ m/s

Acceleration analysis. $\boldsymbol{\alpha}_{AC} = 0, \qquad \boldsymbol{\alpha}_{BD} = \alpha_{BD} \circlearrowright$

$$\mathbf{a}_A = 0 \qquad \mathbf{a}_{P/A} = r\omega_{AB}^2 = (0.14)(20)^2 = 56 \text{ m/s}^2 \downarrow$$

$$\mathbf{a}_P = \mathbf{a}_A + \mathbf{a}_{P/A} = 56 \text{ m/s}^2 \downarrow$$

$$\mathbf{a}_{P'} = [x_P\alpha_{BD} \downarrow] + [e\alpha_B \longrightarrow] + [x_P\omega_{BD}^2 \longleftarrow] + [e\omega_{BD}^2 \downarrow]$$

$$= [0.48\alpha_{BD} \downarrow] + [0.14\alpha_{BD} \longrightarrow] + [(0.48)(3.8080)^2 \longleftarrow]$$

$$+ [(0.14)(3.8080)^2 \downarrow]$$

$$= [0.48\alpha_{BD} \downarrow] + [0.14\alpha_{BD} \longrightarrow] + [6.9604 \text{ m/s}^2 \longleftarrow]$$

$$+ [2.0301 \text{ m/s}^2 \downarrow]$$

$$\mathbf{a}_{P/F} = [\dot{u}\cos\beta \longrightarrow] + [\dot{u}\sin\beta \uparrow]$$

Coriolis acceleration.

$$2\omega_{BD}u = (2)(3.8080)(6.528) = [49.717 \text{ m/s}^2 \searrow \beta]$$

Use $\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + [2\omega_{BD}u \searrow \beta]$ and resolve into components.

$$\xrightarrow{+}: \quad 0 = 0.14\alpha_{BD} - 6.9604 + \dot{u}\cos\beta + 49.717\sin\beta$$

or $\qquad 0.14\alpha_{BD} + \dot{u}\cos\beta = -6.9602 \qquad (3)$

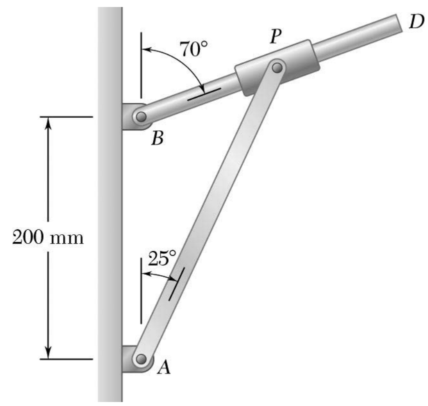
$$+\downarrow: \quad 56 = 0.48\alpha_{BD} + 2.0301 + \dot{u}\sin\beta + 49.717\cos\beta$$

or $\qquad 0.48\alpha_{BD} - \dot{u}\sin\beta = 6.2415 \qquad (4)$

Solving (3) and (4), $\qquad \alpha_{BD} = 8.09 \text{ rad/s}, \qquad \dot{u} = -8.43 \text{ m/s}^2$

(*a*) $\qquad \boldsymbol{\alpha}_{BD} = 8.09 \text{ rad/s}^2 \circlearrowright$ ◀

(*b*) $\qquad \mathbf{a}_{P/F} = 8.43 \text{ m/s}^2 \swarrow 16.26°$ ◀



PROBLEM 15.176

Knowing that at the instant shown the rod attached at A has an angular velocity of 5 rad/s counterclockwise and an angular acceleration of 2 rad/s^2 clockwise, determine the angular velocity and the angular acceleration of the rod attached at B.

SOLUTION

Geometry: Apply the law of sines to the triangle ABP to determine the lengths $\overline{AP}$ and $\overline{BP}$.

$$\text{Angle } PBA = 180° - 70° = 110° \qquad \text{Angle } APB = 180° - 25° - 110° = 45°$$

$$\overline{AB} = 200 \text{ mm} = 0.2 \text{ m} \qquad \frac{0.2}{\sin 45°} = \frac{\overline{AP}}{\sin 110°} = \frac{\overline{BP}}{\sin 25°}$$

$$\overline{AP} = 0.265785 \text{ m} \qquad \overline{BP} = 0.119534$$

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

Relative position vectors:

$$\mathbf{r}_{P/A} = 0.265785(\sin 25°\mathbf{i} + \cos 25°\mathbf{j}) = 0.11233\mathbf{i} - 0.24088\mathbf{j}$$

$$\mathbf{r}_{P/B} = 0.119534(\sin 70°\mathbf{i} + \cos 70°\mathbf{j}) = 0.11233\mathbf{i} + 0.04088\mathbf{j}$$

Angular velocities: $\boldsymbol{\omega}_{AP} = 5 \text{ rad/s} \circlearrowleft = (5 \text{ rad/s})\mathbf{k}$

$$\boldsymbol{\omega}_{BP} = \omega_{BP}\mathbf{k}$$

Angular accelerations: $\boldsymbol{\alpha}_{AP} = 2 \text{ rad/s}^2 \circlearrowright = -(2 \text{ rad/s}^2)\mathbf{k}$

$$\boldsymbol{\alpha}_{BP} = \alpha_{BP}\mathbf{k}$$

Velocity of P:

$$\mathbf{v}_P = \boldsymbol{\omega}_{AP} \times \mathbf{r}_{P/A} = 5\mathbf{k} \times (0.11233\mathbf{i} + 0.24088\mathbf{j})$$
$$= -(1.2044 \text{ m/s})\mathbf{i} + (0.56165 \text{ m/s})\mathbf{j}$$

Acceleration of P:

$$\mathbf{a}_P = \boldsymbol{\alpha}_{AP} \times \mathbf{r}_{P/A} - \omega_{AP}^2\mathbf{r}_{P/A}$$
$$= (-2\mathbf{k}) \times (0.11233\mathbf{i} + 0.24088\mathbf{j}) - (5)^2(0.11233\mathbf{i} + 0.24088\mathbf{j})$$
$$= -(2.3265 \text{ m/s}^2)\mathbf{i} - (6.2467 \text{ m/s}^2)\mathbf{j}$$

Consider the slider P as a particle sliding along the rotating rod BP with a relative velocity

$$\mathbf{v}_{\text{rel}} = u \measuredangle 20° = u(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$$

and a relative acceleration

$$\mathbf{a}_{rel} = \dot{u} \measuredangle 20° = \dot{u}(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$$

Consider the rod BP as a rotating frame of reference.

Motion of Point P' on the rod currently at P.

$$\mathbf{v}_{P'} = \boldsymbol{\omega}_{BP} \times \mathbf{r}_{P/B} = \omega_{BP}\mathbf{k} \times (0.11233\mathbf{i} + 0.04088\mathbf{j})$$
$$= -0.04088\omega_{BP}\mathbf{i} + 0.11233\omega_{BP}\mathbf{j}$$

$$\mathbf{a}_{P'} = \boldsymbol{\alpha}_{BP} \times \mathbf{r}_{P/B} - \omega_{BP}^2\mathbf{r}_{P/B}$$
$$= \alpha_{BP}\mathbf{k} \times (0.11233\mathbf{i} + 0.04088\mathbf{j}) - \omega_{BP}^2(0.11233\mathbf{i} + 0.04088\mathbf{j})$$
$$= -0.04088\alpha_{BP}\mathbf{i} + 0.11233\alpha_{BP}\mathbf{j} - 0.11233\omega_{BP}^2\mathbf{i} - 0.04088\omega_{BP}^2\mathbf{j}$$

Velocity of P: $\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{rel}$

Resolve into components.

i: $-1.2044 = -0.04088\omega_{BP} + u\cos 20°$

j: $0.56165 = 0.11233\omega_{BP} + u\sin 20°$

Solving the simultaneous equations for ω_{BP} and u,

$$\omega_{BP} = 7.8612 \text{ rad/s} \qquad u = -0.93971 \text{ m/s}$$

Angular velocity of BP: $\boldsymbol{\omega}_{BP} = 7.86 \text{ rad/s} \circlearrowleft$ ◀

Relative velocity: $\mathbf{v}_{rel} = -0.93971(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$

Coriolis acceleration:

$$2\boldsymbol{\omega}_{BP} \times \mathbf{v}_{rel} = (2)(7.8612\mathbf{k}) \times (-0.93971\cos 20°\mathbf{i} - 0.93971\sin 20°\mathbf{j})$$
$$= (5.0532 \text{ m/s}^2)\mathbf{i} - (13.8835 \text{ m/s}^2)\mathbf{j}$$

Acceleration of P: $\mathbf{a}_{P'} = \mathbf{a}_{P'} + \mathbf{a}_{rel} + 2\boldsymbol{\omega}_{BP} \times \mathbf{v}_{rel}$

Resolve into components.

i: $-2.3265 = -0.04088\alpha_{BP} - 0.11233\omega_{BP}^2 + \dot{u}\cos 20° + 5.0532$

$$-0.04088\alpha_{BP} + \dot{u}\cos 20° = -0.43788 \quad (1)$$

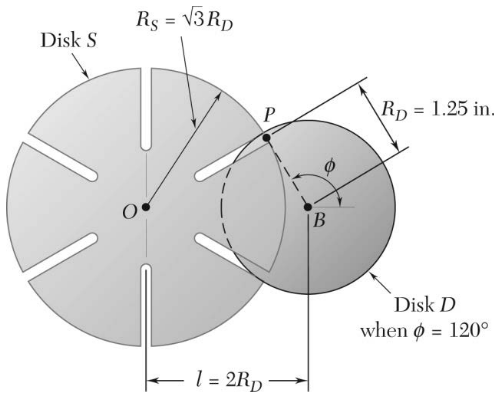
j: $-6.2467 = 0.11233\alpha_{BP} - 0.04088\omega_{BP}^2 + \dot{u}\sin 20° - 13.8835$

$$0.11233\alpha_{BP} + \dot{u}\sin 20° = 10.1631 \quad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$\alpha_{BP} = 81.146 \text{ rad/s}^2 \qquad \dot{u} = 3.0641 \text{ m/s}^2$$

Angular acceleration of BP: $\boldsymbol{\alpha}_{BP} = 81.1 \text{ rad/s}^2 \circlearrowleft$ ◀



PROBLEM 15.177

The Geneva mechanism shown is used to provide an intermittent rotary motion of disk S. Disk D rotates with a constant counterclockwise angular velocity ω_D of 8 rad/s. A pin P is attached to disk D and can slide in one of the six equally spaced slots cut in disk S. It is desirable that the angular velocity of disk S be zero as the pin enters and leaves each of the six slots; this will occur if the distance between the centers of the disks and the radii of the disks are related as shown. Determine the angular velocity and angular acceleration of disk S at the instant when $\phi = 150°$.

SOLUTION

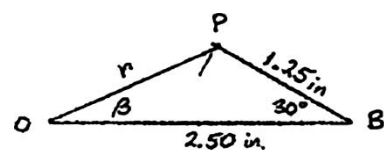
Geometry:

Law of cosines. $r^2 = 1.25^2 + 2.50^2 - (2)(1.25)(2.50)\cos 30°$

$r = 1.54914$ in.

Law of sines. $\dfrac{\sin\beta}{1.25} = \dfrac{\sin 30°}{r}$

$\beta = 23.794°$



Let disk S be a rotating frame of reference.

$$\mathbf{\Omega} = \omega_S \circlearrowright, \quad \dot{\mathbf{\Omega}} = \alpha_S \circlearrowright$$

Motion of coinciding Point P′ on the disk.

$$\mathbf{v}_{P'} = r\omega_S = 1.54914\omega_S \searrow \beta$$

$$\mathbf{a}_{P'} = -\alpha_S \mathrm{k} \times \mathrm{r}_{P/O} - \omega_S^2 \mathrm{r}_{P/O} = [1.54914\alpha_S \searrow \beta] + [1.54914\omega_S^2 \swarrow \beta]$$

Motion relative to the frame.

$$\mathbf{v}_{P/S} = u \swarrow \beta \quad \mathrm{a}_{P/S} = \dot{u} \swarrow \beta$$

Coriolis acceleration. $2\omega_S u \searrow \beta$

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/S} = [1.54914\omega_S \searrow \beta] + [u \swarrow \beta]$$

$$\mathbf{a}_P = \mathbf{a}_P + \mathbf{a}_{P/S} + 2\omega_S u \searrow$$

$$= [1.54914\alpha_S \searrow \beta] + [1.54914\omega_S^2 \swarrow \beta] + [\dot{u} \swarrow \beta] + [2\omega_S u \searrow \beta]$$

Motion of disk D. (Rotation about B)

$$\mathbf{v}_P = (BP)\omega_D = (1.25)(8) = 10 \text{ in./s} \swarrow 30°$$

$$\mathbf{a}_P = [(BP)\alpha_D \swarrow 60°] + [(BP)\omega_S^2 \nwarrow 30°] = 0 + [(1.25)(8)^2 \nwarrow 30°]$$

$$= 80 \text{ in./s}^2 \nwarrow 30°$$

PROBLEM 15.177 (Continued)

Equate the two expressions for $\mathbf{v}_P$ and resolve into components.

$$\searrow \beta: \quad 1.54914\omega_S = 10\cos(30° + \beta)$$

$$\omega_S = \frac{10\cos 53.794°}{1.54914}$$

$$= 3.8130 \text{ rad/s}$$

$\boldsymbol{\omega}_S = 3.81 \text{ rad/s}$ ↻ ◀

$$\swarrow \beta: \quad u = 10\sin(30° + \beta) = 10\sin 53.794° = 8.0690 \text{ in./s}$$

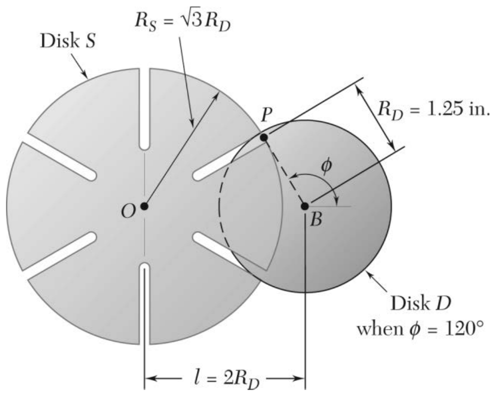
Equate the two expressions for $\mathbf{a}_P$ and resolve into components.

$$\searrow \beta: \quad 1.54914\alpha_S - 2\omega_S u = 80\sin(30° + \beta)$$

$$\alpha_S = \frac{80\sin 53.794° + (2)(3.8130)(8.0690)}{1.54914}$$

$$= 81.4 \text{ rad/s}^2$$

$\boldsymbol{\alpha}_S = 81.4 \text{ rad/s}^2$ ↻ ◀



PROBLEM 15.178

In Problem 15.177, determine the angular velocity and angular acceleration of disk S at the instant when $\phi = 135°$.

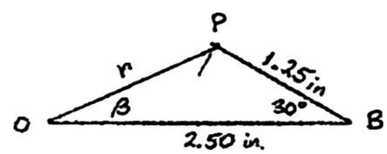
PROBLEM 15.177 The Geneva mechanism shown is used to provide an intermittent rotary motion of disk S. Disk D rotates with a constant counterclockwise angular velocity ω_D of 8 rad/s. A pin P is attached to disk D and can slide in one of the six equally spaced slots cut in disk S. It is desirable that the angular velocity of disk S be zero as the pin enters and leaves each of the six slots; this will occur if the distance between the centers of the disks and the radii of the disks are related as shown. Determine the angular velocity and angular acceleration of disk S at the instant when $\phi = 150°$.

SOLUTION

Geometry:

Law of cosines. $\quad r^2 = 1.25^2 + 2.50^2 - (2)(1.25)(2.50)\cos 45°$

$$r = 1.84203 \text{ in.}$$



Law of sines. $\quad \dfrac{\sin\beta}{1.25} = \dfrac{\sin 45°}{r}$

$$\beta = 28.675°$$

Let disk S be a rotating frame of reference.

$$\mathbf{\Omega} = \omega_S \circlearrowright, \quad \dot{\mathbf{\Omega}} = \alpha_S \circlearrowright$$

Motion of coinciding Point P′ on the disk.

$$\mathbf{v}_{P'} = r\omega_s = 1.84203\omega_s \searrow \beta$$

$$\mathbf{a}_{P'} = -\alpha_S \mathbf{k} \times \mathbf{r}_{P/O} - \omega_S^2 \mathbf{r}_{P/O} = [1.84203\alpha_S \searrow \beta] + [1.84203\omega_S^2 \swarrow \beta]$$

Motion relative to the frame.

$$\mathbf{v}_{P/S} = u \swarrow \beta \qquad \mathbf{a}_{P/S} = \dot{u} \swarrow \beta$$

Coriolis acceleration. $\quad 2\omega_S u \nwarrow \beta$

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/S} = [1.84203\omega_S \searrow \beta] + [u \swarrow \beta]$$

$$\mathbf{a}_P = \mathbf{a}_P + \mathbf{a}_{P/S} + 2\omega_S u \nwarrow$$

$$= [1.84203\alpha_S \searrow \beta] + [1.84203\omega_S^2 \swarrow \beta] + [\dot{u} \swarrow \beta] + [2\omega_S u \nwarrow \beta]$$

PROBLEM 15.178 (Continued)

Motion of disk D. (Rotation about *B*)

$$\mathbf{v}_P = (BP)\omega_D = (1.25)(8) = 10 \text{ in./s} \angle 30^\circ$$

$$\mathbf{a}_P = [(BP)\alpha_D \angle 45^\circ] + [(BP)\omega_S^2 \angle 45^\circ] = 0 + [(1.25)(8)^2 \angle 45^\circ]$$

$$= 80 \text{ in./s}^2 \angle 45^\circ$$

Equate the two expressions for $\mathbf{v}_P$ and resolve into components.

$$\searrow \beta: \quad 1.84203\omega_S = 10\cos(45^\circ + \beta)$$

$$\omega_S = \frac{10\cos 73.675^\circ}{1.84203}$$

$$= 1.52595 \text{ rad/s}$$

$\boldsymbol{\omega}_S = 1.526 \text{ rad/s} \circlearrowright$ ◀

$$\angle \beta: \quad u = 10\sin(45^\circ + \beta) = 10\sin 73.675^\circ = 9.5968 \text{ in./s}$$

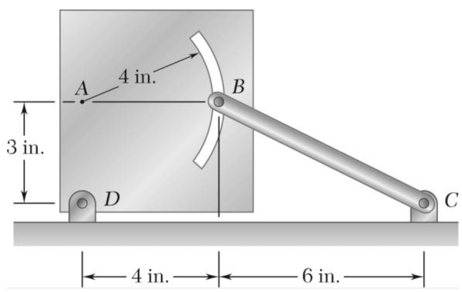
Equate the two expressions for $\mathbf{a}_P$ and resolve into components.

$$\searrow \beta: \quad 1.84203\alpha_S - 2\omega_S u = 80\sin(45^\circ + \beta)$$

$$\alpha_S = \frac{80\sin 73.675^\circ + (2)(1.52595)(9.5968)}{1.84203}$$

$$= 57.6 \text{ rad/s}^2$$

$\boldsymbol{\alpha}_S = 57.6 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 15.179

At the instant shown, bar *BC* has an angular velocity of 3 rad/s and an angular acceleration of 2 rad/s^2, both counterclockwise. Determine the angular acceleration of the plate.

SOLUTION

Relative position vectors.
$$\mathbf{r}_{B/D} = (4 \text{ in.})\mathbf{i} + (3 \text{ in.})\mathbf{j}$$
$$\mathbf{r}_{B/C} = -(6 \text{ in.})\mathbf{i} + (3 \text{ in.})\mathbf{j}$$

Velocity analysis.
$$\boldsymbol{\omega}_{BC} = 3 \text{ rad/s} \circlearrowleft$$

Bar *BC* (Rotation about *C*):
$$\boldsymbol{\omega}_{BC} = (3 \text{ rad/s})\mathbf{k}$$
$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = 3\mathbf{k} \times (-6\mathbf{i} + 3\mathbf{j})$$
$$= -(9 \text{ in./s})\mathbf{i} - (18 \text{ in./s})\mathbf{j}$$

Plate (Rotation about *D*):
$$\omega_P = \omega_P \mathbf{k}$$

Let Point B' be the point in the plate coinciding with *B*.
$$\mathbf{v}_{B'} = \boldsymbol{\omega}_P \times \mathbf{r}_{B/D} = \omega_P \mathbf{k} \times (4\mathbf{i} + 3\mathbf{j})$$
$$= -3\omega_P \mathbf{i} + 4\omega_P \mathbf{j}$$

Let plate be a rotating frame.
$$\mathbf{v}_{B/F} = v_{\text{rel}}\mathbf{j}$$
$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$$
$$= -3\omega_P \mathbf{i} + (4\omega_P + v_{\text{rel}})\mathbf{j}$$

Equate like components of $\mathbf{v}_B$.
$$\mathbf{i}: \quad -9 = -3\omega_P \qquad \boldsymbol{\omega}_P = (3 \text{ rad/s})\mathbf{k}$$
$$\mathbf{j}: \quad -18 = (4)(3) + v_{\text{rel}} \qquad \mathbf{v}_{\text{rel}} = -(30 \text{ in./s})\mathbf{j}$$

Acceleration analysis.
$$\boldsymbol{\alpha}_{BC} = 2 \text{ rad/s}^2 \circlearrowleft$$

Bar *BC*:
$$\boldsymbol{\alpha}_{BC} = (2 \text{ rad/s}^2)\mathbf{k}$$
$$\mathbf{a}_B = \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2 \, \mathbf{r}_{B/C}$$
$$= 2\mathbf{k} \times (-6\mathbf{i} + 3\mathbf{j}) - (3)^2(-6\mathbf{i} + 3\mathbf{j})$$
$$= -12\mathbf{j} - 6\mathbf{i} + 54\mathbf{i} - 27\mathbf{j} = (48 \text{ in./s}^2)\mathbf{i} - (39 \text{ in./s}^2)\mathbf{j}$$

Plate:
$$\boldsymbol{\alpha}_P = \alpha_P \mathbf{k}$$
$$\mathbf{a}_{B'} = \boldsymbol{\alpha}_P \times \mathbf{r}_{B/D} - \omega_P^2 \, \mathbf{r}_{B/D}$$
$$= \alpha_P \mathbf{k} \times (4\mathbf{i} + 3\mathbf{j}) - (3)^2(4\mathbf{i} + 3\mathbf{j})$$
$$= -3\alpha_P \mathbf{i} + 4\alpha_P \mathbf{j} - 36\mathbf{i} - 27\mathbf{j}$$

PROBLEM 15.179 (Continued)

Relative to the frame (plate), the acceleration of pin B is

$$\mathbf{a}_{B/F} = (a_{\text{rel}})_t\,\mathbf{j} - \frac{v_{\text{rel}}^2}{\rho}\mathbf{i} = (a_{\text{rel}})_t\,\mathbf{j} - \frac{30^2}{4}\mathbf{i}$$

$$= -(225 \text{ in./s}^2)\mathbf{i} + (a_{\text{rel}})_t\,\mathbf{j}$$

<u>Coriolis acceleration</u>. $2\boldsymbol{\omega}_P \times \mathbf{v}_{P/F}$

$$\mathbf{a}_c = 2(3\mathbf{k})\times(-30\mathbf{j}) = (180 \text{ in./s}^2)\mathbf{i}$$

Then

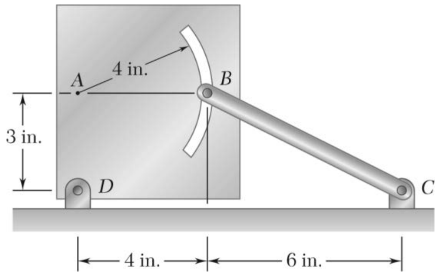
$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + \mathbf{a}_c$$

$$\mathbf{a}_B = -(3\alpha_P + 36)\mathbf{i} + (4\alpha_P - 27)\mathbf{j} - 225\mathbf{i} + (a_{\text{rel}})_t\,\mathbf{j} + 180\mathbf{i}$$

$$\mathbf{a}_B = -(3\alpha_P + 81)\mathbf{i} + [4\alpha_P + (a_{\text{rel}})_t - 27]\mathbf{j}$$

Equate like components of $\mathbf{a}_B$.

$\mathbf{i}$: $48 = -(3\alpha_P + 81)$ $\quad \alpha_P = -43 \text{ rad/s}^2$ $\quad \boldsymbol{\alpha}_P = 43.0 \text{ rad/s}^2$ ↻ ◀



PROBLEM 15.180

At the instant shown bar BC has an angular velocity of 3 rad/s and an angular acceleration of 2 rad/s^2, both clockwise. Determine the angular acceleration of the plate.

SOLUTION

Relative position vectors.

$$\mathbf{r}_{B/D} = (4 \text{ in.})\mathbf{i} + (3 \text{ in.})\mathbf{j}$$
$$\mathbf{r}_{B/C} = -(6 \text{ in.})\mathbf{i} + (3 \text{ in.})\mathbf{j}$$

Velocity analysis.

$$\boldsymbol{\omega}_{BC} = 3 \text{ rad/s} \circlearrowright$$

Bar BC (Rotation about C):

$$\boldsymbol{\omega}_{BC} = -(3 \text{ rad/s})\mathbf{k}$$
$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$
$$= (-3\mathbf{k}) \times (-6\mathbf{i} + 3\mathbf{j})$$
$$= (9 \text{ in./s})\mathbf{i} + (18 \text{ in./s})\mathbf{j}$$

Plate (Rotation about D):

$$\omega_P = \omega_P \mathbf{k}$$

Let Point B' be the point in the plate coinciding with B:

$$\mathbf{v}_{B'} = \boldsymbol{\omega}_P \times \mathbf{r}_{B/D}$$
$$= \omega_P \mathbf{k} \times (4\mathbf{i} + 3\mathbf{j})$$
$$= -3\omega_P \mathbf{i} + 4\omega_P \mathbf{j}$$

Let the plate be a rotating frame.

$$\mathbf{v}_{B/F} = v_{\text{rel}} \mathbf{j}$$
$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$$
$$= -3\omega_P \mathbf{i} + (4\omega_P + v_{\text{rel}})\mathbf{j}$$

Equate like components of $\mathbf{v}_B$.

$\mathbf{i}$: $\quad 9 = -3\omega_P \qquad \boldsymbol{\omega}_P = -(3 \text{ rad/s})\mathbf{k}$

$\mathbf{j}$: $\quad 18 = (4)(3) + v_{\text{rel}} \qquad \mathbf{v}_{\text{rel}} = (30 \text{ in./s})\mathbf{j}$

Acceleration analysis.

$$\boldsymbol{\alpha}_{BC} = 2 \text{ rad/s}^2 \circlearrowright$$

Bar BC:

$$\boldsymbol{\alpha}_{BC} = -(2 \text{ rad/s}^2)\mathbf{k}$$
$$\mathbf{a}_B = \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2 \mathbf{r}_{B/C}$$
$$= (-2\mathbf{k}) \times (-6\mathbf{i} + 3\mathbf{j}) - (3)^2(-6\mathbf{i} + 3\mathbf{j})$$
$$= 12\mathbf{j} + 6\mathbf{i} + 54\mathbf{i} - 27\mathbf{j}$$
$$= (60 \text{ in./s}^2)\mathbf{i} - (15 \text{ in./s}^2)\mathbf{j}$$

PROBLEM 15.180 (Continued)

Plate:

$$\boldsymbol{\alpha}_P = \alpha_P \mathbf{k}$$

$$\mathbf{a}_{B'} = \boldsymbol{\alpha}_P \times \mathbf{r}_{B/D} - \omega_P^2 \mathbf{r}_{B/D}$$

$$= \alpha_P \mathbf{k} \times (4\mathbf{i} + 3\mathbf{j}) - (3)^2 (4\mathbf{i} + 3\mathbf{j})$$

$$= -3\alpha_P \mathbf{i} + 4\alpha_P \mathbf{j} - 36\mathbf{i} - 27\mathbf{j}$$

Relative to the frame (plate), the acceleration of pin B is

$$\mathbf{a}_{B/F} = (a_{\text{rel}})_t \mathbf{j} - \frac{v_{\text{rel}}^2}{\rho} \mathbf{i}$$

$$= (a_{\text{rel}})_t \mathbf{j} - \frac{30^2}{4} \mathbf{i}$$

$$= -(225 \text{ in./s}^2)\mathbf{i} + (a_{\text{rel}})_t \mathbf{j}$$

Coriolis acceleration. $2\boldsymbol{\omega}_P \times \mathbf{v}_{P/F}$

$$\mathbf{a}_c = 2(-3\mathbf{k}) \times (30\mathbf{j}) = (180 \text{ in./s}^2)\mathbf{i}$$

Then

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + \mathbf{a}_c$$

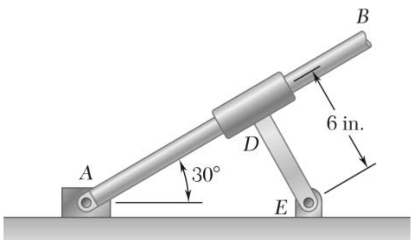
$$\mathbf{a}_B = -(3\alpha_P + 36)\mathbf{i} + (4\alpha_P - 27)\mathbf{j} - 225\mathbf{i} + (a_{\text{rel}})_t \mathbf{j} + 180\mathbf{i}$$

$$\mathbf{a}_B = -(3\alpha_P + 81)\mathbf{i} + [4\alpha_P + (a_{\text{rel}})_t - 27]\mathbf{j}$$

Equate like components of $\mathbf{a}_B$.

$\mathbf{i}$: $60 = -(3\alpha_P + 81)$ $\quad \alpha_P = -47 \text{ rad/s}^2$ $\quad \boldsymbol{\alpha}_P = 47.0 \text{ rad/s}^2$ ↻ ◀

PROBLEM 15.181*



Rod AB passes through a collar which is welded to link DE. Knowing that at the instant shown block A moves to the right at a constant speed of 75 in./s, determine (*a*) the angular velocity of rod AB, (*b*) the velocity relative to the collar of the point of the rod in contact with the collar, (*c*) the acceleration of the point of the rod in contact with the collar. (*Hint:* Rod AB and link DE have the same $\boldsymbol{\omega}$ and the same $\boldsymbol{\alpha}$.)

SOLUTION

Let $\boldsymbol{\omega} = \omega \circlearrowleft$ and $\boldsymbol{\alpha} = \alpha \circlearrowleft$ be the angular velocity and angular acceleration of the link DE and collar rigid body. Let F be a frame of reference moving with this body. The rod AB slides in the collar relative to the frame of reference with relative velocity $\mathbf{u} = u \measuredangle 30°$ and relative acceleration $\dot{\mathbf{u}} = \dot{u} \measuredangle 30°$. Note that this relative motion is a translation that applies to all points along the rod. Let Point A be moving with the end of the rod and A' be moving with the frame. Point E is a fixed point.

Geometry.
$$\mathbf{r}_{A'/E} = \frac{6 \text{ in.}}{\sin 30°} = 12 \text{ in.} \leftarrow$$

Velocity analysis.
$$\mathbf{v}_A = 75 \text{ in./s} \rightarrow$$
$$\mathbf{v}_{A'} = 12\omega \downarrow$$
$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{u} \quad \text{Resolve into components.}$$

$$\xrightarrow{+}: \quad 75 = 0 + u\cos 30° \quad u = \frac{75}{\cos 30°} = 86.603 \text{ in./s}$$

$$+\uparrow: \quad 0 = -12\omega + u\sin 30° \quad \omega = \frac{u \sin 30°}{12} = 3.6085 \text{ rad/s}$$

(*a*) Angular velocity. $\boldsymbol{\omega} = 3.61 \text{ rad/s} \circlearrowleft$ ◀

(*b*) Velocity of rod AB relative to the collar. $\mathbf{u} = 86.6 \text{ in./s} \measuredangle 30°$ ◀

(*c*) Acceleration analysis.
$$\mathbf{a}_A = 0$$
$$\mathbf{a}_{A'} = [12\alpha \downarrow] + [12\omega^2 \rightarrow] = [12\alpha \downarrow] + [156.25 \rightarrow]$$

Coriolis acceleration.

$$\mathbf{a}_c = 2\omega u \searrow 60° = 625.01 \text{ in./s}^2 \searrow 60°$$
$$\mathbf{a}_A = \mathbf{a}_{A'} + \dot{\mathbf{u}} + \mathbf{a}_c \quad \text{Resolve into components.}$$
$$\xrightarrow{+}: \quad 0 = 156.25 + \dot{u}\cos 30° - 625.01\cos 60°$$
$$\dot{u} = 180.43 \text{ in./s}^2$$
$$+\uparrow: \quad 0 = -12\alpha + \dot{u}\sin 30° + 625.01\sin 60°$$
$$\alpha = 52.624 \text{ rad/s}^2$$

For rod AB, $\qquad \boldsymbol{\omega}_{AB} = 3.6085 \text{ rad/s} \circlearrowright$

$$\boldsymbol{\alpha}_{AB} = 52.624 \text{ rad/s}^2 \circlearrowright$$

Let P be the point on AB coinciding with collar D.

$$\mathbf{r}_{P/A} = 12\cos 30° \measuredangle 30° = 10.392 \text{ in.} \measuredangle 30°.$$

$$\mathbf{a}_P = \mathbf{a}_A + (\mathbf{a}_{P/A})_t + (\mathbf{a}_{P/A})_n$$

$$= 0 + [(10.392)(52.624) \searrow 60°] + [(10.392)(3.6085)^2 \swarrow 30°]$$

$$= [546.87 \searrow 60°] + [135.32 \swarrow 30°] = [390.63 \longleftarrow] + [405.94 \downarrow]$$

$$\mathbf{a}_P = 563 \text{ in./s}^2 \swarrow 46.1° \blacktriangleleft$$

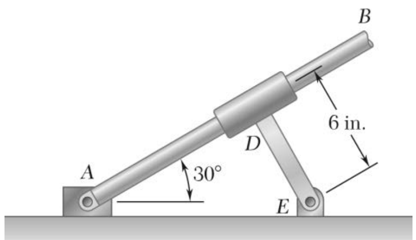
$\mathbf{a}_P$ may also be determined from $\mathbf{a}_P = \mathbf{a}_{P'} + \dot{\mathbf{u}} + \mathbf{a}_c$ using the rotating frame. The already calculated vectors $\dot{\mathbf{u}}$ and $\mathbf{a}_c$ also apply at Points P' and P.

$$\mathbf{a}_{P'} = \mathbf{a}_D = 6\alpha \swarrow 30° + 6\omega \searrow 60°$$

$$= [315.74 \text{ in./s}^2 \swarrow 30°] + [78.13 \text{ in./s}^2 \swarrow 60°]$$

Then $\qquad \mathbf{a}_P = [315.74 \swarrow 30°] + [78.13 \searrow 60°] + (180.43 \measuredangle 30°] + [625.01 \searrow 60°]$

$$= (135.31 \swarrow 30°] + [546.88 \searrow 60°]$$



PROBLEM 15.182*

Solve Problem 15.181, assuming that block A moves to the left at a constant speed of 75 in./s.

PROBLEM 15.181 Rod AB passes through a collar which is welded to link DE. Knowing that at the instant shown block A moves to the right at a constant speed of 75 in./s, determine (*a*) the angular velocity of rod AB, (*b*) the velocity relative to the collar of the point of the rod in contact with the collar, (*c*) the acceleration of the point of the rod in contact with the collar. (*Hint:* Rod AB and link DE have the same $\boldsymbol{\omega}$ and the same $\boldsymbol{\alpha}$.)

SOLUTION

Let $\boldsymbol{\omega} = \omega \circlearrowleft$ and $\boldsymbol{\alpha} = \alpha \circlearrowleft$ be the angular velocity and angular acceleration of the link DE and collar rigid body. Let F be a frame of reference moving with this body. The rod AB slides in the collar relative to the frame of reference with relative velocity $\mathbf{u} = u \measuredangle 30°$ and relative acceleration $\dot{\mathbf{u}} = \dot{u} \measuredangle 30°$. Note that this relative motion is a translation that applies to all points along the rod. Let Point A be moving with the end of the rod and A' be moving with the frame. Point E is a fixed point.

Geometry. $$\mathbf{r}_{A'/E} = \frac{6 \text{ in.}}{\sin 30°} = 12 \text{ in.} \longleftarrow$$

Velocity analysis. $$\mathbf{v}_A = 75 \text{ in./s} \longleftarrow$$

$$\mathbf{v}_{A'} = 12\omega \downarrow$$

$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{u} \quad \text{Resolve into components.}$$

$$\xrightarrow{+}: \quad -75 = 0 + u\cos 30° \quad u = \frac{75}{\cos 30°} = -86.603 \text{ in./s}$$

$$+\uparrow: \quad 0 = -12\omega + u\sin 30° \quad \omega = \frac{u\sin 30°}{12} = -3.6085 \text{ rad/s}$$

(*a*) Angular velocity. $\boldsymbol{\omega} = 3.61 \text{ rad/s} \circlearrowright$ ◀

(*b*) Velocity of rod AB relative to the collar. $\mathbf{u} = 86.6 \text{ in./s} \measuredangle 30°$ ◀

Acceleration analysis. $$\mathbf{a}_A = 0$$

$$\mathbf{a}_{A'} = [12\alpha \downarrow] + [12\omega^2 \longrightarrow] = [12\alpha \downarrow] + [156.25 \longrightarrow]$$

Coriolis acceleration. $$\mathbf{a}_c = 2\omega u \searrow 60° = 625.01 \text{ in./s}^2 \searrow 60°$$

$$\mathbf{a}_A = \tilde{\mathbf{a}}_{A'} + \dot{\mathbf{u}} + \mathbf{a}_c \quad \text{Resolve into components.}$$

$$\xrightarrow{+}: \quad 0 = 156.25 + \dot{u}\cos 30° - 625.01\cos 60°$$

$$\dot{u} = 180.43 \text{ in./s}^2$$

$$+\uparrow: \quad 0 = -12\alpha + \dot{u}\sin 30° + 625.01\sin 60°$$

$$\alpha = 52.624 \text{ rad/s}^2$$

For rod AB, $\quad \boldsymbol{\omega}_{AB} = 3.6085 \text{ rad/s} \circlearrowleft$

$$\boldsymbol{\alpha}_{AB} = 52.624 \text{ rad/s}^2 \circlearrowleft$$

Let P be the point on AB coinciding with collar D.

$$\mathbf{r}_{P/A} = 12\cos 30° \measuredangle 30° = 10.392 \text{ in.} \measuredangle 30°.$$

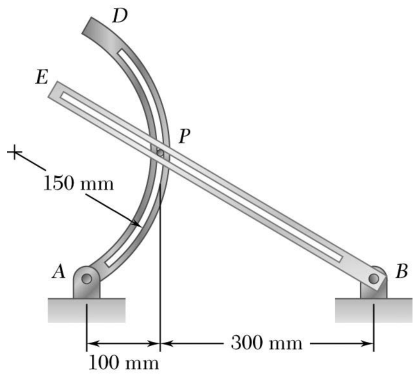
$$\mathbf{a}_P = \mathbf{a}_A + (\mathbf{a}_{P/A})_t + (\mathbf{a}_{P/A})_n$$

$$= 0 + [(10.392)(52.624) \searrow 60°] + [(10.392)(3.6085)^2 \swarrow 30°]$$

$$= [546.87 \searrow 60°] + [135.32 \swarrow 30°] = [390.63 \leftarrow] + [405.94 \downarrow]$$

$$\mathbf{a}_P = 563 \text{ in./s}^2 \swarrow 46.1° \blacktriangleleft$$

PROBLEM 15.183*



In Problem 15.157, determine the acceleration of pin *P*.

PROBLEM 15.157 The motion of pin *P* is guided by slots cut in rods *AD* and *BE*. Knowing that bar *AD* has a constant angular velocity of 4 rad/s clockwise and bar *BE* has an angular velocity of 5 rad/s counterclockwise and is slowing down at a rate of 2 rad/s^2, determine the velocity of *P* for the position shown.

SOLUTION

Units: meters, m/s, m/s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

From the solution of Problem 15.157,

$$\theta = 26.565° \qquad R = 0.100 \text{ m}$$

$$\mathbf{r}_{P/A} = 0.1\mathbf{i} + 0.15\mathbf{j} \qquad \mathbf{r}_{P/B} = -0.3\mathbf{i} + 0.15\mathbf{j}$$

$$\boldsymbol{\omega}_{AD} = -(4 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\omega}_{BE} = (5 \text{ rad/s}^2)\mathbf{k}$$

$$\boldsymbol{\alpha}_{AD} = 0 \qquad \boldsymbol{\alpha}_{BE} = -(2 \text{ rad/s}^2)\mathbf{k}$$

$$\mathbf{v}_{P/AD} = u_1\mathbf{j} = -(1.775 \text{ m/s})\mathbf{j}$$

$$\begin{aligned}\mathbf{v}_{P/BE} &= -u_2\cos\theta\mathbf{i} + u_2\sin\theta\mathbf{j} \\ &= (-1.50935)(-\cos 26.565°\mathbf{i} + \sin 26.565°)\mathbf{j} \\ &= (1.35 \text{ m/s})\mathbf{i} + (0.675 \text{ m/s})\mathbf{j}\end{aligned}$$

Acceleration of Point *P*′ on rod *AD* coinciding with the pin:

$$\begin{aligned}\mathbf{a}_{P'} &= \boldsymbol{\alpha}_{AD} \times \mathbf{r}_{P/A} - \omega_{AD}^2\mathbf{r}_{P/A} \\ &= 0 - (4)^2(0.1\mathbf{i} + 0.15\mathbf{j}) = -1.6\mathbf{i} - 2.4\mathbf{j}\end{aligned}$$

Acceleration of the pin relative to rod *AD*:

$$\begin{aligned}\mathbf{a}_{P/AD} &= \dot{u}_1\mathbf{j} - \frac{u_1^2}{R}\mathbf{i} = \dot{u}_1\mathbf{j} - \frac{(1.775)^2}{0.15}\mathbf{i} \\ &= \dot{u}_1\mathbf{j} - 21.004\mathbf{i}\end{aligned}$$

Coriolis acceleration: $\mathbf{a}_1 = 2\boldsymbol{\omega}_{AD} \times \mathbf{v}_{P/AD}$

$$\mathbf{a}_1 = 2(-4\mathbf{k}) \times (-1.775\mathbf{j}) = -14.2\mathbf{i}$$

Acceleration of *P*: $\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/AD} + \mathbf{a}_1$

$$\mathbf{a}_P = -36.804\mathbf{i} - 2.4\mathbf{j} + \dot{u}_1\mathbf{j}$$

PROBLEM 15.183* (Continued)

Acceleration of Point P'' on rod BE coinciding with the pin

$$\mathbf{a}_{P''} = \boldsymbol{\alpha}_{BE} \times \mathbf{r}_{P/B} - \omega_{BE}^2 \mathbf{r}_{P/E}$$
$$= (-2\mathbf{k}) \times (-0.3\mathbf{i} + 0.15\mathbf{j}) - (5)^2(-0.3\mathbf{i} + 0.15\mathbf{j})$$
$$= 0.6\mathbf{j} + 0.3\mathbf{i} + 7.5\mathbf{i} - 3.75\mathbf{j} = 7.8\mathbf{i} - 3.15\mathbf{j}$$

Acceleration of the pin relative to the rod BE:

$$\mathbf{a}_{P/BE} = \dot{u}_2(-\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$$

Coriolis acceleration:

$$\mathbf{a}_2 = 2\boldsymbol{\omega}_{BE} \times \mathbf{v}_{P/BE}$$
$$= (2)(5\mathbf{k}) \times (1.35\mathbf{i} - 0.675\mathbf{j}) = -13.5\mathbf{j} + 6.75\mathbf{i}$$

Acceleration of P:

$$\mathbf{a}_P = \mathbf{a}_{P''} + \mathbf{a}_{P/BE} + \mathbf{a}_2$$

$$\mathbf{a}_P = 7.8\mathbf{i} - 3.15\mathbf{j} + \dot{u}_2(-\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) + 13.5\mathbf{j} + 6.75\mathbf{i}$$
$$= 14.55\mathbf{i} + 10.35\mathbf{j} - \dot{u}_2\cos\theta\mathbf{i} + \dot{u}_2\sin\theta\mathbf{j}$$

Equating the two expressions for $\mathbf{a}_P$ and resolving into components,

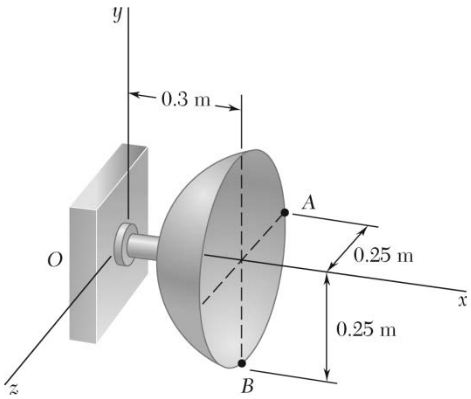
i: $$-36.804 = 14.55 - \dot{u}_2\cos\theta$$
$$\dot{u}_2 = \frac{36.804 + 14.55}{\cos 26.565°} = 57.415 \text{ m/s}^2$$

j: $$-2.4 + \dot{u}_1 = 10.35 + \dot{u}_2\sin\theta$$
$$\dot{u}_1 = 12.75 + 57.415\sin 26.565° = 38.426 \text{ m/s}^2$$

Acceleration of the pin.

$$\mathbf{a}_P = -36.804\mathbf{i} - 2.4\mathbf{j} + 38.427\mathbf{j}$$
$$= -36.804\mathbf{i} + 36.027\mathbf{j}$$

$$\mathbf{a}_P - (36.8 \text{ m/s}^2)\mathbf{i} + (36.0 \text{ m/s}^2)\mathbf{j} = 51.5 \text{ m/s}^2 \measuredangle 44.4° \blacktriangleleft$$



PROBLEM 15.184

At the instant considered, the radar antenna shown rotates about the origin of coordinates with an angular velocity $\boldsymbol{\omega} = \omega_x \mathbf{i} + \omega_y \mathbf{j} + \omega_z \mathbf{k}$. Knowing that $(v_A)_y = 300$ mm/s, $(v_B)_y = 180$ mm/s, and $(v_B)_z = 360$ mm/s, determine (*a*) the angular velocity of the antenna, (*b*) the velocity of Point *A*.

SOLUTION

$$\mathbf{r}_A = (0.3 \text{ m})\mathbf{i} - (0.25 \text{ m})\mathbf{k}$$

$$\mathbf{v}_A = (v_A)_x \mathbf{i} + (0.3 \text{ m/s})\,\mathbf{j} + (v_A)_z \mathbf{k}$$

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_A: \quad (v_A)_x \mathbf{i} + 0.3\mathbf{j} + (v_A)_z \mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & 0 & -0.25 \end{vmatrix}$$

$$(v_A)_x \mathbf{i} + 0.3\mathbf{j} + (v_A)_z \mathbf{k} = -0.25\omega_y \mathbf{i} + (0.3\omega_z + 0.25\omega_x)\mathbf{j} - 0.3\omega_y \mathbf{k}$$

$$\mathbf{i}: \quad (v_A)_x = -0.25\omega_y \qquad (1)$$

$$\mathbf{j}: \quad 0.3 = 0.3\omega_z + 0.25\omega_x \qquad (2)$$

$$\mathbf{k}: \quad (v_A)_z = -0.3\omega_y \qquad (3)$$

$$\mathbf{r}_B = (0.3 \text{ m})\mathbf{i} - (0.25 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = (v_B)_x \mathbf{i} + (0.18 \text{ m/s})\mathbf{j} + (0.36 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_B: \quad (v_B)_x \mathbf{i} + 0.18\mathbf{j} + 0.36\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & -0.25 & 0 \end{vmatrix}$$

$$(v_B)_x \mathbf{i} + 0.18\mathbf{j} + 0.36\mathbf{k} = 0.25\omega_z \mathbf{i} + 0.3\omega_z \mathbf{j} - (0.25\omega_x + 0.3\omega_y)\mathbf{k}$$

$$\mathbf{i}: \quad (v_B)_x = 0.3\omega_z \qquad (4)$$

$$\mathbf{j}: \quad 0.18 = 0.3\omega_z \qquad (5)$$

$$\mathbf{k}: \quad 0.36 = -0.25\omega_x - 0.3\omega_y \qquad (6)$$

From Eq. (5), $\omega_z = 0.6$ rad/s

From Eq. (2), $\omega_x = \frac{1}{0.25}(0.3 - 0.3\omega_z)$

$= 0.48$ rad/s

From Eq. (6), $\omega_y = -\frac{1}{0.3}(0.36 + 0.25\omega_x)$

$= -1.6$ rad/s

(*a*) *Angular velocity*. $\boldsymbol{\omega} = (0.480 \text{ rad/s})\mathbf{i} - (1.600 \text{ rad/s})\mathbf{j} + (0.600 \text{ rad/s})\mathbf{k}$ ◀

From Eq. (1), $(v_A)_x = -0.25\omega_y$

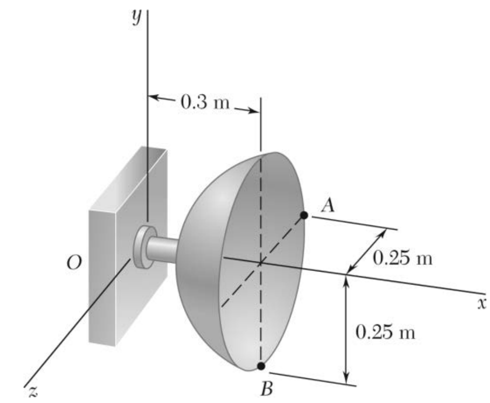
$= 0.400$ m/s

From Eq. (3), $(v_A)_z = -0.3\omega_y$

$= 0.480$ m/s

(*b*) *Velocity of Point A*. $\mathbf{v}_A = (0.400 \text{ m/s})\mathbf{i} + (0.300 \text{ m/s})\mathbf{j} + (0.480 \text{ m/s})\mathbf{k}$

or $\mathbf{v}_A = (400 \text{ mm/s})\mathbf{i} + (300 \text{ mm/s})\mathbf{j} + (480 \text{ mm/s})\mathbf{k}$ ◀



PROBLEM 15.185

At the instant considered the radar antenna shown rotates about the origin of coordinates with an angular velocity $\boldsymbol{\omega} = \omega_x \mathbf{i} + \omega_y \mathbf{j} + \omega_z \mathbf{k}$. Knowing that $(v_A)_x = 100$ mm/s, $(v_A)_y = -90$ mm/s, and $(v_B)_z = 120$ mm/s, determine (*a*) the angular velocity of the antenna, (*b*) the velocity of Point *A*.

SOLUTION

$$\mathbf{r}_A = (0.3 \text{ m})\mathbf{i} - 0.25 \text{ m})\mathbf{k}$$

$$\mathbf{v}_A = (0.1 \text{ m/s})\mathbf{i} - (0.09 \text{ m/s})\mathbf{j} + (v_A)_z \mathbf{k}$$

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_A: \quad 0.1\mathbf{i} - 0.09\mathbf{j} + (v_A)_z \mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & 0 & -0.25 \end{vmatrix}$$

$$0.1\mathbf{i} - 0.09\mathbf{j} + (v_A)_z \mathbf{k} = -0.25\omega_y \mathbf{i} + (0.3\omega_z + 0.25\omega_x)\mathbf{j} - 0.3\omega_y \mathbf{k}$$

$$\mathbf{i}: \quad 0.1 = -0.25\omega_y \tag{1}$$

$$\mathbf{j}: \quad -0.09 = 0.3\omega_z + 0.25\omega_x \tag{2}$$

$$\mathbf{k}: \quad (v_A)_z = -0.3\omega_y \tag{3}$$

$$\mathbf{r}_B = (0.3 \text{ m})\mathbf{i} - (0.25 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = (v_B)_x \mathbf{i} + (v_B)_y \mathbf{j} + (0.12 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_B: \quad (v_B)_x \mathbf{i} + (v_B)_y \mathbf{j} + 0.12\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & -0.25 & 0 \end{vmatrix}$$

$$(v_B)_x \mathbf{i} + (v_B)_y \mathbf{j} + 0.12\mathbf{k} = 0.25\omega_z \mathbf{i} + 0.3\omega_z \mathbf{j} - (0.25\omega_x + 0.3)\omega_y \mathbf{k}$$

$$\mathbf{i}: \quad (v_B)_x = 0.25\omega_z \tag{4}$$

$$\mathbf{j}: \quad (v_B)_y = 0.3\omega_z \tag{5}$$

$$\mathbf{k}: \quad 0.12 = -0.25\omega_x - 0.3\omega_y \tag{6}$$

From Eq. (1),

$$\omega_y = -\frac{0.1}{0.25} = -0.4 \text{ rad/s}$$

PROBLEM 15.185 (Continued)

From Eq. (6),
$$\omega_x = -\frac{1}{0.25}(0.12 + 0.3\omega_y)$$
$$= 0$$

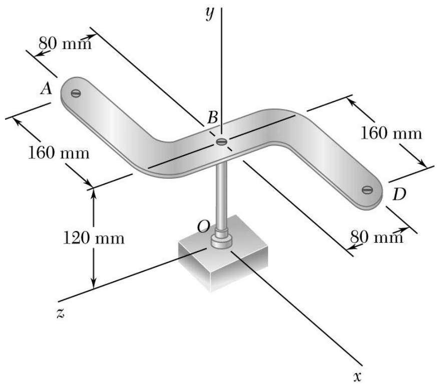
From Eq. (2),
$$\omega_z = -\frac{1}{0.25}(0.09 + 0.25\omega_x)$$
$$= -0.36 \text{ rad/s}$$

From Eq. (3),
$$(v_A)_z = -(0.3)(-0.4)$$
$$= 0.12 \text{ m/s}$$

(*a*) Angular velocity.
$$\boldsymbol{\omega} = -(0.400 \text{ rad/s})\mathbf{j} - (0.360 \text{ rad/s})\mathbf{k} \blacktriangleleft$$

(*b*) Velocity of Point *A*.
$$\mathbf{v}_A = (0.1 \text{ m/s})\mathbf{i} - (0.09 \text{ m/s})\mathbf{j} + (0.12 \text{ m/s})\mathbf{k}$$

or
$$\mathbf{v}_A = (100 \text{ mm/s})\mathbf{i} - (90 \text{ mm/s})\mathbf{j} + (120 \text{ mm/s})\mathbf{k} \blacktriangleleft$$



PROBLEM 15.186

Plate *ABD* and rod *OB* are rigidly connected and rotate about the ball-and-socket joint *O* with an angular velocity $\boldsymbol{\omega} = \omega_x\,\mathbf{i} + \omega_x\,\mathbf{j} + \omega_z\,\mathbf{k}$. Knowing that $\mathbf{v}_A = (80\text{ mm/s})\mathbf{i} + (360\text{ mm/s})\mathbf{j} + (v_A)_z\,\mathbf{k}$ and $\omega_x = 1.5\text{ rad/s}$, determine (*a*) the angular velocity of the assembly, (*b*) the velocity of Point *D*.

SOLUTION

$$\omega_x = 1.5\text{ rad/s} \qquad \boldsymbol{\omega} = (1.5\text{ rad/s})\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

$$\mathbf{r}_A = -(160\text{ mm})\mathbf{i} + (120\text{ mm})\mathbf{j} + (80\text{ mm})\mathbf{k}$$

$$\mathbf{r}_D = +(160\text{ mm})\mathbf{i} + (120\text{ mm})\mathbf{j} - (80\text{ mm})\mathbf{k}$$

(*a*)

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & \omega_y & \omega_z \\ -160 & +120 & +80 \end{vmatrix}$$

$$\mathbf{v}_A = (80\omega_y - 120\omega_z)\mathbf{i} + (-160\omega_z - 120)\mathbf{j} + (180 + 160\omega_y)\mathbf{k}$$

But we are given:

$$\mathbf{v}_A = (80\text{ mm/s})\mathbf{i} + (360\text{ mm/s})\mathbf{j} + (v_A)_z\mathbf{k}$$

$$(v_A)_x:\quad 80\omega_y - 120\omega_z = 80 \qquad (1)$$

$$(v_A)_y:\quad -160\omega_x - 120 = 360 \qquad \omega_z = -3\text{ rad/s} \qquad (2)$$

$$(v_A)_z:\quad 180 + 160\omega_y = (v_A)_z \qquad (3)$$

Substitute $\omega_z = -3.0\text{ rad/s}$ into Eq. (1):

$$80\omega_y - 120(-3) = 80$$

$$\omega_y = -3.5\text{ rad/s}$$

Substitute $\omega_y = -3.5\text{ rad/s}$ into Eq. (3):

$$180 + 160(-3.5) = (v_A)_z$$

$$(v_A)_z = -380\text{ in./s}$$

We have: $\boldsymbol{\omega} = (1.5\text{ rad/s})\mathbf{i} - (3.5\text{ rad/s})\mathbf{j} - (3.0\text{ rad/s})\mathbf{k}$ ◀

PROBLEM 15.186 (Continued)

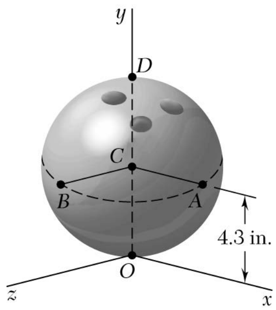
(*b*) Velocity of *D*.

$$\mathbf{v}_D = \boldsymbol{\omega} \times \mathbf{r}_D$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & -3.5 & -3.0 \\ +160 & +120 & -80 \end{vmatrix}$$

$$= (360 + 280)\mathbf{i} + (-480 + 120)\mathbf{j} + (180 + 560)\mathbf{k}$$

$$\mathbf{v}_D = (640 \text{ mm/s})\mathbf{i} - (360 \text{ mm/s})\mathbf{j} + (740 \text{ mm/s})\mathbf{k} \blacktriangleleft$$



PROBLEM 15.187

The bowling ball shown rolls without slipping on the horizontal xz plane with an angular velocity $\omega = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Knowing that $\mathbf{v}_A = (14.4\text{ ft/s})\mathbf{i} - (14.4\text{ ft/s})\mathbf{j} + (10.8\text{ ft/s})\mathbf{k}$ and $\mathbf{v}_D = (28.8\text{ ft/s})\mathbf{i} + (21.6\text{ ft/s})\mathbf{k}$, determine (*a*) the angular velocity of the bowling ball, (*b*) the velocity of its center C.

SOLUTION

Radius of ball: 4.3 in. = 0.35833 ft

At the given instant, the origin is not moving.

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_A: \quad 14.4\mathbf{i} - 14.4\mathbf{j} + 10.8\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.35833 & 0.35833 & 0 \end{vmatrix}$$

$$14.4\mathbf{i} - 14.4\mathbf{j} + 10.8\mathbf{k} = -0.35833\omega_z\mathbf{i} + 0.35833\omega_z\mathbf{j} + 0.35833(\omega_x - \omega_y)\mathbf{k}$$

$$\mathbf{i}: \quad -0.35833\omega_z = 14.4 \qquad \omega_z = -40.186\text{ rad/s}$$

$$\mathbf{j}: \quad 0.35833\omega_z = -14.4 \qquad \omega_z = -40.186\text{ rad/s}$$

$$\mathbf{k}: \quad 0.35833(\omega_x - \omega_y) = 10.8 \qquad \omega_x - \omega_y = 30.140\text{ rad/s}$$

$$\mathbf{v}_D = \boldsymbol{\omega} \times \mathbf{r}_D: \quad 28.8\mathbf{i} + 21.6\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0 & 0.71667 & 0 \end{vmatrix}$$

$$28.8\mathbf{i} + 21.6\mathbf{k} = -0.71667\omega_z\mathbf{i} + 0.71667\omega_x\mathbf{k}$$

$$\mathbf{i}: \quad -0.71667\omega_z = 28.8 \qquad \omega_z = -40.186\text{ rad/s}$$

$$\mathbf{k}: \quad 0.71667\omega_x = 21.6 \qquad \omega_x = 30.140\text{ rad/s}$$

$$\omega_y = \omega_x - 30.140 = 0$$

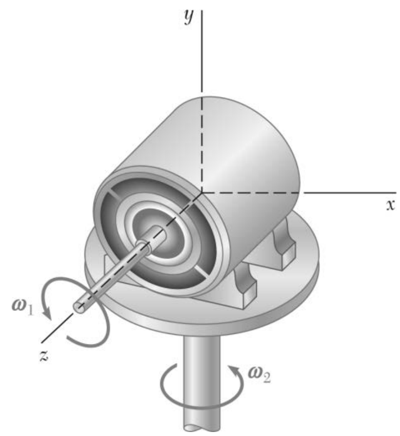
(*a*) *Angular velocity.* $\boldsymbol{\omega} = (30.1\text{ rad/s})\mathbf{i} - (40.2\text{ rad/s})\mathbf{k}$ ◀

(*b*) *Velocity of Point C.*

$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_C = (30.140\mathbf{i} - 40.186\mathbf{k}) \times 0.35833\mathbf{j}$$

$$= 14.4\mathbf{i} + 10.8\mathbf{k}$$

$\mathbf{v}_C = (14.4\text{ ft/s})\mathbf{i} + (10.8\text{ ft/s})\mathbf{k}$ ◀



PROBLEM 15.188

The rotor of an electric motor rotates at the constant rate $\omega_1 = 1800$ rpm. Determine the angular acceleration of the rotor as the motor is rotated about the y axis with a constant angular velocity $\boldsymbol{\omega}_2$ of 6 rpm counterclockwise when viewed from the positive y axis.

SOLUTION

$$\omega_1 = 1800 \text{ rpm}$$
$$= 60\pi \text{ rad/s}$$
$$\omega_2 = 6 \text{ rpm}$$
$$= 0.2\pi \text{ rad/s}$$

Total angular velocity.

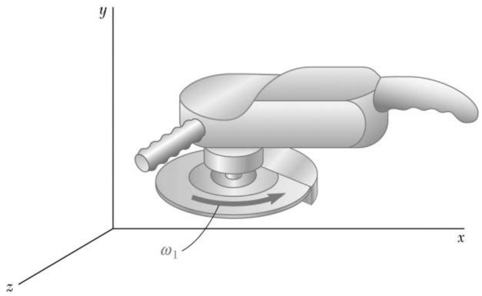
$$\boldsymbol{\omega} = \omega_2\mathbf{j} + \omega_1\mathbf{k}$$
$$\boldsymbol{\omega} = (0.2\pi \text{ rad/s})\mathbf{j} + (60\pi \text{ rad/s})\mathbf{k}$$

Angular acceleration.

Frame $Oxyz$ is rotating with angular velocity $\boldsymbol{\Omega} = \omega_2\mathbf{j}$.

$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}$$
$$= \dot{\boldsymbol{\omega}}_{Oxyz} + \boldsymbol{\Omega}\times\boldsymbol{\omega}$$
$$= 0 + \omega_2\mathbf{j}\times(\omega_2\mathbf{j} + \omega_1\mathbf{k})$$
$$= \omega_2\omega_1\mathbf{i}$$
$$\boldsymbol{\alpha} = (0.2\pi)(60\pi)\mathbf{i}$$
$$= (12\pi^2 \text{ rad/s}^2)\mathbf{i}$$

$\boldsymbol{\alpha} = (118.4 \text{ rad/s}^2)\mathbf{i}$ ◀



PROBLEM 15.189

The disk of a portable sander rotates at the constant rate $\omega_1 = 4400$ rpm as shown. Determine the angular acceleration of the disk as a worker rotates the sander about the z axis with an angular velocity of 0.5 rad/s and an angular acceleration of 2.5 rad/s^2, both clockwise when viewed from the positive z axis.

SOLUTION

Spin rate: $\omega_1 = 4400 \text{ rpm} = 460.77 \text{ rad/s}$

Angular velocity of disk relative to the housing:

$$\boldsymbol{\omega}_1 = (460.77 \text{ rad/s})\,\mathbf{j}$$

Angular motion of the housing:

$$\boldsymbol{\omega}_2 = -(0.5 \text{ rad/s})\mathbf{k} \qquad \dot{\boldsymbol{\omega}}_2 = -(2.5 \text{ rad/s}^2)\mathbf{k}$$

Consider a frame of reference rotating with angular velocity

$$\boldsymbol{\Omega} = \omega_2\mathbf{k} = -(0.5 \text{ rad/s})\mathbf{k}$$

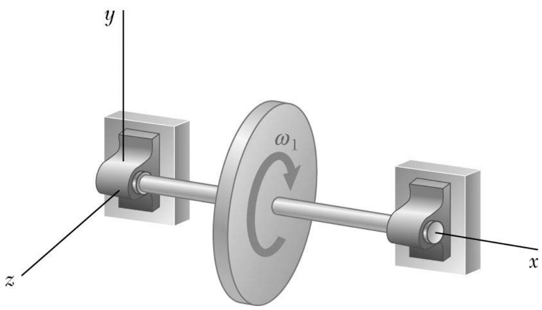
Angular velocity of the disk:

$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 = (460.77 \text{ rad/s})\mathbf{j} - (0.5 \text{ rad/s})\mathbf{k}$$

Angular acceleration of the disk:

$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}_1 + \dot{\boldsymbol{\omega}}_2 + \boldsymbol{\Omega}\times(\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2)$$
$$= 0 - 2.5\mathbf{k} + (-0.5\mathbf{k})\times(460.77\mathbf{j} - 0.5\mathbf{k})$$
$$= (230.38 \text{ rad/s}^2)\mathbf{i} - (2.5 \text{ rad/s}^2)\mathbf{k}$$

$$\boldsymbol{\alpha} = (230 \text{ rad/s}^2)\mathbf{i} - (2.5 \text{ rad/s}^2)\mathbf{k} \blacktriangleleft$$

	**PROBLEM 15.190** Knowing that the turbine rotor shown rotates at a constant rate $\omega_1 = 9000$ rpm, determine the angular acceleration of the rotor if the turbine housing has a constant angular velocity of 2.4 rad/s clockwise as viewed from (*a*) the positive *y* axis, (*b*) the positive *z* axis.

SOLUTION

Spin rate: $\omega_1 = 9000 \text{ rpm} = 942.48 \text{ rad/s}$

Angular velocity of the rotor relative to the axle:

$$\boldsymbol{\omega}_1 = -(942.48 \text{ rad/s})\mathbf{i}$$

(*a*) Axle rotates with angular velocity $\boldsymbol{\omega}_2 = -(2.4 \text{ rad/s})\mathbf{j}$

Consider a frame of reference rotating with angular velocity

$$\boldsymbol{\Omega} = \omega_2 \mathbf{j}$$

Angular acceleration:

$$\begin{aligned}\boldsymbol{\alpha} &= \dot{\boldsymbol{\omega}}_1\mathbf{i} + \dot{\boldsymbol{\omega}}_2\mathbf{j} + \boldsymbol{\Omega}\times(\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2)\\ &= 0 + 0 + \boldsymbol{\Omega}\times\boldsymbol{\omega}_1\\ &= (-2.4\mathbf{j})\times(-942.48\mathbf{i})\end{aligned}$$

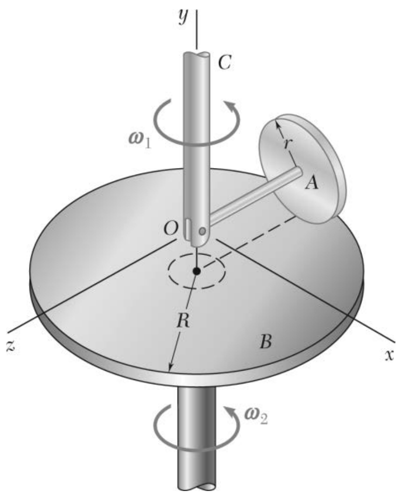
$\boldsymbol{\alpha} = -(2260 \text{ rad/s}^2)\mathbf{k}$ ◀

(*b*) Axle rotates with angular velocity $\boldsymbol{\omega}_2 = -(2.4 \text{ rad/s})\mathbf{k}$.

$$\boldsymbol{\Omega} = -(2.4 \text{ rad/s})\mathbf{k}$$

$$\boldsymbol{\alpha} = \boldsymbol{\Omega}\times\boldsymbol{\omega}_1 = (-2.4\mathbf{k})\times(-942.48\mathbf{i})$$

$\boldsymbol{\alpha} = (2260 \text{ rad/s}^2)\mathbf{j}$ ◀



PROBLEM 15.191

In the system shown, disk A is free to rotate about the horizontal rod OA. Assuming that disk B is stationary $(\omega_2 = 0)$, and that shaft OC rotates with a constant angular velocity ω_1, determine (*a*) the angular velocity of disk A, (*b*) the angular acceleration of disk A.

SOLUTION

Disk A (In rotation about O):

Since

$$\omega_y = \omega_1,$$

$$\boldsymbol{\omega}_A = \omega_x \mathbf{i} + \omega_1 \mathbf{j} + \omega_z \mathbf{k}$$

Point D is point of contact of wheel and disk.

$$\mathbf{r}_{D/O} = -r\mathbf{j} - R\mathbf{k}$$

$$\mathbf{v}_D = \boldsymbol{\omega}_A \times \mathbf{r}_{D/O}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_1 & \omega_z \\ 0 & -r & -R \end{vmatrix}$$

$$\mathbf{v}_D = (-R\omega_1 + r\omega_z)\mathbf{i} + R\omega_1 \mathbf{j} - r\omega_x \mathbf{k}$$

Since $\omega_2 = 0$, $\mathbf{v}_D = 0$.

Each component of $\mathbf{v}_D$ is zero.

$$(v_D)_z = r\omega_x = 0; \quad \omega_x = 0$$

$$(v_D)_x = -R\omega_1 + r\omega_z = 0; \quad \omega_z = \left(\frac{R}{r}\right)\omega_1$$

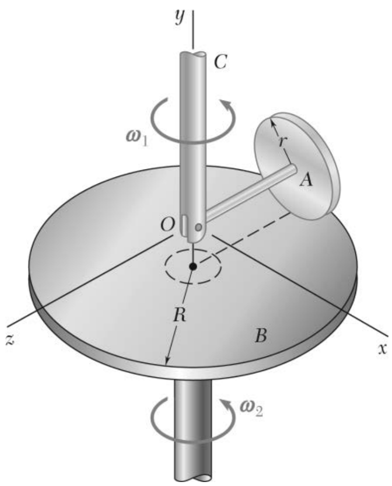
(*a*) Angular velocity.

$$\boldsymbol{\omega}_A = \omega_1 \mathbf{j} + \left(\frac{R}{r}\right)\omega_1 \mathbf{k} \blacktriangleleft$$

(*b*) Angular acceleration. Disk A rotates about y axis at rate ω_1.

$$\boldsymbol{\alpha}_A = \frac{d\boldsymbol{\omega}_A}{dt} = \boldsymbol{\omega}_y \times \boldsymbol{\omega}_A = \omega_1 \mathbf{j} \times \left(\omega_1 \mathbf{j} + \frac{R}{r}\omega_1 \mathbf{k}\right)$$

$$\boldsymbol{\alpha}_A = \frac{R}{r}\omega_1^2 \mathbf{i} \blacktriangleleft$$



PROBLEM 15.192

In the system shown, disk *A* is free to rotate about the horizontal rod *OA*. Assuming that shaft *OC* and disk *B* rotate with constant angular velocities ω_1 and ω_2, respectively, both counterclockwise, determine (*a*) the angular velocity of disk *A*, (*b*) the angular acceleration of disk *A*.

SOLUTION

Disk *A* (in rotation about *O*):

Since $\omega_y = \omega_1$, $\qquad \boldsymbol{\omega}_A = \omega_x\mathbf{i} + \omega_1\mathbf{j} + \omega_z\mathbf{k}$

Point *D* is point of contact of wheel and disk.

$$\mathbf{r}_{D/O} = -r\mathbf{j} - R\mathbf{k}$$

$$\mathbf{v}_D = \boldsymbol{\omega}_A \times \mathbf{r}_{D/O} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_1 & \omega_z \\ 0 & -r & -R \end{vmatrix}$$

$$\mathbf{v}_D = (-R\omega_1 + r\omega_z)\mathbf{i} + R\omega_x\mathbf{j} - r\omega_x\mathbf{k} \qquad (1)$$

Disk *B*: $\qquad \boldsymbol{\omega}_B = \omega_2\mathbf{j}$

$$\mathbf{v}_D = \boldsymbol{\omega}_B \times \mathbf{r}_{D/O} = \omega_2\mathbf{j} \times (-r\mathbf{j} - R\mathbf{k}) = -R\omega_2\mathbf{i} \qquad (2)$$

From Eqs. 1 and 2: $\qquad \mathbf{v}_D = \mathbf{v}_D: \quad (-R\omega_1 - r\omega_z)\mathbf{i} + R\omega_x\mathbf{j} - r\omega_x\mathbf{k} = -R\omega_2\mathbf{i}$

Coefficients of **k**: $\qquad -r\omega_x = 0; \qquad \omega_x = 0$

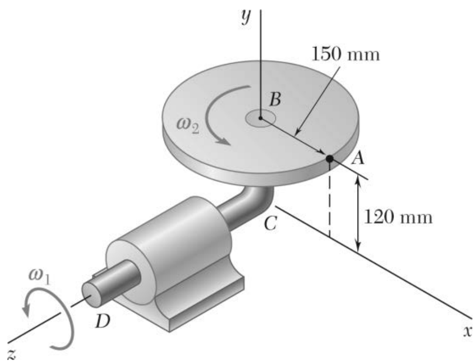
Coefficients of **i**: $\qquad (-R\omega_1 + r\omega_z) = -R\omega_2; \quad \omega_z = \dfrac{R}{r}(\omega_1 - \omega_2)$

(*a*) Angular velocity. $\qquad \boldsymbol{\omega}_A = \omega_1\mathbf{j} + \dfrac{R}{r}(\omega_1 - \omega_2)\mathbf{k}$ ◀

(*b*) Angular acceleration. Disk *A* rotates about *y* axis at rate ω_1.

$$\boldsymbol{\alpha}_A = \frac{d\boldsymbol{\omega}_A}{dt} = \boldsymbol{\omega}_y \times \boldsymbol{\omega}_A = \omega_1\mathbf{j} \times \left[\omega_1\mathbf{j} + \frac{R}{r}(\omega_1 - \omega_2)\mathbf{k}\right] \qquad \boldsymbol{\alpha}_A = \frac{R}{r}\omega_1(\omega_1 - \omega_2)\mathbf{i} \ ◀$$

PROBLEM 15.193



The L-shaped arm *BCD* rotates about the *z* axis with a constant angular velocity ω_1 of 5 rad/s. Knowing that the 150-mm-radius disk rotates about *BC* with a constant angular velocity ω_2 of 4 rad/s, determine (*a*) the velocity of Point *A*, (*b*) the acceleration of Point *A*.

SOLUTION

Total angular velocity.

$$\boldsymbol{\omega} = \omega_2\mathbf{j} + \omega_1\mathbf{k}$$

$$\boldsymbol{\omega} = (4\text{ rad/s})\mathbf{j} + (5\text{ rad/s})\mathbf{k}$$

Angular acceleration.

Frame *Oxyz* is rotating with angular velocity $\boldsymbol{\Omega} = \omega_1\mathbf{k}$.

$$\begin{aligned}\alpha &= \dot{\boldsymbol{\omega}}\\ &= \dot{\boldsymbol{\omega}}_{Oxyz} + \boldsymbol{\Omega}\times\boldsymbol{\omega}\\ &= 0 + \omega_1\mathbf{k}\times(\omega_2\mathbf{j} + \omega_1\mathbf{k})\\ &= -\omega_1\omega_2\mathbf{i}\\ \alpha &= -(5)(4)\mathbf{i}\\ &= -20\mathbf{i}\\ \boldsymbol{\alpha} &= -(20.0\text{ rad/s}^2)\mathbf{i}\end{aligned}$$

(*a*) *Velocity of Point A.*

$$\mathbf{r}_A = (0.15\text{ m})\mathbf{i} + (0.12\text{ m})\mathbf{j}$$

$$\mathbf{v}_A = \boldsymbol{\omega}\times\mathbf{r}_A = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 4 & 5\\ 0.15 & 0.12 & 0\end{vmatrix} = -0.6\mathbf{i} + 0.75\mathbf{j} - 0.6\mathbf{k}$$

$$\mathbf{v}_A = -(0.600\text{ m/s})\mathbf{i} + (0.750\text{ m/s})\mathbf{j} - (0.600\text{ m/s})\mathbf{k} \blacktriangleleft$$

(*b*) *Acceleration of Point A.*

$$\mathbf{a}_A = \boldsymbol{\alpha}\times\mathbf{r}_A + \boldsymbol{\omega}\times\mathbf{v}_A$$

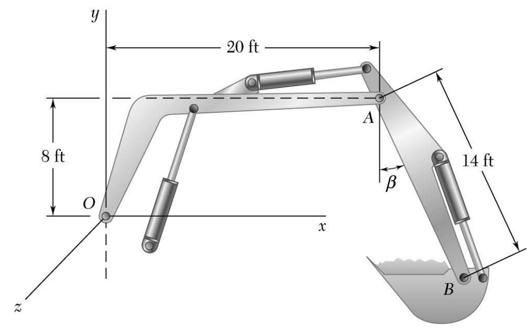
$$= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -20 & 0 & 0\\ 0.15 & 0.6 & 0\end{vmatrix} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 4 & 5\\ -0.6 & 0.75 & -0.6\end{vmatrix}$$

$$= -2.4\mathbf{k} - 6.15\mathbf{i} - 3\mathbf{j} + 2.4\mathbf{k}$$

$$= -6.15\mathbf{i} - 3\mathbf{j}$$

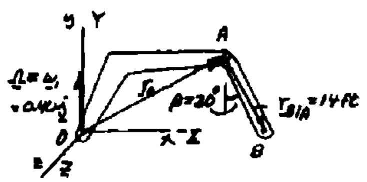
$$\mathbf{a}_A = -(6.15\text{ m/s}^2)\mathbf{i} - (3.00\text{ m/s}^2)\mathbf{j} \blacktriangleleft$$

PROBLEM 15.194



The cab of the backhoe shown rotates with the constant angular velocity $\boldsymbol{\omega}_1 = (0.4 \text{ rad/s})\mathbf{j}$ about the y axis. The arm OA is fixed with respect tot he cab, while the arm AB rotates about the horizontal axle A at the constant rate $\omega_2 = d\beta/dt = 0.6$ rad/s. Knowing that $\beta = 30°$, determine (*a*) the angular velocity and angular acceleration of AB, (*b*) the velocity and acceleration of Point B.

SOLUTION



$$\mathbf{r}_A = 20\mathbf{i} + 8\mathbf{j} \quad \text{(ft)}$$
$$\mathbf{r}_{B/A} = 7\mathbf{i} - 12.12\mathbf{j} \quad \text{(ft)}$$
$$\mathbf{r}_B = 27\mathbf{i} - 4.12\mathbf{j} \quad \text{(ft)}$$

O_{XYZ} is fixed; O_{xyz} rotates with $\boldsymbol{\Omega} = 0.40\mathbf{j}$

<u>Angular velocity of *AB*</u>

With respect to rotating frame: $\boldsymbol{\omega}_2 = +\,(0.60 \text{ rad/s})\mathbf{k}$

With respect to fixed frame: $\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 = (0.40 \text{ rad/s})\mathbf{j} + (0.60 \text{ rad/s})\mathbf{k}$ ◀

<u>Angular acceleration of *AB*</u>

$$\boldsymbol{\alpha} = (\dot{\boldsymbol{\omega}})_{Oxyz} = (\dot{\boldsymbol{\omega}})_{Oxyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega}$$

$$\boldsymbol{\alpha} = 0 + (0.40\mathbf{j}) \times (0.40\mathbf{j} + 0.60\mathbf{k})$$

$\boldsymbol{\alpha} = (0.24 \text{ rad/s}^2)\mathbf{i}$ ◀

Motion of B relative to rotating frame O_{xyz}.

Since A does not move relative to O_{xyz},

$$(\mathbf{v}_{B/F}) = (\dot{\mathbf{r}}_B)_{Oxyz} = (\dot{\mathbf{r}}_A)_{Oxyz} + (\dot{\mathbf{r}}_{B/A})_{Oxyz} = 0 + \boldsymbol{\omega}' \times \mathbf{r}_{B/A}$$
$$= (0.60\mathbf{k}) \times (7\mathbf{i} - 12.12\mathbf{j})$$
$$\mathbf{v}_{B/F} = (7.27 \text{ ft/s})\mathbf{i} + (4.2 \text{ ft/s})\mathbf{j} \qquad (1)$$

$$(\mathbf{a}_{B/F}) = (\dot{\mathbf{r}}_A)_{Oxyz} + (\dot{\mathbf{r}}_{B/A})_{Oxyz} = 0 + \boldsymbol{\omega}' \times (\boldsymbol{\omega}' \times \mathbf{r}_{B/A})$$
$$= (0.60\mathbf{k}) \times (7.27\mathbf{i} - 4.2\mathbf{j})$$
$$\mathbf{a}_{B/F} = (2.52 \text{ ft/s}^2)\mathbf{i} + (4.36 \text{ ft/s}^2)\mathbf{j} \qquad (2)$$

Motion of B' of frame O_{xyz} which coincides with B.

$$\mathbf{v}_{B'} = \boldsymbol{\Omega} \times \mathbf{r}_B = (0.40\mathbf{j}) \times (27\mathbf{i} - 4.12\mathbf{j})$$
$$\mathbf{v}_{B'} = -(10.8 \text{ ft/s})\mathbf{k} \qquad (3)$$

$$\mathbf{a}_{B'} = \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_B) = \boldsymbol{\Omega} \times \mathbf{v}_{B'} = (0.4\mathbf{j}) \times (-10.8\mathbf{k})$$
$$\mathbf{a}_{B'} = -(4.32 \text{ ft/s}^2)\mathbf{i} \qquad (4)$$

PROBLEM 15.194 (Continued)

Velocity of B using Equations (1) and (3):

$$\mathbf{v}_{B'} = \mathbf{v}_{B'} + \mathbf{v}_{B/F} = -10.8\mathbf{k} + 7.27\mathbf{i} + 4.2\mathbf{j}$$

$$\mathbf{v}_B = (7.27 \text{ ft/s})\mathbf{i} + (4.2 \text{ ft/s})\mathbf{j} - (10.8 \text{ ft/s})\mathbf{k} \blacktriangleleft$$

Acceleration of B

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + \mathbf{a}_C$$

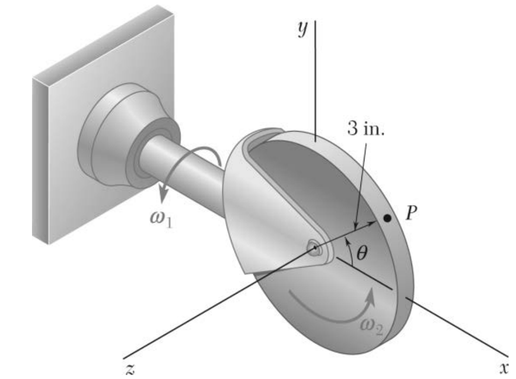
We first compute the Coriolis acceleration. $\mathbf{a}_C$

$$\mathbf{a}_C = 2\mathbf{\Omega} \times \mathbf{v}_{B/F} = 2(0.40\mathbf{j}) \times (7.27\mathbf{i} + 4.2\mathbf{j})$$

Recalling Equations (2) and (4), we now write

$$\mathbf{a}_B = -4.32\mathbf{i} - 2.52\mathbf{i} + 4.36\mathbf{j} - 5.82\mathbf{k}$$

$$\mathbf{a}_B = -(6.84 \text{ ft/s}^2)\mathbf{i} + (4.36 \text{ ft/s}^2)\mathbf{j} - (5.82 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.195

A 3-in.-radius disk spins at the constant rate $\omega_2 = 4$ rad/s about an axis held by a housing attached to a horizontal rod that rotates at the constant rate $\omega_1 = 5$ rad/s. For the position shown, determine (*a*) the angular acceleration of the disk, (*b*) the acceleration of Point *P* on the rim of the disk if $\theta = 0$, (*c*) the acceleration of Point *P* on the rim of the disk if $\theta = 90°$.

SOLUTION

Angular velocity.

$$\boldsymbol{\omega} = \omega_1\mathbf{i} + \omega_2\mathbf{k}$$
$$\boldsymbol{\omega} = (5 \text{ rad/s})\mathbf{i} + (4 \text{ rad/s})\mathbf{k}$$

(*a*) *Angular acceleration.*

Frame *Oxyz* is rotating with angular velocity $\boldsymbol{\Omega} = \omega_1\mathbf{i}.$

$$\begin{aligned}\alpha &= \dot{\boldsymbol{\omega}} \\ &= \dot{\boldsymbol{\omega}}_{Oxyz} + \boldsymbol{\Omega}\times\boldsymbol{\omega} \\ &= 0 + \omega_1\mathbf{i}\times(\omega_1\mathbf{i} + \omega_2\mathbf{k}) \\ &= -\omega_1\omega_2\mathbf{j} \\ &= -(4)(5)\mathbf{j} \\ &= -20\mathbf{j}\end{aligned}$$

$\boldsymbol{\alpha} = -(20.0 \text{ rad/s}^2)\mathbf{j}$ ◀

(*b*) $\theta = 0$. *Acceleration at Point P.*

$$\begin{aligned}\mathbf{r}_P &= (3 \text{ in.})\mathbf{i} \\ &= (0.25 \text{ ft})\mathbf{i} \\ \mathbf{v}_P &= \boldsymbol{\omega}\times\mathbf{r}_P \\ &= (5\mathbf{i} + 4\mathbf{k})\times 0.25\mathbf{i} \\ &= (1 \text{ ft/s})\mathbf{j} \\ \mathbf{a}_P &= \boldsymbol{\alpha}\times\mathbf{r}_P + \boldsymbol{\omega}\times\mathbf{v}_P \\ &= -20\mathbf{j}\times 0.25\mathbf{i} + (5\mathbf{i} + 4\mathbf{k})\times(1 \text{ ft/s})\mathbf{j} \\ &= 5\mathbf{k} + 5\mathbf{k} - 4\mathbf{i} \\ &= -4\mathbf{i} + 10\mathbf{k}\end{aligned}$$

$\mathbf{a}_P - (4.00 \text{ ft/s}^2)\mathbf{i} + (10.00 \text{ ft/s}^2)\mathbf{k}$ ◀

PROBLEM 15.195 (Continued)

(*c*) $\theta = 90°$. *Acceleration at Point P.*

$$\mathbf{r}_P = (0.25 \text{ ft})\mathbf{j}$$

$$\mathbf{v}_P = \boldsymbol{\omega} \times \mathbf{r}_P$$

$$= (5\mathbf{i} + 4\mathbf{k}) \times 0.25\mathbf{j}$$

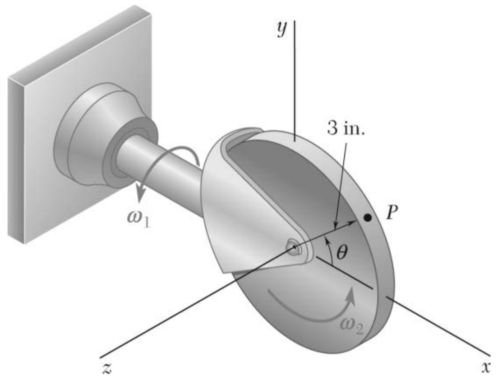
$$= -(1.25 \text{ ft/s})\mathbf{i} + (1 \text{ ft/s})\mathbf{j}$$

$$\mathbf{a}_P = \boldsymbol{\alpha} \times \mathbf{r}_P + \boldsymbol{\omega} \times \mathbf{v}_P$$

$$= -20\mathbf{j} \times 0.25\mathbf{j} + (5\mathbf{i} + 4\mathbf{k}) \times (-1.25\mathbf{i} + \mathbf{j})$$

$$= 0 + 0 - 6.25\mathbf{j} - 4\mathbf{j} + 0$$

$$= -10.25\mathbf{j} \qquad \mathbf{a}_P = -(10.25 \text{ ft/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.196

A 3-in.-radius disk spins at the constant rate $\omega_2 = 4$ rad/s about an axis held by a housing attached to a horizontal rod that rotates at the constant rate $\omega_1 = 5$ rad/s. Knowing that $\theta = 30°$, determine the acceleration of Point P on the rim of the disk.

SOLUTION

Angular velocity.

$$\boldsymbol{\omega} = \omega_1 \mathbf{i} + \omega_2 \mathbf{k}$$
$$\boldsymbol{\omega} = (5 \text{ rad/s})\mathbf{i} + (4 \text{ rad/s})\mathbf{k}$$

Angular acceleration. Frame *Oxyz* is rotating with angular velocity $\Omega = \omega_1 \mathbf{i}$.

$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\omega}}_{Oxyz} + \Omega \times \boldsymbol{\omega}$$
$$= 0 + \omega_1 \mathbf{i} \times (\omega_1 \mathbf{i} + \omega_2 \mathbf{k})$$
$$= -\omega_1 \omega_2 \mathbf{j}$$
$$= -(4)(5)\mathbf{j} = -20\mathbf{j}$$
$$\boldsymbol{\alpha} = -(20.0 \text{ rad/s}^2)\mathbf{j}$$

Geometry. $\theta = 30°$,

$$\mathbf{r}_P = (3 \text{ in.})(\cos 30° \mathbf{i} + \sin 30° \mathbf{j})$$
$$= (0.25 \text{ ft})(\cos 30° \mathbf{i} + \sin 30° \mathbf{j})$$

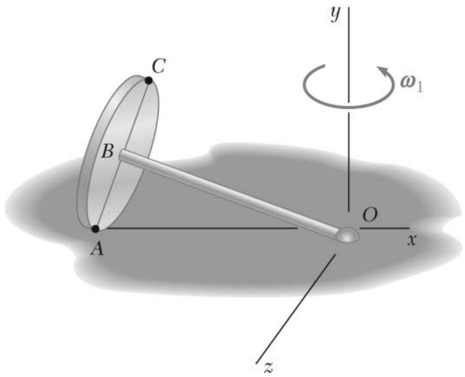
Velocity of Point P.

$$\mathbf{v}_P = \boldsymbol{\omega} \times \mathbf{r}_P$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 4 \\ 0.25\cos 30° & 0.25\sin 30° & 0 \end{vmatrix}$$
$$= -(0.5 \text{ ft/s})\mathbf{i} + (0.86603 \text{ ft/s})\mathbf{j} + (0.625 \text{ ft/s})\mathbf{k}$$

Acceleration of Point P.

$$\mathbf{a}_P = \boldsymbol{\alpha} \times \mathbf{r}_P + \boldsymbol{\omega} \times \mathbf{v}_P$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -20 & 0 \\ 0.25\cos 30° & 0.25\sin 30° & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 4 \\ -0.5 & 0.86603 & 0.625 \end{vmatrix}$$
$$= 4.3301\mathbf{k} - 3.4641\mathbf{i} - 5.125\mathbf{j} + 4.3301\mathbf{k}$$

$$\mathbf{a}_P = -(3.46 \text{ ft/s}^2)\mathbf{i} - (5.13 \text{ ft/s}^2)\mathbf{j} + (8.66 \text{ ft/s}^2)\mathbf{k}$$ ◀

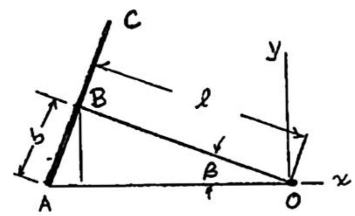


PROBLEM 15.197

A 30 mm-radius wheel is mounted on an axle *OB* of length 100 mm. The wheel rolls without sliding on the horizontal floor, and the axle is perpendicular to the plane of the wheel. Knowing that the system rotates about the *y* axis at a constant rate $\omega_1 = 2.4$ rad/s, determine (*a*) the angular velocity of the wheel, (*b*) the angular acceleration of the wheel, (*c*) the acceleration of Point *C* located at the highest point on the rim of the wheel.

SOLUTION

Geometry.



$$l = 100 \text{ mm} = 0.1 \text{ m}$$
$$b = 30 \text{ mm} = 0.03 \text{ m}$$
$$\tan\beta = \frac{b}{l} = 0.3$$
$$\beta = 16.699°$$
$$\mathbf{r}_A = -l\sec\beta\mathbf{i}$$
$$\mathbf{r}_B = -l\cos\beta\mathbf{i} + b\cos\beta\mathbf{j}$$

(*a*) *Angular velocities.*

For the system, $$\mathbf{\Omega} = \omega_1\mathbf{j} = (2.4 \text{ rad/s})\mathbf{j}$$

For the wheel, $$\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$
$$\mathbf{v}_A = \boldsymbol{\omega}\times\mathbf{r}_A = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k})\times(-l\sec\beta\mathbf{i}) = 0$$
$$-(l\omega_z\sec\beta)\mathbf{j} - (l\omega_y\sec\beta)\mathbf{k} = 0$$
$$\omega_y = 0, \quad \omega_z = 0 \qquad \boldsymbol{\omega} = \omega_x\mathbf{i}$$
$$\mathbf{v}_B = \boldsymbol{\omega}\times\mathbf{r}_B$$
$$= \omega_x\mathbf{i}\times(-l\cos\beta\mathbf{i} + b\cos\beta\mathbf{j})$$
$$= (\omega_x b\cos\beta)\mathbf{k}$$

For the system, $$\mathbf{v}_B = \mathbf{\Omega}\times\mathbf{r}_B$$
$$= \omega_1\mathbf{j}\times(-l\cos\beta\mathbf{i} + b\cos\beta\mathbf{j})$$
$$= (\omega_1 l\cos\beta)\mathbf{k}$$

Matching the two expressions for $\mathbf{v}_B$,

$$\omega_x b\cos\beta = \omega_1 l\cos\beta$$

or $$\omega_x = \frac{\omega_1 l}{b} = \frac{(2.4)(30)}{100} = 8 \text{ rad/s}$$

$\boldsymbol{\omega} = (8.00 \text{ rad/s})\mathbf{i}$ ◀

PROBLEM 15.197 (Continued)

(*b*) *Angular acceleration.*

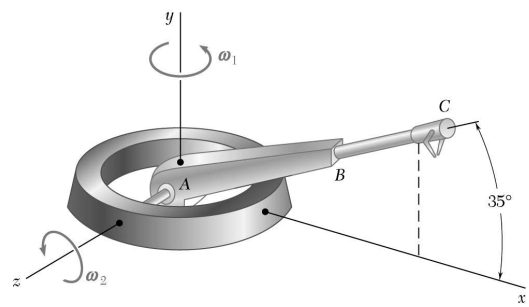
$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}$$
$$= \dot{\boldsymbol{\omega}}_{Oxyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega}$$
$$= (0 + 2.4\mathbf{j}) \times 8\mathbf{i}$$
$$= -(19.2 \text{ rad/s}^2)\mathbf{k}$$

$$\boldsymbol{\alpha} = -(19.20 \text{ rad/s}^2)\mathbf{k} \blacktriangleleft$$

(*c*) *Conditions at Point C.*

$$\mathbf{r}_C = -(l\cos\beta - b\sin\beta)\mathbf{i} + 2b\cos\beta\,\mathbf{j}$$
$$= (-87.162 \text{ mm})\mathbf{i} + (57.47 \text{ mm})\mathbf{j}$$
$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_C$$
$$= 8\mathbf{i} \times (-87.162\mathbf{i} + 57.47\mathbf{j})$$
$$= (459.76 \text{ mm/s})\mathbf{k}$$
$$\mathbf{a}_C = \boldsymbol{\alpha} \times \mathbf{r}_C + \boldsymbol{\omega} \times \mathbf{v}_C$$
$$= -19.2\mathbf{k} \times (-87.162\mathbf{i} + 57.47\mathbf{j}) + 8\mathbf{i} \times 459.76\mathbf{k}$$
$$= (1103.4 \text{ mm/s}^2)\mathbf{i} - (2004.6 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}_C = (1.103 \text{ m/s}^2)\mathbf{i} - (2.005 \text{ m/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.198

At the instant shown, the robotic arm *ABC* is being rotated simultaneously at the constant rate $\omega_1 = 0.15$ rad/s about the *y* axis, and at the constant rate $\omega_2 = 0.25$ rad/s about the *z* axis. Knowing that the length of arm *ABC* is 1 m, determine (*a*) the angular acceleration of the arm, (*b*) the velocity of Point *C*, (*c*) the acceleration of Point *C*.

SOLUTION

Angular velocity:

$$\boldsymbol{\omega} = \omega_1 \mathbf{j} + \omega_2 \mathbf{k}$$
$$= (0.15 \text{ rad/s})\mathbf{j} + (0.25 \text{ rad/s})\mathbf{k}$$

Consider a frame of reference rotating with angular velocity

$$\boldsymbol{\Omega} = \omega_1 \mathbf{j} = (0.15 \text{ rad/s})\mathbf{j}$$

(*a*) *Angular acceleration of the arm.*

$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}_1 + \dot{\boldsymbol{\omega}}_2 + \boldsymbol{\Omega} \times (\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2)$$
$$= 0 + 0 + (0.15\mathbf{j}) \times (0.15\mathbf{j} + 0.25\mathbf{k})$$

$$\boldsymbol{\alpha} = (0.0375 \text{ rad/s}^2)\mathbf{i} \blacktriangleleft$$

Arm *ABC* rotates about the fixed Point *A*.

$$\mathbf{r}_{C/A} = (1 \text{ m})(\cos 35°\mathbf{i} + \sin 35°)\mathbf{j}$$
$$= (0.81915 \text{ m})\mathbf{i} + (0.57358 \text{ m})\mathbf{j}$$

(*b*) *Velocity of Point C.* $\quad \mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_{C/A}$

$$\mathbf{v}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.15 & 0.25 \\ 0.81915 & 0.57358 & 0 \end{vmatrix}$$
$$= -0.14340\mathbf{i} + 0.20479\mathbf{j} - 0.12287\mathbf{k}$$

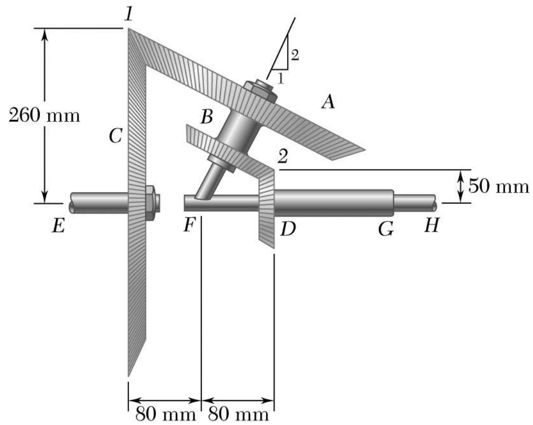
$$\mathbf{v}_C = -(0.1434 \text{ m/s})\mathbf{i} + (0.204 \text{ m/s})\mathbf{j} - (0.1229 \text{ m/s})\mathbf{k} \blacktriangleleft$$

(*c*) *Acceleration of C*: $\quad \mathbf{a}_C = \boldsymbol{\alpha} \times \mathbf{r}_{C/A} + \boldsymbol{\omega} \times \mathbf{v}_C$

$$\mathbf{a}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.0375 & 0 & 0 \\ 0.81915 & 0.57358 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.15 & 0.25 \\ -0.14340 & 0.20479 & -0.12287 \end{vmatrix}$$
$$= 0.021509\mathbf{k} + 0.06962\mathbf{i} + 0.03585\mathbf{j} + 0.02151\mathbf{k}$$

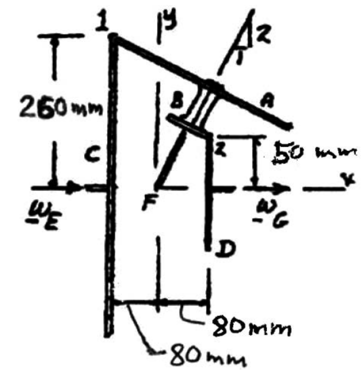
$$\mathbf{a}_C = -(0.0696 \text{ m/s}^2)\mathbf{i} + (0.0359 \text{ m/s}^2)\mathbf{j} + (0.0430 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$

PROBLEM 15.199



In the planetary gear system shown, gears A and B are rigidly connected to each other and rotate as a unit about the inclined shaft. Gears C and D rotate with constant angular velocities of 30 rad/s and 20 rad/s, respectively (both counterclockwise when viewed from the right). Choosing the x axis to the right, the y axis upward, and the z axis pointing out of the plane of the figure, determine (a) the common angular velocity of gears A and B, (b) the angular velocity of shaft FH, which is rigidly attached to the inclined shaft.

SOLUTION



Place origin at F.

Point 1: $\mathbf{r}_1 = -(80\text{ mm})\mathbf{i} + (260\text{ mm})\mathbf{j}$

Point 2: $\mathbf{r}_2 = +(80\text{ mm})\mathbf{i} + (50\text{ mm})\mathbf{j}$

$$\boldsymbol{\omega}_E = +(30\text{ rad/s})\mathbf{i}$$

$$\boldsymbol{\omega}_G = +(20\text{ rad/s})\mathbf{i}$$

$$\mathbf{v}_1 = \boldsymbol{\omega}_E \times \mathbf{r}_1 = (30\mathbf{i}) \times (-80\mathbf{i} + 260\mathbf{j})$$

$$\mathbf{v}_1 = (7800\text{ mm/s})\mathbf{k} \qquad (1)$$

$$\mathbf{v}_2 = \boldsymbol{\omega}_G \times \mathbf{r}_2 = (20\mathbf{i}) \times (80\mathbf{i} + 50\mathbf{j})$$

$$\mathbf{v}_2 = (1000\text{ mm/s})\mathbf{k} \qquad (2)$$

Motion of gear unit AB: $\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$$\mathbf{v}_1 = \boldsymbol{\omega} \times \mathbf{r}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -80 & +260 & 0 \end{vmatrix}$$

$$= -260\omega_z\mathbf{i} - 80\omega_z\mathbf{j} + (260\omega_x + 80\omega_y)\mathbf{k}$$

Recall from Eq. (1) that $\mathbf{v} = 7800\mathbf{k}$.

$$7800\mathbf{k} = -260\omega_z\mathbf{i} - 80\omega_z\mathbf{j} + (260\omega_x + 80\omega_y)\mathbf{k}$$

Equate coefficients of unit vectors.

$$\omega_z = 0$$

$$7800 = 260\omega_x + 80\omega_y \qquad (3)$$

$$\mathbf{v}_2 = \boldsymbol{\omega}\times\mathbf{r}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & 0 \\ 80 & 50 & 0 \end{vmatrix} = (50\omega_x - 80\omega_y)\mathbf{k}$$

Recall from Eq. (2) that $\mathbf{v}_2 = 1000\mathbf{k}$, and write

$$1000 = 50\omega_x - 80\omega_y \qquad (4)$$

Add Eqs. (3) and (4): $8800 = 310\omega_x$ $\qquad \omega_x = 28.387$ rad/s

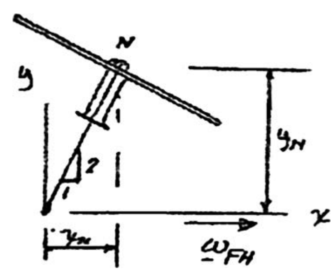
Eq. (4): $1000 = 50(28.387) - 80\omega_y$ $\qquad \omega_y = 5.242$ rad/s

(*a*) <u>Common angular velocity of unit *AB*</u>. $\boldsymbol{\omega} = (28.387 \text{ rad/s})\mathbf{i} + (5.242 \text{ rad/s})\mathbf{j}$

$\boldsymbol{\omega} = (28.4 \text{ rad/s})\mathbf{i} + (5.24 \text{ rad/s})\mathbf{j}$ ◀

(*b*) <u>Angular velocity of shaft *FH*</u>. (See figure in text.)

Point *N* is at nut, which is a part of unit *AB* and also is a part of shaft *GH*.



$$x_N = \frac{1}{2}y_N$$
$$\mathbf{r}_N = x_N\mathbf{i} + y_N\mathbf{j}$$
$$\mathbf{r}_N = \frac{1}{2}y_N\mathbf{i} + y_N\mathbf{j}$$

Nut *N* as a part of unit *AB*: $\boldsymbol{\omega} = (28.387 \text{ rad/s})\mathbf{i} + (5.242 \text{ rad/s})\mathbf{j}$

$$\mathbf{v}_N = \boldsymbol{\omega}\times\mathbf{r}_N = (28.387\mathbf{i} + 5.242\mathbf{j})\times\left(\frac{1}{2}y_N\mathbf{i} + y_N\mathbf{j}\right)$$
$$\mathbf{v}_N = (28.387y_N - 2.621y_N)\mathbf{k} = +(25.766y_N)\mathbf{k} \qquad (5)$$

Nut *N* as a part of shaft *FH*. $\boldsymbol{\omega}_{FH} = \omega_{FH}\mathbf{i}$

$$\mathbf{v}_N = \boldsymbol{\omega}_{FH}\times\mathbf{r}_N = (\omega_{FH}\mathbf{i})\times\left(\frac{1}{2}y_N\mathbf{i} + y_N\mathbf{j}\right) = \omega_{FH}\,y_N\mathbf{k} \qquad (6)$$

Equating expressions for $\mathbf{v}_N$ from Eqs. (5) and (6),

$$+(25.766y_N)\mathbf{k} = \omega_{FH}\,y_N\mathbf{k}$$
$$\boldsymbol{\omega}_{FH} = (25.766 \text{ rad/s})\mathbf{i}$$

$\boldsymbol{\omega}_{FH} = (25.8 \text{ rad/s})\mathbf{i}$ ◀

PROBLEM 15.200

In Problem 15.199, determine (*a*) the common angular acceleration of gears *A* and *B*, (*b*) the acceleration of the tooth of gear *A* which is in contact with gear *C* at Point *I*.

SOLUTION

See the solution to part (*a*) of Problem 15.199 for the calculation of the common angular velocity of unit *AB*.

$$\boldsymbol{\omega} = (28.387 \text{ rad/s})\mathbf{i} + (5.242 \text{ rad/s})\mathbf{j}$$

The angular velocity vector ω rotates about the *x*-axis with angular velocity $\boldsymbol{\omega}_{FH}$. See part (*b*) of Problem 15.199 for the calculation of $\boldsymbol{\omega}_{FH}$.

$$\boldsymbol{\omega}_{FH} = (25.776 \text{ rad/s})\mathbf{i}$$

(*a*) Common angular acceleration of unit *AB*.

$$\begin{aligned}\boldsymbol{\alpha} &= \boldsymbol{\omega}_{FH} \times \boldsymbol{\omega}\\ &= (25.776\mathbf{i}) \times (28.387\mathbf{i} + 5.242\mathbf{j})\\ &= 135.12\mathbf{k}\end{aligned}$$

$\boldsymbol{\alpha} = 135.1 \text{ rad/s}^2\mathbf{k}$ ◀

The position and velocity vectors of a tooth at the contact Point 1 of gears *A* and *C* are

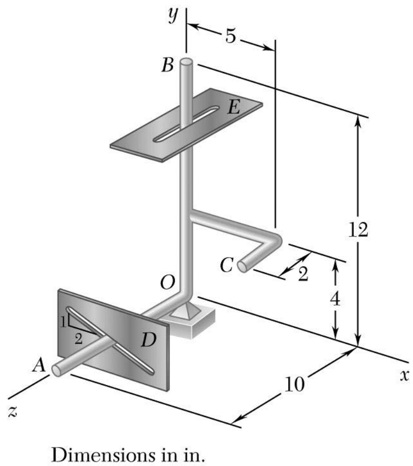
$$\begin{aligned}\mathbf{r}_1 &= -(80 \text{ mm})\mathbf{i} + (260 \text{ mm})\mathbf{j}\\ \mathbf{v}_1 &= (7800 \text{ mm/s})\mathbf{k}\end{aligned}$$

as determined in part (*a*) of Problem 15.199.

(*b*) Acceleration of the tooth at Point 1 of gear *A*.

$$\begin{aligned}\mathbf{a}_1 &= \boldsymbol{\alpha} \times \mathbf{r}_1 + \boldsymbol{\omega} \times \mathbf{v}_1\\ &= (135.12\mathbf{k}) \times (-80\mathbf{i} + 260\mathbf{j}) + (28.387\mathbf{i} + 5.242\mathbf{j}) \times 7800\mathbf{k}\\ &= -10810\mathbf{j} - 35131\mathbf{i} - 221419\mathbf{j} + 40888\mathbf{i}\\ &= (5757 \text{ mm/s}^2)\mathbf{i} - (232229 \text{ mm/s}^2)\mathbf{j}\end{aligned}$$

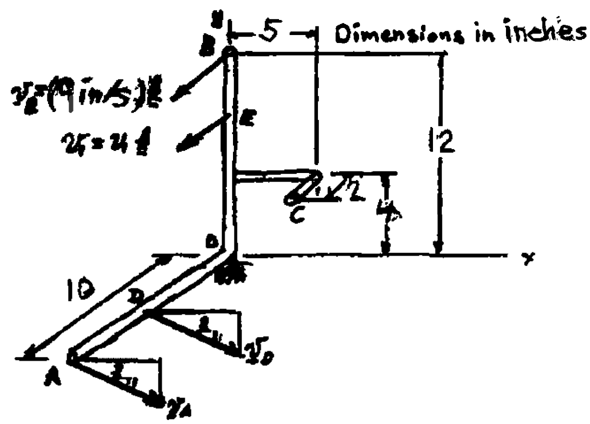
$\mathbf{a}_1 = (5.8 \text{ m/s}^2)\mathbf{i} - (232 \text{ m/s}^2)\mathbf{j}$ ◀



PROBLEM 15.201

Several rods are brazed together to form the robotic guide arm shown which is attached to a ball-and-socket joint at *O*. Rod *OA* slides in a straight inclined slot while rod *OB* slides in a slot parallel to the *z* axis. Knowing that at the instant shown $\mathbf{v}_B = (9 \text{ in./s})\mathbf{k}$, determine (*a*) the angular velocity of the guide arm, (*b*) the velocity of Point *A*, (*c*) the velocity of Point *C*.

SOLUTION



Since rod at *D* slides in slot which is of slope 1:2,

$$(v_D)_x = -2(v_D)_y$$

and

$$(v_A)_x = -2(v_A)_y$$

(*a*) Angular velocity.

$$\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

$$\mathbf{v}_B = \boldsymbol{\omega}\times\mathbf{r}_B: \quad (9 \text{ in./s})\mathbf{k} = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k})\times(12 \text{ in.})\mathbf{j}$$

$$9\mathbf{k} = 12\omega_x\mathbf{k} - 12\omega_z\mathbf{i}$$

Coefficients of **k**: $9 = 12\omega_x \quad \omega_x = 0.75 \text{ rad/s}$

Coefficients of **i**: $0 = -12\omega_z \quad \omega_z = 0$

$$\mathbf{v}_A = \boldsymbol{\omega}\times\mathbf{r}_A$$

$$\mathbf{v}_A = (0.75\mathbf{i} + \omega_y\mathbf{j})\times(10\mathbf{k})$$

$$(v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} + (v_A)_z\mathbf{k} = -7.5\mathbf{j} + 10\omega_y\mathbf{i}$$

PROBLEM 15.201 (Continued)

Coefficients of **j**: $(v_A)_y = -7.5$

Coefficients of **i**: $(v_A)_x = 10\omega_y$

Coefficients of **k**: $(v_A)_z = 0$

Recall the Equations $(v_A)_x = -2(v_A)_y$

and $10\omega_y = -2(-7.5)$

So, $\omega_y = 1.5 \text{ rad/s}$ and $(v_A)_x = 15 \text{ in./s}$

$$\boldsymbol{\omega} = (0.75 \text{ rad/s})\mathbf{i} + (1.5 \text{ rad/s})\mathbf{j} \blacktriangleleft$$

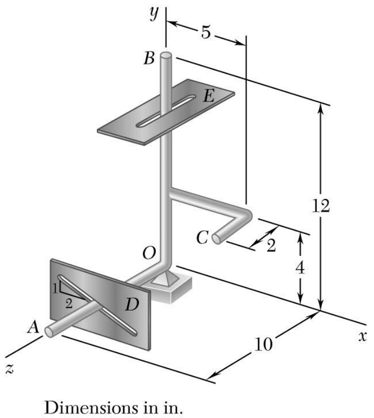
(*b*) Velocity of *A*: $\mathbf{v}_A = (15 \text{ in./s})\mathbf{i} - (7.5 \text{ in./s})\mathbf{j} \blacktriangleleft$

(*c*) Velocity of *C*:

$$\mathbf{r}_C = 5\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.75 & 1.5 & 0 \\ 5 & 4 & 2 \end{vmatrix} = 3\mathbf{i} - 1.5\mathbf{j} + (3 - 7.5)\mathbf{k}$$

$$\mathbf{v}_C = (3 \text{ in./s})\mathbf{i} - (1.5 \text{ in./s})\mathbf{j} - (4.5 \text{ in./s})\mathbf{k} \blacktriangleleft$$

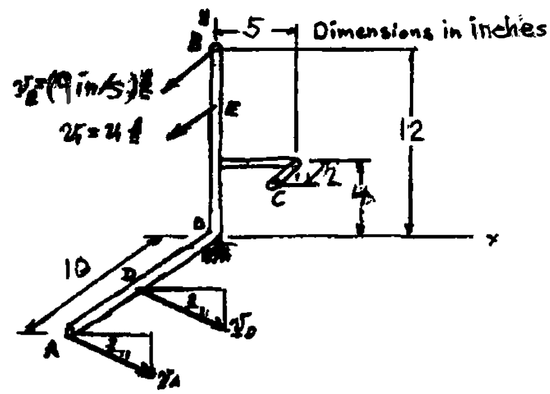


PROBLEM 15.202

In Problem 15.201 the speed of Point B is known to be constant. For the position shown, determine (a) the angular acceleration of the guide arm, (b) the acceleration of Point C.

PROBLEM 15.201 Several rods are brazed together to form the robotic guide arm shown, which is attached to a ball-and-socket joint at O. Rod OA slides in a straight inclined slot while rod OB slides in a slot parallel to the z-axis. Knowing that at the instant shown $\mathbf{v}_B = (9 \text{ in./s})\mathbf{k}$, determine ($a$) the angular velocity of the guide arm, (b) the velocity of Point A, (c) the velocity of Point C.

SOLUTION



Since rod at D slides in slot which is of slope 1:2,

$$(v_D)_x = -2(v_D)_y$$

and

$$(v_A)_x = -2(v_A)_y$$

Angular velocity.

$$\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

$$\mathbf{v}_B = \boldsymbol{\omega}\times\mathbf{r}_B: \quad (9 \text{ in./s})\mathbf{k} = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k})\times(12 \text{ in.})\mathbf{j}$$

$$9\mathbf{k} = 12\omega_x\mathbf{k} - 12\omega_z\mathbf{i}$$

Coefficients of $\mathbf{k}$: $\quad 9 = 12\omega_x \quad \omega_x = 0.75 \text{ rad/s}$

Coefficients of $\mathbf{i}$: $\quad 0 = -12\omega_z \quad \omega_z = 0$

$$\mathbf{v}_A = \boldsymbol{\omega}\times\mathbf{r}_A$$

$$\mathbf{v}_A = (0.75\mathbf{i} + \omega_y\mathbf{j})\times(10\mathbf{k})$$

$$(v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} + (v_A)_z\mathbf{k} = -7.5\mathbf{j} + 10\omega_y\mathbf{i}$$

Coefficients of $\mathbf{j}$: $\quad (v_A)_y = -7.5$

Coefficients of $\mathbf{i}$: $\quad (v_A)_x = 10\omega_y$

Coefficients of $\mathbf{k}$: $\quad (v_A)_z = 0$

PROBLEM 15.202 (Continued)

Recall the Equations $(v_A)_x = -2(v_A)_y$

and $10\omega_y = -2(-7.5)$

So, $\omega_y = 1.5 \text{ rad/s}$ and $(v_A)_x = 15 \text{ in./s}$

$$\boldsymbol{\omega} = (0.75 \text{ rad/s})\mathbf{i} + (1.5 \text{ rad/s})\mathbf{j}$$

Velocity of A: $\mathbf{v}_A = (15 \text{ in./s})\mathbf{i} - (7.5 \text{ in./s})\mathbf{j}$

Velocity of C: $\mathbf{r}_C = 5\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_C$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.75 & 1.5 & 0 \\ 5 & 4 & 2 \end{vmatrix}$$

$$= 3\mathbf{i} - 1.5\mathbf{j} + (3 - 7.5)\mathbf{k}$$

$$\mathbf{v}_C = (3 \text{ in./s})\mathbf{i} - (1.5 \text{ in./s})\mathbf{j} - (4.5 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_B = \boldsymbol{\alpha} \times \mathbf{r}_B + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_C)$$

$$= \boldsymbol{\alpha} \times \mathbf{r}_C + \boldsymbol{\omega} \times \mathbf{v}_B$$

$$\mathbf{a}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 0 & 12 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.75 & 1.5 & 0 \\ 0 & 0 & 9 \end{vmatrix}$$

$$= -12\alpha_z\mathbf{i} + 12\alpha_x\mathbf{k} + 13.5\mathbf{i} - 6.75\mathbf{j}$$

$$\mathbf{a}_B = (13.5 - 12\alpha_z)\mathbf{i} - 6.75\mathbf{j} + 12\alpha_x\mathbf{k} \tag{1}$$

$$(a_B)_x = 13.5 - 12\alpha_z = 0 \qquad \alpha_z = 1.125 \text{ rad/s}^2$$

$$(a_B)_y = -6.75 \qquad (a_B)_y = -6.75 \text{ in./s}^2$$

$$(a_B)_z = 12\alpha_x = 0 \qquad \alpha_x = 0$$

$$\mathbf{a}_A = \boldsymbol{\alpha} \times \mathbf{r}_A + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_A)$$

$$= \boldsymbol{\alpha} \times \mathbf{r}_A + \boldsymbol{\omega} \times \mathbf{v}_A$$

$$\mathbf{a}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \alpha_y & 1.125 \\ 0 & 0 & 10 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.75 & 1.5 & 0 \\ 15 & -7.5 & 0 \end{vmatrix}$$

$$= 10\alpha_y\mathbf{i} + (-5.625 - 22.5)\mathbf{k}$$

$$\mathbf{a}_A = 10\alpha_y\mathbf{i} - 28.125\mathbf{k}$$

Thus, $(a_A)_x = 10\alpha_y \quad (a_A)_y = 0 \quad (a_A)_z = -28.125 \text{ in./s}^2$

But $(a_A)_x = -2(a_A)_y = 0$

Therefore, $(a_A)_x = 10\alpha_y = 0 \quad \alpha_y = 0$

PROBLEM 15.202 (Continued)

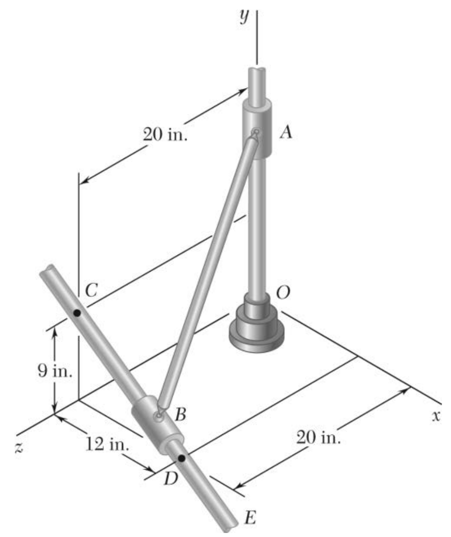
(*a*) Angular acceleration: $\boldsymbol{\alpha} = (1.125 \text{ rad/s}^2)\mathbf{k}$ ◀

(*b*) Acceleration of *C*:

$$\mathbf{a}_C = \boldsymbol{\alpha} \times \mathbf{r}_C + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_C)$$
$$= \boldsymbol{\alpha} \times \mathbf{r}_C + \boldsymbol{\omega} \times \mathbf{v}_C$$
$$\mathbf{r}_C = (5 \text{ in.})\mathbf{i} + (4 \text{ in.})\mathbf{j} + (2 \text{ in.})\mathbf{k}$$

$$\mathbf{a}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1.125 \\ 5 & 4 & 2 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.75 & 1.5 & 0 \\ 3 & -1.5 & -4.5 \end{vmatrix}$$
$$= -4.5\mathbf{i} + 5.625\mathbf{j} - 6.75\mathbf{i} + 3.375\mathbf{j} + (-1.125 - 4.5)\mathbf{k}$$

$$\mathbf{a}_C = -(11.3 \text{ in./s}^2)\mathbf{i} + (9 \text{ in./s}^2)\mathbf{j} - (5.63 \text{ in./s}^2)\mathbf{k}$$ ◀



PROBLEM 15.203

Rod AB of length 25 in. is connected by ball-and-socket joints to collars A and B, which slide along the two rods shown. Knowing that collar B moves toward Point E at a constant speed of 20 in./s, determine the velocity of collar A as collar B passes through Point D.

SOLUTION

Geometry.

$$l_{AB}^2 = x_{A/B}^2 + y_{A/B}^2 + z_{A/B}^2: \quad 25^2 = (-12)^2 + y_{A/B}^2 + (-20)^2$$

$$y_{A/B} = 9 \text{ in.}$$

$$\mathbf{r}_{A/B} = (-12 \text{ in.})\mathbf{i} + (9 \text{ in.})\mathbf{j} - (20 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{D/C} = (12 \text{ in.})\mathbf{i} - (9 \text{ in.})\mathbf{j},$$

$$l_{CD} = \sqrt{(12)^2 + (-9)^2} = 15 \text{ in.}$$

Velocity of collar B.

$$\mathbf{v}_B = v_B \frac{r_{D/C}}{l_{CD}}$$

$$\mathbf{v}_B = (20)\frac{(12\mathbf{i} - 9\mathbf{j})}{15} = (16 \text{ in./s})\mathbf{i} - (12 \text{ in./s})\mathbf{j}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A \mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where

$$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0.$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B})$$

$$= \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or

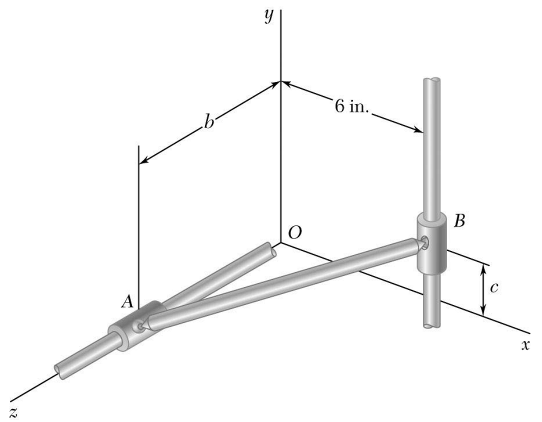
$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

From Eq. (1), $(-12\mathbf{i} + 9\mathbf{j} - 2\,0\mathbf{k}) \cdot (v_A \mathbf{j}) = (-12\mathbf{i} + 9\mathbf{j} - 80\mathbf{k}) \cdot (16\mathbf{i} - 12\mathbf{j})$

$$9v_A = (-12)(16) + (9)(-12)$$

or

$$v_A = -33.333 \text{ in./s} \qquad \mathbf{v}_A = -(33.3 \text{ in./s})\mathbf{j} \blacktriangleleft$$



PROBLEM 15.204

Rod *AB*, of length 11 in., is connected by ball-and-socket joints to collars *A* and *B*, which slide along the two rods shown. Knowing that collar *B* moves downward at a constant speed of 54 in./s, determine the velocity of collar *A* when $c = 2$ in.

SOLUTION

Geometry. $l_{AB}^2 = x_{A/B}^2 + y_{A/B}^2 + z_{A/B}^2: \quad (11)^2 = (6)^2 + (-2)^2 + (z_{A/B})^2$

$$z_{A/B} = 9 \text{ in.} \quad \mathbf{r}_{A/B} = (-6 \text{ in.})\mathbf{i} - (2 \text{ in.})\mathbf{j} + (9 \text{ in.})\mathbf{k}$$

Velocity of collar B. $\mathbf{v}_B = -v_B\mathbf{j} = -(54 \text{ in./s})\mathbf{j}$

Velocity of collar A. $\mathbf{v}_A = v_A\mathbf{k}$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B},$$

where $\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$

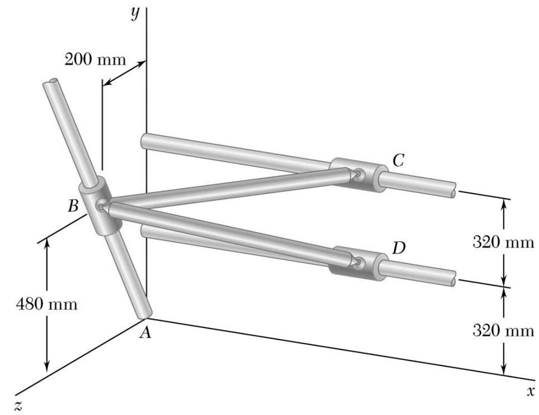
Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0.$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get $\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$

or $\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B$ (1)

From Eq. (1), $(-6\mathbf{i} - 2\mathbf{j} + 9\mathbf{k}) \cdot v_A\mathbf{k} = (-6\mathbf{i} - 2\mathbf{j} + 9\mathbf{k}) \cdot (-54\mathbf{j})$

or $9v_A = 108$ $\mathbf{v}_A = (12.00 \text{ in./s})\mathbf{k}$ ◀



PROBLEM 15.205

Rods *BC* and *BD* are each 840 mm long and are connected by ball-and-socket joints to collars which may slide on the fixed rods shown. Knowing that collar *B* moves toward *A* at a constant speed of 390 mm/s, determine the velocity of collar *C* for the position shown.

SOLUTION

Geometry.

$$l_{BC}^2 = x_{C/B}^2 + y_{C/B}^2 + z_{C/B}^2$$

$$(840)^2 = c^2 + (640-480)^2 + (200)^2 \qquad c = 800 \text{ mm}$$

$$\mathbf{r}_{C/B} = (800 \text{ mm})\mathbf{i} + (160 \text{ mm})\mathbf{j} - (200 \text{ mm})\mathbf{k}$$

Velocity of B.

$$\mathbf{v}_B = v_B\left(-\frac{12}{13}\mathbf{j} - \frac{5}{13}\mathbf{k}\right)$$

$$= (390 \text{ mm/s})\left(-\frac{12}{13}\mathbf{j} - \frac{5}{13}\mathbf{k}\right)$$

$$= -(360 \text{ mm/s})\mathbf{j} - (150 \text{ mm/s})\mathbf{k}$$

Velocity of C.

$$\mathbf{v}_C = v_C\mathbf{i}$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

where

$$\mathbf{v}_{C/B} = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

Noting that $\mathbf{v}_{C/B}$ is perpendicular to $\mathbf{r}_{C/B}$, we get $\mathbf{r}_{C/B} \cdot \mathbf{v}_{C/B} = 0$

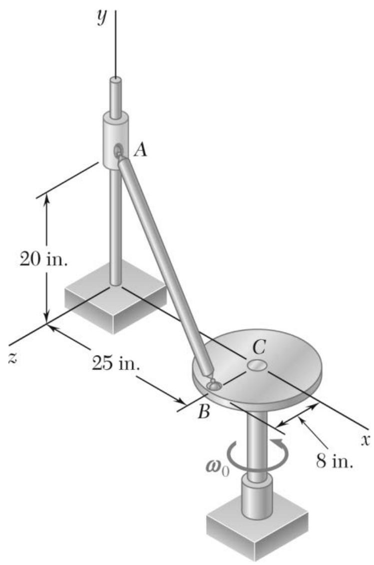
So that

$$\mathbf{r}_{C/B} \cdot \mathbf{v}_C = \mathbf{r}_{C/B} \cdot \mathbf{v}_B$$

$$(800\mathbf{i} + 160\mathbf{j} - 200\mathbf{k}) \cdot (-360\mathbf{j} - 150\mathbf{k}) = (800\mathbf{i} + 160\mathbf{j} - 200\mathbf{k}) \cdot (v_C\mathbf{i})$$

$$(160)(-360) + (-200)(-150) = 800 v_C$$

$$v_C = -34.5 \text{ mm/s} \qquad \mathbf{v}_C = -(34.5 \text{ mm/s})\mathbf{i} \blacktriangleleft$$



PROBLEM 15.206

Rod AB is connected by ball-and-socket joints to collar A and to the 16-in.-diameter disk C. Knowing that disk C rotates counterclockwise at the constant rate $\omega_0 = 3$ rad/s in the zx plane, determine the velocity of collar A for the position shown.

SOLUTION

Geometry.

$$\mathbf{r}_{B/C} = (-8 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{A/B} = -(25 \text{ in.})\mathbf{i} + (20 \text{ in.})\mathbf{j} - (8 \text{ in.})\mathbf{k}$$

Velocity at B.

$$\mathbf{v}_B = \omega_0 \mathbf{j} \times \mathbf{r}_{B/C}$$
$$= 3\mathbf{j} \times (-8\mathbf{k})$$
$$= -(24 \text{ in./s})\mathbf{i}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A \mathbf{j}$$
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where

$$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0.$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B})$$
$$= \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or

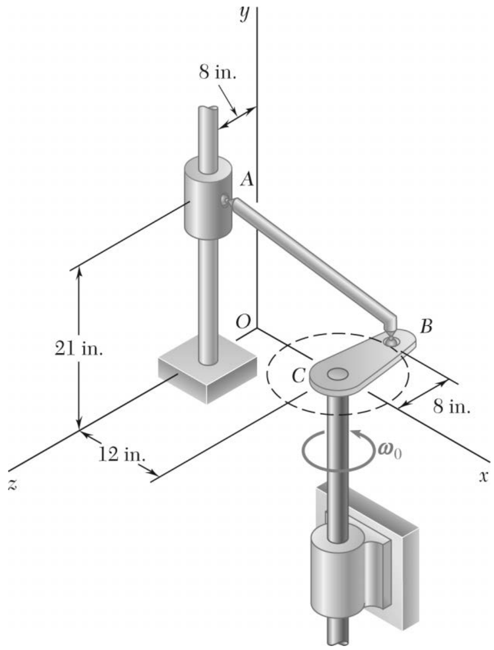
$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

From Eq. (1),

$$(-25\mathbf{i} + 20\mathbf{j} - 8\mathbf{k}) \cdot (v_A \mathbf{j}) = (-25\mathbf{i} + 20\mathbf{j} - 8\mathbf{k}) \cdot (24\mathbf{i})$$
$$20 v_A = -600$$

or

$$v_A = -30 \text{ in./s} \qquad \mathbf{v}_A = -(30.0 \text{ in./s})\mathbf{j} \blacktriangleleft$$



PROBLEM 15.207

Rod AB of length 29 in. is connected by ball-and-socket joints to the rotating crank BC and to the collar A. Crank BC is of length 8 in. and rotates in the horizontal xz plane at the constant rate $\omega_0 = 10$ rad/s. At the instant shown, when crank BC is parallel to the z axis, determine the velocity of collar A.

SOLUTION

Geometry.

$$\mathbf{r}_{B/C} = (-8\text{ in.})\mathbf{k},$$
$$\mathbf{r}_{A/B} = (-12\text{ in.})\mathbf{i} + (21\text{ in.})\mathbf{j} + (16\text{ in.})\mathbf{k}$$

Velocity at B.

$$\mathbf{v}_B = \omega_0\mathbf{j}\times\mathbf{r}_{B/C}$$
$$= 10\mathbf{j}\times(-8\mathbf{k})$$
$$= (-80\text{ in./s})\mathbf{i}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A\mathbf{j}$$
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where

$$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB}\times\mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A}\cdot\mathbf{v}_{B/A} = 0.$

Forming $\mathbf{r}_{A/B}\cdot\mathbf{v}_A$, we get

$$\mathbf{r}_{A/B}\cdot\mathbf{v}_A = \mathbf{r}_{A/B}\cdot(\mathbf{v}_B + \mathbf{v}_{A/B})$$
$$= \mathbf{r}_{A/B}\cdot\mathbf{v}_B + \mathbf{r}_{A/B}\cdot\mathbf{v}_{A/B}$$

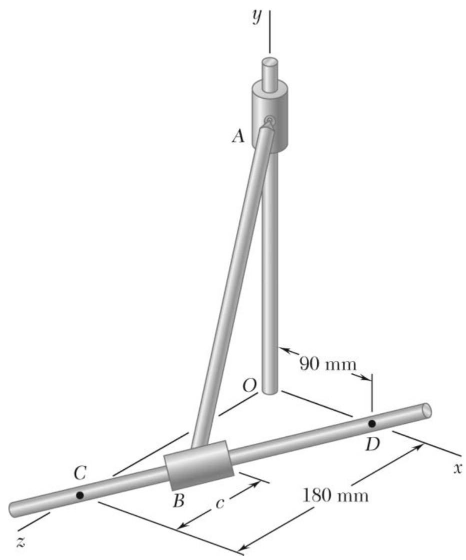
or

$$\mathbf{r}_{A/B}\cdot\mathbf{v}_A = \mathbf{r}_{A/B}\cdot\mathbf{v}_B \qquad (1)$$

From Eq. (1), $(-12\mathbf{i}+21\mathbf{j}+16\mathbf{k})\cdot(v_A\mathbf{j}) = (-12\mathbf{i}+21\mathbf{j}+16\mathbf{k})\cdot(-80\mathbf{i})$

$$21v_A = 960$$

or $v_A = 45.714$ in./s $\qquad \mathbf{v}_A = (45.7\text{ in./s})\mathbf{j}$ ◀



PROBLEM 15.208

Rod AB of length 300 mm is connected by ball-and-socket joints to collars A and B, which slide along the two rods shown. Knowing that collar B moves toward Point D at a constant speed of 50 mm/s, determine the velocity of collar A when $c = 80$ mm.

SOLUTION

Geometry.

$$\mathbf{r}_A = y\mathbf{j},$$
$$\mathbf{r}_D = (90 \text{ mm})\mathbf{i}$$
$$\mathbf{r}_C = (180 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{D/C} = \mathbf{r}_D - \mathbf{r}_C$$
$$= (40 \text{ mm})\mathbf{i} - (180 \text{ mm})\mathbf{k}$$

$$l_{CD} = \sqrt{(90)^2 + (180)^2}$$
$$= 201.246 \text{ mm}$$

$$\mathbf{r}_{B/C} = \frac{c(\mathbf{r}_{D/C})}{180}$$
$$= \frac{80(90\mathbf{i} - 180\mathbf{k})}{180}$$
$$= (40 \text{ mm})\mathbf{i} - (80 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_B = \mathbf{r}_C + \mathbf{r}_{B/C}$$
$$= 180\mathbf{k} + 40\mathbf{i} - 80\mathbf{k}$$
$$= (40 \text{ mm})\mathbf{i} + (100 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{A/B} = \mathbf{r}_A - \mathbf{r}_B$$
$$= -(40 \text{ mm})\mathbf{i} + (y \text{ mm})\mathbf{j} - (100 \text{ mm})\mathbf{k}$$

$$l_{AB}^2 = x_{A/B}^2 + y^2 + z_{A/B}^2: \quad 300^2 = (-40)^2 + y^2 + (-100)^2$$
$$y = 280 \text{ mm},$$
$$\mathbf{r}_{A/B} = (-40 \text{ mm})\mathbf{i} + (280 \text{ mm})\mathbf{j} - (100 \text{ mm})\mathbf{k}$$

Velocity of collar B. $\mathbf{v}_B = v_B \dfrac{\mathbf{r}_{D/C}}{l_{CD}}$

$$\mathbf{v}_B = \frac{(50)(90\mathbf{i} - 180\mathbf{k})}{201.246}$$

$$= (22.3607 \text{ mm/s})\mathbf{i} - (44.7214 \text{ mm/s})\mathbf{k}$$

Velocity of collar A. $\mathbf{v}_A = v_A\mathbf{j}$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where $\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0.$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get $\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B})$

$$= \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

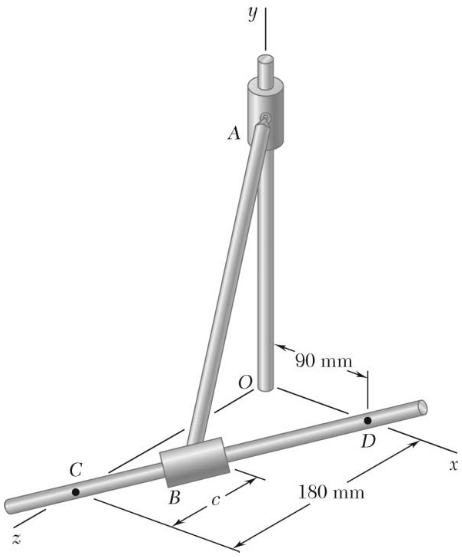
or $\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B$ (1)

From Eq. (1), $(-40\mathbf{i} + 280\mathbf{j} - 100\mathbf{k}) \cdot (v_A\mathbf{j}) = (-40\mathbf{i} + 280\mathbf{j} - 100\mathbf{k}) \cdot (22.3607\mathbf{i} - 44.7214\mathbf{k})$

$$280v_A = (-40)(22.3607) + (-100)(-44.7214)$$

or $v_A = 12.7775$ mm/s $\quad \mathbf{v}_A = (12.78 \text{ mm/s})\mathbf{j}$ ◀

PROBLEM 15.209



Rod *AB* of length 300 mm is connected by ball-and-socket joints to collars *A* and *B*, which slide along the two rods shown. Knowing that collar *B* moves toward Point *D* at a constant speed of 50 mm/s, determine the velocity of collar *A* when $c = 120$ mm.

SOLUTION

Geometry.

$$\mathbf{r}_A = y\mathbf{j},$$
$$\mathbf{r}_D = (90 \text{ mm})\mathbf{i}$$
$$\mathbf{r}_C = (180 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{D/C} = \mathbf{r}_D - \mathbf{r}_C$$
$$= (90 \text{ mm})\mathbf{i} - (180 \text{ mm})\mathbf{k}$$
$$l_{CD} = \sqrt{(90)^2 + (-180)^2}$$
$$= 201.246 \text{ mm}$$
$$\mathbf{r}_{B/C} = \frac{c(\mathbf{r}_{D/C})}{180}$$
$$= \frac{120(90\mathbf{i} - 180\mathbf{k})}{180}$$
$$= (60 \text{ mm})\mathbf{i} - (120 \text{ mm})\mathbf{k}$$
$$\mathbf{r}_B = \mathbf{r}_C + \mathbf{r}_{B/C}$$
$$= 180\mathbf{k} + 60\mathbf{i} - 120\mathbf{k}$$
$$= (60 \text{ mm})\mathbf{i} + (60 \text{ mm})\mathbf{k}$$
$$\mathbf{r}_{A/B} = \mathbf{r}_A - \mathbf{r}_B$$
$$= -60\mathbf{i} + y\mathbf{j} - 60\mathbf{k}$$

$$l_{AB}^2 = x_{A/B}^2 + y^2 + z_{A/B}^2: \quad 300^2 = 60^2 + y^2 + 60^2$$
$$y = 287.75 \text{ mm},$$
$$\mathbf{r}_{A/B} = (-60 \text{ mm})\mathbf{i} + (287.75 \text{ mm})\mathbf{j} - (60 \text{ mm})\mathbf{k}$$

PROBLEM 15.209 (Continued)

Velocity of collar B.

$$\mathbf{v}_B = v_B \frac{\mathbf{r}_{D/C}}{l_{CD}}$$

$$\mathbf{v}_B = \frac{(50)(90\mathbf{i} - 180\mathbf{k})}{201.246}$$

$$= (22.3607 \text{ mm/s})\mathbf{i} - (44.7214 \text{ mm/s})\mathbf{k}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A \mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where

$$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0.$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B})$$

$$= \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

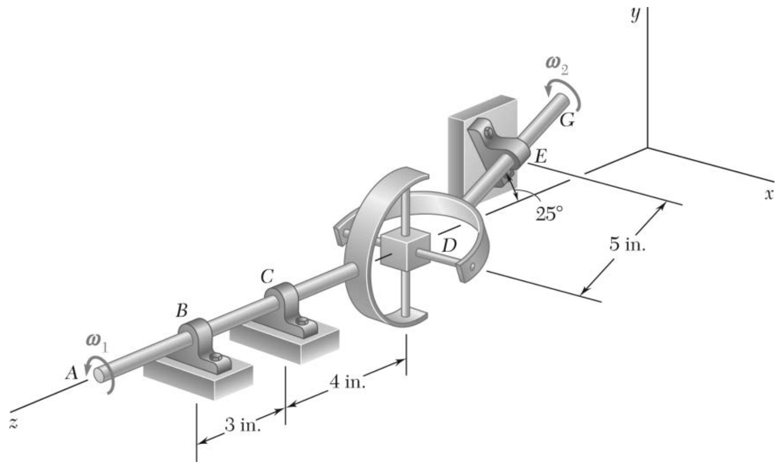
From Eq. (1), $(-60\mathbf{i} + 287.75\mathbf{j} - 60\mathbf{k}) \cdot (v_A \mathbf{j}) = (-60\mathbf{i} + 287.75\mathbf{j} - 60\mathbf{k}) \cdot (22.3607\mathbf{i} - 44.7214\mathbf{j})$

$$287.75 v_A = (-60)(22.3607) + (-60)(-44.7214)$$

or

$$v_A = 4.6626 \text{ mm/s}$$

$\mathbf{v}_A = (4.66 \text{ mm/s})\mathbf{j}$ ◀



PROBLEM 15.210

Two shafts *AC* and *EG*, which lie in the vertical *yz* plane, are connected by a universal joint at *D*. Shaft *AC* rotates with a constant angular velocity ω_1 as shown. At a time when the arm of the crosspiece attached to shaft *AC* is vertical, determine the angular velocity of shaft *EG*.

SOLUTION

Angular velocity of shaft AC. $\boldsymbol{\omega}_{AC} = \omega_1 \mathbf{k}$

Let $\omega_3 \mathbf{j}$ be the angular velocity of body *D* relative to shaft *AD*.

Angular velocity of body D. $\boldsymbol{\omega}_D = \omega_1 \mathrm{k} + \omega_3 \mathbf{j}$

Angular velocity of shaft EG. $\boldsymbol{\omega}_{EG} = \omega_2(\cos 25°\mathbf{k} - \sin 25°\mathbf{j})$

Let $\omega_4 \mathbf{i}$ be the angular velocity of body *D* relative to shaft *EG*.

Angular velocity of body D. $\boldsymbol{\omega}_D = \omega_2(\cos 25°\mathbf{k} - \sin 25°\mathbf{j}) + \omega_4 \mathbf{i}$

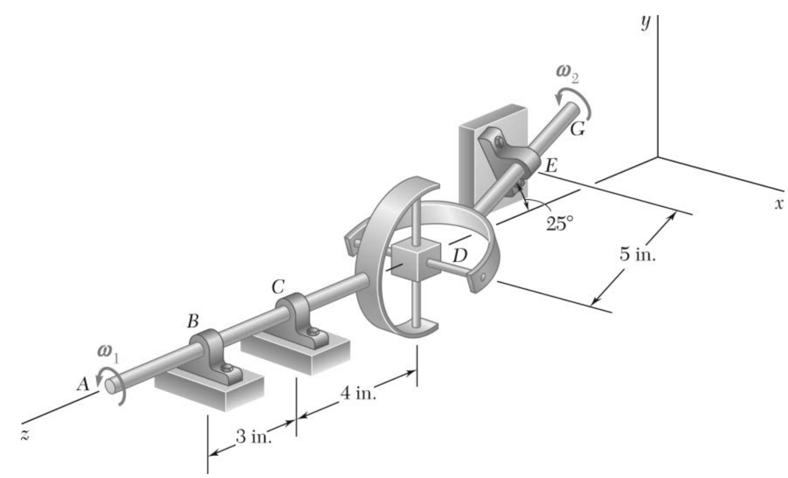
Equate the two expressions for ω_D and resolve into components.

$$\mathbf{i}: \quad 0 = \omega_4 \tag{1}$$

$$\mathbf{j}: \quad \omega_3 = -\omega_2 \sin 25° \tag{2}$$

$$\mathbf{k}: \quad \omega_1 = \omega_2 \cos 25° \tag{3}$$

From Eq. (3), $\omega_2 = \dfrac{\omega_1}{\cos 25°}$ $\quad \boldsymbol{\omega}_{EG} = \dfrac{\omega_2}{\cos 25°}(-\sin 25°\mathbf{j} + \cos 25°\mathbf{k})$ ◀



PROBLEM 15.211

Solve Problem 15.210, assuming that the arm of the crosspiece attached to the shaft *AC* is horizontal.

PROBLEM 15.210 Two shafts *AC* and *EG*, which lie in the vertical *yz* plane, are connected by a universal joint at *D*. Shaft *AC* rotates with a constant angular velocity ω_1 as shown. At a time when the arm of the crosspiece attached to shaft *AC* is vertical, determine the angular velocity of shaft *EG*.

SOLUTION

Angular velocity of shaft AC. $\quad \boldsymbol{\omega}_{AC} = \omega_1 \mathbf{k}$

Let $\omega_3 \mathbf{i}$ be the angular velocity of body *D* relative to shaft *AD*.

Angular velocity of body D. $\quad \boldsymbol{\omega}_D = \omega_1 \mathbf{k} + \omega_3 \mathbf{i}$

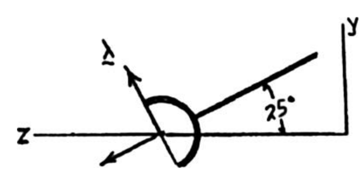
Angular velocity of shaft EG. $\quad \boldsymbol{\omega}_{EG} = \omega_2 (\cos 25° \mathbf{k} - \sin 25° \mathbf{j})$

Let $\omega_4 \boldsymbol{\lambda}$ be the angular velocity of body *D* relative to shaft *EG*,

where $\boldsymbol{\lambda}$ is a unit vector along the clevis axle attached to shaft *EG*.

$$\boldsymbol{\lambda} = \cos 25° \mathbf{j} + \sin 25° \mathbf{k}$$

$$\omega_4 \boldsymbol{\lambda} = \omega_4 \cos 25° \mathbf{j} + \omega_4 \sin 25° \mathbf{k}$$



Angular velocity of body D.

$$\boldsymbol{\omega}_D = \boldsymbol{\omega}_{EG} + \omega_4 \boldsymbol{\lambda}$$

$$\boldsymbol{\omega}_D = (\omega_4 \cos 25° - \omega_2 \sin 25°)\mathbf{j} + (\omega_4 \sin 25° + \omega_2 \cos 25°)\mathbf{k}$$

Equate the two expressions for $\boldsymbol{\omega}_D$ and resolve into components.

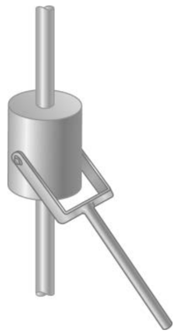
$$\mathbf{i}: \quad \omega_3 = 0 \tag{1}$$

$$\mathbf{j}: \quad 0 = \omega_4 \cos 25° - \omega_2 \sin 25° \tag{2}$$

$$\mathbf{k}: \quad \omega_1 = \omega_4 \sin 25° + \omega_2 \cos 25° \tag{3}$$

From Eqs. (2) and (3), $\quad \omega_2 = \omega_1 \cos 25° \quad \boldsymbol{\omega}_{EG} = \omega_1 \cos 25°(-\sin 25° \mathbf{j} + \cos 25° \mathbf{k})$ ◀

PROBLEM 15.212



In Problem 15.206, the ball-and-socket joint between the rod and collar *A* is replaced by the clevis shown. Determine (*a*) the angular velocity of the rod, (*b*) the velocity of collar *A*.

SOLUTION

<u>Geometry</u>. $\mathbf{r}_{B/C} = (8\text{ in.})\mathbf{k} \quad \mathbf{r}_{A/B} = -(25\text{ in.})\mathbf{i} + (20\text{ in.})\mathbf{j} - (8\text{ in.})\mathbf{k}$

<u>Velocity of collar *B*</u>. $\mathbf{v}_B = \omega_0\mathbf{j}\times\mathbf{r}_{B/C} = -3\mathbf{j}\times 8\mathbf{k} = (24\text{ in./s})\mathbf{i}$

<u>Velocity of collar *A*</u>. $\mathbf{v}_A = \mathbf{v}_A\mathbf{j}$

<u>Angular velocity of collar *A*</u>. $\boldsymbol{\omega}_A = \omega_1\mathbf{j}$

The axle of the clevis at *A* is perpendicular to both the *y* axis and the rod *AB*. A vector **p** along this axle is

$$\mathbf{p} = \mathbf{j}\times\mathbf{r}_{A/B} = \mathbf{j}\times(-25\mathbf{i} + 20\mathbf{j} - 8\mathbf{k}) = -8\mathbf{i} + 25\mathbf{k}$$

$$p = \sqrt{8^2 + 25^2} = 26.2488$$

Unit vector $\boldsymbol{\lambda}$ along axle: $\boldsymbol{\lambda} = \dfrac{\mathbf{p}}{p} = -0.30478\mathbf{i} + 0.95242\mathbf{k}$

Let $\boldsymbol{\omega}_2$ be the angular velocity of the rod *AB* relative to collar *A*.

$$\boldsymbol{\omega}_2 = \omega_2\boldsymbol{\lambda} = -0.30478\omega_2\mathbf{i} + 0.95242\omega_2\mathbf{k}$$

<u>Angular velocity of rod *AB*</u>. $\boldsymbol{\omega}_{AB} = \boldsymbol{\omega}_A + \boldsymbol{\omega}_2$

$$\boldsymbol{\omega}_{AB} = -0.30478\omega_2\mathbf{i} + \omega_1\mathbf{j} + 0.95242\omega_2\mathbf{k} \qquad (1)$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} = \mathbf{v}_B + \boldsymbol{\omega}_{AB}\times\mathbf{r}_{A/B}$$

$$\mathbf{v}_A\mathbf{j} = 24\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.30478\omega_2 & \omega_1 & 0.95242\omega_2 \\ -25 & 20 & -8 \end{vmatrix}$$

Resolving into components,

$$\mathbf{i}: \quad 0 = 24 - 8\omega_1 - 19.0484\omega_2 \qquad (2)$$

$$\mathbf{j}: \quad v_A = -26.2487\omega_2 \qquad (3)$$

$$\mathbf{k}: \quad 0 = 25\omega_1 - 6.0956\omega_2 \qquad (4)$$

PROBLEM 15.212 (Continued)

Solving Eqs. (2), (3) and (4) simultaneously,

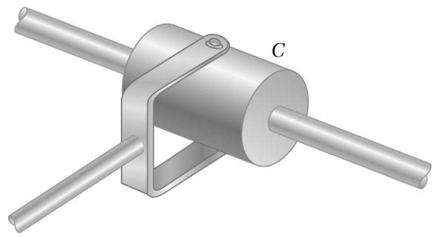
$$v_A = -30 \text{ in./s},$$
$$\omega_1 = 0.27867 \text{ rad/s},$$
$$\omega_2 = 1.1429 \text{ rad/s}$$

(*a*) Angular velocity of rod *AB*.

From Eq. (1) $\boldsymbol{\omega}_{AB} = -(0.30478)(1.1429)\mathbf{i} + 0.27867\mathbf{j} + (0.95242)(1.1429)\mathbf{k}$

$$\boldsymbol{\omega}_{AB} = -(0.348 \text{ rad/s})\mathbf{i} + (0.279 \text{ rad/s})\mathbf{j} + (1.089 \text{ rad/s})\mathbf{k} \blacktriangleleft$$

(*b*) Velocity of *A*. $\mathbf{v}_A = -(30.0 \text{ in./s})\mathbf{j} \blacktriangleleft$



PROBLEM 15.213

In Problem 15.205, the ball-and-socket joint between the rod and collar C is replaced by the clevis connection shown. Determine (*a*) the angular velocity of the rod, (*b*) the velocity of collar C.

SOLUTION

Geometry.

$$\mathbf{r}_C = x_C\mathbf{i} + (640\text{ mm})\mathbf{j},\ \mathbf{r}_B = (480\text{ mm})\mathbf{j} + (200\text{ mm})\mathbf{k}$$

$$l_{AB} = \sqrt{480^2 + 200^2} = 520\text{ mm}$$

$$\mathbf{r}_{C/B} = x_C\mathbf{i} + (160\text{ mm})\mathbf{j} - (200\text{ mm})\mathbf{k}$$

Length of rod BC. $\quad l_{BC}^2 = 840^2 = x_C^2 + 160^2 + 200^2$

Solving for x_C, $\quad x_C = 800\text{ mm}$

$$\mathbf{r}_{C/B} = (800\text{ mm})\mathbf{i} + (160\text{ mm})\mathbf{j} - (200\text{ mm})\mathbf{k}$$

Velocity.

$$\mathbf{v}_B = \frac{390}{520}(-480\mathbf{j} - 200\mathbf{k}) = (-360\text{ mm})\mathbf{j} - (150\text{ mm/s})\mathbf{k}$$

$$\mathbf{v}_C = v_C\mathbf{i}$$

Angular velocity of collar C. $\quad \boldsymbol{\omega}_C = \omega_C\mathbf{i}$

The axle of the clevis at C is perpendicular to the x-axis and to the rod BC.

A vector along this axle is $\quad \mathbf{p} = \mathbf{i} \times \mathbf{r}_{C/B}$

$$\mathbf{p} = \mathbf{i} \times (800\mathbf{i} + 160\mathbf{j} - 200\mathbf{k}) = (200\text{ mm})\mathbf{j} + (160\text{ mm})\mathbf{k}$$

$$p = \sqrt{200^2 + 160^2} = 256.125\text{ mm}$$

Let $\boldsymbol{\lambda}$ be a unit vector along the axle. $\quad \boldsymbol{\lambda} = \dfrac{\mathbf{p}}{p} = 0.78087\mathbf{j} + 0.62470\mathbf{k}$

Let $\boldsymbol{\omega}_s = \omega_s\boldsymbol{\lambda}$ be the angular velocity of rod BC relative to collar C.

$$\boldsymbol{\omega}_s = 0.78087\omega_s\mathbf{j} + 0.62470\omega_s\mathbf{k}$$

Angular velocity of rod BC. $\quad \boldsymbol{\omega}_{BC} = \boldsymbol{\omega}_C + \boldsymbol{\omega}_s$

$$\boldsymbol{\omega}_{BC} = \omega_C\mathbf{i} + 0.78087\omega_s\mathbf{j} + 0.62470\omega_s\mathbf{k}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$v_C\mathbf{i} = -360\mathbf{j} - 150\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_C & 0.78087\omega_s & 0.62470\omega_s \\ 800 & 160 & -200 \end{vmatrix}$$

PROBLEM 15.213 (Continued)

Resolving into components,

$$\mathbf{i}: \quad v_C = -256.126\omega_s \tag{1}$$

$$\mathbf{j}: \quad 0 = -360 + 200\omega_C + 499.76\omega_s \tag{2}$$

$$\mathbf{k}: \quad 0 = -150 + 160\omega_C - 624.70\omega_s \tag{3}$$

Solving the simultaneous equations (1), (2), and (3),

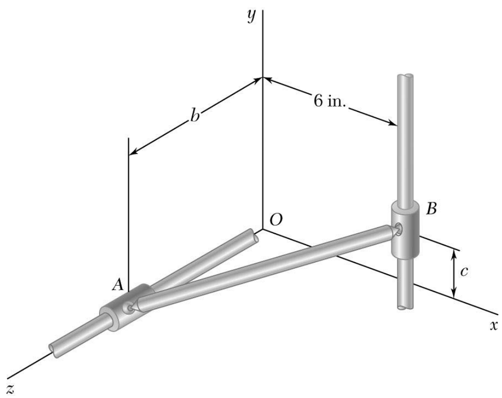
$$\omega_C = 1.4634 \text{ rad/s}, \quad \omega_s = 0.13470 \text{ rad/s}, \quad v_C = -34.50 \text{ mm/s}$$

(*a*) *Angular velocity of rod BC.*

$$\boldsymbol{\omega}_{BC} = 1.4634\mathbf{i} + (0.78087)(0.13470)\mathbf{j} + (0.62470)(0.13470)\mathbf{k}$$

$$\boldsymbol{\omega}_{BC} = (1.463 \text{ rad/s})\mathbf{i} + (0.1052 \text{ rad/s})\mathbf{j} + (0.0841 \text{ rad/s})\mathbf{k} \blacktriangleleft$$

(*b*) *Velocity of collar C.* $\mathbf{v}_C = -(34.5 \text{ mm/s})\mathbf{i} \blacktriangleleft$



PROBLEM 15.214

In Problem 15.204, determine the acceleration of collar A when $c = 2$ in.

PROBLEM 15.204 Rod AB of length 11 in., is connected by ball-and-socket joints to collars A and B, which slide along the two rods shown. Knowing that collar B moves downward at a constant speed of 54 in./s, determine the velocity of collar A when $c = 2$ in.

SOLUTION

Geometry. $$l_{AB}^2 = x_{A/B}^2 + y_{A/B}^2 + z_{A/B}^2: \quad (11)^2 = (6)^2 + (-2)^2 + (z_{A/B})^2$$

$$z_{A/B} = 9 \text{ in.} \quad \mathbf{r}_{A/B} = (-6 \text{ in.})\mathbf{i} - (2 \text{ in.})\mathbf{j} + (9 \text{ in.})\mathbf{k}$$

Velocity of collar B. $$\mathbf{v}_B = -v_B\mathbf{j} = -(54 \text{ in./s})\mathbf{j}$$

$$\mathbf{v}_B = \frac{(2.5)(4.5\mathbf{i} - 9\mathbf{k})}{10.0623} = (1.11803 \text{ in.})\mathbf{i} - (2.23607 \text{ in./s})\mathbf{k}$$

Velocity of collar A. $$\mathbf{v}_A = v_A\mathbf{k}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B},$$

where $$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get $$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or $$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

From Eq. (1), $$(-6\mathbf{i} - 2\mathbf{j} + 9\mathbf{k}) \cdot v_A\mathbf{k} = (-6\mathbf{i} - 2\mathbf{j} + 9\mathbf{k}) \cdot (-54\mathbf{j})$$

or $$9v_A = 108 \qquad v_A = (12.00 \text{ in./s})\mathbf{k}$$

Relative velocity $$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$\mathbf{v}_{A/B} = (54 \text{ in./s})\mathbf{j} + (12.00 \text{ in./s})\mathbf{k}$$

$$(v_{A/B})^2 = (54)^2 + (12.00)^2 = 3060 \text{ in}^2/\text{s}^2$$

Acceleration of collar B. $$\mathbf{a}_B = 0$$

Acceleration of collar A. $$\mathbf{a}_A = a_A\mathbf{k}$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

where $$\mathbf{a}_{A/B} = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B}$$

Noting that $\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{A/B} \cdot \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} = 0$

PROBLEM 15.214 (Continued)

We note also that

$$\mathbf{r}_{A/B} \cdot \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B} = \mathbf{v}_{A/B} \cdot \mathbf{r}_{A/B} \times \boldsymbol{\omega}_{A/B}$$
$$= -\mathbf{v}_{A/B} \cdot \mathbf{v}_{A/B} = -(v_{A/B})^2$$

Then,

$$\mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B} = 0 - (v_{A/B})^2 = -(v_{A/B})^2$$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{a}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot (\mathbf{a}_A + \mathbf{a}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{a}_B + \mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B}$$

or

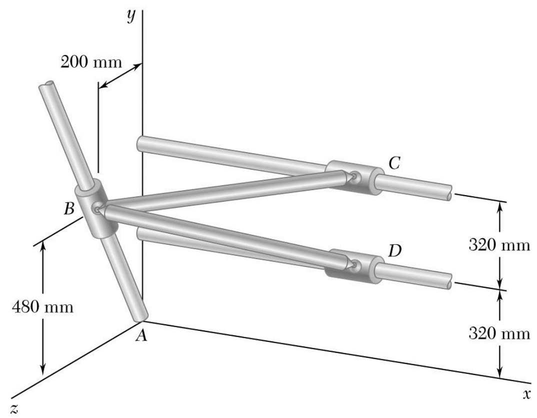
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot \mathbf{a}_B - (v_{A/B})^2 \qquad (2)$$

From Eq. (2)

$$(-6\mathbf{i} - 2\mathbf{i} + 9\mathbf{k}) \cdot a_A\mathbf{k} = 0 - 3060$$

$$9a_A = -3060$$

$\mathbf{a}_A = -(340 \text{ in./s}^2)\mathbf{k}$ ◀



PROBLEM 15.215

In Problem 15.205, determine the acceleration of collar *C*.

PROBLEM 15.205 Rod *BC* and *BD* are each 840 mm long and are connected by ball-and-socket joints to collars which may slide on the fixed rods shown. Knowing that collar *B* moves toward *A* at a constant speed of 390 mm/s, determine the velocity of collar *C* for the position shown.

SOLUTION

Geometry.

$$l_{BC}^2 = x_{C/B}^2 + y_{C/B}^2 + z_{C/B}^2$$

$$(840)^2 = c^2 + (640-480)^2 + (200)^2 \qquad c = 800m$$

$$\mathbf{r}_{C/B} = (800\text{ mm})\mathbf{i} + (160\text{ mm})\mathbf{j} - (200\text{ mm})\mathbf{k}$$

Velocity of B.

$$\mathbf{v}_B = v_B\left(-\frac{12}{13}\mathbf{j} - \frac{5}{13}\mathbf{k}\right)$$

$$= (390\text{ mm/s})\left(-\frac{12}{13}\mathbf{j} - \frac{5}{13}\mathbf{k}\right)$$

$$= -(360\text{ mm/s})\mathbf{j} - (150\text{ mm/s})\mathbf{k}$$

Velocity of C.

$$\mathbf{v}_C = v_C\mathbf{i}$$

where

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$\mathbf{v}_{C/B} = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

Noting that $\mathbf{v}_{C/B}$ is perpendicular to $\mathbf{r}_{C/B}$, we get $\mathbf{r}_{C/B} \cdot \mathbf{v}_C = \mathbf{r}_{C/B} \cdot \mathbf{v}_B$

$$(800\mathbf{i} + 160\mathbf{j} - 200\mathbf{k}) \cdot (-360\mathbf{j} + 150\mathbf{k}) = (800\mathbf{i} + 160\mathbf{j} - 200\mathbf{k}) \cdot (v_C\mathbf{i})$$

$$(160)(-360) + (-200)(-150) = 800v_C$$

$$v_C = -34.5\text{ mm/s} \qquad \mathbf{v}_C = -(34.5\text{ mm/s})\mathbf{i}$$

Relative velocity:

$$\mathbf{v}_{C/B} = \mathbf{v}_C - \mathbf{v}_B$$

$$\mathbf{v}_{C/B} = -(34.5\text{ mm/s})\mathbf{i} + (360\text{ mm/s})\mathbf{j} + (150\text{ mm/s})\mathbf{k}$$

$$v_{C/B}^2 = (34.5)^2 + (360)^2 + (150)^2 = 153290\text{ mm}^2/\text{s}^2$$

Acceleration of collar B:

$$\mathbf{a}_B = 0$$

Acceleration of collar C:

$$\mathbf{a}_C = \mathbf{a}_C\mathbf{i}$$

$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$$

PROBLEM 15.215 (Continued)

where $\mathbf{a}_{C/B} = \boldsymbol{\alpha}_{CB} \times \mathbf{r}_{C/B} + \boldsymbol{\omega}_{CB} \times \mathbf{v}_{C/B}$

Noting that $\boldsymbol{\alpha}_{CB} \times \mathbf{r}_{C/B}$ is perpendicular to $\mathbf{r}_{C/B}$, we get $\mathbf{r}_{C/B} \cdot \boldsymbol{\alpha}_{CB} \times \mathbf{r}_{C/B} = 0$

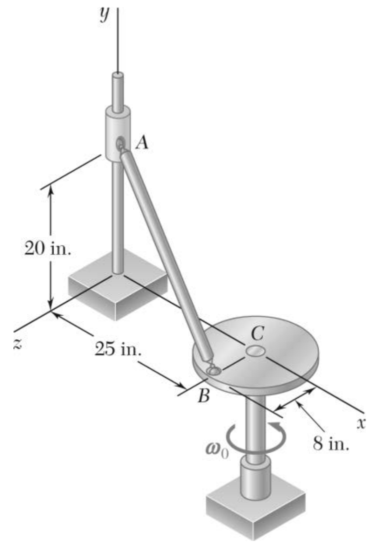
We note also that
$$\mathbf{r}_{B/C} \cdot \boldsymbol{\omega}_{CB} \times \mathbf{v}_{C/B} = \mathbf{v}_{C/B} \cdot \mathbf{r}_{C/B} \times \boldsymbol{\omega}_{C/B}$$
$$= -\mathbf{v}_{C/B} \cdot \mathbf{v}_{C/B} = -(v_{C/B})^2$$

Then,
$$\mathbf{r}_{C/B} \cdot \mathbf{a}_C = 0 - (v_{C/B})^2 = -(v_{C/B})^2$$

Forming $\mathbf{r}_{C/B} \cdot \mathbf{a}_C$, we get
$$\mathbf{r}_{C/B} \cdot \mathbf{a}_C = \mathbf{r}_{C/B} \cdot (\mathbf{a}_B + \mathbf{a}_{C/B}) = \mathbf{r}_{B/C} \cdot \mathbf{a}_B + \mathbf{r}_{B/C} \cdot \mathbf{a}_{C/B}$$

so that
$$\mathbf{r}_{C/B} \cdot \mathbf{a}_C = \mathbf{r}_{C/B} \cdot \mathbf{a}_B - (v_{C/B})^2 \qquad (2)$$

From Equation (2),
$$(800\mathbf{i} + 160\mathbf{j} - 200\mathbf{k}) \cdot a_C\mathbf{i} = 0 - 153290$$
$$800a_C = -153290$$
$$a_C = -191.6 \text{ mm/s}^2 \qquad \mathbf{a}_C = -(191.6 \text{ mm/s}^2)\mathbf{i} \blacktriangleleft$$



PROBLEM 15.216

In Problem 15.206, determine the acceleration of collar *A*.

PROBLEM 15.206 Rod *AB* is connected by ball-and-socket joints to collar *A* and to the 16-in.-diameter disk *C*. Knowing that disk *C* rotates counterclockwise at the constant rate $\omega_0 = 3$ rad/s in the *zx* plane, determine the velocity of collar *A* for the position shown.

SOLUTION

Geometry.

$$\mathbf{r}_{B/C} = (-8 \text{ in.})\mathbf{k}$$
$$\mathbf{r}_{A/B} = -(25 \text{ in.})\mathbf{i} + (20 \text{ in.})\mathbf{j} - (8 \text{ in.})\mathbf{k}$$

Velocity at B.

$$\mathbf{v}_B = \omega_0 \mathbf{j} \times \mathbf{r}_{B/C} = 3\mathbf{j} \times (-8\mathbf{k}) = (24 \text{ in./s})\mathbf{i}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A \mathbf{j}$$
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where

$$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0$.

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \quad (1)$$

From Eq. (1)

$$(-25\mathbf{i} + 20\mathbf{j} - 8\mathbf{k}) \cdot (v_A \mathbf{j}) = (-25\mathbf{i} + 20\mathbf{j} - 8\mathbf{k}) \cdot (24\mathbf{i})$$
$$20 v_A = -600$$

or

$$v_A = -30 \text{ in./s}$$

Relative velocity.

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$
$$\mathbf{v}_{A/B} = (-30 \text{ in./s})\mathbf{j} + (24 \text{ in./s})\mathbf{i}$$
$$(v_{A/B})^2 = (-30)^2 + (24)^2 = 1476 \text{ in}^2/\text{s}^2$$

PROBLEM 15.216 (Continued)

Acceleration at B.

$$\mathbf{a}_B = \omega_0\mathbf{j}\times\mathbf{v}_B$$
$$= 3\mathbf{j}\times 24\mathbf{i}$$
$$= -(72 \text{ in./s}^2)\mathbf{k}$$

Acceleration of collar A.

$$\mathbf{a}_A = a_A\mathbf{j}$$
$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

where

$$\mathbf{a}_{A/B} = \boldsymbol{\alpha}_{AB}\times\mathbf{r}_{A/B} + \boldsymbol{\omega}_{AB}\times\mathbf{v}_{A/B}$$

Noting that $\boldsymbol{\alpha}_{AB}\times\mathbf{r}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{A/B}\cdot\boldsymbol{\alpha}_{AB}\times\mathbf{r}_{A/B} = 0$

We note also that

$$\mathbf{r}_{A/B}\cdot\boldsymbol{\omega}_{AB}\times\mathbf{v}_{A/B} = \mathbf{v}_{A/B}\cdot\mathbf{r}_{A/B}\times\boldsymbol{\omega}_{A/B}$$
$$= -\mathbf{v}_{A/B}\cdot\mathbf{v}_{A/B}$$
$$= -(v_{A/B})^2$$

Then

$$\mathbf{r}_{A/B}\cdot\mathbf{a}_{A/B} = 0 - (v_{A/B})^2$$
$$= -(v_{A/B})^2$$

Forming $\mathbf{r}_{A/B}\cdot\mathbf{a}_A$, we get

$$\mathbf{r}_{A/B}\cdot\mathbf{a}_A = \mathbf{r}_{A/B}\cdot(\mathbf{a}_A + \mathbf{a}_{A/B})$$
$$= \mathbf{r}_{A/B}\cdot\mathbf{a}_B + \mathbf{r}_{A/B}\cdot\mathbf{a}_{A/B}$$

or

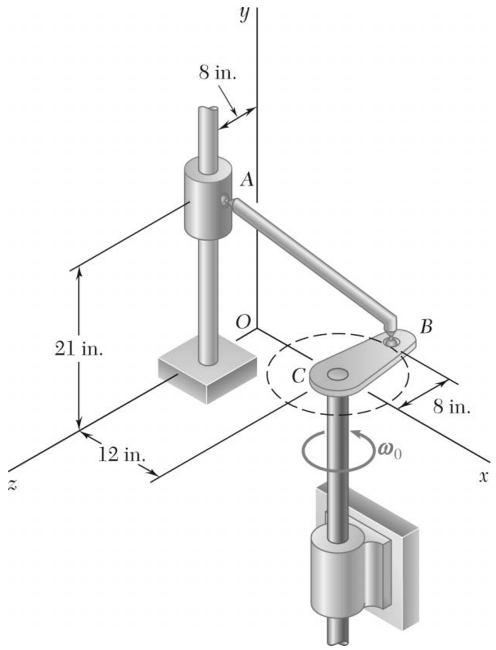
$$\mathbf{r}_{A/B}\cdot\mathbf{a}_A = \mathbf{r}_{A/B}\cdot\mathbf{a}_B - (v_{A/B})^2 \qquad (2)$$

From Eq. (2),

$$(-25\mathbf{i}+20\mathbf{j}-8\mathbf{k})\cdot(a_A\mathbf{j}) = (-25\mathbf{i}+20\mathbf{j}-8\mathbf{k})\cdot(-72\mathbf{k}) - 1476$$

$$20a_A = 576 - 1476$$
$$= -45 \text{ in./s}^2$$

$$\mathbf{a}_A = -(45.0 \text{ in./s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.217

In Problem 15.207, determine the acceleration of collar A.

PROBLEM 15.207 Rod AB of length 29 in. is connected by ball-and-socket joints to the rotating crank BC and to the collar A. Crank BC is of length 8 in. and rotates in the horizontal xz plane at the constant rate $\omega_0 = 10$ rad/s. At the instant shown, when crank BC is parallel to the z axis, determine the velocity of collar A.

SOLUTION

Geometry.

$$\mathbf{r}_{B/C} = (-8 \text{ in.})\mathbf{k},$$
$$\mathbf{r}_{A/B} = (-12 \text{ in.})\mathbf{i} + (21 \text{ in.})\mathbf{j} + (16 \text{ in.})\mathbf{k}$$

Velocity at B.

$$\mathbf{v}_B = \omega_0 \mathbf{j} \times \mathbf{r}_{B/C}$$
$$= 10\mathbf{j} \times (-8\mathbf{k})$$
$$= (-80 \text{ in./s})\mathbf{i}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A \mathbf{j}$$
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where

$$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0.$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B})$$
$$= \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

From Eq. (1)

$$(-12\mathbf{i} + 21\mathbf{j} + 16\mathbf{k}) \cdot (v_A \mathbf{j}) = (-12\mathbf{i} + 21\mathbf{j} - 16\mathbf{k}) \cdot (-80\mathbf{i})$$
$$21 v_A = 960$$

or

$$v_A = 45.714 \text{ in./s} \qquad \mathbf{v}_A = (45.7 \text{ in./s})\mathbf{j} \blacktriangleleft$$

Relative velocity.

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$\mathbf{v}_{A/B} = (45.714 \text{ in./s})\mathbf{j} + (80 \text{ in./s})\mathbf{i}$$

$$(v_{A/B})^2 = (45.714)^2 + (80)^2$$

$$= 8489.8 \text{ (in./s}^2)$$

Acceleration of Point B.

$$\mathbf{a}_B = \omega_0 \mathbf{j} \times \mathbf{v}_B$$

$$= 10\mathbf{j} \times (-80)\mathbf{i}$$

$$= (800 \text{ in./s}^2)\mathbf{k}$$

Acceleration of collar A.

$$\mathbf{a}_A = a_A \mathbf{j}$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

where

$$\mathbf{a}_{A/B} = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B}$$

Noting that $\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{A/B} \cdot \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} = 0.$

We note also that

$$\mathbf{r}_{A/B} \cdot \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B} = \mathbf{v}_{A/B} \cdot \mathbf{r}_{A/B} \times \boldsymbol{\omega}_{A/B}$$

$$= -\mathbf{v}_{A/B} \cdot \mathbf{v}_{A/B}$$

$$= -(v_{A/B})^2$$

Then

$$\mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B} = 0 - (v_{A/B})^2$$

$$= -(v_{A/B})^2$$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{a}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot (\mathbf{a}_A + \mathbf{a}_{A/B})$$

$$= \mathbf{r}_{A/B} \cdot \mathbf{a}_B + \mathbf{r}_{A/B} - \mathbf{a}_{A/B}$$

or

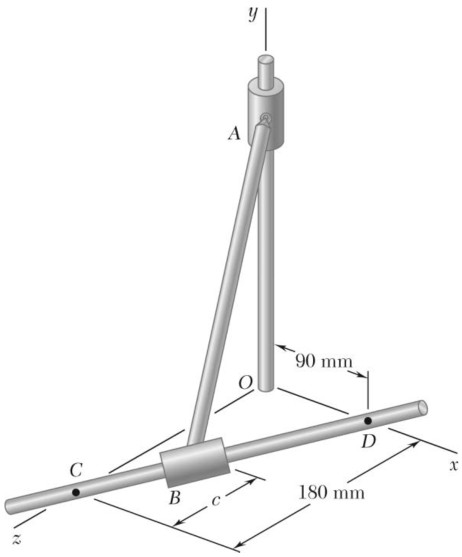
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot \mathbf{a}_B - (v_{A/B})^2 \qquad (2)$$

From Eq. (2) $\quad (-12\mathbf{i} + 21\mathbf{j} + 16\mathbf{k}) \cdot (a_A \mathbf{j}) = (-12\mathbf{i} + 21\mathbf{j} + 16\mathbf{k}) \cdot (800\mathbf{k}) - 8489.8$

$$21a_A = 12{,}800 - 8489.8$$

$\mathbf{a}_A = (205 \text{ in./s}^2)\mathbf{j}$ ◀

PROBLEM 15.218



In Problem 15.208, determine the acceleration of collar *A*.

PROBLEM 15.208 Rod *AB* of length 300 mm is connected by ball-and-socket joints to collars *A* and *B*, which slide along the two rods shown. Knowing that collar *B* moves toward Point *D* at a constant speed of 50 mm/s, determine the velocity of collar *A* when $c = 80$ mm.

SOLUTION

Geometry. $\mathbf{r}_A = y\mathbf{j}, \quad \mathbf{r}_D = (90 \text{ mm})\mathbf{i} \quad \mathbf{r}_C = (180 \text{ mm})\mathbf{k}$

$$\mathbf{r}_{D/C} = \mathbf{r}_D - \mathbf{r}_C = (90 \text{ mm})\mathbf{i} - (180 \text{ mm})\mathbf{k}$$

$$l_{CD} = \sqrt{(90)^2 + (180)^2} = 201.246 \text{ mm}$$

$$\mathbf{r}_{B/C} = \frac{c(\mathbf{r}_{D/C})}{180} = \frac{80(90\mathbf{i} - 180\mathbf{k})}{180} = (40 \text{ mm})\mathbf{i} - (80 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_B = \mathbf{r}_C + \mathbf{r}_{B/C} = 180\mathbf{k} + 40\mathbf{i} - 80\mathbf{k} = (40 \text{ mm})\mathbf{i} + (100 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{A/B} = \mathbf{r}_A - \mathbf{r}_B = -(40 \text{ mm})\mathbf{i} + (y \text{ mm})\mathbf{j} - (100 \text{ mm})\mathbf{k}$$

$$l_{AB}^2 = x_{A/B}^2 + y^2 + z_{A/B}^2: \quad 300^2 = (-40)^2 + y^2 + (-100)^2$$

$$y = 280 \text{ mm},$$

$$\mathbf{r}_{A/B} = (-40 \text{ mm})\mathbf{i} + (280 \text{ mm})\mathbf{j} - (100 \text{ mm})\mathbf{k}$$

Velocity of collar B.

$$\mathbf{v}_B = v_B \frac{\mathbf{r}_{D/C}}{l_{CD}}$$

$$\mathbf{v}_B = \frac{(50)(90\mathbf{i} - 180\mathbf{k})}{201.246}$$

$$= (22.3607 \text{ mm/s})\mathbf{i} - (44.7214 \text{ mm/s})\mathbf{k}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A\mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where $\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0$.

PROBLEM 15.218 (Continued)

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get
$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or
$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \quad (1)$$

From (1)
$$(-40\mathbf{i} + 280\mathbf{j} - 100\mathbf{k}) \cdot (v_A \mathbf{j}) = (-40\mathbf{i} + 280\mathbf{j} - 100\mathbf{k}) \cdot (22.3607\mathbf{i} - 44.7214\mathbf{k})$$
$$280 v_A = (-40)(22.3607) + (-100)(-44.7214)$$

or
$$v_A = 12.7775 \text{ mm/s} \qquad \mathbf{v}_A = (12.7775 \text{ mm/s})\,\mathbf{j}$$

<u>Relative velocity</u>.
$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B = (12.7775 \text{ mm/s})\mathbf{i} - (22.3607 \text{ mm/s})\mathbf{j} + (44.7214 \text{ mm/s})\mathbf{k}$$
$$\mathbf{v}_{A/B} \cdot \mathbf{v}_{A/B} = (12.7775)^2 + (22.3607)^2 + (44.7214)^2 = 2663.3 \text{ mm}^2/\text{s}^2$$

Acceleration of collar B.
$$\mathbf{a}_B = 0$$

Acceleration of collar A.
$$\mathbf{a}_A = a_A \mathbf{j}$$
$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

where
$$\mathbf{a}_{A/B} = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B}$$

Noting that $\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{A/B} \cdot \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} = 0$

We note also that
$$\mathbf{r}_{A/B} \cdot \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B} = \mathbf{v}_{A/B} \cdot \mathbf{r}_{A/B} \times \boldsymbol{\omega}_{A/B} = -\mathbf{v}_{A/B} \cdot \mathbf{v}_{A/B} = -(v_{A/B})^2$$

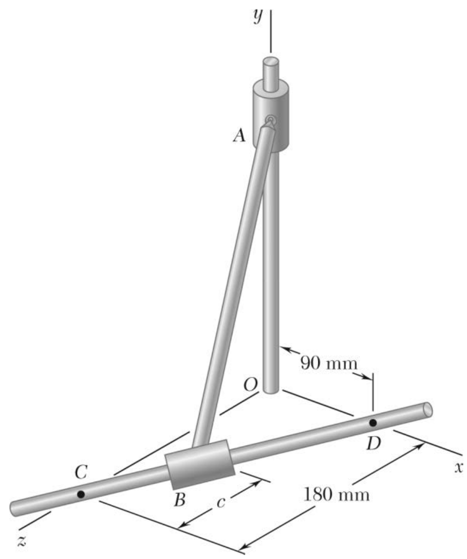
Then
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B} = 0 - (v_{A/B})^2 = -(v_{A/B})^2$$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{a}_A$, we get
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot (\mathbf{a}_A + \mathbf{a}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{a}_B + \mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B}$$

or
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot \mathbf{a}_B - (v_{A/B})^2 \quad (2)$$

From Eq. (2)
$$(-40\mathbf{i} + 280\mathbf{j} - 100\mathbf{k}) \cdot (a_A \mathbf{j}) = 0 - 2663.3$$
$$280 a_A = -2663.3 \qquad a_A = -9.512 \text{ mm/s}^2$$

$$\mathbf{a}_A = -(9.51 \text{ mm/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.219

In Problem 15.209, determine the acceleration of collar *A*.

PROBLEM 15.209 Rod *AB* of length 300 mm is connected by ball-and-socket joints to collars *A* and *B*, which slide along the two rods shown. Knowing that collar *B* moves toward Point *D* at a constant speed of 50 mm/s, determine the velocity of collar *A* when $c = 120$ mm.

SOLUTION

Geometry.

$$\mathbf{r}_A = y\mathbf{j}, \quad \mathbf{r}_D = (90 \text{ mm})\mathbf{i} \quad \mathbf{r}_C = (180 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{D/C} = \mathbf{r}_D - \mathbf{r}_C = (40 \text{ mm})\mathbf{i} - (180 \text{ mm})\mathbf{k}$$

$$l_{CD} = \sqrt{(90)^2 + (-180)^2} = 201.246$$

$$\mathbf{r}_{B/C} = \frac{c(\mathbf{r}_{D/C})}{180} = \frac{120(90\mathbf{i} - 180\mathbf{k})}{180} = (60 \text{ mm})\mathbf{i} - (120 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_B = \mathbf{r}_C + \mathbf{r}_{B/C} = 180\mathbf{k} + 60\mathbf{i} - 120\mathbf{k} = (60 \text{ mm})\mathbf{i} + (60 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{A/B} = \mathbf{r}_A - \mathbf{r}_B = -60\mathbf{i} + y\,\mathbf{j} - 60\mathbf{k}$$

$$l_{AB}^2 = x_{A/B}^2 + y^2 + z_{A/B}^2: \quad 300^2 = 60^2 + y^2 + 60^2$$

$$y = 287.75 \text{ mm},$$

$$\mathbf{r}_{A/B} = (-60 \text{ mm})\mathbf{i} + (287.75 \text{ mm})\mathbf{j} - (60 \text{ mm})\mathbf{k}$$

Velocity of collar B.

$$\mathbf{v}_B = v_B \frac{\mathbf{r}_{D/C}}{l_{CD}}$$

$$\mathbf{v}_B = \frac{(50)(90\mathbf{i} - 180\mathbf{k})}{201.246}$$

$$= (22.3607 \text{ mm/s})\mathbf{i} - (44.7214 \text{ mm/s})\mathbf{k}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A\mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where

$$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0.$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get
$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or
$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \quad (1)$$

From Eq. (1) $$(-60\mathbf{i} + 287.75\mathbf{j} - 60\mathbf{k}) \cdot (v_A\mathbf{j}) = (-60\mathbf{i} + 287.75\mathbf{j} - 60\mathbf{k}) \cdot (1.11803\mathbf{i} - 2.23607\mathbf{j})$$

$$287.75 v_A = (-60)(22.3607) + (-60)(-44.7214)$$

or
$$v_A = 4.6626 \text{ mm/s} \qquad \mathbf{v}_A = (4.6626 \text{ mm/s})\mathbf{j}$$

<u>Relative velocity</u>.
$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B = -(22.3607 \text{ mm/s})\mathbf{i} + (4.6626 \text{ mm/s})\mathbf{j} + (44.7214 \text{ mm/s})\mathbf{k}$$

$$\mathbf{v}_{A/B} \cdot \mathbf{v}_{A/B} = (22.3607)^2 + (4.6626)^2 + (44.7214)^2 = 2521.7$$

Acceleration of collar B. $$\mathbf{a}_B = 0$$

Acceleration of collar A. $$\mathbf{a}_A = a_A\mathbf{j}$$
$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

where
$$\mathbf{a}_{A/B} = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B}$$

Noting that $\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{A/B} \cdot \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} = 0.$

We note also that
$$\mathbf{r}_{A/B} \cdot \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B} = \mathbf{v}_{A/B} \cdot \mathbf{r}_{A/B} \times \boldsymbol{\omega}_{A/B} = -\mathbf{v}_{A/B} \cdot \mathbf{v}_{A/B} = -(v_{A/B})^2$$

Then
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B} = 0 - (v_{A/B})^2 = -(v_{A/B})^2$$

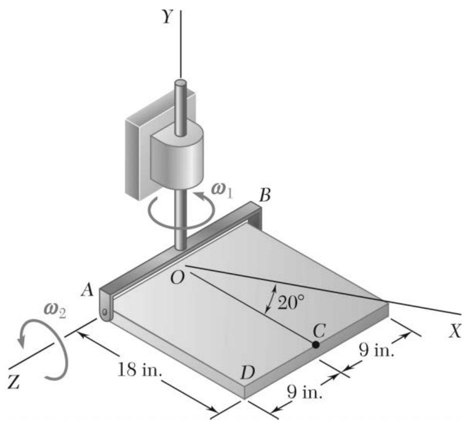
Forming $\mathbf{r}_{A/B} \cdot \mathbf{a}_A$, we get
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot (\mathbf{a}_A + \mathbf{a}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{a}_B + \mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B}$$

or
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot \mathbf{a}_B - (v_{A/B})^2 \quad (2)$$

From Eq. (2), $$(-60\mathbf{i} + 287.75\mathbf{j} - 60\mathbf{k}) \cdot (a_A\mathbf{j}) = 0 - 2521.7$$

$$287.75 a_A = -2521.7 \qquad a_A = -8.764 \text{ mm/s}^2$$

$$\mathbf{a}_A = -(8.76 \text{ mm/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.220

A square plate of side 18 in. is hinged at *A* and *B* to a clevis. The plate rotates at the constant rate $\omega_2 = 4$ rad/s with respect to the clevis, which itself rotates at the constant rate $\omega_1 = 3$ rad/s about the *Y* axis. For the position shown, determine (*a*) the velocity of Point *C*, (*b*) the acceleration of Point *C*.

SOLUTION

Geometry. $\mathbf{r}_C = (18 \text{ in.})(\cos 20°\mathbf{i} - \sin 20°\mathbf{j})$

Let frame *Oxyz* rotate about the *y* axis with angular velocity $\mathbf{\Omega} = \omega_1\mathbf{j}$ and angular acceleration $\dot{\mathbf{\Omega}} = 0$. Then the motion relative to the frame consists of rotation with angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{k}$ and angular acceleration $\boldsymbol{\alpha}_2 = 0$ about the *z* axis.

(*a*)

$$\begin{aligned}
\mathbf{v}_{C'} &= \mathbf{\Omega} \times \mathbf{r}_C \\
&= 3\mathbf{j} \times (18\cos 20°\mathbf{i} - 18\sin 20°\mathbf{j}) \\
&= -54\cos 20°\mathbf{k} \\
\mathbf{v}_{C/F} &= \boldsymbol{\omega}_2 \times \mathbf{r}_C \\
&= 4\mathbf{k} \times (18\cos 20°\mathbf{i} - 18\sin 20°\mathbf{j}) \\
&= 72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j} \\
\mathbf{v}_C &= \mathbf{v}_{C'} + \mathbf{v}_{C/F} \\
&= 72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j} - 54\cos 20°\mathbf{k}
\end{aligned}$$

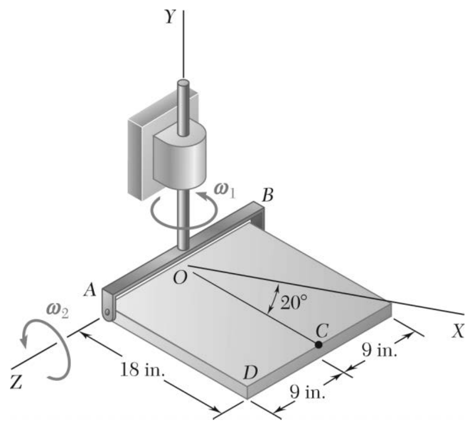
$$\mathbf{v}_C = (24.6 \text{ in./s})\mathbf{i} + (67.7 \text{ in./s})\mathbf{j} - (50.7 \text{ in./s})\mathbf{k} \blacktriangleleft$$

(*b*)

$$\begin{aligned}
\mathbf{a}_{C'} &= \mathbf{\Omega} \times \mathbf{v}_{C'} \\
&= 3\mathbf{j} \times (-54\cos 20°\mathbf{k}) \\
&= -162\cos 20°\mathbf{i} \\
\mathbf{a}_{C/F} &= \boldsymbol{\omega}_2 \times \mathbf{v}_{C/F} \\
&= 4\mathbf{k} \times (72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j}) \\
&= -288\cos 20°\mathbf{i} + 288\sin 20°\mathbf{j} \\
2\mathbf{\Omega} \times \mathbf{v}_{C/F} &= (2)(3\mathbf{j}) \times (72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j}) \\
&= -432\sin 20°\mathbf{k} \\
\mathbf{a}_C &= \mathbf{a}_{C'} + \mathbf{a}_{C/F} + 2\mathbf{\Omega} \times \mathbf{v}_{C/F} \\
&= -(162 + 288)\cos 20°\mathbf{i} + 288\sin 20°\mathbf{j} - 432\sin 20°\mathbf{k}
\end{aligned}$$

$$\mathbf{a}_C = -(423 \text{ in./s}^2)\mathbf{i} + (98.5 \text{ in./s}^2)\mathbf{j} - (147.8 \text{ in./s}^2)\mathbf{k} \blacktriangleleft$$

PROBLEM 15.221



A square plate of side 18 in. is hinged at *A* and *B* to a clevis. The plate rotates at the constant rate $\omega_2 = 4$ rad/s with respect to the clevis, which itself rotates at the constant rate $\omega_1 = 3$ rad/s about the *Y* axis. For the position shown, determine (*a*) the velocity of corner *D*, (*b*) the acceleration of corner *D*.

SOLUTION

Geometry. $\mathbf{r}_D = (18 \text{ in.})(\cos 20°\mathbf{i} - \sin 20°\mathbf{j}) + (9 \text{ in.})\mathbf{k}$

Let frame *Oxyz* rotate about the *y* axis with angular velocity $\boldsymbol{\Omega} = \omega_1\mathbf{j}$ and angular acceleration $\dot{\boldsymbol{\Omega}} = 0$. Then the motion relative to the frame consists of rotation with angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{k}$ and angular acceleration $\boldsymbol{\alpha}_2 = 0$ about the *z* axis.

(*a*)

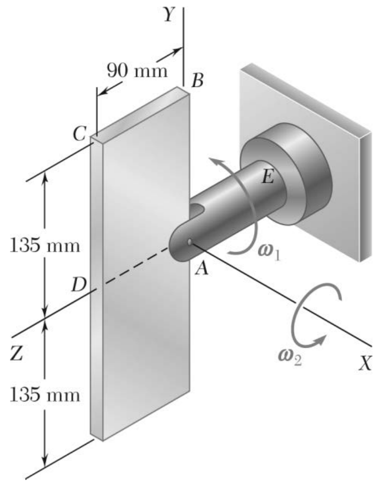
$$\mathbf{v}_{D'} = \boldsymbol{\Omega} \times \mathbf{r}_D$$
$$= 3\mathbf{j} \times (18\cos 20°\mathbf{i} - 18\sin 20°\mathbf{j} + 9\mathbf{k})$$
$$= 27\mathbf{i} - 54\cos 20°\mathbf{k}$$
$$\mathbf{v}_{D/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_D$$
$$= 4\mathbf{k} \times (18\cos 20°\mathbf{i} - 18\sin 20°\mathbf{j} + 9\mathbf{k})$$
$$= 72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j}$$
$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$$
$$= (27 + 72\sin 20°)\mathbf{i} + 72\cos 20°\mathbf{j} - 54\cos 20°\mathbf{k}$$

$$\mathbf{v}_D = (51.6 \text{ in./s})\mathbf{i} + (67.7 \text{ in./s})\mathbf{j} - (50.7 \text{ in./s})\mathbf{k} \blacktriangleleft$$

(*b*)

$$\mathbf{a}_{D'} = \boldsymbol{\Omega} \times \mathbf{v}_{D'}$$
$$= 3\mathbf{j} \times (27\mathbf{i} - 54\cos 20°\mathbf{k})$$
$$= -162\cos 20°\mathbf{i} - 81\mathbf{k}$$
$$\mathbf{a}_{D/F} = \boldsymbol{\omega}_2 \times \mathbf{v}_{D/F}$$
$$= 4\mathbf{k} \times (72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j})$$
$$= -288\cos 20°\mathbf{i} + 288\sin 20°\mathbf{j}$$
$$2\boldsymbol{\Omega} \times \mathbf{v}_{D/F} = (2)(3\mathbf{j}) \times (72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j})$$
$$= -432\sin 20°\mathbf{k}$$
$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\boldsymbol{\Omega} \times \mathbf{v}_{C/F}$$
$$= -(162 + 288)\cos 20°\mathbf{i} + 288\sin 20°\mathbf{j} - (81 + 432\sin 20°)\mathbf{k}$$

$$\mathbf{a}_D = -(423 \text{ in./s}^2)\mathbf{i} + (98.5 \text{ in./s}^2)\mathbf{j} - (229 \text{ in./s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.222

The rectangular plate shown rotates at the constant rate $\omega_2 = 12$ rad/s with respect to arm AE, which itself rotates at the constant rate $w_1 = 9$ rad/s about the Z axis. For the position shown, determine the velocity and acceleration of the point of the plate indicated.

Corner B.

SOLUTION

Geometry. With the origin at A, $\mathbf{r}_B = (0.135 \text{ m})\mathbf{j}$

Let frame $AXYZ$ rotate about the Y axis with constant angular velocity $\mathbf{\Omega} = \omega_1\mathbf{k} = (9 \text{ rad/s})\mathbf{k}$. Then the motion relative to the frame consists of rotation about the X axis with constant angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{i} = (12 \text{ rad/s})\mathbf{i}$.

Motion of coinciding Point B′.

$$\mathbf{v}_{B'} = \mathbf{\Omega} \times \mathbf{r}_B$$
$$= 9\mathbf{k} \times 0.135\mathbf{j}$$
$$= -(1.215 \text{ m/s})\mathbf{i}$$
$$\mathbf{a}_{B'} = \boldsymbol{\alpha} \times \mathbf{r}_B + \mathbf{\Omega} \times \mathbf{v}_{B'}$$
$$= 0 + 9\mathbf{k} \times (-1.215\mathbf{i})$$
$$= -(10.935 \text{ m/s}^2)\mathbf{j}$$

Motion relative to the frame.

$$\mathbf{v}_{B/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_B = 12\mathbf{i} \times 0.135\mathbf{j}$$
$$= (1.62 \text{ m/s})\mathbf{k}$$
$$\mathbf{a}_{B/F} = \boldsymbol{\alpha}_2 \times \mathbf{r}_B + \boldsymbol{\omega}_2 \times \mathbf{v}_{B/F}$$
$$= 0 + 12\mathbf{i} \times 1.62\mathbf{k}$$
$$= -(19.44 \text{ m/s}^2)\mathbf{j}$$

Velocity of Point B. $\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$

$\mathbf{v}_B = -(1.215 \text{ m/s})\mathbf{i} + (1.620 \text{ m/s})\mathbf{k}$ ◀

Coriolis acceleration. $2\mathbf{\Omega} \times \mathbf{v}_{B/F}$

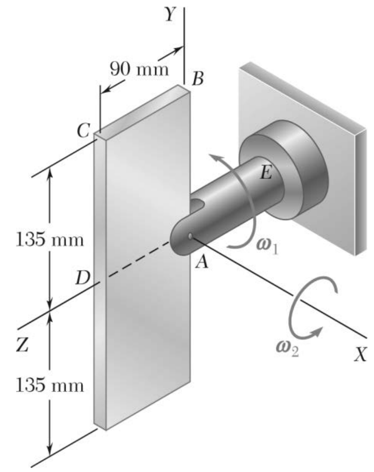
$$2\mathbf{\Omega} \times \mathbf{v}_{B/F} = (2)(9\mathbf{k}) \times 1.62\mathbf{k} = 0$$

Acceleration of Point B.

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\mathbf{\Omega} \times \mathbf{v}_{B/F}$$
$$\mathbf{a}_B = -(30.375 \text{ in./s}^2)\mathbf{j}$$

$\mathbf{a}_B = -(30.4 \text{ m/s}^2)\mathbf{j}$ ◀

PROBLEM 15.223



The rectangular plate shown rotates at the constant rate $\omega_2 = 12$ rad/s with respect to arm *AE*, which itself rotates at the constant rate $\omega_1 = 9$ rad/s about the *Z* axis. For the position shown, determine the velocity and acceleration of the point of the plate indicated.

Corner *C*.

SOLUTION

Geometry. With the origin at *A*, $\mathbf{r}_C = (0.135 \text{ m})\mathbf{j} + (0.09 \text{ m})\mathbf{k}$

Let frame *AXYZ* rotate about the *Y* axis with constant angular velocity $\boldsymbol{\Omega} = \omega_1\mathbf{k} = (9 \text{ rad/s})\mathbf{k}$. Then the motion relative to the frame consists of rotation about the *X* axis with constant angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{i} = (12 \text{ rad/s})\mathbf{i}$.

Motion of coinciding Point C' in the frame.

$$\begin{aligned}\mathbf{v}_{C'} &= \boldsymbol{\Omega} \times \mathbf{r}_C \\ &= 9\mathbf{k} \times (0.135\mathbf{j} + 0.09\mathbf{k}) \\ &= -(1.215 \text{ m/s})\mathbf{i} \\ \mathbf{a}_{C'} &= \boldsymbol{\alpha} \times \mathbf{r}_C + \boldsymbol{\Omega} \times \mathbf{v}_{C'} \\ &= 0 + 9\mathbf{k} \times (1.215\mathbf{i}) \\ &= -(10.935 \text{ m/s}^2)\mathbf{j}\end{aligned}$$

Motion relative to the frame.

$$\begin{aligned}\mathbf{v}_{C/F} &= \boldsymbol{\omega}_2 \times \mathbf{r}_C \\ &= 12\mathbf{i} \times (0.135\mathbf{j} + 0.09\mathbf{k}) \\ &= -(1.08 \text{ m/s})\mathbf{j} + (1.62 \text{ m/s})\mathbf{k} \\ \mathbf{a}_{C/F} &= \boldsymbol{\alpha}_2 \times \mathbf{r}_C + \boldsymbol{\omega}_2 \times \mathbf{v}_{C/F} \\ &= 0 + 12\mathbf{i} \times (-1.08\mathbf{j} + 1.62\mathbf{k}) \\ &= -(19.44 \text{ m/s}^2)\mathbf{j} - (12.96 \text{ m/s}^2)\mathbf{k}\end{aligned}$$

Velocity of Point C. $\mathbf{v}_C = \mathbf{v}_{C'} + \mathbf{v}_{C/F}$

$$\mathbf{v}_C = -(1.215 \text{ m/s})\mathbf{i} - (1.080 \text{ m/s})\mathbf{j} + (1.620 \text{ m/s})\mathbf{k} \blacktriangleleft$$

PROBLEM 15.223 (Continued)

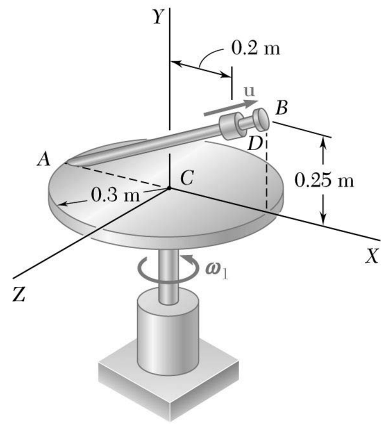
Coriolis acceleration. $2\boldsymbol{\Omega}\times\mathbf{v}_{C/F}$

$$2\boldsymbol{\Omega}\times\mathbf{v}_{C/F} = (2)(9\mathbf{k})\times(-1.08\mathbf{j}+1.62\mathbf{k})$$
$$= (19.44\text{ m/s}^2)\mathbf{i}$$

Acceleration of Point C. $\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/F} + 2\boldsymbol{\Omega}\times\mathbf{v}_{C/F}$

$$\mathbf{a}_C = (19.44\text{ m/s}^2)\mathbf{i} - (30.375\text{ m/s}^2)\mathbf{j} - (12.96\text{ m/s}^2)\mathbf{k}$$

$$\mathbf{a}_C = (19.44\text{ m/s}^2)\mathbf{i} - (30.4\text{ m/s}^2)\mathbf{j} - (12.96\text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.224

Rod AB is welded to the 0.3-m-radius plate, which rotates at the constant rate $\omega_1 = 6$ rad/s. Knowing that collar D moves toward end B of the rod at a constant speed $u = 1.3$ m, determine, for the position shown, (*a*) the velocity of D, (*b*) the acceleration of D.

SOLUTION

Geometry.

$$\mathbf{r}_{B/A} = (0.6\text{ m})\mathbf{i} + (0.25\text{ m})\mathbf{j} \qquad \mathbf{r}_{C/A} = (0.3\text{ m})\mathbf{i}$$

$$\mathbf{r}_{D/A} = \frac{0.5}{0.6}\mathbf{r}_{B/A}$$

$$\mathbf{r}_{D/C} = \mathbf{r}_{D/A} - \mathbf{r}_{C/A}$$

$$= (0.2\text{ m})\mathbf{i} + (0.20833\text{ m})\mathbf{j}$$

$$l_{AB} = \sqrt{0.6^2 + 0.25^2}$$

$$= 0.65\text{ m}$$

Unit vector along AB:

$$\boldsymbol{\lambda}_{AB} = \frac{\mathbf{r}_{B/A}}{l_{AB}}$$

$$= \frac{12}{13}\mathbf{i} + \frac{5}{12}\mathbf{j}$$

Let $Oxyz$ be a frame of reference currently coinciding with $OXYZ$, but rotating with angular velocity

$$\boldsymbol{\Omega} = \omega_1\mathbf{j} = (6\text{ rad/s})\mathbf{j}$$

(*a*) *Velocity of D.*

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/AB}$$

$$\mathbf{v}_{D'} = \boldsymbol{\Omega} \times \mathbf{r}_{D/C}$$

$$= 6\mathbf{j} \times (0.2\mathbf{i} + 0.20833\mathbf{j})$$

$$= -(1.2\text{ m/s})\mathbf{k}$$

$$\mathbf{v}_{D/AB} = u\boldsymbol{\lambda}_{AB}$$

$$= 1.3\left(\frac{12}{13}\mathbf{i} + \frac{5}{13}\mathbf{j}\right)$$

$$= (1.2\text{ m/s})\mathbf{i} + (0.5\text{ m/s})\mathbf{j}$$

$$\mathbf{v}_D = (1.2\text{ m/s})\mathbf{i} + (0.5\text{ m/s})\mathbf{j} - (1.2\text{ m/s})\mathbf{k} \blacktriangleleft$$

PROBLEM 15.224 (Continued)

(*b*) *Acceleration of D.*

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/AB} + 2\mathbf{\Omega}\times\mathbf{v}_{D/AB}$$

$$\mathbf{a}_{D'} = \boldsymbol{\omega}_1 \times (\boldsymbol{\omega}_1 \times \mathbf{r}_{D/C})$$

$$= 6\mathbf{j}\times(6\mathbf{j}\times(0.2\mathbf{i} + 0.20833\mathbf{j}))$$

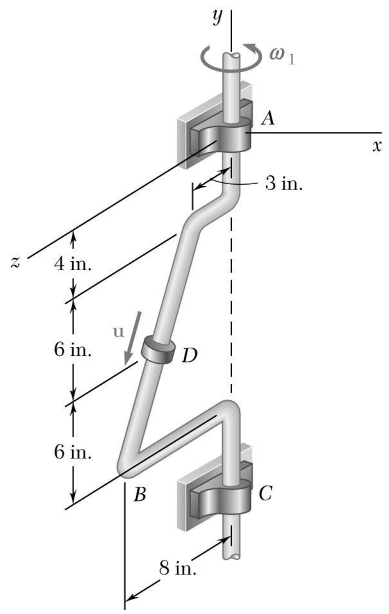
$$= -(7.2 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_{D/AB} = 0$$

$$2\mathbf{\Omega}\times\mathbf{v}_{D/F} = (2)(6\mathbf{j})\times((1.2)\mathbf{i} + (0.5)\mathbf{j})$$

$$= -(14.4 \text{ m/s}^2)\mathbf{k}$$

$$\mathbf{a}_D = -(7.2 \text{ m/s}^2)\mathbf{i} - (14.4 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$

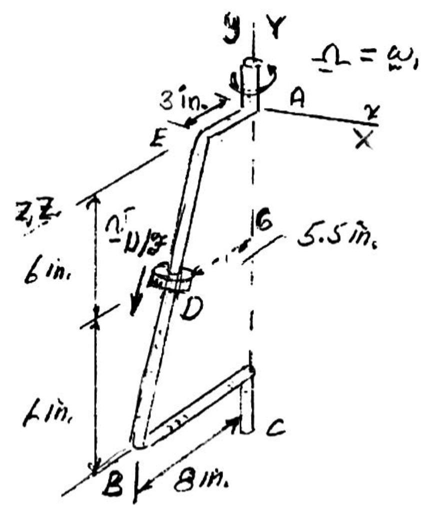


PROBLEM 15.225

The bent rod ABC rotates at the constant rate $\omega_1 = 4$ rad/s. Knowing that collar D moves downward along the rod at a constant relative speed $u = 65$ in./s, determine, for the position shown, (*a*) the velocity of D, (*b*) the acceleration of D.

SOLUTION

Units: inches, in./s, in./s^2



Geometry.

$$\mathbf{r}_E = 3\mathbf{k}$$

$$\mathbf{r}_B = -12\mathbf{j} + 8\mathbf{k}$$

$$\mathbf{r}_{B/E} = -12\mathbf{j} + 5\mathbf{k}$$

$$\mathbf{r}_D = \frac{1}{2}(\mathbf{r}_E + \mathbf{r}_B) = -6\mathbf{j} + 5.5\mathbf{k}$$

$$l_{EB} = \sqrt{12^2 + 5^2} = 13$$

Unit vector along EB:

$$\boldsymbol{\lambda} = \frac{\mathbf{r}_{B/E}}{l_{EB}} = -\frac{12}{13}\mathbf{j} + \frac{5}{13}\mathbf{k}$$

Use a rotating frame of reference that rotates with angular velocity

$$\boldsymbol{\Omega} = \boldsymbol{\omega}_1 = (4 \text{ rad/s})\mathbf{j}$$

Motion of Point D' in the frame currently at D.

$$\mathbf{v}_{D'} = \boldsymbol{\omega}_1 \times \mathbf{r}_D = 4\mathbf{j} \times (-6\mathbf{j} + 5.5\mathbf{k})$$
$$= (22 \text{ in./s})\mathbf{i}$$

$$\mathbf{a}_{D'} = \dot{\boldsymbol{\omega}}_1\mathbf{j} \times \mathbf{r}_D + \boldsymbol{\omega}_1 \times \mathbf{v}_{D'}$$
$$= 0 + (4\mathbf{j}) \times (22\mathbf{i})$$
$$= -(88 \text{ in./s}^2)\mathbf{k}$$

PROBLEM 15.225 (Continued)

Motion of collar D relative to the frame.

$$\mathbf{v}_{D/F} = u\boldsymbol{\lambda} = (65 \text{ in./s})\left(-\frac{12}{13}\mathbf{j} + \frac{5}{13}\mathbf{k}\right)$$

$$= -(60 \text{ in./s})\mathbf{j} + (25 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{D/F} = 0 \qquad \text{(Constant speed on straight path)}$$

(*a*) *Velocity of D.*

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$$

$$\mathbf{v}_D = 22\mathbf{i} - 60\mathbf{j} + 25\mathbf{k}$$

$$\mathbf{v}_D = (22 \text{ in./s})\mathbf{i} - (60 \text{ in./s})\mathbf{j} + (25 \text{ in./s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration.

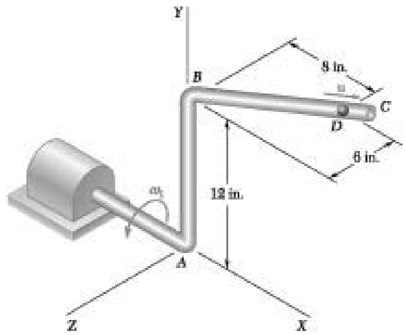
$$2\boldsymbol{\Omega} \times \mathbf{v}_{D/F} = (2)(4\mathbf{j}) \times (-60\mathbf{j} + 25\mathbf{k})$$

$$= (200 \text{ in./s}^2)\mathbf{i}$$

(*b*) *Acceleration of Point D.*

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\boldsymbol{\Omega} \times \mathbf{v}_D$$

$$\mathbf{a}_D = (200 \text{ in./s}^2)\mathbf{i} - (88 \text{ in./s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.226

The bent pipe shown rotates at the constant rate $\omega_1 = 10$ rad/s. Knowing that a ball bearing D moves in portion BC of the pipe toward end C at a constant relative speed $u = 2$ ft/s, determine at the instant shown (*a*) the velocity of D, (*b*) the acceleration of D.

SOLUTION

With the origin at Point A, $\mathbf{r}_D = (8\text{ in.})\mathbf{i} + (12\text{ in.})\mathbf{j} - (6\text{ in.})\mathbf{k}$

$$\mathbf{r}_{C/B} = (8\text{ in.})\mathbf{i} - (6\text{ in.})\mathbf{k},$$

$$l_{BC} = \sqrt{8^2 + 6^2} = 10\text{ in.}$$

Let the frame $Axyz$ rotate with angular velocity $\boldsymbol{\Omega} = \omega_1\mathbf{i} = (10\text{ rad/s})\mathbf{i}$

(*a*) *Velocity of D.*

$$\begin{aligned}\mathbf{v}_{D'} &= \boldsymbol{\Omega} \times \mathbf{r}_D \\ &= 10\mathbf{i} \times (8\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) \\ &= (60\text{ in./s})\mathbf{j} + (120\text{ in./s})\mathbf{k}\end{aligned}$$

$$u = 2\text{ ft/s} = 24\text{ in./s},$$

$$\begin{aligned}\mathbf{v}_{D/F} &= \frac{24}{10}(8\mathbf{i} - 6\mathbf{k}) \\ &= (19.2\text{ in./s})\mathbf{i} - (14.4\text{ in./s})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{v}_D &= \mathbf{v}_{D'} + \mathbf{v}_{D/F} \\ &= (19.2\text{ in./s})\mathbf{i} + (60\text{ in./s})\mathbf{j} + (105.6\text{ in./s})\mathbf{k}\end{aligned}$$

$$\mathbf{v}_D = (1.600\text{ ft/s})\mathbf{i} + (5.00\text{ ft/s})\mathbf{j} + (8.80\text{ ft/s})\mathbf{k} \blacktriangleleft$$

(*b*) *Acceleration of D.*

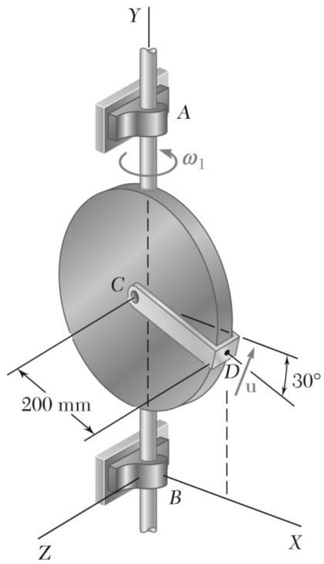
$$\mathbf{a}_{D'} = \boldsymbol{\Omega} \times \mathbf{v}_{D'} = 10\mathbf{i} \times (60\mathbf{j} + 120\mathbf{k}) = -(1200\text{ in./s}^2)\mathbf{j} + (600\text{ in./s}^2)\mathbf{k}$$

$$\mathbf{a}_{D/F} = 0$$

$$2\boldsymbol{\Omega} \times \mathbf{v}_{D/F} = (2)(10\mathbf{i}) \times (19.2\mathbf{i} - 14.4\mathbf{k}) = (288\text{ in./s}^2)\mathbf{j}$$

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\boldsymbol{\Omega} \times \mathbf{v}_{D/F} = -(912\text{ in./s}^2)\mathbf{j} + (600\text{ in./s}^2)\mathbf{k}$$

$$\mathbf{a}_D = -(76.0\text{ ft/s}^2)\mathbf{j} + (50.0\text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.227

The circular plate shown rotates about its vertical diameter at the constant rate $\omega_1 = 10$ rad/s. Knowing that in the position shown the disk lies in the *XY* plane and Point *D* of strap *CD* moves upward at a constant relative speed $u = 1.5$ m/s, determine (*a*) the velocity of *D*, (*b*) the acceleration of *D*.

SOLUTION

Geometry.

$$\mathbf{r}_{D/C} = (0.2\text{ m})(\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$$

$$= (0.1\sqrt{3}\text{ m})\mathbf{i} - (0.1\text{ m})\mathbf{j}$$

Let frame *Cxyz*, which at the instant shown coincides with *CXYZ*, rotate with angular velocity

$$\boldsymbol{\Omega} = \omega_1\mathbf{j} = (10\text{ rad/s})\mathbf{j}.$$

Motion of coinciding Point D' in the frame.

$$\mathbf{v}_{D'} = \boldsymbol{\Omega} \times \mathbf{r}_{D/C}$$

$$= 10\mathbf{j} \times (0.1\sqrt{3}\mathbf{i} + 0.1\mathbf{j})$$

$$= -(\sqrt{3}\text{ m/s})\mathbf{k}$$

$$\mathbf{a}_{D'} = -\Omega^2(r\cos 30°)\mathbf{i}$$

$$= -10^2(0.1\sqrt{3})\mathbf{i}$$

$$= -(10\sqrt{3}\text{ m/s}^2)\mathbf{i}$$

Motion of Point D relative to the frame. $u = 1.5$ m/s

$$\mathbf{v}_{D/F} = u(\sin 30°\mathbf{i} + \cos 30°\mathbf{j})$$

$$= (0.75\text{ m/s})\mathbf{i} + (0.75\sqrt{3}\text{ m/s})\mathbf{j}$$

$$\mathbf{a}_{D/F} = \frac{u^2}{\rho}\cdot(-\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$$

$$= \frac{1.5^2}{0.2}(-\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$$

$$= -(5.625\sqrt{3}\text{ m/s}^2)\mathbf{i} + (5.625\text{ m/s}^2)\mathbf{j}$$

PROBLEM 15.227 (Continued)

(*a*) *Velocity of Point D.*

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$$

$$\mathbf{v}_D = (0.75 \text{ m/s})\mathbf{i} + (0.75\sqrt{3} \text{ m/s})\mathbf{j} - (\sqrt{3} \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_D = (0.750 \text{ m/s})\mathbf{i} + (1.299 \text{ m/s})\mathbf{j} - (1.732 \text{ m/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration. $2\mathbf{\Omega} \times \mathbf{v}_{D/F}$

$$2\mathbf{\Omega} \times \mathbf{v}_{D/F} = (2)(10\mathbf{j}) \times (0.75\mathbf{i} + 0.75\sqrt{3}\mathbf{j})$$

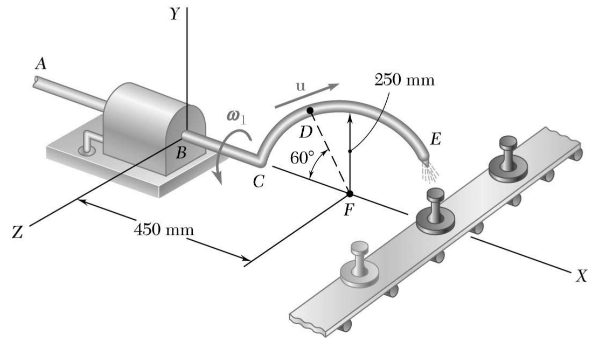
$$= -(15 \text{ m/s}^2)\mathbf{k}$$

(*b*) *Acceleration of Point D.*

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\mathbf{\Omega} \times \mathbf{v}_{D/F}$$

$$\mathbf{a}_D = -(15.625\sqrt{3} \text{ m/s}^2)\mathbf{i} + (5.625 \text{ m/s}^2)\mathbf{j} - (15 \text{ m/s}^2)\mathbf{k}$$

$$\mathbf{a}_D = (27.1 \text{ m/s}^2)\mathbf{i} + (5.63 \text{ m/s}^2)\mathbf{j} - (15.00 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.228

Manufactured items are spray-painted as they pass through the automated work station shown. Knowing that the bent pipe *ACE* rotates at the constant rate $\omega_1 = 0.4$ rad/s and that at Point *D* the paint moves through the pipe at a constant relative speed $u = 150$ mm/s, determine, for the position shown, (*a*) the velocity of the paint at *D*, (*b*) the acceleration of the paint at *D*.

SOLUTION

Use a frame of reference *CE* rotating about the *x*-axis with angular velocity

$$\mathbf{\Omega} = \boldsymbol{\omega}_1 = (0.4 \text{ rad/s})\mathbf{i}$$

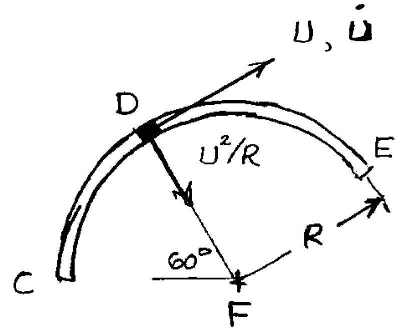
Geometry:

$$\mathbf{r}_{D/F} = (250 \text{ mm})(-\cos 60°\mathbf{i} + \sin 60°\mathbf{j})$$
$$= -(125 \text{ mm})\mathbf{i} + (216.51 \text{ mm})\mathbf{j}$$

Motion of Point *D′* fixed in the frame *CE* but coinciding with Point *D* at the instant considered.

$$\mathbf{v}_{D'} = \Omega \times \mathbf{r}_{D/F} = (0.4\mathbf{i}) \times (-125\mathbf{i} + 216.51\mathbf{j}) = (86.603 \text{ mm/s})\mathbf{k}$$
$$\mathbf{a}_{D'} = \dot{\Omega}\mathbf{i} \times \mathbf{r}_{D/F} + \mathbf{\Omega} \times \mathbf{v}_{D'}$$
$$= 0 + (0.4\mathbf{i}) \times (86.603\mathbf{k}) = -(34.641 \text{ mm/s}^2)\mathbf{j}$$

Motion of *D* relative to the frame *CE*.



$$\mathbf{v}_{D/CE} = u(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = (150 \text{ mm/s})(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$$
$$= (129.90 \text{ mm/s})\mathbf{i} + (75 \text{ mm/s})\mathbf{j}$$
$$\mathbf{a}_{D/CE} = \dot{u}(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) + \frac{u^2}{R}(\sin 30°\mathbf{i} - \cos 30°\mathbf{j})$$
$$= 0 + \frac{(150)^2}{250}(\sin 30°\mathbf{i} - \cos 30°\mathbf{j})$$
$$= (45 \text{ mm/s}^2)\mathbf{i} - (77.94 \text{ mm/s}^2)\mathbf{j}$$

(*a*) *Velocity of D.* $\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/CE}$

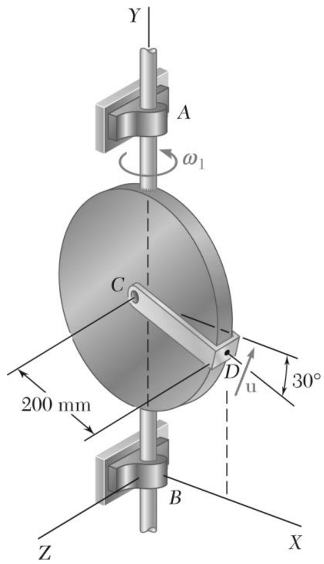
$$\mathbf{v}_D = (129.9 \text{ mm/s})\mathbf{i} + (75.0 \text{ mm/s})\mathbf{j} + (86.6 \text{ mm/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration $2\mathbf{\Omega} \times \mathbf{v}_{D/CE}$

$$2\mathbf{\Omega} \times \mathbf{v}_{D/CE} = (2)(0.4\mathbf{i}) \times (129.90\mathbf{i} + 75\mathbf{j}) = (60 \text{ mm/s}^2)\mathbf{k}$$

(*b*) *Acceleration of D.* $\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/CE} + 2\mathbf{\Omega} \times \mathbf{v}_{D/CE}$

$$\mathbf{a}_D = (45.0 \text{ mm/s}^2)\mathbf{i} - (112.6 \text{ mm/s}^2)\mathbf{j} + (60.0 \text{ mm/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.229

Solve Problem 15.227, assuming that at the instant shown the angular velocity $\boldsymbol{\omega}_1$ of the plate is 10 rad/s and is decreasing at the rate of 25 rad/s^2, while the relative speed u of Point D of strap CD is 1.5 m/s and is decreasing at the rate of 3 m/s^2.

PROBLEM 15.227 The circular plate shown rotates about its vertical diameter at the constant rate $\omega_1 = 10$ rad/s. Knowing that in the position shown the disk lies in the XY plane and Point D of strap CD moves upward at a constant relative speed $u = 1.5$ m/s, determine (a) the velocity of D, (b) the acceleration of D.

SOLUTION

Geometry.

$$\mathbf{r}_{D/C} = (0.2\text{ m})(\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$$
$$= (0.1\sqrt{3}\text{ m})\mathbf{i} - (0.1\text{ m})\mathbf{j}$$

Let frame $Cxyz$, which at the instant shown coincides with $CXYZ$, rotate with angular velocity and angular acceleration

$$\boldsymbol{\Omega} = \omega_1\mathbf{j} = (10\text{ rad/s})\mathbf{j}.$$

$$\dot{\boldsymbol{\Omega}} = -(25\text{ rad/s}^2)\mathbf{j}.$$

Motion of coinciding Point D' in the frame

$$\mathbf{v}_{D'} = \boldsymbol{\Omega} \times \mathbf{r}_{D/C}$$
$$= 10\mathbf{j} \times (0.1\sqrt{3}\mathbf{i} + 0.1\mathbf{j}) = -(\sqrt{3}\text{ m/s})\mathbf{k}$$
$$\mathbf{a}_{D'} = -\Omega^2(r\cos 30°)\mathbf{i} + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{D/C}$$
$$= -10^2(0.1\sqrt{3})\mathbf{i} - 25\mathbf{j} \times (0.1\sqrt{3}\mathbf{i} + 0.1\mathbf{j})$$
$$= -(10\sqrt{3}\text{ m/s}^2)\mathbf{i} + (2.5\sqrt{3}\text{ m/s}^2)\mathbf{k}$$

Motion of Point D relative to the frame. $\quad u = 1.5\text{ m/s} \qquad \dot{u} = -3\text{ m/s}^2$

$$\mathbf{v}_{D/F} = u(\sin 30°\mathbf{i} + \cos 30°\mathbf{j})$$
$$= (0.75\text{ m/s})\mathbf{i} + (0.75\sqrt{3}\text{ m/s})\mathbf{j}$$

PROBLEM 15.229 (Continued)

$$\mathbf{a}_{D/F} = \frac{u^2}{\rho}\cdot(-\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) + \dot{u}(\sin 30°\mathbf{i} + \cos 30°\mathbf{j})$$

$$= \frac{1.5^2}{0.2}(-\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) - 3(\sin 30°\mathbf{i} + \cos 30°\mathbf{j})$$

$$= -(5.625\sqrt{3}\text{ m/s}^2)\mathbf{i} + (5.625\text{ m/s}^2)\mathbf{j} - (1.5\text{ m/s}^2)\mathbf{i} - (1.5\sqrt{3}\text{ m/s}^2)\mathbf{j}$$

(*a*) *Velocity of Point D.*

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$$

$$\mathbf{v}_D = (0.75\text{ m/s})\mathbf{i} + (0.75\sqrt{3}\text{ m/s})\mathbf{j} - (\sqrt{3}\text{ m/s})\mathbf{k}$$

$$\mathbf{v}_D = (0.750\text{ m/s})\mathbf{i} + (1.299\text{ m/s})\mathbf{j} - (1.732\text{ m/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration. $2\mathbf{\Omega}\times\mathbf{v}_{D/F}$

$$2\mathbf{\Omega}\times\mathbf{v}_{D/F} = (2)(10\mathbf{j})\times(0.75\mathbf{i} + 0.75\sqrt{3}\mathbf{j})$$

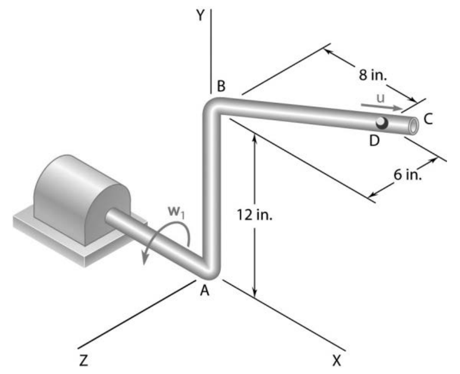
$$= -(15\text{ m/s}^2)\mathbf{k}$$

(*b*) *Acceleration of Point D.*

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\mathbf{\Omega}\times\mathbf{v}_{D/F}$$

$$\mathbf{a}_D = -(10\sqrt{3}\text{ m/s}^2)\mathbf{i} + (2.5\sqrt{3}\text{ m/s}^2)\mathbf{k}$$
$$-(5.625\sqrt{3}\text{ m/s}^2)\mathbf{i} + (5.625\text{ m/s}^2)\mathbf{j}$$
$$-(1.5\text{ m/s}^2)\mathbf{i} - (1.5\sqrt{3}\text{ m/s}^2)\mathbf{j} - (15\text{ m/s}^2)\mathbf{k}$$

$$\mathbf{a}_D = -(28.6\text{ m/s}^2)\mathbf{i} + (3.03\text{ m/s}^2)\mathbf{j} - (10.67\text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.230

Solve Problem 15.226, assuming that at the instant shown the angular velocity ω_1 of the pipe is 10 rad/s and is decreasing at the rate of 15 rad/s^2, while the relative speed *u* of the ball bearing is 2 ft/s and is increasing at the rate of 10 ft/s^2.

PROBLEM 15.226 The bent pipe shown rotates at the constant rate $\omega_1 = 10$ rad/s. Knowing that a ball bearing *D* moves in portion *BC* of the pipe toward end *C* at a constant relative speed $u = 2$ ft/s, determine at the instant shown (*a*) the velocity of *D*, (*b*) the acceleration of *D*.

SOLUTION

With the origin at Point *A*, $\mathbf{r}_D = (8\text{ in.})\mathbf{i} + (12\text{ in.})\mathbf{j} - (6\text{ in.})\mathbf{k}$

$$\mathbf{r}_{C/B} = (8\text{ in.})\mathbf{i} - (6\text{ in.})\mathbf{k},$$

$$l_{BC} = \sqrt{8^2 + 6^2} = 10\text{ in.}$$

Let the frame $Axyz$ rotate with angular velocity $\mathbf{\Omega} = \omega_1\mathbf{i} = (10\text{ rad/s})\mathbf{i}$ and angular acceleration

$\dot{\mathbf{\Omega}} = \dot{\omega}_1\mathbf{i} = -(15\text{ rad/s}^2)\mathbf{i}.$

(*a*) *Velocity of D.*

$$\begin{aligned}\mathbf{v}_{D'} &= \mathbf{\Omega} \times \mathbf{r}_D \\ &= 10\mathbf{i} \times (8\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) \\ &= (60\text{ in./s})\mathbf{j} + (120\text{ in./s})\mathbf{k}\end{aligned}$$

$$u = 2\text{ ft/s} = 24\text{ in./s},$$

$$\begin{aligned}\mathbf{u} &= \frac{24}{10}(8\mathbf{i} - 6\mathbf{k}) \\ &= (19.2\text{ in./s})\mathbf{i} - (14.4\text{ in./s})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{v}_D &= \mathbf{v}_{D'} + \mathbf{u} \\ &= (19.2\text{ in./s})\mathbf{i} + (60\text{ in./s})\mathbf{j} + (105.6\text{ in./s})\mathbf{k}\end{aligned}$$

$$\mathbf{v}_D = (1.600\text{ ft/s})\mathbf{i} + (5.00\text{ ft/s})\mathbf{j} + (8.80\text{ ft/s})\mathbf{k} \blacktriangleleft$$

(*b*) *Acceleration of D.*

$$\begin{aligned}\mathbf{a}_{D'} &= \dot{\mathbf{\Omega}} \times \mathbf{r}_D + \mathbf{\Omega} \times \mathbf{v}_{D'} \\ &= -15\mathbf{i} \times (8\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) + 10\mathbf{i} \times (60\mathbf{j} + 120\mathbf{k}) \\ &= -90\mathbf{j} - 180\mathbf{k} - 1200\mathbf{j} + 600\mathbf{k} \\ &= -(1290\text{ in./s}^2)\mathbf{j} + (420\text{ in./s}^2)\mathbf{k}\end{aligned}$$

PROBLEM 15.230 (Continued)

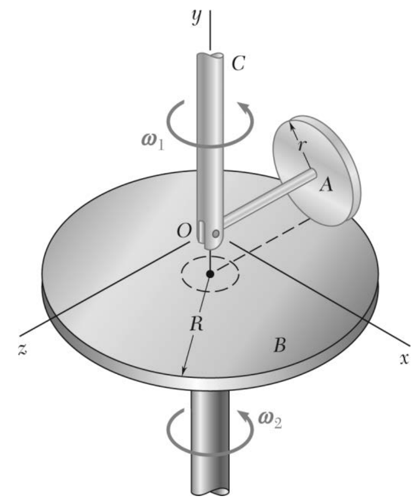
$$a_{\text{rel}} = 10 \text{ ft/s}^2 = 120 \text{ in./s}^2$$

$$\mathbf{a}_{\text{rel}} = \frac{120}{10}(8\mathbf{i} - 6\mathbf{k}) = (96 \text{ in./s}^2)\mathbf{i} - (72 \text{ in./s}^2)\mathbf{k}$$

$$2\boldsymbol{\Omega} \times \mathbf{u} = (2)(10\mathbf{i}) \times (19.2\mathbf{i} - 14.4\mathbf{k}) = (288 \text{ in./s}^2)\mathbf{j}$$

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{\text{rel}} + 2\boldsymbol{\Omega} \times \mathbf{u} = (96 \text{ in./s})\mathbf{i} - (1002 \text{ in./s})\mathbf{j} + (348 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_D = (8.00 \text{ ft/s}^2)\mathbf{i} - (83.5 \text{ ft/s}^2)\mathbf{j} + (29.0 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$

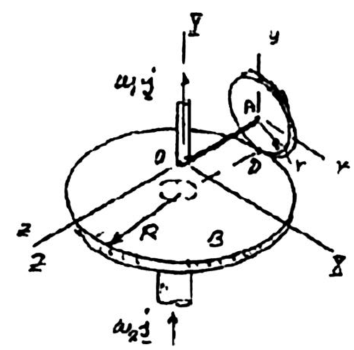


PROBLEM 15.231

Using the method of Section 15.14, solve Problem 15.192.

PROBLEM 15.192 In the system shown, disk *A* is free to rotate about the horizontal rod *OA*. Assuming that shaft *OC* and disk *B* rotate with constant angular velocities ω_1 and ω_2, respectively, both counterclockwise, determine (*a*) the angular velocity of *A*, (*b*) the angular acceleration of disk *A*.

SOLUTION



Moving frame *Axyz* rotates with angular velocity

$$\mathbf{\Omega} = \omega_1 \mathbf{j}$$

$$\boldsymbol{\omega}_{\text{disk}/F} = \omega_x \mathbf{i} + \omega_z \mathbf{k}$$

$$\mathbf{r}_{D/A} = -r\mathbf{j} - R\mathbf{k}$$

(*a*) Total angular velocity of disk *A*:

$$\boldsymbol{\omega} = \omega_1 \mathbf{j} + \boldsymbol{\omega}_{\text{disk}/F}$$

$$= \omega_x \mathbf{i} + \omega_1 \mathbf{j} + \omega_z \mathbf{k} \qquad (1)$$

Denote by *D* point of contact of disks

Consider disk *B*: $\mathbf{v}_D = \omega_2 \mathbf{j} \times (-R\mathbf{k}) = -R\omega_2 \mathbf{i} \qquad (2)$

Consider system *OC*, *OA* and disk *A*.

$$\mathbf{v}_{D'} = \mathbf{\Omega} \times \mathbf{r}_{B/A}$$
$$= \omega_1 \mathbf{j} \times (-r\mathbf{j} - R\mathbf{k})$$
$$= -R\omega_1 \mathbf{i}$$
$$\mathbf{v}_{B/F} = \omega_{\text{disk}/F} \times \mathbf{r}_{D/A}$$
$$= (\omega_2 \mathbf{i} + \omega_z \mathbf{k}) \times (-r\mathbf{j} - R\mathbf{k})$$
$$= -r\omega_x \mathbf{k} + R\omega_x \mathbf{j} + r\omega_z \mathbf{i}$$
$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$$
$$= -R\omega_1 \mathbf{i} - r\omega_x \mathbf{k} + R\omega_x \mathbf{j} + r\omega_2 \mathbf{i} \qquad (3)$$

Equate $\mathbf{v}_D = \mathbf{v}_D$ from Eq. (2) and Eq. (3).

$$-R\omega_2 \mathbf{i} = -R\omega_1 + r\omega_2 + R\omega_x \mathbf{j} - r\omega_x \mathbf{k}$$

PROBLEM 15.231 (Continued)

Coefficient of **j**: $0 = R\omega_x \rightarrow \omega_x = 0$

Coefficient of **i**: $-R\omega_2 = -R\omega_1 + r\omega_z;$

$$\omega_z = \frac{R}{r}(\omega_1 - \omega_2)$$

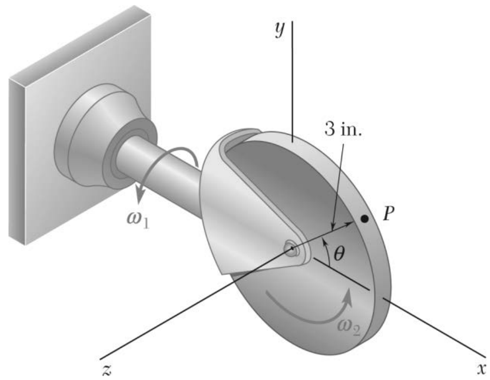
Eq. (3): $\boldsymbol{\omega} = \omega_1\mathbf{j} + \frac{R}{r}(\omega_1 - \omega_2)\mathbf{k}$ ◀

(*b*) Disk *A* rotates about *y* axis at rate $\boldsymbol{\omega}_1$.

$$\boldsymbol{\alpha} = \boldsymbol{\omega}_1 \times \boldsymbol{\omega}$$

$$= \omega_1\mathbf{j} \times \left[\omega\mathbf{j} + \frac{R}{r}(\omega_1 - \omega_2)\right]\mathbf{k}$$

$\boldsymbol{\alpha} = \omega_1(\omega_1 - \omega_2)\frac{R}{r}\mathbf{j}$ ◀



PROBLEM 15.232

Using the method of Section 15.14, solve Problem 15.196.

PROBLEM 15.196 A 3-in.-radius disk spins at the constant rate $\omega_2 = 4$ rad/s about an axis held by a housing attached to a horizontal rod that rotates at the constant rate $\omega_1 = 5$ rad/s. Knowing that $\theta = 30°$, determine the acceleration of Point *P* on the rim of the disk.

SOLUTION

Let frame *Oxyz* rotate with angular velocity

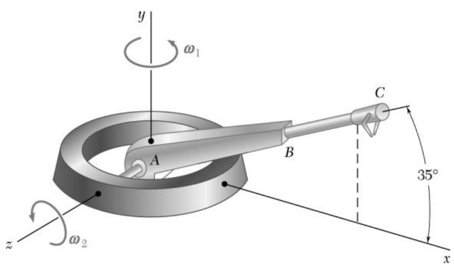
$$\mathbf{\Omega} = \omega_1 \mathbf{i} = (5 \text{ rad/s})\mathbf{i}$$

The motion relative to the frame is the spin

$$\omega_2 \mathbf{k} = (4 \text{ rad/s})\mathbf{k}$$
$$\theta = 30°$$
$$\mathbf{r}_P = (3 \text{ in.})(\cos 30° \mathbf{i} + \sin 30° \mathbf{j})$$
$$\mathbf{v}_{P'} = \omega_1 \mathbf{i} \times \mathbf{r}_P$$
$$= 5\mathbf{i} \times (3\cos 30° \mathbf{i} + 3\sin 30° \mathbf{j})$$
$$= (7.5 \text{ in./s})\mathbf{k}$$
$$\mathbf{v}_{P/F} = \omega_2 \mathbf{k} \times \mathbf{r}_P$$
$$= 4\mathbf{k} \times (3\cos 30° \mathbf{i} + 3\sin 30° \mathbf{j})$$
$$= -(6 \text{ in./s})\mathbf{i} + (10.392 \text{ in./s})\mathbf{j}$$
$$\mathbf{a}_{P'} = \dot{\omega}_1 \mathbf{i} \times \mathbf{r}_P + \omega_1 \mathbf{i} \times \mathbf{v}_{P'}$$
$$= 0 + 5\mathbf{i} \times 7.5\mathbf{k}$$
$$= -(37.5 \text{ in./s}^2)\mathbf{j}$$
$$\mathbf{a}_{P/F} = \dot{\omega}_2 \mathbf{k} \times \mathbf{r}_P$$
$$= \omega_2 \mathbf{k} \times \mathbf{v}_{P/F}$$
$$= 0 + 4\mathbf{k} \times (-6\mathbf{i} + 10.392\mathbf{j})$$
$$= -(41.569 \text{ in./s}^2)\mathbf{i} - (24 \text{ in./s}^2)\mathbf{j}$$
$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{P/F}$$
$$= (2)(5\mathbf{i}) \times (-6\mathbf{i} + 10.392\mathbf{j})$$
$$= (103.92 \text{ in./s}^2)\mathbf{k}$$

Acceleration at Point P. $\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + \mathbf{a}_c$

$$\mathbf{a}_P = -(41.6 \text{ in./s}^2)\mathbf{i} - (61.5 \text{ in./s}^2)\mathbf{j} + (103.9 \text{ in./s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.233

Using the method of Section 15.14, solve Problem 15.198.

PROBLEM 15.198 At the instant shown, the robotic arm *ABC* is being rotated simultaneously at the constant rate $\omega_1 = 0.15$ rad/s about the *y* axis, and at the constant rate $\omega_2 = 0.25$ rad/s about the *z* axis. Knowing that the length of arm *ABC* is 1 m, determine (*a*) the angular acceleration of the arm, (*b*) the velocity of Point *C*, (*c*) the acceleration of Point *C*.

SOLUTION

Geometry: Dimensions in meters.

$$\mathbf{r}_{C/A} = (1.0\cos 35°)\mathbf{i} + (1.0\sin 35°)\mathbf{j} = 0.81915\mathbf{i} + 0.57358\mathbf{j}$$

Angular velocities:

$$\boldsymbol{\omega}_1 = \omega_1\mathbf{j} = (0.15 \text{ rad/s})\mathbf{j} \qquad (\dot{\omega}_1 = 0)$$

$$\boldsymbol{\omega}_2 = \omega_2\mathbf{k} = (0.15 \text{ rad/s})\mathbf{k} \qquad (\dot{\omega}_2 = 0)$$

Use a frame of reference rotating about the *y*-axis.

Its angular velocity is $\quad \boldsymbol{\Omega} = \omega_1\mathbf{j} = (0.15 \text{ rad/s})\mathbf{j}$

(*a*) *Angular acceleration*:

$$\boldsymbol{\alpha} = \dot{\omega}_1\mathbf{j} + \dot{\omega}_2\mathbf{k} + \boldsymbol{\Omega}\times\boldsymbol{\omega}_2$$
$$= 0 + 0 + (0.15\mathbf{j})(0.25\mathbf{k})$$

$$\boldsymbol{\alpha} = (0.0375 \text{ rad/s}^2)\mathbf{i} \blacktriangleleft$$

Motion of coinciding Point C.

$$\mathbf{v}_{C'} = \boldsymbol{\Omega}\times\mathbf{r}_{C/A} = 0.15\mathbf{j}\times(0.81915\mathbf{i} + 0.57358\mathbf{j})$$
$$= -(0.12287 \text{ m/s})\mathbf{k}$$
$$\mathbf{a}_{C'} = \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}_{C/A}) = (0.15\mathbf{j})\times(-0.12287\mathbf{k})$$
$$= -(0.018431 \text{ m/s}^2)\mathbf{i}$$

Motion of C relative to the frame.

$$\mathbf{v}_{C/F} = \omega_2\mathbf{k}\times\mathbf{r}_{C/A} = 0.25\mathbf{k}(0.81915\mathbf{i} + 0.57358\mathbf{j})$$
$$= -(0.14339 \text{ m/s})\mathbf{i} + (0.20479 \text{ m/s})\mathbf{j}$$
$$\mathbf{a}_{C/F} = \omega_1\mathbf{k}\times\mathbf{v}_{C/F} = 0.25\mathbf{k}\times(-0.14339\mathbf{i} + 0.20479\mathbf{j})$$
$$= -(0.051198 \text{ m/s}^2)\mathbf{i} - (0.035848 \text{ m/s}^2)\mathbf{j}$$

PROBLEM 15.233 (Continued)

(*b*) *Velocity of C.* $\mathbf{v}_C = \mathbf{v}_{C'} + \mathbf{v}_{C/F}$

$$\mathbf{v}_C - (0.143 \text{ m/s})\mathbf{i} + (0.205 \text{ m/s})\mathbf{j} - (0.123 \text{ m/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration. $2\mathbf{\Omega} \times \mathbf{v}_{C/F}$

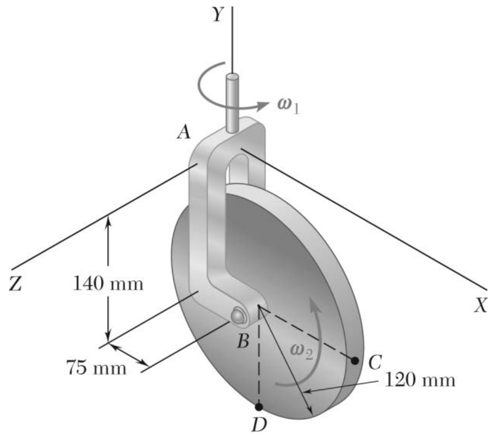
$$2\mathbf{\Omega} \times \mathbf{v}_{C/F} = (2)(0.15\mathbf{j}) \times (-0.14339\mathbf{i} + 0.20479\mathbf{j})$$

$$= (0.043017 \text{ m/s}^2)\mathbf{k}$$

(*c*) *Acceleration of C.* $\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/F} + 2\mathbf{\Omega} \times \mathbf{v}_{C/F}$

$$\mathbf{a}_C = -0.01843\mathbf{i} - 0.051198\mathbf{i} - 0.035848\mathbf{j} + 0.043017\mathbf{k}$$

$$\mathbf{a}_C = -(0.0696 \text{ m/s}^2)\mathbf{i} - (0.0358 \text{ m/s}^2)\mathbf{j} + (0.0430 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$

	**PROBLEM 15.234** A disk of radius 120 mm rotates at the constant rate $\omega_2 = 5$ rad/s with respect to the arm *AB*, which itself rotates at the constant rate $\omega_1 = 3$ rad/s. For the position shown, determine the velocity and acceleration of Point *C*.

SOLUTION

Geometry.

$$\mathbf{r}_{C/A} = (0.195\text{ m})\mathbf{i} - (0.1)\mathbf{j}$$

$$\mathbf{r}_{C/B} = (0.12\text{ m})\mathbf{i}$$

Let frame *Axyz*, which coincides with the fixed frame *AXYZ* at the instant shown, be rotating about the *y* axis with constant angular velocity $\mathbf{\Omega} = \omega_1\mathbf{j} = (3\text{ rad/s})\mathbf{j}$. Then the motion relative to the frame consists of rotation about the axle *B* with a constant angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{k} = (5\text{ rad/s})\mathbf{k}$.

Motion of the coinciding Point C′ in the frame.

$$\mathbf{v}_{C'} = \mathbf{\Omega}\times\mathbf{r}_{C/A}$$
$$= 3\mathbf{j}\times(0.195\mathbf{i} - 0.14\mathbf{j})$$
$$= -(0.585\text{ m/s})\mathbf{k}$$
$$\mathbf{a}_{C'} = \mathbf{\Omega}\times\mathbf{v}_{C'}$$
$$= 3\mathbf{j}\times(-0.585\mathbf{k})$$
$$= -(1.755\text{ m/s}^2)\mathbf{i}$$

Motion relative to the frame.

$$\mathbf{v}_{C/F} = \boldsymbol{\omega}_2\times\mathbf{r}_{C/B}$$
$$= 5\mathbf{k}\times 0.12\mathbf{i}$$
$$= (0.6\text{ m/s})\mathbf{j}$$
$$\mathbf{a}_{C/F} = \boldsymbol{\omega}_2\times\mathbf{v}_{C/F}$$
$$= 5\mathbf{k}\times(-0.6\mathbf{j})$$
$$= -(3\text{ m/s}^2)\mathbf{i}$$

Velocity of Point C.

$$\mathbf{v}_C = \mathbf{v}_{C'} + \mathbf{v}_{C/F}$$

$$\mathbf{v}_C = (0.600\text{ m/s})\mathbf{j} - (0.585\text{ m/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration.

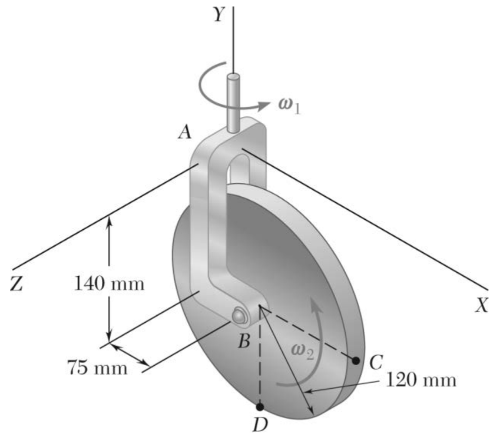
$$2\mathbf{\Omega}\times\mathbf{v}_{C/F} = (2)(3\mathbf{j})\times(0.6\mathbf{j}) = 0$$

Acceleration of Point C.

$$\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/F} + 2\mathbf{\Omega}\times\mathbf{v}_{C/F}$$

$$\mathbf{a}_C = -1.7551\mathbf{i} - 3\mathbf{i} + 0$$

$$\mathbf{a}_C = -(4.76\text{ m/s}^2)\mathbf{i} \blacktriangleleft$$



PROBLEM 15.235

A disk of radius 120 mm rotates at the constant rate $\omega_2 = 5$ rad/s with respect to the arm AB, which itself rotates at the constant rate $\omega_1 = 3$ rad/s. For the position shown, determine the velocity and acceleration of Point D.

SOLUTION

Geometry.

$$\mathbf{r}_{D/A} = (0.075\text{ m})\mathbf{i} - (0.26\text{ m})\mathbf{j}$$

$$\mathbf{r}_{D/B} = -(0.12\text{ m})\mathbf{j}$$

Let frame $AXYZ$, which coincides with the fixed frame $AXYZ$ at the instant shown, be rotating about the y axis with constant angular velocity $\mathbf{\Omega} = \omega_1\mathbf{j} = (3\text{ rad/s})\mathbf{j}$. Then the motion relative to the frame consists of rotation about the axle B with a constant angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{k} = (5\text{ rad/s})\mathbf{k}$.

Motion of the coinciding Point D' in the frame.

$$\begin{aligned}\mathbf{v}_{D'} &= \mathbf{\Omega} \times \mathbf{r}_{D/A} \\ &= 3\mathbf{j} \times (0.075\mathbf{i} - 0.26\mathbf{j}) \\ &= -(0.225\text{ m/s})\mathbf{k} \\ \mathbf{a}_{D'} &= \mathbf{\Omega} \times \mathbf{v}_{D'} \\ &= 3\mathbf{j} \times (-0.225\mathbf{k}) \\ &= -(0.675\text{ m/s}^2)\mathbf{i}\end{aligned}$$

Motion relative to the frame.

$$\begin{aligned}\mathbf{v}_{D/F} &= \boldsymbol{\omega}_2 \times \mathbf{r}_{D/B} \\ &= 5\mathbf{k} \times (-0.12\mathbf{j}) \\ &= (0.6\text{ m/s})\mathbf{i} \\ \mathbf{a}_{D/F} &= \boldsymbol{\omega}_2 \times \mathbf{v}_{C/F} \\ &= 5\mathbf{k} \times (0.6\mathbf{i}) \\ &= (3\text{ m/s}^2)\mathbf{j}\end{aligned}$$

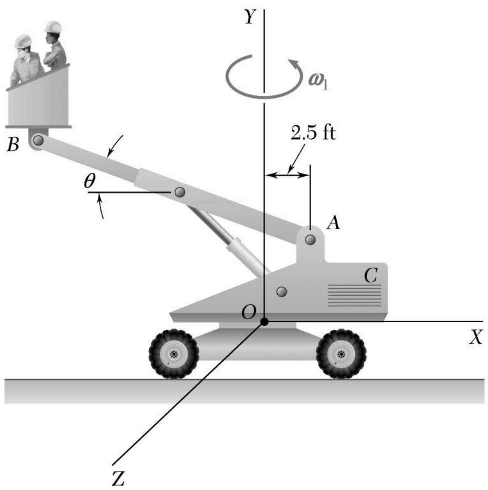
Velocity of Point D. $\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$

$$\mathbf{v}_D = (0.600\text{ m/s})\mathbf{i} - (0.225\text{ m/s})\mathbf{k} \quad \blacktriangleleft$$

Coriolis acceleration. $2\mathbf{\Omega} \times \mathbf{v}_{D/F} = (2)(3\mathbf{j}) \times (0.6\mathbf{i}) = -(3.6\text{ m/s}^2)\mathbf{k}$

Acceleration of Point D. $\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\mathbf{\Omega} \times \mathbf{v}_{D/F}$

$$\mathbf{a}_D = -(0.675\text{ m/s}^2)\mathbf{i} + (3.00\text{ m/s}^2)\mathbf{j} - (3.60\text{ m/s}^2)\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 15.236

The arm AB of length 16 ft is used to provide an elevated platform for construction workers. In the position shown, arm AB is being raised at the constant rate $d\theta/dt = 0.25$ rad/s; simultaneously, the unit is being rotated about the Y axis at the constant rate $\omega_1 = 0.15$ rad/s. Knowing that $\theta = 20°$, determine the velocity and acceleration of Point B.

SOLUTION

Frame of reference. Let moving frame $Axyz$ rotate about the Y axis with angular velocity

$$\mathbf{\Omega} = \omega_1 \mathbf{j}$$
$$= (0.15 \text{ rad/s})\mathbf{j}.$$

Geometry.

$$\mathbf{r}_{B/A} = -16\cos 20°\mathbf{i} + 16\sin 20°\mathbf{j}$$
$$= -(15.035 \text{ ft})\mathbf{i} + (5.4723 \text{ ft})\mathbf{j}$$

Place Point O on Y axis at same level as Point A.

$$\mathbf{r}_{B/O} = \mathbf{r}_{B/A} + \mathbf{r}_{A/O}$$
$$= \mathbf{r}_{B/A} + (2.5 \text{ ft})\mathbf{i}$$
$$= -(12.535 \text{ ft})\mathbf{i} + (5.4723 \text{ ft})\mathbf{j}$$

Motion of corresponding Point B′ in the frame.

$$\mathbf{v}_{B'} = \mathbf{\Omega} \times \mathbf{r}_{B/O}$$
$$= (0.15\mathbf{j}) \times -(12.535\mathbf{i} + 5.4723\mathbf{j})$$
$$= (1.8803 \text{ ft/s})\mathbf{k}$$
$$\mathbf{a}_{B'} = \mathbf{\Omega} \times \mathbf{v}_{B'}$$
$$= (0.15\mathbf{j}) \times (1.8803\mathbf{k})$$
$$= (0.28204 \text{ ft/s}^2)\mathbf{i}$$

Motion of Point B relative to the frame.

$$\boldsymbol{\omega}_2 = -\frac{d\theta}{dt}\mathbf{k}$$

$$= -(0.25 \text{ rad/s})\mathbf{k}$$

$$\mathbf{v}_{B/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_{B/A}$$

$$= (-0.25)\mathbf{k} \times (-15.035\mathbf{i} + 5.4723\mathbf{j})$$

$$= (1.36808 \text{ ft/s})\mathbf{i} + (3.7588 \text{ ft/s})\mathbf{j}$$

$$\mathbf{a}_{B/F} = \boldsymbol{\omega}_2 \times \mathbf{v}_{B/F}$$

$$= (-0.25\mathbf{k}) \times (1.36808\mathbf{i} + 3.7588\mathbf{j})$$

$$= (0.93969 \text{ ft/s}^2)\mathbf{i} - (0.34202 \text{ ft/s}^2)\mathbf{j}$$

Velocity of Point B.

$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$$

$$\mathbf{v}_B = -(1.37 \text{ ft/s})\mathbf{i} + (3.76 \text{ ft/s})\mathbf{j} + (1.88 \text{ ft/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration.

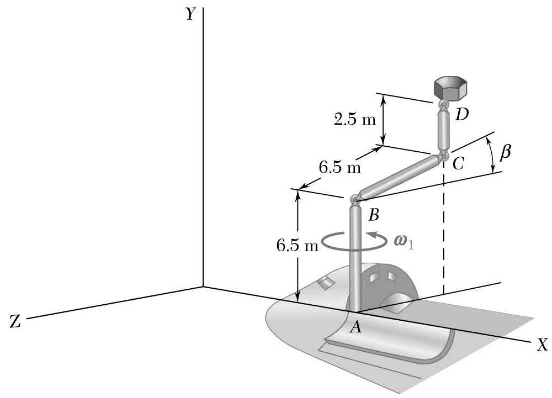
$$2\boldsymbol{\Omega} \times \mathbf{v}_{B/F} = (2)(0.15\mathbf{j}) \times (1.36808\mathbf{i} + 3.7588\mathbf{j})$$

$$= -(0.41042 \text{ ft/s}^2)\mathbf{k}$$

Acceleration of Point B.

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\boldsymbol{\Omega} \times \mathbf{v}_{B/F}$$

$$\mathbf{a}_B = (1.22 \text{ ft/s}^2)\mathbf{i} - (0.342 \text{ ft/s}^2)\mathbf{j} - (0.410 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.237

The remote manipulator system (RMS) shown is used to deploy payloads from the cargo bay of space shuttles. At the instant shown, the whole RMS is rotating at the constant rate $\omega_1 = 0.03$ rad/s about the axis *AB*. At the same time, portion *BCD* rotates as a rigid body at the constant rate $\omega_2 = d\beta/dt = 0.04$ rad/s about an axis through *B* parallel to the *X* axis. Knowing that $\beta = 30°$, determine (*a*) the angular acceleration of *BCD*, (*b*) the velocity of *D*, (*c*) the acceleration of *D*.

SOLUTION

At the instant given, Points *A*, *B*, *C*, and *D* lie in a plane which is parallel to the *YZ* plane. The plane *ABCD* is rotating with angular velocity.

$$\mathbf{\Omega} = \omega_1 \mathbf{j}$$
$$= (0.03 \text{ rad/s})\mathbf{j} \qquad (\dot{\omega}_1 = 0)$$

Body *BCD* is rotating about an axis through *B* parallel to the *x*-axis at angular velocity.

$$\boldsymbol{\omega}_2 = \frac{d\beta}{dt}\mathbf{i} = (0.04 \text{ rad/s})\mathbf{i} \qquad (\dot{\omega}_2 = 0)$$

(*a*) *Angular acceleration of BCD.*

$$\boldsymbol{\alpha}_{BCD} = \dot{\omega}_1\mathbf{j} + \dot{\omega}_2\mathbf{i} + \mathbf{\Omega}\times\boldsymbol{\omega}_2$$
$$= 0 + 0 + (0.03\mathbf{j})\times(0.04\mathbf{i})$$

$$\boldsymbol{\alpha}_{BCD} = -(0.0012 \text{ rad/s}^2)\mathbf{k} \blacktriangleleft$$

Let the plane of *BCD* be a rotating frame of reference rotating about *AB* with angular velocity $\mathbf{\Omega}$.

Geometry: $\beta = 30°$

$$\mathbf{r}_{D/B} = (6.5 \text{ m})(\sin\beta\mathbf{j} - \cos\beta\mathbf{k}) + (2.5 \text{ m})\mathbf{j} = (5.75 \text{ m})\mathbf{j} - (5.6292 \text{ m})\mathbf{k}$$

Motion of Point *D′* in the frame.

$$\mathbf{v}_{D'} = \mathbf{\Omega}\times\mathbf{r}_{D/B} = 0.03\mathbf{j}\times(3.25\mathbf{j} - 5.6292\mathbf{k})$$
$$= -(0.168875 \text{ m/s})\mathbf{i}$$
$$\mathbf{a}_{D'} = \mathbf{\Omega}\times\mathbf{v}_{D'} = (0.03\mathbf{j})\times(-0.168875\mathbf{i})$$
$$= (0.0050662 \text{ m/s}^2)\mathbf{k}$$

PROBLEM 15.237 (Continued)

Motion of D relative to the frame: This motion is a rotation about B with angular velocity.

$$\boldsymbol{\omega}_2 = (0.04 \text{ rad/s})\mathbf{i}$$

$$\begin{aligned}
\mathbf{v}_{D/\text{frame}} &= \boldsymbol{\omega}_2 \times \mathbf{r}_{D/B} \\
&= (0.04\mathbf{i}) \times (5.75\mathbf{j} - 5.6292\mathbf{k}) \\
&= (0.22517 \text{ m/s})\mathbf{j} + (0.23 \text{ m/s})\mathbf{k}
\end{aligned}$$

$$\begin{aligned}
\mathbf{a}_{D/\text{frame}} &= \boldsymbol{\omega}_2 \times r_{D/\text{frame}} \\
&= (0.04\mathbf{i}) \times (0.22517\mathbf{j} + 0.23\mathbf{k}) \\
&= -(0.0092 \text{ m/s}^2)\mathbf{j} + (0.009007 \text{ m/s}^2)\mathbf{k}
\end{aligned}$$

(*b*) *Velocity of D.* $\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/\text{frame}}$

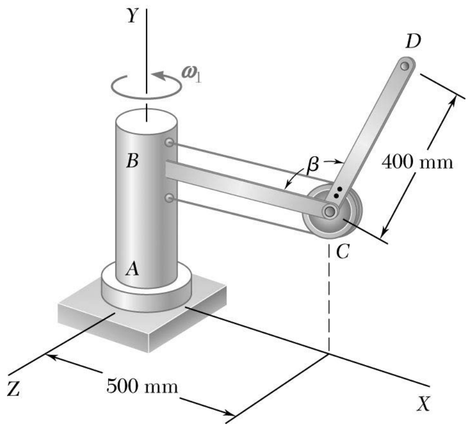
$$\mathbf{v}_D = -(0.169 \text{ m/s})\mathbf{i} + (0.225 \text{ m/s})\mathbf{j} + (0.230 \text{ m/s})\mathbf{k} \quad \blacktriangleleft$$

Coriolis acceleration: $2\boldsymbol{\Omega} \times \mathbf{v}_{D/\text{frame}}$

$$\begin{aligned}
2\boldsymbol{\Omega} \times \mathbf{v}_{D/\text{frame}} &= (2)(0.03\mathbf{j}) \times (0.22517\mathbf{j} + 0.23\mathbf{k}) \\
&= (0.0138 \text{ m/s}^2)\mathbf{i}
\end{aligned}$$

(*c*) *Acceleration of D.* $\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/\text{frame}} + 2\boldsymbol{\Omega} \times \mathbf{v}_{D/\text{frame}}$

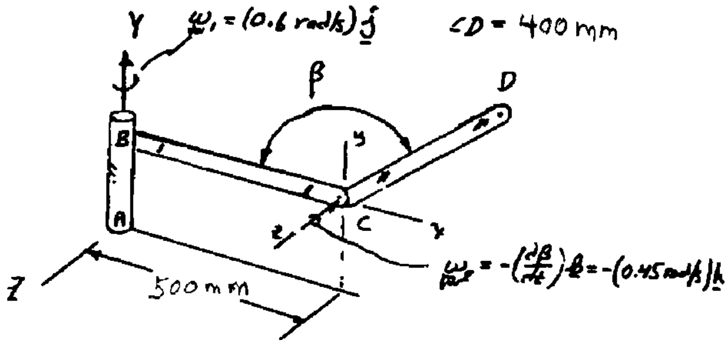
$$\mathbf{a}_D = (0.0138 \text{ m/s}^2)\mathbf{i} - (0.0092 \text{ m/s}^2)\mathbf{j} + (0.0141 \text{ m/s}^2)\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 15.238

The body *AB* and rod *BC* of the robotic component shown rotate at the constant rate $\omega_1 = 0.60$ rad/s about the *Y* axis. Simultaneously a wire-and-pulley control causes arm *CD* to rotate about *C* at the constant rate $\omega_2 = d\beta/dt = 0.45$ rad/s. Knowing that $\beta = 120°$, determine (*a*) the angular acceleration of arm *CD*, (*b*) the velocity of *D*, (*c*) the acceleration of *D*.

SOLUTION



$$\boldsymbol{\Omega} = \boldsymbol{\omega}_1$$
$$= (0.6 \text{ rad/s})\mathbf{j}$$
$$\boldsymbol{\omega}_{D/F} = \boldsymbol{\omega}_2$$
$$= -(0.45 \text{ rad/s})\mathbf{k}$$
$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$$
$$= (0.6 \text{ rad/s})\mathbf{j} - (0.45 \text{ rad/s})\mathbf{k}$$

(*a*) Angular acceleration of *CD*.

$$\boldsymbol{\alpha} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$$
$$= (0.6 \text{ rad/s})\mathbf{j} \times [(0.6 \text{ rad/s})\mathbf{j} - (0.45 \text{ rad/s})\mathbf{k}]$$

$$\boldsymbol{\alpha} = -(0.27 \text{ rad/s}^2)\mathbf{i} \blacktriangleleft$$

For $\beta = 120°$:

$$\mathbf{r}_{D/C} = (400 \text{ mm})\sin 30°\mathbf{i} + (400 \text{ mm})\cos 30°\mathbf{j}$$
$$= (200 \text{ mm})\mathbf{i} + (346.41 \text{ mm})\mathbf{j}$$
$$\mathbf{r}_{D/B} = (500 \text{ mm})\mathbf{i} + \mathbf{r}_{D/C}$$
$$= (700 \text{ mm})\mathbf{i} + (346.41 \text{ mm})\mathbf{j}$$

PROBLEM 15.238 (Continued)

(*b*) Velocity of *D*.

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$$

$$\mathbf{v}_D = \boldsymbol{\Omega}\times\mathbf{r}_{D/B}$$
$$= (0.6 \text{ rad/s})\mathbf{j}\times[(700 \text{ mm})\mathbf{i} + (346.41 \text{ mm})\mathbf{j}]$$
$$= -(420 \text{ mm/s})\mathbf{k}$$

$$\mathbf{v}_{D/F} = \boldsymbol{\omega}_{D/F}\times\mathbf{r}_{D/C}$$
$$= -(0.45 \text{ rad/s})\mathbf{k}\times[(200 \text{ mm})\mathbf{i} + (346.41 \text{ mm})\mathbf{j}]$$
$$= -(90 \text{ mm/s})\mathbf{j} + (155.88 \text{ mm/s})\mathbf{i}$$

$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$: $\mathbf{v}_D = (156 \text{ mm/s})\mathbf{i} - (90 \text{ mm/s})\mathbf{j} - (420 \text{ mm/s})\mathbf{k}$ ◀

(*c*) Acceleration of *D*.

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + \mathbf{a}_c$$

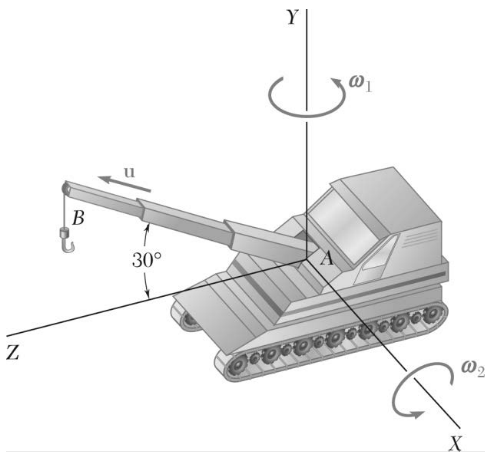
$$\mathbf{a}_{D'} = \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}_{D/B})$$
$$= \boldsymbol{\Omega}\times\mathbf{v}_{D'}$$
$$= (0.6 \text{ rad/s})\mathbf{j}\times(-420 \text{ mm/s})\mathbf{k}$$
$$= -(252 \text{ mm/s}^2)\mathbf{i}$$

$$\mathbf{a}_{D/F} = \boldsymbol{\omega}_{D/F}\times(\boldsymbol{\omega}_{D/F}\times\mathbf{r}_{D/C})$$
$$= \boldsymbol{\omega}_{D/F}\times\mathbf{v}_{D/F}$$
$$= -(0.45 \text{ rad/s})\mathbf{k}\times[-(90)\mathbf{j} + (155.88)\mathbf{i}]$$
$$= -(40.5 \text{ mm/s}^2)\mathbf{i} - (70.148 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}_c = 2\boldsymbol{\Omega}\times\mathbf{v}_{D/F}$$
$$= 2(0.6 \text{ rad/s})\mathbf{j}\times[-(90)\mathbf{j} + (155.88)\mathbf{i}]$$
$$= -(187.06 \text{ mm/s}^2)\mathbf{k}$$

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + \mathbf{a}_c$$
$$= -(252 \text{ mm/s}^2)\mathbf{i} - (40.5 \text{ mm/s}^2)\mathbf{i}$$
$$-(70.148 \text{ mm/s}^2)\mathbf{j} - (187.06 \text{ mm/s}^2)\mathbf{k}$$

$\mathbf{a}_D = -(293 \text{ mm/s}^2)\mathbf{i} - (70.1 \text{ mm/s}^2)\mathbf{j} - (187 \text{ mm/s}^2)\mathbf{k}$ ◀



PROBLEM 15.239

The crane shown rotates at the constant rate $\omega_1 = 0.25$ rad/s; simultaneously, the telescoping boom is being lowered at the constant rate $\omega_2 = 0.40$ rad/s. Knowing that at the instant shown the length of the boom is 20 ft and is increasing at the constant rate $u = 1.5$ ft/s, determine the velocity and acceleration of Point *B*.

SOLUTION

Geometry.

$$\mathbf{r}_{B/A} = \mathbf{r}_B$$
$$= (20\text{ ft})(\sin 30°\,\mathbf{j} + \cos 30°\,\mathbf{k})$$
$$= (10\text{ ft})\mathbf{j} + (10\sqrt{3}\text{ ft})\mathbf{k}$$

Method 1

Let the unextending portion of the boom *AB* be a rotating frame of reference.

Its angular velocity is
$$\boldsymbol{\Omega} = \omega_2\mathbf{i} + \omega_1\mathbf{j}$$
$$= (0.40\text{ rad/s})\mathbf{i} + (0.25\text{ rad/s})\mathbf{j}.$$

Its angular acceleration is
$$\boldsymbol{\alpha} = \omega_1\mathbf{j} \times \omega_2\mathbf{i}$$
$$= -\omega_1\omega_2\mathbf{k}$$
$$= -(0.10\text{ rad/s}^2)\mathbf{k}.$$

Motion of the coinciding Point B′ in the frame.

$$\mathbf{v}_{B'} = \boldsymbol{\Omega} \times \mathbf{r}_B$$
$$= (0.40\mathbf{i} + 0.25\mathbf{j}) \times (10\mathbf{j} + 10\sqrt{3}\mathbf{k})$$
$$= (2.5\sqrt{3}\text{ ft/s})\mathbf{i} - (4\sqrt{3}\text{ ft/s})\mathbf{j} + (4\text{ ft/s})\mathbf{k}$$
$$\mathbf{a}_{B'} = \boldsymbol{\alpha} \times \mathbf{r}_B + \boldsymbol{\Omega} \times \mathbf{v}_{B'}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.10 \\ 0 & 10 & 10\sqrt{3} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.40 & 0.25 & 0 \\ 2.5\sqrt{3} & -4\sqrt{3} & 4 \end{vmatrix}$$
$$\mathbf{i} + \mathbf{i} - 1.6\mathbf{j} - 3.8538\mathbf{k} = (2\text{ ft/s}^2)\mathbf{i} - (1.6\text{ ft/s}^2)\mathbf{j} - (3.8538\text{ ft/s}^2)\mathbf{k}$$

Motion relative to the frame.

$$\mathbf{v}_{B/F} = u(\sin 30°\,\mathbf{j} + \cos 30°\,\mathbf{k})$$
$$= (1.5\text{ ft/s})\sin 30°\,\mathbf{j} + (1.5\text{ ft/s})\cos 30°\,\mathbf{k}$$
$$\mathbf{a}_{B/F} = 0$$

Velocity of Point B. $\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$

$$\mathbf{v}_B = 2.5\sqrt{3}\mathbf{i} - 4\sqrt{3}\mathbf{j} + 4\mathbf{k} + 1.5\sin 30°\mathbf{j} + 1.5\cos 30°\mathbf{k}$$

$$\mathbf{v}_B = (4.33 \text{ ft/s})\mathbf{i} - (6.18 \text{ ft/s})\mathbf{j} + (5.30 \text{ ft/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration. $2\mathbf{\Omega} \times \mathbf{v}_{B/F}$

$$2\mathbf{\Omega} \times \mathbf{v}_{B/F} = (2)(0.40\mathbf{i} + 0.25\mathbf{j}) \times (1.5\sin 30°\mathbf{j} + 1.5\cos 30°\mathbf{k})$$

$$= (0.64952 \text{ ft/s}^2)\mathbf{i} - (1.03923 \text{ ft/s}^2)\mathbf{j} + (0.6 \text{ ft/s}^2)\mathbf{k}$$

Acceleration of Point B. $\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\mathbf{\Omega} \times \mathbf{v}_{B/F}$

$$\mathbf{a}_B = (2 + 0.64952)\mathbf{i} - (1.6 + 1.03923\mathbf{j}) + (-3.8538 + 0.6)\mathbf{k}$$

$$\mathbf{a}_B = (2.65 \text{ ft/s}^2)\mathbf{i} - (2.64 \text{ ft/s}^2)\mathbf{j} - (3.25 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$

Method 2

Let frame *Axyz*, which at the instant shown coincides with *AXYZ*, rotate with an angular velocity $\mathbf{\Omega}_1 = \omega_1\mathbf{j} = (0.25 \text{ rad/s})\mathbf{j}$. Then the motion relative to this frame consists of turning the boom relative to the cab and extending the boom.

Motion of the coinciding Point B′ in the frame.

$$\mathbf{v}_{B'} = \mathbf{\Omega} \times \mathbf{r}_B$$
$$= 0.25\mathbf{j} \times (10\mathbf{j} + 10\sqrt{3}\mathbf{k})$$
$$= (2.5\sqrt{3} \text{ m/s})\mathbf{i}$$
$$\mathbf{a}_{B'} = \mathbf{\Omega} \times \mathbf{v}_{B'}$$
$$= 0.25\mathbf{j} \times (2.5\sqrt{3}\mathbf{i})$$
$$= -(0.625\sqrt{3} \text{ m/s}^2)\mathbf{k}$$

Motion of Point B relative to the frame.

Let the unextending portion of the boom be a rotating frame with constant angular velocity $\mathbf{\Omega}_2 = \omega_2\mathbf{i} = (0.40 \text{ rad/s})\mathbf{i}$. The motion relative to this frame is the extensional motion with speed *u*.

$$\mathbf{v}_{B''} = \mathbf{\Omega}_2 \times \mathbf{r}_B$$
$$= 0.40\mathbf{i} \times (10\mathbf{j} + 10\sqrt{3}\mathbf{k})$$
$$= -(4\sqrt{3} \text{ ft/s})\mathbf{j} + (4 \text{ ft/s})\mathbf{k}$$
$$\mathbf{a}_{B''} = \mathbf{\Omega}_2 \times \mathbf{v}_{B''}$$
$$= 0.40\mathbf{i} \times (-4\sqrt{3}\mathbf{j} + 4\mathbf{k})$$
$$= -(1.6 \text{ ft/s}^2)\mathbf{j} - (1.6\sqrt{3} \text{ ft/s}^2)\mathbf{k}$$
$$\mathbf{v}_{B/\text{boom}} = u(\sin 30°\mathbf{j} + \cos 30°\mathbf{k})$$
$$= (1.5 \text{ ft/s})\sin 30°\mathbf{j} + (1.5 \text{ ft/s})\cos 30°\mathbf{k}$$
$$\mathbf{a}_{B/\text{boom}} = 0$$

$$2\mathbf{\Omega}_2 \times \mathbf{v}_{B/\text{boom}} = (2)(0.40\mathbf{i}) \times (1.5\sin 30°\mathbf{j} + 1.5\cos 30°\mathbf{k})$$

$$= -(1.03923 \text{ ft/s}^2)\mathbf{j} + (0.6 \text{ ft/s}^2)\mathbf{k}$$

$$\mathbf{v}_{B/F} = \mathbf{v}_{B''} + \mathbf{v}_{B/\text{boom}}$$

$$= -4\sqrt{3}\mathbf{j} + 4\mathbf{k} + 1.5\sin 30°\mathbf{j} + 1.5\cos 30°\mathbf{k}$$

$$= -(6.1782 \text{ ft/s})\mathbf{j} + (5.299 \text{ ft/s})\mathbf{k}$$

$$\mathbf{a}_{B/F} = \mathbf{a}_{B''} + \mathbf{a}_{B/\text{boom}} + 2\mathbf{\Omega}_2 \times \mathbf{v}_{B/\text{boom}}$$

$$= -1.6\mathbf{j} - 1.6\sqrt{3}\mathbf{k} + 0 - 1.03923\mathbf{j} + 0.6\mathbf{k}$$

$$= -(2.6392 \text{ ft/s}^2)\mathbf{j} - (2.1713 \text{ ft/s}^2)\mathbf{k}$$

Velocity of Point B.

$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$$

$$\mathbf{v}_B = 2.5\sqrt{3}\mathbf{i} - 6.1782\mathbf{j} + 5.299\mathbf{k}$$

$$\mathbf{v}_B = (4.33 \text{ ft/s})\mathbf{i} - (6.18 \text{ ft/s})\mathbf{j} + (5.30 \text{ ft/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration.

$$2\mathbf{\Omega}_1 \times \mathbf{v}_{B/F}$$

$$2\mathbf{\Omega}_1 \times \mathbf{v}_{B/F} = (2)(0.25\mathbf{j}) \times (-6.1782\mathbf{j} + 5.299\mathbf{k})$$

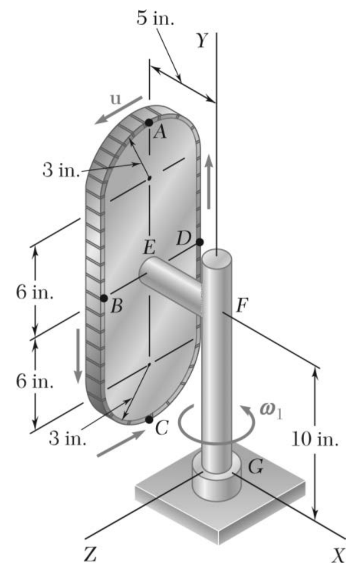
$$= (2.6495 \text{ ft/s}^2)\mathbf{i}$$

Acceleration of Point B.

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\mathbf{\Omega}_1 \times \mathbf{v}_{B/F}$$

$$\mathbf{a}_B = -0.625\sqrt{3}\mathbf{k} - 2.6392\mathbf{j} - 2.1713\mathbf{k} + 2.6495\mathbf{i}$$

$$\mathbf{a}_B = (2.65 \text{ ft/s}^2)\mathbf{i} - (2.64 \text{ ft/s}^2)\mathbf{j} - (3.25 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.240

The vertical plate shown is welded to arm *EFG*, and the entire unit rotates at the constant rate $\omega_1 = 1.6$ rad/s about the *Y* axis. At the same time, a continuous link belt moves around the perimeter of the plate at a constant speed $u = 4.5$ in./s. For the position shown, determine the acceleration of the link of the belt located (*a*) at Point *A*, (*b*) at Point *B*.

SOLUTION

Let the moving frame of reference be the unit, less the pulleys and belt. It rotates about the *Y* axis with constant angular velocity $\mathbf{\Omega} = \omega_1 \mathbf{j} = (1.6 \text{ rad/s})\mathbf{j}.$ The relative motion is that of the pulleys and belt with speed $u = 90$ mm/s.

(*a*) *Acceleration at Point A.*

$$\mathbf{r}_A = -(5 \text{ in.})\mathbf{i} + (19 \text{ in.})\mathbf{j}$$

$$\mathbf{v}_{A'} = \mathbf{\Omega} \times \mathbf{r}_A$$

$$= 1.6\mathbf{j} \times (-5\mathbf{i} + 19\mathbf{j})$$

$$= (8 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{A'} = \mathbf{\Omega} \times \mathbf{v}_{A'}$$

$$= 1.6\mathbf{j} \times 8\mathbf{k}$$

$$= (12.8 \text{ in./s}^2)\mathbf{i}$$

$$\mathbf{v}_{A/F} = u\mathbf{k} = (4.5 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{A/F} = -\left(\frac{u^2}{\rho}\right)\mathbf{j}$$

$$= -\left(\frac{4.5^2}{3}\right)\mathbf{j}$$

$$= -\left(6.75 \text{ in./s}^2\right)\mathbf{j}$$

$$2\mathbf{\Omega} \times \mathbf{v}_{A/F} = (2)(1.6\mathbf{j}) \times (4.5\mathbf{k})$$

$$= (14.4 \text{ in./s}^2)\mathbf{i}$$

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + 2\mathbf{\Omega} \times \mathbf{v}_{A/F}$$

$$= 12.8\mathbf{i} - 6.75\mathbf{j} + 14.4\mathbf{i}$$

$$\mathbf{a}_A = (27.2 \text{ in./s}^2)\mathbf{i} - (6.75 \text{ in./s}^2)\mathbf{j} \blacktriangleleft$$

(*b*) *Acceleration of Point B.*

$$\mathbf{r}_B = -(5\text{ in.})\mathbf{i} + (10\text{ in.})\mathbf{j} + (3\text{ in.})\mathbf{k}$$

$$\mathbf{v}_{B'} = \mathbf{\Omega} \times \mathbf{r}_B$$
$$= 1.6\mathbf{j} \times (-5\mathbf{i} + 10\mathbf{j} + 3\mathbf{k})$$
$$= (4.8\text{ in./s})\mathbf{i} + (8\text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{B'} = \mathbf{\Omega} \times \mathbf{v}_{B'}$$
$$= 1.6\mathbf{j} \times (4.8\mathbf{i} + 8\mathbf{k})$$
$$= (12.8\text{ in./s}^2)\mathbf{i} - (7.68\text{ in./s}^2)\mathbf{k}$$

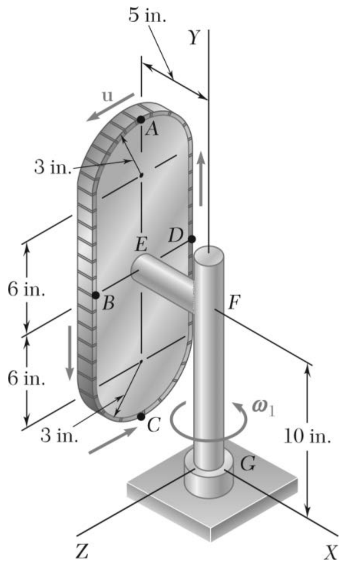
$$\mathbf{v}_{B/F} = -u\mathbf{j}$$
$$= -(3\text{ in./s})\mathbf{j}$$

$$\mathbf{a}_{B/F} = 0$$

$$2\mathbf{\Omega} \times \mathbf{v}_{B/F} = (2)(1.6\mathbf{j}) \times (4.5\mathbf{j}) = 0$$

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\Omega \times \mathbf{v}_{B/F}$$
$$= 12.8\mathbf{i} - 7.68\mathbf{k} + 0 + 0$$

$$\mathbf{a}_B = (12.80\text{ in./s}^2)\mathbf{i} - (7.68\text{ in./s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.241

The vertical plate shown is welded to arm *EFG*, and the entire unit rotates at the constant rate $\omega_1 = 1.6$ rad/s about the *Y* axis. At the same time, a continuous link belt moves around the perimeter of the plate at a constant speed $u = 4.5$ in./s. For the position shown, determine the acceleration of the link of the belt located (*a*) at Point *C*, (*b*) at Point *D*.

SOLUTION

Let the moving frame of reference be the unit, less the pulleys and belt. It rotates about the *Y* axis with constant angular velocity $\mathbf{\Omega} = \omega_1 \mathbf{j} = (1.6 \text{ rad/s})\mathbf{j}$. The relative motion is that of the pulleys and belt with speed $u = 90$ mm/s.

(*a*) *Acceleration at Point C.*

$$\mathbf{r}_C = -(5 \text{ in.})\mathbf{i} + (4 \text{ in.})\mathbf{j}$$

$$\mathbf{v}_{C'} = \mathbf{\Omega} \times \mathbf{r}_A$$
$$= 1.6\mathbf{j} \times (-5\mathbf{i} + 4\mathbf{j})$$
$$= (8 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{C'} = \mathbf{\Omega} \times \mathbf{v}_{A'}$$
$$= 1.6\mathbf{j} \times 8\mathbf{k}$$
$$= (12.8 \text{ in./s}^2)\mathbf{i}$$

$$\mathbf{v}_{C/F} = -u\mathbf{k} = -(4.5 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{C/F} = \left(\frac{u^2}{\rho}\right)\mathbf{j}$$
$$= \left(\frac{4.5^2}{3}\right)\mathbf{j}$$
$$= (6.75 \text{ in./s}^2)\mathbf{j}$$

$$2\mathbf{\Omega} \times \mathbf{v}_{C/F} = (2)(1.6\mathbf{j}) \times (-4.5\mathbf{k})$$
$$= -(14.4 \text{ in./s}^2)\mathbf{i}$$

$$\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/F} + 2\mathbf{\Omega} \times \mathbf{v}_{C/F}$$
$$= 12.8\mathbf{i} + 6.75\mathbf{j} - 14.4\mathbf{i}$$

$$\mathbf{a}_C = -(1.600 \text{ in./s}^2)\mathbf{i} + (6.75 \text{ in./s}^2)\mathbf{j} \blacktriangleleft$$

PROBLEM 15.241 (Continued)

(*b*) *Acceleration at Point D*.

$$\mathbf{r}_D = -(5\text{ in.})\mathbf{i} + (10\text{ in.})\mathbf{j} - (3\text{ in.})\mathbf{k}$$

$$\mathbf{v}_{D'} = \mathbf{\Omega} \times \mathbf{r}_B$$

$$= 1.6\mathbf{j} \times (-5\mathbf{i} + 10\mathbf{j} - 3\mathbf{k})$$

$$= -(4.8\text{ in./s})\mathbf{i} + (8\text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{D'} = \mathbf{\Omega} \times \mathbf{v}_{B'}$$

$$= 1.6\mathbf{j} \times (-4.8\mathbf{i} + 8\mathbf{k})$$

$$= (12.8\text{ in./s}^2)\mathbf{i} + (7.68\text{ in./s}^2)\mathbf{k}$$

$$\mathbf{v}_{D/F} = u\mathbf{j} = (4.5\text{ in./s})\mathbf{j}$$

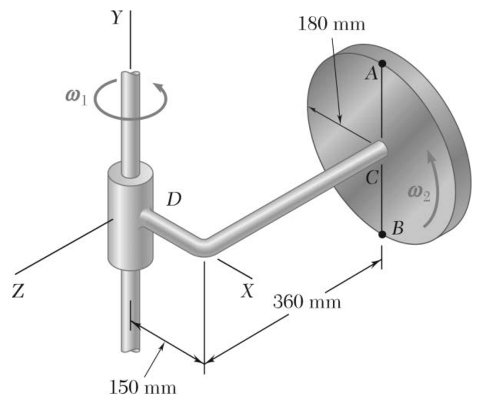
$$\mathbf{a}_{D/F} = 0$$

$$2\mathbf{\Omega} \times \mathbf{v}_{D/F} = (2)(1.6) \times (-4.5\mathbf{j}) = 0$$

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\mathbf{\Omega} \times \mathbf{v}_{D/F}$$

$$= 12.8\mathbf{i} + 7.68\mathbf{k} + 0 + 0$$

$$\mathbf{a}_D = (12.80\text{ in./s}^2)\mathbf{i} + (7.68\text{ in./s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.242

A disk of 180-mm radius rotates at the constant rate $\omega_2 = 12$ rad/s with respect to arm *CD*, which itself rotates at the constant rate $\omega_1 = 8$ rad/s about the *Y* axis. Determine at the instant shown the velocity and acceleration of Point *A* on the rim of the disk.

SOLUTION

Geometry.

$$\mathbf{r}_{A/D} = (0.15\text{ m})\mathbf{i} + (0.18\text{ m})\mathbf{j} - (0.36\text{ m})\mathbf{k}$$

$$\mathbf{r}_{A/C} = (0.18\text{ m})\mathbf{j}$$

Let frame *Dxyz*, which coincides with the fixed frame *DXYZ* at the instant shown, be rotating about the *y* axis with constant angular velocity $\mathbf{\Omega} = \omega_1\mathbf{j} = (8\text{ rad/s})\mathbf{j}$. Then the motion relative to the frame consists of a rotation of the disk *AB* about the bent axle *CD* with constant angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{k} = (12\text{ rad/s})\mathbf{k}$.

Motion of the coinciding Point A′ in the frame.

$$\mathbf{v}_{A'} = \mathbf{\Omega} \times \mathbf{r}_{A/D}$$
$$= 8\mathbf{j} \times (0.15\mathbf{i} + 0.18\mathbf{j} - 0.36\mathbf{k})$$
$$= -(2.88\text{ m/s})\mathbf{i} - (1.2\text{ m/s})\mathbf{k}$$

$$\mathbf{a}_{A'} = \mathbf{\Omega} \times \mathbf{v}_{A'}$$
$$= 8\mathbf{j} \times (-2.88\mathbf{i} - 1.2\mathbf{k})$$
$$= -(9.6\text{ m/s}^2)\mathbf{i} + (23.04\text{ m/s}^2)\mathbf{k}$$

Motion of Point A relative to the frame.

$$\mathbf{v}_{A/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_{A/D}$$
$$= 12\mathbf{k} \times 0.18\mathbf{j}$$
$$= -(2.16\text{ m/s})\mathbf{i}$$

$$\mathbf{a}_{A/F} = \boldsymbol{\omega}_2 \times \mathbf{v}_{A/F}$$
$$= 12\mathbf{k} \times (-2.16\mathbf{i})$$
$$= -(25.92\text{ m/s}^2)\mathbf{j}$$

Velocity of Point A.

$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/F}$$
$$\mathbf{v}_A = -2.88\mathbf{i} - 1.2\mathbf{k} - 2.16\mathbf{i}$$

$$\mathbf{v}_A = -(5.04\text{ m/s})\mathbf{i} - (1.200\text{ m/s})\mathbf{k} \blacktriangleleft$$

PROBLEM 15.242 (Continued)

Coriolis acceleration. $2\boldsymbol{\Omega}\times\mathbf{v}_{A/F}$

$$2\boldsymbol{\Omega}\times\mathbf{v}_{A/F} = (2)(8\mathbf{j})\times(-2.16\mathbf{i})$$

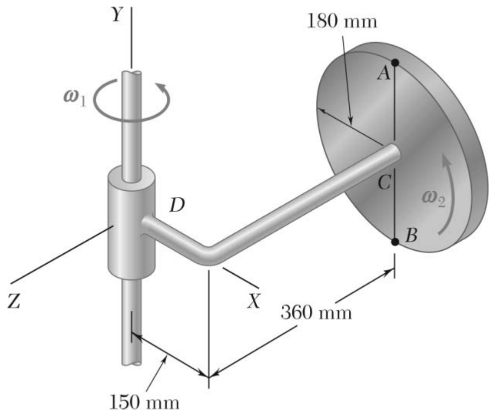
$$= (34.56\text{ m/s}^2)\mathbf{k}$$

Acceleration of Point A.

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + 2\boldsymbol{\Omega}\times\mathbf{v}_{A/F}$$

$$\mathbf{a}_A = -9.6\mathbf{i} + 23.04\mathbf{k} - 25.92\mathbf{j} + 34.56\mathbf{k}$$

$$\mathbf{a}_A = -(9.60\text{ m/s}^2)\mathbf{i} - (25.9\text{ m/s}^2)\mathbf{j} + (57.6\text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.243

A disk of 180-mm radius rotates at the constant rate $\omega_2 = 12$ rad/s with respect to arm *CD*, which itself rotates at the constant rate $\omega_1 = 8$ rad/s about the *Y* axis. Determine at the instant shown the velocity and acceleration of Point *B* on the rim of the disk.

SOLUTION

Geometry.

$$\mathbf{r}_{B/D} = (0.15\text{ m})\mathbf{i} - (0.18\text{ m})\mathbf{j} - (0.36\text{ m})\mathbf{k}$$

$$\mathbf{r}_{B/C} = -(0.18\text{ m})\mathbf{j}$$

Let frame *Dxyz*, which coincides with the fixed frame *DXYZ* at the instant shown, be rotating about the *Y* axis with constant angular velocity $\mathbf{\Omega} = \omega_1\mathbf{j} = (8\text{ rad/s})\mathbf{j}$. Then the motion relative to the frame consists of a rotation of the disk *AB* about the bent axle *CD* with constant angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{k} = (12\text{ rad/s})\mathbf{k}$.

Motion of the coinciding Point B′ in the frame.

$$\mathbf{v}_{B'} = \mathbf{\Omega} \times \mathbf{r}_{B/D}$$
$$= 8\mathbf{j} \times (0.15\mathbf{i} - 0.18\mathbf{j} - 0.36\mathbf{k})$$
$$= -(2.88\text{ m/s})\mathbf{i} - (1.2\text{ m/s})\mathbf{k}$$

$$\mathbf{a}_{B'} = \mathbf{\Omega} \times \mathbf{v}_{B'}$$
$$= 8\mathbf{j} \times (-2.88\mathbf{i} - 1.2\mathbf{k})$$
$$= -(9.6\text{ m/s}^2)\mathbf{i} + (23.04\text{ m/s}^2)\mathbf{k}$$

Motion of Point B relative to the frame.

$$\mathbf{v}_{B/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_{B/D}$$
$$= 12\mathbf{k} \times (-0.18\mathbf{j})$$
$$= (2.16\text{ m/s})\mathbf{i}$$

$$\mathbf{a}_{B/F} = \boldsymbol{\omega}_2 \times \mathbf{v}_{B/F}$$
$$= 12\mathbf{k} \times 2.16\mathbf{i}$$
$$= (25.92\text{ m/s}^2)\mathbf{j}$$

Velocity of Point B.

$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$$
$$\mathbf{v}_B = -2.88\mathbf{i} - 1.2\mathbf{k} + 2.16\mathbf{i}$$

$$\mathbf{v}_B = -(0.720\text{ m/s})\mathbf{i} - (1.200\text{ m/s})\mathbf{k} \blacktriangleleft$$

PROBLEM 15.243 (Continued)

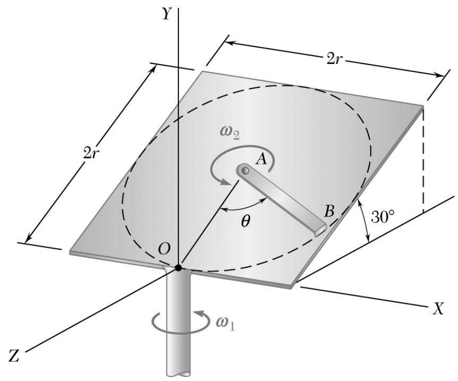
Coriolis acceleration. $2\boldsymbol{\Omega}\times\mathbf{v}_{B/F}$

$$2\boldsymbol{\Omega}\times\mathbf{v}_{B/F} = (2)(8\mathbf{j})\times(2.16\mathbf{i})$$
$$= -(34.56)\mathbf{k}$$

Acceleration of Point B. $\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\boldsymbol{\Omega}\times\mathbf{v}_{B/F}$

$$\mathbf{a}_B = -9.6\mathbf{i} + 23.04\mathbf{k} + 25.92\mathbf{j} - 34.56\mathbf{k}$$

$$\mathbf{a}_B = -(9.60\ \text{m/s}^2)\mathbf{i} + (25.9\ \text{m/s}^2)\mathbf{j} - (11.52\ \text{m/s}^2)\mathbf{k}$$ ◀



PROBLEM 15.244

A square plate of side $2r$ is welded to a vertical shaft which rotates with a constant angular velocity $\boldsymbol{\omega}_1$. At the same time, rod AB of length r rotates about the center of the plate with a constant angular velocity $\boldsymbol{\omega}_2$ with respect to the plate. For the position of the plate shown, determine the acceleration of end B of the rod if (*a*) $\theta = 0$, (*b*) $\theta = 90°$, (*c*) $\theta = 180°$.

SOLUTION

Use a frame of reference moving with the plate.

Its angular velocity is $\boldsymbol{\Omega} = \omega_1 \mathbf{j} \quad (\dot{\boldsymbol{\Omega}} = 0)$

Geometry:

$$\mathbf{r}_{A/O} = r(\sin 30°\mathbf{j} - \cos 30°\mathbf{k})$$

$$\mathbf{r}_{B/O} = \mathbf{r}_{A/O} + \mathbf{r}_{B/A}$$

Acceleration of coinciding Point B' in the frame.

$$\mathbf{a}_{B'} = \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/O})$$

Motion relative to the frame. (Rotation about A with angular velocity $\boldsymbol{\omega}_2$).

$$\boldsymbol{\omega}_2 = \omega_2(\cos 30°\mathbf{j} + \sin 30°\mathbf{k}) \qquad (\dot{\omega}_2 = 0)$$

$$\mathbf{v}_{B/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_{B/A}$$

$$\mathbf{a}_{B/F} = \boldsymbol{\omega}_2 \times \mathbf{v}_{B/F} = -\omega_2^2 \mathbf{r}_{B/A}$$

Coriolis acceleration: $\quad 2\boldsymbol{\Omega} \times \mathbf{v}_{B/F}$

Acceleration of B. $\quad \mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\boldsymbol{\Omega} \times \mathbf{v}_{B/F}$

(*a*) $\quad \theta = 0$

$$\mathbf{r}_{B/A} = r(-\sin 30°\mathbf{j} + \cos 30°\mathbf{k})$$

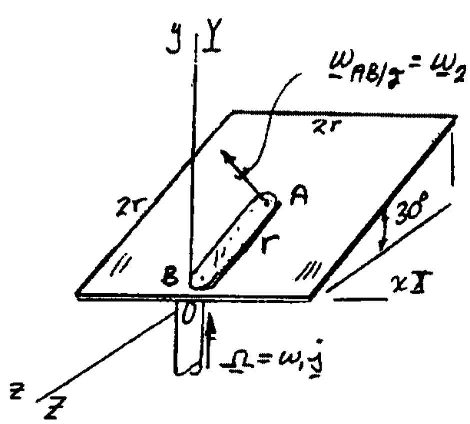
$$\mathbf{r}_{B/O} = 0$$

$$\mathbf{a}_{B'} = 0$$

$$\mathbf{v}_{B/F} = r\omega_2 \mathbf{i}$$

$$\mathbf{a}_{B/F} = -\omega_2^2 \mathbf{r}_{B/A} = r\omega_2^2(\sin 30°\mathbf{j} - \cos 30°\mathbf{k})$$

$$2\boldsymbol{\Omega} \times \mathbf{v}_{B/F} = 2(\omega_1 \mathbf{j}) \times (r\omega_2 \mathbf{i}) = -2r\omega_1\omega_2 \mathbf{k}$$



$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + \mathbf{a}_C$$

$$= 0 + r\omega_2^2(\sin 30°\mathbf{j} - \cos 30°\mathbf{k}) - 2r\omega_1\omega_2 \mathbf{k}$$

$$\mathbf{a}_B = r\omega_2^2 \sin 30°\mathbf{j} - (r\omega_2^2 \cos 30° + 2r\omega_1\omega_2)\mathbf{k} \blacktriangleleft$$

PROBLEM 15.244 (Continued)

(*b*) $\theta = 90°$

$$\mathbf{r}_{B/A} = r\mathbf{i}$$

$$\mathbf{r}_{B/O} = r\mathbf{i} + r\sin 30°\mathbf{j} - r\cos 30°\mathbf{k}$$

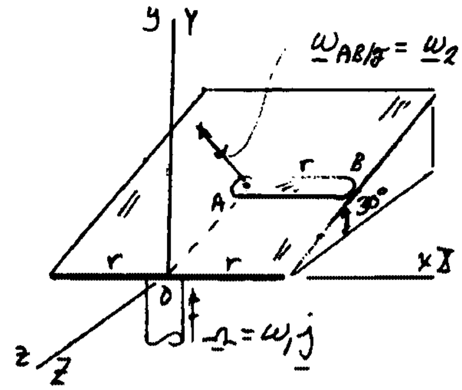
$$a_{B'} = \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r}_{B/O})$$

$$= \omega_1\mathbf{j}\times[\omega_1\mathbf{j}\times(r\mathbf{i} + r\sin 30°\mathbf{j} - r\cos 30°\mathbf{k})]$$

$$= -r\omega_1^2\mathbf{i} + r\omega_1^2\cos 30°\mathbf{k}$$

$$\mathbf{v}_{B/F} = r\omega_2(\sin 30°\mathbf{j} - \cos 30°\mathbf{k})$$

$$\mathbf{a}_{B/F} = -r\omega_2^2\mathbf{i}$$



$$2\mathbf{\Omega}\times\mathbf{v}_{B/F} = 2(\omega_1\mathbf{j})\times(r\omega_2)(\sin 30°\mathbf{j} - \cos 30°\mathbf{k})$$

$$= -2r\omega_1\omega_2\cos 30°\mathbf{i}$$

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + \mathbf{a}_C$$

$$= [-r\omega_1^2\mathbf{i} + r\omega_1^2\cos 30°\mathbf{k}] - r\omega_2^2\mathbf{i} - 2r\omega_1\omega_2\cos 30°\mathbf{i}$$

$$\mathbf{a}_B = -r(\omega_1^2 + \omega_2^2 + 2\omega_1\omega_2\cos 30°)\mathbf{i} + r\omega_1^2\cos 30°\mathbf{k} \blacktriangleleft$$

(*c*) $\theta = 180°$

$$\mathbf{r}_{B/A} = r(\sin 30°\mathbf{j} - \cos 30°\mathbf{k})$$

$$\mathbf{r}_{B/O} = 2r(\sin 30°\mathbf{j} - \cos 30°\mathbf{k})$$

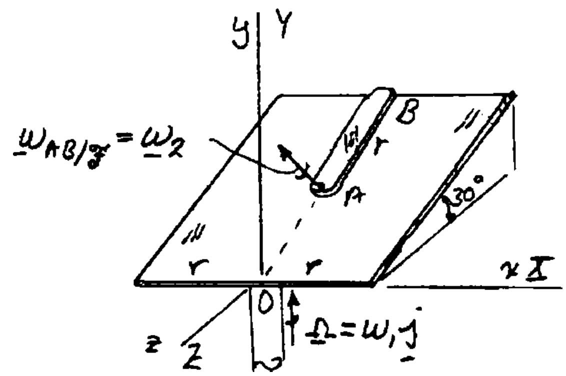
$$\mathbf{a}_{B'} = +(2r\cos 30°)\omega_1^2\mathbf{k} = +2r\omega_1^2\cos 30°\mathbf{k}$$

$$\mathbf{v}_{B/F} = -r\omega_2\mathbf{i}$$

$$\mathbf{a}_{B/F} = r\omega_2^2(-\sin 30°\mathbf{j} + \cos 30°\mathbf{k})$$

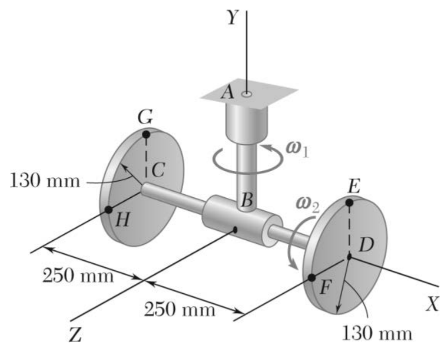
$$2\mathbf{\Omega}\times\mathbf{v}_{B/F} = 2(\omega_1\mathbf{j})\times(-r\omega_2\mathbf{i}) = 2r\omega_1\omega_2\mathbf{k}$$

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\mathbf{\Omega}\times\mathbf{v}_{B/F}$$



$$= 2r\omega_1^2\cos 30°\mathbf{k} + r\omega_2^2(-\sin 30°\mathbf{j} + \cos 30°\mathbf{k}) + 2r\omega_1\omega_2\mathbf{k}$$

$$\mathbf{a}_B = -r\omega_2^2\sin 30°\mathbf{j} + r(2\omega_1^2\cos 30° + \omega_2^2\cos 30° + 2\omega_1\omega_2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.245

Two disks, each of 130-mm radius, are welded to the 500-mm rod *CD*. The rod-and-disks unit rotates at the constant rate $\omega_2 = 3$ rad/s with respect to arm *AB*. Knowing that at the instant shown $\omega_1 = 4$ rad/s, determine the velocity and acceleration of (*a*) Point *E*, (*b*) Point *F*.

SOLUTION

Let the frame of reference *BXYZ* be rotating about the *Y* axis with angular velocity $\mathbf{\Omega} = \omega_2 \mathbf{j} = (4 \text{ rad/s})\mathbf{j}$. The motion relative to this frame is a rotation about the *X* axis with angular velocity $\omega_x \mathbf{i} = (3 \text{ rad/s})\mathbf{i}$.

(*a*) *Point E.*

$$\mathbf{r}_{E/B} = (0.25 \text{ m})\mathbf{i} + (0.13 \text{ m})\mathbf{j}$$

$$\mathbf{r}_{E/D} = (0.13 \text{ m})\mathbf{j}$$

Motion of Point E′ in the frame.

$$\begin{aligned}\mathbf{v}_{E'} &= \mathbf{\Omega} \times \mathbf{r}_{E/B} \\ &= 4\mathbf{j} \times (0.25\mathbf{i} + 0.13\mathbf{j}) \\ &= -(1 \text{ m/s})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{E'} &= \mathbf{\Omega} \times \mathbf{v}_{E'} \\ &= 4\mathbf{j} \times (-\mathbf{k}) \\ &= -(4 \text{ m/s}^2)\mathbf{i}\end{aligned}$$

Motion of Point E relative to the frame.

$$\begin{aligned}\mathbf{v}_{E/F} &= \omega_x \mathbf{i} \times \mathbf{r}_{E/D} \\ &= 3\mathbf{i} \times 0.13\mathbf{j} \\ &= (0.39 \text{ m/s})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{E/F} &= \omega_x \mathbf{i} \times \mathbf{v}_{E/F} \\ &= 3\mathbf{i} \times (0.39\mathbf{k}) \\ &= -(1.17 \text{ m/s}^2)\mathbf{j}\end{aligned}$$

Coriolis acceleration.

$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{E/F}$$

$$\mathbf{a}_c = (2)(4\mathbf{j}) \times (0.39\mathbf{k}) = (3.12 \text{ m/s}^2)\mathbf{i}$$

Velocity of Point E.

$$\mathbf{v}_E = \mathbf{v}_{E'} + \mathbf{v}_{E/F}$$

$$\mathbf{v}_E = -\mathbf{k} + 0.39\mathbf{k} = 0.61\mathbf{k} \qquad \mathbf{v}_E = (0.610 \text{ m/s})\mathbf{k} \blacktriangleleft$$

Acceleration of Point E.

$$\mathbf{a}_E = \mathbf{a}_{E'} + \mathbf{a}_{E/F} + \mathbf{a}_c$$

$$\mathbf{a}_E = -4\mathbf{i} - 1.17\mathbf{j} + 3.12\mathbf{i} \qquad \mathbf{a}_E = -(0.880 \text{ m/s}^2)\mathbf{i} + (1.170 \text{ m/s}^2)\mathbf{j} \blacktriangleleft$$

(*b*) *Point F.*

$$\mathbf{r}_{F/B} = (0.25\text{ m})\mathbf{i} + (0.13\text{ m})\mathbf{k}$$
$$\mathbf{r}_{F/D} = (0.13\text{ m})\mathbf{k}$$

Motion of Point F′ in the frame.

$$\mathbf{v}_{F'} = \mathbf{\Omega} \times \mathbf{r}_{F/B}$$
$$= 4\mathbf{j} \times (0.25\mathbf{i} + 0.13\mathbf{k})$$
$$= (0.52\text{ m/s})\mathbf{i} - (1\text{ m/s})\mathbf{k}$$
$$\mathbf{a}_{F'} = \mathbf{\Omega} \times \mathbf{v}_{F'}$$
$$= (4\mathbf{j}) \times (0.52\mathbf{i} - \mathbf{k})$$
$$= -(4\text{ m/s}^2)\mathbf{i} - (2.08\text{ m/s}^2)\mathbf{k}$$

Motion of Point F relative to the frame.

$$\mathbf{v}_{F/F} = \omega_x \mathbf{i} \times \mathbf{r}_{F/D}$$
$$= 3\mathbf{i} \times (0.13\mathbf{k})$$
$$= -(0.39\text{ m/s})\mathbf{j}$$
$$\mathbf{a}_{F/F} = \omega_x \mathbf{i} \times \mathbf{v}_{F/F}$$
$$= 3\mathbf{i} \times (-0.39\mathbf{j})$$
$$= -(1.17\text{ m/s}^2)\mathbf{k}$$

Coriolis acceleration.

$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{F/F}$$
$$\mathbf{a}_c = (2)(4\mathbf{j}) \times (-0.39\mathbf{j}) = 0$$

Velocity of Point F.

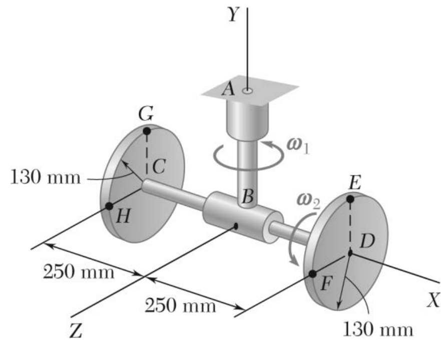
$$\mathbf{v}_F = \mathbf{v}_{F'} + \mathbf{v}_{F/F}$$
$$\mathbf{v}_F = 0.52\mathbf{i} - \mathbf{k} - 0.39\mathbf{j}$$

$$\mathbf{v}_F = (0.520\text{ m/s})\mathbf{i} - (0.390\text{ m/s})\mathbf{j} - (1.000\text{ m/s})\mathbf{k} \blacktriangleleft$$

Acceleration of Point F.

$$\mathbf{a}_F = \mathbf{a}_{F'} + \mathbf{a}_{F/F} + \mathbf{a}_c$$
$$\mathbf{a}_F = -4\mathbf{i} - 2.08\mathbf{k} - 1.17\mathbf{k} + 0$$

$$\mathbf{a}_F = -(4.00\text{ m/s}^2)\mathbf{i} - (3.25\text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.246

In Problem 15.245, determine the velocity and acceleration of (*a*) Point *G*, (*b*) Point *H*.

PROBLEM 15.245 Two disks, each of 130-mm radius, are welded to the 500-mm rod *CD*. The rod-and-disks unit rotates at the constant rate $\omega_2 = 3$ rad/s with respect to arm *AB*. Knowing that at the instant shown $\omega_1 = 4$ rad/s, determine the velocity and acceleration of (*a*) Point *E*, (*b*) Point *F*.

SOLUTION

Let the frame of reference *BXYZ* be rotating about the *Y* axis with angular velocity $\mathbf{\Omega} = \omega_2 \mathbf{j} = (4 \text{ rad/s})\mathbf{j}$. The motion relative to this frame is a rotation about the *X* axis with angular velocity $\omega_x \mathbf{i} = (3 \text{ rad/s})\mathbf{i}$.

(*a*) *Point G.*

$$\mathbf{r}_{G/B} = -(0.25 \text{ m})\mathbf{i} + (0.13 \text{ m})\mathbf{j}$$
$$\mathbf{r}_{G/C} = (0.13 \text{ m})\mathbf{j}$$

Motion of Point G′ in the frame.

$$\mathbf{v}_{G'} = \mathbf{\Omega} \times \mathbf{r}_{G/B}$$
$$= 4\mathbf{j} \times (-0.25\mathbf{i} + 0.13\mathbf{j})$$
$$= (1 \text{ m/s})\mathbf{k}$$
$$\mathbf{a}_{G'} = \mathbf{\Omega} \times \mathbf{v}_{G'}$$
$$= 4\mathbf{j} \times \mathbf{k}$$
$$= (4 \text{ m/s}^2)\mathbf{i}$$

Motion of Point G relative to the frame.

$$\mathbf{v}_{G/F} = \omega_x \mathbf{i} \times \mathbf{r}_{G/C}$$
$$= 3\mathbf{i} \times 0.13\mathbf{j}$$
$$= (0.39 \text{ m/s})\mathbf{k}$$
$$\mathbf{a}_{G/F} = \omega_x \mathbf{i} \times \mathbf{v}_{G/F}$$
$$= 3\mathbf{i} \times (0.39\mathbf{k})$$
$$= -(1.17 \text{ m/s}^2)\mathbf{j}$$

Coriolis acceleration.

$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{G/F}$$
$$\mathbf{a}_c = (2)(4\mathbf{j}) \times (0.39\mathbf{k})$$
$$= (3.12 \text{ m/s}^2)\mathbf{i}$$

Velocity of Point G.

$$\mathbf{v}_G = \mathbf{v}_{G'} + \mathbf{v}_{G/F}$$
$$\mathbf{v}_G = \mathbf{k} + 0.39\mathbf{k}$$

$\mathbf{v}_G = (1.390 \text{ m/s})\mathbf{k}$ ◀

PROBLEM 15.246 (Continued)

Acceleration of Point G. $\mathbf{a}_G = \mathbf{a}_{G'} + \mathbf{a}_{G/F} + \mathbf{a}_c$

$\mathbf{a}_G = 4\mathbf{i} - 1.17\mathbf{j} + 3.12\mathbf{i}$ $\quad \mathbf{a}_G = (7.12 \text{ m/s}^2)\mathbf{i} - (1.170 \text{ m/s}^2)\mathbf{j}$ ◀

(*b*) *Point H.* $\mathbf{r}_{H/B} = -(0.25 \text{ m})\mathbf{i} + (0.13 \text{ m})\mathbf{k}$

$\mathbf{r}_{H/C} = (0.13 \text{ m})\mathbf{k}$

Motion of Point H′ in the frame.

$$\begin{aligned} \mathbf{v}_{H'} &= \mathbf{\Omega} \times \mathbf{r}_{H/B} \\ &= 4\mathbf{j} \times (-0.25\mathbf{i} + 0.13\mathbf{k}) \\ &= (0.52 \text{ m/s})\mathbf{i} + (1 \text{ m/s})\mathbf{k} \\ \mathbf{a}_{H'} &= \mathbf{\Omega} \times \mathbf{v}_{H'} \\ &= 4\mathbf{j} \times (0.52\mathbf{i} + \mathbf{k}) \\ &= (4 \text{ m/s}^2)\mathbf{i} - (2.08 \text{ m/s}^2)\mathbf{k} \end{aligned}$$

Motion of Point H relative to the frame.

$$\begin{aligned} \mathbf{v}_{H/F} &= \omega_x \mathbf{i} \times \mathbf{r}_{H/C} \\ &= 3\mathbf{i} \times (0.13\mathbf{k}) \\ &= -(0.39 \text{ m/s})\mathbf{j} \\ \mathbf{a}_{H/F} &= \omega_x \mathbf{i} \times \mathbf{v}_{H/F} \\ &= 3\mathbf{i} \times (-0.39\mathbf{j}) \\ &= -(1.17 \text{ m/s}^2)\mathbf{k} \end{aligned}$$

Coriolis acceleration. $\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{H/F}$

$\mathbf{a}_c = (2)(4\mathbf{j}) \times (0.39\mathbf{j}) = 0$

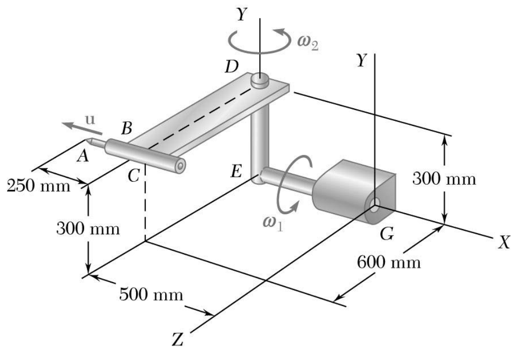
Velocity of Point H. $\mathbf{v}_H = \mathbf{v}_{H'} + \mathbf{v}_{H/F}$

$\mathbf{v}_H = 0.52\mathbf{i} + \mathbf{k} - 0.39\mathbf{j}$

$\mathbf{v}_H = (0.520 \text{ m/s})\mathbf{i} - (0.390 \text{ m/s})\mathbf{j} + (1.000 \text{ m/s})\mathbf{k}$ ◀

Acceleration of Point H. $\mathbf{a}_H = \mathbf{a}_{H'} + \mathbf{a}_{H/F} + \mathbf{a}_c$

$\mathbf{a}_H = 4\mathbf{i} - 2.08\mathbf{k} - 1.17\mathbf{k} + 0$ $\quad \mathbf{a}_H = (4.00 \text{ m/s}^2)\mathbf{i} - (3.25 \text{ m/s}^2)\mathbf{k}$ ◀



PROBLEM 15.247

The position of the stylus tip A is controlled by the robot shown. In the position shown the stylus moves at a constant speed $u = 180$ mm/s relative to the solenoid BC. At the same time, arm CD rotates at the constant rate $\omega_2 = 1.6$ rad/s with respect to component DEG. Knowing that the entire robot rotates about the X axis at the constant rate $\omega_1 = 1.2$ rad/s, determine (a) the velocity of A, (b) the acceleration of A.

SOLUTION

Geometry:

$$\mathbf{r}_{D/G} = -(500 \text{ mm})\mathbf{i} + (300 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{A/D} = -(250 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{A/G} = \mathbf{r}_{A/D} + \mathbf{r}_{D/G} = -(750 \text{ mm})\mathbf{i} + (300 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}$$

Angular velocities:

$$\boldsymbol{\omega}_1 = \omega_1\mathbf{i} = (1.2 \text{ rad/s})\mathbf{i} \qquad (\dot{\omega}_1 = 0)$$

$$\boldsymbol{\omega}_2 = \omega_2\mathbf{j} = (1.6 \text{ rad/s})\mathbf{j} \qquad (\dot{\omega}_2 = 0)$$

Stylus motion:

$$\mathbf{u} = -u\mathbf{i} = -(180 \text{ mm/s})\mathbf{i} \qquad (\dot{u} = 0)$$

Method 1

Let the rigid body BCD be a rotating frame of reference.

Its angular velocity is $\boldsymbol{\omega}_{CD} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 = (1.2 \text{ rad/s})\mathbf{i} + (1.6 \text{ rad/s})\mathbf{j}$

Its angular acceleration is

$$\boldsymbol{\alpha}_{CD} = \boldsymbol{\omega}_1 \times \boldsymbol{\omega}_{CD} = (1.2 \text{ rad/s})\mathbf{i} \times [(1.2 \text{ rad/s})\mathbf{i} + (1.6 \text{ rad/s})\mathbf{j}]$$
$$= (1.92 \text{ rad/s}^2)\mathbf{k}$$

Motion of the coinciding Point A' in the frame.

$$\mathbf{v}_{A'} = \mathbf{v}_D + \mathbf{v}_{A'/D}$$

$$\mathbf{v}_{A'} = \boldsymbol{\omega}_1 \times \mathbf{r}_{D/G} + (\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) \times \mathbf{r}_{A/D}$$
$$= (1.2 \text{ rad/s})\mathbf{i} \times [-(500 \text{ mm})\mathbf{i} + (300 \text{ mm})\mathbf{j}]$$
$$+ [(1.2 \text{ rad/s})\mathbf{i} + (1.6 \text{ rad/s})\mathbf{j}] \times [-(250 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{k}]$$
$$= (360 \text{ mm/s})\mathbf{k} - (720 \text{ mm/s})\mathbf{j} + (400 \text{ mm/s})\mathbf{k} + (960 \text{ mm/s})\mathbf{i}$$

$$\mathbf{v}_{A'} = (960 \text{ mm/s})\mathbf{i} - (720 \text{ mm/s})\mathbf{j} + (760 \text{ mm/s})\mathbf{k}$$

$$\mathbf{a}_D = \omega_1 \times (\boldsymbol{\omega}_1 \times \mathbf{r}_{D/G})$$
$$= (1.2 \text{ rad/s})\mathbf{i} \times \{(1.2 \text{ rad/s})\mathbf{i} \times [-(500 \text{ mm})\mathbf{i} + (300 \text{ mm})\mathbf{j}]\}$$
$$= -(432 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}_{A'/D} = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{A/D} + \boldsymbol{\omega}_{CD} \times (\boldsymbol{\omega}_{CD} \times \mathbf{r}_{A/D})$$
$$= (1.92 \text{ rad/s}^2)\mathbf{k} \times [-(250 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{k}]$$
$$+ \boldsymbol{\omega}_{CD} \times \{[(1.2 \text{ rad/s})\mathbf{i} + (1.6 \text{ rad/s})\mathbf{j}] \times [-(250 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{k}]\}$$

$$\mathbf{a}_{A'/D} = -(480\text{ mm/s}^2)\mathbf{j} + \omega_{CD} \times [-(720\text{ mm/s})\mathbf{j} + (400\text{ mm/s})\mathbf{k} + (960\text{ mm/s})\mathbf{i}]$$

$$= -480\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 1.6 & 0 \\ 960 & -720 & 400 \end{vmatrix}$$

$$= -480\mathbf{j} + 640\mathbf{i} - 480\mathbf{j} - 864\mathbf{k} - 1536\mathbf{k}$$

$$\mathbf{a}_{A'/D} = (640\text{ mm/s}^2)\mathbf{i} - (960\text{ mm/s}^2)\mathbf{j} - (2400\text{ mm/s}^2)\mathbf{k}$$

$$\mathbf{a}_{A'} = \mathbf{a}_D + \mathbf{a}_{A'/D}$$

$$\mathbf{a}_{A'} = (640\text{ mm/s}^2)\mathbf{i} - (1392\text{ mm/s}^2)\mathbf{j} - (2400\text{ mm/s}^2)\mathbf{k}$$

Motion of Point A relative to the frame.

$$\mathbf{v}_{A/F} = \mathbf{u} = -(180\text{ mm/s})\mathbf{i}$$

$$\mathbf{a}_{A/F} = 0$$

(*a*) *Velocity of A.* $\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/F}$

$$\mathbf{v}_A = (960\text{ mm/s})\mathbf{i} - (720\text{ mm/s})\mathbf{j} + (760\text{ mm/s})\mathbf{k} - (180\text{ mm/s})\mathbf{i}$$

$$\mathbf{v}_A = (0.78\text{ m/s})\mathbf{i} - (0.72\text{ m/s})\mathbf{j} + (0.76\text{ m/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration: $\mathbf{a}_c = 2\boldsymbol{\omega}_{CD} \times \mathbf{v}_{A/F}$

$$\mathbf{a}_c = 2[(1.2\text{ rad/s})\mathbf{i} + (1.6\text{ rad/s})\mathbf{j}] \times (-180\text{ mm/s})\mathbf{i}$$

$$= +(576\text{ mm/s}^2)\mathbf{k}$$

(*b*) *Acceleration of A.* $\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + \mathbf{a}_c$

$$\mathbf{a}_A = (640\text{ mm/s}^2)\mathbf{i} - (1392\text{ mm/s}^2)\mathbf{j} - (2400\text{ mm/s}^2)\mathbf{k} + (576\text{ mm/s}^2)\mathbf{k}$$

$$\mathbf{a}_A = (0.64\text{ m/s}^2)\mathbf{i} - (1.392\text{ m/s}^2)\mathbf{j} - (1.824\text{ m/s}^2)\mathbf{k} \blacktriangleleft$$

Method 2

Use a frame of reference rotating about the x axis with angular velocity.

$$\boldsymbol{\omega}_1 = \omega_1\mathbf{i} = (1.2\text{ rad/s})\mathbf{i} \qquad (\dot{\omega}_1 = 0)$$

Motion of coinciding Point A' in the frame.

$$\mathbf{v}_{A'} = \boldsymbol{\omega}_1 \times \mathbf{r}_{A/G} = (1.2\text{ rad/s})\mathbf{i} \times [-(500\text{ mm})\mathbf{i} + (300\text{ mm})\mathbf{j} + (600\text{ mm})\mathbf{k}]$$

$$= (360\text{ mm/s})\mathbf{k} - (720\text{ mm/s})\mathbf{j}$$

$$\mathbf{a}_{A'} = \omega_1 \times (\boldsymbol{\omega}_1 \times \mathbf{r}_{A/G}) = \boldsymbol{\omega}_1 \times \mathbf{v}_{A'}$$

$$= (1.2\text{ rad/s})\mathbf{i} \times [(360\text{ mm/s})\mathbf{k} - (720\text{ mm/s})\mathbf{j}]$$

$$= -(432\text{ mm/s}^2)\mathbf{j} - (864\text{ mm/s}^2)\mathbf{k}$$

PROBLEM 15.247 (Continued)

Motion of Point A relative to the frame.

$$\mathbf{v}_{A/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_{A/D} + \mathbf{u}$$
$$= (1.6 \text{ rad/s})\mathbf{j} \times [-(250 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{k}] - (180 \text{ mm/s})\mathbf{i}$$
$$= (400 \text{ mm/s})\mathbf{k} + (960 \text{ mm/s})\mathbf{i} - (180 \text{ mm/s})\mathbf{i}$$

$\mathbf{a}_{A/F}$: (Since *A* moves on *CD*, which rotates at rate $\boldsymbol{\omega}_2$, we have a Coriolis term here).

$$\mathbf{a}_{A/F} = \boldsymbol{\omega}_2 \times (\boldsymbol{\omega}_2 \times \mathbf{r}_{A/D}) + 2\boldsymbol{\omega}_2 \times \mathbf{u}$$
$$= \boldsymbol{\omega}_2 \times \{(1.6 \text{ rad/s})\mathbf{j} \times [-(250 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{k}]\} + 2\boldsymbol{\omega}_2 \times \mathbf{u}$$
$$= (1.6 \text{ rad/s})\mathbf{j} \times [(400 \text{ mm/s})\mathbf{k} + (960 \text{ mm/s})\mathbf{i}] + 2(1.6 \text{ rad/s})\mathbf{j} \times (-180 \text{ mm})\mathbf{i}$$
$$= (640 \text{ mm/s}^2)\mathbf{i} - (1536 \text{ mm/s}^2)\mathbf{k} + (576 \text{ mm/s}^2)\mathbf{k}$$
$$= (640 \text{ mm/s}^2)\mathbf{i} - (960 \text{ mm/s}^2)\mathbf{k}$$

(*a*) *Velocity of A.*

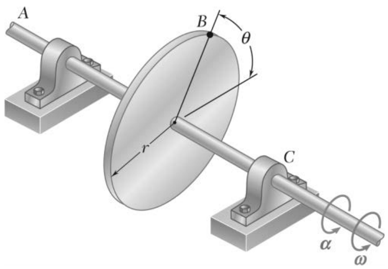
$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/F}$$
$$\mathbf{v}_A = 360\mathbf{k} - 720\mathbf{j} + 400\mathbf{k} + 960\mathbf{i} - 180\mathbf{i}$$
$$\mathbf{v}_A = (0.78 \text{ m/s})\mathbf{i} - (0.72 \text{ m/s})\mathbf{j} + (0.76 \text{ m/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration:

$$\mathbf{a}_c = 2\boldsymbol{\omega}_1 \times \mathbf{v}_{A/F}$$
$$\mathbf{a}_c = 2(1.2 \text{ rad/s})\mathbf{i} \times [(400 \text{ mm})\mathbf{k} + (780 \text{ mm/s})\mathbf{i}]$$
$$= -(960 \text{ mm/s}^2)\mathbf{j}$$

(*b*) *Acceleration of A.*

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + \mathbf{a}_c$$
$$\mathbf{a}_A = -432\mathbf{j} - 864\mathbf{k} + 640\mathbf{i} - 960\mathbf{k} - 960\mathbf{j}$$
$$\mathbf{a}_A = (0.64 \text{ m/s}^2)\mathbf{i} - (1.392 \text{ m/s}^2)\mathbf{j} - (1.824 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.248

The angular acceleration of the 600-mm-radius circular plate shown is defined by the relation $\alpha = \alpha_0 e^{-t}$. Knowing that the plate is at rest when $t = 0$ and that $\alpha_0 = 10 \text{ rad/s}^2$, determine the magnitude of the total acceleration of Point B when (*a*) $t = 0$, (*b*) $t = 0.5$ s, (*c*) $t = \infty$.

SOLUTION

$$\alpha = \frac{d\omega}{dt} = \alpha_0 e^{-t}; \quad \int_0^\omega d\omega = \int_0^t \alpha_0 e^{-t}\,dt$$

$$\omega = \alpha_0 \left| -e^{-t} \right|_0^t \qquad \omega = \alpha_0 (1 - e^{-t})$$

$$a_t = r\alpha = r\alpha_0 e^{-t} = (0.6 \text{ m})(10 \text{ rad/s}^2)e^{-t} = 6e^{-t}$$

$$a_n = r\omega^2 = r\alpha_0^2(1 - e^{-t})^2 = (0.6)(10)^2(1 - e^{-t})^2 = 60(1 - e^{-t})^2$$

(*a*) $t = 0$:

$$a_t = 6e^0 = 6 \text{ m/s}^2$$

$$a_n = 60(1 - e^0)^2 = 0$$

$$a_B^2 = a_t^2 + a_n^2 = 6^2 \qquad a_B = 6.00 \text{ m/s}^2 \blacktriangleleft$$

(*b*) $t = 0.5$ s:

$$a_t = 6e^{-0.5} = 6(0.6065) = 3.639 \text{ m/s}^2$$

$$a_n = 60(1 - e^{-0.5})^2$$

$$= 60(1 - 0.6065)^2$$

$$= 9.289 \text{ m/s}^2$$

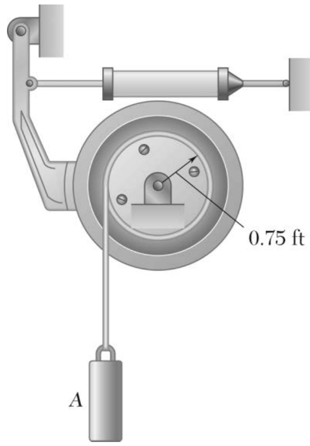
$$a_B^2 = a_t^2 + a_n^2 = (3.639)^2 + (9.289)^2 \qquad a_B = 9.98 \text{ m/s}^2 \blacktriangleleft$$

(*c*) $t = \infty$:

$$a_t = 6e^{-\infty} = 0$$

$$a_n = 60(1 - e^{-\infty})^2 = 60 \text{ m/s}^2$$

$$a_B^2 = a_t^2 + a_n^2 = 0 + 60^2 \qquad a_B = 60.0 \text{ m/s}^2 \blacktriangleleft$$



PROBLEM 15.249

Cylinder A is moving downward with a velocity of 9 ft/s when the brake is suddenly applied to the drum. Knowing that the cylinder moves 18 ft downward before coming to rest and assuming uniformly accelerated motion, determine (a) the angular acceleration of the drum, (b) the time required for the cylinder to come to rest.

SOLUTION

Block A:

$$v^2 - v_0^2 = 2as$$

$$0 - (9 \text{ ft/s})^2 = 2a\,(18 \text{ ft})$$

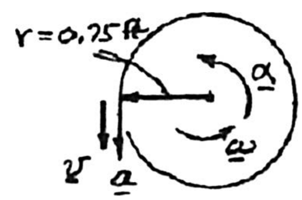
$$a = -2.25 \text{ ft/s}^2 \qquad \mathbf{a} = 2.25 \text{ ft/s}^2 \uparrow$$

Drum:

$$v_A = r\omega_0$$

$$9 \text{ ft/s} = (0.75 \text{ ft})\omega$$

$$\omega_0 = 12 \text{ rad/s}$$



(a) $a = r\alpha$

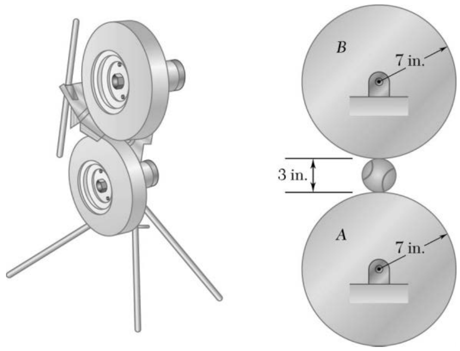
$$-(2.25 \text{ ft/s}^2) = (0.75 \text{ ft})\alpha$$

$$\alpha = -3 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 3.00 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

(b) Uniformly accelerated motion. $\omega = 0$ when $t = t_1$

$$\omega = \omega_0 + \alpha t: \quad 0 = (12 \text{ rad/s}) - (3 \text{ rad/s}^2)t_1 \qquad t_1 = 4.00 \text{ s} \blacktriangleleft$$

PROBLEM 15.250



A baseball pitching machine is designed to deliver a baseball with a ball speed of 70 mph and a ball rotation of 300 rpm clockwise. Knowing that there is no slipping between the wheels and the baseball during the ball launch, determine the angular velocities of wheels *A* and *B*.

SOLUTION

Let Point *G* be the center of the ball, *A* its contact point with wheel *A*, and *B* its contact point with wheel *B*.

Given: $$v_G = \left(70\frac{\text{m}}{\text{h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{5280\text{ ft}}{\text{mi}}\right) = 102.667\text{ ft/s} = 1232\text{ in./s}$$

$$\omega_{\text{ball}} = (300\text{ rpm})\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 10\pi\text{ rad/s}$$

$$\mathbf{v}_G = 1232\text{ in./s} \longrightarrow \qquad \boldsymbol{\omega}_{\text{ball}} = 10\pi\text{ rad/s} \circlearrowright$$

Unit vectors: $\mathbf{i} = 1\longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \mathbf{k} = 1\circlearrowleft$

Relative positions: $$\mathbf{r}_{A/G} = \frac{1}{2}(-3\mathbf{j}) = -(1.5\text{ in.})\mathbf{j}$$

$$\mathbf{r}_{B/G} = \frac{1}{2}(3\mathbf{j}) = (1.5\text{ in.})\mathbf{j}$$

Velocities at *A* and *B*.

$$\mathbf{v}_A = \mathbf{v}_G + \boldsymbol{\omega}_{\text{ball}} \times \mathbf{r}_{A/G} = 1232\mathbf{i} + (-10\pi\mathbf{k}) \times (1.5\mathbf{j})$$
$$= (1232 + 47.12)\mathbf{i} = 1279.12\text{ in./s} \longrightarrow$$

$$\mathbf{v}_B = \mathbf{v}_G + \boldsymbol{\omega}_{\text{ball}} \times \mathbf{r}_{B/G} = 1232\mathbf{i} + (-10\pi\mathbf{k}) \times (1.5\pi\mathbf{j})$$
$$= (1232 - 47.12)\mathbf{i} = 1184.88\text{ in./s} \longrightarrow$$

Angular velocity of A.

$$\omega_A = \frac{v_A}{r_A} = \frac{1184.88}{7} = 169.27\text{ rad/s}$$

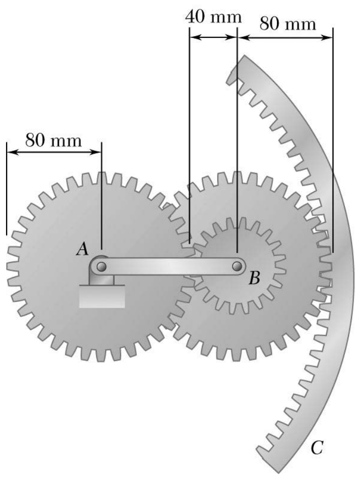
$$\boldsymbol{\omega}_A = 1616\text{ rpm} \circlearrowright \blacktriangleleft$$

Angular velocity of B.

$$\omega_B = \frac{v_B}{r_B} = \frac{1279.12}{7} = 182.73\text{ rad/s} \circlearrowleft$$

$$\boldsymbol{\omega}_B = 1745\text{ rpm} \circlearrowleft \blacktriangleleft$$

PROBLEM 15.251



Knowing that inner gear A is stationary and outer gear C starts from rest and has a constant angular acceleration of 4 rad/s^2 clockwise, determine at $t = 5$ s (*a*) the angular velocity of arm AB, (*b*) the angular velocity of gear B, (*c*) the acceleration of the point on gear B that is in contact with gear A.

SOLUTION

Angular velocity of gear C at $t = 5$ s.

$$\omega_c = \alpha_c t = (4 \text{ rad/s}^2)(5 \text{ s}) = 20 \text{ rad/s} \qquad \boldsymbol{\omega}_c = 20 \text{ rad/s} \circlearrowright$$

Let Point 1 be the contact point between gears A and B. Let Point 2 be the contact point between gears B and C. Points A, B, and C are the centers, respectively, of gears A, B, and C.

Positions: Take x axis along the straight line A1 B2.

$$x_A = x_C = 0$$
$$x_1 = r_A = 80 \text{ mm}$$
$$x_B = (80 \text{ mm} + 40 \text{ mm}) = 120 \text{ mm}$$
$$x_2 = 120 \text{ mm} + 80 \text{ mm} = 200 \text{ mm}$$

Velocity at 1. Since gear A is stationary, $v_1 = 0$.

Velocity at 2. $v_2 = x_2 \omega_C = (200)(20)$

$$\mathbf{v}_2 = 4000 \text{ mm/s} \downarrow$$

Point 1 is the instantaneous center of gear B.

$$v_2 = (x_2 - x_1)\omega_B = (120 \text{ mm})\omega_B$$
$$\omega_B = \frac{v_2}{120} = \frac{4000}{120} = 33.333 \text{ rad/s}$$
$$v_B = (x_B - x_1)\omega_B = (40 \text{ mm})(33.333 \text{ rad/s}) = 1333.33 \text{ mm/s}$$
$$\omega_{AB} = \frac{v_B}{x_B} = \frac{1333.33}{120} = 11.111 \text{ rad/s}$$

(*a*) *Angular velocity of arm AB*: $\boldsymbol{\omega}_{AB} = 11.11 \text{ rad/s} \circlearrowright$ ◀

(*b*) *Angular velocity of gear B*: $\boldsymbol{\omega}_B = 33.3 \text{ rad/s} \circlearrowright$ ◀

PROBLEM 15.251 (Continued)

Calculate tangential accelerations:

$$(a_1)_t = 0$$

$$(a_2)_t = (a_C)_t = r_C\alpha_2 = (200 \text{ mm})(4 \text{ rad/s}^2)$$

$$(a_2)_t = 800 \text{ mm/s}^2 \downarrow$$

$$(a_2)_t = (x_2 - x_1)\alpha_B = (120 \text{ mm})\alpha_B$$

$$\alpha_B = \frac{(a_2)_t}{120} = \frac{800}{120} = 6.667 \text{ rad/s}^2$$

$$(a_B)_t = (x_B - x_1)\alpha_B = (40 \text{ mm})(6.667 \text{ rad/s}^2)$$

$$= 266.67 \text{ mm/s}^2$$

$$\alpha_{AB} = \frac{(a_B)_t}{r_B} = \frac{266.67}{120} = 2.2222 \text{ rad/s}^2$$

Angular accelerations: $\boldsymbol{\alpha}_{AB} = 2.2222 \text{ rad/s}^2 \circlearrowright$ $\quad \boldsymbol{\alpha}_B = 6.667 \text{ rad/s} \circlearrowright$

Acceleration of Point *B*.

$$\mathbf{a}_B = [\alpha_{AB}\ x_B \downarrow] + [\omega_{AB}^2\ x_B \longleftarrow]$$

$$= [(2.2222)(120) \downarrow] + [(11.11)^2(120) \longleftarrow]$$

$$= (266.67 \text{ mm/s}^2) \downarrow + (14815 \text{ mm/s}^2) \longleftarrow$$

(*c*) *Acceleration of Point 1 on gear B.*

$$\mathbf{a}_1 = \mathbf{a}_B + [\alpha_B(x_B - x_1) \uparrow] + [\omega_B^2(x_B - x_1) \longrightarrow]$$

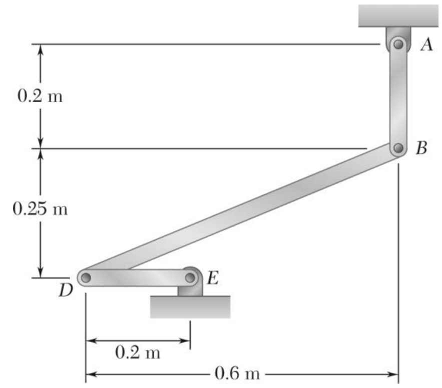
$$= [266.67 \downarrow] + [14815 \longleftarrow] + [(6.667)(40) \uparrow] + [(33.333)^2(40) \longrightarrow]$$

$$= [266.67 \downarrow] + [14815 \longleftarrow] + [266.67 \uparrow] + [44444 \longrightarrow]$$

$$= 29629 \text{ mm/s} \longrightarrow$$

$\mathbf{a}_1 = 29.6 \text{ m/s}^2 \longrightarrow$ ◀

Note that the tangential component of acceleration is zero as expected.

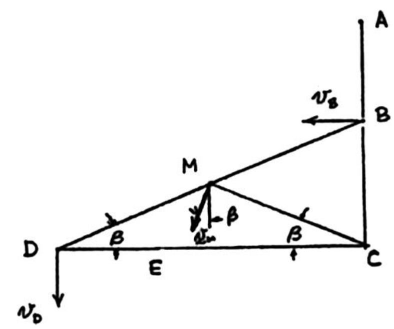


PROBLEM 15.252

Knowing that at the instant shown bar AB has an angular velocity of 10 rad/s clockwise and it is slowing down at a rate of 2 rad/s^2, determine the angular accelerations of bar BD and bar DE.

SOLUTION

Velocity Analysis $\quad \boldsymbol{\omega}_{AB} = 10 \text{ rad/s} \curvearrowright$



$$v_B = (AB)\omega_{AB}$$
$$= (0.200)(10)$$
$$= 2 \text{ m/s}$$

$$\mathbf{v}_B = v_B \longleftarrow$$

$$\mathbf{v}_D = v_D \downarrow$$

Locate the instantaneous center (Point C) of bar BD by noting that velocity directions at Points B and D are known. Draw BC perpendicular to $\mathbf{v}_B$ and DC perpendicular to $\mathbf{v}_D$.

$$\omega_{BD} = \frac{v_B}{BC} = \frac{2}{0.25} = 8 \text{ rad/s} \qquad \boldsymbol{\omega}_{BD} = 8.00 \text{ rad/s} \circlearrowleft$$

$$v_D = (CE)\omega_{BD} = (0.6)(8) = 4.8 \text{ m/s} \qquad \mathbf{v}_D = 4.8 \text{ m/s} \downarrow$$

$$\omega_{DE} = \frac{v_D}{DE} = \frac{4.8}{0.2} = 24 \text{ rad/s} \qquad \boldsymbol{\omega}_{DE} = 24 \text{ rad/s} \circlearrowleft$$

Acceleration Analysis: $\quad \alpha_{AB} = 2 \text{ rad/s}^2 \circlearrowleft, \quad \omega_{AB} = 10 \text{ rad/s} \curvearrowright$

Unit vectors: $\quad \mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1 \uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$
$$= 0 + (2\mathbf{k}) \times (-0.2)\mathbf{j} - (10)^2(-0.2\mathbf{j})$$
$$= (0.4 \text{ m/s}^2)\mathbf{i} + (20 \text{ m/s}^2)\mathbf{j}$$

PROBLEM 15.252 (Continued)

$$\mathbf{a}_D = \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2 \mathbf{r}_{D/B}$$
$$= 0.4\mathbf{i} + 20\mathbf{j} + \alpha_{BD}\mathbf{k} \times (-0.6\mathbf{i} - 0.25\mathbf{j}) - (8)^2(-0.6\mathbf{i} - 0.25\mathbf{j})$$
$$\mathbf{a}_D = (38.8 + 0.25\alpha_{BD})\mathbf{i} + (36 - 0.6\alpha_{BD})\mathbf{j} \quad (1)$$

$$\mathbf{a}_D = \mathbf{a}_E + \boldsymbol{\alpha}_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2 \mathbf{r}_{D/E}$$
$$= 0 + \alpha_{DE}\mathbf{k} \times (-0.2\mathbf{i}) - (24)^2(-0.2\mathbf{i})$$
$$= 115.2\mathbf{i} - 0.2\alpha_{DE}\mathbf{j} \quad (2)$$

Equate like components of $\mathbf{a}_D$ from Equations (1) and (2).

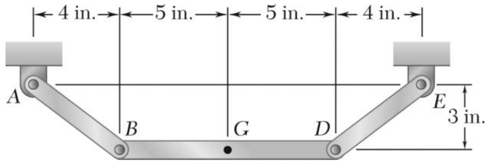
$\mathbf{i}$: $38.8 + 0.25\alpha_{BD} = 115.2 \qquad \alpha_{BD} = 305.6 \text{ rad/s}^2$

$\mathbf{j}$: $36 - 0.6\alpha_{BD} = -0.2\alpha_{DE}$

$$\alpha_{DE} = 3\alpha_{BD} - 180 = 736.8 \text{ rad/s}^2$$

Angular acceleration: $\boldsymbol{\alpha}_{BD} = 306 \text{ rad/s}^2$ ⟲ ◀

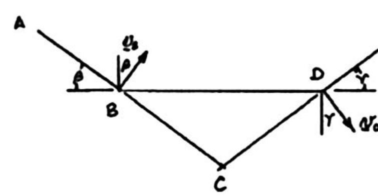
$\boldsymbol{\alpha}_{DE} = 737 \text{ rad/s}^2$ ⟲ ◀



PROBLEM 15.253

Knowing that at the instant shown rod AB has zero angular acceleration and an angular velocity of 15 rad/s counterclockwise, determine (*a*) the angular acceleration of arm DE, (*b*) the acceleration of Point D.

SOLUTION



$$\tan\beta = \frac{3}{4}, \quad \beta = 36.87^\circ$$

$$AB = \frac{4}{\cos\beta} = 5 \text{ in.}$$

$$DE = \frac{4}{\cos\beta} = 5 \text{ in.}$$

$$v_B = (AB)\omega_{AB} = (5)(15)$$

$$= 75 \text{ in./s}$$

$$\mathbf{v}_B = v_B \nearrow \beta, \quad \mathbf{v}_D = v_D \searrow \beta$$

Point C is the instantaneous center of bar BD.

$$CB = \frac{5}{\cos\beta} = 6.25 \text{ in.} \quad \omega_{BD} = \frac{v_B}{CB} = \frac{75}{6.25} = 12 \text{ rad/s} \circlearrowright$$

$$CD = \frac{5}{\cos\beta} = 6.25 \text{ in.} \quad v_D = (CD)\omega_{BD} = (6.25)(12) = 75 \text{ in./s}$$

$$\omega_{DE} = \frac{v_D}{DE} = \frac{75}{5} = 15 \text{ rad/s} \circlearrowleft$$

Acceleration analysis. $\alpha_{AB} = 0$

$$\mathbf{a}_B = [(AB)\alpha_{AB} \nearrow \beta] + [(AB)\omega_{AB}^2 \searrow \beta]$$

$$= 0 + [(5)(15)^2 \searrow \beta] = 1125 \text{ in./s}^2 \searrow \beta$$

$$\mathbf{a}_{D/B} = [(BD)\alpha_{BD} \uparrow] + [(BD)\omega_{BD}^2 \longleftarrow]$$

$$= [10\alpha_{BD} \uparrow] + [(10)(12)^2 \longleftarrow]$$

$$= [10\alpha_{BD} \uparrow] + [1440 \text{ in./s}^2 \longleftarrow]$$

$$\mathbf{a}_D = [(DE)\alpha_{DE} \searrow \beta] + [(DE)\omega_{DE}^2 \swarrow \beta]$$

$$= [5\alpha_{DE} \searrow \beta] + [(5)(15)^2 \swarrow \beta]$$

$$= [5\alpha_{DE} \searrow \beta] + [1125 \text{ in./s}^2 \swarrow \beta]$$

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B}$$ Resolve into components.

PROBLEM 15.253 (Continued)

(*a*) $\xrightarrow{+}$: $5\alpha_{DE}\sin\beta + 1125\cos\beta = -1125\cos\beta - 1440$

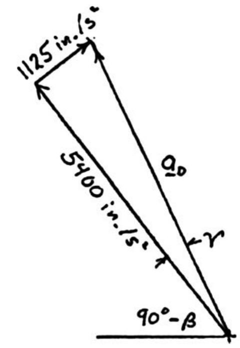
$$\alpha_{DE} = -1080 \text{ rad/s}^2$$

$\boldsymbol{\alpha}_{DE} = 1080 \text{ rad/s}^2$ ↻ ◀

(*b*)

$$\mathbf{a}_D = [(5)(-1080) \searrow \beta] + [1125 \text{ in./s}^2 \nearrow \beta]$$

$$= [5400 \text{ in./s}^2 \nwarrow \beta] + [1125 \text{ in./s}^2 \nearrow \beta]$$



$$\tan\gamma = \frac{1125}{5400}$$

$$\gamma = 11.77°$$

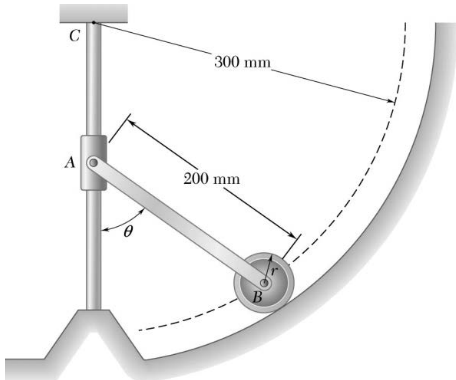
$$a_D = \sqrt{5400^2 + 1125^2}$$

$$= 5516 \text{ in./s}^2$$

$$= 460 \text{ ft/s}^2$$

$$90° - \beta + \gamma = 64.9°$$

$\mathbf{a}_D = 460 \text{ ft/s}^2 \searrow 64.9°$ ◀

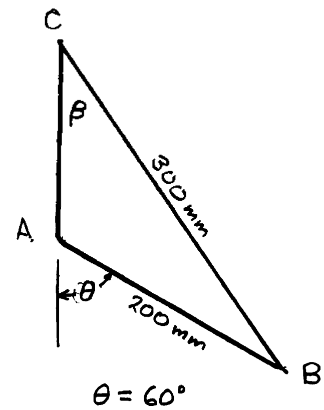


PROBLEM 15.254

Rod AB is attached to a collar at A and is fitted with a wheel at B that has a radius r = 15 mm. Knowing that when $\theta = 60°$ the colar has velocity of 250 mm/s upward and the speed of the collar is decreasing at a rate of 150 mm/s^2, determine (*a*) the angular acceleration of rod AB, (*b*) the angular acceleration of the wheel.

SOLUTION

Geometry. We note that Point B moves on a circle of radius R = 300 mm centered at C. It is useful to use the angle β, angle ACB of the triangle ABC, which indicates the motion of B along this curve. Applying the law of sines to the triangle ABC,



$$\frac{\overline{AB}}{\sin(\pi-\theta)} = \frac{\overline{BC}}{\sin\beta} \qquad (\theta = 60°)$$

or
$$\sin\beta = \frac{\overline{AB}}{\overline{BC}}\sin\theta = \frac{200 \text{ mm}}{300 \text{ mm}}\sin 60°$$
$$\beta = 35.264°$$

With meters as the length unit and the unit vectors defined as

$$\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \text{and } \mathbf{k} = 1 \circlearrowleft$$

The relative position vectors are

$$\mathbf{r}_{B/A} = (0.2 \text{ m})(\sin 60°\mathbf{i} - \cos 60°\mathbf{j})$$
$$\mathbf{r}_{B/C} = (0.3 \text{ m})(\sin\beta\mathbf{i} - \cos\beta\mathbf{j})$$

Let Point P be the contact point where the wheel rolls on the fixed surface.

$$\mathbf{r}_{P/B} = (0.015 \text{ m})(\sin\beta\mathbf{i} - \cos\beta\mathbf{j})$$

Velocity analysis.
$$\mathbf{v}_A = 0.250 \text{ m/s}\uparrow \qquad \mathbf{v}_P = 0$$
$$\mathbf{v}_B = v_B \measuredangle \beta \qquad \boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k} \qquad \boldsymbol{\omega}_{BP} = \omega_{BP}\mathbf{k}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = \mathbf{v}_A + \boldsymbol{\omega}_{AB}\times\mathbf{r}_{B/A}$$
$$= 0.250\mathbf{j} + (\omega_{AB})\mathbf{k}\times(0.2\sin 60°\mathbf{i} - 0.2\cos 60°\mathbf{j})$$
$$v_B(\cos\beta\mathbf{i} + \sin\beta\mathbf{j}) = 0.25\mathbf{j} + 0.1\omega_{AB}\mathbf{i} + 0.173205\omega_{AB}\mathbf{j}$$

PROBLEM 15.254 (Continued)

Resolve into components:

i: $v_B \cos\beta = +0.1\omega_{AB}$ (1)

j: $v_B \sin\beta = 0.25 - 0.173205\omega_{AB}$ (2)

Solving the simultaneous Equations (1) and (2),

$$v_B = -0.29873 \text{ m/s} \qquad \omega_{AB} = -2.43913$$

$$\mathbf{v}_B = 0.29873 \text{ m/s} \nearrow \beta \qquad \boldsymbol{\omega}_{AB} = 2.43913 \text{ rad/s} \circlearrowright$$

Since the wheel rolls without slipping, Point P is its instantaneous center.

$$|v_B| = r\omega_{BP} \qquad \omega_{PB} = \frac{0.29873}{0.15} \qquad \boldsymbol{\omega}_{BP} = 1.992 \text{ rad/s} \circlearrowright$$

Acceleration analysis $\mathbf{a}_A = 150 \text{ mm/s}^2 \downarrow$

$(\mathbf{a}_P)_t = 0$

$$\begin{aligned}
\mathbf{a}_B &= \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_A + \boldsymbol{\alpha}_{AB}\mathbf{k} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} \\
&= -0.150\mathbf{j} + \alpha_{AB}\mathbf{k} \times (0.2\sin 60°\mathbf{i} - 0.2\cos 60°\mathbf{j}) \\
&\quad - (2.43913)^2 (0.2\sin 60°\mathbf{i} - 0.2\cos 60°\mathbf{j}) \\
&= -0.150\mathbf{j} + (0.2\sin 60°)\alpha_{AB}\mathbf{j} + (0.2\cos 60°)\alpha_{AB}\mathbf{i} \\
&\quad - 1.03046\mathbf{i} + 0.59494\mathbf{j} \\
&= (0.2\cos 60°)\alpha_{AB}\mathbf{i} + (0.2\sin 60°)\alpha_{AB}\mathbf{j} - 1.03046\mathbf{i} + 0.44494\mathbf{j}
\end{aligned}$$

Consider the motion of B along its circular path.

$$\begin{aligned}
\mathbf{a}_B &= [(a_B)_t \measuredangle \beta] + \left[\frac{v_B^2}{R} \searrow \beta\right] \\
&= (a_B)_t (\cos\beta\mathbf{i} + \sin\beta\mathbf{j}) + \frac{v_B^2}{R}(-\sin\beta\mathbf{i} + \cos\beta\mathbf{j}) \\
&= (a_B)_t \cos\beta\mathbf{i} + (a_B)_t \sin\beta\mathbf{j} - \frac{(0.29873)^2}{0.3}(-\sin\beta\mathbf{i} + \cos\beta\mathbf{j}) \\
&= (a_B)_t \cos\beta\mathbf{i} + (a_B)_t \sin\beta\mathbf{j} - 0.17174\mathbf{i} + 0.24288\mathbf{j}
\end{aligned}$$

Equate the two expression for $\mathbf{a}_B$ and resolve into components.

i: $(a_B)_t \cos\beta - 0.17174 = (0.2\cos 60°)\alpha_{AB} - 1.03046$ (3)

j: $(a_B)_t \sin\beta + 0.24288 = 0.2\sin 60°\alpha_{AB} + 0.44494$ (4)

Solving Eqs. (3) and (4) simultaneously,

$$(a_B)_t = -2.0187 \text{ m/s}^2 \qquad \alpha_{AB} = -7.8956 \text{ rad/s}^2$$

PROBLEM 15.254 (Continued)

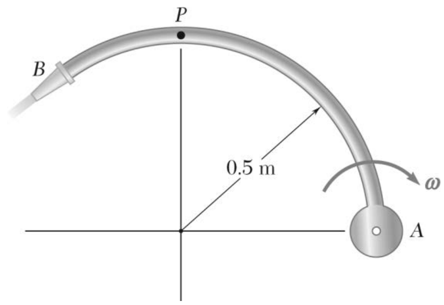
(*a*) *Angular acceleration of AB.* $\alpha_{AB} = 7.90 \text{ rad/s}^2 \circlearrowright$ ◀

(*b*) *Angular acceleration of the wheel.*

$$(a_P)_t = (a_B)_t + r\alpha_{BP} = 0$$

$$\alpha_{BP} = \frac{(a_B)_t}{r} = -\frac{(-2.0187)}{0.015} = 134.6 \text{ rad/s}^2$$

$\boldsymbol{\alpha}_{BP} = 134.6 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 15.255

Water flows through a curved pipe *AB* that rotates with a constant clockwise angular velocity of 90 rpm. If the velocity of the water relative to the pipe is 8 m/s, determine the total acceleration of a particle of water at Point *P*.

SOLUTION

Let the curved pipe be a rotating frame of reference. Its angular velocity is 90 rpm ↻ or $\boldsymbol{\omega} = 9.4248$ rad/s ↻.

Motion of the frame of reference at Point P′.

$$\mathbf{v}_{P'} = (AP)\omega \measuredangle 45° = (0.5\sqrt{2})(9.4248) \measuredangle 45° = 6.6643 \text{ m/s} \measuredangle 45°$$

$$\mathbf{a}_{P'} = (AP)\omega^2 \searrow 45° = (0.5\sqrt{2})(9.4248)^2 \searrow 45° = 62.81 \text{ m/s}^2 \searrow 45°$$

Motion of water relative to the frame at Point P.

$$\mathbf{v}_{P/F} = 8 \text{ m/s} \leftarrow$$

$$\mathbf{a}_{P/F} = \frac{(v_{P/F})^2}{\rho} \downarrow$$

$$= \frac{(8 \text{ m/s})^2}{0.5 \text{ m}} \downarrow$$

$$= 128 \text{ m/s}^2 \downarrow$$

Coriolis acceleration.

$$a_c = 2\omega v_{P/F}$$

$$= (2)(9.4248)(8)$$

$$= 150.797 \text{ m/s}^2$$

$$\mathbf{a}_c = 150.797 \text{ m/s}^2 \uparrow$$

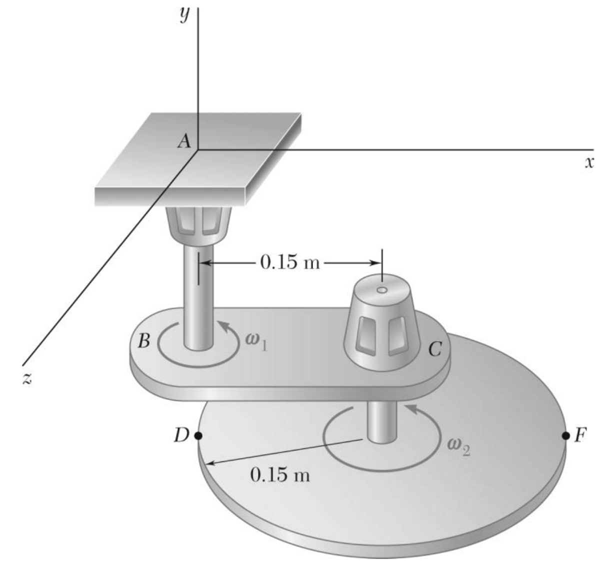
Acceleration of water at Point P.

$$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + \mathbf{a}_c$$

$$\mathbf{a}_P = [62.81 \searrow 45°] + [128 \downarrow] + [150.797 \uparrow]$$

$$= [44.413 \text{ m/s}^2 \rightarrow] + [21.616 \text{ m/s}^2 \downarrow]$$

$$\mathbf{a}_P = 49.4 \text{ m/s}^2 \searrow 26.0° \blacktriangleleft$$



PROBLEM 15.256

A disk of 0.15-m radius rotates at the constant rate ω_2 with respect to plate BC, which itself rotates at the constant rate ω_1 about the y axis. Knowing that $\omega_1 = \omega_2 = 3$ rad/s, determine, for the position shown, the velocity and acceleration (a) of Point D, (b) of Point F.

SOLUTION

Frame *AXYZ* is fixed.

Moving frame, *Exyz*, rotates about $\overline{y}$ axis at

$$\mathbf{\Omega} = \omega_1 \mathbf{j} = (3 \text{ rad/s})\mathbf{j}$$

(a) *Point D*:

$$\boldsymbol{\omega}_2 = \omega_2 \mathbf{j} = (3 \text{ rad/s})\mathbf{j}$$

$$\mathbf{r}_{D/A} = 0$$

$$\mathbf{r}_{D/E} = -(0.15 \text{ m})\mathbf{i}$$

$$\mathbf{v}_{D'} = \mathbf{\Omega} \times \mathbf{r}_{D/A} = 0$$

$$\mathbf{v}_{D/F} = \boldsymbol{\omega}_A \times \mathbf{r}_{D/E}$$

$$= (3 \text{ rad/s})\mathbf{j} \times (-0.15 \text{ m})\mathbf{i}$$

$$= (0.45 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$$

$\mathbf{v}_D = (0.45 \text{ m/s})\mathbf{k}$ ◀

$$\mathbf{a}_{D'} = \mathbf{\Omega} \times \mathbf{v}_{D'} = 0$$

$$\mathbf{a}_{D/F} = \boldsymbol{\omega}_1 \times \mathbf{v}_{D/F}$$

$$= (3 \text{ rad/s})\mathbf{j} \times (0.45 \text{ m/s})\mathbf{k}$$

$$= (1.35 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{D/F}$$

$$= 2(3 \text{ rad/s})\mathbf{j} \times (0.45 \text{ m/s})\mathbf{k} = (2.70 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + \mathbf{a}_c$$

$$= 0 + (1.35 \text{ m/s}^2)\mathbf{i} + (2.70 \text{ m/s}^2)\mathbf{i}$$

$\mathbf{a}_D = (4.05 \text{ m/s}^2)\mathbf{i}$ ◀

PROBLEM 15.256 (Continued)

(*b*) *Point F*:

$$\boldsymbol{\omega}_2 = \omega_2\mathbf{j} = (3 \text{ rad/s})\mathbf{j}$$

$$\mathbf{r}_{F/A} = (0.3 \text{ m})\mathbf{i};$$

$$\mathbf{r}_{F/E} = (0.15 \text{ m})\mathbf{i}$$

$$\mathbf{v}_{F'} = \boldsymbol{\Omega} \times \mathbf{r}_{F/A}$$

$$= (3 \text{ rad/s})\mathbf{j} \times (0.3 \text{ m})\mathbf{i}$$

$$= -(0.9 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_{F/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_{F/E}$$

$$= (3 \text{ rad/s})\mathbf{j} \times (0.15 \text{ m})\mathbf{i}$$

$$= -(0.45 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_F = \mathbf{v}_{F'} + \mathbf{v}_{F/F}$$

$$= -(0.9 \text{ m/s})\mathbf{k} - (0.45 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_F = -(1.35 \text{ m/s})\mathbf{k}$$ ◀

$$\mathbf{a}_{F'} = \boldsymbol{\Omega} \times \mathbf{v}_{F'}$$

$$= (3 \text{ rad/s})\mathbf{j} \times (-0.9 \text{ m/s})\mathbf{k}$$

$$= -(2.7 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_{F/F} = \boldsymbol{\omega}_2 \times \mathbf{v}_{F/F}$$

$$= (3 \text{ rad/s})\mathbf{j} \times (-0.45 \text{ m/s})\mathbf{k}$$

$$= -(1.35 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_c = 2\boldsymbol{\Omega} \times \mathbf{v}_{F/F}$$

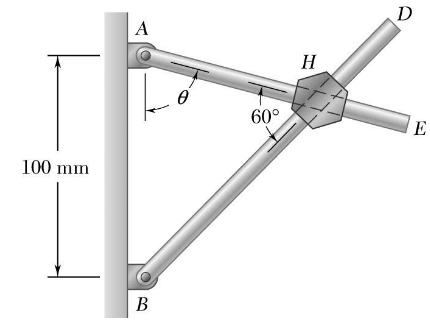
$$= 2(3 \text{ rad/s})\mathbf{j} \times (-0.45 \text{ m/s})\mathbf{k}$$

$$= -(2.7 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_F = \mathbf{a}_{F'} + \mathbf{a}_{F/F} + \mathbf{a}_c$$

$$= -(2.7 \text{ m/s}^2)\mathbf{i} - (1.35 \text{ m/s}^2)\mathbf{i} - (2.7 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_F = -(6.75 \text{ m/s}^2)\mathbf{i}$$ ◀



PROBLEM 15.257

Two rods *AE* and *BD* pass through holes drilled into a hexagonal block. (The holes are drilled in different planes so that the rods will not touch each other.) Knowing that rod *AE* has an angular velocity of 20 rad/s clockwise and an angular acceleration of 4 rad/s^2 counterclockwise when $\theta = 90°$, determine, (*a*) the relative velocity of the block with respect to each rod, (*b*) the relative acceleration of the block with respect to each rod.

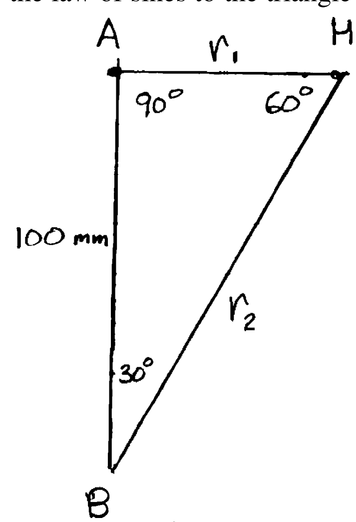
SOLUTION

Geometry: When $\theta = 90°$, Point *H* is located as shown in the sketch. Apply the law of sines to the triangle *ABH*.

$$\frac{100\text{ mm}}{\sin 60°} = \frac{r_1}{\sin 30°} = \frac{r_2}{\sin 90°}$$

$$r_1 = 57.735\text{ mm}$$

$$r_2 = 115.470\text{ mm}$$



The angle at *H* remains at 60° so that rods *AE* and *BD* have a common angular velocity $\boldsymbol{\omega} = \omega \circlearrowleft$ and a common angular acceleration $\boldsymbol{\alpha} = \alpha \circlearrowleft$, where

$$\omega = -20\text{ rad/s}$$

and

$$\alpha = 4\text{ rad/s}$$

Consider the double slider *H* as a particle sliding along the rotating rod *AH* with relative velocity $u_1 \longrightarrow$ and relative acceleration $\dot{u}_1 \longrightarrow$.

Let H' be the point on rod *AE* that coincides with *H*.

$$\mathbf{v}_{H'} = [r_1\omega \uparrow] = [(57.735\text{ mm})(-20\text{ rad/s}) \uparrow] = [1154.7\text{ mm/s} \downarrow]$$

$$\mathbf{a}_{H'} = [r_1\alpha \uparrow] + [r_1\omega^2 \longleftarrow]$$

$$= [(57.735\text{ mm})(4\text{ rad/s}^2) \uparrow] + [(57.735\text{ mm})(20\text{ rad/s})^2 \longleftarrow]$$

$$= [230.9\text{ mm/s}^2 \uparrow] + [23094\text{ mm/s}^2 \longleftarrow]$$

The corresponding Coriolis acceleration is

$$\mathbf{a}_1 = [2\omega u_1 \uparrow] = [(2)(-20)u_1 \uparrow] = 40u_1 \downarrow$$

$$\mathbf{v}_H = \mathbf{v}_{H'} + u_1 \longrightarrow = [1154.7 \text{ mm} \downarrow] + [u_1 \longrightarrow] \quad (1)$$

$$\mathbf{a}_H = a_{H'} + [\dot{u}_1 \longrightarrow] + [40u_1 \downarrow]$$

$$= [230.9 \text{ mm/s} \uparrow] + [23094 \text{ mm/s}^2 \longleftarrow] + [\dot{u}_1 \longrightarrow] + [40u_1 \downarrow] \quad (2)$$

Now consider the double slider H as a particle sliding along the rotating rod BD with relative velocity $u_2 \measuredangle 60°$ and relative acceleration $\dot{u}_2 \measuredangle 60°$.

Let H'' be the point on rod BD that coincides with H.

$$\mathbf{v}_{H''} = r_2\omega \searrow 30° = (115.47 \text{ mm})(-20 \text{ rad/s}) \searrow 30° = 2309.4 \text{ mm/s} \nwarrow 30°$$

$$\mathbf{a}_{H''} = r_2\alpha \searrow 30° + r_2\omega^2 \swarrow 60°$$

$$= (115.47 \text{ mm})(4 \text{ rad/s}^2) \searrow 30° + (115.47 \text{ mm})(20 \text{ rad/s})^2 \swarrow 60°$$

$$= 461.9 \text{ mm/s}^2 \searrow 30° + 46188 \text{ mm/s}^2 \swarrow 60°$$

The corresponding Coriolis acceleration is

$$\mathbf{a}_2 = 2\omega u_2 \searrow 30° = (2)(-20 \text{ rad/s})u_2 \searrow 60° = 40u_2 \nwarrow 30°$$

$$\mathbf{v}_H = \mathbf{v}_{H''} + u_2 \measuredangle 60° = 2309.4 \text{ mm/s} \nwarrow 30° + u_2 \measuredangle 60° \quad (3)$$

$$\mathbf{a}_H = \mathbf{a}_{H''} + \dot{u}_2 \measuredangle 60° + 40u_2 \nwarrow 30°$$

$$= [(461.9 \text{ mm/s}^2) \searrow 30°] + [(46188 \text{ mm/s}^2) \swarrow 60°] + [\dot{u}_2 \measuredangle 60°] + [40u_2 \nwarrow 30°] \quad (4)$$

Equate expression (1) and (3) for $\mathbf{v}_H$ and resolve into components.

$+\uparrow$: $$1154.7 \text{ mm/s} = -(2309.4 \text{ mm/s})\sin 30° + u_2 \sin 60°$$

$$u_2 = \frac{2309.4 \sin 30° - 1154.7}{\sin 60°} \qquad u_2 = 0$$

$\xrightarrow{+}$: $$u_1 = 2309.4 \cos 30° + 0 \qquad u_1 = 2000 \text{ mm/s}$$

Substitute the values for u_1 and u_2 into the Coriolis acceleration terms, and equate expressions (2) and (4) for $\mathbf{a}_H$, and resolve into components.

$+\uparrow$: $$230.9 \text{ mm/s}^2 - (40 \text{ rad/s})(2000 \text{ mm/s})$$

$$= (461.9 \text{ mm/s}^2)\sin 30° + (46188 \text{ mm/s}^2)\sin 60°$$

$$+ \dot{u}_2 \sin 60° - (40)(0)\sin 30°$$

$$\dot{u}_2 = \frac{230.9 - 80000 - 230.9 + 40000}{\sin 60°} \qquad \dot{u}_2 = -46188 \text{ mm/s}^2$$

PROBLEM 15.257 (Continued)

$\xrightarrow{+}$: $\dot{u}_1 - 23094 \text{ mm/s}^2 = -(461.4 \text{ mm/s}^2)\cos 30° - (46188 \text{ mm/s}^2)\cos 60°$
$-(46188 \text{ mm/s}^2)\cos 60° + 0$

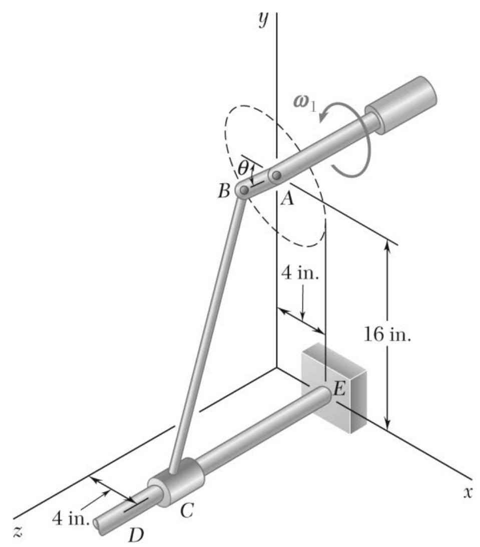
$$\dot{u}_1 = -23494 \text{ mm/s}^2$$

(*a*) *Relative velocities*: *AE*: $u_1 \longrightarrow \; = 2.00 \text{ m/s} \longrightarrow$ ◀

BD: $u_2 \measuredangle 60° = 0$ ◀

(*b*) *Relative accelerations*: *AE*: $\dot{u}_1 = 23.5 \text{ m/s}^2 \longleftarrow$ ◀

BD: $\dot{u}_2 \measuredangle 60° = 46.2 \text{ m/s}^2 \; \swarrow 60°$ ◀

	**PROBLEM 15.258** Rod BC of length 24 in. is connected by ball-and-socket joints to a rotating arm AB and to a collar C that slides on the fixed rod DE. Knowing that the length of arm AB is 4 in. and that it rotates at the constant rate $\omega_1 = 10$ rad/s, determine the velocity of collar C when $\theta = 0$.

SOLUTION

Geometry.

$$l_{BC} = 24 \text{ in.}$$
$$\mathbf{r}_{C/B} = (8 \text{ in.})\mathbf{i} - (16 \text{ in.})\mathbf{j} + z_{C/B}\mathbf{k}$$
$$24^2 = 8^2 + 16^2 + z_{C/B}^2$$
$$z_{C/B} = 16 \text{ in.}$$
$$\mathbf{r}_{C/B} = (8 \text{ in.})\mathbf{i} - (16 \text{ in.})\mathbf{j} + (16 \text{ in.})\mathbf{k}$$
$$\mathbf{r}_{B/A} = -(4 \text{ in.})\mathbf{i}$$

Velocity at B.

$$\mathbf{v}_B = \omega_1\mathbf{k} \times \mathbf{r}_{B/A}$$
$$= 10\mathbf{k} \times (-4\mathbf{i})$$
$$= -(40 \text{ in./s})\mathbf{j}$$

Velocity of collar C.

$$\mathbf{v}_C = v_C\mathbf{k}$$
$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

where

$$\mathbf{v}_{C/B} = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

Noting that $\mathbf{v}_{C/B}$ is perpendicular to $\mathbf{r}_{C/B}$, we get $\mathbf{r}_{C/B} \cdot \mathbf{v}_{C/B} = 0$

Forming $\mathbf{r}_{C/B} \cdot \mathbf{v}_C$, we get

$$\mathbf{r}_{C/B} \cdot \mathbf{v}_C = \mathbf{r}_{C/B} \cdot (\mathbf{v}_B + \mathbf{v}_{C/B})$$
$$= \mathbf{r}_{C/B} \cdot \mathbf{v}_B + \mathbf{r}_{C/B} \cdot \mathbf{v}_{C/B}$$

or

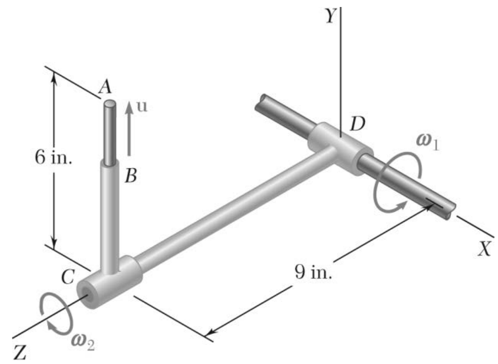
$$\mathbf{r}_{C/B} \cdot \mathbf{v}_C = \mathbf{r}_{C/B}\mathbf{v}_B \qquad (1)$$

From Eq. (1)

$$(8\mathbf{i} - 16\mathbf{j} + 16\mathbf{k}) \cdot (v_C\mathbf{k}) = (8\mathbf{i} - 16\mathbf{j} + 16\mathbf{k}) \cdot (-40\mathbf{j})$$
$$16v_C = (-16)(-40)$$

or

$$v_C = 40 \text{ in./s} \qquad \mathbf{v}_C = (40.0 \text{ in./s})\mathbf{k} \blacktriangleleft$$



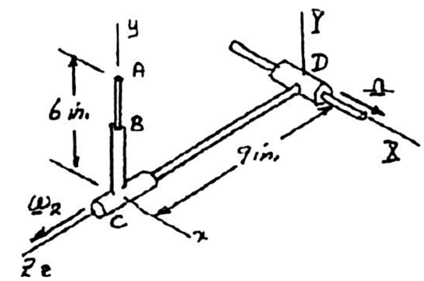
PROBLEM 15.259

In the position shown, the thin rod moves at a constant speed $u = 3$ in./s out of the tube BC. At the same time, tube BC rotates at the constant rate $\omega_2 = 1.5$ rad/s with respect to arm CD. Knowing that the entire assembly rotates about the X axis at the constant rate $\omega_1 = 1.2$ rad/s, determine the velocity and acceleration of end A of the rod.

SOLUTION

Geometry. $\mathbf{r}_A = (6 \text{ in.})\mathbf{j} + (9 \text{ in.})\mathbf{k}$

Method 1



Let the rigid body DCB be a rotating frame of reference.

Its angular velocity is

$$\mathbf{\Omega} = \omega_1\mathbf{i} + \omega_2\mathbf{k}$$
$$= (1.2 \text{ rad/s})\mathbf{i} - (1.5 \text{ rad/s})\mathbf{k}.$$

Its angular acceleration is

$$\boldsymbol{\alpha} = \omega_1\mathbf{i} \times \omega_2\mathbf{k}$$
$$= -\omega_1\omega_2\mathbf{j}$$
$$= (1.8 \text{ rad/s}^2)\mathbf{j}.$$

Motion of the coinciding Point A' in the frame.

$$\mathbf{v}_{A'} = \mathbf{\Omega} \times \mathbf{r}_A$$
$$= (1.2\mathbf{i} - 1.5\mathbf{k}) \times (6\mathbf{j} + 9\mathbf{k})$$
$$= 7.2\mathbf{k} - 10.8\mathbf{j} + 9\mathbf{i}$$
$$= (9 \text{ in./s})\mathbf{i} - (10.8 \text{ in./s})\mathbf{j} + (7.2 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{A'} = \boldsymbol{\alpha} \times \mathbf{r}_A + \mathbf{\Omega} \times \mathbf{v}_{A'}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.8 & 0 \\ 0 & 6 & 9 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 0 & -1.5 \\ 9 & -10.8 & 7.2 \end{vmatrix}$$
$$= 16.2\mathbf{i} - 16.2\mathbf{i} - 22.14\mathbf{j} - 12.96\mathbf{k}$$
$$= -(22.14 \text{ in./s}^2)\mathbf{j} - (12.96 \text{ in./s}^2)\mathbf{k}$$

Motion of Point A relative to the frame.

$$\mathbf{v}_{A/F} = u\mathbf{j} = (3 \text{ in./s})\mathbf{j},$$
$$\mathbf{a}_{A/F} = 0$$

Velocity of Point A.

$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/F}$$

$$\mathbf{v}_A = 9\mathbf{i} - 10.8\mathbf{j} + 7.2\mathbf{k} + 3\mathbf{j}$$

$$\mathbf{v}_A = (9.00 \text{ in./s})\mathbf{i} - (7.80 \text{ in./s})\mathbf{j} + (7.20 \text{ in./s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration.

$$2\boldsymbol{\Omega} \times \mathbf{v}_{A/F} = (2)(1.2\mathbf{i} - 1.5\mathbf{k}) \times 3\mathbf{j}$$

$$= (9 \text{ in./s}^2)\mathbf{i} + (7.2 \text{ in./s}^2)\mathbf{k}$$

Acceleration of Point A.

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + 2\boldsymbol{\Omega} \times \mathbf{v}_{A/F}$$

$$\mathbf{a}_A = -22.14\mathbf{j} - 12.92\mathbf{k} + 9\mathbf{i} + 7.2\mathbf{k}$$

$$\mathbf{a}_A = (9.00 \text{ in./s}^2)\mathbf{i} - (22.1 \text{ in./s}^2)\mathbf{j} - (5.76 \text{ in./s}^2)\mathbf{k} \blacktriangleleft$$

Method 2

Let frame *Dxyz*, which at instant shown coincides with *DXYZ*, rotate with an angular velocity $\boldsymbol{\Omega} = \omega_1\mathbf{i} = 1.2\mathbf{i}$ rad/s. Then the motion relative to the frame consists of the rotation of body *DCB* about the *Z* axis with angular velocity $\omega_2\mathbf{k} = -(1.5 \text{ rad/s})\mathbf{k}$ plus the sliding motion $\mathbf{u} = u\mathbf{i} = (3 \text{ in./s})\mathbf{j}$ of the rod *AB* relative to the body *DCB*.

Motion of the coinciding Point A′ in the frame.

$$\mathbf{v}_{A'} = \boldsymbol{\Omega} \times \mathbf{r}_A$$

$$= 1.2\mathbf{i} \times (6\mathbf{j} + 9\mathbf{k})$$

$$= -(10.8 \text{ in./s})\mathbf{j} + (7.2 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{A'} = \boldsymbol{\Omega} \times \mathbf{v}_{A'}$$

$$= 1.2\mathbf{i} \times (-10.8\mathbf{j} + 7.2\mathbf{k})$$

$$= -(8.64 \text{ in./s}^2)\mathbf{j} - (12.96 \text{ in./s}^2)\mathbf{k}$$

Motion of Point A relative to the frame.

$$\mathbf{v}_{A/F} = \omega_2\mathbf{k} \times \mathbf{r}_A + u\mathbf{j}$$

$$= (-1.5\mathbf{k}) \times (6\mathbf{j} + 9\mathbf{k}) + 3\mathbf{j}$$

$$= (9 \text{ in./s})\mathbf{i} + (3 \text{ in./s})\mathbf{j}$$

$$\mathbf{a}_{A/F} = \alpha_2\mathbf{k} \times \mathbf{r}_A + \omega_2\mathbf{k} \times (\omega_2\mathbf{k} \times \mathbf{r}_A) + \dot{u}\mathbf{j} + 2\omega_2\mathbf{k} \times (u\mathbf{j})$$

$$= 0 + (-1.5\mathbf{k}) \times (9\mathbf{i}) + 0 + (2)(-1.5\mathbf{k}) \times (3\mathbf{j})$$

$$= -13.5\mathbf{j} + 9\mathbf{i}$$

$$= (9 \text{ in./s}^2)\mathbf{i} - (13.5 \text{ in./s}^2)\mathbf{j}$$

Velocity of Point A.

$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/F}$$

$$\mathbf{v}_A = -10.8\mathbf{j} + 7.2\mathbf{k} + 9\mathbf{i} + 3\mathbf{j}$$

$$\mathbf{v}_A = (9.00 \text{ in./s})\mathbf{i} - (7.80 \text{ in./s})\mathbf{j} + (7.20 \text{ in./s})\mathbf{k} \blacktriangleleft$$

PROBLEM 15.259 (Continued)

Coriolis acceleration.

$$2\boldsymbol{\Omega}\times\mathbf{v}_{A/F} = (2)(1.2\mathbf{i})\times(-9\mathbf{i}+3\mathbf{j})$$
$$= (7.2 \text{ in./s}^2)\mathbf{k}$$

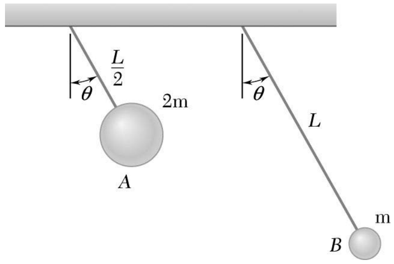
Acceleration of Point A.

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + 2\boldsymbol{\Omega}\times\mathbf{v}_{A/F}$$
$$\mathbf{a}_A = -8.64\mathbf{j} - 12.96\mathbf{k} + 9\mathbf{i} - 13.5\mathbf{j} + 7.2\mathbf{k}$$

$$\mathbf{a}_A = (9.00 \text{ in./s}^2)\mathbf{i} - (22.1 \text{ in./s}^2)\mathbf{j} - (5.76 \text{ in./s}^2)\mathbf{k} \blacktriangleleft$$

CHAPTER 16

PROBLEM 16.CQ1



Two pendulums, *A* and *B*, with the masses and lengths shown are released from rest. Which system has a larger mass moment of inertia about its pivot point?

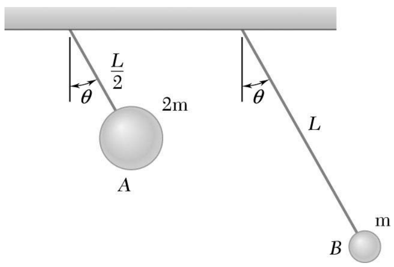
(*a*) *A*

(*b*) *B*

(*c*) They are the same.

SOLUTION

Answer: (*b*)



PROBLEM 16.CQ2

Two pendulums, *A* and *B*, with the masses and lengths shown are released from rest. Which system has a larger angular acceleration immediately after release?

(*a*) *A*

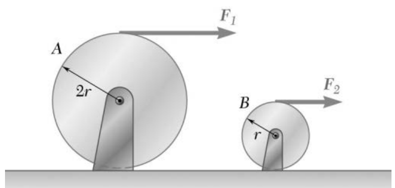
(*b*) *B*

(*c*) They are the same.

SOLUTION

Answer: (*a*)

PROBLEM 16.CQ3



Two solid cylinders, A and B, have the same mass m and the radii $2r$ and r respectively. Each is accelerated from rest with a force applied as shown. In order to impart identical angular accelerations to both cylinders, what is the relationship between F_1 and F_2?

(a) $F_1 = 0.5F_2$

(b) $F_1 = F_2$

(c) $F_1 = 2F_2$

(d) $F_1 = 4F_2$

(e) $F_1 = 8F_2$

SOLUTION

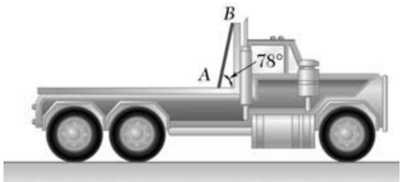
Answer: (c)

$$Fr = I\alpha$$

$$\alpha = \frac{FR}{\frac{1}{2}mR^2} = \frac{F_1(2r)}{\frac{1}{2}m(2r)^2} = \frac{F_2 r}{\frac{1}{2}mr^2}$$

$$\frac{F_1}{mr} = \frac{2F_2}{mr}$$

$\mathbf{F}_1 = 2F_2$ ◀

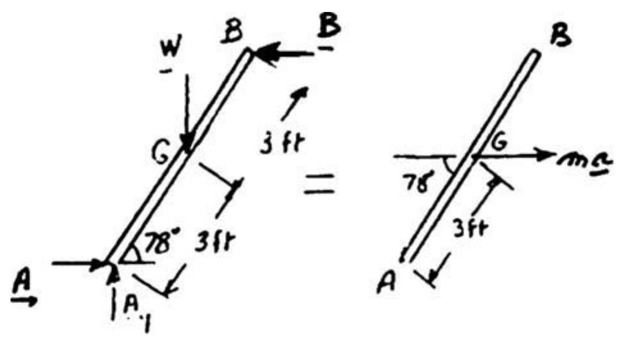


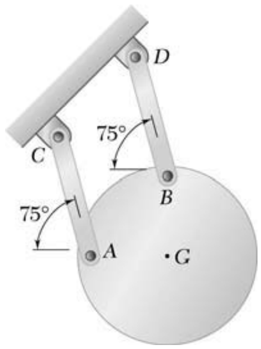
PROBLEM 16.F1

A 6-ft board is placed in a truck with one end resting against a block secured to the floor and the other leaning against a vertical partition. Draw the FBD and KD necessary to determine the maximum allowable acceleration of the truck if the board is to remain in the position shown.

SOLUTION

Answer:



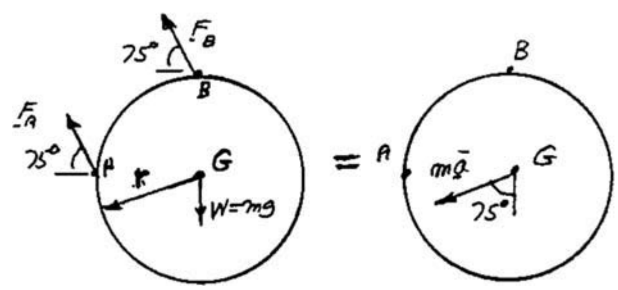


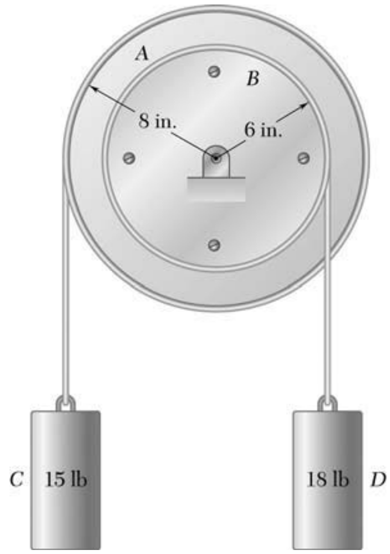
PROBLEM 16.F2

A uniform circular plate of mass 3 kg is attached to two links *AC* and *BD* of the same length. Knowing that the plate is released from rest in the position shown, in which lines joining *G* to *A* and *B* are, respectively, horizontal and vertical, draw the FBD and KD for the plate.

SOLUTION

Answer:



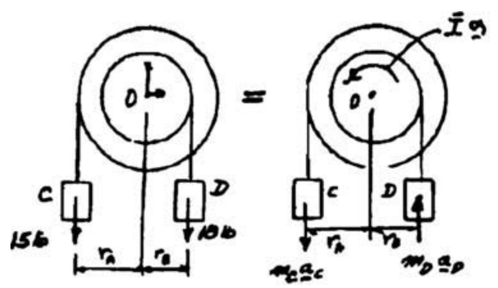


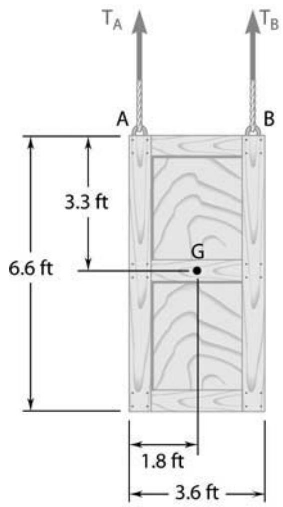
PROBLEM 16.F3

Two uniform disks and two cylinders are assembled as indicated. Disk A weighs 20 lb and disk B weighs 12 lb. Knowing that the system is released from rest, draw the FBD and KD for the whole system.

SOLUTION

Answer:



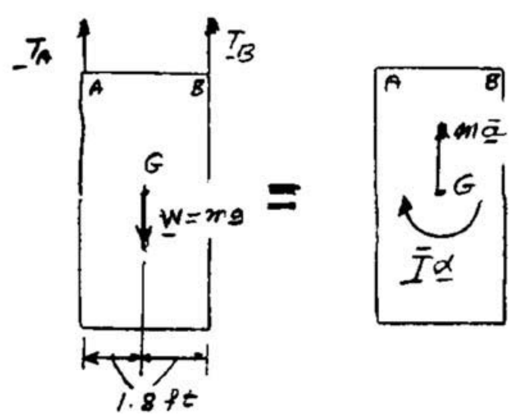


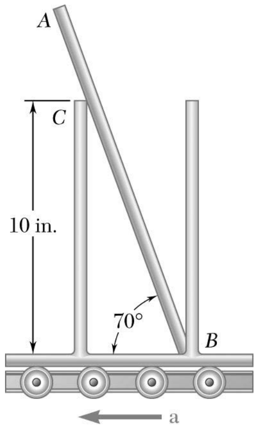
PROBLEM 16.F4

The 400-lb crate shown is lowered by means of two overhead cranes. Knowing the tension in each cable, draw the FBD and KD that can be used to determine the angular acceleration of the crate and the acceleration of the center of gravity.

SOLUTION

Answer:

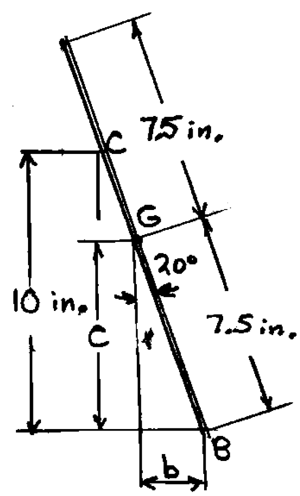




PROBLEM 16.1

A conveyor system is fitted with vertical panels, and a 15-in. rod *AB* weighing 5 lb is lodged between two panels as shown. If the rod is to remain in the position shown, determine the maximum allowable acceleration of the system.

SOLUTION



Geometry:

$$d = \frac{10 \text{ in.}}{\cos 20°} = 10.642 \text{ in.}$$

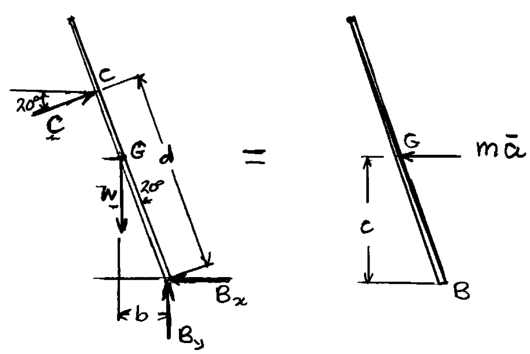
$$b = \frac{1}{2}(15 \text{ in.})\sin 20° = 2.5652 \text{ in.}$$

$$c = \frac{1}{2}(15 \text{ in.})\cos 20° = 7.0477 \text{ in.}$$

Mass:

$$m = \frac{W}{g} = \frac{5 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.15528 \text{ slug}$$

Kinetics:



$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad Cd - Wb = -m\bar{a}c$$

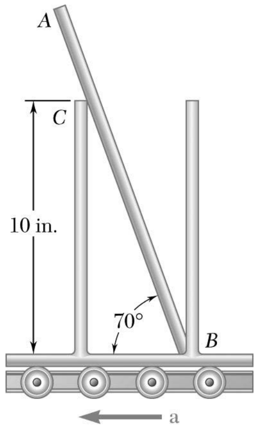
Maximum allowable acceleration.

This occurs at loss of contact when $C = 0$.

$$ma = \frac{Wb}{c} = \frac{(5 \text{ lb})(2.5652 \text{ in.})}{7.0477 \text{ in.}} = 1.8199 \text{ lb}$$

$$a = \frac{ma}{m} = \frac{1.8199 \text{ lb}}{0.15528 \text{ slug}}$$

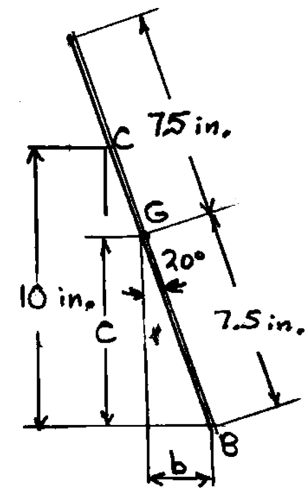
$a = 11.72$ ft/s ◀



PROBLEM 16.2

A conveyor system is fitted with vertical panels, and a 15-in. rod AB weighing 5 lb is lodged between two panels as shown. Knowing that the acceleration of the system is 3 ft/s^2 to the left, determine (*a*) the force exerted on the rod at C, (*b*) the reaction at B.

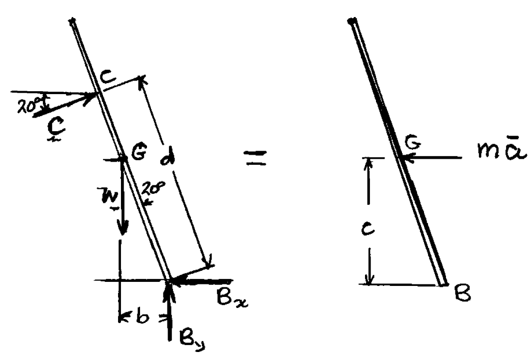
SOLUTION



Geometry:
$$\overline{CB} = d = \frac{10\text{ in.}}{\cos 20°} = 10.642\text{ in.}$$
$$b = \frac{1}{2}(15\text{ in.})\sin 20° = 2.5652\text{ in.}$$
$$c = \frac{1}{2}(15\text{ in.})\cos 20° = 7.0477\text{ in.}$$

Mass:
$$m = \frac{W}{g} = \frac{5\text{ lb}}{32.2\text{ ft/s}^2} = 0.15528\text{ slug}$$

Kinetics:
$$m\bar{\mathbf{a}} = (0.15528\text{ slug})(3\text{ ft/s}^2) = 0.46588\text{ lb} \longleftarrow$$



(*a*) Force at C.

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad Cd - Wb = -m\bar{a}c$$

$$C = \frac{Wb}{d} - \frac{m\bar{a}c}{d} = \frac{(5\text{ lb})(2.5652\text{ in.})}{10.642\text{ in.}} - \frac{(0.46588\text{ lb})(7.0477\text{ in.})}{10.642}$$

$$C = 0.89669\text{ lb}$$

$\mathbf{C} = 0.897\text{ lb} \measuredangle 20°$ ◀

PROBLEM 16.2 (Continued)

(*b*) Reaction at *B*.

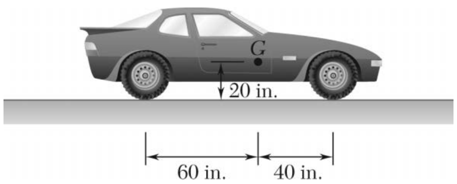
$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad B_y - W + C\sin 20° = 0$

$$B_y = 5 \text{ lb} - (0.89669 \text{ lb})\sin 20° = 4.6933 \text{ lb}$$

$\overset{+}{\longleftarrow} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad B_x - C\cos 20° = ma$

$$B_x = (0.89669 \text{ lb})\cos 20° + 0.46588 = 1.3085 \text{ lb}$$

$$\mathbf{B} = 1.3085 \text{ lb} \longleftarrow + 4.6933 \text{ lb}\uparrow \qquad \mathbf{B} = 4.87 \text{ lb} \nwarrow 74.4° \blacktriangleleft$$

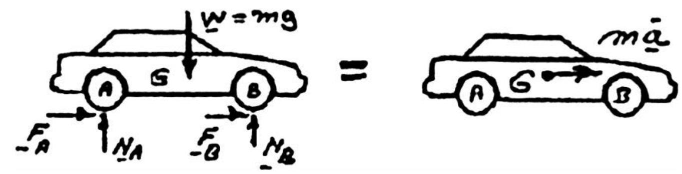


PROBLEM 16.3

Knowing that the coefficient of static friction between the tires and the road is 0.80 for the automobile shown, determine the maximum possible acceleration on a level road, assuming (*a*) four-wheel drive, (*b*) rear-wheel drive, (*c*) front-wheel drive.

SOLUTION

(*a*) Four-wheel drive:



$$+\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0 \qquad N_A + N_B = W = mg$$

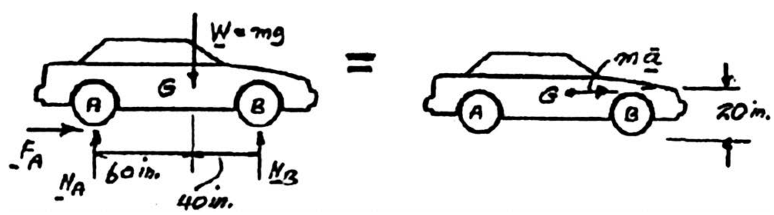
Thus: $$F_A + F_B = \mu_k N_A + \mu_k N_B = \mu_k (N_A + N_B) = \mu_k W = 0.80mg$$

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F_A + F_B = m\bar{a}$$

$$0.80mg = m\bar{a}$$

$$\bar{a} = 0.80g = 0.80(32.2 \text{ ft/s}^2) \qquad \bar{\mathbf{a}} = 25.8 \text{ ft/s}^2 \longrightarrow \blacktriangleleft$$

(*b*) Rear-wheel drive:



$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad (40 \text{ in.})W - (100 \text{ in.})N_A = -(20 \text{ in.})m\bar{a}$$

$$N_A = 0.4W + 0.2m\bar{a}$$

Thus: $$F_A = \mu_k N_B = 0.80(0.4W + 0.2m\bar{a}) = 0.32mg + 0.16m\bar{a}$$

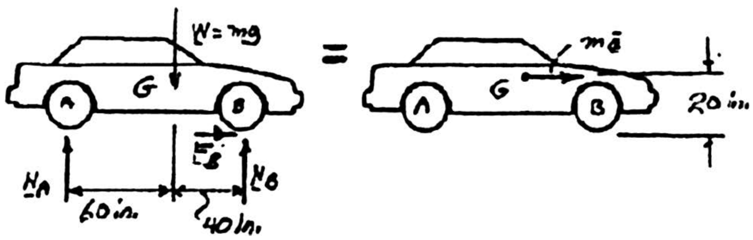
$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F_A = m\bar{a}$$

$$0.32mg + 0.16m\bar{a} = m\bar{a}$$

$$0.32g = 0.84\bar{a}$$

$$\bar{a} = \frac{0.32}{0.84}(32.2 \text{ ft/s}^2) \qquad \bar{\mathbf{a}} = 12.27 \text{ ft/s}^2 \longrightarrow \blacktriangleleft$$

(*c*) Front-wheel drive:



$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad (100 \text{ in.})N_B - (60 \text{ in.})W = -(20 \text{ in.})m\bar{a}$$

$$N_B = 0.6W - 0.2m\bar{a}$$

Thus: $$F_B = \mu_k N_B = 0.80(0.6W - 0.2m\bar{a}) = 0.48mg - 0.16m\bar{a}$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F_B = m\bar{a}$$

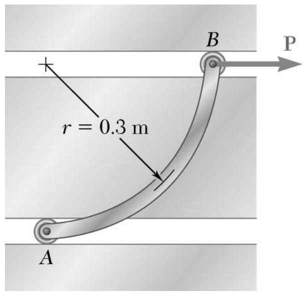
$$0.48mg - 0.16m\bar{a} = m\bar{a}$$

$$0.48g = 1.16\bar{a}$$

$$\bar{a} = \frac{0.48}{1.16}(32.2 \text{ ft/s}^2)$$

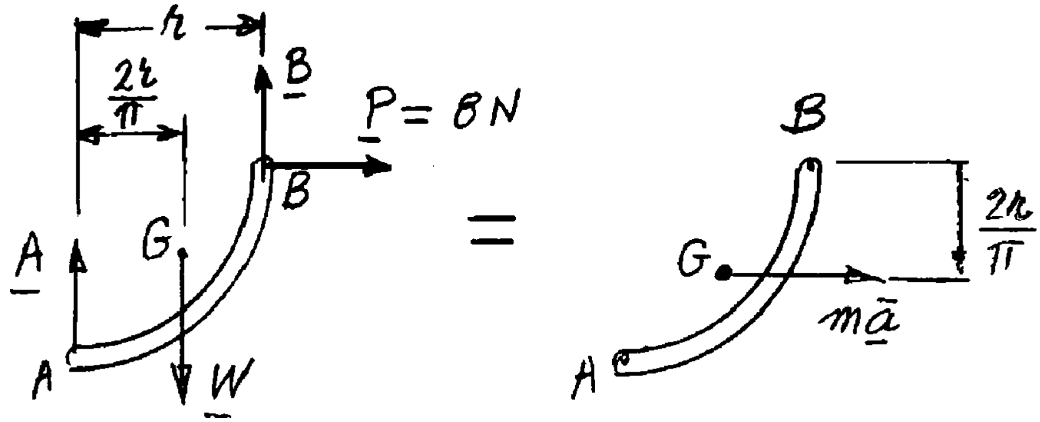
$\bar{\mathbf{a}} = 13.32 \text{ ft/s}^2 \longrightarrow$ ◀

PROBLEM 16.4



The motion of the 2.5-kg rod *AB* is guided by two small wheels which roll freely in horizontal slots. If a force **P** of magnitude 8 N is applied at *B*, determine (*a*) the acceleration of the rod, (*b*) the reactions at *A* and *B*.

SOLUTION



(*a*) $\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad P = m\bar{a}$

$$\bar{a} = \frac{P}{m} = \frac{8 \text{ N}}{2.5 \text{ kg}} = 3.20 \text{ m/s}^2$$

$$\bar{\mathbf{a}} = 3.20 \text{ m/s}^2 \longrightarrow \blacktriangleleft$$

(*b*) $+\!\!\uparrow \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad W\left(r - \frac{2r}{\pi}\right) - Ar = m\bar{a}\left(\frac{2r}{\pi}\right)$

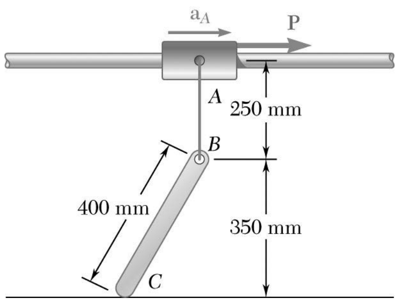
$$A = W\left(1 - \frac{2}{\pi}\right) - m\bar{a}\left(\frac{2}{\pi}\right) = mg\left(1 - \frac{2}{\pi}\right) - P\left(\frac{2}{\pi}\right)$$

$$= (2.5 \text{ kg})(9.81 \text{ m/s}^2)\left(1 - \frac{2}{\pi}\right) - (8 \text{ N})\left(\frac{2}{\pi}\right)$$

$$= 8.912 \text{ N} - 5.093 \text{ N} = 3.819 \text{ N} \qquad \mathbf{A} = 3.82 \text{ N}\uparrow \blacktriangleleft$$

$+\!\!\uparrow \Sigma F_y = 0: \quad A + B - W = 0$

$$B = W - A = (2.5)(9.81) - 3.819, \qquad \mathbf{B} = 20.71 \text{ N}\uparrow \blacktriangleleft$$



PROBLEM 16.5

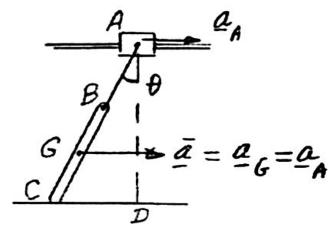
A uniform rod *BC* of mass 4 kg is connected to a collar *A* by a 250-mm cord *AB*. Neglecting the mass of the collar and cord, determine (*a*) the smallest constant acceleration $\mathbf{a}_A$ for which the cord and the rod lie in a straight line, (*b*) the corresponding tension in the cord.

SOLUTION

Geometry and kinematics:

Distance between collar and floor $= AD = 250 \text{ mm} + 350 \text{ mm} = 600 \text{ mm}$

When cord and rod lie in a straight line:



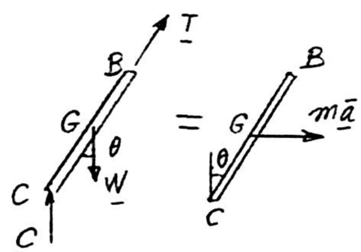
$$AC = AB + BC = 250 \text{ mm} + 400 \text{ mm} = 650 \text{ mm}$$

$$\cos\theta = \frac{AD}{AC} = \frac{600 \text{ mm}}{650 \text{ mm}}$$

$$\theta = 22.62°$$

Kinetics

(*a*) Acceleration at *A*.



$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad W(CG)\sin\theta = m\bar{a}(CG)\cos\theta$$

$$m\bar{a} = mg\tan\theta$$

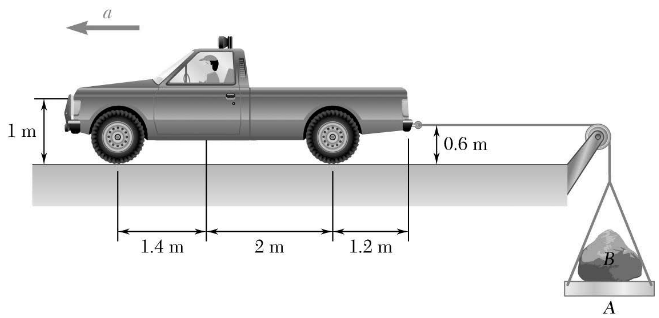
$$\bar{a} = g\tan\theta = (9.81 \text{ m/s}^2)\tan 22.62°$$

$$\mathbf{a}_A = \bar{\mathbf{a}} = 4.09 \text{ m/s}^2 \longrightarrow \blacktriangleleft$$

(*b*) Tension in the cord.

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad T\sin\theta = m\bar{a} = mg\tan\theta$$

$$T = \frac{mg}{\cos\theta} = \frac{(4 \text{ kg})(9.81)}{\cos 22.62°} \qquad T = 42.5 \text{ N} \blacktriangleleft$$



PROBLEM 16.6

A 2000-kg truck is being used to lift a 400-kg boulder B that is on a 50-kg pallet A. Knowing the acceleration of the rear-wheel drive truck is 1 m/s^2, determine (a) the reaction at each of the front wheels, (b) the force between the boulder and the pallet.

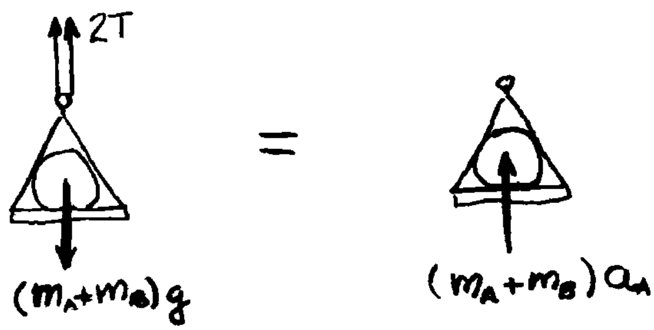
SOLUTION

Kinematics: Acceleration of truck: $\mathbf{a}_T = 1 \text{ m/s}^2 \longleftarrow$.

When the truck moves 1 m to the left, the boulder B and pallet A are raised 0.5 m.

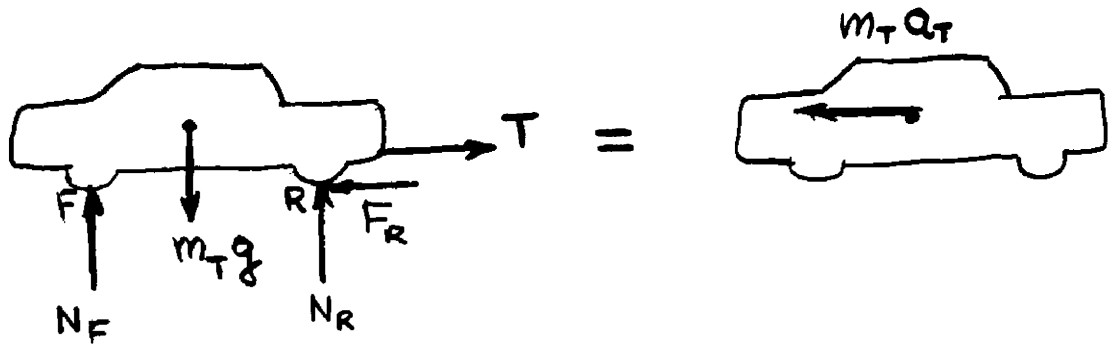
Then, $\mathbf{a}_A = 0.5 \text{ m/s}^2 \uparrow$ $\quad \mathbf{a}_B = 0.5 \text{ m/s}^2 \uparrow$

Kinetics: Let T be the tension in the cable.



Pallet and boulder: $+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad 2T - (m_A + m_B)_g = (m_A + m_B)a_B$

$$2T - (450 \text{ kg})(9.81 \text{ m/s}^2) = (450 \text{ kg})(0.5 \text{ m/s}^2)$$
$$T = 2320 \text{ N}$$



PROBLEM 16.6 (Continued)

Truck: $+\circlearrowleft M_R = \Sigma(M_R)_{\text{eff}}: \quad -N_F(3.4 \text{ m}) + m_T g(2.0 \text{ m}) - T(0.6 \text{ m}) = m_T a_T(1.0 \text{ m})$

$$N_F = \frac{(2.0 \text{ m})(2000 \text{ kg})(9.81 \text{ m/s}^2)}{3.4 \text{ m}} - \frac{(0.6 \text{ m})(2320 \text{ N})}{3.4 \text{ m}} + \frac{(1.0 \text{ m})(2000 \text{ kg})(1.0 \text{ m/s})}{3.4 \text{ m}}$$
$$= 11541.2 \text{ N} - 409.4 \text{ N} - 588.2 \text{ N}$$
$$= 10544 \text{ N}$$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N_F + N_R - m_T g = 0$

$$10544 \text{ N} + N_R - (2000 \text{ kg})(9.81 \text{ m/s}^2) = 0$$
$$N_R = 9076 \text{ N}$$

$\overset{+}{\leftarrow} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F_R - T = m_T a_T$

$$F_R = 2320 \text{ N} + (2000 \text{ kg})(1.0 \text{ m/s}^2)$$
$$= 4320 \text{ N}$$

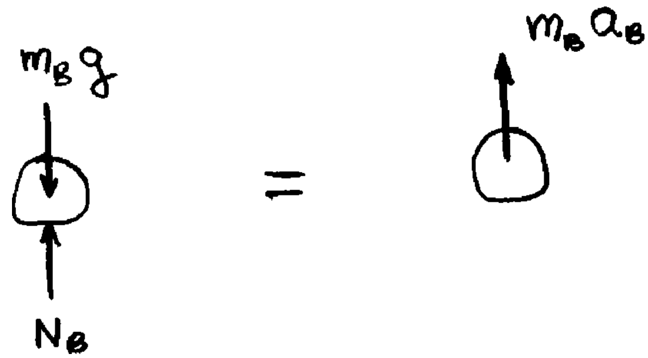
(*a*) *Reaction at each front wheel*:

$\frac{1}{2} N_F \uparrow$: $\quad$ 5270 N$\uparrow$ ◀

Reaction at each rear wheel:

$\frac{1}{2} F_R \leftarrow + \frac{1}{2} N_R \uparrow$ $\quad$ 5030 N ⦫ 64.5°

(*b*) *Force between boulder and pallet*.

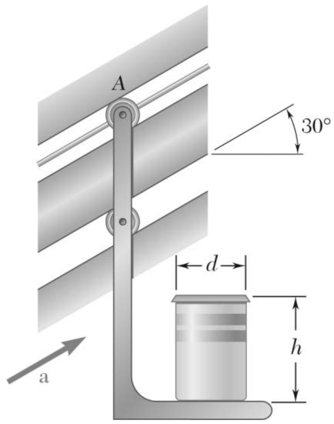


Boulder $\quad +\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N_B + M_B g - m_B a_B$

$$N_B = (400 \text{ kg})(9.81 \text{ m/s}^2) + (400 \text{ kg})(0.5 \text{ m/s}^2)$$
$$= 4124 \text{ N}$$

4120 N (compression) ◀

PROBLEM 16.7



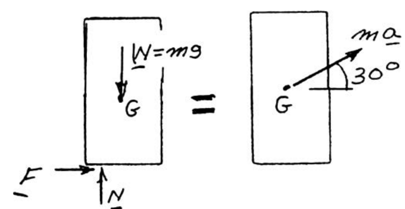
The support bracket shown is used to transport a cylindrical can from one elevation to another. Knowing that $\mu_s = 0.25$ between the can and the bracket, determine (*a*) the magnitude of the upward acceleration **a** for which the can will slide on the bracket, (*b*) the smallest ratio *h*/*d* for which the can will tip before it slides.

SOLUTION

(*a*) Sliding impends

$$\xrightarrow{+} \Sigma F_y = \Sigma (F_x)_{\text{eff}}: \qquad F = ma\cos 30°$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad N - mg = ma\sin 30°$$



$$N = m(g + a\sin 30°)$$

$$\mu_s = \frac{F}{N}$$

$$0.25 = \frac{ma\cos 30°}{m(g + a\sin 30°)}$$

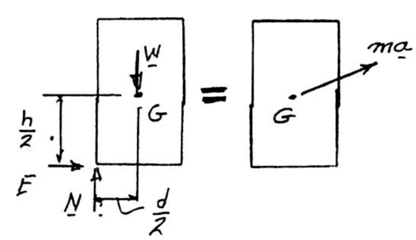
$$g + a\sin 30° = 4a\cos 30°$$

$$\frac{a}{g} = \frac{1}{4\cos 30° - \sin 30°}$$

$\mathbf{a} = 0.337g \measuredangle 30°$ ◀

(*b*) Tipping impends

$$\Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad F\left(\frac{h}{2}\right) - N\left(\frac{d}{2}\right) = 0$$

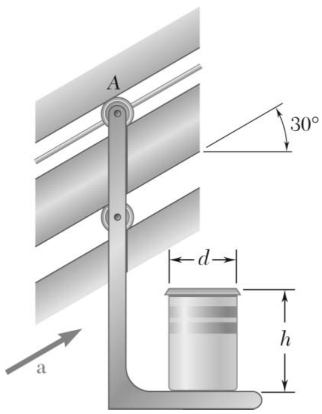


$$\frac{F}{N} = \frac{d}{h}$$

$$\mu = \frac{F}{N}; \quad 0.25 = \frac{d}{h};$$

$\frac{h}{d} = 4.00$ ◀

PROBLEM 16.8

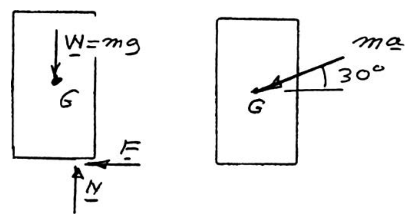


Solve Problem 16.7, assuming that the acceleration **a** of the bracket is directed downward.

PROBLEM 16.7 The support bracket shown is used to transport a cylindrical can from one elevation to another. Knowing that $\mu_s = 0.25$ between the can and the bracket, determine (*a*) the magnitude of the upward acceleration **a** for which the can will slide on the bracket, (*b*) the smallest ratio *h*/*d* for which the can will tip before it slides.

SOLUTION

(*a*) <u>Sliding impends</u>:



$\overset{+}{\leftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \qquad F = ma\cos 30°$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad N - mg = -ma\sin 30°$

$$N = m(g - a\sin 30°)$$

$$\mu_s = \frac{F}{N}$$

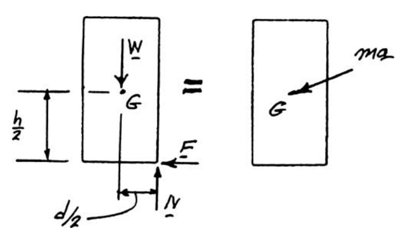
$$0.25 = \frac{ma\cos 30°}{m(g - a\sin 30°)}$$

$$g - a\sin 30° = 4a\cos 30°$$

$$\frac{a}{g} = \frac{1}{4\cos 30° + \sin 30°} = 0.25226$$

$\mathbf{a} = 0.252g \measuredangle 30°$ ◀

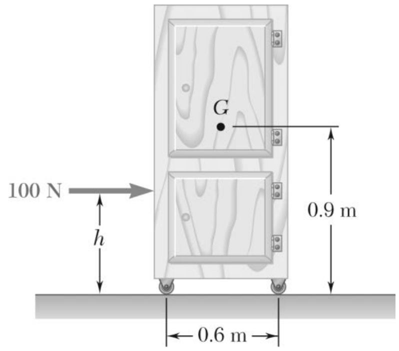
(*b*) <u>Tipping impends</u>:



$\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad F\left(\frac{h}{2}\right) = W\left(\frac{d}{2}\right) \quad \frac{F}{N} = \frac{d}{h}$

$$\mu = \frac{F}{N}; \qquad 0.25 = \frac{d}{h};$$

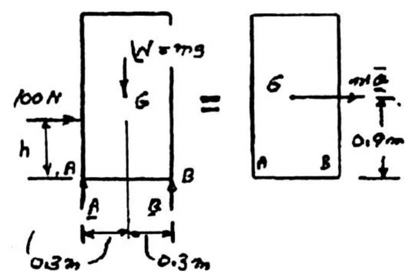
$\frac{h}{d} = 4.00$ ◀



PROBLEM 16.9

A 20-kg cabinet is mounted on casters that allow it to move freely $(\mu = 0)$ on the floor. If a 100-N force is applied as shown, determine (*a*) the acceleration of the cabinet, (*b*) the range of values of h for which the cabinet will not tip.

SOLUTION



(*a*) Acceleration

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad 100\text{ N} = m\bar{a}$$

$$100\text{ N} = (20\text{ kg})\bar{a}$$

$$\bar{\mathbf{a}} = 5.00\text{ m/s}^2 \longrightarrow \blacktriangleleft$$

(*b*) For tipping to impend ↻: $\quad A = 0$

$$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad (100\text{ N})h - mg(0.3\text{ m}) = m\bar{a}(0.9\text{ m})$$

$$(100\text{ N})h - (20\text{ kg})(9.81\text{ m/s}^2)(0.3\text{ m}) = (100\text{ N})(0.9\text{ m})$$

$$h = 1.489\text{ m}$$

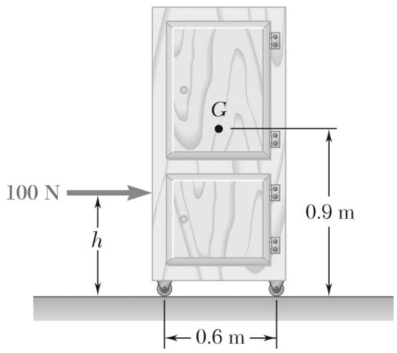
For tipping to impend ↺: $\quad B = 0$

$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad (100\text{ N})h + mg(0.3\text{ m}) = m\bar{a}(0.9\text{ m})$$

$$(100\text{ N})h + (20\text{ kg})(9.81\text{ m/s}^2)(0.3\text{ m}) = (100\text{ N})(0.9\text{ m})$$

$$h = 0.311\text{ m}$$

Cabinet will not tip: $\quad 0.311\text{ m} \le h \le 1.489\text{ m} \quad \blacktriangleleft$



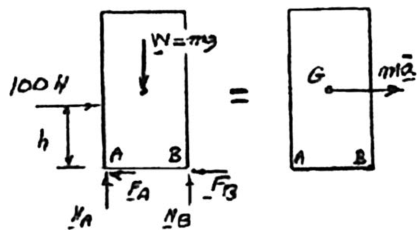
PROBLEM 16.10

Solve Problem 16.9, assuming that the casters are locked and slide on the rough floor $(\mu_k = 0.25)$.

PROBLEM 16.9 A 20-kg cabinet is mounted on casters that allow it to move freely $(\mu = 0)$ on the floor. If a 100-N force is applied as shown, determine (*a*) the acceleration of the cabinet, (*b*) the range of values of *h* for which the cabinet will not tip.

SOLUTION

(*a*) Acceleration.



$$+\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0$$

$$N_A + N_B = mg$$

But, $F = \mu N$, thus $F_A + F_B = \mu(mg)$

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad 100\text{ N} - (F_A + F_B) = m\bar{a}$$

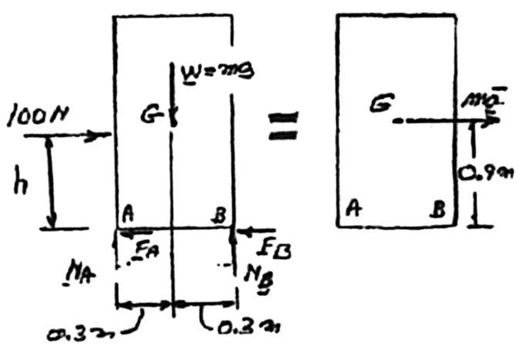
$$100\text{ N} - \mu mg = m\bar{a}$$

$$100\text{ N} - 0.25(20\text{ kg})(9.81\text{ m/s}^2) = (20\text{ kg})\bar{a}$$

$$\bar{a} = 2.548\text{ m/s}^2$$

$\bar{\mathbf{a}} = 2.55\text{ m/s}^2 \longrightarrow$ ◀

(*b*) Tipping of cabinet.



$$W = mg = (20\text{ kg})(9.81\text{ m/s}^2)$$

$$W = 196.2\text{ N}$$

For tipping to impend $\circlearrowright$: $\quad N_A = 0.$

$$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad (100\text{ N})h - W(0.3\text{ m}) = m\bar{a}(0.9\text{ m})$$

$$(100\text{ N})h - (196.2\text{ N})(0.3\text{ m}) = (20\text{ kg})(2.548\text{ m/s}^2)(0.9\text{ m})$$

$$h = 1.047\text{ m}$$

PROBLEM 16.10 (Continued)

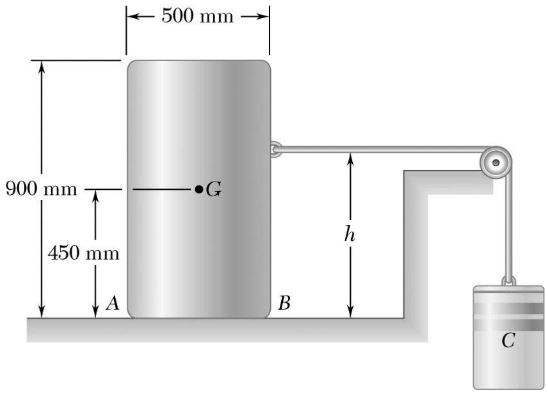
For tipping to impend ↻: $N_B = 0$

$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad (100 \text{ N})h + W(0.3 \text{ m}) = m\bar{a}(0.9 \text{ m})$$

$$(100 \text{ N})h + (196.2 \text{ N})(0.3 \text{ m}) = (20 \text{ kg})(2.548 \text{ m/s}^2)(0.9 \text{ m})$$

$$h = -0.130 \text{ m} \quad \text{(impossible)}$$

Cabinet will not tip: $h \leq 1.047 \text{ m}$ ◀



PROBLEM 16.11

A completely filled barrel and its contents have a combined mass of 90 kg. A cylinder C is connected to the barrel at a height h = 550 mm as shown. Knowing $\mu_s = 0.40$ and $\mu_k = 0.35,$ determine the maximum mass of C so the barrel will not tip.

SOLUTION

Kinematics: Assume that the barrel is sliding, but not tipping.

$$\alpha = 0 \qquad \mathbf{a}_G = a \longrightarrow$$

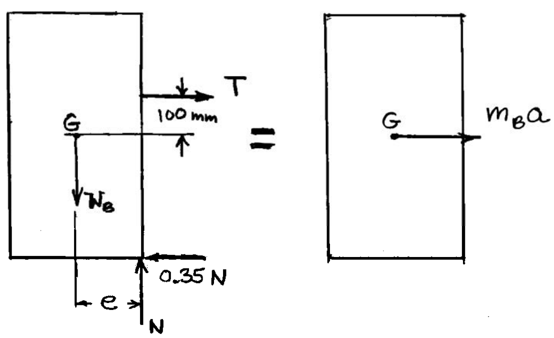
Since the cord is inextensible, $\mathbf{a}_C = a \downarrow$

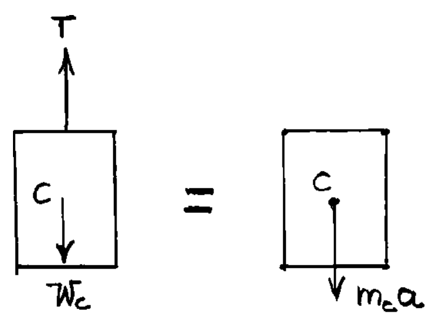
Kinetics: Draw the free body diagrams of the barrel and the cylinder. Let T be the tension in the cord.

The barrel is sliding. $F_F = \mu_k N = 0.35\,N$

Assume that tipping is impending, so that the line of action of the reaction on the bottom of the barrel passes through Point B.

$$e = 250 \text{ mm}$$





For the barrel. $+\uparrow \Sigma F_y = 0{:}\quad N - W_B = 0 \qquad N = W_B = m_B g = 882.90 \text{ N}$

$$+\circlearrowleft \Sigma M_G = 0{:}\quad Ne - 100T - (450)(0.35\,N) = 0$$

$$T = \frac{e - (450)(0.35)}{100} N = \frac{250 - 157.5}{100}(882.90) = 816.68 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = m_B a{:}\quad T - 0.35\,N = m_B a$$

$$a = \frac{816.68}{90} - 0.35\frac{882.90}{90} = 5.6407 \text{ m/s}^2$$

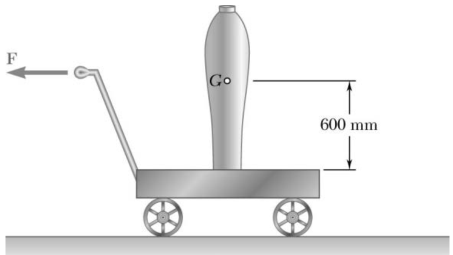
PROBLEM 16.11 (Continued)

For the cylinder: $+\downarrow \Sigma F = m_C a: \quad W_C - T = m_C a$

$$m_C g - T = m_C a$$

$$m_C = \frac{T}{g-a} = \frac{816.68}{9.81 - 5.6407} = 195.88 \text{ kg}$$

$$m_C = 195.9 \text{ kg} \blacktriangleleft$$

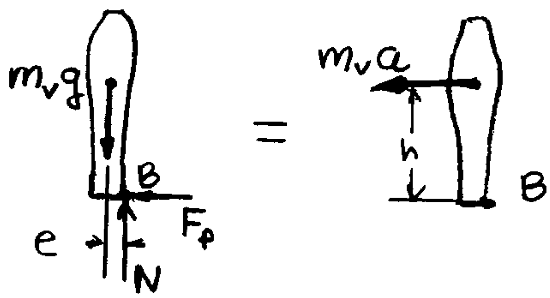


PROBLEM 16.12

A 40-kg vase has a 200-mm-diameter base and is being moved using a 100-kg utility cart as shown. The cart moves freely ($\mu = 0$) on the ground. Knowing the coefficient of static friction between the vase and the cart is $\mu_s = 0.4$, determine the maximum force **F** that can be applied if the vase is not to slide or tip.

SOLUTION

Vase:



$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad N - m_V g = 0$$

$$N = m_V g = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.4 \text{ N}$$

For impending sliding,

$$F_f = \mu_S N$$

$$F_f = (0.4)(392.4 \text{ N}) = 156.96 \text{ N}$$

$$\overset{+}{\longleftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F_f = m_V a$$

$$a = \frac{F_f}{m_V} = \frac{156.96 \text{ N}}{40} = 3.924 \text{ m/s}^2$$

This is the limiting value of a for sliding.

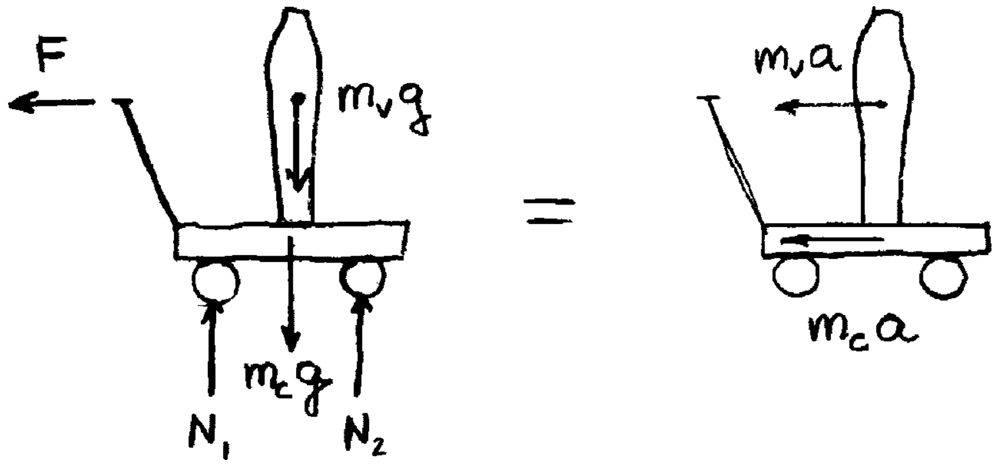
$$+\circlearrowleft M_B = \Sigma (M_B)_{\text{eff:}} \quad m_V g e = m_V a h$$

$$a = \frac{e}{h} g = \frac{100 \text{ mm}}{600 \text{ mm}} (9.81 \text{ m/s}^2) = 1.635 \text{ m/s}^2$$

This is the limiting value of a for tipping.

The smaller value governs. $\quad a = 1.635 \text{ m/s}^2$

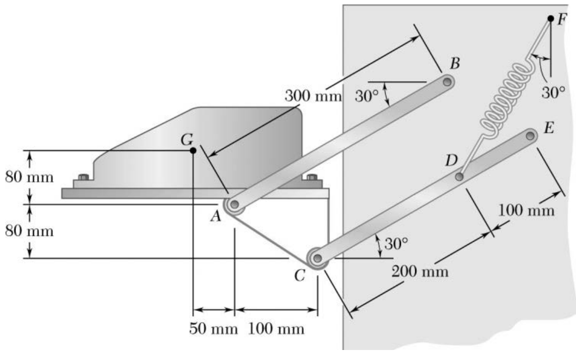
Cart and vase:



$\overset{+}{\longleftarrow}$ $\Sigma F_x = \Sigma (F_x)_{\text{eff}}$: $\quad F = m_C a + m_V a$

$$F = (100 \text{ kg})(1.635 \text{ m/s}^2) + (40 \text{ kg})(1.635 \text{ m/s}^2)$$

$F = 229$ N ◀



PROBLEM 16.13

The retractable shelf shown is supported by two identical linkage-and-spring systems; only one of the systems is shown. A 20-kg machine is placed on the shelf so that half of its weight is supported by the system shown. If the springs are removed and the system is released from rest, determine (*a*) the acceleration of the machine, (*b*) the tension in link *AB*. Neglect the weight of the shelf and links.

SOLUTION

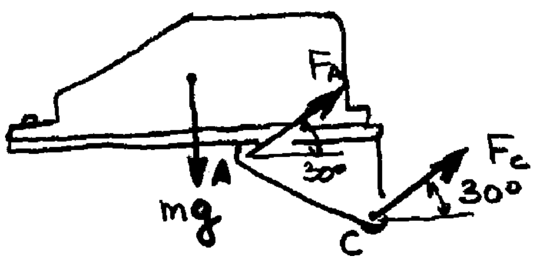
The links *AB* and *CE* keep the line *AC* on the shelf parallel to the fixed line *BE*. Thus the shelf moves in curvilinear translation.

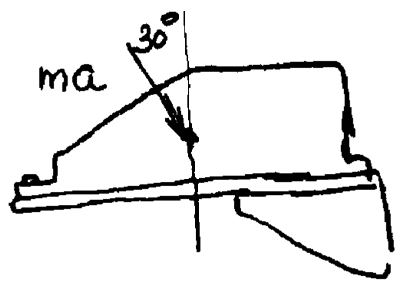
$$\alpha = 0 \qquad \text{and} \qquad \mathbf{a}_G \text{ is perpendicular to } \overrightarrow{AB}$$

Since link *AB* is a massless two-force member, the force at *A* is along link *AB*.

Since link *CDE* is massless and the spring *DF* is removed, the force at *C* is directed along the link *CDE*.

Consider the kinetics of the shelf.





Force perpendicular to link *AB*. ⦣ 30°

$$mg\cos 30° = m\bar{a}$$

$$\bar{a} = g\cos 30° = (9.81 \text{ m/s})\cos 30° = 8.4957 \text{ m/s}^2$$

(*a*) *Acceleration of the machine*: $\bar{\mathbf{a}} = 8.50 \text{ m/s}^2 \measuredangle 60°$ ◀

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (F_A\cos 30°)(0.080) + (F_A\sin 30°)(0.100) - mg(0.150)$$

$$= (ma\sin 30°)(0.160) + (ma\cos 30°)(0.150)$$

$$0.11928F_A - 0.150\,mg = -0.04990\,mg\cos 30°$$

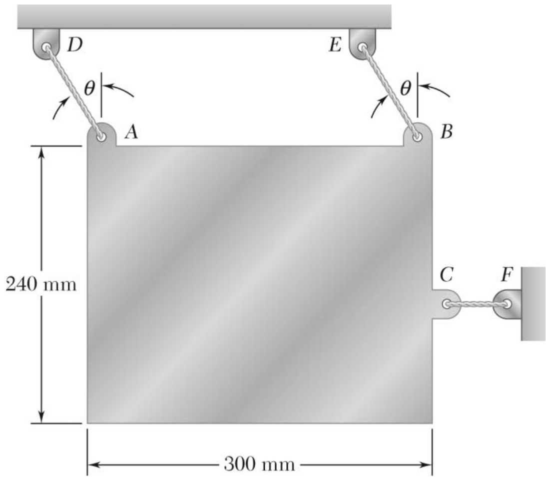
PROBLEM 16.13 (Continued)

(*b*) *Tension in link AB.* $F_A = 89525\, mg$

Taking *mg* to be half the weight of the machine,

$$mg = \frac{1}{2}(20 \text{ kg})(9.81 \text{ m/s}^2) = 98.1 \text{ N}$$

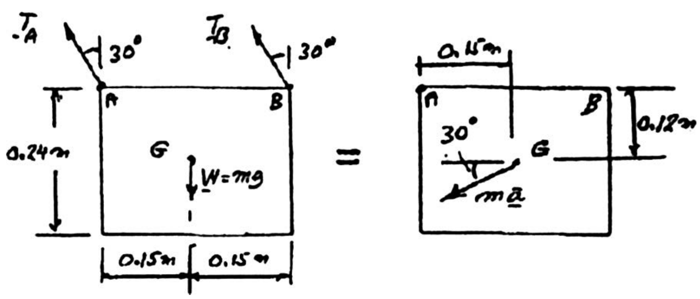
$$F_A = (0.89522)(98.1 \text{ N}) \qquad F = 87.8 \text{ N} \blacktriangleleft$$



PROBLEM 16.14

A uniform rectangular plate has a mass of 5 kg and is held in position by three ropes as shown. Knowing that $\theta = 30°$, determine, immediately after rope CF has been cut, (a) the acceleration of the plate, (b) the tension in ropes AD and BE.

SOLUTION



(a) <u>Acceleration</u> $\quad + \nearrow 30° \Sigma F = \Sigma F_{\text{eff}}: \quad mg \sin 30° = m\bar{a}$

$$\bar{a} = 0.5g = 4.905 \text{ m/s}^2 \qquad \bar{\mathbf{a}} = 4.91 \text{ m/s}^2 \swarrow 30° \blacktriangleleft$$

(b) <u>Tension in ropes</u>

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad (T_B \cos 30°)(0.3 \text{ m}) - mg(0.15 \text{ m}) = -m\bar{a}(\cos 30°)(0.12 \text{ m}) - m\bar{a}(\sin 30°)(0.15 \text{ m})$$

$$0.2598T_B - (5 \text{ kg})(9.81 \text{ m/s}^2)(0.15 \text{ m}) = -(5 \text{ kg})(4.905 \text{ m/s}^2)(0.1039 + 0.075)$$

$$0.2598T_B - 7.3575 = -4.388$$

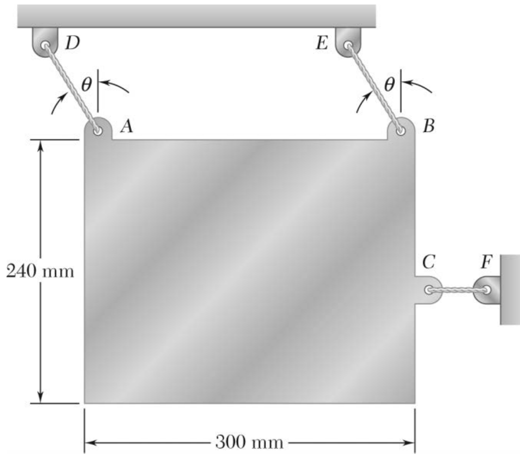
$$T_B = +11.43 \text{ N} \qquad T_{BE} = 11.43 \text{ N} \blacktriangleleft$$

$$+\searrow 10° \Sigma F = \Sigma F_{\text{eff}}: \quad T_A + 11.43 \text{ N} - mg \cos 30° = 0$$

$$T_A + 11.43 \text{ N} - (5 \text{ kg})(9.81) \cos 30° = 0$$

$$T_A + 11.43 \text{ N} - 42.48 \text{ N} = 0$$

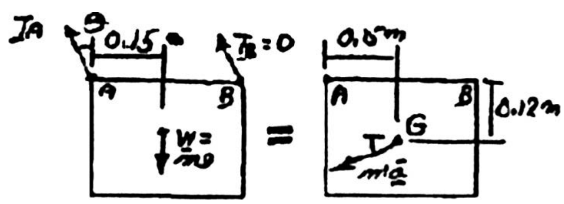
$$T_A = 31.04 \text{ N} \qquad T_{AD} = 31.0 \text{ N} \blacktriangleleft$$



PROBLEM 16.15

A uniform rectangular plate has a mass of 5 kg and is held in position by three ropes as shown. Determine the largest value of θ for which both ropes AD and BE remain taut immediately after rope CF has been cut.

SOLUTION



$$\searrow \Sigma F = \Sigma F_{\text{eff}}: \quad mg\sin\theta = m\bar{a}$$

$$\bar{a} = g\sin\theta$$

$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad mg(0.15\text{ m}) = m\bar{a}\cos\theta(0.12\text{ m}) + m\bar{a}\sin\theta(0.15\text{ m})$$

$$mg(0.15) = m(g\sin\theta)(0.12\cos\theta + 0.15\sin\theta)$$

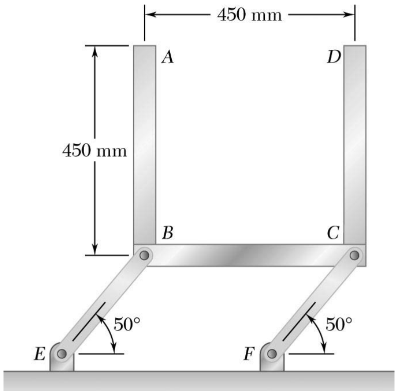
$$1 = 0.8\sin\theta\cos\theta - \sin^2\theta$$

$$1 - \sin^2\theta = 0.8\sin\theta\cos\theta$$

$$\cos^2\theta = 0.8\sin\theta\cos\theta$$

$$1 = 0.8\frac{\sin\theta}{\cos\theta}$$

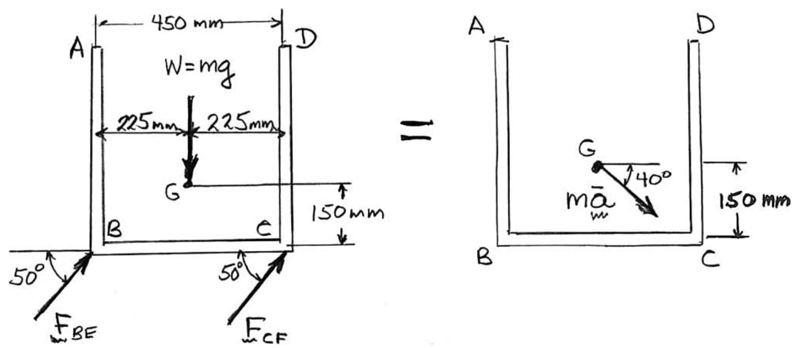
$$\tan\theta = 1.25; \qquad \theta = 51.3° \blacktriangleleft$$



PROBLEM 16.16

Three bars, each of mass 3 kg, are welded together and are pin-connected to two links *BE* and *CF*. Neglecting the weight of the links, determine the force in each link immediately after the system is released from rest.

SOLUTION



Mass center of *ABCD* is at *G*

$$\bar{y} = \frac{\Sigma m_i \bar{y}_i}{\Sigma m_i} = \frac{3(0.225) + 3(0.225) + 3(0)}{9} = 0.15 \text{ m}$$

$$W = mg$$

$$\searrow 40° \quad \Sigma F = \Sigma F_{\text{eff}}: \quad mg \cos 50° = m\bar{a}$$

$$(9.81 \text{ m/s}^2)\cos 50° = \bar{a}$$

$$\bar{\mathbf{a}} = 6.3057 \text{ m/s}^2 \searrow 40°$$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}:$$

$$(F_{CF} \sin 50°)(0.450 \text{ m}) - (9 \text{ kg})(9.81 \text{ m/s}^2)(0.225 \text{ m}) = -m\bar{a} \sin 40°(0.225 \text{ m}) - m\bar{a} \cos 40°(0.150 \text{ m})$$

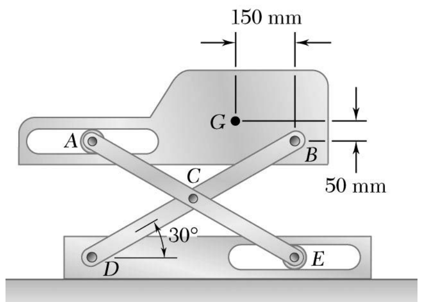
$$0.34472\, F_{CF} - 19.8653 = -m\bar{a}(0.14463 + 0.11491)$$

$$0.34472\, F_{CF} - 19.8653 = -9(6.3057)(0.25953)$$

$$F_{CF} = +14.8998 \text{ N} \qquad F_{CF} = +14.90 \text{ N compression} \blacktriangleleft$$

$$\measuredangle 50° \quad \Sigma F = \Sigma F_{\text{eff}}: \quad F_{BE} + 14.9 \text{ lb} - (9 \text{ kg})(9.81)\sin 50° = 0$$

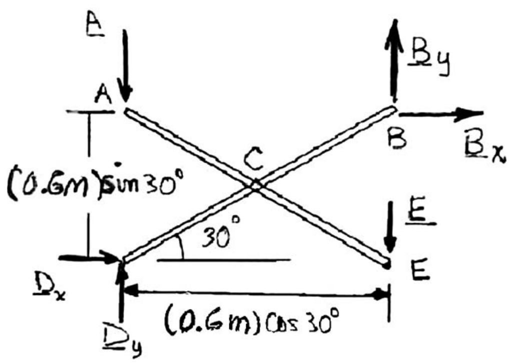
$$F_{BF} = +52.734 \text{ N} \qquad F_{BE} = +52.7 \text{ N compression} \blacktriangleleft$$

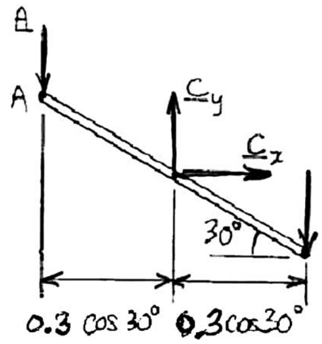


PROBLEM 16.17

Members *ACE* and *DCB* are each 600 mm long and are connected by a pin at *C*. The mass center of the 10-kg member *AB* is located at *G*. Determine (*a*) the acceleration of *AB* immediately after the system has been released from rest in the position shown, (*b*) the corresponding force exerted by roller *A* on member *AB*. Neglect the weight of members *ACE* and *DCB*.

SOLUTION





Analysis of linkage

Since members *ACE* and *DCB* are of negligible mass, their effective forces may also be neglected and the methods of statics may be applied to their analysis.

Free body: *Entire linkage*:

$$+\circlearrowleft \Sigma M_D = 0:$$

$$(B_y - E)(0.6\cos 30°) - B_x(0.6\sin 30°) = 0$$

$$(B_y - E)\cos 30° - B_x \sin 30° = 0 \qquad (1)$$

Free body: *member ACE*

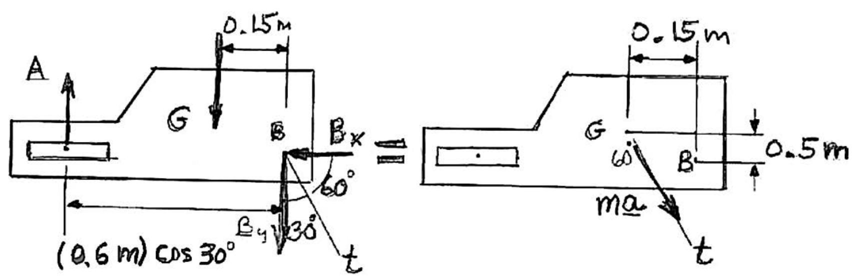
$$+\circlearrowleft \Sigma M_C = 0:$$

$$A(0.3\cos 30°) - E(0.3\cos 30°) = 0; \quad E = A$$

Carrying into Eq. (1):

$$(B_y - A)\cos 30° - B_x \sin 30° = 0 \qquad (2)$$

Equations of Motion for Member AB



(*a*) $+\searrow 60° \quad \Sigma F_t - \Sigma(F_t)_{\text{eff}}:$

$$(B_y - A)\cos 30° - B_x \sin 30° + W\cos 30° = m\overline{a}$$

Recalling equation (2), we have,

$$W\cos 30° = m\overline{a} \qquad \overline{a} = \frac{W}{m}\cos 30° = g\cos 30°$$

$$\overline{a} = (9.81\text{ m/s}^2)\cos 30°$$

$\overline{\mathbf{a}} = 8.50\text{ m/s}^2 \searrow 60°$ ◀

(*b*) $+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}:$

$$W(0.15\text{ m}) - A(0.6\text{ m})\cos 30° = (m\overline{a}\sin 60°)(0.15\text{ m}) - (m\overline{a}\cos 60°)(0.05\text{ m})$$

But, $m\overline{a} = W\cos 30°$

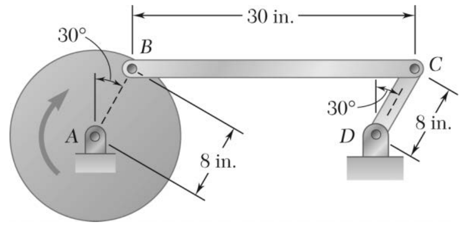
$$0.15W - 0.6A\cos 30° = W\cos 30°(0.15\sin 60° - 0.05\cos 60°)$$

$$A = W\left(\frac{1}{4\cos 30°} - \frac{1}{4}\sin 60° + \frac{1}{12}\cos 60°\right) = 0.11384W = 0.11384mg$$

Recalling that $m = 10$ kg so that $mg = 98.1$ N,

$$A = 0.11384(98.1) = 11.168\text{ N}$$

$\mathbf{A} = 11.17\text{ N}\uparrow$ ◀

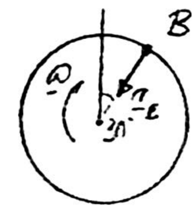


PROBLEM 16.18

The 15-lb rod *BC* connects a disk centered at *A* to crank *CD*. Knowing that the disk is made to rotate at the constant speed of 180 rpm, determine for the position shown the vertical components of the forces exerted on rod *BC* by the pins at *B* and *C*.

SOLUTION

We first determine the acceleration of Point *B* of disk.



$$\omega = 180 \text{ rpm} = 18.85 \text{ rad/s}$$

$$\text{Since } \omega = \text{constant}$$

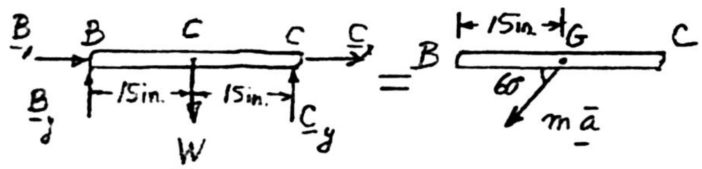
$$a_B = (a_B)_n = r\omega^2$$

$$= \left(\frac{8}{12}\text{ ft}\right)(18.85 \text{ rad/s})^2$$

$$\mathbf{a}_B = 236.9 \text{ ft/s}^2 \measuredangle 60°$$

Since rod *BC* is in translation.

$$\bar{\mathbf{a}} = \bar{\mathbf{a}}_B = 236.9 \text{ ft/s}^2 \measuredangle 60°$$



Vertical components of forces at *B* and *C*.

$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad C_y(30 \text{ in.}) - W(15 \text{ in.}) = -m\bar{a}\sin 60°(15 \text{ in.})$$

Since $\quad W = 15 \text{ lb} \quad \text{and} \quad m\bar{a} = \dfrac{15 \text{ lb}}{32.2}(236.9) = 110.36 \text{ lb}$

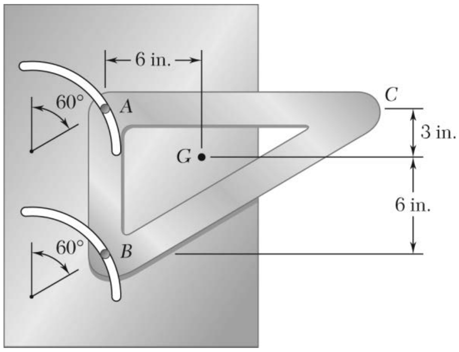
$$30C_y - (15)(15) = -110.36\sin 60°(15) = -95.57(15)$$

$$2C_y = -95.57 + 15 = -80.57, \quad C_y = -40.285 \text{ lb} \qquad \mathbf{C}_y = 40.3 \text{ lb}\downarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \qquad B_y - W + C_y = -m\bar{a}\sin 60°$$

$$B_y = W - C_y - \frac{15}{32.2}(236.9)\sin 60° = 15 + 40.285 - 95.57$$

$$B_y = -40.285 \text{ lb} \qquad \mathbf{B}_y = 40.3 \text{ lb}\downarrow \blacktriangleleft$$

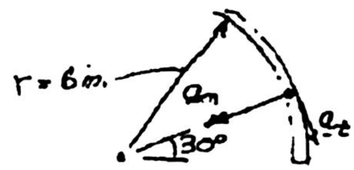


PROBLEM 16.19

The triangular weldment *ABC* is guided by two pins that slide freely in parallel curved slots of radius 6 in. cut in a vertical plate. The weldment weighs 16 lb and its mass center is located at Point *G*. Knowing that at the instant shown the velocity of each pin is 30 in./s downward along the slots, determine (*a*) the acceleration of the weldment, (*b*) the reactions at *A* and *B*.

SOLUTION

Slot: $v = 30 \text{ in./s}$ ↘

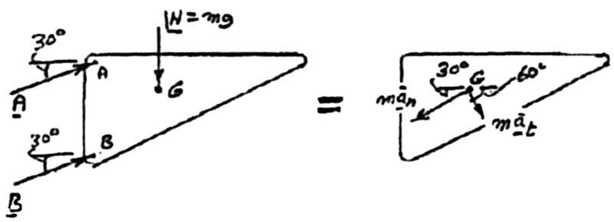


$$a_n = \frac{v^2}{r} = \frac{(30 \text{ in./s})^2}{6 \text{ in.}} = 150 \text{ in./s}^2$$

$$\mathbf{a}_n = 12.5 \text{ ft/s}^2 \nearrow 30°$$

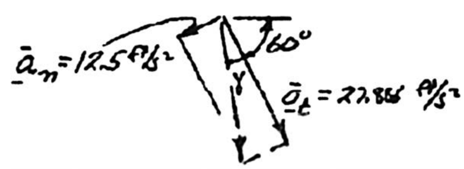
$$\mathbf{a}_t = a_t \searrow 60°$$

Weldment is in translation $\quad \bar{\mathbf{a}}_n = 12.5 \text{ ft/s}^2$



$\searrow 60° \ \Sigma F = \Sigma F_{\text{eff}}: \quad mg\cos 30° = ma_t$

$$\bar{\mathbf{a}}_t = 27.886 \text{ ft/s}^2 \searrow 60°$$



(*a*) Acceleration

$$\beta = \tan^{-1}\frac{\bar{a}_n}{\bar{a}_t} = \tan^{-1}\frac{12.5}{27.886} = 24.14°$$

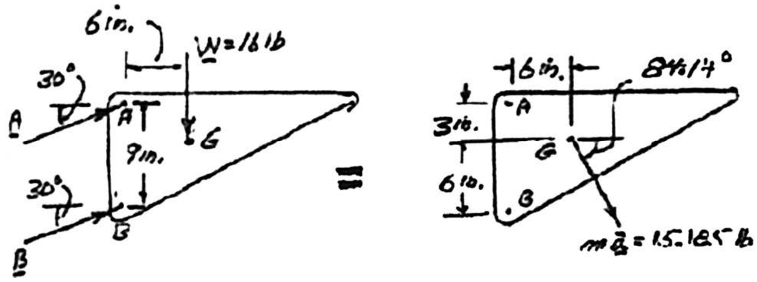
$$\bar{a}^2 = a_t^2 + a_n^2$$

$$= (27.886)^2 + (12.5)^2$$

$$\bar{\mathbf{a}} = 30.56 \text{ ft/s}^2 \searrow 84.1° \qquad \bar{\mathbf{a}} = 30.6 \text{ ft/s}^2 \searrow 84.1° \blacktriangleleft$$

(*b*) <u>Reactions</u>

$$m\bar{a} = \frac{16\ \text{lb}}{32.2\ \text{ft/s}^2}(30.56\ \text{ft/s}^2) = 15.185\ \text{lb}$$



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}:$$

$$B\cos 30°(9\ \text{in.}) - (16\ \text{lb})(6\ \text{in.}) = (15.185\ \text{lb})(\cos 84.14°)(3\ \text{in.}) - (15.185\ \text{lb})(\sin 84.14°)(6\ \text{in.})$$

$$7.794B - 96 = +4.651 - 90.634$$

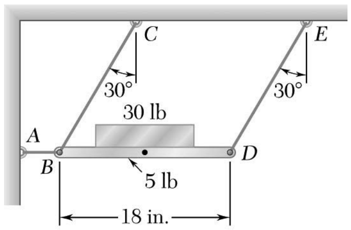
$$B = +1.285\ \text{lb} \qquad \mathbf{B} = 1.285\ \text{lb} \angle 30° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad A\cos 30° + B\cos 30° = m\bar{a}\cos 84.14°$$

$$A\cos 30° + (1.285\ \text{lb})\cos 30° = (15.185\ \text{lb})\cos 84.14°$$

$$A\cos 30° + 1.113\ \text{lb} = 1.550\ \text{lb}$$

$$A = +0.505\ \text{lb} \qquad \mathbf{A} = 0.505\ \text{lb} \angle 30° \blacktriangleleft$$

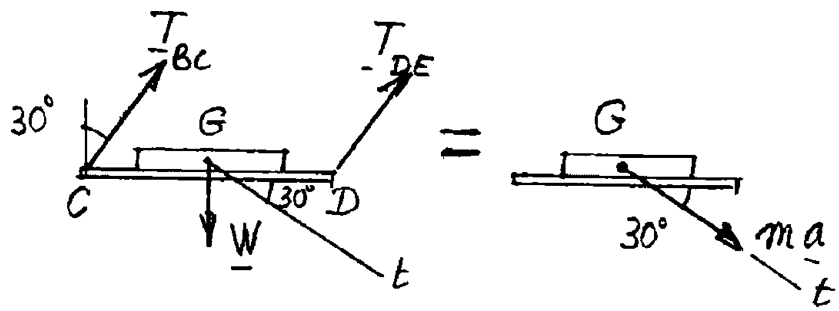


PROBLEM 16.20

The coefficients of friction between the 30-lb block and the 5-lb platform BD are $\mu_s = 0.50$ and $\mu_k = 0.40$. Determine the accelerations of the block and of the platform immediately after wire AB has been cut.

SOLUTION

Assume that the block does not slide relative to the platform. Draw the free body diagram of the platform and block.

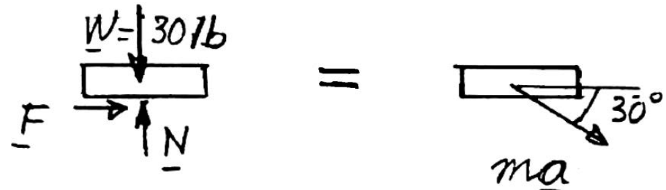


$$+\searrow \Sigma F_t = \Sigma (F_t)_{\text{eff}}: \quad W \sin 30° = ma$$

$$a = \frac{W}{m} \sin 30° = \frac{1}{2} g = 16.1 \text{ ft/s}^2$$

$$a = 16.1 \text{ ft/s}^2 \searrow 30°$$

Check whether or not the block will slide relative to the platform. Draw the free body diagram of the block alone.



$$ma = m\left(\frac{1}{2} g\right) = \frac{1}{2} W = 15 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = (15 \text{ lb}) \cos 30° = 12.99 \text{ lb}$$

$$+\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad 30 \text{ lb} - N = (15 \text{ lb}) \sin 30°$$

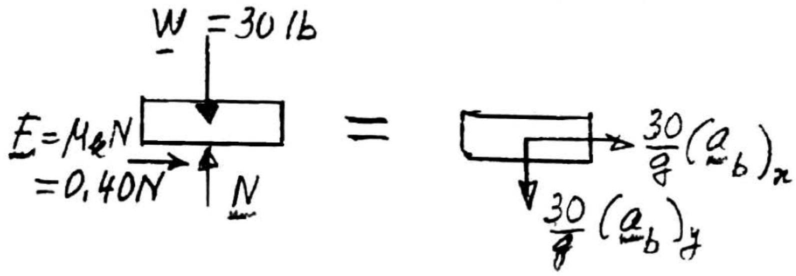
$$N = 30 - 7.5 = 22.5 \text{ lb}$$

Thus, $F_m = \mu_s N = 0.50\ (22.5\ \text{lb}) = 11.25\ \text{lb}$

Since $F > F_m$, <u>the block will slide</u>

Now assume that the block slides relative to the platform.

Equations of motion for block: (assuming sliding)



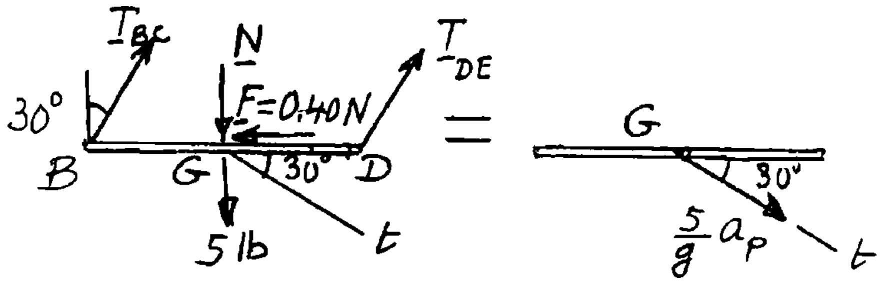
$$+\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad 30 - N = \frac{30}{g}(a_b)_y$$

$$(a_b)_y = g\left(1 - \frac{N}{30}\right) \quad (1)$$

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad 0.40\ \text{N} = \frac{30}{g}(a_b)_x$$

$$(a_b)_x = g\left(\frac{N}{75}\right) \quad (2)$$

Equations of motion for platform.



$$+\searrow \Sigma F_t = \Sigma (F_t)_{\text{eff}}: \quad (N - 5)\sin 30° - (0.40\ \text{N})\cos 30° = \frac{5}{g}a_P$$

$$a_p = g(0.5 + 0.030718\ \text{N}) \quad (3)$$

If contact is maintained between block and platform, we must have

$$(a_b)_y = (a_p)_y = a_p \sin 30° \quad (4)$$

PROBLEM 16.20 (Continued)

Substituting from (1) and (3) into (4):

$$g\left(1-\frac{N}{30}\right)=g(0.5+0.030718\text{ N})\sin 30°$$

$$(0.015359+0.033333)N=0.75$$

$$N=15.403\text{ lb}$$

Substituting for N in (2) and (1):

$$(a_b)_x=(32.2\text{ ft/s}^2)\,\frac{15.403}{75},\quad (\mathbf{a}_b)_x=6.61\text{ ft/s}^2\longrightarrow$$

$$(a_b)_y=(32.2\text{ ft/s}^2)\left(1-\frac{15.403}{30}\right),\quad (\mathbf{a}_b)_y=15.67\text{ ft/s}^2\downarrow$$

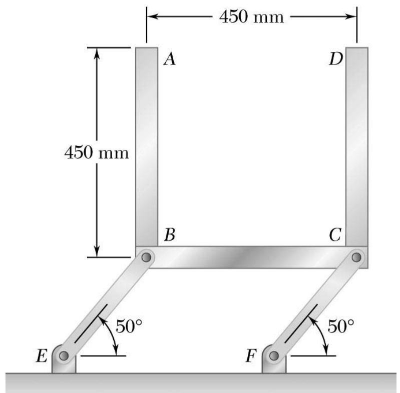
$$\mathbf{a}_b=17.01\text{ ft/s}^2\ \searrow 67.1° \blacktriangleleft$$

Substituting for N in (3):

$$a_P=(32.2\text{ ft/s}^2)(0.5+0.030718\times 15.403)=31.335\text{ ft/s}^2$$

$$\mathbf{a}_P=31.3\text{ ft/s}^2\ \searrow 30° \blacktriangleleft$$

Note: Since $N>0$, we check that contact between block and platform is maintained.



PROBLEM 16.21

Draw the shear and bending-moment diagrams for the vertical rod *AB* of Problem 16.16.

PROBLEM 16.16 Three bars, each of mass 3 kg, are welded together and are pin-connected to two links *BE* and *CF*. Neglecting the weight of the links, determine the force in each link immediately after the system is released from rest.

SOLUTION

From the solution of Problem 16.16, the acceleration of all points of vertical rod *AB* is

$$\mathbf{a} = 6.3057 \text{ m/s}^2 \searrow 40°$$

or

$$\mathbf{a} = 4.8304 \text{ m/s}^2 \longrightarrow +4.0532 \text{ m/s}^2 \downarrow$$

Mass of rod *AB*: $m = 3$ kg

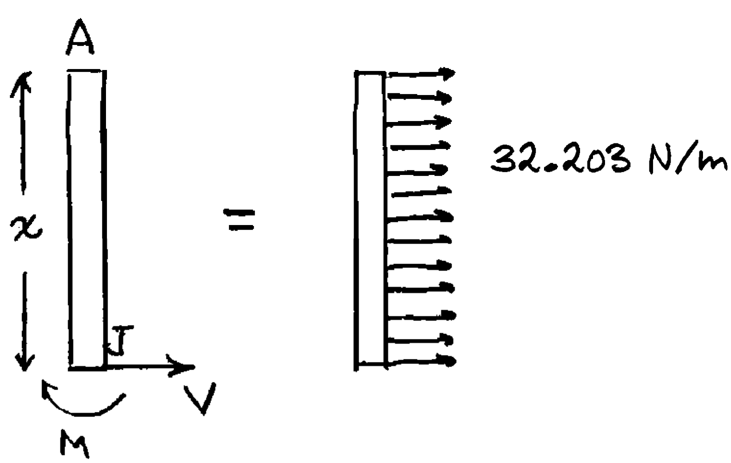
Mass per unit length: $\dfrac{m}{l} = \dfrac{3 \text{ kg}}{0.450 \text{ m}} = 6.6667 \text{ kg/m}$

Effective force per length: $\dfrac{m}{l}\mathbf{a}$

$$(6.6667 \text{ kg/m})(4.8304 \text{ m/s}^2) \longrightarrow + (6.6667 \text{ kg/m})(4.0532 \text{ m/s}^2)$$

$$32.203 \text{ N/m} \longrightarrow + 27.021 \text{ N/m} \downarrow$$

Only the horizontal component contributes to the shear and bending moment. Let *x* be a vertical coordinate (positive down) with its origin at *A*. Draw the free body diagram of the portion of the rod *AB* lying above the section defined by *x*.



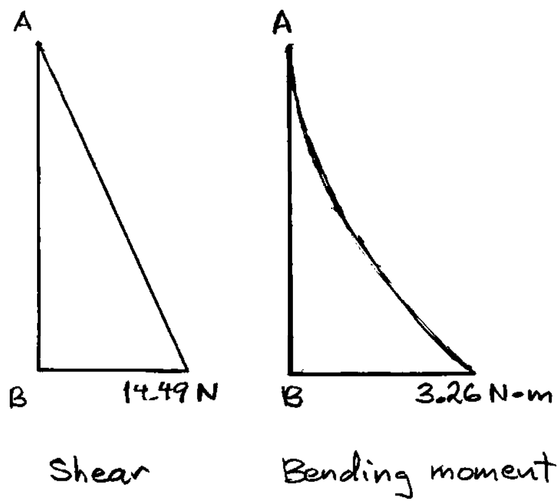
PROBLEM 16.21 (Continued)

$$\xrightarrow{+} \Sigma F = \Sigma (F_j)_{\text{eff}}: \quad V = 32.203x$$

$$\circlearrowleft + \Sigma M_J = \Sigma (M_j)_{\text{eff}}: \quad M = (32.203x)\frac{x}{2}$$

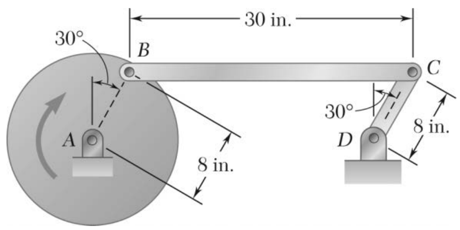
$$= 16.101x^2$$

Shear and bending moment diagrams.



$V_{\text{max}} = (32.203 \text{ N/m})(0.450 \text{ m})$ $\qquad V_{\text{max}} = 14.49 \text{ N}$ ◀

$M_{\text{max}} = (16.101 \text{ N/m})(0.450 \text{ m})^2$ $\qquad M_{\text{max}} = 3.26 \text{ N}\cdot\text{m}$ ◀



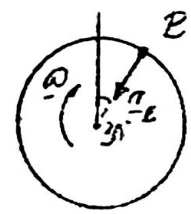
PROBLEM 16.22*

Draw the shear and bending-moment diagrams for the connecting rod *BC* of Problem 16.18.

PROBLEM 16.18 The 15-lb rod *BC* connects a disk centered at *A* to crank *CD*. Knowing that the disk is made to rotate at the constant speed of 180 rpm, determine for the position shown the vertical components of the forces exerted on rod *BC* by the pins at *B* and *C*.

SOLUTION

We first determine the acceleration of Point *B* of disk:



$$\omega = 180 \text{ rpm} = 18.85 \text{ rad/s}$$

Since $\omega = \text{constant}$

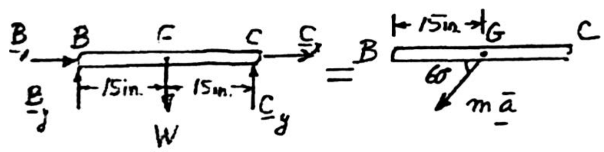
$$a_B = (a_B)_n = r\omega^2$$

$$= \left(\frac{8}{12}\text{ ft}\right)(18.85 \text{ rad/s})^2$$

$$\mathbf{a}_B = 236.9 \text{ ft/s}^2 \measuredangle 60°$$

Since rod *BC* is in translation.

$$\bar{\mathbf{a}} = \mathbf{a}_B = 236.9 \text{ ft/s}^2 \measuredangle 60°$$



$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad C_y(30 \text{ in.}) - W(15 \text{ in.}) = -m\bar{a}\sin 60°(15 \text{ in.})$$

Since $W = 15$ lb and $m\bar{a} = \dfrac{15 \text{ lb}}{32.2}(236.9) = 110.36$ lb:

$$30C_y - (15)(15) = -110.36\sin 60°(15) = -95.57(15)$$

$$2C_y = -95.57 + 15 = -80.57, \quad C_y = -40.285 \text{ lb}$$

$$\mathbf{C}_y = 40.3 \text{ lb} \downarrow$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad B_y - W + C_y = -m\bar{a}\sin 60°$$

$$B_y = W - C_y - \frac{15}{32.2}(236.9)\sin 60° = 15 + 40.285 - 95.57$$

$$B_y = -40.285 \text{ lb} \qquad \mathbf{B}_y = 40.3 \text{ lb} \downarrow$$

$$a_y = 236.7\sin 60° \qquad \mathbf{a}_y = 205.2 \text{ ft/s}^2 \downarrow$$

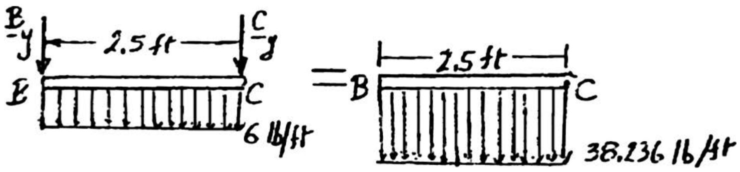
PROBLEM 16.22* (Continued)

$$\text{Distributed weight per unit length } = w = \frac{15 \text{ lb}}{\left(\frac{30}{12}\right)\text{ft}} = 6 \text{ lb/ft}$$

$$\text{Distributed mass per unit length } = \frac{w}{g} = \frac{6}{g} = \frac{6}{32.2} \text{lb} \cdot \text{s}^2/\text{ft}^2$$

$$\text{Vertical component of effective forces } = \frac{w}{g} a_y = \frac{6}{32.2}(205.2)$$

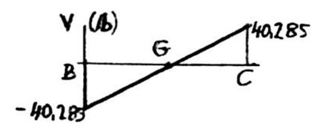
$$= 38.236 \text{ lb/ft}$$



$$+\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad B_y + C_y + (2.5 \text{ ft})(6 \text{ lb/ft}) = (2.5 \text{ ft})(38.236 \text{ lb/ft})$$

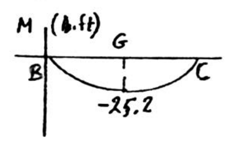
$$B_y + C_y = 80.59 \text{ lb}$$

From symmetry. $B_y = C_y = 40.285$ lb $\quad \mathbf{B}_y = \mathbf{C}_y = 40.3 \text{ lb}\downarrow$



Maximum value of bending moment occurs at G, where $V = 0$:

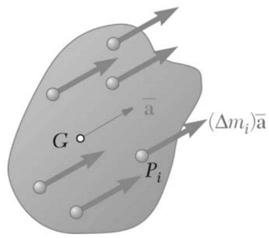
$$|M|_{\max} = \text{Area under } V\text{-diagram from } B \text{ to } G = \frac{1}{2}(40.285 \text{ lb})(1.25 \text{ ft})$$



$|M|_{\max} = 25.2 \text{ lb} \cdot \text{ft}$ ◀

$V_B = -40.3$ lb ◀

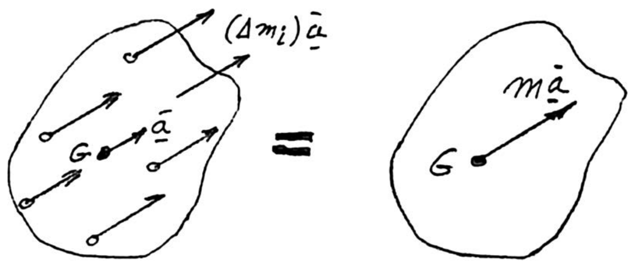
PROBLEM 16.23



For a rigid slab in translation, show that the system of the effective forces consists of vectors $(\Delta m_i)\bar{\mathbf{a}}$ attached to the various particles of the slab, where $\bar{\mathbf{a}}$ is the acceleration of the mass center *G* of the slab. Further show, by computing their sum and the sum of their moments about *G*, that the effective forces reduce to a single vector $m\bar{\mathbf{a}}$ attached at *G*.

SOLUTION

Since slab is in translation, each particle has same acceleration as *G*, namely $\bar{\mathbf{a}}$. The effective forces consist of $(\Delta mi)\bar{\mathbf{a}}$.



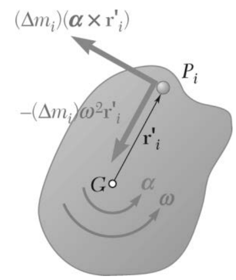
The sum of these vectors is: $\Sigma(\Delta m_i)\bar{\mathbf{a}} = (\Sigma\Delta m_i)\bar{\mathbf{a}}$

or since $\Sigma\Delta m_i = m,$

$$\Sigma(\Delta m_i)\bar{\mathbf{a}} = m\bar{\mathbf{a}}$$

The sum of the moments about *G* is: $\Sigma r_i' \times (\Delta m_i)\bar{\mathbf{a}} = (\Sigma\Delta m_i r_i') \times \bar{\mathbf{a}}$ (1)

But, $\Sigma\Delta m_i r_i' = m\bar{r}' = 0,$ because *G* is the mass center. It follows that the right-hand member of Eq. (1) is zero. Thus, the moment about *G* of $m\bar{a}$ must also be zero, which means that its line of action passes through *G* and that it may be attached at *G*.



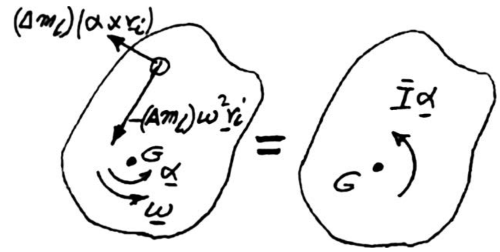
PROBLEM 16.24

For a rigid slab in centroidal rotation, show that the system of the effective forces consists of vectors $-(\Delta m_i)\boldsymbol{\omega}^2\mathbf{r}'_i$ and $(\Delta m_i)(\boldsymbol{\alpha} \times \mathbf{r}'_i)$ attached to the various particles P_i of the slab, where $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ are the angular velocity and angular acceleration of the slab, and where $\mathbf{r}'_i$ denotes the position vector of the particle P_i relative to the mass center G of the slab. Further show, by computing their sum and the sum of their moments about G, that the effective forces reduce to a couple $\bar{I}\boldsymbol{\alpha}$.

SOLUTION

For centroidal rotation: $$\mathbf{a}_i = (\mathbf{a}_i)_t + (\mathbf{a}_i)_n = \boldsymbol{\alpha} \times \mathbf{r}'_i - \omega^2 \mathbf{r}'_i$$

Effective forces are: $$(\Delta m_i)a_i = (\Delta m_i)(\boldsymbol{\alpha} \times \mathbf{r}_i) - (\Delta m_i)\omega^2 r'_i(\Delta m_i)(\alpha \times \mathbf{r}_i)$$



$$\Sigma(\Delta m_i)\mathbf{a}_i = \Sigma(\Delta m_i)(\boldsymbol{\alpha} \times \mathbf{r}'_i) - \Sigma(\Delta m_i)\omega^2 r'_i$$

$$= \alpha \times \Sigma(\Delta m_i)\mathbf{r}'_i - \omega^2 \Sigma(\Delta m_i) r'_i$$

Since G is the mass center, $\Sigma(\Delta m_i) r'_i = 0$

effective forces reduce to a couple, Summing moments about G,

$$\Sigma(\mathbf{r}'_1 \times \Delta m_i \mathbf{a}_i) = \Sigma[\mathbf{r}'_i \times (\Delta m_i)(\boldsymbol{\alpha} \times \mathbf{r}'_i)] - \Sigma \mathbf{r}'_i \times (\Delta m_i)\omega^2 r'_i$$

But, $$\mathbf{r}'_i \times (\Delta m_i)\omega^2 \mathbf{r}'_i = \omega^2 (\Delta m_i)(\mathbf{r}'_i \times \mathbf{r}'_i) = 0$$

and, $$\mathbf{r}'_i \times (\Delta m_i)(\boldsymbol{\alpha} \times \mathbf{r}'_i) = (\Delta m_i) r_1'^2 \boldsymbol{\alpha}$$

Thus, $$\Sigma(\mathbf{r}'_i \times \Delta m_i \mathbf{a}_i) = \Sigma(\Delta m_i) r_1^2 \alpha = \left[\Sigma(\Delta m_i) r_i'^2\right]\alpha$$

Since $\Sigma(\Delta m) r_i'^2 = \bar{I}$, the moment of the couple is $\bar{I}\boldsymbol{\alpha}$.

PROBLEM 16.25

The rotor of an electric motor has an angular velocity of 3600 rpm when the load and power are cut off. The 50-kg rotor, which has a centroidal radius of gyration of 180 mm, then coasts to rest. Knowing that kinetic friction results in a couple of magnitude $3.5\ \text{N}\cdot\text{m}$ exerted on the rotor, determine the number of revolutions that the rotor executes before coming to rest.

SOLUTION

$$\bar{I} = m\bar{k}^2$$
$$= (50)(0.180)^2$$
$$= 1.62\ \text{kg}\cdot\text{m}^2$$

$$M = \bar{I}\alpha: \quad 3.5\ \text{N}\cdot\text{m} = (1.62\ \text{kg}\cdot\text{m}^2)\alpha$$
$$\alpha = 2.1605\ \text{rad/s}^2\ \text{(deceleration)}$$
$$\omega_0 = 3600\ \text{rpm}\left(\frac{2\pi}{60}\right)$$
$$= 120\pi\ \text{rad/s}$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta$$
$$0 = (120\pi\ \text{rad/s})^2 + 2(-2.1605\ \text{rad/s}^2)\theta$$
$$\theta = 32.891\times10^3\ \text{rad}$$
$$= 5235.26\ \text{rev}$$

or $\theta = 5230\ \text{rev}$ ◀

PROBLEM 16.26

It takes 10 min for a 6000-lb flywheel to coast to rest from an angular velocity of 300 rpm. Knowing that the radius of gyration of the flywheel is 36 in., determine the average magnitude of the couple due to kinetic friction in the bearings.

SOLUTION

$$m = \frac{W}{g} = \frac{6000}{32.2} = 186.335 \text{ lb}\cdot\text{s}^2/\text{ft} \qquad \bar{k} = 36 \text{ in.} = 3 \text{ ft}$$

$$\bar{I} = m\bar{k}^2 = (186.336)(3)^2 = 1677 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\omega_0 = 300 \text{ rpm}\left(\frac{2\pi}{60}\right) = 10\pi \text{ rad/s}$$

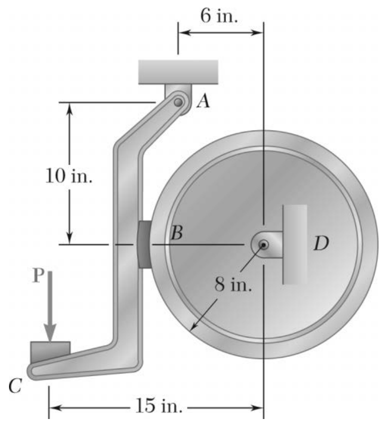
$$\omega = \omega_0 + \alpha t$$

$$0 = 10\pi \text{ rad/s} + \alpha(600 \text{ s})$$

$$\alpha = -0.05236 \text{ rad/s}^2$$

$$M = \bar{I}\alpha = (1677 \text{ lb}\cdot\text{s}^2\cdot\text{ft})(-0.05236 \text{ rad/s}^2) = 87.81 \text{ lb}\cdot\text{ft}$$

$$M = 87.8 \text{ lb}\cdot\text{ft} \blacktriangleleft$$



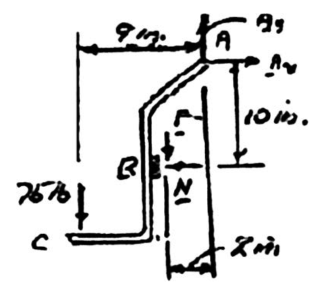
PROBLEM 16.27

The 8-in.-radius brake drum is attached to a larger flywheel that is not shown. The total mass moment of inertia of the drum and the flywheel is $14\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ and the coefficient of kinetic friction between the drum and the brake shoe is 0.35. Knowing that the angular velocity of the flywheel is 360 rpm counterclockwise when a force **P** of magnitude 75 lb is applied to the pedal *C*, determine the number of revolutions executed by the flywheel before it comes to rest.

SOLUTION

Lever *ABC*: Static equilibrium (friction force $\downarrow$)

$$F = \mu_k N = 0.35N$$



$$+\circlearrowleft \Sigma M_A = 0: \quad N(10\text{ in.}) - F(2\text{ in.}) - (75\text{ lb})(9\text{ in.}) = 0$$

$$10N - 2(0.35N) - 675 = 0$$

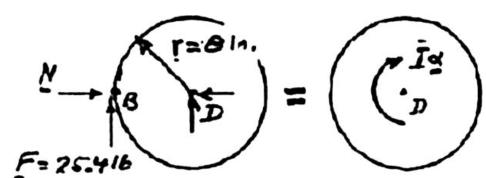
$$N = 72.58\text{ lb}$$

$$F = \mu_k N$$

$$= 0.35(72.58\text{ lb})$$

$$= 25.40\text{ lb}$$

Drum:



$$r = 8\text{ in.} = \frac{2}{3}\text{ ft}$$

$$\omega_0 = 360\text{ rpm}\left(\frac{2\pi}{60}\right)$$

$$\omega_0 = 12\pi\text{ rad/s}$$

$$+\circlearrowleft \Sigma M_D = \Sigma(M_D)_{\text{eff}}: \quad Fr = \bar{I}\alpha$$

$$(25.4\text{ lb})\left(\frac{2}{3}\text{ft}\right) = (14\text{ lb}\cdot\text{ft}\cdot\text{s}^2)\alpha$$

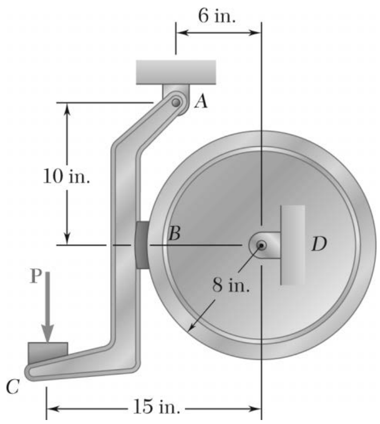
$$\alpha = 1.2097\text{ rad/s}^2 \text{ (deceleration)}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta: \quad 0 = (12\pi\text{ rad/s})^2 + 2(-1.2097\text{ rad/s}^2)\theta$$

$$\theta = 587.4\text{ rad}$$

$$\theta = 587.4\text{ rad}\left(\frac{1}{2\pi}\right) = 93.49\text{ rev}$$

$\theta = 93.5$ rev ◀



PROBLEM 16.28

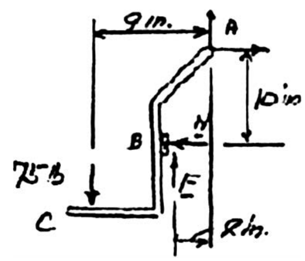
Solve Problem 16.27, assuming that the initial angular velocity of the flywheel is 360 rpm clockwise.

PROBLEM 16.27 The 8-in.-radius brake drum is attached to a larger flywheel that is not shown. The total mass moment of inertia of the drum and the flywheel is $14\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ and the coefficient of kinetic friction between the drum and the brake shoe is 0.35. Knowing that the angular velocity of the flywheel is 360 rpm counterclockwise when a force **P** of magnitude 75 lb is applied to the pedal C, determine the number of revolutions executed by the flywheel before it comes to rest.

SOLUTION

Lever ABC: Static equilibrium (friction force $\uparrow$)

$$F = \mu_k N = 0.35N$$



$$+\circlearrowright \Sigma M_A = 0: \quad N(10\text{ in.}) + F(2\text{ in.}) - (75\text{ lb})(9\text{ in.}) = 0$$

$$10N + 2(0.35N) - 675 = 0$$

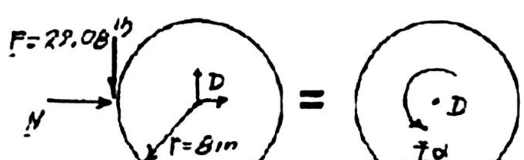
$$N = 63.08\text{ lb}$$

$$F = \mu_k N$$

$$= 0.35(63.08\text{ lb})$$

$$= 22.08\text{ lb}$$

Drum:



$$r = 8\text{ in.} = \frac{2}{3}\text{ft}$$

$$\omega_0 = 360\text{ rpm}\left(\frac{2\pi}{60}\right)$$

$$\omega_0 = 12\pi\text{ rad/s}$$

$$+\circlearrowright \Sigma M_D = \Sigma(M_D)_{\text{eff}} = 0: \quad Fr = \bar{I}\alpha$$

$$(22.08\text{ lb})\left(\frac{2}{3}\text{ft}\right) = (14\text{ lb}\cdot\text{ft}\cdot\text{s}^2)\alpha$$

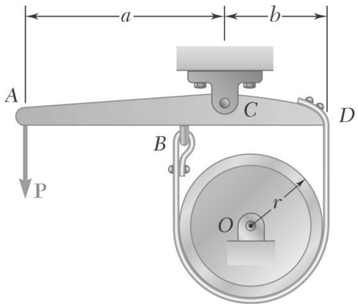
$$\alpha = 1.0515\text{ rad/s}^2 \quad \text{(deceleration)}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta: \quad 0 = (12\pi\text{ rad/s})^2 + 2(-1.0515\text{ rad/s}^2)\theta$$

$$\theta = 675.8\text{ rad}$$

$$\theta = 675.8\text{ rad} = 107.56\text{ rev}$$

$$\theta = 107.6\text{ rev} \blacktriangleleft$$

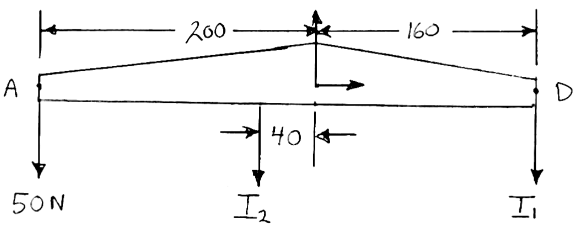


PROBLEM 16.29

The 100-mm-radius brake drum is attached to a flywheel which is not shown. The drum and flywheel together have a mass of 300 kg and a radius of gyration of 600 mm. The coefficient of kinetic friction between the brake band and the drum is 0.30. Knowing that a force **P** of magnitude 50 N is applied at *A* when the angular velocity is 180 rpm counterclockwise, determine the time required to stop the flywheel when $a = 200$ mm and $b = 160$ mm.

SOLUTION

Equilibrium of lever *AD*



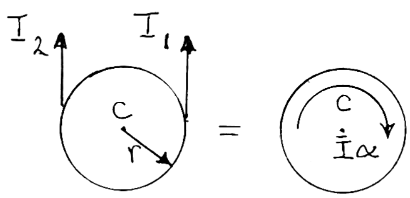
(Dimensions in mm)

$+\circlearrowleft \Sigma M_C = 0;$

$$(50\text{ N})(200) + T_2(40) - T_1(160) = 0$$

$$4T_1 - T_2 = 250\text{ N} \qquad (1)$$

Equation of Motion for flywheel and drum



$$\bar{I} = m\bar{k}^2$$

$$= (300\text{ kg})(0.600\text{ m})^2$$

$$= 108\text{ kg}\cdot\text{m}^2$$

$$r = 0.100\text{ m}$$

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad T_2 r - T_1 r = \bar{I}\alpha$$

$$(T_2 - T_1)(0.100\text{ m}) = 108\,\alpha$$

$$\alpha = (925.93\times 10^{-6})(T_2 - T_1) \qquad (2)$$

Belt Friction

Using μ_k instead of μ_s since the brake band is slipping:

$$\frac{T_2}{T_1} = e^{\mu_k \beta} \quad \text{or} \quad T_2 = T_1 e^{\mu_k \beta} \qquad (3)$$

Making $\mu_k = 0.30$ and $\beta = 180° = \pi$ rad in (3):

$$T_2 = T_1 e^{0.30\pi} \qquad T_2 = 2.5663 T_1 \qquad (4)$$

PROBLEM 16.29 (Continued)

Substituting for T_2 from (4) into (1):

$$4T_1 - 2.5663T_1 = 250 \text{ N} \qquad T_1 = 174.38 \text{ N}$$

From (1): $$T_2 = 4(174.38) - 250 \qquad T_2 = 447.51 \text{ N}$$

Substituting for T_1 and T_2 into (2):

$$\alpha = (925.93 \times 10^{-6})(447.51 - 174.38), \quad \boldsymbol{\alpha} = 0.2529 \text{ rad/s}^2 \circlearrowright$$

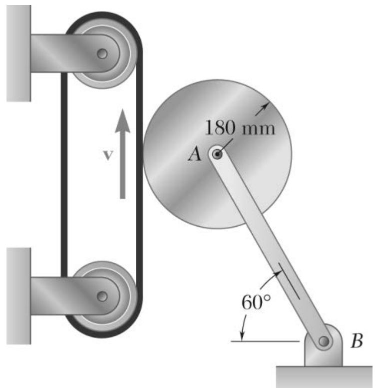
Kinematics

$$\boldsymbol{\omega}_0 = 180 \text{ rpm} \circlearrowleft \qquad \omega_0 = +18.850 \text{ rad/s}$$

$$\boldsymbol{\alpha} = 0.2529 \text{ rad/s}^2 \circlearrowright \qquad \alpha = -0.2529 \text{ rad/s}^2$$

$$\omega = \omega_0 + \alpha t: \quad 0 = 18.850 - 0.2529t$$

$t = 74.5$ s ◀

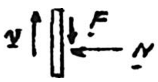


PROBLEM 16.30

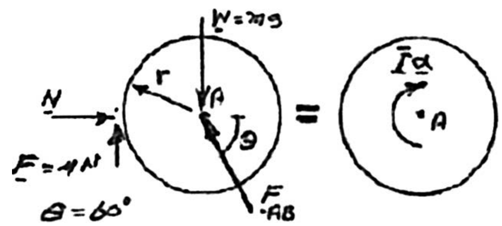
The 180-mm radius disk is at rest when it is placed in contact with a belt moving at a constant speed. Neglecting the weight of the link *AB* and knowing that the coefficient of kinetic friction between the disk and the belt is 0.40, determine the angular acceleration of the disk while slipping occurs.

SOLUTION

Belt: $F = \mu_k N$



Disk:



$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}$: $\quad N - F_{AB}\cos\theta = 0$

$$F_{AB}\cos\theta = N \qquad (1)$$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}$: $\quad \mu_k N + F_{AB}\sin\theta - mg = 0$

$$F_{AB}\sin\theta = mg - \mu_k N \qquad (2)$$

$$\frac{\text{Eq. (2)}}{\text{Eq. (1)}}: \quad \tan\theta = \frac{mg - \mu_k N}{N}$$

$$N\tan\theta = mg - \mu_k N$$

$$N = \frac{mg}{\tan\theta + \mu_k}$$

$$F = \mu_k N = \frac{mg\mu_k}{\tan\theta + \mu_k}$$

PROBLEM 16.30 (Continued)

$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad Fr = \bar{I}\alpha$$

$$\alpha = \frac{r}{I}F$$

$$= \frac{r}{\frac{1}{2}mr^2} \cdot \frac{mg\mu_k}{\tan\theta + \mu_k}$$

$$= \frac{2g}{r} \cdot \frac{\mu_k}{\tan\theta + \mu_k}$$

Data:

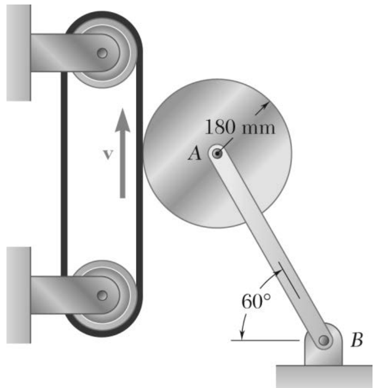
$$r = 0.18 \text{ m}$$

$$\theta = 60°$$

$$\mu_k = 0.40$$

$$\alpha = \frac{2(9.81 \text{ m/s}^2)}{0.18 \text{ m}} \cdot \frac{0.40}{\tan 60° + 0.40}$$

$$\boldsymbol{\alpha} = 20.4 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$



PROBLEM 16.31

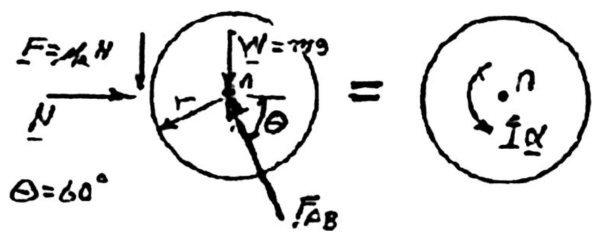
Solve Problem 16.30, assuming that the direction of motion of the belt is reversed.

PROBLEM 16.30 The 180-mm disk is at rest when it is placed in contact with a belt moving at a constant speed. Neglecting the weight of the link *AB* and knowing that the coefficient of kinetic friction between the disk and the belt is 0.40, determine the angular acceleration of the disk while slipping occurs.

SOLUTION

Belt: $F = \mu_k N$

Disk:



$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad N - F_{AB}\cos\theta; \quad F_{AB}\cos\theta = N \qquad (1)$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad F_{AB}\sin\theta - mg - \mu_k N = 0$$

$$F_{AB}\sin\theta = mg + \mu_k N \qquad (2)$$

$$\frac{\text{Eq. (2)}}{\text{Eq. (1)}}: \quad \tan\theta = \frac{mg + \mu_k N}{N}$$

$$N\tan\theta = mg + \mu_k N; \quad N = \frac{mg}{\tan\theta - \mu_k}$$

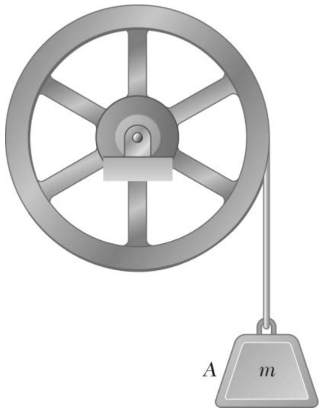
$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad Fr = \bar{I}\alpha$$

$$\alpha = \frac{Fr}{r} = \frac{\mu_k N r}{I} = \frac{\mu_k r}{\frac{1}{2}mr^2} \cdot \frac{mg}{\tan\theta - \mu_k} = \frac{2g}{r} \cdot \frac{\mu_k}{\tan\theta - \mu_k}$$

Data: $r = 0.18 \text{ m}, \quad \theta = 60°, \quad \mu_k = 0.40$

$$\alpha = \frac{2(9.81 \text{ m/s}^2)}{0.18 \text{ m}} \cdot \frac{0.40}{\tan 60° - 0.40}$$

$\alpha = 32.7 \text{ rad/s}^2 \circlearrowright$ ◀

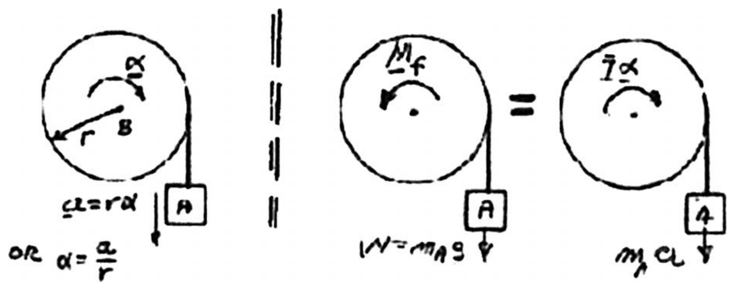


PROBLEM 16.32

In order to determine the mass moment of inertia of a flywheel of radius 600 mm, a 12-kg block is attached to a wire that is wrapped around the flywheel. The block is released and is observed to fall 3 m in 4.6 s. To eliminate bearing friction from the computation, a second block of mass 24 kg is used and is observed to fall 3 m in 3.1 s. Assuming that the moment of the couple due to friction remains constant, determine the mass moment of inertia of the flywheel.

SOLUTION

Kinematics: Kinetics:



$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad (m_A y)r - M_f = \bar{I}\alpha + (m_A a)r$$

$$m_A gr - M_f = \bar{I}\frac{a}{r} + m_A ar \qquad (1)$$

Case 1:

$$y = 3 \text{ m}$$
$$t = 4.6 \text{ s}$$
$$y = \frac{1}{2}at^2$$
$$3 \text{ m} = \frac{1}{2}a(4.6 \text{ s})^2$$
$$a = 0.2836 \text{ m/s}^2$$
$$m_A = 12 \text{ kg}$$

Substitute into Eq. (1)

$$(12 \text{ kg})(9.81 \text{ m/s}^2)(0.6 \text{ m}) - M_f = \bar{I}\left(\frac{0.2836 \text{ m/s}^2}{0.6 \text{ m}}\right) + (12 \text{ kg})(0.2836 \text{ m/s}^2)(0.6 \text{ m})$$

$$70.632 - M_f = \bar{I}(0.4727) + 2.0419 \qquad (2)$$

PROBLEM 16.32 (Continued)

Case 2:

$$y = 3\text{ m}$$
$$t = 3.1\text{ s}$$
$$y = \frac{1}{2}at^2$$
$$3\text{ m} = \frac{1}{2}a(3.1\text{ s})^2$$
$$a = 0.6243\text{ m/s}^2$$
$$m_A = 24\text{ kg}$$

Substitute into Eq. (1):

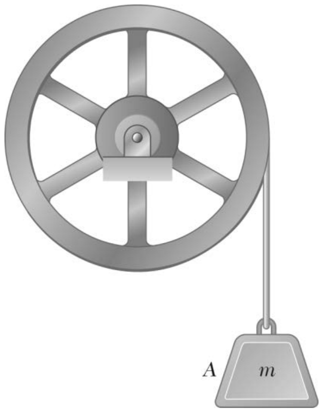
$$(24\text{ kg})(9.81\text{ m/s}^2)(0.6\text{ m}) - M_f = \bar{I}\left(\frac{0.6243\text{ m/s}^2}{0.6\text{ m}}\right) + (24\text{ kg})(0.6243\text{ m/s}^2)(0.6\text{ m})$$

$$141.264 - M_f = \bar{I}(1.0406) + 8.9899 \qquad (3)$$

Subtract Eq. (1) from Eq. (2) to eliminate M_f

$$70.632 = \bar{I}(1.0406 - 0.4727) + 6.948$$
$$63.684 = \bar{I}(0.5679)$$
$$\bar{I} = 112.14\text{ kg}\cdot\text{m}^2$$

$\bar{I} = 112.1\text{ kg}\cdot\text{m}^2$ ◀

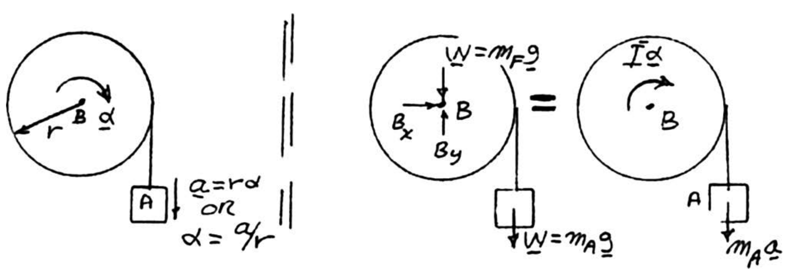


PROBLEM 16.33

The flywheel shown has a radius of 20 in. a weight of 250 lbs, and a radius of gyration of 15 in. A 30-lb block *A* is attached to a wire that is wrapped around the flywheel, and the system is released from rest. Neglecting the effect of friction, determine (*a*) the acceleration of block *A*, (*b*) the speed of block *A* after it has moved 5 ft.

SOLUTION

Kinematics: Kinetics:



$$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad (m_A g)r = \bar{I}\alpha + (m_A a)r$$

$$m_A g r = m_F k^2 \left(\frac{a}{r}\right) + m_A a r$$

$$a = \frac{W_A}{m_A + m_F\left(\frac{k}{r}\right)^2}$$

(*a*) Acceleration of *A*

$$a = \frac{(30 \text{ lb})}{\left(\frac{30 \text{ lb}}{32.2 \text{ ft/s}^2}\right) + \left(\frac{250 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\left(\frac{15 \text{ in.}}{20 \text{ in.}}\right)^2} = 5.6615 \text{ ft/s}^2$$

or $\mathbf{a}_A = 5.66 \text{ ft/s}^2 \downarrow$ ◀

(*b*) Velocity of *A*

$$v_A^2 = (v_A)_0^2 + 2a_A s$$

For $s = 5$ ft

$$v_A^2 = 0 + 2(5.6615 \text{ in./s}^2)(5 \text{ ft})$$

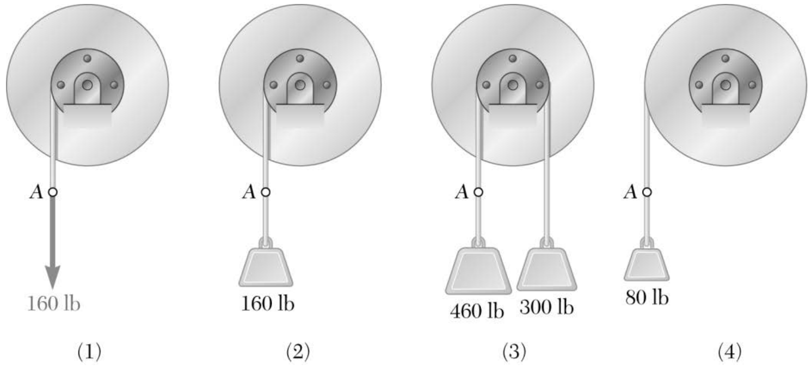
$$= 56.6154 \text{ ft}^2/\text{s}^2$$

$$v_A = 7.5243 \text{ ft/s}$$

or $v_A = 7.52 \text{ ft/s}$ ◀

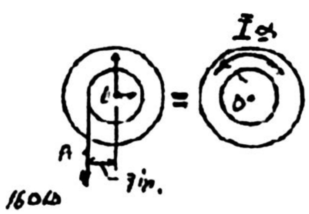
PROBLEM 16.34

Each of the double pulleys shown has a mass moment of inertia of $15 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ and is initially at rest. The outside radius is 18 in., and the inner radius is 9 in. Determine (*a*) the angular acceleration of each pulley, (*b*) the angular velocity of each pulley after Point *A* on the cord has moved 10 ft.



SOLUTION

Case 1:



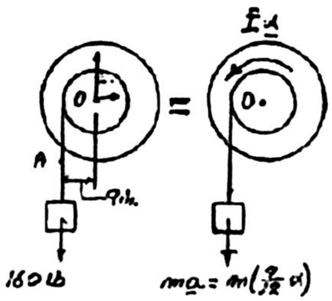
(*a*) $\stackrel{\curvearrowleft}{+}\Sigma M_0 = \Sigma(M_0)_{\text{eff}}: \quad (160 \text{ lb})\left(\dfrac{9}{12}\text{ft}\right) = (15 \text{ lb}\cdot\text{ft}\cdot\text{s}^2)\alpha$

$$\boldsymbol{\alpha} = 8 \text{ rad/s}^2 \curvearrowleft \blacktriangleleft$$

(*b*) $\theta = \dfrac{10 \text{ ft}}{\left(\frac{9}{12}\text{ft}\right)} = 13.333 \text{ rad}$

$$\omega^2 = 2\alpha\theta = 2(8 \text{ rad/s}^2)(13.33 \text{ rad}) \qquad \boldsymbol{\omega} = 14.61 \text{ rad/s} \curvearrowleft \blacktriangleleft$$

Case 2:



(*a*) $\stackrel{\curvearrowleft}{+}\Sigma M_0 = \Sigma(M_0)_{\text{eff}}: \quad (160)\left(\dfrac{9}{12}\right) = 15\alpha + ma\left(\dfrac{9}{12}\right)$

$$120 = 15\alpha + \frac{160}{32.2}\left(\frac{9}{12}\alpha\right)\left(\frac{9}{12}\right)$$

$$120 = (15 + 2.795)\alpha$$

$$\alpha = 6.7435 \text{ rad/s}^2$$

$$\boldsymbol{\alpha} = 6.74 \text{ rad/s}^2 \curvearrowleft \blacktriangleleft$$

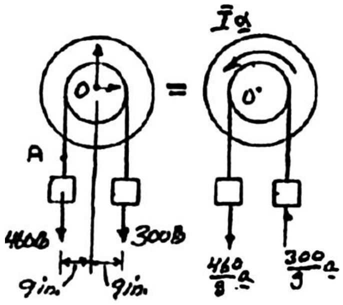
(*b*) $\theta = \dfrac{10 \text{ ft}}{\left(\frac{9}{12}\text{ft}\right)} = 13.333 \text{ rad}$

$$\omega^2 = 2\alpha\theta$$

$$= 2(6.7435 \text{ rad/s}^2)(13.333 \text{ rad}) \qquad \boldsymbol{\omega} = 13.41 \text{ rad/s} \curvearrowleft \blacktriangleleft$$

PROBLEM 16.34 (Continued)

Case 3:



(*a*) $+\circlearrowleft \Sigma M_0 = \Sigma(M_0)_{\text{eff}}:$

$$(460)\left(\frac{9}{12}\right) - (300)\left(\frac{9}{12}\right) = 15\alpha + \frac{460}{32.2}a\left(\frac{9}{12}\right) + \frac{300}{32.2}a\left(\frac{9}{12}\right)$$

$$120 = 15\alpha + \frac{460}{32.2}\left(\frac{9}{12}\right)^2\alpha + \frac{300}{32.2}\left(\frac{9}{12}\right)^2\alpha$$

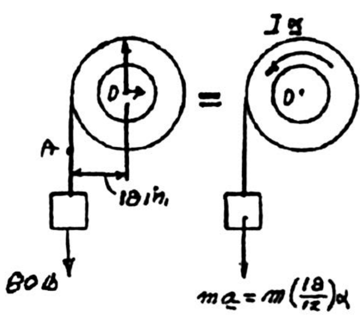
$$\alpha = 4.2437 \text{ rad/s}^2$$

$$\boldsymbol{\alpha} = 4.24 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

(*b*) $\theta = \dfrac{10 \text{ ft}}{\frac{9}{12}} = 13.333 \text{ rad}$

$$\omega^2 = 2\alpha\theta = 2(4.2437)(13.333) \qquad \boldsymbol{\omega} = 10.64 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

Case 4:



(*a*) $+\circlearrowleft \Sigma M_0 = \Sigma(M_0)_{\text{eff}}: \quad (80)\left(\dfrac{18}{12}\right) = 15\alpha + \dfrac{80}{32.2}a\left(\dfrac{18}{12}\right)$

$$120 = 15\alpha + \frac{80}{32.2}\left(\frac{18}{12}\right)^2\alpha$$

$$120 = (15 + 5.5901)\alpha$$

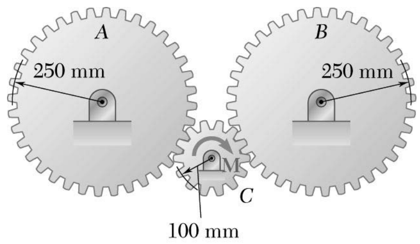
$$\alpha = 5.828 \text{ rad/s}^2$$

$$\boldsymbol{\alpha} = 5.83 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

(*b*) $\theta = \dfrac{10 \text{ ft}}{\left(\frac{18}{12} \text{ ft}\right)} = 6.6667 \text{ rad}$

$$\omega^2 = 2\alpha\theta = 2(5.828 \text{ rad/s}^2)(6.6667 \text{ rad}) \qquad \boldsymbol{\omega} = 8.82 \text{ rad/s} \circlearrowleft \blacktriangleleft$$

PROBLEM 16.35



Each of the gears A and B has a mass of 9 kg and has a radius of gyration of 200 mm; gear C has a mass of 3 kg and has a radius of gyration of 75 mm. If a couple **M** of constant magnitude 5 N-m is applied to gear C, determine (*a*) the angular acceleration of gear A, (*b*) the tangential force which gear C exerts on gear A.

SOLUTION

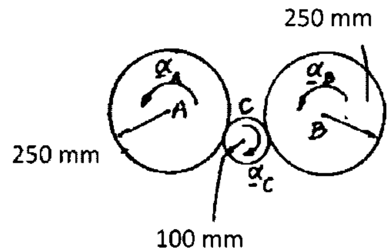
Kinematics:

We express that the tangential components of the accelerations of the gear teeth are equal:

$$a_t = 0.25\alpha_A = 0.25\alpha_B = 0.1\alpha_C$$

$$\alpha_B = \alpha_A$$

$$\alpha_C = 2.5\alpha_A \qquad (1)$$

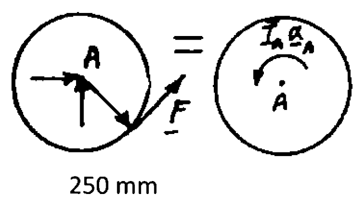


Kinetics:

Gear A: $\bar{I}_A = m_A\bar{k}_A^2 = 9(0.20)^2 = 0.36\ \text{kg}\cdot\text{m}^2$

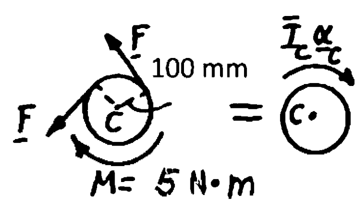
$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad F(0.25) = 0.36\alpha_A$

$$F = 1.44\alpha_A \qquad (2)$$



Because of symmetry, gear C exerts an equal force F on gear B.

Gear C: $\bar{I}_C = m_C\bar{k}_C^2 = 3(0.075)^2 = 0.016875\ \text{kg}\cdot\text{m}^2$



$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad M - 2Fr_C = \bar{I}_C\alpha_C$

$$5\ \text{N}\cdot\text{m} - 2F(0.1\ \text{m}) = 0.016875\alpha_C$$

(*a*) Angular acceleration of gear A.

Substituting for α_C from (1) and for F from (2):

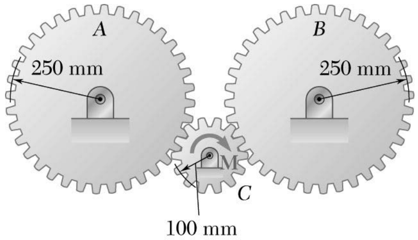
$$5 - 2(1.44\alpha_A)(0.1) = 0.016875(2.5\alpha_A)$$
$$5 - 0.288\alpha_A = 0.04219\alpha_A$$
$$5 = 0.3302\alpha_A$$
$$\alpha_A = 15.143\ \text{rad/s}^2$$

$\boldsymbol{\alpha}_A = 15.14\ \text{rad/s}^2 \circlearrowleft$ ◀

(*b*) Tangential force F.

Substituting for α_A into (2): $F = 1.44(15.143)$

$\mathbf{F}_A = 21.8\ \text{N} \nearrow$ ◀



PROBLEM 16.36

Solve Problem 16.35, assuming that the couple **M** is applied to disk *A*.

PROBLEM 16.35 Each of the gears *A* and *B* has a mass of 9 kg and has a radius of gyration of 200 mm; gear *C* has a mass of 3 kg and has a radius of gyration of 75 mm. If a couple **M** of constant magnitude 5 N-m is applied to gear *C*, determine (*a*) the angular acceleration of gear *A*, (*b*) the tangential force which gear *C* exerts on gear *A*.

SOLUTION

Kinematics: $\boldsymbol{\alpha}_A = \alpha_A \circlearrowright \qquad \boldsymbol{\alpha}_A = \alpha_A \circlearrowleft \qquad \boldsymbol{\alpha}_B = \alpha_B \circlearrowright$

At the contact point between gears *A* and *C*,

$$a_t = r_A\alpha_A = r_C\alpha_C$$

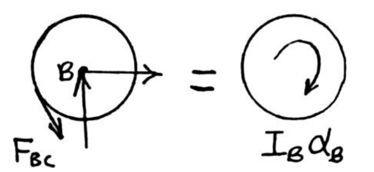
$$\alpha_C = \frac{r_A}{r_C}\alpha_A = \frac{0.25}{0.1}\alpha_A$$

At the contact point between gears *B* and *C*,

$$a_t = r_B\alpha_B = r_C\alpha_C$$

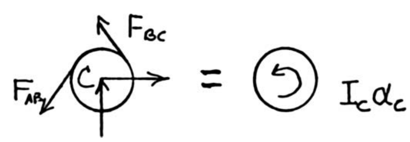
$$\alpha_B = \frac{r_C}{r_A}\alpha_C = \frac{0.1}{0.25}\cdot\frac{0.25}{0.1}\alpha_A = \alpha_A$$

Kinetics:



Gear *B*: $+\circlearrowright \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad F_{BC}r_B = I_B\alpha_B = I_B\alpha_A$

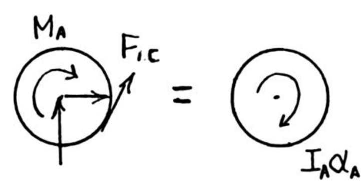
$$F_{BC} = \frac{I_B}{r_B}\alpha_A$$



Gear *C*: $+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad F_{BC}r_C + F_{AB}r_C = I_C\alpha_C$

$$\frac{r_C}{r_B}I_B\alpha_A + F_{AB}r_C = \frac{0.25}{0.1}I_C\alpha_A$$

$$F_{AC} = \frac{1}{r_C}\left(\frac{0.1}{0.25}I_B + \frac{0.25}{0.1}I_C\right)\alpha_A \qquad (1)$$



Gear *A*: $\circlearrowright + M_A = \Sigma (M_A)_{\text{eff}}: \quad M_A - r_A F_{AB} = I_A\alpha_A$

$$M_A - \frac{r_A}{r_C}\left(\frac{0.1}{0.25}I_B + \frac{0.25}{0.1}I_C\right)\alpha_A = I_A\alpha_A$$

$$M_A = \left(I_A + I_B + \left(\frac{0.25}{0.1}\right)^2 I_C\right)\alpha_A \qquad (2)$$

PROBLEM 16.36 (Continued)

Data: From Eq. (2)

$$m_A = m_B = 9 \text{ kg}$$
$$k_A = k_B = 0.2 \text{ m}$$
$$I_A = I_B = m_A k_A^2 = 9(0.2)^2$$
$$= 0.36 \text{ kg} \cdot \text{m}^2$$

$$m_C = 3 \text{ kg}$$
$$k_C = 0.075 \text{ m}$$
$$I_C = 3(0.075)^2 = 0.016875 \text{ kg} \cdot \text{m}^2$$
$$M_A = 5 \text{ N} \cdot \text{m}$$
$$5 = \left[0.36 + 0.36 + \left(\frac{0.25}{0.1}\right)^2 (0.016875)\right]\alpha_A$$

(*a*) Angular acceleration. $\alpha_A = 6.0572 \text{ rad/s}^2$

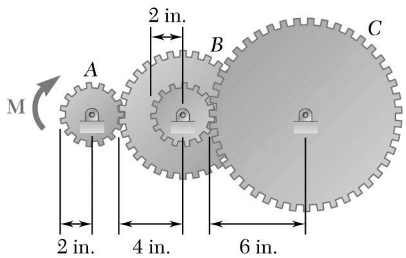
$\boldsymbol{\alpha}_A = 6.06 \text{ rad/s}^2$ ↻ ◀

(*b*) Tangential gear force.

From Eq. (1)

$$F_{AC} = \frac{1}{0.1}\left[\left(\frac{0.1}{0.25}\right)(0.36) + \left(\frac{0.25}{0.1}\right)(0.016875)\right](6.0572)$$
$$= 11.278$$

$\mathbf{F}_{AC} = 11.28 \text{ N}$ ↗ ◀



PROBLEM 16.37

Gear A weighs 1 lb and has a radius of gyration of 1.3 in; Gear B weighs 6 lb and has a radius of gyration of 3 in.; gear C weighs 9 lb and has a radius of gyration of 4.3 in. Knowing a couple **M** of constant magnitude of 40 lb · in is applied to gear A, determine (a) the angular acceleration of gear C, (b) the tangential force which gear B exerts on gear C.

SOLUTION

Masses and moments of inertia.

$$m_A = \frac{1\text{ lb}}{32.2\text{ ft/s}^2} = 0.031056\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_B = \frac{6\text{ lb}}{32.2\text{ ft/s}^2} = 0.18634\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_C = \frac{9\text{ lb}}{32.2\text{ ft/s}^2} = 0.27950\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\bar{I}_A = m_A\bar{k}_A^2 = (0.031056\text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{1.3\text{ in.}}{12\text{ in./ft}}\right)^2$$

$$= 0.36448\times10^{-3}\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\bar{I}_B = m_B\bar{k}_B^2 = (0.18634\text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{3\text{ in.}}{12\text{ in./ft}}\right)^2$$

$$= 11.646\times10^{-3}\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\bar{I}_C = m_C\bar{k}_C^2 = (0.27950\text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{4.3\text{ in.}}{12\text{ in./ft}}\right)^2$$

$$= 35.889\times10^{-3}\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Kinematics. Gear A: $r_A = 2$ in.

Gear B: $r_1 = 4$ in., $r_2 = 2$ in.

Gear C: $r_C = 6$ in.

Point of contact between A and B.

$$a_t = r_A\alpha_A = r_1\alpha_B$$

$$\alpha_A = \frac{r_1}{r_A}\alpha_B = \frac{4\text{ in.}}{2\text{ in.}}\alpha_B$$

PROBLEM 16.37 (Continued)

Point of contact between B and C.

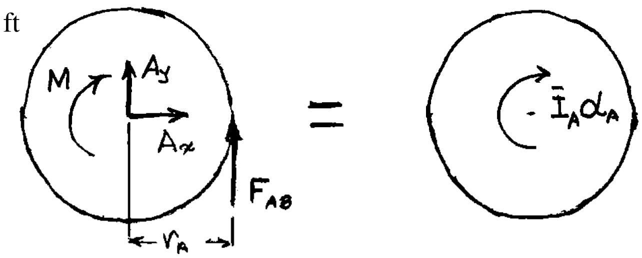
$$a_t = r_2\alpha_B = r_C\alpha_C$$

$$\alpha_B = \frac{r_C}{r_2}\alpha_C = \frac{6\text{ in.}}{2\text{ in.}}\alpha_C$$

Summary. $\alpha_B = 3\alpha_C$ (1)

$\alpha_A = 2\alpha_B = 6\alpha_C$ (2)

Kinetics. Applied couple: $M = 40\text{ lb}\cdot\text{in.} = 3.3333\text{ lb}\cdot\text{ft}$



Gear A:

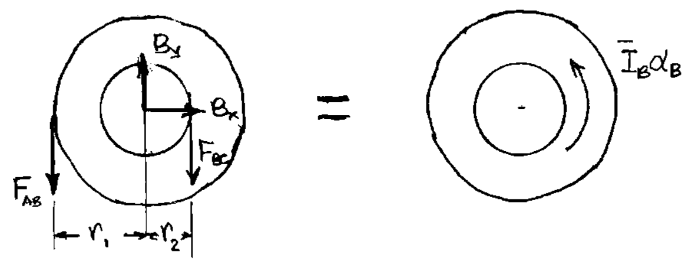
$\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad M - F_{AB}r_A = \bar{I}_A\alpha_A$

$$F_{AB} = \frac{M}{r_A} - \frac{\bar{I}_A(6\alpha_C)}{r_A}$$

$$= \frac{3.3333\text{ lb}\cdot\text{ft}}{(2/12)\text{ ft}} - \frac{(0.36448\times10^{-3})(6)}{(2/12)}\alpha_C$$

$$= 20\text{ lb} - 0.013121\alpha_C \quad (3)$$

Gear B:



$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad F_{AB}r_1 - F_{BC}r_2 = \bar{I}_B\alpha_C$

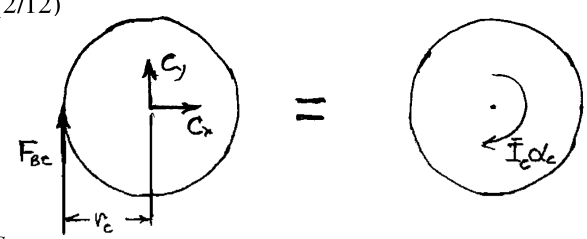
$$F_{BC} = F_{AB}\frac{r_1}{r_2} - \frac{\bar{I}_B\alpha_B}{r_2}$$

$$= 2F_{AB} - \frac{3\bar{I}_B}{r_2}\alpha_C$$

$$= 2[20\text{ lb} - 0.013121\alpha_C] - \frac{(3)(11.646\times10^{-3})}{(2/12)}\alpha_C$$

$$= 40\text{ lb} - 0.23587\alpha_C$$

Gear C:



$\circlearrowleft + \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad F_{BC}r_C = \bar{I}_C\alpha_C$

$$(40 - 0.23587\alpha_C)\left(\frac{6}{12}\right) = (35.889\times10^{-3})\alpha_C$$

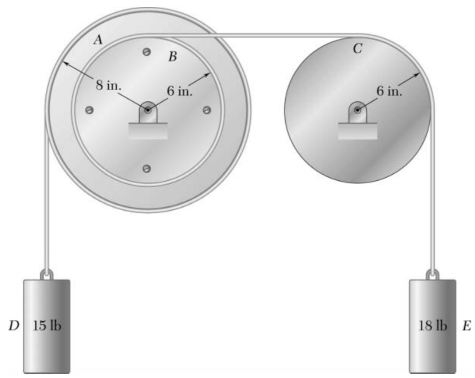
$$20 = 0.153824\alpha_C$$

(*a*) *Angular acceleration of gear C.* $\boldsymbol{\alpha}_C = 130.0\text{ rad/s}^2 \circlearrowright$ ◀

(*b*) *Tangential force which gear B exerts on gear C.*

$$F_{BC} = 40\text{ lb} - (0.23587)(130.0) = 9.33\text{ lb}$$

9.33 lb ↑ ◀



PROBLEM 16.38

Disks A and B are bolted together, and cylinders D and E are attached to separate cords wrapped on the disks. A single cord passes over disks B and C. Disk A weighs 20 lb and disks B and C each weigh 12 lb. Knowing that the system is released from rest and that no slipping occurs between the cords and the disks, determine the acceleration (a) of cylinder D, (b) of cylinder E.

SOLUTION

Masses and moments of inertia.

$$m_D = \frac{W_D}{g} = \frac{15 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.46584 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_E = \frac{W_E}{g} \frac{18 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.55901 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Assume disks have uniform thickness.

$$I_A = \frac{1}{2} m_A r_A^2 = \frac{1}{2} \frac{20 \text{ lb}}{32.2 \text{ ft/s}^2} \left(\frac{8}{12} \text{ ft}\right)^2 = 0.138026 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$I_B = \frac{1}{2} m_B r_B^2 = \frac{1}{2} \frac{12 \text{ lb}}{32.2 \text{ ft/s}^2} \left(\frac{6}{12} \text{ ft}\right)^2 = 0.046584 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$I_C = I_B = 0.046584 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$I_{AB} = I_A + I_B = 0.184610 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

Kinematics: $\mathbf{a}_D = a_D \downarrow$, $\mathbf{a}_E = a_E \uparrow$, $\boldsymbol{\alpha}_{AB} = \alpha_{AB} \circlearrowright$, $\boldsymbol{\alpha}_C = \alpha_C \circlearrowright$

For inextensible cord between disk A and cylinder D,

$$a_D = (a_t)_A = r_1 \alpha_{AB} = \left(\frac{8}{12} \text{ ft}\right) \alpha_{AB} = 0.66667 \alpha_{AB} \quad (1)$$

For inextensible cord between disks B and C,

$$r_2 \alpha_{AB} = r_3 \alpha_C$$

$$\alpha_C = \frac{r_2}{r_3} \alpha_{AB} = \left(\frac{6 \text{ in.}}{6 \text{ in.}}\right) \alpha_{AB} = \alpha_{AB}$$

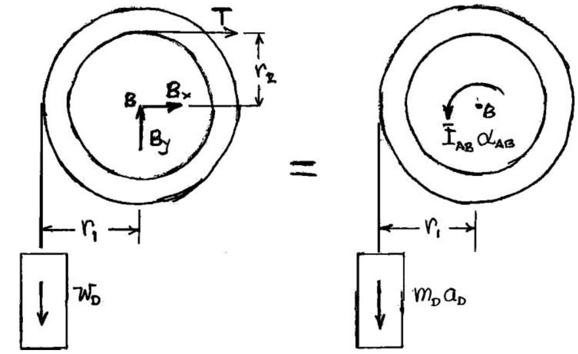
For inextensible cord between disk C and cylinder E,

$$a_E = (a_t)_D = \left(\frac{6}{12} \text{ ft}\right) \alpha_C = 0.5 \alpha_{AB} \quad (2)$$

PROBLEM 16.38 (Continued)

Kinetics.

Let T be the tension in the cord between disks B and C.



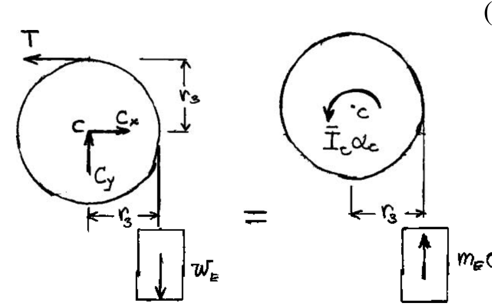
$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad r_1 W_D - r_2 T = I_{AB}\alpha_{AB} + r_1 m_D a_D$$

$$T = \frac{r_1}{r_2}W_D - \frac{I_{AB}\alpha_{AB}}{r_2} - \frac{r_1 m_D a_D}{r_2}$$

$$T = \frac{r_1}{r_2}W_D - \left[\frac{I_{AB}}{r_2} + \frac{m_D r_1^2}{r_2}\right]\alpha_{AB}$$

$$= \frac{8 \text{ in.}}{6 \text{ in.}}(15 \text{ lb}) - \left[\frac{0.184610}{6/12} + \frac{(0.46584)(8/12)^2}{6/12}\right]\alpha_{AB}$$

$$T = 20 \text{ lb} - 0.78330\alpha_{AB} \qquad (3)$$



$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad r_3 T - r_3 W_E = I_C\alpha_C + r_3 m_E a_E$$

$$T = W_E + \left[\frac{I_C}{r_3} + m_E r_3\right]\alpha_{AB}$$

$$T = 18 \text{ lb} + \left[\frac{0.046584}{6/12} + (0.55901)(6/12)\right]\alpha_{AB}$$

$$T = 18 \text{ lb} + 0.37267\alpha_{AB} \qquad (4)$$

Subtracting Eq. (4) from Eq. (3) to eliminate T,

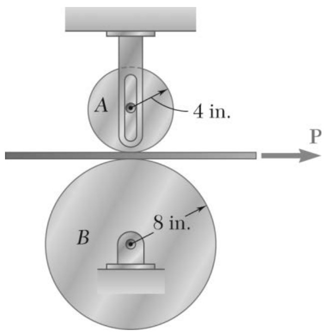
$$0 = 2 \text{ lb} - 1.15597\alpha_{AB} \qquad \alpha_{AB} = 1.7301 \text{ rad/s}^2$$

(*a*) *Acceleration of cylinder D.*

From Eq. (1), $a_D = (0.66667)(7.7301)$ $\qquad \mathbf{a}_D = 1.153 \text{ ft/s}^2 \downarrow$ ◀

(*b*) *Acceleration of cylinder E.*

From Eq. (2), $a_E = (0.5)(1.7301)$ $\qquad \mathbf{a}_E = 0.865 \text{ ft/s}^2 \uparrow$ ◀



PROBLEM 16.39

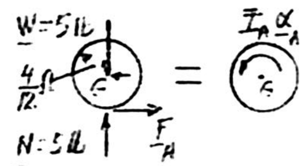
A belt of negligible mass passes between cylinders *A* and *B* and is pulled to the right with a force **P**. Cylinders *A* and *B* weigh, respectively, 5 and 20 lb. The shaft of cylinder *A* is free to slide in a vertical slot and the coefficients of friction between the belt and each of the cylinders are $\mu_s = 0.50$ and $\mu_k = 0.40$. For $P = 3.6$ lb, determine (*a*) whether slipping occurs between the belt and either cylinder, (*b*) the angular acceleration of each cylinder.

SOLUTION

Assume that no slipping occurs.

Then: $$a_{\text{belt}} = (4 \text{ in.})\alpha_A = (8 \text{ in.})\alpha_B \qquad \alpha_B = \frac{1}{2}\alpha_A \tag{1}$$

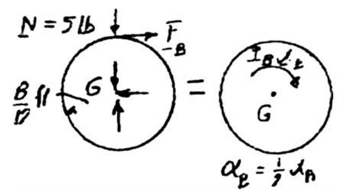
Cylinder *A*. $$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad F_A\left(\frac{4}{12}\text{ft}\right) = \bar{I}_A\alpha_A$$



$$F_A\left(\frac{4}{12}\right) = \frac{1}{2}\frac{5}{g}\left(\frac{4}{12}\right)^2\alpha_A$$

$$F_A = \frac{5}{6}\frac{\alpha_A}{g} \tag{2}$$

Cylinder *B*. $$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad F_B\left(\frac{8}{12}\text{ft}\right) = \bar{I}_B\alpha_B$$

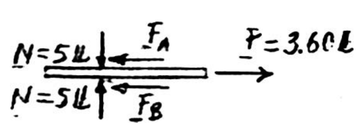


$$F_B\left(\frac{8}{12}\right) = \frac{1}{2}\frac{20}{g}\left(\frac{8}{12}\right)^2\left(\frac{1}{2}\alpha_A\right)$$

$$F_B = \frac{20}{6}\frac{\alpha_A}{g} \tag{3}$$

Belt $$\xrightarrow{+} \Sigma F_A = 0: \quad P - F_A - F_B = 0 \tag{4}$$

$$3.60 - \frac{5}{6}\frac{\alpha_A}{g} - \frac{20}{6}\frac{\alpha_A}{g} = 0$$



$$\alpha_A = \frac{(3.60)6}{25}g$$
$$= 0.864g$$

$$\boldsymbol{\alpha}_A = 27.82 \text{ rad/s}^2 \circlearrowleft$$

PROBLEM 16.39 (Continued)

Check that belt does not slip.

From (2): $F_A = \frac{5}{6}(0.864) = 0.720 \text{ lb}$

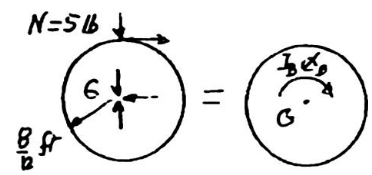
From (4): $F_e = P - F_A = 3.60 - 0.720 = 2.88 \text{ lb}$

But $F_m = \mu_s N = 0.50(5 \text{ lb}) = 2.50 \text{ lb}$

Since $F_e > F_m$, assumption is wrong.

Slipping occurs between disk *B* and the belt. ◀

We redo analysis of *B*, assuming slipping. $\left(\alpha_B \neq \frac{1}{2}\alpha_A\right)$



$$F_B = \mu_k N = 0.40(5 \text{ lb}) = 2 \text{ lb}$$

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad (2 \text{ lb})\left(\frac{8}{12}\text{ ft}\right) = \frac{1}{2}\frac{20}{g}\left(\frac{8}{12}\right)^2 \alpha_B$$

$$\alpha_B = 0.3g, \qquad \boldsymbol{\alpha}_B = 9.66 \text{ rad/s}^2 \circlearrowright ◀$$

Belt Eq. (4):

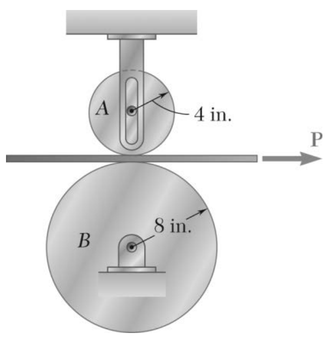
$$P - F_A - F_B = 0$$
$$F_A = P - F_B$$
$$= 3.60 - 2$$
$$= 1.60 \text{ lb}$$

Since $F_A < F_m$, There is no slipping between *A* and the belt. ◀

Our analysis of disk *A*, therefore is valid. Using Eq. (2),

We have $1.60 \text{ lb} = \frac{5}{6}\frac{\alpha_A}{g}$

$$\alpha_A = 1.92g \qquad \boldsymbol{\alpha}_A = 61.8 \text{ rad/s}^2 \circlearrowleft ◀$$



PROBLEM 16.40

Solve Problem 16.39 for $P = 2.00$ lb.

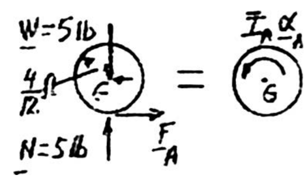
PROBLEM 16.39 A belt of negligible mass passes between cylinders *A* and *B* and is pulled to the right with a force **P**. Cylinders *A* and *B* weigh, respectively, 5 and 20 lb. The shaft of cylinder *A* is free to slide in a vertical slot and the coefficients of friction between the belt and each of the cylinders are $\mu_s = 0.50$ and $\mu_k = 0.40$. For $P = 3.60$ lb, determine (*a*) whether slipping occurs between the belt and either of the cylinders, (*b*) the angular acceleration of each cylinder.

SOLUTION

Assume that no slipping occurs.

Then: $$a_{\text{belt}} = (4\text{ in.})\alpha_A = (8\text{ in.})\alpha_B \qquad \alpha_B = \frac{1}{2}\alpha_A \tag{1}$$

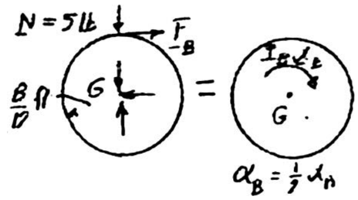
Cylinder *A* $\quad +\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad F_A\left(\frac{4}{12}\text{ft}\right) = \bar{I}_A\alpha_A$



$$F_A\left(\frac{4}{12}\right) = \frac{1}{2}\frac{5}{g}\left(\frac{4}{12}\right)^2\alpha_A$$

$$F_A = \frac{5}{6}\frac{\alpha_A}{g} \tag{2}$$

Cylinder *B* $\quad +\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad F_B\left(\frac{4}{12}\text{ft}\right) = \bar{I}_B\alpha_B$

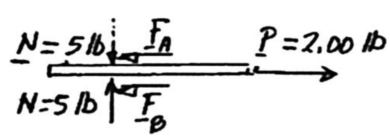


$$F_B\left(\frac{8}{12}\right) = \frac{1}{2}\frac{20}{g}\left(\frac{8}{12}\right)^2\left(\frac{1}{2}\alpha_A\right)$$

$$F_B = \frac{20}{6}\frac{\alpha_A}{g} \tag{3}$$

Belt $\quad \xrightarrow{+} \Sigma F_A = 0: \quad P - F_A - F_B = 0$

$$2.00 - \frac{5}{6}\frac{\alpha_A}{g} - \frac{20}{6}\frac{\alpha_A}{g} = 0$$



$$\alpha_A = \frac{(2.00)6}{25}g$$

$$= 0.480g$$

$$\alpha_A = 15.46\text{ rad/s}^2 \circlearrowright$$

PROBLEM 16.40 (Continued)

From (2): $$F_A = \frac{5}{6}(0.480) = 0.400 \text{ lb}$$

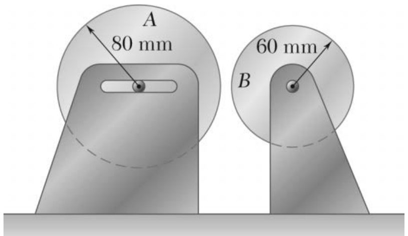
From (4): $$F_B = \frac{20}{6}(0.480) = 1.600 \text{ lb}$$

But $$F_M = \mu_s N = 0.50(5 \text{ lb}) = 2.50 \text{ lb}$$

Thus, F_A and F_B are both $< F_m$. Our assumption was right:

There is no slipping between cylinders and belt $\boldsymbol{\alpha}_A = 15.46 \text{ rad/s}^2 \circlearrowleft$ ◀

From (1): $\boldsymbol{\alpha}_B = 7.73 \text{ rad/s}^2 \circlearrowright$ ◀



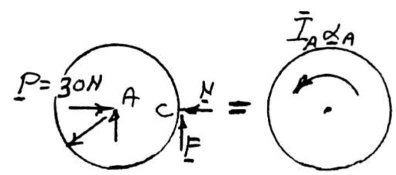
PROBLEM 16.41

Disk A has a mass of 6 kg and an initial angular velocity of 360 rpm clockwise; disk B has a mass of 3 kg and is initially at rest. The disks are brought together by applying a horizontal force of magnitude 20 N to the axle of disk A. Knowing that $\mu_k = 0.15$ between the disks and neglecting bearing friction, determine (a) the angular acceleration of each disk, (b) the final angular velocity of each disk.

SOLUTION

While slipping occurs, a friction force $F\uparrow$ is applied to disk A, and $F\downarrow$ to disk B.

Disk A:



$$I_A = \frac{1}{2} m_A r_A^2$$

$$= \frac{1}{2}(6 \text{ kg})(0.08 \text{ m})^2$$

$$= 0.0192 \text{ kg}\cdot\text{m}^2$$

$$\Sigma F: \quad N = P = 20 \text{ N}$$

$$F = \mu N = 0.15(20) = 3 \text{ N}$$

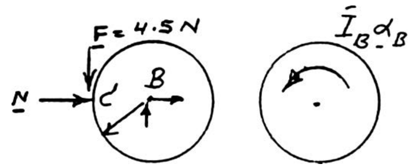
$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad F r_A = \bar{I}_A \alpha_A$$

$$(3 \text{ N})(0.08 \text{ m}) = (0.0192 \text{ kg}\cdot\text{m}^2)\alpha_A$$

$$\alpha_A = 10.227$$

$$\boldsymbol{\alpha}_A = 12.50 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

Disk B:



$$\bar{I}_B = \frac{1}{2} m_B r_B^2$$

$$= \frac{1}{2}(3 \text{ kg})(0.06 \text{ m})^2$$

$$= 0.0054 \text{ kg}\cdot\text{m}^2$$

$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad F r_B = \bar{I}_B \alpha_B$$

$$(3 \text{ N})(0.06 \text{ m}) = (0.0054 \text{ kg}\cdot\text{m}^2)\alpha_B$$

$$\alpha_B = 33.333 \text{ rad/s}^2$$

$$\boldsymbol{\alpha}_B = 33.3 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

$$(\omega_A)_0 = 360 \text{ rpm} = 12\pi \text{ rad/s} \circlearrowright$$

PROBLEM 16.41 (Continued)

Sliding stops when $\mathbf{v}_C = \mathbf{v}_{C'}$ or $\omega_A r_A = \omega_B r_B$

$$r_A[(\omega_A)_0 - \alpha_A t] = r_B \alpha_B t$$

$$(0.08 \text{ m})[12\pi \text{ rad/s} - (12.5 \text{ rad/s}^2)t] = (0.06 \text{ m})(33.333 \text{ rad/s}^2)t$$

$$t = 1.00531 \text{ s}$$

$$+\circlearrowleft\ \omega_A = (\omega_A)_0 - \alpha_A t$$
$$= 12\pi \text{ rad/s} - (12.5 \text{ rad/s}^2)(1.00531 \text{ s})$$
$$= 25.132 \text{ rad/s}$$

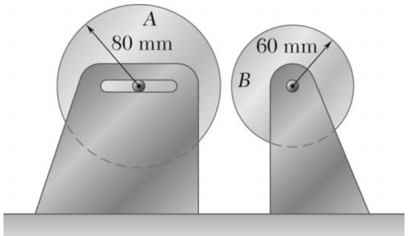
$$\omega_A = (25.132 \text{ rad/s})$$
$$= 240.00 \text{ rpm}$$

or $\boldsymbol{\omega}_A = 240 \text{ rpm} \circlearrowright$ ◀

$$+\circlearrowleft\ \omega_B = \alpha_B t$$
$$= 33.333 \text{ rad/s}^2 (1.00531 \text{ s})$$
$$= 33.510 \text{ rad/s}$$

$$\omega_B = (33.510 \text{ rad/s})$$
$$= 320.00 \text{ rpm}$$

or $\boldsymbol{\omega}_B = 320 \text{ rpm} \circlearrowleft$ ◀



PROBLEM 16.42

Solve Problem 16.41, assuming that initially disk A is at rest and disk B has an angular velocity of 360 rpm clockwise.

PROBLEM 16.41 Disk A has a mass of 6 kg and an initial angular velocity of 360 rpm clockwise; disk B has a mass of 3 kg and is initially at rest. The disks are brought together by applying a horizontal force of magnitude 20 N to the axle of disk A. Knowing that $\mu_k = 0.15$ between the disks and neglecting bearing friction, determine (a) the angular acceleration of each disk, (b) the final angular velocity of each disk.

SOLUTION

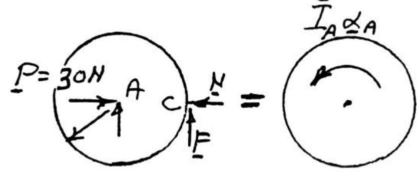
While slipping occurs, a friction force $F\uparrow$ is applied to disk A, and $F\downarrow$ to disk B.

Disk A:

$$I_A = \frac{1}{2} m_A r_A^2$$

$$= \frac{1}{2}(6\text{ kg})(0.08\text{ m})^2$$

$$= 0.0192\text{ kg}\cdot\text{m}^2$$



$$\Sigma F: \quad N = P = 20\text{ N}$$

$$F = \mu N = 0.15(20) = 3\text{ N}$$

$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad Fr_A = \bar{I}_A \alpha_A$$

$$(3\text{ N})(0.08\text{ m}) = (0.0192\text{ kg}\cdot\text{m}^2)\alpha_A$$

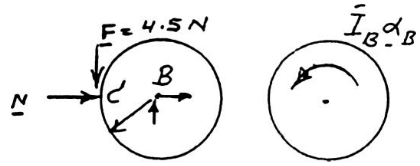
$$\alpha_A = 10.227 \qquad\qquad \boldsymbol{\alpha}_A = 12.50\text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

Disk B:

$$\bar{I}_B = \frac{1}{2} m_B r_B^2$$

$$= \frac{1}{2}(3\text{ kg})(0.06\text{ m})^2$$

$$= 0.0054\text{ kg}\cdot\text{m}^2$$



$$+\circlearrowleft \Sigma M_A = \Sigma(M_B)_{\text{eff}}: \quad Fr_B = \bar{I}_B \alpha_B$$

$$(3\text{ N})(0.06\text{ m}) = (0.0054\text{ kg}\cdot\text{m}^2)\alpha_B$$

$$\alpha_B = 33.333\text{ rad/s}^2 \qquad\qquad \boldsymbol{\alpha}_B = 33.3\text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

PROBLEM 16.42 (Continued)

Sliding starts when $\mathbf{v}_C = \mathbf{v}_{C'}$. That is when

$$\omega_A r_A = \omega_B r_B$$

$$(\alpha_A t) r_A = [(\omega_B)_0 - \alpha_B t] r_B$$

$$[(12.5 \text{ rad/s}^2)t](0.08 \text{ m}) = [12\pi \text{ rad/s} - (33.333 \text{ rad/s}^2)(t)](0.06 \text{ m})$$

$$t = 4.02124 \qquad t = 0.75398 \text{ s}$$

$$\circlearrowleft \; \omega_A = \alpha_A t = (12.5 \text{ rad/s}^2)(0.75398 \text{ s}) = 9.4248 \text{ rad/s}$$

$$+\circlearrowleft \; \omega_A = (9.4248 \text{ rad/s}) = 90.00 \text{ rpm} \circlearrowright$$

or $$\boldsymbol{\omega}_A = 90.0 \text{ rpm} \circlearrowright \blacktriangleleft$$

$$+\circlearrowleft \; \omega_B = (\omega_B)_0 - \alpha_B t$$

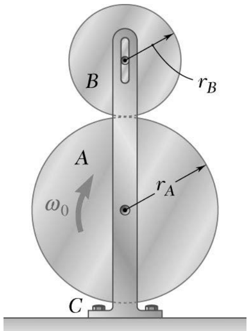
$$= 12\pi \text{ rad/s} - (33.333 \text{ rad/s}^2)(0.75398 \text{ s})$$

$$= 12.566 \text{ rad/s} \circlearrowright$$

$$\omega_B = (12.566 \text{ rad/s})$$

$$= 120.00 \text{ rpm}$$

or $$\boldsymbol{\omega}_B = 120.0 \text{ rpm} \circlearrowright \blacktriangleleft$$

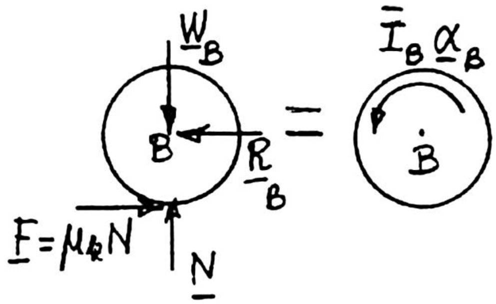


PROBLEM 16.43

Disk A has a mass m_A = 3 kg, a radius r_A = 300 mm, and an initial angular velocity $\omega_0 = 300$ rpm clockwise. Disk B has a mass m_B = 1.6 kg, a radius r_B = 180 mm, and is at rest when it is brought into contact with disk A. Knowing that $\mu_k = 0.35$ between the disks and neglecting bearing friction, determine (a) the angular acceleration of each disk, (b) the reaction at the support C.

SOLUTION

(a) Disk B.



$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - W_B = 0$$

$$N = W_B = m_B g$$

Thus, $$F = \mu_k N = \mu_k m_B g$$

$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad F r_B = \bar{I}_B \alpha_B$$

$$\mu_k m_B g r_B = \frac{1}{2} m_B r_B^2 \alpha_B$$

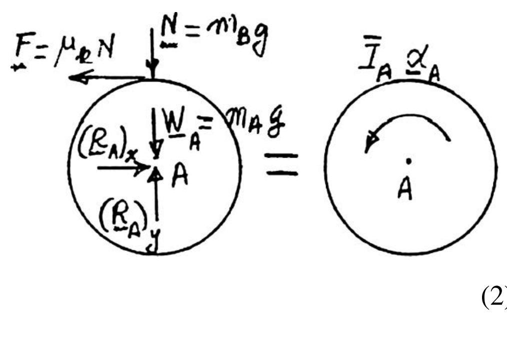
$$\boldsymbol{\alpha}_B = \frac{2\mu_k g}{r_B} \circlearrowright \qquad (1)$$

For the given data: $\alpha_B = \dfrac{2(0.35)(9.81 \text{ m/s}^2)}{0.180 \text{ m}} = 38.15 \text{ rad/s}^2$ $\qquad \boldsymbol{\alpha}_B = 38.2 \text{ rad/s}^2 \circlearrowright$ ◀

$$W_B = m_B g = (1.6 \text{ kg})(9.81 \text{ m/s}^2) = 15.696 \text{ N}$$
$$F = \mu_k m_B g = (0.35)(15.696) = 5.4936 \text{ N}$$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F - R_B = 0$ $\qquad \mathbf{R}_B = 5.4936 \text{ N} \longleftarrow$ ◀

Disk A:



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad F r_A = \bar{I}_A \alpha_A$$

$$\mu_k m_B g r_A = \frac{1}{2} m_A r_A^2 \alpha_A$$

$$\boldsymbol{\alpha}_A = \frac{2\mu_k g}{r_A} \frac{m_B}{m_A} \circlearrowright \qquad (2)$$

PROBLEM 16.43 (Continued)

For the given data: $\alpha_A = \dfrac{2(0.35)(9.81\text{ m/s}^2)}{0.300\text{ m}}\dfrac{1.6\text{ kg}}{4\text{ kg}} = 9.156\text{ rad/s}^2$ $\qquad \boldsymbol{\alpha}_A = 9.16\text{ rad/s}^2 \curvearrowright$ ◀

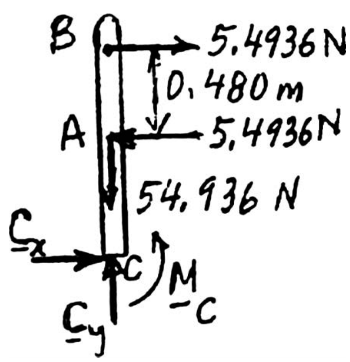
$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad (R_A)_x - F = 0$ $\qquad (\mathbf{R}_A)_x = 5.4936\text{ N} \longrightarrow$

$+\!\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad (R_A)_y - m_A g - m_B g = 0$

$$(R_A)_y = (m_A + m_B)g = (4\text{ kg} + 1.6\text{ kg})(9.81\text{ m/s}^2)$$
$$= 54.936\text{ N}$$

$(\mathbf{R}_A)_y = 54.936\text{ N}\uparrow$

(*b*) *Reaction at C.*

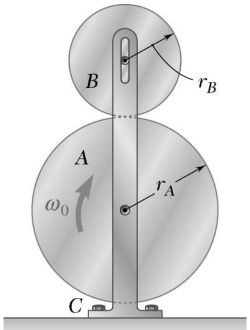


$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$

$+\!\uparrow \Sigma F_y = 0: \quad C_y - 54.936\text{ N} = 0$

$C_y = 54.936\text{ N}$ $\qquad \mathbf{C} = 54.9\text{ N}\uparrow$ ◀

$+\curvearrowright \Sigma M_C = 0: \quad M_C - (5.4936\text{ N})(0.480\text{ m}) = 0$ $\qquad \mathbf{M}_C = 2.64\text{ N}\cdot\text{m} \curvearrowright$ ◀



PROBLEM 16.44

Disk B is at rest when it is brought into contact with disk A, which has an initial angular velocity ω_0. (*a*) Show that the final angular velocities of the disks are independent of the coefficient of friction μ_k between the disks as long as $\mu_k \neq 0$. (*b*) Express the final angular velocity of disk A in terms of ω_0 and the ratio m_A/m_R of the masses of the two disks.

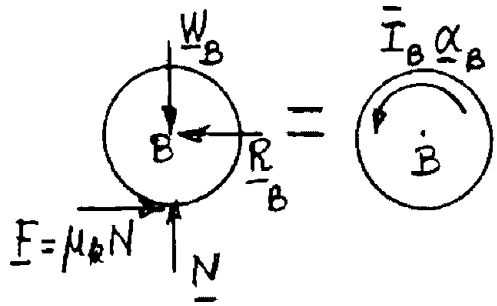
SOLUTION

(*a*) Disk *B*.

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad N - w_B = 0$$

$$N = w_B = m_B g$$

Thus, $$F = \mu_k N = \mu_k m_B g$$

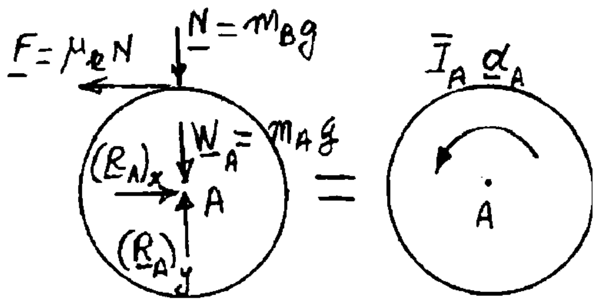


$$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad F r_B = \bar{I}_B \alpha_B$$

$$\mu_k m_B g r_B = \frac{1}{2} m_B r_B^2 \alpha_B$$

$$\boldsymbol{\alpha}_B = \frac{2\mu_k g}{r_B} \circlearrowright \qquad (1)$$

Disk *A*.



$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad F r_A = \bar{I}_A \alpha_A$$

$$\mu_k m_B g r_A = \frac{1}{2} m_A r_A^2 \alpha_A$$

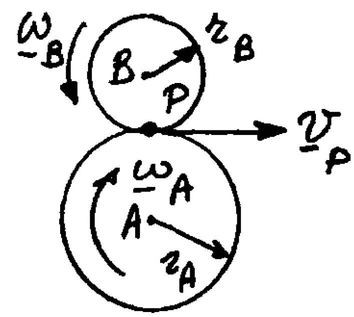
$$\boldsymbol{\alpha}_A = \frac{2\mu_k g}{r_A} \frac{m_B}{m_A} \circlearrowright \qquad (2)$$

Disk *A*. $$\omega_A = \omega_0 - \alpha_A t = \omega_0 - \frac{2\mu_k g}{r_A} \frac{m_B}{m_A} t \circlearrowright \qquad (3)$$

Disk *B*. $$\omega_B = \alpha_B t = \frac{2\mu_k g}{r_B} t \circlearrowright \qquad (4)$$

PROBLEM 16.44 (Continued)

When disks have stopped slipping:



$$v_P = \omega_A r_A = \omega_B r_B$$

$$\omega_0 r_A - (2\mu_k g)\frac{m_B}{m_A}t = 2\mu_k g t$$

$$t = \frac{\omega_0 r_A}{2\mu_k g}\frac{1}{1+\frac{m_B}{m_A}}$$

$$t = \frac{\omega_0 r_A}{2\mu_k g}\frac{m_A}{m_A + m_B} \qquad (5)$$

Substituting for t from (5) into (3) and (4):

$$\omega_A = \omega_0 - \frac{2\mu_k g}{r_A}\frac{m_B}{m_A}\frac{\omega_0 r_A}{2\mu_k g}\frac{m_A}{m_A + m_B} = \omega_0 - \frac{\omega_0 m_B}{m_A + m_B}$$

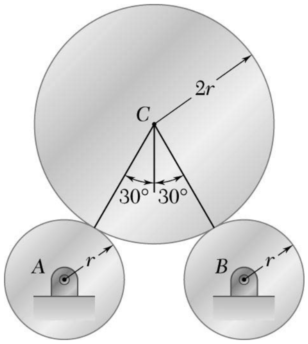
$$\omega_A = \omega_0 \frac{m_A + m_B - m_B}{m_A + m_B} \qquad \omega_A = \omega_0 \frac{m_A}{m_A + m_B} \circlearrowright \qquad (6)$$

$$\omega_B = \frac{2\mu_k g}{r_A}\frac{\omega_0 r_A}{2\mu_k g}\frac{m_A}{m_A + m_B} \qquad \omega_B = \omega_0 \frac{r_A}{r_B}\frac{m_A}{m_A + m_B} \circlearrowleft \qquad (7)$$

(*a*) From Eqs. (6) and (7), it is apparent that ω_A and ω_B are independent of μ_k. However, if $\mu_k = 0$, we have from Eqs. (3) and (4) $\omega_A = \omega_0 \circlearrowright$ and $\omega_B = 0$.

(*b*) We can write (6) in the form $\omega_A = \omega_0/(1 + m_B/m_A) \circlearrowright$ ◀

which shows that ω_A depends only upon ω_0 and m_B/m_A.



PROBLEM 16.45

Cylinder A has an initial angular velocity of 720 rpm clockwise, and cylinders B and C are initially at rest. Disks A and B each weigh 5 lb and have a radius $r = 4$ in. Disk C weighs 20 lb and has a radius of 8 in. The disks are brought together when C is placed gently onto A and B. Knowing that $\mu_k = 0.25$ between A and C and no slipping occurs between B and C, determine (*a*) the angular acceleration of each disk, (*b*) the final angular velocity of each disk.

SOLUTION

Assume Point C, the center of cylinder C, does not move. This is true provided the cylinders remain in contact as shown. Slipping occurs initially between disks A and C and ceases when the tangential velocities at their contact point are equal. We first determine the angular accelerations of each disk while slipping occurs.

Masses and moments of inertia:

$$m_A = m_B = \frac{W_A}{g} = \frac{5\text{ lb}}{32.2\text{ ft/s}^2} = 0.15528\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_C = \frac{W_C}{g} = \frac{20\text{ lb}}{32.2\text{ ft/s}^2} = 0.62112\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$I_A = I_B = \frac{1}{2}m_A r^2 = \frac{1}{2}(0.15528)\left(\frac{4}{12}\right)^2 = 0.0086266\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$I_C = \frac{1}{2}m_C(2r)^2 = \frac{1}{2}(0.62112)\left(\frac{8}{12}\right)^2 = 0.138027\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Kinematics: No slipping at contact BC.

$$(\mathbf{a}_t)_{BC} = (a_t)_{BC} \measuredangle 30°$$

$$(a_t)_{BC} = r\alpha_B = 2r\alpha_C \qquad \alpha_B = 2\alpha_C \tag{1}$$

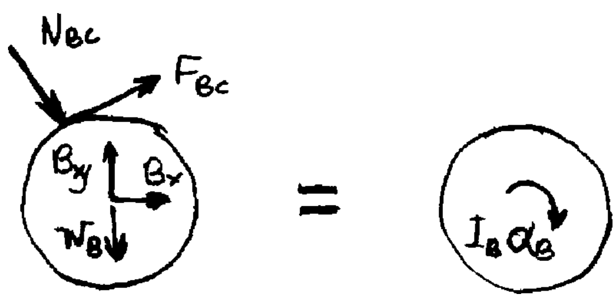
Friction condition:

$$F_{AC} = \mu_k N_{AC} \tag{2}$$

Kinetics:

Disk B:

$$\circlearrowright\, \Sigma F_B = \Sigma(F_B)_{\text{eff}}:$$



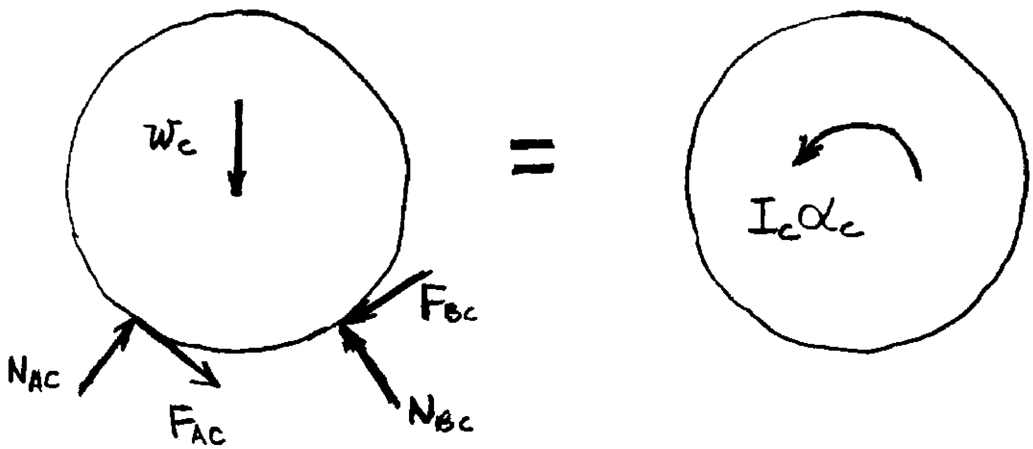
$$F_{BC}r = I_B\alpha_B$$

$$F_{BC} = \frac{I_B}{r}\alpha_B = \frac{2I_B}{r}\alpha_C$$

$$F_{BC} = \frac{(2)(0.0086266)}{4/12}\alpha_C$$

$$= 0.051760\alpha_C \tag{3}$$

PROBLEM 16.45 (Continued)



Disk C:

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad F_{AC}(2r) - F_{BC}(2r) = I_C\alpha_C$$

$$F_{AC} = F_{BC} + \frac{I_C}{2r}\alpha_C$$

$$= 0.051760\alpha_C + \frac{0.138027}{8/12}\alpha_C$$

$$F_{AC} = 0.25880\alpha_C \qquad (4)$$

From Eq. (2), $\quad N_{AC} = \dfrac{F_{AC}}{0.25} = 1.03520\alpha_C \qquad (5)$

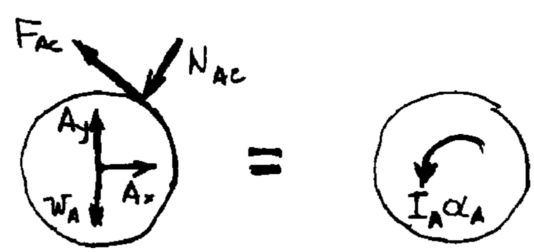
$$+\nearrow 30°\Sigma F = \Sigma F_{\text{eff}}: \quad W_B \sin 30° + F_{BC} - F_{AC}\cos 60° - N_{AC}\sin 60° = 0$$

$$(20)\sin 30° + (0.051760 - 0.25880\cos 60° - 1.03520\sin 60°)\alpha_C = 0$$

$$10 - 0.97415\alpha_C = 0$$

$$\alpha_C = 10.2654 \text{ rad/s}^2$$

$$F_{BC} = (0.051760)(10.2654) = 0.53134 \text{ lb.}$$
$$F_{AC} = (0.25880)(10.2654) = 2.6567 \text{ lb.}$$
$$N_{AC} = (1.03520)(10.2654) = 10.6267 \text{ lb.}$$



Check that $N_{BC} > 0$.

$$+\searrow 60°\Sigma F = \Sigma F_{\text{eff}}: \quad N_{BC} - W_C\cos 30° + N_{AC}\cos 60° - F_{AC}\sin 60° = 0$$

$$N_{BC} = (20 \text{ lb})\cos 30° - (10.6267 \text{ lb})\cos 60° + (2.6567 \text{ lb})\sin 60° = 0$$
$$N_{BC} = 14.3079 \text{ lb.}$$

PROBLEM 16.45 (Continued)

Disk *A*:

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad F_{AC} r = I_A \alpha_A$$

$$\alpha_A = \frac{F_{AC} r}{I_A} = \frac{(2.6567)(4/12)}{0.0086266} = 102.69 \text{ rad/s}^2$$

(*a*) *Angular accelerations of disks.* $\boldsymbol{\alpha}_A = 102.7 \text{ rad/s}^2 \circlearrowleft$ ◀

From Eq. (1), $\boldsymbol{\alpha}_B = 20.5 \text{ rad/s}^2 \circlearrowright$ ◀

$\boldsymbol{\alpha}_C = 10.27 \text{ rad/s}^2 \circlearrowleft$ ◀

(*b*) *Final angular velocities.*

Disk *A*:

$$\omega_0 = 720 \text{ rpm} = 24\pi \text{ rad/s} \circlearrowright$$

$$\omega_A = \omega_0 - \alpha_A t \circlearrowright$$

$$= 24\pi - 102.69t \circlearrowright$$

$$(v_t)_{AC} = r\omega_A = \frac{4}{12}(24\pi - 102.69t)$$

$$(\mathbf{v}_t)_{AC} = (8\pi - 34.23) \text{ ft/s} \searrow 30°$$

Disk *C*:

$$\omega_C = \alpha_C t \circlearrowleft = 10.2654t \circlearrowleft$$

$$(\mathbf{v}_t)_{CA} = 2r\omega_C = \left(\frac{8}{12}\right)(10.2654)t$$

$$(\mathbf{v}_t)_{CA} = 6.8436t \searrow 30°$$

Time when tangential velocities are equal.

$$8\pi - 34.23t = 6.8436t \qquad t = 0.6119 \text{ s}$$

$$\omega_A = 24\pi - (102.69)(0.6119) = 12.562 \text{ rad/s}$$ $\boldsymbol{\omega}_A = 120.0 \text{ rpm} \circlearrowright$ ◀

$$\omega_C = (10.2654)(0.6119) = 6.2813 \text{ rad/s}$$ $\boldsymbol{\omega}_C = 60.0 \text{ rpm} \circlearrowleft$ ◀

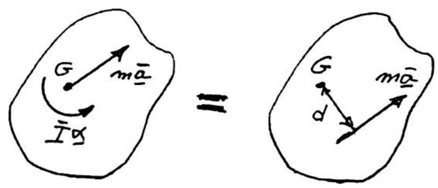
$\boldsymbol{\omega}_B = 120.0 \text{ rpm} \circlearrowright$ ◀

PROBLEM 16.46

Show that the system of the effective forces for a rigid slab in plane motion reduces to a single vector, and express the distance from the mass center G of the slab to the line of action of this vector in terms of the centroidal radius of gyration $\bar{k}$ of the slab, the magnitude $\bar{a}$ of the acceleration of G, and the angular acceleration α.

SOLUTION

We know that the system of effective forces can be reduced to the vector $m\bar{\mathbf{a}}$ at G and the couple $\bar{I}\boldsymbol{\alpha}$. We further know from Chapter 3 on statics that a force-couple system in a plane can be further reduced to a single force.

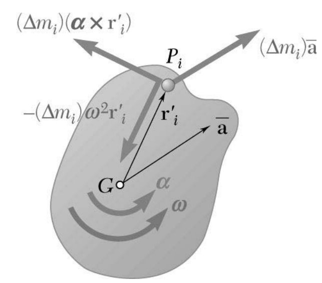


The perpendicular distance d from G to the line of action of the single vector $m\bar{\mathbf{a}}$ is expressed by writing

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad \bar{I}\alpha = (m\bar{a})d$$

$$d = \frac{\bar{I}\alpha}{m\bar{a}} = \frac{m\bar{k}^2\alpha}{m\bar{a}}$$

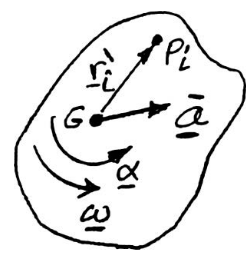
$$d = \frac{\bar{k}^2\alpha}{\bar{a}} \blacktriangleleft$$



PROBLEM 16.47

For a rigid slab in plane motion, show that the system of the effective forces consists of vectors $(\Delta m_i)\,\bar{\mathbf{a}}$, $-(\Delta m_i)\omega^2\mathbf{r}_i'$, and $(\Delta m_i)(\boldsymbol{\alpha}\times\mathbf{r}_i')$ attached to the various particles P_i of the slab, where $\bar{\mathbf{a}}$ is the acceleration of the mass center G of the slab, $\boldsymbol{\omega}$ is the angular velocity of the slab, $\boldsymbol{\alpha}$ is its angular acceleration, and $\mathbf{r}_i'$ denotes the position vector of the particle P_i, relative to G. Further show, by computing their sum and the sum of their moments about G, that the effective forces reduce to a vector $m\bar{\mathbf{a}}$ attached at G and a couple $\bar{I}\boldsymbol{\alpha}$.

SOLUTION



Kinematics: The acceleration of P_L is

$$\mathbf{a}_i = \bar{\mathbf{a}} + \mathbf{a}_{P_i/G}$$

$$\mathbf{a}_i = \bar{\mathbf{a}} + \boldsymbol{\alpha}\times\mathbf{r}_i' + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}_i')$$

$$= \bar{\mathbf{a}} + \boldsymbol{\alpha}\times\mathbf{r}_i' - \omega^2\mathbf{r}_i'$$

Note: that $\boldsymbol{\alpha}\times\mathbf{r}_i'$ is $\perp$ to $\mathbf{r}_i'$

Thus, the effective forces are as shown in Figure P16.47 (also shown above). We write

$$(\Delta m_i)\mathbf{a}_i = (\Delta m_i)\bar{\mathbf{a}} + (\Delta m_i)(\boldsymbol{\alpha}\times\mathbf{r}_i') - (\Delta m_i)\omega^2 r_i'$$

The sum of the effective forces is

$$\Sigma(\Delta m_i)\mathbf{a}_i = \Sigma(\Delta m_i)\bar{\mathbf{a}} + \Sigma(\Delta m_i)(\boldsymbol{\alpha}\times\mathbf{r}_i') - \Sigma(\Delta m_i)\omega^2\mathbf{r}_i'$$

$$\Sigma(\Delta m_i)\mathbf{a}_i = \bar{\mathbf{a}}\,\Sigma(\Delta m_i) + \boldsymbol{\alpha}\times\Sigma(\Delta m_i)\,\mathbf{r}_i' - \omega^2\Sigma(\Delta m_i)\mathbf{r}_i'$$

We note that $\Sigma(\Delta m_i) = m.$ And since G is the mass center,

$$\Sigma(\Delta m_i)\mathbf{r}_i' = m\,\bar{r}_i' = 0$$

Thus, $$\Sigma(\Delta m_i)\mathbf{a}_i = m\bar{\mathbf{a}} \qquad (1)$$

The sum of the moments about G of the effective forces is:

$$\Sigma(\mathbf{r}_i'\times\Delta m_i\mathbf{a}_i) = \Sigma\mathbf{r}_i'\times\Delta m_i\bar{\mathbf{a}} + \Sigma\mathbf{r}_i'\times(\Delta m_i)(\boldsymbol{\alpha}\times\mathbf{r}_i') - \Sigma\mathbf{r}_i'\times(\Delta m_i)\omega^2\mathbf{r}_i'$$

$$\Sigma(\mathbf{r}_i'\times\Delta m_i\mathbf{a}_i) = (\Sigma\mathbf{r}_i'\Delta m_i)\,\bar{\mathbf{a}} + \Sigma[\mathbf{r}_i'\times(\boldsymbol{\alpha}\times\mathbf{r}_i')\Delta m_i] - \omega^2\Sigma(\mathbf{r}_i'\times\mathbf{r}_i')\Delta m_i$$

Since G is the mass center, $\Sigma\mathbf{r}_i'\Delta m_i = 0$

Also, for each particle, $\mathbf{r}_i'\times\mathbf{r}_i' = 0$

Thus, $$\Sigma(\mathbf{r}_i'\times\Delta m_i\mathbf{a}_i) = \Sigma[\mathbf{r}_i'\times(\boldsymbol{\alpha}_i\times\mathbf{r}_i')\Delta m_i]$$

PROBLEM 16.47 (Continued)

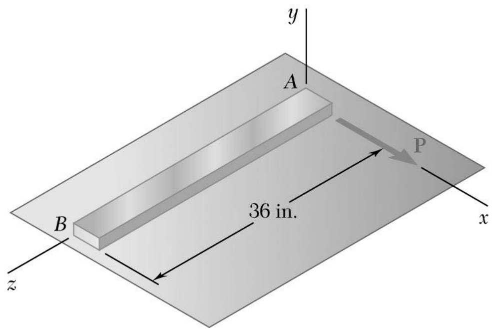
Since $\alpha \perp \mathbf{r}_i'$, we have $\mathbf{r}_i' \times (\boldsymbol{\alpha} \times \mathbf{r}_i') = r_i^2 \alpha$ and

$$\Sigma(\mathbf{r}_{ii}' \times \Delta m_i \mathbf{a}_i) = \Sigma r_i'^2 \, (\Delta m_i)\alpha$$
$$= (\Sigma r_i'^2 \Delta m_i)\boldsymbol{\alpha}$$

Since $\Sigma r_i'^2 \, \Delta m_i = \bar{I}$ $\quad\Sigma(\mathbf{r}_i' \times \Delta m_i \mathbf{a}_i) = \bar{I}\alpha$ (2)

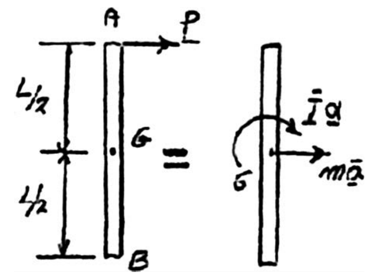
From Eqs. (1) and (2) we conclude that system of effective forces reduce to $m\bar{\mathbf{a}}$ attached at G and a couple $\bar{I}\boldsymbol{\alpha}$.

PROBLEM 16.48



A uniform slender rod *AB* rests on a frictionless horizontal surface, and a force **P** of magnitude 0.25 lb is applied at *A* in a direction perpendicular to the rod. Knowing that the rod weighs 1.75 lb, determine (*a*) the acceleration of Point *A*, (*b*) the acceleration of Point *B*, (*c*) the location of the point on the bar that has zero acceleration.

SOLUTION



$$m = \frac{W}{g}$$

$$\bar{I} = \frac{1}{12}\frac{W}{g}L^2$$

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad P = m\bar{a} = \frac{W}{g}\bar{a}$$

$$\bar{a} = \frac{P}{W}g = \frac{0.25 \text{ lb}}{1.75 \text{ lb}}g = \frac{1}{7}g \qquad \bar{\mathbf{a}} = \frac{1}{7}g \longrightarrow$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad P\frac{L}{2} = \bar{I}\alpha = \frac{1}{12}\frac{W}{g}L^2\alpha$$

$$\alpha = 6\frac{P}{W}\frac{5}{L} = 6\frac{0.25 \text{ lb}}{1.75 \text{ lb}} \cdot \frac{g}{L} = \frac{6}{7}\frac{g}{L} \qquad \boldsymbol{\alpha} = \frac{6}{7}\frac{g}{L} \circlearrowright$$

We calculate the accelerations immediately after the force is applied. After the rod acquires angular velocity, there will be additional normal accelerations.

(*a*) <u>Acceleration of Point *A*.</u>

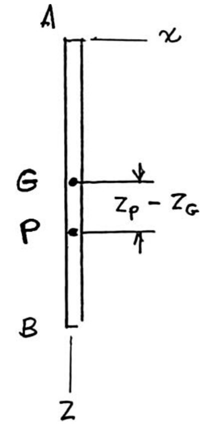
$$\xrightarrow{+} \mathbf{a}_A = \bar{\mathbf{a}} + \frac{L}{2}\alpha = \frac{1}{7}g + \frac{L}{2}\cdot\frac{6}{7}g = \frac{4}{7}g = \frac{4}{7}(32.2 \text{ ft/s}^2) \qquad \mathbf{a}_A = 18.40 \text{ ft/s}^2 \longrightarrow \blacktriangleleft$$

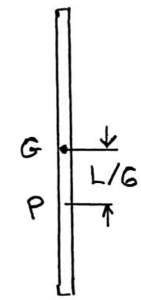
(*b*) <u>Acceleration of Point *B*.</u>

$$\xrightarrow{+} \mathbf{a}_B = \bar{\mathbf{a}} - \frac{L}{2}\alpha = \frac{1}{7}g - \frac{L}{2}\cdot\frac{6}{7}g = -\frac{2}{7}g = -\frac{2}{7}(32.2 \text{ ft/s}^2) \qquad \mathbf{a}_B = 9.20 \text{ ft/s}^2 \longleftarrow \blacktriangleleft$$

PROBLEM 16.48 (Continued)

(c) Point of zero acceleration.





$$a_P = 0$$

$$\bar{a} - (z_P - z_G)\alpha = 0$$

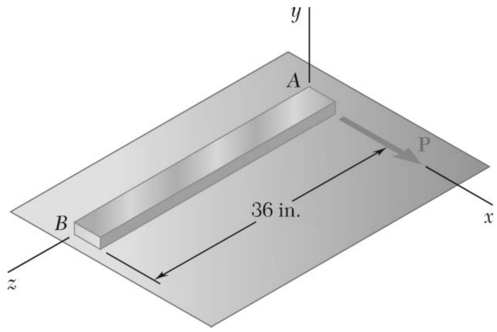
$$z_P - z_G = \frac{\bar{a}}{\alpha} = \frac{\frac{1}{7}g}{\frac{6}{7}\cdot\frac{g}{L}} = \frac{1}{6}L$$

Since $z_G = \frac{1}{2}L$

$$z_P = \frac{1}{2}L + \frac{1}{6}L = \frac{2}{3}L$$

$$z_P = \frac{2}{3}(36 \text{ in.})$$

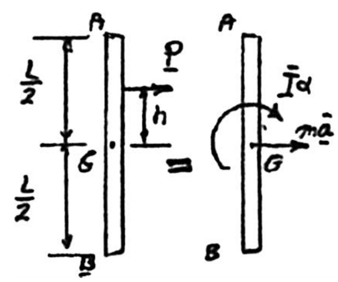
$z_P = 24.0 \text{ in.}$ ◀



PROBLEM 16.49

(*a*) In Problem 16.48, determine the point of the rod *AB* at which the force **P** should be applied if the acceleration of Point *B* is to be zero. (*b*) Knowing that $P = 0.25$ lb, determine the corresponding acceleration of Point *A*.

SOLUTION



$$\xrightarrow{+} \Sigma F_x = \Sigma(F_X)_{\text{eff}}: \quad P = m\bar{a} = \frac{W}{g}\bar{a}$$

$$\mathbf{a} = \frac{P}{W} g \longrightarrow$$

$$\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad Ph = \bar{I}\alpha = \frac{1}{12}\frac{W}{g}L^2\alpha$$

$$\boldsymbol{\alpha} = \frac{12Ph}{WL^2} g \circlearrowright$$

(*a*) <u>Position of force *P*</u>.

$$\xrightarrow{+} \mathbf{a}_B = \bar{\mathbf{a}} - \frac{L}{2}\alpha$$

$$0 = \frac{P}{W}g - \frac{L}{2}\cdot\frac{12Ph}{WL^2}g$$

$$h = \frac{L}{6} = \frac{36 \text{ in.}}{6} = 6 \text{ in.}$$

Thus, *P* is located 12 in. from end *A*. ◀

For $\quad h = \frac{L}{6}: \quad \alpha = \frac{12P\left(\frac{L}{6}\right)}{WL^2} g = 2\frac{P}{W}\cdot\frac{g}{L} \circlearrowright$

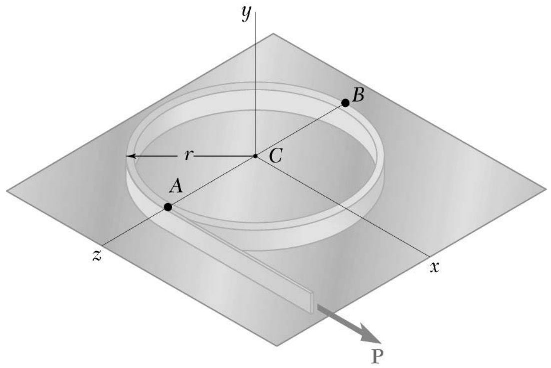
(*b*) <u>Acceleration of Point *A*</u>.

$$\xrightarrow{+} a_A = \bar{a} + \frac{L}{2}\alpha = \frac{P}{W}g + \frac{L}{2}\cdot 2\frac{P}{W}\frac{g}{L} = 2\frac{P}{W}g$$

$$a_A = 2\frac{0.25 \text{ lb}}{1.75 \text{ lb}}(32.2 \text{ ft/s}^2)$$

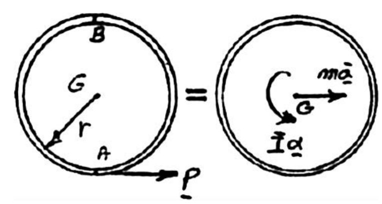
$\mathbf{a}_A = 9.20 \text{ ft/s}^2 \longrightarrow$ ◀

PROBLEM 16.50



A force **P** of magnitude 3 N is applied to a tape wrapped around a thin hoop of mass 2.4 kg. Knowing that the body rests on a frictionless horizontal surface, determine the acceleration of (*a*) Point *A*, (*b*) Point *B*.

SOLUTION



Hoop: $\bar{I} = mr^2$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad P = m\bar{a}$

$$\bar{\mathbf{a}} = \frac{P}{m} \longrightarrow$$

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad Pr = \bar{I}\alpha = mr^2\alpha$

$$\boldsymbol{\alpha} = \frac{P}{mr} \circlearrowleft$$

(*a*) Acceleration of Point *A*.

$$\xrightarrow{+} a_A = \bar{a} + r\alpha = \frac{P}{m} + r\left(\frac{P}{mr}\right) = 2\frac{P}{m}$$

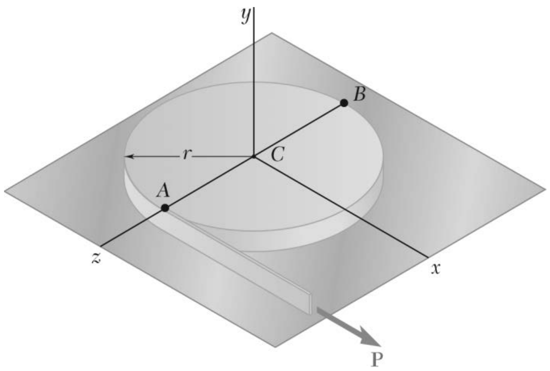
$$a_A = 2\frac{3 \text{ N}}{2.4 \text{ kg}} = 2.5 \text{ m/s}^2$$

$\mathbf{a}_A = 2.50 \text{ m/s}^2 \longrightarrow$ ◀

(*b*) Acceleration of Point *B*.

$$\xrightarrow{+} a_B = \bar{a} - r\alpha = \frac{P}{m} - r\left(\frac{P}{mr}\right) = 0$$

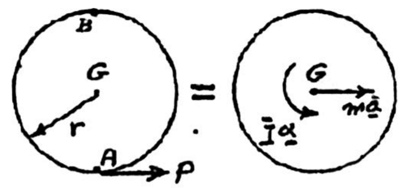
$\mathbf{a}_B = 0$ ◀



PROBLEM 16.51

A force **P** is applied to a tape wrapped around a uniform disk that rests on a frictionless horizontal surface. Show that for each 360° rotation of the disk the center of the disk will move a distance πr.

SOLUTION



Disk: $$\bar{I} = \frac{1}{2}mr^2$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad P = m\bar{a}$$

$$\bar{\mathbf{a}} = \frac{P}{m} \longrightarrow$$

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad Pr = \bar{I}\alpha$$

$$Pr = \frac{1}{2}mr^2\alpha$$

$$\alpha = \frac{2P}{mr} \circlearrowleft$$

Let t_1 be time required for 360° rotation.

$$\theta = \frac{1}{2}\alpha t_1^2$$

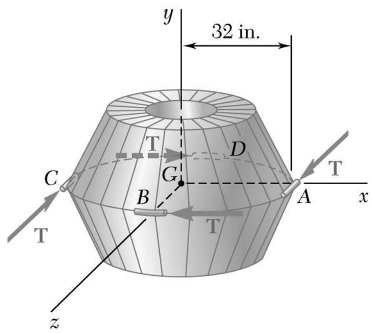
$$2\pi \text{ rad} = \frac{1}{2}\left(\frac{2P}{mr}\right)t_1^2$$

$$t_1^2 = \frac{2\pi mr}{P}$$

Let x_1 = distance G moves during 360° rotation.

$$x_1 = \frac{1}{2}\bar{a}t_1^2 = \frac{1}{2}\frac{P}{m}\left(\frac{2\pi mr}{P}\right)$$

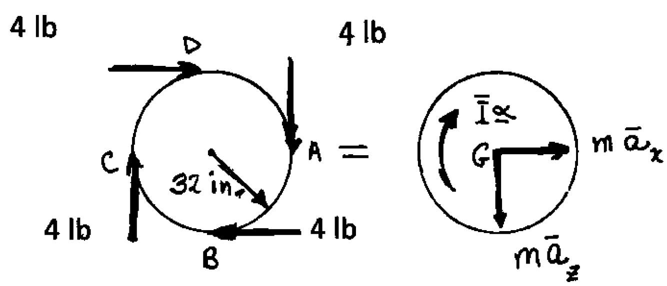
$$x_1 = \pi r$$ Q.E.D. ◀



PROBLEM 16.52

A 250-lb satellite has a radius of gyration of 24 in. with respect to the y axis and is symmetrical with respect to the zx plane. Its orientation is changed by firing four small rockets A, B, C, and D, each of which produces a 4-lb thrust **T** directed as shown. Determine the angular acceleration of the satellite and the acceleration of its mass center G (*a*) when all four rockets are fired, (*b*) when all rockets except D are fired.

SOLUTION



$$W = 250 \text{ lb}, \qquad m = \frac{W}{g} \qquad \bar{I} = mk_y^2 = \left(\frac{250 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\left(\frac{24 \text{ in.}}{12}\right)^2 = 31.056 \text{ slug}\cdot\text{ft}^2$$

(*a*) With all four rockets fired:

$$\Sigma F = \Sigma F_{\text{eff}}: \qquad 0 = m\bar{a} \qquad\qquad \bar{\mathbf{a}} = 0 \blacktriangleleft$$

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad 4Tr = \bar{I}\alpha$$

$$-4(4 \text{ lb})\left(\frac{32 \text{ in.}}{12}\right) = 31.056\alpha \qquad\qquad \boldsymbol{\alpha} = -1.3739 \text{ rad/s}^2 \circlearrowright$$

$$\boldsymbol{\alpha} = -(1.374 \text{ rad/s}^2)\mathbf{j} \blacktriangleleft$$

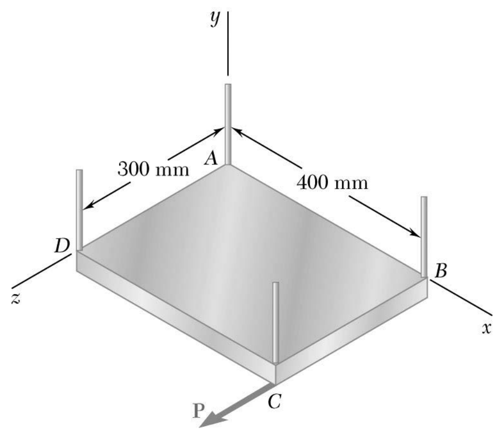
(*b*) With all rockets except D:

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad -4 \text{ lb} = \frac{250}{32.2}\bar{a}_x \qquad \bar{a}_x = -0.51520 \text{ ft/s}^2$$

$$+\downarrow \Sigma F_z = \Sigma(F_z)_{\text{eff}}: \qquad 0 = \frac{250}{32.2}\bar{a} \qquad \bar{a}_z = 0 \qquad\qquad \bar{\mathbf{a}} = -(0.515 \text{ ft/s}^2)\mathbf{i} \blacktriangleleft$$

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad 3Tr = \bar{I}\alpha$$

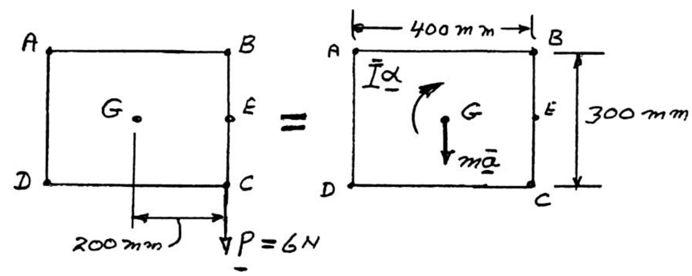
$$-3(4 \text{ lb})\left(\frac{32 \text{ in.}}{12}\right) = 31.056\alpha \qquad\qquad \boldsymbol{\alpha} = -(1.030 \text{ rad/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 16.53

A rectangular plate of mass 5 kg is suspended from four vertical wires, and a force **P** of magnitude 6 N is applied to corner *C* as shown. Immediately after **P** is applied, determine the acceleration of (*a*) the midpoint of edge *BC*, (*b*) corner *B*.

SOLUTION



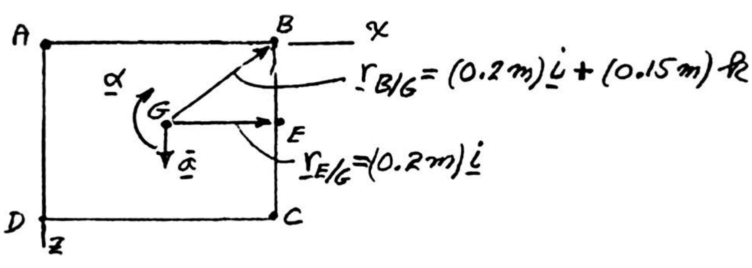
$$\bar{I} = \frac{1}{12}m(b^2 + h^2) = \frac{1}{12}(5\text{ kg})[(0.4\text{ m})^2 + (0.3\text{ m})^2] = 0.10417\text{ kg}\cdot\text{m}^2$$

$$+\downarrow \Sigma F = \Sigma F_{\text{eff}}: \qquad P = m\bar{a}$$

$$6N = (5\text{ kg})\bar{a} \qquad \bar{a} = +(1.2\text{ m/s}^2)\mathbf{k}$$

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad P(0.2\text{ m}) = \bar{I}\alpha$$

$$(6\text{ N})(0.2\text{ m}) = (0.10417\text{ kg}\cdot\text{m}^2)\alpha \qquad \boldsymbol{\alpha} = -(11.52\text{ rad/s}^2)\mathbf{j}$$



(*a*)

$$\mathbf{a}_E = \bar{\mathbf{a}} + \boldsymbol{\alpha} \times \mathbf{r}_{E/G}$$

$$= +(1.2\text{ m/s}^2)\mathbf{k} - (11.52\text{ rad/s}^2)\mathbf{j} \times (0.2\text{ m})\mathbf{i}$$

$$= +(1.2\text{ m/s}^2)\mathbf{k} + (2.304\text{ m/s}^2)\mathbf{k} \qquad \mathbf{a}_E = (3.50\text{ m/s}^2)\mathbf{k} \blacktriangleleft$$

PROBLEM 16.53 (Continued)

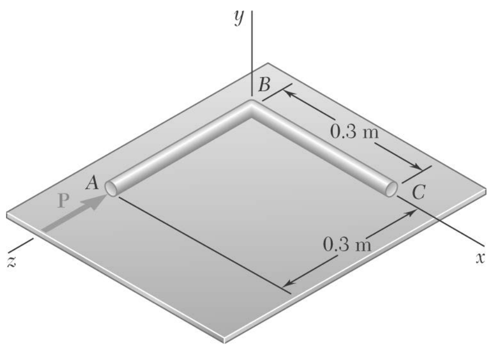
(*b*)

$$\mathbf{a}_B = \overline{\mathbf{a}} + \boldsymbol{\alpha} \times \mathbf{r}_{B/G}$$

$$= +(1.2\text{ m/s}^2)\mathbf{k} - (11.52\text{ rad/s})\mathbf{j} \times [(0.2\text{ m})\mathbf{i} + (0.15\text{ m})\mathbf{k}]$$

$$= +(1.2\text{ m/s}^2)\mathbf{k} + (2.304\text{ m/s}^2)\mathbf{k} + (1.728\text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_B = (1.728\text{ m/s}^2)\mathbf{i} + (3.5\text{ m/s}^2)\mathbf{k}$$ ◀



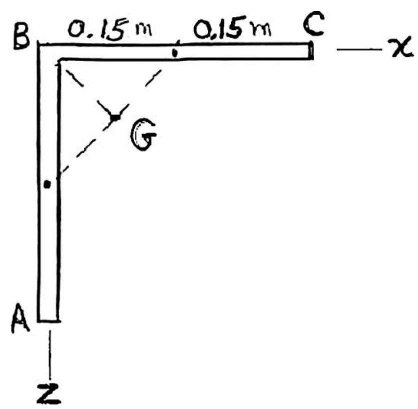
PROBLEM 16.54

A uniform slender L-shaped bar *ABC* is at rest on a horizontal surface when a force **P** of magnitude 4 N is applied at Point *A*. Neglecting friction between the bar and the surface and knowing that the mass of the bar is 2 kg, determine (*a*) the initial angular acceleration of the bar, (*b*) the initial acceleration of Point *B*.

SOLUTION

(*a*)

Mass center at G



$$\bar{x} = \frac{(m/2)\bar{x}_1 + (m/2)\bar{x}_2}{m}$$

$$\bar{x} = \frac{1(0.15) + 1(0)}{2} = 0.075 \text{ m}$$

$$\bar{x} = \bar{z}$$

$$\bar{I} = 2\left[\frac{1}{12}\left(\frac{m}{2}\right)(0.3)^2 + \frac{m}{2}\left((0.075)^2 + (0.075)^2\right)\right] = 0.0375 \text{ kg}\cdot\text{m}^2$$

$$\Sigma F_x = 0 = ma_{Gx}, \qquad a_{Gx} = 0$$

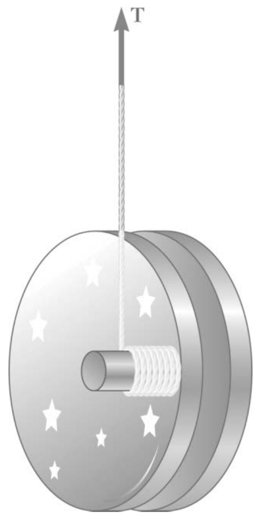
$$\Sigma F_z = -4 = ma_{Gz}, \qquad a_{Gz} = -2 \text{ m/s}^2$$

$$\Sigma M_G = -4(0.075) = 0.0375\alpha,$$

$\boldsymbol{\alpha} = -(8 \text{ rad/s}^2)\mathbf{j}$ ◀

(*b*) $\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G} = -2\mathbf{k} - 8\mathbf{j}\times(-0.075\mathbf{i} - 0.075\mathbf{k})$

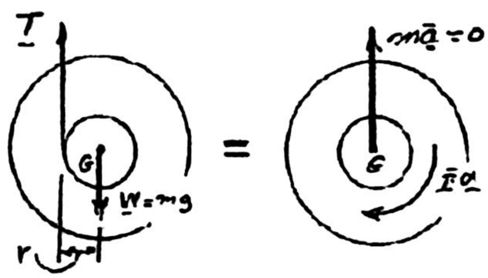
$\mathbf{a}_B = (0.6 \text{ m/s}^2)\mathbf{i} - (2.6 \text{ m/s}^2)\mathbf{k}$ ◀



PROBLEM 16.55

By pulling on the string of a yo-yo, a person manages to make the yo-yo spin, while remaining at the same elevation above the floor. Denoting the mass of the yo-yo by m, the radius of the inner drum on which the string is wound by r, and the centroidal radius of gyration of the yo-yo by $\bar{k}$, determine the angular acceleration of the yo-yo.

SOLUTION

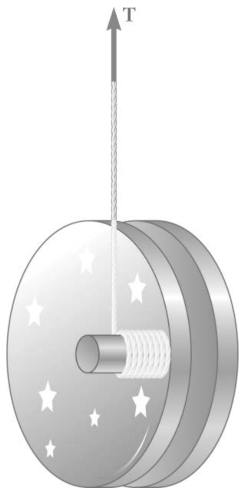


$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad T - mg = 0; \quad T = mg$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \qquad Tr = \bar{I}\alpha$$

$$mgr = m\bar{k}^2\alpha$$

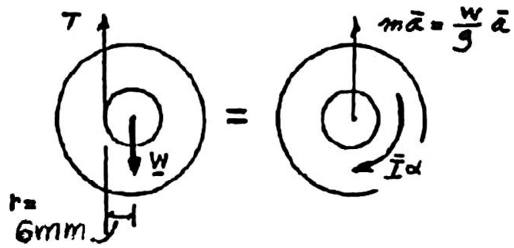
$$\alpha = \frac{rg}{\bar{k}^2} \qquad \boldsymbol{\alpha} = \frac{rg}{\bar{k}^2} \circlearrowright \blacktriangleleft$$



PROBLEM 16.56

The 80-g yo-yo shown has a centroidal radius of gyration of 30 mm. The radius of the inner drum on which a string is wound is 6 mm. Knowing that at the instant shown the acceleration of the center of the yo-yo is 1 m/s^2 upward, determine (*a*) the required tension **T** in the string, (*b*) the corresponding angular acceleration of the yo-yo.

SOLUTION



$$W = mg$$

$$W = 0.080 \text{ kg } (9.81 \text{ m/s}^2) = 0.7848 \text{ N}$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad T - W = \frac{W}{g}\bar{a}$$

$$T - (0.08 \text{ kg})(9.81 \text{ m/s}^2) = (0.08 \text{ kg})(1 \text{ m/s}^2)$$

$$T = 0.8648 \text{ N}$$

(*a*) *Tension in the string.* $T = 0.865 \text{ N}$ ◀

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad Tr = \bar{I}\alpha$$

$$(0.8648 \text{ N})(0.006 \text{ m}) = m\bar{k}^2\alpha$$

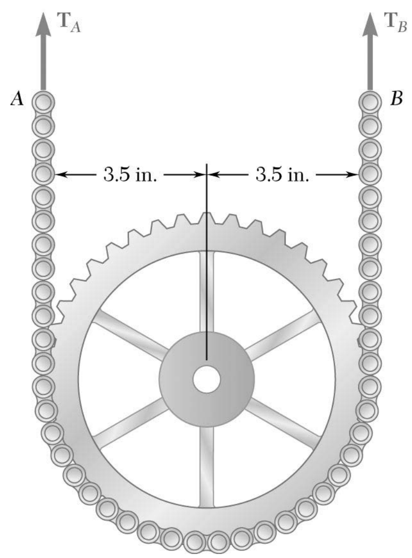
$$5.1888 \times 10^{-3} \text{ N} \cdot \text{m} = (0.08 \text{ kg})(0.03 \text{ m})^2 \alpha$$

(*b*) *Angular acceleration.*

$$\alpha = 72.067 \text{ rad/s}^2$$

$\alpha = 72.1 \text{ rad/s}^2 \circlearrowright$ ◀

PROBLEM 16.57



A 6-lb sprocket wheel has a centroidal radius of gyration of 2.75 in. and is suspended from a chain as shown. Determine the acceleration of Points A and B of the chain, knowing that $T_A = 3$ lb and $T_B = 4$ lb.

SOLUTION

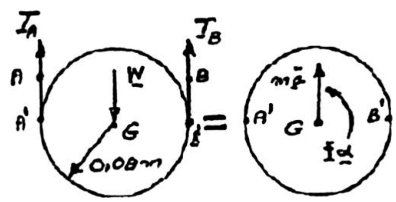
$$m = \frac{W}{g}$$

$$\bar{I} = m\bar{k}^2$$

$$= \left(\frac{6 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\left(\frac{2.75 \text{ in.}}{12 \text{ in./ft}}\right)^2$$

$$= 9.7858 \times 10^{-3} \text{ slug} \cdot \text{ft}^2$$

$$r = 3.5 \text{ in.}$$



$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad T_A + T_B - W = m\bar{a}$$

$$T_A + T_B - 6 \text{ lb} = \left(\frac{(6 \text{ lb})}{32.2 \text{ ft/s}^2}\right)\bar{a}$$

$$+\uparrow \bar{a} = 5.3667(T_A + T_B - 6) \qquad (1)$$

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad T_B\left(\frac{3.5}{12}\text{ft}\right) - T_A\left(\frac{3.5}{12}\text{ft}\right) = \bar{I}\alpha$$

$$(T_B - T_A)\left(\frac{3.5}{12}\text{ft}\right) = (9.7858 \times 10^{-3} \text{ slug} \cdot \text{ft}^2)\alpha$$

$$+\circlearrowleft \alpha = 29.805\ (T_B - T_A) \qquad (2)$$

Given data: $T_A = 3 \text{ lb}, \quad T_B = 4 \text{ lb}$

PROBLEM 16.57 (Continued)

Eq. (1): $\bar{a} = 5.3667(3+4-6) = 5.3667 \text{ ft/s}^2$

Eq. (2): $+\circlearrowleft \alpha = 29.805(4-3) = 29.805 \text{ rad/s}^2$

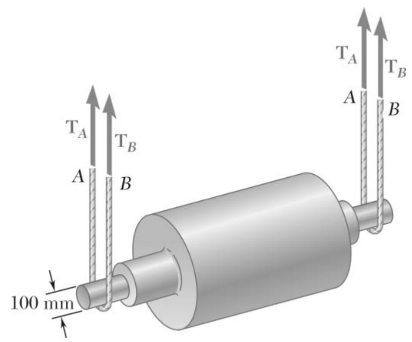
$$+\uparrow \mathbf{a}_A = (a_A)_t$$
$$= \bar{a} + r\alpha$$
$$= 5.3667 - \left(\frac{3.5}{12}\right)(29.805)$$
$$= -3.3264 \text{ ft/s}^2$$

$\mathbf{a}_A = 3.33 \text{ ft/s}^2 \downarrow$ ◀

$$\mathbf{a}_A = (a_A)_t$$
$$= \bar{a} + r\alpha$$
$$= 5.3667 + \left(\frac{3.5}{12}\right)(29.805)$$
$$= +14.06 \text{ ft/s}^2$$

$\mathbf{a}_B = 14.06 \text{ ft/s}^2 \uparrow$ ◀

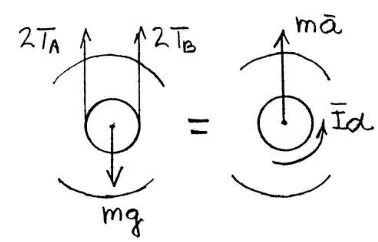
PROBLEM 16.58



The steel roll shown has a mass of 1200 kg, a centriodal radius of gyration of 150 mm, and is lifted by two cables looped around its shaft. Knowing that for each cable $T_A = 3100$ N and $T_B = 3300$ N, determine (*a*) the angular acceleration of the roll, (*b*) the acceleration of its mass center.

SOLUTION

Data:
$$m = 1200 \text{ kg}$$
$$\bar{I} = mk^2 = (1200)(0.150)^2 = 27 \text{ kg} \cdot \text{m}^2$$
$$r = \frac{1}{2}d = \frac{1}{2}(0.100) = 0.050 \text{ m}$$
$$T_A = 3100 \text{ N}$$
$$T_B = 3300 \text{ N}$$



(*a*) Angular acceleration.

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad 2T_B r - 2T_A r = \bar{I}\alpha$$

$$\alpha = \frac{2(T_B - T_A)r}{\bar{I}}$$
$$= \frac{(2)(3300 - 3100)(0.050)}{27}$$

$\boldsymbol{\alpha} = 0.741 \text{ rad/s}^2 \circlearrowright$ ◀

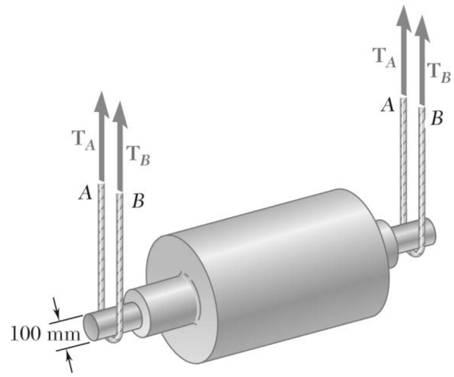
(*b*) Acceleration of mass center.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad 2T_A + 2T_B - mg = m\bar{a}$$

$$\bar{a} = \frac{2(T_A + T_B)}{m} - g$$
$$= \frac{2(3100 + 3300)}{1200} - 9.81$$

$\bar{\mathbf{a}} = 0.857 \text{ m/s}^2 \uparrow$ ◀

PROBLEM 16.59



The steel roll shown has a mass of 1200 kg, has a centriodal radius of gyration of 150 mm, and is lifted by two cables looped around its shaft. Knowing that at the instant shown the acceleration of the roll is 150 mm/s^2 downward and that for each cable $T_A = 3000$ N, determine (*a*) the corresponding tension T_B, (*b*) the angular acceleration of the roll.

SOLUTION

Data:

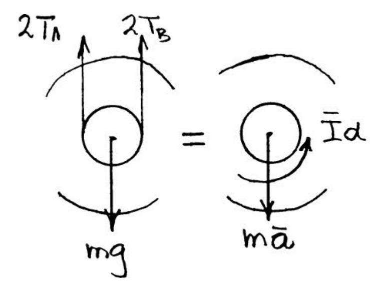
$$m = 1200 \text{ kg}$$

$$\bar{I} = mk^2 = (1200)(0.150)^2 = 27 \text{ kg}\cdot\text{m}^2$$

$$r = \frac{1}{2}d = \frac{1}{2}(0.100) = 0.050 \text{ m}$$

$$T_A = 3000 \text{ N}$$

$$\bar{\mathbf{a}} = 0.150 \text{ m/s}^2 \downarrow$$



(*a*) Tension in cable *B*.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad 2T_A + 2T_B - mg = -m\bar{a}$$

$$T_B = \frac{mg - ma}{2} - T_A$$

$$= \frac{m(g - \bar{a})}{2} - T_A$$

$$= \frac{(1200)(9.81 - 0.150)}{2} - 3000$$

$$= 2796 \text{ N}$$

$T_B = 2800$ N ◀

(*b*) Angular acceleration.

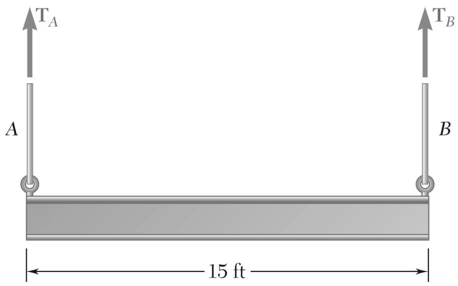
$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad 2T_B r - 2T_A r = \bar{I}\alpha$$

$$\alpha = \frac{2(T_B - T_A)r}{\bar{I}}$$

$$= \frac{(2)(2796 - 3000)}{27}$$

$$= -15.11 \text{ rad/s}^2$$

$\boldsymbol{\alpha} = 15.11 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 16.60

A 15-ft beam weighing 500 lb is lowered by means of two cables unwinding from overhead cranes. As the beam approaches the ground, the crane operators apply brakes to slow the unwinding motion. Knowing that the deceleration of cable A is 20 ft/s^2 and the deceleration of cable B is 2 ft/s^2, determine the tension in each cable.

SOLUTION

Kinematics:

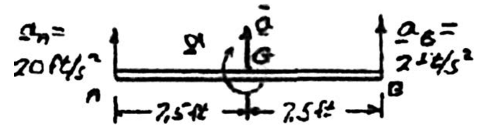
$$a_B = a_A + (15 \text{ ft})\alpha$$

$$2\uparrow = 20\uparrow + 15\alpha$$

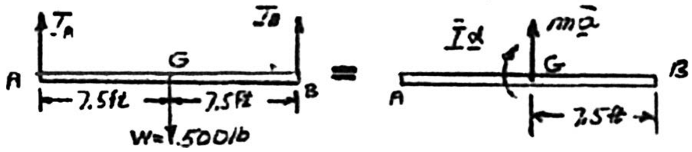
$$\alpha = 1.2 \text{ rad/s}^2 \circlearrowright$$

$$\bar{a} = \frac{1}{2}(a_A + a_B) = \frac{1}{2}(2+20)$$

$$\bar{\mathbf{a}} = 11 \text{ ft/s}^2 \uparrow$$



Kinetics:



$$\bar{I} = \frac{1}{12}mL^2$$

$$= \frac{1}{12}\frac{500}{32.2 \text{ ft/s}^2}(15 \text{ ft})^2$$

$$= 291.15 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad T_A(15 \text{ ft}) - W(2.5 \text{ ft}) = m\bar{a}(7.5 \text{ ft}) + \bar{I}\alpha$$

$$T_A(15 \text{ ft}) - (500 \text{ lb})(7.5 \text{ ft}) = \frac{500 \text{ lb}}{32.2 \text{ ft/s}^2}(11 \text{ ft/s}^2)(7.5 \text{ ft})$$

$$+ (291.15 \text{ lb}\cdot\text{ft/s})(1.2 \text{ rad/s}^2)$$

$$15T_A - 3750 = 1281 + 349.3$$

$$T_A = 358.7 \text{ lb}$$

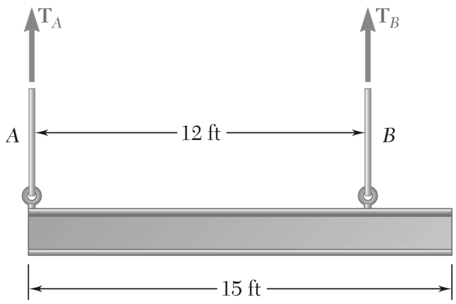
$T_A = 359$ lb ◀

$$+\uparrow \Sigma F = \Sigma F_{\text{eff}}: \quad T_A + T_B - W = m\bar{a}$$

$$358.7 \text{ lb} + T_B - 500 \text{ lb} = \frac{500 \text{ lb}}{32.2 \text{ ft/s}^2}(11 \text{ ft/s}^2)$$

$$T_B = 312.2 \text{ lb}$$

$T_B = 312$ lb ◀



PROBLEM 16.61

A 15-ft beam weighing 500 lb is lowered by means of two cables unwinding from overhead cranes. As the beam approaches the ground, the crane operators apply brakes to slow the unwinding motion. Knowing that the deceleration of cable *A* is 20 ft/s² and the deceleration of cable *B* is 2 ft/s², determine the tension in each cable.

SOLUTION

Kinematics:

$$a_B = a_A + 12\alpha$$

$$2\uparrow = 20\uparrow + 12\alpha$$

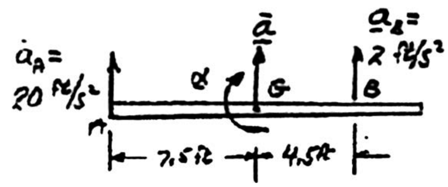
$$\alpha = 1.5 \text{ rad/s}^2 \circlearrowright$$

$$\bar{\mathbf{a}} = a_A + 7.5\alpha$$

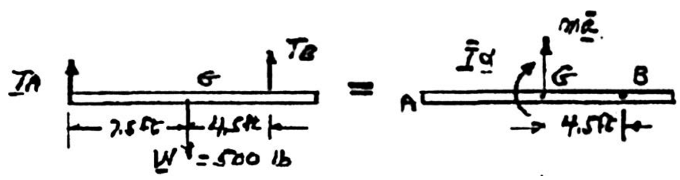
$$= 20\uparrow + (7.5)(1.5)\downarrow$$

$$\bar{\mathbf{a}} = 8.75 \text{ ft/s}^2\uparrow$$

$$\bar{I} = \frac{1}{2}mL^2 = \frac{1}{12}\frac{500}{32.2}(15)^2 = 291.15 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$



Kinetics:



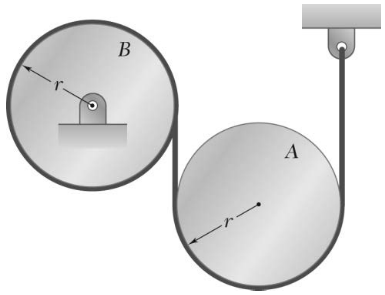
$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad T_A(12 \text{ ft}) + W(4.5 \text{ ft}) = m\bar{a}(4.5 \text{ ft}) + \bar{I}\alpha$$

$$T_A(12 \text{ ft}) - (500 \text{ lb})(4.5 \text{ ft}) = \frac{500 \text{ lb}}{32.2 \text{ ft/s}^2}(8.75 \text{ ft/s}^2)(4.5 \text{ ft}) + (291.15)(1.5 \text{ rad/s}^2)$$

$$12T_A - 2250 = 611.4 + 436.7 \qquad T_A = 275 \text{ lb} \blacktriangleleft$$

$$+\uparrow \Sigma F = \Sigma F_{\text{eff}}: \quad T_A + T_B - W = m\bar{a}$$

$$275 \text{ lb} + T_B - 500 = \frac{500}{32.2}(8.75) \qquad T_B = 361 \text{ lb} \blacktriangleleft$$



PROBLEM 16.62

Two uniform cylinders, each of weight $W = 14$ lb and radius $r = 5$ in., are connected by a belt as shown. If the system is released from rest, determine (*a*) the angular acceleration of each cylinder, (*b*) the tension in the portion of belt connecting the two cylinders, (*c*) the velocity of the center of the cylinder *A* after it has moved through 3 ft.

SOLUTION

Kinematics

Let $\mathbf{a}_A = a_A \downarrow$ be the acceleration of the center of cylinder *A*, $\mathbf{a}_{AB} = a_{AB} \downarrow$ be acceleration of the cord between the disks, $\boldsymbol{\alpha}_A = \alpha_A \circlearrowright$ be the angular acceleration of disk *A*, and $\boldsymbol{\alpha}_B = \alpha_B \circlearrowright$ be the angular acceleration of disk *B*.

$$a_A = r\alpha_A \tag{1}$$

$$a_{AB} = a_A + r\alpha_A = 2r\alpha_A = r\alpha_B \tag{2}$$

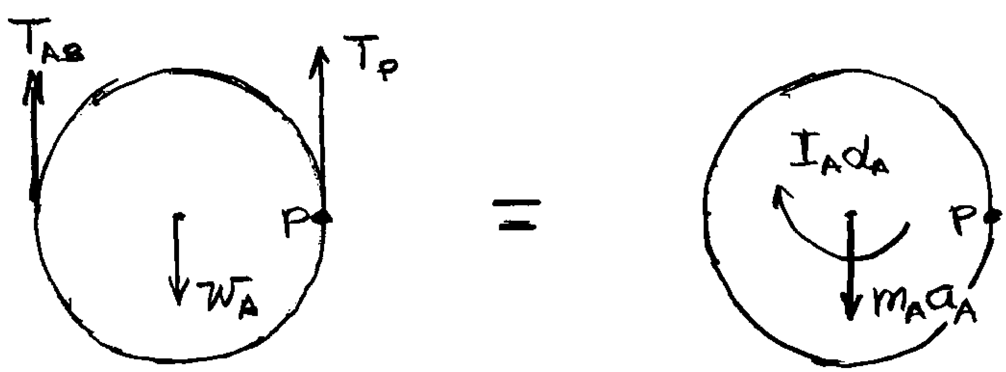
Masses and moments of inertia

$$m_A = m_B = m \tag{3}$$

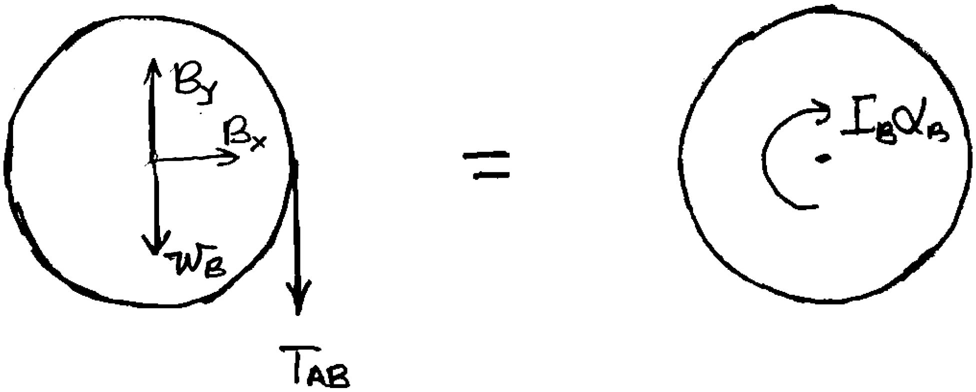
$$\bar{I}_A = \bar{I}_B = \frac{1}{2}mr^2 \tag{4}$$

Kinetics: Let T_{AB} be the tension in the portion of the cable between disks *A* and *B*.

Disk *A*: $\circlearrowright \Sigma M_P = \Sigma(M_P)_{\text{eff}}: \quad rW_A - 2rT_{AB} = rm_A a_A + \bar{I}_A \alpha_A \tag{5}$



Disk *B*: $\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad rT_{AB} = \bar{I}_B \alpha_B$ (6)



Add $2 \times \text{Eq.}(6)$ to Eq. (5) to eliminate T_{AB}.

$$rW_A = rm_A a_A + \bar{I}_A \alpha_A + 2\bar{I}_B \alpha_B \quad (7)$$

Use Eqs. (1) and (2) to eliminate a_A and α_B

$$\begin{aligned} rW_A &= rm_A(r\alpha_A) + \bar{I}_A \alpha_A + 2\bar{I}_B \cdot (2\alpha_A) \\ &= (m_A r^2 + \bar{I}_A + 4\bar{I}_B)\alpha_A \\ &= \left[mr^2 + \frac{1}{2}mr^2 + 4\left(\frac{1}{2}mr^2\right)\right]\alpha_A \\ &= 3.5\, mr^2 \alpha_A \end{aligned}$$

$$\alpha_A = \frac{rW_A}{3.5\, mr^2} = \frac{1g}{3.5r}$$

$$\alpha_B = 2\alpha_A = \frac{2g}{3.5r}$$

From Eq. (2),

$$T_{AB} = \frac{\bar{I}_B \alpha_B}{r} = \frac{\left(\frac{1}{2}mr^2\right)(2g)}{3.5r^2}$$

$$= \frac{mg}{3.5} = \frac{W}{3.5}$$

Data: $W = 14$ lb, $g = 32.2\ \text{ft/s}^2$,

$$r = 5\ \text{in.} = \frac{5}{12}\ \text{ft}$$

PROBLEM 16.62 (Continued)

(*a*) *Angular accelerations.*

$$\alpha_A = \frac{32.2 \text{ ft/s}^2}{(3.5)\left(\frac{5}{12} \text{ ft}\right)} = 22.08 \text{ rad/s}$$

$\boldsymbol{\alpha}_A = 22.1 \text{ rad/s}^2$ ↺ ◀

$$\alpha_B = 2\alpha_A$$

$\boldsymbol{\alpha}_B = 44.2 \text{ rad/s}^2$ ↻ ◀

(*b*) *Tension T_{AB}.*

$$T_{AB} = \frac{W}{3.5} = \frac{14 \text{ lb}}{3.5}$$

$T_{AB} = 4.00 \text{ lb}$ ◀

(*c*) *Velocity of the center of A.*

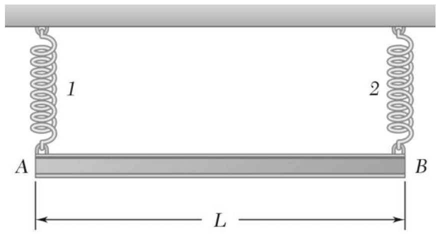
$$a_A = \frac{5}{12}\alpha_A = \frac{5}{12}(22.08) = 9.20 \text{ ft/s}^2$$

$$v_A^2 = [(v_A)_0]^2 + 2a_A d_A$$

$$= 0 + (2)(9.20 \text{ ft/s})(3 \text{ ft}) = 55.2 \text{ ft}^2/\text{s}^2$$

$$v_A = 7.43 \text{ ft/s}$$

$\mathbf{v}_A = 7.43 \text{ ft/s}$ ↓ ◀



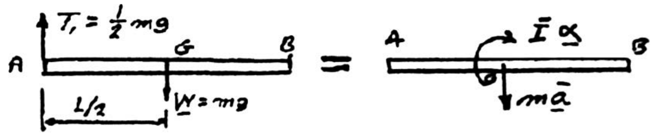
PROBLEM 16.63

A beam *AB* of mass *m* and of uniform cross section is suspended from two springs as shown. If spring 2 breaks, determine at that instant (*a*) the angular acceleration of the bar, (*b*) the acceleration of Point *A*, (*c*) the acceleration of Point *B*.

SOLUTION

Statics: $T_1 = T_2 = \frac{1}{2}W = \frac{1}{2}mg$

(*a*) Angular acceleration:



$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad T\left(\frac{L}{2}\right) = \bar{I}\alpha$$

$$\frac{1}{2}mg\left(\frac{L}{2}\right) = \frac{1}{12}mL^2\alpha$$

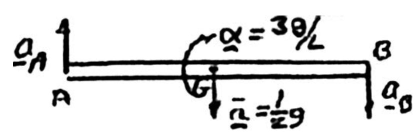
$$\alpha = \frac{3g}{L} \qquad \boldsymbol{\alpha} = \frac{3g}{L} \circlearrowright \blacktriangleleft$$

$$+\downarrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad W - T_1 = m\bar{a}$$

$$mg - \frac{1}{2}mg = m\bar{a}$$

$$\bar{a} = \frac{1}{2}g \quad \bar{\mathbf{a}} = \frac{1}{2}g\downarrow$$

(*b*) Acceleration of *A*:



$$\mathbf{a}_A = \mathbf{a}_G + \mathbf{a}_{A/G}$$

$$+\downarrow \mathbf{a}_A = \frac{1}{2}g - \frac{L}{2}\alpha$$

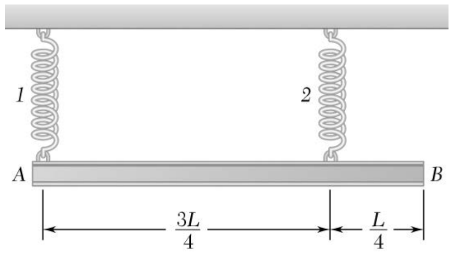
$$= \frac{1}{2}g - \frac{L}{2}\left(\frac{3g}{L}\right)$$

$$a_A = -g \qquad \mathbf{a}_A = g\uparrow \blacktriangleleft$$

(*c*) Acceleration of *B*:

$$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G}$$

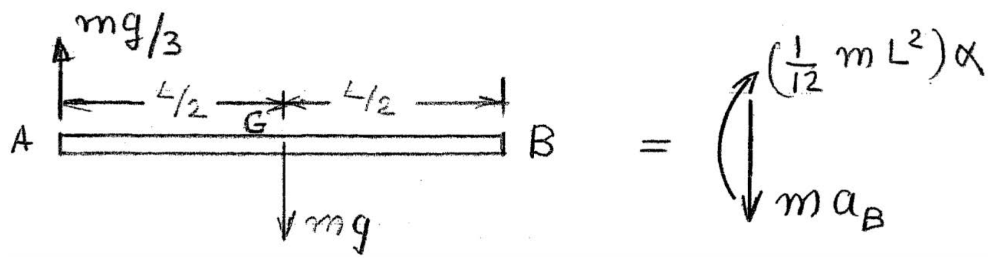
$$+\downarrow a_B = \bar{a} + \frac{L}{2}\alpha = \frac{1}{2}g + \frac{L}{2}\left(\frac{3g}{L}\right) = +2g \qquad \mathbf{a}_B = 2g\downarrow \blacktriangleleft$$



PROBLEM 16.64

A beam *AB* of mass *m* and of uniform cross section is suspended from two springs as shown. If spring 2 breaks, determine at that instant (*a*) the angular acceleration of the beam, (*b*) the acceleration of Point *A*, (*c*) the acceleration of Point *B*.

SOLUTION



$$+\downarrow \Sigma F_y = \frac{2mg}{3} = ma_G$$

$$+\circlearrowright \Sigma M_G = \left(\frac{mg}{3}\right)\frac{L}{2} = \frac{1}{12}mL^2\alpha$$

(*a*) $\mathbf{a}_G = \frac{2g}{3}\downarrow, \qquad \boldsymbol{\alpha} = \frac{2g}{L} \circlearrowright$ ◀

(*b*) $\mathbf{a}_A = \downarrow\frac{2g}{3} + \frac{2g}{L}\frac{L}{2}\uparrow = \frac{g}{3}\uparrow,$

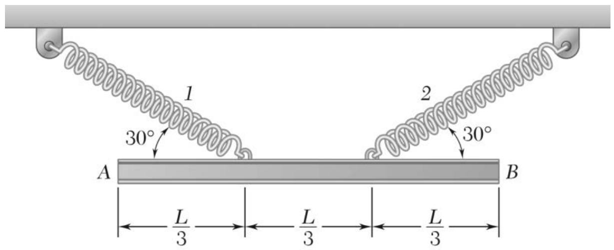
$\mathbf{a}_A = \frac{g}{3}\uparrow$ ◀

(*c*) $\mathbf{a}_B = \downarrow\frac{2g}{3} + \frac{2g}{L}\frac{L}{2}\downarrow = \frac{5g}{3}\downarrow$

$\mathbf{a}_B = \frac{5g}{3}\downarrow$ ◀

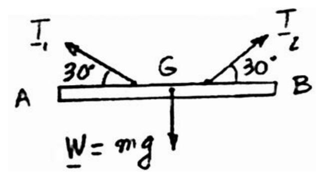
PROBLEM 16.65

A beam *AB* of mass *m* and of uniform cross section is suspended from two springs as shown. If spring 2 breaks, determine at that instant (*a*) the angular acceleration of the bar, (*b*) the acceleration of Point *A*, (*c*) the acceleration of Point *B*.



SOLUTION

Before spring 1 breaks:



$$+\uparrow \Sigma F_y = 0: \quad T_1 \sin 30° + T_2 \sin 30° - W = 0$$

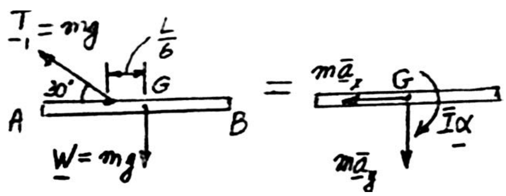
Since $T_1 = T_2$ by symmetry,

$$2T_1 \sin 30° = W = mg$$

$$\mathbf{T}_1 = mg \; \measuredangle \; 30°$$

Immediately after spring 2 breaks, elongation of spring 1 is unchanged. Thus, we still have

$$\mathbf{T}_1 = mg \; \measuredangle \; 30°$$



(*a*) Angular acceleration:

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad (T_1 \sin 30°)\frac{L}{6} = \bar{I}\alpha$$

$$(mg \sin 30°)\frac{L}{6} = \frac{1}{12} mL^2 \alpha \qquad \boldsymbol{\alpha} = \frac{g}{L} \circlearrowright \blacktriangleleft$$

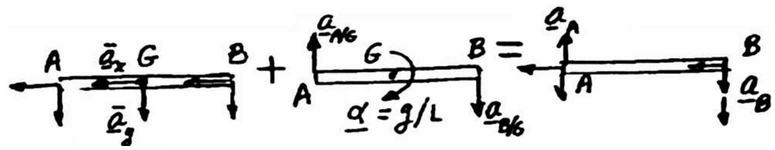
$$\overset{+}{\leftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad T_1 \cos 30° = m\bar{a}_x$$

$$mg \cos 30° = m\bar{a}, \qquad \bar{\mathbf{a}}_x = 0.866g \leftarrow$$

$$+\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad W - T_1 \sin 30° = m\bar{a}_y$$

$$mg - mg \sin 30° = m\bar{a}_y \qquad \bar{\mathbf{a}}_y = 0.5g \downarrow$$

Accelerations of A and B



Translation + Rotation about G

(*b*) Acceleration of A:

$$\mathbf{a}_A = \mathbf{a}_G + \mathbf{a}_{A/G} = [0.866g \longleftarrow] + [0.5g \downarrow] + \left(\frac{g}{L}\right)\left(\frac{L}{2}\right)\uparrow$$

$$= [0.866g \longleftarrow] + 0$$

$\mathbf{a}_A = 0.866g \longleftarrow$ ◀

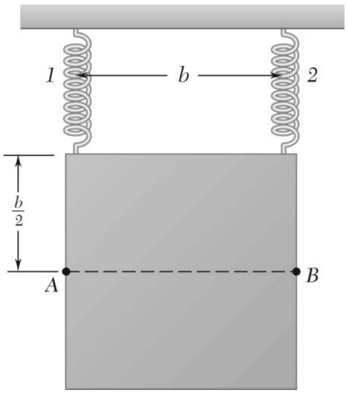
(*c*) Acceleration of B:

$$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G} = [0.866g \longleftarrow] + [0.5g \downarrow] + \left(\frac{g}{L}\right)\left(\frac{L}{2}\right)\downarrow$$

$$= [0.366g \longleftarrow] + [g \downarrow]$$

$\mathbf{a}_B = 1.323g \measuredangle 49.1°$ ◀

PROBLEM 16.66



A thin plate of the shape indicated and of mass m is suspended from two springs as shown. If spring 2 breaks, determine the acceleration at that instant (*a*) of Point A, (*b*) of Point B.

A square plate of side b.

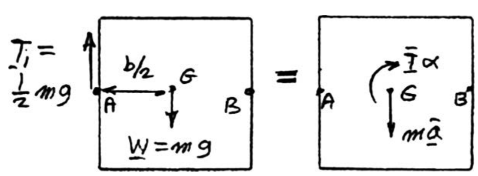
SOLUTION

$$\bar{I} = \frac{1}{12}m(b^2 + b^2)$$

$$\bar{I} = \frac{1}{6}mb^2$$

Statics: $\quad T_1 = T_2 = \frac{1}{2}W = \frac{1}{2}mg$

Kinetics: $\quad +\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad T\frac{b}{2} = \bar{I}\alpha$



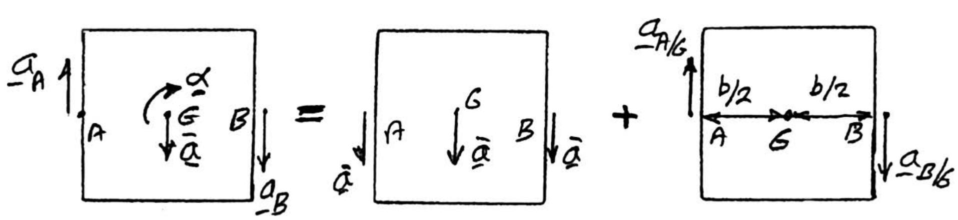
$$\frac{1}{2}mg\left(\frac{b}{2}\right) = \frac{1}{6}mb^2\alpha$$

$$\boldsymbol{\alpha} = \frac{3g}{2b} \circlearrowright$$

$$+\downarrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad W - T_1 = m\bar{a}$$

$$mg - \frac{1}{2}mg = m\bar{a} \qquad \bar{\mathbf{a}} = \frac{1}{2}g\downarrow$$

Kinematics:



Plane motion = Translation + Rotation

PROBLEM 16.66 (Continued)

(*a*) $$\mathbf{a}_A = \mathbf{a}_G + \mathbf{a}_{A/G} = \bar{\mathbf{a}}\downarrow + \frac{b}{2}\alpha\uparrow$$

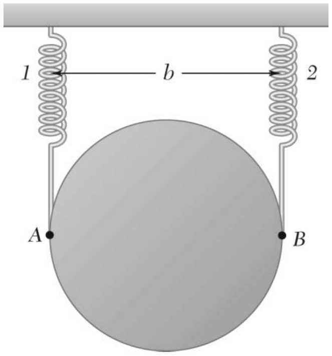
$$\mathbf{a}_A = \frac{g}{2}\downarrow + \frac{b}{2}\left(\frac{3g}{2b}\right)\uparrow = \frac{g}{4}\uparrow$$

$$\mathbf{a}_A = \frac{1}{4}g\uparrow \blacktriangleleft$$

(*b*) $$\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G} = \bar{\mathbf{a}}\downarrow + \frac{b}{2}\alpha\downarrow$$

$$\mathbf{a}_B = \frac{1}{2}g\downarrow + \frac{b}{2}\left(\frac{3g}{2b}\right)\downarrow = \frac{5}{4}g\downarrow$$

$$\mathbf{a}_B = \frac{5}{4}g\downarrow \blacktriangleleft$$



PROBLEM 16.67

A thin plate of the shape indicated and of mass m is suspended from two springs as shown. If spring 2 breaks, determine the acceleration at that instant (a) of Point A, (b) of Point B.

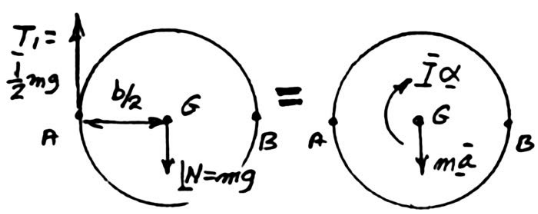
A circular plate of diameter b.

SOLUTION

$$\bar{I} = \frac{1}{2}m\left(\frac{b}{2}\right)^2 = \frac{1}{8}mb^2$$

Statics:
$$T_1 = T_2 = \frac{1}{2}W = \frac{1}{2}mg$$

Kinetics: $+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad T_1\frac{b}{2} = \bar{I}\alpha$



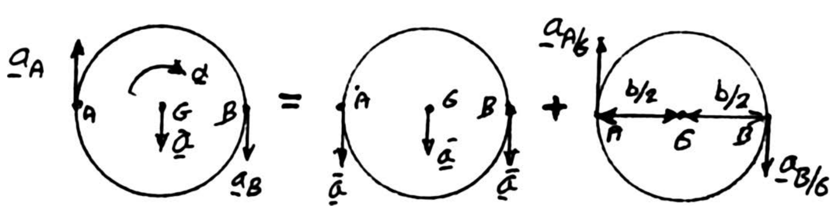
$$\frac{1}{2}mg\left(\frac{b}{2}\right) = \frac{1}{8}mb^2\alpha$$

$$\boldsymbol{\alpha} = 2\frac{g}{b} \circlearrowright$$

$+\downarrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad W - T_1 = m\bar{a}$

$$mg - \frac{1}{2}mg = m\bar{a} \qquad \bar{\mathbf{a}} = \frac{1}{2}g\downarrow$$

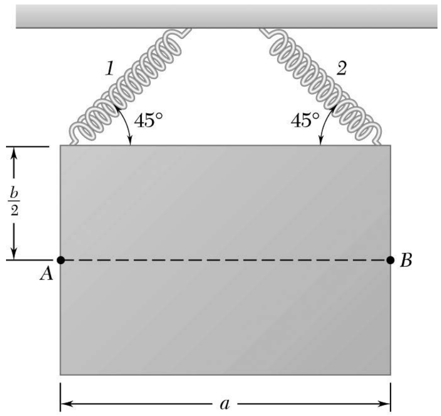
Kinematics:



Plane motion = Translation + Rotation

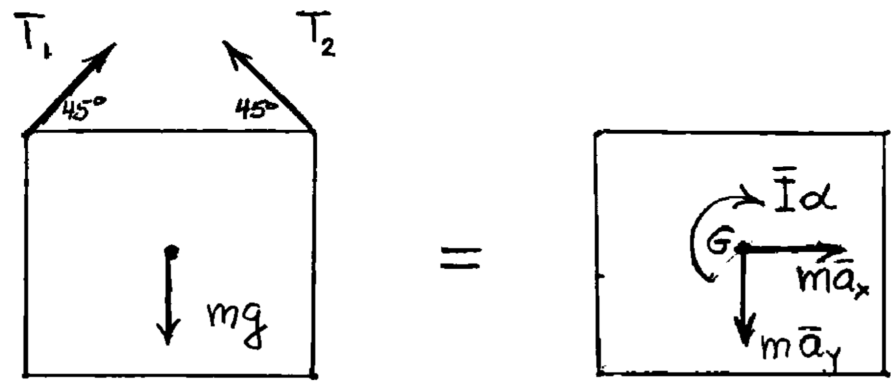
(a) $\mathbf{a}_A = \mathbf{a}_G + \mathbf{a}_{A/G} = \bar{\mathbf{a}}\downarrow + \frac{b}{2}\alpha\uparrow = \frac{1}{2}g\downarrow + \frac{b}{2}\left(2\frac{g}{b}\right)\uparrow$ $\qquad \mathbf{a}_A = \frac{1}{2}g\uparrow$ ◀

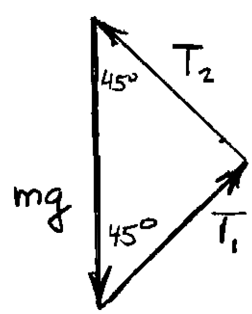
(b) $\mathbf{a}_B = \mathbf{a}_G + \mathbf{a}_{B/G} = \bar{\mathbf{a}}\downarrow + \frac{b}{2}\alpha\downarrow = \frac{1}{2}g\downarrow + \frac{b}{2}\left(2\frac{g}{b}\right)\downarrow$ $\qquad \mathbf{a}_B = \frac{3}{2}g\downarrow$ ◀

	**PROBLEM 16.68** A thin plate of the shape indicated and of mass m is suspended from two springs as shown. If spring 2 breaks, determine the acceleration at that instant (a) of Point A, (b) of Point B. A rectangular plate of height b and width a.

SOLUTION

Moment of inertia. $\qquad I = \frac{1}{12}m(a^2 + b^2)$





Statics: $\qquad a_x = 0, \quad a_y = 0, \quad \alpha = 0$

Draw the force triangle showing equilibrium.

$$T_1 = T_2 = mg\sin 45°$$

Kinetics: $\qquad T_2 = 0$

Since there is no time for displacements to occur, the tension in spring 1 remains equal to

$$T_1 = mg\sin 45°$$

Then $\qquad \mathbf{T}_1 = \left[\frac{1}{2}mg \longrightarrow\right] + \left[\frac{1}{2}mg \uparrow\right]$

PROBLEM 16.68 (Continued)

$$+\Sigma \mathbf{F} = m\bar{\mathbf{a}}: \quad [mg \downarrow] + \left[\frac{1}{2}mg \longrightarrow\right] + \left[\frac{1}{2}mg \uparrow = m\bar{\mathbf{a}}\right]$$

$$\bar{\mathbf{a}} = \left[\frac{1}{2}g \longrightarrow\right] + \left[\frac{1}{2}g \downarrow\right]$$

$$\circlearrowleft + \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad \frac{1}{2}mg\left(\frac{a}{2}\right) + \frac{1}{2}mg\left(\frac{b}{2}\right) = \bar{I}\alpha$$

$$\frac{1}{4}mg(a+b) = \frac{1}{12}m(a^2+b^2)\alpha \qquad \boldsymbol{\alpha} = \frac{3g(a+b)}{a^2+b^2} \circlearrowright$$

In vector notation,

$$\bar{\mathbf{a}} = \frac{1}{2}g(\mathbf{i} - \mathbf{j})$$

$$\boldsymbol{\alpha} = -\frac{3g(a+b)}{a^2+b^2}\mathbf{k}$$

Kinematics.

$$\mathbf{a}_P = \bar{\mathbf{a}} + \boldsymbol{\alpha} \times \mathbf{r}_{P/G} - \omega^2 \mathbf{r}_{P/G}$$

Since there is no time to acquire angular velocity, $\omega^2 = 0$

(*a*) *Acceleration at A.*

$$\mathbf{r}_{A/G} = -\frac{1}{2}a\mathbf{i}$$

$$\mathbf{a}_A = \frac{1}{2}g(\mathbf{i} - \mathbf{j}) + \left[-\frac{3g(a+b)}{a^2+b^2}\mathbf{k}\right] \times \left(-\frac{1}{2}a\mathbf{i}\right)$$

$$\mathbf{a}_A = \frac{1}{2}g(\mathbf{i} - \mathbf{j}) + \frac{3g(a+b)a}{a^2+b^2}\mathbf{j} \blacktriangleleft$$

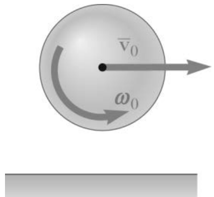
(*b*) *Acceleration at B.*

$$\mathbf{r}_{B/G} = \frac{1}{2}a\mathbf{i}$$

$$\mathbf{a}_B = \frac{1}{2}g(\mathbf{i} - \mathbf{j}) + \left[-\frac{3g(a+b)}{a^2+b^2}\mathbf{k}\right] \times \left(\frac{1}{2}a\mathbf{i}\right)$$

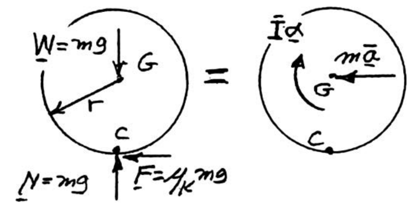
$$\mathbf{a}_B = \frac{1}{2}g(\mathbf{i} - \mathbf{j}) - \frac{3g(a+b)a}{a^2+b^2}\mathbf{j} \blacktriangleleft$$

PROBLEM 16.69



A sphere of radius r and mass m is projected along a rough horizontal surface with the initial velocities indicated. If the final velocity of the sphere is to be zero, express, in terms of v_0, r, and μ_k, (*a*) the required magnitude of $\boldsymbol{\omega}_0$, (*b*) the time t_1 required for the sphere to come to rest, (*c*) the distance the sphere will move before coming to rest.

SOLUTION



Kinetics: $\bar{I} = m\bar{k}^2$

$\overset{+}{\leftarrow} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F = m\bar{a}$

$$\mu_k mg = m\bar{a} \qquad \bar{\mathbf{a}} = \mu_k g \longleftarrow$$

$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad Fr = \bar{I}\alpha$

$$(\mu_k mg)r = m\bar{k}^2\alpha$$

$$\boldsymbol{\alpha} = \frac{\mu_k gr}{\bar{k}^2} \circlearrowright$$

Kinematics: $\overset{+}{\rightarrow} v = v_0 - \bar{a}t$

$$v = v_0 - \mu_k gt$$

For $v = 0$ when $t = t_1$ $\qquad 0 = v_0 - \mu_k gt_1; \qquad t_1 = \dfrac{v_0}{\mu_k g}$ (1)

$$+\circlearrowleft \omega = \omega_0 - \alpha t$$

$$\omega = \omega_0 - \frac{\mu_k gr}{\bar{k}^2}t$$

For $\omega = 0$ when $t = t_1$ $\qquad 0 = \omega_0 - \dfrac{\mu_k gr}{\bar{k}^2}t_1; \qquad t_1 = \dfrac{\bar{k}^2}{\mu_k gr}\omega_0$ (2)

Set Eq. (1) = Eq. (2) $\qquad \dfrac{v_0}{\mu_k g} = \dfrac{\bar{k}^2}{\mu_k gr}\omega_0; \qquad \omega_0 = \dfrac{r}{\bar{k}^2}v_0$ (3)

Distance traveled: $\qquad s_1 = v_0 t_1 - \dfrac{1}{2}\bar{a}t_1^2$

$$s_1 = v_0\left(\frac{v_0}{\mu_k g}\right) - \frac{1}{2}(\mu_k g)\left(\frac{v_0}{\mu_k g}\right)^2; \qquad s_1 = \frac{v_0^2}{2\mu_k g} \qquad (4)$$

PROBLEM 16.69 (Continued)

For a solid sphere $$\bar{k}^2 = \frac{2}{5}r^2$$

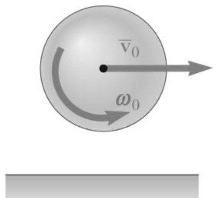
(*a*) Eq. (3): $$\omega_0 = \frac{r}{\frac{2}{5}r^2}v_0 = \frac{5}{2}\frac{v_0}{r}$$

$$\boldsymbol{\omega}_0 = \frac{5}{2}\frac{v_0}{r} \circlearrowleft \blacktriangleleft$$

(*b*) Eq. (1) $$t_1 = \frac{v_0}{\mu_k g} \blacktriangleleft$$

(*c*) Eq. (4) $$s_1 = \frac{v_0^2}{2\mu_k g} \blacktriangleleft$$

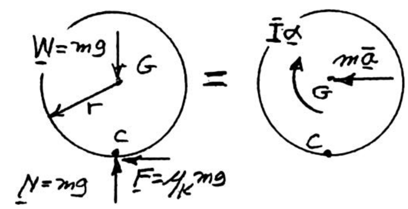
PROBLEM 16.70



Solve Problem 16.69, assuming that the sphere is replaced by a uniform thin hoop of radius r and mass m.

PROBLEM 16.69 A sphere of radius r and mass m is projected along a rough horizontal surface with the initial velocities indicated. If the final velocity of the sphere is to be zero, express, in terms of v_0, r, and μ_k, (*a*) the required magnitude of $\boldsymbol{\omega}_0$, (*b*) the time t_1 required for the sphere to come to rest, (*c*) the distance the sphere will move before coming to rest.

SOLUTION



Kinetics: $I = m\bar{k}^2$

$\overset{+}{\leftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = m\bar{a}$

$\mu_k mg = m\bar{a} \quad \bar{\mathbf{a}} = \mu_k g \longleftarrow$

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad Fr = \bar{I}\alpha$

$(\mu_k mg)r = m\bar{k}^2\alpha$

$$\boldsymbol{\alpha} = \frac{\mu_k g r}{\bar{k}^2} \circlearrowright$$

Kinematics: $\overset{+}{\rightarrow} v = v_0 - \bar{a}t$

$v = v_0 - \mu_k g t$

For $v = 0$ when $t = t_1$ $\quad 0 = v_0 - \mu_k g t_1; \qquad t_1 = \dfrac{v_0}{\mu_k g}$ (1)

$+\circlearrowright \omega = \omega_0 - \alpha t$

$$\omega = \omega_0 - \frac{\mu_k g r}{\bar{k}^2} t$$

For $\omega = 0$ when $t = t_1$ $\quad 0 = \omega_0 - \dfrac{\mu_k g r}{\bar{k}^2} t_1; \qquad t_1 = \dfrac{\bar{k}^2}{\mu_k g r}\omega_0$ (2)

Set Eq. (1) = Eq. (2) $\quad \dfrac{v_0}{\mu_k g} = \dfrac{\bar{k}^2}{\mu_k g r}\omega_0; \qquad \omega_0 = \dfrac{r}{\bar{k}^2} v_0$ (3)

Distance traveled: $\quad s_1 = v_0 t_1 - \dfrac{1}{2}\bar{a}t_1^2$

$s_1 = v_0\left(\dfrac{v_0}{\mu_k g}\right) - \dfrac{1}{2}(\mu_k g)\left(\dfrac{v_0}{\mu_k g}\right)^2; \qquad s_1 = \dfrac{v_0^2}{2\mu_k g}$ (4)

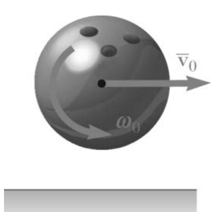
PROBLEM 16.70 (Continued)

For a hoop, $\bar{k} = r$

(*a*) Eq. (3): $\omega_0 = \frac{r}{r^2} r_0 = \frac{v_0}{r}$ $\boldsymbol{\omega}_0 = \frac{v_0}{r}$ ↺ ◀

(*b*) Eq. (1): $t_1 = \frac{v_0}{\mu_k g}$ ◀

(*c*) Eq. (4): $s_1 = \frac{v_0^2}{2\mu_k g}$ ◀

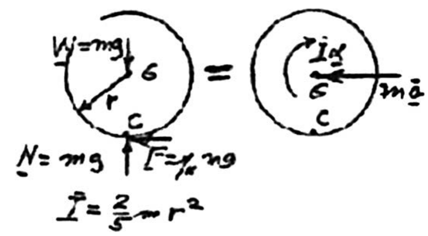


PROBLEM 16.71

A bowler projects an 8-in.-diameter ball weighing 12 lb along an alley with a forward velocity $\mathbf{v}_0$ of 15 ft/s and a backspin $\boldsymbol{\omega}_0$ of 9 rad/s. Knowing that the coefficient of kinetic friction between the ball and the alley is 0.10, determine (*a*) the time t_1 at which the ball will start rolling without sliding, (*b*) the speed of the ball at time t_1, (*c*) the distance the ball will have traveled at time t_1.

SOLUTION

<u>Kinetics</u>:



$$\overset{+}{\leftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad \mu_k mg = m\bar{a}$$

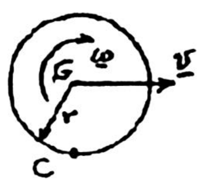
$$\bar{\mathbf{a}} = \mu_k g \longleftarrow$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \qquad Fr = \bar{I}\alpha$$

$$(\mu_k mg)r = \frac{2}{5} mr^2 \alpha$$

$$\boldsymbol{\alpha} = \frac{5}{2}\frac{\mu_k g}{r} \circlearrowright$$

<u>Kinematics</u>: When the ball rolls, the instant center of rotation is at C, and when



$$t = t_1 \quad v = r\omega \tag{1}$$

$$\bar{v} = \bar{v}_0 - \bar{a}t = \bar{v}_0 - \mu_k gt \tag{2}$$

$$\omega = -\omega_0 + \alpha t = -\omega_0 + \frac{5}{2}\frac{\mu_k g}{r} t$$

When $t = t_1$:

Eq. (1): $v = r\omega$:

$$\bar{v}_0 - \mu_k g t_1 = \left(-\omega_0 + \frac{5}{2}\frac{\mu_k g}{r} t_1 \right) r$$

$$\bar{v}_0 - \mu_k g t_1 = -\omega_0 r + \frac{5}{2}\mu_k g t_1$$

$$t_1 = \frac{2}{g}\frac{(\bar{v}_0 + r\omega_0)}{\mu_k g} \tag{3}$$

$$\bar{v}_0 = 15 \text{ ft/s}, \quad \omega_0 = 9 \text{ rad/s}, \quad r = 4 \text{ in.} = \frac{1}{3} \text{ft}$$

(*a*)

$$t_1 = \frac{2}{7}\frac{\left(15 + \frac{1}{3}(9)\right)}{0.1(32.2)} = 1.5972 \text{ s}$$

$t_1 = 1.597$ s ◀

PROBLEM 16.71 (Continued)

(*b*) Eq. (2):

$$\overline{v}_1 = v_0 - \mu_k g t_1$$
$$= 15 - 0.1(32.2)(1.5972)$$
$$\overline{v}_1 = 15 - 5.1429$$
$$= 9.857 \text{ ft/s}$$

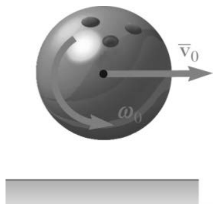
$\overline{v}_1 = 9.86 \text{ ft/s}$ ◀

(*c*)

$$\overline{a} = \mu_k g = 0.1(32.2 \text{ ft/s}^2) = 3.22 \text{ ft/s}^2 \longleftarrow$$

$$\xrightarrow{+} s_1 = \overline{v}_0 t_1 - \frac{1}{2}\overline{a} t_1^2$$
$$= (15 \text{ ft/s})(1.597 \text{ s}) - \frac{1}{2}(3.22 \text{ ft/s}^2)(1.597 \text{ s})^2$$
$$= 23.96 - 4.11 = 19.85 \text{ ft}$$

$s_1 = 19.85 \text{ ft}$ ◀



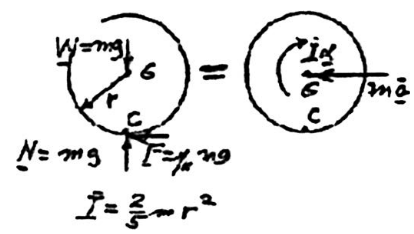
PROBLEM 16.72

Solve Problem 16.71, assuming that the bowler projects the ball with the same forward velocity but with a backspin of 18 rad/s.

PROBLEM 16.71 A bowler projects an 8-in.-diameter ball weighing 12 lb along an alley with a forward velocity $\mathbf{v}_0$ of 15 ft/s and a backspin $\boldsymbol{\omega}_0$ of 9 rad/s. Knowing that the coefficient of kinetic friction between the ball and the alley is 0.10, determine (*a*) the time t_1 at which the ball will start rolling without sliding, (*b*) the speed of the ball at time t_1, (*c*) the distance the ball will have traveled at time t_1.

SOLUTION

<u>Kinetics</u>:



$$\overset{+}{\leftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad \mu_k mg = m\bar{a}$$

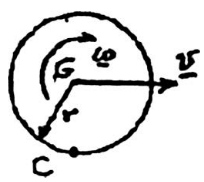
$$\bar{\mathbf{a}} = \mu_k g \longleftarrow$$

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad Fr = \bar{I}\alpha$$

$$(\mu_k mg)r = \frac{2}{5} mr^2 \alpha$$

$$\alpha = \frac{5}{2}\frac{\mu_k g}{r} \circlearrowleft$$

<u>Kinematics</u>: When the ball rolls, the instant center of rotation is at C, and when



$$t = t_1 \quad v = r\omega \qquad (1)$$

$$\bar{v} = \bar{v}_0 - \bar{a}t = \bar{v}_0 - \mu_k g^{t} \qquad (2)$$

$$\omega = -\omega_0 + \alpha t = -\omega_0 + \frac{5}{2}\frac{\mu_k g}{r} t$$

When $t = t_1$:

Eq. (1) $\quad v = r\omega$:

$$\bar{v}_0 - \mu_k g t_1 = \left(-\omega_0 + \frac{5}{2}\frac{\mu_k g}{r} t_1 \right) r$$

$$\bar{v}_0 - \mu_k g t_1 = -\omega_0 r + \frac{5}{2}\mu_k g t_1$$

$$t_1 = \frac{2}{7}\frac{(\bar{v}_0 + r\omega_0)}{\mu_k g} \qquad (3)$$

$$\bar{v}_0 = 15 \text{ ft/s}, \quad \omega_0 = 18 \text{ rad/s}, \quad r = \frac{1}{3} \text{ ft}$$

PROBLEM 16.72 (Continued)

(*a*) Eq. (3): $t_1 = \dfrac{2}{7}\dfrac{\left(15+\frac{1}{3}(18)\right)}{0.1(32.2)} = 1.8634 \text{ s}$ $t_1 = 1.863 \text{ s}$ ◀

(*b*) Eq. (2): $v_1 = v_0 - \mu_k g^t$

$= 15 - 0.1(32.2)(1.8634)$

$v_1 = 15 - 6.000$

$= 9 \text{ ft/s}$ $\overline{v}_1 = 9 \text{ ft/s}$ ◀

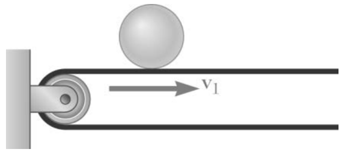
(*c*) $\overline{a} = \mu_k g = 0.1(32.2 \text{ ft/s}^2) = 3.22 \text{ ft/s}^2 \leftarrow$

$\xrightarrow{+} s_1 = \overline{v}_0 t_1 - \dfrac{1}{2}\overline{a} t_1^2$

$= (15 \text{ ft/s})(1.8634 \text{ s}) - \dfrac{1}{2}(3.22 \text{ ft/s}^2)(1.8634 \text{ s})^2$

$= 27.95 - 5.59$

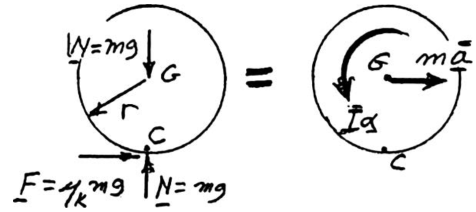
$= 22.36 \text{ ft}$ $s_1 = 22.4 \text{ ft}$ ◀



PROBLEM 16.73

A uniform sphere of radius r and mass m is placed with no initial velocity on a belt that moves to the right with a constant velocity $\mathbf{v}_1$. Denoting by μ_k the coefficient of kinetic friction between the sphere and the belt, determine (a) the time t_1 at which the sphere will start rolling without sliding, (b) the linear and angular velocities of the sphere at time t_1.

SOLUTION



Kinetics: $\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = m\bar{a}$

$$\mu_k mg = m\bar{a}$$

$$\bar{\mathbf{a}} = \mu_k g \longrightarrow$$

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad Fr = \bar{I}\alpha$$

$$(\mu_k mg)r = \frac{2}{5}mr^2\alpha$$

$$\boldsymbol{\alpha} = \frac{5}{2}\frac{\mu_k g}{r} \circlearrowleft$$

Kinematics: $\xrightarrow{+} \bar{v} = \bar{a}t = \mu_k gt \qquad (1)$

$$+\circlearrowleft \omega = \alpha t = \frac{5}{2}\frac{\mu_k g}{r}t \qquad (2)$$

Point C is the point of contact with belt.

$$\xrightarrow{+} v_C = \bar{v} + \omega r = \mu_k gt + \left(\frac{5}{2}\frac{\mu_k g}{r}t\right)r$$

$$v_C = \frac{7}{2}\mu_k gt$$

(a) When sphere starts rolling $(t = t_1)$, we have

$$v_C = v_1$$

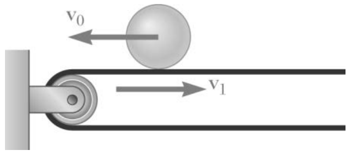
$$v_1 = \frac{7}{2}\mu_k gt_1 \qquad t_1 = \frac{2}{7}\frac{v_1}{\mu_k g} \blacktriangleleft$$

(b) Velocities when $t = t_1$

Eq. (1): $\bar{v} = \mu g\left(\frac{2}{7}\frac{v_1}{\mu_k g}\right) \qquad \bar{\mathbf{v}} = \frac{2}{7}v_1 \longrightarrow \blacktriangleleft$

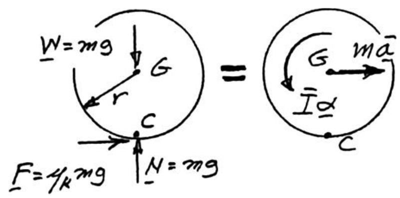
Eq. (2): $\omega = \left(\frac{5}{2}\frac{\mu_k g}{r}\right)\left(\frac{2}{7}\frac{v_1}{\mu_k g}\right) \qquad \boldsymbol{\omega} = \frac{5}{7}\frac{v_1}{r} \circlearrowleft \blacktriangleleft$

PROBLEM 16.74



A sphere of radius r and m has a linear velocity $\mathbf{v}_0$ directed to the left and no angular velocity as it is placed on a belt moving to the right with a constant velocity $\mathbf{v}_1$. If after first sliding on the belt the sphere is to have no linear velocity relative to the ground as it starts rolling on the belt without sliding, determine in terms of v_1 and the coefficient of kinetic friction μ_k between the sphere and the belt (*a*) the required value of v_0, (*b*) time t_1 at which the sphere will start rolling on the belt, (*c*) the distance the sphere will have moved relative to the ground at time t_1.

SOLUTION



Kinetics: $\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = m\bar{a}$

$$\mu_k mg = m\bar{a}$$

$$\bar{\mathbf{a}} = \mu_k g \longrightarrow$$

$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad F_r = \bar{I}\alpha$$

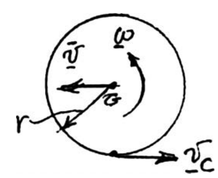
$$(\mu_k mg)r = \frac{5}{2} mr^2 \alpha$$

$$\boldsymbol{\alpha} = \frac{5}{2} \frac{\mu_k g}{r} \circlearrowleft$$

Kinematics: $\xleftarrow{+} \bar{v} = v_0 - \bar{a}t = v_0 - \mu_k g t \qquad (1)$

$$+\circlearrowleft \omega = \alpha t = \frac{5}{2} \frac{\mu_k g}{r} t \qquad (2)$$

Point C is the point of contact with belt.



$$\xrightarrow{+} v_C = -\bar{v} + r\omega$$

$$v_C = -\bar{v} + r \frac{5}{2} \frac{\mu_k g}{r} t$$

$$v_C = -\bar{v} + \frac{5\mu_k g}{2} t \qquad (3)$$

But, when $t = t_1$, $\bar{v} = 0$ and $v_c = v_1$

Eq. (3): $v_1 = \dfrac{5\mu_k g}{2} t_1$ $\qquad t_1 = \dfrac{2v_1}{5\mu_k g}$ ◀

Eq. (1): $\bar{v} = v_0 - \mu_k g t$

When $t = t_1$, $\bar{v} = 0$, $\quad 0 = v_0 - \mu_k g \left(\dfrac{2v_1}{5\mu_k g} \right)$ $\qquad v_0 = \dfrac{2}{5} v_1$ ◀

PROBLEM 16.74 (Continued)

Distance when $t = t_1$:

$$\xleftarrow{+}\ s = v_0 t_1 - \frac{1}{2} a t_1^2$$

$$s = \left(\frac{2}{5} v_1\right)\left(\frac{2v_1}{5\mu_k g}\right) - \frac{1}{2}(\mu_k g)\left(\frac{2v_1}{5\mu_k g}\right)^2$$

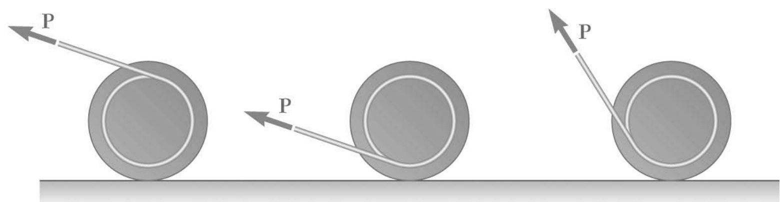
$$s = \frac{v_1^2}{\mu_k g}\left(\frac{4}{25} - \frac{2}{25}\right);$$

$$\mathbf{s} = \frac{2}{25}\frac{v_1^2}{\mu_k g} \longleftarrow \blacktriangleleft$$

PROBLEM 16.CQ4

A cord is attached to a spool when a force **P** is applied to the cord as shown. Assuming the spool rolls without slipping, what direction does the spool move for each case?

Case 1:	(*a*) left	(*b*) right	(*c*) It would not move.
Case 2:	(*a*) left	(*b*) right	(*c*) It would not move.
Case 3:	(*a*) left	(*b*) right	(*c*) It would not move.



SOLUTION

Answer:

Case 1: (*a*)

Case 2: (*a*)

Case 3: (*b*)

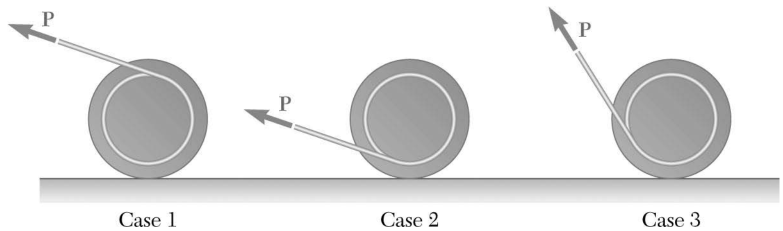
PROBLEM 16.CQ5

A cord is attached to a spool when a force **P** is applied to the cord as shown. Assuming the spool rolls without slipping, in what direction does the friction force act for each case?

Case 1: (*a*) left (*b*) right (*c*) The friction force would be zero.

Case 2: (*a*) left (*b*) right (*c*) The friction force would be zero.

Case 3: (*a*) left (*b*) right (*c*) The friction force would be zero.



SOLUTION

Answer:

Case 1: (*b*)

Case 2: (*b*)

Case 3: (*b*)

PROBLEM 16.CQ6

A front wheel drive car starts from rest and accelerates to the right. Knowing that the tires do not slip on the road, what is the direction of the friction force the road applies to the front tires?

(*a*) left

(*b*) right

(*c*) The friction force is zero.

SOLUTION

Answer: (*b*)

PROBLEM 16.CQ7

A front wheel drive car starts from rest and accelerates to the right. Knowing that the tires do not slip on the road, what is the direction of the friction force the road applies to the rear tires?

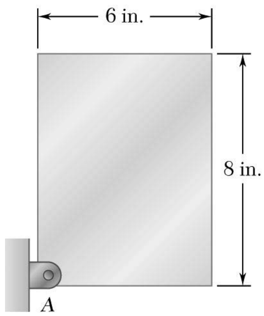
(*a*) left

(*b*) right

(*c*) The friction force is zero.

SOLUTION

Answer: (*a*)

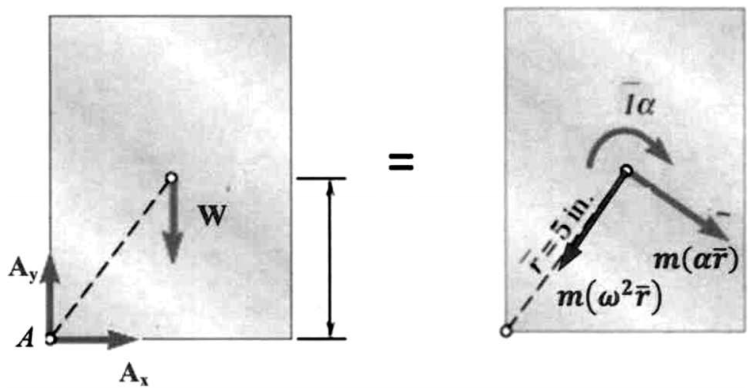


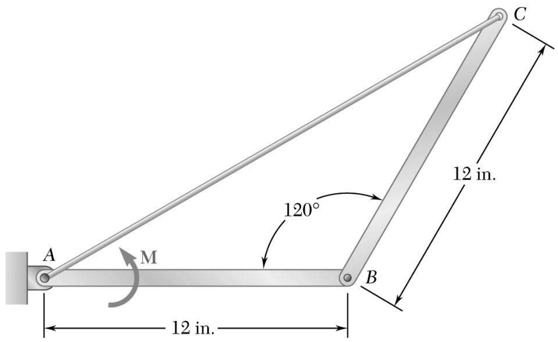
PROBLEM 16.F5

A uniform 6 × 8-in. rectangular plate of mass *m* is pinned at *A*. Knowing the angular velocity of the plate at the instant shown is ω, draw the FBD and KD.

SOLUTION

Answer:



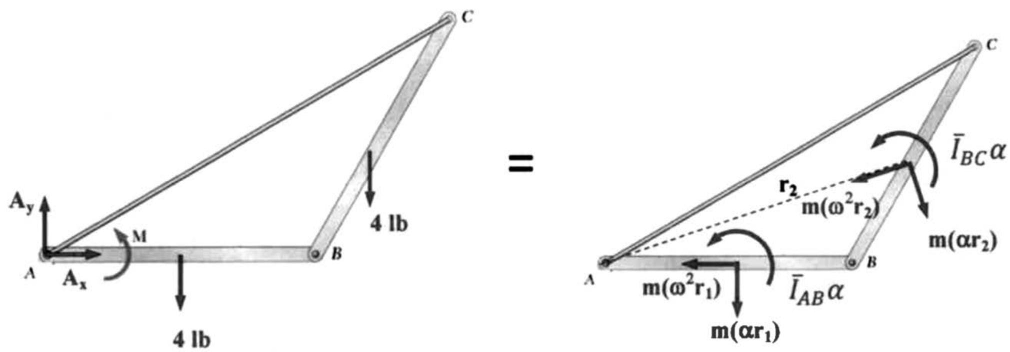


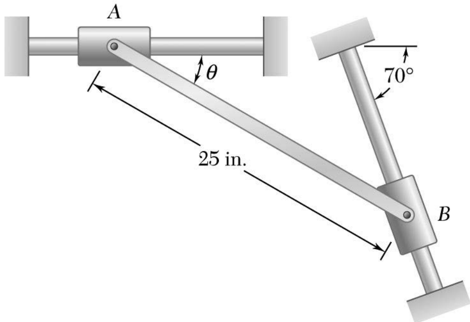
PROBLEM 16.F6

Two identical 4-lb slender rods *AB* and *BC* are connected by a pin at *B* and by the cord *AC*. The assembly rotates in a vertical plane under the combined effect of gravity and a couple **M** applied to rod *AB*. Knowing that in the position shown the angular velocity of the assembly is ω, draw the FBD and KD that can be used to determine the angular acceleration of the assembly and the tension in cord *AC*.

SOLUTION

Answer:



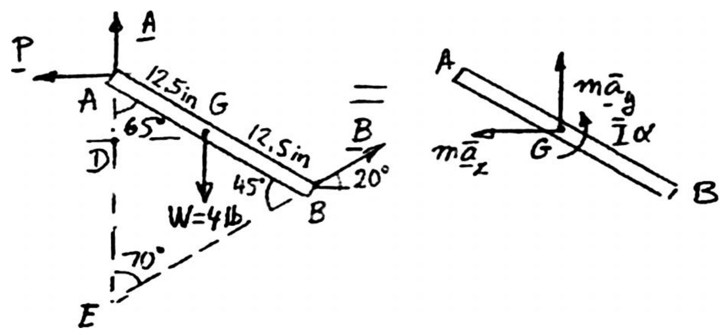


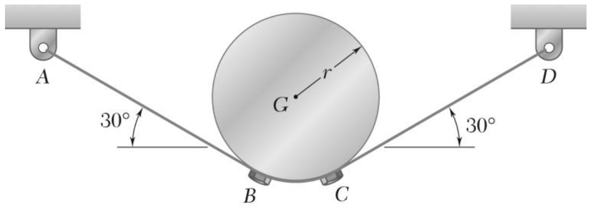
PROBLEM 16.F7

The 4-lb uniform rod *AB* is attached to collars of negligible mass which may slide without friction along the fixed rods shown. Rod *AB* is at rest in the position $\theta = 25°$ when an horizontal force **P** is applied to collar *A* causing it to start moving to the left. Draw the FBD and KD for the rod.

SOLUTION

Answer:



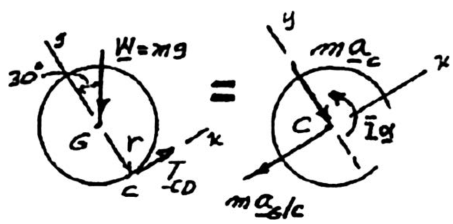


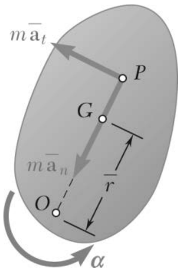
PROBLEM 16.F8

A uniform disk of mass $m = 4$ kg and radius $r = 150$ mm is supported by a belt *ABCD* that is bolted to the disk at *B* and *C*. If the belt suddenly breaks at a point located between *A* and *B*, draw the FBD and KD for the disk immediately after the break.

SOLUTION

Answer:

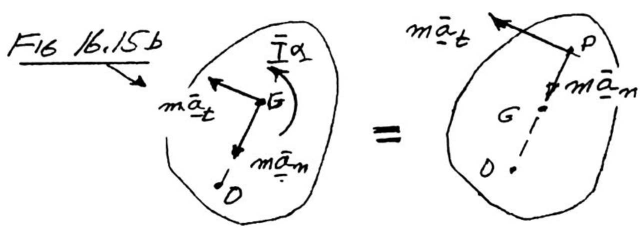




PROBLEM 16.75

Show that the couple $\bar{I}\boldsymbol{\alpha}$ of Figure 16.15 can be eliminated by attaching the vectors $m\bar{\mathbf{a}}_t$ and $m\bar{\mathbf{a}}_n$ at a Point *P* called the *center of percussion,* located on line *OG* at a distance $GP = \bar{k}^2/\bar{r}$ from the mass center of the body.

SOLUTION



$$OG = \bar{r} \qquad \bar{\mathbf{a}}_t = \bar{r}\alpha$$

We first observe that the sum of the vectors is the same in both figures. To have the same sum of moments about *G*, we must have

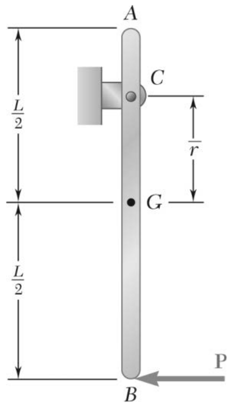
$$+\circlearrowleft \Sigma M_G = \Sigma M_G: \quad \bar{I}\alpha = (m\bar{a}_t)(GP)$$

$$m\bar{k}^2\alpha = m\bar{r}\alpha(GP) \qquad\qquad GP = \frac{\bar{k}^2}{\bar{r}} \quad \text{(Q.E.D.)} \blacktriangleleft$$

Note: The center of rotation and the center of percussion are interchangeable. Indeed, since $OG = \bar{r},$ we may write

$$GP = \frac{\bar{k}^2}{GO} \quad \text{or} \quad GO = \frac{\bar{k}^2}{GP}$$

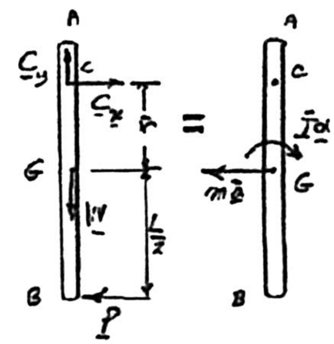
Thus, if Point *P* is selected as center of rotation, then Point *O* is the center of percussion.



PROBLEM 16.76

A uniform slender rod of length L = 900 mm and mass m = 4 kg is suspended from a hinge at C. A horizontal force **P** of magnitude 75 N is applied at end B. Knowing that $\bar{r} = 225$ mm, determine (*a*) the angular acceleration of the rod, (*b*) the components of the reaction at C.

SOLUTION



(*a*) Angular acceleration.

$$\bar{a} = \bar{r}\alpha \qquad \bar{I} = \frac{1}{12}mL^2$$

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad P\left(\bar{r} + \frac{L}{2}\right) = (m\bar{a})\bar{r} + \bar{I}\alpha$$

$$= (m\bar{r}\alpha)\bar{r} + \frac{1}{12}mL^2\alpha$$

$$P\left(\bar{r} + \frac{L}{2}\right) = m\left(\bar{r}^2 + \frac{1}{12}L^2\right)\alpha$$

Substitute data: $$(75\text{ N})\left[0.225\text{ m} + \frac{0.9\text{ m}}{2}\right] = (4\text{ kg})\left[(0.225\text{ m})^2 + \frac{1}{12}(0.9\text{ m})^2\right]\alpha$$

$$50.625 = 0.4725\alpha$$

$$\alpha = 107.14\text{ rad/s}^2$$

$\boldsymbol{\alpha} = 107.1\text{ rad/s}^2 \circlearrowright$ ◀

(*b*) Components of reaction at C.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad C_y - W = 0$$

$$C_y = W = mg = (4\text{ kg})(9.81\text{ m/s}^2)$$

$\mathbf{C}_y = 39.2\text{ N} \uparrow$ ◀

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad C_x - P = -m\bar{a}$$

$$C_x = P - m\bar{a} = P - m(\bar{r}\alpha)$$

$$= 75\text{ N} - (4\text{ kg})(0.225\text{ m})(107.14\text{ rad/s}^2)$$

$$C_x = 75\text{ N} - 96.4\text{ N}$$

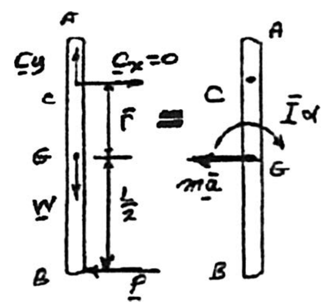
$$= -21.4\text{ N}$$

$\mathbf{C}_x = 21.4\text{ N} \leftarrow$ ◀

PROBLEM 16.77

In Problem 16.76, determine (*a*) the distance $\bar{r}$ for which the horizontal component of the reaction at *C* is zero, (*b*) the corresponding angular acceleration of the rod.

SOLUTION



(*a*) Distance $\bar{r}$.

$$\bar{a} = \bar{r}\alpha$$

$$\bar{I} = \frac{1}{12}mL^2$$

$\overset{+}{\longleftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}$: $P = m\bar{a}$

$$P = m(\bar{r}\alpha)$$

$$\alpha = \frac{P}{m\bar{r}} \qquad (1)$$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}$: $P\frac{L}{2} = \bar{I}\alpha$

$$P\frac{L}{2} = \frac{1}{12}mL^2\alpha$$

$$P\frac{L}{2} = \frac{1}{12}mL^2\left(\frac{P}{m\bar{r}}\right)$$

$$\frac{L}{2} = \frac{L^2}{12\bar{r}}$$

$$\bar{r} = \frac{1}{6}L \quad \bar{r} = \frac{900 \text{ mm}}{6}$$

$\bar{r} = 150 \text{ mm}$ ◀

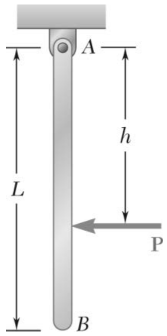
(*b*) Angular acceleration.

Eq. (1):

$$\alpha = \frac{P}{m\bar{r}} = \frac{P}{m\left(\frac{L}{6}\right)} = \frac{6P}{mL}$$

$$\alpha = \frac{6(75 \text{ N})}{(4 \text{ kg})(0.9 \text{ m})} = 125 \text{ rad/s}^2$$

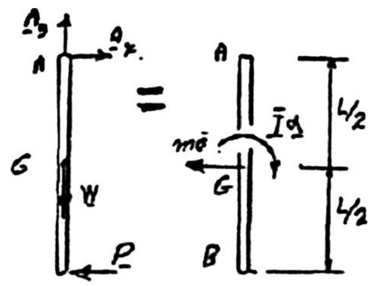
$\boldsymbol{\alpha} = 125 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 16.78

A uniform slender rod of length $L = 36$ in. and weight $W = 4$ lb hangs freely from a hinge at A. If a force **P** of magnitude 1.5 lb is applied at B horizontally to the left ($h = L$), determine (a) the angular acceleration of the rod, (b) the components of the reaction at A.

SOLUTION



$$\bar{a} = \frac{1}{2}\alpha \quad \bar{I} = \frac{1}{12}mL^2$$

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad PL = (m\bar{a})\frac{L}{2} + \bar{I}\alpha$$

$$= \left(m\frac{L}{2}\alpha\right)\frac{L}{2} + \frac{1}{12}mL^2\alpha$$

$$PL = \frac{1}{3}mL^2\alpha$$

(a) Angular acceleration.

$$\alpha = \frac{3P}{mL}$$

$$= \frac{3(1.5 \text{ lb})}{\left(\frac{4 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(3 \text{ ft})}$$

$$= 12.08 \text{ rad/s}^2$$

$$\boldsymbol{\alpha} = 12.08 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

(b) Components of the reaction at A.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad A_y - W = 0$$

$$A_y = W = 4 \text{ lb}$$

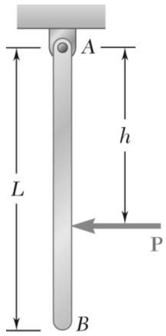
$$\mathbf{A}_y = 4.00 \text{ lb} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad A_x - P = -m\bar{a}$$

$$A_x = P - m\left(\frac{L}{2}\alpha\right) = P - m\frac{L}{2}\left(\frac{3P}{mL}\right) = -\frac{P}{2}$$

$$A_x = -\frac{P}{2} = -\frac{1.5 \text{ lb}}{2} = -0.75 \text{ lb}$$

$$\mathbf{A}_x = 0.750 \text{ lb} \leftarrow \blacktriangleleft$$

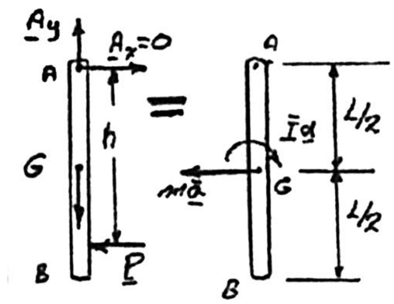


PROBLEM 16.79

In Problem 16.78, determine (*a*) the distance *h* for which the horizontal component of the reaction at *A* is zero, (*b*) the corresponding angular acceleration of the rod.

PROBLEM 16.78 A uniform slender rod of length $L = 36$ in. and weight $W = 4$ lb hangs freely from a hinge at *A*. If a force **P** of magnitude 1.5 lb is applied at *B* horizontally to the left ($h = L$), determine (*a*) the angular acceleration of the rod, (*b*) the components of the reaction at *A*.

SOLUTION



$$\bar{a} = \frac{L}{2}\alpha$$

$$\bar{I} = \frac{1}{12}mL^2$$

$\overset{+}{\leftarrow}\Sigma F_x = \Sigma(F_x)_{\text{eff}}:$ $\quad P = m\bar{a}$

$$P = m\left(\frac{L}{2}\alpha\right)$$

(*b*) Angular acceleration. $\quad \alpha = \dfrac{2P}{mL}$

$$\alpha = \frac{2(1.5\text{ lb})}{\left(\frac{4\text{ lb}}{32.2\text{ ft/s}^2}\right)(3\text{ ft})}$$

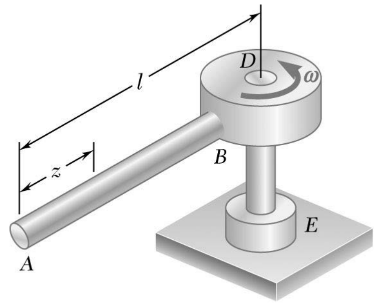
$\boldsymbol{\alpha} = 8.05\text{ rad/s}^2 \circlearrowright$ ◀

$+\circlearrowright\Sigma M_G = \Sigma(M_G)_{\text{eff}}:$ $\quad P\left(h - \dfrac{L}{2}\right) = \bar{I}\alpha: \quad P(h - L)$

$$P\left(h - \frac{L}{2}\right) = \frac{1}{12}mL^2\left(\frac{2P}{mL}\right) = \frac{PL}{6}$$

$$\left(h - \frac{L}{2}\right) = \frac{L}{6}; \quad h = \frac{L}{2} + \frac{L}{6} = \frac{2}{3}L$$

(*a*) Distance *h*. $\quad h = 24$ in. ◀



PROBLEM 16.80

The uniform slender rod AB is welded to the hub D, and the system rotates about the vertical axis DE with a constant angular velocity $\boldsymbol{\omega}$, (a) Denoting by ω the mass per unit length of the rod, express the tension in the rod at a distance z from end A in terms of w, l, z, and $\boldsymbol{\omega}$, (b) Determine the tension in the rod for w = 0.3 kg/m, l = 400 mm, z = 250 mm, and $\boldsymbol{\omega}$ = 150 rpm.

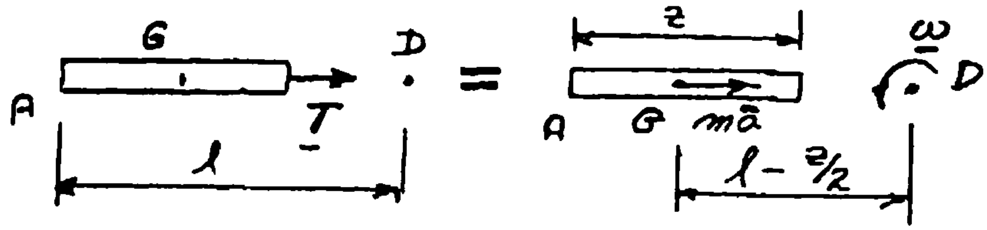
SOLUTION

Consider motion in the horizontal plane. Since ω is constant, the angular acceleration is zero. Only normal acceleration, i.e., along the rod occurs. For a section defined by the coordinate z, the acceleration of the mass center of the portion extending from z to the section is

$$\bar{a} = r\omega^2 = (l - z/2)\omega^2$$

Kinetics. The mass of the section is

$$m = wz$$



(a) $\xrightarrow{+} \Sigma F = \Sigma F_{\text{eff}}:\quad T = m\bar{a}$

$$= (wz)\left(l - \frac{z}{2}\right)\omega^2 \qquad\qquad T = w\left(lz - \frac{z^2}{2}\right)\omega^2 \ ◀$$

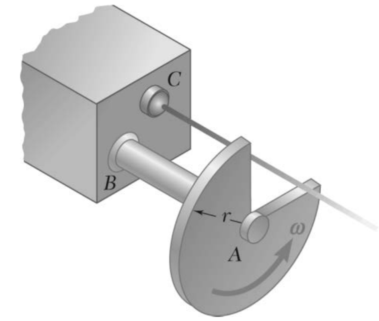
(b) Data: $\omega = 150 \text{ rpm} = \dfrac{(150)(2\pi)}{60} = 5\pi \text{ rad/s}$

$z = 0.250$ m, $l = 0.400$ m, $w = 0.3$ kg/m

$$T = (0.3 \text{ kg/m})\left[(0.4 \text{ m})(0.25 \text{ m}) - \frac{(0.25 \text{ m})^2}{2}\right](5\pi \text{ rad/s})^2 = 5.09 \text{ N}$$

$T = 5.09$ N ◀

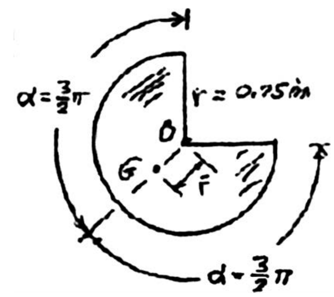
PROBLEM 16.81



The shutter shown was formed by removing one quarter of a disk of 0.75-in. radius and is used to interrupt a beam of light emanating from a lens at C. Knowing that the shutter weighs 0.125 lb and rotates at the constant rate of 24 cycles per second, determine the magnitude of the force exerted by the shutter on the shaft at A.

SOLUTION

See inside front cover for centroid of a circular sector.



$$\bar{r} = \frac{2r\sin\alpha}{3\alpha}$$

$$\bar{r} = \frac{2(0.75\text{ in.})\sin\left(\frac{3}{4}\pi\right)}{3\left(\frac{3}{4}\pi\right)}$$

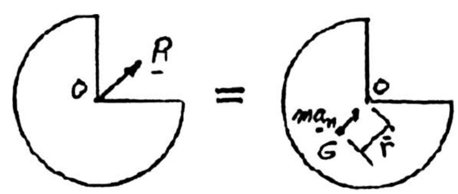
$$\bar{r} = 0.15005\text{ in.}$$

$$a_n = r\omega^2$$

$$\omega = 24\text{ rad/s}$$

$$= 24(2\pi)\text{ rad/s}$$

$$\omega = 150.8\text{ rad/s}$$



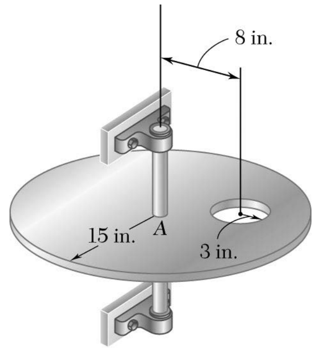
$$+\nearrow \Sigma F = \Sigma F_{\text{eff}}: \quad R = m a_n = m\bar{r}\omega^2$$

$$= \frac{(0.125\text{ lb})}{32.2\text{ ft/s}^2}\left(\frac{0.15005}{12}\text{ ft}\right)(150.8\text{ rad/s})^2$$

$$R = 1.1038\text{ lb} \nearrow$$

Force on shaft is $R = 1.104\text{ lb} \swarrow$

Magnitude: $R = 1.104\text{ lb}$ ◀



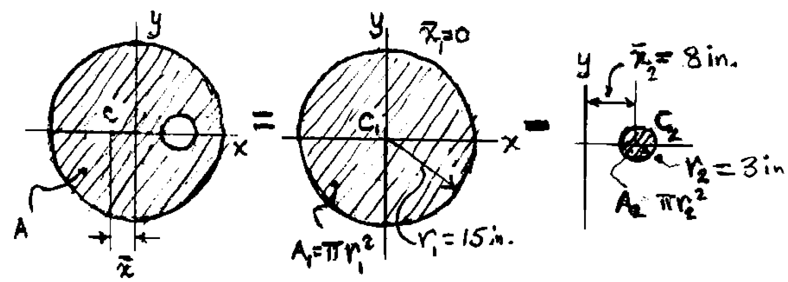
PROBLEM 16.82

A 6-in.-diameter hole is cut as shown in a thin disk of 15-in.-diameter. The disk rotates in a horizontal plane about its geometric center *A* at the constant rate of 480 rpm. Knowing that the disk has a mass of 60 lb after the hole has been cut, determine the horizontal component of the force exerted by the shaft on the disk at *A*.

SOLUTION

Determination of mass center of disk

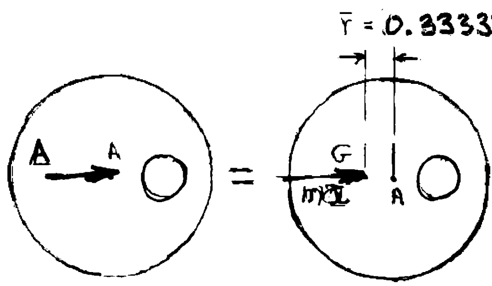
We determine the centroid of the composite area:



$$\bar{x}A = \bar{x}_1 A_1 - \bar{x}_2 A_2 \quad \text{or} \quad \bar{x}(A_1 - A_2) = \bar{x}_1 A_1 - \bar{x}_2 A_2$$

$$\bar{x} = \frac{\bar{x}A_1 - \bar{x}_2 A_2}{A_1 - A_2} = \frac{0 - (8)\pi(3)^2}{\pi(15)^2 - \pi(3)^2} = -\frac{(8)(3)^2}{(15)^2 - (3)^2} = -0.33333 \text{ in.}$$

Kinetics



Mass center *G* coincides with centroid *C*

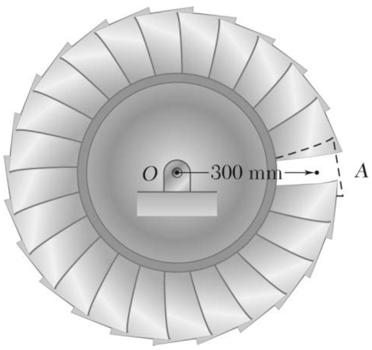
$\omega = 480 \text{ rpm} = 50.265 \text{ rad/s}$

$$\bar{a} = \bar{r}\omega^2 = \left(\frac{0.33333}{12}\right)(50.265 \text{ rad/s})^2 = 70.183 \text{ ft/s}^2$$

$$\Sigma F = \Sigma(F)_{\text{eff}}: \ \mathbf{A} = m\bar{\mathbf{a}} = \left(\frac{60 \text{ lb}}{32.2}\right)(70.183 \text{ ft/s}^2)$$

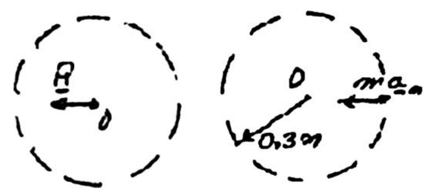
$\mathbf{A} = 130.8 \text{ N} \longrightarrow$ ◀

PROBLEM 16.83



A turbine disk of mass 26 kg rotates at a constant rate of 9600 rpm. Knowing that the mass center of the disk coincides with the center of rotation O, determine the reaction at O immediately after a single blade at A, of mass 45 g, becomes loose and is thrown off.

SOLUTION



$$\omega = 9600 \text{ rpm}\left(\frac{2\pi}{60}\right)$$

$$\omega = 320\pi \text{ rad/s}$$

Consider before it is thrown off.

$$\overset{+}{\longleftarrow} \Sigma F = \Sigma F_{\text{eff}}: \quad R = ma_n = mF\omega^2$$

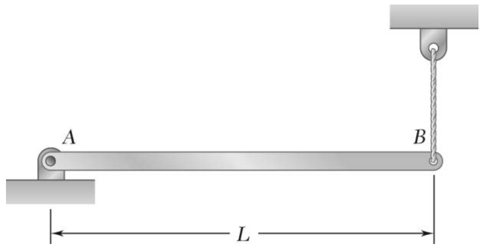
$$= (45\times 10^{-3}\text{ kg})(0.3 \text{ m})(320\,\pi)^2$$

$$R = 13.64 \text{ kN}$$

Before blade was thrown off, the disk was balanced $(R = 0)$. Removing vane at A also removes its reaction, so disk is unbalanced and reaction is

$$\mathbf{R} = 13.64 \text{ kN} \longrightarrow \blacktriangleleft$$

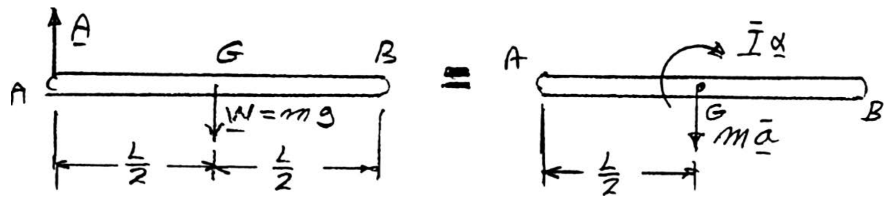
PROBLEM 16.84



A uniform rod of length L and mass m is supported as shown. If the cable attached at end B suddenly breaks, determine (a) the acceleration of end B, (b) the reaction at the pin support.

SOLUTION

$$w = 0 \quad \bar{a} = \frac{L}{2}\alpha$$



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad W\frac{L}{2} = \bar{I}\alpha + m\bar{a}\frac{L}{2}$$

$$mg\frac{L}{2} = \frac{1}{12}mL^2\alpha + m\left(\frac{L}{2}\alpha\right)\frac{L}{2}$$

$$mg\frac{L}{2} = \frac{1}{3}mL^2\alpha \quad \alpha = \frac{3}{2}\frac{g}{L} \circlearrowright$$

(b) Reaction at A.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad A - mg = -m\bar{a} = -m\frac{L}{2}\alpha$$

$$A - mg = -m\left(\frac{L}{2}\right)\left(\frac{3}{2}\frac{g}{L}\right)$$

$$A - mg = -\frac{3}{4}mg$$

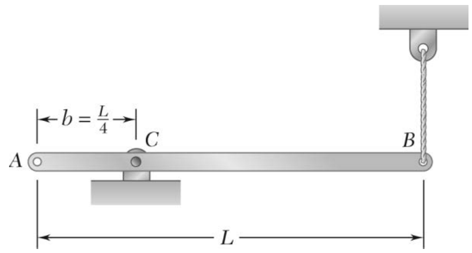
$$A = \frac{1}{4}mg \qquad \mathbf{A} = \frac{1}{4}mg\uparrow \blacktriangleleft$$

(a) Acceleration of B.

$$\mathbf{a}_B = \mathbf{a}_n + \mathbf{a}_{B/A} = 0 + L\alpha\downarrow$$

$$\mathbf{a}_B = L\left(\frac{3}{2}\frac{g}{L}\right) = \frac{3}{2}g\downarrow \qquad \mathbf{a}_B = \frac{3}{2}g\downarrow \blacktriangleleft$$

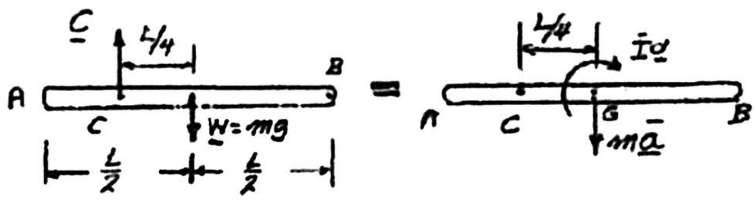
PROBLEM 16.85



A uniform rod of length L and mass m is supported as shown. If the cable attached at end B suddenly breaks, determine (a) the acceleration of end B, (b) the reaction at the pin support.

SOLUTION

$$\omega = 0 \qquad \bar{a} = \frac{L}{4}\alpha$$



$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad W\frac{L}{4} = \bar{I}\alpha + m\bar{a}\frac{L}{4}$$

$$mg\frac{L}{4} = \frac{1}{12}mL^2\alpha + m\left(\frac{L}{4}\alpha\right)\frac{L}{4}$$

$$mg\frac{L}{4} = \frac{7}{48}mL^2\alpha \qquad \boldsymbol{\alpha} = \frac{12g}{7L} \circlearrowright$$

(b) Reaction at C.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad C - mg = -m\bar{a} = -m\frac{L}{4}\alpha$$

$$C - mg = -m\left(\frac{L}{4}\right)\left(\frac{12g}{7L}\right)$$

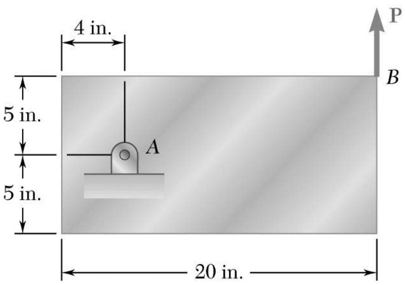
$$C - mg = -\frac{3}{7}mg$$

$$C = \frac{4}{7}mg \qquad \mathbf{C} = \frac{4}{7}mg \uparrow \blacktriangleleft$$

(a) Acceleration of B.

$$a_B = a_C + a_{C/B} = 0 + \frac{3L}{4}\alpha$$

$$a_B = \frac{3L}{4}\left(\frac{12g}{7L}\right) = \frac{9}{7}g \qquad \mathbf{a}_B = \frac{9}{7}g \downarrow \blacktriangleleft$$



PROBLEM 16.86

A 12-lb uniform plate rotates about A in a vertical plane under the combined effect of gravity and of the vertical force **P**. Knowing that at the instant shown the plate has an angular velocity of 20 rad/s and an angular acceleration of 30 rad/s^2 both counterclockwise, determine (a) the force **P**, (b) the components of the reaction at A.

SOLUTION

Kinematics.

$$\bar{a}_t = r\alpha = \left(\frac{6}{12}\text{ ft}\right)(30\text{ rad/s}^2) = 15\text{ ft/s}^2$$

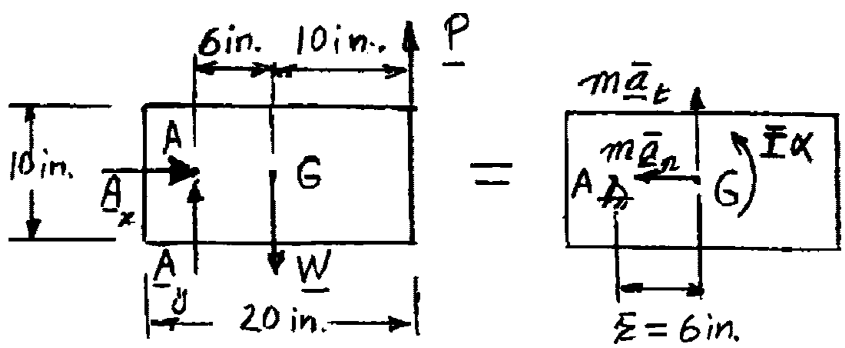
$$\bar{a}_n = r\omega^2 = \left(\frac{6}{12}\text{ ft}\right)(20\text{ rad/s})^2 = 200\text{ ft/s}^2$$

Mass and moment of inertia.

$$m = \frac{W}{g} = \frac{12\text{ lb}}{32.2\text{ ft/s}^2} = 0.37267\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$I = \frac{m}{12}\left[\left(\frac{10}{12}\right)^2 + \left(\frac{20}{12}\right)^2\right] = (0.37267\text{ lb}\cdot\text{s}^2/\text{ft})(0.28935\text{ ft}^2) = 0.10783\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Kinetics.



(a) Force **P**.

$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad P\left(\frac{16}{12}\text{ ft}\right) - W\left(\frac{6}{12}\text{ ft}\right) = m\bar{a}_t\left(\frac{6}{12}\text{ ft}\right) + \bar{I}\alpha$$

$$\frac{4}{3}P = (12)\left(\frac{1}{2}\right) + (0.37267)(15)\left(\frac{1}{2}\right) + (0.10783)(30)$$

$$P = 9.0224\text{ lb.}$$

P $= 9.02$ lb ↑ ◀

PROBLEM 16.86 (Continued)

(*b*) Reaction at *A*.

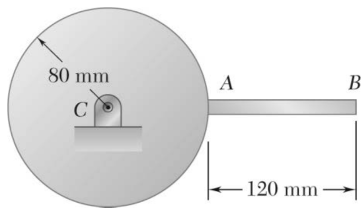
$$\xrightarrow{+} \Sigma F_y = \Sigma (F_x)_{\text{eff}}: \quad A_x = -m\overline{a}_n = -\frac{12}{32.2}(200)$$

$$A_x = -74.53 \text{ lb} \qquad \mathbf{A}_x = 74.5 \text{ lb} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad A_y + P - W = m\overline{a}_t$$

$$A_y = W + m\overline{a}_t - P = 12 + \frac{12}{32.2}(15) - 9.02 = 8.57 \text{ lb}$$

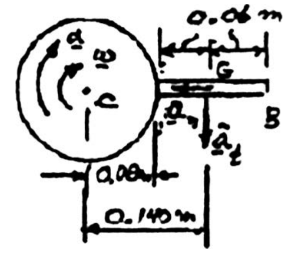
$$\mathbf{A}_y = 8.57 \text{ lb} \uparrow \blacktriangleleft$$



PROBLEM 16.87

A 1.5-kg slender rod is welded to a 5-kg uniform disk as shown. The assembly swings freely about C in a vertical plane. Knowing that in the position shown the assembly has an angular velocity of 10 rad/s clockwise, determine (*a*) the angular acceleration of the assembly, (*b*) the components of the reaction at C.

SOLUTION



<u>Kinematics</u>:

$$\bar{a}_n = (CG)\omega^2 = (0.14 \text{ m})(10 \text{ rad/s}^2)$$

$$\bar{\mathbf{a}}_n = 14 \text{ m/s}^2 \longleftarrow$$

$$\bar{\mathbf{a}}_t = (CG)\alpha = (0.14 \text{ m})\alpha \downarrow$$

<u>Kinetics</u>:

$$\bar{I}_{\text{disk}} = \frac{1}{2} m_{\text{disk}} (CG)^2$$

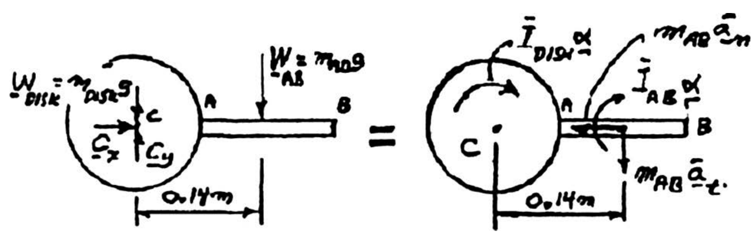
$$= \frac{1}{2}(5 \text{ kg})(0.08 \text{ m})^2$$

$$= 16 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$\bar{I}_{AB} = \frac{1}{12} m_{AB} (AB)^2$$

$$= \frac{1}{12}(1.5 \text{ kg})(0.12 \text{ m})^2$$

$$= 1.8 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$



(*a*) <u>Angular acceleration</u>.

$$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{\text{eff}}:$$

$$W_{AB}(0.14 \text{ m}) = \bar{I}_{\text{disk}}\alpha + m_{AB}\bar{a}_t(0.14 \text{ m}) + \bar{I}_{AB}\alpha$$

$$(1.5 \text{ kg})(9.81 \text{ m/s}^2)(0.14 \text{ m}) = \bar{I}_{\text{disk}}\alpha + (1.5 \text{ kg})(0.14 \text{ m})^2\alpha + \bar{I}_{AB}\alpha$$

$$2.060 \text{ N} \cdot \text{m} = (16 \times 10^{-3} + 29.4 \times 10^{-3} + 1.8 \times 10^{-3})\alpha$$

$$2.060 \text{ N} \cdot \text{m} = (47.2 \times 10^{-3} \text{ kg} \cdot \text{m}^2)\alpha$$

$$\alpha = 43.64 \text{ rad/s}^2$$

$\boldsymbol{\alpha} = 43.6 \text{ rad/s}^2 \circlearrowright$ ◀

PROBLEM 16.87 (Continued)

(*b*) Components of reaction of *C*.

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad C_x = -m_{AB}\bar{a}_n = -(1.5\text{ kg})(14\text{ m/s}^2)$$

$$C_x = -21.0\text{ N} \qquad \mathbf{C}_x = 21.0\text{ N} \longleftarrow \blacktriangleleft$$

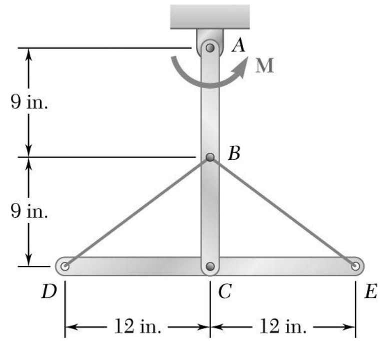
$$+\!\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad a_t = (0.14\text{ m})(\alpha)$$

$$C_y - m_{\text{disk}}g - m_{AB}g = -m_{AB}\bar{a}_t$$

$$C_y - (5\text{ kg})9.81 - (1.5\text{ kg})\,9.81 = -(1.5\text{ kg})(0.14\text{ m})(43.64\text{ rad/s}^2)$$

$$C_y - 49.05\text{ N} - 14.715\text{ N} = -9.164\text{ N}$$

$$C_y = +54.6\text{ N} \qquad \mathbf{C}_y = 54.6\text{ N}\uparrow \blacktriangleleft$$



PROBLEM 16.88

Two uniform rods, *ABC* of weight 6-lb and *DCE* of weight 8-lb, are connected by a pin at *C* and by two cords *BD* and *BE*. The T-shaped assembly rotates in a vertical plane under the combined effect of gravity and of a couple **M** which is applied to rod *ABC*. Knowing that at the instant shown the tension in cord *BE* is 2 lb and the tension in cord *BD* is 0.5 lb, determine (*a*) the angular acceleration of the assembly, (*b*) the couple **M**.

SOLUTION

We first consider the *entire system* and express that the *external* forces are equivalent to the *effective* forces of both rods.

$$(\bar{a}_{AC})_t = 0.75\alpha$$

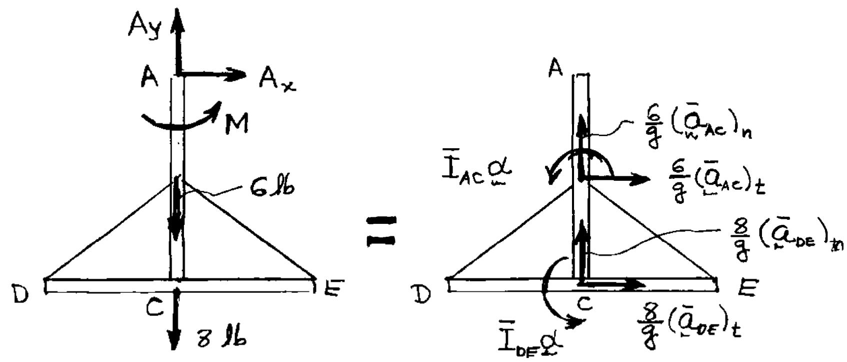
$$(\bar{a}_{DE})_t = 1.5\alpha$$

$$\bar{I}_{AC} = \frac{1}{12}\left(\frac{6\text{ lb}}{32.2}\right)(1.5\text{ ft})^2$$

$$= 34.938 \times 10^{-3}\,\text{slug}\cdot\text{ft}^2$$

$$\bar{I}_{DE} = \frac{1}{12}\left(\frac{8\text{ lb}}{32.2}\right)(2\text{ ft})^2$$

$$= 82.816 \times 10^{-3}\,\text{slug}\cdot\text{ft}^2$$

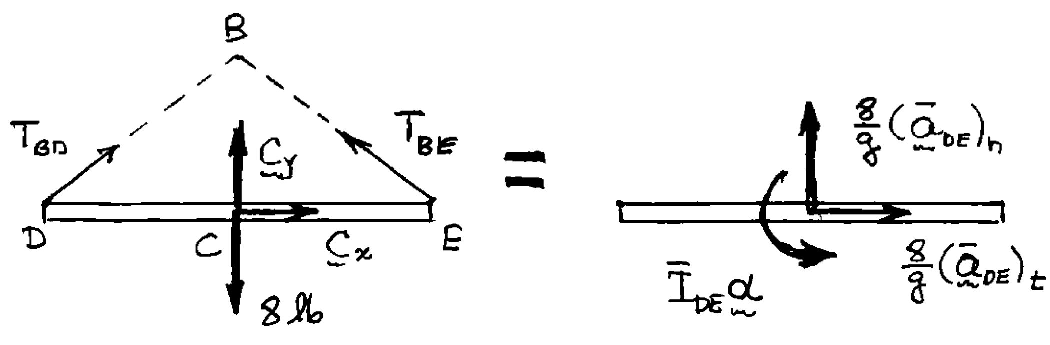


$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad M = \bar{I}_{AC}\alpha + \frac{6}{32.2}(\bar{a}_{AC})_t(0.75) + \bar{I}_{DE}\alpha + \frac{8}{32.2}(\bar{a}_{DE})_t(1.5)$$

$$M = 34.938 \times 10^{-3}\alpha + \frac{6}{32.2}(0.75\alpha)(0.75) + 82.816 \times 10^{-3}\alpha + \frac{8}{32.2}(1.5\alpha)(1.5)$$

$$M = 0.78157\alpha \qquad (1)$$

We now consider rod *DE* alone:



PROBLEM 16.88 (Continued)

$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad \left(\frac{3}{5}T_{BE}\right)(1\text{ ft}) - \left(\frac{3}{5}T_{BD}\right)(1\text{ ft}) = \bar{I}_{DE}\alpha$$

$$0.6(T_{BE} - T_{BD}) = 82.816\times 10^{-3}\alpha$$

$$T_{BE} - T_{BD} = 0.13803\,\alpha \qquad (2)$$

Given data:

$$T_{BD} = 0.5\text{ lb}$$

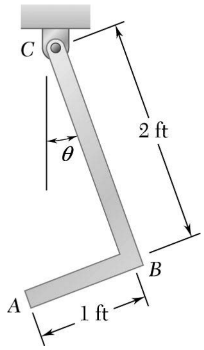
$$T_{BE} = 2\text{ lb}$$

(*a*) Angular acceleration.

Substitute into (2): $2 - 0.5 = 0.13803\alpha$ $\quad\boldsymbol{\alpha} = 10.87\text{ rad/s}^2 \circlearrowleft$ ◀

(*b*) Couple **M**.

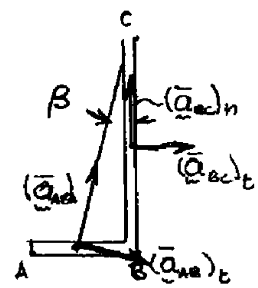
Carry value of α into (1): $M = 0.78157(10.868) = 8.4938\text{ ft}\cdot\text{lb}$ $\quad\mathbf{M} = 8.49\text{ ft}\cdot\text{lb} \circlearrowleft$ ◀



PROBLEM 16.89

The object ABC consists of two slender rods welded together at Point B. Rod AB has a weight of 2 lb and bar BC has a weight of 4 lb. Knowing the magnitude of the angular velocity of ABC is 10 rad/s when $\theta = 0$, determine the components of the reaction at Point C when $\theta = 0$.

SOLUTION



Masses and lengths: $W_{AB} = 2\text{ lb}, \quad L_{AB} = 1\text{ ft}, \quad W_{BC} = 4\text{ lb}, \quad L_{BC} = 2\text{ ft}$

Moments of inertia:

$$\bar{I}_{AB} = \frac{1}{12} m_{AB} L_{AB}^2 = \frac{1}{12}\left(\frac{2}{32.2}\right)(1)^2 = 5.1760 \times 10^{-3}\text{ slug}\cdot\text{ft}^2$$

$$I_{BC} = \frac{1}{12} m_{BC} L_{BC}^2 = \frac{1}{12}\left(\frac{4}{32.2}\right)(2)^2 = 41.408 \times 10^{-3}\text{ slug}\cdot\text{ft}^2$$

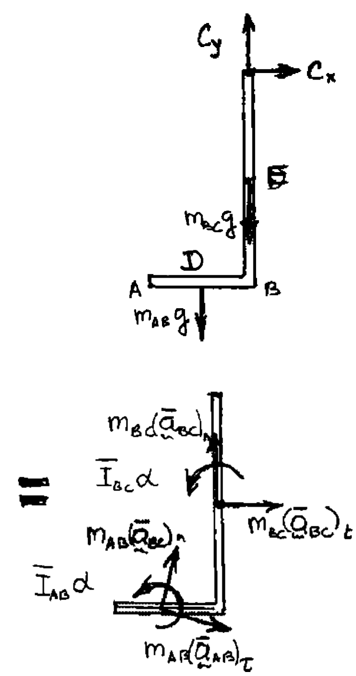
Geometry:

$$r_{CD} = \sqrt{2^2 + 0.5^2} = 2.0616\text{ ft}$$

$$r_{CE} = \frac{1}{2} L_{AB} = 1\text{ ft}$$

$$\tan\beta = \frac{0.5}{2}$$

$$\beta = 14.036°$$



Kinematics: Let $\boldsymbol{\alpha} = \alpha ↺$ be the angular acceleration of object ABC.

$$(\bar{\mathbf{a}}_{AB})_t = r_{CD}\alpha \searrow \beta$$

$$(\bar{\mathbf{a}}_{AB})_n = r_{CD}\omega^2 \nearrow \beta$$

$$(\bar{\mathbf{a}}_{BC})_t = r_{CE}\alpha \longrightarrow$$

$$(\bar{\mathbf{a}}_{BC})_n = r_{CE}\omega^2 \uparrow$$

Kinetics: $+↺\Sigma M_C = +↺\Sigma(M_C)_{\text{eff}}$: $\quad W_{AB}\dfrac{L_{AB}}{2} = \bar{I}_{AB}\alpha + r_{CD} m_{AB}(\bar{a}_{AB})_t$

$$+ \bar{I}_{BC}\alpha + r_{CE} m_{BC}(\bar{a}_{BC})_t$$

$$= \left(\bar{I}_{AB} + m_{AB} r_{CD}^2 + \bar{I}_{BC} + m_{BC} r_{CE}^2\right)\alpha$$

$$(2)(0.5) = \left[(5.1760\times 10^{-3}) + \left(\frac{2}{32.2}\right)(2.0616)^2 + (41.408\times 10^{-3}) + \left(\frac{4}{32.2}\right)(1)^2\right]\alpha$$

$$\boldsymbol{\alpha} = 2.3\text{ rad/s}^2 ↺$$

$\xrightarrow{+} \Sigma F_x = \xrightarrow{+} \Sigma (F_x)_{\text{eff}}$: $\quad C_x = m_{AB}(\bar{a}_{AB})_t \cos\beta + m_{AB}(\bar{a}_{AB})_n \sin\beta + m_{BC}(a_{BC})_t$

$$= m_{AB} r_{CD}(\alpha\cos\beta + \omega^2 \sin\beta) + m_{BC} r_{CE}\alpha$$

$$= \left(\frac{2}{32.2}\right)(2.0616)(2.3\cos 14.035° + 10^2 \sin 14.035°) + \left(\frac{4}{32.2}\right)(1)(2.3)$$

$$= 3.6770 \text{ lb}$$

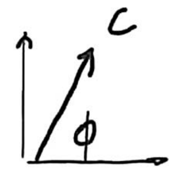
$+\!\uparrow \Sigma F_y = +\!\uparrow \Sigma (F_y)_{\text{eff}}$: $\quad C_y - m_{AB} g - m_{BC} g = -m_{AB}(\bar{a}_{AB})_t \sin\beta + m_{AB}(\bar{a}_{AB})_n \cos\beta + m_{BC}(\bar{a}_{BC})_n$

$$C_y = (W_{AB} + W_{BC}) + m_{AB} r_{CD}(\omega^2 \cos\beta - \alpha\sin\beta) + m_{BC} r_{CE}\,\omega$$

$$C_y = 6 + \left(\frac{2}{32.2}\right)(2.0616)(10^2 \cos 14.035° - 2.3\sin 14.035°)$$

$$+ \left(\frac{4}{32.2}\right)(1)(10)^2$$

$$C_y = 30.773 \text{ lb}$$



Reaction at *C*.

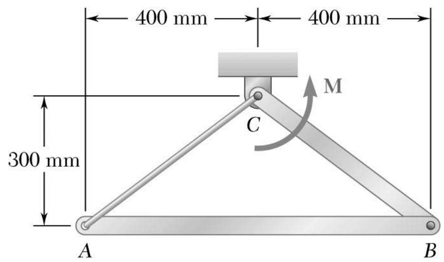
$$C = \sqrt{3.6770^2 + 30.773^2}$$

$$= 30.992 \text{ lb}$$

$$\tan\phi = \frac{30.773}{3.6770}$$

$$\phi = 83.186°$$

$\mathbf{C} = 31.0 \text{ lb} \measuredangle\, 83.2°$ ◀



PROBLEM 16.90

A 3.5-kg slender rod *AB* and a 2-kg slender rod *BC* are connected by a pin at *B* and by the cord *AC*. The assembly can rotate in a vertical plane under the combined effect of gravity and a couple **M** applied to rod *BC*. Knowing that in the position shown the angular velocity of the assembly is zero and the tension in cord *AC* is equal to 25 N, determine (*a*) the angular acceleration of the assembly, (*b*) the magnitude of the couple **M**.

SOLUTION

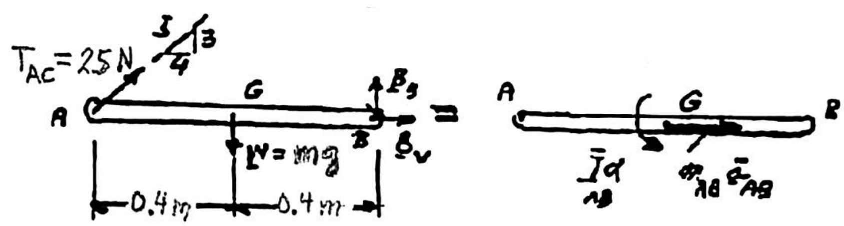
(*a*) Angular acceleration.

Rod *AB*:

$$\bar{I}_{AB} = \frac{1}{12} m_{AB} L_{AB}^2$$

$$= \frac{1}{12} 3.5\ \text{kg}(0.8\ \text{m})^2$$

$$= 0.18667\ \text{kg} \cdot \text{m}^2$$



$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad \frac{3}{5}(25\ \text{N})(0.8\ \text{m}) - (3.5\ \text{kg})(9.81\ \text{m/s}^2)(0.4\ \text{m}) = \bar{I}_{AB}\alpha$$

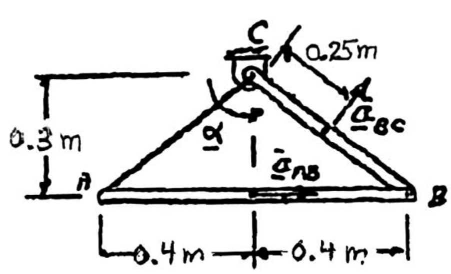
$$\bar{I}_{AB}\alpha = -1.7340\ \text{N} \cdot \text{m} \qquad (1)$$

$$(0.18667\ \text{kg} \cdot \text{m}^2)\alpha = -1.7340\ \text{N} \cdot \text{m}$$

$$\boldsymbol{\alpha} = -9.2893\ \text{rad/s}^2 \qquad \boldsymbol{\alpha} = 9.29\ \text{rad/s}^2 \circlearrowleft \blacktriangleleft$$

Entire assembly: Since *AC* is taut, assembly rotates about *C* as a rigid body.

Kinematics:



$$CB = \sqrt{(0.3)^2 + (0.4)^2} = 0.5\ \text{m}$$

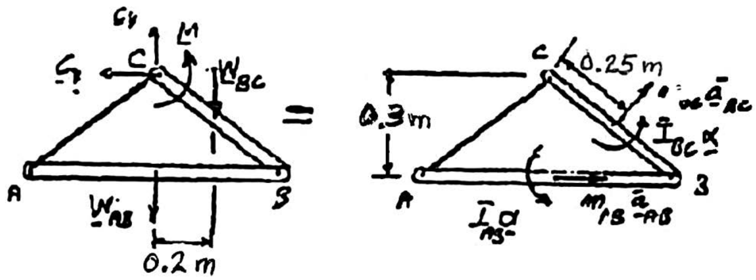
$$CG_{BC} = \frac{1}{2} CB = 0.25\ \text{m}$$

$$\bar{\mathbf{a}}_{BC} = (0.25\ \text{m})\alpha \nearrow$$

$$\bar{\mathbf{a}}_{AB} = (0.3\ \text{m})\alpha \longrightarrow$$

Kinetics:

$$\bar{I}_{BC} = \frac{1}{12} m_{BC}(CB)^2$$
$$= \frac{1}{12}(2\text{ kg})(0.5\text{ m})^2$$
$$= 0.041667\text{ kg}\cdot\text{m}^2$$



(*b*) Couple **M**.

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}:$$

$$M - m_{BC}\,g(0.2\text{ m}) = m_{BC}\bar{a}_{BC}(0.25\text{ m}) + \bar{I}_{BC}\alpha + m_{AB}\bar{a}_{AB}(0.3\text{ m}) + \bar{I}_{AB}\alpha$$
$$M - (2\text{ kg})(9.81\text{ m/s}^2)(0.2\text{ m}) = 2\text{ kg}(0.25\text{ m})^2\alpha + (0.041667\text{ kg}\cdot\text{m}^2)\alpha + 3.5\text{ kg}(0.3\text{ m})^2\alpha + \bar{I}_{AB}\alpha$$

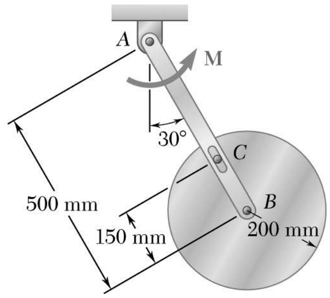
Substitute $\alpha = 9.29\text{ rad/s}^2$ $\quad\bar{I}_{AB}\alpha = -1.7340\text{ N}\cdot\text{m}$

$$M - 3.9240 = (0.125)(9.2893) + (0.041667)(9.2893)$$
$$+ (0.315)(9.2893) + (0.18667)(9.2893)$$
$$M - 3.9240 = 1.1612 + 0.3871 + 2.9261 + 1.7340$$
$$M - 3.9240 = 6.2084$$
$$M = +10.132\text{ N}\cdot\text{m}$$

$$\mathbf{M} = 10.13\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

$$M - m_{BC}\,g(0.2\text{ m}) = I_C\alpha$$

Since C is fixed, we could also use: $M - (3.5\text{ kg})(9.81\text{ m/s}^2)(0.2\text{ m}) = \left(\frac{1}{3}m_{CB}\overline{CB}^2 + \bar{I}_{AB} + 3.5\text{ kg}(0.3\text{ m})^2\right)\alpha$



PROBLEM 16.91

A 9-kg uniform disk is attached to the 5-kg slender rod *AB* by means of frictionless pins at *B* and *C*. The assembly rotates in a vertical plane under the combined effect of gravity and of a couple **M** which is applied to rod *AB*. Knowing that at the instant shown the assembly has an angular velocity of 6 rad/s and an angular acceleration of 25 rad/s^2, both counterclockwise, determine (*a*) the couple **M**, (*b*) the force exerted by pin *C* on member *AB*.

SOLUTION

We first consider the entire system and express that the external forces are equivalent to the effective forces of the disk and the rod.

$$m_R = 5 \text{ kg}, \qquad m_D = 9 \text{ kg}$$

$$W_R = m_D g = (5 \text{ kg})(9.81 \text{ m/s}^2) = 49.05 \text{ N}$$

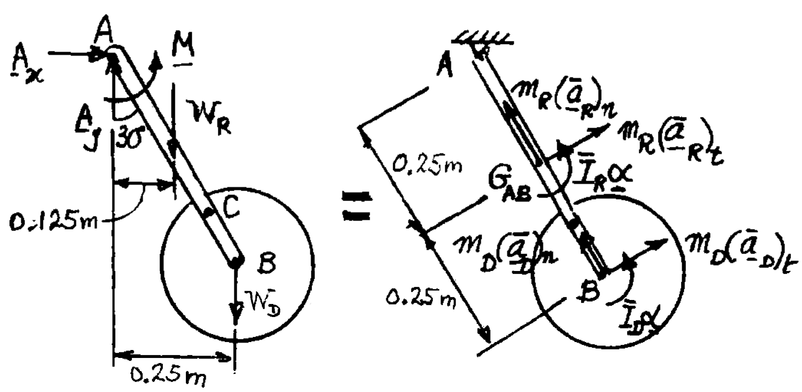
$$W_D = m_D g = (9 \text{ kg})(9.81 \text{ m/s}^2) = 88.29 \text{ N}$$

$$I_R = \frac{1}{12} m_R L_{AB}^2 = \frac{1}{12}(5 \text{ kg})(0.5 \text{ m})^2 = 0.104167 \text{ kg}\cdot\text{m}^2$$

$$I_D = \frac{1}{2} m_D r_D^2 = \frac{1}{2}(9 \text{ kg})(0.2 \text{ m})^2 = 0.18 \text{ kg}\cdot\text{m}^2$$

$$\bar{a}_R = (0.25 \text{ m})\alpha$$

$$\bar{a}_D = (0.5 \text{ m})\alpha$$



(*a*) $\Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad M - W_R(0.125) - W_D(0.25) = \bar{I}_R\alpha + m_R(a_R)_t(0.25) + \bar{I}_D\alpha + m_D(\bar{a}_D)_t(0.5)$

$$M - (49.05)(0.125) - (88.29)(0.25) = 0.104167\alpha + (5)(0.25\alpha)(0.25) + 0.18\alpha + (9)(0.5\alpha)(0.5)$$

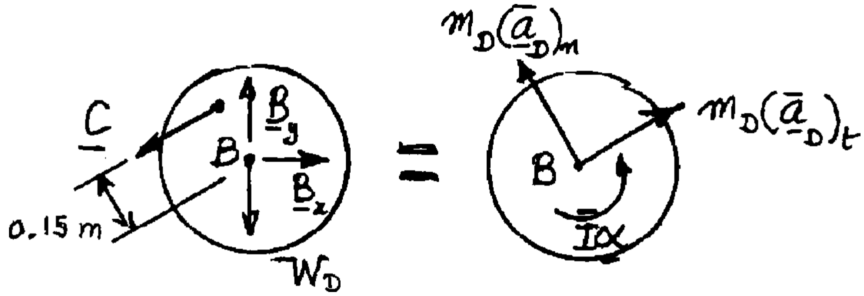
$$M - 6.1312 - 22.073 = (0.10417 + 0.3125 + 0.18 + 2.25)\alpha$$

$$M - 28.204 = (2.8467)(25)$$

$$M = 99.370 \text{ N-m}$$

$M = 99.4$ N-m ↺ ◀

(*b*) Consider now the <u>disk alone</u>:



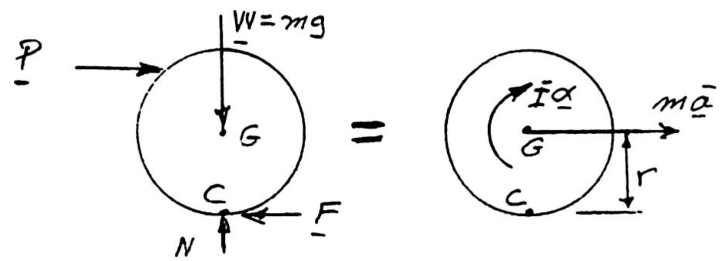
$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad C(0.15) = \bar{I}_D\alpha = (0.18)(25)$$

C = 30.0 N ⦟ 30° ◀

PROBLEM 16.92

Derive the equation $\Sigma M_C = I_C\alpha$ for the rolling disk of Figure 16.17, where ΣM_C represents the sum of the moments of the external forces about the instantaneous center *C*, and I_C is the moment of inertia of the disk about *C*.

SOLUTION



$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad \Sigma M_C = (m\bar{a})r + \bar{I}\alpha = (mr\alpha)\bar{r} + \bar{I}\alpha$$

$$= (mr\alpha)\bar{r} + \bar{I}\alpha$$

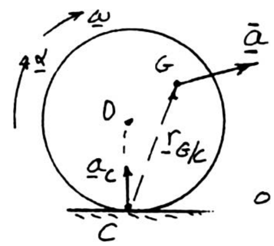
But, we know that $$I_C = mr^2 + \bar{I}$$

Thus: $$\Sigma M_C = I_C\alpha \quad \text{(Q.E.D.)} \blacktriangleleft$$

PROBLEM 16.93

Show that in the case of an unbalanced disk, the equation derived in Problem 16.92 is valid only when the mass center G, the geometric center O, and the instantaneous center C happen to lie in a straight line.

SOLUTION



Kinematics:

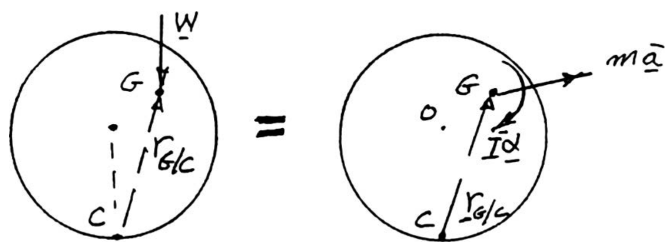
$$\bar{\mathbf{a}} = \mathbf{a}_C + \mathbf{a}_{G/C}$$

$$= \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{G/C} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{G/C})$$

or, since $\boldsymbol{\omega} \perp \mathbf{r}_{G/C}$

$$\bar{\mathbf{a}} = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{G/C} - \omega^2 \mathbf{r}_{G/C} \quad (1)$$

Kinetics:



$$\Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad \Sigma \text{M}_C = \bar{I}\boldsymbol{\alpha} + \mathbf{r}_{G/C} \times m\bar{\mathbf{a}}$$

Recall Eq. (1): $$\Sigma \mathbf{M}_C = \bar{I}\alpha + \mathbf{r}_{G/C} \times m(\mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{G/C} - \omega^2 r_{G/C})$$

$$\Sigma \mathbf{M}_C = \bar{I}\alpha + \mathbf{r}_{G/C} \times m\mathbf{a}_C + m\mathbf{r}_{G/C} \times (\boldsymbol{\alpha} \times \mathbf{r}_{G/C}) - m\omega^2 \mathbf{r}_{G/C} \times \mathbf{r}_{G/C}$$

But: $\mathbf{r}_{G/C} \times \mathbf{r}_{G/C} = 0$ and $\boldsymbol{\alpha} \perp \mathbf{r}_{G/C}$

$$\mathbf{r}_{G/C} \times m(\boldsymbol{\alpha} \times \mathbf{r}_{G/C}) = mr_{G/C}^2 \alpha$$

Thus: $$\Sigma \mathbf{M}_C = \left(\bar{I} + mr_{G/C}^2\right)\alpha + \mathbf{r}_{G/C} \times m\mathbf{a}_C$$

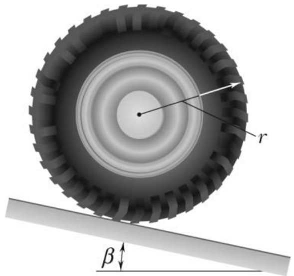
Since $$I_C = \bar{I} + mr_{G/C}^2$$

$$\Sigma \mathbf{M}_C = I_C \alpha + \mathbf{r}_{G/C} \times m\mathbf{a}_C \quad (2)$$

Eq. (2) reduces to $\Sigma M_C = I_C \alpha$ when $\mathbf{r}_{G/C} \times m\mathbf{a}_C = 0$; that is, when $\mathbf{r}_{G/C}$ and $\mathbf{a}_C$ are collinear.

Referring to the first diagram, we note that this will occur only when Points G, O, and C lie in a straight line.

(Q.E.D.) ◀



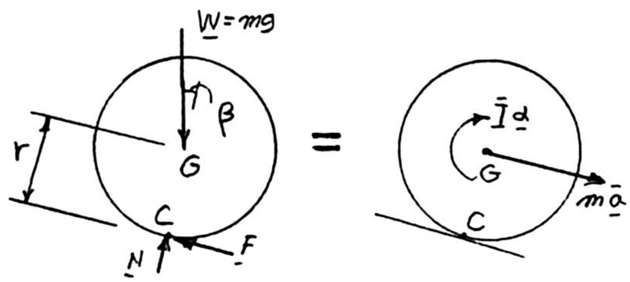
PROBLEM 16.94

A wheel of radius r and centroidal radius of gyration $\bar{k}$ is released from rest on the incline and rolls without sliding. Derive an expression for the acceleration of the center of the wheel in terms of r, $\bar{k}$, β, and g.

SOLUTION

$$\bar{I}\alpha = m\bar{k}^2\alpha$$

$$m\bar{a} = mr\alpha$$



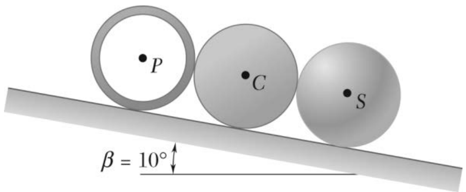
$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (W\sin\beta)r = (m\bar{a})r + \bar{I}\alpha$$

$$(mg\sin\beta)r = (mr\alpha)r + m\bar{k}^2\alpha$$

$$rg\sin\beta = (r^2 + \bar{k}^2)\alpha$$

$$\alpha = \frac{rg\sin\beta}{r^2 + \bar{k}^2}$$

$$\bar{a} = r\alpha = r\frac{rg\sin\beta}{r^2 + k^2} \qquad \bar{a} = \frac{r^2}{r^2 + \bar{k}^2}g\sin\beta \blacktriangleleft$$



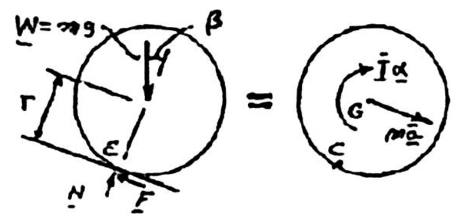
PROBLEM 16.95

A homogeneous sphere S, a uniform cylinder C, and a thin pipe P are in contact when they are released from rest on the incline shown. Knowing that all three objects roll without slipping, determine, after 4 s of motion, the clear distance between (*a*) the pipe and the cylinder, (*b*) the cylinder and the sphere.

SOLUTION

General case: $\bar{I} = m\bar{k}^2 \quad \bar{a} = r\alpha$

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (w\sin\beta)r = \bar{I}\alpha + m\bar{a}r$$



$$mg\sin\beta r = m\bar{k}^2\alpha + mr^2\alpha$$

$$\alpha = \frac{r\theta\sin\beta}{r^2 + \bar{k}^2}$$

$$\bar{a} = r\alpha = r\frac{rg\sin\beta}{r^2+\bar{k}^2} \qquad \bar{a} = \frac{r^2}{r^2+\bar{k}^2}g\sin\beta \tag{1}$$

For pipe: $\bar{k} = r \qquad \bar{a}_P = \dfrac{r^2}{r^2+r^2}g\sin\beta = \dfrac{1}{2}g\sin\beta$

For cylinder: $\bar{k}^2 = \dfrac{1}{2} \qquad a_C = \dfrac{r^2}{r^2+\dfrac{r^2}{2}}g\sin\beta = \dfrac{2}{3}g\sin\beta$

For sphere: $\bar{k}^2 = \dfrac{2}{5} \qquad a_S = \dfrac{r^2}{r^2+\dfrac{2}{5}r^2}g\sin\beta = \dfrac{5}{7}g\sin\beta$

(*a*) Between pipe and cylinder.

$$a_{C/P} = a_C - a_P = \left(\frac{2}{3}-\frac{1}{2}\right)g\sin\beta = \frac{1}{6}g\sin\beta$$

$$x_{C/P} = \frac{1}{2}a_{C/P}t^2 = \frac{1}{2}\left(\frac{1}{6}g\sin\beta\right)t^2$$

SI units: $x_{C/P} = \dfrac{1}{2}\left(\dfrac{1}{6}9.81\text{ m/s}^2\right)\sin 10°(4\text{ s})^2 = 2.27\text{ m}$ ◀

US units: $x_{C/P} = \dfrac{1}{2}\left(\dfrac{1}{6}32.2\text{ ft/s}^2\right)\sin 10°(4\text{ s})^2 = 7.46\text{ ft}$ ◀

PROBLEM 16.95 (Continued)

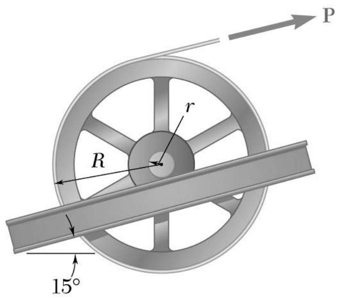
(*b*) Between sphere and cylinder.

$$a_{S/C} = a_S - a_C = \left(\frac{5}{7} - \frac{2}{3}\right) g \sin\beta = \frac{1}{21} g \sin\beta$$

$$x_{S/C} = \frac{1}{2} a_{S/C} t^2 = \frac{1}{2}\left(\frac{1}{21} g \sin\beta\right) t^2$$

SI units: $$x_{S/C} = \frac{1}{2}\left(\frac{1}{21} 9.81\ \text{m/s}^2\right)\sin 10°(4\ \text{s})^2 = 0.649\ \text{m}$$ ◀

US units: $$x_{S/C} = \frac{1}{2}\left(\frac{1}{21} 32.2\ \text{ft/s}^2\right)\sin 10°(4\ \text{s})^2 = 2.13\ \text{ft}$$ ◀



PROBLEM 16.96

A 40-kg flywheel of radius $R = 0.5$ m is rigidly attached to a shaft of radius $r = 0.05$ m that can roll along parallel rails. A cord is attached as shown and pulled with a force **P** of magnitude 150 N. Knowing the centroidal radius of gyration is $\bar{k} = 0.4$ m, determine (*a*) the angular acceleration of the flywheel, (*b*) the velocity of the center of gravity after 5 s.

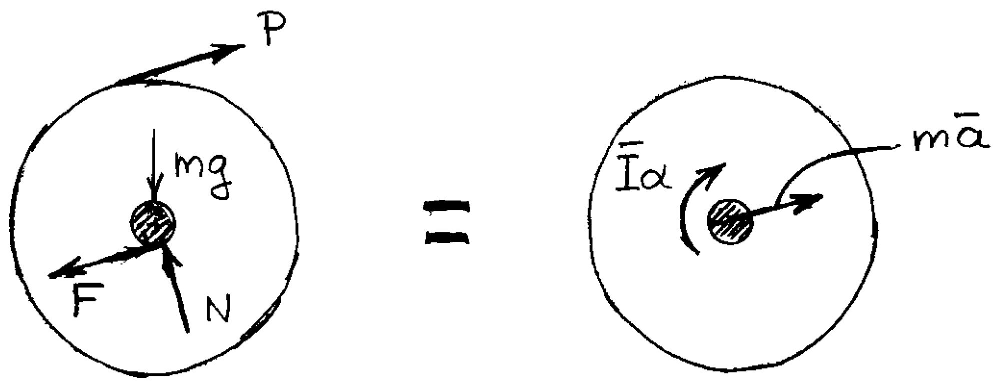
SOLUTION

Mass and moment of inertia: $m = 40$ kg

$$\bar{I} = m\bar{k}^2 = (40\text{ kg})(0.4\text{ m})^2 = 6.4\text{ kg}\cdot\text{m}^2$$

Kinematics: (no slipping) $\bar{\mathbf{a}} = r\alpha \measuredangle 15°$

Kinetics:



Let Point *C* be the contact point between the flywheel and the rails.

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad P(R+r) - mgr\sin 15° = \bar{I}\alpha + (m\bar{a})r$$

$$P(R+r) - mgr\sin 15° = (\bar{I} + mr^2)\alpha$$

(*a*) *Angular acceleration of the flywheel.*

$$\alpha = \frac{P(R+r) - mgr\sin 15°}{\bar{I} + mr^2}$$

$$= \frac{(150\text{ N})(0.55\text{ m}) - (40\text{ kg})(9.81\text{ m/s}^2)(0.05\text{ m})\sin 15°}{(6.4\text{ kg}\cdot\text{m}^2) + (40\text{ kg})(0.05\text{ m})^2}$$

$$= 11.911\text{ rad/s}^2$$

$\boldsymbol{\alpha} = 11.91\text{ rad/s}^2 \circlearrowright$ ◀

PROBLEM 16.96 (Continued)

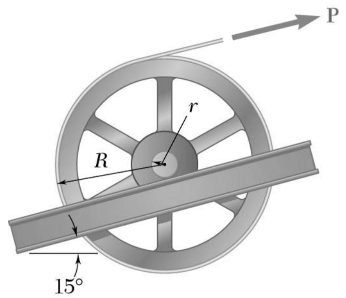
(*b*) *Velocity of center of gravity after* 5 s.

$$\bar{a} = r\alpha = (0.05 \text{ m})(11.911 \text{ rad/s}^2) = 0.59555 \text{ m/s}^2$$

$$\bar{\mathbf{a}} = 0.59555 \text{ m/s}^2 \measuredangle 15^\circ$$

$$\bar{\mathbf{v}} = \bar{\mathbf{v}}_0 + \bar{\mathbf{a}}t = 0 + (0.59555 \text{ m/s}^2)(5 \text{ s})$$

$$\bar{\mathbf{v}} = 2.98 \text{ m/s} \measuredangle 15^\circ \blacktriangleleft$$



PROBLEM 16.97

A 40-kg flywheel of radius R = 0.5 m is rigidly attached to a shaft of radius r = 0.05 m that can roll along parallel rails. A cord is attached as shown and pulled with a force **P**. Knowing the centroidal radius of gyration is $\bar{k} = 0.4\text{ m}$ and the coefficient of static friction is $\mu_s = 0.4$, determine the largest magnitude of force **P** for which no slipping will occur.

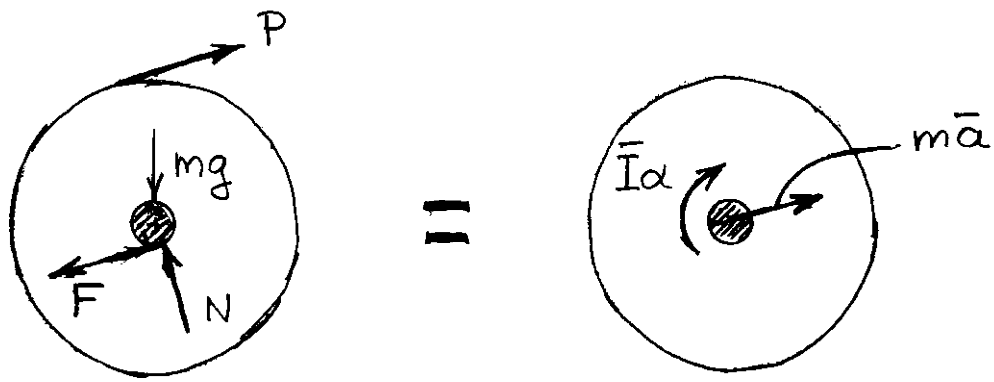
SOLUTION

Mass and moment of inertia: $m = 40\text{ kg}$

$$\bar{I} = m\bar{k}^2 = (40\text{ kg})(0.4\text{ m})^2 = 6.4\text{ kg}\cdot\text{m}^2$$

Kinematics: (no slipping) $\bar{\mathbf{a}} = r\alpha \measuredangle 15°$

Kinetics:



$$+\Sigma F_n = \Sigma(F_n)_{\text{eff}}: \quad N - mg\cos 15° = 0$$

$$N = (40\text{ kg})(9.81\text{ m/s}^2)\cos 15° = 379.03\text{ N}$$

For impending slipping, $F = \mu_s N = (0.4)(379.03)$

$$= 151.61\text{ N}$$

$$+\measuredangle 15°\ \Sigma F = \Sigma(F)_{\text{eff}}: \quad P - F - mg\sin 15° = ma$$

$$P - 151.61\text{ N} = (40\text{ kg})(9.81\text{ m/s}^2)\sin 15° = (40\text{ kg})(0.05\text{ m})\alpha$$

$$P - 253.17\text{ N} = (2\text{ kg}\cdot\text{m})\alpha \qquad (1)$$

PROBLEM 16.97 (Continued)

Let C be the contact point between the flywheel and the rails.

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad P(R+r) - mgr\sin 15° = \bar{I}a + (m\bar{a})r$$

$$P(R+r) - mgr\sin 15° = (\bar{I} + mr^2)\alpha$$

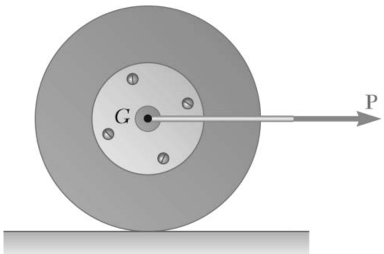
$$(0.55 \text{ m})P - (40 \text{ kg})(9.81 \text{ m/s}^2)(0.05 \text{ m})\sin 15°$$

$$= [6.4 \text{ kg}\cdot\text{m}^2 + (40 \text{ kg})(0.05 \text{ m})^2]\alpha \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$P = 303 \text{ N} \qquad \alpha = 24.8 \text{ rad/s}^2$$

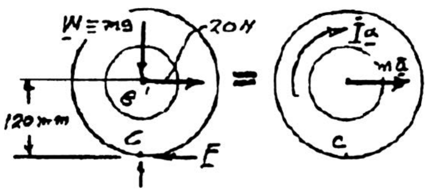
$P = 303$ N ◀



PROBLEM 16.98

A drum of 60-mm radius is attached to a disk of 120-mm radius. The disk and drum have a total mass of 6 kg and a combined radius of gyration of 90 mm. A cord is attached as shown and pulled with a force **P** of magnitude 20 N. Knowing that the disk rolls without sliding, determine (*a*) the angular acceleration of the disk and the acceleration of *G*, (*b*) the minimum value of the coefficient of static friction compatible with this motion.

SOLUTION



$$\bar{a} = r\alpha = (0.12 \text{ m})\alpha$$

$$\bar{I} = m\bar{k}^2$$

$$= (6 \text{ kg})(0.09 \text{ m})^2$$

$$\bar{I} = 48.6\times10^{-3} \text{ kg}\cdot\text{m}^2$$

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (20 \text{ N})(0.12 \text{ m}) = (m\bar{a})r + \bar{I}\alpha$$

$$2.4 \text{ N}\cdot\text{m} = (6 \text{ kg})(0.12 \text{ m})^2\alpha + 48.6\times10^{-3} \text{ kg}\cdot\text{m}^2$$

$$2.4 = 135.0\times10^{-3}\alpha$$

(*a*) $\alpha = 17.778 \text{ rad/s}^2$ $\quad\quad \boldsymbol{\alpha} = 17.78 \text{ rad/s}^2 \circlearrowright$ ◀

$$\bar{a} = r\alpha = (0.12 \text{ m})(17.778 \text{ rad/s}^2)$$

$= 2.133 \text{ m/s}^2$ $\quad\quad \bar{\mathbf{a}} = 2.13 \text{ m/s}^2 \longrightarrow$ ◀

(*b*) $+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - mg = 0$

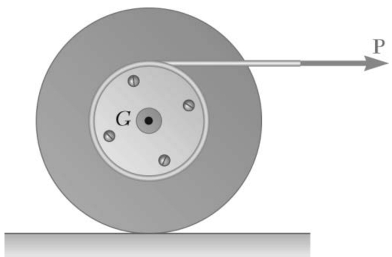
$$N = (6 \text{ kg})(9.81 \text{ m/s}^2) \quad \mathbf{N} = 58.86 \text{ N}\uparrow$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad 20 \text{ N} - F = m\bar{a}$$

$$20 \text{ N} - F = (6 \text{ kg})(2.133 \text{ m/s}^2) \quad \mathbf{F} = 7.20 \text{ N} \longleftarrow$$

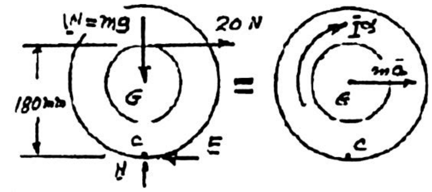
$(\mu_s)_{\min} = \dfrac{F}{N} = \dfrac{7.20 \text{ N}}{58.86 \text{ N}}$ $\quad\quad (\mu_s)_{\min} = 0.122$ ◀

PROBLEM 16.99



A drum of 60-mm radius is attached to a disk of 120-mm radius. The disk and drum have a total mass of 6 kg and a combined radius of gyration of 90 mm. A cord is attached as shown and pulled with a force **P** of magnitude 20 N. Knowing that the disk rolls without sliding, determine (*a*) the angular acceleration of the disk and the acceleration of *G*, (*b*) the minimum value of the coefficient of static friction compatible with this motion.

SOLUTION



$$\bar{a} = r\alpha = (0.12\text{ m})\alpha$$

$$\bar{I} = m\bar{k}^2$$

$$= (6\text{ kg})(0.09\text{ m})^2$$

$$\bar{I} = 48.6\times10^{-3}\text{ kg}\cdot\text{m}^2$$

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (20\text{ N})(0.18\text{ m}) = (m\bar{a})r + \bar{I}\alpha$$

$$3.6\text{ N}\cdot\text{m} = (6\text{ kg})(0.12\text{ m})^2\alpha + 48.6\text{ kg}\cdot\text{m}^2$$

$$3.6 = 135\times10^{-3}\alpha$$

(*a*) $\alpha = 26.667\text{ rad/s}^2$ $\qquad$ $\boldsymbol{\alpha} = 26.7\text{ rad/s}^2 \circlearrowright$ ◀

$$\bar{a} = r\alpha = (0.12\text{ m})(26.667\text{ rad/s}^2)$$

$$= 3.2\text{ m/s}^2$$

$\bar{\mathbf{a}} = 3.20\text{ m/s}^2 \longrightarrow$ ◀

(*b*) $+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - mg = 0$

$$N = (6\text{ kg})(9.81\text{ m/s}^2)\uparrow \qquad \mathbf{N} = 58.86\text{ N}\uparrow$$

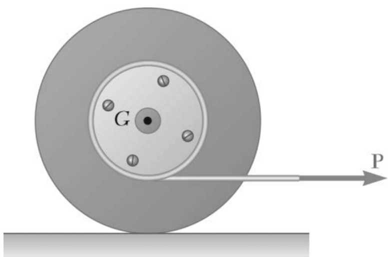
$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad 20\text{ N} - F = m\bar{a}$$

$$20\text{ N} - F = (6\text{ kg})(3.2\text{ m/s}^2) \qquad \mathbf{F} = 0.8\text{ N} \longleftarrow$$

$$(\mu_s)_{\min} = \frac{F}{N} = \frac{0.8\text{ N}}{58.86\text{ N}}$$

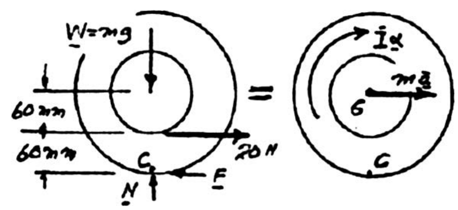
$(\mu_s)_{\min} = 0.0136$ ◀

PROBLEM 16.100



A drum of 60-mm radius is attached to a disk of 120-mm radius. The disk and drum have a total mass of 6 kg and a combined radius of gyration of 90 mm. A cord is attached as shown and pulled with a force **P** of magnitude 20 N. Knowing that the disk rolls without sliding, determine (*a*) the angular acceleration of the disk and the acceleration of *G*, (*b*) the minimum value of the coefficient of static friction compatible with this motion.

SOLUTION



$$\bar{a} = r\alpha = (0.12 \text{ m})\alpha$$
$$\bar{I} = m\bar{k}^2$$
$$= (6 \text{ kg})(0.09 \text{ m})^2$$
$$\bar{I} = 48.6 \text{ kg} \cdot \text{m}^2$$

$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (20 \text{ N})(0.06 \text{ m}) = (m\bar{a})r + \bar{I}\alpha$

$$1.2 \text{ N} \cdot \text{m} = (6 \text{ kg})(0.12 \text{ m})^2 \alpha + 48.6 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$
$$1.2 = 135 \times 10^{-3} \alpha$$

(*a*) $\alpha = 8.889 \text{ rad/s}^2$ $\quad$ $\boldsymbol{\alpha} = 8.89 \text{ rad/s}^2 \circlearrowright$ ◀

$$\bar{a} = r\alpha = (0.12 \text{ m})(8.889 \text{ rad/s}^2)$$
$$= 1.0667 \text{ m/s}^2$$ $\quad$ $\bar{\mathbf{a}} = 1.067 \text{ m/s}^2 \longrightarrow$ ◀

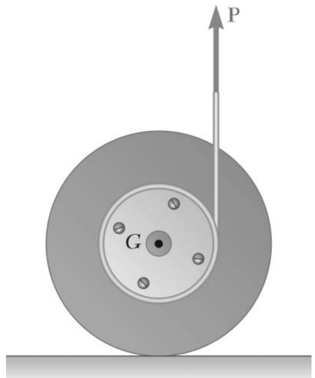
(*b*) $+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - mg = 0$

$$N = (6 \text{ kg})(9.81 \text{ m/s}^2) \qquad \mathbf{N} = 58.86 \text{ N} \uparrow$$

$\xrightarrow{+} \Sigma F_k = \Sigma(F_x)_{\text{eff}}: \quad 20 \text{ N} - F = m\bar{a}$

$$(20 \text{ N}) - F = (6 \text{ kg})(1.0667 \text{ m/s}^2) \qquad \mathbf{F} = 13.6 \text{ N} \longleftarrow$$

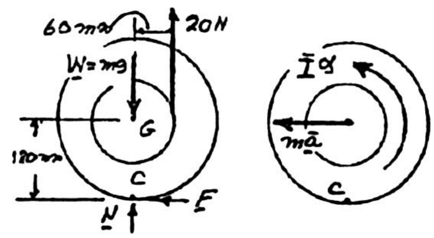
$$(\mu_s)_{\min} = \frac{F}{N} = \frac{13.6 \text{ N}}{58.86 \text{ N}}$$ $\quad$ $(\mu_s)_{\min} = 0.231$ ◀



PROBLEM 16.101

A drum of 60-mm radius is attached to a disk of 120-mm radius. The disk and drum have a total mass of 6 kg and a combined radius of gyration of 90 mm. A cord is attached as shown and pulled with a force **P** of magnitude 20 N. Knowing that the disk rolls without sliding, determine (*a*) the angular acceleration of the disk and the acceleration of *G*, (*b*) the minimum value of the coefficient of static friction compatible with this motion.

SOLUTION



$$\bar{a} = r\alpha = (0.12 \text{ m})\alpha$$

$$\bar{I} = m\bar{k}^2$$

$$= (6 \text{ kg})(0.09 \text{ m})^2$$

$$\bar{I} = 48.6 \text{ kg} \cdot \text{m}^2$$

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (20 \text{ N})(0.06 \text{ m}) = (m\bar{a})r + \bar{I}\alpha$$

$$1.2 \text{ N} \cdot \text{m} = (6 \text{ kg})(0.12 \text{ m})^2\alpha + (48.6 \times 10^{-3} \text{ kg} \cdot \text{m}^2)\alpha$$

$$1.2 = 135 \times 10^{-3}\alpha$$

(*a*) $\alpha = 8.889 \text{ rad/s}^2$ $\quad\quad$ $\boldsymbol{\alpha} = 8.89 \text{ rad/s}^2 \circlearrowleft$ ◀

$$\bar{a} = r\alpha = (0.12 \text{ m})(8.889 \text{ rad/s}^2)$$

$= 1.0667 \text{ m/s}^2$ $\quad\quad$ $\bar{\mathbf{a}} = 1.067 \text{ m/s}^2 \longleftarrow$ ◀

(*b*) $+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N + 20 \text{ N} - mg = 0$

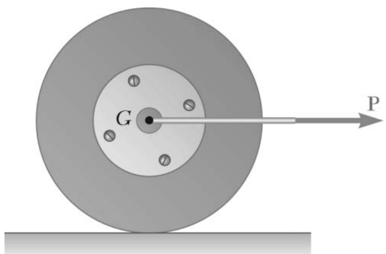
$N + 20 \text{ N} - (6 \text{ kg})(9.81 \text{ m/s}^2) \qquad \mathbf{N} = 38.86 \text{ N} \uparrow$

$\overset{+}{\longleftarrow} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F = m\bar{a}$

$F = (6 \text{ kg})(1.0667 \text{ m/s}^2) \qquad \mathbf{F} = 6.4 \text{ N} \longleftarrow$

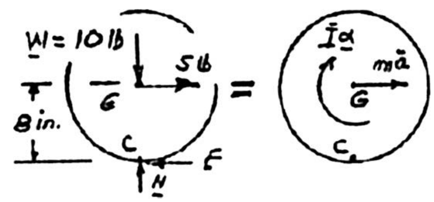
$(\mu_s)_{\min} = \dfrac{F}{N} = \dfrac{6.4 \text{ N}}{38.86 \text{ N}}$ $\quad\quad$ $(\mu_s)_{\min} = 0.165$ ◀

PROBLEM 16.102



A drum of 4-in. radius is attached to a disk of 8-in. radius. The disk and drum have a combined weight of 10 lb and a combined radius of gyration of 6 in. A cord is attached as shown and pulled with a force **P** of magnitude 5 lb. Knowing that the coefficients of static and kinetic friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, respectively, determine (*a*) whether or not the disk slides, (*b*) the angular acceleration of the disk and the acceleration of *G*.

SOLUTION



Assume disk rolls:

$$\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)\alpha$$

$$\bar{I} = m\bar{k}^2 = \frac{10\text{ lb}}{32.2}\left(\frac{6}{12}\text{ ft}\right)^2$$

$$\bar{I} = 0.07764\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad (5\text{ lb})\left(\frac{8}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$$

$$3.333\text{ lb}\cdot\text{ft} = \frac{10\text{ lb}}{32.2}\left(\frac{8}{12}\text{ ft}\right)^2\alpha + 0.07764\alpha$$

$$3.333 = 0.21566\alpha$$

$$\alpha = 15.456\text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 15.46\text{ rad/s}^2 \circlearrowright$$

$$\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)(15.456\text{ rad/s}^2) \qquad \bar{\mathbf{a}} = 10.30\text{ ft/s}^2 \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad -F + 5\text{ lb} = m\bar{a}$$

$$-F + 5\text{ lb} = \frac{10\text{ lb}}{32.2}(10.30\text{ ft/s}^2)$$

$$F = 1.80\text{ lb}$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - 10\text{ lb} = 0 \qquad N = 10\text{ lb}$$

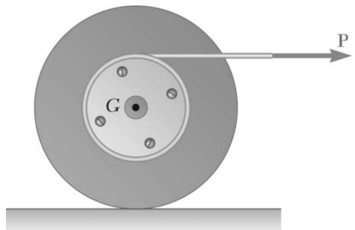
$$F_m = \mu_s N = 0.25(10\text{ lb}) = 2.5\text{ lb}$$

(*a*) Since $F < F_m$, — Disk rolls without sliding ◀

(*b*) Angular acceleration of the disk. — $\boldsymbol{\alpha} = 15.46\text{ rad/s}^2 \circlearrowright$ ◀

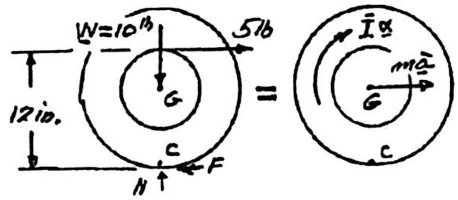
Acceleration of *G*. — $\bar{\mathbf{a}} = 10.30\text{ ft/s}^2 \longrightarrow$ ◀

PROBLEM 16.103



A drum of 4-in. radius is attached to a disk of 8-in. radius. The disk and drum have a combined weight of 10 lb and a combined radius of gyration of 6 in. A cord is attached as shown and pulled with a force **P** of magnitude 5 lb. Knowing that the coefficients of static and kinetic friction are $\mu_s = 0.25$ and $\mu_k = 0.20,$ respectively, determine (*a*) whether or not the disk slides, (*b*) the angular acceleration of the disk and the acceleration of *G*.

SOLUTION



Assume disk rolls: $\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)\alpha$

$$\bar{I} = m\bar{k}^2$$

$$= \frac{10\text{ lb}}{32.2}\left(\frac{6}{12}\text{ ft}\right)^2$$

$$\bar{I} = 0.07764\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}$: $(5\text{ lb})(1\text{ ft}) = (m\bar{a})r + \bar{I}\alpha$

$$5\text{ lb}\cdot\text{ft} = \frac{10\text{ lb}}{32.2}\left(\frac{8}{12}\text{ ft}\right)^2\alpha + 0.07764\alpha$$

$$5 = 0.21566\alpha$$

$$\alpha = 23.184\text{ rad/s}^2$$

$\boldsymbol{\alpha} = 23.2\text{ rad/s}^2 \circlearrowright$

$$\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)(23.184\text{ rad/s}^2)$$

$\bar{\mathbf{a}} = 15.46\text{ ft/s}^2 \longrightarrow$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}$: $-F + 5\text{ lb} = m\bar{a}$

$$-F + 5\text{ lb} = \frac{10\text{ lb}}{32.2}(15.46\text{ ft/s}^2);\quad F = 0.20\text{ lb}$$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}$: $N - 10\text{ lb} = 0$ $\quad N = 10\text{ lb}$

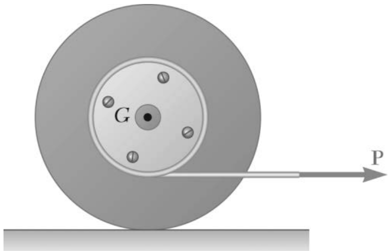
$$F_m = \mu_s N = 0.25(10\text{ lb}) = 2.5\text{ lb}$$

(*a*) Since $F < F_m$, Disk rolls without sliding ◀

(*b*) Angular acceleration of the disk. $\boldsymbol{\alpha} = 23.2\text{ rad/s}^2 \circlearrowright$ ◀

Acceleration of *G*. $\bar{\mathbf{a}} = 15.46\text{ ft/s}^2 \longrightarrow$ ◀

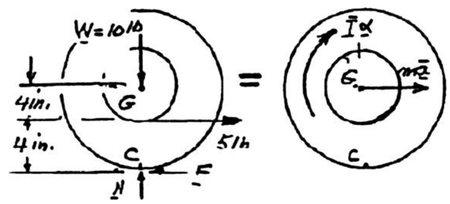
PROBLEM 16.104



A drum of 4-in. radius is attached to a disk of 8-in. radius. The disk and drum have a combined weight of 10 lb and a combined radius of gyration of 6 in. A cord is attached as shown and pulled with a force **P** of magnitude 5 lb. Knowing that the coefficients of static and kinetic friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, respectively, determine (*a*) whether or not the disk slides, (*b*) the angular acceleration of the disk and the acceleration of *G*.

SOLUTION

Assume disk rolls: $\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)\alpha$



$$\bar{I} = m\bar{k}^2 = \frac{10\text{ lb}}{32.2}\left(\frac{6}{12}\text{ ft}\right)^2$$

$$\bar{I} = 0.07764\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}$: $\quad (5\text{ lb})\left(\frac{4}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$

$$1.6667\text{ lb}\cdot\text{ft} = \frac{10\text{ lb}}{32.2}\left(\frac{8}{12}\text{ft}\right)^2\alpha + 0.07764\alpha$$

$$1.6667 = 0.21566\alpha$$

$$\alpha = 7.728\text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 7.73\text{ rad/s}^2 \circlearrowright$$

$$\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)7.728\text{ rad/s}^2 \qquad \bar{\mathbf{a}} = 5.153\text{ ft/s}^2 \longrightarrow$$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}$: $\quad -F + 5\text{ lb} = m\bar{a}$

$$-F + 5\text{ lb} = \frac{10\text{ lb}}{32.2}(5.153\text{ ft/s}^2)$$

$$F = 3.40\text{ lb}$$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}$: $\quad N - 10\text{ lb} = 0 \qquad N = 10\text{ lb}$

$$F_m = \mu_s N = 0.25(10\text{ lb}) = 2.5\text{ lb}$$

(*a*) Since $F < F_m$, Disk slides ◀

Knowing that disk slides

$$F = \mu_k N = 0.20(10\text{ lb}) = 2\text{ lb}$$

PROBLEM 16.104 (Continued)

(*b*) Angular acceleration.

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad F\left(\frac{8}{12}\text{ ft}\right) - (5\text{ lb})\left(\frac{4}{12}\text{ ft}\right) = \bar{I}\alpha$$

$$(2\text{ lb})\left(\frac{8}{12}\text{ ft}\right) - 1.6667\text{ lb}\cdot\text{ft} = (0.07764\text{ lb}\cdot\text{ft}\cdot\text{s}^2)\alpha$$

$$-0.3333 = 0.07764\alpha$$

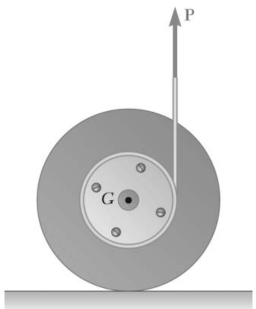
$$\alpha = -4.29\text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 4.29\text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

(*c*) Acceleration of *G*.

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad -F + 5\text{ lb} = m\bar{a}$$

$$-2\text{ lb} + 5\text{ lb} = \frac{10\text{ lb}}{32.2}\bar{a}$$

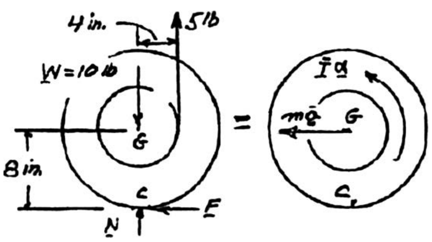
$$\bar{a} = 9.66\text{ ft/s}^2 \qquad \bar{\mathbf{a}} = 9.66\text{ ft/s}^2 \longrightarrow \blacktriangleleft$$



PROBLEM 16.105

A drum of 4-in. radius is attached to a disk of 8-in. radius. The disk and drum have a combined weight of 10 lb and a combined radius of gyration of 6 in. A cord is attached as shown and pulled with a force **P** of magnitude 5 lb. Knowing that the coefficients of static and kinetic friction are $\mu_s = 0.25$ and $\mu_k = 0.20$, respectively, determine (*a*) whether or not the disk slides, (*b*) the angular acceleration of the disk and the acceleration of *G*.

SOLUTION



Assume disk rolls: $\bar{a} = r\alpha = \left(\dfrac{8}{12}\text{ ft}\right)\alpha$

$$\bar{I} = m\bar{k}^2 = \frac{10\text{ lb}}{32.2}\left(\frac{6}{12}\text{ ft}\right)^2$$

$$\bar{I} = 0.07764\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}$: $\quad (5\text{ lb})\left(\dfrac{4}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$

$$1.6667\text{ lb}\cdot\text{ft} = \frac{10\text{ lb}}{32.2}\left(\frac{8}{12}\text{ ft}\right)^2\alpha + 0.07764\alpha$$

$$1.6667 = 0.21566\alpha$$

$$\alpha = 7.728\text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 7.73\text{ rad/s}^2 \circlearrowleft$$

$$\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)(7.728\text{ rad/s}^2) \qquad \bar{\mathbf{a}} = 5.153\text{ ft/s}^2 \longleftarrow$$

$\overset{+}{\longleftarrow} \Sigma F_x = \Sigma(F_x)_{\text{eff}}$: $\quad F = m\bar{a}$

$$F = \frac{10\text{ lb}}{32.2}(5.153\text{ ft/s}^2); \quad F = 1.60\text{ lb}$$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}$: $\quad N - 10\text{ lb} + 5\text{ lb} = 0 \qquad N = 5\text{ lb}$

$$F_m = \mu_s N = 0.25(5\text{ lb}) = 1.25\text{ lb}$$

(*a*) Since $F > F_m$, Disk slides ◀

Knowing that disk slides

$$F = \mu_k N = 0.2(5)$$

$$F = 1.00\text{ lb}$$

PROBLEM 16.105 (Continued)

(*b*) Angular acceleration.

$$+\circlearrowleft \Sigma M_S = \Sigma(M_S)_{\text{eff}}: \quad (5\text{ lb})\left(\frac{4}{12}\text{ ft}\right) - F\left(\frac{8}{12}\text{ ft}\right) = \bar{I}\alpha$$

$$(5\text{ lb})\left(\frac{4}{12}\text{ ft}\right) - (1.00\text{ lb})\left(\frac{8}{12}\text{ ft}\right) = 0.07764\alpha$$

$$1.000 = 0.07764\alpha$$

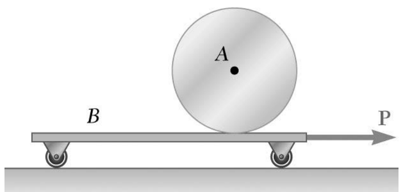
$$\alpha = 12.88\text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 12.88\text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

(*c*) Acceleration of *G*.

$$\overset{+}{\longleftarrow} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad F = m\bar{a}$$

$$1.00\text{ lb} = \frac{10\text{ lb}}{32.2}\bar{a}$$

$$\bar{a} = 3.22\text{ ft/s}^2 \qquad \bar{\mathbf{a}} = 3.22\text{ ft/s}^2 \longleftarrow \blacktriangleleft$$



PROBLEM 16.106

A 12-in.-radius cylinder of weight 16 lb rests on a 6-lb carriage. The system is at rest when a force **P** of magnitude 4 lb is applied. Knowing that the cylinder rolls without sliding on the carriage and neglecting the mass of the wheels of the carriage, determine (*a*) the acceleration of the carriage, (*b*) the acceleration of Point *A*, (*c*) the distance the cylinder has rolled with respect to the carriage after 0.5 s.

SOLUTION

Masses and moments of inertia.

$$m_A = \frac{16 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.49689 \text{ lb}\cdot\text{s}^2/\text{ft} \qquad m_B = \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.18634 \text{ lb}\cdot\text{s}^2/\text{ft}$$

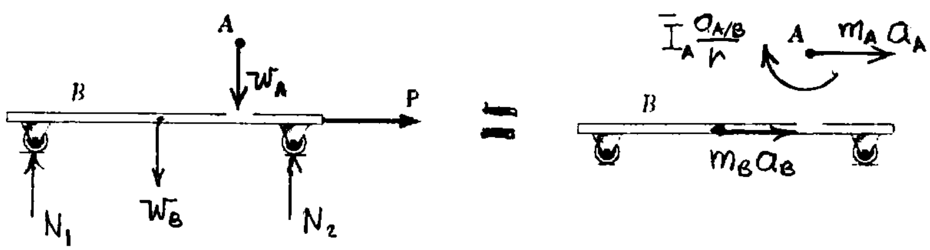
$$\bar{I}_A = \frac{1}{2}m_A r^2 = \frac{1}{2}(0.49689 \text{ lb}\cdot\text{s}^2/\text{ft})(1 \text{ ft})^2 = 0.24895 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Kinematics: Let $\mathbf{a}_A = a_A \longrightarrow$, $\mathbf{a}_B = a_B \longrightarrow$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{A/B} = (a_A - a_B) \longrightarrow \qquad \boldsymbol{\alpha} = \frac{a_{A/B}}{r} \curvearrowright \tag{1}$$

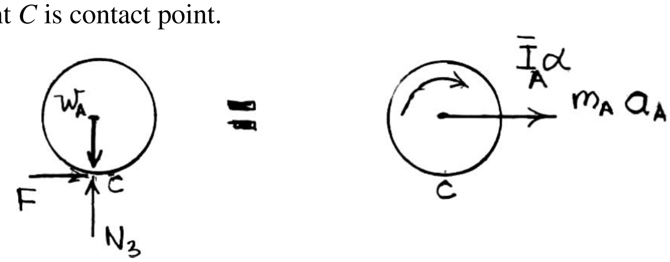
Kinetics: Carriage and cylinder



$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad P = m_A a_A + m_B a_B$$

$$4 \text{ lb} = (0.49689 \text{ lb}\cdot\text{s}^2/\text{ft})\ a_A + (0.18634 \text{ lb}\cdot\text{s}^2/\text{ft})\ a_B \tag{2}$$

Cylinder alone. Point *C* is contact point.



$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad 0 = \bar{I}_A\alpha + m_A a_A r$$

Substituting from Eqs. (1) and (2),

$$0 = \bar{I}_A \frac{a_A - a_B}{r} + m_A a_A r$$

$$0 = \left(\frac{\bar{I}_A}{r} + m_A r\right) a_A - \frac{\bar{I}_A}{r} a_B$$

Data: $$\frac{I_A}{r} = \frac{0.24895 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}}{1 \text{ ft}} = 0.24895 \text{ lb} \cdot \text{s}^2$$

$$m_A r = (0.49689 \text{ lb} \cdot \text{s}^2/\text{ft})(1 \text{ ft}) = 0.49689 \text{ lb} \cdot \text{s}^2$$

$$0 = 0.74584\, a_A - 0.24895\, a_B \qquad (3)$$

Solving Eqs. (2) and (3) simultaneously,

$$a_A = 3.7909 \text{ ft/s}^2 \quad a_B = 11.3574 \text{ ft/s}^2$$

(*a*) *Acceleration of the carriage.* $\mathbf{a}_B = 11.36 \text{ ft/s}^2 \longrightarrow$ ◀

(*b*) *Acceleration of Point A.* $\mathbf{a}_A = 3.79 \text{ ft/s}^2 \longrightarrow$ ◀

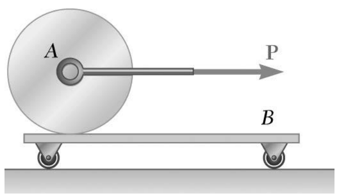
(*c*) *Relative displacement after* 0.5 s.

$$a_{A/B} = 11.3574 \text{ ft/s}^2 - 3.7909 \text{ ft/s}^2 = 7.5665 \text{ ft/s}^2$$

$$x_{A/B} = \frac{1}{2}(a_{A/B})t^2$$

$$= \frac{1}{2}(7.5665 \text{ ft/s})^2 (0.5 \text{ s})^2$$

$\mathbf{x}_{B/A} = 0.946 \text{ ft} \longrightarrow$ ◀



PROBLEM 16.107

A 12-in.-radius cylinder of weight 16 lb rests on a 6-lb carriage. The system is at rest when a force **P** of magnitude 4 lb is applied. Knowing that the cylinder rolls without sliding on the carriage and neglecting the mass of the wheels of the carriage, determine (*a*) the acceleration of the carriage, (*b*) the acceleration of Point *A*, (*c*) the distance the cylinder has rolled with respect to the carriage after 0.5 s.

SOLUTION

Masses and moments of inertia.

$$m_A = \frac{16 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.49689 \text{ lb} \cdot \text{s}^2/\text{ft} \qquad m_B = \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.18634 \text{ lb} \cdot \text{s}^2/\text{ft}$$

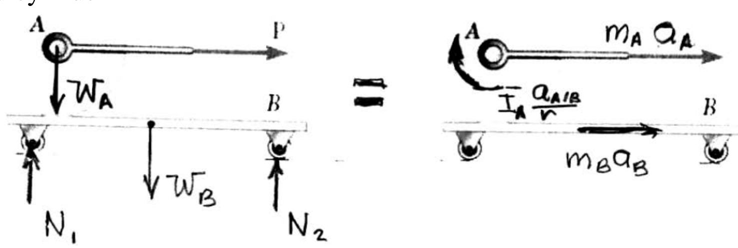
$$\bar{I}_A = \frac{1}{2} m_A r^2 = \frac{1}{2}(0.49689 \text{ lb} \cdot \text{s}^2/\text{ft})(1 \text{ ft})^2 = 0.24895 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

Kinematics: Let $\mathbf{a}_A = a_A \longrightarrow$, $\mathbf{a}_B = a_B \longrightarrow$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{B/A}$$

$$\mathbf{a}_{A/B} = (a_B - a_B) \longrightarrow \qquad \alpha = \frac{a_{A/B}}{r} \circlearrowright \qquad (1)$$

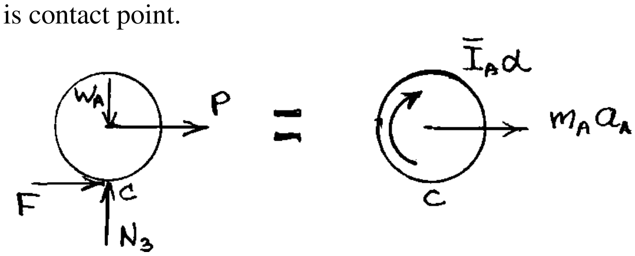
Kinetics: Carriage and cylinder



$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad P = m_A a_A + m_B a_B$$

$$4 \text{ lb} = (0.49689 \text{ lb} \cdot \text{s}^2/\text{ft})\ a_A + (0.18634 \text{ lb} \cdot \text{s}^2/\text{ft}) a_B \qquad (2)$$

Cylinder alone. Point *C* is contact point.



PROBLEM 16.107 (Continued)

$$\circlearrowleft + \Sigma M_C = \Sigma (M_B)_{\text{eff}}: \quad Pr = \bar{I}_A \alpha + m_A a_A r$$

Substituting from Eqs. (1) and (2),

$$Pr = \bar{I}_A \frac{a_A - a_B}{r} + m_A a_A r$$

$$Pr = \left(\frac{\bar{I}_A}{r} + m_A r \right) a_A - \frac{\bar{I}_A}{r} a_B$$

Data:

$$\frac{\bar{I}_A}{r} = \frac{0.24895 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}}{1 \text{ ft}} = 0.24895 \text{ lb} \cdot \text{s}^2$$

$$m_A r = (0.49689 \text{ lb} \cdot \text{s}^2/\text{ft})(1 \text{ ft}) = 0.49689 \text{ lb} \cdot \text{s}^2$$

$$(4 \text{ lb})(1 \text{ ft}) = 0.74584\, a_A - 0.24895\, a_B \qquad (3)$$

Solving Eqs. (2) and (3) simultaneously,

$$a_A = 6.6284 \text{ ft/s}^2 \qquad a_B = 3.7909 \text{ ft/s}^2$$

(*a*) *Acceleration of the carriage.* $\mathbf{a}_B = 3.79 \text{ ft/s}^2 \longrightarrow$ ◀

(*b*) *Acceleration of Point A.* $\mathbf{a}_A = 6.63 \text{ ft/s}^2 \longrightarrow$ ◀

(*c*) *Relative displacement after* 0.5 s.

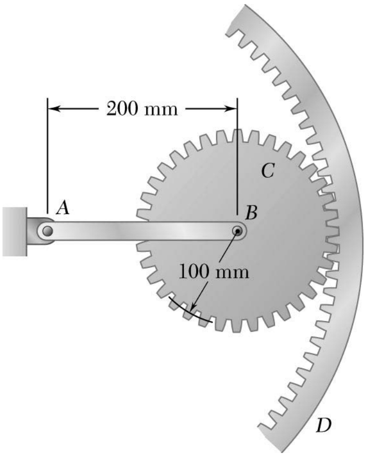
$$a_{A/B} = 6.6284 \text{ ft/s}^2 - 3.7909 \text{ ft/s}^2 = 2.8375 \text{ ft/s}^2$$

$$x_{A/B} = \frac{1}{2}(a_{A/B})t^2$$

$$= \frac{1}{2}(2.8375 \text{ ft/s}^2)(0.5 \text{ s})^2$$

$\mathbf{x}_{A/B} = 0.355 \text{ ft} \longrightarrow$ ◀

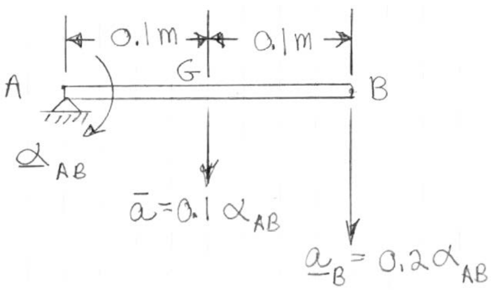
PROBLEM 16.108

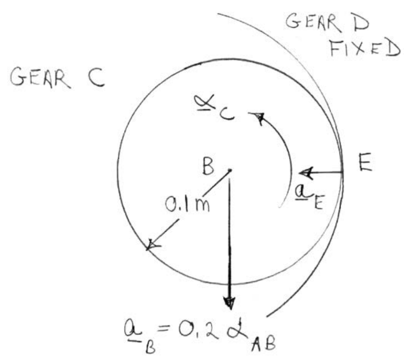


Gear C has a mass of 5 kg and a centroidal radius of gyration of 75 mm. The uniform bar AB has a mass of 3 kg and gear D is stationary. If the system is released from rest in the position shown, determine (*a*) the angular acceleration of gear C, (*b*) the acceleration of Point B.

SOLUTION

Kinematics:





Since gear D is fixed, we have for Point E of gear C: $\downarrow (a_E)_t = 0$

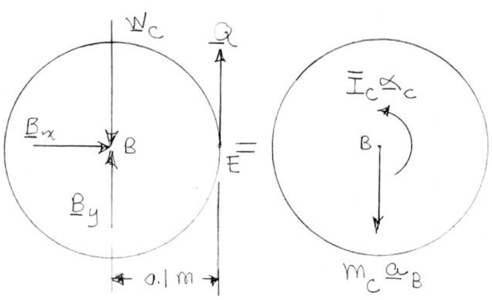
But $\mathbf{a}_E = \mathbf{a}_B + \mathbf{a}_{E/B}$

$+\downarrow (a_E)_t = (a_B)_t + (\mathbf{a}_{E/B})_t$

$0 = 0.2\,\alpha_{AB} - 0.1\,\alpha_C$

$$\alpha_{AB} = \frac{1}{2}\alpha_C \qquad (1)$$

Gear C



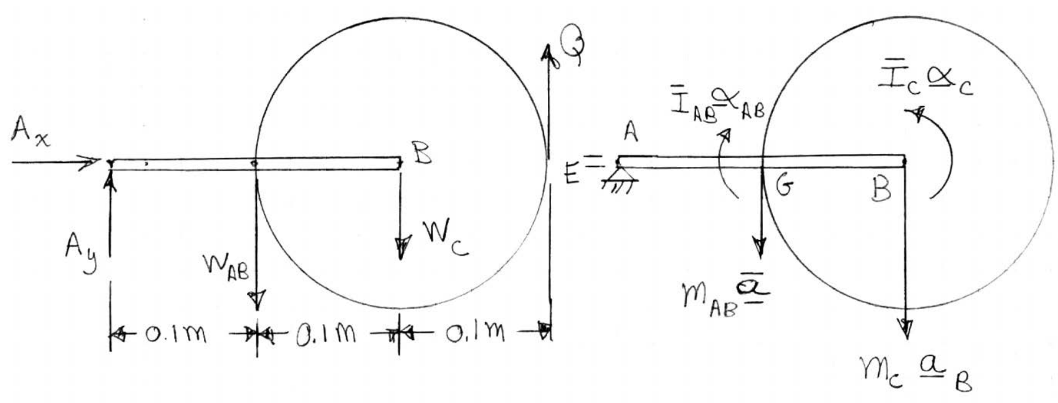
$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}$:

$Q\,(0.1\text{m}) = \bar{I}_C\alpha_C$

$= (5\text{ kg})(0.075\text{ m})^2\,\alpha_C$

$$Q = 0.28125\,\alpha_C \qquad (2)$$

Bar AB and gear C



(*a*) $+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}$:

$$W_{AB}(0.1) + W_C(0.2) - Q(0.3) = (m_{AB}\,\bar{a})\,0.1 + \bar{I}_{AB}\alpha_{AB} + (m_C\,a_B)0.2 - \bar{I}_C\alpha_C$$

$$(3)g(0.1) + (5)g(0.2) - Q(0.3) = 3(0.1\alpha_{AB})0.1 + \frac{1}{12}(3)(0.2)^2\,\alpha_{AB}$$

$$+\,5(0.2\,\alpha_{AB})0.2 - 5(0.075)^2\,\alpha_C$$

$$(1.3)g - 0.3Q = 0.24\alpha_{AB} - 0.028125\alpha_C$$

Substituting for α_{AB} and Q from (2) and (1):

$$1.3g - 0.3(0.28125\alpha_C) = 0.24\left(\frac{1}{2}\alpha_C\right) - 0.028125\,\alpha_C$$

$$1.3g = 0.17625\,\alpha_C \qquad \alpha_C = 7.3759(9.81)$$

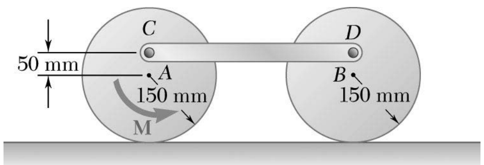
$$\alpha_C = 72.36$$

$\boldsymbol{\alpha}_C = 72.4 \text{ rad/s}^2 \circlearrowright$ ◀

(*b*) $a_B = 0.2\alpha_{AB} = 0.2\left(\frac{1}{2}\alpha_C\right) = 0.1\alpha_C = 0.1(72.36)$

$\mathbf{a}_B = 7.24 \text{ m/s}^2 \downarrow$ ◀

PROBLEM 16.109



Two uniform disks A and B, each of mass of 2 kg, are connected by a 1.5 kg rod CD as shown. A counterclockwise couple **M** of moment 2.5 N-m is applied to disk A. Knowing that the disks roll without sliding, determine (*a*) the acceleration of the center of each disk, (*b*) the horizontal component of the force exerted on disk B by pin D.

SOLUTION

Geometry:

$$r = 150 \text{ mm} = 0.15 \text{ m}$$

$$b = \overline{AC} = \overline{BD} = 50 \text{ mm} = 0.05 \text{ m}$$

$$r + b = 0.20 \text{ m}$$

Masses: $m_A = m_B = m = 2 \text{ kg} \qquad m_{CD} = 1.5 \text{ kg}$

Moment of inertia: $I_A = I_B = I = \dfrac{1}{2}mr^2$

Kinematics: $\alpha_{CD} = 0$

$$\mathbf{a}_A = \mathbf{a}_B = \mathbf{a} = a \longleftarrow$$

Angular accelerations of disks: $\alpha = \dfrac{a}{r} \circlearrowleft$

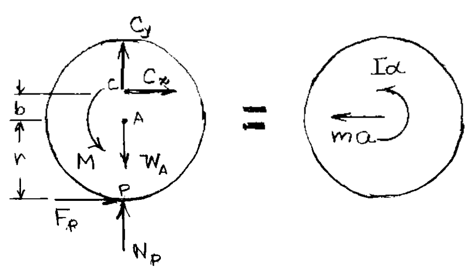
$$\mathbf{a}_C = \mathbf{a}_A + b\alpha \longleftarrow + b\omega^2 \downarrow$$

$$\mathbf{a}_D = \mathbf{a}_B + b\alpha \longleftarrow + b\omega^2 \downarrow$$

For rod CD,

$$\bar{a} = (a + b\alpha) \longleftarrow + b\omega^2 \downarrow$$

$$= \left(1 + \frac{b}{r}\right)a \longleftarrow + b\omega^2 \downarrow$$

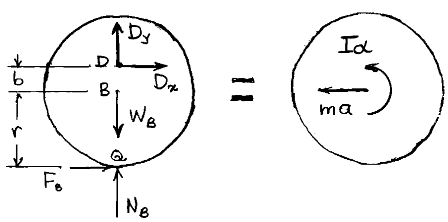


Kinetics:

Disk A: $+\circlearrowleft \Sigma M_P = \Sigma (M_P)_{\text{eff}}$: $\quad M - (r+b)C_x = mar + I\alpha$

$$= \left(mr + \frac{I}{r}\right)(a) \qquad (1)$$

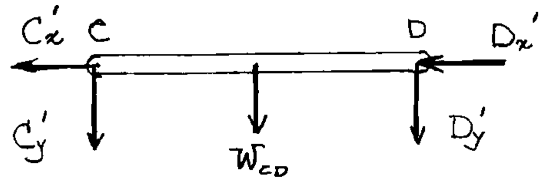
Disk B: $+\circlearrowleft \Sigma M_Q = \Sigma (M_Q)_{\text{eff}}$: $\quad -(r+b)D_x = mar + I\alpha$

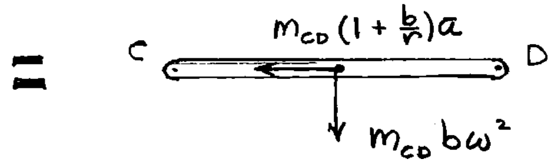


$$= \left(mr + \frac{I}{r}\right)a \qquad (2)$$

PROBLEM 16.109 (Continued)

Rod *CD*: $\overset{+}{\longleftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad C_x + D_x = m_{CD}\left(1+\dfrac{b}{r}\right)a$





Multiply by $(r+b)$

$$(r+b)(C_x + D_x) = m_{CD} r\left(1+\frac{b}{r}\right)^2 a \tag{3}$$

Add Eqs. (1), (2), and (3) to eliminate C_x and D_x

$$M = 2\left(mr + \frac{I}{r}\right)a + m_{CD} r\left(1+\frac{b}{r}\right)^2 a$$

Apply the numerical data. $\quad mr + \dfrac{I}{r} = (2 \text{ kg})(0.15 \text{ m}) + \dfrac{1}{2}(2 \text{ kg})\dfrac{(0.15 \text{ m})^2}{0.15 \text{ m}} = 0.45 \text{ kg}\cdot\text{m}$

$$m_{CD} r\left(1+\frac{b}{r}\right)^2 = (1.5 \text{ kg})(0.15 \text{ m})\left(1+\frac{0.05}{0.15}\right)^{-2} = 0.40 \text{ kg}\cdot\text{m}$$

$$2.5 \text{ N}\cdot\text{m} = 2(0.45 \text{ kg}\cdot\text{m})\,a + (0.40 \text{ kg}\cdot\text{m})\,a$$

$$a = 1.9231 \text{ m/s}^2$$

(*a*) *Acceleration of the center of each disk*:

$$\mathbf{a}_A = \mathbf{a}_B = 1.923 \text{ m/s}^2 \longleftarrow \blacktriangleleft$$

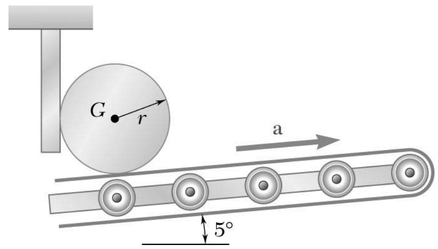
(*b*) *Horizontal component of the force exerted on disk B by pin D*.

From Eq. (2), $\quad D_x = -\dfrac{1}{r+b}\left(mr + \dfrac{I}{r}\right)a$

$$= -\frac{1}{0.20 \text{ m}}(0.45 \text{ kg}\cdot\text{m})(1.9231 \text{ m/s}^2) = -4.33 \text{ N}$$

$$\mathbf{D}_x = 4.33 \text{ N} \longleftarrow \blacktriangleleft$$

PROBLEM 16.110



A 10-lb cylinder of radius $r = 4$ in. is resting on a conveyor belt when the belt is suddenly turned on and it experiences an acceleration of magnitude $a = 6\text{ft/s}^2$. The smooth vertical bar holds the cylinder in place when the belt is not moving. Knowing the cylinder rolls without slipping and the friction between the vertical bar and the cylinder is negligible, determine (*a*) the angular acceleration of the cylinder, (*b*) the components of the force the conveyor belt applies to the cylinder.

SOLUTION

Mass and moment of inertia.

$$m = \frac{W}{g} = \frac{10 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.31056 \text{ lb} \cdot \text{s}^2/\text{ft}$$

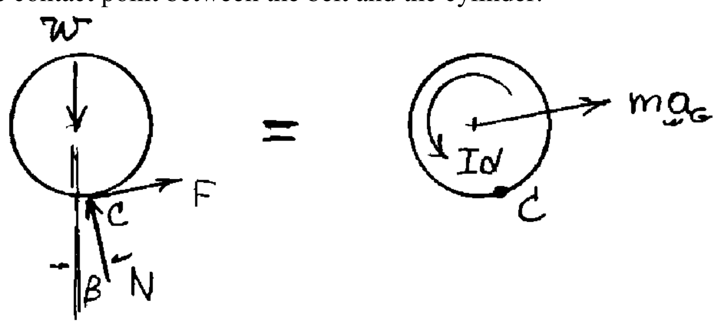
$$I = \frac{1}{2}mr^2 = \frac{1}{2}(0.31056 \text{ lb} \cdot \text{s}^2/\text{ft})\left(\frac{4}{12} \text{ ft}\right)^2 = 0.017253 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

Kinematics: The cylinder rolls without slipping on the belt which is accelerating at $6 \text{ ft/s}^2 \measuredangle 5°$.

$$\mathbf{a}_G = (6 \text{ ft/s}^2 - r\alpha) \measuredangle 5° = \left[6 \text{ ft/s}^2 - \left(\frac{4}{12} \text{ ft}\right)\alpha\right] \measuredangle 5°$$

where $\boldsymbol{\alpha} = \alpha \circlearrowright$ is the angular acceleration of the cylinder.

Kinetics: Let Point C be the contact point between the belt and the cylinder.



$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad Wr\sin\beta = -rma_G + I\alpha$$

$$(10 \text{ lb})\left(\frac{4}{12} \text{ ft}\right)\sin 5° = -\left(\frac{4}{12} \text{ ft}\right)(0.31056 \text{ lb} \cdot \text{s}^2/\text{ft})\left[6 \text{ ft/s}^2 - \left(\frac{4}{12}\text{ft}\right)\alpha\right] + (0.017253 \text{ lb} \cdot \text{s}^2\text{ft})\alpha$$

$$0.29052 \text{ lb} \cdot \text{ft} = -0.62112 \text{ lb} \cdot \text{ft} + (0.051760 \text{ lb} \cdot \text{s}^2 \cdot \text{ft})\alpha$$

PROBLEM 16.110 (Continued)

(*a*) *Angular acceleration.*

$$\alpha = 17.613 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 17.61 \text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

$$a_G = 6 \text{ ft/s}^2 - \left(\frac{4}{12} \text{ ft}\right)(17.613 \text{ rad/s}^2) = 0.129 \text{ ft/s}^2$$

(*b*) *Components of contact force:*

$$\measuredangle 5^\circ\ \Sigma F = \Sigma(F_{\text{eff}}): \quad F - W \sin 5^\circ = ma_G$$

$$F = W \sin 5^\circ + ma_G = (10 \text{ lb}) \sin 5^\circ + (0.31056 \text{ lb} \cdot \text{s}^2/\text{ft})(0.129 \text{ ft/s}^2)$$

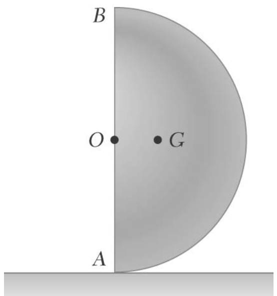
$$\mathbf{F} = 0.912 \text{ lb} \measuredangle 5^\circ \blacktriangleleft$$

$$+\searrow 85^\circ\ \Sigma F = \Sigma F = \Sigma F_{\text{eff}}: \quad N - W \cos 5^\circ = 0$$

$$N = W \cos 5^\circ = (10 \text{ lb}) \cos 5^\circ$$

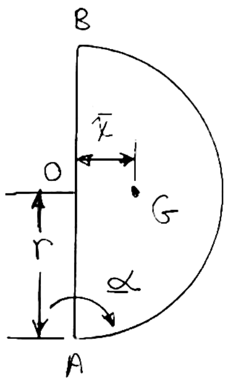
$$\mathbf{N} = 9.96 \text{ lb} \searrow 85^\circ \blacktriangleleft$$

PROBLEM 16.111



A hemisphere of weight W and radius r is released from rest in the position shown. Determine (*a*) the minimum value of μ_s for which the hemisphere starts to roll without sliding, (*b*) the corresponding acceleration of Point B. [*Hint*: Note that $OG = \frac{3}{8}r$ and that, by the parallel-axis theorem, $\overline{I} = \frac{2}{5}mr^2 - m(OG)^2$.]

SOLUTION



Kinematics: $\omega = 0$

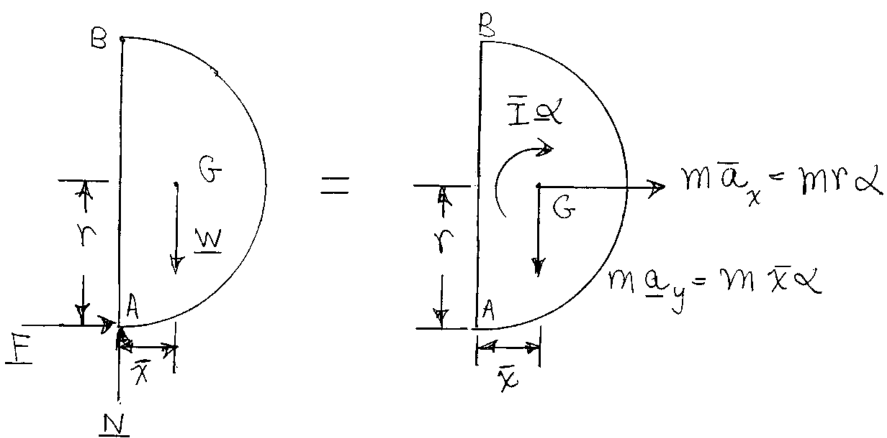
$$\mathbf{a}_O = \mathbf{a}_A + \mathbf{a}_{O/A} = 0 + [r\alpha \longrightarrow]$$

$$\overline{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_O + \mathbf{a}_{G/O}$$

$$= [r\alpha \longrightarrow] + [\overline{x}\alpha \downarrow]$$

Thus, $\overline{\mathbf{a}}_x = r\alpha \longrightarrow, \qquad \overline{\mathbf{a}}_y = \overline{x}\alpha \downarrow$ (1)

Kinetics:



$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad W\overline{x} = (m\overline{a}_x)r + (m\overline{a}_y)\,\overline{x} + \overline{I}\alpha$$

$$mg\overline{x} = (mr\alpha)\,r + (m\overline{x}\alpha)\,\overline{x} + m\overline{k}^2\alpha$$

$$\alpha = \frac{g\overline{x}}{r^2 + \overline{x}^2 + \overline{k}^2} \qquad (2)$$

PROBLEM 16.111 (Continued)

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = ma_x \qquad F = mr\alpha$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad N - W = -m\bar{a}_y \qquad N = mg - m\bar{x}\alpha$$

$$\mu_{\min} = \frac{F}{N} = \frac{mr\alpha}{mg - \bar{x}\alpha} \qquad \mu_{\min} = \frac{r\alpha}{g - \bar{x}\alpha} \tag{3}$$

For a hemisphere:

$$\bar{x} = OG = \frac{3}{8}r \qquad \bar{I} = I_O - m\bar{x}^2 = \frac{2}{5}mr^2 - m\left(\frac{3}{8}r\right)^2$$

$$\bar{I} = \frac{2}{5}mr^2 - \frac{9}{64}mr^2 \qquad \bar{k}^2 = \frac{\bar{I}}{m} = \left(\frac{2}{5} - \frac{9}{64}\right)r^2$$

Substituting into (2)

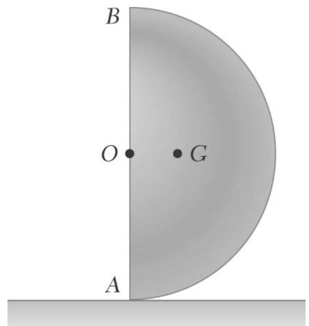
$$\alpha = \frac{g\left(\frac{3}{8}r\right)}{r^2 + \frac{9}{64}r^2 + \left(\frac{2}{5} - \frac{9}{64}\right)r^2} = \frac{\frac{3}{8}}{\frac{7}{5}}\frac{g}{r} \qquad \alpha = \frac{15}{56}\frac{g}{r}$$

(*a*) Substituting into (3)

$$\mu_{\min} = \frac{\frac{15}{56}}{1 - \left(\frac{3}{8}\right)\left(\frac{15}{56}\right)} = \frac{0.26786}{0.89955} \qquad \mu_{\min} = 0.298 \blacktriangleleft$$

(*b*) $a_B = (2r)\,\alpha = (2r)\left(\dfrac{15g}{56r}\right) = \dfrac{30g}{56}$ $\qquad \mathbf{a}_B = 0.536g \longrightarrow \blacktriangleleft$

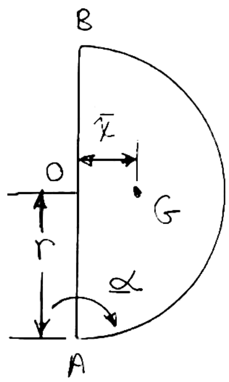
Note: In this problem we *cannot* use the equation $\Sigma M_A = I_A\,\alpha$, since Points *A*, *O*, and *G* are not aligned.



PROBLEM 16.112

Solve Problem 16.111, considering a half cylinder instead of a hemisphere. [*Hint*. Note that $OG = 4r/3\pi$ and that, by the parallel-axis theorem, $\bar{I} = \frac{1}{2}mr^2 - m(OG)^2$.]

SOLUTION



Kinematics: $\omega = 0$

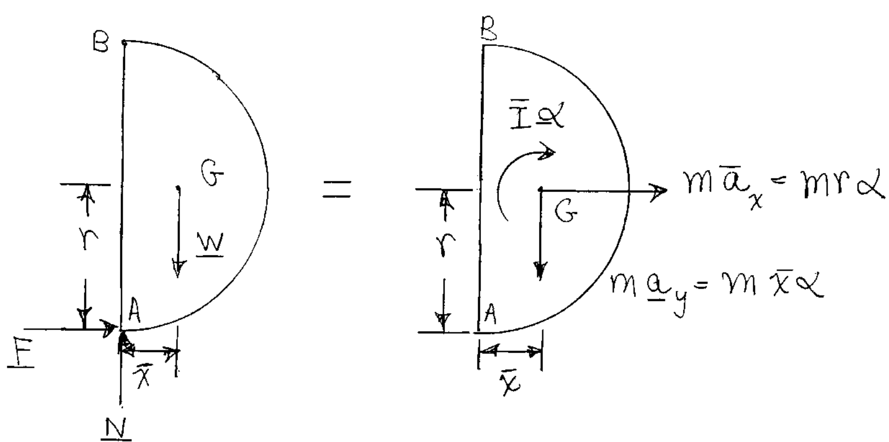
$$\mathbf{a}_O = \mathbf{a}_A + \mathbf{a}_{O/A} = 0 + [r\alpha \longrightarrow]$$

$$\bar{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_O + \mathbf{a}_{G/O}$$

$$= [r\alpha \longrightarrow] + [\bar{x}\alpha \downarrow]$$

Thus, $\mathbf{a}_x = r\alpha \longrightarrow, \quad \mathbf{a}_y = \bar{x}\alpha \downarrow$ (1)

Kinetics:



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad W\bar{x} = (m\bar{a}_x)r + (m\bar{a}_y)\,\bar{x} + \bar{I}\alpha$$

$$mg\bar{x} = (mr\alpha)\,r + (m\bar{x}\alpha)\,\bar{x} + m\bar{k}^2\alpha$$

$$\alpha = \frac{g\bar{x}}{r^2 + \bar{x}^2 + \bar{k}^2} \quad (2)$$

$$\overset{+}{\rightarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad F = ma_x \quad F = mr\alpha$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad N - W = -m\bar{a}_y \quad N = mg - m\bar{x}\alpha$$

$$\mu_{\min} = \frac{F}{N} = \frac{mr\alpha}{mg - \bar{x}\alpha} \qquad \mu_{\min} = \frac{r\alpha}{g - \bar{x}\alpha} \tag{3}$$

For a half cylinder:

$$\bar{x} = OG = \frac{4r}{3\pi} \qquad \bar{I} = I_O - m\bar{x}^2 = \frac{1}{2}mr^2 - m\left(\frac{4r}{3\pi}\right)^2$$

$$\bar{k}^2 = \frac{\bar{I}}{m} = \frac{1}{2}r^2 - \left(\frac{4r}{3\pi}\right)^2$$

Substituting into (2):

$$\alpha = \frac{g\left(\frac{4r}{3\pi}\right)}{r^2 + \left(\frac{4r}{3\pi}\right)^2 + \frac{1}{2}r^2 - \left(\frac{4r}{3\pi}\right)^2} = \frac{\frac{4}{3\pi}}{\frac{3}{2}}\frac{g}{r} \qquad \alpha = \frac{8}{9\pi}\frac{g}{r}$$

(*a*) Substituting into (3):

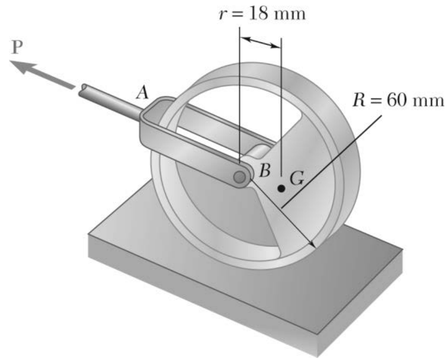
$$\mu_{\min} = \frac{\frac{8}{9\pi}}{1 - \left(\frac{4}{3\pi}\right)\left(\frac{8}{9\pi}\right)} = \frac{0.28294}{0.87992}$$

$\mu_{\min} = 0.322$ ◀

(*b*) $a_B = (2r)\,\alpha = (2r)\left(\dfrac{8g}{9\pi r}\right) = \dfrac{16g}{9\pi}$

$a_B = 0.566g$ ◀

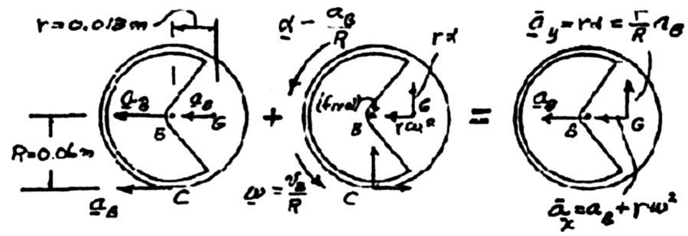
PROBLEM 16.113



The center of gravity G of a 1.5-kg unbalanced tracking wheel is located at a distance $r = 18$ mm from its geometric center B. The radius of the wheel is $R = 60$ mm and its centroidal radius of gyration is 44 mm. At the instant shown the center B of the wheel has a velocity of 0.35 m/s and an acceleration of 1.2 m/s^2, both directed to the left. Knowing that the wheel rolls without sliding and neglecting the mass of the driving yoke AB, determine the horizontal force **P** applied to the yoke.

SOLUTION

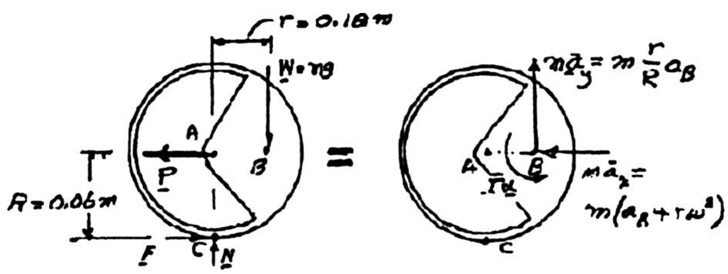
Kinematics: Choose positive $\mathbf{v}_B$ and $\mathbf{a}_B$ to left.



Trans. with B + Rotation about B = Rolling motion

$$\bar{\mathbf{a}} = [a_B + r\omega^2] \longleftarrow + \left[\frac{r}{R} a_B\right] \uparrow$$

Kinetics:



$$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad PR - Wr = (m\bar{a}_y)r + (m\bar{a}_x)R + \bar{I}\alpha$$

$$PR - mgr = m\left(\frac{r}{R} a_B\right) r + m(a_B + r\omega^2)R + m\bar{k}^2 \frac{a_B}{R}$$

$$= ma_B\left(\frac{r^2}{R} + R + \frac{\bar{k}^2}{R}\right) + mr\left(\frac{v_B}{r}\right)^2 R$$

$$P = mg\left(\frac{r}{R}\right) + ma_B\left(1 + \frac{r^2 + \bar{k}^2}{R^2}\right) + m\frac{r}{R^2} v_B^2 \qquad (1)$$

PROBLEM 16.113 (Continued)

Substitute: $m = 1.5$ kg

$r = 0.018$ m

$R = 0.06$ m

$\bar{k} = 0.044$ mm and $g = 9.81$ m/s^2 in Eq. (1)

$$P = 1.5(9.81)\frac{0.018}{0.06} + 1.5(a_B)\left(1 + \frac{0.018^2 + 0.044^2}{0.06^2}\right) + 1.5\frac{0.018}{0.06^2}v_B^2$$

$$P = 4.4145 + 2.4417a_B + 7.5v_B^2 \qquad (2)$$

Data: $\mathbf{v}_B = 0.35$ m/s $\longleftarrow$; $v_B = +0.35$ m/s

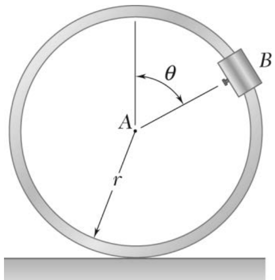
$\mathbf{a}_B = 1.2$ m/s^2 $\longleftarrow$; $a_B = +1.2$ m/s^2

Substitute in Eq. (2): $P = 4.4145 + 2.4417(+1.2) + 7.5(+0.35)^2$

$= 4.4145 + 2.9300 + 0.9188$

$= +8.263$ N

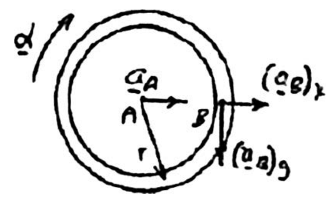
$\mathbf{P} = 8.26$ N $\longleftarrow$ ◀



PROBLEM 16.114

A small clamp of mass m_B is attached at B to a hoop of mass m_h. The system is released from rest when $\theta = 90°$ and rolls without sliding. Knowing that $m_h = 3m_B$, determine (*a*) the angular acceleration of the hoop, (*b*) the horizontal and vertical components of the acceleration of B.

SOLUTION



Kinematics:

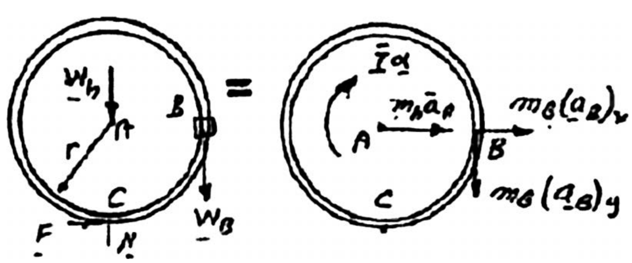
$$\mathbf{a}_A = r\alpha \longrightarrow, \quad a_{B/A} = r\alpha \downarrow$$

$$\mathbf{a}_B = \mathbf{a}_A \longrightarrow + a_{B/A} \downarrow$$

$$\mathbf{a}_B = r\alpha \longrightarrow + r\alpha \downarrow$$

$$(\mathbf{a}_B)_x = r\alpha \longrightarrow \quad (a_B)_y = r\alpha \downarrow$$

Kinetics:



$$m_h = 3m_B$$

$$\bar{I} = m_h r^2 = 3m_B r^2$$

(*a*) Angular acceleration.

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad W_B r = \bar{I}\alpha + m_h \bar{a}_A r + m_B (a_B)_x r + m_B (a_B)_y r$$

$$m_B g r = 3m_B r^2 \alpha + (3m_B) r^2 \alpha + m_B r^2 \alpha + m_B r^2 \alpha$$

$$g r = 8 r^2 \alpha$$

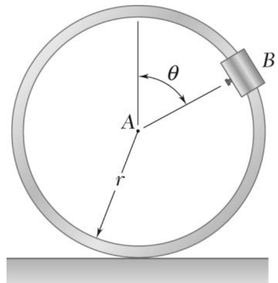
$$\alpha = \frac{1}{8}\frac{g}{r} \circlearrowright \blacktriangleleft$$

(*b*) Components of acceleration of B.

$$(\mathbf{a}_B)_x = r\alpha = \frac{1}{8} g \longrightarrow \quad (\mathbf{a}_B)_y = r\alpha = \frac{1}{8} g \downarrow$$

$$(\mathbf{a}_B)_x = \frac{1}{8} g \longrightarrow \blacktriangleleft$$

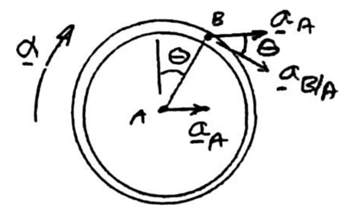
$$(\mathbf{a}_B)_y = \frac{1}{8} g \downarrow \blacktriangleleft$$



PROBLEM 16.115

A small clamp of mass m_B is attached at B to a hoop of mass m_h. Knowing that the system is released from rest and rolls without sliding, derive an expression for the angular acceleration of the hoop in terms of m_B, m_h, r, and θ.

SOLUTION



Kinematics:

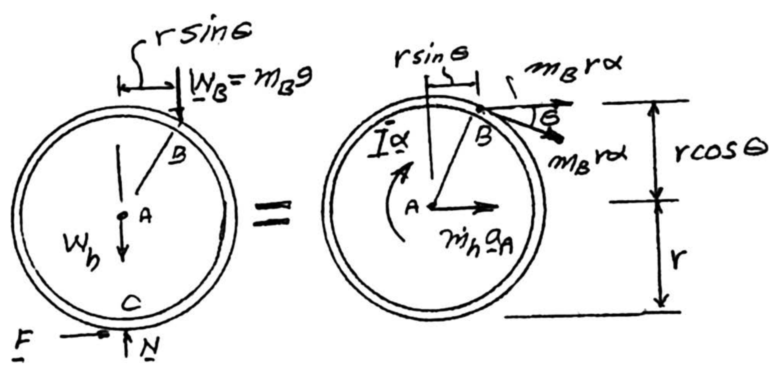
$$\mathbf{a}_A = r\alpha \longrightarrow \quad a_{B/A} = r\alpha \ \searrow \theta$$

$$\mathbf{a}_B = a_A \longrightarrow + \ a_{B/A} \ \searrow \theta$$

$$\mathbf{a}_B = r\alpha \longrightarrow + \ r\alpha \ \searrow \theta$$

Kinetics:

$$\overline{I} = m_h r^2$$



$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad W_B r\sin\theta = \overline{I}\alpha + m_h a_A r + m_B r\alpha(r + r\cos\theta) + m_B r\alpha\sin\theta(r\sin\theta)$$
$$+ m_B r\alpha\cos\theta(r + r\cos\theta)$$

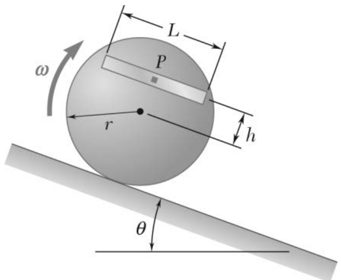
$$m_B g r\sin\theta = m_h r^2\alpha + m_h(r\alpha)r + m_B r\alpha(1+\cos\theta)(r + r\cos\theta)$$
$$+ m_B r\alpha\sin\theta(r\sin\theta)$$

$$m_B g r\sin\theta = 2m_h r^2\alpha + m_B r^2\alpha[(1+\cos\theta)^2 + \sin^2\theta]$$

$$= 2m_h r^2\alpha + m_B r^2\alpha[1 + 2\cos\theta + \cos^2\theta + \sin^2\theta]$$

$$m_B g r\sin\theta = r^2\alpha[2m_h + m_B(2 + 2\cos\theta)] \qquad \alpha = \frac{g}{2r}\frac{m_B\sin\theta}{m_h + m_B(1+\cos\theta)} \ \blacktriangleleft$$

PROBLEM 16.116



A 4-lb bar is attached to a 10-lb uniform cylinder by a square pin, P, as shown. Knowing that $r = 16$ in., $h = 8$ in., $\theta = 20°$, $L = 20$ in. and $\omega = 2$ rad/s at the instant shown, determine the reactions at P at this instant assuming that the cylinder rolls without sliding down the incline.

SOLUTION

Masses and moments of inertia.

Bar: $$m_B = \frac{W_B}{g} = \frac{4\text{lb}}{32.2} = 0.12422 \text{ slug}$$

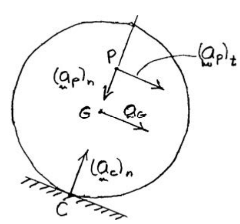
$$\bar{I}_B = \frac{1}{12} m_B L^2 = \frac{1}{12}\left(\frac{4 \text{ lb}}{32.2}\right)\left(\frac{20 \text{ in.}}{12}\right)^2 = 0.028755 \text{ slug}\cdot\text{ft}^2$$

Disk: $$m_D = \frac{W_D}{g} = \frac{10 \text{ lb}}{32.2} = 0.31056 \text{ slug}$$

$$\bar{I}_D = \frac{1}{2} m_C r^2 = \frac{1}{2}\left(\frac{10 \text{ lb}}{32.2}\right)\left(\frac{16 \text{ in}}{12}\right)^2 = 0.27605 \text{ slug}\cdot\text{ft}^2$$

Kinematics. Let Point C be the point of contact between the cylinder and the incline and Point G be the mass center of the cylinder without the bar. Assume that the mass center of the bar lies at Point P.

$$\boldsymbol{\alpha} = \alpha \circlearrowright. \quad \mathbf{a}_G = a_G \searrow 20°$$



For rolling without slipping, $$(a_C)_t = a_G - r\alpha = 0 \quad a_G = r\alpha$$

$$(a_P)_t = a_G + h\alpha$$

$$(a_P)_t = (r+h)\alpha$$

$$(a_P)_h = 0 + h\omega^2 = h\omega^2$$

Kinetics: Using the cylinder plus the bar as a free body,

$$+\circlearrowright \Sigma M_C = +\Sigma (M_C)_{\text{eff}}: \quad m_D g r \sin\theta + m_B g(r+h)\sin\theta$$

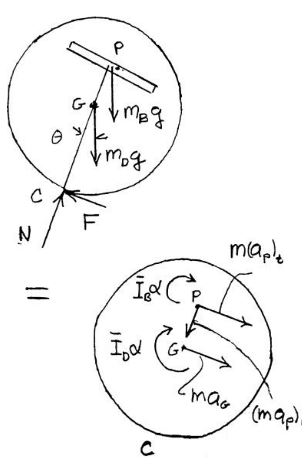
$$= \bar{I}_D\alpha + m_D a_G r + \bar{I}_B\alpha + m_B(a_P)_t(r+h)$$

$$= [\bar{I}_D + m_D r^2 + \bar{I}_B + m_B(r+h)^2]\alpha$$

$$\alpha = \frac{[m_D g r + m_B g(r+h)]\sin\theta}{\bar{I}_D + m_D r^2 + \bar{I}_B + m_B(r+h)^2}$$

$$= \frac{\left[(10)\left(\frac{16}{12}\right) + (4)\left(\frac{24}{12}\right)\right]\sin 20°}{0.27605 + \left(\frac{10}{32.2}\right)\left(\frac{16}{12}\right)^2 + (0.028755) + \left(\frac{4}{32.2}\right)\left(\frac{24}{12}\right)^2}$$

$$= 5.3896 \text{ rad/s}^2$$

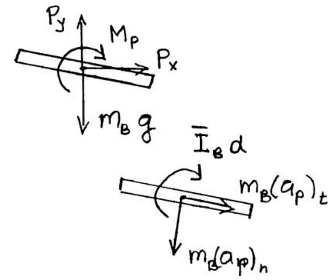


$$(a_P)_t = \left(\frac{24}{12}\right)(5.3896) = 10.7791 \text{ ft/s}^2$$

$$(a_P)_n = \left(\frac{8}{12}\right)(2)^2 = 2.6667 \text{ ft/s}^2$$

PROBLEM 16.116 (Continued)

Using the bar alone as a free body,



$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad P_x = m_B (a_P)_t \cos 20° - m_B (a_P)_n \sin 20°$$

$$= \left(\frac{4}{32.2}\right)(10.779)\cos 20° - \left(\frac{4}{32.2}\right)(2.6667)\sin 20°$$

$$P_x = 1.1450 \text{ lb}$$

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad P_y - m_B g = -m_B (a_P)_t \sin 20° - m_B (a_P)_n \cos 20°$$

$$P_y = (4) - \left(\frac{4}{32.2}\right)(10.779)\sin 20° - \left(\frac{4}{32.2}\right)(2.6667)\cos 20°$$

$$P_y = 3.2307 \text{ lb}$$

$$P = \sqrt{1.145^2 + 3.2307^2} = 3.4276 \text{ lb}$$

$$\tan \beta = \frac{3.2307}{1.145}$$

$$\beta = 70.5°$$

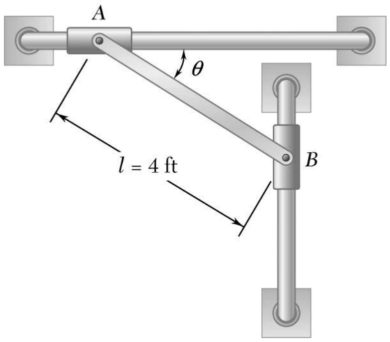
$\mathbf{P} = 3.43 \text{ lb} \measuredangle 70.5°$ ◀

Recognizing that P is the CG of the bar.

$$+\circlearrowright \Sigma M_P = \Sigma (M_P)_{\text{eff}}: \quad M_P = \bar{I}_B \alpha$$

$$= (0.028755)(5.3896)$$

$\mathbf{M}_P = 0.1550 \text{ ft} \cdot \text{lb} \circlearrowright$ ◀

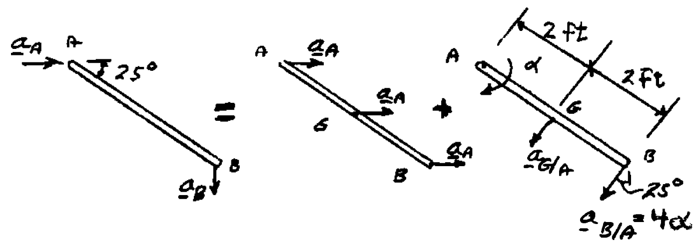


PROBLEM 16.117

The ends of the 20-lb uniform rod AB are attached to collars of negligible mass that slide without friction along fixed rods. If the rod is released from rest when $\theta = 25°$, determine immediately after release (*a*) the angular acceleration of the rod, (*b*) the reaction at A, (*c*) the reaction at B.

SOLUTION

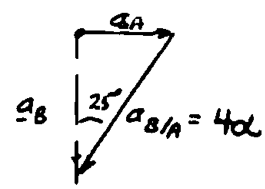
Kinematics: Assume α ↻ $\qquad\qquad \omega = 0$



$$\mathbf{a}_B \downarrow = \mathbf{a}_A + \mathbf{a}_{B/A} = [a_A \longrightarrow] + [4\alpha \swarrow 25°]$$

$$a_B = (4\alpha)\cos 25° = 3.6252\alpha$$

$$a_A = (4\alpha)\sin 25° = 1.6905\alpha$$



$$\mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A} = [a_A \longrightarrow] + [2\alpha \swarrow 25°]$$

$$\mathbf{a}_G = [1.6905\alpha \longrightarrow] + [2\alpha \swarrow 25°]$$

$$\bar{a}_x = (a_G)_x = [1.6905\alpha \longrightarrow] + [0.84524\alpha \longleftarrow]$$

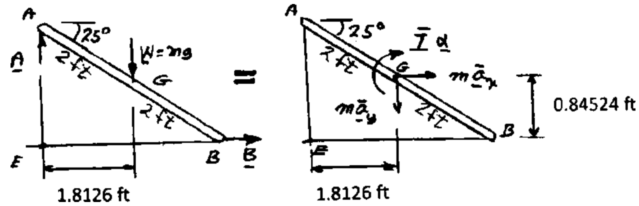
$$\bar{\mathbf{a}}_x = 0.84524\alpha \longrightarrow$$

$$\bar{a}_y = [2\alpha\cos 25° \downarrow] = 1.8126\alpha \downarrow$$

We have found for α ↻ $\qquad \bar{\mathbf{a}}_x = 0.84524\alpha \longrightarrow$

$$\bar{\mathbf{a}}_y = 1.8126\alpha \downarrow$$

Kinetics: $\qquad \bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}m(4\text{ ft})^2$



(*a*) Angular acceleration.

$+\circlearrowright \Sigma M_E = \Sigma(M_E)_{\text{eff}}$: $\quad mg(1.8126 \text{ ft}) = \bar{I}\alpha + m\bar{a}_x(0.84524 \text{ ft}) + m\bar{a}_y(1.8126 \text{ ft})$

$$mg(1.8126) = \frac{1}{12}m(4)^2\alpha + m(0.84524)^2\alpha + m(1.8126)^2\alpha$$

$$g(1.8126) = 5.3333\alpha \qquad \alpha = 0.33988g$$

$\boldsymbol{\alpha} = 10.944 \text{ rad/s}^2 \circlearrowright$ ◀

(*b*) $+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}$: $\quad A - mg = -m\bar{a}_y = m(1.8126\alpha)$

$$A - 20 = -\left(\frac{20}{32.2}\right)(1.8126)(10.944)$$

$$A = 20 - 12.321$$

$$= 7.6791 \text{ lb}$$

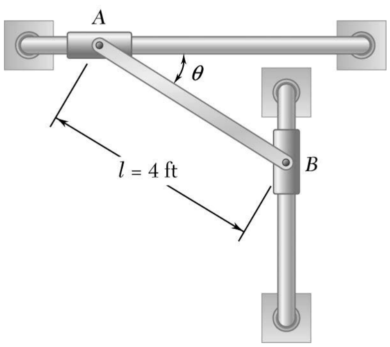
$\mathbf{A} = 7.68 \text{ lb} \uparrow$ ◀

(*c*) $\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}$: $\quad B = m\bar{a}_x = m(0.84524\alpha)$

$$B = \frac{20}{32.2}(0.84524)(10.944)$$

$$B = 5.7453 \text{ lb}$$

$\mathbf{B} = 5.75 \text{ lb} \longrightarrow$ ◀

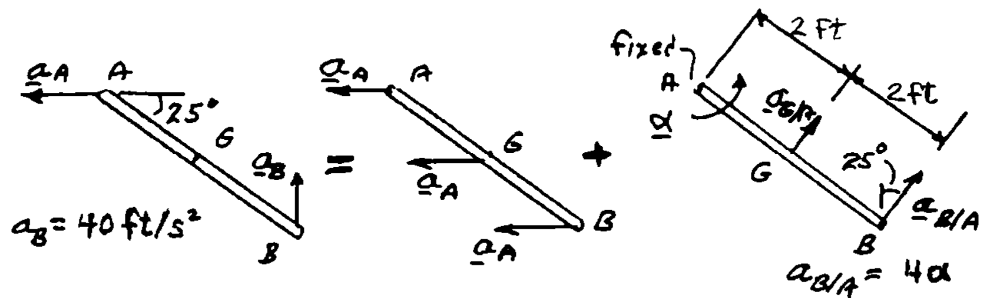


PROBLEM 16.118

The ends of the 20-lb uniform rod AB are attached to collars of negligible mass that slide without friction along fixed rods. A vertical force **P** is applied to collar B when $\theta = 25°$, causing the collar to start from rest with an upward acceleration of 40 ft/s^2. Determine (*a*) the force **P**, (*b*) the reaction at A.

SOLUTION

Kinematics: $\omega = 0$

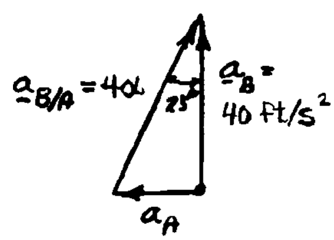


$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}; \quad [40 \text{ ft/s}^2 \uparrow] = [a_A \longleftarrow] + [4\alpha \measuredangle 25°]$$

$$a_B = a_{B/A}\cos 25°$$

$$40 \text{ ft/s}^2 = (4\alpha)\cos 25°$$

$$\boldsymbol{\alpha} = 11.034 \text{ rad/s}^2 \circlearrowleft$$



$$\mathbf{a}_A = 40\tan 25° = 18.6523 \text{ ft/s}^2 \longleftarrow$$

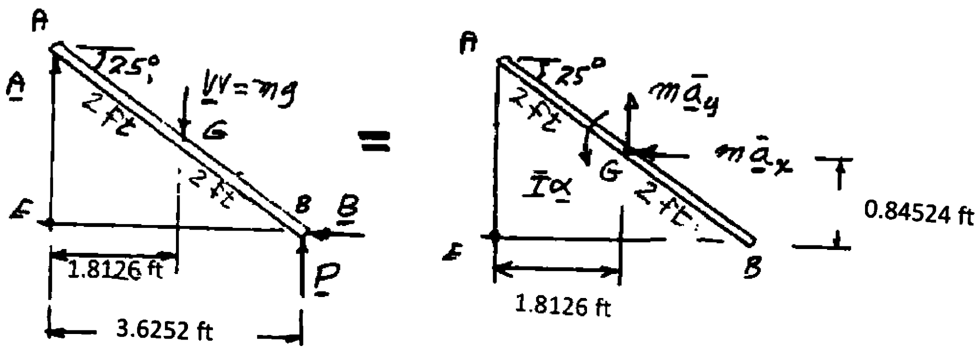
$$\mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A}: \quad \mathbf{a}_G = [18.652 \longleftarrow] + [2\alpha \measuredangle 25°]$$

$$\mathbf{a}_G = [18.652 \longleftarrow] + [2(11.034) \measuredangle 25°]$$

$$\bar{\mathbf{a}}_x = (\mathbf{a}_G)_x = 9.3262 \text{ ft/s}^2 \longleftarrow$$

$$\bar{\mathbf{a}}_y = (\mathbf{a}_G)_y = 20.00 \text{ ft/s}^2 \uparrow$$

<u>Kinetics</u>: $\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}m(4)^2$



(*a*) $+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{\text{eff}}$: $P(3.6252) - W(1.8126) = \bar{I}\alpha + m\bar{a}_x(0.84524) + m\bar{a}_y(1.8126)$

$$W = mg = 20 \text{ lb}$$

$$\bar{I}\alpha = \frac{1}{12}mL^2\alpha$$

$$= \frac{1}{12}\left(\frac{20}{32.2}\right)(4)^2(11.034)$$

$$= 9.1377 \text{ ft}\cdot\text{lb}$$

$$m\bar{a}_x = \left(\frac{20}{32.2}\right)(9.3262) = 5.7926 \text{ lb}$$

$$m\bar{a}_y = \left(\frac{20}{32.2}\right)(20) = 12.4224 \text{ lb}$$

$$P(3.6252) - (20)(1.8126) = 9.1377 + (5.7926)(0.84524) + (12.422)(1.8126)$$

$$P(3.6252) - 36.252 = 9.1377 + 4.8962 + 22.517$$

$$P(3.6252) = 72.8031$$

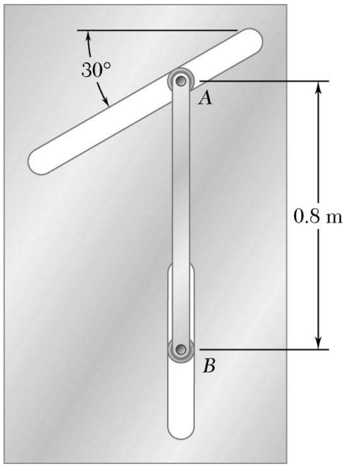
$$P = 20.082 \text{ lb}$$

P = 20.1 lb ↑ ◀

(*b*) $+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}$: $A - W + P = m\bar{a}_y$

$$A - 20 + 20.082 = 12.4224 \text{ lb}$$

$A = 12.34$ lb ↑ ◀

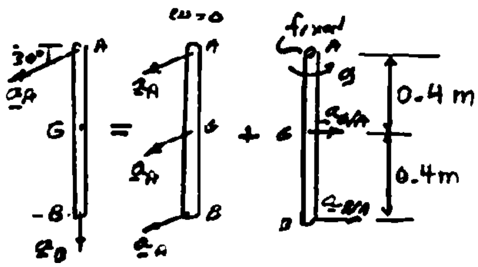


PROBLEM 16.119

The motion of the 3-kg uniform rod AB is guided by small wheels of negligible weight that roll along without friction in the slots shown. If the rod is released from rest in the position shown, determine immediately after release (*a*) the angular acceleration of the rod, (*b*) the reaction at B.

SOLUTION

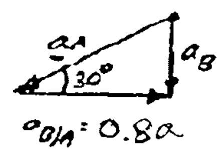
<u>Kinematics</u>:



$$\mathbf{a}_{G/A} = (0.4\text{ m})\alpha \longrightarrow$$

$$\mathbf{a}_{B/A} = (0.8\text{ m})\alpha \longrightarrow$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}: \quad [a_B \downarrow] = [a_A \swarrow 30°] + [0.8\alpha \longrightarrow]$$



$$\mathbf{a}_A = \frac{0.8\alpha}{\cos 30°} = 0.92376\alpha \swarrow 30°$$

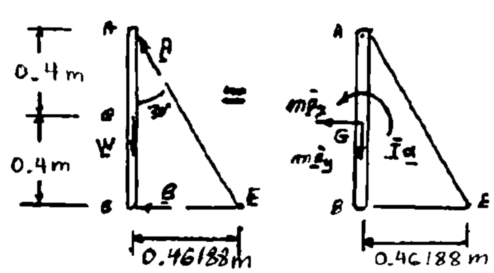
$$\mathbf{a}_B = (0.8\alpha)\tan 30° = 0.46188\alpha \downarrow$$

$$\bar{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A}; \quad \bar{\mathbf{a}} = [0.92376\alpha \swarrow 30°] + [0.4\alpha + \longrightarrow]$$

$$\bar{\mathbf{a}}_x = [0.8\alpha \longleftarrow] + [0.4\alpha \longrightarrow] = 0.4\alpha \longleftarrow$$

$$\mathbf{a}_y = [0.46188\alpha \downarrow] = 0.46188\alpha \downarrow$$

We have:



PROBLEM 16.119 (Continued)

$$\bar{\mathbf{a}}_x = 0.4\alpha \longleftarrow; \quad \mathbf{a}_y = 0.46188\alpha \downarrow$$

$$\bar{I} = \frac{1}{12} \cdot mL^2 = \frac{1}{12} \cdot (3 \text{ kg})(0.8 \text{ m})^2$$

$$\bar{I} = 0.16 \text{ kg} \cdot \text{m}^2$$

(*a*) Angular acceleration.

$$+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{\text{eff}}: \quad mg(0.46188 \text{ m}) = \bar{I}\alpha + m\bar{a}_x(0.4 \text{ m}) + m\bar{a}_y(0.46188 \text{ m})$$

$$3(9.81)(0.46188) = 0.16\alpha + 3(0.4)^2\alpha + 3(0.46188)^2\alpha$$

$$13.593 = 1.28\alpha$$

$$\alpha = 10.620 \text{ rad/s}^2$$

$\boldsymbol{\alpha} = 10.62 \text{ rad/s}^2 \circlearrowleft$ ◀

(*b*) Reaction at *B*.

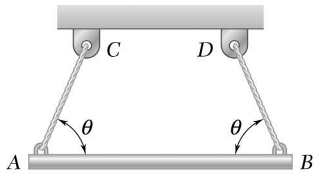
$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad B(0.8 \text{ m}) = -\bar{I}\alpha + m\bar{a}_x(0.4 \text{ m})$$

$$0.8B = -(0.16)(10.620) + 3(0.4)(10.620)(0.4)$$

$$0.8B = -1.6991 + 5.0974$$

$$B = 4.2479 \text{ N}$$

$\mathbf{B} = 4.25 \text{ N} \longleftarrow$ ◀



PROBLEM 16.120

A beam *AB* of length *L* and mass *m* is supported by two cables as shown. If cable *BD* breaks, determine at that instant the tension in the remaining cable as a function of its initial angular orientation θ.

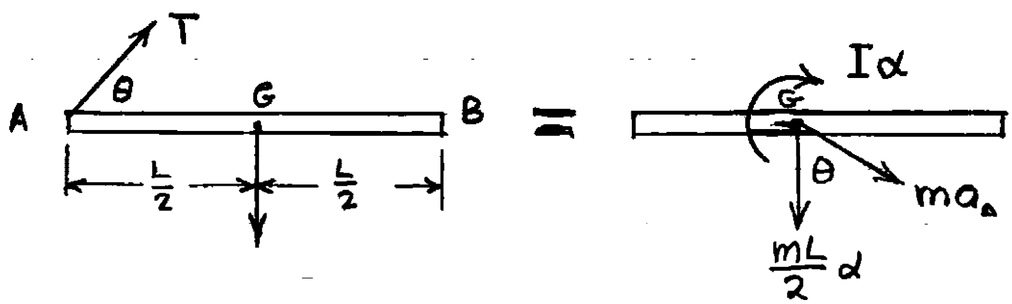
SOLUTION

Kinematics: At the instant just after the cable break,

$$\omega = 0 \qquad \boldsymbol{\alpha} = \alpha \circlearrowright$$

$$\mathbf{a}_G = \mathbf{a}_A + \frac{L}{2}\alpha\downarrow = \alpha_A \searrow \theta + \frac{L}{2}\alpha\downarrow$$

Kinetics: $I = \frac{1}{12}mL^2$



$$\circlearrowleft +\Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad (T\sin\theta)\frac{L}{2} = I\alpha = \frac{1}{12}mL^2\alpha$$

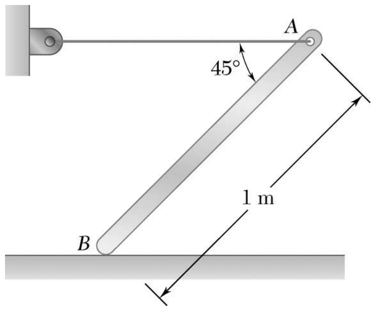
$$\alpha = \frac{6T\sin\theta}{mL}$$

$$\measuredangle\theta \ \Sigma F = \Sigma F_{\text{eff}}: \quad T - mg\sin\theta = -\frac{mL}{2}\alpha$$

$$= -\frac{mL}{2}\cdot\frac{6T\sin\theta}{mL}$$

$$= -3T\sin\theta$$

Solving for T, $$T = \frac{mg\sin\theta}{1+3\sin\theta}$$ ◀



PROBLEM 16.121

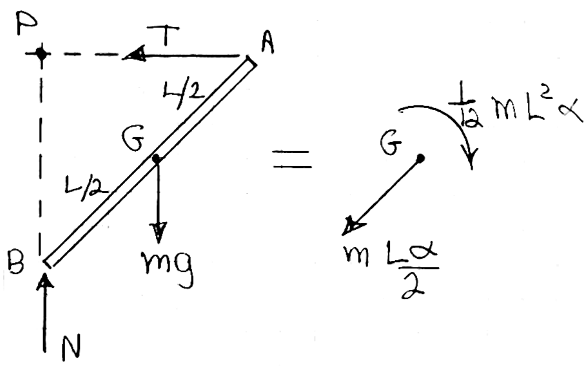
End A of a uniform 10-kg bar is attached to a horizontal rope and end B contacts a floor with negligible friction. Knowing that the bar is released from rest in the position shown, determine immediately after release (*a*) the angular acceleration of the bar, (*b*) the tension in the rope, (*c*) the reaction at B.

SOLUTION

Kinematics: $\boldsymbol{\alpha} = \alpha \circlearrowright$

$$[a_B \leftarrow] = [a_A \downarrow] + [L\alpha \searrow 45°] \qquad \mathbf{a}_A = 0.7071\, L\alpha \downarrow$$

$$\mathbf{a}_G = \mathbf{a}_A + \left[\frac{L\alpha}{2} \searrow 45°\right] = [0.7071\,\alpha \downarrow] + \left[\frac{L\alpha}{2} \searrow 45°\right] = \frac{L\alpha}{2} \searrow 45°$$



$$+\circlearrowright \Sigma M_P = \not{m} g \frac{L}{2}(0.7071)$$

$$= \frac{1}{12} \not{m} L^2 \alpha + \not{m} \frac{L}{2} \alpha \left(\frac{L}{2}\right)$$

$$\alpha = 0.7071 \left(\frac{3g}{2L}\right)$$

$$g = 9.81 \text{ m/s}^2, \qquad L = 1 \text{ m},$$

(*a*) $\boldsymbol{\alpha} = 10.405 \text{ rad/s}^2 \circlearrowright$ ◀

$$\overset{+}{\leftarrow} \Sigma F_x = T = \frac{mL\alpha}{2}(0.7071) = \frac{m\not{L}(0.5)}{2}\left(\frac{3g}{\not{L}}\right) = \frac{3}{8} mg = 36.788 \text{ N}$$

(*b*) $T = 36.8 \text{ N}$ ◀

$$+\uparrow \Sigma F_y = N - mg = -\frac{3}{8} mg, \qquad N = \frac{5}{8} mg = 61.313 \text{ N}$$

(*c*) $N = 61.3 \text{ N} \uparrow$ ◀

Alternate solution:

Kinematics (realizing that the rope constrains Point A to the $\mathbf{j}$-direction at the instant shown):

$$a_A \mathbf{j} = a_B \mathbf{i} + \alpha \mathbf{k}\, x (L\cos 45° \mathbf{i} + L\sin 45° \mathbf{j})$$
$$= a_B \mathbf{i} + (L\cos 45°)\alpha \mathbf{j} - (L\sin 45°)\alpha \mathbf{i}$$

x-components

$$0 = a_B - (L\sin 45°)\alpha$$
$$a_B = (L\sin 45°)\alpha$$

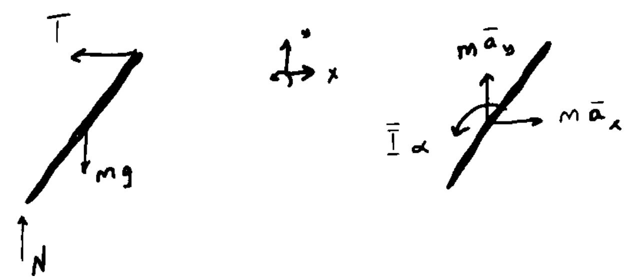
y-components

$$a_A = (L\cos 45°)\alpha$$

Acceleration of *CG*

$$\mathbf{a}_G = \mathbf{a}_B + \boldsymbol{\alpha} \times \mathbf{r}_{G/B} - \omega^2\mathbf{r}_{G/B} = \mathbf{a}_B - \boldsymbol{\alpha} \times \mathbf{r}_{G/B}$$
$$a_G = (L\sin 45°)\alpha\mathbf{i} + \alpha\mathbf{k} \times \left(\frac{L}{2}\cos(45°)\mathbf{i} + \frac{L}{2}\sin(45°)\mathbf{j}\right)$$
$$= (L\sin 45)\alpha\mathbf{i} + \left(\frac{L}{2}\cos(45°)\alpha\mathbf{j} - \frac{L}{2}\sin(45°)\alpha\mathbf{i}\right)$$
$$a_G = \frac{L}{2}\sin 45°\alpha\,\mathbf{i} + \frac{L}{2}\cos 45°\alpha\mathbf{j}$$

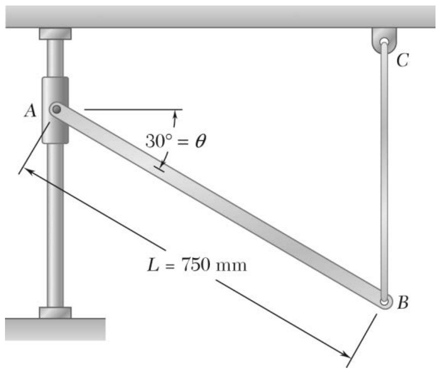
Kinetics:



Equations of motion:

$$\Sigma F_x = m\bar{a}_x$$
$$-T = m\left(\frac{L}{2}\sin 45°\right)\alpha \tag{1}$$
$$\Sigma F_y = m\bar{a}_y$$
$$N - mg = m\left(\frac{L}{2}\cos 45°\right)\alpha$$
$$N = mg + m\left(\frac{L}{2}\cos 45\right)\alpha \tag{2}$$
$$\Sigma M_G = \bar{T}\alpha$$
$$T\left(\frac{L}{2}\right)\sin 45° - N\left(\frac{L}{2}\cos 45°\right) = \frac{1}{12}mL^2\alpha \tag{3}$$

Solving the three simultaneous equations gives the same results as above.

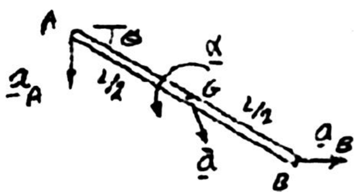


PROBLEM 16.122

End A of the 8-kg uniform rod AB is attached to a collar that can slide without friction on a vertical rod. End B of the rod is attached to a vertical cable BC. If the rod is released from rest in the position shown, determine immediately after release (*a*) the angular acceleration of the rod, (*b*) the reaction at A.

SOLUTION

Kinematics:



$$\omega = 0$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}; \quad [a_B \longrightarrow] = [a_A \downarrow] + [L\alpha \measuredangle \theta]$$

$$+\downarrow 0 = a_A - L\alpha\cos\theta$$

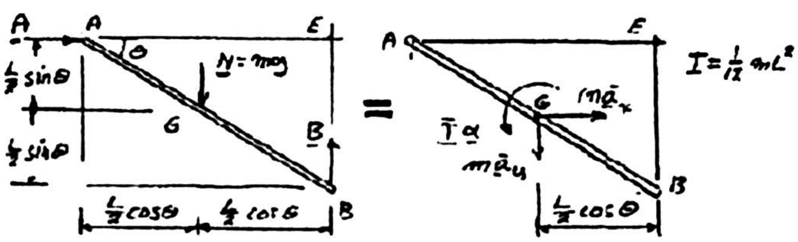
$$\mathbf{a}_A = L\alpha\cos\theta \downarrow$$

$$\bar{\mathbf{a}} = \mathbf{a}_A + \mathbf{a}_{G/A}$$

$$\bar{\mathbf{a}} = [L\alpha\cos\theta \downarrow] + \left[\frac{L}{2}\alpha \measuredangle \theta\right]$$

$$\bar{\mathbf{a}}_x = \frac{L}{2}\alpha\sin\theta \longrightarrow; \quad \mathbf{a}_y = \frac{L}{2}\alpha\cos\theta \uparrow$$

Kinetics:



$$+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{\text{eff}}: \quad mg\frac{L}{2}\cos\theta = I\alpha + m\bar{a}_x\left(\frac{L}{2}\sin\theta\right) + m\bar{a}_y\left(\frac{L}{2}\cos\theta\right)$$

$$mg\frac{L}{2}\cos\theta = \frac{1}{12}mL^2\alpha + m\left(\frac{L}{2}\sin\theta\right)^2\alpha + m\left(\frac{L}{2}\cos\theta\right)^2\alpha$$

$$mg\frac{L}{2}\cos\theta = \frac{1}{3}mL^2\alpha \qquad \boldsymbol{\alpha} = \frac{3}{2}\frac{g}{L}\cos\theta \circlearrowleft$$

PROBLEM 16.122 (Continued)

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad A = m\bar{a}_x = m\frac{L}{2}\alpha\sin\theta$$

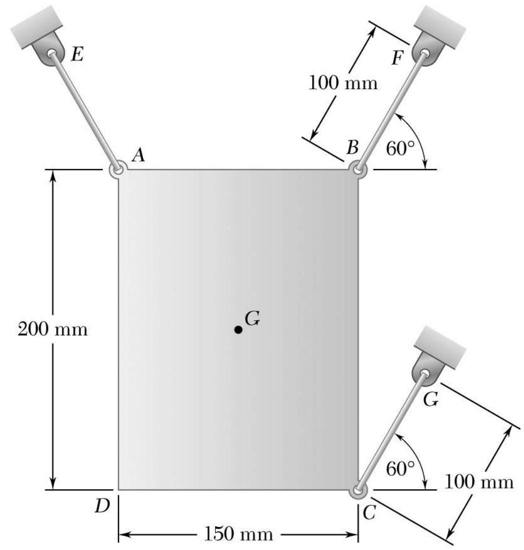
$$A = m\frac{L}{2}\left(\frac{3}{2}\frac{g}{L}\cos\theta\right)\sin\theta$$

$$\mathbf{A} = \frac{3}{4}mg\sin\theta\cos\theta \longrightarrow$$

Data: $m = 8\text{ kg}, \quad \theta = 30°, \quad L = 0.75\text{ m}$

(*a*) Angular acceleration. $\alpha = \frac{3}{2}\frac{9.81\text{ m/s}^2}{0.75\text{ m}}\cos 30°$ $\boldsymbol{\alpha} = 16.99\text{ rad/s}^2 \circlearrowright$ ◀

(*b*) Reaction at *A*. $A = \frac{3}{4}(8\text{ kg})(9.81\text{ m/s}^2)\sin 30°\cos 30°$ $\mathbf{A} = 25.5\text{ N} \longrightarrow$ ◀



PROBLEM 16.123

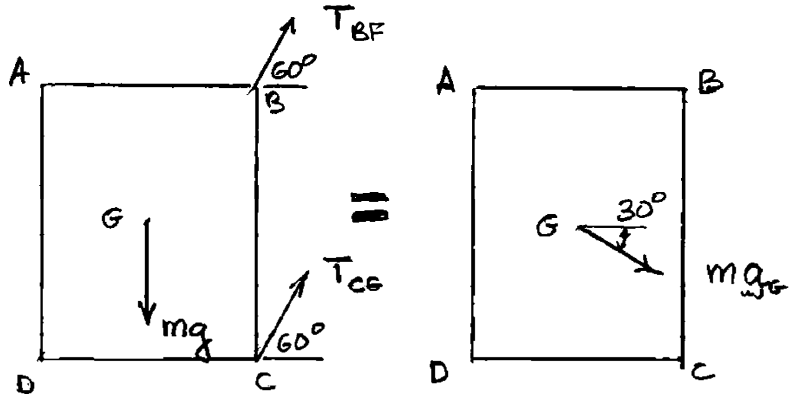
A uniform thin plate *ABCD* has a mass of 8 kg and is held in position by three inextensible cords *AE*, *BF*, and *CG*. If cord *AE* is cut, determine at that instant (*a*) if the plate is undergoing translation or general plane motion, (*b*) the tension in cords *BF* and *CG*.

SOLUTION

Immediately after cord *AE* breaks, $\omega = 0$.

(*a*) Assume that the cords *BF* and *CG* constrain the plate to undergo curvilinear translation, making $\boldsymbol{\alpha} = 0$.

$$\mathbf{a}_G = \bar{a} \searrow 30°$$



$$+\searrow 30° \Sigma F = \Sigma F_{\text{eff}}: \quad mg \sin 30° = ma_G$$

$$a_G = g \sin 30°$$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{\text{eff}}: \quad 0.075mg + 0.200T_{CG} \cos 60° = mg \sin 30°(0.100 + 0.075 \sin 30°)$$

$$0.100T_{CG} = -0.00625mg \quad T_{CG} = -0.0625mg$$

Since T_C is negative, the cord becomes slack so that the plate undergoes general plane motion with $T_{CG} = 0$.

general plane motion ◀

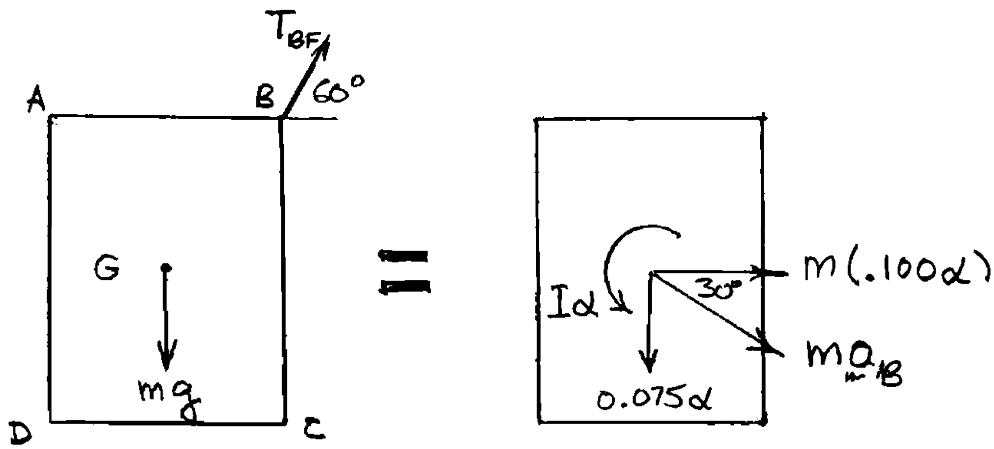
(*b*) *Kinematics*:

$$\mathbf{a}_B = a_A \measuredangle 30°$$

$$\boldsymbol{\alpha} = \alpha \circlearrowleft$$

$$\mathbf{a}_G = \mathbf{a}_B + 0.100\alpha \longrightarrow + 0.075\alpha \downarrow$$

Kinetics:



$m = 8$ kg

$$I = \frac{1}{12}(8\text{ kg})[(0.150\text{ m})^2 + (0.200\text{ m})^2] = 0.041667\text{ kg}\cdot\text{m}^2$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad T_{BF}\cos 60° = (8\text{ kg})(0.100\alpha) + (8\text{ kg})a_B\cos 30° \qquad (1)$$

$$+\!\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad T_{BF}\sin 60° - (8\text{ kg})(9.81\text{ m/s}^2)$$

$$= -(8\text{ kg})(0.075\alpha) - (8\text{ kg})a_B\sin 30° \qquad (2)$$

$$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad 0.075T_{BF}\sin 60° - 0.100T_{BF}\cos 60° = I\alpha$$

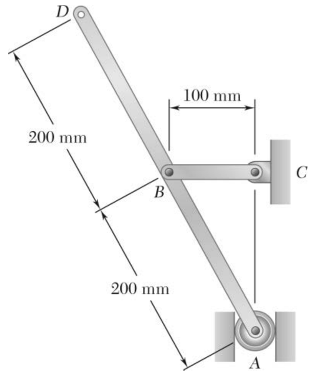
$$0.014952\, T_{BF} = 0.041667\alpha \qquad (3)$$

Solving Eqs. (1), (2), and (3) simultaneously,

$$T_{BF} = 65.168\text{ N}, \quad \alpha = 23.385\text{ rad/s}^2, \quad a_B = 2.0028\text{ m/s}^2$$

Tension in cords: $T_{BF} = 65.2$ N ◀

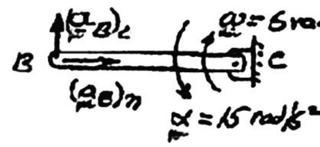
$T_{CG} = 0$ ◀



PROBLEM 16.124

The 4-kg uniform rod *ABD* is attached to the crank *BC* and is fitted with a small wheel that can roll without friction along a vertical slot. Knowing that at the instant shown crank *BC* rotates with an angular velocity of 6 rad/s clockwise and an angular acceleration of 15 rad/s^2 counterclockwise, determine the reaction at *A*.

SOLUTION

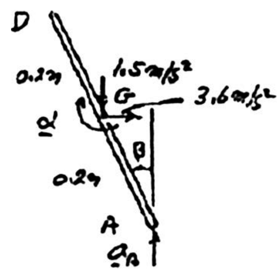


Crank *BC*:

$$BC = 0.1 \text{ m}$$

$$(a_B)_t = (BC)\alpha = (0.1 \text{ m})(15 \text{ rad/s}^2) = 1.5 \text{ m/s}^2 \qquad (\mathbf{a}_B)_t = 1.5 \text{ rad/s}^2 \downarrow$$

$$(a_B)_n = (BC)\omega^2 = (0.1 \text{ m})(6 \text{ rad/s})^2 = 3.6 \text{ m/s}^2 \qquad (\mathbf{a}_B)_n = 3.6 \text{ m/s}^2 \longrightarrow$$



Rod *ABD*:

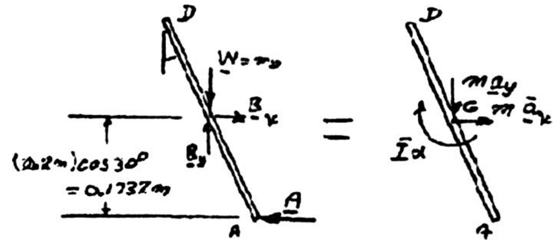
$$\theta = \sin^{-1}\frac{BC}{AB} = \sin^{-1}\frac{0.1 \text{ m}}{0.2 \text{ m}} = 30°$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/G}$$

$$[a_A \uparrow] = [1.5 \downarrow + 3.6 \longrightarrow] + [0.2\alpha \nearrow \beta]$$

$$\xrightarrow{+} \; 0 = 3.6 - (0.2\alpha)\cos\beta$$

$$\alpha = \frac{3.6}{0.2\cos\beta} = \frac{18}{\cos 30°} = 20.78 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 20.78 \text{ rad/s}^2 \circlearrowright$$

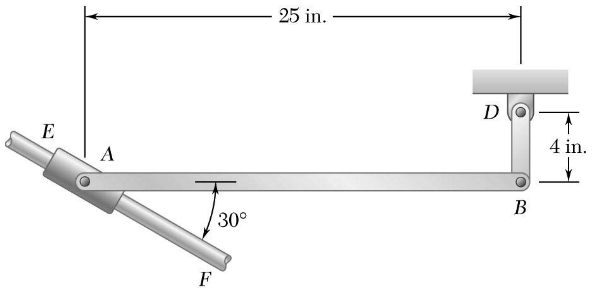


Kinetics:

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad A(0.1732 \text{ m}) = I\alpha = \frac{1}{12}mL^2\alpha$$

$$= \frac{1}{12}(4 \text{ kg})(0.4 \text{ m})^2(20.78 \text{ rad/s}^2)$$

$$A = 6.399 \text{ N} \qquad \mathbf{A} = 6.40 \text{ N} \longleftarrow \blacktriangleleft$$

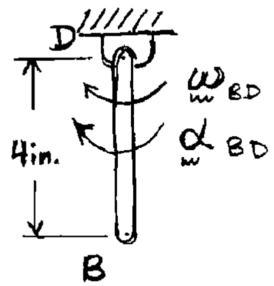


PROBLEM 16.125

The 7-lb uniform rod *AB* is connected to crank *BD* and to a collar of negligible weight, which can slide freely along rod *EF*. Knowing that in the position shown crank *BD* rotates with an angular velocity of 15 rad/s and an angular acceleration of 60 rad/s^2, both clockwise, determine the reaction at *A*.

SOLUTION

Crank BD:



$$\boldsymbol{\omega}_{BD} = 15 \text{ rad/s}, \quad \mathbf{v}_B = \left(\frac{4}{12}\text{ ft}\right)(15 \text{ rad/s}) = 5 \text{ ft/s} \longleftarrow$$

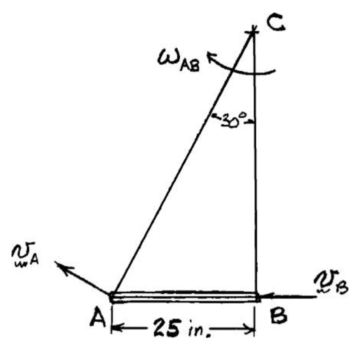
$$\boldsymbol{\alpha}_{BD} = 60 \text{ rad/s}^2$$

$$(\mathbf{a}_B)_x = \left(\frac{4}{12}\text{ ft}\right)(60 \text{ rad/s}^2) = 20 \text{ ft/s}^2 \longleftarrow$$

$$(\mathbf{a}_B)_y = \left(\frac{4}{12}\text{ ft}\right)(15 \text{ rad/s})^2 = 75 \text{ ft/s}^2 \uparrow$$

Rod AB:

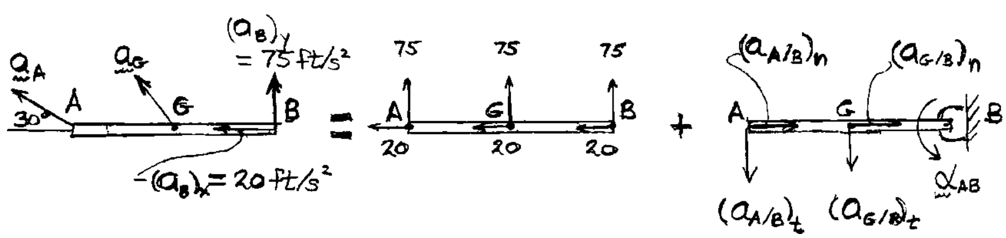
<u>Velocity</u>: Instantaneous center at *C*.



$$CB = \left(\frac{25}{12}\text{ ft}\right)/\tan 30° = 3.6084 \text{ ft}$$

$$\boldsymbol{\omega}_{AB} = \frac{v_B}{CB} = \frac{5 \text{ ft/s}}{3.6084 \text{ ft}} = 1.3856 \text{ rad/s} \circlearrowright$$

<u>Acceleration</u>:



$$(\mathbf{a}_{A/B})_t = (AB)\alpha_{AB} = \frac{25}{12}\alpha_{AB} \downarrow$$

$$(\mathbf{a}_{A/B})_n = (AB)\omega_{AB}^2 = \left(\frac{25}{12}\right)(1.3856)^2 = 4 \text{ ft/s}^2 \longrightarrow$$

$$(\mathbf{a}_{G/B})_t = (GB)\alpha_{AB} \downarrow = \frac{12.5}{12}\alpha_{AB} \downarrow$$

PROBLEM 16.125 (Continued)

$$(\mathbf{a}_{G/B})_n \longrightarrow = (GB)\omega_{AB}^2 = \left(\frac{12.5}{12}\right)(1.3856)^2 = 2 \text{ ft/s}^2 \longrightarrow$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{B/A} = \mathbf{a}_B + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{G/A})_n$$

$$[a_A \searrow 30°] = [20 \longleftarrow] + [75 \uparrow] + \left[\frac{25}{12}\alpha_{AB} \downarrow\right] + [4 \longrightarrow]$$

$$\overset{+}{\longleftarrow}\ a_A \cos 30° = 20 - 4; \qquad \mathbf{a}_A = 18.475 \text{ ft/s}^2 \searrow 30°$$

$$+\uparrow (18.475)\sin 30° = 75 - \frac{25}{12}\alpha_{AB}; \quad \boldsymbol{\alpha}_{AB} = 31.566 \text{ rad/s}^2 \circlearrowright$$

$$\overline{\mathbf{a}} = \mathbf{a}_B + \mathbf{a}_{G/B} = \mathbf{a}_B + (\mathbf{a}_{G/B})_t + (\mathbf{a}_{G/B})_n$$

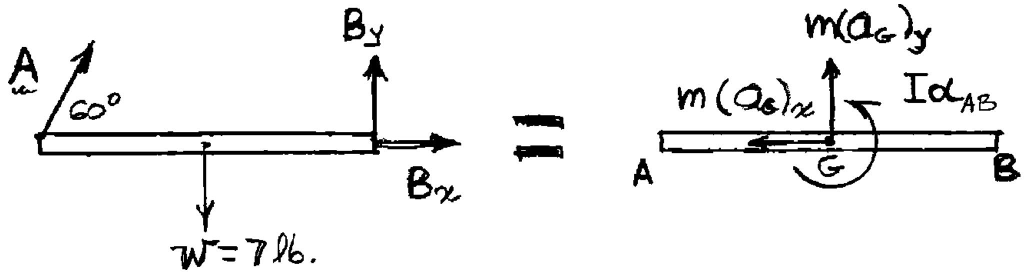
$$\overline{\mathbf{a}} = [20 \longleftarrow] + [75 \uparrow] + [\tfrac{12.5}{12}(31.566) \downarrow] + [2 \longrightarrow]$$

$$\overset{+}{\longleftarrow}\ \overline{a}_x = 20 - 2 = 18; \qquad \overline{\mathbf{a}}_x = 18 \text{ ft/s}^2 \longleftarrow$$

$$+\uparrow\ a_y = 75 - 32.881 = 42.119; \quad \overline{\mathbf{a}}_y = 42.119 \text{ ft/s}^2 \uparrow$$

Kinetics:

$$\overline{I} = \frac{1}{12}m(AB)^2 = \frac{7 \text{ lb}}{12(32.2)}\left(\frac{25}{12}\text{ ft}\right)^2 = 0.078628 \text{ slug}\cdot\text{ft}^2$$

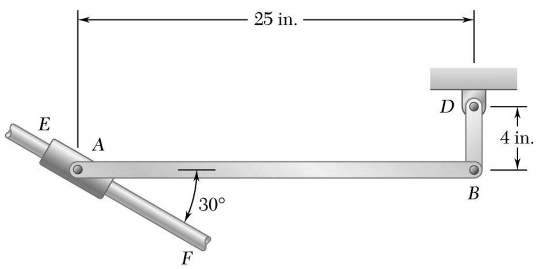


$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad (A\sin 60°)\left(\frac{25}{12}\text{ ft}\right) - mg\left(\frac{12.5}{12}\text{ ft}\right) = -\overline{I}\alpha_{AB} + m\overline{a}_y\left(\frac{12.5}{12}\text{ ft}\right)$$

$$1.8042A - (7 \text{ lb})\left(\frac{12.5}{12}\text{ ft}\right) = -(0.078628 \text{ slug}\cdot\text{ft}^2)(31.566 \text{ rad/s}^2) + \left(\frac{7}{32.2}\text{ slug}\right)(42.119 \text{ ft/s}^2)\left(\frac{12.5}{12}\text{ ft}\right)$$

$$1.8042A - 7.2917 = -2.4820 + 9.5378$$

$$A = 7.9522 \text{ lb} \qquad \mathbf{A} = 7.95 \text{ lb} \measuredangle 60° \blacktriangleleft$$



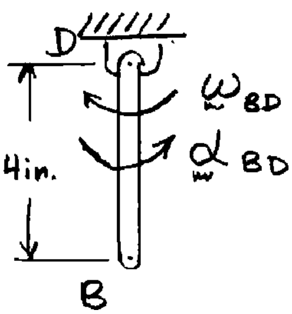
PROBLEM 16.126

In Problem 16.125, determine the reaction at *A*, knowing that in the position shown crank *BD* rotates with an angular velocity of 15 rad/s clockwise and an angular acceleration of 60 rad/s^2 counterclockwise.

PROBLEM 16.125 The 7-lb uniform rod *AB* is connected to crank *BD* and to a collar of negligible weight, which can slide freely along rod *EF*. Knowing that in the position shown crank *BD* rotates with an angular velocity of 15 rad/s and an angular acceleration of 60 rad/s^2, both clockwise, determine the reaction at *A*.

SOLUTION

Crank BD:



$$\boldsymbol{\omega}_{BD} = 15 \text{ rad/s},$$

$$\mathbf{v}_B = \left(\frac{4}{12} \text{ ft}\right)(15 \text{ rad/s}) = 5 \text{ ft/s} \longleftarrow$$

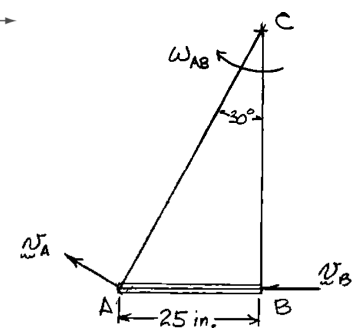
$$\boldsymbol{\alpha}_{BD} = 60 \text{ rad/s}^2$$

$$(\mathbf{a}_B)_x = \left(\frac{4}{12} \text{ ft}\right)(60 \text{ rad/s}^2) = 20 \text{ ft/s}^2 \longrightarrow$$

$$(\mathbf{a}_B)_y = \left(\frac{4}{12} \text{ ft}\right)(15 \text{ rad/s})^2 = 75 \text{ ft/s}^2 \uparrow$$

Rod AB:

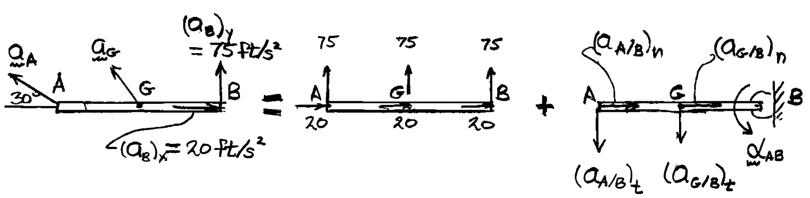
Velocity: Instantaneous center at *C*.



$$CB = \left(\frac{25}{12} \text{ ft}\right)/\tan 30° = 3.6084 \text{ ft}$$

$$\boldsymbol{\omega}_{AB} = \frac{v_B}{CB} = \frac{5 \text{ ft/s}}{3.6084 \text{ ft}} = 1.3856 \text{ rad/s} \circlearrowright$$

Acceleration:



$$(\mathbf{a}_{A/B})_t = (AB)\alpha_{AB} = \frac{25}{12}\alpha_{AB} \downarrow$$

$$(\mathbf{a}_{A/B})_n = (AB)\omega_{AB}^2 = \left(\frac{25}{12}\right)(1.3856)^2 = 4 \text{ ft/s}^2 \longrightarrow$$

PROBLEM 16.126 (Continued)

$$(\mathbf{a}_{G/B})_t = (GB)\alpha_{AB} = \frac{12.5}{12}\alpha_{AB}\downarrow$$

$$(\mathbf{a}_{G/B})_n = (GB)\omega_{AB}^2 = \left(\frac{12.5}{12}\right)(1.3856)^2 = 2\text{ ft/s}^2 \longrightarrow$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{B/A} = \mathbf{a}_B + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$$

$$[\mathbf{a}_A \searrow 30°] = [20 \longrightarrow] + [75\uparrow] + \left[\frac{25}{12}\alpha_{AB}\downarrow\right] + [4\longrightarrow]$$

$\xrightarrow{+}$: $a_A \cos 30° = 20 + 4;$ $\quad \mathbf{a}_A = 27.713\text{ ft/s}^2 \nwarrow 30°$

$+\downarrow$: $(27.713)\sin 30° = -75 + \frac{25}{12}\alpha_{AB};$ $\quad \boldsymbol{\alpha}_{AB} = 42.651\text{ rad/s}^2 \circlearrowright$

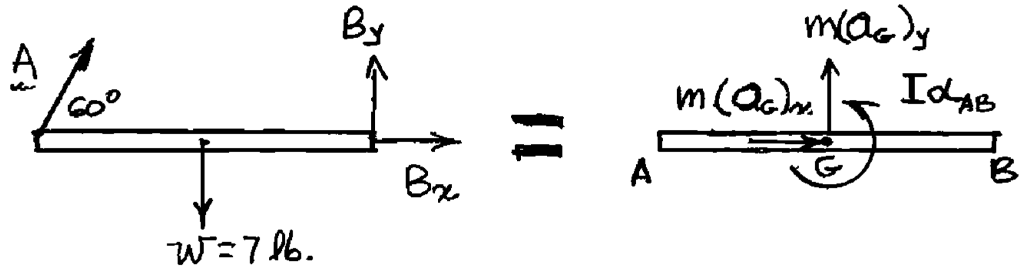
$$\bar{\mathbf{a}} = \mathbf{a}_B + \mathbf{a}_{G/B} = \mathbf{a}_B + (\mathbf{a}_{G/B})_t + (\mathbf{a}_{G/A})_n$$

$$\bar{\mathbf{a}} = [20 \longrightarrow] + [75\uparrow] + \left[\frac{12.5}{12}(42.651)\downarrow\right] + [2\longrightarrow]$$

$\xrightarrow{+}$ $a_x = 20 + 2 = 22;$ $\quad \mathbf{a}_x = 22\text{ ft/s}^2 \longrightarrow$

$+\uparrow$ $a_y = 75 - 44.428 = 30.572;$ $\quad \mathbf{a}_y = 30.6\text{ ft/s}^2\uparrow$

Kinetics: $\quad \bar{I} = \frac{7\text{ lb}}{12(32.2)}\left(\frac{25}{12}\text{ ft}\right)^2 = 0.078628\text{ slug}\cdot\text{ft}^2$



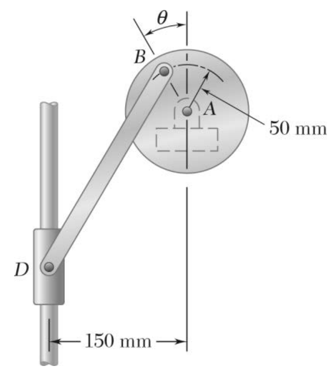
$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}$: $\quad (A\sin 60°)\left(\frac{25}{12}\text{ ft}\right) - mg\left(\frac{12.5}{12}\text{ ft}\right) = -\bar{I}\alpha_{AB} + m\bar{a}_y\left(\frac{12.5}{12}\text{ ft}\right)$

$$1.8042A - (7\text{ lb})\left(\frac{12.5}{12}\text{ ft}\right) = -(0.078628\text{ slug}\cdot\text{ft}^2)(42.651\text{ rad/s}^2) + \left(\frac{7}{32.2}\text{ slug}\right)(30.572\text{ ft/s}^2)\left(\frac{12.5}{12}\text{ ft}\right)$$

$$1.8042A - 7.2917 = -3.3536 + 6.9230$$

$$A = 6.0198\text{ lb}$$

$\mathbf{A} = 6.02\text{ lb} \measuredangle 60°$ ◀

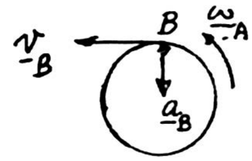


PROBLEM 16.127

The 250-mm uniform rod *BD*, of mass 5 kg, is connected as shown to disk *A* and to a collar of negligible mass, which may slide freely along a vertical rod. Knowing that disk *A* rotates counterclockwise at a constant rate of 500 rpm, determine the reactions at *D* when $\theta = 0$.

SOLUTION

Kinematics: For disk *A*: $(\alpha_A = 0)$

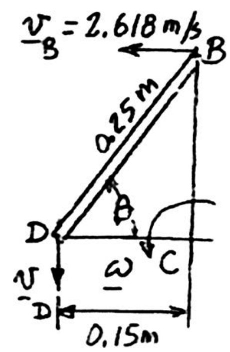


$$\omega_A = 500 \text{ rpm} = 52.36 \text{ rad/s}$$

$$v_B = r\omega_A = (0.05 \text{ m})(52.36 \text{ rad/s})$$
$$= 2.618 \text{ m/s}$$

$$a_B = r\omega_A^2 = (0.05 \text{ m})(52.36 \text{ rad/s}^2)$$
$$= 137.08 \text{ m/s}$$

For rod (velocities)



$$BC = \sqrt{(0.25)^2 - (0.15)^2}$$
$$= 0.20 \text{ m}$$

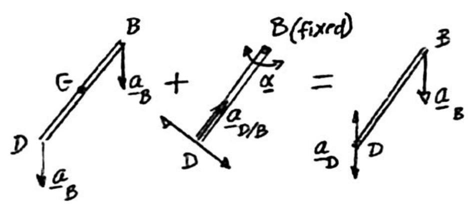
$$\cos\beta = \frac{0.15}{0.25} = \frac{3}{5}$$

$$\sin\beta = \frac{0.20}{0.25} = \frac{4}{5}$$

$$\omega = \frac{v_B}{BC} = \frac{2.618 \text{ m/s}}{0.20 \text{ m}}$$

$$\boldsymbol{\omega} = 13.09 \text{ rad/s} \circlearrowleft$$

Kinematics of rod (accelerations)

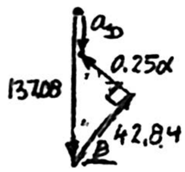


$$\mathbf{a}_B + \mathbf{a}_{D/B} = \mathbf{a}_D$$

$$[a_B \downarrow] + [(a_{D/B})_n \measuredangle \beta] + [(a_{D/B})_t \searrow \beta] = a_D \uparrow$$

$$[137.08 \downarrow] + [0.25(13.09)^2 \measuredangle \beta] + [0.25\alpha \searrow \beta] = a_D \uparrow$$

x components:



$$\frac{3}{5}(42.84) - \frac{4}{5}(0.25\alpha) = 0$$

$$25.704 - 0.2\alpha = 0 \qquad \boldsymbol{\alpha} = 128.52 \text{ rad/s}^2 \circlearrowright$$

$$\bar{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_B + \mathbf{a}_{G/B}$$

$$= [137.08 \downarrow] + [0.125(13.09)^2 \nearrow] + [0.125(128.52) \nwarrow]$$

$$= [137.08 \downarrow] + [21.42 \measuredangle \beta] + [16.065 \nwarrow \beta]$$

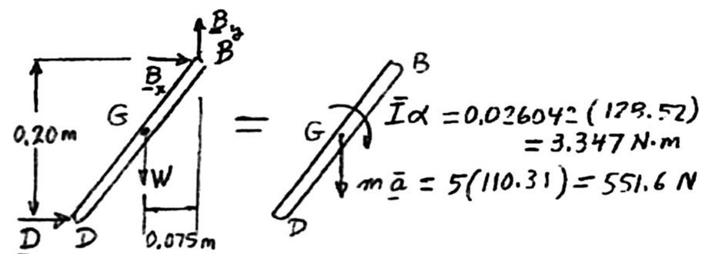
$$\xrightarrow{+} \bar{a}_x = \frac{3}{5}(21.42) - \frac{4}{5}(16.065) = 0$$

$$+\downarrow \bar{a}_y = 137.08 - \frac{4}{5}(21.42) - \frac{3}{5}(16.065) = 110.31 \text{ m/s}^2$$

Kinetics

$$\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}(5 \text{ kg})(0.25 \text{ m})^2$$

$$= 0.026042 \text{ kg} \cdot \text{m}^2$$



$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad D(0.20 \text{ m}) + W(0.075 \text{ m}) = m\bar{a}(0.075) - \bar{I}\alpha$$

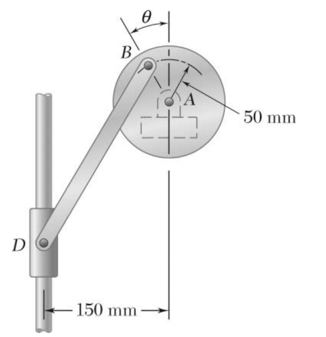
$$0.2D + 5(9.81)(0.075) = 551.6(0.075) - 3.347$$

$$0.2D = 41.370 - 3.347 - 3.679 = 34.344$$

$$D = 171.7 \text{ N}$$

$$\mathbf{D} = 171.7 \text{ N} \longrightarrow \blacktriangleleft$$

PROBLEM 16.128

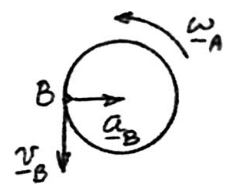


Solve Problem 16.127 when $\theta = 90°$.

PROBLEM 16.127 The 250-mm uniform rod *BD*, of mass 5 kg, is connected as shown to disk *A* and to a collar of negligible mass, which may slide freely along a vertical rod. Knowing that disk *A* rotates counterclockwise at a constants rate of 500 rpm, determine the reactions at *D* when $\theta = 0$.

SOLUTION

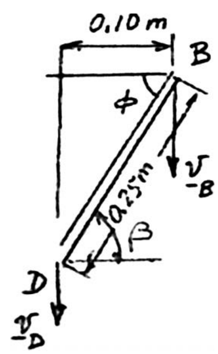
<u>Kinematics</u>: *For disk A*: $(\alpha_A = 0)$



$$\omega_A = 500 \text{ rpm} = 52.36 \text{ rad/s}$$

$$v_B = r\omega_A = (0.05 \text{ m})(52.36 \text{ rad/s}) = 2.618 \text{ m/s}$$

$$a_B = r\omega_A^2 = (0.05 \text{ m})(52.36 \text{ rad/s})^2 = 137.08 \text{ m/s}^2$$



<u>For rod (velocities)</u>

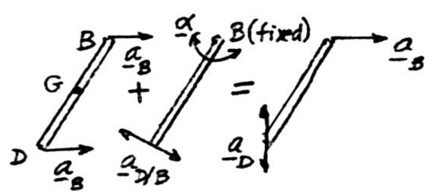
Since $\mathbf{v}_D$ is parallel to, $\mathbf{v}_B$

we have <u>$\omega = 0$</u>

We also note that $\cos\phi = \dfrac{0.10}{0.25}$

$$\phi = 66.42°$$

<u>For rod (accelerations)</u>



$$(a_{D/B})_n = 0$$

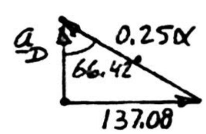
Since $\omega = 0$

$$\mathbf{a}_B + \mathbf{a}_{D/B} = \mathbf{a}_D$$

$$137.08 \rightarrow +0.25\alpha \searrow \phi = a_D \uparrow$$

$$(0.25\alpha)\sin 66.42° = 137.08$$

$$\boldsymbol{\alpha} = 598.3 \text{ rad/s}^2 \circlearrowright$$



$$\bar{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_B + \mathbf{a}_{G/B}$$

$$\xrightarrow{+} \bar{a}_x = 137.08 - 0.125(598.3)\sin 66.42° = +68.54 \text{ m/s}^2$$

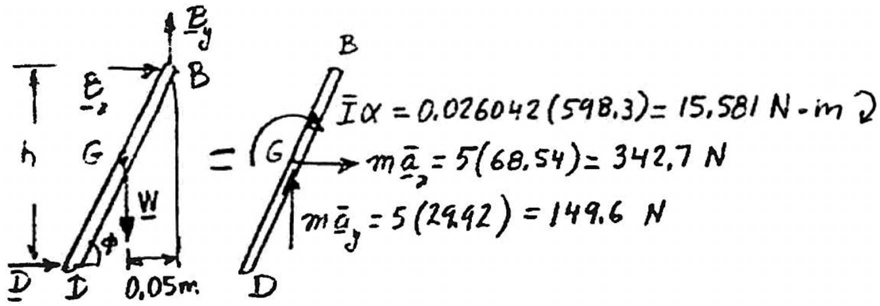
$$+\uparrow \bar{a}_y = 0.125(598.3)\cos 66.42° = +29.92 \text{ m/s}^2$$

PROBLEM 16.128 (Continued)

Summary of kinematics: $\boldsymbol{\alpha} = 598.3 \text{ rad/s}^2$ ↻, $\overline{\mathbf{a}}_x = 68.54 \text{ m/s}^2 \longrightarrow$, $\overline{\mathbf{a}}_y = 29.92 \text{ m/s}^2 \uparrow$

$$\overline{I} = \frac{1}{12}ml^2 = \frac{1}{12}(5 \text{ kg})(0.25 \text{ m})^2$$
$$= 0.026042 \text{ kg} \cdot \text{m}^2$$

Kinetics:



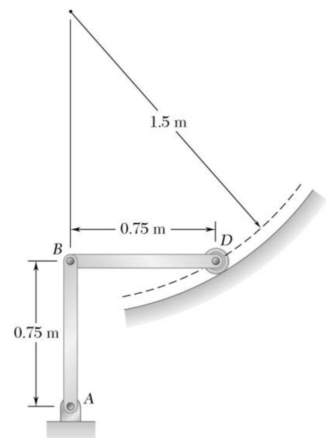
We recall that $\phi = 66.42°$. Thus: $h = (0.25 \text{ m})\sin\phi = 0.2291 \text{ m}$

$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad Dh + W(0.05) = m\overline{a}_x\left(\frac{h}{2}\right) - m\overline{a}_y(0.05) - \overline{I}\alpha$$

$$D(0.2291) + 5(9.81)(0.05) = (342.7)\frac{0.2291}{2} - (149.6)(0.05) - 15.581$$

$$0.2291D = 39.256 - 7.480 - 15.581 - 2.453 = 13.742 \qquad \mathbf{D} = 60.0 \text{ N} \longrightarrow \blacktriangleleft$$

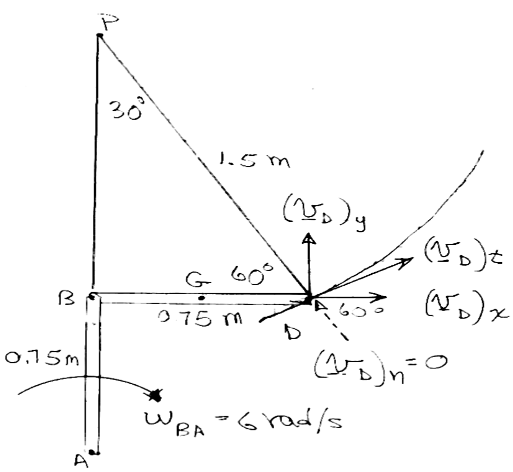
PROBLEM 16.129



The 4-kg uniform slender bar *BD* is attached to bar *AB* and a wheel of negligible mass which rolls on a circular surface. Knowing that at the instant shown bar *AB* has an angular velocity of 6 rad/s and no angular acceleration, determine the reaction at Point *D*.

SOLUTION

Kinematics:



Since $(\mathbf{v}_D)_n = 0,$

$$(v_D)_x \cos 60° = (v_D)_y \cos 30°$$

$$(v_D)_x = (v_B)_x = (AB)\,\omega_{BA} = (0.75 \text{ m})(6 \text{ rad/s})$$

$$(v_D)_y = (v_D)_x \frac{\cos 60°}{\cos 30°}$$

$$= (0.75)(6)\frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{0.75\,(6)}{\sqrt{3}}$$

But

$$(v_D)_y = \omega_{BD}\,(BD)$$

$$\omega_{BD} = \frac{(v_D)_y}{BD} = \frac{0.75\,(6)}{0.75\,\sqrt{3}} = \frac{6}{\sqrt{3}} \text{ rad/s}$$

$$\mathbf{a}_B = AB\,\omega_{BA}^2 = (0.75 \text{ m})(6 \text{ rad/s})^2 = 27 \text{ m/s}^2 \downarrow$$

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B}$$

$$[(\mathbf{a}_D)_t \measuredangle 30°] + [(1.5)(6/\sqrt{3})^2 \searrow 60°] = [27 \downarrow] + [0.75\,\alpha_{AB} \uparrow] + [(0.75)(6/\sqrt{3})^2 \longleftarrow]$$

$$60° \searrow: \quad 0 + 18 = -27\sin 60° + (0.75\sin 60°)\alpha_{BD} + 9\sin 30°$$

$$\alpha_{BD} = \frac{18 + 27\sin 60° - 9\sin 30°}{0.75\sin 60°} = 56.7846 \text{ rad/s}$$

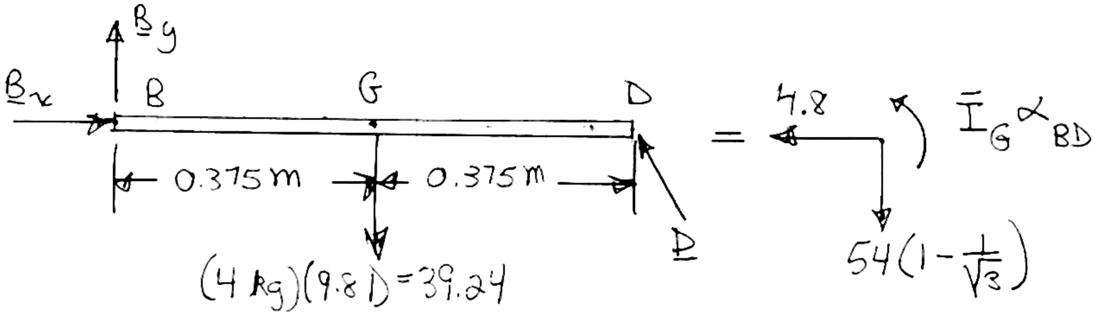
$$\mathbf{a}_G = \mathbf{a}_B + \mathbf{a}_{G/B}$$

$$= [27 \downarrow] + [0.375\alpha_{BD} \uparrow] + [(0.375)(6/\sqrt{3})^2 \longleftarrow]$$

$$= [5.7058 \text{ m/s}^2 \downarrow] + [4.5 \text{ m/s}^2 \longleftarrow]$$

Kinetics:

$$m = 4 \text{ kg} \qquad I_G = \frac{1}{12}mL^2 = \frac{1}{12}(4)(0.75)^2 = 0.1875 \text{ kg}\cdot\text{m}^2$$



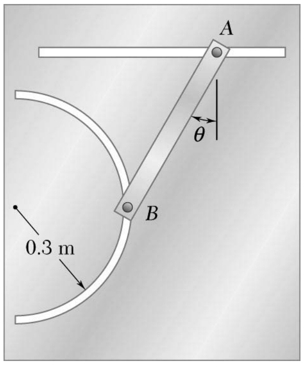
$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}$:

$$-(39.24 \text{ N})(0.375 \text{ m}) + D\cos 30°\,(0.75 \text{ m}) = \left[(0.1875)(56.7846) - (54)(0.375)\left(1-\frac{1}{\sqrt{3}}\right)\right] \text{N}\cdot\text{m}$$

$$0.64952\, D = (14.715 + 10.6471 - 8.5587) \text{ N}\cdot\text{m}$$

$$\mathbf{D} = 25.87 \text{ N} \measuredangle 60°$$

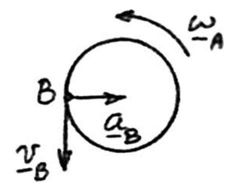
or $\mathbf{D} = 25.9 \text{ N} \measuredangle 60°$ ◀



PROBLEM 16.130

The motion of the uniform slender rod of length $L = 0.5$ m and mass $m = 3$ kg is guided by pins at A and B that slide freely in frictionless slots, circular and horizontal, cut into a vertical plate as shown. Knowing that at the instant shown the rod has an angular velocity of 3 rad/s counter-clockwise and $\theta = 30°$, determine the reactions at Points A and B.

SOLUTION



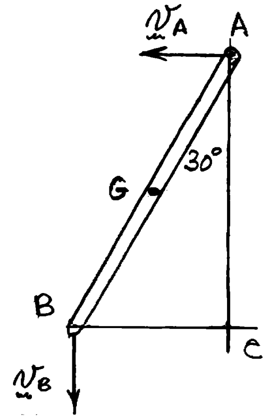
Mass and moment of inertia: $m = 3$ kg

$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(3\text{ kg})(0.5\text{ m})^2 = 0.0625\text{ kg}\cdot\text{m}^2$$

Kinematics: $\boldsymbol{\omega} = 3\text{ rad/s} \circlearrowleft$

$$\mathbf{v}_A = v_A \longleftarrow \qquad \mathbf{v}_B = v_B \downarrow$$

Locate the instantaneous center C by drawing line $\overline{AC}$ perpendicular to $\mathbf{v}_A$ and line $\overline{BC}$ perpendicular to $\mathbf{v}_B$.



$$v_A = (L\cos 30°)\omega = (0.5\text{ m}\cos 30°)(3\text{ rad/s}) = 1.29904\text{ m/s}$$

$$v_B = (L\sin 30°)\omega = (0.5\text{ m}\sin 30°)(3\text{ rad/s}) = 0.75\text{ m/s}$$

$\alpha = \alpha \circlearrowleft$ $\qquad \mathbf{a}_A = a_A \longrightarrow$

$$\mathbf{a}_B = \frac{v_B^2}{R} \longleftarrow + (a_B)_y \uparrow = \frac{(0.75\text{ m/s})^2}{0.3\text{ m}} \longleftarrow (a_B)_y \uparrow$$

$$= 1.875\text{ m/s}^2 \longleftarrow + (a_B)_y \uparrow$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$$

where $\quad (\mathbf{a}_{B/A})_t = L\alpha \nwarrow 30° = 0.5\alpha \nwarrow 30°$

and $\quad (\mathbf{a}_{B/A})_n = L\omega^2 \measuredangle 60° = (0.5)(3)^2 \measuredangle 60° = 4.5\text{ m/s}^2 \measuredangle 60°$

Equating the two expressions for $\mathbf{a}_B$ gives

$$1.875 \text{ m/s}^2 \longleftarrow + (a_B)_y \uparrow = \mathbf{a}_A + 0.5\alpha \measuredangle 30° + 4.5 \text{ m/s}^2 \measuredangle 60°$$

$$\longrightarrow: \quad -1.875 = a_A + 0.5\alpha\cos 30° + 2.25$$

$$a_A = -0.5\alpha\cos 30° - 4.125$$

$$\mathbf{a}_G = \mathbf{a}_A + (\mathbf{a}_{G/A})_t + (\mathbf{a}_{G/A})_n$$

$$= (0.5\alpha\cos 30° + 4.125) \longleftarrow + \frac{L}{2}\alpha \nwarrow 30° + \frac{L}{2}\omega^2 \measuredangle 60°$$

$$= (0.5\alpha\cos 30° + 4.125) \longleftarrow + \frac{0.5}{2}\alpha \nwarrow 30° + \frac{0.5}{2}(3)^2 \measuredangle 60°$$

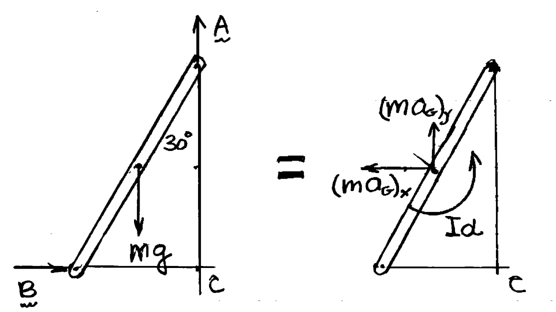
$$= (0.21651\alpha + 3.00) \longleftarrow + (1.94856 - 0.125\alpha) \uparrow$$

Kinetics:

$$m\mathbf{a}_G = (3 \text{ kg})\mathbf{a}_G$$

$$= [0.64952\alpha + 9.00] \longleftarrow + [5.8457 - 0.375\alpha] \uparrow$$

$$I\alpha = 0.0625\alpha$$



$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad mg\frac{L}{2}\sin 30° = I\alpha + (ma_G)_x\frac{L}{2}\cos 30° - (ma_G)_y\frac{L}{2}\sin 30°$$

$$(3)(9.81)\frac{0.5}{2}\sin 30° = 0.0625\alpha$$

$$+ (0.64952\alpha + 9.00)\frac{0.5}{2}\cos 30°$$

$$- (5.8457 - 0.375\alpha)\frac{0.5}{2}\sin 30°$$

$$3.67875 = 0.25\alpha + 1.21784$$

$$\alpha = 9.8436 \text{ rad/s}^2$$

$$m\mathbf{a}_G = [(0.64952)(9.8436) + 9.00] \longleftarrow + [5.8457 - (0.375)(9.8436)] \uparrow$$

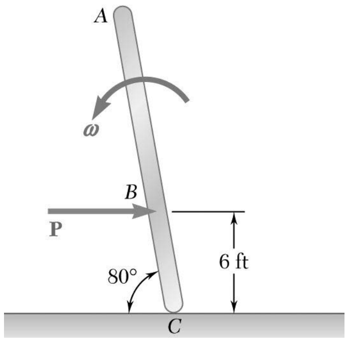
$$= [15.394 \text{ N}] \longleftarrow [2.1544 \text{ N}] \uparrow$$

PROBLEM 16.130 (Continued)

$$\Sigma \mathbf{F} = \Sigma \mathbf{F}_{\text{eff}} = m\mathbf{a}_G: \quad A\uparrow + B \longrightarrow + mg\downarrow = m\mathbf{a}_G$$

$+\uparrow$: $A - (3\text{ kg})(9.81\text{ m/s}) = 2.1544\text{ N}$ $\quad\mathbf{A} = 31.6\text{ N}\uparrow$ ◀

$+\longrightarrow$: $B = -15.894\text{ N}$ $\quad\mathbf{B} = 15.89\text{ N}\longleftarrow$ ◀



PROBLEM 16.131

At the instant shown, the 20 ft long, uniform 100-lb pole *ABC* has an angular velocity of 1 rad/s counterclockwise and Point *C* is sliding to the right. A 120-lb horizontal force **P** acts at *B*. Knowing the coefficient of kinetic friction between the pole and the ground is 0.3, determine at this instant (*a*) the acceleration of the center of gravity, (*b*) the normal force between the pole and the ground.

SOLUTION

Data: $l = 20 \text{ ft}, \quad W = 100 \text{ lb}, \quad P = 120 \text{ lb}, \quad \mu_k = 0.3$

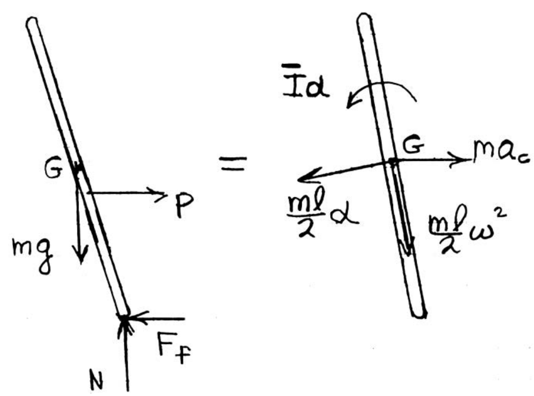
$$\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}\left(\frac{100}{32.2}\right)(20)^2 = 103.52 \text{ slug}\cdot\text{ft}^2$$

Kinematics: $\boldsymbol{\alpha} = \alpha \circlearrowleft \qquad \mathbf{a}_C = a_C \longrightarrow$

$$\mathbf{a}_G = \mathbf{a}_C + (\mathbf{a}_{G/C})_t + (\mathbf{a}_{C/G})_n$$

$$= [a_C \longrightarrow] + \left[\frac{l}{2}\alpha \swarrow 10°\right] + \left[\frac{l}{2}\omega^2 \searrow 80°\right] \qquad (1)$$

Kinetics: Sliding to the right: $F_f = \mu_k N$



$$+\circlearrowleft \Sigma M_G = +\circlearrowleft \Sigma(M_G)_{\text{eff}}: \quad N\frac{l}{2}\sin 10° - F_f\frac{l}{2}\cos 10° + P\left(\frac{l}{2}\cos 10° - h\right) = \bar{I}\alpha$$

$$\bar{I}\alpha - N\frac{l}{2}\sin 10° + \mu_k N\frac{l}{2}\cos 10° = P\left(\frac{l}{2}\cos 10° - h\right)$$

$$103.519\alpha - (10)(\sin 10° - 0.3\cos 10°)N = 120(10\cos 10° - 6) \qquad (2)$$

PROBLEM 16.131 (Continued)

$$+\uparrow \Sigma F_y = +\uparrow \Sigma (F_y)_{\text{eff}}: \quad N - mg = -\left(\frac{ml}{2}\alpha\right)\sin 10° - \frac{ml}{2}\omega^2 \cos 10°$$

$$\left(\frac{ml}{2}\sin 10°\right)\alpha + N = mg - \frac{ml}{2}\omega^2 \cos 10°$$

$$\left[\left(\frac{100}{32.2}\right)(10)\sin 10°\right]\alpha + N = (100) - \left(\frac{100}{32.2}\right)(10)(1)^2 \cos 10° \qquad (3)$$

Solving Eqs. (2) and (3) simultaneously,

$$\alpha = 3.8909 \text{ rad/s}^2 \qquad N = 48.433 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = \xrightarrow{+} \Sigma (F_x)_{\text{eff}}: \quad P - F_f = ma_C - \frac{ml}{2}\alpha \cos 10° + \frac{ml}{2}\omega^2 \sin 10°$$

$$ma_C = P - \mu_k N + \frac{ml}{2}\alpha \cos 10° - \frac{ml}{2}\omega^2 \sin 10°$$

$$\frac{100}{32.2}a_C = 120 - (0.3)(48.433) + \left(\frac{100}{32.2}\right)(10)(3.8909)\cos 10° - \left(\frac{100}{32.2}\right)(10)(1)^2 \sin 10°$$

$$a_C = 70.542 \text{ ft/s}^2$$

Using Eq. (1), $\quad \mathbf{a}_G = 70.542 + [(10)(3.8909) \swarrow 10°] + [(10)(1)^2 \searrow 80°]$

$$(a_G)_x = 70.542 - 38.909\cos 10° + 10\cos 80°$$

$$= 33.961 \text{ ft/s}^2$$

$$(a_G)_y = -38.909\sin 10° - 10\sin 80°$$

$$= -16.605 \text{ ft/s}^2$$

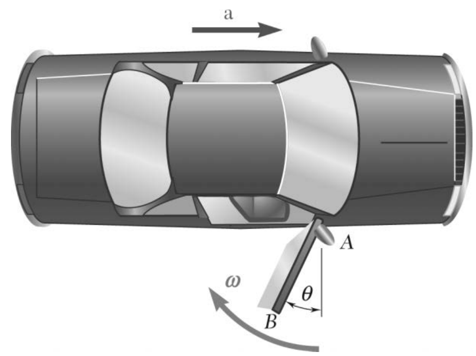
$$a_G = \sqrt{(33.961)^2 + (16.605)^2}$$

$$= 37.803 \text{ ft/s}^2$$

$$\tan\beta = \frac{16.605}{33.961} \qquad \beta = 26.055°$$

(*a*) Acceleration at Point *G*. $\quad \mathbf{a}_G = 37.8 \text{ ft/s}^2 \searrow 26.1°$ ◀

(*b*) Normal force. $\quad \mathbf{N} = 48.4 \text{ lb}\uparrow$ ◀

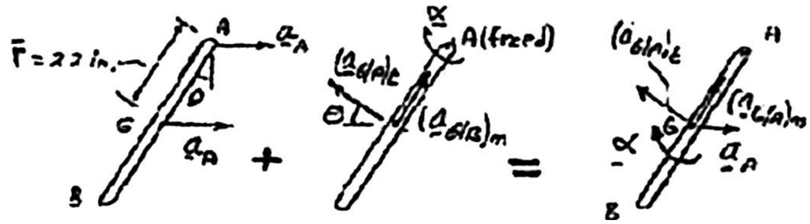


PROBLEM 16.132

A driver starts his car with the door on the passenger's side wide open $(\theta = 0)$. The 80-lb door has a centroidal radius of gyration $\bar{k} = 12.5$ in., and its mass center is located at a distance $r = 22$ in. from its vertical axis of rotation. Knowing that the driver maintains a constant acceleration of 6 ft/s^2, determine the angular velocity of the door as it slams shut $(\theta = 90°)$.

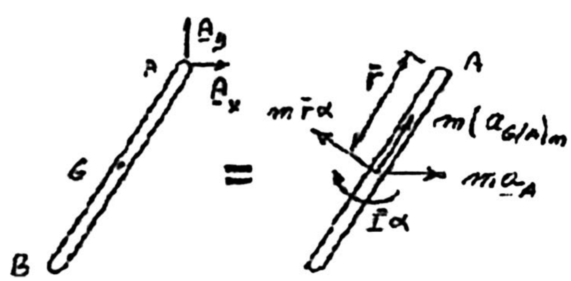
SOLUTION

Kinematics: $\mathbf{a} = a_A$



$$(\mathbf{a}_{G/A})_t = \bar{r}\alpha \searrow \theta$$

Kinetics:



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad 0 = \bar{I}\alpha + (m\bar{r}\alpha)\bar{r} - ma_A(\bar{r}\cos\theta)$$

$$m\bar{k}^2\alpha + m\bar{r}^2\alpha = ma_A\bar{r}\cos\theta$$

$$\alpha = \frac{a_A\bar{r}}{\bar{k}^2 + \bar{r}^2}\cos\theta$$

Setting $\alpha = \omega\frac{d\omega}{d\theta}$, and using $\bar{r} = \frac{22}{12}$ ft, $\bar{k} = \frac{12.5}{12}$ ft

PROBLEM 16.132 (Continued)

$$\omega\frac{d\omega}{d\theta} = \frac{\left(\frac{22}{12}\text{ ft}\right)a_A}{\left[\left(\frac{12.5}{12}\text{ ft}\right)^2 + \left(\frac{22}{12}\text{ ft}\right)^2\right]}\cos\theta$$

$$= 0.41234 a_A \cos\theta$$

$$\omega d\omega = 0.41234 a_A \cos\theta\, d\theta$$

$$\int_0^{\omega_f} \omega d\omega = \int_0^{\pi/2} (0.4124 a_A)\cos\theta\, d\theta$$

$$\left|\frac{1}{2}\omega^2\right|_0^{\omega_f} = 0.41234 a_A |\sin\theta|_0^{\pi/2}$$

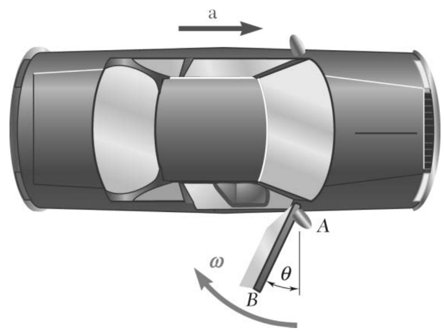
$$\omega_f^2 = 0.82468 a_A \qquad (1)$$

Given data: $\mathbf{a}_A = 6\text{ ft/s}^2 \longrightarrow$

$$\omega_f^2 = 0.82468(6)$$

$$= 4.948\text{ rad}^2/\text{s}^2$$

$\omega_f = 2.22\text{ rad/s}$ ◀

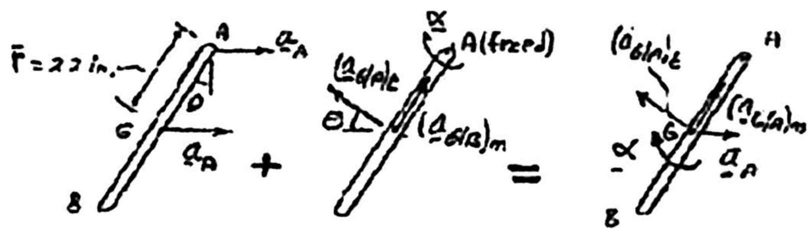


PROBLEM 16.133

For the car of Problem 16.132, determine the smallest constant acceleration that the driver can maintain if the door is to close and latch, knowing that as the door hits the frame its angular velocity must be at least 2 rad/s for the latching mechanism to operate.

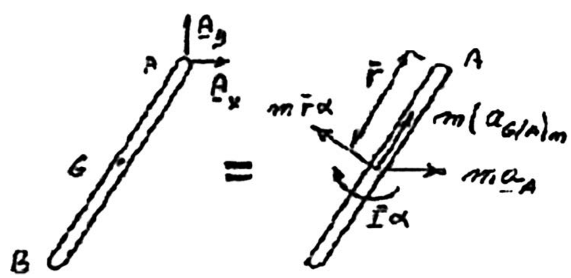
SOLUTION

Kinematics: $\mathbf{a} = a_A$



$$(\mathbf{a}_{G/A})_t = \bar{r}\alpha \searrow \theta$$

Kinetics:



$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad 0 = \bar{I}\alpha + (m\bar{r}\alpha)\bar{r} - ma_A(\bar{r}\cos\theta)$$

$$m\bar{k}^2\alpha + m\bar{r}^2\alpha = ma_A\bar{r}\cos\theta$$

$$\alpha = \frac{a_A\bar{r}}{\bar{k}^2 + \bar{r}^2}\cos\theta$$

Setting $\alpha = \omega\frac{d\omega}{d\theta}$, and using $\bar{r} = \frac{22}{12}\text{ ft}$, $\bar{k} = \frac{12.5}{12}\text{ ft}$

$$\omega\frac{d\omega}{d\theta} = \frac{\left(\frac{22}{12}\text{ ft}\right)a_A}{\left[\left(\frac{12.5}{12}\text{ ft}\right)^2 + \left(\frac{22}{12}\text{ ft}\right)^2\right]}\cos\theta$$

$$= 0.41234 a_A \cos\theta$$

$$\omega d\omega = 0.41234 a_A \cos\theta \, d\theta$$

PROBLEM 16.133 (Continued)

$$\int_0^{\omega_f} \omega d\omega = \int_0^{\pi/2} (0.4124 a_A) \cos\theta \, d\theta$$

$$\left| \frac{1}{2}\omega^2 \right|_0^{\omega_f} = 0.41234 a_A \, |\sin\theta|_0^{\pi/2}$$

$$\omega_f^2 = 0.82468 a_A \qquad (1)$$

Given data: $\omega_f = 2$ rad/s

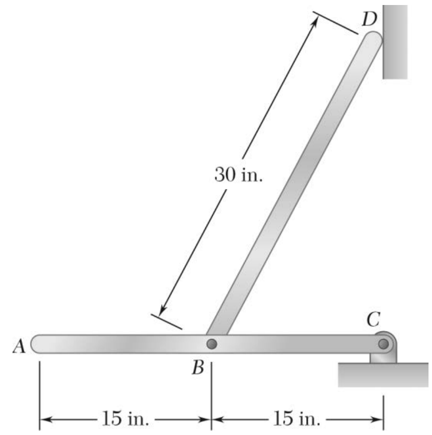
Eq. (1): $\omega_f^2 = 0.82468 a_A$

$$(2)^2 = 0.82468 a_A$$

$$a_A = 4.85 \text{ ft/s}^2$$

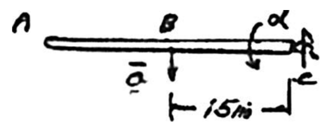
$\mathbf{a}_A = 4.85 \text{ ft/s}^2 \longrightarrow$ ◀

PROBLEM 16.134



Two 8-lb uniform bars are connected to form the linkage shown. Neglecting the effect of friction, determine the reaction at D immediately after the linkage is released from rest in the position shown.

SOLUTION



Kinematics:

Bar AC: Rotation about C

$$\bar{a} = (BC)\alpha = \left(\frac{15}{12}\text{ ft}\right)\alpha$$

$$\bar{\mathbf{a}} = 1.25\alpha \downarrow$$

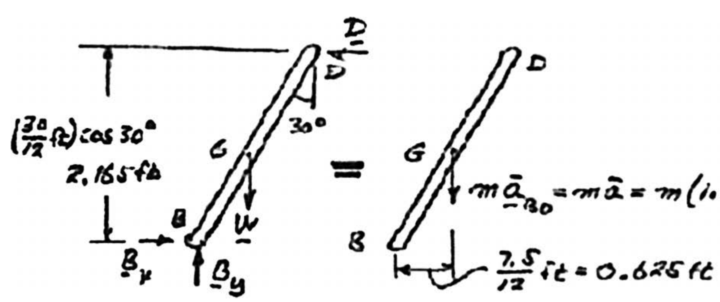
$$\sin\theta = \frac{15\text{ in.}}{30\text{ in.}} \quad \theta = 30°$$

Bar BC: $\mathbf{a}_{D/B} = L\alpha \nwarrow$

Must be zero since $a_D \downarrow$ $\quad \alpha_{BD} = 0 \quad \text{and} \quad \bar{a}_{BD} = \bar{a}$

Kinetics:

Bar BD



$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad B_y - W = -m\bar{a}$$

$$B_y - 8\text{ lb} = -\frac{8\text{ lb}}{32.2}(1.25\alpha)$$

$$B_y = 8 - 0.3105\alpha \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad D(2.165\text{ ft}) - W(0.625\text{ ft}) = -m\bar{a}(0.625\text{ ft})$$

$$D(2.165\text{ ft}) - (8\text{ lb})(0.625\text{ ft}) = -\frac{8\text{ lb}}{32.2}(1.25\alpha)(0.625\text{ ft})$$

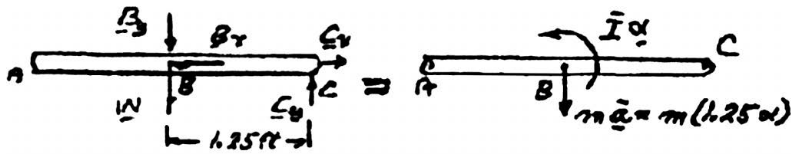
$$D = 2.309 - 0.08965\alpha \qquad (2)$$

Bar AC:

$$\bar{I} = \frac{1}{12}m(AC)^2$$

$$= \frac{1}{12}\frac{8\text{ lb}}{32.2}(2.5\text{ ft})^2$$

$$= 0.1294\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$



$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad W(1.25\text{ ft}) + B_y(1.25\text{ ft}) = \bar{I}\alpha + m(1.25\alpha)(1.25)$$

Substitute from Eq. (1) for B_y

$$8(1.25) + (8 - 0.3105\alpha)(1.25) = (0.1294)\alpha + \frac{8}{32.2}(1.25)^2\alpha$$

$$10 + 10 - 0.3881\alpha = 0.1294\alpha + 0.3882\alpha$$

$$20 = 0.9057\alpha$$

$$\alpha = 22.08\text{ rad/s}^2$$

Eq. (2),

$$D = 2.309 - 0.08965\alpha$$

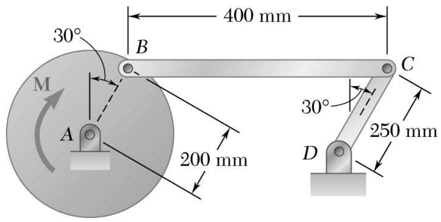
$$= 2.309 - 0.08965(22.08)$$

$$= 2.309 - 1.979$$

$$D = 0.330\text{ lb}$$

$\mathbf{D} = 0.330\text{ lb} \longleftarrow$ ◀

PROBLEM 16.135



The 6-kg rod *BC* connects a 10-kg disk centered at *A* to a 5-kg rod *CD*. The motion of the system is controlled by the couple **M** applied to disk *A*. Knowing that at the instant shown disk *A* has an angular velocity of 36 rad/s clockwise and no angular acceleration, determine (*a*) the couple **M**, (*b*) the components of the force exerted at *C* on rod *BC*.

SOLUTION

Kinematics: *Velocity analysis*. $\boldsymbol{\omega}_{AB} = 36 \text{ rad/s} \circlearrowright$

Disk *AB*: $\mathbf{v}_B = \overline{AB}\ \omega_{AB} \searrow 30^\circ = (0.200)(36) \searrow 30^\circ = 7.2 \text{ m/s} \searrow 30^\circ$

Rod *BC*: $\mathbf{v}_C = v_C \searrow 30^\circ$

Since $\mathbf{v}_C$ is parallel to $\mathbf{v}_B$, bar *BC* is in translation.

$$\mathbf{v}_C = 7.2 \text{ m/s} \searrow 30^\circ \qquad \boldsymbol{\omega}_{BC} = 0$$

Rod *CD*:

$$\omega_{CD} = \frac{v_C}{l_{CD}} = \frac{7.2 \text{ m/s}}{0.25 \text{ m}} = 28.8 \text{ rad/s}$$

$$\boldsymbol{\omega}_{CD} = 28.8 \text{ rad/s} \circlearrowright$$

Acceleration analysis: $\boldsymbol{\alpha}_{AB} = 0$

Disk *AB*:

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= 0 - (36)^2(0.2) \swarrow 60^\circ = 259.2 \text{ m/s}^2 \swarrow 60^\circ$$

Rod *BC*:

$$\boldsymbol{\alpha}_{BC} = \alpha_{BC} \circlearrowleft$$

$$\mathbf{a}_C = \mathbf{a}_B + (\mathbf{a}_{C/B})_t - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$\mathbf{a}_C = 259.2 \swarrow 60^\circ + 0.4\alpha_{AB} \uparrow + 0 \qquad (1)$$

Rod *CD*:

$$\boldsymbol{\alpha}_{CD} = \alpha_{CD} \circlearrowleft$$

$$\boldsymbol{\alpha}_C = (\mathbf{a}_{C/D})_t - \omega_{CD}^2 \mathbf{r}_{C/D} = [0.25\alpha_{CD} \swarrow 30^\circ] - [(28.8)^2(0.25) \swarrow 60^\circ]$$

$$\mathbf{a}_C = [0.25\alpha_{CD} \swarrow 30^\circ] - [207.36 \swarrow 60^\circ] \qquad (2)$$

Equate components of two expressions (1) and (2) for $\mathbf{a}_C$.

$$+\longrightarrow: \quad -259.2\cos 60^\circ = -0.25\alpha_{CD}\cos 30^\circ - 207.36\cos 60^\circ$$

$$\alpha_{CD} = 119.719 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{CD} = 119.719 \text{ rad/s}^2 \circlearrowleft$$

$$+\uparrow: \quad -259.2\sin 60^\circ + 0.4\alpha_{BC} = 0.25\alpha_{CD}\sin 30^\circ - 207.36\sin 60^\circ$$

$$\alpha_{BC} = 149.649 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{BC} = 149.649 \text{ rad/s}^2 \circlearrowleft$$

Accelerations of the mass centers.

Disk *AB*: $\bar{\mathbf{a}}_{AB} = \mathbf{a}_A = 0$

Rod *BC*: Mass center at Point *P*. $\mathbf{r}_{P/B} = (0.2 \text{ m}) \longrightarrow$

$$\mathbf{a}_P = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{P/B} - \omega_{BC}^2 \mathbf{r}_{P/B}$$

$$= [259.2 \nearrow 60°] + [0.2\alpha_{BC} \uparrow] + 0 \quad = [259.2 \nearrow 60°] + [29.9298 \uparrow]$$

Rod *CD*: Mass center at Point *Q*. $\mathbf{r}_{Q/D} = 0.125 \text{ m} \measuredangle 60°$

$$\mathbf{a}_Q = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{Q/D} - \omega_{CD}^2 \mathbf{r}_{Q/D}$$

$$= [0.125\ \alpha_{CD} \searrow 30°] - [(28.8)^2(0.125) \nearrow 60°]$$

$$= [14.964875 \searrow 30°] - [103.60 \nearrow 60°]$$

Masses: $m_{AB} = 10 \text{ kg}, \quad m_{BC} = 6 \text{ kg}, \quad m_{CD} = 5 \text{ kg}$

Effective forces at mass centers.

Disk *AB*: $m_{AB}\mathbf{a}_A = 0$

Rod *BC*: $m_{BC}\mathbf{a}_P = 6\mathbf{a}_P = [1555.2 \text{ N} \nearrow 60°] + [179.58 \text{ N} \uparrow]$

Rod *CD*: $m_{CD}\mathbf{a}_a = 5\mathbf{a}_Q = [74.82 \text{ N} \searrow 30°] + [518 \text{ N} \nearrow 60°]$

Moments of inertia:

Disk *AB*: $\bar{I}_{AB} = \frac{1}{2} m_{AB} r_{AB}^2 = \frac{1}{2}(10)(0.2)^2 = 0.2 \text{ kg} \cdot \text{m}^2$

Rod *BC*: $\bar{I}_{BC} = \frac{1}{12} m_{BC} l_{BC}^2 = \frac{1}{12}(6)(0.4)^2 = 0.08 \text{ kg} \cdot \text{m}^2$

Rod *CD*: $\bar{I}_{CD} = \frac{1}{12} m_{CD} l_{CD}^2 = \frac{1}{12}(5)(0.25)^2 = 0.0260417 \text{ kg} \cdot \text{m}^2$

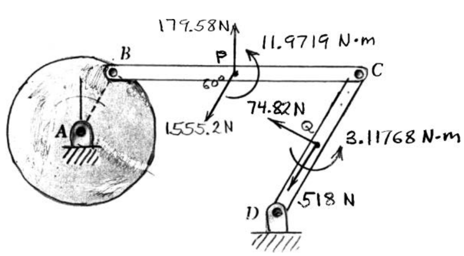
Effective couples at mass centers.

Disk *AB*: $\bar{I}_{AB}\boldsymbol{\alpha}_{AB} = 0$

Rod *BC*: $\bar{I}_{BC}\boldsymbol{\alpha}_{BC} = (0.08)(149.649) \circlearrowright = 11.97192 \text{ N} \cdot \text{m} \circlearrowright$

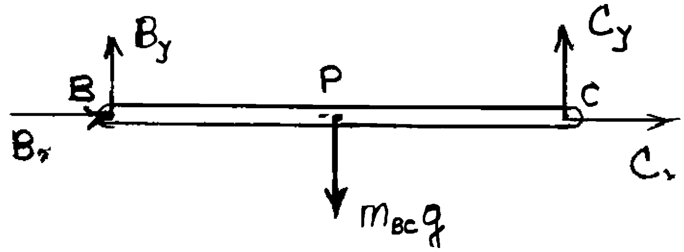
Rod *CD*: $\bar{I}_{CD}\boldsymbol{\alpha}_{CD} = (0.0260417)(119.719) \circlearrowright = 3.11768 \text{ N} \cdot \text{m} \circlearrowright$

Summary of effective forces and couples



Kinetics

Rod *BC*:

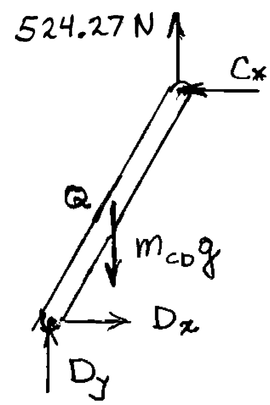


$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad l_{BC}C_y - \frac{1}{2}l_{BC}mg = \Sigma(M_B)_{\text{eff}}$$

$$(0.4 \text{ m})C_y - (0.2 \text{ m})(6 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 11.9719 \text{ N}\cdot\text{m} + (0.2 \text{ m})(179.58 \text{ N})$$
$$-(0.2 \text{ m})(1555.2 \text{ N}) \sin 60°$$

$$C_y = -524.27 \text{ N}$$

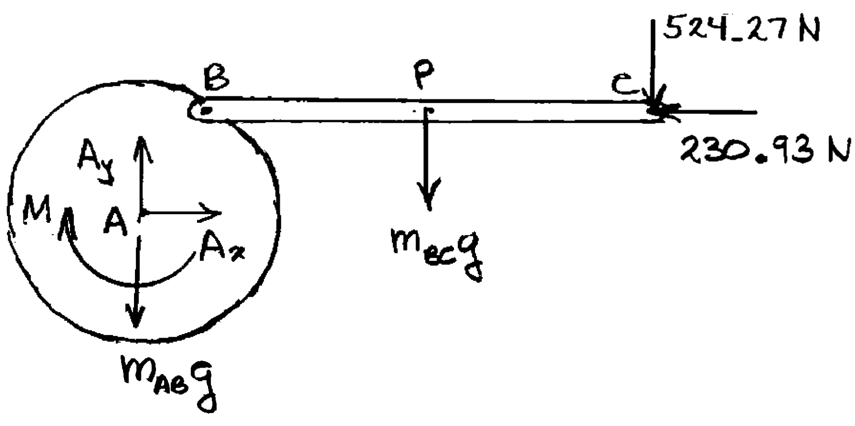
Rod *CD*:



$$+\circlearrowleft \Sigma M_D = \Sigma(M_D)_{\text{eff}}: \quad C_x l_{CD} \cos 30° + (524.27 \text{ N})(0.125 \text{ m})$$
$$-m_{CD}g(0.0625 \text{ m}) = \Sigma(M_D)_{\text{eff}}:$$
$$(0.25 \text{ m})C_x \cos 30° + 65.534 \text{ N}\cdot\text{m}$$
$$-(5 \text{ kg})(9.81 \text{ m/s}^2)(0.0625 \text{ m})$$
$$= 3.11768 \text{ N}\cdot\text{m} + (0.125 \text{ m})(74.82 \text{ N})$$

$$C_x = -230.93 \text{ N}$$

Disk *AB* and rod *BC*:



PROBLEM 16.135 (Continued)

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad -M - (524.27)(0.4 + 0.2\sin 30°) + (230.93)(0.2\cos 30°)$$

$$- (6 \text{ kg})(9.81 \text{ m/s}^2)(0.2 + 0.2\sin 30°)$$

$$= 11.9719 + (179.58)(0.2 + 0.2\sin 30°)$$

$$- (1555.2\cos 30°)(0.2)$$

$$-M - 262.135 + 40.0 - 17.658$$

$$= 11.9719 + 53.874 - 269.369$$

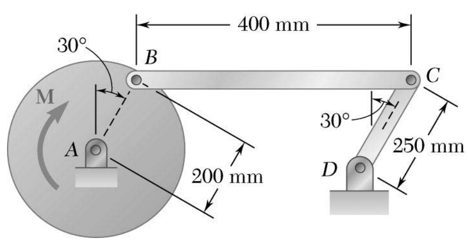
$$-M = 36.27 \text{ N} \cdot \text{m}$$

(*a*) *Couple applied to disk A.*

$$\mathbf{M} = 36.3 \text{ N} \cdot \text{m} \circlearrowleft \blacktriangleleft$$

(*b*) *Components of force exerted at C* on rod *BC*.

$$\mathbf{C} = 231 \text{ N} \longleftarrow + 524 \text{ N} \downarrow \blacktriangleleft$$



PROBLEM 16.136

The 6-kg rod BC connects a 10-kg disk centered at A to a 5-kg rod CD. The motion of the system is controlled by the couple $\mathbf{M}$ applied to disk A. Knowing that at the instant shown disk A has an angular velocity of 36 rad/s clockwise and an angular acceleration of 150 rad/s^2 counterclockwise, determine (*a*) the couple $\mathbf{M}$, (*b*) the components of the force exerted at C on rod BC.

SOLUTION

Kinematics: *Velocity analysis.* $\boldsymbol{\omega}_{AB} = 36 \text{ rad/s} \circlearrowright$

Disk AB: $\mathbf{v}_B = \overline{AB}\omega_{AB} \nwarrow 30° = (0.200)(36) \nwarrow 30° = 7.2 \text{ m/s} \nwarrow 30°$

Rod BC: $\mathbf{v}_C = v_C \nwarrow 30°$

Since $\mathbf{v}_C$ is parallel to $\mathbf{v}_B$, bar BC is in translation

$$\mathbf{v}_C = 7.2 \text{ m/s} \nwarrow 30° \qquad \boldsymbol{\omega}_{BC} = 0$$

Rod CD:
$$\omega_{CD} = \frac{v_C}{l_{CD}} = \frac{7.2 \text{ m/s}}{0.25 \text{ m}} = 28.8 \text{ rad/s}$$

$$\boldsymbol{\omega}_{CD} = 28.8 \text{ rad/s} \circlearrowright$$

Acceleration analysis. $\boldsymbol{\alpha}_{AB} = 0$

Disk AB:
$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$

$$= [(150)(0.2) \searrow 30°] - [(36)^2(0.2) \nearrow 60°]$$

$$= [30 \text{ m/s}^2 \searrow 30°] + [259.2 \text{ m/s}^2 \swarrow 60°]$$

Rod BC:
$$\boldsymbol{\alpha}_{BC} = 150 \text{ rad/s}^2 \circlearrowleft$$

$$\mathbf{a}_C = \mathbf{a}_B + (\mathbf{a}_{C/B})_t - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$\mathbf{a}_C = [30 \searrow] + [259.2 \swarrow 60°] + [0.4\alpha_{AB}] + 0 \qquad (1)$$

Rod CD:
$$\boldsymbol{\alpha}_{CD} = \alpha_{CD} \circlearrowleft$$

$$\mathbf{a}_C = (\mathbf{a}_{C/D})_t - \omega_{CD}^2 \mathbf{r}_{C/D} = [0.25\alpha_{CD} \searrow 30°] - [(28.8)^2(0.25) \nearrow 60°]$$

$$\mathbf{a}_C = [0.25\alpha_{CD} \searrow 30°] - [207.36 \nearrow 60°] \qquad (2)$$

Equate components of two expressions (1) and (2) for $\mathbf{a}_C$.

$$+\longrightarrow: \quad -30\cos 30° - 259.2\cos 60° = -0.25\alpha_{CD}\cos 30° - 207.36\cos 60°$$

$$\alpha_{CD} = 239.719 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{CD} = 239.719 \text{ rad/s}^2 \circlearrowright$$

$$+\uparrow: \quad 30\cos 30° - 259.2\sin 60° + 0.4\alpha_{BC} = 0.25\alpha_{CD}\sin 30° - 207.36\sin 60°$$

$$\alpha_{BC} = 149.649 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{BC} = 149.649 \text{ rad/s}^2 \circlearrowright$$

Accelerations of the mass centers.

Disk *AB*: $\bar{\mathbf{a}}_{AB} = \mathbf{a}_A = 0$

Rod *BC*: Mass center at Point *P*. $\mathbf{r}_{P/B} = (0.2 \text{ m}) \longrightarrow$

$$\mathbf{a}_P = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{P/B} - \omega_{BC}^2 \mathbf{r}_{P/B}$$

$$= [30 \searrow 30°] + [259.2 \swarrow 60°] + [0.2\alpha_{BC} \uparrow] + 0$$

$$= [30 \searrow 30°] + [259.2 \swarrow 60°] + [29.9298 \uparrow]$$

Rod *CD*: Mass center at Point *Q*. $\mathbf{r}_{Q/D} = 0.125 \text{ m} \measuredangle 60°$

$$\mathbf{a}_Q = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{Q/D} - \omega_{CD}^2 \mathbf{r}_{Q/D}$$

$$= [0.125\,\alpha_{CD} \searrow 30°] - [(28.8)^2(0.125) \swarrow 60°]$$

$$= [29.964875 \searrow 30°] - [103.60 \swarrow 60°]$$

Masses: $m_{AB} = 10 \text{ kg}, \quad m_{BC} = 6 \text{ kg}, \quad m_{CD} = 5 \text{ kg}$

Effective forces at mass centers.

Disk *AB*: $m_{AB}\mathbf{a}_A = 0$

Rod *BC*: $m_{BC}\mathbf{a}_P = 6\mathbf{a}_P = [180 \text{ N} \searrow 30°] + [1555.2 \text{ N} \swarrow 60°] + [179.58 \text{ N} \uparrow]$

Rod *CD*: $m_{CD}\mathbf{a}_a = 5\mathbf{a}_Q = [149.82 \text{ N} \searrow 30°] + [518 \text{ N} \swarrow 60°]$

Moments of inertia:

Disk *AB*: $\bar{I}_{AB} = \frac{1}{2} m_{AB} r_{AB}^2 = \frac{1}{2}(10)(0.2)^2 = 0.2 \text{ kg} \cdot \text{m}^2$

Rod *BC*: $\bar{I}_{BC} = \frac{1}{12} m_{BC} l_{BC}^2 = \frac{1}{12}(6)(0.4)^2 = 0.08 \text{ kg} \cdot \text{m}^2$

Rod *CD*: $\bar{I}_{CD} = \frac{1}{12} m_{CD} l_{CD}^2 = \frac{1}{12}(5)(0.25)^2 = 0.0260417 \text{ kg} \cdot \text{m}^2$

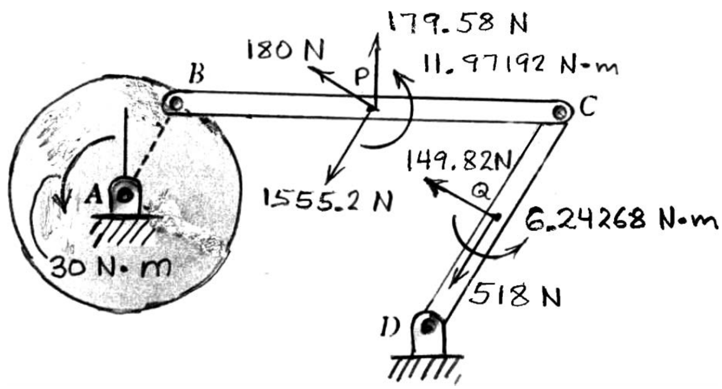
Effective couples at mass centers.

Disk *AB*: $\bar{I}_{AB}\boldsymbol{\alpha}_{AB} = (0.2)(150) \circlearrowleft = 30 \text{ N}\cdot\text{m} \circlearrowleft$

Rod *BC*: $\bar{I}_{BC}\boldsymbol{\alpha}_{BC} = (0.08)(149.649) \circlearrowleft = 11.97192 \text{ N}\cdot\text{m} \circlearrowleft$

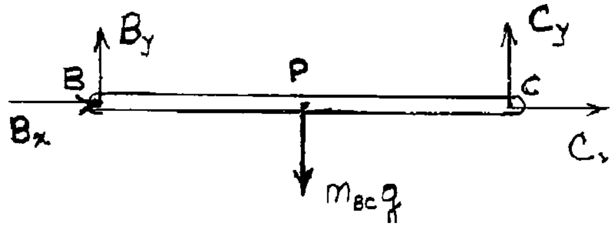
Rod *CD*: $\bar{I}_{CD}\boldsymbol{\alpha}_{CD} = (0.0260417)(239.719) \circlearrowleft = 6.24268 \text{ N}\cdot\text{m} \circlearrowleft$

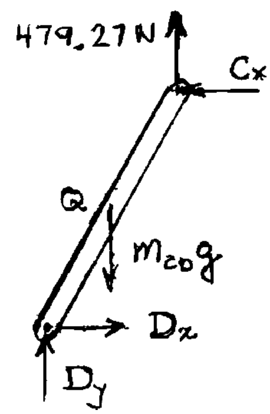
Summary of effective forces and couples.



Kinetics

Rod *BC*:





$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad l_{BC}C_y - \frac{1}{2}l_{BC}mg = \Sigma(M_B)_{\text{eff}}$$

$$(0.4 \text{ m})C_y - (0.2 \text{ m})(6 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= (0.2 \text{ m})(180 \text{ N} \sin 30°) + 11.9719 \text{ N}\cdot\text{m} + (0.2 \text{ m})(179.58 \text{ N})$$

$$-(0.2 \text{ m})(1555.2 \text{ N}) \sin 60°$$

$$C_y = -479.27 \text{ N}$$

Rod *CD*: $+\circlearrowleft \Sigma M_D = \Sigma(M_D)_{\text{eff}}: \quad C_x l_{CD}\cos 30° + (479.27 \text{ N})(0.125 \text{ m})$

$$-m_{CD}g(0.0625 \text{ m}) = \Sigma(M_D)_{\text{eff}}:$$

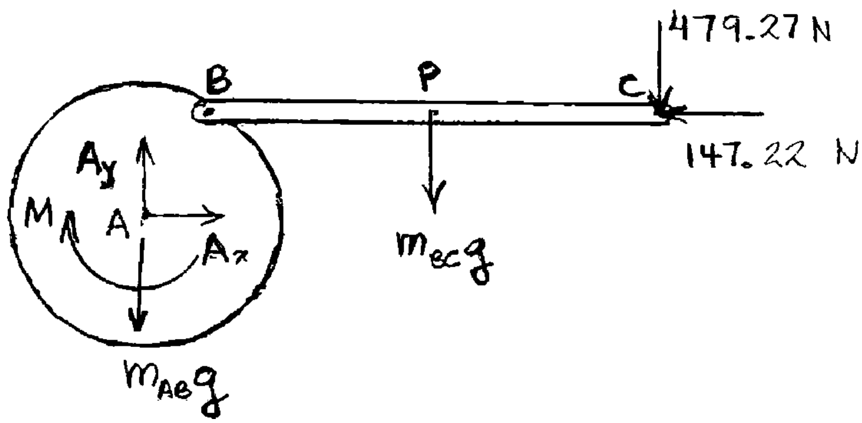
$$(0.25 \text{ m})C_x \cos 30° + 59.909 \text{ N}\cdot\text{m} - (5 \text{ kg})(9.81 \text{ m/s}^2)(0.0625 \text{ m})$$

$$= 6.24268 \text{ N}\cdot\text{m} + (0.125 \text{ m})(149.82 \text{ N})$$

$$C_x = -147.22 \text{ N}$$

PROBLEM 16.136 (Continued)

Disk *AB* and rod *BC*:



$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad -M - (479.27)(0.4 + 0.2\sin 30°) + (147.22)(0.2\cos 30°)$$

$$- (6 \text{ kg})(9.81 \text{ m/s}^2)(0.2 + 0.2\sin 30°)$$

$$= (180)(0.2 + 0.2\sin 30°) + 11.9719 + (179.58)(0.2 + 0.2\sin 30°)$$

$$- (1555.2\cos 30°)(0.2)$$

$$-M - 239.635 + 25.50 - 17.658$$

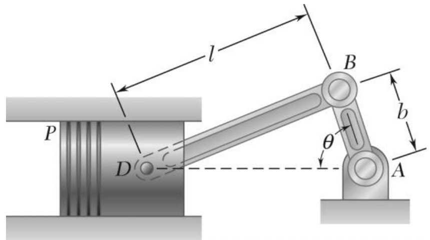
$$= 54 + 11.9719 + 53.874 - 269.369$$

$$-M = 82.27 \text{ N}\cdot\text{m}$$

(*a*) *Couple applied to disk A.* $\mathbf{M} = 82.3 \text{ N}\cdot\text{m} \circlearrowleft$ ◀

(*b*) *Components of force exerted at C* on rod *BC*.

$\mathbf{C} = 147.2 \text{ N} \longleftarrow + 479 \text{ N} \downarrow$ ◀



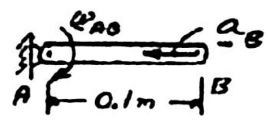
PROBLEM 16.137

In the engine system shown $l = 250$ mm and $b = 100$ mm. The connecting rod *BD* is assumed to be a 1.2-kg uniform slender rod and is attached to the 1.8-kg piston *P*. During a test of the system, crank *AB* is made to rotate with a constant angular velocity of 600 rpm clockwise with no force applied to the face of the piston. Determine the forces exerted on the connecting rod at *B* and *D* when $\theta = 180°$. (Neglect the effect of the weight of the rod.)

SOLUTION

<u>Kinematics</u>: Crank *AB*:

$$\omega_{AB} = 600 \text{ rpm}\left(\frac{2\pi}{60}\right) = 62.832 \text{ rad/s} \curvearrowright$$



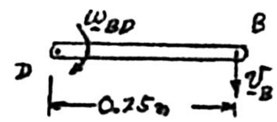
$$a_B = (AB)\omega_{AB}^2 = (0.1 \text{ m})(62.832 \text{ rad/s}^2)$$

$$\mathbf{a}_B = 394.78 \text{ m/s}^2 \longleftarrow$$

Also: $$v_B = (AB)\omega_{AB} = (0.1 \text{ m})(62.832 \text{ rad/s}) = 6.2832 \text{ m/s} \downarrow$$

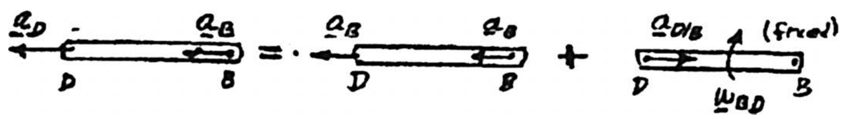
Connecting rod *BD*:

Velocity Instantaneous center at *D*.



$$\omega_{BD} = \frac{v_B}{BD} = \frac{6.2832 \text{ m/s}}{0.25 \text{ m}} = 25.133 \text{ rad/s}$$

Acceleration:



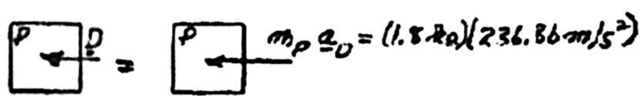
$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} = [a_B \longleftarrow] + [(BD)\omega_{BD}^2 \longrightarrow]$$

$$\mathbf{a}_D = [394.78 \text{ m/s}^2 \longleftarrow] + [(0.25 \text{ m})(25.133 \text{ rad/s})^2 \longrightarrow]$$

$$\mathbf{a}_D = [397.78 \text{ m/s}^2 \longleftarrow] + [157.92 \text{ m/s}^2 \longleftarrow] = 236.86 \text{ m/s}^2 \longleftarrow$$

$$\bar{\mathbf{a}}_{BD} = \frac{1}{2}(\mathbf{a}_B + \mathbf{a}_D) = \frac{1}{2}(394.78 \longleftarrow + 236.86 \longleftarrow) = 315.82 \text{ m/s}^2 \longleftarrow$$

Kinetics of piston



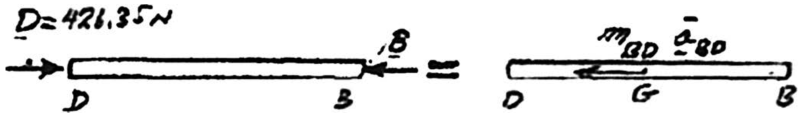
$$\mathbf{D} = 426.35 \text{ N} \longleftarrow$$

PROBLEM 16.137 (Continued)

Force exerted on connecting rod at D is:

$\mathbf{D} = 426.35 \longrightarrow$

Kinetics of connecting rod: (neglect weight)



$$\overset{+}{\longleftarrow} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad B - D = m_{BD}\bar{a}_{BD}$$

$$B - 426.34 \text{ N} = (1.2 \text{ kg})(315.82 \text{ m/s}^2)$$

$$B = 426.35 \text{ N} + 378.48 \text{ N}$$

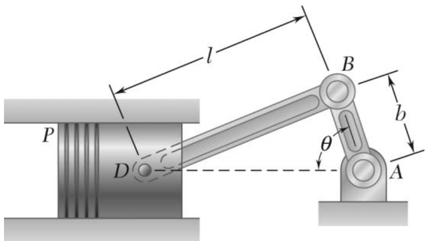
$$= 805.33 \text{ N}$$

Forces exerted on connecting rod.

$\mathbf{B} = 805 \text{ N} \longleftarrow$ ◀

$\mathbf{D} = 426 \text{ N} \longrightarrow$ ◀

PROBLEM 16.138



Solve Problem 16.137 when $\theta = 90°$.

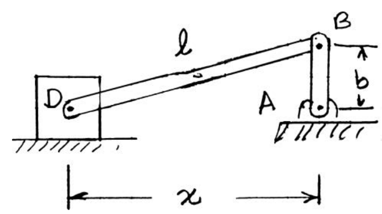
PROBLEM 16.137 In the engine system shown $l = 250$ mm and $b = 100$ mm. The connecting rod *BD* is assumed to be a 1.2-kg uniform slender rod and is attached to the 1.8-kg piston *P*. During a test of the system, crank *AB* is made to rotate with a constant angular velocity of 600 rpm clockwise with no force applied to the face of the piston. Determine the forces exerted on the connecting rod at *B* and *D* when $\theta = 180°$. (Neglect the effect of the weight of the rod.)

SOLUTION

Geometry: $l = 0.25$ m, $b = 0.1$ m, $x = \sqrt{l^2 - b^2} = 0.2291$ m

$$\mathbf{r}_{B/A} = (0.1 \text{ m})\mathbf{j}, \quad \mathbf{r}_{D/B} = -(0.2291 \text{ m})\mathbf{i} - (0.1 \text{ m})\mathbf{j}$$

Kinematics: $\omega_{AB} = 600 \text{ rev/min} = 62.832 \text{ rad/s}$



Velocity:

$$\boldsymbol{\omega}_{AB} = -(62.832 \text{ rad/s})\mathbf{k}$$

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A}$$

$$= -(62.832\mathbf{k}) \times (0.1\mathbf{i})$$

$$= (6.2832 \text{ m/s})\mathbf{i}$$

$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B}$$

$$v_D\mathbf{i} = 6.2832\mathbf{i} + \omega_{BD}\mathbf{k} \times (-0.2291\mathbf{i} - 0.1\mathbf{j})$$

$$= 6.2832\mathbf{i} + 0.1\omega_{BD}\mathbf{j} - 0.2291\omega_{BD}\mathbf{i}$$

Equating components gives

$$v_D = 6.2832 \text{ m/s}, \quad \omega_{BD} = 0.$$

Acceleration:

$$\boldsymbol{\alpha}_{AB} = 0, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k} \quad \mathbf{a}_D = a_D\mathbf{i}$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} = 0 - (62.832)^2(0.1\mathbf{j}) = -(394.78 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_D = \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{B/D} - \omega_{BD}^2\mathbf{r}_{D/B}$$

$$a_D\mathbf{i} = -394.78\mathbf{j} + \alpha_{AB}\mathbf{k} \times (-0.2291\mathbf{i} - 0.1\mathbf{j}) - 0$$

$$= -394.78\mathbf{j} - 0.2291\alpha_{AB}\mathbf{j} + 0.1\alpha_{AB}\mathbf{i}$$

Equate like components.

$$\mathbf{j}: \quad 0 = -394.78 - 0.2291\alpha_{AB} \quad \alpha_{AB} = -1723 \text{ rad/s}^2$$

$$\mathbf{i}: \quad a_D = (0.1)(-1723) = -172.3 \text{ m/s}^2$$

$$\boldsymbol{\alpha}_{AB} = -(1723 \text{ rad/s}^2)\mathbf{k} \qquad \mathbf{a}_D = -(172.3 \text{ m/s}^2)\mathbf{i}$$

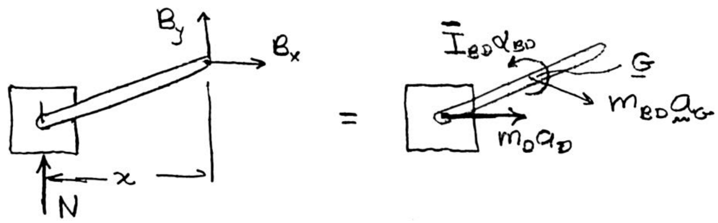
PROBLEM 16.138 (Continued)

Acceleration of mass center G of bar BD

$$\mathbf{r}_{G/D} = \frac{1}{2}(0.2291\mathbf{i} + 0.1\mathbf{j})$$
$$= (0.11455 \text{ m})\mathbf{i} + (0.05 \text{ m})\mathbf{j}$$
$$\mathbf{a}_G = \mathbf{a}_D + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{G/D} - \omega_{BD}^2 \mathbf{r}_{G/D}$$
$$= -172.3\mathbf{i} + (-1723\mathbf{k}) \times (0.11455\mathbf{i} + 0.05\mathbf{j}) - 0$$
$$= -172.3\mathbf{i} - 197.34\mathbf{j} + 86.15\mathbf{i}$$
$$= -(86.15 \text{ m/s}^2)\mathbf{i} - (197.34 \text{ m/s}^2)\mathbf{j}$$

Force on bar BD at P in B.

Use piston + bar BD as a free body



$$m_D\mathbf{a}_D = (1.8)(-172.3\mathbf{i})$$
$$= -(310.14 \text{ N})\mathbf{i}$$
$$m_{BD}\mathbf{a}_G = (1.2)(-86.15\mathbf{i} - 197.34\mathbf{j})$$
$$= -(103.38 \text{ N})\mathbf{i} - (236.88 \text{ N})\mathbf{j}$$
$$\bar{I}_{BD}\alpha_{BD} = \frac{1}{12}(1.2)(0.25)^2(-1723) = -10.769 \text{ N}\cdot\text{m}$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad B_x = m_D a_D + (m_{BD} a_G)_x$$
$$B_x = -310.14 - 103.38 \quad B_x = -413.52 \text{ N}$$

$$+\Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad -xN\mathbf{k} = \bar{I}_{BD}\alpha_{BD}\mathbf{k} + \mathbf{r}_{D/B} \times (m_D\mathbf{a}_D) + \mathbf{r}_{G/B} \times (m_{BD}\mathbf{a}_G)$$
$$-0.2291\, N\mathbf{k} = -10.769\mathbf{k} + (0.1)(-310.14)\mathbf{k} + (-0.11455\mathbf{i} - 0.05\mathbf{j})$$
$$\times(-103.38\mathbf{i} - 236.38\mathbf{j})$$
$$= -10.769\mathbf{k} - 31.014\mathbf{k} + 27.038\mathbf{k} - 5.169\mathbf{k}$$
$$N = 86.923 \text{ N}$$

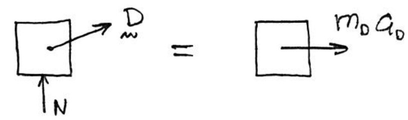
$$+\Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N + B_y = (m_{BD} a_G)_y$$
$$86.923 + B_y = -236.88 \quad B_y = -323.80 \text{ N}$$
$$B = \sqrt{413.52^2 + 323.80^2} = 525.2 \text{ N}$$
$$\tan\beta = \frac{323.80}{413.52} \quad \beta = 38.1°$$

$\mathbf{B} = 525 \text{ N} \swarrow 38.1°$ ◀

PROBLEM 16.138 (Continued)

Force exerted by bar *BD* as piston *D*.

Use piston *D* as a free body $\quad \Sigma\mathbf{F} = \Sigma\mathbf{F}_{\text{eff}}$



$$\mathbf{D} + N\mathbf{j} = m_D a_D \mathbf{i}$$

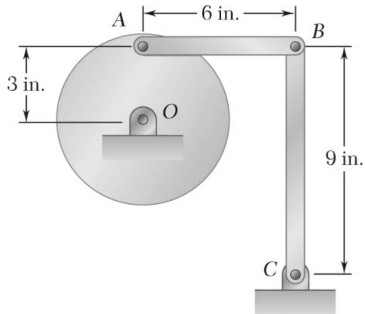
$$\mathbf{D} = m_D a_D \mathbf{i} - N\mathbf{j}$$

$$= -(310.14\text{ N})\mathbf{i} + (86.923\text{ N})\mathbf{j}$$

Force exerted by the piston on bar *BD*. By Newton's third law,

$$\mathbf{D}' = -\mathbf{D} = (310.14\text{ N})\mathbf{i} - (86.923\text{ N})\mathbf{j}$$

$\mathbf{D}' = 322\text{ N} \measuredangle\ 15.7°$ ◀



PROBLEM 16.139

The 4-lb rod AB and the 6-lb rod BC are connected as shown to a disk that is made to rotate in a vertical plane at a constant angular velocity of 6 rad/s clockwise. For the position shown, determine the forces exerted at A and B on rod AB.

SOLUTION

Kinematics:

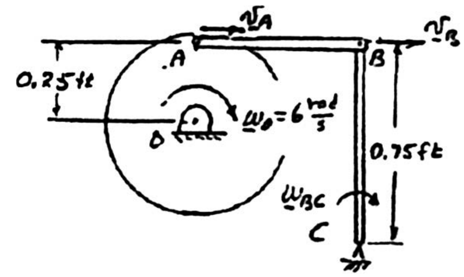
Velocity

$$\omega_{AB} = 0$$

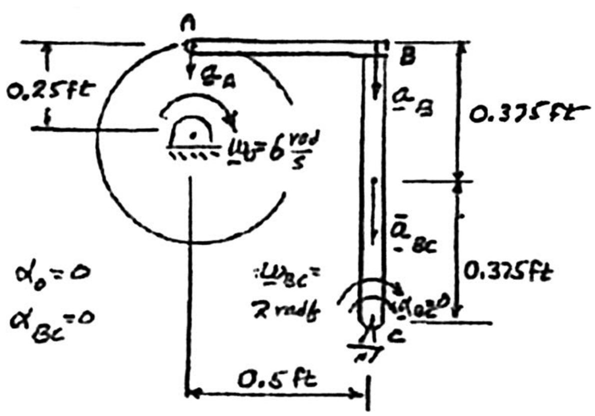
$$v_B = v_A = (0.25\text{ ft})(6\text{ rad/s}) = 1.5\text{ ft/s}^2$$

$$\omega_{BC} = \frac{v_B}{0.75\text{ ft}} = \frac{1.5\text{ m/s}^2}{0.75\text{ ft}}$$

$$\boldsymbol{\omega}_{BC} = 2\text{ rad/s} \circlearrowright$$



Acceleration



$$a_A = (0.25\text{ ft})(6\text{ rad/s})^2$$

$$\mathbf{a}_A = 9\text{ ft/s}^2 \downarrow$$

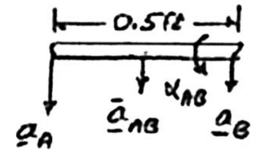
$$\mathbf{a}_B = (0.75\text{ ft})(2\text{ rad/s})^2 = 3\text{ ft/s}^2 \downarrow$$

$$\bar{a}_{BC} = (0.375\text{ ft})(2\text{ rad/s})^2$$

$$\bar{a}_{BC} = 1.5\text{ ft/s}^2$$

PROBLEM 16.139 (Continued)

$$\bar{a}_{AB} = \frac{1}{2}(a_A + a_B) = \frac{1}{2}(9+3)$$



$$\bar{\mathbf{a}}_{AB} = 6 \text{ ft/s}^2 \downarrow$$

$$a_A = a_B + (0.5 \text{ ft})\alpha_{AB}$$

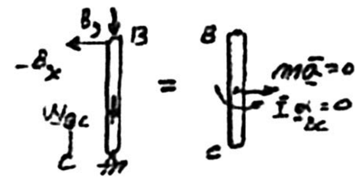
$$9 \text{ ft/s}^2 = 3 \text{ ft/s}^2 + (0.5 \text{ ft})\alpha_{AB}$$

$$\boldsymbol{\alpha}_{AB} = 12 \text{ rad/s}^2 \circlearrowleft$$

Kinetics:

$$\bar{I}_{AB} = \frac{1}{12} m_{AB}(AB)^2 = \frac{1}{12}\frac{4 \text{ lb}}{32.2}(0.5 \text{ ft})^2$$

$$\bar{I}_{AB} = 2.588 \times 10^3 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

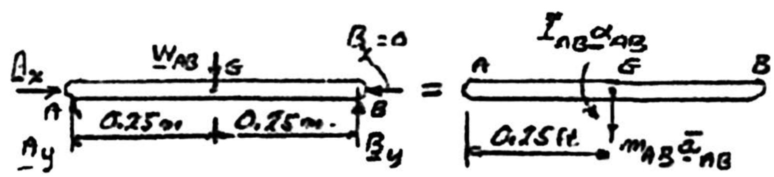


Rod BC:

Since $\alpha_{BC} = 0, \quad \bar{a} = 0$

$$\Sigma M_C = 0 \quad \text{yields} \quad B_x = 0$$

Rod AB:



$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad B_y(0.5 \text{ ft}) - W_{AB}(0.25 \text{ ft}) = \bar{I}_{AB}\alpha_{AB} - m_{AB}\bar{a}_{AB}(0.25 \text{ ft})$$

$$0.5B_y - (4 \text{ lb})(0.25 \text{ ft}) = (2.588 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2)(12 \text{ rad/s})$$

$$-\frac{4 \text{ lb}}{32.2}(6 \text{ ft/s}^2)(0.25 \text{ ft})$$

$$0.5B_y - 1 = 0.03106 - 0.1863$$

$$0.5B_y = 0.8447$$

$$B_y = 1.689 \text{ lb}$$

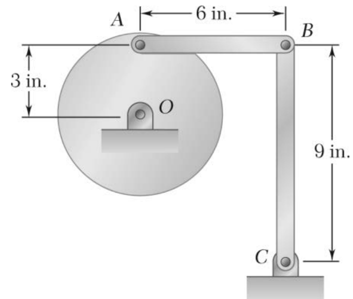
$\mathbf{B} = 1.689 \text{ lb} \uparrow$ ◀

$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad A_y - W_{AB} + B_y = -m_{AB}\bar{a}_{AB}$$

$$A_y - 4 \text{ lb} + 1.689 \text{ lb} = -\frac{4 \text{ lb}}{32.2}(6 \text{ ft/s}^2)$$

$$A_y = 1.565 \text{ lb}$$

$\mathbf{A} = 1.565 \text{ lb} \uparrow$ ◀

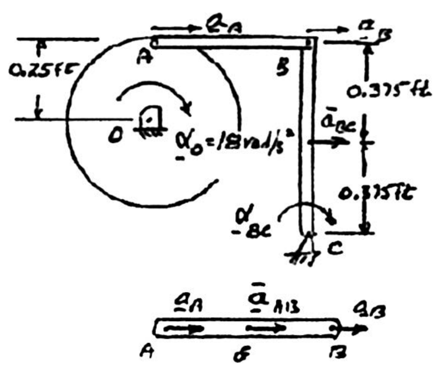


PROBLEM 16.140

The 4-lb rod *AB* and the 6-lb rod *BC* are connected as shown to a disk that is made to rotate in a vertical plane. Knowing that at the instant shown the disk has an angular acceleration of 18 rad/s^2 clockwise and no angular velocity, determine the components of the forces exerted at *A* and *B* on rod *AB*.

SOLUTION

<u>Kinematics</u>: Velocity of all elements $= C$



Acceleration:

$$\mathbf{a}_B = \mathbf{a}_A = (0.25\text{ ft})(18\text{ rad/s}^2) = 4.5\text{ ft/s}^2 \longrightarrow$$

$$\alpha_{BC} = \frac{a_B}{0.75\text{ ft}} = \frac{4.5\text{ ft/s}^2}{0.75\text{ ft}}$$

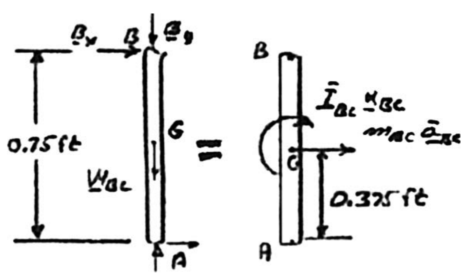
$$\boldsymbol{\alpha}_{BC} = 6\text{ rad/s}^2 \circlearrowright$$

$$\bar{\mathbf{a}}_{BC} = (0.375\text{ ft})(6\text{ rad/s}^2) = 2.25\text{ ft/s}^2 \longrightarrow$$

$$\bar{\mathbf{a}}_{AB} = \mathbf{a}_A = \mathbf{a}_B = 4.5\text{ ft/s}^2 \longrightarrow$$

<u>Kinetics</u>: $\bar{I}_{BC} = \frac{1}{12} m_{BC}(BC)^2 = \frac{1}{12}\frac{6\text{ lb}}{32.2}(0.75\text{ ft})^2$

Rod *BC*: $\bar{I}_{BC} = 8.734 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad B_x(0.75\text{ ft}) = \bar{I}_{BC}\alpha_{BC} + m_{BC}\bar{a}_{BC}(0.375\text{ ft})$$

$$0.75B_y = (8.734\times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2)(6\text{ rad/s}^2) + \left(\frac{6\text{ lb}}{32.2}\right)(2.25\text{ ft/s}^2)(0.375\text{ ft})$$

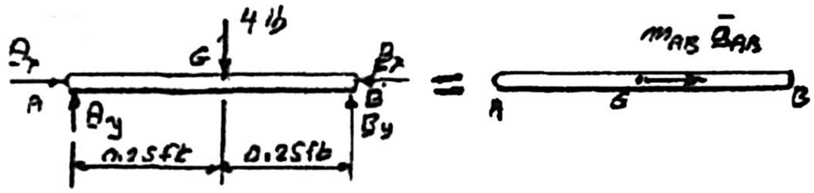
$$0.75B_x = 0.0524 + 0.1572$$

$$B_x = 0.2795\text{ lb}$$

$$(\text{on } AB) \quad \mathbf{B}_x = 0.280\text{ lb} \longleftarrow$$

PROBLEM 16.140 (Continued)

Rod *AB*:



$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad A_x - B_x = m_{AB}\bar{a}_{AB}$$

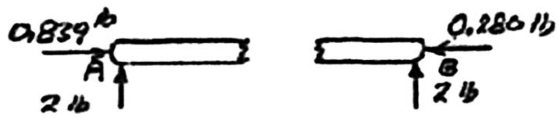
$$A_x - 0.2795 \text{ lb} = \left(\frac{4 \text{ lb}}{32.2}\right)(4.5 \text{ ft/s}^2)$$

$$A_x - 0.2795 \text{ lb} = 0.5590 \text{ lb}$$

$$A_x = 0.8385 \text{ lb} \qquad \mathbf{A}_x = 0.839 \text{ lb} \longrightarrow$$

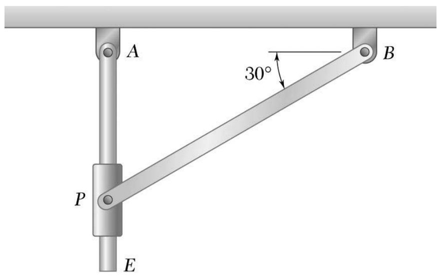
$$\Sigma M_A: \quad \mathbf{B}_y = 2 \text{ lb}\uparrow$$

$$\Sigma M_B: \quad \mathbf{A}_y = 2 \text{ lb}\uparrow$$



◀

PROBLEM 16.141



Two rotating rods in the vertical plane are connected by a slider block P of negligible mass. The rod attached at A has a weight of 1.6 lb and a length of 8 in. Rod BP weighs 2 lb and is 10 in. long and the friction between block P and AE is negligible. The motion of the system is controlled by a couple **M** applied to rod BP. Knowing that rod BP has a constant angular velocity of 20 rad/s clockwise, determine (a) the couple **M**, (b) the components of the force exerted on AE by block P.

SOLUTION

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \; \mathbf{j} = 1 \uparrow, \; \mathbf{k} = 1 \circlearrowleft.$

Geometry:

$$\mathbf{r}_{P/A} = -\left(\frac{10}{12}\sin 30° \text{ ft}\right)\mathbf{j}$$

$$\mathbf{r}_{P/B} = -\left(\frac{10}{12}\cos 30° \text{ ft}\right)\mathbf{i} - \left(\frac{10}{12}\sin 30° \text{ ft}\right)\mathbf{j}$$

$$\mathbf{r}_{P/A} = -\left(\frac{8}{12} \text{ ft}\right)\mathbf{j}$$

Kinematics:

$$\boldsymbol{\omega}_{BP} = -(20 \text{ rad/s})\mathbf{k}, \qquad \boldsymbol{\alpha}_{BP} = 0$$

Velocity analysis.

Rod BP:

$$\mathbf{v}_P = \boldsymbol{\omega}_{BP} \times \mathbf{r}_{P/B}$$

$$= -20\mathbf{k} \times \left(-\frac{10}{12}\cos 30°\mathbf{i} - \frac{10}{12}\sin 30°\mathbf{j}\right)$$

$$= -(8.3333 \text{ ft/s})\mathbf{i} + (14.4338 \text{ ft/s})\mathbf{j}$$

Rod AE: Use a frame of reference rotating with angular velocity $\boldsymbol{\omega}_{AE} = \omega_{AE}\mathbf{k}$. The collar P slides on the rod with relative velocity $\mathbf{v}_{P/A} = u\mathbf{j}$

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/AE} = \boldsymbol{\omega}_{AE} \times \mathbf{r}_{P/A} + u\,\mathbf{j}$$

$$= \omega_{AE}\mathbf{k} \times \left(-\frac{10}{12}\sin 30°\mathbf{j}\right) + u\,\mathbf{j} = 0.41667\,\omega_{AE}\mathbf{i} + u\,\mathbf{j}$$

Equate the two expressions for $\mathbf{v}_P$ and resolve into components.

$\mathbf{i}$: $-8.3333 = 0.41667\,\omega_{AE}$ $\qquad \omega_{AE} = -20 \text{ rad/s}$

$\mathbf{j}$: $14.4388 = u$ $\qquad u = 14.4338 \text{ ft/s}$

Acceleration analysis.

Rod BP:

$$\mathbf{a}_P = \boldsymbol{\alpha}_{BP} \times \mathbf{r}_{P/B} - \omega_{BP}^2\mathbf{r}_{P/B}$$

$$= 0 - (20)^2\left(-\frac{10}{12}\cos 30°\mathbf{i} - \frac{10}{12}\sin 30°\mathbf{j}\right)$$

$$= (288.68 \text{ ft/s}^2)\,\mathbf{i} + (166.67 \text{ ft/s}^2)\,\mathbf{j}$$

Rod *AE*: $\boldsymbol{\alpha}_{AE} = \boldsymbol{\alpha}_{AE}\mathbf{k}, \qquad \mathbf{a}_{P/AE} = \dot{u}\,\mathbf{j}$

$$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/AE} + 2\boldsymbol{\omega}\times\mathbf{v}_{P/AE}$$

where
$$\mathbf{a}_{P'} = \boldsymbol{\alpha}_{AB}\times r_{P/A} - \omega_{AE}^2\mathbf{r}_{P/A}$$
$$= \alpha_{AE}\mathbf{k}\times\left(-\frac{10}{12}\sin 30°\mathbf{j}\right) - (20)^2\left(-\frac{10}{12}\sin 30°\mathbf{j}\right)$$
$$= 0.41667\alpha_{AE}\mathbf{i} + (166.67\text{ ft/s}^2)\mathbf{j}$$

and
$$2\boldsymbol{\omega}_{AE}\times\mathbf{v}_{P/AE} = (2)(-20\mathbf{k})\times(14.4338\,\mathbf{j})$$
$$= 577.35\text{ ft/s}^2\mathbf{i}$$

Equate the two expressions for $\mathbf{a}_P$ and resolve into components.

$$\mathbf{i}:\quad 288.68 = 0.41667\alpha_{AE} + 577.35$$

$$\alpha_{AE} = -692.8\text{ rad/s}^2$$

Summary:
$$\boldsymbol{\omega}_{BP} = -(20\text{ rad/s})\mathbf{k},\quad \boldsymbol{\omega}_{AE} = -(20\text{ rad/s})\mathbf{k}$$
$$\boldsymbol{\alpha}_{BP} = 0,\qquad \boldsymbol{\alpha}_{AE} = -(692.8\text{ rad/s}^2)\mathbf{k}$$

Masses and moments of inertia.

$$m_{AE} = \frac{W_{AE}}{g} = \frac{1.6\text{ lb}}{32.2\text{ ft/s}^2} = 0.049689\text{ lb}\cdot\text{s}^2\text{/ft}$$
$$m_{BP} = \frac{W_{BP}}{g} = \frac{2\text{ lb}}{32.2\text{ ft/s}^2} = 0.062112\text{ lb}\cdot\text{s}^2\text{/ft}$$
$$\bar{I}_{AE} = \frac{1}{12}m_{AE}l_{AE}^2 = \frac{1}{12}(0.049689\text{ lb}\cdot\text{s}^2\text{/ft})\left(\frac{8}{12}\text{ ft}\right)^2 = 1.8403\times10^{-3}\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$
$$\bar{I}_{BP} = \frac{1}{12}m_{BP}l_{BP}^2 = \frac{1}{12}(0.062112\text{ lb}\cdot\text{s}^2\text{/ft})\left(\frac{10}{12}\text{ ft}\right)^2 = 3.5944\times10^{-3}\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Mass centers: Let Point *G* be the mass center of rod *AE* and Point *H* be that of rod *BP*.

$$\mathbf{r}_{G/A} = -\left(\frac{4}{12}\text{ ft}\right)\mathbf{j}$$
$$\mathbf{r}_{H/B} = -\left(\frac{10}{12}\cos 30°\text{ ft}\right)\mathbf{i} - \left(\frac{10}{12}\sin 30°\text{ ft}\right)\mathbf{j}$$

Acceleration of mass centers.

$$\mathbf{a}_G = \boldsymbol{\alpha}_{AE}\times\mathbf{r}_{G/A} - \omega_{AE}^2\mathbf{r}_{G/A}$$
$$= (-692.8\mathbf{k})\times(-0.33333\mathbf{j}) - (20)^2(-0.33333\mathbf{j}) = -(230.93\text{ ft/s}^2)\mathbf{i} + (133.33\text{ ft/s}^2)\mathbf{j}$$
$$\mathbf{a}_H = \boldsymbol{\alpha}_{BP}\times\mathbf{r}_{H/B} - \omega_{BP}^2\mathbf{r}_{H/B}$$
$$= 0 - (20)^2(-0.72169\mathbf{i} - 0.41667\mathbf{j}) = (288.68\text{ ft/s}^2)\mathbf{i} + (166.67\text{ ft/s}^2)\mathbf{j}$$

Effective forces at mass centers.

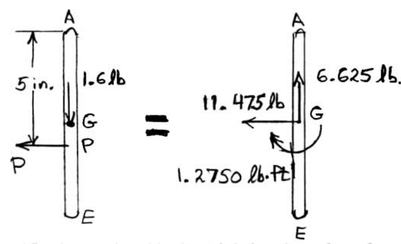
Rod AE: $m_{AE}\mathbf{a}_G = (0.049689)(-230.93\mathbf{i} + 133.33\mathbf{j}) = -(11.475\text{ lb})\mathbf{i} + (6.625\text{ lb})\,\mathbf{j}$

Rod BP: $m_{BP}\mathbf{a}_H = (0.062112)(288.68\mathbf{i} + 166.67\mathbf{j}) = (17.930\text{ lb})\mathbf{i} + (10.352\text{ lb})\,\mathbf{j}$

Effective couples at mass centers.

Rod AE: $\bar{I}_{AE}\boldsymbol{\alpha}_{AE} = (1.8403\times 10^{-3})(-692.8\mathbf{k}) = -(1.2750\text{ lb}\cdot\text{ft})\,\mathbf{k}$

Rod BP: $\bar{I}_{BP}\boldsymbol{\alpha}_{BP} = 0$



Kinetics.

Rod AE: $\Sigma\mathbf{M}_A = \Sigma(\mathbf{M}_A)_{\text{eff}}:\quad \mathbf{r}_{P/A}\times(-P\mathbf{i}) = \mathbf{r}_{G/A}\times(m_{AE}\mathbf{a}_G) + \bar{I}_{AE}\boldsymbol{\alpha}_{AE}$

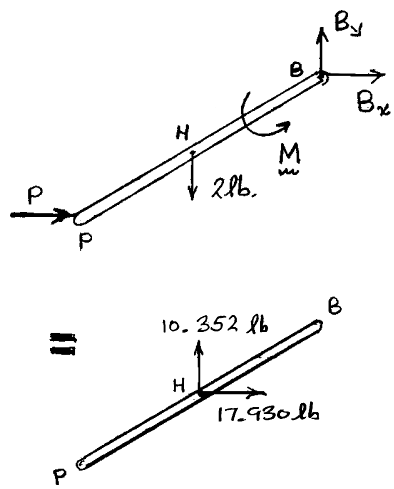
$$\left(-\frac{5}{12}\mathbf{j}\right)\times(-P\mathbf{i}) = \left(-\frac{4}{12}\mathbf{j}\right)\times(-11.475\mathbf{i} + 16.625\mathbf{j}) - 1.2750\mathbf{k}$$

$$-\frac{5}{12}P\mathbf{k} = -3.8249\mathbf{k} - 1.2750\mathbf{k} \qquad P = 12.240\text{ lb}$$

Rod BP: $\Sigma\mathbf{M}_B = \Sigma(\mathbf{M}_B)_{\text{eff:}}\quad \mathbf{r}_{P/B}\times P\mathbf{i} + \mathbf{r}_{H/B}\times(-W_{BP}\mathbf{j}) + M\mathbf{k} = \mathbf{r}_{H/B}\times(m_{BP}\mathbf{a}_H) + \bar{I}_{BP}\boldsymbol{\alpha}_{BP}$

$$\frac{5}{12}P\mathbf{k} + \left(\frac{5\cos 30^\circ}{12}\text{ ft}\right)(2\text{ lb})\mathbf{k} + M\mathbf{k}$$

$$= (-0.36084\mathbf{i} - 0.20833\mathbf{j})\times(17.930\mathbf{i} + 10.352\mathbf{j}) + 0$$

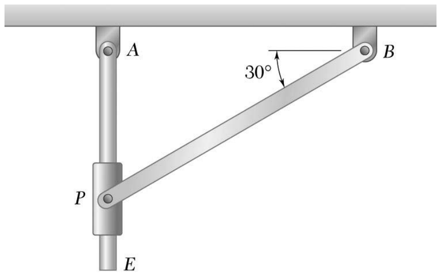


$$5.1\mathbf{k} + 0.72169\mathbf{k} + M\mathbf{k} = -3.7354\mathbf{k} + 3.7354\mathbf{k}$$

$$M = -5.82\text{ lb}\cdot\text{ft}$$

(*a*) *Couple* **M**. $\mathbf{M} = 5.82\text{ lb}\cdot\text{ft}$ ↻ ◀

(*b*) *Force exerted on AE by block P*. $\mathbf{P} = 12.24\text{ lb}$ ⟵ ◀



PROBLEM 16.142

Two rotating rods in the vertical plane are connected by a slider block *P* of negligible mass. The rod attached at *A* has a mass of 0.8 kg and a length of 160 mm. Rod *BP* has a mass of 1 kg and is 200 mm long and the friction between block *P* and *AE* is negligible. The motion of the system is controlled by a couple **M** applied to bar *AE*. Knowing that at the instant shown rod *BP* has an angular velocity of 20 rad/s clockwise and an angular acceleration of 80 rad/s^2 clockwise, determine (*a*) the couple **M**, (*b*) the components of the force exerted on *AE* by block *P*.

SOLUTION

Geometry: $\mathbf{r}_{P/A} = (0.200\text{ m})\sin 30° \downarrow = 0.100\text{ m} \downarrow$

$\mathbf{r}_{P/B} = 0.200\text{ m} \swarrow 30°$

$\mathbf{r}_{E/A} = 0.160\text{ m} \downarrow$

Kinematics: $\boldsymbol{\omega}_{BP} = 20\text{ rad/s} \circlearrowright$

$\boldsymbol{\alpha}_{BP} = 80\text{ rad/s}^2 \circlearrowright$

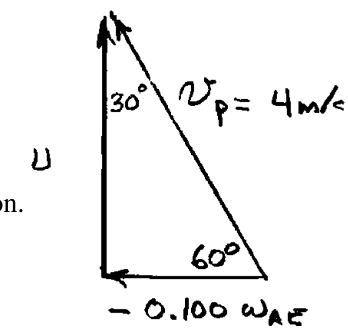
Velocity analysis.

Rod *BP*: $\mathbf{v}_P = \boldsymbol{\omega}_{BP} \times \mathbf{r}_{P/B} = (20\text{ rad/s}) \cdot (0.200\text{ m}) \searrow 60° = 4\text{ m/s} \searrow 60°$

Rod *AE*: Use a frame of reference rotating with angular velocity $\boldsymbol{\omega}_{AE} = \omega_{AE} \circlearrowleft$. The collar slides on the rod with relative velocity $\mathbf{v}_{P/AE} = u \uparrow$.

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/AE} = \omega_{AE} \times \mathbf{r}_{P/A} + u \uparrow$$

$$= 0.100\ \omega_{AE} \longrightarrow + u \uparrow$$



Equate the two expressions for $\mathbf{v}_P$ using a triangle construction for vector addition.

$$-0.100\omega_{AE} = 4\cos 60°$$

$$\omega_{AE} = -20\text{ rad/s} \quad \boldsymbol{\omega}_{AE} = 20\text{ rad/s} \circlearrowright$$

$$u = 4\sin 60° = 3.461\text{ m/s} \qquad \mathbf{v}_{P/AE} = 3.4641\text{ m/s} \uparrow$$

Acceleration analysis. $\boldsymbol{\alpha}_{BP} = \alpha_{BP} \circlearrowleft$

Rod *BP*: $\mathbf{a}_P = \boldsymbol{\alpha}_{BP} \times \mathbf{r}_{P/B} - \omega_{BP}^2 \mathbf{r}_{P/B}$

$$= [(80\text{ rad/s}^2)(0.200\text{ m}) \searrow 60°] + [(20\text{ rad/s})^2 (0.200\text{ m}) \angle 30°]$$

$$= [16\text{ m/s}^2 \searrow 60°] + [80\text{ m/s}^2 \angle 30°]$$

Rod *AE*: $\boldsymbol{\alpha}_{AE} = \alpha_{AE} \circlearrowright, \quad \mathbf{a}_{P/AE} = \dot{u} \uparrow$

$$\boldsymbol{\alpha}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/AE} + 2\boldsymbol{\omega}_{AE} \times \mathbf{v}_{P/AE}$$

where $$\mathbf{a}_{P'} = \boldsymbol{\alpha}_{AE} \times \mathbf{r}_{P/A} - \omega_{AE}^2 \mathbf{r}_{P/A}$$

$$= [(0.100 \text{ m})\alpha_{AE} \longrightarrow] + [(20 \text{ rad/s}^2)(0.100 \text{ m}) \uparrow]$$

$$= [0.100\alpha_{AE} \longrightarrow] + [40 \text{ m/s}^2 \uparrow]$$

and $$2\boldsymbol{\omega}_{AE} \times \mathbf{v}_{P/AE} = [(2)(20 \text{ rad/s})(3.4641 \text{ m/s}) \longrightarrow] = 138.564 \text{ m/s}^2 \longrightarrow$$

Equate the two expressions for $\mathbf{a}_P$ and resolve into components.

$$\longrightarrow: -(16 \text{ m/s}^2)\cos 60° + (80 \text{ m/s}^2)\cos 30°$$
$$= 0.100\alpha_{AE} + 138.564 \text{ m/s}^2$$

$$\alpha_{AE} = -772.82 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{AE} = 772.82 \text{ rad/s}^2 \circlearrowright$$

Masses, weights, and moments of inertia.

$$m_{AE} = 0.8 \text{ kg} \quad W_{AE} = m_{AE}g = (0.8 \text{ kg})(9.81 \text{ m/s}^2) = 7.848 \text{ N}$$
$$m_{BP} = 1.0 \text{ kg} \quad W_{BP} = m_{BP}g = (1.0 \text{ kg})(9.81 \text{ m/s}^2) = 9.81 \text{ N}$$
$$\bar{I}_{AE} = \frac{1}{12}m_{AE}l_{AE}^2 = \frac{1}{12}(0.8 \text{ kg})(0.16 \text{ m})^2 = 1.70667 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$
$$\bar{I}_{BP} = \frac{1}{12}m_{BP}l_{BP}^2 = \frac{1}{12}(1.0 \text{ kg})(0.20 \text{ m})^2 = 3.3333 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Mass centers: Let Point *G* be the mass center of rod *AE* and Point *H* be that of rod *BP*.

$$\mathbf{r}_{G/A} = 0.08 \text{ m} \downarrow \qquad \mathbf{r}_{H/B} = 0.10 \text{ m} \measuredangle 30°$$

Accelerations of mass centers.

$$\mathbf{a}_G = \boldsymbol{\alpha}_{AE} \times \mathbf{r}_{G/A} - \omega_{AE}^2 \mathbf{r}_{G/A}$$

$$= (772.82 \text{ rad/s}^2)(0.08 \text{ m}) \longleftarrow + (20 \text{ rad/s}^2)(0.08 \text{ m}) \uparrow$$

$$= [61.826 \text{ m/s}^2 \longleftarrow] + [32 \text{ m/s}^2 \uparrow]$$

$$\mathbf{a}_H = \boldsymbol{\alpha}_{BP} \times \mathbf{r}_{H/A} - \omega_{BP}^2 \mathbf{r}_{H/B}$$

$$= [(80 \text{ rad/s}^2)(0.10 \text{ m}) \searrow 60°] + [(20 \text{ rad/s})^2(0.10 \text{ m}) \measuredangle 30°]$$

$$= [8 \text{ m/s}^2 \searrow 60°] + [40 \text{ m/s}^2 \measuredangle 30°]$$

Effective forces at mass centers.

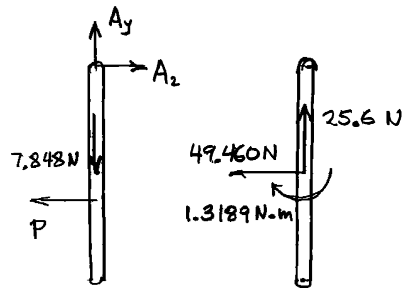
Rod *AE*: $m_{AE}\mathbf{a}_G = [49.460 \text{ N} \longleftarrow] + [25.6 \text{ N} \uparrow]$

Rod *BP*: $m_{BP}\mathbf{a}_H = [8 \text{ N} \searrow 60°] + [40 \text{ N} \measuredangle 30°]$

Effective couples at mass centers.

Rod *AE*: $\bar{I}_{AE}\boldsymbol{\alpha}_{AE} = 1.3189 \text{ N}\cdot\text{m} \circlearrowright$

Rod *BP*: $\bar{I}_{BP}\boldsymbol{\alpha}_{BP} = 0.2667 \text{ N}\cdot\text{m} \circlearrowright$



Kinetics:

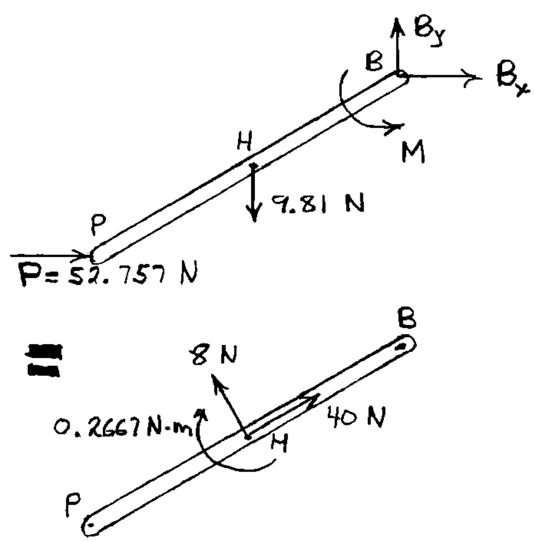
Rod *AE*: $+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad -0.10P = -(0.08)(49.460) - 1.3189$

$$P = 52.757 \text{ N}$$

Rod *BP*: $+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad (52.757 \text{ N})(0.1 \text{ m}) + (9.81 \text{ N})(0.086603 \text{ m}) + M$

$$= -(8 \text{ N})(0.1 \text{ m}) - 0.2667 \text{ N}\cdot\text{m}$$

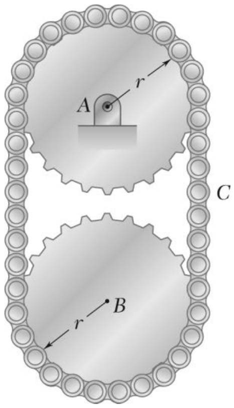
$$M = 7.1919 \text{ N}\cdot\text{m}$$



(*a*) *Couple* **M**. $M = 7.19 \text{ N}\cdot\text{m} \circlearrowright$ ◀

(*b*) *Force exerted on AE by force by block P*. $P = 52.8 \text{ N} \longleftarrow$ ◀

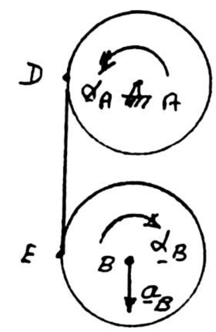
PROBLEM 16.143*



Two disks, each of mass *m* and radius *r* are connected as shown by a continuous chain belt of negligible mass. If a pin at Point *C* of the chain belt is suddenly removed, determine (*a*) the angular acceleration of each disk, (*b*) the tension in the left-hand portion of the belt, (*c*) the acceleration of the center of disk *B*.

SOLUTION

<u>Kinematics</u>: Assume $\alpha_A \circlearrowleft$ and $\alpha_B \circlearrowright$



$$\omega_A = \omega_B = 0$$

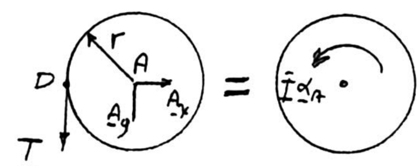
$$\mathbf{a}_D = r\alpha_A \downarrow$$

$$\mathbf{a}_E = a_D = r\alpha_A \downarrow$$

$$\bar{\mathbf{a}}_B = a_E + a_{B/E} = (r\alpha_A + r\alpha_B)$$

$$\bar{\mathbf{a}}_B = r(\alpha_A + \alpha_B) \downarrow$$

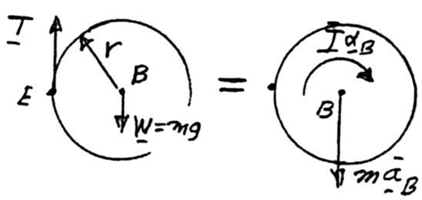
<u>Kinetics</u>: Disk *A*: $+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad Tr = \bar{I}\alpha_A$



$$Tr = \frac{1}{2}mr^2\alpha_A$$

$$\alpha_A = \frac{2T}{mr} \circlearrowleft \qquad (1)$$

Disk *B*: $+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad Tr = \bar{I}\alpha_B$



$$Tr = \frac{1}{2}mr^2\alpha_B$$

$$\alpha_B = \frac{2T}{mr} \circlearrowright \qquad (2)$$

PROBLEM 16.143* (Continued)

From Eqs. (1) and (2) we note that $\alpha_A = \alpha_B$

$$+\circlearrowright \Sigma M_E = \Sigma(M_E)_{\text{eff}}: \quad Wr = \bar{I}\alpha_B + (m\bar{a}_B)r$$

$$Wr = \frac{1}{2}mr^2\alpha_B + mr(\alpha_A + \alpha_B)r$$

$$\alpha_A = \alpha_B: \quad Wr = \frac{5}{2}mr^2\alpha_A$$

$$\boldsymbol{\alpha}_A = \frac{2}{5}\frac{g}{r} \circlearrowleft \blacktriangleleft$$

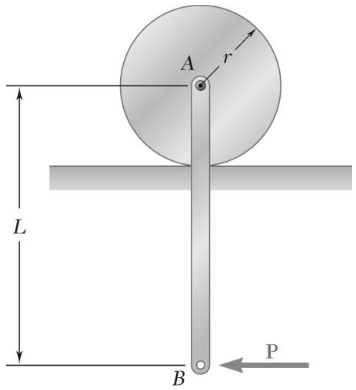
$$\boldsymbol{\alpha}_B = \frac{2}{5}\frac{g}{r} \circlearrowright \blacktriangleleft$$

Substitute for α_A into (1):

$$\frac{2}{5}\frac{g}{r} = \frac{2T}{mr} \qquad T = \frac{1}{5}mg \blacktriangleleft$$

$$a_B = r(\alpha_A + \alpha_B)$$
$$= r(2\alpha_A)$$
$$= 2r\left(\frac{2}{5}\frac{g}{r}\right) \qquad \mathbf{a}_B = \frac{4}{5}g \downarrow \blacktriangleleft$$

PROBLEM 16.144*



A uniform slender bar AB of mass m is suspended as shown from a uniform disk of the same mass m. Neglecting the effect of friction, determine the accelerations of Points A and B immediately after a horizontal force $\mathbf{P}$ has been applied at B.

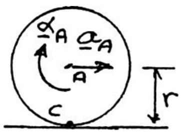
SOLUTION

<u>Kinematics</u>:

$$\omega = 0$$

Cylinder:

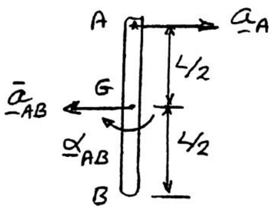
Rolling without sliding $\quad (a_C)_x = 0$



$$\xrightarrow{+} \mathbf{a}_A = (\mathbf{a}_C)_x + \mathbf{a}_{A/C} = 0 + r\alpha_A$$

$$\mathbf{a}_A = r\alpha_A \longrightarrow$$

Rod AB:

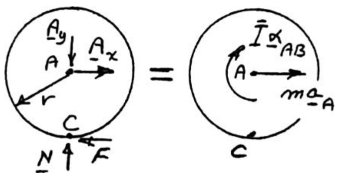


$$\boldsymbol{\alpha}_A = \frac{a_A}{r} \circlearrowright$$

$$\xrightarrow{+} a_A = \frac{L}{2}\alpha_{AB} - \bar{a}_{AB}$$

<u>Kinetics</u>:

Cylinder:



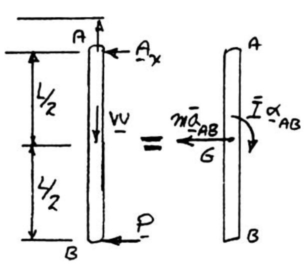
$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad A_x r = m a_A r + \bar{I}\alpha_A$$

$$A_x r = m a_A r + \frac{1}{2} m r^2 \left(\frac{a_A}{r}\right)$$

$$A_x = \frac{3}{2} m a_A$$

$$A_x = \frac{3}{2} m \left(\frac{L}{2}\alpha_{AB} - \bar{a}_{AB}\right) \tag{1}$$

Rod AB:



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad PL = m\bar{a}_{AB}\frac{L}{2} + \bar{I}\alpha_{AB}$$

$$PL = m\bar{a}_{AB}\frac{L}{2} + \frac{m}{12}L^2\alpha_{AB} \tag{2}$$

$$\xleftarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad P + A_x = m\bar{a}_{AB}$$

PROBLEM 16.144* (Continued)

Substitute from (1):

$$P + \frac{3}{2}m\left(\frac{L}{2}\alpha_{AB} - \bar{a}_{AB}\right) = m\bar{a}_{AB}$$

$$P = \frac{5}{2}m\bar{a}_{AB} - \frac{3}{4}mL\alpha_{AB} \qquad (3)$$

Multiply by $\frac{L}{9}$:

$$\frac{1}{9}PL = \frac{5L}{18}m\bar{a}_{AB} - \frac{1}{12}mL^2\alpha_{AB} \qquad (4)$$

(4) + (2):

$$\frac{10}{9}PL = \left(\frac{1}{2} + \frac{5}{18}\right)mL\bar{a}_{AB} = \frac{7}{9}mL\bar{a}_{AB}$$

$$\bar{\mathbf{a}}_{AB} = \frac{10}{7}\frac{P}{m} \leftarrow \qquad (5)$$

(5) ⟶ (3)

$$P = \frac{5}{2}m\left(\frac{10}{7}\frac{P}{m}\right) - \frac{3}{4}mL\alpha_{AB}$$

$$P = \frac{25}{7}P - \frac{3}{4}mL\alpha_{AB}$$

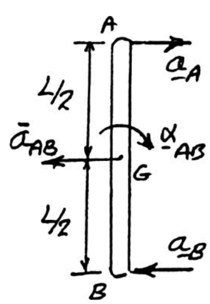
$$-\frac{18}{7}P = \frac{3}{4}mL\alpha_{AB}$$

$$\boldsymbol{\alpha}_{AB} = \frac{24}{7}\frac{P}{mL} \circlearrowright$$

$\xrightarrow{+}$:

$$a_A = \frac{L}{2}\alpha_{AB} - \bar{a}_{AB}$$

$$= \frac{L}{2}\left(\frac{24}{7}\frac{P}{mL}\right) - \frac{10}{7}\frac{P}{m}$$



$$a_A = \left(\frac{12}{7} - \frac{10}{7}\right)\frac{P}{m}$$

$$\mathbf{a}_A = \frac{2}{7}\frac{P}{m} \longrightarrow \blacktriangleleft$$

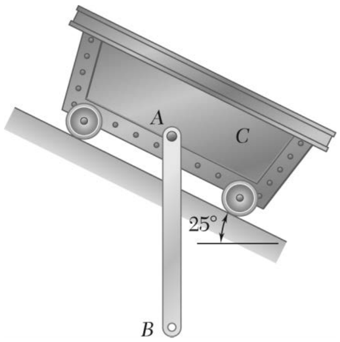
$\xleftarrow{+}$:

$$a_B = \frac{L}{2}\alpha_{AB} + \bar{a}_{AB}$$

$$= \frac{L}{2}\left(\frac{24}{7}\frac{P}{mL}\right) + \frac{10}{7}\frac{P}{m}$$

$$a_B = \left(\frac{12}{7} + \frac{10}{7}\right)\frac{P}{m}$$

$$\mathbf{a}_B = \frac{22}{7}\frac{P}{m} \longleftarrow \blacktriangleleft$$



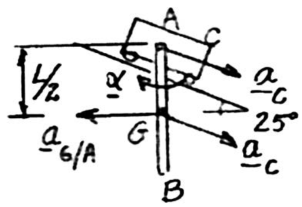
PROBLEM 16.145

A uniform rod *AB*, of mass 15 kg and length 1 m, is attached to the 20-kg cart *C*. Neglecting friction, determine immediately after the system has been released from rest, (*a*) the acceleration of the cart, (*b*) the angular acceleration of the rod.

SOLUTION

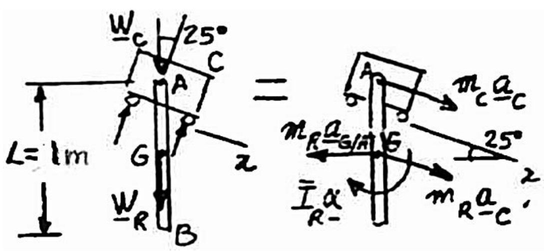
<u>Kinematics</u>: We resolve the acceleration of *G* into the acceleration of the cart and the acceleration of *G* relative to *A*:

$$\bar{\mathbf{a}}_R = \mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A}$$
$$\bar{\mathbf{a}}_R = \mathbf{a}_C + \mathbf{a}_{G/A}$$



where $$a_{G/A} = \frac{1}{2}L\alpha$$

<u>Kinetics</u>: Cart and rod



$$m_R = 15 \text{ kg}$$
$$m_C = 20 \text{ kg}$$
$$L = 1 \text{ m}$$
$$\bar{I}_R = \frac{1}{12} m_R L^2$$
$$a_{G/A} = \frac{1}{2}(1)\alpha = 0.5\alpha$$

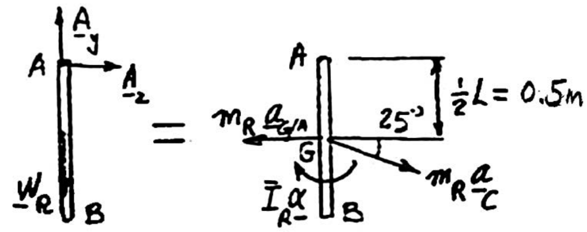
$\searrow^{+}$ $\Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad (m_C + m_R) g \sin 25° = (m_C + m_R)\mathbf{a}_C - m_R a_{G/A} \cos 25°$

$$g \sin 25° = a_C - \left(\frac{m_g}{m_C + m_R}\right)\left(\frac{L}{2}\alpha\right)\cos 25°$$

$$a_C = (9.81)\sin 25° + \left(\frac{15}{20+15}\right)(0.5\cos 25°)\alpha$$

$$a_C = (9.81)\sin 25° + 0.19421\alpha \qquad (1)$$

Rod



$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad 0 = \bar{I}\alpha + (m_R a_{G/R})\frac{L}{2} - (m_R a_C \cos 25°)\frac{L}{2}$$

$$\frac{1}{12}15(1)^2\alpha + 15(0.5\alpha)(0.5) - (15a_C \cos 25°)(0.5) = 0$$

$$1.25\alpha + 3.75\alpha - (7.5\cos 25°)a_C = 0$$

$$\alpha = (1.50\cos 25°)a_C \qquad (2)$$

(*a*) Acceleration of the cart.

Substitute for α from (2) into (1):

$$a_C = (9.81)\sin 25° + 0.19421(1.5\cos 25°)a_C$$

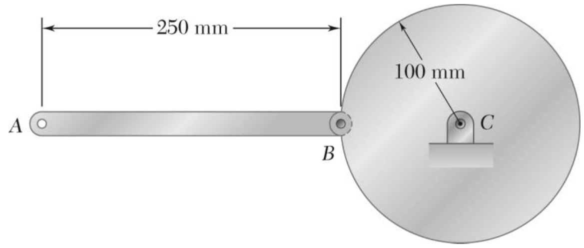
$$a_C = \frac{9.81\sin 25°}{1 - 0.19421(1.5\cos 25°)}$$

$$= 5.6331 \text{ m/s}^2 \qquad \mathbf{a}_C = 5.63 \text{ m/s}^2 \searrow 25° \blacktriangleleft$$

(*b*) Angular acceleration.

From (2): $\alpha = (1.50\cos 25°)(5.6331)$

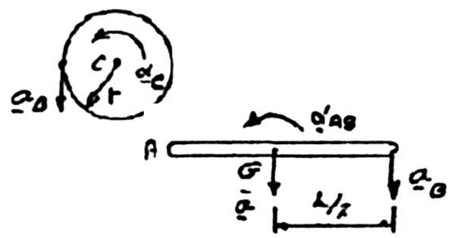
$$= 7.6580 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 7.66 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$



PROBLEM 16.146*

The 5-kg slender rod *AB* is pin-connected to an 8-kg uniform disk as shown. Immediately after the system is released from rest, determine the acceleration of (*a*) Point *A*, (*b*) Point *B*.

SOLUTION



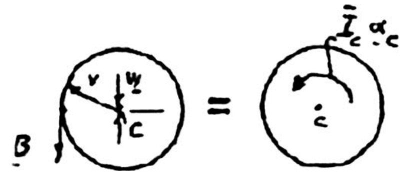
Kinematics:

$$\omega = 0$$

$$\mathbf{a}_B = r\alpha_C \downarrow$$

$$+\downarrow \quad \bar{\mathbf{a}} = \mathbf{a}_B + \mathbf{a}_{G/B}$$

$$\bar{\mathbf{a}} = \left(r\alpha_C + \frac{L}{2}\alpha_{AB} \right)\downarrow$$

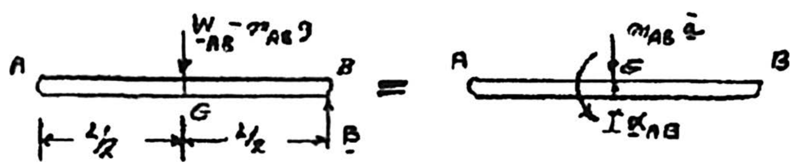


Kinetics: Disk $\quad +\circlearrowleft \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad Br = \bar{I}\,\alpha_C$

$$Br = \frac{1}{2} m_C r^2 \alpha_C$$

$$B = \frac{1}{2} m_C r \alpha_C$$

Rod *AB*:



$$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad B\frac{L}{2} = \bar{I}\,\alpha_{AB}$$

$$\left(\frac{1}{2} m_C r \alpha_C \right) \frac{L}{2} = \frac{1}{12} m_{AB} L^2 \alpha_{AB}$$

$$\alpha_C = \frac{1}{3}\frac{m_{AB}}{m_C} \cdot \frac{L}{r}\alpha_{AB} \qquad (1)$$

$$+\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad m_{AB} g - B = m_{AB}\bar{a}$$

$$m_{AB} g - \frac{1}{2} m_C r \alpha_C = m_{AB}\left(r\alpha_C + \frac{L}{2}\alpha_{AB} \right)$$

$$g = \frac{L}{2}\alpha_{AB} + \left(\frac{1}{2}\frac{m_C}{m_{AB}} + 1 \right) r\alpha_C$$

PROBLEM 16.146* (Continued)

$$\frac{g}{L} = \frac{1}{2}\alpha_{AB} + \left(\frac{1}{2}\frac{m_C}{m_{AB}} + 1\right)\frac{r}{L}\cdot\left(\frac{1}{3}\frac{m_{AB}}{m_C}\cdot\frac{L}{r}\right)\alpha_{AB}$$

$$\frac{g}{L} = \left(\frac{1}{2} + \frac{1}{6} + \frac{1}{3}\frac{m_{AB}}{m_C}\right)\alpha_{AB} = \frac{1}{3}\left(2 + \frac{m_{AB}}{m_C}\right)\alpha_{AB}$$

$$\alpha_{AB} = \frac{3g}{L}\frac{1}{\left(2 + \frac{m_{AB}}{m_C}\right)} \qquad (2)$$

$$m_{AB} = 5\text{ kg}, \quad m_C = 8\text{ kg}, \quad r = 0.1\text{ m}, \quad L = 0.25\text{ m}$$

Eq. (1): $$\boldsymbol{\alpha}_{AB} = \frac{3(9.81)\text{m/s}^2}{0.25\text{ m}}\cdot\frac{1}{2 + \frac{5\text{ kg}}{8\text{ kg}}} = 44.846\text{ rad/s}^2 \circlearrowleft$$

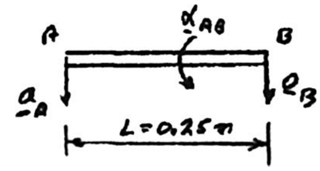
Eq. (2): $$\boldsymbol{\alpha}_C = \frac{1}{3}\frac{5\text{ kg}}{8\text{ kg}}\cdot\frac{0.25\text{ m}}{0.1\text{ m}}\cdot(44.846\text{ rad/s}^2) = 23.357\text{ rad/s}^2 \circlearrowleft$$

(*b*) Acceleration of *B*.

$$a_B = r\alpha_C$$
$$= (0.1\text{ m})(23.357\text{ rad/s}^2)$$
$$= 2.336\text{ m/s}^2 \qquad \mathbf{a}_B = 2.34\text{ m/s}^2 \downarrow \blacktriangleleft$$

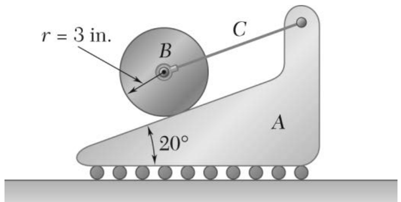
(*a*) Acceleration of *A*.

$$+\downarrow \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{B/A}$$



$$\mathbf{a}_A = \mathbf{a}_B + [L\alpha_{AB}\downarrow]$$
$$a_A = 2.336\text{ m/s}^2 + (0.25\text{ m})(44.846\text{ rad/s}^2)$$
$$a_A = 2.336 + 11.212 \qquad \mathbf{a}_A = 13.55\text{ m/s}^2 \downarrow \blacktriangleleft$$

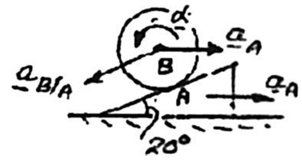
PROBLEM 16.147*



The 6-lb cylinder *B* and the 4-lb wedge *A* are held at rest in the position shown by cord *C*. Assuming that the cylinder rolls without sliding on the wedge and neglecting friction between the wedge and the ground, determine, immediately after cord *C* has been cut, (*a*) the acceleration of the wedge, (*b*) the angular acceleration of the cylinder.

SOLUTION

Kinematics: We resolve $\mathbf{a}_B$ into $\mathbf{a}_A$ and a component parallel to the incline

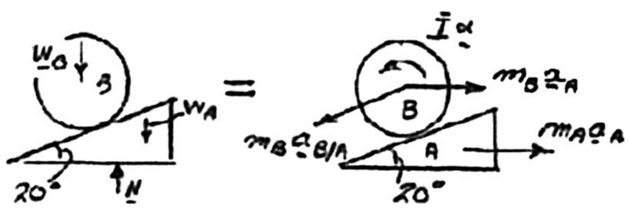


$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Where $a_{B/A} = r\alpha$, since the cylinder rolls on wedge *A*.

$$a_{B/A} = (0.25 \text{ ft})\alpha$$

Kinetics: Cylinder and wedge

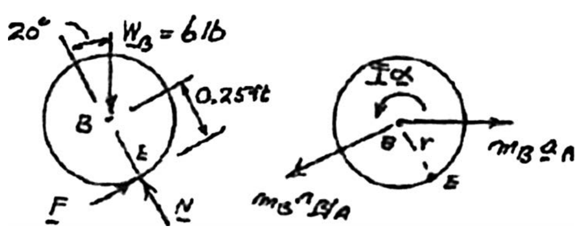


$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad 0 = m_A a_A + m_B a_A - m_B a_{B/A} \cos 20°$$

$$0 = \frac{(4+6)\text{lb}}{g} a_A - \frac{6 \text{ lb}}{g}\left(\frac{3}{12}\text{ft}\right)\alpha \cos 20°$$

$$a_n = (0.15 \cos 20°)\alpha \quad (1)$$

Cylinder



$$\bar{I} = \frac{1}{2}\frac{W}{g}r^2 = \frac{1}{2}\frac{6 \text{ lb}}{g}(0.25 \text{ ft})^2$$

$$\bar{I} = \frac{3}{16g}$$

PROBLEM 16.147* (Continued)

(*b*) Angular acceleration of the cylinder.

$$+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{\text{eff}}: \quad (6\text{ lb})\sin 20°(0.25\text{ ft}) = \bar{I}\alpha + m_B a_{B/A}(0.25\text{ ft}) - m_B a_A \cos 20°(0.25\text{ ft})$$

$$1.5\sin 20° = \frac{3}{16(32.2)}\alpha + \frac{6\text{ lb}}{32.2}(0.25\alpha)(0.25)$$

$$-\frac{6\text{ lb}}{32.2}a_A \cos 20°(0.25)$$

$$0.51303 = 0.00582\alpha + 0.01165\alpha - 0.04378a_A$$

Substitute from (1):

$$0.51303 = 0.01747\alpha - 0.04378(0.15\cos 20°)\alpha$$

$$0.51303 = (0.01747 - 0.00617)\alpha$$

$$\alpha = 45.41\text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 45.4\text{ rad/s}^2 \circlearrowleft \blacktriangleleft$$

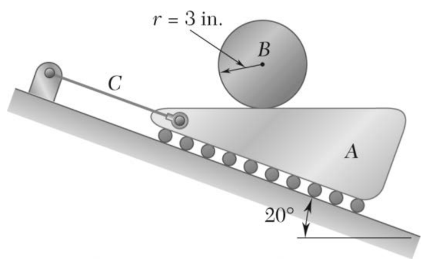
(*a*) Acceleration of the wedge.

Eq. (1):

$$a_A = (0.15\cos 20°)\alpha$$

$$= (0.15\cos 20°)(45.41)$$

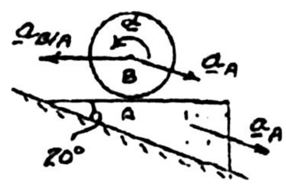
$$a_A = 6.401\text{ ft/s}^2 \qquad \mathbf{a}_A = 6.40\text{ ft/s}^2 \longrightarrow \blacktriangleleft$$



PROBLEM 16.148*

The 6-lb cylinder B and the 4-lb wedge A are held at rest in the position shown by cord C. Assuming that the cylinder rolls without sliding on the wedge and neglecting friction between the wedge and the ground, determine, immediately after cord C has been cut, (*a*) the acceleration of the wedge, (*b*) the angular acceleration of the cylinder.

SOLUTION



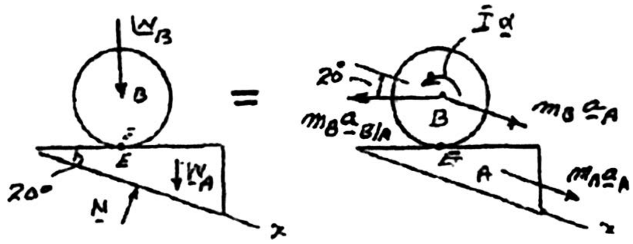
Kinematics: We resolve $\mathbf{a}_B$ into $\mathbf{a}_A$ and a horizontal component $\mathbf{a}_{B/A}$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Where $a_{B/A} = r\alpha$, since the cylinder B rolls on wedge A.

$$a_{B/A} = (0.25 \text{ ft})\alpha$$

Kinetics: Cylinder and wedge:



$$\searrow^{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad (W_A + W_B)\sin 20° = (m_A + m_B)a_A - m_B a_{B/A}\cos 20°$$

$$(10 \text{ lb})\sin 20° = \left(\frac{10}{g}\right)a_A - \left(\frac{6}{g}\right)(0.25\alpha)\cos 20°$$

$$a_A = g\sin 20° + \frac{6}{10}(0.25)\cos 20°\alpha$$

$$a_A = g\sin 20° + 0.15\cos 20°\alpha \qquad (1)$$

Cylinder: $+\circlearrowright \Sigma M_E = \Sigma (M_E)_{\text{eff}}: \quad 0 = \bar{I}\alpha + (m_B a_{B/A})(0.25 \text{ ft}) - (m_B a_A \cos 20°)(0.25 \text{ ft})$

$$0 = \frac{1}{2}\frac{6 \text{ lb}}{g}(0.25 \text{ ft})^2\alpha + \frac{6 \text{ lb}}{g}(0.25\alpha)(0.25)$$

$$-\frac{6 \text{ lb}}{g}a_A\cos 20°(0.25)$$

$$0 = \frac{1}{g}[0.1875\alpha + 0.325\alpha - 1.4095a_A]$$

$$0 = 0.5625\alpha - 1.4095a_A$$

$$\alpha = 2.506a_A \qquad (2)$$

(*a*) Acceleration of the wedge.

Substitute for α from (2) into (1):

$$a_A = g\sin 20° + 0.15\cos 20°(2.506a_A)$$
$$a_A = 11.013 + 0.3532a_A$$
$$(1-0.3532)a_A = 11.013$$
$$a_A = 17.027\ \text{ft/s}^2$$

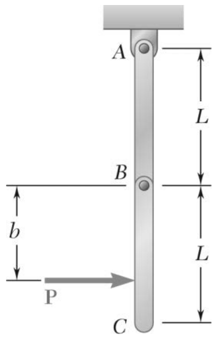
$\mathbf{a}_A = 17.03\ \text{ft/s}^2 \searrow 20°$ ◀

(*b*) Angular acceleration of the cylinder.

Eq. (2):
$$\alpha = 2.506a_A$$
$$= 2.506(17.027)$$
$$\alpha = 42.7\ \text{rad/s}^2$$

$\boldsymbol{\alpha} = 42.7\ \text{rad/s}^2 \circlearrowright$ ◀

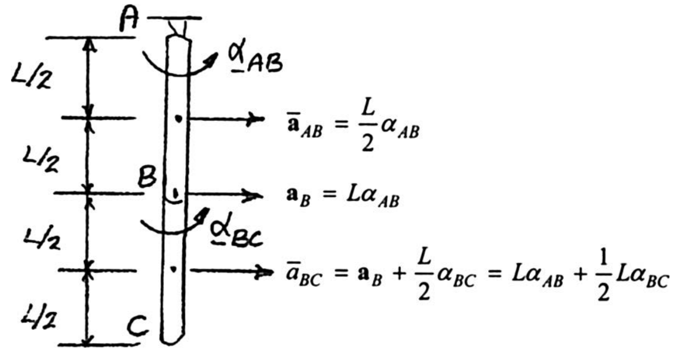
PROBLEM 16.149*



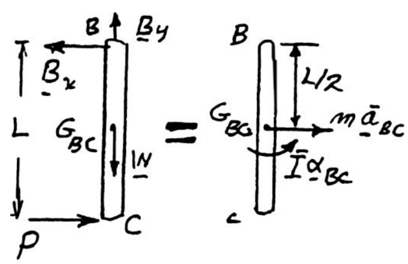
Each of the 3-kg bars *AB* and *BC* is of length $L = 500$ mm. A horizontal force **P** of magnitude 20 N is applied to bar *BC* as shown. Knowing that $b = L$ (**P** is applied at *C*), determine the angular acceleration each bar.

SOLUTION

Kinematics: Assume α_{AB} ↻ α_{BC} ↻ and $\omega_{AB} = \omega_{BC} = 0$



Kinetics: Bar *BC*



$$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad PL = \bar{I}\alpha_{BC} + (m\bar{a}_{BC})\frac{L}{2}$$

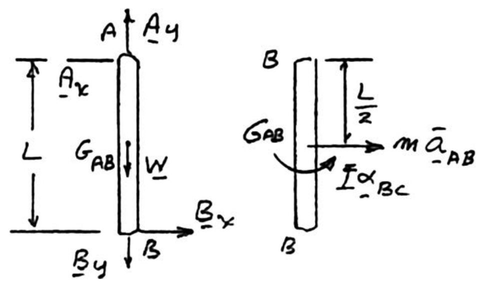
$$= \frac{m}{12}L^2\alpha_{BC} + m\left(L\alpha_{AB} + \frac{L}{2}\alpha_{BC}\right)\frac{L}{2}$$

$$P = \frac{1}{2}mL\alpha_{AB} + \frac{1}{3}mL\alpha_{BC} \quad (1)$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad P - B_x = m\bar{a}_{BC}$$

$$P - B_x = m\left(L\alpha_{AB} + \frac{1}{2}L\alpha_{BC}\right) \quad (2)$$

Bar *AB*:



$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad B_x L = \bar{I}\alpha_{AB} + (m\bar{a}_{AB})\frac{L}{2}$$

$$= \frac{m}{12}L^2\alpha_{AB} + m\left(\frac{L}{2}\alpha_{AB}\right)\frac{L}{2}$$

$$B_x = \frac{1}{3}mL\alpha_{AB} \qquad (3)$$

Add (2) and (3):

$$P = \frac{4}{3}mL\alpha_{AB} + \frac{1}{2}mL\alpha_{BC} \qquad (4)$$

Subtract (1) from (4)

$$0 = \frac{5}{6}mL\alpha_{AB} + \frac{1}{6}mL\alpha_{BC}$$

$$\alpha_{BC} = -5\alpha_{AB} \qquad (5)$$

Substitute for α_{BC} in (1):

$$P = \frac{1}{2}mL\alpha_{AB} + \frac{1}{3}mL(-5\alpha_{AB}) = -\frac{7}{6}mL\alpha_{AB}$$

$$\alpha_{AB} = -\frac{6}{7}\frac{P}{mL} \qquad (6)$$

Eq. (5)

$$\alpha_{BC} = -5\left(-\frac{6}{7}\frac{P}{mL}\right) \qquad \alpha_{BC} = \frac{30}{7}\frac{P}{mL} \qquad (7)$$

Data: $L = 500 \text{ mm} = 0.5 \text{ m}, \quad m = 3 \text{ kg}, \quad P = 20 \text{ N}$

$$\alpha_{AB} = -\frac{6}{7}\frac{P}{mL} = -\frac{6}{7}\frac{20 \text{ N}}{(3 \text{ kg})(0.5 \text{ m})}$$

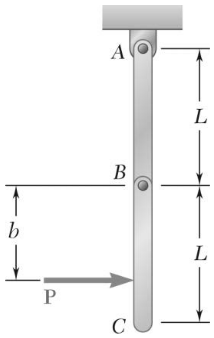
$$= -11.249 \text{ rad/s}^2$$

$\boldsymbol{\alpha}_{AB} = 11.43 \text{ rad/s}^2 \circlearrowright$ ◀

$$\alpha_{BC} = \frac{30}{7}\frac{P}{mL} = \frac{30}{7}\frac{20 \text{ N}}{(3 \text{ kg})(0.5 \text{ m})}$$

$$= 57.143 \text{ rad/s}^2$$

$\boldsymbol{\alpha}_{BC} = 57.1 \text{ rad/s}^2 \circlearrowleft$ ◀

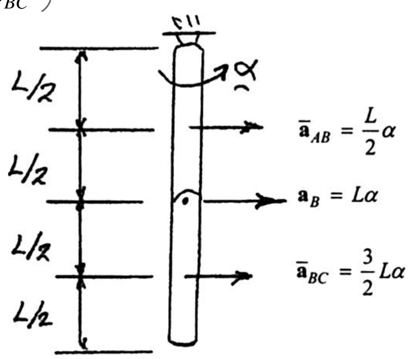


PROBLEM 16.150*

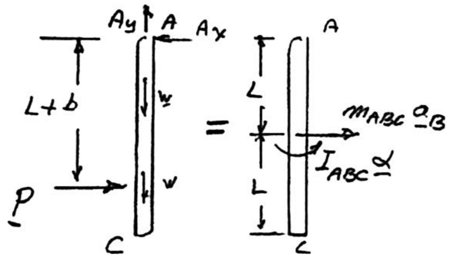
Each of the 3-kg bars AB and BC is of length $L = 500$ mm. A horizontal force **P** of magnitude 20 N is applied to bar BC. For the position shown, determine (a) the distance b for which the bars move as if they formed a single rigid body, (b) the corresponding angular acceleration of the bars.

SOLUTION

Kinematics: We choose $\alpha = \alpha_{AB} = \alpha_{BC}$ ↺



Kinetics: Bars AB and BC (acting as rigid body)



$$m_{ABC} = 2m$$

$$\bar{I} = \frac{1}{12}(2m)(2L)^2$$

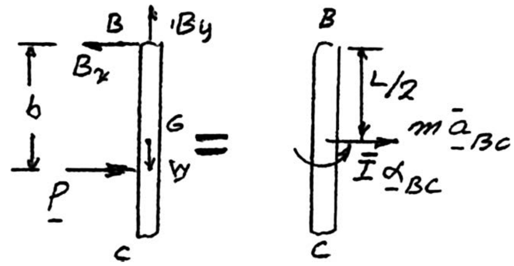
$$\bar{I} = \frac{2}{3}mL^2$$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad P(L+b) = \bar{I}_{ABC}\alpha + m_{ABC}a_B L$$

$$P(L+b) = \frac{2}{3}mL^2\alpha + (2m)(L\alpha)L$$

$$P(L+b) = \frac{8}{3}mL^2\alpha \qquad (1)$$

Bar *BC*:



$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad Pb = \bar{I}_{BC}\alpha + (m\bar{a}_{BC})\frac{L}{2}$$

$$= \frac{m}{12}L^2\alpha + m\left(\frac{3}{2}L\alpha\right)\frac{L}{2}$$

$$Pb = \frac{5}{6}mL^2\alpha$$

$$\alpha = \frac{6}{5}\frac{Pb}{mL^2} \qquad (2)$$

Substitute for α into (1):

$$P(L+b) = \frac{8}{3}mL^2\left(\frac{6}{5}\frac{Pb}{mL^2}\right)$$

$$PL + Pb = \frac{16}{5}Pb$$

$$L = \left(\frac{16}{5} - 1\right)b = \frac{11}{5}b$$

$$b = \frac{5}{11}L$$

Eq. (2)

$$\alpha = \frac{6}{5}\frac{P}{mL^2}\left(\frac{5}{11}L\right) \qquad \alpha = \frac{6}{11}\frac{P}{mL}$$

Data:

$$L = 500 \text{ mm} = 0.5 \text{ m}$$
$$m = 3 \text{ kg}$$
$$P = 20 \text{ N}$$

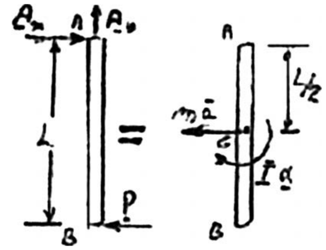
(*a*) $b = \frac{5}{11}L = \frac{5}{11}(0.5) = 0.22727 \text{ m}$ $\qquad b = 227 \text{ mm}$ ◀

(*b*) $\alpha = \frac{6}{11}\frac{P}{mL} = \frac{6}{11}\frac{(20 \text{ N})}{(3 \text{ kg})(0.5 \text{ m})} = 7.2727 \text{ rad/s}^2$ $\qquad \boldsymbol{\alpha} = 7.27 \text{ rad/s}^2 \circlearrowleft$ ◀

PROBLEM 16.151*

(*a*) Determine the magnitude and the location of the maximum bending moment in the rod of Problem 16.78.
(*b*) Show that the answer to Part *a* is independent of the weight of the rod.

SOLUTION



Rod *AB*: $\bar{a} = \frac{L}{2}\alpha$

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad PL = (m\bar{a})\frac{L}{2} + \bar{I}\alpha$$

$$= \left(m\frac{1}{2}\alpha\right)\frac{L}{2} + \frac{1}{12}mL^2\alpha$$

$$\alpha = \frac{3P}{mL} \circlearrowright \quad (1)$$

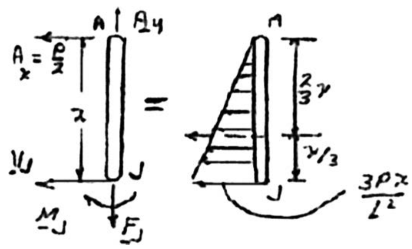
$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad A_x - P = -m\bar{a}$$

$$A_x = P - m\frac{L}{2}\alpha = P - m\frac{L}{2}\left(\frac{3P}{mL}\right) = -\frac{P}{2} \qquad \mathbf{A}_x = \frac{1}{2}P \longleftarrow$$

Portion *AJ* of Rod:

External forces: A_x, W_{AJ}, axial force F_J, shear V_J, and bending moment M_J.

Effective forces: Since acceleration at any point is proportional to distance from *A*, effective forces are linearly distributed. Since mass per unit length is *m*/*L*, at Point *J* we find



$$\left(\frac{m}{L}\right)a_J = \frac{m}{L}(x\alpha)$$

Using (1):

$$\frac{m}{L}a_J = \frac{m}{L}\left(\frac{3P}{mL}\right)$$

$$\frac{m}{L}a_J = \frac{3Px}{L^2}$$

$$+\circlearrowright \Sigma M_J = \Sigma(M_J)_{\text{eff}}: \quad M_J - A_x x = -\frac{1}{2}\left(\frac{3Px}{L^2}\right)\left(\frac{2x}{3}\right)$$

$$M_J = \frac{1}{2}Px - \frac{1}{2}\frac{P}{L^2}x^3 \quad (2)$$

For $M_{\max}$:

$$\frac{dM_J}{dx} = \frac{P}{2} - \frac{3}{2}\frac{P}{L^2}x^2 = 0$$

$$x = \frac{L}{\sqrt{3}} \quad (3)$$

PROBLEM 16.151* (Continued)

Substituting into (2)

$$(M_J)_{\max} = \frac{1}{2}\frac{PL}{\sqrt{3}} - \frac{1}{2}\frac{P}{L^2}\left(\frac{L}{\sqrt{3}}\right)^3 = \frac{1}{2}\frac{PL}{\sqrt{3}}\left(\frac{2}{3}\right)$$

$$(M_J)_{\max} = \frac{PL}{3\sqrt{3}} \qquad (4)$$

Note: Eqs. (3) and (4) are independent of W.

Data: $L = 36 \text{ in.}, \quad P = 1.5 \text{ lb}$

Eq. (3): $x = \frac{L}{\sqrt{3}} = \frac{36 \text{ in.}}{\sqrt{3}} = 20.78 \text{ in.}$

Eq. (4):

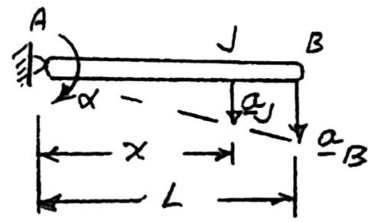
$$(M_J)_{\max} = \frac{(1.5 \text{ lb})(36 \text{ in.})}{3\sqrt{3}}$$
$$= 10.392 \text{ lb} \cdot \text{in.}$$

$M_{\max} = 10.39 \text{ lb} \cdot \text{in.}$ located 20.8 in. below A ◀

PROBLEM 16.152*

Draw the shear and bending-moment diagrams for the beam of Problem 16.84 immediately after the cable at B breaks.

SOLUTION



From answers to Problem 16.84:

$$\mathbf{a}_B = \frac{3}{2}g\downarrow \qquad \mathbf{A} = \frac{1}{4}mg\uparrow$$

We now find

$$\boldsymbol{\alpha} = \frac{a_B}{L}\circlearrowright = \frac{3g}{2L}\circlearrowright$$

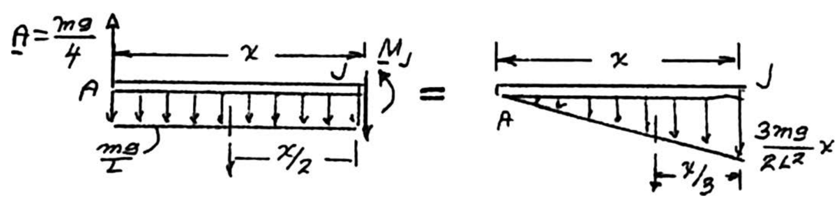
$$\mathbf{a}_J = x\alpha = \frac{3g}{2L}x\downarrow$$

Portion AJ of rod:

External forces: Reaction A, distributed load per unit length mg/L, shear V_J, bending moment M_J.

Effective forces: Since a is proportional to x, the effective forces are linearly distributed. The effective force per unit length at J is:

$$\frac{m}{L}a_J = \frac{m}{L}\cdot\frac{3g}{2L}x = \frac{3mg}{2L^2}x$$



$$+\downarrow\Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad \frac{mg}{L}x - \frac{mg}{4} + V_J = \frac{1}{2}\left(\frac{3mg}{2L^2}x\right)x$$

$$V_J = \frac{mg}{4} - \frac{mg}{L}x + \frac{3}{4}\frac{mg}{L^2}x^2$$

$$+\circlearrowleft\Sigma M_J = \Sigma(M_J)_{\text{eff}}: \quad \left(\frac{mg}{L}x\right)\frac{x}{2} - \frac{mg}{4}x + M_J = \frac{1}{2}\left(\frac{3mg}{2L^2}x\right)x\left(\frac{x}{3}\right)$$

$$M_J = \frac{mg}{4}x - \frac{1}{2}\frac{mg}{L}x^2 + \frac{1}{4}\frac{mg}{L^2}x^3$$

Find $V_{\min}$:

$$\frac{dV_J}{dx} = -\frac{mg}{L} + \frac{3}{2}\frac{mg}{L^2}x = 0; \qquad x = \frac{2}{3}L$$

$$V_{\min} = \frac{mg}{4} - \frac{mg}{L}\left(\frac{2}{3}L\right) + \frac{3}{4}\frac{mg}{L^2}\left(\frac{2}{3}L\right)^2; \qquad V_{\min} = -\frac{mg}{12}$$

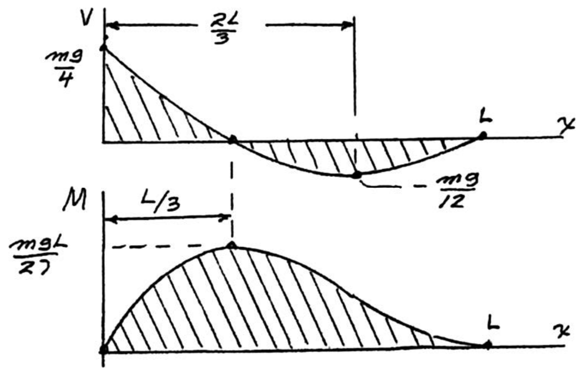
Find $M_{\max}$ where $\quad V_J = 0: \quad V_J = \dfrac{mg}{4} - \dfrac{mg}{L}x + \dfrac{3}{4}\dfrac{mg}{L}x^2 = 0$

$$3x^2 - 4Lx + L^2 = 0$$

$$(3x - L)(x - L) = 0 \qquad x = \frac{L}{3} \quad \text{and} \quad x = L$$

$$M_{\max} = \frac{mg}{4}\left(\frac{L}{3}\right) - \frac{1}{2}\frac{mg}{L}\left(\frac{L}{3}\right)^2 + \frac{1}{4}\left(\frac{mg}{L}\right)\left(\frac{L}{3}\right)^3 = \frac{mgL}{27}$$

$$M_{\min} = \frac{mg}{4}L - \frac{1}{2}\frac{mg}{L}L^2 + \frac{1}{4}\frac{mg}{L}L^3 = 0$$

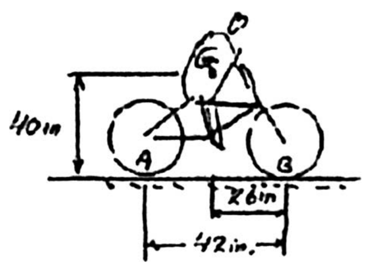


$M_{\max} = \dfrac{mgL}{27}$ at $\dfrac{L}{3}$ from A ◀

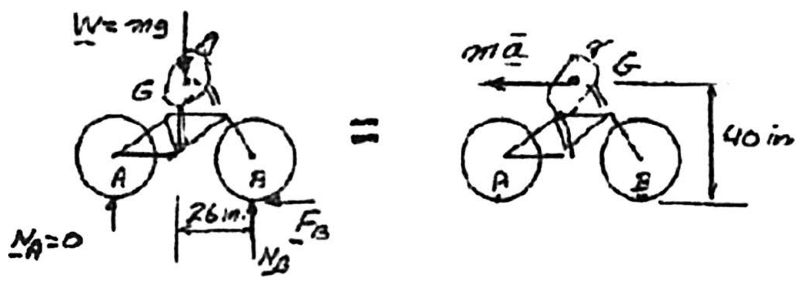
PROBLEM 16.153

A cyclist is riding a bicycle at a speed of 20 mph on a horizontal road. The distance between the axles is 42 in., and the mass center of the cyclist and the bicycle is located 26 in. behind the front axle and 40 in. above the ground. If the cyclist applies the brakes only on the front wheel, determine the shortest distance in which he can stop without being thrown over the front wheel.

SOLUTION



When cyclist is about to be thrown over the front wheel, $N_A = 0$



$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad mg(26\text{ in.}) = m\bar{a}(40\text{ in.})$$

$$a = \frac{26}{40}g = \frac{26}{40}(32.2\text{ ft/s}^2) = 20.93\text{ ft/s}^2$$

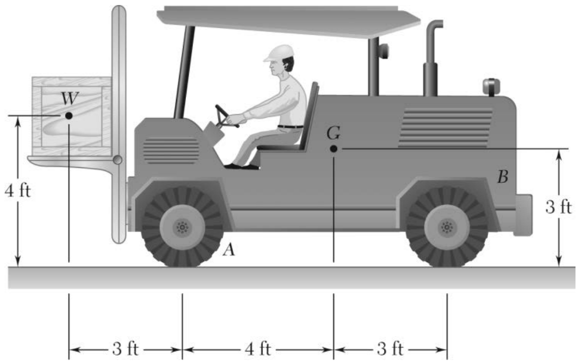
Uniformly accelerated motion:

$$v_0 = 20\text{ mph} = 29.333\text{ ft/s}$$

$$v^2 - v_0^2 = 2as: \quad 0 - (29.333\text{ ft/s})^2 = 2(-20.93\text{ ft/s}^2)s$$

$$s = 20.555\text{ ft}$$

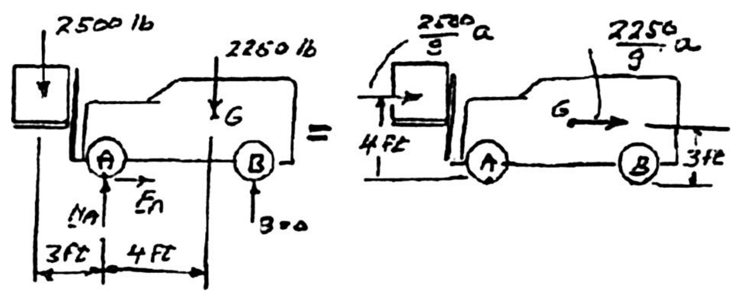
$s = 20.6$ ft ◀



PROBLEM 16.154

The forklift truck shown weighs 2250 lb and is used to lift a crate of weight $W = 2500$ lb. The truck is moving to the left at a speed of 10 ft/s when the brakes are applied on all four wheels. Knowing that the coefficient of static friction between the crate and the fork lift is 0.30, determine the smallest distance in which the truck can be brought to a stop if the crate is not to slide and if the truck is not to tip forward.

SOLUTION



Assume crate does not slide and that tipping impends about A. $(B = 0)$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}:$$

$$(2500 \text{ lb})(3 \text{ ft}) - (2250 \text{ lb})(4 \text{ ft}) = -\left(2500\frac{a}{g}\right)(4 \text{ ft}) - \left(2250\frac{a}{g}\right)(3 \text{ ft})$$

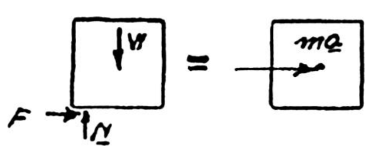
$$7500 - 9000 = -(10{,}000 + 6750)\frac{a}{g}$$

$$\frac{a}{g} = 0.09; \quad a = 0.09(32.2 \text{ ft/s}^2) \qquad \mathbf{a} = 2.884 \text{ ft/s}^2 \longrightarrow$$

Uniformly accelerated motion

$$v^2 = v_0^2 + 2ax; \quad 0 = (10 \text{ ft/s})^2 - 2(2.884 \text{ ft/s}^2)x \qquad x = 17.34 \text{ ft} \blacktriangleleft$$

Check whether crate slides

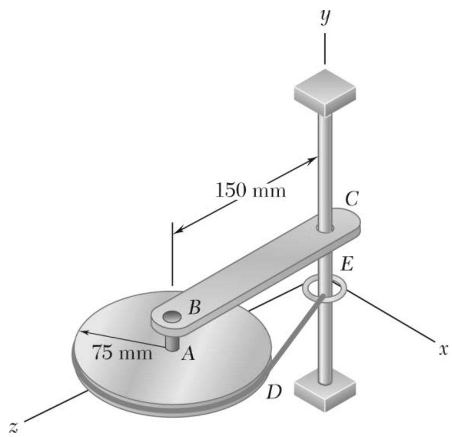


$$N = W$$

$$F = ma = \frac{W}{g}a$$

$$\mu_{\text{req}} = \frac{F}{N} = \frac{a}{g} = \frac{2.884 \text{ ft/s}^2}{32.2 \text{ ft/s}^2}$$

$\mu_{\text{req}} = 0.09 < 0.30.$ The crate does not slide. ◀

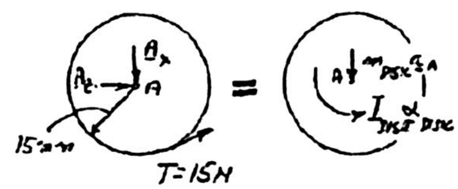


PROBLEM 16.155

A 5-kg uniform disk is attached to the 3-kg uniform rod *BC* by means of a frictionless pin *AB*. An elastic cord is wound around the edge of the disk and is attached to a ring at *E*. Both ring *E* and rod *BC* can rotate freely about the vertical shaft. Knowing that the system is released from rest when the tension in the elastic cord is 15 N, determine (*a*) the angular acceleration of the disk, (*b*) the acceleration of the center of the disk.

SOLUTION

(*a*) Angular acceleration of the disk.



Disk: $\bar{I}_{\text{disk}} = \frac{1}{2} m_{\text{disk}} r^2 = \frac{1}{2}(5\text{ kg})(0.075\text{ m})^2$

$\bar{I}_{\text{disk}} = 14.06 \times 10^{-3} \text{ kg} \cdot \text{m}^2$

$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad (15\text{ N})(0.075\text{ N}) = \bar{I}_{\text{disk}}\, \alpha_{\text{disk}}$

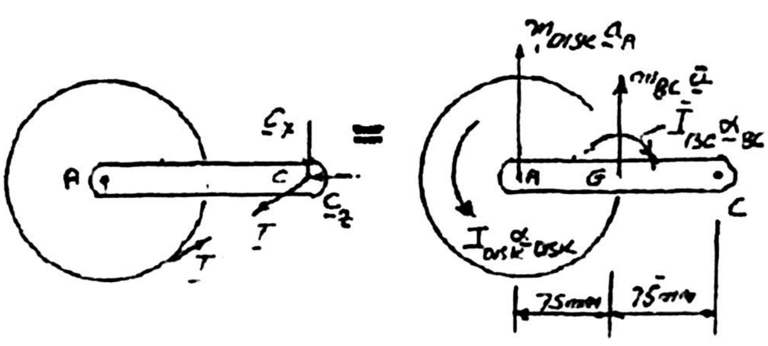
$1.125\text{ N}\cdot\text{m} = (14.06 \times 10^3 \text{ kg}\cdot\text{m}^2)\alpha_{\text{disk}}$

$\alpha_{\text{disk}} = 80.0\text{ rad/s}^2$ $\qquad \boldsymbol{\alpha}_{\text{disk}} = 80.0\text{ rad/s}^2 \circlearrowright$ ◀

(*b*) Acceleration of center of disk.

Entire assembly

$$\bar{I}_{BC} = \frac{1}{12} m_{BC}(BC)^2 = \frac{1}{12}(3\text{ kg})(0.15\text{ m})^2 = 5.625 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$



Assume $\boldsymbol{\alpha}_{BC}$ is ↻ $\quad \mathbf{a}_A = (0.15\text{ m})\alpha_{BC} \uparrow$

$$\bar{a} = (0.075\text{ m})\alpha_{BC}$$

$$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad 0 = I_{\text{disk}}\alpha_{\text{disk}} - m_{\text{disk}}a_A(0.15\text{ m}) - m_{BC}\bar{a}(0.075\text{ m}) - \bar{I}_{BC}\alpha_{BC}$$

$$0 = (14.06\times10^{-3}\text{ kg}\cdot\text{m}^2)(80\text{ rad/s}^2) - (5\text{ kg})(0.15\text{ m})^2\alpha_{BC}$$

$$-(3\text{ kg})(0.075\text{ m})^2\alpha_{BC} - (5.625\times10^3\text{ kg}\cdot\text{m}^2)\alpha_{BC}$$

$$0 = 1.125 - 0.1125\alpha_{BC} - 16.875\times10^{-3}\alpha_{BC} - 5.625\times10^{-3}\alpha_{BC}$$

$$0 = 1.125 - 0.135\alpha_{BC}$$

$$\alpha_{BC} = +8.333\text{ rad/s}^2 \qquad \boldsymbol{\alpha}_{BC} = 8.33\text{ rad/s}^2 \circlearrowright$$

$$a_A = (AC)\alpha_{BC} = (0.15\text{ m})(8.333\text{ rad/s}^2)$$

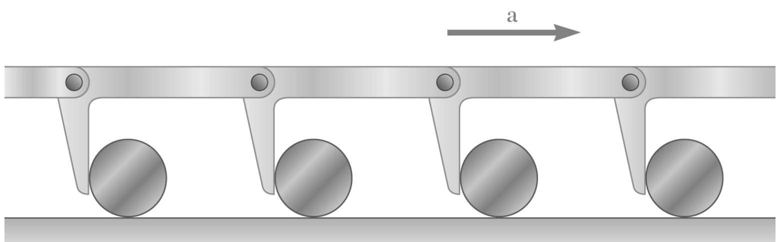
$$a_A = +1.25\text{ m/s}^2 \qquad \mathbf{a}_A = 1.250\text{ m/s}^2 \uparrow \blacktriangleleft$$

Note: Answers can also be written:

$$\boldsymbol{\alpha}_{\text{disk}} = (80\text{ rad/s}^2)\mathbf{j} \qquad \mathbf{a}_A = -(1.25\text{ m/s}^2)\mathbf{i}$$

PROBLEM 16.156

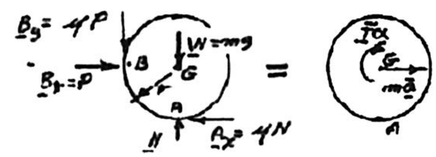
Identical cylinders of mass m and radius r are pushed by a series of moving arms. Assuming the coefficient of friction between all surfaces to be $\mu < 1$ and denoting by a the magnitude of the acceleration of the arms, derive an expression for (a) the maximum allowable value of a if each cylinder is to roll without sliding, (b) the minimum allowable value of a if each cylinder is to move to the right without rotating.



SOLUTION

(a) Cylinder rolls without sliding $a = r\alpha$ or $\alpha = \dfrac{a}{r}$

P is horizontal component of force that the arm exerts on cylinder.



$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \quad Pr - (\mu_k P) r = I\alpha + (m\bar{a})r$$

$$P(1-\mu)r = \frac{1}{2}mr^2\left(\frac{\bar{a}}{r}\right) + (m\bar{a})r$$

$$P = \frac{3}{2}\frac{m\bar{a}}{(1-\mu)} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N - \mu P - mg = 0 \qquad (2)$$

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \quad P - \mu N = m\bar{a} \qquad (3)$$

Solve (2) for N and substitute for N into (3).

$$P - \mu^2 P - \mu mg = m\bar{a}$$

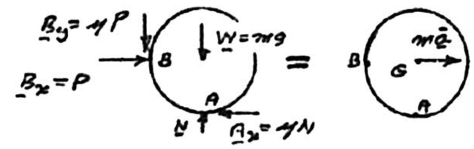
Substitute P from (1): $$(1-\mu^2)\frac{3}{2}\frac{m\bar{a}}{(1-\mu)} - \mu mg = m\bar{a}$$

$$3(1+\mu)\bar{a} - 2\mu g = 2\bar{a}$$

$$\bar{a}(1+3\mu) - 2\mu g = 0$$

$$\bar{a} = \frac{2\mu}{1+3\mu}g \quad \blacktriangleleft$$

(*b*) Cylinder translates: $\alpha = 0$



Sliding occurs at A: $A_x = \mu N$

Assume sliding impends at B: $B_y = \mu P$

$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad Pr - \mu Pr = (m\bar{a})r$$

$$P(1-\mu)r = m\bar{a}r$$

$$P = \frac{m\bar{a}}{1-\mu} \tag{4}$$

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad P - \mu N = m\bar{a} \tag{5}$$

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad N - \mu P - mg = 0 \tag{6}$$

Solve (5) for N and substitute for N into (6).

$$P - P\mu^2 - \mu mg = m\bar{a}$$

Substitute for P from (4):

$$\frac{m\bar{a}}{1-\mu}(1-\mu^2) - \mu mg = m\bar{a}$$

$$\bar{a}(1+\mu) - \mu g = \bar{a}$$

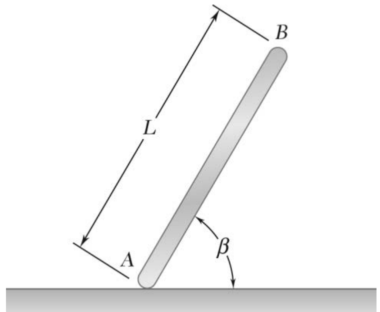
$$\bar{a}\mu - \mu g = 0$$

$\bar{a} = g$ ◀

Summary: $a < \dfrac{2\mu}{1+3\mu} g$: Rolling

$\dfrac{3\mu}{1+3\mu} g < a < g$: Rotating and sliding

$a > g$: Translation

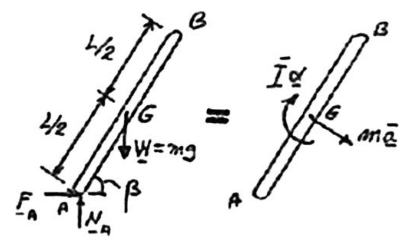


PROBLEM 16.157

The uniform rod AB of weight W is released from rest when $\beta = 70°$. Assuming that the friction force between end A and the surface is large enough to prevent sliding, determine immediately after release (*a*) the angular acceleration of the rod, (*b*) the normal reaction at A, (*c*) the friction force at A.

SOLUTION

We note that rod rotates about A. $\omega = 0$



$$\bar{I} = \frac{1}{12}mL^2$$

$$\bar{a} = \frac{L}{2}\alpha$$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}:$ $$mg\left(\frac{L}{2}\cos\beta\right) = \bar{I}\alpha + (m\bar{a})\frac{L}{2}$$

$$\frac{1}{2}mgL\cos\beta = \frac{1}{12}mL^2\alpha + \left(m\frac{L}{2}\alpha\right)\frac{L}{2}$$

$$= \frac{1}{3}mL^2\alpha$$

$$\alpha = \frac{3}{2}\frac{g\cos\beta}{L} \qquad (1)$$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}:$ $$F_A = m\bar{a}\sin\beta$$

$$F_A = m\frac{L}{2}\alpha\sin\beta = m\frac{L}{2}\left(\frac{3}{2}\frac{g\cos\beta}{L}\right)\sin\beta$$

$$F_A = \frac{3}{4}mg\sin\beta\cos\beta \qquad (2)$$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}:$ $$N_A - mg = -m\bar{a}\cos\beta = -m\left(\frac{L}{2}\alpha\right)\cos\beta$$

$$N_A - mg = -m\frac{L}{2}\left(\frac{3}{2}\frac{g\cos\beta}{L}\right)\cos\beta$$

$$N_A = mg\left(1 - \frac{3}{4}\cos^2\beta\right) \qquad (3)$$

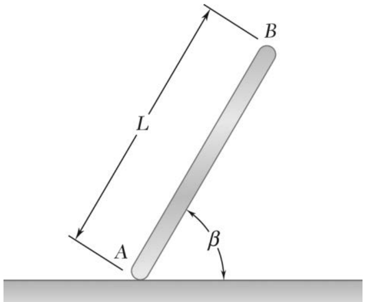
PROBLEM 16.157 (Continued)

For $\beta = 70°$:

(a) Eq. (1): $$\alpha = \frac{3}{2}\frac{g\cos 70°}{L}$$ $$\boldsymbol{\alpha} = 0.513\frac{g}{L} \circlearrowright \blacktriangleleft$$

(b) Eq. (3): $$N_A = mg\left(1 - \frac{3}{4}\cos^2 70°\right)$$ $$\mathbf{N}_A = 0.912mg \uparrow \blacktriangleleft$$

(c) Eq. (2) $$F_A = \frac{3}{4}mg\sin 70°\cos 70°$$ $$\mathbf{F}_A = 0.241mg \longrightarrow \blacktriangleleft$$



PROBLEM 16.158

The uniform rod *AB* of weight *W* is released from rest when $\beta = 70°$. Assuming that the friction force is zero between end *A* and the surface, determine immediately after release (*a*) the angular acceleration of the rod, (*b*) the acceleration of the mass center of the rod, (*c*) the reaction at *A*.

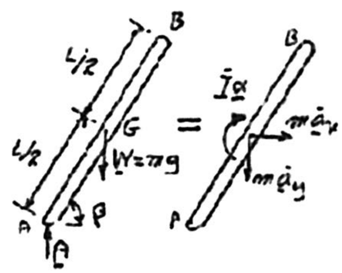
SOLUTION

$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \qquad a = m\bar{\mathbf{a}}_x \quad \bar{\mathbf{a}}_x = 0$$

$$\mathbf{a}_A = \bar{\mathbf{a}}_y + \frac{L}{2}\alpha \searrow \beta$$

$$+\!\downarrow 0 = a_y - \frac{L}{2}\alpha \cos\beta$$

$$\mathbf{a}_y = \frac{L}{2}\alpha\cos\beta \downarrow$$



$$+\!\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \qquad mg - A = m\bar{a}_y = m\left(\frac{L}{2}\alpha\cos\beta\right) \quad (1)$$

$$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{\text{eff}}: \quad A\left(\frac{L}{2}\cos\beta\right) = \bar{I}\alpha = \frac{1}{12}mL^2\alpha$$

$$A = \frac{mL}{6}\frac{\alpha}{\cos\beta} \quad (2)$$

Substitute (2) into (1):

$$mg - \frac{mL}{6}\frac{\alpha}{\cos\beta} = m\frac{L}{2}\alpha\cos\beta$$

$$g = \left(\frac{L}{2}\cos\beta + \frac{L}{6\cos\beta}\right)\alpha$$

$$g = \frac{L}{6}\left(3\cos\beta + \frac{1}{\cos\beta}\right)\alpha$$

$$g = \frac{L}{6}\left(\frac{3\cos^2\beta + 1}{\cos\beta}\right)\alpha$$

$$\alpha = \frac{6g}{L}\left(\frac{\cos\beta}{1 + 3\cos^2\beta}\right) \circlearrowright$$

PROBLEM 16.158 (Continued)

$$\overline{\mathbf{a}} = \frac{L}{2}\alpha\cos\beta = \frac{L}{2}\left(\frac{6g}{L}\cdot\frac{\cos\beta}{1+3\cos^2\beta}\right)\cos\beta = 3g\left(\frac{\cos^2\beta}{1+3\cos^2\beta}\right)\longleftarrow$$

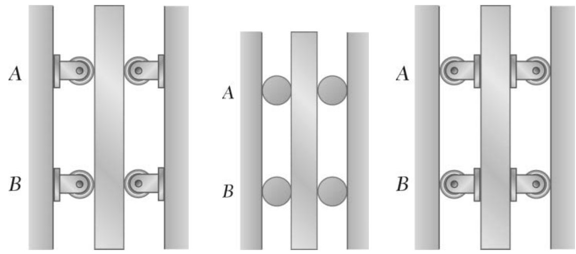
$$\mathbf{A} = \frac{mL}{6}\cdot\frac{\alpha}{\cos\beta} = \frac{mL}{6}\cdot\left(\frac{6g}{L}\cdot\frac{\cos\beta}{1+3\cos^2\beta}\right)\frac{1}{\cos\beta} = mg\frac{1}{1+3\cos^2\beta}\uparrow$$

For $\beta = 70°$:

(*a*) $$\alpha = \frac{6g}{L}\frac{\cos 70°}{1+3\cos^2 70°}$$ $\boldsymbol{\alpha} = 1.519\frac{g}{L}\circlearrowright$ ◀

(*b*) $$\overline{a} = 3g\frac{\cos^2 70}{1+3\cos^2 70°}$$ $\overline{\mathbf{a}} = 0.260g\downarrow$ ◀

(*c*) $$A = mg\frac{1}{1+3\cos^2 70°}$$ $\mathbf{A} = 0.740mg\uparrow$ ◀



(*a*) (*b*) (*c*)

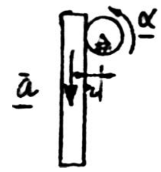
PROBLEM 16.159

A bar of mass $m = 5$ kg is held as shown between four disks, each of mass $m' = 2$ kg and radius $r = 75$ mm. Knowing that the normal forces on the disks are sufficient to prevent any slipping, for each of the cases shown determine the acceleration of the bar immediately after it has been released from rest.

SOLUTION

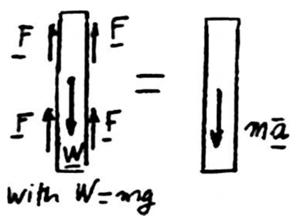
(*a*) Configuration (*a*)

Kinematics:



$$\bar{a} = r\alpha \qquad \alpha = \frac{\bar{a}}{r}$$

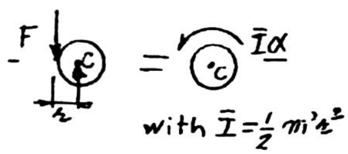
Kinetics of bar



$$+\!\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}:$$

$$W - 4F = m\bar{a} \qquad (1)$$

Kinetics of one disk



$$+\!\circlearrowright \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \quad Fr = \bar{I}\alpha$$

$$Fr = \frac{1}{2} m' r^2 \left(\frac{\bar{a}}{r}\right)$$

$$F = \frac{1}{2} m' \bar{a} \qquad (2)$$

Substitute for F from (2) into (1).

$$mg - 4\left(\frac{1}{2} m' \bar{a}\right) = m\bar{a}$$

$$mg = (m + 2m')\bar{a} \qquad \bar{a} = \frac{mg}{m + 2m'}$$

$$m = 5\text{ kg}, \quad m' = 2\text{ kg}: \quad \bar{a} = \frac{5}{5 + 2(2)} g \qquad \bar{\mathbf{a}} = \frac{5}{9} g \downarrow \blacktriangleleft$$

(*b*) Configuration (*b*)

Kinematics:

Disk is rolling on vertical wall

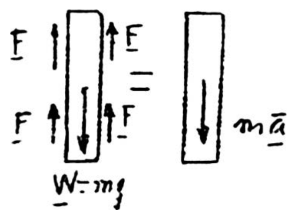
$$\bar{a}' = a_C = r\alpha$$

$$\bar{a} = a_r = 2r\alpha$$

Therefore: $$\alpha = \frac{\bar{a}}{2r}$$

$$\bar{a}' = r\alpha = \frac{1}{2}\bar{a}$$

Kinetics of bar



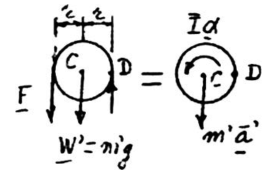
$$+\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}:$$

$$W - 4F = m\bar{a}$$

$$mg - 2(2F) = m\bar{a} \qquad (1)$$

Kinetics of one disk

$$\bar{I} = \frac{1}{2} m' r^2$$



$$+\circlearrowleft \Sigma M_D = \Sigma (M_D)_{\text{eff}}:$$

$$F(2r) + m'gr = \bar{I}\alpha + m'\bar{a}'r$$

$$2Fr = m'gr = \frac{1}{2} m' r^2 \left(\frac{\bar{a}}{2r} \right) + m' \frac{\bar{a}}{2} r$$

$$2F = m' \left(\frac{1}{4}\bar{a} + \frac{1}{2}\bar{a} \right) - m'g \qquad (2)$$

Substitute for $2F$ from (2) into (1):

$$mg - 2m' \left(\frac{3}{4}\bar{a} \right) + 2m'g = m\bar{a}$$

$$\left(m + \frac{3}{2} m' \right) \bar{a} = (m + 2m')g \qquad \bar{a} = \frac{m + 2m'}{m + \frac{3}{2} m'} g$$

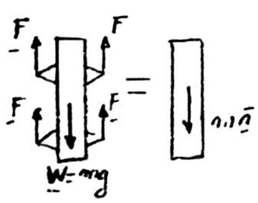
$$m = 5 \text{ kg}, \quad m' = 2 \text{ kg}: \quad \bar{a} = \frac{5+4}{5+3} g \qquad \mathbf{a} = \frac{9}{8} g \downarrow \blacktriangleleft$$

(*c*) Configuration (*c*)

Kinematics:

Disk is rolling on vertical wall $\qquad \bar{a} = \bar{a}' = r\alpha$

Kinetics of bar



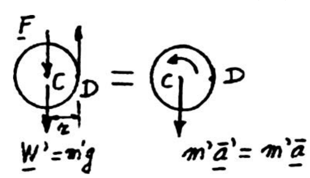
$$+\downarrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}:$$

$$W - 4F = m\bar{a}$$

$$mg - 4F = m\bar{a} \qquad (1)$$

Kinetics of one disk

$$\bar{I}\alpha = \frac{1}{2} m' r^2 \left(\frac{\bar{a}}{r} \right)$$



$$+\circlearrowleft \Sigma M_D = \Sigma (M_D)_{\text{eff}}:$$

$$(F + w')r = \bar{I}\alpha + m'\bar{a}'r$$

$$(F + m'g)r = \frac{1}{2} m' r' \left(\frac{\bar{a}}{r} \right) + m'\bar{a}t$$

$$F = \frac{3}{2} m'\bar{a} - m'g \qquad (2)$$

PROBLEM 16.159 (Continued)

Substitute for F from (2) into (1):

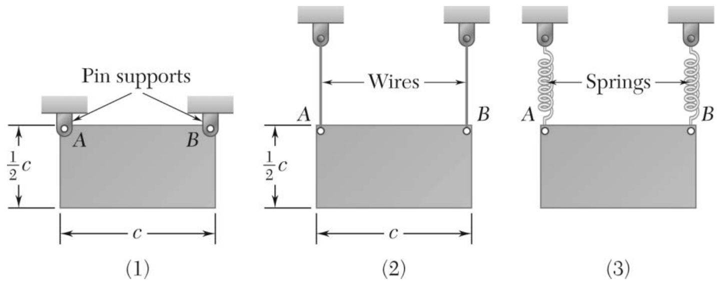
$$mg - 6m'\overline{a} + 4m'g = m\overline{a}$$

$$(m + 6m')\overline{a} = (m + 4m')g \quad \overline{a} = \frac{m + 4m'}{m + 6m'}g$$

$$m = 5 \text{ kg}, \quad m' = 2 \text{ kg}: \quad \overline{a} = \frac{5 + 8}{5 + 12} \qquad \mathbf{\overline{a}} = \frac{13}{17}g \downarrow \blacktriangleleft$$

PROBLEM 16.160

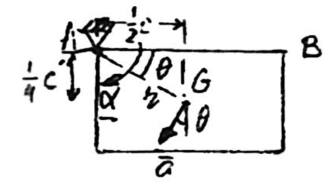
A uniform plate of mass m is suspended in each of the ways shown. For each case determine immediately after the connection at B has been released (a) the angular acceleration of the plate, (b) the acceleration of its mass center.



SOLUTION

(1) Plate attached to pins

Kinematics: Assume α ↻ $(\omega = 0)$



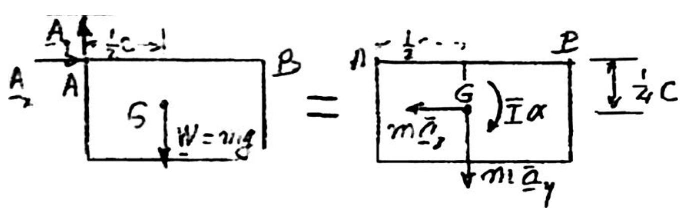
$$\bar{\mathbf{a}} = r\alpha \measuredangle \theta$$

$\overset{+}{\leftarrow}$ x comp: $\bar{a}_x = r\alpha \sin\theta = (r\sin\theta)\alpha$

$+\downarrow$ y comp: $\bar{a}_y = r\alpha\cos\theta = (r\cos\theta)\alpha$

Thus: $\bar{\mathbf{a}}_x = \frac{1}{4}c\alpha \longleftarrow; \quad \bar{\mathbf{a}}_y = \frac{1}{2}c\alpha \downarrow$ (1)

Kinetics: $\bar{I} = \frac{1}{12}m\left[c^2 + \left(\frac{c}{2}\right)^2\right] = \frac{5}{48}mc^2$



(a) $+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad W\left(\frac{c}{2}\right) = \bar{I}\alpha + (m\bar{a}_x)\left(\frac{c}{4}\right) + (m\bar{a}_y)\left(\frac{c}{2}\right)$

$$\frac{1}{2}mgc = \frac{5}{48}mc^2\alpha + m\left(\frac{1}{4}c\alpha\right)\left(\frac{c}{4}\right) + m\left(\frac{1}{2}c\alpha\right)\left(\frac{c}{2}\right)$$

$$\frac{1}{2}mgc = \frac{20}{48}mc^2\alpha \qquad \alpha = 1.2\frac{g}{c}$$

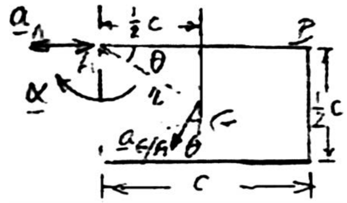
$\boldsymbol{\alpha} = 1.2\frac{g}{c}$ ↻ ◀

(*b*) From (1): $\bar{a}_x = \frac{1}{4}c\alpha = \frac{1}{4}(1.2g)$ $\quad \bar{\mathbf{a}}_x = 0.3g \longleftarrow$

$\bar{a}_y = \frac{1}{2}c\alpha = \frac{1}{2}(1.2g)$ $\quad \bar{\mathbf{a}}_y = 0.6g\downarrow$ $\qquad \bar{\mathbf{a}} = 0.671g \measuredangle 63.4°$ ◀

(2) Plate suspended from wires.

<u>Kinematics</u>: Assume $\alpha \circlearrowright$ $(\boldsymbol{\omega} = 0)$



$$\bar{\mathbf{a}} = \mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{G/A}$$
$$= a_A \leftrightarrow + r\alpha \measuredangle \theta$$

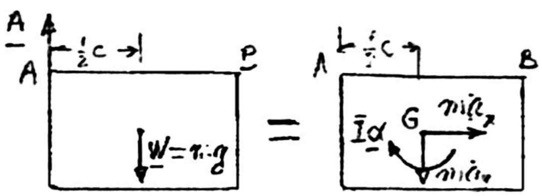
$+\uparrow y$ comp.

$$\bar{a}_y = 0 - r\alpha\cos\theta$$
$$= -(r\cos\theta)\alpha$$

$$r\cos\theta = \frac{1}{2}c$$

Thus: $\bar{a}_y = -\frac{1}{2}c\alpha \quad \bar{\mathbf{a}}_y = \frac{1}{2}c\alpha\downarrow$ (2)

<u>Kinetics</u>: $\bar{I} = \frac{1}{12}m\left[c^2 + \left(\frac{c}{2}\right)^2\right] = \frac{I}{48}mc^2$



$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}$: $\quad 0 = m\bar{a}_2$

$$\bar{a}_x = 0$$

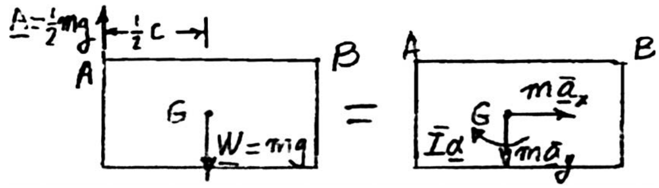
$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}$: $\quad W\left(\frac{1}{2}c\right) = \bar{I}\alpha + (m\bar{a}_y)\left(\frac{1}{2}c\right)$

Recalling (1): $\frac{1}{2}mgc = \frac{5}{48}mc^2\alpha + \left(\frac{1}{2}mc\alpha\right)\left(\frac{1}{2}c\right)$

$\frac{1}{2}mgc = \frac{17}{48}mc^2\alpha \quad \alpha = \frac{24}{17}\frac{g}{c}$ $\qquad \alpha = \frac{24}{17}\frac{g}{c} \circlearrowright$ ◀

$\bar{a}_y = \frac{1}{2}c\alpha = \frac{1}{2}c\left(\frac{24}{17}\frac{g}{c}\right) = \frac{12}{17}g$ $\qquad \bar{\mathbf{a}} = \frac{12}{17}g\downarrow$ ◀

(3) <u>Plate suspended from springs</u>. Immediately after spring B is released, the tension in spring A is still $\frac{1}{2}mg$ since its elongation is unchanged.



(*a*) <u>Angular acceleration</u>.

$$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad \left(\frac{1}{2}mg\right)\left(\frac{1}{2}c\right) = \bar{I}\alpha$$

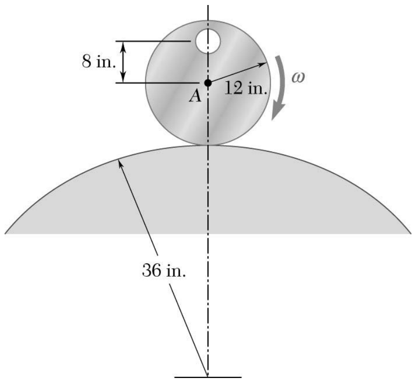
$$\frac{1}{4}mgc = \frac{5}{48}mc^2\alpha \qquad \boldsymbol{\alpha} = 2.4\frac{g}{c} \circlearrowright \blacktriangleleft$$

(*b*) <u>Acceleration at mass center</u>.

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad 0 = m\bar{a}_x \quad \bar{a}_x = 0$$

$$+\downarrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad mg - \frac{1}{2}mg = m\bar{a}_y$$

$$\bar{a}_y = \frac{1}{2}g \qquad \bar{\mathbf{a}} = 0.5g \downarrow \blacktriangleleft$$



PROBLEM 16.161

A cylinder with a circular hole is rolling without slipping on a fixed curved surface as shown. The cylinder would have a weight of 16 lb without the hole, but with the hole it has a weight of 15 lb. Knowing that at the instant shown the disk has an angular velocity of 5 rad/s clockwise, determine (*a*) the angular acceleration of the disk, (*b*) the components of the reaction force between the cylinder and the ground at this instant.

SOLUTION

Geometry: Let the mass center *G* of the cylinder lie a distance *b* below the geometric center for the position shown. Let *C*, the contact point between the cylinder and the fixed curved surface, be the origin of a coordinate system, as shown. The position vector of a point is

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

Let *r* be the radius of the cylinder and *R* that of the fixed curved surface

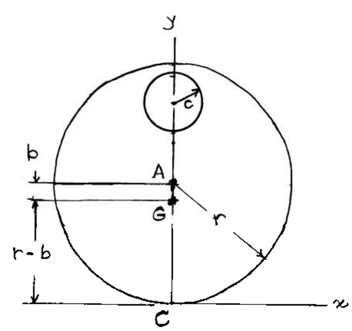
Kinematics:

The acceleration $\mathbf{a}_P$ at a point is given by

$$\mathbf{a}_P = \mathbf{a}_C + \boldsymbol{\alpha}\times\mathbf{r}_{P/C} - \omega^2\mathbf{r}_{P/C}$$

Let

$$\boldsymbol{\alpha} = \alpha \circlearrowright$$



Then, using the coordinate system

$$\mathbf{a}_P = [(a_C)_x \longrightarrow] + [(a_C)_y \uparrow]$$
$$+[y\alpha \longrightarrow] + [x\alpha \downarrow]$$
$$+[y\omega^2 \downarrow] + [x\omega^2 \longleftarrow]$$

Since the cylinder rolls without slipping on a fixed surface,

$$(a_C)_x = 0$$

For Points *G* and *A*,

$$\mathbf{a}_G = [(a_C)_y \uparrow] + [(r-b)\alpha \longrightarrow] + [(r-b)\omega^2 \downarrow] \quad (1)$$

$$\mathbf{a}_A = [(a_C)_y \uparrow] + [r\alpha \longrightarrow] + [r\omega^2 \downarrow] \quad (2)$$

Subtract Eq. (2) from Eq. (1) to eliminate $(a_C)_y$

$$\mathbf{a}_G = \mathbf{a}_A = [b\alpha \longleftarrow] + [b\omega^2 \uparrow]$$

$$\mathbf{a}_G = \mathbf{a}_A + [b\alpha \longleftarrow] + [b\omega^2 \uparrow]$$

$$= [(a_A)_x \longrightarrow] + [(\mathbf{a}_A)_y \uparrow] + [b\alpha \longleftarrow] + [b\omega^2 \uparrow]$$

$$= [(r-b)\alpha \longrightarrow] + [(\mathbf{a}_A)_y \uparrow] + [b\omega^2 \uparrow] \quad (3)$$

Point C is the instantaneous center, so that

$$\mathbf{v}_A = r\omega \longrightarrow$$

Point A is constrained to move on a circle of radius

$$\rho = R + r$$

so its vertical component of acceleration is

$$(\mathbf{a}_A)_y = \frac{v_A^2}{\rho}\downarrow = \frac{r^2\omega^2}{\rho}\downarrow$$

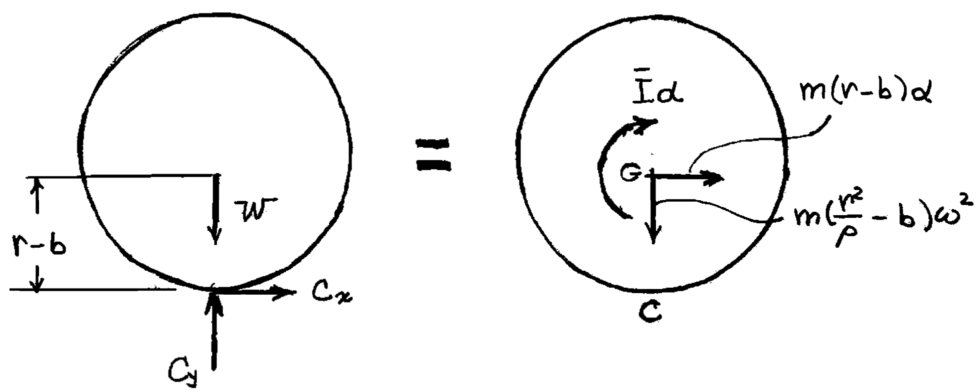
Using Eq. 3,
$$\mathbf{a}_G = [(r-b)\alpha \longrightarrow] + \left(\frac{r^2}{\rho} - b\right)\downarrow$$

The effective force at the mass center is

$$m\mathbf{a}_G = [m(r-b)\alpha \longrightarrow] + \left(\frac{r^2}{\rho} - b\right)\omega^2\downarrow$$

Kinetics:

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad 0 + \bar{I}\alpha + (r-b)m(r-b)\alpha$$
$$= [\bar{I} + m(r-b)^2]\alpha$$



(*a*) *Angular acceleration.* $\boldsymbol{\alpha} = 0$ ◀

(*b*) *Force components at C.*

$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad C_x = m(r-b)\alpha = 0 \qquad \mathbf{C}_x = 0 \blacktriangleleft$$

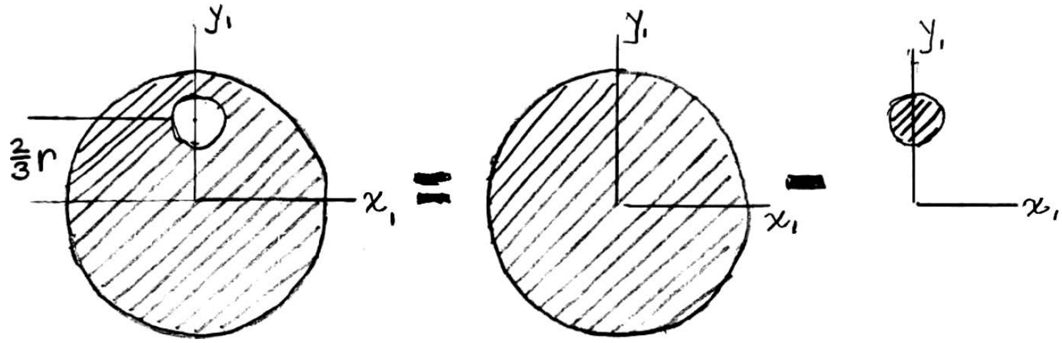
$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad C_y - W = -m\left(\frac{r^2}{\rho} - b\right)\omega^2$$

$$C_y = W - \frac{W}{g}\left(\frac{r^2}{\rho} - b\right)\omega^2$$

PROBLEM 16.161 (Continued)

It remains to determine the distance b from the mass distribution of the cylinder.

The mass center G coincides with the centroid of a circular cylinder of area $A_1 = \pi r^2$ with a circular cut out of area $A_2 = \frac{1}{16}A,$ with its center located $\frac{2}{3}r$ above the center of A_1.



	A_1	$\bar{y}_1$	$A_1\bar{y}_1$
(1)	πr^2	0	0
(2)	$-\dfrac{1}{16}\pi r^2$	$\dfrac{2}{3}r$	$-\dfrac{1}{24}\pi r^3$
Σ	$\dfrac{15}{16}\pi r^2$		$-\dfrac{1}{24}\pi r^3$

$$\bar{Y}\Sigma A = \Sigma A\bar{y}_1$$

$$\frac{15}{16}\pi r^2 \bar{Y} = -\frac{1}{24}\pi r^3$$

$$\bar{Y} = -\frac{2}{45}r$$

$$b = \frac{2}{45}r$$

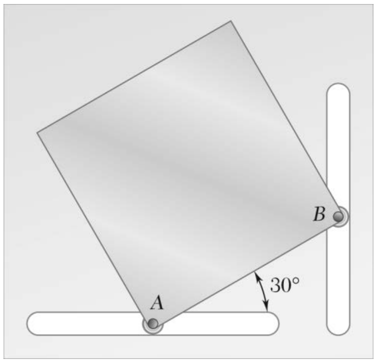
Data: $r = 12 \text{ in.} = 1 \text{ ft}, \quad R = 36 \text{ in.}, \quad \rho = 48 \text{ in.}$

$$b = \frac{2}{45}(12) = 0.53333 \text{ in.}$$

$$\left(\frac{r^2}{\rho} - b\right) = \frac{144}{48} - 0.53333 = 2.4667 \text{ in.} = 0.20556 \text{ ft}$$

$$C_y = 15 \text{ lb} - \frac{15 \text{ lb}}{32.2 \text{ ft/s}^2}(0.20556 \text{ ft})(5 \text{ rad/s})^2$$

$\mathbf{C} = 12.61 \text{ lb}\uparrow$ ◀



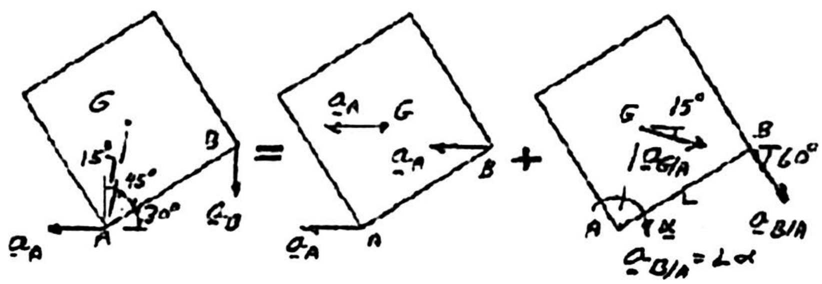
PROBLEM 16.162

The motion of a square plate of side 150 mm and mass 2.5 kg is guided by pins at corners A and B that slide in slots cut in a vertical wall. Immediately after the plate is released from rest in the position shown, determine (a) the angular acceleration of the plate, (b) the reaction at corner A.

SOLUTION

<u>Kinematics</u>:

$$AG = \frac{L}{2}\sqrt{2} = \frac{L}{\sqrt{2}} \quad a_{G/A} = (AB)\alpha = \frac{L\alpha}{\sqrt{2}}$$

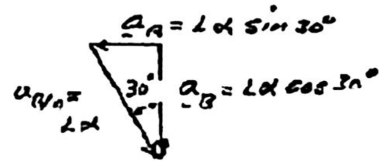


<u>Plane motion</u> = <u>Translation</u> + <u>Rotation</u>

$$a_B \downarrow = a_A \longleftarrow + a_{B/A} \searrow 60^\circ$$

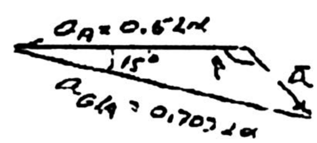
$$a_B \downarrow = a_A \longleftarrow + L\alpha \searrow 60^\circ$$

$$\bar{a} = a_A \longleftarrow + a_{G/A} \searrow 15^\circ$$



$$\bar{\mathbf{a}} = [L\alpha \sin 30^\circ \longleftarrow] + \left[\frac{L\alpha}{\sqrt{2}} \searrow 15^\circ\right] = [0.5L\alpha \longleftarrow] + [0.707L\alpha \searrow 15^\circ]$$

<u>Law of cosines</u>



$$\bar{a}^2 = a_A^2 \cdot a_{G/A}^2 - 2a_A a_{G/A}\cos 15^\circ$$

$$\bar{a}^2 = (0.5L\alpha)^2 + (0.707L\alpha)^2$$

$$-2(0.5L\alpha)(0.707L\alpha)\cos 15^\circ$$

$$\bar{a}^2 = L^2\alpha^2(0.25 + 0.5 - 0.68301)$$

$$a^2 = L^2\alpha^2(0.06699)$$

$$\bar{a} = 0.25882L\alpha$$

Law of sines.

$$\frac{\bar{a}}{\sin 15°} = \frac{a_{G/A}}{\sin \beta}; \quad \sin \beta = \frac{a_{G/A}}{\bar{a}} \sin 15° = \frac{0.707 L\alpha}{0.25950 L\alpha} \sin 15°$$

$$\sin \beta = 0.707; \quad \beta = 135°$$

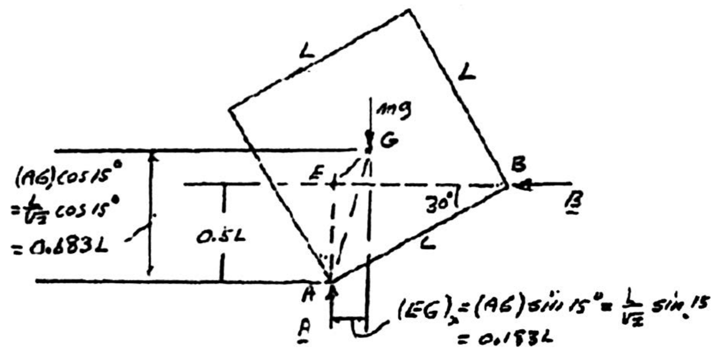
$$\bar{\mathbf{a}} = 0.2583 L\alpha \searrow 45°$$

Kinetics: $(\omega = 0)$

We find the location of Point E where lines of action of A and B intersect.

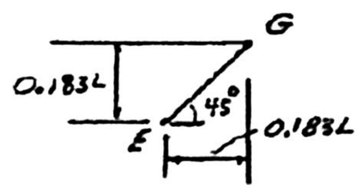
$$MG = \frac{L}{\sqrt{2}}$$

$$\measuredangle EAG = 15°$$

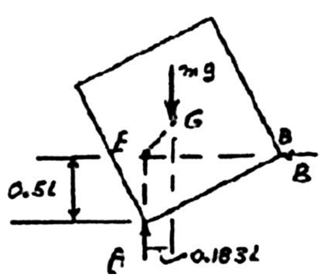


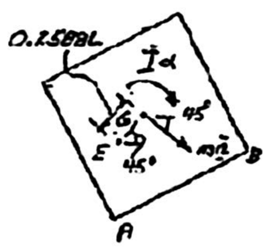
$$(EG)_y = 0.6829L - 0.5L = 0.183L$$

$$(EG) = (0.183L)\sqrt{2} = 0.2588L$$



$$\bar{I} = \frac{1}{6} mL^2$$





PROBLEM 16.162 (Continued)

(*a*) Angular acceleration.

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad mg(0.183L) = \bar{I}\alpha + (m\bar{a})(0.2588L)$$

$$0.183mgL = \frac{1}{6}mL^2\alpha + m(0.2588L\alpha)(0.2588L)$$

$$0.183gL = L^2\alpha\left(\frac{1}{6} + 0.06698\right)$$

$$0.183\frac{g}{L} = 0.2336\alpha; \quad \alpha = 0.7834\frac{g}{L}$$

$$\alpha = 0.7834\frac{9.81 \text{ m/s}^2}{0.15 \text{ m}}$$

$\boldsymbol{\alpha} = 51.2 \text{ rad/s}^2 \circlearrowright$ ◀

(*b*) Reaction at corner *A*.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad A - mg = -m\bar{a}\sin 45°$$

$$= -m(0.2588L\alpha)\sin 45°$$

$$= -m(0.2588L)\left(0.7834\frac{g}{L}\right)\sin 45°$$

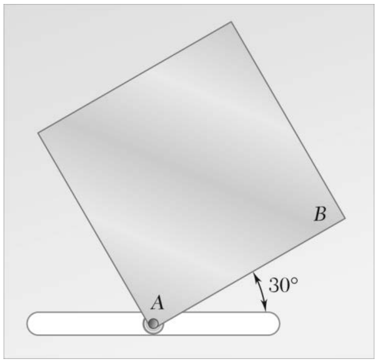
$$A - mg = 0.1434mg$$

$$A = 0.8566mg$$

$$= 0.8566(2.5 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 21.01 \text{ N}$$

$\mathbf{A} = 21.0 \text{ N} \uparrow$ ◀

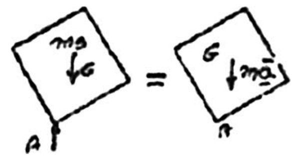


PROBLEM 16.163

Solve Problem 16.162, assuming that the plate is fitted with a single pin at corner *A*.

Problem 16.162 The motion of a square plate of side 150 mm and mass 2.5 kg is guided by pins at corners *A* and *B* that slide in slots cut in a vertical wall. Immediately after the plate is released from rest in the position shown, determine (*a*) the angular acceleration of the plate, (*b*) the reaction of corner *A*.

SOLUTION

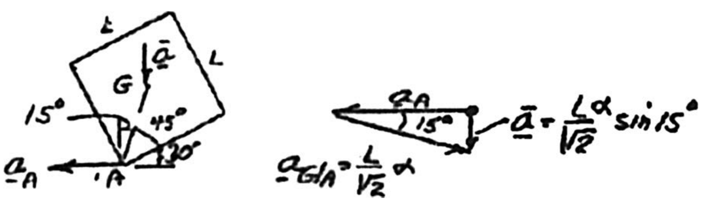


Since both *A* and *mg* are vertical, $\bar{a}_x = 0$ and $\bar{\mathbf{a}}$ is $\downarrow$

Kinematics:

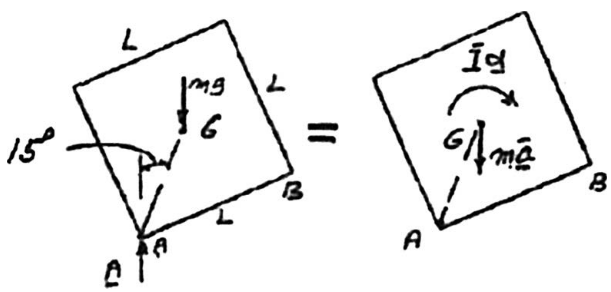
$$AG = \frac{L}{\sqrt{2}} \searrow 15^\circ \quad \mathbf{a}_{G/A} = (AG)\alpha \searrow 15^\circ$$

$$\bar{\mathbf{a}} \downarrow = \mathbf{a}_A \longleftarrow + \mathbf{a}_{G/A} \searrow 15^\circ$$



$$\bar{\mathbf{a}} = 0.183L\alpha \searrow 15^\circ$$

Kinetics: $\bar{I} = \frac{1}{6}mL^2$



PROBLEM 16.163 (Continued)

(*a*) Angular acceleration.

$$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad mg(AG)\sin 15^\circ = \bar{I}\alpha + m\bar{a}(AG)\sin 15^\circ$$

$$mg\left(\frac{L}{\sqrt{2}}\right)\sin 15^\circ = \frac{1}{6}mL^2\alpha + m(0.183L\alpha)\left(\frac{L}{\sqrt{2}}\right)\sin 15^\circ$$

$$0.183\frac{g}{L} = \left(\frac{1}{6} + 0.033494\right)\alpha$$

$$0.183\frac{g}{L} = 0.2002\alpha$$

$$\alpha = 0.9143\frac{g}{L}$$

$$= 0.9143\frac{9.81 \text{ m/s}^2}{0.15 \text{ m}}$$

$\boldsymbol{\alpha} = 59.8 \text{ rad/s}^2 \circlearrowright$ ◀

(*b*) Reaction at corner *A*.

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad A - mg = -m\bar{a}$$

$$A - mg = -m(0.183L\alpha)$$

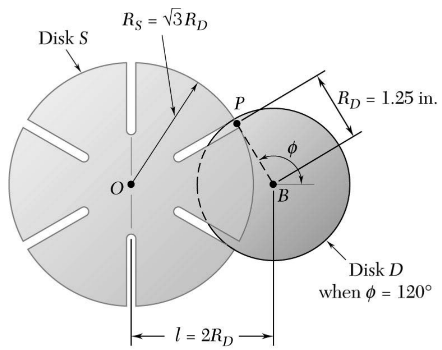
$$= -m(0.183L)\left(0.9143\frac{g}{L}\right)$$

$$A - mg = -0.1673mg$$

$$A = 0.8326mg$$

$$A = 0.8326(2.5 \text{ kg})(9.81 \text{ m/s}^2)$$

$\mathbf{A} = 20.4 \text{ N}\uparrow$ ◀



PROBLEM 16.164

The Geneva mechanism shown is used to provide an intermittent rotary motion of disk *S*. Disk *D* weighs 2 lb and has a radius of gyration of 0.9 in. and disk *S* weighs 6 lb and has a radius of gyration of 1.5 in. The motion of the system is controlled by a couple **M** applied to disk *D*. A pin *P* is attached to disk *D* and can slide in one of the six equally spaced slots cut in disk *S*. It is desirable that the angular velocity of disk *S* be zero as the pin enters and leaves each of the six slots; this will occur if the distance between the centers of the disks and the radii of the disks are related as shown. Knowing disk *D* rotates with a constant counterclockwise angular velocity of 8 rad/s and the friction between the slot and pin *P* is negligible, determine when $\phi = 150°$ (*a*) the couple **M**, (*b*) the magnitude of the force pin *P* applies to disk *S*.

SOLUTION

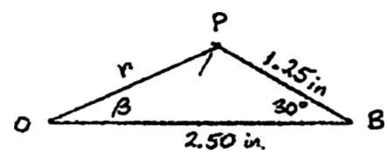
Geometry:

Law of cosines. $$r^2 = 1.25^2 + 2.50^2 - (2)(1.25)(2.50)\cos 30°$$

$$r = 1.54914 \text{ in.}$$

Law of sines. $$\frac{\sin\beta}{1.25} = \frac{\sin 30°}{r}$$

$$\beta = 23.794°$$



Let disk *S* be a rotating frame of reference.

$$\mathbf{\Omega} = \omega_S \circlearrowright, \quad \dot{\mathbf{\Omega}} = \alpha_S \circlearrowright$$

Motion of coinciding Point P′ on the disk.

$$\mathbf{v}_{P'} = r\omega_S = 1.54914\omega_S \searrow \beta$$

$$\mathbf{a}_{P'} = -\alpha_S \mathbf{k} \times \mathbf{r}_{P/O} - \omega_S^2 \mathbf{r}_{P/O} = [1.54914\alpha_S \searrow \beta] + [1.54914\omega_S^2 \swarrow \beta]$$

Motion relative to the frame.

$$\mathbf{v}_{P/S} = u \swarrow \beta \qquad \mathbf{a}_{P/S} = \dot{u} \swarrow \beta$$

Coriolis acceleration. $$2\omega_S u \searrow \beta$$

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/S} = [1.54914\omega_S \searrow \beta] + [u \swarrow \beta]$$

$$\mathbf{a}_P = \mathbf{a}_P + \mathbf{a}_{P/S} + 2\omega_S u \searrow$$

$$= [1.54914\alpha_S \searrow \beta] + [1.54914\omega_S^2 \swarrow \beta] + [\dot{u} \swarrow \beta] + [2\omega_S u \searrow \beta]$$

Motion of disk D. (rotation about *B*)

$$\mathbf{v}_P = (BP)\omega_D = (1.25)(8) = 10 \text{ in./s} \nearrow 30^\circ$$

$$\mathbf{a}_P = [(BP)\alpha_D \nearrow 60^\circ] + [(BP)\omega_S^2 \searrow 30^\circ] = 0 + [(1.25)(8)^2 \searrow 30^\circ]$$

$$= 80 \text{ in./s}^2 \searrow 30^\circ$$

Equate the two expressions for $\mathbf{v}_P$ and resolve into components.

$$\searrow \beta: \quad 1.54914\omega_S = 10\cos(30^\circ + \beta)$$

$$\omega_S = \frac{10\cos 53.794^\circ}{1.54914}$$

$$= 3.8130 \text{ rad/s}$$

$$\nearrow \beta: \quad u = 10\sin(30^\circ + \beta) = 10\sin 53.794^\circ = 8.0690 \text{ in./s}$$

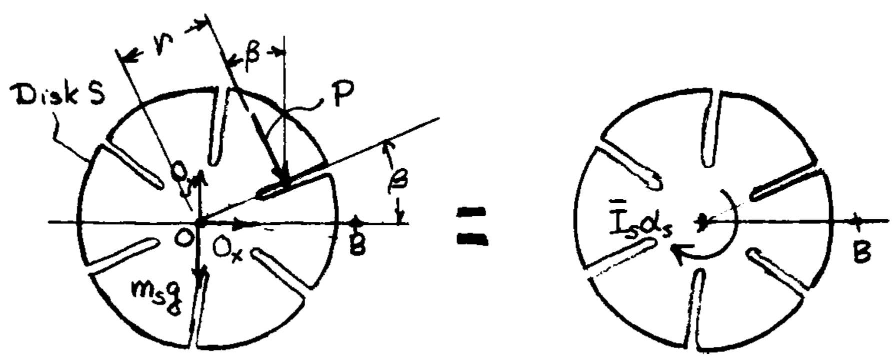
Equate the two expressions for $\mathbf{a}_P$ and resolve into components.

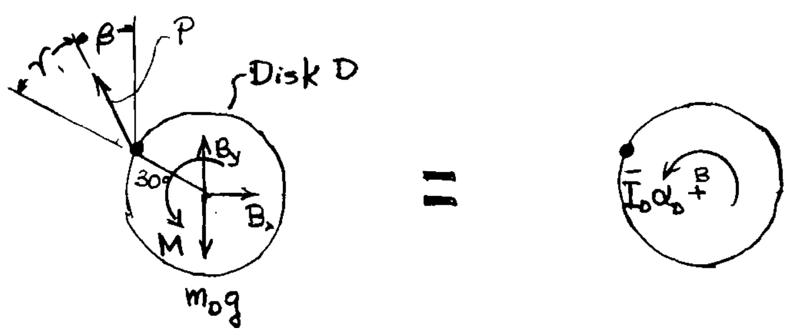
$$\searrow \beta: \quad 1.54914\alpha_S - 2\omega_S u = 80\sin(30^\circ + \beta)$$

$$\alpha_S = \frac{80\sin 53.794^\circ + (2)(3.8130)(8.0690)}{1.54914}$$

$$= 81.391 \text{ rad/s}^2$$

Kinetics:





PROBLEM 16.164 (Continued)

$$\bar{I}_s\alpha_s = \left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)\left(\frac{1.5}{12}\text{ ft}\right)^2 (81.391\text{ rad/s}^2) = 0.23697\text{ lb}\cdot\text{ft}$$

$$\bar{I}\alpha_D = 0 \qquad \text{since} \quad \boldsymbol{\alpha}_D = 0$$

Disk S:

$$\circlearrowright \Sigma M_0 = \Sigma M_{\text{eff}}: \quad Pr = \bar{I}_S\alpha_S$$

$$P\left(\frac{1.54914}{12}\text{ ft}\right) = 0.23697\text{ lb}\cdot\text{ft} \qquad P = 1.8356\text{ lb}$$

Disk D:

$$r = 90° - 30° - \beta = 36.206°$$

$$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad M - (P\sin r)\left(\frac{1.25}{12}\right)\text{ft}$$

$$M = \frac{(1.25)(1.8356\sin 36.206°)}{12} = 0.11294\text{ lb}\cdot\text{ft}$$

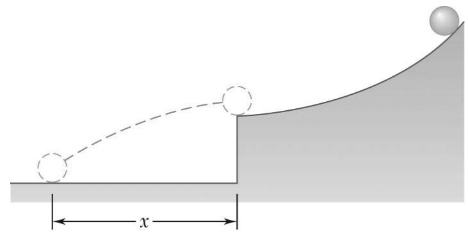
(a) *Couple* **M**. $\mathbf{M} = 1.355\text{ lb}\cdot\text{in}\circlearrowleft$ ◀

(b) *Magnitude of contact force*. $P = 1.836\text{ lb}$ ◀

CHAPTER 17

PROBLEM 17.CQ1

A round object of mass m and radius r is released from rest at the top of a curved surface and rolls without slipping until it leaves the surface with a horizontal velocity as shown. Will a solid sphere, a solid cylinder or a hoop travel the greatest distance c?



(a) A solid sphere

(b) A solid cylinder

(c) A hoop

(d) They will all travel the same distance.

SOLUTION

Answer: (a) It has the smallest mass moment of inertia, so it will have the greatest speed at the bottom of the surface.

PROBLEM 17.CQ2

A solid steel sphere A of radius r and mass m is released from rest and rolls without slipping down an incline as shown. After traveling a distance d the sphere has a speed v. If a solid steel sphere of radius $2r$ is released from rest on the same incline, what will its speed be after rolling a distance d?

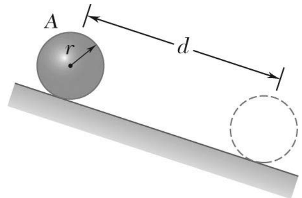
(*a*) 0.25 v

(*b*) 0.5 v

(*c*) v

(*d*) $2v$

(*e*) $4v$



SOLUTION

Answer: (*c*) Using conservation of energy you can show that the speed after traveling a distance d will be independent of the mass and the radius.

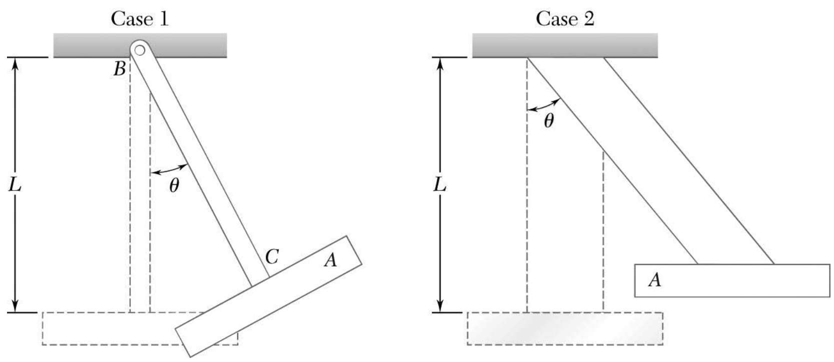
PROBLEM 17.CQ3

Slender bar A is rigidly connected to a massless rod BC in Case 1 and two massless cords in Case 2 as shown. The vertical thickness of bar A is negligible compared to L. In both cases A is released from rest at an angle $\theta = \theta_0$. When $\theta = 0°$ which system will have the larger kinetic energy?

(*a*) Case 1

(*b*) Case 2

(*c*) The kinetic energy will be the same.



SOLUTION

Answer: (*c*)

PROBLEM 17.CQ4

In Problem 17.CQ3, how will the speeds of the centers of gravity compare for the two cases when $\theta = 0°$?

(*a*) Case 1 will be larger.

(*b*) Case 2 will be larger.

(*c*) The speeds will be the same.

SOLUTION

Answer: (*b*) Case 1 will also have rotational kinetic energy, so the speed will be smaller.

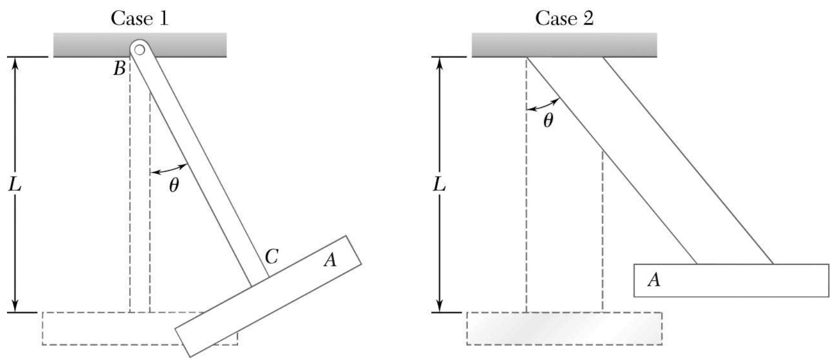
PROBLEM 17.CQ5

Slender bar A is rigidly connected to a massless rod BC in Case 1 and two massless cords in Case 2 as shown. The vertical thickness of bar A is not negligible compared to L. In both cases A is released from rest at an angle $\theta = \theta_0$. When $\theta = \theta°$ which system will have the largest kinetic energy?

(a) Case 1

(b) Case 2

(c) The kinetic energy will be the same.



SOLUTION

Answer: (a) Case 1 will have a greater change in gravitational potential energy, so the kinetic energy will be larger.

PROBLEM 17.1

The rotor of an electric motor has an angular velocity of 3600 rpm when the load and power are cut off. The 50-kg rotor then coasts to rest after 5000 revolutions. Knowing that the kinetic friction of the rotor produces a couple of magnitude 4 N · m, determine the centroidal radius of gyration of the rotor.

SOLUTION

Angular velocities:
$$\omega_1 = 3600\frac{\text{rev}}{\text{min}}\cdot\frac{1\,\text{min}}{60\,\text{s}}\cdot\frac{2\pi\ \text{rad}}{\text{rev}} = 120\pi\ \text{rad/s}$$
$$\omega_2 = 0$$

Angular displacement: $5000\ \text{rev} = 10000\pi\ \text{rad}$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$:

$$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}\bar{I}(120\pi)^2 = 71.061\times 10^3\bar{I}$$
$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 = 0$$
$$U_{1\to 2} = -M\theta = -(4\ \text{N}\cdot\text{m})(10000\pi\ \text{rad}) = -40000\pi\ \text{N}\cdot\text{m}$$
$$71.061\times 10^3\bar{I} - 40000\pi = 0$$
$$\bar{I} = 1.76839\ \text{kg}\cdot\text{m}^2$$
$$\bar{I} = m\bar{k}^2$$

Centroidal radius of gyration.
$$\bar{k} = \sqrt{\frac{\bar{I}}{m}} = \sqrt{\frac{1.76839\ \text{kg}\cdot\text{m}^2}{50\ \text{kg}}} = 0.1881\ \text{m}$$

$\bar{k} = 188.1\ \text{mm}$ ◀

PROBLEM 17.2

It is known that 1500 revolutions are required for the 6000-lb flywheel to coast to rest from an angular velocity of 300 rpm. Knowing that the radius of gyration of the flywheel is 36 in., determine the average magnitude of the couple due to kinetic friction in the bearings.

SOLUTION

Angular velocity:

$$\omega_0 = 300 \text{ rpm}$$
$$= 10\pi \text{ rad/s}$$
$$\omega_2 = 0$$

Moment of inertia:

$$\bar{I} = m\bar{k}^2 = \frac{6000 \text{ lb}}{32.2 \text{ ft/s}^2}(3 \text{ ft})^2$$
$$= 1677 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

Kinetic energy:

$$T_1 = \frac{1}{2}\bar{I}\omega_0^2$$
$$= \frac{1}{2}(1677)(10\pi)^2$$
$$= 827{,}600 \text{ ft}\cdot\text{lb}$$
$$T_2 = 0$$

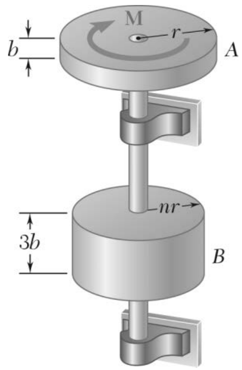
Work:

$$U_{1\to 2} = -M\theta$$
$$= -M(1500 \text{ rev})(2\pi \text{ rad/rev})$$
$$= -9424.7M$$

Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2$$
$$827{,}600 - 9424.7M = 0$$

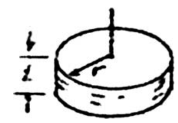
Average friction couple: $M = 87.81 \text{ lb}\cdot\text{ft}$ $\qquad M = 87.8 \text{ lb}\cdot\text{ft}$ ◀



PROBLEM 17.3

Two disks of the same material are attached to a shaft as shown. Disk A has a weight of 30 lb and a radius $r = 5$ in. Disk B is three times as thick as disk A. Knowing that a couple **M** of magnitude 15 lb · ft is to be applied to disk A when the system is at rest, determine the radius nr of disk B if the angular velocity of the system is to be 600 rpm after 4 revolutions.

SOLUTION



For any disk:

$$m = \rho(\pi r^2 t)$$

$$\bar{I} = \frac{1}{2}mr^2$$

$$= \frac{1}{2}\pi\rho t r^4$$

Moment of inertia.

Disk A:

$$I_A = \frac{1}{2}\pi\rho b r^4$$

Disk B:

$$I_B = \frac{1}{2}\pi\rho(3b)(nr)^4$$

$$= 3n^4\left[\frac{1}{2}\pi\rho b r^4\right]$$

$$= 3n^4 I_A$$

$$I_{\text{total}} = I_A + I_B = (1 + 3n^4)I_A \quad (1)$$

Angular velocity:

$$\omega_1 = 0$$

$$\omega_2 = 600 \text{ rpm}$$

$$= 20\pi \text{ rad/s}$$

Rotation:

$$\theta = 4 \text{ rev} = 8\pi \text{ rad}$$

Kinetic energy:

$$T_1 = 0$$

$$T_2 = \frac{1}{2}I_{\text{total}}\ \omega_2^2$$

Work:

$$U_{1\to 2} = M\theta$$

$$= (15 \text{ lb}\cdot\text{ft})(8\pi \text{ rad})$$

$$= 376.99 \text{ lb}\cdot\text{ft}$$

PROBLEM 17.3 (Continued)

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$0 + 376.991 = \frac{1}{2} I_{\text{total}} (20\pi)^2$$

$$I_{\text{total}} = 0.19099 \text{ slug} \cdot \text{ft}^2$$

But,

$$I_A = \frac{1}{2} m_A r_A^2$$

$$= \frac{1}{2}\left(\frac{30 \text{ lb}}{32.2}\right)\left(\frac{5}{12}\text{ft}\right)^2$$

$$= 0.080875 \text{ slug} \cdot \text{ft}^2$$

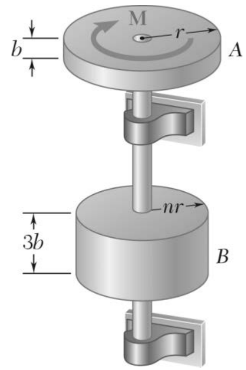
From (1)

$$0.19099 = (1 + 3n^4)(0.080875)$$

$$n^4 = 0.45383$$

$$n = 0.82078$$

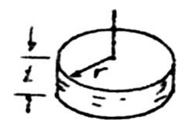
Radius of disk B: $r_B = nr_A = (0.82078)(5 \text{ in.}) = 4.1039 \text{ in.}$ $\quad r_B = 4.10 \text{ in.}$ ◀



PROBLEM 17.4

Two disks of the same material are attached to a shaft as shown. Disk A is of radius r and has a thickness b, while disk B is of radius nr and thickness $3b$. A couple **M** of constant magnitude is applied when the system is at rest and is removed after the system has executed 2 revolutions. Determine the value of n which results in the largest final speed for a point on the rim of disk B.

SOLUTION



For any disk:

$$m = \rho(\pi r^2 t)$$

$$\bar{I} = \frac{1}{2}mr^2$$

$$= \frac{1}{2}\pi\rho t r^4$$

Moment of inertia.

Disk A:

$$I_A = \frac{1}{2}\pi\,\rho\,b\,r^4$$

Disk B:

$$I_B = \frac{1}{2}\pi\,\rho(3b)(nr)^4$$

$$= 3n^4\left[\frac{1}{2}\pi\rho b r^4\right]$$

$$= 3n^4 I_A$$

$$I_{\text{total}} = I_A + I_B$$

$$= (1+3n^4)I_A$$

Work-energy.

$$T_1 = 0 \qquad U_{1\to 2} = M\theta = M(4\pi\,\text{rad})$$

$$T_2 = \frac{1}{2}I_{\text{total}}\,\omega_2^2$$

$$T_1 + U_{1\to 2} = T_2: \quad 0 + M(4\pi) = \frac{1}{2}(1+3n^4)I_A\omega_2^2$$

$$\omega_2^2 = \frac{8\pi M}{(1+3n^4)I_A}$$

For Point D on rim of disk B

$$v_D = (nr)\omega_2 \quad \text{or} \quad v_D^2 = n^2r^2\omega_2^2 = \frac{8\pi M r^2}{I_A}\cdot\frac{n^2}{1+3n^4}$$

PROBLEM 17.4 (Continued)

Value of n for maximum final speed.

For maximum v_D: $\quad \dfrac{d}{dn}\left(\dfrac{n^2}{1+3n^4}\right)=0$

$$\frac{1}{(1+3n^4)^2}[n^2(12n^3)-(1+3n^4)(2n)]=0$$

$$12n^5-2n-6n^5=0$$

$$2n(3n^4-1)=0$$

$n=0$ and $n=\left(\dfrac{1}{3}\right)^{0.25}=0.7598$ $\qquad n=0.760$ ◀

PROBLEM 17.5

The flywheel of a small punch rotates at 300 rpm. It is known that 1800 ft · lb of work must be done each time a hole is punched. It is desired that the speed of the flywheel after one punching be not less that 90 percent of the original speed of 300 rpm. (*a*) Determine the required moment of inertia of the flywheel. (*b*) If a constant 25-lb · ft couple is applied to the shaft of the flywheel, determine the number of revolutions which must occur between each punching, knowing that the initial velocity is to be 300 rpm at the start of each punching.

SOLUTION

Angular velocities: $\omega_1 = 300 \text{ rpm} = 10\pi \text{ rad/s}$

$$\omega_2 = 0.90\omega_1 = 9\pi \text{ rad/s}$$

Principle of work and energy: $T_1 + U_{1\to 2} = T_2$

$$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}\bar{I}(10\pi)^2$$

$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}\bar{I}(9\pi)$$

$$U_{1\to 2} = -1800 \text{ ft} \cdot \text{lb}$$

$$\frac{1}{2}\bar{I}(10\pi)^2 - 1800 = \frac{1}{2}\bar{I}(9\pi)^2$$

(*a*) *Required moment of inertia.*

$$\bar{I} = \frac{2(1800)}{\pi^2(100-81)} = 19.198 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$\bar{I} = 19.20 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$ ◀

(*b*) *Number of revolution between each punching.*

Definition of work: $U_{2\to 1} = M\theta$:

$$1800 \text{ ft} \cdot \text{lb} = (25 \text{ lb} \cdot \text{ft})\theta$$

$$\theta = 72 \text{ rad} = 11.459 \text{ rev}$$

$\theta = 11.46 \text{ rev}$ ◀

PROBLEM 17.6

The flywheel of a punching machine has a mass of 300 kg and a radius of gyration of 600 mm. Each punching operation requires 2500 J of work. (*a*) Knowing that the speed of the flywheel is 300 rpm just before a punching, determine the speed immediately after the punching. (*b*) If a constant 25-N · m couple is applied to the shaft of the flywheel, determine the number of revolutions executed before the speed is again 300 rpm.

SOLUTION

Moment of inertia.

$$I = mk^2 = (300\text{ kg})(0.6\text{ m})^2 = 108\text{ kg}\cdot\text{m}^2$$

Kinetic energy. *Position 1.*

$$\omega_1 = 300\text{ rpm} = 10\pi\text{ rad/s}$$

$$T_1 = \frac{1}{2}I\omega_1^2 = \frac{1}{2}(108)(10\pi)^2 = 53.296\times10^3\text{ J}$$

Position 2.

$$T_2 = \frac{1}{2}I\omega_2^2 = 54\omega_2^2$$

Work.

$$U_{1\to2} = -2500\text{ J}$$

Principle of work and energy for punching.

$$T_1 + U_{1\to2} = T_2: \quad 53.296\times10^3 - 2500 = 54\omega_2^2$$

(*a*)

$$\omega_2^2 = 940.66$$

$$\omega_2 = 30.67\text{ rad/s} \qquad \omega_2 = 293\text{ rpm} \blacktriangleleft$$

Principle of work and energy for speed recovery.

$$T_2 + U_{2\to1} = T_1$$

$$U_{2\to1} = 2500\text{ J}$$

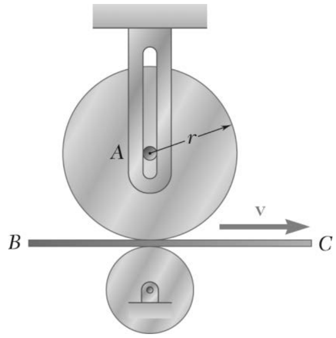
$$M = 25\text{ N}\cdot\text{m}$$

$$U_{2\to1} = M\theta \quad 2500 = 25\theta \qquad \theta = 100\text{ rad}$$

(*b*)

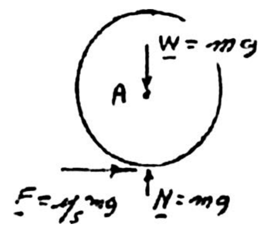
$$\theta = 15.92\text{ rev} \blacktriangleleft$$

PROBLEM 17.7



Disk A, of weight 10 lb and radius $r = 6$ in., is at rest when it is placed in contact with belt BC, which moves to the right with a constant speed $v = 40$ ft/s. Knowing that $\mu_k = 0.20$ between the disk and the belt, determine the number of revolutions executed by the disk before it attains a constant angular velocity.

SOLUTION



Work of external friction force on disk A.

Only force doing work is F. Since its moment about A is $M = rF$, we have

$$U_{1\to 2} = M\theta$$
$$= rF\theta$$
$$= r(\mu_k mg)\theta$$

Kinetic energy of disk A.

Angular velocity becomes constant when $\omega_2 = \dfrac{v}{r}$

$$T_1 = 0$$
$$T_2 = \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$$
$$= \frac{mv^2}{4}$$

Principle of work and energy for disk A.

$$T_1 + U_{1-2} = T_2: \quad 0 + r\mu_k mg\theta = \frac{mv^2}{4}$$

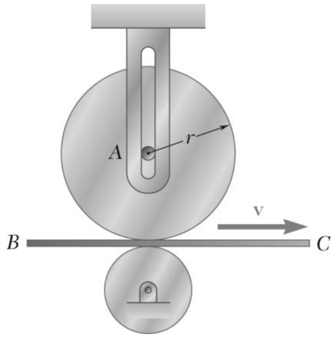
Angle change $\theta = \dfrac{v^2}{4r\mu_k g}$ rad $\qquad \theta = \dfrac{v^2}{8\pi r\mu_k g}$ rev

Data:

$$r = 0.5 \text{ ft}$$
$$\mu_k = 0.20$$
$$v = 40 \text{ ft/s}$$

$$\theta = \frac{(40 \text{ ft/s})^2}{8\pi(0.5 \text{ ft})(0.20)(32.2 \text{ ft/s}^2)}$$

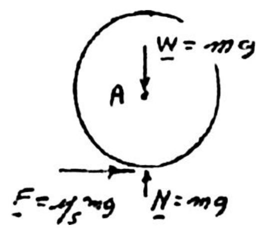
$\theta = 19.77$ rev ◀



PROBLEM 17.8

Disk A is of constant thickness and is at rest when it is placed in contact with belt BC, which moves with a constant velocity $\mathbf{v}$. Denoting by μ_k the coefficient of kinetic friction between the disk and the belt, derive an expression for the number of revolutions executed by the disk before it attains a constant angular velocity.

SOLUTION



Work of external friction force on disk A.

Only force doing work is F. Since its moment about A is $M = rF$, we have

$$U_{1\to 2} = M\theta$$
$$= rF\theta$$
$$= r(\mu_k mg)\theta$$

Kinetic energy of disk A.

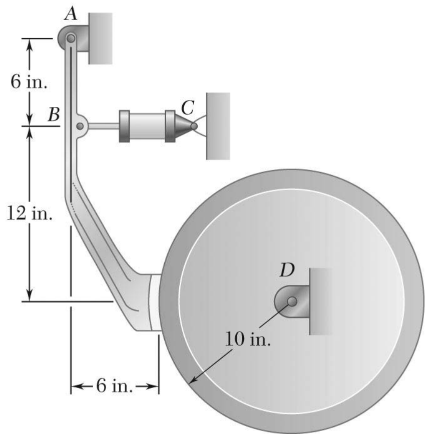
Angular velocity becomes constant when $\omega_2 = \dfrac{v}{r}$

$$T_1 = 0$$
$$T_2 = \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$$
$$= \frac{mv^2}{4}$$

Principle of work and energy for disk A.

$$T_1 + U_{1\to 2} = T_2: \quad 0 + r\mu_k mg\,\theta = \frac{mv^2}{4}$$

Angle change. $\theta = \dfrac{v^2}{4r\mu_k g}$ rad $\qquad \theta = \dfrac{v^2}{8\pi r\mu_k g}$ rev ◀



PROBLEM 17.9

The 10-in.-radius brake drum is attached to a larger flywheel which is not shown. The total mass moment of inertia of the flywheel and drum is 16 lb · ft · s^2 and the coefficient of kinetic friction between the drum and the brake shoe is 0.40. Knowing that the initial angular velocity is 240 rpm clockwise, determine the force which must be exerted by the hydraulic cylinder if the system is to stop in 75 revolutions.

SOLUTION

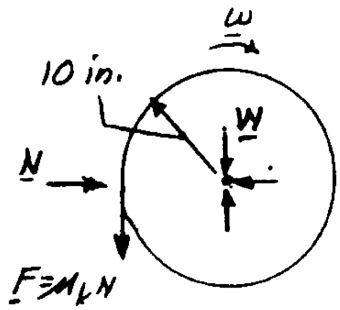
Kinetic energies.

$$\omega_1 = 240 \text{ rpm} = 8\pi \text{ rad/s}$$

$$\bar{I} = 16 \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

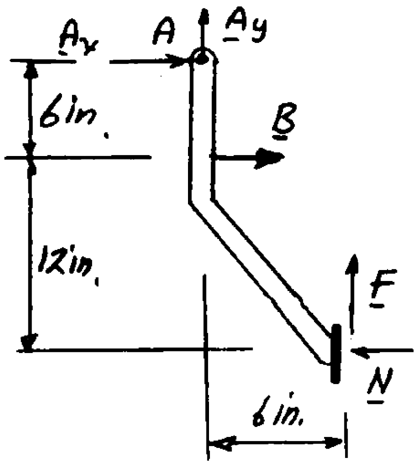
$$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}(16)(8\pi)^2 = 5053 \text{ ft} \cdot \text{lb}$$

$$\omega_2 = 0 \qquad T_2 = 0$$



Angular displacement. $\theta = 75 \text{ rev} = 75(2\pi) = 150\pi \text{ rad}$

Work. $U_{1-2} = -M\theta = -\left[F\left(\frac{10}{12}\text{ft}\right)\right](150\pi \text{ rad}) = -392.7F$



Principle of work and energy. $T_1 + U_{1-2} = T_2:$

$$5053 - 392.7F = 0 \qquad F = 12.868 \text{ lb}$$

$$F = \mu_k N: \quad 12.868 = (0.40)N \qquad N = 32.17 \text{ lb}$$

Free body brake arm:

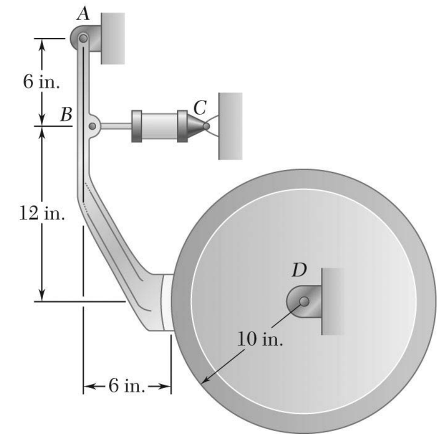
$$+\circlearrowleft \Sigma M_A = 0: \quad B(6 \text{ in.}) + F(6 \text{ in.}) - N(18 \text{ in.}) = 0$$

$$B(6 \text{ in.}) + (12.868 \text{ lb})(6 \text{ in.}) - (32.17 \text{ lb})(18 \text{ in.}) = 0$$

$$B = 83.64 \text{ lb}$$

$B = 83.6$ lb ◀

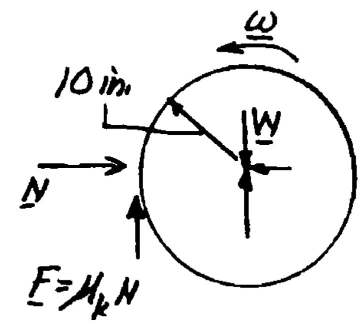
PROBLEM 17.10



Solve Problem 17.9, assuming that the initial angular velocity of the flywheel is 240 rpm counterclockwise.

PROBLEM 17.9 The 10-in.-radius brake drum is attached to a larger flywheel which is not shown. The total mass moment of inertia of the flywheel and drum is $16\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$ and the coefficient of kinetic friction between the drum and the brake shoe is 0.40. Knowing that the initial angular velocity is 240 rpm clockwise, determine the force which must be exerted by the hydraulic cylinder if the system is to stop in 75 revolutions.

SOLUTION



Kinetic energies.

$$\omega_1 = 240\ \text{rpm} = 8\pi\ \text{rad/s}$$

$$\bar{I} = 16\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$$

$$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}(16)(8\pi)^2 = 5053\ \text{ft} \cdot \text{lb}$$

$$\omega_2 = 0 \qquad T_2 = 0$$

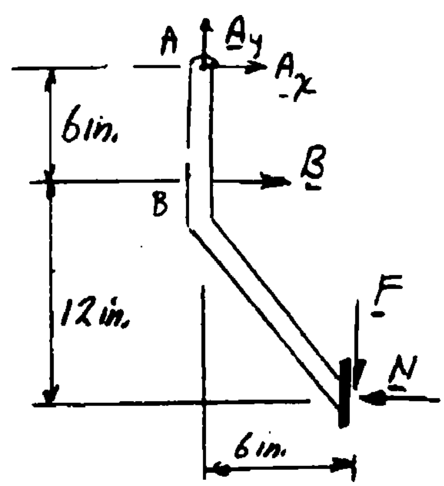
Angular displacement. $\theta = 75\ \text{rev} = 75(2\pi) = 150\pi\ \text{rad}$

Work. $U_{1-2} = -M\theta = -\left[F\left(\frac{10}{12}\text{ft}\right)\right](150\pi\ \text{rad}) = -392.7F$

Principle of work and energy. $T_1 + U_{1-2} = T_2$:

$$5053 - 392.7F = 0 \qquad F = 12.868\ \text{lb}$$

$$F = \mu_k N: \quad 12.868 = (0.40)N \qquad N = 32.17\ \text{lb}$$



Free body brake arm:

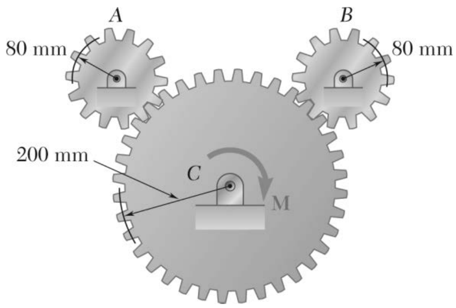
$$+\circlearrowleft \Sigma M_A = 0: \quad B(6\ \text{in.}) - F(6\ \text{in.}) - N(18\ \text{in.}) = 0$$

$$B(6\ \text{in.}) - (12.868\ \text{lb})(6\ \text{in.}) - (32.17\ \text{lb})(18\ \text{in.}) = 0$$

$$B = 109.37\ \text{lb}$$

$B = 109.4\ \text{lb}$ ◀

PROBLEM 17.11



Each of the gears A and B has a mass of 2.4 kg and a radius of gyration of 60 mm, while gear C has a mass of 12 kg and a radius of gyration of 150 mm. A couple **M** of constant magnitude 10 N · m is applied to gear C. Determine (a) the number of revolutions of gear C required for its angular velocity to increase from 100 to 450 rpm, (b) the corresponding tangential force acting on gear A.

SOLUTION

Moments of inertia.

Gears A and B: $I_A = I_B = mk^2 = (2.4)(0.06)^2 = 8.64\times10^{-3}\,\text{kg}\cdot\text{m}^2$

Gear C: $I_C = (12)(0.15)^2 = 270\times10^{-3}\,\text{kg}\cdot\text{m}^2$

Kinematics.

$$r_A\omega_A = r_B\omega_B = r_C\omega_C$$

$$\omega_A = \omega_B = \frac{200}{80}\omega_C = 2.5\omega_C$$

$$\theta_A = \theta_B = 2.5\theta_C$$

Kinetic energy. $T = \frac{1}{2}I\omega^2$:

Position 1.

$$\omega_C = 100\text{ rpm} = \frac{10}{3}\pi\text{ rad/s}$$

$$\omega_A = \omega_B = 250\text{ rpm} = \frac{25}{3}\pi\text{ rad/s}$$

Gear A: $(T_1)_A = \frac{1}{2}(8.64\times10^{-3})\left(\frac{25\pi}{3}\right)^2 = 2.9609\text{ J}$

Gear B: $(T_1)_B = \frac{1}{2}(8.64\times10^{-3})\left(\frac{25\pi}{3}\right)^2 = 2.9609\text{ J}$

Gear C: $(T_1)_C = \frac{1}{2}(270\times10^{-3})\left(\frac{10\pi}{3}\right)^2 = 14.8044\text{ J}$

System: $T_1 = (T_1)_A + (T_1)_B + (T_1)_C = 20.726\text{ J}$

Position 2.

$$\omega_C = 450\text{ rpm} = 15\pi\text{ rad/s}$$

$$\omega_A = \omega_B = 37.5\pi\text{ rad/s}$$

Gear A: $(T_2)_A = \frac{1}{2}(8.64\times10^{-3})(37.5\pi)^2 = 59.957\text{ J}$

PROBLEM 17.11 (Continued)

Gear B: $$(T_2)_B = \frac{1}{2}(8.64\times 10^{-3})(37.5\pi)^2 = 59.957 \text{ J}$$

Gear C: $$(T_2)_C = \frac{1}{2}(270\times 10^{-3})(15\pi)^2 = 299.789 \text{ J}$$

System: $$T_2 = (T_2)_A + (T_2)_B + (T_2)_C = 419.7 \text{ J}$$

Work of couple. $$U_{1\to 2} = M\theta_C = 10\theta_C$$

Principle of work and energy for system.

$$T_1 + U_{1\to 2} = T_2: \quad 20.726 + 10\theta_C = 419.7$$

$$\theta_C = 39.898 \text{ radians}$$

(*a*) Rotation of gear C. $\theta_C = 6.35 \text{ rev}$ ◀

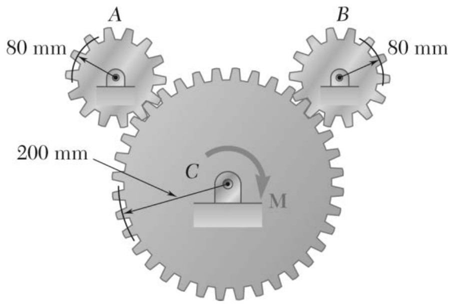
Rotation of gear A. $$\theta_A = (2.5)(39.898) = 99.744 \text{ radians}$$

Principle of work and energy for gear A.

$$(T_1)_A + M_A\theta_A = (T_2)_A: \quad 2.9609 + M_A(99.744) = 59.957$$

$$M_A = 0.57142 \text{ N}\cdot\text{m}$$

(*b*) Tangential force on gear A. $$F_t = \frac{M_A}{r_A} = \frac{0.57142}{0.08}$$ $F_t = 7.14 \text{ N}$ ◀



PROBLEM 17.12

Solve Problem 17.11, assuming that the 10-N · m couple is applied to gear *B*.

PROBLEM 17.11 Each of the gears *A* and *B* has a mass of 2.4 kg and a radius of gyration of 60 mm, while gear *C* has a mass of 12 kg and a radius of gyration of 150 mm. A couple **M** of constant magnitude 10 N · m is applied to gear *C*. Determine (*a*) the number of revolutions of gear *C* required for its angular velocity to increase from 100 to 450 rpm, (*b*) the corresponding tangential force acting on gear *A*.

SOLUTION

Moments of inertia.

Gears *A* and *B*: $I_A = I_B = mk^2 = (2.4)(0.06)^2 = 8.64\times 10^{-3}\,\text{kg}\cdot\text{m}^2$

Gear *C*: $I_C = (12)(0.15)^2 = 270\times 10^{-3}\,\text{kg}\cdot\text{m}^2$

Kinematics. $r_A\omega_A = r_B\omega_B = r_C\omega_C$

$$\omega_A = \omega_B = \frac{200}{80}\omega_C = 2.5\omega_C$$

$$\theta_A = \theta_B = 2.5\theta_C$$

Kinetic energy. $T = \frac{1}{2}I\omega^2$:

Position 1. $\omega_C = 100\text{ rpm} = \frac{10}{3}\pi\text{ rad/s}$

$$\omega_A = \omega_B = 250\text{ rpm} = \frac{25}{3}\pi\text{ rad/s}$$

Gear *A*: $(T_1)_A = \frac{1}{2}(8.64\times 10^{-3})\left(\frac{25\pi}{3}\right)^2 = 2.9609\text{ J}$

Gear *B*: $(T_1)_B = \frac{1}{2}(8.64\times 10^{-3})\left(\frac{25\pi}{3}\right)^2 = 2.9609\text{ J}$

Gear *C*: $(T_1)_C = \frac{1}{2}(270\times 10^{-3})\left(\frac{10\pi}{3}\right)^2 = 14.8044\text{ J}$

System: $T_1 = (T_1)_A + (T_1)_B + (T_1)_C = 20.726\text{ J}$

PROBLEM 17.12 (Continued)

Position 2. $\omega_C = 450 \text{ rpm} = 15\pi \text{ rad/s}$

$\omega_A = \omega_B = 37.5\pi \text{ rad/s}$

Gear A: $(T_2)_A = \frac{1}{2}(8.64\times 10^{-3})(37.5\pi)^2 = 59.957 \text{ J}$

Gear B: $(T_2)_B = \frac{1}{2}(8.64\times 10^{-3})(37.5\pi)^2 = 59.957 \text{ J}$

Gear C: $(T_2)_C = \frac{1}{2}(270\times 10^{-3})(15\pi)^2 = 299.789 \text{ J}$

System: $T_2 = (T_2)_A + (T_2)_B + (T_2)_C = 419.7 \text{ J}$

Work of couple. $U_{1\to 2} = M\theta_B = 10\theta_B$

Principle of work and energy for system.

$$T_1 + U_{1\to 2} = T_2: \quad 20.726 + 10\theta_B = 419.7$$

$$\theta_B = 39.898 \text{ radians}$$

(*a*) Rotation of gear C. $\theta_C = \frac{39.898}{2.5} = 15.959 \text{ radians}$ $\theta_C = 2.54 \text{ rev}$ ◀

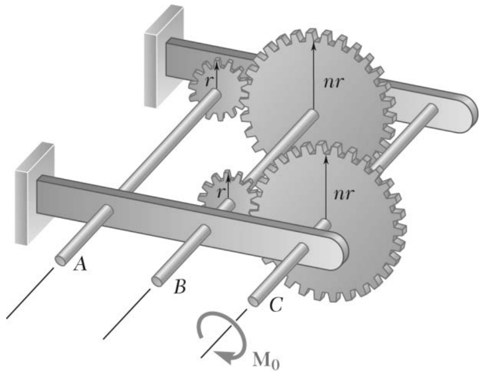
Rotation of gear A. $\theta_A = \theta_B = 39.898 \text{ radians}$

Principle of work and energy for gear A.

$$(T_1)_A + M_A\theta_A = (T_2)_A: \quad 2.9609 + M_A(39.898) = 59.957$$

$$M_A = 1.4285 \text{ N}\cdot\text{m}$$

(*b*) Tangential force on gear A. $F_t = \frac{M_A}{r_A} = \frac{1.4285}{0.08}$ $F_t = 17.86 \text{ N}$ ◀



PROBLEM 17.13

The gear train shown consists of four gears of the same thickness and of the same material; two gears are of radius r, and the other two are of radius nr. The system is at rest when the couple $\mathbf{M}_0$ is applied to shaft C. Denoting by I_0 the moment of inertia of a gear of radius r, determine the angular velocity of shaft A if the couple $\mathbf{M}_0$ is applied for one revolution of shaft C.

SOLUTION

Mass and moment of inertia:

For a disk of radius r and thickness t: $m = \rho(\pi r^2)t = \rho\pi t r^2$

$$\bar{I}_0 = \frac{1}{2}mr^2 = \frac{1}{2}(\rho\pi t r^2)r^2 = \frac{1}{2}\rho\pi t r^4$$

For a disk of radius nr and thickness t, $\bar{I} = \frac{1}{2}\rho\pi t(nr)^4 \qquad \bar{I} = n^4\bar{I}_0$

Kinematics: If for shaft A we have $\omega_A \circlearrowright$

Then, for shaft B we have $\omega_B = \omega_A/n \circlearrowleft$

And, for shaft C we have $\omega_C = \omega_A/n^2 \circlearrowright$

Principle of work-energy:

Couple M_0 applied to shaft C for one revolution. $\theta = 2\pi$ radians, $T_1 = 0$,

$$U_{1-2} = M_0\theta = M_0(2\pi \text{ radians}) = 2\pi M_0$$

$$T_2 = \frac{1}{2}(\bar{I}_{\text{shaft } A})w_A^2 + \frac{1}{2}(\bar{I}_{\text{shaft } B})\omega_B^2 + \frac{1}{2}(\bar{I}_{\text{shaft } C})\omega_C^2$$

$$= \frac{1}{2}\bar{I}_0 w_A^2 + \frac{1}{2}(\bar{I}_0 + n^4\bar{I}_0)\left(\frac{\omega_A}{n}\right)^2 + \frac{1}{2}(n^4\bar{I}_0)\left(\frac{\omega_A}{n^2}\right)^2$$

$$= \frac{1}{2}\bar{I}_0\omega_A^2\left(n^2 + 2 + \frac{1}{n^2}\right)$$

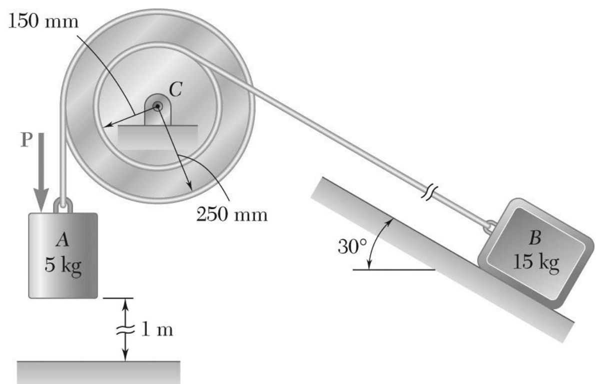
$$= \frac{1}{2}\bar{I}_0\omega_A^2\left(n + \frac{1}{n}\right)^2$$

$$T_1 + U_{1-2} = T_2: \quad 0 + 2\pi M_0 = \frac{1}{2}\bar{I}_0\omega_A^2\left(n + \frac{1}{n}\right)^2$$

Angular velocity. $\omega_A^2 = \dfrac{4\pi M_0}{\bar{I}_0}\dfrac{1}{\left(n + \frac{1}{n}\right)^2}$

$$\omega_A = \frac{2n}{n^2 + 1}\sqrt{\frac{\pi M_0}{\bar{I}_0}} \blacktriangleleft$$

PROBLEM 17.14



The double pulley shown has a mass of 15 kg and a centroidal radius of gyration of 160 mm. Cylinder *A* and block *B* are attached to cords that are wrapped on the pulleys as shown. The coefficient of kinetic friction between block *B* and the surface is 0.2. Knowing that the system is at rest in the position shown when a constant force $P = 200$ N is applied to cylinder *A*, determine (*a*) the velocity of cylinder *A* as it strikes the ground, (*b*) the total distance that block *B* moves before coming to rest.

SOLUTION

Kinematics. Let r_A be the radius of the outer pulley and r_B that of the inner pulley.

$$v_A = r_A\omega_C \qquad v_B = r_B\omega_C = \frac{r_B}{r_A}v_A$$

$$s_A = r_A\theta_C \qquad s_B = \frac{r_B}{r_A}s_A$$

Use the principle of work and energy with position 1 being the initial rest position and position 2 being when cylinder *A* strikes the ground.

$$T_1 + U_{1\to 2} = T_2:$$

where $$T_1 = 0$$

and $$T_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}\bar{I}_C\omega_C^2$$

with $m_A = 5\text{ kg}, \quad m_B = 15\text{ kg}, \quad \bar{I}_C = m_C\bar{k}_C^2 = (15\text{ kg})(0.160\text{ m})^2 = 0.384\text{ kg}\cdot\text{m}^2$

$$\begin{aligned}
T_2 &= \frac{1}{2}\left[m_A + \frac{m_B r_B^2}{r_A^2} + \frac{\bar{I}_C}{r_A^2}\right]v_A^2 \\
&= \frac{1}{2}\left[5\text{ kg} + \frac{(15\text{ kg})(0.150\text{ m})^2}{(0.250\text{ m})^2} + \frac{0.384\text{ kg}\cdot\text{m}^2}{(0.250\text{ m})^2}\right]v_A^2 \\
&= (8.272\text{ kg})v_A^2
\end{aligned}$$

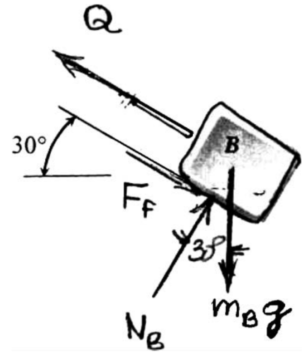
Principle of work and energy applied to the system consisting of blocks *A* and *B* and the double pulley *C*.

Work. $$U_{1\to 2} = Ps_A + m_A g\, s_A - F_F\, s_B - m_B g\, s_B \sin 30°$$

where $$s_A = 1\text{ m}$$

and $$s_B = \frac{r_B}{r_A} s_A = \frac{0.150 \text{ m}}{0.250 \text{ m}}(1 \text{ m}) = 0.6 \text{ m}$$

To find F_f use the free body diagram of block *B*.



$$\measuredangle 60° \; \Sigma F = 0: \quad N_B - m_B g \cos 30° = 0$$

$$N_B = m_B g \cos 30° = (15 \text{ kg})(9.81 \text{ m/s})\cos 30° = 127.44 \text{ N}$$

$$F_f = \mu_k N_B = (0.2)(127.44 \text{ N}) = 25.487 \text{ N}$$

$$U_{1\to 2} = (200 \text{ N})(1 \text{ m}) + (5 \text{ kg})(9.81 \text{ m/s}^2)(1 \text{ m}) - (25.487 \text{ N})(0.6 \text{ m}) - (15 \text{ kg})(9.81 \text{ m/s}^2)(0.6 \text{ m})\sin 30° = 189.613 \text{ J}$$

Work-energy: $0 + 189.613 \text{ J} = (8.272 \text{ kg})v_A^2$

(*a*) *Velocity of A.* $v_A = 4.7877 \text{ m/s}$ $\mathbf{v}_A = 4.79 \text{ m/s} \uparrow$ ◀

when the cylinder strikes the ground,

$$v_B = \frac{r_B}{r_A} v_A = \frac{0.150 \text{ m}}{0.250 \text{ m}}(4.7877 \text{ m/s}) = 2.8726 \text{ m/s}$$

$$\omega_C = \frac{v_A}{r_A} = \frac{4.7877 \text{ m/s}}{0.250 \text{ m}} = 19.1508 \text{ rad/s}$$

After the cylinder strikes the ground use the principle of work and energy applied to a system consisting of block *B* and double pulley *C*.

Let T_3 be its kinetic energy when *A* strikes the ground.

$$T_3 = \frac{1}{2} m_B v_B^2 + \frac{1}{2} \bar{I}_C \omega_C^2 = \frac{1}{2}(15 \text{ kg})(2.8726 \text{ m/s})^2 + \frac{1}{2}(0.384 \text{ kg} \cdot \text{m}^2)(19.1508 \text{ rad/s})^2 = 132.305 \text{ J}$$

When the system comes to rest, $T_4 = 0$

$$U_{3\to 4} = -(25.487 \text{ N})s'_B - (15 \text{ kg})(9.81 \text{ m/s}^2)(s'_B \sin 30°) = -(99.062 \text{ N})s'_B$$

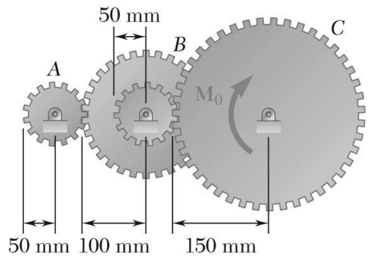
where s'_B is the additional travel of block *B*.

$$T_3 + U_{3\to 4} = T_4: \quad 132.305 \text{ J} - (99.062 \text{ N})s'_B = 0$$

$$s'_B = 1.3356 \text{ m}$$

(*b*) *Total distance:* $s_B + s'_B = 1.936 \text{ m}$ ◀

PROBLEM 17.15



Gear *A* has a mass of 1 kg and a radius of gyration of 30 mm; gear *B* has a mass of 4 kg and a radius of gyration of 75 mm; gear *C* has a mass of 9 kg and a radius of gyration of 100 mm. The system is at rest when a couple $\mathbf{M}_0$ of constant magnitude 4 N · m is applied to gear *C*. Assuming that no slipping occurs between the gears, determine the number of revolutions required for disk *A* to reach an angular velocity of 300 rpm.

SOLUTION

Moments of inertia: $\bar{I} = mk^2$

Gear *A*: $I_A = (1\text{ kg})(0.030\text{ m})^2 = 0.9\times10^{-3}\text{ kg}\cdot\text{m}^2$

Gear *B*: $I_B = (4\text{ kg})(0.075\text{ m})^2 = 22.5\times10^{-3}\text{ kg}\cdot\text{m}^2$

Gear *C*: $I_C = (9\text{ kg})(0.100\text{ m})^2 = 90\times10^{-3}\text{ kg}\cdot\text{m}^2$

Let r_A be the radius of gear *A*, r_1 the outer radius of gear *B*, r_2 the inner radius of gear *B*, and r_C the radius of gear *C*.

$$r_A = 50\text{ mm},\quad r_1 = 100\text{ mm},\quad r_2 = 50\text{ mm},\quad r_C = 150\text{ mm}$$

At the contact point between gears *A* and *B*,

$$r_1\omega_B = r_A\omega_A:\quad \omega_B = \frac{r_A}{r_1}\omega_A = 0.5\omega_A$$

At the contact point between gear *B* and *C*.

$$r_C\omega_C = r_2\omega_B:\quad \omega_C = \frac{r_2}{r_C}\omega_B = 0.33333\omega_B$$

$$\omega_C = 0.16667\omega_A$$

Kinetic energy:
$$T = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2 + \frac{1}{2}I_C\omega_C^2$$

$$T = \frac{1}{2}[0.9\times10^{-3}\omega_A^2 + (22.5\times10^{-3})(0.5\omega_A)^2 + (90\times10^{-3})(0.16667\omega_A)^2]$$

$$= (4.5125\times10^{-3}\text{ kg}\cdot\text{m}^2)\omega_A^2$$

PROBLEM 17.15 (Continued)

Use the principle of work and energy applied to the system of all three gears with position 1 being the initial rest position and position 2 being when $\omega_A = 300$ rpm.

$$\omega_A = \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{min}}{60 \text{ s}} \cdot 300 \frac{\text{rev}}{\text{min}} = 31.416 \text{ rad/s}$$

$$T_1 = 0$$

$$T_2 = (4.5125 \times 10^{-3} \text{ kg} \cdot \text{m}^2)(31.416 \text{ rad/s})^2 = 4.4565 \text{ J}$$

$$U_{1\to 2} = M\theta_C = (4 \text{ N} \cdot \text{m})\theta_C$$

Principle of work and energy.

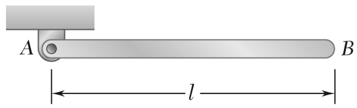
$$T_1 + U_{1\to 2} = T_2: \quad 0 + 4.4565 \text{ J} = 4(\text{N} \cdot \text{m})\theta_C$$

$$\theta_C = 1.11413 \text{ rad}$$

$$\theta_A = \frac{\theta_C}{0.16667} = 6\theta_C = 6.6848 \text{ rad}$$

$$\theta_A = \frac{6.6848 \text{ rad}}{2\pi \text{ rad/rev}}$$

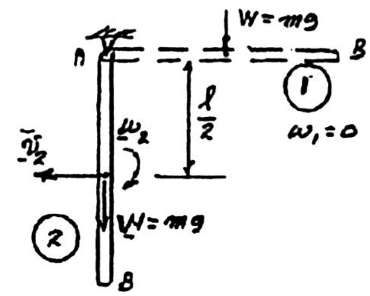
$\theta_A = 1.063$ rev ◀



PROBLEM 17.16

A slender rod of length l and weight W is pivoted at one end as shown. It is released from rest in a horizontal position and swings freely. (*a*) Determine the angular velocity of the rod as it passes through a vertical position and determine the corresponding reaction at the pivot, (*b*) Solve part *a* for $W = 1.8$ lb and $l = 3$ ft.

SOLUTION



Position 1:

$$v_1 = 0$$
$$\omega_1 = 0$$
$$T_1 = 0$$
$$\bar{v}_2 = \frac{l}{2}\omega_2$$

Position 2:

$$T_2 = \frac{1}{2}m\bar{r}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}m\left(\frac{l}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}ml^2\right)\omega_2^2$$
$$T_2 = \frac{1}{6}ml^2\omega_2^2$$

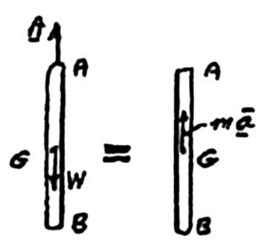
Work: $$U_{1\to 2} = mg\frac{l}{2}$$

Principle of work and energy: $$T_1 + U_{1\to 2} = T_2$$
$$0 + mg\frac{l}{2} = \frac{1}{6}ml^2\omega_2^2$$

(*a*) Expressions for angular velocity and reactions.

$$\omega_2^2 = \frac{3g}{l} \qquad \boldsymbol{\omega}_2 = \sqrt{\frac{3g}{l}} \circlearrowright \blacktriangleleft$$

$$\bar{a} = \frac{l}{2}\omega_2^2 = \frac{l}{2}\cdot\frac{3g}{l} = \frac{3}{2}g$$



$+\uparrow \Sigma F = \Sigma(F)_{\text{eff}}$: $\quad A - W = m\bar{a}$

$$A - mg = m\frac{3}{2}g$$
$$A = \frac{5}{2}mg \qquad \mathbf{A} = \frac{5}{2}W \uparrow \blacktriangleleft$$

PROBLEM 17.16 (Continued)

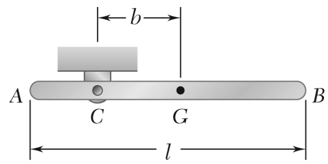
(*b*) Application of data:

$$W = 1.8 \text{ lb}, \quad l = 3 \text{ ft}$$

$$\omega_2^2 = \frac{3g}{l} = \frac{3g}{3} = 32.2 \text{ rad}^2/\text{s}^2 \qquad \boldsymbol{\omega}_2 = 5.67 \text{ rad/s} \circlearrowright \blacktriangleleft$$

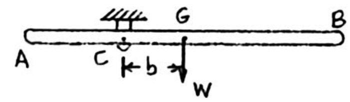
$$A = \frac{5}{2}W = \frac{5}{2}(1.8 \text{ lb}) \qquad \mathbf{A} = 4.5 \text{ lb} \uparrow \blacktriangleleft$$

PROBLEM 17.17



A slender rod of length l is pivoted about a Point C located at a distance b from its center G. It is released from rest in a horizontal position and swings freely. Determine (*a*) the distance b for which the angular velocity of the rod as it passes through a vertical position is maximum, (*b*) the corresponding values of its angular velocity and of the reaction at C.

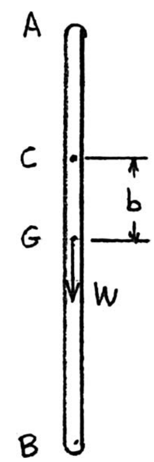
SOLUTION



Position 1. $\bar{v} = 0, \quad \omega = 0 \quad T_1 = 0$

Elevation: $h = 0 \quad V_1 = mgh = 0$

Position 2. $\bar{v}_2 = b\omega_2$



$$I = \frac{1}{12}ml^2$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m\left(b^2 + \frac{1}{12}l^2\right)\omega_2^2$$

Elevation: $h = -b \quad V_2 = -mgb$

Principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \frac{1}{2}m\left(b^2 + \frac{1}{12}l^2\right)\omega_2^2 - mgb$$

$$\omega_2^2 = \frac{2gb}{b^2 + \frac{1}{12}l^2}$$

(*a*) Value of b for maximum ω_2.

$$\frac{d}{db}\left(\frac{b}{b^2 + \frac{1}{12}l^2}\right) = \frac{\left(b^2 + \frac{1}{12}l^2\right) - b(2b)}{\left(b^2 + \frac{1}{12}l^2\right)^2} = 0 \qquad b^2 = \frac{1}{12}l^2$$

$$b = \frac{l}{\sqrt{12}} \blacktriangleleft$$

(*b*) Angular velocity.

$$\omega_2^2 = \frac{2g\frac{l}{\sqrt{12}}}{\frac{l^2}{12} + \frac{l^2}{12}}$$

$$= \sqrt{12}\frac{g}{l}$$

$$\omega_2 = 12^{1/4}\sqrt{\frac{g}{l}}$$

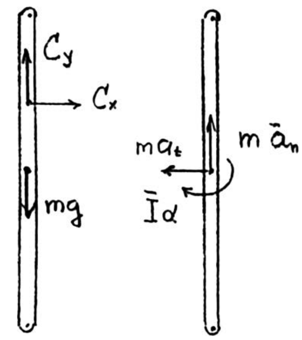
$$\omega_2 = 1.861\sqrt{\frac{g}{l}} \blacktriangleleft$$

Reaction at C.

$$a_n = b\omega_2^2$$

$$= \frac{l}{\sqrt{12}}\sqrt{12}\,\frac{g}{l}$$

$$= g$$



$$+\uparrow \Sigma F_y = ma_n: \quad C_y - mg = mg$$

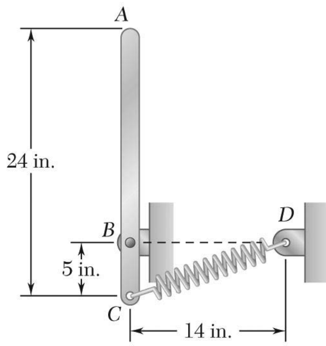
$$C_y = 2mg$$

$$+\circlearrowright \Sigma M_C = mba_t + \bar{I}\alpha: \quad 0 = (mb^2 + \bar{I})\alpha$$

$$\alpha = 0, \quad a_t = 0$$

$$\xrightarrow{+} \Sigma F_x = ma_t: \quad C_x = -ma_t = 0$$

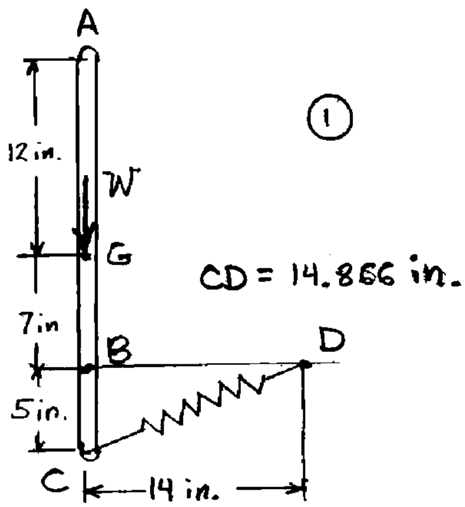
$\mathbf{C} = 2mg \uparrow$ ◀

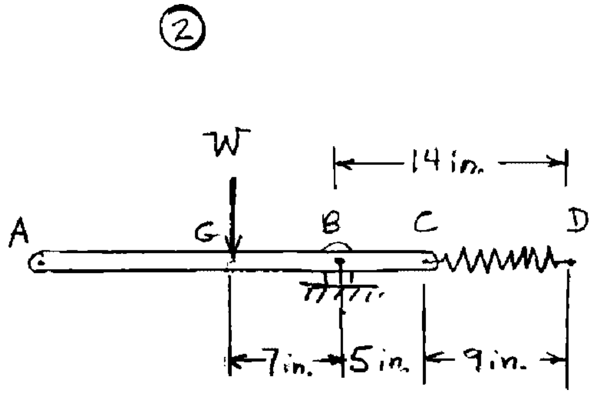


PROBLEM 17.18

A slender 9 lb rod can rotate in a vertical plane about a pivot at *B*. A spring of constant $k = 30$ lb/ft and of unstretched length 6 in. is attached to the rod as shown. Knowing that the rod is released from rest in the position shown, determine its angular velocity after it has rotated through 90°.

SOLUTION





Position 1:

Spring: $x_1 = CD - (\overbrace{6\text{ in.}}^{\text{Unstretched Length}}) = 14.866 - 6 = 8.8661\text{ in.} = 0.73884\text{ ft}$

$$V_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(30\text{ lb/ft})(0.73884)^2 = 8.1882\text{ lb}\cdot\text{ft}$$

Gravity: $V_g = Wh = (9\text{ lb})\left(\frac{7}{12}\text{ft}\right) = 5.25\text{ lb}\cdot\text{ft}$

$$V_1 = V_e + V_g = 8.1882\text{ lb}\cdot\text{ft} + 5.25\text{ lb}\cdot\text{ft} = 13.438\text{ lb}\cdot\text{ft}$$

Kinetic energy: $T_1 = 0$

Position 2:

Spring: $x_2 = 9\text{ in.} - 6\text{ in.} = 3\text{ in.} = 0.25\text{ ft}$

$$V_e = \frac{1}{2}kx_2^2 = \frac{1}{2}(30\text{ lb/ft})(0.25\text{ ft})^2 = 0.9375\text{ lb}\cdot\text{ft}$$

PROBLEM 17.18 (Continued)

Gravity: $V_g = Wh = 0$

$$V_2 = V_e + V_g$$
$$= 0.9375 \text{ lb}\cdot\text{ft}$$

Kinetic energy: $\bar{v}_2 = r\omega_2 = \left(\frac{7}{12}\text{ ft}\right)\omega_2$

$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}\left(\frac{9 \text{ lb}}{32.2}\right)(2 \text{ ft})^2 = 0.093168 \text{ slug}\cdot\text{ft}^2$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

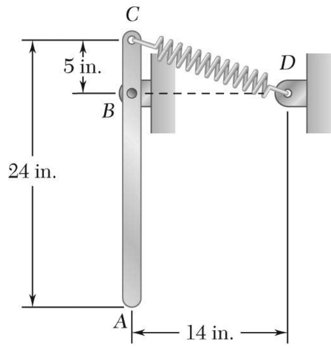
$$= \frac{1}{2}\left(\frac{9 \text{ lb}}{32.2}\right)\left(\left(\frac{7}{12}\text{ ft}\right)\omega_2\right)^2 + \frac{1}{2}(0.093168)\omega_2^2$$

$$T_2 = 0.094138\omega_2^2$$

Conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 13.438 = 0.094138\,\omega_2^2 + 0.9375$$
$$\omega_2^2 = 132.79$$
$$\omega_2 = 11.524 \text{ rad/s}$$

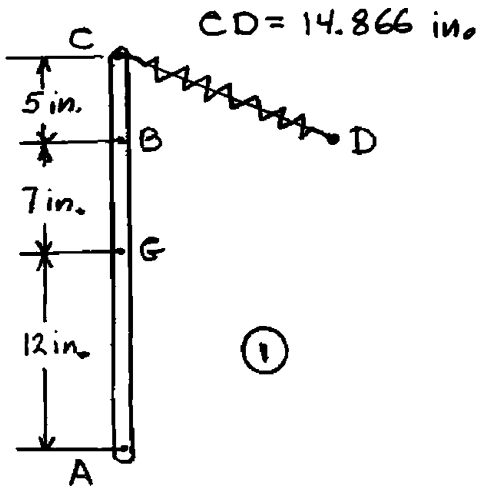
$\boldsymbol{\omega}_2 = 11.52 \text{ rad/s}$ ↺ ◀

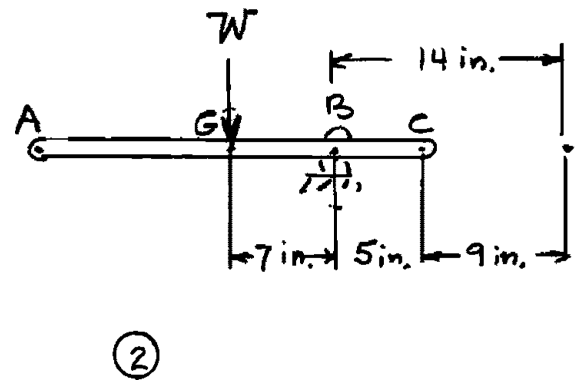


PROBLEM 17.19

A slender 9 lb rod can rotate in a vertical plane about a pivot at *B*. A spring of constant $k = 30$ lb/ft and of unstretched length 6 in. is attached to the rod as shown. Knowing that the rod is released from rest in the position shown, determine its angular velocity after it has rotated through 90°.

SOLUTION





Position 1: $CD = \sqrt{14^2 + 5^2} = 14.866 \text{ in.}$

Spring: $x_1 = CD - (\underbrace{6 \text{ in.}}_{\text{Unstretched Length}}) = 14.866 - 6 = 8.8661 \text{ in.} = 0.73884 \text{ ft}$

$$V_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(30 \text{ lb/ft})(0.73884)^2 = 8.1882 \text{ lb} \cdot \text{ft}$$

Gravity: $V_g = Wh = 9 \text{ lb} = \left(-\frac{7}{12}\text{ft}\right) = -5.25 \text{ lb} \cdot \text{ft}$

$$V_1 = V_e + V_g = 8.1882 \text{ lb} \cdot \text{ft} - 5.25 \text{ lb} \cdot \text{ft} = 2.9382 \text{ lb} \cdot \text{ft}$$

Kinetic energy: $T_1 = 0$

Position 2:

Spring: $x_2 = 9 \text{ in.} - 6 \text{ in.} = 3 \text{ in.} = 0.25 \text{ ft}$

$$V_e = \frac{1}{2}kx_2^2 = \frac{1}{2}(30 \text{ lb/ft})(0.25 \text{ ft})^2 = 0.9375 \text{ lb} \cdot \text{ft}$$

PROBLEM 17.19 (Continued)

Gravity: $V_g = Wh = 0$

$$V_2 = V_e + V_g$$
$$= 0.9375 \text{ lb} \cdot \text{ft}$$

Kinetic energy: $\bar{v}_2 = r\omega_2 = \left(\frac{7}{12}\text{ ft}\right)\omega_2$

$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}\left(\frac{9 \text{ lb}}{32.2}\right)(2 \text{ ft})^2 = 0.093168 \text{ slug} \cdot \text{ft}^2$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\,\omega_2^2$$

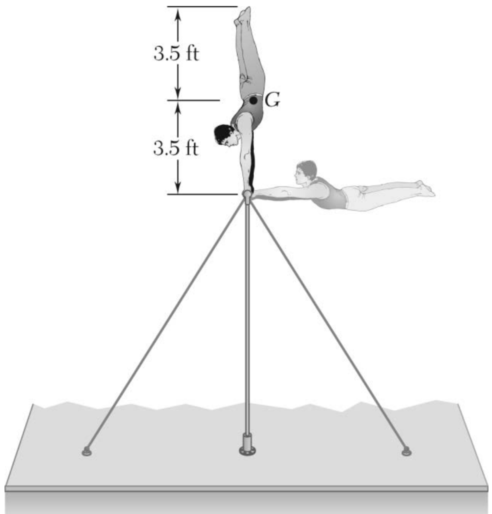
$$= \frac{1}{2}\left(\frac{9 \text{ lb}}{32.2}\right)\left(\left(\frac{7}{12}\text{ ft}\right)\omega_2\right)^2 + \frac{1}{2}(0.093168)\omega_2^2$$

$$T_2 = 0.094138\omega_2^2$$

Conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 2.9382 = 0.094138\omega_2^2 + 0.9375$$
$$\omega_2^2 = 21.253$$
$$\omega_2 = 4.6101 \text{ rad/s}$$

$\boldsymbol{\omega}_2 = 4.61$ rad/s ↻ ◀



PROBLEM 17.20

A 160-lb gymnast is executing a series of full-circle swings on the horizontal bar. In the position shown he has a small and negligible clockwise angular velocity and will maintain his body straight and rigid as he swings downward. Assuming that during the swing the centroidal radius of gyration of his body is 1.5 ft, determine his angular velocity and the force exerted on his hands after he has rotated through (*a*) 90°, (*b*) 180°.

SOLUTION

Position 1. (*Directly above the bar*).

Elevation: $h_1 = 3.5 \text{ ft}$

Potential energy: $V_1 = Wh_1 = (160 \text{ lb})(3.5 \text{ ft}) = 560 \text{ ft} \cdot \text{lb}$

Speeds: $\omega_1 = 0, \quad \bar{v}_1 = 0$

Kinetic energy: $T_1 = 0$

(*a*) *Position 2*. (*Body at level of bar after rotating* 90°).

Elevation: $h_2 = 0.$

Potential energy: $V_2 = 0$

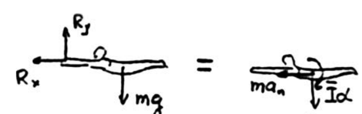
Speeds: $\bar{v}_2 = 3.5\omega_2.$

Kinetic energy:

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}mk^2\omega_2^2$$

$$T_2 = \frac{1}{2}\left(\frac{160}{32.2}\right)(3.5\omega_2)^2 + \frac{1}{2}\left(\frac{160}{32.2}\right)(1.5)^2\omega_2^2$$

$$= 36.025\omega_2^2$$



Principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 560 = 36.025\omega_2^2$$

$$\omega_2^2 = 15.545$$

$\boldsymbol{\omega}_2 = 3.94 \text{ rad/s}$ ↻ ◀

PROBLEM 17.20 (Continued)

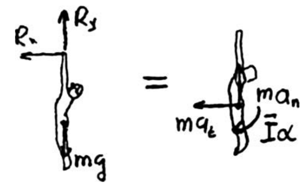
Kinematics: $\bar{a}_t = 3.5\alpha$

$$\bar{a}_n = 3.5\omega_2^2 = (3.5)(15.545) = 54.407 \text{ ft/s}^2 \longleftarrow$$

$$+\circlearrowright \Sigma M_0 = \Sigma(M_0)_{\text{eff}}: \quad (3.5)(160) = \left(\frac{160}{32.2}\right)(3.5)(3.5\alpha) + \left(\frac{160}{32.2}\right)(1.5)^2\alpha$$

$$\alpha = 7.7724 \text{ rad/s}^2 \qquad \bar{a}_t = 27.203 \text{ ft/s}^2 \downarrow$$

$$\overset{+}{\longleftarrow} \Sigma F_x = ma_n: \qquad R_x = \left(\frac{160}{32.2}\right)(54.407) = 270.35 \text{ lb} \longleftarrow$$



$$+\uparrow \Sigma F_y = -ma_t: \qquad R_y - 160 = -\left(\frac{160}{32.2}\right)(27.203)\uparrow$$

$$R_y = 24.83 \text{ lb}\uparrow$$

R = 271 lb ⦨ 5.25° ◀

(*b*) *Position 3*. (*Directly below bar after rotating* 180°).

Elevation: $h_3 = -3.5$ ft.

Potential energy: $V_3 = Wh_3 = (160)(-3.5) = -560 \text{ ft}\cdot\text{lb}$

Speeds: $\bar{v}_3 = 3.5\omega_3.$

Kinetic energy: $T_3 = 36.025\omega_3^2$

Principle of conservation of energy.

$$T_1 + V_1 = T_3 + V_3: \quad 0 + 560 = 36.025\omega_3^2 - 560$$

$$\omega_3^2 = 31.09$$

$\boldsymbol{\omega}_3 = 5.58$ rad/s ↻ ◀

Kinematics: $a_n = (3.5)(31.09) = 108.81 \text{ ft/s}^2\uparrow$

From $\Sigma M_0 = \Sigma(M_0)_{\text{eff}}$ and $\Sigma F_x = 0,$

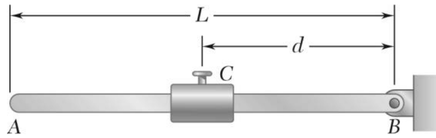
$$\alpha = 0, \qquad a_t = 0 \qquad R_x = 0$$

$$+\uparrow \Sigma F_y = ma_n: \quad R_y - 160 = \left(\frac{160}{32.2}\right)(108.81)$$

$$R_y = 700.62 \text{ lb}$$

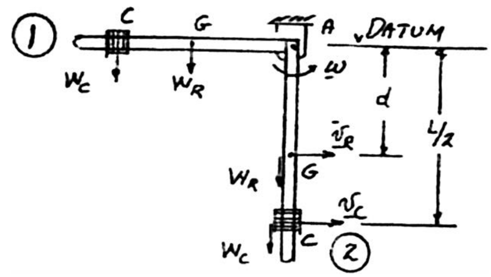
R = 701 lb↑ ◀

PROBLEM 17.21



A collar with a mass of 1 kg is rigidly attached at a distance d = 300 mm from the end of a uniform slender rod AB. The rod has a mass of 3 kg and is of length L = 600 mm. Knowing that the rod is released from rest in the position shown, determine the angular velocity of the rod after it has rotated through 90°.

SOLUTION



Kinematics.

Rod $$v_R = \frac{L}{2}\omega$$

Collar $$v_C = d\omega$$

Position 1. $$\omega = 0$$
$$T_1 = 0 \quad V_1 = 0$$

Position 2. $$T_2 = \frac{1}{2}m_R\bar{v}_R^2 + \frac{1}{2}\bar{I}_R\omega^2 + \frac{1}{2}m_C v_C^2$$
$$= \frac{1}{2}m_R\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}m_R L^2\right)\omega^2 + \frac{1}{2}m_C d^2\omega^2$$
$$= \frac{1}{6}m_R L^2\omega^2 + \frac{1}{2}m_C d^2\omega^2$$
$$V_2 = -W_C d - W_R\frac{L}{2}$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \left(\frac{1}{6}m_R L^2 + \frac{1}{2}m_C d^2\right)\omega^2 - W_C d - W_R\frac{L}{2}$$

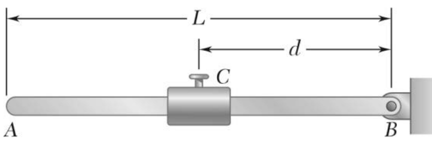
$$\omega^2 = \frac{3(2W_C d + W_C L)}{3m_C d^2 + m_R L^2} = \frac{3g(2m_C d + m_R L)}{3m_C d^2 + m_R L^2} \quad (1)$$

Data: $$m_C = 1\text{ kg}, \quad d = 0.3\text{ m}, \quad m_R = 3\text{ kg}, \quad L = 0.6\text{ m}$$

From Eq. (1), $$\omega^2 = 3(9.81)\left[\frac{(2)(1)(0.3) + 3(0.6)}{3(1)(0.3)^2 + 3(0.6)^2}\right]$$
$$= 52.32$$

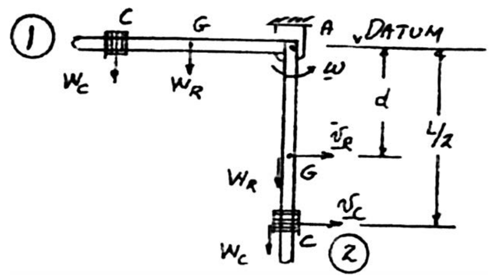
$\bar{\omega} = 7.23$ rad/s ↺ ◀

PROBLEM 17.22



A collar with a mass of 1 kg is rigidly attached to a slender rod AB of mass 3 kg and length $L = 600$ mm. The rod is released from rest in the position shown. Determine the distance d for which the angular velocity of the rod is maximum after it has rotated 90°.

SOLUTION



Kinematics.

Rod $\qquad v_R = \frac{L}{2}\omega$

Collar $\qquad v_C = d\omega$

Position 1. $\qquad \omega = 0$

$$T_1 = 0 \quad V_1 = 0$$

Position 2.

$$T_2 = \frac{1}{2}m_R\bar{v}_R^2 + \frac{1}{2}\bar{I}_R\omega^2 + \frac{1}{2}m_C v_C^2$$

$$= \frac{1}{2}m_R\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}m_R L^2\right)\omega^2 + \frac{1}{2}m_C d^2\omega^2$$

$$= \frac{1}{6}m_R L^2\omega^2 + \frac{1}{2}m_C d^2\omega^2$$

$$V_2 = -W_C d - W_R\frac{L}{2}$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \left(\frac{1}{6}m_R L^2 + \frac{1}{2}m_C d^2\right)\omega^2 - W_C d - W_R\frac{L}{2}$$

$$\omega^2 = \frac{3(2W_C d + W_C L)}{3m_C d^2 + m_R L^2} = \frac{3g(2m_C d + m_R L)}{3m_C d^2 + m_R L^2} \qquad (1)$$

Let $\qquad x = \frac{d}{L}.$

$$\omega^2 = \frac{3g}{L}\cdot\frac{2x + \frac{m_R}{m_C}}{3x^2 + \frac{m_R}{m_C}}$$

Data: $\qquad m_C = 1\text{ kg}, \quad m_R = 3\text{ kg}$

$$\frac{L\omega^2}{3g} = \frac{2x+3}{3x^2+3}$$

$L\omega^2/3g$ is maximum. Set its derivative with respect to x equal to zero.

$$\frac{d}{dx}\left(\frac{L\omega^2}{3g}\right) = \frac{(3x^2+3)(2)-(2x+3)(6x)}{(3x^2+3)^2} = 0$$

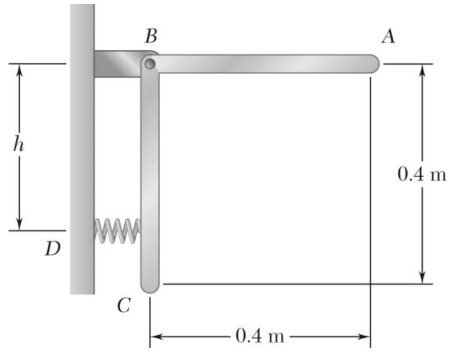
$$-6x^2 - 18x + 6 = 0$$

Solving the quadratic equation

$$x = -3.30 \quad \text{and} \quad x = 0.30278$$

$$\begin{aligned} d &= 0.30278L \\ &= (0.30278)(0.6) \\ &= 0.1817 \text{ m} \end{aligned}$$

$d = 181.7$ mm ◀

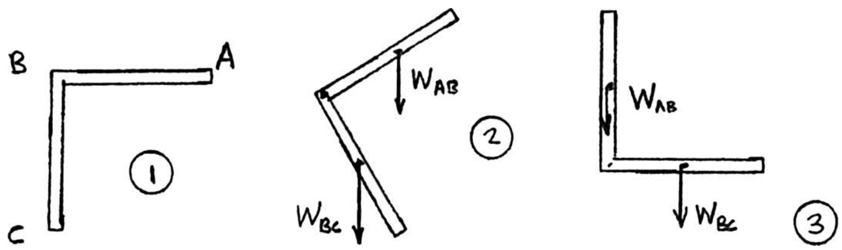


PROBLEM 17.23

Two identical slender rods *AB* and *BC* are welded together to form an L-shaped assembly. The assembly is pressed against a spring at *D* and released from the position shown. Knowing that the maximum angle of rotation of the assembly in its subsequent motion is 90° counterclockwise, determine the magnitude of the angular velocity of the assembly as it passes through the position where rod *AB* forms an angle of 30° with the horizontal.

SOLUTION

Moment of inertia about *B*. $\quad I_B = \frac{1}{3}m_{AB}l^2 + \frac{1}{3}m_{BC}l^2$



Position 2.

$$\theta = 30°$$

$$V_2 = W_{AB}(h_{AB})_2 + W_{BC}(h_{BC})_2$$

$$= W_{AB}\frac{l}{2}\sin 30° + W_{BC}\left(-\frac{l}{2}\cos 30°\right)$$

$$T_2 = \frac{1}{2}I_B\omega_2^2 = \frac{1}{6}(m_{AB} + m_{BC})l^2\omega_2^2$$

Position 3.

$$\theta = 90°$$

$$V_3 = W_{AB}\frac{l}{2} \quad T_3 = 0$$

Conservation of energy.

$$T_2 + V_2 = T_3 + V_3:$$

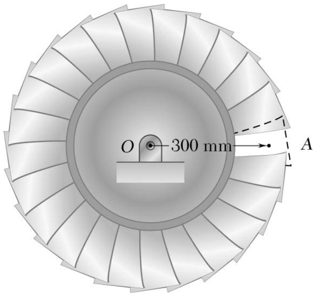
$$\frac{1}{6}(m_{AB} + m_{BC})l^2\omega_2^2 + W_{AB}\frac{l}{2}\sin 30° - W_{BC}\frac{l}{2}\cos 30° = 0 + W_{AB}\frac{l}{2}$$

$$\omega_2^2 = \frac{3}{l}\cdot\frac{W_{AB}(1-\sin 30°) + W_{BC}\cos 30°}{m_{AB} + m_{BC}}$$

$$= \frac{3}{2}\frac{g}{l}[1 - \sin 30° + \cos 30°]$$

$$= 2.049\frac{g}{l} = 2.049\frac{9.81}{0.4} = 50.25$$

$\omega_2 = 7.09$ rad/s ◀



PROBLEM 17.24

The 30-kg turbine disk has a centroidal radius of gyration of 175 mm and is rotating clockwise at a constant rate of 60 rpm when a small blade of weight 0.5 N at Point *A* becomes loose and is thrown off. Neglecting friction, determine the change in the angular velocity of the turbine disk after it has rotated through (*a*) 90°, (*b*) 270°.

SOLUTION

Mass of blade. $m_A = 51 \text{ grams} = 0.051 \text{ kg}$

Weight of blade. $m_A g = (0.051)(9.81) = 0.5 \text{ N}$

Moment of inertia about *O*. $I_O = mk^2 - m_A r_2^2 = 30(0.175)^2 - 51\times 10^{-3}(0.3)^2 = 0.91416 \text{ kg}\cdot\text{m}^2$

Location of mass center for the position shown.

$$(m - m_A)\overline{x} = -m_A r_A \qquad \overline{x} = -\frac{m_A r_A}{m - m_A}$$

Position 1. $\theta = 0°, \quad \omega_1 = 60 \text{ rpm} = 2\pi \text{ rad/s}$

Kinetic energy: $T_1 = \frac{1}{2} I_O \omega_1^2$

Center of gravity lies at the level of Point *O*. $h_1 = 0$

Potential energy: $V_1 = (mg - m_A g)h_1 = 0$

(*a*) *Position 2*. $\theta = 90°$

Kinetic energy: $T_2 = \frac{1}{2} I_O \omega_2^2$

Center of gravity lies a distance $\dfrac{m_A r_A}{m - m_A}$ above Point *O*.

$$h_2 = \frac{m_A r_A}{m - m_A}$$

Potential energy: $V_2 = (mg - m_A g)h_2 = m_A g r_A = (0.5)(0.3) = 0.150 \text{ N}\cdot\text{m}$

Conservation of energy. $T_1 + V_1 = T_2 + V_2$:

$$\frac{1}{2}I_O\omega_1^2 + 0 = \frac{1}{2}I_O\omega_2^2 + V_2$$

$$\omega_2^2 = \omega_1^2 - \frac{2V_2}{I_O} - (2\pi)^2 - \frac{(2)(0.15)}{0.91416} \qquad \omega_2 = 6.257016 \text{ rad/s}$$

$$\Delta\omega = \omega_2 - \omega_1 = 6.257016 - 2\pi = -0.02617 \text{ rad/s}$$

$\Delta\omega = -0.250$ rpm ◀

(*b*) *Position 3*. $\theta = 270°$

Kinetic energy: $T_3 = \frac{1}{2}I_O\omega_3^2$

Center of gravity lies a distance $\frac{m_A r_A}{m - m_A}$ below Point O.

$$h_3 = -\frac{m_A r_A}{m - m_A}$$

Potential energy: $V_3 = (mg - m_A g)h_3 = -m_A g r_A = -(0.5)(0.3) = -0.15 \text{ N}\cdot\text{m}$

Conservation of energy. $T_1 + V_1 = T_3 + V_3$:

$$\frac{1}{2}I_O\omega_1^2 + 0 = \frac{1}{2}I_O\omega_3^2 + V_3$$

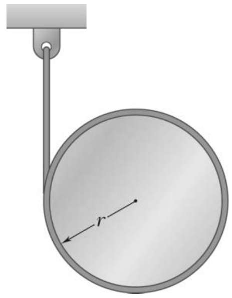
$$\omega_3^2 = \omega_1^2 - \frac{2V_3}{I_O} = (2\pi)^2 - \frac{(2)(-0.15)}{0.91416}$$

$$\omega_3 = 6.309246 \text{ rad/s}$$

$$\Delta\omega = \omega_3 - \omega_1 = 6.309246 - 2\pi = 0.026061 \text{ rad/s}$$

$\Delta\omega = 0.249$ rpm ◀

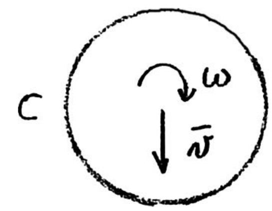
PROBLEM 17.25



A rope is wrapped around a cylinder of radius r and mass m as shown. Knowing that the cylinder is released from rest, determine the velocity of the center of the cylinder after it has moved downward a distance s.

SOLUTION

Point C is the instantaneous center.



$$\bar{v} = r\omega \qquad \omega = \frac{\bar{v}}{r}$$

Position 1. At rest. $\quad T_1 = 0$

Position 2. Cylinder has fallen through distance s.

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\bar{v}}{r}\right)^2$$

$$= \frac{3}{4}m\bar{v}^2$$

Work. $\quad U_{1\to2} = mgs$

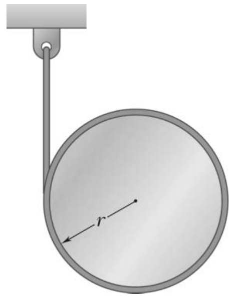
Principle of work and energy.

$$T_1 + U_{1\to2} = T_2: \quad 0 + mgs = \frac{3}{4}m\bar{v}^2$$

$$\bar{v}^2 = \frac{4gs}{3}$$

$$\bar{\mathbf{v}} = \sqrt{\frac{4gs}{3}} \downarrow \blacktriangleleft$$

PROBLEM 17.26

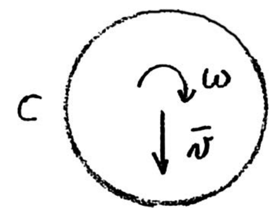


Solve Problem 17.25, assuming that the cylinder is replaced by a thin-walled pipe of radius r and mass m.

PROBLEM 17.25 A rope is wrapped around a cylinder of radius r and mass m as shown. Knowing that the cylinder is released from rest, determine the velocity of the center of the cylinder after it has moved downward a distance s.

SOLUTION

Point C is the instantaneous center.



$$\bar{v} = r\omega \qquad \omega = \frac{\bar{v}}{r}$$

Position 1. At rest. $\quad T_1 = 0$

Position 2. Cylinder has fallen through distance s.

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}(mr^2)\left(\frac{\bar{v}}{r}\right)^2$$

$$= m\bar{v}^2$$

Work. $\quad U_{1\to 2} = mgs$

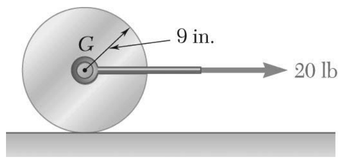
Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2: \quad 0 + mgs = m\bar{v}^2$$

$$\bar{v}^2 = gs$$

$\bar{\mathbf{v}} = \sqrt{gs}\downarrow$ ◀

PROBLEM 17.27



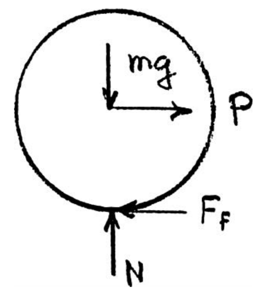
A 45-lb uniform cylindrical roller, initially at rest, is acted upon by a 20-lb force as shown. Knowing that the body rolls without slipping, determine (*a*) the velocity of its center *G* after it has moved 5 ft, (*b*) the friction force required to prevent slipping.

SOLUTION

Since the cylinder rolls without slipping, the point of contact with the ground is the instantaneous center.

Kinematics: $\bar{v} = r\omega$

Position 1. At rest. $T_1 = 0$



Position 2. $s = 5 \text{ ft} \quad v_G = \bar{v} \quad \omega = \dfrac{v_G}{r}$

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv_G^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v_G}{r}\right)^2$$

$$= \frac{3}{4}mv_G^2 = \frac{3}{4}\left(\frac{45}{32.2}\right)v_G^2 = 1.04815v_G^2$$

Work: $U_{1\to 2} = Ps = (20)(5) = 100 \text{ lb}\cdot\text{ft}. \quad F_f$ does no work.

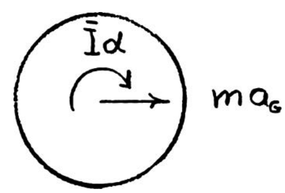
(*a*) Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2: \quad 0 + 100 = 1.0481v_G^2$$

$$v_G^2 = 95.407$$

$\mathbf{v}_G = 9.77 \text{ ft/s} \longrightarrow$ ◀

(*b*) Since the forces are constant, $a_G = \bar{a} = \text{constant}$



$$a_G = \frac{v_G^2}{2s}$$

$$= \frac{95.407}{(2)(5)}$$

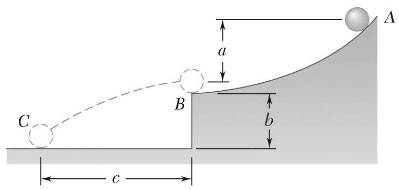
$$= 9.5407 \text{ ft/s}^2$$

$$\overset{+}{\longrightarrow} \Sigma F_x = m\bar{a}: \quad P - F_f = m\bar{a}$$

$$F_f = P - m\bar{a}$$

$$= 20 - \left(\frac{45}{32.2}\right)(9.5407)$$

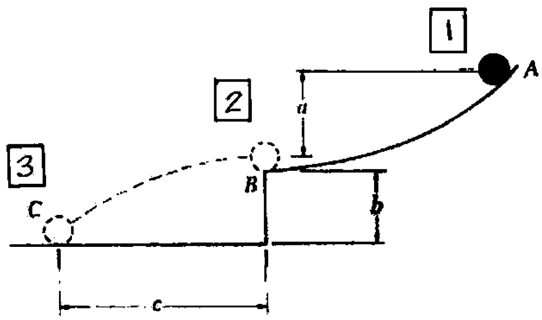
$\mathbf{F}_f = 6.67 \text{ lb} \longleftarrow$ ◀



PROBLEM 17.28

A small sphere of mass m and radius r is released from rest at A and rolls without sliding on the curved surface to Point B where it leaves the surface with a horizontal velocity. Knowing that $a = 1.5$ m and $b = 1.2$ m, determine (a) the speed of the sphere as it strikes the ground at C, (b) the corresponding distance c.

SOLUTION



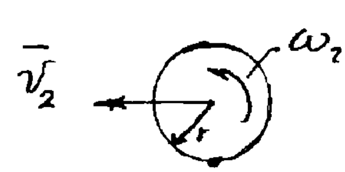
Work: $U_{1\to 2} = mga$

Kinetic energy: $T_1 = 0$

Rolling motion at position 2. $\bar{v}_2 = r\omega$ or $60\omega = \dfrac{v}{r}$

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{7}{10}m\bar{v}^2$$



Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2: \quad 0 + mga = \frac{7}{10}m\bar{v}^2$$

$$\bar{v}^2 = \frac{10ga}{7} = \frac{(10)(9.81\text{ m/s}^2)(1.5\text{ m})}{7} = 21.021\text{ m/s}^2$$

$$\bar{v} = 4.5849\text{ m/s}$$

For path B to C the motion is projectile motion. Let $t = 0$ at Point B. Let $y = 0$ at Point C.

Vertical motion: $v_y = (v_y)_0 - gt = -gt$

$$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$$

At Point C, $0 = b + 0 - \dfrac{1}{2}gt_C^2$

$$t_C = \sqrt{\frac{2b}{g}} = \sqrt{\frac{(2)(1.2\text{ m})}{9.81\text{ m/s}^2}} = 0.49462\text{ s}$$

$$(v_y)_C = -gt_C = -(9.81\text{ m/s}^2)(0.49462\text{ s}) = -4.8522\text{ m/s}$$

PROBLEM 17.28 (Continued)

Horizontal motion: Let the x coordinate point to the left with origin below B.

$$v_x = (v_x)_B = \overline{v} = 4.5849 \text{ m/s}$$

(*a*) *Speed at C.*

$$v_C = \sqrt{(v_x)_C^2 + (v_y)_C^2}$$

$$v_C = \sqrt{(4.5849)^2 + (4.8522)^2}$$

$v_C = 6.68$ m/s ◀

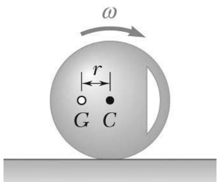
(*b*) *Distance c.*

$$c = v_x t_C$$

$$c = (4.5849 \text{ m/s})(0.49462 \text{ s})$$

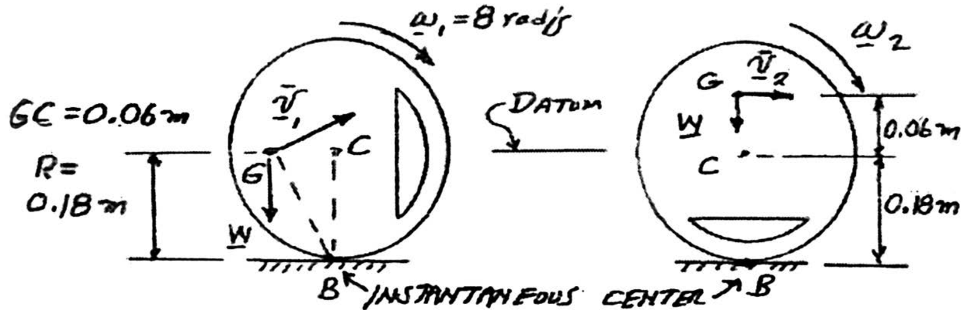
$c = 2.27$ m ◀

PROBLEM 17.29



The mass center G of a 3-kg wheel of radius $R = 180$ mm is located at a distance r = 60 mm from its geometric center C. The centroidal radius of gyration of the wheel is $\bar{k} = 90$ mm. As the wheel rolls without sliding, its angular velocity is observed to vary. Knowing that $\omega = 8$ rad/s in the position shown, determine (*a*) the angular velocity of the wheel when the mass center G is directly above the geometric center C, (*b*) the reaction at the horizontal surface at the same instant.

SOLUTION



$$\bar{v}_1 = (BG)\omega_1$$
$$= \sqrt{(0.18)^2 + (0.06)^2}(8)$$
$$= 8\sqrt{0.036} \text{ m/s}$$
$$\bar{v}_2 = 0.24\omega_2$$
$$m = 3 \text{ kg}$$
$$\bar{k} = 0.09 \text{ m}$$

Position 1.

$$V_1 = 0$$
$$T_1 = \frac{1}{2}m\bar{v}_1^2 + \frac{1}{2}\bar{I}\omega_1^2$$
$$= \frac{1}{2}(3)(8\sqrt{0.036})^2 + \frac{1}{2}(3)(0.09)^2(8)^2$$
$$= 4.2336 \text{ J}$$

Position 2.

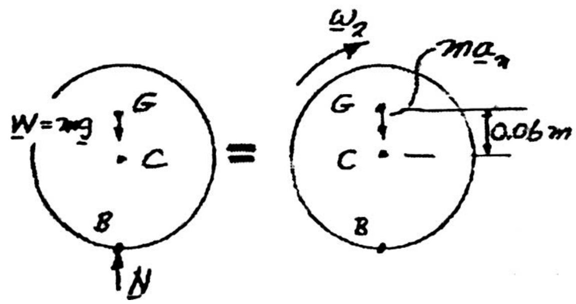
$$V_2 = Wh$$
$$= mgh$$
$$= (3)(9.81)(0.06)$$
$$= 1.7658 \text{ J}$$
$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}(3)(0.24\omega_2)^2 + \frac{1}{2}(3)(0.09)^2\omega_2^2$$
$$= 0.09855\omega_2^2$$

(*a*) Conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$
$$4.2336 \text{ J} + 0 = 0.09855\omega_2^2 + 1.7658 \text{ J}$$
$$\omega_2^2 = 25.041$$
$$\omega_2 = 5.004 \text{ rad/s}$$

$\boldsymbol{\omega}_2 = 5.00$ rad/s ◀

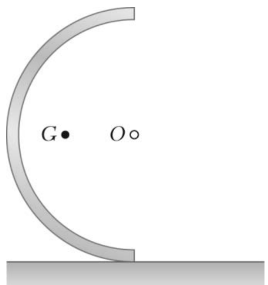
(*b*) Reaction at *B*.



$$ma_n = m(CG)\omega_2^2$$
$$= (3 \text{ kg})(0.06 \text{ m})(5.00 \text{ rad/s})^2$$
$$= 4.5 \text{ N}\downarrow$$

$$+\uparrow \Sigma F_y = ma_y: \quad N - mg = -ma_n$$
$$N - (3)(9.81) = -4.5$$

$\mathbf{N} = 24.9 \text{ N}\uparrow$ ◀

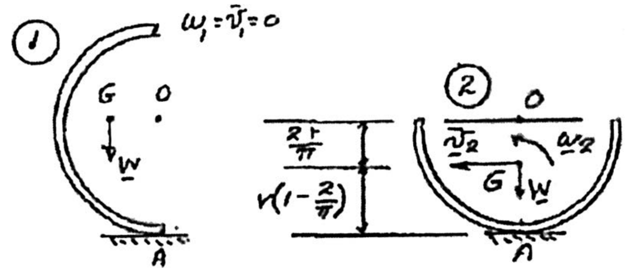


PROBLEM 17.30

A half section of pipe of mass m and radius r is released from rest in the position shown. Knowing that the pipe rolls without sliding, determine (*a*) its angular velocity after it has rolled through 90°, (*b*) the reaction at the horizontal surface at the same instant. [*Hint:* Note that $GO = 2r/\pi$ and that, by the parallel-axis theorem, $\bar{I} = mr^2 - m(GO)^2$.]

SOLUTION

Position 1. $\omega_1 = 0 \quad v_1 = 0 \quad T_1 = 0$



Position 2. Kinematics: $\bar{v}_2 = (AG)\omega_2 = r\left(1 - \frac{2}{\pi}\right)\omega_2$

Moment of inertia: $\bar{I} = mr^2 - m(0.6)^2 = mr^2 - m\left(\frac{2r}{\pi}\right)^2 = mr^2\left(1 - \frac{4}{\pi^2}\right)$

Kinetic energy:

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m\left(1 - \frac{2}{\pi}\right)^2 r^2\omega_2^2 + \frac{1}{2}mr^2\left(1 - \frac{4}{\pi^2}\right)\omega_2^2$$

$$= \frac{1}{2}mr^2\left[\left(1 - \frac{4}{\pi} + \frac{4}{\pi^2}\right) + \left(1 - \frac{4}{\pi^2}\right)\right]$$

$$= \frac{1}{2}mr^2\left(2 - \frac{4}{\pi}\right)$$

Work: $U_{1\to 2} = W(OG) = mg\frac{2r}{\pi} = \frac{2}{\pi}mgr$

Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2$$

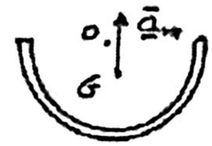
$$0 + mg\frac{2r}{\pi} = \frac{1}{2}mr^2\left(2 - \frac{4}{\pi}\right)\omega_2^2$$

$$\omega_2^2 = \frac{2}{\pi\left(1 - \frac{2}{\pi}\right)} \cdot \frac{g}{r} = 1.7519\frac{g}{r}$$

(*a*) Angular velocity. $\boldsymbol{\omega}_2 = 1.324\sqrt{\dfrac{g}{r}}$ ↻ ◀

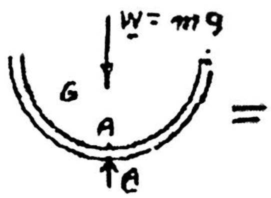
(*b*) Reaction at *A*.

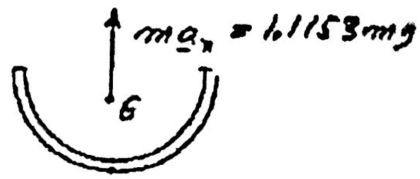
Kinematics: Since *O* moves horizontally, $(a_0)_y = 0$



$$a_n = (0.6)\omega_2^2$$
$$= \frac{2r}{\pi}\left(1.7519\frac{g}{r}\right)$$
$$= 1.1153g \uparrow$$

Kinetics:

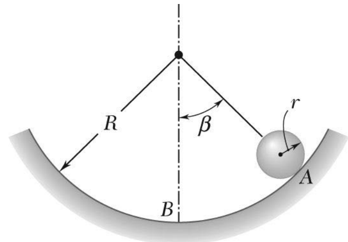




$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad A - mg = 1.1153mg$$

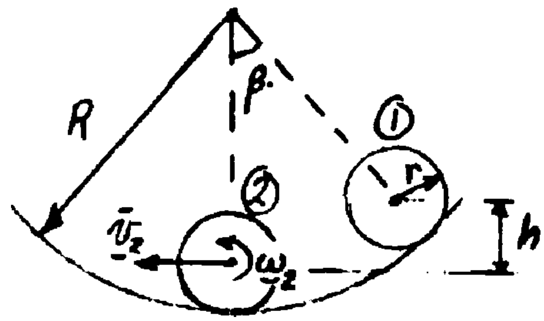
$\mathbf{A} = 2.12mg \uparrow$ ◀

PROBLEM 17.31



A sphere of mass m and radius r rolls without slipping inside a curved surface of radius R. Knowing that the sphere is released from rest in the position shown, derive an expression (a) for the linear velocity of the sphere as it passes through B, (b) for the magnitude of the vertical reaction at that instant.

SOLUTION



Kinematics: The sphere rolls without slipping.

$$\bar{v} = r\omega \quad \omega = \frac{\bar{v}}{r}$$

Kinetic energy.

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{\bar{v}}{r}\right)^2$$

$$T = \frac{7}{10}m\bar{v}^2$$

$$T_1 = 0 \quad T_2 = \frac{7}{10}m\bar{v}_2^2$$

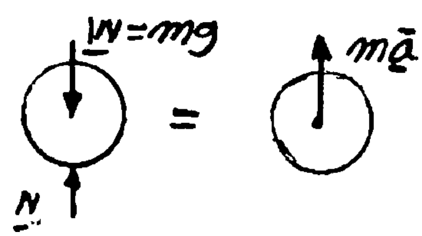
Work. $$U_{1-2} = mgh = mg(R-r)(1-\cos\beta)$$

Principle of work and energy. $T_1 + U_{1-2} = T_2$:

$$0 + mg(R-r)(1-\cos\beta) = \frac{7}{10}m\bar{v}_2^2$$

(a) *Linear velocity at B.* $$\bar{v}_2 = \sqrt{\frac{10}{7}g(R-r)(1-\cos\beta)}$$ ◀

Free body diagram when $\beta = 0$.



$\xrightarrow{+} \Sigma F = ma_t$: $a_t = 0$

$+\circlearrowleft \Sigma M_G = \bar{I}\alpha$: $\alpha = 0$

PROBLEM 17.31 (Continued)

The sphere rolls so that its mass center moves on a circle of radius $\rho = R - r$.

$$\bar{a} = a_n = \frac{\bar{v}_2^2}{R - r} \uparrow$$

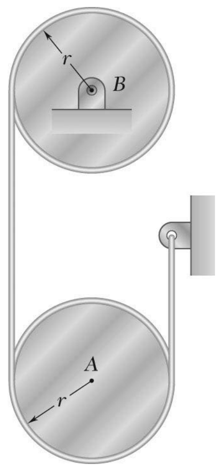
$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad N - mg = m\bar{a}$$

$$N - mg = m\left(\frac{1}{R - r}\right)\left[\frac{10}{7} g(R - r)(1 - \cos\beta)\right]$$

$$N = mg\left[1 + \frac{10}{7}(1 - \cos\beta)\right]$$

(*b*) *Vertical reaction.* $N = \frac{1}{7} mg[17 - 10\cos\beta] \uparrow$ ◀

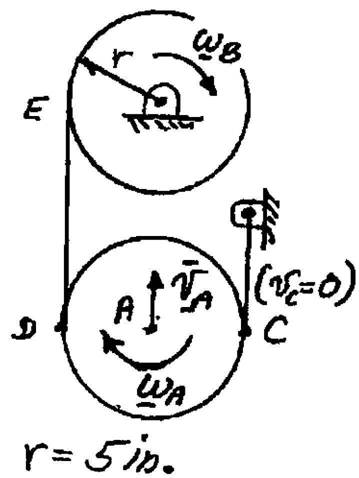
PROBLEM 17.32



Two uniform cylinders, each of weight $W = 14$ lb and radius $r = 5$ in., are connected by a belt as shown. Knowing that at the instant shown the angular velocity of cylinder B is 30 rad/s clockwise, determine (*a*) the distance through which cylinder A will rise before the angular velocity of cylinder B is reduced to 5 rad/s, (*b*) the tension in the portion of belt connecting the two cylinders.

SOLUTION

Kinematics.



$$v_D = v_E = r\omega_B$$

Point C is the instantaneous center of cylinder A.

$$\omega_A = \frac{v_D}{cd} = \frac{r\omega_B}{2r} = \frac{1}{2}\omega_B$$

$$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B$$

$$v_D = 2\bar{v}_A$$

Kinetic energy of the system.

$$T = \frac{1}{2}m\bar{v}_A^2 + \frac{1}{2}\bar{I}\,\omega_A^2 + \frac{1}{2}\bar{I}\,\omega_B^2$$

$$T = \frac{1}{2}m\left(\frac{r}{2}\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{1}{2}\omega_B\right)^2 + \left(\frac{1}{2}mr^2\right)\omega_B^2$$

$$T = \frac{7}{16}mr^2\omega_B^2 \qquad (1)$$

Position 1: $(\omega_B)_1 = 30$ rad/s ↻

Position 2: $(\omega_B)_2 = 5$ rad/s ↻

Work. For the system considered, the only force which does work is the weight of disk A.

$$U_{1-2} = -Wh = -mgh$$

where h is the rise of cylinder A.

PROBLEM 17.32 (Continued)

Principle of work and energy.

$$T_1 + U_{1-2} = T_2: \quad \frac{7}{16}mr^2(\omega_B)_1^2 - mgh = \frac{7}{16}mr^2(\omega_B)_2^2$$

$$h = \frac{7}{16}\frac{r^2}{g}[(\omega_B)_1^2 - (\omega_B)_2^2] \qquad (2)$$

$$h = \frac{7}{16}\left(\frac{5}{12}\text{ft}\right)^2 \frac{1}{32.2\text{ ft/s}^2}[(30\text{ rad/s})^2 - (5\text{ rad/s})^2] = 2.064\text{ ft}$$

(*a*) *Rise of cylinder A.* $h = 2.06$ ft ◀

(*b*) *Tension in cord DE.* Let Q be its value.

Recall that $v_D = 2v_A$ thus D moves twice the distance that A moves, i.e $2h\uparrow$

$$T_1 = \frac{1}{2}\bar{I}(\omega_B)_1^2$$

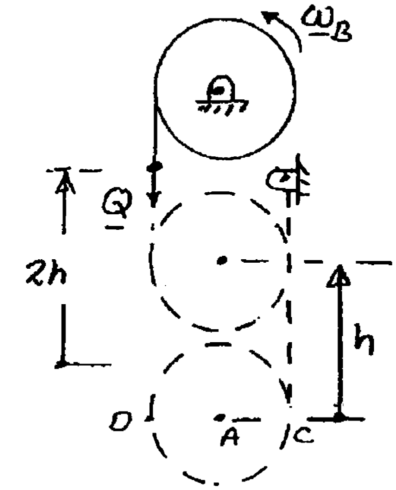
$$T_2 = \frac{1}{2}\bar{I}(\omega_B)_2^2$$

$$U_{1-2} = -Q(2h)$$

$$T_1 + U_{1-2} = T_2$$

$$\frac{1}{2}\bar{I}(\omega_B)_1^2 - 2Qh = \frac{1}{2}\bar{I}(\omega_B)_2^2$$

$$Qh = \frac{1}{4}\bar{I}[(\omega_B)_1^2 - (\omega_B)_2^2] \qquad (3)$$



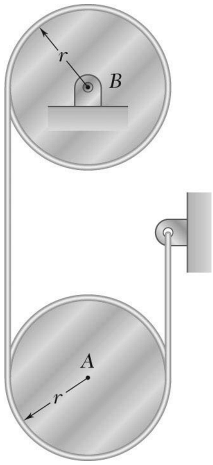
Divide Equation (3) by Equation (2):

$$Q = \frac{1}{4}\bar{I}\frac{16\text{ g}}{7r^2} = \frac{1}{4}\left(\frac{1}{2}mr^2\right)\frac{16\text{ g}}{7r^2} = \frac{2}{7}mg = \frac{2}{7}W \qquad (4)$$

$$Q = \frac{2}{7}(14\text{ lb})$$

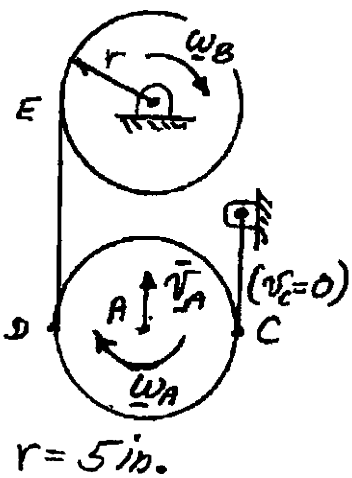
Tension $= Q = 4.00$ lb ◀

PROBLEM 17.33



Two uniform cylinders, each of weight $W = 14$ lb and radius $r = 5$ in., are connected by a belt as shown. If the system is released from rest, determine (*a*) the velocity of the center of cylinder *A* after it has moved through 3 ft, (*b*) the tension in the portion of belt connecting the two cylinders.

SOLUTION



Kinematics.

$$v_D = v_E = r\omega_B$$

Point *C* is the instantaneous center of cylinder *A*.

$$\omega_A = \frac{v_D}{CD} = \frac{r\omega_B}{2r} = \frac{1}{2}\omega_B$$

$$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B$$

$$v_D = 2\bar{v}_A$$

Kinetic energy of the system.

$$T = \frac{1}{2}m\bar{v}_A^2 + \frac{1}{2}\bar{I}\omega_A^2 + \frac{1}{2}\bar{I}\omega_B^2$$

$$T = \frac{1}{2}m\left(\frac{r}{2}\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{1}{2}\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega_B^2$$

$$T = \frac{7}{16}mr^2\omega_B^2 \qquad (1)$$

Position 1: At rest $T_1 = 0$

Position 2: Center of cylinder *C* has moved 3 ft↓.

Work. For the system considered, the only force which does work is the weight of disk *A*.

$$U_{1-2} = Wh = (14 \text{ lb})(3 \text{ ft}) = 42 \text{ ft} \cdot \text{lb}$$

where *h* is the distance that cylinder *A* falls.

PROBLEM 17.33 (Continued)

Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2: \quad 0 + 42 \text{ ft}\cdot\text{lb} = \frac{7}{16}\frac{14 \text{ lb}}{32.2 \text{ ft/s}^2}\left(\frac{5}{12}\text{ ft}\right)^2 (\omega_B)_2^2$$

$$(\omega_B)_2 = 35.662 \text{ rad/s}$$

(*a*) *Velocity of A.* $\quad \bar{v}_A = \frac{1}{2} r\omega_B = \frac{1}{2}\left(\frac{5}{12}\text{ ft}\right)(35.66 \text{ rad/s})$ $\quad \bar{\mathbf{v}}_A = 7.43 \text{ ft/s}\downarrow$ ◀

(*b*) *Tension in cord DE.* Let Q be its value.

Recall that $v_D = 2v_A$ thus D moves twice the distance that A moves, i.e $2h\downarrow$

$$T_1 = 0$$

$$T_2 = \frac{1}{2}\bar{I}(\omega_B)_2^2$$

$$U_{1-2} = Q(2h)$$

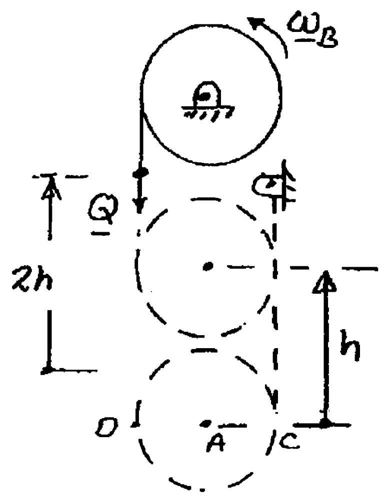
$$T_1 + U_{1-2} = T_2$$

$$0 + Qh = \frac{1}{2}\left(\frac{1}{2}\frac{W}{g}r^2\right)\omega_B^2$$

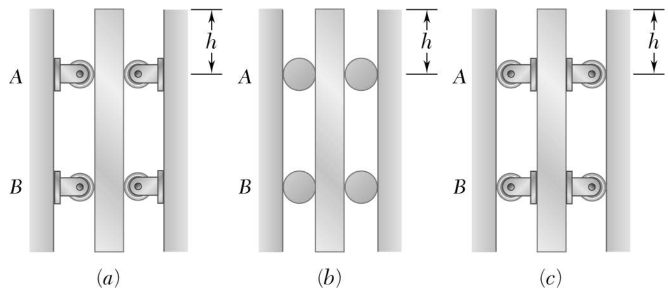
$$Q = \frac{1}{4}\frac{W}{g}r^2\frac{\omega_B^2}{h}$$

$$= \frac{1}{4}\frac{14 \text{ lb}}{32.2 \text{ ft/s}^2}\left(\frac{5}{12}\text{ ft}\right)^2\frac{(35.662 \text{ rad/s})^2}{6 \text{ ft}}$$

$$= 4.00 \text{ lb}$$



$Q = 4.00$ lb. ◀



PROBLEM 17.34

A bar of mass $m = 5$ kg is held as shown between four disks each of mass $m' = 2$ kg and radius $r = 75$ mm. Knowing that the forces exerted on the disks are sufficient to prevent slipping and that the bar is released from rest, for each of the cases shown determine the velocity of the bar after it has moved through the distance h.

SOLUTION

Let **v** be the velocity of the bar ($\mathbf{v} = v\downarrow$), $\overline{\mathbf{v}}'$ be the velocity of the mass center G of the upper left disk, ($\overline{\mathbf{v}}' = \overline{v}'\downarrow$) and $\boldsymbol{\omega}$ be its angular velocity.

For all three arrangements, the magnitudes of mass center velocities are the same for all disks. Likewise, the angular speeds are the same for all disks.

Moment of inertia of one disk. $\quad \overline{I} = \frac{1}{2}m'r^2$

Kinetic energy. $\quad T = \frac{1}{2}mv^2 + 4\left[\frac{1}{2}m'(\overline{v}')^2 + \frac{1}{2}\overline{I}\,\omega^2\right]$

$$T = \frac{1}{2}(5)v^2 + 4\left[\frac{1}{2}(2)(\overline{v}')^2 + \frac{1}{2}\left(\frac{1}{2}\right)(2)r^2\omega^2\right]$$

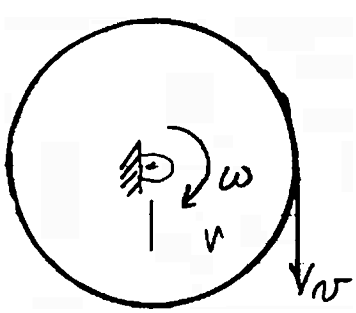
$$= 2.5v^2 + 4(\overline{v}')^2 + 2r^2\omega^2$$

Position 1. Initial at rest position. $T_1 = 0$

Position 2. Bar has moved down a distance h. All the disks move down a distance h'.

Work $\quad U_{1\to 2} = mgh + 4m'gh' = 5gh + 8gh'$

Kinematics and kinetic energy for case (a).



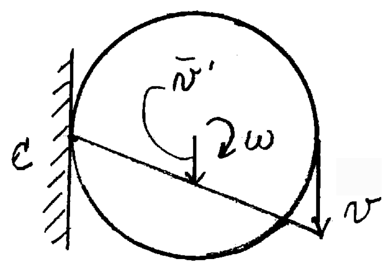
The mass center of each disk is not moving.

$\overline{v}' = 0, \qquad h' = 0$

$\omega = \dfrac{v}{r} \qquad r\omega = v$

$T_{2a} = 2.5v^2 + 0 + 2v^2 = 4.5v^2$

Kinematics and kinetic energy for case (*b*).



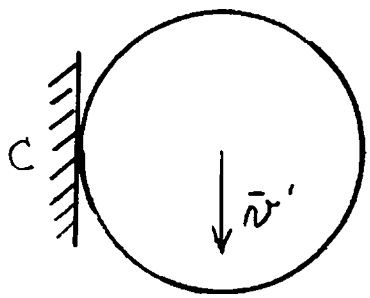
The instantaneous center C of a typical disk lies at its point of contact with the fixed wall.

$$\omega = \frac{v}{2r}$$

$$\bar{v}' = r\omega = \frac{1}{2}v, \qquad h' = \frac{1}{2}h$$

$$T_{2b} = 2.5v^2 + (4)\left(\frac{1}{2}v\right)^2 + 2\left(\frac{1}{2}v\right)^2 = 4.0v^2$$

Kinematics and kinetic energy for case (*c*).



The mass center of each disk moves with the bar.

$$\bar{v}' = v, \qquad h'' = h$$

The instantaneous center C of a typical disk lies at its point of contact with the fixed wall.

$$\bar{v}' = r\omega = v,$$

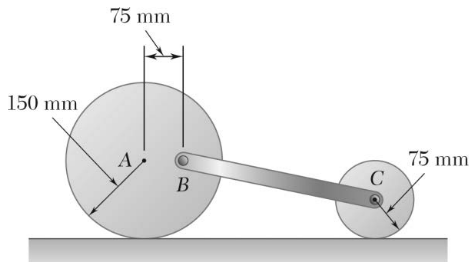
$$T_{2c} = 2.5v^2 + (4)v^2 + 2v^2 = 8.5v^2$$

Principle of Work and Energy. $\quad T_1 + U_{1\to 2} = T_2$

(*a*) $\quad 0 + 5gh + 0 = 4.5v^2$ $\qquad v = 1.054\sqrt{gh}$ ◀

(*b*) $\quad 0 + 5gh + 4gh = 4.0v^2$ $\qquad v = 1.500\sqrt{gh}$ ◀

(*c*) $\quad 0 + 5gh + 8gh = 8.5v^2$ $\qquad v = 1.237\sqrt{gh}$ ◀

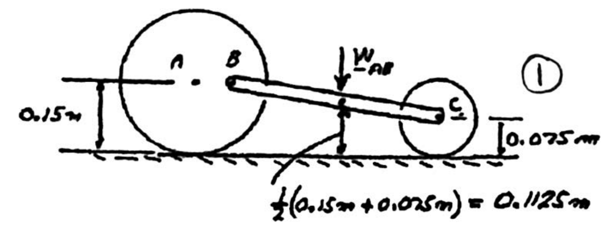


PROBLEM 17.35

The 5-kg rod *BC* is attached by pins to two uniform disks as shown. The mass of the 150-mm-radius disk is 6 kg and that of the 75-mm-radius disk is 1.5 kg. Knowing that the system is released from rest in the position shown, determine the velocity of the rod after disk *A* has rotated through 90°.

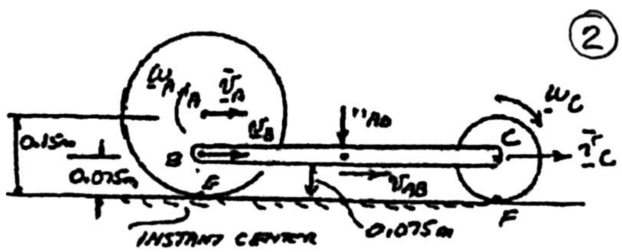
SOLUTION

Position 1. $T_1 = 0$



Position 2.

Kinematics.



$$v_B = v_{AB} \quad \omega_A = \frac{v_B}{BE} = \frac{v_{AB}}{0.075\text{ m}} \quad \overline{v}_A = 2v_B = 2v_{AB}$$

$$\overline{v}_C = v_{AB} \quad \omega_C = \frac{v_C}{CF} = \frac{v_{AB}}{0.075\text{ m}} \quad \omega_{AB} = 0$$

Kinetic energy.

$$T_2 = \frac{1}{2}m_A\overline{v}_A^2 + \frac{1}{2}\overline{I}_A\omega_A^2 + \frac{1}{2}m_{AB}v_{AB}^2 + \frac{1}{2}m_B\overline{v}_B^2 + \frac{1}{2}\overline{I}_B\omega_B^2$$

$$= \frac{1}{2}\left[(6\text{ ft/s})(2v_{AB})^2 + \frac{1}{2}(6\text{ kg})(0.15\text{ m})^2\left(\frac{v_{AB}}{0.075}\right)^2 + (5\text{ kg})v_{AB}^2\right.$$

$$\left. + (1.5\text{ kg})(v_{AB})^2 + \frac{1}{2}(1.5\text{ kg})(0.075\text{ m})^2\left(\frac{v_{AB}}{0.075}\right)^2\right]$$

$$= \frac{1}{2}[24 + 12 + 5 + 1.5 + 0.75]v_{AB}^2$$

$$T_2 = 21.625\, v_{AB}^2$$

PROBLEM 17.35 (Continued)

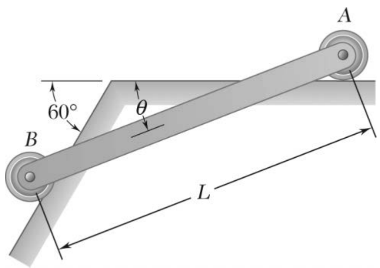
Work:

$$U_{1\to 2} = W_{AB}(0.1125 \text{ m} - 0.075 \text{ m})$$
$$= (5 \text{ kg})(9.81)(0.0375 \text{ m})$$
$$U_{1\to 2} = 1.8394 \text{ J}$$

Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2$$
$$0 + 1.8394 \text{ J} = 21.625\, v_{AB}^2$$
$$v_{AB}^2 = 0.08506$$
$$v_{AB} = 0.2916 \text{ m/s}$$

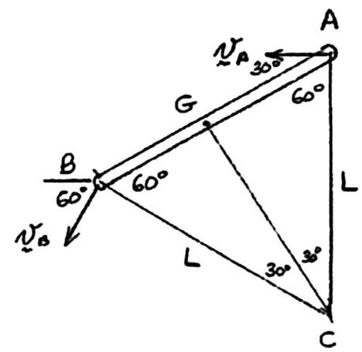
Velocity of the rod. $\mathbf{v}_{AB} = 292 \text{ mm/s} \longrightarrow$ ◀



PROBLEM 17.36

The motion of the uniform rod *AB* is guided by small wheels of negligible mass that roll on the surface shown. If the rod is released from rest when $\theta = 0$, determine the velocities of *A* and *B* when $\theta = 30°$.

SOLUTION



Position 1.

$$\theta = 0$$
$$v_A = v_B = 0$$
$$\omega = 0$$
$$T_1 = 0$$
$$V_1 = 0$$

Position 2.

$$\theta = 30°$$

Kinematics. Locate the instantaneous center *C*. Triangle *ABC* is equilateral.

$$v_A = v_B = L\omega$$
$$v_G = L\omega\cos 30°$$

Moment of inertia.

$$\bar{I} = \frac{1}{12}ml^2$$

Kinetic energy.

$$T_2 = \frac{1}{2}mv_G^2 + \frac{1}{2}\bar{I}\omega^2: \quad T_2 = \frac{1}{2}m(L\omega\cos 30°)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega^2$$
$$= \frac{5}{12}ml^2\omega^2$$

Potential energy.

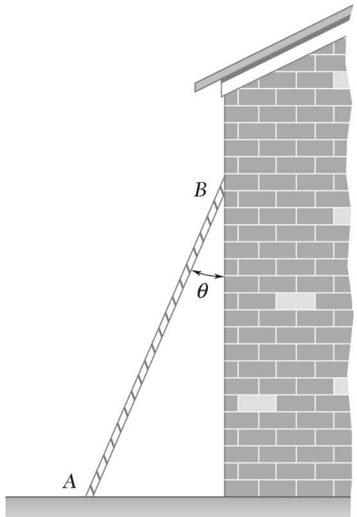
$$V_2 = -mg\frac{L}{2}\sin 30° = -\frac{1}{4}mgL$$

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \frac{5}{12}mL^2\omega^2 - \frac{1}{4}mgL$$
$$\omega^2 = 0.6\frac{g}{L}$$
$$\omega = 0.775\sqrt{\frac{g}{L}}$$
$$v_A = 0.775\sqrt{gL}$$
$$v_B = 0.775\sqrt{gL}$$

$\mathbf{v}_A = 0.775\sqrt{gL} \leftarrow$ ◀

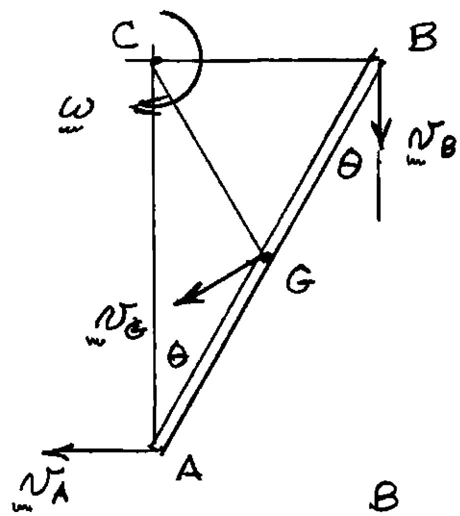
$\mathbf{v}_B = 0.775\sqrt{gL}$ ↙ $60°$ ◀



PROBLEM 17.37

A 5-m long ladder has a mass of 15 kg and is placed against a house at an angle $\theta = 20°$. Knowing that the ladder is released from rest, determine the angular velocity of the ladder and the velocity of A when $\theta = 45°$. Assume the ladder can slide freely on the horizontal ground and on the vertical wall.

SOLUTION



Kinematics:

Let $\mathbf{v}_A = v_A \longleftarrow$, $\mathbf{v}_B = v_B \downarrow$, and $\boldsymbol{\omega} = \omega$ ↻. Locate the instantaneous center C by drawing AC perpendicular to $\mathbf{v}_A$ and BC perpendicular to $\mathbf{v}_B$. Triangle GCB is isosceles.

$GA = GB = GC = L/2$. The velocity of the mass center G is

$$\bar{v} = v_G = L\omega/2$$

Kinetic energy:

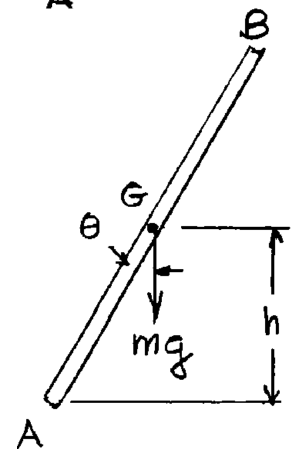
$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\,\omega^2$$

$$= \frac{1}{2}\left(\bar{I} + \frac{1}{4}mL^2\right)\omega^2$$

Since the ladder can slide freely, the friction forces at A and B are zero.

Use the principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2:$$



Potential energy: Use the ground as the datum.

$$V = mgh$$

where $$h = \frac{L}{2}\cos\theta$$

PROBLEM 17.37 (Continued)

Position 1. $\theta = 20°;$ rest $(T_1 = 0)$

Position 2. $\theta = 45°;$ $\omega = ?$

$$0 + mg\frac{L}{2}\cos 20° = \frac{1}{2}\left(\bar{I} + \frac{1}{4}mL^2\right)\omega^2 + mg\frac{L}{2}\cos 45°$$

Data: $m = 15\text{ kg}.\quad L = 5\text{ m}\quad g = 9.81\text{ m/s}^2$

Assume $\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(15\text{ kg})(5\text{ m})^2 = 31.25\text{ kg}\cdot\text{m}^2$

$$\bar{I} + \frac{1}{4}mL^2 = 31.25\text{ kg}\cdot\text{m}^2 + \frac{1}{4}(15\text{ kg})(5\text{ m})^2 = 125\text{ kg}\cdot\text{m}^2$$

$$(15\text{ kg})(9.81\text{ m/s}^2)(2.5\text{ m})(\cos 20° - \cos 45°) = \frac{1}{2}(125\text{ kg}\cdot\text{m}^2)\omega^2$$

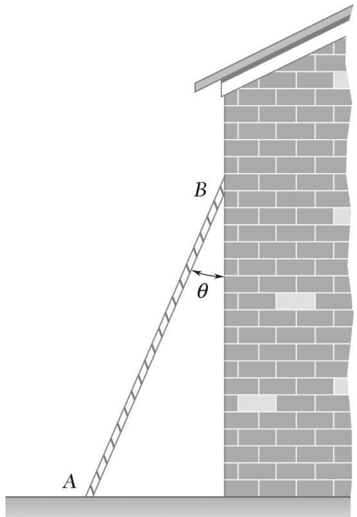
$$\omega^2 = 1.3690\text{ rad}^2/\text{s}^2 \qquad \omega = 1.17004\text{ rad/s}$$

Angular velocity. $\boldsymbol{\omega} = 1.170\text{ rad/s} \circlearrowright$ ◀

Velocity of end A.

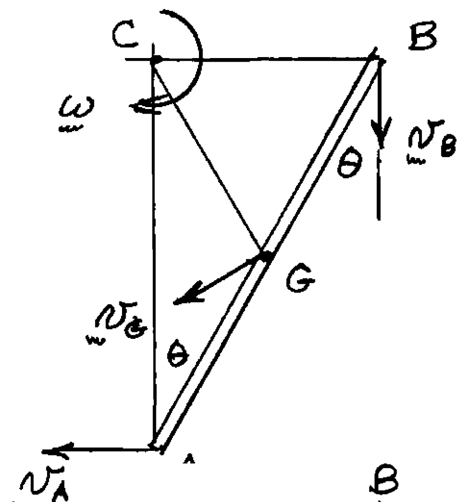
$v_A = \omega L\cos\theta = (1.17004\text{ rad/s})(5\text{ m})\cos 30°$ $\mathbf{v}_A = 5.07\text{ m/s} \longleftarrow$ ◀

PROBLEM 17.38



A long ladder of length l, mass m, and centroidal mass moment of inertia $\bar{I}$ is placed against a house at an angle $\theta = \theta_0$. Knowing that the ladder is released from rest, determine the angular velocity of the ladder when $\theta = \theta_2$. Assume the ladder can slide freely on the horizontal ground and on the vertical wall.

SOLUTION



Kinematics:

Let $\mathbf{v}_A = v_A \longleftarrow$, $\mathbf{v}_B = v_B \downarrow$, and $\boldsymbol{\omega} = \omega \circlearrowright$. Locate the instantaneous center C by drawing AC perpendicular to $\mathbf{v}_A$ and BC perpendicular to $\mathbf{v}_B$. Triangle GCB is isosceles.

$GA = GB = GC = L/2.$ The velocity of the mass center G is

$$\bar{v} = v_G = L\omega/2$$

Kinetic energy:

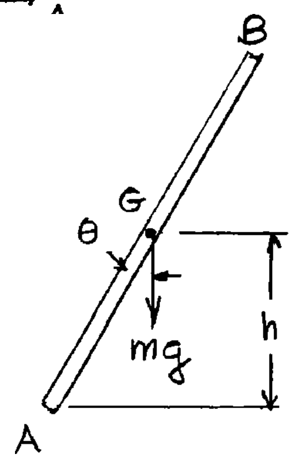
$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}\left(\bar{I} + \frac{1}{4}mL^2\right)\omega^2$$

Since the ladder can slide freely, the friction forces at A and B are zero.

Use the principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2:$$



Potential energy: Use the ground as the datum.

$$V = mgh$$

where

$$h = \frac{L}{2}\cos\theta$$

Position 1. $\theta = \theta_0$; rest $(T_1 = 0)$

PROBLEM 17.38 (Continued)

Position 2. $\theta = \theta_2; \quad \omega = ?$

$$0 + mg\frac{L}{2}\cos\theta = \frac{1}{2}\left(\bar{I} + \frac{1}{4}mL^2\right)\omega^2 + mg\frac{L}{2}\cos\theta_2$$

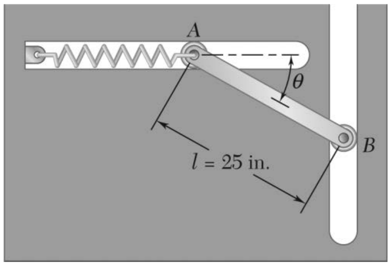
Assume $\bar{I} = \frac{1}{12}mL^2$

$$\bar{I} + \frac{1}{4}mL^2 = \frac{1}{3}mL^2$$

$$\omega^2 = \frac{3g}{L}(\cos\theta_0 - \cos\theta_2)$$

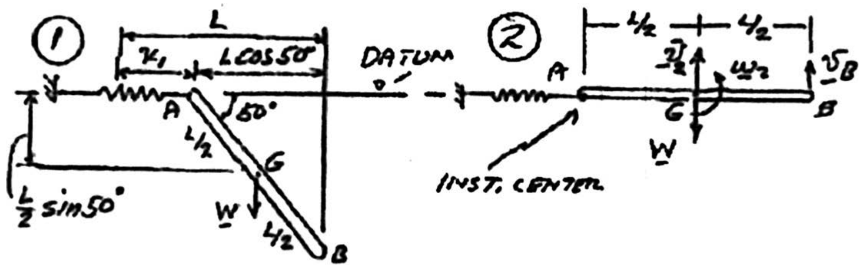
Angular velocity. $\boldsymbol{\omega} = \sqrt{3g(\cos\theta_0 - \cos\theta_2)/L}$ ↻ ◀

PROBLEM 17.39



The ends of a 9-lb rod AB are constrained to move along slots cut in a vertical plate as shown. A spring of constant $k = 3$ lb/in. is attached to end A in such a way that its tension is zero when $\theta = 0$. If the rod is released from rest when $\theta = 50°$, determine the angular velocity of the rod and the velocity of end B when $\theta = 0$.

SOLUTION



$$\bar{v}_2 = \frac{L}{2}\omega_2$$

$$v_B = L\omega_2$$

$$\begin{aligned} x_1 &= L - L\cos 50° \\ &= (25\text{ in.})(1-\cos 50°) \\ &= 8.9303\text{ in.} \end{aligned}$$

Position 1.

$$V_1 = -W\frac{L}{2}\sin 50° + \frac{1}{2}kx_1^2$$

$$\begin{aligned} V_1 &= -(9\text{ lb})\left(\frac{25\text{ in.}}{2}\right)\sin 50° + \frac{1}{2}(3\text{ lb/in.})(8.9303\text{ in.})^2 \\ &= -86.18 + 119.63 \\ &= 33.45\text{ in.}\cdot\text{lb} \\ &= 2.787\text{ ft}\cdot\text{lb.} \end{aligned}$$

$$T_1 = 0$$

Position 2.

$$V_2 = (V_g)_2 + (V_e)_2 = 0$$

$$\begin{aligned} T_2 &= \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 \\ &= \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2 \\ &= \frac{1}{6}mL^2\omega_2^2 = \frac{1}{6}\left(\frac{9\text{ lb}}{32.2}\right)\left(\frac{25\text{ in.}}{12}\right)^2\omega_2^2 = 0.2022\omega_2^2 \end{aligned}$$

PROBLEM 17.39 (Continued)

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$0 + 2.787 \text{ ft} \cdot \text{lb} = 0.2022\omega_2^2$$

$$\omega_2^2 = 13.7849$$

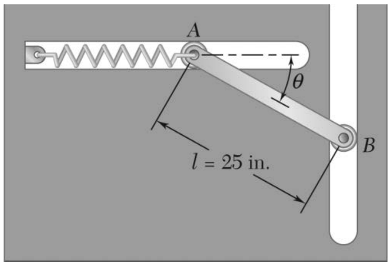
$$\omega_2 = 3.713 \text{ rad/s}$$

$\boldsymbol{\omega}_2 = 3.71 \text{ rad/s} \circlearrowleft$ ◀

Velocity of B: $v_B = L\omega_2 = \left(\frac{25 \text{ in.}}{12}\right)(3.713 \text{ rad/s})$

$$= 7.735 \text{ ft/s}$$

$\mathbf{v}_B = 7.74 \text{ ft/s} \uparrow$ ◀

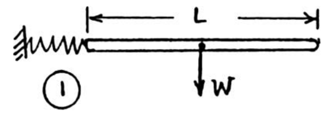


PROBLEM 17.40

The ends of a 9-lb rod AB are constrained to move along slots cut in a vertical plane as shown. A spring of constant $k = 3$ lb/in. is attached to end A in such a way that its tension is zero when $\theta = 0$. If the rod is released from rest when $\theta = 0$, determine the angular velocity of the rod and the velocity of end B when $\theta = 30°$.

SOLUTION

Moment of inertia. Rod. $\qquad \bar{I} = \frac{1}{12}mL^2$



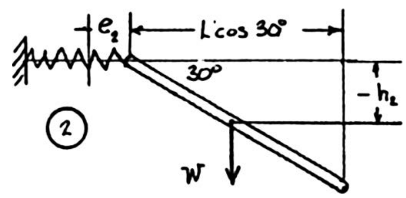
Position 1. $\qquad \theta_1 = 0 \qquad \bar{v}_1 = 0 \qquad \omega_1 = 0$

$h_1 =$ elevation above slot. $\qquad h_1 = 0$

$e_1 =$ elongation of spring. $\qquad e_1 = 0$

$$T_1 = \frac{1}{2}m\bar{v}_1^2 + \frac{1}{2}\bar{I}\omega_1^2 = 0$$

$$V_1 = \frac{1}{2}ke_1^2 + Wh_1 = 0$$



Position 2. $\qquad \theta = 30°$

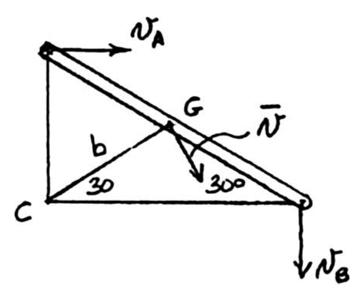
$$e_2 + L\cos 30° = L$$

$$e_2 = L(1 - \cos 30°)$$

$$h_2 = -\frac{L}{2}\sin 30° = -\frac{1}{4}L$$

$$V_2 = \frac{1}{2}ke_2^2 + Wh_2 = \frac{1}{2}kL^2(1 - \cos 30°)^2 - \frac{1}{4}WL$$

Kinematics. Velocities at A and B are directed as shown. Point C is the instantaneous center of rotation. From geometry, $b = \frac{L}{2}$.



$$\bar{v} = b\omega = \frac{L}{2}\omega$$

$$v_B = (L\cos 30°)\omega$$

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2}m\left(\frac{L}{2}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)$$

$$= \frac{1}{6}\frac{W}{g}L^2\omega^2$$

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \frac{1}{6}\frac{W}{g}L^2\omega^2 + \frac{1}{2}kL^2(1-\cos 30°)^2 - \frac{1}{4}WL$$

$$\omega^2 = \frac{3g}{2L} - \frac{3\,kg}{W}(1-\cos 30°)^2$$

Data:

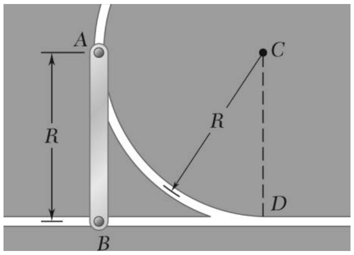
$$W = 9 \text{ lb}$$
$$g = 32.2 \text{ ft/s}^2$$
$$L = 25 \text{ in.} = 2.0833 \text{ ft}$$
$$k = 3 \text{ lb/in.} = 36 \text{ lb/ft}$$

$$\omega^2 = \frac{(3)(32.2)}{(2)(2.0833)} - \frac{(3)(36)(32.2)(1-\cos 30°)^2}{9}$$
$$= 16.2484$$

$\boldsymbol{\omega} = 4.03$ rad/s ↻ ◀

$$v_B = (2.0833)(\cos 30°)(4.03)$$

$\mathbf{v}_B = 7.27$ ft/s ↓ ◀



PROBLEM 17.41

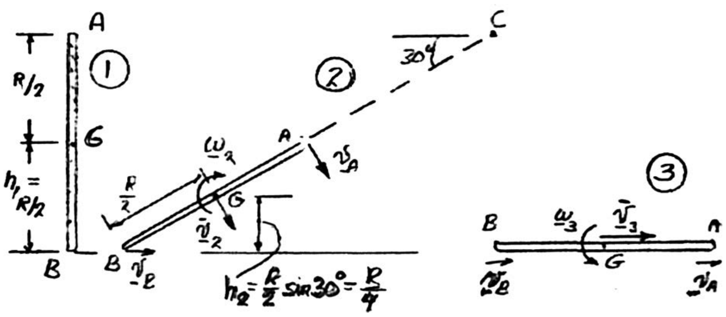
The motion of a slender rod of length R is guided by pins at A and B which slide freely in slots cut in a vertical plate as shown. If end B is moved slightly to the left and then released, determine the angular velocity of the rod and the velocity of its mass center (*a*) at the instant when the velocity of end B is zero, (*b*) as end B passes through Point D.

SOLUTION

The rod AB moves from *Position 1*, where it is nearly vertical, to *Position 2*, where $\mathbf{v}_B = 0$.

In *Position 2*, $\mathbf{v}_A$ is perpendicular to both CA and AB, so CAB is a straight line of length $2L$ and slope angle 30°.

In *Position 3* the end B passes through Point D.



Position 1: $T_1 = 0 \qquad V_1 = Wh = mg\dfrac{R}{2}$

Position 2: Since instantaneous center is at B,

$$v_2 = \frac{1}{2}R\omega_2$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}I\omega_2^2$$

$$= \frac{1}{2}m\left(\frac{1}{2}R\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mR^2\right)\omega_2^2$$

$$= \frac{1}{6}mR^2\omega_2^2$$

$$V_2 = Wh_2 = mg\frac{R}{4}$$

Position 3: $V_3 = 0$

Since both $\mathbf{v}_A$ and $\mathbf{v}_B$ are horizontal, $\omega_3 = 0$ (1)

$$T_3 = \frac{1}{2}m\bar{v}_2^2$$

(*a*) From 1 to 2: Conservation of energy

$$T_1 + V_1 = T_2 + V_2: \quad 0 + \frac{1}{2}mgR = \frac{1}{6}mR^2\omega_2^2 + \frac{1}{4}mgR$$

$$\omega_2^2 = \frac{3}{2}\frac{g}{R}$$

$$\omega_2 = \sqrt{\frac{3}{2}\frac{g}{R}} \qquad \boldsymbol{\omega}_2 = 1.225\sqrt{\frac{g}{R}} \circlearrowright \blacktriangleleft$$

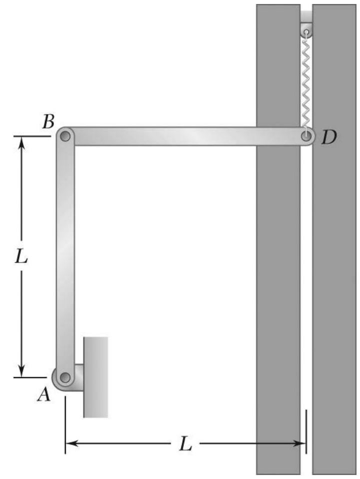
$$\bar{v}_2 = \frac{1}{2}R\omega_2 = \frac{1}{2}\sqrt{\frac{3}{2}gR} = \sqrt{\frac{3}{8}gR} \qquad \mathbf{v}_R = 0.612\sqrt{gR} \searrow 60° \blacktriangleleft$$

(*b*) From 1 to 3: Conservation of energy

From Eq. (1) we have $\boldsymbol{\omega}_3 = 0$ ◀

$$T_1 + V_1 = T_3 + V_3: \quad 0 + \frac{1}{2}mgR = \frac{1}{2}m\bar{v}_3^2$$

$$\bar{v}_3^2 = gR \qquad \mathbf{v}_3 = \sqrt{gR} \longrightarrow \blacktriangleleft$$



PROBLEM 17.42

Each of the two rods shown is of length $L = 1$ m and has a mass of 5 kg. Point D is connected to a spring of constant $k = 20$ N/m and is constrained to move along a vertical slot. Knowing that the system is released from rest when rod BD is horizontal and the spring connected to Point D is initially unstretched, determine the velocity of Point D when it is directly to the right of Point A.

SOLUTION

Moments of inertia. $\qquad \bar{I} = \frac{1}{12}mL^2, \quad I_A = \frac{1}{3}mL^2$

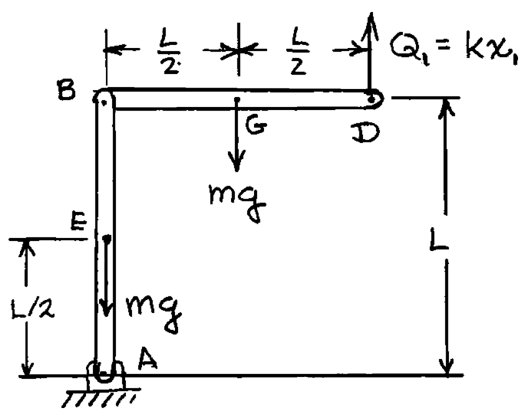
Use the principle of conservation of energy applied to the system consisting of both rods. Use the level at A as the datum for the potential energy of each rod.

Position 1. (no motion)

$$T_1 = 0$$

$$V_1 = mg\left(\frac{1}{2}L\right) + mgL + \frac{1}{2}kx_1^2$$

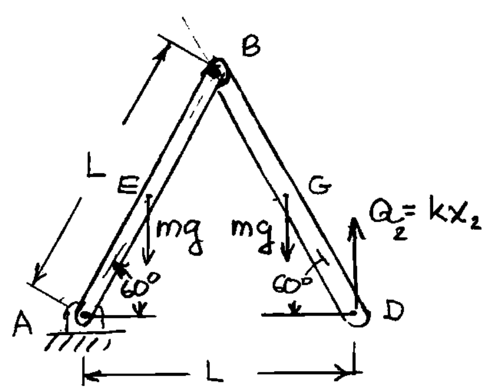
$$= \frac{3}{2}mgL + \frac{1}{2}kx_1^2$$



Position 2.

$$V_2 = mg\frac{L}{2}\sin 60° + mg\frac{L}{2}\sin 60°$$

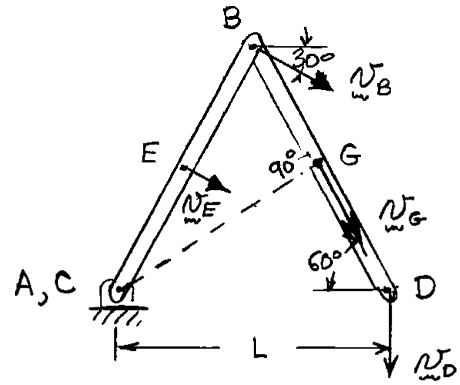
$$= \frac{\sqrt{3}}{2}mgL + \frac{1}{2}kx_2^2$$



Kinematics. $\boldsymbol{\omega}_{AB} = \omega_{AB}$ ↻

$v_B = L\omega_{AB} \quad \mathbf{v}_B = L\omega_{AB} \searrow 30°$

$\mathbf{v}_D = v_D \downarrow$



Locate the instantaneous center *C* of rod *BD* by drawing *BC* perpendicular to $\mathbf{v}_B$ and *DC* perpendicular to $\mathbf{v}_D$. Point *C* coincides with Point *A* in position 2.

Let $\boldsymbol{\omega}_{BD} = \omega_{BD}$ ↻

$$\omega_{BD} = \frac{v_B}{L} = \omega_{AB}$$

$$v_E = \frac{L}{2}\omega_{AB}$$

$$v_G = (L\sin 60°)\omega_{BD} = \frac{\sqrt{3}}{2}L\omega_{AB}$$

$$v_D = L\omega_{BD} = L\omega_{AB} \tag{1}$$

$$T_2 = \frac{1}{2}I_A\omega_{AB}^2 + \frac{1}{2}\bar{I}\omega_{BD}^2 + \frac{1}{2}mv_G^2$$

$$= \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_{AB}^2 + \frac{1}{2}m\left(\frac{\sqrt{3}}{2}\omega_{AB}\right)^2$$

$$= \left(\frac{1}{6} + \frac{1}{24} + \frac{3}{8}\right)mL^2\omega_{AB}^2 = \frac{7}{12}mL^2\omega_{AB}^2$$

Principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + \frac{3}{2}mgL + \frac{1}{2}kx_1^2 = \frac{7}{12}mL^2\omega_{AB}^2 + \frac{\sqrt{3}}{2}mgL + \frac{1}{2}kx_2^2$$

$$\frac{7}{12}mL^2\omega_{AB}^2 = \left(\frac{3}{2} - \frac{\sqrt{3}}{2}\right)mgL - \frac{1}{2}k(x_2^2 - x_1^2) \tag{2}$$

Data: $m = 5\text{ kg}, \quad L = 1\text{ m}, \quad g = 9.81\text{ m/s}^2$

$k = 20\text{ N}\cdot\text{m}, \quad x_1 = 0, \quad x_2 = L = 1\text{ m}$

$$\left(\frac{3}{2} - \frac{\sqrt{3}}{2}\right)mgL = (0.63397)(5\text{ kg})(9.81\text{ m/s}^2)(1\text{ m}) = 31.096\text{ J}$$

$$-\frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}(20\text{ N/m})(1\text{ m})^2 = -10\text{ J}$$

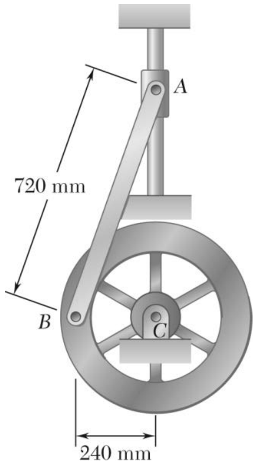
PROBLEM 17.42 (Continued)

By Eq. (2), $$\frac{7}{12}mL^2\omega_{AB}^2 = \left(\frac{35}{12}\,\text{kg}\cdot\text{m}^2\right)\omega_{AB}^2 = 21.096\ \text{J}$$

$$\omega_{AB}^2 = 7.2329\ \text{rad}^2/\text{s}^2 \qquad \omega_{AB} = 2.6894\ \text{rad/s}$$

By Eq. (1), $$v_D = (1\ \text{m})(2.6894\ \text{rad/s})$$ $$\mathbf{v}_D = 2.69\ \text{m/s}\downarrow \blacktriangleleft$$

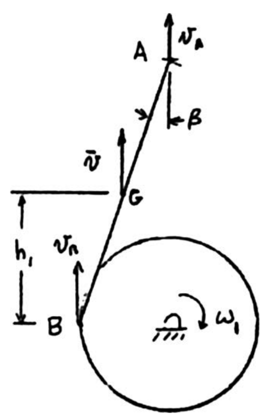
PROBLEM 17.43



The 4-kg rod *AB* is attached to a collar of negligible mass at *A* and to a flywheel at *B*. The flywheel has a mass of 16 kg and a radius of gyration of 180 mm. Knowing that in the position shown the angular velocity of the flywheel is 60 rpm clockwise, determine the velocity of the flywheel when Point *B* is directly below *C*.

SOLUTION

Moments of inertia.



Rod *AB*:

$$\bar{I}_{AB} = \frac{1}{12} m_{AB} L_{AB}^2 = \frac{1}{12}(4 \text{ kg})(0.72 \text{ m})^2 = 0.1728 \text{ kg}\cdot\text{m}^2$$

Flywheel:

$$I_C = m\bar{k}^2 = (16 \text{ kg})(0.18 \text{ m})^2 = 0.5184 \text{ kg}\cdot\text{m}^2$$

Position 1. As shown.

$$\boldsymbol{\omega} = \omega_1 \circlearrowright$$

$$\sin\beta = \frac{0.24}{0.72} \quad \beta = 19.471°$$

$$h_1 = \frac{1}{2}(0.72)\cos\beta = 0.33941 \text{ m}$$

$$V_1 = W_{AB} h_1 = (4)(9.81)(0.33941) = 13.3185 \text{ J}$$

Kinematics.

$$v_B = r\omega_1 = 0.24\omega_1$$

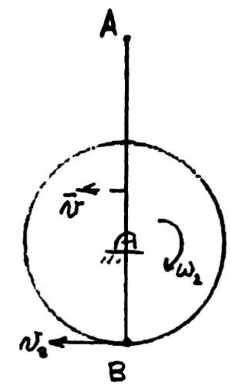
Bar *AB* is in translation.

$$\omega_{AB} = 0, \quad \bar{v} = v_B$$

$$T_1 = \frac{1}{2} m_{AB}\bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2} I_C \omega_1^2 = \frac{1}{2}(4)(0.24\omega_1)^2 + 0 + \frac{1}{2}(0.5184)\omega_1^2 = 0.3744\omega_1^2$$

PROBLEM 17.43 (Continued)

Position 2. Point B is directly below C.



$$h_2 = \frac{1}{2}L_{AB} - r$$
$$= \frac{1}{2}(0.72) - 0.24$$
$$= 0.12 \text{ m}$$
$$V_2 = W_{AB}h_2$$
$$= (4)(9.81)(0.12)$$
$$= 4.7088 \text{ J}$$

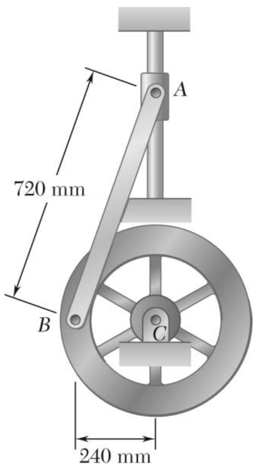
Kinematics. $v_B = r\omega_2 = 0.24\omega_2$

$$\omega_{AB} = \frac{v_B}{0.72} = 0.33333\omega_2$$
$$\bar{v} = \frac{1}{2}v_B = 0.12\omega_2$$
$$T_2 = \frac{1}{2}m_{AB}\bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2}I_C\omega_2^2$$
$$= \frac{1}{2}(4)(0.12\omega_2)^2 + \frac{1}{2}(0.1728)(0.33333\omega_2)^2 + \frac{1}{2}(0.5184)\omega_2^2$$
$$= 0.2976\omega_2^2$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2: \quad 0.3744\omega_1^2 + 13.3185 = 0.2976\omega_2^2 + 4.7088$ (1)

Angular speed data: $\omega_1 = 60 \text{ rpm} = 2\pi \text{ rad/s}$

Solving Equation (1) for ω_2, $\omega_2 = 8.8655 \text{ rad/s}$ $\boldsymbol{\omega}_2 = 84.7 \text{ rpm} ↻$ ◀



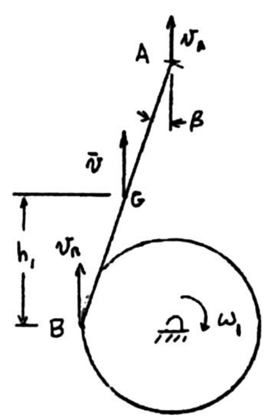
PROBLEM 17.44

If in Problem 17.43 the angular velocity of the flywheel is to be the same in the position shown and when Point *B* is directly above *C*, determine the required value of its angular velocity in the position shown.

PROBLEM 17.43 The 4-kg rod *AB* is attached to a collar of negligible mass at *A* and to a flywheel at *B*. The flywheel has a mass of 16 kg and a radius of gyration of 180 mm. Knowing that in the position shown the angular velocity of the flywheel is 60 rpm clockwise, determine the velocity of the flywheel when Point *B* is directly below *C*.

SOLUTION

Moments of inertia.



Rod *AB*:

$$\bar{I}_{AB} = \frac{1}{12} m_{AB} L_{AB}^2 = \frac{1}{12}(4\text{ kg})(0.72\text{ m})^2 = 0.1728\text{ kg}\cdot\text{m}^2$$

Flywheel:

$$I_C = m\bar{k}^2 = (16\text{ kg})(0.18\text{ m})^2 = 0.5184\text{ kg}\cdot\text{m}^2$$

Position 1. As shown.

$$\boldsymbol{\omega} = \omega_1 \circlearrowright$$

$$\sin\beta = \frac{0.24}{0.72} \quad \beta = 19.471°$$

$$h_1 = \frac{1}{2}(0.72)\cos\beta = 0.33941\text{ m}$$

$$V_1 = W_{AB} h_1 = (4)(9.81)(0.33941) = 13.3185\text{ J}$$

Kinematics.

$$v_B = r\omega_1 = 0.24\omega_1$$

Bar *AB* is in translation.

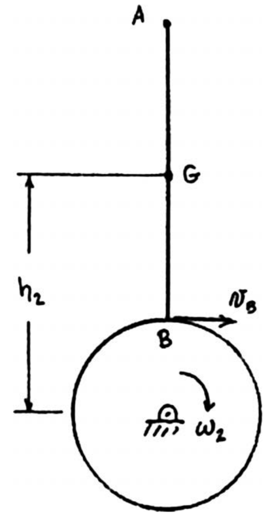
$$\omega_{AB} = 0, \quad \bar{v} = v_B$$

$$T_1 = \frac{1}{2} m_{AB}\bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2} I_C \omega_1^2 = \frac{1}{2}(4)(0.24\omega_1)^2 + 0 + \frac{1}{2}(0.5184)\omega_1^2 = 0.3744\omega_1^2$$

PROBLEM 17.44 (Continued)

Position 2. Point B is directly above C.

$$h_2 = \frac{1}{2}L_{AB} + r$$
$$= \frac{1}{2}(0.72) + 0.24$$
$$= 0.6 \text{ m}$$
$$V_2 = W_{AB}h_2$$
$$= (4)(9.81)(0.6)$$
$$= 23.544 \text{ J}$$



Kinematics. $$v_B = r\omega_2 = 0.24\omega_2$$

$$\omega_{AB} = \frac{v_B}{0.72} = 0.33333\omega_2$$
$$\bar{v} = \frac{1}{2}v_B = 0.12\omega_2$$
$$T_2 = \frac{1}{2}m_{AB}\bar{v}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2}I_C\omega_2^2$$
$$= \frac{1}{2}(4)(0.12\omega_2)^2 + \frac{1}{2}(0.1728)(0.33333\omega_2)^2 + \frac{1}{2}(0.5184)\omega_2^2$$
$$= 0.2976\omega_2^2$$

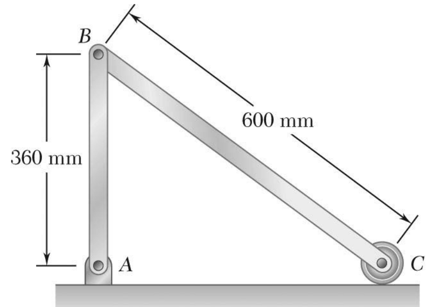
Conservation of energy. $T_1 + V_1 = T_2 + V_2$: $\quad 0.3744\omega_1^2 + 13.3135 = 0.2976\omega_2^2 + 23.544$

Angular speed data: $\omega_2 = \omega_1$

Then, $0.0760\omega_1^2 = +0.4105$

$$\omega_1 = 11.602 \text{ rad/s}$$

$\boldsymbol{\omega}_1 = 110.8$ rpm ↻ ◀



PROBLEM 17.45

The uniform rods *AB* and *BC* of masses 2.4 kg and 4 kg, respectively, and the small wheel at *C* is of negligible mass. If the wheel is moved slightly to the right and then released, determine the velocity of pin *B* after rod *AB* has rotated through 90°.

SOLUTION

Moments of inertia.

Rod *AB*: $$I_A = \frac{1}{3}m_{AB}L_{AB}^2 = \frac{1}{3}(2.4)(0.360)^2 = 0.10368\ \text{kg}\cdot\text{m}^2$$

Rod *BC*: $$\bar{I} = \frac{1}{12}m_{BC}L_{BC}^2 = \frac{1}{12}(4)(0.600)^2 = 0.1200\ \text{kg}\cdot\text{m}^2$$

Position 1. As shown with bar *AB* vertical. Point *G* is the midpoint of *BC*.

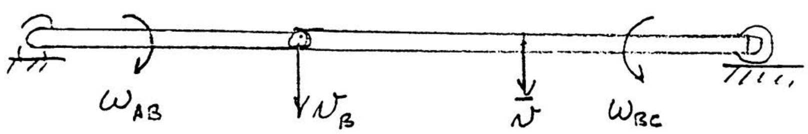
$$V_1 = m_{AB}gh_{AB} + m_{BC}gh_{BC} = (2.4)(9.81)(0.180) + (4)(9.81)(0.180) = 11.3011\ \text{J}$$

Rod *BC* is at rest. $\omega_{BC} = 0$

$$\bar{v} = v_G = v_B = v_C = 0 \qquad \omega_{AB} = \frac{v_B}{L_{AB}} = 0$$

$$T_1 = 0$$

Position 2. Rod *AB* is horizontal.



$$V_2 = 0$$

Kinematics. $$\omega_{AB} = \frac{v_B}{L_{AB}} = \frac{v_B}{0.360} \qquad \omega_{BC} = \frac{v_B}{L_{BC}} = \frac{v_B}{0.600} \qquad \bar{v} = \frac{1}{2}v_B$$

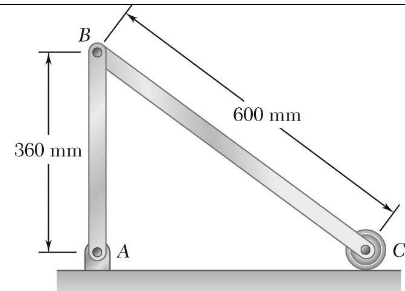
$$T_2 = \frac{1}{2}I_A\omega_{AB}^2 + \frac{1}{2}m_{BC}\bar{v}^2 + \frac{1}{2}\bar{I}\omega_{BC}^2$$

$$= \frac{1}{2}(0.10368)\left(\frac{v_B}{0.360}\right)^2 + \frac{1}{2}(4)\left(\frac{1}{2}v_B\right)^2 + \frac{1}{2}(0.1200)\left(\frac{v_B}{0.600}\right)^2$$

$$= 1.06667v_B^2$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2$: $\quad 0 + 11.3011 = 1.06667v_B^2$

$$v_B = 3.25\ \text{m/s} \qquad \mathbf{v}_B = 3.25\ \text{m/s}\downarrow \blacktriangleleft$$



PROBLEM 17.46

The uniform rods AB and BC of masses 2.4 kg and 4 kg, respectively, and the small wheel at C is of negligible mass. Knowing that in the position shown the velocity of wheel C is 2 m/s to the right, determine the velocity of pin B after rod AB has rotated through 90°.

SOLUTION

Moments of inertia.

Rod AB: $$I_A = \frac{1}{3}m_{AB}L_{AB}^2 = \frac{1}{3}(2.4)(0.36)^2 = 0.10368\ \text{kg}\cdot\text{m}^2$$

Rod BC: $$\overline{I} = \frac{1}{12}m_{BC}L_{BC^2} = \frac{1}{12}(4)(0.600)^2 = 0.1200\ \text{kg}\cdot\text{m}^2$$

Position 1. As shown with rod AB vertical. Point G is the midpoint of BC.

$$\begin{aligned} V_1 &= W_{AB}h_{AB} + W_{BC}h_{BC} \\ &= (2.4)(9.81)(0.180) + (4)(9.81)(0.180) \\ &= 11.301\ \text{J} \end{aligned}$$

Kinematics: At the instant shown in Position 1,

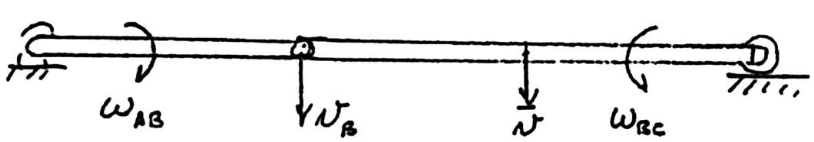
$$\omega_{BC} = 0$$

$$\overline{v} = v_G = v_B = v_C = 2\ \text{m/s}$$

$$\omega_{AB} = \frac{v_B}{L_{AB}} = \frac{2}{0.36} = 5.5556\ \text{rad/s}$$

$$\begin{aligned} T_1 &= \frac{1}{2}I_A\omega_{AB}^2 + \frac{1}{2}m_{BC}\overline{v}^2 + \frac{1}{2}I\omega_{BC}^2 \\ &= \frac{1}{2}(0.10368)(5.5556)^2 + \frac{1}{2}(4)(2)^2 + 0 \\ &= 9.6\ \text{J} \end{aligned}$$

Position 2. Rod AB is horizontal.



$$V_2 = 0$$

PROBLEM 17.46 (Continued)

Kinematics.

$$\omega_{AB} = \frac{v_B}{L_{AB}} = \frac{v_B}{0.36}$$

$$\omega_{BC} = \frac{v_B}{L_{BC}} = \frac{v_B}{0.60}$$

$$\bar{v} = \frac{1}{2} v_B$$

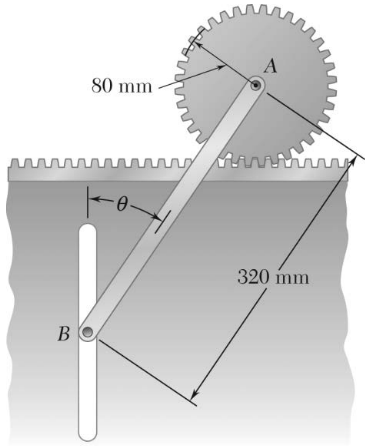
$$T_2 = \frac{1}{2} I_A \omega_{AB}^2 + \frac{1}{2} m_{BC} \bar{v}^2 + \frac{1}{2} \bar{I} \omega_{BC}^2$$

$$= \frac{1}{2}(0.10368)\left(\frac{v_B}{0.36}\right)^2 + \frac{1}{2}(4)\left(\frac{1}{2} v_B\right)^2 + \frac{1}{2}(0.12)\left(\frac{v_B}{0.60}\right)^2$$

$$= 1.0667 v_B^2$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2: \quad 9.6 + 11.301 = 1.0667 v_B^2 + 0$

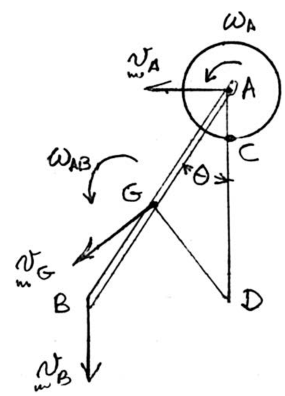
$v_B = 4.4266 \text{ m/s}$ $\qquad \mathbf{v}_B = 4.43 \text{ m/s} \downarrow$ ◀



PROBLEM 17.47

The 80-mm-radius gear shown has a mass of 5 kg and a centroidal radius of gyration of 60 mm. The 4-kg rod *AB* is attached to the center of the gear and to a pin at *B* that slides freely in a vertical slot. Knowing that the system is released from rest when $\theta = 60°$, determine the velocity of the center of the gear when $\theta = 20°$.

SOLUTION



Kinematics. $\mathbf{v}_A = v_A \longleftarrow$

$\mathbf{v}_B = v_B \downarrow$

Point *D* is the instantaneous center of rod *AB*.

$$\omega_{AB} = \frac{v_A}{L\cos\theta}$$

$$v_B = (L\sin\theta)\omega_{AB} = v_A \tan\theta$$

$$v_G = \frac{L}{2}\omega_{AB} = \frac{v_A}{2\cos\theta}$$

Gear *A* effectively rolls without slipping, with Point *C* being the contact point.

$$v_C = 0$$

Angular velocity of gear $\omega_A = \dfrac{v_A}{r}.$

Potential energy: Use the level of the center of gear *A* as the datum.

$$V = -W_{AB}\left(\frac{L}{2}\cos\theta\right) = -\frac{1}{2}m_{AB}gL\cos\theta$$

Kinetic energy: $T = \dfrac{1}{2}m_A v_A^2 + \dfrac{1}{2}I_A\omega_A^2 + \dfrac{1}{2}m_{AB}v_\theta^2 + \dfrac{1}{2}\bar{I}_{AB}\omega_{AB}^2$

Masses and moments of inertia: $m_A = 5\text{ kg}, \quad m_{AB} = 4\text{ kg}$

$$I_A = m_A k^2 = (5)(0.060)^2 = 0.018\text{ kg}\cdot\text{m}^2$$

$$\bar{I}_{AB} = \frac{1}{12}m_{AB}L^2 = \frac{1}{12}(4)(0.320)^2 = 0.03413\text{ kg}\cdot\text{m}^2$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

Position 1: $\theta = 60°$

$v_A = 0 \quad T_1 = 0$

$$V_1 = -\frac{1}{2}(4)(9.81)(0.320)\cos 60°$$
$$= -3.1392 \text{ J}$$

Position 2: $\theta = 20° \quad v_A = ?$

$$T_2 = \frac{1}{2}(5)v_A^2 + \frac{1}{2}(0.018)\left(\frac{v_A}{0.080}\right)^2 + \frac{1}{2}(4)\left(\frac{v_A}{2\cos 20°}\right)^2$$
$$+\frac{1}{2}(0.03413)\left(\frac{v_A}{0.320\cos 20°}\right)^2$$
$$= (2.5 + 1.40625 + 0.56624 + 0.18875)v_A^2$$
$$= 4.66124 v_A^2$$
$$V_2 = -\frac{1}{2}(4)(9.81)(0.320)\cos 20°$$
$$= -5.8998 \text{ J}$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$0 - 3.1392 = 4.66124 v_A^2 - 5.8998$$
$$v_A^2 = 0.59225 \text{ m}^2/\text{s}^2$$
$$v_A = 0.770 \text{ m/s}$$

$\mathbf{v}_A = 0.770 \text{ m/s} \longleftarrow$ ◀

PROBLEM 17.48

Knowing that the maximum allowable couple that can be applied to a shaft is 15.5 kip · in., determine the maximum horsepower that can be transmitted by the shaft at (*a*) 180 rpm, (*b*) 480 rpm.

SOLUTION

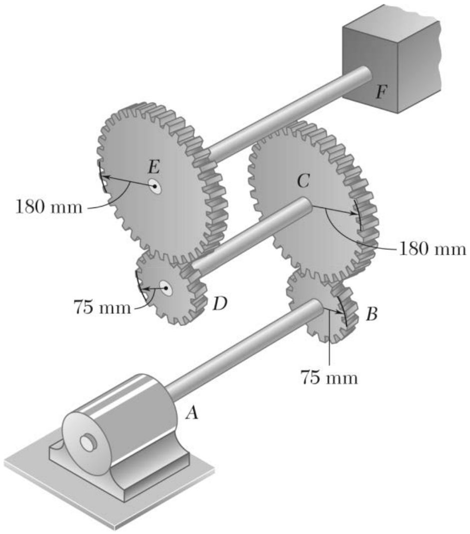
$$M = 15.5 \text{ kip} \cdot \text{in.}$$
$$= 1.2917 \text{ kip} \cdot \text{ft}$$
$$= 1291.7 \text{ lb} \cdot \text{ft}$$

(*a*)

$$\omega = 180 \text{ rpm}$$
$$= 6\pi \text{ rad/s}$$
$$\text{Power} = M\omega$$
$$= (1291.7 \text{ lb} \cdot \text{ft})(6\pi \text{ rad/s})$$
$$= 24348 \text{ ft} \cdot \text{lb/s}$$
$$\text{Horsepower} = \frac{24348}{550}$$
$$= 44.3 \text{ hp} \blacktriangleleft$$

(*b*)

$$\omega = 480 \text{ rpm}$$
$$= 16\pi \text{ rad/s}$$
$$\text{Power} = M\omega$$
$$= (1291.7 \text{ lb} \cdot \text{ft})(16\pi \text{ rad/s})$$
$$= 64930 \text{ ft} \cdot \text{lb/s}$$
$$\text{Horsepower} = \frac{64930}{550}$$
$$= 118.1 \text{ hp} \blacktriangleleft$$



PROBLEM 17.49

Three shafts and four gears are used to form a gear train which will transmit 7.5 kW from the motor at *A* to a machine tool at *F*. (Bearings for the shafts are omitted from the sketch.) Knowing that the frequency of the motor is 30 Hz, determine the magnitude of the couple which is applied to shaft (*a*) *AB*, (*b*) *CD*, (*c*) *EF*.

SOLUTION

Kinematics.

$$\omega_{AB} = 30\text{ Hz} = 30(2\pi)\text{rad/s} = 60\pi\text{ rad/s}$$

Gears *B* and *C*.

$$r_B = 75\text{ mm}$$
$$r_C = 180\text{ mm}$$

$$r_B\omega_{AB} = r_C\omega_{CD}: \quad (75\text{ mm})(60\pi\text{ rad/s}) = (180\text{ mm})(\omega_{CD})$$

Gears *D* and *E*.

$$\omega_{CD} = 25\pi\text{ rad/s}$$
$$r_D = 75\text{ mm}$$
$$r_E = 180\text{ mm}$$

$$r_D\omega_{CD} = r_E\omega_{EF}: \quad (75\text{ mm})(25\pi\text{ rad/s}) = (180\text{ mm})(\omega_{EF})$$

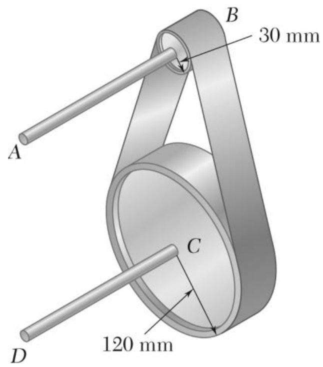
$$\omega_{EF} = 10.4167\pi\text{ rad/s}$$

$$\text{Power} = 7.5\text{ kW}$$

(*a*) Shaft *AB*. Power $= M_{AB}\omega_{AB}$: $7500\text{ W} = M_{AB}(60\pi\text{ rad/s})$ $M_{AB} = 39.8\text{ N}\cdot\text{m}$ ◀

(*b*) Shaft *CD*. Power $= M_{CD}\omega_{CD}$: $7500\text{ W} = M_{CD}(25\pi\text{ rad/s})$ $M_{CD} = 95.5\text{ N}\cdot\text{m}$ ◀

(*c*) Shaft *EF*. Power $= M_{EF}\omega_{EF}$: $7500\text{ W} = M_{EF}(10.4167\pi\text{ rad/s})$ $M_{EF} = 229\text{ N}\cdot\text{m}$ ◀



PROBLEM 17.50

The shaft-disk-belt arrangement shown is used to transmit 2.4 kW from Point A to Point D. Knowing that the maximum allowable couples that can be applied to shafts AB and CD are $25\ \text{N}\cdot\text{m}$ and $80\ \text{N}\cdot\text{m}$, respectively, determine the required minimum speed of shaft AB.

SOLUTION

Power.

$$2.4\ \text{kW} = 2400\ \text{W}$$

$$M_{AB} < 25\ \text{N}\cdot\text{m}$$

$$P = M_{AB}\omega_{AB}$$

$$\min \omega_{AB} = \frac{P}{\max M_{AB}} = \frac{2400}{25} = 96\ \text{rad/s}$$

$$M_{CD} < 80\ \text{N}\cdot\text{m}$$

$$P = M_{CD}\omega_{CD}$$

$$\min \omega_{CD} = \frac{P}{\max M_{CD}} = \frac{2400}{80} = 30\ \text{rad/s}$$

Kinematics.

$$r_A\omega_{AB} = r_C\omega_{CD}$$

$$\min \omega_{AB} = \frac{r_C}{r_A}(\min \omega_{CD})$$

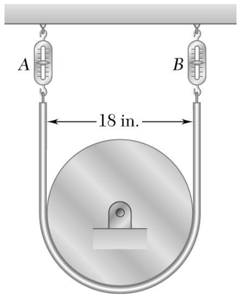
$$= \left(\frac{120}{30}\right)(30)$$

$$= 120\ \text{rad/s}$$

Choose the larger value for $\min \omega_{AB}$. $\min \omega_{AB} = 120\ \text{rad/s}$

$\min \omega_{AB} = 1146\ \text{rpm}$ ◀

PROBLEM 17.51

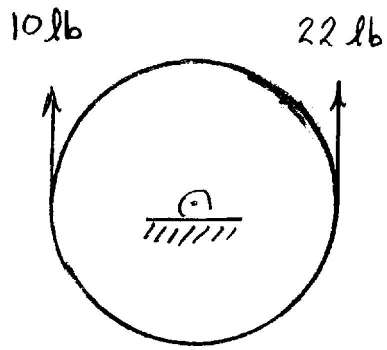


The experimental setup shown is used to measure the power output of a small turbine. When the turbine is operating at 200 rpm, the readings of the two spring scales are 10 and 22 lb, respectively. Determine the power being developed by the turbine.

SOLUTION

Angular velocity. $\omega = 200 \text{ rpm} = 20.944 \text{ rad/s}$

Moments about the fixed axle.



$$M = (22 \text{ lb} - 10 \text{ lb})\left(\frac{9}{12}\text{ft}\right) = 9 \text{ lb}\cdot\text{ft}$$

$$\text{Power} = M\omega = (9)(20.994) = 188.5 \text{ lb}\cdot\text{ft/s}$$

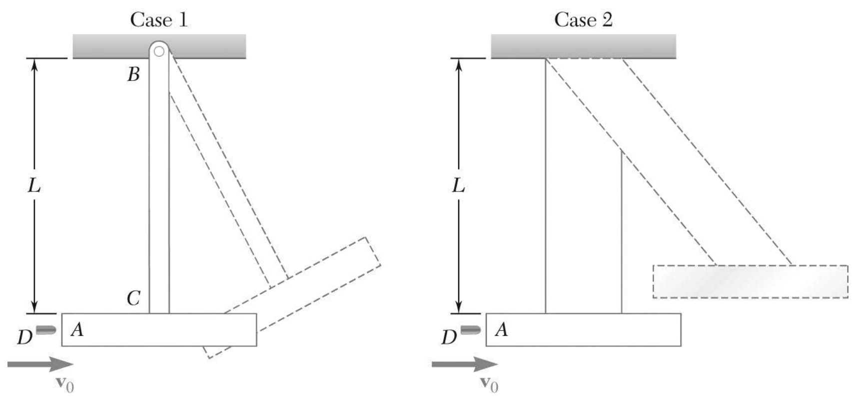
$$\frac{188.5 \text{ lb}\cdot\text{ft/s}}{550 \text{ lb}\cdot\text{ft/s/hp}} =$$

Power = 0.343 hp ◀

PROBLEM 17.CQ6

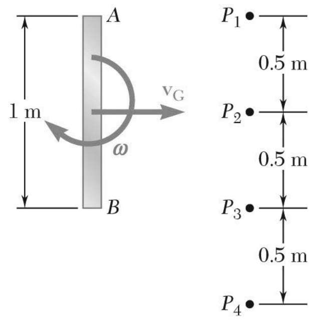
Slender bar *A* is rigidly connected to a massless rod *BC* in Case 1 and two massless cords in Case 2 as shown. The vertical thickness of bar *A* is negligible compared to *L*. If bullet *D* strikes *A* with a speed v_0 and becomes embedded in it, how will the speeds of the center of gravity of *A* immediately after the impact compare for the two cases?

(*a*) Case 1 will be larger.
(*b*) Case 2 will be larger.
(*c*) The speeds will be the same.



SOLUTION

Answer: (*b*)



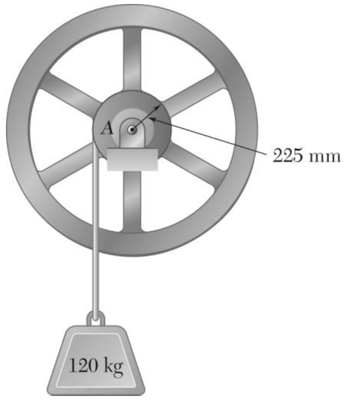
PROBLEM 17.CQ7

A 1-m long uniform slender bar *AB* has an angular velocity of 12 rad/s and its center of gravity has a velocity of 2 m/s as shown. About which point is the angular momentum of *A* smallest at this instant?

(*a*) P_1
(*b*) P_2
(*c*) P_3
(*d*) P_4
(*e*) It is the same about all the points.

SOLUTION

Answer: (*a*)

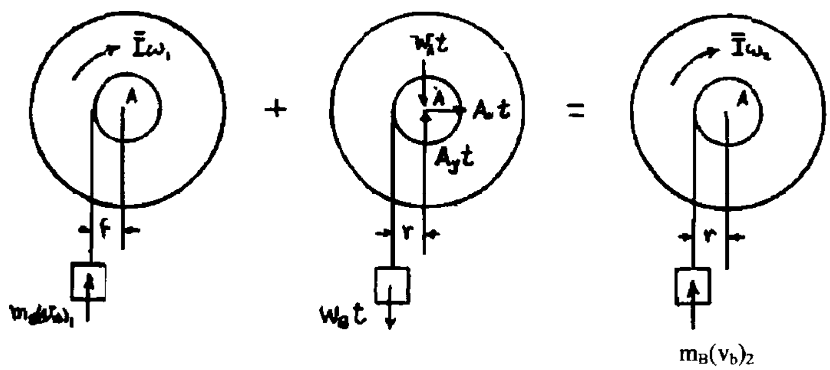


PROBLEM 17.F1

The 350-kg flywheel of a small hoisting engine has a radius of gyration of 600 mm. If the power is cut off when the angular velocity of the flywheel is 100 rpm clockwise, draw an impulse-momentum diagram that can be used to determine the time required for the system to come to rest.

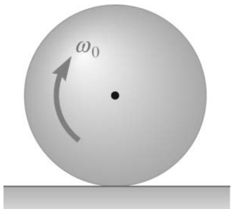
SOLUTION

Answer:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

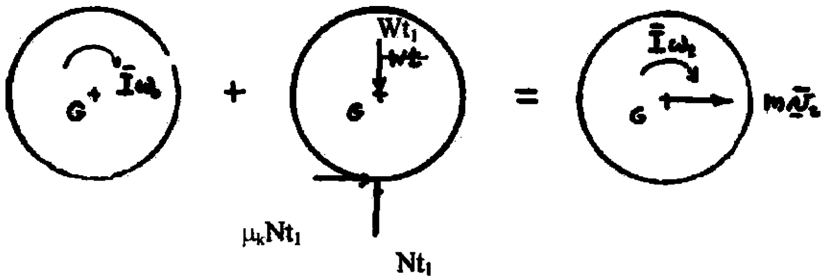
PROBLEM 17.F2



A sphere of radius r and mass m is placed on a horizontal floor with no linear velocity but with a clockwise angular velocity $\boldsymbol{\omega}_0$. Denoting by μ_k the coefficient of kinetic friction between the sphere and the floor, draw the impulse-momentum diagram that can be used to determine the time t_1 at which the sphere will start rolling without sliding.

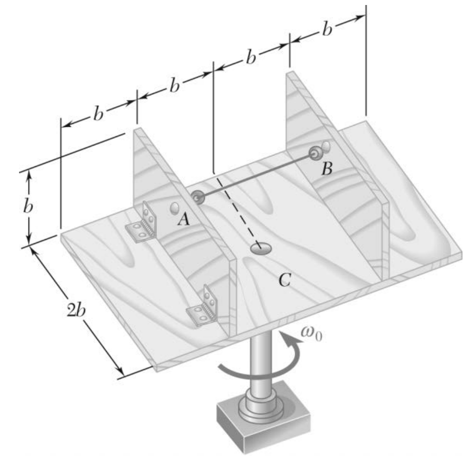
SOLUTION

Answer:



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to2}$ = Syst. Momenta$_2$

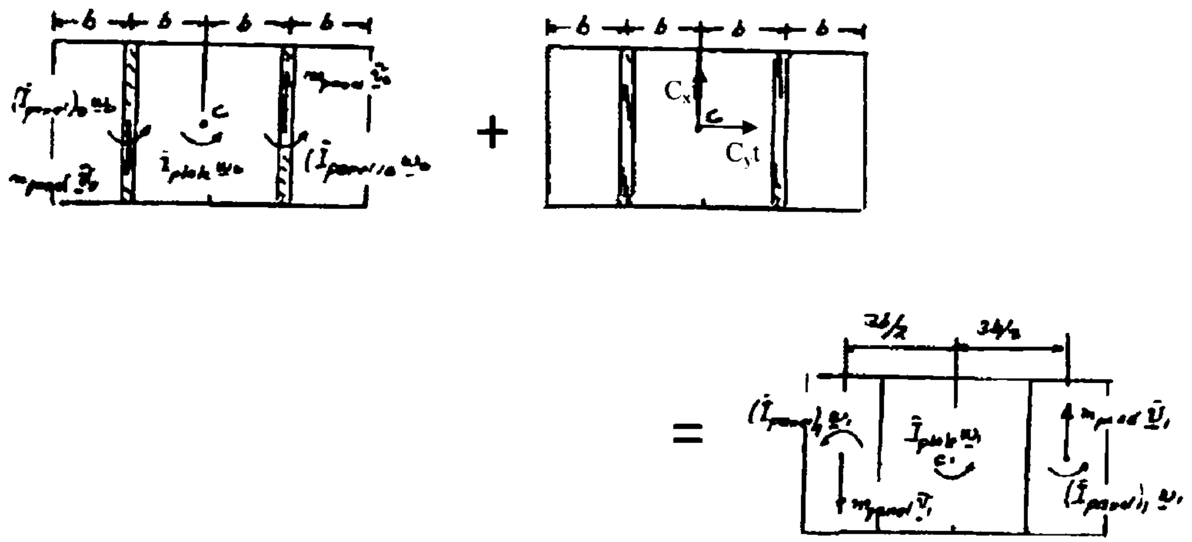
PROBLEM 17.F3



Two panels *A* and *B* are attached with hinges to a rectangular plate and held by a wire as shown. The plate and the panels are made of the same material and have the same thickness. The entire assembly is rotating with an angular velocity ω_0 when the wire breaks. Draw the impulse-momentum diagram that is needed to determine the angular velocity of the assembly after the panels have come to rest against the plate.

SOLUTION

Answer:



PROBLEM 17.52

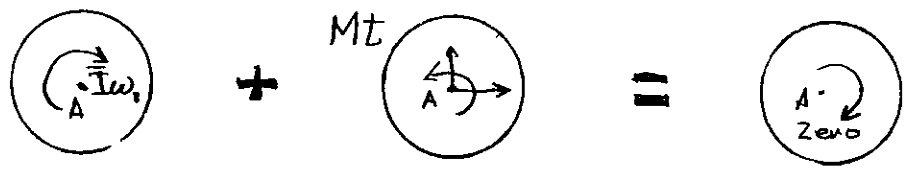
The rotor of an electric motor has a mass of 25 kg, and it is observed that 4.2 min is required for the rotor to coast to rest from an angular velocity of 3600 rpm. Knowing that kinetic friction produces a couple of magnitude 1.2 N·m, determine the centroidal radius of gyration for the rotor.

SOLUTION

Coasting time: $t = 4.2 \text{ min} = 252 \text{ s}$

Initial angular velocity: $\omega_1 = 3600 \text{ rpm} = 120\pi \text{ rad/s}$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **ΣSyst. Impulses.**$_{1\to 2}$ = **Syst. Momenta**$_2$

↺ Moments about axle A: $\bar{I}\omega_1 - Mt = 0$

$$\bar{I} = \frac{Mt}{\omega_1}$$

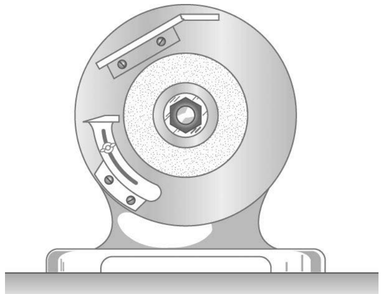
$$= \frac{(1.2 \text{ N}\cdot\text{m})(252 \text{ s})}{120\pi \text{ rad/s}}$$

$$= 0.80214 \text{ kg}\cdot\text{m}^2$$

$$\bar{I} = m\bar{k}^2$$

Radius of gyration: $\bar{k} = \sqrt{\frac{\bar{I}}{m}} = \sqrt{\frac{0.80214 \text{ kg}\cdot\text{m}^2}{25 \text{ kg}}} = 0.1791 \text{ m}$

$\bar{k} = 179.1 \text{ mm}$ ◀



PROBLEM 17.53

A small grinding wheel is attached to the shaft of an electric motor which has a rated speed of 3600 rpm. When the power is turned off, the unit coasts to rest in 70 s. The grinding wheel and rotor have a combined weight of 6 lb and a combined radius of gyration of 2 in. Determine the average magnitude of the couple due to kinetic friction in the bearings of the motor.

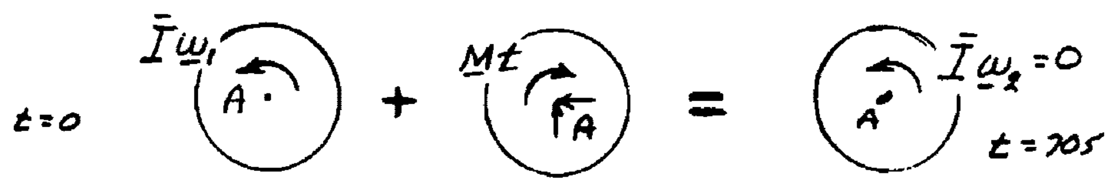
SOLUTION

Use the principle of impulse and momentum applied to the grinding wheel and rotor with

$$t_1 = 0 \qquad t_2 = 70 \text{ s}$$

$$\omega_1 = 3600 \text{ rpm} = 120\pi \text{ rad/s} \qquad \omega_2 = 0$$

Moment of inertia. $$\bar{I} = m\bar{k}^2 = \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2}\left(\frac{2}{12}\text{ft}\right)^2 = 0.00518 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

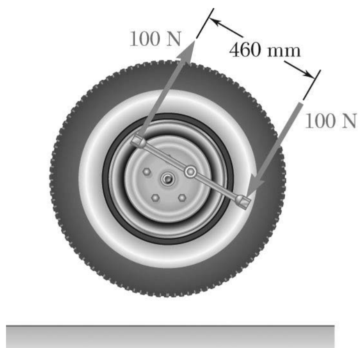
$+\circlearrowleft$ Moments about A: $$\bar{I}\omega_1 - Mt = 0$$

$$(0.00518)(120\pi) - M(70 \text{ s}) = 0$$

$$M = 0.02788 \text{ lb}\cdot\text{ft}$$

$$M = 0.33451 \text{ lb}\cdot\text{in.}$$

$$M = 0.335 \text{ lb}\cdot\text{in.} \blacktriangleleft$$



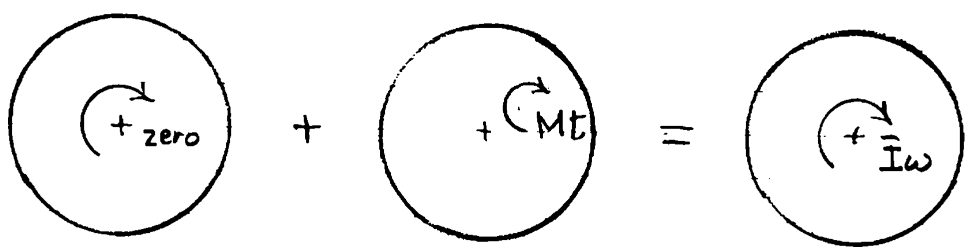
PROBLEM 17.54

A bolt located 50 mm from the center of an automobile wheel is tightened by applying the couple shown for 0.10 s. Assuming that the wheel is free to rotate and is initially at rest, determine the resulting angular velocity of the wheel. The wheel has a mass of 19 kg and has a radius of gyration of 250 mm.

SOLUTION

Moment of inertia. $\bar{I} = m\bar{k}^2 = (19\text{ kg})(0.25\text{ m})^2 = 1.1875\text{ kg-m}^2$

Applied couple. $M = (100\text{ N})(0.460\text{ m}) = 46\text{ N-m}$

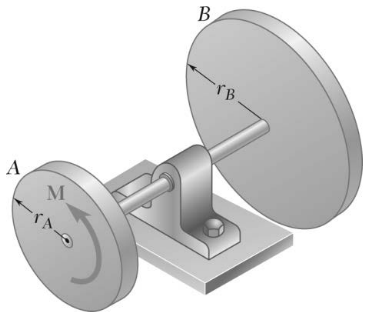


Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Moments about axle: $0 + Mt = \bar{I}\omega$

$$0 + (46\text{ N-m})(0.10\text{ s}) = (1.1875\text{ kg-m}^2)\omega$$

$\omega = 3.87$ rad/s ◀



PROBLEM 17.55

Two disks of the same thickness and same material are attached to a shaft as shown. The 8-lb disk A has a radius $r_A = 3$ in., and disk B has a radius $r_B = 4.5$ in. Knowing that a couple **M** of magnitude 20 lb · in. is applied to disk A when the system is at rest, determine the time required for the angular velocity of the system to reach 960 rpm.

SOLUTION

Weight of disk B.

$$W_B = \left(\frac{r_B}{r_A}\right)^2 W_B$$

$$= \left(\frac{4.5 \text{ in.}}{3 \text{ in.}}\right)^2 (8 \text{ lb})$$

$$= 18 \text{ lb}$$

Moment of inertia.

$$\bar{I} = \bar{I}_A + \bar{I}_B$$

$$= \frac{1}{2}\frac{8 \text{ lb}}{32.2}\left(\frac{3}{12}\text{ft}\right)^2 + \frac{1}{2}\frac{18 \text{ lb}}{32.2}\left(\frac{4.5}{12}\text{ft}\right)^2$$

$$= 0.04707 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

Angular velocity. $\omega_2 = 960 \text{ rpm} = 100.53 \text{ rad/s}$

Moment. $M = 20 \text{ lb}\cdot\text{in.} = 1.667 \text{ lb}\cdot\text{ft}$

Principle of impulse and momentum.

Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

+↻ Moments about C: $0 + Mt = \bar{I}\omega_2$

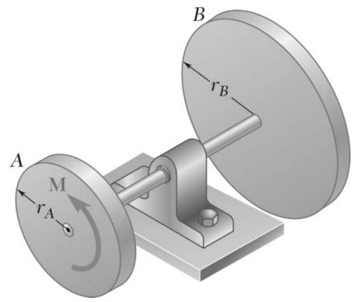
Required time.

$$t = \frac{\bar{I}\omega_2}{M}$$

$$= \frac{(0.04707 \text{ lb}\cdot\text{ft}\cdot\text{s}^2)(100.53 \text{ rad/s})}{1.667 \text{ lb}\cdot\text{ft}}$$

$$t = 2.839 \text{ s}$$

$t = 2.84$ s ◀



PROBLEM 17.56

Two disks of the same thickness and same material are attached to a shaft as shown. The 3-kg disk A has a radius $r_A = 100$ mm, and disk B has a radius $r_B = 125$ mm. Knowing that the angular velocity of the system is to be increased from 200 rpm to 800 rpm during a 3-s interval, determine the magnitude of the couple **M** that must be applied to disk A.

SOLUTION

Mass of disk B.

$$m_B = \left(\frac{r_B}{r_A}\right)^2 m_A$$

$$= \left(\frac{125 \text{ mm}}{100 \text{ mm}}\right)^2 3 \text{ kg}$$

$$= 4.6875 \text{ kg}$$

Moment of inertia.

$$\bar{I} = \bar{I}_A + \bar{I}_B$$

$$= \frac{1}{2}(3 \text{ kg})(0.1 \text{ m})^2 + \frac{1}{2}(4.6875 \text{ kg})(0.125 \text{ m})^2$$

$$= 0.05162 \text{ kg} \cdot \text{m}^2$$

Angular velocities.

$$\omega_1 = 200 \text{ rpm} = 20.944 \text{ rad/s}$$

$$\omega_2 = 800 \text{ rpm} = 83.776 \text{ rad/s}$$

Principle of impulse and momentum.

Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+↻ Moments about B: $\bar{I}\omega_1 + Mt = \bar{I}\omega_2$

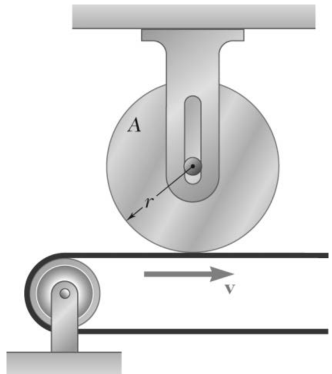
Couple M.

$$M = \frac{\bar{I}}{t}(\omega_2 - \omega_1)$$

$$= \frac{0.05162 \text{ kg} \cdot \text{m}^2}{3 \text{ s}}(83.776 \text{ rad/s} - 20.944 \text{ rad/s})$$

$M = 1.081 \text{ N} \cdot \text{m}$ ◀

PROBLEM 17.57

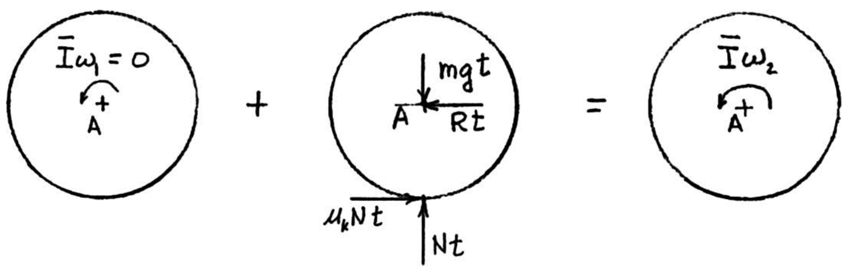


A disk of constant thickness, initially at rest, is placed in contact with a belt that moves with a constant velocity **v**. Denoting by μ_k the coefficient of kinetic friction between the disk and the belt, derive an expression for the time required for the disk to reach a constant angular velocity.

SOLUTION

Moment of inertia. $\bar{I} = \frac{1}{2}mr^2$

Final state of constant angular velocity. $\omega_2 = \frac{v}{r}$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

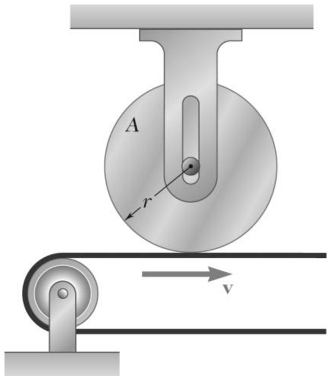
$+\uparrow$ y components: $0 + Nt - mgt = 0 \quad N = mg$

$+\circlearrowleft$ Moments about A: $0 + \mu_k Ntr = \bar{I}\omega_2$

$$t = \frac{\bar{I}\omega_2}{\mu_k mgr} = \frac{\frac{1}{2}mr^2\frac{v}{r}}{\mu_k mgr} = \frac{v}{2\mu_k g}$$

$$t = \frac{v}{2\mu_k g} \blacktriangleleft$$

PROBLEM 17.58

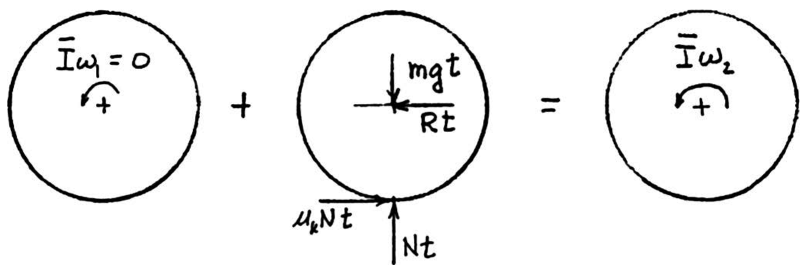


Disk A, of weight 5 lb and radius $r = 3$ in., is at rest when it is placed in contact with a belt which moves at a constant speed $v = 50$ ft/s. Knowing that $\mu_k = 0.20$ between the disk and the belt, determine the time required for the disk to reach a constant angular velocity.

SOLUTION

Moment of inertia. $\bar{I} = \frac{1}{2}mr^2$

Final state of constant angular velocity. $\omega_2 = \frac{v}{r}$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$+\uparrow$ y components: $0 + Nt - mgt = 0 \qquad N = mg$

$+\circlearrowleft$ Moments about A: $0 + \mu_k Ntr = \bar{I}\omega_2$

$$t = \frac{\bar{I}\omega_2}{\mu_k mgr} = \frac{\frac{1}{2}mr^2\frac{v}{r}}{\mu_k mgr} = \frac{v}{2\mu_k g}$$

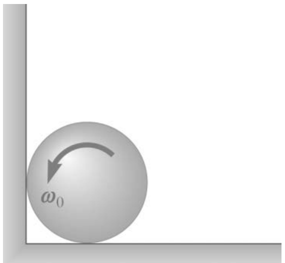
$$t = \frac{v}{2\mu_k g}$$

Data: $v = 50$ ft/s

$\mu_k = 0.20$

$$t = \frac{50}{(2)(0.20)(32.2)}$$

$t = 3.88$ s ◀



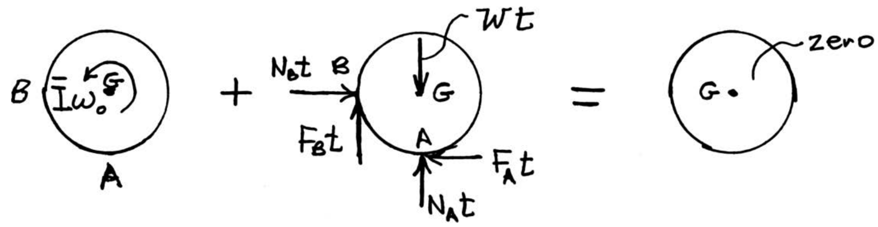
PROBLEM 17.59

A cylinder of radius r and weight W with an initial counterclockwise angular velocity ω_0 is placed in the corner formed by the floor and a vertical wall. Denoting by μ_k the coefficient of kinetic friction between the cylinder and the wall and the floor derive an expression for the time required for the cylinder to come to rest.

SOLUTION

For the cylinder $\quad \bar{I} = \frac{1}{2}mr^2, \quad W = mg$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Linear momentum $\xrightarrow{+}$: $\quad 0 + N_B t - F_A t = 0$

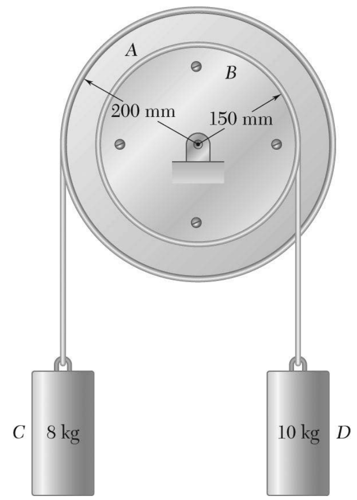
$$N_B = F_A$$

Linear momentum $+\uparrow$: $\quad 0 + N_A t + F_B t - Wt = 0$

$$N_A + F_B = N_A + \mu_k N_B$$
$$= N_A + \mu_k F_A + N_A + \mu_k^2 N_A = W$$
$$N_A = \frac{W}{1+\mu_k^2}$$
$$F_A = \mu_k N_A = \frac{\mu_k W}{1+\mu_k^2}$$
$$N_B = \frac{\mu_k W}{1+\mu_k^2}$$
$$F_B = \frac{\mu_k^2 W}{1+\mu_k^2}$$

$+\circlearrowleft$ Moments about G: $\quad \bar{I}\omega_0 - F_A rt - F_B rt = 0$

$$t = \frac{\bar{I}\omega_0}{(F_A + F_B)r} = \frac{(1+\mu_k^2)\bar{I}\omega_0}{\mu_k(1+\mu_k)Wr} \qquad t = \frac{1+\mu_k^2}{2\mu_k(1+\mu_k)}\frac{r\omega_0}{g} \blacktriangleleft$$



PROBLEM 17.60

Two uniform disks and two cylinders are assembled as indicated. Disk A has a mass of 10 kg and disk B has a mass of 6 kg. Knowing that the system is released from rest, determine the time required for cylinder C to have a speed of 0.5 m/s.

Disks A and B are bolted together and the cylinders are attached to separate cords wrapped on the disks.

SOLUTION

Moments of inertia.

Disk A: $$I_A = \frac{1}{2}m_A r_A^2 = \frac{1}{2}(10\text{ kg})(0.200\text{ m})^2 = 0.2\text{ kg}\cdot\text{m}^2$$

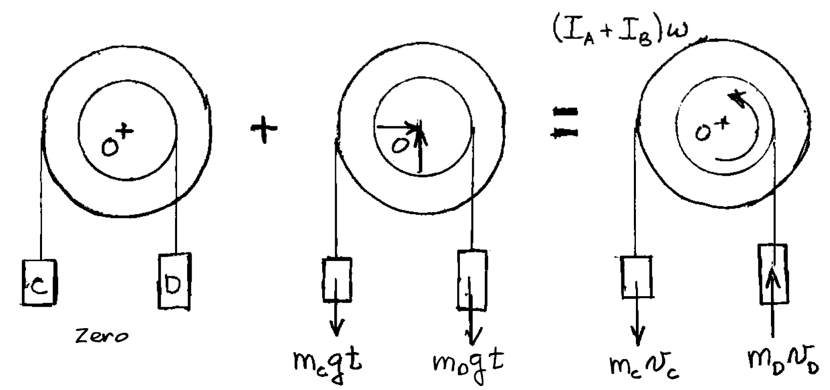
Disk B: $$I_B = \frac{1}{2}m_B r_B^2 = \frac{1}{2}(6\text{ kg})(0.150\text{ m})^2 = 0.0675\text{ kg}\cdot\text{m}^2$$

Kinematics: $$\mathbf{v}_C = v_C \downarrow \qquad \boldsymbol{\omega}_A = \omega_A \circlearrowleft = \frac{v_C}{r_A} \circlearrowleft$$

$$\boldsymbol{\omega}_B = \boldsymbol{\omega}_A = \boldsymbol{\omega}$$

$$\mathbf{v}_D = v_D \uparrow = \omega r_B \uparrow = \frac{r_B}{r_A} v_C \uparrow$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROBLEM 17.60 (Continued)

+↻ Moments about axle 0.

$$0 + m_0 gtr_A - m_D gtr_B = m_C v_C r_A + m_D v_D r_B + (I_A + I_B)\omega \quad (1)$$

Data: $v_C = 0.5\text{ m/s} \qquad t = ?$

$$m_C gr_A = (8\text{ kg})(9.81\text{ m/s}^2)(0.200\text{ m}) = 15.696\text{ N}\cdot\text{m}$$

$$m_D gr_B = (10\text{ kg})(9.81\text{ m/s}^2)(0.150\text{ m}) = 14.715\text{ N}\cdot\text{m}$$

$$m_C gr_A - m_D gr_B = 0.981\text{ N}\cdot\text{m}$$

$$m_C v_C r_A = (8\text{ kg})(0.5\text{ m/s})(0.200\text{ m}) = 0.8\text{ kg}\cdot\text{m}^2/\text{s}$$

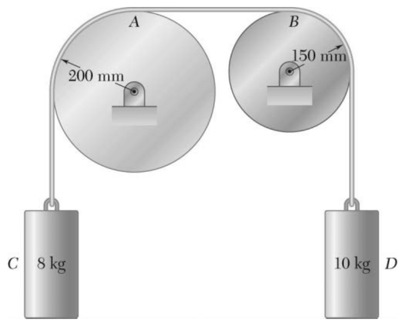
$$m_D v_D r_B = (10\text{ kg})\left(\frac{0.150\text{ m}}{0.200\text{ m}}\right)(0.5\text{ m/s})(0.150\text{ m}) = 0.5625\text{ kg}\cdot\text{m}^2/\text{s}$$

$$(I_A + I_B)\omega = (0.2675\text{ kg}\cdot\text{m}^2)\left(\frac{0.5\text{ m/s}}{0.200\text{ m}}\right) = 0.66875\text{ kg}\cdot\text{m}^2/\text{s}$$

$$m_C v_C r_A + m_D v_D r_B + (I_A + I_B)\omega = 2.03125\text{ kg}\cdot\text{m}^2/\text{s}$$

Solving Eq. (1) for t, $$t = \frac{2.03125\text{ kg}\cdot\text{m}^2/\text{s}}{0.981\text{ N}\cdot\text{m}}$$

$t = 2.07$ s ◀



PROBLEM 17.61

Two uniform disks and two cylinders are assembled as indicated. Disk A has a mass of 10 kg and disk B has a mass of 6 kg. Knowing that the system is released from rest, determine the time required for cylinder C to have a speed of 0.5 m/s.

The cylinders are attached to a single cord that passes over the disks. Assume that no slipping occurs between the cord and the disks.

SOLUTION

Moments of inertia.

Disk A: $I_A = \frac{1}{2}m_A r_A^2 = \frac{1}{2}(10\text{ kg})(0.200\text{ m})^2 = 0.2\text{ kg}\cdot\text{m}^2$

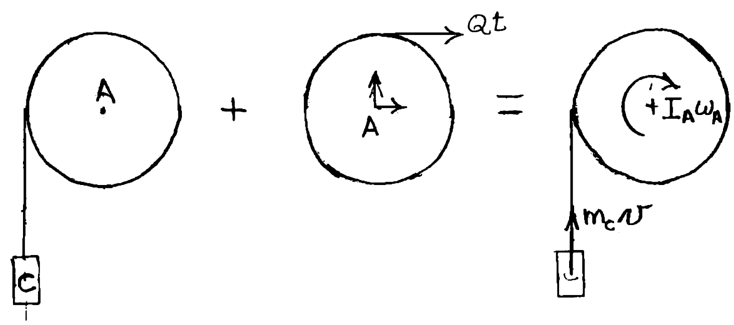
Disk B: $I_B = \frac{1}{2}m_B r_B^2 = \frac{1}{2}(6\text{ kg})(0.150\text{ m})^2 = 0.0675\text{ kg}\cdot\text{m}^2$

Kinematics: $\mathbf{v}_C = v\uparrow \qquad \mathbf{v}_D = v\downarrow$

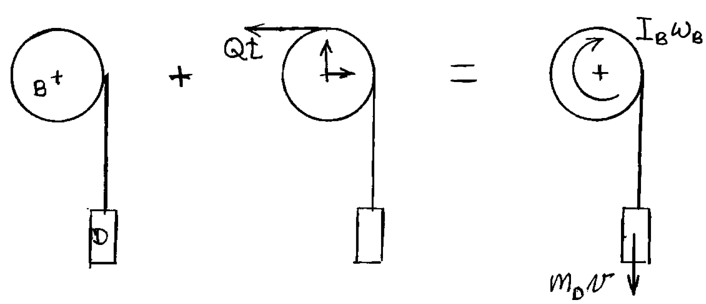
$\boldsymbol{\omega}_A = \frac{v}{r_A}\circlearrowright \qquad \boldsymbol{\omega}_B = \frac{v}{r_B}\circlearrowright$

Principle of impulse and momentum.

Disk A and cylinder C



Disk B and cylinder D



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROBLEM 17.61 (Continued)

Disk A and cylinder C. $\circlearrowleft +$ Moments about A:

$$Qtr_A - m_C gtr_A = m_A vr_A + I_A \omega_A \qquad (1)$$

Disk B and cylinder D. $\circlearrowleft +$ Moments about B:

$$-Qtr_B - m_D gtr_B = m_D r_B v + I_B \omega_B \qquad (2)$$

To eliminate Qt divide Equation (1) by r_A and Equation (2) by r_B, and then add the resulting equations.

$$(m_D - m_C)gt = \left(m_A + \frac{I_A}{r_A^2} + m_B + \frac{I_B}{r_B^2} \right) v \qquad (3)$$

Data: $v = 0.5\text{ m/s}$ $\quad t = ?$

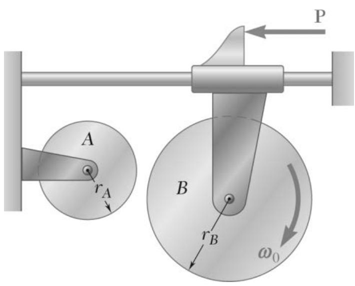
$$(m_D - m_C)g = (2\text{ kg})(9.81\text{ m/s}^2) = 19.62\text{ N}$$

$$m_C + \frac{I_A}{r_A^2} + m_D + \frac{I_B}{r_B^2} = 8\text{ kg} + \frac{0.2\text{ kg}\cdot\text{m}^2}{(0.200\text{ m})^2} + 10\text{ kg} + \frac{0.0675\text{ kg}\cdot\text{m}^2}{(0.150\text{ m})^2} = 26\text{ kg}$$

Equation (3) becomes $\quad (19.62\text{ N})t = (26\text{ kg})(0.5\text{ m})$

$$t = 0.66259\text{ s}$$

$t = 0.663\text{ s}$ ◀



PROBLEM 17.62

Disk B has an initial angular velocity ω_0 when it is brought into contact with disk A which is at rest. Show that the final angular velocity of disk B depends only on ω_0 and the ratio of the masses m_A and m_B of the two disks.

SOLUTION

Let Points A and B be the centers of the two disks and Point C be the contact point between the two disks.
Let ω_A and ω_B be the final angular velocities of disks A and B, respectively, and let v_C be the final velocity at C common to both disks.

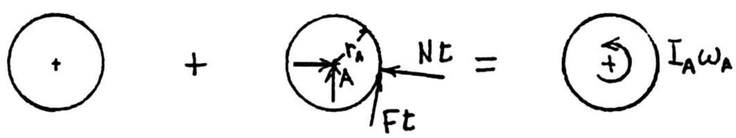
Kinematics: No slipping $\qquad v_C = r_A\omega_A = r_B\omega_B$

Moments of inertia. Assume that both disks are uniform cylinders.

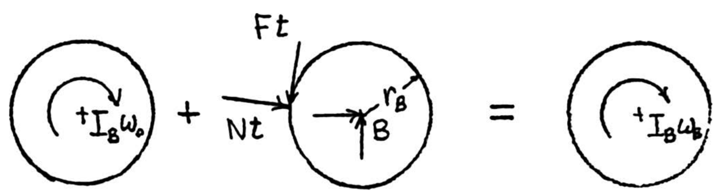
$$I_A = \frac{1}{2}m_A r_A^2 \qquad I_B = \frac{1}{2}m_B r_B^2$$

Principle of impulse and momentum.

Disk A



Disk B



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Disk A: ↺ Moments about A: $\qquad 0 + r_A Ft = I_A\omega_A$

$$Ft = \frac{I_A\omega_A}{r_A} = \frac{1}{2}\frac{m_A r_A^2 v_C}{r_A^2}$$

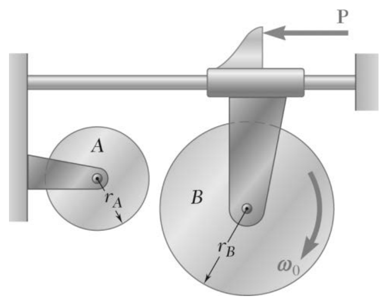
$$= \frac{1}{2}m_A v_C$$

$$= \frac{1}{2}m_A r_B\omega_B$$

Disk B: ↻ Moments about B: $\qquad I_B\omega_0 - r_B Ft = I_B\omega_B$

$$\frac{1}{2}m_B r_B^2\omega_0 - r_B\left(\frac{1}{2}m_A r_B\omega_B\right) = \frac{1}{2}mr_B^2\omega_B$$

$$\omega_B = \frac{\omega_0}{1+\frac{m_A}{m_B}} \quad ◀$$



PROBLEM 17.63

The 7.5-lb disk A has a radius $r_A = 6$ in. and is initially at rest. The 10-lb disk B has a radius $r_B = 8$ in. and an angular velocity ω_0 of 900 rpm when it is brought into contact with disk A. Neglecting friction in the bearings, determine (*a*) the final angular velocity of each disk, (*b*) the total impulse of the friction force exerted on disk A.

SOLUTION

Let Points A and B be the centers of the two disks and Point C be the contact point between the two disks.

Let ω_A and ω_B be the final angular velocities of disks A and B, respectively, and let v_C be the final velocity at C common to both disks.

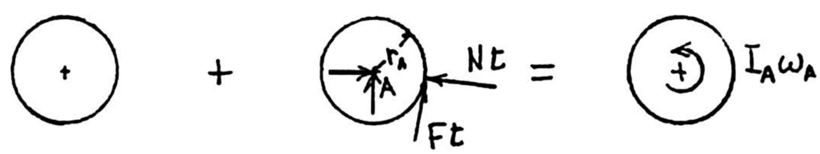
Kinematics: No slipping $\qquad v_C = r_A\omega_A = r_B\omega_B$

Moments of inertia. Assume that both disks are uniform cylinders.

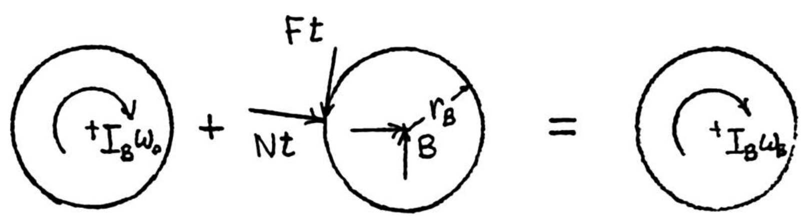
$$I_A = \frac{1}{2}m_A r_A^2 \qquad I_B = \frac{1}{2}m_B r_B^2$$

Principle of impulse and momentum.

Disk A



Disk B



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Disk A: ↻ Moments about A: $\qquad 0 + r_A F t = I_A\omega_A$

$$Ft = \frac{I_A\omega_A}{r_A}$$

$$= \frac{1}{2}\frac{m_A r_A^2 v_C}{r_A^2} = \frac{1}{2}m_A v_C$$

$$= \frac{1}{2}m_A r_B \omega_B$$

Disk B: ↻ Moments about B:

$$I_B\omega_0 - r_B F t = I_B \omega_B$$

$$\frac{1}{2}m_B r_B^2 \omega_0 - r_B\left(\frac{1}{2}m_A r_B \omega_B\right) = \frac{1}{2}m r_B^2 \omega_B$$

$$\omega_B = \frac{\omega_0}{1+\frac{m_A}{m_B}}$$

Data:

$$m_A = \frac{7.5}{32.2} = 0.23292 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\frac{m_A}{m_B} = \frac{W_A}{W_B} = \frac{7.5}{10} = 0.75$$

$$r_B = \frac{8}{12} \text{ ft}$$

$$\frac{r_B}{r_A} = \frac{8}{6}$$

$$\omega_0 = 900 \text{ rpm} = 30\pi \text{ rad/s}$$

(a)

$$\omega_B = \frac{\omega_0}{1+0.75} = \frac{30\pi}{1.75} = 53.856 \text{ rad/s}$$

$$\omega_A = \frac{r_B}{r_A}\omega_B = \left(\frac{8}{6}\right)(53.856) = 71.808 \text{ rad/s}$$

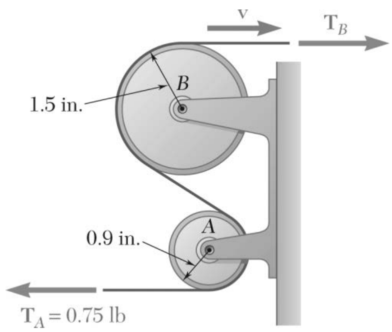
$\boldsymbol{\omega}_A = 686$ rpm ↺ ◄

$\boldsymbol{\omega}_B = 514$ rpm ↻ ◄

(b)

$$Ft = \frac{1}{2}\frac{(0.23292)\left(\frac{8}{12}\right)(30\pi)}{1+0.75}$$

$\mathbf{F}t = 4.18$ lb·s ↑ ◄



PROBLEM 17.64

A tape moves over the two drums shown. Drum A weighs 1.4 lb and has a radius of gyration of 0.75 in., while drum B weighs 3.5 lb and has a radius of gyration of 1.25 in. In the lower portion of the tape the tension is constant and equal to $T_A = 0.75$ lb. Knowing that the tape is initially at rest, determine (*a*) the required constant tension T_B if the velocity of the tape is to be $v = 10$ ft/s after 0.24 s, (*b*) the corresponding tension in the portion of tape between the drums.

SOLUTION

Kinematics. Drums A and B rotate about fixed axes. Let v be the tape velocity in ft/s.

$$v = r_A\omega_A = \frac{0.9}{12}\omega_A \qquad \omega_A = 13.3333v$$

$$v = r_B\omega_B = \frac{1.5}{12}\omega_B \qquad \omega_B = 8v$$

Moments of inertia.

$$\bar{I}_A = m_A\bar{k}_A^2 = \left(\frac{1.4}{32.2}\right)\left(\frac{0.75}{12}\right)^2 = 169.837\times10^{-6}\,\text{lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\bar{I}_B = m_B\bar{k}_B^2 = \left(\frac{3.5}{32.2}\right)\left(\frac{1.25}{12}\right)^2 = 1.17942\times10^{-3}\,\text{lb}\cdot\text{s}^2\cdot\text{ft}$$

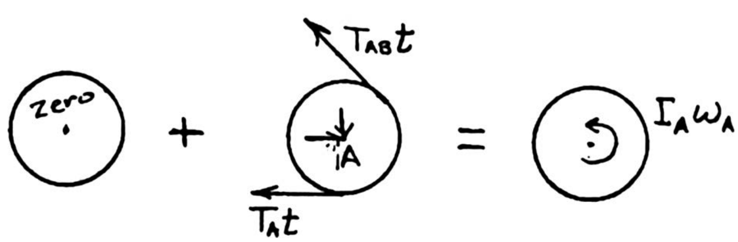
State 1. $t = 0 \qquad v = 0 \qquad \omega_A = \omega_B = 0$

State 2. $t = 0.24$ s, $\qquad v = 10$ ft/s

$$\omega_A = (13.3333)(10) = 133.333 \text{ rad/s} \circlearrowright$$

$$\omega_B = (8)(10) = 80 \text{ rad/s} \circlearrowleft$$

Drum A.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

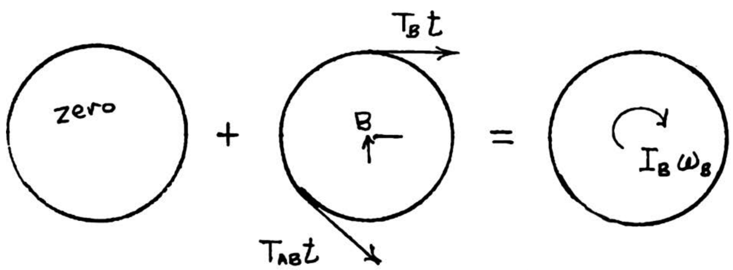
PROBLEM 17.64 (Continued)

+↻ Moments about A: $0 + r_A T_{AB} t - r_A T_A t = I_A \omega_A$

$$0 + \left(\frac{0.9}{12}\right)(T_{AB}t) - \left(\frac{0.9}{12}\right)(0.75)(0.24) = (169.837 \times 10^{-6})(133.333)$$

$$T_{AB}t = 0.48193 \text{ lb} \cdot \text{s}$$

Drum B.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to2}$ = Syst. Momenta$_2$

+↺ Moments about B: $0 + r_B T_B t - r_B T_{AB} t = I_B \omega_B$

$$0 + \frac{1.5}{12}(T_B t) - \left(\frac{1.5}{12}\right)(0.48193) = (1.17942 \times 10^{-3})(80)$$

$$T_B t = 1.23676 \text{ lb} \cdot \text{s}$$

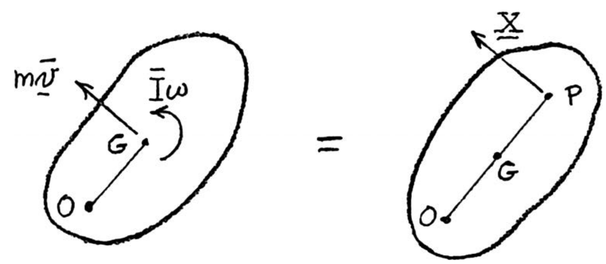
(*a*) $T_B = \frac{T_B t}{t} = \frac{1.23676}{0.24}$ $T_B = 5.15 \text{ lb}$ ◀

(*b*) $T_{AB} = \frac{T_{AB} t}{t} = \frac{0.48193}{0.24}$ $T_{AB} = 2.01 \text{ lb}$ ◀

PROBLEM 17.65

Show that the system of momenta for a rigid slab in plane motion reduces to a single vector, and express the distance from the mass center G to the line of action of this vector in terms of the centroidal radius of gyration $\bar{k}$ of the slab, the magnitude $\bar{v}$ of the velocity of G, and the angular velocity ω.

SOLUTION

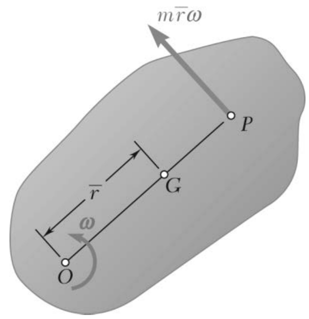


Syst. Momenta = Single Vector

Components parallel to $m\bar{\mathbf{v}}$: $m\bar{\mathbf{v}} = \mathbf{X}$ $\qquad \mathbf{X} = m\bar{\mathbf{v}}$ ◀

Moments about G: $\bar{I}\,\omega = (m\bar{v})\,d$

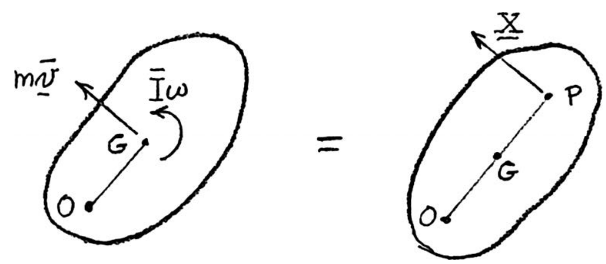
$$d = \frac{\bar{I}\,\omega}{m\bar{v}} = \frac{m\bar{k}^2\omega}{m\bar{v}} \qquad d = \frac{\bar{k}^2\omega}{\bar{v}} \blacktriangleleft$$



PROBLEM 17.66

Show that, when a rigid slab rotates about a fixed axis through O perpendicular to the slab, the system of the momenta of its particles is equivalent to a single vector of magnitude $m\bar{r}\omega$, perpendicular to the line OG, and applied to a Point P on this line, called the *center of percussion*, at a distance $GP = \bar{k}^2/\bar{r}$ from the mass center of the slab.

SOLUTION



Kinematics. Point O is fixed. $\bar{v} = \bar{r}\omega$

System momenta.

Components parallel to $m\bar{\mathbf{v}}$: $X = m\bar{v} = m\bar{r}\omega$ $\qquad X = m\bar{r}\omega$ ◀

Moments about G: $(GP)X = \bar{I}\omega$

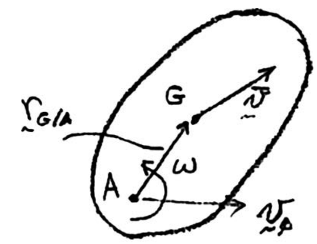
$$(GP)m\,r\,\omega = m\bar{k}^2\omega \qquad (GP) = \frac{\bar{k}^2}{\bar{r}}$$ ◀

PROBLEM 17.67

Show that the sum $\mathbf{H}_A$ of the moments about a Point A of the momenta of the particles of a rigid slab in plane motion is equal to $I_A\omega$, where ω is the angular velocity of the slab at the instant considered and I_A the moment of inertia of the slab about A, if and only if one of the following conditions is satisfied: (*a*) A is the mass center of the slab, (*b*) A is the instantaneous center of rotation, (*c*) the velocity of A is directed along a line joining Point A and the mass center G.

SOLUTION

Kinematics.

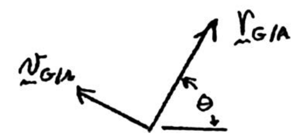


Let $\boldsymbol{\omega} = \omega\mathbf{k}$

and $\mathbf{r}_{G/A} = r_{G/A} \measuredangle\, \theta$

Then, $\mathbf{v}_{G/A} = \boldsymbol{\omega} \times \mathbf{r}_{G/A} = (\omega r_{G/A}) \measuredangle\, \beta$

where $\beta = \theta + 90°$

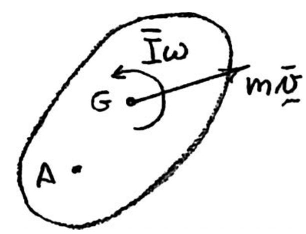


Also $\overline{\mathbf{v}} = \mathbf{v}_A + \mathbf{v}_{G/A}$

Define $\mathbf{h} = \mathbf{r}_{G/A} \times \mathbf{v}_{G/A}$

$$\mathbf{h} = (r_{G/A})(v_{G/A})\mathbf{k} = (r_{G/A})^2\omega\mathbf{k} = (r_{G/A})^2\boldsymbol{\omega}$$

System momenta. Moments about A:



$$\begin{aligned}\mathbf{H}_A &= \mathbf{r}_{G/A} \times m\overline{\mathbf{v}} + \bar{I}\boldsymbol{\omega} \\ &= \mathbf{r}_{G/A} \times m(\mathbf{v}_A + \mathbf{v}_{G/A}) + \bar{I}\,\omega \\ &= \mathbf{r}_{G/A} \times m\mathbf{v}_A + m\mathbf{r}_{G/A} \times \mathbf{v}_{G/A} + \bar{I}\boldsymbol{\omega} \\ &= \mathbf{r}_{G/A} \times m\mathbf{v}_A + m\mathbf{h} + \bar{I}\boldsymbol{\omega} \\ &= \mathbf{r}_{G/A} \times m\mathbf{v}_A + mr_{G/A}^2\boldsymbol{\omega} + \bar{I}\boldsymbol{\omega} \\ &= \mathbf{r}_{G/A} \times m\mathbf{v}_A + (m\,r_{G/A}^2 + \bar{I})\,\boldsymbol{\omega}\end{aligned}$$

The first term on the right hand side is equal to zero if

(*a*) $\mathbf{r}_{G/A} = 0$ (A is the mass center)

or (*b*) $\mathbf{v}_A = 0$ (A is the instantaneous center of rotation)

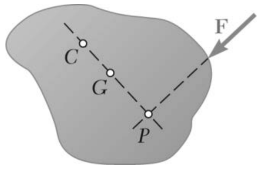
or (*c*) $\mathbf{r}_{G/A}$ is perpendicular to $\mathbf{v}_A$.

In the second term, $mr_{G/A}^2 + \bar{I} = I_A$ by the parallel axis theorem.

Thus, $\mathbf{H}_A = I_A\boldsymbol{\omega}$

when one or more of the conditions (*a*), (*b*) or (*c*) is satisfied.

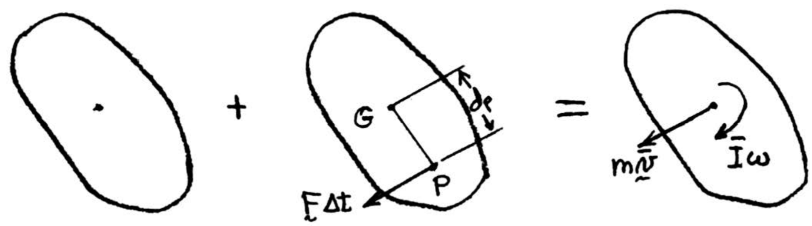
PROBLEM 17.68



Consider a rigid slab initially at rest and subjected to an impulsive force **F** contained in the plane of the slab. We define the *center of percussion P* as the point of intersection of the line of action of **F** with the perpendicular drawn from *G*. (*a*) Show that the instantaneous center of rotation *C* of the slab is located on line *GP* at a distance $GC = \overline{k}^2/GP$ on the opposite side of *G*. (*b*) Show that if the center of percussion were located at *C* the instantaneous center of rotation would be located at *P*.

SOLUTION

(*a*) Locate the instantaneous center *C* corresponding to center of percussion *P*. Let $d_P = GP$.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

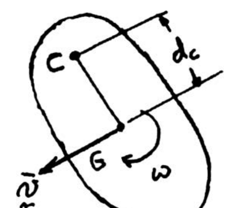
Components parallel to $\mathbf{F}\Delta t$: $0 + F\Delta t = m\bar{v}$

Moments about *G*: $0 + d_P(F\Delta t) = \overline{I}\,\omega$

Eliminate $F\Delta t$ to obtain

$$\frac{\bar{v}}{\omega} = \frac{\overline{I}}{md_P}$$

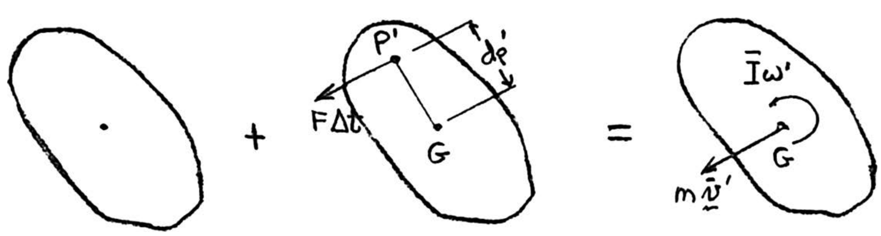
$$= \frac{\overline{k}^2}{d_P}$$



Kinematics. Locate Point *C*. $GC = d_C = \dfrac{\bar{v}}{\omega} = \dfrac{\overline{k}^2}{d_P}$ $\qquad GC = \dfrac{\overline{k}^2}{GP}$ ◀

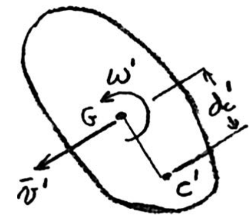
(*b*) Place the center of percussion at $P' = C$. Locate the corresponding instantaneous center C'. Let

$$d_{P'} = GP' = GC = d_C.$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROBLEM 17.68 (Continued)



Components parallel to $\mathbf{F}\Delta t$: $0 + F\Delta t = m\overline{v}'$

Moments about G: $0 + d_{P'}(F\Delta t) = \overline{I}\,\omega'$

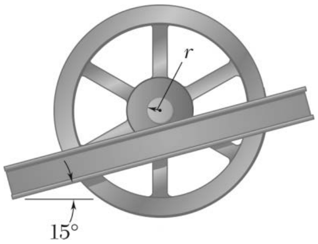
Eliminate $F\Delta t$ to obtain $\dfrac{\overline{v}'}{\omega'} = \dfrac{\overline{I}}{md_{P'}} = \dfrac{\overline{k}^2}{d_{P'}}$

Kinematics. Locate Point C'. $GC' = d_{C'} = \dfrac{\overline{v}'}{\omega'} = \dfrac{\overline{k}^2}{d_{P'}} = \dfrac{\overline{k}^2}{d_C}$

Using $d_C = d_{P'} = \dfrac{\overline{k}^2}{d_P}$ gives $d_{C'} = d_P$ or $GC' = GP$ ◀

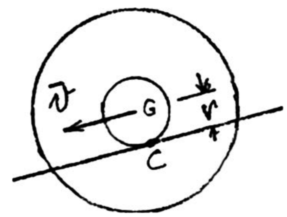
Thus Point C' coincides with Point P.

PROBLEM 17.69



A flywheel is rigidly attached to a 1.5-in.-radius shaft that rolls without sliding along parallel rails. Knowing that after being released from rest the system attains a speed of 6 in./s in 30 s, determine the centroidal radius of gyration of the system.

SOLUTION

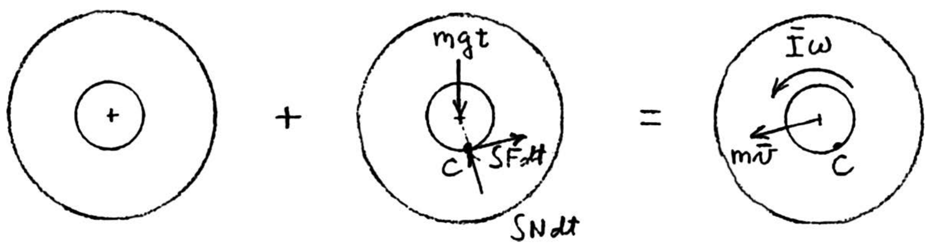


Kinematics. Rolling motion. Instantaneous center at C.

$$\bar{v} = v_G = r\omega$$

Moment of inertia. $\bar{I} = m\bar{k}^2$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

↺ Moments about C: $0 + (mgt)r\sin\beta = m\bar{v}r + \bar{I}\,\omega$

$$mgtr\sin\beta = m\left(r + \frac{\bar{k}^2}{r}\right)\bar{v}$$

Solving for $\bar{k}^2$, $\bar{k}^2 = r^2\left(\frac{gt\sin\beta}{\bar{v}} - 1\right)$

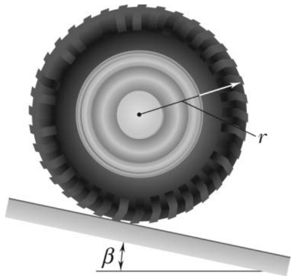
Data:

$$r = 1.5 \text{ in.} = 0.125 \text{ ft}$$
$$g = 32.2 \text{ ft/s}$$
$$t = 30 \text{ s}$$
$$\bar{v} = 6 \text{ in./s} = 0.5 \text{ ft/s}$$

$$\bar{k}^2 = (0.125)^2\left[\frac{(32.2)(30)\sin 15°}{0.5} - 1\right]$$
$$= 7.7974 \text{ ft}^2$$

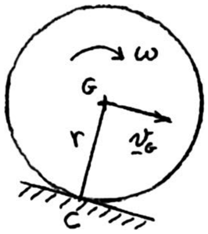
$\bar{k} = 2.79$ ft ◀

PROBLEM 17.70



A wheel of radius *r* and centroidal radius of gyration $\bar{k}$ is released from rest on the incline shown at time $t = 0$. Assuming that the wheel rolls without sliding, determine (*a*) the velocity of its center at time *t*, (*b*) the coefficient of static friction required to prevent slipping.

SOLUTION



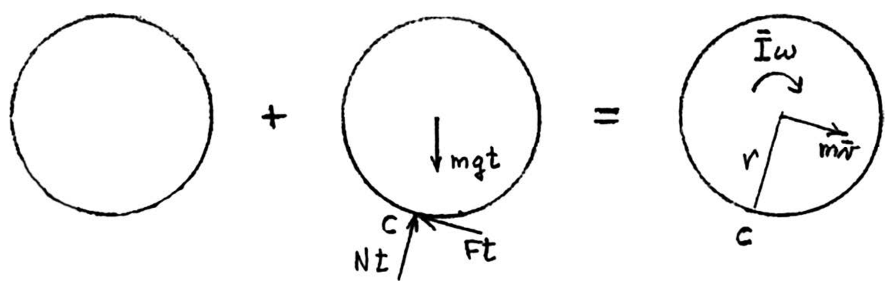
Kinematics. Rolling motion. Instantaneous center at *C*.

$$\bar{v} = v_G = r\omega$$

$$\omega = \frac{\bar{v}}{r}$$

Moment of inertia. $\bar{I} = m\bar{k}^2$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

↷ moments about *C*: $0 + (mgt) r \sin\beta = m\bar{v}r + \bar{I}\omega$

$$(mgr\sin\beta)t = mr\bar{v} + \frac{m\bar{k}^2\bar{v}}{r}$$

(*a*) *Velocity of Point G*. $\bar{\mathbf{v}} = \dfrac{r^2 gt\sin\beta}{r^2 + \bar{k}^2}$ ↘ β ◀

+↘ components parallel to incline:

$$0 + mgt\sin\beta - Ft = m\bar{v}$$

$$Ft = mgt\sin\beta - \frac{mr^2 gt\sin\beta}{r^2 + k^2}$$

$$= \frac{\bar{k}^2 mgt\sin\beta}{r^2 + k^2}$$

$+\nearrow$ components normal to incline:

$$0 + Nt - mgt\cos\beta = 0$$

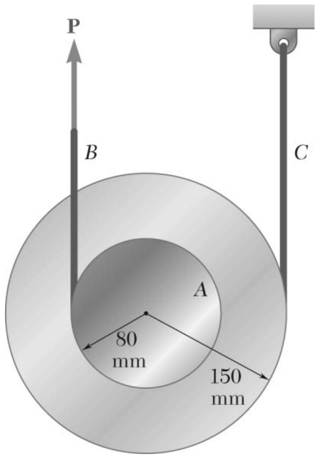
$$Nt = mgt\cos\beta$$

(*b*) *Required coefficient of static friction*.

$$\mu_s \geq \frac{F}{N}$$

$$= \frac{Ft}{Nt}$$

$$= \frac{\bar{k}^2 mgt\sin\beta}{(r^2 + \bar{k}^2)mgt\cos\beta} \qquad \mu_s \geq \frac{\bar{k}^2\tan\beta}{r^2 + \bar{k}^2} \blacktriangleleft$$



PROBLEM 17.71

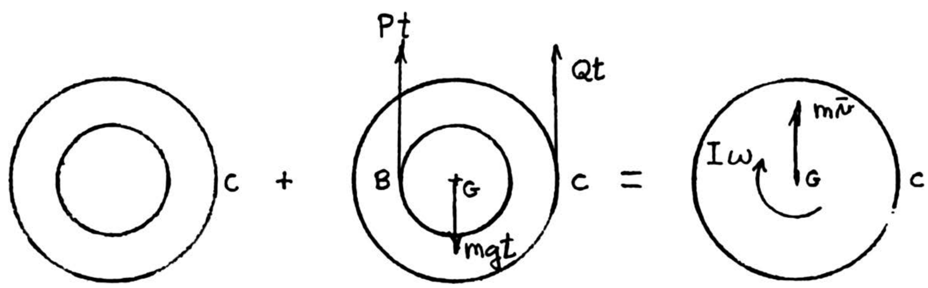
The double pulley shown has a mass of 3 kg and a radius of gyration of 100 mm. Knowing that when the pulley is at rest, a force **P** of magnitude 24 N is applied to cord *B*, determine (*a*) the velocity of the center of the pulley after 1.5 s, (*b*) the tension in cord *C*.

SOLUTION

For the double pulley,

$$r_C = 0.150 \text{ m}$$
$$r_B = 0.080 \text{ m}$$
$$k = 0.100 \text{ m}$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics. Point *C* is the instantaneous center. $\bar{v} = r_C\omega$

↺ Moments about *C*: $0 + Pt(r_C + r_B) - mgtr_C = \bar{I}\omega + m\bar{v}r_C$

$$= mk^2\omega + m(r_C\omega)r_C$$

$$\omega = \frac{Pt(r_C + r_B) - mgtr_C}{m\left(k^2 + r_C^2\right)}$$

$$= \frac{(24)(1.5)(0.230) - (3)(9.81)(1.5)(0.150)}{3(0.100^2 + 0.150^2)}$$

$$= 17.0077 \text{ rad/s}$$

PROBLEM 17.71 (Continued)

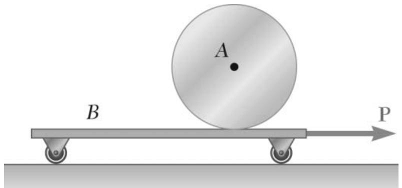
(*a*) $\bar{v} = (0.150)(17.0077) = 2.55115 \text{ m/s}$ $\qquad \bar{\mathbf{v}} = 2.55 \text{ m/s} \uparrow$ ◀

$+\uparrow$ Linear components: $\quad 0 + Pt - mgt + Qt = m\bar{v}$

$$Q = \frac{m\bar{v}}{t} + mg - P$$

$$= \frac{(3)(2.55115)}{1.5} + (3)(9.81) - 24$$

(*b*) *Tension in cord C.* $\qquad Q = 10.53 \text{ N}$ ◀



PROBLEM 17.72

A 9-in.-radius cylinder of weight 18 lb rests on a 6-lb carriage. The system is at rest when a force **P** of magnitude 2.5 lb is applied as shown for 1.2 s. Knowing that the cylinder rolls without sliding on the carriage and neglecting the mass of the wheels of the carriage, determine the resulting velocity of (*a*) the carriage, (*b*) the center of the cylinder.

SOLUTION

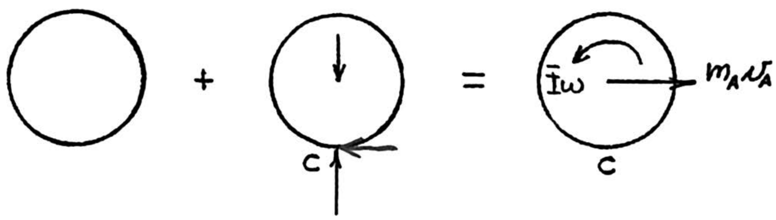
Moment of inertia.

$$\bar{I} = \frac{1}{2} m_A r^2$$

$$= \frac{1}{2}\left(\frac{18 \text{ lb}}{32.2}\right)\left(\frac{9 \text{ in.}}{12}\right)^2$$

$$= 0.15722 \text{ slug}\cdot\text{ft}^2$$

Cylinder alone:

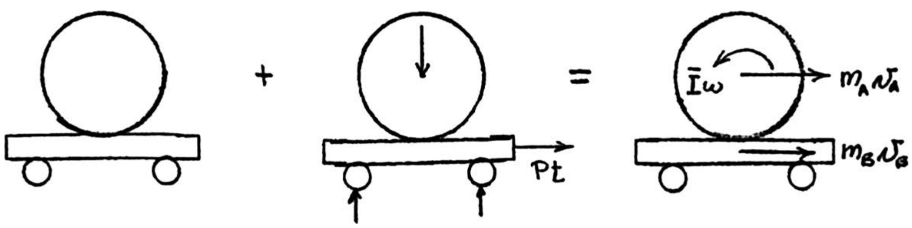


Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

↺ Moments about C: $0 + 0 = \bar{I}\omega - m_A v_A r$

or $$0 = 0.15722\omega - \left(\frac{18}{32.2}\right)\left(\frac{9}{12}\right)v_A \qquad (1)$$

Cylinder and carriage:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\xrightarrow{+}$ Horizontal components: $0 + Pt = m_A v_A + m_B v_B$

or $$0 + (2.5)(1.2) = \left(\frac{18}{32.2}\right)v_A + \left(\frac{6}{32.2}\right)v_B \qquad (2)$$

PROBLEM 17.72 (Continued)

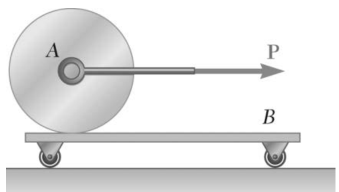
Kinematics. $v_A = v_B - r\omega$

$$v_A = v_B - \left(\frac{9}{12}\right)\omega \quad (3)$$

Solving Equations (1), (2) and (3) simultaneously gives $\boldsymbol{\omega} = 7.16 \text{ rad/s}$ ↺

(*a*) Velocity of the carriage. $\mathbf{v}_B = 8.05 \text{ ft/s} \longrightarrow$ ◀

(*b*) Velocity of the center of the cylinder. $\mathbf{v}_A = 2.68 \text{ ft/s} \longrightarrow$ ◀



PROBLEM 17.73

A 9-in.-radius cylinder of weight 18 lb rests on a 6-lb carriage. The system is at rest when a force **P** of magnitude 2.5 lb is applied as shown for 1.2 s. Knowing that the cylinder rolls without sliding on the carriage and neglecting the mass of the wheels of the carriage, determine the resulting velocity of (*a*) the carriage, (*b*) the center of the cylinder.

SOLUTION

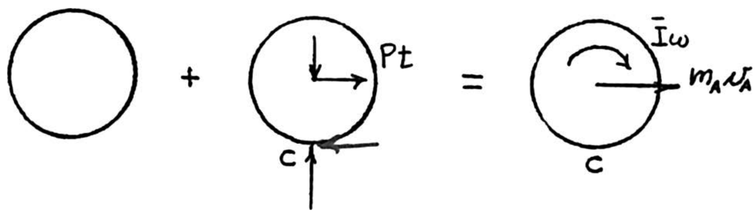
Moment of inertia.

$$\bar{I} = \frac{1}{2} m_A r^2$$

$$= \frac{1}{2}\left(\frac{18 \text{ lb}}{32.2}\right)\left(\frac{9 \text{ in.}}{12}\right)^2$$

$$= 0.15722 \text{ slug} \cdot \text{ft}^2$$

Cylinder alone:

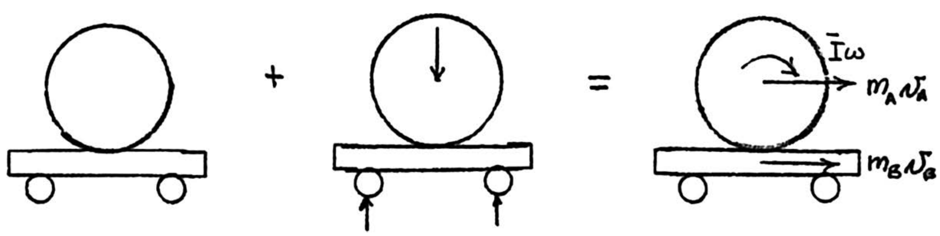


Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

↻ Moments about C: $$0 + Ptr = \bar{I}\omega + m_A v_A r$$

or $$0 + (2.5)(1.2)\left(\frac{9}{12}\right) = 0.15722\omega + \left(\frac{18}{32.2}\right)\left(\frac{9}{12}\right)v_A \quad (1)$$

Cylinder and carriage:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\xrightarrow{+}$ Horizontal components: $$0 + Pt = m_A v_A + m_B v_B$$

or $$0 + (2.5)(1.2) = \left(\frac{18}{32.2}\right)v_A + \left(\frac{6}{32.2}\right)v_B \quad (2)$$

PROBLEM 17.73 (Continued)

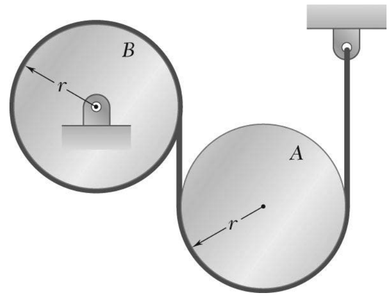
Kinematics. $v_A = v_B + r\omega$

$$v_A = v_B + \left(\frac{9}{12}\right)\omega \qquad (3)$$

Solving Equations (1), (2) and (3) simultaneously gives $\boldsymbol{\omega} = 2.39 \text{ rad/s}$ ↻

(*a*) Velocity of the carriage. $\mathbf{v}_B = 2.68 \text{ ft/s} \longrightarrow$ ◀

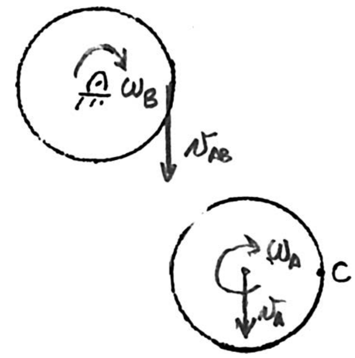
(*b*) Velocity of the center of the cylinder. $\mathbf{v}_A = 4.47 \text{ ft/s} \longrightarrow$ ◀



PROBLEM 17.74

Two uniform cylinders, each of mass $m = 6$ kg and radius $r = 125$ mm, are connected by a belt as shown. If the system is released from rest when $t = 0$, determine (*a*) the velocity of the center of cylinder *B* at $t = 3$ s, (*b*) the tension in the portion of belt connecting the two cylinders.

SOLUTION



Kinematics. $v_{AB} = r\omega_B$

Point *C* is the instantaneous center of cylinder *A*.

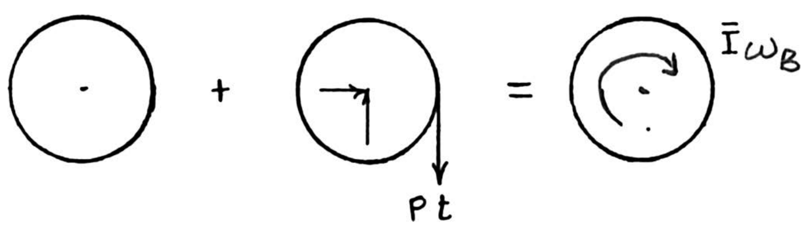
$$\omega_A = \frac{v_{AB}}{2r} = \frac{1}{2}\omega_B$$

$$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B$$

Moment of inertia. $\bar{I} = \frac{1}{2}mr^2$

(*a*) <u>Velocity of the center of *A*</u>.

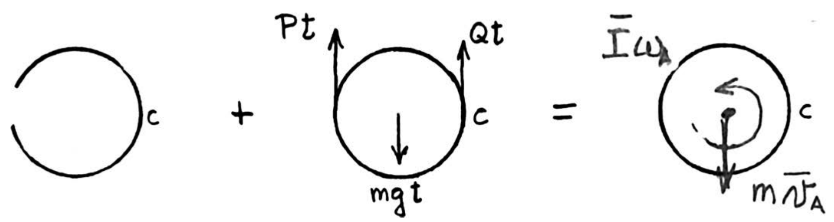
Cylinder B:



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

+↻ Moments about *B*: $0 + Ptr = \bar{I}\omega_B$ (1)

Cylinder. A:



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

PROBLEM 17.74 (Continued)

↻ Moments about C: $0-2Ptr+mgtr=m\bar{v}_A r+\bar{I}\omega_A$

$$0-2\bar{I}\,\omega_B+mgtr=m\left(\frac{1}{2}r\omega_B\right)r+\bar{I}\left(\frac{1}{2}\omega_B\right)$$

$$\left(\frac{5}{2}\bar{I}+\frac{1}{2}mr^2\right)\omega_B=mgrt$$

$$\left(\frac{5}{2}\frac{mr^2}{2}+\frac{1}{2}mr^2\right)\omega_B=mgrt$$

$$\frac{7}{4}r\omega_B=gt$$

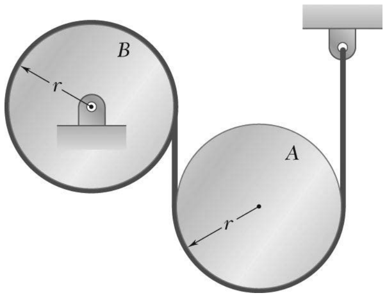
$$\omega_B=\frac{4}{7}\frac{gt}{r} \qquad (2)$$

$$\bar{v}_A=\frac{1}{2}r\omega_B=\frac{2}{7}gt=\frac{2}{7}(9.81)(3) \qquad \bar{\mathbf{v}}_A=8.41\text{ m/s}\downarrow \blacktriangleleft$$

(*b*) <u>Tension in the belt</u>.

From Eqs. (1) and (2), $Ptr=\bar{I}\left(\frac{4}{7}\frac{gt}{r}\right)$

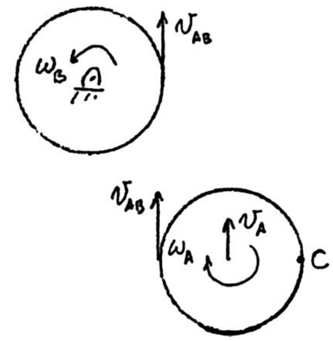
$$P=\frac{1}{tr}\left(\frac{1}{2}mr^2\right)\left(\frac{4}{7}\frac{gt}{r}\right)=\frac{2}{7}mg=\frac{2}{7}(6)(9.81)=16.817\text{ N} \qquad P=16.82\text{ N} \blacktriangleleft$$



PROBLEM 17.75

Two uniform cylinders, each of mass $m = 6$ kg and radius $r = 125$ mm, are connected by a belt as shown. Knowing that at the instant shown the angular velocity of cylinder A is 30 rad/s counterclockwise, determine (a) the time required for the angular velocity of cylinder A to be reduced to 5 rad/s, (b) the tension in the portion of belt connecting the two cylinders.

SOLUTION



Kinematics. $$v_{AB} = r\omega_B$$

Point C is the instantaneous center of cylinder A.

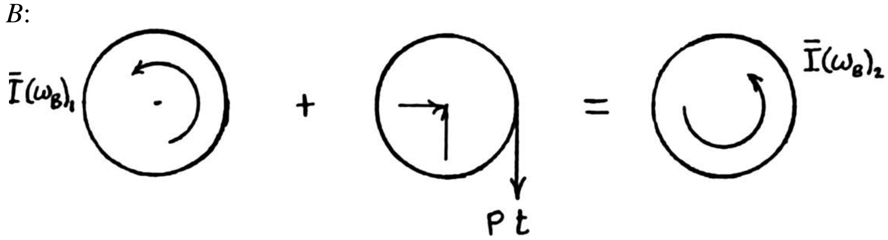
$$\omega_A = \frac{v_{AB}}{2r} = \frac{1}{2}\omega_B$$

$$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B$$

Moment of inertia. $$\bar{I} = \frac{1}{2}\frac{W}{g}r^2$$

(a) *Required time*.

Cylinder B:



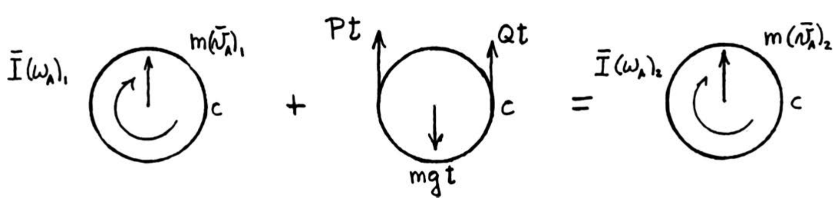
Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

↺ Moments about B: $$\bar{I}(\omega_B)_1 - Ptr = \bar{I}(\omega_B)_2$$

$$Ptr = \bar{I}[(\omega_B)_1 - (\omega_B)_2]$$

$$= \frac{1}{2}mr^2[(\omega_B)_1 - (\omega_B)_2] \quad (1)$$

Cylinder A:



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

PROBLEM 17.75 (Continued)

↻ Moments about C: $\bar{I}(\omega_A)_1 + m(v_A)_1 r + 2Ptr - mgtr = \bar{I}(\omega_A)_2 + m(v_A)_2 r$

$$\frac{1}{2}mr^2[(\omega_A)_1 - (\omega_A)_2 + mr[(\omega_A)_1 - (\omega_A)_2]r + 2Ptr - mgtr = 0$$

$$\frac{3}{2}mr^2\left[\left(\frac{1}{2}\omega_B\right)_1 - \frac{1}{2}(\omega_B)_2\right] + 2\left\{\frac{1}{2}mr^2[(\omega_B)_1 - (\omega_B)_2]\right\} - mgtr = 0$$

$$\frac{7}{4}mr^2[(\omega_B)_1 - (\omega_B)_2] - mgtr = 0$$

$$t = \frac{7r[(\omega_B)_1 - (\omega_B)_2]}{4g} \qquad (2)$$

Data: $m = 6$ kg

$r = 125$ mm $= 0.125$ m

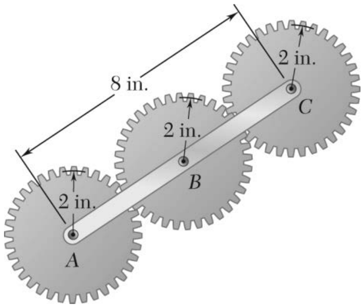
From Equation (2), $t = \dfrac{(7)(0.125)(30-5)}{(4)(9.81)} = 0.55747$ $\qquad t = 0.557$ s ◀

(*b*) *Tension in belt between cylinders.*

From Equation (1), $Ptr = \dfrac{1}{2}(6)(0.125)^2(30-5)$

$= 1.172$ N·m·s

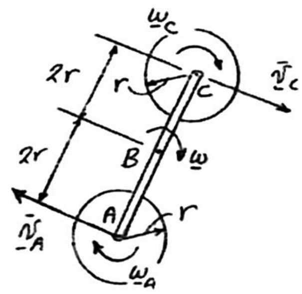
$$P = \frac{Ptr}{tr} = \frac{1.172}{(0.55747)(0.125)} = 16.817 \qquad P = 16.82 \text{ N} ◀$$



PROBLEM 17.76

In the gear arrangement shown, gears A and C are attached to rod ABC, which is free to rotate about B, while the inner gear B is fixed. Knowing that the system is at rest, determine the magnitude of the couple **M** which must be applied to rod ABC, if 2.5 s later the angular velocity of the rod is to be 240 rpm clockwise. Gears A and C weigh 2.5 lb each and may be considered as disks of radius 2 in.; rod ABC weighs 4 lb.

SOLUTION



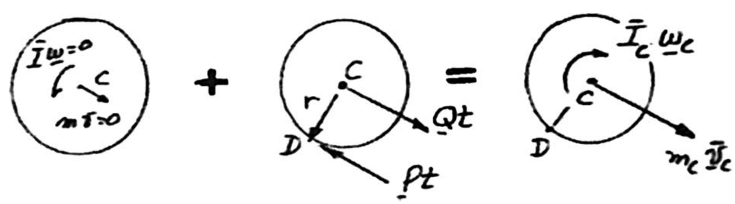
Kinematics of motion

Let $\omega_{ABC} = \omega$ $\qquad\qquad$ $\bar{v}_A = \bar{v}_C = (BC)\omega = 2r\omega$

Since gears A and C roll on the fixed gear B,

$$\omega_A = \omega_C = \frac{v_C}{r} = \frac{2r\omega}{r} = 2\omega$$

Principle of impulse and momentum.

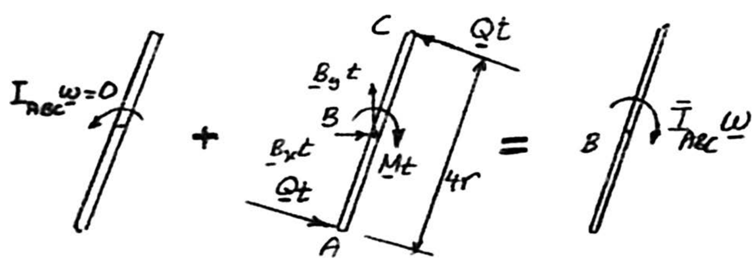


Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+↻ Moments about D: $\qquad 0 + (Qt)r = m_C \bar{v}_C r + \bar{I}_C w_C$

$$(Qt)r = m_C(2r\omega)r + \frac{1}{2}m_C r^2(2\omega)$$

$$Qt = 3m_C r\omega \qquad (1)$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+↻ Moments about B: $$Mt - Qt(4r) = \overline{I}_{ABC}\omega$$

$$Mt - 4(Qt)r = \frac{1}{12} m_{ABC}(4r)^2\omega$$

$$Mt - 4(Qt)r = \frac{4}{3} m_{ABC} r^2 \omega \qquad (2)$$

Substitute for (Qt) from (1) into (2):

$$Mt - 4(3m_C r\omega)r = \frac{4}{3} m_{ABC} r^2 \omega$$

$$Mt = \frac{4}{3} r^2 \omega (m_{ABC} + 9m_C) \qquad (3)$$

Couple M.

Data: $$t = 2.5 \text{ s}$$

$$r = \frac{2}{12} \text{ ft}$$

$$m_{ABC} = \frac{4 \text{ lb}}{32.2 \text{ ft/s}}$$

$$m_C = \frac{2.5 \text{ lb}}{32.2 \text{ ft/s}^2}$$

$$\omega = 240 \text{ rpm}$$

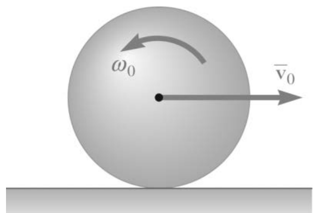
$$= 8\pi \text{ rad/s}$$

Eq. (3): $$M(2.5 \text{ s}) = \frac{4}{3}\left(\frac{2}{12} \text{ ft}\right)^2 (8\pi \text{ rad/s})\left[\frac{4}{32.2} + 9\left(\frac{2.5}{32.2}\right)\right]$$

$$2.5 \text{ M} = 0.76607$$

$$M = 0.3064 \text{ lb} \cdot \text{ft}$$

$\mathbf{M} = 0.306 \text{ lb} \cdot \text{ft}$ ↻ ◀

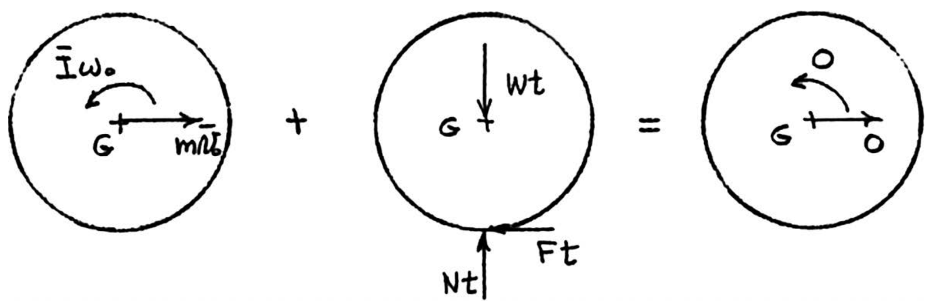


PROBLEM 17.77

A sphere of radius r and mass m is projected along a rough horizontal surface with the initial velocities shown. If the final velocity of the sphere is to be zero, express (*a*) the required magnitude of $\boldsymbol{\omega}_0$ in terms of v_0 and r, (*b*) the time required for the sphere to come to rest in terms of v_0 and coefficient of kinetic friction μ_k.

SOLUTION

Moment of inertia. Solid sphere. $\bar{I} = \frac{2}{5}mr^2$



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to2}$** = **Syst. Momenta$_2$**

$+\uparrow$ y components: $Nt - Wt = 0 \qquad N = W = mg$ (1)

$\xrightarrow{+}$ x components: $m\bar{v}_0 - Ft = 0 \qquad Ft = m\bar{v}_0$ (2)

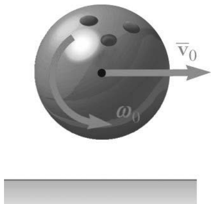
$+\circlearrowleft$ Moments about G: $\bar{I}\omega_0 - Ftr = 0$ (3)

$$\frac{2}{5}mr^2\omega_0 - m\bar{v}_0 r = 0$$

(*a*) *Solving for* ω_0, $\qquad \omega_0 = \frac{5}{2}\frac{\bar{v}_0}{r}$ ◀

(*b*) *Time to come to rest.*

From Equation (2), $\quad t = \frac{mv_0}{F} = \frac{m\bar{v}_0}{\mu_k mg} \qquad t = \frac{\bar{v}_0}{\mu_k g}$ ◀



PROBLEM 17.78

A bowler projects an 8.5-in.-diameter ball weighing 16 lb along an alley with a forward velocity $\mathbf{v}_0$ of 25 ft/s and a backspin ω_0 of 9 rad/s. Knowing that the coefficient of kinetic friction between the ball and the alley is 0.10, determine (*a*) the time t_1 at which the ball will start rolling without sliding, (*b*) the speed of the ball at time t_1.

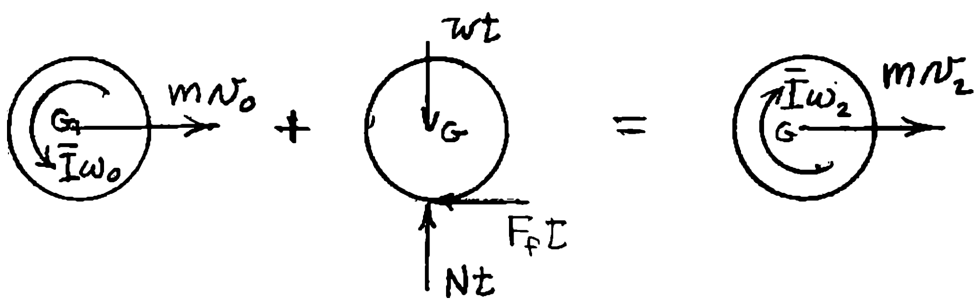
SOLUTION

Radius: $r = \frac{1}{2}d = \frac{1}{2}(8.5 \text{ in.}) = 4.25 \text{ in.} = 0.35417 \text{ ft}$

Mass: $m = \frac{W}{g} = \frac{16 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.49689 \text{ lb} \cdot \text{s}^2/\text{ft}$

Moment of inertia: $\bar{I} = \frac{2}{5}mr^2 = \left(\frac{2}{5}\right)(0.49689)(0.35417)^2 = 0.02493 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$

Use the principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$+\uparrow: Nt - Wt = 0 \qquad N = W = 16 \text{ lb}$

Friction force: $F_F = \mu_k N = (0.10)(16) = 1.6 \text{ lb.}$

$\xrightarrow{+}: mv_0 - F_F t = mv_2$

$$v_2 = v_0 - \frac{F_F t}{m} = 25 \text{ ft/s} - \frac{1.6t}{0.49689}$$
$$= 25 - 3.22t$$

$+\circlearrowright$ Moments about G: $\bar{I}\omega_0 - F_F tr = -\bar{I}\omega_2$

$$\omega_2 = \frac{F_F tr}{\bar{I}} - \omega_0 = \frac{(1.6t)(0.35417)}{0.02493} - 9$$
$$= 22.731t - 9$$

Slipping stops when $v_2 = r\omega_2$

PROBLEM 17.78 (Continued)

(*a*) Time *t* when slipping stops.

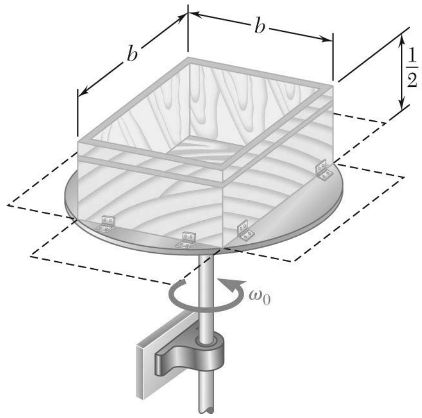
$$(25 - 3.22t) = (0.35417 \text{ ft})(22.731t - 9)$$
$$(25 + 3.1875) = (3.22 + 8.0506)t$$
$$t = 2.501 \text{ s}$$

$t = 2.50$ s ◀

(*b*) Corresponding velocity.

$$v_2 = 25 - 3.22t$$

$v_2 = 16.95$ ft/s ◀



PROBLEM 17.79

Four rectangular panels, each of length b and height $\frac{1}{2}b$, are attached with hinges to a circular plate of diameter $\sqrt{2}b$ and held by a wire loop in the position shown. The plate and the panels are made of the same material and have the same thickness. The entire assembly is rotating with an angular velocity ω_0 when the wire breaks. Determine the angular velocity of the assembly after the panels have come to rest in a horizontal position.

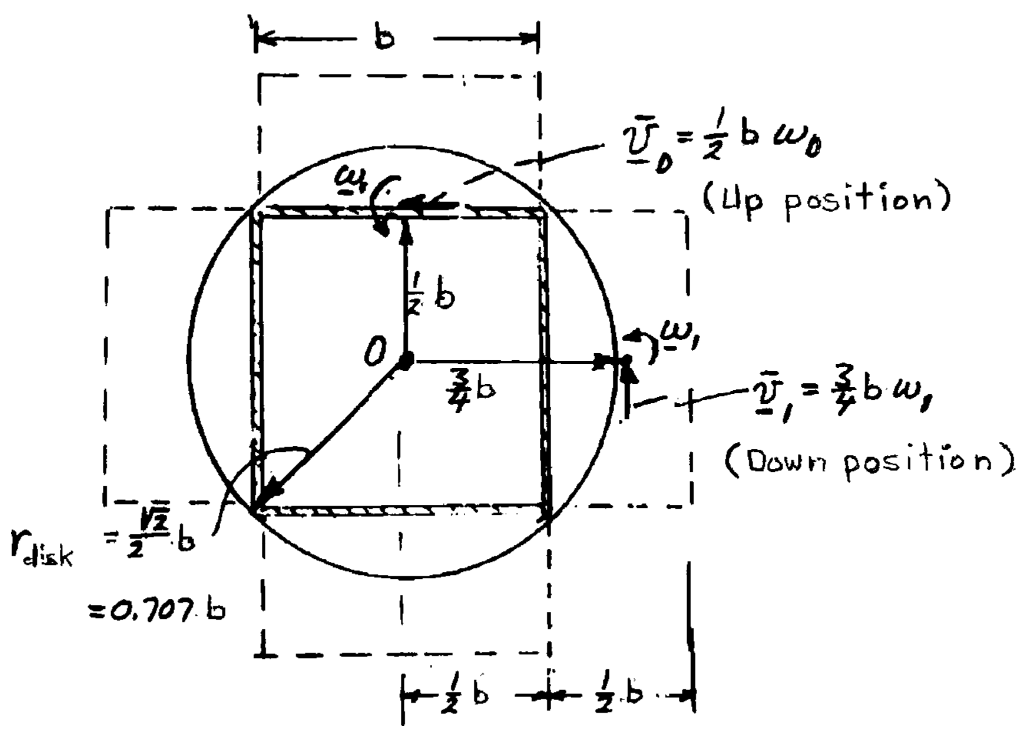
SOLUTION

Kinematics: When the panels are in the up position, the speed of the mass center of each panel is

$$v_0 = \frac{1}{2}b\omega_0$$

When the panels are in the down position the speed of the mass center of each panel is

$$v_1 = \frac{3}{4}b\omega_1$$



Let $\rho =$ mass density of plate and of panels
$t =$ thickness of plate and of panels

Disk: $$m = \rho V = \rho \pi t(0.707b)^2 = \rho t \pi b^2 (0.500)$$

$$I_{\text{disk}} = I_D = \frac{1}{2} m (r_{\text{disk}})^2 = \frac{1}{2} \rho t \pi b^2 (0.500)(0.707b)^2$$

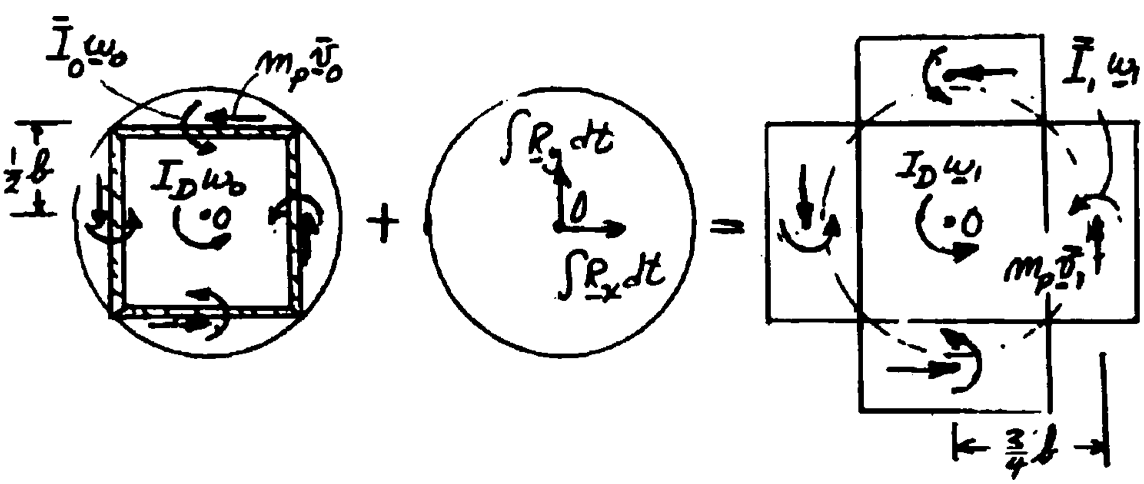
$$I_D = \frac{1}{8} \rho t \pi b^4$$

Each panel: $$m_\rho = b\left(\frac{1}{2}b\right)\rho t = \frac{1}{2}\rho t b^2$$

In up position $$\bar{I}_0 = \frac{1}{12} m_\rho b^2 = \frac{1}{12}\left(\frac{1}{2}\rho t b^2\right) b^2 = \frac{1}{24}\rho t b^4$$

In down position $$\bar{I}_1 = \frac{1}{12} m_\rho \left(b^2 + \left(\frac{1}{2}b\right)^2\right) = \frac{1}{12}\left(\frac{1}{2}\rho t b^2\right)\frac{5}{4}b^2 = \frac{5}{96}\rho t b^4$$

Principle of impulse and momentum.



Syst. Momenta$_0$ + **Syst. Ext. Imp.** $_{0\to 1}$ = **Syst. Momenta**$_1$

In the up position, the angular momentum of one panel about the vertical axle is

$$m_\rho v_0 \left(\frac{1}{2}b\right) + \bar{I}_0 \omega_0$$

In the down position it is

$$m_\rho \bar{v}_1 \frac{3}{4}b + \bar{I}_1 \omega_1$$

Conservation of angular momentum.

PROBLEM 17.79 (Continued)

$$\bar{I}_{\text{disk}}\omega_0 + 4\left[m_\rho v_D\left(\frac{1}{2}b\right) + \bar{I}_0\omega_0\right] = \bar{I}_{\text{disk}}\omega_1 + 4\left[m_\rho \bar{v}_1\left(\frac{3}{4}b\right) + \bar{I}_1\omega_1\right]$$

$$\bar{I}_{\text{disk}}\omega_0 + 4\left[m_\rho\left(\frac{1}{2}b\right)^2 + \bar{I}_0\right]\omega_0 = \bar{I}_{\text{disk}}\omega_1 + 4\left[m_\rho\left(\frac{3}{4}b\right)^2 + \bar{I}_1\right]\omega_1$$

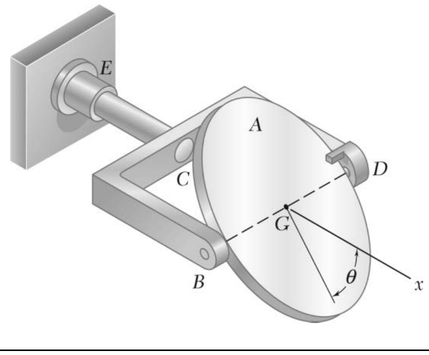
$$\left\{\frac{1}{8}\rho t\pi b^4 + 4\left[\frac{1}{2}\rho tb^2\left(\frac{1}{2}b\right)^2 + \frac{1}{24}\rho tb^4\right]\right\}\omega_0$$

$$= \left\{\frac{1}{8}\rho t\pi b^4 + 4\left[\frac{1}{2}\rho tb^2\left(\frac{3}{4}b\right)^2 + \frac{5}{96}\rho tb^4\right]\right\}\omega_1$$

$$\left\{\frac{\pi}{8} + \frac{1}{2} + \frac{1}{6}\right\}\omega_0 = \left\{\frac{\pi}{8} + \frac{9}{8} + \frac{5}{24}\right\}\omega_1$$

$$\{1.059\}\omega_0 = \{1.726\}\omega_1$$

$$\boldsymbol{\omega}_1 = 0.614\omega_0 \circlearrowleft \blacktriangleleft$$



PROBLEM 17.80

A 2.5-lb disk of radius 4 in. is attached to the yoke *BCD* by means of short shafts fitted in bearings at *B* and *D*. The 1.5-lb yoke has a radius of gyration of 3 in. about the *x* axis. Initially the assembly is rotating at 120 rpm with the disk in the plane of the yoke $(\theta = 0)$. If the disk is slightly disturbed and rotates with respect to the yoke until $\theta = 90°$, where it is stopped by a small bar at *D*, determine the final angular velocity of the assembly.

SOLUTION

Moment of inertia of yoke: $I_C = mk_C^2 = \left(\dfrac{1.5}{32.2}\right)\left(\dfrac{3}{12}\right)^2 = 2.9115\times10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$

Moment of inertia of disk:

$$\theta = 0: \quad I_A = \frac{1}{4}mr^2 = \frac{1}{4}\left(\frac{2.5}{32.2}\right)\left(\frac{4}{12}\right)^2 = 2.15666\times10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\theta = 90°: \quad I_A = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{2.5}{32.2}\right)\left(\frac{4}{12}\right)^2 = 4.3133\times10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

Total moment of inertia about the *x* axis:

$$\theta = 0: \quad (I_x)_1 = I_C + I_A = 5.0682\times10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\theta = 90°: \quad (I_x)_2 = I_C + I_A = 7.2248\times10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

Angular momentum about the *x* axis:

$$\theta = 0: \quad H_1 = (I_x)_1\omega_1 = 5.0682\times10^{-3}\omega_1$$

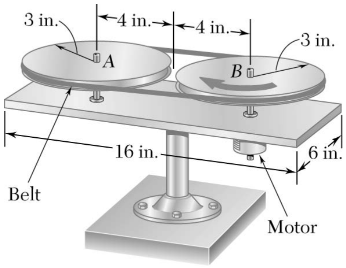
$$\theta = 90°: \quad H_2 = (I_x)_2\omega_2 = 7.2248\times10^{-3}\omega_2$$

Conservation of angular momentum.

$$H_1 = H_2: \quad 5.0682\times10^{-3}\,\omega_1 = 7.2248\times10^{-3}\,\omega_2$$

$$\omega_2 = 0.7015\omega_1 = (0.7015)(120\text{ rpm})$$

$\omega_2 = 84.2\text{ rpm}$ ◀



PROBLEM 17.81

Two 10-lb disks and a small motor are mounted on a 15-lb rectangular platform which is free to rotate about a central vertical spindle. The normal operating speed of the motor is 180 rpm. If the motor is started when the system is at rest, determine the angular velocity of all elements of the system after the motor has attained its normal operating speed. Neglect the mass of the motor and of the belt.

SOLUTION

Kinematics. Motor speed: $\omega_M = 180 \text{ rpm} = 6\pi \text{ rad/s}$

Let ω_A, ω_B, and ω_P be the angular velocities, respectfully, of disk *A*, disk *B* and the platform. Since the motor speed is the angular velocity of disk *B* relative to the platform,

$$\omega_B = \omega_P + \omega_M = \omega_P + 6\pi \quad (1)$$

Since, the disks have the same outer radius, $\omega_B = \omega_A$ (2)

Velocity of the center of disk *A* $v_A = \frac{4}{12}\omega_P$ (3)

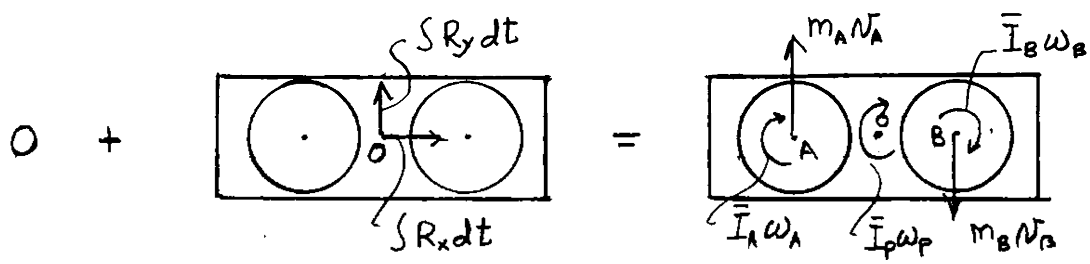
Velocity of the center of disk *B* $v_B = \frac{4}{12}\omega_P$ (4)

Moments of inertia.

Disks *A* and *B*: $\bar{I}_A = \bar{I}_B = \frac{1}{2}\frac{W}{g}r^2 = \frac{1}{2}\left(\frac{10}{32.2}\right)\left(\frac{3}{12}\right)^2 = 9.705 \times 10^{-3} \text{ lb}\cdot\text{s}^2\cdot\text{ft}$

Platform: $\bar{I}_P = \frac{1}{12}\frac{W}{g}(a^2 + b^2) = \frac{1}{12}\left(\frac{15}{32.2}\right)\left[\left(\frac{16}{12}\right)^2 + \left(\frac{6}{12}\right)^2\right] = 78.718 \times 10^{-3} \text{ lb}\cdot\text{s}^2\cdot\text{ft}$

Principle of impulse and momentum for system.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+↻ Moments about O:

$$0 + 0 = \bar{I}_P\omega_P + m_A v_A l_{OA} + \bar{I}_A\omega_A + m_B v_B l_{OB} + \bar{I}_B\omega_B$$

$$= (78.718\times 10^{-3})\omega_P + \left(\frac{10}{32.2}\right)\left(\frac{4}{12}\omega_P\right)\left(\frac{4}{12}\right) + (9.705\times 10^{-3})(\omega_P + 6\pi) + \left(\frac{10}{32.2}\right)\left(\frac{4}{12}\omega_P\right)\left(\frac{4}{12}\right)$$

$$+ (9.705\times 10^{-3})(\omega_P + 6\pi)$$

$$= 167.141\times 10^{-3}\omega_P + 365.87\times 10^{-3}$$

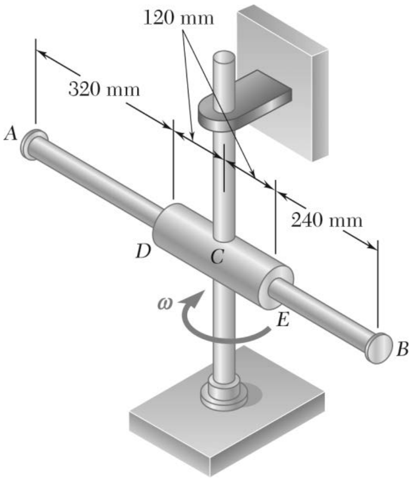
$$\omega_P = -2.189 \text{ rad/s}$$

$$\omega_A = \omega_B = -2.189 + 6\pi = 16.66 \text{ rad/s}$$

Angular velocities. $\omega_A = 159.1$ rpm ↻ ◀

$\omega_B = 159.1$ rpm ↻ ◀

$\omega_P = 20.9$ rpm ↺ ◀

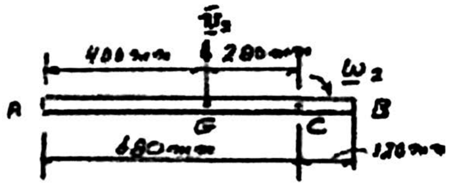


PROBLEM 17.82

A 3-kg rod of length 800 mm can slide freely in the 240-mm cylinder *DE*, which in turn can rotate freely in a horizontal plane. In the position shown the assembly is rotating with an angular velocity of magnitude $\omega = 40$ rad/s and end *B* of the rod is moving toward the cylinder at a speed of 75 mm/s relative to the cylinder. Knowing that the centroidal mass moment of inertia of the cylinder about a vertical axis is $0.025\ \text{kg}\cdot\text{m}^2$ and neglecting the effect of friction, determine the angular velocity of the assembly as end *B* of the rod strikes end *E* of the cylinder.

SOLUTION

Kinematics and geometry.



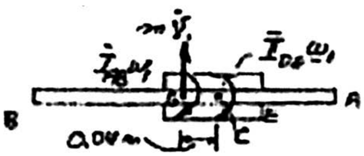
$\bar{v}_1 = (0.04\ \text{m})\omega_1 = (0.4\ \text{m})(40\ \text{rad/s})$

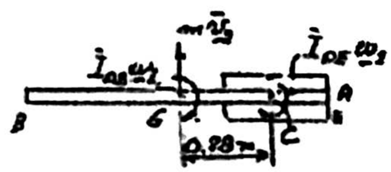
$\bar{v}_1 = 1.6\ \text{m/s}$

$\bar{v}_2 = (0.28\ \text{m})\omega_2$

Initial position — Final position

Conservation of angular momentum about *C*.





$+\circlearrowleft$ Moments about *C*: $\bar{I}_{AB} = \dfrac{1}{12}(3\ \text{kg})(0.8\ \text{m})^2 = 0.16\ \text{kg}\cdot\text{m}^2$

$$\bar{I}_{AB}\omega_1 + m\bar{v}_1(0.04\ \text{m}) + \bar{I}_{DE}\omega_1 = \bar{I}_{AB}\omega_2 + m\bar{v}_2(0.028\ \text{m}) + \bar{I}_{DE}\omega_2$$

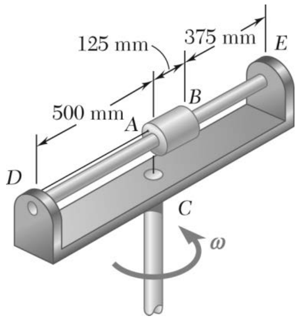
$$(0.16\ \text{kg}\cdot\text{m}^2)(40\ \text{rad/s}) + (3\ \text{kg})(1.6\ \text{m/s})(0.04\ \text{m}) + (0.025\ \text{kg}\cdot\text{m}^2)(40\ \text{rad/s})$$

$$= (0.16\ \text{kg}\cdot\text{m}^2)\omega_2 + (3\ \text{kg})(0.28\omega_2)(0.28) + (0.025\ \text{kg}\cdot\text{m}^2)\omega_2$$

$$(6.4 + 0.192 + 1.00) = (0.16 + 0.2352 + 0.025)\omega_2$$

$$7.592 = 0.4202\omega_2; \quad \omega_2 = 18.068\ \text{rad/s}$$

Angular velocity. $\omega_2 = 18.07\ \text{rad/s}$ ◀



PROBLEM 17.83

A 1.6-kg tube *AB* can slide freely on rod *DE*, which in turn can rotate freely in a horizontal plane. Initially the assembly is rotating with an angular velocity $\omega = 5$ rad/s and the tube is held in position by a cord. The moment of inertia of the rod and bracket about the vertical axis of rotation is $0.30\ \text{kg}\cdot\text{m}^2$ and the centroidal moment of inertia of the tube about a vertical axis is $0.0025\ \text{kg}\cdot\text{m}^2$. If the cord suddenly breaks, determine (*a*) the angular velocity of the assembly after the tube has moved to end *E*, (*b*) the energy lost during the plastic impact at *E*.

SOLUTION

Let Point *C* be the intersection of axle *C* and rod *DE*. Let Point *G* be the mass center of tube *AB*.

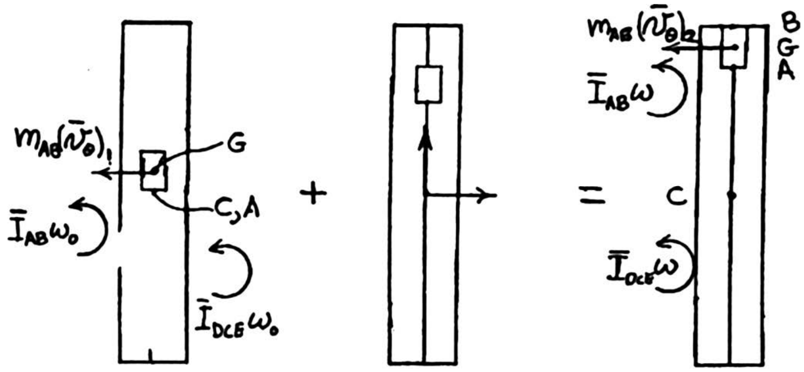
Masses and moments of inertia about vertical axes.

$$m_{AB} = 1.6\ \text{kg}$$
$$\bar{I}_{AB} = 0.0025\ \text{kg}\cdot\text{m}^2$$
$$\bar{I}_{DCE} = 0.30\ \text{kg}\cdot\text{m}^2$$

State 1.
$$(r_{G/A})_1 = \frac{1}{2}(125) = 62.5\ \text{mm}$$
$$\omega_1 = 5\ \text{rad/s}$$

State 2.
$$(r_{G/A})_2 = 500 - 62.5 = 437.5\ \text{mm}$$
$$\omega = \omega_2$$

Kinematics.
$$(v_G)_\theta = \bar{v}_\theta = r_{G/C}\,\omega$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROBLEM 17.83 (Continued)

Moments about C:

$$\bar{I}_{AB}\omega_1 + \bar{I}_{DCE}\omega_1 + m_{AB}(\bar{v}_\theta)_1(r_{G/C})_1 + 0 = \bar{I}_{AB}\omega_2 + \bar{I}_{DCE}\omega_2 + m_{AB}(\bar{v}_\theta)_2(r_{G/C})_2$$

$$\left[\bar{I}_{AB} + \bar{I}_{DCE} + m_{AB}(r_{G/G})_1^2\right]\omega_1 = \left[\bar{I}_{AB} + \bar{I}_{DCE} + m_{AB}(r_{G/C})_2^2\right]\omega_2$$

$$[0.0025 + 0.30 + (1.6)(0.0625)^2](5) = [0.0025 + 0.30 + (1.6)(0.4375)^2]\omega_2$$

$$(0.30875)(5) = 0.60875\omega_2$$

$$\omega_2 = 2.5359 \text{ rad/s}$$

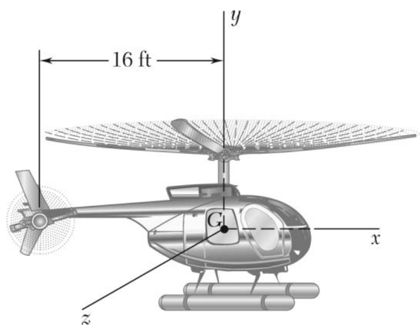
(*a*) *Angular velocity after the plastic impact.* 2.54 rad/s ◀

Kinetic energy. $T = \frac{1}{2}\bar{I}_{AB}\omega^2 + \frac{1}{2}\bar{I}_{DCE}\omega^2 + \frac{1}{2}m_{AB}\bar{v}^2$

$$T_1 = \frac{1}{2}(0.0025)(5)^2 + \frac{1}{2}(0.30)(5)^2 + \frac{1}{2}(1.6)(0.0625)^2(5)^2$$
$$= 3.859375 \text{ J}$$

$$T_2 = \frac{1}{2}(0.0025)(2.5359)^2 + \frac{1}{2}(0.30)(2.5359)^2 + \frac{1}{2}(1.6)(0.4375)^2(2.5359)^2$$
$$= 1.9573 \text{ J}$$

(*b*) *Energy lost.* $T_1 - T_2 = 1.902 \text{ J}$ ◀



PROBLEM 17.84

In the helicopter shown, a vertical tail propeller is used to prevent rotation of the cab as the speed of the main blades is changed. Assuming that the tail propeller is not operating, determine the final angular velocity of the cab after the speed of the main blades has been changed from 180 to 240 rpm. (The speed of the main blades is measured relative to the cab, and the cab has a centroidal moment of inertia of $650\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$. Each of the four main blades is assumed to be a slender 14-ft rod weighting 55 lb.)

SOLUTION

Let $\mathbf{\Omega}$ be the angular velocity of the cab and ω be the angular velocity of the blades relative to the cab. The absolute angular velocity of the blades is $\Omega + \omega$.

$$\omega_1 = 180 \text{ rpm} = 6\pi \text{ rad/s}$$
$$\omega_2 = 240 \text{ rpm} = 8\pi \text{ rad/s}$$

Moments of inertia.

Cab: $\quad I_C = 650\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$

Blades:
$$I_B = 4\left(\frac{1}{3}mL^2\right)$$
$$= (4)\left(\frac{1}{3}\right)\left(\frac{55}{32.2}\right)(14)^2$$
$$= 446.38\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

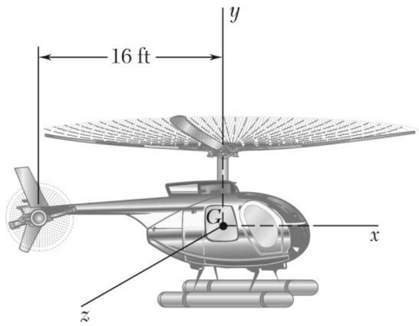
Assume $\Omega_1 = 0$.

Conservation of angular momentum about shaft.

$$I_B(\omega_1 + \Omega_1) + I_C\Omega_1 = I_B(\omega_2 + \Omega_2) + I_C\Omega_2$$

$$\Omega_2 = -\frac{I_B(\omega_2 - \omega_1)}{I_C + I_B}$$
$$= -\frac{(446.38)(8\pi - 6\pi)}{446.38 + 650}$$
$$= -2.5581 \text{ rad/s}$$

$\Omega_2 = -24.4 \text{ rpm}$ ◀



PROBLEM 17.85

Assuming that the tail propeller in Problem 17.84 is operating and that the angular velocity of the cab remains zero, determine the final horizontal velocity of the cab when the speed of the main blades is changed from 180 to 240 rpm. The cab weighs 1250 lb and is initially at rest. Also determine the force exerted by the tail propeller if the change in speed takes place uniformly in 12 s.

SOLUTION

Let $\mathbf{\Omega}$ be the angular velocity of the cab and ω be the angular velocity of the blades relative to the cab. The absolute angular velocity of the blades is $\Omega + \omega$.

$$\omega_1 = 180 \text{ rpm} = 6\pi \text{ rad/s}$$
$$\omega_2 = 240 \text{ rpm} = 8\pi \text{ rad/s}$$

Moments of inertia.

Cab: $I_C = 650 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

Blades: $$I_B = 4\left(\frac{1}{3}mL^2\right) = (4)\left(\frac{1}{3}\right)\left(\frac{55}{32.2}\right)(14)^2$$
$$= 446.38 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

The cab does not rotate. $\Omega_1 = \Omega_2 = 0$

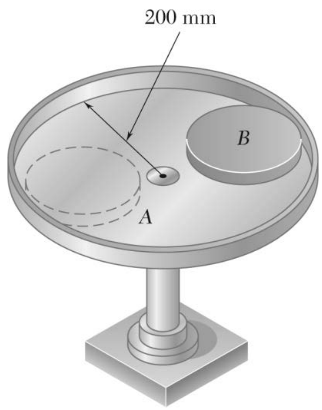
$$\textbf{Syst. Momenta}_1 + \textbf{Syst. Ext. Imp.}_{1\to 2} = \textbf{Syst. Momenta}_2$$

Moments about shaft: $$I_B(\omega_1 + \Omega_1) + I_C\Omega_1 + Frt = I_B(\omega_2 + \Omega_2) + I_C\Omega_2$$
$$Frt = I_B(\omega_2 - \omega_1)$$
$$= (446.38)(8\pi - 6\pi)$$
$$= 2804.7 \text{ lb}\cdot\text{ft}\cdot\text{s}$$
$$Ft = \frac{Frt}{r} = \frac{2804.7}{16} = 175.29 \text{ lb}\cdot\text{s}$$

Linear components: $$mv_1 + Ft = mv_2$$
$$v_2 - v_1 = \frac{Ft}{m} = \frac{175.29}{\frac{1250}{32.2} + (4)\left(\frac{55}{32.2}\right)}$$
$$= 3.8398 \text{ ft/s}$$

(*a*) Assume $v_1 = 0$. $v_2 = 3.84 \text{ ft/s}$ ◀

(*b*) Force. $F = \frac{Ft}{t} = \frac{175.29}{12}$ $F = 14.61 \text{ lb}$ ◀



PROBLEM 17.86

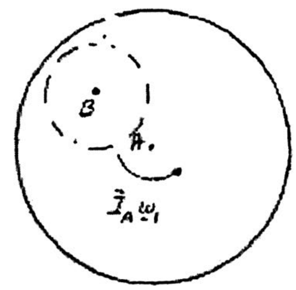
The circular platform A is fitted with a rim of 200-mm inner radius and can rotate freely about the vertical shaft. It is known that the platform-rim unit has a mass of 5 kg and a radius of gyration of 175 mm with respect to the shaft. At a time when the platform is rotating with an angular velocity of 50 rpm, a 3-kg disk B of radius 80 mm is placed on the platform with no velocity. Knowing that disk B then slides until it comes to rest relative to the platform against the rim, determine the final angular velocity of the platform.

SOLUTION

Moments of inertia.

$$\bar{I}_A = m_A k^2 = (5\text{ kg})(0.175\text{ m})^2 = 0.153125\text{ kg}\cdot\text{m}^2$$

$$\bar{I}_B = \frac{1}{2}m_B r_B^2 = \frac{1}{2}(3\text{ kg})(0.08\text{ m})^2 = 9.6\times10^{-3}\text{ kg}\cdot\text{m}^2$$

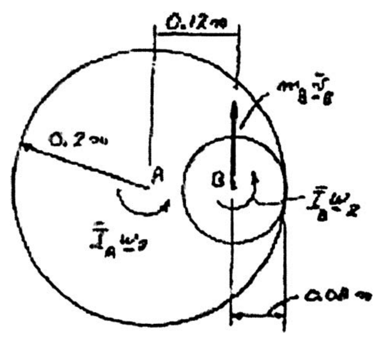


Syst. Momenta$_1$

State 1 Disk B is at rest.

State 2 Disk B moves with platform A.

Kinematics. In State 2, $\bar{v}_B = (0.12\text{ m})\omega_2$



Syst. Momenta$_2$

Principle of conservation of angular momentum.

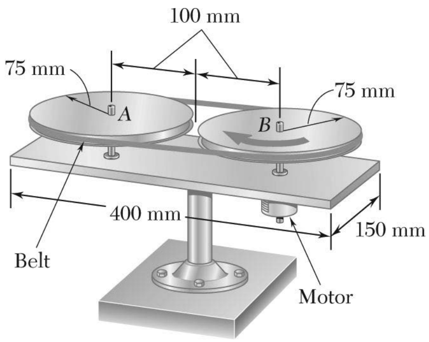
+) Moments about D: $\bar{I}_A\omega_1 = \bar{I}_A\omega_2 + \bar{I}_B\omega_2 + m_B\bar{v}_B(0.12\text{ m})$

$$(0.153125\text{ kg}\cdot\text{m}^2)\omega_1 = (0.153125\text{ kg}\cdot\text{m}^2)\omega_2 + (9.6\times10^{-3}\text{ kg}\cdot\text{m}^2)\omega_2 + (3\text{ kg})(0.12\text{ m})^2\omega_2$$

$$0.153125\omega_2 = 0.20593\omega_1$$

$$\omega_2 = 0.7436\omega_1 = 0.7436(50\text{ rpm})$$

Final angular velocity $\omega_2 = 37.2\text{ rpm}$ ◀



PROBLEM 17.87

Two 4-kg disks and a small motor are mounted on a 6-kg rectangular platform which is free to rotate about a central vertical spindle. The normal operating speed of the motor is 240 rpm. If the motor is started when the system is at rest, determine the angular velocity of all elements of the system after the motor has attained its normal operating speed. Neglect the mass of the motor and of the belt.

SOLUTION

Moments of inertia.

Disks: $$\bar{I}_A = \bar{I}_B = \frac{1}{2}mr^2 = \frac{1}{2}(4\text{ kg})(0.075\text{ m})^2 = 0.01125\text{ kg}\cdot\text{m}$$

Platform: $$\bar{I}_P = \frac{1}{12}m(a^2 + b^2) = \frac{1}{12}(6\text{ kg})[(0.15\text{ m})^2 + (0.4\text{ m})^2] = 0.09125\text{ kg}\cdot\text{m}^2$$

Kinematics:

$$\omega_M = \left(\frac{240\text{ rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) = 8\pi\text{ rad/s}$$

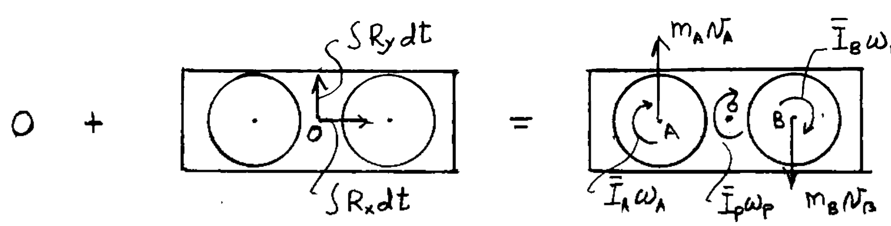
Let ω_A, ω_B and ω_P be the angular velocities of A, B, and the platform. The motor speed is the angular velocity of B relative to the platform.

$$\omega_B = \omega_P + \omega_M = \omega_P + 8\pi$$
$$\omega_A = \omega_B$$

Velocity of center of disk A. $\quad v_A = \omega_P r_{A/O} = 0.1\,\omega_A$

Velocity of center of disk B. $\quad v_B = \omega_P r_{B/O} = 0.1\,\omega_A$

Principle of impulse and momentum for system.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

PROBLEM 17.87 (Continued)

+↻ Moments about O:

$$0 + 0 = \bar{I}_P\omega_P + m_A v_A r_{A/O} + \bar{I}_A\omega_A + m_B v_B r_{B/O} + \bar{I}_B\omega_B$$

$$0 = \bar{I}_P\omega_P + m_A(\omega_P r_{A/O}^2) + \bar{I}_A(\omega_P + 8\pi) + m_B(\omega_P r_{B/O}^2) + \bar{I}_B(\omega_P + 8\pi)$$

$$\omega_P = -\frac{8\pi(\bar{I}_A + \bar{I}_B)}{\bar{I}_P + m_A r_{A/O} + \bar{I}_A + m_B r_{B/O}^2 + \bar{I}_B}$$

$$\omega_P = -\frac{8\pi(0.01125 + 0.01125)}{0.09125 + (4)(0.1)^2 + 0.01125 + 4(0.1)^2 + 0.01125}$$

$$= -2.9186 \text{ rad/s} = -27.87 \text{ rpm}$$

$$\omega_B = \omega_A = -2.9186 + 8\pi = 22.214 \text{ rad/s}$$

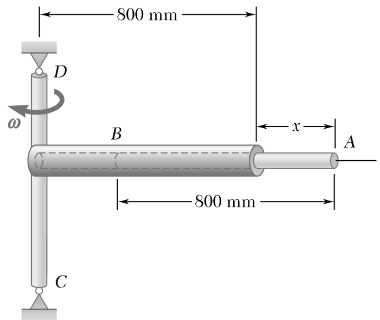
$$= 22.214 \text{ rad/s}\left(\frac{60\,\text{s}}{\text{min}}\right)\left(\frac{1\,\text{rev}}{2\pi}\right) = 212.13 \text{ rpm}$$

Angular velocities.

$\omega_A = 212$ rpm ↻ ◀

$\omega_B = 212$ rpm ↻ ◀

$\omega_P = 27.9$ rpm ↺ ◀



PROBLEM 17.88

The 4-kg rod *AB* can slide freely inside the 6-kg tube *CD*. The rod was entirely within the tube ($x = 0$) and released with no initial velocity relative to the tube when the angular velocity of the assembly was 5 rad/s. Neglecting the effect of friction, determine the speed of the rod relative to the tube when $x = 400$ mm.

SOLUTION

Let l be the length of the tube and the rod and Point O be the point of intersection of the tube and the axle.

Moments of inertia. $\bar{I}_T = \frac{1}{12} m_T l^2, \quad \bar{I}_R = \frac{1}{12} m_R l^2$

Kinematics. $(\bar{v}_\theta)_T = \bar{r}_T \omega = \frac{l}{2}\omega$

$$(\bar{v}_\theta)_R = \bar{r}_R \omega = \left(\frac{l}{2} + x\right)\omega, \qquad (\bar{v}_r)_T = v_r$$

Angular momentum about Point O.

$$H_O = m_T \bar{r}_T (\bar{v}_\theta) + \bar{I}_T \omega + m_R \bar{r}_R (\bar{v}_\theta)_R + \bar{I}_R \omega$$

$$= m_T \frac{l}{2}\left(\frac{l}{2}\omega\right) + \frac{1}{12} m_T l^2 \omega + m_R \left(\frac{l}{2} + x\right)\left(\frac{l}{2} + x\right)\omega + \frac{1}{12} m_R l^2 \omega$$

$$= \left[\frac{1}{3} m_T l^2 + m_R \left(\frac{1}{3} l^2 + lx + x^2\right)\right]\omega$$

Kinetic energy.

$$T = \frac{1}{2} m_T (\bar{v}_\theta)_T^2 + \frac{1}{2} \bar{I}_T \omega^2 + \frac{1}{2} m_R (\bar{v}_\theta{}^2)_R + \frac{1}{2} m_R v_r^2 + \frac{1}{2} \bar{I}_R \omega^2$$

$$= \frac{1}{2}\left[m_T \left(\frac{l}{2}\omega\right)^2 + \frac{1}{12} m_T l^2 \omega^2 + \frac{1}{2} m_R \left(\frac{l}{2} + x\right)^2 \omega^2 + \frac{1}{2} m_R v_r^2 + \frac{1}{12} m_R l^2 \omega^2\right]$$

$$= \frac{1}{2}\left[\frac{1}{3} m_T l^2 + m_R \left(\frac{1}{3} l^2 + lx + x^2\right)\right]\omega^2 + \frac{1}{2} m_R v_r^2 = \frac{1}{2} H_O \omega + \frac{1}{2} m_R v_r^2$$

Potential energy. All motion is horizontal. $V = 0$

State 1. $x = 0,\qquad \omega = \omega_1 = 5\text{ rad/s},\qquad v_r = 0$

$$(H_O)_1 = \frac{1}{3}(m_T + m_R)l^2\omega = \frac{1}{3}(6+4)(0.800)^2(5) = 10.6667\text{ kg}\cdot\text{m}^2\text{/s}$$

$$T_1 = \frac{1}{2}\left[\frac{1}{3}(m_T + m_R)l^2\omega^2\right] + 0 = \frac{1}{2}(H_O)_1\omega = \frac{1}{2}(10.6667)(5) = 26.667\text{ J}$$

$$V_1 = 0$$

State 2. $x = \dfrac{l}{2} = 0.400\text{ m},\qquad \omega = \omega_2 = ?,\qquad v_r = ?$

$$(H_O)_2 = \left[\frac{1}{3}(6)(0.800)^2 + (4)\left\{\frac{1}{3}(0.800)^2 + (0.800)(0.400) + (0.400)^2\right\}\right]\omega_2$$

$$= 4.05333\omega_2$$

$$T_2 = \frac{1}{2}(4.05333\omega_2)\omega_2 + \frac{1}{2}(4)v_r^2 = 2.026667\omega_2^2 + 2v_r^2$$

$$V_2 = 0$$

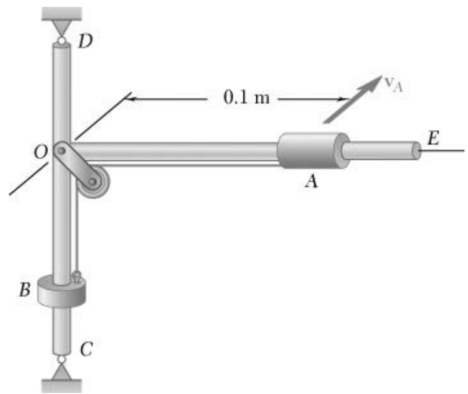
Conservation of angular momentum: $(H_O)_1 = (H_O)_2$

$$10.6667 = 4.05333\,\omega_2 \qquad \omega_2 = 2.6316\text{ rad/s}$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2$

$$26.667 + 0 = (2.02667)(2.6316)^2 + 2v_r^2 + 0$$

$$v_r^2 = 6.3158\text{ m}^2\text{/s}^2 \qquad v_r = 2.51\text{ m/s} ◀$$



PROBLEM 17.89

A 1.8-kg collar A and a 0.7-kg collar B can slide without friction on a frame, consisting of the horizontal rod OE and the vertical rod CD, which is free to rotate about its vertical axis of symmetry. The two collars are connected by a cord running over a pulley that is attached to the frame at O. At the instant shown, the velocity $\mathbf{v}_A$ of collar A has a magnitude of 2.1 m/s and a stop prevents collar B from moving. The stop is suddenly removed and collar A moves toward E. As it reaches a distance of 0.12 m from O, the magnitude of its velocity is observed to be 2.5 m/s. Determine at that instant the magnitude of the angular velocity of the frame and the moment of inertia of the frame and pulley system about CD.

SOLUTION

Components of velocity of collar A. $$v_A^2 = (v_A)_r^2 + (v_A)_\theta^2 \qquad (1)$$

Constraint of rod OE. $$(v_A)_\theta = r_A\omega \qquad (2)$$

Constraint of cable AB. $$\Delta r_A = \Delta y_B, \quad (v_A)_r = v_B \qquad (3)$$

Position 1. $$(\Delta r_A) = 0.1\text{ m}, \quad [(v_A)_r]_1 = 0, \quad (v_A)_1 = 2.1\text{ m/s}$$

From Equation (1), $$(2.1)^2 = 0 + [(v_A)_\theta]_1^2 \qquad [(v_A)_\theta]_1 = 2.1\text{ m/s}$$

From Equation (2), $$(2.1) = 0.1\omega_1 \qquad \omega_1 = 21\text{ rad/s}$$

From Equation (3), $$v_B = 0$$

Potential energy. Take position 1 as datum. $$V_1 = 0 \qquad (4)$$

Angular momentum. $$(H_O)_1 = I\omega_1 + m_A[(v_A)_r]_1(r_A)_1:$$

$$(H_O)_1 = I(21) + (1.8)(2.1)(0.1) \qquad (H_O)_1 = 21I + 0.378 \qquad (5)$$

Kinetic energy. $$T_1 = \frac{1}{2}I\omega_1^2 + \frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B^2:$$

$$T_1 = \frac{1}{2}I(21)^2 + \frac{1}{2}(1.8)(2.1)^2 \qquad T_1 = 220.5I + 3.969 \qquad (6)$$

Position 2. $$(r_A)_2 = 0.12\text{ m}, \quad (v_A)_2 = 2.5\text{ m/s} \qquad \omega = \omega_2$$

From Equation (2), $$[(v_A)_\theta]_2 = 0.12\omega_2$$

From Equation (1), $$[(v_A)_r]_2^2 = (v_A)_2^2 - [(v_A)_\theta]_2^2 = (2.5)^2 - (0.12)^2\omega_2^2$$

$$= 6.25 - 0.0144\omega_2^2$$

From Equation (3), $v_B^2 = 6.25 - 0.0144\omega_2^2$

Change in radial position. $\Delta r_A = (r_A)_2 - (r_A)_1 = 0.02 \text{ m}$

From Equation (3), $\Delta y_B = 0.02 \text{ m}$

Potential energy. $V_2 = m_B g(\Delta y_B) = (0.7)(9.81)(0.02)$

$$V_2 = 0.13734 \text{ J} \qquad (7)$$

Angular momentum. $(H_O)_2 = I\omega_2 + m_A[(v_A)_\theta]_2(r_A)_2$:

$$(H_O)_2 = I\omega_2 + (1.8)(0.12\omega_2)(0.12) \qquad (H_O)_2 = (I + 0.02592)\omega_2 \qquad (8)$$

Kinetic energy. $T_2 = \frac{1}{2}I\omega_2^2 + \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$:

$$T_2 = \frac{1}{2}I\omega_2^2 + \frac{1}{2}(1.8)(2.5)^2 + \frac{1}{2}(0.7)(6.25 - 0.0144\omega_2^2)$$

$$T_2 = (0.5I - 0.00504)\omega_2^2 + 7.8125 \qquad (9)$$

Conservation of angular momentum. $(H_O)_1 = (H_O)_2$:

$$21I + 0.378 = (I + 0.02592)\omega_2$$

Solving for ω_2, $\omega_2 = \frac{21I + 0.378}{I + 0.02592} = \frac{N}{D}$ (10)

Conservation of energy. $T_1 + V_1 = T_2 + V_2$:

$$220.5I + 3.969 = (0.5I - 0.00504)\omega_2^2 + 7.8125 + 0.13734$$

$$220.5I - (0.5I - 0.00504)\frac{N^2}{D^2} - 3.98084 = 0$$

$$220.5ID^2 - 0.5IN^2 + 0.00504N^2 - 3.98084D^2 = 0$$

$$220.5I(I^2 + 0.05184I + 0.0006718464) - 0.5I(441I^2 + 15.876I + 0.142884)$$

$$+ 0.00504(441I^2 + 15.876I + 0.142884) - (3.98084)(I^2 + 0.05184I + 0.0006718464) = 0$$

$$0I^3 + 1.73452I^2 - 0.04965167I - 0.001954378 = 0$$

PROBLEM 17.89 (Continued)

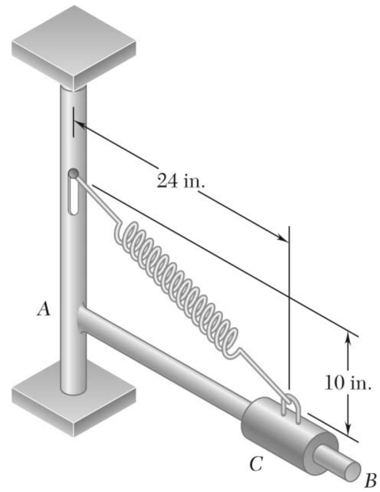
Solving the quadratic equation for I,

$$I = \frac{0.04965167 \pm 0.126590}{3.46904} = 0.050804 \qquad \text{and} \qquad -0.022179$$

Reject the negative root.

From Equation (10), $$\omega_2 = \frac{(21)(0.050804) + 0.378}{0.050804 + 0.02592}$$ $\omega = 18.83 \text{ rad/s}$ ◀

$I = 0.0508 \text{ kg}\cdot\text{m}^2$ ◀



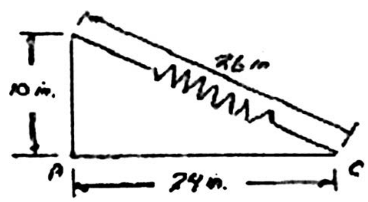
PROBLEM 17.90

A 6-lb collar *C* is attached to a spring and can slide on rod *AB*, which in turn can rotate in a horizontal plane. The mass moment of inertia of rod *AB* with respect to end *A* is 0.35 lb · ft · s^2. The spring has a constant $k = 15$ lb/in. and an undeformed length of 10 in. At the instant shown the velocity of the collar relative to the rod is zero, and the assembly is rotating with an angular velocity of 12 rad/s. Neglecting the effect of friction, determine (*a*) the angular velocity of the assembly as the collar passes through a point located 7.5 in. from end *A* of the rod, (*b*) the corresponding velocity of the collar relative to the rod.

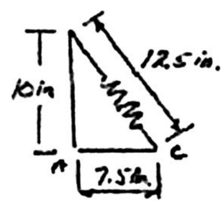
SOLUTION

Potential energy of spring: undeformed length = 10 in.

Position 1:



Position 2:



$$\Delta = 26\text{ in.} - 10\text{ in.} = 16\text{ in.}$$

$$V_1 = \frac{1}{2}k\Delta^2 = \frac{1}{2}(15\text{ lb/in.})(16\text{ in.})^2$$

$$= 1920\text{ in.}\cdot\text{lb}$$

$$V_1 = 160\text{ ft}\cdot\text{lb}$$

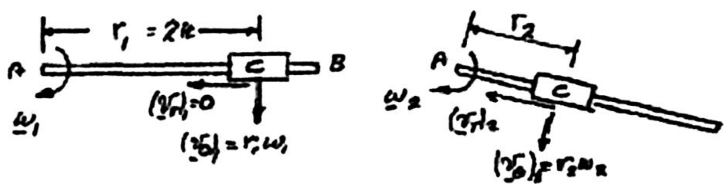
$$\Delta = 12.5\text{ in.} - 10\text{ in.} = 2.5\text{ in.}$$

$$V_2 = \frac{1}{2}k\Delta^2 = \frac{1}{2}(15\text{ lb/in.})(2.5\text{ in.})^2$$

$$= 46.875\text{ in.}\cdot\text{lb}$$

$$V_2 = 3.91\text{ ft}\cdot\text{lb}$$

Kinematics:



Kinetics: Since moments of all forces about shaft at *A* are zero, $(\mathbf{H}_A)_1 = (\mathbf{H}_A)_2$

$$I_R\omega_1 + m_C(v_0)r_1 = I_R\omega_C + m_C(v_0)_2 r_2$$

$$\left(I_R + m_C r_1^2\right)\omega_1 = \left(I_R + m_C r_2^3\right)\omega_2$$

Data:
$$I_R = 0.35\text{ lb}\cdot\text{ft}\cdot\text{s}^2, \quad m_C = \frac{6\text{ lb}}{32.2}$$
$$r_1 = 2\text{ ft}, \quad r_2 = \frac{7.5}{12}\text{ft}, \quad \omega_1 = 12\text{ rad/s}$$

$$\left[0.35\text{ lb}\cdot\text{ft}\cdot\text{s}^2 + \frac{6\text{ lb}}{32.2}(2\text{ ft})^2\right](12\text{ rad/s}) = \left[0.35\text{ lb}\cdot\text{ft}\cdot\text{s}^2 + \frac{6\text{ lb}}{32.2}\left(\frac{7.5}{12}\text{ft}\right)^2\right]\omega_2$$
$$13.1441 = 0.42279\omega_2; \quad \omega_2 = 31.089\text{ rad/s}$$

(*a*) Angular velocity. $\omega_2 = 31.1\text{ rad/s}$ ◀

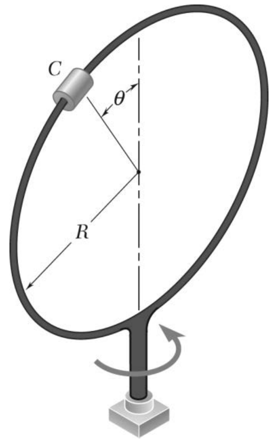
Kinetic energy.
$$T_1 = \frac{1}{2}I_A\omega_1^2 + \frac{1}{2}m_C(v_D)_1^2 + \frac{1}{2}m_C(v_r)_1^2$$
$$= \frac{1}{2}(0.35\text{ lb}\cdot\text{ft}\cdot\text{s}^2)(12\text{ rad/s})^2 + \frac{1}{2}\left(\frac{6\text{ lb}}{32.2}\right)(2\text{ ft})^2(12\text{ rad/s})^2 + 0$$
$$T_1 = 78.865\text{ ft}\cdot\text{lb}$$
$$T_2 = \frac{1}{2}I_R\omega_2^2 + \frac{1}{2}m(v_B)_2^2 + \frac{1}{2}m_2(v_r)_2^2$$
$$= \frac{1}{2}(0.35\text{ lb}\cdot\text{ft}\cdot\text{s}^2)(31.089\text{ rad/s})^2$$
$$+\frac{1}{2}\left(\frac{6\text{ lb}}{32.2}\right)\left(\frac{7.5}{12}\text{ft}\right)^2(31.089\text{ rad/s})^2 + \frac{1}{2}\left(\frac{6\text{ lb}}{32.2}\right)(v_r)_2^2$$
$$T_2 = 204.32 + 0.09317(v_r)_2^2$$

Principle of conservation of energy: $T_1 + V_1 = T_2 + V_2$

Recall: $V_1 = 160\text{ ft}\cdot\text{lb}$ and $V_2 = 3.91\text{ ft}\cdot\text{lb}$

$$78.865 + 160 = 204.32 + 0.09317(v_r)_2^2 + 3.91$$
$$30.638 = 0.09317(v_r)_2^2$$

(*b*) Velocity of collar relative to rod. $(v_r)_2 = 18.13\text{ ft/s}$ ◀

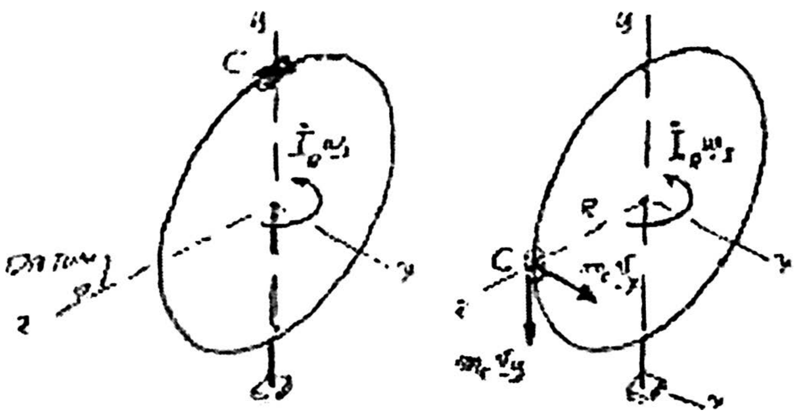


PROBLEM 17.91

A small 4-lb collar C can slide freely on a thin ring of weight 6 lb and radius 10 in. The ring is welded to a short vertical shaft, which can rotate freely in a fixed bearing. Initially the ring has an angular velocity of 35 rad/s and the collar is at the top of the ring $(\theta = 0)$ when it is given a slight nudge. Neglecting the effect of friction, determine (*a*) the angular velocity of the ring as the collar passes through the position $\theta = 90°$, (*b*) the corresponding velocity of the collar relative to the ring.

SOLUTION

Moment of inertia of ring. $$\bar{I}_R = \frac{1}{2} m_R R^2$$



Position 1 Position 2

Position 1. $$\theta = 0$$

$$v_C = 0$$

Position 2. $$\theta = 90°$$

$$(v_C)_y = v_y = R\omega_2$$

Conservation of angular momentum about y axis for system.

$$\bar{I}_R \omega_1 = \bar{I}_R \omega_2 + m_C v_y R$$

$$\frac{1}{2} m_R R^2 \omega_1 = \frac{1}{2} m_R R^2 \omega_2 + m_C R^2 \omega_2$$

$$m_R R^2 \omega_1 = (m_R + 2m_C) R^2 \omega_2$$

$$\omega_2 = \frac{m_R}{m_R + 2m_C} \omega_1 \quad (1)$$

Potential energy. Datum is the center of the ring.

$$V_1 = m_C gR \qquad V_2 = 0$$

Kinetic energy:

$$T_1 = \frac{1}{2}\bar{I}_R\omega_1^2 = \frac{1}{2}\left(\frac{1}{2}m_R R^2\right)\omega_1^2$$

$$= \frac{1}{4}m_R R^2\omega_1^2$$

$$T_2 = \frac{1}{2}\bar{I}_R\omega_2^2 + \frac{1}{2}m_C\left(v_x^2 + v_y^2\right)$$

$$= \frac{1}{4}m_R R^2\omega_2^2 + \frac{1}{2}m_C R^2\omega_2^2 + \frac{1}{2}m_C v_y^2$$

Principle of conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{4}m_R R^2\omega_1^2 + m_C gR = \left(\frac{1}{4}m_R + \frac{1}{2}m_C\right)R^2\omega_2^2 + \frac{1}{2}m_C v_y^2 \qquad (2)$$

Data:

$$W_C = 4 \text{ lb}$$

$$W_R = 6 \text{ lb}$$

$$R = 10 \text{ in.} = 0.83333 \text{ ft}$$

$$\omega_1 = 35 \text{ rad/s}$$

(*a*) Angular velocity.

From Eq. (1),

$$\omega_2 = \frac{\frac{6\text{ lb}}{g}}{\frac{6\text{ lb}}{g} + 2\left(\frac{4\text{ lb}}{g}\right)}(35 \text{ rad/s}) \qquad \omega_2 = 15.00 \text{ rad/s} \blacktriangleleft$$

(*b*) Velocity of collar relative to ring.

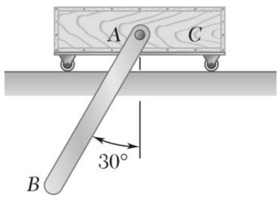
From Eq. (2),

$$\frac{1}{4}\left(\frac{6\text{ lb}}{32.2}\right)\left(\frac{10}{12}\text{ft}\right)^2(35 \text{ rad/s})^2 + (4 \text{ lb})\left(\frac{10}{12}\text{ft}\right)$$

$$= \left[\frac{1}{4}\left(\frac{6\text{ lb}}{32.2}\right) + \frac{1}{2}\left(\frac{4\text{ lb}}{32.2}\right)\right]\left(\frac{10}{12}\text{ft}\right)^2(15 \text{ rad/s})^2 + \frac{1}{2}\left(\frac{4\text{ lb}}{32.2}\right)v_y^2$$

$$39.629 + 3.3333 = 16.984 + 0.062112v_y^2$$

$$v_y^2 = 418.25 \qquad v_y = 20.5 \text{ ft/s} \blacktriangleleft$$



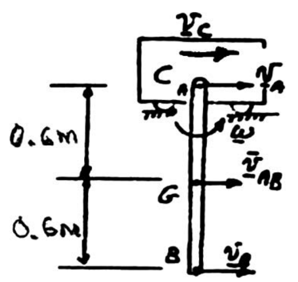
PROBLEM 17.92

A uniform rod AB, of mass 7 kg and length 1.2 m, is attached to the 11-kg cart C. Knowing that the system is released from rest in the position shown and neglecting friction, determine (*a*) the velocity of Point B as rod AB passes through a vertical position (*b*) the corresponding velocity of cart C.

SOLUTION

Kinematics

$$\mathbf{v}_C = \mathbf{v}_A$$



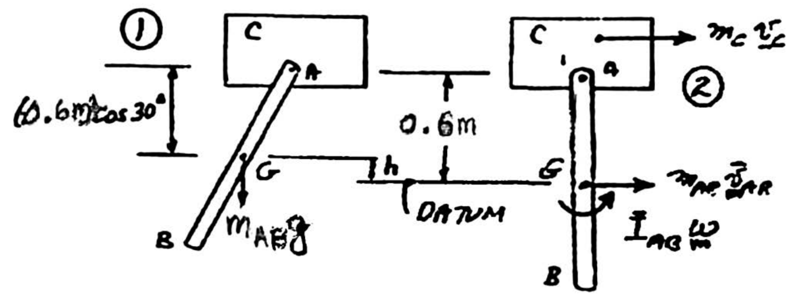
$$\bar{v}_{AB} \longrightarrow\; = v_C \longrightarrow + (0.6\text{ m})\omega \longrightarrow$$

$$v_C = \bar{v}_{AB} - 0.6\omega \qquad (1)$$

$$AB = 1.2\text{ m}$$

Weights.

Kinetics



Linear momentum

$$\xrightarrow{+}: \quad 0 = m_C v_C + m_{AB}\bar{v}_{AB}$$

$$\bar{v}_{AB} = -\frac{m_C}{m_{AB}}v_C = -\frac{(11\text{ kg})}{(7\text{ kg})}v_C, \quad \bar{v}_{AB} = -\frac{11}{7}v_C \qquad (2)$$

Substitute into Eq. (1):

$$v_C = -\frac{11}{7}v_C - 0.6\omega$$

$$\frac{18}{7}v_C = -0.6\omega \qquad v_C = -0.23333\omega \qquad (3)$$

Substitute into Eq. (2):

$$\bar{v}_{AB} = -\frac{11}{7}(-0.23333\omega)$$

$$\bar{v}_{AB} = 0.36667\omega \qquad (4)$$

Kinetic and potential energies.

$$T_1 = 0$$

$$V_1 = m_{AB} g b = (7 \text{ kg})(9.81)(0.6 \text{ m})(1 - \cos 30°)$$
$$= 5.520 \text{ N} \cdot \text{m}$$

$$V_2 = 0$$

$$T_2 = \frac{1}{2} m_C v_C^2 + \frac{1}{2} m_{AB} \overline{v}_{AB}^2 + \frac{1}{2} \overline{I}_{AB} \omega^2$$
$$= \frac{1}{2}(11)(-0.23333\omega)^2 + \frac{1}{2}(7)(0.36667\omega)^2 + \frac{1}{2}\left(\frac{1}{12}(7)(1.2)^2\right)\omega^2$$
$$= (0.29944 + 0.47056 + 0.4200)\omega^2$$
$$= 1.190\omega^2$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$

$$0 + 5.52 = 1.190\omega^2$$

$$\omega^2 = 4.6387 \qquad \omega = 2.1538 \text{ rad/s}$$

(*b*) Velocity of *C*: Eq. (3) $v_C = -0.23333(2.1538)$ $\qquad \mathbf{v}_C = 0.503 \text{ m/s} \longleftarrow$ ◀

(*a*) Velocity of *B*: $\mathbf{v}_B = \mathbf{v}_C + [(1.2)\omega \longrightarrow] = [0.50254 \text{ m/s} \longleftarrow] + [1.2(2.1538) \longrightarrow]$

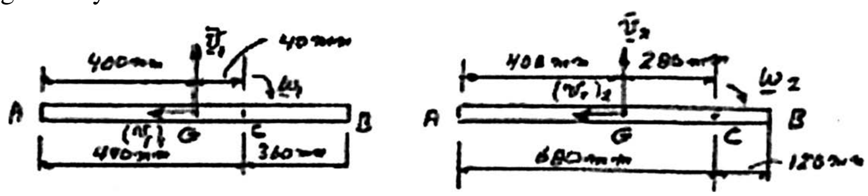
$$\mathbf{v}_B = [0.50254 \longleftarrow] + [2.5845 \longrightarrow]$$ $\qquad \mathbf{v}_B = 2.08 \text{ m/s} \longrightarrow$ ◀

PROBLEM 17.93

In Problem 17.82, determine the velocity of rod *AB* relative to cylinder *DE* as end *B* of the rod strikes end *E* of the cylinder.

SOLUTION

Kinematics and geometry.

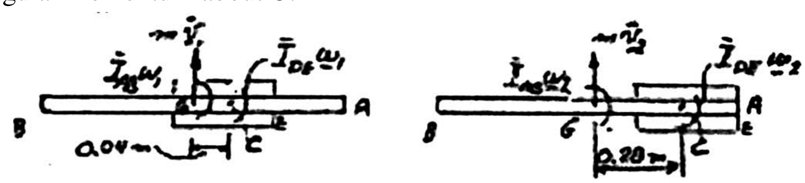


$\bar{v}_1 = (0.04\text{ m})\omega_1 = (0.4\text{ m})(40\text{ rad/s})$ $\qquad$ $\bar{v}_2 = (0.28\text{ m})\omega_2$

$\bar{v}_1 = 1.6\text{ m/s}$

Initial position $\qquad$ Final position

Conservation of angular momentum about *C*.



+↻ Moments about *C*: $\qquad \bar{I}_{AB} = \frac{1}{12}(3\text{ kg})(0.8\text{ m})^2 = 0.16\text{ kg}\cdot\text{m}^2$

$$\bar{I}_{AB}\omega_1 + m\bar{v}_1(0.04\text{ m}) + I_{DE}\omega_1 = I_{AB}\omega_2 + m\bar{v}_2(0.28\text{ m}) + I_{DE}\omega_2$$

$$(0.16\text{ kg}\cdot\text{m}^2)(40\text{ rad/s}) + (3\text{ kg})(1.6\text{ m/s})(0.04\text{ m}) + (0.025\text{ kg}\cdot\text{m}^2)(40\text{ rad/s})$$

$$= (0.16\text{ kg}\cdot\text{m}^2)\omega_2 + (3\text{ kg})(0.28\omega_2)(0.28) + (0.025\text{ kg}\cdot\text{m}^2)\omega_2$$

$$(6.4 + 0.192 + 1.00) = (0.16 + 0.2352 + 0.025)\omega_2$$

$$7.592 = 0.4202\omega_2; \quad \omega_2 = 18.068\text{ rad/s}: \quad \omega_2 = 18.07\text{ rad/s}$$

Conservation of energy $\qquad (v_r) = 0.075\text{ m/s}$

$$V_1 = V_2 = 0$$

$$T_1 = \frac{1}{2}\bar{I}_{DE}\omega_1^2 + \frac{1}{2}\bar{I}_{AB}\omega_1^2 + \frac{1}{2}m_{AB}\bar{v}_1^2 + \frac{1}{2}m_{AB}(v_r)_1^2$$

$$= \frac{1}{2}(0.025\text{ kg}\cdot\text{m}^2)(40\text{ rad/s})^2 + \frac{1}{2}(0.16\text{ kg}\cdot\text{m}^2)(40\text{ rad/s})^2$$

$$+ \frac{1}{2}(3\text{ kg})(1.6\text{ m/s})^2 + \frac{1}{2}(3\text{ kg})(0.075\text{ m/s})^2$$

PROBLEM 17.93 (Continued)

$$T_1 = 20\text{ J} + 128\text{ J} + 3.84\text{ J} + 0.008\text{ J} = 151.85\text{ J}$$

$$\bar{v}_2 = (0.28\text{ m})\omega_2 = (0.28\text{ m})(18.068\text{ rad/s}) = 5.059\text{ m/s}$$

$$T_2 = \frac{1}{2}\bar{I}_{DE}\omega_2^2 + \frac{1}{2}\bar{I}_{AB}\omega_2^2 + \frac{1}{2}m_{AB}v_2^2 + \frac{1}{2}m_{AB}(v_r)_2^2$$

$$= \frac{1}{2}(0.025\text{ kg}\cdot\text{m}^2)(18.068\text{ rad/s})^2$$

$$+\frac{1}{2}(0.16\text{ kg}\cdot\text{m}^2)(18.068\text{ rad/s})^2$$

$$= \frac{1}{2}(3\text{ kg})(5.059\text{ m/s})^2 + \frac{1}{2}(3\text{ kg})(v_r)_2^2$$

$$T_2 = 4.081\text{ J} + 26.116\text{ J} + 38.391\text{ J} + 1.5(v_r)_2^2$$

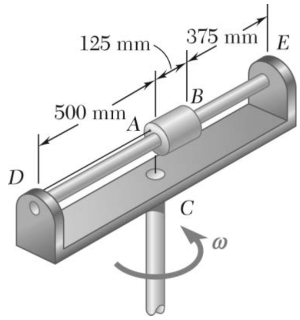
$$T_2 = 68.587\text{ J} + 1.5(v_r)_2^2$$

$$T_1 + V_1 = T_2 + V_2: \quad 151.85\text{ J} + 0 = 68.587\text{ J} + 1.5(v_r)_2^2$$

$$83.263 = 1.5(v_r)_2^2$$

Velocity of rod relative to cylinder. $(v_r)_2 = 7.45\text{ m/s}$ ◀

PROBLEM 17.94



In Problem 17.83 determine the velocity of the tube relative to the rod as the tube strikes end E of the assembly.

PROBLEM 17.83 A 1.6-kg tube AB can slide freely on rod DE which in turn can rotate freely in a horizontal plane. Initially the assembly is rotating with an angular velocity $\omega = 5$ rad/s and the tube is held in position by a cord. The moment of inertia of the rod and bracket about the vertical axis of rotation is $0.30\ \text{kg}\cdot\text{m}^2$ and the centroidal moment of inertia of the tube about a vertical axis is $0.0025\ \text{kg}\cdot\text{m}^2$. If the cord suddenly breaks, determine (*a*) the angular velocity of the assembly after the tube has moved to end E, (*b*) the energy lost during the plastic impact at E.

SOLUTION

Let Point C be the intersection of axle C and rod DE. Let Point G be the mass center of tube AB.

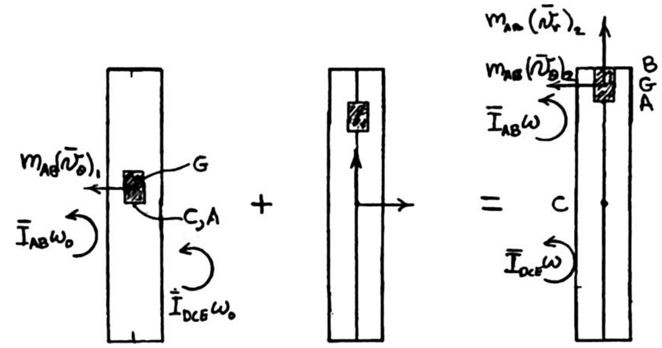
Masses and moments of inertia about vertical axes.

$$m_{AB} = 1.6\ \text{kg}, \qquad \bar{I}_{AB} = 0.0025\ \text{kg}\cdot\text{m}^2, \qquad \bar{I}_{DCE} = 0.30\ \text{kg}\cdot\text{m}^2$$

State 1. $\quad (r_{G/A})_1 = \frac{1}{2}(125) = 62.5\ \text{mm}, \qquad \omega_1 = 5\ \text{rad/s}, \quad (v_r)_1 = 0$

State 2. $\quad (r_{G/A})_2 = 500 - 62.5 = 437.5\ \text{mm}, \qquad \omega = \omega_2, \qquad v_r = (v_r)_2 = 0$

Kinematics. $\quad (v_G)_\theta = \bar{v}_\theta = r_{G/C}\,\omega$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Moments about C:

$$\bar{I}_{AB}\omega_1 + \bar{I}_{DCE}\omega_1 + m_{AB}(\bar{v}_\theta)_1(r_{G/C})_1 + 0 = \bar{I}_{AB}\omega_2 + \bar{I}_{DCE}\omega_2 + m_{AB}(\bar{v}_\theta)_2(r_{G/C})_2$$

$$\left[\bar{I}_{AB} + \bar{I}_{DCE} + m_{AB}(r_{G/C})_1^2\right]\omega_1 = \left[\bar{I}_{AB} + \bar{I}_{DCE} + m_{AB}(r_{G/C})_2^2\right]\omega_2$$

$$[0.0025 + 0.30 + (1.6)(0.0625)^2](5) = [0.0025 + 0.30 + (1.6)(0.4375)^2]\omega_2$$

$$(0.30875)(5) = 0.60875\omega_2 \qquad \omega_2 = 2.5359\ \text{rad/s}$$

PROBLEM 17.94 (Continued)

Kinetic energy.

$$T = \frac{1}{2}\bar{I}_{AB}\omega^2 + \frac{1}{2}\bar{I}_{DCE}\omega^2 + \frac{1}{2}m_{AB}\bar{v}^2$$

$$= \frac{1}{2}\bar{I}_{AB}\omega^2 + \frac{1}{2}\bar{I}_{DCE}\omega^2 + \frac{1}{2}m_{AB}\left(r_{G/C}^2\omega^2 + \bar{v}_r^2\right)$$

$$T_1 = \frac{1}{2}(0.0025)(5)^2 + \frac{1}{2}(0.3)(5)^2 + \frac{1}{2}(1.6)(0.0625)^2 + 0 = 3.859375 \text{ J}$$

$$T_2 = \frac{1}{2}(0.0025)(2.5359)^2 + \frac{1}{2}(0.30)(2.5359)^2$$

$$+\frac{1}{2}(1.6)(0.4375)^2(2.5359)^2 + \frac{1}{2}(1.6)(\bar{v}_r)_2^2$$

$$= 1.95737 + 0.8(\bar{v}_r)_2^2$$

Work. The work of the bearing reactions at C is zero. Since the sliding contact between the rod and the tube is frictionless, the work of the contact force is zero.

$$U_{1\to 2} = 0$$

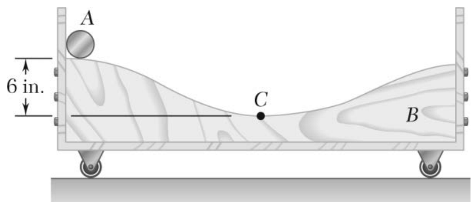
Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2$$

$$3.859375 + 0 = 1.95737 + 0.8(\bar{v}_r)_2^2$$

Velocity of the tube relative to the rod. $(\bar{v}_r)_2 = 1.542 \text{ m/s}$ ◀

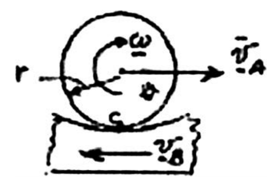
PROBLEM 17.95



The 6-lb steel cylinder A and the 10-lb wooden cart B are at rest in the position shown when the cylinder is given a slight nudge, causing it to roll without sliding along the top surface of the cart. Neglecting friction between the cart and the ground, determine the velocity of the cart as the cylinder passes through the lowest point of the surface at C.

SOLUTION

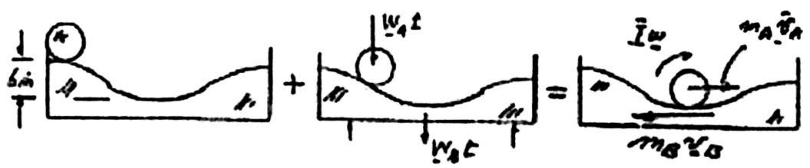
Kinematics (when cylinder is passing C)



$$\overset{+}{\longleftarrow} \quad v_B = v_C = r\omega - \bar{v}_A$$

$$\omega = \frac{\bar{v}_A + v_B}{r}$$

Principle of impulse and momentum.



Syst. of Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\overset{+}{\longrightarrow}$ x components: $\quad m_A\bar{v}_A - m_B v_B = 0$

$$\frac{6\text{ lb}}{g}\bar{v}_A = \frac{10\text{ lb}}{g}v_B; \quad v_B = 0.6\,\bar{v}_A$$

Work:
$$U_{1\to 2} = W_A(6\text{ in.}) = (6\text{ lb})\left(\frac{6}{12}\text{ ft}\right) = 3\text{ ft}\cdot\text{lb}; \quad T_1 = 0$$

Kinetic energy:
$$T_2 = \frac{1}{2}m_A\bar{v}_A^2 + \frac{1}{2}\bar{I}\,\omega^2 + \frac{1}{2}m_B v_3^2$$

$$v_B = 0.6\bar{v}_A; \quad \omega = \frac{\bar{v}_A + v_0}{r} = \frac{v_A + 0.6v_A}{r} = \frac{1.6v_A}{r}$$

$$T_2 = \frac{1}{2}\left(\frac{6\text{ lb}}{g}\right)\bar{v}_A^2 + \frac{1}{2}\left[\frac{1}{2}\frac{6\text{ lb}}{g}r^2\right]\left(\frac{1.6v_A}{r}\right)^2 + \frac{1}{2}\frac{10\text{ lb}}{g}(0.6v_A)^2$$

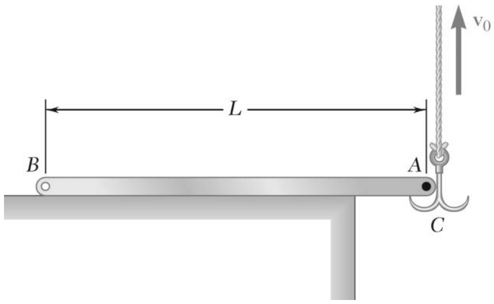
$$= \frac{3}{g}\bar{v}_A^2 + \frac{3.84}{g}\bar{v}_A^2 + \frac{1.8}{g}\bar{v}_A^2 = \frac{8.64}{g}\bar{v}_A^2$$

Principle of work and energy: $\quad T_1 + U_{1\to 2} = T_2$

$$0 + 3\text{ ft}\cdot\text{lb} = \frac{8.64}{32.2}v_A^2$$

$$\bar{v}_A^2 = 11.181 \quad \bar{\mathbf{v}}_A = 3.344\text{ ft/s} \longrightarrow$$

$$v_B = 0.6\bar{v}_A = 0.6(3.344) \qquad \mathbf{v}_B = 2.01\text{ ft/s} \longleftarrow \blacktriangleleft$$

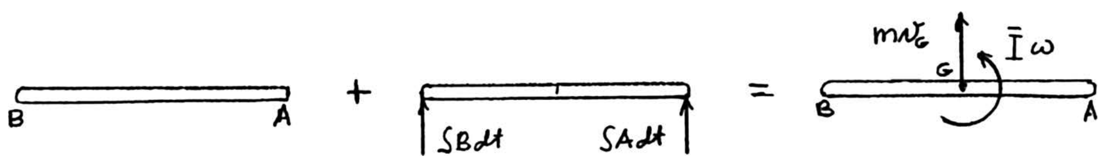


PROBLEM 17.F4

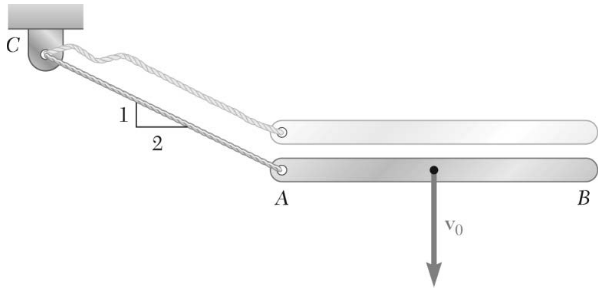
A uniform slender rod AB of mass m is at rest on a frictionless horizontal surface when hook C engages a small pin at A. Knowing that the hook is pulled upward with a constant velocity $\mathbf{v}_0$, draw the impulse-momentum diagram that is needed to determine the impulse exerted on the rod at A and B. Assume that the velocity of the hook is unchanged and that the impact is perfectly plastic.

SOLUTION

Answer:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$



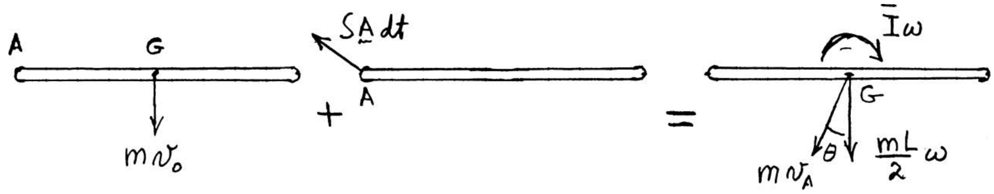
PROBLEM 17.F5

A uniform slender rod AB of length L is falling freely with a velocity $\mathbf{v_0}$ when cord AC suddenly becomes taut. Assuming that the impact is perfectly plastic, draw the impulse-momentum diagram that is needed to determine the angular velocity of the rod and the velocity of its mass center immediately after the cord becomes taut.

SOLUTION

Answer:

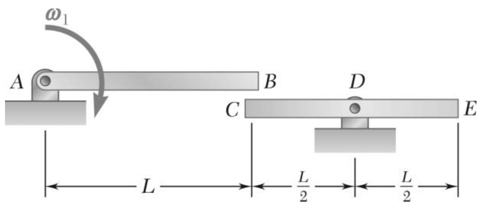
Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Note: For the momentum after the impact a general a_{Gx} and a_{Gy} can be used. These can be related to ω and $\mathbf{v}_A$ using kinematics.

PROBLEM 17.F6

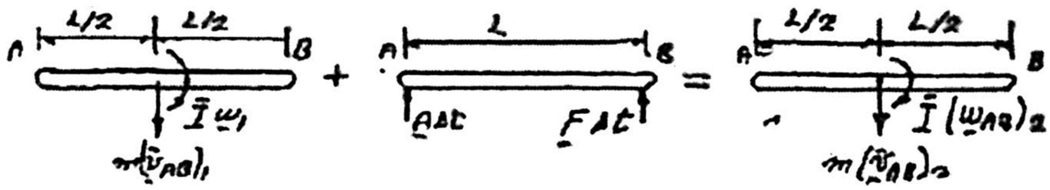


A slender rod *CDE* of length *L* and mass *m* is attached to a pin support at its midpoint *D*. A second and identical rod *AB* is rotating about a pin support at *A* with an angular velocity $\boldsymbol{\omega}_1$ when its end *B* strikes end *C* of rod *CDE*. The coefficient of restitution between the rods is *e*. Draw the impulse-momentum diagrams that are needed to determine the angular velocity of each rod immediately after the impact.

SOLUTION

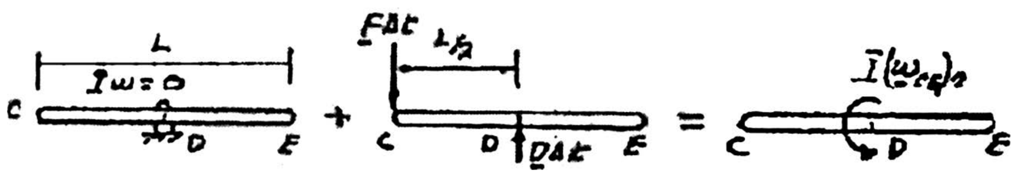
Answer:

Rod *AB*.

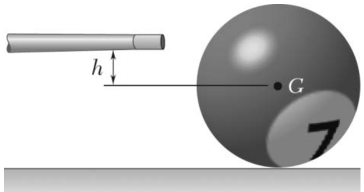


Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\rightarrow 2}$ = **Syst. Momenta**$_2$

Rod *CE*.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\rightarrow 2}$ = **Syst. Momenta**$_2$



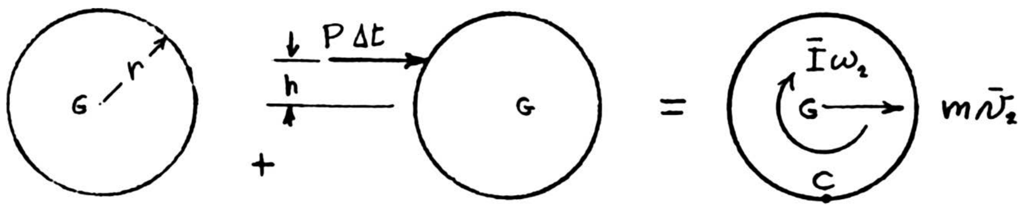
PROBLEM 17.96

At what height h above its center G should a billiard ball of radius r be struck horizontally by a cue if the ball is to start rolling without sliding?

SOLUTION

Moment of inertia. $\bar{I} = \frac{2}{5}mr^2$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics. Rolling without sliding. Point C is the instantaneous center of rotation.

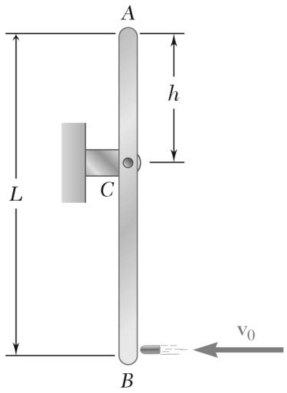
$\xrightarrow{+}$ Linear components: $0 + P\Delta t = m\bar{v}_2$

$$= mr\omega_2$$

$\circlearrowleft$ Moments about G: $0 + hP\Delta t = \bar{I}\omega_2$

$$0 + h(mr\omega_2) = \left(\frac{2}{5}mr^2\right)\omega_2$$

$h = \frac{2}{5}r$ ◀



PROBLEM 17.97

A bullet weighing 0.08 lb is fired with a horizontal velocity of 1800 ft/s into the lower end of a slender 15-lb bar of length $L = 30$ in. Knowing that $h = 12$ in. and that the bar is initially at rest, determine (*a*) the angular velocity of the bar immediately after the bullet becomes embedded, (*b*) the impulsive reaction at *C*, assuming that the bullet becomes embedded in 0.001 s.

SOLUTION

Bar: $L = 30\text{ in.} = 2.5\text{ ft} \quad m = \dfrac{15}{32.2} = 0.46584\text{ lb}\cdot\text{s}^2/\text{ft}$

$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(0.46584)(2.5)^2 = 0.24262\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

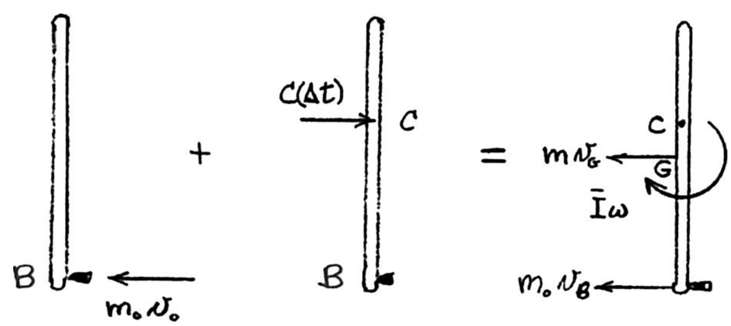
Bullet: $m_0 = \dfrac{0.08}{32.2} = 0.0024845\text{ lb}\cdot\text{s}^2/\text{ft}$

Support location: $h = 12\text{ in.} = 1.0\text{ ft}$

Kinematics. $v_B = (L-h)\omega = (2.5-1.0)\omega = 1.5\omega$

$$v_G = \left(\frac{L}{2} - h\right)\omega = (1.25-1.0)\omega = 0.25\omega$$

Kinetics.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to 2}$ = Syst. Momenta$_2$

↶ Moments about *C*: $m_0 v_0 (L-h) = m_0 v_B (L-h) + m v_0 \left(\dfrac{L}{2} - h\right) + \bar{I}\,\omega$

$$(0.0024845)(1800)(1.5) = (0.0024845)(1.5\omega) + (0.46584)(0.25\omega)(0.25) + (0.24262\omega)$$

PROBLEM 17.97 (Continued)

(*a*) $$6.7082 = 0.27546\omega \quad \text{or} \quad \omega = 24.353$$ $\boldsymbol{\omega} = 24.4 \text{ rad/s}$ ↻ ◀

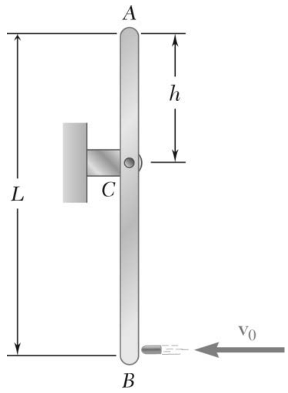
$$v_B = (1.5)(24.353) = 36.53 \text{ ft/s}$$
$$v_G = (0.25)(24.353) = 6.0881 \text{ ft/s}$$

$\xrightarrow{+}$ Horizontal components:

$$-m_0 v_0 + C(\Delta t) = -m_0 v_B - m v_G: \quad C(\Delta t) = m_0(v_0 - v_B) - m v_0$$

$$C(\Delta t) = (0.0024845)(1800 - 36.53) - (0.46584)(6.0881)$$
$$= 1.545 \text{ lb} \cdot \text{s}$$

(*b*) $$C = \frac{C\Delta t}{\Delta t} = \frac{1.545}{0.001}$$ $\mathbf{C} = 1545 \text{ lb} \longrightarrow$ ◀



PROBLEM 17.98

In Problem 17.97, determine (*a*) the required distance h if the impulsive reaction at C is to be zero, (*b*) the corresponding angular velocity of the bar immediately after the bullet becomes embedded.

PROBLEM 17.97 A bullet weighing 0.08 lb is fired with a horizontal velocity of 1800 ft/s into the lower end of a slender 15-lb bar of length $L = 30$ in. Knowing that $h = 12$ in. and that the bar is initially at rest, determine (*a*) the angular velocity of the bar immediately after the bullet becomes embedded, (*b*) the impulsive reaction at C, assuming that the bullet becomes embedded in 0.001 s.

SOLUTION

Bar: $L = 30\text{ in.} = 2.5\text{ ft} \quad m = \dfrac{15}{32.2} = 0.46584\text{ lb}\cdot\text{s}^2/\text{ft}$

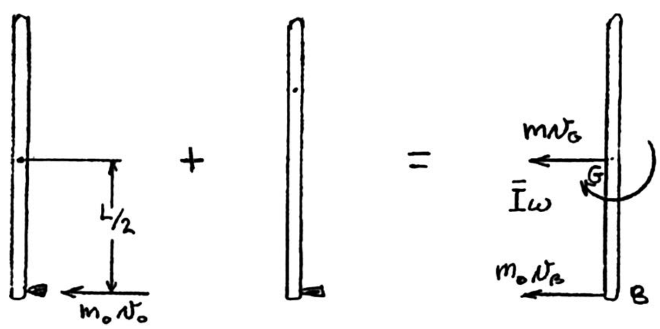
$$\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(0.46584)(2.5)^2 = 2.24262\text{ lb}\cdot\text{s}^2/\text{ft}$$

Bullet: $m_0 = \dfrac{0.08}{32.2} = 0.0024845\text{ lb}\cdot\text{s}^2/\text{ft}$

Kinematics. $v_B = (L-h)\omega = (2.5-h)\omega$

$$v_G = \left(\frac{L}{2}-h\right)\omega = (1.25-h)\omega$$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

↻ moment about B: $0 + 0 = \bar{I}\omega - mv_G\left(\dfrac{L}{2}\right)$

$$0 + 0 = 0.24262\omega - (0.46584)(1.25-h)\omega(1.25)$$

Divide by ω $\quad 0 = 0.24262 - 0.5823(1.25-h)$

PROBLEM 17.98 (Continued)

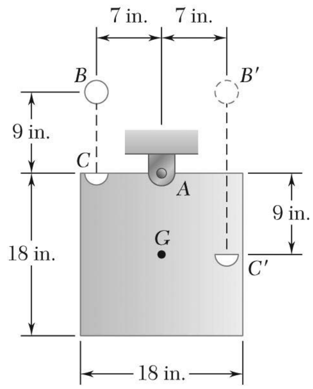
(*a*) $h = 0.8333\text{ ft}$ $h = 10.00\text{ in.}$ ◀

$$v_B = (2.5 - 0.8333)\omega = 1.6667\omega$$
$$v_G = (1.25 - 0.8333)\omega = 0.4167\omega$$

$\overset{+}{\leftarrow}$ Horizontal components: $m_0 v_0 + 0 = m v_G + m_0 v_B$

$$(0.0024845)(1800) + 0 = (0.46584)(0.4167\omega) + (0.0024845)(1.6667\omega)$$

(*b*) $\omega = 22.56$ $\boldsymbol{\omega} = 22.6\text{ rad/s}$ ↻ ◀



PROBLEM 17.99

A 16-lb wooden panel is suspended from a pin support at *A* and is initially at rest. A 4-lb metal sphere is released from rest at *B* and falls into a hemispherical cup *C* attached to the panel at a point located on its top edge. Assuming that the impact is perfectly plastic, determine the velocity of the mass center *G* of the panel immediately after the impact.

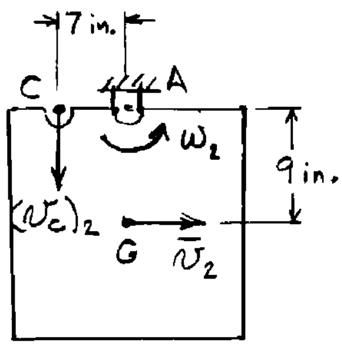
SOLUTION

Mass and moment of inertia

$$W_s = 4 \text{ lb} \quad W_P = 16 \text{ lb}$$

$$\bar{I} = \frac{1}{6} m_P (L)^2 = \frac{1}{6}\left(\frac{16}{32.2}\right)\left(\frac{18}{12}\right)^2 = 0.18634 \text{ slug} \cdot \text{ft}^2$$

Velocity of sphere at *C*. $\quad (v_C)_1 = \sqrt{2gy} = \sqrt{2(32.2 \text{ ft/s}^2)\left(\frac{9}{12}\text{ft}\right)} = 6.9498 \text{ ft/s}$



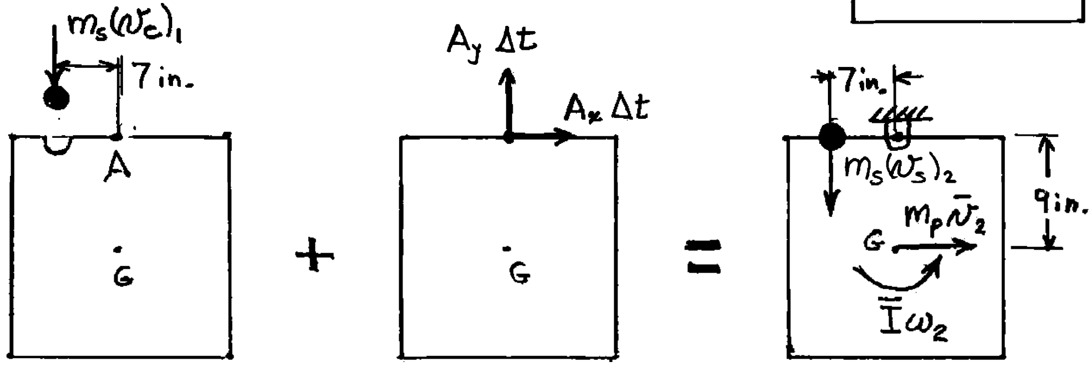
Impact analysis.

Kinematics: Immediately after impact in terms of ω_2

$$\bar{v}_2 = \frac{9}{12}\omega_2$$

$$(v_C)_2 = \frac{7}{12}\omega_2$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

PROBLEM 17.99 (Continued)

$+\circlearrowleft$ Moments about A:

$$m_s(v_C)_1\left(\frac{7}{12}\text{ft}\right)+0=m_s(v_C)_2\left(\frac{7}{12}\text{ft}\right)+\bar{I}\omega_2+m_P\bar{v}_2\left(\frac{9}{12}\text{ft}\right)$$

$$\left(\frac{4\text{ lb}}{32.2}\right)(6.9498\text{ ft/s})\left(\frac{7}{12}\text{ft}\right)=\left(\frac{4\text{ lb}}{32.2}\right)\left(\frac{7}{12}\omega_2\right)\left(\frac{7}{12}\text{ft}\right)+0.18634\omega_2+\left(\frac{16\text{ lb}}{32.2}\right)\left(\frac{9}{12}\omega_2\right)\left(\frac{9}{12}\text{ft}\right)$$

$$0.50361=(0.042271+0.18634+0.2795)\omega_2$$

$$\omega_2=0.99115\text{ rad/s} \quad \boldsymbol{\omega}_2=0.99115\text{ rad/s}\circlearrowleft$$

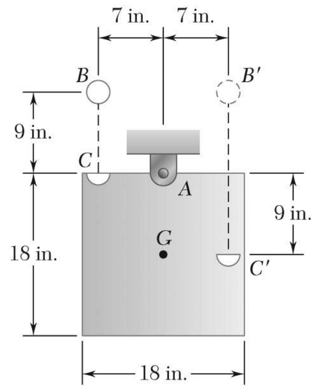
Velocity of the mass center

$$\bar{v}_2=\left(\frac{9}{12}\text{ft}\right)\omega_2=\left(\frac{9}{12}\text{ft}\right)(0.99115\text{ rad/s})$$

$$\bar{v}_2=0.74336\text{ ft/s}$$

$\bar{\mathbf{v}}_2=8.92\text{ in./s}\longrightarrow$ ◀

PROBLEM 17.100



An 16-lb wooden panel is suspended from a pin support at A and is initially at rest. A 4-lb metal sphere is released from rest at B' and falls into a hemispherical cup C' attached to the panel at the same level as the mass center G. Assuming that the impact is perfectly plastic, determine the velocity of the mass center G of the panel immediately after the impact.

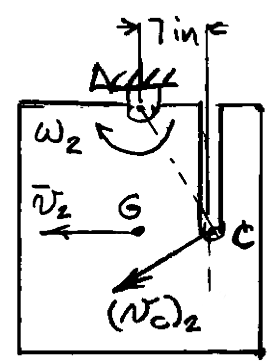
SOLUTION

Mass and moment of inertia.

$$W_s = 4 \text{ lb}$$
$$W_P = 16 \text{ lb}$$
$$\bar{I} = \frac{1}{6} m_P (0.5 \text{ m})^2$$
$$= \frac{1}{6}\left(\frac{16}{32.2}\right)\left(\frac{18}{12}\right)^2$$
$$= 0.18634 \text{ slug}\cdot\text{ft}^2$$

Velocity of sphere at C'.

$$(v_{C'})_1 = \sqrt{2gy}$$
$$= \sqrt{2(32.2 \text{ ft/s}^2)\left(\tfrac{18}{12}\text{ ft}\right)}$$
$$= 9.8285 \text{ ft/s}$$



Impact analysis.

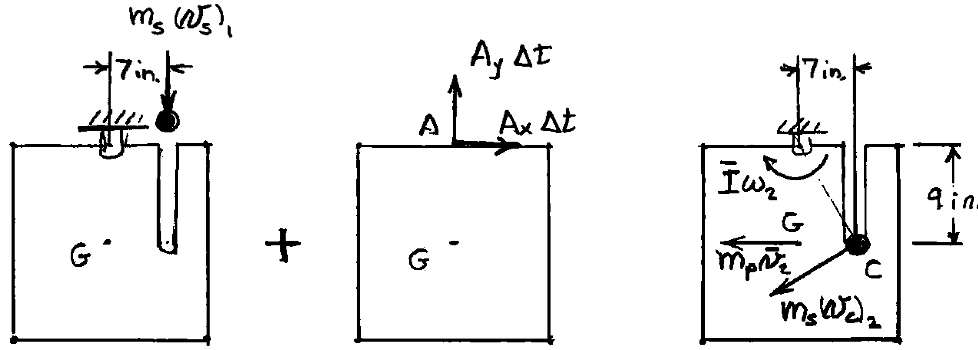
Kinematics: Immediately after impact in terms of ω_2.

$$AC' = \sqrt{\left(\frac{7}{12}\right)^2 + \left(\frac{9}{12}\right)^2} = 0.95015 \text{ ft}$$

$$(\mathbf{v}_{C'})_2 = AC'\omega_2$$
$$= 0.95015\omega_2 \measuredangle \theta \quad \text{(perpendicular to } AC.)$$
$$\bar{v}_2 = \frac{9}{12}\omega_2$$

PROBLEM 17.100 (Continued)

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1-2}$ = **Syst. Momenta**$_2$

+↻ Moments about A:

$$m_s(v_C)_1\left(\frac{7}{12}\text{ ft}\right)+0=m_s(v_C)_2(0.95015\text{ ft})+\bar{I}\omega_2+m_P\bar{v}_2\left(\frac{9}{12}\text{ ft}\right)$$

$$\left(\frac{4\text{ lb}}{32.2}\right)(9.8285\text{ ft/s})\left(\frac{7}{12}\text{ ft}\right)=\left(\frac{4\text{ lb}}{32.2}\right)(0.95015\omega_2)(0.95015\text{ ft})+0.18634\omega_2$$

$$+\left(\frac{16\text{ lb}}{32.2}\right)\left(\frac{9}{12}\omega_2\right)\left(\frac{9}{12}\text{ ft}\right)$$

$$0.71221=(0.11215+0.18634+0.2795)\omega_2$$

$$\omega_2=1.2322$$

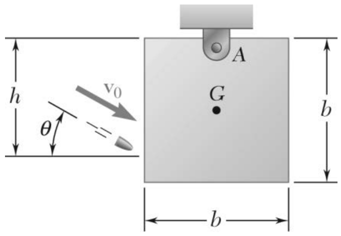
Velocity of the mass center.

$$\bar{v}_2=\left(\frac{9\text{ ft}}{12}\right)\omega_2$$

$$=\left(\frac{9\text{ ft}}{12}\right)(1.2322\text{ rad/s})$$

$$=0.92418\text{ ft/s}$$

$$\bar{\mathbf{v}}_2=11.09\text{ in./s}\longleftarrow \blacktriangleleft$$



PROBLEM 17.101

A 45-g bullet is fired with a velocity of 400 m/s at $\theta = 30°$ into a 9-kg square panel of side $b = 200$ mm. Knowing that $h = 150$ mm and that the panel is initially at rest, determine (*a*) the velocity of the center of the panel immediately after the bullet becomes embedded, (*b*) the impulsive reaction at *A*, assuming that the bullet becomes embedded in 2 ms.

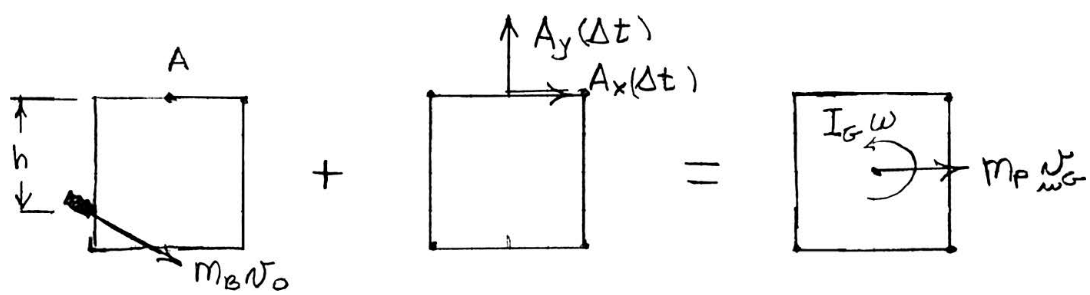
SOLUTION

$$m_B = 0.045 \text{ kg} \quad m_P = 9 \text{ kg} \quad I_G = \frac{1}{6}m_P b^2 = \frac{1}{6}(9)(0.200)^2 = 0.06 \text{ kg}\cdot\text{m}^2$$

Kinematics. After impact, the plate is rotating about the fixed Point *A* with angular velocity $\boldsymbol{\omega} = \omega \circlearrowright$.

$$\mathbf{v}_G = \frac{b}{2}\omega \longrightarrow$$

Principle of impulse and momentum. To simplify the analysis, neglect the mass of the bullet after impact.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

(*a*) $+\circlearrowright$ Moments about *A*:

$$(m_B v_0 \cos 30°)h + m_B v_0 \sin 30°\left(\frac{b}{2}\right) + 0 = I_G\omega + m_P v_G \frac{b}{2}$$

$$m_B v_0\left(h\cos 30° + \frac{b}{2}\sin 30°\right) = \left(I_G + \frac{1}{4}m_P b^2\right)\omega$$

$$(0.045)(400)(0.150\cos 30° + 0.100\sin 30°)$$

$$= \left[0.06 + \frac{1}{4}(9)(0.2)^2\right]\omega = 0.15\omega$$

$$\omega = 21.588 \text{ rad/s}$$

$$v_B = (0.100)(21.588) = 2.1588 \text{ m/s} \qquad \mathbf{v}_G = 2.16 \text{ m/s} \longrightarrow \blacktriangleleft$$

PROBLEM 17.101 (Continued)

(*b*) $\xrightarrow{+}$ Linear momentum: $m_B v_0 \cos 30° + A_x(\Delta t) = m_P v_G$

$$(0.045)(400\cos 30°) + A_x(0.002) = (9)(2.1588)$$

$$A_x = 1920 \text{ N} \qquad \mathbf{A}_x = 1920 \text{ N} \longrightarrow$$

$+\uparrow$ Linear momentum: $-m_B v_0 \sin 30° + A_y(\Delta t) = 0$

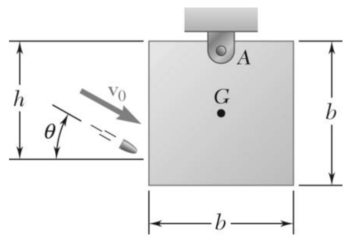
$$-(0.045)(400)\sin 30° + A_y(0.002) = 0$$

$$A_y = 4500 \text{ N} \qquad \mathbf{A}_y = 4500 \text{ N} \uparrow$$

$$A = 4892 \text{ N} = 4.892 \text{ kN} \qquad \tan\beta = \frac{4500}{1920} \quad \beta = 66.9°$$

$\mathbf{A} = 4.87 \text{ kN} \measuredangle 66.9°$ ◀

PROBLEM 17.102



A 45-g bullet is fired with a velocity of 400 m/s at $\theta = 5°$ into a 9-kg square panel of side $b = 200\,\text{mm}$. Knowing that the panel is initially at rest, determine (*a*) the required distance *h* if the horizontal component of the impulsive reaction at *A* is to be zero, (*b*) the corresponding velocity of the center of the panel immediately after the bullet becomes embedded.

SOLUTION

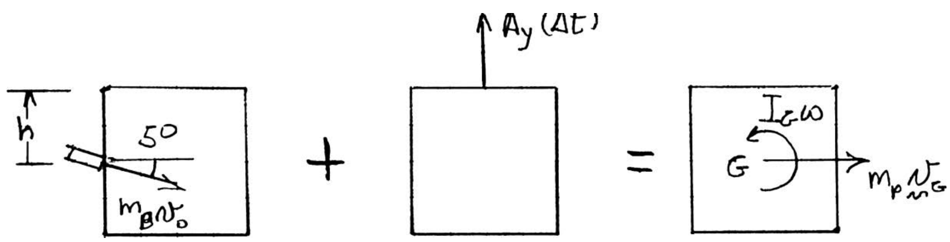
$$m_B = 0.045\text{ kg} \quad m_P = 9\text{ kg} \quad I_G = \frac{1}{6}m_P b^2 = \frac{1}{6}(9)(0.200)^2 = 0.06\text{ kg}\cdot\text{m}^2$$

Kinematics. After impact, the plate is rotating about the fixed Point *A* with angular velocity $\boldsymbol{\omega} = \omega \circlearrowright$.

$$\mathbf{v}_G = \frac{b}{2}\omega \longrightarrow.$$

Principle of impulse and momentum. To simplify the analysis, neglect the mass of the bullet after impact.

Also $\qquad A_X(\Delta t) = 0.$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\xrightarrow{+}$ Linear momentum: $\quad m_B v_0 \cos 5° + 0 = m_P v_G = m_P\left(\frac{b}{2}\omega\right)$

$$(0.045)(400\cos 5°) = (9)(0.100)\omega \quad \omega = 19.9239\text{ rad/s}$$

$$v_G = (0.100)(19.9239) = 1.99239\text{ m/s} \qquad (1)$$

$+\circlearrowright$ Moments about *A*: $\quad (m_B v_0 \cos 5°)h + (m_B v_0 \sin 5°)\frac{b}{2} = I_G\omega + m_P v_G \frac{b}{2}$

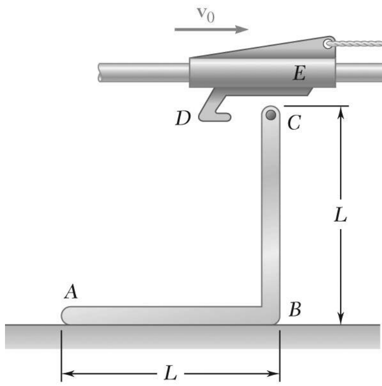
$$m_B v_0\left(h\cos 5° + \frac{b}{2}\sin 5°\right) = \left(I_G + \frac{1}{4}m_P b^2\right)\omega$$

$$(0.045)(400)(h\cos 5° + 0.100\sin 5°) = \left[0.06 + \frac{1}{4}(9)(0.200)^2\right](19.9239)$$

$$17.9315h + 0.1569 = 2.9886$$

(*a*) $\qquad h = 0.15792\text{ m} \qquad h = 158\text{ mm}$ ◀

(*b*) From Eq. (1), $\qquad \mathbf{v}_G = 1.992\text{ m/s} \longrightarrow$ ◀



PROBLEM 17.103

The uniform rods, each of mass m, form the L-shaped rigid body ABC which is initially at rest on the frictionless horizontal surface when hook D of the carriage E engages a small pin at C. Knowing that the carriage is pulled to the right with a constant velocity $\mathbf{v}_0$, determine immediately after the impact (a) the angular velocity of the body, (b) the velocity of corner B. Assume that the velocity of the carriage is unchanged and that the impact is perfectly plastic.

SOLUTION

Kinematics: $\mathbf{v}_B = v_B \longrightarrow, \quad \boldsymbol{\omega} = \omega \circlearrowright$

$$\bar{\mathbf{v}}_{BC} = [v_0 \longrightarrow] + \left[\frac{L}{2}\omega \longleftarrow\right]$$

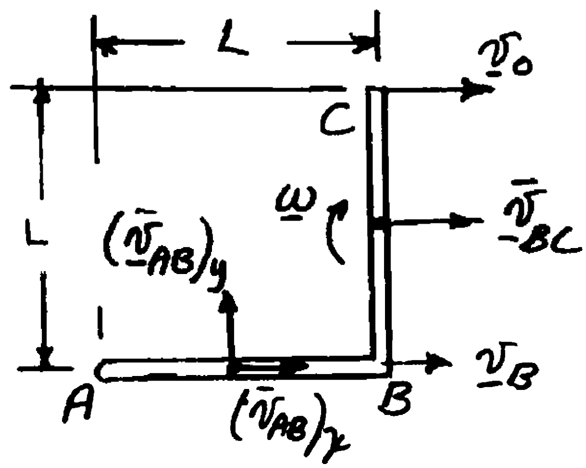
$$\bar{\mathbf{v}}_{BC} = \left[v_0 - \frac{L}{2}\omega\right] \longrightarrow$$

$$\mathbf{v}_B = [v_0 \longrightarrow] + [L\omega \longleftarrow]$$

$$\mathbf{v}_B = [v_0 - L\omega] \longrightarrow$$

$$(\bar{v}_{AB})_x = v_B = [v_0 - L\omega] \longrightarrow$$

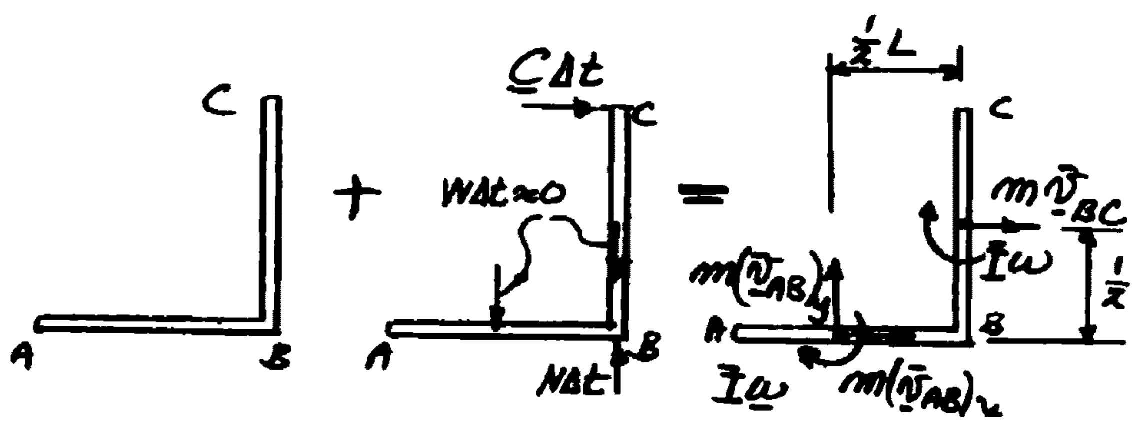
$$(\bar{v}_{AB})_y = (v_B)_y + \frac{L}{2}\omega \uparrow = \frac{L}{2}\omega \uparrow$$



Let m be the mass of each rod.

Moment of inertia of each rod. $\bar{I} = \frac{1}{12}mL^2$

(a) *Principle of impulse and momentum.*



Syst. Momenta$_0$ + **Syst. Ext. Imp.**$_{0-1}$ = **Syst. Momenta**$_1$

PROBLEM 17.103 (Continued)

$\stackrel{+}{\circlearrowright}$ Moments about C:

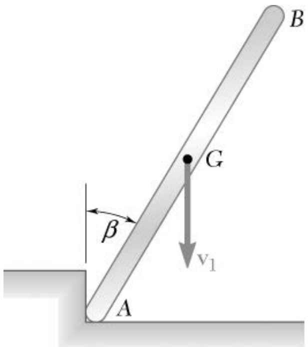
$$0+0 = m(v_{AB})_y\left(\frac{1}{2}L\right) - m(v_{AB})_x L + \bar{I}\omega - m(\bar{v}_{BC})\left(\frac{1}{2}L\right) + \bar{I}\omega$$

$$0 = m\left(\frac{L}{2}\omega\right)\left(\frac{1}{2}L\right) - m(v_0 - L\omega)L + \frac{1}{12}mL^2\omega - m\left(v_0 - \frac{L}{2}\omega\right)\left(\frac{1}{2}L\right) + \frac{1}{12}mL^2\omega$$

$$0 = -\frac{3}{2}mLv_0 + \frac{5}{3}mL^2\omega \qquad \omega = \frac{9}{10}\frac{v_0}{L}$$

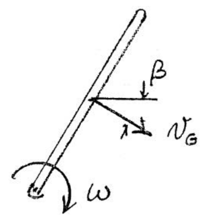
(*a*) *Angular velocity* $\boldsymbol{\omega} = 0.900\ v_0/L \circlearrowright$ ◀

(*b*) *Velocity of B.* $v_B = v_0 - L\omega = \frac{1}{10}v_0$ $\mathbf{v}_B = 0.100\ v_0 \longrightarrow$ ◀

	**PROBLEM 17.104** The uniform slender rod *AB* of weight 5 lb and length 30 in. forms an angle $\beta = 30°$ with the vertical as it strikes the smooth corner shown with a vertical velocity $\mathbf{v}_1$ of magnitude 8 ft/s and no angular velocity. Assuming that the impact is perfectly plastic, determine the angular velocity of the rod immediately after the impact.

SOLUTION

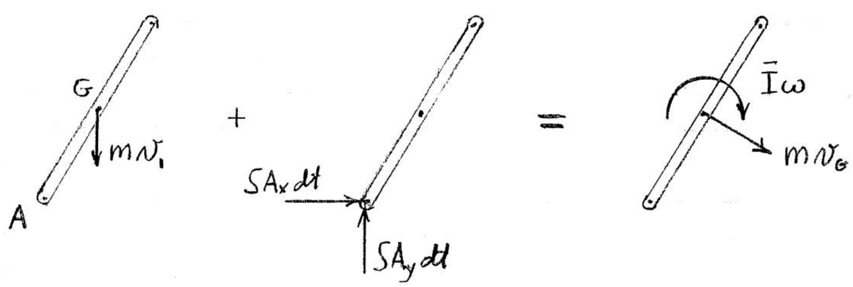
Moment of inertia. $\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}\left(\frac{5}{32.2}\right)\left(\frac{30}{12}\right)^2 = 80.875 \times 10^{-3}\ \text{lb} \cdot \text{s}^2 \cdot \text{ft}$



Kinematics. (Rotation about *A*) $\beta = 30°$

$$v_G = \frac{L}{2}\omega = \frac{15}{12}\omega$$

Kinetics.



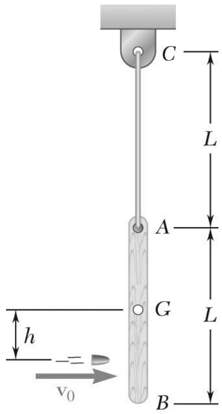
Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to2}$** = **Syst. Momenta$_2$**

↻ moments about *A*: $mv_1\frac{L}{2}\sin\beta + 0 = \bar{I}\omega + mv_G\frac{L}{2}$

$$\left(\frac{5}{32.2}\right)(8)\left(\frac{15}{12}\right)\sin 30° + 0 = 80.875 \times 10^{-3}\omega + \left(\frac{5}{32.2}\right)\left(\frac{15}{12}\omega\right)\left(\frac{15}{12}\right)$$

$$\omega = 2.4\ \text{rad/s}$$

$\boldsymbol{\omega} = 2.40\ \text{rad/s}$ ↻ ◀



PROBLEM 17.105

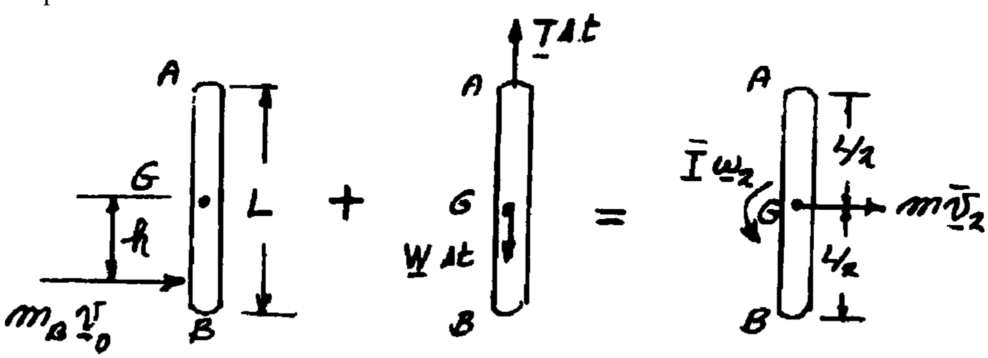
A bullet weighing 0.08 lb is fired with a horizontal velocity of 1800 ft/s into the 15-lb wooden rod AB of length $L = 30$ in. The rod, which is initially at rest, is suspended by a cord of length $L = 30$ in. Determine the distance h for which, immediately after the bullet becomes embedded, the instantaneous center of rotation of the rod is Point C.

SOLUTION

Let m_B be the mass of the bullet and m the mass of the rod. The moment of inertia I of the rod is

$$\bar{I} = \frac{1}{12} mL^2$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

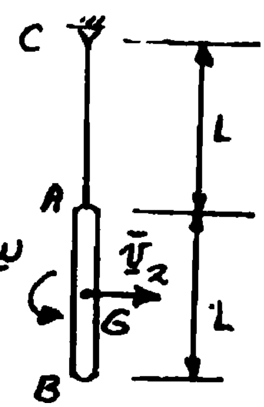
$+\circlearrowright$ Moments about G: $\quad m_B v_0 h = \bar{I}\omega_2 \quad (1)$

$\xrightarrow{+}$ x components: $\quad m_B v_0 = m\bar{v}_2 \quad (2)$

From Eq. (2). $\quad \bar{v}_2 = \frac{m_B}{m} v_0 = \frac{W_B}{W} v_0 \quad (3)$

From Eq. (3). $\quad w_2 = \frac{m_B v_0 h}{\bar{I}} = \frac{\frac{W_B}{g} v_0 h}{\frac{1}{12}\frac{W}{g} L^2} = 12\frac{W_B}{W}\frac{v_0 h}{L^2} \quad (4)$

For the instantaneous center to lie at Point C, $\quad \bar{v}_2 = \frac{3}{2} L\omega_2$



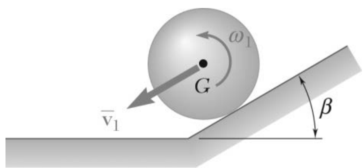
PROBLEM 17.105 (Continued)

Substitute for $\overline{v}_2$ and ω_2 from Equations (3) and (4).

$$\frac{W_B}{W}v_0 = \frac{3}{2}L\left[12\frac{W_B}{W}\cdot\frac{v_0 h}{L^2}\right]$$

$$h = \frac{1}{18}L = \frac{30\text{ in.}}{18}$$

$h = 1.667$ in. ◀



PROBLEM 17.106

A uniform sphere of radius r rolls down the incline shown without slipping. It hits a horizontal surface and, after slipping for a while, it starts rolling again. Assuming that the sphere does not bounce as it hits the horizontal surface, determine its angular velocity and the velocity of its mass center after it has resumed rolling.

SOLUTION

Moment of inertia. Solid sphere. $\bar{I} = \frac{2}{5}mr^2$

Kinematics.

Before After

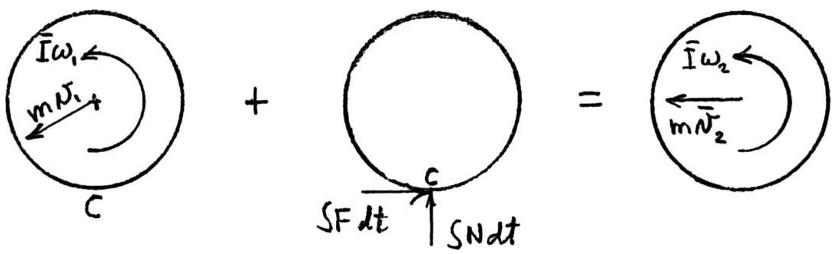
Before impact (rolling).

$$v_1 = r\omega_1$$

After slipping has stopped.

$$\bar{v}_2 = r\omega_2$$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

↻ Moments about C:

$$\bar{I}\omega_1 + mv_1 r\cos\beta + 0 = \bar{I}\omega_2 + m\bar{v}_2 r$$

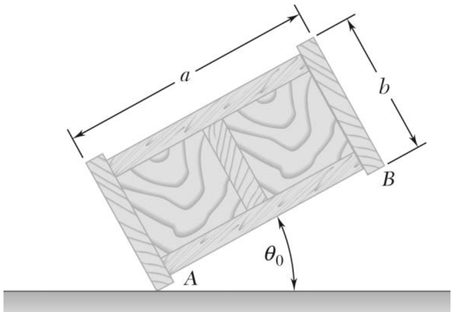
$$\bar{I}\omega_1 + mr^2\omega_1\cos\beta = \bar{I}\omega_2 + mr^2\omega_2$$

$$\omega_2 = \frac{\bar{I} + mr^2\cos\beta}{\bar{I} + mr^2}\omega_1 = \frac{\frac{2}{5}mr^2 + mr^2\cos\beta}{\frac{2}{5}mr^2 + mr^2}\omega_1$$

$$\boldsymbol{\omega}_2 = \frac{1}{7}(2 + 5\cos\beta)\bar{v}_1/r \curvearrowright \blacktriangleleft$$

$$\bar{v}_2 = r\omega_2 = \frac{2 + 5\cos\beta}{7}r\omega_1$$

$$\bar{\mathbf{v}}_2 = \frac{1}{7}(2 + 5\cos\beta)\bar{v}_1 \longleftarrow \blacktriangleleft$$

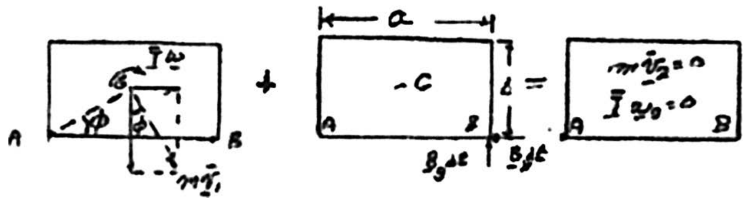


PROBLEM 17.107

A uniformly loaded rectangular crate is released from rest in the position shown. Assuming that the floor is sufficiently rough to prevent slipping and that the impact at B is perfectly plastic, determine the smallest value of the ratio a/b for which corner A will remain in contact with the floor.

SOLUTION

We consider the limiting case when the crate is just ready to rotate about B. At that instant the velocities must be zero and the reaction at corner A must be zero. Use the principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+↻ Moments about B:

$$\bar{I}\omega_1 + (m\bar{v}_1)_2\frac{b}{2} - (m\bar{v}_1)_y\frac{a}{2} + 0 = 0 \qquad (1)$$

Note:
$$\sin\phi = \frac{b}{\sqrt{a^2+b^2}}, \quad \cos\phi = \frac{a}{\sqrt{a^2+b^2}}$$

$$\bar{v}_1 = (AG)v_1 = \frac{1}{2}\sqrt{a^2+b^2}\,\omega_1$$

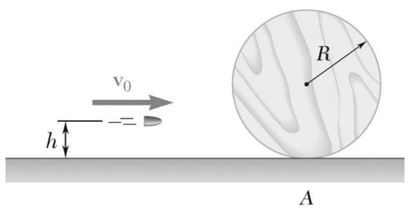
Thus:
$$(m\bar{v}_1)_x = (m\bar{v}_1)\sin\phi = \frac{m}{2}\sqrt{a^2+b^2}\,\omega_1\frac{b}{\sqrt{a^2+b^2}} = \frac{1}{2}mb\omega_1$$

Also,
$$(m\bar{v}_1)_y = (m\bar{v}_1)\cos\phi = \frac{1}{2}ma\omega$$

$$\bar{I} = \frac{1}{12}m(a^2+b^2)$$

From Eq. (1)
$$\frac{1}{12}m(a^2+b^2)\omega_1 + \frac{1}{2}(mb\omega_1)\frac{b}{2} - \frac{1}{2}(ma\omega_1)\frac{a}{2} = 0$$

$$\frac{1}{3}mb^2\omega_1 - \frac{1}{6}ma^2v_1 = 0 \qquad \frac{a^2}{b^2} = 2 \quad \frac{a}{b} = \sqrt{2} \quad \frac{a}{b} = 1.414 \blacktriangleleft$$



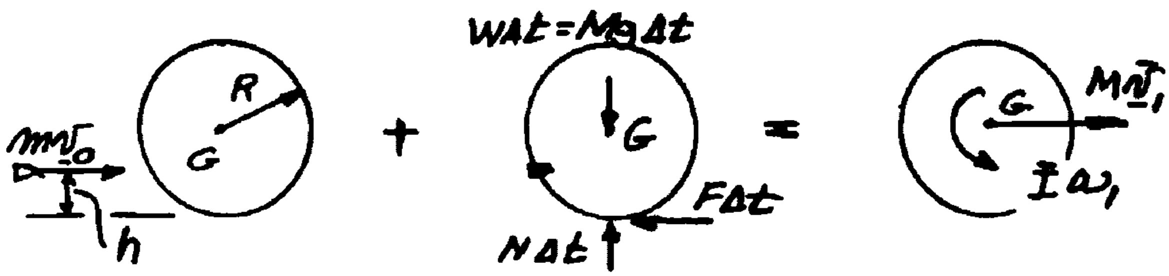
PROBLEM 17.108

A bullet of mass m is fired with a horizontal velocity v_0 and at a height $h = \frac{1}{2}R$ into a wooden disk of much larger mass M and radius R. The disk rests on a horizontal plane and the coefficient of friction between the disk and the plane is finite. (*a*) Determine the linear velocity $\overline{\mathbf{v}}_1$ and the angular velocity $\boldsymbol{\omega}_1$ of the disk immediately after the bullet has penetrated the disk. (*b*) Describe the ensuing motion of the disk and determine its linear velocity after the motion has become uniform.

SOLUTION

(*a*) Conditions immediately after the bullet has penetrated the disk.

Principle of impulse and momentum:



Syst. Momenta$_0$ + **Syst. Ext. Imp.**$_{0\to 1}$ = **Syst. Momenta**$_1$

$+\uparrow$ y components: $\quad 0 + N\Delta t - W\Delta t = 0 \quad N = W$

$\xrightarrow{+}$ x components: $\quad mv_0 - F\Delta t = M\,\overline{v}_1$

$$mv_0 - \mu W\,\Delta t = M\,\overline{v}_1$$

Since $\Delta t \approx 0$, $\quad mv_0 = m\overline{v}_1 \qquad\qquad \overline{\mathbf{v}}_1 = \dfrac{mv_0}{M} \longrightarrow$ (1) ◀

$+\circlearrowright$ Moments about G: $\quad mv_0 = (R-h) - R(\mu W\,\Delta t) = \overline{I}\,\omega_1$

Since $\Delta t \approx 0$, $\quad mv_0 = (R-h) = \dfrac{1}{2}MR^2\omega_1$

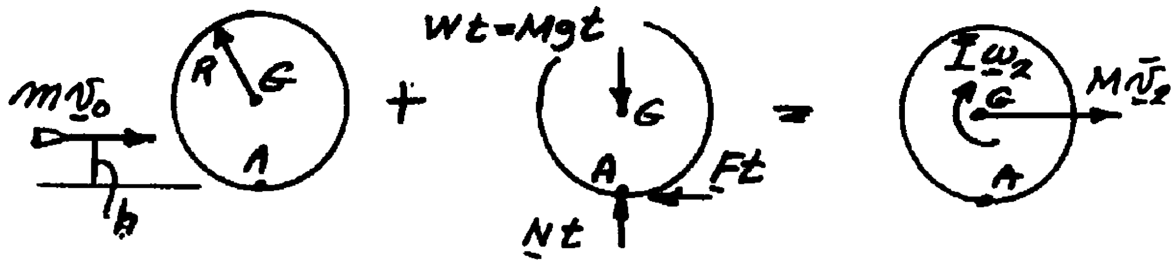
$$\omega_1 = 2\frac{m}{M}\frac{R-h}{R^2}v_0 \circlearrowright \text{ (2)}$$

But $\quad h = \dfrac{1}{2}R$

$$\omega_1 = 2\frac{m}{M}\frac{R-\frac{1}{2}R}{R^2} \qquad\qquad \omega_1 = \frac{mv_0}{MR} \circlearrowright \blacktriangleleft$$

(*b*) After the motion becomes uniform, the disk rolls without slipping.

Kinematics. $\bar{v}_2 = R\omega_2$



Syst. Momenta$_0$ + **Syst. Ext. Imp.**$_{0\to2}$ = **Syst. Momenta**$_2$

$+\circlearrowright$ Moments about A: $mv_0h + 0 = M\,\bar{v}_2 R + \bar{I}\,\omega_2$

Since $h = \frac{1}{2}R$ is given:

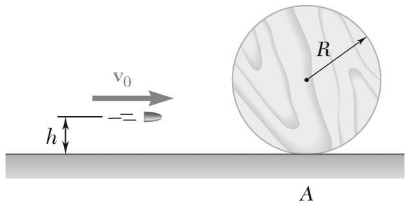
$$mv_0\left(\frac{1}{2}R\right) = (MR\omega_2)R + \frac{1}{2}MR^2\omega_2$$

$$\frac{1}{2}mv_0 = \frac{3}{2}MR\omega_2$$

$$\omega_2 = \frac{mv_0}{3MR} \circlearrowright$$

$$\bar{v}_2 = R\omega_2 \qquad \bar{\mathbf{v}}_2 = \frac{mv_0}{3M} \longrightarrow \blacktriangleleft$$

At first the disk slides $\longrightarrow$ and rotates $\circlearrowleft$, it latter rolls with a constant linear velocity $\bar{\mathbf{v}}_2$ and a constant angular velocity $\boldsymbol{\omega}_2$.

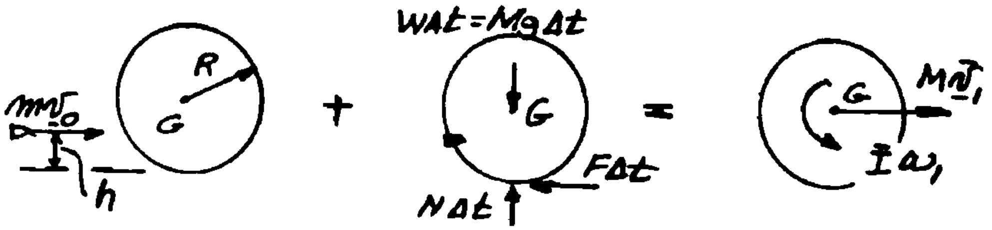


PROBLEM 17.109

Determine the height h at which the bullet of Problem 17.108 should be fired (*a*) if the disk is to roll without sliding immediately after impact, (*b*) if the disk is to slide without rolling immediately after impact.

SOLUTION

Principle of impulse and momentum:



Syst. Momenta$_0$ + **Syst. Ext. Imp.** = **Syst. Momenta$_1$**

$+\uparrow$ y components: $0 + N\Delta t - W\Delta t = 0 \quad N = W$

$\xrightarrow{+}$ x components: $mv_0 - F\Delta t = M\,\bar{v}_1$

$$mv_0 - \mu W\Delta t = M\,\bar{v}_1$$

Since $\Delta t \approx 0$; $mv_0 = m\bar{v}_1$ $\qquad \bar{\mathbf{v}}_1 = \dfrac{mv_0}{M} \longrightarrow$ (1)

$+\circlearrowright$ Moments about G: $mv_0 = (R-h) - R(\mu W\Delta t) = \bar{I}\,\omega_1$

Since $\Delta t \approx 0$; $mv_0 = (R-h) = \dfrac{1}{2}MR^2\omega_1$

$$\omega_1 = 2\frac{m}{M}\frac{R-h}{R^2}v_0 \circlearrowright \qquad (2)$$

(*a*) If disk is to roll without sliding immediately after impact, we must have

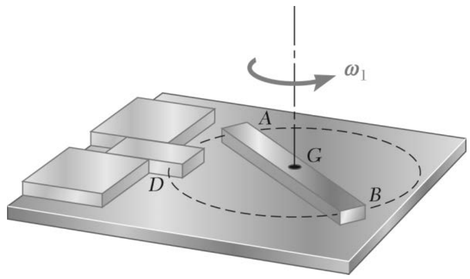
$\omega_1 \circlearrowright$ and $\bar{v}_1 = R\omega_1 \longrightarrow$

$$\frac{mv_0}{M} = -R\left[\frac{2m}{M}\cdot\frac{R-h}{R^2}v_0\right]$$

$$1 = -2\frac{R-h}{R} \qquad h = \frac{3}{2}R \blacktriangleleft$$

(*b*) If disk is to slide without rotating,

$$\omega_1 = \frac{2m}{M}\cdot\frac{R-h}{R^2}v_0 = 0 \qquad h = R \blacktriangleleft$$



PROBLEM 17.110

A uniform slender bar of length $L = 200$ mm and mass $m = 0.5$ kg is supported by a frictionless horizontal table. Initially the bar is spinning about its mass center G with a constant angular velocity $\boldsymbol{\omega}_1 = 6$ rad/s. Suddenly latch D is moved to the right and is struck by end A of the bar. Knowing that the coefficient of restitution between A and D is $e = 0.6$, determine the angular velocity of the bar and the velocity of its mass center immediately after the impact.

SOLUTION

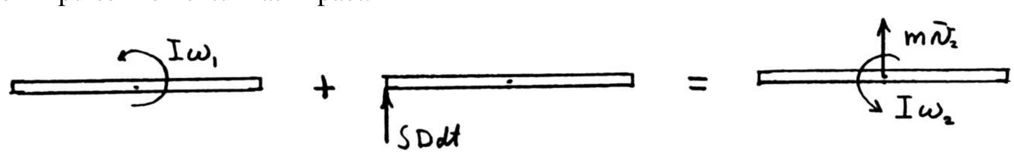
Moment of inertia. $\quad \bar{I} = \frac{1}{12}mL^2$

Before impact. $\quad (\mathbf{v}_A)_1 = \frac{L}{2}\omega_1 \downarrow$

Impact condition. $\quad (\mathbf{v}_A)_2 = -e(\mathbf{v}_A)_1 = \frac{1}{2}eL\omega_1 \uparrow$

Kinematics after impact. $\quad \bar{v}_2 = (v_A)_2 + \frac{L}{2}\omega_2 = \frac{1}{2}eL\omega_1 + \frac{1}{2}L\omega_2$

Principle of impulse-momentum at impact.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

↺ Moments about D: $\quad I\omega_1 + 0 = I\omega_2 + m\bar{v}_2\frac{L}{2}$

$$I\omega_1 = I\omega_2 + m\left(\frac{1}{2}eL\omega_1 + \frac{1}{2}L\omega_2\right)\frac{L}{2}$$

$$\frac{1}{12}mL^2\omega_1 = \frac{1}{12}mL^2\omega_2 + \frac{1}{4}mL^2e\omega_1 + \frac{1}{4}mL^2\omega_2$$

$$\omega_2 = \frac{1}{4}(1-3e)\omega_1$$

$$\bar{v}_2 = \frac{1}{2}Le\omega_1 + \frac{1}{2}L\left(\frac{1}{4}\right)(1-3e)\omega_1 = \frac{1}{8}(1+e)\omega_1 L$$

Coefficient of restitution, $\quad e = 0.6$

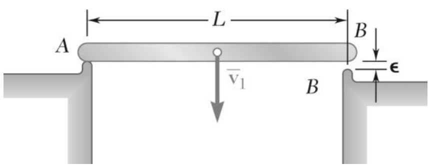
$$\omega_2 = \frac{1}{4}(1-(3)(0.6))(6\text{ rad/s}) = -1.200$$

$\boldsymbol{\omega}_2 = 1.200$ rad/s ↻ ◀

$$v_2 = \frac{1}{8}(1+0.6)(6\text{ rad/s})(0.2\text{ m}) = 0.240\text{ m/s}$$

$\mathbf{v}_2 = 0.240$ m/s ↑ ◀

PROBLEM 17.111

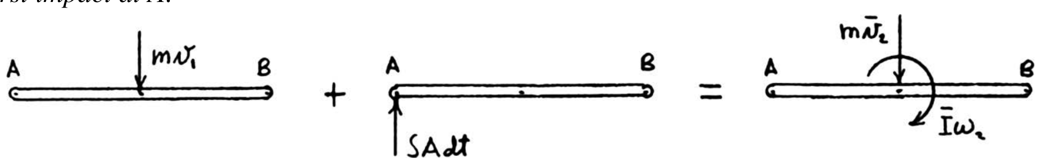


A uniform slender rod of length L is dropped onto rigid supports at A and B. Since support B is slightly lower than support A, the rod strikes A with a velocity $\bar{\mathbf{v}}_1$ before it strikes B. Assuming perfectly elastic impact at both A and B, determine the angular velocity of the rod and the velocity of its mass center immediately after the rod (*a*) strikes support A, (*b*) strikes support B, (*c*) again strikes support A.

SOLUTION

Moment of inertia. $\bar{I} = \frac{1}{12} mL^2$

(*a*) *First impact at A.*



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

Condition of impact: $e = 1{:}\quad (\mathbf{v}_A)_2 = v_1 \uparrow$

Kinematics: $\bar{v}_2 = \frac{L}{2}\omega - (v_A)_2 = \frac{L}{2}\omega - v_1$

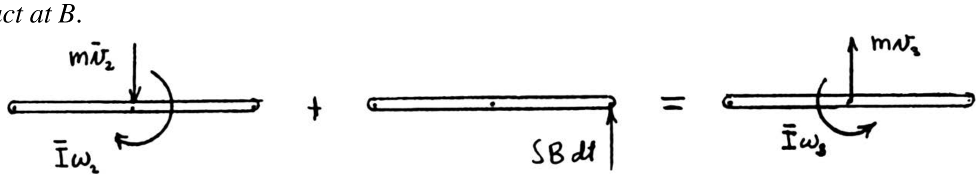
↻ Moments about A: $mv_1 \frac{L}{2} + 0 = m\bar{v}_2 \frac{L}{2} + \bar{I}\omega_2$

$$= m\left(\frac{L}{2}\omega - v_1\right)\frac{L}{2} + \left(\frac{1}{12}mL^2\right)\omega_2 \qquad \boldsymbol{\omega}_2 = \frac{3v_1}{L} \circlearrowright \blacktriangleleft$$

$$\bar{v}_2 = \frac{L}{2}\left(\frac{3v_1}{L}\right) - v_1 = \frac{1}{2}v_1 \qquad \bar{\mathbf{v}}_2 = \frac{1}{2}v_1 \downarrow \blacktriangleleft$$

$$(v_B)_2 = L\omega - (v_A)_2 = 3v_1 - v_1 = 2v_1 \downarrow$$

(*b*) *Impact at B.*



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_3$**

Condition of impact. $e = 1{:}\quad (\mathbf{v}_B)_3 = 2v_1 \uparrow$

Kinematics: $\bar{v}_3 = (v_B)_2 - \frac{L}{2}\omega = 2v_1 - \frac{L}{2}\omega$

↺ Moments about B: $$-m\bar{v}_2\frac{L}{2}+\bar{I}\omega_2+0=m\bar{v}_3\frac{L}{2}-\bar{I}\omega_3$$

$$-m\left(\frac{1}{2}v_1\right)\frac{L}{2}+\left(\frac{1}{12}mL^2\right)\left(\frac{3v_1}{L}\right)+0=m\left(2v_1-\frac{L}{2}\omega_3\right)\frac{L}{2}-\left(\frac{1}{12}mL^2\right)\omega_3$$

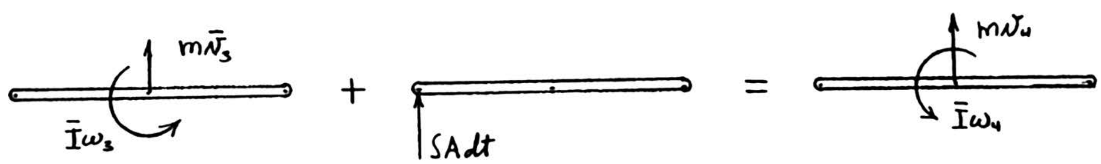
$\boldsymbol{\omega}_3=\frac{3v_1}{L}$ ↻ ◀

$$\bar{v}_3=2v_1-\frac{L}{2}\left(\frac{3v_1}{L}\right)=\frac{1}{2}v_1$$

$\bar{\mathbf{v}}_3=\frac{1}{2}v_1$ ↑ ◀

$$(v_A)_3=L\omega-(v_B)_3=3v_1-2v_1=v_1\downarrow$$

(*c*) *Second impact at A.*



Syst. Momenta$_3$ + **Syst. Ext. Imp.**$_{3\to4}$ = **Syst. Momenta**$_4$

Condition of impact. $e=1$: $(\mathbf{v}_A)_4=v_1\uparrow$

Kinematics: $$\bar{v}_4=(v_A)_4+\frac{L}{2}\omega_4=v_1+\frac{L}{2}\omega_4$$

↻ Moments about A: $$m\bar{v}_3\frac{L}{2}+\bar{I}\omega_3+0=m\bar{v}_4\frac{L}{2}+\bar{I}\omega_4$$

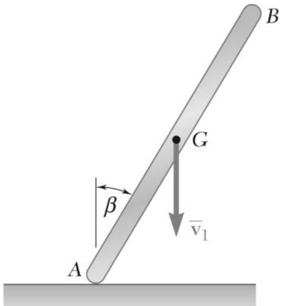
$$m\left(\frac{1}{2}v_1\right)\frac{L}{2}+\left(\frac{1}{12}mL^2\right)\left(\frac{3v_1}{L}\right)+0=m\left(v_1+\frac{L}{2}\omega_4\right)\frac{L}{2}+\left(\frac{1}{12}mL^2\right)\omega_4$$

$\boldsymbol{\omega}_4=0$ ◀

$$\bar{v}_4=v_1+0$$

$\bar{\mathbf{v}}_4=v_1$ ↑ ◀

PROBLEM 17.112



The slender rod AB of length L forms an angle β with the vertical as it strikes the frictionless surface shown with a vertical velocity $\bar{\mathbf{v}}_1$ and no angular velocity. Assuming that the impact is perfectly plastic, derive an expression for the angular velocity of the rod immediately after the impact.

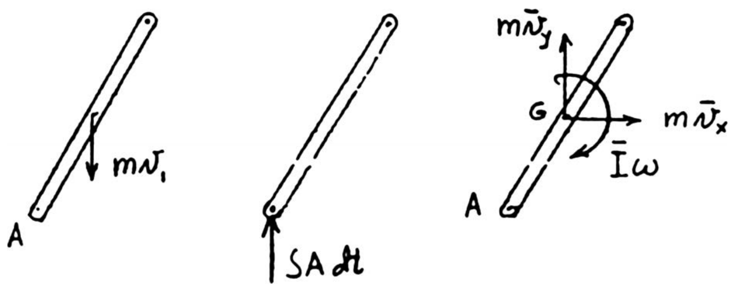
SOLUTION

Moment of inertia. $\quad \bar{I} = \frac{1}{12}mL^2$

Perfectly plastic impact. $\quad e = 0 \qquad [(v_A)_y]_2 = -e(v_A)_{y1} = 0$

$$\mathbf{v}_A = (v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} = (v_A)_x\mathbf{i}$$

Kinetics.



$$\textbf{Syst. Momenta}_1 \quad + \quad \textbf{Syst. Ext. Imp.}_{1\to 2} \quad = \quad \textbf{Syst. Momenta}_2$$

$\xrightarrow{+}$ horizontal components: $\quad 0 + 0 = m\bar{v}_x \qquad m\bar{v}_x = 0$

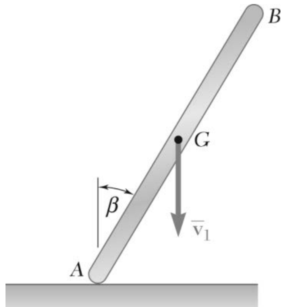
Kinematics. $\quad \mathbf{v}_G = \mathbf{v}_A + \mathbf{v}_{G/A} \qquad [\bar{v}_y \uparrow] = [(v_A)_x \longrightarrow] + \left[\frac{L}{2}\omega \searrow \beta\right]$

Velocity components $\uparrow$: $\quad v_y = -\frac{L}{2}\omega\sin\beta$

$\circlearrowright$ Moments about A: $\quad mv_1\frac{L}{2}\sin\beta + 0 = -m\bar{v}_y\frac{L}{2}\sin\beta + \bar{I}\omega$

$$mv_1\frac{L}{2}\sin\beta = m\left(\frac{L}{2}\omega\sin\beta\right)\frac{L}{2}\sin\beta + \frac{1}{12}mL^2\omega$$

$$\left(\frac{1}{12}mL^2 + \frac{1}{4}mL^2\sin^2\beta\right)\omega = \frac{1}{2}mv_1L\sin\beta \qquad \omega = \frac{6\sin\beta}{1+3\sin^2\beta}\frac{v_1}{L} \blacktriangleleft$$



PROBLEM 17.113

The slender rod AB of length $L = 1$ m forms an angle $\beta = 30°$ with the vertical as it strikes the frictionless surface shown with a vertical velocity $\bar{\mathbf{v}}_1 = 2$ m/s and no angular velocity. Knowing that the coefficient of restitution between the rod and the ground is $e = 0.8$, determine the angular velocity of the rod immediately after the impact.

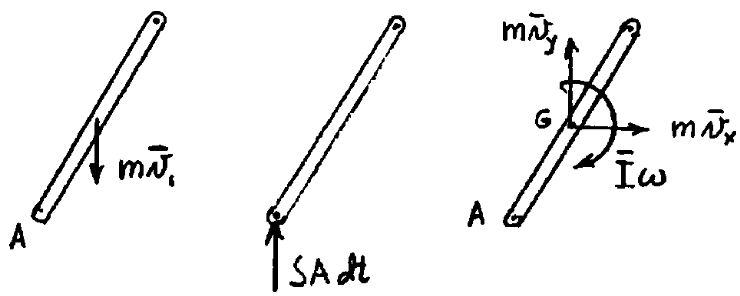
SOLUTION

Moment of inertia. $$\bar{I} = \frac{1}{12}mL^2$$

Apply coefficient of restitution. $$[(v_A)_y]_2 = -e[(v_A)_y]_1 = e\bar{v}_1 \uparrow$$

$$\mathbf{v}_A = (v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} = (v_A)_x\mathbf{i} + e\bar{v}_1\mathbf{j}$$

Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\xrightarrow{+}$ horizontal components: $$0 + 0 = m\bar{v}_x \qquad m\bar{v}_x = 0$$

Kinematics. $$\mathbf{v}_G = \mathbf{v}_A + \mathbf{v}_{G/A} \qquad [\bar{v}_y \uparrow] = [e\bar{v}_1 \uparrow] + [(v_A)_x \longrightarrow] + \left[\frac{L}{2}\omega \searrow \beta\right]$$

Velocity components $\uparrow$: $$v_y = e\bar{v}_1 - \frac{L}{2}\omega\sin\beta$$

$\circlearrowright$ Moments about A: $$mv_1\frac{L}{2}\sin\beta + 0 = -m\bar{v}_y\frac{L}{2}\sin\beta + \bar{I}\omega$$

$$mv_1\frac{L}{2}\sin\beta = m\left(\frac{L}{2}\omega\sin\beta - e\bar{v}_1\right)\frac{L}{2}\sin\beta + \frac{1}{12}mL^2\omega$$

$$\left(\frac{1}{12}mL^2 + \frac{1}{4}mL^2\sin^2\beta\right)\omega = \frac{1+e}{2}mv_1L\sin\beta$$

$$\omega = \frac{6(1+e)\sin\beta}{1+3\sin^2\beta}\frac{\bar{v}_1}{1L}$$

PROBLEM 17.113 (Continued)

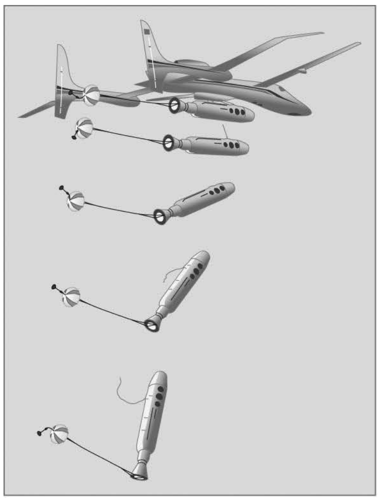
Data: $L = 1\text{ m}, \quad \beta = 30°, \quad \bar{v}_1 = 2\text{ m/s}, \quad e = 0.8$

$$\omega = \frac{(6)(1.8)\sin 30°}{1 + 3\sin^2 30°} \cdot \frac{2\text{ m/s}}{1\text{ m}}$$

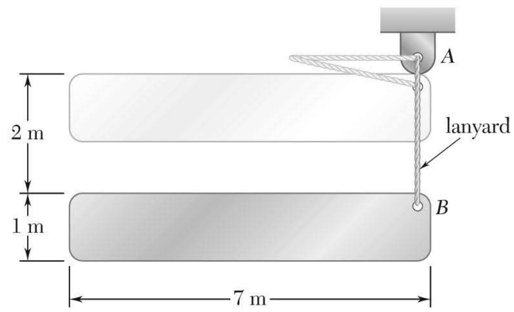
$\boldsymbol{\omega} = 6.17\text{ rad/s}$ ↻ ◀

PROBLEM 17.114

The trapeze/lanyard air drop (t/LAD) launch is a proposed innovative method for airborne launch of a payload-carrying rocket. The release sequence involves several steps as shown in (1) where the payload rocket is shown at various instances during the launch. To investigate the first step of this process where the rocket body drops freely from the carrier aircraft until the 2-m lanyard stops the vertical motion of *B*, a trial rocket is tested as shown in (2). The rocket can be considered a uniform 1-m by 7-m rectangle with a mass of 4000 kg. Knowing that the rocket is released from rest and falls vertically 2 m before the lanyard becomes taut, determine the angular velocity of the rocket immediately after the lanyard is taut.



(1)



(2)

SOLUTION

While the lanyard is slack, the rocket falls freely without rotation. Considering its motion relative to the airplane (a Newtonian frame of reference), its vertical velocity is

$$v_1^2 = v_0^2 + 2gy = 2gy$$

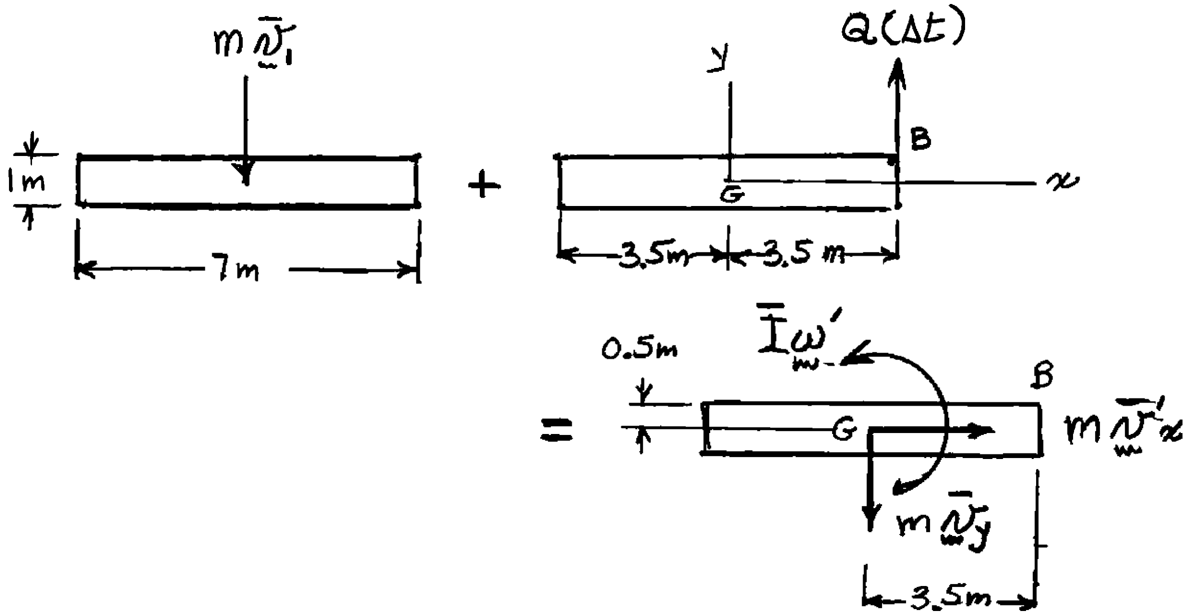
$$v_1 = \sqrt{2gy} = \sqrt{(2)(9.81\text{ m/s})(2\text{ m})} \qquad \mathbf{v}_1 = 6.2642\text{ m/s}\downarrow \qquad (1)$$

Moment of inertia:

$$\bar{I} = \frac{1}{12}m(a^2 + b^2)$$

$$= \frac{1}{12}(4000\text{ kg})[(7\text{ m})^2 + (1\text{ m})^2] = 16.667\text{ kg}\cdot\text{m}^2$$

For impact use the principle of impulse and momentum.



$$\textbf{Syst. Momenta}_1 \quad + \quad \textbf{Syst. Ext. Imp.}_{1\to 2} \quad = \quad \textbf{Syst. Momenta}_2$$

x-components $\longrightarrow$: $\quad 0 + 0 = m\bar{v}'_x \qquad \bar{v}'_x = 0$

$+\circlearrowright$ Moments about B: $\quad mv_1(3.5) + 0 = \bar{I}\omega' + m\bar{v}'_y(3.5) \qquad$ (2)

Kinematics. $\quad \mathbf{v}_B = v_B \longrightarrow$

$$\bar{v} = [\bar{v}_x \longrightarrow] + [\bar{v}_y \downarrow] = [v_B \longrightarrow] + [3.5\omega' \downarrow] + [0.5\omega \longrightarrow]$$

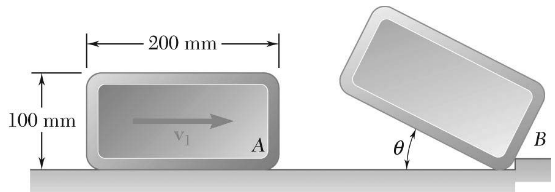
y-components $\downarrow$: $\quad \bar{v}_y = 3.5\omega' \qquad$ (3)

Substitute from Eqs. (1) and (3) into Eq. (2).

$$mv_1(3.5) = [\bar{I} + m(3.5)^2]\omega'$$

$$(4000 \text{ kg})(6.2642 \text{ m/s})(3.5 \text{ m}) = [16667 \text{ kg}\cdot\text{m}^2 + (4000 \text{ kg})(3.5 \text{ m})^2]\omega'$$

Angular velocity. $\quad \boldsymbol{\omega}' = 1.336 \text{ rad/s} \circlearrowright$ ◀



PROBLEM 17.115

The uniform rectangular block shown is moving along a frictionless surface with a velocity $\overline{\mathbf{v}}_1$ when it strikes a small obstruction at B. Assuming that the impact between corner A and obstruction B is perfectly plastic, determine the magnitude of the velocity $\overline{\mathbf{v}}_1$ for which the maximum angle θ through which the block will rotate is 30°.

SOLUTION

Let m be the mass of the block.

Dimensions: $a = 0.200$ m

$b = 0.100$ m

Moment of inertia about the mass center.

$$\overline{I} = \frac{1}{12}m(a^2 + b^2)$$

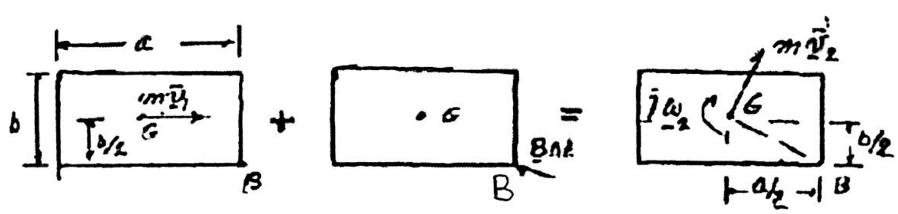
Let d be one half the diagonal. $d = \frac{1}{2}\sqrt{a^2 + b^2} = 0.1118$ m

Kinematics. Before impact $\overline{\mathbf{v}}_1 = v_1 \longrightarrow, \quad \boldsymbol{\omega}_1 = 0$

After impact, the block is rotating about corner at B.

$$\boldsymbol{\omega}_2 = \omega_2 \circlearrowright \qquad \mathbf{v}_2 = d\omega_2 \nearrow$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\circlearrowleft +$ Moments about B:

$$\frac{mv_1 b}{2} + 0 = \overline{I}\omega_2 + mdv_2$$

$$\frac{1}{2}mv_1 b = \frac{1}{12}m(a^2 + b^2)\omega_2 + md^2\omega_2$$

$$= \frac{1}{3}m(a^2 + b^2)\omega_2$$

Angular velocity after impact $\qquad \boldsymbol{\omega}_2 = \dfrac{3v_1 b}{2(a^2+b^2)}$ ↻ $\qquad$ (1)

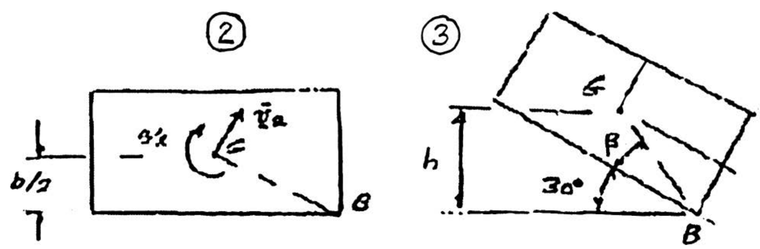
The motion after impact is a rotation about corner *B*.

Position 2 (immediately after impact). $\qquad \bar{v}_2 = d\omega_2$

Position 3 $\quad (\theta = 30°)$. $\qquad \beta = \tan^{-1}\dfrac{b}{a} = \tan^{-1} 0.5 = 26.565°$

$$h = d\sin(\beta + 30°) = 0.11180 \sin 56.565° = 0.093301 \text{ m}$$

$$\omega_3 = 0 \qquad \bar{v}_3 = 0$$



Potential energy: $\qquad V_2 = \dfrac{mgb}{2} \qquad V_3 = mgh$

Kinetic energy: $\qquad T_2 = \dfrac{1}{2}\bar{I}\omega_2^2 + \dfrac{1}{2}m\bar{v}_2^2 = \dfrac{1}{2}(\bar{I} + md^2)\omega_2^2$

$$= \frac{1}{6}m(a^2+b^2)\omega_2^2 \qquad T_3 = 0$$

Principle of conservation of energy:

$$T_2 + V_3 = T_3 + V_3$$

$$\frac{1}{6}m(a^2+b^2)\omega_2^2 + \frac{mgb}{2} = 0 + mgh$$

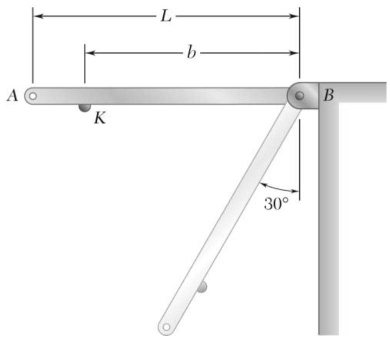
$$\omega_2^2 = \frac{3g(2h-b)}{(a^2+b^2)} = \frac{(3)(9.81)(0.18660 - 0.100)}{(0.200)^2 + (0.100)^2}$$

$$= 50.974 \text{ (rad/s)}^2 \qquad \boldsymbol{\omega}_2 = 7.1396 \text{ rad/s} ↻$$

Magnitude of initial velocity.

Solving Eq. (1) for v_1 $\qquad v_1 = \dfrac{2(a^2+b^2)\omega_2}{3b}$

$$v_1 = \frac{(2)[(0.200)^2 + (0.100)^2](7.1396)}{(3)(0.100)} \qquad v_1 = 2.38 \text{ m/s} ◀$$



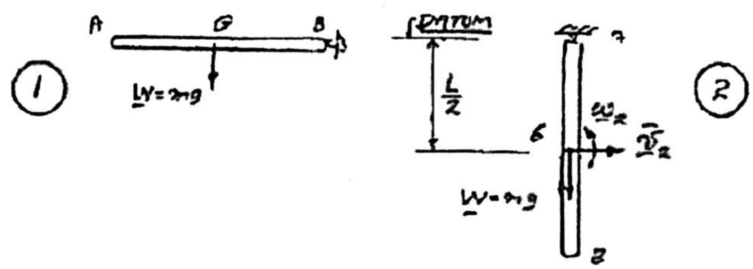
PROBLEM 17.116

A slender rod of length L and mass m is released from rest in the position shown. It is observed that after the rod strikes the vertical surface it rebounds to form an angle of 30° with the vertical. (*a*) Determine the coefficient of restitution between knob K and the surface. (*b*) Show that the same rebound can be expected for any position of knob K.

SOLUTION

For analysis of the downward swing of the rod before impact and for the upward swing after impact use the principle of conservation of energy.

Before impact.



$$V_1 = 0$$

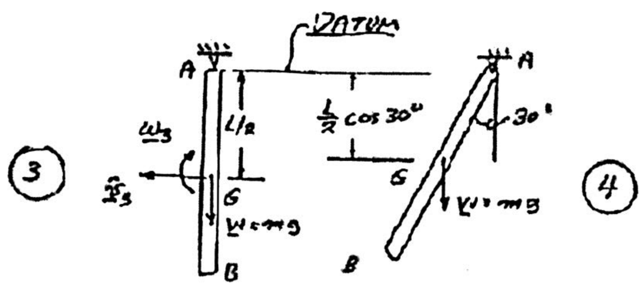
$$V_2 = -W\frac{L}{2} = -mg\frac{L}{2}$$

$$T_1 = 0$$

$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}m\bar{v}_2^2 = \frac{1}{2}\left(\frac{1}{12}mL^2\right)v_2^2 + \frac{1}{2}m\left(\frac{1}{2}\omega_2\right)^2 = \frac{1}{6}mL^2\omega_2^2$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 = \frac{1}{6}mL^2\omega_2^2 = -mg\frac{L}{2}; \quad \omega_2^2 = 3\frac{g}{L} \qquad \boldsymbol{\omega}_2 = 1.73205\sqrt{\frac{g}{L}}$$

After impact.



$$V_3 = -W\frac{L}{2} = -mg\frac{L}{2}$$

$$V_4 = -W\frac{L}{2}\cos 30°$$

$$T_3 = \frac{1}{2}\bar{I}\omega_3^2 + \frac{1}{2}m\bar{v}_3^2$$

$$= \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_3^2 + \frac{1}{2}m\left(\frac{1}{2}\omega_3\right)^2$$

$$= \frac{1}{6}mL^2\omega_3^2$$

$$T_4 = 0$$

$$T_3 + V_3 = T_4 + V_4: \quad \frac{1}{6}mL^2\omega_3^2 - mg\frac{L}{2} = 0 - mg\frac{L}{2}\cos 30°$$

$$\omega_3^2 = 3(1 - \cos 30°)\frac{g}{L} \qquad \boldsymbol{\omega}_3 = 0.63397\sqrt{\frac{g}{L}} \circlearrowright$$

Analysis of impact.

Let r be the distance BK.

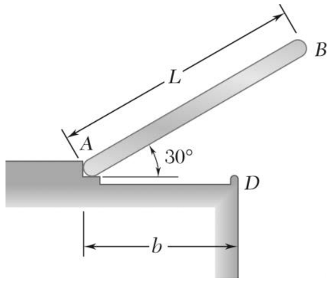
Before impact, $\quad (\mathbf{v}_k)_3 = r\omega_3 \longrightarrow \; = 1.73205r\sqrt{\frac{g}{L}} \longrightarrow$

After impact, $\quad (\mathbf{v}_k)_4 = r\omega_4 \longleftarrow \; = 0.63397r\sqrt{\frac{g}{L}} \longleftarrow$

Coefficient of restitution. $\quad e = \dfrac{|(v_k)_{4n}|}{|(v_k)_{3n}|}$

$$e = \frac{0.63397}{1.73205} \qquad e = 0.366 \blacktriangleleft$$

PROBLEM 17.117

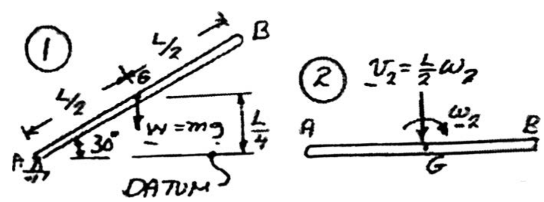


A slender rod of mass m and length L is released from rest in the position shown and hits edge D. Assuming perfectly plastic impact at D, determine for $b = 0.6L$, (*a*) the angular velocity of the rod immediately after the impact, (*b*) the maximum angle through which the rod will rotate after the impact.

SOLUTION

For analysis of the falling motion before impact use the principle of conservation of energy.

Position 1: $T_1 = 0, \quad V_1 = mg\dfrac{L}{4}$



Position 2: $V_2 = 0$

$$T_2 = \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$$

$$T_2 = \frac{1}{6}mL^2\omega_2^2$$

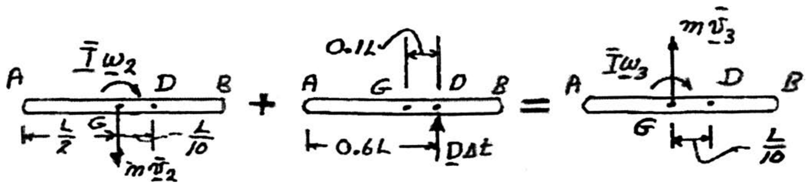
$$T_1 + V_1 = T_2 + V_2: \quad 0 + mg\frac{L}{4} = \frac{1}{6}mL^2\omega_2^2 \qquad \omega_2 = \sqrt{\frac{3g}{2L}}$$

Analysis of impact. Kinematics

Before impact, rotation is about Point A. $\qquad \bar{v}_2 = \dfrac{L}{2}\omega_2$

After impact, rotation is about Point D. $\qquad \bar{v}_3 = \dfrac{L}{10}\omega_3$

Principle of impulse-momentum.



Syst. Momenta$_2$ + Syst. Ext. Imp.$_{2\to3}$ = Syst. Momenta$_3$

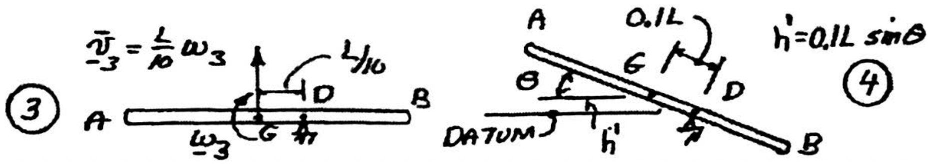
+↻ Moments about D: $\qquad \bar{I}\omega_2 - m\bar{v}_2\left(\dfrac{L}{10}\right) = \bar{I}\omega_3 + m\bar{v}_3\left(\dfrac{L}{10}\right) \qquad (1)$

$$\frac{1}{12}mL^2\omega_2 - m\left(\frac{L}{2}\omega_2\right)\frac{L}{10} = \frac{1}{12}mL^2\omega_3 + m\left(\frac{L}{10}\omega_3\right)\frac{L}{10}$$

$$\left(\frac{1}{12} - \frac{1}{20}\right)\omega_2 = \left(\frac{1}{12} + \frac{1}{100}\right)\omega_3$$

(*a*) Angular velocity. $\omega_3 = \dfrac{5}{14}\omega_2 = \dfrac{5}{14}\sqrt{\dfrac{3g}{2L}}$ $\quad\boldsymbol{\omega}_3 = 0.437\sqrt{\dfrac{g}{L}}$ ◀

For analysis of the rotation about Point D after the impact use the principle of conservation of energy.



Position 3 (just after impact)

$$\bar{v}_3 = \frac{L}{10}\omega_3 \quad V_3 = 0$$

$$T_3 = \frac{1}{2}m\left(\frac{L}{10}\omega_3\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_3^2 = \frac{14}{300}mL^2\omega_3^2$$

$$= \frac{14}{300}mL^2\left(\frac{5}{14}\sqrt{\frac{3g}{2L}}\right)^2 = \frac{mgL}{112}$$

Position 4. θ = maximum rotation angle.

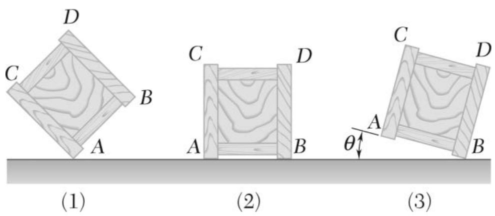
$$h' = \frac{L}{10}\sin\theta$$

$$V_4 = mgh' = \frac{mgL}{10}\sin\theta$$

$$\bar{v}_4 = 0, \quad \omega_4 = 0, \quad T_4 = 0$$

$$T_3 + V_3 = T_4 + V_4; \quad \frac{mgL}{112} + 0 = 0 + \frac{mgL}{10}\sin\theta$$

(*b*) Maximum rotation angle. $\sin\theta = \dfrac{10}{112}$ $\quad\theta = 5.12°$ ◀



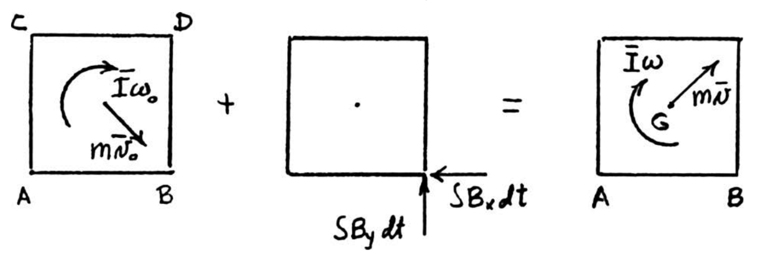
PROBLEM 17.118

A uniformly loaded square crate is released from rest with its corner D directly above A; it rotates about A until its corner B strikes the floor, and then rotates about B. The floor is sufficiently rough to prevent slipping and the impact at B is perfectly plastic. Denoting by $\boldsymbol{\omega}_0$ the angular velocity of the crate immediately before B strikes the floor, determine (a) the angular velocity of the crate immediately after B strikes the floor, (b) the fraction of the kinetic energy of the crate lost during the impact, (c) the angle θ through which the crate will rotate after B strikes the floor.

SOLUTION

Let m be the mass of the crate and c be the length of an edge.

Moment of inertia $$\bar{I} = \frac{1}{12}m(c^2 + c^2) = \frac{1}{6}mc^2$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics: $$\bar{v}_0 = r_{G/A}\omega_0 = \frac{1}{2}\sqrt{2}c\omega_0$$

$$\bar{v} = r_{G/B}\omega = \frac{1}{2}\sqrt{2}c\omega$$

↺ Moments about B: $$\bar{I}\omega_0 + 0 = \bar{I}\omega + r_{G/B}m\bar{v}$$

$$\frac{1}{6}mc^2\omega_0 + 0 = \frac{1}{6}mc^2\omega + \left(\frac{1}{2}\sqrt{2}c\right)m\left(\frac{1}{2}\sqrt{2}c\omega\right) = \frac{2}{3}mc^2\omega$$

(a) Solving for ω, $$\boldsymbol{\omega} = \frac{1}{4}\omega_0 \circlearrowright \blacktriangleleft$$

Kinetic Energy.

Before impact: $$T_1 = \frac{1}{2}\bar{I}\omega_0^2 + \frac{1}{2}m\bar{v}_0^2$$

$$= \frac{1}{2}\left(\frac{1}{6}mc^2\right)\omega_0^2 + \frac{1}{2}m\left(\frac{1}{2}\sqrt{2}c\omega_0\right)^2$$

$$= \frac{1}{3}mc^2\omega_0^2$$

After impact:
$$T_2 = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m\bar{v}^2 = \frac{1}{2}\left(\frac{1}{6}mc^2\right)\omega^2 + \frac{1}{2}m\left(\frac{1}{2}\sqrt{2}c\omega\right)^2$$
$$= \frac{1}{3}mc^2\omega^2 = \frac{1}{3}mc^2\left(\frac{1}{4}\omega_0\right)^2 = \frac{1}{48}mc^2\omega_0$$

(*b*) Fraction of energy lost:
$$\frac{T_1 - T_2}{T_1} = \frac{\frac{1}{3} - \frac{1}{48}}{\frac{1}{3}} = 1 - \frac{1}{16}$$
$\frac{15}{16}$ ◀

Conservation of energy during falling. $T_0 + V_0 = T_1 + V_1$ (1)

Conservation of energy during rising. $T_3 + V_3 = T_2 + V_2$ (2)

Conditions: $T_0 = 0, \quad T_3 = 0 \quad T_2 = \frac{1}{16}T_1$

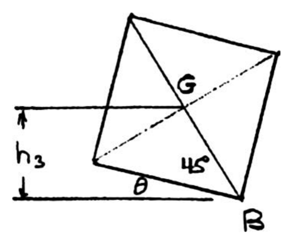
$$V_0 = mg\left(\frac{1}{2}\sqrt{2}c\right) \quad V_1 = V_2 = mg\left(\frac{1}{2}c\right) \quad V_3 = mgh_3$$

From Equation (1), $T_1 = V_0 - V_1 = \frac{1}{2}(\sqrt{2} - 1)mgc$

From Equation (2), $T_2 = V_3 - V_2 = mgh_3 - \frac{1}{2}mgc$

$$\frac{h_3 - \frac{1}{2}c}{\frac{1}{2}\sqrt{2} - 1} = \frac{1}{16} \qquad h_3 = \left[\frac{1}{2} + \frac{1}{16}(\sqrt{2} - 1)\right]c$$

(*c*) From geometry, $h_3 = \frac{1}{2}\sqrt{2}c\sin(\theta + 45°)$

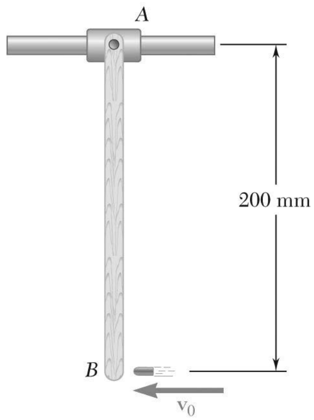


Equating the two expressions for h_3,

$$\sin(45° + \theta) = \frac{\frac{1}{2} + \frac{1}{16}(\sqrt{2} - 1)}{\frac{1}{2}\sqrt{2}}$$

$$45° + \theta = 46.503°$$
$\theta = 1.50°$ ◀

PROBLEM 17.119



A 1-oz bullet is fired with a horizontal velocity of 750 mi/h into the 18-lb wooden beam *AB*. The beam is suspended from a collar of negligible mass that can slide along a horizontal rod. Neglecting friction between the collar and the rod, determine the maximum angle of rotation of the beam during its subsequent motion.

SOLUTION

Mass of bullet. $W' = 1 \text{ ounce} = 0.0625 \text{ lb}$

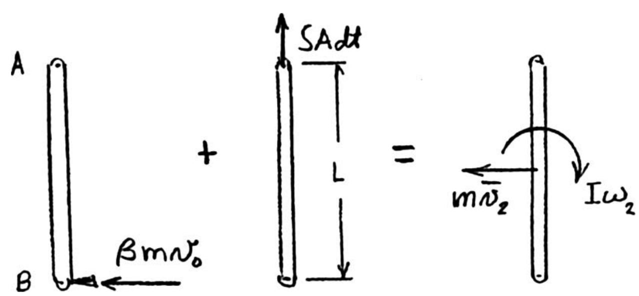
Mass of beam *AB*. $W = 18 \text{ lb}$

Mass ratio. $\beta = \dfrac{W'}{W} = 0.0034722 \qquad W' = \beta W \text{ and } m' = \beta m$

Since β is so small, the mass of the bullet will be neglected in comparison with that of the beam in determining the motion after the impact.

Moment of inertia. $\bar{I} = \dfrac{1}{12} mL^2$

Impact kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\overset{+}{\leftarrow}$ linear components: $-\beta m v_0 + 0 = m\bar{v}_2 \qquad \bar{v}_2 = \beta v_0$

$\curvearrowright$ Moments about *B*: $0 + 0 = \bar{I}\,\omega - m\bar{v}_2 \dfrac{L}{2}$

$$\omega = \frac{m\bar{v}_2 L}{2\bar{I}} = \frac{12 m \beta v_0 L}{2mL^2}$$

$$\omega = \frac{6\beta v_0}{L}$$

Motion during rising. *Position 2*. Just after the impact.

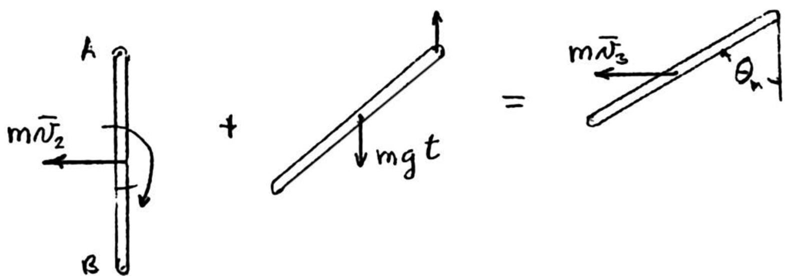
$$V_2 = -mg\frac{L}{2} \quad \text{(datum at level } A\text{)}$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m(\beta v_0)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\left(\frac{6\beta v_0}{L}\right)^2$$

$$= 2\beta^2 m v_0^2$$

Position 3. $\omega = 0, \quad \theta = \theta_m.$



Syst. Momenta$_2$ + Syst. Ext. Imp.$_{2\to3}$ = Syst. Momenta$_3$

$$V_3 = -mg\frac{L}{2}\cos\theta_m$$

$$T_3 = \frac{1}{2}m\bar{v}_3^2$$

$\xleftarrow{+}$ linear components: $m\bar{v}_2 + 0 = m\bar{v}_3 \qquad \bar{v}_3 = \bar{v}_2 = \beta v_0$ where $v_0 = 750$ mi/h $= 1100$ ft/s

Conservation of energy. $T_2 + V_2 = T_3 + V_3: \quad 2\beta^2 m v_0^2 - mg\frac{L}{2} = \frac{1}{2}m(\beta v_0)^2 - mg\frac{L}{2}\cos\theta_m$

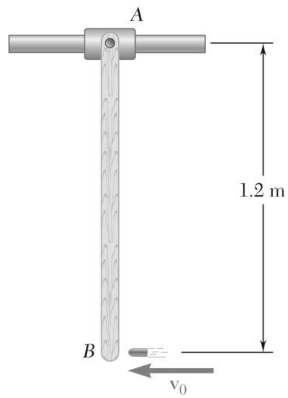
$$\frac{3\beta^2 v_0^2}{gL} = 1 - \cos\theta_m$$

$$\cos\theta_m = 1 - \frac{3\beta^2 v_0^2}{gL}$$

$$= 1 - \frac{(3)(0.0034722)^2(1100)^2}{(32.2)(4)}$$

$$= 0.66021$$

$\theta_m = 48.7° \blacktriangleleft$



PROBLEM 17.120

For the beam of Problem 17.119, determine the velocity of the 1-oz bullet for which the maximum angle of rotation of the beam will be 90°.

PROBLEM 17.119 A 1-oz bullet is fired with a horizontal velocity of 350 m/s into the 18-lb wooden beam *AB*. The beam is suspended from a collar of negligible weight that can slide along a horizontal rod. Neglecting friction between the collar and the rod, determine the maximum angle of rotation of the beam during its subsequent motion.

SOLUTION

Mass of bullet. $W' = 1 \text{ ounce} = 0.0625 \text{ lb}$

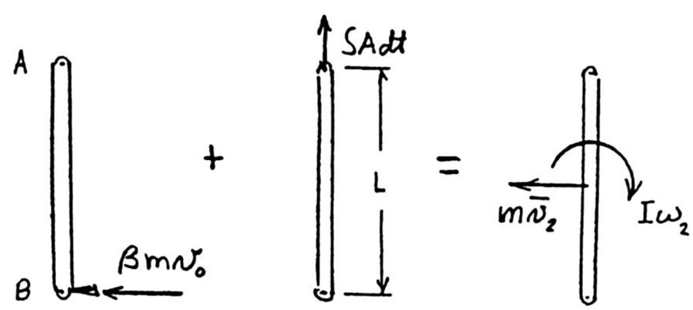
Mass of beam *AB*. $W = 18 \text{ lb}$

Mass ratio. $\beta = \dfrac{W'}{W} = 0.0034722 \qquad W' = \beta W \text{ and } m' = \beta m$

Since β is so small, the mass of the bullet will be neglected in comparison with that of the beam in determining the motion after the impact.

Moment of inertia. $\bar{I} = \dfrac{1}{12} mL^2$

Impact Kinetics.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\xleftarrow{+}$ linear components: $-\beta m v_0 + 0 = m\bar{v}_2 \qquad \bar{v}_2 = \beta v_0$

↻ Moments about *B*: $0 + 0 = \bar{I}\omega - m\bar{v}_2 \dfrac{L}{2}$

$$\omega = \frac{m\bar{v}_2 L}{2\bar{I}} = \frac{12 m\beta v_0 L}{2mL^2}$$

$$\omega = \frac{6\beta v_0}{L}$$

Motion during rising. *Position 2*. Just after the impact.

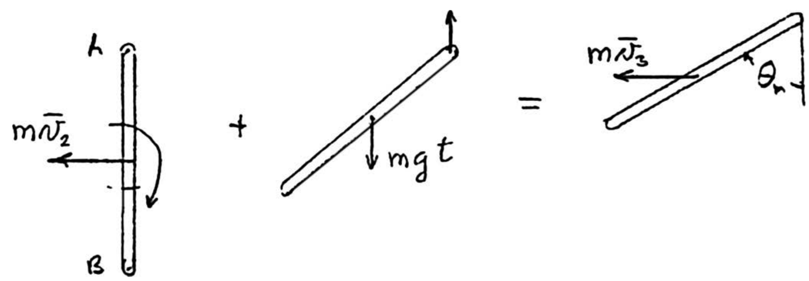
$$V_2 = -mg\frac{L}{2} \quad \text{(datum at level } A\text{)}$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m(\beta v_0)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\left(\frac{6\beta v_0}{L}\right)^2$$

$$= 2\beta^2 m v_0^2$$

Position 3. $\omega = 0, \quad \theta = \theta_m.$



Syst. Momenta$_2$ + **Syst. Ext. Imp.**$_{2\to3}$ = **Syst. Momenta**$_3$

$$V_3 = -mg\frac{L}{2}\cos\theta_m$$

$$T_3 = \frac{1}{2}m\bar{v}_3^2$$

$\overset{+}{\longleftarrow}$ linear components: $m\bar{v}_2 + 0 = m\bar{v}_3 \quad \bar{v}_3 = \bar{v}_2 = \beta v_0$

Conservation of energy.

$$T_2 + V_2 = T_3 + V_3: \quad 2\beta^2 m v_0^2 - mg\frac{L}{2} = \frac{1}{2}m(\beta v_0)^2 - mg\frac{L}{2}\cos\theta_m$$

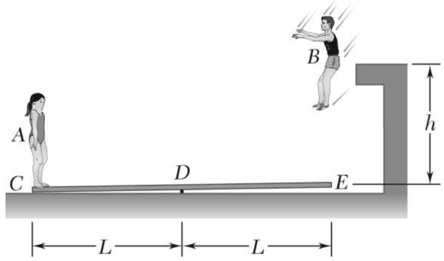
$$\beta v_0 = \sqrt{\frac{1}{3}gL(1-\cos\theta_m)}$$

$$= \sqrt{\left(\frac{1}{3}\right)(32.2)(4)(1-\cos 90^\circ)}$$

$$= 6.5524 \text{ ft/s}$$

$$v_0 = \frac{6.5524}{0.0034722}$$

$v_0 = 1887$ ft/s ◀



PROBLEM 17.121

The plank *CDE* has a mass of 15 kg and rests on a small pivot at *D*. The 55-kg gymnast *A* is standing on the plank at *C* when the 70-kg gymnast *B* jumps from a height of 2.5 m and strikes the plank at *E*. Assuming perfectly plastic impact and that gymnast *A* is standing absolutely straight, determine the height to which gymnast *A* will rise.

SOLUTION

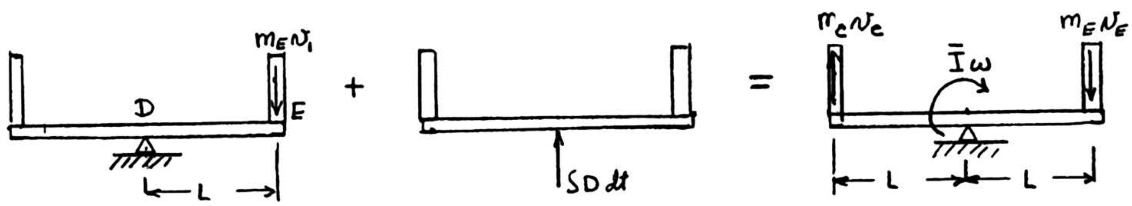
Moment of inertia.

$$\bar{I} = \frac{1}{12} m_P (2L)^2 = \frac{1}{3} m_P L^2$$

Velocity of jumper at *E*.

$$(v)_1 = \sqrt{2gh_1} \quad (1)$$

Principle of impulse-momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics:

$$v_C = L\omega \quad v_D = L\omega$$

↻ Moments about *D*:

$$m_E v_1 L + 0 = m_E v_E L + m_C v_C L + \bar{I}\omega$$

$$= m_E L^2 \omega + m_C L^2 \omega + \frac{1}{3} m_P L^2 \omega$$

$$\omega = \frac{m_E}{m_E + m_C + \frac{1}{3} m_P} \frac{v_1}{L}$$

$$v_C = L\omega = \frac{m_E v_1}{m_E + m_C + \frac{1}{3} m_P} \quad (2)$$

Gymnast (flier) rising.

$$h_C = \frac{v_C^2}{2g} \quad (3)$$

Data:

$$m_E = m_B = 70 \text{ kg}$$
$$m_C = m_A = 55 \text{ kg}$$
$$m_P = 15 \text{ kg}$$
$$h_1 = 2.5 \text{ m}$$

PROBLEM 17.121 (Continued)

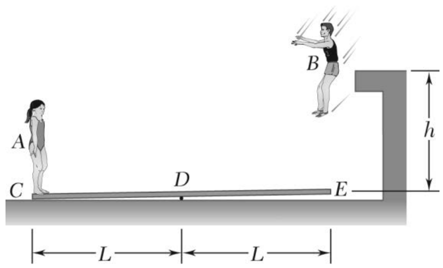
From Equation (1)
$$v_1 = \sqrt{(2)(9.81)(2.5)}$$
$$= 7.0036 \text{ m/s}$$

From Equation (2)
$$v_C = \frac{(70)(7.0036)}{70+55+5}$$
$$= 3.7712 \text{ m/s}$$

From Equation (3)
$$h_2 = \frac{(3.7712)^2}{(2)(9.81)}$$
$$= 0.725 \text{ m}$$

$h_2 = 725 \text{ mm}$ ◀

PROBLEM 17.122



Solve Problem 17.121, assuming that the gymnasts change places so that gymnast A jumps onto the plank while gymnast B stands at C.

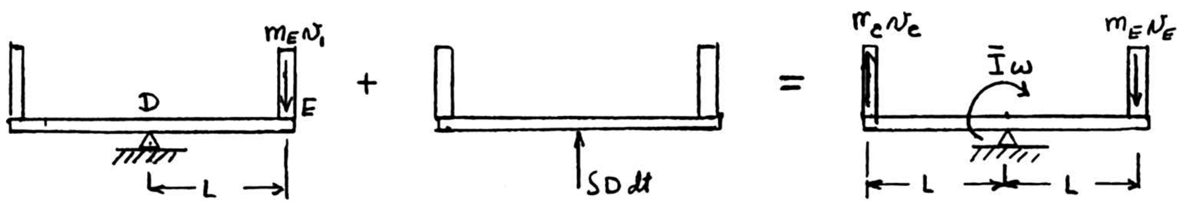
PROBLEM 17.121 The plank CDE has a mass of 15 kg and rests on a small pivot at D. The 55-kg gymnast A is standing on the plank at C when the 70-kg gymnast B jumps from a height of 2.5 m and strikes the plank at E. Assuming perfectly plastic impact and that gymnast A is standing absolutely straight, determine the height to which gymnast A will rise.

SOLUTION

Moment of inertia. $\quad \bar{I} = \frac{1}{12} m_P (2L)^2 = \frac{1}{3} m_P L^2$

Velocity of jumper at E. $\quad (v)_1 = \sqrt{2gh_1} \qquad (1)$

Principle of impulse-momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics: $\quad v_C = L\omega \quad v_D = L\omega$

↻ Moments about D: $\quad m_E v_1 L + 0 = m_E v_E L + m_C v_C L + \bar{I}\omega$

$$= m_E L^2 \omega + m_C L^2 \omega + \frac{1}{3} m_P L^2 \omega$$

$$\omega = \frac{m_E}{m_E + m_C + \frac{1}{3} m_P} \frac{v_1}{L}$$

$$v_C = L\omega = \frac{m_E v_1}{m_E + m_C + \frac{1}{3} m_P} \qquad (2)$$

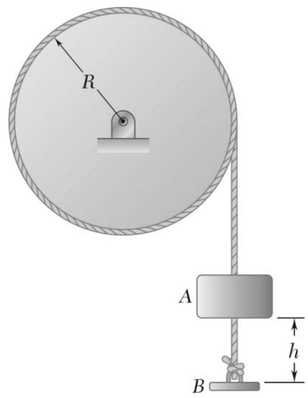
Gymnast (flier) rising. $\quad h_C = \frac{v_C^2}{2g} \qquad (3)$

Data:
$$m_E = m_A = 55 \text{ kg}$$
$$m_C = m_B = 70 \text{ kg}$$
$$m_P = 15 \text{ kg}$$
$$h_1 = 2.5 \text{ m}$$

From Equation (1)
$$v_1 = \sqrt{(2)(9.81)(2.5)}$$
$$= 7.0036 \text{ m/s}$$

From Equation (2)
$$v_C = \frac{(55)(7.0036)}{55 + 70 + 5}$$
$$= 2.9631 \text{ m/s}$$

From Equation (3)
$$h_2 = \frac{(2.9631)^2}{(2)(9.81)}$$
$$= 0.447 \text{ m} \qquad h_2 = 447 \text{ mm} \blacktriangleleft$$



PROBLEM 17.123

A small plate B is attached to a cord that is wrapped around a uniform 8-lb disk of radius $R = 9$ in. A 3-lb collar A is released from rest and falls through a distance $h = 15$ in. before hitting plate B. Assuming that the impact is perfectly plastic and neglecting the weight of the plate, determine immediately after the impact (a) the velocity of the collar, (b) the angular velocity of the disk.

SOLUTION

The collar A falls a distance h. From the principle of conservation of energy.

$$v_1 = \sqrt{2gh}$$

Impact analysis: $e = 0$

Kinematics. Collar A and plate B move together. The cord is inextensible.

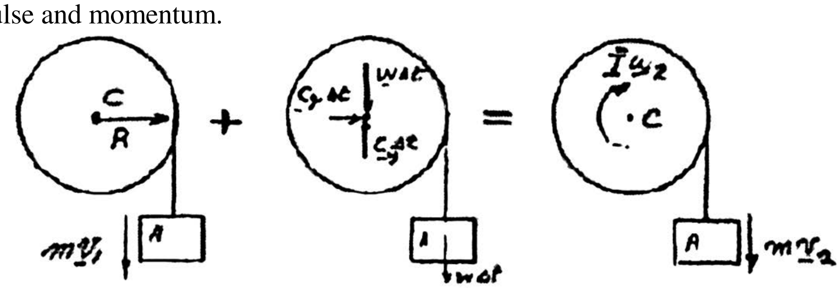
$$\bar{v}_2 = R\omega \quad \text{or} \quad \omega_2 = \frac{\bar{v}_2}{R}$$

Let m = mass of collar A and M = mass of disk.

Moment of inertia of disk: $\bar{I} = \frac{1}{2}MR^2$

Principle of impulse and momentum.

$\bar{I}\,\omega_1 = 0$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\circlearrowright$+ Moments about C: $m v_1 R = \bar{I}\omega_2 + m v_2 R$ (1)

$$m v_1 R = \frac{1}{2}MR^2\left(\frac{v_2}{R}\right) + m v_2 R$$

$$m v_1 = \frac{1}{2}M v_2 + m v_2$$

$$v_2 = \frac{2m}{2m+M}v_1$$

Data:

$$m = 3 \text{ lb/g}$$
$$M = 8 \text{ lb/g}$$
$$h = 15 \text{ in.} = 1.25 \text{ ft}$$
$$R = 9 \text{ in.} = 0.75 \text{ ft}$$

$$v_1 = \sqrt{(2)(32.2)(1.25)}$$
$$= 8.972 \text{ ft/s}$$

(*a*) Velocity of *A*.

$$v_2 = \frac{(2)(3)}{(2)(3)+8} v_1 = \frac{3}{7} v_1$$

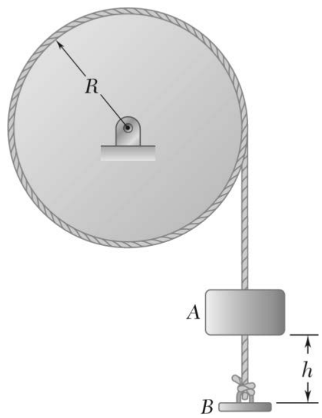
$$v_2 = \frac{3}{7}(8.972) = 3.8452 \text{ ft/s}$$

$\mathbf{v}_2 = 3.85$ ft/s ↓ ◀

(*b*) Angular velocity.

$$\omega_2 = \frac{3.8452}{0.75}$$

$\boldsymbol{\omega}_2 = 5.13$ rad/s ↻ ◀



PROBLEM 17.124

Solve Problem 17.123, assuming that the coefficient of restitution between *A* and *B* is 0.8.

PROBLEM 17.123 A small plate *B* is attached to a cord that is wrapped around a uniform 8-lb disk of radius $R = 9$ in. A 3-lb collar *A* is released from rest and falls through a distance $h = 15$ in. before hitting plate *B*. Assuming that the impact is perfectly plastic and neglecting the weight of the plate, determine immediately after the impact (*a*) the velocity of the collar, (*b*) the angular velocity of the disk.

SOLUTION

$$W_D = 8 \text{ lb}$$

$$m_D = \frac{W_D}{g} = \frac{8}{32.2} = 0.2484 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$R = 9 \text{ in.} = 0.75 \text{ ft}$$

$$I_D = \frac{1}{2} m_D R^2 = 0.06988 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$W_A = 3 \text{ lb}$$

$$m_A = \frac{W_A}{g} = \frac{3}{32.2} = 0.09317 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$h = 15 \text{ in.} = 1.25 \text{ ft}$$

Collar *A* falls through distance *h*. Use conservation of energy.

$$T_1 = 0$$

$$V_1 = W_A h$$

$$T_2 = \frac{1}{2} m_A v_A^2$$

$$V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2: \quad 0 + W_A h = \frac{1}{2} m_A v_A^2 + 0$$

$$v_A^2 = \frac{2 m_A h}{W_A} = 2gh$$

$$= (2)(32.2)(1.25)$$

$$= 80.5 \text{ ft}^2/\text{s}^2$$

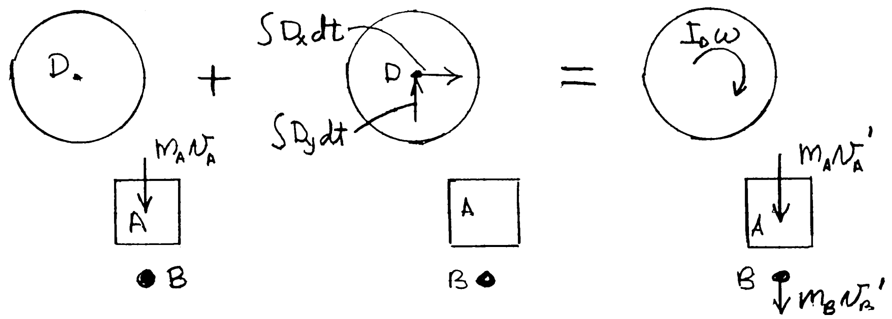
$$\mathbf{v}_A = 8.972 \text{ ft/s}\downarrow$$

Impact. Neglect the mass of plate *B*. Neglect the effect of weight over the duration of the impact.

PROBLEM 17.124 (Continued)

Kinematics. $\boldsymbol{\omega}' = \omega \circlearrowright \qquad \mathbf{v}'_B = R\omega \downarrow = 0.75\omega' \downarrow$

Conservation of momentum.



$+\circlearrowright$ Moments about D: $\quad m_A v_A R + 0 = m_A v'_A R + I_D \omega' + m_B v'_B R$

$$(0.09317)(8.972)(0.75) = (0.09317)(0.75)v'_A + 0.06988\,\omega' \qquad (1)$$

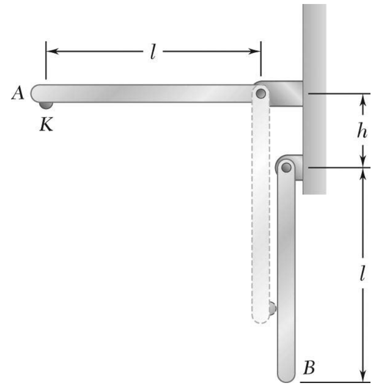
Coefficient of restitution. $\quad v'_B - v'_A = e(v_A - v_B)$

$$0.75\omega' - v'_A = 0.8(8.972 - 0) \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously

(*a*) Velocity of A. $\quad v'_A = -0.25648$ ft/s $\qquad \mathbf{v}'_A = 0.256$ ft/s $\uparrow$ ◀

(*b*) Angular velocity. $\quad \omega' = 9.228$ rad/s $\qquad \boldsymbol{\omega}' = 9.23$ rad/s $\circlearrowright$ ◀



PROBLEM 17.125

Two identical slender rods may swing freely from the pivots shown. Rod A is released from rest in a horizontal position and swings to a vertical position, at which time the small knob K strikes rod B which was at rest. If $h = \frac{1}{2}l$ and $e = \frac{1}{2}$, determine (a) the angle through which rod B will swing, (b) the angle through which rod A will rebound.

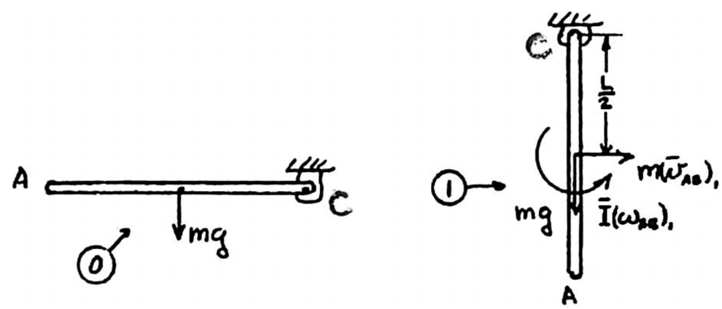
SOLUTION

Let $$m = m_{AC} = m_{BD}$$

Moment of inertia. $$\overline{I}_{AC} = \overline{I} = \frac{1}{12}mL^2$$

$$\overline{I}_{BD} = \frac{1}{12}mL^2$$

Rod AB falls to vertical position.



Position 0. $$V_1 = 0 \qquad T_1 = 0$$

Position 1. $$V_2 = -mg\frac{L}{2}$$

$$(\overline{v}_{AC})_1 = \frac{L}{2}(\omega_{AC})_1$$

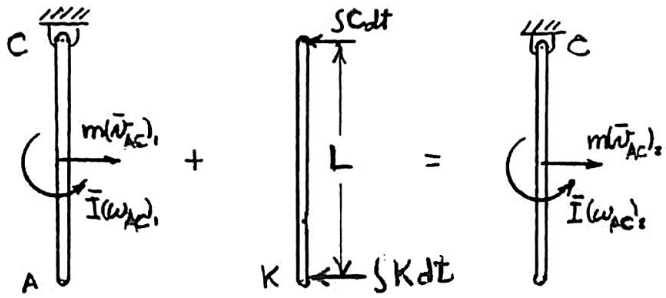
$$T_1 = \frac{1}{2}m(\overline{v}_{AC})_1^2 + \frac{1}{2}I(\omega_{AC})_1^2$$

$$= \frac{1}{6}mL^2(\omega_{AC})_1^2$$

Conservation of energy. $$T_0 + V_0 = T_1 + V_1: \quad 0 + 0 = \frac{1}{6}mL^2(\omega_{AC})_1^2 - \frac{1}{2}mgL$$

$$(\omega_{AC})_1^2 = \frac{3g}{L} \qquad (1)$$

Impact.

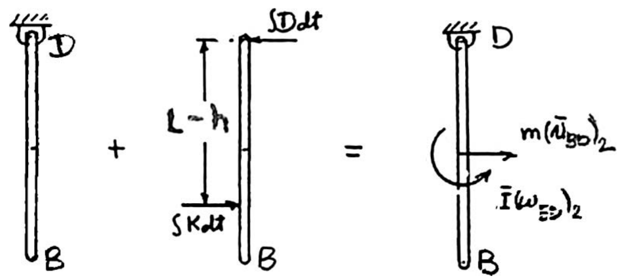


Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics
$$(\bar{v}_{AC})_1 = \frac{L}{2}(\omega_{AC})_1$$
$$(\bar{v}_{AC})_2 = \frac{L}{2}(\omega_{AC})_2$$

↻ Moments about C:
$$m(\bar{v}_{AC})_1\frac{L}{2} + \bar{I}(\omega_{AC})_1 - L\int Kdt = m(\bar{v}_{AC})_2\left(\frac{L}{2}\right) + \bar{I}(\omega_{AC})_2$$

$$\frac{1}{3}mL^2(\omega_{AC})_1 - L\int Kdt = \frac{1}{3}mL^2(\omega_{AC})_2 \qquad (2)$$



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics
$$(\bar{v}_{BD})_2 = \frac{L}{2}(\omega_{BD})_2$$

↻ Moments about D:
$$0 + (L-h)\int Kdt = m(\bar{v}_{BD})_2\frac{L}{2} + \bar{I}(\omega_{BD})_2$$

$$(L-h)\int Kdt = \frac{1}{3}mL^2(\omega_{BD})_2 \qquad (3)$$

Multiply Eq. (2) by $(L-h)$ and Eq. (3) by L and then add to eliminate $\int Kdt$.

$$\frac{1}{3}mL^2(L-h)(\omega_{AC})_1 = \frac{1}{3}mL^2(L-h)(\omega_{AC})_2 + \frac{1}{3}mL^3(\omega_{BD})_2 \qquad (1)$$

Condition of impact: $L(\omega_{AC})_2 - (L-h)(\omega_{BD})_2 = -eL(\omega_{AC})_1$ (2)

For $h = \frac{1}{2}L$ and $e = 0.5$ Eqs. (1) and (2) become

$$\frac{1}{6}mL^3(\omega_{AC})_1 = \frac{1}{6}mL^3(\omega_{AC})_2 + \frac{1}{3}mL^3(\omega_{BD})_2 \tag{3}$$

$$-\frac{1}{2} = 0.5L(\omega_{AB})_1 \tag{4}$$

Dividing Eq. (3) by $\frac{1}{6}mL^3$ and transposing terms gives

$$(\omega_{AC})_2 + 2(\omega_{BD})_2 = (\omega_{AC})_1 \tag{5}$$

Dividing Eq. (4) by $L/2$ and transposing terms gives

$$2(\omega_{AC})_2 - (\omega_{BD})_2 = -(\omega_{AC})_1 \tag{6}$$

Solving Eqs. (5) and (6) simultaneously for $(\omega_{AC})_2$ and $(\omega_{BD})_2$ gives

$$(\omega_{AC})_2 = -0.2(\omega_{AC})_1 \tag{7}$$

$$(\omega_{BD})_2 = 0.6(\omega_{AC})_1 \tag{8}$$

(*a*) *Angle of swing θ_B for rod B.*

Apply the principle of conservation of energy to rod *B*.

$$T_2 + V_2 = T_3 + V_3$$

Position (2): Just after impact.

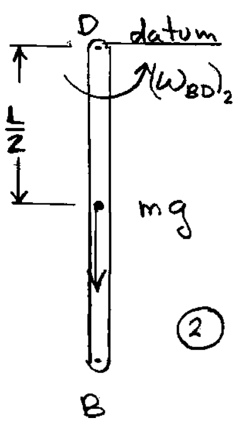
Position (3): At maximum angle of swing.

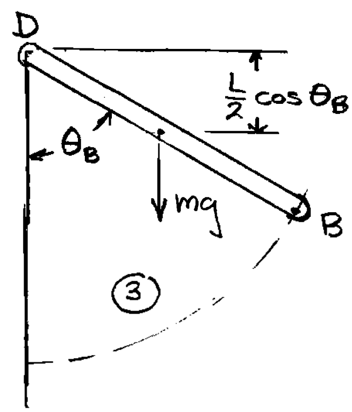
Potential energy.

Use the pivot point *D* as the datum.

$$V_2 = -mg\frac{L}{2}$$

$$V_3 = -mg\frac{L}{2}\cos\theta_B$$





Kinetic energy.

$$T_2 = \frac{1}{2} I_D (\omega_{BD})_2^2 = \frac{1}{6} mL^3 (\omega_{BD})_2^2$$
$$T_3 = 0$$

$$\frac{1}{6} mL^2 (\omega_{BD})_2^2 - mg\frac{L}{2} = 0 - mg\frac{L}{2}\cos\theta_B$$

$$1 - \cos\theta_B = \frac{1}{3}\frac{L}{g}(\omega_{BD})_2^2$$
$$= \frac{1}{3}\frac{L}{g}[0.6(\omega_{AB})_1]^2$$
$$= \left(\frac{1}{3}\right)(0.36)\left(\frac{L}{g}\right)\left(\frac{3g}{L}\right) = 0.36$$

$$\cos\theta_B = 0.64 \qquad \theta_B = 50.2° \blacktriangleleft$$

(*b*) *Angle of rebound θ_A for rod A*.

Apply the principle of conservation of energy to rod *A*.

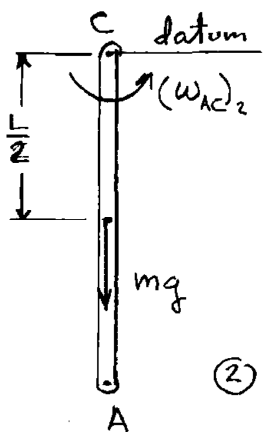
$$T_2 + V_2 = T_4 + V_4$$

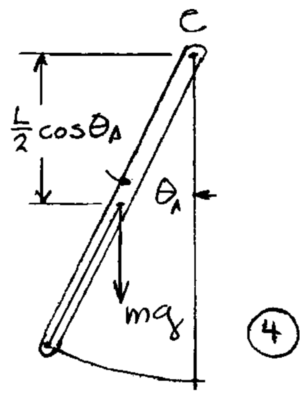
Position (2): Just after impact.

Position (4): At maximum angle of rebound.

Potential energy. Use the pivot Point *C* as the datum.

$$V_2 = -mg\frac{L}{2} \qquad V_4 = -mg\frac{L}{2}\cos\theta_A$$





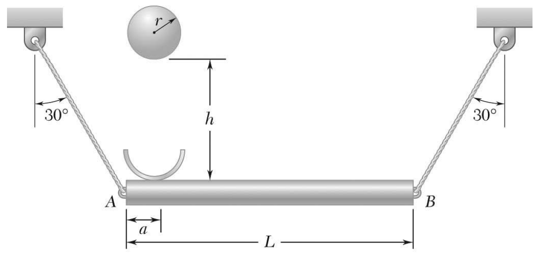
Kinetic energy.

$$T_2 = \frac{1}{2} I_C (\omega_{AB})_2^2 = \frac{1}{6} mL^2 (\omega_{AC})_2^2 \qquad T_4 = 0$$

$$\frac{1}{6} mL^2 (\omega_{AC})_2^2 - mg\frac{L}{2} = 0 - mg\frac{L}{2}\cos\theta_A$$

$$1 - \cos\theta_A = \frac{1}{3}\frac{L}{g}(\omega_{AC})_2^2$$
$$= \frac{1}{3}\frac{L}{g}[-0.2(\omega_{AB})_1]^2$$
$$= \left(\frac{1}{3}\right)(0.04)\left(\frac{L}{g}\right)\left(\frac{3g}{L}\right) = 0.04$$

$$\cos\theta_A = 0.96 \qquad \theta_A = 16.26° \blacktriangleleft$$

	**PROBLEM 17.126** A 2-kg solid sphere of radius $r = 40$ mm is dropped from a height $h = 200$ mm and lands on a uniform slender plank AB of mass 4 kg and length $L = 500$ mm which is held by two inextensible cords. Knowing that the impact is perfectly plastic and that the sphere remains attached to the plank at a distance $a = 40$ mm from the left end, determine the velocity of the sphere immediately after impact. Neglect the thickness of the plank.

SOLUTION

Masses and moments of inertia.

Sphere: $m_S = 2\text{ kg}, \quad r = 40\text{ mm} = 0.040\text{ m}$

$$\bar{I}_S = \frac{2}{5}m_S r^2 = \left(\frac{2}{5}\right)(2\text{ kg})(0.04\text{ m})^2 = 1.28\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

Plank AB: $m_{AB} = 4\text{ kg}, \quad L = 500\text{ mm} = 0.5\text{ m}$

$$\bar{I}_{AB} = \frac{1}{12}m_{AB}L^2 = \left(\frac{1}{12}\right)(4\text{ kg})(0.5\text{ m})^2 = 83.333\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

Velocity of sphere at impact.

$$v_S = \sqrt{2gh} = \sqrt{(2)(9.81\text{ m/s})(0.200\text{ m})} = 1.9809\text{ m/s}$$

Before impact.

Linear momentum: $m_S\mathbf{v}_S = (4\text{ kg})(1.9809\text{ m/s})\downarrow = 7.9236\text{ kg}\cdot\text{m/s}\downarrow$

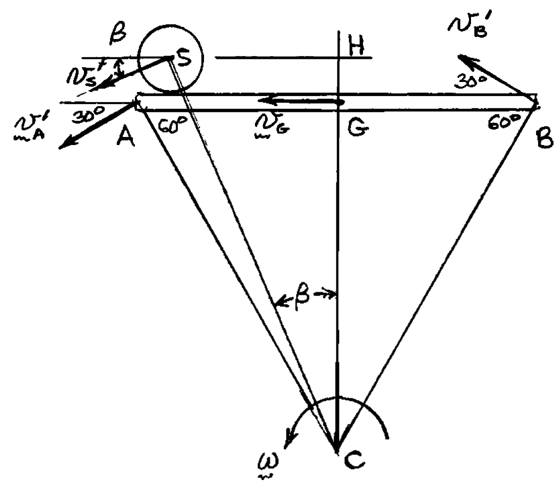
with its line of action lying at distance $\frac{L}{2} - a$ from the midpoint of the plank.

$$\frac{L}{2} - a = 0.25\text{ m} - 0.04\text{ m} = 0.21\text{ m.}$$

After impact. Assume that both cables are taut so $\mathbf{v}_A$ is perpendicular to the cable at A and $\mathbf{v}_B$ is perpendicular to the cable at B.

Kinematics.

To locate the instantaneous center C draw line $\overline{AC}$ perpendicular to $\mathbf{v}_A$ and line $\overline{BC}$ perpendicular to $\mathbf{v}_B$. Let point G be the mass center of the plank AB and Point S be that of the sphere.



$$\overline{CH} = L\cos 30° + r$$
$$= (0.500 \text{ m})\cos 30° + 0.040 \text{ m}$$
$$= 0.47301 \text{ m}$$

$$\overline{HS} = \frac{L}{2} - a = 0.21 \text{ m}$$

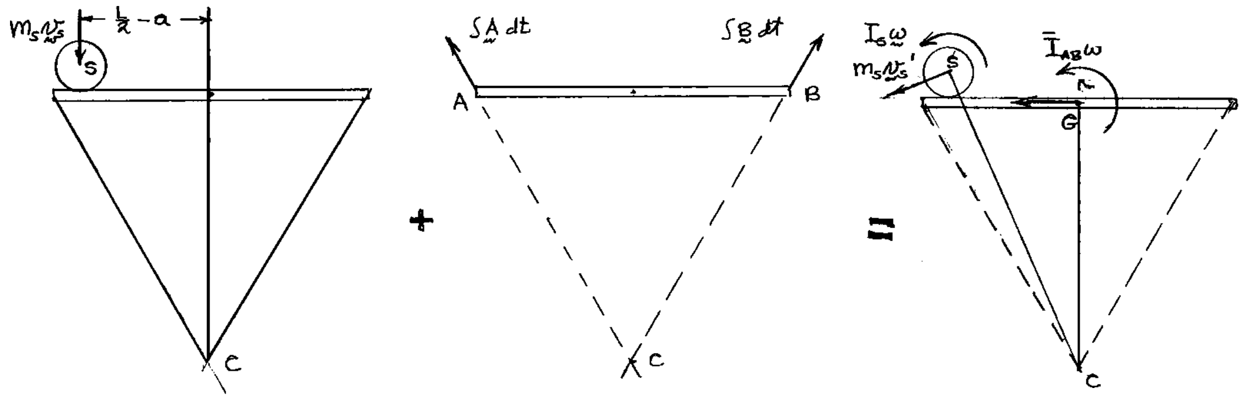
$$\overline{CS} = \sqrt{\overline{CH}^2 + \overline{HS}^2} = 0.51753 \text{ m}$$

$$\tan\beta = \frac{\overline{HS}}{\overline{CH}} = 0.44397 \qquad \beta = 23.94°$$

$$v_S = (\overline{CS})\omega = 0.51753\omega$$

$$v_G = (L\cos 30°)\omega = 0.43301\omega$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+↻ Moments about C:

$$m_S v_S\left(\frac{L}{2} - a\right) + 0 = m_S v_S^1(\overline{CS}) + m_{AB} v_G^1(CH) + \bar{I}_S\omega + \bar{I}_{AB}\omega$$

$$m_S v_S\left(\frac{L}{2} - a\right) = [m_S(\overline{CS})^2 + m_{AB}\overline{CG}^2 + \bar{I}_S + \bar{I}_{AB}]\omega = I_C\omega$$

where $$m_S v_S\left(\frac{L}{2}-a\right)=(2\text{ kg})(1.9809\text{ m/s})(0.21\text{ m})=0.83198\text{ kg}\cdot\text{m}^2/\text{s}$$

and $$I_C=(2\text{ kg})(0.51753\text{ m})^2+(4\text{ kg})(0.43301\text{ m})^2+1.28\times10^{-3}\text{ kg}\cdot\text{m}^2+83.333\times10^{-3}\text{ kg}\cdot\text{m}^2$$
$$=(0.53567+0.75+0.00128+0.08333)\text{ kg}\cdot\text{m}^2=1.37028\text{ kg}\cdot\text{m}^2$$

$$0.83198\text{ kg}\cdot\text{m}^2/\text{s}=(1.37028\text{ kg}\cdot\text{m}^2)\omega \qquad \boldsymbol{\omega}=0.60716\text{ rad/s} \circlearrowleft \blacktriangleleft$$

$$v_S^1=(0.51753\text{ m})(0.60716\text{ rad/s})=0.31422\text{ m/s}$$
$$v_G^1=(0.43301\text{ m})(0.60716\text{ rad/s})=0.26291\text{ m/s}$$

To check that neither cable becomes slack during the impact, we show that $\int A dt$ and $\int B\,dt$ are positive quantities.

$+\uparrow$ components: $$-m_S v_S+(\int Adt+\int Bdt)\cos 30°=-mv_S^1\sin\beta$$
$$\frac{\sqrt{3}}{2}(\int Adt+\int Bdt)=[m_S v_S-m_S v_1^1\sin\beta]/\cos 30°$$
$$=7.9236-(2)(0.31422)\sin 23.94°$$
$$=7.6686\text{ N}\cdot\text{s}$$

$\xleftarrow{+}$ components: $$0+(\int Adt-\int Bdt)\sin 30°=m_{AB}v_G^1+m_S v_S\cos\beta$$
$$\frac{1}{2}(\int Adt-\int Bdt)=(4)(0.26291)+(2)(0.31422)\cos 23.94°$$
$$=1.6260\text{ N}\cdot\text{s}$$

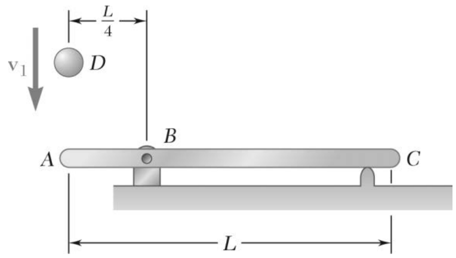
Solving the simultaneous equation gives

$$\int Adt=6.05\text{ N}\cdot\text{s} \qquad \int Bdt=2.80\text{ N}\cdot\text{s}$$

The cables remain taut as assumed.

Velocity of sphere: $\mathbf{v}_S^1=0.314\text{ m/s} \measuredangle\ 23.9° \blacktriangleleft$

PROBLEM 17.127



Member *ABC* has a mass of 2.4 kg and is attached to a pin support at *B*. An 800-g sphere *D* strikes the end of member *ABC* with a vertical velocity $\mathbf{v}_1$ of 3 m/s. Knowing that $L = 750$ mm and that the coefficient of restitution between the sphere and member *ABC* is 0.5, determine immediately after the impact (*a*) the angular velocity of member *ABC*, (*b*) the velocity of the sphere.

SOLUTION

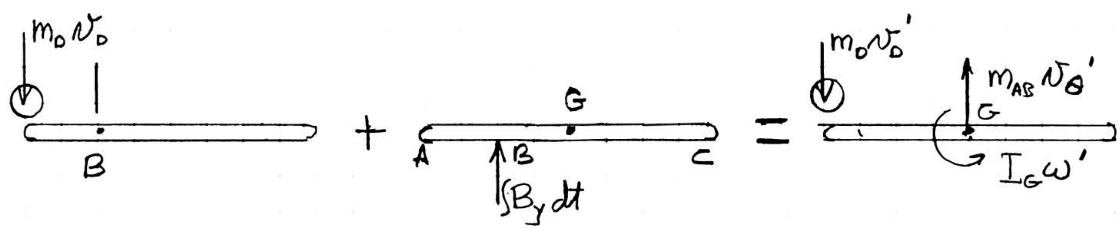
$$m_D = 0.800 \text{ kg}$$
$$L = 0.750 \text{ m}$$
$$\frac{1}{4}L = 0.1875 \text{ m}$$
$$m_{AC} = 2.4 \text{ kg}$$

Let Point *G* be the mass center of member *ABC*.

$$I_G = \frac{1}{12}m_{AC}L^2 = \frac{1}{12}(2.4)(0.750)^2 = 0.1125 \text{ kg}\cdot\text{m}^2$$

Kinematics after impact. $\boldsymbol{\omega}' = \omega' \curvearrowleft$, $\quad \mathbf{v}'_G = \frac{L}{4}\omega' \uparrow$, $\quad \mathbf{v}'_A = \frac{L}{4}\omega' \downarrow$

Conservation of momentum.



$+\curvearrowleft$ Moments about *B*:

$$m_D v_D \frac{L}{2} + 0 = m_D v'_D \frac{L}{2} + I_G\omega' + m_{AC}v'_G\frac{L}{4}$$

$$m_D v_D \frac{L}{4} = m_D v'_D \frac{L}{4} + \left[I_G + m_{AD}\left(\frac{L}{4}\right)^2\right]\omega'$$

$$(0.800)(3)(0.1875) = (0.800)(0.1875)v'_D + [0.1125 + (2.4)(0.1875)^2]\omega'$$

$$0.45 = 0.15v'_D + 0.196875\omega' \qquad (1)$$

PROBLEM 17.127 (Continued)

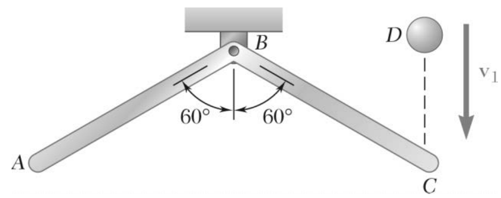
Coefficient of restitution.
$$v'_D - v'_A = v'_D - \frac{L}{4}\omega'$$
$$= -e(v_D - v_A)$$

$$v'_D - 0.1875\omega' = -(0.5)(3-0) \quad (2)$$

Solving Eqs. (1) and (2) simultaneously.

(*a*) Angular velocity. $\omega' = 3$ $\boldsymbol{\omega}' = 3.00 \text{ rad/s} \circlearrowright$ ◀

(*b*) Velocity of *D*. $v'_D = -0.9375$ $v'_D = 0.938 \text{ m/s} \uparrow$ ◀



PROBLEM 17.128

Member *ABC* has a mass of 2.4 kg and is attached to a pin support at *B*. An 800-g sphere *D* strikes the end of member *ABC* with a vertical velocity $\mathbf{v}_1$ of 3 m/s. Knowing that $L = 750\,\text{mm}$ and that the coefficient of restitution between the sphere and member *ABC* is 0.5, determine immediately after the impact (*a*) the angular velocity of member *ABC*, (*b*) the velocity of the sphere.

SOLUTION

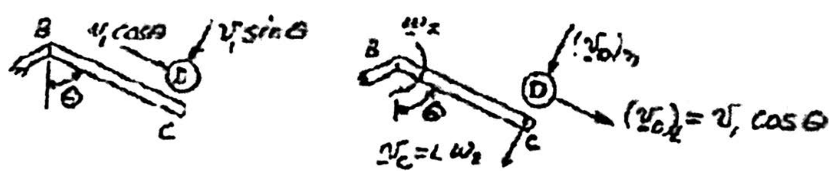
Let *M* be the mass of member *ABC* and $\bar{I}$ its moment of inertia about *B*.

$$M = 2.4\text{ kg} \qquad \bar{I} = \frac{1}{12}M(2L)^2$$

where $\qquad L = 750\text{ mm} = 0.75\text{ m}$

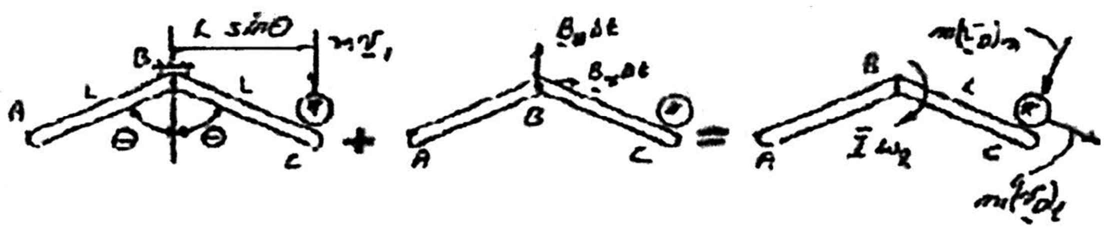
Let *m* be the mass of sphere *D*. $\qquad m = 800\text{ g} = 0.8\text{ kg}$

Impact kinematics and coefficient of restitution.



$$(v_1 \sin\theta)e = L\omega_2 - (v_D)_n: \quad (v_D)_n = L\omega_2 - (v_1\sin\theta)e \qquad (1)$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+↻ Moments about *B*:

$$mv_1 L\sin\theta = \bar{I}\omega_2 + m(v_D)_n L$$

$$mv_1 L\sin\theta = \frac{1}{12}M(2L)^2\omega_2 + m[L\omega_2 - (v_1\sin\theta)e]L$$

$$mv_1\sin\theta = \frac{1}{3}ML\omega_2 - mL\omega_2 - m(v_1\sin\theta)e$$

$$m(1+e)\frac{v_1}{L}\sin\theta = \left(\frac{1}{3}M + m\right)\omega_2$$

(*a*) Angular velocity.

$$\omega_2 = \frac{(3)(1+e)mv_1 \sin\theta}{(M+3m)L}$$

$$\omega_2 = \frac{(3)(1.5)(0.8)(3)\sin 60°}{(2.4+2.4)(0.75)}$$

$$= 2.5981$$

$\boldsymbol{\omega}_2 = 2.60$ rad/s ↻ ◀

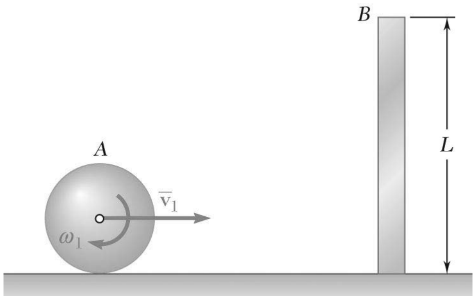
(*b*) Velocity of *D*.

From Eq. (1),

$$(v_D)_n = (0.75)(2.5981) - (3\sin 60°)(0.5)$$
$$= 0.64976 \text{ m/s}$$
$$(v_D)_t = v_1 \cos 60°$$
$$= 3\cos 60°$$
$$= 1.5 \text{ m/s}$$
$$(\mathbf{v}_D)_n = 0.64976 \text{ m/s} \measuredangle 30°$$
$$(\mathbf{v}_D)_t = 1.5 \text{ m/s} \searrow 30°$$
$$v_D = \sqrt{(0.64976)^2 + (1.5)^2}$$
$$= 1.63468 \text{ m/s}$$
$$\tan\theta = \frac{0.64976}{1.5}$$
$$\theta = 23.4°$$
$$\theta + 30° = 53.4°$$

$\mathbf{v}_D = 1.635$ m/s $\searrow$ 53.4° ◀

PROBLEM 17.129



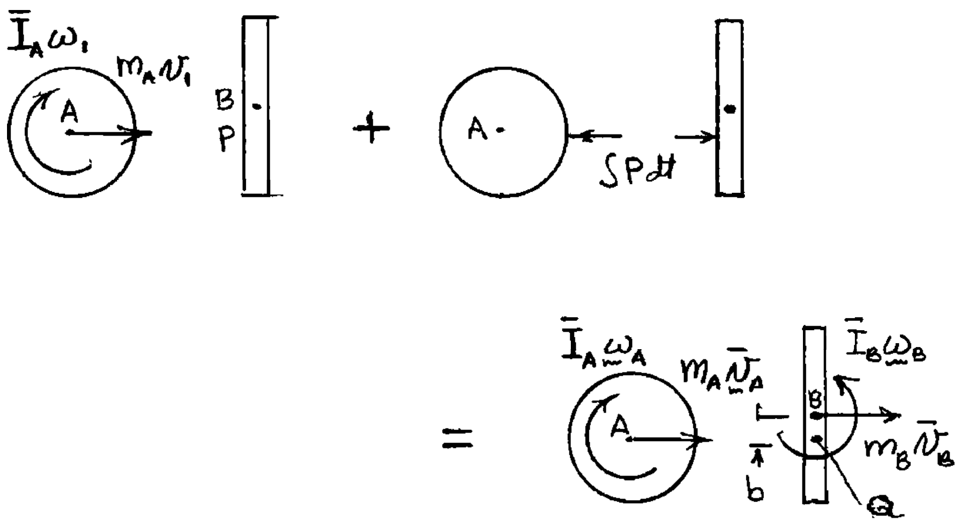
Sphere A of mass $m_A = 2$ kg and radius $r = 40$ mm rolls without slipping with a velocity $\overline{\mathbf{v}}_1 = 2$ m/s on a horizontal surface when it hits squarely a uniform slender bar B of mass $m_B = 0.5$ kg and length $L = 100$ mm that is standing on end and at rest. Denoting by μ_k the coefficient of kinetic friction between the sphere and the horizontal surface, neglecting friction between the sphere and the bar, and knowing the coefficient of restitution between A and B is 0.1, determine the angular velocities of the sphere and the bar immediately after the impact.

SOLUTION

Before impact sphere A rolls without slipping so that its instantaneous center of rotation is its contact point with the floor.

$$\omega_1 = \frac{v_1}{r} = \frac{2 \text{ m/s}}{0.040 \text{ m}} = 50 \text{ rad/s} \qquad \boldsymbol{\omega}_1 = 50 \text{ rad/s} \circlearrowright \blacktriangleleft$$

Analysis of impact. Use the principle of impulse and momentum. Let point A be the center of sphere A, point B be the mass center of bar B, and Points P and Q the contact point between the sphere and the bar, Point P being on sphere A and Point Q being on the bar B.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

$\boldsymbol{\omega}_A = \omega_A \circlearrowright$ $\qquad$ $\boldsymbol{\omega}_B = \omega_B \circlearrowright$

$\overline{\mathbf{v}}_A = \overline{v}_A \longrightarrow$ $\qquad$ $\overline{\mathbf{v}}_B = \overline{v}_B \longrightarrow$, Sphere A alone.

$\circlearrowleft +$ Moments about A: $\quad \bar{I}_A\omega_1 + 0 = \bar{I}_A\omega_A \qquad \omega_A = \omega_1$

$$\boldsymbol{\omega}_A = 50.0 \text{ rad/s} \circlearrowright \blacktriangleleft$$

Kinematics:
$$b = \frac{L}{2} - r = 50 \text{ mm} - 40 \text{ mm} = 10 \text{ mm} = 0.010 \text{ m}$$

$$\mathbf{v}_Q = (\bar{v}_B + b\omega_B) \longrightarrow$$

Condition of impact. $\quad \bar{\mathbf{v}}_A - \mathbf{v}_Q = -e\bar{\mathbf{v}}_1$

$$v_A - v_B - b\omega_B = -e\bar{v}_1 \qquad (1)$$

Bar B alone:

$+\circlearrowright$ Moments about Q:
$$0 + 0 = \bar{I}_B\omega_B - bm_Bv_B$$

$$\frac{1}{12}m_BL^2\omega_B - bm_Bv_B = 0$$

$$bv_B - \frac{1}{12}L^2\omega_B = 0 \qquad (2)$$

Sphere A and bar B together.

$\xrightarrow{+}$ components:

$$m_A\bar{v}_1 + 0 = m_Av_A + m_Bv_B$$
$$m_A\bar{v}_A + m_B\bar{v}_B = m_A\bar{v}_1 \qquad (3)$$

Data:
$$m_A = 2 \text{ kg}, \quad m_B = 0.5 \text{ kg}, \quad e = 0.1$$
$$\bar{v}_1 = 2 \text{ m/s}, \quad L = 0.100 \text{ m}, \quad b = 0.010 \text{ m}$$

$$v_A - v_B - (0.010 \text{ m})\omega_B = -(0.1)(2 \text{ m/s}) \qquad (1)'$$

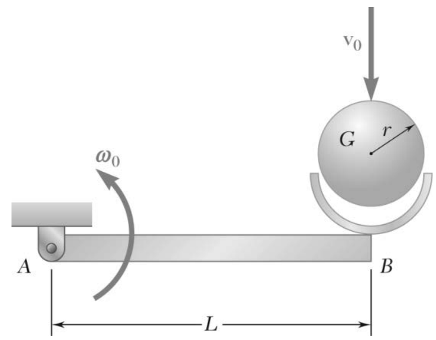
$$(0.010 \text{ m})v_B - \frac{1}{12}(0.100 \text{ m})^2\omega_B = 0 \qquad (2)'$$

$$(2 \text{ kg})v_A + (0.5 \text{ kg})v_B = (2 \text{ kg})(2 \text{ m/s}) \qquad (3)'$$

Solving Eqs. (1)′, (2)′, and (3)′ simultaneously,

$$v_A = 1.599 \text{ m/s} \qquad v_B = 1.606 \text{ m/s} \qquad \omega_B = 19.27 \text{ rad/s}$$

$$\boldsymbol{\omega}_B = 19.27 \text{ rad/s} \circlearrowright \blacktriangleleft$$



PROBLEM 17.130

A large 3-lb sphere with a radius $r = 3$ in. is thrown into a light basket at the end of a thin, uniform rod weighing 2 lb and length $L = 10$ in. as shown. Immediately before the impact the angular velocity of the rod is 3 rad/s counterclockwise and the velocity of the sphere is 2 ft/s down. Assume the sphere sticks in the basket. Determine after the impact (*a*) the angular velocity of the bar and sphere, (*b*) the components of the reactions at *A*.

SOLUTION

Let Point *G* be the mass center of the sphere and Point *C* be that of the rod *AB*.

Rod *AB*: $W_{AB} = 2\text{ lb.} \quad m_{AB} = \dfrac{2}{32.2} = 0.06211\text{ lb}\cdot\text{s}^2/\text{ft}$

$$I_{AB} = \frac{1}{12}m_P L^2 = \frac{1}{12}(0.06211)\left(\frac{10}{12}\right)^2 = 0.003594\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

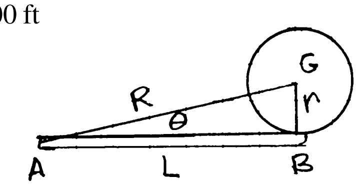
Sphere: $W_S = 3\text{ lb} \quad m_S = \dfrac{3}{32.2} = 0.09317\text{ lb}\cdot\text{s}^2/\text{ft}$

$$I_G = \frac{2}{5}m_S r^2 = \frac{2}{5}(0.09317)\left(\frac{3}{12}\right)^2 = 0.002329\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Impact. Before impact, bar *AB* is rotating about *A* with angular velocity $\boldsymbol{\omega}_0 = \omega_0 \circlearrowleft$ ($\omega_0 = 3$ rad/s) and the sphere is falling with velocity $\mathbf{v}_0 = v_0\downarrow$ ($v_0 = 2$ ft/s). After impact, the rod and the sphere move together, rotating about *A* with angular velocity $\boldsymbol{\omega} = \omega \circlearrowright$.

Geometry. $R = \sqrt{L^2 + r^2} = \sqrt{10^2 + 3^2} = 10.44\text{ in.} = 0.8700\text{ ft}$

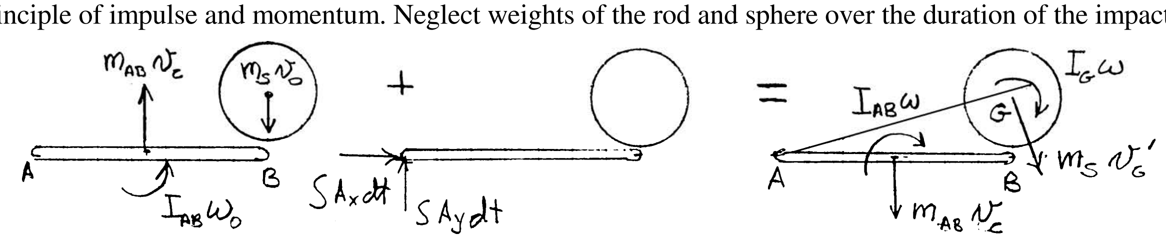
$$\tan\theta = \frac{r}{L} = \frac{3}{10} \quad \theta = 16.7°$$



Kinematics: Before impact, $v_C = \dfrac{L}{2}\omega_0 = \left(\dfrac{5}{12}\right)(3) = 1.25\text{ ft/s}\uparrow$

After impact, $\mathbf{v}_C = \dfrac{L}{2}\omega'\downarrow, \quad \mathbf{v}_G = R\omega' \searrow \theta$

Principle of impulse and momentum. Neglect weights of the rod and sphere over the duration of the impact.



(*a*) $+\circlearrowright$ Moments about A:

$$m_S v_0 L - I_{AB}\omega_0 - m_{AB} v_C \frac{L}{2} + 0 = I_G \omega' + m_S v'_G R + I_{AB}\omega' + m_{AB} v_C \frac{L}{2}$$

or
$$m_S v_0 L - I_{AB}\omega_0^2 - m_{AB} v_C \frac{L}{2} = \left(I_G + m_S R^2 + I_{AB} + \frac{1}{4} m_{AB} L^2\right)\omega' \qquad (1)$$

$$(0.09317)(2)\left(\frac{10}{12}\right) - (0.003594)(3) - (0.06211)(1.25)\left(\frac{5}{12}\right)$$

$$= \left[0.002329 + (0.09317)(0.87)^2 + 0.003594 + \frac{1}{4}(0.06211)\left(\frac{10}{12}\right)^2\right]\omega'$$

$$0.112152 = 0.087226\omega' \qquad \omega' = 1.2858$$

$\boldsymbol{\omega}' = 1.286 \text{ rad/s} \circlearrowright$ ◀

Normal accelerations at C and G.

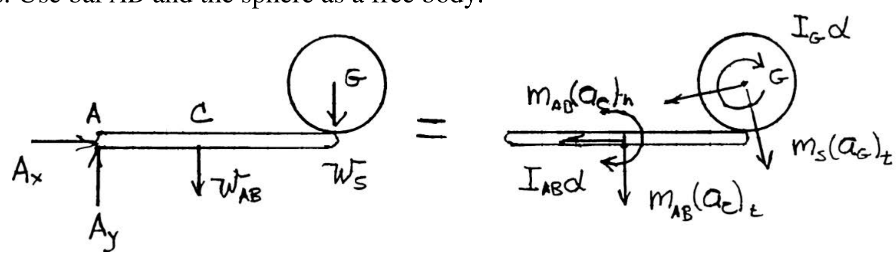
$$(\mathbf{a}_C)_n = \frac{L}{2}(\omega')^2 = \left(\frac{5}{12}\right)(1.2858)^2 = 0.6889 \text{ ft/s}^2 \longleftarrow$$

$$(a_G)_n = R(\omega')^2 = (0.87)(1.2858)^2 = 1.4384 \text{ ft/s}^2 \measuredangle\, 16.7°$$

Tangential accelerations at C and G. $\boldsymbol{\alpha} = \alpha \circlearrowright$

$$(a_C)_t = \frac{L}{2}\alpha = \frac{5}{12}\alpha \downarrow \qquad (a_G)_t = R\alpha = 0.87\alpha \measuredangle\, 16.7°$$

(*b*) Kinetics. Use bar AB and the sphere as a free body.



$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{\text{eff}}:$$

$$W_{AB}\frac{L}{2} + W_S L = I_{AB}\alpha + \frac{L}{2} m_{AB}(a_C)_t + I_G\alpha + m_S(a_G)_t R$$

$$= \left(I_{AB} + \frac{1}{4} m_{AB} L^2 + I_G + m_S R^2\right)\alpha$$

$$(2)\left(\frac{5}{12}\right) + (3)\left(\frac{10}{12}\right) = \left[0.003594 + \frac{1}{4}(0.06211)\left(\frac{10}{12}\right)^2 + 0.002329 + (0.09317)(0.87)^2\right]\alpha$$

$$3.3333 = 0.087226\alpha$$

$\boldsymbol{\alpha} = 38.214 \text{ rad/s}^2 \circlearrowright$

$$(\mathbf{a}_C)_t = \left(\frac{5}{12}\right)(38.214) = 15.923 \text{ ft/s}^2 \downarrow, \quad (\mathbf{a}_G)_t = (0.87)(38.214) = 33.247 \text{ ft/s}^2 \measuredangle\, 16.7°$$

PROBLEM 17.130 (Continued)

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}:$

$$A_x = -m_{AB}(a_C)_n - m_S(a_G)_n \cos 16.7° + m_S(a_G)_t \sin 16.70$$

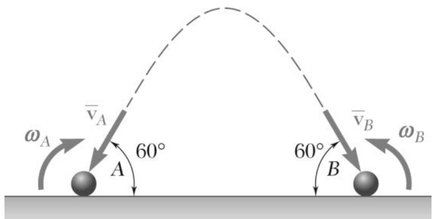
$$A_x = -(0.06211)(0.6889) - (0.09317)(1.4384)\cos 16.7° + (0.09317)(33.247)\sin 16.7°$$

$A_x = 0.719 \text{ lb}$ $\qquad$ $\mathbf{A}_x = 0.719 \text{ lb} \longrightarrow$ ◀

$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \quad A_y - W_{AB} - W_S = -m_{AB}(a_C)_t - m_S(a_G)_t \cos 16.7° - m_S(a_G)_n \sin 16.7°$

$$A_y - 2 - 3 = -(0.06211)(15.923) - (0.09317)(33.247)\cos 16.7° - (0.09317)(1.4384)\sin 16.7°$$

$A_y = 1.006 \text{ lb}$ $\qquad$ $\mathbf{A}_y = 1.006 \text{ lb} \uparrow$ ◀



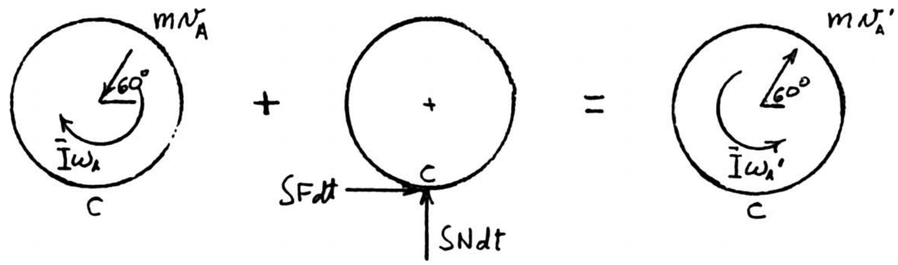
PROBLEM 17.131

A small rubber ball of radius r is thrown against a rough floor with a velocity $\bar{\mathbf{v}}_A$ of magnitude v_0 and a backspin $\boldsymbol{\omega}_A$ of magnitude ω_0. It is observed that the ball bounces from A to B, then from B to A, then from A to B, etc. Assuming perfectly elastic impact, determine the required magnitude ω_0 of the backspin in terms of $\bar{v}_0$ and r.

SOLUTION

Moment of inertia. $\bar{I} = \frac{2}{5}mr^2$ Ball is assumed to be a solid sphere.

Impact at A.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

For the velocity of the ball to be reversed on each impact,

$$v_A' = v_A = v_0$$
$$\omega_A' = \omega_A = \omega_0$$

This is consistent with the assumption of perfectly elastic impact.

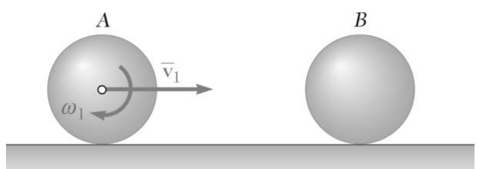
↻ Moments about C: $mv_A r\cos 60° - \bar{I}\,\omega_A + 0 = \bar{I}\,\omega_A' - mv_A' r\cos 60°$

$$mv_0 r\cos 60° - \frac{2}{5}mr^2\omega_0 + 0 = \frac{2}{5}mr^2\omega_0 - mv_0 r\cos 60°$$

$$\frac{2}{5}r\omega_0 = v_0\cos 60°$$

$$\omega_0 = \frac{5}{4}\frac{v_0}{r} \blacktriangleleft$$

PROBLEM 17.132

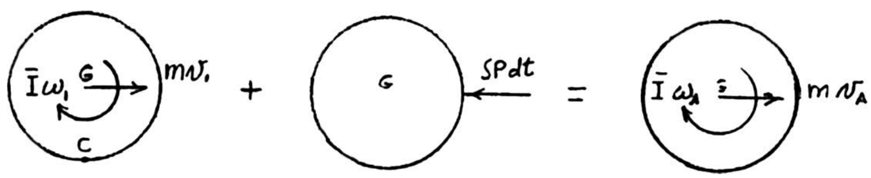


Sphere A of mass m and radius r rolls without slipping with a velocity $\bar{\mathbf{v}}_1$ on a horizontal surface when it hits squarely an identical sphere B that is at rest. Denoting by μ_k the coefficient of kinetic friction between the spheres and the surface, neglecting friction between the spheres, and assuming perfectly elastic impact, determine (*a*) the linear and angular velocities of each sphere immediately after the impact, (*b*) the velocity of each sphere after it has started rolling uniformly.

SOLUTION

Moment of inertia. $\bar{I} = \frac{2}{5}mr^2$

Analysis of impact. Sphere A.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to2}$ = Syst. Momenta$_2$

Kinematics: Rolling without slipping in Position 1.

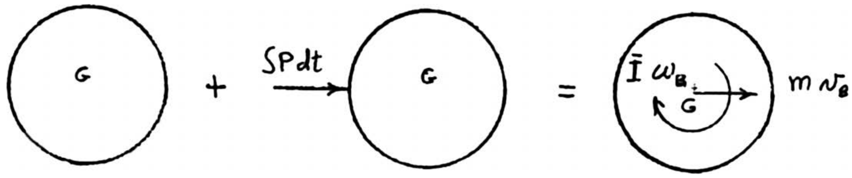
$$\omega_A = \frac{v_1}{r}$$

Moments about G: $\bar{I}\omega_1 + 0 = \bar{I}\omega_A$

$$\omega_A = \omega_1 = \frac{v_1}{r}$$

$\xrightarrow{+}$ Linear components: $mv_1 - \int P dt = mv_A$ (1)

Analysis of impact. *Sphere B*.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to2}$ = Syst. Momenta$_2$

$\xrightarrow{+}$ Linear components: $0 + \int P dt = mv_B$ (2)

Add Equations (1) and (2) to eliminate $\int P dt$.

$$mv_1 = mv_A + mv_B \quad \text{or} \quad v_B + v_A = v_1 \tag{3}$$

Condition of impact. $e = 1$. $\quad v_B - v_A = ev_1 = v_1 \quad (4)$

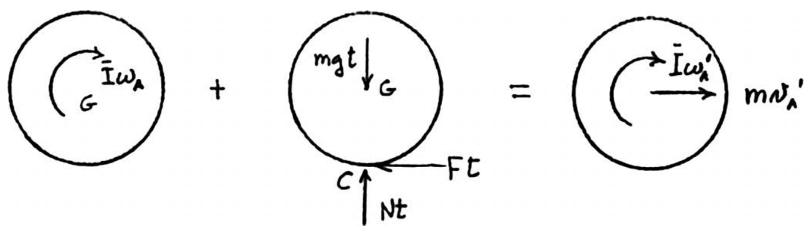
Solving Equations (3) and (4) simultaneously,

$$v_A = 0, \qquad v_B = v_1$$

↺ Moments about G: $\quad 0 + 0 = \bar{I}\,\omega_B \quad \omega_B = 0$

(a) *Velocities after impact.* $\quad \mathbf{v}_A = 0; \quad \boldsymbol{\omega}_A = \dfrac{v_1}{r}$ ↻; $\quad \mathbf{v}_B = v_1$ ⟶; $\quad \boldsymbol{\omega}_B = 0$ ◀

Motion after Impact. *Sphere A*.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

Condition of rolling without slipping: $\quad v'_A = \omega'_A r$

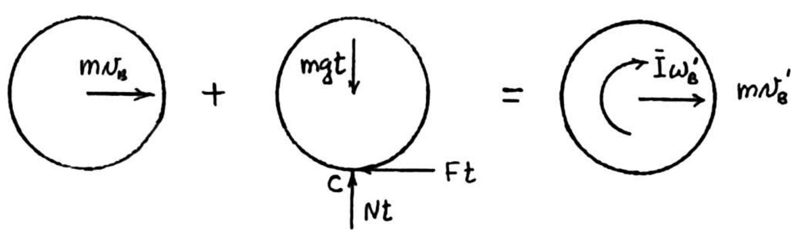
↺ Moments about C: $\quad \bar{I}\,\omega_A + 0 + \bar{I}\,\omega'_A + mv'_A r$

$$\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right) + 0 = \left(\frac{2}{5}mr^2\right)\omega'_A + m(r\omega'_A)r$$

$$\omega'_A = \frac{2}{7}\frac{v_1}{r}$$

$$v'_A = \frac{2}{7}v_1$$

Motion after impact. *Sphere B*.



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to2}$ = **Syst. Momenta**$_2$

PROBLEM 17.132 (Continued)

Condition of rolling without slipping: $v'_B = r\omega'_B$

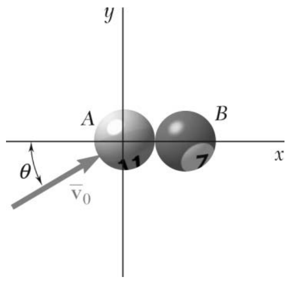
↺ Moments about C: $mv_B r + 0 = \bar{I}\,\omega'_B + mv'_B r$

$$mv_1 r + 0 = \left(\frac{2}{5}mr^2\right)\omega'_B + m(r\omega'_B)r$$

$$\omega'_B = \frac{5}{7}\frac{v_1}{r}$$

$$v'_B = \frac{5}{7}v_1$$

(*b*) *Final Rolling Velocities.* $\mathbf{v}'_A = \frac{2}{7}v_1 \longrightarrow; \quad \mathbf{v}'_B = \frac{5}{7}v_1 \longrightarrow$ ◀



PROBLEM 17.133

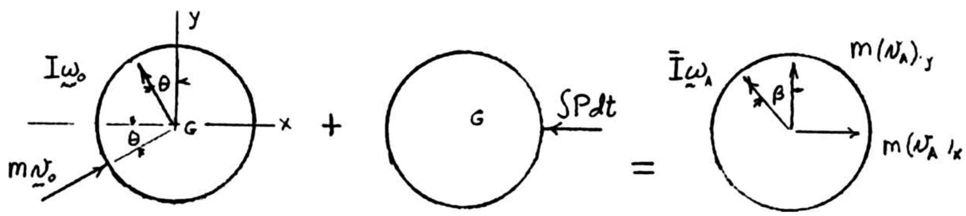
In a game of pool, ball A is rolling without slipping with a velocity $\overline{\mathbf{v}}_0$ as it hits obliquely ball B, which is at rest. Denoting by r the radius of each ball and by μ_k the coefficient of kinetic friction between a ball and the table and assuming perfectly elastic impact, determine (*a*) the linear and angular velocity of each ball immediately after the impact, (*b*) the velocity of ball B after it has started rolling uniformly.

SOLUTION

Moment of inertia. $\quad \overline{I} = \dfrac{2}{5}mr^2$

(*a*) Impact analysis.

Ball A:



$$\textbf{Syst. Momenta}_1 \quad + \quad \textbf{Syst. Ext. Imp.}_{1\to 2} \quad = \quad \textbf{Syst. Momenta}_2$$

Kinematics of rolling: $\quad \omega_0 = \dfrac{v_0}{r}$

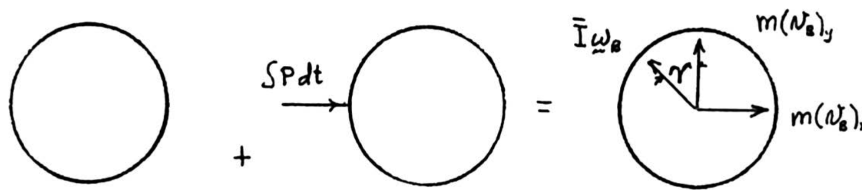
$\xrightarrow{+}$ Linear components: $\quad mv_0\cos\theta - \int P\,dt = m(v_A)_x \quad (1)$

$+\uparrow$ Linear components: $\quad mv_0\sin\theta + 0 = m(v_A)_y \quad (2)$

Moments about y axis: $\quad \overline{I}\,\omega_0\cos\theta + 0 = \overline{I}\,\omega_A\cos\beta \quad (3)$

Moments about x axis: $\quad -\overline{I}\,\omega_0\sin\theta + 0 = -\overline{I}\,\omega_A\sin\beta \quad (4)$

Ball B:



$$\textbf{Syst. Momenta}_1 \quad + \quad \textbf{Syst. Ext. Imp.}_{1\to 2} \quad = \quad \textbf{Syst. Momenta}_2$$

$\xrightarrow{+}$ Linear components: $\quad 0 + \int P\,dt = m(v_B)_x \quad (5)$

$+\uparrow$ Linear components: $\quad 0 + 0 = m(v_B)_y \quad (6)$

Moments about y axis: $\quad 0+0=\bar{I}\,\omega_B \cos\gamma \quad (7)$

Moments about x axis: $\quad 0+0=\bar{I}\,\omega_B \sin\gamma \quad (8)$

Adding Equations (1) and (5) to eliminate $\int P dt$,

$$mv_0 \cos\theta + 0 = m(v_A)_x + m(v_B)_x$$

or $\quad (v_B)_x + (v_A)_x = v_0 \cos\theta \quad (9)$

Condition of impact. $\quad e=1: \quad (v_B)_x - (v_A)_x = ev_0 \cos\theta = v_0 \cos\theta \quad (10)$

Solving Equations (9) and (10) simultaneously,

$$(v_A)_x = 0, \quad (v_B)_x = v_0 \cos\theta$$

From Equations (2) and (6), $\quad (v_A)_y = v_0 \sin\theta, \quad (v_B)_y = 0 \qquad v_A = (v_0 \sin\theta)\mathbf{j}$ ◀

$v_B = (v_0 \cos\theta)\mathbf{i}$ ◀

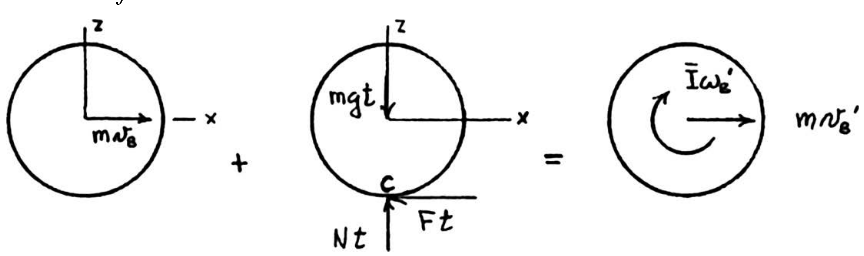
From Equations (3) and (4) simultaneously,

$$\beta = \theta, \quad \omega_A = \omega_0 = \frac{v_0}{r} \qquad \boldsymbol{\omega}_A = \frac{v_0}{r}(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j})$$ ◀

From Equations (7) and (8) simultaneously,

$$\omega_B = 0 \qquad \boldsymbol{\omega}_B = 0$$ ◀

(*b*) *Subsequent motion of ball B*.



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

Kinematics of rolling without slipping. $\quad v_B' = r\omega_B'$

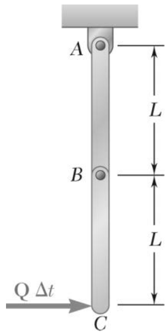
↻ Moments about C: $\quad mv_B r + 0 = \bar{I}\,\omega_B' + mv_B' r$

$$= \frac{2}{5}mr^2\omega_B' + m(r\omega_B')r$$

$$\omega_B' = \frac{5}{7}\frac{v_B'}{r} = \frac{5}{7}\frac{v_1 \cos\theta}{r}$$

$$v_B' = \frac{5}{7}v_1 \cos\theta \qquad \mathbf{v}_B' = \frac{5}{7}(v_0 \cos\theta)\mathbf{i}$$ ◀

PROBLEM 17.134

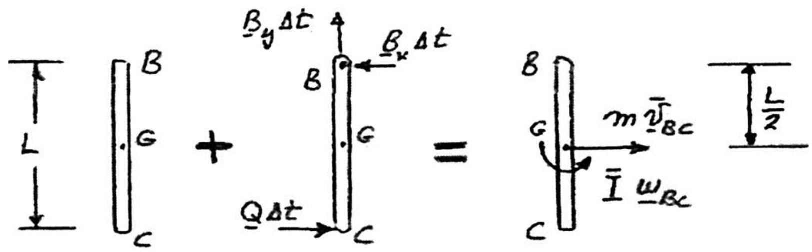


Each of the bars AB and BC is of length $L = 400$ mm and mass $m = 1.2$ kg. Determine the angular velocity of each bar immediately after the impulse $Q\Delta t = (1.5\ \text{N}\cdot\text{s})\mathbf{i}$ is applied at C.

SOLUTION

Principle of impulse and momentum.

Bar BC:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Kinematics $\qquad \bar{v}_{BC} = v_B + \frac{L}{2}\omega_{BC} = L\omega_{AB} + \frac{L}{2}\omega_{BC}$

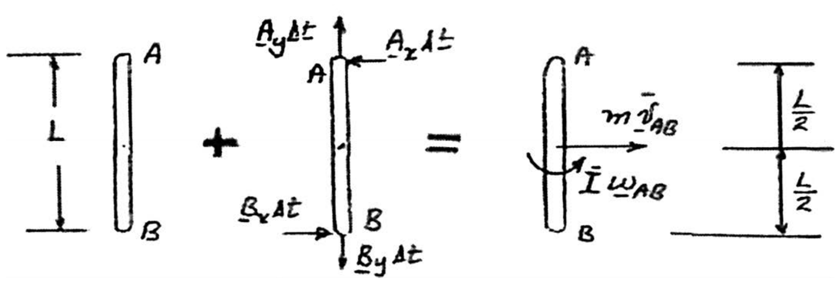
$+\circlearrowleft$ Moments about B: $\qquad 0 + (Q\Delta t)L = \bar{I}\,\omega_{BC} + m\bar{v}_{BC}\frac{L}{2}$

$$(Q\Delta t)L = \frac{1}{12}mL^2\omega_{BC} + m\left(L\omega_{AB} + \frac{L}{2}\omega_{BC}\right)\frac{L}{2}$$

$$Q\Delta t = \frac{1}{2}mL\omega_{AB} + \frac{1}{3}mL\omega_{BC} \tag{1}$$

$\xrightarrow{+}$ x components: $\qquad Q\Delta t - B_x\Delta t = m\left(L\omega_{AB} + \frac{L}{2}\omega_{BC}\right) \qquad (2)$

Bar AB:



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

+↻ Moments about A: $$0+(B_x\Delta t)L=\bar{I}\,\omega_{AB}+m\bar{v}_{AB}\frac{L}{2}$$

$$(B_x\Delta t)L=\frac{1}{12}mL^2\omega_{AB}+m\left(\frac{L}{2}\omega_{AB}\right)\frac{L}{2}$$

$$B_x\Delta t=\frac{1}{3}mL\omega_{AB} \qquad (3)$$

We now have 3 unknowns ($B_x\Delta t_1$, ω_{AB}, and ω_{BC}) and 3 equations.

Add Eqs. (2) and (3): $$Q\Delta t=\frac{4}{3}mL\omega_{AB}+\frac{1}{2}mL\omega_{BC} \qquad (4)$$

Subtract Eq. (1) from Eq. (4): $$0=\frac{5}{6}mL\omega_{AB}+\frac{1}{6}mL\omega_{BC}$$

$$\omega_{BC}=-5\omega_{AB} \qquad (5)$$

Substitute for ω_{BC} in Eq. (1): $$Q\Delta t=\frac{1}{2}mL\omega_{AB}+\frac{1}{3}mL(-5\omega_{AB})$$

$$=-\frac{7}{6}mL\omega_{AB}$$

$$\omega_{AB}=-\frac{6}{7}\frac{Q\Delta t}{mL} \qquad (6)$$

Substituting into Eq. (5): $$\omega_{BC}=-5\left(-\frac{6}{7}\frac{Q\Delta t}{mL}\right)$$

$$\omega_{BC}=\frac{30}{7}\frac{Q\Delta t}{mL} \qquad (7)$$

Given data: $$L=0.400\text{ m}$$

$$Q\Delta t=1.5\text{ N}\cdot\text{s}$$

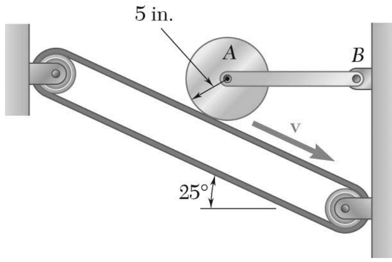
$$m=1.2\text{ kg}$$

<u>Angular velocity of bar AB</u>. $$\omega_{AB}=-\frac{6}{7}\frac{Q\Delta t}{mL}=-\frac{(6)(1.5)}{(7)(1.2)(0.4)}$$

$\boldsymbol{\omega}_{AB}=2.68$ rad/s ↺ ◀

<u>Angular velocity of bar BC</u>. $$\omega_{BC}=\frac{30}{7}\frac{Q\Delta t}{mL}=\frac{(30)(1.5)}{(7)(1.2)(0.4)}$$

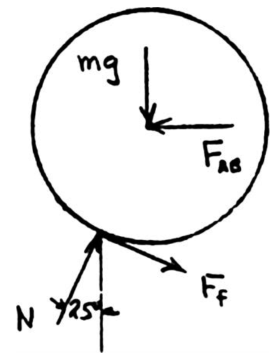
$\boldsymbol{\omega}_{BC}=13.39$ rad/s ↻ ◀



PROBLEM 17.135

A uniform disk of constant thickness and initially at rest is placed in contact with the belt shown, which moves at a constant speed $v = 80$ ft/s. Knowing that the coefficient of kinetic friction between the disk and the belt is 0.15, determine (*a*) the number of revolutions executed by the disk before it reaches a constant angular velocity, (*b*) the time required for the disk to reach that constant angular velocity.

SOLUTION



Kinetic friction. $F_f = \mu_k N = 0.15\text{ N}$

$$+\uparrow \Sigma F_y = N\cos 25° - F_f \sin 25° - mg = 0$$

$$(\cos 25° - \mu_k \sin 25°)N = mg$$

$$N = \frac{mg}{\cos 25° - 0.15\sin 25°}$$

$$= 1.18636\, mg$$

$$F_f = (0.15)(1.18636)mg$$

$$= 0.177954\, mg$$

Final angular velocity. $\omega_2 = \dfrac{v}{r}$

Moment of inertia. $\bar{I} = \dfrac{1}{2}mr^2$

(*a*) *Principle of work and energy.*

$$T_1 + W_{1\to 2} = T_2: \quad T_1 = 0$$

$$W_{1\to 2} = F_f r\theta = 0.177954 mgr\theta$$

$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{4}mv^2$$

$$0 + 0.177954 mgr\theta = \frac{1}{4}mv^2$$

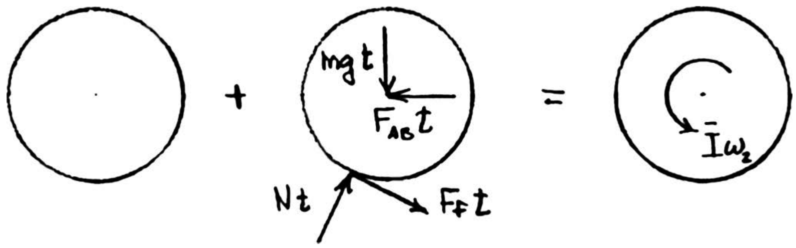
$$\theta = 1.40486\frac{v^2}{gr}$$

$$= \frac{(1.40486)(80)^2}{(32.2)\left(\frac{5}{12}\right)}$$

$$= 670.14 \text{ radians}$$

$\theta = 106.7$ rev ◀

(*b*) *Principle of impulse-momentum.*



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

↺ Moments about *A*: $0 + F_f tr = \bar{I}\omega_2$

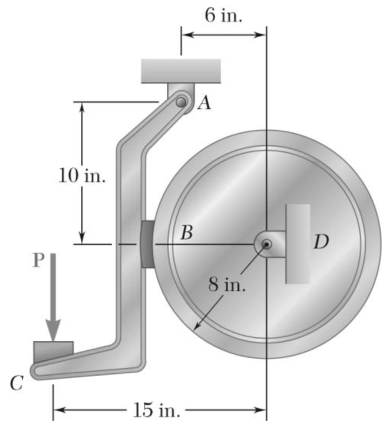
$$t = \frac{\bar{I}\omega_2}{F_f r}$$

$$= \frac{\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)}{0.177954mgr}$$

$$= 2.8097\frac{v}{g}$$

$$= \frac{(2.8097)(80)}{32.2}$$

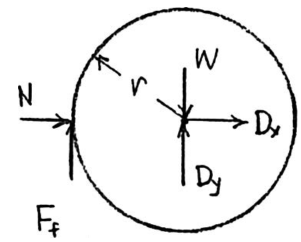
$t = 6.98$ s ◀



PROBLEM 17.136

The 8-in.-radius brake drum is attached to a larger flywheel that is not shown. The total mass moment of inertia of the flywheel and drum is $14\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ and the coefficient of kinetic friction between the drum and the brake shoe is 0.35. Knowing that the initial angular velocity of the flywheel is 360 rpm counterclockwise, determine the vertical force **P** that must be applied to the pedal C if the system is to stop in 100 revolutions.

SOLUTION



Kinetic energy.

$$\omega_1 = 360\ \text{rpm} = 12\pi\ \text{rad/s}$$

$$\omega_2 = 0$$

$$T_1 = \frac{1}{2} I \omega_1^2$$

$$= \frac{1}{2}(14)(12\pi)^2$$

$$= 9.9486 \times 10^3\ \text{ft}\cdot\text{lb}$$

$$T_2 = 0$$

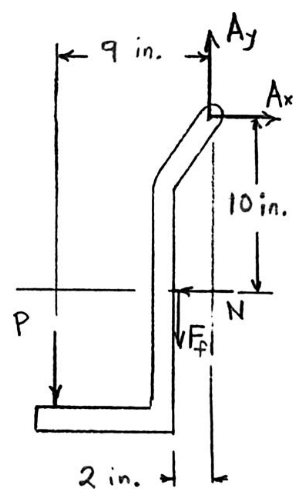
Work.

$$\theta = (100)(2\pi) = 628.32\ \text{rad}$$

$$M_D = F_f r = F_f\left(\frac{8}{12}\right)$$

$$U_{1\to 2} = -M_D\theta = -F_f\left(\frac{8}{12}\right)(628.32)$$

$$= -418.88 F_f$$



Principle of work and energy.

$$T_1 + U_{1\to 2} = T_2: \quad 9.9486 \times 10^3 - 418.88 F_f = 0$$

$$F_f = 23.75\ \text{lb}$$

Kinetic friction force.

$$F_f = \mu_k N$$

$$N = \frac{F_f}{\mu_k} = \frac{23.75}{0.35} = 67.859\ \text{lb}$$

Statics.

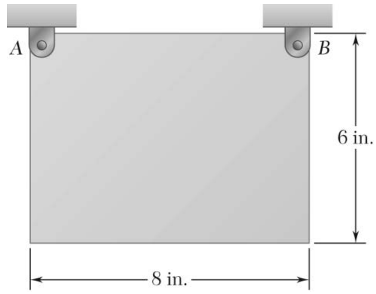
$$+\circlearrowleft \Sigma M_A = 0: \quad (9\ \text{in.})P + (2\ \text{in.})F_f - (10\ \text{in.})N = 0$$

$$9P + (2)(23.75) - (10)(67.859) = 0$$

$$P = 70.12$$

$\mathbf{P} = 70.1\ \text{lb}\downarrow$ ◀

PROBLEM 17.137



A $6 \times 8-$in. rectangular plate is suspended by pins at *A* and *B*. The pin at *B* is removed and the plate swings freely about pin *A*. Determine (*a*) the angular velocity of the plate after it has rotated through 90°, (*b*) the maximum angular velocity attained by the plate as it swings freely.

SOLUTION

Let *m* be the mass of the plate.

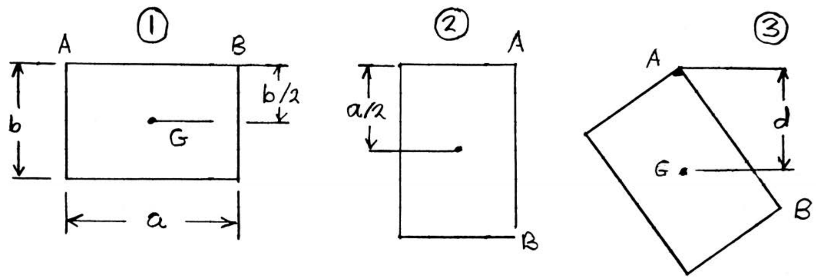
Dimensions: $a = 8 \text{ in.} = 0.66667 \text{ ft} \quad b = 6 \text{ in.} = 0.5 \text{ ft}$

Moment of inertia about *A* $\quad I_A = \frac{1}{3}m(a^2 + b^2)$

Position 1. Initial position. $\quad \omega_1 = 0$

Position 2. Plate has rotated about *A* through 90°.

Position 3. Mass center is directly below pivot *A*.



Potential energy. Use level *A* as datum.

$$V_1 = -\frac{mab}{2} \quad V_2 = -\frac{mga}{2} \quad V_3 = -mgd$$

Where $\quad d = \frac{1}{2}\sqrt{a^2 + b^2} = 0.41667 \text{ ft}$

Kinetic energy. $\quad T_1 = 0 \quad T_2 = \frac{1}{2}I_A\omega_2^2 \quad T_3 = \frac{1}{2}I_A\omega_3^2$

(*a*) 90° rotation. Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 - \frac{mgb}{2} = \frac{1}{2} \cdot \frac{1}{3}m(a^2 + b^2)\omega_2^2 + \frac{mga}{2}$$

$$\omega_2^2 = \frac{3g(a-b)}{a^2 + b^2} = \frac{(3)(32.2)(0.66667 - 0.5)}{(0.66667)^2 + (0.5)^2} = 23.184 (\text{rad/s})^2$$

$\boldsymbol{\omega}_2 = 4.81 \text{ rad/s}$ ↻ ◀

PROBLEM 17.137 (Continued)

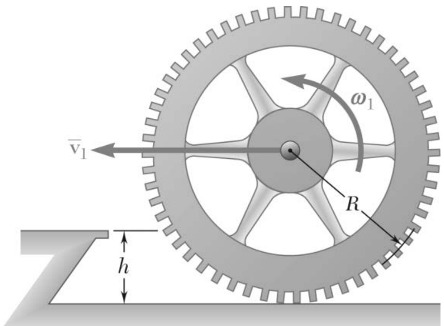
(*b*) $\boldsymbol{\omega}$ is maximum. Conservation of energy.

$$T_1 + V_1 = T_3 + V_3: \quad 0 - \frac{mgb}{2} = \frac{1}{2} \cdot \frac{1}{3} m(a^2 + b^2)\omega_3^2 - mgd$$

$$\omega_3^2 = \frac{g(6d - 3b)}{a^2 + b^2} = \frac{(32.2)(2.5 - 1.5)}{(0.66667)^2 + (0.5)^2} = 46.386 \text{ (rad/s)}^2$$

$\boldsymbol{\omega}_3 = 6.81$ rad/s ↻ ◀

PROBLEM 17.138



The gear shown has a radius R = 150 mm and a radius of gyration $\bar{k} = 125$ mm. The gear is rolling without sliding with a velocity $\bar{\mathbf{v}}_1$ of magnitude 3 m/s when it strikes a step of height h = 75 mm. Because the edge of the step engages the gear teeth, no slipping occurs between the gear and the step. Assuming perfectly plastic impact, determine (a) the angular velocity of the gear immediately after the impact, (*b*) the angular velocity of the gear after it has rotated to the top of the step.

SOLUTION

Part (*a*) Conditions just after impact.

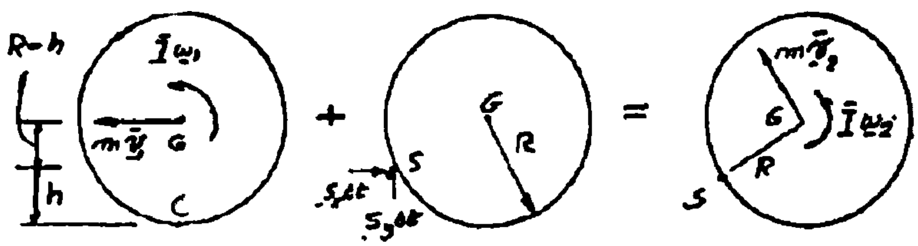
Kinematics. Just before impact, the contact Point *C* with the floor the instantaneous center of rotation of the gear.

$$\bar{\mathbf{v}}_1 = R\omega_1 \longleftarrow$$

Just after impact, Point *S* is the instantaneous center of rotation.

$$\mathbf{v}_2 = R\omega_2 \searrow \theta \quad \text{(perpendicular to } GS\text{)}$$

Principle of impulse and momentum.



+↺ Moments about *S*: $m\bar{v}_2(R-h) + \bar{I}\omega_1 = m\bar{v}_2 R + \bar{I}\omega_2$

$$m(R\omega_1)(R-h) + m\bar{k}^2\omega_1 = m(R\omega_2)R + m\bar{k}^2\omega_2$$

$$[R(R-h) + \bar{k}^2]\omega_1 = (R^2 + \bar{k}^2)\omega_2$$

$$\omega_2 = \frac{R^2 + \bar{k}^2 - Rh}{R^2 + k^2}\omega_1 \qquad \omega_2 = \left[1 - \frac{Rh}{R^2 + k^2}\right]\omega_1 \qquad (1)$$

Data: $R = 150$ mm, $\bar{k} = 125$ mm, $v_1 = 3$ m/s, $h = 75$ mm

$$\omega_1 = \frac{v_1}{R} = \frac{3 \text{ m/s}}{0.150 \text{ m}} = 20 \text{ rad/s}$$

Angular velocity.

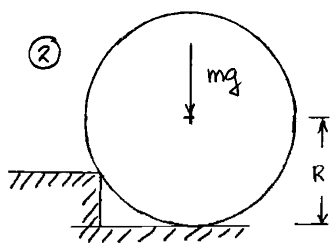
From (1), $$\omega_2 = \left[1 - \frac{(150)(75)}{(150^2 + 125^2)}\right](20 \text{ rad/s}) = 0.7049(20) \qquad \boldsymbol{\omega}_2 = 14.10 \text{ rad/s} \circlearrowleft \blacktriangleleft$$

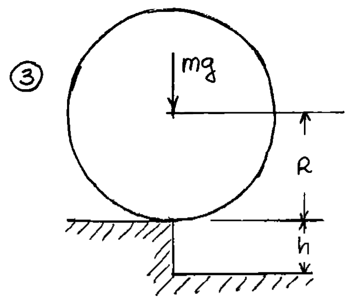
Part (*b*) Conditions at the top of the step.

The gear pivots about the edge of the step. Use the principle of conservation of energy.

Position (2): The gear has just broken contact with the floor.

Position (3): The center of the gear is above the edge of the step.





Kinematics: (Rotation about S) $\bar{v} = \omega R$

Kinetic energy:

$$T = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m\bar{v}^2$$

$$= \frac{1}{2}m\bar{k}^2\omega^2 + \frac{1}{2}mR^2\omega^2$$

$$= \frac{1}{2}m(\bar{k}^2 + R^2)\omega^2$$

Position (2):

$$T_2 = \frac{1}{2}m(\bar{k}^2 + R^2)\omega_2^2$$

$$V_2 = mgR$$

Position (3):

$$T_3 = \frac{1}{2}m(\bar{k}^2 + R^2)\omega_3^2$$

$$V_3 = mg(R + h)$$

Principle of conservation of energy: $T_2 + V_2 = T_3 + V_3$

$$\frac{1}{2}m(\bar{k}^2 + R^2)\omega_2^2 + mgR = \frac{1}{2}m(\bar{k}^2 + R^2)\omega_3^2 + mg(R + h)$$

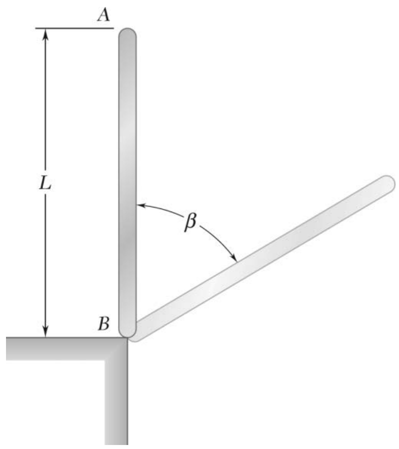
Angular velocity:

$$\omega_3^2 = \omega_3^2 - \frac{2gh}{\bar{k}^2 + R^2}$$

$$= (14.10 \text{ rad/s})^2 - \frac{(2)(9.81 \text{ m/s}^2)(0.075 \text{ m})}{(0.125 \text{ m})^2 + (0.150 \text{ m})^2}$$

$$= 160.21 \text{ rad}^2/\text{s}^2$$

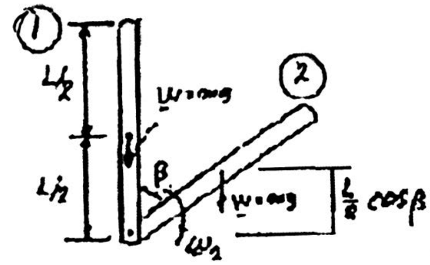
$\omega_3 = 12.66$ rad/s ↻ ◀



PROBLEM 17.139

A uniform slender rod is placed at corner B and is given a slight clockwise motion. Assuming that the corner is sharp and becomes slightly embedded in the end of the rod, so that the coefficient of static friction at B is very large, determine (a) the angle β through which the rod will have rotated when it loses contact with the corner, (b) the corresponding velocity of end A.

SOLUTION



Position 1

$$T_1 = 0$$

$$V_1 = mgh_1 = \frac{mgL}{2}$$

Position 2

$$V_2 = mgh_2 = \frac{mgL\cos\beta}{2}$$

$$T_2 = \frac{1}{2}I\omega_2^2 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_2^2$$

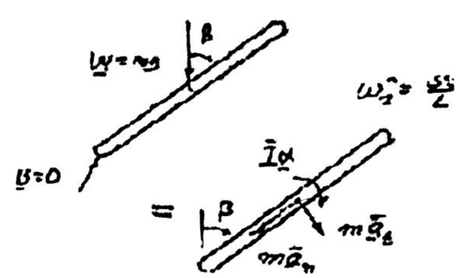
Principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{mgL}{2} = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_2^2 + \frac{mgL\cos\beta}{2}$$

$$\omega_2^2 = \frac{3g}{L}(1-\cos\beta) \qquad (1)$$

Normal acceleration of mass center.



$$a_n = \frac{L}{2}\omega_2^2 = \frac{3}{2}g(1-\cos\beta)$$

$$+\swarrow \Sigma F = +\Sigma F_{\text{eff}} = ma_n$$

$$mg\cos\beta = \frac{3}{2}mg(1-\cos\beta)$$

(a) Angle β.

$$\frac{5}{2}\cos\beta = \frac{3}{2} \qquad \cos\beta = 0.6$$

$\beta = 53.1°$ ◀

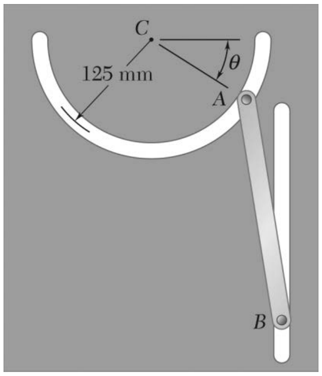
From (1)

$$\omega_2^2 = \frac{3g}{L}(1-0.6) = 1.2\frac{g}{L} \qquad \omega_2 = 1.09545\sqrt{\frac{g}{L}}$$

(b) Velocity of end A

$$v_A = L\omega_2$$

$\mathbf{v}_A = 1.095\sqrt{gL}$ $\searrow$ 53.1° ◀



PROBLEM 17.140

The motion of the slender 250-mm rod AB is guided by pins at A and B that slide freely in slots cut in a vertical plate as shown. Knowing that the rod has a mass of 2 kg and is released from rest when $\theta = 0$, determine the reactions at A and B when $\theta = 90°$.

SOLUTION

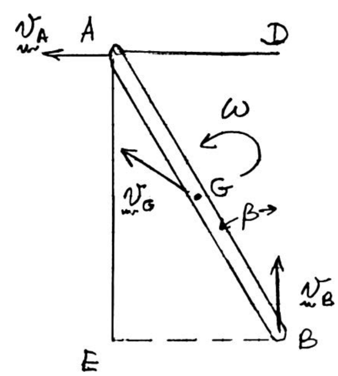
Let Point G be the mass center of rod AB.

$$m = 2 \text{ kg}$$

$$L = 0.25 \text{ m}$$

$$I_G = \frac{1}{12}mL^2 = 0.0104667 \text{ kg}\cdot\text{m}^2$$

Kinematics. $\theta = 90°$



$$\overline{AD} = R = 0.125 \text{ m}$$

$$AB = L = 0.25 \text{ m}$$

$$\sin\beta = \frac{R}{L} = \frac{1}{2} \qquad \beta = 30°$$

$$\overline{AG} = \frac{L}{2} = 0.125 \text{ m}$$

$$\overline{BG} = 0.125 \text{ m}$$

Point E is the instantaneous center of rotation of bar AB.

$$v_G = \frac{L}{2}\omega = 0.125\ \omega$$

$$v_A = (L\cos 30°)\omega = 0.21651\omega$$

$$v_B = (L\sin 30°)\omega = 0.125\omega$$

Use principle of conservation of energy to obtain the velocities when $\theta = 90°$:

Use level A as the datum for potential energy.

Position 1 $\theta = 0$

$$T_1 = 0$$

$$V_1 = -mg\frac{L}{2} = -(2)(9.81)(0.125) = -2.4525 \text{ J}$$

Position 2 $\theta = 90°$

$$T_2 = \frac{1}{2}I_G\omega^2 + \frac{1}{2}mv_G^2$$
$$= \frac{1}{2}(0.0104667)\omega^2 + \frac{1}{2}(2)(0.125\omega)^2$$
$$= 0.0208583\omega^2$$
$$V_2 = -mg\left(R + \frac{L}{2}\cos\beta\right)$$
$$= -(2)(9.81)(0.125 + 0.125\cos 30°)$$
$$= -4.5764 \text{ J}$$

$T_1 + V_1 = T_2 + V_2$: $0 - 2.4525 = 0.0208583\omega^2 - 4.5764$

$$\omega^2 = 101.826 \text{ rad}^2/\text{s}^2$$
$$\omega = 10.091 \text{ rad/s}$$
$$v_A = (0.21651)(10.091)$$
$$= 2.1848 \text{ m/s}$$
$$v_G = (0.125)(10.091)$$
$$= 1.2614 \text{ m/s}$$

More kinematics: For Point A moving in the curved slot,

$$\mathbf{a}_A = (a_C)_x\mathbf{i} + \frac{v_A^2}{R}\mathbf{j}$$
$$= (a_C)_x\mathbf{i} + \frac{(2.1847)^2}{0.125}\mathbf{j}$$
$$= (a_C)_x\mathbf{i} + 38.1833\mathbf{j}$$

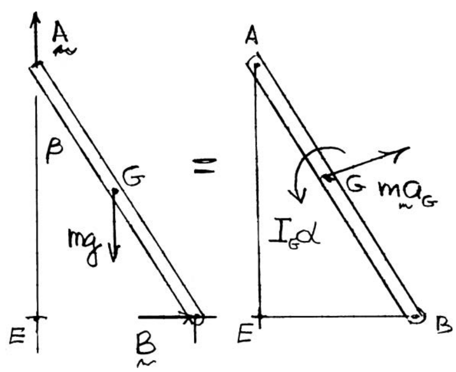
For the rod AB, $\boldsymbol{\alpha} = \alpha\mathbf{k}, \quad \mathbf{v}_B = v_B\mathbf{j}$

$$\mathbf{r}_{A/B} = -L\sin 30°\mathbf{i} + L\cos 30°\mathbf{j}$$
$$= -0.125\mathbf{i} + 0.21651\mathbf{j}$$
$$\mathbf{r}_{G/B} = \frac{1}{2}\mathbf{r}_{A/B}$$
$$= -0.0625\mathbf{i} + 0.108253\mathbf{j}$$
$$\mathbf{a}_A = \mathbf{a}_B + \boldsymbol{\alpha} + \mathbf{r}_{A/B} - \omega^2\mathbf{r}_{A/B}$$
$$= a_B\mathbf{j} + \alpha\mathbf{k} \times (-0.125\mathbf{i} + 0.21651\mathbf{j})$$
$$-(10.091)^2(-0.125\mathbf{i} + 0.21651\mathbf{j})$$
$$= a_B\mathbf{j} - 0.125\alpha\mathbf{j} - 0.21651\alpha\mathbf{i} + 12.7285\mathbf{i} - 22.0468\mathbf{j}$$

Matching vertical components of $\mathbf{a}_A$

$$38.1833 = a_B - 0.125\alpha - 22.0468$$
$$a_B = 0.125\alpha + 60.2301$$
$$\mathbf{a}_G = \mathbf{a}_B + \mathbf{a}_{G/B}$$
$$= \mathbf{a}_B + \alpha\mathbf{k} \times \mathbf{r}_{G/B} - \omega^2 r_{G/B}$$
$$= (0.125\alpha + 60.2324)\mathbf{j} + \alpha\mathbf{k} \times (-0.0625\mathbf{i} + 0.108253\mathbf{j})$$
$$-(10.091)^2(-0.0625\mathbf{i} + 0.108253\mathbf{j})$$
$$= 0.125\alpha\mathbf{j} + 60.2301\mathbf{j} - 0.0625\alpha\mathbf{j} - 0.108253\alpha\mathbf{i}$$
$$+6.3643\mathbf{i} - 11.0232\mathbf{j}$$
$$\mathbf{a}_G = (-0.108253\alpha + 6.3643)\mathbf{i} + (0.0625\alpha + 49.2069)\mathbf{j}$$

Kinetics: Use rod AB as a free body.



$$+\circlearrowleft \Sigma\mathbf{M}_E = \Sigma(\mathbf{M}_E)_{\text{eff}}:$$
$$-mg\frac{L}{2}\sin\beta\mathbf{k} = I_G\boldsymbol{\alpha} + \mathbf{r}_{G/E} \times (m\mathbf{a}_G)$$
$$-(2)(9.81)(0.125)\sin 30°\mathbf{k}$$
$$= 0.0104667\boldsymbol{\alpha} + (0.0625\mathbf{i} + 0.10825\mathbf{j}) \times (m\mathbf{a}_G)$$
$$-1.22625 = 0.0104667\alpha + 0.03125\alpha + 4.7730$$
$$0.0417167\alpha = -5.99925$$
$$\alpha = -143.808 \text{ rad/s}^2$$
$$\mathbf{a}_G = (-21.933 \text{ m/s}^2)\mathbf{i} + (40.2189 \text{ m/s}^2)\mathbf{j}$$

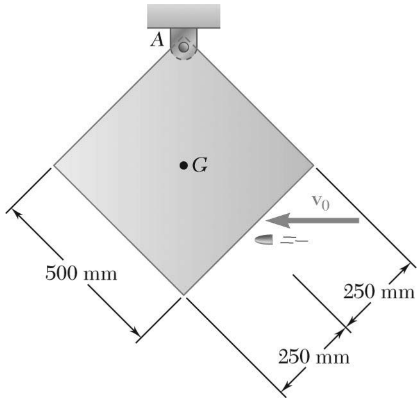
$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}} = m(a_G)_x$: $-B = (2)(-21.932) = 43.864$ N **B** $= 43.9$ N $\longrightarrow$ ◀

$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}} = m(a_G)_y$: $A - mg = (2)(40.4289) = 80.4378$

$$A = (2)(9.81) + 80.4378 = 100.058$$ **A** $= 100.1$ N $\uparrow$ ◀

Check by considering

$$+\circlearrowleft \Sigma M_G = \Sigma M_{G\,\text{eff}}: \quad \circlearrowleft \Sigma M_G = (0.0625)A - 0.108253B = 1.5052 \text{ N}\cdot\text{m} \circlearrowright$$
$$\circlearrowleft \Sigma(M_G)_{\text{eff}} = I_G(-\alpha) = (0.0104667)(143.808) = 1.5052 \text{ N}\cdot\text{m} \circlearrowright$$



PROBLEM 17.141

A 35-g bullet B is fired horizontally with a velocity of 400 m/s into the side of a 3-kg square panel suspended from a pin at A. Knowing that the panel is initially at rest, determine the components of the reaction at A after the panel has rotated 45°.

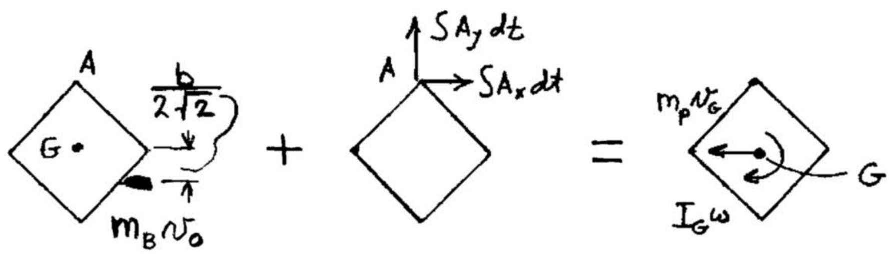
SOLUTION

Masses and moment of inertia.

$$m_B = 0.035 \text{ kg}$$
$$m_P = 3 \text{ kg}$$
$$b = 500 \text{ mm} = 0.5 \text{ m}$$
$$I_G = \frac{1}{6} m_P b^2 = \frac{1}{6}(3)(0.5)^2 = 0.125 \text{ kg} \cdot \text{m}^2$$

Note: The mass of the bullet is neglected in comparison with that of the plate after impact.

Analysis of impact: Use principle of impulse and momentum.



Kinematics: After impact the plate rotates about the pin at A.

$$v_G = \frac{b}{\sqrt{2}}\omega = \frac{0.5}{\sqrt{2}}\omega$$

+↻ Moments about A:

$$m_B v_0 \left(\frac{b}{\sqrt{2}} + \frac{b}{2\sqrt{2}} \right) = I_G \omega + m_p v_G \frac{b}{\sqrt{2}}$$

$$\frac{3}{2\sqrt{2}} m_G v_0 b = (I_G + \frac{1}{2} m_P b^2)\omega$$

$$\frac{3}{2\sqrt{2}}(0.035)(400)(0.5) = \left[0.125 + \frac{1}{2}(3)(0.5)^2 \right] \omega$$

$$\omega = 14.8492 \text{ rad/s}$$

$$v_G = \frac{0.5}{\sqrt{2}}(14.8492) = 5.25 \text{ m/s}$$

PROBLEM 17.141 (Continued)

Corresponding kinetic energy. $T_1 = \frac{1}{2}I_G\omega^2 + \frac{1}{2}m_P v_G^2$

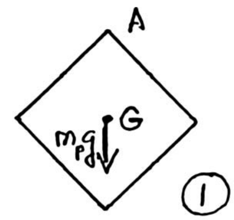
$$T_1 = \frac{1}{2}(0.125)(14.8492)^2 + \frac{1}{2}(3)(5.25)^2$$
$$= 55.125 \text{ J}$$

Plate rotates through $45°$.

Position 1: $\theta = 0°$

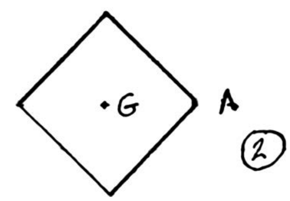
Use Point *A* as the datum for potential energy.

$$V_1 = -m_P g \frac{b}{\sqrt{2}}$$
$$= -(3)(9.81)\frac{0.5}{\sqrt{2}}$$
$$= -10.4051 \text{ J}$$



Position 2: $\theta = 90°$

$V_2 = 0$ since G is at level A.



$$T_2 = \frac{1}{2}I_G\omega_2^2 + \frac{1}{2}m_P(v_G)_2^2$$
$$= \frac{1}{2}(0.125)\omega_2^2 + \frac{1}{2}(3)\left(\frac{0.5}{\sqrt{2}}\omega_2\right)^2$$
$$= 0.25\omega_2^2$$

Principle of conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$
$$55.125 \text{ J} - 10.4051 \text{ J} = 0.25\omega_2^2 + 0$$
$$\omega_2^2 = 178.879 \text{ (rad/s}^2\text{)}$$
$$\omega_2 = 13.3746 \text{ rad/s}$$

Analysis at $90°$ rotation. $\boldsymbol{\alpha} = \alpha$ ↻

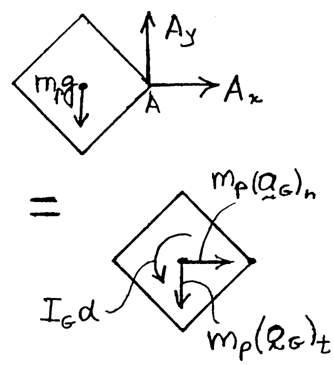
Kinematics: $(a_G)_t = \frac{b}{\sqrt{2}}\alpha = \frac{0.5}{\sqrt{2}}\alpha$ $\qquad (\mathbf{a}_G)_t = 0.35355\alpha \downarrow$

$$(a_G)_n = \frac{b}{\sqrt{2}}\omega^2$$
$$= \frac{(0.5)(178.879)}{\sqrt{2}}$$

$(\mathbf{a}_G)_n = 63.2434 \text{ m/s}^2 \longrightarrow$

PROBLEM 17.141 (Continued)

Kinematics: Use the free body diagram of the plate.



$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad m_P g \frac{b}{\sqrt{2}} = I_G\alpha + m_P(a_G)_t \frac{b}{\sqrt{2}}$$

$$= \left(I_G + \frac{1}{2}m_P b^2\right)\alpha$$

$$\frac{(3)(9.81)(0.5)}{\sqrt{2}} = \left[0.125 + \frac{1}{2}(3)(0.5)^2\right]\alpha$$

$$\boldsymbol{\alpha} = 20.810 \text{ rad/s}^2 \circlearrowleft$$

$$(\mathbf{a}_G)_t = 7.3574 \text{ m/s}^2 \downarrow$$

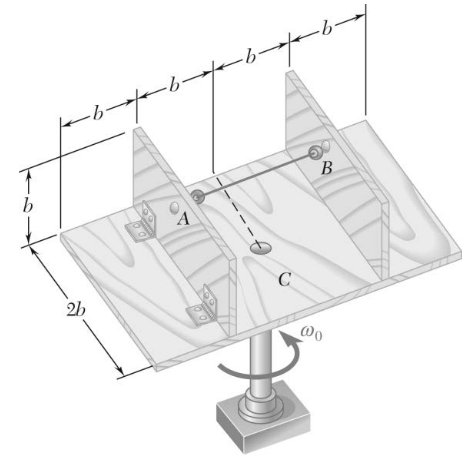
$$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{\text{eff}}: \quad A_x = m_P(a_G)_n = (3)(63.2434)$$

$\mathbf{A}_x = 189.7 \text{ N} \longrightarrow$ ◀

$$+\uparrow \Sigma F_y = \Sigma(F_y)_{\text{eff}}: \quad A_y - m_P g = -m_P(a_G)_y$$

$$A_y - (3)(9.81) = (3)(-7.3574)$$

$\mathbf{A}_y = 7.36 \text{ N} \uparrow$ ◀

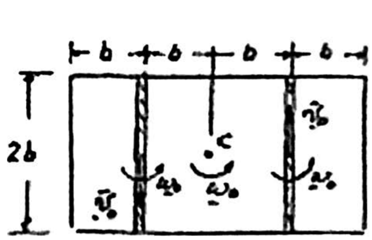


PROBLEM 17.142

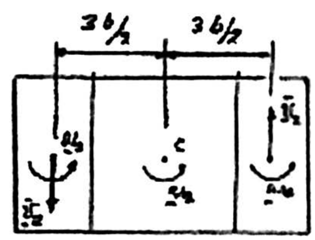
Two panels A and B are attached with hinges to a rectangular plate and held by a wire as shown. The plate and the panels are made of the same material and have the same thickness. The entire assembly is rotating with an angular velocity ω_0 when the wire breaks. Determine the angular velocity of the assembly after the panels have come to rest against the plate.

SOLUTION

Geometry and kinematics:



Panels in up position



Panels in down position

$$\bar{v}_0 = b\omega_0 \qquad \bar{v}_2 = \frac{3}{2}b\omega_0$$

Let ρ = mass density, t = thickness

Plate:

$$m_{\text{plate}} = \rho t(2b)(4b) = 8\rho t b^2$$

$$\bar{I}_{\text{plate}} = \frac{1}{12}(8\rho t b^2)[(2b)^2 + (4b)^2]$$

$$= \frac{160}{12}\rho t b^4$$

$$= \frac{40}{3}\rho t b^4$$

Each panel:

$$m_{\text{panel}} = \rho t(b)(2b) = 2\rho t b^2$$

Panel in up position

$$(\bar{I}_{\text{panel}})_0 = \frac{1}{12}(2\rho t b^2)(2b)^2$$

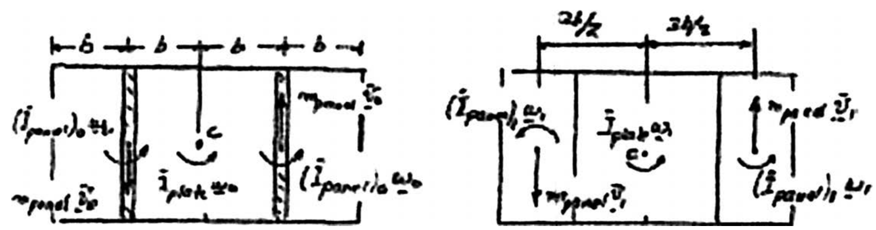
$$= \frac{8}{12}\rho t b^4 = \frac{2}{3}\rho t b^4$$

PROBLEM 17.142 (Continued)

Panel in down position

$$(\bar{I}_{\text{panel}})_1 = \frac{1}{12}(2\rho t b^2)[b^2 + (2b)^2]$$
$$= \frac{10}{12}\rho t b^4$$
$$= \frac{5}{6}\rho t b^4$$

Conservation of angular momentum about the vertical spindle.



Initial momenta **Final momenta**

+↻ Moments about C:

$$\bar{I}_{\text{plate}}\omega_0 + 2[(\bar{I}_{\text{panel}})_0\omega_0 + m_{\text{panel}}v_0(b)] = \bar{I}_{\text{plate}}\omega_1 + 2\left[(\bar{I}_{\text{panel}})_1\omega_1 + m_{\text{panel}}v_1\left(\frac{3b}{2}\right)\right]$$

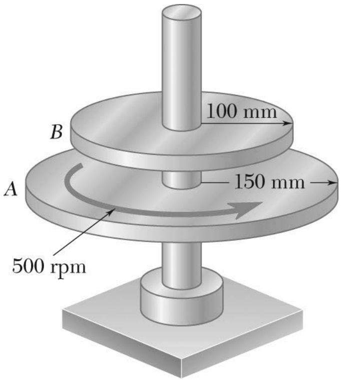
$$\frac{40}{3}\rho t b^4\omega_0 + 2\left[\frac{2}{3}\rho t b^4\omega_0 + (2\rho t b^2)(b\omega_0)b\right] = \frac{40}{3}\rho t b^4\omega_0 + 2\left[\frac{5}{6}\rho t b^4\omega_1 + 2\rho t b^2\left(\frac{3}{2}b\omega_0\right)\left(\frac{3}{2}b\right)\right]$$

$$\left[\frac{40}{3} + \frac{4}{3} + 4\right]\rho t b^4\omega_0 = \left[\frac{40}{3} + \frac{10}{6} + 9\right]\rho t b^4\omega_1$$

$$\frac{56}{3}\omega_0 = 24\omega_1$$

$$\omega_1 = \frac{56}{(3)(24)}$$

$$\omega_1 = \frac{7}{9}\omega_2 \blacktriangleleft$$



PROBLEM 17.143

Disks A and B are made of the same material and are of the same thickness; they can rotate freely about the vertical shaft. Disk B is at rest when it is dropped onto disk A, which is rotating with an angular velocity of 500 rpm. Knowing that disk A has a mass of 8 kg, determine (a) the final angular velocity of the disks, (b) the change is kinetic energy of the system.

SOLUTION

Disk A: $\quad m_A = 8 \text{ kg} \quad r_A = 0.15 \text{ m}$

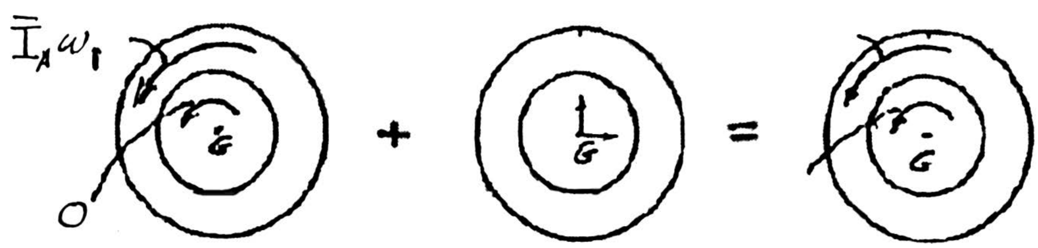
$$\bar{I}_A = \frac{1}{2} m_A r_A^2 = \frac{1}{2}(8)(0.15)^2 = 0.0900 \text{ kg} \cdot \text{m}^2$$

Disk B: $\quad r_B = 0.100 \text{ m}$

$$m_B = m_A \left(\frac{r_B}{r_A}\right)^2 = (8)\left(\frac{0.10}{0.15}\right)^2 = 3.5556 \text{ kg}$$

$$\bar{I}_B = \frac{1}{2} m_B r_B^2 = \frac{1}{2}(3.5556)(0.10)^2 = 0.017778 \text{ kg} \cdot \text{m}^2$$

Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta**$_2$

+↻ Moments about B: $\quad \bar{I}_A \omega_0 + 0 + 0 = \bar{I}_A \omega_2 + \bar{I}_B \omega_2$

$$\omega_2 = \frac{\bar{I}_A}{\bar{I}_A + \bar{I}_B}\omega_1 = \frac{0.09}{0.10778}\omega_1 = 0.83505\omega_1$$

Initial angular velocity of disk A: $\quad \omega_1 = 500 \text{ rpm} = 52.36 \text{ rad/s}$

PROBLEM 17.143 (Continued)

(*a*) Final angular velocity of system: $\omega_2 = (0.83505)(52.36)$

$\omega_2 = 43.723 \text{ rad/s}$ $\omega_2 = 418 \text{ rpm}$ ◀

Initial kinetic energy: $T_1 = \frac{1}{2}\bar{I}_A\omega_1^2$

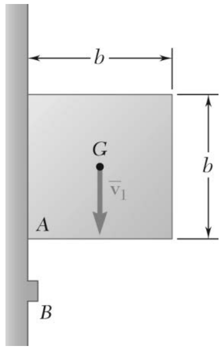
$$T_1 = \frac{1}{2}(0.09)(52.36)^2 = 123.37 \text{ J}$$

Final kinetic energy: $T_2 = \frac{1}{2}(\bar{I}_A + \bar{I}_B)\omega_2^2$

$$T_2 = \frac{1}{2}(0.10778)(43.723)^2 = 103.02 \text{ J}$$

(*b*) Change in energy: $T_2 - T_1 = -20.35 \text{ J}$ $\Delta T = -20.4 \text{ J}$ ◀

PROBLEM 17.144



A square block of mass m is falling with a velocity $\overline{\mathbf{v}}_1$ when it strikes a small obstruction at B. Knowing that the coefficient of restitution for the impact between corner A and the obstruction B is $e = 0.5$, determine immediately after the impact (a) the angular velocity of the block, (b) the velocity of its mass center G.

SOLUTION

Moments of inertia. $\quad \overline{I} = \frac{1}{6}mb^2$

Kinematics. Before impact, block is translating.

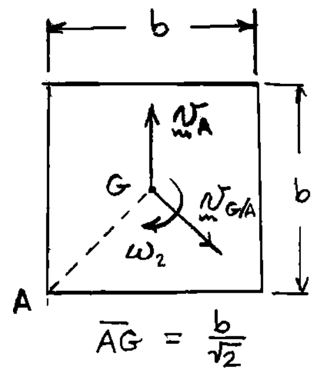
$$\mathbf{v}_1 = v_1 \downarrow \quad \omega_1 = 0$$

After impact,

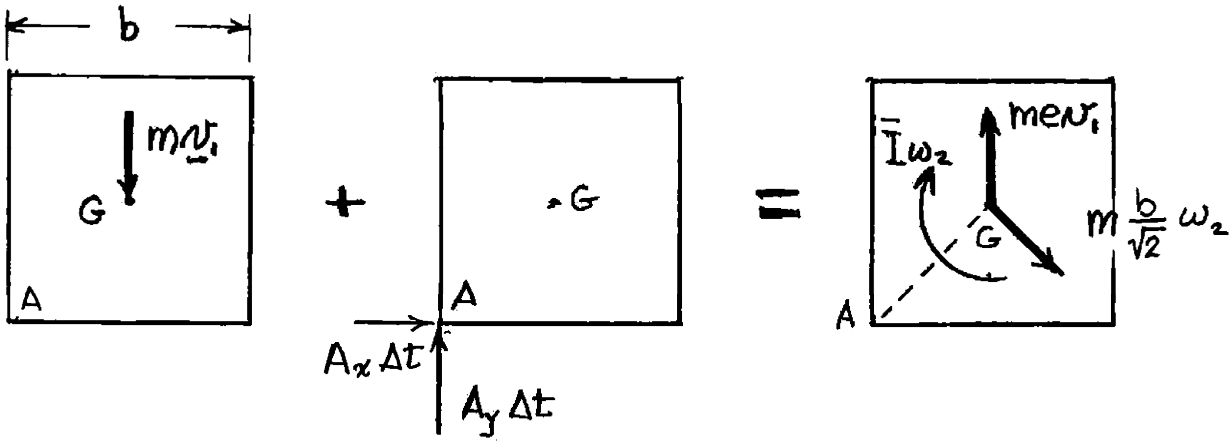
$$\mathbf{v}_A = ev_1 \uparrow$$

$$\overline{\mathbf{v}}_2 = \mathbf{v}_A + \mathbf{v}_{G/A}$$

$$= [ev_1 \uparrow] + \left[\frac{b}{\sqrt{2}}\omega_2 \searrow 45°\right] \quad (1)$$



Principle of impulse and momentum.



Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to2}$ = **Syst. Momenta**$_2$

+↻ Moments about A:

$$m\bar{v}_1\frac{b}{2} = \bar{I}\omega_1 - me\bar{v}_1\frac{b}{2} + m\frac{b}{\sqrt{2}}\omega_2\frac{b}{\sqrt{2}}$$

$$\bar{I} = \frac{1}{6}mb^2$$

$$m\bar{v}_1\frac{b}{2} = \frac{1}{6}mb^2\omega_2 - me\bar{v}_1\frac{b}{2} + m\left(\frac{b}{\sqrt{2}}\right)^2\omega_2$$

$$\frac{(1+e)\bar{v}_1 b}{2} = \frac{2}{3}b^2\omega_2$$

(*a*) Angular velocity. $\omega_2 = \dfrac{3}{4b}(1+e)v_1$ $\quad\boldsymbol{\omega}_2 = 1.125\dfrac{v_1}{b}$ ↻ ◀

(*b*) Velocity of the mass center.

From Eq. (1),

$$\bar{\mathbf{v}}_2 = e\bar{v}_1\uparrow + \left[\frac{b}{\sqrt{2}}\frac{3}{4b}(1+e)v_1 \searrow 45°\right]$$

$$= e\bar{v}_1\uparrow + \left[\frac{3}{4\sqrt{2}}(1+e)v_1\sin 45°\downarrow\right] + \left[\frac{3}{4\sqrt{2}}(1+e)v_1\cos 45° \longrightarrow\right]$$

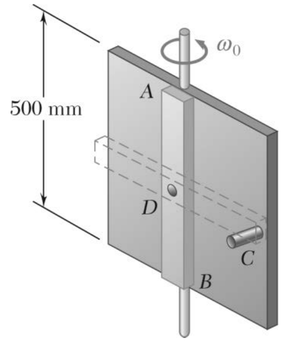
$$= e\bar{v}_1\uparrow + \left[\frac{3}{8}(1+e)v_1\downarrow\right] + \left[\frac{3}{8}(1+e)v_1 \longrightarrow\right]$$

$$= \left[\left(\frac{5}{8}ev_1 - \frac{3}{8}v_1\right)\uparrow\right] + \left[\frac{3}{8}(1+e)v_1 \longrightarrow\right]$$

$$= \left[\left(\frac{5}{8}(0.5) - \frac{3}{8}\right)v_1\uparrow\right] + \frac{3}{8}(1+0.5)v_1 \longrightarrow]$$

$$= [-0.0625\uparrow] + [0.5625 \longrightarrow]$$

$\bar{\mathbf{v}}_2 = 0.566$ m/s $\searrow$ 6.34° ◀



PROBLEM 17.145

A 3-kg bar AB is attached by a pin at D to a 4-kg square plate, which can rotate freely about a vertical axis. Knowing that the angular velocity of the plate is 120 rpm when the bar is vertical, determine (*a*) the angular velocity of the plate after the bar has swung into a horizontal position and has come to rest against pin C, (*b*) the energy lost during the plastic impact at C.

SOLUTION

Moments of inertia about the vertical centroidal axis.

Square plate. $\quad \bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(4)(0.500)^2 = 0.083333 \text{ kg}\cdot\text{m}^2$

Bar AB vertical. $\quad \bar{I} = \text{approximately zero}$

Bar AB horizontal. $\quad \bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}(3)(0.500)^2 = 0.0625 \text{ kg}\cdot\text{m}^2$

Position 1. Bar AB is vertical. $\quad I_1 = 0.083333 \text{ kg}\cdot\text{m}^2$

Angular velocity. $\quad \omega_1 = 120 \text{ rpm} = 4\pi \text{ rad/s}$

Angular momentum about the vertical axis.

$$(H_O)_1 = I_1\omega_1 = (0.083333)(4\pi) = 1.04720 \text{ kg}\cdot\text{m}^2/\text{s}$$

Kinetic energy. $\quad T_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}(0.083333)(4\pi)^2 = 6.5797 \text{ J}$

Position 2. Bar AB is horizontal. $\quad I_2 = 0.145833 \text{ kg}\cdot\text{m}^2$

$$(H_O)_2 = I_2\omega_2 = 0.145833\omega_2$$

Conservation of angular momentum. $\quad (H_O)_1 = (H_O)_2$:

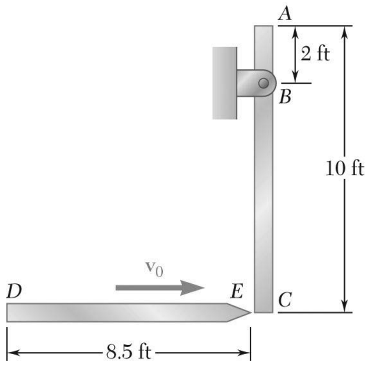
$$1.04720 = 0.145833\omega_2 \qquad \omega_2 = 7.1808 \text{ rad/s}$$

(*a*) Final angular velocity. $\quad \omega_2 = 68.6 \text{ rpm}$ ◀

(*b*) Loss of energy.

$$T_1 - T_2 = T_1 - \frac{1}{2}I_2\omega_2^2 = 6.5797 - \frac{1}{2}(0.145833)(7.1808)^2$$

$T_1 - T_2 = 2.82 \text{ J}$ ◀



PROBLEM 17.146

A 1.8-lb javelin *DE* impacts a 10-lb slender rod *ABC* with a horizontal velocity of $\mathbf{v}_0 = 30$ ft/s as shown. Knowing that the javelin becomes embedded into the end of the rod at Point *C* and does not penetrate very far into it, determine immediately after the impact (*a*) the angular velocity of the rod *ABC* after the impact, (*b*) the components of the reaction at *B*. Assume that the javelin and the rod move as a single body after the impact.

SOLUTION

Masses and moments of inertia.

$$m_{AC} = \frac{W_{AC}}{g} = \frac{10 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.31056 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_{DE} = \frac{W_{DE}}{g} = \frac{1.8 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.05590 \text{ lb} \cdot \text{s}^2/\text{ft}$$

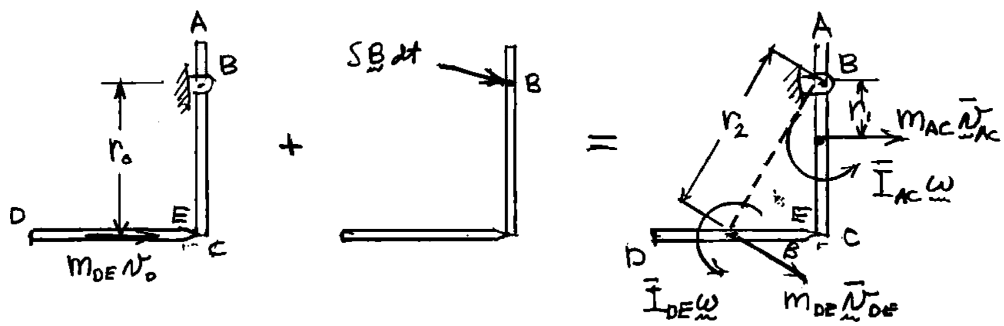
$$\overline{I}_{AC} = \frac{1}{12} m_{AC} L_{AC}^2 = \frac{1}{12}(0.31056)(10)^2 = 2.5880 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$\overline{I}_{DE} = \frac{1}{12} m_{DE} L_{DE}^2 = \frac{1}{12}(0.05590)(8.5)^2 = 0.3365 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

(*a*) *Angular velocity immediately after the impact.*

Principle of impulse and momentum.

$$\boldsymbol{\omega}_{DE} = \boldsymbol{\omega}_{AB} = \boldsymbol{\omega} = \omega \circlearrowright$$



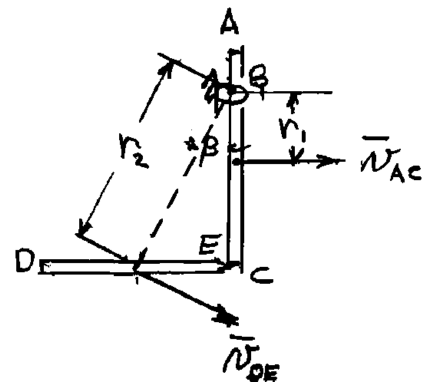
Syst. Momenta$_1$ + **Syst. Ext. Imp.** $_{1\to 2}$ = **Syst. Momenta**$_2$

$+\circlearrowright$ Moments about *B*: $\quad m_{DE} v_0 r_0 + 0 = \overline{I}_{AC}\omega + m_{AC}\overline{v}_{AC} r_1 + \overline{I}_{DE}\omega + m_{DE}\overline{v}_{DE} r_2$

where $\quad v_0 = 30 \text{ ft/s} \quad r_0 = 10 \text{ ft} - 2 \text{ ft} = 8 \text{ ft}$

$$r_1 = 5 \text{ ft} - 2 \text{ ft} = 3 \text{ ft}$$

$$r_2 = \sqrt{(4.25 \text{ ft})^2 + (8 \text{ ft})^2} = 9.0588 \text{ ft}$$



Kinematics: (Rotation about *B*)

$$\bar{v}_{AC} = r_1\omega$$

$$\bar{v}_{DE} = r_2\omega$$

$$\tan\beta = \frac{4.25\text{ ft}}{8\text{ ft}} \qquad \beta = 27.98°$$

$$m_{DE}v_0r_0 = \bar{I}_{AC}\omega + m_{AC}r_1^2\omega + \bar{I}_{DE}\omega + m_{DE}r_2^2\omega$$

$$= I_B\omega$$

where

$$I_B = \bar{I}_{AC} + m_{AC}r_1^2 + \bar{I}_{DE} + m_{DE}r_2^2$$

$$= 2.5880 + (0.31056)(3)^2 + 0.3365 + (0.05590)(9.0588)^2$$

$$= 10.3068\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\omega = \frac{m_{DE}v_0r_0}{I_B} = \frac{(0.05590\text{ lb}\cdot\text{s}^2/\text{ft})(30\text{ ft/s})(8\text{ ft})}{10.3068\text{ lb}\cdot\text{s}^2\cdot\text{ft}}$$

$$= 1.30167\text{ rad/s}$$

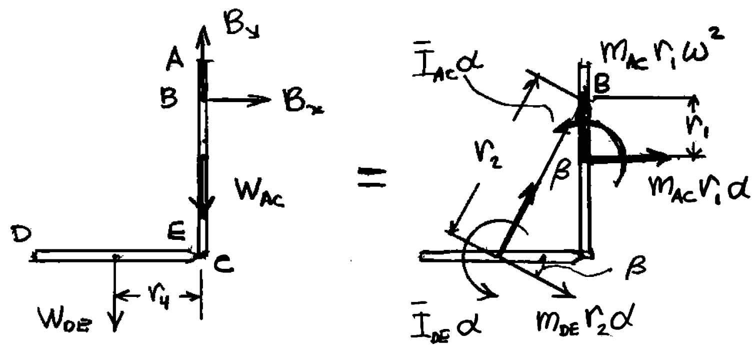
$\boldsymbol{\omega} = 1.302$ rad/s ↻ ◀

Accelerations: $\boldsymbol{\alpha} = \alpha$ ↻

$$\bar{a}_{AD} = [r_1\alpha \longrightarrow] + [r_1\omega^2 \uparrow]$$

$$\bar{a}_{DE} = [r_2\alpha \searrow \beta] + [r_2\omega^2 \nearrow \beta]$$

Free body and kinetic diagrams.



+↻ Moments about *B*:

$$W_{DE}r_4 = \bar{I}_{AC}\alpha + m_{AC}(\bar{a}_{AC})_t r_1 + \bar{I}_{DE}\alpha + m_{DE}(\bar{a}_{DE})_t r_2$$

$$= \bar{I}_{AC}\alpha + m_{AC}r_1^2\alpha + \bar{I}_{DE}\alpha + m_{DE}r_2^2\alpha$$

$$= I_B\alpha$$

$$\alpha = \frac{W_{DE}r_4}{I_B} = \frac{(1.8\text{ lb})(4.25)}{10.3068\text{ lb}\cdot\text{s}^2\cdot\text{ft}}$$

$$= 0.74223\text{ rad/s}^2$$

$\boldsymbol{\alpha} = 0.742$ rad/s² ↻

PROBLEM 17.146 (Continued)

(*b*) *Components of reaction at B.*

$\Sigma F = \Sigma m\bar{a}$:

$$\mathbf{B} + [W_{AC} \downarrow] + [W_{DE} \downarrow] = [m_{AC} r_1 \alpha \longrightarrow] + [m_{AC} r_1 \omega^2 \uparrow] + [m_{DE} r_2 \alpha \searrow \beta] + [m_{DE} r_2 \omega^2 \nearrow \beta]$$

Component $\xrightarrow{+}$:

$$B_x = m_{AC} r_1 \alpha + m_{DE} (r_2 \cos\beta)\alpha + m_{DE} (r_2 \sin\beta)\omega^2$$
$$= (0.31056)(3)(0.74223) + (0.05590)(8)(0.74223) + (0.05590)(4.25)(1.30167)^2$$
$$= 0.6915 + 0.3319 + 0.4025$$

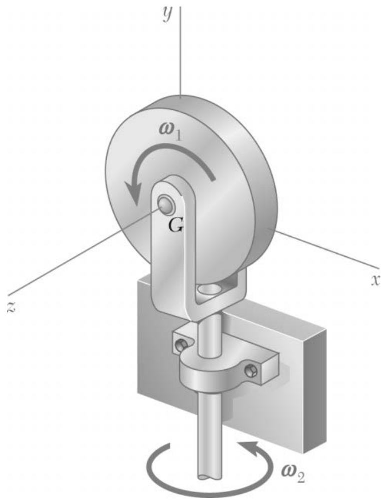
$\mathbf{B}_x = 1.426 \text{ lb} \longrightarrow$ ◀

Component $+\uparrow$:

$$B_y - W_{AC} - W_{DC} = m_{AC} r_1 \omega^2 - m_{DE} (r_1 \sin\beta)\alpha + m_{DE} (r_2 \cos\beta)\omega^2$$
$$B_y - 10 - 1.8 = (0.31056)(3)(1.30167)^2 - (0.05590)(4.25)(0.74223) + (0.05590)(8)(1.30167)^2$$
$$B_y = 11.8 + 1.5785 - 0.1763 + 0.7577$$

$\mathbf{B}_y = 13.96 \text{ lb} \uparrow$ ◀

CHAPTER 18



PROBLEM 18.1

A thin, homogeneous disk of mass m and radius r spins at the constant rate ω_1 about an axle held by a fork-ended vertical rod, which rotates at the constant rate ω_2. Determine the angular momentum $\mathbf{H}_G$ of the disk about its mass center G.

SOLUTION

Angular velocity: $\boldsymbol{\omega} = \omega_2 \mathbf{j} + \omega_1 \mathbf{k}$

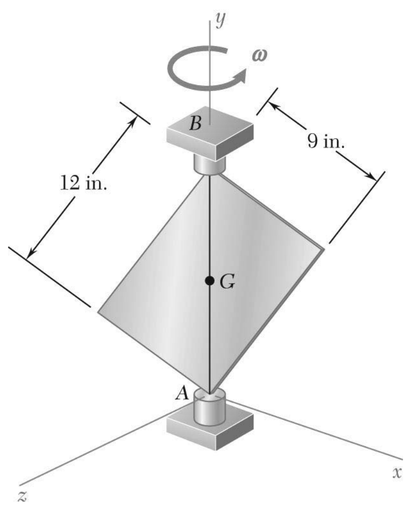
Moments of inertia: $\bar{I}_x = \frac{1}{4}mr^2, \quad \bar{I}_y = \frac{1}{4}mr^2, \quad \bar{I}_z = \frac{1}{2}mr^2$

Products of inertia: by symmetry, $\bar{I}_{xy} = \bar{I}_{yz} = \bar{I}_{zx} = 0$

Angular momentum: $\mathbf{H}_G = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k}$

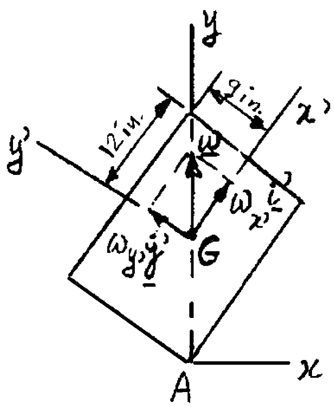
$$\mathbf{H}_G = 0 + \frac{1}{4}mr^2\omega_2\mathbf{j} + \frac{1}{2}mr^2\omega_1\mathbf{k} \qquad \mathbf{H}_G = \frac{1}{4}mr^2\omega_2\mathbf{j} + \frac{1}{2}mr^2\omega_1\mathbf{k} \blacktriangleleft$$

PROBLEM 18.2



A thin rectangular plate of weight 15 lb rotates about its vertical diagonal *AB* with an angular velocity ω. Knowing that the z axis is perpendicular to the plate and that ω is constant and equal to 5 rad/s, determine the angular momentum of the plate about its mass center *G*.

SOLUTION



$$h = \sqrt{(9\text{ in.})^2 + (12\text{ in.})^2} = 15\text{ in.}$$

Resolving ω along the principal axes x', y', z:

$$\omega_{x'} = \frac{12}{15}\omega = 0.8(5\text{ rad/s}) = 4\text{ rad/s}$$

$$\omega_{y'} = \frac{9}{15}\omega = 0.6(5\text{ rad/s}) = 3\text{ rad/s}$$

$$\omega_z = 0$$

Moments of inertia:

$$I_{x'} = \frac{1}{12}\left(\frac{15\text{ lb}}{32.2}\right)\left(\frac{9}{12}\text{ ft}\right)^2 = 0.021836\text{ slug}\cdot\text{ft}^2$$

$$I_{y'} = \frac{1}{12}\left(\frac{15\text{ lb}}{32.2}\right)\left(\frac{12}{12}\text{ ft}\right)^2 = 0.038820\text{ slug}\cdot\text{ft}^2$$

From Eqs. (18.10):

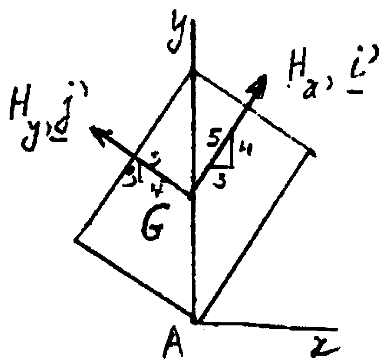
$$H_{x'} = I_{x'}\omega_{x'} = (0.021836)(4) = 0.087345\text{ slug}\cdot\text{ft}^2/\text{s}$$

$$H_{y'} = I_{y'}\omega_{y'} = (0.038820)(3) = 0.11646\text{ slug}\cdot\text{ft}^2/\text{s}$$

$$H_{z'} = I_z\omega_z = 0$$

$$\mathbf{H}_G = (0.087345\text{ slug}\cdot\text{ft}^2/\text{s})\mathbf{i}' + (0.11646\text{ slug}\cdot\text{ft}^2/\text{s})\mathbf{j}'$$

PROBLEM 18.2 (Continued)

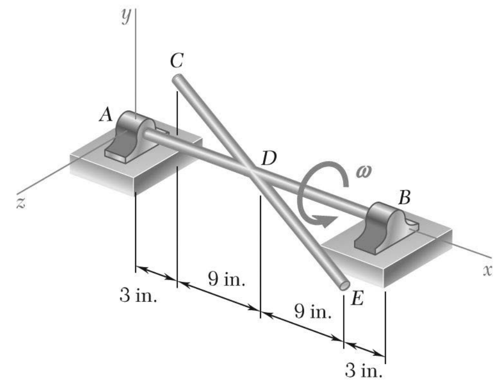


Components along x and y axes:

$$H_x = \frac{3}{5}H_{x'} - \frac{4}{5}H_{y'} = \left(\frac{3}{5}\right)(0.087345) - \left(\frac{4}{5}\right)(0.11646)$$
$$= -0.040761$$

$$H_y = \frac{4}{5}H_{x'} + \frac{3}{5}H_{y'}$$
$$= \left(\frac{4}{5}\right)(0.087345) + \left(\frac{3}{5}\right)(0.11646) = 0.13975$$

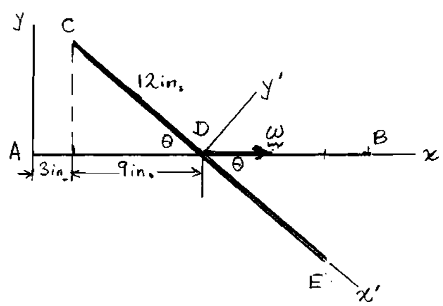
$$\mathbf{H}_G = -(0.0408\ \text{slug}\cdot\text{ft}^2/\text{s})\mathbf{i} + (0.1398\ \text{slug}\cdot\text{ft}^2/\text{s})\mathbf{j} \blacktriangleleft$$



PROBLEM 18.3

Two uniform rods *AB* and *CE*, each of weight 3 lb and length 2 ft, are welded to each other at their midpoints. Knowing that this assembly has an angular velocity of constant magnitude $\omega = 12$ rad/s, determine the magnitude and direction of the angular momentum $\mathbf{H}_D$ of the assembly about *D*.

SOLUTION



$$m = \frac{W}{g} = \frac{3}{32.2} = 0.093168 \text{ lb}\cdot\text{s}^2/\text{ft}, \qquad l = 2 \text{ ft},$$

$$\boldsymbol{\omega} = (12 \text{ rad/s})\mathbf{i}$$

For rod *ADB*, $\mathbf{H}_D = \bar{I}_x \omega \mathbf{i} \approx 0, \quad \text{since } \bar{I}_x \approx 0.$

For rod *CDE*, use principal axes x', y' as shown.

$$\cos\theta = \frac{9}{12}, \qquad \theta = 41.410°$$

$$\omega_{x'} = \omega\cos\theta = 9 \text{ rad/s}^2$$

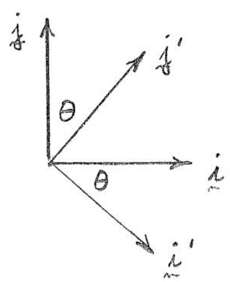
$$\omega_{y'} = \omega\sin\theta = 7.93725 \text{ rad/s}^2$$

$$\omega_{z'} = 0$$

$$\bar{I}_{x'} \approx 0$$

$$\bar{I}_{y'} = \frac{1}{12}ml^2 = \frac{1}{12}(0.093168)(2)^2$$

$$= 0.0310559 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$



$$\mathbf{H}_D = \bar{I}_{x'}\omega_{x'}\mathbf{i}' + \bar{I}_{y'}\omega_{y'}\mathbf{j}' + \bar{I}_{z'}\omega_{z'}\mathbf{k}'$$

$$= 0 + (0.0310559)(7.93725)\mathbf{j}' + 0$$

$$= 0.246498\mathbf{j}'$$

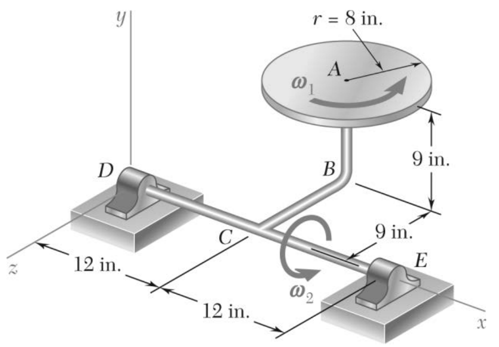
$$H_D = 0.246 \text{ lb}\cdot\text{s}\cdot\text{ft} \blacktriangleleft$$

$$\mathbf{H}_D = 0.246498(\sin\theta\mathbf{i} + \cos\theta\mathbf{j}) = 0.163045\mathbf{i} + 0.184874\mathbf{j}$$

$$\cos\theta_x = \frac{0.163045}{0.246498} \qquad \theta_x = 48.6° \blacktriangleleft$$

$$\cos\theta_y = \frac{0.184874}{0.246498} \qquad \theta_y = 41.4° \blacktriangleleft$$

$$\cos\theta_z = 0 \qquad \theta_z = 90° \blacktriangleleft$$



PROBLEM 18.4

A homogeneous disk of weight $W = 6$ lb rotates at the constant rate $\omega_1 = 16$ rad/s with respect to arm *ABC*, which is welded to a shaft *DCE* rotating at the constant rate $\omega_2 = 8$ rad/s. Determine the angular momentum $\mathbf{H}_A$ of the disk about its center *A*.

SOLUTION

$$\boldsymbol{\omega} = \omega_1\mathbf{j} + \omega_2\mathbf{i} = (8 \text{ rad/s})\mathbf{i} + (16 \text{ rad/s})\mathbf{j}$$

For axes x', y', z' parallel to *x*, *y*, *z* with origin at *A*,

$$m = \frac{W}{g} = \frac{6}{32.2} = 0.186335 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\bar{I}_{x'} = \frac{1}{4}mr^2 = \frac{1}{4}(0.186335)\left(\frac{8}{12}\right)^2$$

$$= 0.020704 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\bar{I}_{z'} = \bar{I}_{x'} = 0.020704 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

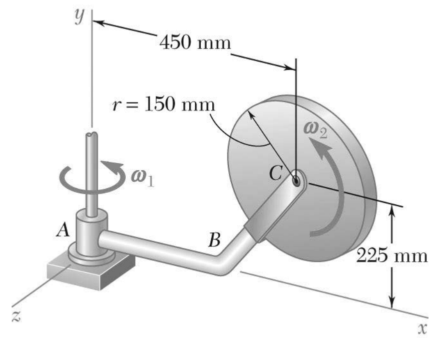
$$\bar{I}_{y'} = \bar{I}_{x'} + \bar{I}_{z'} = 0.041408 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\mathbf{H}_A = \bar{I}_{x'}\omega_{x'}\mathbf{i} + \bar{I}_{y'}\omega_{y'}\mathbf{j} + \bar{I}_{z'}\omega_{z'}\mathbf{k}$$

$$= (0.020704)(8)\mathbf{i} + (0.041408)(16)\mathbf{j}$$

$$= 0.1656\mathbf{i} + 0.6625\mathbf{j}$$

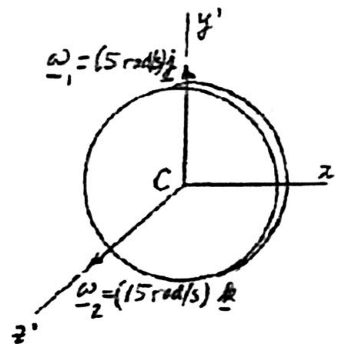
$$\mathbf{H}_A = (0.1656 \text{ lb}\cdot\text{ft}\cdot\text{s})\mathbf{i} + (0.663 \text{ lb}\cdot\text{ft}\cdot\text{s})\mathbf{j} \blacktriangleleft$$



PROBLEM 18.5

A thin disk of mass $m = 4$ kg rotates at the constant rate $\omega_2 = 15$ rad/s with respect to arm *ABC*, which itself rotates at the constant rate $\omega_1 = 5$ rad/s about the *y* axis. Determine the angular momentum of the disk about its center *C*.

SOLUTION



$$r = 150 \text{ mm}$$

Angular velocity of disk:

$$\boldsymbol{\omega} = \omega_1 \mathbf{j} + \omega_2 \mathbf{k}$$
$$= (5 \text{ rad/s})\mathbf{j} + (15 \text{ rad/s})\mathbf{k}$$

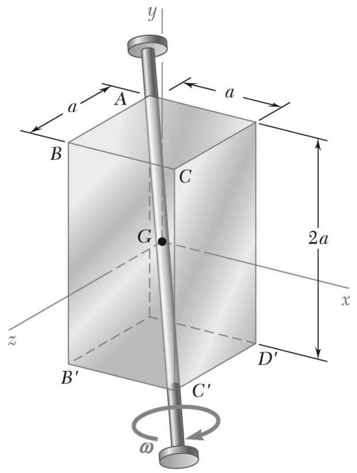
Centroidal moments of inertia:

$$\bar{I}_{x'} = \bar{I}_{y'} = \frac{1}{4} m r^2$$
$$= \frac{1}{4}(4)(0.150 \text{ m})^2 = 0.0225 \text{ kg} \cdot \text{m}^2$$
$$\bar{I}_{z'} = \frac{1}{2} m r^2 = 0.045 \text{ kg} \cdot \text{m}^2$$

<u>Angular momentum about Point *C*</u>.

$$\mathbf{H}_C = \bar{I}_{x'}\omega_{x'}\mathbf{i} + \bar{I}_{y'}\omega_{y'}\mathbf{j} + I_{z'}\omega_{z'}\mathbf{k}$$
$$= 0 + (0.0225)(5)\mathbf{j} + (0.045)(15)\mathbf{k}$$
$$= (0.1125 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{j} + (0.6750 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k}$$

$$\mathbf{H}_C = (0.1125 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{j} + (0.675 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k} \blacktriangleleft$$



PROBLEM 18.6

A solid rectangular parallelepiped of mass m has a square base of side a and a length $2a$. Knowing that it rotates at the constant rate ω about its diagonal AC' and that its rotation is observed from A as counterclockwise, determine (a) the magnitude of the angular momentum $\mathbf{H}_G$ of the parallelepiped about its mass center G, (b) the angle that $\mathbf{H}_G$ forms with the diagonal AC'.

SOLUTION

Body diagonal: $$d = \sqrt{a^2 + (2a)^2 + a^2} = \sqrt{6}a$$

$$\boldsymbol{\omega} = \frac{\omega}{d}(-a\mathbf{i} + 2a\mathbf{j} - a\mathbf{k}) = -\frac{\omega}{\sqrt{6}}\mathbf{i} + \frac{2\omega}{\sqrt{6}}\mathbf{j} - \frac{\omega}{\sqrt{6}}\mathbf{k}$$

$$I_x = \frac{1}{12}m[(2a)^2 + a^2] = \frac{5}{12}ma^2$$

$$I_y = \frac{1}{12}m[a^2 + a^2] = \frac{1}{6}ma^2$$

$$I_z = \frac{1}{12}m[a^2 + (2a)^2] = \frac{5}{12}ma^2$$

(a) $$\mathbf{H}_G = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$= \left(\frac{5}{12}ma^2\right)\left(-\frac{\omega}{\sqrt{6}}\right)\mathbf{i} + \left(\frac{1}{6}ma^2\right)\left(\frac{2\omega}{\sqrt{6}}\right)\mathbf{j} + \left(\frac{5}{12}ma^2\right)\left(-\frac{\omega}{\sqrt{6}}\right)\mathbf{k}$$

$$= \frac{ma^2\omega}{12\sqrt{6}}(-5\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})$$

$$H_G = \frac{ma^2\omega}{12\sqrt{6}}\sqrt{5^2 + 4^2 + 5^2} = \frac{\sqrt{11}ma^2\omega}{12}$$

$\mathbf{H}_G = 0.276\ ma^2\omega$ ◀

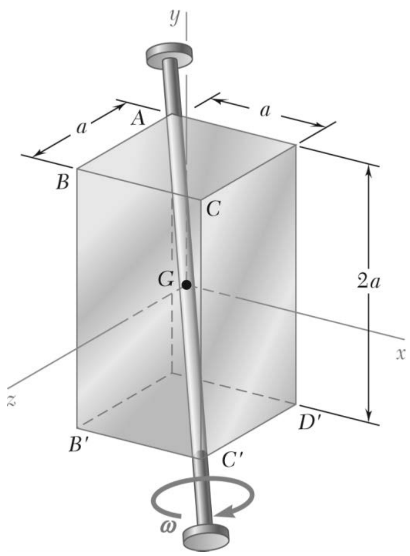
(b) $$\mathbf{H}_G \cdot \boldsymbol{\omega} = \frac{ma^2\omega^2}{(12)(6)}(-5\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}) \cdot (-\mathbf{i} + 2\mathbf{j} - \mathbf{k})$$

$$= \frac{18ma^2\omega^2}{(12)(6)} = \frac{1}{4}ma^2\omega$$

$$H_G\omega = \frac{\sqrt{11}}{12}ma^2\omega^2$$

$$\cos\theta = \frac{\mathbf{H}_G \cdot \boldsymbol{\omega}}{H_G\omega} = \frac{12}{4\sqrt{11}} = 0.90453$$

$\theta = 25.2°$ ◀



PROBLEM 18.7

Solve Problem 18.6, assuming that the solid rectangular parallelepiped has been replaced by a hollow one consisting of six thin metal plates welded together.

PROBLEM 18.6 A solid rectangular parallelepiped of mass m has a square base of side a and a length $2a$. Knowing that it rotates at the constant rate ω about its diagonal AC' and that its rotation is observed from A as counterclockwise, determine (a) the magnitude of the angular momentum $\mathbf{H}_G$ of the parallelepiped about its mass center G, (b) the angle that $\mathbf{H}_G$ forms with the diagonal AC'.

SOLUTION

Body diagonal: $$d = \sqrt{a^2 + (2a)^2 + a^2} = \sqrt{6}a$$

$$\boldsymbol{\omega} = \frac{\omega}{d}(-a\mathbf{i} + 2a\mathbf{j} - a\mathbf{k}) = -\frac{\omega}{\sqrt{6}}\mathbf{i} + \frac{2\omega}{\sqrt{6}}\mathbf{j} - \frac{\omega}{\sqrt{6}}\mathbf{k}$$

$$\text{Total area} = 2(a^2 + 2a^2 + 2a^2) = 10a^2$$

For each square plate: $$m' = \frac{1}{10}m$$

$$I_x = \frac{1}{12}m'a^2 + m'a^2 = \frac{13}{12}m'a^2 = \frac{13}{120}ma^2$$

$$I_y = \frac{1}{6}m'a^2 = \frac{1}{60}ma^2$$

$$I_z = I_x = \frac{13}{120}ma^2$$

For each plate parallel to the *yz* plane: $$m' = \frac{1}{5}m$$

$$I_x = \frac{1}{12}m'[a^2 + (2a)^2] = \frac{5}{12}m'a^2 = \frac{1}{12}ma^2$$

$$I_y = \frac{1}{12}m'a^2 + m'\left(\frac{a}{2}\right)^2 = \frac{1}{3}m'a^2 = \frac{1}{15}ma^2$$

$$I_z = \frac{1}{12}m(2a)^2 + m'\left(\frac{a}{2}\right)^2 = \frac{7}{12}m'a^2 = \frac{7}{60}ma^2$$

PROBLEM 18.7 (Continued)

For each plate parallel to the *xy* plane: $m' = \frac{1}{5}m$

$$I_x = \frac{1}{12}m'(2a)^2 + m'\left(\frac{a}{2}\right)^2 = \frac{7}{12}m'a^2 = \frac{7}{60}ma^2$$

$$I_y = \frac{1}{12}m'a^2 + m'\left(\frac{a}{2}\right)^2 = \frac{1}{3}m'a^2 = \frac{1}{15}ma^2$$

$$I_z = \frac{1}{12}m'[a^2 + (2a)^2] = \frac{5}{12}m'a^2 = \frac{1}{12}ma^2$$

Total moments of inertia:

$$I_x = 2\left(\frac{13}{120} + \frac{1}{12} + \frac{7}{60}\right)ma^2 = \frac{37}{60}ma^2$$

$$I_y = 2\left(\frac{1}{60} + \frac{1}{15} + \frac{1}{15}\right)ma^2 = \frac{3}{10}ma^2$$

$$I_z = 2\left(\frac{13}{120} + \frac{7}{60} + \frac{1}{12}\right)ma^2 = \frac{37}{60}ma^2$$

(*a*)

$$\mathbf{H}_G = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k} = \frac{ma^2\omega}{60\sqrt{6}}(-37\mathbf{i} + 36\mathbf{j} - 37\mathbf{k})$$

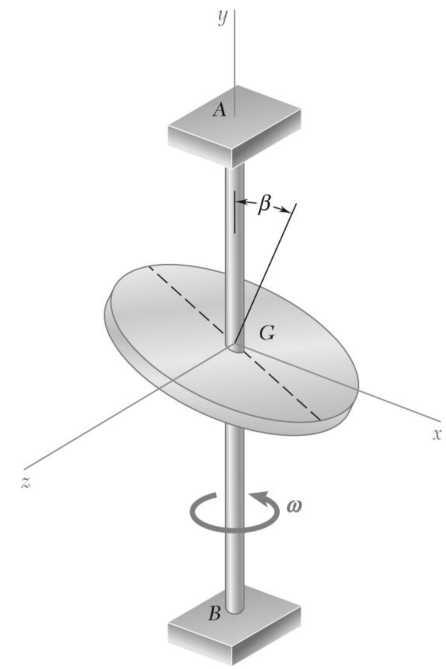
$$H_G = \frac{ma^2\omega}{60\sqrt{6}}\sqrt{(37)^2 + (36)^2 + (37)^2} = 0.43216\ ma^2\omega$$

$$H_G = 0.432\ ma^2\omega \blacktriangleleft$$

(*b*)

$$\mathbf{H}_G \cdot \boldsymbol{\omega} = \frac{ma^2\omega}{(60)(6)}(-37\mathbf{i} + 36\mathbf{j} - 37\mathbf{k}) \cdot (-\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = 0.40556\ ma^2\omega^2$$

$$\cos\theta = \frac{\mathbf{H}_G \cdot \boldsymbol{\omega}}{H_G\omega} = \frac{0.40556}{0.43216} = 0.93844 \qquad \theta = 20.2° \blacktriangleleft$$

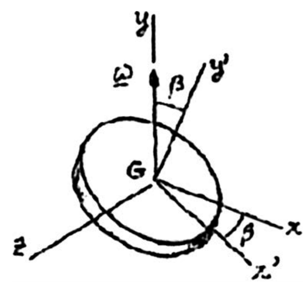


PROBLEM 18.8

A homogeneous disk of mass m and radius r is mounted on the vertical shaft AB. The normal to the disk at G forms an angle $\beta = 25°$ with the shaft. Knowing that the shaft has a constant angular velocity ω, determine the angle θ formed by the shaft AB and the angular momentum $\mathbf{H}_G$ of the disk about its mass center G.

SOLUTION

Use the principal centroidal axes Gx', y' z.



Moments of inertia:

$$\bar{I}_{x'} = \bar{I}_z = \frac{1}{4}mr^2$$

$$\bar{I}_{y'} = \frac{1}{2}mr^2$$

Angular velocities:

$$\omega_{x'} = -\omega \sin\beta$$

$$\omega_{y'} = \omega \cos\beta$$

$$\omega_z = 0$$

Using Eq. (18.10):

$$H_{x'} = \bar{I}_{x'}\omega_{x'} = -\frac{1}{4}mr^2\omega \sin\beta$$

$$H_{y'} = \bar{I}_{y'}\omega_{y'} = \frac{1}{2}mr^2\omega \cos\beta$$

$$H_z = \bar{I}_z\omega_z = 0$$

We have

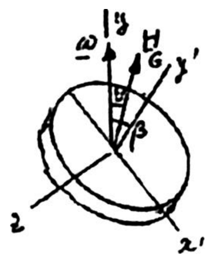
$$\mathbf{H}_G = H_{x'}\mathbf{i}' + H_{y'}\mathbf{j}' + H_z\mathbf{k}$$

PROBLEM 18.8 (Continued)

where $\mathbf{i}'$, $\mathbf{j}'$, $\mathbf{k}$ are the unit vectors along the x', y', z axes.

$$\mathbf{H}_G = -\frac{1}{4}mr^2\omega\sin\beta\mathbf{i}' + \frac{1}{2}mr^2\omega\cos\beta\mathbf{j}' \tag{1}$$

$$\mathbf{H}_G = \frac{1}{4}mr^2\omega(-\sin\beta\mathbf{i}' + 2\cos\beta\mathbf{j}')$$



Forming the scalar product,

$$\mathbf{H}_G \cdot \boldsymbol{\omega} = |\mathbf{H}_G|\,\omega\cos\theta$$

$$\cos\theta = \frac{\mathbf{H}_G \cdot \boldsymbol{\omega}}{|\mathbf{H}_G|\,\omega} \tag{2}$$

But $$\mathbf{H}_G \cdot \boldsymbol{\omega} = \frac{1}{4}mr^2\omega(-\sin\beta\mathbf{i}' + 2\cos\beta\mathbf{j}')\cdot\omega\mathbf{j}$$

or observing that $$\mathbf{i}'\cdot\mathbf{i} = -\sin\beta \quad \text{and} \quad \mathbf{j}'\cdot\mathbf{j} = \cos\beta$$

$$\mathbf{H}_G \cdot \boldsymbol{\omega} = \frac{1}{4}mr^2\omega^2(\sin^2\beta + 2\cos^2\beta)$$

$$= \frac{1}{4}mr^2\omega^2(1+\cos^2\beta) \tag{3}$$

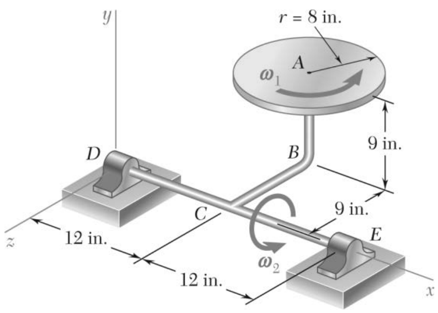
Also, $$|\mathbf{H}_G|\,\omega = \frac{1}{4}mr^2\omega^2\sqrt{\sin^2\beta + 4\cos^2\beta}$$

$$= \frac{1}{4}mr^2\omega^2\sqrt{1+3\cos^2\beta} \tag{4}$$

Substituting from Eqs. (3) and (4) into Eq. (2),

$$\cos\theta = \frac{1+\cos^2\beta}{\sqrt{1+3\cos^2\beta}}$$

For $\beta = 25°$, $\cos\theta = 0.9786$ $\theta = 11.88°$ ◀



PROBLEM 18.9

Determine the angular momentum $\mathbf{H}_D$ of the disk of Problem 18.4 about Point *D*.

PROBLEM 18.4 A homogeneous disk of weight $W = 6$ lb rotates at the constant rate $\omega_1 = 16$ rad/s with respect to arm *ABC*, which is welded to a shaft *DCE* rotating at the constant rate $\omega_2 = 8$ rad/s. Determine the angular momentum $\mathbf{H}_A$ of the disk about its center *A*.

SOLUTION

$$\boldsymbol{\omega} = \omega_2 \mathbf{i} + \omega_1 \mathbf{j} = (8 \text{ rad/s})\mathbf{i} + (16 \text{ rad/s})\mathbf{j}$$

For axes x', y', z' parallel to x, y, z with origin at *A*,

$$m = \frac{W}{g} = \frac{6}{32.2} = 0.186335 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$\bar{I}_{x'} = \frac{1}{4}mr^2 = \frac{1}{4}(0.186335)\left(\frac{8}{12}\right)^2$$

$$= 0.020704 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$\bar{I}_{z'} = \bar{I}_{x'} = 0.020704 \text{ lb} \cdot \text{s}^2 \cdot \text{ft},$$

$$\bar{I}_{y'} = \bar{I}_{x'} + \bar{I}_{z'} = 0.041408 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$\mathbf{H}_A = \bar{I}_{x'}\omega_{x'}\mathbf{i} + \bar{I}_{y'}\omega_{y'}\mathbf{j} + \bar{I}_{z'}\omega_{z'}\mathbf{k}$$

$$= (0.020704)(8)\mathbf{i} + (0.041408)(16)\mathbf{j}$$

$$= (0.1656 \text{ lb} \cdot \text{ft} \cdot \text{s})\mathbf{i} + (0.6625 \text{ lb} \cdot \text{ft} \cdot \text{s})\mathbf{j}$$

Point *A* is the mass center of the disk.

$$\mathbf{r}_{A/D} = (1.0 \text{ ft})\mathbf{i} + (0.75 \text{ ft})\mathbf{j} - (0.75 \text{ ft})\mathbf{k}$$

$$\bar{\mathbf{v}} = \mathbf{v}_A = \omega_2 \mathbf{i} \times \mathbf{r}_{A/D}$$

$$= 8\mathbf{i} \times (1.0\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k})$$

$$= (6 \text{ ft/s})\mathbf{j} + (6 \text{ ft/s})\mathbf{k}$$

$$m\bar{\mathbf{v}} = (1.118 \text{ lb} \cdot \text{s})\mathbf{j} + (1.118 \text{ lb} \cdot \text{s})\mathbf{k}$$

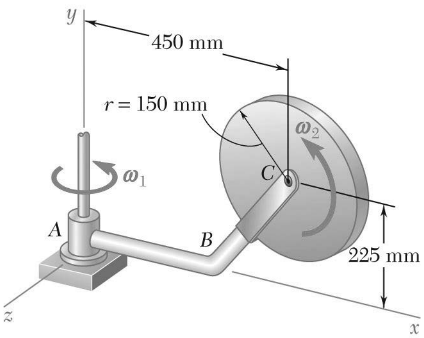
$$\mathbf{r}_{A/D} \times m\bar{\mathbf{v}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.0 & 0.75 & -0.75 \\ 0 & 1.118 & 1.118 \end{vmatrix}$$

$$= (1.677 \text{ lb} \cdot \text{ft} \cdot \text{s})\mathbf{i} - (1.118 \text{ lb} \cdot \text{ft} \cdot \text{s})\mathbf{j} + (1.118 \text{ lb} \cdot \text{ft} \cdot \text{s})\mathbf{k}$$

$$\mathbf{H}_D = \mathbf{H}_A + \mathbf{r}_{A/D} \times m\bar{\mathbf{v}}$$

$$\mathbf{H}_D = (1.843 \text{ lb} \cdot \text{ft} \cdot \text{s})\mathbf{i} - (0.455 \text{ lb} \cdot \text{ft} \cdot \text{s})\mathbf{j} + (1.118 \text{ lb} \cdot \text{ft} \cdot \text{s})\mathbf{k} \blacktriangleleft$$

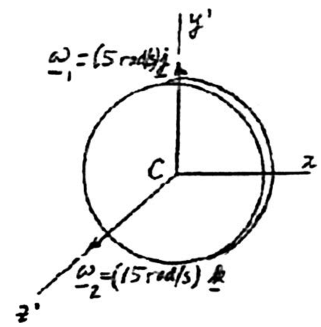
PROBLEM 18.10



Determine the angular momentum of the disk of Problem 18.5 about Point *A*.

PROBLEM 18.5 A thin disk of mass $m = 4$ kg rotates at the constant rate $\omega_2 = 15$ rad/s with respect to arm *ABC*, which itself rotates at the constant rate $\omega_1 = 5$ rad/s about the *y* axis. Determine the angular momentum of the disk about its center *C*.

SOLUTION



$$r = 150 \text{ in.}$$

Angular velocity of disk:

$$\boldsymbol{\omega} = \omega_1 \mathbf{j} + \omega_2 \mathbf{k} = (5 \text{ rad/s})\mathbf{j} + (15 \text{ rad/s})\mathbf{k}$$

Centroidal moments of inertia:

$$\bar{I}_{x'} = \bar{I}_{y'} = \frac{1}{4} m r^2$$

$$= \frac{1}{4}(4)(0.150 \text{ m})^2 = 0.0225 \text{ kg} \cdot \text{m}^2$$

$$I_{z'} = \frac{1}{2} m r^2 = 0.045 \text{ kg} \cdot \text{m}^2$$

Angular momentum about Point *C*.

$$\mathbf{H}_C = \bar{I}_{x'} \omega_{x'} \mathbf{i} + \bar{I}_{y'} \omega_{y'} \mathbf{j} + I_{z'} \omega_{z'} \mathbf{k}$$

$$= 0 + (0.0225)(5)\mathbf{j} + (0.045)(15)\mathbf{k}$$

$$= (0.1125 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{j} + (0.6750 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k}$$

Location of mass center. $\quad \mathbf{r}_{C/A} = (0.450 \text{ m})\mathbf{i} + (0.225 \text{ m})\mathbf{j}$

Velocity of mass center. $\quad \bar{\mathbf{v}} = \boldsymbol{\omega}_1 \times \mathbf{r}_{C/A} = 5\mathbf{j} \times (0.45\mathbf{i} + 0.225\mathbf{j})$

$$= -(2.25 \text{ m/s})\mathbf{k}$$

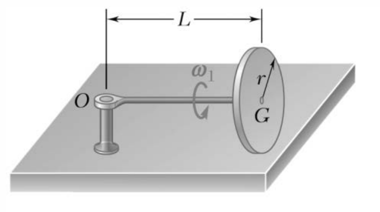
Angular momentum about Point *A*.

$$\mathbf{H}_A = \mathbf{H}_C + \mathbf{r}_{C/A} \times (m\bar{\mathbf{v}})$$

$$\mathbf{H}_A = 0.1125\mathbf{j} + 0.675\mathbf{k} + (0.45\mathbf{i} + 0.225\mathbf{j}) \times [-(4)(2.25)\mathbf{k}]$$

$$= 0.1125\mathbf{j} + 0.675\mathbf{k} + 4.05\mathbf{j} - 2.025\mathbf{i}$$

$$\mathbf{H}_A = -(2.03 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{i} + (4.16 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{j} + (0.675 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k} \blacktriangleleft$$



PROBLEM 18.11

Determine the angular momentum $\mathbf{H}_O$ of the disk of Sample Problem 18.2 from the expressions obtained for its linear momentum $m\overline{\mathbf{v}}$ and its angular momentum $\mathbf{H}_G$ using Eq. (18.11). Verify that the result obtained is the same as that obtained by direct computation.

PROBLEM 18.2 A homogeneous disk of radius r and mass m is mounted on an axle OG of length L and negligible mass. The axle is pivoted at the fixed Point O, and the disk is constrained to roll on a horizontal floor. Knowing that the disk rotates counterclockwise at the rate ω_1 about the axle OG, determine (*a*) the angular velocity of the disk, (*b*) its angular momentum about O, (*c*) its kinetic energy, (*d*) the vector and couple at G equivalent to the momenta of the particles of the disk.

SOLUTION

Using Equation (18.11),

$$\mathbf{H}_O = \overline{\mathbf{r}} \times m\overline{\mathbf{v}} + \overline{\mathbf{H}}_G$$

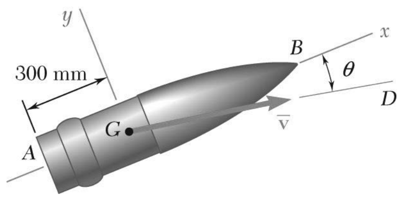
$$= (L\mathbf{i}) \times (mr\omega_1\mathbf{k}) + \frac{1}{2}mr^2\omega_1\left(\mathbf{i} - \frac{r}{2L}\mathbf{j}\right)$$

$$= -mrL\omega_1\mathbf{j} + \frac{1}{2}mr^2\omega_1\mathbf{i} - \frac{1}{4}m\frac{r^3}{L}\omega_1\mathbf{j}$$

$$\mathbf{H}_O = \frac{1}{2}mr^2\omega_1\mathbf{i} - m\left(L^2 + \frac{1}{4}r^2\right)\left(\frac{r\omega_1}{L}\right)\mathbf{j}$$

which is the answer obtained in Part *b* of Sample Problem 18.2.

PROBLEM 18.12



The 100-kg projectile shown has a radius of gyration of 100 mm about its axis of symmetry Gx and a radius of gyration of 250 mm about the transverse axis Gy. Its angular velocity $\boldsymbol{\omega}$ can be resolved into two components; one component, directed along Gx, measures the *rate of spin* of the projectile, while the other component, directed along GD, measures its *rate of precession.* Knowing that $\theta = 6°$ and that the angular momentum of the projectile about its mass center G is $\mathbf{H}_G = (500\text{ g}\cdot\text{m}^2/\text{s})\mathbf{i} - (10\text{ g}\cdot\text{m}^2/\text{s})\mathbf{j}$, determine (*a*) the rate of spin, (*b*) the rate of precession.

SOLUTION

$$m = 100\text{ kg},\quad k_x = 100\text{ mm} = 0.1\text{ m},\quad k_y = 250\text{ mm} = 0.25\text{ m}$$

$$\bar{I}_x = mk_x^2 = (100)(0.1)^2 = 1\text{ kg}\cdot\text{m}^2$$

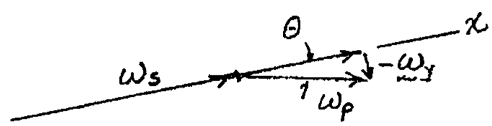
$$\bar{I}_y = \bar{I}_z = mk_y^2 = (100)(0.25)^2 = 6.25\text{ kg}\cdot\text{m}^2$$

$$\mathbf{H}_G = (H_G)_x\mathbf{i} + (H_G)_y\mathbf{j} + (H_G)_z\mathbf{k} = \bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j} + \bar{I}_z\omega_z\mathbf{k}$$

$$\omega_x = \frac{(H_G)_x}{\bar{I}_x} = \frac{0.500\text{ kg}\cdot\text{m}^2/\text{s}}{1\text{ kg}\cdot\text{m}^2} = 0.5\text{ rad/s}$$

$$\omega_y = \frac{(H_G)_y}{\bar{I}_y} = \frac{-0.01\text{ kg}\cdot\text{m}^2/\text{s}}{6.25\text{ kg}\cdot\text{m}^2} = -0.0016\text{ rad/s}$$

$$\omega_z = 0$$



$$\omega_P \sin\theta = -\omega_y$$

$$\omega_P = \frac{-\omega_y}{\sin\theta} = \frac{0.0016}{\sin 6°} = 0.015307\text{ rad/s}$$

(*a*) *Rate of spin.*

$$\omega_x = \omega_s + \omega_P\cos\theta$$

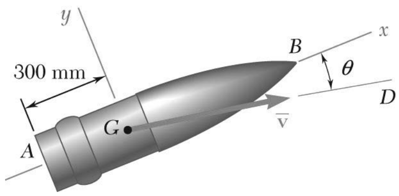
$$\omega_s = \omega_x - \omega_P\cos\theta$$

$$= 0.5 - 0.15307\cos 6°$$

$$= 0.4847$$

$\omega_s = 0.485\text{ rad/s}$ ◀

(*b*) *Rate of precession.* $\omega_P = 0.01531\text{ rad/s}$ ◀



PROBLEM 18.13

Determine the angular momentum $\mathbf{H}_A$ of the projectile of Problem 18.12 about the center A of its base, knowing that its mass center G has a velocity $\overline{\mathbf{v}}$ of 750 m/s. Give your answer in terms of components respectively parallel to the x and y axes shown and to a third axis z pointing toward you.

PROBLEM 18.12 The 100-kg projectile shown has a radius of gyration of 100 mm about its axis of symmetry Gx and a radius of gyration of 250 mm about the transverse axis Gy. Its angular velocity $\boldsymbol{\omega}$ can be resolved into two components; one component, directed along Gx, measures the *rate of spin* of the projectile, while the other component, directed along GD, measures its *rate of precession.* Knowing that $\theta = 6°$ and that the angular momentum of the projectile about its mass center G is $\mathbf{H}_G = (500\ \text{g}\cdot\text{m}^2/\text{s})\mathbf{i} - (10\ \text{g}\cdot\text{m}^2/\text{s})\mathbf{j}$, determine ($a$) the rate of spin, ($b$) the rate of precession.

SOLUTION

$$m = 100\ \text{kg}, \quad \mathbf{r}_{G/A} = (0.300\ \text{m})\mathbf{i}$$

$$\overline{\mathbf{v}} = \overline{v}\cos\theta\mathbf{i} - \overline{v}\sin\theta\mathbf{j}$$

$$\mathbf{H}_G = (0.50\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (0.10\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{j}$$

$$m\overline{\mathbf{v}} = (100)(750)(\cos 6°\mathbf{i} - \sin 6°\mathbf{j})$$

$$= (74589\ \text{kg}\cdot\text{m/s})\mathbf{i} - (7839\ \text{kg}\cdot\text{m/s})\mathbf{j}$$

$$\mathbf{r}_{G/A} \times m\overline{\mathbf{v}} = 0.3\mathbf{i} \times (74589\mathbf{i} - 7839.6\mathbf{j})$$

$$= -(2351.9\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

$$\mathbf{H}_A = \mathbf{H}_G + \mathbf{r}_{G/A} \times m\overline{\mathbf{v}}$$

$$= 0.5\mathbf{i} - 0.1\mathbf{j} + (-2351.9\mathbf{k})$$

$$\mathbf{H}_A = (0.500\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (0.100\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{j} - (2350\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{k} \blacktriangleleft$$

PROBLEM 18.14

(a) Show that the angular momentum $\mathbf{H}_B$ of a rigid body about Point B can be obtained by adding to the angular momentum $\mathbf{H}_A$ of that body about Point A the vector product of the vector $\mathbf{r}_{A/B}$ drawn from B to A and the linear momentum $m\overline{\mathbf{v}}$ of the body:

$$\mathbf{H}_B = \mathbf{H}_A + \mathbf{r}_{A/B} \times m\overline{\mathbf{v}}$$

(b) Further show that when a rigid body rotates about a fixed axis, its angular momentum is the same about any two Points A and B located on the fixed axis $(\mathbf{H}_A = \mathbf{H}_B)$ if, and only if, the mass center G of the body is located on the fixed axis.

SOLUTION

(a) Angular momenta $\mathbf{H}_A$ and $\mathbf{H}_B$ are related to $\mathbf{H}_G$ and $m\overline{\mathbf{v}}$ by

$$\mathbf{H}_A = \mathbf{r}_{G/A} \times m\overline{\mathbf{v}} + \mathbf{H}_G \quad \text{and} \quad \mathbf{H}_B = \mathbf{r}_{G/B} \times m\overline{\mathbf{v}} + \mathbf{H}_G$$

Subtracting,

$$\mathbf{H}_B - \mathbf{H}_A = \mathbf{r}_{G/B} \times m\overline{\mathbf{v}} - \mathbf{r}_{G/A} \times m\overline{\mathbf{v}}$$

$$\mathbf{H}_B = \mathbf{H}_A + (\mathbf{r}_{G/B} - \mathbf{r}_{G/A}) \times m\overline{\mathbf{v}}$$

$$= \mathbf{H}_A + (\mathbf{r}_{G/B} + \mathbf{r}_{A/G}) \times m\overline{\mathbf{v}}$$

$$\mathbf{H}_B = \mathbf{H}_A + \mathbf{r}_{A/B} \times m\overline{\mathbf{v}}$$

(b) It follows that $\mathbf{H}_A = \mathbf{H}_B$ if, and only if

$$\mathbf{r}_{A/B} \times m\overline{\mathbf{v}} = 0$$

With Points A and B located on the fixed axis,

$$\boldsymbol{\omega} = \omega\boldsymbol{\lambda}$$

where $\boldsymbol{\lambda}$ is a unit vector along the fixed axis, and

$$\overline{\mathbf{v}} = \boldsymbol{\omega} \times \mathbf{r}_{G/A} = \omega\boldsymbol{\lambda} \times \mathbf{r}_{G/A}$$

Then

$$\mathbf{r}_{A/B} \times (m\omega\boldsymbol{\lambda} \times \mathbf{r}_{G/A}) = 0$$

but $\mathbf{r}_{A/B}$ is parallel to $\boldsymbol{\lambda}$, hence,

$$\boldsymbol{\lambda} \times (\boldsymbol{\lambda} \times \mathbf{r}_{G/A}) = 0$$

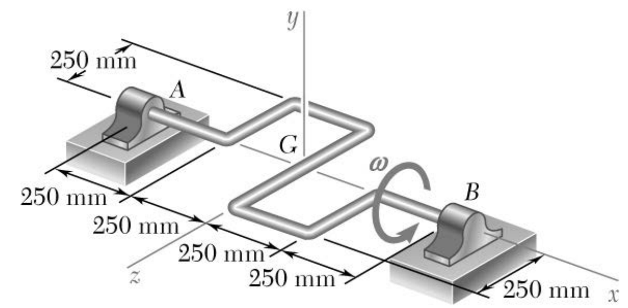
Let $\mathbf{u} = \boldsymbol{\lambda} \times \mathbf{r}_{G/A}$, so that $\boldsymbol{\lambda} \times \mathbf{u} = 0$.

Note that $\mathbf{u}$ must be either perpendicular to λ or equal to zero. But if $\mathbf{u}$ is perpendicular to $\boldsymbol{\lambda}$, $\boldsymbol{\lambda} \times \mathbf{u}$ cannot be equal to zero.

Hence,

$$\mathbf{u} = \boldsymbol{\lambda} \times \mathbf{r}_{G/A} = 0$$

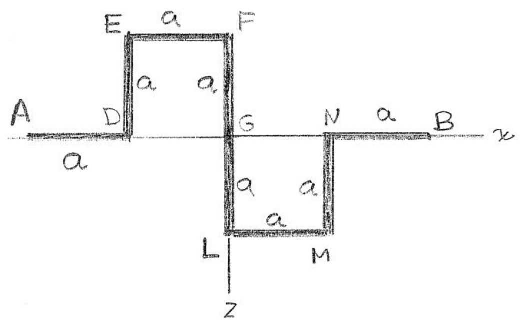
$\mathbf{r}_{G/A}$ is parallel to $\boldsymbol{\lambda}$ and Point G lies on the fixed axis.



PROBLEM 18.15

A 5-kg rod of uniform cross section is used to form the shaft shown. Knowing that the shaft rotates with a constant angular velocity $\boldsymbol{\omega}$ of magnitude 12 rad/s, determine (*a*) the angular momentum $\mathbf{H}_G$ of the shaft about its mass center G, (*b*) the angle formed by $\mathbf{H}_G$ and the axis AB.

SOLUTION



$$\omega = (12 \text{ rad/s})\mathbf{i}, \qquad \omega_y = \omega_z = 0$$

$$(H_G)_x = \bar{I}_x \omega$$

$$(H_G)_y = -\bar{I}_{xy}\omega$$

$$(H_G)_z = -\bar{I}_{xz}\omega$$

The shaft is comprised of 8 sections, each of length $a = 0.25 \text{ m}$ and of mass $m' = \dfrac{m}{8} = 0.625 \text{ kg}$.

$$\bar{I}_x = (4)\left(\frac{1}{3}m'a^2\right) + (2)(m'a^2) = \frac{10}{3}m'a^2 = \frac{10}{3}(0.625)(0.25)^2 = 0.130208 \text{ kg}\cdot\text{m}^2$$

$$\bar{I}_{xy} = 0$$

$$\bar{I}_{xz} = (4)\left(m'a\frac{a}{2}\right) = 2m'a^2 = (2)(0.625)(0.25)^2 = 0.078125 \text{ kg}\cdot\text{m}^2$$

$$(H_G)_x = (0.130208)(12) = 1.5625 \text{ kg}\cdot\text{m}^2\text{/s}$$

$$(H_G)_y = 0$$

$$(H_G)_z = -(0.078125)(12) = -0.9375 \text{ kg}\cdot\text{m}^2\text{/s}$$

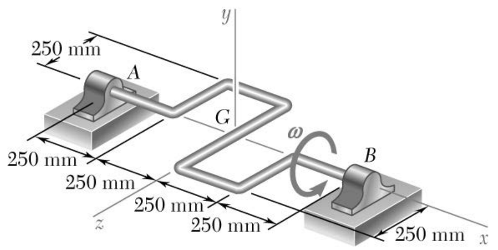
(*a*) $$\mathbf{H}_G = (1.563 \text{ kg}\cdot\text{m}^2\text{/s})\mathbf{i} - (0.938 \text{ kg}\cdot\text{m}^2\text{/s})\mathbf{k} \blacktriangleleft$$

$$H_G = \sqrt{(1.5625)^2 + (0.9375)^2} = 1.82217 \text{ kg}\cdot\text{m}^2\text{/s}$$

(*b*) $$\mathbf{H}_G \cdot \boldsymbol{\omega} = (1.5625\mathbf{i} - 0.9375\mathbf{k}) \cdot 12\mathbf{i} = 18.75 \text{ kg}\cdot\text{m}^2\text{/s}^2$$

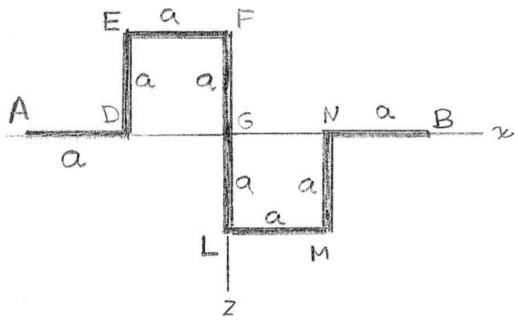
$$\cos\theta = \frac{\mathbf{H}_G \cdot \boldsymbol{\omega}}{H_G \omega} = \frac{18.75}{(1.82217)(12)} = 0.85749 \qquad \theta = 31.0° \blacktriangleleft$$

PROBLEM 18.16



Determine the angular momentum of the shaft of Problem 18.15 about (*a*) Point *A*, (*b*) Point *B*.

SOLUTION



$\omega = (12 \text{ rad/s})\mathbf{i}, \qquad \omega_y = \omega_z = 0, \qquad m = 5 \text{ kg}$

$$(H_G)_x = \bar{I}_x \omega$$

$$(H_G)_y = -\bar{I}_{xy}\omega$$

$$(H_G)_z = -\bar{I}_{xz}\omega$$

The shaft is comprised of 8 sections, each of length $a = 0.25 \text{ m}$ and of mass $m' = \dfrac{m}{8} = 0.625 \text{ kg}$.

$$\bar{I}_x = (4)\left(\frac{1}{3}m'a^2\right) + (2)(m'a^2) = \frac{10}{3}m'a^2 = \frac{10}{3}(0.625)(0.25)^2 = 0.130208 \text{ kg}\cdot\text{m}^2$$

$$\bar{I}_{xy} = 0$$

$$\bar{I}_{xz} = (4)\left(m'a\frac{a}{2}\right) = 2m'a^2 = (2)(0.625)(0.25)^2 = 0.078125 \text{ kg}\cdot\text{m}^2$$

$$(H_G)_x = (0.130208)(12) = 1.5625 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$(H_G)_y = 0$$

$$(H_G)_z = -(0.078125)(12) = -0.9375 \text{ kg}\cdot\text{m}^2/\text{s}$$

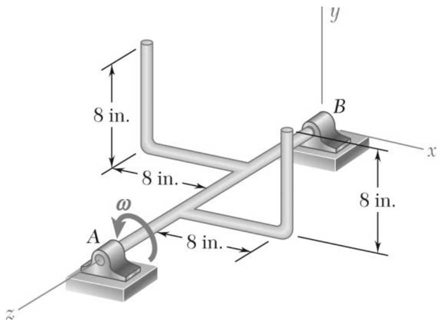
$$\mathbf{H}_G = (1.5625 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (0.9375 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

Since Point *G* lies on the axis of rotation, its velocity is zero.

$$\bar{\mathbf{v}} = \bar{\mathbf{v}}_G = 0$$

(*a*) $\mathbf{H}_A = \mathbf{H}_G + \mathbf{r}_{G/A} \times m\bar{\mathbf{v}} = \mathbf{H}_G$ $\qquad \mathbf{H}_A = (1.563 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (0.938 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$ ◀

(*b*) $\mathbf{H}_B = \mathbf{H}_G + \mathbf{r}_{G/B} \times m\bar{\mathbf{v}} = \mathbf{H}_G$ $\qquad \mathbf{H}_B = (1.563 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (0.938 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$ ◀



PROBLEM 18.17

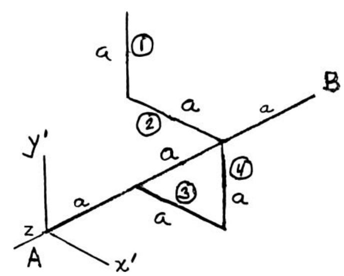
Two L-shaped arms, each weighing 4 lb, are welded at the third points of the 2-ft shaft AB. Knowing that shaft AB rotates at the constant rate $\omega = 240$ rpm, determine (a) the angular momentum of the body about A, (b) the angle formed by the angular momentum and shaft AB.

SOLUTION

$$W = 4 \text{ lb}, \quad m = \frac{4}{32.2} = 0.12422 \text{ lb}\cdot\text{s}^2/\text{ft}, \quad a = 8 \text{ in.} = 0.66667 \text{ ft}$$

$$\omega = \frac{(2\pi)(240)}{60} = 8\pi \text{ rad/s}, \quad \omega_x = 0, \quad \omega_y = 0, \quad \omega_z = 8\pi \text{ rad/s}$$

Use parallel axes x', y', z' with origin at Point A as shown.



$$(H_A)_{x'} = -I_{x'z'}\omega$$

$$(H_A)_{y'} = -I_{y'z'}\omega$$

$$(H_A)_{z'} = I_{z'}\omega$$

Segments 1, 2, 3, and 4, each of mass $m' = 0.06211 \text{ lb}\cdot\text{s}^2/\text{ft}$, contribute to $\bar{I}_{x'z'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'}$.

Part	$I_{x'z'}$	$I_{y'z'}$	$I_{z'}$
①	$2m'a^2$	$-m'a^2$	$\left(\frac{1}{12}+\frac{1}{4}+1\right)m'a^2$
②	$m'a^2$	0	$\frac{1}{3}m'a^2$
③	$-\frac{1}{2}m'a^2$	0	$\frac{1}{3}m'a^2$
④	$-m'a^2$	$-\frac{1}{2}m'a^2$	$\left(\frac{1}{12}+\frac{1}{4}+1\right)m'a^2$
Σ	$\frac{3}{2}m'a^2$	$-\frac{3}{2}m'a^2$	$\frac{10}{3}m'a^2$

PROBLEM 18.17 (Continued)

(*a*) Angular momentum about *A*.

$$(H_A)_{x'} = -I_{x'z'}\omega = -\frac{3}{2}m'a^2\omega$$

$$= -\frac{3}{2}(0.06211)(0.66667)^2(8\pi)$$

$$= -1.04067 \text{ lb}\cdot\text{s}\cdot\text{ft}$$

$$(H_A)_{y'} = -I_{y'z'}\omega = -\left(-\frac{3}{2}m'a^2\right)\omega$$

$$= \frac{3}{2}(0.06211)(0.66667)^2(8\pi)$$

$$= 1.04067 \text{ lb}\cdot\text{ft}\cdot\text{s}$$

$$(H_A)_{z'} = I_{z'}\omega = \frac{10}{3}m'a^2\omega$$

$$= \frac{10}{3}(0.06211)(0.66667)^2(8\pi)$$

$$= 2.3126 \text{ lb}\cdot\text{ft}\cdot\text{s}$$

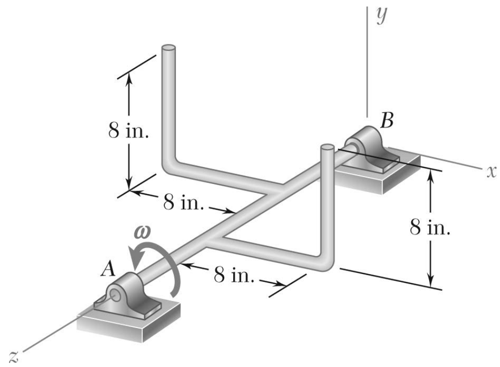
$$\mathbf{H}_A = -(1.041 \text{ lb}\cdot\text{ft}\cdot\text{s})\mathbf{i} + (1.041 \text{ lb}\cdot\text{ft}\cdot\text{s})\mathbf{j} + (2.31 \text{ lb}\cdot\text{ft}\cdot\text{s})\mathbf{k} \blacktriangleleft$$

$$H_A = \sqrt{(1.04067)^2 + (1.04067)^2 + (2.3126)^2} = 2.7412 \text{ lb}\cdot\text{ft}\cdot\text{s}$$

(*b*) Angle formed by $\mathbf{H}_A$ and shaft *AB*.

Unit vector along shaft *AB*: $\boldsymbol{\lambda} = -\mathbf{k}$

$$\cos\theta = \frac{\mathbf{H}_A\cdot\boldsymbol{\lambda}}{H_A} = \frac{-2.3126}{2.7412} = -0.84365 \qquad \theta = 147.5° \blacktriangleleft$$



PROBLEM 18.18

For the body of Problem 18.17, determine (*a*) the angular momentum about *B*, (*b*) the angle formed by the angular momentum about shaft *BA*.

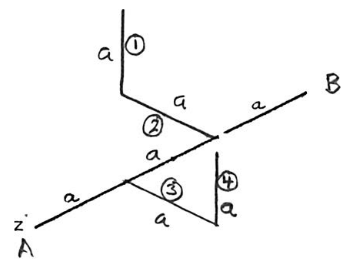
SOLUTION

$$W = 4 \text{ lb.} \quad m = \frac{4}{32.2} = 0.12422 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$a = 8 \text{ in.} = 0.66667 \text{ ft}$$

$$\omega = \frac{(2\pi)(240)}{60} = 8\pi \text{ rad/s}, \quad \omega_x = 0, \quad \omega_y = 0, \quad \omega_z = 8\pi \text{ rad/s}$$

Use parallel axes x', y', z with origin at Point B as shown.



$$(H_B)_x = -I_{xz}\omega$$

$$(H_B)_y = -I_{yz}\omega$$

$$(H_B)_z = I_z\omega$$

Segments 1, 2, 3, and 4, each of mass $m' = 0.06211 \text{ lb}\cdot\text{s}^2/\text{ft}$, contribute to I_{xz}, I_{yz}, and I_z.

Part	I_{xz}	I_{yz}	I_z
①	$-m'a^2$	$\frac{1}{2}m'a^2$	$\left(\frac{1}{12}+\frac{1}{4}+1\right)m'a^2$
②	$-\frac{1}{2}m'a^2$	0	$\frac{1}{3}m'a^2$
③	$m'a^2$	0	$\frac{1}{3}m'a^2$
④	$2m'a^2$	$m'a^2$	$\left(\frac{1}{12}+\frac{1}{4}+1\right)m'a^2$
Σ	$\frac{3}{2}m'a^2$	$\frac{3}{2}m'a^2$	$\frac{10}{3}m'a^2$

(*a*) Angular momentum about *B*.

$$(H_B)_x = -I_{xz}\omega = -\frac{3}{2}m'a^2\omega$$
$$= -\frac{3}{2}(0.06211)(0.66667)^2(8\pi)$$
$$= -1.04067 \text{ lb}\cdot\text{s}\cdot\text{ft}$$

$$(H_B)_y = -I_{yz}\omega = -\frac{3}{2}m'a^2\omega$$
$$= -\frac{3}{2}(0.06211)(0.66667)^2(8\pi)$$
$$= -1.04067 \text{ lb}\cdot\text{s}\cdot\text{ft}$$

$$(H_B)_z = I_z\omega = \frac{10}{3}m'a^2\omega$$
$$= \frac{10}{3}(0.06211)(0.66667)^2(8\pi)$$
$$= 2.3126 \text{ lb}\cdot\text{s}\cdot\text{ft}$$

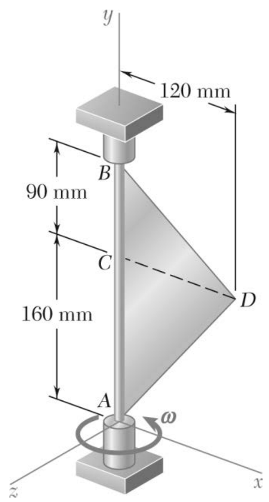
$$\mathbf{H}_B = -(1.041 \text{ lb}\cdot\text{ft}\cdot\text{s})\mathbf{i} - (1.041 \text{ lb}\cdot\text{ft}\cdot\text{s})\mathbf{j} + (2.31 \text{ lb}\cdot\text{ft}\cdot\text{s})\mathbf{k} \blacktriangleleft$$

$$H_B = \sqrt{(1.04067)^2 + (1.04067)^2 + (2.3126)^2} = 2.7412 \text{ lb}\cdot\text{ft}\cdot\text{s}$$

(*b*) Angle formed by $\mathbf{H}_B$ and shaft *BA*.

Unit vector along shaft *BA*: $\boldsymbol{\lambda} = \mathbf{k}$

$$\cos\theta = \frac{\mathbf{H}_B \cdot \boldsymbol{\lambda}}{H_B} = \frac{2.3126}{2.7412} = 0.84365 \qquad \theta = 32.5° \blacktriangleleft$$



PROBLEM 18.19

The triangular plate shown has a mass of 7.5 kg and is welded to a vertical shaft *AB*. Knowing that the plate rotates at the constant rate $\omega = 12$ rad/s, determine its angular momentum about (*a*) Point *C*, (*b*) Point *A*. (*Hint*: To solve part *b* find $\overline{\mathbf{v}}$ and use the property indicated in part *a* of Problem 18.14.)

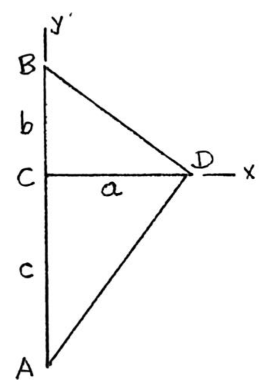
SOLUTION

$$\boldsymbol{\omega} = (12 \text{ rad/s})\mathbf{j}, \quad \omega_x = 0, \quad \omega_y = 12 \text{ rad/s}, \quad \omega_z = 0$$

(*a*) $$(H_C)_x = -I_{xy}\omega, \quad (H_C)_y = I_y\omega, \quad (H_C)_z = -I_{yz}\omega$$

Use axes with origin at *C* as shown. Divide the plate *ABD* into right triangles *ACD* and *CBD*.

For plate *ACD*, the product of inertia of the area is



$$(I_{xy})_{\text{area}} = -\frac{1}{24}a^2c^2$$

For plate *BCD*, it is

$$(I_{xy})_{\text{area}} = \frac{1}{24}a^2b^2$$

For both areas together,

$$(I_{xy})_{\text{area}} = -\frac{1}{24}(c^2 - b^2)a^2$$

Area: $$A = \frac{1}{2}(c+b)a$$

$$(I_{xy})_{\text{mass}} = \frac{m}{A}(I_{xy})_{\text{area}} = -\frac{m(c-b)a}{12}$$

For both areas together, $$(I_y)_{\text{area}} = \frac{1}{12}(c+b)a^3$$

$$(I_y)_{\text{mass}} = \frac{m}{A}(I_y)_{\text{area}} = \frac{1}{6}ma^2$$

$$(I_{xz})_{\text{mass}} \approx 0$$

PROBLEM 18.19 (Continued)

Data: $m = 7.5 \text{ kg} \quad a = 120 \text{ mm} = 0.12 \text{ m}$

$$b = 90 \text{ mm} = 0.09 \text{ m} \quad c = 160 \text{ mm} = 0.16 \text{ m}$$

$$(I_{xy})_{\text{mass}} = -\frac{(7.5)(0.07)(0.12)}{12} = -0.00525 \text{ kg}\cdot\text{m}^2$$

$$(I_y)_{\text{mass}} = \frac{(7.5)(0.12)^2}{6} = 0.018 \text{ kg}\cdot\text{m}^2$$

$$(H_C)_x = -(-0.00525)(12) = 0.063 \text{ kg}\cdot\text{m}^2\text{/s}$$

$$(H_C)_y = (0.018)(12) = 0.216 \text{ kg}\cdot\text{m}^2\text{/s}$$

$$(H_C)_z = 0 \qquad \mathbf{H}_C = (0.063 \text{ kg}\cdot\text{m}^2\text{/s})\mathbf{i} + (0.216 \text{ kg}\cdot\text{m}^2\text{/s})\mathbf{j} \blacktriangleleft$$

Locate the mass center. $\mathbf{r}_{G/C} = \dfrac{a}{3}\mathbf{i} + \overline{y}\mathbf{j}$

Velocity of mass center: $\overline{\mathbf{v}} = \boldsymbol{\omega} \times \mathbf{r}_{G/C}$

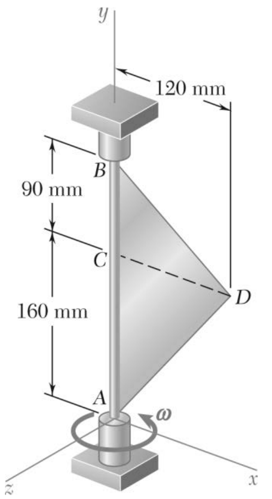
$$\overline{\mathbf{v}} = \omega\mathbf{j} \times \left(\frac{a}{3}\mathbf{i} + \overline{y}\mathbf{j}\right) = -\frac{1}{3}\omega a\mathbf{k} = -\left(\frac{1}{3}\right)(12)(0.12)\mathbf{k} = -(0.48 \text{ m/s})\mathbf{k}$$

$$\mathbf{r}_{C/A} = c\mathbf{j} = (0.16 \text{ m})\mathbf{j}$$

$$\mathbf{r}_{C/A} \times m\overline{\mathbf{v}} = (0.16\mathbf{j}) \times [(7.5)(-0.48\mathbf{k})] = -(0.576 \text{ kg}\cdot\text{m}^2\text{/s})\mathbf{i}$$

(*b*)
$$\mathbf{H}_A = \mathbf{H}_C + \mathbf{r}_{C/A} \times m\overline{\mathbf{v}} = (0.063 - 0.576)\mathbf{i} + 0.216\mathbf{j}$$

$$\mathbf{H}_A = -(0.513 \text{ kg}\cdot\text{m}^2\text{/s})\mathbf{i} + (0.216 \text{ kg}\cdot\text{m}^2\text{/s})\mathbf{j} \blacktriangleleft$$



PROBLEM 18.20

The triangular plate shown has a mass of 7.5 kg and is welded to a vertical shaft *AB*. Knowing that the plate rotates at the constant rate $\omega = 12$ rad/s, determine its angular momentum about (*a*) Point *C,* (*b*) Point *B.* (See hint of Problem 18.19.)

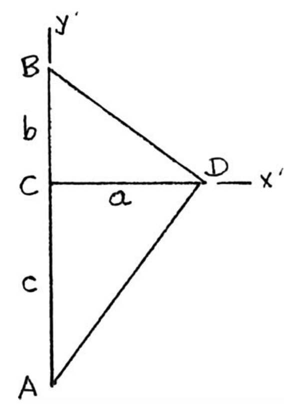
SOLUTION

$$\boldsymbol{\omega} = (12 \text{ rad/s})\mathbf{j}, \quad \omega_x = 0, \quad \omega_y = 12 \text{ rad/s}, \quad \omega_z = 0$$

(*a*) $$(H_C)_x = -I_{xy}\omega, \quad (H_C)_y = I_y\omega, \qquad (H_C)_z = -I_{yz}\omega$$

Use axes with origin at *C* as shown. Divide the plate *ABD* into right triangles *ACD* and *CBD*.

For plate *ACD*, the product of inertia of the area is



$$(I_{xy})_{\text{area}} = -\frac{1}{24}a^2c^2$$

For plate *BCD*, it is

$$(I_{xy})_{\text{area}} = \frac{1}{24}a^2b^2$$

For both areas together,

$$(I_{xy})_{\text{area}} = -\frac{1}{24}(c^2 - b^2)a^2$$

Area: $$A = \frac{1}{2}(c+b)a$$

$$(I_{xy})_{\text{mass}} = \frac{m}{A}(I_{xy})_{\text{area}} = -\frac{m(c-b)a}{12}$$

For both areas together,

$$(I_y)_{\text{area}} = \frac{1}{12}(c+b)a^3$$

$$(I_y)_{\text{mass}} = \frac{m}{A}(I_y)_{\text{area}} = \frac{1}{6}ma^2$$

$$(I_{xz})_{\text{mass}} \approx 0$$

Data: $m = 7.5 \text{ kg} \quad a = 120 \text{ mm} = 0.12 \text{ m}$

$$b = 90 \text{ mm} = 0.09 \text{ m} \quad c = 160 \text{ mm} = 0.16 \text{ m}$$

$$(I_{xy})_{\text{mass}} = -\frac{(7.5)(0.07)(0.12)}{12} = -0.00525 \text{ kg}\cdot\text{m}^2$$

$$(I_y)_{\text{mass}} = \frac{(7.5)(0.12)^2}{6} = 0.018 \text{ kg}\cdot\text{m}^2$$

$$(H_C)_x = -(-0.00525)(12) = 0.063 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$(H_C)_y = (0.018)(12) = 0.216 \text{ kg}\cdot\text{m}^2/\text{s}$$

$$(H_C)_z = 0$$

$$\mathbf{H}_C = (0.063 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} + (0.216 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} \blacktriangleleft$$

Locate the mass center. $\mathbf{r}_{G/C} = \dfrac{a}{3}\mathbf{i} + \bar{y}\mathbf{j}$

Velocity of mass center: $\bar{\mathbf{v}} = \omega \times \mathbf{r}_{G/C}$

$$\bar{\mathbf{v}} = \omega\mathbf{j} \times \left(\frac{a}{3}\mathbf{i} + \bar{y}\,\mathbf{j}\right) = -\frac{1}{3}\omega a\mathbf{k} = -\left(\frac{1}{3}\right)(12)(0.12)\mathbf{k} = -(0.48 \text{ m/s})\mathbf{k}$$

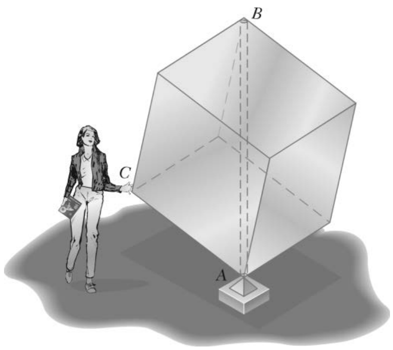
$$\mathbf{r}_{C/B} = -b\mathbf{j} = -(0.09 \text{ m})\mathbf{j}$$

$$\mathbf{r}_{C/B} \times m\bar{\mathbf{v}} = (-0.09\mathbf{j}) \times [(7.5)(-0.48\mathbf{k})] = (0.324 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i}$$

(*b*) $\mathbf{H}_B = \mathbf{H}_C + \mathbf{r}_{C/B} \times m\bar{\mathbf{v}} = (0.063 + 0.324)\mathbf{i} + 0.216\mathbf{j}$

$$\mathbf{H}_B = (0.387 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} + (0.216 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} \blacktriangleleft$$

PROBLEM 18.21



One of the sculptures displayed on a university campus consists of a hollow cube made of six aluminum sheets, each 1.5×1.5 m, welded together and reinforced with internal braces of negligible weight. The cube is mounted on a fixed base at *A* and can rotate freely about its vertical diagonal *AB*. As she passes by this display on the way to a class in mechanics, an engineering student grabs corner *C* of the cube and pushes it for 1.2 s in a direction perpendicular to the plane *ABC* with an average force of 50 N. Having observed that it takes 5 s for the cube to complete one full revolution, she flips out her calculator and proceeds to determine the mass of the cube. What is the result of her calculation? (*Hint:* The perpendicular distance from the diagonal joining two vertices of a cube to any of its other six vertices can be obtained by multiplying the side of the cube by $\sqrt{2/3}$.)

SOLUTION

Let $m' = \frac{1}{6}m$ be the mass of one side of the cube. Choose *x*, *y*, and *z* axes perpendicular to the face of the cube. Let *a* be the side of the cube.

For a side perpendicular to the *x* axis, $(I_x)_1 = \frac{1}{6}m'a^2$.

For a side perpendicular to the *y* or *z* axis, $$(I_x)_2 = \left(\frac{1}{12} + \frac{1}{4}\right)m'a^2 = \frac{1}{3}m'a^2$$

Total moment of inertia: $$I_x = 2(I_x)_1 + 4(I_x)_2 = \frac{5}{3}m'a^2 = \frac{5}{18}ma^2$$

By symmetry, $I_y = I_x$ and $I_z = I_x$.

Since all three moments of inertia are equal, the ellipsoid of inertia is a sphere. All centroidal axes are principal axes.

Moment of inertia about the vertical axis: $$I_v = \frac{5}{18}ma^2$$

Let $b = \sqrt{\frac{2}{3}}a$ be the moment arm of the impulse applied to the corner.

Using the impulse-momentum principle and taking moments about the vertical axis,

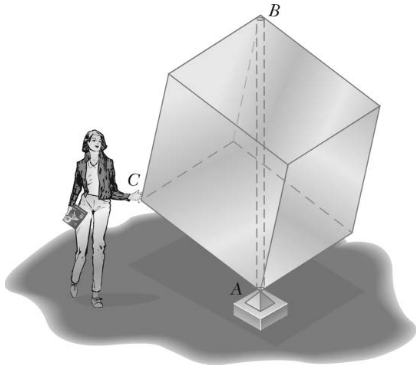
$$bF(\Delta t) = H_v = I_v\omega = \frac{5}{18}ma^2\omega \qquad (1)$$

Data: $$a = 1.5 \text{ m}, \qquad b = \sqrt{\frac{2}{3}}(1.5) = 1.22474 \text{ m}$$

$$\omega = \frac{2\pi}{5} = 1.25664 \text{ rad/s}, \qquad F = 50 \text{ N}, \qquad \Delta t = 1.2 \text{ s}.$$

Solving Equation (1) for *m*, $$m = \frac{18}{5}\frac{bF(\Delta t)}{a^2\omega} = \frac{18}{5}\frac{(1.22474)(50)(1.2)}{(1.5)^2(1.25664)} = 93.563 \text{ kg}$$

$m = 93.6$ kg ◀



PROBLEM 18.22

If the aluminum cube of Problem 18.21 were replaced by a cube of the same size, made of six plywood sheets with mass 8 kg each, how long would it take for that cube to complete one full revolution if the student pushed its corner C in the same way that she pushed the corner of the aluminum cube?

SOLUTION

Let $m' = \frac{1}{6}m$ be the mass of one side of the cube. Choose x, y, and z axes perpendicular to the face of the cube. Let a be the side of the cube.

For a side perpendicular to the x axis, $(I_x)_1 = \frac{1}{6}m'a^2$.

For a side perpendicular to the y or z axis, $$(I_x)_2 = \left(\frac{1}{12} + \frac{1}{4}\right)m'a^2 = \frac{1}{3}m'a^2$$

Total moment of inertia: $$I_x = 2(I_x)_1 + 4(I_x)_2 = \frac{5}{3}m'a^2 = \frac{5}{18}ma^2$$

By symmetry, $I_y = I_x$ and $I_z = I_x$.

Since all three moments of inertia are equal, the ellipsoid of inertia is a sphere. All centroidal axes are principal axes.

Moment of inertia about the vertical axis: $$I_v = \frac{5}{18}ma^2$$

Let $b = \sqrt{\frac{2}{3}}a$ be the moment arm of the impulse applied to the corner.

Using the impulse-momentum principle and taking moments about the vertical axis,

$$bF(\Delta t) = H_v = I_v\omega = \frac{5}{18}ma^2\omega \qquad (1)$$

Data: $$m' = 8 \text{ kg}, \quad m = 6m' = 48 \text{ kg}, \quad a = 1.5 \text{ m},$$

$$b = \sqrt{\frac{2}{3}}a = 1.22474 \text{ m}, \quad F = 50 \text{ N}, \quad \Delta t = 1.2 \text{ s}$$

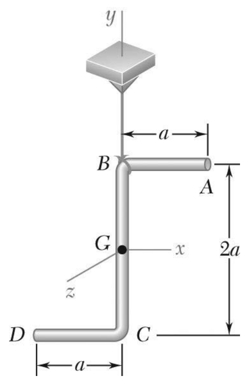
Solving (1) for ω,

$$\omega = \frac{18}{5}\frac{bF(\Delta t)}{ma^2} = \frac{18}{5}\frac{(1.22474)(50)(1.2)}{(48)(1.5)^2} = 2.4495 \text{ rad/s}$$

$$t = \frac{2\pi}{\omega} = \frac{2\pi}{2.4495}$$

$t = 2.57$ s ◀

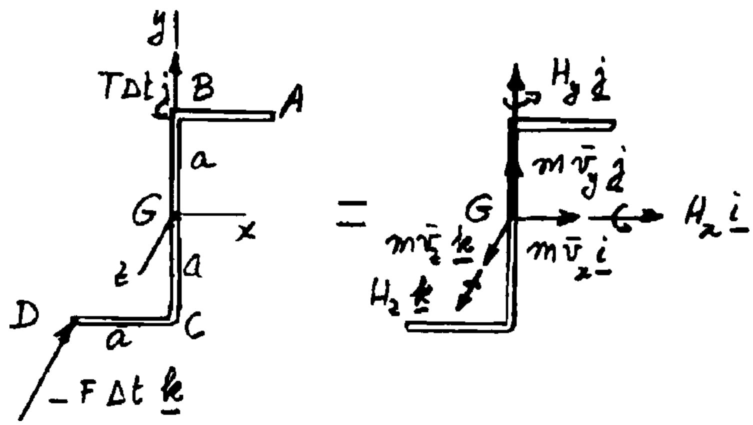
PROBLEM 18.23



A uniform rod of total mass m is bent into the shape shown and is suspended by a wire attached at B. The bent rod is hit at D in a direction perpendicular to the plane containing the rod (in the negative z direction). Denoting the corresponding impulse by $\mathbf{F}\Delta t$, determine (a) the velocity of the mass center of the rod, (b) the angular velocity of the rod.

SOLUTION

We apply the principle of impulse and momentum, considering only the impulsive forces.



(a) *Velocity of mass center*

From constraints: $\bar{v}_y = 0$

$\xrightarrow{+}$ x components: $0 = m\bar{v}_x$ $\qquad \bar{v}_x = 0$

$+\uparrow$ y components: $T\Delta t = m\bar{v}_y = 0$ $\qquad T\Delta t = 0$

$+\swarrow$ z components: $-F\Delta t = m\bar{v}_t$ $\qquad \bar{v}_z = -\dfrac{F\Delta t}{m}$

$$\bar{\mathbf{v}} = -\frac{F\Delta t}{m}\mathbf{k} \blacktriangleleft$$

(b) *Angular velocity*

Equating moments about G:

$$(-a\mathbf{i} - a\,\mathbf{j}) \times (-F\Delta t\,\mathbf{k}) = H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}$$

$$aF\Delta t\,\mathbf{i} - aF\Delta t\,\mathbf{j} = H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}$$

Thus: $$H_x = aF\Delta t, \quad H_y = -aF\Delta t, \quad H_z = 0 \tag{1}$$

To determine angular velocity, we shall use Eqs. (18.7).

First, we determine the <u>moments & products of inertia</u>:

$$\bar{I}_x = \frac{1}{12}\left(\frac{m}{2}\right)(2a)^2 + \frac{m}{4}a^2 + \frac{m}{4}a^2 = \frac{2}{3}ma^2 \tag{2}$$

$$\bar{I}_y = 2\left[\frac{1}{3}\left(\frac{m}{4}\right)a^2\right] = \frac{1}{6}ma^2 \tag{3}$$

$$\bar{I}_{xy} = \frac{m}{4}\left(\frac{a}{2}\right)(a) + \frac{m}{4}\left(-\frac{a}{2}\right)(-a) = +\frac{1}{4}ma^2 \tag{4}$$

$$\bar{I}_{xz} = 0 \qquad \bar{I}_{yz} = 0 \tag{5}$$

We substitute the expressions (1) through (5) into Eqs. (18.7):

$$aF\Delta t = \frac{2}{3}ma^2\omega_x - \frac{1}{4}ma^2\omega_y + 0 \tag{6}$$

$$-aF\Delta t = -\frac{1}{4}ma^2\omega_x + \frac{1}{6}ma^2\omega_y + 0 \tag{7}$$

$$0 = 0 + 0 + \bar{I}_z\omega_z \tag{8}$$

Multiplying Eq. (7) by 3/2 and adding to Eq. (6):

$$-\frac{1}{2}aF\Delta t = \frac{7}{24}ma^2\omega_x \qquad \omega_x = -\frac{12}{7}\frac{F\Delta t}{ma}$$

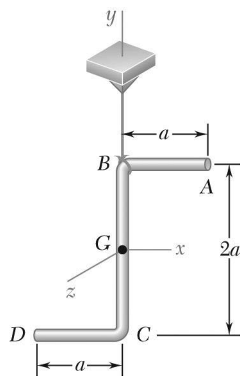
Substituting for ω into (7):

$$-aF\Delta t + \frac{1}{4}\left(-\frac{12^2}{7}\right)aF\Delta t = \frac{1}{6}ma^2\omega_y, \quad \omega_y = -\frac{60}{7}\frac{F\Delta t}{ma}$$

From Eq. (8): $\bar{I}_z\omega_z = 0 \quad \omega_z = 0$

Thus: $$\boldsymbol{\omega} = \frac{12}{7}\frac{F\Delta t}{ma}(-\mathbf{i} - 5\mathbf{j}) \blacktriangleleft$$

PROBLEM 18.24

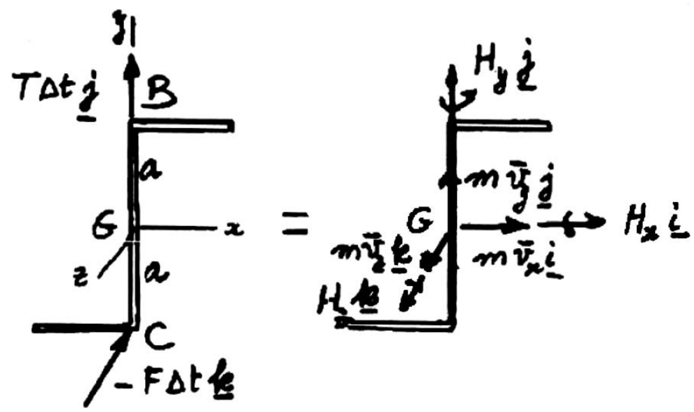


Solve Problem 18.23, assuming that the bent rod is hit at C.

PROBLEM 18.23 A uniform rod of total mass m is bent into the shape shown and is suspended by a wire attached at B. The bent rod is hit at D in a direction perpendicular to the plane containing the rod (in the negative z direction). Denoting the corresponding impulse by $\mathbf{F}\Delta t$, determine (a) the velocity of the mass center of the rod, (b) the angular velocity of the rod.

SOLUTION

We apply the principle of impulse and momentum, consider only impulsive forces.



(a) *Velocity of mass center*

From constraints: $\bar{v}_y = 0$

$\xrightarrow{+}$ x components: $0 = m\bar{v}_x$ $\quad\quad \bar{v}_x = 0$

$+\uparrow$ y components: $T\Delta t = m\bar{v}_y = 0$ $\quad\quad T\Delta t = 0$

$+\swarrow$ z components: $-F\Delta t = m\bar{v}_z$ $\quad\quad \bar{v}_z = -\dfrac{F\Delta t}{m}$

$$\bar{\mathbf{v}} = \frac{F\Delta t}{m}\mathbf{k} \blacktriangleleft$$

(b) *Angular velocity*

Equating moments about G:

$$-a\,\mathbf{j}\times(-F\Delta t\,\mathbf{k}) = H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}$$

$$aF\Delta t\,\mathbf{i} = H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}$$

Thus: $\quad H_x = aF\Delta t, \quad H_y = 0, \quad H_z = 0 \quad\quad (1)$

PROBLEM 18.24 (Continued)

To determine angular velocity, we shall use Eqs. (18.7) first, we determine the moments & products of inertia:

$$\bar{I}_x = \frac{1}{12}\left(\frac{m}{2}\right)(2a)^2 + \frac{m}{4}a^2 + \frac{m}{4}a^2 = \frac{2}{3}ma^2 \tag{2}$$

$$\bar{I}_y = 2\left[\frac{1}{3}\left(\frac{m}{4}\right)a^2\right] = \frac{1}{6}ma^2 \tag{3}$$

$$\bar{I}_{xy} = \frac{m}{4}\left(\frac{a}{2}\right)(a) + \frac{m}{4}\left(-\frac{a}{2}\right)(-a) = +\frac{1}{4}ma^2 \tag{4}$$

$$\bar{I}_{xz} = 0 \qquad \bar{I}_{yz} = 0 \tag{5}$$

$$aF\Delta t = \frac{2}{3}ma^2\omega_x - \frac{1}{4}ma^2\omega_y + 0 \tag{6}$$

$$0 = -\frac{1}{4}ma^2\omega_x + \frac{1}{6}ma^2\omega_y + 0 \tag{7}$$

$$0 = 0 + 0 + \bar{I}_z\omega_z \tag{8}$$

Multiplying Eq. (7) by 3/2 and adding to Eq. (6):

$$aF\Delta t = \left(\frac{2}{3} - \frac{3}{8}\right)ma^2\omega_{x,} \qquad \omega_x = \frac{24}{7}\frac{F\Delta t}{ma}$$

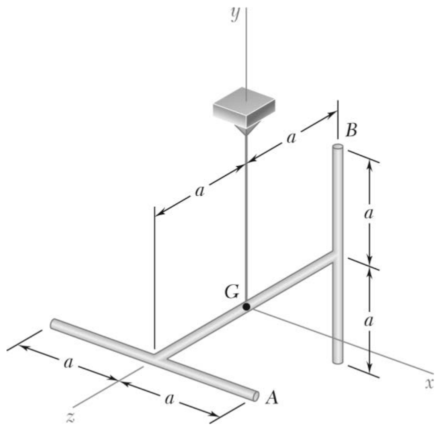
Substituting for ω_x into (7):

$$\omega_y = \frac{3}{2}\omega_x = \frac{3}{2}\frac{24}{7}aF\Delta t, \quad \omega_y = \frac{36}{7}\frac{F\Delta t}{ma}$$

From Eq. (8): $\bar{I}_z\omega_z = 0, \quad \omega_z = 0$

Thus: $\boldsymbol{\omega} = \frac{12}{7}\frac{F\Delta t}{ma}(2\mathbf{i} + 3\mathbf{j})$ ◀

Note that $\omega_y \neq 0$, even though Point *C* where impulse is applied is on the *y* axis.



PROBLEM 18.25

Three slender rods, each of mass m and length $2a$, are welded together to form the assembly shown. The assembly is hit at A in a vertical downward direction. Denoting the corresponding impulse by $\mathbf{F}\,\Delta t,$ determine immediately after the impact (a) the velocity of the mass center G, (b) the angular velocity of the rod.

SOLUTION

Computation of moments and products of inertia.

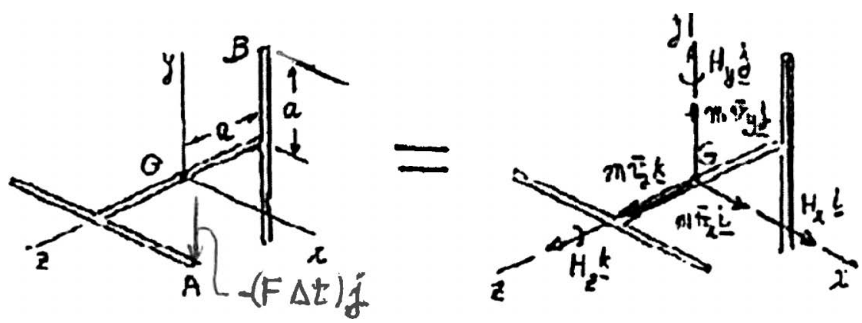
$$\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3 = ma^2 + \frac{1}{3}ma^2 + m\left(a^2 + \frac{a^2}{3}\right) = \frac{8}{3}ma^2$$

$$\bar{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3 = m\left(a^2 + \frac{a^2}{3}\right) + \frac{1}{3}ma^2 + ma^2 = \frac{8}{3}ma^2 \qquad (1)$$

$$\bar{I}_z = (I_z)_1 + (I_z)_2 + (I_z)_3 = \frac{1}{3}ma^2 + 0 + \frac{1}{3}ma^2 = \frac{2}{3}ma^2$$

$$\bar{I}_{xy} = 0, \quad \bar{I}_{yz} = 0, \quad I_{zx} = 0$$

Impulse-momentum principle.



The impulses consist of $-(F\Delta t)\mathbf{j}$ applied at A and $(T\Delta t)\mathbf{j}$ applied at G. Because of constraints, $\bar{v}_y = 0$.

(a) Velocity of mass center.

Equate sums of vectors: $(T\Delta t)\mathbf{j} - (F\Delta t)\mathbf{j} = m\bar{v}_x\mathbf{i} + m\bar{v}_z\mathbf{k}$

Thus, $T\Delta t = F\Delta t, \quad v_x = 0, \quad v_z = 0.$ Since $v_y = 0$ from above, $\bar{\mathbf{v}} = 0$ ◀

PROBLEM 18.25 (Continued)

(*b*) Angular velocity.

Equate moments about G:

$$(a\mathbf{i}+a\mathbf{k})\times(-F\Delta t)\mathbf{j}=H_x\mathbf{i}+H_y\mathbf{j}+H_z\mathbf{k}$$

$$-(aF\Delta t)\mathbf{k}+(aF\Delta t)\mathbf{i}=H_x\mathbf{i}+H_y\mathbf{j}+H_z\mathbf{k}$$

Thus, $$H_x=aF\Delta t,\quad H_y=0,\quad H_z=-aF\Delta t \tag{2}$$

Since the three products of inertia are zero, the x, y, and z axes are principal centroidal axes and we can use Eqs. (18.10). Substituting from Eqs. (1) and (2) into these equations, we have

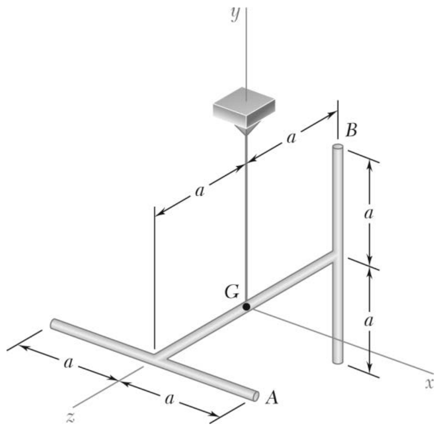
$$H_x=\bar{I}_x\omega_x:\quad aF\Delta t=\frac{8}{3}ma^2\omega_x\quad \omega_x=3F\Delta t/8ma \tag{3}$$

$$H_y=\bar{I}_y\omega_y:\quad 0=\frac{8}{3}ma^2\omega_y\quad \omega_y=0 \tag{4}$$

$$H_z=\bar{I}_z\omega_z:\quad -aF\Delta t=\frac{2}{3}ma^2\quad \omega_z=-3F\Delta t/2ma \tag{5}$$

Therefore,

$$\boldsymbol{\omega}=(3F\Delta t/8ma)(\mathbf{i}-4\mathbf{k})\ \blacktriangleleft$$



PROBLEM 18.26

Solve Problem 18.25, assuming that the assembly is hit at B in a direction opposite to that of the x axis.

PROBLEM 18.25 Three slender rods, each of mass m and length $2a$, are welded together to form the assembly shown. The assembly is hit at A in a vertical downward direction. Denoting the corresponding impulse by $\mathbf{F}\,\Delta t$, determine immediately after the impact (*a*) the velocity of the mass center G, (*b*) the angular velocity of the rod.

SOLUTION

Computation of moments and products of inertia.

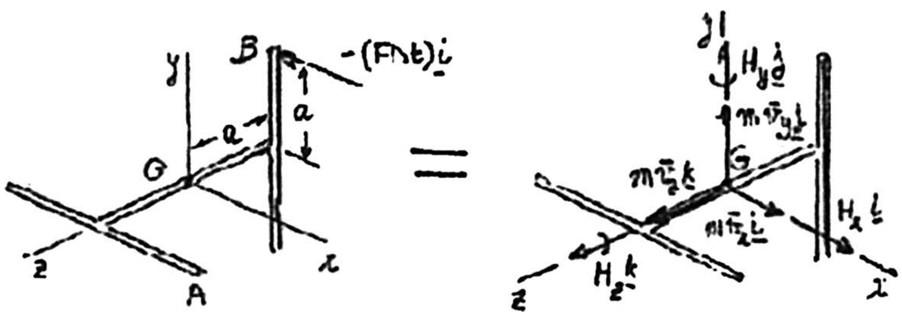
$$\overline{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3 = ma^2 + \frac{1}{3}ma^2 + m\left(a^2 + \frac{a^2}{3}\right) = \frac{8}{3}ma^2$$

$$\overline{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3 = m\left(a^2 + \frac{a^2}{3}\right) + \frac{1}{3}ma^2 + ma^2 = \frac{8}{3}ma^2 \qquad (1)$$

$$\overline{I}_z = (I_z)_1 + (I_z)_2 + (I_z)_3 = \frac{1}{3}ma^2 + 0 + \frac{1}{3}ma^2 = \frac{2}{3}ma^2$$

$$\overline{I}_{xy} = \overline{I}_{yz} = \overline{I}_{zx} = 0$$

Impulse-momentum principle.



The only impulse is $F\Delta t = -(F\Delta t)\mathbf{i}$.

(*a*) Velocity of mass center.

Equate sums of vectors: $\quad -(F\Delta t)\mathbf{i} = m\overline{v}_x\mathbf{i} + mv_y\mathbf{j} + mv_z\mathbf{k}$

Thus, $\quad v_x = -F\Delta t/m, \quad v_y = 0, \quad v_z = 0$

$$\overline{\mathbf{v}} = -(F\Delta t/m)\mathbf{i} \blacktriangleleft$$

PROBLEM 18.26 (Continued)

(*b*) Angular velocity.

Equate moments about G:

$$(a\mathbf{j} - a\mathbf{k}) \times (-F\Delta t)\mathbf{i} = H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}$$

$$(aF\Delta t)\mathbf{k} + (aF\Delta t)\mathbf{j} = H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}$$

Thus, $$H_x = 0, \quad H_y = aF\Delta t, \quad H_z = aF\Delta t \tag{2}$$

Since the three products of inertia are zero, the x, y, and z axes are principal centroidal axes and we can use Eqs. (18.10). Substituting from Eqs. (1) and (2) into these equations, we have

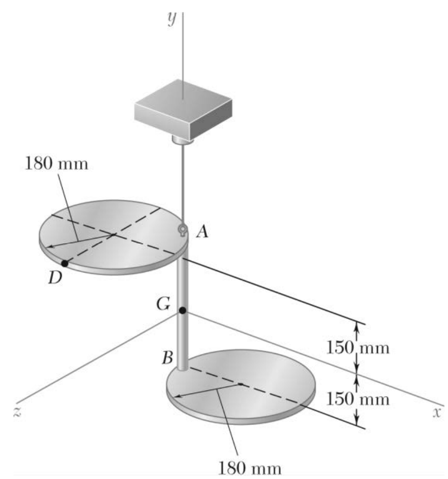
$$H_x = \bar{I}_x\omega_x: \qquad 0 = \frac{8}{3}ma^2\omega_x \quad \omega_x = 0 \tag{3}$$

$$H_y = \bar{I}_y\omega_y: \qquad aF\Delta t = \frac{8}{3}ma^2\omega_y \quad \omega_y = 3F\Delta t/8ma \tag{4}$$

$$H_z = \bar{I}_z\omega_z: \qquad aF\Delta t = \frac{2}{3}ma^2\omega_z \quad \omega_z = 3F\Delta t/2ma \tag{5}$$

Therefore,

$$\boldsymbol{\omega} = (3F\Delta t/8ma)(\mathbf{j} + 4\mathbf{k}) \blacktriangleleft$$



PROBLEM 18.27

Two circular plates, each of mass 4 kg, are rigidly connected by a rod *AB* of negligible mass and are suspended from Point *A* as shown. Knowing that an impulse $\mathbf{F}\,\Delta t = -(2.4\ \text{N}\cdot\text{s})\mathbf{k}$ is applied at Point *D*, determine (*a*) the velocity of the mass center *G* of the assembly, (*b*) the angular velocity of the assembly.

SOLUTION

Moments and products of inertia:

$$I_x = 2\left(\frac{1}{4}mr^2 + mb^2\right) = 2\left[\frac{1}{4}(4)(0.18)^2 + (4)(0.15)^2\right] = 0.2448\ \text{kg}\cdot\text{m}^2$$

$$I_y = 2\left(\frac{1}{2}mr^2 + mr^2\right) = 3mr^2 = (3)(4)(0.18)^2 = 0.3888\ \text{kg}\cdot\text{m}^2$$

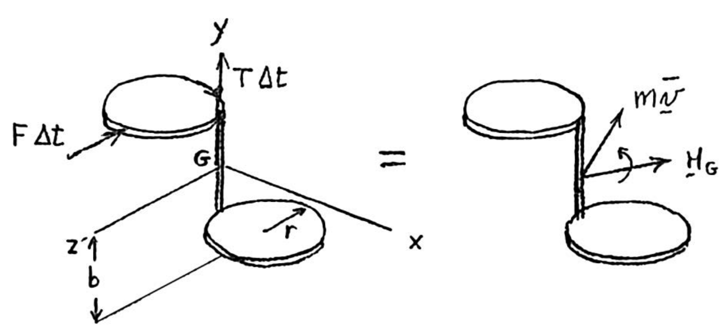
$$I_z = 2\left[\frac{1}{4}mr^2 + m(b^2 + r^2)\right]$$
$$= 2\left[\frac{1}{4}(4)(0.18)^2 + (4)(0.15^2 + 0.18^2)\right] = 0.504\ \text{kg}\cdot\text{m}^2$$

$$I_{xy} = mb(-r) + m(-b)(r) = -2mbr = -(2)(4)(0.15)(0.18) = -0.216\ \text{kg}\cdot\text{m}^2$$

$$I_{xz} = 0, \quad I_{yz} = 0, \qquad \text{total mass} = 2m$$

Constraint of cable: $(v_G)_y = 0 \quad \overline{\mathbf{v}} = \mathbf{v}_G$

Principle of impulse-momentum: $2m\overline{\mathbf{v}} = 0, \quad \mathbf{H}_G = 0$ initially.



PROBLEM 18.27 (Continued)

(*a*) Direct components:

$$\mathbf{F}\Delta t = 2m\bar{\mathbf{v}}$$

$$0 = 2m\bar{v}_x \qquad \bar{v}_x = 0$$

$$T\Delta t = 2m\bar{v}_y = 0$$

$$-F\Delta t = 2m\bar{v}_z$$

$$\bar{v}_z = -\frac{F\Delta t}{2m} = -\frac{2.4}{(2)(4)} = -0.3 \text{ m/s} \qquad \bar{\mathbf{v}} = -(0.300 \text{ m/s})\mathbf{k} \blacktriangleleft$$

(*b*) <u>Moments about *G*.</u> $(b\mathbf{j} - r\mathbf{i}) \times (-F\Delta t\mathbf{k}) = \mathbf{H}_G$

$$-bF(\Delta t)\mathbf{i} - rF\Delta t\mathbf{j} = (I_x\omega_x - I_{xy}\omega_y)\mathbf{i} + (-I_{xy}\omega_x + I_y\omega_y)\mathbf{j} + I_z\omega_z\mathbf{k}$$

Resolve into components and apply the numerical data.

$$\mathbf{i}: \quad -(0.15)(2.4) = 0.2448\omega_x - (-0.216)\omega_y \qquad (1)$$

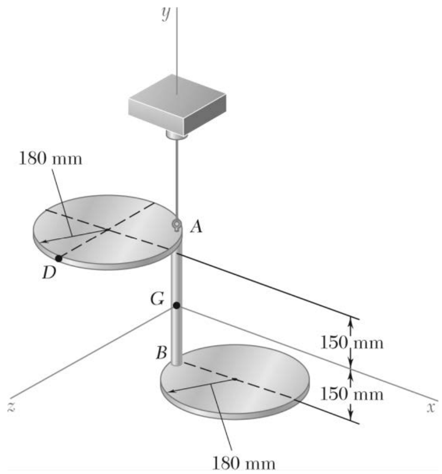
$$\mathbf{j}: \quad -(0.18)(2.4) = -(-0.216)\omega_x + 0.3888\omega_y \qquad (2)$$

$$\mathbf{k}: \quad 0 = 0.504\omega_z \qquad \omega_z = 0$$

Solving Eqs. (1) and (2), $\omega_x = -0.962$ rad/s, $\omega_y = -0.577$ rad/s

$$\boldsymbol{\omega} = -(0.962 \text{ rad/s})\mathbf{i} - (0.577 \text{ rad/s})\mathbf{j} \blacktriangleleft$$

PROBLEM 18.28



Two circular plates, each of mass 4 kg, are rigidly connected by a rod *AB* of negligible mass and are suspended from Point *A* as shown. Knowing that an impulse $\mathbf{F}\Delta t = (2.4\ \text{N}\cdot\text{s})\mathbf{j}$ is applied at Point *D*, determine (*a*) the velocity of the mass center *G* of the assembly, (*b*) the angular velocity of the assembly.

SOLUTION

Moments and products of inertia:

$$I_x = 2\left(\frac{1}{4}mr^2 + mb^2\right) = 2\left[\frac{1}{4}(4)(0.18)^2 + (4)(0.15)^2\right] = 0.2448\ \text{kg}\cdot\text{m}^2$$

$$I_y = 2\left(\frac{1}{2}mr^2 + mr^2\right) = 3mr^2 = (3)(4)(0.18)^2 = 0.3888\ \text{kg}\cdot\text{m}^2$$

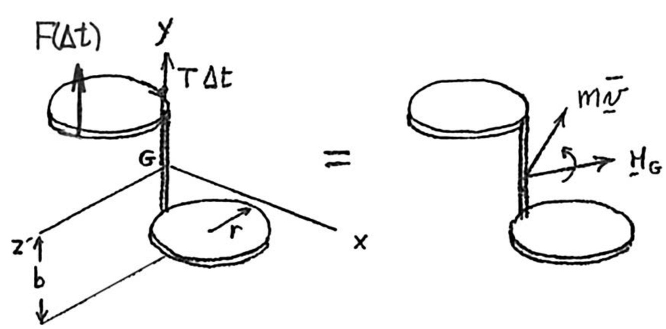
$$I_z = 2\left[\frac{1}{4}mr^2 + m(b^2 + r^2)\right]$$
$$= 2\left[\frac{1}{4}(4)(0.18)^2 + (4)(0.15^2 + 0.18^2)\right] = 0.504\ \text{kg}\cdot\text{m}^2$$

$$I_{xy} = mb(-r) + m(-b)(r) = -2mbr = -(2)(4)(0.15)(0.18) = -0.216\ \text{kg}\cdot\text{m}^2$$

$$I_{xz} = 0, \quad I_{yz} = 0, \qquad \text{total mass} = 2m$$

Constraint of cable: $(v_G)_y = 0 \quad \bar{\mathbf{v}} = \mathbf{v}_G$

Principle of impulse-momentum: $2m\bar{\mathbf{v}} = 0, \quad \mathbf{H}_G = 0$ initially.



(*a*) Direct components:

$$\mathbf{F}\Delta t = 2m\overline{\mathbf{v}}$$

$$0 = 2m\overline{v}_x \qquad \overline{v}_x = 0$$

$$F(\Delta t) + T\Delta t = 2m\overline{v}_y = 0 \qquad \overline{v}_y = \frac{F(\Delta t) + T(\Delta t)}{2m}$$

$$0 = 2m\overline{v}_z \quad v_z = 0$$

Point A moves upward. The cord becomes slack. $T(\Delta t) = 0$

$$v_y = \frac{2.4}{(2)(4)} = 0.300 \text{ m/s} \qquad \overline{\mathbf{v}} = (0.300 \text{ m/s})\mathbf{j} \blacktriangleleft$$

(*b*) <u>Moments about G.</u> $(b\mathbf{j} - r\mathbf{i}) \times (-F\Delta t\mathbf{j}) = \mathbf{H}_G$

$$-rF(\Delta t)\mathbf{i} - rF(\Delta t)\mathbf{k} = (I_x\omega_x - I_{xy}\omega_y)\mathbf{i} + (-I_{xy}\omega_x + I_y\omega_y)\mathbf{j} + I_z\omega_z\mathbf{k}$$

Resolve into components and apply the numerical data.

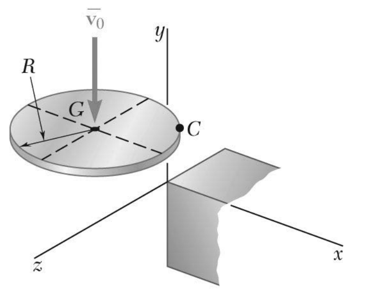
$$\mathbf{i}: \quad -(0.18)(2.4) = 0.2448\omega_x - (-0.216)\omega_y \qquad (1)$$

$$\mathbf{j}: \quad 0 = -(-0.216)\omega_x + 0.3888\omega_y \qquad (2)$$

$$\mathbf{k}: \quad -(0.18)(2.4) = 0.504\omega_z \quad \omega_z = -0.857 \text{ rad/s}$$

Solving Eqs. (1) and (2), $\omega_x = -3.46$ rad/s, $\omega_y = -1.923$ rad/s

$$\boldsymbol{\omega} = -(3.46 \text{ rad/s})\mathbf{i} + (1.923 \text{ rad/s})\mathbf{j} - (0.857 \text{ rad/s})\mathbf{k} \blacktriangleleft$$



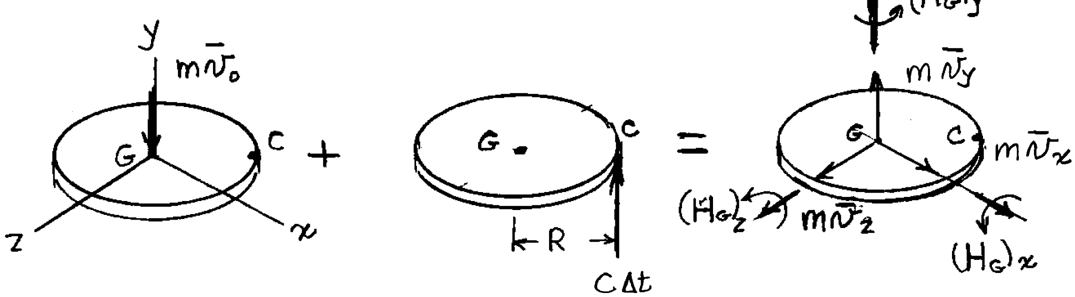
PROBLEM 18.29

A circular plate of mass m is falling with a velocity $\bar{\mathbf{v}}_0$ and no angular velocity when its edge C strikes an obstruction. Assuming the impact to be perfectly plastic $(e = 0)$, determine the angular velocity of the plate immediately after the impact.

SOLUTION

Principal moments of inertia. $\quad \bar{I}_y = \frac{1}{2}mR^2, \quad \bar{I}_x = \bar{I}_z = \frac{1}{4}mR^2$

Principle of impulse and momentum.



Syst. Momenta$_1$ + Syst. Ext. Imp.$_{1\to2}$ = Syst. Momenta$_2$

Linear momentum: $\quad -mv_0\mathbf{j} + C\Delta t\,\mathbf{j} = m\bar{v}_x\mathbf{i} + m\bar{v}_y\mathbf{j} + m\bar{v}_z\mathbf{k}$

$\mathbf{i}$: $\quad 0 = m\bar{v}_x \qquad \bar{v}_x = 0$

$\mathbf{j}$: $\quad -m\bar{\mathbf{v}}_0 + C\Delta t = m\bar{v}_y \qquad C\Delta t = m(\bar{v}_0 + \bar{v}_y) \qquad (1)$

$\mathbf{k}$: $\quad 0 = m\bar{v}_z \qquad \bar{v}_z = 0$

Geometry: $\quad \mathbf{r}_{C/G} = \frac{1}{\sqrt{2}}R(\mathbf{i} - \mathbf{k})$

Condition of impact: $\quad (e = 0) \qquad (v_C)_y = 0$

Kinematics: $\quad \bar{\mathbf{v}}_C = \bar{\mathbf{v}} + \boldsymbol{\omega} \times \mathbf{r}_{C/G}$

$$(v_C)_x\mathbf{i} + (v_C)_z\mathbf{k} = \bar{v}_y\mathbf{j} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times \frac{R}{\sqrt{2}}(\mathbf{i} - \mathbf{k})$$

$$= \bar{v}_y\mathbf{j} + \frac{R}{\sqrt{2}}\omega_x\mathbf{j} - \frac{R}{\sqrt{2}}\omega_y(\mathbf{k} + \mathbf{i}) + \frac{R}{\sqrt{2}}\omega_z\mathbf{j}$$

$\mathbf{j}$: $\quad 0 = \bar{v}_y + \frac{R}{\sqrt{2}}(\omega_x + \omega_z) \qquad \bar{v}_y = -\frac{R}{\sqrt{2}}(\omega_x + \omega_z)$

Moments about G:

$$0 + \mathbf{r}_{C/G} \times C\Delta t\,\mathbf{j} = (H_G)_x\mathbf{i} + (H_G)_y\mathbf{j} + (H_G)_z\mathbf{k}$$

$$\frac{R}{\sqrt{2}}(\mathbf{i} - \mathbf{k}) \times (C\Delta t\,\mathbf{j}) = \bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$\frac{R}{\sqrt{2}}(C\Delta t)(\mathbf{k} + \mathbf{i}) = \frac{1}{4}mR^2\omega_x\mathbf{i} + \frac{1}{2}mR^2\omega_y\mathbf{j} + \frac{1}{4}mR^2\omega_z\mathbf{k} \qquad (2)$$

$\mathbf{i}$: $$\frac{R}{\sqrt{2}}C\Delta t = \frac{1}{4}mR^2\omega_x \qquad (3)$$

$\mathbf{j}$: $$0 = \frac{1}{2}mR^2\omega_y \qquad \omega_y = 0$$

$\mathbf{k}$: $$\frac{R}{\sqrt{2}}C\Delta t = \frac{1}{4}mR^2\omega_z \qquad (4)$$

From Eqs. (1) and (2), $$C\Delta t = m\left[\bar{v}_0 - \frac{R}{\sqrt{2}}(\omega_x + \omega_z)\right]$$

$$\omega_x = 2\sqrt{2}\frac{\bar{v}_0}{R} - 2(\omega_x + \omega_z) \qquad 3\omega_x + 2\omega_z = 2\sqrt{2}\frac{\bar{v}_0}{R} \qquad (5)$$

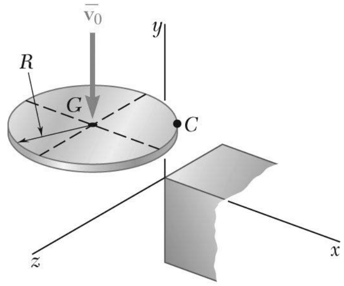
$$\omega_z = 2\sqrt{2}\frac{\bar{v}_0}{R} - 2(\omega_x + \omega_z) \qquad 2\omega_x + 3\omega_z = 2\sqrt{2}\frac{\bar{v}_0}{R} \qquad (6)$$

Solving Eqs. (5) and (6) simultaneously,

$$\omega_x = \omega_z = \frac{2\sqrt{2}}{5}\frac{\bar{v}_0}{R}$$

Angular velocity. $$\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

$$\boldsymbol{\omega} = \frac{2\sqrt{2}}{5}\frac{v_0}{R}(\mathbf{i} + \mathbf{k}) \blacktriangleleft$$



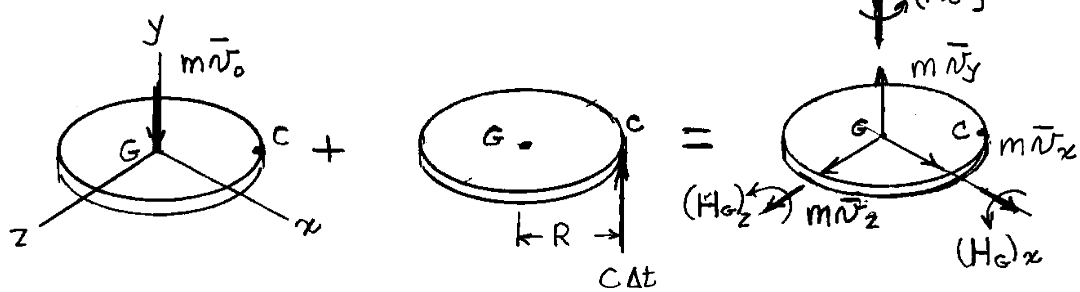
PROBLEM 18.30

For the plate of Problem 18.29, determine (*a*) the velocity of its mass center *G* immediately after the impact, (*b*) the impulse exerted on the plate by the obstruction during the impact.

SOLUTION

Principal moments of inertia. $\bar{I}_y = \frac{1}{2}mR^2, \quad \bar{I}_x = \bar{I}_z = \frac{1}{4}mR^2$

Principle of impulse and momentum.



Syst. Momenta₁ + Syst. Ext. Imp.₁→₂ = Syst. Momenta₂

Linear momentum: $-mv_0\mathbf{j} + C\Delta t\,\mathbf{j} = m\bar{v}_x\mathbf{i} + m\bar{v}_y\mathbf{j} + m\bar{v}_z\mathbf{k}$

$\mathbf{i}$: $\quad 0 = m\bar{v}_x \qquad \bar{v}_x = 0$

$\mathbf{j}$: $\quad -m\bar{v}_0 + C\Delta t = m\bar{v}_y \qquad C\Delta t = m(\bar{v}_0 + \bar{v}_y) \qquad (1)$

$\mathbf{k}$: $\quad 0 = m\bar{v}_z \qquad \bar{v}_z = 0$

Geometry: $\mathbf{r}_{C/G} = \frac{1}{\sqrt{2}}R(\mathbf{i} - \mathbf{k})$

Condition of impact: $(e = 0) \qquad (v_C)_y = 0$

Kinematics: $\bar{\mathbf{v}}_C = \bar{\mathbf{v}} + \boldsymbol{\omega} \times \mathbf{r}_{C/G}$

$$(v_C)_x\mathbf{i} + (v_C)_z\mathbf{k} = \bar{v}_y\mathbf{j} + (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}) \times \frac{R}{\sqrt{2}}(\mathbf{i} - \mathbf{k})$$

$$= \bar{v}_y\mathbf{j} + \frac{R}{\sqrt{2}}\omega_x\mathbf{j} - \frac{R}{\sqrt{2}}\omega_y(\mathbf{k} + \mathbf{i}) + \frac{R}{\sqrt{2}}\omega_z\mathbf{j}$$

$\mathbf{j}$: $\quad 0 = \bar{v}_y + \frac{R}{\sqrt{2}}(\omega_x + \omega_z) \qquad \bar{v}_y = -\frac{R}{\sqrt{2}}(\omega_x + \omega_z)$

Moments about G:

$$0+\mathbf{r}_{C/G}\times C\Delta t\,\mathbf{j}=(H_G)_x\mathbf{i}+(H_G)_y\mathbf{j}+(H_G)_z\mathbf{k}$$

$$\frac{R}{\sqrt{2}}(\mathbf{i}-\mathbf{k})\times(C\Delta t\,\mathbf{j})=\bar{I}_x\omega_x\mathbf{i}+\bar{I}_y\omega_y\mathbf{j}+I_z\omega_z\mathbf{k}$$

$$\frac{R}{\sqrt{2}}(C\Delta t)(\mathbf{k}+\mathbf{i})=\frac{1}{4}mR^2\omega_x\mathbf{i}+\frac{1}{2}mR^2\omega_y\mathbf{j}+\frac{1}{4}mR^2\omega_z\mathbf{k} \qquad (2)$$

$\mathbf{i}$: $$\frac{R}{\sqrt{2}}C\Delta t=\frac{1}{4}mR^2\omega_x \qquad (3)$$

$\mathbf{j}$: $$0=\frac{1}{2}mR^2\omega_y \qquad \omega_y=0$$

$\mathbf{k}$: $$\frac{R}{\sqrt{2}}C\Delta t=\frac{1}{4}mR^2\omega_z \qquad (4)$$

From Eqs. (1) and (2), $$C\Delta t=m\left[\bar{v}_0-\frac{R}{\sqrt{2}}(\omega_x+\omega_z)\right]$$

$$\omega_x=2\sqrt{2}\frac{\bar{v}_0}{R}-2(\omega_x+\omega_z) \qquad 3\omega_x+2\omega_z=2\sqrt{2}\frac{\bar{v}_0}{R} \qquad (5)$$

$$\omega_z=2\sqrt{2}\frac{\bar{v}_0}{R}-2(\omega_x+\omega_z) \qquad 2\omega_x+3\omega_z=2\sqrt{2}\frac{\bar{v}_0}{R} \qquad (6)$$

Solving Eqs. (5) and (6) simultaneously,

$$\omega_x=\omega_z=\frac{2\sqrt{2}}{5}\frac{\bar{v}_0}{R}$$

(*a*) *Velocity of the mass center*.

From Eq. (2), $$\bar{v}_y=-\frac{R}{\sqrt{2}}(\omega_x+\omega_z)=-\frac{4}{5}\bar{v}_0$$

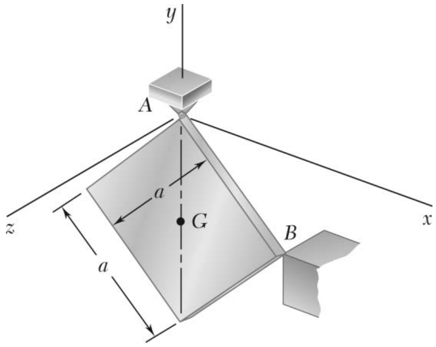
$$\bar{\mathbf{v}}=\bar{v}_x\mathbf{i}+\bar{v}_y\mathbf{j}+\bar{v}_z\mathbf{k}$$

$$\bar{\mathbf{v}}=-\frac{4}{5}\bar{v}_0\mathbf{j} \blacktriangleleft$$

(*b*) *Impulse at C*.

From Eq. (1), $$C\Delta t=m\left(\bar{v}_0-\frac{4}{5}\bar{v}_0\right)=\frac{1}{5}m\bar{v}_0$$

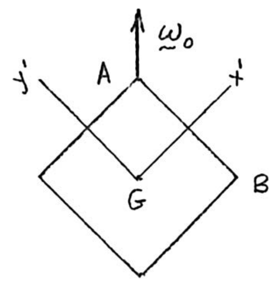
$$\mathbf{C}\Delta t=\frac{1}{5}m\bar{v}_0\mathbf{j} \blacktriangleleft$$



PROBLEM 18.31

A square plate of side a and mass m supported by a ball-and-socket joint at A is rotating about the y axis with a constant angular velocity $\boldsymbol{\omega} = \omega_0 \mathbf{j}$ when an obstruction is suddenly introduced at B in the xy plane. Assuming the impact at B to be perfectly plastic $(e = 0)$, determine immediately after impact (a) the angular velocity of the plate, (b) the velocity of its mass center G.

SOLUTION



For the x' and y' axes shown, the initial angular velocity $\omega_0 \mathbf{j}$ has components

$$\omega_{x'} = \frac{\sqrt{2}}{2}\omega_0, \qquad \omega_{y'} = \frac{\sqrt{2}}{2}\omega_0,$$

Initial angular momentum about the mass center:

$$(\mathbf{H}_G)_0 = \bar{I}_{x'}\omega_{x'}\mathbf{i}' + \bar{I}_{y'}\omega_{y'}\mathbf{j}' = \frac{1}{12}ma^2\frac{\sqrt{2}}{2}\omega_0(\mathbf{i}' + \mathbf{j}')$$

Initial velocity of the mass center: $\bar{\mathbf{v}}_0 = 0$

Let $\boldsymbol{\omega}$ be the angular velocity and $\bar{\mathbf{v}}$ be the velocity of the mass center immediately after impact.
Let $(F\Delta t)\mathbf{k}$ be the impulse at B.

Kinematics: $\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A} = (\omega_{x'}\mathbf{i}' + \omega_{y'}\mathbf{j}' + \omega_{z'}\mathbf{k}') \times (-a\mathbf{j}')$

$$\mathbf{v}_B = a(\omega_{z'}\mathbf{i}' + \omega_{x'}\mathbf{k}')$$

Since the corner B does not rebound, $(v_B)_{z'} = 0$ or $\omega_{x'} = 0$

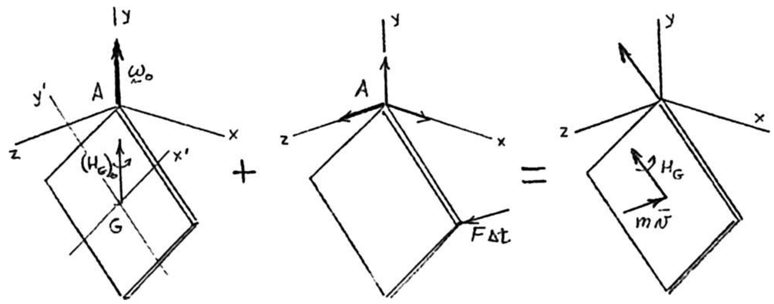
$$\bar{\mathbf{v}} = \boldsymbol{\omega} \times \mathbf{r}_{G/A} = (\omega_{y'}\mathbf{j}' + \omega_{z'}\mathbf{k}') \times \left(\frac{1}{2}a\right)(-\mathbf{i}' - \mathbf{j}')$$

$$= \frac{1}{2}a(\omega_{z'}\mathbf{i}' - \omega_{z'}\mathbf{j} + \omega_{y}\mathbf{k}')$$

Also, $\mathbf{r}_{G/A} \times m\bar{\mathbf{v}} = \frac{1}{4}ma^2(-\omega_{y'}\mathbf{i}' + \omega_{y'}\mathbf{j}' + 2\omega_{z'}\mathbf{k}')$

and $\mathbf{H}_G = I_{x'}\omega_{x'}\mathbf{i}' + I_{y'}\omega_{y'}\mathbf{j}' + \bar{I}_{z'}\omega_{z'}\mathbf{k}' = \frac{1}{12}ma^2\omega_{y'}\mathbf{j}' + \frac{1}{6}ma^2\omega_{z'}\mathbf{k}'$

<u>Principle of impulse-momentum</u>.



Moments about A:
$$(\mathbf{H}_A)_0 + (-a\mathbf{j}) \times (F\Delta t)\mathbf{k} = \mathbf{H}_A$$

$$(\mathbf{H}_G)_0 + \mathbf{r}_{G/A} \times m\bar{\mathbf{v}}_0 - (aF\Delta t)\mathbf{i} = \mathbf{H}_G + \mathbf{r}_{G/A} \times m\bar{\mathbf{v}}$$

Resolve into components.

$$\mathbf{i}': \quad \frac{1}{24}\sqrt{2}ma^2\omega_0 - aF(\Delta t) = -\frac{1}{4}ma^2\omega_{y'}$$

$$\mathbf{j}': \quad \frac{1}{24}\sqrt{2}ma^2\omega_0 = \frac{1}{12}ma^2\omega_{y'} + \frac{1}{4}ma^2\omega_{y'} \quad \omega_{y'} = \frac{\sqrt{2}}{8}\omega_0$$

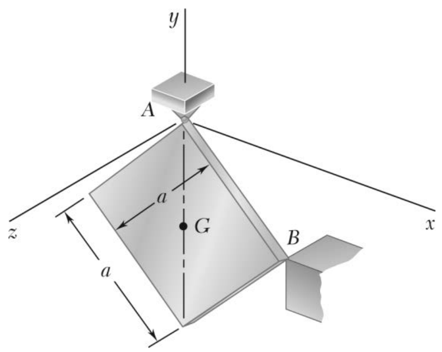
$$\mathbf{k}': \quad 0 = \frac{1}{6}ma^2\omega_{z'} + \frac{1}{2}ma^2\omega_{z'} \quad \omega_{z'} = 0$$

(*a*)
$$\boldsymbol{\omega} = \frac{\sqrt{2}}{8}\omega_0\mathbf{j}' = \frac{1}{8}\sqrt{2}\omega_0\frac{\sqrt{2}}{2}(\mathbf{j} - \mathbf{i})$$

$$\boldsymbol{\omega} = \frac{1}{8}\omega_0(-\mathbf{i} + \mathbf{j}) \blacktriangleleft$$

(*b*)
$$\bar{\mathbf{v}} = \frac{1}{2}a\omega_y\mathbf{k}' = \frac{\sqrt{2}}{16}a\omega_0\mathbf{k}$$

$$\bar{\mathbf{v}} = 0.0884a\omega_0\mathbf{k} \blacktriangleleft$$

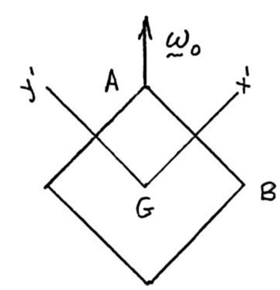


PROBLEM 18.32

Determine the impulse exerted on the plate of Problem 18.31 during the impact by (*a*) the obstruction at *B*, (*b*) the support at *A*.

PROBLEM 18.31 A square plate of side *a* and mass *m* supported by a ball-and-socket joint at *A* is rotating about the *y* axis with a constant angular velocity $\boldsymbol{\omega} = \omega_0 \mathbf{j}$ when an obstruction is suddenly introduced at *B* in the *xy* plane. Assuming the impact at *B* to be perfectly plastic $(e = 0)$, determine immediately after impact (*a*) the angular velocity of the plate, (*b*) the velocity of its mass center *G*.

SOLUTION



For the simpler x' and y' axes, the initial angular velocity $\omega_0 \mathbf{j}$ has components

$$\omega_{x'} = \frac{\sqrt{2}}{2}\omega_0, \qquad \omega_{y'} = \frac{\sqrt{2}}{2}\omega_0,$$

Initial angular momentum about the mass center:

$$(\mathbf{H}_G)_0 = \bar{I}_{x'}\omega_{x'}\mathbf{i}' + \bar{I}_{y'}\omega_{y'}\mathbf{j}' = \frac{1}{12}ma^2\frac{\sqrt{2}}{2}\omega_0(\mathbf{i}' + \mathbf{j}')$$

Initial velocity of the mass center: $\quad \bar{\mathbf{v}}_0 = 0$

Let $\boldsymbol{\omega}$ be the angular velocity and $\bar{\mathbf{v}}$ be the velocity of the mass center immediately after impact.
Let $(F\Delta t)\mathbf{k}$ be the impulse at *B*.

Kinematics:

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A} = (\omega_{x'}\mathbf{i}' + \omega_{y'}\mathbf{j}' + \omega_{z'}\mathbf{k}') \times (-a\mathbf{j}')$$

$$\mathbf{v}_B = a(\omega_{z'}\mathbf{i}' + \omega_{x'}\mathbf{k}')$$

Since the corner *B* does not rebound, $\quad (v_B)_{z'} = 0 \quad$ or $\quad \omega_{x'} = 0$

$$\bar{\mathbf{v}} = \boldsymbol{\omega} \times \mathbf{r}_{G/A} = (\omega_{y'}\mathbf{j}' + \omega_{z'}\mathbf{k}') \times \left(\frac{1}{2}a\right)(-\mathbf{i}' - \mathbf{j}')$$

$$= \frac{1}{2}a(\omega_{z'}\mathbf{i}' - \omega_{z'}\mathbf{j} + \omega_y\mathbf{k}')$$

Also,

$$\mathbf{r}_{G/A} \times m\bar{\mathbf{v}} = \frac{1}{4}ma^2(-\omega_y\mathbf{i}' + \omega_{y'}\mathbf{j}' + 2\omega_{z'}\mathbf{k}')$$

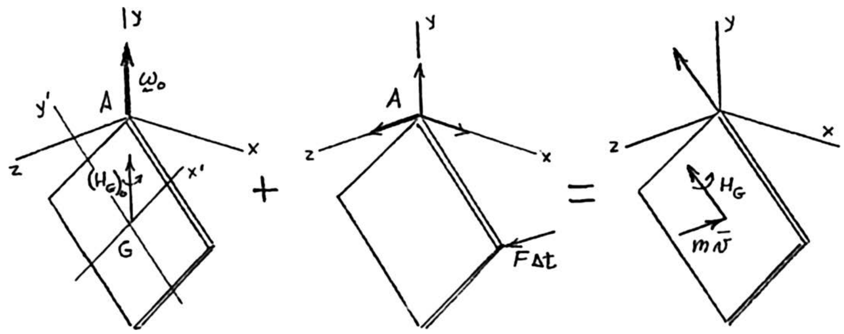
and

$$\mathbf{H}_G = I_{x'}\omega_{x'}\mathbf{i}' + I_{y'}\omega_{y'}\mathbf{j}' + \bar{I}_{z'}\omega_{z'}\mathbf{k}'$$

$$= \frac{1}{12}ma^2\omega_{y'}\mathbf{j}' + \frac{1}{6}ma^2\omega_{z'}\mathbf{k}'$$

PROBLEM 18.32 (Continued)

Principle of impulse-momentum.



Moments about A: $$(\mathbf{H}_A)_0 + (-a\mathbf{j}')\times(F\Delta t)\mathbf{k} = \mathbf{H}_A$$

$$(\mathbf{H}_G)_0 + \mathbf{r}_{G/A}\times m\bar{\mathbf{v}}_0 + aF\Delta t\mathbf{i}' = \mathbf{H}_G + \mathbf{r}_{G/A}\times m\bar{\mathbf{v}}$$

Resolve into components.

$$\mathbf{i}':\quad \frac{1}{24}\sqrt{2}ma^2\omega_0 - aF(\Delta t) = -\frac{1}{4}ma^2\omega_{y'} \qquad (1)$$

$$\mathbf{j}':\quad \frac{1}{24}\sqrt{2}ma^2\omega_0 = \frac{1}{12}ma^2\omega_{y'} + \frac{1}{4}ma^2\omega_{y'} \qquad \omega_{y'} = \frac{1}{8}\sqrt{2}\omega_0$$

$$\mathbf{k}':\quad 0 = \frac{1}{6}ma^2\omega_{z'} + \frac{1}{2}ma^2\omega_{z'} \qquad \omega_{z'} = 0$$

(*a*) From Eq. (1), $$F\Delta t = \frac{1}{24}\sqrt{2}ma\omega_0 + \frac{1}{32}\sqrt{2}ma\omega_0 = \frac{7}{96}\sqrt{2}ma\omega_0$$

$$(F\Delta t)\mathbf{k} = 0.1031ma\omega_0\mathbf{k} \blacktriangleleft$$

$$\bar{\mathbf{v}} = \frac{1}{2}a\omega_y\mathbf{k}' = \frac{1}{16}\sqrt{2}a\omega_0\mathbf{k}'$$

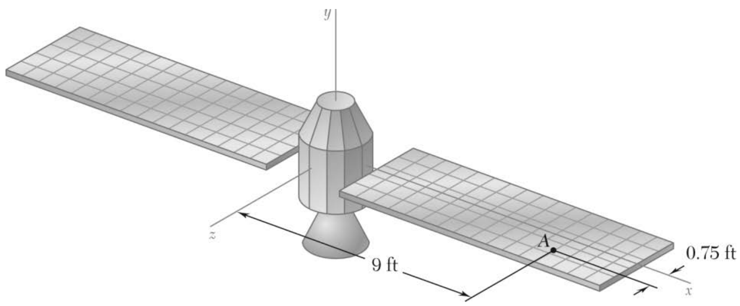
Linear momentum: $$m\bar{\mathbf{v}}_0 + \mathbf{A}\Delta t + F\Delta t\mathbf{k} = m\bar{\mathbf{v}}$$

$$0 + \mathbf{A}\Delta t + \frac{7}{96}\sqrt{2}ma\omega_0\mathbf{k}' = \frac{1}{16}\sqrt{2}ma\omega_0\mathbf{k}'$$

(*b*) $$\mathbf{A}\Delta t = -\frac{1}{96}\sqrt{2}ma\omega_0$$ $$\mathbf{A}\Delta t = -0.01473ma\omega_0\mathbf{k} \blacktriangleleft$$

PROBLEM 18.33

The coordinate axes shown represent the principal centroidal axes of inertia of a 3000-lb space probe whose radii of gyration are $k_x = 1.375$ ft, $k_y = 1.425$ ft, and $k_z = 1.250$ ft. The probe has no angular velocity when a 5-oz meteorite strikes one of its solar panels at Point A with a velocity $\mathbf{v}_0 = (2400 \text{ ft/s})\mathbf{i} - (3000 \text{ ft/s})\mathbf{j} + (3200 \text{ ft/s})\mathbf{k}$ relative to the probe. Knowing that the meteorite emerges on the other side of the panel with no change in the direction of its velocity, but with a speed reduced by 20 percent, determine the final angular velocity of the probe.



SOLUTION

Masses: Space probe: $m' = \dfrac{3000}{32.2} = 93.17 \text{ lb} \cdot \text{s}^2/\text{ft}$

Meteorite: $m = \dfrac{5}{(16)(32.2)} = 0.009705 \text{ lb} \cdot \text{s}^2/\text{ft}$

Point of impact: $\text{r}_A = (9 \text{ ft})\mathbf{i} + (0.75 \text{ ft})\mathbf{k}$

Initial linear momentum of the meteorite, $(\text{lb} \cdot \text{s})$:

$$m\mathbf{v}_0 = (0.009705)(2400\mathbf{i} + 3000\mathbf{j} + 3200\mathbf{k}) = 23.292\mathbf{i} - 29.115\mathbf{j} + 31.056\mathbf{k}$$

Its moment about the origin, $(\text{lb} \cdot \text{ft} \cdot \text{s})$:

$$\mathbf{r}_A \times m\mathbf{v}_0 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 9 & 0 & 0.75 \\ 23.292 & -29.115 & 31.056 \end{vmatrix} = 21.836\mathbf{i} - 262.04\mathbf{j} - 262.04\mathbf{k}$$

Final linear momentum of meteorite and its moment about the origin, $(\text{lb} \cdot \text{s})$ and $(\text{lb} \cdot \text{s} \cdot \text{ft})$:

$$0.8m\mathbf{v}_0 = 18.634\mathbf{i} - 23.292\mathbf{j} + 24.845\mathbf{k}$$

$$\mathbf{r}_A \times (0.8m\mathbf{v}_0) = 17.469\mathbf{i} - 209.63\mathbf{j} - 209.63\mathbf{k}$$

PROBLEM 18.33 (Continued)

Let $\mathbf{H}_A$ be the angular momentum of the probe and m' be its mass. Conservation of angular momentum about the origin for a system of particles consisting of the probe plus the meteorite:

$$\mathbf{r}_A \times m\mathbf{v}_0 = \mathbf{H}_A + \mathbf{r}_A \times (0.8m\mathbf{v}_0)$$

$$\mathbf{H}_A = (4.367 \text{ lb}\cdot\text{s}\cdot\text{ft})\mathbf{i} - (52.41 \text{ lb}\cdot\text{s}\cdot\text{ft})\mathbf{j} - (52.41 \text{ lb}\cdot\text{s}\cdot\text{ft})\mathbf{k}$$

$$I_x\omega_x = (H_A)_x \qquad \omega_x = \frac{(H_A)_x}{m'k_x^2} = \frac{4.367}{(93.17)(1.375)^2} = 0.02479 \text{ rad/s}$$

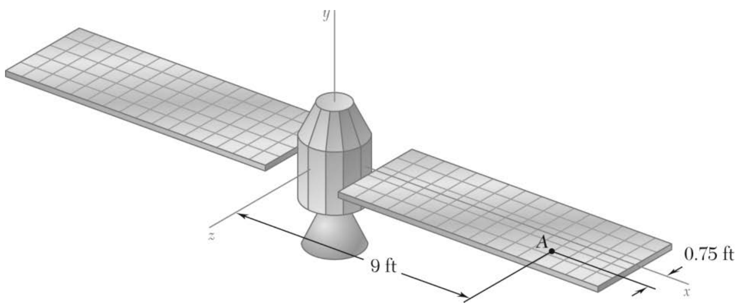
$$I_y\omega_y = (H_A)_y \qquad \omega_y = \frac{(H_A)_y}{m'k_y^2} = \frac{-52.41}{(93.17)(1.425)^2} = -0.2770 \text{ rad/s}$$

$$I_z\omega_z = (H_A)_z \qquad \omega_z = \frac{(H_A)_z}{m'k_z^2} = \frac{-52.41}{(93.17)(1.250)^2} = -0.3600 \text{ rad/s}$$

$$\boldsymbol{\omega} = (0.0248 \text{ rad/s})\mathbf{i} - (0.277 \text{ rad/s})\mathbf{j} - (0.360 \text{ rad/s})\mathbf{k} \blacktriangleleft$$

PROBLEM 18.34

The coordinate axes shown represent the principal centroidal axes of inertia of a 3000-lb space probe whose radii of gyration are $k_x = 1.375$ ft, $k_y = 1.425$ ft, and $k_z = 1.250$ ft. The probe has no angular velocity when a 5-oz meteorite strikes one of its solar panels at Point A and emerges on the other side of the panel with no change in the direction of its velocity, but with a speed reduced by 25 percent. Knowing that the final angular velocity of the probe is $\boldsymbol{\omega} = (0.05 \text{ rad/s})\mathbf{i} - (0.12 \text{ rad/s})\mathbf{j} + \omega_z\mathbf{k}$ and that the x component of the resulting change in the velocity of the mass center of the probe is -0.675 in./s, determine (a) the component ω_z of the final angular velocity of the probe, (b) the relative velocity $\mathbf{v}_0$ with which the meteorite strikes the panel.



SOLUTION

Masses: Space probe: $m' = \dfrac{3000}{32.2} = 93.17 \text{ lb} \cdot \text{s}^2/\text{ft}$

Meteorite: $m = \dfrac{5}{(16)(32.2)} = 0.009705 \text{ lb} \cdot \text{s}^2/\text{ft}$

Point of impact: $\mathbf{r}_A = (9 \text{ ft})\mathbf{i} + (0.75 \text{ ft})\mathbf{k}$

Initial linear momentum of the meteorite, $(\text{lb} \cdot \text{s})$:

$$m\mathbf{v}_0 = (0.009705)(v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k})$$

Its moment about the origin, $(\text{lb} \cdot \text{ft} \cdot \text{s})$:

$$(\mathbf{H}_A)_0 = \mathbf{r}_A \times m\mathbf{v}_0 = 0.009705\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 9 & 0 & 0.75 \\ v_x & v_y & v_z \end{vmatrix}$$

$$= 0.009705[-0.75v_y\mathbf{i} + (0.75v_x - 9v_z)\mathbf{j} + 9v_y\mathbf{k}]$$

Final linear momentum of the meteorite, $(\text{lb} \cdot \text{s})$:

$$0.75m\mathbf{v}_0 = 0.007279(v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k})$$

Its moment about the origin, $(\text{lb} \cdot \text{ft} \cdot \text{s})$:

$$\mathbf{r}_A \times (0.75m\mathbf{v}_0) = 0.007279[-0.75v_y\mathbf{i} + (0.75v_x - 9v_z)\mathbf{j} + 9v_y\mathbf{k}]$$

Initial linear momentum of the space probe, $(\text{lb}\cdot\text{s})$: $m'\mathbf{v}'_0 = 0$

Final linear momentum of the space probe, $(\text{lb}\cdot\text{s})$:

$$m'(v'_x\mathbf{i} + v'_y\mathbf{j} + v'_z\mathbf{k}) = 93.17\left(-\frac{0.675}{12}\mathbf{i} + v'_y\mathbf{j} + v'_z\mathbf{k}\right)$$

Final angular momentum of space probe, $(\text{lb}\cdot\text{ft}\cdot\text{s})$:

$$\mathbf{H}_A = m'\left(k_x^2\omega_x\mathbf{i} + k_y^2\omega_y\mathbf{j} + k_z^2\omega_z\mathbf{k}\right)$$

$$= 93.17[(1.375)^2(0.05)\mathbf{i} + (1.425)^2(-0.12)\mathbf{j} + (1.250)^2\omega_z\mathbf{k}]$$

$$= 8.8075\mathbf{i} - 22.703\mathbf{j} + 145.58\omega_z\mathbf{k}$$

Conservation of linear momentum of the probe plus the meteorite, $(\text{lb}\cdot\text{s})$:

$$0.009705(v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}) = 0.007279(v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}) + 93.17(-0.05625\mathbf{i} + v'_y\mathbf{j} + v'_z\mathbf{k})$$

i: $0.002426v_x = -5.2408$ $\qquad v_x = -2160\text{ ft/s}$

j: $0.002426v_y = 93.17v'_y$

k: $0.002426v_z = 93.17v'_z$

Conservation of angular momentum about the origin, $(\text{lb}\cdot\text{ft}\cdot\text{s})$:

$$(0.009705)[-0.75v_y\mathbf{i} + (0.75v_x - 9v_z)\mathbf{j} + 9v_y\mathbf{k}]$$

$$= (0.007279)[-0.75v_y\mathbf{i} + (0.75v_x - 9v_z)\mathbf{j} + 9v_y\mathbf{k}] + 8.8075\mathbf{i} - 22.703\mathbf{j} + 145.58\omega_z\mathbf{k}$$

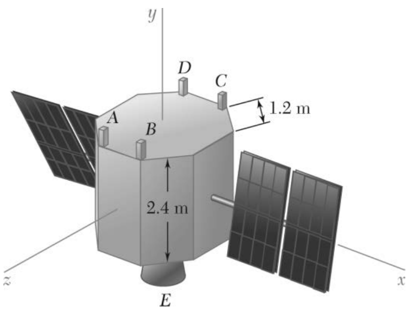
i: $-0.0018195v_y = 8.8075$ $\qquad v_y = 4840.5\text{ ft/s}$

j: $-0.021834v_z + 0.0018195v_x = -22.703$ $\qquad v_z = 0.08333v_x + 1039.8 = 859.8\text{ ft/s}$

k: $-0.021834v_y = 145.58\omega_z$

(*a*) $\omega_z = -149.98\times10^{-6}v_y$ $\qquad \omega_z = -0.726\text{ rad/s}$ ◀

(*b*) $\mathbf{v}_0 = -(2160\text{ ft/s})\mathbf{i} - (4840\text{ ft/s})\mathbf{j} + (860\text{ ft/s})\mathbf{k}$ ◀



PROBLEM 18.35

A 2500-kg probe in orbit about the moon is 2.4 m high and has octagonal bases of sides 1.2 m. The coordinate axes shown are the principal centroidal axes of inertia of the probe, and its radii of gyration are $k_x = 0.98$ m, $k_y = 1.06$ m, and $k_z = 1.02$ m. The probe is equipped with a main 500-N thruster *E* and with four 20-N thrusters *A*, *B*, *C*, and *D* which can expel fuel in the positive *y* direction. The probe has an angular velocity $\boldsymbol{\omega} = (0.040 \text{ rad/s})\mathbf{i} + (0.060 \text{ rad/s})\mathbf{k}$ when two of the 20-N thrusters are used to reduce the angular velocity to zero. Determine (*a*) which of the thrusters should be used, (*b*) the operating time of each of these thrusters, (*c*) for how long the main thruster *E* should be activated if the velocity of the mass center of the probe is to remain unchanged.

SOLUTION

$$\mathbf{H}_G = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k} = m\left(k_x^2\omega_x\mathbf{i} + k_y^2\omega_y^2\mathbf{j} + k_2^2\omega_z\mathbf{k}\right)$$

$$= (2500)[(0.98)^2(0.040)\mathbf{i} + (1.06)^2(0)\mathbf{j} + (1.02)^2(0.060)\mathbf{k}]$$

$$= (96.04 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} + (156.06 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

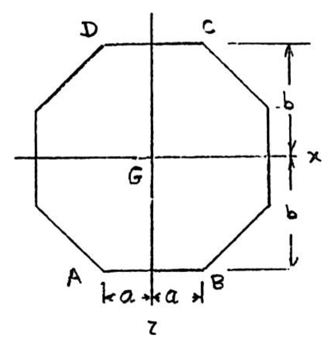
Let $-A\mathbf{j}$, $-B\mathbf{j}$, $-C\mathbf{j}$, and $-D\mathbf{j}$ be the impulses provided by the 20 N thrusters at *A*, *B*, *C*, and *D*, respectively. Let $E\mathbf{j}$ be that provided by the 500 N main thruster.

Position vectors for intersections of the lines of action of the thruster impulses with the *xz* plane:

$$a = \frac{1}{2}(1.2) = 0.6 \text{ m}, \qquad b = 0.6 + 0.6\sqrt{2} = 1.4485 \text{ m}$$

$$\mathbf{r}_A = -a\mathbf{i} + b\mathbf{k}, \qquad \mathbf{r}_B = a\mathbf{i} + b\mathbf{k}$$

$$\mathbf{r}_C = a\mathbf{i} - b\mathbf{k}, \qquad \mathbf{r}_D = -a\mathbf{i} - b\mathbf{k}$$



The final linear and angular momenta are zero.

Principle of impulse-momentum. Moments about *G*:

$$\mathbf{H}_G + \mathbf{r}_A \times (-A\mathbf{j}) + \mathbf{r}_B \times (-B\mathbf{j}) + \mathbf{r}_C \times (-C\mathbf{j}) + \mathbf{r}_D \times (-D\mathbf{j}) = 0$$

$$\mathbf{H}_G + b(A + B - C - D)\mathbf{i} + a(A - B - C + D)\mathbf{k} = 0$$

Resolve into components.

$$\mathbf{i}: \quad A + B - C - D = -\frac{(H_G)_x}{b} = -\frac{96.04}{1.4485} = -66.30 \text{ N}\cdot\text{s} \qquad (1)$$

$$\mathbf{k}: \quad A - B - C + D = -\frac{(H_G)_z}{a} = -\frac{156.06}{0.6} = -260.1 \text{ N}\cdot\text{s} \qquad (2)$$

Of *A*, *B*, *C*, and *D*, two must be zero or positive, the other two zero.

PROBLEM 18.35 (Continued)

Set $A = 0$ and $B - D = N$. Solve the simultaneous equations (1) and (2).

$C = 163.2\ \text{N}\cdot\text{s}$ and $N = 96.9$. Set $D = 0$ and $B = 96.9\ \text{N}\cdot\text{s}$

(*a*) Use thrusters C and B. ◀

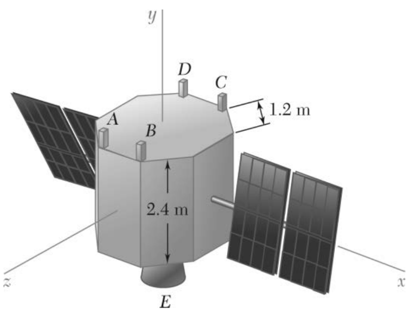
(*b*) $$F_C(\Delta t_C) = C, \qquad \Delta t_C = \frac{C}{F_C} = \frac{163.2}{20}$$ $\Delta t_C = 8.16\ \text{s}$ ◀

$$F_B(\Delta t_B) = B, \qquad \Delta t_B = \frac{B}{F_B} = \frac{96.9}{20}$$ $\Delta t_B = 4.84\ \text{s}$ ◀

(*c*) Linear momentum: $E\mathbf{j} - B\mathbf{j} - C\mathbf{j} = 0, \qquad E = 30.291\ \text{lb}\cdot\text{s}$

$$F_E(\Delta t_E) = E \qquad \Delta t_E = \frac{E}{F_E} = \frac{260.1}{500}$$ $\Delta t_E = 0.520\ \text{s}$ ◀

PROBLEM 18.36



Solve Problem 18.35, assuming that the angular velocity of the probe is $\boldsymbol{\omega} = (0.060 \text{ rad/s})\mathbf{i} - (0.040 \text{ rad/s})\mathbf{k}$.

PROBLEM 18.35 A 2500-kg probe in orbit about the moon is 2.4 m high and has octagonal bases of sides 1.2 m. The coordinate axes shown are the principal centroidal axes of inertia of the probe, and its radii of gyration are $k_x = 0.98$ m, $k_y = 1.06$ m, and $k_z = 1.02$ m. The probe is equipped with a main 500-N thruster *E* and with four 20-N thrusters *A*, *B*, *C*, and *D* which can expel fuel in the positive *y* direction. The probe has an angular velocity $\boldsymbol{\omega} = (0.040 \text{ rad/s})\mathbf{i} + (0.060 \text{ rad/s})\mathbf{k}$ when two of the 20-N thrusters are used to reduce the angular velocity to zero. Determine (*a*) which of the thrusters should be used, (*b*) the operating time of each of these thrusters, (*c*) for how long the main thruster *E* should be activated if the velocity of the mass center of the probe is to remain unchanged.

SOLUTION

$$\mathbf{H}_G = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$= m\left(k_x^2\omega_x\mathbf{i} + k_y^2\omega_y^2\mathbf{j} + k_2^2\omega_z\mathbf{k}\right)$$

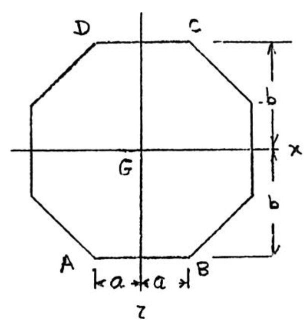
$$= (2500)[(0.98)^2(0.060) + (1.06)^2(0) + (1.02)^2(-0.040)\mathbf{k}]$$

$$= (144.06 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (104.04 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

Let $-A\mathbf{j}, -B\mathbf{j}, -C\mathbf{j},$ and $-D\mathbf{j}$ be the impulses provided by the 20 N thrusters at *A*, *B*, *C*, and *D*, respectively. Let $E\mathbf{j}$ be that provided by the 500 N main thruster.

Position vectors for intersections of the lines of action of the thruster impulses with the *xz* plane:

$$a = \frac{1}{2}(1.2) = 0.6 \text{ m}, \qquad b = 0.6 + 0.6\sqrt{2} = 1.4485 \text{ m}$$



$$\mathbf{r}_A = -a\mathbf{i} + b\mathbf{k}, \qquad \mathbf{r}_B = a\mathbf{i} + b\mathbf{k}$$

$$\mathbf{r}_C = a\mathbf{i} - b\mathbf{k}, \qquad \mathbf{r}_D = -a\mathbf{i} - b\mathbf{k}$$

The final linear and angular momenta are zero.

Principle of impulse-momentum. Moments about *G*:

$$\mathbf{H}_G + \mathbf{r}_A \times (-A\mathbf{j}) + \mathbf{r}_B \times (-B\mathbf{j}) + \mathbf{r}_C \times (-C\mathbf{j}) + \mathbf{r}_D \times (-D\mathbf{j}) = 0$$

$$\mathbf{H}_G + b(A + B - C - D)\mathbf{i} + a(A - B - C + D)\mathbf{k} = 0$$

PROBLEM 18.36 (Continued)

Resolve into components.

$$\mathbf{i}: \quad A + B - C - D = -\frac{(H_G)_x}{b} = -\frac{144.06}{1.4485} = -99.455 \text{ N}\cdot\text{s} \qquad (1)$$

$$\mathbf{k}: \quad A - B - C + D = -\frac{(H_G)_z}{a} = -\frac{-104.04}{0.6} = 173.4 \text{ N}\cdot\text{s} \qquad (2)$$

Of A, B, C, and D, two must be zero or positive, the other two zero.

Set $B = 0$ and $A - C = N$. Solve the simultaneous equations (1) and (2).

$D = 136.43 \text{ N}\cdot\text{s}$ and $N = 36.97 \text{ N}\cdot\text{s}$. Set $C = 0$ and $A = 36.97 \text{ N}\cdot\text{s}$

(*a*) Use thrusters D and A. ◀

(*b*) $F_D(\Delta t_D) = D, \qquad \Delta t_D = \frac{D}{F_D} = \frac{136.43}{20}$ $\qquad \Delta t_D = 6.82 \text{ s}$ ◀

$F_A(\Delta t_A) = A, \qquad \Delta t_A = \frac{A}{F_A} = \frac{36.97}{20}$ $\qquad \Delta t_A = 1.848 \text{ s}$ ◀

(*c*) Linear momentum: $E\mathbf{j} - D\mathbf{j} - A\mathbf{j} = 0 \qquad E = 173.4 \text{ N}\cdot\text{s}$

$F_E(\Delta t_E) = E \qquad \Delta t_E = \frac{E}{F_E} = \frac{173.4}{500}$ $\qquad \Delta t_E = 0.347 \text{ s}$ ◀

PROBLEM 18.37

Denoting, respectively, by $\boldsymbol{\omega}$, $\mathbf{H}_O$, and T the angular velocity, the angular momentum, and the kinetic energy of a rigid body with a fixed Point O, (*a*) prove that $\mathbf{H}_O \cdot \boldsymbol{\omega} = 2T$; (*b*) show that the angle θ between ω and $\mathbf{H}_O$ will always be acute.

SOLUTION

(*a*)

$$\mathbf{H}_0 = (I_x\omega_x - I_{xy}\omega_y - I_{xz}\omega_z)\mathbf{i} + (-I_{xy}\omega_x + I_y\omega_y - I_{yz}\omega_z)\mathbf{j} + (-I_{xz}\omega_x - I_{yz}\omega_y + I_z\omega_z)\mathbf{k}$$

$$\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

$$\mathbf{H}_0 \cdot \boldsymbol{\omega} = I_x\omega_x^2 - I_{xy}\omega_y\omega_x - I_{xz}\omega_z\omega_x - I_{xy}\omega_x\omega_y + I_y\omega_y^2 - I_{yz}\omega_z\omega_y$$

$$-I_{xz}\omega_x\omega_z - I_{yz}\omega_y\omega_z + I_z\omega_z^2$$

$$= (2)\left(\frac{1}{2}\right)\left(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 - 2I_{xy}\omega_x\omega_y - 2I_{yz}\omega_y\omega_z - 2I_{xz}\omega_x\omega_z\right)$$

$$= 2T$$

(*b*)

$$\mathbf{H}_0 \cdot \boldsymbol{\omega} = H_0\omega\cos\theta$$

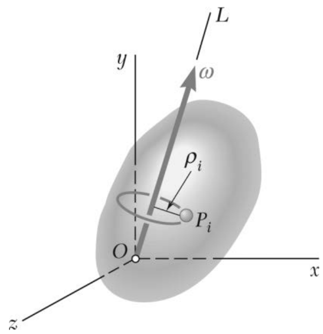
$$2T = H_0\omega\cos\theta$$

$$\cos\theta = \frac{2T}{H_0\omega}$$

But $\quad T > 0,\ H_0 > 0,\ \omega > 0$

$$\cos\theta > 0 \qquad \theta < 90°$$

PROBLEM 18.38



Show that the kinetic energy of a rigid body with a fixed Point O can be expressed as $T = \frac{1}{2} I_{OL} \omega^2$, where ω is the instantaneous angular velocity of the body and I_{OL} is its moment of inertia about the line of action OL of ω. Derive this expression (*a*) from Eqs. (9.46) and (18.19), (*b*) by considering T as the sum of the kinetic energies of particles P_i describing circles of radius ρ_i about line OL.

SOLUTION

(*a*)

$$T = \frac{1}{2}\left(I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2 - 2I_{xy}\omega_x\omega_y - 2I_{yz}\omega_y\omega_z - 2I_{xz}\omega_x\omega_z \right)$$

Let

$$\omega_x = \omega \cos\theta_x = \omega\lambda_x$$

$$\omega_y = \omega \cos\theta_y = \omega\lambda_y$$

$$\omega_z = \omega \cos\theta_z = \omega\lambda_z$$

$$T = \frac{1}{2}\left(I_x \lambda_x^2 + I_y \lambda_y^2 + I_z \lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{xz}\lambda_x\lambda_z \right)\omega^2$$

$$= \frac{1}{2} I_{OL} \omega^2 \qquad T = \frac{1}{2} I_{OL}\omega^2 \blacktriangleleft$$

(*b*) Each particle of mass $(\Delta m)_i$ describes a circle of radius ρ_i.

The speed of the particle is $v_i = \rho_i \omega$.

Its kinetic energy is

$$(\Delta T)_i = \frac{1}{2}(\Delta m_i) v_i^2 = \frac{1}{2}(\Delta m_i)\rho_i^2 \omega^2$$

The kinetic energy of the entire body is

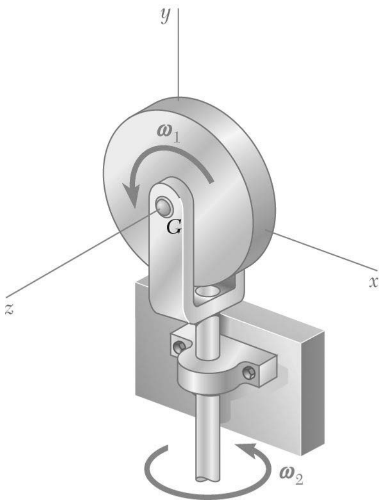
$$T = \Sigma(\Delta T)_i = \frac{1}{2}\Sigma(\Delta m_i)\rho_i^2\omega^2$$

but

$$I_{OL} = \Sigma(\Delta m_i)\rho_i^2$$

Hence,

$$T = \frac{1}{2} I_{OL}\omega^2 \blacktriangleleft$$



PROBLEM 18.39

Determine the kinetic energy of the disk of Problem 18.1.

PROBLEM 18.1 A thin, homogeneous disk of mass m and radius r spins at the constant rate ω_1 about an axle held by a fork-ended vertical rod, which rotates at the constant rate ω_2. Determine the angular momentum $\mathbf{H}_G$ of the disk about its mass center G.

SOLUTION

Angular velocity: $\boldsymbol{\omega} = \omega_2 \mathbf{j} + \omega_1 \mathbf{k}$

Moments of inertia: $\bar{I}_x = \frac{1}{4}mr^2, \quad \bar{I}_y = \frac{1}{4}mr^2, \quad \bar{I}_z = \frac{1}{2}mr^2$

Products of inertia: by symmetry, $\bar{I}_{xy} = \bar{I}_{yz} = \bar{I}_{zx} = 0$

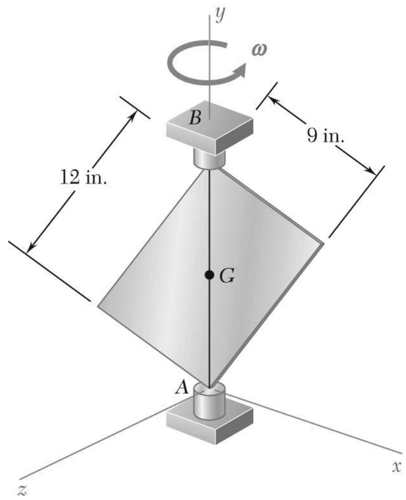
Kinetic energy:

$$T = \frac{1}{2}\left(\bar{I}_x\omega_x^2 + \bar{I}_y\omega_y^2 + \bar{I}_z\omega_z\right)$$

$$T = \frac{1}{2}\left[0 + \left(\frac{1}{4}mr^2\right)\omega_2^2 + \left(\frac{1}{2}mr^2\right)\omega_1^2\right]$$

$$T = \frac{1}{8}mr^2\left(\omega_2^2 + 2\omega_1^2\right) \blacktriangleleft$$

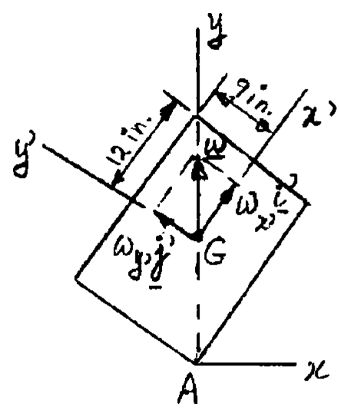
PROBLEM 18.40



Determine the kinetic energy of the plate of Problem 18.2.

PROBLEM 18.2 A thin rectangular plate of weight 15 lb rotates about its vertical diagonal *AB* with an angular velocity ω. Knowing that the *z* axis is perpendicular to the plate and that ω is constant and equal to 5 rad/s, determine the angular momentum of the plate about its mass center *G*.

SOLUTION



$$h = \sqrt{(9\text{ in.})^2 + (12\text{ in.})^2} = 15\text{ in.}$$

Resolving ω along the principal axes x', y', z:

$$\omega_{x'} = \frac{12}{15}\omega = 0.8(5\text{ rad/s}) = 4\text{ rad/s}$$

$$\omega_{y'} = \frac{9}{15}\omega = 0.6(5\text{ rad/s}) = 3\text{ rad/s}$$

$$\omega_z = 0$$

Moments of inertia:

$$I_{x'} = \frac{1}{12}\left(\frac{15\text{ lb}}{32.2}\right)\left(\frac{9}{12}\text{ ft}\right) = 0.021836\text{ slug}\cdot\text{ft}^2$$

$$I_{y'} = \frac{1}{12}\left(\frac{15\text{ lb}}{32.2}\right)\left(\frac{12}{12}\text{ ft}\right)^2 = 0.038820\text{ slug}\cdot\text{ft}^2$$

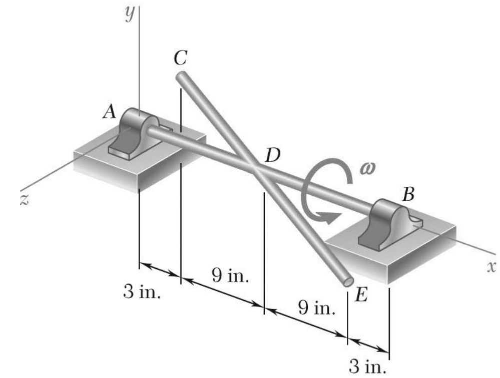
From Eqs. (18.20):

$$T = \frac{1}{2}(\bar{I}_{x'}\omega_{x'}^2 + \bar{I}_{y'}\omega_{y'}^2 + \bar{I}_{z'}\omega_{z'}^2)$$

$$= \frac{1}{2}[(0.021836)(4)^2 + (0.038820)(3)^2 + 0]$$

$$T = 0.34938\text{ ft}\cdot\text{lb}$$

$T = 0.349\text{ ft}\cdot\text{lb}$ ◀

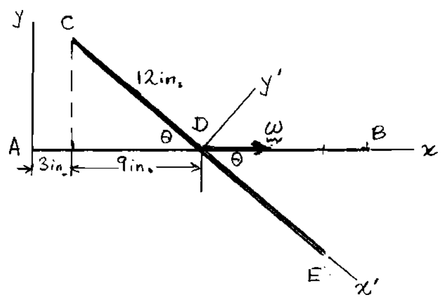


PROBLEM 18.41

Determine the kinetic energy of the assembly of Problem 18.3.

PROBLEM 18.3 Two uniform rods *AB* and *CE*, each of weight 3 lb and length 2 ft, are welded to each other at their midpoints. Knowing that this assembly has an angular velocity of constant magnitude $\omega = 12$ rad/s, determine the magnitude and direction of the angular momentum $\mathbf{H}_D$ of the assembly about *D*.

SOLUTION



$$m = \frac{W}{g} = \frac{3}{32.2} = 0.093168 \text{ lb}\cdot\text{s}^2/\text{ft}, \qquad l = 24 \text{ in.} = 2 \text{ ft},$$

$$\boldsymbol{\omega} = (12 \text{ rad/s})\mathbf{i}$$

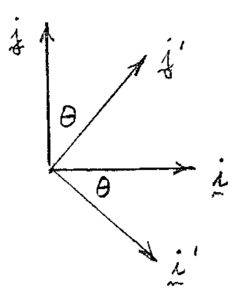
For rod *ADB*, $T = \frac{1}{2} I_x \omega^2 \approx 0,$ since $I_x \approx 0.$

For rod *CDE*, use principal axes x', y' as shown.

$$\cos\theta = \frac{9}{12}, \qquad \theta = 41.410°$$

$$\omega_{x'} = \omega\cos\theta = 9 \text{ rad/s}^2$$

$$\omega_{y'} = \omega\sin\theta = 7.93725 \text{ rad/s}^2$$



$$\omega_{z'} = 0$$

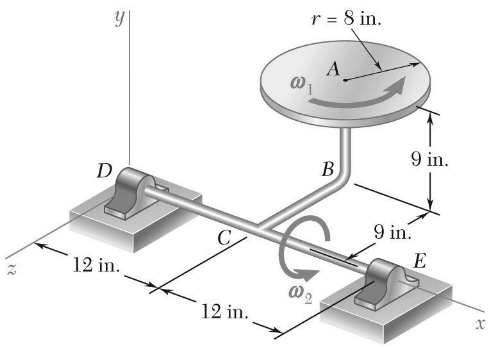
$$\bar{I}_{x'} \approx 0$$

$$\bar{I}_{y'} = \frac{1}{12} ml^2 = \frac{1}{12}(0.093168)(2)^2$$
$$= 0.0310559 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$T = \frac{1}{2} m\bar{v}^2 + \frac{1}{2}\bar{I}_{x'}\omega_{x'}^2 + \frac{1}{2}\bar{I}_{y'}\omega_{y'}^2 + \frac{1}{2}\bar{I}_{z'}\omega_{z'}^2$$

$$= 0 + 0 + \frac{1}{2}(0.0310559)(7.93725)^2 + 0$$

$$= 0.97826 \text{ ft}\cdot\text{lb} \qquad T = 0.978 \text{ ft}\cdot\text{lb} \blacktriangleleft$$



PROBLEM 18.42

Determine the kinetic energy of the disk of Problem 18.4.

PROBLEM 18.4 A homogeneous disk of weight $W = 6$ lb rotates at the constant rate $\omega_1 = 16$ rad/s with respect to arm *ABC*, which is welded to a shaft *DCE* rotating at the constant rate $\omega_2 = 8$ rad/s. Determine the angular momentum $\mathbf{H}_A$ of the disk about its center *A*.

SOLUTION

$$m = \frac{W}{g} = \frac{6}{32.2} = 0.1863 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\boldsymbol{\omega} = \omega_2\mathbf{i} + \omega_1\mathbf{j} = (8 \text{ rad/s})\mathbf{i} + (16 \text{ rad/s})\mathbf{j}$$

For axes x', y', z' parallel to x, y, z with origin at A,

$$\bar{I}_{x'} = \frac{1}{4}mr^2 = \frac{1}{4}(0.1863)\left(\frac{8}{12}\right)^2 = 0.0207 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\bar{I}_{z'} = \bar{I}_{x'} = 0.0207 \text{ lb}\cdot\text{s}^2\cdot\text{ft}, \quad \bar{I}_{y'} = \bar{I}_{x'} + \bar{I}_{z'} = 0.0414 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Point A is the mass center of the disk.

$$\mathbf{r}_{A/C} = (9 \text{ in.})\mathbf{i} - (9 \text{ in.})\mathbf{k} = (0.75 \text{ ft})\mathbf{i} - (0.75 \text{ ft})\mathbf{k}$$

$$\bar{\mathbf{v}} = \mathbf{v}_A = \omega_2\mathbf{i}\times\mathbf{r}_{A/C} = 8\mathbf{i}\times(0.75\mathbf{j} - 0.75\mathbf{k})$$

$$= (6 \text{ ft/s})\mathbf{j} + (6 \text{ ft/s})\mathbf{k}$$

$$\bar{v} = \sqrt{(6)^2 + (6)^2} = 8.4853 \text{ ft/s}$$

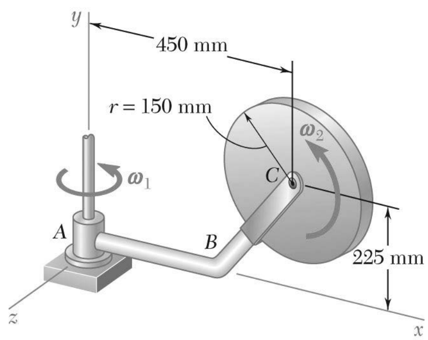
Kinetic energy:

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}_{x'}\omega_x^2 + \frac{1}{2}\bar{I}_{y'}\omega_y^2 + \frac{1}{2}\bar{I}_{z'}\omega_z^2$$

$$T = \frac{1}{2}(0.1863)(8.4853)^2 + \frac{1}{2}(0.0207)(8)^2 + \frac{1}{2}(0.0414)(16)^2 + 0$$

$$= 6.7068 + 0.6624 + 5.2992 = 12.6684 \text{ ft}\cdot\text{lb}$$

$T = 12.67$ ft · lb ◀



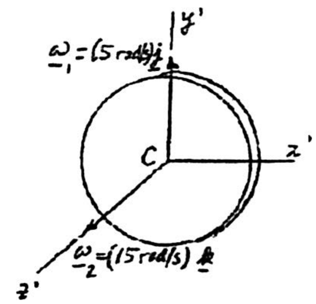
PROBLEM 18.43

Determine the kinetic energy of the disk of Problem 18.5.

PROBLEM 18.5 A thin disk of mass $m = 4\text{ kg}$ rotates at the constant rate $\omega_2 = 15\text{ rad/s}$ with respect to arm *ABC*, which itself rotates at the constant rate $\omega_1 = 5\text{ rad/s}$ about the *y* axis. Determine the angular momentum of the disk about its center *C*.

SOLUTION

$$r = 150\text{ mm}$$



Angular velocity of disk:

$$\boldsymbol{\omega} = \omega_1 \mathbf{j} + \omega_2 \mathbf{k}$$
$$= (5\text{ rad/s})\mathbf{j} + (15\text{ rad/s})\mathbf{k}$$

Centroidal moments of inertia:

$$\bar{I}_{x'} = \bar{I}_{y'} = \frac{1}{4}mr^2$$
$$= \frac{1}{4}(4)(0.150\text{ m})^2 = 0.0225\text{ kg}\cdot\text{m}^2$$
$$\bar{I}_{z'} = \frac{1}{2}mr^2 = 0.045\text{ kg}\cdot\text{m}^2$$

Location of mass center. $\mathbf{r}_{C/A} = (0.450\text{ m})\mathbf{i} + (0.225\text{ m})\mathbf{j}$

Velocity of mass center. $\bar{\mathbf{v}} = \boldsymbol{\omega}_1 \times \mathbf{r}_{C/A} = 5\mathbf{j}\times(0.45\mathbf{i} + 0.225\mathbf{j})$

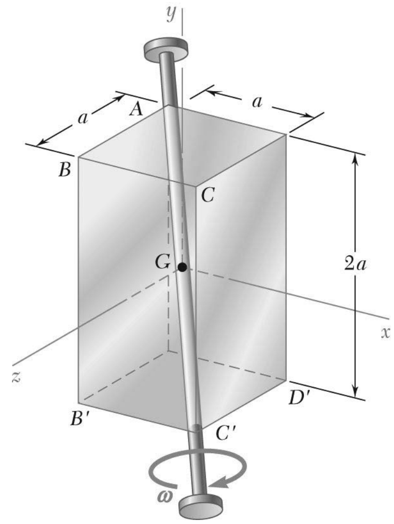
$$= -(2.25\text{ m/s})\mathbf{k}$$

Kinetic energy:

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}_x\omega_x^2 + \frac{1}{2}\bar{I}_y\omega_y^2 + \frac{1}{2}\bar{I}_z\omega_z^2$$
$$= \frac{1}{2}(4)(2.25)^2 + 0 + \frac{1}{2}(0.0225)(5)^2 + \frac{1}{2}(0.0450)(15)^2$$
$$= 10.125 + 0 + 0.28125 + 5.0625$$

$T = 15.47\text{ J}$ ◀

PROBLEM 18.44



Determine the kinetic energy of the solid parallelepiped of Problem 18.6.

PROBLEM 18.6 A solid rectangular parallelepiped of mass m has a square base of side a and a length $2a$. Knowing that it rotates at the constant rate ω about its diagonal AC' and that its rotation is observed from A as counterclockwise, determine (a) the magnitude of the angular momentum $\mathbf{H}_G$ of the parallelepiped about its mass center G, (b) the angle that $\mathbf{H}_G$ forms with the diagonal AC'.

SOLUTION

Body diagonal: $$d = \sqrt{a^2 + (2a)^2 + a^2} = \sqrt{6}a$$

$$\boldsymbol{\omega} = \frac{\omega}{d}(-a\mathbf{i} + 2a\mathbf{j} - a\mathbf{k}) = -\frac{\omega}{\sqrt{6}}\mathbf{i} + \frac{2\omega}{\sqrt{6}}\mathbf{j} - \frac{\omega}{\sqrt{6}}\mathbf{k}$$

$$I_x = \frac{1}{12}m[(2a)^2 + a^2] = \frac{5}{12}ma^2$$

$$I_y = \frac{1}{12}m[a^2 + a^2] = \frac{1}{6}ma^2$$

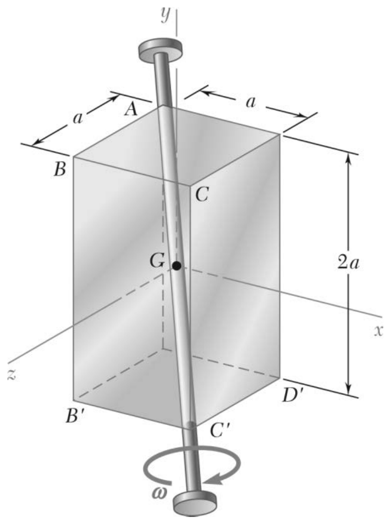
$$I_z = \frac{1}{12}m[a^2 + (2a)^2] = \frac{5}{12}ma^2$$

Axis of rotation passes through the mass center, hence $\bar{\mathbf{v}} = 0$.

Kinetic energy: $$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2$$

$$T = 0 + \frac{1}{2}\left(\frac{5}{12}ma^2\right)\left(\frac{\omega}{\sqrt{6}}\right)^2 + \frac{1}{2}\left(\frac{1}{6}ma^2\right)\left(\frac{2\omega}{\sqrt{6}}\right)^2 + \frac{1}{2}\left(\frac{5}{12}ma^2\right)\left(\frac{\omega}{\sqrt{6}}\right)^2 = \frac{1}{8}ma^2\omega^2$$

$$T = 0.1250\ ma^2\omega^2 \blacktriangleleft$$



PROBLEM 18.45

Determine the kinetic energy of the hollow parallelepiped of Problem 18.7.

PROBLEM 18.7 Solve Problem 18.6, assuming that the solid rectangular parallelepiped has been replaced by a hollow one consisting of six thin metal plates welded together.

SOLUTION

Body diagonal:

$$d = \sqrt{a^2 + (2a)^2 + a^2} = \sqrt{6}a$$

$$\boldsymbol{\omega} = \frac{\omega}{d}(-a\mathbf{i} + 2a\mathbf{j} - a\mathbf{k}) = -\frac{\omega}{\sqrt{6}}\mathbf{i} + \frac{2\omega}{\sqrt{6}}\mathbf{j} - \frac{\omega}{\sqrt{6}}\mathbf{k}$$

$$\text{Total area} = 2(a^2 + 2a^2 + 2a^2) = 10a^2$$

For each square plate,

$$m' = \frac{1}{10}m$$

$$I_x = \frac{1}{12}m'a^2 + m'a^2 = \frac{13}{12}m'a^2 = \frac{13}{120}ma^2$$

$$I_y = \frac{1}{6}m'a^2 = \frac{1}{60}ma^2$$

$$I_z = I_x = \frac{13}{120}ma^2$$

For each plate parallel to the yz plane,

$$m' = \frac{1}{5}m$$

$$I_x = \frac{1}{12}m'[a^2 + (2a)^2] = \frac{5}{12}m'a^2 = \frac{1}{12}ma^2$$

$$I_y = \frac{1}{12}m'a^2 + m'\left(\frac{a}{2}\right)^2 = \frac{1}{3}m'a^2 = \frac{1}{15}ma^2$$

$$I_z = \frac{1}{12}m'(2a)^2 + m'\left(\frac{a}{2}\right)^2 = \frac{7}{12}m'a^2 = \frac{7}{60}ma^2$$

PROBLEM 18.45 (Continued)

For each plate parallel to the *xy* plane, $m' = \frac{1}{5}m$

$$I_x = \frac{1}{12}m'(2a)^2 + m'\left(\frac{a}{2}\right)^2 = \frac{7}{12}m'a^2 = \frac{7}{60}ma^2$$

$$I_y = \frac{1}{12}m'a^2 + m'\left(\frac{a}{2}\right)^2 = \frac{1}{3}m'a^2 = \frac{1}{15}ma^2$$

$$I_z = \frac{1}{12}m'[a^2 + (2a)^2] = \frac{5}{12}m'a^2 = \frac{1}{12}ma^2$$

Total moments of inertia:

$$I_x = 2\left(\frac{13}{120} + \frac{1}{12} + \frac{7}{60}\right)ma^2 = \frac{37}{60}ma^2$$

$$I_y = 2\left(\frac{1}{60} + \frac{1}{15} + \frac{1}{15}\right)ma^2 = \frac{3}{10}ma^2$$

$$I_z = 2\left(\frac{13}{120} + \frac{7}{60} + \frac{1}{12}\right)ma^2 = \frac{37}{60}ma^2$$

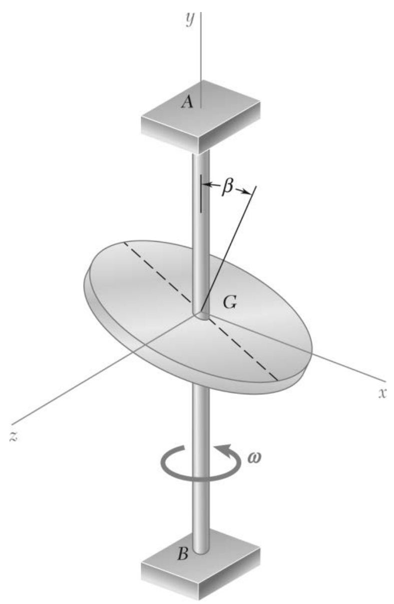
Axis of rotation passes through the mass center, hence $\overline{\mathbf{v}} = 0$.

Kinetic energy:

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2$$

$$T = 0 + \frac{1}{2}\left(\frac{37}{60}ma^2\right)\left(\frac{\omega}{\sqrt{6}}\right)^2 + \frac{1}{2}\left(\frac{3}{10}ma^2\right)\left(\frac{2\omega}{\sqrt{6}}\right)^2 + \frac{1}{2}\left(\frac{37}{60}ma^2\right)\left(\frac{\omega}{\sqrt{6}}\right)^2 = \frac{73}{360}\ ma^2\omega^2$$

$T = 0.203\ ma^2\omega^2$ ◀



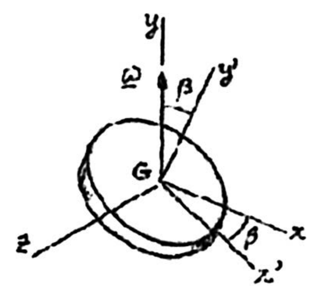
PROBLEM 18.46

Determine the kinetic energy of the disk of Problem 18.8.

PROBLEM 18.8 A homogeneous disk of mass *m* and radius *r* is mounted on the vertical shaft *AB*. The normal to the disk at *G* forms an angle $\beta = 25°$ with the shaft. Knowing that the shaft has a constant angular velocity ω, determine the angle θ formed by the shaft *AB* and the angular momentum $\mathbf{H}_G$ of the disk about its mass center *G*.

SOLUTION

Use the principal centroidal axes $Gx'y'z$.



Moments of inertia.

$$\bar{I}_{x'} = \bar{I}_z = \frac{1}{4}mr^2$$

$$\bar{I}_{y'} = \frac{1}{2}mr^2$$

Angular velocities.

$$\omega_{x'} = -\omega\sin\beta$$

$$\omega_{y'} = \omega\cos\beta$$

$$\omega_z = 0$$

Kinetic energy:

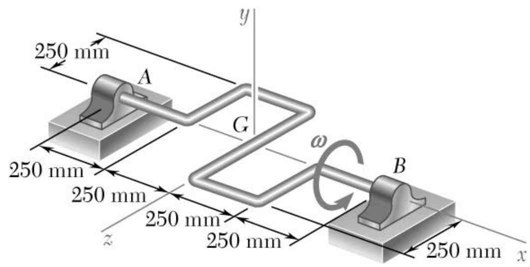
$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}_{x'}^2\omega_{x'}^2 + \frac{1}{2}\bar{I}_{y'}\omega_{y'}^2 + \frac{1}{2}\bar{I}_{z'}\omega_{z'}^2$$

$$= 0 + \frac{1}{2}\cdot\frac{1}{4}mr^2(-\omega\sin\beta)^2 + \frac{1}{2}\cdot\frac{1}{2}mr^2(\omega\cos\beta)^2 + 0$$

$$= \frac{1}{8}mr^2\omega^2(\sin^2\beta + 2\cos^2\beta)$$

$$= \frac{1}{8}mr^2\omega^2(\sin^2 25° + 2\cos^2 25°)$$

$$T = 0.228\, mr^2\omega^2 \blacktriangleleft$$

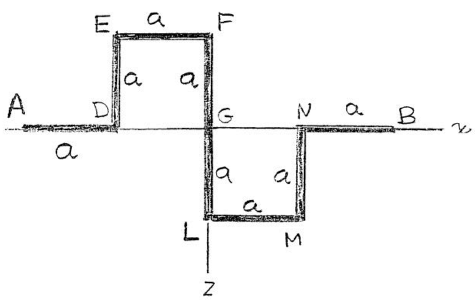


PROBLEM 18.47

Determine the kinetic energy of the shaft of Problem 18.15.

PROBLEM 18.15 A 5-kg rod of uniform cross section is used to form the shaft shown. Knowing that the shaft rotates with a constant angular velocity $\boldsymbol{\omega}$ of magnitude 12 rad/s, determine (*a*) the angular momentum $\mathbf{H}_G$ of the shaft about its mass center *G*, (*b*) the angle formed by $\mathbf{H}_G$ and the axis *AB*.

SOLUTION



$$\omega = (12 \text{ rad/s})\mathbf{i}, \qquad \omega_y = \omega_z = 0$$

$$T = \frac{1}{2}\bar{I}_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2 - I_{xy}\omega_x\omega_y - I_{yz}\omega_y\omega_z - I_{xz}\omega_x\omega_z$$

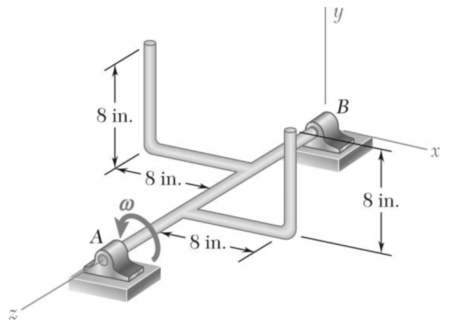
$$= \frac{1}{2}I_x\omega^2$$

The shaft is comprised of eight sections, each of length $a = 0.25\text{ m}$ and of mass $m' = \dfrac{m}{8} = 0.625\text{ kg}.$

$$\bar{I}_x = (4)\left(\frac{1}{3}m'a^2\right) + (2)(m'a^2) = \frac{10}{3}m'a^2 = \frac{10}{3}(0.625)(0.25)^2 = 0.130208\text{ kg}\cdot\text{m}^2$$

$$T = \frac{1}{2}(0.130208)(12)^2 = 9.38\text{ kg}\cdot\text{m}^2/\text{s}^2 = 9.38\text{ N}\cdot\text{m} = 9.38\text{ J}$$

$T = 9.38\text{ J}$ ◀



PROBLEM 18.48

Determine the kinetic energy of the body of Problem 18.17.

PROBLEM 18.17 Two L-shaped arms, each weighing 4 lb, are welded at the third points of the 2-ft shaft *AB*. Knowing that shaft *AB* rotates at the constant rate $\omega = 240$ rpm, determine (*a*) the angular momentum of the body about *A*, (*b*) the angle formed by the angular momentum and shaft *AB*.

SOLUTION

$$W = 4 \text{ lb} \quad m = \frac{4}{32.2} = 0.12422 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$a = 8 \text{ in.} = 0.66667 \text{ ft}$$

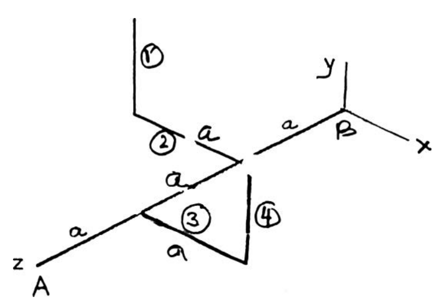
$$\omega = \frac{(2\pi)(240)}{60} = 8\pi \text{ rad/s}, \quad \omega_x = 0, \quad \omega_y = 0, \quad \omega_z = 8\pi \text{ rad/s}$$

For rotation about the fixed Point *B*, the kinetic energy is

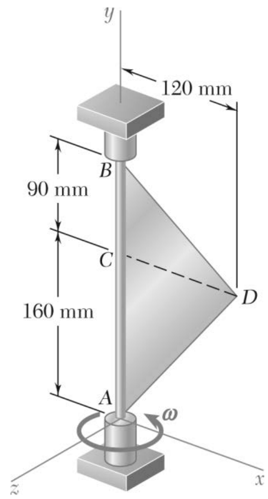
$$T = \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2 - I_{xy}\omega_x\omega_y - I_{xz}\omega_x\omega_z - I_{yz}\omega_y\omega_z = \frac{1}{2}I_z\omega^2$$

Calculation of I_z: Segments 1, 2, 3, and 4, each of mass $m' = 0.06211 \text{ lb} \cdot \text{s}^2/\text{ft}$ contribute to

Part	I_z
①	$\left(\frac{1}{12} + \frac{1}{4} + 1\right)m'a^2$
②	$\frac{1}{3}m'a^2$
③	$\frac{1}{3}m'a^2$
④	$\left(\frac{1}{12} + \frac{1}{4} + 1\right)m'a^2$
Σ	$\frac{10}{3}m'a^2$



Kinetic energy: $T = \frac{1}{2}\left(\frac{10}{3}m'a^2\right)\omega^2 = \frac{1}{2}\left(\frac{10}{3}\right)(0.06211)(0.66667)^2(8\pi)^2$ $\quad T = 29.1 \text{ ft} \cdot \text{lb}$ ◄



PROBLEM 18.49

Determine the kinetic energy of the triangular plate of Problem 18.19.

PROBLEM 18.19 The triangular plate shown has a mass of 7.5 kg and is welded to a vertical shaft *AB*. Knowing that the plate rotates at the constant rate $\omega = 12$ rad/s, determine its angular momentum about (*a*) Point *C*, (*b*) Point *A*. (*Hint*: To solve part *b* find $\bar{\mathbf{v}}$ and use the property indicated in part *a* of Problem 18.13.)

SOLUTION

$$\boldsymbol{\omega} = (12 \text{ rad/s})\mathbf{j}, \quad \omega_x = 0, \quad \omega_y = 12 \text{ rad/s}, \quad \omega_z = 0$$

Kinetic energy:

$$T = \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2 - I_{yz}\omega_y\omega_z - I_{zx}\omega_z\omega_x - I_{xy}\omega_x\omega_y$$

$$= \frac{1}{2}I_y\omega^2$$

For the plate:

$$b = AB = 90 + 160 = 250 \text{ mm} = 0.25 \text{ m}$$

$$h = CD = 120 \text{ mm} = 0.12 \text{ m}$$

Area:

$$A = \frac{1}{2}bh = 15\times 10^{-3} \text{ m}^2$$

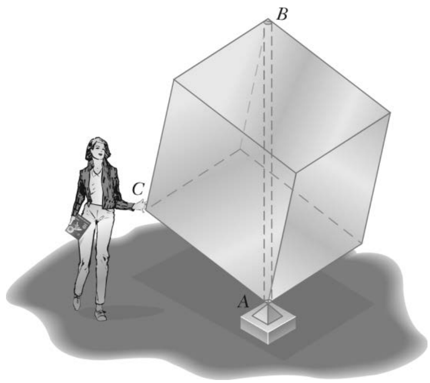
$$(I_y)_{\text{area}} = \frac{1}{12}bh^3 = 36\times 10^{-6} \text{ m}^4$$

$$m = 7.5 \text{ kg}$$

$$(I_y)_{\text{mass}} = \frac{m}{A}(I_y)_{\text{area}} = \frac{(7.5)(36\times 10^{-6})}{15\times 10^{-3}} = 18\times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$T = \frac{1}{2}(18\times 10^{-3})(12)^2 = 1.296 \text{ J}$$

$T = 1.296$ J ◀



PROBLEM 18.50

Determine the kinetic energy imparted to the cube of Problem 18.21.

PROBLEM 18.21 One of the sculptures displayed on a university campus consists of a hollow cube made of six aluminum sheets, each 1.5×1.5 m, welded together and reinforced with internal braces of negligible weight. The cube is mounted on a fixed base at *A* and can rotate freely about its vertical diagonal *AB*. As she passes by this display on the way to a class in mechanics, an engineering student grabs corner *C* of the cube and pushes it for 1.2 s in a direction perpendicular to the plane *ABC* with an average force of 50 N. Having observed that it takes 5 s for the cube to complete one full revolution, she flips out her calculator and proceeds to determine the mass of the cube. What is the result of her calculation? (*Hint:* The perpendicular distance from the diagonal joining two vertices of a cube to any of its other six vertices can be obtained by multiplying the side of the cube by $\sqrt{2/3}$.)

SOLUTION

Let $m' = \frac{1}{6}m$ be the mass of one side of the cube. Choose *x*, *y*, and *z* axes perpendicular to the face of the cube. Let *a* be the side of the cube. For a side perpendicular to the *x* axis, $(I_x)_1 = \frac{1}{6}m'a^2$.

For a side perpendicular to the *y* or *z* axis, $(I_x)_2 = \left(\frac{1}{12} + \frac{1}{4}\right)m'a^2 = \frac{1}{3}m'a^2$

Total moment of inertia: $$I_x = 2(I_x)_1 + 4(I_x)_2 = \frac{5}{3}m'a^2 = \frac{5}{18}ma^2$$

By symmetry, $I_y = I_x$ and $I_z = I_x$.

Since all three moments of inertia are equal, the ellipsoid of inertia is a sphere. All centroidal axes are principal axes.

Moment of inertia about the vertical axis: $$I_v = \frac{5}{18}ma^2$$

Let $b = \sqrt{\frac{2}{3}}a$ be the moment arm of the impulse applied to the corner.

Using the impulse-momentum principle and taking moments about the vertical axis,

$$bF(\Delta t) = H_v = I_v\omega = \frac{5}{18}ma^2\omega \qquad (1)$$

Data: $$a = 1.5 \text{ m}, \quad b = \sqrt{\frac{2}{3}}(1.5) = 1.22474 \text{ m}$$

$$\omega = \frac{2\pi}{5} = 1.25664 \text{ rad/s}, \quad F = 50 \text{ N}, \quad \Delta t = 1.2 \text{ s}.$$

PROBLEM 18.50 (Continued)

Solving Equation (1) for m,

$$m = \frac{18}{5}\frac{bF(\Delta t)}{a^2\omega} = \frac{18}{5}\frac{(1.22474)(50)(1.2)}{(1.5)^2(1.25664)} = 93.563 \text{ kg}$$

$m = 93.6$ kg

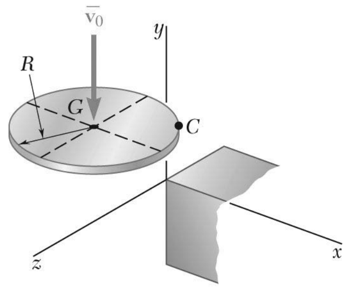
For principal axes,

$$T = \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2$$

$$T = \frac{1}{2}I_v\omega^2 = \frac{1}{2}\left(\frac{5}{18}ma^2\right)\omega^2$$

$$= \frac{1}{2}\frac{5}{18}(93.563)(1.5)^2(1.2566)^2$$

$T = 46.2$ J ◀



PROBLEM 18.51

Determine the kinetic energy lost when edge C of the plate of Problem 18.29 hits the obstruction.

SOLUTION

For principal moments of inertia, the kinetic energy is

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}_x\omega_x^2 + \frac{1}{2}\bar{I}_y\omega_y^2 + \frac{1}{2}\bar{I}_z\omega_z^2$$

Before impact:

$$\bar{v} = \bar{v}_0$$
$$\omega_x = \omega_y = \omega_z = 0$$
$$T_0 = \frac{1}{2}m\bar{v}_0^2$$

After impact:

From Problem 18.30,

$$\bar{v}_x = \bar{v}_z = 0$$
$$\bar{v}_y = -\frac{4}{5}\bar{v}_0$$
$$\bar{v} = \frac{4}{5}\bar{v}_0$$

From Problem 18.29,

$$\omega_x = \omega_z = \frac{2\sqrt{2}}{5}\frac{v_0}{R}$$
$$\omega_y = 0$$
$$\bar{I}_x = \bar{I}_z = \frac{1}{4}mR^2$$

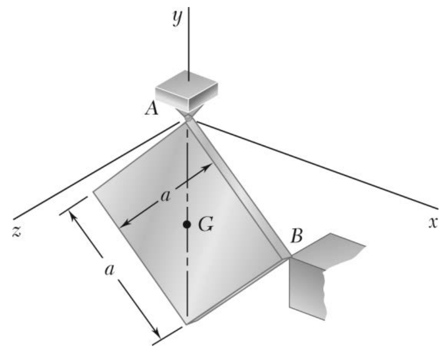
$$T = \frac{1}{2}m\left(\frac{4}{5}v_0\right)^2 + \left(\frac{1}{2}\right)\left(\frac{1}{4}mR^2\right)\left(\frac{2\sqrt{2}}{5}\frac{v_0}{R}\right)^2 + 0 + \left(\frac{1}{2}\right)\left(\frac{1}{4}mR^2\right)\left(\frac{2\sqrt{2}}{5}\frac{\bar{v}_0}{R}\right)^2$$
$$= \frac{1}{2}\left[\frac{16}{25} + \frac{2}{25} + 0 + \frac{2}{25}\right]m\bar{v}_0^2 = \frac{2}{5}m\bar{v}_0^2$$

Energy loss:

$$T_0 - T = \frac{1}{2}m\bar{v}_0^2 - \frac{2}{5}m\bar{v}_0^2$$

$$T_0 - T = \frac{1}{10}m\bar{v}_0^2 \blacktriangleleft$$

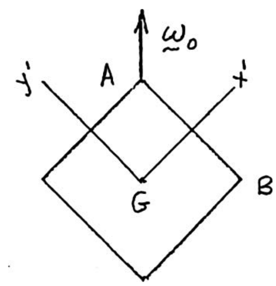
PROBLEM 18.52



Determine the kinetic energy lost when the plate of Problem 18.31 hits the obstruction at *B*.

SOLUTION

For the x' and y' axes shown, the initial angular velocity $\omega_0\mathbf{j}$ has components



$$\omega_{x'} = \frac{\sqrt{2}}{2}\omega_0, \quad \omega_{y'} = \frac{\sqrt{2}}{2}\omega_0$$

Initial angular momentum about the mass center:

$$(\mathbf{H}_G)_0 = \bar{I}_{x'}\omega_{x'}\mathbf{i}' + \bar{I}_{y'}\omega_{y'}\mathbf{j}'$$
$$= \frac{1}{12}ma^2\frac{\sqrt{2}}{2}\omega_0(\mathbf{i}' + \mathbf{j}')$$

Initial velocity of the mass center: $\quad \bar{\mathbf{v}}_0 = 0$

Let $\boldsymbol{\omega}$ be the angular velocity and $\bar{\mathbf{v}}$ be the velocity of the mass center immediately after impact.
Let $(F\Delta t)\mathbf{k}$ be the impulse at *B*.

Kinematics:

$$\mathbf{v}_B = \boldsymbol{\omega}\times\mathbf{r}_{B/A} = (\omega_{x'}\mathbf{i}' + \omega_{y'}\mathbf{j}' + \omega_{z'}\mathbf{k}')\times(-a\mathbf{j}')$$
$$\mathbf{v}_B = a(\omega_{z'}\mathbf{i}' + \omega_{x'}\mathbf{k}')$$

Since the corner *B* does not rebound, $\quad (v_B)_{z'} = 0 \quad \text{or} \quad \omega_{x'} = 0$

$$\bar{\mathbf{v}} = \boldsymbol{\omega}\times\mathbf{r}_{G/A} = (\omega_{y'}\mathbf{j}' + \omega_{z'}\mathbf{k}')\times\left(\frac{1}{2}a\right)(-\mathbf{i}' - \mathbf{j}')$$
$$= \frac{1}{2}a(\omega_{z'}\mathbf{i}' - \omega_{z'}\mathbf{j} + \omega_{y}\mathbf{k}')$$

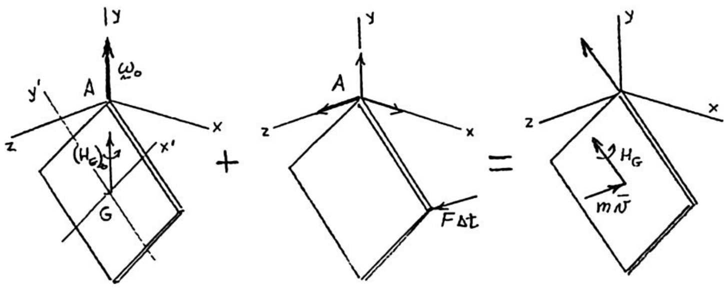
Also,

$$\mathbf{r}_{G/A}\times m\bar{\mathbf{v}} = \frac{1}{4}ma^2(-\omega_{y}\mathbf{i}' + \omega_{y'}\mathbf{j}' + 2\omega_{z'}\mathbf{k}')$$

and

$$\mathbf{H}_G = I_{x'}\omega_{x'}\mathbf{i}' + I_{y'}\omega_{y'}\mathbf{j}' + \bar{I}_{z'}\omega_{z'}\mathbf{k}'$$
$$= \frac{1}{12}ma^2\omega_{y'}\mathbf{j}' + \frac{1}{6}ma^2\omega_{z'}\mathbf{k}'$$

Principle of impulse-momentum.



Moments about A:

$$(\mathbf{H}_A)_0 + (-a\mathbf{j}) \times (F\Delta t)\mathbf{k} = \mathbf{H}_A$$

$$(\mathbf{H}_G)_0 + \mathbf{r}_{G/A} \times m\overline{\mathbf{v}}_0 - (aF\Delta t)\mathbf{i} = \mathbf{H}_G + \mathbf{r}_{G/A} \times m\overline{\mathbf{v}}$$

Resolve into components.

$$\mathbf{i}': \quad \frac{1}{24}\sqrt{2}ma^2\omega_0 - aF(\Delta t) = -\frac{1}{4}ma^2\omega_{y'}$$

$$\mathbf{j}': \quad \frac{1}{24}\sqrt{2}ma^2\omega_0 = \frac{1}{12}ma^2\omega_{y'} + \frac{1}{4}ma^2\omega_{y'} \quad \omega_{y'} = \frac{\sqrt{2}}{8}\omega_0$$

$$\mathbf{k}': \quad 0 = \frac{1}{6}ma^2\omega_{z'} + \frac{1}{2}ma^2\omega_{z'} \quad \omega_{z'} = 0$$

$$\boldsymbol{\omega} = \frac{\sqrt{2}}{8}\omega_0\mathbf{j}' = \frac{1}{8}\sqrt{2}\omega_0\frac{\sqrt{2}}{2}(\mathbf{j} - \mathbf{i})$$

$$\overline{\mathbf{v}} = \frac{1}{2}a\omega_y\mathbf{k}' = \frac{\sqrt{2}}{16}a\omega_0\mathbf{k}$$

Kinetic energy:

$$T = \frac{1}{2}m\overline{v}^2 + \frac{1}{2}\overline{I}_x\omega_{x'}^2 + \frac{1}{2}\overline{I}_y\omega_{y'}^2 + \frac{1}{2}\overline{I}_{z'}\omega_{z'}^2$$

Before impact:

$$\overline{v}_0 = 0, \quad \omega_{x'} = \frac{\sqrt{2}}{2}\omega_0, \quad \omega_{y'} = \frac{\sqrt{2}}{2}\omega_0, \quad \omega_{z'} = 0$$

$$T_0 = 0 + \frac{1}{2}\left(\frac{1}{12}ma^2\right)\left(\frac{\sqrt{2}}{2}\omega_0\right)^2 + \frac{1}{2}\left(\frac{1}{12}ma^2\right)\left(-\frac{\sqrt{2}}{2}\omega_0\right)^2 + 0 = \frac{1}{24}ma^2\omega_0^2$$

After impact:

$$\overline{v} = \frac{\sqrt{2}}{16}a\omega_0, \quad \omega_{x'} = 0, \quad \omega_{y'} = \frac{\sqrt{2}}{8}\omega_0, \quad \omega_{z'} = 0$$

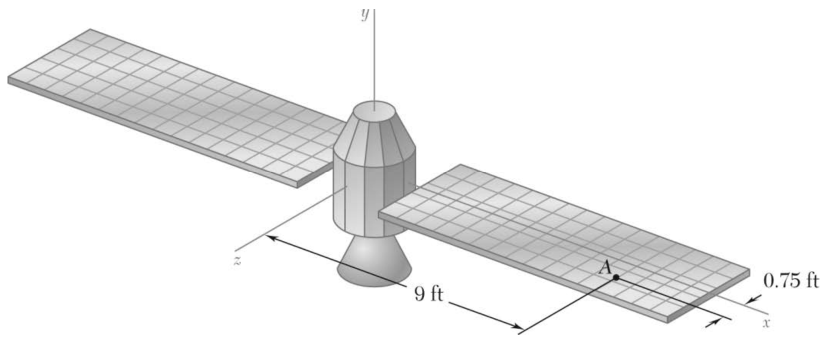
$$T_1 = \frac{1}{2}m\left(\frac{\sqrt{2}}{16}a\omega_0\right)^2 + 0 + \frac{1}{2}\left(\frac{1}{12}ma^2\right)\left(\frac{\sqrt{2}}{8}\omega_0\right)^2 + 0 = \frac{1}{192}ma^2\omega_0^2$$

Kinetic energy lost.

$$T_0 - T_1 = \frac{7}{192}ma^2\omega_0^2 \blacktriangleleft$$

PROBLEM 18.53

Determine the kinetic energy of the space probe of Problem 18.33 in its motion about its mass center after its collision with the meteorite.



SOLUTION

Masses: Space probe: $m' = \dfrac{3000}{32.2} = 93.17 \text{ lb}\cdot\text{s}^2/\text{ft}$

Meteorite: $m = \dfrac{5}{(16)(32.2)} = 0.009705 \text{ lb}\cdot\text{s}^2/\text{ft}$

Point of impact: $\mathbf{r}_A = (9 \text{ ft})\mathbf{i} + (0.75 \text{ ft})\mathbf{k}$

Initial linear momentum of the meteorite, $(\text{lb}\cdot\text{s})$:

$$m\mathbf{v}_0 = (0.009705)(2400\mathbf{i} + 3000\mathbf{j} + 3200\mathbf{k}) = 23.292\mathbf{i} - 29.115\mathbf{j} + 31.056\mathbf{k}$$

Its moment about the origin, $(\text{lb}\cdot\text{ft}\cdot\text{s})$:

$$\mathbf{r}_A \times m\mathbf{v}_0 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 9 & 0 & 0.75 \\ 23.292 & -29.115 & 31.056 \end{vmatrix} = 21.836\mathbf{i} - 262.04\mathbf{j} - 262.04\mathbf{k}$$

Final linear momentum of meteorite and its moment about the origin, $(\text{lb}\cdot\text{s})$ and $(\text{lb}\cdot\text{s}\cdot\text{ft})$:

$$0.8m\mathbf{v}_0 = 18.634\mathbf{i} - 23.292\mathbf{j} + 24.845\mathbf{k}$$
$$\mathbf{r}_A \times (0.8m\mathbf{v}_0) = 17.469\mathbf{i} - 209.63\mathbf{j} - 209.63\mathbf{k}$$

PROBLEM 18.53 (Continued)

Let $\mathbf{H}_A$ be the angular momentum of the probe and m' be its mass. Conservation of angular momentum about the origin for a system of particles consisting of the probe plus the meteorite:

$$\mathbf{r}_A \times m\mathbf{v}_0 = \mathbf{H}_A + \mathbf{r}_A \times (0.8m\mathbf{v}_0)$$

$$\mathbf{H}_A = (4.367 \text{ lb}\cdot\text{s}\cdot\text{ft})\mathbf{i} - (52.41 \text{ lb}\cdot\text{s}\cdot\text{ft})\mathbf{j} - (52.41 \text{ lb}\cdot\text{s}\cdot\text{ft})\mathbf{k}$$

$$I_x\omega_x = (H_A)_x \qquad \omega_x = \frac{(H_A)_x}{m'k_x^2} = \frac{4.367}{(93.17)(1.375)^2} = 0.02479 \text{ rad/s}$$

$$I_y\omega_y = (H_A)_y \qquad \omega_y = \frac{(H_A)_y}{m'k_y^2} = \frac{-52.41}{(93.17)(1.425)^2} = -0.2770 \text{ rad/s}$$

$$I_z\omega_z = (H_A)_z \qquad \omega_z = \frac{(H_A)_z}{m'k_z^2} = \frac{-52.41}{(93.17)(1.250)^2} = -0.3600 \text{ rad/s}$$

Kinetic energy of motion of the probe about its mass center:

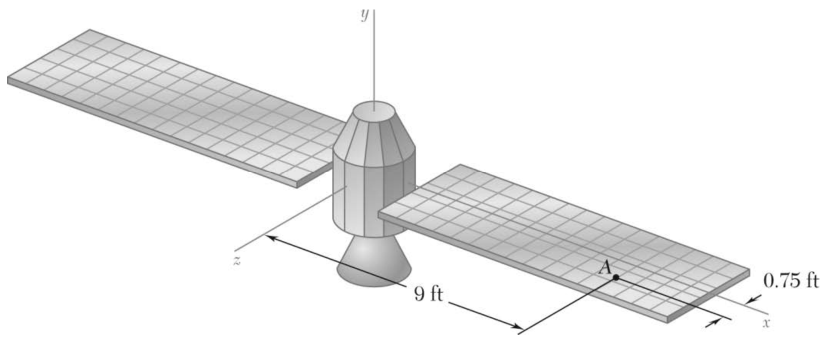
$$T' = \frac{1}{2}\left(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2\right) = \frac{m}{2}\left(k_x^2\omega_x^2 + k_y^2\omega_y^2 + k_z^2\omega_z^2\right)$$

$$= \frac{93.17}{2}[(1.375)^2(0.02479)^2 + (1.425)^2(-0.2770)^2 + (1.250)^2(-0.3600)^2]$$

$$= 16.75 \text{ ft}\cdot\text{lb} \qquad T' = 16.75 \text{ ft}\cdot\text{lb} \blacktriangleleft$$

PROBLEM 18.54

Determine the kinetic energy of the space probe of Problem 18.34 in its motion about its mass center after its collision with the meteorite.



SOLUTION

Masses: Space probe: $m' = \dfrac{3000}{32.2} = 93.17\ \text{lb}\cdot\text{s}^2/\text{ft}$

Meteorite: $m = \dfrac{5}{(16)(32.2)} = 0.009705\ \text{lb}\cdot\text{s}^2/\text{ft}$

Point of impact: $\mathbf{r}_A = (9\ \text{ft})\mathbf{i} + (0.75\ \text{ft})\mathbf{k}$

Initial linear momentum of the meteorite, $(\text{lb}\cdot\text{s})$:

$$m\mathbf{v}_0 = (0.009705)(v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k})$$

Its moment about the origin, $(\text{lb}\cdot\text{ft}\cdot\text{s})$:

$$(\mathbf{H}_A)_0 = \mathbf{r}_A \times m\mathbf{v}_0 = 0.009705\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 9 & 0 & 0.75 \\ v_x & v_y & v_z \end{vmatrix}$$

$$= 0.009705[-0.75v_y\mathbf{i} + (0.75v_x - 9v_z)\mathbf{j} + 9v_y\mathbf{k}]$$

Final linear momentum of the meteorite, $(\text{lb}\cdot\text{s})$:

$$0.75m\mathbf{v}_0 = 0.007279(v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k})$$

Its moment about the origin, $(\text{lb}\cdot\text{ft}\cdot\text{s})$:

$$\mathbf{r}_A \times (0.75m\mathbf{v}_0) = 0.007279[-0.75v_y\mathbf{i} + (0.75v_x - 9v_z)\mathbf{j} + 9v_y\mathbf{k}]$$

Initial linear momentum of the space probe, $(\text{lb}\cdot\text{s})$: $m'\mathbf{v}'_0 = 0$

Final linear momentum of the space probe, $(\text{lb}\cdot\text{s})$:

$$m'(v'_x\mathbf{i}+v'_y\mathbf{j}+v'_z\mathbf{k})=93.17\left(-\frac{0.675}{12}\mathbf{i}+v'_y\mathbf{j}+v'_z\mathbf{k}\right)$$

Final angular momentum of space probe, $(\text{lb}\cdot\text{ft}\cdot\text{s})$:

$$\begin{aligned}\mathbf{H}_A&=m'(k_x^2\omega_x\mathbf{i}+k_y^2\omega_y\mathbf{j}+k_z^2\omega_z\mathbf{k})\\&=93.17[(1.375)^2(0.05)\mathbf{i}+(1.425)^2(-0.12)\mathbf{j}+(1.250)^2\omega_z\mathbf{k}]\\&=8.8075\mathbf{i}-22.703\mathbf{j}+145.58\omega_z\mathbf{k}\end{aligned}$$

Conservation of linear momentum of the probe plus the meteorite $(\text{lb}\cdot\text{s})$:

$$0.009705(v_x\mathbf{i}+v_y\mathbf{j}+v_z\mathbf{k})=0.007279(v_x\mathbf{i}+v_y\mathbf{j}+v_z\mathbf{k})+93.17(-0.05625\mathbf{i}+v'_y\mathbf{j}+v'_z\mathbf{k})$$

$$\mathbf{i}:\quad 0.002426v_x=-5.2408\qquad v_x=-2160\text{ ft/s}$$

$$\mathbf{j}:\quad 0.002426v_y=93.17v'_y$$

$$\mathbf{k}:\quad 0.002426v_z=93.17v'_z$$

Conservation of angular momentum about the origin $(\text{lb}\cdot\text{ft}\cdot\text{s})$:

$$\begin{aligned}(0.009705)[-0.75v_y\mathbf{i}+(0.75v_x-9v_z)\mathbf{j}+9v_y\mathbf{k}]&=(0.007279)[-0.75v_y\mathbf{i}+(0.75v_x-9v_z)\mathbf{j}+9v_y\mathbf{k}]\\&\quad+8.8075\mathbf{i}-22.703\mathbf{j}+145.58\omega_z\mathbf{k}\end{aligned}$$

$$\mathbf{i}:\quad -0.0018195v_y=8.8075\qquad v_y=-4840.5\text{ ft/s}$$

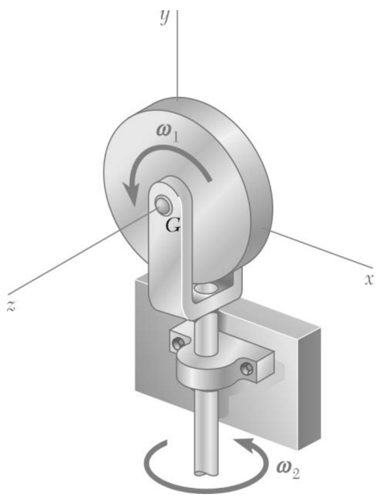
$$\mathbf{k}:\quad -0.021834v_y=145.58\omega_z$$

$$\omega_z=-149.98\times10^{-6}v_y\qquad \omega_z=-0.726\text{ rad/s}$$

Kinetic energy of motion of probe relative to its mass center:

$$\begin{aligned}T'&=\frac{1}{2}\left(I_x\omega_x^2+I_y\omega_y^2+I_z\omega_z^2\right)=\frac{1}{2}m\left(k_x^2\omega_x^2+k_y^2\omega_z^2\omega_z^2\right)\\&=\frac{1}{2}(93.17)[(1.375)^2(0.05)^2+(1.425)^2(-0.12)^2+(1.250)^2(-0.726)^2]\\&=39.9\text{ ft}\cdot\text{lb}\end{aligned}$$

$$T'=39.9\text{ ft}\cdot\text{lb}\ ◀$$



PROBLEM 18.55

Determine the rate of change $\dot{\mathbf{H}}_G$ of the angular momentum $\mathbf{H}_G$ of the disk of Problem 18.1.

PROBLEM 18.1 A thin, homogeneous disk of mass m and radius r spins at the constant rate ω_1 about an axle held by a fork-ended vertical rod, which rotates at the constant rate ω_2. Determine the angular momentum $\mathbf{H}_G$ of the disk about its mass center G.

SOLUTION

Angular velocity: $\boldsymbol{\omega} = \omega_2 \mathbf{j} + \omega_1 \mathbf{k}$

Moments of inertia: $\bar{I}_x = \frac{1}{4}mr^2, \quad \bar{I}_y = \frac{1}{4}mr^2, \quad \bar{I}_z = \frac{1}{2}mr^2$

Products of inertia: by symmetry, $\bar{I}_{xy} = \bar{I}_{yz} = \bar{I}_{zx} = 0$

Angular momentum:

$$\mathbf{H}_G = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k}$$

$$\mathbf{H}_G = 0 + \left(\frac{1}{4}mr^2\right)\omega_2 \mathbf{j} + \frac{1}{2}mr^2 \omega_1 \mathbf{k}$$

$$\mathbf{H}_G = \frac{1}{4}mr^2 \omega_2 \mathbf{j} + \frac{1}{2}mr^2 \omega_1 \mathbf{k}$$

Rate of change of angular momentum. Let the frame of reference $Gxyz$ be rotating with angular velocity

$$\boldsymbol{\Omega} = \omega_1 \mathbf{j}$$

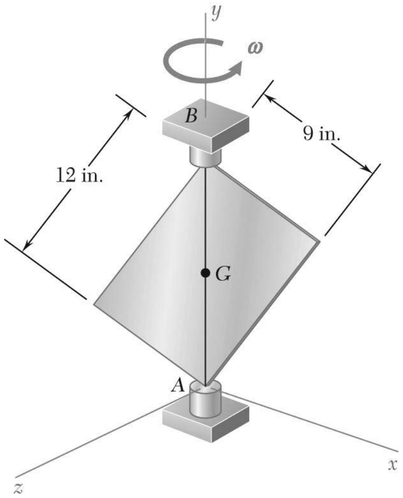
Then

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \boldsymbol{\Omega} \times \mathbf{H}_G$$

$$= 0 + \omega_2 \mathbf{j} \times \left(\frac{1}{4}mr^2 \omega_2 \mathbf{j} + \frac{1}{2}mr^2 \omega_1 \mathbf{k}\right)$$

$$= \frac{1}{2}mr^2 \omega_1 \omega_2 \mathbf{i}$$

$\dot{\mathbf{H}}_G = \frac{1}{2}mr^2 \omega_1 \omega_2 \mathbf{i}$ ◀

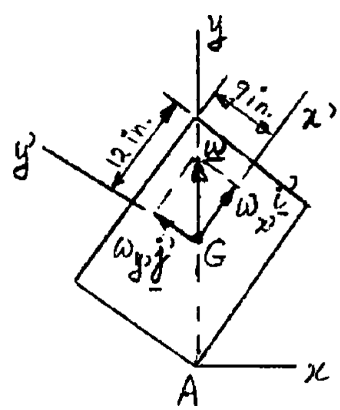


PROBLEM 18.56

Determine the rate of change $\dot{\mathbf{H}}_G$ of the angular momentum $\mathbf{H}_G$ of the plate of Problem 18.2.

PROBLEM 18.2 A thin rectangular plate of weight 15 lb rotates about its vertical diagonal *AB* with an angular velocity ω. Knowing that the *z* axis is perpendicular to the plate and that ω is constant and equal to 5 rad/s, determine the angular momentum of the plate about its mass center *G*.

SOLUTION



$$h = \sqrt{(9 \text{ in.})^2 + (12 \text{ in.})^2} = 15 \text{ in.}$$

Resolving ω along the principal axes x', y', z:

$$\omega_{x'} = \frac{12}{15}\omega = 0.8(5 \text{ rad/s}) = 4 \text{ rad/s}$$

$$\omega_{y'} = \frac{9}{15}\omega = 0.6(5 \text{ rad/s}) = 3 \text{ rad/s}$$

$$\omega_z = 0$$

Moments of inertia:

$$I_{x'} = \frac{1}{12}\left(\frac{15 \text{ lb}}{32.2}\right)\left(\frac{9}{12}\text{ft}\right)^2 = 0.021836 \text{ slug}\cdot\text{ft}^2$$

$$I_{y'} = \frac{1}{12}\left(\frac{15 \text{ lb}}{32.2}\right)\left(\frac{12}{12}\text{ft}\right)^2 = 0.038820 \text{ slug}\cdot\text{ft}^2$$

From Eqs. (18.10):

$$H_{x'} = I_{x'}\omega_{x'} = (0.021836)(4) = 0.087345 \text{ slug ft}^2/\text{s}$$

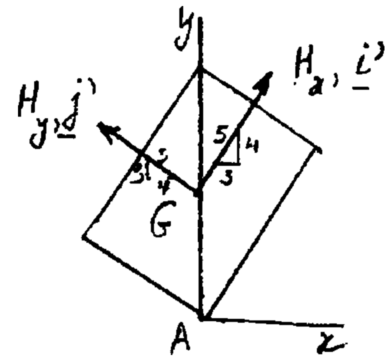
$$H_{y'} = I_{y'}\omega_{y'} = (0.038820)(3) = 0.11646 \text{ slug ft}^2/\text{s}$$

$$H_z = I_z\omega_z = 0$$

$$\mathbf{H}_G = (0.087345 \text{ slug ft}^2/\text{s})\mathbf{i}' + (0.11646 \text{ slug ft}^2/\text{s})\mathbf{j}'$$

PROBLEM 18.56 (Continued)

Components along x and y axes:



$$H_x = \frac{3}{5}H_{x'} - \frac{4}{5}H_{y'}$$
$$= \frac{3}{5}(0.087345) - \frac{4}{5}(0.11646)$$
$$= -0.040761$$
$$H_y = \frac{4}{5}H_{x'} + \frac{3}{5}H_{y'}$$
$$= \frac{4}{5}(0.087345) + \frac{3}{5}(0.11646)$$
$$= 0.13975$$

$$\mathbf{H}_G = (-0.040761 \text{ slug}\cdot\text{ft}^2/\text{s})\mathbf{i} + (0.13975 \text{ slug}\cdot\text{ft}^2/\text{s})\mathbf{j}$$

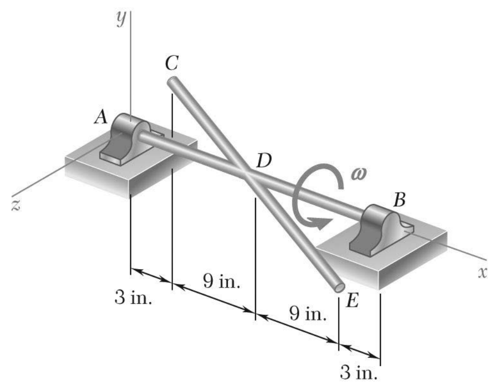
where the frame $Axyz$ rotates with the plate with the angular velocity.

$$\mathbf{\Omega} = \boldsymbol{\omega} = (5 \text{ rod/s})\mathbf{j}$$

We have $(\dot{\mathbf{H}}_G)_{Axyz} = 0.$ Substituting into Eq. (18.22):

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Axyz} + \mathbf{\Omega}\times\mathbf{H}_G = 0 + 5\mathbf{j}\times(-0.040761\mathbf{i} + 0.13975\mathbf{j})$$
$$= (0.20380 \text{ ft/lb})\mathbf{k}$$

$$\dot{\mathbf{H}}_G = (0.204 \text{ ft/lb})\mathbf{k} \blacktriangleleft$$

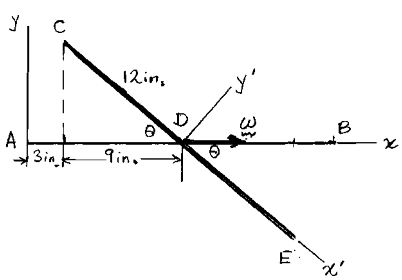


PROBLEM 18.57

Determine the rate of change $\dot{\mathbf{H}}_D$ of the angular momentum $\mathbf{H}_D$ of the assembly of Problem 18.3.

PROBLEM 18.3 Two uniform rods *AB* and *CE*, each of weight 3 lb and length 2 ft, are welded to each other at their midpoints. Knowing that this assembly has an angular velocity of constant magnitude $\omega = 12$ rad/s, determine the magnitude and direction of the angular momentum $\mathbf{H}_D$ of the assembly about *D*.

SOLUTION



$$m = \frac{W}{g} = \frac{3}{32.2} = 0.093168 \text{ lb}\cdot\text{s}^2/\text{ft}, \qquad l = 24 \text{ in.} = 2 \text{ ft},$$

$$\boldsymbol{\omega} = (12 \text{ rad/s})\mathbf{i}$$

For rod *ADB*, $\quad \mathbf{H}_D = I_x\omega\mathbf{i} \approx 0, \quad \text{since } I_x \approx 0.$

For rod *CDE*, use principal axes x', y' as shown.

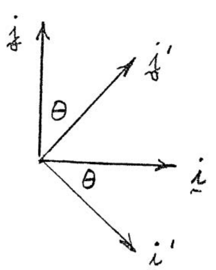
$$\cos\theta = \frac{9}{12}, \qquad \theta = 41.410°$$

$$\omega_{x'} = \omega\cos\theta = 9 \text{ rad/s}^2$$

$$\omega_{y'} = \omega\sin\theta = 7.93725 \text{ rad/s}^2$$

$$\omega_{z'} = 0$$

$$I_{x'} \approx 0$$



$$I_{y'} = \frac{1}{12}ml^2 = \frac{1}{12}(0.093168)(2)^2$$

$$= 0.0310559 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\mathbf{H}_D = I_{x'}\omega_{x'}\mathbf{i}' + I_{y'}\omega_{y'}\mathbf{j}' + I_{z'}\omega_{z'}\mathbf{k}'$$

$$= 0 + (0.0310559)(7.93725)\mathbf{j}' + 0$$

$$= 0.246498\mathbf{j}'$$

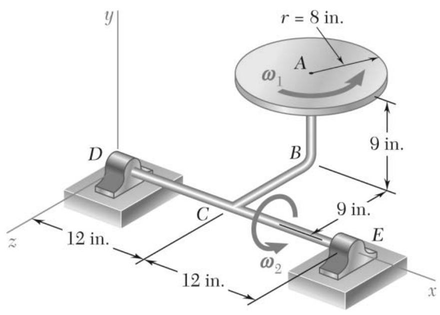
$$\mathbf{H}_D = 0.246498(\sin\theta\mathbf{i} + \cos\theta\mathbf{j}) = 0.163045\mathbf{i} + 0.184874\mathbf{j}$$

Let the frame of reference *Dxyz* be rotating with angular velocity

$$\boldsymbol{\Omega} = \boldsymbol{\omega} = (12 \text{ rad/s})\mathbf{i}$$

Then, $\quad \dot{\mathbf{H}}_D = (\dot{\mathbf{H}}_D)_{Dxyz} + \boldsymbol{\Omega}\times\mathbf{H}_D = 0 + \boldsymbol{\omega}\times\mathbf{H}_D$

$$\dot{\mathbf{H}}_D = 12\mathbf{i}\times(0.163045\mathbf{i} + 0.184874\mathbf{j}) \qquad \dot{\mathbf{H}}_D = (2.22 \text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$



PROBLEM 18.58

Determine the rate of change $\dot{\mathbf{H}}_A$ of the angular momentum $\mathbf{H}_A$ of the disk of Problem 18.4.

PROBLEM 18.4 A homogeneous disk of weight $W = 6\text{ lb}$ rotates at the constant rate $\omega_1 = 16$ rad/s with respect to arm *ABC*, which is welded to a shaft *DCE* rotating at the constant rate $\omega_2 = 8$ rad/s. Determine the angular momentum $\mathbf{H}_A$ of the disk about its center *A*.

SOLUTION

$$\boldsymbol{\omega} = \omega_2\mathbf{i} + \omega_1\mathbf{j} = (8\text{ rad/s})\mathbf{i} + (16\text{ rad/s})\mathbf{j}$$

For axes x', y', z' parallel to x, y, z with origin at A,

$$m = \frac{W}{g} = \frac{6}{32.2} = 0.186335\text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\bar{I}_{x'} = \frac{1}{4}mr^2 = \frac{1}{4}(0.186335)\left(\frac{8}{12}\right)^2 = 0.020704\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\bar{I}_{z'} = \bar{I}_{x'} = 0.020704\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\bar{I}_{y'} = \bar{I}_{x'} + \bar{I}_{z'} = 0.041408\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\mathbf{H}_A = \bar{I}_{x'}\omega_{x'}\mathbf{i}' + \bar{I}_{y'}\,\omega_{y'}\mathbf{j} + I_{z'}\,\omega_{z'}\mathbf{k}$$

$$= (0.020704)(8)\mathbf{i} + (0.041408)(16)\mathbf{j}$$

$$= (0.1656\text{ lb}\cdot\text{s}\cdot\text{ft})\mathbf{i} + (0.6625\text{ lb}\cdot\text{s}\cdot\text{ft})\mathbf{j}$$

Let the frame of reference *Axyz* be rotating with angular velocity

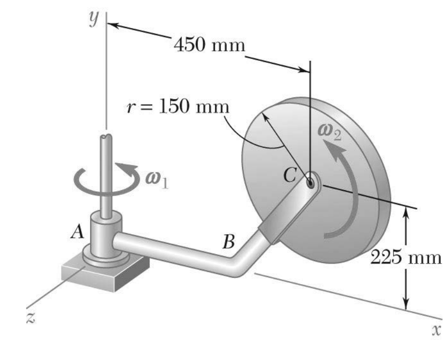
$$\boldsymbol{\Omega} = \omega_2\mathbf{i} = (8\text{ rad/s})\mathbf{i}$$

Then

$$\dot{\mathbf{H}}_A = (\dot{\mathbf{H}}_A)_{Axyz} + \boldsymbol{\Omega}\times\mathbf{H}_A = 0 + \omega_2\mathbf{i}\times\mathbf{H}_A$$

$$\dot{\mathbf{H}}_A = 8\mathbf{i}\times(0.1656\mathbf{i} + 0.6625\mathbf{j}) = (5.30\text{ lb}\cdot\text{ft})\mathbf{k}$$

$$\dot{\mathbf{H}}_A = (5.30\text{ lb}\cdot\text{ft})\mathbf{k}$$ ◀



PROBLEM 18.59

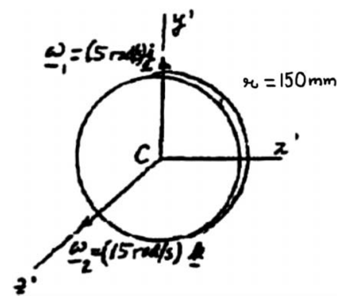
Determine the rate of change $\dot{\mathbf{H}}_C$ of the angular momentum $\mathbf{H}_C$ of the disk of Problem 18.5.

PROBLEM 18.5 A thin disk of mass $m = 4$ kg rotates at the constant rate $\omega_2 = 15$ rad/s with respect to arm *ABC*, which itself rotates at the constant rate $\omega_1 = 5$ rad/s about the *y* axis. Determine the angular momentum of the disk about its center *C*.

SOLUTION

$$r = 150 \text{ mm}$$

Angular velocity of disk:



$$\boldsymbol{\omega} = \omega_1 \mathbf{j} + \omega_2 \mathbf{k}$$
$$= (5 \text{ rad/s})\mathbf{j} + (15 \text{ rad/s})\mathbf{k}$$

Centroidal moments of inertia:

$$\bar{I}_{x'} = \bar{I}_{y'} = \frac{1}{4} m r^2$$
$$= \frac{1}{4}(4)(0.150 \text{ m})^2 = 0.0225 \text{ kg} \cdot \text{m}^2$$
$$\bar{I}_{z'} = \frac{1}{2} m r^2 = 0.045 \text{ kg} \cdot \text{m}^2$$

Angular momentum about *C*.

$$\mathbf{H}_C = \bar{I}_{x'} \omega_{x'} \mathbf{i} + \bar{I}_{y'} \omega_{y'} \mathbf{j} + I_{z'} \omega_{z'} \mathbf{k}$$
$$= 0 + (0.0225)(5)\mathbf{j} + (0.045)(15)\mathbf{k}$$
$$= (0.1125 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{j} + (0.6750 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k}$$

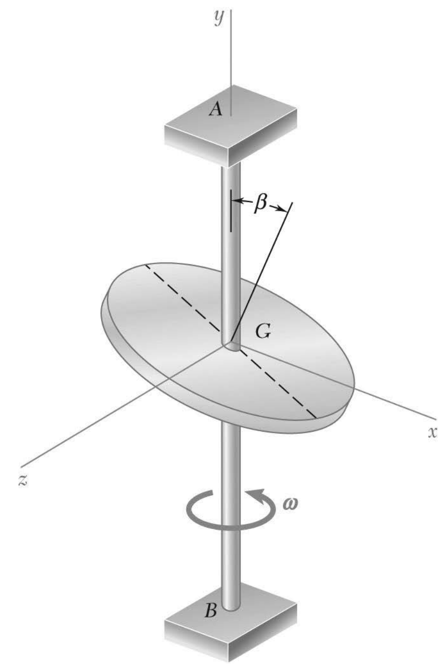
Rate of change of angular momentum. Let the reference frame *Oxyz* be rotating with angular velocity.

$$\boldsymbol{\Omega} = \boldsymbol{\omega}_1 = (5 \text{ rad/s})\mathbf{j}$$

Then

$$\dot{\mathbf{H}}_C = (\dot{\mathbf{H}}_C)_{Oxyz} + \boldsymbol{\Omega} \times \mathbf{H}_C$$
$$= 0 + 5\mathbf{j} \times (0.1125\mathbf{j} + 0.6750\mathbf{k})$$
$$= (3.3750 \text{ N} \cdot \text{m})\mathbf{i}$$

$$\dot{\mathbf{H}}_C = (3.38 \text{ N} \cdot \text{m})\mathbf{i}$$ ◀



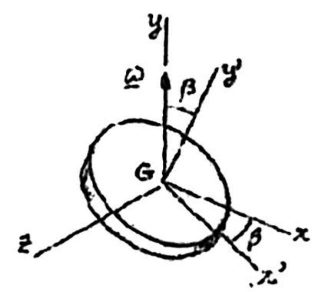
PROBLEM 18.60

Determine the rate of change $\dot{\mathbf{H}}_G$ of the angular momentum $\mathbf{H}_G$ of the disk of Problem 18.8.

PROBLEM 18.8 A homogeneous disk of mass m and radius r is mounted on the vertical shaft AB. The normal to the disk at G forms an angle $\beta = 25°$ with the shaft. Knowing that the shaft has a constant angular velocity ω, determine the angle θ formed by the shaft AB and the angular momentum $\mathbf{H}_G$ of the disk about its mass center G.

SOLUTION

Use the principal centroidal axes $Gx'y'z'$,



Moments of inertia.

$$\bar{I}_{x'} = \bar{I}_z = \frac{1}{4}mr^2$$

$$\bar{I}_{y'} = \frac{1}{2}mr^2$$

Angular velocity.

$$\omega_{x'} = -\omega \sin\beta$$

$$\omega_{y'} = \omega \cos\beta$$

$$\omega_z = 0$$

Angular momentum about G.

Using Eqs. (18.10),

$$H_{x'} = \bar{I}_{x'}\omega_{x'} = -\frac{1}{4}mr^2\omega \sin\beta$$

$$H_{y'} = \bar{I}_{y'}\omega_{y'} = \frac{1}{2}mr^2\omega \cos\beta$$

$$H_z = \bar{I}_z\omega_z = 0$$

$$\mathbf{H}_G = H_{x'}\mathbf{i}' + H_{y'}\mathbf{j}' + H_z\mathbf{k}$$

PROBLEM 18.60 (Continued)

where $\mathbf{i}'$, $\mathbf{j}'$, $\mathbf{k}$ are the unit vectors along the $x'y'z$ axes.

$$\mathbf{H}_G = -\frac{1}{4}mr^2\omega\sin\beta\mathbf{i}' + \frac{1}{2}mr^2\omega\cos\beta\mathbf{j}'$$

<u>Rate of change of angular momentum</u>. Let the frame of reference $Gx'y'z'$ be rotating with angular velocity

$$\mathbf{\Omega} = \omega\mathbf{j} = \omega(-\sin\beta\mathbf{i}' + \cos\beta\mathbf{j}')$$

Then

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \mathbf{\Omega}\times\mathbf{H}_G$$

$$= 0 + \omega(-\sin\beta\mathbf{i}' + \cos\beta\mathbf{j}')\times\left(-\frac{1}{4}mr^2\omega\sin\beta\mathbf{i}' + \frac{1}{2}mr^2\omega\cos\beta\mathbf{j}'\right)$$

$$= \frac{1}{4}mr^2\omega^2\cos\beta\sin\beta\mathbf{k} - \frac{1}{2}mr^2\omega^2\sin\beta\cos\beta\mathbf{k}$$

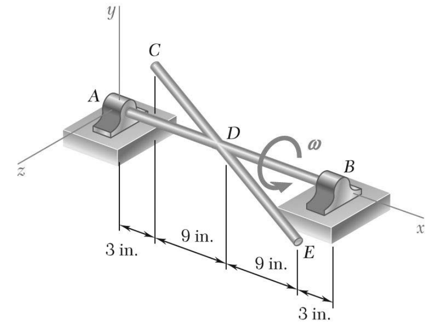
$$= -\frac{1}{4}mr^2\omega^2\sin\beta\cos\beta\mathbf{k}$$

With $\beta = 25°$,

$$\dot{\mathbf{H}}_G = -\frac{1}{4}mr^2\omega^2\sin 25°\cos 25°\mathbf{k}$$

$$\dot{\mathbf{H}}_G = -0.0958mr^2\omega^2\mathbf{k} \blacktriangleleft$$

PROBLEM 18.61



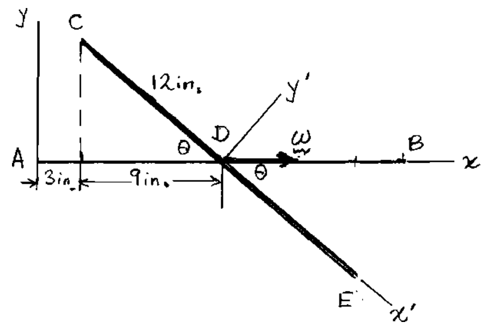
Determine the rate of change $\dot{\mathbf{H}}_D$ of the angular momentum $\mathbf{H}_D$ of the assembly of Problem 18.3, assuming that at the instant considered the assembly has an angular velocity $\boldsymbol{\omega} = (12\text{ rad/s})\mathbf{i}$ and an angular acceleration $\alpha = -(96\text{ rad/s}^2)\mathbf{i}$.

SOLUTION

$$m = \frac{W}{g} = \frac{3}{32.2} = 0.093168\text{ lb}\cdot\text{s}^2/\text{ft}^2, \quad l = 24\text{ in.} = 2\text{ ft}, \quad \boldsymbol{\omega} = (12\text{ rad/s})\mathbf{i}$$

For rod ADB, $\mathbf{H}_D = I_x\omega\mathbf{i} \approx 0$, since $I_x \approx 0$.

For rod CDE, use principal axes x', y' as shown.



$$\cos\theta = \frac{9}{12}, \quad \theta = 41.410°$$

$$\omega_{x'} = \omega\cos\theta = 9\text{ rad/s}^2$$

$$\omega_{y'} = \omega\sin\theta = 7.93725\text{ rad/s}^2$$

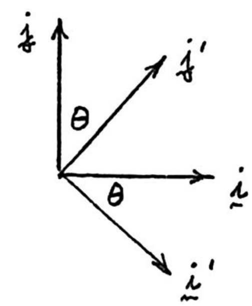
$$\omega_{z'} = 0$$

$$\alpha = -(96\text{ rad/s}^2)\mathbf{i}$$

$$\alpha_{x'} = \alpha\cos\theta = -72\text{ rad/s}$$

$$\alpha_{y'} = \alpha\sin\theta = -63.498\text{ rad/s}^2$$

$$I_{x'} \approx 0$$



$$I_{y'} = \frac{1}{12}ml^2 = \frac{1}{12}(0.93168)(2)^2 = 0.0310559\text{ kg}\cdot\text{m}^2$$

$$I_{z'} = I_{y'}$$

$$\mathbf{H}_D = I_{x'}\omega_{x'}\mathbf{i}' + I_{y'}\omega_{y'}\mathbf{j}' + I_{z'}\omega_{z'}\mathbf{k}'$$

$$= 0 + (0.0310559)(7.93725)\mathbf{j}' + 0 = (0.24698\text{ lb}\cdot\text{s/ft})\mathbf{j}'$$

Let the reference frame $Dx'y'z'$ be rotating with angular velocity

$$\boldsymbol{\Omega} = \omega\mathbf{i} = \omega_{x'}\mathbf{i}' + \omega_{y'}\mathbf{j}' = (9\text{ rad/s})\mathbf{i}' + (7.93725\text{ rad/s})\mathbf{j}'$$

$$\dot{\mathbf{H}}_D = (\dot{\mathbf{H}}_D)_{Dx'y'z'} + \boldsymbol{\Omega}\times\mathbf{H}_D$$

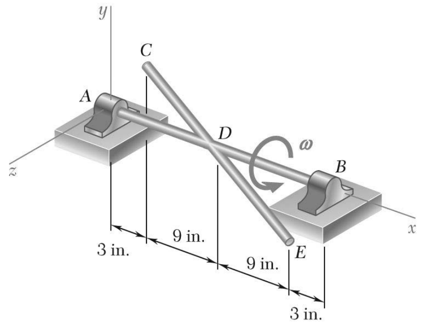
$$= I_{x'}\alpha_{x'}\mathbf{i}' + I_{y'}\alpha_{y'}\mathbf{j}' + I_{z'}\alpha_{z'}\mathbf{k}' + \boldsymbol{\Omega}\times\mathbf{H}_D$$

$$= 0 + (0.0310559)(-63.498)\mathbf{j} + 0 + (9\mathbf{i}' + 7.93725\mathbf{j}')\times(0.246498\mathbf{j}')$$

$$= -1.97199\mathbf{j}' + 2.21848\mathbf{k}$$

$$= -(1.97199\text{ kg}\cdot\text{m}^2/\text{s}^2)(\sin\theta\mathbf{i} + \cos\theta\mathbf{j}) + (2.21848\text{ kg}\cdot\text{m}^2/\text{s}^2)\mathbf{k}$$

$$\dot{\mathbf{H}}_D = -(1.304\text{ N}\cdot\text{m})\mathbf{i} - (1.479\text{ N}\cdot\text{m})\mathbf{j} + (2.22\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 18.62

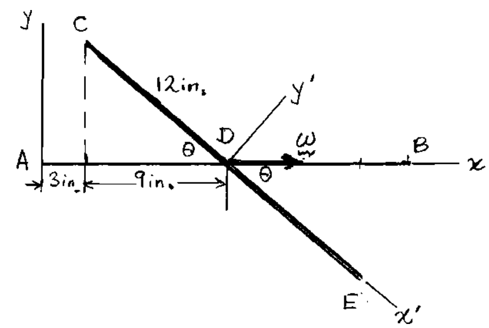
Determine the rate of change $\dot{\mathbf{H}}_D$ of the angular momentum $\mathbf{H}_D$ of the assembly of Problem 18.3, assuming that at the instant considered the assembly has an angular velocity $\boldsymbol{\omega} = (12\text{ rad/s})\mathbf{i}$ and an angular acceleration $\boldsymbol{\alpha} = (96\text{ rad/s}^2)\mathbf{i}$.

SOLUTION

$$m = \frac{W}{g} = \frac{3}{32.2} = 0.093168\text{ lb}\cdot\text{s}^2/\text{ft}, \quad l = 24\text{ in.} = 2\text{ ft}, \quad \boldsymbol{\omega} = (12\text{ rad/s})\mathbf{i}$$

For rod ADB, $\mathbf{H}_D = I_x\omega\mathbf{i} \approx 0$, since $I_x \approx 0$.

For rod CDE, use principal axes x', y' as shown.



$$\cos\theta = \frac{9}{12}, \quad \theta = 41.410°$$

$$\omega_{x'} = \omega\cos\theta = 9\text{ rad/s}^2$$

$$\omega_{y'} = \omega\sin\theta = 7.93725\text{ rad/s}^2$$

$$\omega_{z'} = 0$$

$$\alpha = (96\text{ rad/s}^2)\mathbf{i}$$

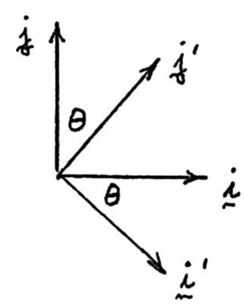
$$\alpha_{x'} = \alpha\cos\theta = 72\text{ rad/s}$$

$$\alpha_{y'} = \alpha\sin\theta = 63.498\text{ rad/s}^2$$

$$I_{x'} \approx 0$$

$$I_{y'} = \frac{1}{12}ml^2 = \frac{1}{12}(0.93168)(2)^2 = 0.0310559\text{ kg}\cdot\text{m}^2$$

$$I_{z'} = I_{y'}$$



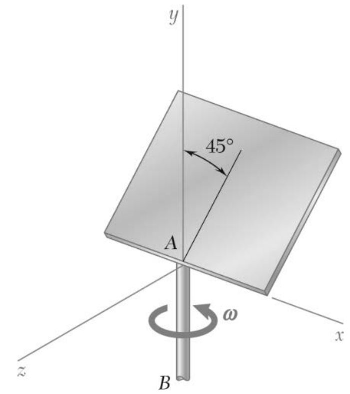
$$\mathbf{H}_D = I_{x'}\omega_{x'}\mathbf{i}' + I_{y'}\omega_{y'}\mathbf{j}' + I_{z'}\omega_{z'}\mathbf{k}'$$
$$= 0 + (0.0310559)(7.93725)\mathbf{j} + 0 = (0.246498\text{ lb}\cdot\text{s/ft})\mathbf{j}'$$

Let the reference frame $Dx'y'z'$ be rotating with angular velocity

$$\boldsymbol{\Omega} = \omega\mathbf{i} = \omega_{x'}\mathbf{i}' + \omega_{y'}\mathbf{j}' = (9\text{ rad/s})\mathbf{i}' + (7.93725\text{ rad/s})\mathbf{j}'$$

$$\dot{\mathbf{H}}_D = (\dot{\mathbf{H}}_D)_{Dx'y'z'} + \boldsymbol{\Omega}\times\mathbf{H}_D$$
$$= I_{x'}\alpha_{x'}\mathbf{i}' + I_{y'}\alpha_{y'}\mathbf{j}' + I_{z'}\alpha_{z'}\mathbf{k}' + \boldsymbol{\Omega}\times\mathbf{H}_D$$
$$= 0 + (0.310559)(63.498)\mathbf{j} + 0 + (9\mathbf{i}' + 7.93725\mathbf{j}')\times(0.246498\mathbf{j}')$$
$$= 1.97199\mathbf{j}' + 2.21848\mathbf{k}$$
$$= (1.97199\text{ kg}\cdot\text{m}^2/\text{s}^2)(\sin\theta\mathbf{i} + \cos\theta\mathbf{j}) + (2.21848\text{ kg}\cdot\text{m}^2/\text{s}^2)\mathbf{k}$$

$$\dot{\mathbf{H}}_D = (1.304\text{ N}\cdot\text{m})\mathbf{i} + (1.479\text{ N}\cdot\text{m})\mathbf{j} + (2.22\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

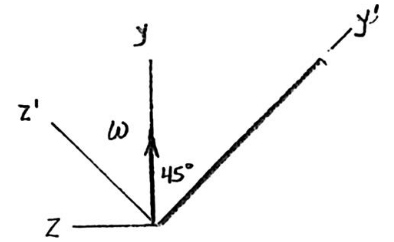


PROBLEM 18.63

A thin homogeneous square of mass m and side a is welded to a vertical shaft AB with which it forms an angle of 45°. Knowing that the shaft rotates with an angular velocity $\boldsymbol{\omega} = \omega \mathbf{j}$ and an angular acceleration $\boldsymbol{\alpha} = \alpha \mathbf{j}$, determine the rate of change $\dot{\mathbf{H}}_A$ of the angular momentum $\mathbf{H}_A$ of the plate assembly.

SOLUTION

Use principal axes y', z' as shown.



$$\omega_{y'} = \omega \cos 45°, \quad \omega_{z'} = \omega \sin 45°$$

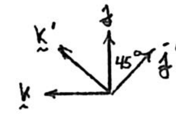
$$\omega_{x'} = 0$$

$$I_{x'} = \frac{1}{3} m a^2, \quad I_{y'} = \frac{1}{12} m a^2$$

$$I_{z'} = I_{x'} + I_{y'} = \frac{5}{12} m a^2$$

$$\mathbf{H}_A = I_{x'} \omega_{x'} \mathbf{i}' + I_{y'} \omega_{y'} \mathbf{j}' + I_{z'} \omega_{z'} \mathbf{k}'$$

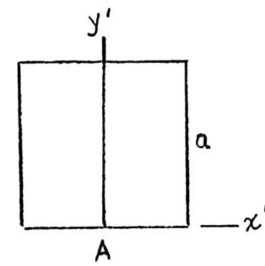
$$= 0 + \left(\frac{1}{12} m a^2 \right) (\omega \cos 45°) \mathbf{j}' + \left(\frac{5}{12} m a^2 \right) (\omega \sin 45°) \mathbf{k}'$$



$$\mathbf{H}_A = \left(\frac{1}{12} m a^2 \right) (\omega \cos 45°)(\cos 45° \mathbf{j} - \sin 45° \mathbf{k})$$

$$+ \left(\frac{5}{12} m a^2 \right) (\omega \sin 45°)(\sin 45° \mathbf{j} + \cos 45° \mathbf{k})$$

$$\mathbf{H}_A = \frac{m a^2 \omega}{12} (3\mathbf{j} + 2\mathbf{k})$$



<u>Angular acceleration</u>. $\boldsymbol{\alpha} = \alpha \mathbf{j}; \quad \dot{\omega}_{y'} = \alpha \cos 45°, \quad \dot{\omega}_{z'} = \alpha \sin 45°$

Let the reference frame $Axyz$ be rotating with angular velocity

$$\boldsymbol{\Omega} = \omega \mathbf{j}$$

With respect to this frame,

$$(\dot{\mathbf{H}}_A)_{Axyz} = I_{x'} \dot{\omega}_{x'} \mathbf{i}' + I_{y'} \dot{\omega}_{y'} \mathbf{j}' + I_{z'} \dot{\omega}_{z} \mathbf{k}'$$

$$= 0 + \left(\frac{1}{12} m a^2 \right) (\alpha \cos 45°) \mathbf{i}' + \left(\frac{5}{12} m a^2 \right) (\alpha \sin 45°) \mathbf{k}'$$

$$= \frac{m a^2 \alpha}{12} (3\mathbf{j} + 2\mathbf{k})$$

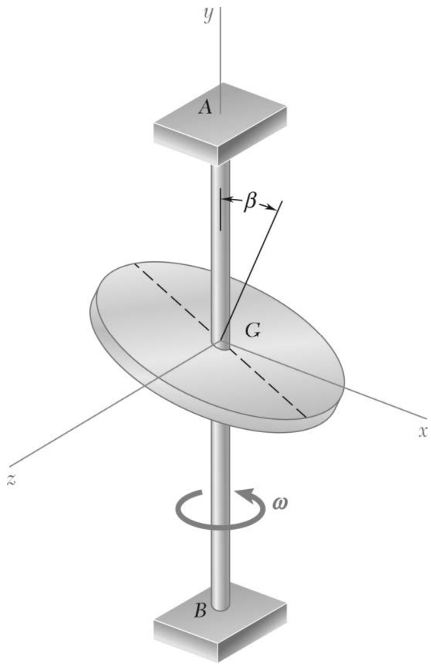
PROBLEM 18.63 (Continued)

With respect to the fixed reference frame,

$$\dot{\mathbf{H}}_A = (\dot{\mathbf{H}}_A)_{Axyz} + \mathbf{\Omega} \times \mathbf{H}_A$$

$$= \frac{ma^2\alpha}{12}(3\mathbf{j} + 2\mathbf{k}) + \omega\mathbf{j} \times \left[\frac{ma^2\omega}{12}(3\mathbf{j} + 2\mathbf{k})\right]$$

$$\dot{\mathbf{H}}_A = \frac{ma^2}{12}(2\omega^2\mathbf{i} + 3\alpha\mathbf{j} + 2\alpha\mathbf{k}) \blacktriangleleft$$



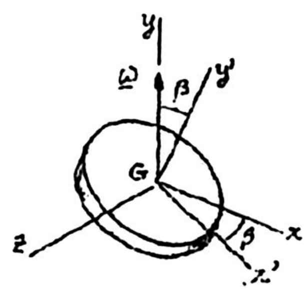
PROBLEM 18.64

Determine the rate of change $\dot{\mathbf{H}}_G$ of the angular momentum $\mathbf{H}_G$ of the disk of Problem 18.8, assuming that at the instant considered the assembly has an angular velocity $\boldsymbol{\omega} = \omega \mathbf{j}$ and an angular acceleration $\boldsymbol{\alpha} = \alpha \mathbf{j}$.

PROBLEM 18.8 A homogeneous disk of mass m and radius r is mounted on the vertical shaft AB. The normal to the disk at G forms an angle $\beta = 25°$ with the shaft. Knowing that the shaft has a constant angular velocity ω, determine the angle θ formed by the shaft AB and the angular momentum $\mathbf{H}_G$ of the disk about its mass center G.

SOLUTION

Use the principal centroidal axes $Gx'y'z'$



Moments of inertia.

$$\bar{I}_{x'} = \bar{I}_{z'} = \frac{1}{4}mr^2$$

$$\bar{I}_{y'} = \frac{1}{2}mr^2$$

Angular velocity.

$$\omega_{x'} = -\omega \sin\beta$$
$$\omega_{y'} = \omega \cos\beta$$
$$\omega_z = 0$$

Angular momentum about G.

Using Eqs. (18.10),

$$H_{x'} = \bar{I}_{x'}\omega_{x'} = -\frac{1}{4}mr^2\omega\sin\beta$$

$$H_{y'} = \bar{I}_{y'}\omega_{y'} = \frac{1}{2}mr^2\omega\cos\beta$$

$$H_z = \bar{I}_z\omega_z = 0$$

$$\mathbf{H}_G = H_{x'}\mathbf{i}' + H_{y'}\mathbf{j} + H_z\mathbf{k}$$

where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are the unit vectors along the x', y', z' axes.

$$\mathbf{H}_G = -\frac{1}{4}mr^2\omega\sin\beta\mathbf{i}' + \frac{1}{2}mr^2\omega\cos\beta\mathbf{j}'$$

<u>Angular acceleration</u>. $\boldsymbol{\alpha} = \alpha\mathbf{j}, \quad \dot{\omega}_{x'} = -\alpha\sin\beta, \quad \dot{\omega}_{y'} = \alpha\cos\beta, \quad \dot{\omega}_{z'} = 0$

<u>Rate of change of angular momentum</u>. Let the reference frame $Gxyz$ be rotating with angular velocity

$$\boldsymbol{\Omega} = \omega\mathbf{j} = -(\omega\sin\beta)\mathbf{i}' + (\omega\cos\beta)\mathbf{j}'$$

With respect to this frame,

$$\begin{aligned}(\dot{\mathbf{H}}_G)_{Gxyz} &= \bar{I}_{x'}\dot{\omega}_{x'}\mathbf{i}' + \bar{I}_{y'}\dot{\omega}_{y'}\mathbf{j}' + \bar{I}_{z'}\dot{\omega}_{z'}\mathbf{k}' \\
&= \left(\frac{1}{4}mr^2\right)(-\alpha\sin\beta)\mathbf{i}' + \left(\frac{1}{2}mr^2\right)(\alpha\cos\beta)\mathbf{j}' + 0 \\
&= -\frac{1}{4}mr^2\alpha\sin\beta\mathbf{i}' + \frac{1}{2}mr^2\alpha\cos\beta\mathbf{j}' \\
&= -\frac{1}{4}mr^2\alpha\sin\beta(\mathbf{i}\cos\beta - \mathbf{j}\sin\beta) + \frac{1}{2}mr^2\alpha\cos\beta(\mathbf{i}\sin\beta + \mathbf{j}\cos\beta) \\
&= \frac{1}{4}mr^2\alpha[\mathbf{i}\sin\beta\cos\beta + \mathbf{j}(2\cos^2\beta + \sin^2\beta)] \\
&= \frac{1}{4}mr^2\alpha[\mathbf{i}\sin 25°\cos 25° + \mathbf{j}(2\cos^2 25° + \sin^2 25°)] \\
&= mr^2\alpha(0.0957556\mathbf{i} + 0.45535\mathbf{j})\end{aligned}$$

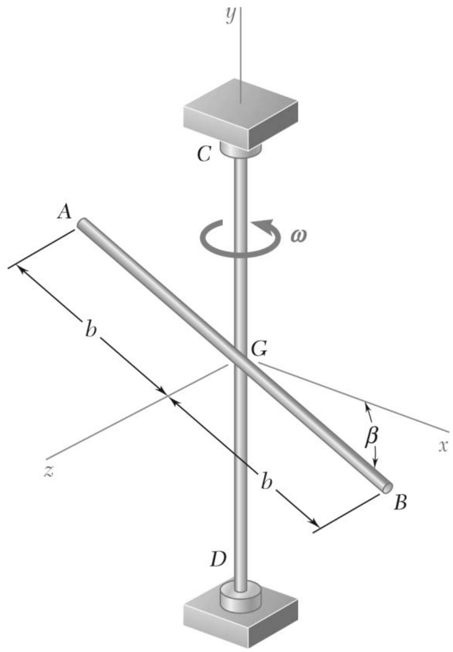
With respect to the fixed reference frame,

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \boldsymbol{\Omega}\times\mathbf{H}_G \quad \text{where}$$

$$\begin{aligned}\boldsymbol{\Omega}\times\mathbf{H}_G &= +(-\omega\sin\beta\mathbf{i}' + \omega\cos\beta\mathbf{j}') \\
&\quad \times\left(-\frac{1}{4}mr^2\omega\sin\beta + \frac{1}{2}mr^2\omega\cos\beta\right) \\
&= \frac{1}{4}mr^2\omega^2\cos\beta\sin\beta\mathbf{k} - \frac{1}{2}mr^2\omega^2\sin\beta\cos\beta\mathbf{k} \\
&= -\frac{1}{4}mr^2\omega^2\sin\beta\cos\beta\mathbf{k} \\
&= -\frac{1}{4}mr^2\omega^2\sin 25°\cos 25°\mathbf{k} = -0.095756mr^2\omega^2\mathbf{k}\end{aligned}$$

Then $\dot{\mathbf{H}}_G = mr^2(0.0958\alpha\mathbf{i} + 0.455\alpha\mathbf{j} - 0.0958\omega^2\mathbf{k})$ ◀

PROBLEM 18.65



A slender, uniform rod AB of mass m and a vertical shaft CD, each of length $2b$, are welded together at their midpoints G. Knowing that the shaft rotates at the constant rate ω, determine the dynamic reactions at C and D.

SOLUTION

Using the principal axes $Gx'y'z$:

$$\bar{I}_{x'} = 0, \quad \bar{I}_{y'} = \bar{I}_z = \frac{1}{3}mb^2$$

$$\omega_{x'} = -\omega \sin\beta, \quad \omega_{y'} = \omega\cos\beta, \quad \omega_z = 0$$

Angular momentum about G. $\quad \mathbf{H}_G = \bar{I}_{x'}\omega_{x'}\mathbf{i}' + \bar{I}_{y'}\omega_{y'}\mathbf{j}' + \bar{I}_z\omega_z\mathbf{k}$

$$\mathbf{H}_G = \frac{1}{3}mb^2\omega\cos\beta\,\mathbf{j}'$$

or, since $\mathbf{j}' = \mathbf{i}\sin\beta + \mathbf{j}\cos\beta$: $\quad \mathbf{H}_G = \frac{1}{3}mb^2\omega\cos\beta(\sin\beta\,\mathbf{i} + \cos\beta\,\mathbf{j}) \quad (1)$

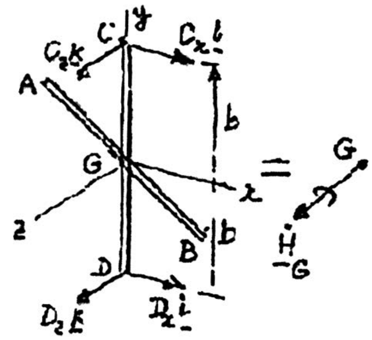
Rate of change of angular momentum.

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \mathbf{\Omega}\times\mathbf{H}_G = 0 + \boldsymbol{\omega}\times\mathbf{H}_B$$

$$= \omega\mathbf{j}\times\left[\frac{1}{3}mb^2\omega\cos\beta(\sin\beta\,\mathbf{i} + \cos\beta\,\mathbf{j})\right]$$

$$= -\frac{1}{3}mb^2\omega^2\sin\beta\cos\beta\,\mathbf{k}$$

PROBLEM 18.65 (Continued)



Equations of motion.

We equate the systems of external and effective forces

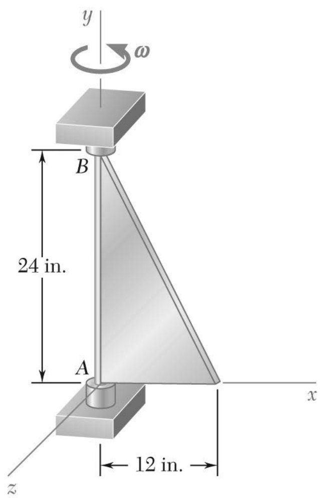
$$\Sigma\mathbf{M}_D = \Sigma(\mathbf{M}_D)_{\text{eff}}: \quad 2b\mathbf{j}\times(C_x\mathbf{i}+C_z\mathbf{k}) = \dot{\mathbf{H}}_G$$

$$-2bC_x\mathbf{k}+2bC_z\mathbf{i} = -\frac{1}{3}mb^2\omega^2\sin\beta\cos\beta\,\mathbf{k}$$

Thus,
$$C_x = \frac{1}{6}mb\omega^2\sin\beta\cos\beta, \quad C_z = 0 \qquad \mathbf{C} = \frac{1}{6}mb\omega^2\sin\beta\cos\beta\,\mathbf{i} \blacktriangleleft$$

$$\Sigma\mathbf{F} = \Sigma\mathbf{F}_{\text{eff}}: \quad \mathbf{C}+\mathbf{D} = 0 \qquad \mathbf{D} = -\frac{1}{6}mb\omega^2\sin\beta\cos\beta\,\mathbf{i} \blacktriangleleft$$

PROBLEM 18.66



A thin homogeneous triangular plate of weight 10 pounds is welded to a light vertical axle supported by bearings at A and B. Knowing that the plate rotates at the constant rate $\omega = 8$ rad/s, determine the dynamic reactions at A and B.

SOLUTION

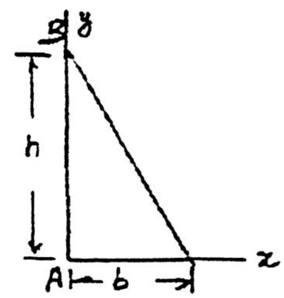
We shall use Eqs. (18.1) and (18.28):

$$\Sigma \mathbf{F} = m\bar{\mathbf{a}} \quad \Sigma \mathbf{M}_A = (\dot{\mathbf{H}}_A)_{Axyz} + \mathbf{\Omega} \times \mathbf{H}_A$$

Computation of $\mathbf{H}_A$:

$$\boldsymbol{\omega} = \omega \mathbf{j}$$

$$\mathbf{H}_A = -I_{xy}\omega\mathbf{i} + I_y\omega\mathbf{j} - I_{yz}\omega\mathbf{k}$$



The moment of inertia of the triangle is

$$(I_y)_{\text{area}} = \frac{1}{12}b^3h, \quad A = \frac{1}{2}bh$$

$$(I_y)_{\text{mass}} = \frac{m}{A}(I_y)_{\text{area}} = \frac{1}{6}mb^2$$

The product of inertia of the triangle is

$$(I_{xy})_{\text{area}} = \frac{1}{24}b^2h^2$$

$$(I_{xy})_{\text{mass}} = \frac{m}{A}(I_{xy})_{\text{area}} = \frac{1}{12}mbh$$

Since the z-coordinate is negligible,

$$(I_{yz})_{\text{mass}} = 0$$

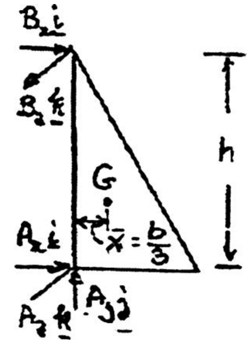
Thus,

$$\mathbf{H}_A = -\frac{1}{12}mbh\omega\mathbf{i} + \frac{1}{6}mb^2\omega\mathbf{j}. \quad (1)$$

where the frame of reference $Axyz$ rotates with the plate with the angular velocity

$$\mathbf{\Omega} = \boldsymbol{\omega} = \omega\mathbf{j}$$

PROBLEM 18.66 (Continued)



<u>Equations of motion</u>. (Weight is omitted for dynamic reactions.)

Eq. (18.28),

$$\Sigma \mathbf{M}_A = (\dot{\mathbf{H}}_A)_{Axyz} + \mathbf{\Omega} \times \mathbf{H}_A.$$

Since $\omega =$ constant, it follows from Eq. (1) that

$$(\dot{\mathbf{H}}_A)_{Axyz} = 0$$

Thus,

$$h\mathbf{j} \times (B_x\mathbf{i} + B_z\mathbf{k}) = 0 + \omega\mathbf{j} \times \left(-\frac{mbh}{12}\omega\mathbf{i} + \frac{mb^2}{6}\omega\mathbf{j}\right) - hB_x\mathbf{k} + h\mathbf{B}_z\mathbf{i}$$

$$= +\frac{1}{12}mbh\omega^2\mathbf{k}$$

Equating coefficients of unit vectors: $\quad B_x = -\frac{1}{12}mb\omega^2, \quad B_z = 0$

$$\mathbf{B} = -\frac{1}{12}mb\omega^2\mathbf{i}$$

The dynamic reactions must also satisfy Eq. (18.1):

$$\Sigma\mathbf{F} = m\bar{\mathbf{a}}: \quad \mathbf{A} + \mathbf{B} = -m\bar{x}\omega^2\mathbf{i} = -m\left(\frac{b}{3}\right)\omega^2\mathbf{i}$$

$$\mathbf{A} = -\frac{1}{3}mb\omega^2\mathbf{i} - \left(-\frac{1}{12}mb\omega^2\mathbf{i}\right), \quad \mathbf{A} = -\frac{1}{4}mb\omega^2\mathbf{i}$$

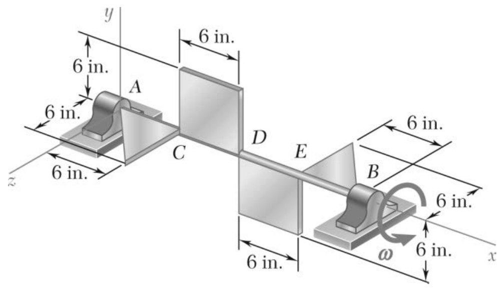
<u>Given data</u>: $\quad W = 10 \text{ lb}, \quad b = 12 \text{ in.} = 1 \text{ ft}, \quad h = 24 \text{ in.} = 2 \text{ ft}, \quad \omega = 8 \text{ rad/s}$

$$\mathbf{A} = -\frac{1}{4}\left(\frac{10 \text{ lbs}}{32.2}\right)(1 \text{ ft})(8 \text{ rad/s})^2\mathbf{i}$$

$\mathbf{A} = -(4.97 \text{ lb})\mathbf{i}$ ◀

$$\mathbf{B} = -\frac{1}{12}\left(\frac{10 \text{ lbs}}{32.2}\right)(1 \text{ ft})(8 \text{ rad/s})^2\mathbf{i}$$

$\mathbf{B} = -(1.656 \text{ lb})\mathbf{i}$ ◀



PROBLEM 18.67

The assembly shown consists of pieces of sheet aluminum of uniform thickness and of total weight 2.7 lb welded to a light axle supported by bearings at *A* and *B*. Knowing that the assembly rotates at the constant rate $\omega = 240\,\text{rpm}$, determine the dynamic reactions at *A* and *B*.

SOLUTION

Mass of sheet metal: $m = \dfrac{2.7}{32.2} = 0.08385\ \text{lb}\cdot\text{s}^2/\text{ft}$

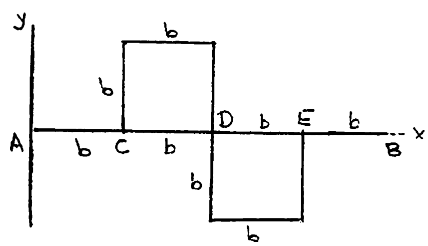
Sheet metal dimension: $b = 6\ \text{in.} = 0.5\ \text{ft}$

Area of sheet metal: $A = \dfrac{1}{2}b^2 + b^2 + b^2 + \dfrac{1}{2}b^2 = 3b^2 = 0.75\ \text{ft}^2$

Let $\rho = \dfrac{m}{A} = \dfrac{0.08385}{0.75} = 0.1118\ \text{lb}\cdot\text{s}^2/\text{ft}^3 = \text{mass per unit area.}$

Moments and products of inertia: $I_{\text{mass}} = \rho I_{\text{area}}$

xy plane (rectangles)



$$I_x = \frac{1}{3}b^4 + \frac{1}{3}b^4 = \frac{2}{3}b^4$$

$$I_x = \frac{2}{3}\rho b^4$$

$$= \frac{2}{3}(0.1118)(0.5)^4$$

$$= 4.658\times 10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

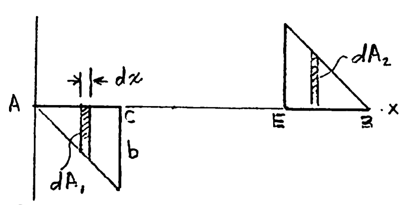
$$I_{xy} = (b^2)\left(\frac{3}{2}b\right)\left(\frac{1}{2}b\right) + (b^2)\left(\frac{5}{2}b\right)\left(-\frac{1}{2}b\right)$$

$$= -\frac{1}{2}b^4$$

$$I_{xy} = -\frac{1}{2}\rho b^4 = -\frac{1}{2}(0.1118)(0.5)^4$$

$$= -3.4938\times 10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

xz plane (triangles)



$$I_x = \frac{1}{12}b^4 + \frac{1}{12}b^4 = \frac{1}{6}b^4$$

$$I_x = \frac{1}{6}\rho b^4 = \frac{1}{6}(0.1118)(0.5)^4$$

$$= 1.1646\times 10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

PROBLEM 18.67 (Continued)

For calculation of I_{xz}, use pairs of elements dA_1 and dA_2:

$$dA_2 = dA_1.$$

$$I_{xz} = \int x\frac{z}{2}dA_1 + \int(4b-x)\left(-\frac{z}{2}\right)dA_2 = -\int(2b-x)z\,dA_1 = -\int_0^b(2b-x)z^2dx$$

but $$z = x.$$

Hence, $$I_{xz} = -\int_0^a(2bx^2-x^3)dx = -\left(\frac{2}{3}b^4-\frac{1}{4}b^4\right) = -\frac{5}{12}b^4$$

$$I_{xz} = -\frac{5}{12}\rho b^4 = -\frac{5}{12}(0.1118)(0.5)^4 = -2.9115\times10^{-3}\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Total for I_x: $$I_x = 4.658\times10^{-3} + 1.1646\times10^{-3} = 5.823\times10^{-3}\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

The mass center lies on the rotation axis, therefore

$$\bar{\mathbf{a}} = 0$$

$$\Sigma\mathbf{F} = \mathbf{A}+\mathbf{B} = m\bar{\mathbf{a}} = 0 \quad \mathbf{A} = -\mathbf{B}$$

$$\mathbf{H}_A = I_x\omega\mathbf{i} - I_{xy}\omega\mathbf{j} - I_{xz}\omega\mathbf{k} \quad \boldsymbol{\omega} = \omega\mathbf{i}, \quad \boldsymbol{\alpha} = \alpha\mathbf{i}$$

Let the frame of reference *Axyz* be rotating with angular velocity

$$\boldsymbol{\Omega} = \boldsymbol{\omega} = \omega\mathbf{i}$$

$$\Sigma\mathbf{M}_A = \dot{\mathbf{H}}_A = (\dot{\mathbf{H}}_A)_{Axyz} + \boldsymbol{\Omega}\times\mathbf{H}_A$$

$$M_0\mathbf{i} + 4b\mathbf{i}\times(B_y\mathbf{j}+B_z\mathbf{k}) = I_x\alpha\mathbf{i} - I_{xy}\alpha\mathbf{j} - I_{xz}\alpha\mathbf{k} + \omega\mathbf{i}\times(I_x\omega\mathbf{i} - I_{xy}\omega\mathbf{j} - I_{xz}\omega\mathbf{k})$$

$$M_0\mathbf{i} - 4bB_z\mathbf{j} + 4bB_y\mathbf{k} = I_x\alpha\mathbf{i} - (I_{xy}\alpha - I_{xz}\omega^2)\mathbf{j} - (I_{xz}\alpha + I_{xy}\omega^2)\mathbf{k}$$

Resolve into components and solve for B_y and B_z.

$\mathbf{i}$: $$M_0 = I_x\alpha$$

$\mathbf{j}$: $$B_z = \frac{(I_{xy}\alpha - I_{xz}\omega^2)}{4b}$$

$\mathbf{k}$: $$B_y = -\frac{(I_{xz}\alpha + I_{xy}\omega^2)}{4b}$$

<u>Data</u>: $$\alpha = 0, \quad \omega = \frac{2\pi(240)}{60} = 25.133\text{ rad/s}, \quad b = 0.5\text{ ft} \quad M_0 = 0$$

$$B_z = \frac{0-(-2.9115\times10^{-3})(25.133)^2}{(4)(0.5)} = 0.91955\text{ lb.}$$

PROBLEM 18.67 (Continued)

$$B_y = \frac{0-(-3.4938\times10^{-3})(25.133)^2}{(4)(0.5)} = 1.10346 \text{ lb.}$$

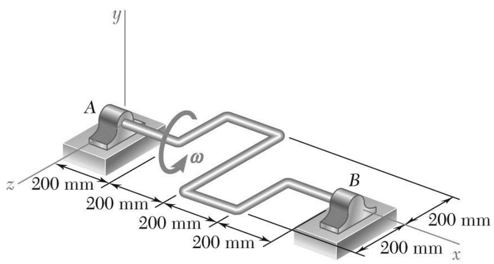
$$A_y = -B_y = -1.10346 \text{ lb.}$$

$$A_z = -B_z = -0.91955 \text{ lb.}$$

$\mathbf{A} = -(1.103 \text{ lb})\mathbf{j} - (0.920 \text{ lb})\mathbf{k}$ ◀

$\mathbf{B} = (1.103 \text{ lb})\mathbf{j} + (0.920 \text{ lb})\mathbf{k}$ ◀

PROBLEM 18.68



The 8-kg shaft shown has a uniform cross section. Knowing that the shaft rotates at the constant rate $\omega = 12$ rad/s, determine the dynamic reactions at A and B.

SOLUTION

Angular velocity: $\boldsymbol{\omega} = \omega \mathbf{i}$

Angular momentum about the mass center G:

$$\mathbf{H}_G = \bar{I}_x \omega \mathbf{i} - \bar{I}_{xy} \omega \mathbf{j} - \bar{I}_{xz} \omega \mathbf{k}$$

Let the reference frame $Axyz$ be rotating with angular velocity

$$\boldsymbol{\Omega} = \omega \mathbf{i}.$$

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Axyz} + \boldsymbol{\Omega} \times \mathbf{H}_G$$

$$= 0 + \omega \mathbf{i} \times (\bar{I}_x \omega \mathbf{i} - \bar{I}_{xy} \omega \mathbf{j} - \bar{I}_{xz} \omega \mathbf{k})$$

$$= \bar{I}_{xz} \omega^2 \mathbf{j} - \bar{I}_{xy} \omega^2 \mathbf{k}$$

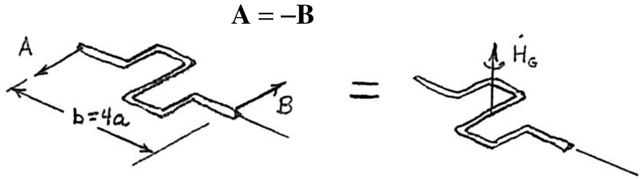
Since the shaft lies in the xz plane, $\bar{I}_{xy} = 0.$

By symmetry, the mass center lies on line AB.

$$m\bar{\mathbf{a}} = 0$$

$\Sigma \mathbf{F} = \Sigma F_{\text{eff}}$: $\mathbf{A} + \mathbf{B} = m\bar{\mathbf{a}} = 0$ $\mathbf{A}$ and $\mathbf{B}$ form a couple.

$$\mathbf{A} = -\mathbf{B}$$



$$b\mathbf{i} \times (B_y \mathbf{j} + B_z \mathbf{k}) = \dot{\mathbf{H}}_G$$

$$-bB_z \mathbf{j} + bB_y \mathbf{k} = \bar{I}_{xz} \omega \mathbf{j} \quad B_z = -\frac{\bar{I}_{xz}\omega^2}{b}, \quad B_y = 0$$

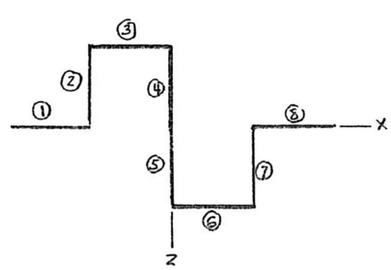
Calculation of $\bar{I}_{xz}$.

Divide the shaft into eight segments, each of length

$$a = 200 \text{ mm} = 0.2 \text{ m}$$

Let m' be the mass of one segment.

$$m' = \frac{8 \text{ kg}}{8} = 1 \text{ kg}$$



For (1), (4), (5), and (8), $I_{xz} = 0$

PROBLEM 18.68 (Continued)

Then

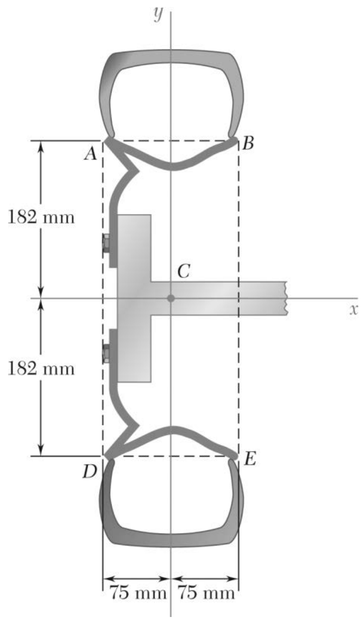
$$I_{xz} = m'(-a)\left(-\frac{a}{2}\right) + m'\left(-\frac{a}{2}\right)(-a) + m'\left(\frac{a}{2}\right)(a) + m'(a)\left(\frac{a}{2}\right) = 2m'a^2$$

$$B_z = -\frac{2m'a^2\omega^2}{4a} = -\frac{(2)(1)(0.2)^2(12)^2}{(4)(0.2)} = -14.4 \text{ N}$$

$$A_y = 0, \qquad A_z = -B_z = 14.4 \text{ N}$$

$\mathbf{A} = (14.4 \text{ N})\mathbf{k}$ ◀

$\mathbf{B} = -(14.4 \text{ N})\mathbf{k}$ ◀



PROBLEM 18.69

After attaching the 18-kg wheel shown to a balancing machine and making it spin at the rate of 15 rev/s, a mechanic has found that to balance the wheel both statically and dynamically, he should use two corrective masses, a 170-g mass placed at *B* and a 56-g mass placed at *D*. Using a right-handed frame of reference rotating with the wheel (with the *z* axis perpendicular to the plane of the figure), determine before the corrective masses have been attached (*a*) the distance from the axis of rotation to the mass center of the wheel and the products of inertia I_{xy} and I_{zx}, (*b*) the force-couple system at *C* equivalent to the forces exerted by the wheel on the machine.

SOLUTION

$$m = 18 \text{ kg}, \quad \boldsymbol{\omega} = 2\pi(15)\mathbf{i} = (94.248 \text{ rad/s})\mathbf{i}$$

$$m_B = 170 \text{ g} = 0.17 \text{ kg}$$

$$m_D = 56 \text{ g} = 0.056 \text{ kg}$$

$$x_B = 75 \text{ mm} = 0.075 \text{ m}, \quad y_B = 182 \text{ mm} = 0.182 \text{ m}$$

$$x_D = -0.075 \text{ m}, \quad y_D = -0.182 \text{ m}$$

(*a*) Balance masses are added to move the mass center to Point *C* and to reduce the products of inertia to zero.

$$m_B y_B + m_D y_D + m\bar{y} = 0$$

$$(0.17)(0.182) + (0.056)(-0.182) + 18\bar{y} = 0$$

$$\bar{y} = -1.15267 \times 10^{-3} \text{ m} \qquad \bar{y} = -1.153 \text{ mm} \blacktriangleleft$$

$$\bar{z} = 0 \qquad \bar{z} = 0 \blacktriangleleft$$

$$m_B x_B y_B + m_D x_D y_D + I_{xy} = 0$$

$$(0.17)(0.075)(0.182) + (0.056)(-0.075)(-0.182) + I_{xy} = 0$$

$$I_{xy} = -3.0848 \times 10^{-3} \qquad I_{xy} = -3.08 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

$$I_{zx} = 0 \blacktriangleleft$$

PROBLEM 18.69 (Continued)

(*b*) Force-couple system exerted on the wheel:

$$\mathbf{F}' = m\bar{\mathbf{a}}_n = -m\bar{\mathbf{r}}\omega^2 = -(18)(-1.15267\times 10^{-3}\,\mathbf{j})(94.248)^2$$
$$= (184.3\text{ N})\mathbf{j}$$

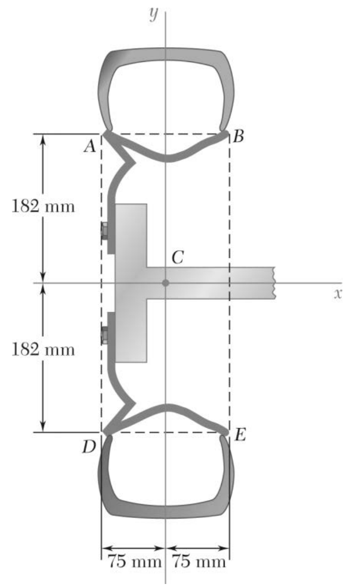
$$\mathbf{H}_C = I_x\omega\mathbf{i} - I_{xy}\omega\mathbf{j} - I_{xz}\omega\mathbf{k}$$

$$\mathbf{M}'_C = \dot{\mathbf{H}}_C = \boldsymbol{\omega}\times\mathbf{H}_C = I_{xz}\omega^2\mathbf{j} - I_{xy}\omega^2\mathbf{k}$$
$$= 0 - (-3.0848\times 10^{-3})(94.248)^2\mathbf{k} = (27.4\text{ N}\cdot\text{m})\mathbf{k}$$

Force-couple system exerted by the wheel on the machine:

$$\mathbf{F} = -\mathbf{F}'$$ $\mathbf{F} = -(184.3\text{ N})\mathbf{j}$ ◀

$$\mathbf{M}_C = -\mathbf{M}'_C$$ $\mathbf{M}_C = -(27.4\text{ N}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 18.70

When the 18-kg wheel shown is attached to a balancing machine and made to spin at a rate of 12.5 rev/s, it is found that the forces exerted by the wheel on the machine are equivalent to a force-couple system consisting of a force $\mathbf{F} = (160\text{ N})\mathbf{j}$ applied at C and a couple $\mathbf{M}_C = (14.7\text{ N}\cdot\text{m})\mathbf{k}$, where the unit vectors form a triad which rotates with the wheel. (*a*) Determine the distance from the axis of rotation to the mass center of the wheel and the products of inertia I_{xy} and I_{zx}. (*b*) If only two corrective masses are to be used to balance the wheel statically and dynamically, what should these masses be and at which of the Points A, B, D, or E should they be placed?

SOLUTION

$$m = 18\text{ kg}, \quad \omega = 2\pi(12.5)\mathbf{i} = (78.54\text{ rad/s})\mathbf{i}$$

(*a*) The force-couple system acting on the wheel is

$$\mathbf{F}' = -(160\text{ N})\mathbf{j}, \quad \mathbf{M}'_C = -(14.7\text{ N}\cdot\text{m})\mathbf{k}$$

$$\mathbf{F}' = m\bar{\mathbf{a}}_n = -m\bar{\mathbf{r}}\omega^2 \qquad \bar{\mathbf{r}} = \bar{y}\mathbf{j} + \bar{z}\mathbf{k} = \frac{-\mathbf{F}'}{m\omega^2}$$

$$\bar{y} = \frac{-F'_y}{m\omega^2} = \frac{160}{(18)(78.54)^2} = 1.441\times10^{-3}\text{ m} \qquad \bar{y} = 1.441\text{ mm}$$

$$\bar{z} = \frac{-F'_z}{m\omega^2} = 0 \qquad \bar{r} = 1.441\text{ mm} \blacktriangleleft$$

$$\mathbf{H}_C = I_x\omega\mathbf{i} - I_{xy}\omega\mathbf{j} - I_{xz}\omega\mathbf{k}$$

$$\mathbf{M}'_C = \dot{\mathbf{H}}_C = \boldsymbol{\omega}\times\mathbf{H}_C = \omega\mathbf{i}\times(I_x\omega\mathbf{i} - I_{xy}\omega\mathbf{j} - I_{xz}\omega\mathbf{k})$$

$$(M'_C)_y\,\mathbf{j} + (M'_C)_z\,\mathbf{k} = I_{xz}\omega^2\mathbf{j} - I_{xy}\omega^2\mathbf{k}$$

$$I_{xy} = -\frac{(M'_C)_z}{\omega^2} = \frac{-(-14.7)}{(78.54)^2} = 2.3831\times10^{-3}\text{ kg}\cdot\text{m}^2$$

$$I_{xy} = 2.38\times10^{-3}\text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

$$I_{xz} = \frac{(M'_C)_y}{\omega^2} = 0 \qquad I_{xz} = 0 \blacktriangleleft$$

PROBLEM 18.70 (Continued)

(*b*) Positions for the balance masses.

$$y_A = y_B = -y_E = -y_D = 182 \text{ mm} = 0.182 \text{ m}$$

$$-x_A = x_B = x_E = -x_D = 75 \text{ mm} = 0.075 \text{ m}$$

Balance masses must be added to move the mass center to Point *C* and to reduce the product of inertia to zero.

$$m_A y_A + m_B y_B + m_E y_E + m_D y_D + m\bar{y} = 0$$

$$(0.182)(m_A + m_B - m_E - m_D) + (18)(1.441\times 10^{-3})$$

$$m_A + m_B - m_E - m_D = -0.1425 \text{ kg} \qquad (1)$$

$$m_A x_A y_A + m_B x_B y_B + m_E x_E y_E + m_D x_D y_D + I_{xy} = 0$$

$$(0.075)(0.182)(-m_A + m_B - m_E + m_D) + 2.3831\times 10^{-3} = 0$$

$$-m_A + m_B - m_E + m_D = -0.17459 \text{ kg} \qquad (2)$$

To solve Eqs. (1) and (2), set $m_B = 0$ and let $m_{AD} = m_A - m_D$.

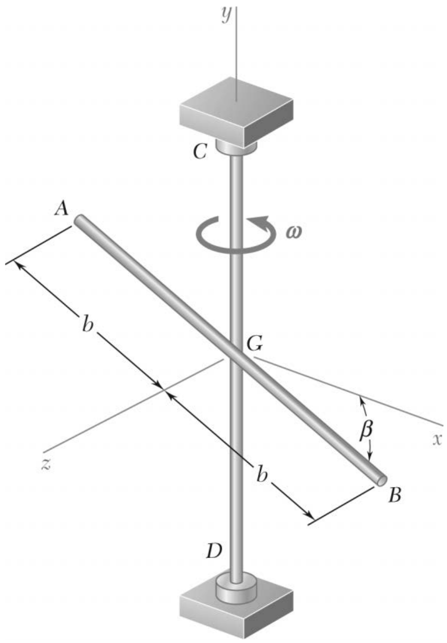
Then $\quad m_{AD} - m_E = -0.1425$

and $\quad -m_{AD} - m_E = -0.17459$

Solving, $\quad m_{AD} = 0.01605 \text{ kg}, \quad m_E = 0.1585 \text{ kg}$

Set $m_D = 0,$ so that $m_A = 0.01605 \text{ kg}$ $\qquad m_E = 158.5 \text{ g}$ ◀

$m_A = 16.05 \text{ g}$ ◀



PROBLEM 18.71

Knowing that the assembly of Problem 18.65 is initially at rest ($\omega = 0$) when a couple of moment $\mathbf{M}_0 = \mathbf{M}_0\mathbf{j}$ is applied to shaft *CD*, determine (*a*) the resulting angular acceleration of the assembly, (*b*) the dynamic reactions at *C* and *D* immediately after the couple is applied.

SOLUTION

Using the principal axes $Gx'y'z$:

$$\bar{I}_{x'} = 0, \quad \bar{I}_{y'} = \bar{I}_z = \frac{1}{3}mb^2$$

$$\omega_{x'} = -\omega \sin\beta, \quad \omega_{y'} = \omega\cos\beta, \quad \omega_z = 0$$

Angular momentum about *G*.

$$\mathbf{H}_G = \bar{I}_{x'}\omega_{x'}\mathbf{i}' + \bar{I}_{y'}\omega_{y'}\mathbf{j}' + I_z\omega_z\mathbf{k}$$

$$\mathbf{H}_G = \frac{1}{3}mb^2\omega\cos\beta\,\mathbf{j}'$$

or, since $\mathbf{j}' = \mathbf{i}\sin\beta + \mathbf{j}\cos\beta$: $\quad \mathbf{H}_G = \frac{1}{3}mb^2\omega\cos\beta(\sin\beta\mathbf{i} + \cos\beta\mathbf{j}) \qquad (1)$

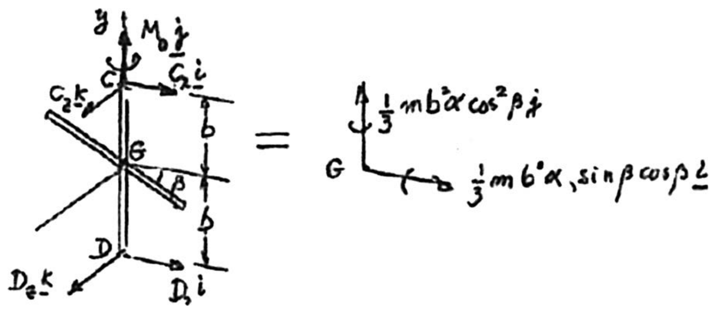
Rate of change of angular momentum.

Eq. (18.22): $\quad \dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \mathbf{\Omega} \times \mathbf{H}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + 0$

Since $\mathbf{\Omega} = \boldsymbol{\omega} = 0$ when couple is applied, thus,

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} = \frac{1}{3}mb^2\alpha\cos\beta(\sin\beta\mathbf{i} + \cos\beta\mathbf{j}) \qquad (2)$$

Equations of motion: Equivalence of applied and effective forces.



$$\Sigma \mathbf{M}_D = \Sigma(\mathbf{M}_D)_{\text{eff}}: \quad 2b\mathbf{j}\times(C_x\mathbf{i}+C_z\mathbf{k})+M_0\mathbf{j} = \frac{1}{3}mb^2\alpha\sin\beta\cos\beta\mathbf{i}+\frac{1}{3}mb^2\alpha\cos^2\beta\mathbf{j}$$

$$-2bC_x\mathbf{k}+2bC_z\mathbf{i}+M_0\mathbf{j} = \frac{1}{3}mb^2\alpha\sin\beta\cos\beta\mathbf{i}+\frac{1}{3}mb^2\alpha\cos^2\beta\mathbf{j}$$

Equating the coefficients of **i**, **j**, **k**:

i: $$2bC_z = \frac{1}{3}mb^2\alpha\sin\beta\cos\beta \qquad (3)$$

j: $$M_0 = \frac{1}{3}mb^2\alpha\cos^2\beta \qquad (4)$$

k: $$C_x = 0 \qquad (5)$$

(*a*) Angular acceleration.

From Eq. (4), $$\alpha = \frac{3M_0}{mb^2\cos^2\beta} \blacktriangleleft$$

(*b*) Initial dynamic reactions.

From Eq. (3),

$$C_z = \frac{1}{6}mb\alpha\sin\beta\cos\beta = \frac{1}{6}mb\sin\beta\cos\beta\left(\frac{3M_0}{mb^2\cos^2\beta}\right)$$

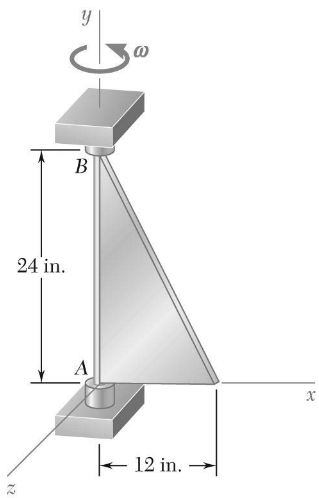
$$C_z = \left(\frac{M_0}{2b}\right)\tan\beta$$

Recalling Eq. (5), $C_x = 0$

$$\mathbf{C} = \left(\frac{M_0}{2b}\right)\tan\beta\mathbf{k} \blacktriangleleft$$

$\Sigma\mathbf{F} = \Sigma(\mathbf{F})_{\text{eff}}$: $\mathbf{C}+\mathbf{D}=0, \quad \mathbf{D}=-\mathbf{C}$

$$\mathbf{D} = -\left(\frac{M_0}{2b}\right)\tan\beta\mathbf{k} \blacktriangleleft$$



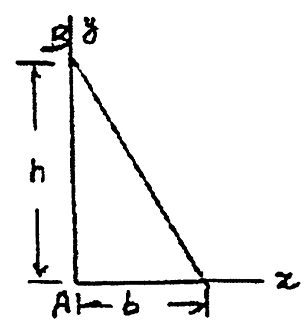
PROBLEM 18.72

Knowing that the plate of Problem 18.66 is initially at rest $(\omega = 0)$ when a couple of moment $\mathbf{M}_0 = (0.75\text{ ft}\cdot\text{lb})\mathbf{j}$ is applied to it, determine (*a*) the resulting angular acceleration of the plate, (*b*) the dynamic reactions *A* and *B* immediately after the couple has been applied.

PROBLEM 18.66 A thin homogeneous triangular plate of weight 10 pounds is welded to a light vertical axle supported by bearings at *A* and *B*. Knowing that the plate rotates at the constant rate $\omega = 8$ rad/s, determine the dynamic reactions at *A* and *B*.

SOLUTION

We shall use Eqs. (18.1) and (18.28):



$$\Sigma\mathbf{F} = m\bar{\mathbf{a}} \quad \Sigma\mathbf{M}_A = (\dot{\mathbf{H}}_A)_{Axyz} + \mathbf{\Omega}\times\mathbf{H}_A$$

Computation of $\mathbf{H}_A$:

$$\boldsymbol{\omega} = \omega\mathbf{j}$$

$$\mathbf{H}_A = -I_{xy}\omega\mathbf{i} + I_y\omega\mathbf{j} - I_{yz}\omega\mathbf{k}$$

The moment of inertia of the triangle is

$$(I_y)_{\text{area}} = \frac{1}{12}b^3h, \quad A = \frac{1}{2}bh$$

$$(I_y)_{\text{mass}} = \frac{m}{A}(I_y)_{\text{area}} = \frac{1}{6}mb^2$$

The product of inertia of the triangle is

$$(I_{xy})_{\text{area}} = \frac{1}{24}b^2h^2$$

$$(I_{xy})_{\text{mass}} = \frac{m}{A}(I_{xy})_{\text{area}} = \frac{1}{12}mbh$$

Since the *z*-coordinate is negligible,

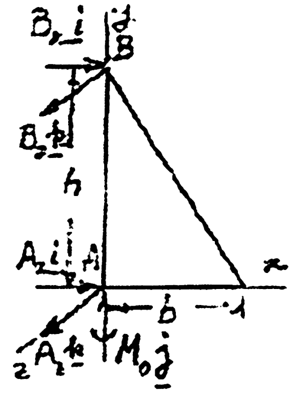
$$(I_{yz})_{\text{mass}} = 0$$

Thus,
$$\mathbf{H}_A = -\frac{1}{12}mbh\omega\mathbf{i} + \frac{1}{6}mb^2\omega\mathbf{j} \qquad (1)$$

where the frame of reference *Axyz* rotates with the plate with the angular velocity

$$\mathbf{\Omega} = \boldsymbol{\omega} = \omega\mathbf{j}$$

PROBLEM 18.72 (Continued)



Equations of motion. We first use Eq. (18.28):

$$\Sigma \mathbf{M}_A = (\dot{\mathbf{H}}_A)_{Axyz} + \mathbf{\Omega} \times \mathbf{H}_A$$

Where $\mathbf{H}_A$ was obtained in Eq. (1) of Problem 18.66:

$$\mathbf{H}_A = -\frac{1}{12} mbh\omega \mathbf{i} + \frac{1}{6} mb^2 \omega \mathbf{j}$$

Differentiating with respect to the rotating frame:

$$(\dot{\mathbf{H}}_A)_{Axyz} = -\frac{1}{12} mbh\alpha \mathbf{i} + \frac{1}{6} mb^2 \alpha \mathbf{j}$$

Substituting for $\mathbf{H}_A$ and $(\dot{\mathbf{H}}_A)_{Axyz}$ into Eq. (18.28), noting that $\omega = 0$, and computing $\Sigma \mathbf{M}_A$ from diagram:

$$M_0 \mathbf{j} + h\mathbf{j} \times (B_x \mathbf{i} + B_z \mathbf{k}) = -\frac{1}{12} mbh\alpha \mathbf{i} + \frac{1}{6} mb^2 \alpha \mathbf{j} + 0$$

$$M_0 \mathbf{j} - hB_x \mathbf{k} + hB_z \mathbf{i} = -\frac{1}{12} mbh\alpha \mathbf{i} + \frac{1}{6} mb^2 \alpha \mathbf{j}$$

Equating the coefficients of the unit vectors:

$$\mathbf{j}: \quad M_0 = \frac{1}{6} mb^2 \alpha \qquad \alpha = \frac{6M_0}{mb^2}$$

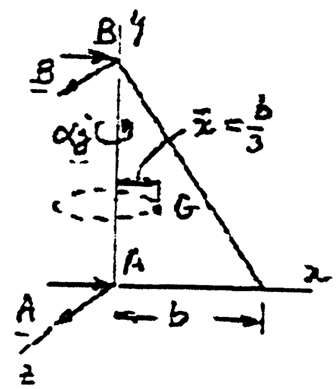
(*a*) $$\alpha = \frac{6(0.75 \text{ lb} \cdot \text{ft})}{(\frac{10 \text{ lb}}{32.2})(1 \text{ ft})^2} = 14.49 \text{ rad/s}^2$$ $\boldsymbol{\alpha} = (14.49 \text{ rad/s}^2)\mathbf{j}$ ◀

$$\mathbf{k}: \quad -hB_x = 0 \qquad B_x = 0$$

$$\mathbf{i}: \quad hB_z = -\frac{1}{12} mbh\alpha, \quad B_z = -\frac{1}{12} mb\alpha = -\frac{1}{12}\left(\frac{10 \text{ lb}}{32.2}\right)(1)(14.49)$$

(*b*) $$B_z = -0.375 \text{ lb}$$ $\mathbf{B} = -(0.375 \text{ lb})\mathbf{k}$ ◀

We shall now apply Eq. (18.1): $\Sigma \mathbf{F} = m\bar{\mathbf{a}}$:



Since $\omega = 0$: $$\bar{\mathbf{a}} = \bar{\mathbf{a}}_t = \alpha \mathbf{j} \times \frac{b}{3} \mathbf{i} = -\frac{1}{3} b\alpha \mathbf{k}$$

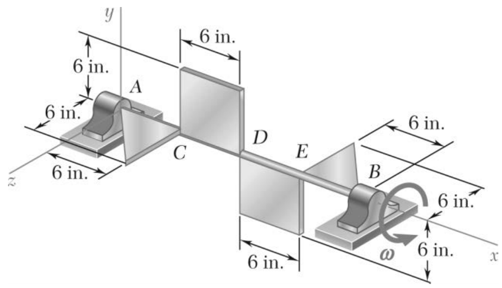
Substituting in Eq. (12.1):

$$\mathbf{A} + \mathbf{B} = -\frac{1}{3} mb\alpha \mathbf{k}$$

$$\mathbf{A} = -\frac{1}{3} mb\alpha \mathbf{k} - B$$

$$= -\frac{1}{3} mb\alpha \mathbf{k} + \frac{1}{12} mb\alpha \mathbf{k}$$

$$\mathbf{A} = -\frac{1}{4} mb\alpha \mathbf{k} = -\frac{1}{4}\left(\frac{10 \text{ lb}}{32.2}\right)(1 \text{ ft})(14.49 \text{ rad/s}^2)\mathbf{k}$$ $\mathbf{A} = -(1.125 \text{ lb})\mathbf{k}$ ◀



PROBLEM 18.73

The assembly of Problem 18.67 is initially at rest ($\omega = 0$) when a couple $\mathbf{M}_0$ is applied to axle AB. Knowing that the resulting angular acceleration of the assembly is $\boldsymbol{\alpha} = (150 \text{ rad/s}^2)\mathbf{i}$, determine (*a*) the couple $\mathbf{M}_0$, (*b*) the dynamic reactions at A and B immediately after the couple is applied.

SOLUTION

Mass of sheet metal: $m = \dfrac{2.7}{32.2} = 0.08385 \text{ lb}\cdot\text{s}^2/\text{ft}$

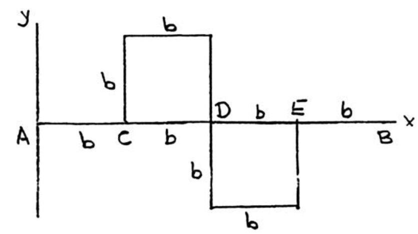
Sheet metal dimension: $b = 6 \text{ in.} = 0.5 \text{ ft}$

Area of sheet metal: $A = \dfrac{1}{2}b^2 + b^2 + b^2 + \dfrac{1}{2}b^2 = 3b^2 = 0.75 \text{ ft}^2$

Let $\rho = \dfrac{m}{A} = \dfrac{0.08385}{0.75} = 0.1118 \text{ lb}\cdot\text{s}^2/\text{ft}^3 = \text{mass per unit area}$

Moments and products of inertia: $(I)_{\text{mass}} = \rho I_{(\text{area})}$

xy plane (rectangles)



$$I_x = \frac{1}{3}b^4 + \frac{1}{3}b^4 = \frac{2}{3}b^4$$

$$I_x = \frac{2}{3}\rho b^4 = \frac{2}{3}(0.1118)(0.5)^4$$

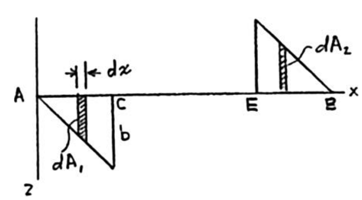
$$= 4.658\times 10^{-3} \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$I_{xy} = (b^2)\left(\frac{3}{2}b\right)\left(\frac{1}{2}b\right) + (b^2)\left(\frac{5}{2}b\right)\left(-\frac{1}{2}b\right) = -\frac{1}{2}b^4$$

$$I_{xy} = -\frac{1}{4}\rho b^4 = -\frac{1}{4}(0.1118)(0.5)^4$$

$$= -3.4938\times 10^{-3} \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

xz plane (triangles)



$$I_x = \frac{1}{12}b^4 + \frac{1}{12}b^4 = \frac{1}{6}b^4$$

$$I_x = \frac{1}{6}\rho b^4 = \frac{1}{6}(0.1118)(0.5)^4$$

$$= 1.1646\times 10^{-3} \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

PROBLEM 18.73 (Continued)

For calculation of I_{xz}, use pairs of elements dA_1 and dA_2: $dA_2 = dA_1$

$$I_{xz} = \int x\frac{z}{2}dA_1 + \int(4b-x)\left(-\frac{z}{2}\right)dA_2 = -\int(2b-x)z\,dA_1 = -\int_0^b(2b-x)z^2dx$$

But $z = x$.

Hence, $$I_{xz} = -\int_0^a(2bx^2 - x^3)dx = -\left(\frac{2}{3}b^4 - \frac{1}{4}b^4\right) = -\frac{5}{12}b^4$$

$$I_{xz} = -\frac{5}{12}\rho b^4 = -\frac{5}{12}(0.1118)(0.5)^4 = -2.9115\times10^{-3}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

Total for I_x: $$I_x = 4.658\times10^{-3} + 1.1646\times10^{-3} = 5.823\times10^{-3}\ \text{kg}\cdot\text{m}^2$$

The mass center lies on the rotation axis, therefore, $\bar{\mathbf{a}} = 0$

$$\Sigma\mathbf{F} = \mathbf{A} + \mathbf{B} = m\bar{\mathbf{a}} = 0 \qquad \mathbf{A} = -\mathbf{B}$$

$$\mathbf{H}_A = I_x\omega\mathbf{i} - I_{xy}\omega\mathbf{j} - I_{xz}\omega\mathbf{k} \qquad \boldsymbol{\omega} = \omega\mathbf{i}, \qquad \boldsymbol{\alpha} = \alpha\mathbf{i}$$

Let the frame of reference *Axyz* be rotating with angular velocity $\boldsymbol{\Omega} = \boldsymbol{\omega} = \omega\mathbf{i}$.

$$\Sigma\mathbf{M}_A = \dot{\mathbf{H}}_A = (\dot{\mathbf{H}}_A)_{Axyz} + \boldsymbol{\Omega}\times\mathbf{H}_A$$

$$M_0\mathbf{i} + 4b\mathbf{i}\times(B_y\mathbf{j} + B_z\mathbf{k}) = I_x\alpha\mathbf{i} - I_{xy}\alpha\mathbf{j} - I_{xz}\alpha\mathbf{k} + \omega\mathbf{i}\times(I_x\omega\mathbf{i} - I_{xy}\omega\mathbf{j} - I_{xz}\omega\mathbf{k})$$

$$M_0\mathbf{i} - 4bB_z\mathbf{j} + 4bB_y\mathbf{k} = I_x\alpha\mathbf{i} - (I_{xy}\alpha - I_{xz}\omega^2)\mathbf{j} - (I_{xz}\alpha + I_{xy}\omega^2)\mathbf{k}$$

Resolve into components and solve for B_y and B_z.

i: $M_0 = I_x\alpha$

j: $$B_z = \frac{(I_{xy}\alpha - I_{xz}\omega^2)}{4b}$$

k: $$B_y = -\frac{(I_{xz}\alpha + I_{xy}\omega^2)}{4b}$$

<u>Data</u>: $\alpha = 150\ \text{rad/s}^2, \quad \omega = 0, \quad b = 0.5\ \text{ft}$

(*a*) $M_0 = (5.823\times10^{-3})(150) = 0.87345\ \text{lb}\cdot\text{ft}$ $\quad M_0 = (0.873\ \text{lb}\cdot\text{ft})\mathbf{i}$ ◀

(*b*) $$B_z = \frac{(-3.4938\times10^{-3})(150) - 0}{(4)(0.5)} = -0.262\ \text{lb}$$

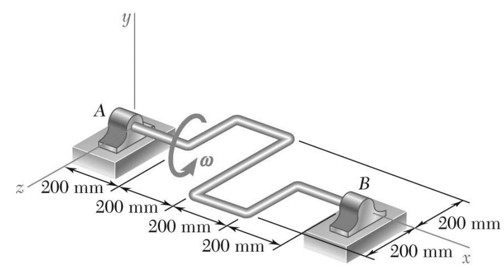
$$B_y = -\frac{(-2.9115\times10^{-3})(150) + 0}{(4)(0.5)} = 0.218\ \text{lb}$$

$$A_y = -B_y = -0.218\ \text{lb}$$

$$A_z = -B_z = 0.262\ \text{lb}$$

$\mathbf{A} = -(0.218\ \text{lb})\mathbf{j} + (0.262\ \text{lb})\mathbf{k}$ ◀

$\mathbf{B} = (0.218\ \text{lb})\mathbf{j} - (0.262\ \text{lb})\mathbf{k}$ ◀



PROBLEM 18.74

The shaft of Problem 18.68 is initially at rest $(\omega = 0)$ when a couple $\mathbf{M}_0$ is applied to it. Knowing that the resulting angular acceleration of the shaft is $\boldsymbol{\alpha} = (20 \text{ rad/s}^2)\mathbf{i}$, determine (*a*) the couple $\mathbf{M}_0$, (*b*) the dynamic reactions at *A* and *B* immediately after the couple is applied.

PROBLEM 18.68 The 8-kg shaft shown has a uniform cross section. Knowing that the shaft rotates at the constant rate $\omega = 12$ rad/s, determine the dynamic reactions at *A* and *B*.

SOLUTION

Angular velocity and angular acceleration: $\dot{\boldsymbol{\omega}} = \dot{\omega}\mathbf{i} = \alpha\mathbf{i}$

Angular momentum about the mass center *G*: $\mathbf{H}_G = \bar{I}_x \omega \mathbf{i} - \bar{I}_{xy}\omega \mathbf{j} - \bar{I}_{xz}\omega \mathbf{k}$

Let the reference frame *Axyz* be rotating with angular velocity $\Omega = \omega \mathbf{i} = 0$

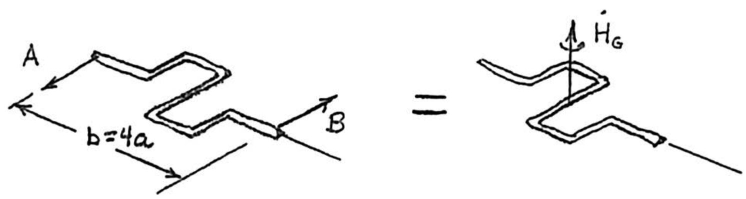
$$\begin{aligned}\dot{\mathbf{H}}_G &= (\dot{\mathbf{H}}_G)_{Axyz} + \boldsymbol{\Omega} \times \mathbf{H}_G \\ &= \bar{I}_x \alpha \mathbf{i} - \bar{I}_{xy}\alpha \mathbf{j} - \bar{I}_{xz}\alpha \mathbf{k} + 0 \\ &= \bar{I}_x \alpha \mathbf{i} - \bar{I}_{xy}\alpha \mathbf{j} - \bar{I}_{xz}\alpha \mathbf{k}\end{aligned}$$

Since the shaft lies in the *xz* plane, $\bar{I}_{xy} = 0$.

By symmetry, the mass center lies on line *AB*. $m\bar{\mathbf{a}} = 0$

$$\Sigma \mathbf{F} = \Sigma F_{\text{eff}}: \quad \mathbf{A} + \mathbf{B} = m\bar{\mathbf{a}} = 0 \qquad \mathbf{A} \text{ and } \mathbf{B} \text{ form a couple.}$$

$$\mathbf{A} = -\mathbf{B}$$



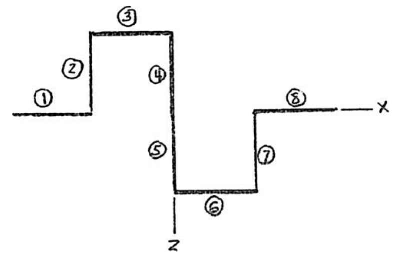
$$M_0 \mathbf{i} + b\mathbf{i} \times (B_y \mathbf{j} + B_z \mathbf{k}) = \dot{\mathbf{H}}_G$$

$$M_0 \mathbf{i} + bB_y \mathbf{k} - bB_z \mathbf{j} = \bar{I}_x \alpha \mathbf{i} - \bar{I}_{xz}\alpha \mathbf{k}$$

$$M_0 = \bar{I}_x \alpha, \quad B_y = -\frac{I_{xz}\alpha}{b}, \quad B_z = 0$$

PROBLEM 18.74 (Continued)

Calculation of $\bar{I}_x$ and I_{xz}.



Divide the shaft into eight segments, each of length

$$a = 200 \text{ mm} = 0.2 \text{ m}$$

Let m' be the mass of one segment.

$$m' = \frac{8 \text{ kg}}{8} = 1 \text{ kg}$$

For ① and ⑧, $I_x = 0$

For ②, ④, ⑤, and ⑦, $I_x = \frac{1}{3}ma^2$

For ③ and ⑥, $I_x = m'a^2$

Total: $\bar{I}_x = \frac{10}{3}m'a^2$

For ①, ④, ⑤, and ⑧, $I_{xz} = 0$

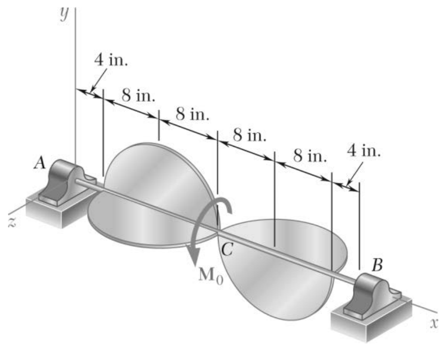
For ②, ③, ⑥, and ⑦, $I_{xz} = m'(-a)\left(-\frac{a}{2}\right) + m'\left(-\frac{a}{2}\right)(-a) + m'\left(\frac{a}{2}\right)(a) + m'(a)\left(\frac{a}{2}\right) = 2m'a^2$

(*a*) $M_0 = \frac{10}{3}m'a^2\alpha = \frac{10}{3}(1)(0.2)^2(20)$ $\quad \mathbf{M}_0 = (2.67 \text{ N}\cdot\text{m})\mathbf{i}$ ◀

(*b*) $B_y = -\frac{2m'a^2\alpha}{b} = -\frac{(2)(1)(0.2)^2(20)}{(4)(0.2)}$ $\quad \mathbf{B} = -(2.00 \text{ N})\mathbf{j}$ ◀

$A_z = 0, \quad A_y = -B_y = 2 \text{ N}$ $\quad \mathbf{A} = (2.00 \text{ N})\mathbf{j}$ ◀

PROBLEM 18.75



The assembly shown weighs 12 lb and consists of 4 thin 16-in.-diameter semicircular aluminum plates welded to a light 40-in.-long shaft AB. The assembly is at rest $(\omega = 0)$ at time $t = 0$ when a couple $\mathbf{M}_0$ is applied to it as shown, causing the assembly to complete one full revolution in 2 s. Determine (*a*) the couple $\mathbf{M}_0$, (*b*) the dynamic reactions at A and B at $t = 0$.

SOLUTION

<u>Fixed axis rotation with constant angular acceleration</u>. $\quad \boldsymbol{\alpha} = \alpha \mathbf{i}, \quad \boldsymbol{\omega} = \omega \mathbf{i}$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2 \qquad \alpha = \frac{2\theta}{t^2} = \frac{2(2\pi)}{2^2} = 3.1416 \text{ rad/s}^2$$

$$\omega = 0$$

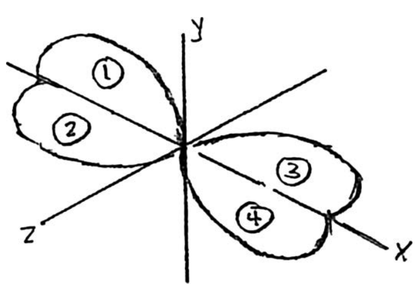
Use centroidal axes x, y, z with origin at C. $\quad \mathbf{H}_C = I_x \omega \mathbf{i} - I_{xy}\omega \mathbf{j} - I_{xz}\omega \mathbf{k}$

Let the reference frame $Cxyz$ rotate with angular velocity $\quad \boldsymbol{\Omega} = \boldsymbol{\omega} = \omega \mathbf{i}$

$$\dot{\mathbf{H}}_C = (\dot{\mathbf{H}}_C)_{Cxyz} + \boldsymbol{\Omega} \times \mathbf{H}_C = I_x \alpha \mathbf{i} - I_{xy}\alpha \mathbf{j} - I_{xz}\alpha \mathbf{k} + I_{xz}\omega^2 \mathbf{j} - I_{xy}\omega^2 \mathbf{k}$$

<u>Required moments and products of inertia</u>. Let $\rho = \dfrac{m}{A}$ = mass per unit area.

$$I_{\text{mass}} = \rho I_{\text{area}} \qquad r = 8 \text{ in.} = 0.66667 \text{ ft} \quad m = \frac{12}{32.2} = 0.37267 \text{ lb} \cdot \text{s}^2/\text{ft}$$



Part	A	I_x	I_{xy}	I_{xz}
①	$\frac{1}{2}\pi r^2$	$\frac{1}{8}\pi r^4$	$-\frac{2}{3}r^4$	0
②	$\frac{1}{2}\pi r^2$	$\frac{1}{8}\pi r^4$	0	$-\frac{2}{3}r^4$
③	$\frac{1}{2}\pi r^2$	$\frac{1}{8}\pi r^4$	0	$-\frac{2}{3}r^4$
④	$\frac{1}{2}\pi r^2$	$\frac{1}{8}\pi r^4$	$-\frac{2}{3}r^4$	0
Σ	$2\pi r^2$	$\frac{1}{2}\pi r^4$	$-\frac{4}{3}r^4$	$-\frac{4}{3}r^4$

PROBLEM 18.75 (Continued)

$$\rho = \frac{m}{2\pi r^2} = \frac{0.37267}{2\pi(0.66667)^2} = 0.13345 \text{ lb} \cdot \text{s}^2/\text{ft}^3$$

$$I_x = (0.13345)\left(\frac{1}{2}\pi\right)(0.66667)^4 = 0.041407 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$I_{xy} = (0.13345)\left(-\frac{4}{3}\right)(0.66667)^4 = -0.035147 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$I_{xz} = (0.13345)\left(-\frac{4}{3}\right)(0.66667)^4 = -0.035147 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

Since the mass center lies on the rotation axis, $\bar{\mathbf{a}} = 0$

$$\Sigma\mathbf{F} = \mathbf{A} + \mathbf{B} = \Sigma\mathbf{F}_{\text{eff}} = m\bar{\mathbf{a}} = 0 \qquad \mathbf{A} = -\mathbf{B}$$

$$\Sigma\mathbf{M}_C = M_0\mathbf{i} + (-b\mathbf{i}) \times \mathbf{A} + b\mathbf{i} \times \mathbf{B} = M_0\mathbf{i} + 2b\mathbf{i} \times (B_y\mathbf{j} + B_z\mathbf{k})$$

$$= M_0\mathbf{i} - 2bB_z\mathbf{j} + 2bB_y\mathbf{k} \qquad \text{where} \qquad 2b = 40 \text{ in.} = 3.3333 \text{ ft}$$

$$\Sigma M_C = \dot{\mathbf{H}}_C \qquad \text{Resolve into components.}$$

(*a*) **i**: $M_0 = I_x\alpha = (0.041407)(3.1416)$ $\qquad \mathbf{M}_0 = (0.1301 \text{ lb} \cdot \text{ft})\mathbf{i}$ ◀

(*b*) **j**: $-2b\,B_z = -I_{xy}\alpha + I_{xz}\omega^2$

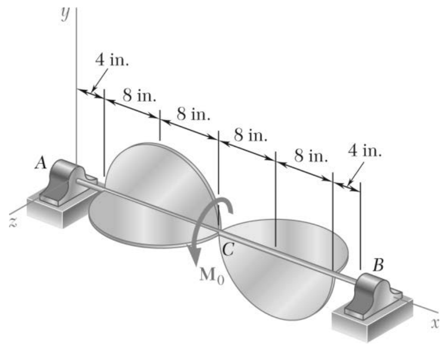
$$B_z = -\frac{-(-0.035147)(3.1416) + 0}{3.3333} = -0.0331 \text{ lb} \qquad A_z = 0.0331 \text{ lb}$$

k: $2bB_y = -I_{xz}\alpha - I_{xy}\omega^2$

$$B_y = -\frac{(-0.035147)(3.1416) + 0}{3.3333} = 0.0331 \text{ lb} \qquad A_y = -0.0331 \text{ lb}$$

$\mathbf{A} = -(0.0331 \text{ lb})\mathbf{j} + (0.0331 \text{ lb})\mathbf{k}$ ◀

$\mathbf{B} = (0.0331 \text{ lb})\mathbf{j} - (0.0331 \text{ lb})\mathbf{k}$ ◀



PROBLEM 18.76

For the assembly of Problem 18.75, determine the dynamic reactions at A and B at $t = 2$ s.

PROBLEM 18.75 The assembly shown weighs 12 lb and consists of four thin 16-in.-diameter semicircular aluminum plates welded to a light 40-in.-long shaft AB. The assembly is at rest $(\omega = 0)$ at time $t = 0$ when a couple $\mathbf{M}_0$ is applied to it as shown, causing the assembly to complete one full revolution in 2 s. Determine (*a*) the couple $\mathbf{M}_0$, (*b*) the dynamic reactions at A and B at $t = 0$.

SOLUTION

<u>Fixed axis rotation with constant angular acceleration</u>. $\quad \boldsymbol{\alpha} = \alpha\mathbf{i}, \quad \boldsymbol{\omega} = \omega\mathbf{i}$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2 \qquad \alpha = \frac{2\theta}{t^2} = \frac{2(2\pi)}{2^2} = 3.1416 \text{ rad/s}^2$$

$$\omega = \omega_0 + \alpha t = \alpha t = (3.1416)(2) = 6.2832 \text{ rad/s}$$

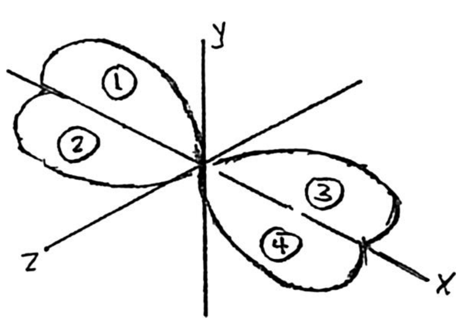
Use centroidal axes x, y, z with origin at C. $\quad \mathbf{H}_C = I_x\omega\mathbf{i} - I_{xy}\omega\mathbf{j} - I_{xz}\omega\mathbf{k}$

Let the reference frame $Cxyz$ rotate with angular velocity $\quad \boldsymbol{\Omega} = \boldsymbol{\omega} = \omega\mathbf{i}$

$$\dot{\mathbf{H}}_C = (\dot{\mathbf{H}}_C)_{Cxyz} + \boldsymbol{\Omega} \times \mathbf{H}_C = I_x\alpha\mathbf{i} - I_{xy}\alpha\mathbf{j} - I_{xz}\alpha\mathbf{k} + I_{xz}\omega^2\mathbf{j} - I_{xy}\omega^2\mathbf{k}$$

<u>Required moments and products of inertia</u>. Let $\rho = \dfrac{m}{A} =$ mass per unit area.

$$I_{\text{mass}} = \rho I_{\text{area}} \qquad r = 8 \text{ in.} = 0.66667 \text{ ft} \quad m = \frac{12}{32.2} = 0.37267 \text{ lb}\cdot\text{s}^2/\text{ft}$$



Part	A	I_x	I_{xy}	I_{xz}
①	$\frac{1}{2}\pi r^2$	$\frac{1}{8}\pi r^4$	$-\frac{2}{3}r^4$	0
②	$\frac{1}{2}\pi r^2$	$\frac{1}{8}\pi r^4$	0	$-\frac{2}{3}r^4$
③	$\frac{1}{2}\pi r^2$	$\frac{1}{8}\pi r^4$	0	$-\frac{2}{3}r^4$
④	$\frac{1}{2}\pi r^2$	$\frac{1}{8}\pi r^4$	$-\frac{2}{3}r^4$	0
Σ	$2\pi r^2$	$\frac{1}{2}\pi r^4$	$-\frac{4}{3}r^4$	$-\frac{4}{3}r^4$

PROBLEM 18.76 (Continued)

$$\rho = \frac{m}{2\pi r^2} = \frac{0.37267}{2\pi(0.66667)^2} = 0.13345 \text{ lb}\cdot\text{s}^2/\text{ft}^3$$

$$I_x = (0.13345)\left(\frac{1}{2}\pi\right)(0.66667)^4 = 0.041407 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$I_{xy} = (0.13345)\left(-\frac{4}{3}\right)(0.66667)^4 = -0.035147 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$I_{xz} = (0.13345)\left(-\frac{4}{3}\right)(0.66667)^4 = -0.035147 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Since the mass center lies on the rotation axis, $\bar{\mathbf{a}} = 0$

$$\Sigma\mathbf{F} = \mathbf{A} + \mathbf{B} = \Sigma\mathbf{F}_{\text{eff}} = m\bar{\mathbf{a}} = 0 \qquad \mathbf{A} = -\mathbf{B}$$

$$\Sigma\mathbf{M}_C = M_0\mathbf{i} + (-b\mathbf{i})\times\mathbf{A} + b\mathbf{i}\times\mathbf{B} = M_0\mathbf{i} + 2b\mathbf{i}\times(B_y\mathbf{j} + B_z\mathbf{k})$$

$$= M_0\mathbf{i} - 2bB_z\mathbf{j} + 2bB_y\mathbf{k} \qquad \text{where} \quad 2b = 40 \text{ in.} = 3.3333 \text{ ft}$$

$$\Sigma M_C = \dot{\mathbf{H}}_C \qquad \text{Resolve into components.}$$

i: $M_0 = I_x\alpha = (0.041407)(3.1416)$ $\qquad M_0 = 0.1301 \text{ lb}\cdot\text{ft}$

j: $-2b\,B_z = -I_{xy}\alpha + I_{xz}\omega^2$

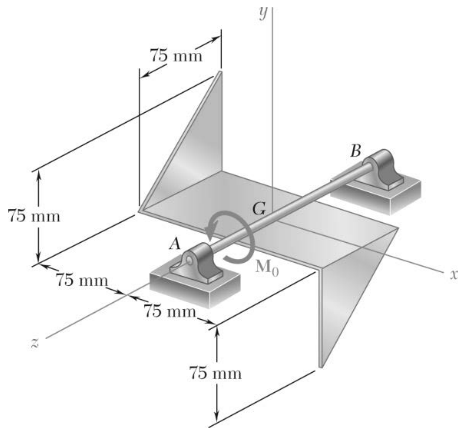
$$B_z = -\frac{-(-0.035147)(3.1416) + (-0.035147)(6.2832)^2}{3.3333} = 0.383 \text{ lb}, \qquad A_z = -0.383 \text{ lb}$$

k: $2bB_y = -I_{xz}\alpha - I_{xy}\omega^2$

$$B_y = -\frac{(-0.035147)(3.1416) + (-0.035147)(6.2832)^2}{3.3333} = 0.449 \text{ lb}, \qquad A_y = -0.449 \text{ lb}$$

$\mathbf{A} = -(0.449 \text{ lb})\mathbf{j} - (0.383 \text{ lb})\mathbf{k}$ ◀

$\mathbf{B} = (0.449 \text{ lb})\mathbf{j} + (0.383 \text{ lb})\mathbf{k}$ ◀



PROBLEM 18.77

The sheet-metal component shown is of uniform thickness and has a mass of 600 g. It is attached to a light axle supported by bearings at A and B located 150 mm apart. The component is at rest when it is subjected to a couple $\mathbf{M}_0$ as shown. If the resulting angular acceleration is $\boldsymbol{\alpha} = (12\ \text{rad/s}^2)\mathbf{k}$, determine ($a$) the couple $\mathbf{M}_0$, (b) the dynamic reactions at A and B immediately after the couple has been applied.

SOLUTION

The sheet metal component rotates about the fixed z axis, so that Equations (18.29) of the textbook apply. These are relisted below as equations (1), (2), and (3).

$$\Sigma M_x = -I_{xz}\alpha + I_{yz}\omega^2 \tag{1}$$

$$\Sigma M_y = -I_{yz}\alpha - I_{xz}\omega^2 \tag{2}$$

$$\Sigma M_z = I_z\alpha \tag{3}$$

Calculation of the required moment and products of inertia.

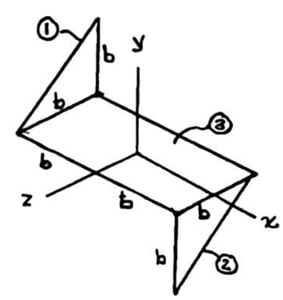
Total mass: $m = 600\ \text{g} = 0.6\ \text{kg}$

Total area: $A = \frac{1}{2}(0.075)^2 + (0.150)(0.075) + \frac{1}{2}(0.075)^2 = 16.875 \times 10^{-3}\ \text{m}^2$

Let ρ be the mass per unit area. $\rho = \frac{m}{A} = 35.556\ \text{kg/m}^2$

The component is comprised of 3 parts: triangle ①, triangle ②, and rectangle ③ as shown. Let the lengths of 75 mm be labeled b.

Triangle ①. The equation of the upper edge is



$$y = \frac{b}{2} - z$$

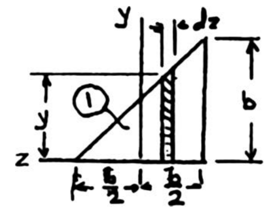
Use elements of width dz and height y.

$$dm = \rho y dz \qquad (d\bar{I}_z)_{\text{el}} = \frac{1}{12}y^2 dm$$

$$(d\bar{I}_{xz})_{\text{el}} = 0 \qquad (d\bar{I}_{yz})_{\text{el}} = 0$$

Coordinates of the element mass center:

$$\bar{x}_{el} = -b, \qquad \bar{y}_{el} = \frac{1}{2}y, \qquad \bar{z}_{el} = z$$



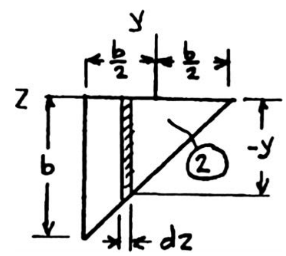
$$dI_{xz} = (d\bar{I}_{xz})_{el} + \bar{x}_{el}\bar{z}_{el}dm$$

$$= 0 + (-b)z(\rho y d_z) = -\rho bz\left(\frac{b}{2} - z\right)dz$$

$$I_{xz} = -\rho b\int_{-b/2}^{b/2} z\left(\frac{b}{2} - z\right)dz = \frac{1}{12}\rho b^4$$

$$dI_{yz} = (d\bar{I}_{yz})_{el} + \bar{y}_{el}\bar{z}_{el}dm$$

$$= 0 + \left(\frac{1}{2}y\right)z(\rho y dz) = \frac{1}{2}\rho\left(\frac{b}{2} - z\right)^2 zdz$$



$$I_{yz} = \frac{1}{2}\rho\int_{-b/2}^{b/2}\left(\frac{b}{2} - z\right)^2 zdz = -\frac{1}{24}\rho b^4$$

$$dI_z = (d\bar{I}_z)_{el} + (\bar{x}_{el}^2 + \bar{y}_{el}^2)dm$$

$$= \left(\frac{1}{12}y^2 + b^2 + \frac{1}{4}y^2\right)\rho ydz = \rho\left(b^2 y + \frac{1}{3}y^3\right)dz$$

$$= \rho\left(\frac{13}{24}b^3 - \frac{5}{4}b^2 z + \frac{1}{2}bz^2 - \frac{1}{3}z^3\right)dz$$

$$I_z = \rho\int_{-b/2}^{b/2}\left(\frac{13}{24}b^3 - \frac{5}{4}b^2 z + \frac{1}{2}bz^2 - \frac{1}{3}z^3\right)dz = \frac{7}{12}\rho b^4$$

Applying the data,

$$I_{xz} = \frac{1}{12}(35.556)(0.075)^4 = 93.75\times10^{-6}\ \text{kg}\cdot\text{m}^2$$

$$I_{yz} = -\frac{1}{24}(35.556)(0.075)^4 = -46.875\times10^{-6}\ \text{kg}\cdot\text{m}^2$$

$$I_z = \frac{7}{12}(35.556)(0.075)^4 = 656.25\times10^{-6}\ \text{kg}\cdot\text{m}^2$$

Triangle ②. The equation of the lower edge is $y = -\left(\frac{b}{2} - z\right)$.

Use elements of width dz and height $-y$.

$$dm = -\rho y dz, \qquad (d\bar{I}_z)_{\text{el}} = \frac{1}{12} y^2 dm, \qquad (d\bar{I}_{zx})_{\text{el}} = 0, \qquad (d\bar{I}_{yz})_{\text{el}} = 0$$

Coordinates of the element mass center:

$$\bar{x}_{\text{el}} = b, \qquad \bar{y}_{\text{el}} = \frac{1}{2} y, \qquad \bar{z}_{\text{el}} = z$$

The integrals for I_{xz}, I_{yz}, and I_z turn out to be the same as those of triangle ①.

$$I_{xz} = 93.75 \times 10^{-6} \text{ kg} \cdot \text{m}^2, \quad I_{yz} = -46.875 \times 10^{-6} \text{ kg} \cdot \text{m}^2, \quad I_z = 656.25 \times 10^{-6} \text{ kg} \cdot \text{m}^2$$

Rectangle ③. Area: $A = (0.150)(0.075) = 11.25 \times 10^{-3} \text{ m}^2$

Mass: $m = \rho A = 400 \times 10^{-3}$ kg

$$I_{xz} = 0, \qquad I_{yz} = 0$$

$$I_z = \frac{1}{12} m(2b)^2 = \frac{1}{12}(400 \times 10^{-3})(0.150)^2 = 750 \times 10^{-6} \text{ kg} \cdot \text{m}^2$$

Totals.

$$I_{xz} = \Sigma I_{xz} = 187.5 \times 10^{-6} \text{ kg} \cdot \text{m}^2$$

$$I_{yz} = \Sigma I_{yz} = -93.75 \times 10^{-6} \text{ kg} \cdot \text{m}^2$$

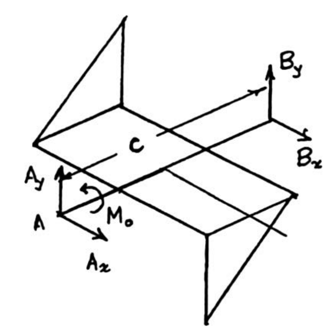
$$I_z = \Sigma I_z = 2062.5 \times 10^{-6} \text{ kg} \cdot \text{m}^2$$

Since the mass center lies on the fixed axis, the acceleration $\bar{\mathbf{a}}$ of the mass center is zero.

$$\Sigma \mathbf{F} = m\bar{\mathbf{a}} = 0$$

The reactions at A and B form a couple.

$$\mathbf{B} = -\mathbf{A}$$



Let $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$

Resultant couple acting on the body:

$$\mathbf{M} = c\mathbf{k} \times (A_x\mathbf{i} + A_y\mathbf{j}) + M_0\mathbf{k}$$

$$= -cA_y\mathbf{i} + cA_x\mathbf{j} + M_0\mathbf{k}$$

(*a*) Moment M_0. Using Equation (3),

$$\Sigma M_z = M_0 = I_z\alpha = (2062.5 \times 10^{-6})(12)$$

$M_0 = 24.8 \times 10^{-3}$ N · m ◀

PROBLEM 18.77 (Continued)

(*b*) Reactions at A and B for the case $\omega = 0$.

$$\Sigma M_x = -cA_y = -I_{xz}\alpha - I_{yz}\omega^2 = -I_{xz}\alpha$$

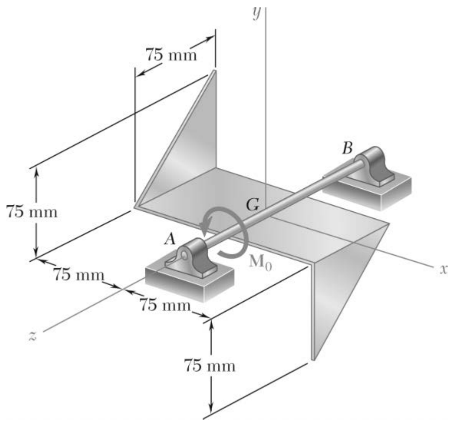
$$A_y = \frac{I_{xz}\alpha}{c} = \frac{(187.5\times 10^{-6})(12)}{0.150} = 15\times 10^{-3}\text{ N}$$

$$\Sigma M_y = cA_x = -I_{yz}\alpha - I_{xz}\omega^2 = -I_{yz}\alpha$$

$$A_x = -\frac{I_{yz}\alpha}{c} = -\frac{(-93.75\times 10^{-6})(12)}{0.150} = 7.5\times 10^{-3}\text{ N}$$

$$\mathbf{A} = (7.50\times 10^{-3}\text{ N})\mathbf{i} + (15.00\times 10^{-3}\text{ N})\mathbf{j}$$ ◀

$$\mathbf{B} = -(7.50\times 10^{-3}\text{ N})\mathbf{i} - (15.00\times 10^{-3}\text{ N})\mathbf{j}$$ ◀



PROBLEM 18.78

For the sheet-metal component of Problem 18.77, determine (*a*) the angular velocity of the component 0.6 s after the couple $\mathbf{M}_0$ has been applied to it, (*b*) the magnitude of the dynamic reactions at *A* and *B* at that time.

SOLUTION

The sheet metal component rotates about the fixed *z* axis with angular acceleration $\boldsymbol{\alpha} = (12 \text{ rad/s}^2)\mathbf{k}$.

(*a*) *Angular velocity at* $t = 0.6$ s.

$$\omega = \omega_0 + \alpha t = 0 + (12)(0.6) = 7.2 \text{ rad/s} \qquad \boldsymbol{\omega} = (7.20 \text{ rad/s})\mathbf{k} \blacktriangleleft$$

(*b*) *Dynamic reactions.*

Equations (18.29) of the textbook apply. These are relisted below as equations (1), (2), and (3).

$$\Sigma M_x = -I_{xz}\alpha + I_{yz}\omega^2 \quad (1)$$

$$\Sigma M_y = -I_{yz}\alpha - I_{xz}\omega^2 \quad (2)$$

$$\Sigma M_z = I_z\alpha \quad (3)$$

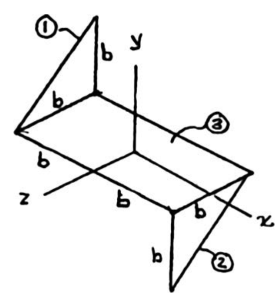
Calculation of the required moment and products of inertia.

Total mass: $m = 600 \text{ g} = 0.6 \text{ kg}$

Total area: $A = \frac{1}{2}(0.075)^2 + (0.150)(0.075) + \frac{1}{2}(0.075)^2 = 16.875 \times 10^{-3} \text{ m}^2$

Let ρ be the mass per unit area. $\rho = \frac{m}{A} = 35.556 \text{ kg/m}^2$

The component is comprised of 3 parts: triangle ①, triangle ②, and rectangle ③ as shown. Let the lengths of 75 mm be labeled *b*.



Triangle ①. The equation of the upper edge is

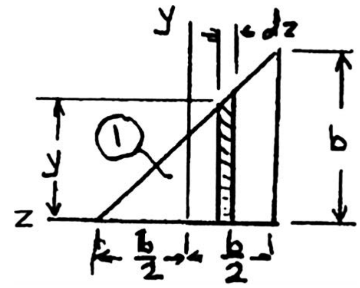
$$y = \frac{b}{2} - z$$

Use elements of width *dz* and height *y*.

$$dm = \rho y dz \qquad (d\bar{I}_z)_{\text{el}} = \frac{1}{12} y^2 dm$$

$$(d\bar{I}_{xz})_{\text{el}} = 0 \qquad (d\bar{I}_{yz})_{\text{el}} = 0$$

Coordinates of the element mass center:



$$\bar{x}_{\text{el}} = -b, \ \bar{y}_{\text{el}} = \frac{1}{2}y, \quad \bar{z}_{\text{el}} = z$$

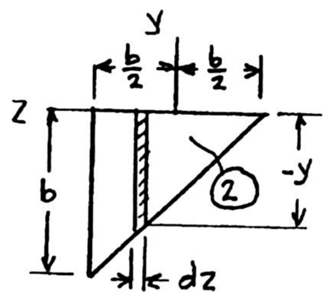
$$dI_{xz} = (d\bar{I}_{xz})_{\text{el}} + \bar{x}_{\text{el}}\bar{z}_{\text{el}}dm$$

$$= 0 + (-b)z(\rho y dz) = -\rho bz\left(\frac{b}{2} - z\right)dz$$

$$I_{xz} = -\rho b\int_{-b/2}^{b/2} z\left(\frac{b}{2} - z\right)dz = \frac{1}{12}\rho b^4$$

$$dI_{yz} = (d\bar{I}_{yz})_{\text{el}} + \bar{y}_{\text{el}}\bar{z}_{\text{el}}dm$$

$$= 0 + \left(\frac{1}{2}y\right)z(\rho y dz) = \frac{1}{2}\rho\left(\frac{b}{2} - z\right)^2 zdz$$



$$I_{yz} = \frac{1}{2}\rho\int_{-b/2}^{b/2}\left(\frac{b}{2} - z\right)^2 zdz = -\frac{1}{24}\rho b^4$$

$$dI_z = (d\bar{I}_z)_{\text{el}} + \left(\bar{x}_{\text{el}}^2 + \bar{y}_{\text{el}}^2\right)dm$$

$$= \left(\frac{1}{12}y^2 + b^2 + \frac{1}{4}y^2\right)\rho y_i dz = \rho\left(b^2 y + \frac{1}{3}y^3\right)dz$$

$$= \rho\left(\frac{13}{24}b^3 - \frac{5}{4}b^2 z + \frac{1}{2}bz^2 - \frac{1}{3}z^3\right)dz$$

$$I_z = \rho\int_{-b/2}^{b/2}\left(\frac{13}{24}b^3 - \frac{5}{4}b^2 z + \frac{1}{2}bz^2 - \frac{1}{3}z^3\right)dz = \frac{7}{12}\rho b^4$$

Applying the data,

$$I_{xz} = \frac{1}{12}(35.556)(0.075)^4 = 93.75\times 10^{-6}\text{ kg}\cdot\text{m}^2$$

$$I_{yz} = -\frac{1}{24}(35.556)(0.075)^4 = -46.875\times 10^{-6}\text{ kg}\cdot\text{m}^2$$

$$I_z = \frac{7}{12}(35.556)(0.075)^4 = 656.25\times 10^{6}\text{ kg}\cdot\text{m}^2$$

Triangle ②. The equation of the lower edge is $y = -\left(\frac{b}{2} - z\right).$

Use elements of width dz and height $-y$.

$$dm = -\rho y dz, \quad (d\bar{I}_z)_{\text{el}} = \frac{1}{12}y^2 dm, \quad (d\bar{I}_{zx})_{\text{el}} = 0, \quad (d\bar{I}_{yz})_{\text{el}} = 0$$

Coordinates of the element mass center:

$$\bar{x}_{\text{el}} = b, \qquad \bar{y}_{\text{el}} = \frac{1}{2}y, \qquad \bar{z}_{\text{el}} = z$$

The integrals for I_{xz}, I_{yz}, and I_z turn out to be the same as those of triangle ①.

$$I_{xz} = 93.75\times10^{-3}\ \text{kg}\cdot\text{m}^2, \quad I_{yz} = -46.875\times10^{-6}\ \text{kg}\cdot\text{m}^2, \quad I_z = 656.25\times10^{-6}\ \text{kg}\cdot\text{m}^2$$

Rectangle ③. Area: $A = (0.150)(0.075) = 11.25\times10^{-3}\ \text{m}^2$

Mass: $m = \rho A = 400\times10^{-3}\ \text{kg}$

$$I_{xz} = 0, \qquad I_{yz} = 0$$

$$I_z = \frac{1}{12}m(2b)^2 = \frac{1}{12}(400\times10^{-3})(0.150)^2 = 750\times10^{-6}\ \text{kg}\cdot\text{m}^2$$

Totals.

$$I_{xz} = \Sigma I_{xz} = 187.5\times10^{-6}\ \text{kg}\cdot\text{m}^2$$

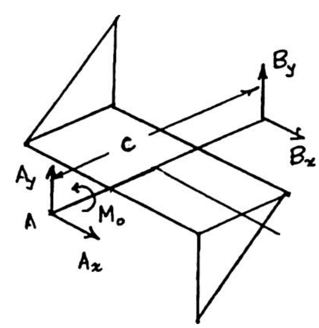
$$I_{yz} = \Sigma I_{yz} = -93.75\times10^{-6}\ \text{kg}\cdot\text{m}^2$$

$$I_z = \Sigma I_z = 2062.5\times10^{-6}\ \text{kg}\cdot\text{m}^2$$

Since the mass center lies on the fixed axis, the acceleration $\bar{\mathbf{a}}$ of the mass center is zero.

$$\Sigma\mathbf{F} = m\bar{\mathbf{a}} = 0$$

The reactions at A and B form a couple.



$$\mathbf{B} = -\mathbf{A}$$

Let $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$

Resultant couple acting on the body:

$$\mathbf{M} = c\mathbf{k}\times(A_x\mathbf{i} + A_y\mathbf{j}) + M_0\mathbf{k}$$

$$= -cA_y\mathbf{i} + cA_x\mathbf{j} + M_0\mathbf{k}$$

From Eq. (1), $-cA_y = -I_{xz}\alpha + I_{yz}\omega^2$

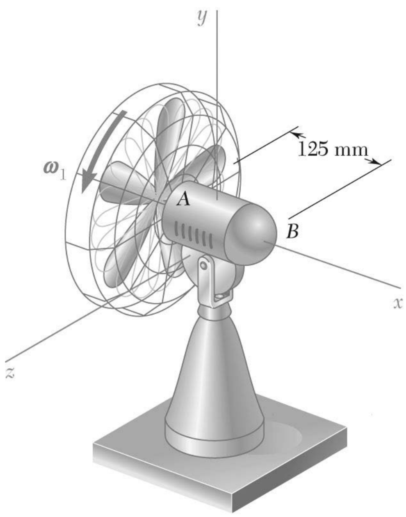
$$A_y = \frac{I_{xz}\alpha}{c} - \frac{I_{yz}\omega^2}{c} = \frac{(187.5\times10^{-6})(12)}{0.150} - \frac{(-93.75\times10^{-6})(7.2)^2}{0.150} = 47.4\times10^{-3}\ \text{N}$$

From Eq. (2), $cA_x = -I_{yz}\alpha - I_{xz}\omega^2$

$$A_x = -\frac{I_{yz}\alpha}{c} - \frac{I_{xz}\omega^2}{c} = -\frac{(-93.75\times10^{-6})(12)}{0.150} - \frac{(187.5\times10^{-6})(7.2)^2}{0.150} = -57.3\times10^{-3}\ \text{N}$$

$$\mathbf{A} = -(57.3\times10^{-3}\ \text{N})\mathbf{i} + (47.3\times10^{-3}\ \text{N})\mathbf{j}\ ◀$$

$$\mathbf{B} = (57.3\times10^{-3}\ \text{N})\mathbf{i} - (47.3\times10^{-3}\ \text{N})\mathbf{j}\ ◀$$



PROBLEM 18.79

The blade of an oscillating fan and the rotor of its motor have a total weight of 300 g and a combined radius of gyration of 75 mm. They are supported by bearings at A and B, 125 mm apart, and rotate at the rate $\omega_1 = 1800$ rpm. Determine the dynamic reactions at A and B when the motor casing has an angular velocity $\omega_2 = (0.6 \text{ rad/s})\mathbf{j}$.

SOLUTION

$$\boldsymbol{\omega}_1 = \omega_1 \mathbf{i}$$

$$\omega_1 = \frac{2\pi(1800)}{60}$$

$$= 188.5 \text{ rad/s}$$

Angular velocity: $\boldsymbol{\omega} = \omega_1 \mathbf{i} + \omega_2 \mathbf{j}$

Angular momentum: $\mathbf{H}_G = I_x \omega_1 \mathbf{i} + I_y \omega_2 \mathbf{j}$

Let the reference frame be rotating with angular velocity $\boldsymbol{\Omega} = \omega_2 \mathbf{j}$.

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \boldsymbol{\Omega} \times \mathbf{H}_G$$

$$= 0 + \omega_2 \mathbf{j} \times (I_x \omega_1 \mathbf{i} + I_y \omega_2 \mathbf{j})$$

$$= -I_x \omega_1 \omega_2 \mathbf{k}$$

Assume that the acceleration of the mass center is negligible. Then the dynamic reactions at A and B reduce to a couple.

$$\mathbf{A} = -\mathbf{B}$$

$$\mathbf{M} = b\mathbf{i} \times \mathbf{B}$$

$$= b\mathbf{i} \times (B_y \mathbf{j} + B_z \mathbf{k})$$

$$= -b B_z \mathbf{j} + b B_y \mathbf{k}$$

$\mathbf{M} = \dot{\mathbf{H}}_G$ Resolve into components.

$\mathbf{j}$: $\quad -b B_z = 0 \qquad B_z = 0, \qquad A_z = 0$

$\mathbf{k}$: $\quad b B_y = -I_x \omega_1 \omega_2 \qquad B_y = -\dfrac{I_x \omega_1 \omega_2}{b} = -\dfrac{m k_x^2 \omega_1 \omega_2}{b}$

PROBLEM 18.79 (Continued)

Data:

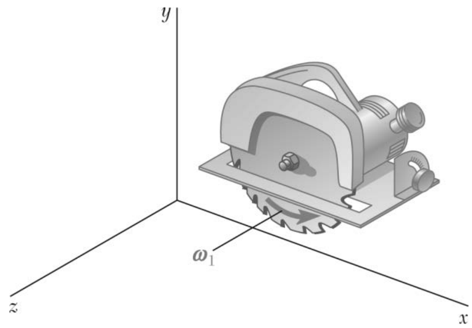
$$m = 0.300 \text{ kg}$$
$$k_x = 0.075 \text{ m}$$
$$b = 0.125 \text{ m}$$

$$B_y = -\frac{(0.300)(75\times 10^{-3})^2(188.5)(0.6)}{0.125} = -1.527 \text{ N}$$

$$A_y = 1.527 \text{ N}$$

$\mathbf{A} = (1.527 \text{ N})\mathbf{j}$ ◀

$\mathbf{B} = -(1.527 \text{ N})\mathbf{j}$ ◀



PROBLEM 18.80

The blade of a portable saw and the rotor of its motor have a total weight of 2.5 lb and a combined radius of gyration of 1.5 in. Knowing that the blade rotates as shown at the rate $\omega_1 = 1500$ rpm, determine the magnitude and direction of the couple **M** that a worker must exert on the handle of the saw to rotate it with a constant angular velocity $\boldsymbol{\omega}_2 = -(2.4 \text{ rad/s})\mathbf{j}$.

SOLUTION

$$\boldsymbol{\omega}_1 = \frac{(2\pi)(1500)}{60}$$
$$= 157.08 \text{ rad/s}$$
$$\boldsymbol{\omega}_1 = \omega_1 \mathbf{k}$$

Angular velocity: $\boldsymbol{\omega} = \omega_2 \mathbf{j} + \omega_1 \mathbf{k}$

Angular momentum of rotor: $\mathbf{H}_G = I_y \omega_2 \mathbf{j} + I_z \omega_1 \mathbf{k}$

Let the reference frame *Gxyz* be rotating with angular velocity $\boldsymbol{\Omega} = \omega_2 \mathbf{j}$.

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \boldsymbol{\Omega} \times \mathbf{H}_G$$
$$= 0 + \omega_2 \mathbf{j} \times (I_y \omega_2 \mathbf{j} + I_z \omega_1 \mathbf{k})$$
$$= I_z \omega_1 \omega_2 \mathbf{i}$$

Couple exerted on the saw:

$$\mathbf{M} = \dot{\mathbf{H}}_G$$
$$= I_z \omega_1 \omega_2 \mathbf{i}$$
$$= m k_z^2 \omega_1 \omega_2 \mathbf{i}$$

<u>Data</u>:

$$W = 2.5 \text{ lb}$$
$$m = \frac{2.5}{32.2}$$
$$= 0.07764 \text{ lb} \cdot \text{s}^2/\text{ft}$$
$$k_z = 1.5 \text{ in.}$$
$$= 0.125 \text{ ft}$$
$$\mathbf{M} = (0.07764)(0.125)^2(157.08)(-2.4)\mathbf{i}$$

$\mathbf{M} = -(0.457 \text{ lb} \cdot \text{ft})\mathbf{i}$ ◀

PROBLEM 18.81

The flywheel of an automobile engine, which is rigidly attached to the crankshaft, is equivalent to a 400-mm-diameter, 15-mm-thick steel plate. Determine the magnitude of the couple exerted by the flywheel on the horizontal crankshaft as the automobile travels around an unbanked curve of 200-m radius at a speed of 90 km/h, with the flywheel rotating at 2700 rpm. Assume the automobile to have (*a*) a rear-wheel drive with the engine mounted longitudinally, (*b*) a front-wheel drive with the engine mounted transversely. (Density of steel $= 7860\ \text{kg/m}^3$.)

SOLUTION

Let the x axis be a horizontal axis directed along the engine mounting, i.e., longitudinally for rear-wheel drive and transversely for front-wheel drive.

Let the y axis be vertical.

The angular velocity of the automobile, $\boldsymbol{\omega}_2$, is equal to $(v/\rho)\mathbf{j}$, where

$$v = 90\ \text{km/h} = 25\ \text{m/s} \ \text{ and } \ \rho = 200\ \text{m}.$$

$$\boldsymbol{\omega}_2 = \frac{25}{200}\mathbf{j} = (0.125\ \text{rad/s})\mathbf{j}$$

Angular velocity of the fly wheel relative to the automobile: $\boldsymbol{\omega}_1 = \omega_1 \mathbf{i}$

where $$\omega_1 = \frac{2\pi(2700)}{60} = 282.74\ \text{rad/s}$$

Angular momentum of fly wheel: $\mathbf{H}_G = I_x\omega_1\mathbf{i} + I_y\omega_y\mathbf{j}$

Let the reference frame $Gxyz$ be rotating with angular velocity $\boldsymbol{\Omega} = \omega_2\mathbf{j}$

$$\begin{aligned}\dot{\mathbf{H}}_G &= (\dot{\mathbf{H}}_G)_{Gxyz} + \boldsymbol{\Omega}\times\mathbf{H}_G \\ &= 0 + \omega_2\mathbf{j}\times(I_x\omega_1\mathbf{i} + I_y\omega_2\mathbf{j}) \\ &= -I_x\omega_1\omega_2\mathbf{k}\end{aligned}$$

Couple exerted by the shaft on the fly wheel: $\mathbf{M} = -I_x\omega_1\omega_2\mathbf{k}$

Couple exerted by the fly wheel on the shaft: $\mathbf{M}' = -\mathbf{M} = I_x\omega_1\omega_2\mathbf{k}$ (1)

Data for fly wheel: $$m = \rho\left(\frac{\pi}{4}d^2\right)t = (7860)\frac{\pi}{4}(0.4)^2(0.015) = 14.816\ \text{kg}$$

For a circular plate, $$I_x = \frac{1}{2}mr^2 = \frac{1}{2}(14.816)(0.2)^2 = 0.29632\ \text{kg}\cdot\text{m}^2$$

Using Equation (1), $$\mathbf{M}' = (0.29632)(282.74)(0.125)\mathbf{k} = (10.47\ \text{N}\cdot\text{m})\mathbf{k}$$

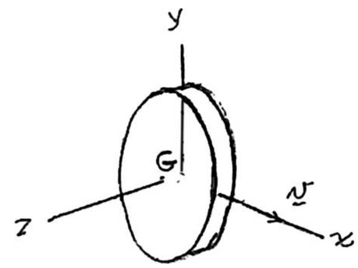
(*a*) Magnitude of couple for rear-wheel drive: $M' = 10.47\ \text{N}\cdot\text{m}$ ◀

(*b*) For front-wheel drive: $M' = 10.47\ \text{N}\cdot\text{m}$ ◀

PROBLEM 18.82

Each wheel of an automobile has a mass of 22 kg, a diameter of 575 mm, and a radius of gyration of 225 mm. The automobile travels around an unbanked curve of radius 150 m at a speed of 95 km/h. Knowing that the transverse distance between the wheels is 1.5 m, determine the additional normal force exerted by the ground on each outside wheel due to the motion of the car.

SOLUTION



For each wheel, $v = 95 \text{ km/h} = 26.389 \text{ m/s}$

$$\omega_x = 0$$

$$\omega_y = \frac{v}{\rho} = \frac{26.389}{150} = 0.17593 \text{ rad/s}$$

$$r = \frac{d}{2} = \frac{575}{2} = 287.5 \text{ mm} = 0.2875 \text{ m}$$

$$\bar{I}_z = mk^2 = (22)(0.225)^2 = 1.11375 \text{ kg} \cdot \text{m}^2$$

$$\omega_z = -\frac{v}{r} = -\frac{26.389}{0.2875} = -91.787 \text{ rad/s}$$

$$\mathbf{H}_G = \bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j} + \bar{I}_z\omega_z\mathbf{k}$$
$$= \bar{I}_y\omega_y\mathbf{j} + \bar{I}_z\omega_z\mathbf{k}$$

Let reference frame $Gxyz$ be rotating with angular velocity $\mathbf{\Omega} = \omega_y\mathbf{j}$.

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \mathbf{\Omega} \times \mathbf{H}_G$$
$$= 0 + \omega_y\mathbf{j} \times (\bar{I}_y\omega_y\mathbf{j} + \bar{I}_z\omega_z\mathbf{k})$$
$$= \bar{I}_z\omega_z\omega_y\mathbf{i}$$

$$\dot{\mathbf{H}}_G = (1.11375)(-91.787)(0.17593)\mathbf{i}$$
$$= -(17.985 \text{ N} \cdot \text{m})\mathbf{i}$$

Let O be the point at the center of the axle. For the two wheels plus the axle,

$$\dot{\mathbf{H}}_O = \dot{\mathbf{H}}_G + \dot{\mathbf{H}}_G = -(35.97 \text{ N} \cdot \text{m})\mathbf{i}$$

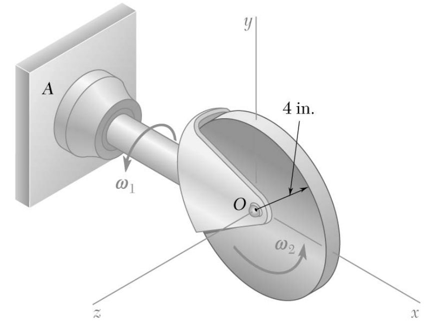
The distance between the wheels is 1.5 m.

$$\mathbf{M}_O = 1.5\mathbf{k} \times F_y\mathbf{j} = -1.5F_y\mathbf{i}$$

Set $\mathbf{M}_O = \dot{\mathbf{H}}_O$ and solve for F_y.

$$F_y = \frac{-35.97}{-1.5} = 23.98 \text{ N}$$

$\mathbf{F}_y = 24.0 \text{ N} \uparrow$ ◀



PROBLEM 18.83

The uniform thin 5-lb disk spins at a constant rate $\omega_2 = 6$ rad/s about an axis held by a housing attached to a horizontal rod that rotates at the constant rate $\omega_1 = 3$ rad/s. Determine the couple which represents the dynamic reaction at the support *A*.

SOLUTION

Angular velocity: $\omega_x = \omega_1, \qquad \omega_y = 0, \qquad \omega_z = \omega_2.$

$$\boldsymbol{\omega} = \omega_1 \mathbf{i} + \omega_2 \mathbf{k}$$

Angular momentum:

$$\mathbf{H}_O = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k}$$

$$= \bar{I}_x \omega_1 \mathbf{i} + \bar{I}_z \omega_2 \mathbf{k}$$

Let frame *Oxyz* be rotating with angular velocity $\boldsymbol{\Omega} = \omega_1 \mathbf{i}$.

Rate of change of angular momentum.

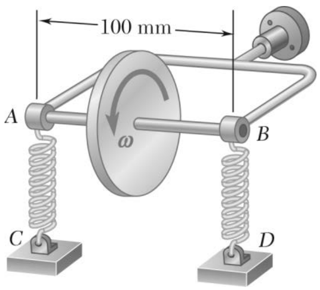
$$\dot{\mathbf{H}}_O = (\dot{\mathbf{H}}_O)_{Oxyz} + \boldsymbol{\Omega} \times \mathrm{H}_O$$
$$= \bar{I}_x \dot{\omega}_1 \mathbf{i} + \bar{I}_z \dot{\omega}_2 \mathbf{k} + \omega_1 \mathbf{i} \times (\bar{I}_x \omega_1 \mathbf{i} + \bar{I}_z \omega_2 \mathbf{k})$$
$$= 0 + 0 + 0 - \bar{I}_z \omega_1 \omega_2 \mathbf{j} = -\frac{1}{2} m r^2 \omega_1 \omega_2 \mathbf{j}$$

Dynamic reaction couple:

$$\mathbf{M} = \dot{\mathbf{H}}_O$$

$$\mathbf{M} = -\frac{1}{2} m r^2 \omega_1 \omega_2 \mathbf{j} = -\frac{1}{2}\left(\frac{5}{32.2}\right)\left(\frac{4}{12}\right)^2 (3)(6)\mathbf{j}$$

$$\mathbf{M} = -(0.1553 \text{ lb}\cdot\text{ft})\mathbf{j} \blacktriangleleft$$

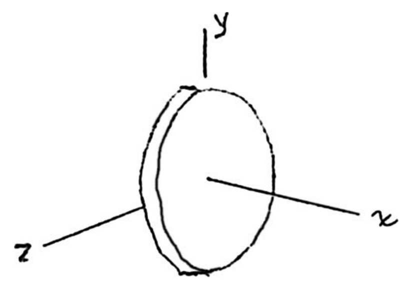


PROBLEM 18.84

The essential structure of a certain type of aircraft turn indicator is shown. Each spring has a constant of 500 N/m, and the 200-g uniform disk of 40-mm radius spins at the rate of 10,000 rpm. The springs are stretched and exert equal vertical forces on yoke *AB* when the airplane is traveling in a straight path. Determine the angle through which the yoke will rotate when the pilot executes a horizontal turn of 750-m radius to the right at a speed of 800 km/h. Indicate whether Point *A* will move up or down.

SOLUTION

Let the *x* axis lie along the axle *AB* and the *y* axis be vertical.



$$\omega_x = \frac{2\pi(10,000)}{60} = 1047.2 \text{ rad/s}$$

$$v = 800 \text{ km/h} = 222.22 \text{ m/s}$$

$$\rho = 750 \text{ m}$$

$$\omega_y = -\frac{v}{\rho} = -\frac{22.222}{750} = -0.2963 \text{ rad/s}$$

$$\omega_z = 0$$

Angular momentum:

$$\mathbf{H}_G = \bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j} + \bar{I}_z\omega_z\mathbf{k}$$

$$= I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j}$$

Let the reference frame *Gxyz* be turning about the *y* axis with angular velocity $\mathbf{\Omega} = \omega_y\mathbf{j}$.

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \mathbf{\Omega}\times\mathbf{H}_G$$

$$= \omega_y\mathbf{j}\times(\bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j})$$

$$= -I_x\omega_x\omega_y\mathbf{k}$$

Data for the disk:

$$m = 200 \text{ g} = 0.2 \text{ kg}$$

$$I_x = \frac{1}{2}mr^2$$

$$= \frac{1}{2}(0.2)(0.040)^2$$

$$= 160\times10^{-6} \text{ kg}\cdot\text{m}^2$$

$$\mathbf{M}_G = \dot{\mathbf{H}}_G$$

$$= -(160\times10^{-6})(1047.2)(-0.2963)\mathbf{k}$$

$$= (0.049646 \text{ N}\cdot\text{m})\mathbf{k}$$

The spring forces $\mathbf{F}_A$ and $\mathbf{F}_B$ exerted on the yoke provide the couple $\mathbf{M}_G$. The force exerted by spring B is upward.

Let $\mathbf{F}_B = F\mathbf{j}$

Then $\mathbf{F}_A = -F\mathbf{j}$

$$\mathbf{M}_G = \mathbf{r}_{B/A}\mathbf{i} \times F\mathbf{j} = 0.100\mathbf{i} \times F\mathbf{j} = 0.1F\mathbf{k}$$

From $\mathbf{M}_G = \dot{\mathbf{H}}_G$,

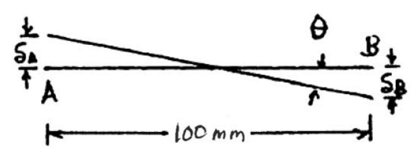
$$0.1F = 0.049646$$

$$F = 0.49646 \text{ N.}$$

Compression of spring B:

$$\delta_B = \frac{F}{k} = \frac{0.49646}{500} = 0.99291 \times 10^{-3}\text{ m} = 0.9929 \text{ mm}$$

Point B moves 0.9929 mm *down*. Point A moves 0.9929 mm *up*.

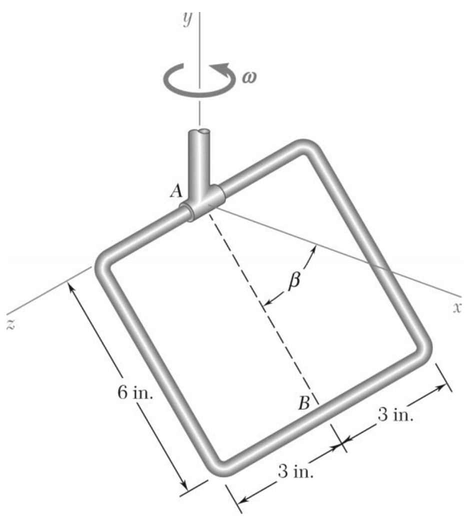


Turning angle for yoke:

$$\theta = \frac{0.9929 + 0.9929}{100} = 0.019858 \text{ rad}$$

$\boldsymbol{\theta} = 1.138°$ ◀

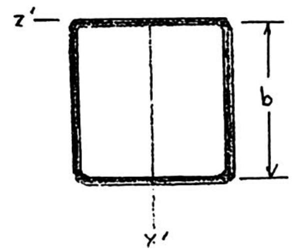
Point A moves *up*. ◀



PROBLEM 18.85

A slender rod is bent to form a square frame of side 6 in. The frame is attached by a collar at A to a vertical shaft which rotates with a constant angular velocity $\boldsymbol{\omega}$. Determine the value of ω for which line AB forms an angle $\beta = 48°$ with the horizontal x axis.

SOLUTION

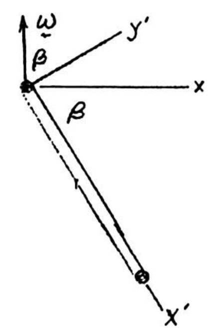


Choose principal axes x', y', z' with origin at the fixed Point A.

$$I_{x'} = 2\left(\frac{m}{4}\right)\left(\frac{1}{12}a^2\right) + 2\left(\frac{m}{4}\right)\left(\frac{a}{2}\right)^2 = \frac{1}{6}ma^2$$

$$I_{z'} = 2\left(\frac{m}{4}\right)\left(\frac{1}{3}a^2\right) + \left(\frac{m}{4}\right)(a)^2 = \frac{5}{12}ma^2$$

$$I_{y'} = I_{x'} + I_{z'} = \frac{7}{12}ma^2$$

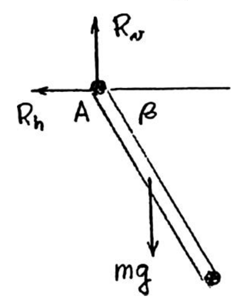


Angular velocity: $\boldsymbol{\omega} = -\omega \sin\beta\,\mathbf{i}' + \omega\cos\beta\,\mathbf{j}'$

Angular momentum about A: $\mathbf{H}_A = -I_{x'}\omega\sin\beta\,\mathbf{i}' + I_{y'}\omega\cos\beta\,\mathbf{j}'$

Let the reference from $Axyz$ be rotating with angular velocity $\boldsymbol{\Omega} = \boldsymbol{\omega}$.

$$\begin{aligned}\dot{\mathbf{H}}_A &= (\dot{\mathbf{H}}_A)_{Axyz} + \boldsymbol{\Omega}\times\mathbf{H}_A = 0 + \boldsymbol{\omega}\times\mathbf{H}_A \\ &= (-\omega\sin\beta\,\mathbf{i}' + \omega\cos\beta\,\mathbf{j}') \\ &\quad\times(-I_{x'}\,\omega\sin\beta\,\mathbf{i}' + I_{y'}\,\omega\cos\beta\,\mathbf{j}') \\ &= -(I_y - I_z)\,\omega^2\sin\beta\cos\beta\,\mathbf{k} \\ &= -\frac{5}{12}ma^2\omega^2\sin\beta\cos\beta\,\mathbf{k}\end{aligned}$$

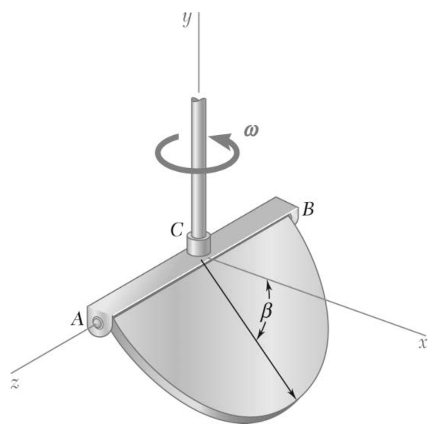


$$\Sigma\mathbf{M}_A = -mg\frac{a}{2}\cos\beta\,\mathbf{k} = \dot{\mathbf{H}}_A$$

$$-mg\frac{a}{2}\cos\beta = -\frac{5}{12}ma^2\omega^2\sin\beta\cos\beta$$

$$\omega^2 = \frac{6}{5}\frac{g}{a\sin\beta} = \frac{(6)(32.2)}{(5)(0.5)\sin 48°} = 103.99\,(\text{rad/s})^3$$

$\omega = 10.20$ rad/s ◀



PROBLEM 18.86

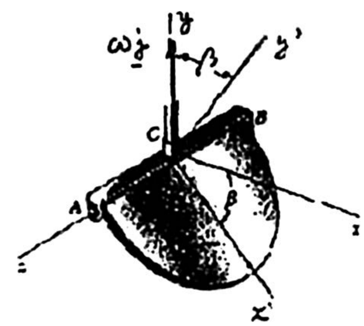
A uniform semicircular plate of radius 120 mm is hinged at A and B to a clevis which rotates with a constant angular velocity $\boldsymbol{\omega}$ about a vertical axis. Determine (a) the angle β that the plate forms with the horizontal x axis when $\omega = 15$ rad/s, (b) the largest value of ω for which the plate remains vertical ($\beta = 90°$).

SOLUTION

Moments and products of inertia.

We use the axes $Cx'y'z$ shown.

We note that $I_{x'}$ and $I_{y'}$ are half those for a circular plate, and so is the mass m. Thus,



$$I_{x'} = \frac{1}{4}mr^2$$

$$I_{y'} = \frac{1}{2}mr^2$$

Because of symmetry, all products of inertia are equal to zero:

$$I_{x'y'} = I_{y'z} = I_{zx'} = 0$$

Angular momentum about C.

$$\mathbf{H}_C = I_{x'}\,\omega_{x'}\mathbf{i}' + I_{y'}\omega_{y'}\mathbf{j}'$$

$$= \frac{1}{4}mx^2(-\omega \sin \beta)\mathbf{i}' + \frac{1}{2}mr^2(\omega \cos \beta)\mathbf{j}'$$

$$= \frac{1}{4}mx^2\omega(-\sin \beta\mathbf{i}' + 2 \cos \beta\mathbf{j}')$$

Since C is a fixed point, we can use Equation (18.28):

$$\Sigma\mathbf{M}_C = (\dot{\mathbf{H}}_C)_{Cx'y'z} + \boldsymbol{\Omega}\times\mathbf{H}_C = 0 + \omega\mathbf{j}\times\mathbf{H}_C$$

Or, since $\mathbf{j} = -\mathbf{i}'\sin\beta + \mathbf{j}'\cos\beta$:

$$\Sigma\mathbf{M}_C = \omega(-\mathbf{i}' \sin \beta + \mathbf{j}' \cos \beta)\times\frac{1}{4}mx^2\omega(-\sin \beta\mathbf{i}' + 2 \cos \beta\mathbf{j}')$$

$$= \frac{1}{4}mr^2\omega^2(-2 \sin \beta \cos \beta\mathbf{k} + \cos \beta \sin \beta\mathbf{k})$$

$$\Sigma\mathbf{M}_C = -\frac{1}{4}mr^2\omega^2 \sin \beta \cos \beta\mathbf{k} \qquad (1)$$

PROBLEM 18.86 (Continued)

But $$\Sigma \mathbf{M}_C = -mg\overline{x}' \cos \beta \mathbf{k}$$

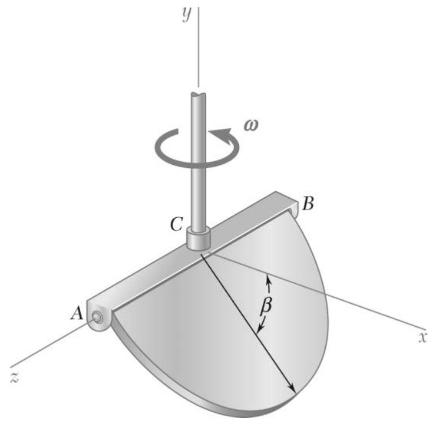
$$= -mg \frac{4r}{3\pi} \cos \beta \mathbf{k} \qquad (2)$$

Equating (1) and (2): $$\frac{1}{4} mr^2 \omega^2 \sin \beta \cos \beta = \frac{4mgr}{3\pi} \cos \beta$$

$$\omega^2 \sin \beta = \frac{16}{3\pi} \frac{g}{r} = \frac{16}{3\pi} \frac{9.81 \text{ m/s}^2}{0.12 \text{ m}} \qquad \omega^2 \sin \beta = 138.78 \text{ s}^{-2} \quad (3)$$

(*a*) Let $\omega = 15$ rad/s in Eq. (3): $\sin \beta = \frac{138.78}{(15)^2} = 0.61681$ $\quad \beta = 38.1° ◀$

(*b*) Let $\beta = 90°$ in Eq. (3): $\omega^2 = 138.78 \text{ s}^{-2}$ $\quad \omega = 11.78$ rad/s ◀



PROBLEM 18.87

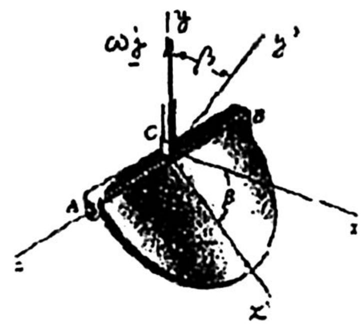
A uniform semicircular plate of radius 120 mm is hinged at A and B to a clevis which rotates with a constant angular velocity $\boldsymbol{\omega}$ about a vertical axis. Determine the value of ω for which the plate forms an angle $\beta = 50°$ with the horizontal x axis.

SOLUTION

Moments and products of inertia.

We use the axes $Cx'y'z$ shown.

We note that $I_{x'}$ and $I_{y'}$ are half those for a circular plate, and so is the mass m. Thus,



$$I_{x'} = \frac{1}{4}mr^2$$

$$I_{y'} = \frac{1}{2}mr^2$$

Because of symmetry, all products of inertia are equal to zero:

$$I_{x'y'} = I_{y'z} = I_{zx'} = 0$$

Angular momentum about C.

$$\mathbf{H}_C = I_{x'}\omega_{x'}\mathbf{i}' + I_{y'}\omega_{y'}\mathbf{j}'$$

$$= \frac{1}{4}mr^2(-\omega\sin\beta)\mathbf{i}' + \frac{1}{2}mr^2(\omega\cos\beta)\mathbf{j}'$$

$$= \frac{1}{4}mr^2\omega(-\sin\beta\mathbf{i}' + 2\cos\beta\mathbf{j}')$$

Since C is a fixed point, we can use Equation (18.28):

$$\Sigma\mathbf{M}_C = (\dot{\mathbf{H}}_C)_{Cx'y'z} + \boldsymbol{\Omega}\times\mathbf{H}_C$$

$$= 0 + \omega\mathbf{j}\times\mathbf{H}_C$$

Or, since $\mathbf{j} = -\mathbf{i}'\sin\beta + \mathbf{j}'\cos\beta$:

$$\Sigma\mathbf{M}_C = \omega(-\mathbf{i}'\sin\beta + \mathbf{j}'\cos\beta)\times\frac{1}{4}mr^2\omega(-\sin\beta\mathbf{i}' + 2\cos\beta\mathbf{j}')$$

$$= \frac{1}{4}mr^2\omega^2(-2\sin\beta\cos\beta\mathbf{k} + \cos\beta\sin\beta\mathbf{k})$$

$$\Sigma\mathbf{M}_C = -\frac{1}{4}mr^2\omega^2\sin\beta\cos\beta\mathbf{k} \qquad (1)$$

PROBLEM 18.87 (Continued)

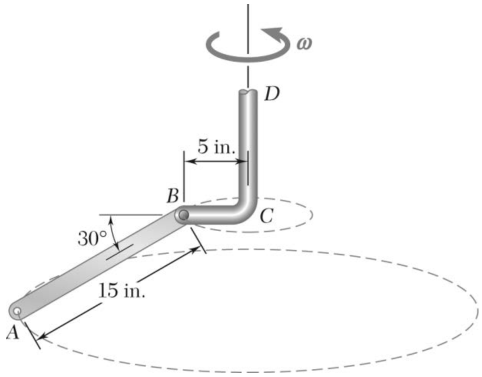
But $$\Sigma \mathbf{M}_C = -mg\overline{x}' \cos\beta\,\mathbf{k}$$

$$= -mg\frac{4r}{3\pi}\cos\beta\,\mathbf{k} \qquad (2)$$

Equating (1) and (2): $$\frac{1}{4}mr^2\omega^2 \sin\beta\cos\beta = \frac{4mgr}{3\pi}\cos\beta$$

$$\omega^2 \sin\beta = \frac{16}{3\pi}\frac{g}{r} = \frac{16}{3\pi}\frac{9.81\ \text{m/s}^2}{0.12\ \text{m}} \qquad \omega^2 \sin\beta = 138.78\ \text{s}^{-2} \quad (3)$$

Let $\beta = 50°$ in Equation (3): $$\omega^2 = \frac{138.78\ \text{s}^{-2}}{\sin 50°} = 181.17\ \text{s}^{-2} \qquad \omega = 13.46\ \text{rad/s} \ ◀$$



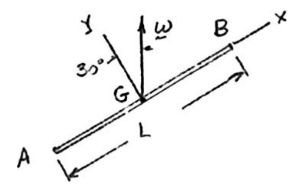
PROBLEM 18.88

The slender rod AB is attached by a clevis to arm BCD which rotates with a constant angular velocity $\boldsymbol{\omega}$ about the centerline of its vertical portion CD. Determine the magnitude of the angular velocity $\boldsymbol{\omega}$.

SOLUTION

Let $AB = L = 15 \text{ in.} = 1.25 \text{ ft}, \quad \text{and} \quad BC = b = 5 \text{ in.} = 0.41667 \text{ ft}$

Choose x, y, z axes as shown. $\bar{I}_x \approx 0, \quad \bar{I}_y = \bar{I}_z = \frac{1}{12}mL^2$



Angular velocity: $\boldsymbol{\omega} = \omega \sin 30°\mathbf{i} + \omega \cos 30°\mathbf{j}$

Angular momentum of rod AB about its mass center G:

$$\mathbf{H}_G = \bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j} + \bar{I}_z\omega_z\mathbf{k} = \bar{I}_y\omega\cos 30°\mathbf{j}$$

Let the reference frame $Gxyz$ be rotating with angular velocity $\boldsymbol{\Omega} = \boldsymbol{\omega}$.

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \boldsymbol{\Omega} \times \mathbf{H}_G$$

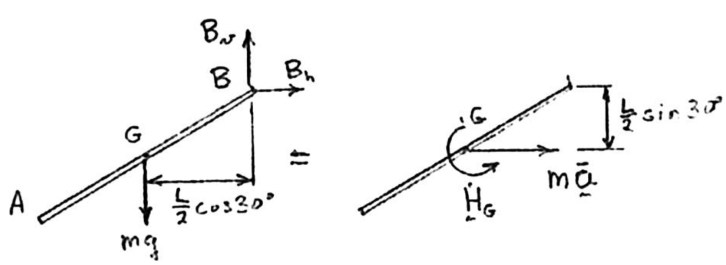
$$= 0 + (\omega\sin 30°\mathbf{i} + \omega\cos 30°\mathbf{j}) \times \bar{I}_y\omega\cos 30°\mathbf{j}$$

$$= \bar{I}_y\omega^2 \sin 30° \cos 30°\mathbf{k} = \frac{\sqrt{3}}{48}mL^2\omega^2\mathbf{k}$$

Radius of circular path of Point G: $r = \frac{L}{2}\cos 30° + b = 0.95793 \text{ ft}$

Acceleration of the mass center: $\bar{\mathbf{a}} = r\omega^2 \longrightarrow$

Equations of motion:



PROBLEM 18.88 (Continued)

$$+\circlearrowleft \Sigma \mathbf{M}_B = mg\frac{L}{2}\cos 30°\mathbf{k} = \left(\frac{L}{2}\sin 30°\right)mr\omega^2\mathbf{k} + \dot{\mathbf{H}}_G$$

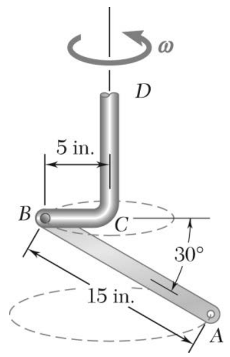
$$\frac{\sqrt{3}}{4}mgL\mathbf{k} = \left(\frac{1}{4}mLr + \frac{\sqrt{3}}{48}mL^2\right)\omega^2\mathbf{k}$$

$$\frac{\sqrt{3}}{4}g = \left(\frac{1}{4}r + \frac{\sqrt{3}}{48}L\right)\omega^2$$

$$\frac{\sqrt{3}}{4}(32.2) = \left[\frac{1}{4}(0.95793) + \frac{\sqrt{3}}{48}(1.25)\right]\omega^2$$

$$\omega^2 = 48.994$$

$\omega = 7.00$ rad/s ◀

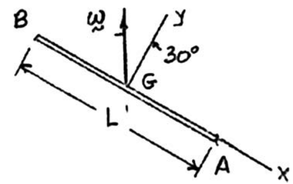


PROBLEM 18.89

The slender rod *AB* is attached by a clevis to arm *BCD*, which rotates with a constant angular velocity ω about the centerline of its vertical portion *CD*. Determine the magnitude of the angular velocity ω.

SOLUTION

Let $AB = L = 15 \text{ in.} = 1.25 \text{ ft}$, and $BC = b = 5 \text{ in.} = 0.41667 \text{ ft}$



Choose x, y, z axes as shown. $\bar{I}_x \approx 0, \quad \bar{I}_y = \bar{I}_z = \dfrac{1}{12} mL^2$

Angular velocity: $\boldsymbol{\omega} = -\omega \sin 30° \mathbf{i} + \omega \cos 30° \mathbf{j}$

Angular momentum of rod *AB* about its mass center *G*:

$$\mathbf{H}_G = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k} = \bar{I}_y \omega \cos 30° \mathbf{j}$$

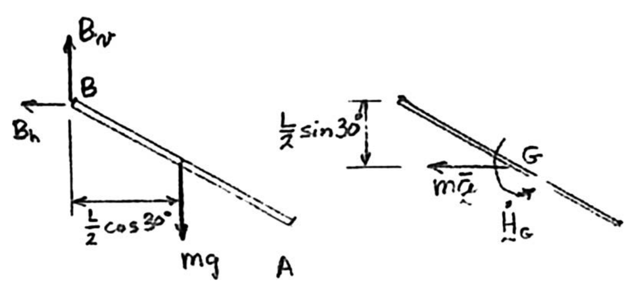
Let the reference frame *Gxyz* be rotating with angular velocity $\boldsymbol{\Omega} = \boldsymbol{\omega}$.

$$\begin{aligned}\dot{\mathbf{H}}_G &= (\dot{\mathbf{H}}_G)_{Gxyz} + \boldsymbol{\Omega} \times \mathbf{H}_G \\ &= 0 + (-\omega \sin 30° \mathbf{i} + \omega \cos 30° \mathbf{j}) \times \bar{I}_y \omega \cos 30° \mathbf{j} \\ &= -\bar{I}_y \omega^2 \sin 30° \cos 30° \mathbf{k} = -\frac{\sqrt{3}}{48} mL^2 \omega^2 \mathbf{k}\end{aligned}$$

Radius of circular path of Point *G*: $r = \dfrac{L}{2} \cos 30° - b = 0.1246 \text{ ft}$

Acceleration of the mass center: $\bar{\mathbf{a}} = r\omega^2 \longleftarrow$

Equations of motion:



PROBLEM 18.89 (Continued)

$$+\circlearrowleft \Sigma \mathbf{M}_B = -mg\frac{L}{2}\cos 30°\mathbf{k} = -\left(\frac{L}{2}\sin 30°\right)mr\omega^2\mathbf{k} + \dot{\mathbf{H}}_G$$

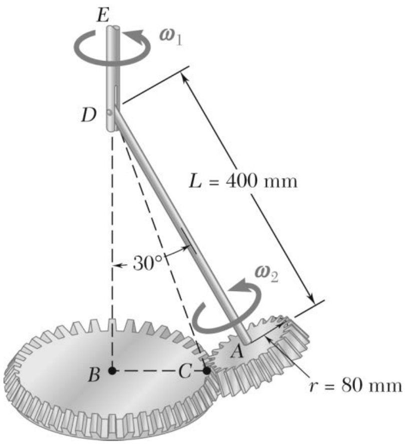
$$-\frac{\sqrt{3}}{4}mgL\mathbf{k} = -\left(\frac{1}{4}mLr + \frac{\sqrt{3}}{48}mL^2\right)\omega^2\mathbf{k}$$

$$\frac{\sqrt{3}}{4}g = \left(\frac{1}{4}r + \frac{\sqrt{3}}{48}L\right)\omega^2$$

$$\frac{\sqrt{3}}{4}(32.3) = \left[\frac{1}{4}(0.1246) + \frac{\sqrt{3}}{48}(1.25)\right]\omega^2$$

$$\omega^2 = 182.85$$

$\omega = 13.52$ rad/s ◀



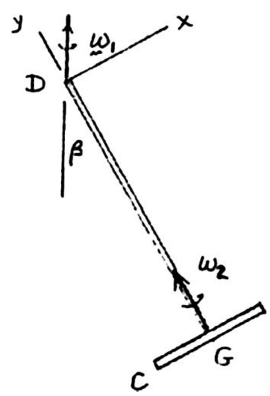
PROBLEM 18.90

The 950-g gear A is constrained to roll on the fixed gear B, but is free to rotate about axle AD. Axle AD, of length 400 mm and negligible mass, is connected by a clevis to the vertical shaft DE, which rotates as shown with a constant angular velocity $\boldsymbol{\omega}_1$. Assuming that gear A can be approximated by a thin disk of radius 80 mm, determine the largest allowable value of ω_1 if gear A is not to lose contact with gear B.

SOLUTION

$$\beta = 30^\circ$$
$$L = 400 \text{ mm} = 0.4 \text{ m}$$
$$r = 80 \text{ mm} = 0.08 \text{ m}$$

Choose principal axes x, y, z as shown.



Kinematics:

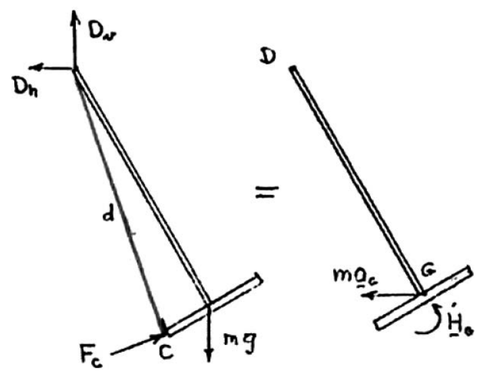
$$\boldsymbol{\omega}_1 = \omega_1 \sin\beta\mathbf{i} + \omega_1 \cos\beta\mathbf{j}$$
$$\boldsymbol{\omega}_2 = \omega_2\mathbf{j}$$
$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$$
$$= \omega_1 \sin\beta\mathbf{i} + (\omega_1 \cos\beta + \omega_2)\mathbf{j}$$
$$\omega_x = \omega_1 \sin\beta$$
$$\omega_y = \omega_1 \cos\beta + \omega_2$$
$$\mathbf{v}_G = \boldsymbol{\omega}_1 \times \mathbf{r}_{G/D} = \boldsymbol{\omega}_1\mathbf{j} \times \mathbf{r}_{G/D}$$
$$= -\omega_1 L \sin\beta\mathbf{k}$$
$$\mathbf{a}_G = \boldsymbol{\omega}_1 \times \mathbf{v}_G = \omega_1^2 L \sin\beta \longleftarrow$$
$$\mathbf{v}_C = \mathbf{v}_G + \boldsymbol{\omega} \times \mathbf{r}_{C/G}$$
$$0 = -\omega_1 L \sin\beta\mathbf{k} + (\omega_x\mathbf{i} + \omega_y\mathbf{j}) \times (-r\mathbf{i})$$
$$= -\omega_1 L \sin\beta\mathbf{k} + \omega_y r\mathbf{k}$$
$$\omega_y = \omega_1 \frac{L}{r} \sin\beta$$

Angular momentum:

$$\mathbf{H}_G = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j}$$

Let the reference frame *Dxyz* be rotating with angular velocity $\mathbf{\Omega} = \boldsymbol{\omega}_1$.

$$\begin{aligned}\dot{\mathbf{H}}_G &= (\dot{\mathbf{H}}_G)_{Gxyz} + \mathbf{\Omega}\times\mathbf{H}_G \\ &= 0 + (\omega_1 \sin\beta\mathbf{i} + \omega_1\cos\beta\mathbf{j})\times(\bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j}) \\ &= (\bar{I}_y\omega_y\omega_1\sin\beta - \bar{I}_x\omega_x\omega_1\cos\beta)\mathbf{k} \\ &= \left(\frac{1}{2}mr^2\omega_y\sin\beta - \frac{1}{4}mr^2\omega_x\cos\beta\right)\omega_1\mathbf{k}\end{aligned}$$



Moments about *D*: $\mathbf{M}_D = Fd\mathbf{k} - mgL\sin\beta\mathbf{k}$

where $d = (DC) = \sqrt{L^2 + r^2}$

$$\begin{aligned}(\mathbf{M}_D)_{\text{eff}} &= \dot{\mathbf{H}}_G + \mathbf{r}_{G/D}\times m\mathbf{a}_G \\ &= \dot{\mathbf{H}}_G + m\omega_1^2L^2\sin\beta\cos\beta\mathbf{k}\end{aligned}$$

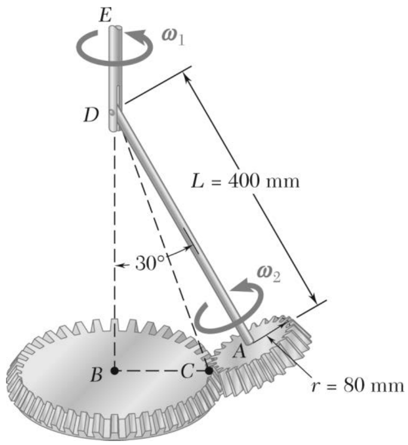
Equating $\mathbf{M}_D = (\mathbf{M}_D)_{\text{eff}}$ and taking the *z* component,

$$\begin{aligned}F_Cd - mgL\sin\beta &= \left(\frac{1}{2}mr^2\frac{L}{r}\sin^2\beta - \frac{1}{4}mr^2\sin\beta\cos\beta\right)\omega_1^2 + m\omega_1^2L^2\sin\beta\cos\beta \\ &= m\omega_1^2\sin\beta\left(\frac{1}{2}rL\sin\beta - \frac{1}{4}r^2\cos\beta - L^2\cos\beta\right)\end{aligned}$$

Set $F_C = 0$ and solve for ω_1^2:

$$\begin{aligned}\omega_1^2 &= \frac{gL}{L^2\cos\beta + \frac{1}{4}r^2\cos\beta - \frac{1}{2}rL\sin\beta} \\ &= \frac{(9.81)(0.4)}{(0.4)^2\cos30^\circ + \frac{1}{4}(0.08)^2\cos30^\circ - \frac{1}{2}(0.08)(0.4)\sin30^\circ} \\ &= 29.739\end{aligned}$$

$\omega_2 = 5.45\text{ rad/s}$ ◀

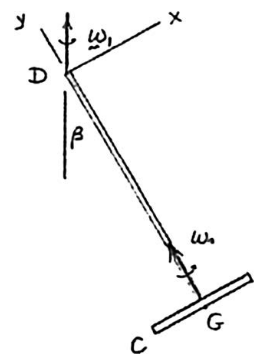


PROBLEM 18.91

Determine the force **F** exerted by gear *B* on gear *A* of Problem 18.90 when shaft *DE* rotates with the constant angular velocity $\omega_1 = 4$ rad/s. (*Hint:* The force **F** must be perpendicular to the line drawn from *D* to *C*.)

SOLUTION

$$\beta = 30°$$
$$L = 400 \text{ mm} = 0.4 \text{ m}$$
$$r = 80 \text{ mm} = 0.08 \text{ m}$$



Choose principal axes *x*, *y*, *z* as shown.

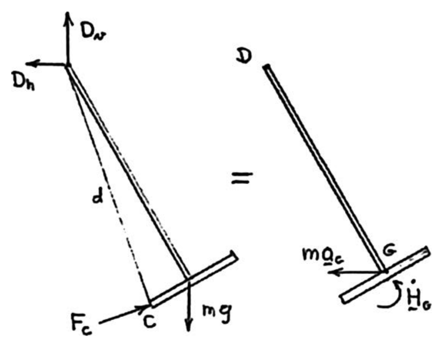
Kinematics:
$$\boldsymbol{\omega}_1 = \omega_1 \sin\beta \mathbf{i} + \omega_1 \cos\beta \mathbf{j}$$
$$\boldsymbol{\omega}_2 = \omega_2 \mathbf{j}$$
$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$$
$$= \omega_1 \sin\beta \mathbf{i} + (\omega_1 \cos\beta + \omega_2)\mathbf{j}$$
$$\omega_x = \omega_1 \sin\beta$$
$$\omega_y = \omega_1 \cos\beta + \omega_2$$
$$\mathbf{v}_G = \boldsymbol{\omega}_1 \times \mathbf{r}_{G/D}$$
$$= \boldsymbol{\omega}_1 \mathbf{j} \times \mathbf{r}_{G/D}$$
$$= -\omega_1 L \sin\beta \mathbf{k}$$
$$\mathbf{a}_G = \boldsymbol{\omega}_1 \times \mathbf{v}_G = \omega_1^2 L \sin\beta \longleftarrow$$
$$\mathbf{v}_C = \mathbf{v}_G + \boldsymbol{\omega} \times \mathbf{r}_{C/G}$$
$$0 = -\omega_1 L \sin\beta \mathbf{k} + (\omega_x \mathbf{i} + \omega_y \mathbf{j}) \times (-r\mathbf{i})$$
$$= -\omega_1 L \sin\beta \mathbf{k} + \omega_y r \mathbf{k}$$
$$\omega_y = \omega_1 \frac{L}{r} \sin\beta$$

Angular momentum:
$$\mathbf{H}_G = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j}$$

PROBLEM 18.91 (Continued)

Let the reference frame $Dxyz$ be rotating with angular velocity $\mathbf{\Omega} = \boldsymbol{\omega}_1$.

$$\begin{aligned}\dot{\mathbf{H}}_G &= (\dot{\mathbf{H}}_G)_{Gxyz} + \mathbf{\Omega}\times\mathbf{H}_G \\ &= 0 + (\omega_1 \sin\beta\mathbf{i} + \omega_1\cos\beta\mathbf{j})\times(\bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j}) \\ &= (\bar{I}_y\omega_y\omega_1\sin\beta - \bar{I}_x\omega_x\omega_1\cos\beta)\mathbf{k} \\ &= \left(\frac{1}{2}mr^2\omega_y\sin\beta - \frac{1}{4}mr^2\omega_x\cos\beta\right)\omega_1\mathbf{k}\end{aligned}$$



Moments about D: $\quad \mathbf{M}_D = Fd\mathbf{k} - mgL\sin\beta\mathbf{k}$

where $\quad d = (DC) = \sqrt{L^2 + r^2}$

$$(\mathbf{M}_D)_{\text{eff}} = \dot{\mathbf{H}}_G + \mathbf{r}_{G/D}\times m\mathbf{a}_G = \dot{\mathbf{H}}_G + m\omega_1^2 L^2\sin\beta\cos\beta\mathbf{k}$$

Equating $\mathbf{M}_D = (\mathbf{M}_D)_{\text{eff}}$ and taking the z component,

$$\begin{aligned}F_C d - mgL\sin\beta &= \left(\frac{1}{2}mr^2\frac{L}{r}\sin^2\beta - \frac{1}{4}mr^2\sin\beta\cos\beta\right)\omega_1^2 + m\omega_1^2L^2\sin\beta\cos\beta \\ &= m\omega_1^2\sin\beta\left(\frac{1}{2}rL\sin\beta - \frac{1}{4}r^2\cos\beta - L^2\cos\beta\right)\end{aligned}$$

Solving for F_C, $\quad F_C = \dfrac{m\sin\beta}{d}\left[gL - \omega_1^2\left(L^2\cos\beta + \dfrac{1}{4}r^2\cos\beta - \dfrac{1}{2}rL\sin\beta\right)\right]$

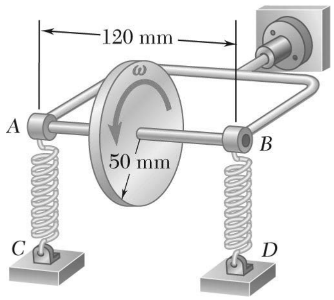
Additional data: $\quad m = 950g = 0.95 \text{ kg}, \quad \omega_1 = 4 \text{ rad/s}$

$$d = \sqrt{(0.4)^2 + (0.08)^2} = 0.40792 \text{ m}$$

$$F_C = \frac{(0.95)\sin 30^\circ}{0.40792}\left\{(9.81)(0.4) - (4)^2\left[(0.4)^2\cos 30^\circ + \frac{1}{4}(0.08)^2\cos 30^\circ - \frac{1}{2}(0.4)(0.08)\sin 30^\circ\right]\right\}$$
$$= 2.11 \text{ N}$$

$$\alpha = \beta - \tan^{-1}\frac{r}{L} = 30^\circ - \tan^{-1}\left(\frac{4}{20}\right)$$

$\alpha = 18.7^\circ$ $\qquad$ $2.11 \text{ N} \measuredangle 18.7^\circ$ ◀



PROBLEM 18.92

The essential structure of a certain type of aircraft turn indicator is shown. Springs *AC* and *BD* are initially stretched and exert equal vertical forces at *A* and *B* when the airplane is traveling in a straight path. Each spring has a constant of 600 N/m and the uniform disk has a mass of 250 g and spins at the rate of 12,000 rpm. Determine the angle through which the yoke will rotate when the pilot executes a horizontal turn of 800-m radius to the right at a speed of 720 km/h. Indicate whether point *A* will move up or down.

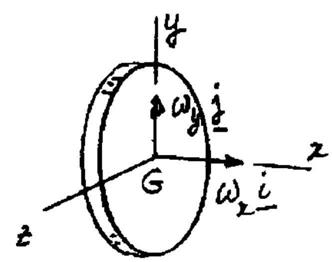
SOLUTION

Aircraft speed: $v = 720 \text{ km/h} = 200 \text{ m/s}$

Radius of turn: $\rho = 800 \text{ m}$

Angular velocity: $\omega_x = 12000 \text{ rpm} = 1256.6 \text{ rad/s}$

ω_y is negative since the aircraft is turning to the right.



$$\omega_y = -\frac{v}{\rho} = -\frac{200 \text{ m/s}}{800 \text{ m}} = -0.25 \text{ rad/s}$$

$$\omega_z = 0$$

Angular momentum: $\mathbf{H}_G = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j}$

where the reference frame *Gxyz* is turning with the aircraft with an angular velocity

$$\mathbf{\Omega} = \omega_y \mathbf{j}$$

Rate of change of angular momentum:

Since ω_x and ω_y are constant, $(\dot{\mathbf{H}}_G)_{Gxyz} = 0$ and Eq. (18.22) yields

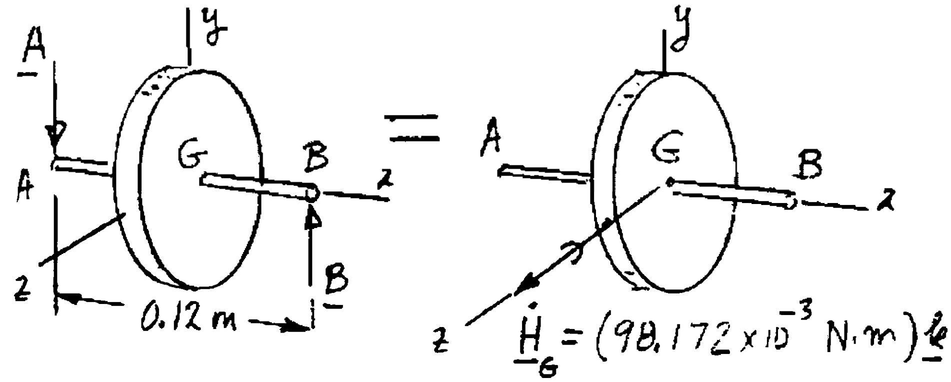
$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \mathbf{\Omega} \times \mathbf{H}_G = 0 + \omega_y \mathbf{j} \times (\bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j})$$

$$= -\bar{I}_x \omega_x \omega_y \mathbf{k} = -\left(\frac{1}{2} m r^2\right) \omega_x \omega_y \mathbf{k}$$

$$= -\frac{1}{2}(0.25 \text{ kg})(0.05 \text{ m})^2 (1256.6 \text{ rad/s})(-0.250 \text{ rad/s})\mathbf{k}$$

$$\dot{\mathbf{H}}_G = +(98.172 \times 10^{-3} \text{ N} \cdot \text{m})\mathbf{k}$$

Free Body and kinetic diagrams



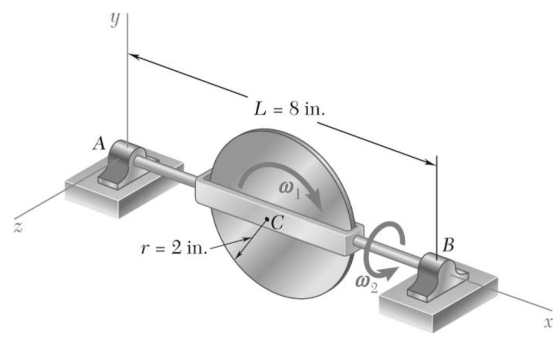
The forces A and B exerted by the springs must be equivalent to the couple $\dot{\mathbf{H}}_G$. They must therefore be directed as shown, which means that the spring at A will stretch and A will move up. ◀

We have $A = B$, $(0.12 \text{ m})A = 98.172\times 10^{-3} \text{ N}\cdot\text{m}$

$$F = A = 0.81810 \text{ N}$$

$$\text{Deflection of spring } = x = \frac{F}{k} = \frac{0.81810 \text{ N}}{600 \text{ N/m}} = 1.3635\times 10^{-3} \text{ m}$$

$$\text{Angle of rotation } = \frac{x}{GA} = \frac{1.3635\times 10^{-3} \text{ m}}{0.06 \text{ m}} = 0.022725 \text{ rad} = 1.30°$$ ◀



PROBLEM 18.93

The 10-oz disk shown spins at the rate $\omega_1 = 750$ rpm, while axle AB rotates as shown with an angular velocity ω_2 of 6 rad/s. Determine the dynamic reactions at A and B.

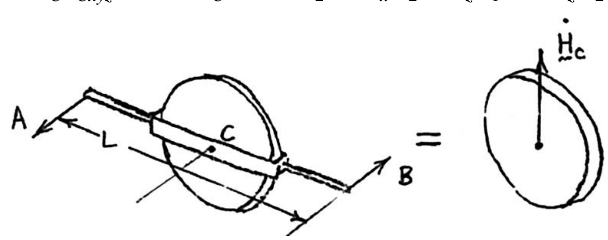
SOLUTION

Angular velocity: $\boldsymbol{\omega} = \omega_2\mathbf{i} - \omega_1\mathbf{k}$

Angular momentum: $\mathbf{H}_C = \bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j} + \bar{I}_z\omega_z\mathbf{k} = \bar{I}_x\omega_2\mathbf{i} - \bar{I}_z\omega_1\mathbf{k}$

Let the reference frame $Cxyz$ be rotating with angular velocity $\boldsymbol{\Omega} = \omega_2\mathbf{i}$.

$$\dot{\mathbf{H}}_C = (\dot{\mathbf{H}}_C)_{Cxyz} + \boldsymbol{\Omega}\times\mathbf{H}_C = 0 + \omega_2\mathbf{i}\times(I_x\omega_2\text{i} - \bar{I}_z\omega_1\mathbf{k}) = \bar{I}_z\omega_2\omega_1\mathbf{j}$$



Acceleration of mass center:

$$\bar{\mathbf{a}} = 0$$

$$\Sigma\mathbf{F} = m\bar{\mathbf{a}}$$

$$A\mathbf{k} - B\mathbf{k} = 0$$

$$A = B$$

$$\mathbf{M}_C = \dot{\mathbf{H}}_C$$

$$LB\mathbf{j} = \bar{I}_z\omega_2\omega_1\mathbf{j} \quad B = \frac{\bar{I}_z\omega_2\omega_1}{L}$$

Data: $$m = \frac{W}{g} = \frac{10}{(16)(32.2)} = 0.01941 \text{ lb}\cdot\text{s}^2/\text{ft} \quad r = 2 \text{ in.} = 0.16667 \text{ ft}$$

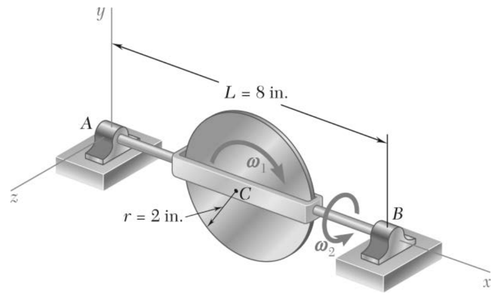
$$\bar{I}_Z = \frac{1}{2}mr^2 = \frac{1}{2}(0.01941)(0.16667)^2 = 269.6\times10^{-6} \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\omega_1 = \frac{2\pi(750)}{60} = 25\pi \text{ rad/s}, \quad \omega_2 = 6 \text{ rad/s}, \quad L = 8 \text{ in.} = 0.66667 \text{ ft}$$

$$A = B = \frac{(269.6\times10^{-6})(6)(25\pi)}{0.66667} = 0.1906 \text{ lb} \qquad \mathbf{A} = (0.1906 \text{ lb})\mathbf{k} \blacktriangleleft$$

$$\mathbf{B} = -(0.1906 \text{ lb})\mathbf{k} \blacktriangleleft$$

PROBLEM 18.94



The 10-oz disk shown spins at the rate $\omega_1 = 750$ rpm, while axle AB rotates as shown with an angular velocity ω_2. Determine the maximum allowable magnitude of ω_2 if the dynamic reactions at A and B are not to exceed 0.25 lb each.

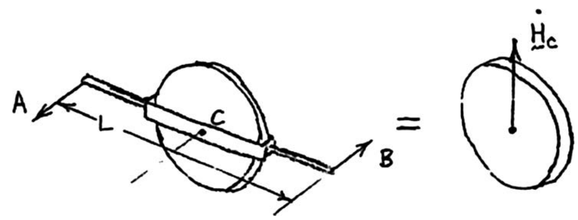
SOLUTION

Angular velocity: $\boldsymbol{\omega} = \omega_2 \mathbf{i} - \omega_1 \mathbf{k}$

Angular momentum: $\mathbf{H}_C = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k} = \bar{I}_x \omega_2 \mathbf{i} - \bar{I}_z \omega_1 \mathbf{k}$

Let the reference frame $Cxyz$ be rotating with angular velocity $\boldsymbol{\Omega} = \omega_2 \mathbf{i}$.

$$\dot{\mathbf{H}}_C = (\dot{\mathbf{H}}_C)_{Cxyz} + \boldsymbol{\Omega} \times \mathbf{H}_C = 0 + \omega_2 \mathbf{i} \times (I_x \omega_2 \mathbf{i} - \bar{I}_z \omega_1 \mathbf{k}) = \bar{I}_z \omega_2 \omega_1 \mathbf{j}$$



Acceleration of mass center:

$$\bar{\mathbf{a}} = 0$$
$$\Sigma \mathbf{F} = m\bar{\mathbf{a}}$$
$$A\mathbf{k} - B\mathbf{k} = 0$$
$$A = B$$
$$\mathbf{M}_C = \dot{\mathbf{H}}_C$$
$$LB\mathbf{j} = \bar{I}_z \omega_2 \omega_1 \mathbf{j} \quad B = \frac{\bar{I}_z \omega_2 \omega_1}{L}$$

Data: $m = \dfrac{W}{g} = \dfrac{10}{(16)(32.2)} = 0.01941 \text{ lb}\cdot\text{s}^2/\text{ft} \quad r = 2 \text{ in.} = 0.16667 \text{ ft}$

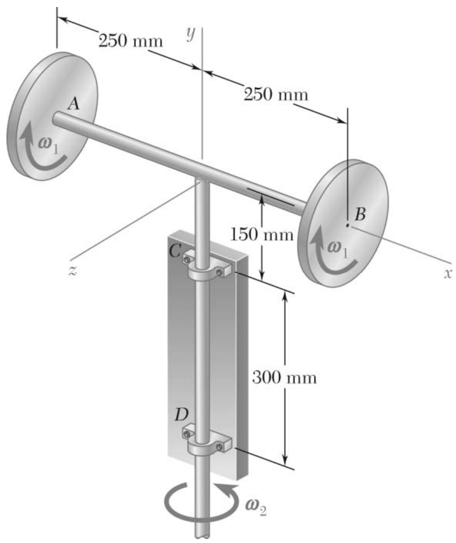
$$\bar{I}_z = \frac{1}{2} m r^2 = \frac{1}{2}(0.01941)(0.16667)^2 = 269.6 \times 10^{-6} \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\omega_1 = \frac{2\pi(750)}{60} = 25\pi \text{ rad/s}, \quad L = 8 \text{ in.} = 0.66667 \text{ ft}$$

$$A = B = 0.25 \text{ lb}$$

$$\omega_2 = \frac{LB}{\bar{I}_z \omega_1} = \frac{(0.66667)(0.25)}{(269.6 \times 10^{-6})(25\pi)}$$

$\omega_2 = 7.87$ rad/s ◀

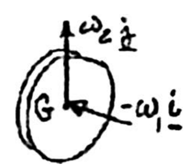


PROBLEM 18.95

Two disks, each of mass 5 kg and radius 100 mm, spin as shown at the rate $\omega_1 = 1500$ rpm about a rod AB of negligible mass which rotates about a vertical axis at the rate $\omega_2 = 45$ rpm. (*a*) Determine the dynamic reactions at C and D. (*b*) Solve part *a*, assuming that the direction of spin of disk B is reversed.

SOLUTION

<u>Angular momentum of each disk about its mass center</u>.



$$\mathbf{H}_G = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} = -\frac{1}{2} m r^2 \omega_1 \mathbf{i} + \frac{1}{4} m r^2 \omega_2 \mathbf{j}$$

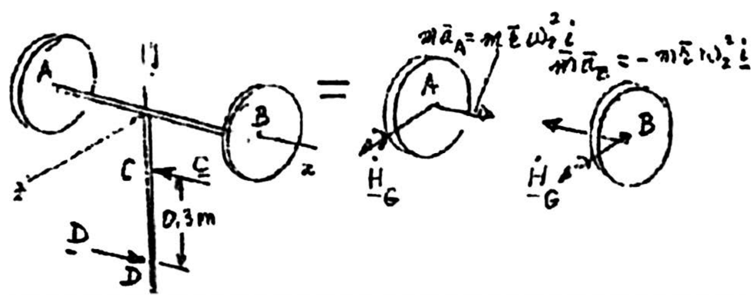
$$\mathbf{H}_G = \frac{1}{4} m r^2 (-2\omega_1 \mathbf{i} + \omega_2 \mathbf{j}) \qquad (1)$$

Eq. (18.22):

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \mathbf{\Omega} \times \mathbf{H}_G = 0 + \omega_2 \mathbf{j} \times \frac{1}{4} m r^2 (-2\omega_1 \mathbf{i} + \omega_2 \mathbf{j})$$

$$\dot{\mathbf{H}}_B = +\frac{1}{2} m r^2 \omega_1 \omega_2 \mathbf{k} \qquad (2)$$

<u>Equations of motion</u>.



Since $m\bar{\mathbf{a}}_A$ and $m\bar{\mathbf{a}}_B$ cancel out, effective forces reduce to couple $2\dot{\mathbf{H}}_G = m r^2 \omega_1 \omega_2 \mathbf{k}$.

It follows that the reactions form an equivalent couple with

$$-\mathbf{C} = \mathbf{D} = \left(\frac{m r^2 \omega_1 \omega_2}{0.3 \text{ m}} \right) \mathbf{i} \qquad (3)$$

PROBLEM 18.95 (Continued)

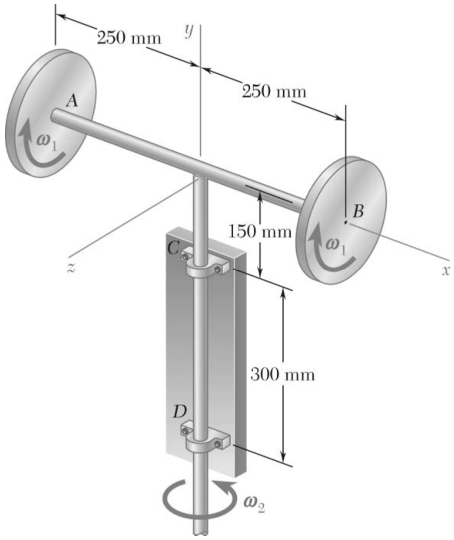
(*a*) With $m = 5$ kg, $r = 0.1$ m, $\omega_1 = 1500$ rpm $= 50\pi$ rad/s, and $\omega_2 = 45$ rpm $= 1.5\pi$ rad/s, Eq. (3) yields

$$C = D = (5 \text{ kg})(0.1 \text{ m})^2 (50\pi \text{ rad/s})\left(\frac{1.5\pi \text{ rad/s}}{0.3 \text{ m}}\right) = 123.37 \text{ N}$$

$\mathbf{C} = -(123.4 \text{ N})\mathbf{i}$; $\mathbf{D} = (123.4 \text{ N})\mathbf{i}$ ◀

(*b*) With direction of spin of *B* reversed, its angular momentum will also be reversed and the effective forces (and thus, the applied forces) reduce to zero:

$\mathbf{C} = \mathbf{D} = 0$ ◀

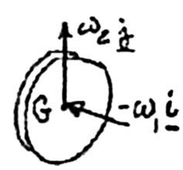


PROBLEM 18.96

Two disks, each of mass 5 kg and radius 100 mm, spin as shown at the rate $\omega_1 = 1500$ rpm about a rod AB of negligible mass which rotates about a vertical axis at a rate ω_2. Determine the maximum allowable value of ω_2 if the dynamic reactions at C and D are not to exceed 250 N each.

SOLUTION

Angular momentum of each disk about its mass center.



$$\mathbf{H}_G = \bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j} = -\frac{1}{2}mr^2\omega_1 i + \frac{1}{4}mr^2\omega_2\mathbf{j}$$

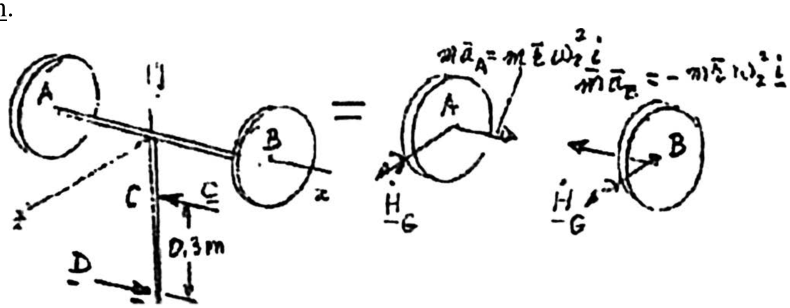
$$\mathbf{H}_G = \frac{1}{4}mr^2(-2\omega_1\mathbf{i} + \omega_2\mathbf{j}) \quad (1)$$

Eq. (18.22):

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \mathbf{\Omega}\times\mathbf{H}_G = 0 + \omega_2\mathbf{j}\times\frac{1}{4}mr^2(-2\omega_1\mathbf{i} + \omega_2\mathbf{j})$$

$$\dot{\mathbf{H}}_B = +\frac{1}{2}mr^2\omega_1\omega_2\mathbf{k} \quad (2)$$

Equations of motion.



Since $m\bar{\mathbf{a}}_A$ and $m\bar{\mathbf{a}}_B$ cancel out, effective forces reduce to couple $2\dot{\mathbf{H}}_G = mr^2\omega_1\omega_2\mathbf{k}$. It follows that the reactions form an equivalent couple with

$$-\mathbf{C} = \mathbf{D} = \left(\frac{mr^2\omega_1\omega_2}{0.3 \text{ m}}\right)\mathbf{i} \quad (3)$$

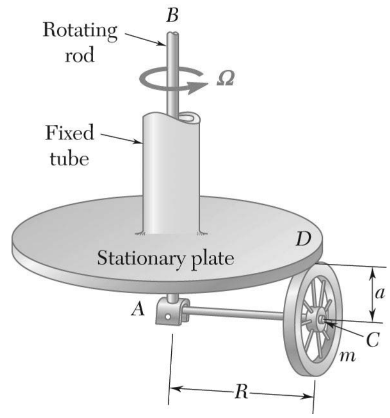
PROBLEM 18.96 (Continued)

Making $C = D - 250$ N in Eq. (3) yields

$$\frac{mr^2\omega_1\omega_2}{0.3 \text{ m}} = 250 \text{ N}$$

With $m = 5$ kg, $r = 0.1$ m, $\omega_1 = 1500 \text{ rpm} = 50\pi$ rad/s

We have $$\omega_2 = \frac{(250 \text{ N})(0.3 \text{ m})}{(5 \text{ kg})(0.1 \text{ m})^2(50\pi \text{ rad/s})} = 9.5493 \text{ rad/s}$$ $\omega_2 = 91.2$ rpm ◀

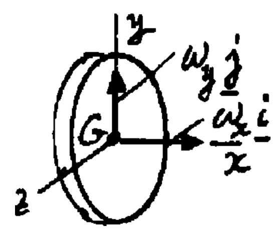


PROBLEM 18.97

A stationary horizontal plate is attached to the ceiling by means of a fixed vertical tube. A wheel of radius a and mass m is mounted on a light axle AC which is attached by means of a clevis at A to a rod AB fitted inside the vertical tube. The rod AB is made to rotate with a constant angular velocity $\mathbf{\Omega}$ causing the wheel to roll on the lower face of the stationary plate. Determine the minimum angular velocity $\mathbf{\Omega}$ for which contact is maintained between the wheel and the plate. Consider the particular cases (a) when the mass of the wheel is concentrated in the rim, (b) when the wheel is equivalent to a thin disk of radius a.

SOLUTION

Angular momentum $\mathbf{H}_G$ of wheel: $\omega_y = \Omega, \quad \omega_x = \dfrac{R}{a}\Omega, \quad \omega_z = 0$



$$\mathbf{H}_G = \bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j} + \bar{I}_z\omega_z\mathbf{k}$$

$$\mathbf{H}_G = \bar{I}_x\frac{R}{a}\Omega\mathbf{i} + \bar{I}_y\Omega\mathbf{j}$$

Rate of change of angular momentum:

Since ω_x and ω_y are constant, and observing that the frame $Gxyz$ rotates with the angular velocity $\mathbf{\Omega} = \Omega\mathbf{i}$:

$$(\dot{\mathbf{H}}_G)_{Gxyz} = 0$$

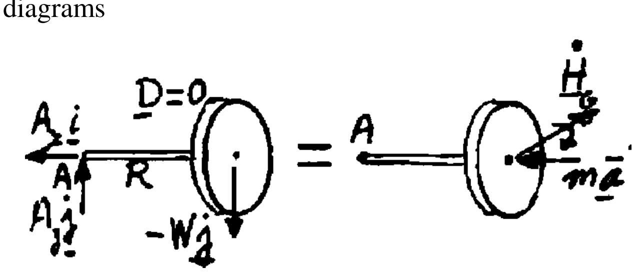
Eq. (18.22):

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \mathbf{\Omega}\times\mathbf{H}_G$$

$$\dot{\mathbf{H}}_G = 0 + \Omega\mathbf{j}\times\left(\bar{I}_x\frac{R}{a}a\mathbf{i} + \bar{I}_y\Omega\mathbf{j}\right)$$

$$\dot{\mathbf{H}}_G = -\bar{I}_z\frac{R}{a}\Omega^2\mathbf{k} \qquad (1)$$

The free body and kinetic diagrams



PROBLEM 18.97 (Continued)

Equating moments about A:

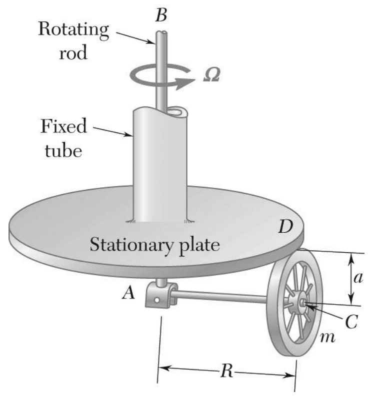
$$R\,\mathbf{i}\times(-W\,\mathbf{j}) = \dot{\mathbf{H}}_G$$

$$-Rmg\,\mathbf{k} = -\bar{I}_z\,\frac{R}{a}\Omega^2\mathbf{k}$$

$$\Omega = \sqrt{\frac{mga}{\bar{I}_x}}$$

(a) mass in rim: $\bar{I}_x = ma^2$ $\Omega = \sqrt{g/a}$ ◀

(b) thin disk: $\bar{I}_x = \frac{1}{2}ma^2$ $\Omega = \sqrt{2g/a}$ ◀



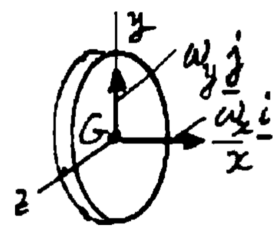
PROBLEM 18.98

Assuming that the wheel of Problem 18.97 weights 8 lb, has a radius $a = 4$ in. and a radius of gyration of 3 in., and that $R = 20$ in., determine the force exerted by the plate on the wheel when $\Omega = 25$ rad/s.

PROBLEM 18.97 A stationary horizontal plate is attached to the ceiling by means of a fixed vertical tube. A wheel of radius a and mass m is mounted on a light axle AC which is attached by means of a clevis at A to a rod AB fitted inside the vertical tube. The rod AB is made to rotate with a constant angular velocity $\mathbf{\Omega}$ causing the wheel to roll on the lower face of the stationary plate. Determine the minimum angular velocity $\mathbf{\Omega}$ for which contact is maintained between the wheel and the plate. Consider the particular cases (a) when the mass of the wheel is concentrated in the rim, (b) when the wheel is equivalent to a thin disk of radius a.

SOLUTION

<u>Angular momentum $\mathbf{H}_G$ of wheel</u>: $\omega_y = \Omega, \quad \omega_x = \frac{R}{a}\Omega, \quad \omega_z = 0$



$$\mathbf{H}_G = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k}$$

$$\mathbf{H}_G = \bar{I}_x \frac{R}{a}\Omega \mathbf{i} + \bar{I}_y \Omega \mathbf{j}$$

<u>Rate of change of angular momentum</u>:

Since ω_x and ω_y are constant, and observing that the frame $Gxyz$ rotates with the angular velocity $\mathbf{\Omega} = \Omega\mathbf{j}$:

$$(\dot{\mathbf{H}}_G)_{Gxyz} = 0$$

Eq. (18.22):

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \mathbf{\Omega} \times \mathbf{H}_G$$

$$\dot{\mathbf{H}}_G = 0 + \Omega\mathbf{j} \times \left(\bar{I}_x \frac{R}{a} a\mathbf{i} + \bar{I}_y \Omega \mathbf{j} \right)$$

$$\dot{\mathbf{H}}_G = -\bar{I}_z \frac{R}{a}\Omega^2 \mathbf{k} \qquad (1)$$

With

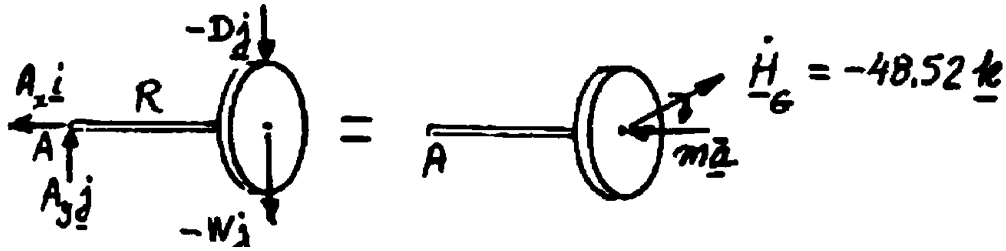
$$R = 20 \text{ in.}, \quad a = 4 \text{ in.}, \quad \Omega = 25 \text{ rad/s}, \quad m = \frac{8}{g}$$

and

$$\bar{I}_x = m\bar{k}_x^2 = \frac{8}{g}\left(\frac{3}{12}\text{ft}\right)^2 = 15.53 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$\dot{\mathbf{H}}_G = -15.53 \times 10^{-3} \left(\frac{20}{4}\right)(25)^2 \mathbf{k} = 48.52\mathbf{k}$$

PROBLEM 18.98 (Continued)



Taking moments about A:

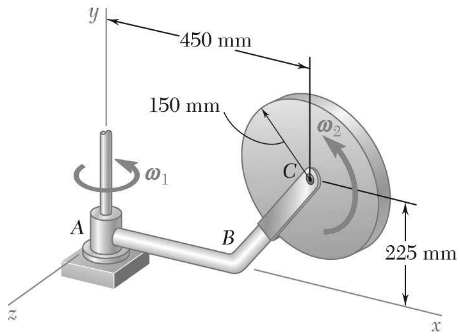
$$R\mathbf{i}\times(-D\mathbf{j}-W\mathbf{j})=\dot{\mathbf{H}}_G$$

$$\frac{20}{12}\mathbf{i}\times(-D\mathbf{j}-8\mathbf{j})=-48.52\mathbf{k}$$

$$-\frac{5}{3}(D+8)\mathbf{k}=-48.52\mathbf{k}$$

$$D=\frac{3}{5}(48.52)-8=21.1\text{ lb}$$

$\mathbf{D}=21.1\text{ lb}\downarrow$ ◀



PROBLEM 18.99

A thin disk of mass $m = 4$ kg rotates with an angular velocity ω_2 with respect to arm *ABC*, which itself rotates with an angular velocity ω_1 about the *y* axis. Knowing that $\omega_1 = 5$ rad/s and $\omega_2 = 15$ rad/s and that both are constant, determine the force-couple system representing the dynamic reaction at the support at *A*.

SOLUTION

Angular velocity of the disk. $\boldsymbol{\omega} = \omega_1 \mathbf{j} + \omega_2 \mathbf{k} = (5 \text{ rad/s})\mathbf{j} + (15 \text{ rad/s})\mathbf{k}$

Moments of inertia about principal axes passing through the mass center.

$$\bar{I}_{x'} = \bar{I}_{y'} = \frac{1}{4} mr^2$$

$$= \frac{1}{4}(4)(0.150 \text{ m})^2 = 0.0225 \text{ kg}\cdot\text{m}^2$$

$$\bar{I}_{z'} = \frac{1}{2} mr^2 = 0.045 \text{ kg}\cdot\text{m}^2$$

Angular momentum about mass center C.

$$\mathbf{H}_C = \bar{I}_{x'}\omega_{x'}\mathbf{i} + \bar{I}_{y'}\omega_{y'}\mathbf{j} + \bar{I}_{z'}\omega_{z'}\mathbf{k}$$

$$= 0 + (0.0225)5\mathbf{j} + (0.045)15\mathbf{k}$$

$$\mathbf{H}_C = (0.1125 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (0.6750 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

Rate of change of $\mathbf{H}_C$. Let the frame *Axyz* be turning with angular velocity $\boldsymbol{\Omega} = \omega_1 \mathbf{j}$.

$$\dot{\mathbf{H}}_C = (\dot{\mathbf{H}}_C)_{Axyz} + \boldsymbol{\Omega}\times\mathbf{H}_C = 0 + \boldsymbol{\Omega}\times\mathbf{H}_C$$

$$= 5\mathbf{j}\times(0.1125\mathbf{j} + 0.675\mathbf{k}) = (3.375 \text{ N}\cdot\text{m})\mathbf{i}$$

Position vector of Point C.

$$\mathbf{r}_{C/A} = (0.450 \text{ m})\mathbf{i} + (0.225 \text{ m})\mathbf{j}$$

Velocity of Point C, the mass center of the disk.

$$\mathbf{v}_C = \omega_1 \times \mathbf{r}_{C/A} = 5\mathbf{j}\times(0.45\mathbf{i} + 0.225\mathbf{j})$$

$$= -(2.25 \text{ m/s})\mathbf{k}$$

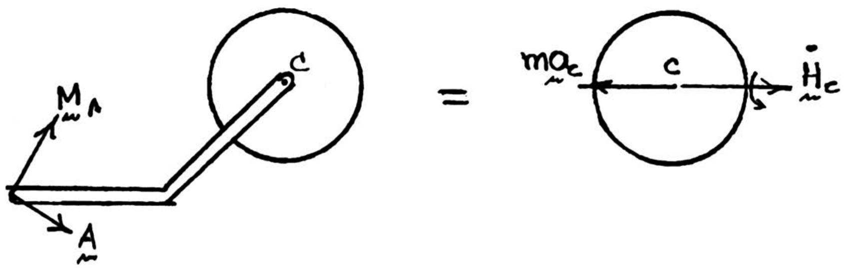
Acceleration of Point C.

$$\mathbf{a}_C = \alpha_1\mathbf{j}\times\mathbf{r}_{C/A} + \omega_1\mathbf{j}\times\mathbf{v}_C = 0 + 5\mathbf{j}\times(-2.25\mathbf{k}) = -(11.25 \text{ m/s}^2)\mathbf{i}$$

PROBLEM 18.99 (Continued)

$$m\mathbf{a}_C = (4)(-11.25\mathbf{i}) = -(45\text{ N})\mathbf{i}$$

Free body and kinetic diagrams

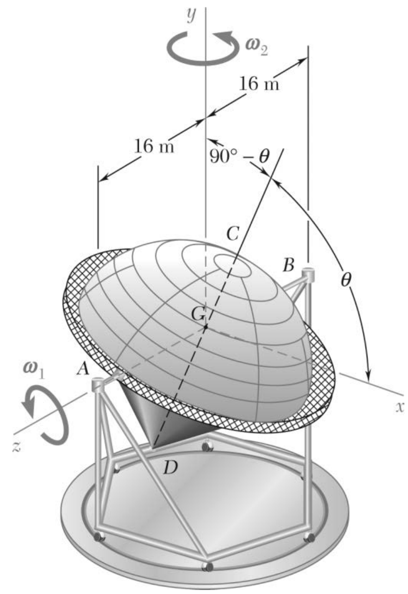


Linear components: $\mathbf{A} = m\mathbf{a}_C$ $\quad$ $\mathbf{A} = -(45\text{ N})\mathbf{i}$ ◀

<u>Moments about</u> A.

$$\mathbf{M}_A = \mathbf{r}_{C/A} \times m\mathbf{a}_C + \dot{\mathbf{H}}_C$$
$$\mathbf{M}_A = (0.450\mathbf{i} + 0.225\mathbf{j}) \times (-45\mathbf{i}) + 3.375\mathbf{i}$$

$$\mathbf{M}_A = (3.38\text{ N}\cdot\text{m})\mathbf{i} + (10.13\text{ N}\cdot\text{m})\mathbf{k}$$ ◀



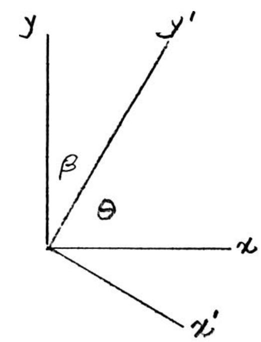
PROBLEM 18.100

An experimental Fresnel-lens solar-energy concentrator can rotate about the horizontal axis *AB*, which passes through its mass center *G*. It is supported at *A* and *B* by a steel framework, which can rotate about the vertical *y* axis. The concentrator has a mass of 30 Mg, a radius of gyration of 12 m about its axis of symmetry *CD*, and a radius of gyration of 10 m about any transverse axis through *G*. Knowing that the angular velocities ω_1 and ω_2 have constant magnitudes equal to 0.20 rad/s and 0.25 rad/s, respectively, determine for the position $\theta = 60°$ (*a*) the forces exerted on the concentrator at *A* and *B*, (*b*) the couple $M_2\mathbf{k}$ applied to the concentrator at that instant.

SOLUTION

Let the *y* axis be vertical and the y' axis be the symmetry axis.

Let the *z* axis be directed along axle *BA* as shown and the x' axis be the transverse axis perpendicular to *BA*.



Unit vectors.

$$\beta = 90° - \theta$$
$$\mathbf{i}' = \mathbf{i}\cos\beta - \mathbf{j}\sin\beta$$
$$\mathbf{j}' = \mathbf{i}\sin\beta + \mathbf{j}\cos\beta$$
$$\mathbf{i} = \mathbf{i}'\cos\beta + \mathbf{j}'\sin\beta$$
$$\mathbf{j} = -\mathbf{i}'\sin\beta + \mathbf{j}\cos\beta$$

Angular velocity.

$$\boldsymbol{\omega} = \omega_2\mathbf{j} + \omega_1\mathbf{k}$$
$$\omega = -(\omega_2\sin\beta)\mathbf{i}' + (\omega_2\cos\beta)\mathbf{j}' + \omega_1\mathbf{k}$$
$$= -\omega_2\cos\theta\mathbf{i} + \omega_2\sin\theta\mathbf{j} + \omega_1\mathbf{k}$$

$$\omega_{x'} = \omega_2\cos\theta \qquad \omega_{y'} = \omega_2\sin\theta \qquad \omega_z = \omega_1$$

$$\dot{\theta} = \omega_1 \qquad \omega_1 \text{ and } \omega_2 \text{ are constant.}$$
$$\dot{\omega}_{x'} = \omega_2\omega_1\sin\theta$$
$$\dot{\omega}_{y'} = \omega_2\omega_1\cos\theta, \quad \dot{\omega}_2 = 0$$

Radii of gyration:

$$k_{x'} = 10 \text{ m}$$
$$k_{y'} = 12 \text{ m}$$
$$k_z = 10 \text{ m}$$

Moments of inertia:

$$I = mk^2$$

PROBLEM 18.100 (Continued)

Angular momentum about Point G.

$$\mathbf{H}_G = I_{x'}\omega_{x'}\mathbf{i}' + I_y\omega_{y'}\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$\mathbf{H}_G = m[(10)^2\omega_{x'}\mathbf{i}' + (12)^2\omega_{y'}\mathbf{j}' + (10)^2\omega_z\mathbf{k}]$$

$$= m(100\omega_z\cos\theta\mathbf{i}' + 144\omega_2\sin\theta\mathbf{j}' + 100\omega_1\mathbf{k})$$

where m is the mass. $\quad m = 30{,}000\text{ kg}$

Acceleration of the mass center.

Since the mass center lies at the center of the axle BA, $\quad \bar{\mathbf{a}} = 0$

Rate of change of angular momentum.

Let the reference frame $G_{x'y'z}$ be turning with angular velocity $\mathbf{\Omega} = \boldsymbol{\omega}$

$$\dot{\mathbf{H}}_G = \dot{\mathbf{H}}_{Gx'y'z} + \mathbf{\Omega}\times\mathbf{H}_G$$

where

$$\dot{\mathbf{H}}_{Gx'y'z} = I_{x'}\dot{\omega}_{x'}\mathbf{i}' + I_{y'}\dot{\omega}_y\mathbf{j}' + I_z\dot{\omega}_z\mathbf{k}$$

$$= m[(10)^2\omega_2\omega_1\sin\theta\mathbf{i}' + (12)^2\omega_2\omega_1\cos\theta\mathbf{j}' + 0]$$

$$= m[(100)(0.25)(0.20)\sin 60°\mathbf{i}' + (144)(0.25)(0.20)\cos 60°\mathbf{j}']$$

$$= m(4.3301\mathbf{i}' + 3.6\mathbf{j}')$$

and

$$\mathbf{\Omega}\times\mathbf{H}_G = m\begin{vmatrix} \mathbf{i}' & \mathbf{j}' & \mathbf{k} \\ -\omega_2\cos\theta & \omega_2\sin\theta & \omega_1 \\ -100\omega_2\cos\theta & 144\omega_2\sin\theta & 100\omega_1 \end{vmatrix}$$

$$= m[-44\omega_1\omega_2\sin\theta\mathbf{i}' - 44\omega_2^2\sin\theta\cos\theta\mathbf{k}]$$

$$= m[-(44)(0.20)(0.25)\sin 60°\mathbf{i}'$$

$$+ (44)(0.25)^2\sin 60°\cos 60°\mathbf{k}]$$

$$= m(-1.9053\mathbf{i}' + 1.1908\mathbf{k})$$

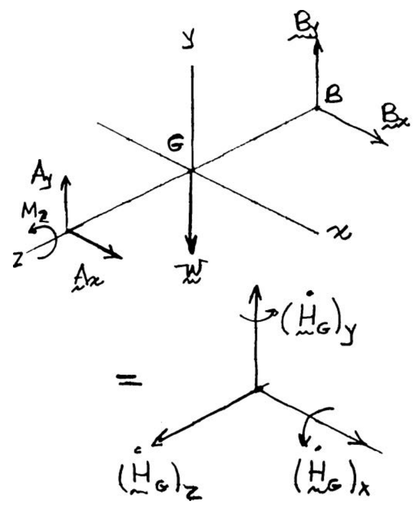
$$\dot{\mathbf{H}}_G = m[2.4248\mathbf{i}' + 3.6\mathbf{j}' - 1.1908\mathbf{k}]$$

$$= (30{,}000)[2.4248(\mathbf{i}\cos 30° - \mathbf{j}\sin 30°)$$

$$+ 3.6(\mathbf{i}\sin 30° + \mathbf{j}\cos 30°) - 1.1908\mathbf{k}]$$

$$= (117\times 10^3\text{ N}\cdot\text{m})\mathbf{i} + (57.158\times 10^3\text{ N}\cdot\text{m})\mathbf{j}$$

$$- (35.724\times 10^3\text{ N}\cdot\text{m})\mathbf{k}$$



Weight:

$$\mathbf{W} = -mg\mathbf{j} = -(30\times 10^3)(9.81)\mathbf{j}$$

$$= -(294.3\times 10^3\text{ N})\mathbf{j}$$

Equations of motion.

$$\Sigma\mathbf{M}_B = \Sigma(\mathbf{M}_B)_{\text{eff}}: \quad \mathbf{r}_{A/B}\times(A_x\mathbf{i} + B_y\mathbf{j}) + \mathbf{r}_{G/B}\times\mathbf{W}) + M_2\mathbf{k} = \dot{\mathbf{H}}_G$$

$$32\mathbf{k}\times(A_x\mathbf{i} + A_y\mathbf{j}) + 16\mathbf{k}\times(-294.3\times 10^3)\mathbf{j} + M_2\mathbf{k} = \dot{\mathbf{H}}_G$$

$$(-32A_y + 4.7088\times 10^6)\mathbf{i} + 32A_x\mathbf{j} + M_2\mathbf{k} = \dot{\mathbf{H}}_G$$

Equate like components:

$$\mathbf{i}: \quad -32A_y + 4.7088\times10^6 = 117\times10^3 \qquad A_y = 143.5\times10^3 \text{ N}$$

$$\mathbf{j}: \quad 32A_x = 57.158\times10^3 \qquad A_x = 1.786\times10^3 \text{ N}$$

$$\mathbf{k}: \quad M_2 = -35.724\times10^3$$

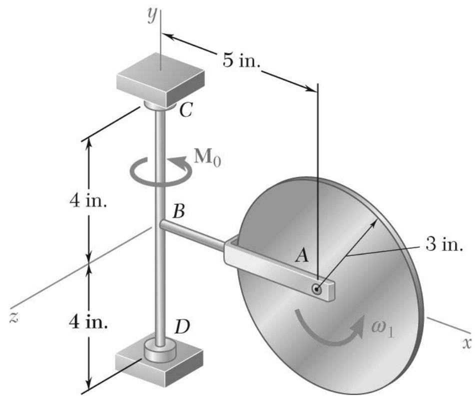
(*a*) Reduction at *A*. $\mathbf{A} = (1.786 \text{ kN})\mathbf{i} + (143.5 \text{ kN})\mathbf{j}$ ◀

$$\Sigma\mathbf{F} = \Sigma\mathbf{F}_{\text{eff}}: \quad \mathbf{B} + \mathbf{A} + \mathbf{W} = 0$$

$$\mathbf{B} = -\mathbf{W} - \mathbf{A} = 294.3\times10^3\mathbf{j} - \mathbf{A}$$

Reaction at *B*. $\mathbf{B} = -(1.786 \text{ kN})\mathbf{i} + (150.8 \text{ kN})\mathbf{j}$ ◀

(*b*) Couple $M_2\mathbf{k}$: $M_2\mathbf{k} = -(35.7 \text{ kN}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 18.101

A 6-lb homogeneous disk of radius 3 in. spins as shown at the constant rate $\omega_1 = 60$ rad/s. The disk is supported by the fork-ended rod *AB*, which is welded to the vertical shaft *CBD*. The system is at rest when a couple $\mathbf{M}_0 = (0.25 \text{ ft}\cdot\text{lb})\mathbf{j}$ is applied to the shaft for 2 s and then removed. Determine the dynamic reactions at *C* and *D* after the couple has been removed.

SOLUTION

Angular velocity of shaft *CBD* and arm *AB*: $\boldsymbol{\Omega} = \omega_2 \mathbf{j}$

Angular velocity of disk *A*: $\boldsymbol{\omega} = \omega_2 \mathbf{j} + \omega_1 \mathbf{k}$

Its angular momentum about *A*: $\mathbf{H}_A = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k} = \bar{I}_y \omega_2 \mathbf{j} + \bar{I}_z \omega_1 \mathbf{k}$

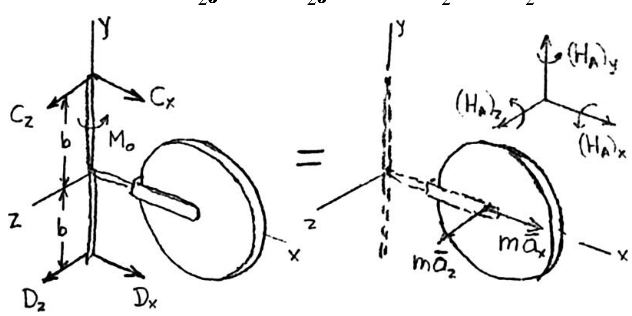
Let the reference frame *Bxyz* be rotating with angular velocity $\boldsymbol{\Omega}$.

$$\begin{aligned}\dot{\mathbf{H}}_A &= (\dot{\mathbf{H}}_A)_{Axyz} + \boldsymbol{\Omega} \times \mathbf{H}_A \\ &= \bar{I}_y \dot{\omega}_2 \mathbf{j} + \bar{I}_z \dot{\omega}_1 \mathbf{k} + \omega_2 \mathbf{j} \times (\bar{I}_y \omega_2 \mathbf{j} + \bar{I}_z \omega_1 \mathbf{k}) \\ &= \bar{I}_z \omega_1 \omega_2 \mathbf{i} + \bar{I}_y \dot{\omega}_2 \mathbf{j} + \bar{I}_z \dot{\omega}_1 \mathbf{k} \\ &= \frac{1}{2} m r^2 \omega_1 \omega_2 \mathbf{i} + \frac{1}{4} m r^2 \dot{\omega}_2 \mathbf{j} + \frac{1}{2} m r^2 \dot{\omega}_1 \mathbf{k}\end{aligned}$$

Velocity and acceleration of the mass center *A* of the disk:

$$\bar{\mathbf{v}} = \omega_2 \mathbf{j} \times c\mathbf{i} = -c\omega_2 \mathbf{k},$$

$$\bar{\mathbf{a}} = \dot{\omega}_2 \mathbf{j} \times c\mathbf{i} + \omega_2 \mathbf{j} \times \bar{\mathbf{v}} = -c\dot{\omega}_2 \mathbf{k} - c\omega_2^2 \mathbf{i}$$



$$\Sigma \mathbf{F} = \Sigma \mathbf{F}_{\text{eff}} = m\bar{\mathbf{a}}$$

$$C_x \mathbf{i} + C_y \mathbf{k} + D_x \mathbf{i} + D_z \mathbf{k} = m\bar{\mathbf{a}}$$

Resolve into components.

$$C_x + D_x = -mc\omega_2^2$$

$$C_z + D_z = -mc\dot{\omega}_2$$

PROBLEM 18.101 (Continued)

$$\Sigma \mathbf{M}_D = \dot{\mathbf{H}}_D = \dot{\mathbf{H}}_A + \mathbf{r}_{A/D} \times m\bar{\mathbf{a}}$$

$$= \dot{\mathbf{H}}_A + (c\mathbf{i} + b\mathbf{j}) \times (-c\dot{\omega}_2\mathbf{k} - c\omega_2^2\mathbf{i})$$

$$M_0\mathbf{j} + 2b\mathbf{j} \times (C_x\mathbf{i} + C_z\mathbf{k}) = \dot{\mathbf{H}}_A - mbc\dot{\omega}_2\mathbf{i} + mc^2\dot{\omega}_2\mathbf{j} + mbc\omega_2^2\mathbf{k}$$

$$2bC_z\mathbf{i} + M_0\mathbf{j} - 2bC_x\mathbf{k} = m\left(\frac{1}{2}r^2\omega_1\omega_2 - bc\dot{\omega}_2\right)\mathbf{i} + m\left(\frac{1}{4}r^2 + c^2\right)\dot{\omega}_2\mathbf{j} + m\left(\frac{1}{2}r^2\dot{\omega}_1 + bc\omega_2^2\right)\mathbf{k}$$

$$\mathbf{j}: \quad M_0 = m\left(\frac{1}{4}r^2 + c^2\right)\dot{\omega}_2 \qquad (1)$$

$$\mathbf{k}: \quad C_x = -\frac{m}{2b}\left(\frac{1}{2}r^2\dot{\omega}_1 + bc\omega_2^2\right) \quad D_x = -\frac{m}{2b}\left(-\frac{1}{2}r^2\dot{\omega}_1 + bc\omega_2^2\right) \qquad (2)$$

$$\mathbf{i}: \quad C_z = \frac{m}{2b}\left(\frac{1}{2}r^2\omega_1\omega_2 - bc\dot{\omega}_2\right) \quad D_z = -\frac{m}{2b}\left(\frac{1}{2}r^2\omega_1\omega_2 + bc\dot{\omega}_2\right) \qquad (3)$$

Data: $W = 6$ lb, $r = 3$ in. $= 0.25$ ft, $b = 4$ in. $= 0.33333$ ft, $c = 5$ in. $= 0.41667$ ft, $\omega_1 = 60$ rad/s, $\dot{\omega}_1 = 0$

While the couple is applied, $M_0 = 0.25$ ft·lb

Rearranging Equation (1)

$$\dot{\omega}_2 = \frac{M_0}{m\left(\frac{1}{4}r^2 + c^2\right)} = \frac{0.25}{\left(\frac{6}{32.2}\right)\left[\frac{1}{4}(0.25)^2 + (0.41667)^2\right]} = 7.0899 \text{ rad/s}^2$$

At $t = 2$ s, $\omega_2 = (\omega_2)_0 + \dot{\omega}_2 t = 0 + (7.0899)(2) = 14.18$ rad/s

For $t > 2$ s, $M_0 = 0, \quad \dot{\omega}_2 = 0$

From Equations (2), (3)

$$C_x = \frac{\frac{6}{32.2}}{(2)(0.33333)}[0 + (0.33333)(0.41667)(14.1798)^2] = -7.8054 \text{ lb}$$

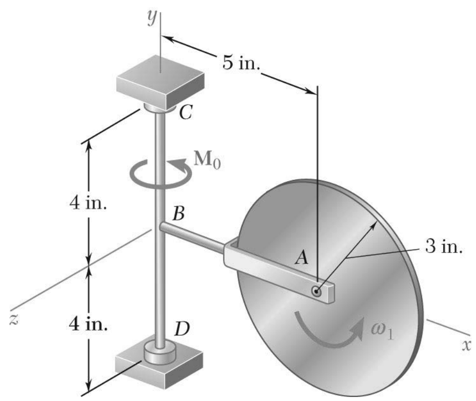
$$D_x = \frac{\frac{6}{32.2}}{(2)(0.33333)}[0 + (0.33333)(0.41667)(14.1798)^2] = -7.8054 \text{ lb}$$

$$C_z = \frac{\frac{6}{32.2}}{(2)(0.33333)}\left[\frac{1}{2}(0.25)^2(60)(14.1798) - 0\right] = 7.4312 \text{ lb}$$

$$D_z = \frac{\frac{6}{32.2}}{(2)(0.33333)}\left[-\frac{1}{2}(0.25)^2(60)(14.1798) - 0\right] = -7.4312 \text{ lb}$$

$\mathbf{C} = -(7.81 \text{ lb})\mathbf{i} + (7.43 \text{ lb})\mathbf{k}$ ◀

$\mathbf{D} = -(7.81 \text{ lb})\mathbf{i} - (7.43 \text{ lb})\mathbf{k}$ ◀



PROBLEM 18.102

A 6-lb homogeneous disk of radius 3 in. spins as shown at the constant rate $\omega_1 = 60$ rad/s. The disk is supported by the fork-ended rod *AB*, which is welded to the vertical shaft *CBD*. The system is at rest when a couple $\mathbf{M}_0$ is applied as shown to the shaft for 3 s and then removed. Knowing that the maximum angular velocity reached by the shaft is 18 rad/s, determine (*a*) the couple $\mathbf{M}_0$, (*b*) the dynamic reactions at *C* and *D* after the couple has been removed.

SOLUTION

Angular velocity of shaft *CBD* and arm *AB*: $\boldsymbol{\Omega} = \omega_2 \mathbf{j}$

Angular velocity of disk *A*: $\boldsymbol{\omega} = \omega_2 \mathbf{j} + \omega_1 \mathbf{k}$

Its angular momentum about *A*: $\mathbf{H}_A = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k} = \bar{I}_y \omega_2 \mathbf{j} + \bar{I}_z \omega_1 \mathbf{k}$

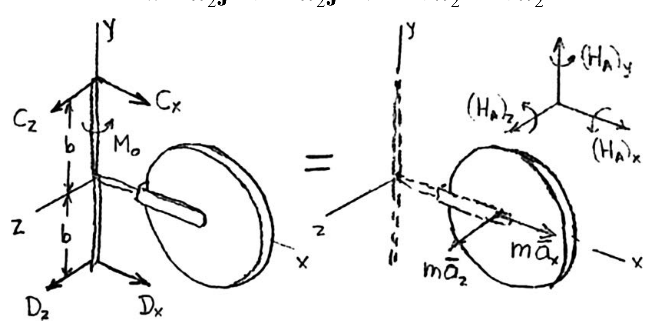
Let the reference frame *Bxyz* be rotating with angular velocity $\boldsymbol{\Omega}$.

$$\begin{aligned}
\dot{\mathbf{H}}_A &= (\dot{\mathbf{H}}_A)_{Axyz} + \boldsymbol{\Omega} \times \mathbf{H}_A \\
&= \bar{I}_y \dot{\omega}_2 \mathbf{j} + \bar{I}_z \dot{\omega}_1 \mathbf{k} + \omega_2 \mathbf{j} \times (\bar{I}_y \omega_2 \mathbf{j} + \bar{I}_z \omega_1 \mathbf{k}) \\
&= \bar{I}_z \omega_1 \omega_2 \mathbf{i} + \bar{I}_y \dot{\omega}_2 \mathbf{j} + \bar{I}_z \dot{\omega}_1 \mathbf{k} \\
&= \frac{1}{2} m r^2 \omega_1 \omega_2 \mathbf{i} + \frac{1}{4} m r^2 \dot{\omega}_2 \mathbf{j} + \frac{1}{2} m r^2 \dot{\omega}_1 \mathbf{k}
\end{aligned}$$

Velocity and acceleration of the mass center *A* of the disk:

$$\bar{\mathbf{v}} = \omega_2 \mathbf{j} \times c\mathbf{i} = -c\omega_2 \mathbf{k},$$

$$\bar{\mathbf{a}} = \dot{\omega}_2 \mathbf{j} \times c\mathbf{i} + \omega_2 \mathbf{j} \times \bar{\mathbf{v}} = -c\dot{\omega}_2 \mathbf{k} - c\omega_2^2 \mathbf{i}$$



$$\Sigma \mathbf{F} = \Sigma \mathbf{F}_{\text{eff}} = m\bar{\mathbf{a}}$$

$$C_x \mathbf{i} + C_y \mathbf{k} + D_x \mathbf{i} + D_z \mathbf{k} = m\bar{\mathbf{a}}$$

Resolve into components.

$$C_x + D_x = -mc\omega_2^2$$

$$C_z + D_z = -mc\dot{\omega}_2$$

$$\Sigma \mathbf{M}_D = \dot{\mathbf{H}}_D = \dot{\mathbf{H}}_A + \mathbf{r}_{A/D} \times m\bar{\mathbf{a}}$$
$$= \dot{\mathbf{H}}_A + (c\mathbf{i} + b\mathbf{j}) \times (-c\dot{\omega}_2\mathbf{k} - c\omega_2^2\mathbf{i})$$
$$M_0\mathbf{j} + 2b\mathbf{j} \times (C_x\mathbf{i} + C_z\mathbf{k}) = \dot{\mathbf{H}}_A - mbc\dot{\omega}_2\mathbf{i} + mc^2\dot{\omega}_2\mathbf{j} + mbc\omega_2^2\mathbf{k}$$
$$2bC_z\mathbf{i} + M_0\mathbf{j} - 2bC_x\mathbf{k} = m\left(\frac{1}{2}r^2\omega_1\omega_2 - bc\dot{\omega}_2\right)\mathbf{i} + m\left(\frac{1}{4}r^2 + c^2\right)\dot{\omega}_2\mathbf{j} + m\left(\frac{1}{2}r^2\dot{\omega}_1 + bc\omega_2^2\right)\mathbf{k}$$

$$\mathbf{j}: \quad M_0 = m\left(\frac{1}{4}r^2 + c^2\right)\dot{\omega}_2 \qquad (1)$$

$$\mathbf{k}: \quad C_x = -\frac{m}{2b}\left(\frac{1}{2}r^2\dot{\omega}_1 + bc\omega_2^2\right) \quad D_x = -\frac{m}{2b}\left(-\frac{1}{2}r^2\dot{\omega}_1 + bc\omega_2^2\right) \qquad (2)$$

$$\mathbf{i}: \quad C_z = \frac{m}{2b}\left(\frac{1}{2}r^2\omega_1\omega_2 - bc\dot{\omega}_2\right) \quad D_z = -\frac{m}{2b}\left(\frac{1}{2}r^2\omega_1\omega_2 + bc\dot{\omega}_2\right) \qquad (3)$$

<u>Data</u>: $W = 6$ lb, $r = 3$ in. $= 0.25$ ft, $b = 4$ in. $= 0.33333$ ft,
$c = 5$ in. $= 0.41667$ ft, $\omega_1 = 60$ rad/s, $\dot{\omega}_1 = 0$

(*a*) While the couple is applied $\dot{\omega}_2 = \dfrac{\omega_2}{t} = \dfrac{18}{3} = 6 \text{ rad/s}^2$

From Equation (1) $M_0 = m\left(\frac{1}{4}r^2 + c^2\right)\dot{\omega}_2$

$$= \left(\frac{6}{32.2}\right)\left[\frac{1}{4}(0.25)^2 + (0.41667)^2\right](6)$$

$M_0 = (0.212 \text{ ft}\cdot\text{lb})\mathbf{j}$ ◀

(*b*) For $t > 3$ s, $M_0 = 0, \quad \dot{\omega}_2 = 0$

From Equations (2), (3)

$$C_x = \frac{\frac{6}{32.2}}{(2)(0.33333)}[0 + (0.33333)(0.41667)(18)^2] = -12.578 \text{ lb}$$

$$D_x = \frac{\frac{6}{32.2}}{(2)(0.33333)}[0 + (0.33333)(0.41667)(18)^2] = -12.578 \text{ lb}$$

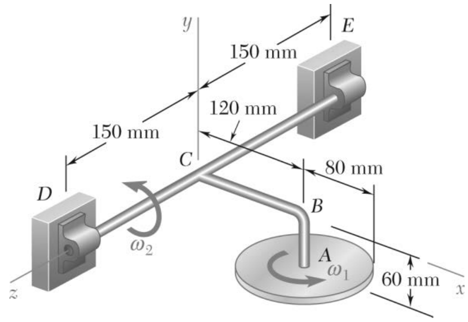
$$C_z = \frac{\frac{6}{32.2}}{(2)(0.33333)}\left[\frac{1}{2}(0.25)^2(60)(18) - 0\right] = 9.4332 \text{ lb}$$

$$D_z = \frac{\frac{6}{32.2}}{(2)(0.33333)}\left[-\frac{1}{2}(0.25)^2(60)(18) - 0\right] = -9.4332 \text{ lb}$$

$\mathbf{C} = -(12.58 \text{ lb})\mathbf{i} + (9.43 \text{ lb})\mathbf{k}$ ◀

$\mathbf{D} = -(12.58 \text{ lb})\mathbf{i} - (9.43 \text{ lb})\mathbf{k}$ ◀

PROBLEM 18.103



A 2.5 kg homogeneous disk of radius 80 mm rotates with an angular velocity $\boldsymbol{\omega}_1$ with respect to arm *ABC*, which is welded to a shaft *DCE* rotating as shown at the constant rate $\omega_2 = 12$ rad/s. Friction in the bearing at *A* causes ω_1 to decrease at the rate of 15 rad/s^2. Determine the dynamic reactions at *D* and *E* at a time when ω_1 has decreased to 50 rad/s.

SOLUTION

Angular velocity of shaft *DCE* and arm *CBA*: $\boldsymbol{\Omega} = \omega_2 \mathbf{k}$

Angular velocity of disk *A*: $\boldsymbol{\omega} = \omega_1 \mathbf{j} + \omega_2 \mathbf{k}$

Its angular momentum about *A*: $\mathbf{H}_A = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k} = \bar{I}_y \omega_1 \mathbf{j} + \bar{I}_z \omega_2 \mathbf{k}$

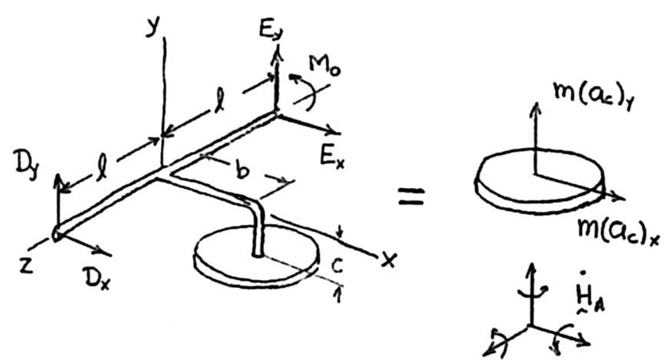
Let the reference frame *Cxyz* be rotating with angular velocity $\boldsymbol{\Omega}$.

$$\begin{aligned}
\dot{\mathbf{H}}_A &= (\dot{\mathbf{H}}_A)_{Axyz} + \boldsymbol{\Omega} \times \mathbf{H}_A \\
&= \bar{I}_y \dot{\omega}_1 \mathbf{j} + \bar{I}_z \dot{\omega}_2 \mathbf{k} + \omega_2 \mathbf{k} \times (\bar{I}_y \omega_1 \mathbf{j} + \bar{I}_z \omega_2 \mathbf{k}) \\
&= -\bar{I}_y \omega_2 \omega_1 \mathbf{i} + \bar{I}_y \dot{\omega}_1 \mathbf{j} + \bar{I}_z \dot{\omega}_2 \mathbf{k} \\
&= -\frac{1}{2} m r^2 \omega_2 \omega_1 \mathbf{i} + \frac{1}{2} m r^2 \dot{\omega}_1 \mathbf{j} + \frac{1}{4} m r^2 \dot{\omega}_2 \mathbf{k}
\end{aligned}$$

Velocity and acceleration of the mass center *A* of the disk:

$$\mathbf{v}_A = \boldsymbol{\omega}_2 \mathbf{k} \times \mathbf{r}_{A/C} = \omega_2 \mathbf{k} \times (b\mathbf{i} - c\mathbf{j}) = c\omega_2 \mathbf{i} + b\omega_2 \mathbf{j}$$

$$\mathbf{a}_A = \dot{\omega}_2 \mathbf{k} \times \mathbf{r}_{A/C} + \omega_2 \mathbf{k} \times \mathbf{v}_A = (c\dot{\omega}_2 - b\omega_2^2)\mathbf{i} + (b\dot{\omega}_2 + c\omega_2^2)\mathbf{j}$$



$$\Sigma \mathbf{F} = \Sigma \mathbf{F}_{\text{eff}}$$

$$D_x \mathbf{i} + D_y \mathbf{j} + E_x \mathbf{i} + E_y \mathbf{j} = m\mathbf{a}_A$$

Resolve into components.

$$D_x + E_x = m(c\dot{\omega}_2 - b\omega_2^2)$$
$$D_y + E_y = m(b\dot{\omega}_2 + c\omega_2^2)$$
$$\Sigma \mathbf{M}_E = \dot{\mathbf{H}}_E = \dot{\mathbf{H}}_A + \mathbf{r}_{A/E} \times m\mathbf{a}_A = \dot{\mathbf{H}}_A + (b\mathbf{i} - c\mathbf{j} + l\mathbf{k}) \times m\mathbf{a}_A$$
$$2l\mathbf{k} \times (D_x\mathbf{i} + D_y\mathbf{j}) + M_0\mathbf{k} = \dot{\mathbf{H}}_A + m(bl\dot{\omega}_2 - cl\omega_2^2)\mathbf{i} + m(cl\dot{\omega}_2 + bl\omega_2^2)\mathbf{j} + m(b^2 + c^2)\dot{\omega}_2\mathbf{k}$$
$$-2lD_y\mathbf{i} + 2lD_x\mathbf{j} + M_0\mathbf{k} = m\left(-\frac{1}{2}r^2\omega_2\omega_1 + bl\dot{\omega}_2 - cl\omega_2^2\right)\mathbf{i}$$
$$+ m\left(\frac{1}{2}r^2\dot{\omega}_1 + cl\dot{\omega}_2 - bl\omega_2^2\right)\mathbf{j} + m\left(\frac{1}{4}r^2 + b^2 + c^2\right)\dot{\omega}_2\mathbf{k}$$

$$\mathbf{k}: \quad M_0 = m\left(\frac{1}{4}r^2 + b^2 + c^2\right)\dot{\omega}_2$$
$$\mathbf{j}: \quad D_x = \frac{m}{2l}\left(\frac{1}{2}r^2\dot{\omega}_1 + cl\dot{\omega}_2 - bl\omega_2^2\right) \quad E_x = \frac{m}{2l}\left(-\frac{1}{2}r^2\dot{\omega}_1 + cl\dot{\omega}_2 - bl\omega_2^2\right)$$
$$\mathbf{i}: \quad D_y = \frac{m}{2l}\left(\frac{1}{2}r^2\omega_2\omega_1 + bl\dot{\omega}_2 + cl\omega_2^2\right) \quad E_y = \frac{m}{2l}\left(-\frac{1}{2}r^2\omega_2\omega_1 + bl\dot{\omega}_2 + cl\omega_2^2\right)$$

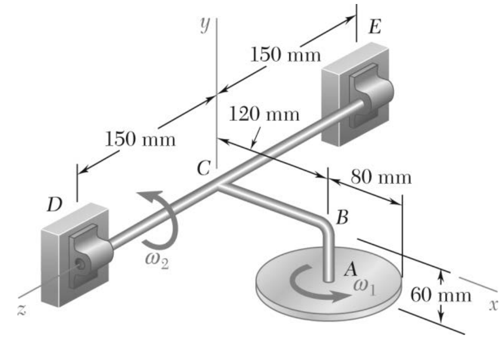
Data: $m = 2.5$ kg, $r = 80$ mm $= 0.08$ m
$b = 120$ mm $= 0.12$ m, $c = 60$ mm $= 0.06$ m, $l = 150$ mm $= 0.15$ m
$\omega_1 = 50$ rad/s, $\dot{\omega}_1 = -15$ rad/s^2, $\omega_2 = 12$ rad/s, $\dot{\omega}_2 = 0$

$$D_x = \frac{2.5}{(2)(0.15)}\left[\frac{1}{2}(0.08)^2(-15) + 0 - (0.12)(0.15)(12)^2\right] = -22.0 \text{ N}$$
$$D_y = \frac{2.5}{(2)(0.15)}\left[\frac{1}{2}(0.08)^2(12)(50) + 0 + (0.06)(0.15)(12)^2\right] = 26.8 \text{ N}$$

$$\mathbf{D} = -(22.0 \text{ N})\mathbf{i} + (26.8 \text{ N})\mathbf{j} \blacktriangleleft$$

$$E_x = \frac{2.5}{(2)(0.15)}\left[-\frac{1}{2}(0.08)^2(-15) + 0 - (0.12)(0.15)(12)^2\right] = -21.2 \text{ N}$$
$$E_y = \frac{2.5}{(2)(0.15)}\left[-\frac{1}{2}(0.08)^2(12)(50) + 0 + (0.06)(0.15)(12)^2\right] = -5.20 \text{ N}$$

$$\mathbf{E} = -(21.2 \text{ N})\mathbf{i} - (5.20 \text{ N})\mathbf{j} \blacktriangleleft$$



PROBLEM 18.104

A 2.5-kg homogeneous disk of radius 80 mm rotates at the constant rate $\omega_1 = 50$ rad/s with respect to arm *ABC*, which is welded to a shaft *DCE*. Knowing that at the instant shown shaft *DCE* has an angular velocity $\omega_2 = (12 \text{ rad/s})\mathbf{k}$ and an angular acceleration $\alpha_2 = (8 \text{ rad/s}^2)\mathbf{k}$, determine (*a*) the couple which must be applied to shaft *DCE* to produce that acceleration, (*b*) the corresponding dynamic reactions at *D* and *E*.

SOLUTION

Angular velocity of shaft *DCE* and arm *CBA*: $\mathbf{\Omega} = \omega_2 \mathbf{k}$

Angular velocity of disk *A*: $\boldsymbol{\omega} = \omega_1 \mathbf{j} + \omega_2 \mathbf{k}$

Its angular momentum about *A*: $\mathbf{H}_A = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k} = \bar{I}_y \omega_1 \mathbf{j} + \bar{I}_z \omega_2 \mathbf{k}$

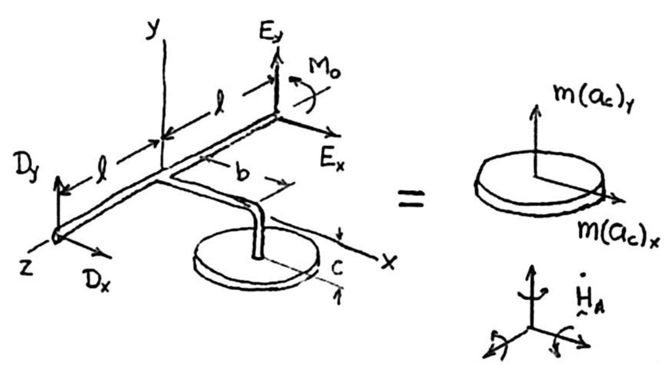
Let the reference frame *Cxyz* be rotating with angular velocity $\mathbf{\Omega}$.

$$\begin{aligned}
\dot{\mathbf{H}}_A &= (\dot{\mathbf{H}}_A)_{Axyz} + \mathbf{\Omega} \times \mathbf{H}_A \\
&= \bar{I}_y \dot{\omega}_1 \mathbf{j} + \bar{I}_z \dot{\omega}_2 \mathbf{k} + \omega_2 \mathbf{k} \times (\bar{I}_y \omega_1 \mathbf{j} + \bar{I}_z \omega_2 \mathbf{k}) \\
&= -\bar{I}_y \omega_2 \omega_1 \mathbf{i} + \bar{I}_y \dot{\omega}_1 \mathbf{j} + \bar{I}_z \dot{\omega}_2 \mathbf{k} \\
&= -\frac{1}{2} m r^2 \omega_2 \omega_1 \mathbf{i} + \frac{1}{2} m r^2 \dot{\omega}_1 \mathbf{j} + \frac{1}{4} m r^2 \dot{\omega}_2 \mathbf{k}
\end{aligned}$$

Velocity and acceleration of the mass center *A* of the disk:

$$\mathbf{v}_A = \boldsymbol{\omega}_2 \mathbf{k} \times \mathbf{r}_{A/C} = \omega_2 \mathbf{k} \times (b\mathbf{i} - c\mathbf{j}) = c\omega_2 \mathbf{i} + b\omega_2 \mathbf{j}$$

$$\mathbf{a}_A = \dot{\omega}_2 \mathbf{k} \times \mathbf{r}_{A/C} + \omega_2 \mathbf{k} \times \mathbf{v}_A = (c\dot{\omega}_2 - b\omega_2^2)\mathbf{i} + (b\dot{\omega}_2 + c\omega_2^2)\mathbf{j}$$



$$\Sigma \mathbf{F} = \Sigma \mathbf{F}_{\text{eff}}$$

$$D_x \mathbf{i} + D_y \mathbf{j} + E_x \mathbf{i} + E_y \mathbf{j} = m\mathbf{a}_A$$

Resolve into components.

$$D_x + E_x = m(c\dot{\omega}_2 - b\omega_2^2)$$
$$D_y + E_y = m(b\dot{\omega}_2 + c\omega_2^2)$$
$$\Sigma\mathbf{M}_E = \dot{\mathbf{H}}_E = \dot{\mathbf{H}}_A + \mathbf{r}_{A/E} \times m\mathbf{a}_A = \dot{\mathbf{H}}_A + (b\mathbf{i} - c\mathbf{j} + l\mathbf{k}) \times m\mathbf{a}_A$$
$$2l\mathbf{k} \times (D_x\mathbf{i} + D_y\mathbf{j}) + M_0\mathbf{k} = \dot{\mathbf{H}}_A + m(bl\dot{\omega}_2 - cl\omega_2^2)\mathbf{i} + m(cl\dot{\omega}_2 + bl\omega_2^2)\mathbf{j} + m(b^2 + c^2)\dot{\omega}_2\mathbf{k}$$
$$-2lD_y\mathbf{i} + 2lD_x\mathbf{j} + M_0\mathbf{k} = m\left(-\frac{1}{2}r^2\omega_2\omega_1 + bl\dot{\omega}_2 - cl\omega_2^2\right)\mathbf{i}$$
$$+ m\left(\frac{1}{2}r^2\dot{\omega}_1 + cl\dot{\omega}_2 - bl\omega_2^2\right)\mathbf{j} + m\left(\frac{1}{4}r^2 + b^2 + c^2\right)\dot{\omega}_2\mathbf{k}$$

$$\mathbf{k}: \quad M_0 = m\left(\frac{1}{4}r^2 + b^2 + c^2\right)\dot{\omega}_2$$
$$\mathbf{j}: \quad D_x = \frac{m}{2l}\left(\frac{1}{2}r^2\dot{\omega}_1 + cl\dot{\omega}_2 - bl\omega_2^2\right) \quad E_x = \frac{m}{2l}\left(-\frac{1}{2}r^2\dot{\omega}_1 + cl\dot{\omega}_2 - bl\omega_2^2\right)$$
$$\mathbf{i}: \quad D_y = \frac{m}{2l}\left(\frac{1}{2}r^2\omega_2\omega_1 + bl\dot{\omega}_2 + cl\omega_2^2\right) \quad E_y = \frac{m}{2l}\left(-\frac{1}{2}r^2\omega_2\omega_1 + bl\dot{\omega}_2 + cl\omega_2^2\right)$$

Data: $m = 2.5 \text{ kg}, \quad r = 80 \text{ mm} = 0.08 \text{ m}$

$b = 120 \text{ mm} = 0.12 \text{ m}, \quad c = 60 \text{ mm} = 0.06 \text{ m}, \quad l = 150 \text{ mm} = 0.15 \text{ m}$

$\omega_1 = 50 \text{ rad/s}, \quad \dot{\omega}_1 = 0, \quad \omega_2 = 12 \text{ rad/s}, \quad \dot{\omega}_2 = \alpha_2 = 8 \text{ rad/s}^2$

(*a*)
$$M_0 = (2.5)\left[\frac{1}{4}(0.08)^2 + (0.12)^2 + (0.06)^2\right](8)$$

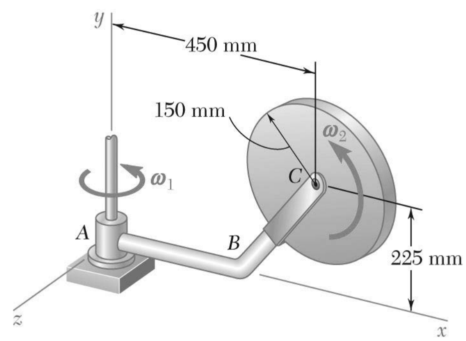
$\mathbf{M}_0 = (0.392 \text{ N}\cdot\text{m})\mathbf{k}$ ◀

(*b*)
$$D_x = \frac{2.5}{(2)(0.15)}[0 + (0.06)(0.15)(8) - (0.12)(0.15)(12)^2] = -21.0 \text{ N}$$
$$D_y = \frac{2.5}{(2)(0.15)}\left[\frac{1}{2}(0.08)^2(12)(50) + (0.12)(0.15)(8) + (0.06)(0.15)(12)^2\right] = 28.0 \text{ N}$$

$\mathbf{D} = -(21.0 \text{ N})\mathbf{i} + (28.0 \text{ N})\mathbf{j}$ ◀

$$E_x = \frac{2.5}{(2)(0.15)}[-0 + (0.06)(0.15)(8) - (0.12)(0.15)(12)^2] = -21.0 \text{ N}$$
$$E_y = \frac{2.5}{(2)(0.15)}\left[-\frac{1}{2}(0.08)^2(12)(50) + (0.12)(0.15)(8) + (0.06)(0.15)(12)^2\right]$$
$$= -4.00 \text{ N}$$

$\mathbf{E} = -(21.0 \text{ N})\mathbf{i} - (4.00 \text{ N})\mathbf{j}$ ◀



PROBLEM 18.105

For the disk of Problem 18.99, determine (*a*) the couple $M_1\mathbf{j}$ which should be applied to arm *ABC* to give it an angular acceleration $\alpha_1 = -(7.5 \text{ rad/s}^2)\mathbf{j}$ when $\omega_1 = 5$ rad/s, knowing that the disk rotates at the constant rate $\omega_2 = 15$ rad/s, (*b*) the force-couple system representing the dynamic reaction at *A* at that instant. Assume that *ABC* has a negligible mass.

PROBLEM 18.99 A thin disk of mass $m = 4$ kg rotates with an angular velocity ω_2 with respect to arm *ABC*, which itself rotates with an angular velocity ω_1 about the *y* axis. Knowing that $\omega_1 = 5$ rad/s and $\omega_2 = 15$ rad/s and that both are constant, determine the force-couple system representing the dynamic reaction at the support at *A*.

SOLUTION

Angular velocity of the disk. $\boldsymbol{\omega} = \omega_1\mathbf{j} + \omega_2\mathbf{k} = (5 \text{ rad/s})\mathbf{j} + (15 \text{ rad/s})\mathbf{k}$

Moments of inertia about principal axes passing through the mass center.

$$\bar{I}_x = \bar{I}_y = \frac{1}{4}mr^2$$
$$= \frac{1}{4}(4)(0.150 \text{ m})^2 = 0.0225 \text{ kg}\cdot\text{m}^2$$
$$\bar{I}_z = \frac{1}{2}mr^2 = 0.045 \text{ kg}\cdot\text{m}^2$$

Angular momentum about mass center C.

$$\mathbf{H}_C = \bar{I}_{x'}\omega_{x'}\mathbf{i} + \bar{I}_{y'}\omega_{y'}\mathbf{j} + I_{z'}\omega_{z'}\mathbf{k}$$
$$= 0 + (0.0225)5\mathbf{j} + (0.045)15\mathbf{k}$$
$$= (0.1125 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (0.6750 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

Rate of change of $\mathbf{H}_C$. Let the frame *Axyz* be turning with angular velocity $\boldsymbol{\Omega} = \omega_1\mathbf{j}$.

$$\dot{\mathbf{H}}_C = (\dot{\mathbf{H}}_C)_{Axyz} + \boldsymbol{\Omega}\times\mathbf{H}_C = \bar{I}_x\dot{\omega}_x\mathbf{i} + \bar{I}_y\dot{\omega}_y\mathbf{j} + \bar{I}_z\dot{\omega}_z\mathbf{k} + \boldsymbol{\Omega}\times\mathbf{H}_C$$
$$= 0 + (0.0225)(-7.5)\mathbf{j} + 0 + 5\mathbf{j}\times(0.1125\mathbf{j} + 0.675\mathbf{k})$$
$$= -(0.16875 \text{ N}\cdot\text{m})\mathbf{j} + (3.375 \text{ N}\cdot\text{m})\mathbf{i}$$

Position vector of Point C. $\mathbf{r}_{C/A} = (0.450 \text{ m})\mathbf{i} + (0.225 \text{ m})\mathbf{j}$

Velocity of Point C, the mass center of the disk.

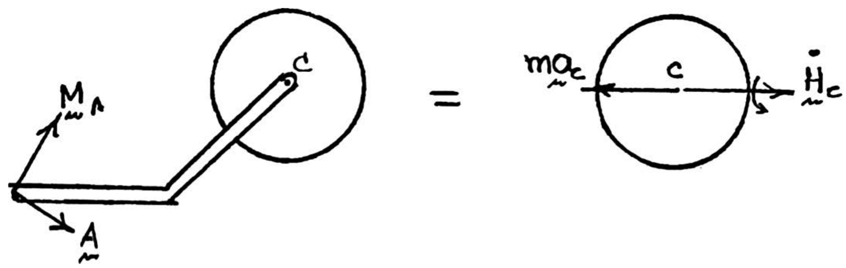
$$\mathbf{v}_C = \omega_1\times\mathbf{r}_{C/A} = 5\mathbf{j}\times(0.45\mathbf{i} + 0.225\mathbf{j}) = -(2.25 \text{ m/s})\mathbf{k}$$

Acceleration of Point C.

$$\mathbf{a}_C = \alpha_1\mathbf{j}\times\mathbf{r}_{C/A} + \omega_1\mathbf{j}\times\mathbf{v}_C = (-7.5\mathbf{j})\times(0.45\mathbf{i}+0.225\mathbf{j}) + 5\mathbf{j}\times(-2.25\mathbf{k})$$
$$= (3.3750 \text{ m/s}^2)\mathbf{k} - (11.25 \text{ m/s}^2)\mathbf{i}$$

$$m\mathbf{a}_C = (4)(-11.25\mathbf{i} + 3.3750\mathbf{k})$$
$$= -(45 \text{ N})\mathbf{i} + (13.5 \text{ N})\mathbf{k}$$

Free body and kinetic diagrams



Linear components: $\mathbf{A} = m\mathbf{a}_C \qquad \mathbf{A} = -(45 \text{ N})\mathbf{i} + (13.5 \text{ N})\mathbf{k}$

Moments about A. $\mathbf{M}_A = \mathbf{r}_{C/A}\times m\mathbf{a}_C + \dot{\mathbf{H}}_C$

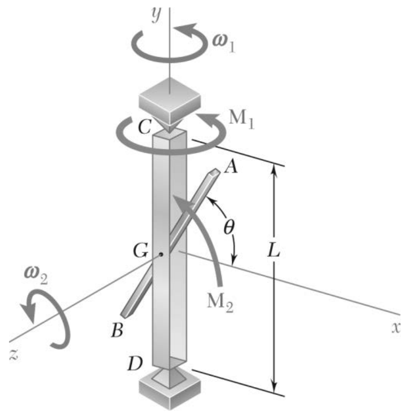
$$\mathbf{M}_A = (0.450\mathbf{i} + 0.225\mathbf{j})\times(-45\mathbf{i} + 13.5\mathbf{k}) - 0.16875\mathbf{j} + 3.375\mathbf{i}$$
$$= -6.0750\mathbf{j} + 10.125\mathbf{k} + 3.0375\mathbf{i} - 0.16875\mathbf{j} + 3.375\mathbf{i}$$
$$= 6.4125\mathbf{i} - 6.2438\mathbf{j} + 10.125\mathbf{k}$$

(*a*) Required couple. $\mathbf{M}_1\mathbf{j} = -(6.24 \text{ N}\cdot\text{m})\mathbf{j}$ ◀

(*b*) Dynamic reaction. $\mathbf{A} = -(45.0 \text{ N})\mathbf{i} + (13.50 \text{ N})\mathbf{k}$ ◀

$\mathbf{M}_A = (6.41 \text{ N}\cdot\text{m})\mathbf{i} + (10.13 \text{ N}\cdot\text{m})\mathbf{k}$ ◀

PROBLEM 18.106*



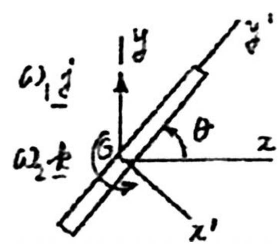
A slender homogeneous rod AB of mass m and length L is made to rotate at the constant rate ω_2 about the horizontal z axis, while frame CD is made to rotate at the constant rate ω_1 about the y axis. Express as a function of the angle θ (*a*) the couple $\mathbf{M}_1$ required to maintain the rotation of the frame, (*b*) the couple $\mathbf{M}_2$ required to maintain the rotation of the rod, (*c*) the dynamic reactions at the supports C and D.

SOLUTION

Angular momentum $\mathbf{H}_G$:

We resolve the angular velocity $\boldsymbol{\omega} = \omega_1\mathbf{j} + \omega_2\mathbf{k}$ into components along the principal axes $Gx'y'z$:

$$\omega_{x'} = -\omega_1 \cos\theta$$
$$\omega_{y'} = \omega_1 \sin\theta$$
$$\omega_z = \omega_2$$



Moments of inertia:

$$\bar{I}_{x'} = \bar{I}_z = \frac{1}{12}mL^2 \quad \bar{I}_{y'} = 0$$

We have
$$H_{x'} = \bar{I}_{x'}\,\omega_{x'} = -\frac{1}{12}mL^2\,\omega_1 \cos\theta$$
$$H_{y'} = \bar{I}_{y'}\,\omega_{y'} = 0$$
$$H_z = \bar{I}_z\,\omega_z = \frac{1}{12}mL^2\,\omega_2$$

Computing the components of $\mathbf{H}_G$ along the *x*, *y*, *z* axes:

$$H_x = H_{x'}\sin\theta = -\frac{1}{12}mL^2\,\omega_1\cos\theta\sin\theta = -\frac{1}{24}mL^2\,\omega_1\sin 2\theta$$
$$H_y = -H_{x'}\cos\theta = +\frac{1}{12}mL^2\,\omega_1\cos^2\theta$$
$$H_z = \frac{1}{12}mL^2\,\omega_2$$

The angular momentum is therefore

$$\mathbf{H}_G = -\frac{1}{24}mL^2\omega_1\sin 2\theta\mathbf{i} + \frac{1}{12}mL^2\omega_1\cos^2\theta\mathbf{j} + \frac{1}{12}mL^2\omega_2\mathbf{k}$$

where the reference frame *Gxyz* rotates with the angular velocity

$$\boldsymbol{\Omega} = \omega_1\mathbf{j}$$

<u>Rate of change of $\mathbf{H}_G$</u>. We note that ω_1 and ω_2 are constant, while θ varies with t, with $\dot{\theta} = \omega_2$.
Eq. (18.22) yields

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \mathbf{\Omega} \times \mathbf{H}_G = -\frac{1}{24} mL^2 \omega_1 (2\cos 2\theta\dot{\theta})\mathbf{i} + \frac{1}{12} mL^2 \omega_1 (-2\cos\theta \sin\theta\dot{\theta})\mathbf{j}$$

$$+ \omega_1 \mathbf{j} \times \left(-\frac{1}{24} mL^2 \omega_1 \sin 2\theta \mathbf{i} + \frac{1}{12} mL^2 \omega_2 \mathbf{k} \right)$$

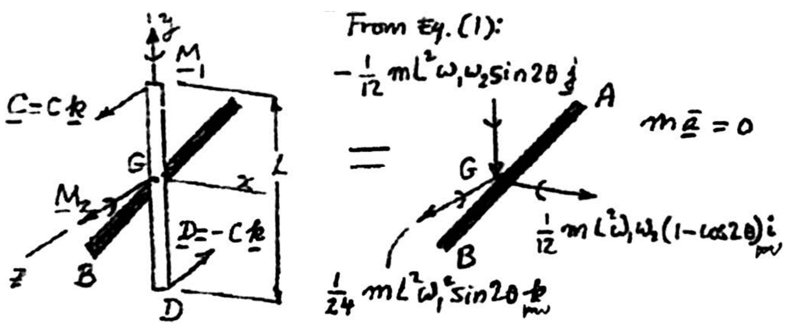
$$= -\frac{1}{12} mL^2 \omega_1 \omega_2 \cos 2\theta \mathbf{i} - \frac{1}{12} mL^2 \omega_1 \omega_2 \sin 2\theta \mathbf{j}$$

$$+ \frac{1}{24} mL^2 \omega_1^2 \sin 2\theta \mathbf{k} + \frac{1}{12} mL^2 \omega_1 \omega_2 \mathbf{i}$$

$$\dot{\mathbf{H}}_G = \frac{1}{12} mL^2 \omega_1 \omega_2 (1 - \cos 2\theta)\mathbf{i} - \frac{1}{12} mL^2 \omega_1 \omega_2 \sin 2\theta \mathbf{j}$$

$$+ \frac{1}{24} mL^2 \omega_1^2 \sin 2\theta \mathbf{k} \qquad (1)$$

<u>Equivalence of external and effective forces</u>.



Equating the moments of the variable couples:

$$L\mathbf{j} \times C\mathbf{k} + M_1\mathbf{j} + M_2\mathbf{k} = \dot{\mathbf{H}}_G$$

$$LC\mathbf{i} + M_1\mathbf{j} + M_2\mathbf{k} = \frac{1}{12} mL^2 \omega_1 \omega_2 (1 - \cos 2\theta)\mathbf{i} - \frac{1}{12} mL^2 \omega_1 \omega_2 \sin 2\theta \mathbf{j}$$

$$+ \frac{1}{24} mL^2 \omega_1^2 \sin 2\theta \mathbf{k}$$

Equating the coefficients of the unit vectors:

$$\mathbf{j}: \quad M_1 = -\frac{1}{12} mL^2 \omega_1 \omega_2 \sin 2\theta$$

(*a*) Couple M_1: $\mathbf{M}_1 = -\frac{1}{12} mL^2 \omega_1 \omega_2 \sin 2\theta \mathbf{j}$ ◀

$$\mathbf{k}: \quad M_2 = \frac{1}{24} mL^2 \omega_1^2 \sin 2\theta$$

PROBLEM 18.106* (Continued)

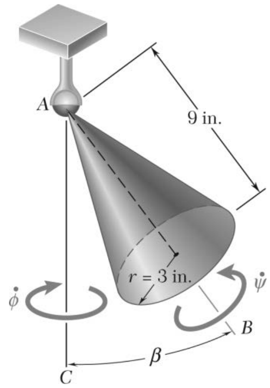
(*b*) Couple M_2: $$\mathbf{M}_2 = \frac{1}{24} mL^2 \omega_1^2 \sin 2\theta \mathbf{k} \blacktriangleleft$$

$$\mathbf{i}: \quad LC = \frac{1}{12} mL^2 \omega_1 \omega_2 (1 - \cos 2\theta)$$

$$C = \frac{1}{12} mL\omega_1 \omega_2 (1 - \cos 2\theta) = \frac{1}{6} mL\omega_1 \omega_2 \sin^2 \theta$$

Dynamic reactions.

$$\mathbf{C} = \frac{1}{6} mL\omega_1 \omega_2 \sin^2 \theta \mathbf{k}; \ \mathbf{D} = -\frac{1}{6} mL\omega_1 \omega_2 \sin^2 \theta \mathbf{k} \blacktriangleleft$$

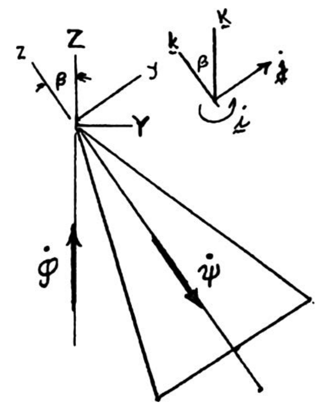


PROBLEM 18.107

A solid cone of height 9 in. with a circular base of radius 3 in. is supported by a ball-and-socket joint at A. Knowing that the cone is observed to precess about the vertical axis AC at the constant rate of 40 rpm in the sense indicated and that its axis of symmetry AB forms an angle $\beta = 40°$ with AC, determine the rate at which the cone spins about the axis AB.

SOLUTION

Use principal axes xyz with origin at A as shown.



For the solid cone,

$$r = 3\text{ in.} = 0.25\text{ ft}$$

$$h = 9\text{ in.} = 0.75\text{ ft} \quad c = \frac{3h}{4} = 0.5625\text{ ft}$$

$$I = \frac{3}{10}mr^2 \quad I' = \frac{3}{5}m\left(h^2 + \frac{1}{4}r^2\right)$$

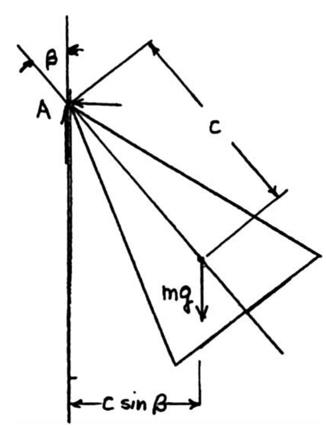
$$I' - I = \frac{3}{5}m\left(h^2 - \frac{1}{4}r^2\right)$$

Angular velocity.

spin: $\dot{\psi}$ about negative z axis

precession: $\dot{\phi}$ about positive Z axis

$$\boldsymbol{\omega} = \dot{\phi}\mathbf{K} - \dot{\psi}\mathbf{k}$$
$$= \dot{\phi}(\cos\beta\mathbf{k} + \sin\beta\mathbf{j}) - \dot{\psi}\mathbf{k}$$
$$\omega_x = 0, \quad \omega_y = \dot{\phi}\sin\beta, \quad \omega_z = \dot{\phi}\cos\beta - \dot{\psi}$$



Angular momentum about fixed Point A.

$$\mathbf{H}_A = I'\omega_x\mathbf{i} + I'\omega_y\mathbf{j} + I\omega_z\mathbf{k}$$
$$= I'\dot{\phi}\sin\beta\mathbf{j} + I(\dot{\phi}\cos\beta - \dot{\psi})\mathbf{k}$$

Let frame $Axyz$ be rotating with angular velocity $\boldsymbol{\Omega}$.

$$\boldsymbol{\Omega} = \dot{\phi}\mathbf{K} = \dot{\phi}\cos\beta\mathbf{j} + \dot{\phi}\sin\beta\,\mathbf{k}$$

Rate of change of $\mathbf{H}_A$.

$$\dot{\mathbf{H}}_A = \boldsymbol{\Omega}\times\mathbf{H}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \dot{\phi}\sin\beta & \dot{\phi}\cos\beta \\ 0 & I'\dot{\phi}\sin\beta & I(\dot{\phi}\cos\beta - \dot{\psi}) \end{vmatrix}$$
$$= -[(I' - I)\dot{\phi}^2\cos\beta\sin\beta + I\dot{\phi}\dot{\psi}\sin\beta]\mathbf{i}$$

PROBLEM 18.107 (Continued)

Moment about A.

$$\mathbf{M}_A = -mgc\sin\beta\mathbf{i}$$

$\mathbf{M}_A = \dot{\mathbf{H}}_A$ leads to

$$gc = \frac{I' - I}{m}\dot{\phi}^2 \cos\beta + \frac{I}{m}\dot{\phi}\dot{\psi}$$

$$= \frac{3}{5}\left(h^2 - \frac{1}{4}r^2\right)\dot{\phi}^2 \cos\beta + \frac{3}{10}r^2\dot{\phi}\dot{\psi}$$

$$20gc = (12h^2 - 3r^2)\dot{\phi}^2 \cos\beta + 6r^2\dot{\phi}\dot{\psi}$$

$$\dot{\psi} = \frac{20gc - (12h^2 - 3r^2)\dot{\phi}^2 \cos\beta}{6r^2\dot{\phi}}$$

Data:

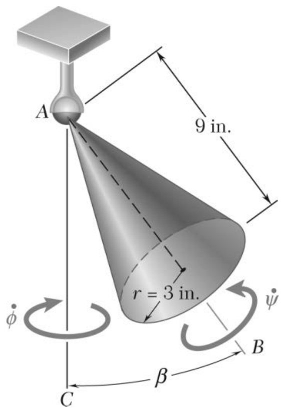
$$\beta = 40°$$

$$\dot{\phi} = 40 \text{ rpm} = 4.1888 \text{ rad/s}$$

$$\dot{\psi} = \frac{(20)(32.2)(0.5625) - [(12)(0.75)^2 - (3)(0.25)^2]4.1888^2 \cos 40°}{(6)(0.25)^2(4.1888)}$$

$$= 174.46 \text{ rad/s}$$

$\dot{\psi} = 1666$ rpm ◀

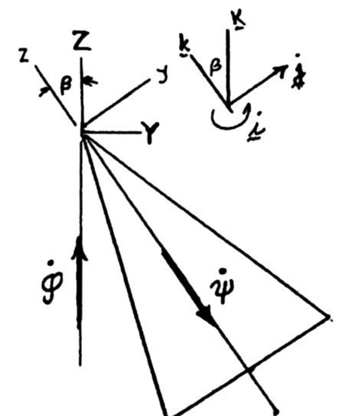


PROBLEM 18.108

A solid cone of height 9 in. with a circular base of radius 3 in. is supported by a ball-and-socket joint at *A*. Knowing that the cone is spinning about its axis of symmetry *AB* at the rate of 3000 rpm and that *AB* forms an angle $\beta = 60°$ with the vertical axis *AC*, determine the two possible rates of steady precession of the cone about the axis *AC*.

SOLUTION

Use principal axes *xyz* with origin at *A* as shown.



For the solid cone,

$$r = 3 \text{ in.} = 0.25 \text{ ft}$$

$$h = 9 \text{ in.} = 0.75 \text{ ft} \quad c = \frac{3h}{4} = 0.5625 \text{ ft}$$

$$I = \frac{3}{10}mr^2 \quad I' = \frac{3}{5}m\left(h^2 + \frac{1}{4}r^2\right)$$

$$I' - I = \frac{3}{5}m\left(h^2 - \frac{1}{4}r^2\right)$$

<u>Angular velocity</u>.

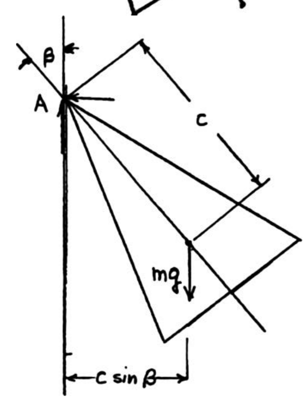
spin: $\dot{\psi}$ about negative *z* axis

precession: $\dot{\phi}$ about positive *Z* axis

$$\boldsymbol{\omega} = \dot{\phi}\mathbf{K} - \dot{\psi}\mathbf{k}$$

$$= \dot{\phi}(\cos\beta\,\mathbf{k} + \sin\beta\,\mathbf{j}) - \dot{\psi}\mathbf{k}$$

$$\omega_x = 0, \quad \omega_y = \dot{\phi}\sin\beta, \quad \omega_z = \dot{\phi}\cos\beta - \dot{\psi}$$



<u>Angular momentum about fixed Point *A*</u>.

$$\mathbf{H}_A = I'\omega_x\mathbf{i} + I'\omega_y\mathbf{j} + I\omega_z\mathbf{k}$$

$$= I'\dot{\phi}\sin\beta\,\mathbf{j} + I(\dot{\phi}\cos\beta - \dot{\psi})\mathbf{k}$$

Let frame *Axyz* be rotating with angular velocity $\boldsymbol{\Omega}$.

$$\boldsymbol{\Omega} = \dot{\phi}\mathbf{K} = \dot{\phi}\cos\beta\,\mathbf{j} + \dot{\phi}\sin\beta\,\mathbf{k}$$

<u>Rate of change of $\mathbf{H}_A$</u>.

$$\dot{\mathbf{H}}_A = \boldsymbol{\Omega} \times \mathbf{H}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \dot{\phi}\sin\beta & \dot{\phi}\cos\beta \\ 0 & I'\dot{\phi}\sin\beta & I(\dot{\phi}\cos\beta - \dot{\psi}) \end{vmatrix}$$

$$= -[(I' - I)\dot{\phi}^2\cos\beta\sin\beta + I\dot{\phi}\dot{\psi}\sin\beta]\mathbf{i}$$

PROBLEM 18.108 (Continued)

Moment about A. $\mathbf{M}_A = -mgc\sin\beta\mathbf{i}$

$\mathbf{M}_A = \dot{\mathbf{H}}_A$ leads to

$$gc = \frac{I' - I}{m}\dot{\phi}^2 \cos\beta + \frac{I}{m}\dot{\phi}\dot{\psi}$$

$$= \frac{3}{5}\left(h^2 - \frac{1}{4}r^2\right)\dot{\phi}^2 \cos\beta + \frac{3}{10}r^2\dot{\phi}\dot{\psi}$$

$$20gc = (12h^2 - 3r^2)\dot{\phi}^2 \cos\beta + 6r^2\dot{\phi}\dot{\psi}$$

$$(12h^2 - 3r^2)\dot{\phi}^2 \cos\beta + 6r^2\dot{\psi}\,\dot{\phi} - 20gc = 0$$

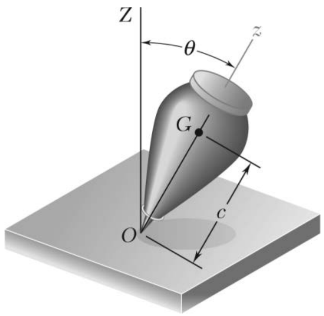
Data: $\dot{\psi} = 3000 \text{ rpm} = 314.16 \text{ rad/s}$

$\beta = 60°$

$$[(12)(0.75)^2 - (3)(0.25)^2](\cos 60°)\dot{\phi}^2 + (6)(0.25)^2(314.16)\,\dot{\phi} - (20)(32.2)(0.5625) = 0$$

$$3.28125\dot{\phi}^2 + 117.81\dot{\phi} - 362.25 = 0$$

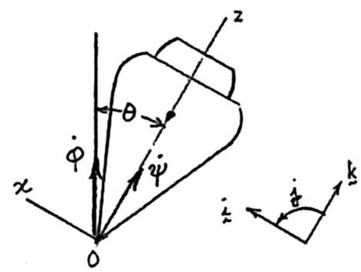
Solving the quadratic equation, $\dot{\phi} = 2.8488 \text{ rad/s}, \quad -38.753 \text{ rad/s}$ $\qquad \dot{\phi} = 27.2 \text{ rpm}, \quad -370 \text{ rpm}$ ◀



PROBLEM 18.109

The 85-g top shown is supported at the fixed Point O. The radii of gyration of the top with respect to its axis of symmetry and with respect to a transverse axis through O are 21 mm and 45 mm, respectively. Knowing that $c = 37.5$ mm and that the rate of spin of the top about its axis of symmetry is 1800 rpm, determine the two possible rates of steady precession corresponding to $\theta = 30°$.

SOLUTION



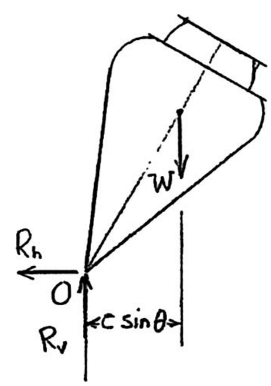
Use principal axes x, y, z with origin at O.

Angular velocity: $\boldsymbol{\omega} = \dot{\varphi}\sin\theta\mathbf{i} + (\dot{\psi} + \dot{\varphi}\cos\theta)\mathbf{k}$

$$\omega_x = \dot{\varphi}\sin\theta, \quad \omega_y = 0, \quad \omega_z = \dot{\psi} + \dot{\varphi}\cos\theta$$

Angular momentum about O:

$$\mathbf{H}_O = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$
$$= I'\dot{\varphi}\sin\theta\mathbf{i} + I\omega_z\mathbf{k}$$



Let the reference frame $Oxyz$ be rotating with angular velocity $\boldsymbol{\Omega} = \dot{\varphi}\sin\theta\mathbf{i} + \dot{\varphi}\cos\theta\mathbf{k}$.

$$\dot{\mathbf{H}}_O = \boldsymbol{\Omega} \times \mathbf{H}_O$$
$$= (\dot{\varphi}\sin\theta\mathbf{i} + \dot{\varphi}\cos\theta\mathbf{k}) \times (I'\dot{\varphi}\sin\theta + I\omega_z\mathbf{k})$$
$$= (I'\dot{\varphi}\sin\theta\cos\theta - I\omega_z\sin\theta)\dot{\varphi}\mathbf{j}$$

$$\Sigma\mathbf{M}_O = \dot{\mathbf{H}}_O$$

$$-Wc\sin\theta\mathbf{j} = (I'\dot{\varphi}\sin\theta\cos\theta - I\omega_z\sin\theta)\mathbf{j}$$

$$Wc = (I\omega_z - I'\dot{\varphi}\cos\theta)\dot{\varphi}$$

$$Wc = [I\dot{\psi} - (I' - I)\dot{\varphi}\cos\theta]\dot{\varphi} \qquad (1)$$

Data:

$$m = 85 \text{ g} = 0.085 \text{ kg}$$
$$W = mg = (0.085)(9.81) = 0.83385 \text{ N}$$

$$I = mk_z^2 = (0.085)(0.021)^2 = 37.485 \times 10^{-6} \text{ kg} \cdot \text{m}^2$$
$$I' = mk_x^2 = (0.085)(0.045)^2 = 172.125 \times 10^{-6} \text{ kg} \cdot \text{m}^2$$

$$c = 37.5 \text{ mm} = 0.0375 \text{ m}, \quad \dot{\psi} = 1800 \text{ rpm} = 188.496 \text{ rad/s}$$

$$\theta = 30°$$

PROBLEM 18.109 (Continued)

Substituting into Eq. (1),

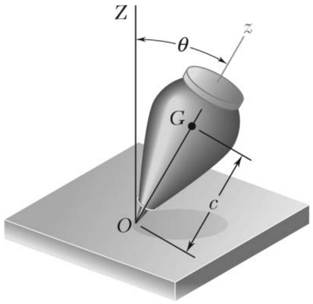
$$(0.83385)(0.0375) = [(37.485\times10^{-6})(188.496) - (134.64\times10^{-6})\dot{\varphi}\cos 30°]\dot{\varphi}$$

$$116.602\times10^{-6}\dot{\varphi}^2 - 7.0658\times10^{-3}\dot{\varphi} + 31.269\times10^{-3} = 0$$

$$\dot{\varphi} = 30.299 \pm 25.492 \qquad \dot{\varphi} = 4.807 \text{ rad/s}, \quad 55.791 \text{ rad/s}$$

$$\dot{\varphi} = 45.9 \text{ rpm}, \quad 533 \text{ rpm} \blacktriangleleft$$

PROBLEM 18.110

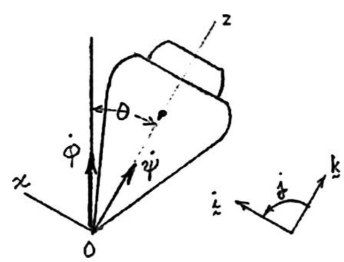


The top shown is supported at the fixed Point O and its moments of inertia about its axis of symmetry and about a transverse axis through O are denoted, respectively, by I and I'. (a) Show that the condition for steady precession of the top is

$$(I\omega_z - I'\dot{\phi}\cos\theta)\dot{\phi} = Wc$$

where $\dot{\phi}$ is the rate of precession and ω_z is the rectangular component of the angular velocity along the axis of symmetry of the top. (b) Show that if the rate of spin $\dot{\psi}$ of the top is very large compared with its rate of precession $\dot{\phi}$, the condition for steady precession is $I\dot{\psi}\dot{\phi} \approx Wc$. ($c$) Determine the percentage error introduced when this last relation is used to approximate the slower of the two rates of precession obtained for the top of Problem 18.109.

SOLUTION



Use principal axes x, y, z with origin at O.

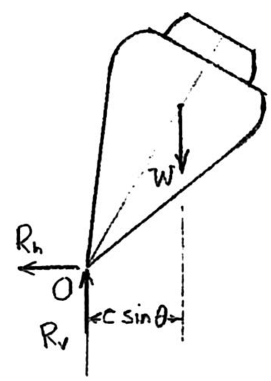
Angular velocity: $\boldsymbol{\omega} = \dot{\varphi}\sin\theta\mathbf{i} + (\dot{\psi} + \dot{\varphi}\cos\theta)\mathbf{k}$

$$\omega_x = \dot{\varphi}\sin\theta, \quad \omega_y = 0, \quad \omega_z = \dot{\psi} + \dot{\varphi}\cos\theta$$

Angular momentum about O:

$$\mathbf{H}_O = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k} = I'\dot{\varphi}\sin\theta\mathbf{i} + I\omega_z\mathbf{k}$$

Let the reference frame $Oxyz$ be rotating with angular velocity $\mathbf{\Omega} = \dot{\varphi}\sin\theta\mathbf{i} + \dot{\varphi}\cos\theta\mathbf{k}$.



$$\begin{aligned}\dot{\mathbf{H}}_O &= \mathbf{\Omega}\times\mathbf{H}_O \\ &= (\dot{\varphi}\sin\theta\mathbf{i} + \dot{\varphi}\cos\theta\mathbf{k})\times(I'\dot{\varphi}\sin\theta + I\omega_z\mathbf{k}) \\ &= (I'\dot{\varphi}\sin\theta\cos\theta - I\omega_z\sin\theta)\dot{\varphi}\mathbf{j}\end{aligned}$$

$$\Sigma\mathbf{M}_O = \dot{\mathbf{H}}_O$$

$$-Wc\sin\theta\mathbf{j} = (I'\dot{\varphi}\sin\theta\cos\theta - I\omega_z\sin\theta)\mathbf{j}$$

(a) $\qquad Wc = (I\omega_z - I'\dot{\varphi}\cos\theta)\dot{\varphi}$ ◀

$$Wc = [I\dot{\psi} - (I' - I)\dot{\varphi}\cos\theta]\dot{\varphi} \qquad (1)$$

(b) For $|\dot{\varphi}| << \dot{\psi}$, $\qquad Wc \approx I\dot{\psi}\dot{\varphi} \qquad$ (2) ◀

(c) Data:

$$m = 85 \text{ g} = 0.085 \text{ kg}$$
$$W = mg = (0.085)(9.81) = 0.83385 \text{ N}$$
$$I = mk_z^2 = (0.085)(0.021)^2 = 37.485\times 10^{-6} \text{ kg}\cdot\text{m}^2$$
$$I = mk_x^2 = (0.085)(0.045)^2 = 172.125\times 10^{-6} \text{ kg}\cdot\text{m}^2$$
$$c = 37.5 \text{ mm} = 0.0375 \text{ m}, \quad \dot{\psi} = 1800 \text{ rpm} = 188.496 \text{ rad/s}$$
$$\theta = 30°$$

Substituting into Eq. (1),

$$(0.83385)(0.0375) = [(37.485\times 10^{-6})(188.496) - (134.64\times 10^{-6})\dot{\varphi}\cos 30°]\dot{\varphi}$$

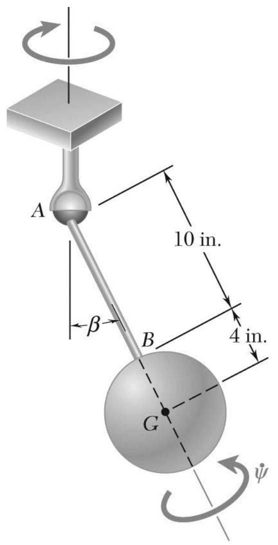
$$116.602\times 10^{-6}\dot{\varphi}^2 - 7.0658\times 10^{-3}\dot{\varphi} + 31.269\times 10^{-3} = 0$$

$$\dot{\varphi} = 30.299 \pm 25.492 \qquad \dot{\varphi} = 4.807 \text{ rad/s, } 55.791 \text{ rad/s}$$

From Eq. (2),

$$\dot{\varphi} = \frac{Wc}{I\dot{\psi}} = \frac{(0.83385)(0.0375)}{(37.485\times 10^{-6})(188.496)} = 4.425 \text{ rad/s}$$

$$\% \text{ error} = \frac{4.425 - 4.807}{4.807}\times 100\% \qquad \% \text{ error} = -7.95\% \blacktriangleleft$$

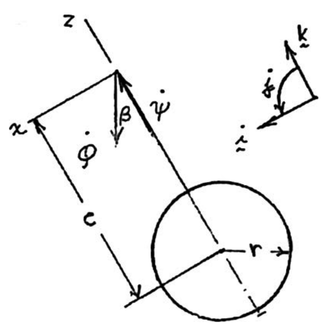


PROBLEM 18.111

A solid aluminum sphere of radius 4 in. is welded to the end of a10-in.-long rod *AB* of negligible mass which is supported by a ball-and-socket joint at *A*. Knowing that the sphere is observed to precess about a vertical axis at the constant rate of 60 rpm in the sense indicated and that rod *AB* forms an angle $\beta = 20°$ with the vertical, determine the rate of spin of the sphere about line *AB*.

SOLUTION

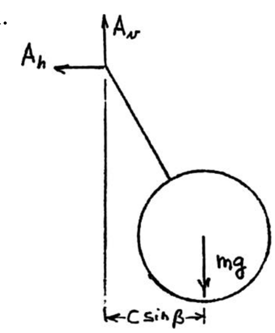
Use principal axes *x*, *y*, *z* with origin at *A*.



Angular velocity:
$$\boldsymbol{\omega} = \dot{\varphi}\sin\beta\mathbf{i} + (\dot{\psi} - \dot{\varphi}\cos\beta)\mathbf{k}$$
$$\omega_x = \dot{\varphi}\sin\beta, \quad \omega_y = 0, \quad \omega_z = \dot{\psi} - \dot{\varphi}\cos\beta$$

Angular momentum about *A*:
$$\mathbf{H}_A = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$
$$= I'\dot{\varphi}\sin\beta\mathbf{i} + I(\dot{\psi} - \dot{\varphi}\cos\beta)\mathbf{k}$$

Let the reference frame *Axyz* be rotating with angular velocity $\boldsymbol{\Omega} = \dot{\varphi}\sin\beta\mathbf{i} - \dot{\varphi}\cos\beta\mathbf{k}$.



$$\dot{\mathbf{H}}_A = \boldsymbol{\Omega} \times \mathbf{H}_A$$
$$= -\dot{\varphi}[I\dot{\psi}\sin\beta + (I' - I)\dot{\varphi}\sin\beta\cos\beta]\mathbf{j}$$
$$\Sigma\mathbf{M}_A = \dot{\mathbf{H}}_A$$
$$-mgc\sin\beta\mathbf{j} = -\dot{\varphi}[I\dot{\psi}\sin\beta + (I' - I)\dot{\varphi}\sin\beta\cos\beta]\mathbf{j}$$
$$mgc = I\dot{\varphi}\dot{\psi} - (I' - I)\dot{\varphi}^2\cos\beta$$
$$\frac{I\dot{\varphi}\dot{\psi}}{mgc} - \frac{(I' - I)\dot{\varphi}^2\cos\beta}{mgc} + 1 = 0 \qquad (1)$$

<u>Data</u>:
$$r = 4\text{ in.} = 0.333\text{ ft}$$
$$c = 10\text{ in.} + 4\text{ in.}$$
$$= 14\text{ in.} = 1.1667\text{ ft},$$
$$g = 32.2\text{ ft/s}^2$$

$$\frac{I}{m} = \frac{2}{5}r^2 = \frac{2}{5}(0.3333)^2 = 0.04444 \text{ ft}^2$$

$$\frac{I'}{m} = \frac{2}{5}r^2 + c^2 = \frac{2}{5}(0.3333)^2 + (1.16667)^2 = 1.4056 \text{ ft}^2$$

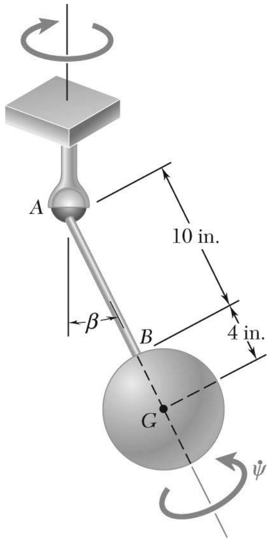
$$\beta = 20°, \quad \dot{\varphi} = 60 \text{ rpm} = 6.2832 \text{ rad/s}$$

Substituting into Eq. (1),

$$\frac{(0.04444)(6.2832)\dot{\psi}}{(32.2)(1.1667)} - \frac{(1.4056 - 0.04444)(6.2832)^2 \cos 20°}{(32.2)(1.1667)} + 1 = 0$$

$$7.4335 \times 10^{-3}\dot{\psi} - 1.34412 + 1 = 0$$

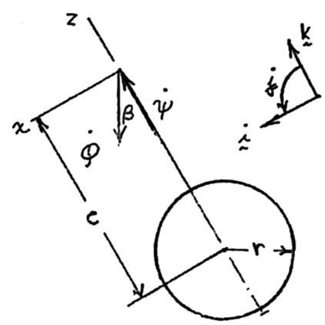
$$\dot{\psi} = 46.293 \text{ rad/s} = 442 \text{ rpm} \qquad \dot{\psi} = 442 \text{ rpm} \blacktriangleleft$$



PROBLEM 18.112

A solid aluminum sphere of radius 4 in. is welded to the end of a 10-in.-long rod *AB* of negligible mass which is supported by a ball-and-socket joint at *A*. Knowing that the sphere spins as shown about line *AB* at the rate of 600 rpm, determine the angle β for which the sphere will precess about a vertical axis at the constant rate of 60 rpm in the sense indicated.

SOLUTION



Use principal axes *x*, *y*, *z* with origin at *A*.

Angular velocity:

$$\boldsymbol{\omega} = \dot{\varphi}\sin\beta\,\mathbf{i} + (\dot{\psi} - \dot{\varphi}\cos\beta)\mathbf{k}$$

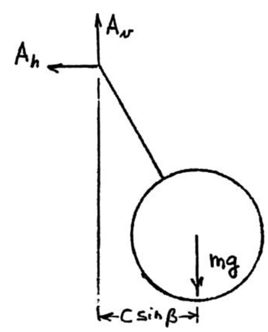
$$\omega_x = \dot{\varphi}\sin\beta, \quad \omega_y = 0, \quad \omega_z = \dot{\psi} - \dot{\varphi}\cos\beta$$

Angular momentum about *A*:

$$\mathbf{H}_A = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$= I'\dot{\varphi}\sin\beta\,\mathbf{i} + I(\dot{\psi} - \dot{\varphi}\cos\beta)\mathbf{k}$$

Let the reference frame *Axyz* be rotating with angular velocity $\boldsymbol{\Omega} = \dot{\varphi}\sin\beta\,\mathbf{i} - \dot{\varphi}\cos\beta\,\mathbf{k}$.



$$\dot{\mathbf{H}}_A = \boldsymbol{\Omega}\times\mathbf{H}_A$$

$$= -\dot{\varphi}[I\dot{\psi}\sin\beta + (I' - I)\dot{\varphi}\sin\beta\cos\beta]\mathbf{j}$$

$$\Sigma\mathbf{M}_A = \dot{\mathbf{H}}_A$$

$$-mgc\sin\beta\,\mathbf{j} = -\dot{\varphi}[I\dot{\psi}\sin\beta + (I' - I)\dot{\varphi}\sin\beta\cos\beta]\mathbf{j}$$

$$mgc = I\dot{\varphi}\dot{\psi} - (I' - I)\dot{\varphi}^2\cos\beta$$

$$\frac{I\dot{\varphi}\dot{\psi}}{mgc} - \frac{(I' - I)\dot{\varphi}^2\cos\beta}{mgc} + 1 = 0 \qquad (1)$$

Data:

$$r = 4\text{ in.} = \frac{1}{3}\text{ ft}$$

$$c = 10 + 4 = 14\text{ in.}$$

$$= \frac{14}{12} = \frac{7}{6}\text{ ft,}$$

$$g = 32.2\text{ ft/s}^2$$

PROBLEM 18.112 (Continued)

$$\frac{I}{m} = \frac{2}{5}r^2 = \frac{2}{5}\left(\frac{1}{3}\right)^2 = 0.04444 \text{ ft}^2$$

$$\frac{I'}{m} = \frac{2}{5}r^2 + c^2 = \frac{2}{5}\left(\frac{1}{3}\right)^2 + \left(\frac{7}{6}\right)^2 = 1.4056 \text{ ft}^2$$

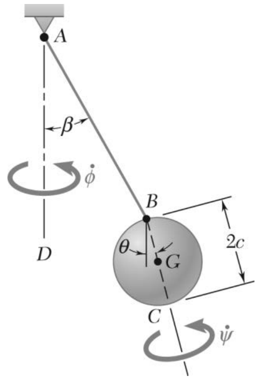
$$\dot{\psi} = 600 \text{ rpm} = 62.832 \text{ rad/s} \qquad \dot{\varphi} = -60 \text{ rpm} = -6.2832 \text{ rad/s}$$

Substituting into Eq. (1),

$$\frac{(0.044444)(-6.2832)(62.832)}{(32.2)(1.16667)} - \frac{(1.40556 - 0.04444)(-6.2832)^2}{(32.2)(1.16667)}\cos\beta + 1 = 0$$

$$-0.46706 - 1.43038\cos\beta + 1 = 0$$

$$\cos\beta = 0.37259 \qquad \beta = 68.1° \blacktriangleleft$$

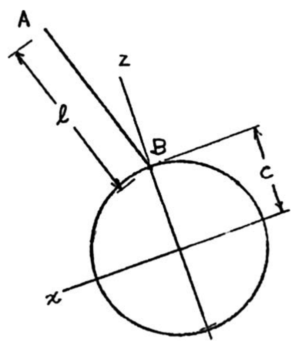


PROBLEM 18.113

A solid sphere of radius $c = 3$ in. is attached as shown to cord AB. The sphere is observed to precess at the constant rate $\dot{\phi} = 6$ rad/s about the vertical axis AD. Knowing that $\beta = 40°$, determine the angle θ that the diameter BC forms with the vertical when the sphere (*a*) has no spin, (*b*) spins about its diameter BC at the rate $\dot{\psi} = 50$ rad/s, (*c*) spins about BC at the rate $\dot{\psi} = -50$ rad/s.

SOLUTION

Use principal centroidal axes *x*, *y*, *z* as shown.



Angular velocity: $\boldsymbol{\omega} = -\dot{\varphi}\sin\theta\mathbf{i} + (\dot{\psi} + \dot{\varphi}\cos\theta)\mathbf{k}$

Angular momentum about the mass center *G*:

$$\mathbf{H}_G = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$= -I'\dot{\varphi}\sin\theta\mathbf{i} + I(\dot{\psi} + \dot{\varphi}\cos\theta)\mathbf{k}$$

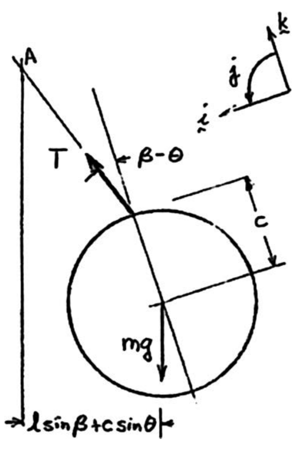
Let the reference frame $Gxyz$ be rotating with angular velocity

$$\boldsymbol{\Omega} = -\dot{\varphi}\sin\theta\mathbf{i} + \dot{\varphi}\cos\theta\mathbf{k}$$

$$\dot{\mathbf{H}}_G = \boldsymbol{\Omega} \times \mathbf{H}_G$$

$$= [I\dot{\varphi}\dot{\psi}\sin\theta - (I' - I)\dot{\varphi}^2\sin\theta\cos\theta]\mathbf{j}$$

Acceleration of the mass center:



$$\bar{\mathbf{a}} = (l\sin\beta + c\sin\theta)\dot{\varphi}^2 \leftarrow$$

$$\Sigma\mathbf{F} = \Sigma\mathbf{F}_{\text{eff}}:$$

$$+\uparrow: \quad T\cos\beta - mg = 0, \qquad T = \frac{mg}{\cos\beta} \tag{1}$$

$$+\leftarrow: \quad T\sin\beta = m\bar{a}$$

$$g\tan\beta = (l\sin\beta + c\sin\theta)\dot{\varphi}^2 \tag{2}$$

$$+\circlearrowleft \Sigma M_G = Tc\sin(\beta - \theta) = \dot{H}_G$$

$$\frac{mgc\sin(\beta - \theta)}{\cos\beta} = I\dot{\psi}\dot{\varphi}\sin\theta - (I' - I)\dot{\varphi}^2\sin\theta\cos\theta \tag{3}$$

<u>Data</u>: $c = 3\text{ in.} = 0.25\text{ ft}, \quad \beta = 40°, \quad \dot{\varphi} = 6\text{ rad/s}$

$$\frac{I}{m} = \frac{I'}{m} = \frac{2}{5}c^2 = 0.025\text{ ft}^2 \quad I' - I = 0$$

PROBLEM 18.113 (Continued)

Substituting into Eq. (1),

$$\frac{(32.2)(0.25)\sin(\beta-\theta)}{\cos\beta}=(0.025)(6)\dot{\psi}\sin\theta$$

$$8.05(\sin\beta\cos\theta-\cos\beta\sin\theta)=0.15\dot{\psi}\sin\theta\cos\beta$$

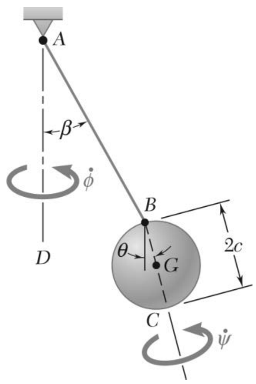
$$\tan\theta=\frac{\tan\beta}{1+0.018634\dot{\psi}}$$

(*a*) $\dot{\psi}=0,$ $\tan\theta=\tan\beta \quad \theta=\beta$ $\theta=40.0°$ ◀

(*b*) $\dot{\psi}=50\text{ rad/s},$ $\tan\theta=\frac{\tan 40°}{1+(0.018634)(50)}$ $\theta=23.5°$ ◀

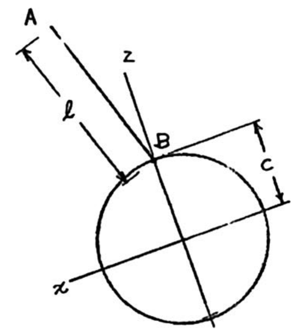
(*c*) $\dot{\psi}=-50\text{ rad/s},$ $\tan\theta=\frac{\tan 40°}{1+(0.018634)(-50)}$ $\theta=85.3°$ ◀

PROBLEM 18.114



A solid sphere of radius $c = 3$ in. is attached as shown to a cord AB of length 15 in. The sphere spins about its diameter BC and precesses about the vertical axis AD. Knowing that $\theta = 20°$ and $\beta = 35°$, determine (*a*) the rate of spin of the sphere, (*b*) its rate of precession.

SOLUTION



Use principal centroidal axes x, y, z as shown.

Angular velocity: $\boldsymbol{\omega} = -\dot{\varphi}\sin\theta\mathbf{i} + (\dot{\psi} + \dot{\varphi}\cos\theta)\mathbf{k}$

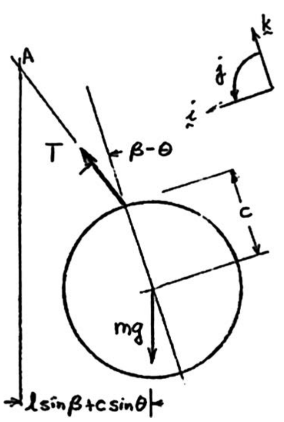
Angular momentum about the mass center G:

$$\mathbf{H}_G = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$
$$= -I'\dot{\varphi}\sin\theta\mathbf{i} + I(\dot{\psi} + \dot{\varphi}\cos\theta)\mathbf{k}$$

Let the reference frame $Gxyz$ be rotating with angular velocity

$$\boldsymbol{\Omega} = -\dot{\varphi}\sin\theta\mathbf{i} + \dot{\varphi}\cos\theta\mathbf{k}$$
$$\dot{\mathbf{H}}_G = \boldsymbol{\Omega}\times\mathbf{H}_G$$
$$= [I\dot{\varphi}\dot{\psi}\sin\theta - (I' - I)\dot{\varphi}^2\sin\theta\cos\theta]\mathbf{j}$$

Acceleration of the mass center:



$$\bar{\mathbf{a}} = (l\sin\beta + c\sin\theta)\dot{\varphi}^2 \longleftarrow$$

$$\Sigma\mathbf{F} = \Sigma\mathbf{F}_{\text{eff}}:$$

$$+\uparrow: \quad T\cos\beta - mg = 0, \qquad T = \frac{mg}{\cos\beta} \tag{1}$$

$$+\longleftarrow: \quad T\sin\beta = m\bar{a}$$

$$g\tan\beta = (l\sin\beta + c\sin\theta)\dot{\varphi}^2 \tag{2}$$

$$+\circlearrowleft\ \Sigma M_G = Tc\sin(\beta - \theta) = \dot{H}_G$$

$$\frac{mgc\sin(\beta - \theta)}{\cos\beta} = I\dot{\psi}\dot{\varphi}\sin\theta - (I' - I)\dot{\varphi}^2\sin\theta\cos\theta \tag{3}$$

Data:

$$c = 3\text{ in.} = 0.25\text{ ft}, \quad l = 15\text{ in.} = 1.25\text{ ft}$$
$$g = 32.2\text{ ft/s}^2$$
$$\frac{I}{m} = \frac{I'}{m} = \frac{2}{5}c^2 = 0.025\text{ ft}^2 \quad I' - I = 0,$$
$$\beta = 35°, \quad \theta = 20°$$

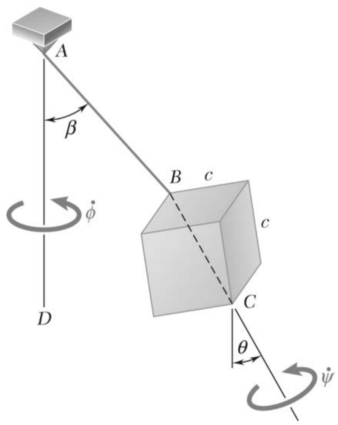
From Equation (2),

$$(32.2)\tan 35° = (1.25\sin 35° + 0.25\sin 20°)\dot{\varphi}^2$$
$$\dot{\varphi}^2 = 28.096$$
$$\dot{\varphi} = 5.3006 \text{ rad/s}$$

From Equation (3), $$\frac{(32.2)(0.25)\sin 15°}{\cos 35°} = (0.025)\dot{\psi}(5.3006)\sin 20°$$

(*a*) $\dot{\psi} = 56.119$ rad/s $\quad$ $\dot{\psi} = 56.1$ rad/s ◀

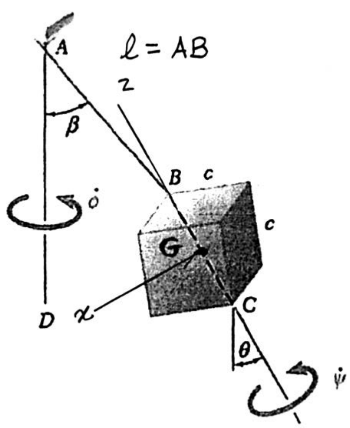
(*b*) $\dot{\varphi} = 5.30$ rad/s ◀

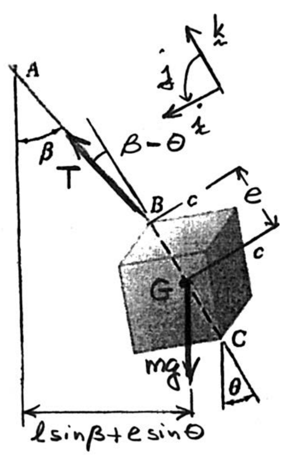


PROBLEM 18.115

A solid cube of side $c = 80$ mm is attached as shown to cord *AB*. It is observed to spin at the rate $\dot{\psi} = 40$ rad/s about its diagonal *BC* and to precess at the constant rate $\dot{\phi} = 5$ rad/s about the vertical axis *AD*. Knowing that $\beta = 30°$, determine the angle ϕ that the diagonal *BC* forms with the vertical. (*Hint:* The moment of inertia of a cube about an axis through its center is independent of the orientation of that axis.)

SOLUTION





Use centroidal axes *x*, *y*, *z* such that the *z* axis lies along the body diagonal *BC* and the *x* axis lies in the plane of *A*, *B*, *C*, and *D*. Let *e* be the length *GB*.

$$e = \frac{\sqrt{3}}{2}c$$

Moment of inertia: $$I_x = I_y = I_z = \frac{1}{6}mc^2$$

Angular velocity: $$\boldsymbol{\omega} = -\dot{\varphi}\sin\theta\mathbf{i} + (\dot{\psi} + \dot{\varphi}\cos\theta)\mathbf{k}$$

Angular momentum about the mass center *G*:

$$\mathbf{H}_G = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k} = -I'\dot{\varphi}\sin\theta\mathbf{i} + I(\dot{\psi} + \dot{\varphi}\cos\theta)\mathbf{k}$$

Let the reference frame *Gxyz* be rotating with angular velocity

$$\boldsymbol{\Omega} = -\dot{\varphi}\sin\theta\mathbf{i} + \dot{\varphi}\cos\theta\mathbf{k}$$

$$\dot{\mathbf{H}}_G = \boldsymbol{\Omega}\times\mathbf{H}_G$$

$$= [I\dot{\varphi}\dot{\psi}\sin\theta - (I' - I)\dot{\varphi}^2\sin\theta\cos\theta]\mathbf{j}$$

Acceleration of the mass center:

$$\bar{\mathbf{a}} = (l\sin\beta + e\sin\theta)\dot{\varphi}^2 \longleftarrow$$

$$\Sigma\mathbf{F} = \Sigma\mathbf{F}_{\text{eff}}:$$

$$+\uparrow: \quad T\cos\beta - mg = 0, \quad T = \frac{mg}{\cos\beta} \qquad (1)$$

$$+\longleftarrow: \quad T\sin\beta = m\bar{a}$$

$$g\tan\beta = (l\sin\beta + e\sin\theta)\dot{\varphi}^2 \qquad (2)$$

$$+\circlearrowleft \Sigma M_G = Te\sin(\beta - \theta) = \dot{H}_G$$

$$\frac{mge\sin(\beta - \theta)}{\cos\beta} = I\dot{\psi}\dot{\varphi}\sin\theta - (I' - I)\dot{\varphi}^2\sin\theta\cos\theta \qquad (3)$$

PROBLEM 18.115 (Continued)

Data:

$$c = 80 \text{ mm} = 0.08 \text{ m} \quad e = \frac{\sqrt{3}}{2}(0.08) = 0.069282 \text{ m}$$

$$\frac{I}{m} = \frac{I'}{m} = \frac{1}{6}c^2 = 1.06667 \times 10^{-3} \text{ m}^2 \quad I' - I = 0$$

$$\beta = 30° \quad \dot{\psi} = 40 \text{ rad/s} \quad \dot{\varphi} = 5 \text{ rad/s}$$

Substituting into Eq. (3),

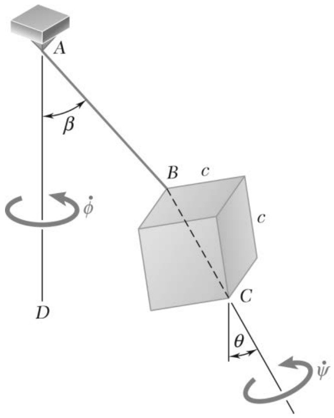
$$\frac{(9.81)(0.069282)\sin(\beta - \theta)}{\cos\beta} = (1.06667 \times 10^{-3})(40)(5)\sin\theta$$

$$0.67966(\sin\beta\cos\theta - \cos\beta\sin\theta) = 0.21333\sin\theta\cos\beta$$

$$0.89299\sin\theta\cos\beta = 0.67966\sin\beta\cos\theta$$

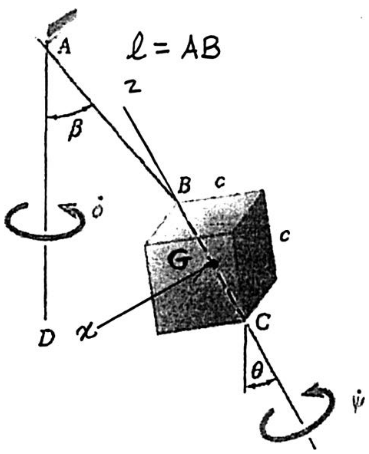
$$\tan\theta = 0.76111\tan\beta = 0.76111\tan 30° = 0.43942 \qquad \theta = 23.7° \blacktriangleleft$$

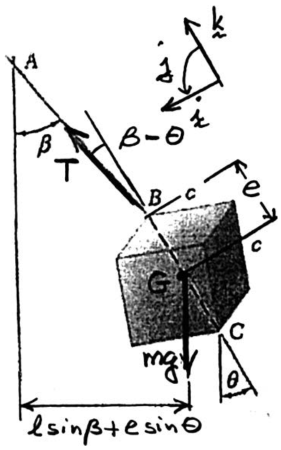
PROBLEM 18.116



A solid cube of side $c = 120$ mm is attached as shown to a cord AB of length 240 mm. The cube spins about its diagonal BC and precesses about the vertical axis AD. Knowing that $\theta = 25°$ and $\beta = 40°$, determine (*a*) the rate of spin of the cube, (*b*) its rate of precession. (See hint of Problem 18.115.)

SOLUTION





Use centroidal axes x, y, z such that the z axis lies along the body diagonal BC and the x axis lies in the plane of A, B, C, and D. Let e be the length GB.

$$e = \frac{\sqrt{3}}{2}c$$

Moment of inertia:
$$I_x = I_y = I_z = \frac{1}{6}mc^2$$

Angular velocity:
$$\boldsymbol{\omega} = -\dot{\varphi}\sin\theta\mathbf{i} + (\dot{\psi} + \dot{\varphi}\cos\theta)\mathbf{k}$$

Angular momentum about the mass center G:

$$\mathbf{H}_G = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k} = -I'\dot{\varphi}\sin\theta\mathbf{i} + I(\dot{\psi} + \dot{\varphi}\cos\theta)\mathbf{k}$$

Let the reference frame $Gxyz$ be rotating with angular velocity

$$\boldsymbol{\Omega} = -\dot{\varphi}\sin\theta\mathbf{i} + \dot{\varphi}\cos\theta\mathbf{k}$$

$$\dot{\mathbf{H}}_G = \boldsymbol{\Omega} \times \mathbf{H}_G$$

$$= [I\dot{\varphi}\dot{\psi}\sin\theta - (I' - I)\dot{\varphi}^2\sin\theta\cos\theta]\mathbf{j}$$

Acceleration of the mass center:

$$\bar{\mathbf{a}} = (l\sin\beta + e\sin\theta)\dot{\varphi}^2 \longleftarrow$$

$$\Sigma\mathbf{F} = \Sigma\mathbf{F}_{\text{eff}}:$$

$$+\uparrow: \quad T\cos\beta - mg = 0, \quad T = \frac{mg}{\cos\beta} \tag{1}$$

$$+\longleftarrow: \quad T\sin\beta = m\bar{a}$$

$$g\tan\beta = (l\sin\beta + e\sin\theta)\dot{\varphi}^2 \tag{2}$$

$$+\circlearrowleft \Sigma M_{G.} = Te\sin(\beta - \theta) = \dot{H}_G$$

$$\frac{mge\sin(\beta - \theta)}{\cos\beta} = I\dot{\psi}\dot{\varphi}\sin\theta - (I' - I)\dot{\varphi}^2\sin\theta\cos\theta \tag{3}$$

PROBLEM 18.116 (Continued)

Data:

$$c = 120 \text{ mm} = 0.12 \text{ m}$$

$$e = \frac{\sqrt{3}}{2}(0.12) = 0.103923 \text{ m}$$

$$l = 240 \text{ mm} = 0.24 \text{ m}$$

$$\frac{I}{m} = \frac{I'}{m} = \frac{1}{6}c^2 = \frac{1}{6}(0.12)^2 = 2.4\times10^3 \text{ m}^2 \quad I' - I = 0$$

$$\theta = 25° \quad \beta = 40° \quad \theta - \beta = 15°$$

From Eq. (2), $$(9.81)\tan 40° = (0.24\sin 40° + 0.103923\sin 25°)\dot{\varphi}^2$$

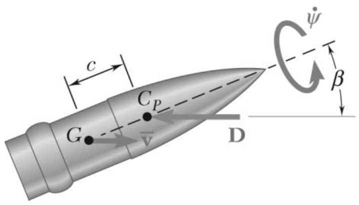
$$\dot{\varphi}^2 = 41.534 \qquad \dot{\varphi} = 6.4447 \text{ rad/s}$$

From Eq. (3), $$\frac{(9.81)(0.103923)\sin 15°}{\cos 40°} = (2.4\times10^{-3})\dot{\psi}(6.4447)\sin 25°$$

(*a*) $$\dot{\psi} = 52.694 \text{ rad/s}$$ $\dot{\psi} = 52.7 \text{ rad/s}$ ◀

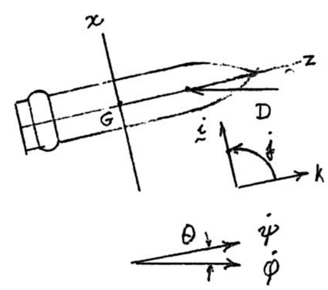
(*b*) $\dot{\varphi} = 6.44 \text{ rad/s}$ ◀

PROBLEM 18.117



A high-speed photographic record shows that a certain projectile was fired with a horizontal velocity $\bar{\mathbf{v}}$ of 2000 ft/s and with its axis of symmetry forming an angle $\beta = 3°$ with the horizontal. The rate of spin $\dot{\psi}$ of the projectile was 6000 rpm, and the atmospheric drag was equivalent to a force **D** of 25 lb acting at the center of pressure C_P located at a distance $c = 6$ in. from G. (*a*) Knowing that the projectile has a weight of 45 lb and a radius of gyration of 2 in. with respect to its axis of symmetry, determine its approximate rate of steady precession. (*b*) If it is further known that the radius of gyration of the projectile with respect to a transverse axis through G is 8 in., determine the exact values of the two possible rates of precession.

SOLUTION



Choose principal centroidal axes x, y, z as shown.

The symmetry (spin) axis is the z axis.

Reduce the drag force to a force-couple system at the mass center.

$$\mathbf{F} = (D\sin\beta - W\cos\beta)\mathbf{i} - (D\cos\beta - W\sin\beta)\mathbf{k}$$

$$\mathbf{M}_G = Dc\sin\beta\,\mathbf{j}$$

For the occurrence of steady precession, the precession axis must be parallel to the drag force.

Thus, $\theta = \beta = 3°.$

(*a*) Using Equation (18.44), $\Sigma M_G = \dot{\mathbf{H}}_G$

$$Dc\sin\beta = (I\omega_z - I'\dot{\varphi}\cos\theta)\dot{\varphi}\sin\theta$$

Using $\beta = \theta$ and $\omega_z = \dot{\psi} + \dot{\varphi}\cos\theta$ gives

$$I\dot{\psi}\dot{\varphi} - (I' - I)\dot{\varphi}^2\cos\theta = Dc \qquad (1)$$

Neglecting the quadratic term in $\dot{\varphi}$,

$$I\dot{\psi}\dot{\varphi} \approx Dc \qquad \dot{\varphi} \approx \frac{Dc}{I\dot{\psi}}$$

<u>Data</u>: $W = 45$ lb, $m = \dfrac{W}{g} = \dfrac{45}{32.2} = 1.3975$ slug

$$I = mk_z^2 = (1.3975)\left(\frac{2}{12}\right)^2 = 0.0388205 \text{ slug}\cdot\text{ft}^2$$

$$\dot{\psi} = 6000 \text{ rpm} = 628.32 \text{ rad/s}, \quad c = \frac{6}{12} \text{ in.} = 0.5 \text{ ft}$$

$$\dot{\varphi} \approx \frac{(25)(0.5)}{(0.0388205)(628.32)} = 0.51248 \text{ rad/s}$$

$\dot{\varphi} \approx 4.89$ rpm ◀

(*b*) $$I' = mk_x^2 = (1.3975)\left(\frac{8}{12}\right)^2 = 0.62112 \text{ slug} \cdot \text{ft}^2$$

$$I' - I = 0.58230 \text{ slug} \cdot \text{ft}^2$$

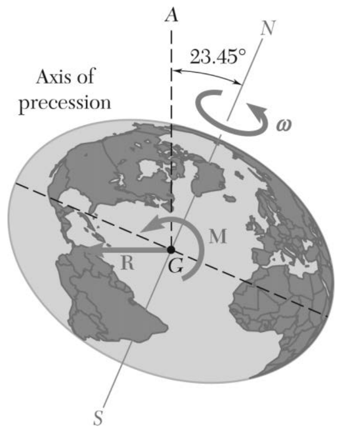
Substituting into Equation (1),

$$(0.0388205)(628.32)\dot{\varphi} - (0.58230)\dot{\varphi}^2 \cos 3° = (25)(0.5)$$

$$0.58150\dot{\varphi}^2 - 24.3912\dot{\varphi} + 12.5 = 0$$

$$\dot{\varphi} = 20.9727 \pm 20.4538$$

$$= 0.51890 \text{ rad/s}, \quad 41.427 \text{ rad/s} \qquad \dot{\varphi} = 4.96 \text{ rpm}, 396 \text{ rpm} \blacktriangleleft$$



PROBLEM 18.118

If the earth were a sphere, the gravitational attraction of the sun, moon, and planets would at all times be equivalent to a single force **R** acting at the mass center of the earth. However, the earth is actually an oblate spheroid and the gravitational system acting on the earth is equivalent to a force **R** and a couple **M**. Knowing that the effect of the couple **M** is to cause the axis of the earth to precess about the axis *GA* at the rate of one revolution in 25,800 years, determine the average magnitude of the couple **M** applied to the earth. Assume that the average density of the earth is 5.51 g/cm^3, that the average radius of the earth is 6370 km, and that $\bar{I} = \frac{2}{5}mR^2$. (*Note:* This forced precession is known as the precession of the equinoxes and is not to be confused with the free precession discussed in Problem 18.123.)

SOLUTION

$$25{,}800 \text{ years} = (25{,}800 \text{ yr})(365.24 \text{ day/yr})(24 \text{ h/day})(3600 \text{ s/h})$$
$$= 814.16\times10^9 \text{ s}$$
$$\dot{\varphi} = \frac{2\pi}{814.16\times10^9} = 7.7173\times10^{-12} \text{ rad/s}$$
$$\dot{\psi} = \frac{2\pi}{(23.93 \text{ h})(3600 \text{ s/h})} = 72.935\times10^{-6} \text{ rad/s}$$

Mass density of Earth: $\rho = 5.51 \text{ g/cm}^3 = 5510 \text{ kg/m}^3$

Radius of Earth: $R = 6370 \text{ km} = 6.370\times10^6 \text{ m}$

Mass of Earth: $\frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi(6.370\times10^6)^3(5510) = 5.9657\times10^{24} \text{ kg}$

$$\bar{I} = \frac{2}{5}mR^2 = \frac{2}{5}(5.9657\times10^{24})(6.370\times10^6)^2$$
$$= 96.827\times10^{36} \text{ kg}\cdot\text{m}^2$$

Using Equation (18.44),

$$M = (I\omega_z - I'\dot{\varphi}\cos\theta)\dot{\varphi}\sin\theta$$
$$= [I\dot{\psi} + (I - I')\dot{\varphi}\cos\theta]\dot{\varphi}\sin\theta$$
$$= I\dot{\psi}\dot{\varphi}\sin\theta$$
$$= (96.827\times10^{36})(72.935\times10^{-6})(7.7173\times10^{-12})\sin 23.45°$$
$$= 21.688\times10^{21} \text{ N}\cdot\text{m}$$

$$M = 21.7\times10^{21} \text{ N}\cdot\text{m} \blacktriangleleft$$

PROBLEM 18.119

Show that for an axisymmetrical body under no force, the rates of precession and spin can be expressed, respectively, as

$$\dot{\varphi} = \frac{H_G}{I'}$$

and

$$\dot{\psi} = \frac{H_G \cos\theta (I' - I)}{II'}$$

where H_G is the constant value of the angular momentum of the body.

SOLUTION

By Equations (18.48), $\omega_x = -\frac{H_G \sin\theta}{I'}, \quad \omega_y = 0, \quad \omega_z = \frac{H_G \cos\theta}{I}$

By Equation (18.35) with $\dot{\theta} = 0,$

$$\omega_x = -\dot{\varphi}\sin\theta, \quad \omega_y = 0, \quad \omega_z = \dot{\psi} + \dot{\varphi}\cos\theta$$

Eliminating ω_x and ω_z,

$$\dot{\varphi} = \frac{H_G}{I'}$$

$$\dot{\psi} + \dot{\varphi}\cos\theta = \frac{H_G \cos\theta}{I}$$

$$\dot{\psi} = \frac{H_G \cos\theta}{I} - \dot{\varphi}\cos\theta = \frac{H_G \cos\theta}{I} - \frac{H_G \cos\theta}{I'} = \frac{H_G (I' - I)\cos\theta}{II'}$$

PROBLEM 18.120

(*a*) Show that for an axisymmetrical body under no force, the rate of precession can be expressed as

$$\dot{\phi} = \frac{I\omega_z}{I'\cos\theta}$$

where ω_z is the rectangular component of $\boldsymbol{\omega}$ along the axis of symmetry of the body. (*b*) Use this result to check that the condition (18.44) for steady precession is satisfied by an axisymmetrical body under no force.

SOLUTION

(*a*) Angular velocity of the body: $\boldsymbol{\omega} = -\dot{\varphi}\sin\theta\mathbf{i} + \omega_2\mathbf{k}$

Its angular momentum about *G*: $\mathbf{H}_G = -I'\dot{\varphi}\sin\theta\mathbf{i} + I\omega_2\mathbf{k}$

Let the reference frame *Gxyz* be rotating with angular velocity $\boldsymbol{\Omega}$, where

$$\boldsymbol{\Omega} = -\dot{\varphi}\sin\theta\mathbf{i} + \dot{\varphi}\cos\theta\mathbf{k}$$

$$\begin{aligned}\dot{\mathbf{H}}_G &= (\dot{\mathbf{H}}_G)_{Gxyz} + \boldsymbol{\Omega}\times\mathbf{H}_G \\ &= 0 + (-\dot{\varphi}\sin\theta\mathbf{i} + \dot{\varphi}\cos\theta\mathbf{k})\times(-I'\dot{\varphi}\sin\theta\mathbf{i} + I\omega_z\mathbf{k}) \\ &= I\dot{\varphi}\omega_z\sin\theta - I'\dot{\varphi}^2\sin\theta\cos\theta\end{aligned}$$

For no force, $\dot{\mathbf{H}}_G = 0$

Hence, $(I\omega_z - I'\dot{\varphi}\cos\theta)\dot{\varphi}\sin\theta = 0$ or $I\omega_z - I'\dot{\varphi}\cos\theta = 0$ (1)

Solving for $\dot{\varphi}$,

$$\dot{\varphi} = \frac{I\omega_z}{I'\cos\theta}$$

(*b*) Comparing Equation (1) with Equation (18.44) yields $\Sigma\mathbf{M}_0 = 0$, which is the condition for no force.

PROBLEM 18.121

Show that the angular velocity vector $\boldsymbol{\omega}$ of an axisymmetrical body under no force is observed from the body itself to rotate about the axis of symmetry at the constant rate

$$n = \frac{I' - I}{I'}\omega_z$$

where ω_z is the rectangular component of $\boldsymbol{\omega}$ along the axis of symmetry of the body.

SOLUTION

Angular velocity of the body: $\boldsymbol{\omega} = -\dot{\varphi}\sin\theta\mathbf{i} + (\dot{\varphi}\cos\theta + \dot{\psi})\mathbf{k}$

Let the reference frame $Gxyz$ be rotating with angular velocity $\boldsymbol{\Omega}$, where

$$\boldsymbol{\Omega} = -\dot{\varphi}\sin\theta\mathbf{i} + \dot{\varphi}\cos\theta\mathbf{j}$$

Angular acceleration of the body: $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}_{Gxyz} + \boldsymbol{\Omega}\times\boldsymbol{\omega}$

$$\boldsymbol{\alpha} = 0 + \dot{\varphi}\dot{\psi}\sin\theta\mathbf{j}$$

The rate of change of angular velocity as observed from the body is $-\boldsymbol{\alpha}$.

Assume that $-\boldsymbol{\alpha}$ may be represented as the angular velocity vector rotating with angular velocity $l\mathbf{i} + m\mathbf{j} + n\mathbf{k}$.

$$\begin{aligned} -\boldsymbol{\alpha} &= (l\mathbf{i} + m\mathbf{j} + n\mathbf{k})\times(\omega_x\mathbf{i} + \omega_z\mathbf{k}) \\ &= m\omega_z\mathbf{i} + (n\omega_x - l\omega_z)\mathbf{j} - m\omega_x\mathbf{k} \end{aligned}$$

Matching components: $\mathbf{i}$: $0 = m\omega_z \quad m = 0$

$\mathbf{j}$: $-\dot{\varphi}\dot{\psi}\sin\theta = n\omega_x - l\omega_z$ (1)

$\mathbf{k}$: $0 = -m\omega_x \quad m = 0$

From Equation (1), $-\dot{\varphi}\dot{\psi}\sin\theta = -n\dot{\varphi}\sin\theta - l(\dot{\varphi}\cos\theta + \dot{\psi})$

from which $l = 0$ and $n = \dot{\psi}$.

Using Equation (18.44) with $\Sigma\mathbf{M}_0 = 0$ yields $I\omega_z - I'\dot{\varphi}\cos\theta = 0$

or $\dot{\varphi}\cos\theta = \dfrac{I\omega_z}{I'}$

But $\omega_z = \dot{\psi} + \dot{\varphi}\cos\theta = \dot{\psi} + \dfrac{I\omega_z}{I'}$

Solving for $\dot{\psi}$, $\dot{\psi} = \dfrac{I' - I}{I}\omega_z$

Using $\omega_z = \omega_2$ and $n = \dot{\psi}$ yields $n = \dfrac{I' - I}{I'}\omega_z$. ◀

PROBLEM 18.122

For an axisymmetrical body under no force, prove (*a*) that the rate of retrograde precession can never be less than twice the rate of spin of the body about its axis of symmetry, (*b*) that in Figure 18.24 the axis of symmetry of the body can never lie within the space cone.

SOLUTION

For no force, $\mathbf{M}_G = 0$

(*a*) Using Equation (18.44), $0 = (\bar{I}\,\omega_z - \bar{I}'\dot{\varphi}\cos\theta)\dot{\varphi}\sin\theta$

$$\bar{I}\,\omega_z - \bar{I}'\dot{\varphi}\cos\theta = 0$$

Using $\omega_z = \dot{\psi} + \dot{\varphi}\cos\theta$ yields $\bar{I}\,\dot{\psi} - (\bar{I}' - I)\dot{\varphi}\cos\theta = 0$

$$\dot{\varphi} = \frac{\bar{I}\,\dot{\psi}}{(\bar{I}' - \bar{I}\,)\cos\theta}$$

$$|\dot{\varphi}| = \frac{\bar{I}\,\dot{\psi}}{(\bar{I} - \bar{I}')}\sec\theta$$

For a flat disk, $\bar{I}' = \frac{1}{2}\bar{I}$

For any other shape, $\bar{I}' > \frac{1}{2}\bar{I}$

Hence, $\bar{I} - \bar{I}' < \frac{1}{2}I$ and $\dfrac{\bar{I}}{\bar{I} - \bar{I}'} > 2$

Also, $\sec\theta > 1$: $|\dot{\varphi}| > 2\dot{\psi}$

(*b*) $\tan\gamma = \dfrac{I}{I'}\tan\theta$ or $\tan\theta = \dfrac{I'}{I}\tan\gamma > \dfrac{1}{2}\tan\gamma$

Angle of surface of space cone is $\gamma - \theta$.

Angle of axis is θ.

$$\tan\theta > \frac{1}{2}\tan[\theta + (\gamma - \theta)] = \frac{1}{2}\frac{\tan\theta + \tan(\gamma - \theta)}{1 + \tan\theta\tan(\gamma - \theta)}$$

$$[1 + \tan\theta\tan(\gamma - \theta)]\tan\theta > \frac{1}{2}\tan\theta + \frac{1}{2}\tan(\gamma - \theta)$$

PROBLEM 18.122 (Continued)

$$\tan\theta + \tan^2\theta\tan(\gamma-\theta) > \frac{1}{2}\tan\theta + \frac{1}{2}\tan(\gamma-\theta)$$

$$\frac{1}{2}\tan\theta > \frac{1}{2}\tan(\gamma-\theta) - \tan^2\theta\tan(\gamma-\theta)$$

$$\frac{1}{2}\tan\theta > \frac{1}{2}\tan(\gamma-\theta)$$

$$\tan\theta > \tan(\gamma-\theta)$$

$$\theta > (\gamma-\theta)$$

The axis lies outside the space cone. ◀

PROBLEM 18.123

Using the relation given in Problem 18.121, determine the period of precession of the north pole of the earth about the axis of symmetry of the earth. The earth may be approximated by an oblate spheroid of axial moment of inertia I and of transverse moment of inertia $I' = 0.9967I$. (*Note:* Actual observations show a period of precession of the north pole of about 432.5 mean solar days; the difference between the observed and computed periods is due to the fact that the earth is not a perfectly rigid body. The free precession considered here should not be confused with the much slower precession of the equinoxes, which is a forced precession. See Problem 18.118.)

SOLUTION

Angular velocity of the body: $\boldsymbol{\omega} = -\dot{\varphi}\sin\theta\mathbf{i} + (\dot{\varphi}\cos\theta + \dot{\psi})\mathbf{k}$

Let the reference frame $Gxyz$ be rotating with angular velocity $\boldsymbol{\Omega}$, where

$$\boldsymbol{\Omega} = -\dot{\varphi}\sin\theta\mathbf{i} + \dot{\varphi}\cos\theta\mathbf{j}$$

Angular acceleration of the body: $\boldsymbol{\alpha} = \dot{\omega}_{Gxyz} + \boldsymbol{\Omega}\times\boldsymbol{\omega}$

The rate of change of angular velocity as observed from the body is $-\boldsymbol{\alpha}$.

Assume that $-\alpha$ may be represented as the angular velocity vector rotating with angular velocity

$$l\mathbf{i} + m\mathbf{j} + n\mathbf{k}.$$

$$-\boldsymbol{\alpha} = (l\mathbf{i} + m\mathbf{j} + n\mathbf{k})\times(\omega_x\mathbf{i} + \omega_z\mathbf{k})$$
$$= -m\omega_z\mathbf{i} + (n\omega_x - l\omega_z\mathbf{j}) - m\omega_x\mathbf{k}$$

Matching components: $\mathbf{i}:\quad 0 = m\omega_z \qquad m = 0$

$$\mathbf{j}:\qquad -\dot{\varphi}\dot{\psi}\sin\theta = n\omega_x - l\omega_z \qquad (1)$$

$$\mathbf{k}:\quad 0 = -m\omega_x \qquad m = 0$$

From Equation (1), $-\dot{\varphi}\dot{\psi}\sin\theta = -n\dot{\varphi}\sin\theta - l(\dot{\varphi}\cos\theta + \psi)$

From which $l = 0$ and $n = \dot{\psi}$.

Using Equation (18.44) with $\Sigma\mathbf{M}_0 = 0$ yields $I\omega_z - I'\dot{\varphi}\cos\theta = 0$

or $\dot{\varphi}\cos\theta = \dfrac{I\omega_z}{I'}$

But $\omega_z = \dot{\psi} + \dot{\varphi}\cos\theta = \dot{\psi} + \dfrac{I\omega_z}{I'}$

Solving for $\dot{\psi}$, $\dot{\psi} = \dfrac{I' - I}{I}\omega_z$

PROBLEM 18.123 (Continued)

Using $\omega_z = \omega_2$ and $n = \dot{\psi}$ yields $n = \frac{I' - I}{I'}\omega_2$

Data for Earth:

$$I' = 0.9967I$$

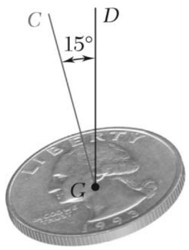
$$I' - I = -0.0033I,$$

$$n = -\frac{0.0033}{0.9967}\omega_2 = -0.003311\omega_2$$

$$\text{period} = \frac{2\pi}{|n|} = \frac{1}{0.003311}\frac{2\pi}{\omega_2} = 302.03\frac{2\pi}{\omega_2}$$

$$= (302.03)(24\text{ h}) = 7248\text{ h}$$

period = 302 days ◀



PROBLEM 18.124

A coin is tossed into the air. It is observed to spin at the rate of 600 rpm about an axis *GC* perpendicular to the coin and to precess about the vertical direction *GD*. Knowing that *GC* forms an angle of 15° with *GD*, determine (*a*) the angle that the angular velocity $\boldsymbol{\omega}$ of the coin forms with *GD*, (*b*) the rate of precession of the coin about *GD*.

SOLUTION

Moments of inertia: $I = \frac{1}{2}mr^2$

$I' = \frac{1}{4}mr^2$

Euler angle θ for steady precession: $\theta = 15°$

For axisymmetric body under no force, Equation (18.49) gives for the body cone angle:

$$\tan\gamma = \frac{I}{I'}\tan\theta = 2\tan 15° \quad \gamma = 28.187°$$

(*a*) Angular velocity: $\boldsymbol{\omega} = \omega(-i\sin\gamma + \mathbf{k}\cos\gamma)$

Its projection onto the vertical direction is

$$\omega\cos\beta = \boldsymbol{\omega}\cdot\mathbf{k} = \omega(-i\sin\gamma + \mathbf{k}\cos\gamma)\cdot(-\sin\theta\mathbf{i} + \cos\theta\mathbf{k})$$
$$= \omega(\sin\gamma\sin\theta + \cos\gamma\cos\theta) = \omega\cos(\gamma - \theta)$$

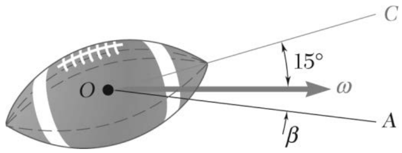
$$\cos\beta = \cos(\gamma-\theta) \qquad \beta = |\gamma - \theta| = 28.187° - 15° = 13.187°$$

Angle between $\boldsymbol{\omega}$ and vertical direction *GD*: $\beta = 13.19°$ ◀

(*b*)
$$\tan\gamma = -\frac{\omega_x}{\omega_z} = \frac{\dot{\varphi}\sin\theta}{\dot{\varphi}\cos\theta + \dot{\psi}}$$

from which
$$\dot{\varphi} = \frac{\dot{\psi}\sin\gamma}{\sin(\theta-\gamma)} = \frac{\dot{\psi}\sin 28.187°}{-\sin 13.187°} = -2.0705\dot{\psi}$$

With $\dot{\psi} = 600$ rpm, $\dot{\varphi} = (-2.0705)(600)$ $\quad |\dot{\varphi}| = 1242$ rpm (retrograde) ◀



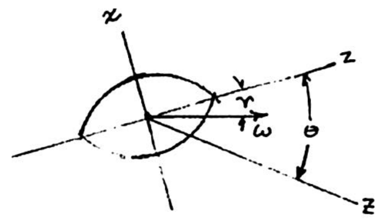
PROBLEM 18.125

The angular velocity vector of a football which has just been kicked is horizontal, and its axis of symmetry OC is oriented as shown. Knowing that the magnitude of the angular velocity is 200 rpm and that the ratio of the axis and transverse moments of inertia is $I/I' = \frac{1}{3}$, determine (*a*) the orientation of the axis of precession OA, (*b*) the rates of precession and spin.

SOLUTION

$$\tan\gamma = -\frac{\omega_x}{\omega_z}$$

$$\gamma = 15°$$



For steady precession with no force,

$$\tan\theta = \frac{I'}{I}\tan\gamma$$

$$= 3\tan 15°$$

$$\theta = 38.794°$$

(*a*) $\beta = \theta - \gamma = 38.794 - 15°$ $\beta = 23.8°$ ◀

(*b*) $\omega_x = -\dot{\varphi}\sin\theta = -\omega\sin\gamma$

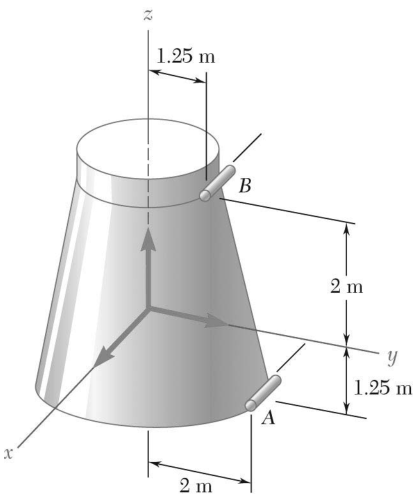
$$\dot{\varphi} = \frac{\omega\sin\gamma}{\sin\theta} = \frac{(200 \text{ rpm})\sin 15°}{\sin(38.794°)}$$

$= 82.621$ rpm *precession*: $\dot{\varphi} = 82.6$ rpm ◀

$$\omega_z = \dot{\psi} + \dot{\varphi}\cos\theta = \omega\cos\gamma$$

$$\dot{\psi} = \omega\cos\gamma - \dot{\varphi}\cos\theta$$

$= 200\cos 15° - 82.621\cos 38.794°$ *spin*: $\dot{\psi} = 128.8$ rpm ◀



PROBLEM 18.126

The space capsule has no angular velocity when the jet at *A* is activated for 1 s in a direction parallel to the *x* axis. Knowing that the capsule has a mass of 1000 kg, that its radii of gyration are $\bar{k}_z = \bar{k}_y = 1.00$ m and $\bar{k}_z = 1.25$ m, and that the jet at *A* produces a thrust of 50 N, determine the axis of precession and the rates of precession and spin after the jet has stopped.

SOLUTION

Initial angular momentum about the mass center:

$$\mathbf{H}_G = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k} = 0$$

Applied impulse at *A*: $\mathbf{A}\,\Delta t = (50\text{ N})(1\text{ s})\mathbf{i} = (50\text{ N}\cdot\text{s})\mathbf{i}$

Its moment about the mass center *G*:

$$\mathbf{r}_{A/G} \times \mathbf{A}\,\Delta t = [(2\text{ m})\mathbf{j} - (1.25\text{ m})\mathbf{k}] \times (50\text{ N}\cdot\text{s})\mathbf{i}$$
$$= -(100\text{ N}\cdot\text{m}\cdot\text{s})\mathbf{k} - (62.5\text{ N}\cdot\text{m}\cdot\text{s})\mathbf{j}$$

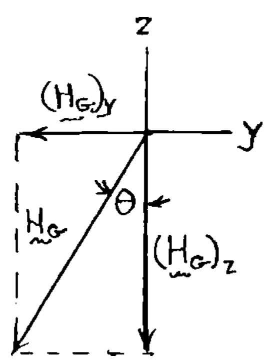
Principle of impulse and momentum. (Moments about *G*)

$$(\mathbf{H}_G)_0 = \mathbf{r}_{A/G} \times \mathbf{A}\,\Delta t = \mathbf{H}_G$$

where $\mathbf{H}_G$ is the final angular momentum about *G*.

$$0 + \mathbf{r}_{G/A} \times \mathbf{A}\,\Delta t = (H_G)_x \mathbf{i} + (H_G)_y \mathbf{j} + (H_G)_z \mathbf{k}$$

Angular momentum vector components:



i: $0 = (H_G)_x$

j: $-62.5\text{ N}\cdot\text{m}\cdot\text{s} = (H_G)_y = -62.5\text{ kg}\cdot\text{m}^2/\text{s}$

k: $-100\text{ N}\cdot\text{m}\cdot\text{s} = (H_G)_z = -100\text{ kg}\cdot\text{m}^2/\text{s}$

$$\tan\theta = \frac{(H_G)_y}{(H_G)_z} = 0.625$$

$\theta = 32.005°$ ◀

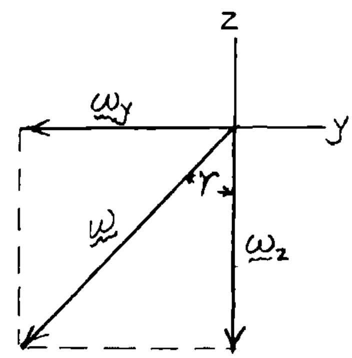
PROBLEM 18.126 (Continued)

Moments of inertia:

$$\bar{I}_x = m\bar{k}_k^2 = (1000\text{ kg})(1\text{ m})^2 = 1000\text{ kg}\cdot\text{m}^2$$
$$\bar{I}_y = m\bar{k}_y^2 = (1000\text{ kg})(1\text{ m})^2 = 1000\text{ kg}\cdot\text{m}^2$$
$$\bar{I}_z = m\bar{k}_z^2 = (1000\text{ kg})(1.25\text{ m})^2 = 1562.5\text{ kg}\cdot\text{m}^2$$

Angular velocity vector components:

$$\omega_x = \frac{(H_G)_x}{\bar{I}_x} = 0$$
$$\omega_y = \frac{(H_G)_y}{\bar{I}_y} = \frac{-62.5}{1000} = -0.0625\text{ rad/s}$$
$$\omega_z = \frac{(H_G)_z}{\bar{I}_z} = \frac{-100}{1562.5} = -0.0640\text{ rad/s}$$
$$\tan\gamma = \frac{\omega_y}{\omega_z} = 0.97656 \qquad \gamma = 44.321°$$
$$\omega = \sqrt{\omega_y^2 + \omega_z^2} = 0.089455\text{ rad/s.}$$



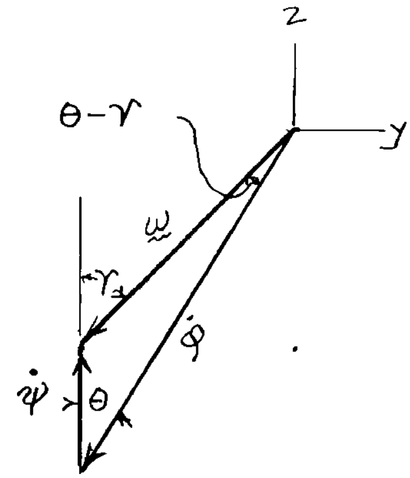
Rates of precession and spin.

The angular velocity is resolved into a component $\dot{\varphi}$ (rate of precession) parallel to $\mathbf{H}_G$ and $\dot{\psi}$ (rate of spin) parallel to the axis of symmetry.

$$\gamma - \theta = 12.316°$$

Law of sines:

$$\frac{\dot{\varphi}}{\sin\gamma} = \frac{\dot{\psi}}{\sin(\gamma-\theta)} = \frac{\omega}{\sin\theta}$$
$$\dot{\varphi} = \frac{\omega\sin\gamma}{\sin\theta} = \frac{0.089455\sin 44.321°}{\sin 32.005°}$$
$$\dot{\psi} = \frac{\omega\sin(\gamma-\theta)}{\sin\theta} = \frac{0.089455\sin 12.316°}{\sin 32.005°}$$

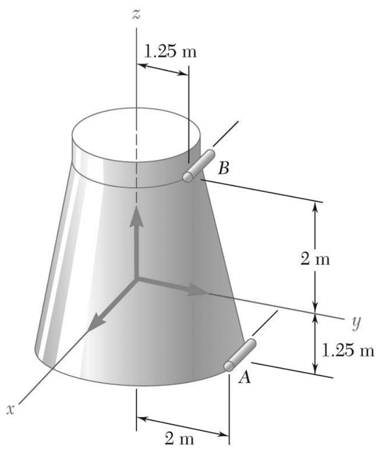


Rate of precession: $\dot{\varphi} = 0.1179\text{ rad/s}$ $\dot{\varphi} = 1.126\text{ rpm}$ ◀

Rate of spin: $\dot{\psi} = 0.0360\text{ rad/s}$ $\dot{\psi} = 0.344\text{ rpm}$ ◀

Since $\gamma > \theta$, the precession is <u>retrograde</u>.

PROBLEM 18.127



The space capsule has an angular velocity $\omega = (0.02 \text{ rad/s})\mathbf{j} + (0.10 \text{ rad/s})\mathbf{k}$ when the jet at *B* is activated for 1 s in a direction parallel to the *x* axis. Knowing that the capsule has a mass of 1000 kg, that its radii of gyration are $\bar{k}_z = \bar{k}_y = 1.00 \text{ m}$ and $\bar{k}_z = 1.25 \text{ m}$, and that the jet at *B* produces a thrust of 50 N, determine the axis of precession and the rates of precession and spin after the jet has stopped.

SOLUTION

Moments of inertia:

$$\bar{I}_x = m\bar{k}_k^2 = (1000 \text{ kg})(1 \text{ m})^2 = 1000 \text{ kg}\cdot\text{m}^2$$
$$\bar{I}_y = m\bar{k}_y^2 = (1000 \text{ kg})(1 \text{ m})^2 = 1000 \text{ kg}\cdot\text{m}^2$$
$$\bar{I}_z = m\bar{k}_z^2 = (1000 \text{ kg})(1.25 \text{ m})^2 = 1562.5 \text{ kg}\cdot\text{m}^2$$

Initial angular velocity: $(\omega_x)_0 = 0$

$(\omega_y)_0 = 0.02 \text{ rad/s}$

$(\omega_z)_0 = 0.10 \text{ rad/s}$

Initial angular momentum about *G*:

$$(\mathbf{H}_G)_0 = \bar{I}_x(\omega_x)_0\mathbf{i} + \bar{I}_y(\omega_y)_0\mathbf{j} + \bar{I}_z(\omega_z)_0\mathbf{k}$$
$$= (1000)(0)\mathbf{i} + (1000)(0.02)\mathbf{j} + (1562.5)(0.10)\mathbf{k}$$
$$= (20 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (156.25 \text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

Applied impulse at *B*: $\mathbf{B}\,\Delta t = (50 \text{ N})(1 \text{ s})\mathbf{i} = (50 \text{ N}\cdot\text{s})\mathbf{i}$

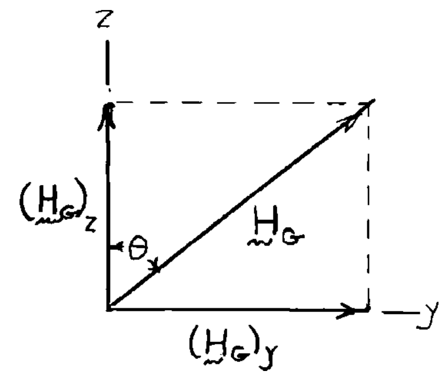
Its moment about the mass center *G*:

$$\mathbf{r}_{B/G} \times \mathbf{B}\,\Delta t = [(1.25 \text{ m})\mathbf{j} + (2 \text{ m})\mathbf{k}] \times (50 \text{ N}\cdot\text{s})\mathbf{i}$$
$$= (100 \text{ N}\cdot\text{m}\cdot\text{s})\mathbf{j} - (62.5 \text{ N}\cdot\text{m}\cdot\text{s})\mathbf{k}$$

Principle of impulse and momentum. (Moments about *G*):

$$(\mathbf{H}_G)_0 + \mathbf{r}_{B/G} \times \mathbf{B}\,\Delta t = \mathbf{H}_G$$

where $\mathbf{H}_G$ is the final angular momentum about *G*.



Angular momentum vector components:

$$\mathbf{i}: \quad 0 + 0 = (H_G)_x = 0$$

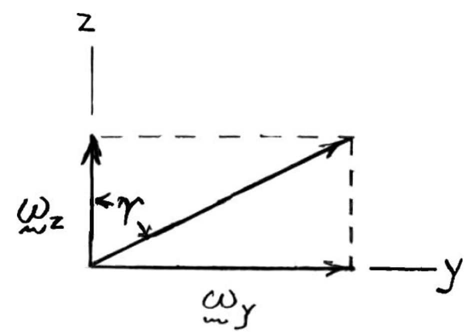
$$\mathbf{j}: \quad 20\ \text{kg}\cdot\text{m}^2/\text{s} + 100\ \text{N}\cdot\text{m}\cdot\text{s} = (H_G)_y = 120\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$\mathbf{k}: \quad 156.25\ \text{kg}\cdot\text{m}^2/\text{s} - 62.5\ \text{N}\cdot\text{m}\cdot\text{s} = (H_G)_z = 93.75\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$\tan\theta = \frac{(H_G)_y}{(H_G)_z} = 1.28$$

$\theta = 52.001°$ ◀

Angular velocity vector components:



$$\omega_x = \frac{(H_G)_x}{\bar{I}_x} = 0$$

$$\omega_y = \frac{(H_G)_y}{\bar{I}_y} = \frac{120}{1000} = 0.12\ \text{rad/s}$$

$$\omega_z = \frac{(H_G)_z}{\bar{I}_z} = \frac{93.75}{1562.5} = 0.060\ \text{rad/s}$$

$$\tan\gamma = \frac{\omega_y}{\omega_z} = 2.0000 \quad \gamma = 63.435°$$

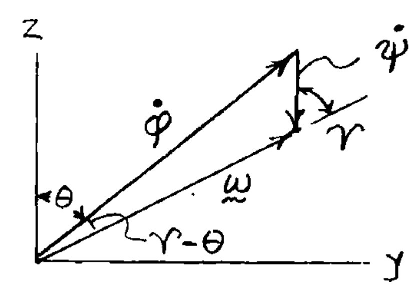
$$\omega = \sqrt{\omega_y^2 + \omega_z^2} = 0.134164\ \text{rad/s}$$

Rates of precession and spin.

The angular velocity is resolved into a component $\dot{\varphi}$ (rate of precession) parallel to $\mathbf{H}_G$ and $\dot{\psi}$ (rate of spin) parallel to the axis of symmetry.

$$\gamma - \theta = 11.434°$$

Law of sines:



$$\frac{\dot{\varphi}}{\sin\gamma} = \frac{\dot{\psi}}{\sin(\gamma-\theta)} = \frac{\omega}{\sin\theta}$$

$$\dot{\varphi} = \frac{\omega\sin\gamma}{\sin\theta} = \frac{0.134164\sin 63.435°}{\sin 52.001°}$$

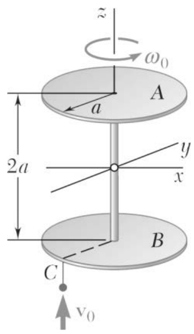
$$\dot{\psi} = \frac{\omega\sin(\gamma-\theta)}{\sin\theta} = \frac{0.134164\sin 11.434°}{\sin 52.001°}$$

Rate of precession: $\dot{\varphi} = 0.1523\ \text{rad/s}$ ◀

Rate of spin: $\dot{\psi} = 0.0338\ \text{rad/s}$ ◀

Since $\gamma > \theta$, the precession is <u>retrograde</u>.

PROBLEM 18.128



Solve Sample Problem 18.6, assuming that the meteorite strikes the satellite at C with a velocity $\mathbf{v}_0 = (2000 \text{ m/s})\mathbf{i}$.

PROBLEM 18.6 A space satellite of mass m is known to be dynamically equivalent to two thin disks of equal mass. The disks are of radius a = 800 mm and are rigidly connected by a light rod of length $2a$. Initially the satellite is spinning freely about its axis of symmetry at the rate $\omega_0 = 60$ rpm. A meteorite, of mass $m_0 = m/1000$ and traveling with a velocity $\mathbf{v}_0$ of 2000 m/s relative to the satellite, strikes the satellite and becomes embedded at C. Determine (a) the angular velocity of the satellite immediately after impact, (b) the precession axis of the ensuing motion, (c) the rates of precession and spin of the ensuing motion.

SOLUTION

(a) *Angular velocity after impact*.

From Sample Problem 18.6:

$$I = I_z = \frac{1}{2}ma^2 \quad I' = I_x = I_y = \frac{5}{4}ma^2$$

Conservation of angular momentum: Angular momentum after impact:

$$\begin{aligned}\mathbf{H}_G &= \mathbf{r}_C \times m_0\mathbf{v}_0 + I\omega_0\mathbf{k} \\ &= (-a\mathbf{j} - a\mathbf{k}) \times m_0 v_0 \mathbf{i} + I\omega_0\mathbf{k} \\ &= -am_0v_0\mathbf{j} + (I\omega_0 + am_0v_0)\mathbf{k}\end{aligned}$$

Data: $a = 800 \text{ mm} = 0.8 \text{ m}$

$$\omega_0 = 60 \text{ rpm} = 2\pi \text{ rad/s}$$

$$v_0 = 2000 \text{ m/s}, \quad m_0 = \frac{m}{1000}$$

$$\omega_x = \frac{(H_G)_x}{I_x} = 0$$

$$\omega_y = \frac{(H_G)_y}{I_y} = -\frac{am_0v_0}{\frac{5}{4}ma^2} = -\frac{4}{5}\frac{m_0}{m}\frac{v_0}{a} = -\frac{4}{5}\left(\frac{1}{1000}\right)\frac{2000}{0.8} = -2 \text{ rad/s}$$

$$\omega_z = \frac{(H_G)_z}{I_z} = \omega_0 + \frac{am_0v_0}{\frac{1}{2}ma^2} = \omega_0 + 2\frac{m_0}{m}\frac{v_0}{a}$$

$$= 2\pi + 2\left(\frac{1}{1000}\right)\frac{2000}{0.8} = 11.2832 \text{ rad/s}$$

$$\boldsymbol{\omega} = -(2.00 \text{ rad/s})\mathbf{j} + (11.28 \text{ rad/s})\mathbf{k}$$

$$\omega = \sqrt{(2)^2 + (11.2832)^2} = 11.4591 \text{ rad/s} = 109.426 \text{ rpm}$$

$\omega = 109.4 \text{ rpm}$ ◀

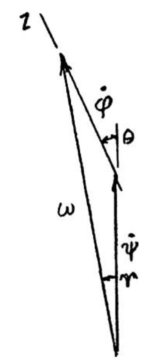
PROBLEM 18.128 (Continued)

$$\tan\gamma = \left|\frac{\omega_y}{\omega_z}\right| = \frac{2}{11.2832} \qquad \gamma = 10.0515° \qquad \gamma_x = 90°, \quad \gamma_y = 100.05°, \quad \gamma_z = 10.05° \blacktriangleleft$$

(*b*) *Precession axis.*

$$\tan\theta = \frac{I'}{I}\tan\gamma$$
$$= \left(\frac{5}{4}\right)(2)\frac{2}{11.2832}$$
$$\theta = 23.900° \qquad \theta_x = 90°, \quad \theta_y = 113.9°, \quad \theta_z = 23.9° \blacktriangleleft$$

(*c*) *Rates of precession and spin.*



$$\theta - \gamma = 13.8484°$$

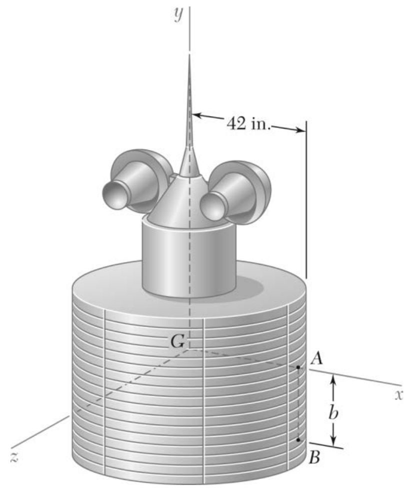
Law of sines.

$$\frac{\dot\varphi}{\sin\gamma} = \frac{\dot\psi}{\sin(\theta-\gamma)} = \frac{\omega}{\sin\theta}$$
$$\dot\varphi = \frac{\omega\sin\gamma}{\sin\theta} = \frac{109.4\sin 10.05°}{\sin 23.9°}$$

precession: $\dot\varphi = 47.1$ rpm ◀

$$\dot\psi = \frac{\omega\sin(\theta-\gamma)}{\sin\theta} = \frac{109.4\sin 13.85°}{\sin 23.9°}$$

spin: $\dot\psi = 64.6$ rpm ◀



PROBLEM 18.129

An 800-lb geostationary satellite is spinning with an angular velocity $\omega_0 = (1.5 \text{ rad/s})\mathbf{j}$ when it is hit at B by a 6-oz meteorite traveling with a velocity $\mathbf{v}_0 = -(1600 \text{ ft/s})\mathbf{i} + (1300 \text{ ft/s})\mathbf{j} + (4000 \text{ ft/s})\mathbf{k}$ relative to the satellite. Knowing that $b = 20$ in. and that the radii of gyration of the satellite are $\bar{k}_x = \bar{k}_z = 28.8$ in. and $\bar{k}_y = 32.4$ in., determine the precession axis and the rates of precession and spin of the satellite after the impact.

SOLUTION

Mass of satellite: $m = \dfrac{W}{g} = \dfrac{800}{32.2} = 24.845 \text{ lb} \cdot \text{s}^2/\text{ft}$

Principal moments of inertia: $\bar{I}_x = m k_x^2 = (24.845)\left(\dfrac{28.8}{12}\right)^2$

$$= 143.106 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$\bar{I}_y = m k_y^2 = (24.845)\left(\frac{32.4}{12}\right)^2$$

$$= 181.118 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$\bar{I}_z = \bar{I}_x = 143.106 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

Mass of meteorite: $m' = \dfrac{6}{(16)(32.2)} = 0.011649 \text{ lb} \cdot \text{s}^2/\text{ft}$

Initial momentum of meteorite:

$$m'\mathbf{v}_0 = (0.011649)(-1600\mathbf{i} + 1300\mathbf{j} + 4000\mathbf{k})$$
$$= -(18.633 \text{ lb} \cdot \text{s})\mathbf{i} + (15.140 \text{ lb} \cdot \text{s})\mathbf{j} + (46.584 \text{ lb} \cdot \text{s})\mathbf{k}$$

Assume that the position of the mass center of the satellite plus the meteorite is essentially that of the satellite alone.

Position of Point B relative to the mass center:

$$\mathbf{r}_{B/G} = \left(\frac{42}{12}\mathbf{i} - \frac{20}{12}\mathbf{j}\right)$$
$$= (3.5 \text{ ft})\mathbf{i} - (1.66667 \text{ ft})\mathbf{j}$$

Angular velocity of satellite before impact:

$$\boldsymbol{\omega}_0 = (1.5 \text{ rad/s})\mathbf{j} \qquad (\omega_0)_x = (\omega_0)_z = 0, \quad (\omega_0)_y = 1.5 \text{ rad/s}$$

Angular momentum of satellite-meteorite system before impact:

$$(\mathbf{H}_G)_0 = \bar{I}_y\omega_0\mathbf{j} + \mathbf{r}_{B/G} \times (m'\mathbf{v}_0)$$

$$= (181.118)(1.5)\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.5 & -1.66667 & 0 \\ -18.633 & 15.140 & 46.584 \end{vmatrix}$$

$$= -(77.64 \text{ lb}\cdot\text{s}\cdot\text{ft})\mathbf{i} + (108.637 \text{ lb}\cdot\text{s}\cdot\text{ft})\mathbf{j} + (21.935 \text{ lb}\cdot\text{s}\cdot\text{ft})\mathbf{k}$$

Principle of impulse and momentum for satellite-meteorite system. Moments about G:

$$(\mathbf{H}_G)_1 = (\mathbf{H}_G)_0 = \mathbf{H}_G$$

Angular velocity immediately after impact.

$$\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

Neglect the mass of the meteorite.

$$\mathbf{H}_G = \bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j} + \bar{I}_z\omega_z\mathbf{k}$$

$$\omega_x = \frac{(H_G)_x}{\bar{I}_x} = \frac{-77.64}{143.106} = -0.54253 \text{ rad/s}$$

$$\omega_y = \frac{(H_G)_y}{\bar{I}_y} = \frac{108.637}{181.118} = 0.59981 \text{ rad/s}$$

$$\omega_z = \frac{(H_G)_z}{\bar{I}_z} = \frac{21.934}{143.106} = 0.15327 \text{ rad/s}$$

$$\boldsymbol{\omega} = -(0.54253 \text{ rad/s})\mathbf{i} + (0.59981 \text{ rad/s})\mathbf{j} + (0.15327 \text{ rad/s})\mathbf{k}$$

$$\omega = \sqrt{(0.54253)^2 + (0.59981)^2 + (0.15327)^2} = 0.82317 \text{ rad/s}$$

$$H_G = \sqrt{(77.64)^2 + (108.637)^2 + (21.935)^2} = 135.319 \text{ lb}\cdot\text{s}\cdot\text{ft}$$

Motion after impact. Since the moments of inertia $\bar{I}_x$ and $\bar{I}_z$ are equal, the body moves as an axisymmetrical body with the y axis as the symmetry axis.

Moment of inertia about the symmetry axis: $I = \bar{I}_y = 181.118 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$

Moment of inertia about a transverse axis through G: $I' = \bar{I}_x = \bar{I}_z = 143.106 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$

The motion is a steady precession $\dot{\phi}$ about the precession axis together with a steady spin $\dot{\psi}$ about the spin or symmetry axis. Since $I > I'$, the precession is retrograde.

Precession axis. The precession axis is directed along the angular momentum vector $\mathbf{H}_G$, which remains fixed. Immediately after impact, its direction cosines relative to the body axes x, y, z are:

$$\cos\theta_x = \frac{(H_G)_x}{H_G} = \frac{-77.64}{135.319} = -0.57376 \qquad \theta_x = 125.0° ◀$$

$$\cos\theta_y = \frac{(H_G)_y}{H_G} = \frac{108.637}{135.319} = 0.80282 \qquad \theta_y = 36.6° ◀$$

$$\cos\theta_z = \frac{(H_G)_z}{H_G} = \frac{21.935}{135.319} = 0.16210 \qquad \theta_z = 80.7° ◀$$

The angle θ between the spin axis (y axis) and the precession axis remains constant.

$$\theta = \theta_y = 36.600°$$

The angle γ between the angular velocity vector and the spin axis (y axis) is

$$\cos\gamma = \frac{\omega_y}{\omega} = \frac{0.59981}{0.82317} \qquad \gamma = 43.226°$$

The angle γ could also have been calculated from

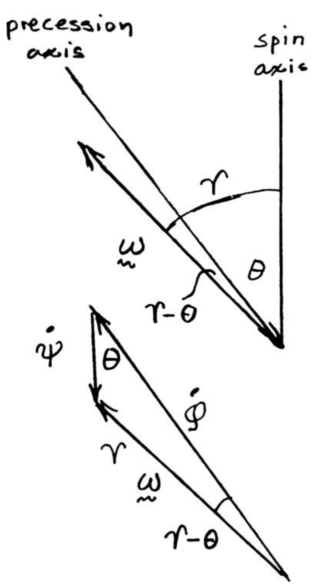
$$\tan\gamma = \frac{I}{I'}\tan\beta = \frac{181.118}{143.106}\tan 36.600°$$

The angle between the precession axis and the angular velocity vector is

$$\gamma - \theta = 6.626°$$

Rates of precession and spin.

Set up the triangle of vector addition for the components of angular velocity. Apply the law of sines.



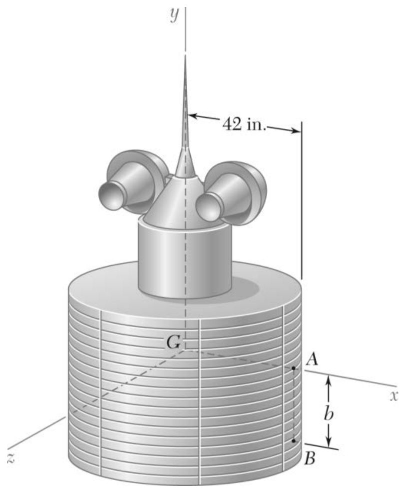
$$\frac{\omega}{\sin\theta} = \frac{\dot\psi}{\sin(\gamma-\theta)} = \frac{\dot\varphi}{\sin\gamma}$$

$$\frac{0.82317}{\sin 36.600°} = \frac{\dot\psi}{\sin 6.626°} = \frac{\dot\varphi}{\sin 43.226°}$$

Rate of precession: $\dot\varphi = 0.946$ rad/s ◀

Rate of spin: $\dot\psi = 0.1593$ rad/s ◀

Since $\gamma > \theta$, precession is retrograde. ◀



PROBLEM 18.130

Solve Problem 18.129, assuming that the meteorite hits the satellite at *A* instead of *B*.

PROBLEM 18.129 An 800-lb geostationary satellite is spinning with an angular velocity $\omega_0 = (1.5 \text{ rad/s})\mathbf{j}$ when it is hit at *B* by a 6-oz meteorite traveling with a velocity $\mathbf{v}_0 = -(1600 \text{ ft/s})\mathbf{i} + (1300 \text{ ft/s})\mathbf{j} + (4000 \text{ ft/s})\mathbf{k}$ relative to the satellite. Knowing that $b = 20$ in. and that the radii of gyration of the satellite are $\bar{k}_x = \bar{k}_z = 28.8$ in. and $\bar{k}_y = 32.4$ in., determine the precession axis and the rates of precession and spin of the satellite after the impact.

SOLUTION

Mass of satellite: $m = \dfrac{W}{g} = \dfrac{800}{32.2} = 24.845 \text{ lb}\cdot\text{s}^2/\text{ft}$

Principal moments of inertia:

$$\bar{I}_x = m k_x^2 = (24.845)\left(\frac{28.8}{12}\right)^2 = 143.106 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\bar{I}_y = m k_y^2 = (24.845)\left(\frac{32.4}{12}\right)^2 = 181.118 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$\bar{I}_z = \bar{I}_x = 143.106 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Mass of meteorite: $m' = \dfrac{6}{(16)(32.2)} = 0.011649 \text{ lb}\cdot\text{s}^2/\text{ft}$

Initial momentum of meteorite:

$$m'\mathbf{v}_0 = (0.011649)(-1600\mathbf{i} + 1300\mathbf{j} + 4000\mathbf{k})$$
$$= -(18.633 \text{ lb}\cdot\text{s})\mathbf{i} + (15.140 \text{ lb}\cdot\text{s})\mathbf{j} + (46.584 \text{ lb}\cdot\text{s})\mathbf{k}$$

Assume that the position of the mass center of the satellite plus the meteorite is essentially that of the satellite alone.

Position of Point *A* relative to the mass center:

$$\mathbf{r}_{A/G} = \frac{42}{12}\mathbf{i} = (3.5 \text{ ft})\mathbf{i}$$

Angular velocity of satellite before impact:

$$\boldsymbol{\omega}_0 = (1.5 \text{ rad/s})\mathbf{j}, \quad (\omega_0)_x = (\omega_0)_z = 0, \quad (\omega_0)_y = 1.5 \text{ rad/s}$$

Angular momentum of satellite-meteorite system before impact:

$$(\mathbf{H}_G)_0 = \bar{I}_y\omega_0\mathbf{j} + \mathbf{r}_{A/G} \times (m'\mathbf{v}_0)$$

$$= (181.118)(1.5)\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.5 & 0 & 0 \\ -18.633 & 15.140 & 46.584 \end{vmatrix}$$

$$= (108.637 \text{ lb}\cdot\text{s}\cdot\text{ft})\mathbf{j} + (52.99 \text{ lb}\cdot\text{s}\cdot\text{ft})\mathbf{k}$$

Principle of impulse and momentum for satellite-meteorite system. Moments about G:

$$(\mathbf{H}_G)_1 = (\mathbf{H}_G)_0 = \mathbf{H}_G$$

Angular velocity immediately after impact.

$$\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

Neglect the mass of the meteorite.

$$\mathbf{H}_G = \bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j} + \bar{I}_z\omega_z\mathbf{k}$$

$$\omega_x = \frac{(H_G)_x}{\bar{I}_x} = 0$$

$$\omega_y = \frac{(H_G)_y}{\bar{I}_y} = \frac{108.637}{181.118} = 0.59981 \text{ rad/s}$$

$$\omega_z = \frac{(H_G)_z}{\bar{I}_z} = \frac{52.99}{143.106} = 0.37028 \text{ rad/s}$$

$$\boldsymbol{\omega} = (0.59981 \text{ rad/s})\mathbf{j} + (0.37028 \text{ rad/s})\mathbf{k}$$

$$\omega = \sqrt{(0.59981)^2 + (0.37028)^2} = 0.70490 \text{ rad/s}$$

$$H_G = \sqrt{(108.637)^2 + (52.99)^2} = 120.872 \text{ lb}\cdot\text{s}\cdot\text{ft}$$

Motion after impact. Since the moments of inertia $\bar{I}_x$ and $\bar{I}_z$ are equal, the body moves as an axisymmetrical body with the y axis as the symmetry axis.

Moment of inertia about the symmetry axis:

$$I = \bar{I}_y = 181.118 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

Moment of inertia about a transverse axis through G:

$$I' = \bar{I}_x = \bar{I}_z = 143.106 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

The motion is a steady precession $\dot{\phi}$ about the precession axis together with a steady spin $\dot{\psi}$ about the spin or symmetry axis. Since $I > I'$, the precession is retrograde.

Precession axis. The precession axis is directed along the angular momentum vector $\mathbf{H}_G$, which remains fixed. Immediately after impact, its direction cosines relative to the body axes x, y, z are:

$$\cos\theta_x = \frac{(H_G)_x}{H_G} = 0 \qquad \theta_x = 90.0° \blacktriangleleft$$

$$\cos\theta_y = \frac{(H_G)_y}{H_G} = \frac{108.637}{120.872} = 0.89878 \qquad \theta_y = 26.0° \blacktriangleleft$$

$$\cos\theta_z = \frac{(H_G)_z}{H_G} = \frac{52.99}{120.872} = 0.43840 \qquad \theta_z = 64.0° \blacktriangleleft$$

The angle θ between the spin axis (y axis) and the precession axis remains constant.

$$\theta = \theta_y = 26.002°$$

The angle γ between the angular velocity vector and the spin axis (y axis) is

$$\cos\gamma = \frac{\omega_y}{\omega} = \frac{0.59981}{0.70490} \qquad \gamma = 31.689°$$

The angle γ could also have been calculated from

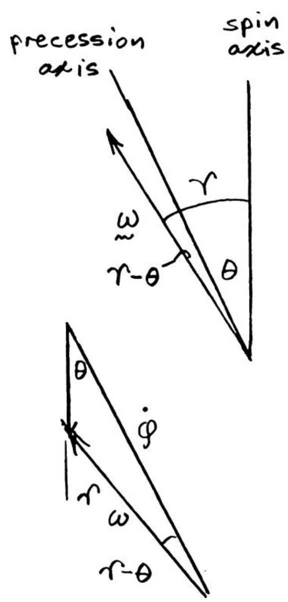
$$\tan\gamma = \frac{I}{I'}\tan\theta = \frac{181.118}{143.106}\tan 26.002°$$

The angle between the precession axis and the angular velocity vector is

$$\gamma - \theta = 5.687°$$

Rates of precession and spin.

Set up the triangle of vector addition for the components of angular velocity. Apply the law of sines.



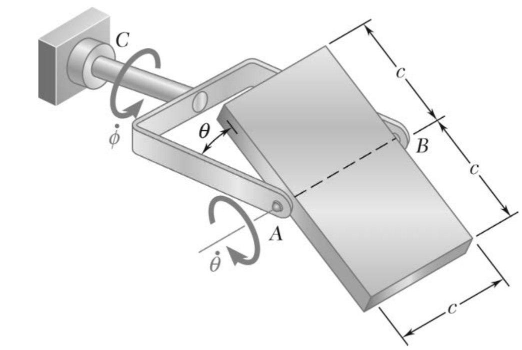
$$\frac{\omega}{\sin\theta} = \frac{\dot\psi}{\sin(\gamma-\theta)} = \frac{\dot\varphi}{\sin\gamma}$$

$$\frac{0.70490}{\sin 26.002°} = \frac{\dot\psi}{\sin 5.687°} = \frac{\dot\varphi}{\sin 31.689°}$$

Rate of precession: $\dot\varphi = 0.844$ rad/s $\blacktriangleleft$

Rate of spin: $\dot\psi = 0.1593$ rad/s $\blacktriangleleft$

Since $\gamma > \theta$, the precession is retrograde. $\blacktriangleleft$



PROBLEM 18.131

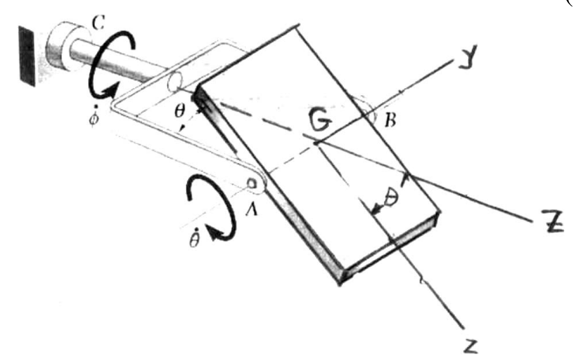
A homogeneous rectangular plate of mass m and sides c and $2c$ is held at A and B by a fork-ended shaft of negligible mass, which is supported by a bearing at C. The plate is free to rotate about AB, and the shaft is free to rotate about a horizontal axis through C. Knowing that, initially, $\theta_0 = 40°$, $\dot{\theta}_0 = 0$, and $\dot{\phi}_0 = 10$ rad/s, determine for the ensuring motion (*a*) the range of values of θ, (*b*) the minimum value of $\dot{\phi}$, (*c*) the maximum value of $\dot{\theta}$.

SOLUTION

Let the fixed Z axis lie along the axle of the fork-ended shaft. Let the axes $Gxyz$ be attached at the mass center with x perpendicular to the plate, y along the axle AB and z parallel to the long edges of the plate.

Angular velocity vector: $\dot{\boldsymbol{\omega}} = \dot{\varphi}\mathbf{k} + \dot{\theta}\mathbf{j}$

$$= \dot{\varphi}\cos\theta\mathbf{i} + \dot{\theta}\mathbf{j} + \dot{\varphi}\sin\theta\mathbf{k}$$



Conservation of angular momentum.

Since plate is free to rotate about Z axis,

$$H_Z = \text{constant} \tag{1}$$

But $H_Z = H_x\cos\theta + H_z\sin\theta$

$$H_Z = I_x\omega_x\cos\theta + I_z\omega_z\sin\theta$$

$$= \frac{1}{12}mc^2\dot{\phi}\cos^2\theta + \frac{5}{12}mc^2\dot{\phi}\sin^2\theta$$

$$= \frac{1}{12}mc^2\dot{\phi}(\cos^2\theta + 5\sin^2\theta)$$

$$= \frac{1}{12}mc^2\dot{\phi}(1 + 4\sin^2\theta)$$

Using the initial conditions, Eq. (1) yields

$$\dot{\phi}(1 + 4\sin^2\theta) = \dot{\phi}_0(1 + 4\sin^2\theta_0) \tag{2}$$

Conservation of energy.

Since no work is done, we have $T = \text{constant}$ (3)

where

$$T = \frac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2)$$

$$T = \frac{1}{2}\left(\frac{1}{12}mc^2\dot{\phi}^2\cos^2\theta + \frac{1}{3}mc^2\dot{\theta}^2 + \frac{5}{12}mc^2\dot{\phi}^2\sin^2\theta\right)$$

$$= \frac{1}{24}mc^2[4\dot{\theta}^2 + \dot{\phi}^2(\cos^2\theta + 5\sin^2\theta)]$$

$$= \frac{1}{24}mc^2[4\dot{\theta}^2 + \dot{\phi}^2(1 + 4\sin^2\theta)]$$

PROBLEM 18.131 (Continued)

Using the initial conditions, including $\dot{\theta}_0 = 0$, Eq. (3) yields

$$4\dot{\theta}^2 + \dot{\phi}^2(1 + 4\sin^2\theta) = \dot{\phi}_0^2(1 + 4\sin^2\theta_0) \tag{4}$$

(*a*) With $\theta_0 = 40°$ and $\dot{\phi}_0 = 10$ rad/s in. Eqs. (2) and (4),

$$\dot{\phi}(1 + 4\sin^2\theta) = 26.527$$

$$\dot{\phi} = \frac{26.527}{1 + 4\sin^2\theta} \tag{2'}$$

$$4\dot{\theta}^2 + \dot{\phi}^2(1 + 4\sin^2\theta) = 265.27$$

Eliminate $\dot{\phi}$ and solve for $\dot{\theta}^2$:

$$4\dot{\theta}^2 = 265.27 - \frac{(26.527)^2}{1 + 4\sin^2\theta} \tag{5}$$

For $\dot{\theta}^2 = 0, \quad 1 + 4\sin^2\theta = \frac{(26.527)^2}{265.27} = 2.6527$

$$\sin^2\theta = 0.4132 \quad \sin\theta = 0.6428$$

From which $\theta = 40°$ and $140°$ $\qquad 40° < \theta < 140°$ ◀

(*b*) From Eq. (2′), $\dot{\phi}$ is minimum for $\theta = 90°$.

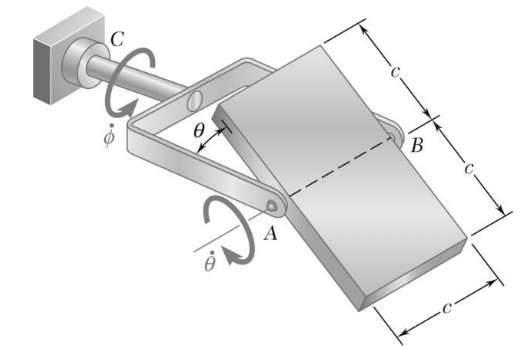
$\dot{\phi}_{min} = 5.3054$ $\qquad \dot{\phi}_{min} = 5.31$ rad/s ◀

(*c*) From Eq. (5),

$$\dot{\theta}^2 = 66.318 - \frac{175.92}{1 + 4\sin^2\theta}$$

$\dot{\theta}^2$ is maximum for $\theta = 90°$.

$\dot{\theta}_{max}^2 = 31.134$ rad^2/s^2 $\qquad \dot{\theta}_{max} = 5.58$ rad/s ◀



PROBLEM 18.132

A homogeneous rectangular plate of mass m and sides c and $2c$ is held at A and B by a fork-ended shaft of negligible mass which is supported by a bearing at C. The plate is free to rotate about AB, and the shaft is free to rotate about a horizontal axis through C. Initially the plate lies in the plane of the fork $(\theta_0 = 0)$ and the shaft has an angular velocity $\dot{\phi}_0 = 10\text{ rad/s}$. If the plate is slightly disturbed, determine for the ensuring motion (a) the minimum value of $\dot{\phi}$, (b) the maximum value of $\dot{\theta}$.

SOLUTION

Let the fixed Z axis lie along the axle of the fork-ended shaft. Let the axes $Gxyz$ be attached at the mass center with x perpendicular to the plate, y along the axle AB and z parallel to the long edges of the plate.

Angular velocity vector: $\quad \dot{\boldsymbol{\omega}} = \dot{\varphi}\mathbf{k} + \dot{\theta}\mathbf{j}$

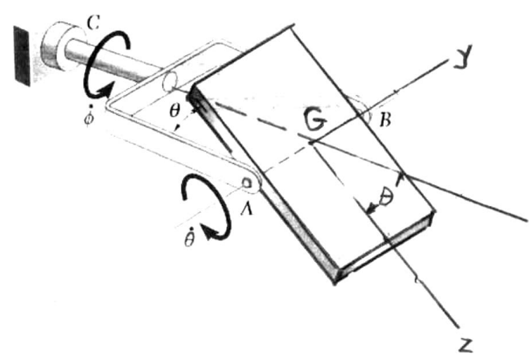
$$= \dot{\varphi}\cos\theta\mathbf{i} + \dot{\theta}\mathbf{j} + \dot{\varphi}\sin\theta\mathbf{k}$$

Conservation of angular momentum.

Since plate is free to rotate about Z axis,

$$H_Z = \text{constant} \tag{1}$$

But $\quad H_Z = H_x\cos\theta + H_z\sin\theta$



$$H_Z = I_x\omega_x\cos\theta + I_z\omega_z\sin\theta$$

$$= \frac{1}{12}mc^2\dot{\phi}\cos^2\theta + \frac{5}{12}mc^2\dot{\phi}\sin^2\theta$$

$$= \frac{1}{12}mc^2\dot{\phi}(\cos^2\theta + 5\sin^2\theta)$$

$$= \frac{1}{12}mc^2\dot{\phi}(1 + 4\sin^2\theta)$$

Using the initial conditions, Eq. (1) yields

$$\dot{\phi}(1 + 4\sin^2\theta) = \dot{\phi}_0(1 + 4\sin^2\theta_0) \tag{2}$$

Conservation of energy.

Since no work is done, we have $T = \text{constant}$ (3)

where $\quad T = \frac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2)$

$$T = \frac{1}{2}\left(\frac{1}{12}mc^2\dot{\phi}^2\cos^2\theta + \frac{1}{3}mc^2\dot{\theta}^2 + \frac{5}{12}mc^2\dot{\phi}^2\sin^2\theta\right)$$

$$= \frac{1}{24}mc^2[4\dot{\theta}^2 + \dot{\phi}^2(\cos^2\theta + 5\sin^2\theta)]$$

$$= \frac{1}{24}mc^2[4\dot{\theta}^2 + \dot{\phi}^2(1 + 4\sin^2\theta)]$$

Using the initial conditions, including $\dot{\theta}_0 = 0$, Eq. (3) yields

$$4\dot{\theta}^2 + \dot{\phi}^2(1+4\sin^2\theta) = \dot{\phi}_0^2(1+4\sin^2\theta_0) \tag{4}$$

(*a*) With $\theta_0 = 0, \quad \dot{\phi}_0 = 10 \text{ rad/s}$

Eq. (2) yields $$\dot{\phi} = \frac{10}{1+4\sin^2\theta}$$

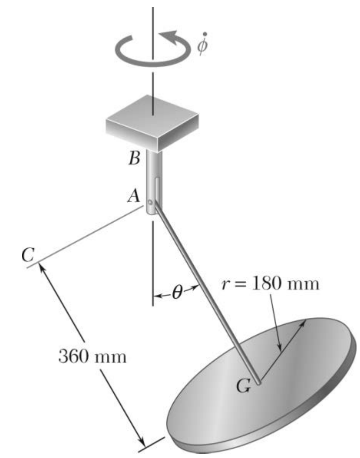
$\dot{\phi}$ is minimum for $\theta = 90°$: $\dot{\phi}_{min} = 2.00 \text{ rad/s}$ ◀

(*b*) Eq. (4) yields $$4\dot{\theta}^2 = 100 - \dot{\phi}^2(1+4\sin^2\theta) = 100\left(1 - \frac{1}{1+4\sin^2\theta}\right)$$

$\dot{\theta}^2$ is largest for $\theta = 90°$:

$$4\dot{\theta}_{max}^2 = 100\left(1 - \frac{1}{5}\right)$$

$$\dot{\theta}_{max}^2 = 20$$ $\dot{\theta}_{max} = 4.47 \text{ rad/s}$ ◀



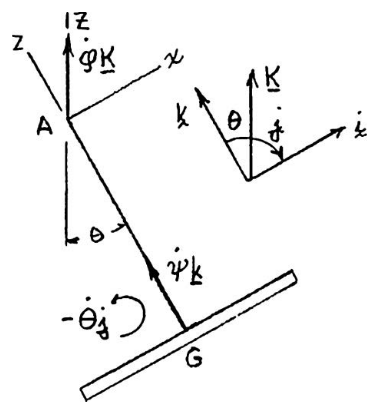
PROBLEM 18.133

A homogeneous disk of radius 180 mm is welded to a rod *AG* of length 360 mm and of negligible mass which is connected by a clevis to a vertical shaft *AB*. The rod and disk can rotate freely about a horizontal axis *AC*, and shaft *AB* can rotate freely about a vertical axis. Initially, rod *AG* is horizontal $(\theta_0 = 90°)$ and has no angular velocity about *AC*. Knowing that the maximum value $\dot{\phi}_m$ of the angular velocity of shaft *AB* in the ensuing motion is twice its initial value $\dot{\phi}_0$, determine (*a*) the minimum value of θ, (*b*) the initial angular velocity $\dot{\phi}_0$ of shaft *AB*.

SOLUTION

Let the *Z* axis be vertical.

For principal axes *xyz* with origin at *A*, the principal moments of inertia are



$$I' = I_x = I_y$$

$$= m\left[\frac{1}{4}a^2 + (2a)^2\right] = \frac{17}{4}ma^2$$

$$I = I_z = \frac{1}{2}ma^2$$

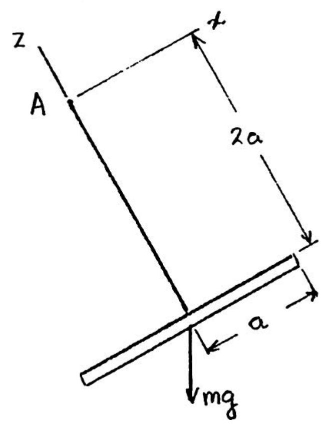
Angular velocity components: $\omega_x = \dot{\varphi}\sin\theta$

$$\omega_y = -\dot{\theta}$$

$$\omega_z = \dot{\psi} + \dot{\varphi}\cos\theta$$

Angular momentum about *A*: $\mathbf{H}_A = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$

$$= I'\dot{\varphi}\sin\theta\mathbf{i} - I'\dot{\theta}\mathbf{j} + I\omega_z\mathbf{k}$$



Kinetic energy: $T = \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2$

$$T = \frac{1}{2}I'(\dot{\varphi}^2\sin^2\theta + \dot{\theta}^2) + \frac{1}{2}I\omega_z^2$$

Potential energy: $V = -2mga\cos\theta$

Conservation of angular momentum about fixed *Z* axis:

$$\mathbf{H}_A \cdot \mathbf{K} = \mathbf{H}_A \cdot (\mathbf{i}\sin\theta + \mathbf{k}\cos\theta)$$

$$= I'\dot{\varphi}\sin^2\theta + I\omega_z\cos\theta$$

$$= \frac{17}{4}ma^2\dot{\varphi}\sin^2\theta + \frac{1}{2}ma^2\omega_z\cos\theta = \alpha \quad (1)$$

where α is a constant.

PROBLEM 18.133 (Continued)

Conservation of energy: $T + V = E$, where E is a constant.

$$\frac{17}{8}ma^2(\dot{\varphi}^2 \sin^2\theta + \dot{\theta}^2) + \frac{1}{4}ma^2\omega_z^2 - 2mga\cos\theta = E \quad (2)$$

Constraint of clevis: $\dot{\psi} = 0 \qquad \omega_z = \dot{\varphi}\cos\theta$

(*a*) From Eq. (1), $\frac{17}{4}ma^2\dot{\varphi}_m \sin^2\theta_m + \frac{1}{2}ma^2\dot{\varphi}_m\cos^2\theta_m = \frac{17}{4}ma^2\dot{\varphi}_0\sin^2 90° + \frac{1}{2}ma^2\dot{\varphi}_0\cos^2 90°$

$$\frac{17}{4}\sin^2\theta_m + \frac{1}{2}\cos^2\theta_m = \frac{17}{4}\frac{\dot{\varphi}_0}{\dot{\varphi}_m} = \left(\frac{17}{4}\right)\left(\frac{1}{2}\right) = \frac{17}{8}$$

$$\frac{17}{4}\sin^2\theta_m + \frac{1}{2}(1 - \sin^2\theta_m) = \frac{17}{8}$$

$$\sin^2\theta_m = \frac{13}{30}$$

$$\sin\theta_m = 0.65828$$

$$\theta_m = 41.169°$$

$\theta_m = 41.2°$ ◀

(*b*) At the minimum value of θ, $\dot{\theta} = 0$

From Eq. (2), $\frac{17}{8}ma^2(\dot{\varphi}_m^2\sin^2\theta_m + 0) + \frac{1}{4}ma^2\dot{\varphi}_m^2\cos^2\theta_m - 2mga\cos\theta_m$

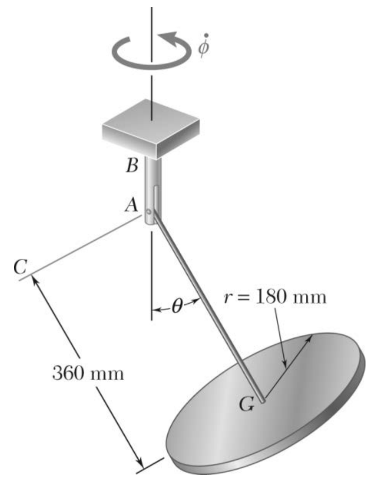
$$= \frac{17}{8}ma^2(\dot{\varphi}_0^2\sin^2\theta_0 + 0) + \frac{1}{4}ma^2\dot{\varphi}_0^2\cos^2 90° - 2mga\cos 90°$$

$$\left[\left(\frac{17}{8}\right)(2)^2\sin^2\theta_m + \left(\frac{1}{4}\right)(2)^2\cos^2\theta_m - \frac{17}{8}\right]ma^2\dot{\varphi}_0^2 = 2mga\cos\theta_m$$

$$2.1250\ ma^2\dot{\varphi}_0^2 = 1.5055mga$$

$$\dot{\varphi}_0^2 = 0.70849\frac{g}{a} = 0.70849\frac{9.81}{0.18} = 38.613$$

$\dot{\varphi}_0 = 6.21$ rad/s ◀



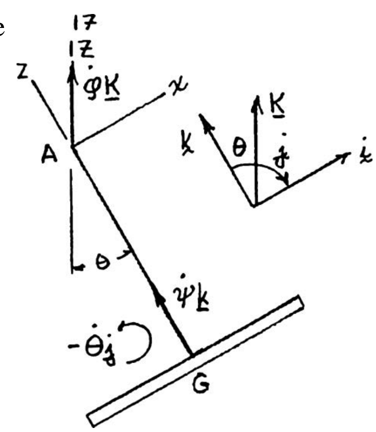
PROBLEM 18.134

A homogeneous disk of radius 180 mm is welded to a rod *AG* of length 360 mm and of negligible mass which is connected by a clevis to a vertical shaft *AB*. The rod and disk can rotate freely about a horizontal, axis *AC*, and shaft *AB* can rotate freely about a vertical axis. Initially, rod *AG* is horizontal $(\theta_0 = 90°)$ and has no angular velocity about *AC*. Knowing that the smallest value of θ in the ensuing motion is 30°, determine (*a*) the initial angular velocity of shaft *AB*, (*b*) its maximum angular velocity.

SOLUTION

Let the *Z* axis be vertical.

For principal axes *x*, *y*, *z* with origin at *A*, the principal moments of inertia are



$$I' = I_x = I_y$$

$$= m\left[\frac{1}{4}a^2 + (2a)^2\right] = \frac{17}{4}ma^2$$

$$I = I_z = \frac{1}{2}ma^2$$

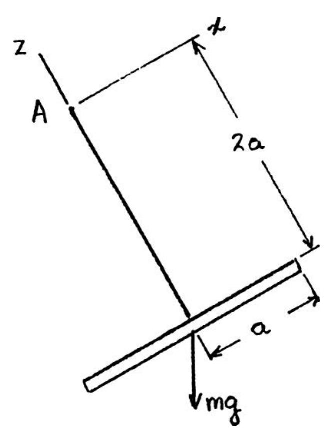
Angular velocity components: $\omega_x = \dot{\varphi}\sin\theta$

$$\omega_y = -\dot{\theta}$$

$$\omega_z = \dot{\psi} + \dot{\varphi}\cos\theta$$

Angular momentum about *A*: $\mathbf{H}_A = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$

$$= I'\dot{\varphi}\sin\theta\mathbf{i} - I'\dot{\theta}\mathbf{j} + I\omega_z\mathbf{k}$$



Kinetic energy: $T = \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2$

$$T = \frac{1}{2}I'(\dot{\varphi}^2\sin^2\theta + \dot{\theta}^2) + \frac{1}{2}I\omega_z^2$$

Potential energy: $V = -2mga\cos\theta$

Conservation of angular momentum about fixed *Z* axis:

$$\mathbf{H}_A \cdot \mathbf{K} = \mathbf{H}_A \cdot (\mathbf{i}\sin\theta + \mathbf{k}\cos\theta)$$

$$= I'\dot{\varphi}\sin^2\theta + I\omega_z\cos\theta$$

$$= \frac{17}{4}ma^2\dot{\varphi}\sin^2\theta + \frac{1}{2}ma^2\omega_z\cos\theta = \alpha \quad (1)$$

where α is a constant

Conservation of energy: $T + V = E$, where E is a constant.

$$\frac{17}{8}ma^2(\dot{\varphi}^2 \sin^2\theta + \dot{\theta}^2) + \frac{1}{4}ma^2\omega_z^2 - 2mga\cos\theta = E \qquad (2)$$

Constraint of clevis: $\dot{\psi} = 0 \qquad \omega_z = \dot{\varphi}\cos\theta$

(*a*) At $\theta = \theta_m = 30°$ and at $\theta = \theta_0 = 90°$, $\dot{\theta} = 0$.

From Eq. (1), $$\frac{17}{4}ma^2\dot{\varphi}_m \sin^2\theta_m + \frac{1}{2}ma^2\dot{\varphi}_m\cos^2\theta_m = \frac{17}{4}ma^2\dot{\varphi}_0\sin^2\theta_0 + 0$$

$$\left[\left(\frac{17}{4}\right)\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)^2\right]ma^2\dot{\varphi}_m = \frac{17}{4}ma^2\dot{\varphi}_0$$

$$\dot{\varphi}_m = \frac{68}{23}\dot{\varphi}_0$$

From Eq. (2), $$\frac{17}{8}ma^2\left(\frac{68}{23}\dot{\varphi}_0\right)^2\sin^2 30° + \frac{1}{4}ma^2\left(\frac{68}{23}\dot{\varphi}_0\right)^2\cos^2 30° - 2mga\cos 30°$$

$$= \frac{17}{32}ma^2\dot{\varphi}_0^2\sin^2 90° + \frac{1}{4}ma^2\dot{\varphi}_0^2\cos^2 90° - 2mga\cos 90°$$

$$\left[\left(\frac{17}{32} + \frac{3}{16}\right)\left(\frac{68}{23}\right)^2 - \frac{17}{8}\right]ma^2\dot{\varphi}_0^2 = 2mga\cos 30°$$

$$\dot{\varphi}_0^2 = 0.41660\frac{g}{a}$$

$$= 0.41660\ \frac{9.81}{0.18}$$

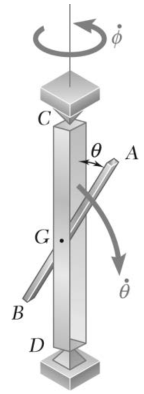
$$= 22.705$$

$$\dot{\varphi}_0 = 4.7649 \text{ rad/s}$$

$\dot{\varphi}_0 = 4.76$ rad/s ◀

(*b*) $$\dot{\varphi}_m = \frac{68}{23}(4.7649)$$

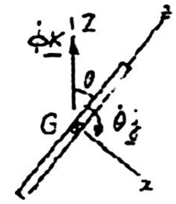
$\dot{\varphi}_m = 14.09$ rad/s ◀



PROBLEM 18.135

The slender homogeneous rod AB of mass m and length L is free to rotate about a horizontal axle through its mass center G. The axle is supported by a frame of negligible mass which is free to rotate about the vertical CD. Knowing that, initially, $\theta = \theta_0$, $\dot{\theta} = 0$, and $\dot{\phi} = \dot{\phi}_0$, show that the rod will oscillate about the horizontal axle and determine (*a*) the range of values of angle θ during this motion, (*b*) the maximum value of $\dot{\theta}$, (*c*) the minimum value of $\dot{\phi}$.

SOLUTION



Angular velocity.

Using the coordinate axes x, y, z shown (with y running *into* the paper), we have

$$\boldsymbol{\omega} = -\dot{\phi}\sin\theta\mathbf{i} + \dot{\theta}\mathbf{j} + \dot{\phi}\cos\theta\mathbf{k} \qquad (1)$$

Moments of inertia.

For slender rod of length L are mass m:

$$\bar{I}_x = \bar{I}_y = \frac{1}{12}mL^2$$

$$\bar{I}_z = 0 \qquad (2)$$

Conservation of energy.

Since the x, y, z axes are principal axis, we use

$$T = \frac{1}{2}\left(\bar{I}_x\omega_x^2 + \bar{I}_y\omega_y^2 + \bar{I}_z\omega_z^2\right) = \frac{1}{2}\left(\frac{1}{12}mL^2\dot{\phi}^2\sin^2\theta + \frac{1}{12}mL^2\dot{\theta}^2 + 0\right)$$

$$T = \frac{1}{24}mL^2(\dot{\phi}^2\sin^2\theta + \dot{\theta}^2) \qquad (3)$$

Using a datum through G, we have $V = 0$.

$$T + V = \text{constant}: \quad \frac{1}{24}mL^2(\dot{\phi}^2\sin^2\theta + \dot{\theta}^2) = \text{constant}$$

Recalling the initial conditions $\theta = \theta_0$, $\dot{\theta} = 0$, $\dot{\varphi} = \dot{\phi}_0$, we determine the constant and write

$$\dot{\phi}^2\sin^2\theta + \dot{\theta}^2 = \dot{\phi}_0^2\sin^2\theta_0 \qquad (4)$$

Conservation of angular momentum.

Since the only forces exerted on the rod are its weight and the reaction at G, we have $\Sigma \mathbf{M}_G = 0.$

Using a fixed reference frame $GXYZ$, with Z directed vertically upward, we have from Eq. (18.2)

$$\dot{\mathbf{H}}_G = \Sigma \mathbf{M}_G = 0$$

or, integrating with respect to the frame $GXYZ$,

$$\mathbf{H}_G = \text{constant}$$

Considering the vertical component of $\mathbf{H}_G$,

$$H_Z = \text{constant}$$

But
$$H_Z = H_z \cos\theta - H_x \sin\theta = \bar{I}_z \omega_z \cos\theta - \bar{I}_x \omega_x \sin\theta$$
$$= 0 - \frac{1}{12} mL^2 (-\dot{\phi} \sin\theta) \sin\theta$$

Thus,
$$H_Z = \frac{1}{12} m L^2 \dot{\phi} \sin^2\theta = \text{constant}$$

Recalling the initial conditions, we obtain

$$\dot{\phi} \sin^2\theta = \dot{\phi}_0 \sin^2\theta_0 \qquad (5)$$

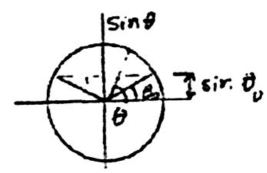
Solving Eq. (5) for $\dot{\phi}$ and substituting into Eq. (4):

$$\left(\frac{\dot{\phi}_0 \sin^2\theta_0}{\sin^2\theta}\right)^2 \sin^2\theta + \dot{\theta}^2 = \dot{\phi}_0 \sin^2\theta_0$$

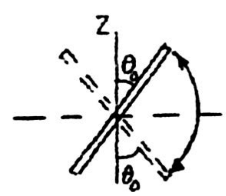
$$\dot{\theta}^2 = \dot{\phi}_0^2 \sin^2\theta_0 \left(1 - \frac{\sin^2\theta_0}{\sin^2\theta}\right) \qquad (6)$$

(*a*) Range of values of θ.

Since $\dot{\theta} \geq 0$, we must have



$$1 - \frac{\sin^2\theta_0}{\sin^2\theta} \geq 0$$
$$\sin^2\theta \geq \sin^2\theta_0$$
$$|\sin\theta| \geq |\sin\theta_0|$$



From trigonometric circle, we conclude that the range is

$$\theta_0 \leq \theta \leq 180° - \theta_0 \blacktriangleleft$$

Rod oscillates about axle and about horizontal line (dashed line).

PROBLEM 18.135 (Continued)

(*b*) Maximum value of $\dot{\theta}$.

Referring to Eq. (6), we note that this occurs when $\sin\theta = 1$, that is, when $\theta = 90°$. We have

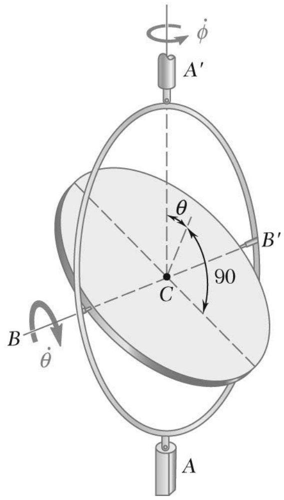
$$\dot{\theta}_{max}^2 = \dot{\phi}_0^2 \sin^2\theta_0 (1 - \sin^2\theta_0)$$
$$= \dot{\phi}_0^2 \sin^2\theta_0 \cos^2\theta_0 \qquad \dot{\theta}_{max} = \dot{\phi}_0 \sin\theta_0 \cos\theta_0 \blacktriangleleft$$

(*c*) Minimum value of $\dot{\phi}_0$.

Referring to Eq. (5), we note that $\dot{\phi}$ is minimum when $\sin\theta = 1$, that is, when $\theta = 90°$.
We have

$$\dot{\phi}_{min} = \dot{\phi}_0 \sin^2\theta_0 \blacktriangleleft$$

PROBLEM 18.136



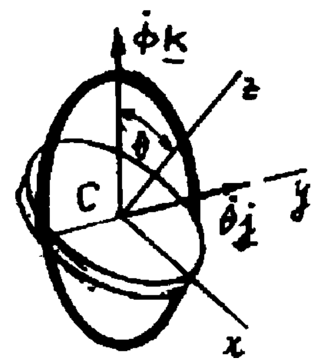
The gimbal $ABA'B'$, is of negligible mass and may rotate freely about the vertical AA'. The uniform disk of radius a and mass m may rotate freely about its diameter BB', which is also the horizontal diameter of the gimbal. (*a*) Applying the principle of conservation of energy, and observing that, since $\Sigma M_{AA'} = 0,$ the component of the angular momentum of the disk along the fixed axis AA' must be constant, write two first-order differential equations defining the motion of the disk. (*b*) Given the initial conditions $\theta_0 \neq 0, \dot{\phi}_0 = 0,$ and $\dot{\theta}_0 = 0,$ express the rate of nutation $\dot{\theta}$ as a function of θ. (*c*) Show that the angle θ will never be larger than θ_0 during the ensuing motion.

SOLUTION

We use a reference frame *Cxyz* attached to the disk as shown. The angular velocity of the disk is

$$\omega = \dot{\phi}\mathbf{k} + \dot{\theta}\mathbf{j}$$

$$\omega = -\dot{\phi}\sin\theta\,\mathbf{i} + \dot{\theta}\mathbf{j} + \dot{\phi}\cos\theta\,\mathbf{k}$$



(*a*) Conservation of energy: $T + V = \text{constant}$

Since $V = \text{constant}$, we have

$$T = \text{constant}.$$

For principal centroidal axes and $\bar{v} = 0$, the kinetic energy is given by

$$T = \frac{1}{2}(I_z\omega_z^2 + I_y\omega_y^2 + I_z\omega_z^2)$$

But $$I_x = I_y = \frac{1}{4}ma^2, \quad I_z = \frac{1}{2}ma^2$$

Using the components of $\boldsymbol{\omega}$ computed above:

$$T = \frac{1}{8}ma^2\dot{\phi}^2\sin^2\theta + \frac{1}{8}ma^2\dot{\theta}^2 + \frac{1}{4}ma^2\omega s^2\theta$$

$$T = \frac{1}{8}ma^2[(1+\cos^2\theta)\dot{\phi}^2 + \dot{\theta}^2]$$

We have therefore $$(1+\cos^2\theta)\dot{\phi}^2 + \dot{\theta}^2 = \text{constant} \qquad (1) ◀$$

PROBLEM 18.136 (Continued)

We now determine the angular momentum $\mathbf{H}_C$:

$$\mathbf{H}_C = I_z\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$= -\frac{1}{4}ma^2\dot{\phi}\sin\theta\mathbf{i} + \frac{1}{4}ma^2\dot{\theta}\mathbf{j} + \frac{1}{2}ma^2\dot{\phi}\cos\theta\mathbf{k}$$

Since $H_z = \text{constant}$, we write

$$H_z = \mathbf{H}_C \cdot \mathbf{K} = \text{constant}$$

$$\frac{1}{4}ma^2(-\dot{\phi}\sin\theta\mathbf{i} + \dot{\theta}\mathbf{j} + 2\dot{\phi}\cos\theta\mathbf{k}) \cdot \mathbf{K} = \text{constant}$$

Since $\mathbf{i}\cdot\mathbf{K} = -\sin\theta$, $\mathbf{j}\cdot\mathbf{K} = 0$, $\mathbf{k}\cdot\mathbf{K} = \cos\theta$, we have

$$\dot{\phi}\sin^2\theta + 2\dot{\phi}\cos^2\theta = \text{constant}$$

$$\dot{\phi}(1+\cos^2\theta) = \text{constant} \qquad (2) \blacktriangleleft$$

(*b*) We determine the constants in (1) and (2) from the initial conditions $\theta_0, \dot{\phi}_0, \dot{\theta}_0 = 0$, and write

$$(1+\cos^2\theta)\dot{\phi}^2 + \dot{\theta}^2 = (1+\cos^2\theta_0)\dot{\phi}_0^2 \qquad (1')$$

$$\dot{\phi}(1+\cos^2\theta) = \dot{\phi}_0(1+\cos^2\theta_0) \qquad (2')$$

Solving (2′) for $\dot{\phi}$:

$$\dot{\phi} = \dot{\phi}_0\frac{1+\cos^2\theta_0}{1+\cos^2\theta}$$

Substituting into (1′):

$$\frac{(1+\cos^2\theta_0)^2}{1+\cos^2\theta}\dot{\phi}_0^2 + \dot{\theta}^2 = (1+\cos^2\theta_0)\dot{\phi}_0^2$$

$$\dot{\theta}^2 = \dot{\phi}_0^2(1+\cos^2\theta_0)\left(1 - \frac{1+\cos^2\theta_0}{1+\cos^2\theta_0}\right)$$

$$\dot{\theta} = \dot{\phi}_0\sqrt{\frac{(1+\cos^2\theta_0)(\cos^2\theta - \cos^2\theta_0)}{1+\cos^2\theta}} \qquad \blacktriangleleft$$

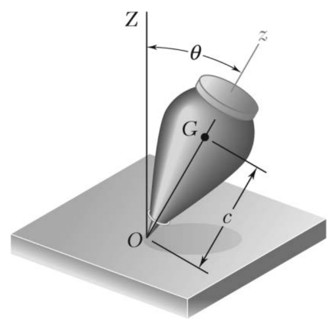
(*c*) For $\dot{\theta}$ to be real, we need $\cos^2\theta_0 - \cos^2\theta_0 \geq 0$

Thus $|\cos\theta| \geq |\cos\theta_0|$

Assuming that the axes have been chosen so that $\theta_0 \leq 90°$, we must have

$$\cos\theta \geq \cos\theta_0 \qquad \theta \leq \theta_0 \blacktriangleleft$$

PROBLEM 18.137*



The top shown is supported at the fixed Point O. Denoting by ϕ, θ, and ψ the Eulerian angles defining the position of the top with respect to a fixed frame of reference, consider the general motion of the top in which all Eulerian angles vary.

(*a*) Observing that $\Sigma M_Z = 0$ and $\Sigma M_z = 0$, and denoting by I and I', respectively, the moments of inertia of the top about its axis of symmetry and about a transverse axis through O, derive the two first-order differential equations of motion

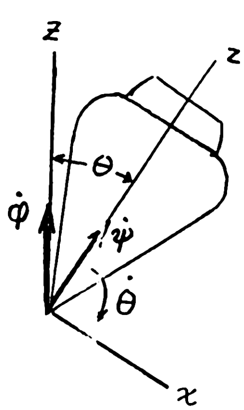
$$I'\dot{\phi}\sin^2\theta + I(\dot{\psi} + \dot{\phi}\cos\theta)\cos\theta = \alpha$$

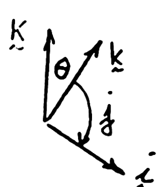
$$I(\dot{\psi} + \dot{\phi}\cos\theta) = \beta$$

where α and β are constants depending upon the initial conditions. These equations express that the angular momentum of the top is conserved about both the Z and z axes, i.e., that the rectangular component of $\mathbf{H}_O$ along each of these axes is constant.

(*b*) Use Eqs. (1) and (2) to show that the rectangular component ω_z of the angular velocity of the top is constant and that the rate of precession $\dot{\phi}$ depends upon the value of the angle of nutation θ.

SOLUTION





Use a rotating frame of reference with the y axis pointing into the paper.

Angular velocity of the frame:

$$\mathbf{\Omega} = -\dot{\phi}\sin\theta\mathbf{i} + \dot{\theta}\mathbf{j} + \dot{\phi}\cos\theta\mathbf{k}$$

Angular velocity of the top:

$$\boldsymbol{\omega} = -\dot{\phi}\sin\theta\mathbf{i} + \dot{\theta}\mathbf{j} + (\dot{\psi} + \dot{\phi}\cos\theta)\mathbf{k}$$

Its angular momentum about O:

$$\mathbf{H}_O = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$= -I'\dot{\phi}\sin\theta\mathbf{i} + I'\dot{\theta}\mathbf{j} + I(\dot{\psi} + \dot{\phi}\cos\theta)\mathbf{k}$$

where $I_x = I_y = I'$ and $I_z = I.$

The moment $\mathbf{M}_O$ about O is due to the weight mg.

$$\mathbf{M}_0 = mgc\sin\theta\mathbf{j}$$

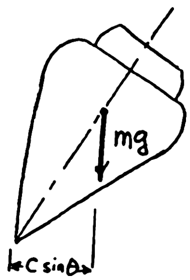
(*a*) Since the fixed Z axis refers to a Newtonian frame of reference and $(M_O)_Z = 0,$ it follows that $(H_O)_Z$ is constant. Thus,

$$(H_O)_Z = \mathbf{H}_O \cdot \mathbf{K} = \mathbf{H}_O \cdot (-\mathbf{i}\sin\theta + \mathbf{k}\cos\theta)$$

$$= I'\dot{\phi}\sin^2\theta + I(\dot{\psi} + \dot{\phi}\cos\theta)\cos\theta = \alpha \qquad (1) ◀$$

where α is a constant.

PROBLEM 18.137* (Continued)



$$\mathbf{M}_O = \dot{\mathbf{H}}_O = (\dot{\mathbf{H}}_O)_{Oxyz} + \mathbf{\Omega} \times \mathbf{H}_O$$

$$mgc\sin\theta\mathbf{j} = -I'\frac{d}{dt}(\dot{\phi}\sin\theta)\,\mathbf{i} + I'\ddot{\theta}\mathbf{j} + I\frac{d}{dt}(\dot{\psi} + \dot{\phi}\cos\theta)\,\mathbf{k}$$
$$+[I(\dot{\psi} + \dot{\phi}\cos\theta)\,\dot{\theta} - I'\dot{\phi}\dot{\theta}\cos\theta]\mathbf{i}$$
$$+[I(\dot{\psi} + \dot{\phi}\cos\theta)\,\dot{\phi}\sin\theta - I'\dot{\phi}^2\sin\theta\cos\theta]\mathbf{j}$$

z-components: $\quad 0 = \dfrac{d}{dt}(\dot{\psi} + \dot{\phi}\cos\theta)$

Integrating, $\quad I(\dot{\psi} + \dot{\phi}\cos\theta) = \beta \quad (2)$ ◀

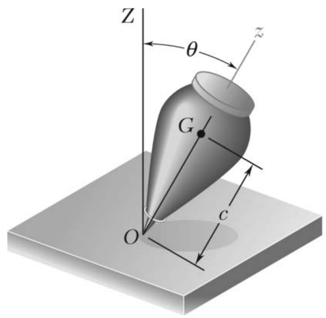
where β is a constant.

(*b*) Since $\quad \omega_z = \dot{\psi} + \dot{\phi}\cos\theta \qquad \omega_z = \dfrac{\beta}{I} = \text{constant} \quad (3)$ ◀

From Eqs. (1) and (2), $\quad I'\dot{\phi}\sin^2\theta + \beta\cos\theta = \alpha \qquad \dot{\phi} = \dfrac{\alpha - \beta\cos\theta}{I'\sin^2\theta} \quad (4)$ ◀

which is a function of θ.

PROBLEM 18.138*



(*a*) Applying the principle of conservation of energy, derive a third differential equation for the general motion of the top of Problem 18.137.

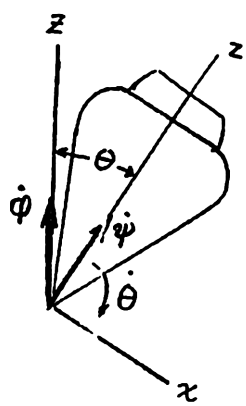
(*b*) Eliminating the derivatives $\dot{\phi}$ and $\dot{\psi}$ from the equation obtained and from the two equations of Problem 18.139, show that the rate of nutation $\dot{\theta}$ is defined by the differential equation $\dot{\theta}^2 = f(\theta)$, where

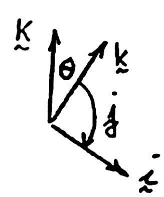
$$f(\theta) = \frac{1}{I'}\left(2E - \frac{\beta^2}{I} - 2mgc\cos\theta\right) - \left(\frac{\alpha - \beta\cos\theta}{I'\sin\theta}\right)^2$$

(*c*) Further show, by introducing the auxiliary variable $x = \cos\theta$, that the maximum and minimum values of θ can be obtained by solving for x the cubic equation

$$\left(2E - \frac{\beta^2}{I} - 2mgcx\right)(1 - x^2) - \frac{1}{I'}(\alpha - \beta x)^2 = 0$$

SOLUTION





(*a*) Angular velocity of the top:

$$\boldsymbol{\omega} = -\dot{\varphi}\sin\theta\mathbf{i} + \dot{\theta}\mathbf{j} + (\dot{\psi} + \dot{\varphi}\cos\theta)\mathbf{k}$$

Kinetic energy:

$$T = \frac{1}{2}I_x\omega_x^2 + \frac{1}{2}I_y\omega_y^2 + \frac{1}{2}I_z\omega_z^2$$

$$= \frac{1}{2}I'(\dot{\varphi}\sin\theta)^2 + \frac{1}{2}I'\dot{\theta}^2 + \frac{1}{2}I_z\omega_z^2$$

Potential energy: $V = mgc\cos\theta$

Principle of conservation of energy: $T + V = E$

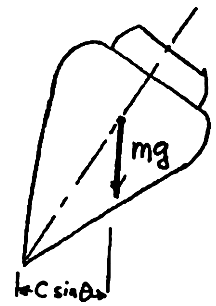
$$\frac{1}{2}I'(\dot{\varphi}\sin\theta)^2 + \frac{1}{2}I'\dot{\theta}^2 + \frac{1}{2}I\omega_z^2 + mgc\cos\theta = E \blacktriangleleft$$

(*b*) Solving for $\dot{\theta}^2$,

$$\dot{\theta}^2 = \frac{1}{I'}(2E - I_z\omega_z^2 - 2mgc\cos\theta) - (\dot{\varphi}\sin\theta)^2 \qquad \text{(A)}$$

Equation (2) of Problem 18.137, with $\omega_z = \dot{\psi} + \dot{\varphi}\cos\theta$ gives

$$I_z\omega_z^2 = \frac{\beta^2}{I} \qquad \text{(B)}$$



Equation (1) of Problem 18.137 gives

$$I'\dot{\varphi}\sin^2\theta + \beta\cos\theta = \alpha \qquad \text{(C)}$$

$$\dot{\varphi} = \frac{\alpha - \beta\cos\theta}{I'\sin^2\theta} \qquad \text{(D)}$$

Substituting Equations (D) and (B) into Equation (A),

$$\dot{\theta}^2 = f(\theta)$$

where $$f(\theta) = \frac{1}{I'}\left(2E - \frac{\beta^2}{I} - 2mgc\cos\theta\right) - \left(\frac{\alpha - \beta\cos\theta}{I'\sin\theta}\right)^2 \qquad (1) ◀$$

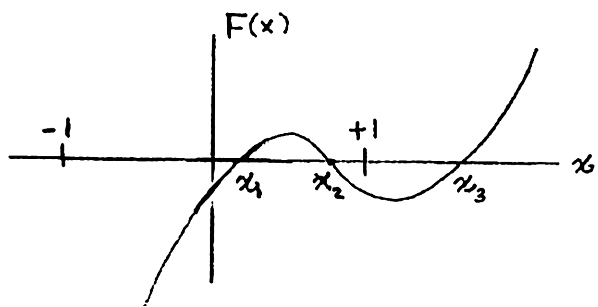
(*c*) Maximum and minimum values of θ occur when $f(\theta) = 0$. Setting $\cos\theta = x$ and $\sin^2\theta = 1 - x^2$ in Equation (1), and letting $f(\theta) = 0$ gives

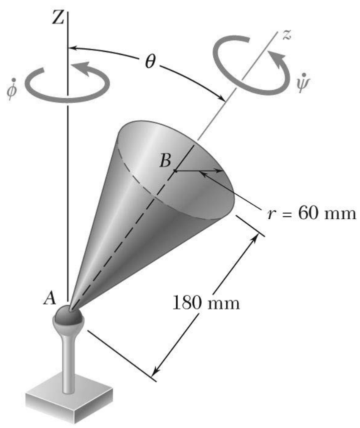
$$\frac{1}{I'}\left(2E - \frac{\beta^2}{I} + 2mgcx\right) - \frac{(\alpha - \beta x)^2}{(I')^2(1 - x^2)} = 0$$

Multiplying by $I'(1 - x^2)$ gives the cubic equation $F(x) = 0$:

$$\left(2E - \frac{\beta^2}{I} - 2mgcx\right)(1 - x^2) - \frac{1}{I'}(\alpha - \beta x)^2 = 0 \qquad (2) ◀$$

Solving this equation will yield three values of x. The two values lying between -1 and $+1$ correspond to the maximum and minimum values of θ.





PROBLEM 18.139

A solid cone of height 180 mm with a circular base of radius 60 mm is supported by a ball and socket at A. The cone is released from the position $\theta_0 = 30°$ with a rate of spin $\dot\psi_0 = 300$ rad/s, a rate of precession $\dot\phi_0 = 20$ rad/s, and a zero rate of nutation. Determine (*a*) the maximum value of θ in the ensuing motion, (*b*) the corresponding values of the rates of spin and precession. [*Hint:* Use Eq. (2) of Prob. 18.138; you can either solve this equation numerically or reduce it to a quadratic equation, since one of its roots is known.]

SOLUTION

Data: $r = 60 \text{ mm} = 0.06 \text{ m}, \quad h = 180 \text{ mm} = 0.18 \text{ m}, \quad c = \frac{3}{4}h = 0.135 \text{ m},$

$$\theta_0 = 30°, \quad \dot\psi_0 = 300 \text{ rad/s}, \quad \dot\phi_0 = 20 \text{ rad/s}, \quad \dot\theta_0 = 0$$

Calculate the following:

$$\frac{I}{m} = \frac{3}{10}r^2 = \frac{3}{10}(0.06)^2 = 1.08 \times 10^{-3} \text{ m}^2$$

$$\frac{I'}{m} = \frac{3}{5}\left(\frac{1}{4}r^2 + h^2\right) = \frac{3}{5}\left[\frac{1}{4}(0.06)^2 + (0.18)^2\right] = 19.98 \times 10^{-3} \text{ m}^2$$

Initially, $\omega_x = -\dot\phi_0 \sin\theta_0 = -20\sin 30° = -10 \text{ rad/s}, \qquad \omega_y = \dot\theta_0 = 0$

$$\omega_z = \dot\psi_0 + \dot\phi_0 \cos\theta_0 = 300 + 20\cos 30° = 317.32 \text{ rad/s}$$

$$\frac{T}{m} = \frac{1}{2}\frac{I'}{m}(\omega_x^2 + \omega_y^2) + \frac{1}{2}\frac{I}{m}\omega_z^2$$

$$= \frac{1}{2}(19.98 \times 10^{-3})(10^2 + 0) + \frac{1}{2}(1.08 \times 10^{-3})(317.32)^2 = 55.3728 \text{ m}^2/\text{s}^2$$

$$\frac{V}{m} = gc\cos\theta_0 = (9.81)(0.135)\cos 30° = 1.1469 \text{ m}^2/\text{s}^2$$

$$\frac{2E}{m} = 2(55.3728 + 1.1469) = 113.0394 \text{ m}^2/\text{s}^2$$

$$\frac{\beta}{m} = \frac{I}{m}\omega_z = (1.08 \times 10^{-3})(317.32) = 0.342706 \text{ m}^2/\text{s}$$

$$\frac{\alpha}{m} = \frac{I'}{m}\dot\phi_0 \sin^2\theta_0 + \beta\cos\theta_0$$

$$= (19.98 \times 10^{-3})(20)\sin^2 30° + 0.342706\cos 30°$$

$$= 0.396692 \text{ m}^2/\text{s}$$

PROBLEM 18.139 (Continued)

After dividing by *m*, Equation (2) of Problem 18.138 becomes

$$F(x) = \left(\frac{2E}{m} - \frac{\frac{\beta^2}{m^2}}{\frac{I}{m}} - 2gcx \right)(1 - x^2) - \frac{m}{I'}\left(\frac{\alpha}{m} - \frac{\beta x}{m} \right)^2 = 0$$

$$\left[113.0394 - \frac{(0.342706)^2}{1.08 \times 10^{-3}} - (2)(9.81)(0.135)x \right](1 - x^2) - \frac{(0.396692 - 0.342706x)^2}{19.98 \times 10^{-3}} = 0$$

$$(4.29198 - 2.6487x)(1 - x^2) - 5.87825(1.157529 - x)^2 = 0$$

(*a*) Roots are: $x = \cos\theta = 0.68170, \quad 0.86603, \quad 2.2919$

$$\theta_{\max} = \cos^{-1}(0.68170) = 47.023°$$

$\theta_{\max} = 47.0°$ ◀

(*b*) By Equation (4) of Problem 18.137,

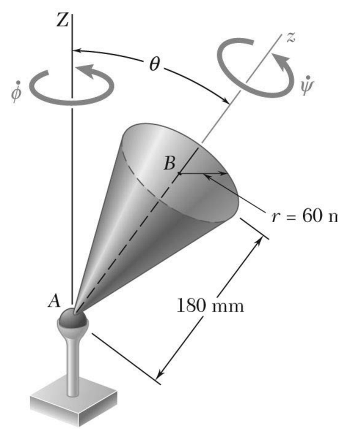
$$\dot{\phi} = \frac{\alpha - \beta\cos\theta}{I'\sin^2\theta} = \frac{0.396692 - (0.342706)(0.68170)}{(19.98 \times 10^{-3})\sin^2 47.023°} = 15.2474$$

$$\omega_z = 317.32 \text{ rad/s}$$

$$\dot{\psi} = \omega_z - \dot{\phi}\cos\theta = 317.32 - (15.2474)(0.68170)$$

spin: $\dot{\psi} = 307$ rad/s ◀

precession: $\dot{\phi} = 15.25$ rad/s ◀



PROBLEM 18.140

A solid cone of height 180 mm with a circular base of radius 60 mm is supported by a ball and socket at *A*. The cone is released from the position $\theta_0 = 30°$ with a rate of spin $\dot\psi_0 = 300$ rad/s, a rate of precession $\dot\phi_0 = -4$ rad/s, and a zero rate of nutation. Determine (*a*) the maximum value of θ in the ensuing motion, (*b*) the corresponding values of the rates of spin and precession, (*c*) the value of θ for which the sense of the precession is reversed. (See hint of Problem 18.139.)

SOLUTION

Data: $r = 60 \text{ mm} = 0.06 \text{ m}, \quad h = 180 \text{ mm} = 0.18 \text{ m}, \quad c = \frac{3}{4}h = 0.135 \text{ m},$

$$\theta_0 = 30°, \quad \dot\psi_0 = 300 \text{ rad/s}, \quad \dot\phi_0 = -4 \text{ rad/s}, \quad \dot\theta_0 = 0$$

Calculate the following:

$$\frac{I}{m} = \frac{3}{10}r^2 = \frac{3}{10}(0.06)^2 = 1.08\times 10^{-3} \text{ m}^2$$

$$\frac{I'}{m} = \frac{3}{5}\left(\frac{1}{4}r^2 + h^2\right) = \frac{3}{5}\left[\frac{1}{4}(0.06)^2 + (0.18)^2\right] = 19.98\times 10^{-3} \text{ m}^2$$

Initially, $\omega_x = -\dot\phi_0 \sin\theta_0 = -(-4)\sin 30° = 2 \text{ rad/s}, \quad \omega_y = \dot\theta_0 = 0$

$$\omega_z = 300 + (-4)\cos 30° = 296.536 \text{ rad/s}$$

$$\frac{T}{m} = \frac{1}{2}\frac{I'}{m}(\omega_x^2 + \omega_y^2) + \frac{1}{2}\frac{I}{m}\omega_z^2$$

$$= \frac{1}{2}(19.98\times 10^{-3})((2)^2 + 0) + \frac{1}{2}(1.08\times 10^{-3})(296.536)^2 = 47.5241 \text{ m}^2/\text{s}^2$$

$$\frac{V}{m} = gc\cos\theta = (9.81)(0.135)\cos 30° = 1.1469 \text{ m}^2/\text{s}^2$$

$$\frac{2E}{m} = 2(47.5241 + 1.1469) = 97.3419 \text{ m}^2/\text{s}^2$$

$$\frac{\beta}{m} = \frac{I}{m}\omega_z = (1.08\times 10^{-3})(296.536) = 0.320259 \text{ m}^2/\text{s}$$

$$\frac{\alpha}{m} = \frac{I'}{m}\dot\phi_0 \sin^2\theta_0 + \beta\cos\theta_0$$

$$= (19.98\times 10^{-3})(-4)\sin^2 30° + 0.320259\cos 30°$$

$$= 0.257372 \text{ m}^2/\text{s}$$

PROBLEM 18.140 (Continued)

After dividing by m, Equation (2) of Problem 18.138 becomes

$$F(x) = \left(\frac{2E}{m} - \frac{\frac{\beta^2}{m^2}}{\frac{I}{m}} - 2gcx \right)(1 - x^2) - \frac{m}{I'}\left(\frac{\alpha}{m} - \frac{\beta x}{m} \right)^2 = 0$$

$$\left[97.3419 - \frac{(0.320259)^2}{1.08 \times 10^{-3}} - (2)(9.81)(0.135)x \right](1 - x^2) - \frac{(0.257372 - 0.320259x)^2}{19.98 \times 10^{-3}} = 0$$

$$(2.37324 - 2.6487x)(1 - x^2) - 5.13342(0.80364 - x)^2 = 0$$

(*a*) Roots are: $x = \cos\theta = 0.23732, \quad 0.86603, \quad 1.73$

$$\theta_{\max} = \cos^{-1}(0.23732) = 76.272°$$

$\theta_{\max} = 76.3°$ ◀

(*b*) By Equation (4) of Problem 18.137,

$$\dot{\phi} = \frac{\alpha - \beta\cos\theta}{I'\sin^2\theta} = \frac{0.257372 - (0.320259)(0.23732)}{(19.98 \times 10^{-3})\sin^2 76.272°} = 9.6192 \text{ rad/s}$$

$$\omega_z = 296.536 \text{ rad/s}$$

$$\dot{\psi} = \omega_z - \dot{\phi}\cos\theta = 296.536 - (9.6192)(0.23732)$$

spin: $\dot{\psi} = 294$ rad/s ◀

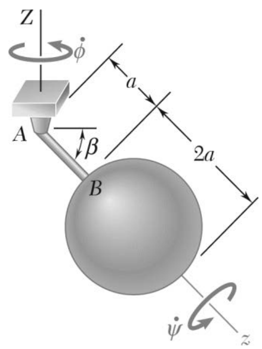
precession: $\dot{\phi} = 9.62$ rad/s ◀

(*c*)

$$\dot{\phi} = \frac{\alpha - \beta\cos\theta}{I'\sin^2\theta} = 0$$

$$\cos\theta = \frac{\alpha}{\beta} = \frac{0.257372}{0.320259}$$

$\theta = 36.5°$ ◀



PROBLEM 18.141*

A homogeneous sphere of mass m and radius a is welded to a rod AB of negligible mass, which is held by a ball-and-socket support at A. The sphere is released in the position $\beta = 0$ with a rate of precession $\dot{\phi}_0 = \sqrt{17g/11a}$ with no spin or nutation. Determine the largest value of β in the ensuing motion.

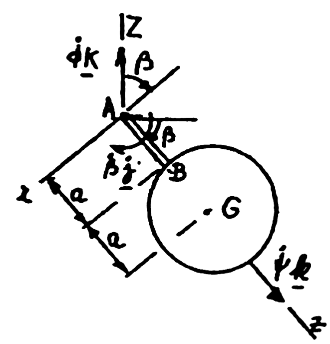
SOLUTION

<u>Conservation of angular momentum about the Z and z axes.</u>

Since the only external forces are the weight of the sphere and the reaction at A, a reasoning similar to that used in Problem 18.109 shows that the angular momentum is conserved about the Z and z axes.

Choosing the principal axes $Axyz$ shown (with y horizontal and pointing into the paper), we have

$$\boldsymbol{\omega} = -\dot{\phi}\cos\beta\mathbf{i} + \dot{\beta}\mathbf{j} + (\dot{\psi} - \dot{\phi}\sin\beta)\mathbf{k}$$



The moments of inertia are

$$I_z = \frac{2}{5}ma^2$$

$$I_x = I_y = \frac{2}{5}ma^2 + m(2a)^2 = \frac{22}{5}ma^2$$

Angular momentum about A:

$$\mathbf{H}_A = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$\mathbf{H}_A = -\frac{22}{5}ma^2\dot{\phi}\cos\beta\mathbf{i} + \frac{22}{5}ma^2\dot{\beta}\mathbf{j} + \frac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)\mathbf{k}$$

We write $\mathbf{H}_z = \text{constant}$, or $\mathbf{H}_A \cdot \mathbf{K} = \text{constant}$

Since $\mathbf{i}\cdot\mathbf{K} = -\cos\beta$, $\quad \mathbf{j}\cdot\mathbf{K} = 0$, $\quad \mathbf{k}\cdot\mathbf{K} = -\sin\beta$

$$\mathbf{H}_0 \cdot \mathbf{K} = -\frac{22}{5}ma^2\dot{\phi}\cos\beta(-\cos\beta)$$
$$+\frac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)(-\sin\beta) = \text{constant}$$

With the initial conditions $\dot{\phi} = \dot{\phi}_0$, $\dot{\psi} = \dot{\beta} = 0$, $\beta = 0$, we find that the constant is $\frac{22}{5}ma^2\dot{\phi}_0$. Thus,

$$11\dot{\phi}\cos^2\beta - (\dot{\psi} - \dot{\phi}\sin\beta)\sin\beta = 11\dot{\phi}_0 \qquad (1)$$

We now write $H_z = \text{constant}$:

$$H_z = \frac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta) = \text{constant}$$

and, from the initial conditions, we find that the constant is zero. Thus,

$$\dot{\psi} - \dot{\phi}\sin\beta = 0 \tag{2}$$

Conservation of energy.

We have

$$T = \frac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2)$$

$$T = \frac{1}{2}\left[\frac{22}{5}ma^2\dot{\phi}^2\cos^2\beta + \frac{22}{5}ma^2\dot{\beta}^2 + \frac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)^2\right]$$

and selecting the datum at $\beta = 0$:

$$V = -2mga\sin\beta$$

$$T + V = \text{constant:} \quad \frac{1}{2}\left[\frac{22}{5}ma^2\dot{\phi}^2\cos^2\beta + \frac{22}{5}ma^2\dot{\beta}^2 + \frac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)^2\right] - 2mga\sin\beta = \text{constant}$$

From the initial conditions $\dot{\phi} = \dot{\phi}_0$, $\dot{\psi} = \dot{\beta} = 0$, $\beta = 0$, we find that the constant is $\frac{11}{5}ma^2\dot{\phi}_0^2$. Thus,

$$11\dot{\phi}^2\cos^2\beta + 11\dot{\beta}^2 + (\dot{\psi} - \dot{\phi}\sin\beta)^2 - 10\frac{g}{a}\sin\beta = 11\dot{\phi}_0^2 \tag{3}$$

Substituting for $\dot{\psi} - \dot{\phi}\sin\beta$ from Eq. (2) into Eqs. (1) and (3):

Eq. (1): $$11\dot{\phi}\cos^2\beta = 11\dot{\phi}_0 \tag{1$'$}$$

Eq. (3): $$11\dot{\phi}^2\cos^2\beta + 11\dot{\beta}^2 - 10\frac{g}{a}\sin\beta = 11\dot{\phi}_0^2 \tag{3$'$}$$

Solving (1′) for $\dot{\phi}$,

$$\dot{\phi} = \dot{\phi}_0\sec^2\beta \tag{4}$$

Substituting for $\dot{\phi}$ from Eq. (4) into Eq. (3′):

$$11(\dot{\phi}_0\sec^2\theta)^2\cos^2\beta + 11\dot{\beta}^2 - 10\frac{g}{a}\sin\beta = 11\dot{\phi}_0^2 \tag{5}$$

For the maximum value of β, we have $\dot{\beta} = 0$ and Eq. (5) yields

$$\dot{\phi}_0^2\left(\frac{1}{\cos^2\theta} - 1\right) = \frac{10}{11}\frac{g}{a}\sin\beta$$

$$\dot{\phi}_0^2 = \frac{10}{11}\frac{g}{a}\frac{\cos^2\beta}{\sin\beta} \tag{6}$$

PROBLEM 18.141* (Continued)

Given data: $\dot{\phi}_0^2 = \frac{17}{11}\frac{g}{a}\cdot$ Substituting into Eq. (6),

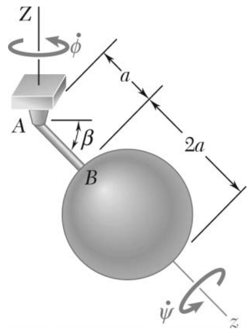
$$\frac{17}{11}\frac{g}{a} = \frac{10}{11}\frac{g}{a}\frac{\cos^2\beta}{\sin\beta} \quad \cos^2\beta = 1.7\sin\beta$$

Letting $\cos^2\beta = 1 - \sin^2\beta$, we have

$$\sin^2\beta + 1.7\sin\beta - 1 = 0$$

Solving the quadratic,

$$\sin\beta = -2.162 \text{ (impossible)} \quad \text{and} \quad \sin\beta = 0.46244 \qquad \beta_{\max} = 27.5° \blacktriangleleft$$



PROBLEM 18.142*

A homogeneous sphere of mass m and radius a is welded to a rod AB of negligible mass, which is held by a ball-and-socket support at A. The sphere is released in the position $\beta = 0$ with a rate of precession $\dot{\phi} = \dot{\phi}_0$ with no spin or nutation. Knowing that the largest value of β in the ensuing motion is 30°, determine (*a*) the rate of precession $\dot{\phi}_0$ of the sphere in its initial position, (*b*) the rates of precession and spin when $\beta = 30°$.

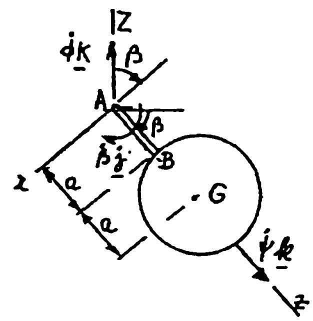
SOLUTION

<u>Conservation of angular momentum about the Z and z axes.</u>

Since the only external forces are the weight of the sphere and the reaction at A, a reasoning similar to that used in Problem 18.109 shows that the angular momentum is conserved about the Z and z axes.

Choosing the principal axes $Axyz$ shown (with y horizontal and pointing into the paper), we have

$$\boldsymbol{\omega} = -\dot{\phi}\cos\beta\mathbf{i} + \dot{\beta}\mathbf{j} + (\dot{\psi} - \dot{\phi}\sin\beta)\mathbf{k}$$



The moments of inertia are

$$I_z = \frac{2}{5}ma^2$$

$$I_x = I_y = \frac{2}{5}ma^2 + m(2a)^2 = \frac{22}{5}ma^2$$

Angular momentum about A:

$$\mathbf{H}_A = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$\mathbf{H}_A = -\frac{22}{5}ma^2\dot{\phi}\cos\beta\mathbf{i} + \frac{22}{5}ma^2\dot{\beta}\mathbf{j} + \frac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)\mathbf{k}$$

We write $\mathbf{H}_Z = \text{constant}$, or $\mathbf{H}_A \cdot \mathbf{K} = \text{constant}$

Since $\mathbf{i}\cdot\mathbf{K} = -\cos\beta$, $\quad \mathbf{j}\cdot\mathbf{K} = 0$, $\quad \mathbf{k}\cdot\mathbf{K} = -\sin\beta$

$$\mathbf{H}_0 \cdot \mathbf{K} = -\frac{22}{5}ma^2\dot{\phi}\cos\beta(-\cos\beta)$$

$$+\frac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)(-\sin\beta) = \text{constant}$$

With the initial conditions $\dot{\phi} = \dot{\phi}_0$, $\dot{\psi} = \dot{\beta} = 0$, $\beta = 0$, we find that the constant is $\frac{22}{5}ma^2\dot{\phi}_0$. Thus,

$$11\dot{\phi}\cos^2\beta - (\dot{\psi} - \dot{\phi}\sin\beta)\sin\beta = 11\dot{\phi}_0 \qquad (1)$$

We now write $H_z = \text{constant}$:

$$H_z = \frac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta) = \text{constant}$$

and, from the initial conditions, we find that the constant is zero. Thus,

$$\dot{\psi} - \dot{\phi}\sin\beta = 0 \qquad (2)$$

<u>Conservation of energy</u>.

We have

$$T = \frac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2)$$

$$T = \frac{1}{2}\left[\frac{22}{5}ma^2\dot{\phi}^2\cos^2\beta + \frac{22}{5}ma^2\dot{\beta}^2 + \frac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)^2\right]$$

and, selecting the datum at $\beta = 0$:

$$V = -2mga\sin\beta$$

$$T + V = \text{constant:}\quad \frac{1}{2}\left[\frac{22}{5}ma^2\dot{\phi}^2\cos^2\beta + \frac{22}{5}ma^2\dot{\beta}^2 + \frac{2}{5}ma^2(\dot{\psi} - \dot{\phi}\sin\beta)^2\right] - 2mga\sin\beta = \text{constant}$$

From the initial conditions $\dot{\phi} = \dot{\phi}_0,\ \dot{\psi} = \dot{\beta} = 0,\ \beta = 0,$ we find that the constant is $\frac{11}{5}ma^2\dot{\phi}_0^2$. Thus,

$$11\dot{\phi}^2\cos^2\beta + 11\dot{\beta}^2 + (\dot{\psi} - \dot{\phi}\sin\beta)^2 - 10\frac{g}{a}\sin\beta = 11\dot{\phi}_0^2 \qquad (3)$$

Substituting for $\dot{\psi} - \dot{\phi}\sin\beta$ from Eq. (2) into Eqs. (1) and (3),

Eq. (1): $$11\dot{\phi}\cos^2\beta = 11\dot{\phi}_0 \qquad (1')$$

Eq. (3): $$11\dot{\phi}^2\cos^2\beta + 11\dot{\beta}^2 - 10\frac{g}{a}\sin\beta = 11\dot{\phi}_0^2 \qquad (3')$$

Solving $(1')$ for $\dot{\phi}$,

$$\dot{\phi} = \dot{\phi}_0\sec^2\beta \qquad (4)$$

Substituting for $\dot{\phi}$ from Eq. (4) into Eq. $(3')$,

$$11(\dot{\phi}_0\sec^2\theta)^2\cos^2\beta + 11\dot{\beta}^2 - 10\frac{g}{a}\sin\beta = 11\dot{\phi}_0^2 \qquad (5)$$

For the maximum value of β, we have $\dot{\beta} = 0$ and Eq. (5) yields

$$\dot{\phi}_0^2\left(\frac{1}{\cos^2\theta} - 1\right) = \frac{10}{11}\frac{g}{a}\sin\beta$$

$$\dot{\phi}_0^2 = \frac{10}{11}\frac{g}{a}\frac{\cos^2\beta}{\sin\beta} \qquad (6)$$

PROBLEM 18.142* (Continued)

(*a*) Making $\beta = 30°$ in Eq. (6), we have

$$\dot{\phi}_0^2 = \frac{10}{11}\frac{g}{a}\frac{0.75}{0.5} = \frac{15}{11}\frac{g}{a} \qquad \dot{\phi}_0 = \sqrt{\frac{15}{11}\frac{g}{a}} \blacktriangleleft$$

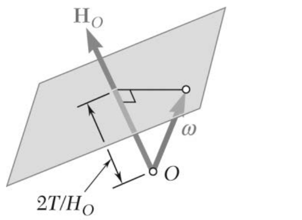
(*b*) Substituting for $\dot{\phi}_0$ in Eq. (4), and making $\beta = 30°$:

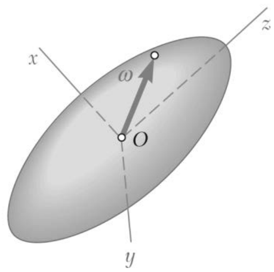
$$\dot{\phi} = \dot{\phi}_0 \sec^2 30° = \sqrt{\frac{15}{11}\frac{g}{a}}\left(\frac{4}{3}\right) = \sqrt{\frac{15(16)}{11(9)}\frac{g}{a}} \qquad \dot{\phi} = 2\sqrt{\frac{20}{33}\frac{g}{a}} \blacktriangleleft$$

Substituting for in Eq. (2), and making $\beta = 30°$:

$$\dot{\psi} = \dot{\phi}\sin 30° = \frac{1}{2}\dot{\phi} \qquad \dot{\psi} = \sqrt{\frac{20}{33}\frac{g}{a}} \blacktriangleleft$$

PROBLEM 18.143*





Consider a rigid body of arbitrary shape which is attached at its mass center O and subjected to no force other than its weight and the reaction of the support at O.

(a) Prove that the angular momentum $\mathbf{H}_O$ of the body about the fixed Point O is constant in magnitude and direction, that the kinetic energy T of the body is constant, and that the projection along $\mathbf{H}_O$ of the angular velocity $\boldsymbol{\omega}$ of the body is constant.

(b) Show that the tip of the vector $\boldsymbol{\omega}$ describes a curve on a fixed plane in space (called the *invariable plane*), which is perpendicular to $\mathbf{H}_O$ and at a distance $2T/H_O$ from O.

(c) Show that with respect to a frame of reference attached to the body and coinciding with its principal axes of inertia, the tip of the vector $\boldsymbol{\omega}$ appears to describe a curve on an ellipsoid of equation

$$I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 = 2T = \text{constant}$$

The ellipsoid (called the *Poinsot ellipsoid*) is rigidly attached to the body and is of the same shape as the ellipsoid of inertia, but of a different size.

SOLUTION

(a) From Equation (18.27), $\Sigma\mathbf{M}_O = \dot{\mathbf{H}}_O$

Since $\Sigma\mathbf{M}_O = 0, \quad \dot{\mathbf{H}}_O = 0.$ $\qquad \mathbf{H}_O = \text{constant} \quad (1)$ ◀

Conservation of energy: $T + V = \text{constant}$

Since $V = 0,$ $\qquad T = \text{constant} \quad (2)$ ◀

For a rigid body rotating about Point O,

$$T = \frac{1}{2}\left(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2\right)$$

$$\mathbf{H}_O = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$$

$$\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

$$\mathbf{H}_O \cdot \omega = I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 = 2T$$

Let β be the angle between the vectors $\mathbf{H}_O$ and $\boldsymbol{\omega}$.

$$\mathbf{H}_O \cdot \boldsymbol{\omega} = H_O\omega\cos\beta$$

The projection of $\boldsymbol{\omega}$ along $\mathbf{H}_O$ is $\omega\cos\beta$

$$\omega\cos\beta = \frac{2T}{H_O} = \text{constant}$$ $\qquad \omega\cos\beta = \text{constant} \quad (3)$ ◀

PROBLEM 18.143* (Continued)

(*b*) $\omega \cos \beta$ is the perpendicular distance from the invariable plane. This distance is equal to $\frac{2T}{H_O}$.

(*c*) For a frame of reference attached to the body, the moments of inertia with respect of orthogonal axes of the frame do not change.

$$I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2 = 2T \tag{4}$$

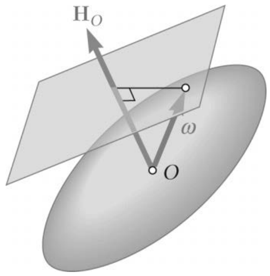
Let

$$a_1 = \sqrt{\frac{2T}{I_x}}, \quad b_1 = \sqrt{\frac{2T}{I_y}}, \quad c_1 = \sqrt{\frac{2T}{I_z}} \tag{5}$$

Then

$$\frac{\omega_x^2}{a_1^2} + \frac{\omega_y^2}{b_1^2} + \frac{\omega_z^2}{c_1^2} = 1 \tag{6}$$

which is the equation of an ellipsoid.



PROBLEM 18.144*

Referring to Problem 18.143, (*a*) prove that the Poinsot ellipsoid is tangent to the invariable plane, (*b*) show that the motion of the rigid body must be such that the Poinsot ellipsoid appears to roll on the invariable plane. [*Hint:* In part *a,* show that the normal to the Poinsot ellipsoid at the tip of $\boldsymbol{\omega}$ is parallel to $\mathbf{H}_O$. It is recalled that the direction of the normal to a surface of equation $F(x,\ y,\ z) = \text{constant}$ at a Point *P* is the same as that of **grad** *F* at Point *P*.]

SOLUTION

(*a*) From Problem 18.143, the equation of the Poinsot ellipsoid is

$$I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 = 2T = \text{constant}$$

Let

$$F(\omega_x, \omega_y, \omega_z) = I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2$$

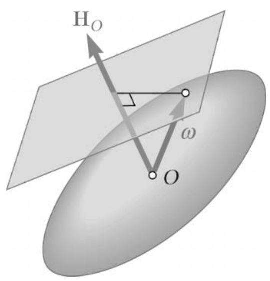
$$\textbf{grad}\ F = \frac{\partial F}{\partial \omega_x}\mathbf{i} + \frac{\partial F}{\partial \omega_y}\mathbf{j} + \frac{\partial F}{\partial \omega_z}\mathbf{k}$$

$$= 2I_x\omega_x\mathbf{i} + 2I_y\omega_y\mathbf{j} + 2I_z\omega_z\mathbf{k} = 2\mathbf{H}_O$$

The direction of the normal at a point on the surface of the ellipsoid is parallel to **grad** *F*, which in turn is parallel to $\mathbf{H}_O$. Since $\mathbf{H}_O$ is normal also to the invariable plane, it follows that the Poinsot ellipsoid is tangent to the invariable plane at the point common to the plane and the ellipsoid.

(*b*) The Poinsot elipsoid moves with the body. Thus, its angular velocity is $\boldsymbol{\omega}$, the angular velocity of the body. Since Point *O* is regarded as fixed, the angular velocity vector lies along the axis of rotation, or the locus of points of zero velocity. Thus, the velocity of the point of contact of the Poinsot ellipsoid with the invariable plane is zero. The Poinsot ellipsoid rolls without slipping on the invariable plane.

PROBLEM 18.145*



Using the results obtained in Problems 18.143 and 18.144, show that for an axisymmetrical body attached at its mass center O and under no force other than its weight and the reaction at O, the Poinsot ellipsoid is an ellipsoid of revolution and the space and body cones are both circular and are tangent to each other. Further show that (*a*) the two cones are tangent externally, and the precession is direct, when $I < I'$, where I and I' denote, respectively, the axial and transverse moment of inertia of the body, (*b*) the space cone is inside the body cone, and the precession is retrograde, when $I > I'$.

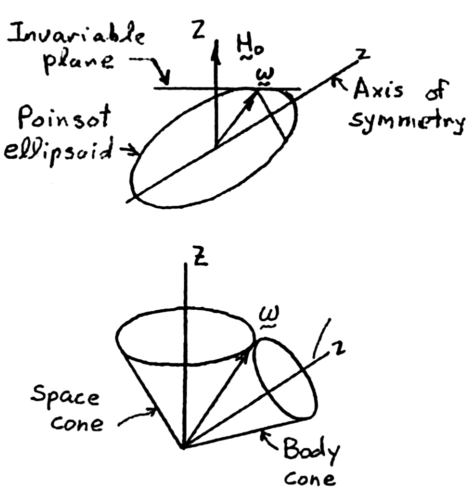
SOLUTION

Let $I_x = I_y = I'$ and $I_z = I$ so that the z axis is the symmetry axis. Then, the equation of the Poinsot ellipsoid (Equation (4) of Problem 18.143) becomes

$$I'\left(\omega_x^2 + \omega_y^2\right) + I\omega_z^2 = 2T = \text{constant}$$

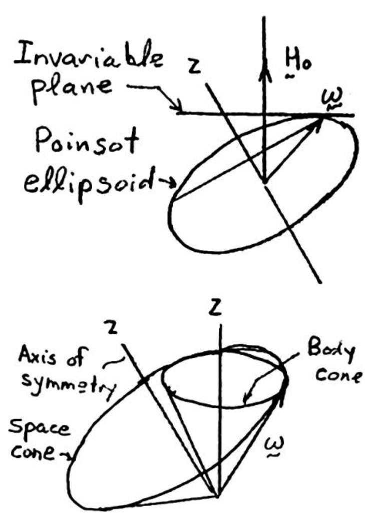
which is the equation of an *ellipsoid of revolution*. It follows that the tip of $\boldsymbol{\omega}$ describes circles on both the Poinsot ellipsoid and on the invariable plane, and that the vector $\boldsymbol{\omega}$ itself describes *circular body and space cones.* The Poinsot ellipsoid, the invariable plane, and the body and space cones are shown below for cases *a* and *b*.

(*a*) $I < I'$



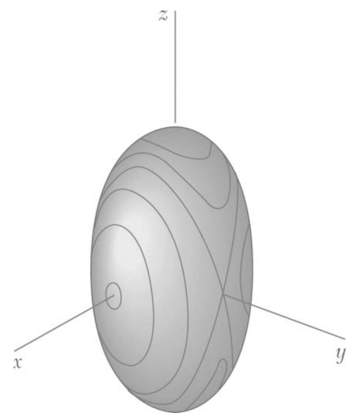
Direct Precession

(*b*) $I > I'$



Retrograde Precession

PROBLEM 18.146*



Refer to Problems 18.143 and 18.144.

(*a*) Show that the curve (called *polhode*) described by the tip of the vector $\boldsymbol{\omega}$ with respect to a frame of reference coinciding with the principal axes of inertia of the rigid body is defined by the equations

$$I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 = 2T = \text{constant} \qquad (1)$$

$$I_x^2\omega_x^2 + I_y^2\omega_y^2 + I_z^2\omega_z^2 = H_O^2 = \text{constant} \qquad (2)$$

and that this curve can, therefore, be obtained by intersecting the Poinsot ellipsoid with the ellipsoid defined by Eq. (2).

(*b*) Further show, assuming $I_x > I_y > I_z$, that the polhodes obtained for various values of H_O have the shapes indicated in the figure.

(*c*) Using the result obtained in part *b*, show that a rigid body under no force can rotate about a fixed centroidal axis if, and only if, that axis coincides with one of the principal axes of inertia of the body, and that the motion will be stable if the axis of rotation coincides with the major or minor axis of the Poinsot ellipsoid (*z* or *x* axis in the figure) and unstable if it coincides with the intermediate axis (*y* axis).

SOLUTION

(*a*) Equation (1) expresses conservation of energy as shown in the solution to Problem 18.143. It is the equation of the Poinsot ellipsoid.

Let $$a_1 = \sqrt{\frac{2T}{I_x}}, \quad b_1 = \sqrt{\frac{2T}{I_y}}, \quad c_1 = \sqrt{\frac{2T}{I_z}}$$

Then $$\frac{\omega_x^2}{a_1^2} + \frac{\omega_y^2}{b_1^2} + \frac{\omega_z^2}{c_1^2} = 1 \qquad (3)$$

which is the equation of an ellipsoid.

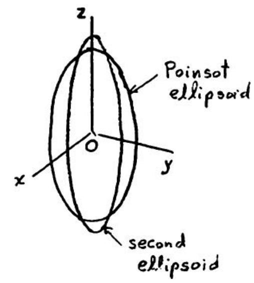
Equation (2) expresses the constancy of $H_O^2 = (H_O)_x^2 + (H_O)_y^2 + (H_O)_z^2$, the square of the magnitude of the angular momentum vector.

Let $$a_2 = \frac{H_O}{I_x}, \quad b_2 = \frac{H_O}{I_y}, \quad c_2 = \frac{H_O}{I_z}$$

Then $$\frac{\omega_x^2}{a_2^2} + \frac{\omega_y^2}{b_2^2} + \frac{\omega_z^2}{c_2^2} = 1 \qquad (4)$$

which is the equation of a second ellipsoid.

Since the coordinates ω_x, ω_y, ω_z of the tip of the vector $\boldsymbol{\omega}$ must satisfy both Equations (1) and (2), the curve described by the tip of $\boldsymbol{\omega}$ is the intersection of the two ellipsoids.

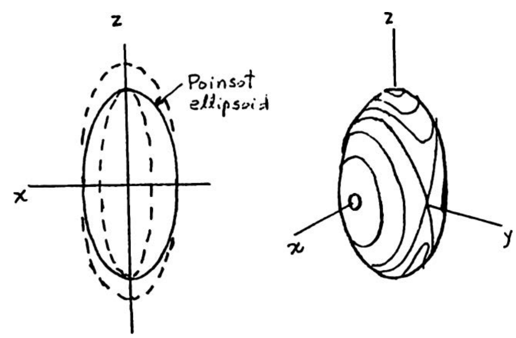


(*b*) Assume $I_x > I_y > I_z$.

Then $a_1 < b_1 < c_1$ and $a_2 < b_2 < c_2$.

Thus, for both ellipsoids, the minor axis is directed along the *x* axis, the intermediate axis along the *y* axis, and the major axis along the *z* axis. However, because the ratio of the major to minor semiaxis is $\sqrt{\frac{I_x}{I_z}}$ for the Poinsot ellipsoid and is $\frac{I_x}{I_z}$ for the second ellipsoid, the deviation from a spherical shape is more pronounced in the second ellipsoid.

The largest ellipsoid of the second type to be in contact with the Poinsot ellipsoid will lie outside that ellipsoid and touch it at its points of intersection with the *x* axis, and the smallest will lie inside the Poinsot ellipsoid and touch it at its points of intersection with the *z* axis (see left-hand sketch). All ellipsoids of the second type comprised between these two will intersect the Poinsot ellipsoid along the curves called polhodes, as shown in the right-hand figure.

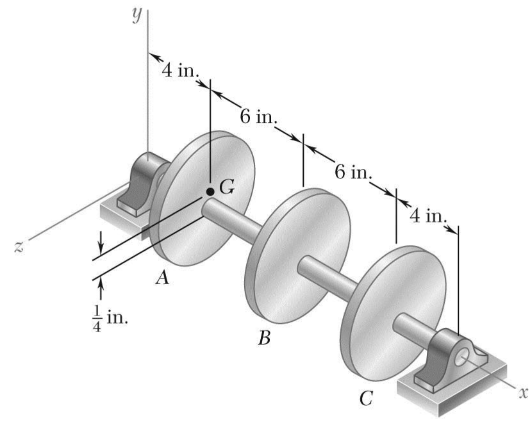


Note that the ellipsoid of the second type, which has the same intermediate axis as the Poinsot ellipsoid, intersects that ellipsoid along two ellipses whose planes contain the *y* axis. These curves are not polhodes, since the tip of $\boldsymbol{\omega}$ will not describe them, but they separate the polhodes into four groups. Two groups loop around the minor axis (*x* axis) and the other two around the major axis (*z* axis).

(*c*) If the body is set to spin about one of the principal axes, the Poinsot ellipsoid will remain in contact with the invariable plane at the same point (on the *x*, *y*, or *z* axis); the rotation is steady. In any other case, the point of contact will be located on one of the polhodes, and the tip of $\boldsymbol{\omega}$ will start describing that polhode, while the Poinsot ellipsoid rolls on the invariable plane.

A rotation about the *minor* or the *major* axis (*x* or *z* axis) is *stable*. If that motion is disturbed, the tip of $\boldsymbol{\omega}$ will move to a very small polhode surrounding that axis and stay close to its original position.

On the other hand, a rotation about the *intermediate* axis (*y* axis) is *unstable.* If that motion is disturbed, the tip of $\boldsymbol{\omega}$ will move to one of the polhodes located near that axis and start describing it, departing completely from its original position and causing the body to tumble.

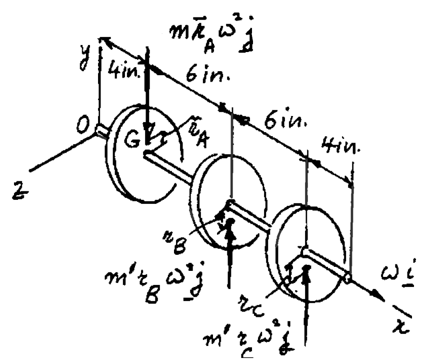


PROBLEM 18.147

Three 25-lb rotor disks are attached to a shaft which rotates at 720 rpm. Disk *A* is attached eccentrically so that its mass center is $\frac{1}{4}$in. from the axis of rotation, while disks *B* and *C* are attached so that their mass centers coincide with the axis of rotation. Where should 2-lb weights be bolted to disks *B* and *C* to balance the system dynamically?

SOLUTION

The system is dynamically balanced if the effective forces are equivalent to zero. Let Points *A*, *B*, and *C* be the points where the disks are attached to the shaft and let Point *G* be the mass center of disk *A*. Let *m* be the mass of each rotor and *m*′ the magnitude of each added mass.



Since $\omega = \text{constant},$

$$\dot{H}_G = 0$$

for each disk. We treat the added masses as particles so that their moments of inertia about their mass centers are negligible. The effective force of disk *A* is.

$$(F_A)_{\text{eff}} = -m\bar{r}_A\omega^2\mathbf{j}$$

The effective forces of the added masses are

$$m'r_B\omega^2\mathbf{j} \quad \text{and} \quad m'r_C\omega^2\mathbf{j},$$

respectively for the masses added to disks *B* and *C*.

$$\Sigma F_{\text{eff}} = 0: \quad -m\bar{r}_A\omega^2\mathbf{j} + m'r_B\omega^2\mathbf{j} + m'r_C\omega^2\mathbf{j} = 0$$

$$r_B + r_C = \frac{m}{m'}\bar{r}_A \qquad (1)$$

$$\Sigma(M_0)_{\text{eff}} = 0 \quad 4\mathbf{i}\times(-m\bar{r}_A\omega^2\mathbf{j}) + 10\mathbf{i}\times(m'r_B\omega^2\mathbf{j}) + 16\mathbf{i}\times(m'r_C\omega^2\mathbf{j})$$

$$10r_B + 16r_2 = 4\frac{m}{m'}\bar{r}_A \qquad (2)$$

PROBLEM 18.147 (Continued)

Data: $\overline{r}_A = \frac{1}{4}\text{ in.} = 0.25\text{ in.} \qquad \frac{m}{m'} = \frac{25\text{ lb}}{2\text{ lb}} = 12.5$

$$r_A + r_B = 3.125\text{ in.} \tag{1'}$$

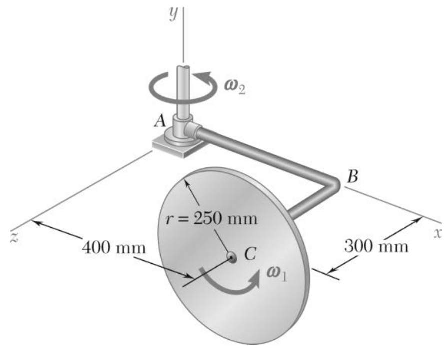
$$10r_A + 16r_B = 12.5\text{ in.} \tag{2'}$$

Solving the simultaneous equations.

$$r_B = 6.25\text{ in.} \qquad r_C = -3.125$$

Placement of added masses.

On B: $6\frac{1}{4}$ in. below shaft. On C: $3\frac{1}{8}$ in. above shaft ◀

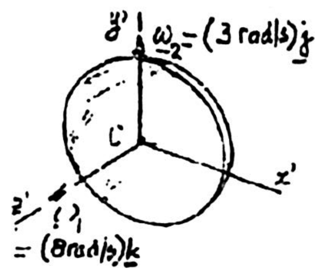


PROBLEM 18.148

A homogeneous disk of mass $m = 5$ kg rotates at the constant rate $\omega_1 = 8$ rad/s with respect to the bent axle *ABC*, which itself rotates at the constant rate $\omega_2 = 3$ rad/s about the *y* axis. Determine the angular momentum $\mathbf{H}_C$ of the disk about its center *C*.

SOLUTION

Using frame $Cx'y'z'$:



$$\bar{I}_{x'} = \bar{I}_{y'} = \frac{1}{4}mr^2$$

$$\bar{I}_{z'} = \frac{1}{2}mr^2$$

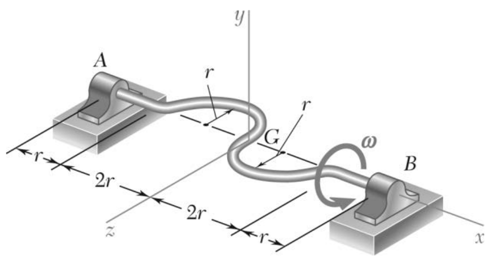
$$\mathbf{H}_C = \bar{I}_{y'}\omega_2\mathbf{j} + I_{z'}\omega_1\mathbf{k}$$

$$= \frac{1}{4}mr^2(\omega_2\mathbf{j} + 2\omega_1\mathbf{k})$$

$$\mathbf{H}_C = \frac{1}{4}(5\text{ kg})(0.25\text{ m})^2[(3\text{ rad/s})\mathbf{j} + 2(8\text{ rad/s})\mathbf{k}]$$

$$\mathbf{H}_C = (0.234\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (1.250\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k} \quad ◀$$

PROBLEM 18.149



A rod of uniform cross section is used to form the shaft shown. Denoting by m the total mass of the shaft and knowing that the shaft rotates with a constant angular velocity $\boldsymbol{\omega}$, determine (*a*) the angular momentum $\mathbf{H}_G$ of the shaft about its mass center G, (*b*) the angle formed by $\mathbf{H}_G$ and the axis AB, (*c*) the angular momentum of the shaft about Point A.

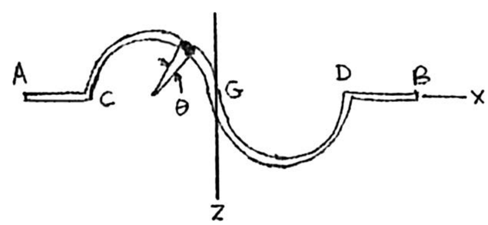
SOLUTION

Length of rod: $L = 2r + 2\pi r = 8.2832r$

Angular velocity: $\boldsymbol{\omega} = \omega\mathbf{i}$

Angular momentum: $\mathbf{H}_G = \bar{I}_x\omega\mathbf{i} - \bar{I}_{xz}\omega\mathbf{k}$

Calculation of $\bar{I}_x$ and $\bar{I}_{xz}$:



Let $\rho = \text{mass per unit length} = \dfrac{m}{L}.$

For portions AC and DB,

$$I_x = 0, \quad I_{xz} = 0$$

For portion CG, use polar coordinate θ.

$$x = -r(1-\cos\theta), \quad z = -r\sin\theta \quad dm = \rho r d\theta$$

$$I_x = \int z^2 dm = \int_0^{\pi} r^2 \sin^2\theta \rho r d\theta = \frac{\pi}{2}\rho r^3$$

$$I_{xz} = \int xz dm = \int_0^{\pi} r^2 (1-\cos\theta)\sin\theta \rho r d\theta = 2\rho r^3$$

Likewise, for portion GD, $\quad I_x = \dfrac{\pi}{2}\rho r^3, \quad I_{xz} = 2\rho r^3$

Total: $\quad \bar{I}_x = \pi\rho r^3 = \dfrac{\pi m r^3}{L}, \quad \bar{I}_{xz} = 4\rho r^3 = \dfrac{4mr^3}{L}$

(*a*) <u>Angular momentum $\mathbf{H}_G$.</u> $\quad \mathbf{H}_G = \dfrac{\pi m r^3}{L}\omega\mathbf{i} - \dfrac{4mr^3}{L}\omega\mathbf{k} = mr^2\omega(0.37927\mathbf{i} - 0.48291\mathbf{k})$

$$\mathbf{H}_G = mr^2\omega(0.379\mathbf{i} - 0.483\mathbf{k}) \blacktriangleleft$$

$$H_G = mr^2\omega(0.37927^2 + 0.48291^2)^{1/2} = 0.61404\, mr^2\omega$$

PROBLEM 18.149 (Continued)

(*b*) Angle formed by $\mathbf{H}_G$ and the axis AB.

$$\mathbf{H}_G \cdot \mathbf{i} = 0.37927mr^2\omega$$

$$\cos\theta = \frac{\mathbf{H}_G \cdot \mathbf{i}}{H_G} = \frac{0.37927mr^2\omega}{0.61404mr^2\omega} = 0.61766$$

$\theta = 51.9°$ ◀

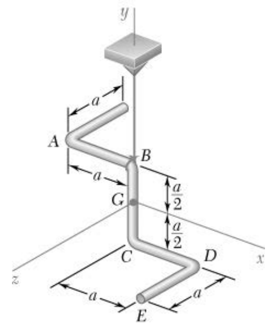
(*c*) Angular momentum about Point A:

$$\mathbf{H}_A = \mathbf{H}_G + \mathbf{r}_{G/A} \times (m\overline{v})$$

$$\mathbf{H}_A = \mathbf{H}_G + 0$$

$\mathbf{H}_A = mr^2\omega(0.379\mathbf{i} - 0.483\mathbf{k})$ ◀

PROBLEM 18.150



A uniform rod of mass m and length $5a$ is bent into the shape shown and is suspended from a wire attached at Point B. Knowing that the rod is hit at Point A in the negative y direction and denoting the corresponding impulse by $-(F\Delta t)\mathbf{j}$, determine immediately after the impact (*a*) the velocity of the mass center G, (*b*) the angular velocity of the rod.

SOLUTION

Moments and products of inertia:

Part	m	I_x	I_y	I_z	I_{xy}
OA	$\frac{1}{5}m$	$\left(\frac{1}{5}m\right)\times\left(\frac{1}{12}a^2+\frac{1}{4}a^2+\frac{1}{4}a^2\right)$	$\left(\frac{1}{5}m\right)\times\left(\frac{1}{12}a^2+a^2+\frac{1}{4}a^2\right)$	$\left(\frac{1}{5}m\right)\left(a^2+\frac{1}{4}a^2\right)$	$\left(\frac{1}{5}m\right)\left(-\frac{1}{2}a^2\right)$
AB	$\frac{1}{5}m$	$\left(\frac{1}{5}m\right)\left(\frac{1}{4}a^2\right)$	$\left(\frac{1}{5}m\right)\left(\frac{1}{3}a^2\right)$	$\left(\frac{1}{5}m\right)\times\left(\frac{1}{12}a^2+\frac{1}{4}a^2+\frac{1}{4}a^2\right)$	$\left(\frac{1}{5}m\right)\left(-\frac{1}{4}a^2\right)$
BC	$\frac{1}{5}m$	$\left(\frac{1}{5}m\right)\left(\frac{1}{12}a^2\right)$	0	$\left(\frac{1}{5}m\right)\left(\frac{1}{12}a^2\right)$	0
CD	$\frac{1}{5}m$	$\left(\frac{1}{5}m\right)\left(\frac{1}{4}a^2\right)$	$\left(\frac{1}{5}m\right)\left(\frac{1}{3}a^2\right)$	$\left(\frac{1}{5}m\right)\times\left(\frac{1}{12}a^2+\frac{1}{4}a^2+\frac{1}{4}a^2\right)$	$\left(\frac{1}{5}m\right)\left(-\frac{1}{4}a^2\right)$
DE	$\frac{1}{5}m$	$\left(\frac{1}{5}m\right)\times\left(\frac{1}{12}a^2+\frac{1}{4}a^2+\frac{1}{4}a^2\right)$	$\left(\frac{1}{5}m\right)\times\left(\frac{1}{12}a^2+a^2+\frac{1}{4}a^2\right)$	$\left(\frac{1}{5}m\right)\left(a^2+\frac{1}{4}a^2\right)$	$\left(\frac{1}{5}m\right)\left(-\frac{1}{2}a^2\right)$
Σ	m	$0.35ma^2$	$0.66667ma^2$	$0.75ma^2$	$-0.3ma^2$

$$I_{xz}=\left(\frac{1}{5}m\right)\left(\frac{1}{2}a^2\right)+\left(\frac{1}{5}m\right)\left(\frac{1}{2}a^2\right)=0.2ma^2$$

$$I_{yz}=\left(\frac{1}{5}m\right)\left(-\frac{1}{4}a^2\right)+\left(\frac{1}{5}m\right)\left(-\frac{1}{4}a^2\right)=-0.1ma^2$$

Angular momentum about the mass center.

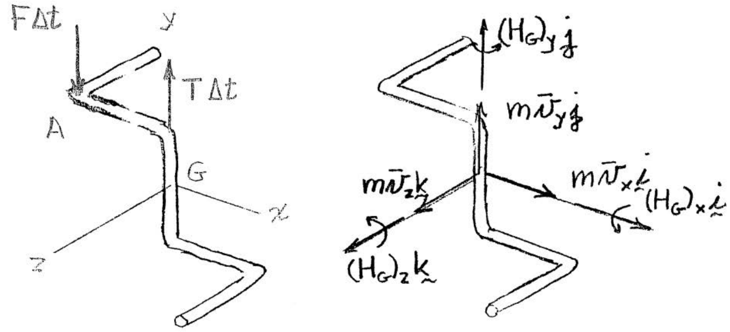
$$(H_G)_x=I_x\omega_x-I_{xy}\omega_y-I_{xz}\omega_z=0.35ma^2\omega_x+0.3ma^2\omega_y-0.2ma^2\omega_z$$

$$(H_G)_y=-I_{xy}\omega_x+I_y\omega_y-I_{yz}\omega_z=0.3ma^2\omega_x+0.66667ma^2\omega_y+0.1ma^2\omega_z$$

$$(H_G)_z=-I_{xz}\omega_x-I_{yz}\omega_z+I_z\omega_z=-0.2ma^2\omega_x+0.1ma^2\omega_y+0.75ma^2\omega_z$$

Constraint of the supporting cable: $\bar{v}_y = 0$

Impulse-momentum principle: Before impact, $\bar{\mathbf{v}} = 0, \quad \mathbf{H}_G = 0.$



(*a*) Linear momentum: $\mathbf{F}(\Delta t) + T\Delta t\mathbf{j} = m\bar{\mathbf{v}}$ Resolve into components.

$$0 = m\bar{v}_x, \qquad -F\Delta t + T\Delta t\mathbf{j} = 0, \qquad 0 = m\bar{v}_z$$

$$\bar{v}_x = 0, \qquad T\Delta t = F\Delta t, \qquad \bar{v}_z = 0$$

$\bar{\mathbf{v}} = 0$ ◀

(*b*) Angular momentum, moments about *G*: $\mathbf{r}_{A/G} \times \mathbf{F}\Delta t = \mathbf{H}_G$

$$\left(\frac{a}{2}\mathbf{j} - a\mathbf{i}\right) \times [-(F\Delta t)\mathbf{j}] = (aF\Delta t)\mathbf{k} = (H_G)_x\mathbf{i} + (H_G)_y\mathbf{j} + (H_G)_z\mathbf{k}$$

Using expressions for $(H_G)_x$, $(H_G)_y$, and $(H_G)_z$ and resolving into components,

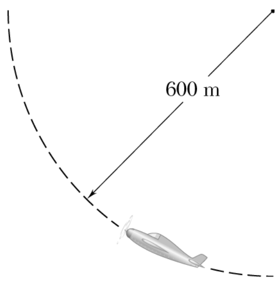
$$\mathbf{i}: \quad 0 = 0.35ma^2\omega_x + 0.3ma^2\omega_y - 0.2ma^2\omega_z$$

$$\mathbf{j}: \quad 0 = 0.3ma^2\omega_x + 0.66667ma^2\omega_y + 0.1ma^2\omega_z$$

$$\mathbf{k}: \quad aF\Delta t = -0.2ma^2\omega_x + 0.1ma^2\omega_y + 0.75ma^2\omega_z$$

Solving, $$\omega_x = 2.5\frac{(F\Delta t)}{ma}, \qquad \omega_y = -1.454\frac{(F\Delta t)}{ma}, \qquad \omega_z = 2.19\frac{(F\Delta t)}{ma}$$

$$\boldsymbol{\omega} = \left(\frac{F\Delta t}{ma}\right)(2.50\mathbf{i} - 1.454\mathbf{j} + 2.19\mathbf{k})$$ ◀

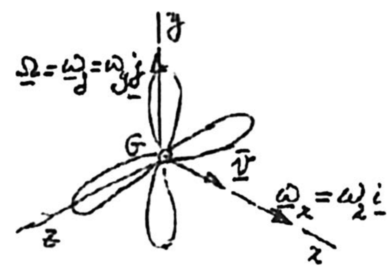


PROBLEM 18.151

A four-bladed airplane propeller has a mass of 160 kg and a radius of gyration of 800 mm. Knowing that the propeller rotates at 1600 rpm as the airplane is traveling in a circular path of 600-m radius at 540 km/h, determine the magnitude of the couple exerted by the propeller on its shaft due to the rotation of the airplane.

SOLUTION

We assume senses shown for $\boldsymbol{\omega}_{x,}$ $\boldsymbol{\omega}_y$, and $\mathbf{v}$.



$$\bar{v} = 540 \text{ km/h} = 150 \text{ m/s}$$

$$\omega_x = 1600 \text{ rpm}\left(\frac{2\pi \text{ rad}}{60 \text{ s}}\right)$$

$$= 167.55 \text{ rad/s}$$

$$\omega_y = \frac{\bar{v}}{\rho} = \frac{150 \text{ m/s}}{600 \text{ m}} = 0.25 \text{ rad/s}$$

$$\bar{I}_x = m\bar{k}^2 = (160 \text{ kg})(0.8 \text{ m})^2 = 102.4 \text{ kg}\cdot\text{m}^2$$

Angular momentum about G:

$$\mathbf{H}_G = \bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j}$$

Eq. (18.22),

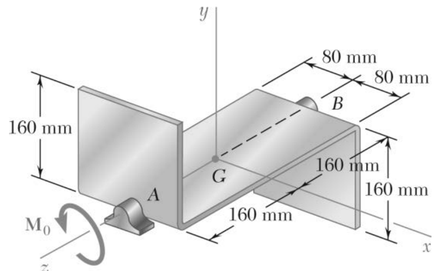
$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \boldsymbol{\Omega}\times\mathbf{H}_G = 0 + \omega_y\mathbf{j}\times(\bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j})$$

$$\dot{\mathbf{H}}_G = -\bar{I}_x\omega_x\omega_y\mathbf{k} = -(102.4 \text{ kg}\cdot\text{m}^2)(167.55 \text{ rad/s})(0.25 \text{ rad/s})\mathbf{k}$$

$$\dot{\mathbf{H}}_G = -(4289 \text{ N}\cdot\text{m})\mathbf{k} = -(4.29 \text{ kN}\cdot\text{m})\mathbf{k}$$

The couple exerted *on* the propeller, therefore, must be $\mathbf{M} = \dot{\mathbf{H}}_G = -(4.29 \text{ kN}\cdot\text{m})\mathbf{k}$, and the couple exerted *by* the propeller on its shaft is $-\mathbf{M} = (4.29 \text{ kN}\cdot\text{m})\mathbf{k}$.

Magnitude of couple. $4.29 \text{ kN}\cdot\text{m}$ ◀



PROBLEM 18.152

A 2.4-kg piece of sheet steel with dimensions 160×640 mm was bent to form the component shown. The component is at rest $(\omega = 0)$ when a couple $\mathbf{M}_0 = (0.8\ \text{N}\cdot\text{m})\mathbf{k}$ is applied to it. Determine (*a*) the angular acceleration of the component, (*b*) the dynamic reactions at *A* and *B* immediately after the couple is applied.

SOLUTION

$$m = 2.4\ \text{kg}, \quad b = 160\ \text{mm} = 0.16\ \text{m}$$

Area of sheet metal:

$$A = b^2 + (2b)b + b^2 = 4b^2 = 0.1024\ \text{m}^2$$

Let

$$\rho = \frac{m}{A} = \frac{2.4}{0.1024}$$
$$= 23.4375\ \text{kg/m}^2$$
$$= \text{mass per unit area}$$

Angular velocity and angular acceleration:

$$\boldsymbol{\omega} = \omega\mathbf{k}$$
$$\alpha = \alpha\mathbf{k}$$

Angular momentum about *G*:

$$\mathbf{H}_G = -I_{xz}\omega\mathbf{i} - I_{yz}\omega\mathbf{j} + I_z\omega\mathbf{k}$$
$$= -I_{yz}\omega\mathbf{j} + I_z\omega\mathbf{k}$$

Let the frame of reference *Gxyz* be rotating with angular velocity $\boldsymbol{\Omega} = \boldsymbol{\omega} = \omega\mathbf{k}$

$$\dot{\mathbf{H}}_G = (\dot{\mathbf{H}}_G)_{Gxyz} + \boldsymbol{\Omega} \times \mathbf{H}_G$$
$$= -I_{yz}\alpha\mathbf{j} + I_z\alpha\mathbf{k} + I_{yz}\omega^2\mathbf{i}$$

Required moment and product of inertia: $I_{\text{mass}} = \rho I_{\text{area}}$

Part	A	I_z	I_{yz}
plate *A*	b^2	$\frac{1}{6}b^2 + b^2\left(\frac{1}{2}b\right)^2$	$b^2(b)\left(\frac{1}{2}b\right)$
plate *AB*	$2b^2$	$\frac{1}{12}(2b)b^3$	0
plate *B*	b^2	$\frac{1}{6}b^2 + b^2\left(\frac{1}{2}b\right)^2$	$b^2(-b)\left(-\frac{1}{2}b\right)$
Σ	$4b^2$	b^4	b^4

$$I_z = \rho b^4$$
$$= (23.4375)(0.16)^4$$
$$= 0.01536\ \text{kg}\cdot\text{m}^2$$
$$I_{yz} = \rho b^4$$
$$= 0.01536\ \text{kg}\cdot\text{m}^2$$

PROBLEM 18.152 (Continued)

Since the mass center lies on the rotation axis, $\bar{\mathbf{a}} = 0$

$$\Sigma \mathbf{F} = \mathbf{A} + \mathbf{B} = m\bar{\mathbf{a}}$$

$$\mathbf{B} = -\mathbf{A}$$

$$\begin{aligned}\Sigma \mathbf{M}_G &= M_0\mathbf{k} + b\mathbf{k} \times \mathbf{A} + (-b\mathbf{k}) \times \mathbf{B} \\ &= M_0\mathbf{k} + 2b\mathbf{k} \times (A_x\mathbf{i} + A_y\mathbf{j}) \\ &= M_0\mathbf{k} + 2bA_x\mathbf{j} - 2bA_y\mathbf{i}\end{aligned}$$

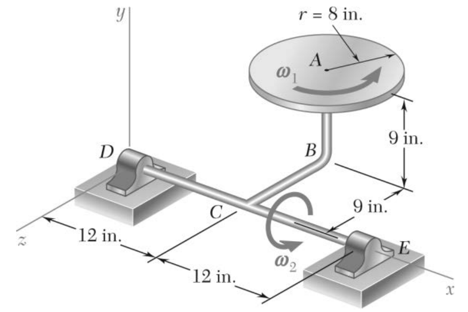
$\Sigma \mathbf{M}_G = \dot{\mathbf{H}}_G$ Resolve into components.

(*a*) $\mathbf{k}$: $M_0 = I_z\alpha \quad \alpha = \dfrac{M_0}{I_z} = \dfrac{0.8}{0.01536} = 52.083 \text{ rad/s}^2$ $\alpha = 52.1 \text{ rad/s}^2$ ◀

(*b*) $\mathbf{j}$: $2bA_x = -I_{yz}\alpha \quad A_x = -\dfrac{I_{yz}\alpha}{2b} = -\dfrac{(0.01536)(52.083)}{(2)(0.16)} = -2.50 \text{ N}$

$\mathbf{i}$: $-2bA_y = I_{yz}\omega^2 = 0 \qquad A_y = 0$ $\mathbf{A} = -(2.50 \text{ N})\mathbf{i}$ ◀

$\mathbf{B} = (2.50 \text{ N})\mathbf{i}$ ◀



PROBLEM 18.153

A homogeneous disk of weight $W = 6$ lb rotates at the constant rate $\omega_1 = 16$ rad/s with respect to arm *ABC*, which is welded to a shaft *DCE* rotating at the constant rate $\omega_2 = 8$ rad/s. Determine the dynamic reactions at *D* and *E*.

SOLUTION

Angular velocity of shaft *DE* and arm *CBA*: $\mathbf{\Omega} = \omega_2 \mathbf{i}$

Angular velocity of disk *A*: $\boldsymbol{\omega} = \omega_2 \mathbf{i} + \omega_1 \mathbf{j}$

Angular velocity about its mass center *A*:

$$\mathbf{H}_A = \bar{I}_x \omega_x \mathbf{i} + \bar{I}_y \omega_y \mathbf{j} + \bar{I}_z \omega_z \mathbf{k}$$
$$= \bar{I}_x \omega_2 \mathbf{i} + \bar{I}_y \omega_1 \mathbf{j}$$
$$= \frac{1}{4} m r^2 \omega_2 \mathbf{i} + \frac{1}{2} m r^2 \omega_1 \mathbf{j}$$

Let the reference frame *Oxyz* be rotating with angular velocity $\mathbf{\Omega} = \omega_2 \mathbf{i}$.

$$\dot{\mathbf{H}}_A = (\dot{\mathbf{H}}_A)_{Oxyz} + \mathbf{\Omega} \times \mathbf{H}_A$$
$$= \frac{1}{4} m r^2 \dot{\omega}_2 \mathbf{i} + \frac{1}{2} m r^2 \dot{\omega}_1 \mathbf{j} + \omega_2 \mathbf{i} \times \mathbf{H}_A$$
$$= \frac{1}{4} m r^2 \dot{\omega}_2 \mathbf{i} + \frac{1}{2} m r^2 \dot{\omega}_1 \mathbf{j} + \frac{1}{2} m r^2 \omega_1 \omega_2 \mathbf{k}$$

Velocity of Point *A*:

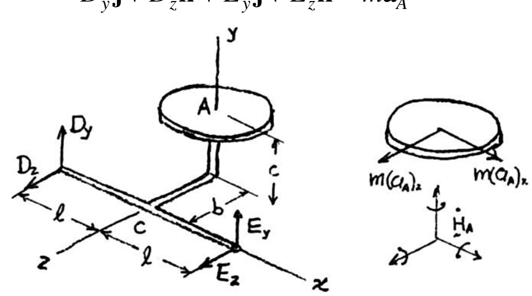
$$\mathbf{v}_A = \omega_2 \mathbf{i} \times \mathbf{r}_{A/O}$$
$$= \omega_2 \mathbf{i} \times (-b\mathbf{k} + c\mathbf{j})$$
$$= b\omega_2 \mathbf{j} + c\omega_2 \mathbf{k}$$

Acceleration of Point *A*:

$$\mathbf{a}_A = \dot{\omega}_2 \mathbf{j} \times \mathbf{r}_{A/O} + \omega_2 \mathbf{j} \times \mathbf{v}_A$$
$$\mathbf{a}_A = (b\dot{\omega}_2 - c\omega_2^2)\mathbf{j} + (c\dot{\omega}_2 + b\omega_2)\mathbf{k}$$

Consider the system of particles consisting of the shaft, the arm, and the disk. Neglect the mass of the arm.

$$\Sigma \mathbf{F} = m\mathbf{a}_A$$
$$D_y \mathbf{j} + D_z \mathbf{k} + E_y \mathbf{j} + E_z \mathbf{k} = m\mathbf{a}_A$$



Resolve into components.

$$D_y + E_y = m(b\dot{\omega}_2 - c\omega_2^2)$$

$$D_z + E_z = m(c\dot{\omega}_2 + b\omega_2^2)$$

$$\Sigma\mathbf{M}_D = \dot{\mathbf{H}}_D = \dot{\mathbf{H}}_A + \mathbf{r}_{A/D} \times m\mathbf{a}_A$$

$$(M_0)\mathbf{i} + 2l\mathbf{i} \times (E_y\mathbf{j} + E_z\mathbf{k}) = \dot{\mathbf{H}}_A + (l\mathbf{i} + c\mathbf{j} - b\mathbf{k}) \times m\mathbf{a}_A$$

$$(M_0)\mathbf{i} - 2lE_z\mathbf{j} + 2lE_y\mathbf{k} = m\left(\frac{1}{4}r^2 + b^2 + c^2\right)\dot{\omega}_2\mathbf{i} + m\left(\frac{1}{2}r^2\dot{\omega}_1 - lc\dot{\omega}_2 - lb\omega_2^2\right)\mathbf{j}$$

$$+ m\left(\frac{1}{2}r^2\omega_1\omega_2 + lb\dot{\omega}_2 - lc\omega_2^2\right)\mathbf{k}$$

$\mathbf{i}$: $$M_0 = m\left(\frac{1}{4}r^2 + b^2 + c^2\right)\dot{\omega}_2$$

$\mathbf{k}$: $$E_y = \frac{m}{2l}\left(\frac{1}{2}r^2\omega_1\omega_2 + lb\dot{\omega}_2 - lc\omega_2^2\right)$$

$$D_y = \frac{m}{2l}\left(-\frac{1}{2}r^2\omega_1\omega_2 + lb\dot{\omega}_2 - lc\omega_2^2\right)$$

$\mathbf{j}$: $$E_z = \frac{m}{2l}\left(lc\dot{\omega}_2 + lb\omega_2^2 - \frac{1}{2}r^2\dot{\omega}_1\right)$$

$$D_z = \frac{m}{2l}\left(lc\dot{\omega}_2 + lb\omega_2^2 + \frac{1}{2}r^2\dot{\omega}_1\right)$$

<u>Data</u>: $W = 6$ lb. $\quad m = \dfrac{6}{32.2} = 0.186335 \quad r = 8 \text{ in.} = 0.66667 \text{ ft}$

$b = c = 9 \text{ in.} = 0.75 \text{ ft} \quad l = 12 \text{ in.} = 1.0 \text{ ft}$

$\omega_1 = 16$ rad/s, $\quad \dot{\omega}_1 = 0, \quad \omega_2 = 8$ rad/s, $\quad \dot{\omega}_2 = 0$

$$D_y = \frac{0.186335}{(2)(1.0)}\left[-\left(\frac{1}{2}\right)(0.66667)^2(16)(8) + 0 - (1.0)(0.75)(8)^2\right] = -7.12 \text{ lb}$$

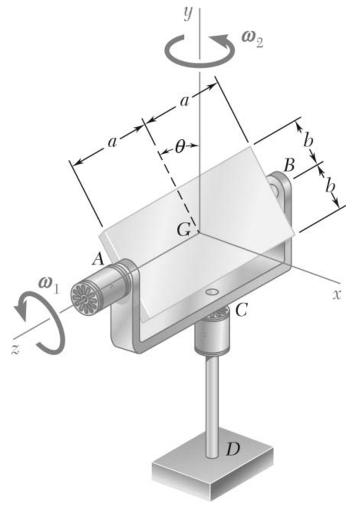
$$D_z = \frac{0.186335}{(2)(1.0)}[0 + (1.0)(0.75)(8)^2 + 0] = 4.47 \text{ lb}$$

$$\mathbf{D} = -(7.12 \text{ lb})\mathbf{j} + (4.47 \text{ lb})\mathbf{k} \blacktriangleleft$$

$$E_y = \frac{0.186335}{(2)(1.0)}\left[\left(\frac{1}{2}\right)(0.66667)^2(16)(8) + 0 - (1.0)(0.75)(8)^2\right] = -1.822 \text{ lb}$$

$$E_z = \frac{0.186335}{(2)(1.0)}[0 + (1.0)(0.75)(8)^2 + 0] = 4.47 \text{ lb}$$

$$\mathbf{E} = -(1.822 \text{ lb})\mathbf{j} + (4.47 \text{ lb})\mathbf{k} \blacktriangleleft$$

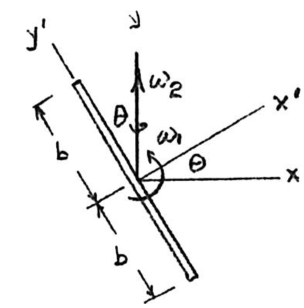


PROBLEM 18.154

A 48-kg advertising panel of length $2a = 2.4$ m and width $2b = 1.6$ m is kept rotating at a constant rate ω_1 about its horizontal axis by a small electric motor attached at A to frame ACB. This frame itself is kept rotating at a constant rate ω_2 about a vertical axis by a second motor attached at C to the column CD. Knowing that the panel and the frame complete a full revolution in 6 s and 12 s, respectively, express, as a function of the angle θ, the dynamic reaction exerted on column CD by its support at D.

SOLUTION

Use principal axes x', y', z' as shown.



Moments of inertia: $\bar{I}_{x'} = \frac{1}{3}m(a^2 + b^2)$

$$\bar{I}_{y'} = \frac{1}{3}ma^2, \quad \bar{I}_{z'} = \frac{1}{3}mb^2$$

Kinematics: $\dot{\theta} = \omega_1$

$$\omega_{x'} = \omega_2 \sin\theta, \quad \omega_{y'} = \omega_2 \cos\theta, \quad \omega_{z'} = \omega_1$$

$$\dot{\omega}_{x'} = \omega_1\omega_2 \cos\theta, \quad \dot{\omega}_{y'} = -\omega_1\omega_2 \sin\theta, \quad \dot{\omega}_{z'} = 0$$

Since the acceleration of the mass center is zero, the resultant force acting on the column CD is zero.

$\mathbf{R} = 0$ ◀

Euler's equations of motion for the plate:

$$\Sigma M_{x'} = \bar{I}_{x'}\dot{\omega}_{x'} - (\bar{I}_{y'} - \bar{I}_{z'})\omega_{y'}\omega_{z'}$$
$$= \bar{I}_{x'}\omega_1\omega_2 \cos\theta - (\bar{I}_{y'} - \bar{I}_{z'})\omega_1\omega_2 \cos\theta$$
$$= (\bar{I}_{x'} + \bar{I}_{z'} - \bar{I}_{y'})\omega_1\omega_2 \cos\theta = \frac{2}{3}mb^2\omega_1\omega_2 \cos\theta$$

$$\Sigma M_{y'} = \bar{I}_{y'}\dot{\omega}_{y'} - (\bar{I}_{z'} - \bar{I}_{x'})\omega_{x'}\omega_{z'}$$
$$= -\bar{I}_y\omega_1\omega_2 \sin\theta - (\bar{I}_z - \bar{I}_x)\omega_1\omega_2 \sin\theta$$
$$= (\bar{I}_{x'} - \bar{I}_{y'} - \bar{I}_{z'})\omega_1\omega_2 \sin\theta = 0$$

$$\Sigma M_{z'} = I_{z'}\dot{\omega}_{z'} - (\bar{I}_{x'} - \bar{I}_y)\omega_{x'}\omega_{y'}$$
$$= 0 - (\bar{I}_{x'} - \bar{I}_y)\omega_2^2 \sin\theta\cos\theta$$
$$= -\frac{1}{3}mb^2\omega_2^2 \sin\theta\cos\theta$$

PROBLEM 18.154 (Continued)

$$\Sigma\mathbf{M} = \frac{2}{3}mb^2\omega_1\omega_2\cos\theta\mathbf{i}' - \frac{1}{3}mb^2\omega_2^2\sin\theta\cos\theta\mathbf{k}$$

$$= \frac{2}{3}mb^2\omega_1\omega_2\cos\theta(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) - \frac{1}{3}mb^2\omega_2^2\sin\theta\cos\theta\mathbf{k}$$

Resolve into components: $\Sigma M_x = \frac{2}{3}mb^2\omega_1\omega_2\cos^2\theta$

$$\Sigma M_y = \frac{2}{3}mb^2\omega_1\omega_2\sin\theta\cos\theta$$

$$\Sigma M_z = -\frac{1}{3}mb^2\omega_2^2\sin\theta\cos\theta$$

Data: $m = 48$ kg, $b = 0.8$ m

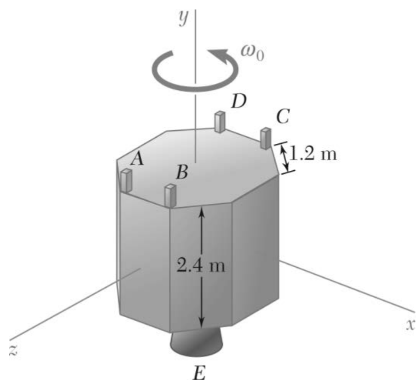
$$\omega_1 = \frac{2\pi}{6} = 1.0472 \text{ rad/s}, \quad \omega_2 = \frac{2\pi}{12} = 0.5236 \text{ rad/s}$$

$$\Sigma M_x = \frac{2}{3}(48)(0.8)^2(1.0472)(0.5236)\cos^2\theta$$
$$= 11.23\cos^2\theta \text{ N}\cdot\text{m}$$

$$\Sigma M_y = \frac{2}{3}(48)(0.8)^2(1.0472)(0.5236)\sin\theta\cos\theta$$
$$= 11.23\sin\theta\cos\theta \text{ N}\cdot\text{m}$$

$$\Sigma M_z = -\frac{1}{3}(48)(0.8)^2(0.5236)^2\sin\theta\cos\theta$$
$$= -2.81\sin\theta\cos\theta \text{ N}\cdot\text{m}$$

$$\mathbf{M}_D = (11.23 \text{ N}\cdot\text{m})\cos^2\theta\mathbf{i} + (11.23 \text{ N}\cdot\text{m})\sin\theta\cos\theta\mathbf{j} - (2.81 \text{ N}\cdot\text{m})\sin\theta\cos\theta\mathbf{k} \blacktriangleleft$$



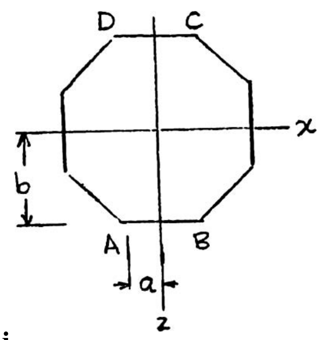
PROBLEM 18.155

A 2500-kg satellite is 2.4 m high and has octagonal bases of sides 1.2 m. The coordinate axes shown are the principal centroidal axes of inertia of the satellite, and its radii of gyration are $k_x = k_z = 0.90$ m and $k_y = 0.98$ m. The satellite is equipped with a main 500-N thruster *E* and four 20-N thrusters *A*, *B*, *C*, and *D*, which can expel fuel in the positive *y* direction. The satellite is spinning at the rate of 36 rev/h about its axis of symmetry G_y, which maintains a fixed direction in space, when thrusters *A* and *B* are activated for 2 s. Determine (*a*) the precession axis of the satellite, (*b*) its rate of precession, (*c*) its rate of spin.

SOLUTION

$$\omega_0 = (36\text{ rev/h})\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$$
$$= 0.062832\text{ rad/s}$$
$$m = 2500\text{ kg}$$
$$I_x = mk_x^2 = (2500)(0.90)^2 = 2025\text{ kg}\cdot\text{m}^2$$
$$I_z = I_x = 2025\text{ kg}\cdot\text{m}^2$$
$$I_y = mk_y^2 = (2500)(0.98)^2 = 2401\text{ kg}\cdot\text{m}^2$$
$$(\mathbf{H}_G)_0 = I_y\omega_0\mathbf{j} = (2401)(0.062832)\mathbf{j} = (150.86\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j}$$
$$a = 0.6\text{ m},\quad b = 0.6 + 1.2\cos 45° = 1.4485$$

When thrusters *A* and *B* are activated,



$$\mathbf{M}_G = -b(F_A + F_B)\mathbf{i}$$
$$= -(1.4485)(40)\mathbf{i}$$
$$= -(57.941\text{ N}\cdot\text{m})\mathbf{i}$$

Angular momentum after 2 s:

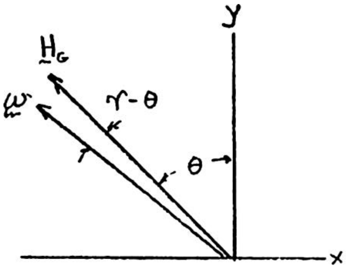
$$\mathbf{H}_G = (\mathbf{H}_G)_0 + \mathbf{M}_G(\Delta t)$$
$$= 150.86\mathbf{j} + (-57.941)(2)\mathbf{i}$$
$$= -(115.88\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} + (150.86\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j}$$
$$\omega_x = \frac{H_x}{I_x} = -\frac{115.88}{2025} = -0.057225\text{ rad/s} = -32.788\text{ rev/h}$$
$$\omega_y = \frac{H_y}{I_y} = 36\text{ rev/h}$$
$$\omega_z = \frac{H_z}{I_z} = 0$$

(*a*) Precession axis:

$$\tan\theta = -\frac{H_x}{H_y} = \frac{115.88}{150.86} \quad \theta = 37.529°$$

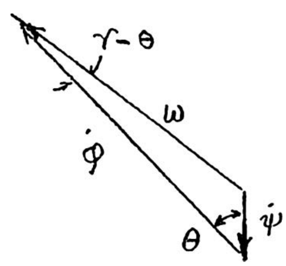
$$\theta_x = 52.5°,\ \theta_y = 37.5°,\ \theta_z = 90° \blacktriangleleft$$

$$\tan\gamma = -\frac{\omega_x}{\omega_y}$$
$$= \frac{32.788}{36}$$
$$\gamma = 42.327°$$
$$\gamma - \theta = 4.798°$$
$$\omega = \sqrt{\omega_x^2 + \omega_y^2}$$
$$= 48.693 \text{ rev/h}$$



Law of sines.

$$\frac{\dot{\varphi}}{\sin\gamma} = \frac{\dot{\psi}}{\sin(\gamma-\theta)} = \frac{\omega}{\sin\theta}$$



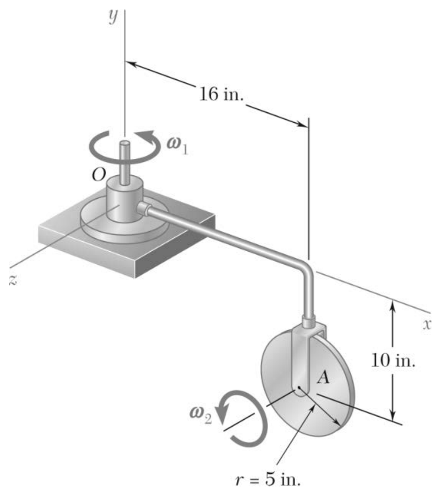
(*b*) $$\dot{\varphi} = \frac{48.693\sin 42.327°}{\sin 37.529°}$$

$$\dot{\varphi} = 53.8 \text{ rev/h} \blacktriangleleft$$

(*c*) $$\dot{\psi} = \frac{48.693\sin 4.798°}{\sin 37.529°}$$

$$\dot{\psi} = 6.68 \text{ rev/h} \blacktriangleleft$$

PROBLEM 18.156



A thin disk of weight $W = 8$ lb rotates with an angular velocity ω_2 with respect to arm OA, which itself rotates with an angular velocity ω_1 about the y axis. Determine (a) the couple $M_1\mathbf{j}$ which should be applied to arm OA to give it an angular acceleration $\boldsymbol{\alpha}_1 = (6 \text{ rad/s}^2)\mathbf{j}$ with $\omega_1 = 4$ rad/s, knowing that the disk rotates at the constant rate $\omega_2 = 12$ rad/s, (b) the force-couple system representing the dynamic reaction at O at that instant. Assume that arm OA has a negligible mass.

SOLUTION

Angular velocity of arm OA: $\quad \boldsymbol{\Omega} = \omega_1\mathbf{j}$

Angular velocity of disk A: $\quad \boldsymbol{\omega} = \omega_1\mathbf{j} + \omega_2\mathbf{k}$

Angular momentum about its mass center A:

$$\begin{aligned}\mathbf{H}_A &= \bar{I}_x\omega_x\mathbf{i} + \bar{I}_y\omega_y\mathbf{j} + \bar{I}_z\omega_z\mathbf{k} \\ &= \bar{I}_y\omega_1\mathbf{j} + \bar{I}_z\omega_2\mathbf{k} \\ &= \frac{1}{4}mr^2\omega_1\mathbf{j} + \frac{1}{2}mr^2\omega_2\mathbf{k}\end{aligned}$$

Let the reference $Oxyz$ be rotating with angular velocity $\boldsymbol{\Omega} = \omega_2\mathbf{j}$.

$$\begin{aligned}\dot{\mathbf{H}}_A &= (\dot{\mathbf{H}}_A)_{Oxyz} + \boldsymbol{\Omega}\times\mathbf{H}_A \\ &= \frac{1}{4}mr^2\dot{\omega}_1\mathbf{j} + \frac{1}{2}mr^2\dot{\omega}_2\mathbf{k} + \omega_1\mathbf{j}\times\mathbf{H}_A \\ &= \frac{1}{2}mr^2\omega_1\omega_2\mathbf{i} + \frac{1}{4}mr^2\dot{\omega}_1\mathbf{j} + \frac{1}{2}mr^2\dot{\omega}_2\mathbf{k}\end{aligned}$$

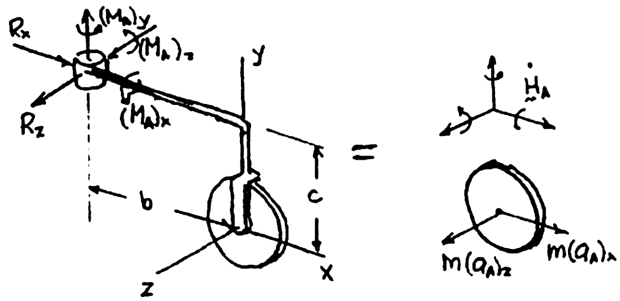
Velocity of Point A:

$$\begin{aligned}\mathbf{v}_A &= \omega_1\mathbf{j}\times\mathbf{r}_{A/O} \\ &= \omega_1\mathbf{j}\times(b\mathbf{i} - c\mathbf{j}) \\ &= -b\omega_1\mathbf{k}\end{aligned}$$

Acceleration of Point A:

$$\begin{aligned}\mathbf{a}_A &= \dot{\omega}_1\mathbf{j}\times\mathbf{r}_{A/O} + \omega_1\mathbf{j}\times\mathbf{v}_A \\ &= -b\omega_1^2\mathbf{i} - b\dot{\omega}_1\mathbf{k}\end{aligned}$$

Consider the system of particles consisting of the arm OA and disk A. Neglect the mass of the arm.



$$\Sigma\mathbf{F} = m\mathbf{a}_A:$$

$$R_x\mathbf{i} + R_z\mathbf{k} = -mb\omega_1^2\mathbf{i} - mb\dot{\omega}_1\mathbf{k}$$

$$R_x = -mb\omega_1^2$$

$$R_z = -mb\dot{\omega}_1$$

$$\Sigma\mathbf{M}_O = \dot{\mathbf{H}}_O = \dot{\mathbf{H}}_A + \mathbf{r}_{A/O} \times m\mathbf{a}_A$$

$$(M_O)_x\mathbf{i} + (M_O)_y\mathbf{j} + (M_O)_z\mathbf{k} = m\left(\frac{1}{2}r^2\omega_1\omega_2 + bc\dot{\omega}_1\right)\mathbf{i}$$

$$+m\left(\frac{1}{4}r^2 + b^2\right)\dot{\omega}_1\mathbf{j} + m\left(\frac{1}{2}r^2\dot{\omega}_2 - bc\omega_1^2\right)\mathbf{k}$$

Data:

$$W = 8 \text{ lb}$$

$$m = \frac{8}{32.2} = 0.24845 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$r = 5 \text{ in.} = 0.41667 \text{ ft}$$

$$b = 16 \text{ in.} = 1.33333 \text{ ft}$$

$$c = 10 \text{ in.} = 0.83333 \text{ ft}$$

$$\omega_1 = 4 \text{ rad/s}$$

$$\dot{\omega}_1 = 6 \text{ rad/s}^2$$

$$\omega_2 = 12 \text{ rad/s}$$

$$\dot{\omega}_2 = 0$$

(*a*) <u>Required couple</u>: The required couple is the y-component of the couple at Point O.

$$(\mathbf{M}_O)_y = (0.24845)\left[\frac{1}{4}(0.41667)^2 + (1.33333)^2\right](6)\mathbf{j} \qquad (\mathbf{M}_O)_y = (2.71 \text{ lb}\cdot\text{ft})\mathbf{j} \blacktriangleleft$$

PROBLEM 18.156 (Continued)

(*b*) Dynamic reaction at Point *O*.

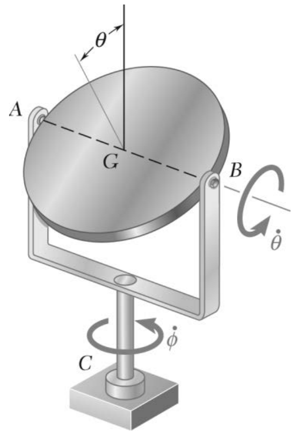
$$R_x = -(0.24845)(1.33333)(4)^2 = -5.30 \text{ lb}, \qquad R_y = 0$$
$$R_z = -(0.24845)(1.33333)(6) = -1.988 \text{ lb}$$

$$\mathbf{R} = -(5.30 \text{ lb})\mathbf{i} - (1.988 \text{ lb})\mathbf{k} \blacktriangleleft$$

$$(M_O)_x = (0.24845)\left[\left(\frac{1}{2}\right)(0.41667)^2(4)(12) + (1.33333)(0.83333)(6)\right]$$
$$= 2.69 \text{ lb}\cdot\text{ft}$$
$$(M_O)_z = (0.24845)[0 - (1.33333)(0.83333)(4)^2]$$
$$= -4.42 \text{ lb}\cdot\text{ft}$$

$$\mathbf{M}_O = (2.69 \text{ lb}\cdot\text{ft})\mathbf{i} - (4.42 \text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$

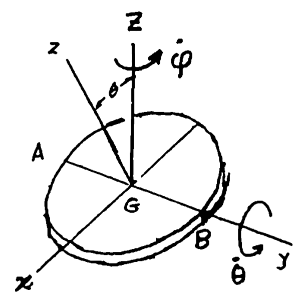
PROBLEM 18.157

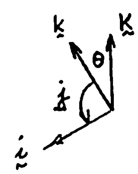


A homogeneous disk of mass m connected at A and B to a fork-ended shaft of negligible mass which is supported by a bearing at C. The disk is free to rotate about its horizontal diameter AB and the shaft is free to rotate about a vertical axis through C. Initially, the disk lies in a vertical plane $(\theta_0 = 90°)$ and the shaft has an angular velocity $\dot{\phi}_0 = 8$ rad/s. If the disk is slightly disturbed, determine for the ensuring motion (*a*) the minimum value of $\dot{\phi}$, (*b*) the maximum value of $\dot{\theta}$.

SOLUTION

Place the origin at the center of mass and let *Oxyz* be a principal axis frame of reference with the *y* axis directed along the moving axle *AB*. Let the *Z* axis lie along the fixed axle. Useful unit vectors are **i**, **j** and **k** along the *x*, *y*, *z* axes and **K** along the *Z* axis.





$$\mathbf{K} = -\mathbf{i}\sin\theta + \mathbf{k}\cos\theta$$

Angular velocity: $\boldsymbol{\omega} = \dot{\varphi}\mathbf{k} + \dot{\theta}\mathbf{j}$

$$\boldsymbol{\omega} = -\dot{\varphi}\sin\theta\mathbf{i} + \dot{\theta}\mathbf{j} + \dot{\varphi}\cos\theta\mathbf{k}$$

Moments of inertia: $I_x = e_1 I_y, \qquad I_z = e_2 I_y$

<u>Angular momentum about *O*.</u> $\mathbf{H}_O = I_x\omega_x\mathbf{i} + I_y\omega_y\mathbf{j} + I_z\omega_z\mathbf{k}$

$$= I_y(-e_1\dot{\varphi}\sin\theta\mathbf{i} + \dot{\theta}\mathbf{j} + e_2\dot{\varphi}\cos\theta\mathbf{k})$$

The moment about the fixed *Z* axis is zero, hence, $\mathbf{H}_O \cdot \mathbf{K} = \text{constant}$.

$$\mathbf{H}_O \cdot \mathbf{K} = I_y(e_1\sin^2\theta + e_2\cos^2\theta)\dot{\varphi} = I_y C_1$$

$$\dot{\varphi} = \frac{C_1}{e_1\sin^2\theta + e_2\cos^2\theta}$$

$$C_1 = (e_1\sin^2\theta_0 + e_2\cos^2\theta_0)\dot{\varphi}_0$$

Twice the kinetic energy: $2T = I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 = \text{constant}$

$$2T = I_y[(e_1\sin^2\theta + e_2\cos^2\theta)\dot{\varphi}^2 + \dot{\theta}^2]$$

$$= I_y(C_1\dot{\varphi} + \dot{\theta}^2)$$

$$= I_y C_2$$

$$\dot{\theta}^2 = C_2 - C_1\dot{\varphi}$$

$$C_2 = \dot{\theta}_0^2 + C_1\dot{\varphi}_0$$

PROBLEM 18.157 (Continued)

Data:

$$e_1 = 1$$
$$e_2 = 2$$
$$e_1 \sin^2\theta + e_2 \cos^2\theta = 1 + \cos^2\theta$$
$$\theta_0 = 90°$$
$$\dot{\theta}_0 = 0$$
$$\dot{\varphi}_0 = 16 \text{ rad/s}$$

$$C_1 = (1+\cos^2 90°)(8)$$
$$= 8 \text{ rad/s}$$
$$\dot{\varphi} = \frac{8}{1+\cos\theta}$$

(*a*)
$$\dot{\varphi}_{\min} = \frac{8}{1+\cos\theta} = 4$$

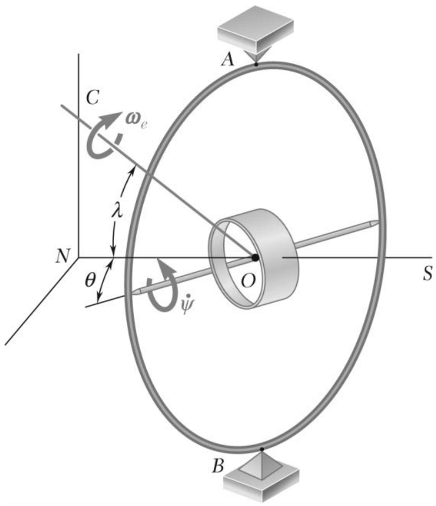
$\dot{\varphi}_{\min} = 4.00 \text{ rad/s}$ ◀

$$C_2 = 0 + (8)(8) = 64 \text{ (rad/s)}^2$$
$$\dot{\theta}^2 = C_2 - C_1\dot{\varphi}$$

(*b*)
$$(\dot{\theta}^2)_{\max} = C_2 - C_1\dot{\varphi}_{\min}$$
$$= 64 - (8)(4)$$
$$= 32 \text{ (rad/s)}^2$$

$\dot{\theta}_{\max} = 5.66 \text{ rad/s}$ ◀

PROBLEM 18.158



The essential features of the gyrocompass are shown. The rotor spins at the rate $\dot{\psi}$ about an axis mounted in a single gimbal, which may rotate freely about the vertical axis AB. The angle formed by the axis of the rotor and the plane of the meridian is denoted by θ, and the latitude of the position on the earth is denoted by λ. We note that line OC is parallel to the axis of the earth, and we denote by $\boldsymbol{\omega}_e$ the angular velocity of the earth about its axis.

(*a*) Show that the equations of motion of the gyrocompass are

$$I'\ddot{\theta} + I\omega_z\omega_e \cos\lambda \sin\theta - I'\omega_e^2 \cos^2\lambda \sin\theta\cos\theta = 0$$

$$I\dot{\omega}_z = 0$$

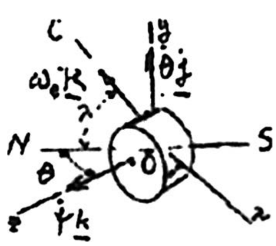
where ω_z is the rectangular component of the total angular velocity $\boldsymbol{\omega}$ along the axis of the rotor, and I and I' are the moments of inertia of the rotor with respect to its axis of symmetry and a transverse axis through O, respectively.

(*b*) Neglecting the term containing ω_e^2, show that for small values of θ, we have

$$\ddot{\theta} + \frac{I\omega_z\omega_e\cos\lambda}{I'}\theta = 0$$

and that the axis of the gyrocompass oscillates about the north-south direction.

SOLUTION



(*a*) Angular momentum about *O*.

We select a frame of reference *Oxyz* attached to the gimbal. The angular velocity of *Oxyz* with respect to a Newtonian frame is $\boldsymbol{\Omega} = \omega_e\mathbf{K} + \dot{\theta}\mathbf{j}$

where $\quad \mathbf{K} = -\cos\lambda\sin\theta\mathbf{i} + \sin\lambda\mathbf{j} + \cos\lambda\cos\theta\mathbf{k}$

Thus, $\quad \boldsymbol{\Omega} = -\omega_e\cos\lambda\sin\theta\mathbf{i} + (\dot{\theta} + \omega_e\sin\lambda)\mathbf{j} + \omega_e\cos\lambda\cos\theta\mathbf{k} \qquad (1)$

The angular velocity $\boldsymbol{\omega}$ of the rotor is obtained by adding its spin $\dot{\psi}\mathbf{k}$ to $\boldsymbol{\Omega}$. Setting

$$\dot{\psi} + \omega_e\cos\lambda\cos\theta = \omega_z.$$

We have $\quad \boldsymbol{\omega} = -\omega_e\cos\lambda\sin\theta\mathbf{i} + (\dot{\theta} + \omega_e\sin\lambda)\mathbf{j} + \omega_z\mathbf{k} \qquad (2)$

The angular momentum $\mathbf{H}_O$ of the rotor is

$$\mathbf{H}_O = I_x \omega_z \mathbf{i} + I_y \omega_y \mathbf{j} + I_z \omega_z \mathbf{k}$$

where $I_x = I_y = I'$ and $I_z = I$. Recalling Eq. (2), we write

$$\mathbf{H}_O = -I'\omega_e \cos\lambda \sin\theta \mathbf{i} + I'(\dot{\theta} + \omega_e \sin\lambda)\mathbf{j} + I\omega_z \mathbf{k} \qquad (3)$$

<u>Equations of motion</u>.

Eq. (18.28): $\Sigma\mathbf{M}_O = (\dot{\mathbf{H}}_O)_{Oxyz} + \mathbf{\Omega} \times \mathbf{H}_O$ or, from Eqs. (1) and (3):

$$\Sigma\mathbf{M}_O = -I'\omega_e \cos\lambda \cos\theta\dot{\theta}\mathbf{i} + I'\ddot{\theta}\mathbf{j} + I\dot{\omega}_z\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\omega_e \cos\lambda \sin\theta & \dot{\theta} + \omega_e \sin\lambda & \omega_e \cos\lambda \cos\theta \\ -I'\omega_e \cos\lambda \sin\theta & I'(\dot{\theta} + \omega_e \sin\lambda) & I\omega_z \end{vmatrix} \qquad (4)$$

We observe that the rotor is free to spin about the z axis and free to rotate about the y axis. Therefore, the y and z components of $\Sigma\mathbf{M}_O$ must be zero. It follows that the coefficients of **j** and **k** at the right-hand member of Eq. (4) must also be zero.

Setting the coefficient of **j** in the right-hand member of Eq. (4) equal to zero,

$$I'\ddot{\theta} + (-I'\omega_e \cos\lambda \sin\theta)(\omega_e \cos\lambda \cos\theta) - (-\omega_e \cos\lambda \sin\theta) I\omega_z = 0$$

$$I'\ddot{\theta} + I\omega_z\omega_e \cos\lambda \sin\theta - I'\omega_e^2 \cos^2\lambda \sin\theta \cos\theta = 0 \qquad (5) \blacktriangleleft$$

Q. E. D

Setting the coefficient of **k** equal to zero,

$$I\dot{\omega}_z + (-\omega_e \cos\lambda \sin\theta) I'(\theta + \omega_e \sin\lambda) - (-I'\omega_e \cos\lambda \sin\theta)(\dot{\theta} + \omega_e \sin\lambda) = 0$$

Observing that the last two terms cancel out, we have

$$I\dot{\omega}_z = 0 \qquad \text{Q. E. D.} \qquad (6) \blacktriangleleft$$

(*b*) It follows from Eq. (6) that $\omega_z = \text{constant}$ (7)

Rewrite Eq. (5) as follows: $I'\ddot{\theta} + (I\omega_z - I'\omega_e \cos\lambda \cos\theta)\omega_e \cos\lambda \sin\theta = 0$

It is evident that $\omega_z >>> \omega_e$. We can therefore neglect the second term in the parenthesis and write

$$I'\ddot{\theta} + I\omega_z\omega_e \cos\lambda \sin\theta = 0$$

or

$$\ddot{\theta} + \frac{I\omega_z\omega_e \cos\lambda}{I'} \sin\theta = 0 \qquad (8)$$

PROBLEM 18.158 (Continued)

where the coefficient of sin θ is a constant. The rotor, therefore, oscillates about the line *NS* as a simple pendulum. For small oscillations, $\sin\theta \approx \theta,$ and Eq. (8) yields

$$\ddot{\theta} + \frac{I\omega_z\omega_e \cos\lambda}{I'}\theta = 0 \qquad \text{Q. E. D.} \qquad (9) ◀$$

Eq. (9) is the equation of simple harmonic motion with period

$$\tau = 2\pi\sqrt{\frac{I'}{I\omega_z\omega_e \cos\lambda}} \qquad (10)$$

Since its rotor oscillates about the line *NS*, the gyrocompass can be used to determine the direction of that line. We should note, however, that for values of λ close to 90° or –90°, the period of oscillation becomes very large and the line about which the rotor oscillates cannot be determined. The gyrocompass, therefore, cannot be used in the polar regions.

CHAPTER 19

PROBLEM 19.1

Determine the maximum velocity and maximum acceleration of a particle which moves in simple harmonic motion with an amplitude of 3 mm and a frequency of 20 Hz.

SOLUTION

Frequency: $f = 20 \text{ Hz}$

$$\omega_n = 2\pi f = (2\pi)(20) = 125.66 \text{ rad/s}$$

Amplitude: $x_m = 3 \text{ mm}$

Simple harmonic motion: $x = x_m \sin(\omega_n t + \phi)$

$$v = \dot{x} = \omega_n x_m \cos(\omega_n t + \phi)$$
$$a = \dot{v} = \ddot{x} = -\omega_n^2 x_m \sin(\omega_n t + \phi)$$

Maximum velocity:
$$v_m = \omega_n x_m = (125.66 \text{ rad/s})(3 \text{ mm})$$
$$= 377 \text{ mm/s}$$

$$v_m = 0.377 \text{ m/s} \blacktriangleleft$$

Maximum acceleration:
$$a_m = \omega_n^2 x_m = (125.66 \text{ rad/s})^2 (3 \text{ mm})$$
$$= 47.3 \times 10^3 \text{ mm/s}^2$$

$$a_m = 47.3 \text{ m/s}^2 \blacktriangleleft$$

PROBLEM 19.2

A particle moves in simple harmonic motion. Knowing that the amplitude is 15 in. and the maximum acceleration is 15 ft/s^2, determine the maximum velocity of the particle and the frequency of its motion.

SOLUTION

Simple harmonic motion.

$$x = x_m \sin(\omega_n t + \phi)$$

$$x_m = 15 \text{ in.} = 1.25 \text{ ft}$$

$$\dot{x} = v = x_m \omega_n \cos(\omega_n t + \phi)$$

$$\ddot{x} = a = -x_m \omega_n^2 \sin(\omega_n t + \phi)$$

$$a_m = -x_m \omega_n^2$$

$$|a_m| = 15 \text{ ft/s}^2 = (1.25 \text{ ft})\omega_n^2$$

Natural frequency

$$\omega_n = 3.4641 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = 0.55133 \text{ Hz}$$

$$f_n = 0.551 \text{ Hz} \blacktriangleleft$$

Maximum velocity

$$v_m = x_m \omega_n = (1.25 \text{ ft})(3.4641 \text{ rad/s})$$

$$= 4.3301 \text{ ft/s}$$

$$v_m = 4.33 \text{ ft/s} \blacktriangleleft$$

PROBLEM 19.3

Determine the amplitude and maximum velocity of a particle which moves in simple harmonic motion with a maximum acceleration of 15 ft/s^2 and a frequency of 8 Hz.

SOLUTION

Simple harmonic motion $\quad x = x_m \sin(\omega_n t + \phi)$

$$\omega_n = 2\pi f_n = 2\pi(8\text{ Hz}) = 16\pi \text{ rad/s}$$

$$\dot{x} = v = x_m \omega_n \cos(\omega_n t + \phi)$$

$$v_m = x_m \omega_n$$

$$\ddot{x} = a = -x_m \omega_n^2 \sin(\omega_n t + \phi)$$

$$a_m = x_m \omega_n^2$$

$$15\text{ ft/s}^2 = x_m (16\pi \text{ rad/s})^2$$

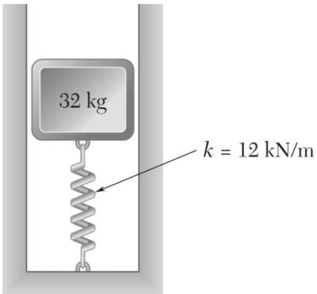
Maximum displacement. $\quad x_m = 0.005937\text{ ft} = 0.0712\text{ in.}$

$$x_m = 0.0712\text{ in.} \blacktriangleleft$$

Maximum velocity. $\quad v_m = x_m \omega_n = (0.005937\text{ ft})(16\pi \text{ rad/s})$

$$= 0.2984\text{ ft/s} = 3.58\text{ in./s}$$

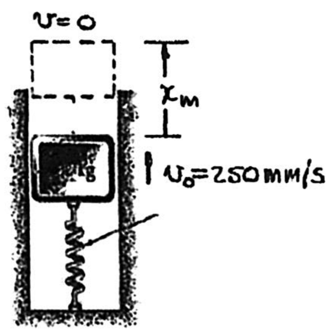
$$v_m = 3.58\text{ in./s} \blacktriangleleft$$



PROBLEM 19.4

A 32-kg block is attached to a spring and can move without friction in a slot as shown. The block is in its equilibrium position when it is struck by a hammer, which imparts to the block an initial velocity of 250 mm/s. Determine (*a*) the period and frequency of the resulting motion, (*b*) the amplitude of the motion and the maximum acceleration of the block.

SOLUTION



(*a*)

$$x = x_m \sin(\omega_n t + \phi)$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{12\times 10^3 \text{ N/m}}{32 \text{ kg}}}$$

$$\omega_n = 19.365 \text{ rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_n}$$

$$\tau_n = \frac{2\pi}{19.365} \qquad \tau_n = 0.324 \text{ s} \blacktriangleleft$$

$$f_n = \frac{1}{\tau_n} = \frac{1}{0.324} = 3.08 \text{ Hz} \blacktriangleleft$$

(*b*) At $t = 0$, $x_0 = 0$, $\dot{x}_0 = v_0 = 250$ mm/s

Thus, $x_0 = 0 = x_m \sin(\omega_n(0) + \phi)$

and $\phi = 0$

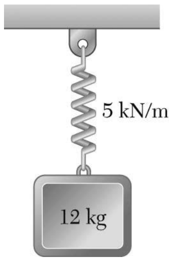
$$\dot{x}_0 = v_0 = x_m \omega_n \cos(\omega_n(0) + 0) = x_m \omega_n$$

$$v_0 = 0.250 \text{ m/s} = x_m (19.365 \text{ rad/s})$$

$$x_m = \frac{(0.250 \text{ m/s})}{(19.365 \text{ rad/s})}$$

$$x_m = 12.91\times 10^{-3} \text{ m} \qquad x_m = 12.91 \text{ mm} \blacktriangleleft$$

$$a_m = x_m \omega_n^2 = (12.91\times 10^{-3} \text{ m})(19.365 \text{ rad/s})^2 \qquad a_m = 4.84 \text{ m/s}^2 \blacktriangleleft$$

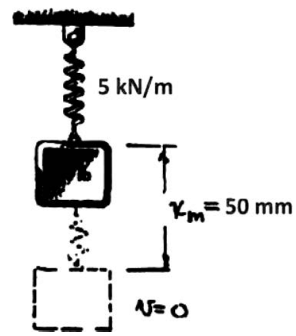


PROBLEM 19.5

A 12-kg block is supported by the spring shown. If the block is moved vertically downward from its equilibrium position and released, determine (*a*) the period and frequency of the resulting motion, (*b*) the maximum velocity and acceleration of the block if the amplitude of its motion is 50 mm.

SOLUTION

(*a*) Simple harmonic motion. $x = x_m \sin(\omega_n t + \phi)$



Natural frequency. $\omega_n = \sqrt{\dfrac{k}{m}} \qquad k = 5 \text{ kN/m} = 5000 \text{ N/m}$

$$\omega_n = \sqrt{\frac{(5000 \text{ N/m})}{12 \text{ kg}}}$$

$$\omega_n = 20.412 \text{ rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_n}$$

$$\tau_n = \frac{2\pi}{20.412} = 0.30781 \text{ s} \qquad \tau_n = 0.308 \text{ s} \blacktriangleleft$$

$$f_n = \frac{1}{\tau_n} = \frac{1}{0.30781} = 3.25 \text{ Hz} \blacktriangleleft$$

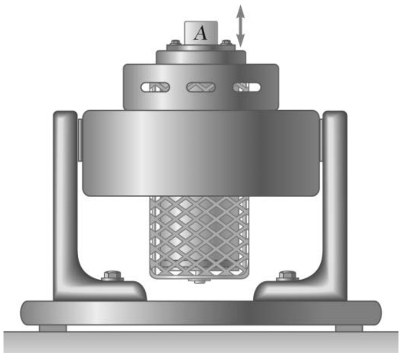
(*b*)

$$x_m = 50 \text{ mm} = 0.05 \text{ m}$$

$$x = 0.05 \sin(20.412t + \phi)$$

Maximum velocity. $v_m = x_m \omega_n = (0.05 \text{ m})(20.412 \text{ rad/s})$ $\qquad v_m = 1.021 \text{ m/s} \blacktriangleleft$

Maximum acceleration. $a_m = x_m \omega_n^2 = (0.05 \text{ m})(20.412 \text{ rad/s})^2$ $\qquad a_m = 20.8 \text{ m/s}^2 \blacktriangleleft$



PROBLEM 19.6

An instrument package A is bolted to a shaker table as shown. The table moves vertically in simple harmonic motion at the same frequency as the variable-speed motor which drives it. The package is to be tested at a peak acceleration of 150 ft/s^2. Knowing that the amplitude of the shaker table is 2.3 in., determine (a) the required speed of the motor in rpm, (b) the maximum velocity of the table.

SOLUTION

In simple harmonic motion,

$$a_{\max} = x_{\max}\omega_n^2$$

$$150 \text{ ft/s}^2 = \left(\frac{2.3}{12}\text{ ft}\right)\omega_n^2$$

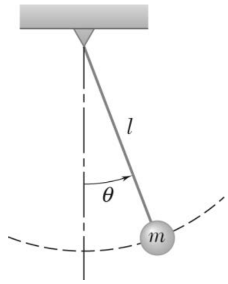
$$\omega_n^2 = (782.6 \text{ rad/s})^2$$

$$\omega_n = 27.98 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{27.98}{2\pi} = 4.452 \text{ Hz (cycles per second)}$$

(a) Motor speed. $(4.452 \text{ rev/s})(60 \text{ s/min})$ speed = 267 rpm ◄

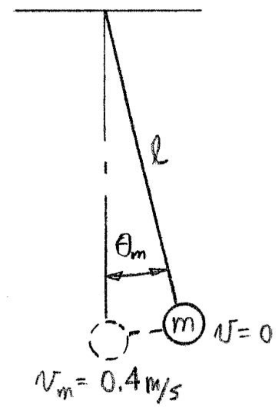
(b) Maximum velocity. $v_{\max} = x_{\max}\omega_n = \left(\frac{2.3}{12}\text{ft}\right)(27.98 \text{ rad/s})$ $v_{\max} = 5.36 \text{ ft/s}$ ◄



PROBLEM 19.7

A simple pendulum consisting of a bob attached to a cord oscillates in a vertical plane with a period of 1.3 s. Assuming simple harmonic motion and knowing that the maximum velocity of the bob is 0.4 m/s, determine (*a*) the amplitude of the motion in degrees, (*b*) the maximum tangential acceleration of the bob.

SOLUTION



(*a*) Simple harmonic motion $\theta = \theta_m \sin(\omega_n t + \phi)$

$$\omega_n = \frac{2\pi}{\tau_n} = \frac{2\pi}{(1.3\text{ s})}$$
$$= 4.8332 \text{ rad/s}$$
$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$
$$\dot{\theta}_m = \theta_m \omega_n$$
$$v_m = l\dot{\theta}_m = l\theta_m \omega_n$$

Thus, $$\theta_m = \frac{v_m}{l\omega_n} \quad (1)$$

For a simple pendulum, $$\omega_n = \sqrt{\frac{g}{l}}$$

Thus, $$l = \frac{g}{\omega_n^2} = \frac{9.81 \text{ m/s}^2}{(4.8332 \text{ rad/s})^2}$$
$$= 0.41995 \text{ m}$$

Amplitude from (1), $$\theta_m = \frac{v_m}{l\omega_n} = \frac{0.4 \text{ m/s}}{(0.42 \text{ m})(4.833 \text{ rad/s})}$$
$$= 0.19707 \text{ rad}$$
$$= 11.291°$$

$\theta_m = 11.29°$ ◀

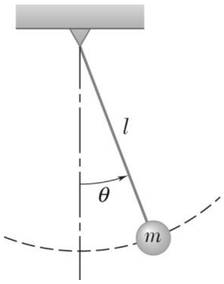
(*b*) Maximum tangential acceleration $a_t = l\ddot{\theta}$

The maximum tangential acceleration occurs when $\ddot{\theta}$ is maximum.

$$\ddot{\theta} = -\theta_m \omega_n^2 \sin(\omega_n t + \phi)$$
$$\ddot{\theta}_m = \theta_m \omega_n^2$$
$$(a_t)_m = l\theta_m \omega_n^2$$
$$(a_t)_m = (0.41995 \text{ m})(0.19707 \text{ rad})(4.8332 \text{ rad/s})^2$$
$$= 1.933 \text{ m/s}^2$$

$(a_t)_m = 1.933 \text{ m/s}^2$ ◀

PROBLEM 19.8



A simple pendulum consisting of a bob attached to a cord of length $l = 800$ mm oscillates in a vertical plane. Assuming simple harmonic motion and knowing that the bob is released from rest when $\theta = 6°$, determine (*a*) the frequency of oscillation, (*b*) the maximum velocity of the bob.

SOLUTION

(*a*) Frequency.

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{(9.81 \text{ m/s}^2)}{(0.8 \text{ m})}}$$

$$\omega_n = 3.502 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{(3.502 \text{ rad/s})}{2\pi}$$

$f_n = 0.557$ Hz ◄

(*b*) Simple harmonic motion.

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

where

$$\theta_m = 6° = 0.10472 \text{ rad}$$

Maximum velocity.

$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

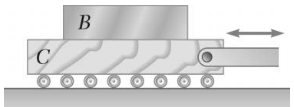
$$\dot{\theta}_m = \theta_m \omega_n$$

$$v_m = l\dot{\theta}_m = l\theta_m \omega_n = (0.8 \text{ m})(0.10472)(3.502)$$

$$v_m = 293.4 \times 10^{-3} \text{ m/s}$$

$v_m = 293$ mm/s ◄

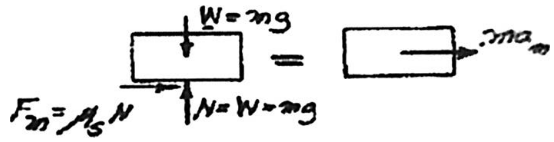
PROBLEM 19.9



An instrument package B is placed on the shaking table C as shown. The table is made to move horizontally in simple harmonic motion with a frequency of 3 Hz. Knowing that the coefficient of static friction is $\mu_s = 0.40$ between the package and the table, determine the largest allowable amplitude of the motion if the package is not to slip on the table. Given the answers in both SI and U.S. customary units.

SOLUTION

Maximum allowable acceleration of B.



$$\mu_s = 0.40$$

$\xrightarrow{+} \Sigma F = ma:$

$$F_m = ma_m$$

$$\mu_s mg = ma_m$$

$$a_m = \mu_s g \qquad a_m = 0.40g$$

Simple harmonic motion.

$$f_n = 3 \text{ Hz} = \frac{\omega_n}{2\pi}$$

$$\omega_n = 6\pi \text{ rad/s}$$

$$a_m = x_m \omega_n^2$$

$$0.40g = x_m (6\pi \text{ rad/s})^2$$

$$x_m = 1.1258 \times 10^{-3} g$$

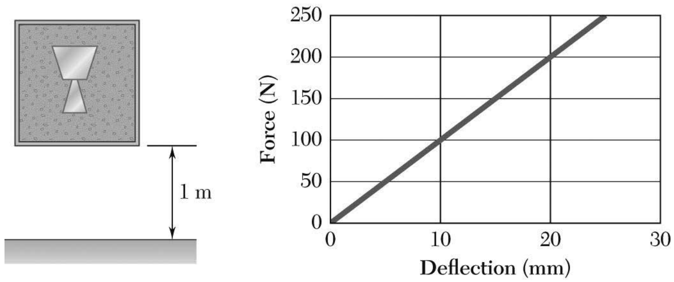
Largest allowable amplitude.

SI: $x_m = 1.1258 \times 10^{-3}(9.81) = 11.044 \times 10^{-3}$ m $\qquad x_m = 11.04$ mm ◀

U.S.: $x_m = 1.1258 \times 10^{-3}(32.2) = 0.03625$ ft $\qquad x_m = 0.435$ in. ◀

PROBLEM 19.10

A 5-kg fragile glass vase is surrounded by packing material in a cardboard box of negligible weight. The packing material has negligible damping and a force-deflection relationship as shown. Knowing that the box is dropped from a height of 1 m and the impact with the ground is perfectly plastic, determine (*a*) the amplitude of vibration for the vase, (*b*) the maximum acceleration the vase experiences in g's.

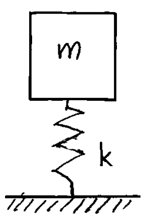


SOLUTION

Velocity at end of free fall: $v = \sqrt{2gh}$

$$v = \sqrt{(2)(9.81 \text{ m/s}^2)(1 \text{ m})} = 4.4294 \text{ m/s}$$

Assume that the spring is unstretched during the free fall. Use a simple spring-mass model for the motion of the vase and the packing material.



$$m = 5 \text{ kg}$$

$$k = \frac{100 \text{ N}}{10 \text{ mm}} \quad \text{(slope from graph)}$$

$$k = 10 \text{ N/m} = 10000 \text{ N/m}$$

Natural frequency: $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{10000 \text{ N/m}}{5 \text{ kg}}} = 44.721 \text{ rad/s}$

Simple harmonic motion:

$$x = x_m \sin(\omega_n t + \phi)$$

$$v = \dot{x} = \omega_n x_m \cos(\omega_n t + \phi)$$

Let $t = 0$ at the instant when the box bottom hits the ground.

Then, at $t = 0$, $x = 0$ and $v = 4.4294$ m/s

from which $\phi = 0$

and $\omega_n x_m = 4.4294$ m/s

PROBLEM 19.10 (Continued)

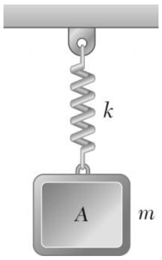
(*a*) Amplitude: $x_m = \dfrac{4.4294 \text{ m/s}}{44.721 \text{ rad/s}} = 0.099045 \text{ m}$

$$x_m = 99.0 \text{ mm} \blacktriangleleft$$

(*b*) Maximum acceleration:

$$a_m = \omega_n^2 x_m = (44.721 \text{ rad/s})^2 (0.099045 \text{ m})$$
$$= 198.087 \text{ m/s}^2 = (20.192)(9.81 \text{ m/s}^2)$$

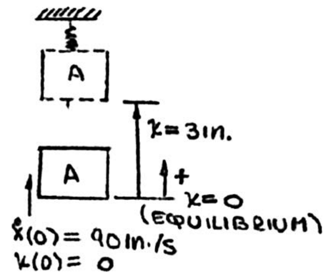
$$a_m = 20.2\, g \blacktriangleleft$$



PROBLEM 19.11

A 3-lb block is supported as shown by a spring of constant $k = 2$ lb/in. which can act in tension or compression. The block is in its equilibrium position when it is struck from below by a hammer which imparts to the block an upward velocity of 90 in./s. Determine (*a*) the time required for the block to move 3 in. upward, (*b*) the corresponding velocity and acceleration of the block.

SOLUTION



Simple harmonic motion. $x = x_m \sin(\omega_n t + \phi)$

Natural frequency. $\omega_n = \sqrt{\frac{k}{m}}, \quad k = 2 \text{ lb/in.} = 24 \text{ lb/ft}$

$$\omega_n = \sqrt{\frac{24 \text{ lb/ft}}{\left(\frac{3 \text{ lb}}{32.2 \text{ ft/s}^2}\right)}}$$

$$\omega_n = 16.05 \text{ rad/s}$$

$$x(0) = 0 = x_m \sin(0 + \phi)$$

$$\phi = 0$$

$$\dot{x}(0) = x_m \omega_n \cos(0 + 0)$$

$$\dot{x}(0) = \frac{90}{12} = 7.5 \text{ ft/s}$$

$$7.5 = x_m(16.05) \qquad x_m = 0.4673 \text{ ft}$$

$$x = (0.4673)\sin(16.05t)(\text{ft/s}) \tag{1}$$

(*a*) Time at $x = 3$ in. ($x = 0.25$ ft)

$$0.25 = 0.4673\sin(16.05t)$$

$$t = \frac{\sin^{-1}\left(\frac{0.25}{0.4673}\right)}{16.05}$$

$t = 0.0352$ s ◀

(*b*) Velocity and acceleration.

$$\dot{x} = x_m \omega_n \cos(\omega_n t)$$

$$\ddot{x} = -x_m \omega_n^2 \sin \omega_n t$$

$$t = 0.0352$$

$$\dot{x} = (0.4673)(16.05)\cos[(16.05)(0.0352)]$$

$$\dot{x} = 6.34 \text{ ft/s}$$

$\mathbf{v} = 6.34$ ft/s ↑ ◀

$$\ddot{x} = -(0.4673)(16.05)^2 \sin[(16.05)(0.0352)]$$

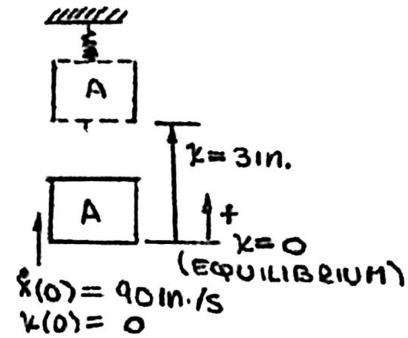
$$= -64.4 \text{ ft/s}^2$$

$\mathbf{a} = 64.4$ ft/s^2 ↓ ◀

PROBLEM 19.12

In Problem 19.11, determine the position, velocity, and acceleration of the block 0.90 s after it has been struck by the hammer.

SOLUTION



Simple harmonic motion. $x = x_m \sin(\omega_n t + \phi)$

Natural frequency. $\omega_n = \sqrt{\dfrac{k}{m}}, \quad k = 2 \text{ lb/in.} = 24 \text{ lb/ft}$

$$\omega_n = \sqrt{\frac{24 \text{ lb/ft}}{\left(\frac{3 \text{ lb}}{32.2 \text{ ft/s}^2}\right)}}$$

$$\omega_n = 16.05 \text{ rad/s}$$

$$x(0) = 0 = x_m \sin(0 + \phi)$$

$$\phi = 0$$

$$\dot{x}(0) = x_m \omega_n \cos(0 + 0) \qquad \dot{x}(0) = \frac{90}{12} = 7.5 \text{ ft/s}$$

$$7.5 = x_m (16.05) \qquad x_m = 0.4673 \text{ ft}$$

$$x = (0.4673)\sin(16.05t)(\text{ft/s})$$

Simple harmonic motion.

$$x = x_m \sin(\omega_n t + \phi)$$

$$\dot{x} = x_m \omega_n \cos(\omega_n t + \phi)$$

$$\ddot{x} = -x_m \omega_n^2 \sin(\omega_n t + \phi)$$

At 0.90 s:

$$x = (0.4673)\sin[(16.05)(0.90)] = 0.445 \text{ ft}$$

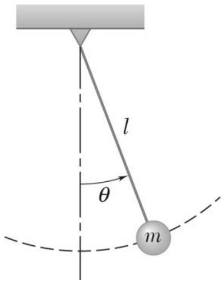
x = 0.445 ft ↑ ◀

$$\dot{x} = (0.4673)(16.05)\cos[(16.05)(0.90)] = -2.27 \text{ ft/s}$$

v = 2.27 ft/s ↓ ◀

$$\ddot{x} = -(0.4673)(16.05)^2 \sin[(16.05)(0.90)] = -114.7 \text{ ft/s}^2$$

a = 114.7 ft/s² ↓ ◀



PROBLEM 19.13

The bob of a simple pendulum of length $l = 40$ in. is released from rest when $\theta = +5°$. Assuming simple harmonic motion, determine 1.6 s after release (*a*) the angle θ, (*b*) the magnitudes of the velocity and acceleration of the bob.

SOLUTION

For simple harmonic motion and $l = 40\text{ in.} = 3.333\text{ ft}$:

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{32.2\text{ ft/s}^2}{3.333\text{ ft}}} = 3.1082\text{ rad/s}$$

Angular displacement: $\theta = \theta_m \sin(\omega_n t + \phi)$

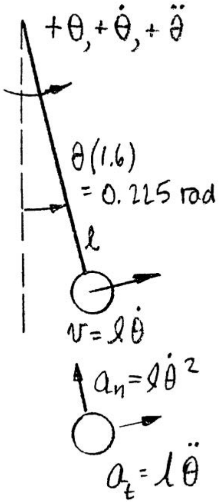
Initial conditions: $\theta(0) = 5° = 0.08727\text{ rad}, \quad \text{and} \quad \dot{\theta}(0) = 0$:

$$\dot{\theta}(0) = 0 = \theta_m \omega_n \cos(0 + \phi) \qquad \phi = \frac{\pi}{2}$$

$$\theta_m = \theta(0) = \frac{5\pi}{180} = 0.08727\text{ rad}$$

$$\theta = \frac{5\pi}{180}\sin\left[(3.1082\text{ rad/s})t + \frac{\pi}{2}\right]$$

$$= (0.08727\text{ rad})\sin\left[(3.1082\text{ rad/s})t + \frac{\pi}{2}\right]$$



(*a*) At $t = 1.6$ s.

$$\theta = \frac{5\pi}{180}\sin\left[(3.1082\text{ rad/s})(1.6\text{ s}) + \frac{\pi}{2}\right]$$
$$= 0.022496\text{ rad} = 1.288°$$

$\theta = 1.288°$ ◀

(*b*) Velocity:

$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$
$$= \frac{5\pi}{180}(3.1082\text{ rad/s})\cos\left[(3.1082\text{ rad/s})(1.6\text{ s}) + \frac{\pi}{2}\right]$$
$$= 0.262074\text{ rad/s}$$
$$v = l\dot{\theta} = (3.3333\text{ ft})(0.262074\text{ rad/s}) = 0.874\text{ ft/s}$$

$v = 0.874\text{ ft/s}$ ◀

PROBLEM 19.13 (Continued)

Angular acceleration:

$$\ddot{\theta} = -\theta_m \omega_n^2 \sin(\omega_n t + \phi) = -\frac{5\pi}{180}(3.1082 \text{ rad/s})^2 \cos\left[(3.1082 \text{ rad/s})(1.6 \text{ s}) + \frac{\pi}{2}\right]$$

$$= -0.21733 \text{ rad/s}^2$$

Acceleration:

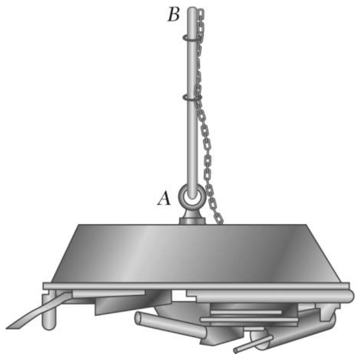
$$a = \sqrt{(a_n)^2 + (a_t)^2}$$

$$a_n = \frac{v^2}{l} = l\dot{\theta}^2 = (3.3333 \text{ ft})(0.26207 \text{ rad/s})^2 = 0.22894 \text{ ft/s}^2$$

$$a_t = l\ddot{\theta} = (3.333 \text{ ft})(-0.21733 \text{ rad/s}^2)^2 = -0.72443 \text{ m/s}^2$$

$$a = 0.75974 \text{ ft/s}^2$$

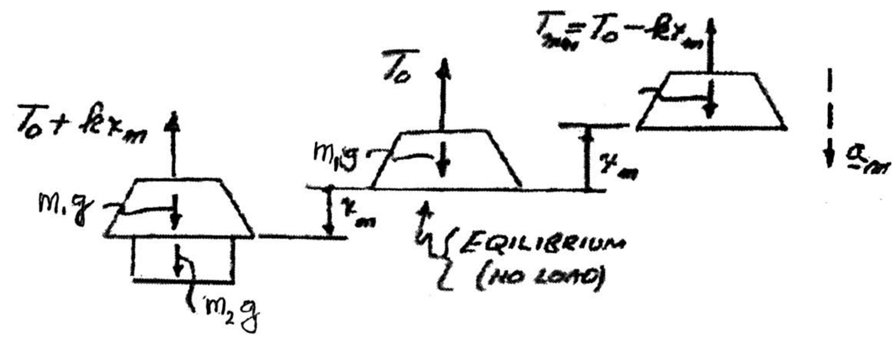
$a = 0.760 \text{ ft/s}^2$ ◀



PROBLEM 19.14

A 150-kg electromagnet is at rest and is holding 100 kg of scrap steel when the current is turned off and the steel is dropped. Knowing that the cable and the supporting crane have a total stiffness equivalent to a spring of constant 200 kN/m, determine (*a*) the frequency, the amplitude, and the maximum velocity of the resulting motion, (*b*) the minimum tension which will occur in the cable during the motion, (*c*) the velocity of the magnet 0.03 s after the current is turned off.

SOLUTION



Data: $m_1 = 150$ kg $\quad m_2 = 100$ kg $\quad k = 200\times10^3$ N/m

From the first two sketches, $\quad T_0 + kx_m = (m_1 + m_2)g \qquad (1)$

$$T_0 = m_1 g \qquad (2)$$

Subtracting Eq. (2) from Eq. (1), $\quad kx_m = m_2 g$

$$x_m = \frac{m_2 g}{k} = \frac{(100)(9.81)}{200\times10^3} = 4.905\times10^{-3}\ \text{m} = 4.91\ \text{mm}$$

Natural circular frequency: $\quad \omega_n = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{200\times10^3}{150}} = 36.515$ rad/s

Natural frequency: $\quad f_n = \frac{\omega_n}{2\pi} = \frac{36.515}{2\pi} \qquad f_n = 5.81$ Hz

Maximum velocity: $\quad v_m = \omega_n x_m = (36.515)(4.905\times10^{-3}) = 0.1791$ m/s

(*a*) Resulting motion: amplitude $x_m = 4.91$ mm ◀

frequency $\quad f_n = 5.81$ Hz ◀

maximum velocity $\quad v_m = 0.1791$ m/s ◀

PROBLEM 19.14 (Continued)

(*b*) Minimum value of tension occurs when $x = -x_m$.

$$T_{\min} = T_0 - kx_m$$
$$= m_1 g - m_2 g$$
$$= (m_1 - m_2)g$$
$$= (50)(9.81)$$

$T_{\min} = 491 \text{ N}$ ◀

The motion is given by $x = x_m \sin(\omega_n t + \varphi)$

$$\dot{x} = \omega_n x_m \cos(\omega_n t + \varphi)$$

Initally, $x_0 = -x_m$ or $\sin\varphi = -1$

$$\dot{x}_0 = 0 \quad \text{or} \quad \cos\varphi = 0$$

$$\varphi = -\frac{\pi}{2}$$

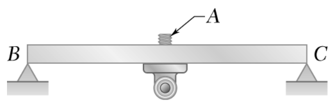
$$\dot{x} = \omega_n x_m \cos\left(\omega_n t - \frac{\pi}{2}\right)$$

(*c*) Velocity at $t = 0.03$ s.

$$\omega_n t = (36.515)(0.03) = 1.09545 \text{ rad}$$
$$\omega_n t - \varphi = -0.47535 \text{ rad}$$
$$\cos(\omega_n t - \varphi) = 0.88913$$
$$\dot{x} = (36.515)(4.905 \times 10^{-3})(0.88913)$$

$\dot{\mathbf{x}} = 0.1592 \text{ m/s} \uparrow$ ◀

PROBLEM 19.15



A variable-speed motor is rigidly attached to beam *BC*. The rotor is slightly unbalanced and causes the beam to vibrate with a frequency equal to the motor speed. When the speed of the motor is less than 600 rpm or more than 1200 rpm, a small object placed at *A* is observed to remain in contact with the beam. For speeds between 600 rpm and 1200 rpm, the object is observed to "dance" and actually to lose contact with the beam. Determine the amplitude of the motion of *A* when the speed of the motor is (*a*) 600 rpm, (*b*) 1200 rpm. Give answers in both SI and U.S. customary units.

SOLUTION

At both 600 rpm and 1200 rpm, the maximum acceleration is just equal to g.

(*a*) $\omega = 600 \text{ rpm} = 62.832 \text{ rad/s}$

Eq. (19.15): $a_m = x_m\omega^2 \qquad x_m = \dfrac{g}{(62.832)^2}$

SI: $x_m = \dfrac{9.81}{(62.832)^2} = 2.4849 \times 10^{-3} \text{ m}$ $\qquad x_m = 2.48 \text{ mm}$ ◀

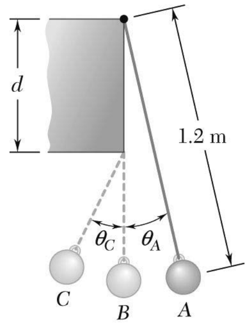
US: $x_m = \dfrac{32.2}{(62.832)^2} = 0.008156 \text{ ft}$ $\qquad x_m = 0.0979 \text{ in.}$ ◀

(*b*) $\omega = 1200 \text{ rpm} = 125.664 \text{ rad/s}$

Eq. (19.15): $a_m = x_m\omega^2 \qquad x_m = \dfrac{g}{(125.664)^2}$

SI: $x_m = \dfrac{9.81}{(125.664)^2} = 621.2 \times 10^{-6} \text{ m}$ $\qquad x_m = 0.621 \text{ mm}$ ◀

US: $x_m = \dfrac{32.2}{(125.664)^2} = 0.002039 \text{ ft}$ $\qquad x_m = 0.0245 \text{ in.}$ ◀



PROBLEM 19.16

A small bob is attached to a cord of length 1.2 m and is released from rest when $\theta_A = 5°$. Knowing that $d = 0.6$ m, determine (*a*) the time required for the bob to return to Point *A*, (*b*) the amplitude θ_C.

SOLUTION

As the pendulum moves between Points *A* and *B*, the length of the pendulum is $l = l_{AB} = 1.2$ m.

$$\omega_n = \omega_{n1} = \sqrt{\frac{g}{l_{AB}}} = \sqrt{\frac{9.81\ \text{m/s}^2}{1.2\ \text{m}}} = 2.8592\ \text{rad/s}$$

$$\tau_1 = \frac{2\pi}{\omega_{n1}} = \frac{2\pi}{2.8592\ \text{rad/s}} = 2.1975\ \text{s}$$

The falling from *A* to *B* is one quarter period.

$$\tau_{AB} = \frac{1}{4}\tau_1 = 0.54938\ \text{s.}$$

As the pendulum moves between Points *B* and *C*, the length of the pendulum is $l = l_{BC} = 1.2\ \text{m} - 0.6\ \text{m} = 0.6\ \text{m}$.

$$\omega_n = \omega_{n2} = \sqrt{\frac{g}{l_{BC}}} = \sqrt{\frac{9.81\ \text{m/s}^2}{0.6\ \text{m}}} = 4.0435\ \text{rad/s}$$

$$\tau_2 = \frac{2\pi}{\omega_{n2}} = \frac{2\pi}{4.0435\ \text{rad/s}} = 1.55389\ \text{s}$$

The motion from *B* to *C* and back to *B* is one half period

$$\tau_{BCB} = \frac{1}{2}\tau_2 = 0.77695\ \text{s}$$

As the pendulum moves from *B* to *A*, the length is again 1.2 meters.

$$\tau_{BA} = \frac{1}{4}\tau_1 = 0.54938\ \text{s}$$

(*a*) Time required to return to *A*.

$$\tau = \tau_{AB} + \tau_{BCB} + \tau_{BA}$$
$$\tau = 1.87571\ \text{s}$$

$\tau = 1.876$ s ◀

PROBLEM 19.16 (Continued)

For falling from A to B, $\theta_m = \theta_A$

At B, $\dot{\theta}_B = \dot{\theta}_m = \omega_{n1}\theta_A$

$$v_B = l_{AB}\dot{\theta}_B = l_{AB}\omega_{n1}\theta_A$$

For rising from B to C,

$$\dot{\theta}_B = \frac{v_B}{l_{BC}} = \frac{l_{AB}}{l_{BC}}\omega_{n1}\theta_A = \dot{\theta}_{\max}$$

$$\theta_C = \theta_{\max} = \frac{\dot{\theta}_{\max}}{\omega_{n2}} = \frac{l_{AB}\omega_{n1}}{l_{BC}\omega_{n2}}\theta_A$$

$$\theta_C = \frac{(1.2 \text{ m})(2.8592 \text{ rad/s})}{(0.6 \text{ m})(4.0435 \text{ rad/s})}\theta_A = 1.4142\,\theta_A$$

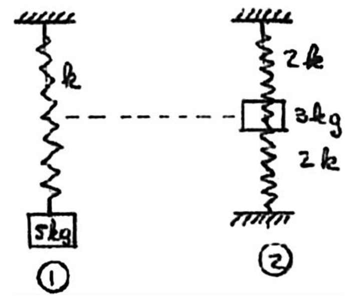
(*b*) Amplitude θ_C:

With $\theta_A = 5°$, $\theta_C = 7.07°$ ◀

PROBLEM 19.17

A 5-kg block, attached to the lower end of a spring whose upper end is fixed, vibrates with a period of 6.8 s. Knowing that the constant k of a spring is inversely proportional to its length, determine the period of a 3-kg block which is attached to the center of the same spring if the upper and lower ends of the spring are fixed.

SOLUTION



Equivalent spring constant.

$$k' = 2k + 2k = 4k \qquad \text{(Deflection of each spring is the same.)}$$

For case ①,

$$\tau_{n1} = 6.8 \text{ s}$$

$$\omega_{n1} = \frac{2\pi}{\tau_{n1}} = \frac{2\pi}{6.8} = 0.924 \text{ rad/s}$$

$$\omega_n^2 = \frac{k}{m_1}$$

$$k = m_1\omega_{n1}^2 = (5)(0.924)^2 = 4.2689 \text{ N/m}$$

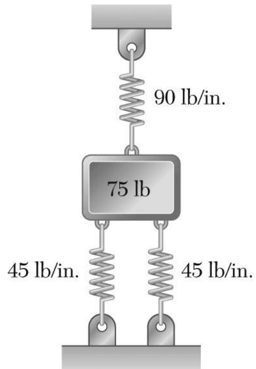
For case ②,

$$\omega_{n2}^2 = \frac{4k}{m_2} = \frac{(4)(4.2689)}{3} = 5.6918 \text{ (rad/s)}^2$$

$$\omega_{n2} = 2.3857 \text{ rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_{n2}} = \frac{2\pi}{2.3857}$$

$\tau_{n2} = 2.63$ s ◀

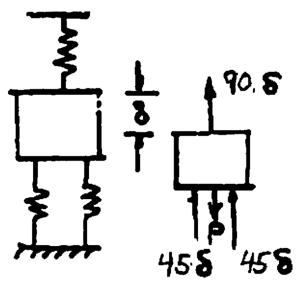


PROBLEM 19.18

A 75-lb block is supported by the spring arrangement shown. The block is moved vertically downward from its equilibrium position and released. Knowing that the amplitude of the resulting motion is 2 in., determine (*a*) the period and frequency of the motion, (*b*) the maximum velocity and maximum acceleration of the block.

SOLUTION

(*a*) Determine the constant k of a single spring equivalent to the three springs



$$P = k\delta$$
$$k\delta = 90\delta + 45\delta + 45\delta$$
$$k = 180 \text{ lb/in.} = 2160 \text{ lb/ft}$$

Natural frequency.

$$\omega_n = \sqrt{\frac{k}{m}}$$
$$= \sqrt{\frac{2160 \text{ lb/ft}}{\frac{75 \text{ lb}}{32.2 \text{ ft/s}^2}}}$$
$$\omega_n = 30.453 \text{ rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{30.453} = 0.20633 \text{ s} \qquad \tau_n = 0.206 \text{ s} \blacktriangleleft$$

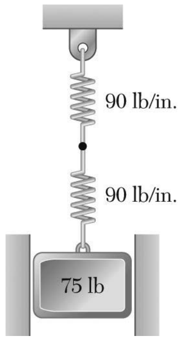
$$f_n = \frac{1}{\tau_n} \qquad f_n = 4.85 \text{ Hz} \blacktriangleleft$$

(*b*)

$$x = x_m \sin(\omega_n t + \phi) \qquad x_0 = 2 \text{ in.} = 0.16667 \text{ ft} = x_m$$

$$\omega_n = 30.453 \text{ rad/s}$$

$$x = 0.16667 \sin(30.453t + \phi)$$
$$\dot{x} = (0.16667)(30.453)\cos(30.453t + \phi) \qquad v_{max} = 5.08 \text{ ft/s} \blacktriangleleft$$
$$\ddot{x} = -(0.16667)(30.453)^2 \sin(30.453t + \phi) \qquad a_{max} = 154.6 \text{ ft/s}^2 \blacktriangleleft$$

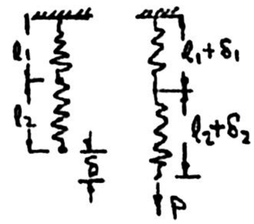


PROBLEM 19.19

A 75-lb block is supported by the spring arrangement shown. The block is moved vertically downward from its equilibrium position and released. Knowing that the amplitude of the resulting motion is 2 in., determine (*a*) the period and frequency of the motion, (*b*) the maximum velocity and maximum acceleration of the block.

SOLUTION

(*a*) Determine the constant *k* of a single spring equivalent to the two springs shown.



$$\delta = \delta_1 + \delta_2 = \frac{P}{90 \text{ lb/in.}} + \frac{P}{90 \text{ lb/in.}} = \frac{P}{k}$$

$$\frac{1}{k} = \frac{1}{90} + \frac{1}{90} \qquad k = 45 \text{ lb/in.} = 540 \text{ lb/ft}$$

Period of the motion.

$$\tau_n = \frac{2\pi}{\sqrt{\frac{k}{m}}} = \frac{2\pi}{\sqrt{\frac{540}{75/32.2}}} = 0.41265 \text{ s} \qquad \tau_n = 0.413 \text{ s} \blacktriangleleft$$

$$f_n = \frac{1}{\tau_n} = \frac{1}{0.41265} = 2.42 \text{ Hz} \blacktriangleleft$$

(*b*)

$$x = x_m \sin(\omega_n t + \phi) \; x_0 = 2 \text{ in.} = 0.16667 \text{ ft} = x_m$$

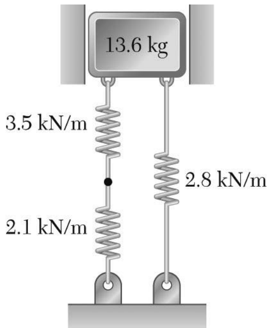
$$\omega_n = 2\pi f_n = 2\pi(2.4233) = 15.226 \text{ rad/s}$$

$$x = 0.16667 \sin(15.226t + \phi)$$

$$\dot{x} = (0.16667)(15.226)\cos(15.226t + \phi)$$

$$v_{max} = 2.54 \text{ ft/s} \blacktriangleleft$$

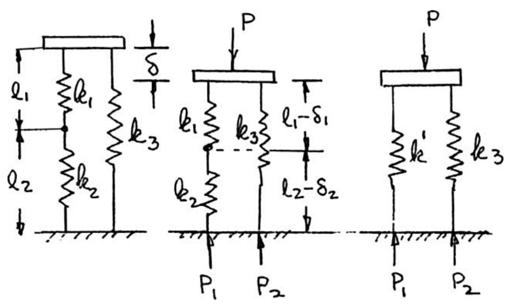
$$\ddot{x} = -(0.16667)(15.226)^2 \sin(15.226t + \phi) \qquad a_{max} = 38.6 \text{ ft/s}^2 \blacktriangleleft$$



PROBLEM 19.20

A 13.6-kg block is supported by the spring arrangement shown. If the block is moved from its equilibrium position 44 mm vertically downward and released, determine (*a*) the period and frequency of the resulting motion, (*b*) the maximum velocity and acceleration of the block.

SOLUTION



Determine the constant k of a single spring equivalent to the three springs shown.

Springs 1 and 2: $\delta = \delta_1 + \delta_2$, and $\dfrac{P_1}{k'} = \dfrac{P_1}{k_1} + \dfrac{P_1}{k_2}$

Hence, $k' = \dfrac{k_1 k_2}{k_1 + k_2}$

where k' is the spring constant of a single spring equivalent of springs 1 and 2.
Springs k' and 3: (Deflection in each spring is the same).

So $P = P_1 + P_2$, and $P = k\delta$, $P_1 = k'\delta$, $P_2 = k_3\delta$

Now $k\delta = k'\delta + k_3\delta$

$$k = k' + k_3 = \frac{k_1 k_2}{k_1 + k_2} + k_3$$

or $k = \dfrac{(3.5\text{ kN/m})(2.1\text{ kN/m})}{(3.5\text{ kN/m}) + (2.1\text{ kN/m})} + 2.8\text{ kN/m} = 4.11\text{ kN/m} = 4.11\times 10^3\text{ N/m}$

(*a*) Period and frequency: $\tau_n = \dfrac{2\pi}{\sqrt{\frac{k}{m}}} = \dfrac{2\pi}{\sqrt{\frac{4.11\times 10^3\text{ N/m}}{13.6\text{ kg}}}}$ $\qquad t_n = 0.361\text{ s}$ ◀

$f_n = \dfrac{1}{\tau_n} = \dfrac{1}{0.3614\text{ s}}$ $\qquad f_n = 2.77\text{ Hz}$ ◀

PROBLEM 19.20 (Continued)

(*b*) Displacement:

$$x = x_m \sin(\omega_n t + \phi)$$

$$x_m = 44 \text{ mm} = 0.044 \text{ m}$$

$$\omega_n = 2\pi f_n = (2\pi)(2.77 \text{ Hz}) = 17.384 \text{ rad/s}$$

$$x = (0.044 \text{ m})\sin[(17.384 \text{ rad/s})t + \phi]$$
$$\dot{x} = (0.044 \text{ m})(17.384 \text{ rad/s})\cos[(17.384 \text{ rad/s})t + \phi]$$

Velocity: $$v_{\max} = (0.044 \text{ m})(17.384 \text{ rad/s}) = 0.765 \text{ m/s}$$

$$\ddot{x} = -(0.044 \text{ m})(17.384 \text{ rad/s})^2 \sin[(17.384 \text{ rad/s})t + \phi]$$

Acceleration: $$a_{\max} = (0.044 \text{ m})(17.384 \text{ rad/s})^2 = 13.30 \text{ m/s}^2$$

$v_{\max} = 0.765 \text{ m/s}$ ◀

$a_{\max} = 13.30 \text{ m/s}^2$ ◀

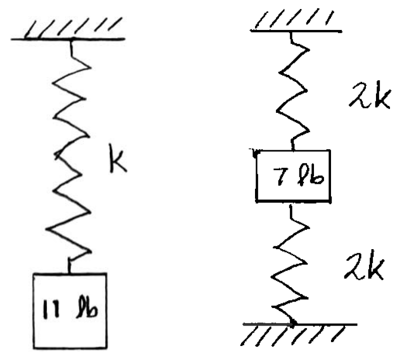
PROBLEM 19.21

A 11-lb block, attached to the lower end of a spring whose upper end is fixed, vibrates with a period of 7.2 s. Knowing that the constant k of a spring is inversely proportional to its length (e.g., if you cut a 10 lb/in. spring in half, the remaining two springs each have a spring constant of 20 lb/in.), determine the period of a 7-lb block which is attached to the center of the same spring if the upper and lower ends of the spring are fixed.

SOLUTION

<u>Equivalent spring constant</u>.

$$k' = 2k + 2k = 4k \qquad \text{(Deflection of each spring is the same.)}$$



For case ①, $\tau_{n1} = 7.2 \text{ s}$

$$\omega_{n1} = \frac{2\pi}{\tau_{n1}} = \frac{2\pi}{7.2} = 0.87266 \text{ rad/s}$$

$$\omega_n^2 = \frac{k}{m_1}$$

$$k = m_1\omega_{n1}^2 = \left(\frac{11}{32.2}\right)(0.87266)^2 = 0.26015 \text{ lb/ft}$$

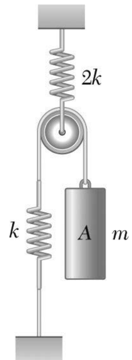
For case ②, $\omega_{n2}^2 = \frac{4k}{m_2} = \frac{(4)(0.26015)}{\frac{7}{32.2}} = 4.7868 \text{ (rad/s)}^2$

$$\omega_{n2} = 2.1879 \text{ rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_{n2}} = \frac{2\pi}{2.1879}$$

$\tau_{n2} = 2.87 \text{ s}$ ◀

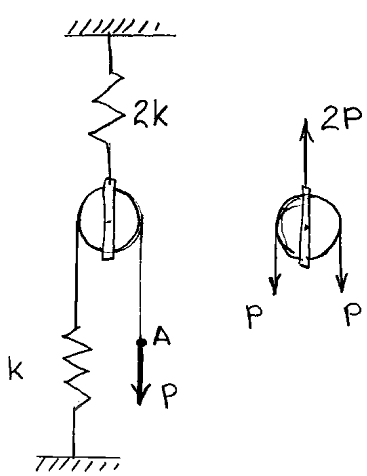
PROBLEM 19.22



Block *A* of mass *m* is supported by the spring arrangement as shown. Knowing that the mass of the pulley is negligible and that the block is moved vertically downward from its equilibrium position and released, determine the frequency of the motion.

SOLUTION

We first determine the constant k_{eq} of a single spring equivalent to the spring and pulley system supporting the block by finding the total displacement δ_A of the end of the cable under a given static load **P**. Owing to the force $2P$ in the upper spring the pulley moves down a distance



$$\delta_1 = \frac{2P}{2k}$$

Owing to the force P in the lower spring, Point A moves down an additional distance

$$\delta_2 = \frac{P}{k}$$

The total displacement is

$$\delta_A = \delta_1 + \delta_2 = \frac{2P}{2k} + \frac{P}{k} = \frac{2P}{k}$$

But $\quad \delta_A = \dfrac{P}{k_{eq}}$ so that

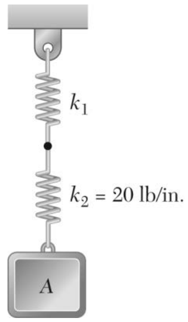
$$k_{eq} = \frac{k}{2}$$

Natural frequency:

$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$

$$\omega_n = \sqrt{\frac{k}{2\,m}} \blacktriangleleft$$

PROBLEM 19.23



The period of vibration of the system shown is observed to be 0.2 s. After the spring of constant $k_2 = 20$ lb/in. is removed and block A is connected to the spring of constant k_1, the period is observed to be 0.12 s. Determine (a) the constant k_1 of the remaining spring, (b) the weight of block A.

SOLUTION

Equivalent spring constant for springs in series.

$$k_e = \frac{k_1 k_2}{(k_1 + k_2)}$$

For k_1 and k_2,

$$\tau = \frac{2\pi}{\sqrt{\frac{k_e}{m_A}}} = \frac{2\pi}{\sqrt{\frac{(k_1 k_2)}{(m_A)(k_1+k_2)}}}$$

For k_1 alone,

$$\tau' = \frac{2\pi}{\sqrt{\frac{k_1}{m_A}}}$$

(a)

$$\frac{\tau}{\tau'} = \sqrt{\frac{(k_1 + k_2)(k_1)}{(k_1 k_2)}} = \sqrt{\frac{k_1 + k_2}{k_2}}$$

$$k_2 \left(\frac{\tau}{\tau'}\right)^2 = k_1 + k_2$$

$$\frac{\tau}{\tau'} = \frac{0.2 \text{ s}}{0.12 \text{ s}} = 1.6667$$

$$k_2 = 20 \text{ lb/in.}$$

$$(20 \text{ lb/in.})(1.6667)^2 = k_1 + 20 \text{ lb/in.}$$

$k_1 = 35.6$ lb/in. ◀

(b)

$$\tau' = \frac{2\pi}{\sqrt{\frac{k_1}{m_A}}} \qquad m_A = \frac{W_A}{g}$$

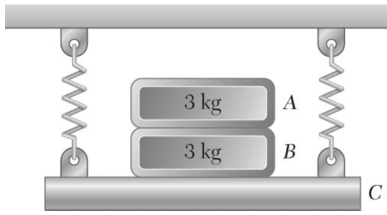
$$m_A = \frac{(\tau')^2 k_1}{(2\pi)^2}$$

$$k_1 = 35.6 \text{ lb/in.} = 426.7 \text{ lb/ft}$$

$$W_A = \frac{(32.2 \text{ ft/s}^2)(0.12 \text{ s})^2 (426.7 \text{ lb/ft})}{(2\pi)^2}$$

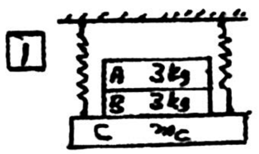
$W_A = 5.01$ lb ◀

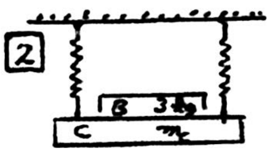
PROBLEM 19.24



The period of vibration of the system shown is observed to be 0.8 s. If block A is removed, the period is observed to be 0.7 s. Determine (a) the mass of block C, (b) the period of vibration when both blocks A and B have been removed.

SOLUTION





$$m_1 = m_C + 6 \text{ kg} \qquad \tau_1 = 0.8 \text{ s}$$

$$\omega_1 = \frac{2\pi}{\tau_1} = \frac{2\pi}{0.8 \text{ s}} = \frac{2\pi}{0.8} \text{ rad/s}$$

$$\omega_1^2 = \frac{k}{m_1}; \quad k = m_1\omega_1^2 = (m_C + 6)\left(\frac{2\pi}{0.8}\right)^2 \qquad (1)$$

$$m_2 = m_C + 3 \text{ kg} \quad \tau_2 = 0.7 \text{ s}$$

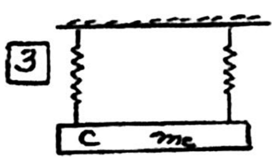
$$\omega_2 = \frac{2\pi}{\tau_2} = \frac{2\pi}{0.7 \text{ s}} = \frac{2\pi}{0.7} \text{ rad/s}$$

$$\omega_2^2 = \frac{k}{m_2}; \quad k = m_2\omega_2^2 = (m_C + 3)\left(\frac{2\pi}{0.7}\right)^2 \qquad (2)$$

Equating the expressions found for k in Eqs. (1) and (2):

$$(m_C + 6)\left(\frac{2\pi}{0.8}\right)^2 = (m_C + 3)\left(\frac{2\pi}{0.7}\right)^2$$

$$\frac{m_C + 6}{m_C + 3} = \left(\frac{0.8}{0.7}\right)^2; \quad \text{Solve for } m_C: \qquad m_C = 6.80 \text{ kg} \blacktriangleleft$$



$$\omega_3 = \frac{2\pi}{\tau_3}$$

$$\omega_3^2 = \frac{k}{m_C}; \quad k = m_C\omega_3^2 = m_C\left(\frac{2\pi}{\tau_3}\right)^2 \qquad (3)$$

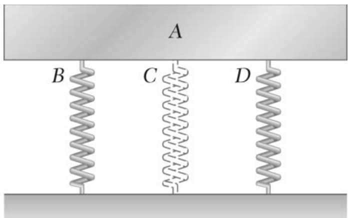
Equating expressions for k from Eqs. (2) and (3),

$$(m_C + 3)\left(\frac{2\pi}{0.7}\right)^2 = m_C\left(\frac{2\pi}{\tau_3}\right)^2$$

Recall $m_C = 6.8$ kg:

$$(6.8 + 3)\left(\frac{2\pi}{0.7}\right)^2 = 6.8\left(\frac{2\pi}{\tau_3}\right)^2$$

$$\left(\frac{\tau_3}{0.7}\right)^2 = \frac{6.8}{9.8}; \quad \frac{\tau_3}{0.7} = 0.833 \qquad \tau_3 = 0.583 \text{ s} \blacktriangleleft$$



PROBLEM 19.25

The 100-lb platform A is attached to springs B and D, each of which has a constant $k = 120$ lb/ft. Knowing that the frequency of vibration of the platform is to remain unchanged when an 80-lb block is placed on it and a third spring C is added between springs B and D, determine the required constant of spring C.

SOLUTION

<u>Frequency of the original system</u>.

Springs B and D are in parallel.

$$k_e = k_B + k_D = 2(120 \text{ lb/ft}) = 240 \text{ lb/ft}$$

$$\omega_n^2 = \frac{k_e}{m_A} = \frac{240 \text{ lb/ft}}{\left(\frac{100 \text{ lb}}{32.2 \text{ ft/s}^2}\right)}$$

$$\omega_n^2 = 77.28\,(\text{rad/s})^2$$

<u>Frequency of new system</u>.

Springs A, B, and C are in parallel.

$$k_e' = k_B + k_D + k_C = (2)(120) + k_e$$

$$(\omega_n')^2 = \frac{k_e'}{m_A + m_B} = \frac{(240 + k_C)(32.2 \text{ ft/s}^2)}{(100 \text{ lb} + 80 \text{ lb})}$$

$$(\omega_n')^2 = (0.1789)(240 + k_C)$$

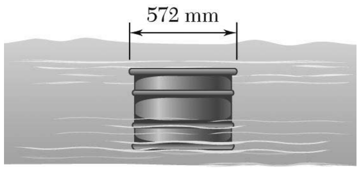
$$\omega_n^2 = (\omega_n')^2$$

$$77.28 = (0.1789)(240 + k_C)$$

$$k_C = 191.97 \text{ lb/ft}$$

$k_C = 192.0$ lb/ft ◀

PROBLEM 19.26



The period of vibration for a barrel floating in salt water is found to be 0.58 s when the barrel is empty and 1.8 s when it is filled with 55 gallons of crude oil. Knowing that the density of the oil is 900 kg/m^3, determine (*a*) the mass of the empty barrel, (*b*) the density of the salt water, ρ_{sw}. [*Hint*: the force of the water on the bottom of the barrel can be modeled as a spring with constant $k = \rho_{sw} g A$.]

SOLUTION

Area of bottom of barrel: $$A = \frac{\pi D^2}{4} = \frac{\pi(0.572 \text{ m})^2}{4} = 0.2570 \text{ m}^2$$

Mass of oil: $$m_{oil} = (55 \text{ gal})\left(\frac{1 \text{ m}^3}{264.172 \text{ gal}}\right)(900 \text{ kg/m}^3) = 187.378 \text{ kg}$$

Barrel empty: $$\tau_1 = 0.58 \text{ s}$$

$$\omega_{n1} = \frac{2\pi}{\tau_1} = \frac{2\pi}{0.58 \text{ s}} = 10.833 \text{ rad/s}$$

$$\omega_{n1} = \sqrt{\frac{k}{m_b}} \qquad (1)$$

Barrel full: $$\tau_2 = 1.8 \text{ s}$$

$$\omega_{n2} = \frac{2\pi}{\tau_2} = \frac{2\pi}{1.8 \text{ s}} = 3.4907 \text{ rad/s}$$

$$\omega_{n2} = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{m_{oil} + m_b}} \qquad (2)$$

(*a*) Mass m_b of empty barrel.

Divide Eq. (1) by Eq. (2) and square both sides.

$$\frac{\omega_{n1}^2}{\omega_{n2}^2} = \frac{(10.833)^2}{(3.4907)^2} = 9.6310 = \frac{m_{oil} + m_b}{m_b}$$

$$9.6310\, m_b = m_{oil} + m_b$$

$$m_b = \frac{m_{oil}}{9.6310 - 1} = \frac{187.378 \text{ kg}}{8.6310} = 21.710 \text{ kg}$$

$$m_b = 21.7 \text{ kg} \blacktriangleleft$$

Spring constant: $$k = m_b \omega_{n1}^2 = (21.710)(10.833)^2 = 2.5477 \times 10^3 \text{ N/m}$$

PROBLEM 19.26 (Continued)

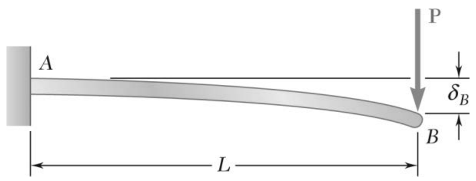
(*b*) Density of the salt water.

$$k = \rho_{sw} gA$$

$$\rho_{sw} = \frac{k}{gA} = \frac{2.5477 \times 10^3 \text{ N/m}}{(9.81 \text{ m/s}^2)(0.2570 \text{ m}^2)}$$

$$\rho_{sw} = 1011 \text{ kg/m}^3 \blacktriangleleft$$

PROBLEM 19.27



From mechanics of materials it is known that for a cantilever beam of constant cross section, a static load **P** applied at end B will cause a deflection $\delta_B = PL^3/3EI$, where L is the length of the beam, E is the modulus of elasticity, and I is the moment of inertia of the cross-sectional area of the beam. Knowing that $L = 10$ ft, $E = 29\times10^6$ lb/in.^{2}, and $I = 12.4$ in.^{4}, determine (*a*) the equivalent spring constant of the beam, (*b*) the frequency of vibration of a 520-lb block attached to end B of the same beam.

SOLUTION

(*a*) Equivalent spring constant.

$$k_e = \frac{P}{\delta_B}$$

$$P = k_e\delta_B$$

$$\delta_B = \frac{PL^3}{3EI}$$

$$P = \left(\frac{3EI}{L^3}\right)\delta_B$$

$$k_e = \frac{3EI}{L^3}$$

$$= \frac{(3)(29\times10^6\text{ lb/in.}^2)(12.4\text{ in.}^4)}{(10\times12\text{ in.})^3}$$

$$k_e = 624.3\text{ lb/in.}$$

$k_e = 624.3$ lb/in. ◀

(*b*) Natural frequency.

$$f_n = \frac{\sqrt{\frac{k_e}{m}}}{2\pi}$$

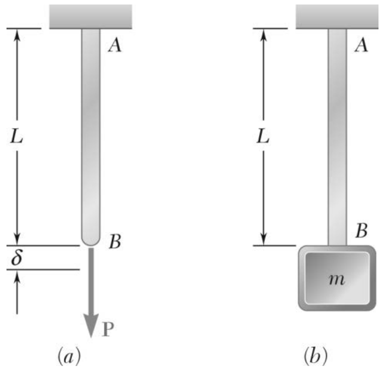
$$k_e = 624.3\text{ lb/in.}$$

$$= 7.492\times10^3\text{ lb/ft}$$

$$f_n = \frac{\sqrt{\frac{(7.492\times10^3\text{ lb/ft})}{\left(\frac{(520\text{ lb})}{(32.2\text{ ft/s}^2)}\right)}}}{2\pi}$$

$$f_n = 3.428\text{ Hz}$$

$f_n = 3.43$ Hz ◀



PROBLEM 19.28

From mechanics of materials it is known that when a static load **P** is applied at the end *B* of a uniform metal rod fixed at end *A*, the length of the rod will increase by an amount $\delta = PL/AE$, where *L* is the length of the undeformed rod. *A* is its cross-sectional area, and *E* is the modulus of elasticity of the metal. Knowing that $L = 450$ mm and $E = 200$ GPa and that the diameter of the rod is 8 mm, and neglecting the mass of the rod, determine (*a*) the equivalent spring constant of the rod, (*b*) the frequency of the vertical vibrations of a block of mass $m = 8$ kg attached to end *B* of the same rod.

SOLUTION

(*a*)

$$P = k_e\delta$$

$$\delta = \frac{PL}{AE}$$

$$P = \left(\frac{AE}{L}\right)\delta$$

$$k_e = \frac{AE}{L}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi(8\times10^{-3}\text{ m})^2}{4}$$

$$A = 5.027\times10^{-5}\text{ m}^2$$

$$L = 0.450\text{ m}$$

$$E = 200\times10^9\text{ N/m}^2$$

$$k_e = \frac{(5.027\times10^{-5}\text{ m}^2)(200\times10^9\text{ N/m}^2)}{(0.450\text{ m})}$$

$$k_e = 22.34\times10^6\text{ N/m}$$

$k_e = 22.3$ MN/m ◀

(*b*)

$$f_n = \frac{\sqrt{\frac{k_e}{m}}}{2\pi}$$

$$= \frac{\sqrt{\frac{22.3\times10^6}{8}}}{2\pi}$$

$$= 265.96\text{ Hz}$$

$f_n = 266$ Hz ◀

PROBLEM 19.29

Denoting by δ_{st} the static deflection of a beam under a given load, show that the frequency of vibration of the load is

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{st}}}$$

Neglect the mass of the beam, and assume that the load remains in contact with the beam.

SOLUTION

$$k = \frac{W}{\delta_{st}}$$

$$m = \frac{W}{g}$$

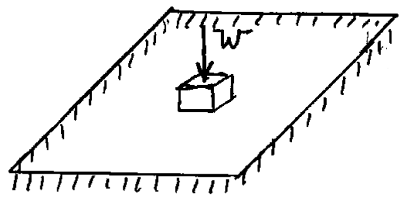
$$\omega_n^2 = \frac{k}{m} = \frac{\frac{W}{\delta_{st}}}{\frac{W}{g}} = \frac{g}{\delta_{st}}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_{st}}} \blacktriangleleft$$

PROBLEM 19.30

A 40-mm deflection of the second floor of a building is measured directly under a newly installed 3500-kg piece of rotating machinery, which has a slightly unbalanced rotor. Assuming that the deflection of the floor is proportional to the load it supports, determine (*a*) the equivalent spring constant of the floor system, (*b*) the speed in rpm of the rotating machinery that should be avoided if it is not to coincide with the natural frequency of the floor-machinery system.

SOLUTION



(*a*) Equivalent spring constant.

$$W = k_e \delta_s$$

$$k_e = \frac{mg}{\delta}$$

$$= \frac{3500(9.81)\text{ N}}{40\text{ mm}}$$

$$k_e = 858\text{ N/mm} \blacktriangleleft$$

(*b*) Natural frequency.

$$f_n = \frac{\sqrt{\frac{k_e}{m}}}{2\pi}$$

$$= \frac{\sqrt{\frac{(858.38\times 1000\text{ N/m})}{(3500\text{ kg})}}}{2\pi}$$

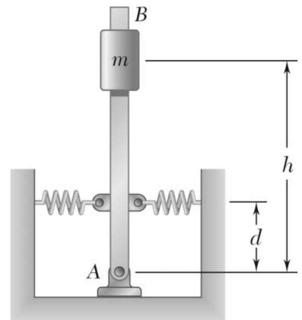
$$f_n = 2.4924\text{ Hz}$$

$$1\text{ Hz} = 1\text{ cycle/s}$$

$$= 60\text{ rpm}$$

$$\text{Speed} = (2.424\text{ Hz})\frac{(60\text{ rpm})}{\text{Hz}}$$

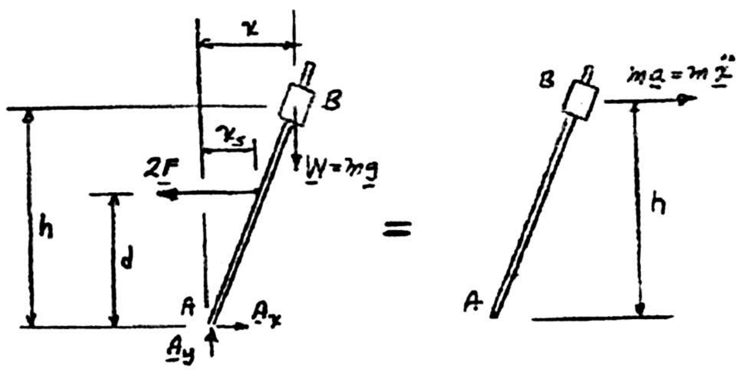
$$\text{Speed} = 149.5\text{ rpm} \blacktriangleleft$$



PROBLEM 19.31

If $h = 700$ mm and $d = 500$ mm and each spring has a constant $k = 600$ N/m, determine the mass m for which the period of small oscillations is (*a*) 0.50 s, (*b*) infinite. Neglect the mass of the rod and assume that each spring can act in either tension or compression.

SOLUTION



$$x_s = x\frac{d}{h}$$

$$2F = 2kx_s = 2k\frac{d}{h}x$$

$$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad 2Fd - mgx = -(m\ddot{x})h$$

$$2k\left(\frac{d}{h}x\right)d - mgx = -m\ddot{x}h$$

$$\ddot{x} + \left[\frac{2kd^2}{mh^2} - \frac{g}{h}\right]x = 0$$

$$\omega_n^2 = \left[\frac{2kd^2}{mh^2} - \frac{g}{h}\right]$$

$$\omega_n^2 = \frac{2k}{m}\left(\frac{d}{h}\right)^2 - \frac{g}{h} \qquad (1)$$

Data:

$$d = 0.5 \text{ m}$$

$$h = 0.7 \text{ m}$$

$$k = 600 \text{ N/m}$$

PROBLEM 19.31 (Continued)

(*a*) For $\tau = 0.5$ s: $\tau = \dfrac{2\pi}{\omega_n}; \quad 0.5 = \dfrac{2\pi}{\omega_n} \qquad \omega_n = 4\pi$

Eq. (1): $$(4\pi)^2 = \frac{2(600)}{m}\left(\frac{0.5}{0.7}\right)^2 - \frac{9.81}{0.7}$$

$m = 3.561$ kg $\qquad m = 3.56$ kg ◀

(*b*) For $\tau =$ infinite: $\tau = \dfrac{2\pi}{\omega_n} \quad \omega_n = 0$

Eq. (1): $$0 = \frac{2(600)}{m}\left(\frac{0.5}{0.7}\right)^2 - \frac{9.81}{0.7}$$

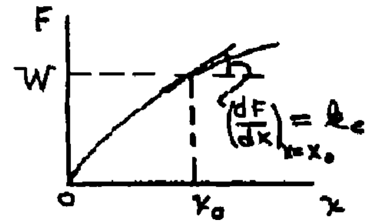
$m = 43.69$ kg $\qquad m = 43.7$ kg ◀

PROBLEM 19.32

The force-deflection equation for a nonlinear spring fixed at one end is $F = 1.5x^{1/2}$ where F is the force, expressed in newtons, applied at the other end, and x is the deflection expressed in meters. (*a*) Determine the deflection x_0 if a 4-oz block is suspended from the spring and is at rest. (*b*) Assuming that the slope of the force-deflection curve at the point corresponding to this loading can be used as an equivalent spring constant, determine the frequency of vibration of the block if it is given a very small downward displacement from its equilibrium position and released.

SOLUTION

(*a*) Deflection x_0.



$$W = 4 \text{ oz} = 0.25 \text{ lb}$$

$$F = W$$

$$= 1.5x_0^{1/2}$$

$$x_0 = \left(\frac{0.25}{1.5}\right)^2$$

$$= 0.027778 \text{ ft}$$

$x_0 = 0.333$ in. ◀

Equivalent spring constant.

At x_0,

$$\left(\frac{dF}{dx}\right)_{x_0} = \frac{1.5}{2}(x_0)^{-1/2} = \frac{1.5}{2}(0.027778)^{-1/2}$$

$$\left(\frac{dF}{dx}\right)_{x_0} = 4.5 \text{ lb/ft}$$

$$k_e = 4.5 \text{ lb/ft}$$

(*b*) Natural frequency.

$$f_n = \frac{\sqrt{\frac{k_e}{m}}}{2\pi}$$

$$= \frac{\sqrt{\frac{(4.5 \text{ lb/ft})}{(0.25/32.2)}}}{2\pi}$$

$$f_n = 3.8316 \text{ Hz}$$

$f_n = 3.83$ Hz ◀

PROBLEM 19.33*

Expanding the integrand in Equation (19.19) of Section 19.4 into a series of even powers of $\sin\phi$ and integrating, show that the period of a simple pendulum of length l may be approximated by the formula

$$\tau = 2\pi\sqrt{\frac{l}{g}}\left(1+\frac{1}{4}\sin^2\frac{\theta_m}{2}\right)$$

where θ_m is the amplitude of the oscillations.

SOLUTION

Using the Binomial Theorem, we write

$$\frac{1}{\sqrt{1-\sin^2\left(\frac{\theta_m}{2}\right)\sin^2\phi}} = \left[1-\sin^2\left(\frac{\theta_m}{2}\right)\sin\phi\right]^{-1/2}$$

$$= 1+\frac{1}{2}\sin^2\frac{\theta_m}{2}\sin^2\phi+\cdots$$

Neglecting terms of order higher than 2 and setting $\sin^2\phi = \frac{1}{2}(1-\cos 2\phi)$, we have

$$\tau_n = 4\sqrt{\frac{l}{g}}\int_0^{\pi/2}\left\{1+\frac{1}{2}\sin^2\frac{\theta_m}{2}\left[\frac{1}{2}(1-\cos 2\phi)\right]\right\}d\phi$$

$$= 4\sqrt{\frac{l}{g}}\int_0^{\pi/2}\left\{1+\frac{1}{4}\sin^2\frac{\theta_m}{2}-\frac{1}{4}\sin^2\frac{\theta_m}{2}\cos 2\phi\right\}d\phi$$

$$= 4\sqrt{\frac{l}{g}}\left[\phi+\frac{1}{4}\left(\sin^2\frac{\theta_m}{2}\right)\phi-\frac{1}{8}\sin^2\frac{\theta_m}{2}\sin 2\phi\right]_0^{\pi/2}$$

$$= 4\sqrt{\frac{l}{g}}\left[\frac{\pi}{2}+\frac{1}{4}\left(\sin^2\frac{\theta_m}{2}\right)\frac{\pi}{2}+0\right]$$

$$\tau_n = 2\pi\sqrt{\frac{l}{g}}\left(1+\frac{1}{4}\sin^2\frac{\theta_m}{2}\right) \blacktriangleleft$$

PROBLEM 19.34*

Using the formula given in Problem 19.33, determine the amplitude θ_m for which the period of a simple pendulum is $\frac{1}{2}$ percent longer than the period of the same pendulum for small oscillations.

SOLUTION

For small oscillations, $$(\tau_n)_0 = 2\pi\sqrt{\frac{l}{g}}$$

We want $$\tau_n = 1.005(\tau_n)_0$$
$$= 1.005\; 2\pi\sqrt{\frac{l}{g}}$$

Using the formula of Problem 19.33, we write

$$\tau_n = (\tau_n)_0\left(1+\frac{1}{4}\sin^2\frac{\theta_m}{2}\right)$$
$$= 1.005(\tau_n)_0$$
$$\sin^2\frac{\theta_m}{2} = 4[1.005-1] = 0.02$$
$$\sin\frac{\theta_m}{2} = \sqrt{0.02}$$
$$\frac{\theta_m}{2} = 8.130°$$

$\theta_m = 16.26°$ ◀

PROBLEM 19.35*

Using the data of Table 19.1, determine the period of a simple pendulum of length $l = 750\text{ mm}$ (*a*) for small oscillations, (*b*) for oscillations of amplitude $\theta_m = 60°$, (*c*) for oscillations of amplitude $\theta_m = 90°$.

SOLUTION

(*a*) $\tau_n = 2\pi\sqrt{\dfrac{l}{g}}$ (Equation 19.18 for small oscillations):

$$\tau_n = 2\pi\sqrt{\frac{0.750\text{ m}}{9.81\text{ m/s}^2}}$$

$$= 1.737\text{ s}$$

$\tau_n = 1.737\text{ s}$ ◀

(*b*) For large oscillations (Eq. 19.20),

$$\tau_n = \left(\frac{2K}{\pi}\right)\left(2\pi\sqrt{\frac{l}{g}}\right)$$

$$= \frac{2K}{\pi}(1.737\text{ s})$$

For $\theta_m = 60°$, $K = 1.686$ (Table 19.1)

$$\tau_n(60°) = \frac{2(1.686)(1.737\text{ s})}{\pi}$$

$$= 1.864\text{ s}$$

$\tau_n = 1.864\text{ s}$ ◀

(*c*) For $\theta_m = 90°$, $K = 1.854$ $\tau_n = \dfrac{2(1.854)(1.737\text{ s})}{\pi} = 2.05\text{ s}$ ◀

PROBLEM 19.36*

Using the data of Table 19.1, determine the length in inches of a simple pendulum which oscillates with a period of 2 s and an amplitude of 90°.

SOLUTION

For large oscillations (Eq. 19.20),

$$\tau_n = \left(\frac{2K}{\pi}\right)\left(2\pi\sqrt{\frac{l}{g}}\right)$$

for $$\theta_m = 90°$$

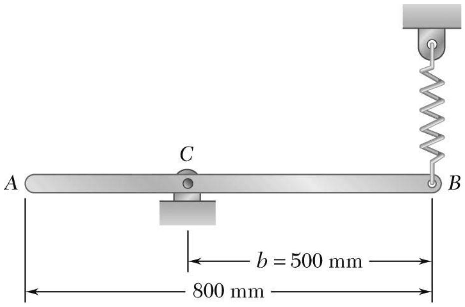
$$K = 1.854 \text{ (Table 19.1)}$$

$$(2\text{ s}) = (2)(1.854)(2)\sqrt{\frac{l}{32.2\text{ ft/s}^2}}$$

$$l = \frac{(2\text{ s})^2(32.2\text{ ft/s}^2)}{[(4)(1.854)]^2}$$

$$= 2.342\text{ ft}$$

$l = 28.1$ in. ◀

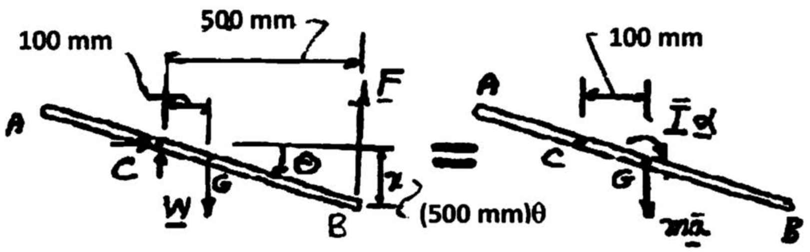


PROBLEM 19.37

The uniform rod shown has mass 6 kg and is attached to a spring of constant $k = 700$ N/m. If end B of the rod is depressed 10 mm and released, determine (a) the period of vibration, (b) the maximum velocity of end B.

SOLUTION

$$k = 700 \text{ N/m}$$
$$W = mg$$



where
$$F = k(x + \delta_{st})$$
$$= k(0.5\theta + \delta_{st})$$
$$m\bar{a} = m\bar{r}\alpha = 6(0.1 \text{ m})\ddot{\theta} = 0.6\ddot{\theta}$$
$$\bar{I}\alpha = \frac{1}{12}(6)(0.8 \text{ m})^2\ddot{\theta}$$
$$= 0.32\ddot{\theta}$$

(a) Equation of motion.

$$+\circlearrowright \Sigma M_C = \bar{I}\alpha + m\bar{a}d: \quad W(0.1 \text{ m}) - F(0.5 \text{ m}) = \bar{I}\alpha + m\bar{a}(0.1 \text{ m})$$
$$W(0.1) - k(0.5\theta + \delta_{st})(0.5 \text{ m}) = 0.32\ddot{\theta} + 0.6\ddot{\theta}(0.1)$$

But in equilibrium, we have $+\circlearrowright W(0.1 \text{ m}) - k\delta_{st}(0.5 \text{ m}) = 0$

Thus, $-k(0.5)^2\theta = [0.32 + 0.06]\ddot{\theta}$
$$-(700 \text{ N/m})(0.5)^2\theta = 0.38\ddot{\theta}$$
$$\ddot{\theta} + (460.53)\theta = 0$$

PROBLEM 19.37 (Continued)

Natural frequency and period.

$$\omega_n^2 = 460.53$$

$$\omega_n = 21.46 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{21.46 \text{ rad/s}}$$

$\tau = 0.293 \text{ s}$ ◀

(*b*) At end *B*.

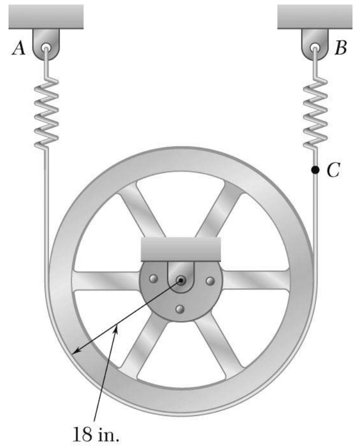
$$x_m = 0.010 \text{ m}$$

$$v_m = x_m \omega_n$$

$$= (10 \text{ mm})(21.46 \text{ rad/s})$$

$$= 214.6 \text{ mm/s}$$

$v_m = 0.215 \text{ m/s}$ ◀

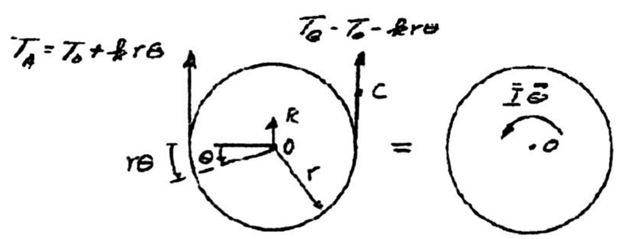


PROBLEM 19.38

A belt is placed around the rim of a 500-lb flywheel and attached as shown to two springs, each of constant $k = 85$ lb/in. If end C of the belt is pulled 1.5 in. down and released, the period of vibration of the flywheel is observed to be 0.5 s. Knowing that the initial tension in the belt is sufficient to prevent slipping, determine (*a*) the maximum angular velocity of the flywheel, (*b*) the centroidal radius of gyration of the flywheel.

SOLUTION

Denote the initial tension by T_0.



Equation of motion. $+\circlearrowleft \Sigma M_0 = \bar{I}\ddot{\theta}: \quad -T_A r + T_B r = \bar{I}\ddot{\theta}$

$$-(T_0 + kr\theta)r + (T_0 - kr\theta)\theta = \bar{I}\ddot{\theta}$$

$$\ddot{\theta} + \frac{2kr^2}{\bar{I}}\theta = 0$$

$$\omega_n^2 = \frac{2kr^2}{\bar{I}} \qquad (1)$$

Data: $m = \frac{W}{g} = \frac{500 \text{ lb}}{32.2} \qquad k = 85 \text{ lb/in.} = 1020 \text{ lb/ft}$

$$\tau = 0.5 \text{ s} \qquad r = 18 \text{ in.} = 1.5 \text{ ft}$$

$$\tau = \frac{2\pi}{\omega_n}; \qquad \omega_n = \frac{2\pi}{\tau} = \frac{2\pi}{0.5} = 4\pi \text{ rad/s}$$

(*a*) Maximum angular velocity. If Point C is pulled down 1.5 in. and released,

$$\theta_m = \theta_{\max} = \left(\frac{1.5 \text{ in.}}{18 \text{ in.}}\right) = 83.333\times 10^{-3} \text{ rad}$$

$$\dot{\theta}_m = \theta_m\omega_n = (83.333\times 10^{-3} \text{ rad})(4\pi \text{ rad/s}) \qquad \dot{\theta}_m = 1.047 \text{ rad/s} \blacktriangleleft$$

PROBLEM 19.38 (Continued)

(*b*) Centroidal radius of gyration.

$$\omega_n^2 = \frac{2kr^2}{I}$$

$$(4\pi \text{ rad/s})^2 = \frac{2(1020 \text{ lb/ft})(1.5 \text{ ft})^2}{\overline{I}}$$

$$\overline{I} = 29.067 \text{ slug} \cdot \text{ft}^2$$

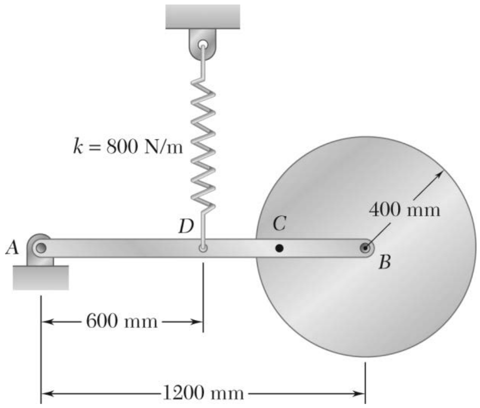
or since

$$\overline{I} = m\overline{k}^2$$

$$\left(\frac{500 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\overline{k}^2 = 29.067 \text{ slug} \cdot \text{ft}^2$$

$$\overline{k} = 1.3682 \text{ ft}$$

$\overline{k} = 16.42$ in. ◀

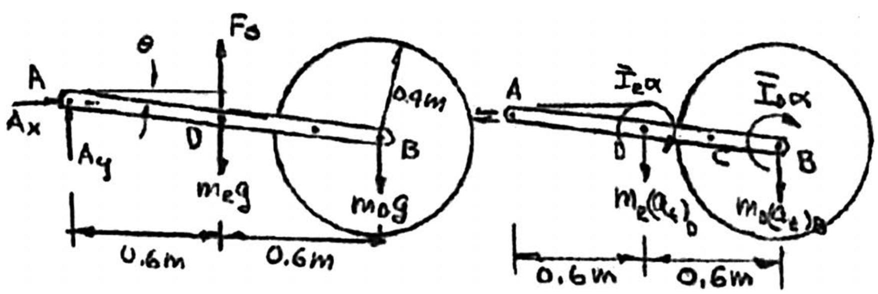


PROBLEM 19.39

An 8-kg uniform rod AB is hinged to a fixed support at A and is attached by means of pins B and C to a 12-kg disk of radius 400 mm. A spring attached at D holds the rod at rest in the position shown. If Point B is moved down 25 mm and released, determine (*a*) the period of vibration, (*b*) the maximum velocity of Point B.

SOLUTION

(*a*)



Equation of motion. $\Sigma M_A = (\Sigma M_A)_{\text{eff}}: \quad F_S = k(0.6\theta + \delta_{\text{st}})$

$$+\circlearrowleft\ 0.6(m_R g - F_S) + 1.2 m_D g = (\bar{I}_R + \bar{I}_D)\alpha + 0.6(m_R)(a_t)_D + 1.2(m_D)(a_t)_B \quad (1)$$

At equilibrium $(\theta = 0)$, $\quad F_S = k\delta_{\text{st}}$

$$\Sigma M_A = 0 = 0.6(m_R g - k(\delta_{\text{st}})) + 1.2 m_D g \quad (2)$$

Substituting Eq. (2) into Eq. (1), $\quad (\bar{I}_R + \bar{I}_D)\alpha + 0.6\, m_R (a_t)_D + 1.2 m_D (a_t)_B + (0.6)^2 k\theta = 0$

$$\alpha = \ddot{\theta}$$

$$(a_t)_B = 0.6\ddot{\theta}$$

$$(a_t)_D = 1.2\ddot{\theta}$$

$$\bar{I}_R = \frac{1}{12} m_R l^2 = \frac{1}{12}(8)(1.2)^2$$

$$= 0.960 \text{ kg} \cdot \text{m}$$

$$\bar{I}_D = \frac{1}{2} m_D R^2 = \frac{1}{2}(12)(0.4)^2 = 0.960 \text{ kg} \cdot \text{m}$$

$$[0.960 + 0.960 + (0.6)^2(8) + (1.2)^2(12)]\ddot{\theta} + (0.6)^2(800)\theta = 0$$

$$\ddot{\theta} + \frac{288 \text{ N} \cdot \text{m}}{(22.08 \text{ kg} \cdot \text{m}^2)}\theta = 0$$

PROBLEM 19.39 (Continued)

(*a*) Natural frequency and period.

$$\omega_n = \sqrt{\frac{288}{22.08}}$$

$$= 3.6116 \text{ rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.6116} \qquad \tau_n = 1.740 \text{ s} \blacktriangleleft$$

(*b*) Maximum velocity at *B*.

$$(v_B)_{\max} = (1.2)(\dot{\theta}_{\max})$$

$$\theta_m = \frac{y_B}{1.2}$$

$$\theta_m = \frac{0.025}{1.2} = 0.02083 \text{ rad}$$

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

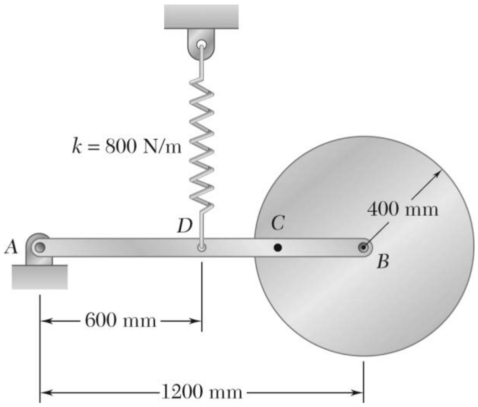
$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

$$\dot{\theta}_{\max} = \theta_m \omega_n = (0.02083)(3.612) = 0.07524 \text{ rad/s}$$

$$(v_B)_{\max} = (1.2)(\dot{\theta}_{\max}) = (1.2 \text{ m})(0.07524) \text{ rad/s}$$

$$(v_B)_{\max} = 0.09029 \text{ m/s} \qquad (v_B)_{\max} = 90.3 \text{ mm/s} \blacktriangleleft$$

PROBLEM 19.40

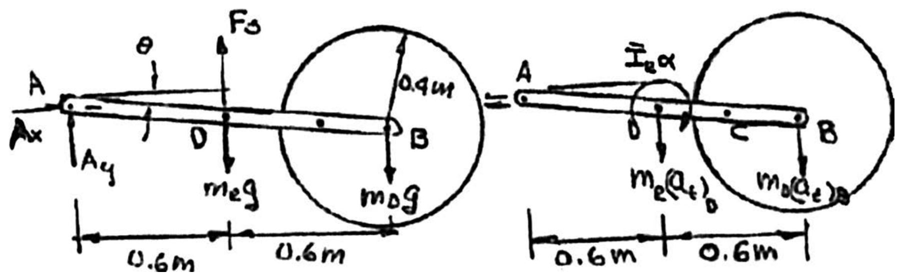


Solve Problem 19.39, assuming that pin C is removed and that the disk can rotate freely about pin B.

PROBLEM 19.39 An 8-kg uniform rod AB is hinged to a fixed support at A and is attached by means of pins B and C to a 12-kg disk of radius 400 mm. A spring attached at D holds the rod at rest in the position shown. If Point B is moved down 25 mm and released, determine (a) the period of vibration, (b) the maximum velocity of Point B.

SOLUTION

(a)



Note: This problem is the same as Problem 19.39, *except* that the disk does not rotate, so that the effective moment $I_D\alpha = 0$.

Equation of motion. $\Sigma M_A = (\Sigma M_A)_{\text{eff}}$: $F_S = k(0.60 + \delta_{\text{st}})$

$$\curvearrowright + \; (0.6)(m_R g - F_S) + 1.2 m_D g = \bar{I}_R\alpha + (0.6)(m_R)(a_t)_D + 1.2(m_D)(a_t)_B \quad (1)$$

At equilibrium $(\theta = 0)$, $F_S = k\delta_{\text{st}}$

$$\curvearrowright + \; \Sigma M_A = 0 = 0.6(m_R g - \delta_{\text{st}}) + 1.2\, m_D g \quad (2)$$

Substituting Eq. (2) into Eq. (1), $I_R\alpha + 0.6\, m_R(a_t)_D + 1.2 m_D (a_t)_B + (0.6)^2 k\theta = 0$

$$\alpha = \ddot{\theta}$$

$$(a_t)_B = 0.6\ddot{\theta}$$

$$(a_t)_D = 1.2\ddot{\theta}$$

$$I_R = \frac{1}{12} m_R l^2 = \frac{1}{12}(8)(1.2)^2$$

$$= 0.960 \text{ kg}\cdot\text{m}$$

$$[0.960 + (0.6)^2(8) + (1.2)^2(12)]\ddot{\theta} + (0.6)^2(800)\theta = 0$$

$$\ddot{\theta} + \frac{(288 \text{ N}\cdot\text{m})}{21.12 \text{ kg}\cdot\text{m}^2}\theta = 0$$

PROBLEM 19.40 (Continued)

(*a*) Natural frequency and period.

$$\omega_n = \sqrt{\frac{288}{21.12}}$$

$$= 3.693 \text{ rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.693} \qquad \tau_n = 1.701 \text{ s} \blacktriangleleft$$

(*b*) Maximum velocity at *B*.

$$(v_B)_{\max} = (1.2)(\dot{\theta})_{\max}$$

$$\theta_m = \frac{y_B}{1.2} = \frac{0.025}{1.20} = 0.02083 \text{ rad}$$

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

$$\ddot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

$$\ddot{\theta}_{\max} = \theta_m \omega_n$$

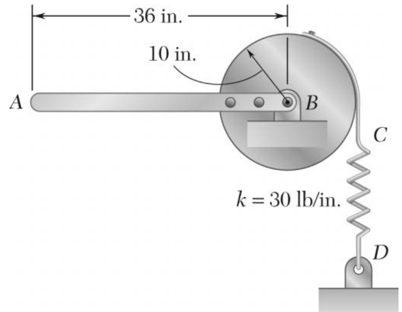
$$= (0.02083)(3.693)$$

$$= 0.07694 \text{ rad/s}$$

$$(v_B)_{\max} = (1.2)(\dot{\theta}_{\max})$$

$$= (1.2)(0.07694)$$

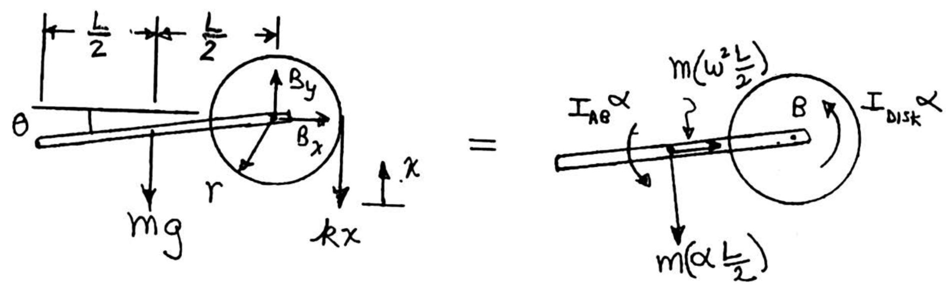
$$= 0.09233 \text{ m/s} \qquad (v_B)_{\max} = 92.3 \text{ mm/s} \blacktriangleleft$$



PROBLEM 19.41

A 15-lb slender rod *AB* is riveted to a 12-lb uniform disk as shown. A belt is attached to the rim of the disk and to a spring which holds the rod at rest in the position shown. If end *A* of the rod is moved 0.75 in. down and released, determine (*a*) the period of vibration, (*b*) the maximum velocity of end *A*.

SOLUTION



Equation of motion. $+\circlearrowright \Sigma M_B = \Sigma (M_B)_{\text{eff}}:\quad mg\dfrac{L}{2}\cos\theta - kxr = I_{AB}\alpha + m\left(\alpha\dfrac{L}{2}\right)\left(\dfrac{L}{2}\right) + I_{\text{disk}}\alpha$ (1)

where $x = r\theta + \delta_{\text{st}}$ and from statics, $mg\dfrac{L}{2} = k\delta_{\text{st}} r$

Assuming small angles $(\cos\theta \approx 1)$, Equation (1) becomes

$$\cancel{mg\frac{L}{2}}\theta - kr^2\theta - \cancel{kr\delta_{\text{st}}} = \left(I_{AB} + m\left(\frac{L}{2}\right)^2 + I_{\text{disk}}\right)\alpha$$

$$\left(I_{AB} + \frac{mL^2}{4} + I_{\text{disk}}\right)\ddot{\theta} + kr^2\theta = 0$$

Data:

$$m = \frac{15}{32.2} = 0.46584 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_{\text{disk}} = \frac{12}{32.2} = 0.37267 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$L = 36 \text{ in.} = 3.0 \text{ ft}$$

$$r = 10 \text{ in.} = 0.83333 \text{ ft}$$

$$k = 30 \text{ lb/in.} = 360 \text{ lb/ft}$$

$$I_{AB} = \frac{1}{12}mL^2$$
$$= \frac{1}{12}(0.46584)(3.0)^2$$
$$= 0.34938 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$
$$I_{\text{disk}} = \frac{1}{2}m_{\text{disk}}r^2$$
$$= \frac{1}{2}(0.37267)(0.83333)^2$$
$$= 0.1294 \text{ lb} \cdot \text{s}^2 \cdot \text{ft}$$

$$\left[0.34935 + \frac{1}{4}(0.46584)(3.0)^2 + 0.1294\right]\ddot{\theta} + (360)(0.83333)^2\theta = 0$$

$$1.5269\ddot{\theta} + 250\theta = 0 \quad \text{or} \quad \ddot{\theta} + 163.73\theta = 0$$

(*a*) Natural frequency and period. $\omega_n^2 = 163.73 \text{ (rad/s)}^2$

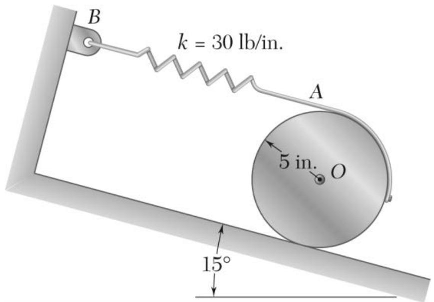
$$\omega_n = 12.796 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{12.796}$$

$\tau = 0.491 \text{ s}$ ◀

(*b*) Maximum velocity. $v_m = \omega_n x_m = (12.796)(0.75)$

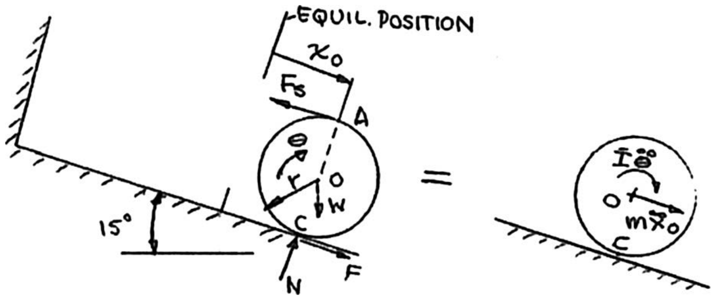
$v_m = 9.60 \text{ in./s}$ ◀



PROBLEM 19.42

A 30-lb uniform cylinder can roll without sliding on a 15°-incline. A belt is attached to the rim of the cylinder, and a spring holds the cylinder at rest in the position shown. If the center of the cylinder is moved 2 in. down the incline and released, determine (*a*) the period of vibration, (*b*) the maximum acceleration of the center of the cylinder.

SOLUTION



Spring deflection.

$$x_A = x_0 + x_{A/0}$$

$$x_{A/0} = r\theta$$

$$\theta = \frac{x_0}{r}$$

$$x_A = 2x_0$$

$$F_S = k(x_A + \delta_{\text{st}}) = k(2x_0 + \delta_{\text{st}})$$

$$\curvearrowright + \quad \Sigma M_C = (\Sigma M)_{\text{eff}}: \quad -2rk(2x_0 + \delta_{\text{st}}) + rW\sin 15° = rm\ddot{x}_0 + \bar{I}\ddot{\theta} \qquad (1)$$

But in equilibrium, $x_0 = 0$

$$\Sigma M_C = 0 = -2rk\delta_{\text{st}} + rW\sin 15° \qquad (2)$$

Substituting Eq. (2) into Eq. (1) and noting that $\theta = \frac{x_0}{r}, \quad \ddot{\theta} = \frac{\ddot{x}_0}{r}$

$$rm\ddot{x}_0 + \bar{I}\frac{\ddot{x}_0}{r} + 4rkx_0 = 0$$

$$\bar{I} = \frac{1}{2}mr^2$$

$$\frac{3}{2}mr\ddot{x}_0 + 4rkx_0 = 0$$

$$\ddot{x}_0 + \left(\frac{8}{3}\frac{k}{m}\right)x_0 = 0$$

PROBLEM 19.42 (Continued)

Natural frequency. $\omega_n = \sqrt{\frac{8}{3}\frac{k}{m}} = \sqrt{\frac{(8)(30\times 12 \text{ lb/ft})}{(3)\frac{(30 \text{ lb})}{(32.2 \text{ ft/s}^2)}}} = 32.1 \text{ s}^{-1}$

(*a*) Period. $\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{32.1} = 0.1957 \text{ s}$ $\tau_n = 0.1957 \text{ s}$ ◀

(*b*) $x_0 = (x_0)_m \sin(\omega_n t + \phi)$

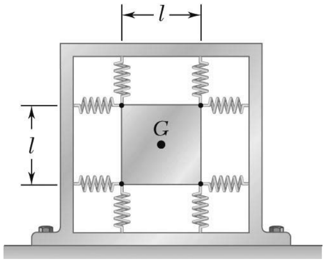
At $t = 0$, $x_0 = \frac{2}{12} \text{ ft} \qquad \dot{x}_0 = 0$

$$\dot{x}_0 = (x_0)_m \omega_n \cos(\omega_n t + \phi)$$
$$t = 0$$
$$0 = (x_0)_m \omega_n \cos\phi$$

Thus, $\phi = \frac{\pi}{2}$

$$t = 0$$
$$x_0(0) = \frac{1}{6} \text{ ft} = (x_0)_m \sin\phi = (x_0)_m (1)$$
$$(x_0)_m = \frac{1}{6} \text{ ft}$$
$$\ddot{x}_0 = -(x_0)_m \omega_n^2 \sin(\omega_n t + \phi)$$
$$(a_0)_{\max} = (\ddot{x}_0)_{\max}$$
$$= -(x_0)_m \omega_n^2$$
$$= -\left(\frac{1}{6} \text{ ft}\right)(32.1 \text{ s}^{-1})^2$$
$$= 171.7 \text{ ft/s}^2$$

$(a_0)_{\max} = 171.7 \text{ ft/s}^2$ ◀



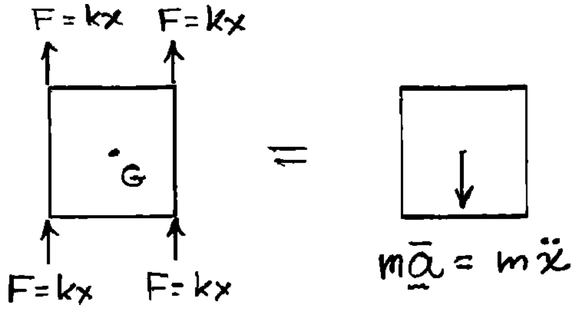
PROBLEM 19.43

A square plate of mass m is held by eight springs, each of constant k. Knowing that each spring can act in either tension or compression, determine the frequency of the resulting vibration (a) if the plate is given a small vertical displacement and released, (b) if the plate is rotated through a small angle about G and released.

SOLUTION

(a) Small vertical displacement.

Let the plate be displaced downward a distance x from the equilibrium position. Each corner moves downward a distance x and the four vertical springs exert additional forces kx for each spring. The horizontal springs exert negligible change.



$$+\downarrow \Sigma F = ma: \quad -4kx = m\ddot{x}$$

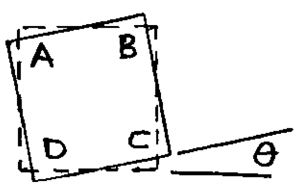
$$\ddot{x} + \frac{4k}{m}x = 0$$

$$\omega_n^2 = \frac{4k}{m}$$

Frequency: $\quad f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{4k}{m}}$ $\qquad f = 0.318\sqrt{\frac{k}{m}}$ ◀

(b) Small rotation about G.

Let the plate be rotated through a small counterclockwise angle θ from the equilibrium position. The corners A, B, C, and D move as indicated below:



A: $\quad (l/2)\theta\downarrow + (l/2)\theta \leftarrow$

B: $\quad (l/2)\theta \leftarrow + (l/2)\theta\uparrow$

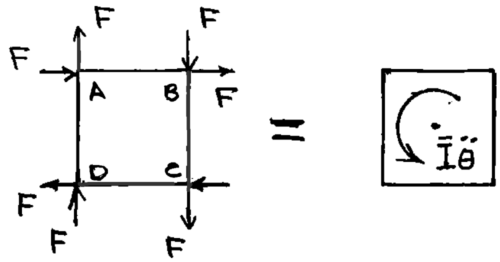
C: $\quad (l/2)\theta\uparrow + (l/2)\theta \leftarrow$

D: $\quad (l/2)\theta \rightarrow + (l/2)\theta\downarrow$

The additional force exerted by each of the eight springs is $F = (kl/2)\theta$ and directed as shown on the free body diagram. The eight forces reduce to four clockwise couples, each of magnitude Fl. For a square plate

$$\bar{I} = \frac{1}{6}ml^2$$

PROBLEM 19.43 (Continued)



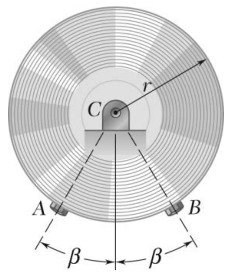
$$+\circlearrowleft M_G = \bar{I}\alpha: \quad 4Fl = \bar{I}\ddot{\theta} - 4(kl^2/2)\theta$$

$$= \frac{1}{6} ml^2 \ddot{\theta}$$

$$\ddot{\theta} + \frac{12k}{\text{m}}\theta = 0$$

$$\omega_n^2 = \frac{12k}{\text{m}}$$

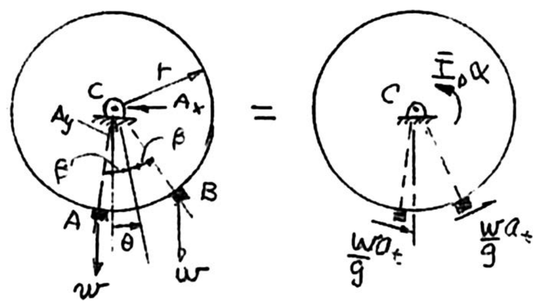
Frequency: $f = \dfrac{\omega_n}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{12k}{m}}$ $\qquad f = 0.551\sqrt{\dfrac{k}{m}}$ ◀



PROBLEM 19.44

Two small weights w are attached at A and B to the rim of a uniform disk of radius r and weight W. Denoting by τ_0 the period of small oscillations when $\beta = 0,$ determine the angle β for which the period of small oscillations is $2\tau_0$.

SOLUTION



$$\alpha = \ddot{\theta}$$

$$a_t = r\alpha = r\ddot{\theta}$$

$$\bar{I}_D = \frac{1}{2}\frac{W}{g}r^2$$

Equation of motion.

$$\Sigma M_C = (\Sigma M_C)_{\text{eff}}: \quad wr\sin(\beta-\theta) - wr\sin(\beta+\theta) = \frac{2w}{g}ra_t + \bar{I}\alpha$$

$$wr[\sin(\beta-\theta) - \sin(\beta+\theta)] = -2wr\sin\theta\cos\beta$$

$$\sin\theta \approx \theta$$

$$\left(\frac{2w}{g}r^2 + \frac{W}{2g}r^2\right)\ddot{\theta} + (2wr\cos\beta)\theta = 0$$

Natural frequency.

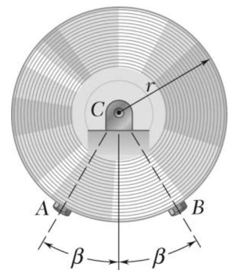
$$\omega_n = \sqrt{\frac{2wg\cos\beta}{\left(2w+\frac{W}{2}\right)r}} = \sqrt{\frac{4g\cos\beta}{(4+\frac{W}{w})r}} \qquad (1)$$

$$\beta = 0 \qquad \tau_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{\frac{4g}{(4+\frac{W}{w})r}}}$$

$$\tau_n = \frac{2\pi}{\sqrt{\frac{\cos\beta}{(4+\frac{W}{w})r}}} = 2\tau_0 = \frac{4\pi}{\sqrt{\frac{4g}{(4+\frac{W}{w})r}}}$$

$$\cos\beta = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

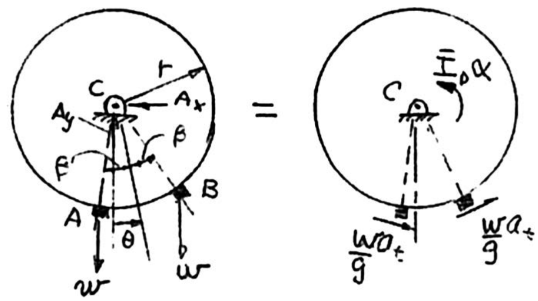
$\beta = 75.5°$ ◀



PROBLEM 19.45

Two 40-g weights are attached at *A* and *B* to the rim of a 1.5-kg uniform disk of radius $r = 100$ mm. Determine the frequency of small oscillations when $\beta = 60°$.

SOLUTION



$$\alpha = \ddot{\theta}$$

$$a_t = r\alpha = r\ddot{\theta}$$

$$\bar{I}_D = \frac{1}{2}m_D r^2$$

Equation of motion.

$$\Sigma M_C = \bar{I}\alpha + m\bar{a}d: \quad wr\sin(\beta-\theta) - wr\sin(\beta+\theta) = \frac{2w}{g}ra_t + \bar{I}\alpha$$

$$wr[\sin(\beta-\theta) - \sin(\beta+\theta)] = -2wr\sin\theta\cos\beta$$

$$\sin\theta \approx \theta$$

$$\left(\frac{2w}{g}r^2 + \frac{W}{2g}r^2\right)\ddot{\theta} + (2wr\cos\beta)\theta = 0$$

Natural frequency.

$$\omega_n = \sqrt{\frac{2wg\cos\beta}{\left(2w+\frac{W}{2}\right)r}} = \sqrt{\frac{4g\cos\beta}{(4+\frac{W}{w})r}} \qquad (1)$$

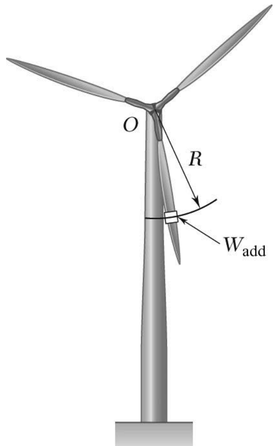
Data:

$$w = mg = 0.04g \qquad W = m_D g = 1.5g \qquad \frac{W}{w} = \frac{1.5g}{0.04g} = 37.5$$

$$r = 0.100 \text{ m} \qquad \beta = 60°$$

$$\omega_n = \sqrt{\frac{(4)(9.81)\cos 60°}{(4+37.5)(0.10)}} = 2.1743 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{2.1743}{2\pi} \qquad f_n = 0.346 \text{ Hz} \blacktriangleleft$$

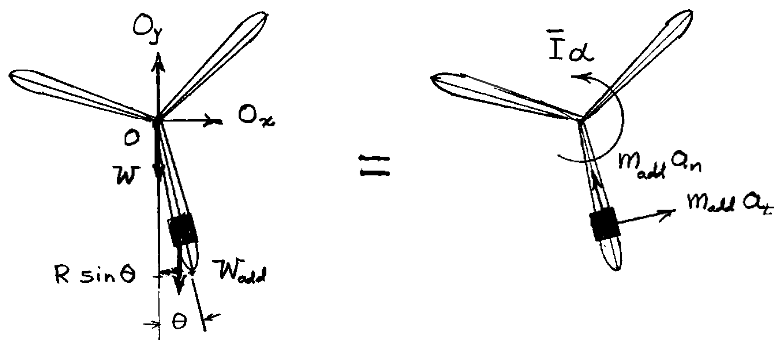


PROBLEM 19.46

A three-bladed wind turbine used for research is supported on a shaft so that it is free to rotate about O. One technique to determine the centroidal mass moment of inertia of an object is to place a known weight at a known distance from the axis of rotation and to measure the frequency of oscillations after releasing it from rest with a small initial angle. In this case, a weight of $W_{add} = 50$ lb is attached to one of the blades at a distance $R = 20$ ft from the axis of rotation. Knowing that when the blade with the added weight is displaced slightly from the vertical axis, the system is found to have a period of 7.6 s, determine the centroidal mass moment of inertia of the 3-bladed rotor.

SOLUTION

Let the turbine rotor be turned counterclockwise through a small angle θ. The moment of the added weight about Point O is



$$M = -W_{add} R \sin\theta$$

$$+\circlearrowleft \Sigma M_0 = \Sigma(M_0)_{eff}: \quad -W_{add} R \sin\theta = \bar{I}\alpha + m_{add} R a_a$$

$$= (\bar{I} + m_{add} R^2)\alpha$$

$$= (\bar{I} + m_{add} R^2)\ddot{\theta}$$

$$\ddot{\theta} + \frac{W_{add} R}{\bar{I} + m_{add} R^2} \sin\theta = 0$$

Using $\sin\theta \approx \theta$ gives

$$\ddot{\theta} + \frac{W_{add} R}{\bar{I} + m_{add} R^2} \theta = 0$$

$$\ddot{\theta} + \omega_n^2 \theta = 0 \qquad \omega_n^2 = \frac{W_{add} R}{\bar{I} + m_{add} R^2}$$

PROBLEM 19.46 (Continued)

Solving for $\bar{I}$,

$$\bar{I} = \frac{W_{\text{add}} R}{\omega_n^2} - m_{\text{add}} R^2 \qquad (1)$$

Data: $R = 20 \text{ ft}, \qquad W_{\text{add}} = 50 \text{ lb}$

$$m_{\text{add}} = \frac{W_{\text{add}}}{g} = \frac{50 \text{ lb}}{32.2 \text{ ft/s}^2} = 1.5528 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Period and frequency: $\tau = 7.6 \text{ s}$

$$f = \frac{1}{\tau} = \frac{1}{7.6} \text{Hz}$$

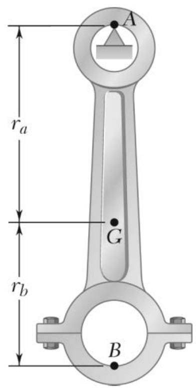
$$\omega_n = 2\pi f = \frac{2\pi}{7.6} = 0.82673 \text{ rad/s}$$

From Eq. (1),

$$\bar{I} = \frac{(50 \text{ lb})(20 \text{ ft})}{(0.82673 \text{ rad/s})^2} - (1.5528 \text{ lb} \cdot \text{s}^2/\text{ft})(20 \text{ ft})^2$$

$$= 1463.10 - 621.12$$

$$\bar{I} = 842 \text{ lb} \cdot \text{s}^2 \cdot \text{ft} \blacktriangleleft$$

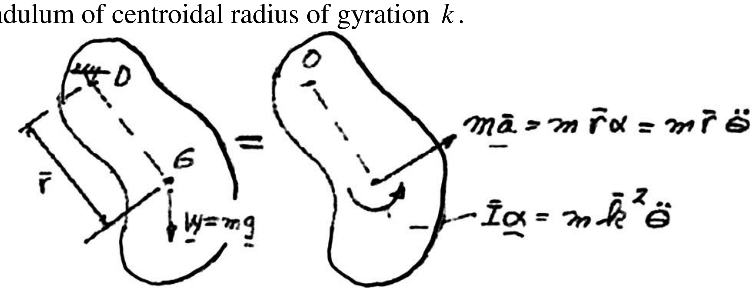


PROBLEM 19.47

A connecting rod is supported by a knife-edge at Point *A*; the period of its small oscillations is observed to be 0.87 s. The rod is then inverted and supported by a knife-edge at Point *B* and the period of small oscillations is observed to be 0.78 s. Knowing that $r_a + r_b = 10$ in. determine (*a*) the location of the mass center *G*, (*b*) the centroidal radius of gyration $\bar{k}$.

SOLUTION

Consider general pendulum of centroidal radius of gyration $\bar{k}$.



<u>Equation of motion</u>. $\quad +\circlearrowleft \Sigma M_0 = \Sigma(M_0)_{\text{eff}}: \quad -mg\bar{r}\sin\theta = (m\bar{r}\ddot{\theta})\bar{r} + m\bar{k}^2\ddot{\theta}$

$$\ddot{\theta} + \left[\frac{g\bar{r}}{\bar{r}^2 + \bar{k}^2}\right]\sin\theta = 0$$

For small oscillations, $\sin\theta \approx \theta$, we have

$$\ddot{\theta} + \left[\frac{g\bar{r}}{\bar{r}^2 + \bar{k}^2}\right]\theta = 0$$

$$\omega_n^2 = \frac{g\bar{r}}{\bar{r}^2 + \bar{k}^2}$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{\bar{r}^2 + \bar{k}^2}{g\bar{r}}}$$

For rod suspended at *A*,

$$\tau_A = 2\pi\sqrt{\frac{r_a^2 + \bar{k}^2}{g r_a}}$$

$$g\tau_A^2 r_a = 4\pi^2\left(r_a^2 + \bar{k}^2\right) \tag{1}$$

For rod suspended at *B*,

$$\tau_B = 2\pi\sqrt{\frac{r_b^2 + \bar{k}^2}{g r_b}}$$

$$g\tau_B^2 r_b = 4\pi^2\left(r_b^2 + \bar{k}^2\right) \tag{2}$$

PROBLEM 19.47 (Continued)

(*a*) Value of r_a.

Subtracting Eq. (2) from Eq. (1), $$g\tau_A^2 r_a - g\tau_B^2 r_b = 4\pi^2\left(r_a^2 - r_b^2\right)$$

$$g\tau_A^2 r_a - g\tau_B^2 r_b = 4\pi^2(r_a + r_b)(r_a - r_b)$$

Applying the numerical data with $r_a + r_b = 10\text{ in.} = 0.83333\text{ ft}$

$$(32.2)(0.87)^2 r_a - (32.2)(0.78)^2 r_b = 4\pi^2(0.83333)(r_a - r_b)$$

$$24.372 r_a - 19.590 r_b = 32.899(r_a - r_b)$$

$$13.309 r_b = 8.527 r_a \qquad r_b = 0.6407 r_a$$

$$0.83333 = r_a + 0.6407 r_a \qquad r_a = 0.5079\text{ ft}$$

$r_a = 6.09\text{ in.}$ ◀

$$r_b = 0.83333 - 0.5079 \qquad r_b = 0.32543\text{ ft}$$

$r_b = 3.91\text{ in.}$ ◀

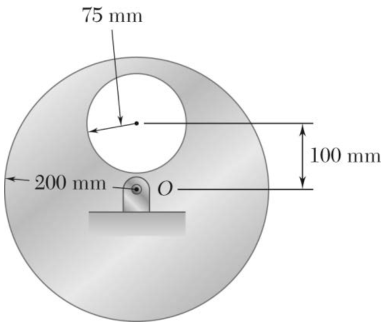
(*b*) Centroidal radius of gyration.

From Eq. (1), $$4\pi^2 \bar{k}^2 = g\,\tau_A^2\, r_a - 4\pi^2 r_a^2$$

$$= (32.2)(0.87)^2(0.5079) - 4\pi^2(0.5079)^2 = 2.1947\text{ ft}^2$$

$$\bar{k} = 0.2398\text{ ft}$$

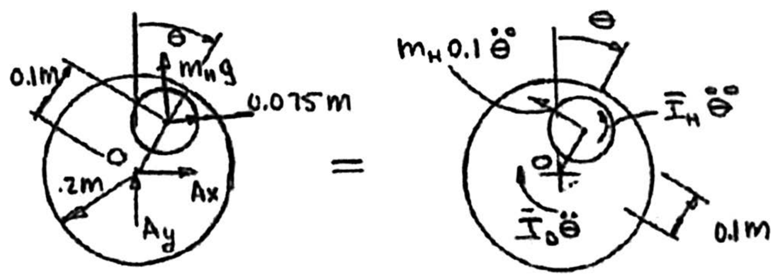
$\bar{k} = 2.83\text{ in.}$ ◀



PROBLEM 19.48

A 75-mm-radius hole is cut in a 200-mm-radius uniform disk, which is attached to a frictionless pin at its geometric center O. Determine (a) the period of small oscillations of the disk, (b) the length of a simple pendulum which has the same period.

SOLUTION



<u>Equation of motion</u>.

$$\Sigma M_0 = (\Sigma M_0)_{\text{eff}}: \quad \circlearrowleft + \; -m_H g(0.1)\sin\theta = \bar{I}_D \theta - I_H \ddot{\theta} - (0.1)^2 m_H \ddot{\theta}$$

$$\begin{aligned} m_D &= \rho t \pi R^2 \\ &= (\rho t \pi)(0.2)^2 \\ &= (0.04)\pi\rho t \\ m_H &= \rho t \pi r^2 \\ &= (\rho t \pi)(0.075)^2 \\ &= (0.005625)\pi\rho t \\ I_D &= \frac{1}{2} m_D R^2 = \frac{1}{2}(0.04\pi\rho t)(.2)^2 \\ &= 800 \times 10^{-6} \pi\rho t \\ I_H &= \frac{1}{2} m_H r^2 \\ &= \frac{1}{2}(0.005625\pi\rho t)(0.75)^2 \\ &= 15.82 \times 10^{-6} \pi\rho t \end{aligned}$$

PROBLEM 19.48 (Continued)

Small angles. $\sin\theta \approx \theta$

$$[800\times10^{-6}\pi\rho t - 15.82\times10^{-6}\pi\rho t - (0.1)(0.005625)\pi\rho t]\ddot{\theta}$$
$$+ (0.005625\ \pi\rho t)\ (9.81)(0.1)\theta = 0$$

$$727.9\times10^{-6}\ddot{\theta} + 5.518\times10^{-3}\theta = 0$$

(*a*) Natural frequency and period.

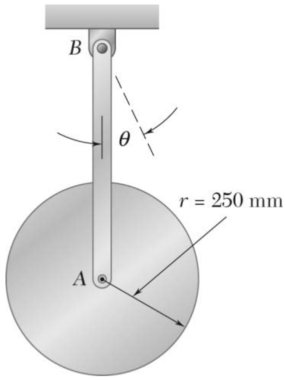
$$\omega_n^2 = \frac{5.518\times10^{-3}}{727.9\times10^{-6}}$$
$$= 7.581$$
$$\omega_n = 2.753 \text{ rad/s}$$
$$\tau_n = \frac{2\pi}{\omega_n}$$

$$\tau_n = \frac{2\pi}{2.753} = 2.28 \text{ s} \blacktriangleleft$$

(*b*) Length and period of a simple pendulum.

$$\tau_n = 2\pi\sqrt{\frac{l}{g}}$$
$$l = \left(\frac{\tau_n}{2\pi}\right)^2 g$$
$$l = \left[\frac{(2.753)}{2\pi}\right]^2 (9.81 \text{ m/s}^2)$$

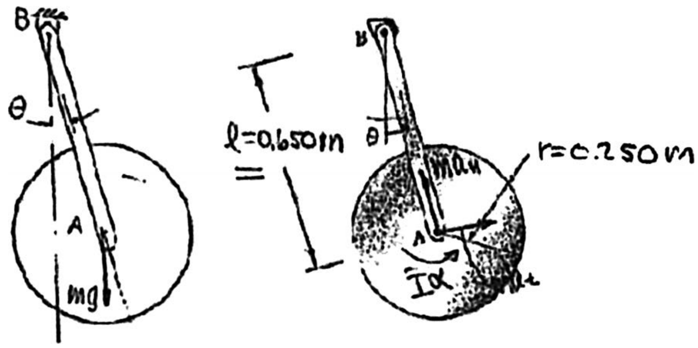
$$l = 1.294 \text{ m} \blacktriangleleft$$



PROBLEM 19.49

A uniform disk of radius $r = 250$ mm is attached at A to a 650-mm rod AB of negligible mass, which can rotate freely in a vertical plane about B. Determine the period of small oscillations (*a*) if the disk is free to rotate in a bearing at A, (*b*) if the rod is riveted to the disk at A.

SOLUTION



$$\bar{I} = \frac{1}{2}mr^2$$

$$= \frac{1}{2}(0.250)^2 m = \frac{m}{32}$$

$$a_t = l\alpha = 0.650\alpha$$

$$\alpha = \ddot{\theta}$$

(*a*) The disk is free to rotate and is in curvilinear translation.

Thus, $\bar{I}\alpha = 0$

$$\Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad +\!\circlearrowleft -mgl\sin\theta = lma_t \qquad \sin\theta \approx \theta$$

$$ml^2\ddot{\theta} - mgl\theta = 0$$

$$\omega_n^2 = \frac{g}{l} = \frac{9.81\text{ m/s}^2}{0.650\text{ m}}$$

$$= 15.092$$

$$\omega_n = 3.885\text{ rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.885}$$

$\tau_n = 1.617$ s ◀

PROBLEM 19.49 (Continued)

(*b*) When the disk is riveted at A, it <u>rotates</u> at an angular acceleration α.

$$\Sigma M_B = \Sigma(M_B)_{\text{eff}}: \quad \circlearrowleft + -mgl\sin\theta = \bar{I}\alpha + lma_t \qquad I = \frac{1}{2}mr^2$$

$$\left(\frac{1}{2}mr^2 + ml^2\right)\ddot{\theta} + mgl\theta = 0$$

$$\omega_n^2 = \frac{gl}{\left(\frac{r^2}{2} + l^2\right)}$$

$$= \frac{(9.81\text{ m/s}^2)(0.650\text{ m})}{\left[\left(\frac{0.250^2}{2}\right) + (0.650)^2\right]}$$

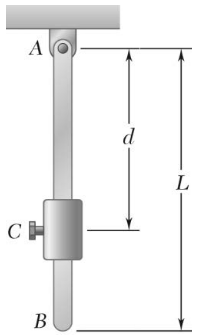
$$= 14.053$$

$$\omega_n = 3.749\text{ rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_n}$$

$$= \frac{2\pi}{3.749}$$

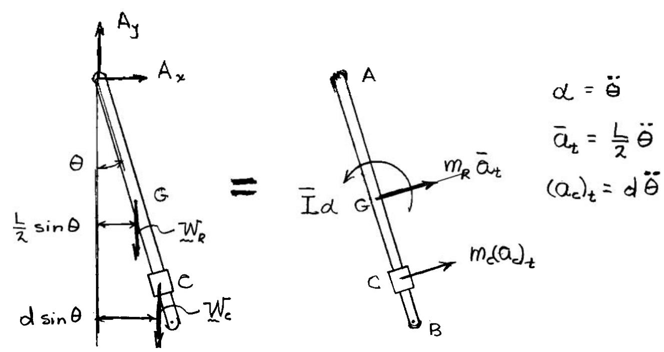
$\tau_n = 1.676\text{ s}$ ◀



PROBLEM 19.50

A small collar of mass 1 kg is rigidly attached to a 3-kg uniform rod of length $L = 750$ mm. Determine (*a*) the distance d to maximize the frequency of oscillation when the rod is given a small initial displacement, (*b*) the corresponding period of oscillation.

SOLUTION



Equation of motion.

$$+\Sigma M_A = (\Sigma M_A)_{\text{eff}}: \quad -W_R\frac{L}{2}\sin\theta - W_C d\sin\theta = \bar{I}_R\alpha + m_R\frac{L}{2}(\bar{a}_t)_R + m_C d(a_t)_C$$

$$\sin\theta \approx \theta \qquad \alpha = \ddot{\theta}, \quad (\bar{a}_t)_R = \frac{L}{2}\alpha = \frac{L}{2}\ddot{\theta}, \quad (a_t)_C = d\alpha = d\ddot{\theta}$$

$$\left(\bar{I}_R + m_R\left(\frac{L}{2}\right)^2 + m_C d^2\right)\ddot{\theta} + \left(m_R g\frac{L}{2} + m_C gd\right)\theta = 0$$

$$\bar{I}_R = \frac{1}{12}m_R L^2$$

$$\bar{I}_R + m_R\left(\frac{L}{2}\right)^2 = \frac{m_R L^2}{3}$$

$$\left(\frac{m_R L^2}{3} + m_C d^2\right)\ddot{\theta} + \left(m_R g\frac{L}{2} + m_C gd\right)\theta = 0$$

$$\ddot{\theta} + \frac{\left(\frac{L}{2} + \frac{m_C}{m_R}d\right)g}{\left(\frac{L^2}{3} + \frac{m_C}{m_R}d^2\right)}\theta = 0$$

$$\frac{m_C}{m_R} = \frac{1}{3}$$

Natural frequency. $$\omega_n^2 = \frac{\left(\frac{L}{2} + \frac{m_C}{m_R} d\right) g}{\frac{L^2}{3} + \frac{m_C}{m_R} d^2} = \frac{\left(\frac{3L}{2} + d\right) g}{(L^2 + d^2)}$$

(*a*) To maximize the frequency, we need to take the derivative with respect to *d* and set it equal to zero.

$$\frac{1}{g}\frac{d\left(\omega_n^2\right)}{d(d)} = \frac{(L^2 + d^2)(1) - \left(\frac{3L}{2} + d\right)(2d)}{(L^2 + d^2)^2} = 0$$

$$d^2 + L^2 - 3Ld - 2d^2 = 0$$

$$d^2 + 3Ld - L^2 = 0$$

Solve for d knowing that $L = 0.75$ m $\quad d = 0.22708 \quad \text{or} \quad -2.4771$

$$d = 0.22708 \text{ m}$$

$d = 227$ mm ◀

$$\omega_n^2 = \frac{\left(\frac{3(0.75)}{2} + 0.22708\right)9.81}{(0.75^2 + 0.22708^2)}$$

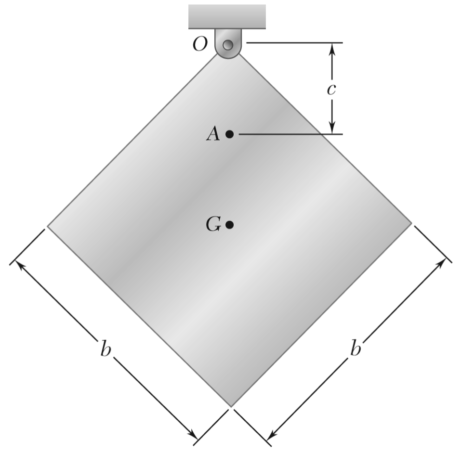
$$= 21.6$$

$\omega_n = 4.6476$ rad/s

(*b*) Period of oscillation. $$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{4.6476}$$

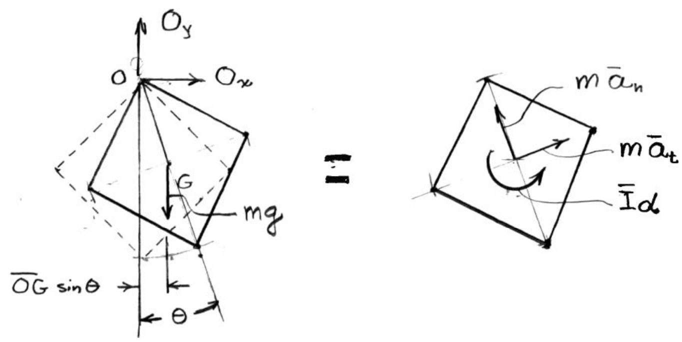
$\tau_n = 1.352$ s ◀

PROBLEM 19.51



For the uniform square plate of side $b = 12$ in, determine (a) the period of small oscillations if the plate is suspended as shown, (b) the distance c from O to a Point A from which the plate should be suspended for the period to be a minimum.

SOLUTION



(a) Equation of motion.

$$\Sigma M_0 = \bar{I}\alpha + m\bar{a}d: \quad \alpha = \ddot{\theta}$$

$$\bar{I} = \frac{1}{6}mb^2$$

$$a_t = (OG)(\alpha)$$

$$OG = b\frac{\sqrt{2}}{2}$$

$$a_t = \left(b\frac{\sqrt{2}}{2}\right)\ddot{\theta}$$

$$\circlearrowright^{+} (OG)(\sin\theta)(mg) = -(OG)ma_t - \bar{I}\alpha \qquad \sin\theta \approx \theta$$

$$\left(b\frac{\sqrt{2}}{2}\right)m\left(b\frac{\sqrt{2}}{2}\right)\ddot{\theta} + \frac{1}{6}mb^2\,\ddot{\theta} + \left(b\frac{\sqrt{2}}{2}\right)mg\,\theta = 0$$

$$(b)\left(\frac{1}{2}+\frac{1}{6}\right)m\ddot{\theta} + \left(\frac{\sqrt{2}}{2}\right)mg\,\theta = 0$$

PROBLEM 19.51 (Continued)

$$\ddot{\theta}+\frac{\left(\frac{\sqrt{2}}{2}\right)g}{\left(\frac{2}{3}\right)b}\theta=0 \quad \text{or} \quad \ddot{\theta}+\frac{3\sqrt{2}}{4}\frac{g}{b}=0$$

<u>Natural frequency and period</u>.

$$\omega_{n0}^2=\frac{3\sqrt{2}}{4}\frac{g}{b}=\frac{3\sqrt{2}(32.2)}{(4)(1)}=34.153$$

$$\omega_{n0}=5.8441 \text{ rad/s}$$

$$\tau_{n0}=\frac{2\pi}{\omega_{n0}} \qquad \tau_{n0}=1.075 \text{ s} \blacktriangleleft$$

(*b*) <u>Suspended about *A*</u>. Let $e=(OG-c)$

$$a_t=e\alpha$$

<u>Equation of motion</u>.

$$\circlearrowleft^{+}\Sigma M_A=\bar{I}\alpha+m\bar{a}d: \quad mge\sin\theta=-ema_t-\bar{I}\alpha=-(me^2+\bar{I})\alpha$$

$$m\left(e^2+\frac{1}{6}b^2\right)\ddot{\theta}+mge\theta=0$$

<u>Frequency and period</u>.

$$\omega_n^2=\frac{eg}{e^2+\frac{1}{6}b^2}$$

$$\tau_n^2=\frac{4\pi^2}{\omega_n^2}=\frac{4\pi^2(e^2+\frac{1}{6}b^2)}{eg}$$

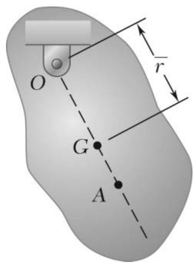
$$\tau_n^2=\frac{4\pi^2}{g}\left(e+\frac{b^2}{6e}\right)$$

For τ_n to be minimum, $\frac{d}{de}\left(e+\frac{b^2}{6e}\right)=0$

$$1-\frac{b^2}{6e^2}=0 \qquad \frac{b^2}{e^2}=6 \qquad e=\frac{b}{\sqrt{6}}$$

$$c=OG-e=\frac{\sqrt{2}}{2}b-\frac{b}{\sqrt{6}}=0.29886b$$

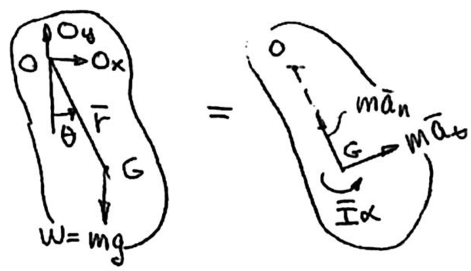
$$c=(0.29886)(12 \text{ in.}) \qquad c=3.59 \text{ in.} \blacktriangleleft$$



PROBLEM 19.52

A *compound pendulum* is defined as a rigid slab which oscillates about a fixed Point *O*, called the center of suspension. Show that the period of oscillation of a compound pendulum is equal to the period of a simple pendulum of length *OA*, where the distance from *A* to the mass center *G* is $GA = \bar{k}^2/\bar{r}$. Point *A* is defined as the center of oscillation and coincides with the center of percussion defined in Problem 17.66.

SOLUTION

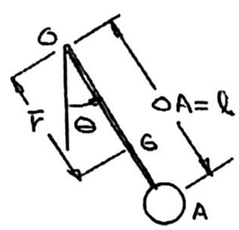


$$+\circlearrowleft \Sigma M_0 = \Sigma(M_0)_{\text{eff}}: \quad -W\bar{r}\sin\theta = \bar{I}\alpha + m\bar{a}_t\bar{r}$$

$$-mg\bar{r}\sin\theta = m\bar{k}^2\ddot{\theta} + m\bar{r}^2\ddot{\theta}$$

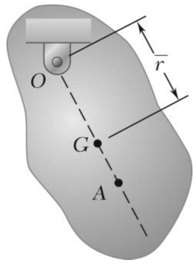
$$\ddot{\theta} + \frac{g\bar{r}}{\bar{r}^2 + \bar{k}^2}\sin\theta = 0 \qquad (1)$$

For a simple pendulum of length $OA = l$,



$$\ddot{\theta} + \frac{g}{l}\theta = 0 \qquad (2)$$

Comparing Equations (1) and (2), $\quad l = \dfrac{\bar{r}^2 + \bar{k}^2}{\bar{r}} \qquad GA = l - \bar{r} = \dfrac{\bar{k}^2}{\bar{r}}$ Q.E.D. ◀



PROBLEM 19.53

A rigid slab oscillates about a fixed Point O. Show that the smallest period of oscillation occurs when the distance $\bar{r}$ from Point O to the mass center G is equal to $\bar{k}$.

SOLUTION

See Solution to Problem 19.52 for derivation of

$$\ddot{\theta} + \frac{g\bar{r}}{\bar{r}^2 + \bar{k}^2}\sin\theta = 0$$

For small oscillations, $\sin\theta \approx \theta$ and

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{\bar{r}^2 + \bar{k}^2}{g\bar{r}}} = \frac{2\pi}{\sqrt{g}}\sqrt{\bar{r} + \frac{\bar{k}^2}{\bar{r}}}$$

For smallest τ_n, we must have $\bar{r} + \frac{\bar{k}^2}{\bar{r}}$ as a minimum:

$$\frac{d\left(\bar{r} + \frac{\bar{k}^2}{\bar{r}}\right)}{d\bar{r}} = 1 - \frac{\bar{k}^2}{\bar{r}^2} = 0$$

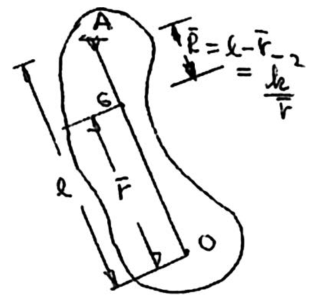
$$\bar{r}^2 = \bar{k}^2$$

$\bar{r} = \bar{k}$ Q.E.D. ◀

PROBLEM 19.54

Show that if the compound pendulum of Problem 19.52 is suspended from A instead of O, the period of oscillation is the same as before and the new center of oscillation is located at O.

SOLUTION



Same derivation as in Problem 19.52 with $\bar{r}$ replaced by $\bar{R}$.

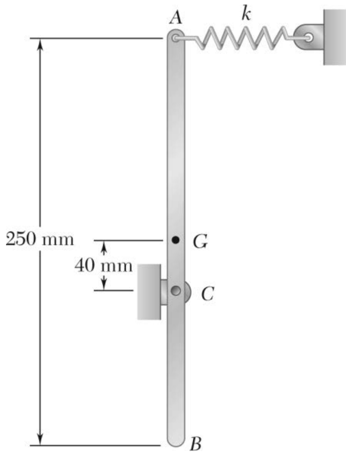
Thus, $$\ddot{\theta} + \frac{g\bar{R}}{\bar{R}^2 + \bar{k}}\theta = 0$$

Length of the equivalent simple pendulum is

$$L = \frac{\bar{R}^2 + \bar{k}^2}{\bar{R}} = \bar{R} + \frac{\bar{k}^2}{\bar{R}}$$

$$L = (l - \bar{r}) + \frac{\bar{k}^2}{\frac{\bar{k}^2}{\bar{r}}} = l$$

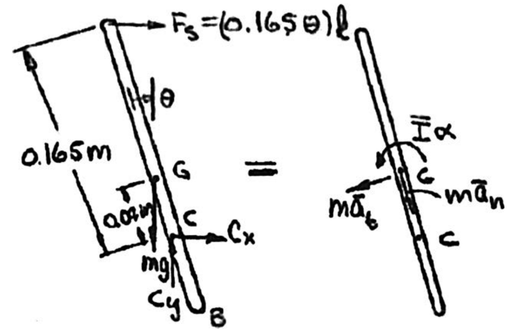
Thus, the length of the equivalent simple pendulum is the same as in Problem 19.52. It follows that the period is the same and that the new center of oscillation is at O. Q.E.D.



PROBLEM 19.55

The 8-kg uniform bar AB is hinged at C and is attached at A to a spring of constant $k = 500$ N/m. If end A is given a small displacement and released, determine (a) the frequency of small oscillations, (b) the smallest value of the spring constant k for which oscillations will occur.

SOLUTION



$$\bar{I} = \frac{1}{12}ml^2 = \left(\frac{1}{12}\right)(8)(0.250)^2$$

$$\bar{I} = 0.04167 \text{ kg}\cdot\text{m}^2$$

$$\alpha = \ddot{\theta}$$

$$a_t = 0.04\alpha = 0.04\ddot{\theta}$$

$$\sin\theta \approx \theta$$

Equation of motion.

$$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{\text{eff}}: \quad -(0.165)^2 k\theta + 0.04mg\theta = \bar{I}\ddot{\theta} + (0.04)^2 m\ddot{\theta}$$

$$(0.04167 + 0.01280)\ddot{\theta} + (0.02722k - 0.32g)\theta = 0 \qquad (1)$$

(a) Frequency if $k = 500$ N·m.

$$0.05447\ddot{\theta} + (10.47)\theta = 0$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{\left(\sqrt{\frac{10.47}{0.05447}}\right)}{2\pi}$$

$f_n = 2.21$ Hz ◄

(b) For $\tau_n \longrightarrow \infty$ or $\omega_n \longrightarrow 0$, oscillations will not occur.

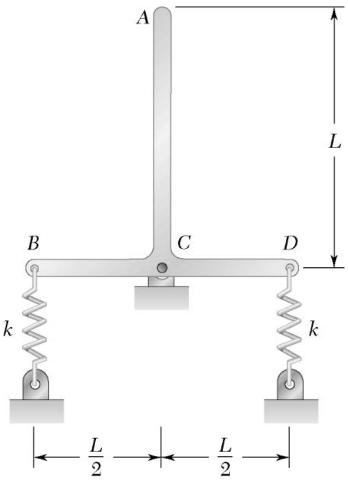
From Equation (1),

$$\omega_n^2 = \frac{0.02722k - 0.32g}{(0.05447)} = 0$$

$$k = \frac{0.32g}{0.02722} = \frac{(0.32)(9.81)}{(0.02722)}$$

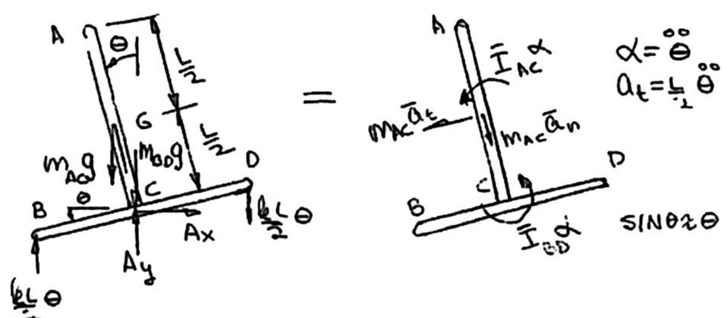
$k = 115.3$ N/m ◄

PROBLEM 19.56



Two uniform rods, each of mass $m = 12$ kg and length L = 800 mm, are welded together to form the assembly shown. Knowing that the constant of each spring is $k = 500$ N/m and that end A is given a small displacement and released, determine the frequency of the resulting motion.

SOLUTION



Equation of motion. $\circlearrowleft + \Sigma M_0 = \Sigma(M_0)_{\text{eff}}$: $\left[m_{AC}\, g\frac{L}{2} - 2k\left(\frac{L}{2}\right)^2\right]\theta = (\bar{I}_{AC} + \bar{I}_{BD})\alpha + m_{AC}\left(\frac{L}{2}\right)^2\alpha$

$$m_{BD} = m_{AC} = m$$

$$\bar{I}_{BD} = \bar{I}_{AC} = \bar{I} = \frac{1}{2}mL^2$$

$$\left(\frac{1}{6} + \frac{1}{4}\right)mL^2\ddot{\theta} + \left[2k\left(\frac{L}{2}\right)^2 - mg\frac{L}{2}\right]\theta = 0$$

$$\frac{10}{24}mL^2\ddot{\theta} + \left[\frac{kL^2}{2} - \frac{mgL}{2}\right]\theta = 0 \Rightarrow \omega_n^2 = \frac{6(kL - mg)}{5mL}$$

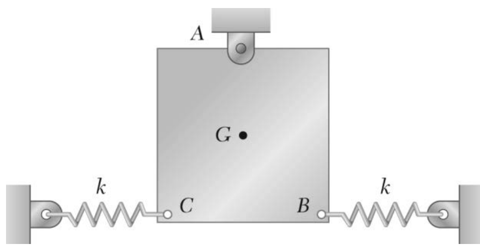
Data: $L = 800\text{ mm} = 0.8\text{m}, \quad m = 12\text{ kg}, \quad k = 500\text{ N/m}$

Frequency. $$\omega_n^2 = \frac{6[(500)(0.8) - (12)(9.81)]}{(5)(12)(0.8)} = 35.285$$

$$\omega_n = 5.9401\text{ rad/s} \qquad f_n = \frac{\omega_n}{2\pi}$$

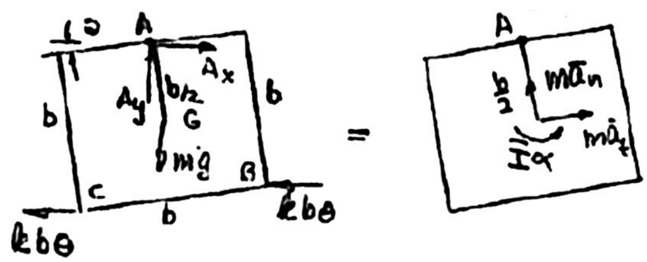
$f_n = 0.945$ Hz ◀

PROBLEM 19.57



A 45-lb uniform square plate is suspended from a pin located at the midpoint A of one of its 1.2-ft edges and is attached to springs, each of constant $k = 8$ lb/in. If corner B is given a small displacement and released, determine the frequency of the resulting vibration. Assume that each spring can act in either tension or compression.

SOLUTION



$$\alpha = \ddot{\theta}$$

$$a_t = \frac{b}{2}\alpha - \frac{b}{2}\ddot{\theta}$$

$$\sin\theta \approx \theta$$

Equation of motion. $\circlearrowleft + \Sigma M_0 = \Sigma (M_0)_{\text{eff}}: \quad -mg\frac{b}{2}\theta + 2kb^2\theta = \bar{I}\alpha + \left(\frac{b}{2}\right)^2 m\alpha$

$$\bar{I} + m\left(\frac{b}{2}\right)^2 = \frac{1}{6}mb^2 + m\frac{b^2}{4} = \frac{5}{12}mb^2$$

$$\frac{5}{12}mb^2\ddot{\theta} + \left(mg\frac{b}{2} + 2kb\right)\theta = 0$$

$$\ddot{\theta} + \left(\frac{12}{10}\frac{g}{b} + \frac{24k}{5mb}\right)\theta = 0$$

Data: $b = 1.2$ ft; $\quad m = \dfrac{45}{32.2} = 1.3975$ lb$\cdot$s^2/ft

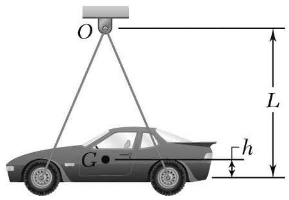
$$k = 8 \text{ lb/in.} = 96 \text{ lb/ft}$$

$$\ddot{\theta} + \left[\frac{(12)(32.2)}{(10)(1.2)} + \frac{(24)(96)}{(5)(1.3975)(1.2)}\right]\theta = 0$$

$$\ddot{\theta} + 306.98\theta = 0$$

$$\omega_n^2 = 306.98 \quad \omega_n = 17.521 \text{ rad/s}$$

Frequency. $f_n = \dfrac{\omega_n}{2\pi} = \dfrac{17.521}{2\pi}$ $\qquad f_n = 2.79$ Hz ◀



PROBLEM 19.58

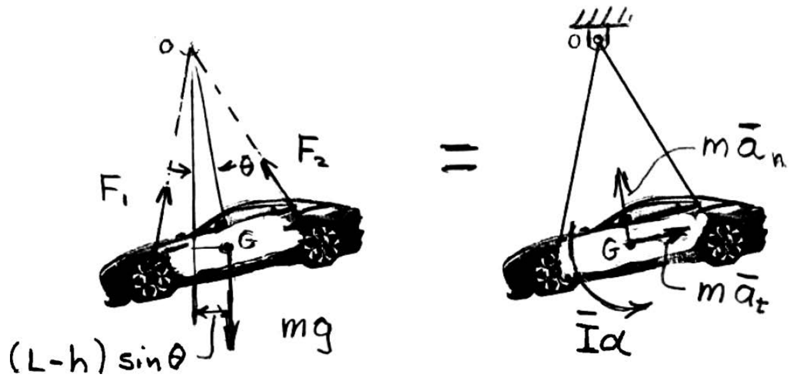
A 1300-kg sports car has a center of gravity G located a distance h above a line connecting the front and rear axles. The car is suspended from cables that are attached to the front and rear axles as shown. Knowing that the periods of oscillation are 4.04 s when $L = 4$ m and 3.54 s when $L = 3$ m, determine h and the centroidal radius of gyration.

SOLUTION

Let the mass center of the car be displaced a small distance x to the right. The mass center is moves on a circular arc of radius $L - h$, so that $x = (L - h)\sin\theta$, where θ is the counterclockwise rotation of the car. From kinematics

$$\alpha = \ddot{\theta} \qquad \bar{a}_t = (L-h)\ddot{\theta}$$

The moment of the weight force about O is



$$M_0 = -mg(L-h)\sin\theta$$

$$+\circlearrowleft M_0 = \bar{I}\alpha + (L-h)m\bar{a}_t$$

$$-mg(L-h)\sin\theta = \bar{I}\ddot{\theta} + m(L-h)^2\ddot{\theta}$$

Dividing by m and transposing terms yields

$$[\bar{k}^2 + (L-h)^2]\ddot{\theta} + g(L-h)\sin\theta = 0$$

For small angle θ, $\quad \sin\theta \approx \theta$

$$\ddot{\theta} + \frac{g(L-h)}{\bar{k}^2 + (L-h)^2}\theta = 0$$

$$\ddot{\theta} + \omega_n^2\theta = 0 \qquad \omega_n^2 = \frac{g(L-h)}{\bar{k}^2 + (L-h)^2}$$

$$\bar{k}^2 + (L-h)^2 = \frac{g}{\omega_n^2}(L-h)$$

PROBLEM 19.58 (Continued)

Using the two different values (L_1 and L_2) for L,

$$\bar{k}^2 + (L_1 - h)^2 = \frac{g}{\omega_{n1}^2}(L_1 - h) \qquad (1)$$

$$\bar{k}^2 + (L_2 - h)^2 = \frac{g}{\omega_{n2}^2}(L_2 - h) \qquad (2)$$

Subtracting Eq. (2) from Eq. (1) to eliminate $\bar{k}^2$,

$$(L_1 - h)^2 - (L_2 - h)^2 = \frac{gL_1}{\omega_{n1}^2} - \frac{gL_2}{\omega_{n2}^2} - \left(\frac{g}{\omega_{n1}^2} - \frac{g}{\omega_{n2}^2}\right)h$$

$$\left(L_1^2 - L_2^2\right) - 2(L_1 - L_2)h = A - Bh$$

where $$A = \frac{gL_1}{\omega_{n1}^2} - \frac{gL_2}{\omega_{n2}^2}$$

and $$B = \frac{g}{\omega_{n1}^2} - \frac{g}{\omega_{n2}^2}$$

$$h = \frac{L_1^2 - L_2^2 - A}{2(L_1 - L_2) - B}$$

Data: $L_1 = 4\text{ m}, \quad L_2 = 3\text{ m}, \quad g = 9.81\text{ m/s}^2$

$$\omega_{n1} = \frac{2\pi}{\tau_1} = \frac{2\pi}{4.04\text{ s}} = 1.55524\text{ rad/s}$$

$$\omega_{n2} = \frac{2\pi}{\tau_2} = \frac{2\pi}{3.54\text{ s}} = 1.77491\text{ rad/s}$$

$$A = \frac{(9.81)(4)}{(1.55524)^2} - \frac{(9.81)(3)}{(1.77491)^2} = 6.8812\text{ m}^2$$

$$B = \frac{9.81}{(1.55524)^2} - \frac{9.81}{(1.77491)^2} = 0.94190\text{ m}$$

$$h = \frac{(4)^2 - (3)^2 - 6.8812}{2(4-3) - 0.94190} = 0.11228\text{ m}$$

$h = 0.1123\text{ m}$ ◀

$$L_1 - h = 3.88772\text{ m} \qquad L_2 - h = 2.88772\text{ m}$$

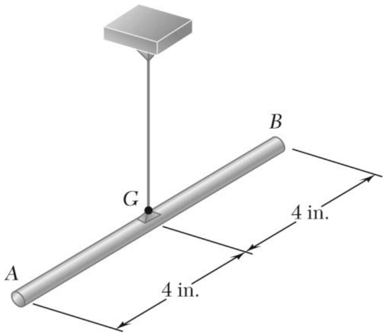
From Eq. (1), $$\bar{k}^2 + (3.88772)^2 = \frac{(9.81)(3.88772)}{(1.55524)^2}$$

$$\bar{k}^2 = 0.65336\text{ m}^2$$

$\bar{k} = 0.808\text{ m}$ ◀

Checking, using Eq. (2), $$\bar{k}^2 + (2.88772)^2 = \frac{(9.81)(2.88772)}{(1.77491)^2}$$

$$\bar{k}^2 = 0.65339\text{ m}^2$$



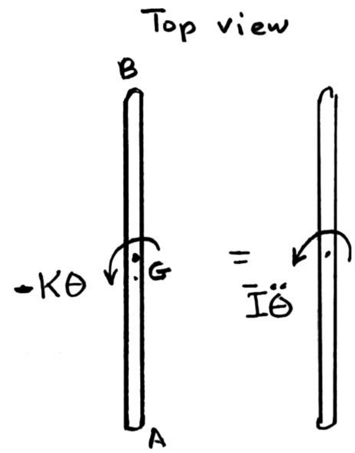
PROBLEM 19.59

A 6-lb slender rod is suspended from a steel wire which is known to have a torsional spring constant $K = 1.5$ ft·lb/rad. If the rod is rotated through 180° about the vertical and released, determine (*a*) the period of oscillation, (*b*) the maximum velocity of end *A* of the rod.

SOLUTION

Equation of motion. $\Sigma M_G = \Sigma(M_G)_{\text{eff}}:\quad -K\theta = \bar{I}\ddot{\theta} \qquad \ddot{\theta} + \frac{K}{\bar{I}}\theta = 0$

$$\ddot{\theta} + \omega_n^2\theta = 0 \qquad \omega_n^2 = \frac{K}{\bar{I}}$$



Data:

$$W = 6 \text{ lb.}$$

$$m = \frac{W}{g} = \frac{6}{32.2} = 0.186335 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$l = 8 \text{ in.} = \frac{2}{3}\text{ft}$$

$$\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}(0.186335)\left(\frac{2}{3}\right)^2$$

$$= 0.006901 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$K = 1.5 \text{ lb}\cdot\text{ft/rad}$$

$$\omega_n^2 = \frac{1.5}{0.006901} = 217.35$$

$$\omega_n = 14.743 \text{ rad/s}$$

(*a*) Period of oscillation. $\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{14.743}$ $\qquad \tau_n = 0.426 \text{ s}$ ◀

Simple harmonic motion:

$$\theta = \theta_m \sin(\omega_n t + \varphi)$$

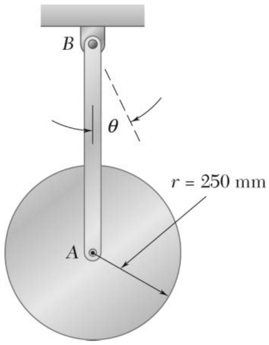
$$\dot{\theta} = \omega_n\theta_m \cos(\omega_n t + \varphi)$$

$$\dot{\theta}_m = \omega_n\theta_m$$

$$(v_A)_m = \frac{l}{2}\dot{\theta}_m = \frac{1}{2}l\omega_A\theta_m$$

$$\theta_m = 180° = \pi \text{ radians}$$

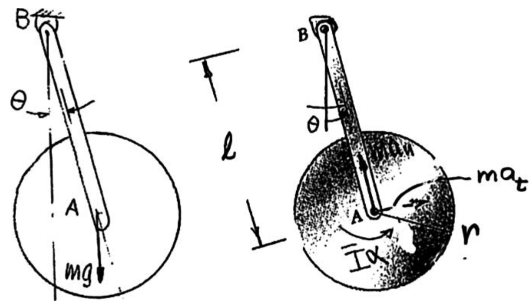
(*b*) Maximum velocity at end *A*. $(v_A)_m = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)(14.743)(\pi)$ $\qquad (v_A)_m = 15.44 \text{ ft/s}$ ◀



PROBLEM 19.60

A uniform disk of radius $r = 250$ mm is attached at A to a 650-mm rod AB of negligible mass which can rotate freely in a vertical plane about B. If the rod is displaced 2° from the position shown and released, determine the magnitude of the maximum velocity of Point A, assuming that the disk (a) is free to rotate in a bearing at A, (b) is riveted to the rod at A.

SOLUTION



$$\bar{I} = \frac{1}{2}mr^2$$

Kinematics:

$$\alpha = \ddot{\theta}$$

$$a_t = l\alpha = l\ddot{\theta}$$

(a) The disk is free to rotate and is in curvilinear translation.

Thus, $\bar{I}\alpha = 0$

Equation of motion. $\Sigma M_B = (\Sigma M_B)_{\text{eff}}: \quad -mgl\sin\theta = lma_t, \quad \sin\theta \approx \theta$

$$ml^2\ddot{\theta} + mgl\theta = 0$$

Frequency.

$$\omega_n^2 = \frac{g}{l} = \frac{9.81}{0.650} = 15.092$$

$$\omega_n = 3.8849 \text{ rad/s}$$

$$\theta_m = 2° = 0.034906 \text{ rad}$$

$$\dot{\theta}_m = \omega_n \theta_m = (3.8849)(0.034906) = 0.13561 \text{ rad/s}$$

$$(v_A)_m = l\dot{\theta}_m = (0.650)(0.13561)$$

$$(v_A)_m = 0.0881 \text{ m/s} \blacktriangleleft$$

(*b*) When the disk is riveted at *A*, it rotates at an angular acceleration α.

Equation of motion. $\Sigma M_B = (\Sigma M_B)_{\text{eff}}: \quad -mgl\sin\theta = \bar{I}\alpha + lma_t, \quad \bar{I} = \frac{1}{2}mr^2, \quad \sin\theta \approx \theta$

$$\left(\frac{1}{2}mr^2 + ml^2\right)\ddot{\theta} + mgl\theta = 0$$

Frequency.

$$\omega_n^2 = \frac{gl}{\left(\frac{r^2}{2} + l^2\right)}$$

$$= \frac{(9.81)(0.650)}{\frac{1}{2}(0.250)^2 + (0.650)^2}$$

$$= 14.053$$

$$\omega_n = 3.7487 \text{ rad/s}$$

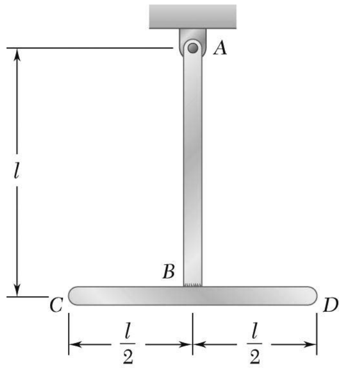
$$\theta_m = 2° = 0.034906 \text{ rad}$$

$$\dot{\theta}_m = \omega_n\theta_m = (3.7487)(0.034906) = 0.13085 \text{ rad/s}$$

$$(v_A)_m = l\dot{\theta}_m = (0.650)(0.13085)$$

$$(v_A)_m = 0.0851 \text{ m/s} \blacktriangleleft$$

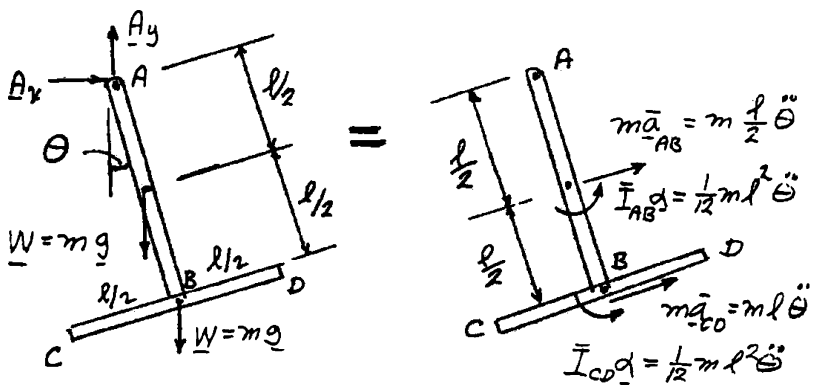
PROBLEM 19.61



Two uniform rods, each of mass m and length l, are welded together to form the T-shaped assembly shown. Determine the frequency of small oscillations of the assembly.

SOLUTION

Let the assembly be rotated counterclockwise through the small angle θ about the fixed Point A.



Equation of motion. $\quad +\circlearrowleft \Sigma M_A = \Sigma(M_A)_{\text{eff}}: \quad -mg\dfrac{l}{2}\sin\theta - mgl\sin\theta = \bar{I}_{AB}\alpha + m\bar{a}_{AB}\dfrac{l}{2} + \bar{I}_{CD}\alpha + m\bar{a}_{CD}l$

$$-\frac{3}{2}mgl\sin\theta = \frac{1}{12}ml^2\ddot{\theta} + m\left(\frac{l}{2}\right)^2\ddot{\theta} + \frac{1}{12}ml^2\ddot{\theta} + ml^2\ddot{\theta}$$

$$-\frac{3}{2}mgl\sin\theta = \frac{17}{12}ml^2\ddot{\theta}$$

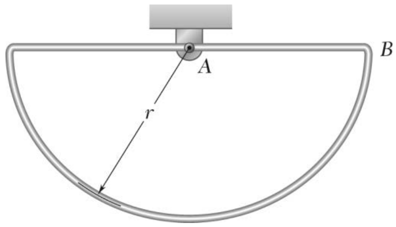
$$\ddot{\theta} + \frac{18}{17}\frac{g}{l}\sin\theta = 0$$

For small oscillations, $\quad \sin\theta \approx \theta$

$$\ddot{\theta} + \frac{18}{17}\frac{g}{l}\theta = 0$$

$$\omega_n^2 = \frac{18}{17}\frac{g}{l} \qquad \omega_n = \sqrt{\frac{18g}{17l}}$$

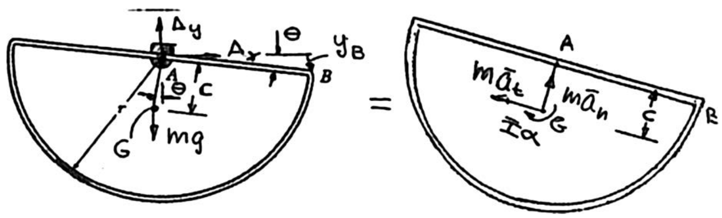
Frequency. $\quad f = \dfrac{\omega_n}{2\pi} \quad f = \dfrac{1}{2\pi}\sqrt{\dfrac{18g}{17l}} \qquad f = 0.1638\sqrt{\dfrac{g}{l}}$ ◀



PROBLEM 19.62

A homogeneous wire bent to form the figure shown is attached to a pin support at A. Knowing that $r = 220$ mm and that Point *B* is pushed down 20 mm and released, determine the magnitude of the velocity of *B*, 8 s later.

SOLUTION



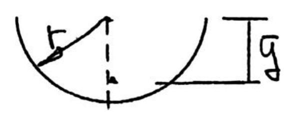
Determine location of the centroid *G*.

Let $\qquad \rho =$ mass per unit length

Then total mass $\qquad m = \rho(2r + \pi r) = \rho r(2+\pi)$

About *C*: $\qquad mgc = 0 + \pi r\rho\left(\dfrac{2r}{\pi}\right)g = 2r^2\rho g$

$$\overline{y} = \frac{2r}{\pi} \quad \text{for a semicircle}$$



$$\rho r(2+\pi)c = 2r^2\rho, \quad c = \frac{2r}{(2+\pi)}$$

<u>Equation of motion</u>. $\qquad \circlearrowleft\!+\ \Sigma M_0 = \Sigma(M_0)_{\text{eff}}: \quad \alpha = \ddot{\theta} \quad a_t = c\alpha = c\ddot{\theta}$

$$-mgc\sin\theta = \overline{I}\alpha + mca_n \qquad \sin\theta \approx \theta$$

$$(\overline{I} + mc^2)\ddot{\theta} + mgc\theta = 0 \qquad I_0\ddot{\theta} + mgc\theta = 0$$

<u>Moment of inertia</u>. $\qquad \overline{I} + mc^2 = I_0$

$$I_0 = (I_0)_{\text{semicirc.}} + (I_0)_{\text{line}} = m_{\text{semicirc.}}r^2 + m_{\text{line}}\frac{(2r)^2}{12}$$

$$m_{\text{semicirc.}} = \rho\pi r \qquad m_{\text{line}} = 2\rho r \qquad \rho = \frac{m}{(2+\pi)r}$$

$$I_0 = \rho\left[\pi r^2 \cdot r + 2r\cdot\frac{r^2}{3}\right] = \frac{mr^2}{(2+\pi)}\left[\pi + \frac{2}{3}\right]$$

$$\frac{mr^2}{(2+\pi)}\left[\pi + \frac{2}{3}\right]\ddot{\theta} + mg\frac{2r}{(2+\pi)}\theta = 0$$

PROBLEM 19.62 (Continued)

Frequency.
$$\omega_n^2 = \frac{2g}{\left(\pi + \frac{2}{3}\right)r} = \frac{(2)(9.81)}{\left(\pi + \frac{2}{3}\right)(0.220)}$$

$$\omega_n^2 = 23.418 \text{ s}^{-2} \quad \omega_n = 4.8392 \text{ rad/s}$$

$$\theta = \theta_m \sin(\omega_n t + \phi) \quad y_B = r\theta$$

At $t = 0,$
$$y_B = 20 \text{ mm}, \quad \dot{y}_B = 0$$

$$\dot{y}_B = 0 = (y_B)_m \omega_n \cos(0 + \phi), \quad \phi = \frac{\pi}{2}$$

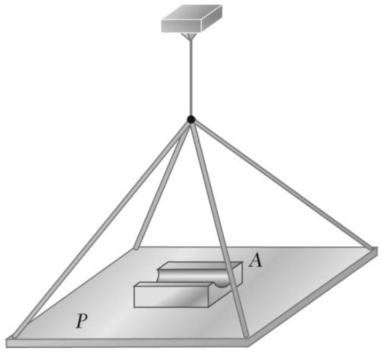
$$y_B = 20 \text{ mm } = (y_B)_m \sin\left(0 + \frac{\pi}{2}\right), \quad (y_B)_m = 20 \text{ mm}$$

$$y_B = (20 \text{ mm}) \sin\left(\omega_n t + \frac{\pi}{2}\right) \quad \omega_n = 4.8392 \text{ rad/s}$$

$$\dot{y}_B = 20\omega \cos\left(\omega_n t + \frac{\pi}{2}\right) = -(20 \text{ mm})\omega_n \sin \omega_n t$$

At $t = 8$ s,
$$\dot{y}_B = -(20)(4.8392)\sin[(4.8392)(8)] = -(96.78)(0.8492)$$

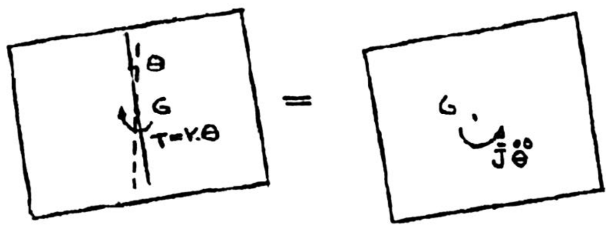
$$= -82.2 \text{ mm/s} \qquad v_B = 82.2 \text{ mm/s} \blacktriangleleft$$



PROBLEM 19.63

A horizontal platform P is held by several rigid bars which are connected to a vertical wire. The period of oscillation of the platform is found to be 2.2 s when the platform is empty and 3.8 s when an object A of unknown moment of inertia is placed on the platform with its mass center directly above the center of the plate. Knowing that the wire has a torsional constant $K = 27\ \text{N}\cdot\text{m/rad}$, determine the centroidal moment of inertia of object A.

SOLUTION



Equation of motion. $\Sigma M_G = \bar{I}\alpha$: $\quad -K\theta = \bar{I}\ddot{\theta}$

$$\ddot{\theta} + \frac{K}{\bar{I}}\theta = 0 \qquad \omega_n^2 = \frac{K}{\bar{I}}$$

Case 1. The platform is empty. $\omega_{n1} = \dfrac{2\pi}{\tau_1} = \dfrac{2\pi}{2.2} = 2.856\ \text{rad/s}$

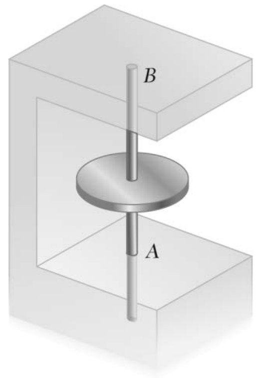
$$\bar{I}_1 = \frac{K}{\omega_{n1}^2} = \frac{27}{(2.856)^2} = 3.31\ \text{kg}\cdot\text{m}^2$$

Case 2. Object A is on the platform. $\omega_{n2} = \dfrac{2\pi}{\tau_2} = \dfrac{2\pi}{3.8} = 1.653\ \text{rad/s}$

$$\bar{I}_2 = \frac{K}{\omega_{n2}^2} = \frac{27}{(1.653)^2} = 9.8814\ \text{kg}\cdot\text{m}^2$$

Moment of inertia of object A. $\bar{I}_A = \bar{I}_2 - \bar{I}_1$ $\qquad \bar{I}_A = 6.57\ \text{kg}\cdot\text{m}^2$ ◀

PROBLEM 19.64



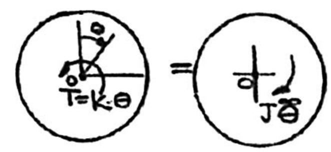
A uniform disk of radius $r = 120$ mm is welded at its center to two elastic rods of equal length with fixed ends at *A* and *B*. Knowing that the disk rotates through an $8°$ angle when a 500-mN · m couple is applied to the disk and that it oscillates with a period of 1.3 s when the couple is removed, determine (*a*) the mass of the disk, (*b*) the period of vibration if one of the rods is removed.

SOLUTION

Torsional spring constant.

$$k = \frac{T}{\theta} = \frac{0.5 \text{ N} \cdot \text{m}}{(8)\left(\frac{\pi}{180}\right)}$$

$$k = 3.581 \text{ N} \cdot \text{m/rad}$$



Equation of motion.

$$\Sigma M_0 = \Sigma(M_0)_{\text{eff}}: \quad -K\theta = I\ddot{\theta} \quad \ddot{\theta} + \frac{K}{I}\theta = 0$$

Natural frequency and period. $\omega_n^2 = \dfrac{K}{I}$

Period. $\tau_n = \dfrac{2\pi}{\omega_n} = 2\pi\sqrt{\dfrac{I}{K}}$

Mass moment of inertia. $I = \dfrac{\tau_n^2 K}{(2\pi)^2} = \dfrac{(1.35)^2(3581 \text{ N} \cdot \text{m/r})}{(2\pi)^2}$

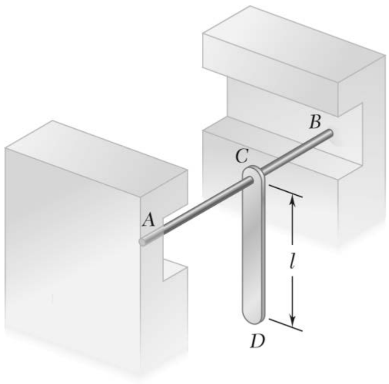
$$I = 0.1533 \text{ N} \cdot \text{m} \cdot \text{s}^2 = \frac{1}{2}mr^2 = \frac{1}{2}m(0.120 \text{ m})^2$$

(*a*) Mass of the disk. $m = \dfrac{(0.1533 \text{ N} \cdot \text{m} \cdot \text{s}^2)(2)}{(0.120 \text{ m})^2}$ $m = 21.3$ kg ◀

(*b*) With one rod removed: $K' = \dfrac{K}{2} = \dfrac{3.581}{2} = 1.791 \text{ N} \cdot \text{m/rad}$

Period. $\tau = 2\pi\sqrt{\dfrac{I}{K'}} = 2\pi\sqrt{\dfrac{(0.1533 \text{ N} \cdot \text{m} \cdot \text{s}^2)}{1.791 \text{ N} \cdot \text{m/rad}}}$ $\tau = 1.838$ s ◀

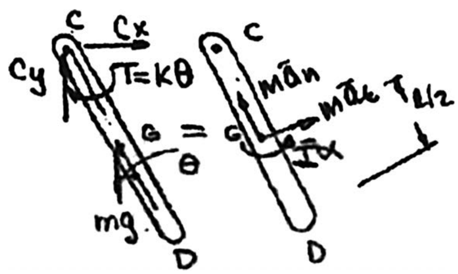
PROBLEM 19.65



A 5-kg uniform rod *CD* of length $l = 0.7$ m is welded at *C* to two elastic rods, which have fixed ends at *A* and *B* and are known to have a combined torsional spring constant $K = 24\ \text{N}\cdot\text{m/rad}$. Determine the period of small oscillation, if the equilibrium position of *CD* is (*a*) vertical as shown, (*b*) horizontal.

SOLUTION

(*a*) Equation of motion.



$$\alpha = \ddot{\theta} \qquad a_t = \frac{l}{2}\alpha = \frac{l}{2}\ddot{\theta}$$

$$+\circlearrowleft \Sigma M_C = \bar{I}\alpha + m\bar{a}d: \quad -K\theta - (mg)\frac{l}{2}\sin\theta = \bar{I}\alpha + \frac{l}{2}(ma_t)$$

$$-K\theta - \frac{1}{2}mgl\sin\theta = \bar{I}\ddot{\theta} + \frac{1}{4}ml^2\ddot{\theta}$$

$$\left(\bar{I} + \frac{1}{4}ml^2\right)\ddot{\theta} + \frac{1}{2}mgl\sin\theta + K\theta = 0$$

$$\left(\frac{1}{12}ml^2 + \frac{1}{4}ml^2\right)\ddot{\theta} + K\theta + \frac{1}{2}mgl\theta = 0$$

$$\frac{1}{3}ml^2\ddot{\theta} + \left(K + \frac{1}{2}mgl\right)\theta = 0$$

$$\ddot{\theta} + \left(\frac{3K}{ml^2} + \frac{3g}{2l}\right)\theta = 0$$

PROBLEM 19.65 (Continued)

Data: $K = 24\text{ N}\cdot\text{m/rad}, \quad m = 5\text{ kg}, \quad l = 0.7\text{ m}$

$$\ddot{\theta} + \left[\frac{(3)(24)}{(5)(0.7)^2} + \frac{(3)(9.81)}{(2)(0.7)}\right]\theta = 0$$

$$\ddot{\theta} + 50.409\theta = 0$$

Frequency. $\omega_n^2 = 50.409 \qquad \omega_n = 7.1\text{ rad/s}$

Period. $\tau = \dfrac{2\pi}{\omega_n} = \dfrac{2\pi}{7.1}$ $\qquad \tau_n = 0.885\text{ s}$ ◀

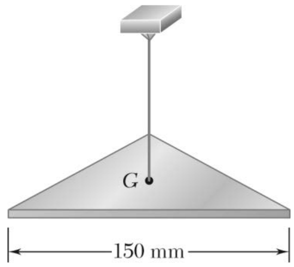
(*b*) If the rod is horizontal, the gravity term is not present and the equation of motion is

$$\ddot{\theta} + \frac{3K}{ml^2}\theta = 0$$

$$\omega_n^2 = \frac{3K}{ml^2} = \frac{(3)(24)}{(5)(0.7)^2} = 29.388$$

$\omega_n = 5.4210\text{ rad/s} \qquad \tau = \dfrac{2\pi}{\omega_n} = \dfrac{2\pi}{5.4210}$ $\qquad \tau_n = 1.159\text{ s}$ ◀

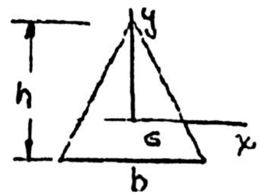
PROBLEM 19.66



A 1.8-kg uniform plate in the shape of an equilateral triangle is suspended at its center of gravity from a steel wire which is known to have a torsional constant $K = 35\ \text{mN}\cdot\text{m/rad}$. If the plate is rotated 360° about the vertical and then released, determine (*a*) the period of oscillation, (*b*) the maximum velocity of one of the vertices of the triangle.

SOLUTION

Mass moment of inertia of plate about a vertical axis:



$$h = \frac{\sqrt{3}}{2}b \qquad A = \frac{1}{2}bh = \frac{\sqrt{3}}{4}b^2$$

For area,

$$\bar{I}_x = \bar{I}_y = \frac{1}{36}bh^3 = \frac{\sqrt{3}b^4}{96}$$

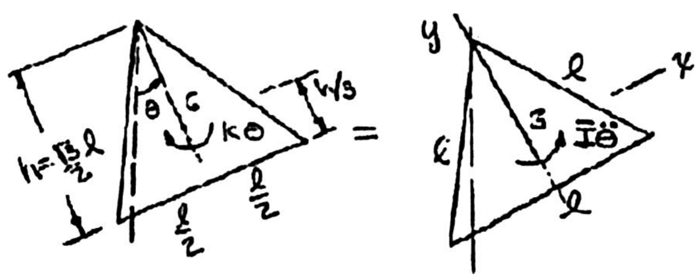
$$\bar{I}_z = \bar{I}_x + \bar{I}_y = \frac{\sqrt{3}}{48}b^4$$

For mass,

$$\bar{I} = \frac{m}{A}(\bar{I}_z)_{\text{area}}$$

$$= \left(\frac{4\,m}{\sqrt{3}b}\right)\left(\frac{\sqrt{3}}{48}b^4\right) = \frac{1}{12}mb^2$$

$$\bar{I} = \frac{1}{12}(1.8)(0.150)^2 = 3.375\times 10^{-3}\ \text{kg}\cdot\text{m}^2$$



Equation of motion.

$$\circlearrowleft + \Sigma M_G = \Sigma(M_G)_{\text{eff}}: \quad -K\theta = \bar{I}\ddot{\theta}$$

$$\ddot{\theta} + \frac{K}{\bar{I}}\theta = 0$$

Frequency.

$$\omega_n^2 = \frac{K}{\bar{I}} = \frac{35\times 10^{-3}}{3.375\times 10^{-3}} = 10.37$$

$$\omega_n = 3.2203\ \text{rad/s}$$

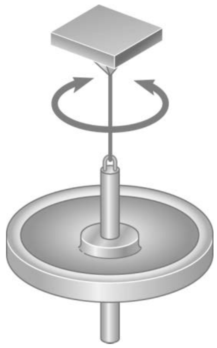
PROBLEM 19.66 (Continued)

(*a*) Period.
$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.2203}$$
$\tau = 1.951\text{ s}$ ◀

Maximum rotation. $\theta_m = 360° = 2\pi\text{ rad}$

Maximum angular velocity. $\dot{\theta}_m = \omega_n \theta_m = (3.2203)(2\pi) = 20.234\text{ rad/s}$

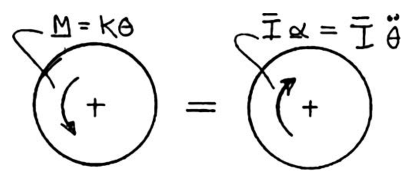
(*b*) Maximum velocity at a vertex.
$$v_m = r\dot{\theta}_m = \frac{2}{3}h\dot{\theta}_m = \frac{2}{3}\frac{\sqrt{3}}{2}b = \left(\frac{2}{3}\right)\left(\frac{\sqrt{3}}{2}\right)(0.150)(20.234)$$
$v_m = 1.752\text{ m/s}$ ◀



PROBLEM 19.67

A period of 6.00 s is observed for the angular oscillations of a 4-oz gyroscope rotor suspended from a wire as shown. Knowing that a period of 3.80 s is obtained when a 1.25-in.-diameter steel sphere is suspended in the same fashion, determine the centroidal radius of gyration of the rotor. (Specific weight of steel = 490 lb/ft^3.)

SOLUTION



$$\circlearrowleft \Sigma M = \Sigma(M)_{\text{eff}}: \quad -K\theta = \bar{I}\ddot{\theta}$$

$$\ddot{\theta} + \frac{K}{\bar{I}}\theta = 0$$

$$\omega_n^2 = \frac{K}{\bar{I}}$$

$$\tau = 2\pi\sqrt{\frac{\bar{I}}{K}} \qquad (1)$$

$$K = \frac{4\pi^2\bar{I}}{\tau^2} \qquad (2)$$

$$\bar{I} = \frac{K\tau^2}{4\pi^2} \qquad (3)$$

For the sphere, $r = \dfrac{d}{2} = 0.625 \text{ in.} = 52.083\times10^{-3} \text{ ft}$

Volume: $V_s = \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi(52.083\times10^{-3})^3$

$= 591.81\times10^{-6} \text{ ft}^3$

Weight: $W_s = \gamma V_s$

$= (490 \text{ lb/ft}^3)(591.81\times10^{-6} \text{ ft}^3)$

$= 0.28999 \text{ lb}$

Mass: $m_s = \dfrac{W_s}{g} = \dfrac{0.28999}{32.2}$

$= 9.0059\times10^{-3} \text{ lb}\cdot\text{s}^2/\text{ft}$

Moment of inertia: $\bar{I} = \dfrac{2}{5}m_s r^2 = \dfrac{2}{5}(9.0059\times10^{-3})(52.083\times10^{-3})^2$

$= 9.7719\times10^{-6} \text{ lb}\cdot\text{s}^2\cdot\text{ft}$

Period: $\tau_s = 3.80 \text{ s}$

PROBLEM 19.67 (Continued)

From Eq. (2):

$$K = \frac{4\pi^2(9.7719\times10^{-6})}{(3.80)^2}$$
$$= 26.716\times10^{-6}\ \text{lb}\cdot\text{ft/rad}$$

For the rotor,

$$m = \frac{W}{g} = \left(\frac{4}{16}\right)\left(\frac{1}{32.2}\right)$$
$$= 7.764\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$
$$\tau = 6.00\ \text{s}$$

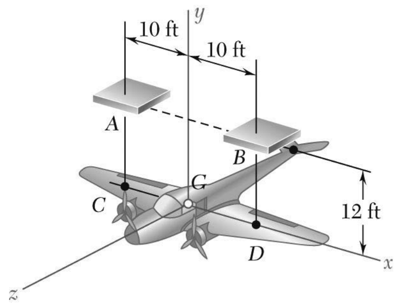
From Eq. (3):

$$\bar{I} = (26.716\times10^{-6})\frac{(6.00)^2}{4\pi^2}$$
$$= 24.362\times10^{-6}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

Radius of gyration.

$$\bar{I} = m\bar{k}^2$$
$$\bar{k} = \sqrt{\frac{\bar{I}}{m}}$$
$$= \sqrt{\frac{24.362\times10^{-6}}{7.764\times10^{-3}}}$$
$$= 0.056016\ \text{ft}$$

$\bar{k} = 0.672\ \text{in.}$ ◀



PROBLEM 19.68

The centroidal radius of gyration $\bar{k}_y$ of an airplane is determined by suspending the airplane by two 12-ft-long cables as shown. The airplane is rotated through a small angle about the vertical through G and then released. Knowing that the observed period of oscillation is 3.3 s, determine the centroidal radius of gyration $\bar{k}_y$.

SOLUTION

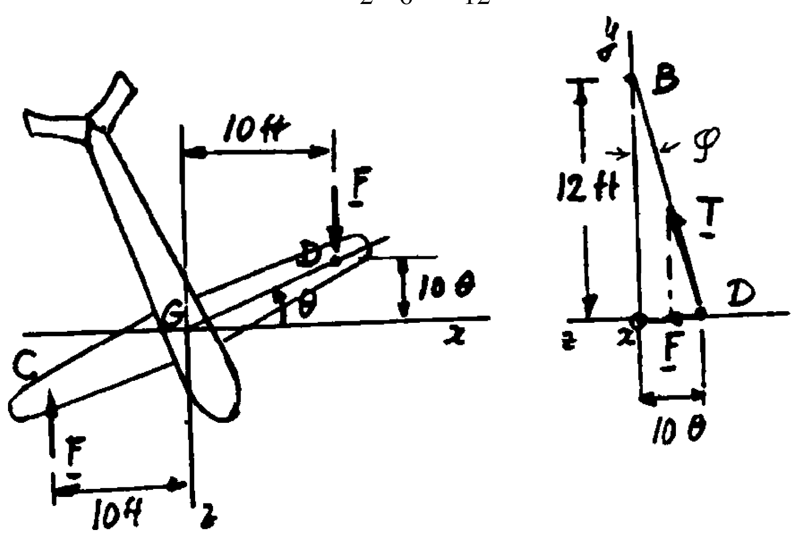
Let the airplane rotate through the small angle θ about a vertical axis. Suspension Points C and D on the airplane each move horizontally a distance (10 ft) sin θ $\approx$ (10 ft) θ. Let φ be the angle between a cable and the vertical direction. Then, $\sin\varphi = (10\text{ ft})\theta/(12\text{ ft}) = \frac{5}{6}\theta.$

$$+\uparrow\ \Sigma F = 0: \quad 2T\cos\varphi - W = 0$$

$$T = \frac{W}{2\cos\varphi} \approx \frac{W}{2}$$

Let F be the horizontal component of T.

$$F = T\sin\varphi \approx \frac{W}{2}\cdot\frac{5}{6}\theta = \frac{5}{12}W\theta$$



The two forces F form a couple of moment

$$M = -(20\text{ ft})F = -(20)\left(\frac{5}{12}\right)W\theta$$

Equation of motion:

$$+\circlearrowleft\ \Sigma M_y = \bar{I}_y\alpha: \quad -20\left(\frac{5W}{12}\theta\right) = \frac{W}{g}\bar{k}_y^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{(20)(5)g}{12\bar{k}_y^2}\theta = 0$$

PROBLEM 19.68 (Continued)

Natural frequency:

$$\omega_n^2 = \frac{(20)(5)g}{12\,\bar{k}_y^2} = \frac{(20)(5)(32.2)}{12\,\bar{k}_y^2} = \frac{268.33}{\bar{k}_y^2}$$

$$\bar{k}_y^2 = \frac{268.33}{\omega_n^2}$$

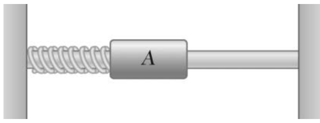
$$\bar{k}_y = \frac{16.381}{\omega_n} = \frac{16.381}{2\pi f}$$

$$= \frac{2.607}{f} = 2.607\tau$$

With $\tau = 3.3$ s, $\bar{k}_y = (2.607)(3.3)$

$\bar{k}_y = 8.60$ ft ◀

PROBLEM 19.69

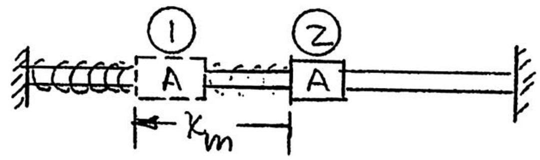


A 1.8-kg collar *A* is attached to a spring of constant 800 N/m and can slide without friction on a horizontal rod. If the collar is moved 70 mm to the left from its equilibrium position and released, determine the maximum velocity and maximum acceleration of the collar during the resulting motion.

SOLUTION

Datum at ①:

Position ①



$$T_1 = 0 \qquad V_1 = \frac{1}{2}kx_m^2$$

Position ②

$$T_2 = \frac{1}{2}mv_2^2 \qquad V_2 = 0 \qquad v_2 = \dot{x}_m$$

$$\dot{x}_m = \omega_n x_m$$

$$T_1 + V_1 = T_2 + V_2 \qquad 0 + \frac{1}{2}kx_m^2 = \frac{1}{2}m\dot{x}_m^2 + 0$$

$$\frac{1}{2}kx_m^2 = \frac{1}{2}m\omega_n^2 x_m^2 \qquad \omega_n^2 = \frac{k}{m} = \frac{800 \text{ N/m}}{1.8 \text{ kg}}$$

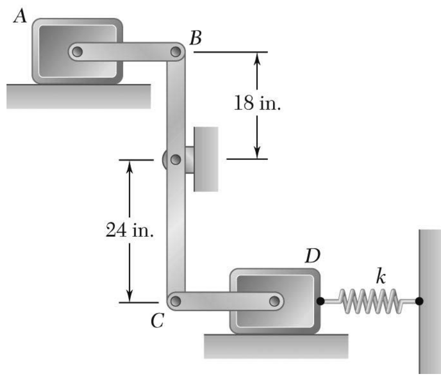
$$\omega_n^2 = 444.4 \text{ s}^{-2} \qquad \omega_n = 21.08 \text{ rad/s}$$

$$\dot{x}_m = \omega_n x_m = (21.08 \text{ s}^{-1})(0.070 \text{ m})$$

$\dot{x}_m = 1.476 \text{ m/s}$ ◀

$$\ddot{x}_m = \omega_n^2 x_m = (21.08 \text{ s}^{-2})(0.070 \text{ m})$$

$\dot{x}_m = 31.1 \text{ m/s}^2$ ◀



PROBLEM 19.70

Two blocks, each of weight 3 lb, are attached to links which are pin-connected to bar BC as shown. The weights of the links and bar are negligible, and the blocks can slide without friction. Block D is attached to a spring of constant $k = 4$ lb/in. Knowing that block A is moved 0.5 in. from its equilibrium position and released, determine the magnitude of the maximum velocity of block D during the resulting motion.

SOLUTION

$$T = \frac{1}{2}m(b^2 + c^2)\dot{\theta}^2$$

$$V = \frac{1}{2}kc^2\theta^2$$

$$\omega_n^2 = \frac{kc^2}{m(b^2 + c^2)}$$

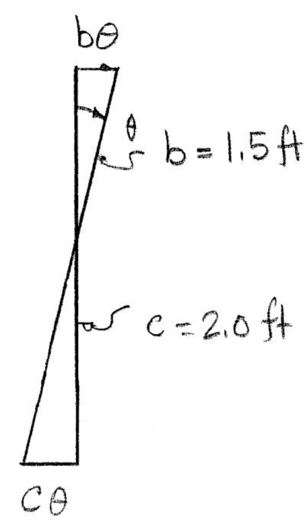
$$k = 48 \text{ lb/ft}$$

$$m = \frac{3 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.093167 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\omega_n^2 = \frac{(48)(2)^2}{(0.093167)(1.5^2 + 2^2)} = 329.73$$

$$\omega_n = 18.158 \text{ rad/s}$$

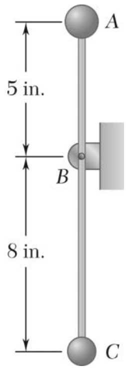
$$\theta_0 = \frac{\frac{0.5 \text{ in.}}{12 \text{ in./ft}}}{1.5 \text{ ft}} = 0.0277 \text{ rad}$$



$$|v_D|_m = c\omega_n\theta_0$$

$$= (2)(18.158)(0.02778) = 1.009 \text{ ft}$$

$|v_D|_m = 12.11$ in./s ◀

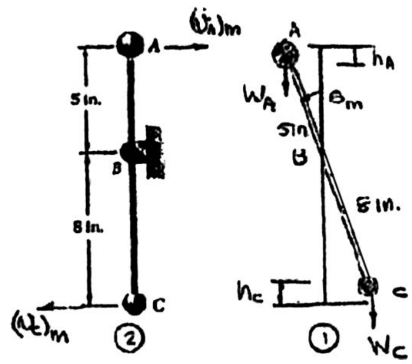


PROBLEM 19.71

A 14-oz sphere A and a 10-oz sphere C are attached to the ends of a rod AC of negligible weight which can rotate in a vertical plane about an axis at B. Determine the period of small oscillations of the rod.

SOLUTION

Datum at ①:



Position ①

$$T_1 = 0$$
$$V_1 = W_C h_C - W_A h_A$$
$$h_C = BC(1-\cos\theta_m)$$
$$h_A = BA(1-\cos\theta_m)$$

Small angles.

$$1-\cos\theta_m \approx \frac{\theta_m^2}{2}$$

$$V_1 = [(W_C)(BC)-(W_A)(BA)]\frac{\theta_m^2}{2}$$

$$V_1 = \left[\left(\frac{10}{16}\text{lb}\right)\left(\frac{8}{12}\text{ft}\right)-\left(\frac{14}{16}\text{lb}\right)\left(\frac{5}{12}\text{ft}\right)\right]\frac{\theta_m^2}{2}$$

$$V_1 = (0.4167-0.3646)\frac{\theta_m^2}{2} = 0.05208\frac{\theta_m^2}{2}$$

Position ②

$$V_2 = 0$$
$$(v_C)_m = \frac{8}{12}\dot{\theta}_m \qquad (v_A)_m = \frac{5}{12}\dot{\theta}_m$$
$$m_C = \frac{W_C}{g} = \frac{10}{(16)(32.2)} = 0.019410 \text{ lb}\cdot\text{s}^2/\text{ft}$$
$$m_A = \frac{W_A}{g} = \frac{14}{(16)(32.2)} = 0.027174 \text{ lb}\cdot\text{s}^2/\text{ft}$$

PROBLEM 19.71 (Continued)

$$T_2 = \frac{1}{2}m_C(v_C)_m^2 + \frac{1}{2}m_A(v_A)_m^2$$

$$= \frac{1}{2}\left[(0.019410)\left(\frac{8}{12}\right)^2 + \frac{1}{2}(0.027174)\left(\frac{5}{12}\right)^2\right]\dot{\theta}_m^2$$

$$= 0.013344\frac{\dot{\theta}_m^2}{2}$$

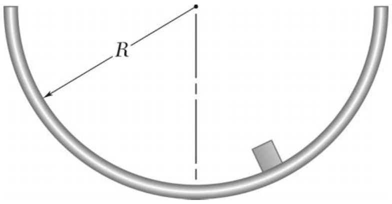
$$= 0.013344\frac{\omega_n^2\theta_m^2}{2}$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2: \quad 0 + 0.05208\frac{\theta_m^2}{2} = 0.013344\frac{\omega_n^2\theta_m^2}{2} + 0$

$$\omega_n^2 = \frac{0.05208}{0.013344} = 3.902$$

$$\omega_n = 1.9755 \text{ rad/s}$$

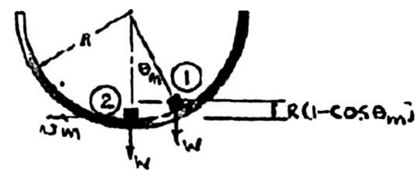
Period of oscillations. $\tau_n = \dfrac{2\pi}{\omega_n}$ $\qquad \tau_n = 3.18 \text{ s}$ ◀



PROBLEM 19.72

Determine the period of small oscillations of a small particle which moves without friction inside a cylindrical surface of radius *R*.

SOLUTION



Datum at ②:

Position ①

$$T_1 = 0$$
$$V_1 = WR(1-\cos\theta_m)$$

Small oscillations:

$$(1-\cos\theta_m) = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_1 = \frac{WR\theta_m^2}{2}$$

Position ②

$$v_m = R\dot{\theta}_m \quad T_2 = \frac{1}{2}mv_m^2 = \frac{1}{2}mR^2\dot{\theta}_m^2$$

$$V_2 = 0$$

Conservation of energy. $T_1 + Y_1 = T_2 + V_2$

$$0 + WR\frac{\theta_m^2}{2} = \frac{1}{2}mR^2\dot{\theta}_m^2 + 0 \qquad \dot{\theta}_m = \omega_n\theta_m$$

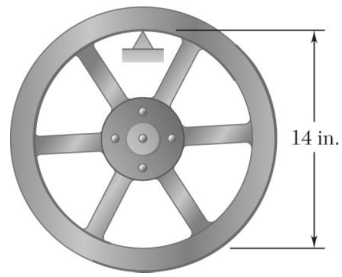
$$W = mg$$

$$mgR\frac{\theta_m^2}{2} = \frac{1}{2}mR^2\omega_n^2\theta_m^2$$

$$\omega_n = \sqrt{\frac{g}{R}}$$

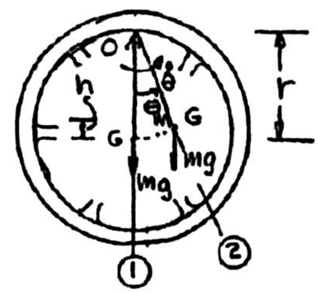
Period of oscillations. $\tau_n = \dfrac{2\pi}{\omega_n} = 2\pi\sqrt{\dfrac{R}{g}}$ ◀

PROBLEM 19.73



The inner rim of an 85-lb flywheel is placed on a knife edge, and the period of its small oscillations is found to be 1.26 s. Determine the centroidal moment of inertia of the flywheel.

SOLUTION



Datum at ①:

Position ①

$$T_1 = \frac{1}{2} I_0 \dot{\theta}_m^2 \quad V_1 = 0$$

Position ②

$$T_2 = 0 \quad V_2 = mgh$$

$$h = r(1 - \cos\theta_m) = r\left(2\sin^2\frac{\theta_m}{2}\right)$$

$$\approx r\frac{\theta_m^2}{2}$$

$$V_2 = mgr\frac{\theta_m^2}{2}$$

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad \frac{1}{2} I_0 \dot{\theta}_m^2 + 0 = 0 + mgr\frac{\theta_m^2}{2}$$

For simple harmonic motion, $\dot{\theta}_m = \omega_n \theta_m$

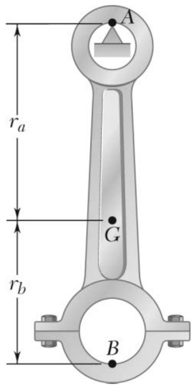
$$I_0 \omega_n^2 \theta_m^2 = mgr\,\theta_m^2 \qquad \omega_n^2 = \frac{mgr}{I_0} \qquad \tau_n^2 = \frac{4\pi^2}{\omega_n^2} = \frac{(4\pi^2)I_0}{mgr}$$

Moment of inertia. $I_0 = \bar{I} + mr^2 \quad \bar{I} + mr^2 = \dfrac{\left(\tau_n^2\right)(mgr)}{4\pi^2}$

$$\bar{I} = \frac{\left(\tau_n^2\right)(mgr)}{4\pi^2} - mr^2 = \frac{(1.26\text{ s})^2(85\text{ lb})}{4\pi^2}\left(\frac{7}{12}\text{ ft}\right) - \frac{(85\text{ lb})}{(32.2\text{ ft/s}^2)}\left(\frac{7}{12}\text{ ft}\right)^2$$

$$\bar{I} = 1.994 - 0.8983$$

$$\bar{I} = 1.096\text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$



PROBLEM 19.74

A connecting rod is supported by a knife edge at Point A; the period of its small oscillations is observed to be 1.03 s. Knowing that the distance r_a is 6 in. determine the centroidal radius of gyration of the connecting rod.

SOLUTION

Position ① Displacement is maximum.

$$T_1 = 0, \quad V_1 = mgr_a(1-\cos\theta_m) \approx \frac{1}{2}mgr_a\theta_m^2$$

Position ② Velocity is maximum.

$$(v_G)_m = r_a\dot{\theta}_m$$

$$T_2 = \frac{1}{2}mv_G^2 + \frac{1}{2}\bar{I}\dot{\theta}_m^2 = \frac{1}{2}mr_a^2\dot{\theta}_m^2 + \frac{1}{2}m\bar{k}^2\dot{\theta}_m^2$$

$$= \frac{1}{2}m\left(r_a^2 + \bar{k}^2\right)\dot{\theta}_m^2$$

$$V_2 = 0$$

For simple harmonic motion, $\dot{\theta}_m = \omega_n\theta_m$

Conservation of energy. $T_1 + V_1 = T_2 + V_2$

$$0 + \frac{1}{2}mgr_a\theta_m^2 = \frac{1}{2}m\left(r_a^2 + \bar{k}^2\right)\omega_n^2\,\theta_m^2 + 0$$

$$\omega_n^2 = \frac{gr_a}{r_a^2 + \bar{k}^2} \quad \text{or} \quad \bar{k}^2 = \frac{gr_a}{\omega_n^2} - r_a^2$$

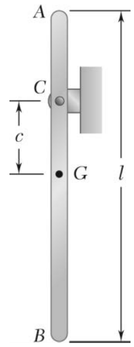
Data: $\tau_n = 1.03\text{ s} \qquad \omega_n = \frac{2\pi}{\tau_n} = \frac{2\pi}{1.03} = 6.1002\text{ rad/s}$

$$r_a = 6\text{ in.} = 0.5\text{ ft} \qquad g = 32.2\text{ ft/s}^2$$

$$\bar{k}^2 = \frac{(32.2)(0.5)}{(6.1002)^2} - (0.5)^2 = 0.43265 - 0.25 = 0.18265\text{ ft}^2$$

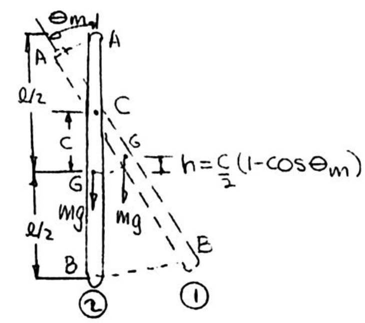
$\bar{k} = 0.42738\text{ ft}$ $\qquad \bar{k} = 5.13\text{ in.}$ ◀

PROBLEM 19.75



A uniform rod AB can rotate in a vertical plane about a horizontal axis at C located at a distance c above the mass center G of the rod. For small oscillations, determine the value of c for which the frequency of the motion will be maximum.

SOLUTION



Find ω_n as a function of c.

Datum at ②:

Position ①

$$T_1 = 0 \qquad V_1 = mgh$$

$$V_1 = mgc(1-\cos\theta_m)$$

$$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_1 = mgc\frac{\theta_m^2}{2}$$

Position ②

$$T_2 = \frac{1}{2}I_C\dot{\theta}_m^2$$

$$I_C = \overline{I} + mc^2 = \frac{1}{12}ml^2 + mc^2$$

$$T_2 = \frac{1}{2}m\left(\frac{l^2}{12}+c^2\right)\dot{\theta}_m^2 \qquad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2 \quad 0 + mgc\frac{\theta_m^2}{2} = m\left(\frac{l^2}{12}+c^2\right)\frac{\dot{\theta}_m^2}{2} + 0$$

$$\dot{\theta}_m = \omega_n\theta_m$$

$$gc = m\left(\frac{l^2}{12}+c^2\right)\omega_n^2$$

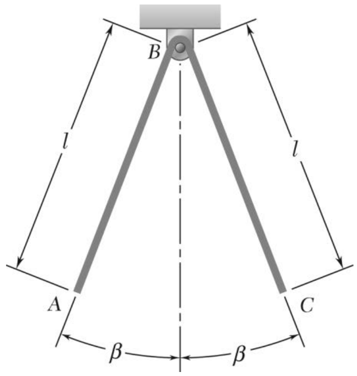
$$\omega_n^2 = \frac{gc}{\left(\frac{l^2}{12}+c^2\right)}$$

Maximum c, when

$$\frac{d\omega_n^2}{dc} = 0 = \frac{g\left(\frac{l^2}{12}+c^2\right)-2c^2g}{\left(\frac{l^2}{12}+c^2\right)} = 0$$

$$\frac{l^2}{12} - c^2 = 0 \qquad\qquad c = \frac{l}{\sqrt{12}} \blacktriangleleft$$

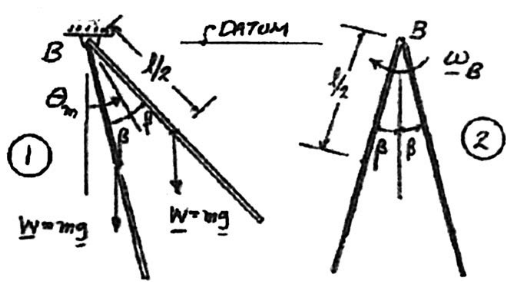
PROBLEM 19.76



A homogeneous wire of length $2l$ is bent as shown and allowed to oscillate about a frictionless pin at B. Denoting by τ_0 the period of small oscillations when $\beta = 0$, determine the angle β for which the period of small oscillations is $2\tau_0$.

SOLUTION

We denote by m the mass of half the wire.



Position ① Maximum deflections:

$$T_1 = 0, \quad V_1 = -mg\frac{l}{2}\cos(\theta_m - \beta) - mg\frac{l}{2}\cos(\theta_m + \beta)$$

$$= -mg\frac{l}{2}(\cos\theta_m\cos\beta + \sin\theta_m\sin\beta + \cos\theta_m\cos\beta - \sin\theta_m\sin\beta)$$

$$V_1 = -mgl\cos\beta\cos\theta_m$$

For small oscillations, $\cos\theta_m \approx 1 - \frac{1}{2}\theta_m^2$

$$V_1 = -mgl\cos\beta + \frac{1}{2}mgl\cos\beta\,\theta_m^2$$

Position ② Maximum velocity:

$$T_2 = \frac{1}{2}I_B\dot{\theta}_m^2 \quad \text{but} \quad I_B = 2\left(\frac{1}{3}ml^2\right)$$

Thus, $$T_2 = \frac{1}{2}\left(\frac{2}{3}ml^2\right)\dot{\theta}_m^2$$

$$V_2 = -2mg\left(\frac{l}{2}\cos\beta\right) = -mgl\cos\beta$$

PROBLEM 19.76 (Continued)

Conservation of energy. $T_1 + V_1 = T_2 + V_2$

$$-mgl\cos\beta + \frac{1}{2}mgl\cos\beta\,\theta_m^2 = \frac{1}{3}ml^2\dot{\theta}_m^2 - mgl\cos\beta$$

Setting $\dot{\theta}_m = \theta_m\omega_n$, $\frac{1}{2}mgl\cos\beta\,\theta_m^2 = \frac{1}{3}ml^2\,\theta_m^2\omega_n^2$

$$\omega_n^2 = \frac{3}{2}\frac{g}{l}\cos\beta \qquad \tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{2l}{3g\cos\beta}} \qquad (1)$$

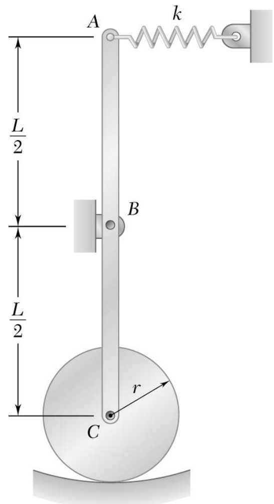
But for $\beta = 0$, $\tau_0 = 2\pi\sqrt{\frac{2l}{3g}}$

For $\tau = 2\tau_0$, $2\pi\sqrt{\frac{2l}{3g\cos\beta}} = 2\left(2\pi\sqrt{\frac{2l}{3g}}\right)$

Squaring and reducing,

$$\frac{1}{\cos\beta} = 4 \qquad \cos\beta = \frac{1}{4}$$

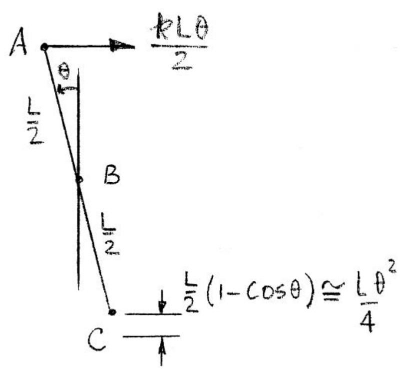
$\beta = 75.5°$ ◀



PROBLEM 19.77

A uniform disk of radius r and mass m can roll without slipping on a cylindrical surface and is attached to bar ABC of length L and negligible mass. The bar is attached to a spring of constant k and can rotate freely in the vertical plane about Point B. Knowing that end A is given a small displacement and released, determine the frequency of the resulting oscillations in terms of m, L, k, and g.

SOLUTION



$$V = \frac{1}{2}k\left(\frac{L\theta}{2}\right)^2 + mg\frac{L\theta^2}{4}$$

$$T = \frac{1}{2}m\left(\frac{L^2\dot{\theta}^2}{4}\right) + \frac{1}{2}\frac{mr^2}{2}\left(\frac{L^2\dot{\theta}^2}{4r^2}\right)$$

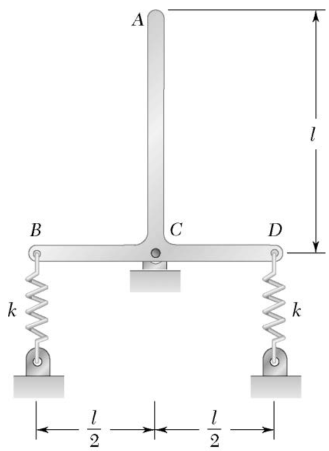
$$= \frac{3mL^2\dot{\theta}^2}{16}$$

$$\omega_n^2 = \frac{\frac{kL^2}{8} + \frac{mgL}{4}}{\frac{3mL^2}{16}}$$

$$= \frac{2}{3}\left(\frac{k}{m} + \frac{2g}{L}\right)$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{2k}{3m} + \frac{4g}{3L}} \blacktriangleleft$$

PROBLEM 19.78

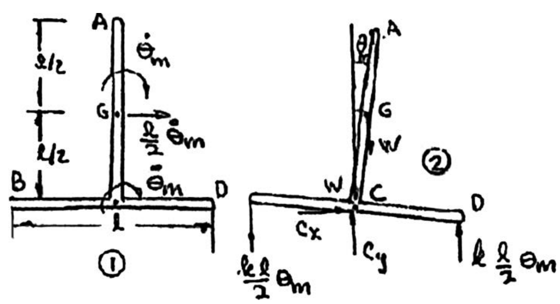


Two uniform rods, each of weight $W = 1.2$ lb and length $l = 8$ in., are welded together to form the assembly shown. Knowing that the constant of each spring is $k = 0.6$ lb/in. and that end A is given a small displacement and released, determine the frequency of the resulting motion.

SOLUTION

Mass and moment of inertia of one rod.
$$m = \frac{W}{g} = \frac{1.2}{32.2} = 0.037267 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}(0.037267)\left(\frac{8}{12}\right)^2 = 1.38026\times10^{-3} \text{ lb}\cdot\text{s}^2\cdot\text{ft}$$



Approximation.

$$\sin\theta_m \approx \tan\theta_m \approx \theta_m$$
$$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{1}{2}\theta_m^2$$

Spring constant: $k = 0.6 \text{ lb/in.} = 7.2 \text{ lb/ft}$

Position ①
$$T_1 = 2\left(\frac{1}{2}\bar{I}\dot{\theta}_m^2\right) + \frac{1}{2}m\left(\frac{l}{2}\dot{\theta}_m\right)^2$$
$$= (2)\left(\frac{1}{2}\right)(1.38026\times10^{-3})\dot{\theta}_m^2 + \left(\frac{1}{2}\right)(0.037267)\left(\frac{4}{12}\dot{\theta}_m\right)^2$$
$$= 3.4506\times10^{-3}\dot{\theta}_m^2$$
$$V_1 = 0$$

PROBLEM 19.78 (Continued)

Position ②

$$T_2 = 0$$

$$V_2 = -\frac{Wl}{2}(1-\cos\theta_m) + 2\left(\frac{1}{2}\right)k\left(\frac{l}{2}\theta_m\right)^2$$

$$\approx -\frac{(1.2)(0.66667)}{2}\left(\frac{1}{2}\theta_m^2\right) + (2)\left(\frac{1}{2}\right)(7.2)\left(\frac{0.66667}{2}\theta_m\right)^2$$

$$\approx 0.6\theta_m^2$$

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2$$

$$3.4506\times10^{-3}\dot{\theta}_m^2 + 0 = 0 + 0.6\theta_m^2$$

$$\dot{\theta}_m = 13.186\theta_m$$

Simple harmonic motion.

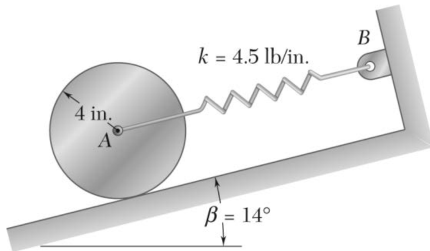
$$\dot{\theta}_m = \omega_n\theta_m$$

$$\omega_n = 13.186 \text{ rad/s}$$

Frequency.

$$f_n = \frac{\omega_n}{2\pi}$$

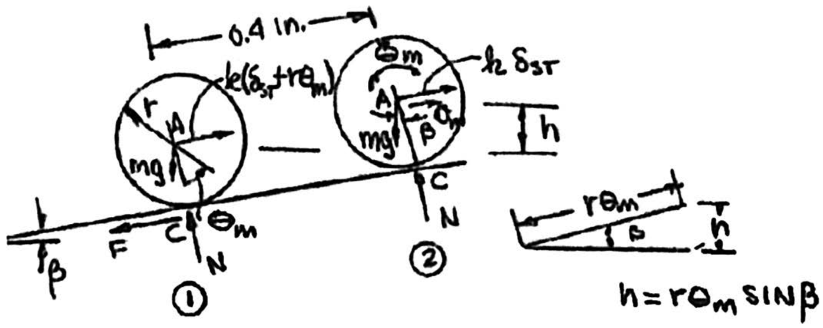
$f_n = 2.10$ Hz ◀



PROBLEM 19.79

A 15-lb uniform cylinder can roll without sliding on an incline and is attached to a spring *AB* as shown. If the center of the cylinder is moved 0.4 in. down the incline and released, determine (*a*) the period of vibration, (*b*) the maximum velocity of the center of the cylinder.

SOLUTION



(*a*) Position ①

$$T_1 = 0 \quad V_1 = \frac{1}{2}k(\delta_{st} + r\theta_m)^2$$

Position ②

$$T_2 = \frac{1}{2}\bar{I}\dot{\theta}_m^2 + \frac{1}{2}m\bar{v}_m^2$$

$$V_2 = mgh + \frac{1}{2}k(\delta_{st})^2$$

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + \frac{1}{2}k(\delta_{st} + r\theta_m)^2 = \frac{1}{2}\bar{I}\dot{\theta}_m^2 + \frac{1}{2}m\bar{v}_m^2 + mgh + \frac{1}{2}k(\delta_{st})^2$$

$$k\delta_{st}^2 + 2k\delta_{st}r\theta_m + kr^2\theta_m^2 = \bar{I}\dot{\theta}_m^2 + mv_m^2 + 2mgh + k\delta_{st}^2 \tag{1}$$

When the disk is in equilibrium,

$$\circlearrowleft + \Sigma M_c = 0 = mg\sin\beta r - k\delta_{st}r$$

Also, $h = r\sin\beta\,\theta_m$

Thus,

$$mgh - k\delta_{st}r = 0 \tag{2}$$

Substituting Eq. (2) into Eq. (1)

$$kr^2\theta_m^2 = \bar{I}\dot{\theta}_m^2 + m\bar{v}_m^2$$

$$\dot{\theta}_m = \omega_n\theta_m \quad v_m = r\dot{\theta}_m = r\omega_n\theta_m$$

$$kr^2\theta_m^2 = (\bar{I} + mr^2)\theta_m^2\omega_n^2$$

$$\omega_n^2 = \frac{kr^2}{\bar{J} + mr^2} \qquad \bar{I} = \frac{1}{2}mr^2$$

$$\omega_n^2 = \frac{kr^2}{\frac{1}{2}mr^2 + mr^2} = \frac{2}{3}\frac{k}{m}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{2}{3}\frac{(4.5\times 12\text{ lb/ft})}{\left(\frac{15\text{ lb}}{32.2}\text{ ft/s}^2\right)}}} \qquad \tau_n = 0.715\text{ s} \blacktriangleleft$$

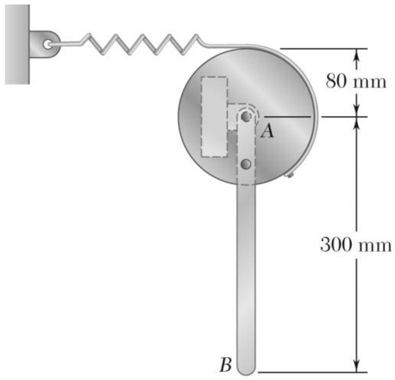
(*b*) <u>Maximum velocity</u>.

$$v_m = r\dot{\theta}_m$$

$$\dot{\theta}_m = \theta_m\omega_n$$

$$v_m = r\theta_m\omega_n \qquad r\theta_m = \frac{0.4}{12}\text{ ft}$$

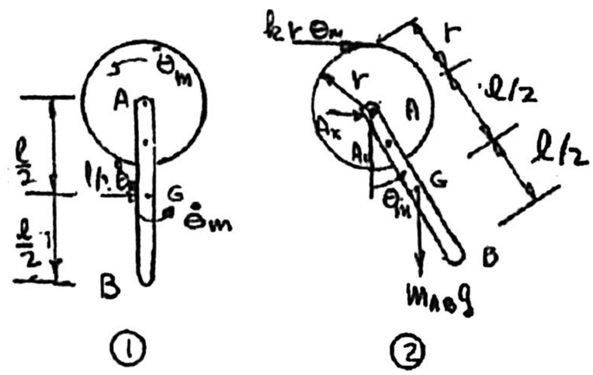
$$v_m = \left(\frac{0.4}{12}\text{ ft}\right)\left(\frac{2\pi}{0.715\text{ s}}\right) \qquad v_m = 0.293\text{ ft/s} \blacktriangleleft$$



PROBLEM 19.80

A 3-kg slender rod *AB* is bolted to a 5-kg uniform disk. A spring of constant 280 N/m is attached to the disk and is unstretched in the position shown. If end *B* of the rod is given a small displacement and released, determine the period of vibration of the system.

SOLUTION



$$r = 0.08 \text{ m}$$
$$l = 0.3 \text{ m}$$

<u>*Position* ①</u>

$$T_1 = \frac{1}{2}\bar{I}_{\text{disk}}\dot{\theta}_m^2 + \frac{1}{2}(I_A)_{\text{rod}}\dot{\theta}_m^2$$

$$V_1 = 0$$

$$I_{\text{disk}} = \frac{1}{2}m_0 r^2$$

$$I_{A\text{ rod}} = \frac{1}{3}m_{AB}l^2$$

<u>*Position* ②</u>

$$T_2 = 0$$

$$V_2 = \frac{1}{2}k(r\theta_m)^2 + m_{AB}g\frac{l}{2}(1-\cos\theta_m)$$

$$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_2 = \frac{1}{2}kr^2\theta_m^2 + \frac{m_{AB}g\left(\frac{l}{2}\right)}{2}\theta_m^2$$

Conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad \frac{1}{2}\left(\frac{1}{2}m_0 r^2 + \frac{1}{3}m_{AB} l^2\right)\dot{\theta}_m^2 + 0 = 0 + \frac{1}{2}kr^2\theta_m^2 + \frac{1}{2}m_{AB}g\frac{l}{2}\theta_m^2$$

For simple harmonic motion,

$$\dot{\theta}_m = \omega_n \theta_m$$

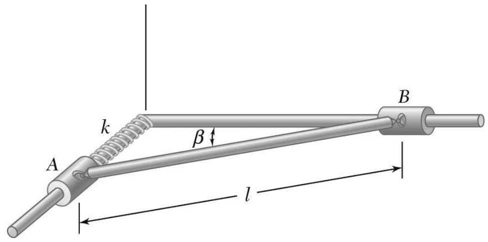
$$\left(\frac{1}{2}m_0 t^2 + \frac{1}{3}m_{AB} l^2\right)\omega_n^2\theta_m^2 = \left(kr^2 + m_{AB}g\frac{l}{2}\right)\theta_m^2$$

$$\omega_n^2 = \frac{kr^2 + m_{AB}gl}{\frac{1}{2}m_0 r^2 + \frac{1}{3}m_{AB} l^2}$$

$$\omega_n^2 = \frac{(280 \text{ N/m})(0.08 \text{ m})^2 + (3 \text{ kg})(9.81 \text{ m/s}^2)\left(\frac{0.3}{2}\text{ m}\right)}{\frac{1}{2}(5 \text{ kg})(0.08 \text{ m})^2 + \frac{1}{3}(3 \text{ kg})(0.300 \text{ m})^2}$$

$$\omega_n^2 = \frac{6.207}{0.106} = 58.55$$

Period of vibration. $\quad \tau_n = \dfrac{2\pi}{\omega_n} = \dfrac{2\pi}{\sqrt{58.55}}$ $\quad \tau_n = 0.821 \text{ s}$ ◀



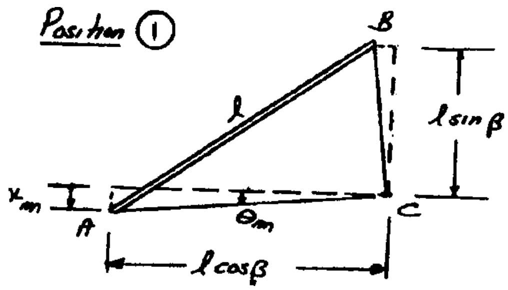
PROBLEM 19.81

A slender rod AB of mass m and length l is connected to two collars of negligible mass in a horizontal plane as shown. Collar A is attached to a spring of constant k. Knowing that the collars can slide freely on their respective rods and the system is in equilibrium in the position shown, determine the period of vibration if collar A is given a small displacement and released.

SOLUTION

Moment of inertia: $\bar{I} = \frac{1}{12} ml^2$

Position ① Maximum deflection: Let collar A be moved a small distance x_m as shown. Since the movement is horizontal, there is no change in gravitational potential energy.



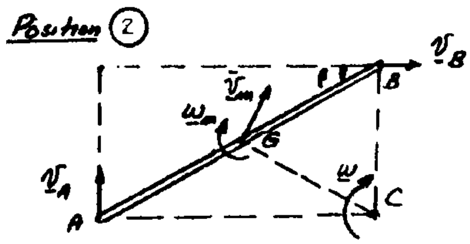
$$x_m = (l\cos\beta)\theta_m$$

$$V_1 = \frac{1}{2}kx_m^2 = \frac{1}{2}k(l\cos\beta\theta_m)^2$$

$$V_1 = \frac{1}{2}kl^2\cos^2\beta\theta_m^2$$

$$T_1 = 0$$

Position ② Maximum velocity: The instantaneous center of rotation lies at Point C, the intersection of lines perpendicular, respectively, to $\mathbf{v}_A$ and $\mathbf{v}_B$.



$$\bar{v}_m = (GC)\omega_m = \frac{1}{2}l\omega_m$$

$$T_2 = \frac{1}{2}m\bar{v}_m^2 + \frac{1}{2}\bar{I}\omega_m^2$$

$$= \frac{1}{2}m\left(\frac{1}{2}l\omega_m\right)^2 + \frac{1}{2}\left(\frac{1}{12}ml^2\right)\omega_m^2$$

$$T_2 = \frac{1}{6}ml^2\omega_m^2$$

But, $\omega_m = -\dot{\theta}_m$

so that $T_2 = \frac{1}{6}ml^2\dot{\theta}_m^2$

$$V_2 = 0$$

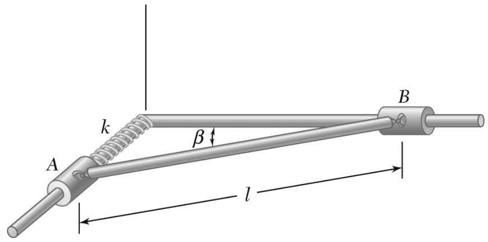
PROBLEM 19.81 (Continued)

For simple harmonic motion, $\dot{\theta}_m = \omega_n \theta_m$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$: $0 + \frac{1}{2} k l^2 \cos^2 \beta \theta_m^2 = \frac{1}{6} m l^2 \omega_n^2 \theta_m^2 + 0$

Natural frequency: $\omega_n^2 = \frac{3k}{m} \cos^2 \beta$

Period of vibration: $\tau = \frac{2\pi}{\omega_n}$ $\tau = 2\pi \sqrt{m/3k \cos^2 \beta}$ ◀



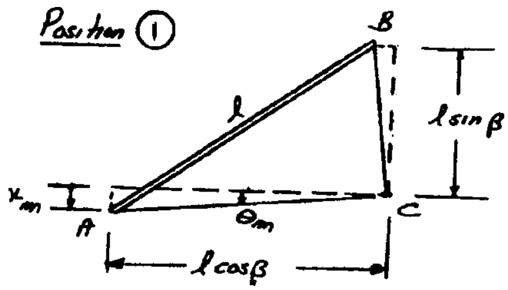
PROBLEM 19.82

A slender rod AB of mass m and length l is connected to two collars of mass m_C in a horizontal plane as shown. Collar A is attached to a spring of constant k. Knowing that the collars can slide freely on their respective rods and the system is in equilibrium in the position shown, determine the period of vibration if collar A is given a small displacement and released.

SOLUTION

Moment of inertia of rod: $\bar{I} = \frac{1}{12}ml^2$

Position ① Maximum deflection: Let collar A be moved a small distance x_m as shown. Since the movement is horizontal, there is no change in gravitational potential energy.



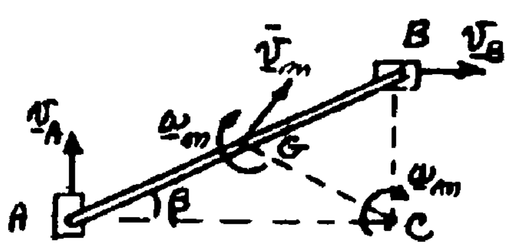
$$x_m = (l\cos\beta)\theta_m$$

$$V_1 = \frac{1}{2}kx_m^2 = \frac{1}{2}k(l\cos\beta\theta_m)^2$$

$$V_1 = \frac{1}{2}kl^2\cos^2\beta\theta_m^2$$

$$T_1 = 0$$

Position ② Maximum velocity: The instantaneous center of rotation lies at Point C, the intersection of lines perpendicular, respectively, to $\mathbf{v}_A$ and $\mathbf{v}_B$.



$$v_A = (AC)\omega_m = l\cos\beta\omega_m$$

$$v_B = (BC)\omega_m = l\sin\beta\omega_m$$

$$\bar{v}_m = (CG)\omega_m = \frac{1}{2}l\omega_m$$

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega_m^2 + \frac{1}{2}m_C v_A^2 + \frac{1}{2}m_C v_B^2$$

$$= \frac{1}{2}m\left(\frac{1}{2}lv_m\right)^2 + \frac{1}{2}\left(\frac{1}{12}ml^2\right)\omega_m^2$$

$$+\frac{1}{2}m_C(l\sin\beta\omega_m)^2 + \frac{1}{2}m_C(l\cos\beta\omega_m)^2$$

$$= \frac{1}{2}\frac{1}{3}ml^2\omega_m^2 + \frac{1}{2}m_C l^2(\sin^2\beta + \cos^2\beta)\omega_m^2$$

$$T_2 = \frac{1}{2}\left(\frac{1}{3}m + m_C\right)l^2\omega_m^2$$

PROBLEM 19.82 (Continued)

But, $\omega_m = -\dot{\theta}_m$

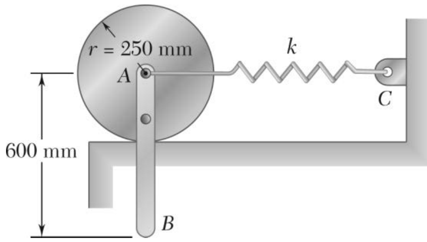
so that $T_2 = \frac{1}{2}\left(\frac{1}{3}m + m_C\right)l^2\dot{\theta}_m^2$

$V_2 = 0$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$: $\quad 0 + \frac{1}{2}kl^2\cos^2\beta\theta_m^2 = \frac{1}{2}\left(\frac{1}{3}m + m_C\right)l^2\omega_n^2\theta_m^2$

Natural frequency: $\omega_n^2 = \dfrac{k\cos^2\beta}{\frac{m}{3} + m_C}$

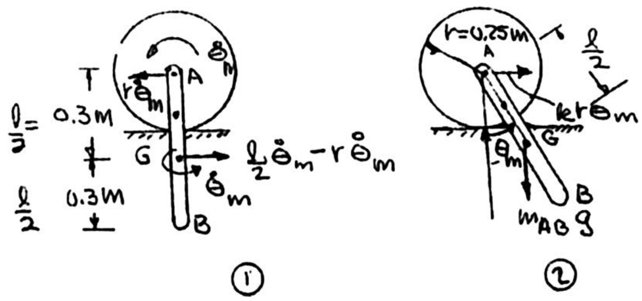
Period of vibration: $\tau = \dfrac{2\pi}{\omega_n}$ $\qquad \tau = 2\pi\sqrt{\left(\frac{m}{3} + m_C\right)/k\cos^2\beta}$ ◀



PROBLEM 19.83

An 800-g rod AB is bolted to a 1.2-kg disk. A spring of constant $k = 12$ N/m is attached to the center of the disk at A and to the wall at C. Knowing that the disk rolls without sliding, determine the period of small oscillations of the system.

SOLUTION



<u>*Position* ①</u>

$$T_1 = \frac{1}{2}(\bar{I}_G)_{AB}\dot{\theta}_m^2 + \frac{1}{2}m_{AB}\left(\frac{l}{2}-r\right)^2\dot{\theta}_m + \frac{1}{2}(\bar{I}_G)_{\text{disk}}\dot{\theta}_m^2 + \frac{1}{2}m_{\text{disk}}r^2\dot{\theta}_m^2$$

$$(\bar{I}_G)_{AB} = \frac{1}{12}ml^2 = \frac{1}{12}(0.8)(0.6)^2 = 0.024 \text{ kg}\cdot\text{m}^2$$

$$m_{AB}\left(\frac{l}{2}-r\right)^2 = (0.8)(0.3-0.25)^2 = 0.002 \text{ kg}\cdot\text{m}^2$$

$$(I_G)_{\text{disk}} = \frac{1}{2}m_{\text{disk}}r^2 = \frac{1}{2}(1.2)(0.25)^2 = 0.0375 \text{ kg}\cdot\text{m}^2$$

$$m_{\text{disk}}r^2 = 1.2(0.25)^2 = 0.0750 \text{ kg}\cdot\text{m}^2$$

$$T_1 = \frac{1}{2}\left[0.024 + 0.002 + 0.0375 + 0.0750\right]\dot{\theta}_m^2$$

$$T_1 = \frac{1}{2}[0.1385]\dot{\theta}_m^2$$

$$V_1 = 0$$

<u>*Position* ②</u>

$$T_2 = 0$$

$$V_2 = \frac{1}{2}k(r\theta_m)^2 + m_{AB}g\frac{l}{2}(1-\cos\theta_m)$$

$$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2} \text{ (small angles)}$$

$$V_2 = \frac{1}{2}(12 \text{ N/m})(0.25 \text{ m})^2\theta_m^2 + (0.8 \text{ kg})(9.81 \text{ m/s}^2)\left(\frac{0.6 \text{ m}}{2}\right)\frac{\theta_m^2}{2}$$

PROBLEM 19.83 (Continued)

$$V_2 = \frac{1}{2}[0.750 + 2.354]\theta_m^2$$

$$= \frac{1}{2}(3.104)\ \theta_m^2\ \text{N}\cdot\text{m}$$

$$T_1 + V_1 = T_2 + V_2$$

$$\dot{\theta}_m^2 = \omega_n^2 \theta_m^2$$

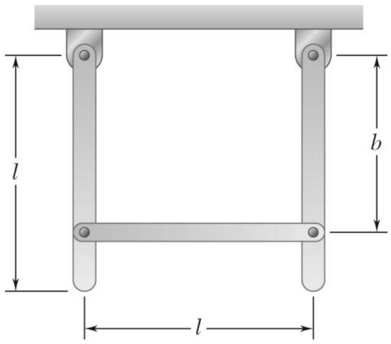
$$\frac{1}{2}(0.1385)\theta_m^2 \omega_n^2 + 0 = 0 + \frac{1}{2}(3.104)\theta_m^2$$

$$\omega_n^2 = \frac{(3.104\ \text{N}\cdot\text{m})}{(0.1385\ \text{kg}\cdot\text{m}^2)}$$

$$= 22.41\ \text{s}^{-2}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{22.41}}$$

$\tau_n = 1.327\ \text{s}$ ◀



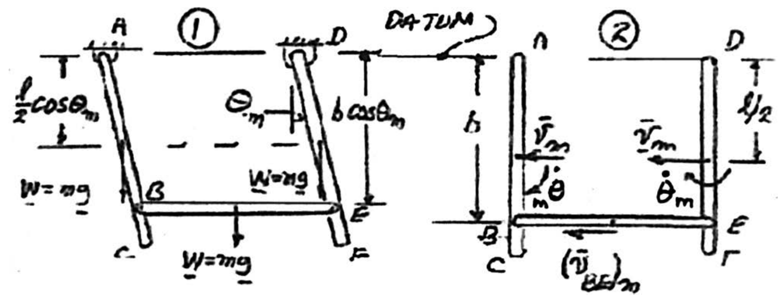
PROBLEM 19.84

Three identical rods are connected as shown. If $b = \frac{3}{4}l$, determine the frequency of small oscillations of the system.

SOLUTION

$l =$ length of each rod

$m =$ mass of each rod



Kinematics:

$$\bar{v}_m = \frac{l}{2}\dot{\theta}_m$$

$$(\bar{v}_{BE})_m = b\dot{\theta}_m$$

<u>*Position* ①</u>

$$T_1 = 0$$

$$V_1 = -2mg\frac{l}{2}\cos\theta_m - mgb\cos\theta_m$$

$$V_1 = -mg(l+b)\cos\theta_m$$

<u>*Position* ②</u>

$$V_2 = -2mg\frac{l}{2} - mgb$$

$$= -mg(l+b)$$

$$T_2 = 2\left[\frac{1}{2}\bar{I}\dot{\theta}_m^2 + \frac{1}{2}m\bar{v}_m^2\right] + \frac{1}{2}m(\bar{v}_{BE})_m^2$$

$$= \frac{1}{12}ml^2\dot{\theta}_m^2 + m\left(\frac{l}{2}\dot{\theta}_m\right)^2 + \frac{1}{2}m(b\dot{\theta}_m)^2$$

$$T_2 = \left(\frac{1}{3}l^2 + \frac{1}{2}b^2\right)m\dot{\theta}_m^2$$

PROBLEM 19.84 (Continued)

Conservation of energy. $T_1 + V_1 = T_2 + V_2$: $0 - mg(l+b)\cos\theta_m = \left(\frac{1}{3}l^2 + \frac{1}{2}b^2\right)m\dot{\theta}_m^2 - mg(l+b)$

$$mg(l+b)(1-\cos\theta_m) = \left(\frac{1}{3}l^2 + \frac{1}{2}b^2\right)m\dot{\theta}_m^2$$

For small oscillations, $(1-\cos\theta_m) = \frac{1}{2}\theta_m^2$

$$\frac{1}{2}mg(l+b)\theta_m^2 = \left(\frac{1}{3}l^2 + \frac{1}{2}b^2\right)m\dot{\theta}_m^2$$

But for simple harmonic motion, $\dot{\theta}_m = \omega_n\theta_m$: $\frac{1}{2}mg(l+b)\theta_m^2 = \left(\frac{1}{3}l^2 + \frac{1}{2}b^2\right)m(\omega_n\theta_m)^2$

$$\omega_n^2 = \frac{1}{2}g\frac{l+b}{\frac{1}{3}l^2 + \frac{1}{2}b^2}$$

or

$$\omega_n^2 = 3g\frac{l+b}{2l^2 + 3b^2} \qquad (1)$$

For $b = \frac{3}{4}l$, we have

$$\omega_n^2 = 3g\frac{l + \frac{3}{4}l}{2l^2 + 3\left(\frac{3}{4}l\right)^2}$$

$$= 3g\frac{\frac{7}{4}l}{\frac{59}{16}l^2}$$

$$= 1.4237\frac{g}{l}$$

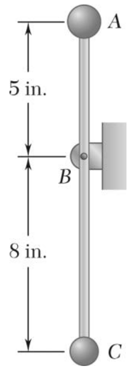
$$\omega_n = 1.1932\sqrt{\frac{g}{l}}$$

$$f_n = \frac{\omega_n}{2\pi}$$

$$= \frac{1.1932}{2\pi}\sqrt{\frac{g}{l}}$$

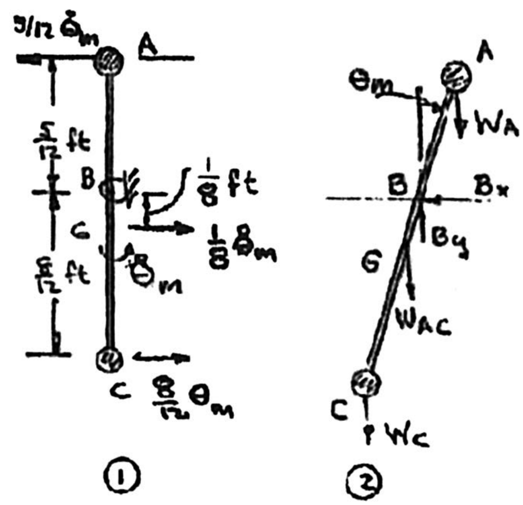
$f_n = 0.1899\sqrt{\frac{g}{l}}$ ◀

PROBLEM 19.85



A 14-oz sphere A and a 10-oz sphere C are attached to the ends of a 20-oz rod AC which can rotate in a vertical plane about an axis at B. Determine the period of small oscillations of the rod.

SOLUTION



Position ①

$$T_1 = \frac{1}{2}\frac{W_A}{g}\left(\frac{5}{12}\dot{\theta}_m\right)^2 + \frac{1}{2}\frac{W_C}{g}\left(\frac{8}{12}\dot{\theta}_m\right)^2 + \frac{1}{2}\frac{W_{AC}}{g}\left(\frac{1}{8}\dot{\theta}_m\right)^2 + \frac{1}{2}\overline{I}_{AC}\dot{\theta}_m^2$$

$$\overline{I}_{AC} = \frac{1}{12}\frac{W_{AC}}{g}\left(\frac{13}{12}\right)^2$$

$$T_1 = \frac{1}{2g}\left[\frac{14}{16}\left(\frac{5}{12}\right)^2 + \frac{10}{16}\left(\frac{8}{12}\right)^2 + \frac{20}{16}\left(\frac{1}{8}\right)^2 + \frac{1}{12}\left(\frac{20}{16}\right)\left(\frac{13}{12}\right)^2\right]\dot{\theta}_m^2$$

$$T_1 = \frac{1}{2(32.2 \text{ ft/s}^2)}[0.1519 + 0.2778 + 0.01953 + 0.1223]\dot{\theta}_m^2$$

$$T_1 = \frac{1}{2}\left(\frac{0.5715 \text{ lb}\cdot\text{ft}^2}{32.2 \text{ ft/s}^2}\right)\dot{\theta}_m^2 = \frac{1}{2}(0.01775)\dot{\theta}_m^2 (\text{lb}\cdot\text{ft})$$

$$V_1 = 0$$

Position ②

$$T_2 = 0$$

$$V_2 = -W_A \frac{5}{12}(1-\cos\theta_m) + W_C \frac{8}{12}(1-\cos\theta_m) + W_{AC}\frac{1}{8}(1-\cos\theta_m)$$

$$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_2 = \left[-\left(\frac{14}{16}\right)\left(\frac{5}{12}\right) + \left(\frac{10}{16}\right)\left(\frac{8}{12}\right) + \left(\frac{20}{16}\right)\left(\frac{1}{8}\right)\right]\frac{\theta_m^2}{2}\ (\text{lb}\cdot\text{ft})$$

$$V_2 = [-0.3646 + 0.4167 + 0.1563]\frac{\theta_m^2}{2}$$

$$V_2 = \frac{0.2084\theta_m^2}{2}$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2$: $\frac{1}{2}(0.01775)\dot{\theta}_m^2 + 0 = 0 + \frac{0.2084}{2}\theta_m^2$

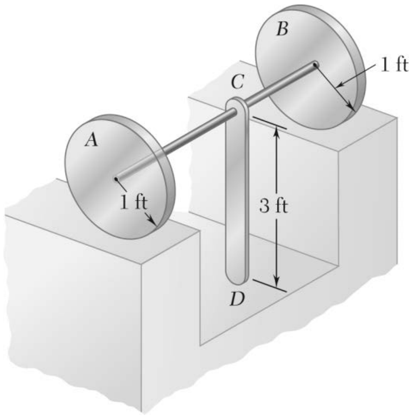
Simple harmonic motion.

$$\dot{\theta}_m = \omega_n\theta_m$$

$$\omega_n^2 = \frac{0.2084}{0.01775} = 11.738$$

$$\tau_n = \frac{2\pi}{\sqrt{\omega_n}} = \frac{2\pi}{\sqrt{11.738}}$$

$\tau_n = 1.834\text{ s}$ ◀

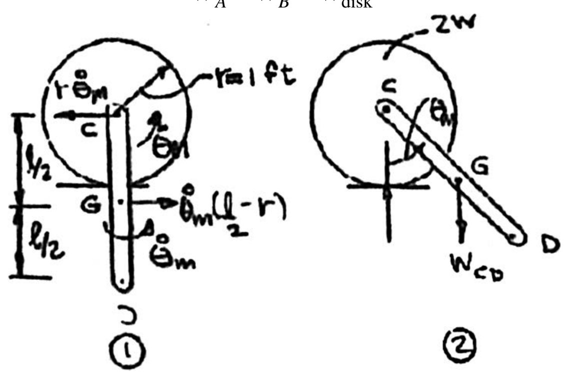


PROBLEM 19.86

A 10-lb uniform rod *CD* is welded at *C* to a shaft of negligible mass which is welded to the centers of two 20-lb uniform disks *A* and *B*. Knowing that the disks roll without sliding, determine the period of small oscillations of the system.

SOLUTION

$$W_A = W_B = W_{\text{disk}}$$



<u>*Position* ①</u>

$$T_1 = \frac{1}{2}2(\bar{I}_A)_{\text{disk}}\,\dot{\theta}_m^2 + \frac{1}{2}\left(\frac{2W_{\text{disk}}}{g}\right)(r\dot{\theta}_m)^2$$

$$+\frac{1}{2}\bar{I}_{CD}\dot{\theta}_m^2 + \frac{1}{2}\frac{W_{CD}}{g}\left(\frac{l}{2}-r\right)^2\dot{\theta}_m^2$$

$$(\bar{I}_A)_{\text{disk}} = \frac{1}{2}\frac{W_{\text{disk}}}{g}r^2 = \frac{1}{2}\frac{(20)}{g}(1)^2 = \frac{10}{g}$$

$$\bar{I}_{CD} = \frac{1}{12}\frac{W_{CD}}{g}l^2 = \frac{1}{12}\frac{(10)}{g}(3)^2 = \frac{15}{2g}$$

$$T_1 = \frac{1}{2g}\left[20+40+\frac{15}{2}+\frac{5}{2}\right]\dot{\theta}_m^2$$

$$T_1 = \frac{1}{2}g(70)\dot{\theta}_m^2 \qquad V_1 = 0$$

<u>*Position* ②</u>

$$T_2 = 0$$

$$V_2 = W_{CD}\frac{l}{2}(1-\cos\theta_m)$$

PROBLEM 19.86 (Continued)

Small angles: $$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_2 = \frac{1}{2}W_{CD}l\frac{\theta_m^2}{2}$$

$$= \frac{1}{2}(10)(1.5)\theta_m^2$$

$$= \frac{1}{2}(15)\theta_m^2$$

Conservation of energy and simple harmonic motion.

$$T_1 + V_1 = T_2 + V_2$$

$$\dot{\theta}_m = \omega_n\theta_m$$

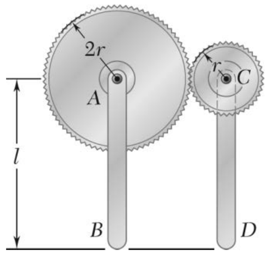
$$\frac{1}{2g}(70)\omega_n^2\theta_m^2 + 0 = 0 + \frac{1}{2}(15)\theta_m^2$$

$$\omega_n^2 = \frac{15g}{70}$$

Period of oscillations. $$\tau_n = \frac{2\pi}{\sqrt{\omega_n}} = 2\pi\sqrt{\frac{70}{(15)(32.2)}}$$

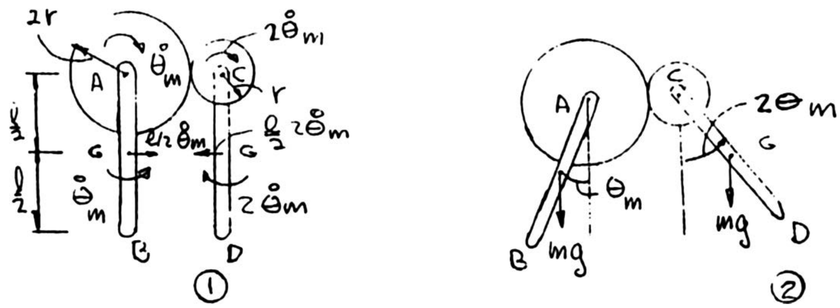
$\tau_n = 2.39 \text{ s}$ ◀

PROBLEM 19.87



Two uniform rods *AB* and *CD*, each of length *l* and mass *m*, are attached to gears as shown. Knowing that the mass of gear *C* is *m* and that the mass of gear *A* is 4*m*, determine the period of small oscillations of the system.

SOLUTION



Kinematics:

$$2r\theta_A = r\theta_C$$
$$2\theta_A = \theta_C$$
$$2\dot{\theta}_A = \dot{\theta}_C$$

Let

$$\theta_A = \theta_m$$
$$2\theta_m = (\theta_C)_m$$
$$2\dot{\theta}_m = (\dot{\theta}_C)_m$$

Position ①

$$T_1 = \frac{1}{2}\bar{I}_A\dot{\theta}_m^2 + \frac{1}{2}\bar{I}_C(2\dot{\theta}_m)^2 + \frac{1}{2}\bar{I}_{AB}\dot{\theta}_m^2 + \frac{1}{2}\bar{I}_{CD}(2\dot{\theta}_m)^2 + \frac{1}{2}m_{AB}\left(\frac{l}{2}\dot{\theta}_m\right)^2 + \frac{1}{2}m_{CD}\left(\frac{l}{2}2\dot{\theta}_m\right)^2$$

$$\bar{I}_A = \frac{1}{2}(4m)(2r)^2 = 8mr^2$$

$$\bar{I}_C = \frac{1}{2}(m)(r)^2 = \frac{1}{2}mr^2$$

$$\bar{I}_{AB} = \frac{1}{12}ml^2 \qquad \bar{I}_{CD} = \frac{1}{12}ml^2$$

$$T_1 = \frac{1}{2}m\left[8r^2 + \left(\frac{r^2}{2}\right)4 + \frac{l^2}{12} + \frac{l^2}{3} + \frac{l^2}{4} + l^2\right]$$

$$T_1 = \frac{1}{2}m\left[10r^2 + \frac{5}{3}l^2\right]\dot{\theta}_m^2$$

$$V_1 = 0$$

PROBLEM 19.87 (Continued)

<u>*Position* ②</u>

$$T_1 = 0$$

$$V_1 = mg\frac{l}{2}(1-\cos\theta_m) + \frac{mgl}{2}(1-\cos 2\theta_m)$$

For small angles,

$$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$1-\cos 2\theta_m = 2\sin^2\theta_m \approx 2\theta_m^2$$

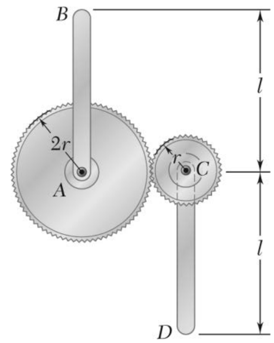
$$V_1 = \frac{1}{2}mgl\left(\frac{\theta_m^2}{2} + 2\theta_m^2\right) = \frac{1}{2}mgl\frac{5\theta_m^2}{2}$$

$$T_1 + V_1 = T_2 + V_2 \qquad \dot{\theta}_m^2 = \omega_n^2\theta_m^2$$

$$\frac{1}{2}m\left[10r^2 + \frac{5}{3}l^2\right]\omega_n^2\theta_m^2 + 0 = 0 + \frac{1}{2}mgl\frac{5\theta_m^2}{2}$$

$$\omega_n^2 = \frac{\frac{5}{2}gl}{10r^2 + \frac{5}{3}l^2}$$

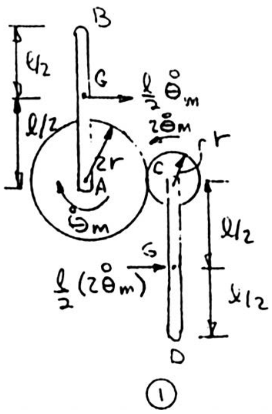
$$= \frac{3gl}{12r^2 + 2l^2} \qquad \tau_n = \frac{2\pi}{\sqrt{\omega_n}} = 2\pi\sqrt{\frac{12r^2+2l^2}{3gl}} \blacktriangleleft$$

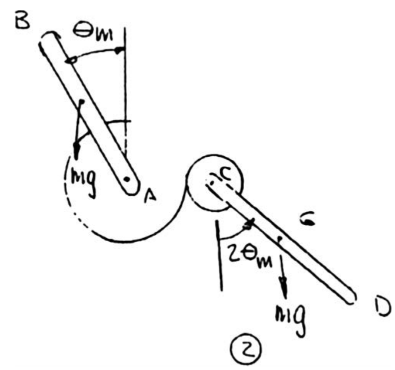


PROBLEM 19.88

Two uniform rods *AB* and *CD*, each of length *l* and mass *m*, are attached to gears as shown. Knowing that the mass of gear *C* is *m* and that the mass of gear *A* is 4*m*, determine the period of small oscillations of the system.

SOLUTION





Kinematics: $2r\theta_A = r\theta_C \qquad 2\theta_A = \theta_C$

$$2\dot{\theta}_A = \dot{\theta}_C$$

Let $\theta_A = \theta_m \qquad 2\theta_m = (\theta_C)_m$

$$2\dot{\theta}_m = (\dot{\theta}_C)_m$$

<u>*Position ①*</u>

$$T_1 = \frac{1}{2}\bar{I}_A\dot{\theta}_m^2 + \frac{1}{2}\bar{I}_C(2\dot{\theta}_m)^2 + \frac{1}{2}\bar{I}_{AB}\dot{\theta}_m^2 + \frac{1}{2}\bar{I}_{CD}(2\dot{\theta}_m)^2 + \frac{1}{2}m_{AB}\left(\frac{l}{2}\dot{\theta}_m\right)^2 + \frac{1}{2}m_{CD}\left(\frac{l}{2}2\dot{\theta}_m\right)^2$$

$$\bar{I}_A = \frac{1}{2}(4m)(2r)^2 = 8mr^2$$

$$\bar{I}_C = \frac{1}{2}(m)(r^2) = \frac{1}{2}mr^2$$

$$\bar{I}_{AB} = \frac{1}{12}ml^2 \qquad \bar{I}_{CD} = \frac{1}{12}ml^2$$

$$T_1 = \frac{1}{2}m\left[8r^2 + \left(\frac{r^2}{2}\right)4 + \frac{l^2}{12} + \frac{l^2}{3} + \frac{l^2}{4} + l^2\right]\dot{\theta}_m^2$$

$$T_1 = \frac{1}{2}m\left[10r^2 + \frac{5}{3}l^2\right]\dot{\theta}_m^2 \qquad V_1 = 0$$

PROBLEM 19.88 (Continued)

Position ②

$$T_2 = 0$$

$$V_2 = -mg\frac{l}{2}(1-\cos\theta_m) + \frac{mgl}{2}(1-\cos 2\theta_m)$$

For small angles,

$$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$1-\cos 2\theta_m = 2\sin^2\theta_m \approx 2\theta_m^2$$

$$V_2 = -mg\frac{l}{2}\frac{\theta_m^2}{2} + \frac{mgl}{2}2\theta_m^2$$

$$= \frac{1}{2}mgl\frac{3}{2}\theta_m^2$$

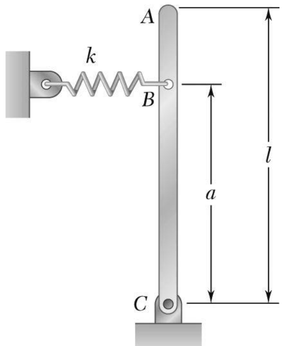
$$T_1 + V_1 = T_2 + V_2 \qquad \dot{\theta}_m = \omega_n\theta_m$$

$$\frac{1}{2}m\left[10r^2 + \frac{5}{3}l^2\right]\theta_m^2\omega_n^2 + 0 = 0 + \frac{1}{2}mgl\frac{3}{2}\theta_m^2$$

$$\omega_n^2 = \frac{\frac{3}{2}gl}{10r^2 + \frac{5}{3}l^2}$$

$$= \frac{9gl}{60r^2 + 10l^2}$$

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{60r^2 + 10l^2}{9gl}} \blacktriangleleft$$



PROBLEM 19.89

An inverted pendulum consisting of a rigid bar *ABC* of length *l* and mass *m* is supported by a pin and bracket at *C*. A spring of constant *k* is attached to the bar at *B* and is undeformed when the bar is in the vertical position shown. Determine (*a*) the frequency of small oscillations, (*b*) the smallest value of *a* for which these oscillations will occur.

SOLUTION

Moment of inertia: $\bar{I} = \frac{1}{12}ml^2$

Position ① Maximum deflection. Let rod *AC* rotate through angle θ_m. The spring stretches an amount

$$x_m = a\sin\theta_m$$

and the center of gravity moves down an amount

$$-y_m = \frac{l}{2}(1-\cos\theta_m)$$

$$V_1 = \frac{1}{2}kx_m^2 + mgy_m$$

$$= \frac{1}{2}k(a\sin\theta_m)^2 - mg\frac{l}{2}(1-\cos\theta_m)$$

$$\approx \frac{1}{2}ka^2\theta_m^2 - mg\left(\frac{l}{2}\right)\left(\frac{1}{2}\theta_m^2\right)$$

$$= \frac{1}{2}\left(ka^2 - \frac{1}{2}mgl\right)\theta_m^2$$

$$T_1 = 0$$

Position ② Maximum velocity:

For simple harmonic motion, $\dot{\theta} = -\omega_n\theta_m$

Velocity of the mass center of the rod: $\bar{v} = \frac{l}{2}\dot{\theta}$

PROBLEM 19.89 (Continued)

Kinetic energy:

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\dot{\theta}^2$$

$$= \frac{1}{2}\left[m\left(\frac{l\dot{\theta}}{2}\right)^2 + \frac{1}{12}ml^2\dot{\theta}^2\right]$$

$$= \frac{1}{2}\left[\frac{1}{3}ml^2\dot{\theta}^2\right]$$

$$= \frac{1}{2}\left(\frac{1}{3}ml^2\omega_n^2\theta_m^2\right)$$

$$V_2 = 0$$

Conservation of energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}\left(ka^2 - \frac{1}{2}mgl\right)\theta_m^2 = \frac{1}{2}\left(\frac{1}{3}ml^2\omega_n^2\theta_m^2\right)$$

$$\omega_n^2 = \frac{6\,ka^2 - 3mgl}{2ml^2}$$

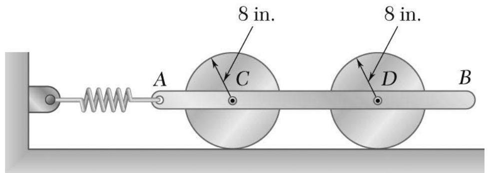
(*a*) Frequency:

$$f = 2\pi\omega_n$$

$$f = 2\pi\sqrt{(6ka^2 - 3mgl)/2ml^2} \quad ◀$$

(*b*) Smallest value of *a* for oscillations. *f* is real for $6\,ka^2 > 3mgl$

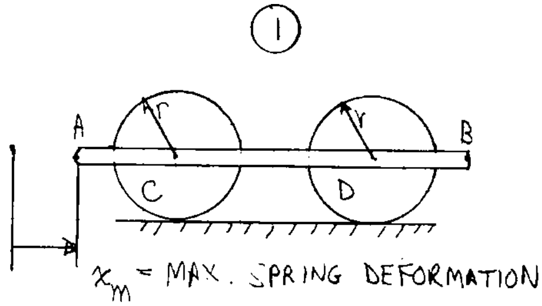
$$a > \sqrt{\frac{mgl}{2k}} \qquad a_{\min} = \sqrt{\frac{mgl}{2k}} \quad ◀$$

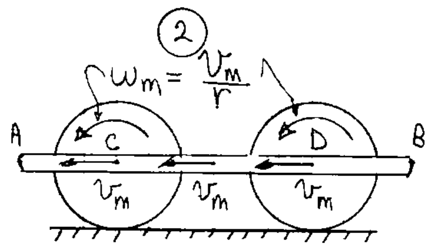


PROBLEM 19.90

Two 12-lb uniform disks are attached to the 20-lb rod AB as shown. Knowing that the constant of the spring is 30 lb/in. and that the disks roll without sliding, determine the frequency of vibration of the system.

SOLUTION





Position 1: $T_1 = 0, \qquad V_1 = \frac{1}{2}kx_m^2$

Position 2: $V_2 = 0, \qquad T_2 = \frac{1}{2}m_{AB}v_m^2 + 2\left(\frac{1}{2}\bar{I}\omega_m^2 + \frac{1}{2}m_{\text{disk}}\bar{v}_m^2\right)$

$$T_2 = \frac{1}{2}m_{AB}v_m^2 + \left(\frac{1}{2}m_{\text{disk}}r^2\right)\left(\frac{v_m}{r}\right)^2 + m_{\text{disk}}v_m^2$$

$$T_2 = \frac{1}{2}(m_{AB} + 3m_{\text{disk}})v_m^2$$

Conservation of energy

$$T_1 + V_1 = T_2 + V_2: \qquad 0 + \frac{1}{2}kx_m^2 = \frac{1}{2}(m_{AB} + 3m_{\text{disk}})v_m^2$$

But for simple harmonic motion, $v_m = \omega_n x_m$:

$$\frac{1}{2}kx_m^2 = \frac{1}{2}(m_{AB} + 3m_{\text{disk}})(\omega_n x_m)^2$$

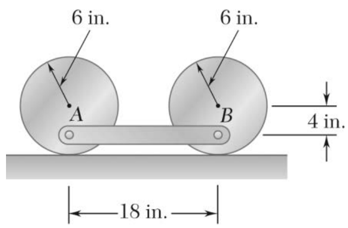
$$\omega_n^2 = \frac{k}{m_{AB} + 3m_{\text{disk}}}$$ Note: Result is independent of r

Data: $k = 30$ lb/in. $\quad W_{AB} = 20$ lb $\quad W_{\text{disk}} = 12$ lb $\quad m_{AB} = \frac{W_{AB}}{g} \quad m_{\text{disk}} = \frac{W_{\text{disk}}}{g}$

$$\omega_n^2 = \frac{30(12)\text{ lb/ft}}{20\text{ lb}/32.2 + 3(12/32.2)} = 207 \qquad \omega_n = 14.387\text{ rad/s}$$

$$f = \frac{\omega_n}{2\pi} = \frac{14.387\text{ rad/s}}{2\pi}$$

$f = 2.29$ Hz ◀



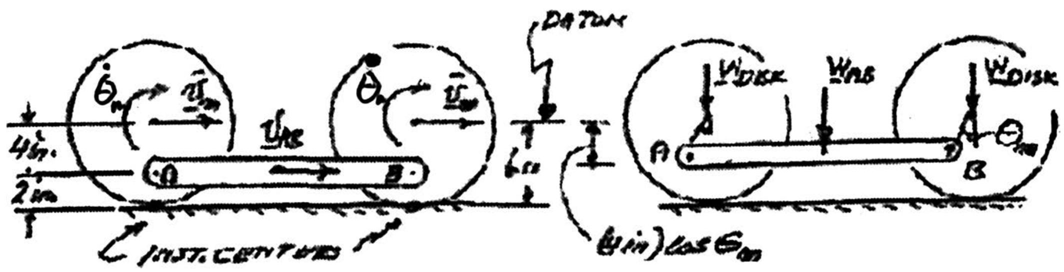
PROBLEM 19.91

The 20-lb rod *AB* is attached to two 8-lb disks as shown. Knowing that the disks roll without sliding, determine the frequency of small oscillations of the system.

SOLUTION

Position ② | *Position ①*

$r = 6$ in.



Masses and moments of inertia.

$$m_A = m_B = \frac{8}{32.2} = 0.24845 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\overline{I}_A = \overline{I}_B = \frac{1}{2}m_A r_A^2 = \frac{1}{2}(0.24845)\left(\frac{6}{12}\right)^2$$

$$= 0.031056 \text{ lb}\cdot\text{s}\cdot\text{ft}$$

$$m_{AB} = \frac{20}{32.2} = 0.62112 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Kinematics:

$$\overline{v}_m = r_A\dot{\theta}_m = \frac{6}{12}\dot{\theta}_m = 0.5\dot{\theta}_m$$

$$v_{AB} = \left(\frac{2}{12}\right)\dot{\theta}_m = \frac{1}{6}\dot{\theta}_m$$

Position ① (Maximum displacement)

$$T_1 = 0$$

$$V_1 = -W_{AB}\left(\frac{4}{12}\cos\theta_m\right) = -\frac{80}{12}\cos\theta_m$$

Position ② (Maximum speed)

$$T_2 = \frac{1}{2}m_A v_m^2 + \frac{1}{2}\overline{I}_A\dot{\theta}_m^2 + \frac{1}{2}m_B v_m^2 + \frac{1}{2}\overline{I}_B\dot{\theta}_m^2 + \frac{1}{2}m_{AB}v_{AB}^2$$

$$= 2\left[\frac{1}{2}(0.24845)(0.5\dot{\theta}_m)^2 + \frac{1}{2}(0.031056)\dot{\theta}_m^2\right] + \frac{1}{2}(0.62112)\left(\frac{1}{6}\dot{\theta}_m\right)^2$$

$$= 0.101795\dot{\theta}_m^2$$

$$V_2 = -W_{AB}\left(\frac{4}{12}\right) = -\frac{80}{12}$$

PROBLEM 19.91 (Continued)

Conservation of energy. $T_1 + V_1 = T_2 + V_2$

$$0 - \frac{80}{12}\cos\theta_m = 0.101795\dot{\theta}_m^2 - \frac{80}{12}$$

$$\dot{\theta}_m^2 = 65.491(1 - \cos\theta_m)$$

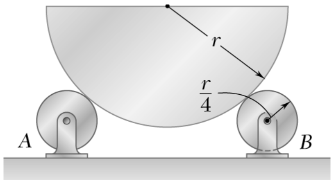
$$\approx 65.491\left(\frac{1}{2}\theta_m^2\right)$$

$$= 32.745\theta_m^2$$

$$\theta_m = 5.7224\,\theta_m$$

Simple harmonic motion. $\dot{\theta}_m = \omega_n\theta_m$ $\qquad \omega_n = 5.7224$ rad/s

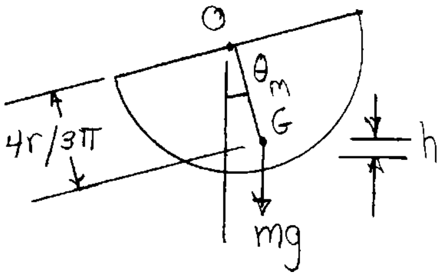
Frequency. $f_n = \dfrac{\omega_n}{2\pi} = \dfrac{5.7224}{2\pi}$ $\qquad f_n = 0.911$ Hz ◀



PROBLEM 19.92

A half section of a uniform cylinder of radius r and mass m rests on two casters A and B, each of which is a uniform cylinder of radius $r/4$ and mass $m/8$. Knowing that the half cylinder is rotated through a small angle and released and that no slipping occurs, determine the frequency of small oscillations.

SOLUTION



$$V_1 = mgh = mg\left(\frac{4r}{3\pi}\right)(1-\cos\theta)$$

$$1-\cos\theta \approx \frac{\theta_m^2}{2}$$

$$V_1 = 2mgr\,\frac{\theta_m^2}{3\pi}$$

$$T_2 = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2$$

Where $\quad I_A = I_B = \frac{1}{2}\left(\frac{m}{8}\right)\left(\frac{r}{4}\right)^2 = \frac{mr^2}{256}$

and $\quad \omega_A = \omega_B = 4\omega$

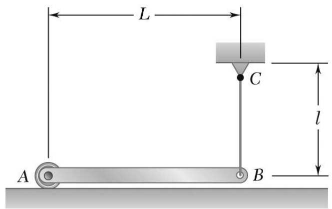
$$\therefore T_2 = \left(\frac{mr^2}{16} + \frac{mr^2}{4}\right)\omega^2 = \frac{5mr^2\omega^2}{16}$$

$$V_1 = T_2, \quad \frac{2\not{m}gr\not{\theta}_m^2}{3\pi} = \frac{5\not{m}r^2\omega_n^2\not{\theta}_m^2}{16}$$

$$\omega_n^2 = \frac{32g}{15\pi r},$$

$$f_n = \left(\frac{1}{2\pi}\right)\sqrt{\frac{32g}{15\pi r}}$$

$$f_n = 0.1312\sqrt{\frac{g}{r}} \blacktriangleleft$$



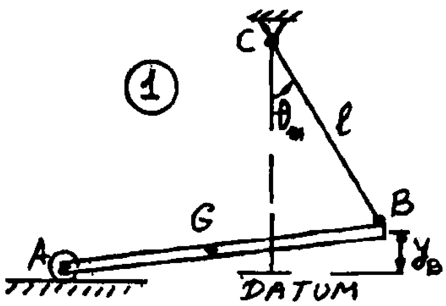
PROBLEM 19.93

The motion of the uniform rod AB is guided by the cord BC and by the small roller at A. Determine the frequency of oscillation when the end B of the rod is given a small horizontal displacement and released.

SOLUTION

Position ①. (Maximum deflection):

Let θ_m be the small angle between the cord CB and the vertical. As the rod is moved from the equilibrium position the center of gravity G moves up an amount $\bar{y}_m$.



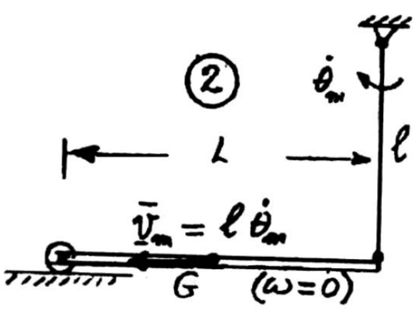
$$y_B = l(1-\cos\theta_m) \approx l\left(1-1+\frac{1}{2}\theta_m^2\right) = \frac{1}{2}l\theta_m^2$$

$$\bar{y}_m = y_G = \frac{1}{2}y_B = \frac{1}{4}l\theta_m^2$$

$$V_1 = mg\bar{y}_m = \frac{1}{4}mgl\theta_m^2$$

$$T_1 = 0,$$

Position ②. (Maximum velocity): At the equilibrium position the motion of the rod is a translation.



$$\bar{v}_m = l\omega = l\dot{\theta}_m$$

$$T_2 = \frac{1}{2}m\bar{v}_m^2 = \frac{1}{2}ml^2\dot{\theta}_m^2$$

$$V_2 = 0$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$: $\frac{1}{4}mgl\theta_m^2 = \frac{1}{2}ml^2\dot{\theta}_m^2$

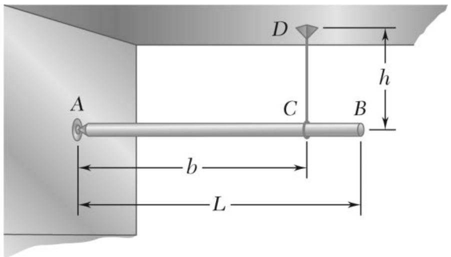
For simple harmonic motion; $\dot{\theta}_m = \omega_n\theta_m$ so that

$$\frac{1}{4}mgl\theta_m^2 = \frac{1}{2}ml^2\omega_n^2\theta_m^2$$

Natural frequency: $\omega_n^2 = \frac{g}{2l}$

$$f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{2l}}$$

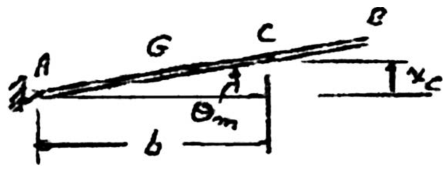
$f = 0.1125\sqrt{\frac{g}{l}}$ ◀



PROBLEM 19.94

A uniform rod of length L is supported by a ball-and-socket joint at A and by a vertical wire CD. Derive an expression for the period of oscillation of the rod if end B is given a small horizontal displacement and then released.

SOLUTION

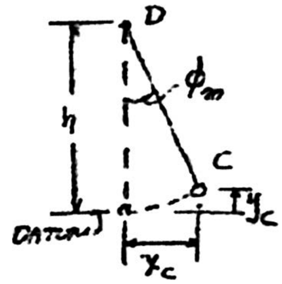


Position ① (Maximum deflection)

Looking from above:

Horizontal displacement of C: $x_C = b\theta_m$

Looking from right: $\phi_m = \dfrac{x_C}{h} = \dfrac{b}{h}\theta_m$



$$y_C = h(1-\cos\phi_m) \approx \frac{1}{2}h\phi_m^2$$

$$y_C = \frac{1}{2}h\left(\frac{b}{h}\theta_m\right)^2 = \frac{1}{2}\frac{b^2}{h}\theta_m^2$$

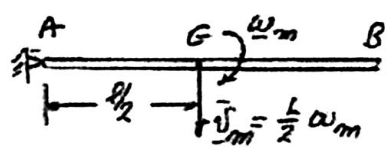
$$\bar{y}_m = y_G = \frac{AG}{AC} - y_C = \frac{\frac{1}{2}L}{b}\left(\frac{1}{2}\frac{b^2}{h}\theta_m^2\right)$$

$$\bar{y}_m = \frac{1}{4}\cdot\frac{bL}{h}\theta_m^2$$

We have $T_1 = 0$

$$V_1 = mgy_m = \frac{1}{4}\frac{mgbL}{h}\theta_m^2$$

Position ② (Maximum velocity)



Looking from above: $T_2 = \dfrac{1}{2}\bar{I}\dot{\theta}_m^2 + \dfrac{1}{2}m\bar{v}_m^2$

$$= \frac{1}{2}\left(\frac{1}{12}mL^2\right)\dot{\theta}_m^2 + \frac{1}{2}m\left(\frac{L}{2}\dot{\theta}\right)^2$$

$$T_2 = \frac{1}{6}mL^2\dot{\theta}_m^2$$

$$V_2 = 0$$

Conservation of energy. $T_1 + V_1 = T_2 + V_2$: $\quad 0 + \dfrac{1}{4}\dfrac{mgbL}{h}\theta_m^2 = \dfrac{1}{6}mL^2\dot{\theta}_m^2$

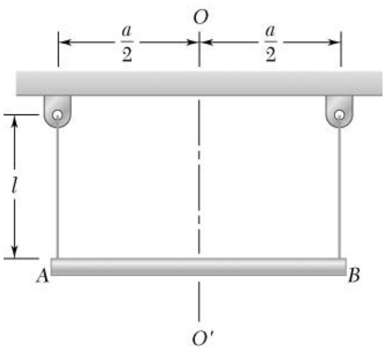
PROBLEM 19.94 (Continued)

But for simple harmonic motion, $\dot{\theta}_m = \omega_n \theta_m$

$$\frac{1}{4}\frac{mgbL}{h}\theta_m^2 = \frac{1}{6}mL^2(\omega_n\theta_m)^2$$

$$\omega_n^2 = \frac{3}{2}\frac{bg}{hL}$$

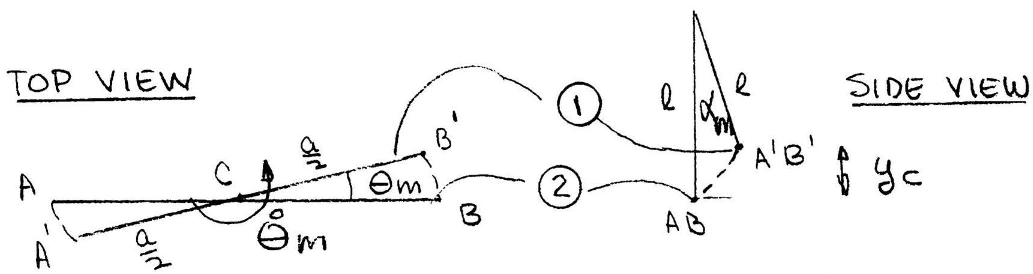
Period of vibration. $\tau_n = \dfrac{2\pi}{\omega_n}$ $\tau_n = 2\pi\sqrt{\dfrac{2hL}{3bg}}$ ◀



PROBLEM 19.95

A section of uniform pipe is suspended from two vertical cables attached at A and B. Determine the frequency of oscillation when the pipe is given a small rotation about the centroidal axis OO' and released.

SOLUTION



$$AA' = BB' = \frac{a}{2}\theta_m = l\alpha_m \qquad \alpha_m = \frac{a}{2l}\theta_m$$

Position ①

$$T_1 = 0 \qquad V_1 = mgy_c = mgl(1 - \cos\alpha)$$

For small angles

$$1 - \cos\alpha_m = 2\sin\frac{\alpha_m}{2} \approx \frac{\alpha_m^2}{2} = \frac{a^2}{8l^2}\theta_m^2$$

$$V_1 = mgl\left(\frac{a^2}{8l^2}\right)\theta_m^2$$

Position ②

$$T_2 = \frac{1}{2}\bar{I}\dot{\theta}_m^2 = \frac{1}{2}\left(\frac{1}{12}ma^2\right)\dot{\theta}_m^2 \qquad V_2 = 0$$

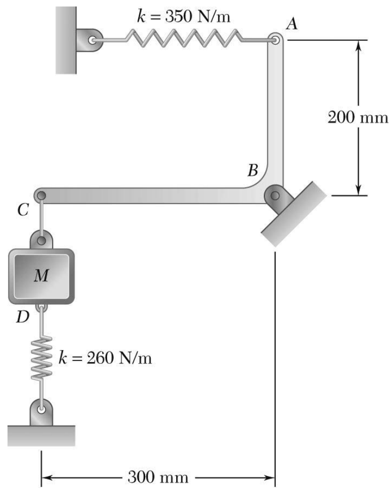
$$\dot{\theta}_m = \omega_n\theta_m$$

$$T_1 + V_1 = T_2 + V_2$$

$$mgl\left(\frac{a^2}{8l^2}\right) + 0 + \frac{1}{24}ma^2\omega_n^2\theta_m^2$$

$$\omega_n^2 = \frac{3g}{l}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{3g}{l}} \blacktriangleleft$$



PROBLEM 19.96

A 0.6-kg uniform arm *ABC* is supported by a pin at *B* and is attached to a spring at *A*. It is connected at *C* to a 1.4-kg mass M which is attached to a spring. Knowing that each spring can act in tension or compression, determine the frequency of small oscillations of the system when the weight is given a small vertical displacement and released.

SOLUTION

Data:

$$k_A = 260 \text{ N/m} \qquad k_C = 350 \text{ N/m}$$

$$l_{AB} = 0.200 \text{ m} \qquad l_{BC} = 0.300 \text{ m}$$

$$l_{ABC} = l_{BA} + l_{BA} = 0.500 \text{ m}$$

$$m_{ABC} = 0.6 \text{ kg} \qquad m_C = 1.4 \text{ kg}$$

$$m_{BA} = \frac{0.200}{0.500} m_{ABC} = \frac{2}{5}(0.6 \text{ kg}) = 0.24 \text{ kg}$$

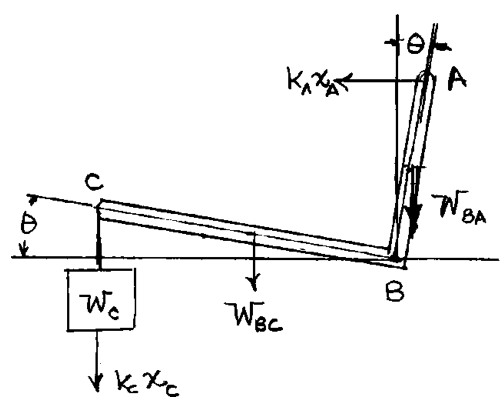
$$m_{BC} = \frac{0.300}{0.500} m_{ABC} = \frac{3}{5}(0.6 \text{ kg}) = 0.36 \text{ kg}$$

$$W_{BA} = m_{BA} g = (0.24 \text{ kg})(9.81 \text{ m/s}^2) = 2.3544 \text{ N}$$

$$W_{BC} = m_{BC} g = (0.36 \text{ kg})(9.81 \text{ m/s}^2) = 3.5316 \text{ N}$$

$$W_C = m_C g = (1.4 \text{ kg})(9.81 \text{ m/s}^2) = 13.734 \text{ N}$$

Let x_A and x_C be the amounts of stretch from their zero force lengths of the springs at locations *A* and *C*, respectively. Let θ be the small clockwise rotation of arm *ABC* about the fixed Point *B*, measured from the equilibrium position. Let $\overline{y}_{BA}$ and $\overline{y}_{BC}$ be the upward movement of the mass centers of portions *BA* and *BC* of the arm *ABC*.



Potential energy:

$$V = W_C y_C + W_{BC}\bar{y}_{BC} + W_{BA}\bar{y}_{BA} + \frac{1}{2}k_A x_A^2 + \frac{1}{2}k_B x_B^2$$

$$= W_C l_{BC}\sin\theta + W_{BC}\left(\frac{1}{2}l_{BC}\sin\theta\right) - W_{BA}\left[\frac{1}{2}l_{BA}(1-\cos\theta)\right]$$

$$+\frac{1}{2}k_A(l_{BA}\sin\theta + \delta_A)^2 + \frac{1}{2}k_C(l_{BC}\sin\theta + \delta_C)^2$$

$$= W_C l_{BC}\sin\theta + \frac{1}{2}W_{BC}l_{BC}\sin\theta - \frac{1}{2}W_{BA}l_{BA}(1-\cos\theta)$$

$$+\frac{1}{2}k_A l_{BA}^2\sin^2\theta + k_A l_{BA}\delta_A\sin\theta + \frac{1}{2}k_A\delta_A^2$$

$$+\frac{1}{2}k_C l_{BC}^2\sin^2\theta + k_C l_{BC}\delta_C\sin\theta + \frac{1}{2}k_C\delta_C^2 \qquad (1)$$

where δ_A and δ_C are the spring elongations at the equilibrium position.

In the static equilibrium position,

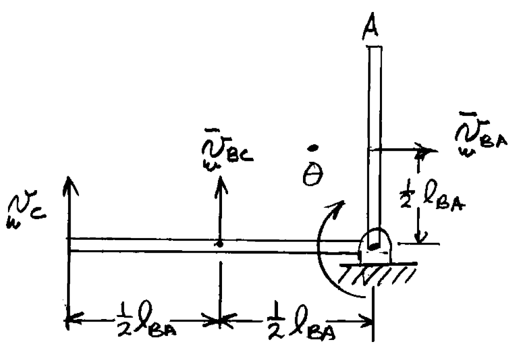
$$+\circlearrowright \Sigma M_B = 0: \quad W_C l_{BC} + \frac{1}{2}W_{BC}l_{BC} + (k_A\delta_A)l_{BA} + (k_C\delta_C)l_{BC} = 0 \qquad (2)$$

Substiting Eq. (2) into Eq. (1) gives

$$V = -\frac{1}{2}W_{BA}l_{BA}(1-\cos\theta) + \frac{1}{2}k_A l_{BA}^2\sin^2\theta$$

$$+\frac{1}{2}k_C l_{BC}^2\sin^2\theta + \frac{1}{2}k_A\delta_A^2 + \frac{1}{2}k_C\delta_C^2 \qquad (3)$$

Kinematics for position with $\theta = 0$.



$$v_C = l_{BC}\dot{\theta}$$

$$\bar{v}_{BC} = \frac{1}{2}l_{BC}\dot{\theta}$$

$$\bar{v}_{BA} = \frac{1}{2}l_{BA}\dot{\theta}$$

Kinetic energy:

$$T = \frac{1}{2}m_C v_C^2 + \frac{1}{2}m_{BC}\bar{v}_{BC}^2 + \frac{1}{2}\bar{I}_{BC}\dot{\theta}^2 + \frac{1}{2}m_{BA}\bar{v}_{BA}^2 + \frac{1}{2}\bar{I}_{BA}\dot{\theta}^2$$

$$= \frac{1}{2}m_C l_{BC}^2\dot{\theta}^2 + \frac{1}{2}m_{BC}\left(\frac{1}{2}l_{BC}\dot{\theta}\right)^2 + \frac{1}{2}\left(\frac{1}{12}m_{BC}l_{BC}^2\right)\dot{\theta}^2$$

$$+\frac{1}{2}m_{BA}\left(\frac{1}{2}l_{BA}\dot{\theta}\right)^2 + \frac{1}{2}\left(\frac{1}{12}m_{BA}l_{BA}^2\right)\dot{\theta}$$

$$= \frac{1}{2}\left(m_C l_{BC}^2 + \frac{1}{3} m_{BC} l_{BC}^2 + \frac{1}{3} m_B l_{BA}^2 \right)\dot{\theta}^2 \quad (4)$$

Conservation of energy: $T_1 + V_1 = T_2 + V_2$ (5)

Position ①. (Maximum deflection) $\theta = \theta_m$

$$T_1 = 0 \quad (6)$$

$$V_1 = -\frac{1}{2} W_{AB} l_{AB} (1 - \cos\theta_m) + \frac{1}{2} k_A l_{BA}^2 \sin^2\theta_m$$
$$+ \frac{1}{2} k_C l_{BC}^2 \sin^2\theta_m + \frac{1}{2} k_A \delta_A^2 + \frac{1}{2} k_C \delta_C^2$$

For small angle θ_m, $\sin\theta_m \approx \theta_m$

$$1 - \cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{1}{2}\theta_m^2$$

$$V_1 \approx \frac{1}{2}\left(-\frac{1}{2} W_{BA} l_{BA} + k_A l_{BA}^2 + k_C l_{BC}^2 \right)\theta_m^2$$

$$+ \frac{1}{2} k_A \delta_A^2 + \frac{1}{2} k_C \delta_C^2 \quad (7)$$

Position ②: Maximum velocity. $\theta = 0$

For simple harmonic motion $\dot{\theta} = \omega_n \theta_m$ (8)

$$T_2 = \frac{1}{2}\left(m_{BC} l_{BC}^2 + \frac{1}{3} m_{BC} l_{BC}^2 + \frac{1}{3} m_{BA} l_{BA}^2 \right)\omega_n^2 \theta_m^2 \quad (9)$$

Substituting Eqs. (6), (7), (8), and (9) into Eq. (5) and noting that the terms containing δ_A and δ_C cancel,

$$0 + \frac{1}{2}\left(-\frac{1}{2} W_{BA} l_{BC} + k_A l_{BA}^2 + k_C l_{BC}^2 \right)\theta_m^2$$
$$= \frac{1}{2}\left(m_C l_{BC}^2 + \frac{1}{3} m_{BC} l_{BC}^2 + \frac{1}{3} m_{BA} l_{BA}^2 \right)\omega_n^2 \theta_m^2 + 0$$

Applying the numerical data: $-\frac{1}{2} W_{BA} l_{BA} + k_A l_{BA}^2 + k_C l_{BC}^2$

$$= -\frac{1}{2}(2.3544)(0.2) + (350)(0.2)^2 + (260)(0.3)^2$$
$$= -0.23544 + 14.0 + 23.4 = 37.165 \text{ N} \cdot \text{m}$$

PROBLEM 19.96 (Continued)

$$m_C l_{BC}^2 + \frac{1}{3} m_{BC} l_{BC}^2 + \frac{1}{3} m_{BA} l_{BA}^2$$

$$= (1.4)(0.3)^2 + \frac{1}{3}(0.36)(0.3)^2 + \frac{1}{3}(0.24)(0.2)^2$$

$$= 0.126 + 0.0108 + 0.0032 = 0.1400 \text{ kg} \cdot \text{m}^2$$

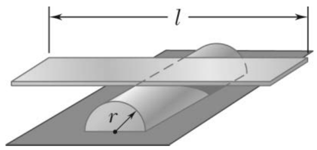
Then, $$\frac{1}{2}(37.165)\,\theta_m^2 = \frac{1}{2}(0.1400)\,\omega_n^2 \theta_m^2$$

Natural frequency: $$\omega_n^2 = \frac{37.165}{0.1400} = 265.46$$

$$\omega_n = 16.293 \text{ rad/s}$$

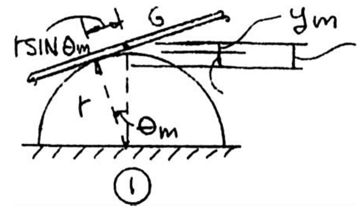
$$f = \frac{\omega_n}{2\pi}$$ $f = 2.59$ Hz ◀

PROBLEM 19.97*



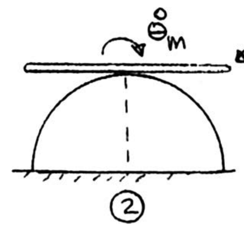
A thin plate of length l rests on a half cylinder of radius r. Derive an expression for the period of small oscillations of the plate.

SOLUTION



$$(r\sin\theta_m)\sin\theta_m \approx r\theta_m^2$$

$$r(1-\cos\theta_m) \approx r\frac{\theta_m^2}{2}$$



Position ① (Maximum deflection)

$$T_1 = 0$$

$$V_1 = Wy_m$$

$$= mgr\frac{\theta_m^2}{2}$$

Position ② $(\theta = 0)$:

$$T_2 = \frac{1}{2}\bar{I}\dot{\theta}_m^2$$

$$= \frac{1}{2}\left(\frac{1}{12}\right)ml^2\dot{\theta}_m^2$$

$$\dot{\theta}_m = \omega_n\theta_m$$

$$T_2 = \frac{1}{2}\left(\frac{1}{12}\right)ml^2\omega_n^2\theta_m^2$$

$$T_1 + V_1 = T_2 + V_2$$

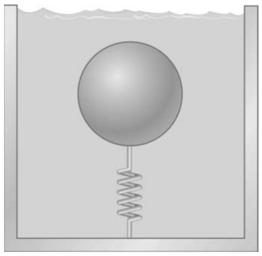
$$0 + \frac{1}{2}mgr\theta_m^2 = \frac{1}{2}\left(\frac{1}{12}\right)ml^2\omega_n^2\theta_m^2$$

$$\omega_n^2 = \frac{12gr}{l^2}$$

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{l^2}{12gr}}$$

$$\tau_n = \frac{\pi l}{\sqrt{3gr}}$$ ◀

PROBLEM 19.98*



As a submerged body moves through a fluid, the particles of the fluid flow around the body and thus acquire kinetic energy. In the case of a sphere moving in an ideal fluid, the total kinetic energy acquired by the fluid is $\frac{1}{4}\rho V v^2$, where ρ is the mass density of the fluid, V is the volume of the sphere, and v is the velocity of the sphere. Consider a 500-g hollow spherical shell of radius 80 mm, which is held submerged in a tank of water by a spring of constant 500 N/m. (*a*) Neglecting fluid friction, determine the period of vibration of the shell when it is displaced vertically and then released. (*b*) Solve Part *a,* assuming that the tank is accelerated upward at the constant rate of 8 m/s^2.

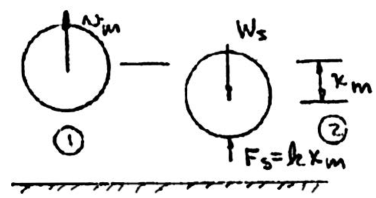
SOLUTION

This is not a damped vibration. However, the kinetic energy of the fluid must be included.

(*a*) *Position ②*

$$T_2 = 0$$

$$V_2 = \frac{1}{2}kx_m^2$$



Position ①

$$T_1 = T_{\text{spere}} + T_{\text{fluid}} = \frac{1}{2}m_s v_m^2 + \frac{1}{4}\rho V v_m^2$$

$$V_1 = 0$$

Conservation of energy and simple harmonic motion.

$$T_1 + V_1 = T_2 + V_2: \quad \frac{1}{2}m_s v_m^2 + \frac{1}{4}\rho V v_m^2 + 0 = 0 + \frac{1}{2}kx_m^2$$

$$v_m = \dot{x}_m = x_m \omega_n$$

$$\frac{1}{2}\left(m_s + \frac{1}{2}\rho V\right)x_m^2\omega_n^2 = \frac{1}{2}kx_m^2$$

$$\omega_n^2 = \frac{k}{m_s + \frac{1}{2}\rho V}$$

$$\omega_n^2 = \frac{500 \text{ N/m}}{(0.5 \text{ kg}) + \left(\frac{1}{2}\rho V\right)}$$

$$\frac{1}{2}\rho V = \frac{1}{2}(1000 \text{ kg/m}^3)\left(\frac{4}{3}\pi(0.08 \text{ m})^3\right)$$

$$\frac{1}{2}\rho V = 1.0723 \text{ kg}$$

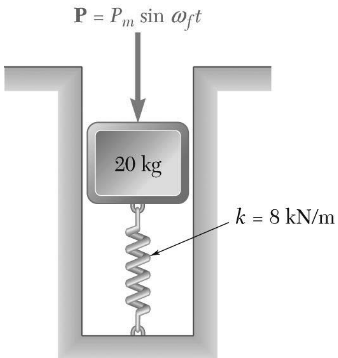
$$\omega_n^2 = \frac{500 \text{ N/m}}{(0.5 \text{ kg}) + (1.0723 \text{ kg})} = 318 \text{ s}^{-2}$$

Period of vibration.

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{318}}$$

$\tau_n = 0.352$ s ◀

(*b*) Acceleration does not change mass. $\tau_n = 0.352$ s ◀



PROBLEM 19.99

A 20-kg block is attached to a spring of constant $k = 8$ N/m and can move without friction in a vertical slot as shown. The block is acted upon by a periodic force of magnitude $P = P_m \sin \omega_f t$, where $P_m = 100$ N. Determine the amplitude of the motion of the block if (*a*) $\omega_f = 10$ rad/s, (*b*) $\omega_f = 19$ rad/s, (*c*) $\omega_f = 30$ rad/s.

SOLUTION

Equation of motion: $m\ddot{x} + kx = P_m \sin \omega_f t$

The steady state response is
$$x_m = \frac{P_m/k}{1 - \left(\frac{\omega_f}{\omega_n}\right)^2}$$

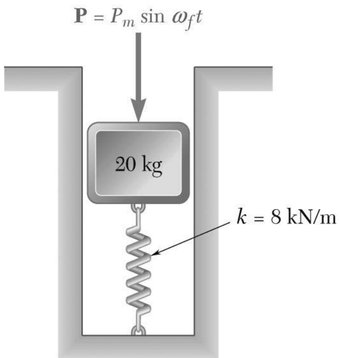
where
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{8000 \text{ N/m}}{20 \text{ kg}}} = 20 \text{ rad/s}$$

and
$$P_m/k = \frac{100 \text{ N}}{8000 \text{ N/m}} = 0.0125 \text{ m}$$

(*a*) $\omega_f = 10$ rad/s:
$$\frac{\omega_f}{\omega_n} = \frac{10}{20} = 0.5$$
$$x_m = \frac{0.0125}{1 - (0.5)^2} = 0.01667 \text{ m}$$
$x_m = 166.7$ mm ◀
(in-phase)

(*b*) $\omega_f = 19$ rad/s:
$$\frac{\omega_f}{\omega_n} = \frac{19}{20} = 0.95$$
$$x_m = \frac{0.0125}{1 - (0.95)^2} = 0.1282 \text{ m}$$
$x_m = 128.2$ mm ◀
(in-phase)

(*c*) $\omega_f = 30$ rad/s:
$$\frac{\omega_f}{\omega_n} = \frac{30}{20} = 1.5$$
$$x_m = \frac{0.0125}{1 - (1.5)^2} = -0.0100 \text{ m}$$
$x_m = 10.00$ mm ◀
(out-of-phase)



PROBLEM 19.100

A 20-kg block is attached to a spring of constant $k = 8$ kN/m and can move without friction in a vertical slot as shown. The block is acted upon by a periodic force of magnitude $P = P_m \sin \omega_f t$, where $P_m = 10$ N. Knowing that the amplitude of the motion is 3 mm, determine the value of ω_f.

SOLUTION

Equation of motion: $m\ddot{x} + kx = P_m \sin \omega_f t$

The steady state response is
$$x_m = \frac{P_m/k}{1 - \left(\frac{\omega_f}{\omega_n}\right)^2}$$

where
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{8000 \text{ N/m}}{20 \text{ kg}}} = 20 \text{ rad/s}$$

and
$$P_m/k = \frac{10 \text{ N}}{8000 \text{ N/m}} = 0.00125 \text{ m}$$

Solve for ω_f/ω_n:
$$\frac{\omega_f}{\omega_n} = \left(1 - \frac{P_m}{kx_m}\right)^{1/2}$$

The amplitude is 3 mm so that $x_m = \pm 0.003$ m.

so that
$$\frac{P_m}{kx_m} = \frac{0.00125 \text{ m}}{\pm 0.003} = \pm 0.41667$$

For the in-phase motion,
$$\frac{\omega_f}{\omega_n} = (1 - 0.41667)^{1/2} = 0.76376$$

$$\omega_f = (0.76376)(20 \text{ rad/s})$$

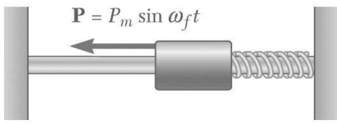
$\omega_f = 15.28$ rad/s ◀

For the out-of-phase motion,
$$\frac{\omega_f}{\omega_n} = (1 + 0.41667)^{1/2} = 1.19024$$

$$\omega_f = (1.19024)(20 \text{ rad/s})$$

$\omega_f = 23.8$ rad/s ◀

PROBLEM 19.101



A 9-lb collar can slide on a frictionless horizontal rod and is attached to a spring of constant k. It is acted upon by a periodic force of magnitude $P = P_m \sin \omega_f t$, where $P_m = 2$ lb and $\omega_f = 5$ rad/s. Determine the value of the spring constant k knowing that the motion of the collar has an amplitude of 6 in. and is (*a*) in phase with the applied force, (*b*) out of phase with the applied force.

SOLUTION

Eq. (19.33): $$x_m = \frac{\frac{P_m}{k}}{1-\left(\frac{\omega_f}{\omega_n}\right)^2} \qquad \omega_n^2 = \frac{k}{m}$$

$$x_m = \frac{P_m}{k - m\omega_f^2}$$

$$k = \frac{P_m}{x_m} + m\omega_f^2$$

<u>Data</u>: $$P_m = 2 \text{ lb}, \qquad m = \frac{W}{g} = \frac{9}{32.2} = 0.2795 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$\omega_f = 5 \text{ rad/s}$$

$$k = \frac{P_m}{x_m} + (0.2795)(5)^2$$

$$= \frac{P_m}{x_m} + 6.9876$$

(*a*) (In phase) $$x_m = 6 \text{ in.} = 0.5 \text{ ft}$$

$$k = \frac{2}{0.5} + 6.9876$$

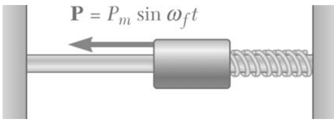
$k = 10.99$ lb/ft ◀

(*b*) (Out of phase) $$x_m = -6 \text{ in.} = -0.5 \text{ ft}$$

$$k = \frac{2}{-0.5} + 6.9876$$

$k = 2.99$ lb/ft ◀

PROBLEM 19.102



A collar of mass m which slides on a frictionless horizontal rod is attached to a spring of constant k and is acted upon by a periodic force of magnitude $P = P_m \sin \omega_f t$. Determine the range of values of ω_f for which the amplitude of the vibration exceeds two times the static deflection caused by a constant force of magnitude P_m.

SOLUTION

Circular natural frequency. $\omega_n = \sqrt{\dfrac{k}{m}}$

For forced vibration, the equation of motion is

$$m\ddot{x} + kx = P_m \sin(\omega_f t + \varphi)$$

The amplitude of vibration is

$$x_m = \frac{\frac{P_m}{k}}{\left|1 - \left(\frac{\omega_f}{\omega_n}\right)^2\right|} = \frac{\delta_{st}}{\left|\left(1 - \frac{\omega_f}{\omega_n}\right)^2\right|}$$

For $\omega_f < \omega_n$ and $x_m = 2\,\delta_{st}$, we have

$$2\,\delta_{st} = \frac{\delta_{st}}{1 - \left(\frac{\omega_f}{\omega_n}\right)^2} \quad \text{or} \quad 1 - \left(\frac{\omega_f}{\omega_n}\right)^2 = \frac{1}{2}$$

$$\omega_f^2 = \frac{1}{2}\omega_n^2 = \frac{1}{2}\frac{k}{m} \qquad \omega_f = \sqrt{\frac{k}{2m}} \qquad (1)$$

For $\sqrt{\dfrac{k}{2m}} < \omega_f \le \omega_n,$ $|x_m|$ exceeds $2\delta_{st}$

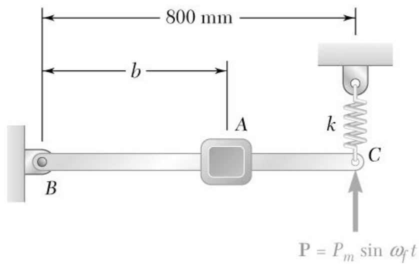
For $\omega_f > \omega_n$ and $x_m = 2\delta_{st}$, we have

$$2\delta_{st} = \frac{\delta_{st}}{(\omega_f - \omega_n)^2 - 1} \quad \text{or} \quad \frac{\omega_f^2}{\omega_n^2} - 1 = \frac{1}{2}$$

$$\omega_f^2 = \frac{3}{2}\omega_n^2 = \frac{3}{2}\frac{k}{m} \qquad \omega_f = \sqrt{\frac{3k}{2m}} \qquad (2)$$

For $\omega_n \le \omega_f \le \sqrt{\dfrac{3k}{2m}}$ $|x_m|$ exceeds $2\delta_{st}$

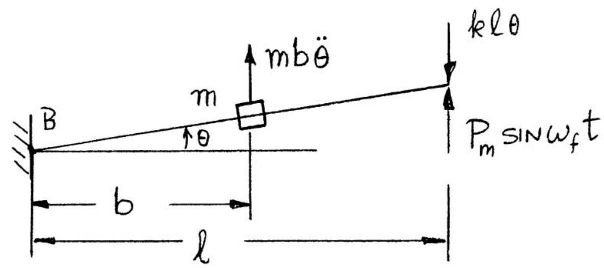
From Eqs. (1) and (2), Range: $\sqrt{\dfrac{k}{2m}} < \omega_f < \sqrt{\dfrac{3k}{2m}}$ ◀



PROBLEM 19.103

A small 20-kg block A is attached to the rod BC of negligible mass which is supported at B by a pin and bracket and at C by a spring of constant $k = 2$ kN/m. The system can move in a vertical plane and is in equilibrium when the rod is horizontal. The rod is acted upon at C by a periodic force **P** of magnitude $P = P_m \sin \omega_f t$, where $P_m = 6$ N. Knowing that $b = 200$ mm, determine the range of values of ω_f for which the amplitude of vibration of block A exceeds 3.5 mm.

SOLUTION



$$+\circlearrowleft \Sigma M_B = mb^2\ddot{\theta} = -kl^2\theta + P_m l \sin \omega_f t$$

$$mb^2\ddot{\theta} + kl^2\theta = P_m l \sin \omega_f t$$

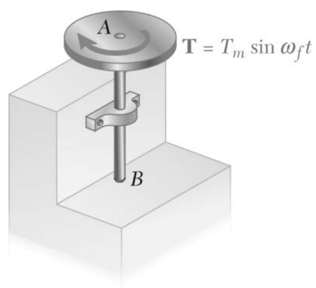
$$\omega_n = \sqrt{\frac{kl^2}{mb^2}} = 40 \text{ rad/s}, \ \theta = \theta_m \sin \omega_f t$$

$$\theta_m = \frac{\pm 3.5 \text{ mm}}{b} = \pm 0.0175 \text{ rad} = \frac{\frac{P_m l}{mb^2}}{\omega_n^2 - \omega_f^2} = \frac{6}{1600 - \omega_f^2}$$

Lower frequency: $6 = 0.0175\left(1600 - \omega_f^2\right), \quad \omega_f = 35.5 \text{ rad/s}$

Upper frequency: $6 = -0.0175\left(1600 - \omega_f^2\right), \quad \omega_f = 44.1 \text{ rad/s}$

$35.5 \text{ rad/s} < \omega_f < 44.1 \text{ rad/s}$ ◀



PROBLEM 19.104

An 8-kg uniform disk of radius 200 mm is welded to a vertical shaft with a fixed end at B. The disk rotates through an angle of $3°$ when a static couple of magnitude $50 \text{ N}\cdot\text{m}$ is applied to it. If the disk is acted upon by a periodic torsional couple of magnitude $T = T_m \sin \omega_f t$, where $T_m = 60 \text{ N}\cdot\text{m}$, determine the range of values of ω_f for which the amplitude of the vibration is less than the angle of rotation caused by a static couple of magnitude T_m.

SOLUTION

Mass moment of inertia: $$\bar{I} = \frac{1}{2}mr^2 = \frac{1}{2}(8)(0.200)^2 = 0.16 \text{ kg}\cdot\text{m}^2$$

Torsional spring constant: $$K = \frac{T}{\theta}$$

$$T = 50 \text{ N}\cdot\text{m}$$

$$\theta = 3° = 0.05236 \text{ rad}$$

$$K = \frac{50}{0.05236}$$

$$= 954.93 \text{ N}\cdot\text{m/rad}$$

Natural circular frequency: $$\omega_n = \sqrt{\frac{K}{I}} = \sqrt{\frac{954.93}{0.16}} = 77.254 \text{ rad/s}$$

For forced vibration, $$\theta_m = \frac{\frac{T_m}{K}}{1 - \left(\frac{\omega_f}{\omega_n}\right)^2} = \frac{\theta_{\text{st}}}{1 - \left(\frac{\omega_f}{\omega_n}\right)^2}$$

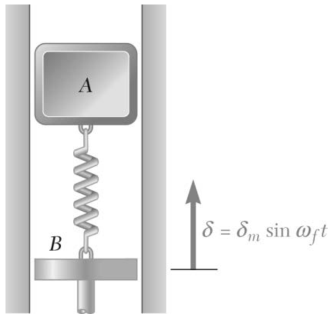
For the amplitude $|\theta_m|$ to be less than θ_{st}, we must have $\omega_f > \omega_n$.

Then $$|\theta_m| = \frac{\theta_{\text{st}}}{\left(\frac{\omega_f}{\omega_n}\right)^2 - 1} < \theta_{\text{st}}$$

$$\left(\frac{\omega_f}{\omega_n}\right)^2 - 1 > 1$$

$$\left(\frac{\omega_f}{\omega_n}\right)^2 > 2 \qquad \omega_f > \sqrt{2}\omega_n = (\sqrt{2})(77.254)$$

$$\omega_f > 109.3 \text{ rad/s} \blacktriangleleft$$



PROBLEM 19.105

An 18-lb block A slides in a vertical frictionless slot and is connected to a moving support B by means of a spring AB of constant $k = 10$ lb/in. Knowing that the displacement of the support is $\delta = \delta_m \sin \omega_f t$, where $\delta_m = 6$ in., determine the range of values of ω_f for which the amplitude of the fluctuating force exerted by the spring on the block is less than 30 lb.

SOLUTION

Natural circular frequency:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10\times 12}{\frac{18\text{ lb}}{32.2}}} = 14.652 \text{ rad/s}$$

Eq. (19.33′):

$$x_m = \frac{\delta_m}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}$$

Spring force:

$$F_m = -k(x_m - \delta_m) = -k\delta_m\left[1 - \frac{1}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}\right]$$

$$= k\delta_m \frac{\left(\frac{\omega_f}{\omega_n}\right)^2}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}$$

$$= (120)(0.50)\frac{\left(\frac{\omega_f}{\omega_n}\right)^2}{1-\left(\frac{\omega_f}{\omega_n}\right)^2} = 60\frac{\left(\frac{\omega_f}{\omega_n}\right)^2}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}$$

Limit on spring force: $|F_m| < 30$ lb

$$60\left|\frac{\left(\frac{\omega_f}{\omega_n}\right)^2}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}\right| < 30 \quad \text{or} \quad \left|\frac{\left(\frac{\omega_f}{\omega_n}\right)^2}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}\right| < \frac{1}{2}$$

In phase motion.

$$\frac{\left(\frac{\omega_f}{\omega_n}\right)^2}{1-\left(\frac{\omega_f}{\omega_n}\right)^2} < \frac{1}{2}$$

$$\left(\frac{\omega_f}{\omega_n}\right)^2 < \frac{1}{2} - \frac{1}{2}\left(\frac{\omega_f}{\omega_n}\right)^2$$

$$\frac{3}{2}\left(\frac{\omega_f}{\omega_n}\right)^2 < \frac{1}{2} \qquad \frac{\omega_f}{\omega_n} > \frac{1}{3}$$

$$\omega_f < \frac{1}{\sqrt{3}}\omega_n$$

$\omega_f < 8.46$ rad/s ◀

Out of phase motion.

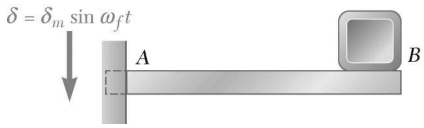
$$\frac{\left(\frac{\omega_f}{\omega_n}\right)^2}{\left(\frac{\omega_f}{\omega_n}\right)^2 - 1} < \frac{1}{2}$$

$$\left(\frac{\omega_f}{\omega_n}\right)^2 < \frac{1}{2}\left(\frac{\omega_f}{\omega_n}\right)^2 - \frac{1}{2}$$

$$\left(\frac{\omega_f}{\omega_n}\right)^2 < -\frac{1}{2}$$

No solution for ω_f.

PROBLEM 19.106



A cantilever beam AB supports a block which causes a static deflection of 8 mm at B. Assuming that the support at A undergoes a vertical periodic displacement $\delta = \delta_m \sin \omega_f t,$ where $\delta_m = 2$ mm, determine the range of values of ω_f for which the amplitude of the motion of the block will be less than 4 mm. Neglect the weight of the beam and assume that the block does not leave the beam.

SOLUTION

For the static condition. $$mg = k\delta_{st}$$

Natural circular frequency. $$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta_{st}}}$$

$$g = 9.81 \text{ m/s}, \quad \delta_{st} = 8 \text{ mm} = 0.008 \text{ m}$$

$$\omega_n = \sqrt{\frac{9.81}{0.008}} = 35.018 \text{ rad/s}$$

From Eqs. (19.31 and 19.33′): $$(x_m)_B = \frac{\delta_m}{1 - \left(\frac{\omega_f}{\omega_n}\right)^2}$$

Conditions: $$|x_m|_B < 4 \text{ mm} \qquad \delta_m = 2 \text{ mm}$$

<u>In phase motion</u>.

$$\frac{\delta_m}{1 - \left(\frac{\omega_f}{\omega_n}\right)^2} < x_m$$

$$\frac{1 - \left(\frac{\omega_f}{\omega_n}\right)^2}{\delta_m} > \frac{1}{x_m}$$

$$1 - \left(\frac{\omega_f}{\omega_n}\right)^2 > \frac{\delta_m}{x_m}$$

$$1 - \frac{\delta_m}{x_m} > \left(\frac{\omega_f}{\omega_n}\right)^2$$

$$\omega_f < \left(\sqrt{1 - \frac{\delta_m}{x_m}}\right)\omega_n$$

$$\omega_f < \left(\sqrt{1 - \frac{2}{4}}\right)(35.018)$$

$\omega_f < 24.8$ rad/s ◀

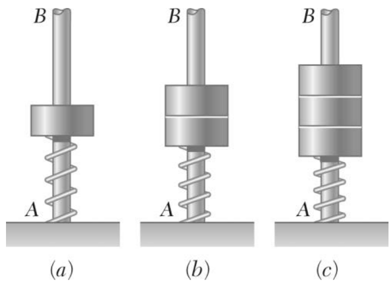
PROBLEM 19.106 (Continued)

Out of phase motion.

$$\frac{\delta_m}{\left(\frac{\omega_f}{\omega_n}\right)^2 - 1} < x_m$$

$$\frac{\left(\frac{\omega_f}{\omega_n}\right)^2 - 1}{\delta_m} > \frac{1}{x_m} \qquad \left(\frac{\omega_f}{\omega_n}\right)^2 - 1 > \frac{\delta_m}{x_m}$$

$$\left(\frac{\omega_f}{\omega_n}\right)^2 > 1.5 \qquad \omega_f > \sqrt{1.5}\,\omega_n \qquad \omega_f > 42.9 \text{ rad/s} \blacktriangleleft$$

	**PROBLEM 19.107** Rod *AB* is rigidly attached to the frame of a motor running at a constant speed. When a collar of mass *m* is placed on the spring, it is observed to vibrate with an amplitude of 15 mm. When two collars, each of mass *m*, are placed on the spring, the amplitude is observed to be 18 mm. What amplitude of vibration should be expected when three collars, each of mass *m*, are placed on the spring? (Obtain two answers.)

SOLUTION

(*a*) *One collar*: $(x_m)_1 = 15 \text{ mm} \qquad (\omega_n)_1^2 = \dfrac{k}{m}$

(*b*) *Two collars*: $(x_m)_2 = 18 \text{ mm} \qquad (\omega_n)_2^2 = \dfrac{k}{2m} = \dfrac{1}{2}(\omega_n)_1^2$

$$\left(\frac{\omega}{\omega_n}\right)_2 = \sqrt{2}\left(\frac{\omega}{\omega_n}\right)_1$$

(*c*) *Three collars*:

$$(x_m)_3 = \text{unknown}, \quad (\omega_n)_3^2 = \frac{k}{3m} = \frac{1}{3}(\omega_n)_1^2, \qquad \left(\frac{\omega}{\omega_n}\right)_3 = \sqrt{3}\left(\frac{\omega}{\omega_n}\right)_1$$

We also note that the amplitude δ_m of the displacement of the base remains constant.

Referring to Section 19.7, Figure 19.9, we note that, since $(x_m)_2 > (x_m)_1$ and $\frac{\omega}{(\omega_n)_2} > \frac{\omega}{(\omega_n)_1}$, we must have $\frac{\omega}{(\omega_n)_1} < 1$ and $(x_m)_1 > 0$. However, $\frac{\omega}{(\omega_n)_2}$ may be either < 1 or > 1, with $(x_m)_2$ being correspondingly either > 0 or < 0.

1. *Assuming* $(x_m)_2 > 0$:

For one collar,

$$(x_m)_1 = \frac{\delta_m}{1 - \left(\frac{\omega}{\omega_n}\right)_1^2} \qquad +15 \text{ mm} = \frac{\delta_m}{1 - \left(\frac{\omega}{\omega_n}\right)_1^2} \tag{1}$$

For two collars,

$$(x_m)_2 = \frac{\delta_m}{1 - \left(\frac{\omega}{\omega_n}\right)_2^2} \qquad +18 \text{ mm} = \frac{\delta_m}{1 - 2\left(\frac{\omega}{\omega_n}\right)_1^2} \tag{2}$$

Dividing Eq. (2) by Eq. (1), member by member:

$$1.2 = \frac{1-\left(\frac{\omega}{\omega_n}\right)_1^2}{1-2\left(\frac{\omega}{\omega_n}\right)_1^2}; \quad \text{we find } \left(\frac{\omega}{\omega_n}\right)_1^2 = \frac{1}{7}$$

Substituting into Eq. (1), $\delta_m = (15 \text{ mm})\left(1-\frac{1}{7}\right) = \frac{90}{7} \text{ mm}$

For three collars,

$$(x_m)_3 = \frac{\delta_m}{1-3\left(\frac{\omega}{\omega_n}\right)_1^2} = \frac{\left(\frac{90}{7}\right) \text{ mm}}{1-3\left(\frac{1}{7}\right)} = \frac{90}{4} \text{ mm}, \qquad (x_m)_3 = 22.5 \text{ mm} \blacktriangleleft$$

2. *Assuming* $(x_m)_2 < 0$:

For two collars, we have $-18 \text{ mm} = \dfrac{\delta_m}{1-2\left(\frac{\omega}{\omega_n}\right)_1^2}$ (3)

Dividing Eq. (3) by Eq. (1), member by member:

$$-1.2 = \frac{1-\left(\frac{\omega}{\omega_n}\right)_1^2}{1-2\left(\frac{\omega}{\omega_n}\right)_1^2}$$

$$-1.2 + 2.4\left(\frac{\omega}{\omega_n}\right)_1^2 = 1-\left(\frac{\omega}{\omega_n}\right)_1^2$$

$$\left(\frac{\omega}{\omega_n}\right)_1^2 = \frac{2.2}{3.4} = \frac{1.1}{1.7}$$

Substitute into Eq. (1), $\delta_m = (15 \text{ mm})\left(1-\frac{1.1}{1.7}\right) = \frac{9}{1.7} \text{ mm}$

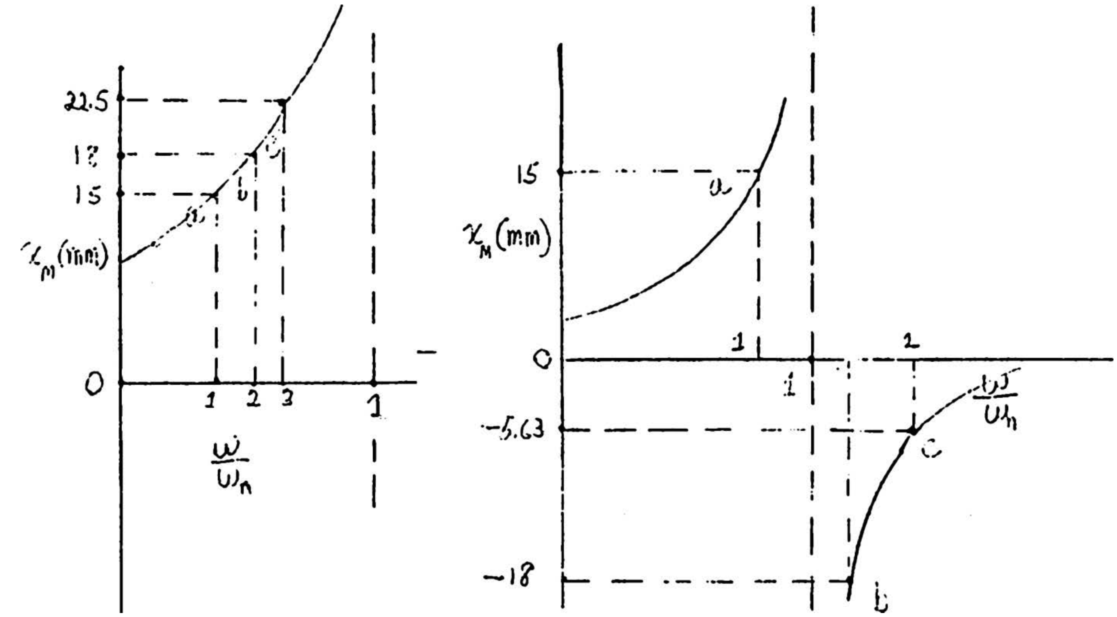
For three collars,

$$(x_m)_3 = \frac{\delta_m}{1-3\left(\frac{\omega}{\omega_n}\right)_1^2} = \frac{\left(\frac{9}{1.7}\right)}{1-3\left(\frac{1.1}{1.7}\right)} = \frac{9 \text{ mm}}{-1.6}, \qquad (x_m)_3 = -5.63 \text{ mm} \blacktriangleleft$$

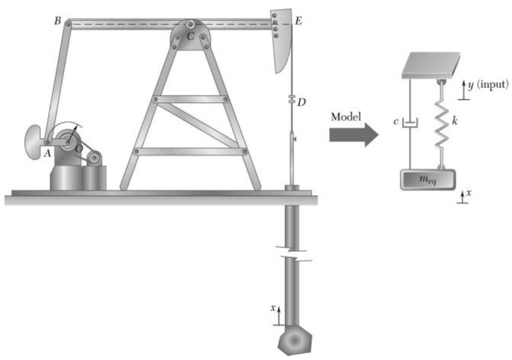
(out of phase)

PROBLEM 19.107 (Continued)

Points corresponding to the two solutions are indicated below:



PROBLEM 19.108



The crude-oil-pumping rig shown in the accompanying figure is driven at 20 rpm. The inside diameter of the well pipe is 2 in., and the diameter of the pump rod is 0.75 in. The length of the pump rod and the length of the column of oil lifted during the stroke are essentially the same, and equal to 6000 ft. During the downward stroke, a valve at the lower end of the pump rod opens to let a quantity of oil into the well pipe, and the column of oil is then lifted to obtain a discharge into the connecting pipeline. Thus, the amount of oil pumped in a given time depends upon the stroke of the lower end of the pump rod. Knowing that the upper end of the rod at *D* is essentially sinusoidal with a stroke of 45 in. and the specific weight of crude oil is 56.2 lb/ft^3, determine (*a*) the output of the well in ft^3/min if the shaft is rigid, (*b*) the output of the well in ft^3/min if the stiffness of the rod is 2210 N/m, the equivalent mass of the oil and shaft is 290 kg and damping is negligible.

SOLUTION

Forcing frequency: $\omega_f = 20 \text{ rpm} = 2.0944 \text{ rad/s}$

Cross sectional area of the flow chamber

$$A_{\text{oil}} = \frac{\pi}{4}\left[(2 \text{ in.})^2 - (0.75 \text{ in.})^2\right] = 2.6998 \text{ in}^2 = 0.018749 \text{ ft}^2$$

Let *s* be the stroke at the lower end of the pump in feet. Stroke is twice the amplitude. $s = 2x_m$

Volume of oil pumped per revolution:

$$V_{\text{oil}} = A_{\text{oil}} s = 0.018749 s$$

Amplitude of motion at top of shaft:

$$\delta_m = \frac{1}{2}(45 \text{ in.}) = 22.5 \text{ in.} = 1.875 \text{ ft}$$

Amplitude of motion at bottom of shaft:

$$x_m = \frac{\delta_m}{1 - \left(\frac{\omega_f}{\omega_n}\right)^2}$$

(*a*) Rigid shaft:

$$\omega_n = \infty$$

$$x_m = \delta_m = 1.875 \text{ ft}$$

$$s = (2)(1.875) = 3.75 \text{ ft}$$

$$V_{\text{oil}} = (0.018749 \text{ ft}^2)(3.75 \text{ ft}) = 0.070309 \text{ ft}^3/\text{rev}$$

PROBLEM 19.108 (Continued)

output rate: $(0.070309 \text{ ft}^3/\text{rev})(20 \text{ rev/min})$ $1.406 \text{ ft}^3/\text{min}$ ◄

(*b*) Flexible shaft.

$$k = 2210 \text{ N/m} \qquad m_{\text{eq}} = 290 \text{ kg}$$

$$\omega_n = \sqrt{\frac{k}{m_{\text{eq}}}} = \sqrt{\frac{2210 \text{ N/m}}{290 \text{ kg}}} = 2.7606 \text{ rad/s}$$

$$\frac{\omega_f}{\omega_n} = \frac{2.0944}{2.7606} = 0.75869$$

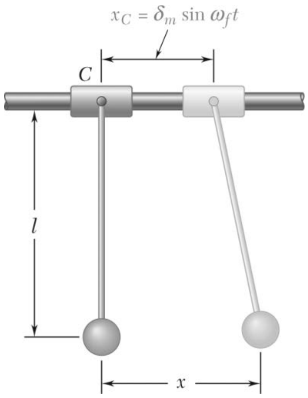
$$x_m = \frac{1.875}{1-(0.75869)^2} = 4.4178 \text{ ft}$$

$$s = (2)(4.4178) = 8.8358 \text{ ft}$$

$$V_{\text{oil}} = (0.018749 \text{ ft}^2)(8.8358 \text{ ft}) = 0.16566 \text{ ft}^3/\text{rev}$$

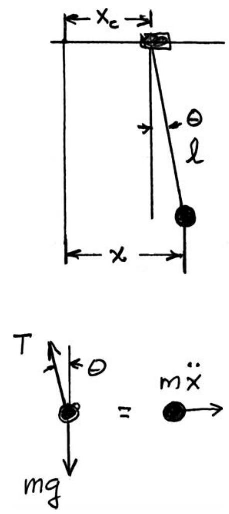
output rate: $(0.16566 \text{ ft}^3/\text{rev})(20 \text{ rev/min})$ $3.31 \text{ ft}^3/\text{min}$ ◄

PROBLEM 19.109



A simple pendulum of length l is suspended from a collar C which is forced to move horizontally according to the relation $x_C = \delta_m \sin \omega_f t$. Determine the range of values of ω_f for which the amplitude of the motion of the bob is less than δ_m. (assume that δ_m is small compared with the length l of the pendulum).

SOLUTION



Geometry. $$x = x_C + l \sin\theta$$

$$\sin\theta = \frac{x - x_C}{l}$$

$$+\uparrow \Sigma F_y = ma_y \approx 0: \quad T\cos\theta - mg = 0 \qquad T \approx mg$$

$$\xrightarrow{+} \Sigma F_x = ma_x: \qquad -T\sin\theta = m\ddot{x}$$

$$m\ddot{x} + \frac{mg(x - x_C)}{l} = 0$$

$$\ddot{x} + \frac{g}{l}x = \frac{g}{l}x_C$$

Using the given motion of x_C,

$$\ddot{x} + \frac{g}{l}x = \frac{g}{l}\delta_m \sin\omega_f t$$

Circular natural frequency. $$\omega_n = \sqrt{\frac{g}{l}}$$

$$\ddot{x} + \omega_n^2 x = \omega_n^2 \delta_m \sin\omega_f t$$

The steady state response is $$x_m = \frac{\delta_m}{1 - \left(\frac{\omega_f}{\omega_n}\right)^2}$$

$$x_m^2 = \frac{\delta_m^2}{\left[1 - \left(\frac{\omega_f}{\omega_n}\right)^2\right]^2} \le \delta_m^2$$

Consider $x_m^2 = \delta_m^2$.

PROBLEM 19.109 (Continued)

Then
$$\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 = 1-2\left(\frac{\omega_f}{\omega_n}\right)^2+\left(\frac{\omega_f}{\omega_n}\right)^4 = 1$$

$$\left(\frac{\omega_f}{\omega_n}\right)^2 = 0 \quad \text{and} \quad \left(\frac{\omega_f}{\omega_n}\right)^2 = 2$$

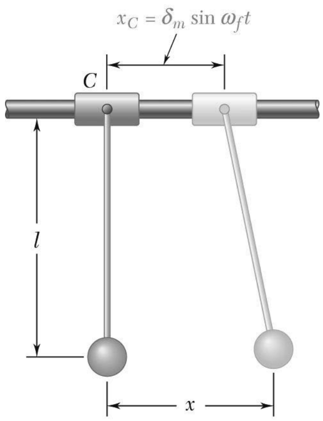
$$\left(\frac{\omega_f}{\omega_n}\right) = 0 \quad \text{and} \quad \frac{\omega_f}{\omega_n} = \sqrt{2}$$

For
$$0 < \frac{\omega_f}{\omega_n} < \sqrt{2}, \quad |x_m| > \delta_m$$

For
$$\frac{\omega_f}{\omega_n} > \sqrt{2}, \quad |x_m| < \delta_n$$

Then
$$\omega_f > \sqrt{2}\,\omega_n = \sqrt{\frac{2g}{l}}$$

$$\omega_f > \sqrt{\frac{2g}{l}} \blacktriangleleft$$

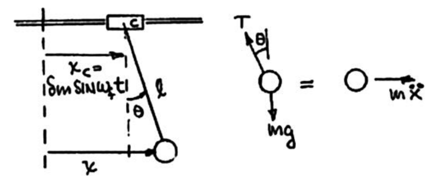


PROBLEM 19.110

The 2.75-lb bob of a simple pendulum of length $l = 24$ in. is suspended from a 3-lb collar C. The collar is forced to move according to the relation $x_C = \delta_m \sin \omega_f t$, with an amplitude $\delta_m = 0.4$ in. and a frequency $f_f = 0.5$ Hz. Determine (*a*) the amplitude of the motion of the bob, (*b*) the force that must be applied to collar C to maintain the motion.

SOLUTION

(*a*)



$$\Sigma F_x = ma_x$$
$$-T \sin\theta = m\ddot{x}$$
$$\Sigma F_y = T\cos\theta - mg = 0$$

For small angles $\cos\theta \approx 1$. Acceleration in the y direction is second order and is neglected.

$$T = mg$$
$$m\ddot{x} = -mg\sin\theta$$
$$\sin\theta = \frac{x - x_c}{l}$$
$$m\ddot{x} + \frac{mg}{l}x = \frac{g}{l}x_c = \frac{mg}{l}\delta_m \sin\omega_f t$$
$$\omega_n^2 = \frac{g}{l}$$
$$\ddot{x} + \omega_n^2 x = \omega_n^2 \delta_m \sin\omega_f t$$

From Equation (19.33′):

$$x_m = \frac{\delta_m}{1 - \frac{\omega_f^2}{\omega_n^2}}$$

So $\quad \omega_f^2 = (2\pi f_f)^2 = 4\pi^2(0.5)^2 = \pi^2\ s^{-2}$

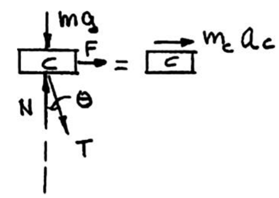
$$\omega_n^2 = \frac{g}{l} = \frac{32.2\ \text{ft/s}^2}{2\,\text{ft}} = 16.1\,\text{s}^{-2}$$

$$x_m = \frac{\frac{0.4}{12}\ \text{ft}}{1 - \frac{\pi^2}{16.1}} = 0.086137\ \text{ft}$$

$x_m = 1.034$ in. ◀

PROBLEM 19.110 (Continued)

(*b*)



$$a_c = \ddot{x}_c = -\delta_m \omega_f^2 \sin \omega_f t$$

$$\xrightarrow{+} \Sigma F_x = m_c a_c$$

$$F - T \sin\theta = m_c a_c$$

From Part (*a*): $T = mg, \quad \sin\theta = \dfrac{x - x_c}{l}$

Thus, $F = -mg\left[\dfrac{x - x_c}{l}\right] + m_c \ddot{x}_c$

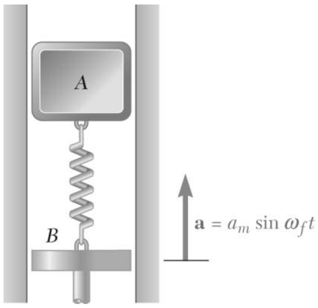
$$= -m\omega_n^2 x + m\omega_n^2 x_c + m_c \ddot{x}_c$$

$$= -m\omega_n^2 x_m \sin\omega_f t + m\omega_n^2 \delta_m \sin\omega_f t - m_c \omega_f^2 \delta_m \sin\omega_f t$$

$$= \left[-\left(\frac{2.75 \text{ lb}}{32.2}\right)(16.1)(0.086137) + \left(\frac{2.75 \text{ lb}}{32.2}\right)(16.1)\left(\tfrac{0.4}{12}\right) - \left(\frac{3 \text{ lb}}{32.2}\right)\pi^2\left(\tfrac{0.4}{12}\right)\right]\sin \pi t$$

$$= -0.10326 \sin \pi t$$

$F = -0.1033 \sin \pi t$ (lb) ◀

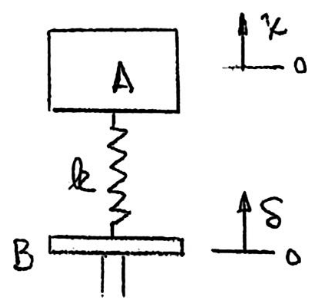


PROBLEM 19.111

An 18-lb block A slides in a vertical frictionless slot and is connected to a moving support B by means of a spring AB of constant $k = 8$ lb/ft. Knowing that the acceleration of the support is $a = a_m \sin \omega_f t$, where $a_m = 5$ ft/s^2 and $\omega_f = 6$ rad/s, determine (a) the maximum displacement of block A, (b) the amplitude of the fluctuating force exerted by the spring on the block.

SOLUTION

(a) Support motion.



$$a = \ddot{\delta} = a_m \sin \omega_f t$$

$$\delta = -\left(\frac{a_m}{\omega_f^2}\right)\sin \omega_f t$$

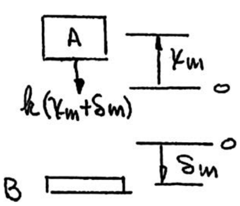
$$\delta_m = \frac{-a_m}{\omega_f^2} = -\frac{5 \text{ ft/s}^2}{(6 \text{ rad/s})^2} = -0.13889 \text{ ft}$$

From Equations (19.31 and 19.33′):

$$x_m = \frac{\delta_m}{1 - \frac{\omega_f^2}{\omega_n^2}} \qquad \omega_n^2 = \frac{k}{m} = \frac{8 \text{ lb/ft}}{\frac{18}{32.2}} = 14.311 \text{ (rad/s)}^2$$

$$x_m = \frac{-0.13889}{1 - \left(\frac{36}{14.311}\right)} = 0.091643 \text{ ft}$$

$x_m = 1.100$ in. ◀



(b) x is out of phase with δ for $\omega_f = 6$ rad/s.

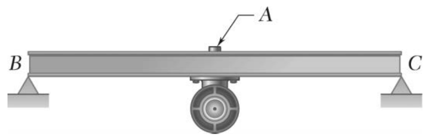
Thus,

$$F_m = k(x_m + \delta_m) = 8 \text{ lb/ft}(0.091643 \text{ ft} + 0.13889 \text{ ft})$$

$$= 1.8443 \text{ lb}$$

$F_m = 1.844$ lb ◀

PROBLEM 19.112



A variable-speed motor is rigidly attached to a beam *BC*. When the speed of the motor is less than 600 rpm or more than 1200 rpm, a small object placed at *A* is observed to remain in contact with the beam. For speeds between 600 and 1200 rpm the object is observed to "dance" and actually to lose contact with the beam. Determine the speed at which resonance will occur.

SOLUTION

Let m be the unbalanced mass and $\bar{r}$ the eccentricity of the unbalanced mass. The vertical force exerted on the beam due to the rotating unbalanced mass is

$$P = m\bar{r}\omega_f^2 \sin\omega_f t = P_m \sin\omega_f t$$

Then from Eq. 19.33,

$$x_m = \frac{\frac{P_m}{k}}{1-\left(\frac{\omega_f}{\omega_n}\right)^2} = \frac{\frac{m\bar{r}\omega_f^2}{k}}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}$$

For simple harmonic motion, the acceleration is

$$a_m = -\omega_f^2 x_m = \frac{\frac{m\bar{r}\omega_f^4}{k}}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}$$

When the object loses contact with the beam, the acceleration $|a_m|$ is greater than g.
Let $\omega_1 = 600 \text{ rpm} = 62.832 \text{ rad/s}$.

$$|a_m|_1 = \frac{\frac{m\bar{r}\omega_1^4}{k}}{1-\left(\frac{\omega_1}{\omega_n}\right)^2} = \frac{\frac{mr\omega_n^4U^4}{k}}{1-U^2} \qquad (1)$$

where $$U = \frac{\omega_1}{\omega_n}.$$

Let $$\omega_2 = 1200 \text{ rpm} = 125.664 \text{ rad/s} = 2\omega_1$$

$$|a_m|_2 = \frac{\frac{m\bar{r}\omega_2^4}{k}}{\left(\frac{\omega_2}{\omega_n}\right)^2 - 1} = \frac{\frac{mr\omega_n^4(2U)^4}{k}}{4U^2-1} \qquad (2)$$

PROBLEM 19.112 (Continued)

Dividing Eq. (1) by Eq. (2),

$$1 = \frac{4U^2 - 1}{16(1-U^2)} \quad \text{or} \quad 16 - 16U^2 = 4U^2 - 1$$

$$20U^2 = 17 \qquad U = \sqrt{\frac{17}{20}}$$

$$\frac{\omega_1}{\omega_n} = \sqrt{\frac{17}{20}} \qquad \omega_n = \sqrt{\frac{20}{17}}\omega_1 = 1.08465\,\omega_1$$

$$\omega_n = (1.08465)(600 \text{ rpm})$$

$\omega_n = 651 \text{ rpm}$ ◀

PROBLEM 19.113

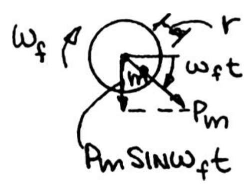
A motor of mass M is supported by springs with an equivalent spring constant k. The unbalance of its rotor is equivalent to a mass m located at a distance r from the axis of rotation. Show that when the angular velocity of the motor is ω_f, the amplitude x_m of the motion of the motor is

$$x_m = \frac{r\left(\frac{m}{M}\right)\left(\frac{\omega_f}{\omega_n}\right)^2}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}$$

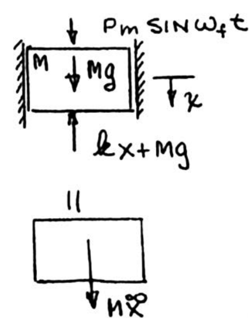
where $\omega_n = \sqrt{\frac{k}{M}}$.

SOLUTION

Rotor



Motor



$$+\downarrow \Sigma F = ma \quad P_m \sin \omega_f t - kx = M\ddot{x}$$

$$M\ddot{x} + kx = P_m \sin \omega_f t$$

$$\ddot{x} + \frac{k}{M}x = \frac{P_m}{M}\sin \omega_f t$$

$$\omega_n^2 = \frac{k}{M}$$

From Equation (19.33):

$$x_m = \frac{\frac{P_m}{k}}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}$$

But

$$\frac{P_m}{k} = \frac{mr\omega_f^2}{k} \qquad k = M\omega_n^2$$

$$\frac{P_m}{k} = r\left(\frac{m}{M}\right)\left(\frac{\omega_f}{\omega_n}\right)^2$$

Thus,

$$x_m = \frac{r\left(\frac{m}{M}\right)\left(\frac{\omega_f}{\omega_n}\right)^2}{1-\left(\frac{\omega_f}{\omega_n}\right)^2} \quad \text{Q.E.D.} \blacktriangleleft$$

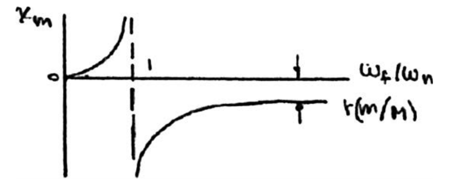
PROBLEM 19.114

As the rotational speed of a spring-supported 100-kg motor is increased, the amplitude of the vibration due to the unbalance of its 15-kg rotor first increases and then decreases. It is observed that as very high speeds are reached, the amplitude of the vibration approaches 3.3 mm. Determine the distance between the mass center of the rotor and its axis of rotation. (*Hint:* Use the formula derived in Problem 19.113.)

SOLUTION

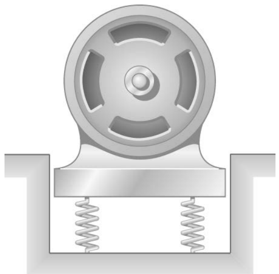
Use the equation derived in Problem 19.113.

$$x_m = \frac{r\left(\frac{m}{M}\right)\left(\frac{\omega_f}{\omega_n}\right)^2}{1-\left(\frac{\omega_f}{\omega_n}\right)^2} = \frac{r\left(\frac{m}{M}\right)}{\frac{1}{\left(\frac{\omega_f}{\omega_n}\right)^2}-1}$$



For very high speeds, $\quad \frac{1}{\left(\frac{\omega_f}{\omega_n}\right)^2} \longrightarrow 0 \quad$ and $\quad x_m \longrightarrow \frac{rm}{M}$,

thus, $\quad 3.3 \text{ mm} = r\left(\frac{15}{100}\right) \qquad r = 22 \text{ mm}$ ◀



PROBLEM 19.115

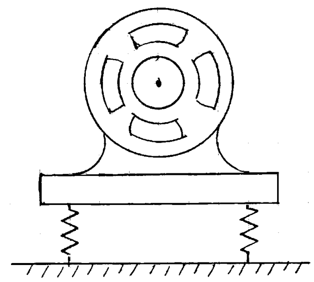
A motor of weight 40 lb is supported by four springs, each of constant 225 lb/in. The motor is constrained to move vertically, and the amplitude of its motion is observed to be 0.05 in. at a speed of 1200 rpm. Knowing that the weight of the rotor is 9 lb, determine the distance between the mass center of the rotor and the axis of the shaft.

SOLUTION

W = 40 lb

Four springs each of constant 225 lb/in.

We note that the motor is constrained to move vertically.



$$4(225 \text{ lb/in.}) = 900 \text{ lb/in.} = 10800 \text{ lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{10800 \text{ lb/ft}}{(40 \text{ lb}/32.2)}} = 93.242 \text{ rad/s}$$

For $\omega = 1200 \text{ rpm} = 125.664 \text{ rad/s}$ we have

$$x_m = 0.05 \text{ in.} = 4.1667 \times 10^{-3} \text{ ft}$$

Eq. (19.33): $$x_m = \frac{\frac{P_m}{k}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

Thus: $$\frac{P_m}{k} = x_m\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]$$

$$= (4.1667 \times 10^{-3} \text{ ft})\left[1 - \left(\frac{125.664}{93.242}\right)^2\right] = -3.4015 \times 10^{-3} \text{ ft (out of phase)}$$

$$P_m = (10800 \text{ lb/ft})(3.4015 \times 10^{-3} \text{ ft}) = 36.736 \text{ lb}$$

We have found: $P_m = 36.736$ lb

For an unbalanced rotor of weight $W_R = 9$ lb, rotating at $\omega = 1200 \text{ rpm} = 125.664 \text{ rad/s}$, with the mass center at a distance $\bar{r}$ from the axis of rotation, we have,

$$P_m = m_R \bar{r} \omega^2$$

$$\bar{r} = \frac{P_m}{m_R \omega^2} = \frac{36.736 \text{ lb}}{(9 \text{ lb}/32.2)(125.664 \text{ rad/s})^2} = 8.3231 \times 10^{-3} \text{ ft}$$

$\bar{r} = 0.0999$ in. ◀

PROBLEM 19.116

A motor weighing 400 lb is supported by springs having a total constant of 1200 lb/in. The unbalance of the rotor is equivalent to a 1-oz weight located 8 in. from the axis of rotation. Determine the range of allowable values of the motor speed if the amplitude of the vibration is not to exceed 0.06 in.

SOLUTION

Let M = mass of motor, m = unbalance mass, r = eccentricity

$$M = \frac{400}{32.2} = 12.4224 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m = \left(\frac{1}{16}\right)\left(\frac{1}{32.2}\right) = 0.001941 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$r = 8 \text{ in.} = 0.66667 \text{ ft} \quad k = 1200 \text{ lb/in.} = 14,400 \text{ lb/ft}$$

Natural circular frequency:
$$\omega_n = \sqrt{\frac{k}{M}} = \sqrt{\frac{14,400}{12.4224}} = 34.047 \text{ rad/s}$$

$$\frac{rm}{M} = \frac{(0.66667)(0.001941)}{12.4224}$$
$$= 0.00014017 \text{ ft} = 0.00125 \text{ in.}$$

From the derivation given in Problem 19.113,

$$x_m = \frac{\left(\frac{rm}{M}\right)\left(\frac{\omega_f}{\omega_n}\right)^2}{1-\left(\frac{\omega_f}{\omega_n}\right)^2} = \frac{0.00125\left(\frac{\omega_f}{\omega_n}\right)^2}{1-\left(\frac{\omega_f}{\omega_n}\right)^2} \text{ in.}$$

In phase motion with $|x_m| < 0.06$ in.

$$\frac{0.00125\left(\frac{\omega_f}{\omega_n}\right)^2}{1-\left(\frac{\omega_f}{\omega_n}\right)^2} < 0.06$$

$$0.00125\left(\frac{\omega_f}{\omega_n}\right)^2\left(\frac{\omega_f}{\omega_n}\right)^2 < 0.06 - 0.06\left(\frac{\omega_f}{\omega_n}\right)^2$$

$$0.06125\left(\frac{\omega_f}{\omega_n}\right)^2 < 0.06$$

$$\frac{\omega_f}{\omega_n} < \sqrt{\frac{0.06}{0.06125}} = 0.98974$$

$$\omega_f < (0.98974)(34.047) = 33.698 \text{ rad/s}$$ $\omega_f < 322$ rpm ◀

PROBLEM 19.116 (Continued)

Out of phase motion with $|x_m| = 0.06$ in.

$$\frac{0.00125\left(\frac{\omega_f}{\omega_n}\right)^2}{\left(\frac{\omega_f}{\omega_n}\right)^2 - 1} < 0.06$$

$$0.00125\left(\frac{\omega_f}{\omega_n}\right)^2 < 0.06\left(\frac{\omega_f}{\omega_n}\right)^2 - 0.06$$

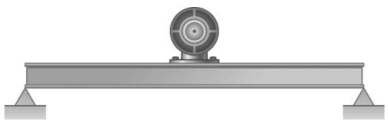
$$0.06 < 0.05875\left(\frac{\omega_f}{\omega_n}\right)^2$$

$$\frac{\omega_f}{\omega_n} > \sqrt{\frac{0.06}{0.05875}} = 1.01058$$

$$\omega_f > (1.01058)(34.047) = 34.407 \text{ rad/s}$$

$\omega_f > 329$ rpm ◀

PROBLEM 19.117



A 180-kg motor is bolted to a light horizontal beam. The unbalance of its rotor is equivalent to a 28-g mass located 150 mm from the axis of rotation, and the static deflection of the beam due to the weight of the motor is 12 mm. The amplitude of the vibration due to the unbalance can be decreased by adding a plate to the base of the motor. If the amplitude of vibration is to be less than 60 μm for motor speeds above 300 rpm, determine the required mass of the plate.

SOLUTION

Before the plate is added, $M_1 = 180 \text{ kg}, \quad m = 28\times 10^{-3} \text{ kg}$

$$r = 150 \text{ mm} = 0.150 \text{ m}$$

Equivalent spring constant: $k = \dfrac{W_1}{\delta_{\text{st}}} = \dfrac{M_1 g}{\delta_{\text{st}}}$

$$k = \frac{(180)(9.81)}{12\times 10^{-3}} = 147.15\times 10^3 \text{ N/m}$$

Let M_2 be the mass of motor plus the plate.

Natural circular frequency. $\omega_n = \sqrt{\dfrac{k}{M_2}}$

Forcing frequency: $\omega_f = 300 \text{ rpm} = 31.416 \text{ rad/s}$

$$\left(\frac{\omega_f}{\omega_n}\right)^2 = \frac{\omega_f^2 M_2}{k} = \frac{(31.416)^2 M_2}{147.15\times 10^3} = 0.006707 M_2$$

From the derivation in Problem 19.113,

$$x_m = \frac{\left(\frac{rm}{M_2}\right)\left(\frac{\omega_f}{\omega_n}\right)^2}{1 - \left(\frac{\omega_f}{\omega_n}\right)^2}$$

For out of phase motion with $x_m = -60\times 10^{-6}$ m,

$$-60\times 10^{-6} = \frac{\left[\frac{(0.150)(28\times 10^{-3})}{M_2}\right](0.006707 M_2)}{1 - 0.006707 M_2}$$

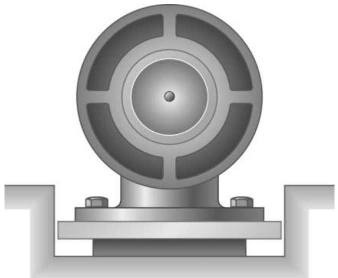
$$-60\times 10^{-6} + (60\times 10^{-6})(0.006707) M_2 = 28.170\times 10^{-6}$$

$$402.49\times 10^{-9} M_2 = 88.170\times 10^{-6}$$

$$M_2 = 219.10 \text{ kg}$$

Added mass: $\Delta M = M_2 - M_1 = 219.10 - 180$ $\quad \Delta M = 39.1 \text{ kg}$ ◄

PROBLEM 19.118



The unbalance of the rotor of a 400-lb motor is equivalent to a 3-oz weight located 6 in. from the axis of rotation. In order to limit to 0.2 lb the amplitude of the fluctuating force exerted on the foundation when the motor is run at speeds of 100 rpm and above, a pad is to be placed between the motor and the foundation. Determine (*a*) the maximum allowable spring constant *k* of the pad, (*b*) the corresponding amplitude of the fluctuating force exerted on the foundation when the motor is run at 200 rpm.

SOLUTION

Mass of motor. $$M = \frac{400}{32.2} = 12.422 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Unbalance mass. $$m = \left(\frac{3}{16}\right)\left(\frac{1}{32.2}\right) = 0.005823 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Eccentricity. $$r = 6 \text{ in.} = 0.5 \text{ ft}$$

Equation of motion: $$M\ddot{x} + kx = P_m \sin\omega_f t = mr\omega_f^2 \sin\omega_f t$$

$$(-M\omega_f^2 + k)x_m = mr\omega_f^2$$

$$x_m = \frac{mr\omega_f^2}{k - M\omega_f^2}$$

Transmitted force. $$F_m = kx_m = \frac{kmr\omega_f^2}{k - M\omega_f^2}$$

For out of phase motion, $$|F_m| = \frac{kmr\omega_f^2}{M\omega_f^2 - k} \quad (1)$$

(*a*) Required value of *k*.

Solve Eq. (1) for *k*. $$|F_m|(M\omega_f^2 - k) = kmr\omega_f^2$$

$$k(mr\omega_f^2 + |F_m|) = |F_m| M\omega_f^2$$

$$k = \frac{|F_m| M\omega_f^2}{mr\omega_f^2 + |F_m|}$$

Data: $|F_m| = 0.2$ lb $\quad \omega_f = 100 \text{ rpm} = 10.472 \text{ rad/s}$

$$k = \frac{(0.2)(12.422)(10.472)^2}{(0.005823)(0.5)(10.472)^2 + 0.2} = 524.65$$

$k = 525$ lb/ft ◀

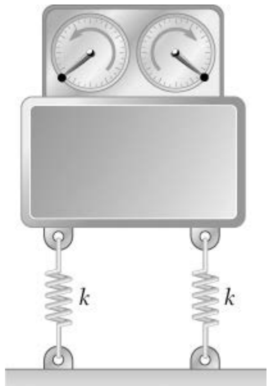
PROBLEM 19.118 (Continued)

(*b*) Force amplitude at 200 rpm. $\omega_f = 20.944$ rad/s

From Eq. (1),

$$|F_m| = \frac{(524.65)(0.005823)(0.5)(20.944)^2}{(12.422)(20.944)^2 - 524.65}$$

$|F_m| = 0.1361\text{ lb}$ ◀



PROBLEM 19.119

A counter-rotating eccentric mass exciter consisting of two rotating 100-g masses describing circles of radius r at the same speed but in opposite senses is placed on a machine element to induce a steady-state vibration of the element. The total mass of the system is 300 kg, the constant of each spring is $k = 600$ kN/m, and the rotational speed of the exciter is 1200 rpm. Knowing that the amplitude of the total fluctuating force exerted on the foundation is 160 N, determine the radius r.

SOLUTION

$$P_m = 2mr\omega_f^2, \qquad x_m = \frac{\frac{2mr\omega_f^2}{2k}}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}, \qquad \omega_n^2 = \frac{2k}{M}$$

With
$$2kx_m = 160\text{ N} = \pm\frac{2mr\omega_f^2}{1-\frac{M\omega_f^2}{2k}}, \qquad \omega_f = 40\pi\text{ rad/s}$$

Solving for r,

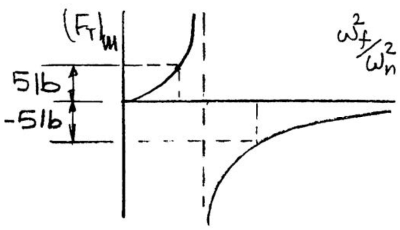
$$r = \pm\frac{160\text{ N}\left[1-\frac{(300\text{ kg})(40\pi\text{ s}^{-1})^2}{1200000\text{ N/m}}\right]}{2(0.1\text{ kg})(40\pi\text{ s}^{-1})^2} = 0.1493\text{ m}$$

$r = 149.3$ mm ◀

PROBLEM 19.120

A 360-lb motor is supported by springs of total constant 12.5 kips/ft. The unbalance of the rotor is equivalent to a 0.9-oz weight located 7.5 in. from the axis of rotation. Determine the range of speeds of the motor for which the amplitude of the fluctuating force exerted on the foundation is less than 5 lb.

SOLUTION



From Problem 19.113 $$x_m = \frac{r\left(\frac{m}{M}\right)\left(\frac{\omega_f}{\omega_n}\right)^2}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}$$

And $$(F_T)_m = kx_m, \quad \frac{k}{M} = \omega_n^2, \quad (F_T)_m = \frac{rm\omega_f^2}{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]}$$

Then $$rm = \left(\frac{7.5}{12}\text{ ft}\right)\left(\frac{\left(\frac{0.9}{16}\text{ lb}\right)}{32.2\text{ ft/s}^2}\right) = 0.0010918\text{ lb}\cdot\text{s}^2$$

$$\omega_n^2 = \frac{k}{M} = \frac{12500\text{ lb/ft}}{\frac{360\text{ lb}}{32.2\text{ ft/s}^2}} = 1118.1\text{ s}^{-2}$$

$$(F_T)_m = (0.0010918\text{ lb}\cdot\text{s}^2)\frac{\omega_f^2}{1-\frac{\omega_f^2}{1118.1}}$$

or $$(F_T)_m\left[1-\frac{\omega_f^2}{1118.1}\right] = (0.0010918\text{ lb}\cdot\text{s}^2)\omega_f^2$$

$$(F_T)_m = \left[\frac{(F_T)_m}{1118.1} + (0.0010918)\right]\omega_f^2$$

Then $$\omega_f^2 = \frac{1118.1(F_T)_m}{(F_T)_m + 1.2207}$$

(*a*) $$(F_T)_m = +5: \quad \omega_f^2 = \frac{1118.1(5)}{5+1.2207} = 898.69\text{ s}^{-2},$$

$$\omega_f \le 29.978\text{ rad/s}$$

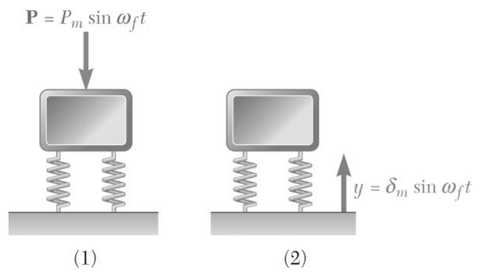
$$\le 286.26\text{ rpm} \qquad \omega_f \le 286\text{ rpm} ◀$$

(*b*) $$(F_T)_m = -5: \quad \omega_f^2 = \frac{1118.1\,(-5)}{-5+1.2207} = 1479.2\text{ s}^{-2},$$

$$\omega_f > 38.461\text{ rad/s}$$

$$> 367.27\text{ rpm} \qquad \omega_f > 367\text{ rpm} ◀$$

PROBLEM 19.121



Figures (1) and (2) show how springs can be used to support a block in two different situations. In Figure (1), they help decrease the amplitude of the fluctuating force transmitted by the block to the foundation. In Figure (2), they help decrease the amplitude of the fluctuating displacement transmitted by the foundation to the block. The ratio of the transmitted force to the impressed force or the ratio of the transmitted displacement to the impressed displacement is called the *transmissibility*. Derive an equation for the transmissibility for each situation. Give your answer in terms of the ratio ω_f/ω_n of the frequency ω_f of the impressed force or impressed displacement to the natural frequency ω_n of the spring-mass system. Show that in order to cause any reduction in transmissibility, the ratio ω_f/ω_n must be greater than $\sqrt{2}$.

SOLUTION

(1) From Equation (19.33):

$$x_m = \frac{\frac{P_m}{k}}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}$$

Force transmitted:

$$(P_T)_m = kx_m = k\left[\frac{\frac{P_m}{k}}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}\right]$$

Thus,

$$\text{Transmissibility} = \frac{(P_T)_m}{P_m} = \frac{1}{1-\left(\frac{\omega_f}{\omega_n}\right)^2} \blacktriangleleft$$

(2) From Equation (19.33′):

Displacement transmitted:

$$x_m = \frac{\delta_m}{1-\left(\frac{\omega_f}{\omega_n}\right)^2}$$

$$\text{Transmissibility} = \frac{x_m}{\delta_m} = \frac{1}{1-\left(\frac{\omega_f}{\omega_n}\right)^2} \blacktriangleleft$$

For $\frac{(P_T)_m}{P_m}$ or $\frac{x_m}{\delta_m}$ to be less than 1,

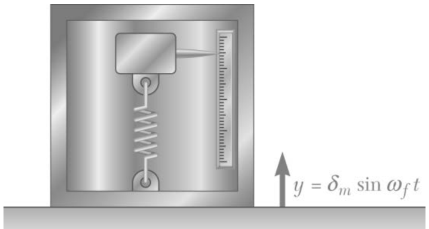
$$\frac{1}{\left|1-\left(\frac{\omega_f}{\omega_n}\right)^2\right|} < 1$$

$$1 < \left|1-\left(\frac{\omega_f}{\omega_n}\right)^2\right|$$

$$\left(\frac{\omega_f}{\omega_n}\right)^2 > 2$$

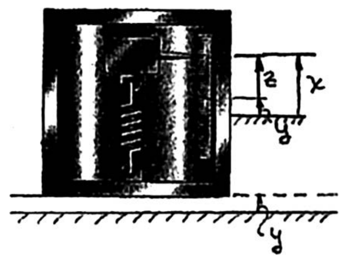
$$\frac{\omega_f}{\omega_n} > \sqrt{2} \quad \text{Q.E.D.} \blacktriangleleft$$

PROBLEM 19.122



A vibrometer used to measure the amplitude of vibrations consists essentially of a box containing a mass-spring system with a known natural frequency of 120 Hz. The box is rigidly attached to a surface, which is moving according to the equation $y = \delta_m \sin \omega_f t$. If the amplitude z_m of the motion of the mass relative to the box is used as a measure of the amplitude δ_m of the vibration of the surface, determine (*a*) the percent error when the frequency of the vibration is 600 Hz, (*b*) the frequency at which the error is zero.

SOLUTION



$$x = \left(\frac{\delta_m}{1 - \frac{\omega_f^2}{\omega_n^2}}\right) \sin \omega_f t$$

$$y = \delta_m \sin \omega_f t$$

$z =$ relative motion

$$z = x - y = \left[\frac{\delta_m}{1 - \frac{\omega_f^2}{\omega_n^2}} - \delta_m\right] \sin \omega_f t$$

$$z_m = \delta_m \left[\frac{1}{1 - \frac{\omega_f^2}{\omega_n^2}} - 1\right] = \frac{\delta_m \frac{\omega_f^2}{\omega_n^2}}{1 - \frac{\omega_f^2}{\omega_n^2}}$$

(*a*)

$$\frac{z_m}{\delta_m} = \frac{\frac{\omega_f^2}{\omega_n^2}}{1 - \frac{\omega_f^2}{\omega_n^2}} = \frac{\left(\frac{600}{120}\right)^2}{1 - \left(\frac{600}{120}\right)^2} = \frac{25}{24} = 1.0417 \quad ◄$$

Error = 4.17%

(*b*)

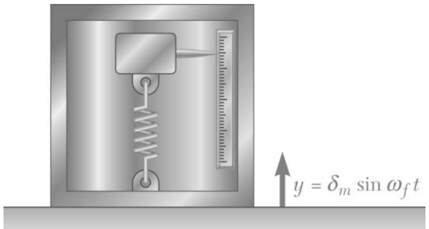
$$\frac{z_m}{\delta_m} = 1 = \frac{\frac{\omega_f^2}{\omega_n^2}}{1 - \frac{\omega_f^2}{\omega_n^2}}$$

$$1 = 2\frac{\omega_f^2}{\omega_n^2}$$

$$f_f = \frac{\sqrt{2}}{2} f_n = \frac{\sqrt{2}}{2}(120) = 84.853 \text{ Hz}$$

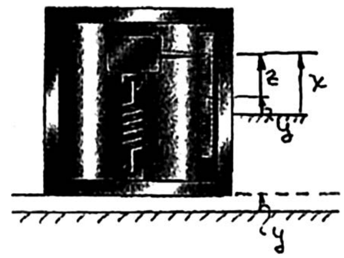
$f_n = 84.9$ Hz ◄

PROBLEM 19.123



A certain accelerometer consists essentially of a box containing a mass-spring system with a known natural frequency of 2200 Hz. The box is rigidly attached to a surface, which is moving according to the equation $y = \delta_m \sin \omega_f t$. If the amplitude z_m of the motion of the mass relative to the box times a scale factor ω_n^2 is used as a measure of the maximum acceleration $\alpha_m = \delta_m \omega_f^2$ of the vibrating surface, determine the percent error when the frequency of the vibration is 600 Hz.

SOLUTION



$$x = \left(\frac{\delta_m}{1 - \frac{\omega_f^2}{\omega_n^2}} \right) \sin \omega_f t$$

$$y = \delta_m \sin \omega_f t$$

$$z = \text{relative motion}$$

$$z = x - y = \left[\frac{\delta_m}{1 - \frac{\omega_f^2}{\omega_n^2}} - \delta_m \right] \sin \omega_f t$$

$$z_m = \delta_m \left[\frac{1}{1 - \frac{\omega_f^2}{\omega_n^2}} - 1 \right] = \frac{\delta_m \frac{\omega_f^2}{\omega_n^2}}{1 - \frac{\omega_f^2}{\omega_n^2}}$$

The actual acceleration is $a_m = -\omega_f^2 \delta_m$

The measurement is proportional to $z_m \omega_n^2$.

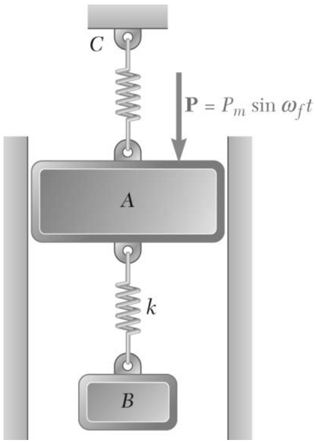
Then
$$\frac{z_m \omega_n^2}{a_m} = \frac{z_m}{\delta_m} \left(\frac{\omega_n}{\omega_f} \right)^2$$

$$= \frac{1}{1 - \left(\frac{\omega_f}{\omega_n} \right)^2}$$

$$= \frac{1}{1 - \left(\frac{600}{2200} \right)^2}$$

$$= 1.0804$$

Error = 8.04% ◀



PROBLEM 19.124

Block A can move without friction in the slot as shown and is acted upon by a vertical periodic force of magnitude $P = P_m \sin \omega_f t$, where $\omega_f = 2$ rad/s and $P_m = 20$ N. A spring of constant k is attached to the bottom of block A and to a 22-kg block B. Determine (a) the value of the constant k which will prevent a steady-state vibration of block A, (b) the corresponding amplitude of the vibration of block B.

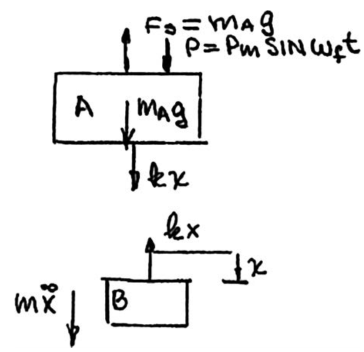
SOLUTION

In steady state vibration, block A does not move and therefore, remains in its original equilibrium position.

Block A: $+\downarrow \Sigma F = 0$

$$kx = -P_m \sin \omega_f t \tag{1}$$

Block B: $+\downarrow \Sigma F = m_B \ddot{x}$



$$m_B \ddot{x} + kx = 0$$

$$x = x_m \sin \omega_n t$$

$$\omega_n^2 = k/m_B$$

From Eq. (1): $kx_m \sin \omega_n t = -P_m \sin \omega_f t$

$$\omega_n = \omega_f = 2 \text{ rad/s}$$

$$kx_m = -P_m$$

$$\omega_n = \sqrt{\frac{k}{m_B}}$$

$$k = m_B \omega_n^2$$

(a) Required spring constant. $k = (22)(2)^2$ $\qquad k = 88.0$ N/m ◀

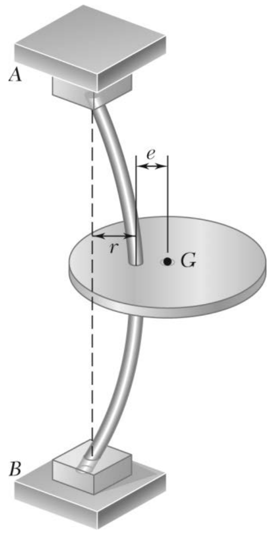
(b) Corresponding amplitude of vibration of B.

$$kx_m = -P_m$$

$$x_m = -\frac{P_m}{k}$$

$$x_m = -\frac{20 \text{ N}}{88 \text{ N/m}}$$

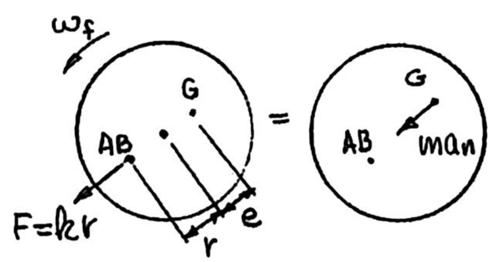
$x_m = -0.227$ m ◀



PROBLEM 19.125

A 60-lb disk is attached with an eccentricity $e = 0.006$ in. to the midpoint of a vertical shaft *AB*, which revolves at a constant angular velocity ω_f. Knowing that the spring constant *k* for horizontal movement of the disk is 40,000 lb/ft, determine (*a*) the angular velocity ω_f at which resonance will occur, (*b*) the deflection *r* of the shaft when $\omega_f = 1200$ rpm.

SOLUTION



G describes a circle about the axis *AB* of radius $r + e$.

Thus, $$a_n = (r+e)\omega_f^2$$

Deflection of the shaft is $$F = kr$$

Thus, $$F = ma_n$$

$$kr = m(r+e)\omega_f^2$$

$$\omega_n^2 = \frac{k}{m} \qquad m = \frac{k}{\omega_n^2}$$

$$\not{k}r = \frac{\not{k}}{\omega_n^2}(r+e)\omega_f^2$$

$$r = \frac{e\frac{\omega_f^2}{\omega_n^2}}{1 - \frac{\omega_f^2}{\omega_n^2}}$$

PROBLEM 19.125 (Continued)

(*a*) Resonance occurs when $\omega_f = \omega_n$, i.e., $r \longrightarrow \infty$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{W}}$$

$$= \sqrt{\frac{(40,000)(32.2)}{60}}$$

$$= 146.52 \text{ rad/s}$$

$$= 1399.1 \text{ rpm}$$

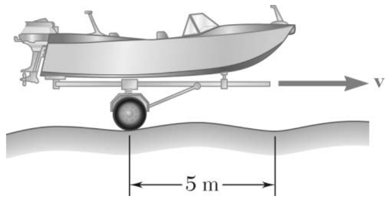
$\omega_n = \omega_f = 1399 \text{ rpm}$ ◀

(*b*)
$$r = \frac{(0.006 \text{ in.})\left(\frac{1200}{1399.1}\right)^2}{1-\left(\frac{1200}{1399.1}\right)^2}$$

$$= 0.01670 \text{ in.}$$

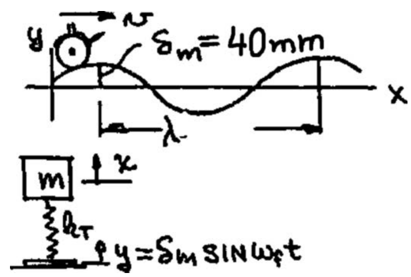
$r = 0.01670$ in. ◀

PROBLEM 19.126



A small trailer and its load have a total mass of 250-kg. The trailer is supported by two springs, each of constant 10 kN/m, and is pulled over a road, the surface of which can be approximated by a sine curve with an amplitude of 40 mm and a wavelength of 5 m (i.e., the distance between successive crests is 5 m and the vertical distance from crest to trough is 80 mm). Determine (*a*) the speed at which resonance will occur, (*b*) the amplitude of the vibration of the trailer at a speed of 50 km/h.

SOLUTION



Total spring constant $k = 2(10\times10^3 \text{ N/m})$

$$= 20\times10^3 \text{ N/m}$$

(*a*) $$\omega_n^2 = \frac{k}{m} = \frac{20\times10^3 \text{ N/m}}{250 \text{ kg}} = 80 \text{ s}^{-2}$$

$$\lambda = 5 \text{ m}$$

$$\delta_m = 40 \text{ mm} = 40\times10^{-3} \text{ m}$$

$$y = \delta_m \sin\frac{2\pi x}{\lambda} \quad \text{where} \quad x = vt$$

$$y = \delta_m \sin\omega_f t$$

$$\omega_f = \frac{2\pi v}{\lambda}$$

$$\delta_m = 40 \text{ mm} = 40\times10^{-3} \text{ m}$$

From Equation (19.33′): $$x_m = \frac{\delta_m}{\left(1-\frac{\omega_f^2}{\omega_n^2}\right)}$$

Resonance: $$\omega_f = \frac{2\pi v}{5} = \omega_n = \sqrt{80} \text{ s}^{-1},$$

$v = 7.1176$ m/s $\qquad$ $v = 25.6$ km/h ◀

(*b*) Amplitude at $v = 50$ km/h $= 13.8889$ m/s

$$\omega_f = \frac{2\pi(13.8889)}{5} = 17.4533 \text{ rad/s}$$

$$\omega_f^2 = 304.60 \text{ s}^{-2}$$

$$x_m = \frac{40\times10^{-3}}{1-\frac{304.62}{80}} = -14.246\times10^{-3} \text{ m}$$

$x_m = -14.25$ mm ◀

PROBLEM 19.127

Show that in the case of heavy damping $(c > c_c)$, a body never passes through its position of equilibrium O (a) if it is released with no initial velocity from an arbitrary position or (b) if it is started from O with an arbitrary initial velocity.

SOLUTION

Since $c > c_c$, we use Equation (19.42), where

$$\lambda_1 < 0,\ \lambda_2 < 0$$

$$x = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \quad (1)$$

$$v = \frac{dx}{dt} = C_1\lambda_1 e^{\lambda_1 t} + C_2\lambda_2 e^{\lambda_2 t} \quad (2)$$

(a) $t = 0,\ x = x_0,\quad v = 0$:

From Eqs. (1) and (2):

$$x_0 = C_1 + C_2$$
$$0 = C_1\lambda_1 + C_2\lambda_2$$

Solving for c_1 and c_2,

$$C_1 = \frac{\lambda_2}{\lambda_2 - \lambda_1} x_0$$

$$C_2 = \frac{-\lambda_1}{\lambda_2 - x_1} x_0$$

Substituting for C_1 and C_2 in Eq. (1), $x = \dfrac{x_2}{\lambda_2 - \lambda_1}\left[\lambda_2 e^{\lambda_1 t} - \lambda_1 e^{\lambda_2 t}\right]$

For $x = 0$: when $t \neq \infty$, we must have

$$\lambda_1 e^{\lambda_2 t} - \lambda_2 e^{\lambda_1 t} = 0 \qquad \frac{\lambda_2}{\lambda_1} = e^{(\lambda_2 - \lambda_1)t} \quad (3)$$

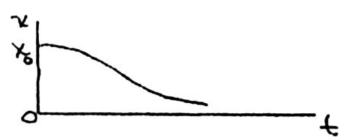
Recall that

$\lambda_1 < 0,\ \lambda_2 < 0.$ Choosing λ_1 and λ_2 so that $\lambda_1 < \lambda_2 < 0,$ we have

$$0 < \frac{\lambda_2}{\lambda_1} < 1 \quad \text{and} \quad \lambda_2 - \lambda_1 > 0$$

Thus a positive solution for $t > 0$ for Equation (3) cannot exist, since it would require that e raised to a positive power be less than 1, which is impossible. Thus, x is never 0.

The $x - t$ curve for this case is as shown.



◀

(*b*) $t = 0,\ x = 0,\ v = v_0$: Equations (1) and (2) yield

$$0 = C_1 + C_2$$
$$v_0 = C_1\lambda_1 + C_2\lambda_2$$

Solving for C_1 and C_2,

$$C_1 = -\frac{v_0}{\lambda_2 - \lambda_1}$$

$$C_2 = \frac{v_2}{\lambda_2 - \lambda_1}$$

Substituting into Eq. (1),

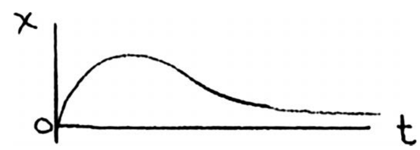
$$x = \frac{v_0}{\lambda_2 - \lambda_1}[e^{\lambda_2 t} - e^{\lambda_1 t}]$$

For $x = 0$, and $t > 0$

$$e^{\lambda_2 t} = e^{\lambda_1 t}$$

For $c > c_c$, $\lambda_1 \neq \lambda_2$; thus, no solution can exist for *t*, and *x* is never 0 when $t > 0$.

The $x - t$ curve for this motion is as shown.



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PROBLEM 19.128

Show that in the case of heavy damping $(c > c_c)$, a body released from an arbitrary position with an arbitrary initial velocity cannot pass more than once through its equilibrium position.

SOLUTION

Substitute the initial conditions, $t = 0$, $x = x_0$, $v = v_0$ in Equations (1) and (2) of Problem 19.127.

$$x_0 = C_1 + C_2 \qquad v_0 = C_1\lambda_1 + C_2\lambda_2$$

Solving for C_1 and C_2,

$$C_1 = -\frac{(v_0 - \lambda_2 x_0)}{\lambda_2 - \lambda_1}$$

$$C_2 = \frac{(v_0 - \lambda_1 x_0)}{\lambda_2 - \lambda_1}$$

And substituting in Eq. (1)

$$x = \frac{1}{\lambda_2 - \lambda_1}\left[(v_0 - \lambda_1 x_0)e^{\lambda_2 t} - (v_0 - \lambda_2 x_0)e^{\lambda_1 t}\right]$$

For $x = 0$, $t \neq \infty$:

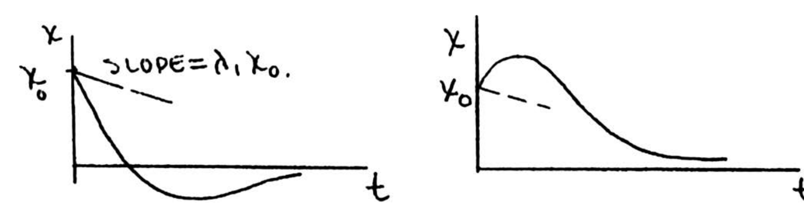
$$(v_0 - \lambda_1 x_0)e^{\lambda_2 t} = (v_0 - \lambda_2 x_0)e^{\lambda_1 t}$$

$$e^{(\lambda_2 - \lambda_1)t} = \frac{(v_0 - \lambda_2 x_0)}{(v_0 - \lambda_1 x_0)}$$

$$t = \frac{1}{(\lambda_2 - \lambda_1)}\ln\frac{v_0 - \lambda_2 x_0}{v_0 - \lambda_1 x_0}$$

This defines one value of t only for $x = 0$, which will exist if the argument of the natural logarithm is positive, i.e., if $\frac{v_0 - \lambda_2 x_0}{v_0 - \lambda_1 x_0} > 1$. Assuming $\lambda_1 < \lambda_2 < 0$,

this occurs if $v_0 < \lambda_1 x_0$.



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PROBLEM 19.129

In the case of light damping, the displacements x_1, x_2, x_3, shown in Figure 19.11 may be assumed equal to the maximum displacements. Show that the ratio of any two successive maximum displacements x_n and x_{n+1} is a constant and that the natural logarithm of this ratio, called the *logarithmic decrement*, is

$$\ln\frac{x_n}{x_{n+1}} = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$$

SOLUTION

For light damping,

Equation (19.46): $$x = x_0 e^{-\left(\frac{c}{2m}\right)t}\sin(\omega_0 t + \phi)$$

At given maximum displacement, $$t = t_n, \quad x = x_n$$

$$\sin(\omega_0 t_n + \phi) = 1$$

$$x_n = x_0 e^{-\left(\frac{c}{2m}\right)t_n}$$

At next maximum displacement, $$t = t_{n+1}, \quad x = x_{n+1}$$

$$\sin(\omega_0 t_{n+1} + \phi) = 1$$

$$x_{n+1} = x_0 e^{-\left(\frac{c}{2m}\right)t_{n+1}}$$

But $$\omega_D t_{n+1} - \omega_D t_n = 2\pi$$

$$t_{n+1} - t_n = \frac{2\pi}{\omega_D}$$

Ratio of successive displacements: $$\frac{x_n}{x_{n+1}} = \frac{x_0 e^{-\frac{c}{2m}t_n}}{x_0 e^{-\frac{c}{2m}t_{n+1}}}$$

$$= e^{-\frac{c}{2m}\left(t_n - t_{n+1}\right)} = e^{+\frac{c}{2m}\frac{2\pi}{\omega_D}}$$

Thus, $$\ln\frac{x_n}{x_{n+1}} = \frac{c\pi}{m\omega_D} \qquad (1)$$

From Equations (19.45) and (19.41): $$\omega_D = \omega_n\sqrt{1-\left(\frac{c}{c_c}\right)^2}$$

$$\omega_D = \frac{c_c}{2m}\sqrt{1-\left(\frac{c}{c}\right)^2}$$

Thus, $$\ln\frac{x_n}{x_{n+1}} = \frac{c\pi}{m}\frac{2m}{c_c}\frac{1}{\sqrt{1-\left(\frac{c}{c_c}\right)^2}}$$

$$\ln\frac{x_n}{x_{n+1}} = \frac{2\pi\left(\frac{c}{c_c}\right)}{\sqrt{1-\left(\frac{c}{c_c}\right)^2}}$$ Q.E.D. ◀

PROBLEM 19.130

In practice, it is often difficult to determine the logarithmic decrement of a system with light damping defined in Problem 19.129 by measuring two successive maximum displacements. Show that the logarithmic decrement can also be expressed as $(1/k)\ln(x_n/x_{n+k})$, where k is the number of cycles between readings of the maximum displacement.

SOLUTION

As in Problem 19.129, for maximum displacements x_n and x_{n+k} at t_n and t_{n+k}, $\sin(\omega_0 t_n + \phi) = 1$

and $\sin(\omega_n t_{n+k} + \phi) = 1$.

$$x_n = x_0 e^{-\left(\frac{c}{2m}\right)t_n}$$

$$x_{n+k} = x_0 e^{-\left(\frac{c}{2m}\right)(t_{n+k})}$$

Ratio of maximum displacements:

$$\frac{x_n}{x_{n+k}} = \frac{x_0 e^{\left(\frac{-c}{2m}\right)t_n}}{x_0 e^{\left(\frac{-c}{2m}\right)t_{n+k}}} = e^{\frac{-c}{2m}(t_n - t_{n+k})}$$

But

$$\omega_D t_{n+k} - \omega_D t_n = k(2\pi)$$

$$t_n - t_{n+k} = k\frac{2\pi}{\omega_D}$$

Thus,

$$\frac{x_n}{x_{n+k}} = +\frac{c}{2m}\left(\frac{2k\pi}{\omega_D}\right)$$

$$\ln\frac{x_n}{x_{n+k}} = k\frac{c\pi}{m\omega_D} \qquad (2)$$

But from Problem 19.129, Equation (1):

$$\log\ \text{decrement} = \ln\frac{x_n}{x_{n+1}} = \frac{c\pi}{m\omega_D}$$

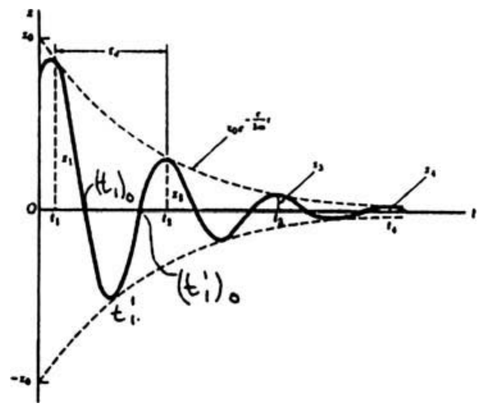
Comparing with Equation (2),

$$\log\ \text{decrement} = \frac{1}{k}\ln\frac{x_n}{x_{n+k}} \quad \text{Q.E.D.} \blacktriangleleft$$

PROBLEM 19.131

In a system with light damping $(c < c_c)$, the period of vibration is commonly defined as the time interval $\tau_d = 2\pi/\omega_d$ corresponding to two successive points where the displacement-time curve touches one of the limiting curves shown in Figure 19.11. Show that the interval of time (*a*) between a maximum positive displacement and the following maximum negative displacement is $\frac{1}{2}\tau_d$, (*b*) between two successive zero displacements is $\frac{1}{2}\tau_d$, (*c*) between a maximum positive displacement and the following zero displacement is greater than $\frac{1}{4}\tau_d$.

SOLUTION



Equation (19.46): $$x = x_0 e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi)$$

(*a*) Maxima (positive or negative) when $\dot{x} = 0$:

$$\dot{x} = x_0 \left(\frac{-c}{2m}\right) e^{-\left(\frac{c}{2m}\right)t} \sin(\omega_d t + \phi) + x_0 \omega_d e^{-\left(\frac{c}{2m}\right)t} \cos(\omega_d t + \phi)$$

Thus, zero velocities occur at times when

$$\dot{x} = 0, \quad \text{or} \quad \tan(\omega_d t + \phi) = \frac{2m\omega_d}{c} \tag{1}$$

The time to the first zero velocity, t_1, is

$$t_1 = \frac{\left[\tan^{-1}\left(\frac{2m\omega_d}{c}\right) - \phi\right]}{\omega_d} \tag{2}$$

The time to the next zero velocity where the displacement is negative is

$$t_1' = \frac{\left[\tan^{-1}\left(\frac{2m\omega_d}{c}\right) - \phi + \pi\right]}{\omega_d} \tag{3}$$

Subtracting Eq. (2) from Eq. (3),

$$t_1' - t_1 = \frac{\pi}{\omega_d} = \frac{\pi \cdot \tau_d}{2\pi} = \frac{\tau_d}{2} \quad \text{Q.E.D.}$$

PROBLEM 19.131 (Continued)

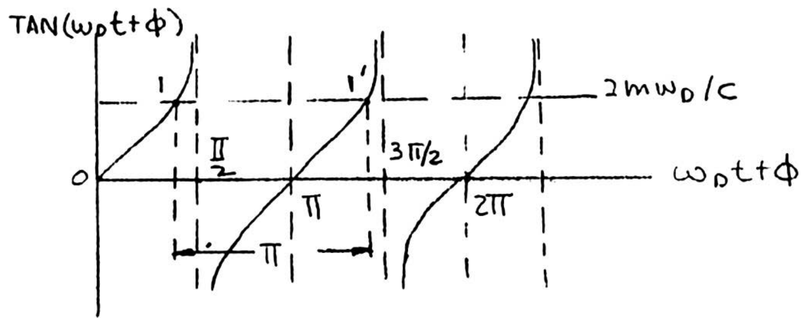
(*b*) Zero displacements occur when

$$\sin(\omega_\theta t + \phi) = 0 \quad \text{or at intervals of}$$

$$\omega_d t + \phi = \pi,\ 2\pi\ n\pi$$

Thus, $$(t_1)_0 = \frac{(\pi - \phi)}{\omega_d} \quad \text{and} \quad (t_1')_0 = \frac{(2\pi - \phi)}{\omega_d}$$

Time between $$0_s' = (t_1')_0 - (t_1)_0 = \frac{2\pi - \pi}{\omega_d} = \frac{\pi\tau_d}{2\pi} = \frac{\tau_d}{2} \quad \text{Q.E.D.}$$



Plot of Equation (1)

(*c*) The first maximum occurs at 1: $(\omega_d t_1 + \phi)$

The first zero occurs at $(\omega_d (t_1)_0 + \phi) = \pi$

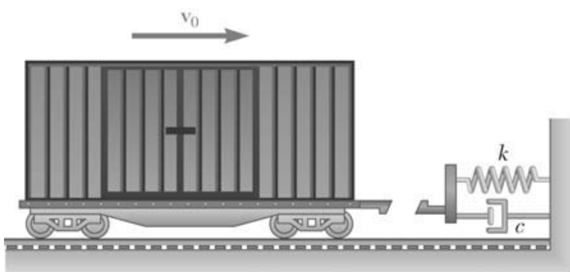
From the above plot, $$(\omega_d (t_1)_0 + \phi) - (\omega_D t_1 + \phi) > \frac{\pi}{2}$$

or $$(t_1)_0 - t_1 > \frac{\pi}{2\omega_d} \quad (t_1)_0 - t_1 > \frac{\tau_d}{4} \quad \text{Q.E.D.}$$

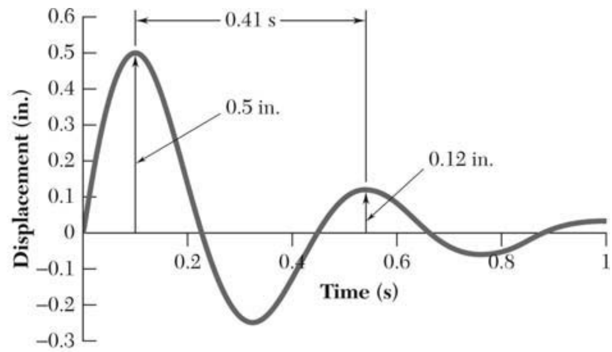
Similar proofs can be made for subsequent maximum and minimum.

PROBLEM 19.132

A loaded railroad car weighing 30,000 lb is rolling at a constant velocity $\mathbf{v}_0$ when it couples with a spring and dashpot bumper system (Figure 1). The recorded displacement-time curve of the loaded railroad car after coupling is as shown (Figure 2). Determine (*a*) the damping constant, (*b*) the spring constant. (*Hint:* Use the definition of logarithmic decrement given in Problem 19.129.)



(1)



(2)

SOLUTION

Mass of railroad car:
$$m = \frac{W}{g} = \frac{30,000}{32.2}$$
$$= 931.67 \text{ lb}\cdot\text{s}^2/\text{ft}$$

The differential equation of motion for the system is

$$m\ddot{x} + c\dot{x} + kx = 0$$

For light damping, the solution is given by Eq. (19.44):

$$x = e^{-\left(\frac{c}{2m}\right)t}(C_1 \sin \omega_d t + C_2 \cos \omega_d t)$$

From the displacement versus time curve,

$$\tau_d = 0.41 \text{ s}$$
$$\omega_d = \frac{2\pi}{\tau_d} = \frac{2\pi}{0.41} = 15.325 \text{ rad/s}$$

At the first peak, $x_1 = 0.5$ in. and $t = t_1$.

At the second peak, $x_2 = 0.12$ in. and $t = t_1 + \tau_d$.

Forming the ratio $\frac{x_2}{x_1}$,
$$\frac{x_2}{x_1} = \frac{e^{-\left(\frac{c}{2m}\right)(t_1+\tau_d)}}{e^{-\left(\frac{c}{2m}\right)t_1}} = e^{-\left(\frac{c}{2m}\right)\tau_d} \qquad (1)$$
$$\frac{x_1}{x_2} = e^{\frac{c\tau_d}{2m}}$$

PROBLEM 19.132 (Continued)

(*a*) Damping constant.

From Eq. (1):

$$\frac{c\tau_d}{2m} = \ln\left(\frac{x_1}{x_2}\right)$$

$$c = \frac{2m}{\tau_d}\ln\frac{x_1}{x_2}$$

$$= \frac{(2)(931.67)}{0.41}\ln\frac{0.5}{0.12}$$

$$= 6485.9 \text{ lb}\cdot\text{s/ft} \qquad c = 6.49 \text{ kip}\cdot\text{s/ft} \blacktriangleleft$$

(*b*) Spring constant.

Equation for ω_d:

$$\omega_d^2 = \frac{k}{m} - \left(\frac{c}{2m}\right)^2$$

$$k = m\omega_d^2 + \frac{c^2}{4m}$$

$$= (931.67)(15.325)^2 + \frac{(6485.9)^2}{(4)(931.67)}$$

$$= 230\times 10^3 \text{ lb/ft} \qquad k = 230 \text{ kips/ft} \blacktriangleleft$$

PROBLEM 19.133

A torsional pendulum has a centroidal mass moment of inertia of 0.3 kg-m^2 and when given an initial twist and released is found to have a frequency of oscillation of 200 rpm. Knowing that when this pendulum is immersed in oil and given the same initial condition it is found to have a frequency of oscillation of 180 rpm, determine the damping constant for the oil.

SOLUTION

Let the mass be rotated through the small angle θ from the equilibrium position.

Couples acting on the mass: Shaft: $-K\theta$

Oil: $-C\dot{\theta}$

Equation of motion: $\Sigma M = \bar{I}\ddot{\theta}: -K\theta - C\dot{\theta} = \bar{I}\ddot{\theta}$

$$\bar{I}\ddot{\theta} + C\dot{\theta} + K\theta = 0$$

Solution for light damping: $\theta = e^{-\lambda t}(C_1 \sin \omega_d t + C_2 \sin \omega_d t)$

where $\lambda = \dfrac{C}{2\bar{I}}$

$$\omega_n = \sqrt{\frac{K}{\bar{I}}}$$

$$\omega_d = \sqrt{\omega_n^2 - \lambda^2}$$

When there is no oil, assume $C \approx 0.$

$$\omega_n = 200 \text{ rpm} = 20.944 \text{ rad/s}$$

When oil is present,

$$\omega_d = 180 \text{ rpm} = 18.8496 \text{ rad/s}$$

$$\lambda = \sqrt{\omega_n^2 - \omega_d^2} = 9.1293 \text{ s}^{-1}$$

Damping constant for oil.

$$C = 2I\lambda = (2)(0.3 \text{ kg}\cdot\text{m}^2)(9.1293 \text{ s}^{-1})$$

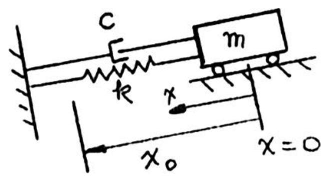
$C = 5.48 \text{ N}\cdot\text{m}\cdot\text{s}$ ◀

PROBLEM 19.134

The barrel of a field gun weighs 1500 lb and is returned into firing position after recoil by a recuperator of constant $c = 1100$ lb·s/ft. Determine (*a*) the constant *k* which should be used for the recuperator to return the barrel into firing position in the shortest possible time without any oscillation, (*b*) the time needed for the barrel to move back two-thirds of the way from its maximum-recoil position to its firing position.

SOLUTION

(*a*) A critically damped system regains its equilibrium position in the shortest time.



$$c = c_c = 1100 = 2m\sqrt{\frac{k}{m}}$$

Then $$k = \frac{\left(\frac{c_c}{2}\right)^2}{m} = \frac{\left(\frac{1100}{2}\right)^2}{\frac{1500 \text{ lb}}{32.2 \text{ ft/s}^2}} = 6494 \text{ lb/ft}$$ $k = 6490$ lb/ft ◀

(*b*) For a critically damped system, Equation (19.43):

$$x = (C_1 + C_2 t)e^{-\omega_n t}$$

We take $t = 0$ at maximum deflection x_0.

Thus, $$\dot{x}(0) = 0$$ $$x(0) = x_0$$

Using the initial conditions, $x(0) = x_0 = (C_1 + 0)e^0$, so $C_1 = x_0$

$$x = (x_0 + C_2 t)e^{-\omega_n t}$$

and $$\dot{x} = -\omega_n (x_0 + C_2 t)e^{-\omega_n t} + C_2 e^{-\omega_n t}$$
$$\dot{x}(0) = 0 = -\omega_n x_0 + C_2, \quad \text{so } C_2 = \omega_n x_0$$

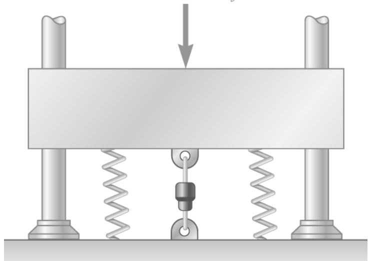
Thus, $$x = x_0(1 + \omega_n t)e^{-\omega_n t}$$

For $$x = \frac{x_0}{3}, \quad \frac{1}{3} = (1 + \omega_n t)e^{-\omega_n t}$$

Solving by trial for $\omega_n t$ gives: $\omega_n t = 2.289$

But $$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{6494 \text{ lb/ft}}{\left(\frac{1500 \text{ lb}}{32.2 \text{ ft/s}^2}\right)}} = 11.807 \text{ s}^{-1}$$

Then $$t = \frac{\omega_n t}{\omega_n} = \frac{2.289}{11.807} = 0.19387$$ $t = 0.1939$ s ◀



PROBLEM 19.135

A platform of weight 200 lb, supported by two springs each of constant k = 250 lb/in., is subjected to a periodic force of maximum magnitude equal to 125 lb. Knowing that the coefficient of damping is 12 lb·s/in., determine (*a*) the natural frequency in rpm of the platform if there were no damping, (*b*) the frequency in rpm of the periodic force corresponding to the maximum value of the magnification factor, assuming damping, (*c*) the amplitude of the actual motion of the platform for each of the frequencies found in parts *a* and *b*.

SOLUTION

(*a*) *No Damping*:

$$k = 2(250 \text{ lb/in.}) = 500 \text{ lb/in.} = 6000 \text{ lb/ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{6000 \text{ lb/ft}}{(2000 \text{ lb})/(32.2 \text{ ft/s}^2)}} = 31.08 \text{ rad/s}$$

$$f = 4.947 \text{ Hz}$$

$$f = 297 \text{ rpm} \blacktriangleleft$$

(*b*) *Damped Motion*:

$$c = 12 \text{ lb}\cdot\text{s/in.} = 144 \text{ lb}\cdot\text{s/ft}$$

$$c_c = 2m\omega_n = (2)\left(\frac{200}{32.2}\right)(31.08) = 386.09 \text{ lb}\cdot\text{s/ft}$$

$$\frac{c}{c_c} = \frac{144 \text{ lb}\cdot\text{s/ft}}{386.09 \text{ lb}\cdot\text{s/ft}} = 0.37297$$

From Eq. (19.53):

$$\frac{x_m}{\delta_m} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right] + \left[2\frac{c}{c_c}\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

For maximum amplitude we set equal to zero the derivative with respect to $\left(\frac{\omega}{\omega_n}\right)$ of the square of the denominator.

$$2\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]\left[-2\left(\frac{\omega}{\omega_n}\right)\right] + 8\left(\frac{c}{c_c}\right)^2\left(\frac{\omega}{\omega_n}\right) = 0$$

PROBLEM 19.135 (Continued)

Rearranging, we obtain

$$4\left(\frac{\omega}{\omega_n}\right)\left[\left(\frac{\omega}{\omega_n}\right)^2 - 1 + 2\left(\frac{c}{c_c}\right)^2\right] = 0$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = 1 - 2\left(\frac{c}{c_c}\right)^2$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = 1 - 2\left(\frac{c}{c_c}\right)^2 = 1 - 2(0.37297)^2 = 0.72179$$

$$\frac{\omega}{\omega_n} = 0.84958 \qquad \omega = (0.84958)\omega_n = (0.84958)(31.08 \text{ rad/s})$$

$$\omega = 26.405 \text{ rad/s} \qquad f = 4.2025 \text{ Hz} \qquad f = 252 \text{ rpm} \blacktriangleleft$$

(*c*) *Amplitude*:

From Eq. (19.53):
$$x_m = \frac{\frac{P_m}{k}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

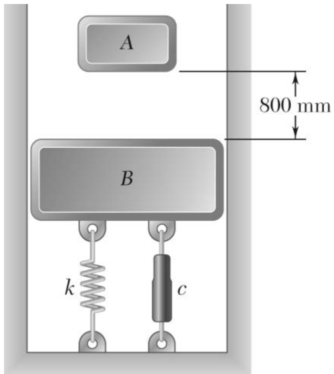
For part (*a*) with $P_m = 125$ lb and $\omega = \omega_n$

$$x_m = \frac{\frac{125 \text{ lb}}{6000 \text{ lb/ft}}}{\sqrt{[1-1]^2 + [2(0.37297)(1)]^2}} = 0.02793 \text{ ft} \qquad x_m = 0.335 \text{ in.} \blacktriangleleft$$

For part (*b*) with $P_m = 125$ lb and $\left(\frac{\omega}{\omega_n}\right) = 0.84958$

$$x_m = \frac{\frac{125 \text{ lb}}{6000 \text{ lb/ft}}}{\sqrt{[1 - (0.84958)^2]^2 + [2(0.84958)(0.37297)]^2}} = 0.0301 \text{ ft}$$

$$x_m = 0.361 \text{ in.} \blacktriangleleft$$



PROBLEM 19.136

A 4-kg block A is dropped from a height of 800 mm onto a 9-kg block B which is at rest. Block B is supported by a spring of constant $k = 1500$ N/m and is attached to a dashpot of damping coefficient $c = 230$ N·s/m. Knowing that there is no rebound, determine the maximum distance the blocks will move after the impact.

SOLUTION

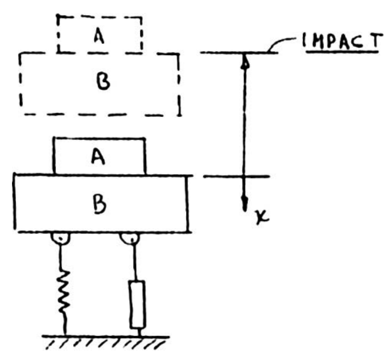
Velocity of Block A just before impact.

$$v_A = \sqrt{2gh}$$
$$= \sqrt{2(9.81)(0.8)}$$
$$= 3.962 \text{ m/s}$$

Velocity of Blocks A and B immediately after impact.

Conservation of momentum.

$$m_A v_A + m_B v_B = (m_A + m_B)v'$$
$$(4)(3.962) + 0 = (4 + 9)v'$$
$$v' = 1.219 \text{ m/s}$$
$$\dot{x}_0 = +1.219 \text{ m/s} \downarrow = \dot{x}_0$$



Static deflection (Block A):

$$x_0 = -\frac{m_A g}{k}$$
$$= -\frac{(4)(9.82)}{1500}$$
$$= -0.02619 \text{ m}$$

$x = 0$, Equilibrium position for both blocks:

$$c_c = 2\sqrt{km}$$
$$= 2\sqrt{(1500)(13)}$$
$$= 279.3 \text{ N}\cdot\text{s/m}$$

Since $c < c_c$, Equation (19.44):

$$x = e^{-\left(\frac{c}{2m}\right)t}[C_1 \sin \omega_d t + C_2 \cos \omega_d t]$$

$$\frac{c}{2m} = \frac{230}{(2)(13)}$$
$$= 8.846 \text{ s}^{-1}$$

PROBLEM 19.136 (Continued)

Expression for ω_d:
$$\omega_d^2 = \frac{k}{m} - \left(\frac{c}{2m}\right)^2$$

$$\omega_d = \sqrt{\frac{1500}{13} - \left(\frac{230}{(2)(13)}\right)^2}$$
$$= 6.094 \text{ rad/s}$$
$$x = e^{-8.846t}(C_1 \sin 6.094t + C_2 \cos 6.094t)$$

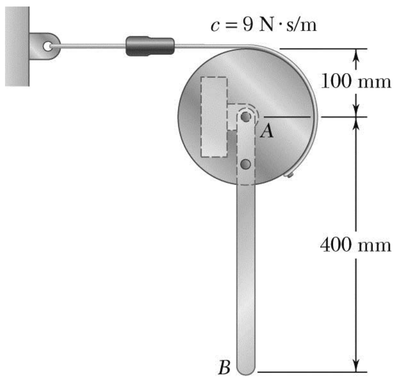
Initial conditions:
$$x_0 = -0.02619 \text{ m}$$
$$(t = 0) \quad \dot{x}_0 = +1.219 \text{ m/s}$$
$$x_0 = -0.02619 = e^0[C_1(0) + C_2(1)]$$
$$C_2 = -0.02619$$
$$\dot{x}(0) = -8.846e^{(-8.846)0}[C_1(0) + (-0.02619)(1)]$$
$$+e^{(-8.846)(0)}[6.094C_1(1) + C_2(0)] = 1.219$$

$$1.219 = (-8.846)(-0.02619) + 6.094C_1$$
$$C_1 = 0.16202$$
$$x = e^{-8.846t}(0.16202 \sin 6.094t - 0.02619 \cos 6.094t)$$

Maximum deflection occurs when $\dot{x} = 0$
$$\dot{x} = 0 = -8.846e^{-8.846t_m}(0.16202 \sin 6.094t_m - 0.02619 \cos 6.094t_m)$$
$$+ e^{-8.846t_m}[6.094][0.1620 \cos 6.094t_m + 0.02619 \sin 6.094t_m]$$
$$0 = [(-8.846)(0.16202) + (6.094)(0.02619)] \sin 6.094t_m$$
$$+[(-8.846)(-0.02619) + (6.094)(0.1620)] \cos 6.094t_m$$
$$0 = -1.274 \sin 6.094t_m + 1.219 \cos 6.094t_m$$
$$\tan 6.094t_m = \frac{1.219}{1.274} = 0.957$$

$$\text{Time at maximum deflection} = t_m = \frac{\tan^{-1} 0.957}{6.094} = 0.1253 \text{ s}$$
$$x_m = e^{-(8.846)(0.1253)}[0.1620 \sin(6.094)(0.1253)$$
$$-0.02619 \cos(6.094)(0.1253)]$$
$$x_m = (0.3301)(0.1120 - 0.0189) = 0.0307 \text{ m}$$

Blocks move, static deflection + x_m Total distance $= 0.02619 + 0.0307 = 0.0569 \text{ m} = 56.9 \text{ mm}$ ◀



PROBLEM 19.137

A 3-kg slender rod AB is bolted to a 5-kg uniform disk. A dashpot of damping coefficient $c = 9\ \text{N}\cdot\text{s/m}$ is attached to the disk as shown. Determine (*a*) the differential equation of motion for small oscillations, (*b*) the damping factor c/c_c.

SOLUTION

Data: $r = 100\ \text{mm} = 0.100\ \text{m},\ \ l = 400\ \text{mm} = 0.400\ \text{m}$

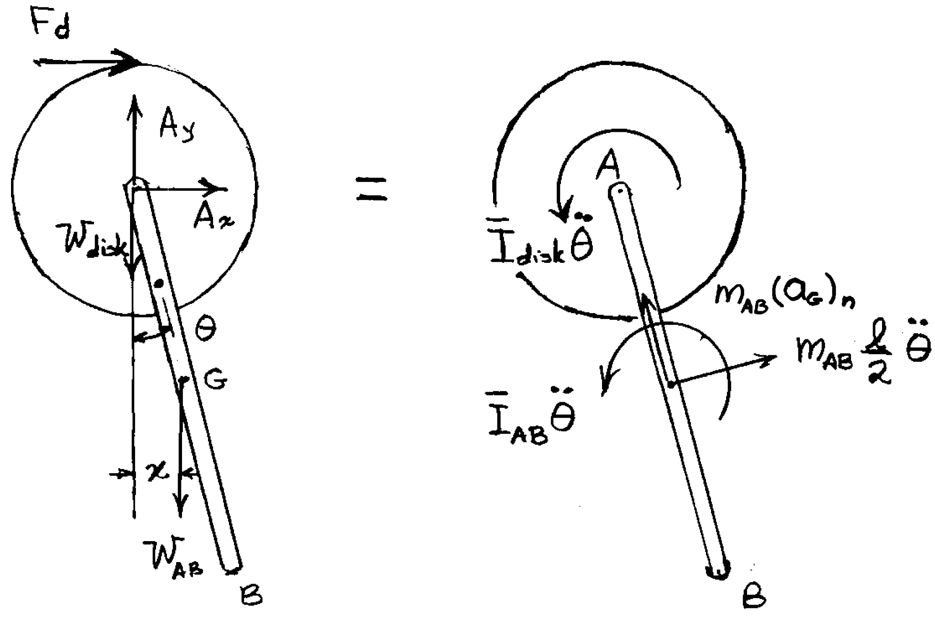
$$\bar{I}_{\text{disk}} = \frac{1}{2}m_{\text{disk}}r^2 = \frac{1}{2}(5\ \text{kg})(0.100\ \text{m})^2 = 0.025\ \text{kg}\cdot\text{m}^2$$

$$\bar{I}_{AB} = \frac{1}{2}m_{AB}l^2 = \frac{1}{12}(3\ \text{kg})(0.400\ \text{m})^2 = 0.040\ \text{kg}\cdot\text{m}^2$$

$$W_{AB} = m_{AB}g = (3\ \text{kg})(9.81\ \text{m/s}^2) = 29.43\ \text{N}$$

$$c = 9\ \text{N}\cdot\text{s/m}$$

Equation of motion: Let the disk and rod assembly be rotated through a small counterclockwise angle θ



$$\Sigma M_A = \Sigma(M_A)_{\text{eff}}:$$

$$-W_{AB}x - F_d r = \bar{I}_{\text{disk}}\ddot{\theta} + \bar{I}_{AB}\ddot{\theta} + m_{AB}\left(\frac{l}{2}\ddot{\theta}\right)\frac{l}{2}$$

PROBLEM 19.137 (Continued)

where $W_{AB}x = -W_{AB}\dfrac{l}{2}\sin\theta$

$= (29.43 \text{ N})(0.2 \text{ m}) \sin \theta$

$= 5.886 \sin\theta \text{ N} \cdot \text{m}$

$\approx -5.886\,\theta$

Damping force: $F_d = cr\dot{\theta}$

$$F_d r = cr^2\dot{\theta} = (9 \text{ N}\cdot\text{s}\cdot\text{m})(0.100 \text{ m})^2\dot{\theta} = 0.09\,\dot{\theta} = C\dot{\theta}$$

Inertia: $\bar{I}_{\text{disk}} + \bar{I}_{AB} + m_{AB}\left(\dfrac{l}{2}\right)^2 = 0.025 + 0.040 + (3)(0.2)^2 = 0.185 \text{ kg}\cdot\text{m}^2$

$$-5.886\theta - 0.09\dot{\theta} = 0.185\ddot{\theta}$$

$$0.185\ddot{\theta} + 0.09\dot{\theta} + 5.886\theta = 0$$

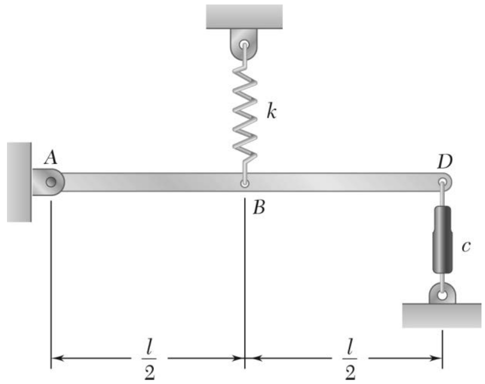
$$M\ddot{\theta} + C\dot{\theta} + K\theta = 0$$

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{5.886}{0.185}} = 5.6406 \text{ rad/s}$$

$$C_c = 2\,M\,\omega_2 = (2)(0.185)(5.6406) = 2.087$$

$$\frac{c}{c_c} = \frac{C}{C_c} = \frac{0.09}{2.087}$$

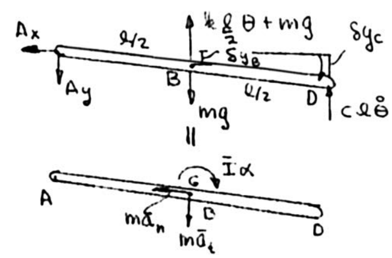
$\dfrac{c}{c_c} = 0.0431$ ◀



PROBLEM 19.138

A uniform rod of mass m is supported by a pin at A and a spring of constant k at B and is connected at D to a dashpot of damping coefficient c. Determine in terms of m, k, and c, for small oscillations, (*a*) the differential equation of motion, (*b*) the critical damping coefficient c_c.

SOLUTION



In equilibrium, the force in the spring is mg.

For small angles, $\sin\theta \approx \theta \qquad \cos\theta \approx 1$

$$\delta y_B = \frac{l}{2}\theta$$

$$\delta y_C = l\theta$$

(*a*) Newton's Law: $\Sigma M_A = (\Sigma M_A)_{\text{eff}}$

$$\frac{mgl}{2} - \left(k\frac{l}{2}\theta + mg\right)\frac{l}{2} - cl\dot{\theta}l = \bar{I}\alpha + m\bar{a}_t\frac{l}{2}$$

Kinematics: $\alpha = \ddot{\theta}$

$$\bar{a}_t = \frac{l}{2}\alpha = \frac{l}{2}\ddot{\theta}$$

$$\left[\bar{I} + m\left(\frac{l}{2}\right)^2\right]\ddot{\theta} + cl^2\dot{\theta} + k\left(\frac{l}{2}\right)^2\theta = 0$$

$$\bar{I} + m\left(\frac{l}{2}\right)^2 = \frac{1}{3}ml^2$$

$$\ddot{\theta} + \left(\frac{3c}{m}\right)\dot{\theta} + \left(\frac{3k}{4m}\right)\theta = 0 \blacktriangleleft$$

(*b*) Substituting $\theta = e^{\lambda t}$ into the differential equation obtained in (*a*), we obtain the characteristic equation,

$$\lambda^2 + \left(\frac{3c}{m}\right)\lambda + \frac{3k}{4m} = 0$$

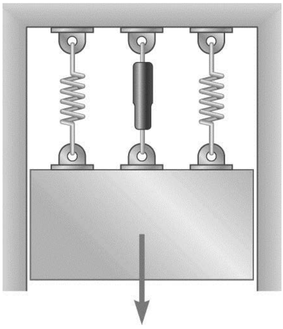
and obtain the roots

$$\lambda = \frac{\frac{-3c}{m} \mp \sqrt{\left(\frac{3c}{m}\right)^2 - \left(\frac{3k}{m}\right)}}{2}$$

The critical damping coefficient, c_c, is the value of c, for which the radicand is zero.

Thus, $\left(\frac{3c_c}{m}\right)^2 = \frac{3k}{m}$

$$c_c = \sqrt{\frac{km}{3}} \blacktriangleleft$$



PROBLEM 19.139

A machine element weighing 800 lb is supported by two springs, each having a constant of 200 lb/in. A periodic force of maximum value 30 lb is applied to the element with a frequency of 2.5 cycles per second. Knowing that the coefficient of damping is $8 \text{ lb}\cdot\text{s/in.}$, determine the amplitude of the steady-state vibration of the element.

SOLUTION

Equivalent spring: $k = 2(200) = 400 \text{ lb/in.} = 4800 \text{ lb/ft}$

Undamped natural frequency: $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{4800}{800/32.2}} = 13.90 \text{ rad/s}$

$$f_n = \frac{\omega_n}{2\pi} = \frac{13.90}{2\pi} = 2.212 \text{ Hz}$$

Critical damping coefficient: $c_c = 2\, m\omega_n = 2\left(\dfrac{800}{32.2}\right)(13.90) = 691 \text{ lb}\cdot\text{s/ft}$

Damping coefficient: $c = 8 \text{ lb}\cdot\text{s/in.} = 96 \text{ lb}\cdot\text{s/ft}$

Damping ratio: $\dfrac{c}{c_c} = \dfrac{96}{691} = 0.1390$

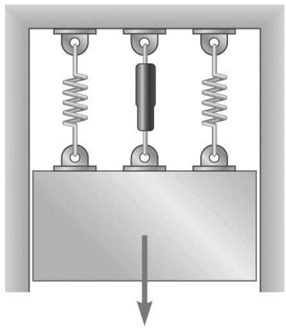
Amplitude:

$$x_m = \frac{P_m/k}{\sqrt{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left[2\frac{c}{c_c}\frac{\omega_f}{\omega_n}\right]^2}} \qquad (1)$$

where $\dfrac{\omega_f}{\omega_n} = \dfrac{f_f}{f_n} = \dfrac{2.5 \text{ Hz}}{2.212 \text{ Hz}} = 1.130$

Substituting into Eq. (1) with $P_m = 30$ lb, we have

$$x_m = \frac{30 \text{ lb/400 lb/in.}}{\sqrt{[1-(1.130)^2]^2 + [2(0.1390)(1.130)]^2}} \qquad x_m = 0.1791 \text{ in.} \blacktriangleleft$$



PROBLEM 19.140

In Problem 19.139, determine the required value of the coefficient of damping if the amplitude of the steady-state vibration of the element is to be 0.15 in.

PROBLEM 19.139 A machine element weighing 800 lb is supported by two springs, each having a constant of 200 lb/in. A periodic force of maximum value 30 lb is applied to the element with a frequency of 2.5 cycles per second. Knowing that the coefficient of damping is $8 \text{ lb}\cdot\text{s/in.}$, determine the amplitude of the steady-state vibration of the element.

SOLUTION

Equivalent spring: $k = 2(200) = 400 \text{ lb/in.} = 4800 \text{ lb/ft}$

Undamped natural frequency:
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4800}{800/32.2}} = 13.90 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{13.90}{2\pi} = 2.212 \text{ Hz}$$

Critical damping coefficient:
$$c_c = 2\,m\omega_n = 2\left(\frac{800}{32.2}\right)(13.90) = 691 \text{ lb}\cdot\text{s/ft}$$

Amplitude:
$$x_m = \frac{P_m/k}{\sqrt{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left[2\frac{c}{c_c}\frac{\omega_f}{\omega_n}\right]^2}} \qquad (1)$$

where
$$\frac{\omega_f}{\omega_n} = \frac{f_f}{f_n} = \frac{2.5 \text{ Hz}}{2.212 \text{ Hz}} = 1.130$$

Using $x_m = 0.15$ in., $P_m = 30$ lb, and $k = 400$ lb/in.

$$0.15 \text{ in.} = \frac{30 \text{ lb}/400 \text{ lb/in.}}{\sqrt{[1-(1.130)^2]^2 + \left[2\frac{c}{c_c}(1.130)\right]^2}}$$

Solving for $\frac{c}{c_c}$, we find $\frac{c}{c_c} = 0.1842.$

Since $c_c = 691 \text{ lb}\cdot\text{s/ft}$, we have

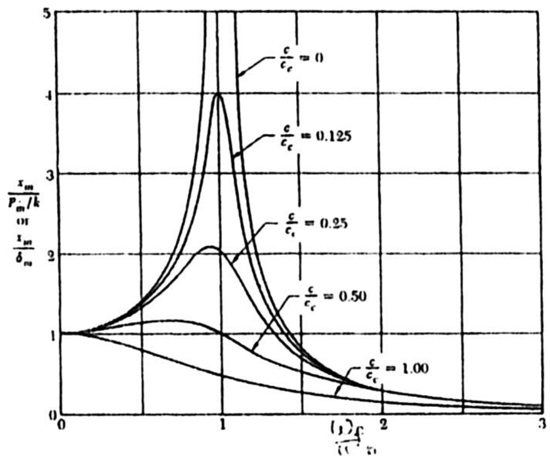
$$c = (0.1842)(691) \qquad c = 1273 \text{ lb}\cdot\text{s/ft}$$

or $c = 10.61 \text{ lb}\cdot\text{s/in.}$ ◀

PROBLEM 19.141

In the case of the forced vibration of a system, determine the range of values of the damping factor c/c_c for which the magnification factor will always decrease as the frequency ratio ω_f/ω_n increases.

SOLUTION



From Eq. $(19.53)'$:

Magnification factor:

$$\frac{x_m}{\frac{P_m}{k}} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)\right]^2}}$$

Find value of $\frac{c}{c_c}$ for which there is no maximum for $\frac{x_m}{\frac{P_m}{k}}$ as $\frac{\omega_f}{\omega_n}$ increases.

$$\frac{d\left(\frac{x_m}{\frac{P_m}{k}}\right)^2}{d\left(\frac{\omega_f}{\omega_n}\right)^2} = \frac{-\left[2\left(1-\left(\frac{\omega_f}{\omega_n}\right)^2\right)(-1)+4\frac{c^2}{c_c^2}\right]}{\left\{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)^2\right]\right\}^2} = 0$$

$$-2+2\left(\frac{\omega_f}{\omega_n}\right)^2 + 4\frac{c^2}{c_c^2} = 0$$

$$\left(\frac{\omega_f}{\omega_n}\right)^2 = 1-2\frac{c^2}{c_c^2}$$

For $\frac{c^2}{c_c^2} \geq \frac{1}{2}$, there is no maximum for $\frac{x_m}{\left(\frac{P_m}{k}\right)}$ and the magnification factor will decrease as $\frac{\omega_f}{\omega_n}$ increases.

$$\frac{c}{c_c} \geq \frac{1}{\sqrt{2}}$$

$$\frac{c}{c_c} \geq 0.707 \blacktriangleleft$$

PROBLEM 19.142

Show that for a small value of the damping factor c/c_c, the maximum amplitude of a forced vibration occurs when $\omega_f \approx \omega_n$ and that the corresponding value of the magnification factor is $\frac{1}{2}(c/c_c)$.

SOLUTION

From Eq. (19.53′):

$$\text{Magnification factor} = \frac{x_m}{\frac{P_m}{k}} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)\right]^2}}$$

Find value of $\dfrac{\omega_f}{\omega_n}$ for which $\dfrac{x_m}{\frac{P_m}{k}}$ is a maximum.

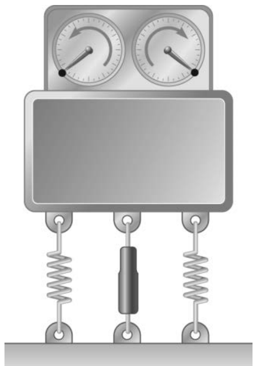
$$0 = \frac{d\left(\frac{x_m}{\frac{P_m}{k}}\right)^2}{d\left(\frac{\omega_f}{\omega_n}\right)^2} = -\frac{\left[2\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right](-1) + 4\left(\frac{c^2}{c_c^2}\right)\right]}{\left\{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)\right]^2\right\}^2}$$

$$-2 + 2\left(\frac{\omega_f}{\omega_n}\right)^2 + 4\left(\frac{c}{c_c}\right)^2 = 0$$

For small $\dfrac{c}{c_c}$, $\qquad \dfrac{\omega_f}{\omega_n} \approx 1 \qquad \omega_f \approx \omega_n$

For $\qquad \dfrac{\omega_f}{\omega_n} = 1$

$$\frac{x_m}{\frac{P_m}{k}} = \frac{1}{\sqrt{[1-1]^2 + \left[2\left(\frac{c}{c_c}\right)1\right]^2}} \qquad \frac{x_m}{\left(\frac{P_m}{k}\right)} = \frac{1}{2}\frac{c_c}{c} \blacktriangleleft$$



PROBLEM 19.143

A counter-rotating eccentric mass exciter consisting of two rotating 14-oz masses describing circles of 6-in. radius at the same speed but in opposite senses is placed on a machine element to induce a steady-state vibration of the element and to determine some of the dynamic characteristics of the element. At a speed of 1200 rpm a stroboscope shows the eccentric masses to be exactly under their respective axes of rotation and the element to be passing through its position of static equilibrium. Knowing that the amplitude of the motion of the element at that speed is 0.6 in. and that the total mass of the system is 300 lb, determine (*a*) the combined spring constant *k*, (*b*) the damping factor c/c_c.

SOLUTION

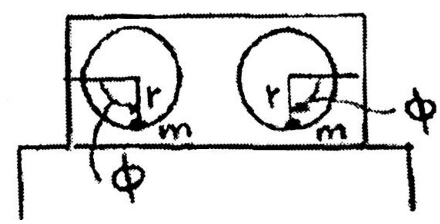
Forcing frequency: $\omega_f = 1200 \text{ rpm} = 125.664 \text{ rad/s}$

Unbalance of one mass: $w = 14 \text{ oz} = 0.875 \text{ lb}$

$r = 6 \text{ in.} = 0.5 \text{ ft}$

Shaking force:

$$P = 2\, mr\omega_f^2 \sin \omega_f t$$
$$= (2)\left(\tfrac{0.875}{32.2}\right)(0.5)(125.664)^2 \sin \omega_f t$$
$$= 429.11 \sin \omega_f t$$
$$P_m = 429 \text{ lb}$$



Total weight: $W = 300 \text{ lb}$

By Eqs. (19.48) and (19.52), the vibratory response of the system is

$$x = x_m \sin(\omega_f t - \varphi)$$

where

$$x_m = \frac{P_m}{\sqrt{\left(k - M\omega_f^2\right)^2 + (c\omega_f)^2}} \qquad (1)$$

and

$$\tan\varphi = \frac{c\omega_f}{k - M\omega_f^2} \qquad (2)$$

Since $\varphi = 90° = \dfrac{\pi}{2}$, $\tan\varphi = \infty$ and $k - M\omega_f^2 = 0.$

(*a*) Combined spring constant.

$$k = M\omega_f^2$$
$$= \left(\tfrac{300}{32.2}\right)(125.664)^2$$
$$= 147.12 \times 10^3 \text{ lb/ft}$$

$k = 147 \text{ kip/ft}$ ◀

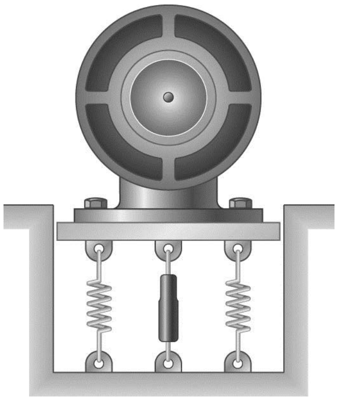
The observed amplitude is $x_m = 0.6 \text{ in.} = 0.05 \text{ ft}$

From Eq. (1):
$$c = \frac{1}{\omega_f}\sqrt{\left(\frac{P_m}{x_m}\right)^2 - (k - M\,\omega_f)^2} = \frac{P_m}{\omega_f\, x_m}$$
$$= \frac{429.11}{(125.664)(0.05)}$$
$$= 68.296 \text{ lb}\cdot\text{s/ft}$$

Critical damping coefficient:
$$c_c = 2\sqrt{kM}$$
$$= 2\sqrt{(147.12\times 10^3)\left(\tfrac{300}{32.2}\right)}$$
$$= 2.3416\times 10^3 \text{ lb}\cdot\text{s/ft}$$

(*b*) Damping factor.
$$\frac{c}{c_c} = \frac{68.296}{2.3416\times 10^3}$$
$$\frac{c}{c_c} = 0.0292 \blacktriangleleft$$

PROBLEM 19.144



A 15-kg motor is supported by four springs, each of constant 40 kN/m. The unbalance of the motor is equivalent to a mass of 20 kg located 125 mm from the axis of rotation. Knowing that the motor is constrained to move vertically and that the damping factor c/c_c is equal to 0.4, determine the range of frequencies for which the amplitude of the steady-state vibration of the motor is less than 0.2 mm.

SOLUTION

Equivalent spring: $k = (4)(40\times 10^3 \text{ N/m}) = 160\times 10^3 \text{ N/m}$

Mass: $m = 15 \text{ kg}$

Natural frequency: $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{160\times 10^3}{15}} = 103.280 \text{ rad/s}$

Unbalanced force:

$$P_m = m_{\text{eq}} r\omega_f^2 = m_{\text{eq}} r\omega_n^2 \left(\frac{\omega_f}{\omega_n}\right)^2$$

$$= (0.020 \text{ kg})(0.125 \text{ m})(130.280 \text{ rad/s})^2$$

$$= 26.667\left(\frac{\omega_f}{\omega_n}\right)^2 \text{ N}$$

At steady state,

$$x_m = \frac{P_m/k}{\left[\left(1-\left(\frac{\omega_f}{\omega_n}\right)^2\right)^2 + \left(2\frac{c}{c_c}\frac{\omega_f}{\omega_n}\right)^2\right]^{1/2}}$$

$$\left[\left(1-\left(\frac{\omega_f}{\omega_n}\right)^2\right)^2 + \left(2\frac{c}{c_c}\frac{\omega_f}{\omega_n}\right)^2\right]^{1/2} = \frac{P_m}{kx_m}$$

$$1-2\left(\frac{\omega_f}{\omega}\right)^2 + \left(\frac{\omega_f}{\omega_n}\right)^4 + \left(2\,\frac{c}{c_c}\frac{\omega_f}{\omega_n}\right)^2 = \left(\frac{P_m}{kx_m}\right)^2 \qquad (1)$$

Amplitude: $x_m = 0.2 \text{ mm} = 0.2\times 10^{-3} \text{ m}$

$$\frac{P_m}{kx_m} = \frac{26.667}{(160\times 10^3)(0.2\times 10^{-3})}\left(\frac{\omega_f}{\omega_n}\right)^2 = 0.83333\left(\frac{\omega_f}{\omega_n}\right)^2$$

Damping factor: $\dfrac{c}{c_c} = 0.4$

PROBLEM 19.144 (Continued)

Substituting into Eq. (1),

$$1-2\left(\frac{\omega_f}{\omega_n}\right)^2+\left(\frac{\omega_f}{\omega_n}\right)^4+\left[(2)(0.4)\frac{\omega_f}{\omega_n}\right]^2=\left(0.83333\frac{\omega_f^2}{\omega_n^2}\right)^2$$

$$[(1-(0.83333)^2]\left(\frac{\omega_f}{\omega_n}\right)^4-[2-(2)^2(0.4)^2]\left(\frac{\omega_f}{\omega_n}\right)^2+1=0$$

$$0.30556\left(\frac{\omega_f}{\omega_n}\right)^4-1.36\left(\frac{\omega_f}{\omega_n}\right)^2+1=0$$

Solving the quadratic equation for $\left(\frac{\omega_f}{\omega_n}\right)^2$,

$$\left(\frac{\omega_f}{\omega_n}\right)^2=3.5216 \quad \text{and} \quad 0.92934$$

$$\frac{\omega_f}{\omega_n}=1.8766 \quad \text{and} \quad 0.96402$$

$$\omega_f=(1.8766)(103.280 \text{ rad/s})=193.815 \text{ rad/s}$$

and

$$\omega_f=(0.96402)(103.280 \text{ rad/s})=99.564 \text{ rad/s}$$

For $x_m < 0.2$ m, the forcing frequency must satisfy

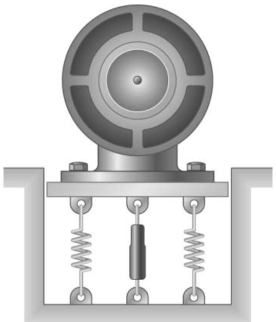
$$\omega_f>193.8 \text{ rad/s} \quad \text{and} \quad \omega_f<99.6 \text{ rad/s}$$

Since

$$f_f=\frac{\omega_f}{2\pi},$$

$f_f > 30.8$ Hz and $f_f < 15.85$ Hz ◀

PROBLEM 19.145



A 220-lb motor is supported by four springs, each of constant 500 lb/in., and is connected to the ground by a dashpot having a coefficient of damping $c = 35 \text{ lb} \cdot \text{s/in.}$ The motor is constrained to move vertically, and the amplitude of its motion is observed to be 0.08 in. at a speed of 1200 rpm. Knowing that the weight of the rotor is 30 lb, determine the distance between the mass center of the rotor and the axis of the shaft.

SOLUTION

Forcing frequency: $\omega_f = 1200 \text{ rpm} = 125.664 \text{ rad/s}$

Equivalent spring: $k = (4)(500) = 2000 \text{ lb/in.} = 24000 \text{ lb/ft}$

Mass: $$m = \frac{W}{g} = \frac{220 \text{ lb}}{32.2 \text{ ft/s}^2} = 6.8323 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Natural frequency: $$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{24000}{6.8323}} = 59.268 \text{ rad/s}$$

$$\frac{\omega_f}{\omega_n} = \frac{125.664 \text{ rad/s}}{59.268 \text{ rad/s}} = 2.12026$$

Critical damping coefficient: $c_c = 2m\omega_n$

$$c_c = (2)(6.8323)(59.268) = 809.87 \text{ lb} \cdot \text{s/ft}$$

Damping coefficient: $c = 35 \text{ lb} \cdot \text{s/in.} = 420 \text{ lb} \cdot \text{s/ft}$

Damping factor: $$\frac{c}{c_c} = \frac{420}{809.87} = 0.51860$$

Amplitude: $x_m = 0.08 \text{ in.} = 6.6667 \times 10^{-3} \text{ ft}$

$$x_m = \frac{P_m/k}{\left[\left(1 - \left(\frac{\omega_f}{\omega_n}\right)^2\right)^2 + \left(2\frac{c}{c_c}\frac{\omega_f}{\omega_n}\right)^2\right]^{1/2}}$$

Unbalanced force: $$P_m = kx_m\left[\left(1 - \left(\frac{\omega_f}{\omega_n}\right)^2\right)^2 + \left(2\frac{c}{c_c}\frac{\omega_f}{\omega_n}\right)^2\right]^{1/2}$$

$$= kx_m[(1 - 4.4955)^2 + ((2)(0.51860)(2.12026))^2]^{1/2}$$

$$= kx_m\ [12.2185 + 4.8362]^{1/2}$$

$$= 4.1297\,kx_m = (4.1297)(24000)(6.6667 \times 10^{-3})$$

$$= 660.76 \text{ lb}$$

PROBLEM 19.145 (Continued)

But, $$P_m = m'e\omega_f^2$$

where m' is the mass of the rotor and e is the distance between the mass center of the rotor and the axis of the shaft.

$$m' = \frac{30 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.93168 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$e = \frac{P_m}{m'\omega_f^2} = \frac{660.76 \text{ lb}}{(0.93168 \text{ lb} \cdot \text{s}^2/\text{ft})(125.664 \text{ rad/s})^2}$$

$$= 0.044911 \text{ ft}$$

$e = 0.539$ in. ◀

PROBLEM 19.146

A 100-lb motor is directly supported by a light horizontal beam which has a static deflection of 0.2 in. due to the weight of the motor. The unbalance of the rotor is equivalent to a mass of 3.5 oz located 3 in. from the axis of rotation. Knowing that the amplitude of the vibration of the motor is 0.03 in. at a speed of 400 rpm, determine (*a*) the damping factor c/c_c, (*b*) the coefficient of damping c.

SOLUTION

Spring constant: $$k = \frac{W}{\delta_{st}} = \frac{100}{\frac{0.2}{12}} = 6000 \text{ lb/ft}$$

Natural undamped circular frequency: $$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{6000}{\frac{100}{32.2}}} = 43.955 \text{ rad/s}$$

Unbalance: $$m' = \frac{w}{g} = \frac{\frac{3.5}{16}}{32.2} 6.7935\times 10^{-3} \text{ slug}$$

$$r = 3 \text{ in.} = 0.25 \text{ ft}$$

Forcing frequency: $$\omega_f = 400 \text{ rpm} = 41.888 \text{ rad/s}$$

Unbalance force: $$P_m = m' r \omega_f^2 = (6.7935\times 10^{-3})(0.25)(41.888)^2 = 2.98 \text{ lb}$$

Static deflection: $$\delta_{st} = \frac{P_m}{k} = \frac{2.98}{6000} = 0.49666\times 10^{-3} \text{ ft}$$

Amplitude: $$x_m = 0.03 \text{ in.} = 2.5\times 10^{-3} \text{ ft}$$

Frequency ratio: $$\frac{\omega_f}{\omega_n} = \frac{41.888}{43.955} = 0.95298$$

Eq. (19.53): $$x_m = \frac{\delta_{st}}{\sqrt{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)\right]^2}}$$

$$\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)\right]^2 = \left(\frac{\delta_{st}}{x_m}\right)^2$$

$$[1-(0.95298)^2]^2 + \left[2\left(\frac{c}{c_c}\right)(0.95298)\right]^2 = \left[\frac{0.49666\times 10^{-3}}{2.5\times 10^{-3}}\right]^2$$

$$0.0084326 + 3.6327\left(\frac{c}{c_c}\right)^2 = 0.039467$$

$$\left(\frac{c}{c_c}\right)^2 = 0.0085431$$

PROBLEM 19.146 (Continued)

(*a*) Damping factor. $\frac{c}{c_c} = 0.092429$ $\frac{c}{c_c} = 0.0924$ ◀

Critical damping factor. $c_c = 2\sqrt{km}$

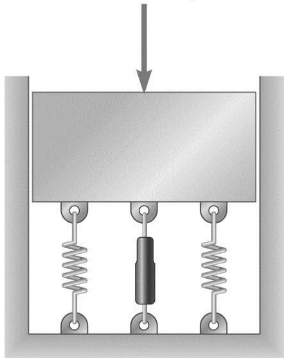
$$= 2\sqrt{(6000)\left(\tfrac{100}{32.2}\right)}$$

$$= 273.01 \text{ lb} \cdot \text{s/ft}$$

(*b*) Coefficient of damping. $c = \left(\frac{c}{c_c}\right)c_c$

$$= (0.092429)(273.01)$$

$c = 25.2 \text{ lb} \cdot \text{s/ft}$ ◀



PROBLEM 19.147

A machine element is supported by springs and is connected to a dashpot as shown. Show that if a periodic force of magnitude $P = P_m \sin \omega_f t$ is applied to the element, the amplitude of the fluctuating force transmitted to the foundation is

$$F_m = P_m \sqrt{\frac{1+\left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)\right]^2}{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2+\left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2}}$$

SOLUTION

From Equation (19.48), the motion of the machine is $\quad x = x_m \sin(\omega_f t - \phi)$

The force transmitted to the foundation is

Springs: $\quad F_s = kx = kx_m \sin(\omega_f t - \phi)$

Dashpot: $\quad F_d = c\dot{x} = cx_m\omega_f \cos(\omega_f t - \phi)$

$$F_t = x_m[k\sin(\omega_f t - \phi) + c\omega_f \cos(\omega_f t - \phi)]$$

or recalling the identity,

$$A\sin y + B\cos y = \sqrt{A^2 + B^2}\sin(y + \psi)$$

$$\sin\psi = \frac{B}{\sqrt{A^2 + B^2}}$$

$$\cos\psi = \frac{A}{\sqrt{A^2 + B^2}}$$

$$F_t = \left[x_m\sqrt{k^2 + (c\omega_f)^2}\right]\sin(\omega_f t - \phi + \psi)$$

Thus, the amplitude of F_t is $\quad F_m = x_m\sqrt{k^2 + (c\omega_f)^2} \qquad (1)$

From Equation (19.53):

$$x_m = \frac{\frac{P_m}{k}}{\sqrt{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2+\left[2\left(\frac{c}{c_c}\right)\frac{\omega_f}{\omega_n}\right]^2}}$$

Substituting for x_m in Equation (1),

$$F_m = \frac{P_m\sqrt{1+\left(\frac{c\omega_f}{k}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2+\left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n^2}\right)\right]}} \qquad (2)$$

$$\omega_n^2 = \frac{k}{m}$$

and Equation (19.41),

$$c_c = 2m\omega_n$$

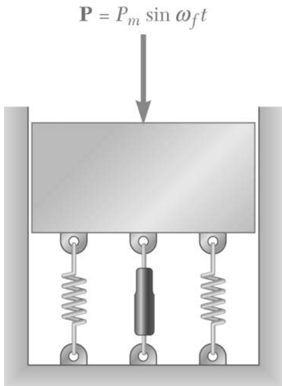
$$m = \frac{c\omega_n}{2}$$

$$\frac{c\omega_f}{k} = \frac{c\omega_f}{m\omega_n^2} = 2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)$$

Substituting in Eq. (2),

$$F_m = \frac{P_m\sqrt{1+\left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)\right]^2}}{\sqrt{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2+\left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)\right]^2}}$$

Q.E.D. ◀



PROBLEM 19.148

A 91-kg machine element supported by four springs, each of constant $k = 175$ N/m, is subjected to a periodic force of frequency 0.8 Hz and amplitude 89 N. Determine the amplitude of the fluctuating force transmitted to the foundation if (*a*) a dashpot with a coefficient of damping $c = 365$ N · s/m is connected to the machine element and to the ground, (*b*) the dashpot is removed.

SOLUTION

Forcing frequency: $\omega_f = 2\pi f_f = (2\pi)(0.8) = 1.6\pi$ rad/s

Exciting force amplitude: $P_m = 89$ N

Equivalent spring constant: $k = (4)(175 \text{ N/m}) = 700$ N/m

Natural frequency:
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{700}{91}}$$
$$= 2.7735 \text{ rad/s}$$

Frequency ratio:
$$\frac{\omega_f}{\omega_n} = \frac{1.6\pi}{2.7735}$$
$$= 1.8123$$

Critical damping coefficient:
$$c_c = 2\sqrt{km}$$
$$= 2\sqrt{(700)(91)}$$
$$= 504.78 \text{ N}\cdot\text{s/m}$$

From the derivation given in Problem 19.147, the amplitude of the force transmitted to the foundation is

$$F_m = \frac{P_m\sqrt{1+\left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)\right]^2}}{\sqrt{\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2+\left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)\right]^2}} \qquad (1)$$

$$1-\left(\frac{\omega_f}{\omega_n}\right)^2 = 1-(1.8123)^2 = -2.2844$$

PROBLEM 19.148 (Continued)

(*a*) F_m when $c = 365\ \text{N}\cdot\text{s/m}$:

$$\frac{c}{c_c} = \frac{365}{504.78}$$
$$= 0.72309$$
$$2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right) = (2)(0.72309)(1.8123)$$
$$= 2.6209$$

From Eq. (1):

$$F_m = \frac{89\sqrt{1+(2.6209)^2}}{\sqrt{(-2.2844)^2+(2.6209)^2}}$$
$$= \frac{89\sqrt{7.8692}}{\sqrt{12.088}}$$

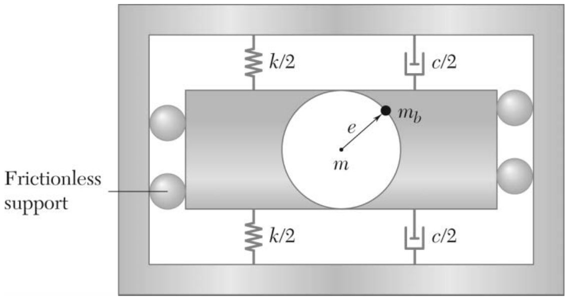
$F_m = 71.8\ \text{N}$ ◀

(*b*) F_m when $c = 0$:

$$F_m = \frac{P_m}{\left|1-\left(\frac{\omega_f}{\omega_n}\right)^2\right|} = \frac{89}{2.2844}$$

$F_m = 39.0\ \text{N}$ ◀

PROBLEM 19.149



A simplified model of a washing machine is shown. A bundle of wet clothes forms a weight w_b of 20 lb in the machine and causes a rotating unbalance. The rotating mass is 40 lb (including m_b) and the radius of the washer basket e is 9 in. Knowing the washer has an equivalent spring constant k = 70 lb/ft and damping ratio $\zeta = c/c_c = 0.05$ and during the spin cycle the drum rotates at 250 rpm, determine the amplitude of the motion and the magnitude of the force transmitted to the sides of the washing machine.

SOLUTION

Forced circular frequency: $\omega_f = \dfrac{(2\pi)(250)}{60} = 26.18 \text{ rad/s}$

System mass: $m = \dfrac{W}{g} = \dfrac{40 \text{ lb}}{32.2}$

Spring constant: $k = 70 \text{ lb/ft}$

Natural circular frequency: $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{70}{\frac{40}{32.2}}} = 7.5067 \text{ rad/s}$

Critical damping constant: $c_c = 2\sqrt{km} = 2\sqrt{(70)\left(\frac{40}{32.2}\right)} = 18.650 \text{ lb}\cdot\text{s/ft}$

Damping constant: $c = \left(\dfrac{c}{c_c}\right)c_c = (0.05)(18.650) = 0.9325 \text{ lb}\cdot\text{s/ft}$

Unbalance force: $m_b = \dfrac{w_b}{g}$

$$P_m = m_b e \omega_f^2$$

$$P_m = \left(\frac{20 \text{ lb}}{32.2}\right)\left(\frac{9}{12}\text{ft}\right)(26.18 \text{ rad/s})^2 = 319.28 \text{ lb}$$

The differential equation of motion is

$$m\ddot{x} + c\dot{x} + kx = P_m \sin \omega_f t$$

The steady state response is

$$x = x_m \sin(\omega_f t - \varphi) \qquad \dot{x} = \omega_f x_m \cos(\omega_f t - \varphi)$$

PROBLEM 19.149 (Continued)

where
$$x_m = \frac{P_m}{\sqrt{\left(k - m\omega_f^2\right)^2 + (c\omega_f)^2}}$$
$$= \frac{319.28}{\sqrt{[70 - \left(\frac{40}{32.2}\right)(26.18)^2]^2 + [(0.9325)(26.18)]^2}}$$
$$= \frac{319.28}{\sqrt{(-781.42)^2 + (24.413)^2}} = \frac{319.28}{781.796} = 0.40839 \text{ ft}$$

(*a*) Amplitude of vibration. $x_m = 4.90$ in. ◀

$$x = 0.40839 \sin(\omega_f t - \varphi)$$
$$\dot{x} = (26.18)(0.40839)\cos(\omega_f t - \varphi)$$
$$= 10.6917 \cos(\omega_f t - \varphi)$$

Spring force:
$$kx = (70)(0.40839)\sin(\omega_f t - \varphi)$$
$$= 28.588 \sin(\omega_f t - \varphi)$$

Damping force:
$$c\dot{x} = (0.9325)(10.6917)\cos(\omega_f t - \varphi)$$
$$= 9.9701 \cos(\omega_f t - \varphi)$$

(*b*) Total force:
$$F = 28.588 \sin(\omega_f t - \varphi) + 9.9701 \cos(\omega_f t - \varphi)$$

Let
$$F = F_m \cos\psi \sin(\omega_f t - \varphi) + F_m \sin\psi \sin(\omega_f t - \varphi)$$
$$= F_m \sin(\omega_f t - \varphi + \psi)$$

Maximum force.
$$F_m^2 = F_m^2 \cos^2\psi + F_m^2 \sin^2\psi$$
$$= (28.588)^2 + (9.9701)^2$$
$$= 916.65$$

$F_m = 30.3$ lb ◀

PROBLEM 19.150*

For a steady-state vibration with damping under a harmonic force, show that the mechanical energy dissipated per cycle by the dashpot is $E = \pi c x_m^2 \omega_f$, where c is the coefficient of damping, x_m is the amplitude of the motion, and ω_f is the circular frequency of the harmonic force.

SOLUTION

Energy is dissipated by the dashpot.

From Equation (19.48), the deflection of the system is

$$x = x_m \sin(\omega_f t - \varphi)$$

The force on the dashpot.

$$F_d = c\dot{x}$$

$$F_d = c x_m \omega_f \cos(\omega_f t - \varphi)$$

The work done in a complete cycle with

$$\tau_f = \frac{2\pi}{\omega_f}$$

$$E = \int F_d dx \text{ (i.e., force} \times \text{distance)}$$

$$dx = x_m \omega_f \cos(\omega_f t - \varphi) dt$$

$$E = \int_0^{2\pi/\omega_f} c x_m^2 \omega_f^2 \cos^2(\omega_f t - \varphi) dt$$

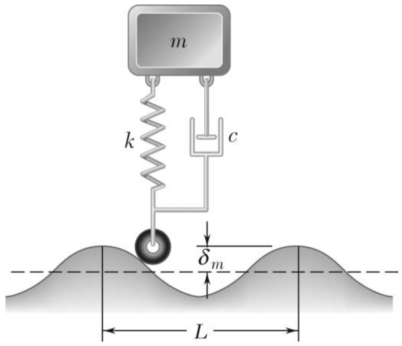
$$\cos^2(\omega_D t - \varphi) = \frac{[1 - 2\cos(\omega_f t - \phi)]}{2}$$

$$E = c x_m^2 \omega_f^2 \int_0^{2\pi/\omega_f} \frac{1 - 2\cos(\omega_f t - \phi)}{2} dt$$

$$E = \frac{c x_m^2 \omega_f^2}{2} \left[t - \frac{2\sin(\omega_f t - \phi)}{\omega_f} \right]_0^{2\pi/\omega_f}$$

$$E = \frac{c x_m^2 \omega_f^2}{2} \left[\frac{2\pi}{\omega_f} - \frac{2}{\omega_f} (\sin(2\pi - \varphi) + \sin\varphi) \right]$$

$E = \pi c x_m^2 \omega_f$ Q.E.D. ◀

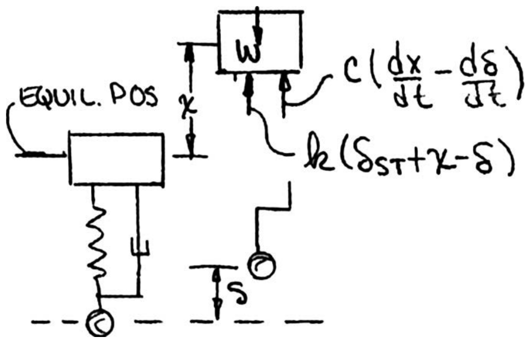


PROBLEM 19.151*

The suspension of an automobile can be approximated by the simplified spring-and-dashpot system shown. (*a*) Write the differential equation defining the vertical displacement of the mass *m* when the system moves at a speed *v* over a road with a sinusoidal cross section of amplitude δ_m and wave length *L*. (*b*) Derive an expression for the amplitude of the vertical displacement of the mass *m*.

SOLUTION

(*a*)



$$+\downarrow \Sigma F = ma: \quad W - k(\delta_{st} + x - \delta) - c\left(\frac{dx}{dt} - \frac{d\delta}{dt}\right) = m\frac{d^2x}{dt^2}$$

Recalling that $W = k\delta_{st}$, we write

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = k\delta + c\frac{d\delta}{dt} \quad (1)$$

Motion of wheel is a sine curve, $\delta = \delta_m \sin \omega_f t.$ The interval of time needed to travel a distance *L* at a speed *v* is $t = \frac{L}{v}.$

Thus,
$$\omega_f = \frac{2\pi}{\tau_f} = \frac{2\pi}{\frac{L}{v}} = \frac{2\pi v}{L}$$

and
$$\delta = \delta_m \sin \omega_f t \quad \frac{d\delta}{dt} = \frac{\delta_m 2\pi}{\frac{L}{v}} \cos \omega_f t$$

Thus, Equation (1) is

$$m\frac{d^2x}{dt} + c\frac{dx}{dt} + kx = (k \sin \omega_f t + c\omega_f \cos \omega_f t)\delta_m \blacktriangleleft$$

PROBLEM 19.151* (Continued)

(*b*) From the identity

$$A\sin y + B\cos y = \sqrt{A^2 + B^2}\,\sin(y+\psi)$$

$$\sin\psi = \frac{B}{\sqrt{A^2 + B^2}}$$

$$\cos\psi = \frac{A}{\sqrt{A^2 + B^2}}$$

We can write the differential equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = \delta_m\sqrt{k^2 + (c\omega_f)^2}\,\sin(\omega_f t + \psi)$$

$$\psi = \tan^{-1}\frac{c\omega_f}{k}$$

The solution to this equation is analogous to Equations 19.47 and 19.48, with

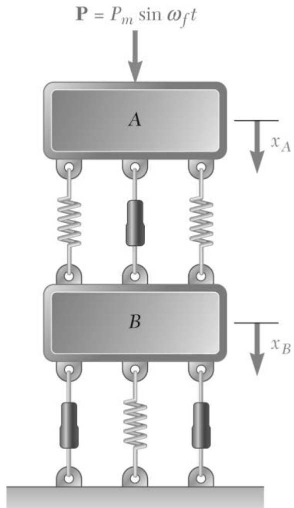
$$P_m = \delta_m\sqrt{k^2 + (c\omega_f)^2}$$

$$x = x_m\sin(\omega_f t - \varphi + \psi) \text{ (where analogous to Equations (19.52))} \blacktriangleleft$$

$$x_m = \frac{\delta_m\sqrt{k^2 + (c\omega_f)^2}}{\sqrt{\left(k - m\omega_f^2\right)^2 + (c\omega_f)^2}} \blacktriangleleft$$

$$\tan\varphi = \frac{c\omega_f}{k - m\omega_f^2} \blacktriangleleft$$

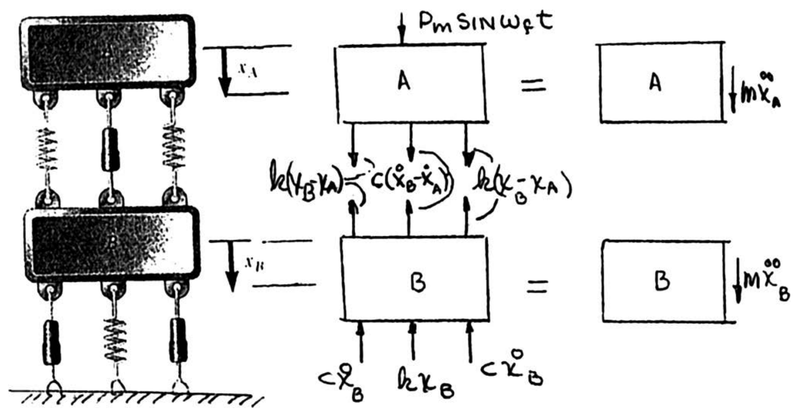
$$\tan\psi = \frac{c\omega_f}{k} \blacktriangleleft$$



PROBLEM 19.152*

Two blocks A and B, each of mass m, are supported as shown by three springs of the same constant k. Blocks A and B are connected by a dashpot, and block B is connected to the ground by two dashpots, each dashpot having the same coefficient of damping c. Block A is subjected to a force of magnitude $P = P_m \sin \omega_f t$. Write the differential equations defining the displacements x_A and x_B of the two blocks from their equilibrium positions.

SOLUTION



Since the origins of coordinates are chosen from the equilibrium position, we may omit the initial spring compressions and the effect of gravity

For load A,

$$+\downarrow \Sigma F = ma_A: \quad P_m \sin \omega_f t + 2k(x_B - x_A) + c(\dot{x}_B - \dot{x}_A) = m\ddot{x}_A \qquad (1)$$

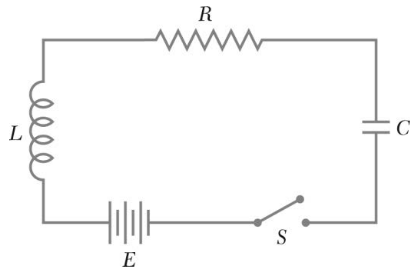
For load B,

$$+\downarrow \Sigma F = ma_B: \quad -2k(x_B - x_A) - c(\dot{x}_B - \dot{x}_A) - kx_B - 2c\dot{x}_B = m\ddot{x}_B \qquad (2)$$

Rearranging Equations (1) and (2), we find:

$$m\ddot{x}_A + c(\dot{x}_A - \dot{x}_B) + 2k(x_A - x_B) = P_m \sin \omega_f t \blacktriangleleft$$

$$m\ddot{x}_B + 3c\dot{x}_B - c\dot{x}_A + 3kx_B - 2kx_A = 0 \blacktriangleleft$$



PROBLEM 19.153

Express in terms of L, C, and E the range of values of the resistance R for which oscillations will take place in the circuit shown when switch S is closed.

SOLUTION

For a mechanical system, oscillations take place if $c < c_c$ (lightly damped).

But from Equation (19.41), $$c_c = 2m\sqrt{\frac{k}{m}} = 2\sqrt{km}$$

Therefore, $$c < 2\sqrt{km} \qquad (1)$$

From Table 19.2:

$$c \longrightarrow R$$

$$m \longrightarrow L$$

$$k \longrightarrow \frac{1}{C} \qquad (2)$$

Substituting in Eq. (1) the analogous electrical values in Eq. (2), we find that oscillations will take place if

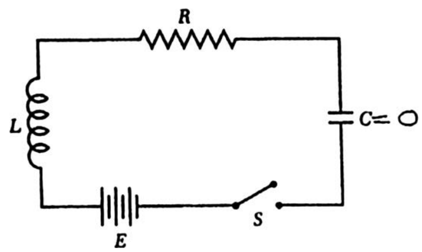
$$R < 2\sqrt{\left(\frac{1}{C}\right)(L)} \qquad R < 2\sqrt{\frac{L}{C}} \blacktriangleleft$$

PROBLEM 19.154

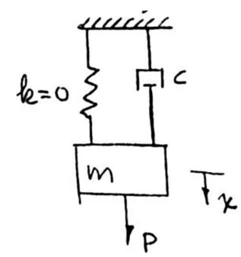
Consider the circuit of Problem 19.153 when the capacitor C is removed. If switch S is closed at time $t = 0$, determine (a) the final value of the current in the circuit, (b) the time t at which the current will have reached $(1 - 1/e)$ times its final value. (The desired value of t is known as the *time constant* of the circuit.)

SOLUTION

Electrical system

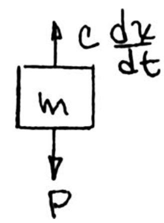


Mechanical system



The mechanical analogue of closing a switch S is the sudden application of a constant force of magnitude P to the mass.

(a) Final value of the current corresponds to the final velocity of the mass, and since the capacitance is zero, the spring constant is also zero



$$+\downarrow \Sigma F = ma: \quad P - c\frac{dx}{dt} = m\frac{d^2x}{dt^2} \tag{1}$$

Final velocity occurs when $\dfrac{d^2x}{dt^2} = 0$

$$P - c\left.\frac{dx}{dt}\right|_{\text{final}} = 0 \qquad \left.\frac{dx}{dt}\right|_{\text{final}} = v_{\text{final}}$$

$$v_{\text{final}} = \frac{P}{c}$$

From Table 19.2: $\quad v \longrightarrow i, \quad P \longrightarrow E, \quad c \longrightarrow R$

Thus, $\quad i_{\text{final}} = \dfrac{E}{R}$ ◀

PROBLEM 19.154 (Continued)

(*b*) Rearranging Equation (1), we have

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}=P$$

Substitute
$$\frac{dx}{dt}=Ae^{-\lambda t}+\frac{P}{c};\qquad \frac{d^2x}{dt}=-A\lambda e^{-\lambda t}$$

$$m\left[-A\lambda e^{-\lambda t}\right]+c\left[Ae^{-\lambda t}+\frac{P}{c}\right]=P$$

$$-m\lambda+c=0\qquad \lambda=\frac{c}{m}$$

Thus,
$$\frac{dx}{dt}=Ae^{-(c/m)t}+\frac{P}{c}$$

At $t=0$,
$$\frac{dx}{dt}=0\quad 0=A+\frac{P}{c}\qquad A=-\frac{P}{c}$$

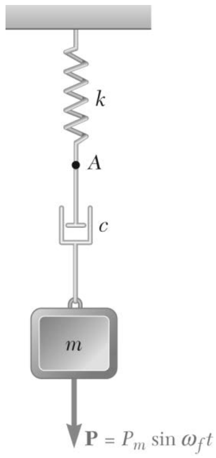
$$v=\frac{dx}{dt}=\frac{p}{c}\left[1-e^{-(c/m)t}\right]$$

From Table 19.2:
$$v\longrightarrow i,\quad P\longrightarrow E,\quad c\longrightarrow R,\quad m\longrightarrow L$$

$$L=\frac{E}{R}\left[1-e^{-(R/L)t}\right]$$

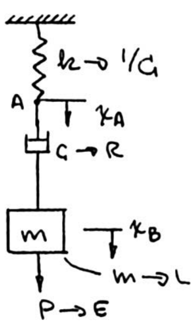
For $i=\left(\frac{E}{R}\right)\left(1-\frac{1}{e}\right)$,
$$\left(\frac{R}{L}\right)t=1$$
$$t=\frac{L}{R}\ ◀$$

PROBLEM 19.155



Draw the electrical analogue of the mechanical system shown. (*Hint:* Draw the loops corresponding to the free bodies m and A.)

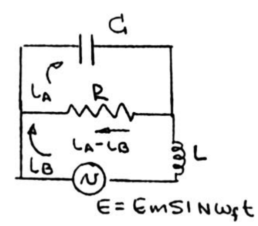
SOLUTION



We note that both the spring and the dashpot affect the motion of Point A. Thus, one loop in the electrical circuit should consist of a capacitor $\left(k \Rightarrow \frac{1}{C}\right)$ and a resistance $(c \Rightarrow R)$.

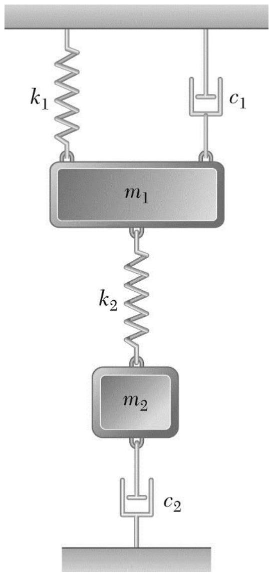
The other loop consists of $(P_m \sin \omega_f t \longrightarrow E_m \sin \omega_f t)$, an inductor $(m \longrightarrow L)$ and the resistor $(c \longrightarrow R)$.

Since the resistor is common to both loops, the circuit is



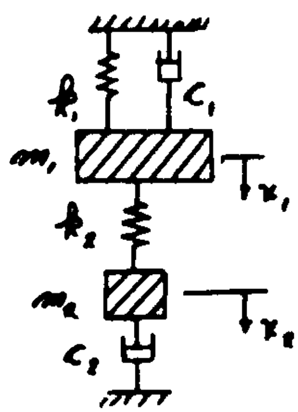
◀

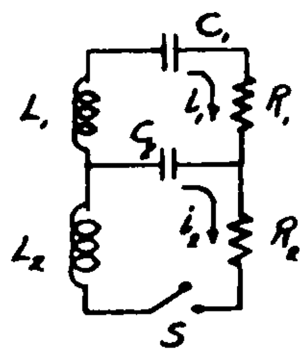
PROBLEM 19.156



Draw the electrical analogue of the mechanical system shown. (*Hint:* Draw the loops corresponding to the free bodies *m* and *A*.)

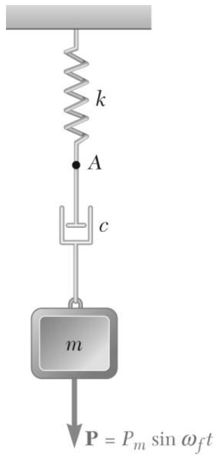
SOLUTION





Loop 1 (Mass 1)	Loop 2 (Mass 2)
$k_1 \longrightarrow 1/C_1$	$k_2 \longrightarrow 1/C_2$
$c_1 \longrightarrow R_1$	$c_2 \longrightarrow R_2$
$m_1 \longrightarrow L_1$	$m_2 \longrightarrow L_2$
$x_1 \longrightarrow q_1$	$x_2 \longrightarrow q_2$
$\dot{x}_1 \longrightarrow \dot{x}_1$	$\dot{x}_2 \longrightarrow \dot{x}_2$

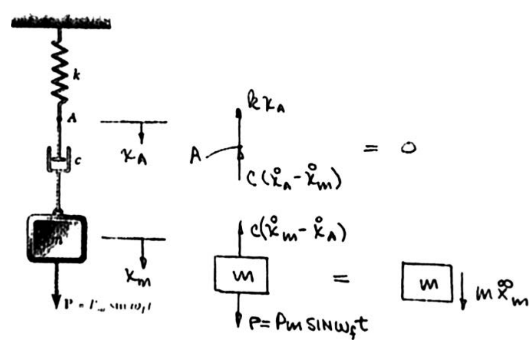
k_2 is connected to both masses, so C_2 is common to both loops.



PROBLEM 19.157

Write the differential equations defining (*a*) the displacements of the mass *m* and of the Point *A*, (*b*) the charges on the capacitors of the electrical analogue.

SOLUTION



(*a*) Mechanical system.

Point *A*: $+\uparrow \Sigma F = 0$: $$c\frac{d}{dt}(x_A - x_m) + kx_A = 0 \quad ◀$$

Mass *m*:

$$+\uparrow \Sigma F = ma: \quad c\frac{d}{dt}(x_m - x_A) - P_m \sin \omega_f t = -m\frac{d^2 x_m}{dt^2}$$

$$m\frac{d^2 x_m}{dt^2} + c\frac{d}{dt}(x_m - x_A) = P_m \sin \omega_f t \quad ◀$$

(*b*) Electrical analogue.
From Table 19.2:

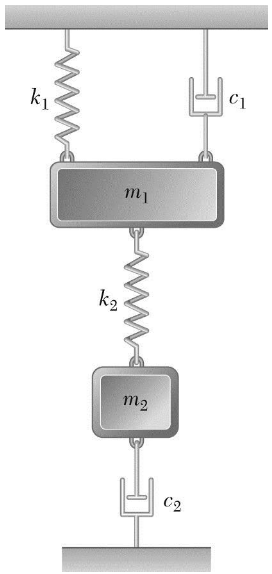
$$m \longrightarrow L$$
$$c \longrightarrow R$$
$$k \longrightarrow \frac{1}{C}$$
$$x \longrightarrow q$$
$$P \longrightarrow E$$

Substituting into the results from Part (*a*), the analogous electrical characteristics,

$$R\frac{d}{dt}(q_A - q_m) + \left(\frac{1}{C}\right)q_n = 0 \blacktriangleleft$$

$$L\frac{d^2 q_m}{dt^2} + R\frac{d}{dt}(q_m - q_A) = E_m \sin \omega_f t \blacktriangleleft$$

Note: These equations can also be obtained by summing the voltage drops around the loops in the circuit of Problem 19.155.



PROBLEM 19.158

Write the differential equations defining (*a*) the displacements of the masses m_1 and m_2, (*b*) the charges on the capacitors of the electrical analogue.

SOLUTION

(*a*) Displacements at masses m_1 and m_2

$$m_1 \frac{d^2 x_1}{dt^2} + c_1 \frac{dx_1}{dt} + k_1 x_1 + k_2(x_1 - x_2) = 0 \quad \blacktriangleleft$$

$$m_2 \frac{d^2 x_2}{dt^2} + c_2 \frac{dx_2}{dt} + k_2(x_2 - x_1) = 0 \quad \blacktriangleleft$$

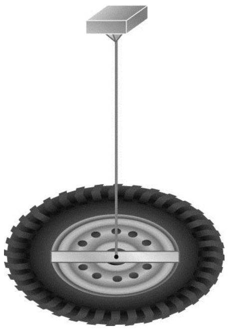
(*b*) Electrical analogues.

We let: $q_1 = \int i_1 dt \qquad q_2 = \int i_2 dt$

Thus, $i_1 = \frac{dq_1}{dt} \qquad i_2 = \frac{dq_2}{dt}$

$$L_1 \frac{d^2 q_1}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{q_1}{C_1} + \frac{(q_1 - q_2)}{C_2} = 0 \quad \blacktriangleleft$$

$$L_2 \frac{d^2 q_2}{dt^2} + R_2 \frac{dq_2}{dt} + \frac{q_2 - q_1}{C_2} = 0 \quad \blacktriangleleft$$



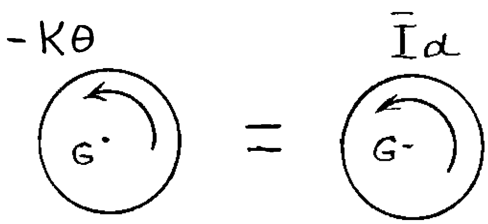
PROBLEM 19.159

An automobile wheel-and-tire assembly of total weight 47 lb is attached to a mounting plate of negligible weight which is suspended from a steel wire. The torsional spring constant of the wire is known to be $K = 0.40$ lb · in./rad. The wheel is rotated through 90° about the vertical and then released. Knowing that the period of oscillation is observed to be 30 s, determine the centroidal mass moment of inertia and the centroidal radius of gyration of the wheel-and-tire assembly.

SOLUTION

Torsional spring constant: $K = 0.40 \text{ lb}\cdot\text{in/rad} = 33.333\times10^{-3} \text{ lb}\cdot\text{ft/rad}$

Let the wheel-and-tire assembly be rotated through the small angle θ. The moment that the wire exerts on the assembly is



$$M = -K\theta$$

$$\Sigma M = \Sigma M_{\text{eff}} = \bar{I}\alpha: \quad -K\theta = \bar{I}\alpha = \bar{I}\ddot{\theta}$$

$$\ddot{\theta} + \frac{K}{\bar{I}}\theta = 0$$

$$\omega_n^2 = \frac{K}{\bar{I}} \qquad (1)$$

Frequency: $f = \dfrac{1}{\tau} = \dfrac{1}{30 \text{ s}} = 0.033333 \text{ H}_z$

$$\omega_n = 2\pi f = (2\pi)(0.033333) = 0.20944 \text{ rad/s}$$

From Eq. (1), $\bar{I} = \dfrac{K}{\omega_n^2} = \dfrac{33.333\times10^{-3} \text{ lb}\cdot\text{ft/rad}}{(0.20944 \text{ rad/s})^2}$

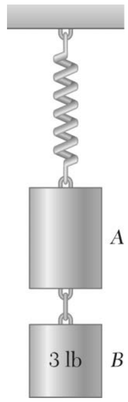
Centroidal mass moment of inertia: $\bar{I} = 0.75990 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$ $\qquad \bar{I} = 0.760 \text{ lb}\cdot\text{s}^2\cdot\text{ft}$ ◀

Mass: $m = \dfrac{W}{g} = \dfrac{47 \text{ lb}}{32.2 \text{ ft/s}^2} = 1.4596 \text{ lb}\cdot\text{s}^2/\text{ft}$

Centroidal radius of gyration: $\bar{k}^2 = \dfrac{\bar{I}}{m} = \dfrac{0.75990 \text{ lb}\cdot\text{s}^2\cdot\text{ft}}{1.4596 \text{ lb}\cdot\text{s}^2/\text{ft}} = 0.52061 \text{ ft}^2$

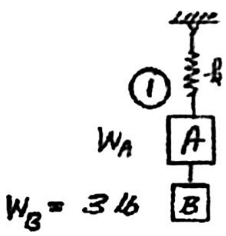
$\bar{k} = 0.7215 \text{ ft}$ $\qquad \bar{k} = 8.66 \text{ in.}$ ◀

PROBLEM 19.160



The period of vibration of the system shown is observed to be 0.6 s. After cylinder *B* has been removed, the period is observed to be 0.5 s. Determine (*a*) the weight of cylinder *A*, (*b*) the constant of the spring.

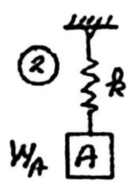
SOLUTION



$$m_1 = \frac{W_A + 3}{g} \qquad \tau_1 = 0.6 \text{ s}$$

$$\tau_1 = \frac{2\pi}{\omega_1} \qquad \omega_1 = \frac{2\pi}{\tau_1} = \frac{2\pi}{0.6} = 3.333\pi \text{ rad/s}$$

$$\omega_1^2 = \frac{k}{m_1} \qquad k = m_1\omega_1^2 = \left(\frac{W_A + 3}{g}\right)(3.333\pi)^2 \qquad (1)$$



$$m_2 = \frac{W_A}{g} \qquad \tau_2 = 0.5 \text{ s}$$

$$\tau_2 = \frac{2\pi}{\omega_2} \qquad \omega_2 = \frac{2\pi}{\tau_2} = \frac{2\pi}{0.5} = 4\pi \text{ rad/s}$$

$$\omega_2^2 = \frac{k}{m_2} \qquad k = m_2\omega_2^2 = \frac{W_A}{g}(4\pi)^2 \qquad (2)$$

(*a*) Equating the expressions found for *k* in Eqs. (1) and (2):

$$\frac{W_A + 3}{g}(3.333\pi)^2 = \frac{W_A}{g}(4\pi)^2$$

$$(11.111)(W_A + 3) = 16W_A$$

$$4.889W_A = 33.333$$

$$W_A = 6.818 \text{ lb}$$

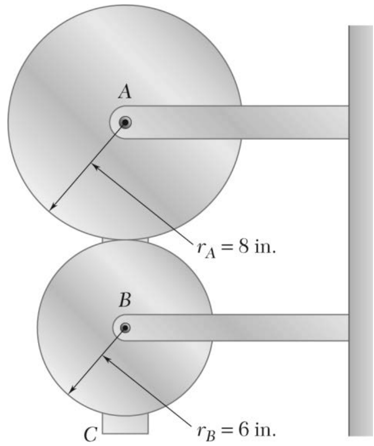
$W_A = 6.82$ lb ◀

(*b*) Eq. (1):

$$k = \frac{6.818 \text{ lb} + 3 \text{ lb}}{32.2 \text{ ft/s}^2}(3.333\pi \text{ rad/s})^2$$

$$k = 33.44 \text{ lb/ft}$$

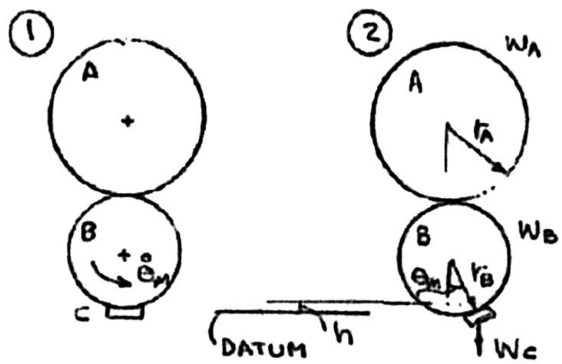
$k = 33.4$ lb/ft ◀



PROBLEM 19.161

Disks A and B weigh 30 lb and 12 lb, respectively, and a small 5-lb block C is attached to the rim of disk B. Assuming that no slipping occurs between the disks, determine the period of small oscillations of the system.

SOLUTION



Small oscillations: $$h = r_B(1-\cos\theta_m) \approx \frac{r_B\theta_B^2}{2}$$

<u>*Position* ①</u> $$r_B\dot{\theta}_B = r_A\dot{\theta}_A$$

$$T_1 = \frac{1}{2}m_C(r_B\dot{\theta}_m)^2 + \frac{1}{2}\overline{I}_B\dot{\theta}_m^2 + \frac{1}{2}\overline{I}_A\left(\frac{r_B}{r_A}\dot{\theta}_m\right)^2$$

$$\overline{I}_B = \frac{m_Br_B^2}{2}$$

$$\overline{I}_A = \frac{m_Ar_A^2}{2}$$

$$T_1 = \frac{1}{2}\left[m_Cr_B^2 + \frac{m_Br_B^2}{2} + \left(\frac{m_Ar_A^2}{2}\right)\left(\frac{r_B}{r_A}\right)^2\right]\dot{\theta}_m^2$$

$$T_1 = \frac{1}{2}\left[\left(m_C + \frac{m_B}{2} + \frac{m_A}{2}\right)\right]r_B^2\dot{\theta}_m^2$$

$$V_1 = 0$$

PROBLEM 19.161 (Continued)

Position ②

$$T_2 = 0$$

$$V_2 = m_C g h$$

$$= \frac{m_C g \dot{\theta}_m^2}{2}$$

Conservation of energy and simple harmonic motion.

$$T_1 + V_1 = T_2 + V_2$$

$$\dot{\theta}_m = \omega_n \theta_m$$

$$\frac{1}{2}\left[\left(m_C + \frac{m_B}{2} + \frac{m_A}{2}\right)\right] r_B^2 \omega_n^2 \theta_m^2 + 0 = 0 + \frac{m_C g r_B \theta_m^2}{2}$$

$$\omega_n^2 = \frac{m_C}{m_C + \frac{(m_B + m_A)}{2}} \frac{g}{r_B}$$

$$\omega_n^2 = \frac{5}{5 + \frac{(12+30)}{2}} \frac{(32.2 \text{ ft/s}^2)}{\left(\frac{6}{12}\right)\text{ft}}$$

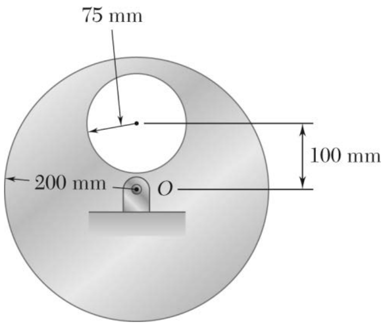
$$\omega_n^2 = 12.39 \text{ s}^{-2}$$

Period of small oscillations.

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{12.39}}$$

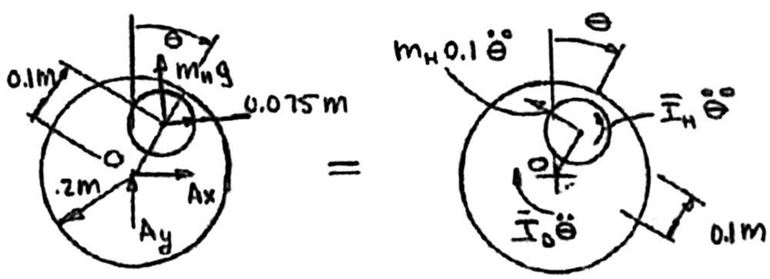
$\tau_n = 1.785$ s ◀

PROBLEM 19.162



A 75-mm-radius hole is cut in a 200-mm-radius uniform disk, which is attached to a frictionless pin at its geometric center O. Determine (*a*) the period of small oscillations of the disk, (*b*) the length of a simple pendulum which has the same period.

SOLUTION



Equation of motion.

$$\Sigma M_0 = (\Sigma M_0)_{\text{eff}}: \quad \circlearrowleft^{+} -m_H g(0.1)\sin\theta = \bar{I}_D\theta - I_H\ddot{\theta} - (0.1)^2 m_H\ddot{\theta}$$

$$\begin{aligned} m_D &= \rho t\pi R^2 \\ &= (\rho t\pi)(0.2)^2 \\ &= (0.04)\pi\rho t \\ m_H &= \rho t\pi r^2 \\ &= (\rho t\pi)(0.075)^2 \\ &= (0.005625)\pi\rho t \\ I_D &= \frac{1}{2}m_D R^2 = \frac{1}{2}(0.04\pi\rho t)(0.2)^2 \\ &= 800\times10^{-6}\pi\rho t \\ I_H &= \frac{1}{2}m_H r^2 \\ &= \frac{1}{2}(0.005625\pi\rho t)(0.075)^2 \\ &= 15.82\times10^{-6}\pi\rho t \end{aligned}$$

Small angles. $\sin\theta \approx \theta$

$$[(800\times10^{-6}\not\pi - 15.82\times10^{-6}\not\pi - (0.1)^2(0.005625\not\pi)]\rho\not t\,\ddot{\theta} + (0.005625)\not\pi\,\rho\not t\,(9.81)(0.1)\theta = 0$$

$$727.9\times10^{-6}\ddot{\theta} + 5.518\times10^{-3}\theta = 0$$

PROBLEM 19.162 (Continued)

(*a*) Natural frequency and period.

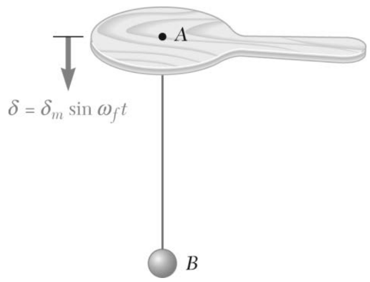
$$\omega_n^2 = \frac{5.518\times10^{-3}}{727.9\times10^{-6}}$$
$$= 7.581$$
$$\omega_n = 2.753 \text{ rad/s}$$
$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{2.753}$$

$\tau_n = 2.28$ s ◀

(*b*) Length and period of a simple pendulum.

$$\tau_n = 2\pi\sqrt{\frac{l}{g}}$$
$$l = \left(\frac{\tau_n}{2\pi}\right)^2 g$$
$$l = \left[\frac{(2.753)}{2\pi}\right]^2 (9.81 \text{ m/s}^2)$$

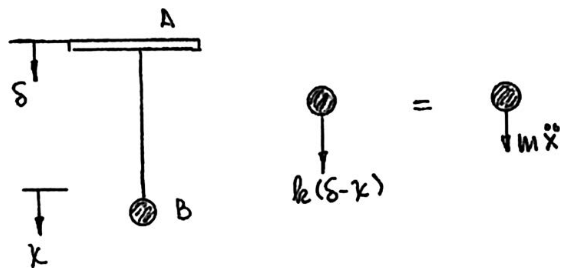
$l = 1.294$ m ◀



PROBLEM 19.163

An 0.8-lb ball is connected to a paddle by means of an elastic cord *AB* of constant $k = 5$ lb/ft. Knowing that the paddle is moved vertically according to the relation $\delta = \delta_m \sin \omega_f t$, where $\delta_m = 8$ in., determine the maximum allowable circular frequency ω_f if the cord is not to become slack.

SOLUTION



$$\Sigma F = ma \qquad k(\delta - x) = m\ddot{x} \qquad \ddot{x} + \left(\frac{k}{m}\right)x = \delta$$

From Equation (19.31 and 19.33′):

$$x_m = \frac{\delta_m}{\left(1 - \frac{\omega_f^2}{\omega_n^2}\right)}$$

<u>Data</u>:

$$m = \frac{W}{g} = \frac{0.8}{32.2} = 0.024845 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$k = 5 \text{ lb/ft} \qquad \delta_m = 8 \text{ in.} = 0.66667 \text{ ft}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{5}{0.024845}} = 14.186 \text{ rad/s}$$

The cord becomes slack if $x_m - \delta_m$ exceeds δ_{st}, where

$$\delta_{st} = \frac{W}{k} = \frac{0.8 \text{ lb}}{5 \text{ lb/ft}} = 0.16 \text{ ft}$$

PROBLEM 19.163 (Continued)

Then $$\frac{0.66667}{1-\left(\frac{\omega_f}{\omega_n}\right)^2} - 0.66667 < 0.16$$

$$0.66667 - 0.66667 + 0.66667\left(\frac{\omega_f}{\omega_n}\right)^2 < 0.16 - 0.16\left(\frac{\omega_f}{\omega_n}\right)^2$$

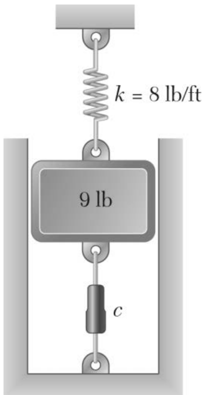
$$0.82667\left(\frac{\omega_f}{\omega_n}\right)^2 < 0.16$$

$$\frac{\omega_f}{\omega_n} < \sqrt{\frac{0.16}{0.82667}} = 0.43994$$

Maximum allowable circular frequency.

$$\omega_f < (0.43994)(14.186 \text{ rad/s})$$

$\omega_f < 6.24$ rad/s ◀



PROBLEM 19.164

The block shown is depressed 1.2 in. from its equilibrium position and released. Knowing that after 10 cycles the maximum displacement of the block is 0.5 in., determine (*a*) the damping factor *c/c*, (*b*) the value of the coefficient of viscous damping. (*Hint:* See Problems 19.129 and 19.130.)

SOLUTION

From Problems 19.130 and 19.129:

$$\left(\frac{1}{k}\right)\ln\left(\frac{x_n}{x_{n+k}}\right) = \frac{2\pi \frac{c}{c_c}}{\sqrt{1-\left(\frac{c}{c_c}\right)^2}}$$

where k = number of cycles = 10

(*a*) First maximum is $x_1 = 1.2$ in.

Thus, $n = 1$

$$\frac{x_1}{x_{1+10}} = \frac{1.2}{0.5} = 2.4$$

$$\frac{1}{10}\ln 2.4 = 0.08755$$

$$= \frac{2\pi \frac{c}{c_c}}{\sqrt{1-\left(\frac{c}{c_c}\right)^2}}$$

<u>Damping factor</u>.

$$1-\left(\frac{c}{c_c}\right)^2 = \left(\frac{2\pi}{0.08755}\right)^2\left(\frac{c}{c_c}\right)^2$$

$$\left(\frac{c}{c_c}\right)^2\left[\left(\frac{2\pi}{0.08755}\right)^2 + 1\right] = 1$$

$$\left(\frac{c}{c_c}\right)^2 = \frac{1}{(5150+1)}$$

$$= 0.0001941$$

$$\frac{c}{c_c} = 0.01393 \blacktriangleleft$$

PROBLEM 19.164 (Continued)

(*b*) Critical damping coefficient. $c_c = 2m\sqrt{\frac{k}{m}}$ (Eq. 19.41)

or $c_c = 2\sqrt{km}$

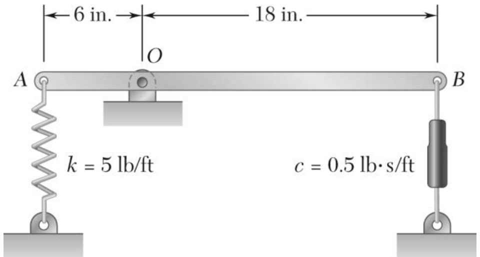
$$c_c = 2\sqrt{(8 \text{ lb/ft})\left(\frac{9 \text{ lb}}{32.2 \text{ ft/s}^2}\right)}$$

$$c_c = 2.991 \text{ lb} \cdot \text{s/ft}$$

From Part (*a*), $\frac{c}{c_c} = 0.01393$

$$c = (0.01393)(2.991)$$

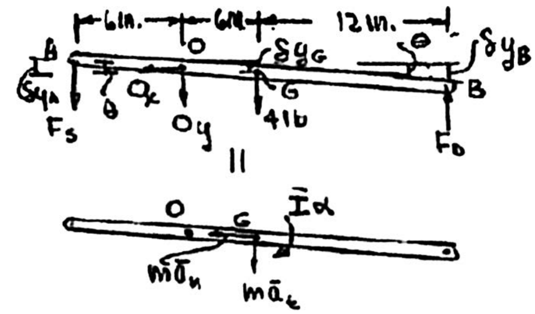
Coefficient of viscous damping. $c = 0.0417 \text{ lb} \cdot \text{s/ft}$ ◀



PROBLEM 19.165

A 4-lb uniform rod is supported by a pin at O and a spring at A, and is connected to a dashpot at B. Determine (*a*) the differential equation of motion for small oscillations, (*b*) the angle that the rod will form with the horizontal 5 s after end B has been pushed 0.9 in. down and released.

SOLUTION



Small angles: $\sin\theta \approx \theta, \quad \cos\theta \approx 1$

$$\delta y_A = \left(\frac{6}{12}\text{ ft}\right)\theta = \frac{\theta}{2}$$

$$\delta y_C = \left(\frac{6}{12}\text{ ft}\right)\theta = \frac{\theta}{2}$$

$$\delta y_B = \left(\frac{18}{12}\text{ ft}\right)\theta = \frac{3\theta}{2}$$

(*a*) Newton's Law: $\Sigma M_0 = (\Sigma M_0)_{\text{eff}}$

$$+\curvearrowleft -\left(\frac{6}{12}\text{ ft}\right)F_s + \left(\frac{6}{12}\text{ft}\right)(4) - \left(\frac{18}{12}\text{ ft}\right)F_D$$

$$= \bar{I}\alpha + \left(\frac{6}{12}\text{ ft}\right)m\bar{a}_t \tag{1}$$

$$F_s = k(\delta y_A + (\delta_{\text{st}})_A) = k\left(\frac{\theta}{2} + (\delta_{\text{st}})_A\right)$$

$$F_D = c\delta\dot{y}_B = c\frac{3}{2}\dot{\theta}$$

$$\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}m\left(\frac{24}{12}\text{ft}\right)^2 = \frac{1}{3}m$$

Kinematics: $\alpha = \ddot{\theta}, \quad \bar{a}_t = \left(\frac{6}{12}\text{ft}\right)\alpha = \frac{\ddot{\theta}}{2}$

Thus, from Eq. (1),

$$\left[\frac{m}{3} + \frac{m}{4}\right]\ddot{\theta} + \left(\frac{3}{2}\right)^2 c\dot{\theta} + \left(\frac{k}{2}\right)\left(\frac{\theta}{2} + (\delta_{\text{st}})_A\right) - 2 = 0 \tag{2}$$

But in equilibrium, $\Sigma M_0 = 0$

$$+\curvearrowleft k(\delta_{\text{st}})_A\left(\frac{6}{12}\right) - (4)\left(\frac{6}{12}\right) = 0, \quad \frac{k}{2}(\delta_{\text{st}})_A = 2$$

Equation (2) becomes

$$\left(\frac{7}{12}\right)m\ddot{\theta}+\left(\frac{9}{4}\right)\left(\ddot{\theta}+\frac{k}{4}\right)\theta=0$$

$$\frac{7}{12}m=\left(\frac{7}{12}\right)\left(\frac{4}{32.2}\right)=0.07246$$

$$\frac{9}{4}c=\left(\frac{9}{4}\right)(0.15)=0.3375$$

$$\frac{k}{4}=\frac{5}{4}=1.25$$

$$0.07246\ddot{\theta}+0.3375\dot{\theta}+1.25\theta=0 \quad \blacktriangleleft$$

(*b*) Substituting $e^{\lambda t}$ into the above differential equation,

$$0.07246\lambda^2+0.3375\lambda+1.25=0$$

$$\lambda=\frac{(-0.3375\mp\sqrt{(0.3375)^2-4(.07246)(1.25)})}{(2)(0.07246)}$$

$$\lambda=\frac{(-0.3375\mp\sqrt{(-0.2484)})}{(2)(0.07246)}$$

$$\lambda=-2.329\pm 3.439i$$

Since the roots are complex and conjugate (light damping), the solution to the differential equation is (Eq. 19.46):

$$\theta=\theta_0 e^{-2.329t}\sin(3.939t+\phi) \qquad (3)$$

Initial conditions. $(\delta y_B)(0)=0.9$ in.

$$\theta(0)=\frac{(\delta y_B)}{18\text{ in.}}=\frac{0.9}{18}$$

$$\theta(0)=0.05\text{ rad}$$

$$\dot{\theta}(0)=0$$

From Eq. (3):

$$\theta(0)=0.05=\theta_0\sin\phi$$

$$\dot{\theta}(0)=0=-2.329\theta_0\sin\phi+3.439\theta_0\cos\phi$$

$$\tan\phi=\frac{3.439}{2.329}$$

$$\phi=0.9755\text{ rad}$$

$$\theta_0=\frac{0.05}{\sin(0.9755)}=0.06039\text{ rad}$$

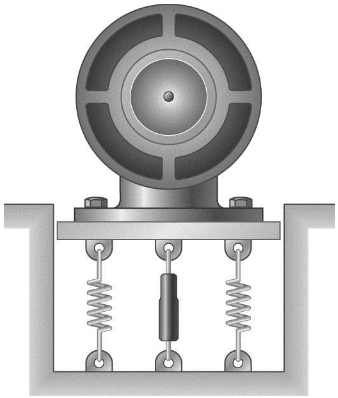
PROBLEM 19.165 (Continued)

Substituting into Eq. (3), $\theta = 0.06039e^{-2.329t}\sin(3.439t + 0.9752)$

At $t = 5$ s,

$$\begin{aligned}\theta &= 0.06039e^{-(2.329)(5)}\sin[(3.439)(5) + 0.9752]\\ &= 0.06039e^{-11.645}\sin(18.1702)\\ &= (0.06039)(8.7627\times 10^{-6})(-0.6283)\\ &= -0.332\times 10^{-6}\text{ rad}\end{aligned}$$

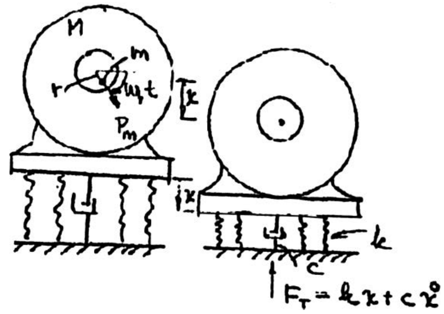
$\theta = -19.05\times 10^{-6}$ degrees ◀



PROBLEM 19.166

A 400-kg motor supported by four springs, each of constant 150 kN/m, and a dashpot of constant $c = 6500\ \text{N}\cdot\text{s/m}$ is constrained to move vertically. Knowing that the unbalance of the rotor is equivalent to a 23-g mass located at a distance of 100 mm from the axis of rotation, determine for a speed of 800 rpm (*a*) the amplitude of the fluctuating force transmitted to the foundation, (*b*) the amplitude of the vertical motion of the motor.

SOLUTION



Total mass: $M = 400\ \text{kg}$

Unbalance: $m = 23\ \text{g} = 0.023\ \text{kg}$

$r = 100\ \text{mm} = 0.100\ \text{m}$

Forcing frequency: $\omega_f = 800\ \text{rpm}$

$= 83.776\ \text{rad/s}$

Spring constant: $(4)(150\times10^3\ \text{N/m}) = 600\times10^3\ \text{N/m}$

Natural frequency: $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{600\times10^3}{400}}$

$= 38.730\ \text{rad/s}$

Frequency ratio: $\dfrac{\omega_f}{\omega_n} = 2.1631$

Viscous damping coefficient: $c = 6500\ \text{N}\cdot\text{s/m}$

Critical damping coefficient: $c_c = 2\sqrt{kM} = 2\sqrt{(600\times10^3)(400)}$

$= 30{,}984\ \text{N}\cdot\text{s/m}$

Damping factor: $\dfrac{c}{c_c} = 0.20978$

Unbalance force: $P_m = mr\omega_f^2 = (0.023)(0.100)(83.776)^2$

$= 16.1424\ \text{N}$

Static deflection: $\delta_{\text{st}} = \dfrac{P_m}{k} = \dfrac{16.1424}{600\times10^3}$

$= 26.904\times10^{-6}\ \text{m}$

Amplitude of vibration. Use Eq. (19.53).

$$x_m = \frac{\frac{P_m}{k}}{\sqrt{1\left[1-\left(\frac{\omega_f}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right)\right]^2}}$$

Where $$1-\left(\frac{\omega_f}{\omega_n}\right)^2 = 1-(2.1631)^2 = -3.679$$

and $$2\left(\frac{c}{c_c}\right)\left(\frac{\omega_f}{\omega_n}\right) = (2)(0.20978)(2.1631) = 0.90755$$

$$x_m = \frac{26.904\times 10^{-6}}{\sqrt{(-3.679)^2 + (0.90755)^2}}$$
$$= 7.1000\times 10^{-6} \text{ m}$$

Resulting motion: $$x = x_m \sin(\omega_f t - \varphi)$$
$$\dot{x} = \omega_f x_m \cos(\omega_f t - \varphi)$$

Spring force: $$F_s = kx = kx_m \sin(\omega_f t - \varphi) = 4.26 \sin(\omega_f t - \varphi)$$

Damping force: $$F_d = c\dot{x} = c\omega_f x_m \cos(\omega_f t - \varphi) = 3.8663\cos(\omega_f t - \varphi)$$

Let $F_s = F_m \cos\psi \sin(\omega_f t - \varphi)$ and $F_d = F_m \sin\psi \cos(\omega_f t - \varphi)$

Total force: $$F = F_m \cos\psi \sin(\omega_f t - \varphi) + F_m \sin\psi \cos(\omega_f t - \varphi)$$
$$= F_m \sin(\omega_f t - \varphi + \psi)$$

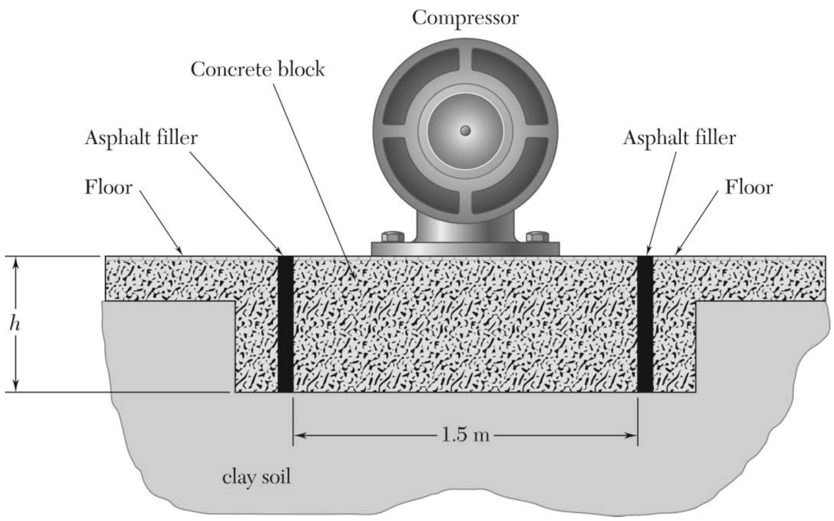
(*a*) Force amplitude. $$F_m = \sqrt{(F_m \cos\psi)^2 + (F_m \sin\psi)^2}$$
$$F_m = \sqrt{(kx_m)^2 + (c\omega_f x_m)^2}$$
$$= \sqrt{(4.26)^2 + (3.8663)^2}$$

$F_m = 5.75$ N ◀

(*b*) Amplitude of vibration. $x_m = 0.00710$ mm ◀

PROBLEM 19.167

The compressor shown has a mass of 250 kg and operates at 2000 rpm. At this operating condition the force transmitted to the ground is excessively high and is found to be $mr\omega_f^2$ where mr is the unbalance and ω_f is the forcing frequency. To fix this problem, it is proposed to isolate the compressor by mounting it on a square concrete block separated from the rest of the floor as shown. The density of concrete is 2400 kg/m^3 and the spring constant for the soil is found to be 80×10^6 N/m. The geometry of the compressor leads to choosing a block that is 1.5 m by 1.5 m. Determine the depth h that will reduce the force transmitted to the ground by 75%.



SOLUTION

Forced circular frequency corresponding to 2000 rpm.

$$\omega_f = \frac{(2\pi)(2000)}{60} = 209.44 \text{ rad/s}$$

In the first case the natural frequency is very large so that the transmitted force is $mr\omega_f^2$.

After the problem is fixed, the transmitted force is

$$P = kx_m = \frac{P_m}{\left|1 - \left(\frac{\omega_f}{\omega_n}\right)^2\right|}$$

Since the motion is out-of-phase,

$$P = \frac{P_m}{\left(\frac{\omega_f}{\omega_n}\right)^2 - 1} = \frac{mr\omega_f^2}{\left(\frac{\omega_f}{\omega_n}\right)^2 - 1} \qquad (1)$$

But $$P = (1 - 0.75)mr\omega_f^2 = 0.25\, mr\omega_f^2 \qquad (2)$$

Equating expressions (1) and (2) dividing by $mr\omega_f^2$,

$$\frac{1}{\left(\frac{\omega_f}{\omega_n}\right)^2 - 1} = 0.25$$

$$\left(\frac{\omega_f}{\omega_n}\right)^2 - 1 = 4$$

$$\frac{\omega_f}{\omega_n} = \sqrt{5}$$

$$\omega_n = \frac{1}{\sqrt{5}}\omega_f = \frac{1}{\sqrt{5}}(209.44) = 93.664 \text{ rad/s}$$

$$\sqrt{\frac{k}{m}} = \omega_n$$

$$m = \frac{k}{\omega_n^2} = \frac{80\times 10^6 \text{ N/m}}{(93.664 \text{ rad/s})^2} = 9119 \text{ kg}$$

Required properties of the attached concrete block.

Mass: $m - 250 \text{ kg} = 8869 \text{ kg}$

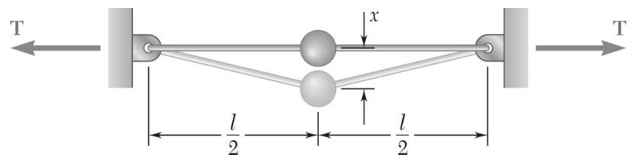
$$\text{volume} = \frac{\text{mass}}{\text{density}} = \frac{8869 \text{ kg}}{2400 \text{ kg/m}^3} = 3.6954 \text{ m}^3$$

$$\text{area} = 1.5 \text{ m}\times 1.5 \text{ m} = 2.25 \text{ m}^2$$

$$\text{depth} = \frac{\text{volume}}{\text{area}} = \frac{3.6954 \text{ m}^3}{2.25 \text{ m}^2}$$

$h = 1.642$ m ◀

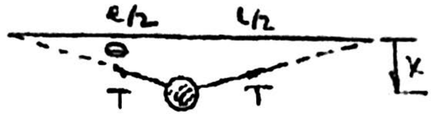
PROBLEM 19.168



A small ball of mass m attached at the midpoint of a tightly stretched elastic cord of length l can slide on a horizontal plane. The ball is given a small displacement in a direction perpendicular to the cord and released. Assuming the tension T in the cord to remain constant, (*a*) write the differential equation of motion of the ball, (*b*) determine the period of vibration.

SOLUTION

(*a*) Differential equation of motion.



$$+\uparrow \Sigma F = ma: \quad 2T\sin\theta = -m\ddot{x}$$

For small x,

$$\sin\theta \approx \tan\theta = \frac{x}{\left(\frac{l}{2}\right)} = \frac{2x}{l}$$

$$m\ddot{x} + (2T)\left(\frac{2x}{l}\right) = 0 \quad \blacktriangleleft$$

$$m\ddot{x} + \left(\frac{4T}{l}\right)x = 0$$

Natural circular frequency.

$$\omega_n^2 = \frac{4T}{ml}$$

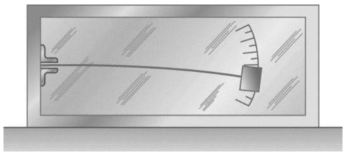
$$\omega_n = 2\sqrt{\frac{T}{ml}}$$

(*b*) Period of vibration.

$$\tau_n = \frac{2\pi}{\omega_n}$$

$$\tau_n = \pi\sqrt{\frac{ml}{T}} \quad \blacktriangleleft$$

PROBLEM 19.169



A certain vibrometer used to measure vibration amplitudes consists essentially of a box containing a slender rod to which a mass *m* is attached; the natural frequency of the mass-rod system is known to be 5 Hz. When the box is rigidly attached to the casing of a motor rotating at 600 rpm, the mass is observed to vibrate with an amplitude of 0.06 m. relative to the box. Determine the amplitude of the vertical motion of the motor.

SOLUTION

Natural frequency:

$$f_n = 5 \text{ Hz}$$

$$\omega_n = 2\pi f_n = 31.416 \text{ rad/s}$$

Forcing frequency:

$$f_f = 600 \text{ rpm} = 10 \text{ Hz}$$

$$\omega_f = 2\pi f_f = 62.832 \text{ rad/s}$$

Ratio:

$$\frac{\omega_f}{\omega_n} = 2.000$$

$$x_m = \frac{\delta_m}{1 - \left(\frac{\omega_f}{\omega_n}\right)^2} = \frac{\delta_m}{1 - (2)^2} = \frac{\delta_m}{-3}$$

$$x_m = -\frac{1}{3}\delta_m$$

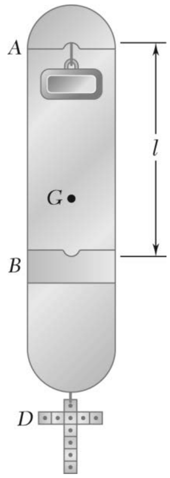
Relative motion:

$$y_m = x_m - \delta_m = \frac{4}{3}x_m$$

The observed relative motion is

$$y_m = 0.06 \text{ in.}$$

$$x_m = \frac{3}{4}y_m = \frac{3}{4}(0.06 \text{ in.}) \qquad x_m = 0.045 \text{ in.} ◀$$

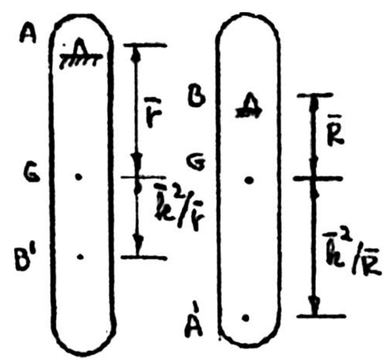


PROBLEM 19.170

If either a simple or a compound pendulum is used to determine experimentally the acceleration of gravity g, difficulties are encountered. In the case of the simple pendulum, the string is not truly weightless, while in the case of the compound pendulum, the exact location of the mass center is difficult to establish. In the case of a compound pendulum, the difficulty can be eliminated by using a reversible, or Kater, pendulum. Two knife edges A and B are placed so that they are obviously not at the same distance from the mass center G, and the distance l is measured with great precision. The position of a counterweight D is then adjusted so that the period of oscillation τ is the same when either knife edge is used. Show that the period τ obtained is equal to that of a true simple pendulum of length l and that $g = 4\pi^2 l/\tau^2$.

SOLUTION

From Problem 19.52, the length of an equivalent simple pendulum is:



$$l_A = \bar{r} + \frac{\bar{k}^2}{\bar{r}}$$

and $$l_B = \bar{R} + \frac{\bar{k}^2}{\bar{R}}$$

But $$\tau_A = \tau_B$$

$$2\pi\sqrt{\frac{l_A}{g}} = 2\pi\sqrt{\frac{l_B}{g}}$$

Thus, $$l_A = l_B$$

For $$l_A = l_B$$

$$\bar{r} + \frac{\bar{k}^2}{\bar{r}} = \bar{R} + \frac{\bar{k}^2}{\bar{R}}$$

$$\bar{r}^2\bar{R} + \bar{k}^2\bar{R} = \bar{r}\,\bar{R}^2 + \bar{k}^2\bar{r}$$

$$\bar{r}\,\bar{R}[\bar{r} - \bar{R}] = \bar{k}^2[\bar{r} - \bar{R}]$$

$$(\bar{r} - \bar{R}) \neq 0$$

Thus, $$\bar{r}\,\bar{R} = \bar{k}^2$$

or $$\bar{r} = \frac{\bar{k}^2}{\bar{R}}$$

$$\bar{R} = \frac{\bar{k}^2}{\bar{r}}$$

PROBLEM 19.170 (Continued)

Thus, $AG = GA' \quad \text{and} \quad BG = GB'$

That is, $A = A' \quad \text{and} \quad B = B'$

Noting that $l_A = l_B = l$

$$\tau = 2\pi\sqrt{\frac{l}{g}}$$

or
$$g = \frac{4\pi^2 l}{\tau^2}$$