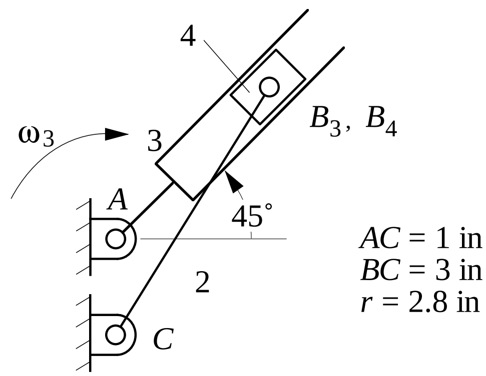
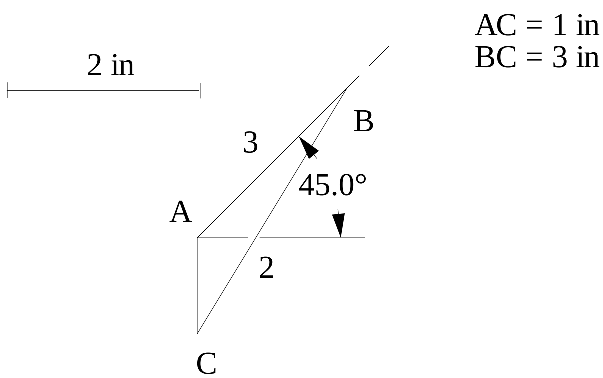
Solutions to Chapter 3 Exercise Problems

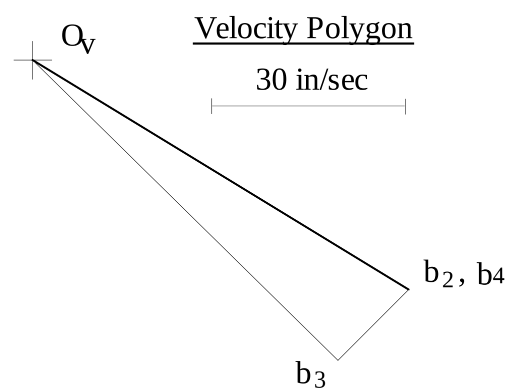
Problem 3.1

In the figure below, points A and C have the same horizontal coordinate, and ω_3 = 30 rad/s. Draw and dimension the velocity polygon. Identify the sliding velocity between the block and the slide, and find the angular velocity of link 2.



Position Analysis: Draw the linkage to scale.





Velocity Analysis:

$$^1\mathbf{v}_{B_3} = {}^1\mathbf{v}_{B_3/A_3} = {}^1\omega_3 \times \mathbf{r}_{B_3/A_3} \Rightarrow \left|{}^1\mathbf{v}_{B_3}\right| = \left|{}^1\omega_3\right|\left|\mathbf{r}_{B_3/A_3}\right| = 30(2.2084) = 66.252 \text{ in/sec}$$

$$^1\mathbf{v}_{B_4} = {}^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{B_3} + {}^1\mathbf{v}_{B_4/B_3} \qquad (1)$$

$^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{B_2/C_2} = {}^1\omega_2 \times \mathbf{r}_{B_2/C_2}$

Now,

$^1\mathbf{v}_{B_3} = 66.252 \text{in/sec}$ in the direction of $\mathbf{r}_{B/A}$

$^1\mathbf{v}_{B_2} = {}^1\omega_2 \times \mathbf{r}_{B/C}$ ($\perp$ to $\mathbf{r}_{B/C}$)

$^1\mathbf{v}_{B_4/B_3}$ is on the line of AB

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$^1\mathbf{v}_{B_4/B_3} = 15.63 \text{in/sec}$

Also,

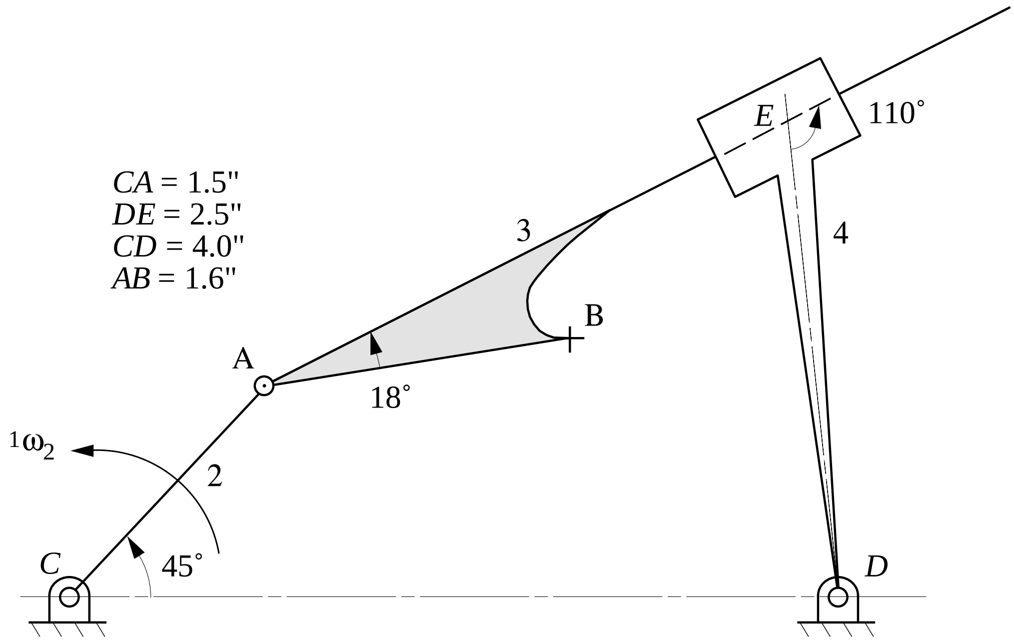
$$\left|{}^1\omega_2\right| = \frac{\left|{}^1\mathbf{v}_{B_2/C_2}\right|}{\left|\mathbf{r}_{B/C}\right|} = \frac{68.829}{3} = 22.943 \text{ rad/sec}$$

From the directions given in the position and velocity polygons

$^1\omega_2 = 22.943 \text{ rad/sec CW}$

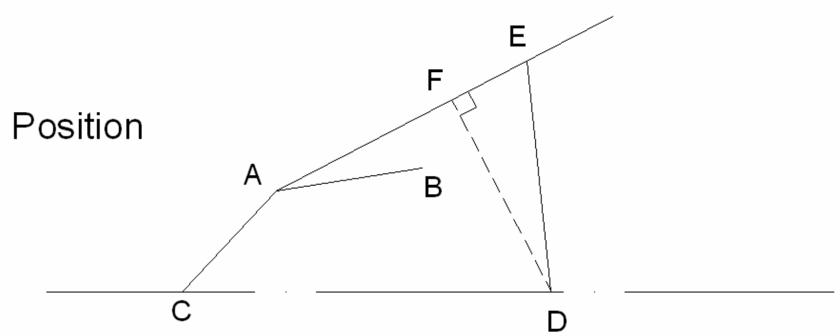
Problem 3.2

If $\omega_2 = 10$ rad/s CCW, find the velocity of point B_3.



Position Analysis

Draw the linkage to scale.



Velocity Analysis

$$ {}^1\mathbf{v}_{A_2} = {}^1\mathbf{v}_{C_2} + {}^1\mathbf{v}_{A_2/C_2} = 0 + {}^1\omega_2 \times \mathbf{r}_{A/C} \Rightarrow \left|{}^1\mathbf{v}_{A_2}\right| = \left|{}^1\omega_2\right|\left|\mathbf{r}_{A/C}\right| = 10(1.5) = 15 \text{ in/s} \quad (1) $$

$$ {}^1\mathbf{v}_{E_3} = {}^1\mathbf{v}_{A_3} + {}^1\mathbf{v}_{E_3/A_3} = {}^1\mathbf{v}_{A_2} + {}^1\mathbf{v}_{E_3/A_3} $$

$$ {}^1\mathbf{v}_{E_3} = {}^1\mathbf{v}_{E_4} + {}^1\mathbf{v}_{E_3/E_4} $$

$$ {}^1\mathbf{v}_{E_4} = {}^1\mathbf{v}_{D_4} + {}^1\mathbf{v}_{E_4/D_4} = 0 + {}^1\omega_4 \times \mathbf{r}_{E/D} $$

Now,

$$ {}^1\mathbf{v}_{A_2} = 15 \; in/\text{s} \quad (\perp \text{ to } \mathbf{r}_{A/C}) $$

$$ {}^1\mathbf{v}_{E_3/A_3} = {}^1\omega_3 \times \mathbf{r}_{E/A} \quad (\perp \text{ to } \mathbf{r}_{E/A}) $$

$$ {}^1\mathbf{v}_{E_4/D_4} = {}^1\omega_4 \times \mathbf{r}_{E/D} \quad (\perp \; to \; \mathbf{r}_{E/D}) $$

and $\omega_3 = \omega_4$, need to get ω_3 to find ${}^1\mathbf{v}_{B_3}$.

Define the point F where $\overline{AF} \perp \overline{DF}$ in position polygon.

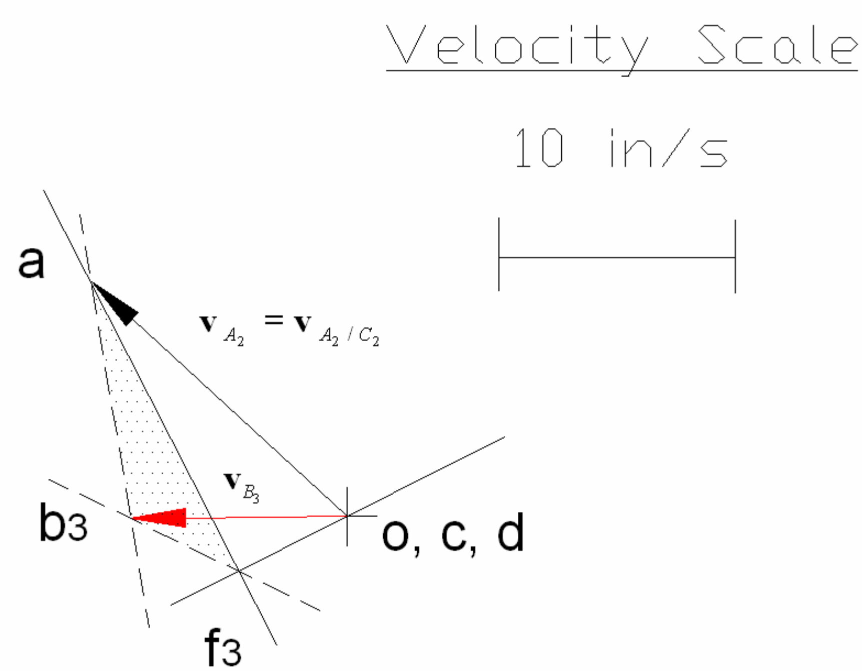
$$ {}^1\mathbf{v}_{F_3} = {}^1\mathbf{v}_{A_3} + {}^1\mathbf{v}_{F_3/A_3} $$

$$ {}^1\mathbf{v}_{F_3} = {}^1\mathbf{v}_{F_4} + {}^1\mathbf{v}_{F_3/F_4} $$

$$ {}^1\mathbf{v}_{F_4} = {}^1\mathbf{v}_{F_3} + {}^1\mathbf{v}_{F_4/F_3} $$

$$ {}^1\mathbf{v}_{F_4} = {}^1\mathbf{v}_{F_4/D_4} $$

Solve Eq. (1) graphically with a velocity polygon.



After finding point "f3", construct the velocity image to find the point "b3"

a line $\perp$ *to* $\overline{AB}$ through the point "a"

a line $\perp$ *to* $\overline{BF}$ through the point "f3"

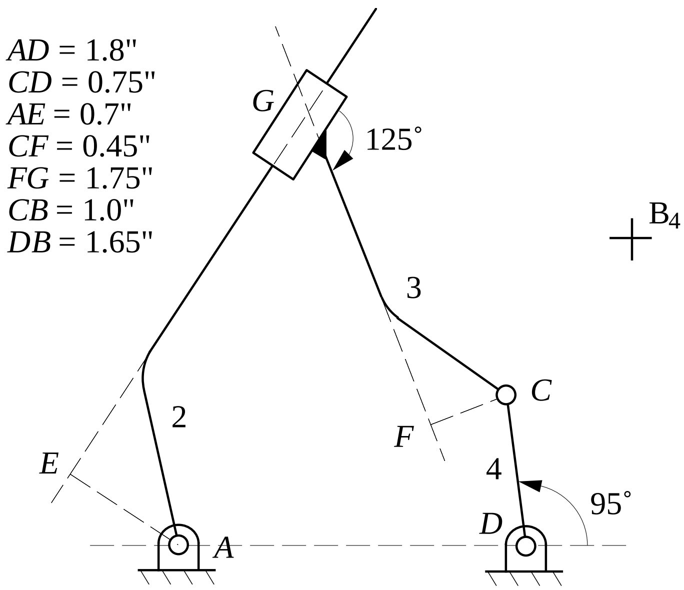
fine the point "b3"

From the polygon,

$^1\mathbf{v}_{B_3}$ =9.4 in/s

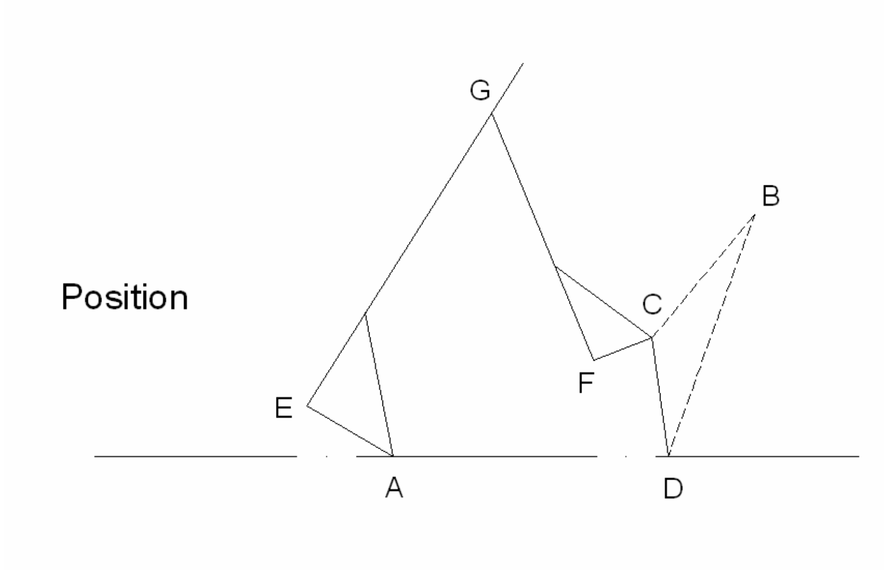
Problem 3.3

If ω_2 = 100 rad/s CCW, find $\boldsymbol{v}_{B_4}$.



<u>**Position Analysis**</u>

Draw the linkage to scale.



Velocity Analysis

$$^1\mathbf{v}_{G_2} = {}^1\mathbf{v}_{A_2} + {}^1\mathbf{v}_{G_2/A_2} = 0 + {}^1\omega_2 \times \mathbf{r}_{G/A} \qquad (1)$$

$$^1\mathbf{v}_{G_3} = {}^1\mathbf{v}_{C_3} + {}^1\mathbf{v}_{G_3/C_3} = {}^1\mathbf{v}_{C_4} + {}^1\omega_3 \times \mathbf{r}_{G/C}$$

$$^1\mathbf{v}_{G_3} = {}^1\mathbf{v}_{G_2} + {}^1\mathbf{v}_{G_3/G_2}$$

$$^1\mathbf{v}_{C_4} = {}^1\mathbf{v}_{C_3} = {}^1\mathbf{v}_{D_4} + {}^1\mathbf{v}_{C_4/D_4} = 0 + {}^1\omega_4 \times \mathbf{r}_{C/D}$$

Now,

$$\left|{}^1\mathbf{v}_{G_2}\right| = \left|{}^1\omega_2\right|\left|\mathbf{r}_{G/A}\right| = 100(3.44) = 344 \text{ in/s} \quad (\perp \textit{ to } \mathbf{r}_{G/A})$$

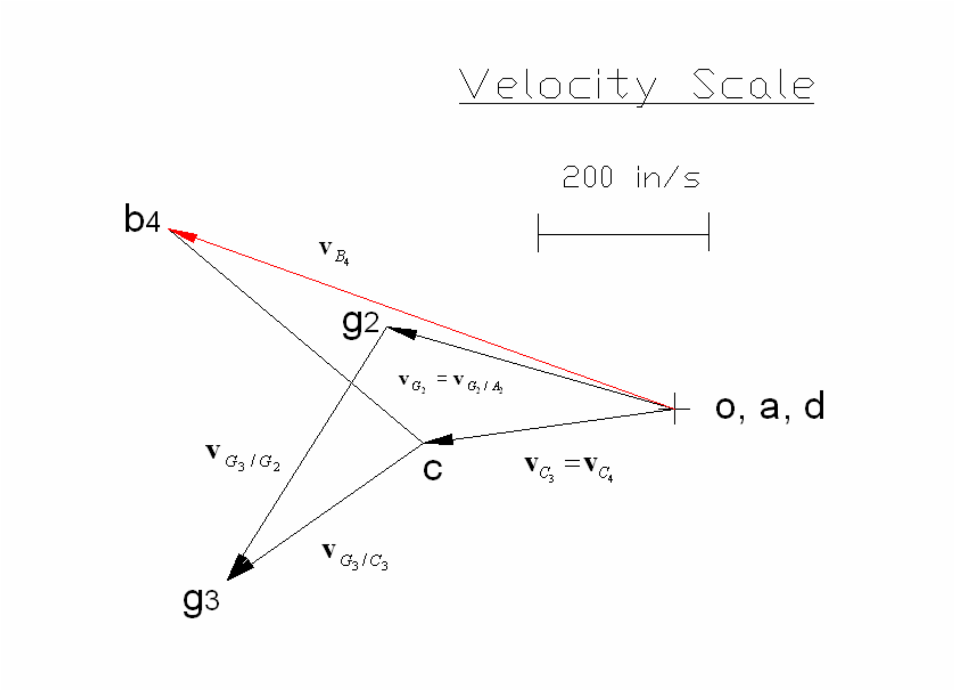
$$^1\omega_3 = {}^1\omega_2$$

$$\left|{}^1\mathbf{v}_{G_3/C_3}\right| = \left|{}^1\omega_3\right|\left|\mathbf{r}_{G/C}\right| = 100(2.65) = 265 \text{ in/s} \quad (\perp \textit{ to } \mathbf{r}_{G/C})$$

$^1\mathbf{v}_{G_3/G_2}$ is on the line of EG

$$^1\mathbf{v}_{C_4/D_4} = {}^1\omega_4 \times \mathbf{r}_{C/D} \quad (\perp \textit{ to } \mathbf{r}_{C/D})$$

Solve Eq. (1) graphically with a velocity polygon.

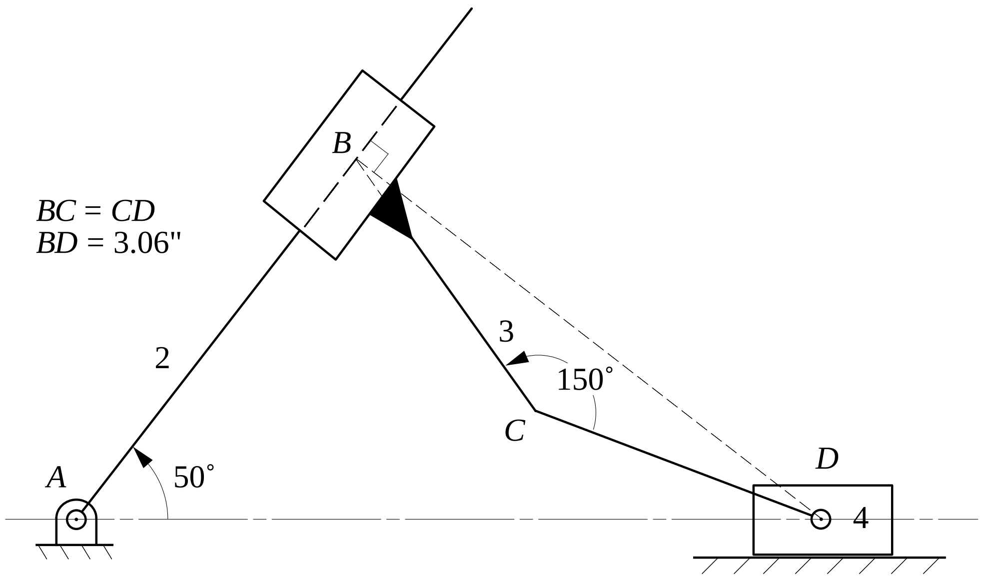


To find the point "b_4" use velocity polygon

$$\left| {}^{1}\mathbf{v}_{C_4/D_4} \right| = 612.14 \ in/s$$

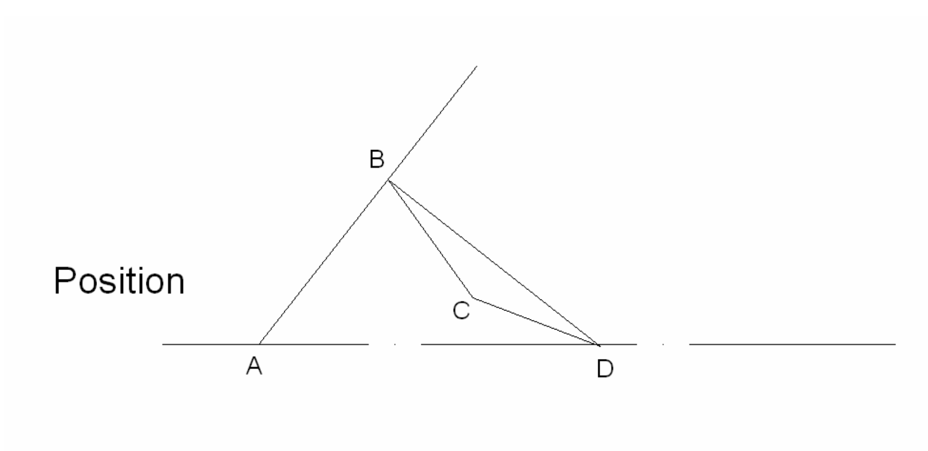
Problem 3.4

If ω_2 = 50 rad/s CCW, find $\boldsymbol{v}_{D4}$.



Position Analysis

Draw the linkage to scale.



Velocity Analysis

$$^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{A_2} + {}^1\mathbf{v}_{B_2/A_2} = 0 + {}^1\omega_2 \times \mathbf{r}_{B/A} \qquad (1)$$

$$^1\mathbf{v}_{B_3} = {}^1\mathbf{v}_{D_3} + {}^1\mathbf{v}_{B_3/D_3} = {}^1\mathbf{v}_{D_4} + {}^1\omega_3 \times \mathbf{r}_{B/D}$$

$$^1\mathbf{v}_{B_3} = {}^1\mathbf{v}_{B_2} + {}^1\mathbf{v}_{B_3/B_2}$$

$$^1\mathbf{v}_{D_4} = {}^1\mathbf{v}_{D_3}$$

Now,

$$\left|{}^1\mathbf{v}_{B_2}\right| = \left|{}^1\omega_2\right|\left|\mathbf{r}_{B/A}\right| = 50(2.39) = 119.5 \text{ in/s} \quad (\perp \textit{ to } \mathbf{r}_{B/A})$$

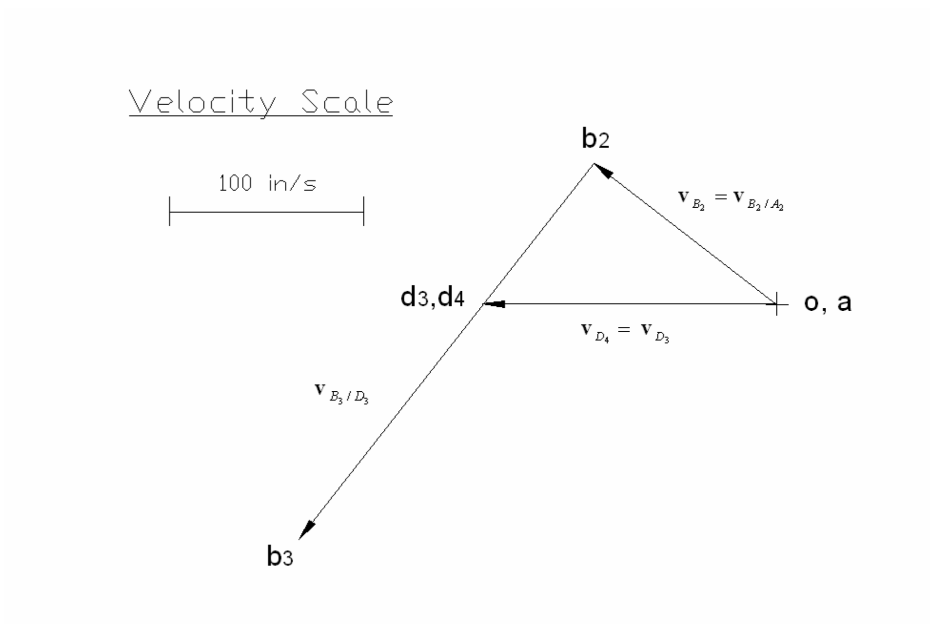
$$^1\omega_3 = {}^1\omega_2$$

$$\left|{}^1\mathbf{v}_{B_3/D_3}\right| = \left|{}^1\omega_3\right|\left|\mathbf{r}_{B/D}\right| = 50(3.06) = 153 \text{ in/s} \quad (\perp \textit{ to } \mathbf{r}_{B/D})$$

$^1\mathbf{v}_{B_3/B_2}$ is on the line of AB

$$^1\mathbf{v}_{D_4} = {}^1\mathbf{v}_{D_3} \quad (// \textit{ to } \mathbf{r}_{D/A})$$

Solve Eq. (1) graphically with a velocity polygon.

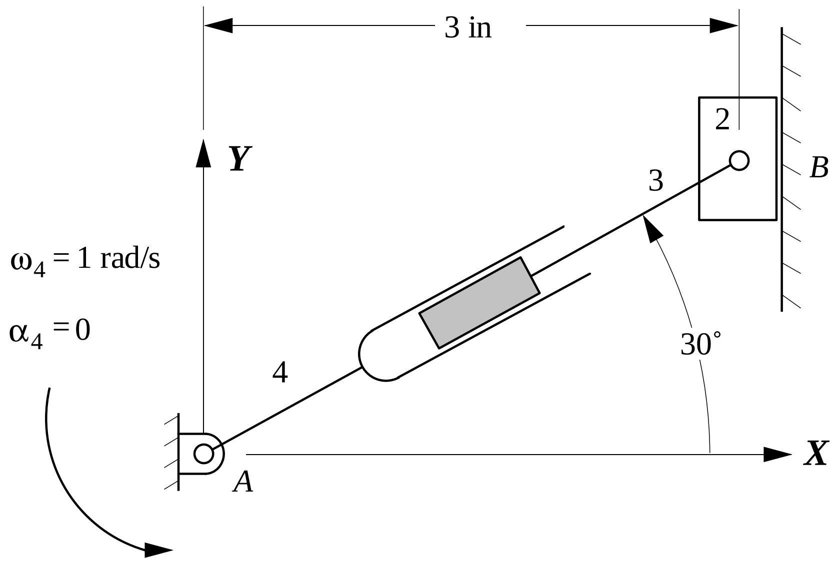


From the velocity polygon

$$\left|{}^1\mathbf{v}_{D_4}\right| = 164.34 \ \ in/s$$

Problem 3.5

Determine the velocity and acceleration of point *B* on link 2.



Position Analysis

Draw the mechanism to scale.

Velocity Analysis

$$|\mathbf{r}_{B/A}| = 3/\cos 30° = 3.4641$$

The velocity of B_2 is given by

$$` \; {}^1\mathbf{v}_{B_2/A_1} = {}^4\mathbf{v}_{B_2/A_1} + {}^1\boldsymbol{\omega}_4 \times \mathbf{r}_{B/A} \tag{1}$$

and

$$|{}^1\boldsymbol{\omega}_4 \times \mathbf{r}_{B/A}| = 1 \cdot 3.4642 = 3.4642$$ ($\perp$ to $\mathbf{r}_{B/A}$ in the direction indicated by the cross product)

The direction for the velocity ${}^1\mathbf{v}_{B_2/A_1}$ must be vertical. Equation (1) can be solved for the unknowns. From the polygon,

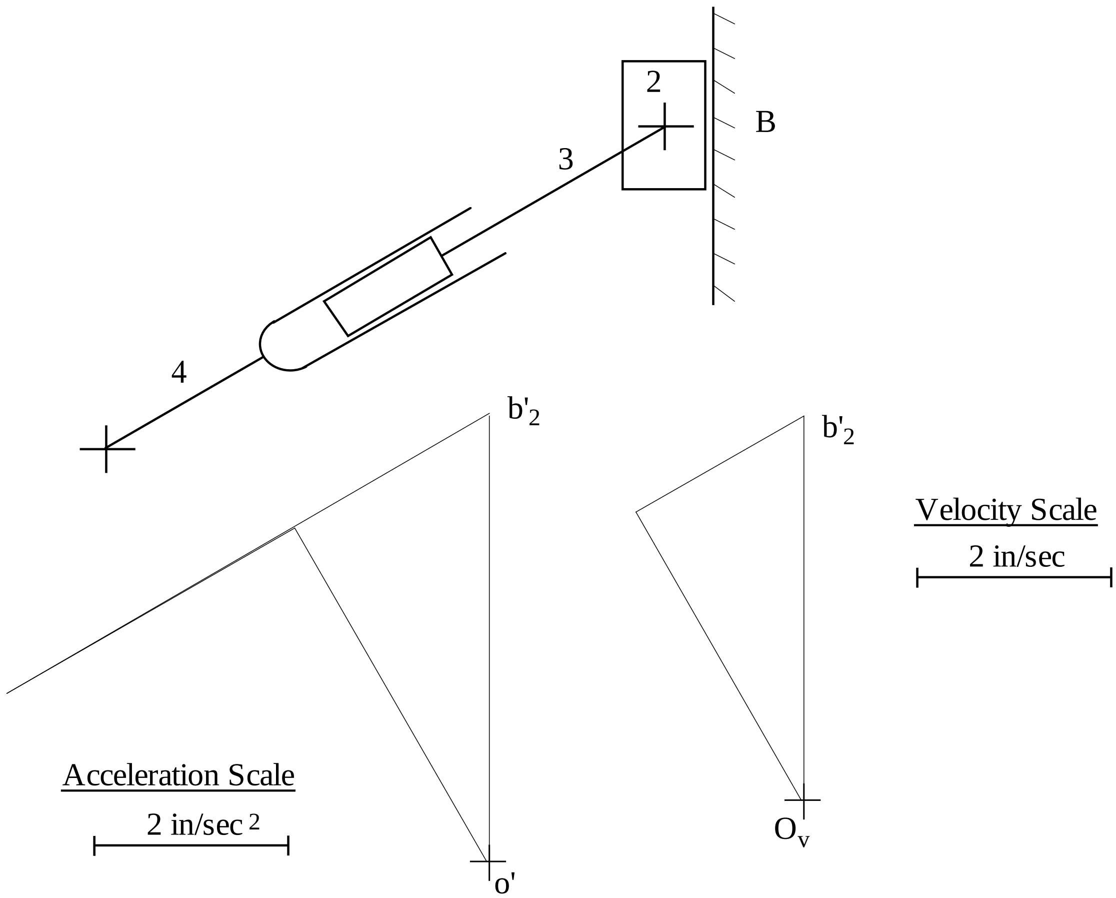
$${}^1\mathbf{v}_{B_2/A_1} = 4.0278 \text{ in/sec} \uparrow$$

$${}^4\mathbf{v}_{B_2/A_1} = 2.0148 \text{ in/sec}$$

Acceleration Analysis

The acceleration of B_2 is given by

$$ {}^1\mathbf{a}_{B_2/A_1} = {}^4\mathbf{a}_{B_2/A_1} + {}^1\alpha_4 \times \mathbf{r}_{B/A} + {}^1\omega_4 \times ({}^1\omega_4 \times \mathbf{r}_{B/A}) + 2\,{}^1\omega_4 \times {}^4\mathbf{v}_{B_2/A_1} \tag{2} $$



The individual vectors are:

$\left| {}^1\omega_4 \times ({}^1\omega_4 \times \mathbf{r}_{B/A}) \right| = (1)^2 \cdot 3.4642 = 3.4642$ in/sec^2 (opposite to $\mathbf{r}_{B/A}$)

${}^1\alpha_4 \times \mathbf{r}_{B/A} = 0 \times \mathbf{r}_{B/A} = 0$

$2\,{}^1\omega_4 \times {}^4\mathbf{v}_{B_2/A_1} = 2(1)2.0148 = 4.0296$ in / sec^2 ($\perp$ to ${}^4\mathbf{v}_{B_2/A_1}$ in the direction up and to left)

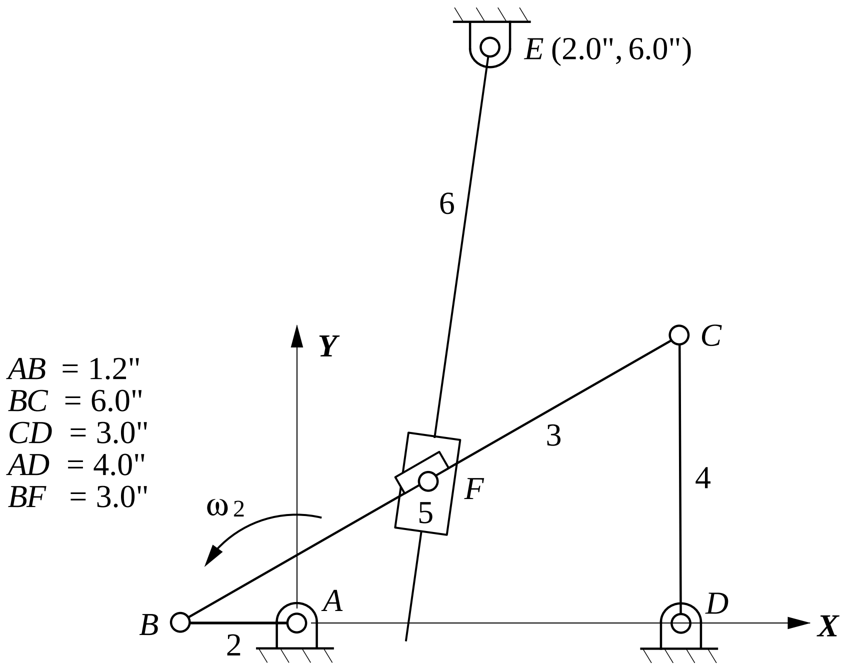
${}^1\alpha_4 \times \mathbf{r}_{B/A}$ (vector $\perp$ to $\mathbf{r}_{B/A}$)

${}^4\mathbf{a}_{B_2/A_1}$ (vector along the slot)

Equation (2) has only two unknowns and can be solved. From the polygon, the acceleration of B_2 is 4.666 in/sec^2 upward

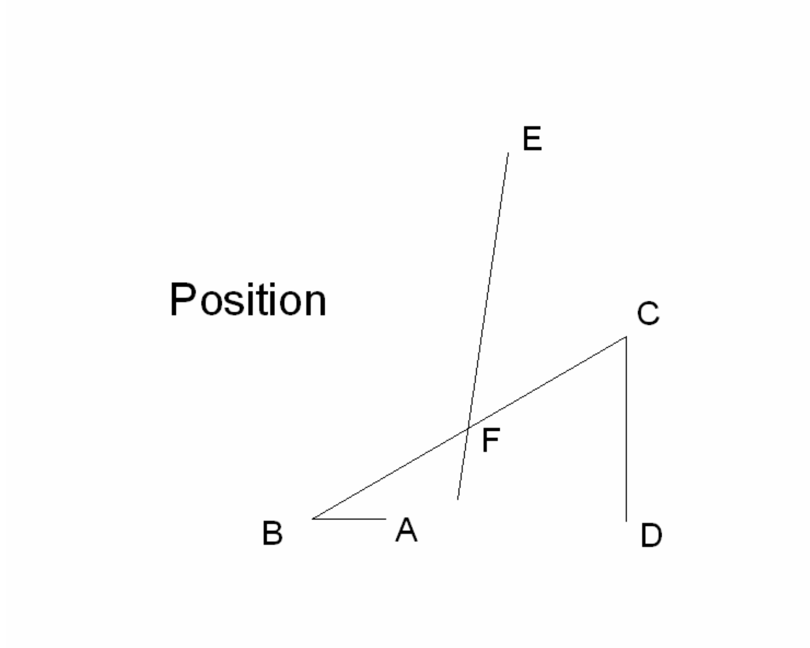
Problem 3.6

If ${}^1\omega_2$ = 100 rad/s CCW, find ${}^1\omega_6$.



Position Analysis

Draw the linkage to scale.



Velocity Analysis

$$ {}^{1}\mathbf{v}_{B_2} = {}^{1}\mathbf{v}_{B_2/A_2} = {}^{1}\omega_2 \times \mathbf{r}_{B/A} \Rightarrow \left| {}^{1}\mathbf{v}_{B_2} \right| = \left| {}^{1}\omega_2 \right| \left| \mathbf{r}_{B/A} \right| = 100(1.2) = 120 \text{ in/s} \tag{1} $$

$$ {}^{1}\mathbf{v}_{C_3} = {}^{1}\mathbf{v}_{B_3} + {}^{1}\mathbf{v}_{C_3/B_3} = {}^{1}\mathbf{v}_{B_2} + {}^{1}\omega_3 \times \mathbf{r}_{C/B} $$

$$ {}^{1}\mathbf{v}_{C_4} = {}^{1}\mathbf{v}_{D_4} + {}^{1}\mathbf{v}_{C_4/D_4} = 0 + {}^{1}\omega_4 \times \mathbf{r}_{C/D} $$

$$ {}^{1}\mathbf{v}_{F_6} = {}^{1}\mathbf{v}_{F_5} + {}^{1}\mathbf{v}_{F_6/F_5} $$

$$ {}^{1}\mathbf{v}_{F_6} = {}^{1}\mathbf{v}_{E_6} + {}^{1}\mathbf{v}_{F_6/E_6} = 0 + {}^{1}\omega_6 \times \mathbf{r}_{F/E} $$

Now,

$$^{1}\mathbf{v}_{B_2} = 120\ in/\mathrm{s}\ \ (\perp \text{ to } \mathbf{r}_{B/A})$$

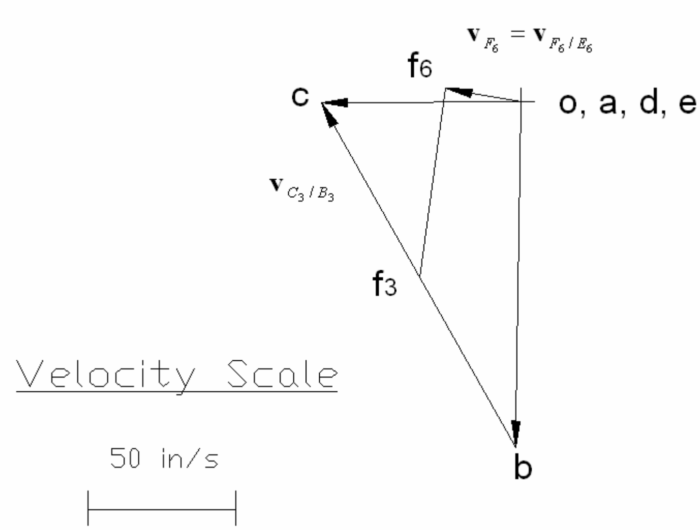
$$^{1}\mathbf{v}_{C_3/B_3} = {}^{1}\omega_3 \times \mathbf{r}_{C/B} \quad (\perp \text{ to } \mathbf{r}_{C/B})$$

$$^{1}\mathbf{v}_{C_4/D_4} = {}^{1}\omega_4 \times \mathbf{r}_{C/D} \quad (\perp\ to\ \mathbf{r}_{C/D})$$

$$^{1}\mathbf{v}_{F_6/F_5} \ \ (\ //\ to\ \mathbf{r}_{E/F})$$

$$^{1}\mathbf{v}_{F_6/E_6} = {}^{1}\omega_6 \times \mathbf{r}_{F/E} \quad (\perp\ to\ \mathbf{r}_{F/E})$$

Solve Eq. (1) graphically with a velocity polygon.



To find the point " f_3 " construct the velocity image by

$$\overline{BC} : \overline{BF} = \overline{bc} : \overline{bf}$$

and $\quad {}^1\mathbf{v}_{F_3} = {}^1\mathbf{v}_{F_5}$

From the polygon,

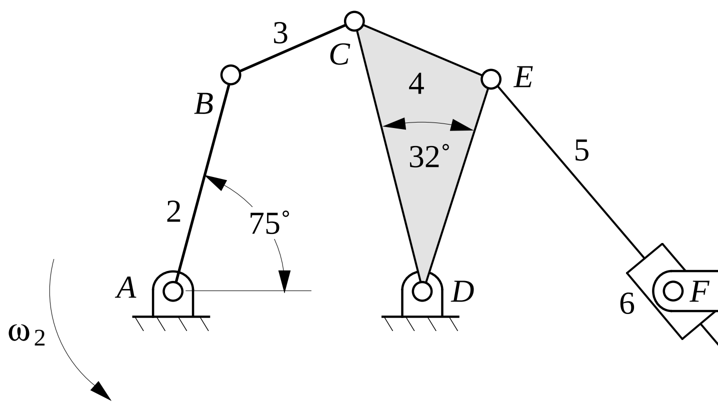
$${}^1\mathbf{v}_{F_6/E_6} = 25.2 \ in/\sec$$

and

$$\left|{}^1\omega_6\right| = \frac{\left|{}^1\mathbf{v}_{F_6/E_6}\right|}{\left|\mathbf{r}_{F/E}\right|} = \frac{25.2}{4.549} = 5.54 \ rad/\sec \ CW$$

Problem 3.7

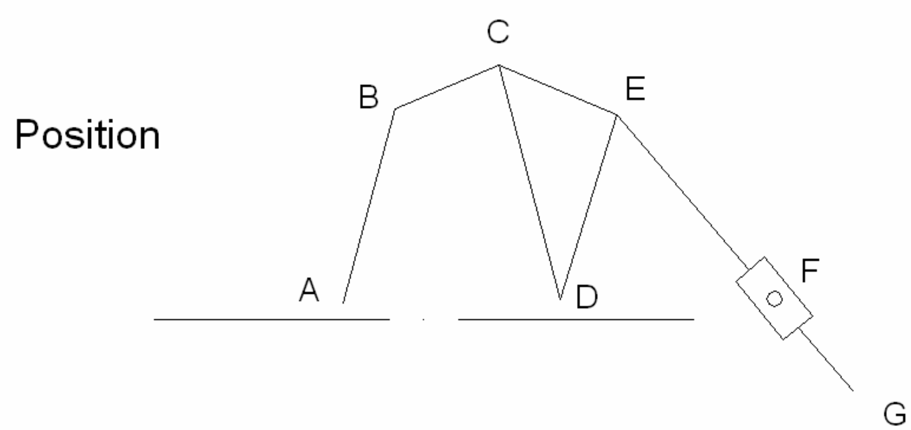
If ω_2 = 50 rad/s CCW, find the velocity of point G_5.



AB = 1.16"
BC = 0.70"
CD = 1.45"
DE = 1.16"
AD = 1.30"
DF = 1.30
EG = 2.20"

Position Analysis

Draw the linkage to scale.



Velocity Analysis

$$ {}^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{B_2/A_2} = {}^1\omega_2 \times \mathbf{r}_{B/A} \Rightarrow \left|{}^1\mathbf{v}_{B_2}\right| = \left|{}^1\omega_2\right|\left|\mathbf{r}_{B/A}\right| = 50(1.16) = 58 \text{ in/s} \qquad (1)$$

$$ {}^1\mathbf{v}_{C_3} = {}^1\mathbf{v}_{B_3} + {}^1\mathbf{v}_{C_3/B_3} = {}^1\mathbf{v}_{B_2} + {}^1\omega_3 \times \mathbf{r}_{C/B}$$

$$ {}^1\mathbf{v}_{C_4} = {}^1\mathbf{v}_{C_3} = {}^1\mathbf{v}_{D_4} + {}^1\mathbf{v}_{C_4/D_4} = 0 + {}^1\omega_4 \times \mathbf{r}_{C/D}$$

$$ {}^1\mathbf{v}_{E_4} = {}^1\mathbf{v}_{D_4} + {}^1\mathbf{v}_{E_4/D_4} = 0 + {}^1\omega_4 \times \mathbf{r}_{E/D}$$

$$ {}^1\mathbf{v}_{G_5} = {}^1\mathbf{v}_{E_5} + {}^1\mathbf{v}_{G_5/E_5} = {}^1\mathbf{v}_{E_4} + {}^1\omega_5 \times \mathbf{r}_{G/E}$$

$$ {}^1\mathbf{v}_{F_5} = {}^1\mathbf{v}_{F_6} + {}^1\mathbf{v}_{F_5/F_6} = 0 + {}^1\mathbf{v}_{F_5/F_6}$$

Now,

$$ {}^1\mathbf{v}_{B_2} = 58 \; in/\text{s} \;\; (\perp \text{ to } \mathbf{r}_{B/A})$$

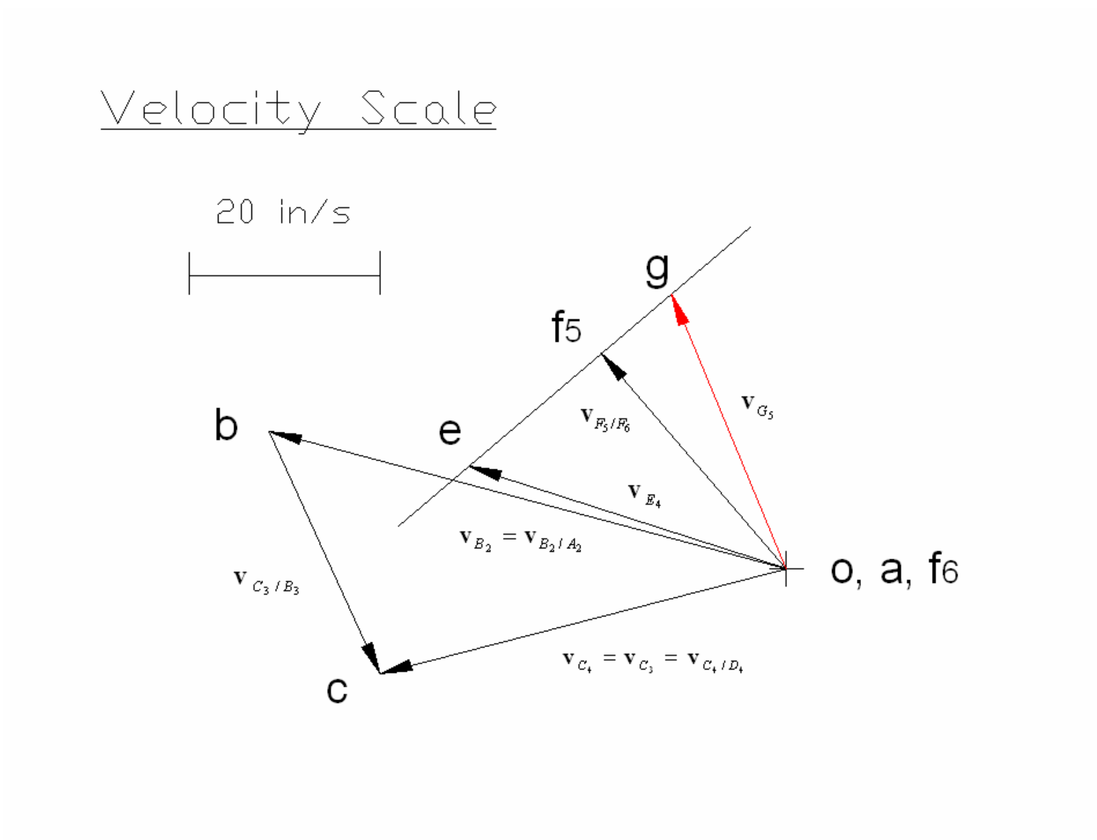
$$ {}^1\mathbf{v}_{C_3/B_3} = {}^1\omega_3 \times \mathbf{r}_{C/B} \quad (\perp \text{ to } \mathbf{r}_{C/B})$$

$$^{1}\mathbf{v}_{C_4/D_4} = {}^{1}\omega_4 \times \mathbf{r}_{C/D} \quad (\perp \textit{ to } \mathbf{r}_{C/D})$$

$$^{1}\mathbf{v}_{G_5/E_5} = {}^{1}\omega_5 \times \mathbf{r}_{G/E} \quad (\perp \textit{ to } \mathbf{r}_{G/E})$$

$$^{1}\mathbf{v}_{F_5/F_6} \quad (\textit{// to } \mathbf{r}_{G/E})$$

Solve Eq. (1) graphically with a velocity polygon.



From the polygon,

$$^{1}\mathbf{v}_{C_4/D_4} = 46\ in/\sec$$

and

$$\left|{}^{1}\omega_4\right| = \frac{\left|{}^{1}\mathbf{v}_{C_4/D_4}\right|}{\left|\mathbf{r}_{C/D}\right|} = \frac{46}{1.45} = 31.72\ rad/\sec\ CCW$$

To find the point "g" construct the velocity image by

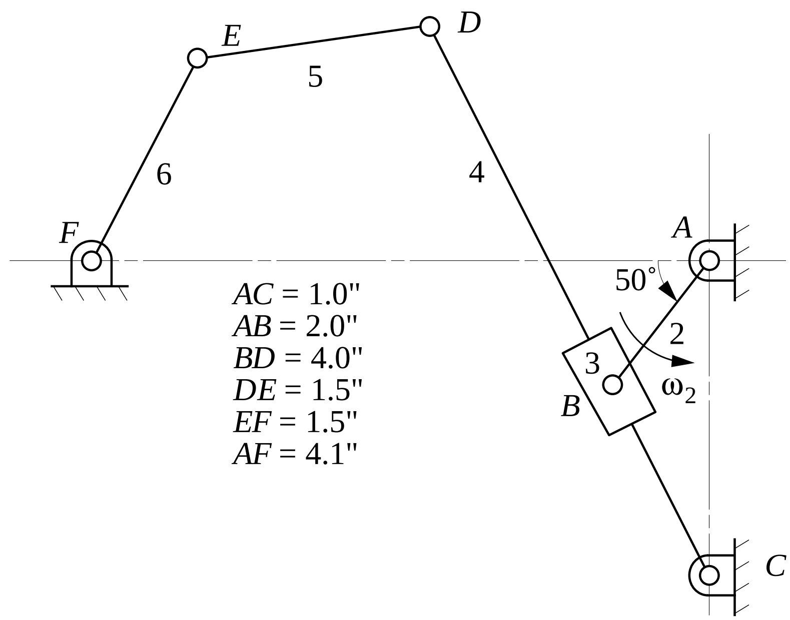
$$\overline{EF} : \overline{EG} = \overline{ef_5} : \overline{eg}$$

From the polygon,

$$\left|{}^{1}\mathbf{v}_{G_5}\right| = 32.5\ in/\sec$$

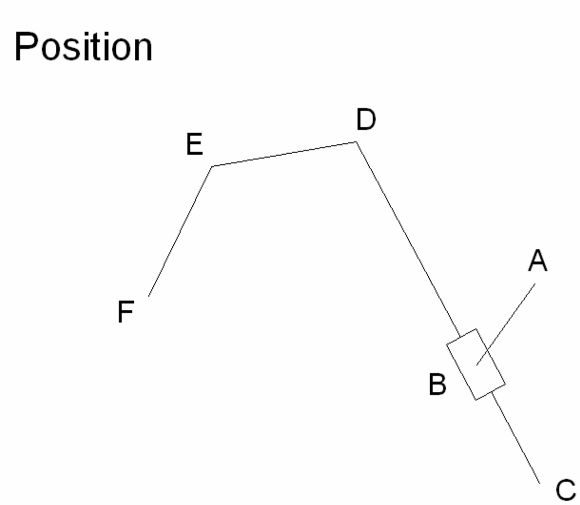
Problem 3.8

If ω_2 = 5 rad/s CCW, find ω_6.



Position Analysis

Draw the linkage to scale.



Velocity Analysis

$${}^{1}\mathbf{v}_{B_2} = {}^{1}\mathbf{v}_{B_2/A_2} = {}^{1}\omega_2 \times \mathbf{r}_{B/A} \Rightarrow \left|{}^{1}\mathbf{v}_{B_2}\right| = \left|{}^{1}\omega_2\right|\left|\mathbf{r}_{B/A}\right| = 5(1) = 5 \text{ in/s} \tag{1}$$

$${}^{1}\mathbf{v}_{B_4} = {}^{1}\mathbf{v}_{B_3} + {}^{1}\mathbf{v}_{B_4/B_3}$$

$${}^{1}\mathbf{v}_{B_4} = {}^{1}\mathbf{v}_{C_4} + {}^{1}\mathbf{v}_{B_4/C_4}$$

Now,

$${}^{1}\mathbf{v}_{B_2} = \; 5 \; in/\text{s} \;\; (\perp \text{ to } \mathbf{r}_{B/A})$$

$${}^{1}\mathbf{v}_{B_4/C_4} = {}^{1}\omega_4 \times \mathbf{r}_{B/C} \quad (\perp \text{ to } \mathbf{r}_{B/C})$$

Solve Eq. (1) graphically with a velocity polygon.

$$\left|\mathbf{r}_{B/C}\right| = 1.33 \; in$$

From the polygon,

$${}^{1}\mathbf{v}_{B_4/C_4} = 4.5 \; in/\text{sec}$$

and

$$\left|{}^{1}\omega_4\right| = \frac{\left|{}^{1}\mathbf{v}_{B_4/C_4}\right|}{\left|\mathbf{r}_{B/C}\right|} = \frac{4.5}{1.33} = 3.38 \; rad/\text{sec} \; CW$$

With the value of ω_4

$${}^{1}\mathbf{v}_{D_4} = {}^{1}\mathbf{v}_{C_4} + {}^{1}\mathbf{v}_{C_4/D_4} = {}^{1}\omega_4 \times \mathbf{r}_{C/D} \Rightarrow \left|{}^{1}\mathbf{v}_{D_4}\right| = \left|{}^{1}\omega_4\right|\left|\mathbf{r}_{C/D}\right| = 3.38(3.88) = 13.12 \text{ in/s}$$

$${}^{1}\mathbf{v}_{D_5} = {}^{1}\mathbf{v}_{D_4}$$

$${}^{1}\mathbf{v}_{D_5} = {}^{1}\mathbf{v}_{E_5} + {}^{1}\mathbf{v}_{D_5/E_5}$$

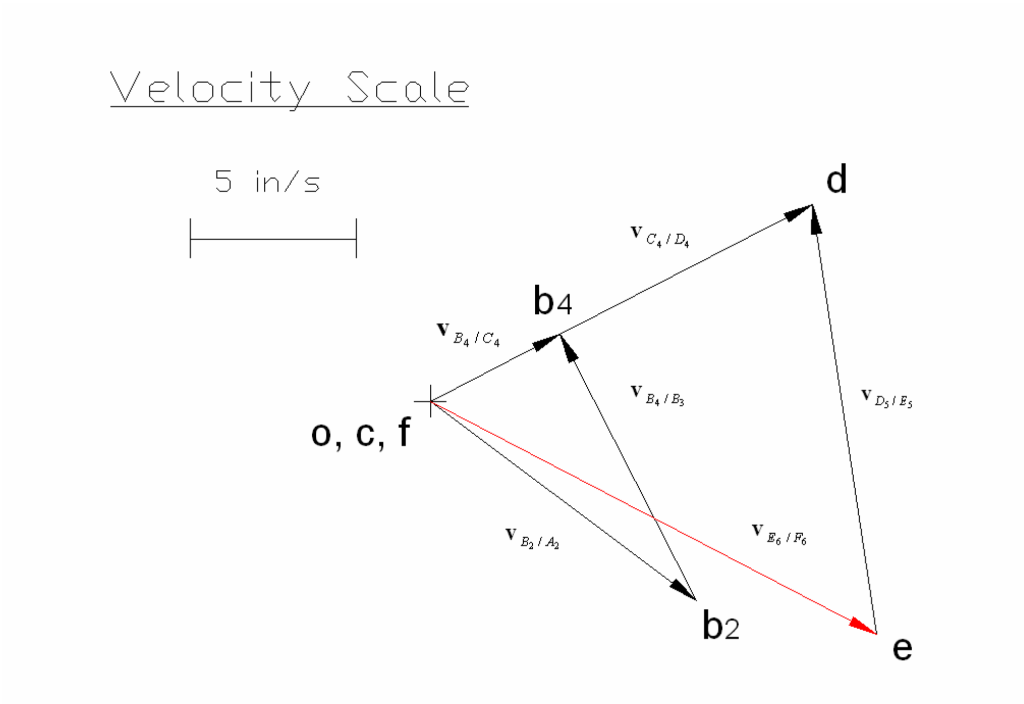
$${}^{1}\mathbf{v}_{E_6} = {}^{1}\mathbf{v}_{E_5}$$

$${}^{1}\mathbf{v}_{E_6} = {}^{1}\mathbf{v}_{F_6} + {}^{1}\mathbf{v}_{E_6/F_6} = 0 + {}^{1}\mathbf{v}_{E_6/F_6}$$

$^{1}\mathbf{v}_{D_5/E_5} \quad (\perp \ to \ \mathbf{r}_{D/E})$

$^{1}\mathbf{v}_{E_6/F_6} \quad (\perp \ to \ \mathbf{r}_{E/F})$

Solve graphically with a velocity polygon.



From the polygon,

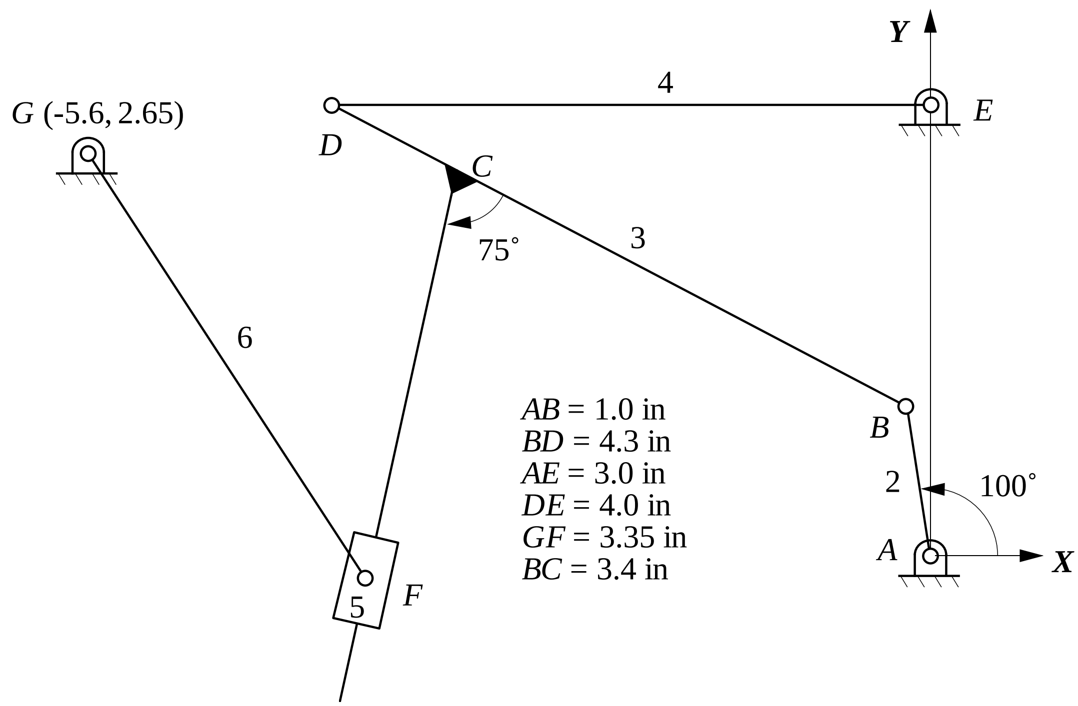
$$^{1}\mathbf{v}_{E_6/F_6} = 15.4 \ in/\sec$$

and

$$\left|^{1}\omega_6\right| = \frac{\left|^{1}\mathbf{v}_{E_6/F_6}\right|}{\left|\mathbf{r}_{E/F}\right|} = \frac{15.4}{1.5} = 10.267 \ rad/\sec \ CW$$

Problem 3.9

In the mechanism below, ω_2 = 10 rad/s. Write the velocity equations and determine the following: $\mathbf{v}_{D_4}$, ω_4, $\mathbf{v}_{F_6}$, ω_6.



Position Analysis

Draw the linkage to scale. First locate the pivots A, E, and G. Next draw link 2 and locate B. Then locate point D and point C. Draw the line CF as shown, and finally locate point F.

Velocity Analysis:

$$ {}^1\mathbf{v}_{B_3} = {}^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{B_2/A_2} $$

$$ {}^1\mathbf{v}_{D_4} = {}^1\mathbf{v}_{D_3} = {}^1\mathbf{v}_{D_4/E_4} = {}^1\mathbf{v}_{B_3} + {}^1\mathbf{v}_{D_3/B_3} \tag{1} $$

$$ {}^1\mathbf{v}_{F_5} = {}^1\mathbf{v}_{F_6} = {}^1\mathbf{v}_{F_6/G_6} = {}^1\mathbf{v}_{F_3} + {}^1\mathbf{v}_{F_5/F_3} \tag{2} $$

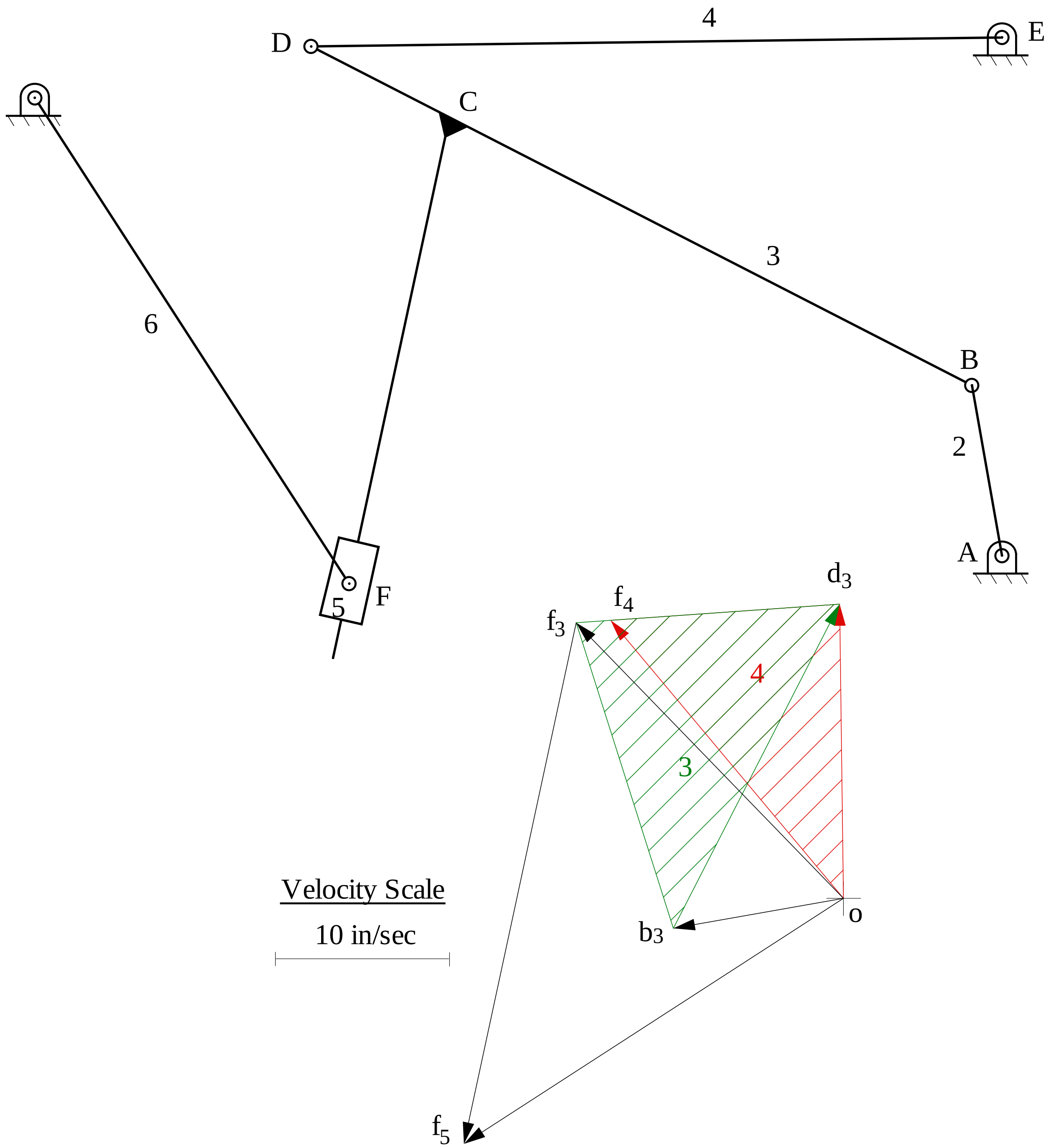
Now,

$$ {}^1\mathbf{v}_{D_4/E_4} = {}^1\omega_4 \times \mathbf{r}_{D_4/E_4} \Rightarrow |{}^1\mathbf{v}_{D_4/E_4}| = |{}^1\omega_4| \cdot |\mathbf{r}_{D_4/E_4}| \, (\perp \text{ to } \mathbf{r}_{D_4/E_4}) $$

$$ {}^1\mathbf{v}_{B_2/A_2} = {}^1\omega_2 \times \mathbf{r}_{B_2/A_2} \Rightarrow |{}^1\mathbf{v}_{B_2/A_2}| = |{}^1\omega_2| \cdot |\mathbf{r}_{B_2/A_2}| = 10 \cdot 1 = 10 \text{ in / sec } (\perp \text{ to } \mathbf{r}_{B_2/A_2}) $$

$$ {}^1\mathbf{v}_{D_3/B_3} = {}^1\omega_3 \times \mathbf{r}_{D_3/B_3} \Rightarrow |{}^1\mathbf{v}_{D_3/B_3}| = |{}^1\omega_3| \cdot |\mathbf{r}_{D_3/B_3}| \, (\perp \text{ to } \mathbf{r}_{D_3/B_3}) $$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,



$$ {}^{1}\mathbf{v}_{D4} = 17.034 \text{ in/sec} $$

or

$$ |{}^{1}\omega_{4}| = \frac{|{}^{1}\mathbf{v}_{D4/E4}|}{|\mathbf{r}_{D4/E4}|} = \frac{17.034}{4} = 4.259 \text{ rad/sec CW} $$

Also, using velocity image

$$ {}^1\mathbf{v}_{F_4} = 20.994 \text{ in / sec} $$

Now,

$$ {}^1\mathbf{v}_{F_3} = 22.227 \text{ in / sec (using velocity polygon)} $$

$$ {}^1\mathbf{v}_{F_6/G_6} = {}^1\omega_6 \times \mathbf{r}_{F_6/G_6} \Rightarrow |{}^1\mathbf{v}_{F_6/G_6}| = |{}^1\omega_6| \cdot |\mathbf{r}_{F_6/G_6}| \ (\perp \text{ to } \mathbf{r}_{F_6/G_6}) $$

$$ {}^1\mathbf{v}_{F_6/G_6} \text{ along the slot} $$

Solve Eq. (2) graphically with a velocity polygon. From the polygon,

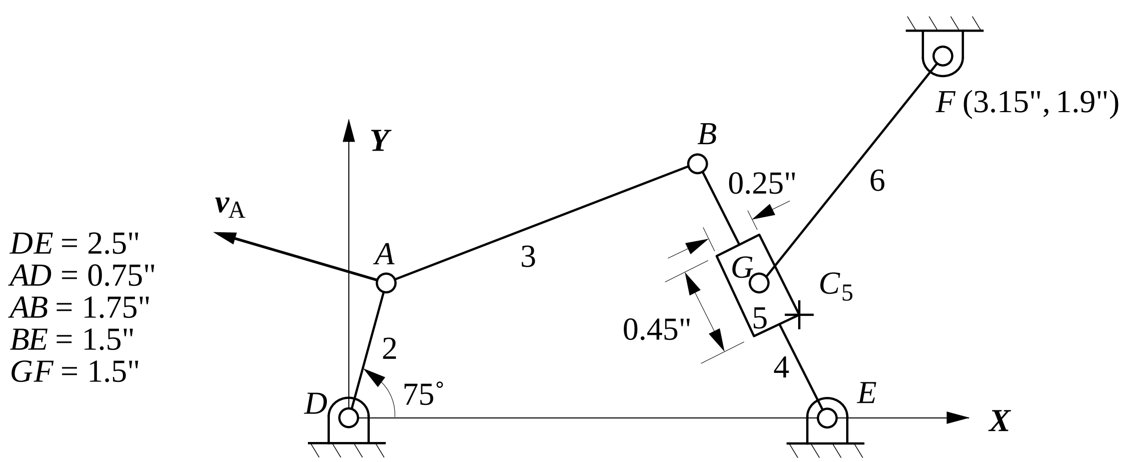
$$ {}^1\mathbf{v}_{F_6/G_6} = 26.171 \text{ in / sec} $$

or

$$ |{}^1\omega_6| = \frac{|{}^1\mathbf{v}_{F_6/G_6}|}{|\mathbf{r}_{F_6/G_6}|} = \frac{26.171}{3.35} = 7.812 \text{ rad / sec CW} $$

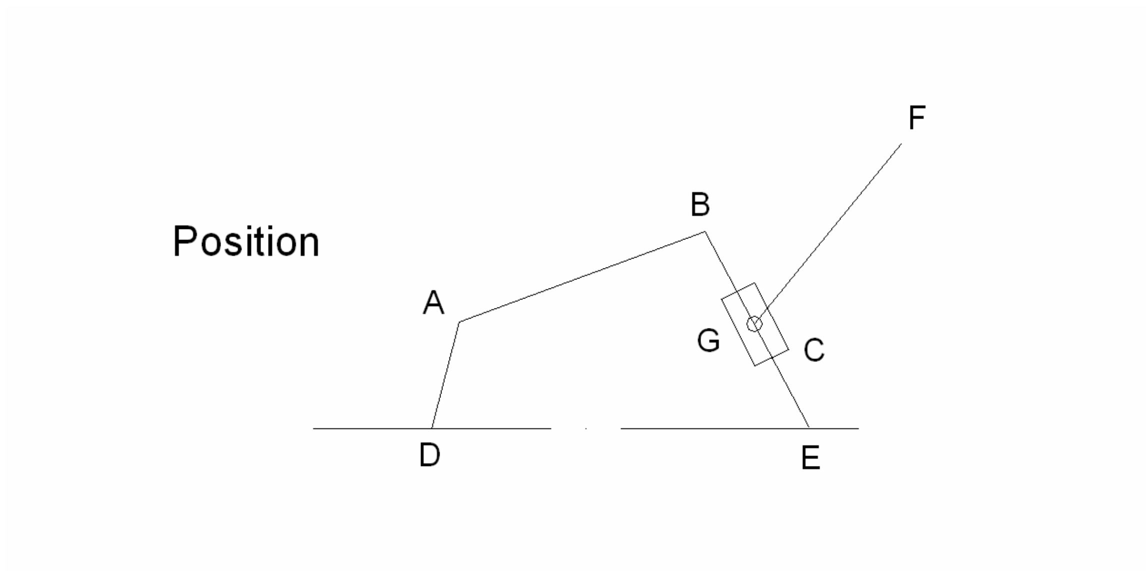
Problem 3.10

If the velocity of point *A* on link 2 is 10 in/s as shown, find the velocity of point *C* on link 5.



Position Analysis

Draw the linkage to scale.



Velocity Analysis

$$^{1}\mathbf{v}_{B_3} = {}^{1}\mathbf{v}_{A_3} + {}^{1}\mathbf{v}_{B_3/A_3} = {}^{1}\mathbf{v}_{A_2} + {}^{1}\omega_3 \times \mathbf{r}_{B/A} \ (1)$$

$$^{1}\mathbf{v}_{B_4} = {}^{1}\mathbf{v}_{B_3} = {}^{1}\mathbf{v}_{B_4/E_4} = {}^{1}\omega_4 \times \mathbf{r}_{B/E}$$

$$^{1}\mathbf{v}_{G_5} = {}^{1}\mathbf{v}_{G_4} + {}^{5}\mathbf{v}_{G_5/G_4}$$

$$^{1}\mathbf{v}_{G_6} = {}^{1}\mathbf{v}_{G_5} = {}^{1}\mathbf{v}_{G_6/F_6} = {}^{1}\omega_6 \times \mathbf{r}_{G/F}$$

$$^{1}\mathbf{v}_{C_5} = {}^{1}\mathbf{v}_{G_5} + {}^{1}\mathbf{v}_{C_5/G_5} = {}^{1}\mathbf{v}_{G_6} + {}^{1}\omega_5 \times \mathbf{r}_{C/G}$$

Now,

$$ {}^1\mathbf{v}_{A_2} = 10\ in/\mathrm{s} \quad (\perp \text{ to } \mathbf{r}_{A/D}) $$

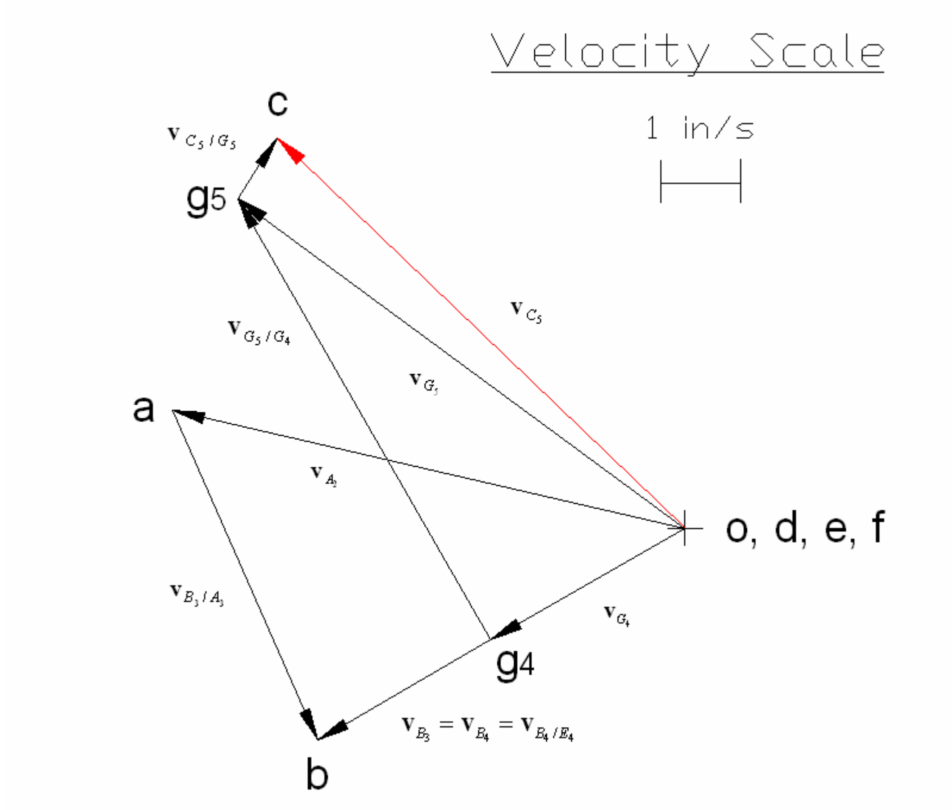
$$ {}^1\mathbf{v}_{B_3/A_3} = {}^1\omega_3 \times \mathbf{r}_{B/A} \quad (\perp \text{ to } \mathbf{r}_{B/A}) $$

$$ {}^1\mathbf{v}_{B_4/E_4} = {}^1\omega_4 \times \mathbf{r}_{B/E} \quad (\perp\ to\ \mathbf{r}_{B/E}) $$

$$ {}^1\mathbf{v}_{G_5/F_6} = {}^1\omega_6 \times \mathbf{r}_{G/F} \quad (\perp\ to\ \mathbf{r}_{G/F}) $$

$$ {}^5\mathbf{v}_{G_5/G_4} \quad (\ //\ to\ \mathbf{r}_{B/E}) $$

Solve Eq. (1) graphically with a velocity polygon.



From the polygon,

$$ {}^1\mathbf{v}_{B_4/E_4} = 8.1\ in/\sec $$

and

$$ \left|{}^1\omega_4\right| = \frac{\left|{}^1\mathbf{v}_{B_4/E_4}\right|}{\left|\mathbf{r}_{B/E}\right|} = \frac{8.1}{1.5} = 5.4\ rad/\sec \quad CCW $$

With ${}^1\omega_5 = {}^1\omega_4$

$$\left|{}^1\mathbf{v}_{C_5/G_5}\right| = \left|{}^1\omega_5\right|\left|\mathbf{r}_{C/G}\right| = 5.4(0.26) = 1.404 \text{ in/s} \quad (\perp \text{ to } \mathbf{r}_{C/G})$$

To find the point "g_4" construct the velocity image by

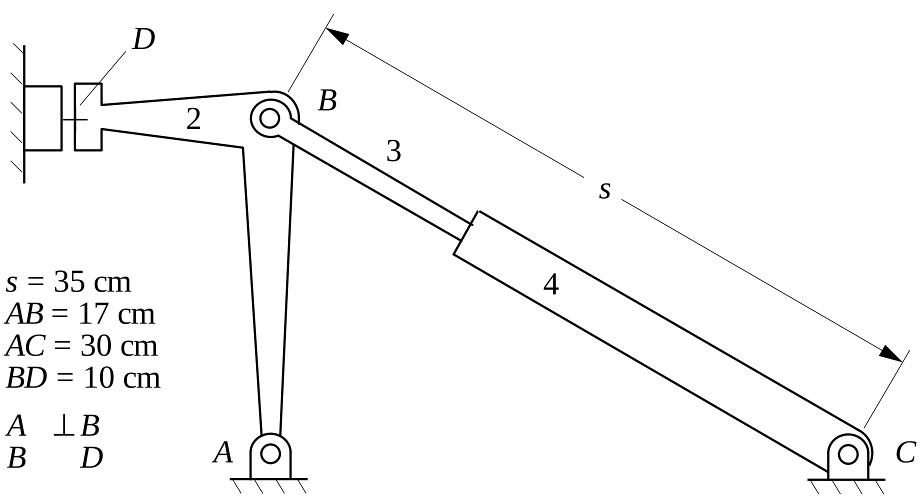
$$\overline{BE} : \overline{EG} = \overline{be} : \overline{eg_4}$$

From the polygon,

$$\left|{}^1\mathbf{v}_{C_5}\right| = 10.75 \; in/\sec$$

Problem 3.11

In the clamping device shown, links 3 and 4 are an air cylinder. If the opening rate of the air cylinder is 5 cm/s and the opening acceleration of the cylinder is 2 cm/s², find the angular velocity and acceleration of link 2, and the linear velocity and acceleration of point *D* on Link 2.



Position Analysis

Draw the linkage to scale. Start by locating the pivots A and C. Then locate point B

Velocity Analysis

Consider the points at location B.

$${}^1\boldsymbol{v}_{B_2} = {}^1\boldsymbol{v}_{B_3} = {}^1\boldsymbol{v}_{B_2/A_2}$$

$${}^1\boldsymbol{v}_{B_3} = {}^1\boldsymbol{v}_{B_4} + {}^1\boldsymbol{v}_{B_3/B_4} \tag{1}$$

Where

$${}^1\boldsymbol{v}_{B_2/A_2} = {}^1\boldsymbol{\omega}_2 \times \boldsymbol{r}_{B/A} \ (\perp \text{ to } \boldsymbol{r}_{B/A})$$

$${}^1\boldsymbol{v}_{B_4} = {}^1\boldsymbol{v}_{B_4/C_4} = {}^1\boldsymbol{\omega}_4 \times \boldsymbol{r}_{B/C} \ (\perp \text{ to } \boldsymbol{r}_{B/C})$$

$$\left|{}^1\boldsymbol{v}_{B_3/B_4}\right| = 5 \text{ cm/s along } \boldsymbol{r}_{B/C}$$

Solve Eq. (1) using a velocity polygon, and determine the velocity of D_2 by image.

$${}^1\boldsymbol{v}_{D_2} = 5.79 \text{ cm/s}$$

$$\left|{}^1\boldsymbol{\omega}_2\right| = \frac{\left|{}^1\boldsymbol{v}_{B_2/A_2}\right|}{\left|\boldsymbol{r}_{B/A}\right|} = \frac{5.81}{17} = 0.342 \text{ rad/s CCW}$$

and

$$\left|{}^1\boldsymbol{\omega}_4\right| = \frac{\left|{}^1\boldsymbol{v}_{B_4/C_4}\right|}{\left|\boldsymbol{r}_{B/C}\right|} = \frac{3.018}{35} = 0.0862 \text{ rad/s CCW}$$

<u>Acceleration Analysis</u>

Again, consider the points at location B.

$${}^1\boldsymbol{\alpha}_{B_2} = {}^1\boldsymbol{\alpha}_{B_3} = {}^1\boldsymbol{\alpha}_{B_2/A_2} = {}^1\boldsymbol{\alpha}^r_{B_2/A_2} + {}^1\boldsymbol{\alpha}^t_{B_2/A_2}$$

$${}^1\boldsymbol{\alpha}_{B_3} = {}^1\boldsymbol{\alpha}_{B_4/A_4} + {}^1\boldsymbol{\alpha}_{B_3/B_4} = {}^1\boldsymbol{\alpha}^r_{B_4/A_4} + {}^1\boldsymbol{\alpha}^t_{B_4/A_4} + {}^4\boldsymbol{\alpha}^t_{B_3/B_4} + {}^4\boldsymbol{\alpha}^n_{B_3/B_4} + {}^1\boldsymbol{\alpha}^c_{B_3/B_4}$$

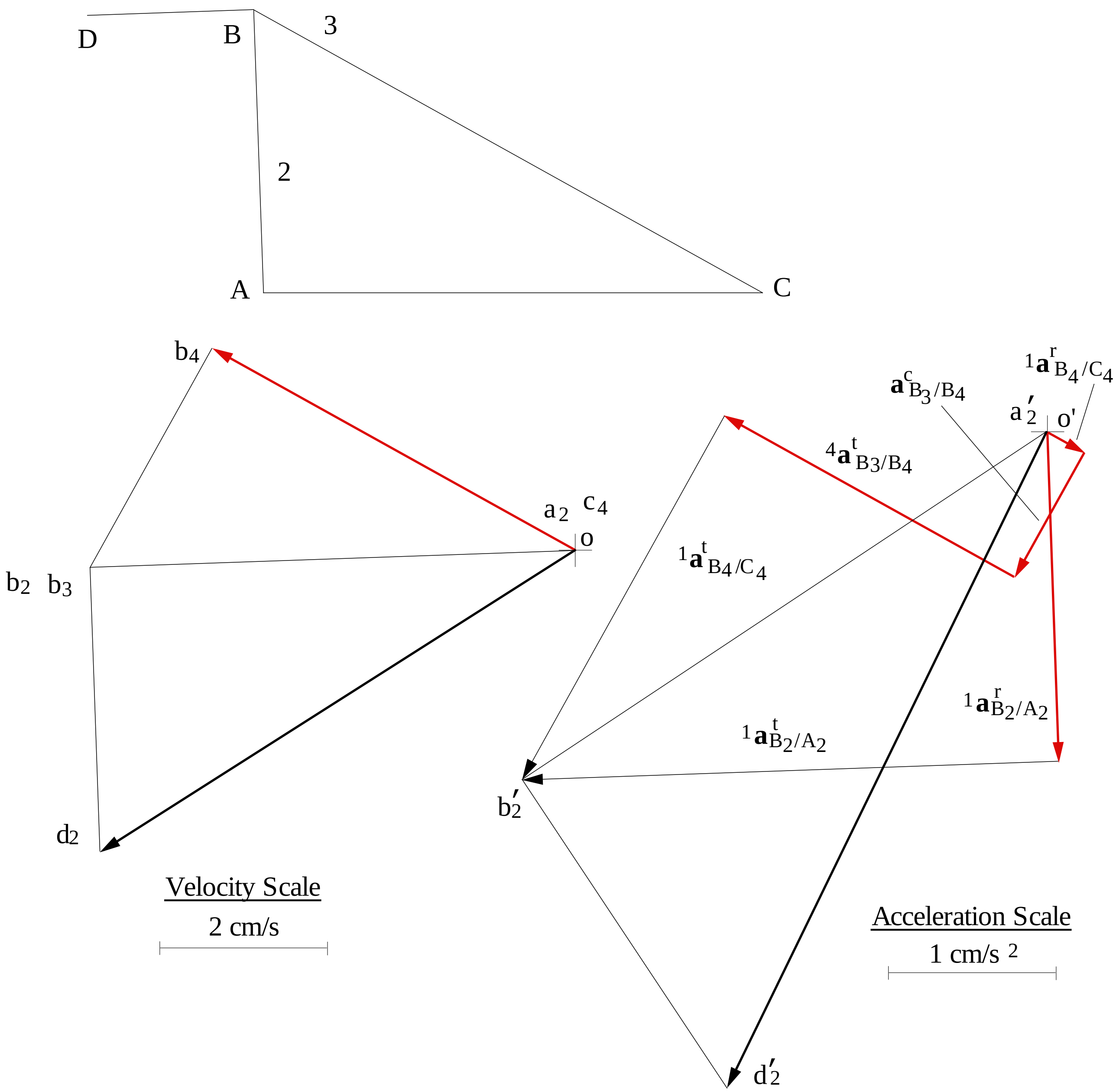
Combining the equations,

$${}^1\boldsymbol{\alpha}^r_{B_2/A_2} + {}^1\boldsymbol{\alpha}^t_{B_2/A_2} = {}^1\boldsymbol{\alpha}^r_{B_4/A_4} + {}^1\boldsymbol{\alpha}^t_{B_4/A_4} + {}^4\boldsymbol{\alpha}^t_{B_3/B_4} + {}^4\boldsymbol{\alpha}^n_{B_3/B_4} + {}^1\boldsymbol{\alpha}^c_{B_3/B_4} \tag{2}$$

Where

$$\left|{}^1\boldsymbol{\alpha}^r_{B_2/A_2}\right| = \left|{}^1\boldsymbol{\omega}_2\right|^2 \left|\boldsymbol{r}_{B/A}\right| = 0.342^2(17) = 1.99 \text{ cm/s}^2 \text{ (opposite to } \boldsymbol{r}_{B/A})$$

$${}^1\boldsymbol{\alpha}^t_{B_2/A_2} = {}^1\boldsymbol{\alpha}_2 \times \boldsymbol{r}_{B/A} \ (\perp \text{ to } \boldsymbol{r}_{B/A})$$



$$\left|{}^1\alpha^r_{B_4/C_4}\right| = \left|{}^1\omega_4\right|^2\left|\boldsymbol{r}_{B/C}\right| = 0.0862^2(35) = 0.260 \text{ cm/s}^2 \text{ (opposite to } \boldsymbol{r}_{B/C})$$

$${}^1\alpha^t_{B_4/A_4} = {}^1\alpha_4 \times \boldsymbol{r}_{B/C} \ (\perp \text{ to } \boldsymbol{r}_{B/C})$$

$$\left|{}^4\alpha^t_{B_3/B_4}\right| = 10 \text{ cm/s}^2 \text{ (along } \boldsymbol{r}_{B/C})$$

$$\left|{}^4\alpha^n_{B_3/B_4}\right| = \frac{\left|{}^1\boldsymbol{v}_{B_3/B_4}\right|^2}{\infty} = 0$$

$${}^1\alpha^c_{B_3/B_4} = 2\cdot{}^1\omega_4 \times {}^4\boldsymbol{v}_{B_3/B_4} \Rightarrow \left|{}^1\alpha^c_{B_3/B_4}\right| = 2\left|{}^1\omega_4\right|\left|{}^4\boldsymbol{v}_{B_3/B_4}\right| = 2(0.0862)(5) = 0.862$$

The direction for $^1\boldsymbol{\alpha}^c_{B_3/B_4}$ is perpendicular to BC and in the direction defined by rotating $^4\boldsymbol{v}_{B_3/B_4}$ 90˚ in the direction of $^1\boldsymbol{\omega}_4$. This direction is generally down and to the left.

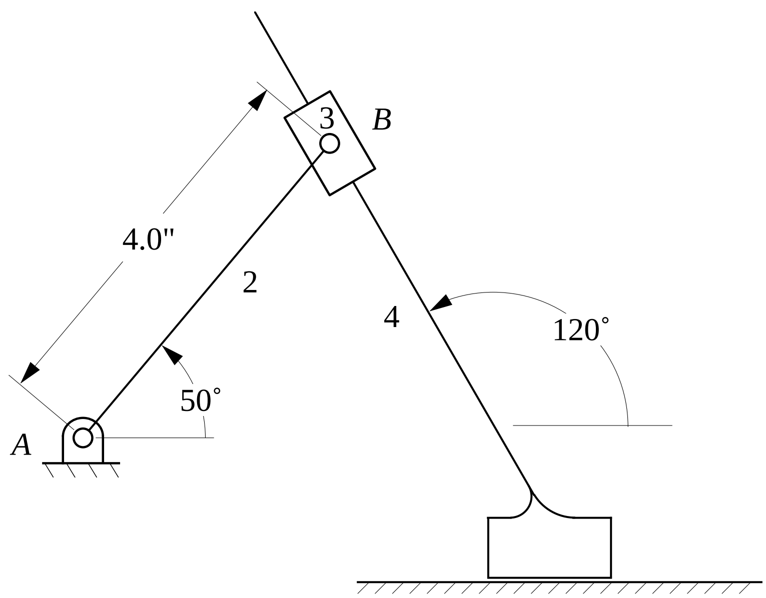
Solve Eq. (2) using an acceleration polygon, and determine the acceleration of D_2 by image.

$$^1\boldsymbol{a}_{D_2} = 4.39\ cm/s^2$$

$$\left|{}^1\alpha_2\right| = \frac{\left|{}^1\boldsymbol{\alpha}^t_{B_2/A_2}\right|}{\left|\boldsymbol{r}_{B/A}\right|} = \frac{3.23}{17} = 0.190\ \text{rad}/\text{s}^2\ \text{CCW}$$

Problem 3.12

In the mechanism shown, link 4 moves to the left with a velocity of 8 in/s and the acceleration is 80 in/s^2 to the left. Draw the velocity and acceleration polygons and solve for the angular velocity and acceleration of link 2.



Position Analysis

Draw link 2 at 45˚ to the horizontal lineand 4" long. Construct link 2 at an angle of 120˚ to the horizontal and through point B.

Velocity Analysis:

$$^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{B_4} + {}^1\mathbf{v}_{B_2/B_4} = {}^1\mathbf{v}_{B_4} + {}^4\mathbf{v}_{B_2/B_4} \tag{1}$$

$$^1\mathbf{v}_{B_2} = {}^1\omega_2 \times \mathbf{r}_{B_2/A_2} \Rightarrow \left|{}^1\mathbf{v}_{B_2}\right| = \left|{}^1\omega_2\right|\left|\mathbf{r}_{B_2/A_2}\right|$$

Now,

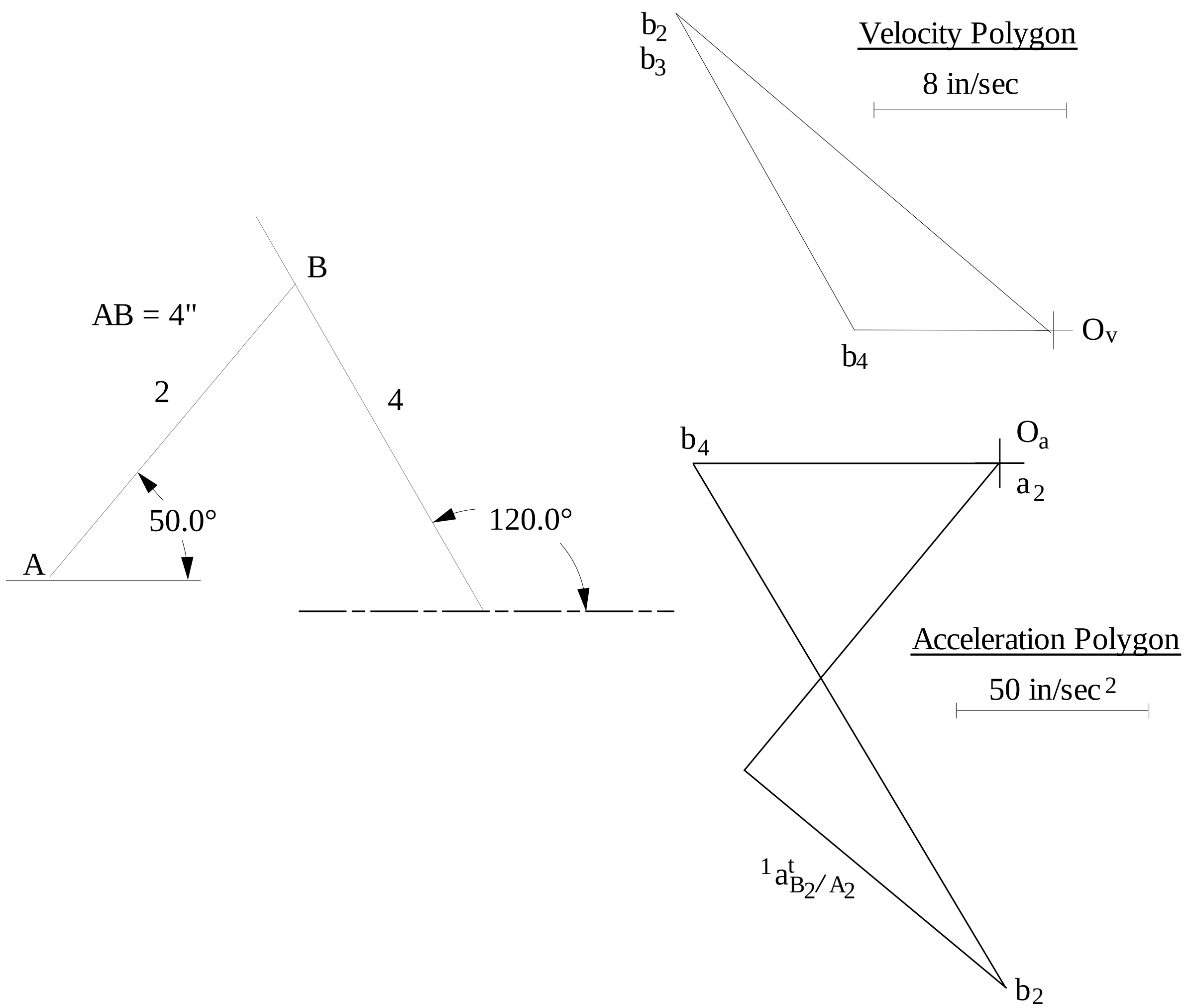
$$^1\mathbf{v}_{B_4} = 8\ \text{in}/\text{sec}\ \text{in the horizontal direction to the left}$$

$$^{4}\mathbf{v}_{B_2/B_4} = {}^{1}\mathbf{v}_{B_2/B_4} \text{ along the link 4}$$

$$^{1}\mathbf{v}_{B_2} = {}^{1}\omega_2 \times \mathbf{r}_{B/A} \quad (\perp \text{ to } \mathbf{r}_{B/A})$$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$$^{1}\mathbf{v}_{B_2/B_3} = {}^{4}\mathbf{v}_{B_2/B_3} = 15.2 \ \text{ in/sec}$$



Also,

$$|{}^{1}\omega_2| = \frac{|{}^{1}\mathbf{v}_{B_2/A_2}|}{|\mathbf{r}_{B/A}|} = \frac{20.62}{4} = 5.1572 \, \text{rad/sec}$$

From the directions given in the position and velocity polygons

$$^{1}\omega_2 = 5.1572 \text{ rad/sec CCW}$$

<u>Acceleration Analysis:</u>

$$ {}^1\mathbf{a}_{B2} = {}^1\mathbf{a}_{B4} + {}^1\mathbf{a}_{B2/B4} = {}^1\mathbf{a}_{B4} + {}^4\mathbf{a}_{B2/B4} + 2\,{}^1\omega_4 \times {}^4\mathbf{v}_{B2/B4} \tag{2} $$

But, ${}^1\omega_4 = 0$. Therefore,

$$ {}^1\mathbf{a}_{B2} = {}^1\mathbf{a}_{B4} + {}^4\mathbf{a}_{B2/B4} $$

Also,

$$ {}^1\mathbf{a}_{B2} = {}^1\mathbf{a}_{B2/A2} = {}^1\mathbf{a}^t_{B2/A2} + {}^1\mathbf{a}^r_{B2/A2} $$

Therefore,

$$ {}^1\mathbf{a}^t_{B2/A2} + {}^1\mathbf{a}^r_{B2/A2} = {}^1\mathbf{a}_{B4} + {}^4\mathbf{a}_{B2/B4} $$

Now,

$$ {}^1\mathbf{a}_{B4} = 80 \text{ in / sec}^2 \text{ in the horizontal direction to the left} $$

$$ {}^4\mathbf{a}_{B2/B4} \text{ is along the link 4} $$

$$ {}^1\mathbf{a}^r_{B2/A2} = |{}^1\omega_2|^2 |\mathbf{r}_{B/A}| \text{ (opposite to } \mathbf{r}_{B/A}) $$

$$ {}^1a^t_{B2/A2} = {}^1\alpha_2 \times \mathbf{r}_{B/A} \text{ and is } \perp \text{ to } \mathbf{r}_{B/A} $$

From the acceleration polygon,

$$ {}^1a^t_{B2/A2} = 89 \text{ in / sec}^2 $$

Therefore,

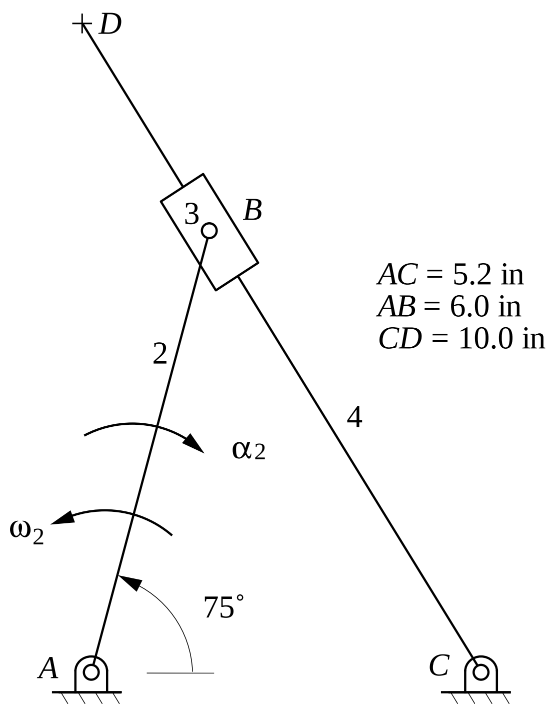
$$ |{}^1\alpha_2| = \frac{|{}^1a^t_{B2/A2}|}{|\mathbf{r}_{B/A}|} = \frac{89}{4} = 22.25 \text{ rad / sec}^2 $$

From the directions given on the acceleration and position polygons,

$$ {}^1\alpha_2 = 22.25 \text{ rad / sec}^2 \text{ CW} $$

Problem 3.13

In the mechanism below, the angular velocity of Link 2 is 2 rad/s CCW and the angular acceleration is 5 rad/s^2 CW. Determine the following: $\mathbf{v}_{B4}$, $\mathbf{v}_{D4}$, $\boldsymbol{\omega}_4$, $\mathbf{a}_{B4}$, $\mathbf{a}_{D4}$, $\boldsymbol{\alpha}_4$,.



Position Analysis

Draw the mechanism to scale. Locate the pivots A and C. Draw link 2 and locate point B. Then draw line CBD.

Velocity Analysis:

$$^1\mathbf{v}_{B_2} = {^1\mathbf{v}_{B_3}} = {^1\mathbf{v}_{B_2/A_2}} = {^1\mathbf{v}_{B_4}} + {^1\mathbf{v}_{B_2/B_4}} \tag{1}$$

$$^1\mathbf{v}_{B_4} = {^1\mathbf{v}_{B_4/C_4}}$$

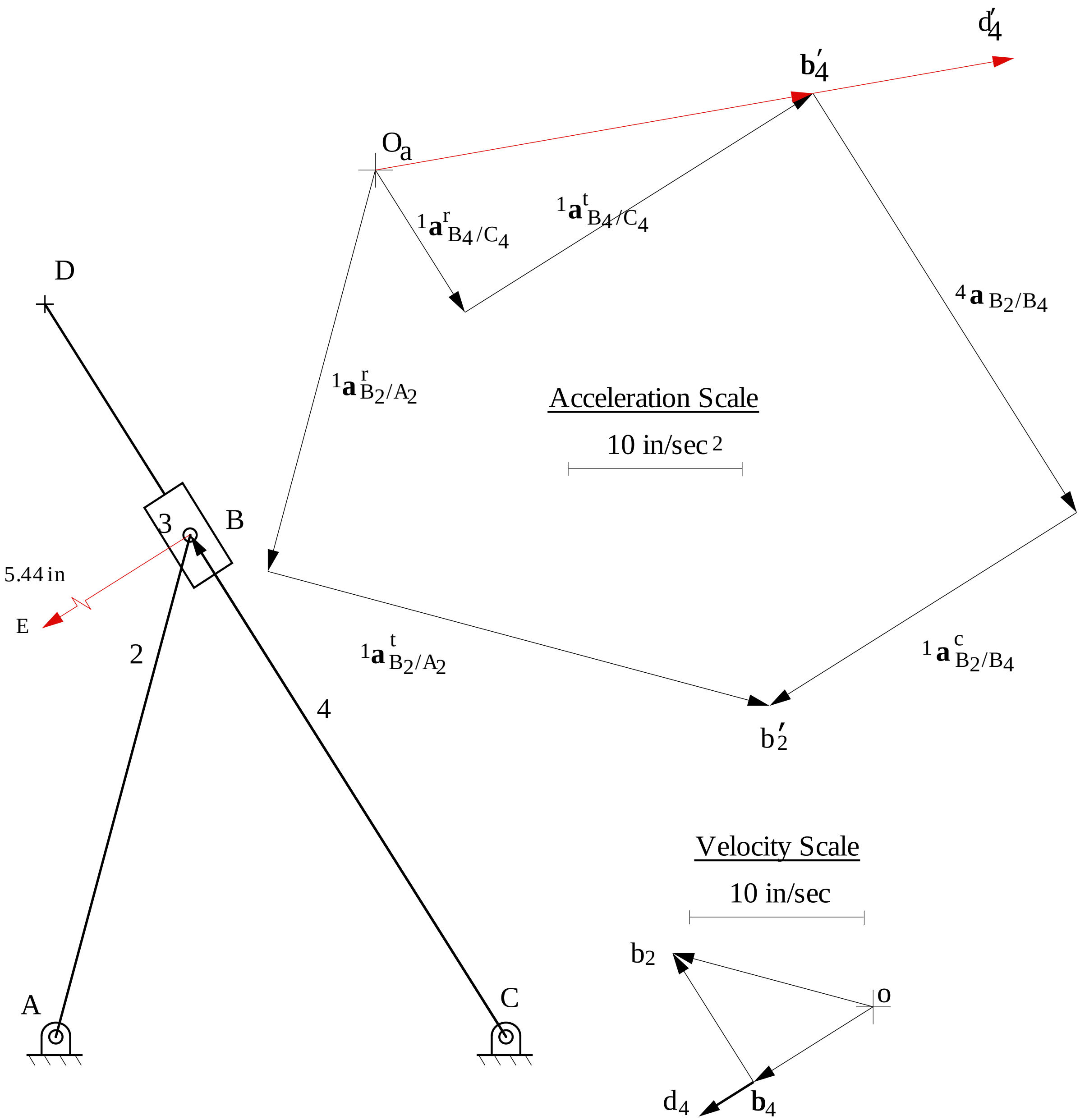
Now,

$$^1\mathbf{v}_{B_2/A_2} = {^1\boldsymbol{\omega}_2} \times \mathbf{r}_{B/A} \Rightarrow \left|^1\mathbf{v}_{B_2/A_2}\right| = \left|^1\boldsymbol{\omega}_2\right|\left|\mathbf{r}_{B/A}\right| = 2\cdot 6 = 12 \text{ in / sec } (\perp \text{ to } \mathbf{r}_{B/A})$$

$$^1\mathbf{v}_{B_4/C_4} = {^1\boldsymbol{\omega}_4} \times \mathbf{r}_{B/C} \Rightarrow \left|^1\mathbf{v}_{B_4/C_4}\right| = \left|^1\boldsymbol{\omega}_4\right|\left|\mathbf{r}_{B/C}\right| (\perp \text{ to } \mathbf{r}_{B/C})$$

$^1\mathbf{v}_{B_2/B_4}$ in the direction of $\mathbf{r}_{B/C}$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,



$$^1\mathbf{v}_{B_4} = 8.156 \text{ in / sec}$$

or

$$|{}^1\omega_4| = \frac{|{}^1\mathbf{v}_{B_4/C_4}|}{|\mathbf{r}_{B/C}|} = \frac{8.156}{6.8476} = 1.191 \text{ rad / sec CCW}$$

Also,

$$^1\mathbf{v}_{B_2/B_4} = 8.802 \text{ in / sec}$$

Also,

$$^1\mathbf{v}_{D_4} = 11.91 \text{ in/sec}$$

Acceleration Analysis:

$$^1\mathbf{a}_{B_2} = {}^1\mathbf{a}_{B_3} = {}^1\mathbf{a}_{B_2/A_2} = {}^1\mathbf{a}_{B_4} + {}^1\mathbf{a}_{B_2/B_4}$$

$$^1\mathbf{a}_{B_4} = {}^1\mathbf{a}_{B_4/C_4}$$

$$\mathbf{a}^r_{B_2/A_2} + {}^1\mathbf{a}^t_{B_2/A_2} = {}^1\mathbf{a}^r_{B_4/C_4} + {}^1\mathbf{a}^t_{B_4/C_4} + {}^4\mathbf{a}_{B_2/B_4} + 2\cdot{}^1\omega_4 \times {}^4\mathbf{v}_{B_4/C_4} \qquad (2)$$

Now,

$$^1\mathbf{a}^r_{B_2/A_2} = {}^1\omega_2 \times ({}^1\omega_2 \times \mathbf{r}_{B/A}) \Rightarrow \left|{}^1\mathbf{a}^r_{B_2/A_2}\right| = \left|{}^1\omega_2\right|^2 \cdot \left|\mathbf{r}_{B/A}\right| = 2^2 \cdot 6 = 24 \text{ in/sec}^2$$

in the direction opposite to $\mathbf{r}_{B_2/A_2}$

$$^1\mathbf{a}^t_{B_2/A_2} = {}^1\alpha_2 \times \mathbf{r}_{B/A} \Rightarrow \left|{}^1\mathbf{a}^t_{B_2/A_2}\right| = \left|{}^1\alpha_2\right| \cdot \left|\mathbf{r}_{B/A}\right| = 5 \cdot 6 = 30 \text{ in/sec}^2 \; (\perp \text{ to } \mathbf{r}_{B/A})$$

$$^1\mathbf{a}^r_{B_4/C_4} = {}^1\omega_4 \times ({}^1\omega_4 \times \mathbf{r}_{B/C}) \Rightarrow \left|{}^1\mathbf{a}^r_{B_4/C_4}\right| = \left|{}^1\omega_4\right|^2 \cdot \left|\mathbf{r}_{B/C}\right| = 1.191^2 \cdot 6.848 = 9.713 \text{ in/sec}^2$$

in the direction opposite to $\mathbf{r}_{B_3/C_3}$

$$^1\mathbf{a}^t_{B_4/C_4} = {}^1\alpha_4 \times \mathbf{r}_{B/C} \Rightarrow \left|{}^1\mathbf{a}^t_{B_4/C_4}\right| = \left|{}^1\alpha_4\right| \cdot \left|\mathbf{r}_{B/C}\right| \; (\perp \text{ to } \mathbf{r}_{B/C})$$

$^4\mathbf{a}_{B_2/B_4}$ in the direction of $\mathbf{r}_{B_4/C_4}$

$$2\cdot{}^1\omega_4 \times {}^4\mathbf{v}_{B_2/B_4} = 2 \cdot 1.191 \cdot 8.802 = 20.966 \text{ in/sec}^2$$

Solve Eq. (2) graphically with an acceleration polygon. From the polygon,

$$^1\mathbf{a}^t_{B_4/C_4} = 23.75 \text{ in/sec}^2$$

or

$$\left|{}^1\alpha_4\right| = \frac{\left|{}^1\mathbf{a}^t_{B_4/C_4}\right|}{\left|\mathbf{r}_{B_4/C_4}\right|} = \frac{23.752}{6.8476} = 3.469 \text{ rad/sec}^2 \text{ CW}$$

Also,

$$^1\mathbf{a}_{B_4} = 25.66 \text{ in/sec}^2$$

Also,

$$^1\mathbf{a}_{D_4} = 37.47 \text{ in/sec}^2$$

To find the center of the curvature of the path that B_4 traces on link 2, we must find an expression which involves that the radius of curvature of the path. This term is ${}^2\mathbf{a}^n_{B4/B2}$ and it can be evaluated from the following:

$$ {}^1\mathbf{a}_{B2/B4} = -{}^1\mathbf{a}_{B4/B2} $$

therefore,

$$ {}^1\mathbf{a}^t_{B2/B4} = -{}^1\mathbf{a}^t_{B4/B2} $$

and

$$ {}^1\mathbf{a}^n_{B2/B4} = -{}^1\mathbf{a}^n_{B4/B2} $$

Also,

$$ {}^3\mathbf{a}^n_{B2/B4} + 2\cdot{}^1\boldsymbol{\omega}_4 \times {}^1\mathbf{v}_{B2/B4} = -{}^2\mathbf{a}^n_{B4/B2} - 2\cdot{}^1\boldsymbol{\omega}_2 \times {}^1\mathbf{v}_{B4/B2} \tag{3} $$

For our purpose, we should arrange Eq. (3) as

$$ {}^2\mathbf{a}^n_{B4/B2} = -\left({}^3\mathbf{a}^n_{B2/B4} + 2\cdot{}^1\boldsymbol{\omega}_2 \times {}^1\mathbf{v}_{B4/B2} + 2\cdot{}^1\boldsymbol{\omega}_4 \times {}^1\mathbf{v}_{B2/B4}\right) $$

Now,

$$ {}^3\mathbf{a}^n_{B2/B4} = \frac{\left|{}^3\mathbf{v}^n_{B2/B4}\right|}{\infty} = 0 $$

$$ 2\cdot{}^1\boldsymbol{\omega}_4 \times {}^1\mathbf{v}_{B2/B4} = 2\cdot 1.191\cdot 8.802 = 20.97 \text{ in/sec}^2 \ (\perp \text{ to } DC) \text{ down and to the left.} $$

$$ 2\cdot{}^1\boldsymbol{\omega}_2 \times {}^1\mathbf{v}_{B4/B2} = 2\cdot 2\cdot 8.802 = 35.20 \text{ in/sec}^2 \ (\perp \text{ to } DC) \text{ up and to the right} $$

Let E be the location of the center of curvature of B_4 on link 2. If we choose up and to the right as the positive direction,

$$ {}^2\mathbf{a}^n_{B4/B2} = \frac{\left|{}^1\mathbf{v}_{B4/B2}\right|^2}{\mathbf{r}_{B/E}} = -(35.20 - 20.97 + 0) = -14.24 \text{ in/sec}^2 $$

Because ${}^2\mathbf{a}^n_{B4/B2}$ is negative, ${}^2\mathbf{a}^n_{B4/B2}$ points down and to the left which is the direction of E. The magnitude of the distance is given by

$$ \left|\mathbf{r}_{B/E}\right| = \frac{\left|{}^1\mathbf{v}_{B4/B2}\right|^2}{14.24} = \frac{8.802^2}{14.24} = 5.44 \text{ in} $$

The direction of E is shown on the drawing.

Problem 3.14

Resolve Problem 3.13 if ω_2 = 2 rad/sec (constant)

Position Analysis

Draw the mechanism to scale. Locate the pivots A and C. Draw link 2 and locate point B. Then draw line CBD.

Velocity Analysis:

$$ {}^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{B_3} = {}^1\mathbf{v}_{B_2/A_2} = {}^1\mathbf{v}_{B_4} + {}^1\mathbf{v}_{B_2/B_4} \qquad (1) $$

$$ {}^1\mathbf{v}_{B_4} = {}^1\mathbf{v}_{B_4/C_4} $$

Now,

$$ {}^1\mathbf{v}_{B_2/A_2} = {}^1\omega_2 \times \mathbf{r}_{B/A} \Rightarrow \left|{}^1\mathbf{v}_{B_2/A_2}\right| = \left|{}^1\omega_2\right|\left|\mathbf{r}_{B/A}\right| = 2\cdot 6 = 12 \text{ in / sec} \; (\perp \text{ to } \mathbf{r}_{B/A}) $$

$$ {}^1\mathbf{v}_{B_4/C_4} = {}^1\omega_4 \times \mathbf{r}_{B/C} \Rightarrow \left|{}^1\mathbf{v}_{B_4/C_4}\right| = \left|{}^1\omega_4\right|\left|\mathbf{r}_{B/C}\right| \; (\perp \text{ to } \mathbf{r}_{B/C}) $$

$$ {}^1\mathbf{v}_{B_2/B_4} \text{ in the direction of } \mathbf{r}_{B/C} $$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$$ {}^1\mathbf{v}_{B_4} = 8.156 \text{ in / sec} $$

or

$$ \left|{}^1\omega_4\right| = \frac{\left|{}^1\mathbf{v}_{B_4/C_4}\right|}{\left|\mathbf{r}_{B/C}\right|} = \frac{8.156}{6.8476} = 1.191 \text{ rad / sec CCW} $$

Also,

$$ {}^1\mathbf{v}_{B_2/B_4} = 8.802 \text{ in / sec} $$

Also,

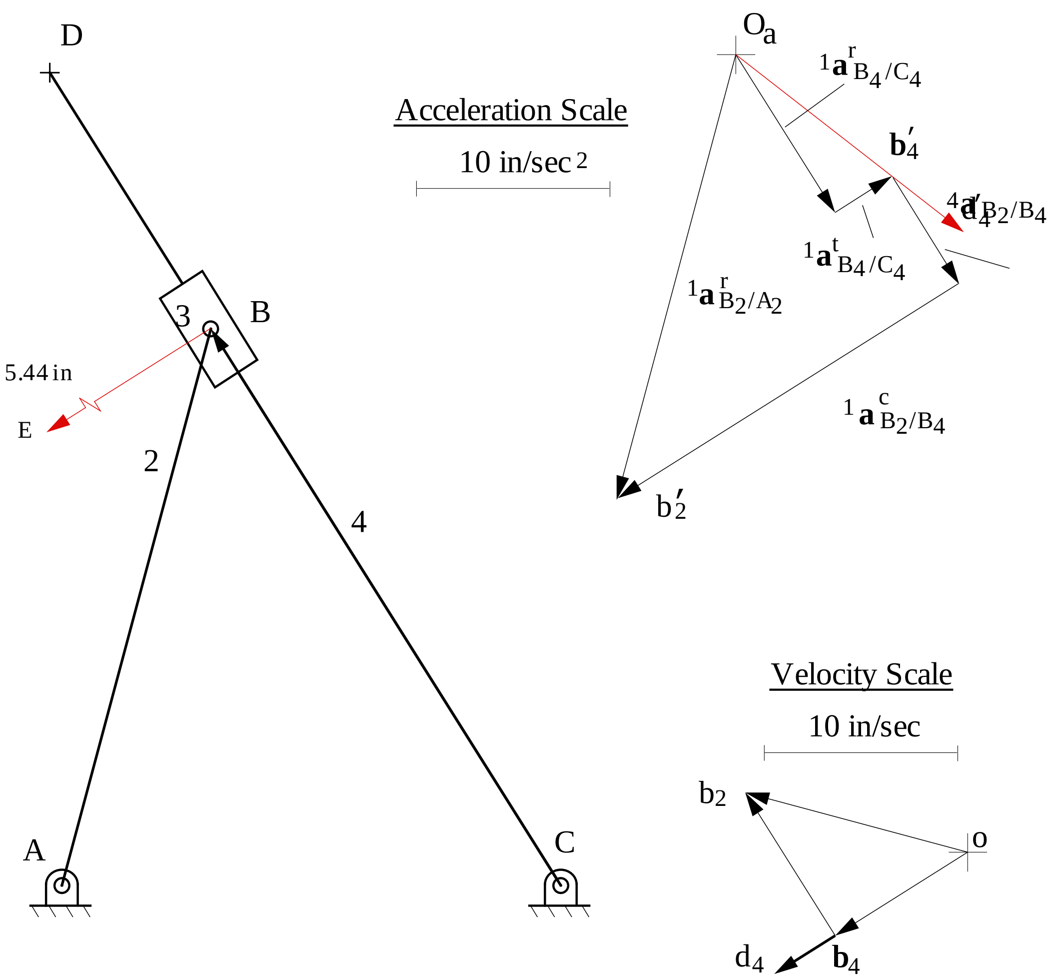
$$ {}^1\mathbf{v}_{D_4} = 11.91 \text{ in / sec} $$

Acceleration Analysis:

$$ {}^1\mathbf{a}_{B_2} = {}^1\mathbf{a}_{B_3} = {}^1\mathbf{a}_{B_2/A_2} = {}^1\mathbf{a}_{B_4} + {}^1\mathbf{a}_{B_2/B_4} $$

$$ {}^1\mathbf{a}_{B_4} = {}^1\mathbf{a}_{B_4/C_4} $$

$$ \mathbf{a}^r_{B_2/A_2} + {}^1\mathbf{a}^t_{B_2/A_2} = {}^1\mathbf{a}^r_{B_4/C_4} + {}^1\mathbf{a}^t_{B_4/C_4} + {}^4\mathbf{a}_{B_2/B_4} + 2\cdot{}^1\omega_4 \times {}^4\mathbf{v}_{B_4/C_4} \qquad (2) $$



Now,

$${}^1\mathbf{a}^r_{B_2/A_2} = {}^1\omega_2 \times ({}^1\omega_2 \times \mathbf{r}_{B/A}) \Rightarrow \left|{}^1\mathbf{a}^r_{B_2/A_2}\right| = |{}^1\omega_2|^2 \cdot |\mathbf{r}_{B/A}| = 2^2 \cdot 6 = 24 \text{ in/sec}^2$$

in the direction opposite to $\mathbf{r}_{B_2/A_2}$

$${}^1\mathbf{a}^t_{B_2/A_2} = {}^1\alpha_2 \times \mathbf{r}_{B/A} \Rightarrow \left|{}^1\mathbf{a}^t_{B_2/A_2}\right| = |{}^1\alpha_2| \cdot |\mathbf{r}_{B/A}| = 0 \cdot 6 = 0$$

$${}^1\mathbf{a}^r_{B_4/C_4} = {}^1\omega_4 \times ({}^1\omega_4 \times \mathbf{r}_{B/C}) \Rightarrow \left|{}^1\mathbf{a}^r_{B_4/C_4}\right| = |{}^1\omega_4|^2 \cdot |\mathbf{r}_{B/C}| = 1.191^2 \cdot 6.848 = 9.713 \text{ in/sec}^2$$

in the direction opposite to $\mathbf{r}_{B_3/C_3}$

$${}^1\mathbf{a}^t_{B_4/C_4} = {}^1\alpha_4 \times \mathbf{r}_{B/C} \Rightarrow \left|{}^1\mathbf{a}^t_{B_4/C_4}\right| = |{}^1\alpha_4| \cdot |\mathbf{r}_{B/C}| \; (\perp \text{ to } \mathbf{r}_{B/C})$$

${}^4\mathbf{a}_{B_2/B_4}$ in the direction of $\mathbf{r}_{B_4/C_4}$

$$2 \cdot {}^1\omega_4 \times {}^4\mathbf{v}_{B_2/B_4} = 2 \cdot 1.191 \cdot 8.802 = 20.966 \text{ in/sec}^2$$

Solve Eq. (2) graphically with an acceleration polygon. From the polygon,

$$ {}^{1}\mathbf{a}^{t}_{B4/C4} = 3.515 \text{ in / sec}^2 $$

or

$$ \left|{}^{1}\alpha_4\right| = \frac{\left|{}^{1}\mathbf{a}^{t}_{B4/C4}\right|}{\left|\mathbf{r}_{B4/C4}\right|} = \frac{3.515}{6.8476} = 0.513 \text{ rad / sec}^2 \text{ CW} $$

Also,

$$ {}^{1}\mathbf{a}_{B4} = 10.29 \text{ in / sec}^2 $$

Also,

$$ {}^{1}\mathbf{a}_{D4} = 15.06 \text{ in / sec}^2 $$

To find the center of the curvature of the path that B_4 traces on link 2, we must find an expression which involves that the radius of curvature of the path. This term is ${}^{2}\mathbf{a}^{n}_{B4/B2}$ and it can be evaluated from the following:

$$ {}^{1}\mathbf{a}_{B2/B4} = -{}^{1}\mathbf{a}_{B4/B2} $$

therefore,

$$ {}^{1}\mathbf{a}^{t}_{B2/B4} = -{}^{1}\mathbf{a}^{t}_{B4/B2} $$

and

$$ {}^{1}\mathbf{a}^{n}_{B2/B4} = -{}^{1}\mathbf{a}^{n}_{B4/B2} $$

Also,

$$ {}^{3}\mathbf{a}^{n}_{B2/B4} + 2\cdot{}^{1}\boldsymbol{\omega}_4 \times {}^{1}\mathbf{v}_{B2/B4} = -{}^{2}\mathbf{a}^{n}_{B4/B2} - 2\cdot{}^{1}\boldsymbol{\omega}_2 \times {}^{1}\mathbf{v}_{B4/B2} \tag{3} $$

For our purpose, we should arrange Eq. (3) as

$$ {}^{2}\mathbf{a}^{n}_{B4/B2} = -\left({}^{3}\mathbf{a}^{n}_{B2/B4} + 2\cdot{}^{1}\boldsymbol{\omega}_2 \times {}^{1}\mathbf{v}_{B4/B2} + 2\cdot{}^{1}\boldsymbol{\omega}_4 \times {}^{1}\mathbf{v}_{B2/B4}\right) $$

Now,

$$ {}^{3}\mathbf{a}^{n}_{B2/B4} = \frac{\left|{}^{3}\mathbf{v}^{n}_{B2/B4}\right|}{\infty} = 0 $$

$$ 2\cdot{}^{1}\boldsymbol{\omega}_4 \times {}^{1}\mathbf{v}_{B2/B4} = 2\cdot 1.191\cdot 8.802 = 20.97 \text{ in / sec}^2 \ (\perp \text{ to DC}) $$ down and to the left.

$$ 2\cdot{}^{1}\boldsymbol{\omega}_2 \times {}^{1}\mathbf{v}_{B4/B2} = 2\cdot 2\cdot 8.802 = 35.20 \text{ in / sec}^2 \ (\perp \text{ to DC}) $$ up and to the right

Let E be the location of the center of curvature of B_4 on link 2. If we choose up and to the right as the positive direction,

$$ {}^2\mathbf{a}^n_{B_4/B_2} = \frac{|{}^1\mathbf{v}_{B_4/B_2}|^2}{\mathbf{r}_{B/E}} = -(35.20 - 20.97 + 0) = -14.24 \text{ in / sec}^2 $$

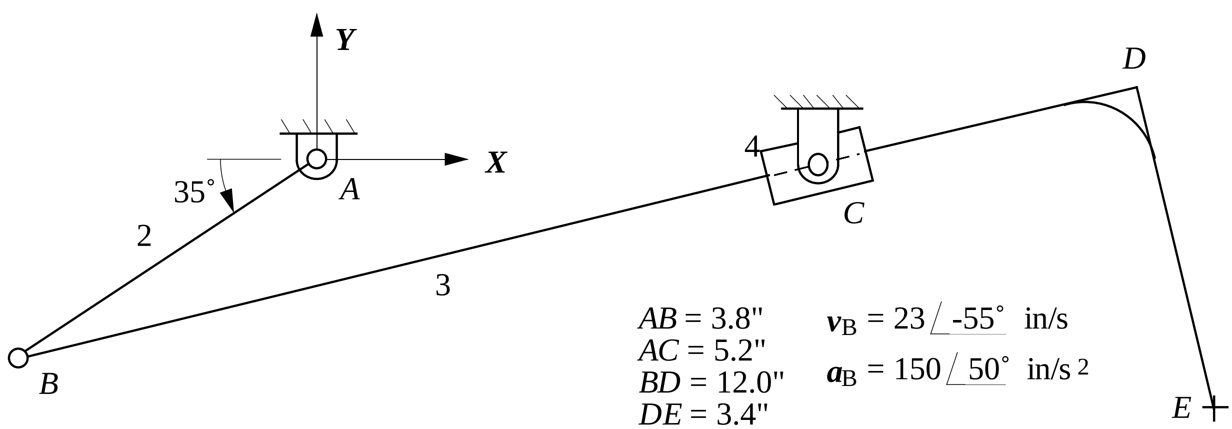
Because ${}^2\mathbf{a}^n_{B_4/B_2}$ is negative, ${}^2\mathbf{a}^n_{B_4/B_2}$ points down and to the left which is the direction of E. The magnitude of the distance is given by

$$ |\mathbf{r}_{B/E}| = \frac{|{}^1\mathbf{v}_{B_4/B_2}|^2}{14.24} = \frac{8.802^2}{14.24} = 5.44 \text{ in} $$

The direction of E is shown on the drawing. Note that the location of E does not depend on the acceleration of link 2.

Problem 3.15

In the mechanism below, the velocity and acceleration of Point *B* are given. Determine the angular velocity and acceleration of Links 3 and 4. On the velocity and acceleration diagrams, locate the velocity and acceleration of Point E on Link 3.



Position Analysis

Draw the linkage to scale. Locate both pivots and start with link 2. Locate point B and draw line BC. Then locate point D. Construct point E on a line perpendicular to the line BD.

Velocity Analysis:

$$ {}^1\boldsymbol{v}_{B_3} = {}^1\boldsymbol{v}_{B_2} $$

$$ {}^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{C_4} + {}^1\boldsymbol{v}_{C_3/C_4} $$

$$ {}^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{B_3} + {}^1\boldsymbol{v}_{C_3/B_3} \qquad (1) $$

Now,

$${}^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{C_3/C_4} \text{ in the direction of } \boldsymbol{r}_{C_3/B_3}$$

$${}^1\boldsymbol{v}_{C_3/B_3} = {}^1\omega_3 \times \boldsymbol{r}_{C_3/B_3} \Rightarrow \left|{}^1\boldsymbol{v}_{C_3/B_3}\right| = \left|{}^1\omega_3\right| \cdot \left|\boldsymbol{r}_{C_3/B_3}\right| \ (\perp \text{ to } \boldsymbol{r}_{C_3/B_3})$$

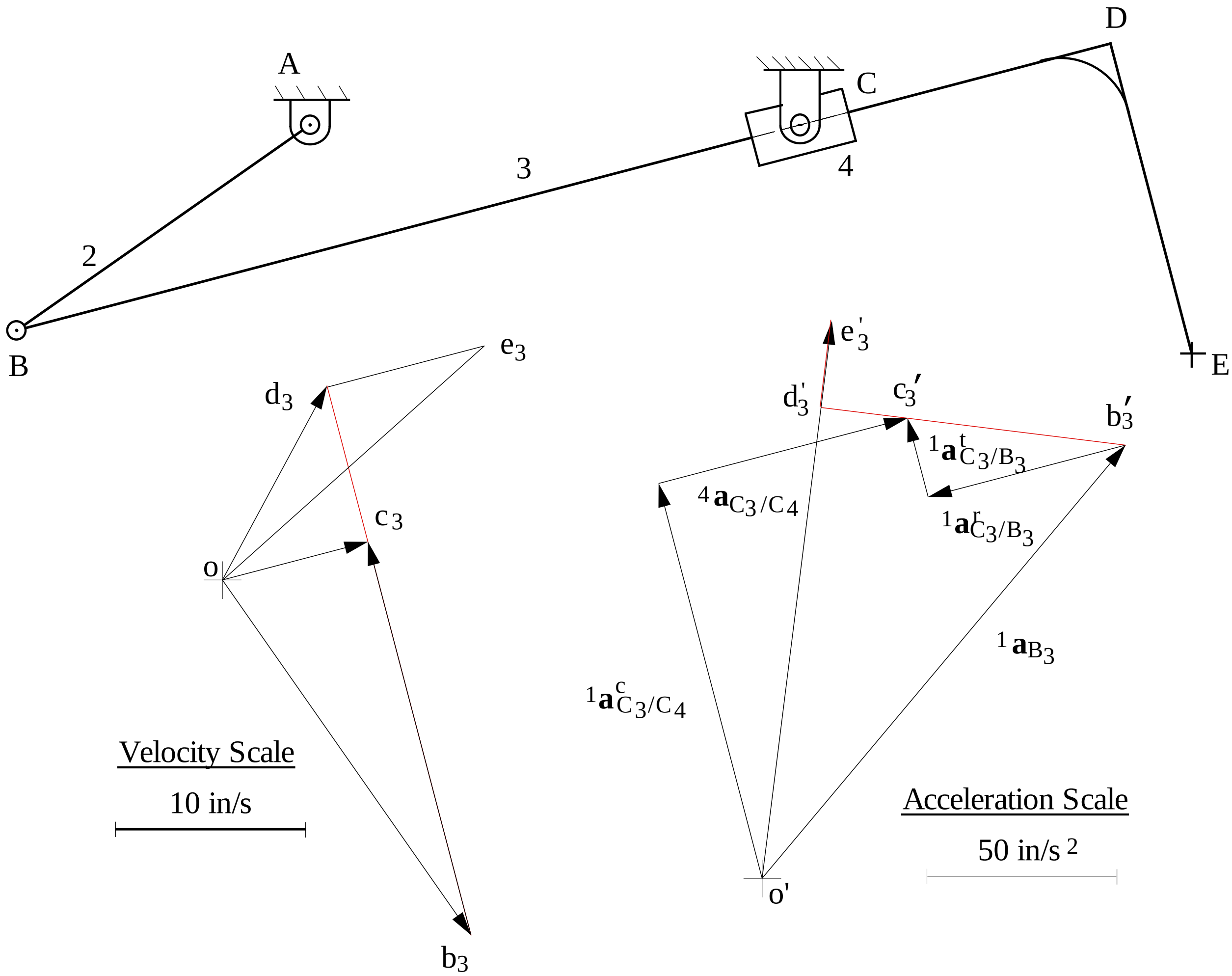
Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$${}^1\boldsymbol{v}_{C_3/B_3} = 21.6 \text{ in/s}$$

or

$$\left|{}^1\omega_3\right| = \frac{\left|{}^1\boldsymbol{v}_{C_3/B_3}\right|}{\left|\boldsymbol{r}_{C_3/B_3}\right|} = \frac{21.6}{8.59} = 2.51 \text{ rad/s CCW}$$

Also,



$${}^1\omega_3 = {}^1\omega_4$$

The velocity of E_3 can be found by image. The magnitude is

$${}^1\boldsymbol{v}_{E_3} = 18.6 \text{ in/s}$$

Acceleration Analysis:

$$ {}^1\boldsymbol{a}_{B_2} = {}^1\boldsymbol{a}_{B_3} $$

$$ {}^1\boldsymbol{a}_{C_3} = {}^1\boldsymbol{a}_{C_4} + {}^1\boldsymbol{a}_{C_3/C_4} $$

$$ {}^1\boldsymbol{a}_{C_3} = {}^4\boldsymbol{a}_{C_3/C_4} + 2 \cdot {}^1\omega_4 \times {}^4\boldsymbol{v}_{C_3/C_4} $$

$$ {}^1\boldsymbol{a}_{C_3} = {}^1\boldsymbol{a}_{B_3} + {}^1\boldsymbol{a}_{C_3/B_3} $$

$$ {}^4\boldsymbol{a}_{C_3/C_4} + 2 \cdot {}^1\omega_4 \times {}^4\boldsymbol{v}_{C_3/C_4} = {}^1\boldsymbol{a}_{B_3} + {}^1\boldsymbol{a}^r_{C_3/B_3} + {}^1\boldsymbol{a}^t_{C_3/B_3} \qquad (2) $$

Now,

$$ {}^4\boldsymbol{a}_{C_3/C_4} \text{ in the direction of } \boldsymbol{r}_{C_3/B_3} $$

$$ 2 \cdot {}^1\omega_4 \times {}^1\boldsymbol{v}_{C_3/C_4} = 2 \cdot 2.51 \cdot 21.6 = 108 \text{ in}/\text{s}^2 $$

$$ {}^1\boldsymbol{a}^r_{C_3/B_3} = {}^1\boldsymbol{\omega}_3 \times \left({}^1\boldsymbol{\omega}_3 \times \boldsymbol{r}_{C_3/B_3}\right) \Rightarrow \left|{}^1\boldsymbol{a}^r_{C_3/B_3}\right| = \left|{}^1\boldsymbol{\omega}_3\right|^2 \cdot \left|\boldsymbol{r}_{C_3/B_3}\right| = 2.51^2 \cdot 8.59 = 54.1 \text{ in}/\text{s}^2 $$

$$ {}^1\boldsymbol{a}^t_{C_3/B_3} = {}^1\boldsymbol{\alpha}_3 \times \boldsymbol{r}_{C_3/B_3} \Rightarrow \left|{}^1\boldsymbol{a}^t_{C_3/B_3}\right| = \left|{}^1\boldsymbol{\alpha}_3\right| \cdot \left|\boldsymbol{r}_{C_3/B_3}\right| \left(\perp \textit{ to } \boldsymbol{r}_{C_3/B_3}\right) $$

Solve Eq. (2) graphically with an acceleration polygon. From the polygon,

$$ {}^1\boldsymbol{a}^t_{C_3/B_3} = 43.2 \; in/s^2 $$

or

$$ \left|{}^1\alpha_3\right| = \frac{\left|{}^1\boldsymbol{a}^t_{C_3/B_3}\right|}{\left|\boldsymbol{r}_{C_3/B_3}\right|} = \frac{43.2}{8.59} = 5.03 \; rad/s^2 $$

Also,

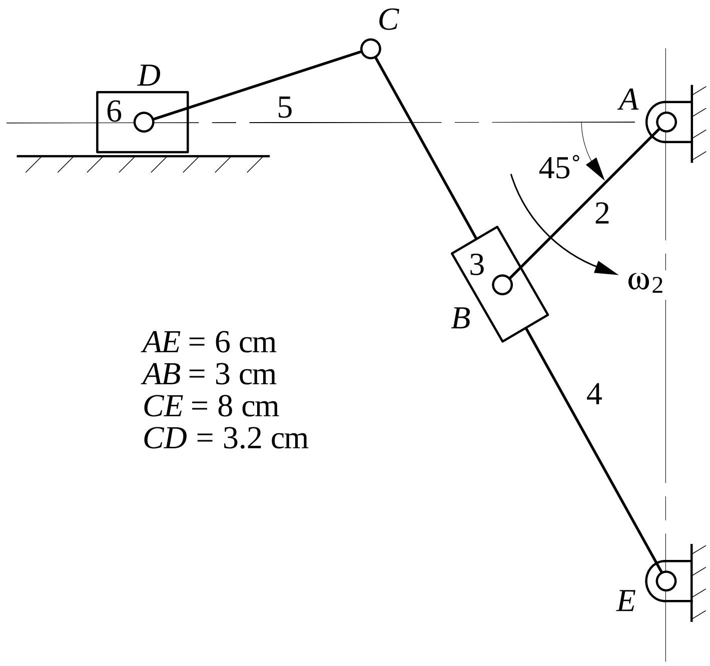
$$ {}^1\boldsymbol{\alpha}_3 = {}^1\boldsymbol{\alpha}_4 $$

The acceleration of point E_3 is given by image. The magnitude is

$$ {}^1\boldsymbol{a}_{E_3} = 149 \text{ in}/\text{s}^2 $$

Problem 3.16

In the figure below, ω_2 = 500 rad/s CCW (constant). Find ω_4, ${}^2\omega_4$, ω_3, ${}^6\omega_5$, ${}^3\omega_5$, $\boldsymbol{v}_D$, α_4 ${}^2\alpha_4$, α_3, ${}^6\alpha_5$, and $\boldsymbol{a}_D$.



AE = 6 cm
AB = 3 cm
CE = 8 cm
CD = 3.2 cm

Position Analysis

Locate the relative position of points A and E and the line of motion of point D. Next locate point B. Draw the line EB and locate point C. Then locate point D by drawing an arc centered at C and with a radius of 3.2.

Velocity Analysis:

$${}^1\boldsymbol{v}_{B_3} = {}^1\boldsymbol{v}_{B_2} = {}^1\boldsymbol{v}_{B_2/A_2} = {}^1\boldsymbol{v}_{B_4} + {}^1\boldsymbol{v}_{B_2/B_4} \tag{1}$$

$${}^1\boldsymbol{v}_{B_4} = {}^1\boldsymbol{v}_{B_4/E_4}$$

$${}^1\boldsymbol{v}_{D_5} = {}^1\boldsymbol{v}_{D_6} = {}^1\boldsymbol{v}_{C_5} + {}^1\boldsymbol{v}_{D_5/C_5} \tag{2}$$

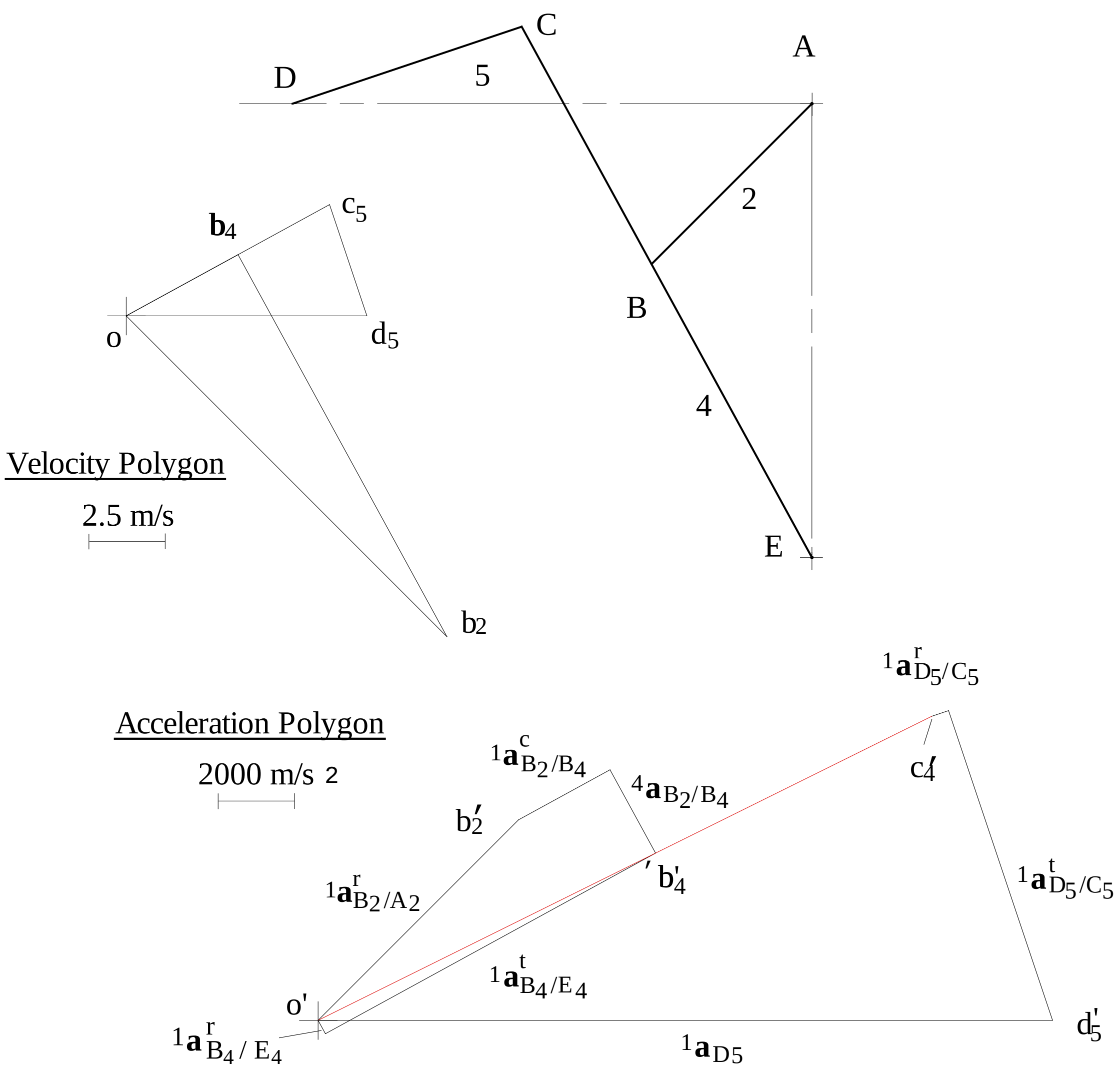
$${}^1\boldsymbol{v}_{C_5} = {}^1\boldsymbol{v}_{C_4} = {}^1\boldsymbol{v}_{C_4/E_4}$$

Now,

$${}^1\boldsymbol{v}_{B_2/A_2} = {}^1\omega_2 \times \boldsymbol{r}_{B_2/A_2} \Rightarrow \left|{}^1\boldsymbol{v}_{B_2/A_2}\right| = \left|{}^1\omega_2\right| \cdot \left|\boldsymbol{r}_{B_2/A_2}\right| = 500 \cdot 0.03 = 15\, m/s \ (\perp \text{ to } \boldsymbol{r}_{B_2/A_2})$$

$${}^1\boldsymbol{v}_{B_4/E_4} = {}^1\omega_4 \times \boldsymbol{r}_{B_4/E_4} \Rightarrow \left|{}^1\boldsymbol{v}_{B_4/E_4}\right| = \left|{}^1\omega_4\right| \cdot \left|\boldsymbol{r}_{B_4/E_4}\right| \ (\perp \text{ to } \boldsymbol{r}_{B_4/E_4})$$

${}^1\boldsymbol{v}_{B_2/B_4}$ in the direction of $\boldsymbol{r}_{B_4/E_4}$



Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$$ {}^1\boldsymbol{v}_{B_2/B_4} = 14.4 \text{ m/s} $$

Also,

$$ {}^1\boldsymbol{v}_{B_4/E_4} = 4.22\, m / \text{sec} $$

or

$$ \left|{}^1\omega_4\right| = \frac{\left|{}^1\boldsymbol{v}_{B_4/E_4}\right|}{\left|\boldsymbol{r}_{B_4/E_4}\right|} = \frac{4.22}{0.044} = 95.9 \text{ rad/s CW} $$

Also,

$$ {}^1\boldsymbol{\omega}_3 = {}^1\boldsymbol{\omega}_4 $$

Now,

$$ {}^1\boldsymbol{v}_{C_4/E_4} = {}^1\omega_4 \times \boldsymbol{r}_{C_4/E_4} \Rightarrow \left|{}^1\boldsymbol{v}_{C_4/E_4}\right| = \left|{}^1\omega_4\right| \cdot \left|\boldsymbol{r}_{C_4/E_4}\right| = 95.8 \cdot 0.08 = 7.66 \text{ m/s } (\perp \text{ to } \boldsymbol{r}_{C_4/E_4}) $$

$$ {}^1\boldsymbol{v}_{D_5} \text{ in horizontal direction} $$

$$ {}^1\boldsymbol{v}_{D_5/C_5} = {}^1\omega_5 \times \boldsymbol{r}_{D_5/C_5} \Rightarrow \left|{}^1\boldsymbol{v}_{D_5/C_5}\right| = \left|{}^1\omega_5\right| \cdot \left|\boldsymbol{r}_{D_5/C_5}\right| \; (\perp \text{ to } \boldsymbol{r}_{D_5/C_5}) $$

Solve Eq. (2) graphically with a velocity polygon. From the polygon,

$$ {}^1\boldsymbol{v}_{D_5/C_5} = 3.88 \text{ m/s} $$

or

$$ \left|{}^1\omega_5\right| = \frac{\left|{}^1\boldsymbol{v}_{D_5/C_5}\right|}{\left|\boldsymbol{r}_{D_5/C_5}\right|} = \frac{3.88}{0.032} = 121 \text{ rad/s CCW} $$

And,

$$ {}^1\boldsymbol{v}_{D_5} = {}^1\boldsymbol{v}_{D_6} = 3.18 \text{ m/s} $$

For the relative angular velocities,

$$ {}^2\omega_4 = {}^1\omega_4 - {}^1\omega_2 = 95.8\, CW - 500\, CCW = 595.8\, CW $$

$$ {}^6\omega_5 = {}^1\omega_5 - {}^1\omega_6 = 121\, CCW - 0 = 121\, CCW $$

$$ {}^3\omega_5 = {}^1\omega_5 - {}^1\omega_3 = 121\, CCW - 95.8\, CW = 216.8\, CCW $$

<u>Acceleration Analysis:</u>

$$ {}^1\boldsymbol{a}_{B_3} = {}^1\boldsymbol{a}_{B_2} = {}^1\boldsymbol{a}_{B_2/A_2} = {}^1\boldsymbol{a}_{B_4} + {}^1\boldsymbol{a}_{B_2/B_4} $$

$$ {}^1\boldsymbol{a}_{B_4} = {}^1\boldsymbol{a}_{B_4/E_4} $$

$$ {}^1\boldsymbol{a}^r_{B_2/A_2} + {}^1\boldsymbol{a}^t_{B_2/A_2} = {}^1\boldsymbol{a}^r_{B_4/E_4} + {}^1\boldsymbol{a}^t_{B_4/E_4} + {}^4\boldsymbol{a}_{B_2/B_4} + 2 \cdot {}^1\omega_4 \times {}^4\boldsymbol{v}_{B_2/B_4} \tag{3} $$

$$ {}^1\boldsymbol{a}_{D_5} = {}^1\boldsymbol{a}_{D_6} = {}^1\boldsymbol{a}_{C_5} + {}^1\boldsymbol{a}_{D_5/C_5} $$

$$ {}^1\boldsymbol{a}_{C_5} = {}^1\boldsymbol{a}_{C_4} = {}^1\boldsymbol{a}_{C_4/E_4} $$

$$ {}^1\boldsymbol{a}_{D_5} = {}^1\boldsymbol{a}^r_{C_4/E_4} + {}^1\boldsymbol{a}^t_{C_4/E_4} + {}^1\boldsymbol{a}^r_{D_5/C_5} + {}^1\boldsymbol{a}^t_{D_5/C_5} \tag{4} $$

Now,

$$ {}^1\boldsymbol{a}^r_{B_2/A_2} = {}^1\omega_2 \times ({}^1\omega_2 \times \boldsymbol{r}_{B_2/A_2}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{B_2/A_2}\right| = \left|{}^1\omega_2\right|^2 \cdot \left|\boldsymbol{r}_{B_2/A_2}\right| = 500^2 \cdot 0.03 = 7500 \text{ m/s}^2 $$

in the direction opposite to $\boldsymbol{r}_{B_2/A_2}$

$$ {}^1\boldsymbol{a}^t_{B_2/A_2} = {}^1\alpha_2 \times \boldsymbol{r}_{B_2/A_2} \Rightarrow \left|{}^1\boldsymbol{a}^t_{B_2/A_2}\right| = \left|{}^1\alpha_2\right| \cdot \left|\boldsymbol{r}_{B_2/A_2}\right| = 0 $$

$$ {}^1\boldsymbol{a}^r_{B_4/E_4} = {}^1\omega_4 \times ({}^1\omega_4 \times \boldsymbol{r}_{B_4/E_4}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{B_4/E_4}\right| = \left|{}^1\omega_4\right|^2 \cdot \left|\boldsymbol{r}_{B_4/E_4}\right| = 95.8^2 \cdot 0.044 = 404 \text{ m/s}^2 $$

in the direction opposite to $\boldsymbol{r}_{B_4/E_4}$

$$ {}^1\boldsymbol{a}^t_{B_4/E_4} = {}^1\alpha_4 \times \boldsymbol{r}_{B_4/E_4} \Rightarrow \left|{}^1\boldsymbol{a}^t_{B_4/E_4}\right| = \left|{}^1\alpha_4\right| \cdot \left|\boldsymbol{r}_{B_4/E_4}\right| \; (\perp \text{ to } \boldsymbol{r}_{B_4/E_4}) $$

${}^4\boldsymbol{a}_{B_2/B_4}$ in the direction opposite to $\boldsymbol{r}_{B_4/E_4}$

$${}^1\boldsymbol{a}^c_{B_2/B_4} = 2\,{}^1\omega_4 \times {}^4\boldsymbol{v}_{B_2/B_4} = 2\cdot 95.8\cdot 14.4 = 2760 \text{ m/s}^2 \, (\perp \text{ to } \boldsymbol{r}_{B_4/E_4})$$

Solve Eq. (3) graphically with an acceleration polygon. From the polygon,

$${}^1\boldsymbol{a}^t_{B_4/E_4} = 9960 \text{ m/s}^2$$

or

$$\left|{}^1\alpha_4\right| = \frac{\left|{}^1\boldsymbol{a}^t_{B_4/E_4}\right|}{\left|\boldsymbol{r}_{B_4/E_4}\right|} = \frac{9960}{0.044} = 226{,}000 \text{ rad/s}^2 \text{ CW}$$

Also,

$${}^2\alpha_4 = {}^1\alpha_4 - {}^1\alpha_2 = 226{,}000 - 0 = 226{,}000 \text{ rad/s}^2 \text{ CW}$$

And,

$${}^1\alpha_4 = {}^1\alpha_3$$

Also, using acceleration image

$${}^1\boldsymbol{a}_{C_4} = 18{,}100 \text{ m/s}^2$$

Now,

${}^1\boldsymbol{a}_{D_5}$ in horizontal direction

$${}^1\boldsymbol{a}^r_{D_5/C_5} = {}^1\omega_5 \times ({}^1\omega_5 \times \boldsymbol{r}_{D_5/C_5}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{D_5/C_5}\right| = \left|{}^1\omega_5\right|^2 \cdot \left|\boldsymbol{r}_{D_5/C_5}\right| = 121^2 \cdot 0.032 = 470 \text{ m/s}^2$$

in the direction of $-\boldsymbol{r}_{D_5/C_5}$

$${}^1\boldsymbol{a}^t_{D_5/C_5} = {}^1\alpha_5 \times \boldsymbol{r}_{D_5/C_5} \Rightarrow \left|{}^1\boldsymbol{a}^t_{D_5/C_5}\right| = \left|{}^1\alpha_5\right| \cdot \left|\boldsymbol{r}_{D_5/C_5}\right| \, (\perp \text{ to } \boldsymbol{r}_{D_5/C_5})$$

Solve Eq. (4) graphically with an acceleration polygon. From the polygon,

$${}^1\boldsymbol{a}^t_{D_5/C_5} = 8640 \text{ m/s}^2$$

or

$$\left|{}^1\alpha_5\right| = \frac{\left|{}^1\boldsymbol{a}^t_{D_5/C_5}\right|}{\left|\boldsymbol{r}_{D_5/C_5}\right|} = \frac{8640}{0.032} = 270{,}000 \text{ rad/s}^2 \text{ CCW}$$

Also,

$${}^6\alpha_5 = {}^1\alpha_5 - {}^1\alpha_6 = 270{,}000\ CCW - 0 = 270{,}000 \text{ rad/s}^2 \text{ CCW}$$

And,

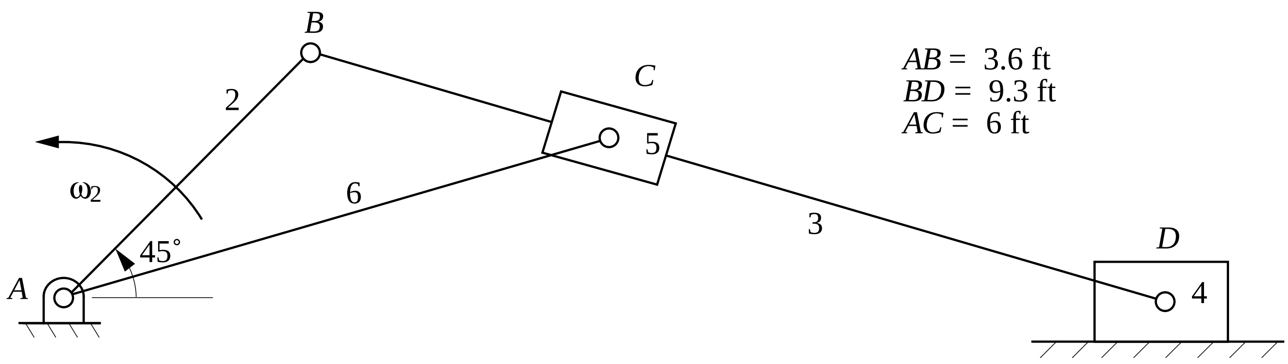
$${}^3\alpha_5 = {}^1\alpha_5 - {}^1\alpha_3 = 270{,}000\ CCW - 226{,}000 CW = 496{,}000 \text{ rad/s}^2 \text{ CCW}$$

Also,

$${}^1\boldsymbol{a}_{D_5} = 19{,}400 \text{ m/s}^2$$

Problem 3.17

In the mechanism below, the angular velocity of Link 2 is 60 rpm CCW (constant). Determine the acceleration of Point C_6 and the angular velocity of Link 6.



Position Analysis

First locate pivot A and the line of action (through A) of the slider. Next draw link 2 and locate point B. Then locate point D and finally locate point C.

Velocity Analysis:

$${}^1\boldsymbol{v}_{B_2} = {}^1\boldsymbol{v}_{B_2/A_2} = {}^1\boldsymbol{v}_{B_3} = {}^1\boldsymbol{\omega}_2 \times \boldsymbol{r}_{B/A}$$

$${}^1\boldsymbol{v}_{D_3} = {}^1\boldsymbol{v}_{D_4} = {}^1\boldsymbol{v}_{B_3} + {}^1\boldsymbol{v}_{D_3/B_3} \tag{1}$$

Compute ${}^1\boldsymbol{v}_{C_3}$ by inage.

$${}^1\boldsymbol{v}_{C_6} = {}^1\boldsymbol{v}_{C_5} = {}^1\boldsymbol{v}_{C_6/A_6} = {}^1\boldsymbol{v}_{C_3} + {}^1\boldsymbol{v}_{C_6/C_3} \tag{2}$$

Now,

$$\left|{}^1\boldsymbol{\omega}_2\right| = 60\, rpm = 60\frac{2\pi}{60}\, rad/s = 6.28\ rad/s$$

$$\left|{}^1\boldsymbol{v}_{B_3}\right| = \left|{}^1\boldsymbol{\omega}_2\right|\left|\boldsymbol{r}_{B/A}\right| = 6.28 \cdot 3.6 = 22.6 \text{ ft/s} (\perp \text{ to } \boldsymbol{r}_{B/A})$$

${}^1\mathbf{v}_{D3}$ in horizontal direction

$${}^1\boldsymbol{v}_{D_3/B_3} = {}^1\boldsymbol{\omega}_3 \times \boldsymbol{r}_{D/B} \Rightarrow \left|{}^1\boldsymbol{v}_{D_3/B_3}\right| = \left|{}^1\boldsymbol{\omega}_3\right|\left|\boldsymbol{r}_{D/B}\right| (\perp \text{ to } \boldsymbol{r}_{D/B})$$

$${}^1\boldsymbol{v}_{C_6/A_6} = {}^1\boldsymbol{\omega}_6 \times \boldsymbol{r}_{C/A} \Rightarrow \left|{}^1\boldsymbol{v}_{C_6/A_6}\right| = \left|{}^1\boldsymbol{\omega}_6\right|\left|\boldsymbol{r}_{C/A}\right| (\perp \text{ to } \boldsymbol{r}_{C/A})$$

${}^1\boldsymbol{v}_{C_6/C_3}$ along BD

Solve Eqs. (1) and (2) graphically with a velocity polygon. From the polygon,

$${}^1\boldsymbol{v}_{C_6} = 5.85 \text{ ft/s}$$

$${}^1\boldsymbol{v}_{D_3/B_3} = 16.5 \text{ ft/s}$$

$${}^1\boldsymbol{v}_{C_6/C_3} = 16.7 \text{ ft/s}$$

and

$$\left|{}^1\omega_3\right| = \frac{\left|{}^1\boldsymbol{v}_{D_3/B_3}\right|}{\left|\boldsymbol{r}_{D/B}\right|} = \frac{16.5}{9.3} = 1.77 \ \text{rad / s CW}$$

<u>Acceleration Analysis:</u>

$${}^1\boldsymbol{a}_{B_2} = {}^1\boldsymbol{a}_{B_2/A_2} = {}^1\boldsymbol{a}_{B_3} = {}^1\boldsymbol{a}^r_{B_2/A_2} + {}^1\boldsymbol{a}^t_{B_2/A_2}$$

$${}^1\boldsymbol{a}_{D_3} = {}^1\boldsymbol{a}_{D_4} = {}^1\boldsymbol{a}_{B_3} + {}^1\boldsymbol{a}_{D_3/B_3} = {}^1\boldsymbol{a}_{B_3} + {}^1\boldsymbol{a}^r_{D_3/B_3} + {}^1\boldsymbol{a}^t_{D_3/B_3} \tag{3}$$

Compute ${}^1\boldsymbol{a}_{C_3}$ by image.

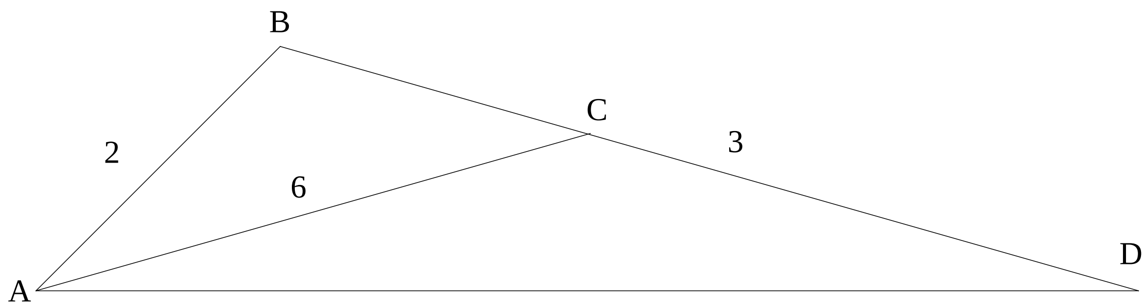
$${}^1\boldsymbol{a}_{C_6} = {}^1\boldsymbol{a}_{C_5} = {}^1\boldsymbol{a}_{C_6/A_6} = {}^1\boldsymbol{a}_{C_3} + {}^1\boldsymbol{a}_{C_5/C_3}$$

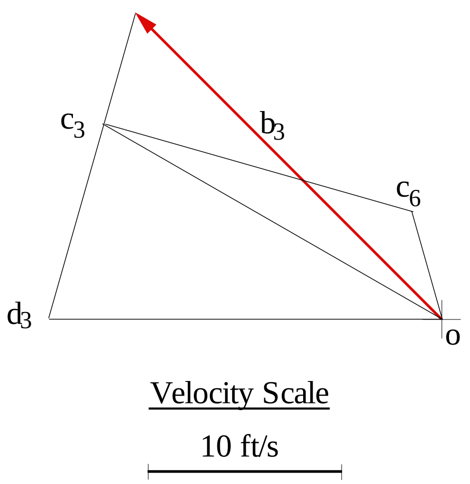
or

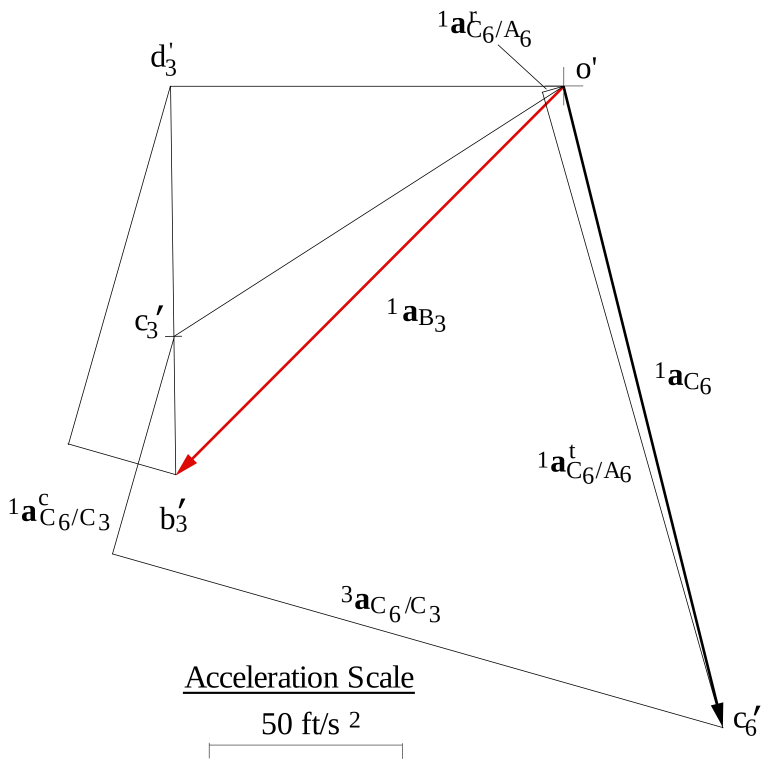
$${}^1\boldsymbol{a}^r_{C_6/A_6} + {}^1\boldsymbol{a}^t_{C_6/A_6} = {}^1\boldsymbol{a}_{C_3} + {}^1\boldsymbol{a}^c_{C_5/C_3} + {}^3\boldsymbol{a}^t_{C_5/C_3} + {}^3\boldsymbol{a}^n_{C_5/C_3} \tag{4}$$

Now,

${}^1\boldsymbol{a}_{D_3}$ in horizontal direction







$${}^1\boldsymbol{a}^r_{B_2/A_2} = {}^1\omega_2 \times ({}^1\omega_2 \times \boldsymbol{r}_{B/A}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{B_2/A_2}\right| = \left|{}^1\omega_2\right|^2 \cdot \left|\boldsymbol{r}_{B/A}\right| = 6.28^2 \cdot 3.6 = 142 \text{ ft/s}^2$$

in the direction opposite to $\boldsymbol{r}_{B/A}$

$${}^1\boldsymbol{a}^t_{B_2/A_2} = {}^1\alpha_2 \times \boldsymbol{r}_{B/A} \Rightarrow \left|{}^1\boldsymbol{a}^t_{B_2/A_2}\right| = \left|{}^1\alpha_2\right| \cdot \left|\boldsymbol{r}_{B/A}\right| = 0 \cdot 3.6 = 0 \text{ ft/s}^2$$

$${}^1\boldsymbol{a}^t_{D_3/B_3} = {}^1\alpha_3 \times \boldsymbol{r}_{D/B} \Rightarrow \left|{}^1\boldsymbol{a}^t_{D_3/B_3}\right| = \left|{}^1\alpha_2\right| \cdot \left|\boldsymbol{r}_{A/D}\right| (\perp \text{ to } \boldsymbol{r}_{A/D})$$

$${}^1\boldsymbol{a}^r_{D_3/B_3} = {}^1\omega_2 \times ({}^1\omega_2 \times \boldsymbol{r}_{D/B}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{D_3/B_3}\right| = \frac{\left|{}^1\boldsymbol{v}_{D_3/B_3}\right|^2}{\left|\boldsymbol{r}_{D/B}\right|} = \frac{16.46^2}{9.3} = 29.1 \text{ ft/s}^2$$

in the direction opposite to $\boldsymbol{r}_{D/B}$

$${}^1\boldsymbol{a}^t_{C_6/A_6} = {}^1\alpha_6 \times \boldsymbol{r}_{C/A} \Rightarrow \left|{}^1\boldsymbol{a}^t_{C_6/A_6}\right| = \left|{}^1\alpha_6\right| \cdot \left|\boldsymbol{r}_{C/A}\right| (\perp \text{ to } \boldsymbol{r}_{C/A})$$

$$ {}^1\boldsymbol{a}^r_{C_6/A_6} = {}^1\boldsymbol{\omega}_6 \times ({}^1\boldsymbol{\omega}_6 \times \boldsymbol{r}_{C/A}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{C_6/A_6}\right| = \frac{\left|{}^1\boldsymbol{v}_{C_6/A_6}\right|^2}{\left|\boldsymbol{r}_{C/A}\right|} = \frac{5.85^2}{6} = 5.70 \ \text{ft}/\text{s}^2 $$

in the direction opposite to $\boldsymbol{r}_{C/A}$

$$ {}^1\boldsymbol{a}^c_{C_5/C_3} = 2 \cdot {}^1\boldsymbol{\omega}_3 \times {}^3\boldsymbol{v}_{C_5/C_3} \Rightarrow \left|{}^1\boldsymbol{a}^c_{C_5/C_3}\right| = 2\left|{}^1\boldsymbol{\omega}_3\right| \cdot \left|{}^3\boldsymbol{v}_{C_5/C_3}\right| = 2(1.77)(16.66) = 58.97 \text{ ft}/\text{s}^2 $$

in the direction perpendicular to BD and in the direction obtained by rotating ${}^3\boldsymbol{v}_{C_5/C_3}$ 90˚ in the direction of ${}^1\boldsymbol{\omega}_3$. The direction is shown on the acceleration polygon.

${}^3\boldsymbol{a}^t_{C_5/C_3}$ is along the slide (line BD)

$$ \left|{}^3\boldsymbol{a}^n_{C_5/C_3}\right| = \frac{\left|{}^1\boldsymbol{v}_{C_5/C_3}\right|}{\infty} = 0 $$

Solve Eqs. (3) and (4) graphically with an acceleration polygon. From the polygon,

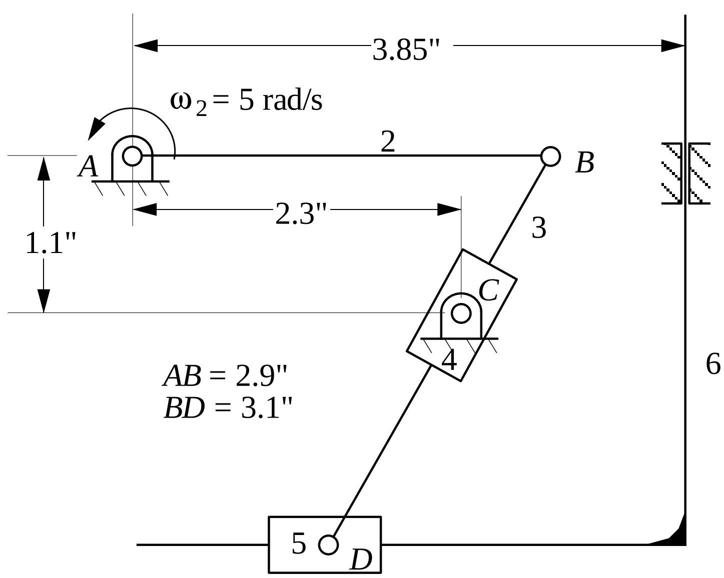
$$ {}^1\boldsymbol{a}_{C_6} = 172 \text{ ft}/\text{s}^2 $$

Also,

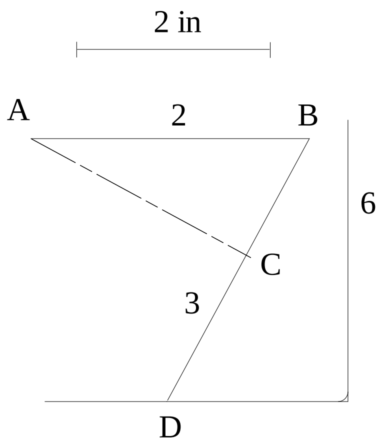
$$ \left|{}^1\alpha_6\right| = \frac{\left|{}^1\boldsymbol{a}^t_{C_6/A_6}\right|}{\left|\boldsymbol{r}_{C/A}\right|} = \frac{171}{6} = 28.5 \ \text{rad}/\text{s}^2 \text{ CW} $$

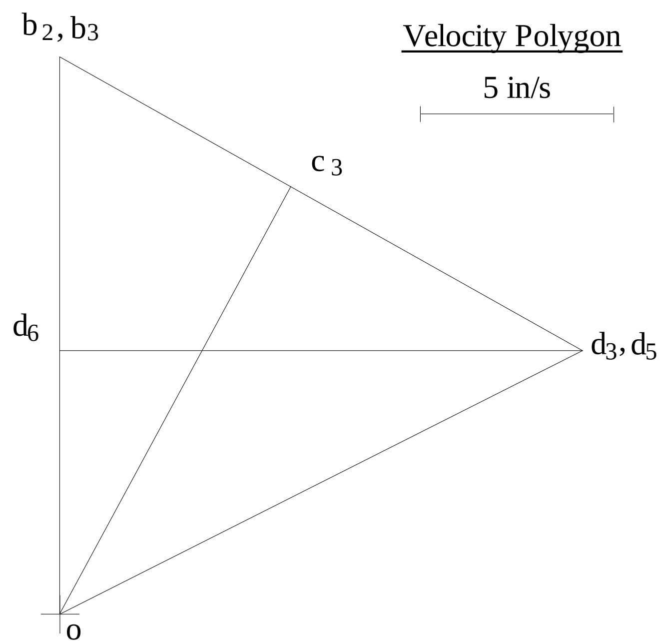
Problem 3.18

In the position shown AB is horizontal. Draw the velocity diagram to determine the sliding velocity of link 6. Determine a new position for point C (between B and D) so that the velocity of link 6 would be equal and opposite to the one calculated for the original position of point C.



Position Analysis: Draw the linkage to scale.





Velocity Analysis:

The equations needed for the analysis are:

$$ {}^1\boldsymbol{v}_{B_2} = {}^1\boldsymbol{v}_{B_2/A_2} = {}^1\omega_2 \times \boldsymbol{r}_{B_2/A_2} \Rightarrow \left|{}^1\boldsymbol{v}_{B_2}\right| = \left|{}^1\omega_2\right|\left|\boldsymbol{r}_{B_2/A_2}\right| = 5(2.9) = 14.5 \text{ in / s} $$

$$ {}^1\boldsymbol{v}_{B_3} = {}^1\boldsymbol{v}_{B_2} $$

$$ {}^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{B_3} + {}^1\boldsymbol{v}_{C_3/B_3} \qquad (1) $$

$$ {}^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{C_4} + {}^1\boldsymbol{v}_{C_3/C_4} $$

$$ {}^1\boldsymbol{v}_{C_4} = 0 $$

$$ {}^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{C_3/C_4} = {}^4\boldsymbol{v}_{C_3/C_4} = {}^4\boldsymbol{v}_{C_3} $$

$$ {}^1\boldsymbol{v}_{D_3} = {}^1\boldsymbol{v}_{D_5} = {}^1\boldsymbol{v}_{B_3} + {}^1\boldsymbol{v}_{D_3/B_3} $$

$$ {}^1\boldsymbol{v}_{D_5} = {}^1\boldsymbol{v}_{D_6} + {}^1\boldsymbol{v}_{D_5/D_6} \qquad (2) $$

Now,

$$ {}^1\boldsymbol{v}_{B_2} = 14.5 \text{ in / s} \quad (\perp \text{ to } \boldsymbol{r}_{B/A}) $$

$$ {}^1\boldsymbol{v}_{C_3/B_3} = {}^1\boldsymbol{\omega}_3 \times \boldsymbol{r}_{C/B} \quad (\perp \text{ to } \boldsymbol{r}_{C/B}) $$

${}^1\boldsymbol{v}_{C_3}$ is on line of $\boldsymbol{r}_{C/B}$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$$^1\boldsymbol{v}_{C_3/B_3} = 6.90 \text{ in/s}$$

or

$$\left|{}^1\omega_3\right| = \frac{\left|{}^1\boldsymbol{v}_{C_3/B_3}\right|}{\left|\boldsymbol{r}_{C/B}\right|} = \frac{6.90}{1.37} = 5.04 \text{ rad/s}$$

From the directions given in the position and velocity polygons

$$^1\omega_3 = 5.04 \text{ rad/s CCW}$$

Also,

$$^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{C_3/C_4} = 12.6 \text{ in/s}$$

Using velocity image theorem,

$$^1\boldsymbol{v}_{D_3} = {}^1\boldsymbol{v}_{D_5} = 15.25 \text{ in/s}$$

Now,

$^1\boldsymbol{v}_{D_6}$ is on the vertical axis,

$^1\boldsymbol{v}_{D_5/D_6}$ is on the horizantal axis.

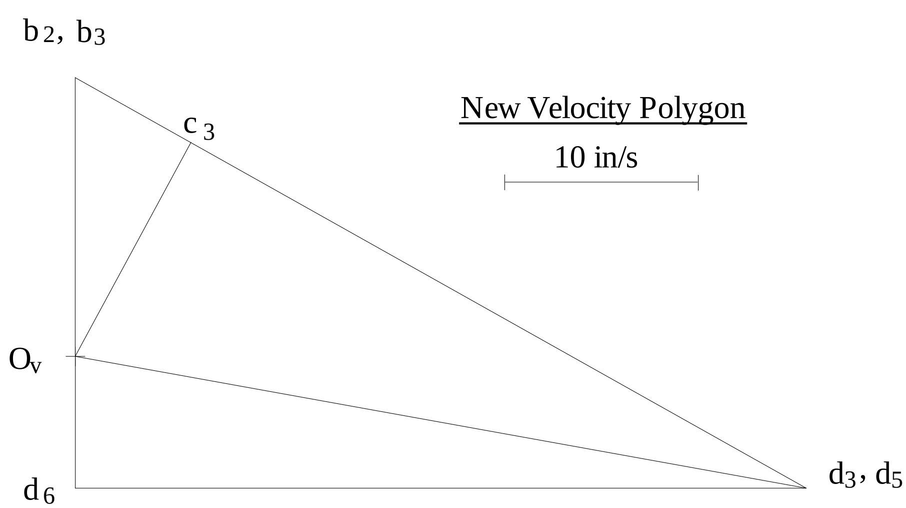
Solve Eq. (2) graphically with a velocity polygon. From the polygon,

$$^1\boldsymbol{v}_{D_6} = 6.85 \text{ in/s}$$

also,

$$^1\boldsymbol{v}_{D_5/D_6} = 13.62 \text{ in/s}$$

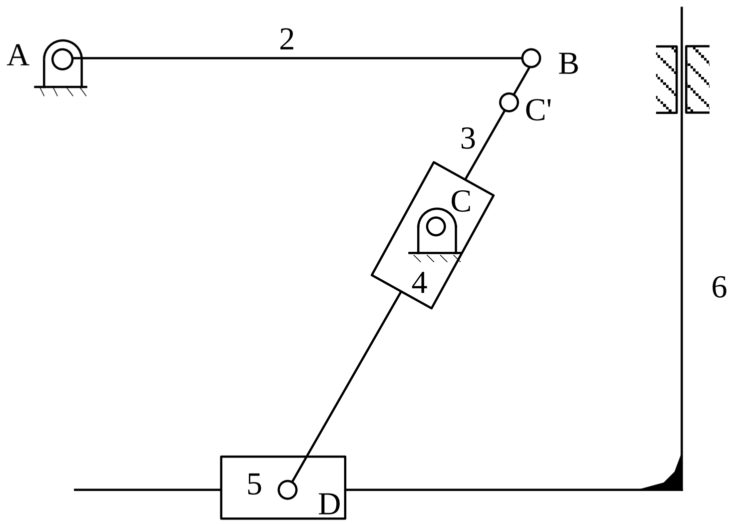
To find the new location of point *C* which will make the velocity of link 6 change signs, plot a new velocity polygon with the velocity of d_6 in the direction indicated.



using velocity image theorem,

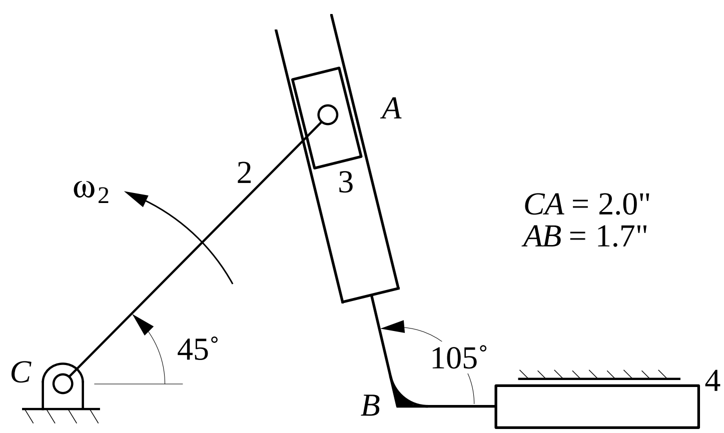
$$|\mathbf{r}_{C/B}| = |\mathbf{r}_{D/B}| \cdot \frac{|{}^1\mathbf{v}_{C_3/B_3}|}{|{}^1\mathbf{v}_{D_3/B_3}|} = (3.1) \cdot \frac{6.90}{43.7} = 0.48 \text{ in}$$

The location of C is shown on the following figure.

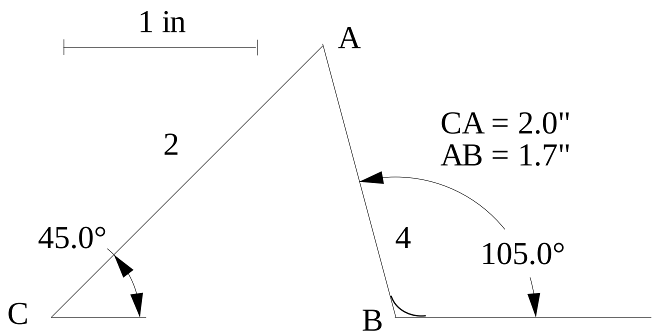


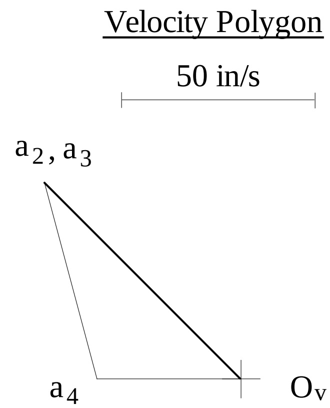
Problem 3.19

The scotch-yoke mechanism is driven by crank 2 at ω_2 = 36 rad/s (CCW). Link 4 slides horizontally. Find the velocity of point *B* on Link 4.



Position Analysis: Draw the linkage to scale.





Velocity Analysis:

$${}^1\boldsymbol{v}_{A_2} = {}^1\boldsymbol{v}_{A_2/C_2} = {}^1\omega_2 \times \boldsymbol{r}_{A_2/C_2} \Rightarrow \left|{}^1\boldsymbol{v}_{A_2}\right| = \left|{}^1\omega_2\right|\left|\boldsymbol{r}_{A_2/C_2}\right| = 36 \cdot (2) = 72 \text{ in/s}$$

$${}^1\boldsymbol{v}_{A_3} = {}^1\boldsymbol{v}_{A_4} + {}^1\boldsymbol{v}_{A_3/A_4} \qquad (1)$$

$${}^1\boldsymbol{v}_{A_3} = {}^1\boldsymbol{v}_{A_2}$$

Now,

$${}^1\boldsymbol{v}_{A_2} = 72 \text{ in/s} \quad (\perp \text{ to } \boldsymbol{r}_{A/C})$$

${}^1\boldsymbol{v}_{A_3/A_4}$ is along the slider

${}^1\boldsymbol{v}_{A_4}$ moves on a horizontal axis, and because 4 is a rigid body

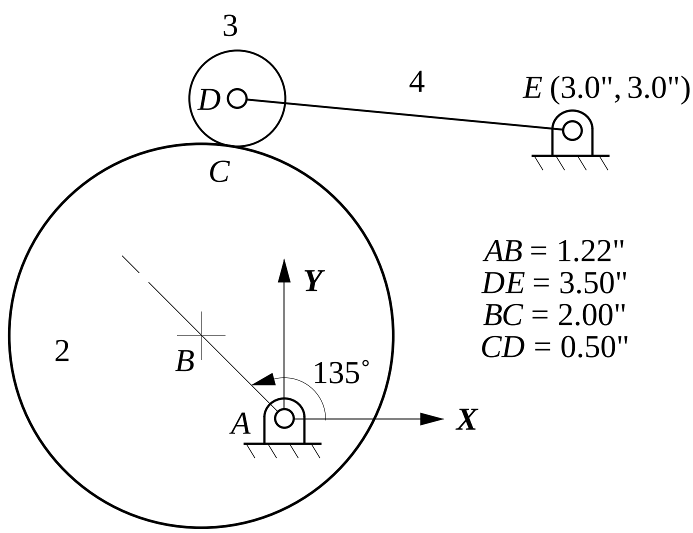
$${}^1\boldsymbol{v}_{A_4} = {}^1\boldsymbol{v}_{B_4}$$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$${}^1\boldsymbol{v}_{A_4} = {}^1\boldsymbol{v}_{B_4} = 37.5 \text{ in/s}$$

Problem 3.20

The circular cam shown is driven at an angular velocity ω_2 = 15 rad/s (CW) and α_2 = 100 rad/s^2 (CW). There is rolling contact between the cam and the roller, link 3. Find the angular velocity and angular acceleration of the oscillating follower, link 4.



Position Analysis:

Draw the linkage to scale. Note that because of rolling contact and because we are to find the velocity and acceleration of link 4 only, we can model the system as a four-bar linkage. If we were asked for the kinematic properties of link 3, we would have to model the system using rolling contact directly.

Velocity Analysis:

$$^1\boldsymbol{v}_{B_2} = {}^1\boldsymbol{v}_{B_2/A_2} = {}^1\omega_2 \times \boldsymbol{r}_{B/A} \Rightarrow \left|{}^1\boldsymbol{v}_{B_2}\right| = \left|{}^1\omega_2\right|\left|\boldsymbol{r}_{B/A}\right| = 15\cdot(1.22) = 18.3 \text{ in / s}$$

$$^1\boldsymbol{v}_{B_5} = {}^1\boldsymbol{v}_{B_2}$$

$$^1\boldsymbol{v}_{D_5} = {}^1\boldsymbol{v}_{B_5} + {}^1\boldsymbol{v}_{D_5/B_5} \tag{1}$$

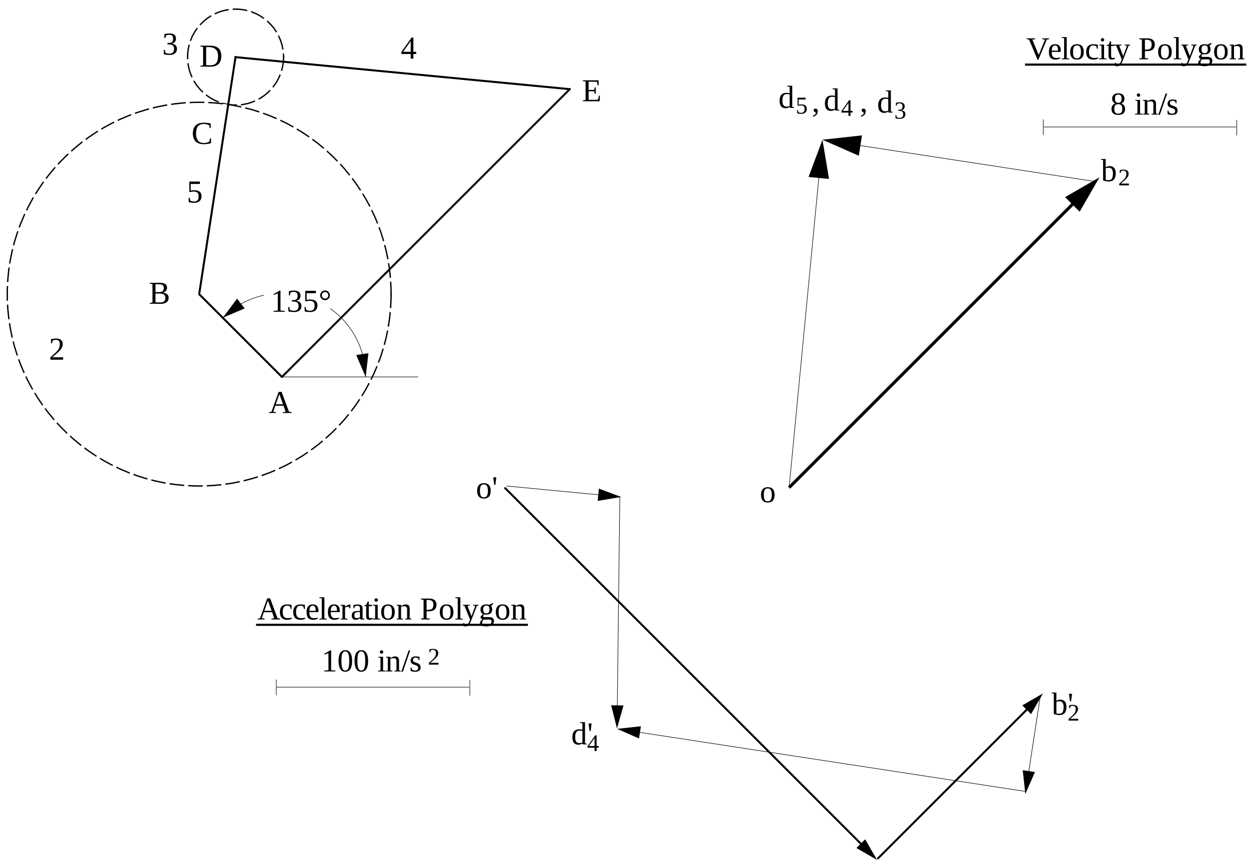
$$^1\boldsymbol{v}_{D_5} = {}^1\boldsymbol{v}_{D_4} = {}^1\omega_4 \times \boldsymbol{r}_{D/E} \Rightarrow \left|{}^1\boldsymbol{v}_{D_4}\right| = \left|{}^1\omega_4\right|\left|\boldsymbol{r}_{D/E}\right|$$

Now,

$$^1\boldsymbol{v}_{D_5/B_5} = {}^1\omega_5 \times \boldsymbol{r}_{D/B} \quad (\perp \text{ to } \boldsymbol{r}_{D/B})$$

$$^1\boldsymbol{v}_{D_5} = {}^1\omega_4 \times \boldsymbol{r}_{D4/E4} \quad (\perp \text{ to } \boldsymbol{r}_{D/E})$$

Solve Eq. (1) graphically with a velocity polygon. The velocity directions can be gotten directly from the polygon. The magnitudes are given by:



$$\left|{}^{1}\boldsymbol{v}_{D_5/B_5}\right| = 11.4 \text{ in/s} \Rightarrow \left|{}^{1}\omega_5\right| = \frac{\left|{}^{1}\boldsymbol{v}_{D_5/B_5}\right|}{\left|\boldsymbol{r}_{D/B}\right|} = \frac{11.4}{2.5} = 4.56 \text{ rad/s}$$

From the directions given in the position and velocity polygons

$${}^{1}\boldsymbol{\omega}_5 = 4.56 \text{ rad/s } CCW$$

Also,

$$\left|{}^{1}\boldsymbol{v}_{D_4/E_4}\right| = 14.6 \text{ in/s} \Rightarrow \left|{}^{1}\omega_4\right| = \frac{\left|{}^{1}\boldsymbol{v}_{D_4/E_4}\right|}{\left|\boldsymbol{r}_{D/E}\right|} = \frac{14.6}{3.5} = 4.17 \text{ rad/s}$$

From the directions given in the position and velocity polygons

$${}^{1}\boldsymbol{\omega}_4 = 4.17 \text{ rad/s } CW$$

Acceleration Analysis:

For the acceleration analysis, use the same points in the same order as was done in the velocity analysis.

$${}^1\boldsymbol{a}_{B_2} = {}^1\boldsymbol{a}_{B_2/A_2} = {}^1\boldsymbol{a}^r_{B_2/A_2} + {}^1\boldsymbol{a}^t_{B_2/A_2}$$

$${}^1\boldsymbol{a}_{B_5} = {}^1\boldsymbol{a}_{B_2}$$

$${}^1\boldsymbol{a}_{D_5} = {}^1\boldsymbol{a}_{B_5} + {}^1\boldsymbol{a}_{D_5/B_5} = {}^1\boldsymbol{a}^r_{B_2/A_2} + {}^1\boldsymbol{a}^t_{B_2/A_2} + {}^1\boldsymbol{a}^r_{D_5/B_5} + {}^1\boldsymbol{a}^t_{D_5/B_5}$$

$${}^1\boldsymbol{a}_{D_5} = {}^1\boldsymbol{a}_{D_4} = {}^1\boldsymbol{a}_{D_4/E_4} = {}^1\boldsymbol{a}^r_{D_4/E_4} + {}^1\boldsymbol{a}^t_{D_4/E_4}$$

Therefore,

$${}^1\boldsymbol{a}^r_{D_4/E_4} + {}^1\boldsymbol{a}^t_{D_4/E_4} = {}^1\boldsymbol{a}^r_{B_2/A_2} + {}^1\boldsymbol{a}^t_{B_2/A_2} + {}^1\boldsymbol{a}^r_{D_5/B_5} + {}^1\boldsymbol{a}^t_{D_5/B_5} \tag{2}$$

Now,

$${}^1\boldsymbol{a}^r_{B_2/A_2} = |{}^1\omega_2|^2 \cdot |r_{B/A}| = 15^2 \cdot 1.22 = 274.5 \text{ in/s}^2$$ in a direction opposite to $\mathbf{r}_{B/A}$.

$${}^1\boldsymbol{a}^r_{D_4/E_4} = |{}^1\omega_4|^2 \cdot |r_{D/E}| = 4.17^2 \cdot 3.5 = 60.8 \text{ in/s}^2$$ in a direction opposite to $\mathbf{r}_{D/E}$.

$${}^1\boldsymbol{a}^r_{D_5/B_5} = |{}^1\omega_5|^2 \cdot |r_{D/B}| = 4.56^2 \cdot 2.5 = 52.0 \text{ in/s}^2$$ in a direction opposite to $\mathbf{r}_{D/B}$.

$${}^1\boldsymbol{a}^t_{B_2/A_2} = {}^1\boldsymbol{\alpha}_2 \times \boldsymbol{r}_{B/A} \Rightarrow |{}^1\boldsymbol{a}^t_{B_2/A_2}| = |{}^1\boldsymbol{\alpha}_2| \cdot |\boldsymbol{r}_{B/A}| = 100 \cdot 1.22 = 122 \text{ in/s}^2 \ (\perp \text{ to } r_{B/A})$$

$${}^1\boldsymbol{a}^t_{D_4/E_4} = {}^1\boldsymbol{\alpha}_4 \times \boldsymbol{r}_{D/E} \Rightarrow |{}^1\boldsymbol{a}^t_{D_4/E_4}| = |{}^1\boldsymbol{\alpha}_4| \cdot |\boldsymbol{r}_{D/E}| \ (\perp \text{ to } r_{D/E})$$

$${}^1\boldsymbol{a}^t_{D_5/B_5} = {}^1\boldsymbol{\alpha}_5 \times \boldsymbol{r}_{D/B} \Rightarrow |{}^1\boldsymbol{a}^t_{D_5/B_5}| = |{}^1\boldsymbol{\alpha}_5| \cdot |\boldsymbol{r}_{D/B}| \ (\perp \text{ to } r_{D/B})$$

Solve Eq. (2) graphically with an acceleration polygon. The acceleration directions can be gotten directly from the polygon. The magnitudes are given by:

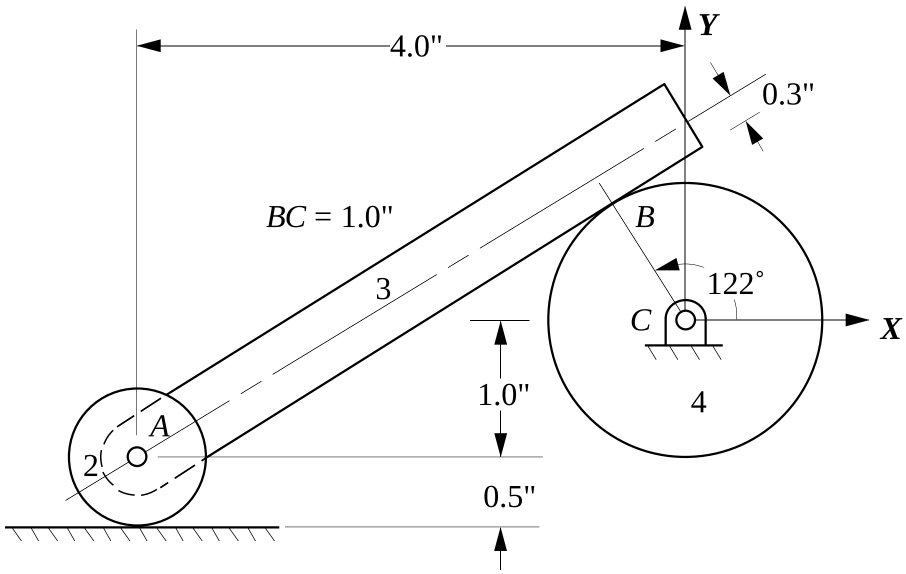
$$|{}^1\mathbf{a}^t_{D4/E4}| = 120.6 \text{ in/s}^2 \Rightarrow |{}^1\alpha_4| = \frac{|{}^1\mathbf{a}^t_{D4/E4}|}{|\mathbf{r}_{D/E}|} = \frac{120.6}{3.5} = 34.5 \text{ rad/s}^2$$

From the directions given in the position and acceleration polygons

$${}^1\boldsymbol{\alpha}_{D_4/E_4} = 34.5 \text{ rad/s}^2 \ CCW$$

Problem 3.21

For the mechanism shown, assume that link 2 rolls on the frame (link 1) and link 4 rolls on Link 3. Assume that link 2 is rotating CW with a constant angular velocity of 100 rad/s. Determine the angular acceleration of link 3 and link 4.



Position Analysis

Draw the linkage to scale. Start with link 2 and locate point A. Locate point C and draw link 4. Then draw a line corresponding to the path of point C. Then locate point C and draw a circle 1.3" in radius. Draw a line from point A tangent to the circle centered at C. Then locate point B on the radial line from the tangent point to C.

Velocity Analysis:

Find angular velocity of link 2,

$$^1\mathbf{v}_{A_2} = {}^1\mathbf{v}_{A_2/D_2} = {}^1\mathbf{v}_{A_3} = {}^1\boldsymbol{\omega}_2 \times \mathbf{r}_{A/D}$$

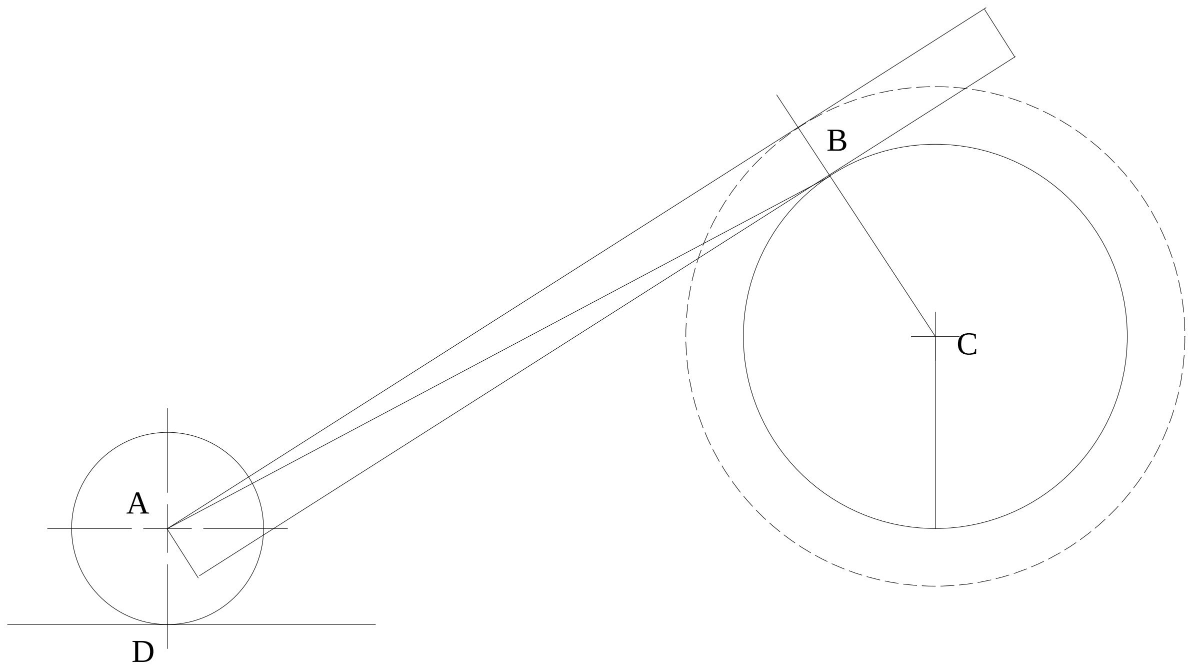
$$^1\mathbf{v}_{B_3} = {}^1\mathbf{v}_{A_3} + {}^1\mathbf{v}_{B_3/A_3} = {}^1\mathbf{v}_{B_4} = {}^1\mathbf{v}_{B_4/C_4} \tag{1}$$

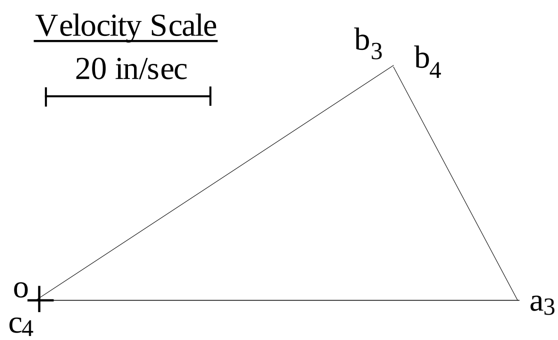
Now,

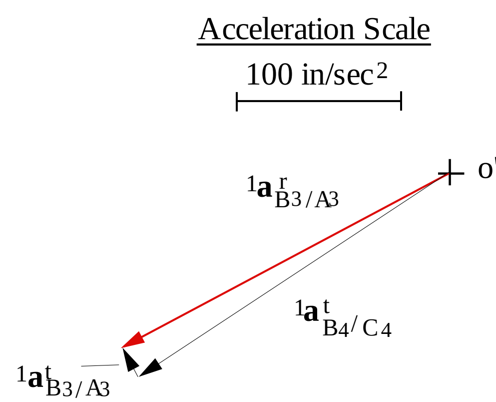
$$\left|{}^1\mathbf{v}_{A_2}\right| = \left|{}^1\boldsymbol{\omega}_2\right|\left|\mathbf{r}_{A/D}\right| = 100 \cdot 0.5 = 50 \text{ in/s } (\perp \text{ to } \mathbf{r}_{A/D})$$

$$^1\mathbf{v}_{B_3/A_3} = {}^1\boldsymbol{\omega}_3 \times \mathbf{r}_{B/A} \Rightarrow \left|{}^1\mathbf{v}_{B_3/A_3}\right| = \left|{}^1\boldsymbol{\omega}_3\right|\left|\mathbf{r}_{B/A}\right| \; (\perp \text{ to } \mathbf{r}_{B/A})$$

$$^1\mathbf{v}_{B_4/C_4} = {}^1\boldsymbol{\omega}_4 \times \mathbf{r}_{B/C} \Rightarrow \left|{}^1\mathbf{v}_{B_4/C_4}\right| = \left|{}^1\boldsymbol{\omega}_4\right|\left|\mathbf{r}_{B/C}\right| \; (\perp \; to \; \mathbf{r}_{B/C})$$







Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$$^1\boldsymbol{v}_{B_3/A_3} = 27.5 \text{ in/s}$$

$$^1\boldsymbol{v}_{B_4/C_4} = 44.6 \text{ in/s}$$

Then

$$|{}^1\omega_3| = \frac{|{}^1\boldsymbol{v}_{B_3/A_3}|}{|\boldsymbol{r}_{B/A}|} = \frac{27.5}{3.91} = 7.03 \text{ rad/s}$$

$$|{}^1\omega_4| = \frac{|{}^1\boldsymbol{v}_{B_4/C_4}|}{|\boldsymbol{r}_{B/C}|} = \frac{44.6}{1} = 44.6\, rad/s$$

To determine the direction of ${}^1\omega_3$, determine the direction that $\mathbf{r}_{B/A}$ must be rotated to be parallel to ${}^1\boldsymbol{v}_{B_3/A_3}$. This direction is clearly counterclockwise.

To determine the direction of ${}^1\omega_4$, determine the direction that $\mathbf{r}_{B/C}$ must be rotated to be parallel to ${}^1\boldsymbol{v}_{B4/C4}$. This direction is clearly clockwise.

Acceleration Analysis:

$${}^1\boldsymbol{a}_{A3} = {}^1\boldsymbol{a}_{A2} = {}^1\boldsymbol{a}_{D2/D1} + {}^1\boldsymbol{a}_{A2/D2}$$

Because of rolling contact on a flat surface,

$${}^1\boldsymbol{a}_{D2/D1} = {}^1\boldsymbol{a}^n_{D2/D1} = {}^1\boldsymbol{a}^r_{D2/A2}$$

Also,

$${}^1\boldsymbol{a}_{A2/D2} = {}^1\boldsymbol{a}^r_{A2/D2} + {}^1\boldsymbol{a}^t_{A2/D2}$$

Combining the equations,

$${}^1\boldsymbol{a}_{A2} = {}^1\boldsymbol{a}^r_{D2/A2} + {}^1\boldsymbol{a}^r_{A2/D2} + {}^1\boldsymbol{a}^t_{A2/D2} = {}^1\boldsymbol{a}^t_{A2/D2} = {}^1\boldsymbol{\alpha}_2 \times \boldsymbol{r}_{A/D} = 0$$

Going to point B,

$${}^1\boldsymbol{a}_{B3} = {}^1\boldsymbol{a}_{A3} + {}^1\boldsymbol{a}_{B3/A3} = {}^1\boldsymbol{a}_{A3} + {}^1\boldsymbol{a}^r_{B3/A3} + {}^1\boldsymbol{a}^t_{B3/A3}$$

also,

$${}^1\boldsymbol{a}_{B3} = {}^1\boldsymbol{a}_{B4} + {}^1\boldsymbol{a}_{B3/B4} = {}^1\boldsymbol{a}_{B4/C4} + {}^1\boldsymbol{a}_{B3/B4}$$

Then,

$${}^1\boldsymbol{a}_{B4/C4} + {}^1\boldsymbol{a}_{B3/B4} = {}^1\boldsymbol{a}_{A3} + {}^1\boldsymbol{a}_{B3/A3}$$

Expanding the terms,

$${}^1\boldsymbol{a}^r_{B4/C4} + {}^1\boldsymbol{a}^t_{B4/C4} + {}^1\boldsymbol{a}^n_{B3/B4} = {}^1\boldsymbol{a}_{A3} + {}^1\boldsymbol{a}^r_{B3/A3} + {}^1\boldsymbol{a}^t_{B3/A3}$$

Expanding ${}^1\boldsymbol{a}^n_{B3/B4}$ recognizing that there is rolling contact between a circle and flat surface, and that ${}^1\mathbf{a}_{A3} = 0$,

$${}^1\mathbf{a}^r_{B4/C4} + {}^1\mathbf{a}^t_{B4/C4} + {}^1\mathbf{a}^n_{C4/B4} = {}^1\mathbf{a}^r_{B3/A3} + {}^1\mathbf{a}^t_{B3/A3}$$

which simplifies to

$${}^1\mathbf{a}^t_{B4/C4} = {}^1\mathbf{a}^r_{B3/A3} + {}^1\mathbf{a}^t_{B3/A3} \qquad (3)$$

Now,

$${}^1\boldsymbol{a}^r_{B3/A3} = {}^1\boldsymbol{\omega}_3 \times ({}^1\boldsymbol{\omega}_3 \times \boldsymbol{r}_{B/A}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{B3/A3}\right| = \left|{}^1\boldsymbol{\omega}_3\right|^2 \cdot \left|\boldsymbol{r}_{B/A}\right| = 7.04^2 \cdot 3.91 = 194 \text{ in}/\text{s}^2$$

in the direction opposite to $\boldsymbol{r}_{B/A}$

$${}^1\boldsymbol{a}^t_{B3/A3} = {}^1\boldsymbol{\alpha}_3 \times \boldsymbol{r}_{B/A} \; (\perp \textit{ to } \boldsymbol{r}_{B/A})$$

$${}^1\boldsymbol{a}^t_{B4/C4} = {}^1\boldsymbol{\alpha}_4 \times \boldsymbol{r}_{B/C} \; (\perp \textit{ to } \boldsymbol{r}_{B/C})$$

Solve Eq. (3) graphically with an acceleration polygon. From the polygon,

$$\left|{}^1\boldsymbol{a}^t_{B_3/A_3}\right| = 17.2 \text{ in/s}^2$$

$$\left|{}^1\boldsymbol{a}^t_{B_4/C_4}\right| = 194 \text{ in/s}^2$$

Then

$$\left|{}^1\alpha_3\right| = \frac{\left|{}^1\boldsymbol{a}^t_{B_3/A_3}\right|}{\left|\boldsymbol{r}_{B/A}\right|} = \frac{17.2}{3.91} = 4.41 \text{ rad/s}^2$$

and

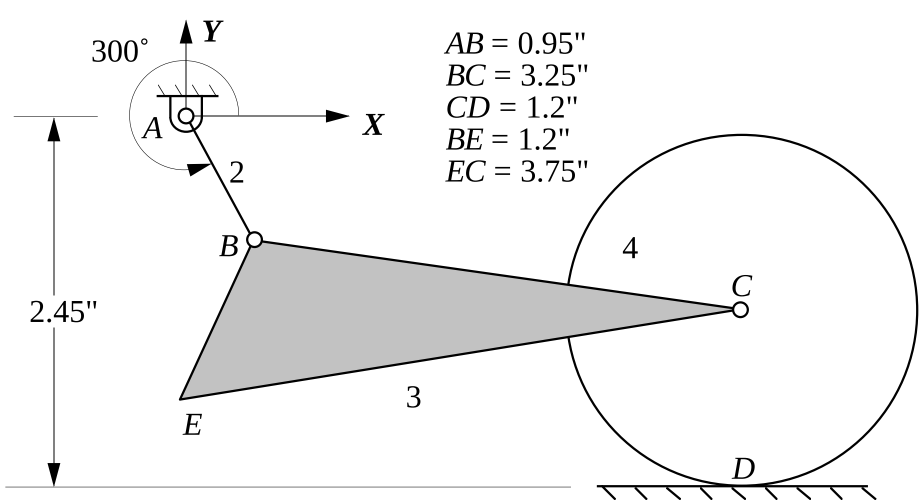
$$\left|{}^1\alpha_4\right| = \frac{\left|{}^1\boldsymbol{a}^t_{B_4/C_4}\right|}{\left|\boldsymbol{r}_{B/C}\right|} = \frac{194}{1} = 194 \text{ rad/s}^2$$

To determine the direction of ${}^1\alpha_3$, determine the direction that $\boldsymbol{r}_{B/A}$ must be rotated to be parallel to ${}^1\boldsymbol{a}^t_{B_3/A_3}$. This direction is clearly counterclockwise.

To determine the direction of ${}^1\alpha_4$, determine the direction that $\boldsymbol{r}_{B/C}$ must be rotated to be parallel to ${}^1\boldsymbol{a}^t_{B_4/C_4}$. This direction is clearly counterclockwise.

Problem 3.22

For the mechanism shown, assume that link 4 rolls on the frame (link 1). If link 2 is rotating CW with a constant angular velocity of 10 rad/s, determine the angular accelerations of links 3 and 4 and the acceleration of point E on link 3.



Position Analysis

Draw the linkage to scale. Start with link 2 and locate point B. Then draw a line corresponding to the path of point C. Then locate point C and draw link 4. Then locate point E.

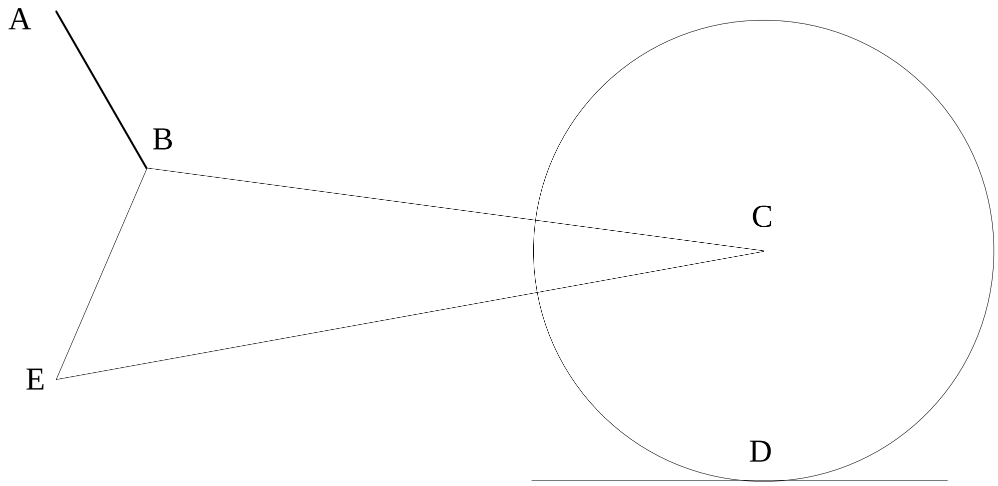
Velocity Analysis:

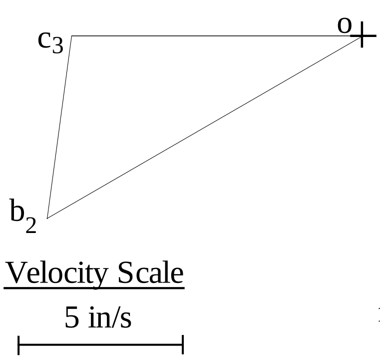
Find angular velocity of link 2,

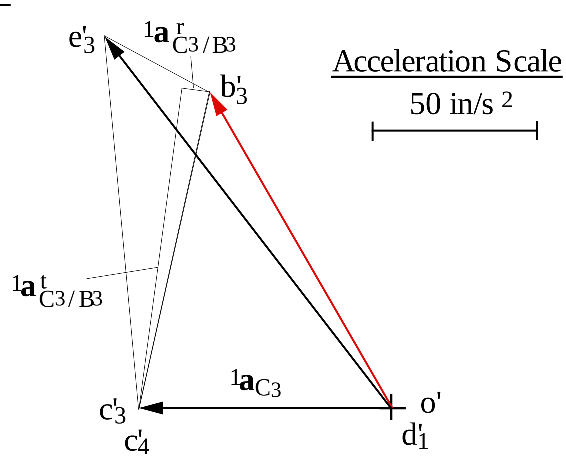
$${}^1\boldsymbol{v}_{B_2} = {}^1\boldsymbol{v}_{B_2/A_2} = {}^1\boldsymbol{v}_{B_3} = {}^1\boldsymbol{\omega}_2 \times \boldsymbol{r}_{B/A}$$

$$^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{B_3} + {}^1\boldsymbol{v}_{C_3/B_3} \tag{1}$$

$$^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{C_4}$$







Now,

$$\left|{}^1\boldsymbol{v}_{B_2}\right| = \left|{}^1\omega_2\right|\left|\boldsymbol{r}_{B/A}\right| = 10 \cdot 0.95 = 9.5 \text{ in / s} \left(\perp \text{ to } \boldsymbol{r}_{B/A}\right)$$

$^1\boldsymbol{v}_{C_3}$ along the line of motion of C.

$$^1\boldsymbol{v}_{C_3/B_3} = {}^1\omega_3 \times \boldsymbol{r}_{C/B} \Rightarrow \left|{}^1\boldsymbol{v}_{C_3/B_3}\right| = \left|{}^1\omega_3\right|\left|\boldsymbol{r}_{C/B}\right| \left(\perp \text{ to } \boldsymbol{r}_{C/B}\right)$$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$$^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{C_4} = 7.53 \text{ in / s}$$

Also,

$$^1\boldsymbol{v}_{C_3/B_3} = 4.81 \text{ in / s}$$

or

$$\left|{}^1\omega_3\right| = \frac{\left|{}^1\boldsymbol{v}_{C_3/B_3}\right|}{\left|\boldsymbol{r}_{C/B}\right|} = \frac{4.81}{3.25} = 1.48 \text{ rad/s}$$

To determine the direction of ${}^1\omega_3$, determine the direction that $\boldsymbol{r}_{B/A}$ must be rotated to be parallel to ${}^1\boldsymbol{v}_{B_3/A_3}$. This direction is clearly counterclockwise.

For link 4,

$${}^1\boldsymbol{v}_{C_4} = {}^1\boldsymbol{v}_{D_4} + {}^1\boldsymbol{v}_{C_4/D_4} = {}^1\boldsymbol{v}_{C_4/D_4} = {}^1\omega_4 \times \boldsymbol{r}_{C/D}$$

or

$$\left|{}^1\omega_4\right| = \frac{\left|{}^1\boldsymbol{v}_{C_4/D_4}\right|}{\left|\boldsymbol{r}_{C/D}\right|} = \frac{7.53}{1.2} = 6.28\, rad/s$$

To determine the direction of ${}^1\omega_4$, determine the direction that $\mathbf{r}_{C/D}$ must be rotated to be parallel to ${}^1\boldsymbol{v}_{C_4/D_4}$. This direction is clearly counterclockwise.

<u>Acceleration Analysis:</u>

$${}^1\boldsymbol{a}_{B_2} = {}^1\boldsymbol{a}_{B_3} = {}^1\boldsymbol{a}^r_{B_2/A_2} + {}^1\boldsymbol{a}^t_{B_2/E_2}$$

$${}^1\boldsymbol{a}_{C_3} = {}^1\boldsymbol{a}_{C_4} = {}^1\boldsymbol{a}_{B_3} + {}^1\boldsymbol{a}_{C_3/B_3}$$

$${}^1\boldsymbol{a}_{C_3} = {}^1\boldsymbol{a}^r_{B_2/A_2} + {}^1\boldsymbol{a}^t_{B_2/E_2} + {}^1\boldsymbol{a}^r_{C_3/B_3} + {}^1\boldsymbol{a}^t_{C_3/B_3} \tag{3}$$

$${}^1\boldsymbol{a}_{D_4} = {}^1\boldsymbol{a}_{C_4} + {}^1\boldsymbol{a}_{D_4/C_4} = {}^1\boldsymbol{a}_{D_4/D_1} = {}^1\boldsymbol{a}^n_{D_4/D_1}$$

$${}^1\boldsymbol{a}^n_{D_4/D_1} = {}^1\boldsymbol{a}_{C_4} + {}^1\boldsymbol{a}^r_{D_4/C_4} + {}^1\boldsymbol{a}^t_{D_4/C_4} = {}^1\boldsymbol{a}^n_{D_4/C_4} + {}^1\boldsymbol{a}^n_{C_4/F_1} + {}^1\boldsymbol{a}^n_{F_1/D_1}$$

Where F is the center of curvature of the line. Consequently, F is at infinity and both ${}^1\boldsymbol{a}^n_{C_4/F_1}$ and ${}^1\boldsymbol{a}^n_{F_1/D_1}$ are zero. Also, ${}^1\boldsymbol{a}^n_{D_4/C_4}$ and ${}^1\boldsymbol{a}^r_{D_4/C_4}$ cancel. Therefore, the acceleration equation reduces to

$${}^1\boldsymbol{a}_{C_4} + {}^1\boldsymbol{a}^t_{D_4/C_4} = 0$$

or

$${}^1\boldsymbol{a}^t_{D_4/C_4} = -{}^1\boldsymbol{a}_{C_4}$$

Now,

${}^1\boldsymbol{a}_{C_3}$ along the line of motion of point C.

$${}^1\boldsymbol{a}^r_{B_2/A_2} = {}^1\omega_2 \times ({}^1\omega_2 \times \boldsymbol{r}_{B/A}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{B_2/A_2}\right| = \left|{}^1\omega_2\right|^2 \cdot \left|\boldsymbol{r}_{B/A}\right| = 10^2 \cdot 0.95 = 95 \text{ in/s}^2$$

in the direction opposite to $\boldsymbol{r}_{B/A}$

$$ {}^1\boldsymbol{a}^t_{B_2/A_2} = {}^1\boldsymbol{\alpha}_2 \times \boldsymbol{r}_{B/A} \Rightarrow \left|{}^1\boldsymbol{a}^t_{B_2/A_2}\right| = \left|{}^1\boldsymbol{\alpha}_2\right| \cdot \left|\boldsymbol{r}_{B/A}\right| = 0 \cdot 1 = 0 \text{ in / s}^2 $$

$$ {}^1\boldsymbol{a}^r_{C_3/B_3} = {}^1\boldsymbol{\omega}_3 \times ({}^1\boldsymbol{\omega}_3 \times \boldsymbol{r}_{C/B}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{C_3/B_3}\right| = \left|{}^1\boldsymbol{\omega}_3\right|^2 \cdot \left|\boldsymbol{r}_{C/B}\right| = 1.48^2 \cdot 3.25 = 7.12 \text{ in / s}^2 $$

in the direction opposite to $\boldsymbol{r}_{C/B}$

$$ {}^1\boldsymbol{a}^t_{C_3/B_3} = {}^1\boldsymbol{\alpha}_3 \times \boldsymbol{r}_{C/B} \Rightarrow \left|{}^1\boldsymbol{a}^t_{C_3/B_3}\right| = \left|{}^1\boldsymbol{\alpha}_3\right| \cdot \left|\boldsymbol{r}_{C/B}\right| (\perp \textit{ to } \boldsymbol{r}_{C/B}) $$

Solve Eq. (3) graphically with an acceleration polygon. From the polygon,

$$ {}^1\boldsymbol{a}_{C_3} = 65.8 \text{ in / s}^2 $$

and by image,

$$ {}^1\boldsymbol{a}_{E_3} = 122 \text{ in / s}^2 $$

in the directions shown. Also,

$$ {}^1\boldsymbol{a}^t_{C_3/B_3} = 84.2 \text{ in / s}^2 $$

or

$$ \left|{}^1\boldsymbol{\alpha}_3\right| = \frac{\left|{}^1\boldsymbol{a}^t_{C_3/B_3}\right|}{\left|\boldsymbol{r}_{C/B}\right|} = \frac{84.2}{3.25} = 25.9 \text{ rad / s}^2 $$

To determine the direction of ${}^1\boldsymbol{\alpha}_3$, determine the direction that $\boldsymbol{r}_{C/B}$ must be rotated to be parallel to ${}^1\boldsymbol{a}^t_{C_3/B_3}$. This direction is clearly clockwise.

Also,

$$ {}^1\boldsymbol{a}^t_{D_4/C_4} = {}^1\boldsymbol{\alpha}_4 \times \boldsymbol{r}_{D/C} = -{}^1\boldsymbol{a}_{C_4} \Rightarrow \left|{}^1\boldsymbol{a}^t_{D_4/C_4}\right| = \left|{}^1\boldsymbol{\alpha}_4\right| \cdot \left|\boldsymbol{r}_{D/C}\right| $$

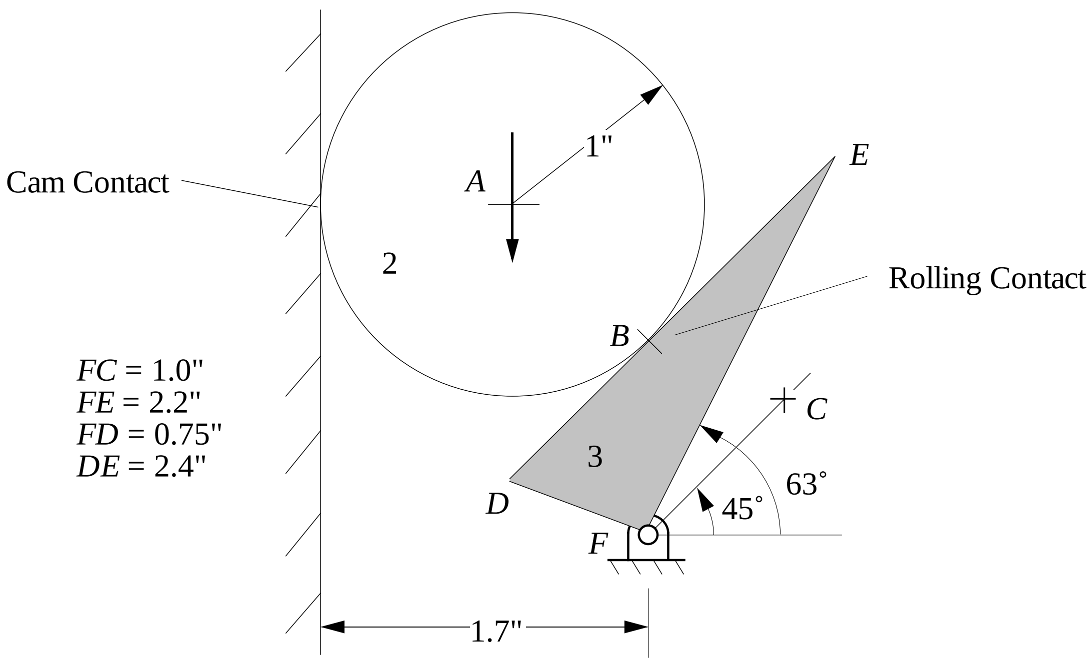
or

$$ \left|{}^1\boldsymbol{\alpha}_4\right| = \frac{\left|{}^1\boldsymbol{a}^t_{D_4/C_4}\right|}{\left|\boldsymbol{r}_{D/C}\right|} = \frac{65.8}{1.2} = 54.8 \text{ rad / s}^2 $$

To determine the direction of ${}^1\boldsymbol{\alpha}_4$, determine the direction that $\boldsymbol{r}_{D/C}$ must be rotated to be parallel to ${}^1\boldsymbol{a}^t_{D_4/C_4}$. This direction is clearly counterclockwise.

Problem 3.23

If $\boldsymbol{v}_{A_2}$ = 10 in/s (constant) downward, find $\boldsymbol{\omega}_3$, $\boldsymbol{\alpha}_3$, $\boldsymbol{v}_{C_3}$, and $\boldsymbol{a}_{C_3}$.



Velocity Analysis

$$^1\boldsymbol{v}_{B_2} = {}^1\boldsymbol{v}_{A_2} + {}^1\boldsymbol{v}_{B_2/A_2} = {}^1\boldsymbol{v}_{A_2} + {}^1\omega_2 \times \boldsymbol{r}_{B/A}$$

$$^1\boldsymbol{v}_{B_3} = {}^1\boldsymbol{v}_{B_2} + {}^1\boldsymbol{v}_{B_3/B_2} = {}^1\boldsymbol{v}_{B_3/F_3} = {}^1\omega_3 \times \boldsymbol{r}_{B/F}$$

Therefore,

$$^1\boldsymbol{v}_{B_3/F_3} = {}^1\boldsymbol{v}_{A_2} + {}^1\boldsymbol{v}_{B_2/A_2} + {}^1\boldsymbol{v}_{B_3/B_2} \tag{1}$$

Because of rolling contact,

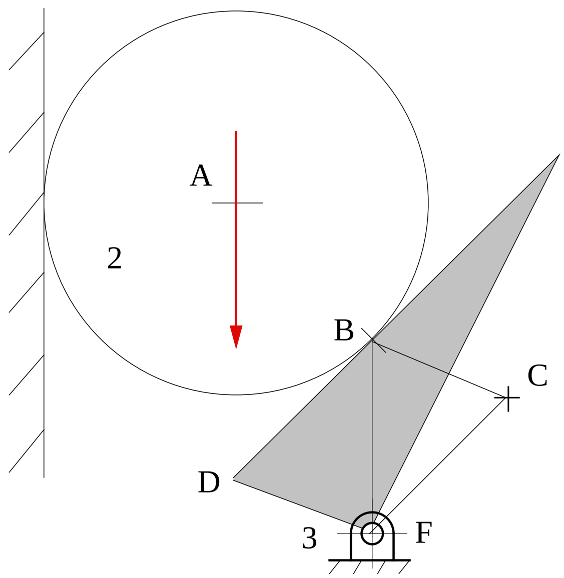
$$^1\boldsymbol{v}_{B_3/B_2} = 0$$

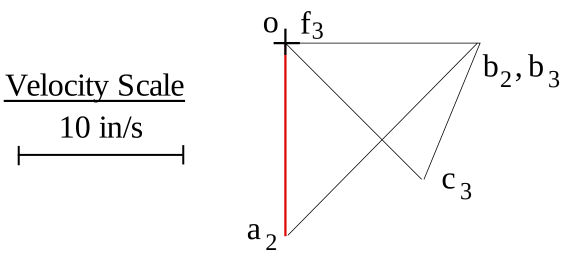
Also,

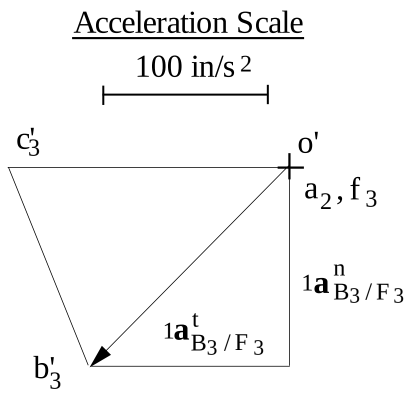
$$^1\boldsymbol{v}_{B_3/F_3} = {}^1\omega_3 \times \boldsymbol{r}_{B/F} \ (\perp \text{ to } \boldsymbol{r}_{B/F})$$

$$^1\boldsymbol{v}_{B_2/A_2} = {}^1\omega_2 \times \boldsymbol{r}_{B/A} \ (\perp \text{ to } \boldsymbol{r}_{B/A})$$

Therefore, we can solve Eq. (1) using the velocity polygon. Using the velocity polygon,







$$\left|{}^1\omega_2\right| = \frac{\left|{}^1\boldsymbol{v}_{B_2/A_2}\right|}{\left|\boldsymbol{r}_{B/A}\right|} = \frac{14.14}{1.0} = 14.14 \text{ rad / s}$$

To determine the direction of ${}^1\omega_2$, determine the direction that we must rotate $\boldsymbol{r}_{B/A}$ 90° to get the direction of ${}^1\boldsymbol{v}_{B_2/A_2}$. This is CCW. Also,

$$\left|{}^1\omega_3\right| = \frac{\left|{}^1\boldsymbol{v}_{B_3/F_3}\right|}{\left|\boldsymbol{r}_{B/F}\right|} = \frac{10.14}{1.0} = 10.14 \text{ rad / s}$$

To determine the direction of ${}^1\boldsymbol{\omega}_3$, determine the direction that we must rotate $\mathbf{r}_{B/F}$ 90° to get the direction of ${}^1\boldsymbol{v}_{B_3/F_3}$. This is CW.

The velocity of point C_3 is found by image. The magnitude is

$$\left|{}^1\boldsymbol{v}_{C_3}\right| = 10 \text{ in / s}$$

and the direction is given by the velocity polygon.

Acceleration Analysis

$$^1\boldsymbol{a}_{B_2} = {}^1\boldsymbol{a}_{A_2} + {}^1\boldsymbol{a}_{B_2/A_2} = {}^1\boldsymbol{a}_{A_2} + {}^1\boldsymbol{a}^t_{B_2/A_2} + {}^1\boldsymbol{a}^r_{B_2/A_2} = {}^1\boldsymbol{a}_{A_2} + {}^1\boldsymbol{\alpha}_2 \times \boldsymbol{r}_{B/A} + {}^1\boldsymbol{\omega}_2 \times \left({}^1\boldsymbol{\omega}_2 \times \boldsymbol{r}_{B/A}\right)$$

$$^1\boldsymbol{a}_{B_3} = {}^1\boldsymbol{a}_{B_2} + {}^1\boldsymbol{a}^n_{B_3/B_2}$$

and

$$^1\boldsymbol{a}_{B_3} = {}^1\boldsymbol{a}_{B_3/F_3} = {}^1\boldsymbol{a}^t_{B_3/F_3} + {}^1\boldsymbol{a}^r_{B_3/F_3} = {}^1\boldsymbol{\alpha}_3 \times \boldsymbol{r}_{B/F} + {}^1\boldsymbol{\omega}_3 \times \left({}^1\boldsymbol{\omega}_3 \times \boldsymbol{r}_{B/F}\right)$$

Compute the normal component of acceleration at the rolling contact point.

$$^1\boldsymbol{a}^n_{B_3/B_2} = {}^1\boldsymbol{a}^r_{B_3/O_3} + {}^1\boldsymbol{a}^r_{O_3/A_2} + {}^1\boldsymbol{a}^r_{A_2/B_2}$$

where O_3 is at infinity in the direction of AB. Then,

$$\begin{aligned} {}^1\boldsymbol{a}^t_{B_3/F_3} + {}^1\boldsymbol{a}^r_{B_3/F_3} &= {}^1\boldsymbol{a}_{A_2} + {}^1\boldsymbol{a}^t_{B_2/A_2} + {}^1\boldsymbol{a}^r_{B_2/A_2} + {}^1\boldsymbol{a}^r_{B_3/O_3} + {}^1\boldsymbol{a}^r_{O_3/A_2} + {}^1\boldsymbol{a}^r_{A_2/B_2} \\ &= {}^1\boldsymbol{a}_{A_2} + {}^1\boldsymbol{a}^t_{B_2/A_2} + {}^1\boldsymbol{a}^r_{B_3/O_3} + {}^1\boldsymbol{a}^r_{O_3/A_2} \end{aligned}$$

The known information can be summarized as follows:

$$^1\boldsymbol{a}_{A_2} = 0$$

$$^1\boldsymbol{a}^t_{B_2/A_2} = {}^1\boldsymbol{\alpha}_2 \times \boldsymbol{r}_{B/A} \; (\perp \text{to } \boldsymbol{r}_{B/A})$$

$$^1\boldsymbol{a}^r_{B_3/O_3} = \frac{\left|{}^1\boldsymbol{v}_{B_3/O_3}\right|^2}{\left|\boldsymbol{r}_{B/O_3}\right|} = \frac{\left|{}^1\boldsymbol{v}_{B_3/O_3}\right|^2}{|\infty|} = 0$$

$$^1\boldsymbol{a}^r_{O_3/A_2} = \frac{\left|{}^1\boldsymbol{v}_{O_3/A_2}\right|^2}{\left|\boldsymbol{r}_{O_3/A_2}\right|} = \frac{\left|{}^1\boldsymbol{v}_{O_3/A_2}\right|^2}{|\infty|} = 0$$

$$^1\boldsymbol{a}^t_{B_3/F_3} = {}^1\boldsymbol{\alpha}_3 \times \boldsymbol{r}_{B/F} \; (\perp \text{to } \boldsymbol{r}_{B/F})$$

$$\left|{}^1\boldsymbol{a}^r_{B_3/F_3}\right| = \frac{\left|{}^1\boldsymbol{v}_{B_3/F_3}\right|^2}{\left|\boldsymbol{r}_{B/F}\right|} = \frac{10.14^2}{1} = 103 \text{ in/s}^2 \text{ opposite to } \boldsymbol{r}_{B/F}$$

Note that ${}^1\boldsymbol{a}^r_{A_2/B_2}$ and ${}^1\boldsymbol{a}^r_{B_2/A_2}$ are in opposite directions and cancel each other. Therefore, the acceleration equations can be combined into the following simple equation

$$^1\boldsymbol{a}^t_{B_3/F_3} + {}^1\boldsymbol{a}^r_{B_3/F_3} = {}^1\boldsymbol{a}^t_{B_2/A_2} \tag{2}$$

and solved for the unknown magnitudes of ${}^1\boldsymbol{a}^t_{B_3/F_3}$ and ${}^1\boldsymbol{a}^t_{B_2/A_2}$. Using values from the acceleration polygon,

$$\left|{}^1\boldsymbol{\alpha}_3\right| = \frac{\left|{}^1\boldsymbol{a}^t_{B_3/F_3}\right|}{\left|\boldsymbol{r}_{B/F}\right|} = \frac{104}{1.0} = 104 \text{ rad/s}^2$$

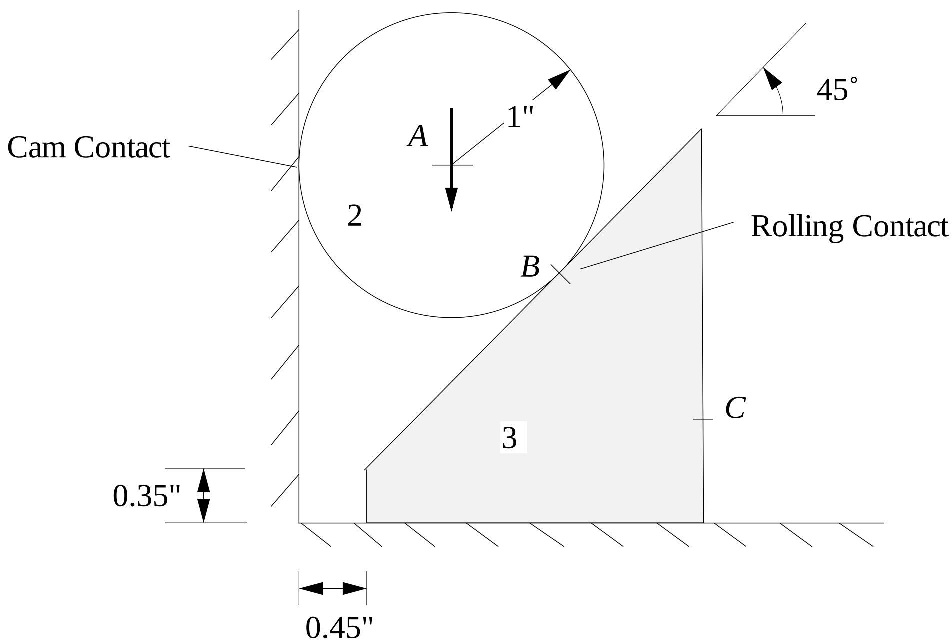
To determine the direction of ${}^1\alpha_3$, determine the direction that we must rotate $\boldsymbol{r}_{B/F}$ 90° to get the direction of ${}^1\boldsymbol{a}^t_{B_3/F_3}$. This is CCW. The acceleration of point C_3 is found by image. The magnitude is

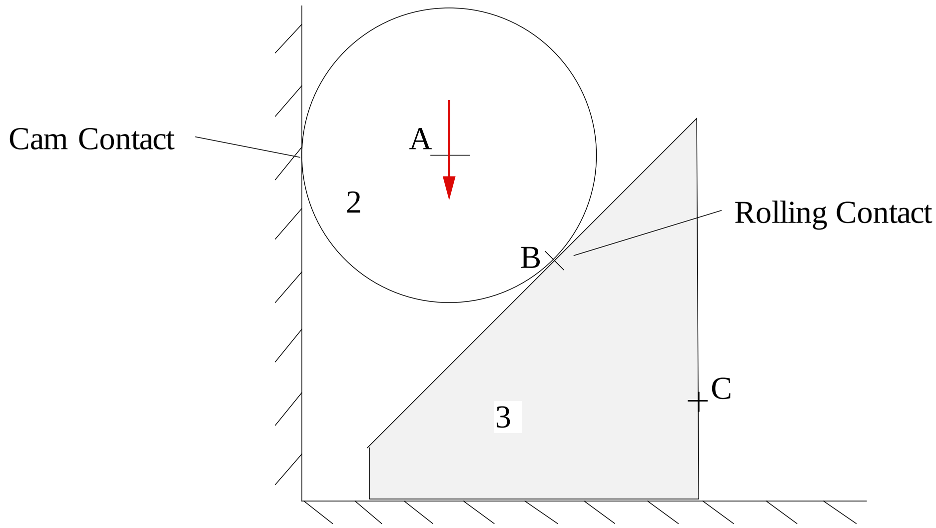
$$\left|{}^1\mathbf{a}_{C_3}\right| = 148 \text{ in/s}^2$$

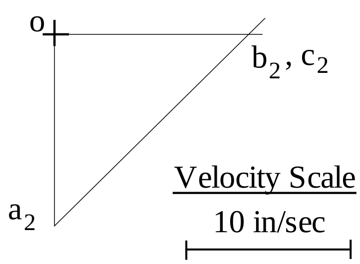
and the direction is given by the polygon.

Problem 3.24

In the figure shown below, points *A, B,* and *C* are collinear. If $\boldsymbol{v}_{A_2}$ = 10 in/s (constant) downward, find $\boldsymbol{v}_{C_3}$, and $\boldsymbol{a}_{C_3}$.







Velocity Analysis

$$^1\mathbf{v}_{C_3} = {}^1\mathbf{v}_{B_3} = {}^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{A_2} + {}^1\mathbf{v}_{B_2/A_2} \tag{1}$$

This can be solved using velocity polygons as shown since we know the directions of $^1\mathbf{v}_{A_2}$ and $^1\mathbf{v}_{B_2}$. Therefore, $^1\mathbf{v}_{C_2} = 10$ in/sec.

Acceleration Analysis

Differentiate Eq. (1) to solve for $^1\mathbf{a}_{C_2}$.

$$^1\mathbf{a}_{C_3} = {}^1\mathbf{a}_{B_3}$$

and

$$^1\mathbf{a}_{B_3} = {}^1\mathbf{a}_{A_2} + {}^1\mathbf{a}^r_{B_2/A_2} + {}^1\mathbf{a}^t_{B_2/A_2} + {}^1\mathbf{a}^r_{B_3/O_3} + {}^1\mathbf{a}^r_{O_3/A_2} + {}^1\mathbf{a}^r_{A_2/B_2} \tag{2}$$

where O_3 is at infinity in the direction of AB.

$$^1\mathbf{a}_{A_2} = 0$$

$$^1\mathbf{a}^r_{B_2/A_2} = \frac{|{}^1\mathbf{v}_{B_2/A_2}|^2}{|\mathbf{r}_{B/A}|} \text{ opposite to } \mathbf{r}_{B/A}$$

$${}^{1}\mathbf{a}^{t}_{B_2/A_2} = {}^{1}\alpha_2 \times \mathbf{r}_{B/A} \;\; (\perp \text{to } \mathbf{r}_{B/A})$$

$${}^{1}\mathbf{a}^{r}_{B_3/O_3} = \frac{\left|{}^{1}v_{B_3/O_3}\right|^2}{\left|\mathbf{r}_{B/O_3}\right|} = \frac{\left|{}^{1}v_{B_3/O_3}\right|^2}{|\infty|} = 0$$

$${}^{1}\mathbf{a}^{r}_{O_3/A_2} = \frac{\left|{}^{1}v_{O_3/A_2}\right|^2}{\left|\mathbf{r}_{O_3/A_2}\right|} = \frac{\left|{}^{1}v_{O_3/A_2}\right|^2}{|\infty|} = 0$$

$${}^{1}\mathbf{a}^{r}_{A_2/B_2} = \frac{\left|{}^{1}v_{A_2/B_2}\right|^2}{\left|\mathbf{r}_{A/B}\right|}$$

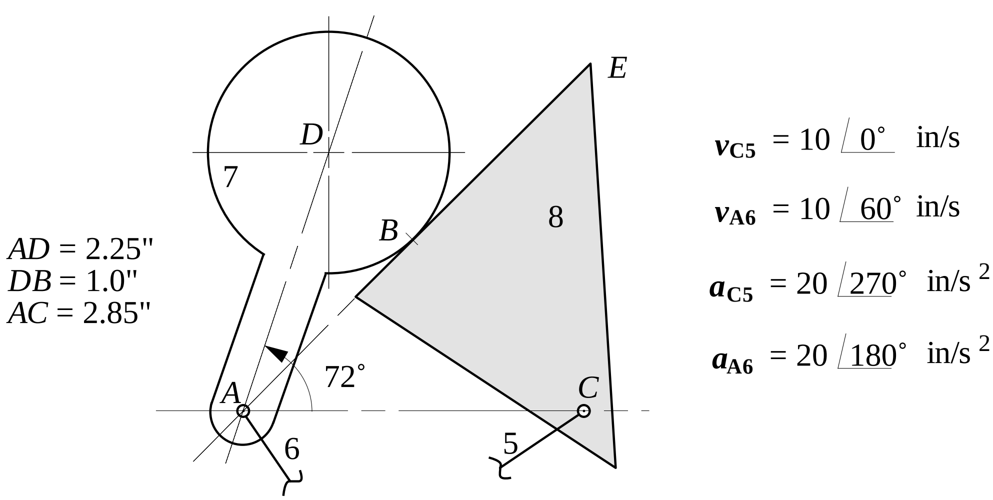
Note that ${}^{1}\mathbf{a}^{r}_{A_2/B_2}$ and ${}^{1}\mathbf{a}^{r}_{B_2/A_2}$ are in opposite directions and cancel each other. Therefore, all of the terms on the right hand side of Eq. (2) are either zero or cancel each other except for ${}^{1}\mathbf{a}^{t}_{B_2/A_2}$. Therefore,

$${}^{1}\mathbf{a}_{B_3} = {}^{1}\mathbf{a}^{t}_{B_2/A_2}$$

However, ${}^{1}\mathbf{a}_{B_3}$ must be horizontal and ${}^{1}\mathbf{a}^{t}_{B_2/A_2}$ is perpendicular to $\mathbf{r}_{B/A}$ which is not horizontal. Because the directions are different, the magnitudes must be zero. Therefore, ${}^{1}\mathbf{a}_{B_3}$ must be zero.

Problem 3.25

Part of an eight-link mechanism is shown in the figure. There is rolling contact at location B and the velocity and acceleration of points A_6 and C_5 are as shown. Find ω_8 and α_7 for the position given. Also find the velocity of E_7 by image.



Position Analysis

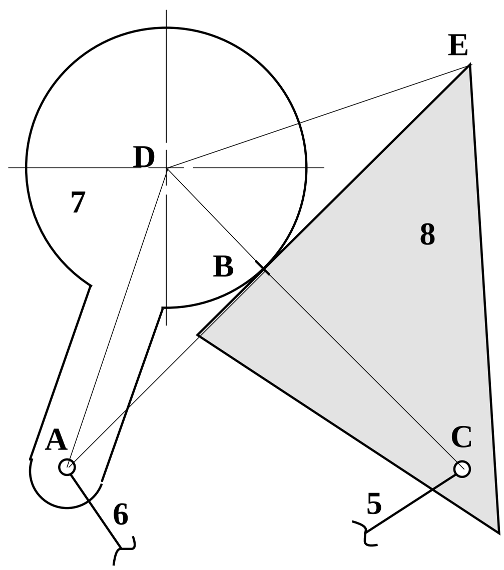
Draw the mechanism to scale. Vectors are:

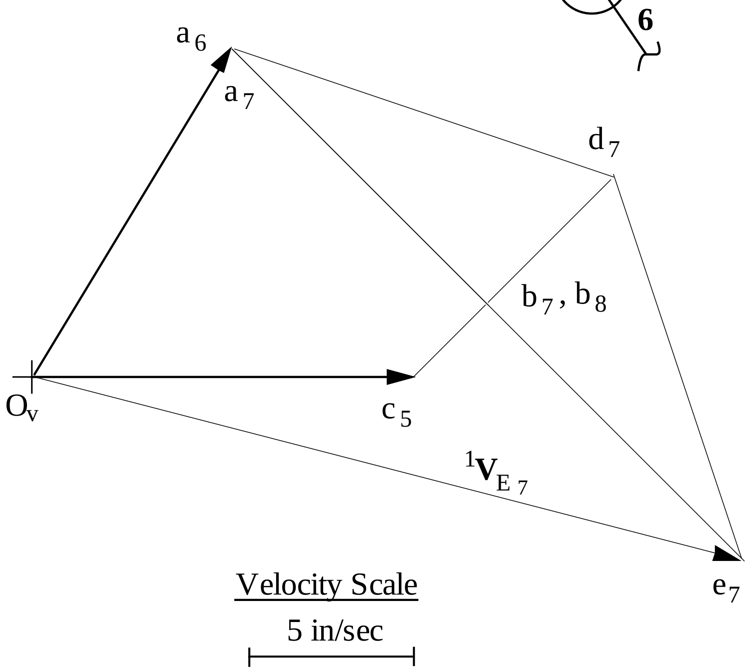
$\mathbf{r}_{B/C} = 2 \text{ in}$

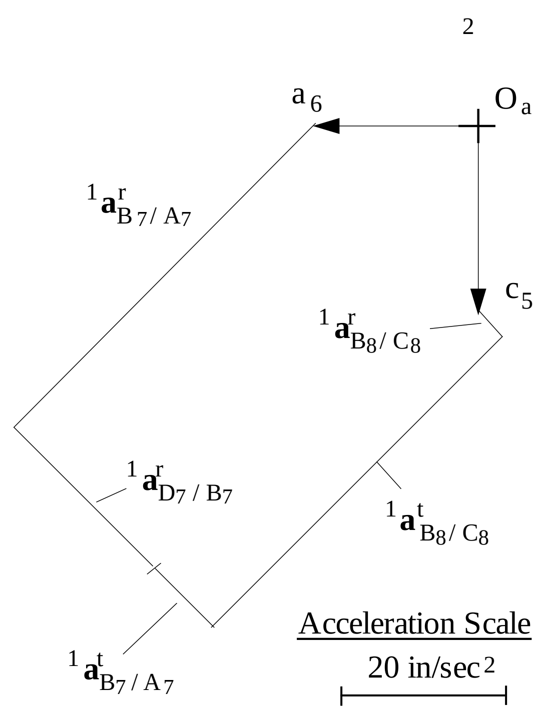
$\mathbf{r}_{B/A} = 2 \text{ in}$

$\mathbf{r}_{D/B} = 1 \text{ in}$

Velocity Analysis







Velocity analysis:

The basic equations are:

$$^1\mathbf{v}_{B7} = {}^1\mathbf{v}_{B7/A7} + {}^1\mathbf{v}_{A7}$$
$$^1\mathbf{v}_{B8} = {}^1\mathbf{v}_{B8/C8} + {}^1\mathbf{v}_{C8}$$
$$^1\mathbf{v}_{B8} = {}^1\mathbf{v}_{B7} + {}^1\mathbf{v}_{B8/B7}$$

Combining the equations,

$$^1\mathbf{v}_{B8/C8} + {}^1\mathbf{v}_{C8} = {}^1\mathbf{v}_{B7/A7} + {}^1\mathbf{v}_{A7} + {}^1\mathbf{v}_{B8/B7}$$

where

$$^1\mathbf{v}_{B8/C8} = {}^1\omega_8 \times \mathbf{r}_{B/C} \ (\perp \text{ to BC})$$

$$^1\mathbf{v}_{B7/A7} = {}^1\omega_7 \times \mathbf{r}_{B/A} \ (\perp \text{ to BA})$$

$$^1\mathbf{v}_{B8/B7} = 0 \text{ (rolling contact)}$$

$$^1\mathbf{v}_{C8} = {}^1\mathbf{v}_{C5} = 10\angle 0°$$

$$^1\mathbf{v}_{A7} = {}^1\mathbf{v}_{A6} = 10\angle 60°$$

The solution is given on the polygon. The velocity of E_7 is found by image. Then,

$$^1\mathbf{v}_{E7} = 18.89 \text{ in/sec}$$

$$^1\mathbf{v}_{B8/C8} = 2.687 \text{ in/sec}$$

$$^1\mathbf{v}_{B7/A7} = 9.428 \text{ in/sec}$$

Therefore,

$$|{}^1\omega_8| = \frac{|{}^1\mathbf{v}_{B8/C8}|}{|\mathbf{r}_{B/C}|} = \frac{2.687}{2} = 1.34 \text{ rad/sec}$$

To determine the direction of ${}^1\omega_8$, determine the direction that we must rotate $\mathbf{r}_{B/C}$ 90° to get the direction of ${}^1\mathbf{v}_{B8/C8}$. This is CW.

Acceleration Analysis

$$^1\mathbf{a}_{B7} = {}^1\mathbf{a}_{B7/A7} + {}^1\mathbf{a}_{A7}$$
$$^1\mathbf{a}_{B8} = {}^1\mathbf{a}_{B8/C8} + {}^1\mathbf{a}_{C8}$$
$$^1\mathbf{a}_{B8} = {}^1\mathbf{a}_{B7} + {}^1\mathbf{a}_{B8/B7}$$

Combining the equations,

$$^1\mathbf{a}_{B8/C8} + {}^1\mathbf{a}_{C8} = {}^1\mathbf{a}_{B7/A7} + {}^1\mathbf{a}_{A7} + {}^1\mathbf{a}_{B8/B7}$$

In component form:

$$ {}^1\mathbf{a}^r_{B_8/C_8} + {}^1\mathbf{a}^t_{B_8/C_8} + {}^1\mathbf{a}_{C_8} = {}^1\mathbf{a}^r_{B_7/A_7} + {}^1\mathbf{a}^t_{B_7/A_7} + {}^1\mathbf{a}_{A_7} + {}^1\mathbf{a}^r_{B_8/C_8} + {}^1\mathbf{a}^r_{C_8/D_7} + {}^1\mathbf{a}^r_{D_7/B_7} $$

Computing the individual terms,

$$ {}^1\mathbf{a}_{C_8} = {}^1\mathbf{a}_{C_5} = 20\angle 270° $$

$$ {}^1\mathbf{a}_{A_7} = {}^1\mathbf{a}_{A_6} = 20\angle 180° $$

$$ {}^1\mathbf{a}^r_{B_8/C_8} = \frac{\left|{}^1\mathrm{v}_{B_8/C_8}\right|^2}{\left|\mathrm{r}_{B/C}\right|} = \frac{(2.687)^2}{2} = 3.610\frac{\mathrm{in}}{\mathrm{sec}^2} \text{ B to C} $$

$$ {}^1\mathbf{a}^r_{B_7/A_7} = \frac{\left|{}^1\mathrm{v}_{B_7/A_7}\right|^2}{\left|{}^1\mathrm{r}_{B_7/A_7}\right|} = \frac{(9.428)^2}{2} = 44.444\frac{\mathrm{in}}{\mathrm{sec}^2} \text{ B to A} $$

$$ {}^1\mathbf{a}^r_{B_8/C_8} = \frac{\left|{}^1\mathrm{v}_{B_8/C_8}\right|^2}{|\infty|} = 0 $$

$$ {}^1\mathbf{a}^r_{C_8/D_7} = \frac{\left|{}^1\mathrm{v}_{C_8/D_7}\right|^2}{|\infty|} = 0 $$

$$ {}^1\mathbf{a}^r_{D_7/B_7} = \frac{\left|{}^1\mathrm{v}_{D_7/B_7}\right|^2}{\left|\mathrm{r}_{D/B}\right|} = \frac{(4.544)^2}{1} = 20.653\frac{\mathrm{in}}{\mathrm{sec}^2} \text{ D to B} $$

$$ {}^1\mathbf{a}^t_{B_8/C_8} = {}^1\alpha_8 \times \mathbf{r}_{B/C} \ (\perp \text{ to BC}) $$

$$ {}^1\mathbf{a}^t_{B_7/A_7} = {}^1\alpha_7 \times \mathbf{r}_{B/A} \ (\perp \text{ to BA}) $$

From the acceleration polygon,

$$ {}^1\mathbf{a}^t_{B_7/A_7} = {}^1\alpha_7 \times \mathbf{r}_{B/A} = 7.81 \text{ in/sec}^2 $$

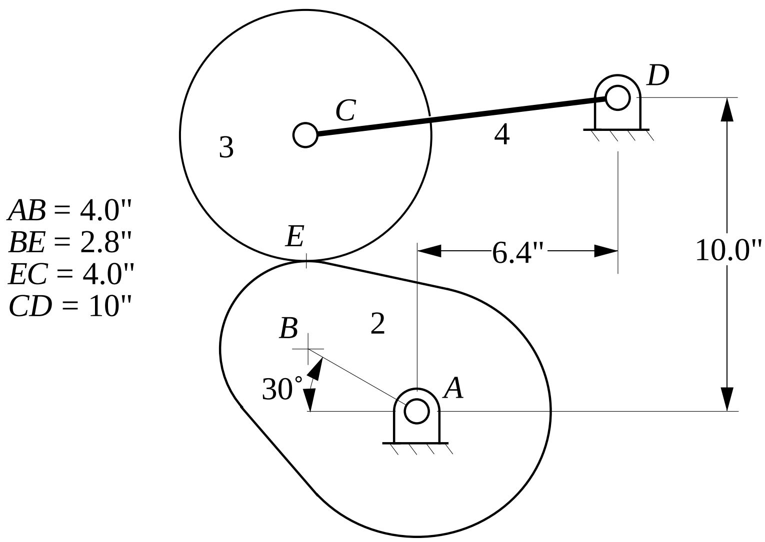
Then,

$$ \left|{}^1\alpha_7\right| = \frac{\left|{}^1\mathbf{a}^t_{B_7/A_7}\right|}{\left|\mathbf{r}_{B/A}\right|} = \frac{7.81}{2} = 3.91 \text{ rad/sec}^2 $$

To determine the direction of ${}^1\alpha_7$, determine the direction that we must rotate $\mathbf{r}_{B/A}$ 90° to get the direction of ${}^1\mathbf{a}^t_{B_7/A_7}$. This is CW.

Problem 3.26

In the mechanism shown below, link 2 is turning CW at the rate of 20 rad/s, and link 3 rolls on link 2. Draw the velocity and acceleration polygons for the mechanism, and determine $\boldsymbol{a}_{C3}$ and α_3.



Position Analysis

Draw the linkage by scale. Locate the relative positions of A and D. Locate point B and then draw the circle arc centered at B. Locate point C by finding the intersection of a circle arc of 10 inches centered at D and a second circle of 6.8 inches centered at B. Finally draw the circle centered at C and of radius 4.0 inches.

Velocity Analysis:

The equations required for the velocity analysis are:

$$
\begin{aligned}
{}^1\boldsymbol{v}_{E_2} &= {}^1\boldsymbol{v}_{E_2/A_2} \\
{}^1\boldsymbol{v}_{E_3} &= {}^1\boldsymbol{v}_{E_2} + {}^1\boldsymbol{v}_{E_3/E_2} \\
{}^1\boldsymbol{v}_{E_3} &= {}^1\boldsymbol{v}_{C_3} + {}^1\boldsymbol{v}_{E_3/C_3} \qquad (1) \\
{}^1\boldsymbol{v}_{C_3} &= {}^1\boldsymbol{v}_{C_4} = {}^1\boldsymbol{v}_{C_4/D_4}
\end{aligned}
$$

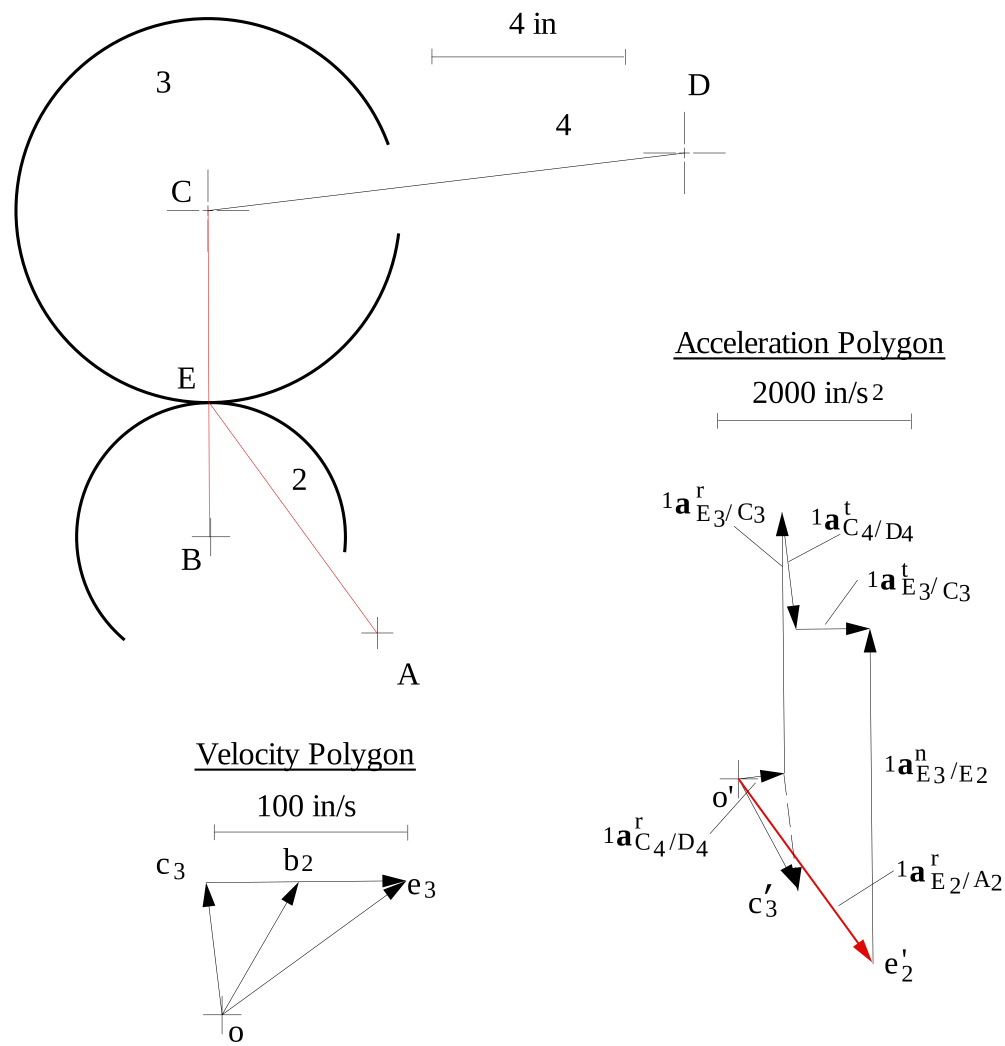
Because ${}^1\boldsymbol{v}_{E_3/E_2} = 0$, ${}^1\boldsymbol{v}_{E_3} = {}^1\boldsymbol{v}_{E_2}$ and

$$
{}^1\boldsymbol{v}_{E_2/A_2} = {}^1\omega_2 \times \boldsymbol{r}_{E_2/A_2} \Rightarrow \left|{}^1\boldsymbol{v}_{E_2/A_2}\right| = \left|{}^1\omega_2\right| \cdot \left|\boldsymbol{r}_{E/A}\right| = 20 \cdot 5.94 = 118.8 \text{ in / s} (\perp \text{ to } \boldsymbol{r}_{E/A})
$$

$$ {}^1\mathbf{v}_{E_3/C_3} = {}^1\omega_3 \times \mathbf{r}_{E_3/C_3} \Rightarrow |{}^1\mathbf{v}_{E_3/C_3}| = |{}^1\omega_3| \cdot |\mathbf{r}_{E_3/C_3}| \; (\perp \text{ to } \mathbf{r}_{E_3/C_3}) $$

$$ {}^1\mathbf{v}_{C_4/D_4} = {}^1\omega_4 \times \mathbf{r}_{C_4/D_4} \Rightarrow |{}^1\mathbf{v}_{C_4/D_4}| = |{}^1\omega_4| \cdot |\mathbf{r}_{C_4/D_4}| \; (\perp \text{ to } \mathbf{r}_{C_4/D_4}) $$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,



$$ {}^1v_{E_3/C_3} = 104 \text{ in / s} $$

or

$$ |{}^1\omega_3| = \frac{|{}^1v_{E_3/C_3}|}{|r_{E_3/C_3}|} = \frac{104}{4} = 26.0 \text{ rad / s CCW} $$

Also,

$$ {}^1v_{C_4/D_4} = 69.33 \text{ in / s} $$

or

$$ |{}^1\omega_4| = \frac{|{}^1v_{C_4/D_4}|}{|r_{C_4/D_4}|} = \frac{69.3}{10} = 6.93 \text{ rad / s CW} $$

Acceleration Analysis:

The equations required for the acceleration analysis are:

$$^1\boldsymbol{a}_{E_2} = {}^1\boldsymbol{a}_{E_2/A_2}$$

$$^1\boldsymbol{a}_{E_3} = {}^1\boldsymbol{a}_{E_2} + {}^1\boldsymbol{a}_{E_3/E_2}$$

$$^1\boldsymbol{a}_{E_3} = {}^1\boldsymbol{a}_{C_3} + {}^1\boldsymbol{a}_{E_3/C_3}$$

$$^1\boldsymbol{a}_{C_3} = {}^1\boldsymbol{a}_{C_4} = {}^1\boldsymbol{a}_{C_4/D_4}$$

$$^1\boldsymbol{a}^r_{E_2/A_2} + {}^1\boldsymbol{a}^n_{E_3/E_2} = {}^1\boldsymbol{a}^r_{C_4/D_4} + {}^1\boldsymbol{a}^t_{C_4/D_4} + {}^1\boldsymbol{a}^r_{E_3/C_3} + {}^1\boldsymbol{a}^t_{E_3/C_3} \tag{2}$$

Now,

$$^1\boldsymbol{a}^r_{E_2/A_2} = {}^1\omega_2 \times ({}^1\omega_2 \times \boldsymbol{r}_{E_2/A_2}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{E_2/A_2}\right| = \left|{}^1\omega_2\right|^2 \cdot \left|\boldsymbol{r}_{E_2/A_2}\right| = 20^2 \cdot 5.94 = 2360 \text{ in / s}^2$$

in the direction opposite to $\boldsymbol{r}_{E_2/A_2}$

$$^1\boldsymbol{a}^n_{E_3/E_2} = {}^1\boldsymbol{a}^n_{E_3/C_3} + {}^1\boldsymbol{a}^n_{C_3/B_2} + {}^1\boldsymbol{a}^n_{B_2/E_2}$$

$$\left|{}^1\boldsymbol{a}^n_{E_3/E_2}\right| = \frac{\left|{}^1\boldsymbol{v}_{E_3/C_3}\right|^2}{\left|\boldsymbol{r}_{E_3/C_3}\right|} - \frac{\left|{}^1\boldsymbol{v}_{C_3/B_2}\right|^2}{\left|\boldsymbol{r}_{C_3/B_2}\right|} + \frac{\left|{}^1\boldsymbol{v}_{B_2/E_2}\right|^2}{\left|\boldsymbol{r}_{B_2/E_2}\right|} = \frac{104^2}{4} - \frac{48.3^2}{6.8} + \frac{56^2}{2.8} = 3500 \text{ in / s}^2$$

in the direction opposite to $\boldsymbol{r}_{E_3/C_3}$

$$^1\boldsymbol{a}^r_{C_4/D_4} = {}^1\omega_4 \times ({}^1\omega_4 \times \boldsymbol{r}_{C_4/D_4}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{C_4/D_4}\right| = \left|{}^1\omega_4\right|^2 \cdot \left|\boldsymbol{r}_{C_4/D_4}\right| = 6.93^2 \cdot 10 = 480 \ in / s^2$$

in the direction opposite to $\mathbf{r}_{C_4/D_4}$

$$^1\mathbf{a}^t_{C_4/D_4} = {}^1\alpha_4 \times \mathbf{r}_{C_4/D_4} \Rightarrow \left|{}^1\mathbf{a}^t_{C_4/D_4}\right| = \left|{}^1\alpha_4\right| \cdot \left|\mathbf{r}_{C_4/D_4}\right| (\perp \text{ to } \mathbf{r}_{C_4/D_4})$$

$$^1\boldsymbol{a}^r_{E_3/C_3} = {}^1\omega_3 \times ({}^1\omega_3 \times \boldsymbol{r}_{E_3/C_3}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{E_3/C_3}\right| = \left|{}^1\omega_3\right|^2 \cdot \left|\boldsymbol{r}_{E_3/C_3}\right| = 26.0^2 \cdot 4 = 2700 \text{ in / s}^2$$

in the direction opposite to $\boldsymbol{r}_{E_3/C_3}$

$$^1\boldsymbol{a}^t_{E_3/C_3} = {}^1\alpha_3 \times \boldsymbol{r}_{E_3/C_3} \Rightarrow \left|{}^1\boldsymbol{a}^t_{E_3/C_3}\right| = \left|{}^1\alpha_3\right| \cdot \left|\boldsymbol{r}_{E_3/C_3}\right| (\perp \ to \ \boldsymbol{r}_{E_3/C_3})$$

Solve Eq. (2) graphically with an acceleration polygon. From the polygon,

$$^1\boldsymbol{a}^t_{E_3/C_3} = 770 \text{ in / s}^2$$

or

$$\left|{}^1\alpha_3\right| = \frac{\left|{}^1\boldsymbol{a}^t_{E_3/C_3}\right|}{\left|\boldsymbol{r}_{E_3/C_3}\right|} = \frac{770}{4} = 192 \text{ rad / s}^2$$

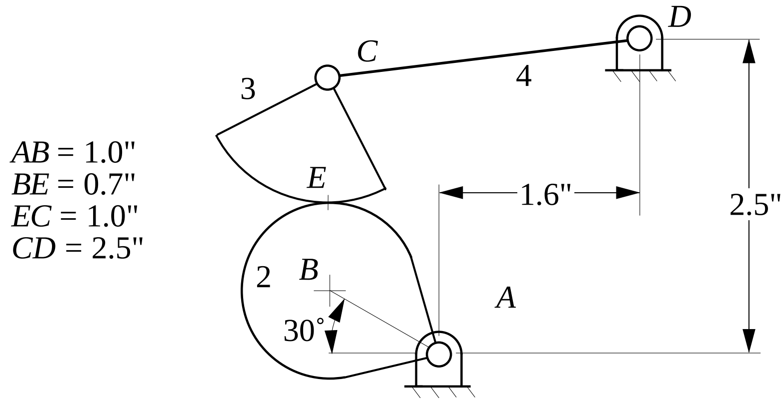
To find the direction, determine the direction that $\mathbf{r}_{E_3/C_3}$ must be rotated 90° to get the direction of ${}^1\boldsymbol{a}^t_{E_3/C_3}$. The direction is clearly CCW.

Also from the polygon,

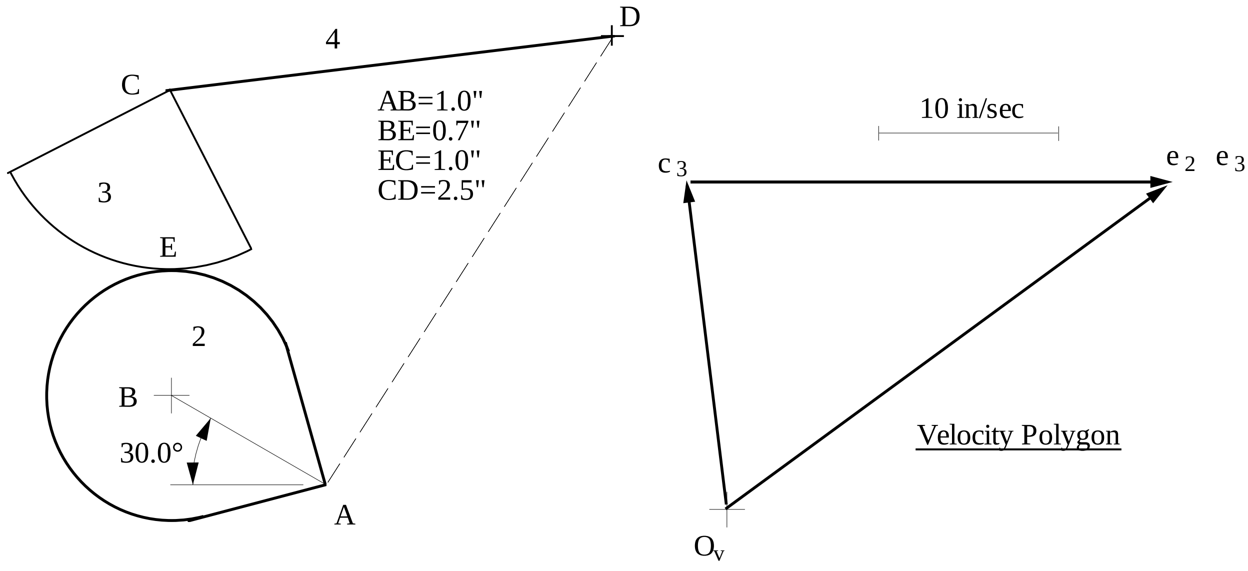
$$^1a_{C_3} = 1290 \text{ in / s}^2$$

Problem 3.27

In the mechanism shown below, Link 2 is turning CW at the rate of 200 rpm. Draw the velocity polygon for the mechanism, and determine $\boldsymbol{v}_{C_3}$ and $\boldsymbol{\omega}_3$.



Position Analysis: Draw the linkage to scale.



Velocity Analysis:

$$\left|{}^1\omega_2\right| = 200\frac{2\pi}{60} = 20.94 \text{ rad / s}$$

$$^1\boldsymbol{v}_{B_2} = {}^1\boldsymbol{v}_{B_2/A_2} = {}^1\omega_2 \times \boldsymbol{r}_{B_2/A_2} \Rightarrow \left|{}^1\boldsymbol{v}_{B_2}\right| = \left|{}^1\omega_2\right|\left|\boldsymbol{r}_{B_2/A_2}\right| = 20.94 \cdot (1) = 20.94 \text{ in / s}$$

$${}^1\boldsymbol{v}_{E_2} = {}^1\boldsymbol{v}_{E_2/A_2} = {}^1\boldsymbol{\omega}_2 \times \boldsymbol{r}_{E_2/A_2} \Rightarrow \left|{}^1\boldsymbol{v}_{E_2}\right| = \left|{}^1\boldsymbol{\omega}_2\right|\left|\boldsymbol{r}_{E_2/A_2}\right| = 20.94 \cdot (1.46) = 30.57 \text{ in/s}$$

$${}^1\boldsymbol{v}_{E_3} = {}^1\boldsymbol{v}_{E_2}$$

$${}^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{E_3} + {}^1\boldsymbol{v}_{C_3/E_3} \tag{1}$$

Now,

$${}^1\boldsymbol{v}_{E_3} = 30.57 \text{ in/s} \quad (\perp \text{ to } \boldsymbol{r}_{E/A})$$

$${}^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{C_4} = {}^1\boldsymbol{\omega}_4 \times \boldsymbol{r}_{C/D} \quad (\perp \text{ to } \boldsymbol{r}_{C/D})$$

$${}^1\boldsymbol{v}_{E_3/C_3} = {}^1\boldsymbol{\omega}_3 \times \boldsymbol{r}_{E/C} \quad (\perp \text{ to } \boldsymbol{r}_{E/C}) \text{ tangent to two circles.}$$

Solve Eq. (1) graphically with a velocity polygon. The velocity directions can be gotten directly from the polygon. The magnitudes are given by:

$$\left|{}^1\boldsymbol{v}_{E_3/C_3}\right| = 27 \text{ in/s} \Rightarrow \left|{}^1\boldsymbol{\omega}_3\right| = \frac{\left|{}^1\boldsymbol{v}_{E_3/C_3}\right|}{\left|\boldsymbol{r}_{E/C}\right|} = \frac{27}{1} = 27 \text{ rad/s}$$

From the directions given in the position and velocity polygons

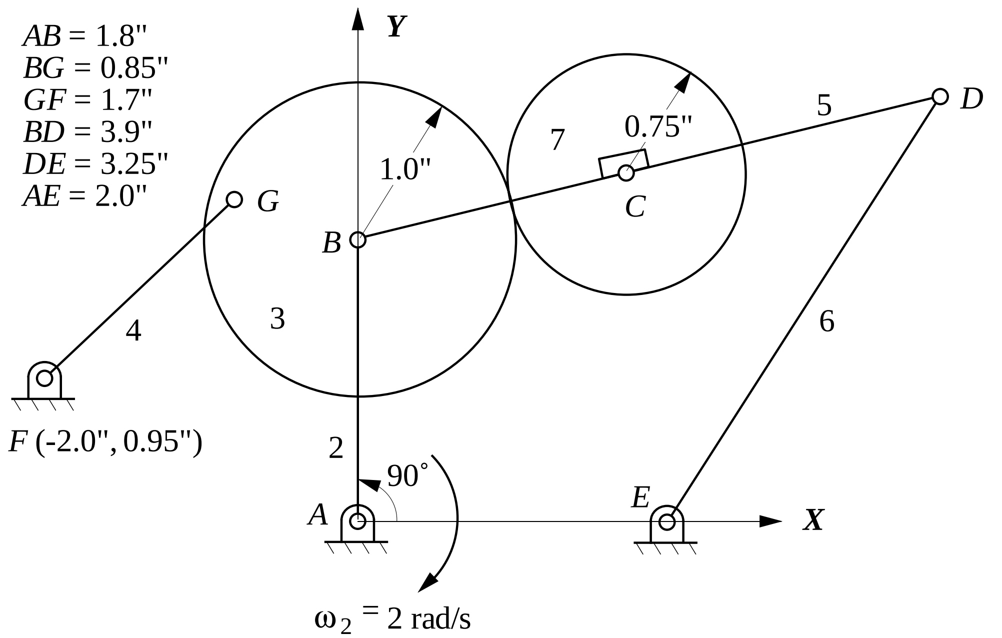
$${}^1\boldsymbol{\omega}_3 = 27 \text{ rad/s CCW}$$

Also, from the velocity polygon,

$$\left|{}^1\boldsymbol{v}_{C_3}\right| = 18.29 \text{ in/s}$$

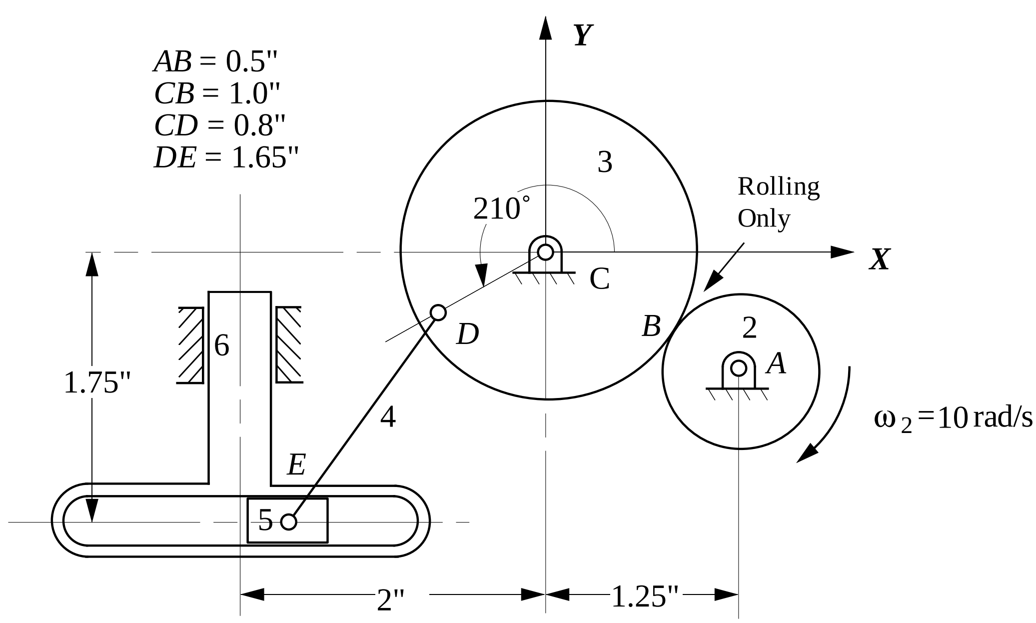
Problem 3.28

Assume that link 7 rolls on link 3 without slipping and find ω_7.



Problem 3.29

In the two degree-of-freedom mechanism shown, ω_2 is given as 10 rad/s CCW. What should the linear velocity of link 6 be so that ω_4 = 5 rad/s CCW?



Position Analysis:

Draw the linkage to scale. Locate A and C first. Then draw the two circles and locate point D. Draw the horizontal line on which E is located and locate E 1.65" from D.

Velocity Analysis:

Compute the velocity of points B_3 and D_3.

$$ {}^1\boldsymbol{v}_{B_2} = {}^1\omega_2 \times \boldsymbol{r}_{B_2/A_2} \Rightarrow \left|{}^1\boldsymbol{v}_{B_2}\right| = \left|{}^1\omega_2\right| \cdot \left|\boldsymbol{r}_{B_2/A_2}\right| = 10 \cdot 0.5 = 5 \text{ in / s} $$

$$ {}^1\boldsymbol{v}_{B_3} = {}^1\boldsymbol{v}_{B_2} = {}^1\omega_3 \times \boldsymbol{r}_{B_3/C_3} \Rightarrow \left|{}^1\omega_3\right| = \frac{\left|{}^1\boldsymbol{v}_{B_3}\right|}{\left|\boldsymbol{r}_{B_3/C_3}\right|} = \frac{5}{1} = 5 \text{ rad / s} $$

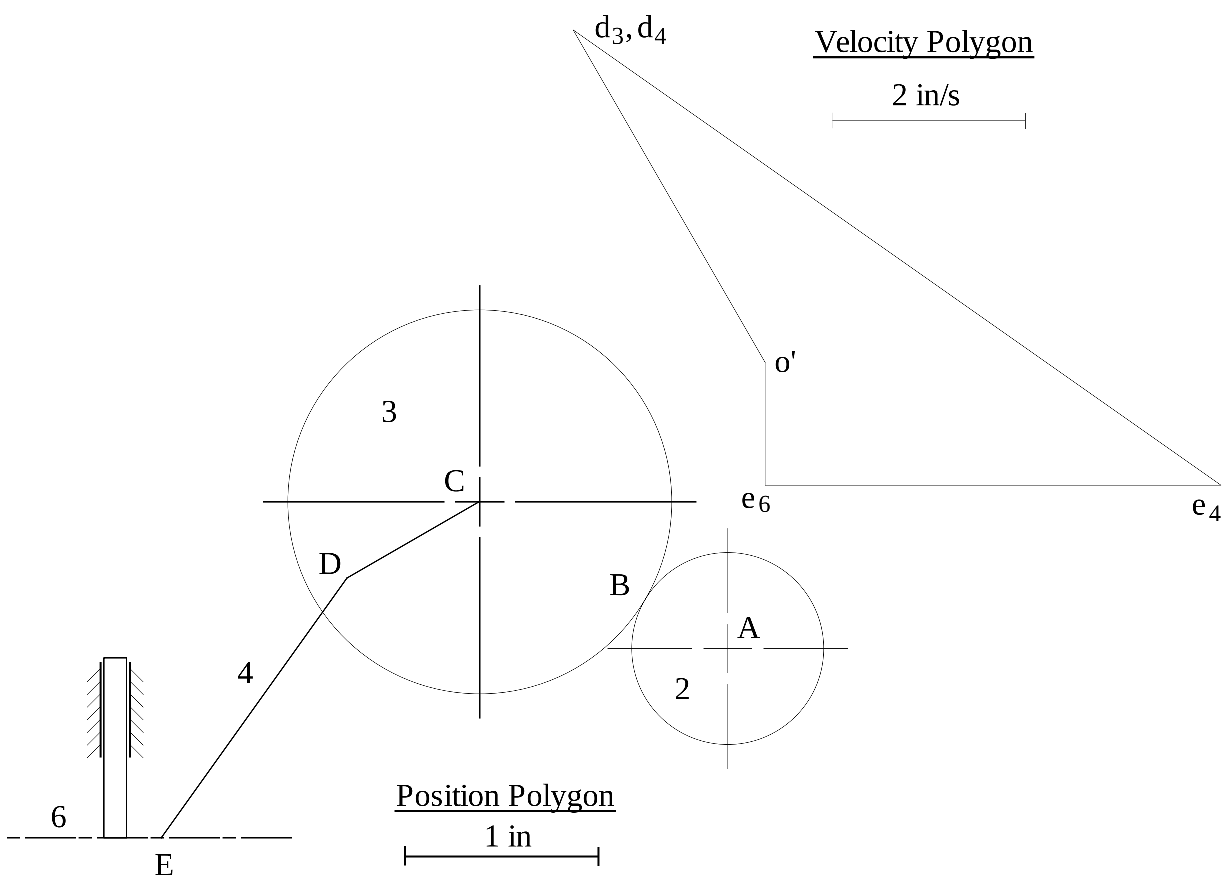
$$ {}^1\boldsymbol{v}_{D_3} = {}^1\boldsymbol{v}_{D_4} = {}^1\omega_3 \times \boldsymbol{r}_{D_3/C_3} \Rightarrow \left|{}^1\boldsymbol{v}_{D_4}\right| = \left|{}^1\omega_3\right| \cdot \left|\boldsymbol{r}_{D_3/C_3}\right| = 5 \cdot 0.8 = 4 \text{ in / s} $$

Next consider the coincident points at E.

$$ {}^1\boldsymbol{v}_{E_4} = {}^1\boldsymbol{v}_{E_5} = {}^1\boldsymbol{v}_{D_4} + {}^1\boldsymbol{v}_{E_4/D_4} $$

$$ {}^1\boldsymbol{v}_{E_4} = {}^1\boldsymbol{v}_{E_6} + {}^1\boldsymbol{v}_{E_4/E_6} $$

$$ {}^1\boldsymbol{v}_{E_6} + {}^1\boldsymbol{v}_{E_4/E_6} = {}^1\boldsymbol{v}_{D_4} + {}^1\boldsymbol{v}_{E_4/D_4} \tag{1} $$



Now,

$${}^1\boldsymbol{v}_{D_4} = 4 \text{ in/s} \ (\perp \text{ to } \boldsymbol{r}_{D_3/C_3})$$

$$\left|{}^1\boldsymbol{v}_{E_4/D_4}\right| = \left|{}^1\omega_4\right| \cdot \left|\boldsymbol{r}_{E_4/D_4}\right| = 5 \cdot 1.65 = 8.25 \text{ in/s} \ (\perp \text{ to } \boldsymbol{r}_{E_4/D_4})$$

${}^1\mathbf{v}_{E_6}$ is in the vertical direction,

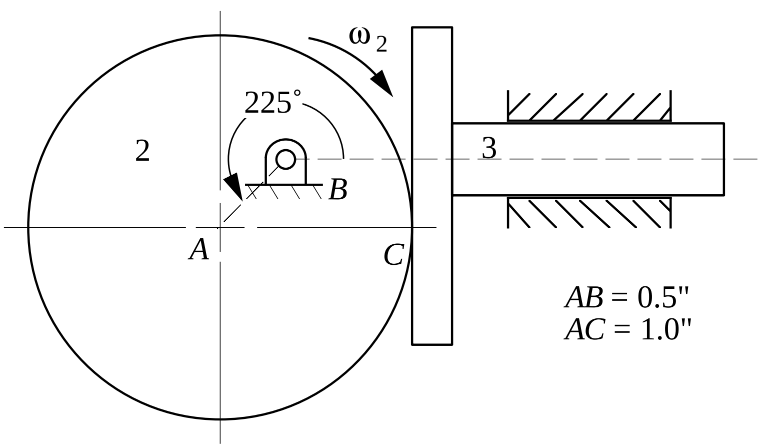
${}^1\mathbf{v}_{E_4/E_6}$ is in the horizontal direction.

Solve Eq. (1) graphically with a velocity polygon. From the polygon

$${}^1\boldsymbol{v}_{E_6} = 1.28 \text{ in/s} \ \text{ in the direction shown on the polygon.}$$

Problem 3.30

In the mechanism shown, ω_2 = 10 rad/s. Determine $\boldsymbol{v}_{C_3/C_2}$ and $\boldsymbol{v}_{C_3}$ using two approaches: 1) Equivalent linkages and 2) Coincident points at C



Solution (Equivalent Linkage)

The equivalent linkage is shown below. For the equivalent linkage, we need only find the velocity and acceleration of point B_2.

The velocity equations which must be solved are:

$${}^1\boldsymbol{v}_{A_2} = {}^1\boldsymbol{v}_{A_2/B_2} = {}^1\boldsymbol{\omega}_2 \times \boldsymbol{r}_{A/B}$$

and

$${}^1\boldsymbol{v}_{A_2} = {}^1\boldsymbol{v}_{A_3} + {}^1\boldsymbol{v}_{A_2/A_3} \qquad (1)$$

Here we have written the velocity equation in terms of the velocity of B_2 relative to B_3 rather than vice versa because we can easily identify the direction of the velocity of B_2 relative to B_3. We also know the direction for the velocity (and acceleration) of B_3.

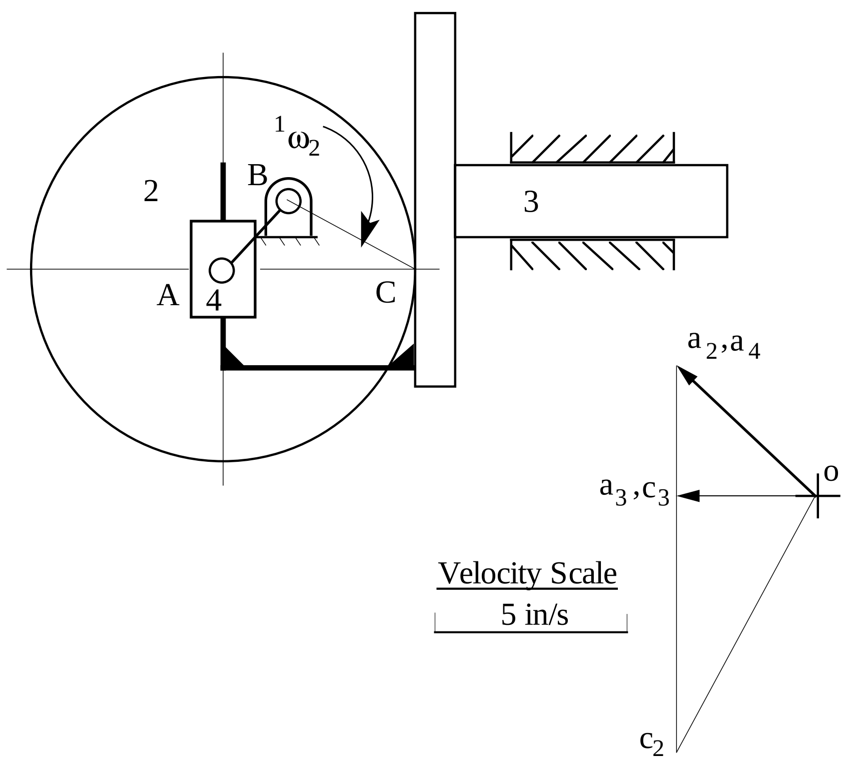
$${}^1\boldsymbol{v}_{A_2} = 10(0.5) = 5 \text{ in/s} (\perp \text{to } AB)$$

${}^1\boldsymbol{v}_{A_3}$ is along the slide direction between link 3 and the frame

${}^1\boldsymbol{v}_{A_2/A_3}$ is along the slide direction between link 4 and link 3

From the velocity polygon,

${}^1\boldsymbol{v}_{A_3} = 3.61$ in/s in the direction shown.



Solution (Direct Approach)

To analyze the problem, we can determine the velocity and acceleration of any point on link 3 because all points on link 3 have the same velocity and the same acceleration. The point to choose is the contact point C_3. To solve for the velocity and acceleration of C_3, first find the velocity of point C_2. Then write the relative velocity expression between points C_2 and C_3 and solve for the velocity of C_3.

Velocity Analysis

The relevant equations are:

$${}^1\boldsymbol{v}_{C_2} = {}^1\boldsymbol{v}_{C_2/B_2} = {}^1\boldsymbol{\omega}_2 \times \boldsymbol{r}_{C/B}$$

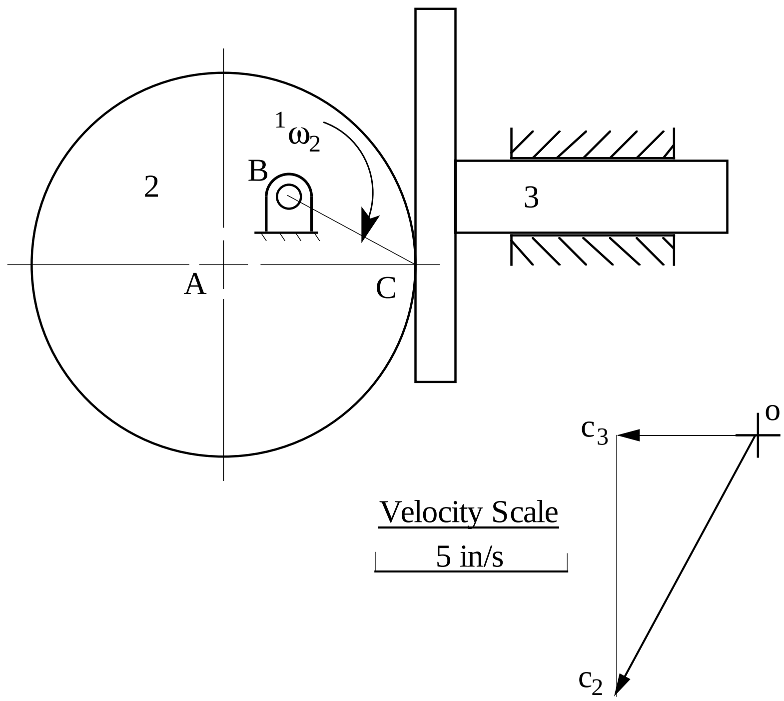
and

$^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{C_2} + {}^1\boldsymbol{v}_{C_3/C_2}$

$^1\boldsymbol{v}_{C_2/B_2} = 10(0.737) = 7.37 \text{ in/s } (\perp \text{ to } AC)$

$^1\boldsymbol{v}_{C_3}$ is along the slide direction between link 3 and the frame

$^1\boldsymbol{v}_{C_3/C_2}$ is along the face of link 3

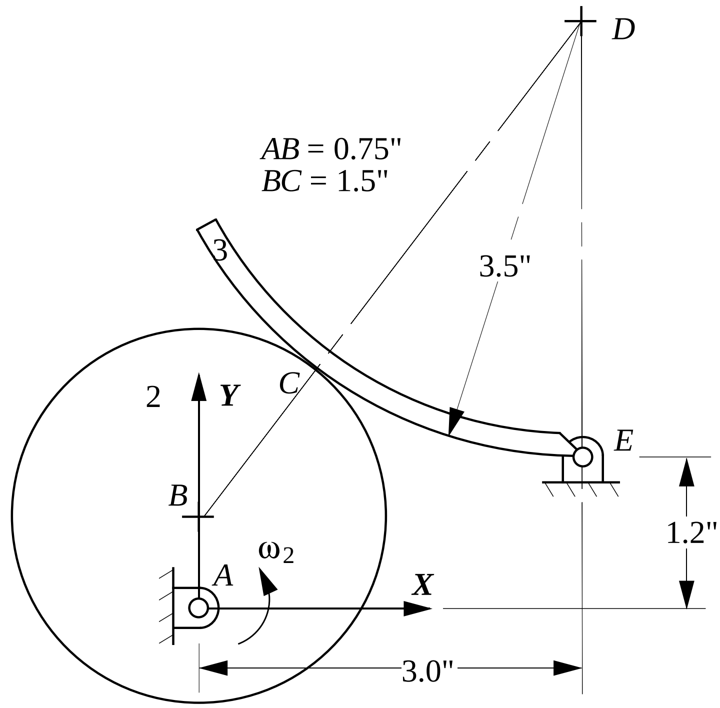


From the velocity polygon,

$\boldsymbol{v}_{C_3}$ = 35 in/s in the direction shown.

Problem 3.31

In the mechanism shown, ω_2 = 20 rad/s CCW. At the instant shown, point D, the center of curvature of link 3, lies directly above point E, and point B lies directly above point A. Determine v_{C_3/C_2} and ω_3 using: 1) equivalent linkages and 2) Coincident points at C.



Position Analysis

Draw the linkage to scale. First locate pivots A and E. Then locate point B and draw link 2. Next locate point D and draw link 3.

Solution (Equivalent Linkage)

The equivalent linkage is a fourbar linkage involving points A, B, D, and E. For the equivalent linkage, we need only find the velocity of point B_2.

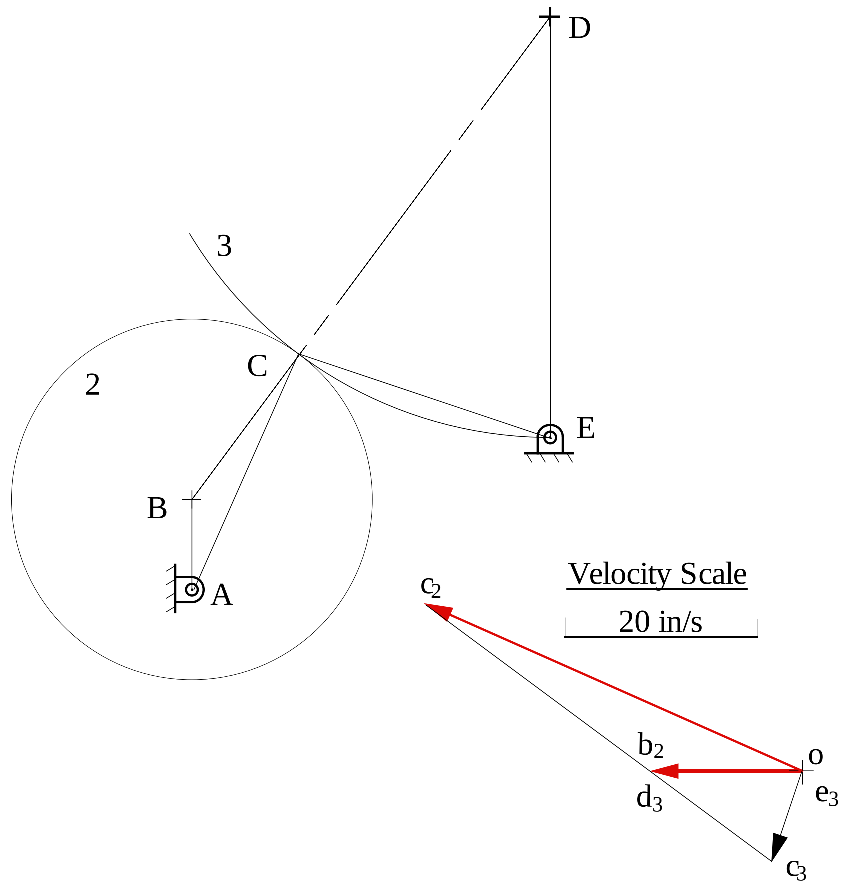
The velocity equations which must be solved are:

$$^1v_{B_2} = {}^1v_{B_2/A_2} = {}^1\omega_2 \times r_{B/A}$$

$$^1v_{D_x} = {}^1v_{B_x} + {}^1v_{D_x/B_x} = {}^1v_{D_3} = {}^1v_{E_3} + {}^1v_{D_3/E_3}$$

Where x is the imaginary link between points B and D. Simplifying and combining terms

$$^1v_{D_3/E_3} = {}^1v_{B_2} + {}^1v_{D_x/B_x} \tag{1}$$



Now

$$\left|{}^1\boldsymbol{v}_{B_2}\right| = \left|{}^1\omega_2\right|\left|\boldsymbol{r}_{B/A}\right| = 20(0.75) = 15 \text{ in/s} \ (\perp \text{ to } AB)$$

$${}^1\boldsymbol{v}_{D_3/E_3} = {}^1\boldsymbol{\omega}_3 \times \boldsymbol{r}_{D/E} \text{ is perpendicular to DE}$$

$${}^1\boldsymbol{v}_{D_x/B_x} \text{ is perpendicular to BD}$$

From the velocity polygon,

$${}^1\boldsymbol{v}_{D_3/E_3} = {}^1\boldsymbol{v}_{B_2} = 15 \text{ in/s} \ (\perp \text{ to } AB)$$

Therefore,

$$\left|{}^1\omega_3\right| = \frac{\left|{}^1\boldsymbol{v}_{D_3/E_3}\right|}{\left|\boldsymbol{r}_{D/E}\right|} = \frac{15}{3.5} = 4.28 \text{ rad/s}$$

To determine the direction of $\left|{}^1\omega_3\right|$, determine the direction that we must rotate $\boldsymbol{r}_{D/E}$ 90˚ to get the direction of ${}^1\boldsymbol{v}_{D_3/E_3}$. This is counterclockwise

To find ${}^1\boldsymbol{v}_{C_3/C_2}$, compute ${}^1\boldsymbol{v}_{C_2}$ and ${}^1\boldsymbol{v}_{C_3}$ and use

$$ {}^1\boldsymbol{v}_{C_3/C_2} = {}^1\boldsymbol{v}_{C_3} - {}^1\boldsymbol{v}_{C_2} $$

Both ${}^1\boldsymbol{v}_{C_2}$ and ${}^1\boldsymbol{v}_{C_3}$ may be determined by velocity image (or computed directly). From the polygon,

$$ {}^1\boldsymbol{v}_{C_3/C_2} = 44.8 \text{ in/s} $$

in the direction indicated by the polygon.

<u>Solution (Direct Approach)</u>

First find the velocity of point C_2. Then write the relative velocity expression between points C_2 and C_3 and solve for the velocity of C_3.

The relevant equations are:

$$ {}^1\boldsymbol{v}_{C_2} = {}^1\boldsymbol{v}_{C_2/A_2} = {}^1\boldsymbol{\omega}_2 \times \boldsymbol{r}_{C/A} $$

and

$$ {}^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{C_2} + {}^1\boldsymbol{v}_{C_3/C_2} = {}^1\boldsymbol{v}_{C_3/E_3} $$

Where

$$ {}^1\boldsymbol{v}_{C_2/A_2} = 20(2.158) = 43.16 \text{ in/s } (\perp \text{ to } AC) $$

${}^1\boldsymbol{v}_{C_3/E_3}$ is perpendicular to CE

${}^1\boldsymbol{v}_{C_3/C_2}$ is along the tangent to the contact point (perpendicular to BD)

From the velocity polygon,

$$ {}^1\boldsymbol{v}_{C_3/C_2} = 44.8 \text{ in/s} $$

and

$$ {}^1\boldsymbol{v}_{C_3/E_3} = 9.84 \text{ in/s} $$

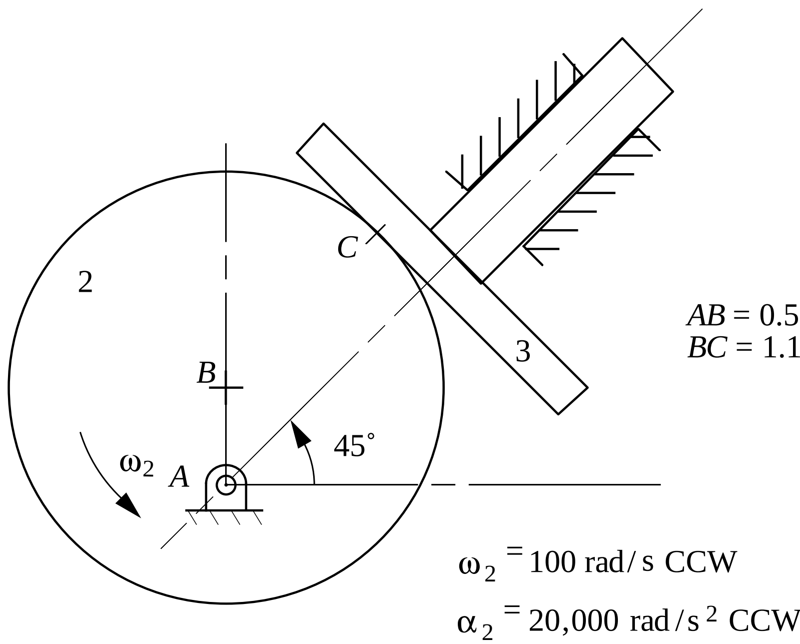
Therefore,

$$ |{}^1\boldsymbol{\omega}_3| = \frac{|{}^1\boldsymbol{v}_{C_3/E_3}|}{|\boldsymbol{r}_{C/E}|} = \frac{9.84}{2.30} = 4.28 \text{ rad/s} $$

To determine the direction of $|{}^1\boldsymbol{\omega}_3|$, determine the direction that we must rotate $\boldsymbol{r}_{C/E}$ 90° to get the direction of ${}^1\boldsymbol{v}_{C_3/E_3}$. This is counterclockwise

Problem 3.32

In the position shown, find the velocity and acceleration of link 3 using: 1) equivalent linkages and 2) Coincident points at *C*



Solution (Equivalent Linkage)

This problem is similar to Example 2.11 except for a nonzero value for the acceleration. The equivalent linkage is shown below. For the equivalent linkage, we need only find the velocity and acceleration of point B_2.

Velocity Analysis

The velocity equations which must be solved are:

$${}^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{B_2/A_2} = {}^1\boldsymbol{\omega}_2 \times \mathbf{r}_{B/A}$$

and

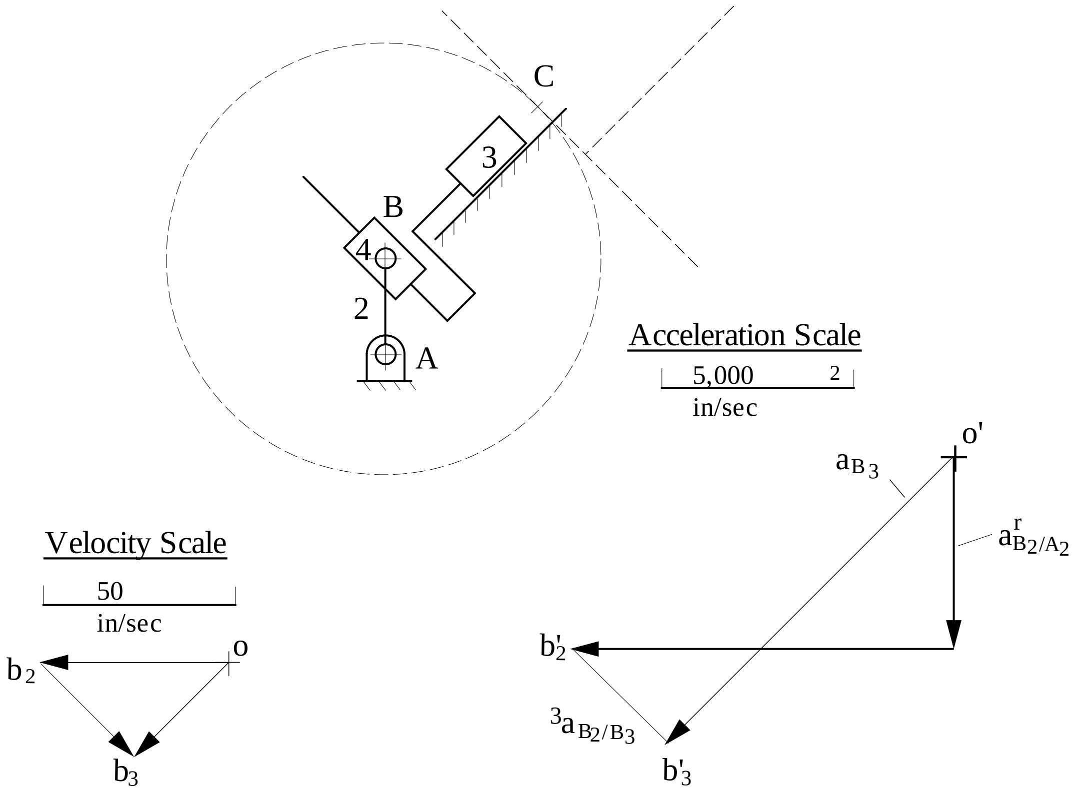
$${}^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{B_3} + {}^1\mathbf{v}_{B_2/B_3} \tag{1}$$

Here we have written the velocity equation in terms of the velocity of B_2 relative to B_3 rather than vice versa because we can easily identify the direction of the velocity of B_2 relative to B_3. We also know the direction for the velocity (and acceleration) of B_3.

$${}^1\mathbf{v}_{B_2} = 100(0.5) = 50 \ \text{in/s} \ (\perp \text{to AB})$$

${}^1\mathbf{v}_{B_3}$ is along the slide direction between link 3 and the frame

${}^1\mathbf{v}_{B_2/B_3}$ is along the slide direction between link 4 and link 3



From the velocity polygon,

$\mathbf{v}_{B3}$ = 35 in/s in the direction shown.

Acceleration Analysis

The acceleration equations which must be solved are:

$$ {}^{1}\mathbf{a}_{B2} = {}^{1}\mathbf{a}_{B2/A2} = {}^{1}\mathbf{a}^{r}_{B2/A2} + {}^{1}\mathbf{a}^{t}_{B2/A2}\,, $$

$$ {}^{1}\mathbf{a}_{B2} = {}^{1}\mathbf{a}_{B3} + {}^{1}\mathbf{a}_{B2/B3} = {}^{1}\mathbf{a}_{B3} + \mathbf{a}^{c}_{B2/B3} + {}^{3}\mathbf{a}_{B2/B3}\,, $$

and

$$ \mathbf{a}^{c}_{B2/B3} = 2\ {}^{1}\boldsymbol{\omega}_{3} \times {}^{3}\mathbf{v}_{B2/B3} = 0 $$

The Coriolis term is a function of velocities only and can be computed; however, Links 3 and 4 simply translate making ${}^{1}\boldsymbol{\omega}_{3} = 0$. Therefore, the acceleration expression becomes

$$ {}^{1}\mathbf{a}_{B2} = {}^{1}\mathbf{a}_{B3} + {}^{3}\mathbf{a}_{B2/B3} $$

Therefore, the equation has only two unknowns (once ${}^{1}\mathbf{a}_{B2}$ is computed), and the equation can be solved for ${}^{1}\mathbf{a}_{B3}$ and ${}^{3}\mathbf{a}_{B2/B3}$.

$$ \left|{}^{1}\mathbf{a}^{r}_{B2/A2}\right| = \left|{}^{1}\boldsymbol{\omega}_{2}\right|^{2}\left|\mathbf{r}_{B/A}\right| = 100^{2} \cdot 0.5 = 5000 \text{ in/sec}^{2} \text{ opposite to } \mathbf{r}_{B/A} $$

$$ \left|{}^{1}\mathbf{a}^{t}_{B2/A2}\right| = \left|{}^{1}\boldsymbol{\alpha}_{2}\right|\left|\mathbf{r}_{B/A}\right| = 20000 \cdot 0.5 = 10000 \text{ in/sec}^{2} \ (\perp \text{ to AB}) $$

$^1\mathbf{a}_{B3}$ is along the slide direction between link 3 and the frame

$^3\mathbf{a}_{B2/B3}$ is along the slide direction between link 4 and link 3

Therefore, the equation has only two unknowns (once $^1\mathbf{a}_{B2}$ is computed), and the equation can be solved for $^1\mathbf{a}_{B3}$ and $^3\mathbf{a}_{B2/B3}$.

The result is shown in the acceleration polygon. From the polygon,

a_{B3} = 10,200 in/sec^2 in the direction shown.

<u>Solution (Direct Approach)</u>

This problem is similar to Example 2.10 except for a nonzero value for the acceleration. To analyze the problem, we can determine the velocity and acceleration of any point on link 3 because <u>all</u> points on link 3 have the same velocity and the same acceleration. The point to choose is the contact point C_3. To solve for the velocity and acceleration of C_3, first find the velocity of point C_2. Then write the relative velocity expression between points C_2 and C_3 and solve for the velocity of C_3.

<u>Velocity Analysis</u>

The relevant equations are:

$$^1\mathbf{v}_{C2} = {}^1\mathbf{v}_{C2/A2} = {}^1\boldsymbol{\omega}_2 \times \mathbf{r}_{C/A}$$

and

$$^1\mathbf{v}_{C3} = {}^1\mathbf{v}_{C2} + {}^1\mathbf{v}_{C3/C2}$$

$$^1\mathbf{v}_{C2} = 100(1.52) = 152 \text{ in/sec} (\perp \text{ to AC})$$

$^1\mathbf{v}_{C3}$ is along the slide direction between link 3 and the frame

$^1\mathbf{v}_{C3/C2}$ is along the face of link 3

Solve the velocity equation and then solve for the velocity of B_2 by image. This will be needed for the acceleration analysis.

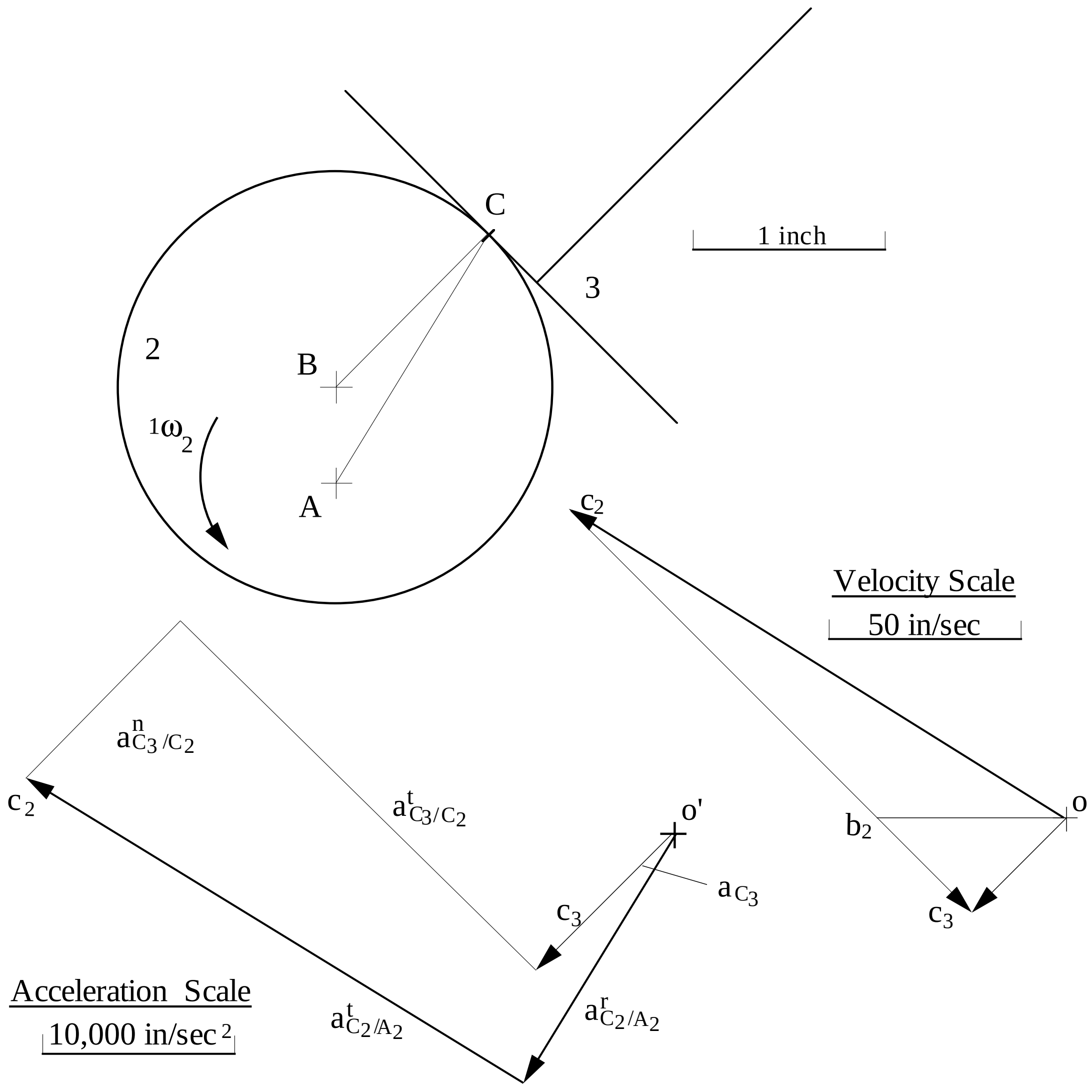
$\mathbf{v}_{C3}$ = 35 in/s in the direction shown.

<u>Acceleration Analysis</u>

The acceleration equations which must be solved are:

$$^1\mathbf{a}_{C2} = {}^1\mathbf{a}_{C2/A2} = {}^1\mathbf{a}^n_{C2/A2} + {}^1\mathbf{a}^t_{C2/A2},$$

$$^1\mathbf{a}_{C3} = {}^1\mathbf{a}_{C2} + {}^1\mathbf{a}_{C3/C2} = {}^1\mathbf{a}_{C2} + {}^1\mathbf{a}^t_{C3/C2} + {}^1\mathbf{a}^n_{C3/C2},$$



and

$$
{}^1\mathbf{a}^n_{C_3/C_2} = {}^1\mathbf{a}^n_{C_3/D_3} + {}^1\mathbf{a}^n_{D_3/B_2} + {}^1\mathbf{a}^n_{B_2/C_2} = \frac{|{}^1\mathbf{v}_{C_3/D_3}|^2}{|\infty|} + \frac{|{}^1\mathbf{v}_{D_3/B_2}|^2}{|\infty|} + \frac{|{}^1\mathbf{v}_{B_2/C_2}|^2}{|\mathbf{r}_{B/C}|} = \frac{|{}^1\mathbf{v}_{B_2/C_2}|^2}{|\mathbf{r}_{B/C}|}
$$

Where D_3 is the center of curvature of the cam follower surface and is located at infinity. The final equation which must be solved is

$$
{}^1\mathbf{a}_{C_3} = {}^1\mathbf{a}^n_{C_2/A_2} + {}^1\mathbf{a}^t_{C_2/A_2} + {}^1\mathbf{a}^t_{C_3/C_2} + {}^1\mathbf{a}^n_{B_2/C_2}
$$

where

$$
\left|{}^1\mathbf{a}^n_{C_2/A_2}\right| = |{}^1\omega_2|^2 |\mathbf{r}_{C/A}| = 100^2 \cdot 1.52 = 15{,}200 \text{ in / sec}^2 \text{ opposite to } \mathbf{r}_{C/A}
$$

$$
\left|{}^1\mathbf{a}^t_{C_2/A_2}\right| = |{}^1\alpha_2| |\mathbf{r}_{C/A}| = 20000 \cdot 1.52 = 30{,}400 \text{ in / sec}^2 \ (\perp \text{ to AC})
$$

${}^1\mathbf{a}_{C_3}$ is along the slide direction between link 3 and the frame

$$\left|{}^1\mathbf{a}^n_{B_2/C_2}\right| = \frac{\left|{}^1\mathbf{v}_{B_2/C_2}\right|^2}{\left|\mathbf{r}_{B/C}\right|} = \frac{|112.9|^2}{1.11} = 11{,}480 \text{ in}/\text{s}^2 \text{ from B to C}$$

${}^1\mathbf{a}^t_{C_3/C_2}$ is along the along the face of link 3

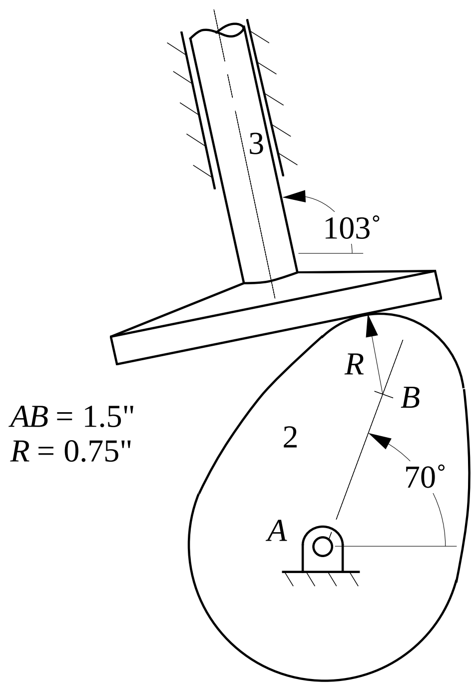
Therefore, the equation has only two unknowns (once ${}^1\mathbf{a}_{C2}$ is computed), and the equation can be solved for ${}^1\mathbf{a}_{C3}$ and ${}^1\mathbf{a}^t_{C_3/C_2}$.

The result is shown in the acceleration polygon. From the polygon,

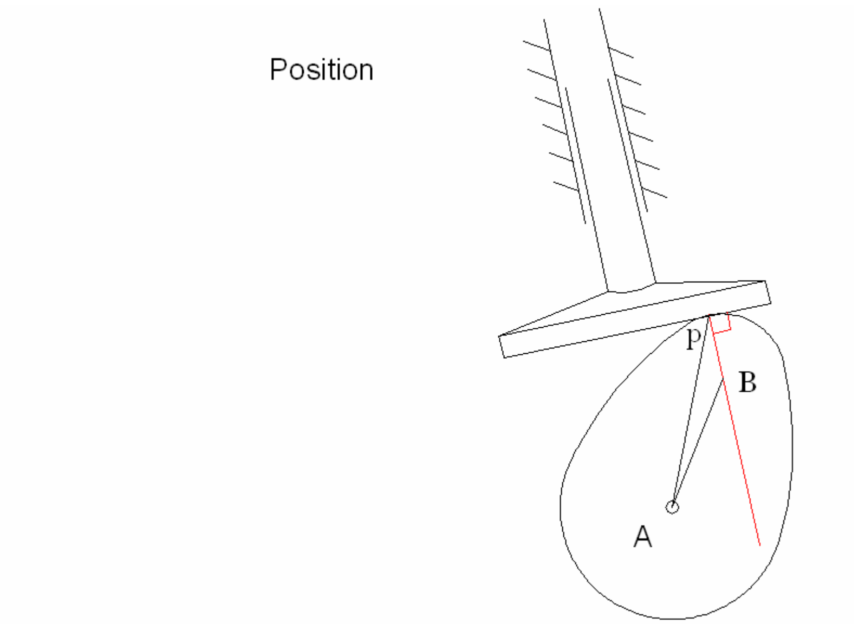
a_{C_3} = 10,200 in/sec^2 in the direction shown.

Problem 3.33

Locate all of the instant centers in the mechanism shown below. If the cam (link 2) is turning CW at the rate of 900 rpm, determine the linear velocity of the follower.



Position



Velocity of the Follower

Convert the angular velocity from "rpm" to "rad/s"

$$ {}^{1}\omega_{2} = 900\ rpm = \frac{900(2\pi)}{60\sec} = 94.25\ rad/s \quad CW $$

and

$$ \mathbf{v}_{P_2} = \mathbf{v}_{A_2} + \mathbf{v}_{P_2/A_2} $$

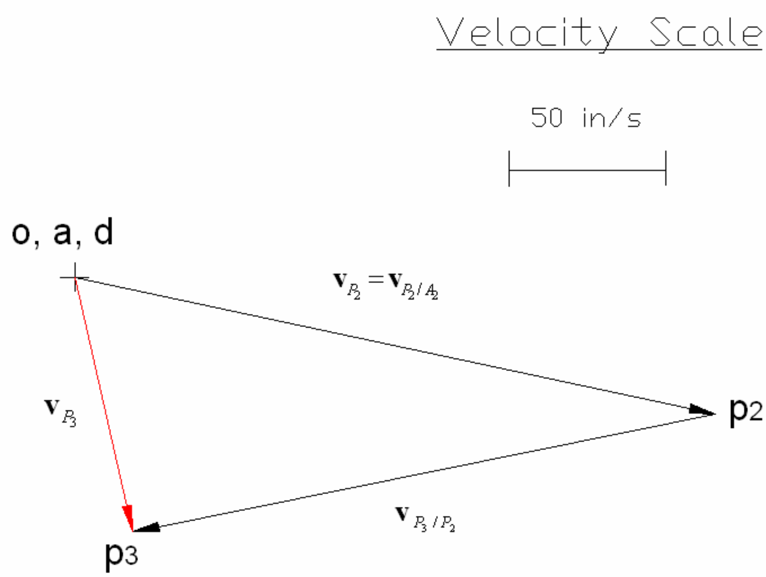
$$ \mathbf{v}_{P_3} = \mathbf{v}_{P_2} + \mathbf{v}_{P_3/P_2} $$

Now,

$$\mathbf{v}_{P_2} = \mathbf{v}_{A_2} + \mathbf{v}_{P_2/A_2} = 0 + \omega_2 \times \mathbf{r}_{P/A} = (94.25\ rad/s)(2.18\,in) = 205.47\ in/s \quad (\perp \text{ to } \mathbf{r}_{P/A})$$

$\mathbf{v}_{P_3}$: $\perp$ to the follower face

$\mathbf{v}_{P_3/P_2}$: // to the follower face

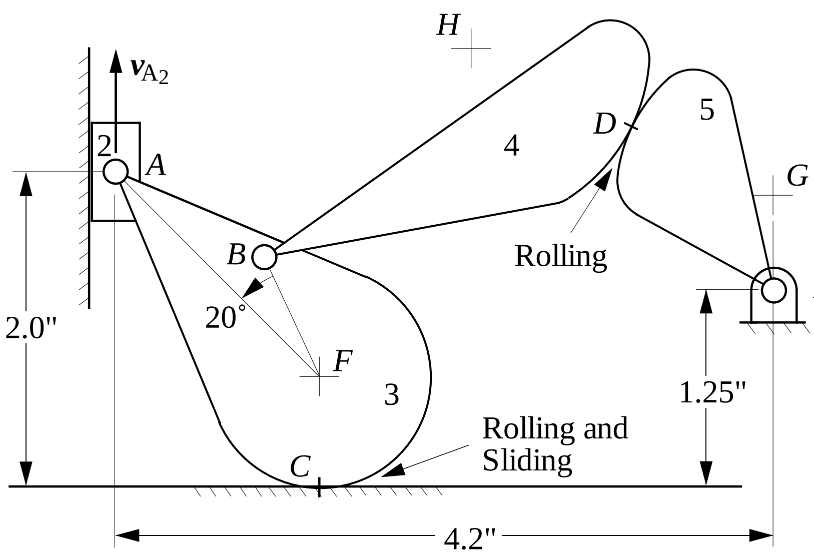


From the polygon,

$$\left|\mathbf{v}_{P_3}\right| = 81.17\, in/s$$

Problem 3.34

In the mechanism shown, $\boldsymbol{v}_{A_2} = 20$ in/s. Find $\boldsymbol{\omega}_5$ and ${}^3\boldsymbol{\omega}_4$. Indicate on link 4 the point which has zero velocity. In the drawing, *H* and *G* are the centers of curvature of links 4 and 5, respectively, corresponding to location *D*. *F* is the center of curvature of link 3 corresponding to *C*. Also, point *G* lies exactly above point *E*.



$CF = 0.7"$
$BF = 0.84"$
$AF = 1.85"$
$BH = 1.88"$
$DH = 1.14"$
$DG = 1"$
$EG = 0.6"$

Position Analysis

Locate point points A and E. Draw a line 1.3 inches below A and locate point F at a distance of 1.85" from A. Next locate C by drawing a circle arc about F. Then locate B at an angle of 20° from the line AF. Locate center G and then locate center H. Draw the two circle arcs to locate D.

Velocity Analysis:

$${}^1\boldsymbol{v}_{A_3} = {}^1\boldsymbol{v}_{A_2}$$

$${}^1\boldsymbol{v}_{F_3} = {}^1\boldsymbol{v}_{A_3} + {}^1\boldsymbol{v}_{F_3/A_3} \tag{1}$$

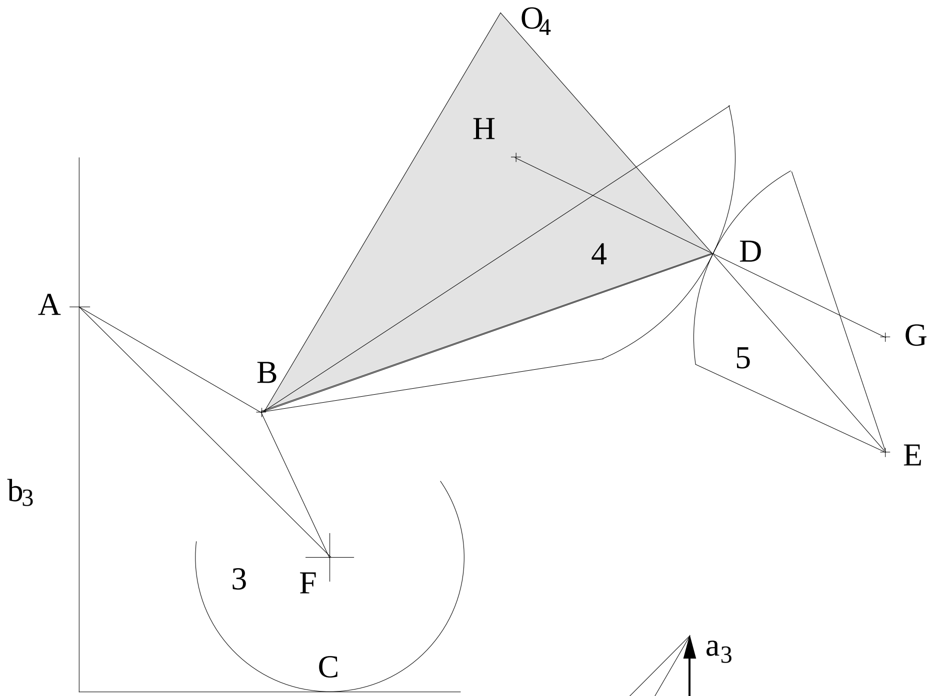
Now,

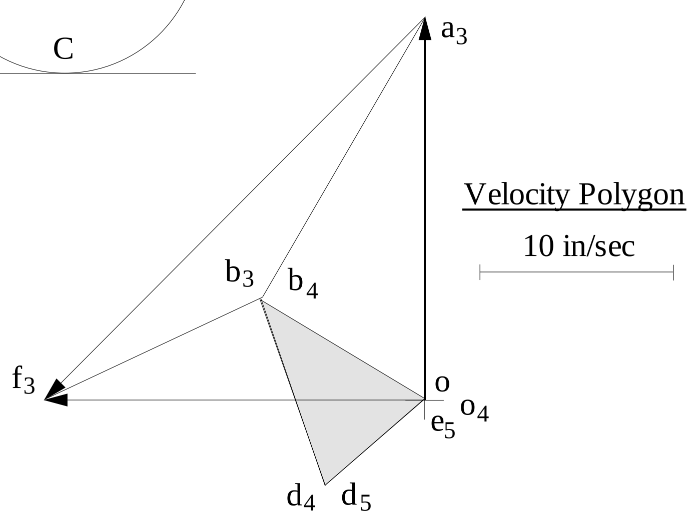
${}^1\boldsymbol{v}_{F_3}$ in horizontal direction

$${}^1\boldsymbol{v}_{F_3/A_3} = {}^1\boldsymbol{\omega}_3 \times |\boldsymbol{r}_{F/A}| \Rightarrow |{}^1\boldsymbol{v}_{F_3/A_3}| = |{}^1\boldsymbol{\omega}_3| \cdot |\boldsymbol{r}_{F/A}| \; (\perp \; to \; \boldsymbol{r}_{A/F})$$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$$|{}^1\boldsymbol{v}_{F_3/A_3}| = 28.11 \text{ in / s}$$





Continuing with the analysis

$${}^1\mathbf{v}_{D4} = {}^1\mathbf{v}_{D5}$$

$${}^1\mathbf{v}_{D4} = {}^1\mathbf{v}_{B4} + {}^1\mathbf{v}_{D4/B4}$$

$${}^1\mathbf{v}_{D5} = {}^1\mathbf{v}_{E5} + {}^1\mathbf{v}_{D5/E5}$$

and

$${}^1\mathbf{v}_{D5/E5} = {}^1\mathbf{v}_{B4} + {}^1\mathbf{v}_{D4/B4} \tag{2}$$

Now,

$${}^1\boldsymbol{v}_{D4/B4} = {}^1\boldsymbol{\omega}_4 \times \boldsymbol{r}_{D/B} \Rightarrow \left|{}^1\boldsymbol{v}_{D4/B4}\right| = \left|{}^1\boldsymbol{\omega}_4\right| \cdot \left|\boldsymbol{r}_{D/B}\right| \; (\perp \text{ to } \boldsymbol{r}_{D/B})$$

$${}^1\boldsymbol{v}_{D_5/E_5} = {}^1\boldsymbol{\omega}_5 \times \boldsymbol{r}_{D/E} \Rightarrow \left|{}^1\boldsymbol{v}_{D_5/E_5}\right| = \left|{}^1\boldsymbol{\omega}_5\right| \cdot \left|\boldsymbol{r}_{D/E}\right| \ (\perp \text{ to } \boldsymbol{r}_{D/E})$$

Solve Eq. (2) graphically with a velocity polygon. From the polygon,

$$\left|{}^1\boldsymbol{v}_{D_4/B_4}\right| = 10.30 \text{ in / s}$$

$$\left|{}^1\boldsymbol{v}_{D_5/E_5}\right| = 6.91 \text{ in / s}$$

Solving for the angular velocities

$$\left|{}^1\boldsymbol{\omega}_3\right| = \frac{\left|{}^1\boldsymbol{v}_{F_3/A_3}\right|}{\left|\boldsymbol{r}_{F/A}\right|} = \frac{28.11}{1.85} = 15.19 \text{ rad / s}$$

$$\left|{}^1\boldsymbol{\omega}_4\right| = \frac{\left|{}^1\boldsymbol{v}_{D_4/B_4}\right|}{\left|\boldsymbol{r}_{D/B}\right|} = \frac{10.3}{2.495} = 4.128 \text{ rad / s}$$

$$\left|{}^1\boldsymbol{\omega}_5\right| = \frac{\left|{}^1\boldsymbol{v}_{D_5/E_5}\right|}{\left|\boldsymbol{r}_{D/E}\right|} = \frac{6.91}{1.354} = 5.10 \text{ rad / s}$$

To determine the direction for ${}^1\boldsymbol{\omega}_3$, determine the direction that $\mathbf{r}_{F/A}$ must be rotated to be in the direction of ${}^1\boldsymbol{v}_{F_3/A_3}$. From the polygon, this direction is CW.

To determine the direction for ${}^1\boldsymbol{\omega}_4$, determine the direction that $\mathbf{r}_{D/B}$ must be rotated to be in the direction of ${}^1\boldsymbol{v}_{D_4/B_4}$. From the polygon, this direction is CW.

To determine the direction for ${}^1\boldsymbol{\omega}_5$, determine the direction that $\mathbf{r}_{D/E}$ must be rotated to be in the direction of ${}^1\boldsymbol{v}_{D_5/E_5}$. From the polygon, this direction is CCW.

To find ${}^3\boldsymbol{\omega}_4$ use the chain rule for angular velocities. Then,

$${}^3\boldsymbol{\omega}_4 = {}^1\boldsymbol{\omega}_4 - {}^1\boldsymbol{\omega}_3 = 4.128\ CW - 15.19\ CW = -11.06 \text{ rad / s } \ CW$$

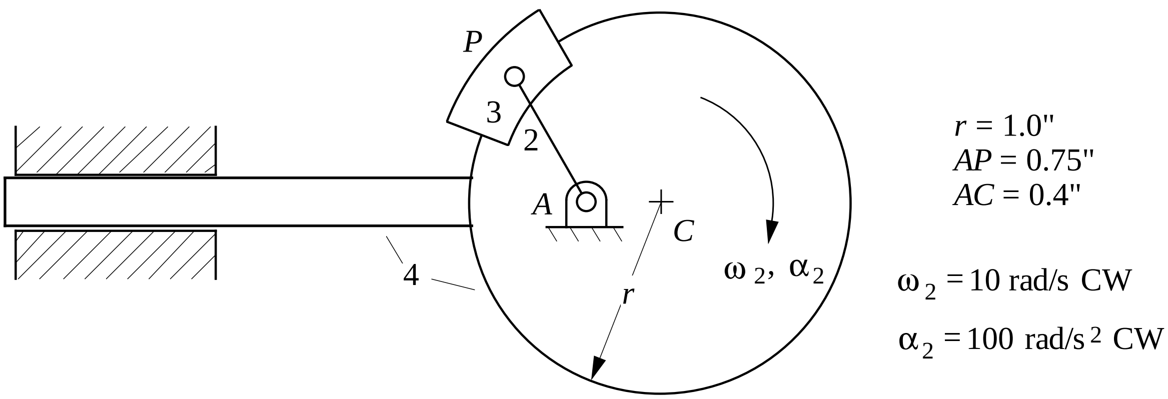
or

$${}^3\boldsymbol{\omega}_4 = 11.06 \text{ rad / s CCW}$$

To find the point on link 4 which has zero velocity, use the velocity image. The resulting location is shown in the figure.

Problem 3.35

On the mechanism shown, link 4 slides on link 1, and link 3 slides on link 4 around the circle arc. Link 2 is pinned to links 1 and 3 as shown. Determine the location of the center of curvature of the path that point P_4 traces on link 2.



Velocity Analysis:

The center of curvature of the path involves only velocity and position information. Therefore, we need perform only a velocity analysis.

$$^1\boldsymbol{v}_{P_2} = {}^1\boldsymbol{v}_{P_3} = {}^1\boldsymbol{v}_{P_2/A_2} = {}^1\boldsymbol{v}_{P_4} + {}^1\boldsymbol{v}_{P_2/P_4} \quad (1)$$

Now,

$$^1\boldsymbol{v}_{P_2/A_2} = {}^1\boldsymbol{\omega}_2 \times \boldsymbol{r}_{P_2/A_2} \Rightarrow \left|{}^1\boldsymbol{v}_{P_2/A_2}\right| = \left|{}^1\omega_2\right|\left|\boldsymbol{r}_{P/A}\right| = 10\cdot 0.75 = 7.5 \text{ in / s} (\perp \text{ to } \boldsymbol{r}_{P/A})$$

$^1\boldsymbol{v}_{P_4}$ in the horizontal direction

$^1\boldsymbol{v}_{P_2/P_4}$ ($\perp$ to $\boldsymbol{r}_{P/C}$)

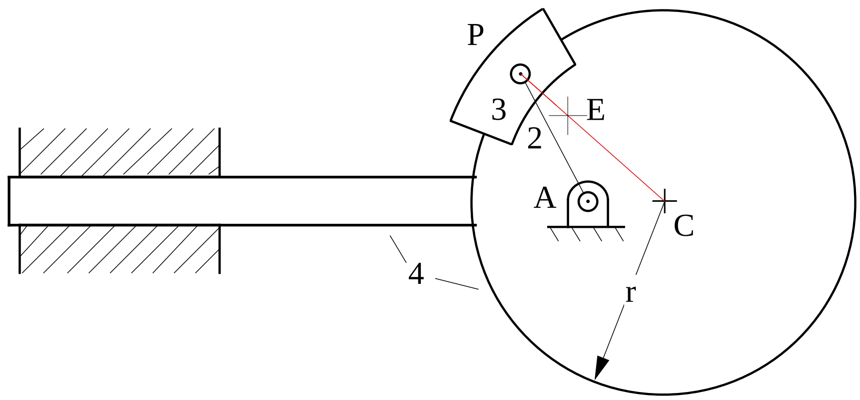
Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$$^1\boldsymbol{v}_{P_2/P_4} = 4.675 \text{ in / s}$$

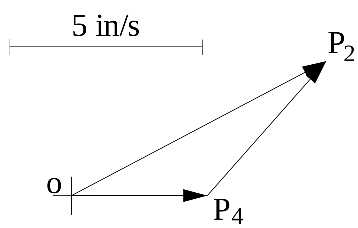
Also,

$$^1\boldsymbol{v}_{P_4} = 3.536 \text{ in / s}$$

To find the center of the curvature of the path that P_4 traces on link 2, we must find an expression which involves that the radius of curvature of the path. This term is $^1\boldsymbol{a}^n_{P_4/P_2}$ and it can be evaluated from the following:



Velocity Polygon



$$^1\boldsymbol{a}_{P_4/P_2} = -{}^1\boldsymbol{a}_{P_2/P_4}$$

therefore,

$$^1\boldsymbol{a}^t_{P_4/P_2} = -{}^1\boldsymbol{a}^t_{P_2/P_4}$$

and

$$^1\boldsymbol{a}^n_{P_4/P_2} = -{}^1\boldsymbol{a}^n_{P_2/P_4}$$

Also,

$$^2\boldsymbol{a}^n_{P_4/P_2} + 2\cdot{}^1\omega_2 \times {}^1\boldsymbol{v}_{P_4/P_2} = -{}^4\boldsymbol{a}^n_{P_2/P_4} - 2\cdot{}^1\omega_4 \times {}^1\boldsymbol{v}_{P_2/P_4} \qquad (3)$$

For our purpose, we should arrange Eq. (3) as

$$^2\boldsymbol{a}^n_{P_4/P_2} = -\left({}^4\boldsymbol{a}^n_{P_2/P_4} + 2\cdot{}^1\omega_2 \times {}^1\boldsymbol{v}_{P_4/P_2} + 2\cdot{}^1\omega_4 \times {}^1\boldsymbol{v}_{P_2/P_4}\right)$$

Now,

$$^4\boldsymbol{a}^n_{P_2/P_4} = \frac{\left|{}^1\boldsymbol{v}_{P_2/P_4}\right|^2}{\boldsymbol{r}_{P/C}} = \frac{(4.675)^2}{1} = 21.86 \text{ in}/\text{s}^2 \text{ (from } P \text{ to } C\text{)}$$

$$2\cdot{}^1\omega_2 \times {}^1\boldsymbol{v}_{P_4/P_2} = 2\cdot 10\cdot 4.675 = 93.5 \text{ in}/\text{s}^2 \text{ (from } C \text{ to } P\text{)}$$

$$2\cdot{}^1\omega_4 \times {}^1\boldsymbol{v}_{P_2/P_4} = 2\cdot 0\cdot 4.675 = 0 \text{ in}/\text{s}^2$$

If we choose $\mathbf{r}_{C/P}$ as the positive direction,

$$^2\boldsymbol{a}^n_{P_4/P_2} = \frac{\left|{}^1\boldsymbol{v}_{P_4/P_2}\right|^2}{\boldsymbol{r}_{B/E}} = -(21.86 - 93.5) = 66.9 \text{ in}/\text{s}^2$$

So,

$$\left|\mathbf{r}_{B/E}\right| = \frac{\left|{}^{1}\mathbf{v}_{P_4/P_2}\right|^2}{66.9} = \frac{(4.675)^2}{66.9} = 0.327 \text{ in}$$

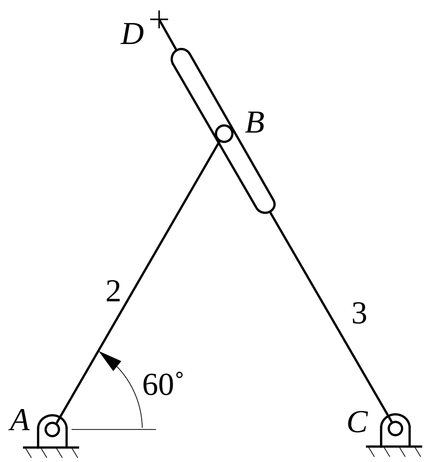
Therefore, the center of the curvature of the path that B_3 traces on link 2 is in same direction of $\mathbf{r}_{C/P}$ as shown in the drawing.

Problem 3.36

For the mechanism shown, find $\boldsymbol{\omega}_2$, $\boldsymbol{\alpha}_2$, $\mathbf{v}_{B_2}$, $\mathbf{a}_{B_2}$, $\mathbf{v}_{D_3}$, $\mathbf{a}_{D_3}$, and the location of the center of curvature of the path that point B_3 traces on link 2.

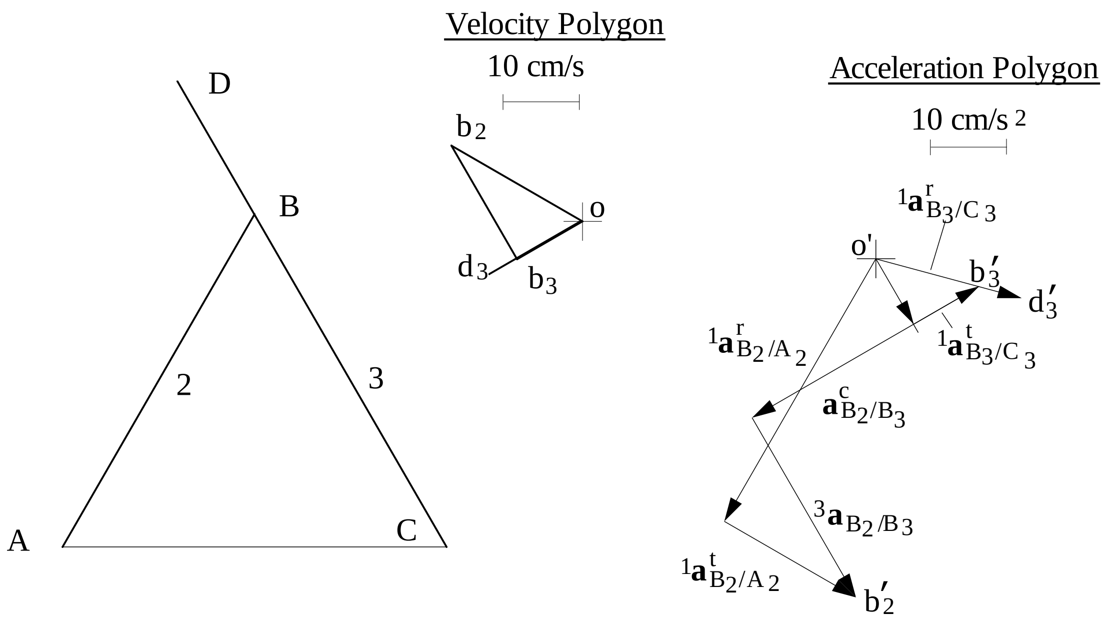
$AB = AC = 10 \text{ cm}$ $\quad CD = 14 \text{ cm}$

$\omega_3 = 1 \text{ rad/s CCW}$ $\quad \alpha_3 = 1 \text{ rad/s}^2 \text{ CW}$



Position Analysis

Locate points A and C. Construct link 2 at an angle of 60° from the line AC. Draw the line CD from C and through B.



Velocity Analysis:

$$^1\boldsymbol{v}_{B_2} = {}^1\boldsymbol{v}_{B_3} + {}^1\boldsymbol{v}_{B_2/B_3} \qquad (1)$$

Now,

$$^1\boldsymbol{v}_{B_2} = {}^1\omega_2 \times \boldsymbol{r}_{B/A} \quad (\perp \text{ to } \boldsymbol{r}_{B/A})$$

$$^1\boldsymbol{v}_{B_3} = {}^1\omega_3 \times \boldsymbol{r}_{B/C} \Rightarrow \left|{}^1\boldsymbol{v}_{B_3}\right| = \left|{}^1\omega_3\right| \cdot \left|\boldsymbol{r}_{B/C}\right| = 1 \cdot 0.1 = 0.10 \text{ m/s } (\perp \text{ to } \boldsymbol{r}_{B/C})$$

$^1\boldsymbol{v}_{B_2/B_3}$ in the direction of $\boldsymbol{r}_{B/C}$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$$^1\boldsymbol{v}_{B_2} = 0.200 \text{ m/s}$$

or

$$\left|{}^1\omega_2\right| = \frac{\left|{}^1\boldsymbol{v}_{B_2}\right|}{\left|\boldsymbol{r}_{B/A}\right|} = \frac{0.200}{0.1} = 2.00 \text{ rad/s}$$

From the directions given in the position and velocity polygons

$$^1\boldsymbol{\omega}_2 = 2.00 \text{ rad/s CCW}$$

Also,

$$^1\boldsymbol{v}_{B_2/B_3} = 0.173 \text{ m/s}$$

The velocity of D_3 can be computed directly because it is located on the driver link.

$$ {}^1\boldsymbol{v}_{D_3} = {}^1\boldsymbol{\omega}_3 \times \boldsymbol{r}_{D/C} \Rightarrow \left|{}^1\boldsymbol{v}_{D_3}\right| = \left|{}^1\omega_3\right| \cdot \left|\boldsymbol{r}_{D/C}\right| = 1 \cdot 0.14 = 0.14 \text{ m/s } (\perp \text{ to } \boldsymbol{r}_{D/C}) $$

<u>Acceleration Analysis:</u>

$$ {}^1\boldsymbol{a}_{B_2} = {}^1\boldsymbol{a}_{B_3} + {}^1\boldsymbol{a}_{B_2/B_3} $$

$$ {}^1\boldsymbol{a}^r_{B_2/A_2} + {}^1\boldsymbol{a}^t_{B_2/A_2} = {}^1\boldsymbol{a}^r_{B_3/C_3} + {}^1\boldsymbol{a}^t_{B_3/C_3} + {}^3\boldsymbol{a}_{B_2/B_3} + 2 \cdot {}^1\omega_3 \times {}^3\boldsymbol{v}_{B_2/B_3} \tag{2} $$

Now,

$$ {}^1\boldsymbol{a}^r_{B_2/A_2} = {}^1\omega_2 \times \left({}^1\omega_2 \times \boldsymbol{r}_{B/A}\right) \Rightarrow \left|{}^1\boldsymbol{a}^r_{B_2/A_2}\right| = \left|{}^1\omega_2\right|^2 \cdot \left|\boldsymbol{r}_{B/A}\right| = 2.00^2 \cdot 0.1 = 0.400 \text{ m/s}^2 $$

in the direction of $-\boldsymbol{r}_{B/A}$

$$ {}^1\boldsymbol{a}^t_{B_2/A_2} = {}^1\alpha_2 \times \boldsymbol{r}_{B/A} \Rightarrow \left|{}^1\boldsymbol{a}^t_{B_2/A_2}\right| = \left|{}^1\alpha_2\right| \cdot \left|\boldsymbol{r}_{B/A}\right| (\perp \text{ to } \boldsymbol{r}_{B/A}) $$

$$ {}^1\boldsymbol{a}^r_{B_3/C_3} = {}^1\omega_3 \times \left({}^1\omega_3 \times \boldsymbol{r}_{B/C}\right) \Rightarrow \left|{}^1\boldsymbol{a}^r_{B_3/C_3}\right| = \left|{}^1\omega_3\right|^2 \cdot \left|\boldsymbol{r}_{B/C}\right| = 1^2 \cdot 0.1 = 0.1 \text{ m/s}^2 $$

in the direction of $-\boldsymbol{r}_{B/C}$

$$ {}^1\boldsymbol{a}^t_{B_3/C_3} = {}^1\alpha_3 \times \boldsymbol{r}_{B/C} \Rightarrow \left|{}^1\boldsymbol{a}^t_{B_3/C_3}\right| = \left|{}^1\alpha_3\right| \cdot \left|\boldsymbol{r}_{B/C}\right| = 1 \cdot 0.1 = 0.1 \text{ m/s}^2 (\perp \text{ to } \boldsymbol{r}_{B/C}) $$

$$ {}^1\boldsymbol{a}^c_{B_2/B_3} = 2 \cdot {}^1\omega_3 \times {}^3\boldsymbol{v}_{B_2/B_3} = 2 \cdot 1 \cdot 0.173 = 0.346 \text{ m/s}^2 (\perp \text{ to } \boldsymbol{r}_{B/C}) $$

${}^3\boldsymbol{a}_{B_2/B_3}$ in the direction of $\boldsymbol{r}_{B/C}$

Solve Eq. (2) graphically with a acceleration polygon. From the polygon,

$$ {}^1\boldsymbol{a}^t_{B_2/A_2} = 0.201 \text{ m/s}^2 $$

and

$$ \left|{}^1\alpha_2\right| = \frac{\left|{}^1\boldsymbol{a}^t_{B_2/A_2}\right|}{\left|\boldsymbol{r}_{B/A}\right|} = \frac{0.201}{0.1} = 2.01 \text{ rad/s}^2 $$

From directions given in the polygon,

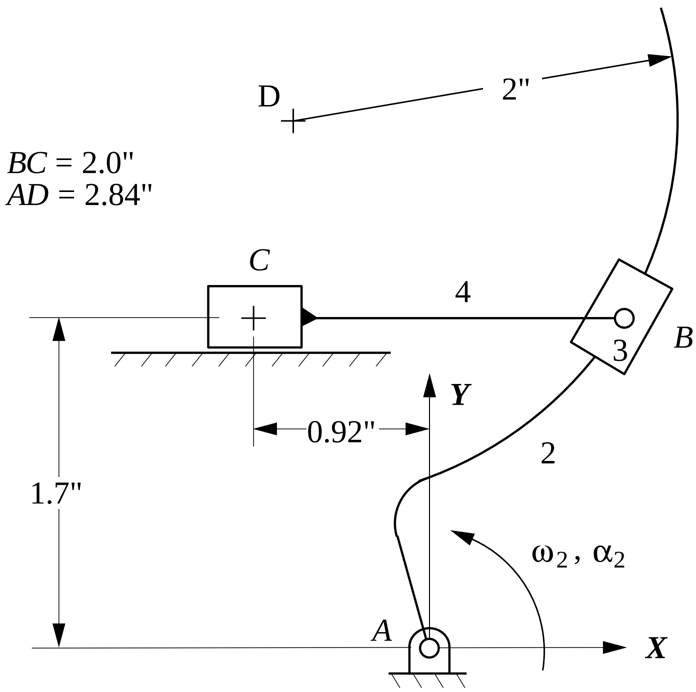
$$ {}^1\alpha_2 = 2.01 \text{ rad/s}^2 \; CW $$

The acceleration of D_3 can be computed by image or directly since it is on the driver link. The magnitude of the acceleration is

$$ {}^1\boldsymbol{a}_{D_3} = 0.200 \text{ m/s}^2 $$

Problem 3.37

If ω_2 = 10 rad/s (constant), find $\mathbf{v}_{B_2}$, $\mathbf{v}_{B_3}$, $\mathbf{a}_{B_3}$, and $\mathbf{a}_{C_4}$.



Position Analysis

Draw the mechanism to scale. Start by drawing points *A* and *C*. Then locate point *B*. Locate point *D* to be 2.84 inches above *A* and 2 inches from *B*.

Velocity Analysis:

$${}^1\boldsymbol{v}_{B_2} = {}^1\omega_2 \times \boldsymbol{r}_{B/A} \;\Rightarrow\; \left|{}^1\boldsymbol{v}_{B_2}\right| = \left|{}^1\omega_2\right| \cdot \left|\boldsymbol{r}_{B/A}\right| = 10 \cdot 2.01 = 20.1 \text{ in/s}$$

$${}^1\boldsymbol{v}_{B_3} = {}^1\boldsymbol{v}_{B_4} = {}^1\boldsymbol{v}_{B_2} + {}^1\boldsymbol{v}_{B_3/B_2} \qquad (1)$$

Now,

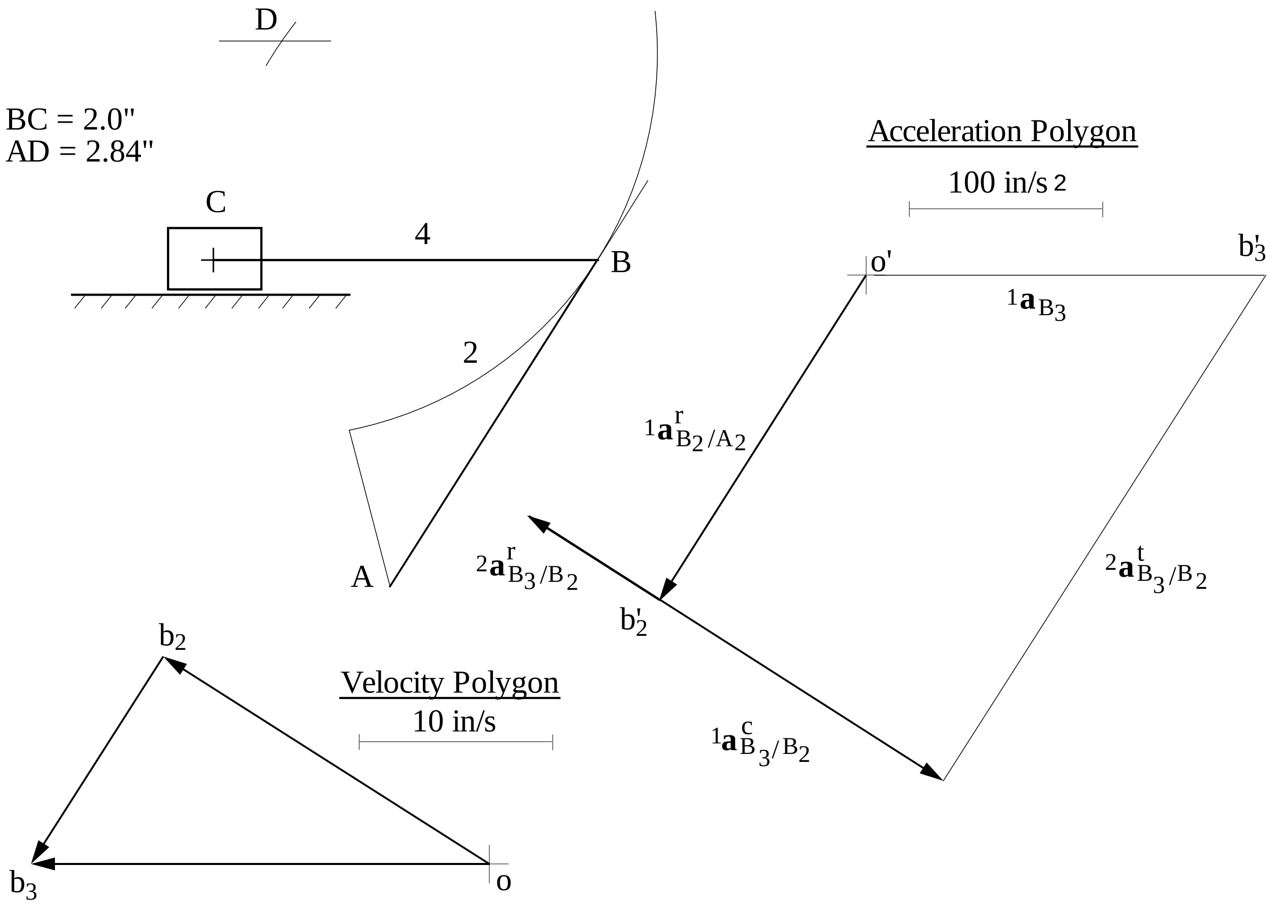
$${}^1\boldsymbol{v}_{B_2} = 20.133 \text{ in/s} (\perp \text{ to } \boldsymbol{r}_{B/A})$$

${}^1\boldsymbol{v}_{B_3}$ in the direction of $\boldsymbol{r}_{B/C}$

${}^1\boldsymbol{v}_{B_3/B_2}$ $(\perp \text{ to } \boldsymbol{r}_{B/D})$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$${}^1\boldsymbol{v}_{B_3} = 23.9 \text{ in/s}$$



Also,

$${}^1\boldsymbol{v}_{B_3/B_2} = 12.8 \text{ in/s}$$

<u>Acceleration Analysis:</u>

$${}^1\boldsymbol{a}_{B_3} = {}^1\boldsymbol{a}_{B_4}$$

$${}^1\boldsymbol{a}_{B_3} = {}^1\boldsymbol{a}_{B_2} + {}^1\boldsymbol{a}_{B_3/B_2}$$

$${}^1\boldsymbol{a}_{B_3} = {}^1\boldsymbol{a}^r_{B_2/A_2} + {}^1\boldsymbol{a}^t_{B_2/A_2} + {}^2\boldsymbol{a}^r_{B_3/B_2} + {}^2\boldsymbol{a}^t_{B_3/B_2} + 2\cdot{}^1\omega_2 \times {}^1\boldsymbol{v}_{B_3/B_2} \qquad (2)$$

Now,

${}^1\boldsymbol{a}_{B_3} = {}^1\boldsymbol{a}_{B_4}$ in the direction of $\boldsymbol{r}_{B/C}$

${}^1\boldsymbol{a}^r_{B_2/A_2} = {}^1\omega_2 \times ({}^1\omega_2 \times \boldsymbol{r}_{B/A}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{B_2/A_2}\right| = \left|{}^1\omega_2\right|^2 \cdot \left|\boldsymbol{r}_{B/A}\right| = 10^2 \cdot 2.01 = 201 \text{ in/s}^2$ in the direction of $-\boldsymbol{r}_{B/A}$

${}^1\boldsymbol{a}^t_{B_2/A_2} = {}^1\alpha_2 \times \boldsymbol{r}_{B/A} \Rightarrow \left|{}^1\boldsymbol{a}^t_{B_2/A_2}\right| = \left|{}^1\alpha_2\right| \cdot \left|\boldsymbol{r}_{B/A}\right| = 0$ ($\perp$ to $\boldsymbol{r}_{B/A}$)

$$\left|{}^2\boldsymbol{a}^r_{B_3/B_2}\right| = \frac{\left|\boldsymbol{v}_{B_3/B_2}\right|^2}{\left|\boldsymbol{r}_{B/D}\right|} = \frac{12.8^2}{2} = 81.9 \ \text{in}/\text{s}^2 \text{ in the direction of } -\boldsymbol{r}_{B/D}$$

${}^2\boldsymbol{a}^t_{B_3/B_2}$ tangent to the path that B_3 traces on link 2.

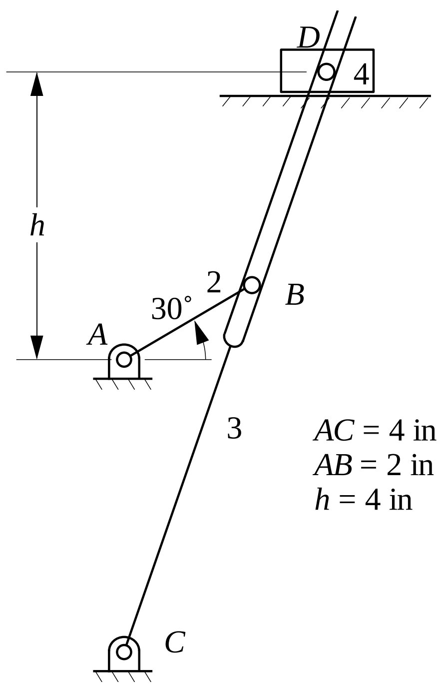
$$ {}^1\boldsymbol{a}^c_{B_3/B_2} = 2 \cdot {}^1\omega_2 \times {}^1\boldsymbol{v}_{B_3/B_2} = 2 \cdot 10 \cdot 12.8 = 256 \ \text{in}/\text{s}^2 \text{ in the direction of } \boldsymbol{r}_{B/D}$$

Solve Eq. (2) graphically with a acceleration polygon. From the polygon,

$$ {}^1\boldsymbol{a}_{B_3} = {}^1\boldsymbol{a}_{B_4} = {}^1\boldsymbol{a}_{C_4} = 203 \ \text{in}/\text{s}^2$$

Problem 3.38

In the mechanism shown, ω_2 = 10 rad/s CW (constant). Determine the angular acceleration of link 3.



Position Analysis

Draw the linkage to scale. Locate the pivots A and C and the line of motion of point D. Next draw link 2 and locate B. Then draw link 3 and locate point D.

Velocity Analysis:

$$ {}^1\boldsymbol{v}_{B_2} = {}^1\boldsymbol{v}_{B_3} + {}^1\boldsymbol{v}_{B_2/B_3} \qquad (1)$$

Now,

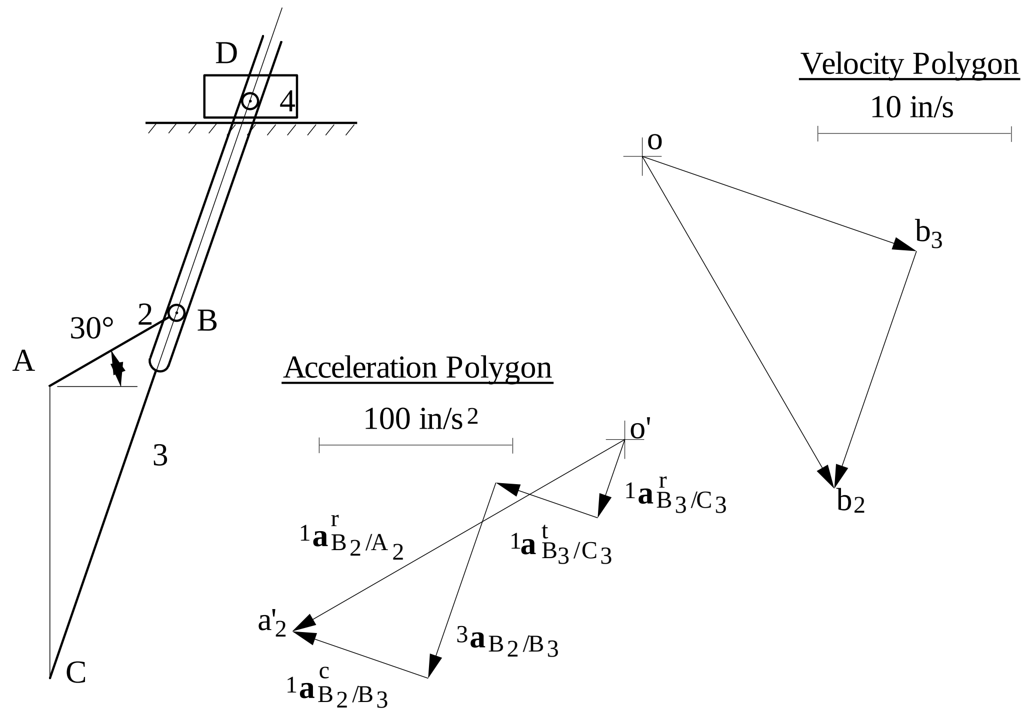
$${}^1\boldsymbol{v}_{B_2} = {}^1\omega_2 \times \boldsymbol{r}_{B/A} \quad (\perp \text{ to } \boldsymbol{r}_{B/A}) \Rightarrow \left|{}^1\boldsymbol{v}_{B_2}\right| = \left|{}^1\omega_2\right| \cdot \left|\boldsymbol{r}_{B/A}\right| = 10 \cdot 2 = 20 \text{ in/s}$$

$${}^1\boldsymbol{v}_{B_3} = {}^1\omega_3 \times \boldsymbol{r}_{B/C} \quad (\perp \text{ to } \boldsymbol{r}_{B/C})$$

${}^1\boldsymbol{v}_{B_2/B_3}$ in the direction of $\boldsymbol{r}_{B/C}$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$${}^1\boldsymbol{v}_{B_3} = 15.1 \text{ in/s}$$



or

$$\left|{}^1\omega_3\right| = \frac{\left|{}^1\boldsymbol{v}_{B_3}\right|}{\left|\boldsymbol{r}_{B/C}\right|} = \frac{15.1}{5.29} = 2.85 \text{ rad/s CW}$$

Also,

$${}^1\boldsymbol{v}_{B_2/B_3} = 13.1 \text{ in/s}$$

Acceleration Analysis:

$${}^1\boldsymbol{a}_{B_2} = {}^1\boldsymbol{a}_{B_3} + {}^1\boldsymbol{a}_{B_2/B_3}$$

$${}^1\boldsymbol{a}^r_{B_2/A_2} + {}^1\boldsymbol{a}^t_{B_2/A_2} = {}^1\boldsymbol{a}^r_{B_3/C_3} + {}^1\boldsymbol{a}^t_{B_3/C_3} + {}^3\boldsymbol{a}_{B_2/B_3} + 2 \cdot {}^1\omega_3 \times {}^3\boldsymbol{v}_{B_2/B_3} \tag{2}$$

where

$${}^1\boldsymbol{a}^r_{B_2/A_2} = {}^1\omega_2 \times ({}^1\omega_2 \times \boldsymbol{r}_{B/A}) \Rightarrow \left|{}^1\boldsymbol{a}^r_{B_2/A_2}\right| = \left|{}^1\omega_2\right|^2 \cdot \left|\boldsymbol{r}_{B/A}\right| = 10^2 \cdot 2 = 200 \text{ in/s}^2$$

in the direction of $\boldsymbol{r}_{B/A}$

$$ {}^1\boldsymbol{a}^t_{B_2/A_2} = {}^1\alpha_2 \times \boldsymbol{r}_{B/A} \Rightarrow \left|{}^1\boldsymbol{a}^t_{B_2/A_2}\right| = \left|{}^1\alpha_2\right| \cdot \left|\boldsymbol{r}_{B/A}\right| = 0 \cdot 2 = 0 \text{ in/s}^2 $$

$$ {}^1\boldsymbol{a}^r_{B_3/C_3} = {}^1\omega_3 \times \left({}^1\omega_3 \times \boldsymbol{r}_{B/C}\right) \Rightarrow \left|{}^1\boldsymbol{a}^r_{B_3/C_3}\right| = \left|{}^1\omega_3\right|^2 \cdot \left|\boldsymbol{r}_{B/C}\right| = 2.85^2 \cdot 5.29 = 43.2 \text{ in/s}^2 $$

in the direction of $\boldsymbol{r}_{B/C}$

$$ {}^1\boldsymbol{a}^t_{B_3/C_3} = {}^1\alpha_3 \times \boldsymbol{r}_{B/C} \Rightarrow \left|{}^1\boldsymbol{a}^t_{B_3/C_3}\right| = \left|{}^1\alpha_3\right| \cdot \left|\boldsymbol{r}_{B/C}\right| (\perp \text{ to } \boldsymbol{r}_{B/C}) $$

${}^3\boldsymbol{a}_{B_2/B_3}$ in the direction of $\boldsymbol{r}_{B/C}$

$$ {}^1\boldsymbol{a}^c_{B_2/B_3} = 2 \cdot {}^1\omega_3 \times {}^3\boldsymbol{v}_{B_2/B_3} = 2 \cdot 2.85 \cdot 13.1 = 74.9 \text{ in/s}^2 $$

Solve Eq. (2) graphically with a velocity polygon. From the polygon,

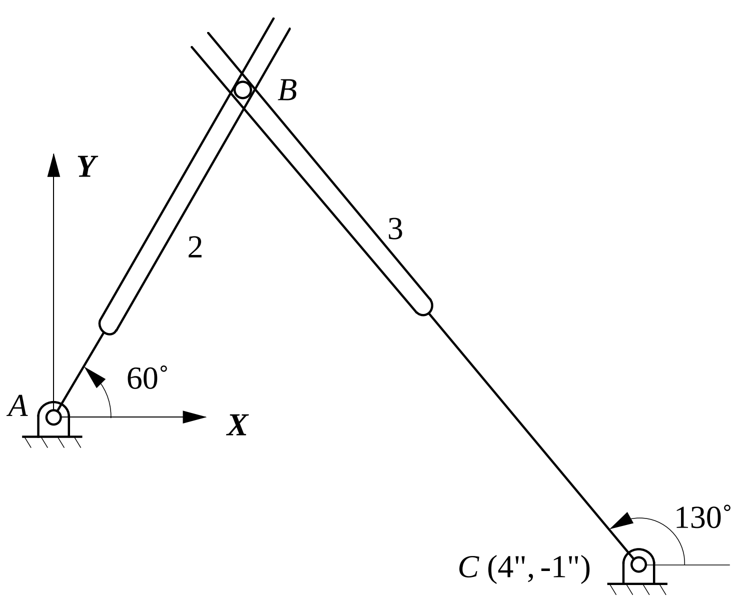
$$ \left|{}^1\alpha_3\right| = \frac{\left|{}^1\boldsymbol{a}^t_{B_3/C_3}\right|}{\left|\boldsymbol{r}_{B/C}\right|} = \frac{56.1}{5.29} = 10.6 \text{ rad/s}^2 $$

therefore,

$$ {}^1\alpha_3 = 10.6 \text{ rad/s}^2 \text{ CCW} $$

Problem 3.39

In the mechanism shown, slotted links 2 and 3 are independently driven at angular velocities of 30 and 20 rad/s CW and have angular accelerations of 900 and 400 rad/s^2 CW, respectively. Determine the acceleration of point B, the center of the pin carried at the intersection of the two slots.



Position Analysis

Locate the pivots A and C. Then draw links 2 and 3 in the orientations shown.

$\boldsymbol{r}_{B/A} = 2.58$ in

$\boldsymbol{r}_{B/C} = 4.23$ in

Velocity Analysis

Consider the points at location B.

$${}^1\boldsymbol{v}_{B_2} = {}^1\boldsymbol{v}_{B_2/A_2} = {}^1\omega_2 \times \boldsymbol{r}_{B/A} \Rightarrow \left|{}^1\boldsymbol{v}_{B_2/A_2}\right| = \left|{}^1\omega_2\right| \cdot \left|\boldsymbol{r}_{B/A}\right| = 30(2.58) = 77.4 \text{ in / s} (\perp \text{ to } \boldsymbol{r}_{B/A})$$

$${}^1\boldsymbol{v}_{B_3} = {}^1\boldsymbol{v}_{B_3/C_3} = {}^1\omega_3 \times \boldsymbol{r}_{B/C} \Rightarrow \left|{}^1\boldsymbol{v}_{B_3/C_3}\right| = \left|{}^1\omega_2\right| \cdot \left|\boldsymbol{r}_{B/C}\right| = 20(4.23) = 84.6 \text{ in / s} (\perp \text{ to } \boldsymbol{r}_{B/C})$$

Call the pin link 4. Then,

$${}^1\boldsymbol{v}_{B_4} = {}^1\boldsymbol{v}_{B_3} + {}^1\boldsymbol{v}_{B_4/B_3} = {}^1\boldsymbol{v}_{B_2} + {}^1\boldsymbol{v}_{B_4/B_2} \tag{1}$$

Where,

${}^1\boldsymbol{v}_{B_4/B_2}$ is along $\boldsymbol{r}_{B/A}$

${}^1\boldsymbol{v}_{B_4/B_3}$ is along $\boldsymbol{r}_{B/C}$

Solve Eq. (1) using the velocity polygon. Then,

${}^1\boldsymbol{v}_{B_4/B_2}$=118 in/s

${}^1\boldsymbol{v}_{B_4/B_3}$=113 in/s

Acceleration Analysis

Consider the points at location B.

$${}^1\boldsymbol{a}_{B_2} = {}^1\boldsymbol{a}_{B_2/A_2} = {}^1\boldsymbol{a}^r_{B_2/A_2} + {}^1\boldsymbol{a}^t_{B_2/A_2}$$

$${}^1\boldsymbol{a}_{B_3} = {}^1\boldsymbol{a}_{B_3/C_3} = {}^1\boldsymbol{a}^r_{B_3/C_3} + {}^1\boldsymbol{a}^t_{B_3/C_3}$$

Call the pin link 4. Then,

$${}^1\boldsymbol{a}_{B_4} = {}^1\boldsymbol{a}_{B_3} + {}^1\boldsymbol{a}_{B_4/B_3} = {}^1\boldsymbol{a}_{B_2} + {}^1\boldsymbol{a}_{B_4/B_2}$$

or

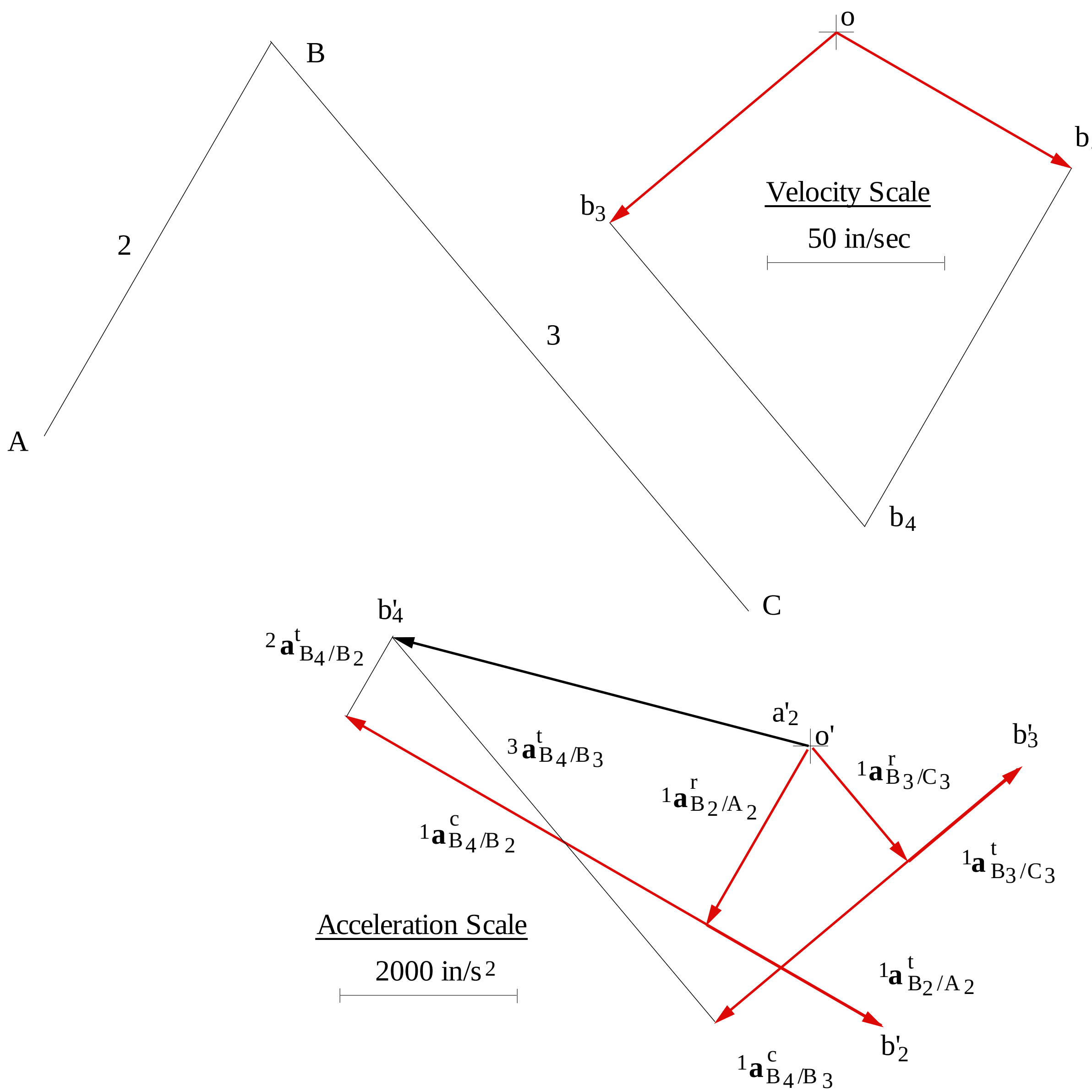
$${}^1\boldsymbol{a}^r_{B_3/C_3} + {}^1\boldsymbol{a}^t_{B_3/C_3} + {}^3\boldsymbol{a}^t_{B_4/B_3} + {}^3\boldsymbol{a}^r_{B_4/B_3} + \boldsymbol{a}^c_{B_4/B_3} = {}^1\boldsymbol{a}^r_{B_2/A_2} + {}^1\boldsymbol{a}^t_{B_2/A_2} + {}^3\boldsymbol{a}^t_{B_4/B_2} + {}^3\boldsymbol{a}^r_{B_4/B_2} + \boldsymbol{a}^c_{B_4/B_2} \tag{2}$$

Where,

$$\left|{}^1\boldsymbol{a}^r_{B_3/C_3}\right| = \left|{}^1\omega_3\right|^2\left|\boldsymbol{r}_{B/C}\right| = 20^2(4.23) = 1690 \text{ in / s}^2 \text{ (opposite } \boldsymbol{r}_{B/C})$$

$$\left|{}^1\boldsymbol{a}^t_{B_3/C_3}\right| = \left|{}^1\alpha_3\right|\cdot\left|\boldsymbol{r}_{B/C}\right| = 400(4.23) = 1690 \text{ in / s}^2 \ (\perp \text{ to } \boldsymbol{r}_{B/C})$$

${}^3\boldsymbol{a}^t_{B_4/B_3}$ along $\boldsymbol{r}_{B/C}$



Sol

$$\left|{}^3\boldsymbol{a}^r_{B_4/B_3}\right| = \frac{\left|{}^3\boldsymbol{v}_{B_4/B_3}\right|^2}{\infty} = 0$$

$$\boldsymbol{a}^c_{B_4/B_3} = 2\cdot{}^1\boldsymbol{\omega}_3 \times {}^3\boldsymbol{v}_{B_4/B_3} \Rightarrow \left|\boldsymbol{a}^c_{B_4/B_3}\right| = 2\left|{}^1\omega_3\right|\cdot\left|{}^3\boldsymbol{v}_{B_4/B_3}\right| = 2(20)(113) = 4520 \text{ in / s}^2 \ (\perp \text{ to } \boldsymbol{r}_{B/C})$$

$$\left|{}^1\boldsymbol{a}^r_{B_2/A_2}\right| = \left|{}^1\omega_2\right|^2\left|\boldsymbol{r}_{B/A}\right| = 30^2(2.58) = 2320 \text{ in / s}^2 \text{ (opposite } \boldsymbol{r}_{B/A})$$

$$\left|{}^1\boldsymbol{a}^t_{B_2/A_2}\right| = \left|{}^1\alpha_2\right|\cdot\left|\boldsymbol{r}_{B/A}\right| = 900(2.58) = 2320 \text{ in / s}^2 \ (\perp \text{ to } \boldsymbol{r}_{B/A})$$

${}^1\boldsymbol{a}^t_{B_2/A_2}$ along $\boldsymbol{r}_{B/A}$

$$\left|{}^2\boldsymbol{a}^r_{B_4/B_2}\right| = \frac{\left|{}^2\boldsymbol{v}_{B_4/B_2}\right|^2}{\infty} = 0$$

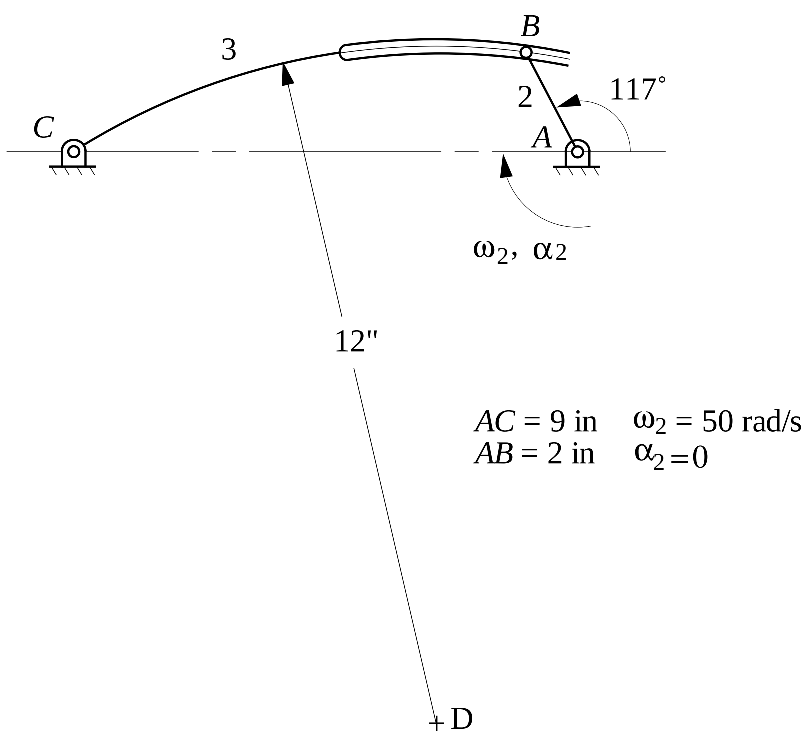
$$\boldsymbol{a}^c_{B_4/B_2} = 2\cdot{}^1\boldsymbol{\omega}_2 \times {}^2\boldsymbol{v}_{B_4/B_2} \Rightarrow \left|\boldsymbol{a}^c_{B_4/B_2}\right| = 2\left|{}^1\boldsymbol{\omega}_2\right|\cdot\left|{}^2\boldsymbol{v}_{B_4/B_2}\right| = 2(30)(118) = 7080 \text{ in}/\text{s}^2 \ (\perp \text{ to } \boldsymbol{r}_{B/A})$$

Solve Eq. (2) using the acceleration polygon. Then,

${}^1\boldsymbol{a}_{B_4} = 4890 \text{ in}/\text{s}^2$ in the direction shown.

Problem 3.40

For the mechanism shown, find $\boldsymbol{\omega}_3$, $\boldsymbol{\alpha}_3$, $\boldsymbol{a}_{B_3}$, and the location of the center of curvature of the path that point B_3 traces on link 2.



Velocity Analysis:

$${}^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{B_2/A_2} = {}^1\mathbf{v}_{B_3} + {}^1\mathbf{v}_{B_2/B_3} \tag{1}$$

$${}^1\mathbf{v}_{B_3} = {}^1\mathbf{v}_{B_3/C_3}$$

Now,

$${}^1\mathbf{v}_{B_2/A_2} = {}^1\omega_2 \times \mathbf{r}_{B_2/A_2} \Rightarrow \left|{}^1\mathbf{v}_{B_2/A_2}\right| = \left|{}^1\omega_2\right|\left|\mathbf{r}_{B/A}\right| = 50\cdot 2 = 100 \text{ in}/\text{sec} \ (\perp \text{ to } \mathbf{r}_{B/A})$$

$${}^1\mathbf{v}_{B_3/C_3} = {}^1\omega_3 \times \mathbf{r}_{B/C} \Rightarrow |{}^1\mathbf{v}_{B_3/C_3}| = |{}^1\omega_3||\mathbf{r}_{B/C}| \;(\perp \text{ to } \mathbf{r}_{B/C})$$

${}^1\mathbf{v}_{B_2/B_3}$ tangent to the curve

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$${}^1\mathbf{v}_{B_3} = 60.78 \text{ in/sec}$$

or

$$|{}^1\omega_3| = \frac{|{}^1\mathbf{v}_{B_3/C_3}|}{|\mathbf{r}_{B_3/C_3}|} = \frac{60.78}{8.286} = 7.335 \text{ rad/sec CCW}$$

Also,

$${}^1\mathbf{v}_{B_2/B_3} = 103.12 \text{ in/sec}$$

<u>Acceleration Analysis:</u>

$${}^1\mathbf{a}_{B_2} = {}^1\mathbf{a}_{B_2/A_2} = {}^1\mathbf{a}_{B_3} + {}^1\mathbf{a}_{B_2/B_3}$$

$${}^1\mathbf{a}_{B_3} = {}^1\mathbf{a}_{B_3/C_3}$$

$$\mathbf{a}^r_{B_2/A_2} + {}^1\mathbf{a}^t_{B_2/A_2} = {}^1\mathbf{a}^r_{B_3/C_3} + {}^1\mathbf{a}^t_{B_3/C_3} + {}^3\mathbf{a}^n_{B_2/B_3} + {}^3\mathbf{a}^t_{B_2/B_3} + 2\cdot{}^1\omega_3 \times {}^3\mathbf{v}_{B_2/B_3} \qquad (2)$$

Now,

$${}^1\mathbf{a}^r_{B_2/A_2} = {}^1\omega_2 \times ({}^1\omega_2 \times \mathbf{r}_{B_2/A_2}) \Rightarrow |{}^1\mathbf{a}^r_{B_2/A_2}| = |{}^1\omega_2|^2 \cdot |\mathbf{r}_{B_2/A_2}| = 50^2 \cdot 2 = 5000 \text{ in/sec}^2$$

in the direction opposite to $\mathbf{r}_{B_2/A_2}$

$${}^1\mathbf{a}^t_{B_2/A_2} = {}^1\alpha_2 \times \mathbf{r}_{B_2/A_2} \Rightarrow |{}^1\mathbf{a}^t_{B_2/A_2}| = |{}^1\alpha_2| \cdot |\mathbf{r}_{B_2/A_2}| = 0 \cdot 2 = 0 \text{ in/sec}^2 \;(\perp \text{ to } \mathbf{r}_{B_2/A_2})$$
$${}^1\mathbf{a}^r_{B_3/C_3} = {}^1\omega_3 \times ({}^1\omega_3 \times \mathbf{r}_{B_3/C_3}) \Rightarrow |{}^1\mathbf{a}^r_{B_3/C_3}| = |{}^1\omega_3|^2 \cdot |\mathbf{r}_{B/C}| = 7.335^2 \cdot 8.286 = 445.805 \text{ in/sec}^2$$

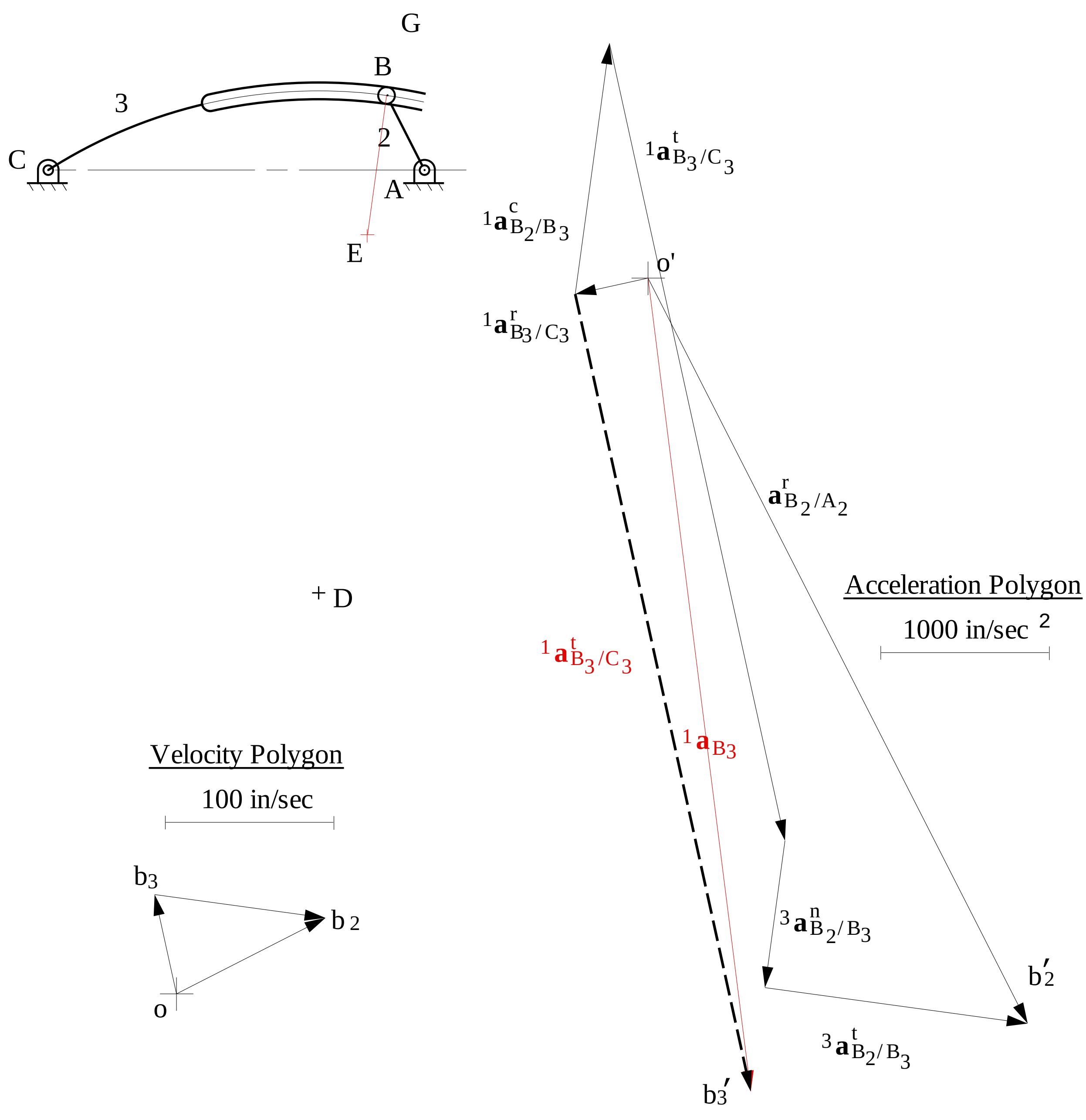
in the direction opposite to $\mathbf{r}_{B/C}$

$${}^1\mathbf{a}^t_{B_3/C_3} = {}^1\alpha_3 \times \mathbf{r}_{B_3/C_3} \Rightarrow |{}^1\mathbf{a}^t_{B_3/C_3}| = |{}^1\alpha_3| \cdot |\mathbf{r}_{B_3/C_3}| \;(\perp \text{ to } \mathbf{r}_{B_3/C_3})$$

$$|{}^3\mathbf{a}^n_{B_2/B_3}| = \frac{|{}^1\mathbf{v}_{B_2/B_3}|^2}{|\mathbf{r}_{B/D}|} = \frac{103.12^2}{12} = 886.145 \text{ in/sec}^2 \text{ (in the direction of } \mathbf{r}_{B/D})$$

$${}^3\mathbf{a}^t_{B_2/B_3} \;(\perp \text{ to } \mathbf{r}_{B/D})$$

$$2\cdot{}^1\omega_3 \times {}^3\mathbf{v}_{B_2/B_3} = 2 \cdot 7.335 \cdot 103.12 = 1513 \text{ in/sec}^2$$



Solve Eq. (2) graphically with an acceleration polygon. From the polygon,

$${}^{1}\mathbf{a}^{t}_{B_3/C_3} = 4880 \text{ in/sec}^2$$

or

$$|{}^{1}\alpha_3| = \frac{|{}^{1}\mathbf{a}^{t}_{B_3/C_3}|}{|\mathbf{r}_{B/C}|} = \frac{4880}{8.286} = 589 \text{ rad/sec}^2 \text{ CW}$$

Also, to find the ${}^{1}\mathbf{a}_{B_3}$, add ${}^{1}\mathbf{a}^{t}_{B_3/C_3}$ in the acceleration polygon to ${}^{1}\mathbf{a}^{r}_{B_3/C_3}$,

$${}^{1}\mathbf{a}_{B_3} = 4900 \text{ in/sec}^2$$

To find the center of the curvature of the path that B_3 traces on link 2, we must find an expression which involves that the radius of curvature of the path. This term is ${}^2\mathbf{a}^n_{B_3/B_2}$ and it can be evaluated from the following:

$${}^1\mathbf{a}_{B_2/B_3} = -{}^1\mathbf{a}_{B_3/B_2}$$

therefore,

$${}^1\mathbf{a}^t_{B_2/B_3} = -{}^1\mathbf{a}^t_{B_3/B_2}$$

and

$$\mathbf{a}^n_{B_2/B_3} = -{}^1\mathbf{a}^n_{B_3/B_2}$$

Also,

$${}^3\mathbf{a}^n_{B_2/B_3} + 2\cdot{}^1\omega_3 \times {}^1\mathbf{v}_{B_2/B_3} = -{}^2\mathbf{a}^n_{B_3/B_2} - 2\cdot{}^1\omega_2 \times {}^1\mathbf{v}_{B_3/B_2} \qquad (3)$$

For our purpose, we should arrange Eq. (3) as

$${}^2\mathbf{a}^n_{B_3/B_2} = -\left({}^3\mathbf{a}^n_{B_2/B_3} + 2\cdot{}^1\omega_2 \times {}^1\mathbf{v}_{B_3/B_2} + 2\cdot{}^1\omega_3 \times {}^1\mathbf{v}_{B_2/B_3}\right)$$

Now,

$${}^3\mathbf{a}^n_{B_2/B_3} = 886 \text{ in / sec}^2 \text{ (from B to D)}$$

$$2\cdot{}^1\omega_3 \times {}^1\mathbf{v}_{B_2/B_3} = 2\cdot 7.335\cdot 103.12 = 1513 \text{ in / sec}^2 \text{ (from D to B)}$$

$$2\cdot{}^1\omega_2 \times {}^1\mathbf{v}_{B_3/B_2} = 2\cdot 50\cdot 103.12 = 10310 \text{ in / sec}^2 \text{ (from D to B)}$$

Let E be the location of the center of curvature of B_3 on link 2. If we choose $\mathbf{r}_{D/B}$ as the positive direction,

$${}^2\mathbf{a}^n_{B_3/B_2} = \frac{\left|{}^1\mathbf{v}_{B_3/B_2}\right|^2}{\mathbf{r}_{B/E}} = -(886 - 1513 - 10310) = 10{,}937 \text{ in / sec}^2$$

Because ${}^2\mathbf{a}^n_{B_3/B_2}$ points from B toward E, point E must lie on the same side of B as D does. The magnitude of the distance is given by

$$\left|\mathbf{r}_{B/E}\right| = \frac{\left|{}^1\mathbf{v}_{B_3/B_2}\right|^2}{10{,}937} = \frac{103.12^2}{10{,}937} = 0.972 \text{ in}$$

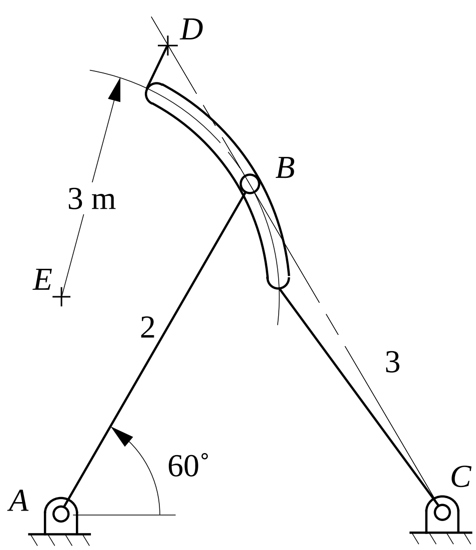
The location of E is shown on the drawing.

Problem 3.41

For the mechanism shown, points *C, B* and *D* are collinear. Point B_2 moves in a curved slot on link 3. For the position given, find $\boldsymbol{\omega}_3$, $\boldsymbol{\alpha}_3$, $\boldsymbol{v}_{B_3}$,$\boldsymbol{a}_{B_3}$, $\boldsymbol{v}_{D_3}$,$\boldsymbol{a}_{D_3}$, and the location of the center of curvature of the path that point B_3 traces on Link 2.

$AB = AC = 5\,\text{m}$ $\quad CD = 7\,\text{m}$ $\quad CE = 5.7\,\text{m}$

$\omega_2 = 2\ \text{rad/s CCW}$ $\quad \alpha_2 = 3\ \text{rad/s}^2\ \text{CCW}$



Position Analysis:

Draw the linkage to scale. Start by locating the pivots A and C. Next draw link 2 and locate point B. Next find point E and draw the arc at B. Also locate point D.

Velocity Analysis:

$$^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{B_2/A_2} = {}^1\boldsymbol{\omega}_2 \times \mathbf{r}_{B/A} \Rightarrow |{}^1\mathbf{v}_{B_2}| = |{}^1\boldsymbol{\omega}_2||\mathbf{r}_{B/A}| = 2\cdot(5) = 10\,\text{m/sec}$$

$$^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{B_3} + {}^1\mathbf{v}_{B_2/B_3} = {}^1\mathbf{v}_{B_3} + {}^3\mathbf{v}_{B_2/B_3} \qquad (1)$$

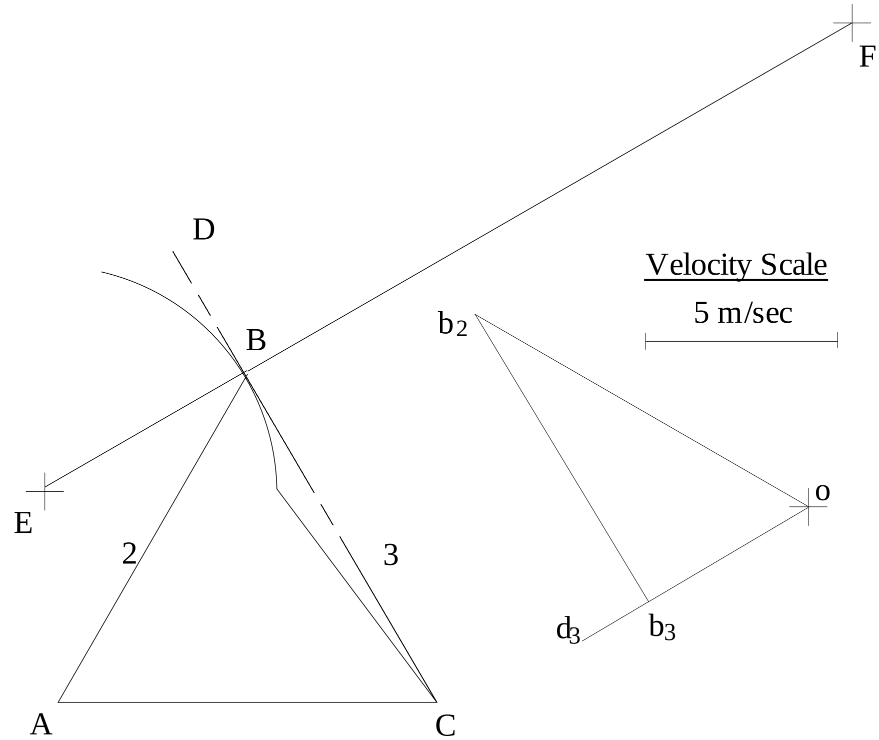
or

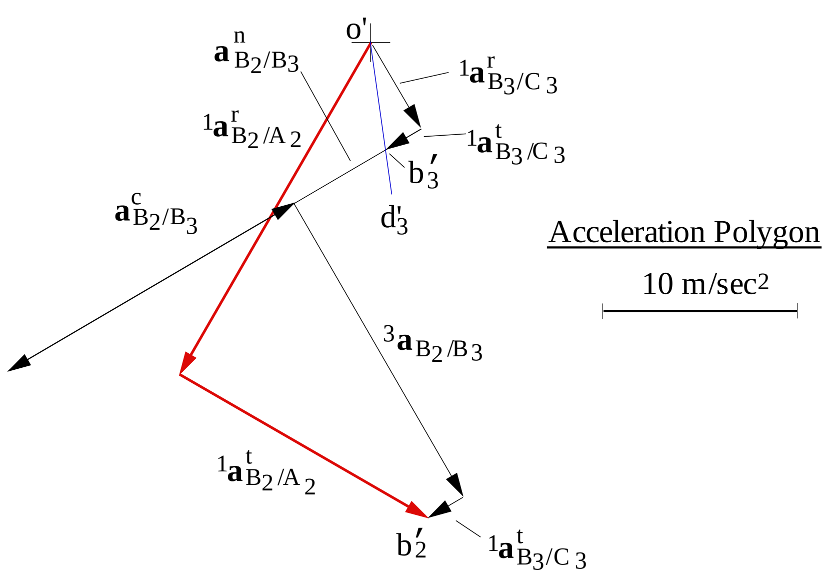
$$^1\mathbf{v}_{B_3} = {}^1\mathbf{v}_{B_2} + {}^1\mathbf{v}_{B_3/B_2} = {}^1\mathbf{v}_{B_2} + {}^2\mathbf{v}_{B_3/B_2} \qquad (2)$$

$$^1\mathbf{v}_{B_3} = {}^1\mathbf{v}_{B_3/C_3} = {}^1\boldsymbol{\omega}_3 \times \mathbf{r}_{B_3/C_3}$$

$$^1\mathbf{v}_{D_3} = {}^1\mathbf{v}_{D_3/C_3} = {}^1\boldsymbol{\omega}_3 \times \mathbf{r}_{D_3/C_3}$$

Now, to find ${}^1\mathbf{v}_{B3}$, we can use either equ.1 or equ.2. However, to see ${}^1\mathbf{v}_{B3/B2}$, it is better to begin with Eq..(1).





$|{}^1\mathbf{v}_{B2}| = |{}^1\omega_2||\mathbf{r}_{B2/A2}| = 2\cdot(5) = 10\,m/sec$ in the direction of $\mathbf{r}_{B/A}$

${}^1\mathbf{v}_{B3} = {}^1\omega_3 \times \mathbf{r}_{B/C}$ ($\perp$ to $\mathbf{r}_{B/C}$)

${}^1\mathbf{v}_{B2/B3}$ is tangent to the curvature on point B

Solve Eq. (1) graphically with a velocity polygon. The velocity directions can be gotten directly from the polygon. The magnitudes are given by:

$$|{}^1\mathbf{v}_{B2/B3}| = 8.66\,\mathrm{m/sec}.$$

Also,

$$|{}^1\mathbf{v}_{B3}| = 5\ \mathrm{m/sec} \Rightarrow |{}^1\omega_3| = \frac{|{}^1\mathbf{v}_{B3}|}{|\mathbf{r}_{B/C}|} = \frac{5}{5} = 1\,\mathrm{rad/sec}$$

From the directions given in the position and velocity polygons

$${}^1\omega_3 = 1\ \mathrm{rad/sec\ CCW}$$

To compute the velocity of D_3,

$${}^1\mathbf{v}_{D3} = {}^1\mathbf{v}_{D3/C3} = {}^1\omega_3 \times \mathbf{r}_{D3/C3} \Rightarrow |{}^1\mathbf{v}_{D3}| = |{}^1\omega_3||\mathbf{r}_{B3/C3}| = 1\cdot(7) = 7\,\mathrm{m/sec}$$

<u>Acceleration Analysis:</u>

$${}^1\alpha_{B2} = {}^1\alpha_{B2/A2} = {}^1\alpha^t_{B2/A2} + {}^1\alpha^r_{B2/A2}$$

$${}^1\alpha_{B2} = {}^1\alpha_{B3} + {}^1\alpha_{B2/B3}$$

also

$${}^1\alpha_{B2/A2} = {}^1\alpha_{B3/C3} + {}^1\alpha_{B2/B3}$$

Expanding the equation,

$${}^1\alpha^t_{B2/A2} + {}^1\alpha^r_{B2/A2} = {}^1\alpha^r_{B3/C3} + {}^1\alpha^t_{B3/C3} + {}^3\alpha_{B2/B3} + {}^1\alpha^n_{B2/B3} + {}^1\alpha^c_{B2/B3}$$

Or

$${}^1\mathbf{a}^r_{B2/A2} + {}^1\mathbf{a}^t_{B2/A2} = {}^1\mathbf{a}^r_{B3/C3} + {}^1\mathbf{a}^t_{B3/C3} + {}^3\mathbf{a}_{B2/B3} + 2\cdot{}^1\omega_3 \times {}^3\mathbf{v}_{B2/B3} + {}^1\mathbf{a}^n_{B2/B3} \qquad (2)$$

Now,

$${}^1\mathbf{a}^r_{B2/A2} = {}^1\omega_2 \times ({}^1\omega_2 \times \mathbf{r}_{B/A}) \Rightarrow |{}^1\mathbf{a}^r_{B2/A2}| = |{}^1\omega_2|^2 \cdot |\mathbf{r}_{B/A}| = 2.0^2\cdot 5 = 20\ \mathrm{m/sec^2}$$

in the direction of $\mathbf{r}_{B/A}$

$${}^1\mathbf{a}^t_{B2/A2} = {}^1\alpha_2 \times \mathbf{r}_{B/A} \Rightarrow |{}^1\mathbf{a}^t_{B2/A2}| = |{}^1\alpha_2|\cdot|\mathbf{r}_{B/A}| = 3.0\cdot 5 = 15\ \mathrm{m/sec^2} (\perp \text{ to } \mathbf{r}_{B/A})$$

$${}^1\mathbf{a}^r_{B3/C3} = {}^1\omega_3 \times ({}^1\omega_3 \times \mathbf{r}_{B/C}) \Rightarrow |{}^1\mathbf{a}^r_{B3/C3}| = |{}^1\omega_3|^2\cdot|\mathbf{r}_{B/C}| = 1^2\cdot 5 = 5\ \mathrm{m/sec^2}$$

in the direction opposite to $\mathbf{r}_{B/C}$

$${}^{1}\mathbf{a}^{t}_{B_3/C_3} = {}^{1}\alpha_3 \times \mathbf{r}_{B/C} \Rightarrow \left|{}^{1}\mathbf{a}^{t}_{B_3/C_3}\right| = \left|{}^{1}\alpha_3\right| \cdot \left|\mathbf{r}_{B/C}\right| \ (\perp \text{ to } \mathbf{r}_{B/C})$$

$${}^{1}\mathbf{a}^{c}_{B_2/B_3} = 2 \cdot {}^{1}\omega_3 \times {}^{3}\mathbf{v}_{B_2/B_3} = 2 \cdot 1 \cdot 8.66 = 17.32 \text{ m/sec}^2 \ (\perp \text{ to } \mathbf{r}_{B/C})$$

$${}^{1}\mathbf{a}^{n}_{B_2/B_3} = \frac{\left|{}^{3}\mathbf{v}_{B_2/B_3}\right|^2}{\left|\mathbf{r}_{B/E}\right|} = \frac{8.66^2}{3} = 25 \text{m/sec}^2 \ \text{ from B to E}$$

${}^{3}\mathbf{a}_{B_2/B_3}$ in the direction of $\mathbf{r}_{B/C}$

Solve Eq. (2) graphically with a acceleration polygon. From the polygon,

$$\left|{}^{1}\alpha_3\right| = \frac{\left|{}^{1}\mathbf{a}^{t}_{B_3/C_3}\right|}{\left|\mathbf{r}_{B/C}\right|} = \frac{0.2131}{5} = 0.0426 \text{ rad/sec}^2$$

From directions given in the polygon,

$${}^{1}\alpha_3 = 0.0426 \text{ rad/sec}^2 \text{ CCW}$$

The accelerations of D_3 and B_3 can be computed by image. The direction of the accelerations are given in the polygon and the magnitudes of the accelerations are

$${}^{1}\mathbf{a}_{D_3} = 8.00 \text{ m/sec}^2$$

$${}^{1}\mathbf{a}_{B_3} = 5.71 \text{ m/sec}^2$$

To find the center of the curvature of the path that B_3 traces on link 2, we must find an expression which involves that the radius of curvature of the path. This term is ${}^{2}\mathbf{a}^{n}_{B_3/B_2}$ and it can be evaluated from the following:

$${}^{1}\mathbf{a}_{B_2/B_3} = -{}^{1}\mathbf{a}_{B_3/B_2}$$

therefore,

$${}^{1}\mathbf{a}^{t}_{B_2/B_3} = -{}^{1}\mathbf{a}^{t}_{B_3/B_2}$$

and

$$\mathbf{a}^{n}_{B_2/B_3} = -{}^{1}\mathbf{a}^{n}_{B_3/B_2}$$

Also,

$${}^{3}\mathbf{a}^{n}_{B_2/B_3} + 2 \cdot {}^{1}\omega_3 \times {}^{1}\mathbf{v}_{B_2/B_3} = -{}^{2}\mathbf{a}^{n}_{B_3/B_2} - 2 \cdot {}^{1}\omega_2 \times {}^{1}\mathbf{v}_{B_3/B_2} \tag{3}$$

For our purpose, we should arrange Eq. (3) as

$${}^{2}\mathbf{a}^{n}_{B_3/B_2} = -\left({}^{3}\mathbf{a}^{n}_{B_2/B_3} + 2 \cdot {}^{1}\omega_2 \times {}^{1}\mathbf{v}_{B_3/B_2} + 2 \cdot {}^{1}\omega_3 \times {}^{1}\mathbf{v}_{B_2/B_3}\right)$$

Now,

$^3\mathbf{a}^n_{B_2/B_3} = 25\ \mathrm{m/sec^2}$ (from B to E)

$2\cdot{}^1\omega_3 \times {}^3\mathbf{v}_{B_2/B_3} = 2\cdot 1\cdot 8.66 = 17.32\ \mathrm{m/sec^2}$ (from B to E)

$2\cdot{}^1\omega_2 \times {}^1\mathbf{v}_{B_3/B_2} = 2\cdot 2\cdot 8.66 = 34.64\ \mathrm{m/sec^2}$ (from E to B)

Let F be the location of the center of curvature of B_3 on link 2. If we choose $\mathbf{r}_{D/B}$ as the positive direction,

$$^2\mathbf{a}^n_{B_3/B_2} = \frac{|{}^1\mathbf{v}_{B_3/B_2}|^2}{\mathbf{r}_{B/F}} = -(25 + 17.32 - 34.64) = 7.68\ \mathrm{m/sec^2}\ \text{(from E to B)}$$

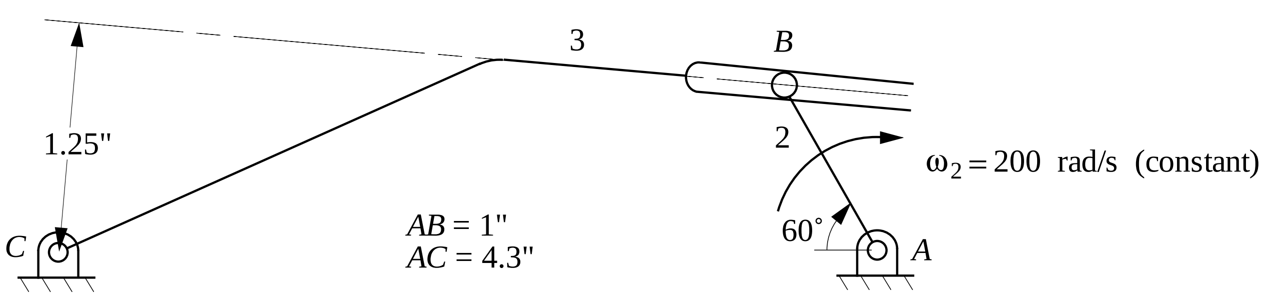
Because $^2\mathbf{a}^n_{B_3/B_2}$ points from F toward B, point E must lie on the opposite same side of B than F. The magnitude of the distance is given by

$$|\mathbf{r}_{B/F}| = \frac{|{}^1\mathbf{v}_{B_3/B_2}|^2}{7.68} = \frac{8.66^2}{7.68} = 9.765\mathrm{m}$$

The location of F is shown on the drawing.

Problem 3.42

If the mechanism shown is drawn full scale, find ω_3, α_3, and the location of the center of curvature of the path that point B_3 traces on Link 2. Assume that Link 2 is driven at constant velocity.



Velocity Analysis:

$$^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{B_2/A_2} = {}^1\mathbf{v}_{B_3} + {}^1\mathbf{v}_{B_2/B_3} \tag{1}$$

$$^1\mathbf{v}_{B_3} = {}^1\mathbf{v}_{B_3/C_3}$$

Now,

$$^1\mathbf{v}_{B_2/A_2} = {}^1\omega_2 \times \mathbf{r}_{B_2/A_2} \Rightarrow |{}^1\mathbf{v}_{B_2/A_2}| = |{}^1\omega_2||\mathbf{r}_{B/A}| = 200\cdot 1 = 200\ \mathrm{in/sec}\ (\perp \text{ to } \mathbf{r}_{B_2/A_2})$$

$${}^{1}\mathbf{v}_{B_3/C_3} = {}^{1}\omega_3 \times \mathbf{r}_{B_3/C_3} \Rightarrow \left|{}^{1}\mathbf{v}_{B_3/C_3}\right| = \left|{}^{1}\omega_3\right|\left|\mathbf{r}_{B_3/C_3}\right| \;(\perp \text{ to } \mathbf{r}_{B_3/C_3})$$

${}^{1}\mathbf{v}_{B_2/B_3}$ linearly along the slot

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$${}^{1}\mathbf{v}_{B_3} = 123.66 \text{ in/sec}$$

or

$$\left|{}^{1}\omega_3\right| = \frac{\left|{}^{1}\mathbf{v}_{B_3/C_3}\right|}{\left|\mathbf{r}_{B_3/C_3}\right|} = \frac{123.66}{3.8977} = 31.726 \text{ rad/sec CCW}$$

Also,

$${}^{1}\mathbf{v}_{B_2/B_3} = 201.77 \text{ in/sec}$$

<u>Acceleration Analysis:</u>

$${}^{1}\mathbf{a}_{B_2} = {}^{1}\mathbf{a}_{B_2/A_2} = {}^{1}\mathbf{a}_{B_3} + {}^{1}\mathbf{a}_{B_2/B_3}$$

$${}^{1}\mathbf{a}_{B_3} = {}^{1}\mathbf{a}_{B_3/C_3}$$

$$\mathbf{a}^{r}_{B_2/A_2} + {}^{1}\mathbf{a}^{t}_{B_2/A_2} = {}^{1}\mathbf{a}^{r}_{B_3/C_3} + {}^{1}\mathbf{a}^{t}_{B_3/C_3} + {}^{3}\mathbf{a}_{B_2/B_3} + 2\cdot{}^{1}\omega_3 \times {}^{3}\mathbf{v}_{B_2/B_3} \tag{2}$$

Now,

$${}^{1}\mathbf{a}^{r}_{B_2/A_2} = {}^{1}\omega_2 \times ({}^{1}\omega_2 \times \mathbf{r}_{B/A}) \Rightarrow \left|{}^{1}\mathbf{a}^{r}_{B_2/A_2}\right| = \left|{}^{1}\omega_2\right|^2 \cdot \left|\mathbf{r}_{B/A}\right| = 200^2 \cdot 1 = 40000 \text{ in/sec}^2$$

in the direction opposite to $\mathbf{r}_{B/A}$

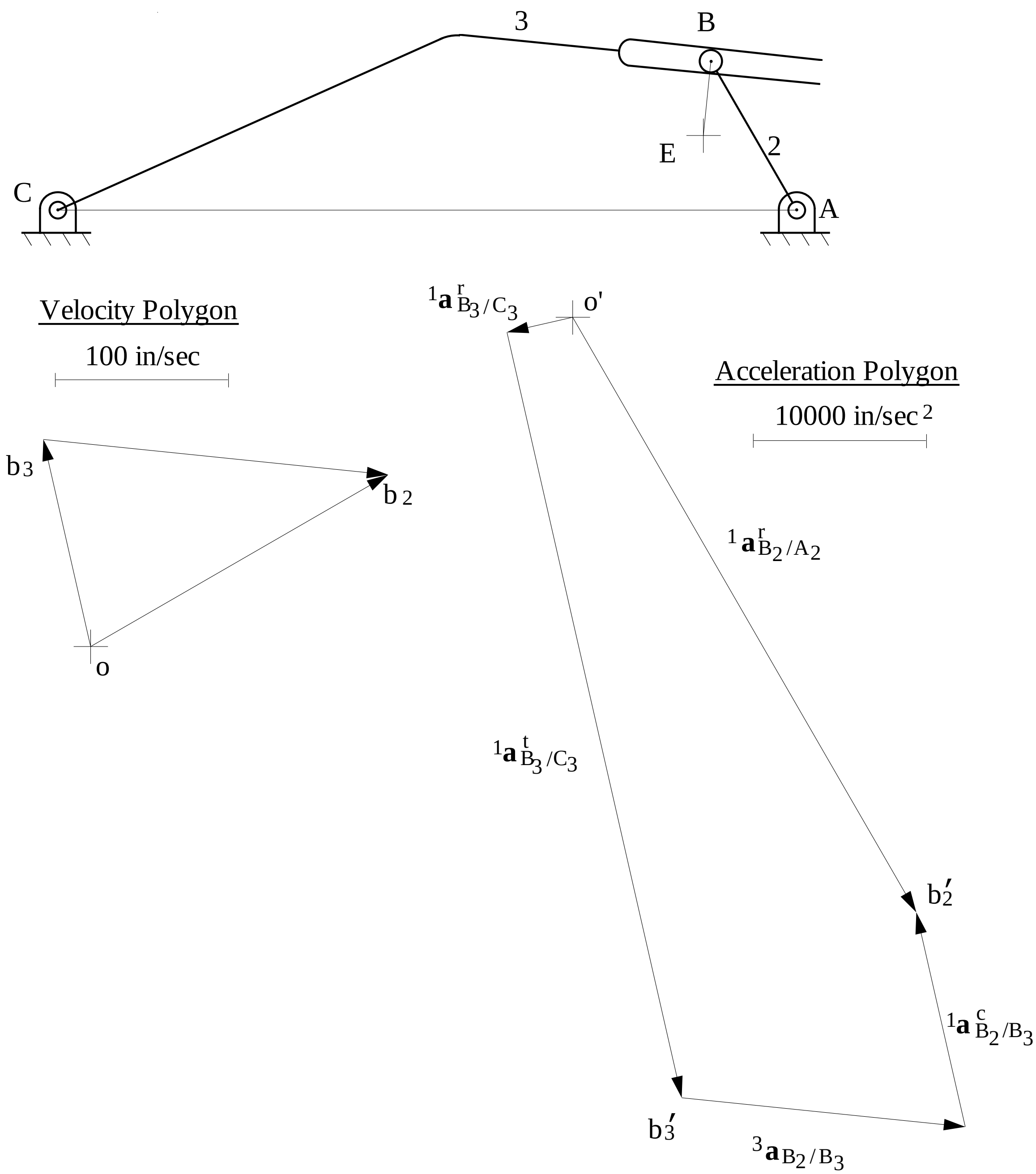
$${}^{1}\mathbf{a}^{t}_{B_2/A_2} = {}^{1}\alpha_2 \times \mathbf{r}_{B_2/A_2} \Rightarrow \left|{}^{1}\mathbf{a}^{t}_{B_2/A_2}\right| = \left|{}^{1}\alpha_2\right| \cdot \left|\mathbf{r}_{B/A}\right| = 0 \cdot 1 = 0 \text{ in/sec}^2$$

$${}^{1}\mathbf{a}^{r}_{B_3/C_3} = {}^{1}\omega_3 \times ({}^{1}\omega_3 \times \mathbf{r}_{B_3/C_3}) \Rightarrow \left|{}^{1}\mathbf{a}^{r}_{B_3/C_3}\right| = \left|{}^{1}\omega_3\right|^2 \cdot \left|\mathbf{r}_{B/C}\right| = 31.726^2 \cdot 3.8977 = 3923 \text{ in/sec}^2$$

in the direction opposite to $\mathbf{r}_{B_3/C_3}$

$${}^{1}\mathbf{a}^{t}_{B_3/C_3} = {}^{1}\alpha_3 \times \mathbf{r}_{B_3/C_3} \Rightarrow \left|{}^{1}\mathbf{a}^{t}_{B_3/C_3}\right| = \left|{}^{1}\alpha_3\right| \cdot \left|\mathbf{r}_{B_3/C_3}\right| \;(\perp \text{ to } \mathbf{r}_{B_3/C_3})$$

${}^{3}\mathbf{a}_{B_2/B_3}$ lies along the slot



$$2 \cdot {}^1\omega_3 \times {}^3\mathbf{v}_{B_2/B_3} = 2 \cdot 31.726 \cdot 201.77 = 12803 \text{ in/sec}^2 \ (\perp \text{ to slot})$$

Solve Eq. (2) graphically with an acceleration polygon. From the polygon,

$$^1\mathbf{a}^t_{B_3/C_3} = 45706 \text{ in/sec}^2$$

or

$$|{}^1\alpha_3| = \frac{|{}^1\mathbf{a}^t_{B_3/C_3}|}{|\mathbf{r}_{B_3/C_3}|} = \frac{45706}{3.8977} = 11726 \text{ rad/sec}^2 \text{ CW}$$

To find the center of the curvature of the path that B_3 traces on link 2, we must find an expression which involves that the radius of curvature of the path. This term is ${}^2\mathbf{a}^n_{B3/B2}$ and it can be evaluated from the following:

$$ {}^1\mathbf{a}_{B2/B3} = -{}^1\mathbf{a}_{B3/B2} $$

therefore,

$$ {}^1\mathbf{a}^t_{B2/B3} = -{}^1\mathbf{a}^t_{B3/B2} $$

and

$$ \mathbf{a}^n_{B2/B3} = -{}^1\mathbf{a}^n_{B3/B2} $$

Also,

$$ {}^3\mathbf{a}^n_{B2/B3} + 2\cdot{}^1\omega_3 \times {}^1\mathbf{v}_{B2/B3} = -{}^2\mathbf{a}^n_{B3/B2} - 2\cdot{}^1\omega_2 \times {}^1\mathbf{v}_{B3/B2} \qquad (3) $$

For our purpose, we should arrange Eq. (3) as

$$ {}^2\mathbf{a}^n_{B3/B2} = -\left({}^3\mathbf{a}^n_{B2/B3} + 2\cdot{}^1\omega_2 \times {}^1\mathbf{v}_{B3/B2} + 2\cdot{}^1\omega_3 \times {}^1\mathbf{v}_{B2/B3}\right) $$

Now,

$$ {}^3\mathbf{a}^n_{B2/B3} = \frac{\left|{}^1\mathbf{v}_{B2/B3}\right|^2}{\infty} = 0 \text{ m/sec}^2 $$

$$ 2\cdot{}^1\omega_3 \times {}^3\mathbf{v}_{B2/B3} = 2\cdot 31.726\cdot 201.77 = 12803 \text{ in/sec}^2 \text{ (}\perp\text{ toslot and generally upward)} $$

$$ 2\cdot{}^1\omega_2 \times {}^3\mathbf{v}_{B3/B2} = 2\cdot 200\cdot 201.77 = 80708 \text{ in/sec}^2 \text{ (}\perp\text{ toslot and generally upward)} $$

Let E be the location of the center of curvature of B_3 on link 2. If we choose "gnerally upward" as the positive direction,

$$ {}^2\mathbf{a}^n_{B3/B2} = \frac{\left|{}^1\mathbf{v}_{B3/B2}\right|^2}{\mathbf{r}_{B/E}} = -(0 + 12803 + 80708) = -93511 \text{ in/sec}^2 $$

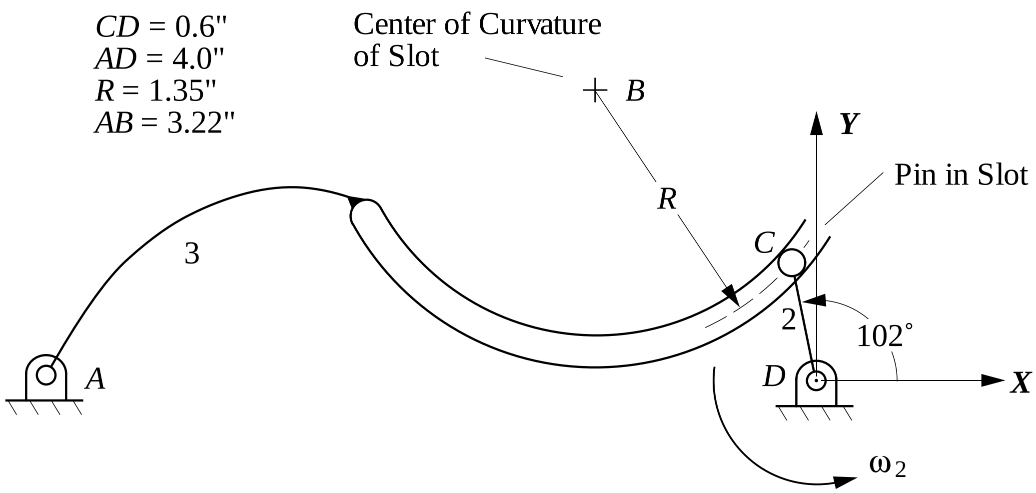
Because ${}^2\mathbf{a}^n_{B3/B2}$ points from B toward E, point E must lie in the "generally downward" (opposite "generally upward") direction from B in a direction perpendicular to the slot. The magnitude of the distance is given by

$$ \left|\mathbf{r}_{B/E}\right| = \frac{\left|{}^1\mathbf{v}_{B3/B2}\right|^2}{1016} = \frac{201.77^2}{93511} = 0.435 \text{ in} $$

The location of E is shown on the drawing.

Problem 3.43

If ω_2 = 20 rad/s (constant), find ω_3, α_3, and the center of curvature of the path that C_3 traces on Link 2.



Position Analysis

Locate the pivots A and D. Then draw link 2 and locate C. Next locate B and draw the circle arc through C.

Velocity Analysis:

$$^1\mathbf{v}_{C_2} = {}^1\mathbf{v}_{C_2/D_2} = {}^1\mathbf{v}_{C_3} + {}^1\mathbf{v}_{C_2/C_3} \tag{1}$$

$$^1\mathbf{v}_{C_3} = {}^1\mathbf{v}_{C_3/A_3}$$

Now,

$$^1\mathbf{v}_{C_2/D_2} = {}^1\boldsymbol{\omega}_2 \times \mathbf{r}_{C_2/D_2} \Rightarrow |{}^1\mathbf{v}_{C_2/D_2}| = |{}^1\omega_2| \cdot |\mathbf{r}_{C_2/D_2}| = 20 \cdot 0.6 = 12 \text{ in/sec } (\perp \text{ to } \mathbf{r}_{C_2/D_2})$$

$$^1\mathbf{v}_{C_3/A_3} = {}^1\boldsymbol{\omega}_3 \times \mathbf{r}_{C_3/A_3} \Rightarrow |{}^1\mathbf{v}_{C_3/A_3}| = |{}^1\omega_3| \cdot |\mathbf{r}_{C_3/A_3}| \; (\perp \text{ to } \mathbf{r}_{C_3/A_3})$$

$^1\mathbf{v}_{C_2/C_3}$ along the slot

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$$^1\mathbf{v}_{C_3} = 9.37 \text{ in/sec}$$

or

$$|{}^1\omega_3| = \frac{|{}^1\mathbf{v}_{C_3/A_3}|}{|\mathbf{r}_{C_3/A_3}|} = \frac{9.37}{3.9194} = 2.391 \text{ rad/sec CCW}$$

Also,

$^{1}\mathbf{v}_{C2/C3} = 15.66 \text{ in/sec}$

<u>Acceleration Analysis:</u>

$$^{1}\mathbf{a}_{C_2} = {}^{1}\mathbf{a}_{C_2/D_2} = {}^{1}\mathbf{a}_{C_3} + {}^{1}\mathbf{a}_{C_2/C_3}$$

$$^{1}\mathbf{a}_{C_3} = {}^{1}\mathbf{a}_{C_3/A_3}$$

$$^{1}\mathbf{a}^{r}_{C_2/D_2} + {}^{1}\mathbf{a}^{t}_{C_2/D_2} = {}^{1}\mathbf{a}^{r}_{C_3/A_3} + {}^{1}\mathbf{a}^{t}_{C_3/A_3} + {}^{3}\mathbf{a}^{r}_{C_2/C_3} + {}^{3}\mathbf{a}^{t}_{C_2/C_3} + 2 \cdot {}^{1}\omega_3 \times {}^{3}\mathbf{v}_{C_2/C_3} \qquad (2)$$

Now,

$$^{1}\mathbf{a}^{r}_{C_2/D_2} = {}^{1}\omega_2 \times ({}^{1}\omega_2 \times \mathbf{r}_{C_2/D_2}) \Rightarrow \left|{}^{1}\mathbf{a}^{r}_{C_2/D_2}\right| = \left|{}^{1}\omega_2\right|^2 \cdot \left|\mathbf{r}_{C_2/D_2}\right| = 20^2 \cdot 0.6 = 240 \text{ in/sec}^2$$

in the direction of $\mathbf{r}_{C_2/D_2}$

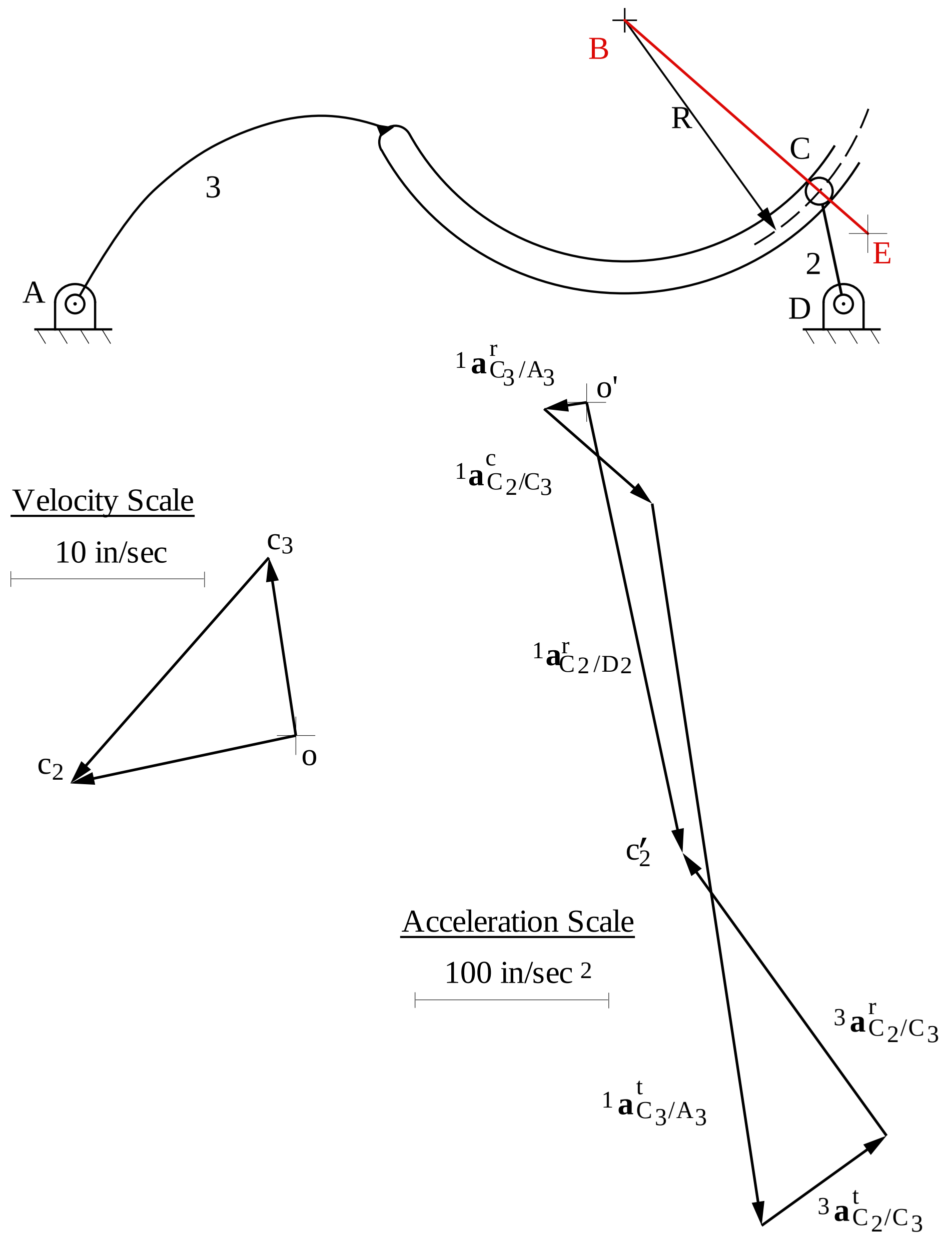
$$^{1}\mathbf{a}^{t}_{C_2/D_2} = {}^{1}\alpha_2 \times \mathbf{r}_{C_2/D_2} \Rightarrow \left|{}^{1}\mathbf{a}^{t}_{C_2/D_2}\right| = \left|{}^{1}\alpha_2\right| \cdot \left|\mathbf{r}_{C_2/D_2}\right| = 0 \cdot 0.6 = 0 \text{ in/sec}^2$$

$$^{1}\mathbf{a}^{r}_{C_3/A_3} = {}^{1}\omega_3 \times ({}^{1}\omega_3 \times \mathbf{r}_{C_3/A_3}) \Rightarrow \left|{}^{1}\mathbf{a}^{r}_{C_3/A_3}\right| = \left|{}^{1}\omega_3\right|^2 \cdot \left|\mathbf{r}_{C_3/A_3}\right| = 2.391^2 \cdot 3.9194 = 22.407 \text{ in/sec}^2$$

in the direction of $\mathbf{r}_{C_3/A_3}$

$$^{1}\mathbf{a}^{t}_{C_3/A_3} = {}^{1}\alpha_3 \times \mathbf{r}_{C_3/A_3} \Rightarrow \left|{}^{1}\mathbf{a}^{t}_{C_3/A_3}\right| = \left|{}^{1}\alpha_3\right| \cdot \left|\mathbf{r}_{C_3/A_3}\right| \; (\perp \text{ to } \mathbf{r}_{C_3/A_3})$$

$$\left|{}^{3}\mathbf{a}^{r}_{C_2/C_3}\right| = \frac{\left|{}^{1}\mathbf{v}_{C_2/C_3}\right|^2}{\left|\mathbf{r}_{C_3/B_3}\right|} = \frac{15.66^2}{1.35} = 181.656 \text{ in/sec}^2 \quad \text{in the direction of } \mathbf{r}_{C_3/B_3}$$



$${}^3\mathbf{a}^t_{C_2/C_3} \quad (\perp \text{ to } \mathbf{r}_{C_3/B_3})$$

$$2 \cdot {}^1\omega_3 \times {}^3\mathbf{v}_{C_2/C_3} = 2 \cdot 2.391 \cdot 15.66 = 74.886 \text{ in / sec}^2$$

Solve Eq. (2) graphically with an acceleration velocity polygon. From the polygon,

$${}^1\mathbf{a}^t_{C_3/A_3} = 380.49 \text{ in / sec}^2$$

or

$$\left|{}^1\alpha_3\right| = \frac{\left|{}^1\mathbf{a}^t_{C_3/A_3}\right|}{\left|\mathbf{r}_{C_3/A_3}\right|} = \frac{380.49}{3.9194} = 97.079 \text{ rad / sec}^2 \text{ CW}$$

To find the center of the curvature of the path that C_3 traces on link 2, we must find an expression which involves that the radius of curvature of the path. This term is ${}^2\mathbf{a}^n_{C_3/C_2}$ and it can be evaluated from the following:

$$ {}^1\mathbf{a}_{C_3/C_2} = -{}^1\mathbf{a}_{C_2/C_3} $$

therefore,

$$ {}^1\mathbf{a}^t_{C_3/C_2} = -{}^1\mathbf{a}^t_{C_2/C_3} $$

and

$$ {}^1\mathbf{a}^n_{C_3/C_2} = -{}^1\mathbf{a}^n_{C_2/C_3} $$

Also,

$$ {}^2\mathbf{a}^n_{C_3/C_2} + 2\cdot{}^1\omega_2 \times {}^1\mathbf{v}_{C_3/C_2} = -{}^3\mathbf{a}^n_{C_2/C_3} - 2\cdot{}^1\omega_3 \times {}^1\mathbf{v}_{C_2/C_3} \tag{3} $$

For our purpose, we should arrange eq.3 as

$$ {}^2\mathbf{a}^n_{C_3/C_2} = -\left({}^3\mathbf{a}^n_{C_2/C_3} + 2\cdot{}^1\omega_3 \times {}^1\mathbf{v}_{C_2/C_3} + 2\cdot{}^1\omega_2 \times {}^1\mathbf{v}_{C_3/C_2}\right) $$

Now,

$$ {}^3\mathbf{a}^n_{C_2/C_3} = \frac{\left|{}^1\mathbf{v}_{C_2/C_3}\right|^2}{\mathbf{r}_{C_3/B_3}} = \frac{15.66^2}{1.35} = 181.656\ \text{in}/\sec^2 $$

$$ 2\cdot{}^1\omega_3 \times {}^3\mathbf{v}_{C_2/C_3} = 2\cdot 2.391\cdot 15.66 = 74.886\ \text{in}/\sec^2 $$

$$ 2\cdot{}^1\omega_2 \times {}^2\mathbf{v}_{C_3/C_2} = 2\cdot 20\cdot 15.66 = 626.4\ \text{in}/\sec^2 $$

If we choose BC as positive direction,

$$ {}^2\mathbf{a}^n_{C_3/C_2} = \frac{\left|{}^1\mathbf{v}_{C_3/C_2}\right|^2}{\mathbf{r}_{C_3/E_3}} = -(181.656 - 74.886 + 626.4) = -733.17\ \text{in}/\sec^2 $$

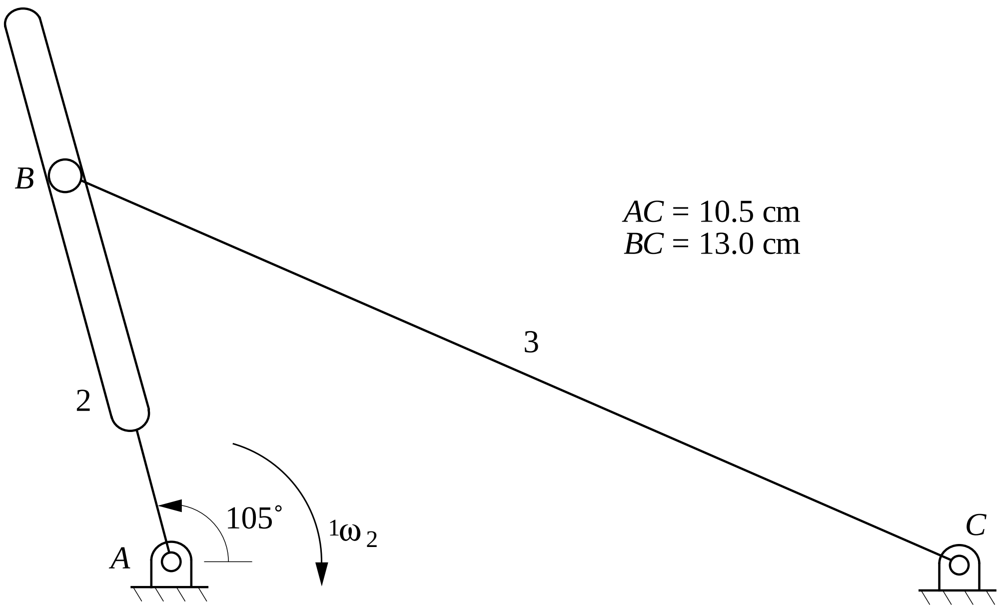
So,

$$ \left|\mathbf{r}_{C_3/E_3}\right| = \frac{\left|{}^1\mathbf{v}_{C_3/C_2}\right|^2}{-733.17} = \frac{15.66^2}{-733.17} = -0.334\ \text{in} $$

Therefore, the center of the curvature of the path that B_3 traces on link 2 is in opposite direction of BC as shown in the sketch.

Problem 3.44

If ω_2 = 10 rad/s (constant), find α_3.



Position Analysis

Locate points A and C. Then draw the line AB. Next locate point B at a distance of 13 cm from point C.

Velocity Analysis:

$$^1\mathbf{v}_{B_3} = {^1}\mathbf{v}_{B_2} + {^1}\mathbf{v}_{B_3/B_2} \tag{1}$$

Now,

$$^1\mathbf{v}_{B_2} = {^1}\omega_2 \times \mathbf{r}_{B/A} \Rightarrow |{^1}\mathbf{v}_{B_2}| = |{^1}\omega_2| \cdot |\mathbf{r}_{B/A}| = 10 \cdot 5.4088 = 54.088 \text{ cm/sec } (\perp \text{ to } \mathbf{r}_{B/A})$$

$$^1\mathbf{v}_{B_3} = {^1}\omega_3 \times \mathbf{r}_{B/C} \Rightarrow |{^1}\mathbf{v}_{B_3}| = |{^1}\omega_4| \cdot |\mathbf{r}_{B/C}| \quad (\perp \text{ to } \mathbf{r}_{B/C})$$

$^1\mathbf{v}_{B_3/B_2}$ in the direction of $\mathbf{r}_{B/A}$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$$^1\mathbf{v}_{B_3} = 86.581 \text{ cm/sec}$$

or

$$|{^1}\omega_3| = \frac{|{^1}\mathbf{v}_{B_3}|}{|\mathbf{r}_{B/C}|} = \frac{86.581}{13} = 6.66 \text{ rad/sec}$$

From the directions given in the position and velocity polygons

$$^1\omega_3 = 6.66 \text{ rad/sec CW}$$

Also,

$$^1\mathbf{v}_{B_3/B_2} = 67.532 \text{ cm/sec}$$

<u>Acceleration Analysis:</u>

$$^1\mathbf{a}_{B_3} = {}^1\mathbf{a}_{B_2} + {}^1\mathbf{a}_{B_3/B_2}$$

$$^1\mathbf{a}^r_{B_3/C_3} + {}^1\mathbf{a}^t_{B_3/C_3} = {}^1\mathbf{a}^r_{B_2/A_2} + {}^1\mathbf{a}^t_{B_2/A_2} + {}^2\mathbf{a}_{B_3/B_2} + 2\,{}^1\omega_2 \times {}^1\mathbf{v}_{B_3/B_2}$$

Now,

$$^1\mathbf{a}^r_{B_3/C_3} = {}^1\omega_3 \times ({}^1\omega_3 \times \mathbf{r}_{B/C}) \Rightarrow \left|{}^1\mathbf{a}^r_{B_3/C_3}\right| = \left|{}^1\omega_3\right|^2 \cdot \left|\mathbf{r}_{B/C}\right| = 6.66^2 \cdot 13 = 576.623 \text{ cm/sec}^2$$

in the direction of $\mathbf{r}_{B/C}$

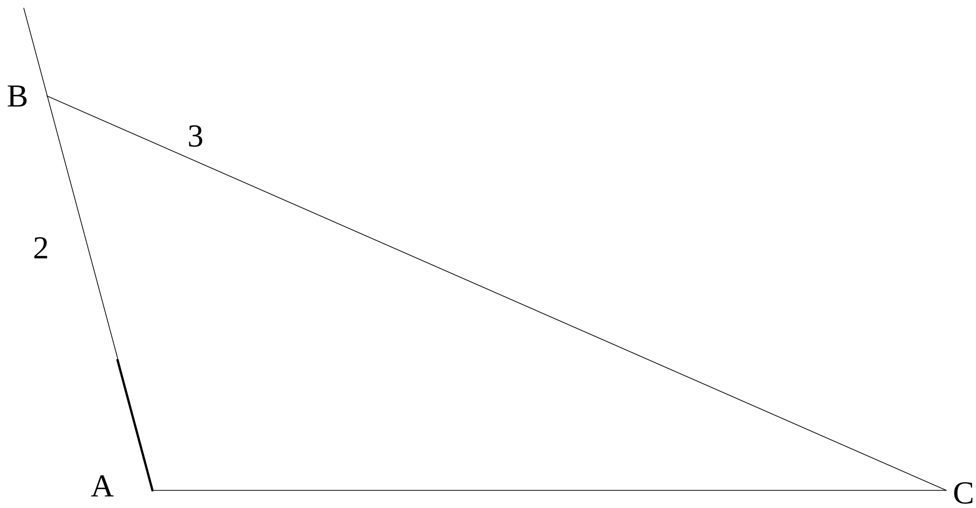
$$^1\mathbf{a}^t_{B_3/C_3} = {}^1\alpha_3 \times \mathbf{r}_{B/C} \Rightarrow \left|{}^1\mathbf{a}^t_{B_3/C_3}\right| = \left|{}^1\alpha_3\right| \cdot \left|\mathbf{r}_{B/C}\right| (\perp \text{ to } \mathbf{r}_{B/C})$$

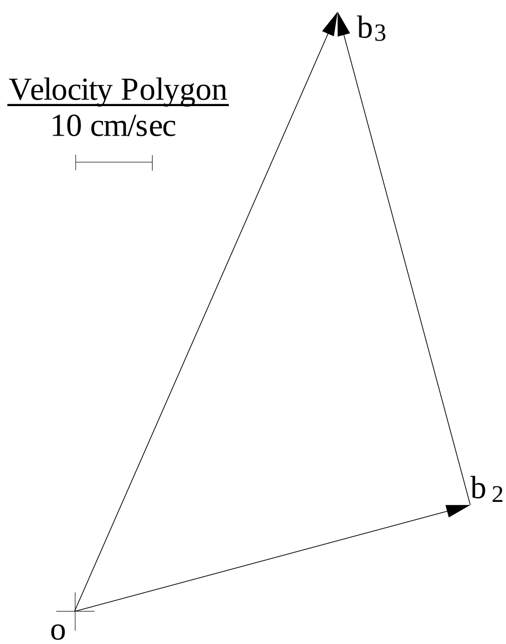
$$^1\mathbf{a}^r_{B_2/A_2} = {}^1\omega_2 \times ({}^1\omega_2 \times \mathbf{r}_{B/A}) \Rightarrow \left|{}^1\mathbf{a}^r_{B_2/A_2}\right| = \left|{}^1\omega_2\right|^2 \cdot \left|\mathbf{r}_{B/A}\right| = 10^2 \cdot 5.4088 = 540.88 \text{ cm/sec}^2$$

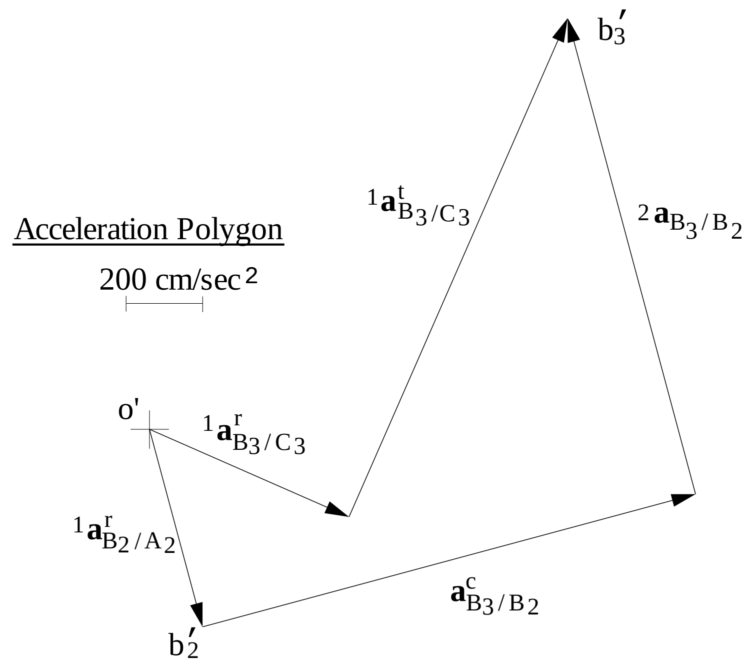
in the direction of $\mathbf{r}_{B/A}$

$$^1\mathbf{a}^t_{B_2/A_2} = {}^1\alpha_2 \times \mathbf{r}_{B/A} \Rightarrow \left|{}^1\mathbf{a}^t_{B_2/A_2}\right| = \left|{}^1\alpha_2\right| \cdot \left|\mathbf{r}_{B/A}\right| = 0$$

$$^1\mathbf{a}^c_{B_3/B_2} = 2\,{}^1\omega_2 \times {}^1\mathbf{v}_{B_3/B_2} = 2 \cdot 10 \cdot 67.532 = 1351 \text{ cm/sec}^2\ (\perp \text{ to } \mathbf{r}_{B/A})$$







$^2\mathbf{a}_{B_3/B_2}$ in the direction of $\mathbf{r}_{B/A}$

Solve Eq. (2) graphically with a acceleration polygon. From the polygon,

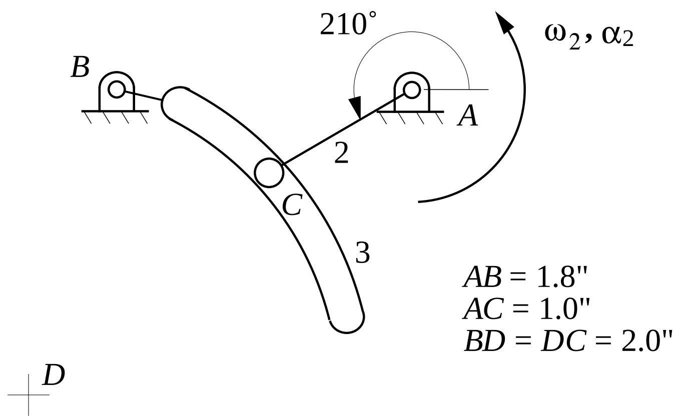
$$\left|{}^1\alpha_3\right| = \frac{\left|{}^1\mathbf{a}^t_{B_3/C_3}\right|}{\left|\mathbf{r}_{B/C}\right|} = \frac{1440.2}{13} = 110.785 \text{ rad/sec}^2$$

From directions given in the polygon,

$${}^1\alpha_3 = 110.785 \text{ rad/sec}^2 \text{ CW}$$

Problem 3.45

For the linkage shown, ω_2 = 10 rad/s CCW and α_2 = 100 rad/s^2 CCW. Determine ω_3, and α_3.



Position Analysis

Draw the mechanism to scale. First locate the relative position of points A and B. Next locate point C. Locate point D at the intersection of two circle arcs: one centered at C and with a 2" radius, the other centered at point B with a 2" radius. Next draw a circle arc through C and centered at D and of radius 2"

Velocity Analysis:

$${}^1\boldsymbol{v}_{C_2} = {}^1\boldsymbol{v}_{C_2/A_2} = {}^1\boldsymbol{v}_{C_3} + {}^1\boldsymbol{v}_{C_2/C_3} \tag{1}$$

$${}^1\boldsymbol{v}_{C_3} = {}^1\boldsymbol{v}_{C_3/B_3}$$

Now,

$${}^1\boldsymbol{v}_{C_2/A_2} = {}^1\mathrm{w}_2 \times \boldsymbol{r}_{C_2/A_2} \Rightarrow \left|{}^1\boldsymbol{v}_{C_2/A_2}\right| = \left|{}^1\omega_2\right| \cdot \left|\boldsymbol{r}_{C_2/A_2}\right| = 10 \cdot 1 = 10 \text{ in / s} (\perp \text{ to } \boldsymbol{r}_{C_2/A_2})$$

$${}^1\boldsymbol{v}_{C_3/B_3} = {}^1\boldsymbol{\omega}_3 \times \boldsymbol{r}_{C_3/B_3} \Rightarrow \left|{}^1\boldsymbol{v}_{C_3/B_3}\right| = \left|{}^1\omega_3\right| \cdot \left|\boldsymbol{r}_{C_3/B_3}\right| (\perp \text{ to } \boldsymbol{r}_{C_3/B_3})$$

${}^1\boldsymbol{v}_{C_2/C_3}$ along the path of slot

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

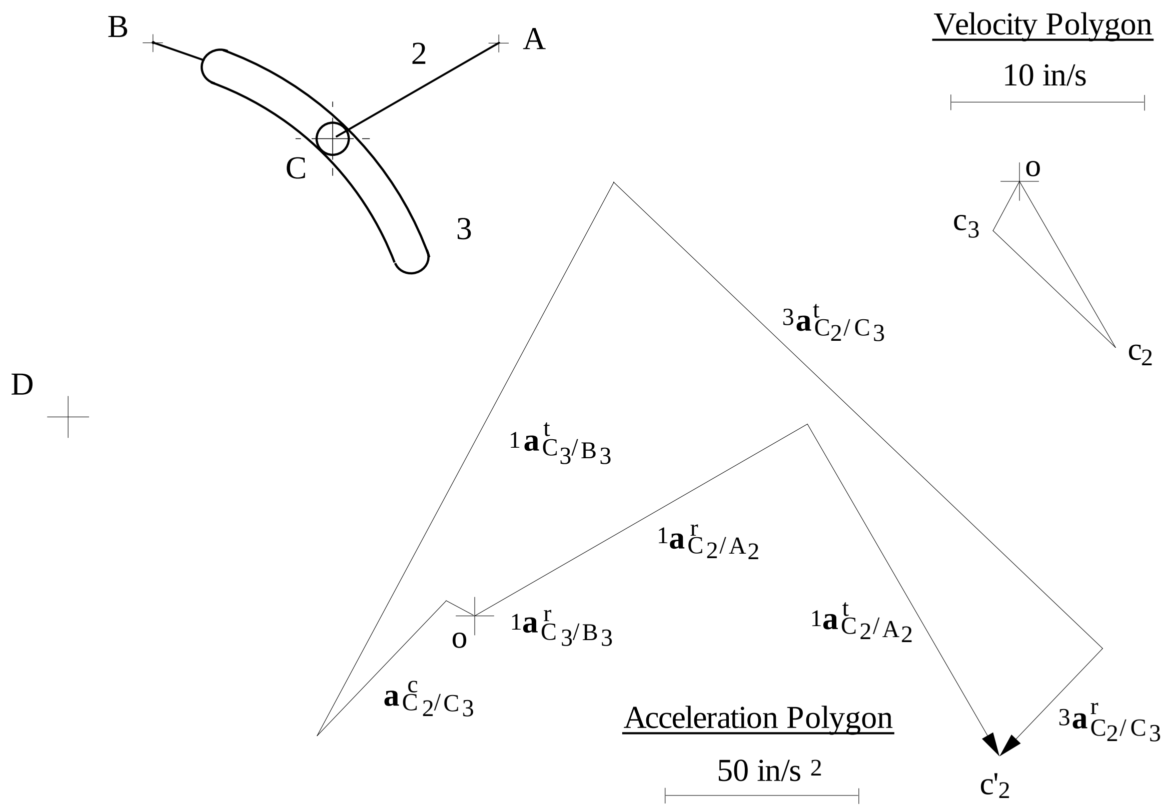
$${}^1\boldsymbol{v}_{C_2/C_3} = 8.817 \text{ in / s}$$

Also,

$${}^1\boldsymbol{v}_{C_3/B_3} = 2.917 \text{ in / s}$$

or

$$\left|{}^1\omega_3\right| = \frac{\left|{}^1\boldsymbol{v}_{C_3/B_3}\right|}{\left|\boldsymbol{r}_{C_3/B_3}\right|} = \frac{2.917}{1.0594} = 2.753 \text{ rad / s CW}$$



Accelration Analysis:

$${}^1\boldsymbol{a}_{C_2} = {}^1\boldsymbol{a}_{C_2/A_2} = {}^1\boldsymbol{a}_{C_3} + {}^1\boldsymbol{a}_{C_2/C_3}$$

$${}^1\boldsymbol{a}_{C_3} = {}^1\boldsymbol{a}_{C_3/B_3}$$

$${}^1\boldsymbol{a}^r_{C_2/A_2} + {}^1\boldsymbol{a}^t_{C_2/A_2} = {}^1\boldsymbol{a}^r_{C_3/B_3} + {}^1\boldsymbol{a}^t_{C_3/B_3} + {}^3\boldsymbol{a}^r_{C_2/C_3} + {}^3\boldsymbol{a}^t_{C_2/C_3} + 2\,{}^1\boldsymbol{\omega}_3 \times {}^1\boldsymbol{v}_{C_2/C_3} \qquad (2)$$

Now,

$${}^1\boldsymbol{a}^r_{C_2/A_2} = {}^1\boldsymbol{\omega}_2 \times \left({}^1\boldsymbol{\omega}_2 \times \boldsymbol{r}_{C_2/A_2}\right) \Rightarrow \left|{}^1\boldsymbol{a}^r_{C_2/A_2}\right| = \left|{}^1\boldsymbol{\omega}_2\right|^2 \cdot \left|\boldsymbol{r}_{C_2/A_2}\right| = 10^2 \cdot 1 = 100 \text{ in / s}^2$$

in the direction opposite to $\boldsymbol{r}_{C_2/A_2}$

$${}^1\boldsymbol{a}^t_{C_2/A_2} = {}^1\boldsymbol{\alpha}_2 \times \boldsymbol{r}_{C_2/A_2} \Rightarrow \left|{}^1\boldsymbol{a}^t_{C_2/A_2}\right| = \left|{}^1\boldsymbol{\alpha}_2\right| \cdot \left|\boldsymbol{r}_{C_2/A_2}\right| = 100 \cdot 1 = 100 \text{ in / s}^2 \ (\perp \text{ to } \boldsymbol{r}_{C_2/A_2})$$

$${}^1\boldsymbol{a}^r_{C_3/B_3} = {}^1\boldsymbol{\omega}_3 \times \left({}^1\boldsymbol{\omega}_3 \times \boldsymbol{r}_{C_3/B_3}\right) \Rightarrow \left|{}^1\boldsymbol{a}^r_{C_3/B_3}\right| = \left|{}^1\boldsymbol{\omega}_3\right|^2 \cdot \left|\boldsymbol{r}_{C_3/B_3}\right| = 2.753^2 \cdot 1.0594 = 8.029 \text{ in / s}^2$$

in the direction opposite $\boldsymbol{r}_{C_3/B_3}$

$${}^1\boldsymbol{a}^t_{C_3/B_3} = {}^1\boldsymbol{\alpha}_3 \times \boldsymbol{r}_{C_3/B_3} \Rightarrow \left|{}^1\boldsymbol{a}^t_{C_3/B_3}\right| = \left|{}^1\boldsymbol{\alpha}_3\right| \cdot \left|\boldsymbol{r}_{C_3/B_3}\right| \ (\perp \text{ to } \boldsymbol{r}_{C_3/B_3})$$

$$\left|{}^3\boldsymbol{a}^r_{C_2/C_3}\right| = \frac{\left|{}^1\boldsymbol{v}_{C_2/C_3}\right|^2}{\left|\boldsymbol{r}_{C_3/D_3}\right|} = \frac{8.817^2}{2} = 38.87\ in/s^2 \text{ in the direction of } -\boldsymbol{r}_{C_3/D_3}$$

$${}^3\boldsymbol{a}^t_{C_2/C_3} \quad (\perp \text{ to } \boldsymbol{r}_{C_3/D_3})$$

$$2\,{}^1\boldsymbol{\omega}_3 \times {}^1\boldsymbol{v}_{C_2/C_3} = 2 \cdot 2.753 \cdot 8.817 = 48.546 \text{ in/s}^2 \text{ in the direction of } -\boldsymbol{r}_{C_3/D_3}$$

Solve Eq. (2) graphically with an acceleration polygon. From the polygon,

$${}^1\boldsymbol{a}^t_{C_3/B_3} = 164 \text{ in/s}^2$$

or

$$\left|{}^1\alpha_3\right| = \frac{\left|{}^1\boldsymbol{a}^t_{C_3/B_3}\right|}{\left|\boldsymbol{r}_{C_3/B_3}\right|} = \frac{164}{1.0594} = 154.8 \text{ rad/s}^2 \text{ CCW}$$

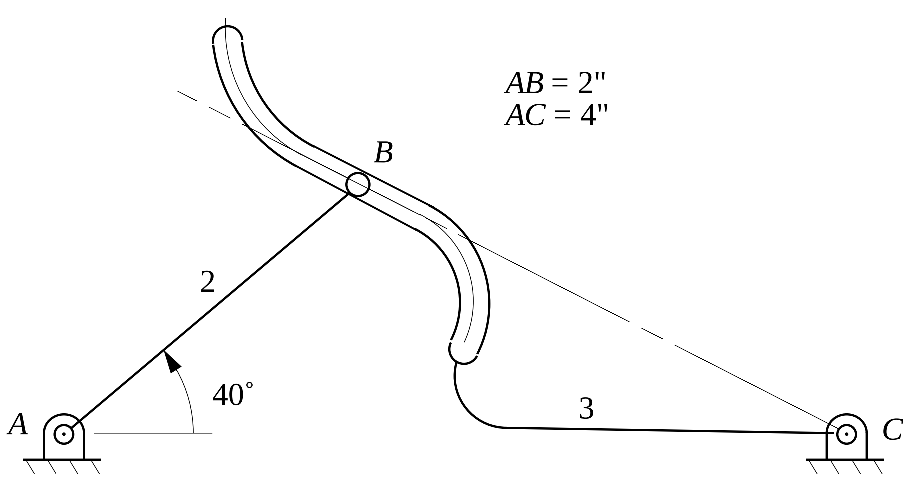
Problem 3.46

If ω_2 = 10 rad/s CW (constant), find

a) ω_3

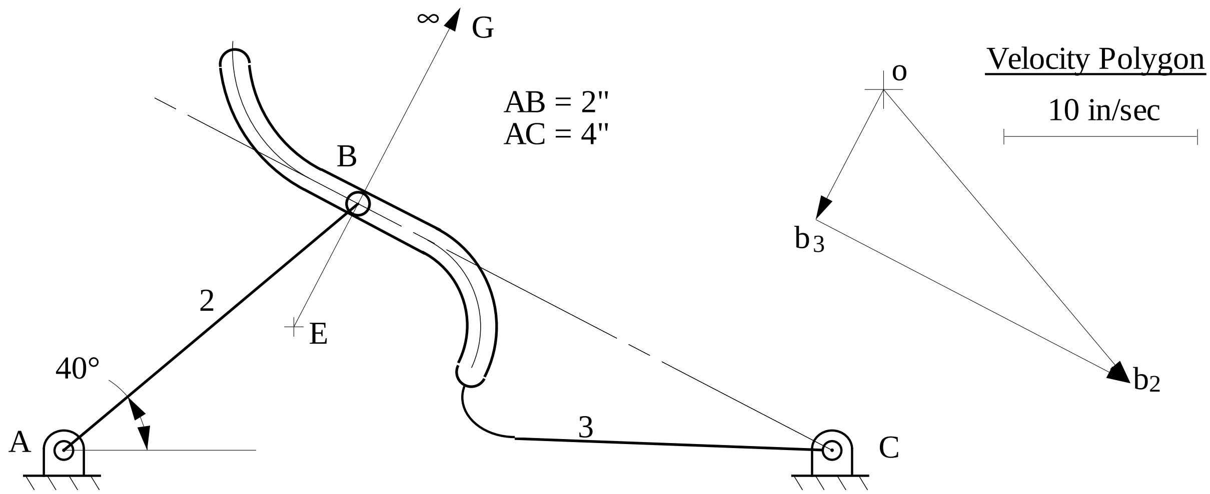
b) The center of curvature of the path that B_2 traces on link 3 (show on drawing).

c) The center of curvature of the path that B_3 traces on link 2 (show on drawing).



Position Analysis

Draw the linkage to scale. Start by locating points B and C relative to A. The line BC gives the direction of travel of B relative to link 3.



Velocity Analysis:

$$^1\mathbf{v}_{B_2} = {}^1\mathbf{v}_{B_3} + {}^1\mathbf{v}_{B_2/B_3} \tag{1}$$

Now,

$$^1\mathbf{v}_{B_2} = {}^1\omega_2 \times \mathbf{r}_{B/A} \quad (\perp \text{ to } \mathbf{r}_{B/A}) \Rightarrow \left|{}^1\mathbf{v}_{B_2}\right| = \left|{}^1\omega_2\right| \cdot \left|\mathbf{r}_{B/A}\right| = 10 \cdot 2 = 20 \text{ in/sec}$$

$$^1\mathbf{v}_{B_3} = {}^1\omega_3 \times \mathbf{r}_{B/C} \quad (\perp \text{ to } \mathbf{r}_{B/C})$$

$^1\mathbf{v}_{B_2/B_3}$ in the direction of $\mathbf{r}_{B/C}$

Solve Eq. (1) graphically with a velocity polygon. From the polygon,

$$^1\mathbf{v}_{B_3} = 7.647 \text{ in/sec}$$

or

$$\left|{}^1\omega_3\right| = \frac{\left|{}^1\mathbf{v}_{B_3}\right|}{\left|\mathbf{r}_{B/C}\right|} = \frac{7.647}{2.7827} = 2.748 \text{ rad/sec}$$

From the directions given in the position and velocity polygons

$$^1\boldsymbol{\omega}_3 = 2.748 \text{ rad/sec CCW}$$

Also,

$$^1\mathbf{v}_{B_2/B_3} = 18.48 \text{ in/sec}$$

Also, the center of curvature (Point G) of the path that B_2 traces on link 3 is at infinity and is perpendicular to $\mathbf{r}_{B/C}$.

To find the center of the curvature (Point E) of the path that B_3 traces on link 2, we must find an expression which involves that the radius of curvature of the path. This term is $\mathbf{a}^n_{B_3/B_2}$ and it can be evaluated from the following:

$$^1\mathbf{a}_{B_2/B_3} = -^1\mathbf{a}_{B_3/B_2}$$

therefore,

$$^1\mathbf{a}^t_{B_2/B_3} = -^1\mathbf{a}^t_{B_3/B_2}$$

and

$$^1\mathbf{a}^n_{B_2/B_3} = -^1\mathbf{a}^n_{B_3/B_2}$$

Also,

$$^3\mathbf{a}^n_{B_2/B_3} + 2\,^1\omega_3 \times {}^1\mathbf{v}_{B_2/B_3} = -^2\mathbf{a}^n_{B_3/B_2} - 2\,^1\omega_2 \times {}^1\mathbf{v}_{B_3/B_2} \tag{2}$$

Because the center of the curvature of the path that B_2 traces on link 3 is at infinity, the first term of the left hand side of Eq (2) is 0. Therefore,

$$^2\mathbf{a}^n_{B_3/B_2} = -2\,^1\omega_3 \times {}^1\mathbf{v}_{B_2/B_3} - 2\,^1\omega_2 \times {}^1\mathbf{v}_{B_3/B_2} \tag{3}$$

Now, If we choose BG as the positive direction,

$$^2\mathbf{a}^n_{B_3/B_2} = \frac{\left|^1\mathbf{v}_{B_3/B_2}\right|^2}{\mathbf{r}_{B/E}} = -(101.566 + 369.6) = -471.166 \text{ in / sec}^2$$

So,

$$\left|\mathbf{r}_{B/E}\right| = \frac{\left|^1\mathbf{v}_{B_3/B_2}\right|^2}{-471.166} = \frac{18.48^2}{-471.166} = -0.724 \text{ in}$$

Therefore, the center of the curvature of the path that B_3 traces on link 2 is in the opposite direction of BG as shown in the sketch.